The Acceleration of the Gravitational Wave Events Through the Circumbinary Disk Around Two Inspiraling Black Holes

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Abstract. The observation of gravitational wave opens a new window for both physics and astrophysics. Among the models of the formation of binary black holes (BHs), some suggest the existence of circumbinary disk, which may provide the material to ignite the electromagnetic radiation observable to modern astronomical facilities. In this work, we investigate the consequences of such circumbinary disk, with an emphasis on the reduction in the BH-BH merger timescale, i.e. the total time necessary for the gravitational wave events to happen. We find that, the circumbinary disk excites a tidal torque onto the BH-BH system, and it dominates the angular momentum transfer (outward) when the BH-BH separation \( a \gg \text{xxx} \). Theoretical implications of this result are discussed.

1. Introduction

Gravitational wave astronomy is part of observational astronomy. In general relativity, Einstein predicted the existence of gravitational waves, but for the next century they were unproven. It wasn’t until September 14, 2015, that the U.S. LIGO detectors in Washington and Louisiana detected a gravitational wave signal. The signal is the result of a merger of two black holes. A black hole twenty-nine times the mass of the sun and a black hole thirty-six times the mass of the sun merge to form a black hole sixty-two times the mass of the sun. The three times the mass of the sun is lost and converted into gravitational wave energy. On October 6, 2017, the U.S. LIGO detector teamed up with the Italian VIRGO detector to detect gravitational waves from the merger of two neutron stars for the first time.

Gravitational wave detectors work by sending a laser beam back and forth in a 3-4km vacuum chamber. When gravitational waves sweep through the detector, the ripples in space-time cause tiny changes in length, which the researchers judge to be very slight changes in the length of the laser arm. As exploration progresses, LIGO and other detectors around the world are being upgraded to increase the accuracy and sensitivity of the probes. For example, doubling the power of the mechanism, replacing the reflector, and reducing the level of quantum noise make the observation distance become longer. Japan's upgraded gravitational wave detector, KAGRA, has also begun sharing observations around the world. In fact, the first detection of gravitational waves in 2015 ushered in a whole new era of physical and astronomical observation. Human beings have officially entered the era of gravitational wave astronomy. In the future, gravitational wave events will be detected more and more frequently, and people will have more new discoveries.

Accretion disks are gaseous disks of large celestial bodies such as black holes and neutron stars. They are composed of diffuse matter. Due to viscous action, these materials will have angular velocity different from that of the central body, and they will rotate angular difference. We use standard thin
accretion disk in our research. The disk is long-lived and well approximated by hydrostatic equilibrium, thermal equilibrium, and local ionization equilibrium. It has a low mass compared to black hole, heated from star and dominated by gas pressure. Our topic is an attempt to explore the effects of existence of circumbinary gas disks on binary black holes merger. When there are two black holes in the center of gaseous disk, the disk tidal interaction will be around. We expect that the presence of a gas disk will reduce the timescale for black holes’ merger. We use numerical simulation to put the available data into the procedure we create to run. This procedure is based on derivation of the classical hydrodynamical equations of accretion theory and combination of effects due to the disk-binary torque and the gravitational-wave radiation. The data available and our procedure models are used to calculate the time for binary black holes merge in the condition under the circumbinary gas disk and in the condition just involved by gravitational wave radiational decay. We compare the results of the two sets of data and come to a conclusion. But for the exact merge time, the results are still open.

2. Theoretical model

We in this section describe the circumbinary disk model, which consists a binary of BHs and a coplanar circular circumbinary disk that orbits the mass center of the two BHs. The two black holes, with masses $M_p$ (primary) and $M_s$ (secondary) individually, are separated by a distance $a$. The total BH mass is then $M_{bin} = M_p + M_s$ and the mass ratio of the primary to the secondary is $q = M_s/M_p$ (obviously $q \leq 1$). Since the mass ratio of the 4 binary BH mergers currently detected by LIGO/VIRGO is $q \sim 1$, we thus in our work adopt a cylinder coordinate $(R, \phi, Z)$ centered at the mass centroid of the two BHs. Before the eventual merger, the two BHs will lose angular momentum due to gravitational waves and tidal torque due to the interaction with circumbinary material, and such inspiral phase is quasi-circular thus only relates to separation $a$.

We assume the disk to be two-phase, i.e. a MRI-unactivated dead zone, sandwiched by two active layers. The dead zone has a surface density $\Sigma_d$ and a temperature $T_d$. Similarly, the two active layers above and below the dead zone has a total surface density $\Sigma_a$ and $T_a$. The total surface density is then $\Sigma_{tot} = \Sigma_d + \Sigma_a$. Following the thin disk approximation (Frank, King & Raine 2002), we can estimate the scale height of the dead zone and the active layers as $H_d = c_s, d/\Omega_k$ and $H_a = c_s, a/\Omega_k^2$, where $c_s = \sqrt{P_{tot}/\rho}$ is the isothermal sound speed and $\Omega_k = \sqrt{GM_{bin}/R^3}$ is Keplerian angular velocity around the binary BHs. For the total pressure $P_{tot}$, the radiation pressure is negligible because of extremely low temperature [1], we thus include gas and magnetic pressure. At temperature $T$, the sound speed of gas can be expressed as

$$c_s^2(T) = \frac{k_B T}{\mu m_H} \left[ \frac{1}{\beta} \right]$$

(1)

Here the plasma $\beta$ parameter, defined as gas to magnetic pressure ratio, is fixed to $\beta = 10$ throughout this work. $k_B$ is the Boltzmann constant, $m_H$ is the Hydrogen mass. $\mu \approx 2.36$ is the mean molecular weight. The “mid-plane” density of dead zone and two active layers can then be evaluated as

$$\rho_d = \Sigma_d/\sqrt{2\pi}H_a$$

(2)

Below we provide the dynamics and energy balance of the circumbinary disk. The evolution of the disc surface density around binary BHs is described by the classical hydrodynamical equations of accretion theory [2, 3]. We adopt a cylinder coordinate $(R, \phi, Z)$ which is centered at the mass centroid of the two BHs. The equation for the evolution of total surface density is [2, 4],

$$\frac{\partial \Sigma_{tot}}{\partial t} = \frac{3}{R} \frac{\partial}{\partial R} \left[ R^{1/2} \frac{\partial}{\partial R} \left( \frac{1}{2} \frac{\partial}{\partial R} \left( \frac{1}{2} \Sigma_d \frac{\partial v_d}{\partial R} + \frac{\partial \Sigma_d}{\partial R} \right) \right) \right] - \frac{2}{R} \frac{\partial}{\partial \phi} \left[ \frac{\partial \Sigma_{tot}}{\partial \Omega_k} \right]$$

(3)

We adopt the conventional $\alpha$-description [5] for the viscosity in the MRI active layer, i.e.

$$v_a = \alpha \Omega_k c_s^2 \Sigma_d/\Omega_k$$

(4)

gravity turbulent process. One important process worth emphasizing is that, when the Toomre parameter $Q = c_s d/\pi G \Sigma_d$ is less than a critical value $Q_{crit} \sim 2$, the self-gravitational instability operates. We assume this only operates in the dead zone. There are several possible outcomes [6]. For example, the disk may exhibit large amplitude bursts of accretion or it may fragment and break up into distinct bound
objects. In this work, we adopt the possibility that the disk will settle into a quasi-steady state of self-gravitating turbulence, in which trailing spiral arms yield an outward transport of angular momentum via gravitational torques [7]. Following Lin & Pringle (1987) [7], we approximate the turbulence driven by self-gravitational instability through the “effective α viscosity” description, i.e. vsg takes the form

\[ v_{sg} = \alpha_{sg} \frac{c_{s}^{2}a}{\Omega_{k}} \left( \frac{Q_{\text{crit}}}{Q} \right)^{2} - 1 \]  

For \( Q < Q_{\text{crit}} \) and \( v_{g} = 0 \) otherwise. In this work, we omit other possible turbulence in the dead zone [4] and take \( v_{d} = v_{sg} \frac{\Delta r}{r} \) is the specific tidal torque exerted by the binary BHs on the disc.

Armitage, Livio & Pringle 2001 [9]: dead zone and self-gravity disc. The vertically-averaged velocity, and the radial velocities of active layer and dead zone, can then be evaluated as

\[ \bar{V}_{\delta} = -\frac{3}{r \Sigma_{\text{tot}}} \frac{\partial}{\partial r} \left[ R^{2} (v_{a} \Sigma_{a} + v_{d} \Sigma_{d}) \right] + \frac{2 \Delta r}{r \Omega_{k}} \]  

We adopt the following simplified energy equation [8],

\[ \frac{\partial T_{c}}{\partial t} = \frac{2 (Q^{+} - Q^{-})}{c_{p} \Sigma_{\text{tot}}} - \frac{\partial T_{c}}{\partial r} V_{r} \]  

Where \( T_{c} \) is the mid-plane temperature and \( c_{p} \) is the specific heat capacity of the gas. For the typical temperature range considered in this work, \( c_{p} \approx 2.7 k_{B}/\mu m_{H} \). The second term on the right-hand side represents the energy advection in radial direction. \( Q^{+} \) and \( Q^{-} \) are respectively. The total heating and cooling rate per unit surface.

It is known that the tidal barrier is not perfect, i.e. when the gas is relatively hot (e.g., disk aspect ratio \( H/R \geq 0.1 \)), a non-negligible fraction of gas flows from the edge of the cavity, form smaller disk around individual BHs and eventually be accreted [10, 11, 12, 1]. The details of such process, e.g., dependence on mass ratio \( q \) and aspect ratio \( H/R \) (aka. gas temperature), awaits further investigation. Through 3D SPH simulations, Ragusa et al. (2016) [13] found that, when \( H/R \ll 1 \), this “leaked” accretion rate is highly reduced compared to that of the referenced single-BH accretion (the accretion rate can be expressed as \( \dot{M}_{\text{single}} \approx 3 \pi (v_{a} \Sigma_{a} + v_{d} \Sigma_{d}) \)), and it saturates to \( \dot{M}_{\text{ref}} \) \( H/R \geq 0.1 \) (but see Shi et al. 2012) [12], i.e. it reads \( \dot{M}_{\text{leak,bin}} = \min(1, 10 H/R) \dot{M}_{\text{leak, single}} \).

Here normalization factor \( f_{\text{leak}} \) is introduced to generalize the result of Ragusa et al. (2016) [13]. We will adopt this formulate to estimate the “leaked” mass accretion rate, where the reference accretion rate \( \dot{M}_{\text{single}} \) is estimated by disk properties at \( 10 a \).

The binary separation evolution is determined by the combination of effects due to the disk-binary torque and the gravitational-wave radiation, i.e.

\[ \frac{d a}{d t} = (\frac{d a}{d t})_{T} + (\frac{d a}{d t})_{gw} \]  

We admit that, since only the shrink in separation a is included for the evolution of the binary BHs, we actually assume the binary BHs to orbit circularly. In reality, the binary is likely to be eccentric to certain degrees [14]. If the disk is massive, then the eccentricity will be low due to efficient disk-binary coupling.

3. Basic results

First, let’s consider merger time of the binary BHs without a disk. In this case, the change in separation \( (a_{\text{sep}}) \) depends solely on Gravitational Wave (GW) radiational decay. Evolution is defined by

\[ (\frac{d a}{d t})_{gw} = -\frac{64 c}{5} g (\frac{R_{g}}{a})^{3} \]  

With \( g \) being defined as

\[ g = \frac{q}{(1+q)^{2}} \]
\( q \) is the ratio of mass between the two BHs, i.e. \( q = \frac{M_p}{M_s} \). Based on current BBH merger events detected by LIGO/VERGO, \( q \sim 1 \). We adopt \( q \) to be 0.8 in our calculation. The merger time can thus be expressed as [14]

\[
t_{gw} = \frac{1}{4} \frac{a}{\left| \frac{da}{dt} \right|_{gw}} = 6.3 \times 10^8 \text{yr} s \ g^{-1} a^4_{\text{gw}} M_{\text{bin,2}}
\]  

(11)

If there’s an accretion disk around the BBHs, the actual merger time will be influenced. To find the impact quantitatively using mathematical method, we can model the whole system as the primary stage of planetary migration, as the evolution is highly similar to the Type II migration in planetary system: plant and gas disk interact gravitationally, launching a spiral density wave, result in angular momentum exchange. When mass of the planet in this model exceeds certain value, the gravitational interaction will succeed in opening up a gap, which is definite in the case of binary BHs, mass of the secondary BH will always exceed the critical value. After opening up of the gap, some gases continue to flow in and add up to the mass of the planet, and the rate of mass inflow, by assuming the accretion disk as a standard thin accretion disk, is given by:

\[
\dot{M} = -2\pi r v_r \Sigma
\]

(12)

\( \Sigma \) in this equation represents the surface density of the disk, whose evolution can be calculated using a diffusion equation:

\[
\frac{\partial \Sigma}{\partial t} = \frac{3}{\mu} \frac{\partial}{\partial R} \left\{ R^{1/2} \frac{\partial}{\partial R} \left[ \nu \Sigma R^{1/2} \right] \right\}
\]

(13)

\( \Sigma \) is the surface density, \( \nu \) is viscosity of the disc

\[
\nu = \alpha c_s H = \alpha c_s^2 / \Omega_K
\]

(14)

\( \alpha \) is adopted from the conventional \( \alpha \)-prescription for viscosity in MRI layer. \( C_s \) is the speed of the sound and \( H \) is the scale height.

For calculation simplicity, the circumbinary disk is considered to have even mass distribution and far away from the BBH at the beginning, i.e. \( R_{in} > a_\text{sep0} \), with \( R_{in} \) being the inner radius of the disk and \( a_\text{sep0} \) being the initial separation of the binary BHs. The disk is considered to have low mass compared with BHs, \( f_{disk} \) is the ratio of the mass of the disk and mass of the BBH. \( f_{disk} \approx 0.01 \) in the general case.

When the disk interacts with the BHs, they exchange their angular momentum by disk mass inflow to the BH. The evolution is thus highly similar to the Type II migration in planetary system: plant and gas disk interact gravitationally, launching a spiral density wave, result in torques. When mass of the planet exceeds certain value, the gravitational interaction will succeed in opening up a gap. Some gases continue to flow in and add up to the mass of the planet. The overall exchange of angular momentum is thus determined by viscous evolution of the disk, rate of change of surface density so to say. The change in surface density is defined by a general diffusion equation of standard thin accretion disk:

\[
\frac{\partial \Sigma}{\partial t} = \frac{3}{\mu} \frac{\partial}{\partial R} \left\{ R^{1/2} \frac{\partial}{\partial R} \left[ \nu \Sigma R^{1/2} \right] \right\}
\]

(15)

\( \Sigma \) is the surface density, \( \nu \) is viscosity of the disk

\[
\nu = \alpha c_s H = \alpha c_s^2 / \Omega_K
\]

(16)

\( \alpha \) is adopted from the conventional \( \alpha \)-prescription for viscosity in MRI layer. \( C_s \) is the speed of the sound and \( H \) is the scale height.

Viscosity of the disk \( \nu \) is also related to the energy evolution. Energy change is mainly due to viscous frictional heating and radiation cooling, the rates are

\[
Q_{vis} = \frac{9}{8} \nu \Sigma \Omega_K^2
\]

(17)

\[
Q_{rad} = \frac{8 \sigma T^4}{3 \kappa \Sigma}
\]

(18)

respectively [15]. The disc is assumed to be in thermal equilibrium, \( Q_{vis} = Q_{rad} \). Therefore, by setting an initial temperature of the disk, we can get the thermal evolution of the disk at any instant.

From equations above, change in separation caused by tidal torque can be defined as
Thus the merger time due to torque driven by the circumbinary disk can be expressed as:

$$ t_A = \frac{a}{|\frac{da}{dt}|} = 1.2 \times 10^7 \text{ yrs } g^{-1} f^{-1} a_5^{-3/2} M_{\text{bin},2} \left( \frac{M_{\text{disk,eff}}}{M_{\text{bin}}} \right)^{-1/6} $$

(20)

The total separation evolution is the combination of disk-binary torque and GW radiation decay

$$ \frac{da}{dt} = \left( \frac{da}{dt} \right)_A + \left( \frac{da}{dt} \right)_{gw} $$

(21)

Only decrease in separation is considered in the evolution of the binary BH system, as the result of the process will always be a merger.

From figure 1, we can see that at a moderate separation ($a \sim 10^6$), tidal torque plays a great role in terms of transporting angular momentum. It’s reasonable since from equation (1) and (2), we can see explicitly that

$$ t_{gw} \propto a^4 $$

(22)

$$ t_A \propto a^{3/2} $$

(23)

Tidal torque at this distance has a much larger effect to the change in separation of the BBH compared with GW decay at the separation roughly larger than $\sim 10^5 R_g$. As a result, actual merger timescale will be reduced dramatically to be less than $\sim 10^9$ yrs.

4. Summary and discussion

Since the discovery of gravitational wave, a new window has been opened for both physics and astrophysics. Among the models of the formation of binary black holes (BHs), some suggest the existence of circumbinary disk, which may provide the material to ignite the electromagnetic radiation observable to modern astronomical facilities. In this work, we investigate the consequences of such circumbinary disk, with an emphasis on the reduction in the BH-BH merger timescale, i.e. the total time necessary for the gravitational wave events to happen.

The far shorter merger time scale brought by a circumbinary disk around the BBH system may have several impacts on our vision about current observatory data. Firstly, the BBH merger events that we have already observed may not come from the binary black hole systems formed in the early universe. Therefore, electromagnetic or optical signals detected from such events should, responsively, have smaller red-shift values. Moreover, since merger event can happen easier or faster than anticipated before, then we should also expect more observation of gravitational waves from a BBH merger.
In recent years, we have detected a large amount of GW from BBH, which agrees with hypothesis of our theory.

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