IMPURITIES IN “ODD-PAIRING” SUPERCONDUCTORS

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We present the results of theoretical analysis of normal impurities effects in superconductors with the gap being an odd function of $k - k_F$. This model proposed by Mila and Abrahams leads to the possibility of pairing in the presence of an arbitrarily strong short-range repulsion between electrons and may be applied to high-$T_c$ oxides. However, we demonstrate that normal impurities lead to rather strong suppression of this type of pairing, which is actually stronger than in the case of magnetic impurities in traditional superconductors. Relative stability of high-$T_c$ cuprates to disordering makes this model a rather unlikely candidate for the pairing mechanism in these systems.

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In a recent paper Mila and Abrahams proposed an interesting model, which allows the existence of superconducting pairing even in the case of infinitely strong point-like repulsion between electrons [1]. Naturally this model is of great interest as a basis for a possible mechanism of high-temperature superconductivity in metallic oxides. The model is based upon the demonstration of the existence of nontrivial solution of BCS-like gap equation:

\[
\Delta(\xi) = -N(0) \int_{-\omega_c}^{\omega_c} d\xi' \frac{\Delta(\xi')}{2\sqrt{\xi'^2 + \Delta^2(\xi')}} \frac{h}{2T} \sqrt{\xi'^2 + \Delta^2(\xi')} (1)
\]

with the gap function \(\Delta(\xi) = -\Delta(-\xi)\) (i.e. odd in \(k - k_F, \xi = v_F(k - k_F)\)) in case of the presence in \(V(\xi, \xi')\) of an attractive interaction \(-V_2(\xi, \xi') < 0\) (which is non-zero for \(|\xi|, |\xi'| < \omega_c\) and \(|\xi - \xi'| < \omega_c\)) despite the existence of a strong (infinite) point-like repulsion \(V_1(\xi, \xi') = U > 0\) (for \(|\xi|, |\xi'| < E_F\)). In case of the odd gap function \(\Delta(\xi)\) the repulsive interaction in Eq. (1) drops out, while the attractive part \(V_2(\xi, \xi')\) may produce pairing with unusual properties (gap function is zero at the Fermi surface, which leads to the gapless superconductivity).

If the normal (nonmagnetic) impurities are present the equations for normal and anomalous Green’s functions take the usual form [2], which is valid in case of weak scattering:

\[
G(\omega \xi) = -\frac{i \omega + \xi}{\omega^2 + \xi^2 + |\Delta(\xi)|^2}
\]

\[
F(\omega \xi) = \frac{\tilde{\Delta}^*(\xi)}{\omega^2 + \xi^2 + |\Delta(\xi)|^2}
\]

\[
\omega = (2n + 1)\pi T,
\]

\[
\tilde{\omega} = \omega - \frac{\gamma}{\pi} \int_{-\infty}^{\infty} d\xi \frac{\tilde{\omega}}{\omega^2 + \xi^2 + |\Delta(\xi)|^2}
\]

\[
\tilde{\Delta}(\xi) = \Delta(\xi) + \frac{\gamma}{\pi} \int_{-\infty}^{\infty} d\xi \frac{\tilde{\Delta}(\xi)^*}{\omega^2 + \xi^2 + |\Delta(\xi)|^2} = \Delta(\xi) (3)
\]

Here \(\gamma = \pi c V_0^2 N(0)\)—is the scattering rate due to point-like impurities with potential \(V_0\), chaotically distributed in space with concentration \(c\). The integral term in the second equation vanishes due to the odd nature of \(\Delta(\xi)\) and the gap renormalization is absent. This
fact explains the strong impurity suppression of the “odd” pairing. Note that the similar behavior exists in the case of anisotropic pairing e.g. of the $d$-wave type [3,4].

The gap equation takes now the following form:

$$\Delta(\xi) = N(0)T \sum_{\omega_n} \int_{-\infty}^{\infty} d\xi' V_2(\xi, \xi') \frac{\Delta^*(\xi')}{\omega^2 + \xi'^2 + |\Delta^2(\xi')|^2}$$

(4)

Close to the transition temperature $T_c$ Eqs. (3) and (4) may be linearized over $\Delta(\xi)$, and after the standard calculations we obtain the following linear gap equation, which determines $T_c$:

$$\Delta(\xi) = N(0) \int_{-\infty}^{\infty} d\xi' V_2(\xi, \xi') \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{\xi'} \text{th} \left( \frac{\omega + \xi'}{2T_c} \right) \frac{\gamma}{\omega^2 + \gamma^2} \Delta(\xi')$$

(5)

In the following we shall use the model interaction:

$$V_2(\xi, \xi') = \begin{cases} V[\cos \frac{\pi}{2} \frac{\xi - \xi'}{\omega_c} + 1] & |\xi - \xi'| < \omega_c \\ 0 & \text{for } |\xi|, |\xi'| > \omega_c; |\xi - \xi'| > \omega_c \\ \end{cases}$$

(6)

The main attractive property of this choice is that it allows the reduction of the integral gap equation to a simple transcendental equation which can be easily solved. Model potentials used in Ref. [1] do not allow such a reduction and in most cases have no other serious preferences. The main qualitative results obtained below do not depend on the choice of the model potential.

The gap function takes now the following form:

$$\Delta(\xi) = \Delta_0(T) \sin \left( \frac{\pi}{2} \frac{\xi}{\omega_c} \right) \text{ for } |\xi| < \omega_c$$

(7)

with $\Delta(\xi) = 0$ for $|\xi| > \omega_c$. The $T_c$-equation reduces to:

$$1 = N(0)V \int_{0}^{\omega_c} \frac{d\xi'}{\xi'} \sin^2 \left( \frac{\pi}{2} \frac{\xi'}{\omega_c} \right) \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \text{th} \left( \frac{\omega + \xi'}{2T_c} \right) \frac{\gamma}{\omega^2 + \gamma^2}$$

(8)

In the “pure” limit of ($\gamma \rightarrow 0$) we get the $T_c$ dependence on the pairing coupling constant $g = N(0)V$, which is shown in Fig. 1. Pairing exists for $g > g_c = 1.213$. In Fig. 2 we show the dependence of $T_c$ on $\gamma$ for a number of characteristic values of the pairing constant $g$. It is
clearly seen that normal impurities strongly suppress the “odd” pairing. Superconductivity vanishes for $\gamma \sim T_{c0}$ and this suppression is even stronger than in case of magnetic impurities in traditional superconductors [5]. This is reflected in particular by the disappearance of superconductivity region on the “phase diagram” in Fig. 2 for $g \to g_c$ and the absence of the universal behavior which is characteristic for the case of magnetic impurities.

The critical scattering rate $\gamma_c$, corresponding to $T_c(\gamma \to \gamma_c) \to 0$, is determined, according to Eq. (5), by the following equation:

$$\Delta(\xi) = N(0) \int_{-\infty}^{\infty} d\xi' V_2(\xi, \xi') \frac{1}{\pi \xi} \arctan \left( \frac{\xi'}{\gamma_c} \right) \Delta(\xi')$$

(9)

which for the model interaction of Eq. (6) reduces to:

$$1 = \frac{2}{\pi} N(0) V \int_0^{\omega_c} \frac{d\xi'}{\xi'} \sin^2 \left( \frac{\pi \xi'}{2 \omega_c} \right) \arctan \left( \frac{\xi'}{\gamma_c} \right)$$

(10)

It is easily shown that for $g \gg g_c$ we have the universal result: $\gamma_c/T_{c0} = 4/\pi \approx 1.273$. It is also not difficult to see that this result as well as the dependence of $T_c$ on $\gamma$ for $g \gg g_c$ do not depend at all on the choice of the model potential $V_2(\xi, \xi')$. For $g \to g_c$ we always obtain the dependence $\gamma_c \sim (g - g_c) \to 0$. This behavior is clearly seen in Fig. 2.

We have already noted that the model under discussion is attractive as a possible microscopic approach for the explanation of high-temperature superconductivity in metallic oxides [1]. It is well known that high-$T_c$ state in these systems is very sensitive to the structural disordering [3]. However, from the existing experimental data [3] it follows that superconductivity in oxides is destroyed close to the metal-insulator transition induced by disordering i.e. for $\gamma \sim E_F$, but not for $\gamma \sim T_{c0} \ll E_F$. This fact makes the model of an “odd” pairing rather improbable candidate for the explanation of high-$T_c$ superconductivity in cuprates.

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Figure captions:

Fig. 1. $T_{c0}$ dependence on the pairing constant $g = N(0)V$ for the model interaction of Eq. (6).

Fig. 2. $T_c$ dependence on the scattering rate $\gamma$ for the different values of pairing constant $g$:

1—$g = 1.22$; 2—1.24; 3—1.30; 4—1.5; 5—2.0; 6—5.0, 7—10.0.