Research Article

An Iterative Algorithm for Solving $n$-Order Fractional Differential Equation with Mixed Integral and Multipoint Boundary Conditions

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In this paper, we consider the iterative algorithm for a boundary value problem of $n$-order fractional differential equation with mixed integral and multipoint boundary conditions. Using an iterative technique, we derive an existence result of the uniqueness of the positive solution, then construct the iterative scheme to approximate the positive solution of the equation, and further establish some numerical results on the estimation of the convergence rate and the approximation error.

1. Introduction

In this paper, we focus on the iterative algorithm for the following $n$-order fractional equation involving mixed integral and multipoint boundary conditions:

$$
-D_0^\alpha x(t) = f(t, x(t), x(t)), \quad 0 < t < 1,
$$

$$
x^{(i)}(0) = 0, \quad i = 0, 1, 2, \ldots, n-2,
$$

$$
x(1) = \sum_{i=1}^{m-2} \beta_i \int_0^{\eta_i} x(s) ds + \sum_{i=1}^{m-2} \gamma_i x(\eta_i),
$$

where $D_0^\alpha$ is the standard fractional derivative of order $\alpha$ satisfying $n-1 < \alpha \leq n$ with $m, n \geq 3$ and $m, n \in N^+$, $0 < \eta_1 < \eta_2 < \cdots < \eta_{m-2} < 1$, $\beta_i, \gamma_i > 0$, $1 \leq i \leq m-2$, and $f(t, u, v)$ may be singular at $v=0$ and $t=0, 1$.

Based on the wide range applications of calculus, in recent years, the study for various differential equations has become a frontier issue of nonlinear field and many mathematical methods and techniques, such as iterative techniques [1–17], dual approach and perturbed techniques [18–23], fixed-point theorems [24–50], lower-upper solution method [51–53], variational method [54–68], numerical methods and stability analysis [69–81], which were developed by many researchers to handle various nonlinear problems. In particular, in describing and modeling viscoelasticity and nonlocal problems in complex analysis, environmental issue, chemistry physics, and statistical physics, a large amount of work [18, 30, 64, 82–111] have shown that fractional differential equations possess greater advantage than classical integer differential equations. In recent work [95], Salam, by using the Hahn-Banach fixed point theorem, studied the following multipoint boundary value problem:

$$
-D_0^\alpha x(t) = q(t) f(t, x(t)), \quad 0 < t < 1, n-1 < \alpha \leq n, n \geq 3,
$$

$$
x^{(i)}(0) = 0, \quad 0 \leq k \leq n-2,
$$

$$
x(1) = \sum_{i=1}^{m-2} \zeta_i x(\eta_i).
$$

The weakly continuous solution for the above nonlinear boundary value problem of fractional type was derived. And then, Xie et al. [96] studied the following nonlinear fractional differential equation with a three-point nonlinear boundary condition:
By using the method of upper and lower solutions as well as the monotone iterative technique, the results about the existence of extremal solutions were obtained, where the iterative process can start from the fixed upper and lower solutions.

Among various techniques of dealing with differential equations, upper and lower solutions’ method and fixed-point methods have been verified to be the most efficient approaches. However, the efficiency of those methods depends essentially on the monotonicity and compactness of the operator. So, how to overcome the requirement of the monotonicity and compactness is a challenge, since such qualities are not naturally available and difficult to prove which lead to the complexity to solve some BVP, especially for the fractional nonlinear boundary value problems.

Different from [96], in this paper, we develop the new iterative algorithm to overcome the requirement of the compactness for the nonlinear operator. Our work has three major features. Firstly, equation (1) possesses a more general nonlinear term which has two space variables and the boundary condition is a mixed integral and multipoint boundary condition. Secondly, the nonlinear term can be singular in the time variables and the second space variables. In the end, our results are more refined, that is, we not only construct a new iterative process which can perform from any initial value but also obtain the uniform convergence of the iterative sequences; at the same time, the estimation of the convergence rate and the approximation error are also given, which imply that more strong results are established under the relatively weaker condition than that of [96].

The paper is organized as follows. In Section 2, we recall some definitions and lemmas. In Section 3, the unique of positive solutions to BVP (1) are obtained. Finally, in Section 4, an illustrative example is also presented.

2. Preliminaries

In this section, we first list some notations and recall related definitions and lemmas to be used in our proofs later.

**Definition 1** (see [5]). Let \( p > 0 \), the Riemann–Liouville standard fractional integral derivative of order \( p > 0 \) of a function \( f : (0, \infty) \to \mathbb{R} \) is given by

\[
D^n_0 f(t) = \frac{1}{\Gamma(p)} \int_0^t f(t) \left( t-s \right)^{-p} \, ds,
\]

where \( n = [p] + 1 \), \([p]\) denotes the integer part of the real number \( p \).

**Lemma 1** (see [5]). Suppose that \( n - 1 < \alpha \leq n \) with \( n \geq 3 \) and \( h \in L^1[0,1] \); then, the boundary value problem

\[
-D^n_0 x(t) = h(t), \quad 0 < t < 1, \\
x^{(1)}(0) = 0, \quad i = 0, 1, 2, \ldots, n - 2, \\
x(1) = \sum_{i=1}^{m-2} \beta_i \int_0^{\eta_i} x(s) \, ds + \sum_{i=1}^{m-2} \gamma_i x(\eta_i),
\]

is given by

\[
x(t) = \int_0^1 G(t, s) h(s) \, ds + \frac{t^{\alpha-1}}{\xi} \sum_{i=1}^{m-2} \int_0^1 G(t, s) \beta_i \int_0^{\eta_i} x(s) \, ds + \sum_{i=1}^{m-2} \gamma_i x(\eta_i),
\]

where \( \xi = 1 - (1/\alpha) \sum_{i=1}^{m-2} \beta_i \eta_i^{\alpha-1} - \sum_{i=1}^{m-2} \gamma_i \eta_i^{\alpha-1} > 0 \), and

\[
G(t, s) = \frac{1}{\Gamma(\alpha)} \begin{cases} 
\frac{t^{\alpha-1} (1-s)^{\alpha-1} - (t-s)^{\alpha-1}}{(1-s)^{\alpha-1}}, & 0 \leq s \leq t \leq 1, \\
\frac{t^{\alpha-1} (1-s)^{\alpha-1}}{(1-s)^{\alpha-1}}, & 0 \leq t \leq s \leq 1,
\end{cases}
\]

\[
H(t, s) = \frac{1}{\Gamma(\alpha + 1)} \begin{cases} 
\frac{t^{\alpha-1} (1-s)^{\alpha-1} - (t-s)^{\alpha-1}}{(1-s)^{\alpha-1}}, & 0 \leq s \leq t \leq 1, \\
\frac{t^{\alpha-1} (1-s)^{\alpha-1}}{(1-s)^{\alpha-1}}, & 0 \leq t \leq s \leq 1.
\end{cases}
\]

**Lemma 2** (see [5]). For all \( t, s \in [0,1] \), the functions \( G(t, s) \) and \( H(t, s) \) in Lemma 1 satisfy the following properties:

1. \( G(t, s) \) and \( H(t, s) \) are continuous and nonnegative
2. \((a-1)/\Gamma(a)\alpha^{\alpha-1}(1-t)(1-s)^{a-1} \leq G(t, s) \leq (1/\Gamma(a))\alpha^{\alpha-1}(1-s)^{\alpha-1}
3. \((a-1)/\Gamma(a+1)\alpha^{\alpha-1}(1-t)(1-s)^{a-1} \leq H(t, s) \leq (1/\Gamma(a+1))\alpha^{\alpha-1}(1-s)^{\alpha-1}

In this paper, we will work in the space \( E = C[0,1] \). Define a set \( P \) in \( E \) and an operator \( T : E \times E \to E \) as follows:

\[
P = \left\{ x \in E \mid \text{there exists a positive constant} \ l_x \in (0,1), \text{ such that} \right\},
\]

\[
T(x, y)(t) = \int_0^1 G(t, s) f(s, x(s), y(s)) \, ds \\
+ \frac{t^{\alpha-1}}{\xi} \sum_{i=1}^{m-2} \int_0^{\eta_i} \left[ \beta_i \eta_i^{\alpha-1} + \gamma_i \eta_i^{\alpha-1} \right] f(s, x(s), y(s)) \, ds.
\]
Obviously, \((t^{a-1}, t^{-1}) \in P \times P\), so \(P \times P\) is not empty.

3. Main Results

Before claiming our main results, we first introduce the following notations:

(i) \(M = \xi \alpha + (1 + \alpha)(m - 2)\)

(ii) \(N = \sum_{i=1}^{m-2} (\beta_i + \alpha \gamma_i)\eta_i^{-1}(1 - \eta_i)\)

Theorem 1. Assume that

\((H_1)\) \(f(t, u, v) \in C([0, 1] \times [0, +\infty) \times (0, +\infty); (0, +\infty))\) and for \((t, u, v) \in (0, 1) \times [0, +\infty) \times (0, +\infty), f\) is increasing with respect to \(u\), decreasing with respect to \(v\.

\((H_2)\) For \((t, u, v) \in (0, 1) \times [0, +\infty) \times (0, +\infty)\) and \(k \in (0, 1)\), there exists a constant \(\mu \in (0, 1)\) such that \(f(t, ku, k^{-1}v) \geq k^\mu f(t, u, v)\).

\((H_3)\) \(0 < \int_0^1 (1 - s)^{a-2}f(s, s^{a-1}, s^{a-1})ds < \infty\)

Then, the BVP (1) has unique positive solution \(x^*(t) \in P\), and there exists a constant \(0 < l < 1\) satisfying

\[
T(x, y)(t) = \int_0^1 G(t, s)f(s, x(s), y(s))ds + \frac{t^{-1}}{\Gamma(\alpha)} \sum_{i=1}^{m-2} \left[ \frac{\beta_i H(\eta_i, s) + \gamma_i G(\eta_i, s)}{\Gamma(\alpha + 1) + \xi \alpha} \right] J^\alpha \Bigg[ \int_0^1 (1 - s)^a f(s, (t_*)^{-1} s^{-1}, (t_*)^{-1} s^{-1})ds \Bigg] ds
\]

\[
\leq \int_0^1 \frac{(t_*)^{-\mu} \xi \alpha + (1 + \alpha)(m - 2)}{\xi \alpha \Gamma(\alpha)} \int_0^1 (1 - s)^a f(s, s^{a-1}, s^{a-1})ds < \infty.
\]

Proof. Firstly, it is easy to know that \(x^*\) is the solution of the BVP (1) if and only if \(x^*\) satisfies \(T(x^*, x^*) = x^*\).

Next, it follows from \((H_1)\) that the operator \(T: P \times P \to P\) is nondecreasing with respect to \(x\) and nonincreasing with respect to \(y\); thus, by \((H_1), (H_2),\) and \((H_3),\) for any \((x, y) \in P \times P\) and \(t \in (0, 1),\) there exist two constants \(0 < l_x < 1, 0 < l_y < 1\) such that

\[
l_x t^{-1} \leq x(t) \leq (l_x)^{-1} t^{-1},
\]

\[
l_y t^{-1} \leq y(t) \leq (l_y)^{-1} t^{-1}.
\]

Denote \(l_x = \min\{l_x, l_y\};\) then, we have

\[
l_x t^{-1} \leq x(t) \leq (l_x)^{-1} t^{-1},
\]

\[
l_y t^{-1} \leq y(t) \leq (l_y)^{-1} t^{-1}.
\]

Consequently,

\[
l x(t) \leq x^*(t) \leq l^{-1} x(t), \quad t \in [0, 1].
\]
Let

\[
I_{Tx} = \min \left\{ \frac{1}{2} \frac{(\alpha - 1)N(I^*_x)^{\alpha}}{\xi \alpha! (\alpha)} \int_0^1 s (1 - s)^{\alpha - 1} f(s, s^{\alpha - 1}, s^{\alpha - 1}) ds, \right. \\
\left. \frac{M(I^*_x)^{\alpha - \mu}}{\xi \alpha! (\alpha)} \int_0^1 (1 - s)^{\alpha - 2} f(s, s^{\alpha - 1}, s^{\alpha - 1}) ds \right\}^{-1}.
\]  \quad (14)

Then, there exists a constant \(0 < I_{Tx} < 1\) such that

\[
l_{Tx}^\alpha - 1 \leq (T(x, y))(t) \leq (I_{Tx})^{-1} l_{Tx}^\alpha, \quad t \in (0, 1),
\]  \quad (15)

which implies that the operator \(T: P \times P \to P\) is well defined. According to the Arzela-Ascoli theorem, it is easy to know that \(T: P \times P \to P\) is completely continuous.

Now, take \(h(t) = t^{\alpha - 1}\), then \((h, h) \in P \times P\), it follows from (12) and (13) that \(T(h, h) \in P\). Thus, by the definition of \(P\), there exists a constant \(0 < l_{Th} < 1\) such that

\[
l_{Th}^\alpha - 1 \leq T(h, h)(t) \leq (I_{Th})^{-1} l_{Th}^\alpha.
\]  \quad (16)

Take

\[
0 < \lambda \leq l_{Th}^{(1/1-\mu)}.
\]  \quad (17)

and let

\[
x_0 = \lambda h(t),
\]

\[
y_0 = \lambda^{-1} h(t).
\]  \quad (18)

Now, construct the following iterative sequence:

\[
x_n = T(x_{n-1}, y_{n-1}),
y_n = T(y_{n-1}, x_{n-1}), \quad n = 1, 2, \ldots
\]  \quad (19)

We assert

\[
x_0 \leq x_1 \leq \cdots \leq x_n \leq \cdots \leq y_1 \leq \cdots \leq y_n \leq y_0.
\]  \quad (20)

In fact, it follows from \(0 < \lambda < 1\) and (18) that \(x_0, y_0 \in P\) and \(x_0 \leq y_0\). Moreover,

\[
x_1 = T(x_0, y_0)(t) = \int_0^1 G(t, s) f(s, \lambda h(t), \lambda^{-1} h(t)) ds \\
+ \frac{t^\alpha - 1}{\xi} \sum_{i=1}^{m-2} \int_0^1 \left[ \beta_i H(\eta_i, s) + \gamma_i G(\eta_i, s) \right] f(s, \lambda h(t), \lambda^{-1} h(t)) ds \\
\geq \lambda^\mu \int_0^1 G(t, s) f(s, h(t), h(t)) ds \\
+ \frac{\lambda^\mu l_{Th}^\alpha - 1}{\xi} \sum_{i=1}^{m-2} \int_0^1 \left[ \beta_i H(\eta_i, s) + \gamma_i G(\eta_i, s) \right] f(s, h(t), h(t)) ds \\
= \lambda^\mu T(h, h)(t) \geq \lambda^\mu l_{Th} h(t) \geq \lambda^\mu \lambda^{-1-\mu} h(t) = x_0,
\]

\[
y_1 = T(y_0, x_0)(t) = \int_0^1 G(t, s) f(s, \lambda^{-1} h(t), \lambda h(t)) ds \\
+ \frac{t^\alpha - 1}{\xi} \sum_{i=1}^{m-2} \int_0^1 \left[ \beta_i H(\eta_i, s) + \gamma_i G(\eta_i, s) \right] f(s, \lambda^{-1} h(t), \lambda h(t)) ds \\
\leq \lambda^{-\mu} \int_0^1 G(t, s) f(s, h(t), h(t)) ds \\
+ \frac{\lambda^{-\mu} l_{Th}^\alpha - 1}{\xi} \sum_{i=1}^{m-2} \int_0^1 \left[ \beta_i H(\eta_i, s) + \gamma_i G(\eta_i, s) \right] f(s, h(t), h(t)) ds \\
= \lambda^{-\mu} T(h, h)(t) \leq \lambda^{-\mu} (I_{Th})^{-1} h(t) \leq \lambda^{-\mu} \lambda^\mu h(t) = y_0.
\]  \quad (21)
On the other hand, it follows from $x_0 \leq y_0$ and the fact of $T$ being nondecreasing with respect to second variable and nonincreasing with respect to third variable that $x_1 \leq y_1$. Thus, according to the above fact, we have (20) holds.

Notice that, for any nature number $n$,

$$x_n = T^n(x_{n-1}, y_{n-1}) = T^n(y_0, x_0) = T^n(\lambda h(t), \lambda^{-1} h(t)) = T^n(\lambda^{-1} h(t), \lambda^{-2} \lambda h(t)) \geq (\lambda^2)^n T^n(\lambda^{-1} h(t), \lambda h(t)) = c^{n} y_n,$$

where $c = \lambda^2$. So, for any nature numbers $n$ and $n^*$, we have

$$0 \leq x_{n+n^*} - x_n \leq y_{n+n^*} - y_n \leq (1 - c^{n^*}) y_n \leq (1 - c^n) \lambda^{-1} h(t) \to 0, \quad n \to + \infty,$$

which implies that there exists $x^* \in P$ such that

$$x_n(t) \to x^*(t), \quad t \in (0, 1).$$

uniformly on $(0, 1)$. By the same method, we can also prove that

$$y_n(t) \to y^*(t), \quad t \in (0, 1).$$

uniformly on $(0, 1)$. In view of the continuous of $T$, take the limits in $x_n = T(x_{n}, y_{n}),$ we have $x^* = T^*(x^*, y^*)$. So, $x^*$ is a positive solution of BVP (1). Since $x^* \in P$, for any $t \in (0, 1)$, there exists a constant $l \in (0, 1)$ such that

$$lt^{n-1} \leq x^*(t) \leq l^{-1} t^{n-1} \quad (26)$$

holds.

Finally, we show that the uniqueness of the positive solution. Let $y^*(t)$ be another positive solution of BVP (1); then, for any $t \in (0, 1)$, there exists a constant $m \in (0, 1)$ such that

$$mt^{n-1} \leq y^*(t) \leq m^{-1} t^{n-1}. \quad (27)$$

Taking $\lambda$ defined in (17) be small enough such that $\lambda < m$. So,

$$x_0(t) \leq y^*(t) \leq y_0(t), \quad t \in (0, 1). \quad (28)$$

According to $T(y^*, y^*) = y^*$, using the nondecreasing of $T$, we can show that

$$x_n(t) \leq y^*(t) \leq y_n(t), \quad t \in (0, 1). \quad (29)$$

Taking limits to the both sides of (29), we have $x^* = y^*$. It follows that the solution of BVP (1) is unique. The proof of Theorem 1 is completed.

In the following, we consider the error estimation between unique solution and iterative value.

**Theorem 2.** Let conditions $(H_1)$, $(H_2)$, and $(H_3)$ be satisfied. Then, for any initial value $z_0 \in P$, there exists a sequence $z_n(t)$ that uniformly converges to the unique positive solution $x^*(t)$ with the following error estimation:

$$\max \{|z_n(t) - x^*(t)|, |z^*_n(t) - y^*(t)|\} = 0. \quad (30)$$

where c $\in (0, 1)$ is determined by $z_0$.

**Proof.** By Theorem 1, we know that the positive solution $x^*$ is unique. For any $z_0 \in P$, there exists a constant $l_{z_0} \in (0, 1)$, such that

$$l_{z_0} t^{n-1} \leq z_0(t) \leq l^{-1} t^{n-1}. \quad (31)$$

Similar to Theorem 1, we can take $\lambda < \min \{l_{z_0}, l_{x_0}^{(1/1 - p)}\}$ as a fixed number. It follows that

$$x_0(t) \leq z_0(t) \leq y_0(t), \quad t \in (0, 1). \quad (32)$$

Let $z_n(t) = T(z_{n-1}, z_{n-1}),$ $n = 1, 2, \ldots$; then, by monotonicity of the operator $T$, we have

$$y_1(t) \leq z_1(t) \leq y_1(t), \quad t \in (0, 1). \quad (33)$$

Thus, it follows from mathematical induction that

$$x_n(t) \leq z_n(t) \leq y_n(t), \quad t \in (0, 1). \quad (34)$$

Take limits for the above inequality, we get that $\{z_n(t)\}$ uniformly converges to the unique positive solution $x^*$ of BVP (1). Using (23), we can now derive the error estimation (30), which implies that the error estimation is the same order infinitesimal of $(1 - c^n)$, where $c = \lambda^2$ and determined by $z_0$. This completes the proof of Theorem 2. \qed

### 4. Example

Let us illustrate the main results with an example.

**Example 1.** Let $\alpha = (7/2), \quad m = 4, \eta_1 = (1/3), \quad \eta_2 = (2/3), \quad \beta_1 = (3/2), \quad \beta_2 = 4, \quad \gamma_1 = (5/2),$ and $\gamma_2 = 2$. We consider the following BVP:

$$D_{0+}^{(7/2)} x(t) + a(t) x^{(1/8)} + b(t) y^{-(1/5)} = 0, \quad 0 < t < 1,$$

$$x'(0) = x''(0) = 0,$$

$$x(1) = \frac{3}{2} \int_0^{(1/3)} x(s)ds + 4 \int_0^{(2/3)} x(s)ds + \frac{5}{2} x^{(1/3)}(1) + 2 x^{(2/3)}(1). \quad (35)$$

where $f(t, x, y) = a(t)x^{(1/8)} + b(t)y^{-(1/5)}$. For any $k \in (0, 1)$, take $\mu = (1/4)$, and it is easy to verify that $f(t, kx, k^{-1} y) \geq k^\mu f(t, x, y). \quad (36)$

According to the expression of $f$ and the above inequality, it follows that $(H_1)$ and $(H_2)$ are held. In addition,
0 < \int_0^1 (1 - t)^{(3/2)} f(t, t^{(5/2)}, t^{(5/2)}) dt < \infty. \quad (37)

So, all of the assumptions of Theorem 1 are satisfied. As a result, BVP (35) has a unique positive solution $x^*$ and for any initial value $x_0 \in P$, the successive iterative sequence $\{x_n(t)\}$ is generated by

$$x_n(t) = \int_0^1 G(t, s)f(s, x_{n-1}(s), x_{n-1}(s))ds$$

$$+ \frac{t^{\alpha - 1}}{\xi} \sum_{i=1}^{m-2} \left( \beta_i H(\eta_i, s) + y_i G(\eta_i, s) \right) f(s, x_{n-1}(s), x_{n-1}(s))ds, \quad n = 1, 2, \ldots ,$$

and uniformly converges to the unique positive solution $x^*$ on $(0, 1)$. We also obtain the error estimation

$$\max \left\| x_n(t) - x^*(t) \right\| = o\left(1 - c^{(1/4)r}\right), \quad (39)$$

where $c \in (0, 1)$ is a constant and determined by the initial value $x_0$. Moreover, for any $t \in (0, 1)$, there exists a constant $l \in (0, 1)$ which the iterative process in some previous work such as [31, 33, 35] cannot be performed, which implies our developed iterative technique in this paper can be suitable for a wider range of functions; in particular, even if $f(t, x, y)$ is reduced to only have one space variable $f(t, x)$, our results is more general than those of [96].

5. Result and Discussion

In this paper, we obtain the results of the existence solutions of the $n$-order fractional equation involving mixed integral and multipoint boundary conditions by using a new iterative algorithm. The efficiency of those methods depends essentially on the monotonicity and compactness of the operator. Different from [96], the iterative process does not need to start with the fixed upper and lower solution. We can not only construct a new iterative process which can perform from any initial value but also obtain the uniform convergence of the iterative sequences; at the same time, the estimation of the convergence rate and the approximation error are also given, which imply that the more strong results are established under the relatively weaker condition than that of [96].

Data Availability

The calculating data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors’ Contributions

The study was carried out in collaboration with all authors. All authors read and approved the final manuscript.

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References

[1] X. Zhang, L. Liu, Y. Wu, and Y. Cui, “A sufficient and necessary condition of existence of blow-up radial solutions for a $k$-Hessian equation with a nonlinear operator,” Nonlinear Analysis: Modelling and Control, vol. 25, no. 1, pp. 126–143, 2020.

[2] J. He, X. Zhang, L. Liu, Y. Wu, and Y. Cui, “A singular fractional Kelvin–Voigt model involving a nonlinear operator and their convergence properties,” Boundary Value Problems, vol. 2019, no. 1, p. 112, 2019.

[3] Y. Cui and Y. Zou, “Monotone iterative method for differential systems with coupled integral boundary value problems,” Boundary Value Problems, 2013, no. 1, pp. 245, 2013.

[4] X. Zhang, Y. Wu, and Y. Cui, “Existence and nonexistence of blow-up solutions for a Schrödinger equation involving a nonlinear operator,” Applied Mathematics Letters, vol. 82, pp. 85–91, 2018.

[5] G. Wang, “Twin iterative positive solutions of fractional $q$-difference Schrödinger equations,” Applied Mathematics Letters, vol. 76, pp. 103–109, 2018.

[6] X. Zhang, J. Xu, J. Jiang, Y. Wu, and Y. Cui, “The convergence analysis and uniqueness of blow-up solutions for a Dirichlet problem of the general $k$-Hessian equations,” Applied Mathematics Letters, vol. 102, p. 106124, 2020.

[7] S. Liu, J. Wang, D. Shen, and D. O’Regan, “Iterative learning control for differential inclusions of parabolic type with noninstantaneous impulses,” Applied Mathematics and Computation, vol. 350, pp. 48–59, 2019.

[8] J. Mao, Z. Zhao, and C. Wang, “The exact iterative solution of fractional differential equation with nonlocal boundary value conditions,” Journal of Function Spaces, vol. 2018, Article ID 8346398, 6 pages, 2018.

[9] J. Mao, Z. Zhao, and C. Wang, “The unique positive solutions for singular Hadamard fractional boundary value problem,” Journal of Function Spaces, vol. 2019, Article ID 5923490, 6 pages, 2019.

[10] T. Ren, S. Li, X. Zhang, and L. Liu, “Maximum and minimum solutions for a nonlocal $p$-Laplacian fractional differential system from eco-economical processes,” Boundary Value Problems, vol. 2017, no. 1, p. 118, 2017.

[11] X. Zhang, L. Liu, and Y. Wu, “The uniqueness of positive solution for a fractional order model of turbulent flow in a porous medium,” Applied Mathematics Letters, vol. 37, pp. 26–33, 2014.
[12] F. Wang, L. Liu, and Y. Wu, "Iterative unique positive solutions for a new class of nonlinear singular higher order fractional differential equations with mixed-type boundary value conditions," Journal of Inequalities and Applications, vol. 2019, no. 1, p. 210, 2019.

[13] T. Ren, H. Xiao, Z. Zhou et al., "The iterative scheme and the convergence analysis of unique solution for a singular fractional differential equation from the eco-economic complex systems co-evolution process," Complexity, vol. 2019, Article ID 9278056, 15 pages, 2019.

[14] X. Zhang, L. Liu, and Y. Wu, "The uniqueness of positive solution for a singular fractional differential system involving derivatives," Communications in Nonlinear Science and Numerical Simulation, vol. 18, no. 6, pp. 1400–1409, 2013.

[15] X. Zhang, J. Jiang, L. Liu, and Y. Wu, "Extremal solutions for a class of tempered fractional turbulent flow equations in a porous medium," Mathematical Problems in Engineering, vol. 2020, Article ID 2492193, 11 pages, 2020.

[16] S. Liu, J. Wang, D. Shen, and D. O’Regan, "Iterative learning control for noninstantaneous impulsive fractional-order systems with varying trial lengths," International Journal of Robust and Nonlinear Control, vol. 28, no. 18, pp. 6202–6238, 2018.

[17] X. Zhang, L. Liu, Y. Wu, and Y. Lu, "The iterative solutions of nonlinear fractional differential equations," Applied Mathematics and Computation, vol. 219, no. 9, pp. 4680–4691, 2013.

[18] Z. You, M. Feckan, and J. Wang, "Relative controllability of fractional delay differential equations via delayed perturbation of Mittag-Leffler functions," Journal of Computational and Applied Mathematics, vol. 378, p. 112939, 2020.

[19] X. Zhang, L. Liu, Y. Wu, and Y. Cui, "The existence and nonexistence of entire large solutions for a quasilinear Schrödinger elliptic system by dual approach," Journal of Mathematical Analysis and Applications, vol. 464, no. 2, pp. 1089–1106, 2018.

[20] X. Zhang, L. Liu, and Y. Wu, "The entire large solutions for a quasilinear Schrödinger elliptic equation by the dual approach," Applied Mathematics Letters, vol. 55, pp. 1–9, 2016.

[21] X. Zhang, L. Liu, Y. Wu, and Y. Cui, "Entire blow-up solutions for a quasilinear p-Laplacian Schrödinger equation with a non-square diffusion term," Applied Mathematics Letters, vol. 74, pp. 85–93, 2017.

[22] X. Zhang, L. Liu, Y. Wu, and Y. Cui, "Existence of infinitely solutions for a modified nonlinear Schrodinger equation via dual approach," Electronic Journal of Differential Equations, vol. 2018, no. 147, pp. 1–15, 2018.

[23] X. Zhang, J. Jiang, Y. Wu, and Y. Cui, "The existence and nonexistence of entire large solutions for a quasilinear Schrödinger elliptic system by dual approach," Applied Mathematics Letters, vol. 100, p. 106018, 2020.

[24] Y. Wang, "Existence and multiplicity of positive solutions for a class of singular fractional nonlocal boundary value problems," Boundary Value Problems, vol. 2019, no. 1, p. 92, 2019.

[25] X. Zhang, L. Yu, J. Jiang, and Y. Wu, "Positive solutions for a weakly singular Hadamard-type fractional differential equation with changing-sign nonlinearity," Journal of Function Spaces, vol. 2020, Article ID 5623589, 10 pages, 2020.

[26] J. Liu and Z. Zhao, "Multiple solutions for impulsive problems with non-autonomous perturbations," Applied Mathematics Letters, vol. 64, pp. 143–149, 2017.

[27] J. He, X. Zhang, L. Liu, and Y. Wu, "Existence and nonexistence of radial solutions of the Dirichlet problem for a class of general k-Hessian equations," Nonlinear Analysis: Modelling and Control, vol. 23, no. 4, pp. 475–492, 2018.

[28] X. Zhang, J. Jiang, Y. Wu, and Y. Cui, "Existence and asymptotic properties of solutions for a nonlinear Schrödinger elliptic equation from geophysical fluid flows," Applied Mathematics Letters, vol. 90, pp. 229–237, 2019.

[29] J. Wang and M. Feckan, "Periodic solutions and stability of linear evolution equations with noninstantaneous impulses," Miskolc Mathematical Notes, vol. 20, no. 2, pp. 1299–1313, 2019.

[30] F. Yan, M. Zuo, and X. Hao, "Positive solution for a fractional singular boundary value problem with p-Laplacian operator," Boundary Value Problems, vol. 2018, no. 1, p. 51, 2018.

[31] X. Zhang, L. Liu, Y. Wu, and B. Wiwatanapataphee, "The spectral analysis for a singular fractional differential equation with a signed measure," Applied Mathematics and Computation, vol. 257, pp. 252–263, 2015.

[32] L. Liu, F. Sun, X. Zhang, and Y. Wu, "Bifurcation analysis for a singular differential system with two parameters via topological degree theory," Nonlinear Analysis: Modelling and Control, vol. 22, no. 1, pp. 31–50, 2017.

[33] F. Wang, L. Liu, and Y. Wu, "A numerical algorithm for a class of fractional BVPs with p-Laplacian operator and singularity-the convergence and dependence analysis," Applied Mathematics and Computation, vol. 382, p. 125339, 2020.

[34] B. Zhu, L. Liu, and Y. Wu, "Existence and uniqueness of global mild solutions for a class of nonlinear fractional reaction-diffusion equations with delay," Computers & Mathematics with Applications, vol. 78, no. 6, pp. 1811–1818, 2019.

[35] J. Zhao, Y. Zhang, and Y. Xu, "Implicit Runge-Kutta and spectral Galerkin methods for Riesz space fractional/distributed-order diffusion equation," Computational and Applied Mathematics, vol. 39, no. 2, p. 47, 2020.

[36] M. Feckan, M. Pospisil, and J. Wang, "Note on weakly fractional differential equations," Advances in Difference Equations, vol. 2019, no. 1, p. 143, 2019.

[37] Y. Wang and L. Liu, "Positive solutions for a class of fractional infinite-point boundary value problems," Boundary Value Problems, vol. 2018, no. 1, p. 118, 2018.

[38] Y. Wang, "Positive solutions for a class of two-term fractional differential equations with multipoint boundary value conditions," Advances in Difference Equations, vol. 2019, no. 1, p. 304, 2019.

[39] X. Hao, H. Wang, L. Liu, and Y. Cui, "Positive solutions for a system of nonlinear fractional nonlocal boundary value problems with parameters and p-Laplacian operator," Boundary Value Problems, vol. 2017, no. 1, p. 182, 2017.

[40] X. Zhang, L. Liu, and Y. Wu, "Multiple positive solutions of a singular fractional differential equation with negatively perturbed term," Mathematical and Computer Modelling, vol. 55, no. 3-4, pp. 1263–1274, 2012.

[41] Y. Wang and L. Liu, "Positive solutions for a class of fractional 3-point boundary value problems at resonance," Advances in Difference Equations, vol. 2017, no. 1, p. 7, 2017.

[42] Y. Wang and L. Liu, "Necessary and sufficient condition for the existence of positive solution to singular fractional differential equations," Advances in Difference Equations, vol. 2015, p. 207, 2015.
[43] Y. Wang, “Necessary conditions for the existence of positive solutions to fractional boundary value problems at resonance,” *Applied Mathematics Letters*, vol. 97, pp. 34–40, 2019.

[44] M. Ahmad, J. Jiang, A. Zada, S. O. Shah, and J. Xu, “Analysis of coupled system of implicit fractional differential equations involving Katugampola-Caputo fractional derivative,” *Complexity*, vol. 2020, Article ID 9285686, 11 pages, 2020.

[45] F. Sun, L. Liu, X. Zhang, and Y. Wu, “Spectral analysis for a singular differential system with integral boundary conditions,” *Mediterranean Journal of Mathematics*, vol. 13, no. 6, pp. 4763–4782, 2016.

[46] Y. Wang, L. Liu, X. Zhang, and Y. Wu, “Positive solutions of an abstract fractional semipositone differential system model for bioprocesses of HIV infection,” *Applied Mathematics and Computation*, vol. 258, pp. 312–324, 2015.

[47] L. Liu, D. Min, and Y. Wu, “Existence and multiplicity of positive solutions for a new class of singular higher-order fractional differential equations with Riemann–Stieltjes integral boundary value conditions,” *Advances in Difference Equations*, vol. 2020, no. 1, p. 442, 2020.

[48] T. Wang and Z. Hao, “Existence and uniqueness of positive solutions for singular nonlinear fractional differential equation via mixed monotone operator method,” *Journal of Function Spaces*, vol. 2020, Article ID 2354927, 9 pages, 2020.

[49] X. Zhang, L. Liu, and Y. Wu, “Existence results for multiple positive solutions of nonlinear higher order perturbed fractional differential equations with derivatives,” *Applied Mathematics and Computation*, vol. 219, no. 4, pp. 1420–1433, 2012.

[50] P. Yang, J. Wang, and M. Fečkan, “Periodic nonautonomous differential equations with noninstantaneous impulsive effects,” *Mathematical Methods in the Applied Sciences*, vol. 42, no. 10, pp. 3700–3720, 2019.

[51] J. He, X. Zhang, L. Liu, Y. Wu, and Y. Cui, “Existence and asymptotic analysis of positive solutions for a singular fractional differential equation with nonlocal boundary conditions,” *Boundary Value Problems*, vol. 2018, no. 1, p. 189, 2018.

[52] X. Zhang, L. Liu, and Y. Wu, “The eigenvalue problem for a singular higher order fractional differential equation involving fractional derivatives,” *Applied Mathematics and Computation*, vol. 218, no. 17, pp. 8526–8536, 2012.

[53] X. Zhang, L. Liu, B. Wiwatanapataphee, and Y. Wu, “The eigenvalue for a class of singular p-Laplacian fractional differential equations involving the Riemann–Stieltjes integral boundary condition,” *Applied Mathematics and Computation*, vol. 235, pp. 412–422, 2014.

[54] J. Mao, Z. Zhao, and A. Qian, “Laplace’s equation with concave and convex boundary nonlinearities on an exterior region,” *Boundary Value Problems*, vol. 2019, no. 1, p. 51, 2019.

[55] J. Liu and Z. Zhao, “An application of variational methods to second-order impulsive differential equation with derivative dependence,” *Electronic Journal of Differential Equations*, vol. 2014, no. 62, 2014.

[56] J. Sun and T.-F. Wu, “Steep potential well may help Kirchhoff type equations to generate multiple solutions,” *Nonlinear Analysis*, vol. 190, pp. 111609, 2020.

[57] A. Mao and X. Zhu, “Existence and multiplicity results for Kirchhoff problems,” *Mediterranean Journal of Mathematics*, vol. 14, no. 2, p. 58, 2017.

[58] M. Shao and A. Mao, “Multiplicity of solutions to Schrödinger-Poisson system with concave-convex nonlinearities,” *Applied Mathematics Letters*, vol. 83, pp. 212–218, 2018.

[59] J. Sun, T. Wu, and Z. Feng, “Non-autonomous Schrödinger-Poisson system in R^n,” *Discrete & Continuous Dynamical Systems*, vol. 38, no. 4, pp. 1889–1933, 2018.

[60] J. Liu and Z. Zhao, “Existence of positive solutions to a singular boundary-value problem using variational methods,” *Electronic Journal of Differential Equations*, vol. 2014, no. 135, pp. 1–9, 2014.

[61] X. Zhang, L. Liu, and Y. Wu, “Variational structure and multiple solutions for a fractional advection-dispersion equation,” *Computers & Mathematics with Applications*, vol. 68, no. 12, pp. 1794–1805, 2014.

[62] D. Ma, L. Liu, and Y. Wu, “Existence of nontrivial solutions for a system of fractional advection–dispersion equations,” *Revista de la Real Academia de Ciencias Exactas, Físicas y Naturales. Serie A. Matemáticas*, vol. 113, no. 2, pp. 1041–1057, 2019.

[63] J. Zhang, Z. Lou, Y. Ji, and W. Shao, “Ground state of Kirchhoff type fractional Schrödinger equations with critical growth,” *Journal of Mathematical Analysis and Applications*, vol. 462, no. 1, pp. 57–83, 2018.

[64] X. Zhang, L. Liu, Y. Wu, and Y. Cui, “New result on the critical exponent for solution of an ordinary fractional differential problem,” *Journal of Function Spaces*, vol. 2017, Article ID 3976469, 4 pages, 2017.

[65] A. Mao and W. Wang, “Nontrivial solutions of nonlocal fourth order elliptic equation of Kirchhoff type in R^3,” *Journal of Mathematical Analysis and Applications*, vol. 459, no. 1, pp. 556, 2018.

[66] G. Wang, X. Ren, Z. Bai, and W. Hou, “Radial symmetry of standing waves for nonlinear fractional Laplacian Hardy–Schrödinger systems,” *Applied Mathematics Letters*, vol. 110, pp. 106560, 2020.

[67] L. Zhang and W. Hou, “Standing waves of nonlinear fractional p-Laplacian Schrödinger equation involving logarithmic nonlinearity,” *Applied Mathematics Letters*, vol. 102, pp. 106149, 2020.

[68] G. Wang, X. Ren, Z. Bai, and W. Hou, “Radial symmetry of standing waves for nonlinear fractional Hardy-Schrödinger equation,” *Applied Mathematics Letters*, vol. 96, pp. 131–137, 2019.

[69] B. Zhang, Y. Xia, L. Zhu, H. Liu, and L. Gu, “Global stability of fractional order coupled systems with impulses via a graphic approach,” *Mathematics*, vol. 7, no. 8, pp. 744, 2019.

[70] S. Song, B. Zhang, X. Song, Y. Zhang, Z. Zhang, and W. Li, “Fractional-order adaptive neuro-fuzzy sliding mode $H_\infty$ control for fuzzy singularly perturbed systems,” *Journal of the Franklin Institute*, vol. 356, no. 10, pp. 5027–5048, 2019.

[71] K. Liu, M. Fečkan, D. O’Regan, and J. Wang, “Hyers-Ulam stability and existence of solutions for differential equations with Caputo-Fabrizio fractional derivative,” *Mathematics*, vol. 7, no. 4, pp. 333, 2019.

[72] Q. Feng and F. Meng, “Traveling wave solutions for fractional partial differential equations arising in mathematical physics by an improved fractional Jacobi elliptic equation method,” *Mathematical Methods in the Applied Sciences*, vol. 40, no. 10, pp. 3676–3686, 2017.

[73] H. Liu and R. Xu, “The oscillatory of linear conformable fractional differential equations of Kamenev type,” *Discrete Dynamics in Nature and Society*, vol. 2020, Article ID 3857592, 8 pages, 2020.
operators,” *Mathematical Methods in the Applied Sciences*, vol. 43, no. 5, pp. 2548–2557, 2020.

[75] X. Wang, J. Wang, and M. Fečkan, “Controllability of conformable differential systems,” *Nonlinear Analysis: Modelling and Control*, vol. 25, no. 4, pp. 658–674, 2020.

[76] J. Zhao, Y. Zhang, and Y. Xu, “Implicit Runge-Kutta and spectral Galerkin methods for the two-dimensional non-linear Riesz space fractional diffusion equation,” *Applied Mathematics and Computation*, vol. 386, p. 125505, 2020.

[77] J. Wang, A. Zada, and H. Waheed, “Stability analysis of a coupled system of nonlinear implicit fractional anti-periodic boundary value problem,” *Mathematical Methods in the Applied Sciences*, vol. 42, no. 18, pp. 6706–6732, 2019.

[78] J. Zhao, Y. Zhang, and Y. Xu, “Implicit Runge-Kutta and spectral Galerkin methods for the two-dimensional non-linear Riesz space distributed-order diffusion equation,” *Applied Numerical Mathematics*, vol. 157, pp. 223–235, 2020.

[79] M. M. A. Khater, R. A. M. Attia, A.-H. Abdel-Aty, W. Alharbi, and D. Lu, “Abundant analytical and numerical solutions of the fractional microbiological densities model in bacteria cell as a result of diffusion mechanisms,” *Chaos, Solitons & Fractals*, vol. 136, p. 109824, 2020.

[80] X. Hao, L. Zhang, and L. Liu, “Positive solutions of higher order fractional integral boundary value problem with a parameter,” *Nonlinear Analysis: Modelling and Control*, vol. 24, no. 2, pp. 210–223, 2019.

[81] X. Hao, H. Sun, and L. Liu, “Existence results for fractional integral boundary value problem involving fractional derivatives on an infinite interval,” *Mathematical Methods in the Applied Science*, vol. 41, no. 16, pp. 6984–6996, 2018.

[82] J. Jiang, D. O’Regan, J. Xu, and Z. Fu, “Positive solutions for a system of nonlinear Hadamard fractional differential equations involving coupled integral boundary conditions,” *Journal of Inequalities and Applications*, vol. 2019, p. 204, 2019.

[83] Y. Ding, J. Jiang, D. O’Regan, and J. Xu, “Positive solutions for a system of Hadamard-type fractional differential equations with semipositive nonlinearities,” *Complexity*, vol. 2020, Article ID 9742418, 14 pages, 2020.

[84] J. Wu, X. Zhang, L. Liu, Y. Wu, and Y. Cui, “The convergence analysis and error estimation for unique solution of a p-Laplacian fractional differential equation with singular decreasing nonlinearity,” *Boundary Value Problems*, vol. 2018, no. 1, p. 82, 2018.

[85] L. Ren, J. Wang, and M. Fečkan, “Periodic mild solutions of impulsive fractional evolution equations,” *AIMS Mathematics*, vol. 5, no. 1, pp. 497–506, 2019.

[86] X. Zhang, C. Mao, L. Liu, and Y. Wu, “Exact iterative solution for an abstract fractional dynamic system model for bioprocess,” *Qualitative Theory of Dynamical Systems*, vol. 16, no. 1, pp. 205–222, 2017.

[87] K. Liu, J. Wang, and D. O’Regan, “On the Hermite-Hadamard type inequality for Ψ-Riemann-Liouville fractional integrals via convex functions,” *Journal of Inequalities and Applications*, vol. 2019, p. 27, 2019.

[88] J. Wu, X. Zhang, L. Liu, Y. Wu, and Y. Cui, “Convergence analysis of iterative scheme and error estimation of positive solution for a fractional differential equation,” *Mathematical Modeling and Analysis*, vol. 23, pp. 611–626, 2018.

[89] J. Wang, A. G. Ibrahim, and D. O’Regan, “Global attracting solutions to Hilfer fractional differential inclusions of Sobolev type with noninstantaneous impulses and nonlocal conditions,” *Nonlinear Analysis: Modelling and Control*, vol. 24, no. 5, pp. 775–803, 2019.

[90] Y. Chen and J. Wang, “Continuous dependence of solutions of integer and fractional order non-instantaneous impulsive equations with random impulsive and junction points,” *Mathematics*, vol. 7, no. 4, p. 331, 2019.

[91] X. Zhang, L. Liu, Y. Wu, and B. Wiwatanapatapee, “Nontrivial solutions for a fractional advection dispersion equation in anomalous diffusion,” *Applied Mathematics Letters*, vol. 66, pp. 1–8, 2017.

[92] J. Wang, A. G. Ibrahim, and D. O’Regan, “Nonemptiness and compactness of the solution set for fractional evolution inclusions with non-instantaneous impulses,” *Electronic Journal of Differential Equations*, vol. 37, pp. 1–17, 2019.

[93] S. Xu, H. Lv, H. Liu, and A. Liu, “Robust control of disturbed fractional-order economical chaotic systems with uncertain parameters,” *Complexity*, vol. 2019, Article ID 7567695, 13 pages, 2019.

[94] M. Fečkan, T. Sathiyaraj, and J. Wang, “Synchronization of butterfly fractional order chaotic system,” *Mathematics*, vol. 8, no. 3, p. 446, 2020.

[95] H. A. H. Salm, “On the fractional order p-point boundary value problem in reflexive Banach spaces and weak topologies,” *Computers & Mathematics with Applications*, vol. 224, no. 2, pp. 565–572, 2009.

[96] W. Xie, J. Xiao, and Z. Luo, “Existence of extremal solutions for nonlinear fractional differential equation with nonlinear boundary conditions,” *Applied Mathematics Letters*, vol. 41, pp. 46–51, 2015.

[97] F. Wang and Z. Zheng, “Quasi-projective synchronization of fractional order chaotic systems under input saturation,” *Physica A: Statistical Mechanics and Its Applications*, vol. 534, p. 122132, 2019.

[98] S. Ha, H. Liu, S. Li, and A. Liu, “Backstepping-based adaptive fuzzy synchronization control for a class of fractional-order chaotic systems with input saturation,” *International Journal of Fuzzy Systems*, vol. 21, no. 5, pp. 1571–1584, 2019.

[99] M. Li and J. Wang, “Exploring delayed Mittag-Leffler type matrix functions to study finite time stability of fractional delay differential equations,” *Applied Mathematics and Computation*, vol. 324, pp. 254–265, 2018.

[100] M. Li and J. Wang, “Representation of solution of a Riemann-Liouville fractional differential equation with pure delay,” *Applied Mathematics Letters*, vol. 85, pp. 118–124, 2018.

[101] F. Wang and Y. Yang, “Quasi-synchronization for fractional-order delayed dynamical networks with heterogeneous nodes,” *Applied Mathematics and Computation*, vol. 339, pp. 1–14, 2018.

[102] X. Wu, J. Wang, and J. Zhang, “Hermite-Hadamard-type inequalities for convex functions via the fractional integrals with exponential kernel,” *Mathematics*, vol. 7, p. 845, 2018.

[103] P. Yang, J. Wang, and Y. Zhou, “Representation of solution for a linear fractional delay differential equation of Hadamard type,” *Advances in Difference Equations*, vol. 2019, p. 300, 2019.

[104] I. Jiang, D. O’Regan, J. Xu, and Y. Cui, “Positive solutions for a Hadamard fractional p-Laplacian three-point boundary value problem,” *Mathematics*, vol. 7, no. 5, p. 439, 2019.

[105] X. Zhang, L. Yu, J. Jiang, Y. Wu, and Y. Cui, “Solutions for a singular Hadamard-type fractional differential equation by the spectral construct analysis,” *Journal of Function Spaces*, vol. 2020, Article ID 8392397, 12 pages, 2020.

[106] W. Liu, L. Liu, and Y. Wu, “Existence of solutions for integral boundary value problems of singular Hadamard-type
fractional differential equations on infinite interval,” Advances in Difference Equations, vol. 2020, no. 1, p. 274, 2020.

[107] R. H. Tian, L. Fu, Y. W. Ren, and H. W. Yang, “(3+1)-dimensional time-fractional modified Burgers equation for dust ion-acoustic waves as well as its exact and numerical solutions,” Mathematical Methods in the Applied Sciences, vol. 2019, pp. 1–20, In press.

[108] C. Yue, M. M. A. Khater, R. A. M. Attia, and D. C. Lu, “The plethora of explicit solutions of the fractional KS equation through liquid-gas bubbles mix under the thermodynamic conditions via Atangana-Baleanu derivative operator,” Advances in Difference Equations, vol. 2020, p. 62, 2020.

[109] X. Hao, H. Wang, C. Park, and D. Lu, “Positive solutions of semipositone singular fractional differential systems with a parameter and integral boundary conditions,” Open Mathematics, vol. 16, pp. 581–596, 2018.

[110] X. Hao, “Positive solution for singular fractional differential equations involving derivatives,” Advances in Difference Equations, vol. 2016, no. 1, p. 139, 2016.

[111] J. Mao, Z. Zhao, and C. Wang, “The unique iterative positive solution of fractional boundary value problem with q-difference,” Applied Mathematics Letters, vol. 100, p. 106002, 2020.