Variety of Stock Returns in Normal and Extreme Market Days: The August 1998 Crisis.

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\textbf{Abstract.} We investigate the recently introduced variety of a set of stock returns traded in a financial market. This investigation is done by considering daily and intraday time horizons in a 15-day time period centered at the August 31st, 1998 crash of the S&P500 index. All the stocks traded at the NYSE during that period are considered in the present analysis. We show that the statistical properties of the variety observed in analyses of daily returns also hold for intraday returns. In particular the largest changes of the variety of the return distribution turns out to be most localized at the opening or (to a less degree) at the closing of the market.

\section{Introduction}

In recent years physicists started to model financial markets as complex systems (Anderson et al. 1988) within their academic research activity. This triggered the interest of a group of physicists towards the analysis and modeling of price dynamics in financial markets performed by using paradigms and tools of statistical and theoretical physics (Li 1991, Mantegna 1991, Takayasu 1992, Bak 1993, Bouchaud 1994, Mantegna and Stanley 1995, Mantegna 1999b, Bouchaud et al. 2000b). One target of these researches is to implement a new stochastic model of price dynamics in financial markets which reproduces the statistical properties observed in the time evolution of financial stocks (Mantegna and Stanley 2000, Bouchaud and Potters 2000). In the last few years physicists performed several empirical researches investigating the statistical properties of price and volatility time series of a single stock (or of an index) at different time horizons (Müller et al. 1995, Mantegna and Stanley 1995, Lux 1996, Gopikrishnan et al. 1998). Such a kind of analysis does not take into account any interaction of the considered financial stock with other stocks which are traded simultaneously in the same market. It is known that the synchronous price returns time series of different stocks are pair correlated (Elton and Gruber 1995, Campbell et al. 1997) and several researches has been performed also by physicists in order to extract information from the correlation properties of a set of stocks (Mantegna 1999a, Laloux et al 1999, Plerou et al. 1999). A precise characterization of collective movements in a
financial market is of key importance in understanding the market dynamics and in controlling the risk associated to a portfolio of stocks. The present study presents some of the results obtained by our group about the collective behavior of an ensemble of stocks in normal and extreme days of market activity. This is done by discussing the main concepts recently introduced and by presenting them by using a case study focused on the August 1998 crisis of the New York Stock Exchange (NYSE).

Some properties of the collective behavior of stocks traded simultaneously in a market are studied by considering the ensemble properties of a set of stocks. Specifically, we investigate the stock returns of an ensemble of $n$ stocks simultaneously traded in a financial market at a given day. With this approach we quantify what we have called the variety of the financial market at a given trading day (Lillo and Mantegna 2000a, Lillo and Mantegna 2000b). The variety provides statistical information about the amount of different behavior observed in stock returns in a given ensemble of stocks at a given trading time horizon (in the present study, we obtain empirical results by investigating time horizons ranging from one trading day down to 5 trading minutes). The shape and parameters characterizing the ensemble return distribution are relatively stable during normal phases of the market activity whereas become time dependent in the periods subsequent to crashes.

The statistical properties of variety are sensitive to the composition of the portfolio investigated (especially to the capitalization of the considered stocks) and a simple model such as the single-index model is not able to reproduce the statistical properties empirically observed. In this paper we present the results obtained by our group about the synchronous analysis of the daily and high-frequency returns of all the stocks traded in the NYSE during a period of time centered around a significant market crash. The time period selected is a 15 trading days period centered at the August 31st, 1998 crisis. At this day the S&P500 experienced a -6.79% drop, the fourth biggest one-day crash of the last 50 years.

The paper is organized as follow. In Section 2 we illustrate the statistical properties of the daily variety. Section 3 is devoted to study in detail the intraday behavior of the variety. In Section 4, we present a brief discussion of the obtained results.

2 The Variety of an Ensemble of Stocks Simultaneously Traded

For presentation purposes we first start our analysis with a one day time horizon and we then consider a high-frequency analysis of a period of crisis. The first investigation is performed by extracting the $n$ returns $R_i$ of the $n = 2798$ stocks traded in the NYSE for each trading day $t$ of our database covering the period from August 20th to September 10th 1998. The distribution of these returns $P_t(R)$ provides information about the kind of activity occur-
Fig. 1. Daily ensemble return distribution of all the equities traded in the New York Stock Exchange for the extreme trading days August 31st (top panel), and September 8th, 1998 (bottom panel). The August 31st is the worst performing day of 1998 (-6.79% of S&P500), and the 8th September 1998 is the best rally day (+5.09% of S&P500). The skewness of the distribution is negative in crash (top), and positive in rally (bottom) days.

ring in the market at the selected trading day $t$ belonging to a period of high volatility such as the one of August and September 1998. A study covering an 11-year time period is published in ref. (Lillo and Mantegna 2000b) where it has been shown that a customary profile of the ensemble return distribution exists for typical market days. However, this profile is not observed during days of large absolute market averages (Lillo and Mantegna 2000c).

The time period investigated in the present study is a period of large absolute market averages. Hence we expect that the ensemble return distribution bears the properties of asymmetry observed for the first time in ref. (Lillo and Mantegna 2000c). Figure 1 shows the empirical return probability density function (pdf) for two days representative of extreme market days. In this figure we show the interval of daily returns from $-20\%$ to $20\%$. The two distributions refer to the largest drop and increase of the S&P500 observed in
the investigated time period. Consistently with the results published in ref. (Lillo and Mantegna 2000c), we observe that the symmetry of the distribution is not conserved during extreme market days. Moreover, the stylized fact of observing a negatively skewed distribution (top panel of Fig. 1) during a crash and a positively skewed distribution (bottom panel of Fig. 1) during a rally is fully confirmed.

In order to characterize the ensemble return distribution at each day \( t \) of the investigated period we determine both the pdfs and the first two central moments for each of the 15 investigated trading days. In Fig. 2 we show the 15 daily pdfs in a 3D plot both from the perspective of the crashes (bottom panel of the figure) and from the perspective of the rallies (top panel of the figure). During the trading days immediately before the crises (August 31st 1998 labeled as day 8 in the figure) we observe a pdf of returns which is approximately symmetric and characterized by a non Gaussian shape. This shape is close to the “typical” profile observed during normal market days (Lillo and Mantegna 2000b).

The abrupt change of the shape and parameters of the pdf occurring at the crisis and immediately after is documented in this figure. The most extreme deformation of the symmetry and parameters of the pdf is observed at the 8th day and at the 13th day of of the investigated period. These days correspond to August 31st and September 8th 1998 which are two days when the S&P500 experienced a variation of -6.79% and +5.09% respectively.

3 Intraday Behavior of the Variety During a Crisis

The daily investigation summarized in Fig. 2 does not provide any information about the intraday behavior of the variety and of the ensemble return distribution. We use the Trade and Quote database of NYSE to investigate the high frequency behavior of the ensemble return distribution and of its mean and standard deviation (the variety). This is done by investigating the return of 2798 stocks traded in the NYSE by using time horizons of 55 and 5 minutes.

A problem experienced by investigating the returns of an ensemble of stocks at high frequency is that not all the investigated stocks are traded at each investigated time interval. The number of stocks that are not traded becomes relevant at very short time horizons. Before we perform our analysis we calculate the percentage of stocks traded for each investigated time interval. The results are summarized in Fig. 3 where we show the percentage of stocks traded for each investigated time interval. The results are summarized in Fig. 3 where we show the percentage of stocks traded for each investigated time interval. The number of stocks that are not traded becomes relevant at very short time horizons. Before we perform our analysis we calculate the percentage of stocks traded for each investigated time interval. The results are summarized in Fig. 3 where we show the percentage of stocks traded for each investigated time interval. The number of stocks that are not traded becomes relevant at very short time horizons. Before we perform our analysis we calculate the percentage of stocks traded for each investigated time interval. The results are summarized in Fig. 3 where we show the percentage of stocks traded for each investigated time interval. The number of stocks that are not traded becomes relevant at very short time horizons. Before we perform our analysis we calculate the percentage of stocks traded for each investigated time interval. The results are summarized in Fig. 3 where we show the percentage of stocks traded for each investigated time interval. The number of stocks that are not traded becomes relevant at very short time horizons. Before we perform our analysis we calculate the percentage of stocks traded for each investigated time interval. The results are summarized in Fig. 3 where we show the percentage of stocks traded for each investigated time interval. The number of stocks that are not traded becomes relevant at very short time horizons. Before we perform our analysis we calculate the percentage of stocks traded for each investigated time interval. The results are summarized in Fig. 3 where we show the percentage of stocks traded for each investigated time interval.
Fig. 2. Natural logarithm of the daily pdf of 2798 stock returns traded in the NYSE. The investigated time period covers the 15 trading days period starting at August 20th 1998. The crash of the investigated crisis occurs at August 31st (8th day of the figure), while the most effective rally occurs at September 8th (13th day of the figure). The 3D figures is provided from two different perspectives to direct observe both crashes (bottom panel) and rallies (top panel). The return pdfs for trading days occurring before the crash are characterized by an approximately symmetric profile. During crashes and rallies the return pdf becomes asymmetric showing negative (8th day of the figure) and positive (13th day of the figure) skewness respectively.

of trading increases to values between 80% and 95% when time intervals of 55 minutes are considered (middle panel of the figure). This percentage further
Fig. 3. Percentage of the 2798 stocks listed in the NYSE having performed at least one transaction in a time window of 5 minutes (top panel), 55 minutes (middle) panel and 390 minutes equal to one trading day (bottom panel). Vertical lines indicates the closing and opening of trading days. 

increases to values close to 100% when a one day time interval is considered (bottom panel of the figure).

To make our investigation at different time horizons consistent we performed all our analyses over the set of stocks that in each time horizon are effectively traded. In other words, we include each stock in our analysis if during the time interval considered is traded at least one time. In our analysis we also consistently remove all the overnight returns.

In Fig. 4 we show the contour plot of the 55 minutes time horizon ensemble return distribution. The figure is drawn by using a gray scale. Each gray level refers to an interval between two contour plots of the logarithm of the ensemble pdf. Vertical lines are indicating the closing (and opening) of each trading day. The contour plot shows that the larger broadening, distortion and swing of the return pdf is observed close to the opening and closing of each trading day.
Fig. 4. Contour plot of the logarithm of the return pdf for the 15-day investigated time period. The intraday time horizon used to compute the return pdf is 55 trading minutes. The contour plot is obtained for equidistant intervals of the logarithmic probability density. The central brightest area of the contour plot corresponds to the most probable value. The darker regions correspond to less probable. The contour lines are obtained by considering the natural logarithm of the return pdf. The numbers of the gray scale are given in this unit. Each vertical line indicates the closing of a market day. After the 8th day (the crash day of August 31st) the contour plot becomes more distorted. The strongest distortion is observed immediately after the opening and occasionally near the closing.

The end of summer 1998 was a period of high variety (see, for example, Fig. 4 of ref. (Lillo and Mantegna 2000b)). Even in the presence of a generalized high level of variety, Fig. 4 shows that after the end of August crisis, consistently with similar results observed in different market periods (Lillo and Mantegna 2001), there is a relative increase of the variety during the days immediately after the August 31st drop.

To provide a more explicit tracking of the variety of the market observed during the selected time period, we directly calculate it in parallel with the market average $\mu(t)$. Specifically, we consider the average and the standard deviation defined as

$$\mu(t) = \frac{1}{n} \sum_{i=1}^{n} R_i(t),$$

$$\sigma(t) = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (R_i(t) - \mu(t))^2},$$

where $n$ indicates the number of stocks effectively traded in the investigated period.
Fig. 5. Time series of the market average $\mu(t)$ (top panel) computed by using a 55 minutes time horizon. The largest absolute values of the market averages are occurring near the opening or closing of the market. Each vertical line refers to the closing of a market day. The first trading day is August 20th 1998 and the 8th trading day is the crash day of August 31st. In the bottom panel we show the time series of the variety $\sigma(t)$ determined under the same conditions as in the top panel. Again the largest variety is observed near the opening or closing of a market day. After the onset of the crisis (August 31st) the average level of the variety increases in the market.

The mean of price returns $\mu(t)$ quantifies the general trend of the market at day $t$. The standard deviation $\sigma(t)$ is the variety of the market and gives a measure of the width of the ensemble return distribution. A large value of the variety $\sigma(t)$ indicates that different stocks are characterized by rather different returns at day $t$. In fact, in days of high variety, some stocks perform great gains whereas others have great losses. The mean and the standard deviation of price returns are not constant and fluctuate in time.

In Fig. 5 we show the market average $\mu(t)$ (top panel) and the variety $\sigma(t)$ (bottom panel) computed with a 55 minutes time horizon in the investigated period. By using the same presentation scheme of Fig. 3, vertical lines indicates the closing and opening of trading days. By inspecting Fig. 5 one
proves quantitatively the relative increases of the variety observed at the August 31st crisis and immediately after. There is also additional information concerning the intraday localization of the moments of highest variety. Spikes of variety are localized at the opening of the market and to a less degree near the closing of the market day. Large values of variety are associated with large values of absolute market average. The relation between these two variables has been worked out within the framework of the single-index model (Lillo and Mantegna 2001).

The single-index model (Elton and Gruber 1995, Campbell et al. 1997) assumes that $R_i(t)$ can be written as:

$$R_i(t) = \alpha_i + \beta_i R_m(t) + \epsilon_i(t), \quad (3)$$

where $\alpha_i$ is the expected value of the component of security $i$’s return that is independent of the market’s performance, $\beta_i$ is a coefficient usually close to unity, $R_m(t)$ is the market factor and $\epsilon_i(t)$ is called the idiosyncratic return, by construction uncorrelated to the market.

Indeed, under the assumptions that $\alpha_i$ parameters can be neglected, the relation between the variety and the market average return is well approximated as

$$\sigma(t) \simeq \sqrt{\frac{\langle \epsilon_i^2(t) \rangle + \langle \beta_i^2 \rangle - \langle \beta_i \rangle^2}{\langle \beta_i \rangle^2}} \mu^2(t) \quad (4)$$

where $\langle \epsilon_i^2(t) \rangle$ is the mean square value of idiosyncratic terms and the symbol $\langle \cdot \rangle$ indicates the average over all stocks $i$ of the considered parameter.

The single-index model explains the general relation between variety and market average. However the quantitative comparison of the theoretical predictions of Eq. (4) with the empirical results is not satisfactory for the time intervals characterized by a large absolute market average and variety. This empirical observation is summarized in Fig. 6 where we present for each 55-minute interval of our investigated time period (corresponding to each open circle) the variety versus the market average. In the same figure, we also show the theoretical prediction of the single-index model of Eq. (4) as a solid line. This theoretical prediction is obtained by using the market average as a market factor. From Fig. 6 it is evident that the single-index market underestimates the market variety empirically observed in the presence of large absolute values of the market average.

This is not the only empirical property which is not well described by the single-index model. In fact the single-index model also fails in describing the asymmetry of the return pdf detected in the presence of large values of the absolute market average (Lillo and Mantegna 2000c, Cizeau et al. 2001). In the following we investigate the daily and high frequency asymmetry $A$ of the return pdf defined as

$$A_{\Delta t}(t) = \mu_{\Delta t}(t) - \mu^*_{\Delta t}(t) \quad (5)$$
Fig. 6. Variety $\sigma(t)$ of the return pdf as a function of the market average $\mu(t)$ for each 55-minute intraday time intervals of the 15-day investigated time period. Each circle refers to a 55-minute intraday time interval. The solid line is the theoretical prediction of Eq. (4) with the parameters detected by least square procedures with the market average used as market factor. Specifically, $<\epsilon_i^2(t)> = 2.17 \cdot 10^{-4}$ and $(<\beta_i^2> - <\beta_i>)^2 / <\beta_i>^2 = 0.6225$. The variety determined in the presence of large values of the absolute market average is underestimated by the single-index model.

where $\mu_{\Delta t}(t)$ is the market average at the time $t$ computed by using a time horizon $\Delta t$ and $\mu_{\Delta t}^*(t)$ is the median of the pdf at the same time and time horizon. In our study $\Delta t$ is set equal to 1 trading day, 55 and 5 trading minutes. We use an asymmetry measure based on the lowest possible moments because the use of asymmetry parameters based on higher moments (such as, for example, the skewness) would provide an estimation heavily dependent on the most rare events. With our choice of an asymmetry measure based on the mean and the median of the return pdf we obtain empirical measure of the asymmetry which is statistically robust.

In Fig. 7 we show the asymmetry $A_{\Delta t}$ for daily, 55 and 5 minutes time horizons. For daily returns (top left panel), we observe that the sigmoid shape of the asymmetry curve, already detected in (Lillo and Mantegna 2000c), is
Fig. 7. Asymmetry $A_{\Delta t}$ measured as the mean minus the median of the return pdf as a function of the market average $\mu_{\Delta t}$ for different trading intervals of the investigated time period. In the top left panel we show results obtained by using a daily time horizon during the time period of the present investigation (from August 20th to September 10th 1998). A 55 minutes time horizon is used to obtain the top right panel, and the bottom panel is obtained by investigating the time period with a 5 minutes time horizon. The results obtained for all three the time horizons show that empirical results always cluster on a typical pattern. The asymmetry in the return pdf is empirically detected down to a time interval as short as 5 trading minutes. This pattern assumes a sigmoid shape for longer time horizons. It is worth noting that this empirical behavior cannot be modeled with a simple single-index model.

also observed in the investigated 15-day time interval of the 1998 crisis. By investigating shorter time horizons, it is worth noting that the value of the asymmetry $A$ also depends on the absolute value of the market average $\mu$ when intraday time horizons are used. In fact the top right panel (55 minutes time horizon) and the bottom panel (5 minutes time horizon) of Fig. 7 show that the change in the asymmetry of the return pdf occurs down a time horizon as short as 5 trading minutes.
4 Discussion

The variety of an ensemble of stocks simultaneously traded in a financial market provides a direct tool allowing to monitor the overall behavior of a market in a simple and direct way.

In spite of its simplicity (variety can be straightforwardly estimated from market data), the variety is providing information about the status of the market which cannot be taken into account by widely used market models such as the single-index model. The variety (and indeed the return pdf) can be tracked very frequently down to very short time horizons (in the present study we reached the 5 minutes time horizon) providing information on the ability of a single-index model to describe the dynamics of an ensemble of stocks simultaneously traded.

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References

1. Anderson, P. W., Arrow, K. J., Pines, D., (eds) (1988) The Economy as an Evolving Complex System. Addison-Wesley, Redwood City
2. Bak, P., Chen, K., Scheinkman, J., Woodford, M. (1993) Aggregate Fluctuations from Independent Sectoral Shocks: Self-Organized Criticality in a Model of Production and Inventory Dynamics. Ricerche Economiche 47, 3–30
3. Bouchaud, J.-P., Sornette, D. (1994) The Black & Scholes Option Pricing Problem in Mathematical Finance: Generalization and Extensions for a Large Class of Stochastic Processes. J. Phys. I France 4, 863–881
4. Bouchaud J.-P., Potters, M. (2000) Theory of financial risk. Cambridge University Press, Cambridge, UK
5. Bouchaud, J.-P., Lauritsen, K., Alstrom, P., (eds) (2000b) Proceedings of the International Conference on Application of Physics in Financial Analysis. Int. J. Theor. Appl. Finance 3, 309–608
6. Campbell, J. Y., Lo, A. W., MacKinlay, A. C. (1997) The Econometrics of Financial Markets. Princeton University Press, Princeton.
7. Cizeau, P., M. Potters, M., Bouchaud, J.-P. (2001) Correlation structure of extreme stock returns. Quantitative Finance 1, 217-222
8. Elton, E. J., Gruber, M. J. (1995) Modern Portfolio Theory and Investment Analysis. J. Wiley & Sons, New York
9. Gopikrishnan, P., Meyer, M., Amaral, L. A. N., Stanley, H. E. (1998) Inverse cubic law for the distribution of stock price variations. Eur. Phys. J. B 3 139-140.
10. Laloux, L., Cizeau, P., Bouchaud, J.-P., Potters, M. (1999) Noise Dressing of Financial Correlation Matrices. Phys. Rev. Lett. 83, 1467–1470
11. Li, W., (1991) Absence of 1/f Spectra in Dow Jones Daily Average. Int’l J. Bifurcations and Chaos 1, 583–597
12. Lillo, F., Mantegna, R. N. (2000a) Statistical Properties of Statistical Ensembles of Stock Returns. Int. J. Theor. Appl. Finance 3, 405–408
13. Lillo, F., Mantegna, R. N. (2000b) Variety and Volatility in Financial Markets. Phys. Rev. E 62 6126-6134
14. Lillo, F., Mantegna, R. N. (2000c) Symmetry alteration of ensemble return distribution in crash and rally days of financial market. Eur. Phys. J. B 15 603-606.
15. Lillo, F., Mantegna, R. N. (2001) Empirical properties of the variety of a financial portfolio and the single-index model, Eur. Phys. J. B, in press
16. Lux, T. (1996) The stable Paretoian hypothesis and the frequency of large returns: an examination of major German stocks, Applied Financial Economics 6 463-475.
17. Mantegna, R. N. (1991) Lévy Walks and Enhanced Diffusion in Milan Stock Exchange. Physica A 179, 232–242
18. Mantegna, R. N., Stanley, H. E. (1995) Scaling Behaviour in the Dynamics of an Economic Index. Nature 376, 46–49
19. Mantegna, R. N. (1999a) Hierarchical Structure in Financial Markets. Eur. Phys. J. B 11, 193–197
20. Mantegna, R. N., (ed) (1999b) Proceedings of the International Workshop on Econophysics and Statistical Finance. Physica A 269, 1–188
21. Mantegna, R. N., Stanley, H. E. (2000) An Introduction to Econophysics: Correlations and Complexity in Finance. Cambridge University Press, Cambridge, UK
22. Müller, U. A., Dacorogna, M. M., Olsen, R. B., Pictet, O. V., Schwarz, M. (1995) Statistical Study of Foreign Exchange Rates, Empirical Evidence of a Price Change Scaling Law and Intraday Analysis. J. Banking and Finance 14 1189-1208.
23. Plerou, V., Gopikrishnan, P., Rosenow, B., Amaral L. A. N., Stanley, H. E. (1999) Universal and Nonuniversal Properties of Cross Correlations in Financial Time Series. Phys. Rev. Lett. 83, 1471-1474
24. Takayasu, H., Miura, H., Hirabayashi, T., Hamada, K. (1992) Statistical Properties of Deterministic Threshold Elements – The Case of Market Price. Physica A 184, 127–134