Effect of interleaving on the performances of the MAP turbo decoder controlled by the IHDA and CE Stopping criteria

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Abstract. In this article, we present the effect of interleaver size on the performance of a turbo decoder using a stopping criterion to reduce complexity and decoding time. We choose the two criteria CE and IHDA. The interleavers used are pseudo-random interleavers, S-Random, of sizes 5120 and 1280. The simulation results show that the IHDA criterion uses the lowest average iteration number in medium and high Signal-to-Noise Ratios SNR. Increasing the size of the interleaver also provides significant coding gain.

1. Introduction
Turbo codes [1] have shown that they are powerful codes for digital communications. They have been adopted in several modern telecommunications systems. The decoding quality improves as the number of iterations is increased. However, there is no need to continue the iterative processing when the frame is decoded. For this, several stopping criteria for turbo decoding are proposed in the literature [2-14]. Among these criteria, we can cite the famous Cross-Entropy criterion CE [2] [14] and the Improved Hard Decision Aided IHDA [5] criterion which offer acceptable performances in turbo decoding with reasonable complexity.

Turbo-codes are parallel concatenated codes, approaching the Shannon limit. Their performances are highly dependent on the size of the used interleaver. A large interleaver always offers low Bit Error Rates BER. The decoding quality is degraded when using small interleavers. When we integrate a stopping criterion to the turbo decoder, the behaviour differs depending on the size of the interleaver. In this article we try to verify this behaviour by considering the two stopping rules CE and IHDA.

The article is structured as follows: In section 2 and 3, we present turbo decoder and some existing stopping techniques. In section 4, we present the Cross-Entropy CE stopping rule. In section 5, we briefly describe the IHDA criterion. In section 6, we explain the construction principle of S-random interleavers [15]. The simulation results are presented in section 7.

2. Turbo decoder
The major advantage of the concatenation [1] of two codes is mainly to obtain a code of high minimum distance, and therefore powerful, while maintaining a reasonable complexity of coding and especially decoding.

We consider two Recursive Systematic Convolutional ‘RSC’ codes concatenated in parallel. The frame of information bits \( \{u(k)\}, k = 1, ..., N \), is coded by this turbo code. Each information bit, after coding gives a systematic bit \( u(k) \), and two redundant bits \( x1(k) \) and \( x2(k) \). After transmission over a Gaussian channel using BPSK modulation, the received samples are \( \{yu(k), y1(k), y2(k)\} \).
Figure 1. Turbo decoder

The received frame is passed to the turbo decoder of the figure 2. INT and DEI represent respectively interleaver and the deinterleaver. We use the MAP algorithm [16] for the decoders 1 and 2 (Figure 1).

3. Existing stopping techniques
Before presenting the two criteria CE and IHDA, we describe some stopping rules proposed for turbo codes.

The first is the Sign Change Ratio criterion SCR [3]. It consists of counting the number of sign changes $C(i)$ of extrinsic information produced by the second decoder between the iteration ‘i’ and ‘i-1’. Simulations show that we can stop the iterations when [3]:

$$C(i) \leq (0.0005 \sim 0.03) N$$

where $N$ is the length of information frames before coding.

This criterion makes it possible to stop the turbo process with about the same average number of iterations and the same performances of the CE rule.

The second rule HDA (Hard-Decision-Aided) [3], compares the hard decisions at the output of the second decoder with those of the previous iteration. Decoding is stopped if all hard decisions remain the same. The overall performances of the simplified variants are close to that obtained by the original CE rule.

Another stopping criterion called Sign Difference Ratio (SDR) [4] is a variant of the SCR. In this case, we count the number of times $D_{ji}$ where the signs of the a priori information and the extrinsic information of the same decoder differ at the iteration ‘i’. The turbo process is stopped if:

$$D_{ji} \leq p N$$

where ‘p’ is a threshold that represents the sign difference ratio SDR, and:

$$10^{-3} \leq p \leq 10^{-2}$$

$D_{ji}$ is also the number of sign differences between the extrinsic informations of the two decoders.

SDR achieves similar performance of SCR in terms of BER, FER, and the average number of iterations, while requiring lower complexity.

Another contribution [6] uses a CRC code to check if the frame has been corrected, but it penalizes the information rate by adding redundancy. Other method use the identification of undecodable blocks for stopping the turbo process [7]. We cite other criteria that also use the LLR (or decisions) [8-14].
4. Cross-entropy CE criterion

The famous criterion Cross-Entropy CE uses the Log-Likelihood Ratio LLR at the outputs of the decoders [2]. Consider a Turbo encoder made up of two Recursive Systematic Convolutional RSC encoders concatenated in parallel, and separated by an interleaver. The frame of information bits \{d(k)\}, \(k=1,\ldots,N\), is coded to give \(3N\) bits. Each bit of information, after coding gives a systematic bit \(d(k)\), and two redundancy bits \(x_1(k)\) and \(x_2(k)\). After transmission over a Gaussian channel using BPSK (Binary Phase Shift Keying) modulation, the received samples are \(\{y_1(k), y_2(k)\}\). The received frame is decoded by a MAP turbo decoder (Maximum A Posteriori [16]).

Let \(l_1^{(i)}(d(k))\) be the LLR of bit \(d(k)\) at the output of decoder ‘m’ of iteration ‘i’ at time ‘k’, and \(L_e^{(i)}(d(k))\) its extrinsic information. The cross-entropy of the iteration ‘i’ is approximated by

\[
CE(i) \approx \sum_k \frac{|L_e^{(i)}(d(k))|^2}{e^{|L_e^{(i+1)}(d(k))|}}. 
\]

where

\[
\Delta L_e^{(i)}(d(k)) = L_e^{(i)}(d(k)) - L_e^{(i-1)}(d(k)).
\]

This is also equivalent to

\[
\Delta L_e^{(i)}(d(k)) = L_2^{(i)}(d(k)) - L_1^{(i)}(d(k)).
\]

The decoding is considered as converged and stopped when the cross-Entropy \(CE(i)\) is below a threshold ‘\(\varepsilon\)’:

\[
CE(i) < \varepsilon
\]

[2] affirms that a threshold between \(10^{-2} \leq CE(1) \leq 10^{-4}\) is appropriate to stop the iterations. This criterion makes it possible to stop the Turbo process after decoding the frames with very little degradation in performance. We choose

\[
\varepsilon = 10^{-3} CE(1).
\]

5. IHDA criterion

This rule uses the fact that extrinsic information keeps increasing with increasing of number of iterations, which causes refinement of LLRs [5]. Its principle is to compare hard decisions based on \(\left[\left(\frac{2}{\sigma^2}\right)y_d^1 (k) + L_{e1}(d(k))\right]\) with the decisions based on the LLR \(L_2^{(i)}(d(k))\) at the output of the second decoder.

1. Decode iteration i (decoder 1, decoder 2).
2. Apply a decision on \(\left[\left(\frac{2}{\sigma^2}\right)y_d^1 (k) + L_{e1}(d(k))\right]\).
3. Apply a decision on \(L_2^{(i)}(d(k))\).
4. Compare the two decisions,
   - If both decisions give the same frame, stop Turbo decoding.
   - Otherwise, go to iteration \(i+1\).

6. S-random interleaver

S-random interleavers were first proposed by Divsalar and Pollard [15]. The adjacent information bits are separated by a distance greater than a previously set parameter S. It is constructed as follows:

1. Generation of an integer \(n, n<M\), where \(M\) is the size of the interleaver.
2. Calculate the distance between \(n\) and the \(S\) integers generated previously.
3. If this distance is equal to at least ±S, then the integer n is retained.

4. Otherwise, the generated integer is rejected, repeat step 1.

Usually, $S < \sqrt{\frac{M}{2}}$ where M is the length of the Interleaver.

7. Simulation results

We use a turbo encoder consisting of two RSC encoders of polynomials [1, 35/23], followed by a pseudo-random S-Random interleaver. The $d(k)$ information bits are encoded with this turbo encoder, mapped using a BPSK constellation and then transmitted through a Gaussian channel without Inter-Symbol Interference ISI.

After demodulation and detection, the received samples feed a MAP turbo decoder. It uses a maximum of 10 iterations. It is driven by the Cross-Entropy CE or IHDA stopping criteria. For the CE criterion, the turbo decoder is stopped at iteration i when the criterion is verified. That is, when $CE(i) < 10^{-3} CE(1)$.

The IHDA criterion does not use a threshold. It only compares decisions. The number of frames transmitted is 3000.

Figure 2. BER of MAP turbo decoders coupled with CE and IHDA criteria using two interleavers.

The simulation results (Figures 2 and 3) show that using a large interleaver provides the best performance. The gain at BER = $10^{-3}$ between the interleaver of size 5120 and that of size 1280 is 0.28 dB. For the interleaver of small size (1280), IHDA is better than CE for high Signal to Noise Ratios SNR. But the two criteria are equivalent at low and medium SNR. For the interleaver of size 5120, the two criteria are equivalent. Except at high SNR ratio, the CE criterion gives weak BER and FER.
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Figure 3. FER of MAP turbo decoders coupled with CE and IHDA criteria.

Regarding complexity, Figure 4 plots the average number of iterations of the MAP turbo decoder. For high SNRs, the IHDA criterion is better for both interleavers. But for low SNR, the turbo decoder using small interleaver (1280), uses a low number of iterations for both IHDA and CE. At medium SNR, the IHDA criterion with the large interleaver (5120) ensures the lowest complexity.

Figure 4. Average number of iterations of the MAP turbo decoder using the CE and IHDA criteria.

8. Conclusion
In this article, we have verified the behaviour of the MAP turbo decoder coupled to the IHDA and CE stopping criteria for interleaver sizes 1280 and 5120. The simulation results have shown a significant coding gain between the turbo decoder using the size 5120 interleaver and the one using size 1280 for
both IHDA and CE criteria. In addition, the two criteria ensure the same performance in terms of BER and FER in low and medium SNR. At high SNRs, the CE criterion is better for the large size 5120 interleaver. For the size 1280 interleaver, we observe that IHDA is better. IHDA uses the least complexity at high and medium SNRs.

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