Kalman Filter estimation of angular acceleration

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Abstract. The Kalman filter is a commonly used tool to eliminate measurement noise occurring in the data most often taken from various sensors. The most common application is data filtration from accelerometers and gyroscopes, which are found, among others in smartphones and other modern devices. The Kalman filter is also an estimator, which allows the estimation of parameters that are not directly measured by the sensors but depend on the measurement quantities. This article will present the case of measuring angular velocity and linear acceleration measured during activity on a stationary bike. The sensor was placed on the leg of the person exercising. On the basis of angular velocity, the tests will determine angular acceleration not measured directly by sensors and comparison of the resultant linear acceleration with the resultant acceleration measured by sensors.

1. Introduction

This article presents methods for estimating angular acceleration based on measurements of rotational speed and linear acceleration read from XSENS sensors, including digital gyroscope and magnetometer. The measurements were made on a stationary bike and the sensor was placed at the height of the ankle joint. Angular velocity and linear acceleration values were measured.

The purpose of this article is to present the method of estimating angular acceleration based on measurements of angular velocity and linear acceleration using the Kalman filter [1-5]. The Kalman filter is a frequently used tool to eliminate measuring noise, in particular in mobile devices such as smartphones, where gyroscopes and accelerometers are used, to detect the rotation of the device and to change the screen orientation. In this work, one of the Kalman Filter properties is used, which is the estimation of quantities not measured based on the parameters measured by the sensors. The main element of the work will be the presentation of the model for estimation of angular acceleration. Then, based on the measured linear acceleration, the result of the estimated measurement will be compared using the theorem of circular motion, i.e. with the calculation of centripetal and tangential acceleration. The determination of angular acceleration may be a useful parameter in the application of Denavit-Hartenberg notation [6].

The presented work is closely related with other works concerned with the analysis of signals in dynamical systems. Among others it could be utilized in analysis of control signals in virtual systems [7,8]. The other potential area relates to control of complex technical systems [9-11] as well as to investigation of the influence of material aspects on the results of signal detection [12,13]. Hence these investigations are developed in cooperation with other centers.
2. Estimation of angular acceleration

Figures 1 and 2 show measurement data obtained from XSENS sensors - angular velocities (gyroscope) and linear accelerations (accelerometer).

**Figure 1.** Angular velocity values from XSENS sensors (Octave program). There is little noise in the course of angular velocity, which allows you to choose the Kalman Filter parameters well.

**Figure 2.** Resultant linear acceleration from XSENS sensors after subtracting the acceleration due to gravity (Octave program). Noise occurs in the course of angular velocity, which hinders the selection of Kalman Filter parameters.
The figure shows that the values of the resultant linear acceleration (from the X and Y axes with the removal of gravitational acceleration measured by digital gyroscopes) are in the range of about $-5 \frac{m}{s^2}$ to about $5 \frac{m}{s^2}$. The minus sign means the opposite return of the linear acceleration vector, therefore figure 3 shows the values without taking into account the return of the vector – the resultant values, i.e. the square, will be determined.

Figure 3. Skipping the validity of returning the vector of the linear acceleration result (Octave Program). In the course of angular velocity, there are significant measuring noise, which makes the selection of Kalman Filter parameters difficult.

To determine the value of angular acceleration, a linear Kalman filter with the following model was used:

\[
\begin{align*}
\dot{\omega}_{i+1} &= \frac{\Delta \omega_i}{\Delta t} + w_i \\
\dot{y}_i &= \frac{\Delta \omega_i}{\Delta t} + v_i
\end{align*}
\]

(1)

where:

- $\Delta$ - difference between the previous and current state $\Delta = \frac{\omega_{i+1}}{\omega_i} - 1$,
- $dt$ – time resulting from sensor sampling 100Hz.

Figure 4 shows the results of the estimation of angular acceleration with a Kalman filter.
Figure 4. Estimation of the angular acceleration value (Octave program). The determined values are characterized by low measurement noise - this is affected by the angular velocity values.

In order to determine the correctness of the results from figure 4, the relationships used in circular motion and therefore for tangential and normal acceleration [14] were used:

\[
\begin{align*}
\alpha_t &= \varepsilon R \\
\alpha_n &= \omega^2 R 
\end{align*}
\]  

The resultant of the tangential and normal acceleration was calculated and compared with the resultant value of linear acceleration from figure 3. The comparison is shown in figure 5.

Figure 5. Comparison of accidental linear accelerations (Octave program). The values on both graphs have similar values, the difference in the shape of the graphs is due to the last unanalyzed sample from the Kalman Filter.
To compare the two accelerations, the DTW algorithm was used [15-19], and the results of the comparison are shown in figure 6.

![Figure 6. Comparison of two resultant linear accelerations by using the DTW algorithm - red is the distance calculated according to DTW (Octave program). The difference between the estimated and measured values is small, which may indicate a good selection of Kalman Filter parameters.](image)

Based on the DTW algorithm, the average distance / average error between the graphs was calculated \(0.32 \text{m/s}^2\). Given the range of linear acceleration values from \(0 \text{m/s}^2\) to about \(5 \text{m/s}^2\), the designated model (1) is acceptable.

### 3. Conclusions

In this article, Kalman Filter models for estimating angular acceleration values based on speed measurements were tested. Measurements were made using XSENS sensors. Angular velocity and linear acceleration of the system were measured. A Kalman filter model was formulated to estimate angular acceleration values. The results of the adopted model were compared with the resultant value of linear acceleration obtained from measurements with a digital accelerometer. The comparison was made on the basis of determining the resultant acceleration from the tangent and normal components of acceleration in a circular motion, using the values of angular velocity and angular acceleration. The DTW algorithm was used to compare the results. The average distance between the compared charts was \(0.32 \text{m/s}^2\) and at the same time it is the average error of a particular model.

Further work on parameter estimation will use the Extended Kalman Filter, which will allow for more accurate estimation of the tested model.

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