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An annual energy production estimation methodology for onshore wind farms over complex terrain using a RANS model with actuator discs

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Abstract.
This work presents a new methodology for the interpolation and extrapolation of the wind power generated by each wind turbine in a wind farm at different wind speeds for a given wind direction, running only three RANS-CFD simulations, two with and one without wind turbines. The wind turbines are modeled as uniformly loaded actuator discs with an adaptation for wake interaction cases. Three RANS turbulence models are used, standard $k-\varepsilon$, $k-\varepsilon-f_P$ and the realizable $k-\varepsilon$ model modified to account for Coriolis forces and with appropriate coefficients for the simulation of ABL flows. The turbulence models are validated against wake deficit and power output measurements from the wind turbine test site Wieringermeer. Then, the new methodology proposed to predict the wind turbine power is detailed and evaluated with simulation results and measurements for a large onshore wind farm, particularly for a strong wake interaction case. It is observed that the power predictions using the proposed strategy are in good agreement with the obtained from different wind velocity simulations and power measurements. The methodology is a useful tool for wind farm annual energy production estimation.

1. Introduction
The prediction of annual energy production (AEP) of a wind farm over complex terrain requires its simulation with Computational Fluid Dynamics (CFD) models, involving the resolution of the turbulent Atmospheric Boundary Layer (ABL) and the effects induced by wind turbines (WT), including wind speed deficit and interaction among wakes. Commonly, WT\textsc{s} are modeled as uniformly loaded actuator discs [1], an option which allows a compromise between a reasonable model accuracy and a low computational cost. It is well known that the standard $k-\varepsilon$ model overestimates Reynolds stresses [2] behind actuator discs, resulting on a significant underestimation of wind velocities. In the present work we use the $k-\varepsilon-f_P$ model [3], and the $k-\varepsilon$ realizable model proposed by Shih et al.[4] modified to account for Coriolis forces and with appropriate coefficients for the simulation of ABL flows [5]. These two models enhance the wake predictions of actuator discs respect to standard $k-\varepsilon$ model. The realizable model is known to enhance the wind simulation in complex terrain, and it does not need to calibrate any parameter to solve properly WT wakes. The CFD model has been implemented in Alya [6], an in-house High
Performance Computing (HPC) parallel solver that uses the stabilized finite element method. The mesh was generated by the inhouse mesh generator code, which is described in [7, 5].

The prediction of the AEP over an onshore wind farm using annual wind measurements in one meteorological mast requires the simulation at different wind directions and at different wind velocities. This is because the strength of the wake generated by a WT depends on the thrust coefficient $C_t$, which is related to the incoming wind speed. Murcia et al [8] developed a methodology to minimize the number of model evaluations required to predict the AEP based on polynomial chaos techniques. In the present work we propose a new methodology for the interpolation and extrapolation of the wind power generated by each WT in the wind farm at different wind mast velocities for a single wind direction. This methodology has the advantage that the wind farm with WTs is simulated using CFD at only two wind mast velocities for each wind direction. Later, a wind over terrain simulation without WT is also necessary to extrapolate the power output for higher wind velocities.

With respect to the organization of this work, in Section 2 is presented the RANS turbulence models and the actuator disc strategy. Then, in Section 3 is carried out a validation with wake and power output measurements from the wind turbine test site Wieringermeer. In Section 4 the interpolation and extrapolation methodology is presented and evaluated for a WT row. Finally, in Section 5 a large onshore wind farm over complex terrain is presented, showing WT power outputs for a wind mast direction corresponding to a strong wake interaction. The proposed methodology is used to obtain the WT power output for different wind mast velocities, comparing with measurements. It can be seen that this strategy is able to obtain the power output in complex wake cases for one wind mast direction and many wind mast velocities with a reasonably low error.

2. RANS models for ABL flows, governing equations.
Considering the flow as incompressible and isothermal (neutral stability), the modified $k$-$\varepsilon$ RANS model accounting for Coriolis forces and using the Apsley and Castro correction for the mixing length limitation [9] reads

\[ \nabla \cdot \mathbf{u} = 0 \quad (1) \]

\[ \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} - \nabla p + 2 \omega \times \mathbf{u} + \frac{1}{2} \frac{C_t}{\Delta} U^2_{\infty} n_d = 0 \quad (2) \]

\[ \frac{\partial k}{\partial t} + \mathbf{u} \cdot \nabla k - \nabla \cdot \left( \frac{\nu_t}{\sigma_k} \nabla k \right) + \frac{C_\mu k^2}{\nu_t} = P_k \quad (3) \]

\[ \frac{\partial \varepsilon}{\partial t} + \mathbf{u} \cdot \nabla \varepsilon - \nabla \cdot \left( \frac{\nu_t}{\sigma_\varepsilon} \nabla \varepsilon \right) + \frac{C_2 \varepsilon^2}{k} = C'_1 C_{\mu} S \quad (4) \]

\[ \nu_t = C_{\mu} \frac{k^2}{\varepsilon} \quad (5) \]

where the unknowns are the velocity field $\mathbf{u}$, pressure $p$, turbulent kinetic energy $k$, dissipation rate of turbulent kinetic energy $\varepsilon$, and turbulent viscosity $\nu_t$ (computed with the diagnostic equation (5)). The fifth term on the left hand side (LHS) of momentum equation (2) models the Coriolis force $\omega$ being the Earth’s angular velocity. The sixth term on the LHS of equation (2) is the actuator disc force, which is active only inside the disc volume, where $C_t$ is the thrust coefficient, $U_{\infty}$ is the free-stream velocity at hub height, $n_d$ is the disc normal unit vector (pointing opposite to inflow), and $\Delta$ is the thickness of the disc, around 6% of the disc diameter [1]. The forces inside each disc volume are uniformly distributed, like an step function. In the turbulence equations (3)-(4), the term $P_k = \nu_t S$ is the kinetic energy production, with $S = \nabla^* \mathbf{u} : \nabla^* \mathbf{u}$ (denotes the strain tensor). The $k$-$\varepsilon$ model coefficients used in the present
implementation were proposed by Panofsky and Dutton [10] for the solution of atmospheric flows, being $C_\mu = 0.0333; C_1 = 1.176; C_2 = 1.92; \sigma_k = 1.0; \sigma_\varepsilon = \kappa^2/(C_\mu^{1/2}(C_2 - C_1))$, where $\kappa$ is the von Karman constant.

The coefficient $C_1'$ in the RHS of the dissipation equation (4) is a modified coefficient, originally proposed by Apsley and Castro [9], to prevent the increase of mixing length $l_m = C_\mu^{3/4}k^{3/2}/\varepsilon$ above a maximum value $l_{\text{max}}$ when accounting for Coriolis forces.

### 2.1. Realizable $k-\varepsilon$ model with Coriolis forces

The realizable $k-\varepsilon$ model proposed by Shih et al. [4] presents the advantage of satisfying realizability conditions on the Reynolds stresses. This model is known to improve the accuracy of flows involving detachment and re-circulations, and specifically, to enhance the prediction of wakes when using actuator disc models [1]. These properties make the Realizable model optimal for the simulation of onshore wind farms. The realizable model shares the same turbulent kinetic energy equation with the standard $k-\varepsilon$ model. However, differences exist for the dissipation rate equations. In order to use model coefficients appropriate for ABL flows the coefficients in the dissipation equation have been modified. The dissipation equation is also modified to account for Coriolis forces using the Apsley and Castro limitation model. The equations of the used realizable model are described in [5].

### 2.2. Actuator disc model

The force exerted by the WT over the flow is modeled as a uniformly loaded disc by means of the force term $\frac{1}{2} \rho \omega^2 \pi \delta^2 \mathbf{n}_d$ in the momentum equation (2). The thrust coefficient $C_t$ is supplied by the manufacturers as a thrust coefficient curve depending on the undisturbed wind velocity $U_\infty$. However, in the case of wind farms, there is no obvious approach to estimate the free stream velocity $U_\infty$ (and therefore $C_t$) because the WT power and thrust curves are usually provided for single-machine operation rather than operation in the wake of another WT. In the present work the free stream velocity $U_\infty$ is obtained using a calibration procedure of single WT wake simulations, tabulating the obtained local velocity average $U_{\text{disk}}$ around the actuator disk volume for different free stream wind velocities $U_\infty$. This procedure is similar to the variable scaling method proposed by van der Laan et al. [11]. The use of this calibration $U_\infty-U_{\text{disk}}$ procedure allows to calculate the thrust exerted by the WT as function of the local velocity average $U_{\text{disk}}$, avoiding the errors associated with the use of one dimensional momentum theory.

### 3. Validation of the method simulating Wieringermeer test site

The purpose in this section is to validate the turbulence models presented in the previous section, standard $k-\varepsilon$, $k-\varepsilon-f_p$ and realizable $k-\varepsilon$, and the inclusion of the Coriolis effect, comparing with wind mast and WT power output measurements. Its presented three cases from the wind turbine test site Wieringermeer, explained in detail in [11, 12]. It consists in a meteorological mast and five 2.500 kW Nordex N80 WT, with a hub height and diameter of 80 m, aligned with wind direction 275°. The Eastern and Western cases correspond to wind measurements inside a single wake for different downwind distances and ambient turbulence intensities at hub height ($I_{\text{CFD,H,} \infty} = \sqrt{2k/3}/U$) for the CFD, detailed in Table 1. The Farm case, Table 2, corresponds to the power output for the WT when aligned with the wind direction, being important because this is the lower power output case for a WT row arrangement in a wind farm.

The measurements and simulations results are shown in Figure 1, with wake values centered in the main wind direction, for the Eastern and Western cases. It can be noticed that without Coriolis force the standard $k-\varepsilon$ has the worst accuracy, showing a weaker wake, while $k-\varepsilon-f_p$ and realizable $k-\varepsilon$ models have similar results and a good agreement with measurements. When Coriolis force is considered the three turbulence models have a larger velocity deficit and the
wind velocity distributions get closer to the data, being an important improvement specially for the standard \( k-\varepsilon \) model. The main contribution of modeling Coriolis forces is the global turbulence length scale limiter which delays the wake recovery, this is also shown in [13]. The global mixing length scale limiter of Apsley and Castro is double counting the limitation of eddy viscosity introduced by the \( k-\varepsilon \)-\( f_P \) and realizable models through the \( f_P \) limiter and the modified \( C_{\mu} \) coefficients respectively. The authors have not observed significant differences in the obtained results when switching off the global scale limiter in the wake region to avoid double counting, as proposed in [13].

In Figure 2 are compared the different models for the farm case, in which the power output measurements for the perturbed downwind WT are similar, with a lower power output for the second WT. In the results corresponding to the turbulence models without Coriolis forces is seen that \( k-\varepsilon \)-\( f_P \) model shows the best agreement with measurements, while the standard \( k-\varepsilon \) model overestimates the power output for all the WTs and realizable \( k-\varepsilon \) has a tendency to underestimate the power output for the most downwind WTs. When Coriolis forces are introduced, the result for the first perturbed WT is improved, but the power outputs of the downwind WTs are underestimated. The \( k-\varepsilon \)-\( f_P \) model without Coriolis forces shows the best agreement against measurements, both for single and multiple WTs. In the next section the RANS models will be run without Coriolis forces.

4. Methodology for the prediction of wind turbine power at different wind velocities

On an onshore wind farm with \( N \) wind turbines, when the wind mast velocity is \( U_{\text{mast}} \) each wind turbine \( i \) has a free stream velocity \( U_{\infty,i} \). The WT power is \( P_i(U_{\infty,i}) = \frac{1}{2} \rho C_{p,i} U_{\infty,i}^3 A_i \), where \( \rho \) is the air density, and \( C_{p,i} \) and \( A_i \) are the power coefficient and the swept area of the rotor of WT \( i \) respectively. Therefore, the wind power of each WT can be determined if we know the free stream velocity \( U_{\infty,i} \). To obtain the annual energy production of a wind farm it is necessary to know the wind rose distribution at site. The wind distribution function can be obtained from mast measurements during one or more years. Let \( f(U_{\text{mast}}, \theta) \) be the wind velocity and directional distribution function at mast, where \( U_{\text{mast}} \) is the mast velocity modulus and \( \theta \) the wind direction, normalized such \( \int_0^\infty \int_0^{2\pi} dU_{\text{mast}} d\theta f(U_{\text{mast}}, \theta) = 1 \). Then, the mean annual wind power generated by WT \( i \) depends on wind mast velocity distribution, integrating

### Table 1. Parameters for the Eastern and Western Wieringermeer test cases

| Test case | Main wind direction (°) | Downwind distance (D) | \( U_{H,\infty} \) (m/s) | \( I_{CFD,H,\infty} \) |
|-----------|-------------------------|-----------------------|------------------------|----------------|
| Eastern   | 31                      | 2.5                   | 10.9                   | 0.06          |
| Western   | 315                     | 3.5                   | 10.7                   | 0.08          |

### Table 2. Parameters for the farm Wieringermeer test cases

| Measurement Data (°) | Spacing (D) | \( U_{H,\infty} \) (m/s) | \( I_{CFD,H,\infty} \) |
|----------------------|-------------|------------------------|----------------|
| 275 +/- 15           | 3.8         | 8.35                   | 0.1          |
Figure 1. Wieringermeer Eastern (top) and Western (bottom) test cases. Results with (right) and without (left) Coriolis force.

Figure 2. Wieringermeer Farm test case. Results with (right) and without (left) Coriolis force.

all mast velocities and directions as.

\[ P_i = \int_0^\infty dU_{\text{mast}} \int_0^{2\pi} d\theta P_i(U_{\infty,i}) f(U_{\text{mast}}, \theta) \]

Therefore, to obtain the annual energy production, the wind farm needs to be simulated for several wind directions and several wind velocities. The wake characteristics of each WT depends on its thrust coefficient \( C_t \), which strongly depends on wind velocity.

In order to save computational time, we propose a methodology to interpolate and to extrapolate the dependence of the free stream velocity \( U_{\infty,i} \) in terms of mast velocity \( U_{\text{mast}} \).

For each wind direction \( N_{VEL} \) CFD wind simulations are run with different wind mast velocities \( U_{\text{mast},CFD,k} \) (with \( k = 1 \) to \( N_{VEL} \)). The obtained free stream wind velocity for
each WT $i$ corresponding to the CFD simulation with mast velocity $U_{\text{mast},C F D,i,k}$ is denoted as $U_{\infty,C F D,i,k}$. And the obtained speed ups are denoted as $spd_{C F D,i,k} = U_{\infty,C F D,i,k}/U_{\text{mast},C F D,k}$

The free stream velocity of a WT $i$ can be expressed as

$$U_{\infty,i} = \frac{U_{\infty,i}}{U_{\text{mast}}}$$

(6)

Therefore, when the speed up function $spd_i(U_{\text{mast}}) = U_{\infty,i}/U_{\text{mast}}$ is known for each WT in terms of mast velocity, the generated wind power can be determined. The speed up function $U_{\infty,i}/U_{\text{mast}}$ depends on how the WT is affected by the wake of another WT. When the speed up of a WT is only affected by terrain and not by the wake of another WT, it is a good approximation to consider a constant speed up $U_{\infty,i}/U_{\text{mast}}$. Therefore when no wake interaction exists it is a good approximation to have $N_{C F D} = 1$ and to consider $U_{\infty,i}/U_{\text{mast}} = U_{\infty,C F D,i}/U_{\text{mast},C F D}$ for all $U_{\text{mast}}$ to find the free stream velocity using Eq. (6).

When the WT are placed in the wake of another WT the dependence of the speed up $spd_i$ is not known a priori. In Figure 3 is shown the speed up function $spd_i = U_{\infty,i}/U_{\text{mast}}$ for a selection of 5 WT in a 16 turbines row separated by 2 diameters. The mast is located 2 diameters upstream the first WT and no terrain height difference is placed between the mast and the WTs. In Figure 3 is also plotted the speed up function $spd_i$ for a selection of 5 WTs in a 16 row but separated by a larger distance of 5 diameters. The modeled WTs are a General Electric model with a diameter $D = 77$ m and a maximum power of 1500 kW, with a thrust coefficient $C_t > 1$ for free stream velocities lower than 4 m/s.

The speed ups are plotted in terms of the mast velocity and the free stream velocity of each WT. It is seen that the first turbine $T1$ has a constant speed up equal to one. The WT placed downwind have a speed up dependence that is flat at lower wind velocities and then increases with wind velocity. The speed up function is different for each WT, but all of them have a similar pattern. This pattern gets more similar between the different WTs when plotting against the free stream velocity of each WT (right of Figure 3).

The proposed interpolation and extrapolation method to calculate the speed up dependence of each WT for different mast velocities is developed for 3 CFD runs for each wind direction $N_{C F D} = 3$. One CFD run is done without WTs and the other two with WTs at different wind velocities. The CFD results are denoted with subindexes $i = 1, 2$ when simulating WTs. The subindex $i = 1$ refers to the lower velocity, and $i = 2$ to the higher velocity. The subindex $i = 3$ is employed for the simulation without WTs. This last simulation without WTs is computationally less expensive. It is assumed that the asymptotic value for the speed up of each WT at high wind velocities is the speed up without WTs, $spd_i(U_{\text{mast}} \to \infty) = spd_{C F D,i,3}$. This speed up is calculated as $spd_{C F D,i,3} = U_{\text{hub},i}/U_{\text{mast}}$ the ratio between wind velocity at hub height of WT $i$ and mast velocity.

The proposed interpolation and extrapolation method to calculate the speed up of each WT $i$ for a mast velocity $U_{\text{mast}}$ reads:

$$spd_i = \begin{cases} 
spd_{C F D,i,1} & U_{\infty,i} < U_{\infty,C F D,i,1} \\
spd_{C F D,i,1}(U_{\infty,C F D,i,2} - U_{\infty,i}) + spd_{C F D,i,2}(U_{\infty,i} - U_{\infty,C F D,i,1}) & U_{\infty,C F D,i,1} < U_{\infty,i} < U_{\infty,C F D,i,2} \\
spd_{C F D,i,2}(U_{\infty,C F D,i,3} - U_{\infty,i}) + spd_{C F D,i,3}(U_{\infty,i} - U_{\infty,C F D,i,2}) & U_{\infty,C F D,i,2} < U_{\infty,i} < U_{\infty,thres} \\
spd_{C F D,i,3} & U_{\infty,i} > U_{\infty,thres}
\end{cases}$$

The method is iterative to interpolate in terms of the obtained $U_{\infty,i}$ (using (6)).

We apply the proposed methodology to predict the wind power in an array of 16 wind turbines separated by 2 diameters. This short distance is used due to the high wake effect.
Figure 3. Speed up function $U_{\infty}/U_{\text{mast}}$ in terms of $U_{\text{mast}}$ (left) and $U_{\infty}$ (right), for a selection of five wind turbines in a array of 16 wind turbines separated by 2 (top) and 5 (bottom) diameters.

In Figure 4 is shown how the proposed methodology is applied to predict the wind power of the three selected turbines $T_2$, $T_9$ and $T_{16}$. These three WTs cover different wake behavior in the WT array. The CFD results used to interpolate the wind power were obtained with mast velocities $U_{\text{mast,CFD,1}} = 7$ m/s and $U_{\text{mast,CFD,2}} = 13$ m/s, and the threshold velocity was set to $U_{\infty,\text{thres}} = 21$ m/s. The used RANS model is the $k$-$\varepsilon$-$f_P$ without Coriolis forces. In the plots on the top of Figure 4 is observed that the interpolated speed ups differ from the previously simulated. However, in the plots on the bottom of Figure 4 it is observed that the interpolated wind power matches very well the obtained using CFD simulation. The error of the present methodology between the interpolated power $P_{k,pred}$ and the CFD power $P_{k,CFD}$ for each velocity $U_{\text{mast,CFD,k}}$ is larger for the last WT of the row, $T_{16}$, being

$$L_2\text{-err} = \left(\frac{\sum_k (P_{k,pred} - P_{k,CFD})^2}{\sum_k P_{k,CFD}^2}\right)^{1/2} = 0.019$$

that is, only 1.9% applying the proposed methodology.

The proposed methodology is tested to predict the wind power in an array of 16 WTs, now separated by 5 diameters. The obtained interpolation values are shown in Figure 5. The CFD mast velocity values used for the interpolation are the same that were used in the previous test, $U_{\text{mast,CFD,1}} = 7$ m/s and $U_{\text{mast,CFD,2}} = 13$ m/s. The threshold velocity is $U_{\infty,\text{thres}} = 21$ m/s. On the top of Figure 5 it is observed that the interpolated speed ups differ from the simulated. However, the predicted power output matches very well with the obtained using CFD simulations for all wind velocities. The higher $L_2$-error between the interpolated and the CFD power occurs for the second WT $T_2$, with an error $L_2\text{-err} = 1.0\%$. When the turbines are separated by a distance of 5 diameters they are subjected to a lower wake effect, obtaining
Figure 4. Obtained speed up (top) and wind power (bottom) for different wind velocities, using the proposed methodology applied to three WTs in a 16 WT array separated by 2 diameters.

larger speed ups (comparing Figure 4 and Figure 5). In this lower wake situation the proposed methodology yields a lower error.

5. Wind power prediction of each wind turbine for an onshore wind farm
In this section the methodology proposed in section 4 is applied to an onshore wind farm located in Spain. The wind farm is composed of 165 WTs with 77 m diameter and a hub height of 80 m. The computational domain is $17 \times 14$ km$^2$. The mesh was generated by the inhouse mesh generator code, which is described in [7, 5]. The topography is meshed with a resolution of 25 m. The generated mesh is composed by 16.5 Million elements (hexahedra, tetrahedra and pyramids). The first vertical element has a 1 m height, and the boundary layer has a geometrical growing ratio of 1.15. The actuator discs have been meshed with 16 hexahedral elements per diameter, and a thickness of 5.0 m ($\approx 6.5\%$ of diameter) to obtain a negligible mesh error.

On the left of Figure 6 is depicted the wind farm layout. The mesh topography, and some Paraview results are shown and discussed in [5]. Large arrays of WTs are located in the farm running from North to South, with a distance between WTs of around 2 diameters. On the left of Figure 6 the first WT array is named L01. In order to select a proper RANS model the wind farm is simulated for a wind direction of $5^\circ$. On the right of Figure 6 are shown the wind power results obtained using three RANS models without Coriolis forces along the L01 WT array, composed by 33 WTs. The obtained results are compared against power measurements when the wind at mast has a direction of $5^\circ \pm 5^\circ$ N, and a mast velocity of $8 \pm 0.5$ m/s. It is observed
Figure 5. Obtained speed up (top) and wind power (bottom) for different wind velocities, using the proposed methodology applied to three WTs in a 16 WT array separated by 5 diameters.

Figure 6. Wind farm layout (left) turbine power in the large wind farm array L01 (right). Comparison between the three proposed $k$-$\varepsilon$ turbulence models and power measurements for wind direction $5^\circ \pm 5^\circ$ and wind velocity $8 \pm 0.5$ m/s at mast.

on the right of Figure 6 that the standard $k$-$\varepsilon$ model gives the less accurate results and that the $k$-$\varepsilon$-f$_P$ model gives the most accurate results, when comparing against power measurements. All models fail after the WT 20, where the array turns its direction. It is known that any RANS model is able to model with reasonably accuracy the wind velocity 2 diameters downwind the rotor.

The proposed methodology in section 4 is applied to the wind farm for a wind direction of
Figure 7. Obtained speed up (top) and wind power (bottom) for all the wind turbines in a wind farm at two different wind velocities. The obtained results using the proposed and a constant speed up methodologies are compared against measurements and CFD data.

5°, when the wakes have a higher effect over the WTs. The authors only have access for the power data of 132 WTs. On the Figure 7 is shown the speed up and wind power for all the WTs in the wind farm for mast velocities $U_{\text{mast,1}} = 8.7 \text{ m/s}$ and $U_{\text{mast,2}} = 17.7 \text{ m/s}$. This figure compares the predicted speed up and wind power against CFD results using the current methodology and the constant speed up approximation. Only for the wind mast $U_{\text{mast}} = 8.7 \text{ m/s}$ the power output measurements are shown, due to the lack of data for high wind velocities at this wind direction. The simulated mast velocities used for the interpolation method are $U_{\text{mast,CFD,1}} = 6.51$ and $U_{\text{mast,CFD,2}} = 10.92$, another simulation was carried out without WTs to approximate the threshold velocity behavior when $U_{\text{infty,thres}} = 21 \text{ m/s}$. The methodology using the constant speed up assumes that the speed up is the same as the CFD speed up when mast velocity is $U_{\text{mast,CFD,2}} = 10.92$.

It is observed that for the lower wind mast velocity $U_{\text{mast,1}} = 8.7 \text{ m/s}$, the proposed and the constant speed up methodologies obtain a speed up in close agreement with the CFD results, giving both methodologies a similar power output, which are in good agreement with data measurements. When the wind mast velocity is larger, $U_{\text{mast,2}} = 17.7 \text{ m/s}$, the obtained speed up using the proposed methodology and the CFD data have larger values showing a lower wake effect. The proposed methodology obtains wind power values that are much closer to CFD results than those obtained using the constant speed up approximation. Note that the maximum power error is around 5% using the proposed methodology and around 25% when using the constant speed up approximation method.
6. Conclusions and future work

The accuracy of the standard $k$-$\varepsilon$, $k$-$\varepsilon$-$f_P$ and the modified realizable $k$-$\varepsilon$ models have been evaluated comparing the obtained wind velocity and wind power values against measurements from the wind turbine test site Wieringermeer. When the RANS models account for the Coriolis force, the mixing length limitation model produces too deep wakes and the results are less accurate than when Coriolis force is not considered. The $k$-$\varepsilon$-$f_P$ model obtained the most accurate results when compared against measurements for single and multiple wake interaction cases, and for a wind farm sited over moderate complex terrain.

A new methodology for the interpolation and extrapolation of the wind farm power output at different wind velocities has been presented. The proposed methodology uses only three CFD simulations per wind direction, one of them without WTs. The methodology has been evaluated against simulation results in a row of 16 WTs with separations of 2 and 5 diameters, having an $L_2$-error of less than 2 % accounting for all wind velocities.

Finally, the power prediction methodology is compared against wind farm power measurements and CFD simulations for different wind velocities and a single wind direction. The power predictions were in good agreement with the obtained from CFD simulations and power measurements. The greatest error was around 5%, showing the capability of the proposed methodology to accurately estimate the wind power of the wind farm in terms of the wind speed. Due to the good agreement with CFD results and the reduced computational cost, the proposed methodology is presented as a potential useful tool for the prediction of annual energy production of wind farms sited over complex terrain. For future works, it is necessary to evaluate the presented methodology for the estimation of the annual energy production of the wind farm, running different wind directions and comparing against annual measurements.

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References

[1] Cabezón D, Migoya E and Crespo A 2011 Wind Energy 14 909–921
[2] Réthoré P E 2009 Wind Turbine Wake in Atmospheric Turbulence Ph.D. thesis Riso National Laboratory for Sustanaible Energy Technical University of Denmark
[3] van der Laan M P, Sørensen N N, Réthoré P E, Mann J, Kelly M C, Trolldborg N, Schepers J G and Machefaux E 2015 Wind Energy 18 889–907 ISSN 1099-1824 URL http://dx.doi.org/10.1002/we.1736
[4] Shih T H, Lio W W, Shabbir A, Yang Z and Zhu J 1995 Computers & Fluids 24 227–238
[5] Avila M, Gargallo-Peiro A and Folch A 2017 Journal of Physics: Conference Series 854 012002
[6] Vazquez M 2016 Journal of Computational Science 14 15 – 27 the Route to Exascale: Novel Mathematical Methods, Scalable Algorithms and Computational Science Skills
[7] Gargallo- Peiró A, Avila M, Owen H, Prieto L and Folch A 2015 Procedia Engineering 124 239–251
[8] Murcia J P, Réthoré P E, A N and D S J 2015 Journal of Physics: Conference Series 625 012030
[9] Apsley D and Castro I 1997 Boundary-Layer Meteorology 83 75–98
[10] Panofsky H A and Dutton J 1984 Atmospheric turbulence, models and methods for engineering applications (Wiley)
[11] van der Laan M P, Sørensen N N, Réthoré P E, Mann J, Kelly M C and Trolldborg N 2015 Wind Energy 18 2223–2240 ISSN 1099-1824
[12] van der Laan P M, Sørensen N N, Réthoré P E, Mann J, Kelly M C, Trolldborg N, Hansen K S and Leon J P M 2015 Wind Energy 18
[13] van der Laan M P, Hansen K S, Sørensen N N and Réthoré P E 2015 Journal of Physics: Conference Series 625 012026