Damping of collective modes in ultracold trapped Fermi gases with in-medium cross section

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Abstract. We study the damping of the quadrupole and scissors modes of ultracold trapped Fermi gases with large negative scattering length at temperatures above the superfluid critical temperature $T_c$, within the framework of the Boltzmann equation. The in-medium scattering cross-section is strongly enhanced at temperatures slightly above $T_c$. This has strong effects on the collisional damping of the collective modes and the transition from hydrodynamic to collisionless behaviour as a function of temperature. Within the method of phase-space moments of second order, it was found that the resulting damping rates are in strong disagreement with experimental data from the Innsbruck group, which seemed to favour the free cross section. However, the comparison with numerical solutions of the Boltzmann equation revealed that the method of second-order phase-space moments is insufficient. Here we show that the agreement between the calculation using the in-medium cross-section and the data can be clearly improved by including also fourth-order moments.

1. Introduction

The study of many-body physics with ultracold atoms has become possible in the last few years due to the enormous progress in trapping and cooling neutral atoms in magneto-optical traps combining lasers and magnetic fields. From the theorist’s point of view, the trap is simply an external potential, which can be considered as harmonic near the minimum, $V_{\text{trap}}(r) = \frac{1}{2} \sum_{i=x,y,z} \omega_i^2 r_i^2$. Atoms are loaded into this potential and cooled down by laser and evaporative cooling and are then ready for the actual experiment. These systems have the unique feature that almost all properties (particle number, density, interaction strength, statistics, dimensionality) can be varied in the experiments. Here, we will consider the case of fermionic atoms with two equally occupied spin states $\uparrow$ and $\downarrow$ with an attractive interaction, i.e., a negative scattering length $a < 0$. At temperatures below a certain critical temperature $T_c$, this system undergoes a phase transition to a superfluid phase, but we will concentrate on the normal phase ($T > T_c$).

A couple of experiments were done to study the collective modes of these systems. Here we are interested in interpreting the results obtained by the Innsbruck group \cite{1, 2}. In these
Figure 1. Schematic representation of different collective modes, as seen in a cut through the $z = 0$ plane: (a) radial breathing mode, (b) radial quadrupole mode, (c) scissors mode.

experiments, the trap was cigar shaped ($\omega_x \approx \omega_y \gg \omega_z$) and the following collective modes in the radial direction were studied: (a) the radial breathing mode (Fig. 1a), whose frequency depends on the equation of state of the system; (b) the radial quadrupole mode (Fig. 1b), whose frequency is different depending on whether the system behaves hydrodynamically (superfluid or normal fluid) or collisionless; (c) the scissors mode (Fig. 1c), in which the cloud rotates back and forth against the triaxial trap potential ($\omega_x \neq \omega_y$).

The question we are interested in here concerns the transition from collisionally hydrodynamic to collisionless behaviour as function of interaction strength and temperature. With increasing temperature, the system expands and gets more and more dilute, so that the collision rate decreases. In the other limit, at very low temperature, the collision rate goes down, too, due to Pauli blocking. In the regime of a strong attractive interaction, $-1 \lesssim 1/k_F a \leq 0$ ($k_F$ being the Fermi momentum at the trap center), the system behaves more or less hydrodynamically at $T \approx T_C$ (for a summary of the experimental results, see Fig. 4 of Ref. [1]).

2. Formalism

Our starting point is the Boltzmann equation $\dot{f} + \frac{p}{m} \cdot \nabla_r f - \nabla_r V \cdot \nabla_p f = -I[f]$, where $f(r, p)$ denotes the Wigner function (phase-space distribution function), $V(r) = V_{\text{trap}}(r) + U$ is the sum of trap and mean-field potentials, and $I$ is the collision integral [3]. The collision integral contains the cross section $\sigma$ and Pauli-blocking factors for the final states.

In free space, the cross section is given by $\sigma_0 = 4\pi a^2/[1 + (qa)^2]$, where $q$ denotes the momentum of the atoms in the center-of-mass frame ($q = |p_1 - p_2|/2$). The cross section in the medium can be calculated from the in-medium T matrix in ladder approximation as shown in Fig. 2a. At $T = T_C$, the in-medium T matrix develops a pole at the Fermi surface (Thouless criterion for $T_C$). As a precursor effect, the in-medium cross section is strongly enhanced as compared to the free one at temperatures close to $T_C$ [2, 3], as shown in Fig. 2b. In the context of nuclear physics, this effect was already discussed in Ref. [4].

For the study of collective oscillations, the Boltzmann equation is linearised around equilibrium. Approximate solutions of the linearised equation can be obtained with the method of moments. One writes the distribution function as $f(r, p, t) = f_{\text{eq}}(r, p) + (df_{\text{eq}}/d\mu)\Psi(r, p, t),\quad \sigma_{\text{free}} = 4\pi a^2/[1 + (qa)^2]$. 

Figure 2. (a) Diagrams for the in-medium T matrix in ladder approximation. (b) Ratio of in-medium to free cross section for zero total momentum as function of the relative momentum $q$ for different temperatures $T$ ($T_F = \text{Fermi temperature}$).
where the equilibrium distribution function \( f_{eq} \) is a Fermi function with chemical potential \( \mu \), and \( \Phi \) is supposed to be a smooth function in phase space (contrary to \( f \)). Then one makes the ansatz that \( \Phi \) is a polynomial in \( r \) and \( p \) with time-dependent coefficients. The time dependence of the coefficients can be determined by taking moments of the Boltzmann equation. In the previous literature [2, 3], only polynomials of second order in \( r \) and \( p \) were included. In this case, the method is equivalent to the widely used scaling ansatz. Only very recently, the method of moments has been applied to higher-order polynomials in \( r \) and \( p \) [5].

3. Results
Let us start by discussing results obtained within the method of second-order moments. As an example, we display in Fig. 3 the temperature dependence of the frequency and damping rate of the scissors mode for the parameters of the experiment by Wright et al. [1]. Note that the inclusion of the in-medium cross section, \( \sigma \), instead of the free one, \( \sigma_0 \), completely spoils the agreement with the data. Identical results, except for the mean field effects, were found earlier in calculations by Bruun and Smith [2].

Having obtained such a disappointing result, one might worry that the whole Boltzmann approach is invalid, e.g., because there are no good quasiparticles in the strongly correlated pseudogap regime. But before drawing such pessimistic conclusions, one should first check whether the problem persists if one finds a more accurate solution of the Boltzmann equation.

Our first idea was to solve the Boltzmann equation numerically using the method of test particles (like in the simulation of heavy-ion collisions). So far, we were able to do so only for the case of an isotropic trap with a reduced number of atoms, with the free cross section and without mean field effects. Nevertheless, these calculations revealed that the method of second-order moments systematically overestimates the effects of collisions [5]. As a typical example, we show in Fig. 4 the strength function (excitation spectrum) of the quadrupole mode. The result obtained within the second-order moment method deviates clearly from the numerical

**Figure 3.** Frequency \( \omega \) (left) and damping rate \( \Gamma \) (right) of the scissors mode as functions of temperature. The data are from Ref. [1], the theoretical curves were obtained within the second-order moment method.

**Figure 4.** Strength function of the quadrupole mode obtained by a test-particle simulation, by the method of moments up to second order, and by the extended method of moments up to fourth-order (parameters: \( N = 10000 \) atoms in a spherical trap, \( T/T_F = 0.4 \), \( 1/k_F a = -0.5 \)).
What is wrong with the second-order moment method? Let us look at the ansatz for the function $\Phi$ of the quadrupole mode, which reads $\Phi = c_1(x^2 - y^2) + c_2(p_x^2 - p_y^2) + c_3(xp_x - yp_y)$. The deformation of the Fermi sphere is determined by the second term ($\propto p_x^2 - p_y^2$), which is independent of $r$. In reality, however, collisions are much more frequent in the centre than in the low-density regions of the cloud. Therefore, the Fermi surface deformation should be much smaller in the centre ($r = 0$) than at large $r$.

Following these arguments, we concluded that the inclusion of higher order moments should help to improve the results. For instance, in the case of the quadrupole mode, a term of the form $\propto r^2(p_x^2 - p_y^2)$ in the ansatz for $\Phi$ would account for the position dependence of the Fermi-surface deformation. We therefore decided to extend the method of moments to fourth order [5]. The corresponding result is also shown in Fig. 4, and one can clearly see that the agreement with the numerical simulation is now much better than within the second-order moment method.

Encouraged by this result, we applied the fourth-order moment method to the realistic parameters, including the in-medium cross section (but not yet the mean field). First results for the case of the radial quadrupole mode, with parameters corresponding to the conditions of the experiment by Riedl et al. [2], are displayed in Fig. 5. It can be clearly seen that the fourth order results are in much better agreement with the data than the second-order ones.

4. Conclusion
In the normal-fluid phase, the collective modes of trapped Fermi gases show a smooth transition from hydrodynamic to collisionless behaviour as function of temperature. Previous analyses indicated that the data concerning this transition contradict the theoretical results including the in-medium enhancement of the cross section. However, we have shown that this conclusion was at least partly an artefact of the approximate solution of the Boltzmann equation (second-order moments) and that the agreement with the data can be greatly improved by including fourth-order moments, too. In conclusion, at the present stage, the available solutions of the Boltzmann equation are not reliable enough to decide whether the dynamics of these systems can be described by the Boltzmann equation or not.

References
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Figure 5. Frequency $\omega$ (left) and damping rate $\Gamma$ (right) of the radial quadrupole mode as functions of temperature. The data are from Ref. [2], the theoretical curves were obtained within the second- and fourth-order moment method, respectively.