The dynamical simulations of the planets orbiting GJ 876

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ABSTRACT

In this paper we have performed simulations to investigate the dynamics of the M dwarf star GJ 876 in an attempt to reveal any stabilizing mechanism for sustaining the system. We simulated different coplanar and non-coplanar configurations of two-planet system and other cases. From the simulations, we found that the 2:1 mean motion resonance between two planets can act as an effective mechanism of maintaining the stability of the system. This result is explained by a proposed analytical model. By means of the model, we still studied the region of motion of the inner planet by varying the parameters of the system and detected that the analytical results are well consistent with the numerical simulations.

Subject headings: dynamical simulations, stellar dynamics - methods: N-body simulations, planetary systems - stars: individual GJ 876

1. Introduction

At present more and more extrasolar planetary systems are being discovered (Marcy et al. 2000; Butler et al. 2000), and many research groups throughout the world are

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now devoting themselves to surveying these planets. From the Extrasolar Planets Encyclopedia website (http://www.obspm.fr/planets), we have known that \( \sim 60 \) planets around main-sequence stars are confirmed and there are many others yet to be confirmed. Most of the extrasolar planets are found to either orbit very close to their host stars or travel on much more eccentric paths than any of the major planets in our solar system, which is an enormous challenge for standard planetary formation theories as well as for the dynamical stability of the planetary system.

Marcy et al. (2001) pointed out that the two planets orbiting the M4 main-sequence star GJ 876 are now apparently locked in the state of 2:1 resonance, with orbital periods 30.1 d and 61.0 d, and semimajor axes 0.13 AU and 0.21 AU. They suggested that stability might be sustained by the above mean-motion resonance. In their paper, they gave GJ 876 an estimated mass of \( 0.32 \pm 0.05 M_\odot \), and this value was used in our numerical simulations. The masses of the two planets given in Table 1 (Laughlin & Chambers 2001) were derived with an inclination \( \sin i = 0.55 \), where \( i \) is the inclination of the orbit relative to sky plane. The masses of the planets are always utilized except for special comments.

Laughlin & Chambers (2001) showed that short-term perturbations among massive planets in multiple planet system can give rise to radial velocity variations of the central star that differ from those which consider the planets move as Keplerian ellipse. They also implied that the configurations given in the paper are stable for at least \( 10^7 \) yr as the mean motion resonances and secular resonances appeared during the long-term orbital evolution. Recently Rivera & Lissauer (2000) studied the planetary system orbiting \( \nu \) Andromedae, which consists of three Jovian-mass planets with the orbital periods range from \( \sim 4 \) d to \( \sim 4 \) yr. They found that some configurations are stable for at least \( 10^9 \) yr, but others can be ejected into the stellarspace because of the excitation of the eccentricity. Due to their great efforts, now we have more helpful knowledge about the dynamics of the extrasolar systems, however in this article we shall present our substantial research of GJ 876 by simulations.

In the rest of this paper is organized as follows: In Section 2, we briefly introduce the numerical setup of the dynamical simulations. In Section 3, we present the results of the simulations of the GJ 876 system. In Section 4, we describe an analytical model that helps to explain the resonant mechanism of the studied system. In Section 5, we give a brief discussion.
2. Numerical setup

We have developed a purely gravitational N-body scheme for numerical simulation, in which general relativistic effect of the central body is not considered at present. We adopted RKF7(8)(Fehlberg 1968) to carry out the integrations. We used time step of 0.3d (\(\sim\) one percent of the orbital period of the inner planet) when integrating two planets in our simulations. The integrator was also optimized for close encounters during the orbital evolution. The integration was automatically ceased if either of the planets is deemed to be too far from the central star. We also effectively controlled the numerical errors, with the local truncation error \(10^{-14}\), over our time span of integration of \(10^6\) yr (\(\sim 10^7\) orbital period of the inner planet). Additionally, as the energy of the system is dissipative by using traditional algorithm, so we again used symplectic algorithm (Feng 1986; Wisdom & Holman 1991), which has many advantages such as holding the symplectic structure of the Hamiltonian system and remaining the periodical variations of the error of the energy, to examine several examples and then to confirm the results given by RKF7(8). Therefore, on the basis of these, we began to prepare the simulations of the system under study.

On the whole, we carried out three groups of the simulation. In the first runs, we aim to explore the likely resonance of the planetary system. The two planets are considered to locate at the same orbital plane, and we present herein one set of the parameters of numerical simulations of planetary configurations which are partly taken from Laughlin & Chambers (2001) (see Table 1). It is clear that six orbital elements are respectively semimajor axis \(a\), eccentricity \(e\), inclination \(i\), nodal longitude \(\Omega\), periastron \(\omega\) and mean anomaly \(M\). However, in the table the inclinations of the planets referred to reference plane are constructed as well as the nodal longitudes. We note that Fig. 1 shows the initial configuration of the two companions. Moreover, in the simulations, we fixed the semimajor axes, eccentricities and inclinations of the two planets and developed codes to randomly generate other three angles of each planet to furnish initial elements for integration.

For the second runs, we integrated the mutual inclined orbits to investigate the dynamical characteristic of the system. In the simulations, the cloned orbits of the outer planet are generated by changing its inclination. Finally, we examined the cases that the system is assumed to be composed of three planets. Rivera & Lissauer (2000) investigated the simulations of the test bodies of \(\nu\) Andromedae system. By comparison, our codes differ from theirs in that we take the assumed planet as a massive body that travels outside the orbit of the outer planet, which would be expected to provide certain information to detect unknown planets of the extrasolar planetary systems.
3. Results of dynamical simulations

In this section, we will present the leading results of our dynamical simulations. Our goal is to explore for different planetary configurations, the long-term orbital evolution of GJ 876 and to attempt to reveal any mechanism that helps to maintain the stability of the planetary system. Although simulations may differ from reality, nevertheless they can be supposed to exhibit some rough picture of the system under study.

3.1. Two-planet coplanar system

As is well known, most of the major planets in the solar system have small inclinations with regard to the essential reference plane. Thus at first we are very interested in the simulations of the coplanar cases of the system. We performed an ensemble of systems by combining the angles of $\Omega$, $\omega$ and $M$, which the two planets have small inclinations, but the semimajor axes, eccentricities and inclinations still remained unchanged for all cases when we started to separately integrate these systems in which the time span covers 1 Myr. In particular, Table 1 are listed one set of the initial orbital parameters selected from many examples.

By analyzing the integration results, we found that nearly 1/4 of hundreds of systems in our simulations remain stable over a time span of 1 Myr. It is not difficult to understand that the stability of a system is sensitive to its initial planetary configuration. The results of the simulations show that there is a tendency for many systems to self-destruct in $10^2$-$10^3$ yr, and the lifetime is even shorter for those unstable cases. Marcy et al. (2001) tried the similar simulations with different values of the initial epoch of the integration to point out that the initial epoch for the integration is an important factor in determining the system stability. However, we are particularly concerned with the stable cases. Table 2 lists the computational results by using the initial parameters produced by Table 1. In the table are presented the variations of the semimajor axis $a$, eccentricity $e$ and inclination $i$ of the two planets for 1 Myr, one can observe that these orbital parameters do not change dramatically and just remain bounded instead. In addition, Fig. 2 displays the orbital variations of the inner and outer planets for a stable case; we particularly see that the semimajor axis, eccentricity and inclination undergo small oscillations for the whole time span. We should point out that the evolution shown in Figure 2 and the results of Table 2 are typical of all the stable cases.

The stabilizing mechanism of the system is really our interest, so we will next focus on the resonant mechanism. In the usual notation of celestial mechanics, the critical argument $\sigma$ for the mean motion resonance is
\[ \sigma = \lambda_1 - 2\lambda_2 + \tilde{\omega}_1, \]  
\[ \lambda_1 = \omega_1 + \Omega_1 + M_1, \]  
\[ \lambda_2 = \omega_2 + \Omega_2 + M_2, \]  
\[ \tilde{\omega}_1 = \omega_1 + \Omega_1. \]

Here \( \lambda_1, \lambda_2 \) are, respectively, the mean longitudes of the inner and outer planets, \( \tilde{\omega}_1 \) denotes the longitudes of the periastron of the inner (hereafter subscript 1 denotes the inner planet, 2 the outer planet). For the whole integration time span, Fig. 3 exhibits the critical argument \( \sigma \) librates with an amplitude of \( \pm 70^\circ \). It is easy to understand that because of the libration of the critical argument, when the two planets are in conjunction, they are far from each other and hence are protected from close encounters, resulting in their mutual perturbations being weaker than in the non-resonant case.

Again, we illustrate the motion of the inner planet in phase space. The semimajor axis (action) of the inner planet is shown as a function of the resonant argument \( \sigma \) in Fig. 4. The action-angle coupling is characteristic of the so-called ideal resonance. Notice that the equilibrium point appears to be \( (0.130, 0) \), and this confirms the fitting result given by Marcy et al. (2001) (ref. Table 3 in Marcy et al. 2001). Besides, the figure reveals the fact that the 2:1 mean motion resonance between two planets corresponds to a moderate resonance in which the semimajor axis is well locked, and further indicates that the existing resonance is an effective mechanism to maintain the stability of the system. For the sake of better understanding, we shall construct an analytical model to explain this in the next section.

Meanwhile, for two-planet coplanar case, we carried out an extended, forward integration to check the dynamical evolution of the system. The result again suggested the 2:1 mean motion resonance. We also integrated some other cases in order to examine whether the system could be stable when the masses of the planets changed simultaneously. The amplitude in the variation in radial velocity of the star (Laughlin & Chambers 2001) suggested minimum masses of \( 0.56M_J \) for the inner planet and \( 1.89M_J \) for the outer one; we used the coupled values for the masses and the orbital elements still remained unchanged to repeat the integration. The results again showed that the 2:1 mean motion resonance still exists even when the initial masses were varied. Therefore, it seems that this resonance is possibly the most important dynamical feature of the system.

However, one may ask whether there exist other likely mechanisms to sustain the system, the answer is that the probability indeed could not be ignored. The dynamics of the asteroids in our solar system (Morbidelli & Nesvorny 1999; Malhotra et al. 1996, 1998, 2000; Duncan...
& Levison 1997) shows that the secular resonances with the major planets (such as Jupiter, Saturn, Neptune, etc) are very important for the long-term orbital evolution of the asteroids, which can excite their eccentricities and inclinations over tens of millions years. The leading viewpoints are that the asteroids in the main belt can be transported to near-earth space and the bodies of the Kuiper Belt can be as a reservoir of the jovian short-period comets through the complicated interaction of the mean motion resonance and secular resonances. In addition, there is a special secular resonance, so-called Kozai resonance (Kozai 1962), which corresponds to the coupled oscillations of the inclination and eccentricity of the planet, unlike the precession of the longitudes of the bodies’ perihelia (or periastra) or ascending nodes. The discovery of the extrasolar planets provide abundant sources to investigate the dynamics of the bodies. Laughlin & Chambers (2001) performed dynamical simulations to suggestion a secular resonance of the GJ 876 system. Hence, we were determined to focus on these secular resonances, as a result, we were capable of discovering such systems from the simulations. Fig.5 displays that the periastron of the inner planet temporarily librates around 0° or 180°, correspondingly, the eccentricity is extremely excited to pump up to the value of unity, which is associated with Kozai resonance. Additionally, the diagram implies that this kind of secular resonance plays significant part in the long-term dynamical evolution of the planet.

3.2. Two-planet highly inclined orbits system

For other simulations, we again performed tests of highly inclined orbits configuration to examine the dynamics of the GJ 876 system. Regarding our solar system, we recollect the fact that most of the major planets have low inclinations and eccentricities, accompanying the sun, therefore in a viewpoint of dynamics the investigations of the non-coplanar cases can further help understanding the dynamical evolution of the extrasolar systems.

For the sake of simplicity, we only varied the inclination of the outer planet but remained that of the inner planet. For each case, the inclination of the outer planet was separately increased by 5°, 10°, 15°, 20°, 25° and 30° with respect to that of the inner planet. Additionally, in these experiments, we only adopted other orbital elements given by Table 1 rather than combined the angles. The numerical integrations reveal the fact that for some cases, though the inclination was greatly changed but the two planets are still in the mean motion resonance (MMR), which indicates that to some extent this resonant mechanism can make the system stable, while for unstable cases we found that the inner planet abruptly leaves the system on account of the accumulation of perturbations by the outer planet. Table 3 reports the details. However, these simulations partly show some of dynamical features of the system, we can not simply believe that highly inclined orbits will lead to the unstable
systems by common judgement, while the stability of the system is still relative to the given initial conditions. Our conclusions are consistent with the dynamical simulations of Marcy et al. (2001), who announced the finding that the two planets were locked in a 2:1 mean motion resonance for all the stable configurations.

Still, we also considered the cases of three-planet system, and details of the problem will be examined in a later paper (Ji & Liu, in preparation).

4. Analytical model

In Section 3, we have taken into account several groups of the planetary configurations by numerical simulations, then we are strongly impressed that the above-mentioned mean motion resonance between two companions could play important role in the stabilization of the system, then it is essential for one to clarify the mechanism of the problem. Hence, we propose an analytical model to explain the mechanism of the mean motion resonance of the explored system.

4.1. Ideal resonant model of the problem

From Table 1, we underline that these two planets have very low inclinations with respect to the fundamental plane and the outer planet has a small eccentricity, and this reminded us of the ideal resonant model (Garfinkel 1966; Liu et al. 1985a, 1985b) under the dynamical framework of a star-plus-two planets system. For simplicity, if we take the mass of the central star $M_c$ as the unit of mass, the semimajor axis of the outer planet $a_2$ as the unit of length, and define the unit of time by $[T] = (a_2^3/G M_c)^{1/2}$, then the gravitational constant $G$ will have the value unity.

In the beginning, we introduce Delaunay variables commonly used in the celestial mechanics,

\[
\begin{align*}
L &= \sqrt{a} \\
G &= L \sqrt{1 - e^2} \\
H &= G \cos i
\end{align*}
\]

In the next step, we conveniently introduce the following transformation,
\[
\begin{align*}
\begin{cases}
\tilde{L} = L/q, & \tilde{l} = ql - pl + p(\tilde{\omega} - \tilde{\omega}_2), \\
\tilde{G} = G - p\bar{L}, & \tilde{g} = g, \\
\tilde{H} = H - p\bar{L}, & \tilde{h} = h.
\end{cases}
\end{align*}
\]

(6)

where \( p \) and \( q \) are integers, and we let \( q : p = 1 : 2 \) in our paper.

Following our earlier paper (Ji, Liu & Liao 2000), we will have the action-angle variables (hereafter subscript 1 is omitted for simplicity):

\[
\begin{align*}
\begin{cases}
\tilde{L} = \sqrt{a} \\
\tilde{l} = \sigma = \lambda - 2\lambda_2 + \tilde{\omega}
\end{cases}
\end{align*}
\]

(7)

Then in terms of the action-angle variables of Eqs. (6) and (7), the relevant Hamiltonian function can be written as

\[
F = \tilde{F}_0(\tilde{L}) + \tilde{F}_c(\tilde{L}, \tilde{G}) + F_{2/1} \cos \tilde{l}
\]

(8)

where

\[
\begin{align*}
\begin{cases}
\tilde{F}_0(\tilde{L}) = & \frac{1}{2\tilde{L}^2} + 2n_2\tilde{L} \\
\tilde{F}_c(\tilde{L}, \tilde{G}) = & \mu_2a^2 \left[ \frac{1}{4} (1 + \frac{3}{2}e^2) + \frac{9}{64}a^2 (1 + 5e^2) \right] \\
\tilde{F}_1 = F_{2/1} = & -\mu_2a^2 \left( \frac{9}{4} + \frac{5}{4}a^2 \right) e \\
\tilde{L} = L, & \tilde{G} = G - 2\bar{L}.
\end{cases}
\end{align*}
\]

(9)

where in Eq. 9, \( \mu_2 \) and \( n_2 \), respectively, are the mass and mean motion rate of the outer planet. We should point out that short period terms have been averaged, and that high eccentricity terms are neglected when deriving Eqs. 8 and 9.

As mentioned in the earlier papers (Liu et al. 1985a, 1985b; Ji, Liu & Liao 2000), the Hamiltonian \( F \) does not explicitly involve \( \tilde{g}, \tilde{h} \) and \( t \), therefore there exist the following integrals:

\[
\tilde{G} = \tilde{G}_0, \quad \tilde{H} = \tilde{H}_0,
\]

(10)

and

\[
F = \bar{h}.
\]

(11)
where $\bar{h}$ is a constant of integration, i.e., the total energy of the system.

After the above treatment, finally we have the Hamiltonian of a one degree of freedom system:

$$ F = \tilde{F}_0 \left( \tilde{L} \right) + \tilde{F}_c \left( \tilde{L}, \tilde{G}_0 \right) + \tilde{F}_1 \cos \tilde{l}, \quad (12) $$

and we have the canonical Hamiltonian equations of motion,

$$
\begin{align*}
\dot{\tilde{L}} &= \frac{\partial F}{\partial \tilde{l}} = -\tilde{F}_1 \sin \tilde{l} \\
\dot{\tilde{l}} &= -\frac{\partial F}{\partial \tilde{L}} = -\left( \frac{\partial \tilde{F}_0}{\partial \tilde{L}} + \frac{\partial \tilde{F}_c}{\partial \tilde{L}} \cos \tilde{l} \right).
\end{align*}
$$

According to Eq. 13, we theoretically investigated the motion of the inner planet using the initial values given in Table 1. Fig. 6 illustrates the results of this analytical model. The resonant argument $\sigma$ librates with an amplitude of $\pm 70^\circ$, and from the figure we are able to estimate the equilibrium point to be about $(0.131, 0)$. Moreover, Fig.6 also indicates that the inner planet is really in the 2:1 resonant state with the outer planet. Hence, in comparison with Fig.4, we come to the conclusion that the structure of phase space given by our analytical model is consistent with the leading results of our numerical simulations (see Fig. 4), and this model can help explaining the resonant mechanism of GJ 876, which can maintain the stability of the system.

### 4.2. The region of motion of the inner planet

From the numerical simulations, we have learned that the stability of the system greatly depends on the initial configurations, however in the perspective of the analytics, we still hope to get knowledge about the dynamical evolution of the system as we change other parameters by means of the proposed resonant model.

Rivera & Lissauer (2001) provided the fitting results of the planetary masses expressed by the product $M_p \sin i$ according to their dynamical models, but in a sense the actual masses are really hard to determine, so at first we aim to understand the dynamics of the system if we gradually change the mass of the outer planet $\mu_2$. In accordance with Eq. 13, we set to carry out calculations to study the region of motion of the inner planet by varying $\mu_2$. In our experiments, we let $\mu_2 = 0.05, 0.15, 0.30, 0.32, 0.332, 0.35, 0.55, 1.06, 1.695$ and $3.39 \, M_J$ separately and other parameters are still chosen from Table 1. Fig. 7 exhibits the portrait of the phase space of the computations, then the semimajor axis $a$ is plotted
against the resonant argument $\sigma$ for different masses. From the phase diagram, we note that when $\mu_2 \leq 0.32M_J$, we have the circulation of the system; for $3.39 \leq \mu_2 \leq 0.332M_J$, in contrast, the system undergoes the state of libration instead. The figure indicates that the perturbation of the outer planet may greatly influence on the regime of motion of the inner planet and the resonance becomes much stronger as the mass of the outer planet $\mu_2$ increases, so that the region of motion of the inner planet is much wider. Furthermore, we shall seek the quantitative explanation. In fact, in terms of Eq. 13, if we fix $e, a_0$ and $\tilde{l}_0$ and drive $\mu_2$ augment, then $\dot{\tilde{L}}$ increases, but $\dot{l}$ decreases, vice versa.

On the other hand, we studied the system by varying the starting angle variable $\tilde{l}_0$ in Eq. 7, which corresponds to different cases of the initial motion of the two planets. For detailed calculations, we took $\tilde{l}_0$ as $0^0, 30^0, 60^0, 90^0, 120^0, 135^0, 144^0, 145^0, 150^0$ and $180^0$, respectively. The dominant results are presented by Fig. 8. The figure displays that the intensity of the resonance grows less weaker as the angle becomes much larger, which the dynamical behavior of the system varies from libration to circulation; especially for $\tilde{l}_0=145^0$, the trajectory lies on the side of the separatrix, on the edge of the resonance. In addition, for smaller $\tilde{l}_0$, we can easily detect that the equilibrium point turns to be $(0.131, 0)$, which again confirms those of the numerical simulations. In a word, these quantitative study may somewhat sketch rough outline of the dynamics of the GJ 876 system.

5. Discussion

In this paper, we have mainly explored the dynamics of the GJ 876 system by simulating different cases of the planetary configurations. Moreover, we also proposed an analytical model to make quantitative explanation of the studied system.

In the end, we summarize some conclusions:

For two-planet coplanar systems, we found that nearly 1/4 of hundreds of systems in our simulations remain stable over a time span of 1Myr. Moreover, the stability of a system is sensitive to its initial planetary configuration. And the numerical results suggested the existence of a 2:1 resonance for the stable systems, and this kind of resonant mechanism can also be found for those stable cases of the two-planet highly inclined orbit systems. Therefore, according to the resulting simulations, we should emphasize that the 2:1 mean motion resonance between two planets can act as an effective mechanism for maintaining the stability of the system.

From another direction, we have proposed an analytical model to explain the resonant mechanism found in the simulations and observations. In a qualitative viewpoint, the model
again demonstrates that the 2:1 mean motion resonance plays the major role of sustaining the system. Using the model, we then studied the region of motion of the inner planet by changing different parameters involved in the dynamics of the system and presented the helpful results.

However, the actual dynamics of the GJ 876 system should be rather complicated beyond imagination. As mentioned before, the secular resonance might also have effect on the long-term dynamical evolution of the system, thus we can safely reach the conclusion that in a sense the mixture of the different resonances are responsible for the dynamics of the system. As we still do not know much about such systems, we shall make further study of the dynamics of the extrasolar planets to better understand their origin and evolution.

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Figure Captions

Fig.1 The initial configuration of two companions projected on the plane, the red filled circle represents the main star; the blue, the inner planet; the magenta, the outer planet. The orbital periods of are 30.1d and 61.0d, respectively.

Fig.2 The variations of the semimajor axis $a$, eccentricity and inclination of the inner and outer planets for the time span of 1Myr. The unit of the semimajor axis is AU, and degree for the inclination. The figure shows that the orbital parameters do not change dramatically, and the semi-major axes $a$ librate about 0.130 AU and 0.210 AU, respectively, then the mean motion resonance between two planets maintains during the dynamical evolution. The evolution shown in the figure is typical of all the stable cases.

Fig.3 The critical argument $\sigma$ is plotted against time, note that the argument librates with an amplitude of $\pm 70^\circ$ over the time span of 1Myr.

Fig.4 The motion of the phase space of the inner planet is given by numerical simulations, the semimajor axis is plotted as a function of resonant argument $\sigma$, note that the equilibrium point is around (0.130, 0).

Fig.5 The case of the secular resonance of the inner planet, the upper panel exhibits that the eccentricity changes with time and the lower panel shows that the periastron $\omega$ as a function of time, temporarily librating about $0^\circ$ or $180^\circ$. In final, the inner planet escapes from its orbit because of the excitation of the eccentricity.

Fig.6 The motion of the phase space of the inner planet is presented by analytical method, the semimajor axis is plotted as a function of resonant argument $\sigma$, which also librates with an amplitude of $\pm 70^\circ$. In comparison with Fig.4, the structure of phase space is in good agreement with each other.

Fig.7 The motion of the phase space of the inner planet for different masses of the outer planet, for $\mu_2 \leq 0.32 M_J$, the system in the circulation; for $3.39 \leq \mu_2 \leq 0.332 M_J$, the system in the libration, the resonance becomes much stronger as we increase the mass of the outer planet, so that the region of motion of the inner planet is much wider.

Fig.8 The motion of the phase space of the inner planet for different initial phases of two planets, note that for $\tilde{l}_0=145^\circ$, the trajectory lies on the side of the separatrix, on the
edge of the resonance. For smaller $\tilde{l}_0$, notice that the equilibrium point turns to be $(0.131, 0)$. 
Table 1. The initial osculating orbital elements of two planets

| Parameter          | Inner   | Outer   |
|--------------------|---------|---------|
| Mass ($M_{Jup}$)   | 1.06    | 3.39    |
| Periods (days)     | 29.995  | 62.092  |
| $a$ (AU)           | 0.1294  | 0.2108  |
| Eccentricity       | 0.314   | 0.051   |
| Inclination (deg)  | 0.5     | 0.5     |
| $\Omega$ (deg)     | 59.2    | 20.0    |
| $\omega$ (deg)     | 51.8    | 40.0    |
| Mean anomaly (deg) | 289.0   | 340.0   |
Table 2. The orbital elements variation of two planets

| Inner | Upper limit | Lower limit | Outer | Upper limit | Lower limit |
|-------|-------------|-------------|-------|-------------|-------------|
| $a$ (AU) | 0.1353 | 0.1248 | $a$ (AU) | 0.2148 | 0.2013 |
| $e$ | 0.3676 | 0.2419 | $e$ | 0.0631 | 0.0001 |
| $i$ (°) | 0.7852 | 0.1806 | $i$ (°) | 0.5560 | 0.4097 |
Table 3. The status of GJ 876 system for different inclinations of the outer planet over the time span of 1Myr

| Incl. of inner planet | Incl. of outer planet | status of the system                          |
|-----------------------|-----------------------|-----------------------------------------------|
| 0.5                   | 5.5                   | Stable, MMR                                   |
| \(\cdots\)            | 10.5                  | \(T > 413,560\) yr , inner planet escape      |
| \(\cdots\)            | 15.5                  | \(T > 158,880\) yr , inner planet escape      |
| \(\cdots\)            | 20.5                  | \(T > 553,172\) yr , inner planet escape      |
| \(\cdots\)            | 25.5                  | Stable, MMR                                   |
| \(\cdots\)            | 30.5                  | Stable, MMR                                   |