Bounded Noises Estimation Based On Cognitive Radio In Distributed Fusion System

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ABSTRACT In distributed fusion system, the close interaction between the network and the physical world is emphasized, and the cognitive radio (CR) of wireless terminal has sufficient intelligence or cognitive ability to effectively improve the communication quality by detecting the history and current conditions of the surrounding wireless environment. Based on the Kalman filter, a channel sensing and conversion mechanism is studied and a new necessary and sufficient condition for MSE stability is deduced. Stable gain is received based on the stable requirements. And the average estimated error covariance is deduced. Finally, the average estimated error covariance with the mechanism is at least 63% less than that without this mechanism by using MATLAB simulation.

1. INTRODUCTION
As a smart radio communication system, cognitive radio technology dynamically detects spatial spectrum through spectrum sensing technology, thereby improving spectrum utilization efficiency [1]. There are many delimiting of cognitive radio. It is generally believed that cognitive radio is an intelligent wireless communication system with the learning ability of artificial intelligence technology, so it can recognize the external environment change. By changing its own working parameters in real time, such as modulation technology, operating frequency, transmission power, etc.

It allows the internal state to adapt the exchange of wireless transmission environment. That is to say, it can make full use of spectrum resources to meet the purpose of reliable communication at any time and any place [2]. One of the significant characteristic of cognitive radio is that it allows unauthorized users use holes opportunistically, by which without interfering the authorized communication.

As we know, the CR devices can be turned into different frequencies and allow the extended links to operate on different bands. This special CR feature will help to reduce co-band interference between
the extended links so that the end-to-end throughput may be improved. A number of documents have been published and various aspects of CR network have been studied such as optimal sensing Algorithm, a spectrum selection and scheduling algorithm based on the opportunistic capacity concept, network protocols, and network security.

In this article, we consider the distributed fusion estimation issue for the networked time varying systems with bounded noises based on cognitive radio. Multisensor fusion estimation is one of the most important studies in the area of the state estimation fusion. More recently, networked multisensor fusion estimation has found applications in a wide range of areas, such as networked filtering in wireless sensor networks and distributed fusion systems. Under the networked fusion framework, two problems must be taken into account: 1) communication delays and packet dropouts; 2) restraints of sensor energy and sensor communication bandwidth. We can see that information loss is ineluctable because of the above constraints, and for this fusion estimation with incomplete information will degrade the estimation performance. In this article, we follow the interest of the constraint of sensor energy and the sensor communication bandwidth.

For the lower communication traffic meets the bandwidth constraint, the main idea is to reduce the size of data packets. From this significance we can say that to guarantee a significant fusion estimation performance and various quantization reduction methods have been proposed in [3]. Meanwhile, when designing a estimator in energy-constrained sensor networks, it is not inevitable for sensors to transmit messages at every sampling instants [4]. Although this method cannot handle the bandwidth constrain problem, it can be combined with the key point of packet size reduction to design the fusion estimator in bandwidth and energy constrained sensor network [5]. As we all known, Kalman filtering is the most impactful way to find the optimal estimation of unknown state for the dynamic systems [6]. Figure 1 and Figure 2 show packet losses significantly affecting the estimation performance.

The issue of state estimation stability under a variety of factors such as bounded noises has attracted intense research attention [7]-[14]. [15] Bo Chen et al. study a new local estimator with time-varying gain is designed by solving a class of convex optimization problems such that the square error of the estimator is bounded. The information fusion noise statistics estimators are presented by averaging the local estimators of noise statistics in [16]. More generally, a class packet loss models in the view of semi-Markov chains is studied in [17]. We use CR over multiple channels to enhance the state estimation performance. We propose a CR based channel sensing and switching mechanism (CSSM) for state estimation. We focus on answering whether and how the state estimation can be increased by the new mechanism and the constraint of sensor energy and communication bandwidth. We get the conditions under which the proposed mechanism can improve the estimation performance. A couple of upper and lower bounds for mean square fusion estimation error is authenticated. And the performance melioration is analyzed.

Notations: The \( \mathbb{R}^n \) is \( n \)-dimensional Euclidean space. \( \mathbb{E}[\cdot] \) and \( \mathbb{P}[\cdot] \) manifest the expectation and likelihood of random variable, respectively. For any matrix \( M \), \( M' \) and \( M'' \) manifest \( Tr(\cdot) \) and \( \rho(\cdot) \) manifest the trace and spectral radius of a square matrix, respectively. \( \lambda_{\text{max}}(Y) \) is the maximum eigenvalues, and \( \lambda_{\text{min}}(X) \) is the minimum eigenvalues.
2. PROBLEM STATEMENT

2.1 System Modeling

Consider a linear discrete-time state dynamics described by the following state-space model:

$$x_{k+1} = Ax_k + Bu_k + w_k$$  \hspace{1cm} (1)

$$y_k = Cx_k + v_k$$  \hspace{1cm} (2)

where \(x_k \in \mathbb{R}^n\) manifest the system state, \(u_k \in \mathbb{R}^m\) manifest the control action, \(w_k \in \mathbb{R}^r\) manifest the system noises, \(v_k \in \mathbb{R}^r\) manifest the meterage noise. \(A, B, C\) are constant matrices with corresponding dimensions.

Kalman Filtering based State Estimation: we define a random variable \(\gamma_k \in [0,1]\) to model the wireless communication reliability , \(\gamma_k = 1\) indicates the measurement packet is successfully received, \(\gamma_k = 0\) indicates the packet is lost.

Based on the measurements, the local estimator is given by following recursive form:
Based on Equation (3), the predicted error covariance meets the following modified algebraic Riccati equation.

\[
P_k = A P_{k-1} A^T - \gamma_k A P_{k-1} C \left( C P_{k-1} C^T + R \right)^{-1} C P_{k-1} A^T + Q
\]  

(4)

2.2 CSSM

The sensor is equipped with an antenna that can sense the signals in the channels. The sensor scans the authorized channels following the ordered channel index set \( \mathcal{O} \). When sensed idle, it will stop sensing and transmit packet by the first channel. Let \( m \in \mathcal{O} \). For each \( m \in \mathcal{O} \), it indicates the state of \( iCH \) at the time when sensor senses \( iCH \) in step \( k \) as \( s_i = 0 \) or \( 1 \). Let \( \phi_{ij} \) manifests the perception results. If \( CH_i \) is sensed busy, so \( \phi_{ij} = 1 \), \( \phi_{ij} = 0 \) otherwise. Once a transmission is completed, the sensor will turn back to \( CH_0 \), in order to minimize probable interference to PU.

The following proposition originates from probability transition matrix \( \Phi \) of \( CH_i \) where \( \{\phi_{ij}\} = P[s_i = l| s_{i-1} = j-1] \).

Proposition 1: \( \{s_i\}_{i=0} \) comprises a homogeneous Markov chain with the transition probability matrix

\[
\Phi_i = \begin{bmatrix}
1 - \alpha_i & \alpha_i \\
\beta_i & 1 - \beta_i
\end{bmatrix}
\]

(5)

where \( \alpha_i = \frac{\phi_{ij} - \phi_{ij} \left( 1 - e^{-c_i \lambda_s s_i} \right)}{\phi_{ij} + \phi_{ij} \left( 1 - e^{-c_i \lambda_s s_i} \right)} \), \( \beta_i = \frac{\phi_{ij} \lambda_s s_i}{\phi_{ij} + \phi_{ij} \left( 1 - e^{-c_i \lambda_s s_i} \right)} \).

To describe the channel sensing accuracy, we define two probabilities. \( p_{ij} = P[\phi_{ij} = 0 | s_i = 0] \) manifests the correct detection probabilities, \( p_{ij} = P[\phi_{ij} = 0 | s_i = 0] \) manifests the false detection probabilities.

3. CHANNEL CASE

In this portion, the conditions for estimation stability will be discussed. Then we show how the CSSM will expand estimation stability region. Let us define

3.1 The Critical Matrix \( \Phi\Psi \)

Delimiting \( B_a(i) \) as follows. \( \forall b \in \{1, \ldots, 2^m\} \), there exists the only binary vector \( B_a(b) = [h_1, \ldots, h_m] \) such that \( b = \sum_{i=1}^{m} 2^i h_i + 1 \). Define channel state vector \( s_i = [s_{i1}, \ldots, s_{im}] \), \( \psi_s = [\rho_{s1}, \ldots, \rho_{sm}] \). Based on \( \psi_s \) and the hypothesis of the inter-channel independencies.

\[
\Phi_s \psi_s = B_a(i) \psi_s = B_a(i) \]

(6)

where \( i, j \in \{1, \ldots, 2^m\} \). The sensing matrix \( \Psi_s \) is in form Diag \( \{\psi_1, \ldots, \psi_{2^m}\} \) where \( \forall b \in \{1, \ldots, 2^m\}, \psi_b = P[\phi_b = 1 | \phi_b = B_a(b)] \). \( \epsilon_i \) indicates the packet loss rate on \( CH_i \) when it is in state \( s : \epsilon_i = 1 \) if \( s = 1 \) and \( \epsilon_i = 0 \) otherwise.
$$\psi_s = P_{[s_{i1}, \ldots, s_{iJ}]} = B_s(b)$$

$$= P_{[s_{i1} = 0, s_{i2} = h_2, \ldots, s_{iJ} = h_J]} + \sum_{s_{i2}} P_{[s_{i2} = 0, s_{i3} = h_3, \ldots, s_{iJ} = h_J]} + \cdots + P_{[s_{iJ} = 0, s_{i1} = h_1]}$$

$$\times P_{[s_{i1} = 0, s_{i2} = h_2, \ldots, s_{iJ} = h_J]}$$

$$\times \cdots \times P_{[s_{iJ} = 0, s_{i1} = h_1]}$$

$$\times \cdots \times P_{[s_{i2} = 0, s_{i3} = h_3, \ldots, s_{iJ} = h_J]}$$

$$+ \cdots + P_{[s_{iJ} = 0, s_{i1} = h_1]}$$

$$= \sum_{s_{i1}} \cdots \sum_{s_{iJ}} P_{[s_{i1} = 0, s_{i2} = h_2, \ldots, s_{iJ} = h_J]}$$

$$\times \cdots \times P_{[s_{iJ} = 0, s_{i1} = h_1]}$$

$$= \sum_{s_{i1}} \cdots \sum_{s_{iJ}} \prod_{j=1}^J P_{[s_{ij} = 0 | s_{i1} = h_1]} \prod_{j=1}^J P_{[s_{ij} = 0 | s_{i1} = h_1]}$$

$$= \prod_{j=1}^J \prod_{k=1}^J P_{[s_{ij} = 0 | s_{ik} = h_k]}$$

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$$= \prod_{j=1}^J \prod_{k=1}^J P_{[s_{ij} = 0 | s_{ik} = h_k]}$$

$$(7)$$

Where the second equality is based upon the definition of the sending sequence $\Omega$.  

### 3.2 Stability Analysis

**Theorem 1:** For system (1) with above channel sending schedule $\Omega$, a necessary condition for MSE stability of fusion estimation process with CSSM is

$$\rho(\Phi \Psi) \rho(A)^2 < 1$$

Moreover, (8) is sufficient if $\mathcal{C}$ has full column rank.  

### 3.3 Performance Analysis

**Theorem 2:** Let $\ell_{\min} = \min\{\ell_1, \ell_2, \ldots, \ell_n\}$. With sensing schedule $\Omega$, stability gain $\eta$ satisfies:

$$\eta = \sqrt{\ell_1} \sqrt{\rho(\Phi \Psi)} \leq \frac{1}{\sqrt{\prod_{i=1}^n (1 - \beta^i)}} \sqrt{\ell_{\min}}$$

Let $\tau^* = \max\{\omega(\Phi \Psi)u, \forall i \in \{1, 2, \ldots, n\}\}$, where $\omega$ is a $2^n \times 1$ vector with $[\omega_i] = 1$ and $[\omega_j] = 0$ for all $j \neq i$, $u = [1, \ldots, 1]$.  

**Theorem 3:** If $\tau^* \leq \ell_{\min}$, and initial conditions satisfy $P'_0 = P_0$, then $Tr(E[P'_0]) \leq Tr(E[P_0])$.  

**Theorem 4:** For all $k > 0$, $P_k \leq P_{k+1}$, $E[P_k] \leq E[P_{k+1}]$. Moreover

$$Tr(E[P_k]) = \sum_{i=1}^n \nu(\omega(\Phi \Psi)u) Tr(E(\omega(\Phi \Psi)T))$$

$$Tr(E[P_{k+1}]) = \sum_{i=1}^n \nu(\omega(\Phi \Psi)u) Tr(E(\omega(\Phi \Psi)T))$$

$$\nu$$ manifests a changeless state of the vector $p_k$, $a = 1 - \nu(1 - \Phi \Psi)u$.  

Remark: Our results can apply to a wide range of scenarios. Considering another channel sensing which differs from $\Omega$, in which the sensor can directly transmit packets through $CH_i$ if $CH_{i-1}$ is sensed busy. If we hold $CH_i$ as a primordial channel; the scenario the original channel is Markovian and the CSSM is not used can be viewed as special single-authorized-channel case. So the results will be renewed.  

### 4. SIMULATION EXAMPLE

Considering a maneuvering target with the physical process satisfies (1). We use the same parameters as in [18]: $A = \begin{bmatrix} 1.25 & 0 \\ 1 & 1.1 \end{bmatrix}$, $Q = 20I_{2x2}$, and the sensor meterages are described by Equation (2), where $C = I_{2x2}$, $R = 2.5I_{2x2}$, the sample period $T = 1$, $\alpha = 0.537$, $\beta = 0.461$, $\lambda = 0.05$. So the largest eigenvalue of critical matrix $\Phi \Psi$ is $\sigma_2 = 0.415 < \ell_{\min} \leq \frac{1}{\rho(A)^2}$, the state fusion estimation based on the Kalman filter is stable in the mean square sense. However, the case completes different performance according to $Tr(E[P_k])$. As shown in Figure 3, the CSSM brings about much more fusion estimation performance.
Firstly, we observe the bounds are tight in the simulation case. Without CSSM, it is widely known that fusion estimation error covariance diverges only if $\ell_0$ is larger than $\frac{1}{\rho(4)^{\frac{1}{2}}} \approx 0.64$. In contrast, with $CH_1$ and CSSM, the critical value increases obviously, i.e., the demands for the estimation stability is relaxed. From Figure 4 we demonstrate that CSSM improves estimation performance when $\ell_0 \geq 0.34$. When each local estimate $\hat{x}_k(t)$ is sent to the FC due to finite bandwidth and the limited sensor energy. In reality, when $\ell_0 \geq 0.34$, both $\sigma_2 < \ell_0$ and $\psi_0 (0 - \beta) + \psi_0 \beta \leq \ell_0$ are true. Thus, the fusion estimation capability is assured to be enhanced. However, the delineation also manifest that there is a performance descend when $\ell_0 < 0.34$ such that quantity of $CH_0$ is better than that of the $CH_1$. This can be finished off by strengthening the channel sensing accuracy.

More authorized channels can provide higher opportunities for sensor to triumphantly transmit its measurement packets. The performance bounds and the performance ratio obtained in Theorem 4, where $m = 0$ demonstrates the case without CSSM. We can see that: 1) With CSSM, the upper bound and relaxed upper bounds are quite close in all situations. 2) The worst situation property ratio is less than 36.68%. 3) The property is clearly improved by introducing to the CSSM more authorized channels.

5. CONCLUSION
In this paper, the fusion estimation problem with linear process state dynamics has been investigated for networked time-varying fusion system with bounded noises. Based on cognitive radio technology, we propose CSSM mechanism for sensor to opportunistically enter authorized spectrum in data transfer. We develop a new necessary and sufficient condition for fusion estimation stability in mean square sense. At last, simulation results manifest that fusion estimation property is remarkably improved by CSSM.

![Figure 3. Performance comparison.](image-url)
Figure 4. Performance bound comparison.

REFERENCES

[1] Lee W Y, Akyildiz I F. Optimal spectrum sensing framework for cognitive radio networks[J]. IEEE Transactions on Wireless Communications, 2008, 7(10):3845-3857.

[2] Pan M, Li P, Fang Y. Cooperative Communication Aware Link Scheduling for Cognitive Vehicular Networks[J]. IEEE Journal on Selected Areas in Communications, 2012, 30(4):760-768.

[3] Xia Y F, Zhu Y M, Huang K N. Scheme for observation quantization in information estimation fusion[J]. Journal of Sichuan University, 2009, 41(2):211-215.

[4] Pei Y, Liang Y C, Teh K C, et al. Energy-Efficient Design of Sequential Channel Sensing in Cognitive Radio Networks: Optimal Sensing Strategy, Power Allocation, and Sensing Order[J]. IEEE Journal on Selected Areas in Communications, 2011, 29(8):1648-1659.

[5] Wu X, Tian Z. Optimized Data Fusion in Bandwidth and Energy Constrained Sensor Networks[C]// IEEE International Conference on Acoustics, Speech and Signal Processing, 2006. ICASSP 2006 Proceedings. IEEE, 2006:IV-IV.

[6] Sinopoli B, Schenato L, Franceschetti M, et al. Kalman filtering with intermittent observations[J]. IEEE Transactions on Automatic Control, 2004, 1(9):1453-1464.

[7] Guerra P, Puig V, Ingimundarson A. Robust fault detection using a consistency-based state estimation test considering unknown but bounded noise and parametric uncertainty[C]// Control Conference. IEEE, 2015:1595-1601.

[8] Becis-Aubry Y, Boutayeb M, Darouach M. An ellipsoidal state estimation algorithm for nonlinear systems subject to bounded disturbances[C]// European Control Conference. IEEE, 2015.

[9] Foo Y K, Soh Y C, Moayed M. Linear set-membership state estimation with unknown but bounded disturbances[J]. International Journal of Systems Science, 2012, 43(4):715-730.

[10] Xu X B, Zhang Z, Zheng J, et al. State estimation method based on evidential reasoning rule[C]// Advanced Information Technology, Electronic and Automation Control Conference. IEEE, 2016:610-617.

[11] Chen B, Ho D W C, Zhang W A, et al. Networked Fusion Estimation with Bounded Noises[J]. IEEE Transactions on Automatic Control, 2017, PP(99):1-1.

[12] Xie W, Xia Y. Recursive parameter estimation with bounded noises in the presence of missing outputs and outliers[C]// Control Conference. IEEE, 2014:5167-5172.

[13] Qu X, Zhou J, Tan W. Robust decentralized estimation fusion in energy-constrained wireless sensor networks with correlated noises[J]. Digital Signal Processing, 2018.
[14] Jin X B, Bao J, Zheng H J. Centralized robust fusion estimation in estimation of paper basis weight based on norm-bounded parameter uncertain model[J]. Proceedings of SPIE - The International Society for Optical Engineering, 2010, 7820(1):485-490.
[15] Chen B, Yu L, Zhang W A, et al. Robust Information Fusion Estimator for Multiple Delay-Tolerant Sensors With Different Failure Rates[J]. IEEE Transactions on Circuits & Systems I Regular Papers, 2013, 60(2):401-414.
[16] Zhang M, Xiao-Ling F U, Cui P. Multi-sensor optimal information fusion for time-delay systems with multiplicative noise[J]. Journal of Shandong University, 2010.
[17] Censi A. Kalman Filtering With Intermittent Observations: Convergence for Semi-Markov Chains and an Intrinsic Performance Measure[J]. Automatic Control IEEE Transactions on, 2011, 56(2):376-381.
[18] Sinopoli B, Schenato L, Franceschetti M, et al. Kalman filtering with intermittent observations[J]. IEEE Transactions on Automatic Control, 2004, 1(9):1453-1464.
[19] Wang Z S, Liu W J, Zhen Z Y. Design of Optimal Tracking Controller for Nonlinear Discrete System with Input Delay Using Information Fusion Estimation Method[C]// IEEE International Conference on Control and Automation. IEEE, 2007:2535-2538.
[20] Zhang W A, Ni H, Song H, et al. Distributed information fusion estimation for sensor networks with nonuniform sampling rates[C]// International Conference on Mechatronics and Control. IEEE, 2015:502-505.