Constraining new physics in the Tev range by the recent BNL measurement of $(g - 2)_\mu$

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Abstract
In this paper we study the implications of the recent high precision measurement of $(g - 2)_\mu$ by BNL [1] on new heavy physics beyond the SM in a model independent way. We find that if the new physics responsible for the muon anomaly is due to d=6 direct operators then they could arise from the following following three broad classes of new physics a) new particles in the few hundred Gev range with weak gauge coupling b) strongly interacting particles and resonances in the few Tev range and c) massive Kaluza-Klein modes of the graviton in the Tev range with couplings to SM particles of the order of $E_{(Tev)}$. 

The effect of new heavy physics beyond the SM appearing above some high energy scale $\Lambda$ at energies much small compared to $\Lambda$ can be expressed by non-renormalizable operators constructed out of SM fields. These operators can be expressed in a systematic power series expansion in $\frac{1}{\Lambda}$. The structure of these operators is completely determined by a) the fields that are dynamical at the relevant energy scale b) the residual gauge symmetry at scales much small compared to $\Lambda$ and c) the global symmetries respected by the low energy theory.

The muon $(g - 2)_\mu$ collaboration has reported a new improved measurement of the positive muon anomaly [1]

$$a_\mu(\text{expt}) = 11659202(14)(6) \times 10^{-10}. \quad (1)$$

The value currently expected in the SM is [2]

$$a_\mu(\text{SM}) = 11659159.6(6.7) \times 10^{-10}. \quad (2)$$

The world-average experimenta value of $a_\mu$ shows a discrepancy of 2.6 $\sigma$ from the SM value [1]

$$\delta a_\mu = a_\mu(\text{expt}) - a_\mu(\text{SM}) = 43(16) \times 10^{-10}. \quad (3)$$

The new measurement of the muon anomaly by BNL has produced quite some interest and activity in this area. The discrepancy between the SM and the experimental value reported by BNL has been used to put bounds on the unknown parameters in a variety of new physics scenario namely extra gauge bosons, exotic fermions, compositeness, supersymmetry and leptoquarks [3].

In this report we shall study the effect of new heavy physics appearing at some high energy scale $\Lambda$ on the anomalous magnetic moment of the muon. Such effects can be expressed by non-renormalizable operators constructed out of SM fields. The operators
must be invariant under the SM gauge group $SU(3)_c \times SU(2)_l \times U(1)_y$ which is the relevant gauge symmetry below $\Lambda$. We shall assume that the SM gauge symmetry is linearly realized on the SM fields. This will correspond to an elementary or a light composite higgs scalar. It then follows that the lowest dimension operator invariant under the SM gauge group that contributes to the muon magnetic moment anomaly is six. Here we shall consider two such operators that contribute directly to $a_\mu^{np}$ and determine the lower bound on the scale associated with them from the new physics contribution to muon anomaly reported by BNL. A detailed discussion of effective Lagrangian analysis of muon anomaly can be found in Ref[4]. The two direct operators of dimension six that contributes to $a_\mu$ are [5]

\begin{align}
O_1 &= (\bar{l} \sigma^{\mu\nu} \tau_a \mu_R) \phi W^a_{\mu\nu} + h.c. \\
&= -\frac{1}{\sqrt{2}}(\bar{\mu} \sigma^{\mu\nu} \mu)(c_w Z_{\mu\nu} + s_w F_{\mu\nu})(v + H) + .. \\ 
\end{align}

(4)

and

\begin{align}
O_2 &= (\bar{l} \sigma^{\mu\nu} \mu_R) \phi B_{\mu\nu} \\
&= \frac{1}{\sqrt{2}}(c_w F_{\mu\nu} - s_w Z_{\mu\nu})(v + H) + .. \\
\end{align}

(5)

The low energy effective Lagrangian relevant for us is therefore

\begin{equation}
L_{eff} = \frac{C_1}{\Lambda^2} O_1 + \frac{C_2}{\Lambda^2} O_2. \\
\end{equation}

(6)

The coefficients $C_1$ and $C_2$ can arise from three broad classes of new physics a) weakly coupled gauge theory b) strongly coupled gauge theory c) theories in extra space-time dimensions. The size of the coefficients will depend upon from which kind of new physics it arises. a) If the new physics that give rise to $O_1$ and $O_2$ is a weakly coupled gauge theory then the coefficients $C_1$ and $C_2$ can be estimated by explicitly evaluating the loop
diagrams made of virtual states of new heavy particles perturbatively. Typically \( \frac{e}{\sqrt{2}} C_1 \) and \( \frac{c_w}{\sqrt{2}} C_2 \) are expected to be of the order of \( e \frac{g^2}{16\pi^2} \xi \) where \( g \) is some weak coupling that appears at the scale \( \Lambda \). \( \xi \approx \frac{m_F}{\Lambda} \) where \( m_F \) is the mass of some internal fermion line. The parameter \( \xi \) carries the information that the operators \( O_1 \) and \( O_2 \) break chiral symmetry. The muon anomaly due to \( O_1 \) in the weakly coupled scenario is therefore given by

\[
a_{\mu}^{np} = \frac{m_\mu m_F}{\Lambda^2} \frac{g^2}{8\pi^2}.
\]  

We would like to note that the interaction that gives rise to anomalous magnetic moment of muon will also contribute to the muon mass. The shift in the muon mass will be given by \( \delta m_\mu \approx \frac{g^2}{16\pi^2} m_F \ln \frac{M}{m_\mu} \) where \( M \) is the mass scale which gives the dominant contribution to the loop integral. In the weakly coupled case the coupling \( g \) must be small enough so that \( \delta m_\mu \ll m_\mu \) and the muon mass is protected from receiving large radiative corrections from the new physics scale. The best known example of weakly coupled new physics that also satisfies the criteria of naturalness is the the supersymmetric version of the SM. In such a scenario the muon gets its mass from yukawa coupling to one of the higgs doublets. However the dynamics that gives rise to the phenomenological yukawa couplings is assumed to take place at an absurdly high energy, certainly much higher than the mass scale of the weakly interacting new particles that give rise to the muon anomaly. Elementary leptoquarks constitute a non-supersymmetric example weakly coupled new physics. If the muon anomaly is due to a second generation leptoquark then we have \( m_F = m_c \) (mass of charm quark) and \( \delta m_\mu \approx \frac{g^2}{16\pi^2} m_c \ln \frac{M}{m_c} \approx 5.1 \text{MeV} \ll m_\mu \) if the coupling \( g \) of the leptoquark to quark-lepton pair is of the order of electromagnetic coupling. Further in this case the muon anomaly is given by \( a_{\mu}^{np} \approx \frac{g^2}{8\pi} \frac{m_\mu m_c}{\Lambda^2} \). The new BNL value of the muon anomaly is important because of two reasons. Firstly the average value of the anomaly is large (2.6 \( \sigma \) effect). Secondly the error in the new value is one third of the combined previous data. Both these factors can be taken into account by determining 95\% CL limits on \( \Lambda \). For the leptoquark case we find that for \( g \approx e \), \( \Lambda \) must
satisfy the following bounds: 160 Gev ≤ Λ ≤ 378 Gev. In contrast the previous data [2] (δa_µ ≈ 45(46) × 10^{-10}) would have given us a central value of 350 Gev and a lower limit of 105 Gev for Λ. Clearly the new BNL value allows a much more precise determination of the scale of new physics. In general for weakly coupled scenario (due to the small coupling and loop suppression factor ) we expect \( \frac{g_2}{\sqrt{2}} C_1 \) to be much smaller than \( \frac{m_\mu}{v} \). Assuming a typical suppression factor of .01-.04 we expect new particles to appear in the few hundred Gev range with couplings of the order of electromagnetic coupling or even weaker.

b.) In the strongly coupled case (as for example in composite models) on the other hand, the underlying physics that give rise to the mass of the muon appears at the scale relevant for the muon anomaly itself. This happens for example in extended technicolor models. Hence the results for this case can be obtained by setting the expression for δm_µ or vC_1 given above equal to m_µ. In that limit the expression for the muon anomaly due to O_1 becomes \( a_\mu^{np} \approx \frac{m_\mu^2}{\Lambda^2} \). To justify that this expression is correct consider a nonabelian gauge theory where the small muon mass arises from the strong binding of very massive preons then the anomalous magnetic moment of the muon is expected to be of the order of \( \delta \mu \approx e \frac{m_\mu}{\Lambda} \) [6]. The mass of the muon must appear in the numerator since a non zero \( \delta \mu \) (anomalous magnetic moment) implies chiral symmetry breaking in the light composite muon. The above expression for \( \delta a_\mu \) also arises in theories where the muon gets its mass from extended technicolor interactions (ETC) [7]. To see that consider the loop diagram with an exchange of ETC gauge boson that gives rise to an anomalous magnetic moment of the muon. A simple calculation shows that the anomalous magnetic moment would be given by

\[
\delta \mu \approx e \frac{g_\text{etc}^2}{16\pi^2} \frac{\langle \bar{T}T \rangle}{M_{\text{etc}}}.
\]

Here \( g_{\text{etc}} \) is the ETC gauge coupling, \( M_{\text{etc}} \) is the mass of the extended technicolor gauge boson and \( \langle \bar{T}T \rangle \) is the technifermion condensate renormalized at \( M_{\text{etc}} \). The exchange of the same ETC gauge boson will also generate the muon mass and will be given by \( m_\mu \approx \frac{g_\text{etc}^2}{16\pi^2} \frac{\langle \bar{T}T \rangle}{M_{\text{etc}}} \). From these two equations it follows that the muon anomaly due to ETC interaction will be given by \( a_\mu^{np} \approx \frac{m_\mu^2}{M_{\text{etc}}^2} \). Using the new BNL value for muon...
anomaly we get the following 95% CL range for $\Lambda$ in the strongly coupled scenario: $1.2 \text{ Tev} \leq \Lambda \leq 3.2 \text{ Tev}$. The ultimate goal of the experiment is to reduce the error to $\pm 4 \times 10^{-10}$ about a factor of 3.5 times better than the present result. Even the inclusion of already existing data from 2000 run would improve the statistical error by a factor of 2. If the central value and other errors are unaffected, the 95% CL bounds will become: $1.3 \text{ Tev} \leq \Lambda \leq 2.3 \text{ Tev}$.

c.) Theories in extra space-time dimensions: Recently theories in extra dimension have been proposed to explain the hierarchy problem [8]. In this section we shall estimate the coefficient $C_1$ associated with the direct operator $O_1$ assuming that it arises from a higher dimensional model. In these models the SM fields are assumed to be localized on a 3 brane but gravity is allowed to propagate in the bulk. From the point of view of an observer in the visible four dimensional world the effect of having gravity in the bulk is described by a tower of Kaluza-Klein (KK) modes of the graviton with a level spacing of a few Tev. The zero mode is the usual graviton and it couples to the SM fields with a strength proportional to $\frac{E}{M_\mu}$ where $M_\mu$ is the Planck mass. But the higher KK modes lie in the Tev range and they couple to SM fields with a strength proportional to $\frac{E}{\Lambda}$ where $\Lambda$ is of the order of a Tev. The couplings of the KK modes of the graviton to SM particles are therefore strong for energies in the Tev range. Consider now a muon self energy diagram with an exchange of KK graviton. Attach a photon and a higgs field to the muon line. On integrating over the fluctuations of all the massive KK graviton modes such a diagram will generate the operator $O_1$. Pulling out the photon momentum out of the integral and evaluating the loop integral (which receives largest contribution from loop momenta of the order of the cut off $\Lambda$) we find that

$$C_1 O_1 = \frac{e}{16\pi^2} \frac{m_\mu}{v} \ln \frac{\Lambda^2}{m_\mu^2} \bar{\mu}\sigma_{\mu\nu}\mu F^{\mu\nu}/h + ...$$

(8)

Hence $\sqrt{s} \approx \frac{e}{16\pi^2} \frac{m_\mu}{v} \ln \frac{\Lambda^2}{m_\mu^2}$. This will generate a muon anomaly of the order of $a_\mu^{np} \approx \frac{.24}{\Lambda \Lambda} \approx 2.4 \times 10^{-9}$. Here we have assumed that $\Lambda \approx 1 \text{ Tev}$. This is of the
right order of magnitude to give rise to the observed BNL muon anomaly. Hence extra
dimension scenarios with massive KK modes of the graviton that couple to SM fields with
a strength inversely proportional to the Tev scale, can also give rise to the direct operators
with coefficients of the right size to generate the BNL muon anomaly

The operators $O_1$ and $O_2$ are the only two operators among the d=6 operators which
contribute directly to $a_{np}^{\nu\rho}$ at the tree level. However there are many d=6 operators that
contribute indirectly to $\delta a_\mu$ through loops. Here we shall consider only those indirect
operators (with an elementary or a light composite higgs scalar) whose effects on the
muon anomaly have not been considered in Ref.[9].

a) Effect of indirect operators made of gauge bosons and scalars: Consider the two
operators [5] $O_3 = (\phi^+ D_\mu \phi) (D_\mu \phi^+ \phi)$ and $O_4 = (\phi^+ \phi) (D_\mu \phi^+ D_\mu \phi)$. These operators cause
$O(\frac{v^2}{\Lambda^2})$ mixing between $W_{3\mu}$ and $B_\mu$. They also shift the physical $W(Z)$ boson masses by an
amount $\delta m_{w(z)}^2 \approx \frac{v^2}{\Lambda^2} m_{w(z)}^2$. If this operator appears in the low energy effective Lagrangian
with a coefficient of the order of one, then it would cause a shift in the $\rho$ parameter by
an amount $\delta \rho_{new} \approx -O(\frac{v^2}{\Lambda^2})$. The LEP constraint $|\delta \rho_{new}| \leq 4\%$ implies that $\frac{(v^2)}{\Lambda^2}$ can
be at most of the order of one percent. If the operator $O_3$ is introduced on a weak gauge
boson line of a loop diagram that contributes to $a_{\mu}^{ew}$ then the change $a_{\mu}^{np}$ due to $O_3$ will
be given by $a_{\mu}^{np} \approx O(\frac{\delta m_{w(z)}^2}{m_{z}^2}) a_{\mu}^{ew} \approx \frac{v^2}{\Lambda^2} a_{\mu}^{ew} \approx 10^{-11}$. This contribution is much smaller than
the ultimate precision $4 \times 10^{-10}$ that the BNL collaboration aims to achieve. The $O(\frac{v^2}{\Lambda^2})$
mixing between $W_{3\mu}$ and $B_\mu$ will also affect the direct operators $O_1$ and $O_2$. When $W_{3\mu}$
and $B_\mu$ are expressed in terms of physical states $Z_\mu$ and $A_\mu$ the shift in $a_{\mu}^{np}$ will be of order
$\frac{v^2}{\Lambda^2} a_{\mu}^{direct}$. Here $a_{\mu}^{direct}$ is the contribution to $a_{\mu}^{np}$ due to $O_1$ and $O_2$. Hence the effect of
this operator on the muon anomaly is too small to be observed with the present precision.

b) Effect of indirect operators made of gauge bosons, fermions and scalars: Consider
the operator [5]

$$O_5 = (\bar{l}_\mu D_{\mu} \mu) D^\mu \phi$$

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\[
= - \frac{w}{2\sqrt{2}} (g^2 + g'^2) \frac{1}{2} Z_\mu \bar{\mu}_l \partial^\mu \mu_r + \ldots
\] (9)

Since the operator \( O_4 \) breaks the chiral symmetry of the muon the dimensionless coefficient associated with it in the low energy effective Lagrangian must be proportional to the Yukawa coupling \( \frac{m_\mu}{v} \) of the muon. This would guarantee that in the limit of vanishing muon mass the operator \( O_4 \) vanishes and the chiral symmetry of the muon is recovered. Hence the relevant term in the effective Lagrangian becomes

\[
\frac{C_4}{\Lambda^2} O_4 \approx - \frac{w}{2\sqrt{2}} (g^2 + g'^2) \frac{1}{2} Z_\mu \bar{\mu}_l \partial^\mu \mu_r + \ldots
\] (9).

Consider now a loop diagram with Z boson exchange that contributes to \( a_\mu^{ew} \) in the SM. Replace one of the SM vertices of Z by the above nonrenormalizable effective vertex. The resulting diagram will give the contribution of \( O_4 \) to \( a_\mu^{np} \). Pulling out the photon momentum out of the integral and evaluating the resulting integral we get

\[
a_\mu^{np} \approx \frac{m_\mu^2 \sqrt{2} \Lambda^2}{16\pi^2 e_w^2} \frac{e^2}{c_w^2} \ln \frac{\Lambda}{m_z}.
\] (10).

This contribution is of the order of .001 \( \frac{m_\mu^2}{\Lambda^2} \) and hence much smaller than the direct contribution considered in this paper.

The dimension less coefficient associated with the chirality flipping direct operators \( O_1 \) and \( O_2 \) must be order of the yukawa coupling of the muon and hence these operators will not produce any significant effect at a high energy \( \mu^+\mu^- \) collider. However the new physics that give rise to the chiral symmetry breaking operators can also give rise to chiral symmetry conserving operators. The coefficient of these operators in the low energy effective Lagrangian could be of order one. Consider for example the operator \( O_6 = i[(\phi^+ D_\mu \phi) - (D_\mu \phi^+ \phi)]\bar{\gamma}^\mu l \). Besides shifting the Z coupling to LH muons this operator could also give rise to anomalous contributions to the process \( \mu^+\mu^- \rightarrow hZ \). Although the contribution of this indirect operator to the muon anomaly is suppressed [9] compared
to that of the direct operators, its collider signatures are stronger than that of the direct operators. It will be interesting to study the collider implications of this and other d=6 indirect operators which are not chirality suppressed for the values of Λ presented in this paper. This would help in revealing the complementary nature of the new physics responsible for the muon anomaly. In fact this idea was used by Eitchen et al in Ref[10] to propose the study of four fermion contact interactions at a high energy $e^+e^-$ collider to determine the bounds on the compositeness scale Λ. Until then the muon anomaly was considered as providing the best bound on the compositeness scale. Motivated by this complementary search strategy we have done some rough estimates of the effect of the operator $O_5$ on the process $\mu^+\mu^- \rightarrow hz$. We find that at $\sqrt{s}=500$ Gev, $m_h = 150$Gev and $\Lambda=2$ Tev, $\sigma_{total}=80$ fb if $O_6$ interferes constructively with the SM contribution. This is to be compared with the SM contribution of 52 Gev. Hence unless Λ is much higher we could expect to see large new physics effects in the process $\mu^+\mu^- \rightarrow hZ$. Similarly the operator $O_5$ can make important new physics contributions to higgs production on resonance at a 500 Gev $\mu^+\mu^-$ collider.

In conclusion in this paper we have shown that the scale associated with the direct operators that can explain the BNL muon anomaly can naturally arise from the following three distinct scenarios: a) weakly coupled gauge theories b) strongly coupled gauge theories and c) theories in extra space time dimension. Remarkably all the three scenarios namely weak scale supersymmetry, technicolor and theories in extra dimensions provide solutions to the hierarchy problem. If the direct operators arise from a weakly coupled underlying gauge theory, the scale of new physics typically turns out to be a few hundred Gev. Here we expect weakly coupled new particles (leptoquarks or supersymmetric partners of the SM particles) with a mass of the order of few hundred Gev. On the other hand if $O_1$ and $O_2$ arise from a strongly coupled gauge theory, the scale of new physics turns out to be a few Tev. In this scenario we expect strongly coupled new particles (technihadrons and technimesons ) and other resonances with a mass in the Tev range. Finally if the the
direct operators arise from extra dimension theories with gravity living in the bulk then
we expect to see the massive KK modes of the graviton in the Tev range which couple to
energy momentum tensor of SM fields with only Tev scale suppression. Interestingly all
the three scenarios for new physics will be accessible at LHC and other future colliders for
detailed study.

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