Basic properties of Gamma-ray loud blazars

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Abstract. In this paper, a method is proposed to determine the basic properties of γ-ray loud blazars, among them the central black hole mass, \(M\), the Doppler factor, \(\delta\), the propagation angle of the γ-rays with respect to the symmetric axis of a two-temperature accretion disk, \(\Phi\), and the distance (i.e. the height above the accretion disk), \(d\) at which the γ-rays are created, for seven γ-ray loud blazars with available GeV variability timescales and in which the absorption effect of a γ-ray and the beaming effect have been taken into account. Our results indicate that, if we take the intrinsic γ-ray luminosity to be \(\lambda\) times the Eddington luminosity, \(L_{\gamma}^{in} = \lambda L_{Edd}\), the masses of the blazars are in the range of 

\[4 \times 10^7 M_\odot \leq M \leq 83 \times 10^7 M_\odot (\lambda = 1.0)\]

\[6 \times 10^7 M_\odot \leq M \leq 131 \times 10^7 M_\odot (\lambda = 0.1)\]

the Doppler factors (\(\delta\)) lie in the range of 

\[0.57 \leq \delta \leq 3.72 (\lambda = 1.0)\]

\[0.79 \leq \delta \leq 5.33 (\lambda = 0.1)\]

the angle (\(\Phi\)) is in the range of 

\[14^\circ \leq \Phi \leq 43^\circ (\lambda = 1.0)\]

\[13^\circ \leq \Phi \leq 39^\circ (\lambda = 0.1)\]

and the distance (\(d\)) is in the range of 

\[28R_g \leq d \leq 411R_g (\lambda = 1.0)\]

\[26R_g \leq d \leq 366R_g (\lambda = 0.1)\]

For 3C279, the case of a uniformly-bright disk is also adopted to determine the basic parameters, which are compared with those obtained for a two-temperature disk as well as those obtained by Becker & Kafatos (1995). Our model results are independent of γ-ray emission mechanisms but they do depend on the X-ray emission mechanism of the accretion disk.

Key words: γ-rays : Active Galactic Nuclei - Jet - Black Hole Mass

1. Introduction

One of the most important results of the CGRO/EGRET instrument in the field of extragalactic astronomy has been the discovery of blazars (i.e. flat-spectrum radio quasars–(FSRQs) and BL Lac objects) which emit most of their bolometric luminosity in the high energy range of γ-rays ( \(E > 100\) MeV). Many of the γ-ray emitters are also superluminal radio sources (von Montigny et al. 1995). The common properties of these
EGRET-detected AGNs are the following: the \( \gamma \)-ray flux is dominant over the flux in lower energy bands; the \( \gamma \)-ray luminosity above 100 MeV ranges from \( \sim 3 \times 10^{44} \) erg \( s^{-1} \) to \( \sim 10^{49} \) erg \( s^{-1} \) (assuming isotropic emission); many of the sources are strongly variable in the \( \gamma \)-ray band on timescales ranging from days to months (Mukherjee et al. 1997), but large flux variability on short timescales of \(< 1 \) day is also detected (see below). These facts suggest that the \( \gamma \)-ray emission is likely to arise from the jet of a blazar.

Various models for \( \gamma \)-ray emission from AGNs have been proposed. Generally, they are of two kinds: leptonic and hadronic models. In the leptonic model, high energy \( \gamma \)-rays are produced by the inverse Compton scattering of high energy electrons in the soft photon field. The soft photons may be emitted from the nearby accretion disk (Dermer et al. 1992; Coppi et al. 1993; Zhang & Cheng, 1997a) or they may arise from disk radiation reprocessed in some region of AGNs (e.g. a broad emission line region) (Sikora et al. 1994; Blandford & Levinson 1995; Xie et al. 1997, 1998); or they may come from the synchrotron emission in the jet (Maraschi et al. 1992; Zdziarski & Krolak 1993; Bloom & Marscher 1996; Marscher & Travis 1996), or from a differential rotating flux tube near the inner edge of the accretion disk (Cheng, Yu & Ding 1993). In the hadronic model, high energy \( \gamma \)-rays are produced by the synchrotron emission from ultrarelativistic electrons and positrons created in a proton-induced cascade (\( \text{PIC} \)) (Mannheim & Biermann 1992; Mannheim 1993; Cheng & Ding 1994). TeV radiation has been observed from 4 X-ray-selected BL Lacertae objects (XBLs): Mkn 421 (Punch et al. 1992), Mkn 501 (Quinn et al. 1996) and IES 2344+514 (Catanese et al. 1998), PKS 2155-304 (Chadwick et al. 1999). But there is no consensus yet on the dominant emission process (see von Montigny et al. (1997) for 3C273, Ghisellini et al. (1996) for 3C279, Comastri et al. (1997) for 0836+710, Böttcher & Collmar 1998 for PKS 0528+134).

It is generally believed that the escape of high energy \( \gamma \)-rays from an AGN depends on \( \gamma - \gamma \) pair production process because there are lots of soft photons around the central black hole. Becker & Kafatos (1995) have calculated the \( \gamma \)-ray optical depth in the X-ray field of an accretion disk. They found that the \( \gamma \)-rays should escape preferentially along the symmetric axis of the disk, due to the strong angular dependence of the pair production cross section. The phenomenon of \( \gamma - \gamma \) ”focusing” is related to the more general issue of \( \gamma - \gamma \) transparency, which sets a minimum distance between the central black hole and the site of \( \gamma \)-ray production (Bednarek 1993, Dermer & Schlickeiser 1994, Becker & Kafatos 1995, Romero et al. 2000; Zhang & Cheng 1997b). Therefore the \( \gamma \)-rays are focused in a small solid angle, \( \Omega = 2\pi(1 - \cos\Phi) \), suggesting that the apparent observed luminosity should be expressed as \( L_\gamma = \Omega D^2 (1 + z)^{\alpha_\gamma - 1} F_{\gamma \, \text{obs}}^{\geq 100\, \text{MeV}} \), where \( F_{\gamma \, \text{obs}}^{\geq 100\, \text{MeV}} \) is the observed energy flux of the \( \gamma \)-rays, \( D \) the distance to the AGN, and \( z \) the redshift. The observed \( \gamma \)-rays from the AGN require that the jet almost points
to us and the optical depth $\tau \leq 1.0$. In this sense, both the absorption and beaming (boosting) effects should be considered when the properties of a $\gamma$-ray loud blazar are discussed, which is the focus of the present paper. The paper is arranged as follows: In section 2 we summarize the observed results of seven $\gamma$-ray loud blazars with available GeV variable timescale. In section 3, we present our method which is used to estimate the black hole mass, the Doppler factor, the propagation angle of the $\gamma$-rays, and their emission distance above the accretion disk. In section 4, we discuss the results and give a brief summary.

$H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$, and $q_0 = 0.5$ are adopted throughout the paper.

2. Observed data

Since we are interested in the variability timescale, we consider here only those $\gamma$-ray loud blazars with short timescales of variation, detected in the $\gamma$-ray region. Because the variability timescale corresponds to a different variation amplitude for different sources and/or different observation periods, we use the doubling timescale, $\Delta T_D = (F_{\text{minimum}}/\Delta F)\Delta T$, as the variability timescale, where $\Delta F = F_{\text{maximum}} - F_{\text{minimum}}$ is the variation of the flux over the time $\Delta T$. There are few simultaneous observations of the X-ray and $\gamma$-ray bands available, the data considered here are not simultaneous. The $\gamma$-ray data are from those periods corresponding to variability. The X-ray data are from recent publications, particularly the paper by Comastri et al. (1997).

2.1. PKS0208-512

PKS0208-512 is an HPQ ($P_{\text{opt}} = 11.3\%$, see Impey and Tapia, 1988). It was visible to EGRET during the 1991 (Bertsch et al. 1993) and 1994/1995 (Stacy et al. 1996) observation periods. The 1991 Sep./Oct. observations can be expressed well by a power law with a photon spectral index of $\alpha_{\gamma} = 1.689$ and an isotropic luminosity of $L(>100\text{MeV}) = 2 \times 10^{48} \text{ erg s}^{-1}$. The 1991 observations show evidence for a variability of at least a factor of 3 on a time scale of tens of days. During the 1994/1995 observation period a variation of a factor of 2.5 over a time scale of a two-week EGRET point was detected (Stacy et al. 1996) suggesting $\Delta T_D = 5.6 \text{ days}$. $f_{1\text{K}eV} = 0.61 \mu\text{Jy}$ and $\alpha_X = 1.04$ are available in the paper by Comastri et al. (1997).

2.2. PKS 0528+134

PKS 0528+134, $z = 2.07$ (Hunter et al. 1993), is one of the most luminous examples of blazars. It is observed by EGRET, COMPTE and OSSE aboard the CGRO (see Hunter et al. 1993, McNaron-Brown et al. 1995, Mukherjee et al. 1996, Collmar et al. 1997; Sambruna et al. 1997).
During 23-29 March 1993, $F(>100\ \text{MeV}) = (0.23\pm0.12 - 3.08\pm0.35) \times 10^{-6}\ \text{photon cm}^{-2}\ \text{s}^{-1}$, with a photon spectral index $\alpha_{ph} = 2.21\pm0.10$. In the 1993 observation, a variation of order of 100% over a timescale of $\sim 2$ days was detected (see Wagner et al. 1997), which suggests a doubling time scale of $\Delta T_D = 1\text{ day}$. X-ray data of $f_{1\text{KeV}} = 0.65\ \mu\text{Jy}$ and $\alpha_X = 0.54\pm0.29$ are reported in the paper by Comastri et al. (1997).

### 2.3. PKS 0537-441

PKS 0537-441, $z = 0.896$, a candidate for a gravitational lens (Surpi et al. 1996), is a violently variable object (Romero et al. 1995; Fan & Lin 1999). The $\gamma$-ray flux varies from $(1.83\pm0.91)$ to $(8.98\pm1.45) \times 10^{-6}\ \text{photon cm}^{-2}\ \text{s}^{-1}$ (Mukherjee et al. 1997). A flare of a factor of $\sim 3$ from 0.35 to $2.0 \times 10^{-6}\ \text{photon cm}^{-2}\ \text{s}^{-1}$ over a time scale of $\sim 2$ days can be seen from Fig. 3 in Hartman’s paper (Hartman 1996), $\Delta T_D = 16\ \text{hrs}$. $f_{1\text{KeV}} = 0.81\ \mu\text{Jy}$ and $\alpha_X = 1.16 \pm 0.09$ (Comastri et al. 1997).

### 2.4. 1253-055 (3C279)

3C279 is a well known member of the OVV subclass of blazars. It is perhaps the prototypical superluminal radio source (Moffet et al. 1972) and the first quasar detected at energies greater than 1 GeV by EGRET/CGRO. The simultaneous variability in X-rays and $\gamma$-rays ($>100\ \text{MeV}$) suggests for the first time that they are approximately cospatial (McHardy 1996). The $\gamma$-ray flux varies from $1.28$ to $28.7 \times 10^{-6}\ \text{photon cm}^{-2}\ \text{s}^{-1}$ (Mukherjee et al. 1997). Two $\gamma$-ray flares were detected (see Kniffen et al. 1993; Hartman et al. 1996; McHardy 1996; Wehrle et al. 1998). The 16-28 June 1991 flare showed: $F(>100\ \text{MeV}) = (2.8 \pm 0.4) \times 10^{-6}\ \text{photon cm}^{-2}\ \text{s}^{-1}$ with a photon spectral index $\alpha_{ph} = 1.89\pm0.06$. A variation of a factor of 4 over 2 days was seen. The quasi-simultaneous X-ray data detected in June 1991 can be described by $J(E) = (4.0 \pm 0.34) \times 10^{-3} E^{-(1.68\pm0.05)}\ \text{photon cm}^{-2}\ \text{s}^{-1}\ \text{Kev}^{-1}$ (Hartman et al. 1996).

The January-February 1996 flare intensity was (see McHardy 1996; Wehrle et al. 1998), $F(>100\ \text{MeV}) = (11.0 \pm 1.) \times 10^{-6}\ \text{photon cm}^{-2}\ \text{s}^{-1}$ with a photon spectral index of $\alpha_{ph} = 1.97\pm0.07$. During this flare, a variation of a factor of 4~5 in a day was observed, $\Delta T = 6\ \text{hrs}$ (Wehrle et al. 1998). During the 1996 January observation period, 3C279 was detected at a level of $3 \times 10^{-3}\ \text{photon cm}^{-2}\ \text{s}^{-1}\ \text{KeV}^{-1}$ at 1 KeV with a spectral index $\alpha_X = 0.78$ (Lawson & McHardy, 1998).

### 2.5. PKS 1622-297

For PKS 1622-297, $z=0.815$, we have very little information in the lower energy bands. But it is one of the most luminous objects in the $\gamma$-ray region. A peak flux of $(17\pm3) \times 10^{-6}$
photon cm$^{-2}$ s$^{-1}$ (E$>100$ MeV) and a flux increase by a factor of 2 in 9.7 hours were observed (Mattox et al. 1997). $f_{1\text{KeV}} = 0.08\mu$ Jy (Mattox et al. 1997).

2.6. Q1633+382 (4C 38.41)

Quasar 1633+382, $z = 1.814$, is an LPQ ($P_{\text{opt}} = 2.6\%$, Moore & Stockman 1984). During the 1992 November 17 - December 1 observation period, it was detected with a flux of $F(>100\text{ MeV}) = (0.30 \pm 0.06) \times 10^{-6}$ photon cm$^{-2}$ s$^{-1}$, with a photon spectral index $\alpha_{\text{ph}} = 1.87 \pm 0.07$. The flux varied by a factor of 1.5 within 24 hrs, $\Delta T_D = 16\text{ hrs}$, while the spectral index did not change. The $\gamma$-ray luminosity is at least two orders of magnitude larger than the maximum ever observed in any other band (see Mattox et al. 1993). $f_{1\text{KeV}} = 0.42\mu$ Jy and $\alpha_X = 0.53 \pm 0.08$ (Comastri et al. 1997).

2.7. 2200+420 (BL Lacertae)

2200+420 is the prototype of the BL Lacertae class. It is variable at all wavelengths (see Fan et al. 1998a, 1998b; Bloom et al. 1997; Böttcher & Bloom 1998; Madejski et al. 1998). A 14-year period was found in the optical light curve (Fan et al. 1998b). During 1995 January 24 - February 14, BL Lacertae showed a flux of $F(>100\text{ MeV}) = (40 \pm 12) \times 10^{-8}$ photon cm$^{-2}$ s$^{-1}$ with a photon spectral index $\alpha_{\text{ph}} = 2.2 \pm 0.3$. The upper limit on the flux at higher energy is $F(>300\text{ GeV}) < 0.53 \times 10^{-11}$ photon cm$^{-2}$ s$^{-1}$ (Catanese et al. 1997). During the 1997 January 15/22 observation period, it was detected at a flux of $F(>100\text{ MeV}) = (171 \pm 42) \times 10^{-8}$ photon cm$^{-2}$ s$^{-1}$, with a photon spectral index $\alpha_{\text{ph}} = 1.68 \pm 0.16$ and a dramatic increase of a factor of 2.5 within a timescale of 8hrs, $\Delta T_D = 3.2\text{ hrs}$. Besides, simultaneous optical and $\gamma$-ray flares were observed ruling out external scattering models (see Bloom et al. 1997). The observations from the object show that the spectrum of BL Lacertae hardens when the $\gamma$-ray flux increases. $f_{2\text{KeV}} = 0.749\mu$ Jy and $\alpha_{OX} = 1.31$ are reported in the paper by Perlman et al. (1996).

3. Method and results

3.1. Method

Now we describe our method of estimating the basic parameters ($M$, $\delta$, $\Phi$ and $d$) of the blazars with short timescale variabilities in the $\gamma$-ray band. As mentioned above, high energy $\gamma$-rays can escape only when the optical depth of $\gamma$-$\gamma$ pair production is not larger than unity. Based on Becker & Kafatos (1995), we can obtain an approximate relation for the optical depth at an arbitrary angle, $\Phi$,

$$
\tau_{\gamma\gamma}(M_7, \Phi, d) = \frac{1}{3} (51 - 8 \omega) \times \Phi^{2.5} \left( \frac{d}{R_g} \right)^{2\alpha_X + 3} + kM_7^{-1} \left( \frac{d}{R_g} \right)^{-2\alpha_X - 3}, \tag{1}
$$

where $M_7 = M/10^7 M_\odot$, $\alpha_X = \alpha_{\text{X}} - 3$, $k$ is a constant, and $R_g$ is the gravitational radius. The right hand side of (1) is the total optical depth $\tau_{\gamma\gamma}$ for the $\gamma$-ray band.
where \( k \) is given by

\[
k = 4.50 \times 10^9 \frac{\Psi(\alpha_X)(2-\omega)(1+z)^{3+\alpha_X} F_0^\alpha(1+z+\sqrt{1+z})^2}{(2\alpha_X+4-\omega)(2\alpha_X+3)} \times \\
\frac{[\frac{d}{d\omega}]_{\alpha_X=3}^{2\alpha_X + 4 - \omega} \Phi(\omega)}{(\frac{d}{d\omega})_{\alpha_X=3}^{2\alpha_X + 4 - \omega}} \left( E_{\text{iso}} \right)^{\alpha_X/2}
\]

(2)

\( \Psi(\alpha_X) \) is a function of the X-ray spectral index, \( \alpha_X \), \( F_0 \) the X-ray flux parameter in units of \( \text{cm}^{-2} \text{s}^{-1} \), \( m_e \) the electron mass, \( c \) the speed of light, \( R_g = \frac{2GM}{c^2} \) the Schwarzschild radius, \( E_\gamma \) the average energy of the \( \gamma \)-rays, and \( R_0 \) and \( R_{ms} \) are, respectively, the outer and inner radii of the accretion disk. \( \omega \) is a free parameter, \( \omega = 3 \) is for a two-temperature disk while \( \omega = 0 \) is for a uniformly bright disk.

From Eq. (1), the optical depth depends on \( d, \Phi \) and \( M \). At first, \( d \) can be determined if the variability timescale \( (\Delta T_D) \) for a blazar is observed, it is given by

\[
\frac{d}{R_g} = 1.73 \times 10^3 \frac{\Delta T_D}{1+z} M_7^{-1} \quad (3)
\]

Furthermore, using the observed \( \gamma \)-ray flux, \( F_\gamma^{obs}(>100\text{MeV}) \) in units of \( \text{ergs cm}^{-2} \text{s}^{-1} \), the relationship among the intrinsic luminosity, \( L_{in} \), the Doppler factor, \( \delta \), the mass of the central black hole, \( M \), and the propagation angle, \( \Phi \), is given by \( F_\gamma^{obs}(>100\text{MeV}) = (1+z)^{1-\alpha}\delta^{\alpha+4} L_{in}/\Omega D^2 \). We can define an isotropic luminosity as \( L_{iso} = 4\pi D^2 (1+z)^{\alpha+1} F_\gamma^{obs}(>100\text{MeV}) \) in units of \( 10^{48} \text{ergs s}^{-1} \), it can be expressed as

\[
L_{iso}^{48} = \frac{\lambda 2.52 10^{-3} \delta^{\alpha+4}}{1-\cos \Phi} M_7^{-1}
\]

(4)

where \( L_{in} = \lambda L_{Edd} = \lambda 1.26 \times 10^{45} M_7 \), and \( \lambda \) is a parameter depending on the specific \( \gamma \)-ray emission model.

Substituting Eqs. (3) and (4) into Eq. (1), we obtain a function of \( M \) and \( \Phi \). From this equation, a minimum value of \( \tau_{\gamma\gamma} \) for a given mass, \( M \), can be determined by \( \frac{\partial \tau}{\partial \Phi} \big|_{M = 0} = 0 \), i.e. solving

\[
\frac{2.5}{3} (51 - 8\omega) \Phi^{1.5} (1-\cos \Phi) - \frac{1}{3} (51 - 8\omega) \times \frac{2\alpha_X + 3}{2\alpha_X + 8} \Phi^{2.5} \sin \Phi \\
- \frac{2\alpha_X + 3}{\alpha_X + 4} k M_7^{-1} A \times \frac{2\alpha_X + 3}{2\alpha_X + 8} \sin \Phi = 0
\]

(5)

where

\[
A = 1.73 \times 10^3 \frac{\Delta T_D}{1+z} M_7^{-1} \frac{\lambda 2.52}{L_{iso}^{45}} \frac{L_{iso}^{45}}{\lambda 2.52}
\]

Finally, letting the minimum of \( \tau(M_7, \Phi) \) equal to 1.0, we have

\[
\frac{1}{3} (51 - 8\omega) \times \Phi^{2.5} \left( \frac{\frac{d}{R_g}}{2\alpha_x + 3} \right) k M_7^{-1} \left( \frac{\frac{d}{R_g}}{2\alpha_x + 3} \right)^{-2\alpha_x - 3} = 1
\]

(6)

For a source with available data in the X-ray and \( \gamma \)-ray bands, the masses of the central black holes, \( M_7 \), the Doppler factor, \( \delta \), the distance (height), \( d \), and the propagation angle with respect to the axis of the accretion disk, \( \Phi \), can be derived from Eqs. (3), (4), (5) and (6), where \( R_{ms} = 6 R_g, R_0 = 30 R_g, E_\gamma = 1\text{GeV} \) and \( \omega = 3 \) (a two-temperature disk) are used.
3.2. Results

Since we do not know the intrinsic $\gamma$-ray luminosity, we assume it is close to the Eddington luminosity, say $\lambda L_{Edd}$. Using the available X-ray and $\gamma$-ray data (see Table 1), we estimate the four parameters ($M_7$, $\Phi$, $\delta$, $d$) and find that the derived values of the four parameters are not sensitive to the value of $\lambda$. The results are shown in Table 2.

In Table 1 Col. 1, gives the name; Col. 2, the redshift; Col. 3, the X-ray flux density in units of $\mu$ Jy, Col. 4, the X-ray spectral index, $\alpha_X$. The averaged value of $<\alpha_X>$ = 0.67 (Comastri et al. 1997) is adopted for PKS 1622-297, and we take $\alpha_{OX} = 1.31$ for $\alpha_X$ for BL Lacertae, as did Ghisellini et al. (1998). Col. 5, the flux $F(>100\text{MeV})$ in units of $10^{-6}$ photon cm$^{-2}$ s$^{-1}$, $\sigma$ is the uncertainty; Col. 6, $\gamma$-ray spectral index, $\alpha_\gamma = 1.0$ is adopted for 0537-441 (see Fan et al. 1998c); Col. 7, the doubling time scale in units of hours; Col. 8, the observed isotropic luminosity in units of $10^{48}\text{erg s}^{-1}$. In Table 2, Col. 1 gives the name, Col. 2 the Doppler factor ($\lambda = 1.0$); Col. 3 the Doppler factor ($\lambda = 0.1$); Col. 4, the central black hole mass in units of $10^7M_\odot$ ($\lambda = 1.0$); Col. 5, the central black hole mass ($\lambda = 0.1$); Col. 6, the propagation angle, $\Phi$ in the units of degree($) ($\lambda = 1.0$); Col. 7, the propagation angle ($\lambda = 0.1$), Col. 8, the distance (height), $d_Rg$, where the $\gamma$-rays are created ($\lambda = 1.0$); Col. 9, the distance (height) ($\lambda = 0.1$); Col. 10, the central black hole mass estimated by the method of Dermer & Gehrels (1995), in units of $10^7M_\odot$; Col. 11, the mass estimated directly from the Eddington limit in units of $10^{10}M_\odot$ (cf. section 4.1).

For comparison, we also considered a uniformly-bright disk for 3C279. The derived basic parameters are listed in Table 3, in which Col. 1 gives the flare time, Col. 2 the parameter $\lambda$, Col. 3 the Doppler factor, Col. 4 the masses of the central black hole, 5 the propagation angle, and Col. 6 the distance, $d_Rg$. There is not much difference between the parameters obtained for a two-temperature disk ($\omega = 3$) and those for a uniformly-bright disk ($\omega = 0$) although the masses are a little smaller, the propagation angles are wider, and the distance is farther as compared with those obtained for $\omega = 3$.

In the paper by Becker & Kafatos (1995), basic parameters are also determined for the 1991 flare of 3C279. Using $M = 10^9M_\odot$, $R_0/R_g = 30$, $R_{\text{ms}}/R_g = 6$, and $\omega = 3$, Becker & Kafatos obtained a variability time scale of $t_{\gamma\gamma} = 2.57$ days, $d/R_g = 45.2$ and $\Delta\phi = 9.5^\circ$. But from our definition of the variability time scale, $t_{\gamma\gamma}$ should be 0.5 days for the 1991 flare. In this sense, our results should correspond to the first case presented in the first line in Table 1 (Becker & Kafatos 1995). From the 3C279 1991 flare, the masses obtained here are $(0.667 \sim 1.062) \times 10^9M_\odot$, which are consistent with the value of $10^9M_\odot$ (Becker & Kafatos 1995), but our results for the distance, $d/R_g$, and the propagation angle, $\Phi$, are larger than theirs. These differences do not suggest that our results are not consistent with theirs because the methods used are different.
Table 1. Observation data for γ-ray loud blazars

| Name       | z     | $f_{1KeV}$ | Ref | $\alpha_X$ | Ref | $F(\sigma)$ | Ref | $\alpha_\gamma$ | Ref | $\Delta T_D$ | Ref | $L_{iso}^{48}$ |
|------------|-------|------------|-----|------------|-----|-------------|-----|-----------------|-----|-------------|-----|--------------|
| 0208-512   | 1.003 | 0.61       | C97 | 1.04(0.04) | C97 | 9.1(0.4)    | B93 | 0.69           | B93 | 5.6         | S96 | 2.0          |
| 0528+134   | 2.07  | 0.65       | C97 | 0.54(0.29) | C97 | 3.08(0.35)  | W97 | 1.21           | W97 | 24.          | W97 | 18.4         |
| 0537-441   | 0.894 | 0.81       | C97 | 1.16(0.09) | C97 | 2.0(0.4)    | H96 | 1.0            | F98 | 16.          | H96 | 3.01         |
| 1253-055   | 0.537 | 2.43       | H96b| 0.68       | H96b| 2.8(0.4)    | K93 | 1.02           | K93 | 12.          | K93 | 1.34         |
| 1253-055   | 0.538 | 2.0        | L98 | 0.78       | L98 | 11.(1.)     | M96 | 0.97           | M96 | 6.          | W98 | 5.75         |
| 1622-297   | 0.815 | 0.08       | M97 | 0.67       | C97 | 17.(3.)     | M97 | 0.87           | M97 | 4.85        | M97 | 26.9         |
| 1633+382   | 1.814 | 0.42       | C97 | 0.53(0.08) | C97 | 0.96(0.08)  | M93 | 0.86           | M93 | 16.          | M93 | 9.72         |
| 2200+420   | 0.07  | 1.84       | P96 | 1.31       | P96 | 1.71(0.42)  | B97 | 0.68           | B97 | 3.2         | B97 | 0.019        |

Notes to Table 1

Col. 1, gives the name; Col. 2, the redshift; Col. 3, the X-ray flux density in units of $\mu$Jy; Col. 4, reference for Col. 3; Col. 5, the X-ray spectral index, $\alpha_X$. The averaged value of $<\alpha_X>$ = 0.67 (Comastri et al. 1997) is adopted to PKS 1622-297, and $\alpha_{OX}$ = 1.31 is used for BL Lacertae as did Ghisellini et al. (1998); Col. 6, references for Col. 5; Col. 7, the flux $F(\sigma)$ greater than 100 MeV in units of $10^{-6}$ photon cm$^{-2}$ s$^{-1}$, $\sigma$ is the uncertainty; Col. 8, the reference for Col. 7; Col. 9, γ-ray spectral index, $\alpha_\gamma$ = 1.0 is adopted for 0537-441 (see Fan et al. 1998c); Col. 10, reference for Col. 9; Col. 11, the doubling time scale in units of hours; Col. 12, references for Col. 11; Col. 13, the observed isotropic luminosity in units of $10^{48}$ ergs s$^{-1}$ calculated in the present paper. B93: Bertsch et al. 1993; B97: Bloom et al. 1997; C97: Comastri et al. 1997; F98: Fan et al. 1998c; H96: Hartman 1996; H96b: Hartman et al. 1996; K93: Kniffen et al. 1993; L98: Lawson & McHardy 1998; M93: Mattox et al. 1993; M96: McHardy 1996; M97: Mattox et al. 1997; P96: Perlman et al. 1996; S96: Stacy et al. 1996; W97: Wagner et al. 1997; W98: Wehrle et al. 1998.

4. Discussion

Rapid γ-ray variability has been detected in 10 γ-ray loud objects including 2 known TeV emitting objects, but we only discuss the absorption effect of X-rays on the GeV γ-ray emissions. Neither TeV emitting object is included in the present paper since there is no evidence of GeV γ-ray variability for them even though rapid TeV variabilities have been detected from them (Gaidos et al. 1996; Quinn et al. 1996). PKS 1406-074 shows rapid GeV γ-ray variability (Wagner et al. 1995), but there is no X-ray data available in the literature. So, only seven objects are considered here. It is important to note that our model results are independent of the emission mechanisms of γ-rays although they are dependent on the X-ray emission mechanism from the accretion disk. For X-ray emission, many authors have discussed the processes. Apart from the above mentioned process, the synchrotron/inverse Compton process (see Konigl 1981; Kubo et al. 1998; Padovani et al. 1997; Pian et al. 1998; Makino et al. 1997; Sambruna et al. 1996; Urry et al. 1986) and the annihilation of $e^\pm$ (Robson 1986) are also important mechanisms of X-ray production.
Table 2. Determined Results for 7 γ-ray loud Blazars

| Name     | δ (λ = 1.0) | δ (λ = 0.1) | $M_7$ (λ = 1.0) | $M_7$ (λ = 0.1) | Φ (λ = 1.0) | Φ (λ = 0.1) | $\Phi_{\text{gy}}$ (λ = 1.0) | $\Phi_{\text{gy}}$ (λ = 0.1) | $d_{\text{gy}}$ (λ = 1.0) | $d_{\text{gy}}$ (λ = 0.1) | $M_{KN}$ | $M_{10}$ |
|-----------|-------------|-------------|-----------------|-----------------|-------------|-------------|-----------------|-----------------|----------------|----------------|----------|---------|
| 0208-512  | 1.00        | 1.33        | 82.94           | 131.5           | 25.5        | 21.7        | 71.9            | 61.3            | 14.4          | 1.59          |          |         |
| 0528+134  | 3.72        | 5.33        | 5.09            | 8.21            | 43.0        | 39.2        | 411             | 366             | 22.98         | 14.6          |          |         |
| 0537-441  | 1.83        | 2.51        | 12.56           | 19.02           | 38.1        | 35.3        | 86.7            | 80.3            | 3.48          | 2.39          |          |         |
| 1253-055  | 1.43        | 2.03        | 6.67            | 10.62           | 21.3        | 19.2        | 121.0           | 107.0           | 1.92          | 1.06          |          |         |
| 1253-055  | 2.11        | 3.00        | 5.40            | 8.47            | 23.8        | 21.7        | 110.0           | 99.6            | 7.53          | 4.56          |          |         |
| 1622-297  | 2.42        | 3.45        | 5.71            | 9.06            | 14.8        | 13.5        | 81.0            | 73.0            | 25.0          | 21.35         |          |         |
| 1633+382  | 3.22        | 4.60        | 3.81            | 6.15            | 36.8        | 33.5        | 347.0           | 309.0           | 5.74          | 7.71          |          |         |
| 2200+420  | 0.57        | 0.79        | 4.45            | 6.63            | 14.1        | 13.0        | 27.7            | 25.8            | 0.02          | 0.015         |          |         |

Notes to Table 2:
Col. 1 gives the name, Col. 2 Doppler factor (λ = 1.0); Col. 3 Doppler factor (λ = 0.1); Col. 4, the central black hole mass in units of $10^7 M_\odot$ (λ = 1.0); Col. 5, the central black hole mass (λ = 0.1); Col. 6, propagation angle, Φ in the units of degree (λ = 1.0), Col. 7, propagation angle (λ = 0.1), Col. 8, the distance (height), $d_{\text{gy}}$, where the γ-rays are created (λ = 1.0); Col. 9, the distance (height) (λ = 0.1); Col. 10, the central black hole mass estimated from the method of Dermer & Gehrels (1995), in units of $10^7 M_\odot$; Col. 11, the mass estimated directly from Eddington limit in units of $10^{10} M_\odot$ (cf. section 4.1).

Table 3. Determined Results for 3C 279 (1253-055) for a uniformly-bright disk

| Flare | λ | δ | $M_7$ | Φ | $\Phi_{\text{gy}}$ |
|-------|---|---|-------|---|-----------------|
|       | (1) | (2) | (3) | (4) | (5) |
| 1991  | 0.1 | 2.1 | 7.64 | 20. | 154. |
| 1991  | 1.0 | 1.5 | 4.93 | 21.8 | 171.7 |
| 1996  | 0.1 | 3.14 | 6.19 | 23.15 | 143 |
| 1996  | 1.0 | 2.23 | 4.0 | 25.2 | 157 |

Notes to Table 3:
Col. 1 gives the flare time, Col. 2 the parameter of λ, Col. 3 the Doppler factor, Col. 4 the masses of the central black holes, Col. 5 the propagation angle, and Col. 6 the distance, $\Phi_{\text{gy}}$.

In addition, fluorescent and thermal bremsstrahlung emissions are important for X-rays (Rybicki & Lightman 1979; Makino 1999).

4.1. Mass

Assuming that the observed γ-ray luminosity is isotropic without a beaming effect and equals the Eddington-luminosity, one can estimate the central black hole mass,

$$M_{10} \geq \frac{L_T}{1.26 \times 10^{48} \text{ergs}^{-1}},$$

(7)
where $L_T$ is the bolometric luminosity for emission in the Thomson region and $M_{10}$ is the central black hole mass in units of $10^{10}M_\odot$. The derived masses are as high as $10^{11}M_\odot$ for some $\gamma$-ray loud blazars, PKS 0528+134 and PKS1622-297 for instance (see Col. 11 in Table 2).

Dermer & Gehrels (1995) considered the Klein-Nishina effect without the beaming effect and obtained an expression for the black hole mass,

$$M_{8}^{KN} \geq \frac{3\pi d^2_l (m_e c^2)}{2 \times 1.26 \times 10^{46} \text{ergs}^{-1}} \frac{F(\varepsilon_l, \varepsilon_u)}{1 + z} \ln[2\varepsilon_l(1 + z)]$$

(8)

where $F(\varepsilon_l, \varepsilon_u)$ is the integrated photon flux in units of $10^{-6}$ photon cm$^{-2}$ s$^{-1}$ between photon energies $\varepsilon_l$ and $\varepsilon_u$ (in units of 0.511MeV). For the objects considered here, $M_{7}^{KN}$ is obtained and shown in Col. 10 in table 2. Table 2 shows that there are some differences between the masses obtained from our method and those estimated from the method of Dermer & Gehrels (1995) for PKS 0208-512, PKS0528+134, PKS 1622-297 and BL Lacertae. The reason is that they considered the Klein-Nishina effect but not the beaming effect nor the actual solid angle. In addition, other reasons can also cause the difference. For PKS1622-297, if we adopt a flux density of $2.45 \times 10^{-6}$ photon cm$^{-2}$ s$^{-1}$ instead of the peak value as did Muhkerjee et al. (1997) and Fan et al. (1998c), then the isotropic luminosity is $3.87 \times 10^{48}$ erg s$^{-1}$ suggesting a mass of $6.1 M_\gamma$ from our calculation and a mass of $3.61 M_\gamma$ from the method of Dermer & Gehrels (1995). Both masses are quite similar in this case. For 3C279, our results show that the estimated central black hole masses, $6.67 M_\gamma$ and $5.40 M_\gamma$ if $\lambda = 1$ is adopted, and $10.62 M_\gamma$ and $8.47 M_\gamma$ if $\lambda = 0.1$ is adopted, for the 1991 and 1996 flares respectively, are almost the same. The difference in masses obtained from our method when $\lambda$ changes by an order of one magnitude is only a factor of $\sim 1.5$, suggesting that the estimated mass is not sensitive to the choice of $\lambda$. The masses obtained here are in a range of $(\sim 4 - 83) \times 10^7 M_\odot$ ($\lambda = 1.0$) or $(\sim 6 - 130) \times 10^7 M_\odot$ ($\lambda = 0.1$).

For 3C279, to fit the multiwavelength energy spectrum corresponding to the 1991 $\gamma$-ray flare, Hartman et al. (1996) used an accreting black hole of $10^8 M_\odot$ while Becker & Kafatos (1995) also obtained $M = 10^8 - 10^9 M_\odot$; our result of $M = (6.67 - 10.62) \times 10^7 M_\odot$ is consistent with theirs. For PKS 0537-441, Romero et al. (2000) obtained a mass of $8 \times 10^7 M_\odot$ for the central black hole, our result of $12.56 \times 10^7 M_\odot$ ($\lambda = 1.0$) does not conflict with theirs, the slight difference is from that fact that we used an outer radius of a two-temperature disk $R_0 = 30 R_g$ while they adopted $R_0 = 100 R_g$.

### 4.2. Beaming factors

To explain the extremely high and violently variable luminosity of AGNs, a beaming model has been proposed. To let the optical depth ($\tau_{\gamma\gamma}$) be less than unity, a Doppler
factor in the γ-ray region has been obtained for some objects by other authors. \( \delta \geq 7.6 \) for Q1633+382 (Mattox et al., 1993); \( \delta \geq 6.3 \sim 8.5 \) for the 3C279 1996 flare (Wehrle et al., 1998) and \( \delta \geq 3.9 \) for the 3C279 1991 flare (Mattox et al. 1993), \( \delta \sim 5 \) is also obtained by Henri et al. (1993); \( \delta \geq 6.6 \sim 8.1 \) for PKS 1622-297 (Mattox et al. 1997). The Doppler factors obtained here are smaller than those obtained by others. The reason is that we believe that the γ-rays are from a solid angle \( \Omega = 2\pi(1 - \cos \Phi) \) while others assume that the γ-rays are isotropic, i.e., \( \Omega = 4\pi \). Therefore, different assumptions result in different Doppler factors. The Doppler factor derived under the former assumption is \((\frac{1}{2(\cos \Phi)})^{1/(4+\alpha)}\) times the factor derived under the latter assumption.

4.3. Propagation angle, \( \Phi \)

Generally, the observed luminosity is calculated assuming the emissions are isotropic. From the arguments (Becker & Kafatos, 1995 and reference therein), we know that only the γ-rays within the propagation angle are visible, i.e., \( \tau_{\gamma\gamma} \leq 1.0 \). Our calculations show that the \( \Phi \)'s are in the range of 14° to 43° (\( \lambda = 1.0 \)) or 13° to 39° (\( \lambda = 0.1 \)); the average value is \( < \Phi > = 27°.5 \) (\( \lambda = 1.0 \)) or \( < \Phi > = 24°.6 \) (\( \lambda = 0.1 \)). These values of \( \Phi \) are consistent with the X-ray cone (\( \Phi = 15° \sim 40° \)) of BL Lac objects (Maraschi & Rovetti, 1994).

4.4. Distance, \( \frac{d}{R_g} \)

The variability time scale places some constraints on the size of the emission region. By fitting the energy spectrum of 3C279, Hartman et al. (1996) obtained that the γ-rays in 3C279 were from 100\( R_g \). From the time scale and the estimated black hole mass of Mkn421, Xie et al. (1998b) found that the γ-rays in Mkn421 are from 205\( R_g \). Recently, Celotti & Ghisellini (1998) argued that the γ-rays are from a distance of hundreds of Schwarzschild radii. From our method, the distance is from 25\( \sim 28 R_g \) for BL Lacertae to 366\( \sim 411 R_g \) for PKS0528+134. The average distance is 160\( R_g \) (\( \lambda = 1.0 \)) or 143 \( R_g \) (\( \lambda = 0.1 \)) for the seven objects.

For the X-ray emission regions, the simultaneous variability of X-rays and γ-rays during 1996 flare of 3C 279 suggests that the time scales can be taken the same for both bands. Then the size of the X-ray emission region should not be much smaller than the size of γ-ray emission region. For BL Lacertae, there is an X-ray variation up to 30% on a time scale of hours (Kawai et al. 1991) suggesting that the size of the X-ray emission region is 130\( R_g \) if we take \( M = (4.45 \sim 6.63) \times 10^7 M_\odot \), which is much larger than the size of the γ-ray emission region. For two other objects (PKS 0528+134 and PKS 0537-441), variations in the X-ray region are detected but there is no variability timescale of days reported in the literature (see Ghisellini et al. 1999; Treves et al. 1993) except for a
variation of 50% in 2 weeks for 0528+134 (Ghisellini et al. 1999). For the remaining three objects, there are no X-ray timescales. Anyway, the available X-ray variability timescales of the three objects (3C279, BL Lacerte, and 0528+134) imply that the size of the X-ray emission region is not much smaller than the size of the $\gamma$-ray emission region.

### 4.5. Summary

In this paper, the optical depth of a $\gamma$-ray travelling in the field of a two-temperature disk including the beaming effect have been used to determine the central mass, $M$, Doppler factor, $\delta$, propagation angle, $\Phi$, and the distance, $d$ for seven $\gamma$-ray loud blazars with available short $\gamma$-ray timescales. The masses obtained are compared with those obtained by the method of Dermer & Gehrels (1995). The distance (height) on average is $140 \sim 160 \, R_g$ from the center for the seven objects, the propagation angle $\Phi$ is, on average, $24^\circ \sim 27^\circ$. For 3C279, a uniformly-bright disk is also considered to determine the basic parameters, the results are compared with those by Becker & Kafatos (1995), the masses obtained from the two flares being almost the same.

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