Process modeling of gas displacement of oil on the example of oil fields in Bashkortostan

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Abstract. The article presents the results of the process modeling of gas displacement of oil. The proposed model takes into account the features of the state and changes in the physical and chemical state of formation systems on exposure to formation fluids with various gases-carbon dioxide and hydrocarbons. Numerical experiments made it possible to predict changes in technological parameters of impact objects and determine the incremental value in the oil recovery factor for Devonian fields of several oil-fields in Bashkortostan.

1. Introduction
It is known that when oil-pool developing under natural conditions, the oil recovery factor (ORF) is on average 10-15%. When using secondary and tertiary methods, the values of this indicator increase to 60-70%. In some cases, this increase is absent, and if the wrong method of increasing oil recovery (EOR) is chosen, the ORF may be lower than when oil-pool developing under natural conditions, other things being equal [1-5].

In this regard, the justification and choice of a particular recovery method should be justified and should take into account the features of the geological structure of fields.

Methods based on the oil displacement from a porous medium by gas are considered to be one of the most promising EOR.

The oil displacement processes from a porous medium by gas, solvent and gas banks, water-gas mixture, carbon dioxide (CO₂) banks and water are accompanied by complex phenomena. Individual phases move at different speeds. The formation fluid composition changes in the collaborative filtering area. The mixture components are redistributed between the phases. This leads to composition changes, hence to the physical and chemical properties of the phases and under certain conditions to phase transitions (appearance, development or disappearance of individual phases). This process is described by the seepage theory of multicomponent mixtures [6].

To calculate the three-phase filtration process, the asymmetric Masket-Meres model is often used, in which the oil phase is represented by two components: gas (the volatile oil part) and oil (the non-
volatile part) [7, 8]. This model is not suitable in our case. It is based on the assumption that the saturation pressure and the volume factors are constant and phase viscosities and solubility of the gas in oil depending on the pressure.

When solving development problems using methods of gas injection on the bank, the use of the Masket-Meres model leads to significant errors [9, 10].

Compositional models describe the process most fully [11], but they are usually used for solving one-dimensional problems.

2. Materials and methods
To describe special cases of the studying process are executed symmetrical models of three-phase filtration of ternary mixtures, which are a generalization of the Masket-Meres model for the case with the time value of saturation pressure [11]. In our case, models are not suitable for calculations for several reasons: they have a limited dimension; they do not take into account the irregularity of distributions of geological and physical parameters; they also neglect the gravity segregation of phases. And oil is represented by only two pseudo-components: high volatile (gas) and nonvolatile (commercial oil).

In this paper, we propose a general three-dimensional model of three-phase filtration of quaternary mixtures, which, in turn, is a special case of the compositional model. The model allows you to take into account complex geological and physical, physical and chemical parameters and the technology of formation stimulation:
- heterogeneity in terms of permeability, porosity, the saturation thickness with formation fluids;
- pressure-temperature reservoir conditions, occurrence depth and total thickness of the field;
- presence fully or partially isolated interlayers in the formation;
- area of marginal and bottom water;
- presence of a gas cap;
- elastic properties of formation fluids and gas and porous medium;
- three-dimensional three-phase filtration of a multicomponent mixture with phase transitions and changes in the physical and chemical properties of these phases;
- non-stationary injection of water or gas, or a water-gas mixture with an arbitrary gas proportion in the mixture;
- non-stationary withdrawal of fluid and gas;
- different technologies for injection banks of gas, solvents, CO$_2$, and water;
- continuous, sequential, alternating, and joint.

The hydrodynamic filtration model of multicomponent mixtures can be divided into two parts. The first is based on the laws of material balance and fractional flows, the second - the laws of the phase behavior of the mixture. The conservation equations and phase equilibria are supplemented with initial and limiting conditions that reflect the initial state of the formation system and the technology of influencing it during development.

Let's make the following assumptions. Let filtration take place under isothermal conditions, and phase equilibria in each selected element (cell) of the filtering area occur instantly. We believe that the movement speed of individual phases obeys the generalized Darcy law, and the number of jointly filtered phases can be from one to three. We neglect the diffusion and capillary forces.

We consider a three-dimensional flow in a heterogeneous stratum. Let's choose a rectangular coordinate system. Let $x$, $y$ be horizontal, and $z$ be a vertical coordinate that coincides with the vector direction of the gravity acceleration $g$. The heterogeneity of the formation thickness is taken into account by specifying the distribution of absolute permeability $k = k(x, y, z)$, porosity $m = m(x, y, z)$, no-flow boundaries within the filtration area, initial pressure distributions, saturation and phase compositions.

A complex mixture of hydrocarbon and non-hydrocarbon components (formation oil) is represented as a mixture of three pseudo – components: I – high volatile pseudo-component (methane, nitrogen) $k = 2$-volatile NGLs, condensate, CO$_2$), $k = 3$-low-volatile (actually commercial oil is higher
boiling hydrocarbons). We believe that the second pseudo-component can be contained in oil and gas, the first – in all three, and the third (water) - only in the water phase. Taking into account the accepted assumptions and designations, based on the general seepage theory of multicomponent mixtures, the process under consideration can be represented as the following simultaneous equations:

$$\nabla \sum_{k=1}^{3} k_{\alpha} \mu_{\alpha} c_{k} \rho_{\alpha} (\nabla P + \rho_{\alpha} g \nabla z) + \sum_{k=1}^{3} \rho_{\alpha} c_{k} \rho_{\alpha} \delta (\vec{r} - \vec{r}_{j}) = \frac{\partial}{\partial t} m U c_{k}$$

(1)

$$\sum_{k=1}^{3} C_{k} = 1,$$

(2)

where \(k_{\alpha}\) – relative permeability of the phase \(\alpha\); \(\mu_{\alpha}\) – phase viscosity \(\alpha\); \(c_{ak}\) – weight content of the \(k\) component in the phase \(\alpha\); \(P\) – pressure; \(p_{\alpha}\) – phase density \(\alpha\); \(q_{\alpha}\) – source density (flow) of the phase \(\alpha\); \(\sigma\) – Dirac delta function; \(r_{j}\)-coordinates of the \(j\) source (flow) (\(j\)-sources number); \(C_{k}\) – weight component of the \(k\) component in the mixture; \(L, B, H\)-characteristic length, width and thickness of the formation element; \(k\) – component index, \(k = 1, 2, 3\) – pseudo – components of the oil and gas part of the mixture, \(K = 4\) – water; \(\alpha\) – phase index, \(\alpha = 1-3\) – gas, oil, and water, respectively.

The equation coefficients \((1)k_{\alpha}, \mu_{\alpha}, p_{\alpha}, c_{ak}\) are pressure functions, compositions, and phase saturation, which are uniquely determined from the solution of the equations of phase equilibria, as will be shown below, by pressure setting and the ratio of the mixture. Therefore, the relations are valid for the values:

$$P_{\alpha}, p_{\alpha}, c_{ak}$$

(3)

where \(S_{\alpha}\) is the saturation of the porous medium with the phase \(\alpha\), and

$$\sum_{\alpha=1}^{3} S_{\alpha} = 1.$$  

(4)

Thus, the equation system \((1), (2)\) taking into account the relations \((3)\) is formally complete concerning the independent variables \(P\) and \(C_{k}\) \((k = 1.4)\).

The formality is that direct obtaining of partial characteristics \((3)\) is possible only in some exceptional cases (for example, when filtering a single-phase fluid, by piston swabbing one fluid by another).

In general, to construct dependencies \((3)\), it is necessary to have an equation system that relates the pressure and ratio of the mixture to the composition and volume ratios of the phases. This property has an equation system describing phase equilibria.

The thermodynamical equilibrium condition of a multiphase mixture at plane interfaces and in the absence of external influence is the minimum of the Gibbs free energy. This means a mechanical (pressure balance in the phases), thermal (temperature balance) and chemical (chemical potentials balance or volatility of each component in the phases) equilibrium [6]. Under the above assumptions, the first two conditions are met, and the third can be written in the equivalent form:

$$x_{k1} = x_{k2} x_{k3}, \quad k = 1, 2,$$

$$x_{12} = x_{12} x_{13},$$

(5)

where \(x_{ka}\) is the mole fraction of the \(k\) component in the \(\alpha\) phase; \(x_{ka}\) is the distribution coefficient (equilibrium constant) of the \(k\) component between the \(\alpha\) and \(\beta\) phases.

The use of relations \((5)\) is convenient if the values of \(x_{ka}\) can be determined experimentally or computationally (for example, using methods based on the principle of corresponding states [6] or state equations [6]). Note that calculating the characteristics of phase equilibria based on the ratios \((5)\) reduces the volume of calculations by about two orders of magnitude in comparison with the state equations. System \((5)\) should be supplemented with the following restrictive relations:

$$\sum_{k=1}^{3} x_{k2} = 1, \quad x_{13} + x_{33} = 1,$$

(6)

$$\sum_{\alpha=1}^{3} L_{\alpha} = 1, \quad Z_{k} = \sum_{\alpha=1}^{3} x_{ka} L_{\alpha}, \quad k = 1, 2, 3.$$  

(7)
where $L_\alpha$ is the mole fraction of the phase and in the mixture, $Z_k Z_k$ is the mole fraction of component k in the mixture.

Mole and weight fractions are related by the following ratios:

$$Z_k = \frac{c_k}{M_k} \left( \sum_{l=1}^{4} \frac{c_l}{M_l} \right),$$  \hspace{1cm} \text{(8)}

$$c_{k\alpha} = x_{k\alpha} M_k / \left( \sum_{l=1}^{4} x_{l\alpha} M_l \right),$$  \hspace{1cm} \text{(9)}

where $M_k$ is the molecular weight of the component k.

Thus, the mole fractions of the components in the mixture are calculated first from the ratio (8) based on the pressure setpoints and the mixture weight composition. Then, using a given binary phase diagram using the ratios (5)–(7), the mole fractions and phase compositions are calculated. Finally, the component weight fractions in the phases are calculated from the ratios (9).

Phase densities and viscosities are calculated either using known dependencies [6] or directly from experimental data. In the latter case, the experimental data can be approximated as follows.

The phase density:

$$\rho_1 = \rho_{10} \frac{P}{P_{st}} T \frac{T}{T_{st}} \frac{1}{ZZ(P, C_{11})} + \rho_{20} 3^{-b_2 (P - P_{st})}, \text{ for } P \leq P_{sat}^*;$$

$$\rho_2 = \left\{ \begin{array}{ll}
\rho_2 (P_{sat}^*) + \beta_2 (P - P_{sat}^*), \text{ for } P > P_{sat}^*; \\
\rho_3 = \rho_{30} [1 + \beta_3 (P - P_{st})],
\end{array} \right.$$

where $\rho_{st}$– phase density under standard conditions; $P_{st}$–standard pressure and temperature, $P_{sat}$ = 0.1013 MPa; $T_{st}$ = 288.06 K.; $T_{fm}$-formation temperature, K; $ZZ$ – real gas factor; $B_2, \beta_2, \beta_3$ - experimentally determined coefficients, MPa⁻¹; $P_{sat}$ – current saturation pressure, a function of the mixture composition (determined by a binary phase diagram).

The phase viscosity:

$$\mu_1 = \mu_{10} (P - P_{st}^A);$$

$$\mu_2 = \left\{ \begin{array}{ll}
\mu_{20} e^{-a_2 (P - P_{st})}, \text{ for } P \leq P_{sat}^*; \\
\mu_3 = \mu_{30}, \text{ for } P > P_{sat}^*;
\end{array} \right.$$

where $\mu_{st}$– phase viscosity under standard conditions; $A_1, A_2, A_3$ are experimentally determined coefficients, MPa⁻¹.

The mass density is related to the mole (p*) ratio

$$\rho^*_\alpha = \rho_{\alpha} / M_\alpha,$$  \hspace{1cm} \text{(12)}

where $M_\alpha$ is the molecular weight of the phase $\alpha$, which can be calculated using the well-known k-rule:

$$M_\alpha = \sum_{k=1}^{3} x_{k\alpha} M_k,$$  \hspace{1cm} \text{(13)}

where $M_k$ is the molecular weight of the component k.

The volume ratio (saturation) of phases are calculated from the ratio:

$$S_\alpha = \frac{V_\alpha}{\rho_{\alpha}^*} \sum_{k=1}^{3} \frac{V_k}{\rho_k^*}, \alpha = 1, 2, 3.$$  \hspace{1cm} \text{(14)}

To problem solution of fluids and gas filtration on a limited formation volume, you must set the initial and boundary conditions.

As initial conditions, it is advisable to set the pressure distribution, compositions, and phase saturation:

$$P(x, y, z, o) = P^0(x, y, o) + \frac{H}{\rho_{\alpha}^*} \left( \sum_{\alpha=1}^{3} \rho_\alpha S_\alpha \right) g dz;$$

$$c_{k\alpha}(x, y, z, o) = c_{k\alpha}^0(x, y, z, o);$$

$$S_\alpha(x, y, z, o) = S_\alpha^0(x, y, z, o);$$
\[ x \in [0, L], \ y \in [0, B], \ z \in [0, H], \]

where \( P^\circ \) is the initial pressure on the reference surface.

Boundary conditions are determined, as a rule, by the impermeability conditions of the roof and sole of the oil formation. The lateral surfaces of the formation can be isolated (discharge, wedging) or can communicate with a marginal water basin. The property of conditional isolation of the lateral surfaces has a so-called symmetrical section of the formation, which is characterized by zero pressure gradient at the border with neighboring elements (there are no flows between the elements). The impermeability condition of surfaces is a limiting condition of the 2nd kind, it can be written as follows:

\[ q^*_\alpha = \frac{K_k \mu_\alpha}{\mu_a} \left( \frac{\partial P}{\partial n} \right)_G = 0, \quad \alpha = 1, 2, 3, \quad (16) \]

where \( q^*_\alpha \) is the phase flow \( \alpha \) through a certain surface \( G \).

At the contact of an oil formation with a strong aquifer, the condition described by the limiting condition of the 3rd kind is fulfilled:

\[ -a \frac{\partial P}{\partial n} \bigg|_{G=0} = b \langle P > -P \rangle \bigg|_{G=0}, \quad (17) \]

where \( a \) is the mobility ratio; \( b \) is the transmission coefficient; \( \langle P \rangle \) is the medium pressure in the strong aquifer.

Boundary conditions on the well can be of the 1st kind (set pressure) and the 2nd kind (set flow rate).

In the 1st case, using the well-known Dupuis formula, we can write:

\[ q^* = \frac{2nHK_k \chi}{\mu_a} \frac{p(r_c,t) - p(r_o,t)}{\ln(r_c/r_o)} , \quad \alpha = 1, 2, 3, \quad (18) \]

where \( \chi \) is the participation share of the well in the selected formation element; \( r_c \) is the radius of the downhole area (cell) [11]; \( r_c, r_o \) is the well radius (absolute and reduced, respectively).

In the second case the phase flow rates can be approximated by the formula:

\[ q^*_\alpha = q^* \frac{K_k}{\mu_a} \left( \sum_{\beta=1}^{n^3} \frac{K_k}{\mu_\beta} \right) \quad \alpha = 1, 2, 3, \quad (19) \]

where \( q^* \) is the total production rate (injection capacity) of the well.

The source density (flow) appears in an equation (1) is equal to the ratio of the flow rate (injection capacity) to the volume of the selected near-well area:

\[ q^*_\alpha = \frac{q^*_\alpha}{\pi(r_c^2 - r_e^2)H}, \quad (20) \]

The nonlinearity of 2nd-order partial differential equations describing the filtration of multicomponent mixtures, the three-dimensional problem formulation, and complex boundary conditions determine the possibility of problem solution only by numerical methods. One of the most effective of them is, in this case, the finite-difference method [9].

### 3. Results and Discussion

To solve this problem, a finite-difference scheme is constructed, which differs from the known ones [6, 9] in the following features. A completely conservative finite-difference scheme that approximates the filtration equations belongs to the tin “implicit pressure – explicit saturation”. It is calculated based on the effective Newton iteration method not only for pressure but also for the composition of the mixture. Also, the minimum derivative principle is used for approximating the mobility ratios, a ternary phase diagram for calculating the partial characteristics of phase equilibria, and a block for automatic step selection of the time integration. All this significantly improves the efficiency of the numerical model.

To predict the technological efficiency of oil formation development with terrigenous, carbonate (reef) reservoirs, the influence methods of hydrocarbon and carbon dioxide on the displacement of hard-to-recover and remaining oil reserves are considered.

Theoretical researches to determine the optimal parameters of the hydrocarbon gas exposure technology was performed on two formation models D_I and D_{II} of the Tuymazinskij fields-
without taking into account vertical flows of formation fluids. The research results after the corresponding generalization were as follows: the total flow gas 27-51% of reservoir void space, the specific technological efficiency of 0.31 tons of oil per 1 thousand m$^3$ of gas, the increase in ORF – 5.8–6.1 points.

Studies on the displacement of remaining oil of formations of the Vvedenovsky field have established that the maximum value of the displacement coefficient will reach 0.92 UF when: pressure build-up to the initial values of 13 MPa by pumping hydrocarbon gas, which will significantly restore the original physical and chemical properties of the oil; pumping the bank of the NGLs with a volume of 5.1 % of the pore volume; the displacement rate should not exceed the critical speed of phase separation.

Numerical modeling of the oil displacement process has shown that when dry gas is injected into the formation, the agent's influence covers almost the entire object. The central and middle parts of the object are produced until the residual oil saturation, and the peripheral parts are produced only partially. Calculations have shown that the technological efficiency will reach 2.1 t / t; the increase in ORF will be 9.5–10.1 points.

Determination of the optimal parameters of the CO$_2$+ water injection technology and calculation of the development parameters on the terrigenous reservoirs of the formations D$_{KH}$, D$_{I}$, D$_{II}$, D$_{IV}$ of the Sergeyevsky field under normal (basic) flooding and using the method is performed using the proposed numerical model. As a result of calculations, it is established that the ratio of bank volumes of CO$_2$ and water on average should be 1:1.23; the total flow of CO$_2$ is 14% of the reservoir void space. The application of the proposed method will increase the ORF by 8.3–9.7 points, the specific efficiency will be 0.51 t/t.

4. Conclusion

Thus, the mathematical model of gas injection on hard-to-recover and remaining oil reserves was adapted taking into account the geological and field conditions for applying the impact, which allowed reducing the amount of calculations.

The results of mathematical modeling of the oil displacement process from the porous medium of reservoir rocks of the Republic of Bashkortostan fields due to the use of gas injection are presented. The incremental value in recoverable reserves for Devonian production facilities is in the range of 5-10 %.

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