Fitting to data of superluminal neutrinos with phenomenological scenarios

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We test several phenomenological scenarios of faster-than-light neutrinos, by fitting to the experimental data from OPERA, MINOS and Fermilab\textsuperscript{79}. Our purpose is to see, from the perspective of the current data, whether or not the speed of the superluminal neutrino depends on its energy. We show that the Coleman-Glashow scenario in which the velocity of the neutrino is free of the energy fits the data best. However, the result of SN1987A cannot be explained by this model. We find that a power-law model with the power close to zero can simultaneously explain the results of SN1987A and OPERA+MINOS+Fermilab\textsuperscript{79}.

The OPERA collaboration reported the superluminal muon neutrino ($\nu_\mu$) data \cite{1}, recently. The neutrinos with average energy $\sim 17$ GeV arrive earlier by
\begin{equation}
\delta t = (60.7 \pm 6.9 \text{ (stat.)} \pm 7.4 \text{ (sys.)}) \text{ ns}
\end{equation}
\textsuperscript{(1)}
than photons, from CERN to Gran Sasso Laboratory with distance about 730 km. This indicates that neutrinos are superluminal, with an excess of the speed than light (in vacuum)
\begin{equation}
v - 1 = (2.48 \pm 0.28 \text{ (stat.)} \pm 0.30 \text{ (sys.)}) \times 10^{-5},
\end{equation}
\textsuperscript{(2)}
with significance level of 6$\sigma$. Note that throughout the paper we use the natural units with $c = 1$. It was also found by OPERA that the velocity difference $v - 1$ is almost independent of energy, by dividing the events into two groups with energies below or above 20 GeV: $v - 1 = (2.16 \pm 0.76 \text{ (stat.)} \pm 0.30 \text{ (sys.)})$ for $\langle E \rangle = 13.9$ GeV and $v - 1 = (2.74 \pm 0.74 \text{ (stat.)} \pm 0.30 \text{ (sys.)})$ for $\langle E \rangle = 42.9$ GeV.

It should also be pointed out that earlier experiments ever obtained similar results of superluminal neutrinos, though with lower significance. For example, the MINOS experiment at Fermilab in 2007 \cite{2} and even earlier experiments at Fermilab in 1979 \cite{3} reported the results of $v - 1 \sim 10^{-5}$, similar to that of OPERA. These results are summarized in Table I. However, the supernova neutrinos from SN1987A place a stringent velocity constraint, $v - 1 < 2 \times 10^{-9}$, for electron neutrinos with energies from 5 to 40 MeV, seemingly inconsistent with the superluminal neutrino result of OPERA.

At first glance, the result of that the neutrino travels faster than light is too shocking, and many people instinctively reject such a result. Indeed, there might be some systematic errors or some other unknown factors in the experiment that have not been taken into account. For example, it was suspected that there may be some serious problems in the measurement of time and distance in the experiment \cite{4}. And, in the theoretical respect, Cohen and Glashow \cite{5} argued that superluminal neutrinos with high energies would lose energy rapidly via bremsstrahlung of electron-positron pairs, causing the beam to be depleted of higher energy neutrinos, and so they refuted the superluminal interpretation of the OPERA result. However, on the other hand, if the nature of neutrinos is indeed superluminal, then the modern physics would be revised enormously. Thus, before the independent experiments, such as BOREXINO, ICARUS, and MINOS, confirm (or negate) the OPERA result, it is a rational way for us to suppose that the data and conclusion.

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TABLE I: The summary of superluminal neutrinos from OPERA, MINOS, and Fermilab79.

| Experiment    | Velocity constraint | Energy range | Reference |
|---------------|---------------------|--------------|-----------|
| OPERA         | $v - 1 = (2.48 \pm 0.58) \times 10^{-5} (6.0\,\sigma)$ | $\sim 17$ GeV | 1         |
| MINOS         | $v - 1 = (5.1 \pm 2.9) \times 10^{-5}$ (68% CL) | $\sim 3$ GeV | 2         |
| Fermilab79    | $|v - 1| < 4 \times 10^{-5}$ (95% CL) | 30 to 200 GeV | 3         |

of OPERA are right and try to look for the possible clues of the new revolution of physics via the superluminal neutrino experiments.

In recent days, many theoretical explanations for the superluminal neutrinos have been put forward. For example, it has been suggested that such a phenomenon might originate from a violation of Lorentz invariance [6–9]. Nevertheless, it is hard to understand why the Lorentz violation happens at such low energies, far below the energy scale of quantum gravity. Tachyonic neutrino scenario is another attractive explanation. However, this scenario is clearly disfavored by the data [6]. Besides, there are also many other possible mechanisms, such as, extra-dimension model [10], neutrino dark energy model [11], and so on.

One important point should be clarified, in our opinion, before we make deeper theoretical analysis: whether or not the superluminal phenomenon of neutrinos is energy-dependent? In fact, it was proposed by Li and Wang [7] that the Lorentz violation might be energy-dependent, only occurring in a window of particular energies (including $3 - 200$ GeV), thus reconciling the results of OPERA and SN1987A. In addition, Amelino-Camelia et al. [6] also tackled the issue of the energy-dependence of the speed of neutrinos. In [6] several cases of violation or deformation of Lorentz invariance were considered, and the authors showed that the Coleman-Glashow (CG) scenario and the Doubly Special Relativity (DSR) scenario with linear-dependence are more favored, while the tachyon scenario and DSR model with quadratic-dependence are clearly disfavored, by the data. Of course, it seems that the data-favored models are inconsistent with the result of SN1987A. In this paper, we will make a more detailed analysis of current data with some phenomenological scenarios. We will show whether the superluminal phenomenon of neutrinos is energy-dependent, according to the current experimental data, and which phenomenological scenario can fit the data well and be consistent with the result of SN1987A at the same time.

We show the data of OPERA+MINOS+Fermilab79 in the left panel of Fig. 1. As discussed in [3, 6], when using the data of Fermilab79, a bias correction, downward shift of data with $b_{1979} = 0.5 \times 10^{-4}$, should be considered. The data with the above correction (for Fermilab79) are shown in the right panel of Fig. 1.

In the ultra-relativistic regime, the Einstein’s mass-energy relation gives the following description of the dependence of speed on energy:

$$v - 1 = -\frac{1}{2} \frac{m^2}{E^2}.$$  \hspace{1cm} (3)

Obviously, in the theory of relativity a normal particle with real mass $m$ can never travel faster than light in the vacuum. Now, let us estimate the order of magnitude of the velocity difference for the neutrino by using Eq. (3). If the mass of neutrino is supposed to be $m \sim eV$ and the neutrino energy $E$ under consideration ranges from 3 GeV to 200 GeV, we obtain $m^2/2E^2 \sim 10^{-18} - 10^{-22}$. So, even if we consider a tachyon model in which the neutrino has an imaginary mass, $M^2 = -m^2$, the theoretical value obtained is many order of magnitude less than the experimental result, $O(10^{-5})$.

Supposing that there is a theoretical mechanism that can explain the experiments such as OPERA, the above relation can be modified, phenomenologically, as

$$v(E) - 1 = -\frac{1}{2} \frac{m^2}{E^2} + f(E) \approx f(E),$$  \hspace{1cm} (4)
FIG. 1: The superluminal neutrino data from OPERA, MINOS and Fermilab79. The left panel shows the raw data, and the right panel shows the data corrected by \(b_{1979} = 0.5 \times 10^{-4}\) for Fermilab79.

where the form of \(f(E)\) is determined by the underlying theoretical mechanism that is unknown for us now but can be modeled phenomenologically. We can view that \(f(E)\) comes from a Lorentz violation, though we cannot understand why the energy scale of such a Lorentz violation is so low. Alternatively, we can even consider a mass-running tachyonic neutrino scenario with mass \(\mathcal{M}(E)^2 = -m(E)^2 = 2E^2 f(E)\).

In what follows, we take a totally phenomenological perspective, assuming the possible forms of \(f(E)\). First, we consider the possible form inspired by Coleman and Glashow [12, 13],

\[
f(E) = \delta,
\]

where \(\delta\) is a constant parameter. Also, we follow Amelino-Camelia et al. [6], taking the DSR-type cases, \(f(E) = \ell_1 E\) and \(f(E) = \ell_2 E^2\). We, furthermore, consider a more general form, a combination of the above possible cases,

\[
f(E) = \alpha E^2 + \beta E + \delta.
\]

We will use the superluminal neutrino data currently available, namely, the combination of OPERA, MINOS and Fermilab79, to tell us which scenario works well. To fit the data, we use a \(\chi^2\) statistic,

\[
\chi^2(\theta) = \sum_{i=1}^{11} \frac{(v_{\text{mod}}(E_i; \theta) - v_{\text{exp}}(E_i))^2}{\sigma(E_i)^2},
\]

where \(\sigma(E_i)\) is the 1\(\sigma\) error for each datum of \(v(E_i)\), and \(\theta\) denotes the model parameters. By minimizing \(\chi^2\), we can find the best-fitted parameters of the models, and further obtain the probability contours in the parameter-planes.

For the CG scenario, we get \(\chi^2_{\text{min}} = 56.94\). The DSR-type cases, \(f(E) = \ell_1 E\) and \(f(E) = \ell_2 E^2\), are both clearly disfavored by the data, leading to much bigger \(\chi^2_{\text{min}}\), being 195.46 and 368.18, respectively. The combination form, \(f(E) = \alpha E^2 + \beta E + \delta\), gives the least \(\chi^2_{\text{min}}\), 56.77, among these scenarios. However, it is unwise to use the \(\chi^2\) statistic alone to compare competing models since the number of parameters is different for the models. In general, a model with more parameters tends to give a lower \(\chi^2_{\text{min}}\), so instead one may employ the information criteria (IC) to assess different models. In this paper, we use the BIC (Bayesian information criterion) and AIC (Akaike information criterion) as model selection criteria, defined respectively as

\[
BIC = \chi^2_{\text{min}} + k \ln N, \quad AIC = \chi^2_{\text{min}} + 2k;
\]
TABLE II: The fit results of the phenomenological scenarios.

| $f(E)$ scenario | $\chi^2_{\text{min}}$ | $k$ | $\Delta\text{BIC}$ | $\Delta\text{AIC}$ |
|-----------------|-----------------------|-----|---------------------|---------------------|
| $\delta$        | 56.94                 | 1   | 0                   | 0                   |
| $\ell E$        | 195.46                | 1   | 138.52              | 138.52              |
| $\ell^2 E^2$    | 368.18                | 1   | 311.24              | 311.24              |
| $\alpha E^2 + \beta E + \delta$ | 56.77 | 3 | 4.62 | 3.83 |
| $\xi(E/\text{GeV})^\lambda$ | 56.78 | 2 | 2.24 | 1.84 |

where $k$ is the number of parameters, and $N$ is the data points used in the fit. Note that the absolute value of the criterion is not of interest, only the relative value between different models, $\Delta\text{BIC}$ or $\Delta\text{AIC}$, is useful. For the details about the BIC and AIC, especially their applications in a cosmological context, see, e.g., [14, 15]. According to the fitting result, we see that the CG scenario has the minimal values of BIC and AIC. The scenario of $f(E) = \alpha E^2 + \beta E + \delta$ yields $\Delta\text{BIC} = 4.62$ and $\Delta\text{AIC} = 3.83$, relative to the CG case; see Table II. So, according to the principle of Occam’s razor, “entities must not be multiplied beyond necessity,” the CG scenario, $f(E) = \delta$, is preferred by the data. Figure 2 shows the fit result of the CG scenario, with the left panel the one-dimensional likelihood distribution of the parameter $\delta$, and the right panel the best-fitted case of $v(E)$ comparing to the data. Figure 3 shows the fit result of the scenario of $f(E) = \alpha E^2 + \beta E + \delta$. From this figure, we can see that, though the parameters $\alpha$ and $\beta$ are both around zero at the best fit, there are some evident degeneracies between the parameters. From the probability contours in the parameter-planes, we find that, $\alpha$ and $\beta$ are anti-correlated, $\beta$ and $\delta$ are also anti-correlated, and so $\alpha$ and $\delta$ are in positive correlation. The best-fitted $v(E)$ curve is similar to a horizontal line, but with a slight slope in the right-hand region.

![Figure 2: Likelihood distribution of parameter $\delta$ for the Coleman-Glashow scenario, $f(E) = \delta$.](image)

While the CG scenario can fit the superluminal neutrino data well, it cannot simultaneously explain the result of SN1987A. Can we find a phenomenological scenario that can reconcile the SN1987A with the superluminal neutrino experiments such as OPERA, MINOS and Fermilab79? For this purpose, we test the following power-law scenario,

$$f(E) = \xi E^\lambda,$$

where the energy $E$ is in units of GeV. We now fit to the experimental data of OPERA, MINOS and Fermilab79 with this model. From Table II we see that the power-law model yields a lower
FIG. 3: Probability contours at 68.3%, 95.4% and 99.7% confidence level in $\alpha - \beta$, $\alpha - \delta$ and $\beta - \delta$ planes for the scenario, $f(E) = aE^2 + bE + \delta$.

$\chi^2_{\text{min}}$, 56.78, than that of the CG model. However, when considering the number of parameters, the power-law model produces slightly larger BIC and AIC values than the CG model, namely, $\Delta \text{BIC} = 2.24$ and $\Delta \text{AIC} = 1.84$. This indicates that the power-law scenario can also fit the data very well. We show the probability contours in $\xi - \lambda$ plane in the left panel of Fig. 4. It can be seen from this figure that the best-fitted $\lambda$ is a tiny positive number, in the vicinity of zero. In addition, the parameters $\xi$ and $\lambda$ are in evident degeneracy, strongly anti-correlated. The right panel of Fig. 4 shows the best-fitted $v(E)$ curve of the power-law scenario. We can see that this scenario behaves just like the CG scenario, and the only difference lies in the region of energy approaching zero, i.e., in the power-law model the speed of neutrino will rapidly decrease to the speed of light as the energy goes to zero, and so the result of SN1987A can also be successfully accommodated in this scenario.

In summary, we have tested several phenomenological scenarios of superluminal neutrinos, according to the fits of the experimental data of OPERA, MINOS and Fermilab79. The purpose of this work is to see whether or not the speed of the faster-than-light neutrino is dependent on the energy, from an analysis on the current data. We adopted a phenomenological perspective that the only focus is on the data but not the deeper theoretical mechanism. We have shown that the CG scenario in which the velocity of neutrino is free of the energy can fit the data best. Nevertheless, this model cannot meanwhile provide an explanation for the result of SN1987A. We found that a power-law scenario with the power being a tiny positive number can simultaneously explain the results of SN1987A and OPERA+MINOS+Fermilab79. The scenario may deserve a
FIG. 4: Probability contours at 68.3%, 95.4% and 99.7% confidence level in $\xi - \lambda$ plane for the power-law scenario, $f(E) = \xi E^\lambda$.

further investigation.

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