Dynamical Instabilities and Deterministic Chaos in Ballistic Electron Motion in Semiconductor Superlattices

Kirill N. Alekseev\textsuperscript{a,b}, Gennady P. Berman\textsuperscript{a,b}, and David K. Campbell\textsuperscript{c}

\textsuperscript{a}Center for Nonlinear Studies, Los Alamos National Laboratory, Los Alamos, New Mexico 87545
\textsuperscript{b}Theory of Nonlinear Processes Laboratory, Kirensky Institute of Physics, Krasnoyarsk 660036, Russia
\textsuperscript{c}Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green St., Urbana, IL 61801-3080

We consider the motion of ballistic electrons within a superlattice miniband under the influence of an alternating electric field. We show that the interaction of electrons with the self-consistent electromagnetic field generated by the electron current may lead to the transition from regular to chaotic dynamics. We estimate the conditions for the experimental observation of this deterministic chaos and discuss the similarities of the superlattice system with the other condensed matter and quantum optical systems.

I. INTRODUCTION

Recent progress in the fabrication of semiconductor superlattices has made investigations of the physical consequences of ballistic electron transport in these systems an area of considerable and growing interest. Studies of the interaction between ballistic electrons in superlattices and the electromagnetic field have already established that nonlinear phenomena may appear even for small field strength \cite{1}. Theoretical investigations in this area have examined oscillations of electrons due to their interaction with constant \cite{2} and alternating \cite{3} electric fields, self-consistent electron-field oscillations \cite{4,5}, propagation of electromagnetic solitons through the superlattice \cite{6}, and other problems. Reviews of the early theoretical investigations of nonlinear effects in superlattice-field interactions have already appeared \cite{7,8}. Recently, oscillations of electrons in superlattices under the influence of constant electromagnetic field (Bloch oscillations) were demonstrated experimentally \cite{9}.

It is well known, that for most of nonlinear dynamical systems with more than 1.5 degrees of freedom, transitions from the regular to the chaotic motion can take place \cite{10}. Among quantum mesoscopic systems, for example, the possibility that nonlinear effects can lead to instabilities and, in particular, to dynamical chaos has been widely studied in Josephson junctions under the presence of an rf-field in a variety of papers involving theoretical modeling, numerical simulations, and experiments (for a review see, e.g. \cite{11}). Several articles have also studied the possible occurrence of chaotic dynamics in superlattice-field interactions \cite{12-15}. In particular, chaotic motion of ballistic electrons in a 2D superlattice in a constant magnetic field was studied within the classical approximation \cite{12}, and recently this approach has been used to explain the experiments on magnetotransport in antidot arrays \cite{13}. Other studies suggest that the transition to dynamical chaos can occur for superlattice electrons under the influence of a constant magnetic field applied perpendicular to the axis of superlattice and for either electromagnetic waves of constant amplitude \cite{14} or for electromagnetic solitons \cite{15}. Chaotic Larmor oscillations of the superlattice electrons \cite{14,15} appear, when the carriers are initially populated rather close to the top of the superlattice miniband.

In this paper we discuss another possible manifestation of dynamical chaos in superlattices: namely, the transition to chaotic behavior for ballistic electrons moving through a superlattice and interacting with spontaneously generated self-consistent field and with the external alternating electric field. We consider the specifically the case in which this transition to chaos may occur for electrons with energies belonging to the bottom of superlattice miniband.

II. BASIC EQUATIONS

Consider the motion of electrons within the miniband of 1D superlattice under the influence of an external alternating electric field applied along the superlattice axis \( z \). Neglecting the inter-miniband transitions, the electron dynamics for weak fields may be described semiclassically using the effective Hamiltonian approach \cite{8,16}

\begin{equation}
H = \epsilon(p) + U(r,t), \quad p = P + \frac{e}{c}A(r,t),
\end{equation}

\begin{equation}
\frac{dP}{dt} = -\frac{\partial H}{\partial r}, \quad \mathbf{v} \equiv \frac{d\mathbf{r}}{dt} = \frac{\partial H}{\partial P}.
\end{equation}
where $\epsilon(p)$ is the (mini)band energy, $P$ is the canonical quasi-momentum of the electron, $e$ is the modulus of the electron charge, $v$ is the electron velocity, and $A(r,t)$ is the vector potential due to the electromagnetic field. The crystal momentum of the electron $\hbar k$ is related to the canonical electron momentum $P$ by

$$\hbar k = P + \frac{e}{c}A(r,t). \quad (3)$$

The potential energy $U(r,t)$ in (1) is connected with the alternating electric field applied along $z$ direction

$$E_{\text{ext}}(t) = E_0 \cos(\Omega t). \quad (4)$$

In (4) $\Omega$ is the frequency of the alternating electric field. The motion of electrons produces a current density

$$j_z = -eN v_z, \quad (5)$$

where $N$ is the number of carriers per unit volume. Consequently, a “self-consistent” field, $A(r,t)$, is generated and can be described by the Maxwell equation

$$\nabla^2 A_z - \frac{1}{c^2} \frac{\partial^2 A_z}{\partial t^2} = -\frac{4\pi}{c} j_z. \quad (6)$$

At the same time, according to (1) and (2) this field affects the electron motion.

We assume a standard dispersion relation for electron motion within the miniband corresponding to the tight-binding approximation [7,16], so that

$$\epsilon(p) = \frac{p_x^2 + p_y^2}{2m^*} + \frac{\Delta}{2} \left[ 1 - \cos \left( \frac{p_z a}{\hbar} \right) \right], \quad (7)$$

where $m^*$ is an effective electron mass, $a$ is a superlattice period and $\Delta$ is a miniband width.

From (1),(2),(4),(5)-(7) one can derive a set of equations describing the interaction of electrons with the self-consistent field and with the alternating electric field

$$\nabla^2 A_z - \frac{1}{c^2} \frac{\partial^2 A_z}{\partial t^2} = -\frac{4\pi}{c} j_z,$nabla^2 A_z - \frac{1}{c^2} \frac{\partial^2 A_z}{\partial t^2} = -\frac{4\pi}{c} j_z,$nabla^2 A_z - \frac{1}{c^2} \frac{\partial^2 A_z}{\partial t^2} = -\frac{4\pi}{c} j_z.$

$$j_z = -\frac{eN a \Delta}{2\hbar} \sin \left( \frac{a}{\hbar} P_z + \frac{ea}{\hbar c} A_z \right), \quad (8)$$

Consider a sample with a characteristic size shorter than the wavelength of the field $A_z$. For typical mesoscopic samples, this leads to fields in the microwave or far-IR domain. Then in a first approximation one can neglect the spatial derivatives in (8) and rewrite this set of equations as one equation describing a parametrically forced pendulum

$$\ddot{\Phi} + \omega^2 E \sin \left( \Phi - \frac{\omega S}{\Omega} \sin \Omega t \right) = 0, \quad (9)$$

where we have introduced the following notations :

$$\Phi = \frac{ea}{\hbar c} A_z, \quad \omega_S = \frac{ea}{\hbar} E_0, \quad \omega_E = \left[ \frac{2\pi e^2 Na^2 \Delta}{\hbar^2} \right]^{1/2}. \quad (10)$$

The frequency $\omega_S$ characterizes Bloch oscillations when the self-consistent field is absent and is often called the Stark frequency [7,8]. As first predicted by Epshtein [4], in the absence of the alternating external field ($\omega_S = 0$), a periodic energy transfer between the field and the electrons is possible with the characteristic frequency $\omega_E$. We note that nonlinear oscillations of this type in different semiconductors with nonparabolic dispersion laws were previously considered by Vatova [17].
Before examining in detail the nonlinear dynamics implied by Eq. (9), let us consider some instructive analogies with other systems. Eq. (9) is exactly the same as the equation describing a Josephson junction subjected to an rf field in the non-dissipative limit where the feedback mechanism is formed by an external circuit. As noted early, it is well-known that in the perturbed Josephson junction various dynamical instabilities and even dynamical chaos can be observed [11]. Quite recently Dunlap et al. [18] suggested connecting a superlattice in series to a capacitor and subjecting the system to an rf field. The circuit produces the feedback mechanism, and as a result, the system [18] demonstrates nonlinear properties, including the possibility of converting ac frequency to dc voltage.

In these two references, the feedback mechanism is extrinsic, in that it is formed by the external circuit. In contrast, in our present system, the feedback mechanism is intrinsic and is formed through the influence of the self-consistent field on the electrons. This situation is analogous to the quantum optical system which consists of an ensemble of 2-level atoms interacting with a self-consistent electromagnetic field and with an external time-periodic field [19-21]. The strength of the external field is characterized by the Rabi frequency, which is an analog of the Stark frequency ω_S defined in (10). The self-consistent oscillations of the field have a characteristic frequency — the so-called “cooperative frequency” — which is an analog of the Epstein frequency ω_E (10), and is also proportional to square root of the number of particles N^{1/2}. The miniband width in the frequency units Δ/h is equivalent to the 2-level transition frequency and, pursuing the analogy to the end, the value ea formally corresponds to the dipole moment of the 2-level transition. The nonlinear dynamics of the system “2-level atoms plus self-consistent field plus external field” in some reasonable approximations can also be described by the equation of forced pendulum [19-21]. As we shall see below, these previous studies of analogous systems analogies from the viewpoint of nonlinear dynamics and transition to chaos will provide useful insights into the a priori complicated superlattice electron dynamics.

We now turn to a detailed description of the nonlinear dynamics governed by equation (9). Using an expansion in terms of Bessel functions, one can rewrite equation (9) as

\[ \ddot{\Phi} + \omega_E^2 \sum_{n=-\infty}^{\infty} (-1)^n J_n(G) \sin(\Phi + n\Omega t) = 0, \]  

where \( G \equiv \omega_S/\Omega \) and \( J_n(x) \) is the standard Bessel function. The case when the external field is absent (\( \omega_S = 0 \)) was first considered by Epshtein in [4]. It was shown in [4] that if the self-consistent field is initially present, then the nonlinear energy exchange between the ballistic electrons and the field is governed by the pendulum equation. The case of high-frequency perturbation \( \Omega \gg \omega_E \) has been also considered by Epshtein in [5]. In this case one can neglect all terms with \( n \neq 0 \) in the expansion (11). Thus the only effect of the high-frequency perturbation is the renormalization of the frequency \( \omega_E \rightarrow \omega_E \sqrt{J_0(G)} \). In our case, the natural initial conditions for the eq. (9) are those for which both the self-consistent field and its vector potential at \( t = 0 \) are absent: \( E_{sc}(0) = A_{sc}(0) = 0 \). (Without loss of generality we assume \( A_{sc}(0) = \text{const.} = 0 \). Taking into account that \( E = (-1/e)A \), we have \( \Phi(0) = \dot{\Phi}(0) = 0 \). These initial conditions correspond to the elliptic stable fixed point of a pendulum without perturbation (\( \omega_S = 0 \)). In the remainder of this paper, we shall consider only these initial conditions. In the absence of the external field, the self-consistent field is not generated and can’t influence on the electron motion. But for \( \omega_S \neq 0 \), the self-consistent field can be generated spontaneously, due to instabilities of the motion for some values of the perturbation frequency \( \Omega \). We first consider the case of \( G \lesssim 1 \) and retain in the Bessel expansion in (11) only the terms up to \( |n| \leq 2 \). Then, we have from (11) after linearization (\( \Phi \ll 1 \))

\[ \ddot{\Phi} + \omega_E^2 J_0(G) \left[ 1 + \frac{2J_0(G)}{J_0(G)} \cos 2\Omega t \right] \Phi = \omega_E^2 J_1(G) \sin \Omega t. \]  

From (12) it is evident that at \( \Omega \approx \omega_E \) and \( \Omega \approx (2\omega_E)/l \) (with \( l \) an integer), instabilities exist corresponding to linear and to parametric resonances. The strongest resonances occur at \( \Omega \approx \omega_E \) and at \( \Omega \approx 2\omega_E \). Of course, the growth of the self-consistent field \( \dot{\Phi} \) due to the linear instabilities will eventually saturate at an amplitude at which the previously neglected nonlinear terms become significant.

Let us now return to equations (9), (11) and consider the case \( G \gg 1 \). This is the well-known problem of nonlinear resonance crossing, which was one of the starting points in the investigations of the Hamiltonian chaos [22]. In another physical context, analysis of the same mathematical problem using the method of nonlinear resonance overlap (the “Chirikov criterion”) [22,23] showed [24] that at
the nonlinear dynamics becomes chaotic for the majority of initial conditions, and the maximal amplitude of the generated self-consistent field $E_{sc}^{max}$ in the units of frequency is

$$|E_{sc}^{max}| = |\Phi^{max}| \sim G \Omega = \omega_S.$$  

Therefore, the maximal amplitude of the chaotic field is of the same order as the amplitude of the external field $|E_{sc}^{max}| \sim E_0$. In contrast, at $K < 1$ the nonlinear dynamics is regular, and the maximal amplitude of the generated self-consistent field is of the order of or less than the width of a single nonlinear resonance

$$|E_{sc}^{max}| = |\Phi^{max}| \lesssim \omega_E.$$  

As one can see, the basic equation (9) can be transformed to the equation of the periodically forced pendulum

$$\ddot{\Phi} + \omega_E^2 \sin \Phi = g \sin \Omega t,$$

by the substitutions $\Psi = \Phi - G \sin \Omega t$ and $g \equiv \omega_S \Omega$.

As we noted above, in the present physical context we should solve eq. (9) (or (16)) for the initial conditions $\Phi(0) = \dot{\Phi}(0) = 0$. Previously, it has been shown ([20]), for the quantum optical analog of the model described by eq. (16), that at $\Omega/\omega_E \lesssim 1$ dynamical chaos is possible even when at $t = 0$ the self-consistent field is absent. This situation can be realized if the dimensionless perturbation parameter $g/\omega_E^2$ is larger than some critical value of order one.

So, for slow external perturbation ($\Omega/\omega_E \lesssim 1$) the self-consistent field can be generated spontaneously and be chaotic. It should be noticed that in this case the amplitude of the self-consistent field is of the same order as or larger than the field amplitude for the regular motion.

The dynamical behavior of the system (9) is illustrated in Figs. 1-4. The chaotic time-dependence of the spontaneously generated self-consistent field is shown in Fig. 1a for $\Omega/\omega_E = 0.5$. In contrast, for the same amplitude of the external field, but for $\Omega/\omega_E \gtrsim 1$, the dynamics of the self-consistent field is regular (Fig. 1b). It is seen from comparison of Figs. 1a and 1b that the amplitude of the chaotically generated self-consistent field is several times larger than the field amplitude of the regular motion. Fig. 2 demonstrates the regions of regular motion and of strong chaotic dynamics in the plane of dimensionless parameters $\Omega/\omega_E$ and $\omega_S/\omega_E$, and at fixed initial conditions: $\Phi(0) = \dot{\Phi}(0) = 0$. Figs. 3 and 4 illustrate the modification of the nonlinear oscillations of the self-consistent field under the variation of the amplitudes of the external field but at fixed frequency $\Omega/\omega_E$. Fig. 3 shows the nonlinear dynamics at rather slow frequency, $\Omega/\omega_E = 0.1$. In this case, the chaotic dynamics exhibits intermittent behavior (see Fig. 3a). In the region of high-frequency external field ($\Omega/\omega_E \gtrsim 1$), the transition from regular to chaotic dynamics is rather sensitive to variations of the parameters (see Fig. 2). The chaotic dynamics in this case also reveals the character of intermittency, which is demonstrated in Fig. 4a.

IV. CONCLUSION

Our analysis establishes that the motion of ballistic electrons through a semiconductor superlattice can, when one takes account of the generation of a self-consistent field, demonstrate both linear and nonlinear instabilities and deterministic chaos. We have argued that this behavior is quite analogous to the quantum optical model [20] describing nonlinear dynamics of 2-level atoms interacting with the self-consistent and with the external time-periodic fields.

It is important to examine the condition of validity of our approach. From [7,8], we conclude that this condition requires that all characteristic frequencies are much less then the miniband width. In our case, this condition takes the explicit form

$$\max\{\Omega, \omega_E, \omega_S\} \ll \Delta/h.$$  

For $\Delta \sim 10^{-2}$eV, $N \sim 10^{14}$cm$^{-3}$, $a \sim 10^{-6}$cm and $eaE_0 \sim 1$meV [3,7], the Stark frequency ($\omega_S$) is much less than $\Delta/h$ but comparable to the collective Epshtein frequency $\omega_E$ (10) and thus belongs to the terahertz region. In this case, the criterion for chaotic dynamics may be satisfied. So, one can see that this first general condition for
a transition to chaos in the case of ballistic electrons in superlattices are close to those needed for the observation of Bloch oscillations and related phenomena in superlattices [2,3,7-9,18]. For applications to realistic experimental systems, it is essential to understand and model the role of dissipation caused by the collisions of the ballistic electrons with impurities and phonons. In our present considerations, the neglect of these effects means that our model nonlinear dynamics actually represents Hamiltonian chaos, whereas in realistic systems, the dynamics would likely be dominated by dissipative effects and consequently attractors. Nonetheless, for parameters similar to those discussed above, we expect dissipative chaos and resulting strange attractors to emerge. A more detailed discussion of these dissipative effects, of analytic estimates of the boundary of the region of chaos, and of potential applications to experiments will be presented elsewhere [25].

ACKNOWLEDGMENTS

We acknowledge fruitful discussions with Mark Sherwin and James Bayfield. KNA and GPB thank Don Cohen of The Center for Nonlinear Studies, Los Alamos National Laboratory, for the hospitality. This work was partially supported by the Grant 94-02-04410 of the Russian Fund for Basic Research and by the Linkage Grant 93-1602 from the NATO Special Programme Panel on Nanotechnology.

* E-mail: kna@iph.krasnoyarsk.su
E-mail: kna@vist.krascience.rssi.ru

1. L. Esaki and R. Tsu, “Superlattice and Negative Differential Conductivity in Semiconductors”, IBM J. Res. Div. 14, 61 (1970).
2. A. M. Bouchard and M. Luban, “Semiconductor Superlattices as Terahertz Generators”, Phys. Rev. B 47, 6815 (1993), and references therein.
3. A. A. Ignatov, K. F. Renk and E. P. Dodin, “Esaki-Tsu Superlattice Oscillator: Josephson-Like Dynamics of Carriers”, Phys. Rev. Lett. 70, 1996 (1993).
4. E. M. Epshtein, “Nonlinear Plasma Oscillations in a Semiconductor with a Superlattice”, Fiz. Tekh. Polupr. 11, 1386 (1977) [Sov. Phys. Semicond., 11, 814 (1977)].
5. E. M. Epshtein, “Nonlinear Plasma Oscillations in a Superlattice in the Presence of a High-Frequency Electric Field”, Fiz. Tekh. Polupr. 12, 985 (1978) [Sov. Phys. Semicond., 12, 583 (1978)].
6. E. M. Epstein, “Solitons in a Superlattice”, Fiz. Tverd. Tela (Leningrad) 19, 3456 (1977) [Sov. Phys. Solid State 19, 2020 (1977)].
7. F. G. Bass and A. P. Tetervov, “High-Frequency Phenomena in Semiconductor Superlattices”, Phys. Rep. 140, 237 (1986).
8. F. G. Bass, A. A. Bulgakov and A. P. Tetervov, High-Frequency Properties of Semiconductors with Superlattices, Topics in Physics of Semiconductors and Semiconductor Devices, (Nauka, Moscow, 1989), in Russian.
9. J. Feldmann, K. Leo, J. Shah, D. A. B. Miller, J. E. Cunningham, T. Meier, G. von Plessen, A. Schulze, P. Thomas, and S. Schmitt-Rink, “Optical Investigation of Bloch Oscillations in a Semiconductor Superlattice”, Phys. Rev. B 46, 7252 (1992); K. Leo, P. H. Bolivar, F. Brüggemann, R. Schwedler and K. Köhler, “Observation of Bloch Oscillations in a Semiconductor Superlattice”, Solid State Commun. 84, 943 (1992).
10. A. J. Lichtenberg and M. A. Liberman, Regular and Stochastic Motion, (Springer-Verlag, New York, 1983).
11. U. Krüger, J. Kurkijärvi, M. Bauer, W. Martienssen, “Chaos and Nonlinear Effects in Josephson Junctions and Devices”, in Nonlinear Dynamics in Solids, Ed. by H. Thomas, (Springer-Verlag, Berlin, 1992).
12. J. Wagenhuber, T. Geisel, P. Niebauer, and G. Obermair, “Chaos and Anomalous Diffusion of Ballistic Electrons in Lateral Surface Superlattices”, Phys. Rev. B 45, 4372 (1992); R. Fleischmann, T. Geisel, and R. Ketzmerick, “Quenched and Negative Hall Effect in Periodic Media: Application to Antidot Superlattices”, Europhys. Lett. 25, 219 (1994), and references therein.
13. D. Weiss, K. Richter, A. Menschig, R. Bergmann, H. Schweizer, K. von Klitzing, and G. Weimann “Quantized Periodic Orbits in Large Antidot Arrays”, Phys. Rev. Lett. 70, 4118 (1993); D. Weiss, K. Richter, E. Vasiliadou, and G. Lütjering, “Magnetotransport in Antidot Arrays”, Surface Science 305, 408 (1994).
14. F. G. Bass, V. V. Konotop, and A. P. Panchekha, “Dynamic Stochasticization of Electrons in Semiconductors with Superlattices”, JETP Lett. 48, 114 (1988); “Electron Stochasticity in Semiconductors with Nonparabolic Dispersion Law”, Sov. Phys. JETP 69, 1055 (1989).
15. F. G. Bass and A. P. Panchekha, “Nonlinear Electromagnetic Waves in a Stochastized Electron Gas with a Nonquadratic Dependence of Energy on Momentum”, Sov. Phys. JETP 72, 955 (1991).
16. N. W. Ashcroft and N. D. Mermin, Solid State Physics, (Holt, Rinehart and Winston Publ., 1976).
17. L. B. Vatova, “Propagation of Nonlinear Plasma Waves in Semiconductors Obeying a Nonquadratic Dispersion Law”, Fiz. Tverd. Tela (Leningrad) 15, 2468 (1973) [Sov. Phys. Solid State, 15, 1639 (1974)].
18. D. H. Dunlap, V. Kovanis, R. V. Duncan and J. Simmons, “Frequency-To-Voltage Convertor Based on Bloch Oscillations in a Capacitively Coupled GaAs−Ga_{1−x}Al_{x}As Quantum Well”, Phys. Rev. B 48, 7975 (1993).
19. K. N. Alekseev and G.P.Berman, “Dynamic Chaos in the Interaction Between External Monochromatic Radiation and Two-Level Medium, with Allowance for Cooperative Effects”, Sov. Phys. JETP 65, 1115 (1987).
20. K. N. Alekseev and G.P.Berman, “Stochastic Mechanism of Generation of Optical Radiation”, Sov. Phys. JETP 67, 1762 (1988).
21. D. Holm and G. Kovačič, “Homoclinic Chaos in a Laser-Matter System”, Physica D 56, 270 (1992).
22. B. V. Chirikov, Dissertation “Research Concerning the Theory of Nonlinear Resonance and Stochasticity”, Novosibirsk 1969,[English translation: CERN, report 71-40, Geneva, 1979].
23. B. V. Chirikov, “A Universal Instability of Many-Dimensional Oscillator Systems”, Phys. Rep. 52, 263 (1979).
24. B. V. Chirikov and D. L. Shepelyansky, “Diffusion at Multiple Crossing of Nonlinear Resonance”, Zh. Tekh. Fiz. 52, 238 (1982) [Sov. Phys. Tech. Phys. 27, 156 (1982)].
25. K. N. Alekseev, et al., in preparation (cond-mat/9604173, cond-mat/9709026).

FIGURE CAPTIONS

Fig. 1 The dependence of the dimensionless amplitude of the self-consistent field $\tilde{E} \equiv e\Phi/(\hbar\omega)$ on the dimensionless time $\tau = \omega t$: (a) chaotic dynamics for $\omega_S/\omega = 0.8$, and $\Omega/\omega = 0.5$; (b) regular dynamics for $\omega_S/\omega = 0.8$, and $\Omega/\omega = 2$. For both cases, the initial conditions are $\Phi(0) = \dot{\Phi}(0) = 0$.

Fig. 2 The regions with strong chaotic and with regular dynamics in the space of the dimensionless parameters: the frequency of the external field ($\Omega/\omega$), and the amplitude of the external field ($\omega_S/\omega$). The initial conditions are $\Phi(0) = \dot{\Phi}(0) = 0$.

Fig. 3 The same as in Fig. 1 but for the parameters: (a) chaotic dynamics for $\omega_S/\omega = 1.8$, and $\Omega/\omega = 0.1$; (b) regular dynamics for $\omega_S/\omega = 1.2$, and $\Omega/\omega = 0.1$.

Fig. 4 The same as in Fig. 1 but for the parameters: (a) chaotic dynamics for $\omega_S/\omega = 1.2$, and $\Omega/\omega = 1.5$; (b) regular dynamics for $\omega_S/\omega = 1.5$, and $\Omega/\omega = 1.5$. 
