$Z_N$ orbifold compactifications in $AdS_6$ with

**Gauss-Bonnet term**

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Abstract

We present a general setup for junctions of semi-infinite 4-branes in $AdS_6$ with the Gauss-Bonnet term. The 3-brane tension at the junction of 4-branes can be nonzero. Using the brane junctions as the origin of the $Z_N$ discrete rotation symmetry, we identify 3-brane tensions at three fixed points of the orbifold $T^2/Z_3$ in terms of the 4-brane tensions. As a result, the three 3-brane tensions can be simultaneously positive, which enables us to explain the mass hierarchy by taking one of two branes apart from the hidden brane as the visible brane, and hence does not introduce a severe cosmological problem.

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I. INTRODUCTION

The Higgs mass problem of the Standard Model (SM) includes (i) the quadratic divergence of the Higgs mass squared and (ii) the quartic divergence of vacuum energy density, which render power law sensitivities to the unknown ultraviolet physics. Supersymmetry in 4 dimensional(4D) spacetime has been thought as a possible solution of this hierarchy problem. Recently, however, extra dimensional scenarios have been suggested as alternative solutions of the gauge hierarchy problem with the idea that the SM particles are confined to a 3-brane embedded in a higher dimensional spacetime [1,2].

In particular, the Randall-Sundrum (RS) I model [2] with two 3-branes embedded in a slice of $AdS_5$, the gauge hierarchy problem can be solved by taking the brane at the weak scale(TeV brane) as the visible brane through the warp factor decreasing away from the hidden brane(Planck brane). As a result, the stability of the gauge hierarchy becomes the mechanism of radius stabilization at appropriate separation of the Planck and TeV branes [3]. On the other hand, the RS II model [4] cannot serve as a solution of the gauge hierarchy problem, but presumably renders a more important paradigm toward an alternative to compactification with a single 3-brane embedded in noncompact $AdS_5$. It is based on the fact that the linearized Einstein gravity can be reproduced on the brane even for the noncompact extra dimension of the RS II model [5]. Even with these interesting results of the RS models, the notorious cosmological constant problem is not solved, but recast only into fine-tunings between brane and bulk cosmological constants [6]. However, it has been shown that it is possible to have a flat solution without fine-tunings of input parameters in the RS II model with the addition of the bulk three form field [7], signaling a possible existence of the solution of the cosmological constant problem.

One more thing to note is that in the RS I model the TeV brane takes a negative tension while the Planck brane takes a positive tension, which introduced some cosmological difficulties toward a smooth transition to the standard big bang cosmology [8]. This cosmological problem in the RS I model can be solved by taking into account the radius stabilization [9].
But it is worthwhile to search for models with the visible brane taking a positive tension or branes taking positive tensions only.

In this context, when the higher curvature term is included as a Gauss-Bonnet invariant in the RS I model, one can find that there exists another flat solution of the RS type under the condition that the visible brane should have a positive tension \[10\]. However, it is necessary to deal with both the Einstein-Hilbert term and the Gauss-Bonnet term on the equal footing, but not the Gauss-Bonnet term as being subdominant compared to the Einstein-Hilbert term. Other investigations of the higher curvature gravity concentrated mainly on the Gauss-Bonnet term in the brane background can be also found in the literature \[11–14\].

On the other hand, we can find more diverse possibilities in higher dimensional extensions of the RS model \[12–17\]. In fact, in dimensions higher than \(D = 5\) the intersecting brane worlds were considered as a direct generalization of the RS models \[15\], but the visible universe does not have a nonzero brane tension, just regarded as the location where branes with lower codimensions intersect. The nonzero 3-brane tension in \(D > 5\) is allowed if the 3-brane is assumed to be made of topological defects supported by a bulk scalar field \[16\]. In that case, it is shown that there does not arise fine-tuning condition between brane and bulk cosmological constants as in the RS model but fine-tuning conditions appear differently \[16\].

It is also shown that it is possible to have a nonzero 3-brane tension in \(AdS_6\) intersecting brane worlds by adding the Gauss-Bonnet term in the bulk action \[13\]. On compactifying the extra dimensions on an orbifold \(T^2/(Z_2 \times Z_2)\), the mass hierarchy can be explained by taking a 3-brane with positive tension as the visible brane \[13\]. In fact, the \(Z_2 \times Z_2\) orbifold symmetry with \(Z_2\) acting once on each extra dimension is nothing but the \(Z_2\) orbifold symmetry belonging to the rotation group around the origin. A higher dimensional \((d > 6)\) generalization of the intersecting branes in the existence of the Gauss-Bonnet term have been dealt with in Ref. \[14\].

Two extra dimension, i.e. 6D, is of particular interest since the orbifold compactification
toward standard-like models in string theory compactify three two-extra dimensions \[18,19\]. The \( Z_3 \) orbifold compactification has been extensively studied \[20\], which assumed vanishing bulk cosmological constant. Thus, it is of interest to consider \( Z_N \) orbifold compactification in 6D with negative bulk cosmological constant, in the hope of obtaining a more general string compactification in the future. Already, there exists an example that all the SM matter fields are located at the orbifold fixed points \[21\], the kind of which can be generalized to the RS I type models.

In this paper, therefore, we extend the orbifold symmetry of the brane junction in \( AdS_6 \) to the \( Z_N \) case. Then, we also consider a six-dimensional compactification on another orbifold \( T^2/Z_3 \). In that case, there exist three fixed points where reside 3-branes corresponding to centers of the \( Z_3 \) symmetric brane junctions. We show that the 3-brane tensions at these fixed points are all positive for the Gauss-Bonnet coupling \( \alpha > 0 \). Then, we can explain the mass hierarchy by regarding two 3-branes, apart from the 3-brane at the origin, as the visible brane. The results can be generalized to the orbifold \( T^2/Z_N \).

This paper is organized as follows. In Section II, we set up a general formalism for junctions of semi-infinite 4-branes in \( AdS_6 \) in the presence of the Gauss-Bonnet term. In Sec. III we derive the consistency conditions for the \( Z_N \) symmetric brane junctions and apply the results to the \( Z_3 \) case. In Sec. IV we compactify the extra dimensional space with the \( Z_3 \) symmetric brane junction on a torus and determine the warp factor and the 3-brane tension located at the fixed points in terms of two independent 4-brane tensions. Sec. V is a conclusion.

**II. GENERAL SETUP WITH GAUSS-BONNET TERM**

The Gauss-Bonnet(GB) term is the only consistent higher curvature term in the RS models since it does not give rise to higher derivative terms of the metric beyond the second \[10\]. In 6D, it was also shown that the theory with the GB term can accomodate 3-branes with nonzero tensions in the 6D spacetime \[13\]. Thus, when we include the GB term as the
next leading-order higher curvature interaction on top of the conventional Einstein-Hilbert term, the 6D bulk action reads

$$S_6 = \int d^4x dz_1 dz_2 \sqrt{-g} \left[ \frac{M^4}{2} R - \Lambda_b + \frac{1}{2} \alpha M^2 (R^2 - 4 R_{MN} R^{MN} + R_{MNPQ} R^{MNPQ}) \right]$$

(1)

where $M$ is the six dimensional gravitational constant, $\Lambda_b$ is the bulk cosmological constant, $\alpha$ is the effective coupling.

If we assume the metric ansatz as

$$ds^2_6 = A^2(z_1, z_2) (\eta_{\mu\nu} dx^\mu dx^\nu + dz_1^2 + dz_2^2),$$

(2)

where $(\eta_{\mu\nu}) = \text{diag.}(-1, +1, +1, +1)$, it has been shown [13] that the bulk solution of the warp factor is

$$A^{-1}(z_1, z_2) = \vec{k} \cdot \vec{z} + c_0$$

(3)

where $\vec{k} = (k_{z_1}, k_{z_2})$, $\vec{z} = (z_1, z_2)$ and $c_0$ is an integration constant. In each 4-brane patch, one $\vec{k}$ is defined. Thus, when we consider multi branes, we will denote $N$ such vectors as $\vec{k}(l)(l = 1, 2, \cdots N)$. However, the magnitude of $\vec{k}$ is fixed in terms of the bulk cosmological constant $\Lambda_b$ and the effective coupling $\alpha$ as

$$k_{z_1}^2 + k_{z_2}^2 = \frac{M^2}{12\alpha} \left[ 1 \pm \sqrt{1 + \frac{12\alpha \Lambda_b}{5M^6}} \right] \equiv k^2_{\pm}.$$  

(4)

When we introduce singular brane sources in the bulk, the boundary conditions at a 4-brane with tension $\Lambda$ and a 3-brane with tension $\lambda$ become [13],

$$4 \left( 1 - \frac{12\alpha k_{\pm}^2}{M^2} \right) \left( \frac{A'}{A^2} \right)_-^+ = -\frac{\Lambda}{M^4} (4 - \text{brane})$$

(5)

$$\frac{24\alpha}{M^2} \left( \frac{A'}{A^2} \right)_-^+ \left( \frac{\dot{A}}{A^2} \right)_-^+ = \frac{\lambda}{M^4} (3 - \text{brane}),$$

(6)

respectively, where the prime for the case of the 4-brane denotes the derivative with respect to the bulk coordinate normal to the 4-brane and the prime and dot for the case of the 3-brane denote the derivatives with respect to any set of two orthogonal bulk coordinates for
the case of two orthogonally intersecting 4-branes [13]. Note that here \( \cdot \) is not a derivative with respect to \( t \).

Let us consider a junction of semi-infinite \( L \) 4-branes and one 3-brane residing on the brane junction in the presence of the GB term as shown in Fig. 1. In order to obtain the solutions of the Einstein equations in the presence of singular brane sources in the extra dimensions, we only have to glue the patches of different \( A \)'s between 4-branes such that the metric is continuous at the locations of the branes and the discontinuities of the derivatives reproduce the energy momentum tensor of the branes. Note that we assume the same bulk parameters in all patches and thus \( \vec{k} = (k_{z_1}, k_{z_2}) \) in each patch is constrained by Eq. (4).

Then we can write the warp factor in a compact form for a space composed of \( L \) AdS patches

\[
A^{-1} = c_0 + \sum_{l=1}^{L} (\vec{k}_l \cdot \vec{z}) \theta(\vec{n}_{l-1} \cdot \vec{z}) \theta(-\vec{n}_l \cdot \vec{z}),
\]

(7)

where \( \vec{n}_l = (-\sin \varphi_l, \cos \varphi_l) \) is a unit vector in the \( z_1 - z_2 \) plane normal to the \( l^{th} \) 4-brane, and \( \varphi_l \) is the angle between the \( l^{th} \) 4-brane and the \( z_1 \) axis. We can always set \( \varphi_L = 0 \) up to the overall rotation of the configuration.

Let us turn to the energy momentum tensor of the configuration of \( L \) AdS patches separated by 4-branes at the junction of which a 3-brane is located as in Fig. 1. We recall that the bulk energy momentum tensor is assumed to be the same for all bulk spaces as

\[
T_{MN}^{\text{bulk}} = -\Lambda_6 g_{MN}.
\]

On the other hand, the energy momentum tensor for branes is

\[
T_{MN}^{\text{brane}} = \sum_{l=1}^{L} T_{MN}^{4-\text{brane},l} + T_{MN}^{3-\text{brane}},
\]

(8)

where
\[ T_{4-\text{brane},l}^{M,N} = \Lambda_l A(z_1, z_2) \delta(\vec{n} \cdot \vec{z}) \left( \begin{array}{cccc} 1 & -1 & -1 & -\cos^2 \varphi_l - \sin \varphi_l \cos \varphi_l \\ -1 & 1 & -1 & -\sin \varphi_l \cos \varphi_l \\ -1 & -1 & -1 & -\sin^2 \varphi_l \\ -\cos \varphi_l & -\sin \varphi_l \cos \varphi_l & -\sin \varphi_l \cos \varphi_l & -\sin^2 \varphi_l \end{array} \right), \]  

(9)

and

\[ T_{3-\text{brane}}^{M,N} = \lambda_1 \delta(z_1) \delta(z_2) \eta_{\mu\nu} \delta^\mu_M \delta^\nu_N. \]  

(10)

Note that the warp factors satisfy

\[ \frac{A'}{A^2} = -k_{z_1}, \quad \frac{\dot{A}}{A^2} = -k_{z_2} \]  

(11)

Then, on inspecting Eqs. (5) and (6), we find the boundary conditions matching the discontinuities of derivatives with brane singularities as

\[ \gamma \Delta \vec{k}_l = \gamma (\vec{k}_{l+1} - \vec{k}_l) = \frac{\Lambda_l}{4M^4} \vec{m}_l, \quad (l = 1, 2, \ldots, L), \quad \text{with} \quad \vec{k}_{L+1} = \vec{k}_1, \]  

(12)

\[ (\vec{k}_1 - \vec{k}_{[L/2]})_{z_1} (\vec{k}_{[L/4]} - \vec{k}_{[3L/4]})_{z_2} = \frac{\lambda_1}{24\alpha M^2}, \]  

(13)

where

\[ \gamma = 1 - \frac{12\alpha k^2}{M^2} \]  

(14)

and \([x]\) denotes the largest integer not exceeding \(x\). Here we note that the polygonal integration near the origin of the 3-brane is assumed in deriving the boundary condition (13) but it can be easily shown that it is equivalent to the square integration as in the case of two orthogonally intersecting 4-branes. After summing up Eq. (12), we obtain a condition for the brane sources,

\[ \sum_{l=1}^{L} \Lambda_l \vec{n}_l = 0 \quad \text{or} \quad \sum_{l=1}^{L} \vec{A}_l = 0, \]  

(15)
where $\vec{\Lambda}_l \equiv (\Lambda_l \cos \varphi_l, \Lambda_l \sin \varphi_l)$ is defined to point to the brane direction with angle $\varphi_l$ with respect to the $L^{th}$ brane.

Consequently, there arise $3L + 1$ equations (Eqs. (4), (12), and (13)) with respect to the $3L - 1$ parameters $(k_{lz}, k_{l\bar{z}}$ and $\varphi_l$) of the ansatz (4) and (7), which would give rise to two fine-tuning conditions for consistency.

III. THE $Z_N$ SYMMETRIC BRANE JUNCTIONS IN 6D

Let us now impose the discrete rotation symmetry $Z_N$ on the brane junction solutions obtained in the previous section. In this case, we must investigate the additional requirements arising from the brane junctions $Z_N$ symmetry. For this purpose, we let the AdS patches between 4-branes to be equally spaced, i.e., $\varphi_l = 2\pi l/L$ with $l = 1, 2, \ldots, L$.

For convenience, we can rewrite the warp factor (7) for the brane junction by using complex numbers as

$$A^{-1} = c_0 + \sum_{l=1}^{L} \frac{1}{2} (k_l z + k_l \bar{z}) \theta \left( \frac{e^{-i \varphi_l - 1} z - e^{i \varphi_l - 1} \bar{z}}{2i} \right) \theta \left( \frac{e^{-i \varphi_l} z - e^{i \varphi_l} \bar{z}}{-2i} \right)$$

(16)

where $k_l = k_{l,1} + ik_{l,2}$ and $z = z_1 + iz_2$. In order to make the brane junction invariant under a $Z_N$ rotation, $z \rightarrow e^{i(2\pi n/N)} z$ with $n = 1, 2, \ldots, N$, we obtain the complex number $k_l$ transforms as

$$k_l' = e^{-i(2\pi n/N)} k_l = k_l'$$

(17)

with the angle rotated to

$$\varphi_l' = \varphi_l - \frac{2\pi n}{N} = \varphi_l'$$

(18)

from which $l' = l - nL/N$. Therefore, we obtain the following consistency conditions for the $Z_N$ symmetric brane junction:

$$k_l = e^{i(2\pi n/N)} k_{l-\bar{r}n}, \ (l = 1, 2, \ldots, L, \ n = 1, 2, \ldots, N, \ l - \bar{r}n > 0)$$

(19)

where $r = L/N$ is assumed to be a natural number.
For instance, let us consider the case with the $Z_3$ symmetric brane junction. We also take the number of AdS patches or 4-branes as

Case (A) $L = 12$, and

Case (B) $L = 6$

for future use in considering the $T^2/Z_3$ orbifold. We include 3-brane tensions at the brane junctions for both cases.

To begin with, for Case (A) with $L = 12$, from Eq. (19), we obtain the consistency condition for the $Z_3$ symmetry as

$$k_l = e^{i(2\pi n/3)} k_{l-4n}$$

with $l = 1, 2, \ldots, L$ with $L = 12$ and $n = 1, 2, 3$, which implies only four independent AdS patches or 4-branes. Then, from Eq. (12), the boundary conditions at the four independent 4-branes ($\Lambda_1, \Lambda_2, \Lambda_3$ and $\Lambda_{12}$) become

$$\gamma(\vec{k}_2 - \vec{k}_1) = \frac{\Lambda_1}{8M^4}(-1, \sqrt{3}),$$

$$\gamma(\vec{k}_3 - \vec{k}_2) = \frac{\Lambda_2}{8M^4}(-\sqrt{3}, 1),$$

$$\gamma(\vec{k}_4 - \vec{k}_3) = \frac{\Lambda_3}{8M^4}(-2, 0),$$

$$\gamma(\vec{k}_1 - \vec{k}_{12}) = \frac{\Lambda_{12}}{8M^4}(0, 2),$$

where $k_{12}$ is related to $k_4$ by the $Z_3$ symmetric condition, Eq. (19), as

$$k_{12} = e^{i(4\pi/3)} k_4.$$  

Then, we can rewrite the $k_l$’s in terms of $k_1$ and the 4-brane tensions as

$$\vec{k}_2 = \vec{k}_1 + \frac{\Lambda_1}{8\gamma M^4}(-1, \sqrt{3}),$$

$$\vec{k}_3 = \vec{k}_1 + \frac{1}{8\gamma M^4}(-\Lambda_1 - \sqrt{3}\Lambda_2, \sqrt{3}\Lambda_1 + \Lambda_2),$$

$$\vec{k}_4 = \vec{k}_1 + \frac{1}{8\gamma M^4}(-\Lambda_1 - \sqrt{3}\Lambda_2 - 2\Lambda_3, \sqrt{3}\Lambda_1 + \Lambda_2),$$

$$\vec{k}_{12} = \vec{k}_1 + \frac{\Lambda_{12}}{8\gamma M^4}(0, -2),$$

where $k_{12}$ has additional relations as
\[ k_{12,z_1} = \frac{1}{2}(-k_{1,z_1} + \sqrt{3}k_{4,z_2}) \]
\[ = \frac{1}{2}(-k_{1,z_1} + \sqrt{3}k_{1,z_2}) + \frac{1}{8\gamma M^4}(2\Lambda_1 + \sqrt{3}\Lambda_2 + \Lambda_3), \quad (29) \]
\[ k_{12,z_2} = \frac{1}{2}(-\sqrt{3}k_{4,z_1} - k_{4,z_2}) \]
\[ = \frac{1}{2}(-\sqrt{3}k_{1,z_1} - k_{1,z_2}) + \frac{1}{8\gamma M^4}(\Lambda_2 + \sqrt{3}\Lambda_3). \quad (30) \]

In view of Eq. (4), we have a universal \( \vec{k}_1^2 = k_1^2 = k_{\pm}^2 \) for all \( l \). Thus, taking the square of each of Eqs. (25)-(28) on both sides gives, respectively,

\[ \Lambda_1 \left( -k_{1,z_1} + \sqrt{3}k_{1,z_2} + \frac{\Lambda_1}{4\gamma M^4} \right) = 0, \quad (31) \]
\[ \Lambda_2 \left( -\sqrt{3}k_{1,z_1} + k_{1,z_2} + \frac{1}{4\gamma M^4}(\sqrt{3}\Lambda_1 + \Lambda_2) \right) = 0, \quad (32) \]
\[ \Lambda_3 \left( -k_{1,z_1} + \frac{1}{8\gamma M^4}(\Lambda_3 + \Lambda_1 + \sqrt{3}\Lambda_2) \right) = 0, \quad (33) \]
\[ \Lambda_{12} \left( -k_{1,z_2} + \frac{\Lambda_{12}}{8\gamma M^4} \right) = 0. \quad (34) \]

Therefore, there are three cases consistent with the \( Z_3 \) symmetry

(i) : \( \Lambda_3 = \Lambda_1 \neq 0, \quad \Lambda_{12} = \Lambda_2 \neq 0, \quad (35) \)

(ii) : \( \Lambda_3 = \Lambda_1 = 0, \quad \Lambda_{12} \neq 0, \quad \Lambda_2 \neq 0, \quad (36) \)

(iii) : \( \Lambda_{12} = \Lambda_2 = 0, \quad \Lambda_3 \neq 0, \quad \Lambda_1 \neq 0. \quad (37) \)

Here we keep both the cases (ii) and (iii), which will be useful for the \( T^2/Z_3 \) orbifold in the next section even though those configurations themselves are equivalent to each other up to a rotation. Then, we determine \( k_1 \) by solving the equations (31)-(34) consistently with Eqs. (28)-(30) for each case as

(i) : \( k_{1,z_1} = \frac{1}{8\gamma M^4}(2\Lambda_1 + \sqrt{3}\Lambda_2), \quad k_{1,z_2} = \frac{\Lambda_2}{8\gamma M^4}, \quad (38) \)

(ii) : \( k_{1,z_1} = \frac{1}{8\sqrt{3}\gamma M^4}(\Lambda_{12} + 2\Lambda_2), \quad k_{1,z_2} = \frac{\Lambda_{12}}{8\gamma M^4} \quad (39) \)

(iii) : \( k_{1,z_1} = \frac{1}{8\gamma M^4}(\Lambda_1 + \Lambda_3), \quad k_{1,z_2} = \frac{1}{8\sqrt{3}\gamma M^4}(\Lambda_3 - \Lambda_1). \quad (40) \)
On the other hand, from Eq. (13), the boundary condition at the 3-brane becomes

\[(k_1 - k_6)z_1(k_4 - k_9)z_2 = \frac{\lambda_1}{24\alpha M^2}.\]  

(41)

Since \(k_6 = e^{i(2\pi/3)}k_2\) and \(k_9 = e^{i(4\pi/3)}k_1\), using Eq. (25), (27) and (38)-(40), we obtain a fine-tuning relation involving both 4-brane and 3-brane tensions for each case as

(i) \(\lambda_1 = \frac{3\alpha}{2\gamma^2 M^6}(2\Lambda_1 + \sqrt{3}\Lambda_2)(\sqrt{3}\Lambda_1 + 2\Lambda_2),\)  

(42)

(ii) \(\lambda_1 = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6}(\Lambda_{12} + \Lambda_2)^2,\)  

(43)

(iii) \(\lambda_1 = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6}(\Lambda_1 + \Lambda_3)^2.\)  

(44)

Thus we find that the 3-brane located at the junction of 4-branes with \(L = 12\) for any case of (33)-(37) should have a positive tension for \(\alpha > 0\) (from the positivity of \(k_{1,21}\) and \(k_{1,22}\) in Eq. (40) for the case (i)).

For Case (B) with \(L = 6\), we also obtain the \(Z_3\) symmetric condition as \(k'_l = e^{i(2\pi n/3)}k'_{l-2n}\) with \(l = 1, 2, \ldots, 6\) and \(n = 1, 2, 3\), which implies only two independent \(AdS\) patches or 4-branes \((V_1\) and \(V_6\)). Therefore, we do not need to do another calculation for that since Case (B) with \(L = 6\) corresponds to Case (A)–(ii) when a nonzero 4-brane tension is taken to coincide with the positive half of the \(z_1\) axis. We also determine \(k'_1\) and the 3-brane tension located at the brane junction, respectively,

\[k'_{1,z_1} = \frac{1}{8\sqrt{3}\gamma M^4}(V_6 + 2V_1), \quad k'_{1,z_2} = \frac{V_6}{8\gamma M^4},\]  

(45)

\[\lambda_1 = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6}(V_1 + V_6)^2.\]  

(46)

Here we also observe that the 3-brane located at the junction of 4-branes with \(L = 6\) should a positive tension for \(\alpha > 0\).

**IV. MASS HIERARCHY WITH THE ORBITFOLD \(T^2/Z_3\)**

In the previous section, we dealt with the \(Z_N\) symmetric brane junction solutions for the non-compact extra dimensions. We can also consider the case with compact extra dimensions
of the $Z_N$ invariance through taking into account the $Z_N$ symmetric brane junction solutions.

In this section, for the purpose of using the exact solutions for the $Z_3$ symmetric brane junction solutions obtained in the previous section, we take the orbifold $T^2/Z_3$ as a geometry of extra dimensions. The case with flat extra dimensions was investigated in the orbifold constructions of superstring compactifications [18].

When we consider a torus with $Z_3$ invariance, we need to identify the extra dimensions in the following way:

$$T^2/Z_3: \quad z \approx z + a(n_1 + n_2 e^{i(2\pi/3)})$$

where $z = z_1 + iz_2$, $a$ is the size of each extra dimension and $(n_1, n_2)$ is an integer-valued lattice vector. Then, there appear three fixed points with $z = \frac{ka}{\sqrt{3}} e^{i\pi/6}$ with $k = 0, 1, 2$. Identification of the orbifold $T^2/Z_3$ is shown in Fig. 2. Essentially, we find that the extra dimension space is composed of one brane junction (around •) with $L = 12$ AdS patches (Case (A)) and two brane junctions (around ■ and around ▲) with $L = 6$ AdS patches (Case (B)), each of which is shown to be $Z_3$ symmetric around its center corresponding to one of fixed points. [The similar things happen for other orbifolds : two $Z_4$ symmetric brane junctions with $L = 8$ and one $Z_4$ symmetric brane junction with $L = 4$ for an $T^2/Z_4$ orbifold while one $Z_6$ symmetric brane junction with $L = 12$ and one $Z_3$ symmetric brane junction with $L = 6$ for an $T^2/Z_6$ orbifold.] Thus, we can easily identify 4-brane and 3-brane tensions in the fundamental region by using results on the $Z_3$ brane junction solutions obtained in the previous section. We denote independent 4-brane tensions with $(\Lambda_1, \Lambda_2)$ around •, $(V_1', V_6')$ around ■ and $(V_1'', V_6'')$ around ▲. And hereafter we use a simple notation $(k_1, k_2)$ for $k_{1,6}$, for example, in Eqs. (38)-(40).

However, since we chose the coordinate system such that a 4-brane coincides with the positive half of one of bulk coordinates, in the common region of Cases (A) and (B), we need to write $k_1'$ and $k_1''$ defined in Case (B) in terms of $k_l$ defined in Case (A). To begin with, using the solution of Case (A), we can find the warp factor in the regions of Case (B) by the orbifold symmetry. Particularly, when the warp factor in the region (I) of Fig. 2 is
given by the solution of Case (A), the warp factor in the regions (II) and (III) of Fig. 2 are also written in bulk coordinates adopted for Cases (A) and (B). The results read

\begin{align}
(\text{I}) : & \quad A^{-1} = k_1 z_1 + k_2 z_2 + 1, \quad (48)\\
(\text{II}) : & \quad A^{-1} = -k_1 z_1 + k_2 z_2 + k_1 a + 1 = k_1' z_1' + k_2' z_2' + c_0', \quad (49)\\
(\text{III}) : & \quad A^{-1} = k_1 z_1 - k_2 z_2 + 1 = k_1'' z_1'' + k_2'' z_2'' + c_0'' \quad \text{for (A)–(i),(ii)}, \quad (50)\\
& \quad A^{-1} = k_1 z_1 + k_2 z_2 + 1 = k_1'' z_1'' + k_2'' z_2'' + c_0'' \quad \text{for (A)–(iii)}, \quad (51)
\end{align}

where \(-z_1'\) and \(z_1''\) axes are chosen to coincide with \(z_1 = a/2\). Since \(z_1 = z_2' = -z_2''\) and \(z_2 = -z_1' = z_1''\), we obtain \(k_1' = \pm k_1'' = -k_2\) (+ for Cases (A)–(i),(ii), and − for Case (A)–(iii)) and \(k_2' = k_2'' = -k_1\). Thus we can regard the warp factor in the extra dimensions as being determined only by \((k_1, k_2)\), which are given by 4-brane tensions belonging to Case (A) in view of Eqs. (38)-(40) as follows

\begin{align}
(\text{i}) & : \quad k_1 = \frac{1}{8\gamma M^4}(2\Lambda_1 + \sqrt{3}\Lambda_2), \quad k_2 = \frac{\Lambda_2}{8\gamma M^4}, \quad (52)\\
(\text{ii}) & : \quad k_1 = \frac{1}{8\sqrt{3}\gamma M^4}(\Lambda_{12} + 2\Lambda_2), \quad k_2 = \frac{\Lambda_{12}}{8\gamma M^4}, \quad (53)\\
(\text{iii}) & : \quad k_1 = \frac{1}{8\gamma M^4}(\Lambda_1 + \Lambda_3), \quad k_2 = \frac{1}{8\sqrt{3}\gamma M^4}(\Lambda_3 - \Lambda_1). \quad (54)
\end{align}

Therefore, it is easy to show that the 4-brane tensions belonging to Case (B) do not remain independent from Eq. (45) but they should be related to the 4-brane tensions belonging to Case (A) as

\begin{align}
(\text{i}) & : \quad V_6' = V_6'' = -(2\Lambda_1 + \sqrt{3}\Lambda_2), \quad V_1' = V_1'' = \Lambda_1, \quad (55)\\
(\text{ii}) & : \quad V_6' = V_6'' = -\frac{1}{\sqrt{3}}(\Lambda_{12} + 2\Lambda_2), \quad V_1' = V_1'' = \frac{1}{\sqrt{3}}(\Lambda_2 - \Lambda_{12}) = 0, \quad (56)\\
(\text{iii}) & : \quad V_6' = V_6'' = -(\Lambda_1 + \Lambda_3), \quad V_1' = \Lambda_1, \quad V_1'' = \Lambda_3, \quad (57)
\end{align}

where we used \(V_1' = \Lambda_1 = 0\) in Eq. (56) from the \(Z_3\) identification in Fig. 2. On the other hand, the 3-brane tensions at the three fixed points are determined in terms of 4-brane tensions from Eqs. (42) and (46) for Case (A)–(i) to begin with
\[
\begin{align*}
(i) & : \lambda_1 = \frac{3\alpha}{2\gamma^2 M^6} (2\Lambda_1 + \sqrt{3}\Lambda_2)(\sqrt{3}\Lambda_1 + 2\Lambda_2), \\
\lambda_2 & = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6} (V'_1 + V'_6)^2 = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6} (\Lambda_1 + \sqrt{3}\Lambda_2)^2, \\
\lambda_3 & = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6} (V''_1 + V''_6)^2 = \lambda_2, \\
\end{align*}
\]

and for Cases (A)–(ii) and (iii) as

\[
\begin{align*}
(ii) & : \lambda_1 = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6} (\Lambda_{12} + \Lambda_2)^2, \quad \lambda_2 = \frac{9\sqrt{3}\alpha}{4\gamma^2 M^6} \Lambda_2^2 = \lambda_3, \\
(iii) & : \lambda_1 = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6} (\Lambda_1 + \Lambda_3)^2, \quad \lambda_2 = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6} \Lambda_3^2, \quad \lambda_3 = \frac{3\sqrt{3}\alpha}{4\gamma^2 M^6} \Lambda_1^2. \\
\end{align*}
\]

Moreover, in view of Eqs. (48)-(51), we also need to have another 3-brane tension at a non-fixed point \((z_1, z_2) = (a/2, 0)\) (and at its \(Z_3\) transformed points) only for Cases (A)–(i) and (ii)

\[
\begin{align*}
(i) & : \lambda_4 = -\frac{3\alpha}{2\gamma^2 M^6} \Lambda_2 (2\Lambda_1 + \sqrt{3}\Lambda_2), \\
(ii) & : \lambda_4 = -\frac{3\sqrt{3}\alpha}{2\gamma^2 M^6} \Lambda_2^2. \\
\end{align*}
\]

Here we find that the 3-brane tensions at the fixed points are all positive for \(\alpha > 0\) but all negative for \(\alpha < 0\) (from the positiveness of \(k_1\) and \(k_2\) in Eq. (52) for case (i)). In addition, there should exist an extra 3-brane (the \(\lambda_1\)-brane) which is not located at the fixed points only for Cases (A)–(i) and (ii) but there does not arise such an additional 3-brane for Case (A)–(iii). Note that in order to explain the large mass hierarchy with extra dimensions compactified on the \(T^2/Z_3\) orbifold, we may take the \(\lambda_2\) or \(\lambda_3\) branes as the visible brane with positive tension for \(\alpha > 0\) while the \(\lambda_1\) brane can be considered as the hidden brane at the Planck scale.

For our purpose of showing the generation of the large mass hierarchy, let us rewrite the metric as

\[
\begin{align*}
\text{ds}_6^2 = A^2(z_1, z_2)(\eta_{\mu\nu}dx^\mu dx^\nu + dz_1^2 + dz_2^2) \\
&= A^2(y_1, y_2)\eta_{\mu\nu}dx^\mu dx^\nu + B^2(y_1, y_2)dy_1^2 + C^2(y_1, y_2)dy_2^2 \\
\end{align*}
\]
by the following bulk coordinate transformations:

\[ dz_1 = \frac{B}{A} dy_1, \quad dz_2 = \frac{C}{A} dy_2. \]  

(66)

For example, \( k_1 z_1 = e^{k_1 y_1} - 1, \) \( k_2 z_2 = e^{k_2 y_2} - 1 \) for \( A^{-1} = k_1 z_1 + k_2 z_2 + 1. \) Then, we can have the warp factor in the new coordinate: \( A = (e^{k_1 y_1} + e^{k_2 y_2} - 1)^{-1}, \) \( B = e^{k_1 y_1} A \) and \( C = e^{k_2 y_2} A. \)

In this new coordinate, let us consider the action for the Higgs scalar field at the \( \lambda_2 \) brane connecting to the patch (I) for all cases of (A)

\[ S_{vis} \supset \int d^4 x \sqrt{-g^{(vis)}} \left[ \bar{g}^{\mu\nu} \partial_\mu H \partial_\nu H - (H^2 - m_0^2)^2 \right], \]

\[ = \int d^4 x \sqrt{-g^{(4)}} A^4 \left[ A^{-2} (\partial H)^2 - (H^2 - m_2^2)^2 \right] \]  

(67)

where \( m_0 \) is of order the Planck scale. Then, redefining the scalar field as \( \bar{H} = A H \) gives

\[ \int d^4 x \sqrt{-g^{(4)}} \left[ (\partial \bar{H})^2 - (\bar{H}^2 - m_2^2)^2 \right] \]  

(68)

where the Higgs mass parameter on the \( \lambda_2 \) brane is given by

\[ m_2 = A m_0 = (e^{k_1 b_1} + e^{k_2 b_2} - 1)^{-1} m_0 \]  

(69)

with

\[ b_1 = \frac{1}{k_1} \log \left( \frac{1}{2} k_1 a + 1 \right), \]  

(70)

\[ b_2 = \frac{1}{k_2} \log \left( \frac{\sqrt{3}}{6} k_2 a + 1 \right). \]  

(71)

Similarly, we obtain the effective mass scale on the \( \lambda_3 \) brane connecting to the patch (III)

(i), (ii) : \[ m_3 = (e^{k_1 b_1} + e^{k_2 b_2} - 1)^{-1} m_0 = m_2, \]  

(iii) : \[ m_3 = (e^{k_1 b_1} + e^{-k_2 b_2} - 1)^{-1} m_0. \]  

(72)

(73)

As a result, we can obtain two weak scale branes with positive tensions located at two fixed points of the \( T^2/Z_3 \) orbifold. Therefore, the hierarchy problem related to the Higgs mass can be explained by regarding one of two weak scale branes as the visible brane while the
A λ_1 brane with the Planck scale at the origin is taken as the hidden brane. These two scales can be used to solve the hierarchy problem as in the RS I case. Or we can use it for the third generation and the first two generations mass hierarchy problem (the third generation matter located at the λ_1-brane) if the gauge hierarchy problem is solved by supersymmetry. Except for the Case (iii), the λ_4-brane can be used to locate the untwisted matter fields if it is required to put matter fields at the 3-branes. For Case (iii), however, it is not required to put a λ_4-brane and hence the string models with matter arising only from twisted sectors belong to this category \[21\].

As in the case of \(T^2/Z_2\) discussed in Ref. [13] and \(T^2/Z_3\) orbifolds considered in this section, it turns out that other possible orbifolds with discrete rotation symmetry such as \(T^2/Z_4\) and \(T^2/Z_6\) can give rise to similar candidates for the visible brane with positive tension to solve the gauge hierarchy or flavor hierarchy problem. However, only for the \(T^2/Z_3\) case we can take all the 3-brane tensions at the fixed points to be positive.

V. CONCLUSION

We formulated a \(Z_N\) symmetric RS models in \(AdS_6\) with the Gauss-Bonnet term. Firstly, we considered a junction of semi-infinite 4-branes with the \(Z_N\) discrete rotation symmetry in \(AdS_6\) with a Gauss-Bonnet term. The Gauss-Bonnet term gives rise to more divergent terms than the case with 4-branes only to accommodate a nonzero 3-brane tension at the junction of 4-branes. Then the extra dimensional space with the \(Z_N\) discrete symmetry due to the brane junction is compactified on a torus, we find that the bulk space is separated into a number of distinguishable regions (for example a, b, c, and d of Fig. 2 for the \(Z_3\) orbifold) by the brane junctions with discrete symmetries around their centers. Our particular interest is on the \(Z_3\) orbifold which renders a possible solution of the three family problem \[18\]. Hence, we identified the 3-brane tensions at the fixed points of an \(T^2/Z_3\) orbifolds by using the results on the \(Z_3\) symmetric brane junctions. As a result, the three 3-brane tensions can be positive with a positive Gauss-Bonnet coupling (\(\alpha > 0\)). We also observed that the
gauge hierarchy can be explained by regarding one of two weak scale 3-branes as the visible
brane. The results can be generalized to the case of the $T^2/Z_N$ orbifold but it is the case
only for the $T^2/Z_3$ orbifold that we can take all the 3-brane tensions at the fixed points to
be positive, circumventing the cosmological problem of the negative visible-brane tension in
the RS model.

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Fig. 1. A junction of semi-infinite 4-branes where a 3-brane resides.

Fig. 2. The $T^2/Z_3$ orbifold. We distinguish the regions of the torus under $Z_3$ symmetry as $a$, $b$, $c$, and $d$. There are three fixed points depicted by •, □ and ▲.