Heavy Quark Initiated Contributions to
Deep Inelastic Structure Functions

S. Kretzer and I. Schienbein

Institut für Physik, Universität Dortmund
D-44221 Dortmund, Germany

Abstract

We present $\mathcal{O}(\alpha_s^1)$ corrections to deep inelastic scattering amplitudes on massive quarks obtained within the scheme of Aivazis, Collins, Olness and Tung (ACOT). After identifying the correct subtraction term the convergence of these contributions towards the analogous coefficient functions for massless quarks, obtained within the modified minimal subtraction scheme (MS), is demonstrated. Furthermore, the quantitative relevance of the contributions to neutral current (NC) and charged current (CC) structure functions is investigated for several choices of the factorization scale.
1 Introduction

Leptoproduction of heavy quarks has become a subject of major interest in QCD phenomenology both for experimental and theoretical reasons. Heavy quark contributions are an important component of measured neutral current (NC) and charged current (CC) deep inelastic (DI) structure functions at lower values of Bjorken-\(x\), accessible to present experiments. Charm tagging in NC and CC deep inelastic scattering (DIS) offers the possibility to pin down the nucleon’s gluon and strange sea density, respectively, both of which are nearly unconstrained by global fits to inclusive DI data. Theoretically it is challenging to understand the production mechanism of heavy quarks within perturbative QCD. The cleanest and most predictive method of calculating heavy quark contributions to structure functions seems to be fixed order perturbation theory (FOPT) where heavy quarks are produced exclusively by operators built from light quarks (u,d,s) and gluons (g) and no initial state heavy quark lines show up in any Feynman diagram. Heavy quarks produced via FOPT are therefore also called 'extrinsic' since no contractions of heavy quark operators with the nucleon wavefunction are considered (which in turn would be characteristic for 'intrinsic' heavy quarks). Besides FOPT much effort has been put into formulating variable flavor number schemes (VFNS’s) which aim at resumming the quasi-collinear logs [\(\ln(Q^2/m^2); Q\) and \(m\) being the virtuality of the mediated gauge boson and the heavy quark mass, respectively] arising at any order in FOPT. All these schemes have in common that extrinsic FOPT induces the boundary condition for an intrinsic heavy quark density, which then undergoes massless renormalization group (RG) evolution. Apart from their theoretical formulation VFNS’s have to be well understood phenomenologically for a comparison with FOPT and with heavy quark tagged DI data. We will concentrate here on the scheme developed by Aivazis, Collins, Olness and Tung (ACOT). In the ACOT scheme full dependence on the heavy quark mass is kept in graphs containing heavy quark lines. This gives rise to the above mentioned quasi-collinear logs as well as to power suppressed terms of \(O[(m^2/Q^2)^k]\). While the latter give mass
corrections to the massless, dimensionally regularized, standard coefficient functions (e.g. in the $\overline{\text{MS}}$ scheme), the former are removed by numerical subtraction since the collinear region of phase space is already contained in the RG evolution of the heavy quark density. Up to now explicit expressions in this scheme exist for DIS on a heavy quark at $\mathcal{O}(\alpha_s^0)$ \[9\] as well as for the production of heavy quarks via virtual boson gluon fusion (GF) at $\mathcal{O}(\alpha_s^1)$ \[10\]. In section 2 we will give expressions which complete the scheme up to $\mathcal{O}(\alpha_s^1)$ and calculate DIS on a heavy quark at first order in the strong coupling, i.e. $B^*Q_1 \rightarrow Q_2g$ (incl. virtual corrections to $B^*Q_1 \rightarrow Q_2$) with general couplings of the virtual boson $B^*$ to the heavy quarks, keeping all dependence on the masses $m_{1,2}$ of the quarks $Q_{1,2}$. It is unclear whether (heavy) quark scattering (QS) and GF at $\mathcal{O}(\alpha_s^1)$ should be considered on the same level in the perturbation series. Due to its extrinsic prehistory QS\(^{(1)}\) (bracketed upper indices count powers\(^{1}\) of $\alpha_s$) includes a collinear subgraph of GF\(^{(2)}\), e.g. $\gamma^*c \rightarrow cg$ contains the part of $\gamma^*g \rightarrow c\bar{c}g$, where the gluon splits into an almost on-shell $c\bar{c}$ pair. Therefore QS at $\mathcal{O}(\alpha_s^1)$ can be considered on the level of GF at $\mathcal{O}(\alpha_s^2)$. On the other hand the standard counting for light quarks is in powers of $\alpha_s$ and heavy quarks should fit in. We therefore suggest that the contributions obtained in section 2 should be included in complete experimental and theoretical NLO-analyses which make use of the ACOT scheme. Theoretically the inclusion is required for a complete renormalization of the heavy quark density at $\mathcal{O}(\alpha_s^1)$. However, we leave an ultimate decision on that point to numerical relevance and present numerical results in section 3. Not surprisingly they will depend crucially on the exact process considered (e.g. NC or CC) and the choice of the factorization scale. Finally, in section 4 we draw our conclusions. Appendices A and B outline the calculation of real gluon emission and virtual corrections, respectively, and some longish formulae are presented in Appendix C.

\(^{1}\) For the reasons given here we refrain in most cases from using the standard terminology of 'leading' and 'next-to-leading' contributions and count explicit powers of $\alpha_s$. 

2 Heavy quark contributions to structure functions

In this section we will present all contributions to heavy quark structure functions up to \( \mathcal{O}(\alpha_s^4) \). They are presented analytically in their fully massive form together with the relevant numerical subtraction terms which are needed to remove the collinear divergences in the high \( Q^2 \) limit. Section 2.1 and 2.3 contain no new results and are only included for completeness. In section 2.2 we present our results for the massive analogue of the massless \( \overline{\text{MS}} \) coefficient functions \( C_{i,\overline{\text{MS}}}^q \).

2.1 DIS on a massive quark at \( \mathcal{O}(\alpha_s^0) \)

The \( \mathcal{O}(\alpha_s^0) \) results for \( B^* Q_1 \rightarrow Q_2 \), including mass effects, have been obtained in [9] within a helicity basis for the hadronic/partonic structure functions. For completeness and in order to define our normalization we repeat these results here within the standard tensor basis implying the usual structure functions \( F_{i=1,2,3} \). The helicity basis seems to be advantageous since in the tensor basis partonic structure functions mix to give hadronic structure functions in the presence of masses [9]. However, the mixing matrix is diagonal [9] for the experimental relevant structure functions \( F_{i=1,2,3} \) and only mixes \( F_4 \) with \( F_5 \) which are both suppressed by two powers of the lepton mass. We neglect target (nucleon) mass corrections which are important at larger values of Bjorken-\( x \) [3] where heavy quark contributions are of minor importance.

We consider DIS of the virtual Boson \( B^* \) on the quark \( Q_1 \) with mass \( m_1 \) producing the quark \( Q_2 \) with mass \( m_2 \). At order \( \mathcal{O}(\alpha_s^0) \) this proceeds through the parton model diagram in Fig. 1 (a). Finite mass corrections to the massless parton model expressions are taken into account by adopting the Ansatz given in Eq. (4) of [3]

\[
W^{\mu\nu} = \int \frac{d\xi}{\xi} \, Q_1(\xi, \mu^2) \, \hat{\omega}^{\mu\nu}|_{p_i^+=\xi P^+} .
\]

(1)

\( W^{\mu\nu} \) is the usual hadronic tensor and \( \hat{\omega}^{\mu\nu} \) is its partonic analogue. Here as in the following a hat on partonic quantities refers to unsubtracted amplitudes, i.e. expressions which still
contain mass singularities in the massless limit. $p_1^+$ and $P^+$ are the light-cone momentum components of the incident quark $Q_1$ and the nucleon, respectively. Generally the '$+\'$ light-cone component of a vector $v$ is given by $v^+ \equiv (v^0 + v^3)/\sqrt{2}$.

Contracting the convolution in Eq. (1) with the projectors in Appendix A gives the individual hadronic structure functions $F_{i=1,2,3}$. In leading order (LO) the latter are given by [3]

$$
F_1^{QS(0)}(x,Q^2) = \frac{S_+ \Sigma_{++} - 2m_1m_2S_-}{2\Delta} \ Q_1(\chi,Q^2)
$$

$$
F_2^{QS(0)}(x,Q^2) = \frac{S_+ \Delta}{2Q^2} \ 2x \ Q_1(\chi,Q^2)
$$

$$
F_3^{QS(0)}(x,Q^2) = 2R_+ \ Q_1(\chi,Q^2)
$$

(2)

with

$$
\Sigma_{\pm\pm} = Q^2 \pm m_2^2 \pm m_1^2 .
$$

(3)

In Eq. (2) we use the shorthand $\Delta \equiv \Delta[m_1^2, m_2^2, -Q^2]$, where the usual triangle function is defined by

$$
\Delta[a,b,c] = \sqrt{a^2 + b^2 + c^2 - 2(ab + bc + ca)} .
$$

(4)

The vector ($V$) and axial vector ($A$) couplings of the $\overline{Q}_2 \gamma_\mu (V - A\gamma_5) Q_1$ quark current enter via the following combinations:

$$
S_\pm = VV' \pm AA'
$$

$$
R_\pm = (VA' \pm V'A)/2
$$

(5)

where $V, A \equiv V', A'$ in the case of pure $B$ scattering and $V, A \neq V', A'$ in the case of $B, B'$ interference (e.g. $\gamma, Z^0$ interference in the standard model). The scaling variable $\chi$ generalizes the usual Bjorken-$x$ in the presence of parton masses and is given by [3]:

$$
\chi = \frac{x}{2Q^2} \ (\Sigma_{++} + \Delta) .
$$

(6)
The mass dependent structure functions in Eq. (2) motivate the following definitions

\[
\begin{align*}
F_1 &= \frac{2\Delta}{S_+\Sigma_+ - 2m_1 m_2 S_-} F_1 \\
F_2 &= \frac{2Q_2}{S_+\Delta} \frac{1}{2x} F_2 \\
F_3 &= \frac{1}{2R_+} F_3
\end{align*}
\]

such that \( F_i - F_j, i, j = 1, 2, 3 \), will be finite of \( O(\alpha_s) \) in the limit \( m_{1,2} \to 0 \).

2.2 DIS on a massive quark at \( O(\alpha_s^1) \)

At \( O(\alpha_s^1) \) contributions from real gluon emission [Fig. 1 (b)] and virtual corrections [Fig. 1 (c)] have to be added to the \( O(\alpha_s^0) \) results of section 2.1. The vertex correction with general masses and couplings [Fig. 1 (c)] does to our knowledge not exist in the literature and is presented in some detail in Appendix B. The final result (virtual+real) can be cast into the following form:

\[
\begin{align*}
\hat{F}_{Q_i}^{S(0+1)}(x, Q^2, \mu^2) &= \hat{F}_{Q_i}^{S(0)}(x, \mu^2) + \hat{F}_{Q_i}^{S(1)}(x, Q^2, \mu^2) \\
&= Q_1(\chi, \mu^2) + \frac{\alpha_s(\mu^2)}{2\pi} \int_{\chi}^{1} \frac{d\xi'}{\xi'} \times \left[ Q_1 \left( \frac{\chi}{\xi'}, \mu^2 \right) \hat{H}_i^q(\xi', m_1, m_2) \right], \xi' = \frac{\chi}{\xi}
\end{align*}
\]

with

\[
\hat{H}_i^q(\xi', m_1, m_2) = C_F \left[ (S_i + V_i) \delta(1 - \xi') + \frac{1 - \xi'}{(1 - \xi')^+} \frac{s - m_2^2}{8s} N_i^{-1} \hat{f}_i^Q(\xi') \right]
\]

where \( s = (p_1 + q)^2 \) and the \( S_i, V_i, N_i \) and \( \hat{f}_i^Q \) are given in Appendix C. The factorization scale \( \mu^2 \) will be taken equal to the renormalization scale throughout. The ‘+’ distribution in Eq. (9) is a remnant of the cancellation of the soft divergencies from the real and virtual contributions. It is defined as usual:

\[
\int_0^1 d\xi' \ f(\xi') \ [g(\xi')]_+ \equiv \int_0^1 d\xi' \ [f(\xi') - f(1)] \ g(\xi')
\]

As indicated by the hat on \( \hat{H}_i^q \), the full massive convolution in Eq. (8) still contains the mass singularity arising from quasi-collinear gluon emission from the initial state quark.
leg. The latter has to be removed by subtraction in such a way that in the asymptotic limit \( Q^2 \to \infty \) the well known massless \( \overline{\text{MS}} \) expressions are recovered. The \( \overline{\text{MS}} \) limit is mandatory since all modern parton distributions – and therefore all available heavy quark densities – are defined in this particular scheme (or in the DIS scheme \([18]\), which can be straightforwardly derived from \( \overline{\text{MS}} \)). The correct subtraction term can be obtained from the following limit

\[
\lim_{m_1 \to 0} \int_1^1 \frac{d\xi'}{\xi'} Q_1 \left( \frac{\chi}{\xi'}, m_1, m_2, \mu^2 \right) \hat{H}^q_i (\xi', m_1, m_2) = \int_1^1 \frac{d\xi'}{\xi'} Q_1 \left( \frac{x}{\chi, \mu_2^2} \right) \left\{ H^q_\text{MS} (\xi', \mu_2, \lambda) + C_F \left[ \frac{1 + \xi'}{1 - \xi'} \left( \ln \frac{\mu_2^2}{m_1^2} - 1 - 2 \ln(1 - \xi') \right) \right]_+ \right\} + O \left( \frac{m_1^2}{Q^2} \right)
\]

(11)

where \( \lambda = Q^2/(Q^2 + m_2^2) \), \( x/\lambda = \chi|_{m_1 = 0} \) and the \( H^q_\text{MS} \) can be found in Appendix A of \([15]\). Obviously the \( \overline{\text{MS}} \) subtraction term for a 'heavy quark inside a heavy quark' is given not only by the splitting function \( P_{qq}^{(0)} = C_F [(1 + \xi'^2)/(1 - \xi')]_+ \) times the collinear log \( \ln(\mu^2/m_1^2) \) but also comprises a constant term. Herein we agree with Eq. (3.15) in \([16]\), where this was first pointed out in the framework of perturbative fragmentation functions for heavy quarks. We therefore define

\[
\mathcal{F}^{\text{SUB}}_i (x, Q^2, \mu_2^2) = \frac{\alpha_s(\mu_2^2)}{2\pi} C_F \int_1^1 \frac{d\xi'}{\xi'} \left[ \frac{1 + \xi'^2}{1 - \xi'} \left( \ln \frac{\mu_2^2}{m_1^2} - 1 - 2 \ln(1 - \xi') \right) \right]_+ Q_1 \left( \frac{x}{\chi, \mu_2^2} \right)
\]

(12)

such that

\[
\lim_{m_1 \to 0} \left\{ \mathcal{F}^{Q^1 (i)}_i (x, Q^2, \mu^2) - \mathcal{F}^{\text{SUB}}_i (x, Q^2, \mu_2^2) \right\} = \mathcal{F}^{Q^1 (i), \text{MS}}_i (x, Q^2, \mu_2^2)
\]

(13)

where the superscript \( Q_1 \) on \( \mathcal{F}^{Q^1 (i), \text{MS}}_i \) refers to that part of the inclusive structure function \( \mathcal{F}_i \) which is initiated by the heavy quark \( Q_1 \), i.e. which is obtained from a convolution with the heavy quark parton density. Note that the limit in Eq. (11) guarantees that Eq. (13) is also fulfilled when \( m_1 = m_2 \to 0 \) (e.g. NC lepton production of charm) since

\[
\lim_{m_2 \to 0} H^q_\text{MS} (\xi', \mu^2, \lambda) = C^q_\text{MS} (\xi', \mu^2) + O \left( \frac{m_2^2}{Q^2} \right)
\]

(14)

\[2\] We also agree with the quark initiated coefficient functions in \([17]\) where quark masses have been used as regulators.
where \( C_{i}^{q,\overline{\text{MS}}} \) are the standard massless coefficient functions in the \( \overline{\text{MS}} \) scheme, e.g. in [18, 19].

### 2.2.1 Comparison to existing NC and CC results

We have performed several cross checks of our results against well known calculations that exist in the literature [15, 17, 18, 19, 20]. The checks can be partly inferred from section 2.2. Nevertheless we present here a systematic list for the reader’s convenience and in order to hint at several errors which we uncovered in [20].

In the charged current case \( V = A = 1 \) our results in Eq. (8) reduce in the limit \( m_1 \rightarrow 0 \) to the corresponding expressions in [17], or in [15] if the scheme dependent term represented by Eq. (12) is taken into account. The latter agrees with Eq. (3.15) in [16]. For \( m_{1,2} \rightarrow 0 \) we reproduce the well known \( \overline{\text{MS}} \) coefficient functions e.g. in [18, 19]. The vertex correction in Appendix C is implicitly tested because it contributes to any of the final results. However, as an independent cross check the well known QED textbook result can be reproduced for \( m_1 = m_2, A = 0 \).

Initial state parton mass effects in NC DIS at \( \mathcal{O}(\alpha_s^1) \) have been first considered in [20] within the scenario [21] of intrinsic nonperturbative \( c\bar{c} \) pairs stemming from fluctuations of the nucleon Fock space wavefunction. Although we do not consider such a scenario here we note that our results could be easily transferred to corresponding applications [22]. The main difference would be an inclusion of kinematical target mass effects which are important at larger \( x \) [9] where a possible nonperturbative charm component is expected [21] to reside. Apart from obvious typos, we uncovered some errors in [20] such that our results differ from those in [20] by terms that vanish in the massless limit. Eq. (C.16) in [20] should be multiplied by \((1 + 4\lambda z^2)\). The typo propagates into the final result in Eq. (51). Furthermore the longitudinal cross section, i.e. Eq. (C.17) in [20] seems to have been obtained as a residual of \( \sigma_{R}^{(1)} \) and the (wrong) \( \sigma_{R}^{(2)} \). Constructing \( \sigma_{R}^{(1)} \) from Eqs. (C.16), (C.17) via \( \sigma_{R}^{(1)} = -2\sigma_{R}^{(L)} + \sigma_{R}^{(2)} \) reproduces up to a constant of normalization the part...
∼ f_i^Q of our result for $F_1$ in Eq. (9). Given the amount of successful independent tests of our results we regard the disagreement with [20] as a clear evidence that the results in [20] should be updated by our calculation.

2.3 Gluon fusion contributions at $\mathcal{O}(\alpha_s^1)$

The gluon fusion contributions to heavy quark structure functions ($B^* g \rightarrow \bar{Q}_1 Q_2$) are known for a long time [23, 24] and have been reinterpreted in [10] within the helicity basis for structure functions. Here we only briefly recall the corresponding formulae in the tensor basis for completeness. The GF component of DI structure functions is given by

$$F_{1,3}^{GF}(x, Q^2) = \int_{a_x}^{1} \frac{d\xi'}{\xi'} g(\xi', \mu^2) f_{1,3} \left( \frac{x}{\xi'}, Q^2 \right)$$
$$F_2^{GF}(x, Q^2) = \int_{a_x}^{1} \frac{d\xi'}{\xi'} \xi' g(\xi', \mu^2) f_2 \left( \frac{x}{\xi'}, Q^2 \right)$$

(15)

where $a_x = [1 + (m_1 + m_2)^2/Q^2] x$ and the $f_i$ can be found for general masses and couplings in [23]. The corresponding $F_{i}^{GF}$ are obtained from the $F_{i}^{GF}$ by using the same normalization factors as in Eq. (7). Along the lines of [10] the GF contributions coexist with the QS contributions which are calculated from the heavy quark density, which is evolved via the massless RG equations in the $\overline{\text{MS}}$ scheme. As already pointed out in section 2.2 the quasi-collinear log of the fully massive GF term has to be subtracted since the corresponding mass singularities are resummed to all orders in the massless RG evolution.

The subtraction term for the GF contribution is given by [10]

$$F_{SUB}^i(x, Q^2, \mu^2) = \sum_k \frac{\alpha_s(\mu^2)}{2\pi} \ln \frac{\mu^2}{m_k^2} \int_{x}^{1} \frac{d\xi'}{\xi'} P_{qg}^{(0)}(\xi') \ g \left( \frac{x}{\xi'}, \mu^2 \right)$$

(16)

where $P_{qg}^{(0)}(\xi') = 1/2 \ [\xi'^2 + (1 - \xi')^2]$. Note that Eq. (16) as well as Eq. (12) are defined relative to the $F_i$ in Eq. (7) and not with respect to the experimental structure functions $F_i$. The sum in Eq. (16) runs over the indices of the quarks $Q_k$ for which the quasi-collinear logs are resummed by massless evolution of a heavy quark density, i.e. $k = 1$, $k = 2$ or $k = 1, 2$. 

8
2.4 ACOT structure functions at $O(\alpha_s^1)$

As already mentioned in the introduction it is not quite clear how the perturbation series should be arranged for massive quarks, i.e. whether the counting is simply in powers of $\alpha_s$ as for light quarks or whether an intrinsic heavy quark density carries an extra power of $\alpha_s$ due to its prehistory as an extrinsic particle produced by pure GF. We are here interested in the $Q\!S^{(1)}$ component of heavy quark structure functions. Usually the latter is neglected in the ACOT formalism since it is assumed to be suppressed by one order of $\alpha_s$ with respect to the GF contribution as just explained above. It has however been demonstrated in [25, 26] within $\overline{\text{MS}}$ that this naive expectation is quantitatively not supported in the special case of semi-inclusive production of charm (dimuon events) in CC DIS. We therefore want to investigate the numerical relevance of the $Q\!S^{(1)}$ contribution to general heavy quark structure functions. In this article we present results for the fully inclusive case, relevant for inclusive analyses and fits to inclusive data. We postpone experimentally more relevant semi-inclusive ($z$-dependent) results to a future publication [27]. Our results at full $O(\alpha_s^1)$ will be given by

$$F_i^{(1)} = F_i^{Q\!S^{(0+1)}} + F_i^{GF} - F_i^{SUB_q} - F_i^{SUB_g}$$  \hspace{1cm} (17)

with $F_i^{Q\!S^{(0+1)}}$, $F_i^{GF}$, $F_i^{SUB_q}$ and $F_i^{SUB_g}$ given in Eqs. (8), (13), (12), and (16), respectively. Furthermore, we will also consider a perturbative expression for $F_i$ which is constructed along the expectations of the original formulation of the ACOT scheme, i.e. $Q\!S^{(1)}$ is neglected and therefore $F_i^{SUB_q}$ need not be introduced

$$F_i^{(0)+GF-SUB_g} = F_i^{Q\!S^{(0)}} + F_i^{GF} - F_i^{SUB_g}.$$  \hspace{1cm} (18)

3 Results for NC and CC structure functions

In this section we present results which clarify the numerical relevance of $Q\!S^{(1)}$ contributions to inclusive heavy quark structure functions in the ACOT scheme. We will restrict ourselves to NC and CC production of charm since bottom contributions are insignificant
to present DI data. Our canonical parton distributions for the NC case will be CTEQ4M
\cite{28} (Figs. 4 and 5 below), which include ‘massless heavy partons’ $Q_k$ above the scale
$Q^2 = m_k^2$. Figs. 2 and 3, however, have been obtained from the older GRV92 \cite{29} dis-
tributions. The newer GRV94 \cite{30} parametrizations do not include a resummed charm
density since they are constructed exclusively along FOPT. GRV94 is employed in the CC
section. The radiative strange sea of GRV94 seems to be closest to presently available CC
charm production data \cite{25}. Furthermore, the low input scale of GRV94 allows for a wide
range of variation of the factorization scale around the presently relevant experimental
scales, which are lower for CC DIS than for NC DIS. Qualitatively all our results do not
depend on the specific set of parton distributions chosen.

\subsection{NC structure functions}

For our qualitative analysis we are only considering photon exchange and we neglect the
$Z^0$. The relevant formulae are all given in section 2 with the following identifications:

\begin{align*}
Q_{1,2} & \rightarrow c \\
m_{1,2} & \rightarrow m_c = 1.6 \ (1.5) \ \text{GeV} \ \text{for CTEQ4 (GRV92)} \\
V = V', A = A' & \rightarrow \frac{2}{3}, 0
\end{align*}

and we use $\mu^2 = Q^2$ if not otherwise noted. We consider contributions from charmed
quarks and anti-quarks which are inseparably mixed by the GF contribution. This means
that in Eq. (16) the sum runs over $k = 1, 2$ and the relevant expressions of section 2.1
and 2.2 have to be doubled [since $c(x, \mu^2) = \bar{c}(x, \mu^2)$].

First we investigate the importance of finite mass corrections to the limit in Eq. (13). In
Fig. 2 the difference $F_2^{QS^{(1)}} - F_2^{SUB_q}$ can be compared to its $\overline{\text{MS}}$ analogue which is

\begin{equation}
F_2^{(c+\bar{c})^{(1)}, \overline{\text{MS}}} = \frac{4}{9} x \frac{\alpha_s(\mu^2)}{2\pi} \left[ (c + \bar{c})(\mu^2) \otimes C_{2, \overline{\text{MS}}}^q \left( \frac{Q^2}{\mu^2} \right) \right] (x, Q^2)
\end{equation}

where $\otimes$ denotes the usual (massless) convolution. From Fig. 2 it is obvious that the
relative difference between ACOT and $\overline{\text{MS}}$ depends crucially on $x$. It can be large and
only slowly convergent to the asymptotic $\overline{\text{MS}}$ limit as can be inferred from Fig. 3. Note that the solid curves in Figs. 2, 3 are extremely sensitive to the precise definition of the subtraction term in Eq. (12), e.g. changing $\chi \to x$ – which also removes the collinear singularity in the high $Q^2$ limit – can change the ACOT result by about a factor of 5 around $Q^2 \sim 5 \text{ GeV}^2$. This is an example of the ambiguities in defining a variable flavor number scheme which have been formulated in a systematic manner in [11].

The relative difference between the subtracted $QS^{(1)}$ contribution calculated along ACOT and the corresponding $\overline{\text{MS}}$ contribution in Eq. (19) appears, however, phenomenologically irrelevant if one considers the significance of these contributions to the total charm structure function in Fig. 4. The complete $\mathcal{O}(\alpha_s^1)$ result (solid line) is shown over a wide range of $Q^2$ together with its individual contributions from Eq. (17). It can be clearly seen that the full massive $QS^{(1)}$ contribution is almost completely canceled by the subtraction term $SUB_q$ (Indeed the curves for $QS^{(1)}$ and $SUB_q$ are hardly distinguishable on the scale of Fig. 4). The subtracted quark correction is numerically negligible and turns out to be indeed suppressed compared to the gluon initiated contribution, which is also shown in Fig. 4. Note, however, that the quark initiated corrections are not unimportant because they are intrinsically small. Rather the large massive contribution $QS^{(1)}$ is perfectly canceled by the subtraction term $SUB_q$ provided that $\mu^2 = Q^2$ is chosen. This is not necessarily the case for different choices of $\mu^2$ as we will now demonstrate.

In Fig. 5 we show the dependence of the complete structure function and its components on the arbitrary factorization scale $\mu^2$. Apart from the canonical choice $\mu^2 = Q^2$ (which was used for all preceding figures) also different scales have been proposed like the maximum transverse momentum of the outgoing heavy quark which is approximately given by $(p_T^{\text{max}})^2 \approx (1/x - 1) Q^2/4$. For low values of $x$, where heavy quark structure functions are most important, the scale $(p_T^{\text{max}})^2 \gg Q^2$. The effect of choosing a $\mu^2$ which differs much from $Q^2$ can be easiest understood for the massless coefficient functions $C_{q,g,\overline{\text{MS}}}^{(1)}$ which contain an unresummed $\ln(Q^2/\mu^2)$. The latter is of course absent.

\[3 \text{ The subtracted gluon contribution GF changes by about a factor of 2 under the same replacement.}\]
for \( \mu^2 = Q^2 \) but becomes numerically increasingly important, the more \( \mu^2 \) deviates from \( Q^2 \). This logarithmic contribution cannot be neglected since it is the unresummed part of the collinear divergence which is necessary to define the scale dependence of the charm density. This expectation is confirmed by Fig. 5. For larger values of \( \mu^2 \) the subtracted \( QS^{(1)} \) contribution is indeed still suppressed relative to the subtracted \( GF \) contribution. Nevertheless, its contribution to the total structure function becomes numerically significant and reaches the \( \sim 20\% \) level around \( (p_T^{\text{max}})^2 \). Note that in this regime the involved formulae of section 2.2 may be safely approximated by the much simpler convolution in Eq. (19) because they are completely dominated by the universal collinear logarithm and the finite differences ACOT – \( \overline{\text{MS}} \) from Figs. 2 and 3 become immaterial. In practice it is therefore always legitimate to approximate the ACOT results of section 2.2 by their \( \overline{\text{MS}} \) analogues because both are either numerically inessential or logarithmically dominated.

Finally we confirm that the scale dependence of the full \( \mathcal{O}(\alpha_s^1) \) structure function \( F_2^{(1)} \) in Eq. (17) is larger than the scale dependence of \( F_i^{(0)+GF}\text{-SUB} \) in Eq. (18) which was already pointed out in [33]. Nevertheless, the subtracted \( QS^{(1)} \) contribution should be respected for theoretical reasons whenever \( \alpha_s \ln(Q^2/\mu^2) \ll 1 \).

### 3.2 CC structure functions

Charm production in CC DIS is induced by an \( s \to c \) transition at the \( W \)-Boson vertex. The strange quark is not really a heavy quark in the sense of the ACOT formalism, i.e., the production of strange quarks cannot be calculated reliably at any scale using FOPT because the strange quark mass is too small. It is nevertheless reasonable to take into account possible finite \( m_s \) effects into perturbative calculations using ACOT since the subtraction terms remove all long distance physics from the coefficient functions. Indeed the ACOT formalism has been used for an experimental analysis of CC charm production in order to extract the strange sea density of the nucleon [7]. Along the assumptions of ACOT \( QS^{(1)} \) contributions have not been taken into account. This procedure is obviously questionable and has been shown not to be justified within the \( \overline{\text{MS}} \) scheme [25, 26].
With our results in section 2.2 we can investigate the importance of quark initiated $\mathcal{O}(\alpha_s^1)$ corrections within the ACOT scheme for inclusive CC DIS. As already mentioned above, results for the experimentally more important case of semi-inclusive ($z$-dependent) DIS will be presented in a future publication [27]. In the following we only introduce subtraction terms for collinear divergencies correlated with the strange mass and treat all logarithms of the charm mass along FOPT. We do so for two reasons, one theoretical and one experimental: First, at present experimental scales of CC charm production $\ln(Q^2/m_c^2)$ terms can be safely treated along FOPT and no introduction of an a priori unknown charm density is necessary. Second, the introduction of a subtraction term for the mass singularity of the charm quark would simultaneously require the inclusion of the $c \rightarrow s$ QS-transition at the $W$-vertex with no spectator-like $\bar{c}$-quark as in $GF$. This contribution must, however, be absent when experiments tag on charm in the final state. CC DIS on massive charm quarks without final-state charm tagging has been studied in [34].

The numerics of this section can be obtained by the formulae of section 2 with the following identifications:

$$Q_1 \rightarrow s, \quad Q_2 \rightarrow c$$

$$m_2 \rightarrow m_c = 1.5 \text{ GeV (GRV94)}$$

$$V = V', A = A' \rightarrow 1, 1$$

and the strange mass $m_1 = m_s$ will be varied in order to show its effect on the structure function $F_2^c$.

In Fig. 6 we show the structure function $F_2^c$ and its individual contributions for two experimental values of $x$ and $Q^2$ [3] under variation of the factorization scale $\mu^2$. Like in the NC case we show the complete $\mathcal{O}(\alpha_s^1)$ result as well as $F_2^{(0)+GF-SUB}$ where $QS^{(1)}$ has been neglected. The thick curves in Fig. 6 (a) have been obtained with a regularizing strange mass of 10 MeV. They are numerically indistinguishable from the $m_s = 0$ MS results along the lines of [14]. For the thin curves a larger strange mass of 500 MeV
has been assumed as an upper limit. Finite mass effects can therefore be inferred from the difference between the thin and the thick curves. Obviously they are very small for all contributions and can be safely neglected. For the higher $Q^2$ value of Fig. 6 (b) they would be completely invisible, so we only show the $m_s = 10$ MeV results $(\equiv \overline{\text{MS}})$. Since the finite mass corrections within the ACOT scheme turn out to be negligible as compared to massless $\overline{\text{MS}}$ it is not surprising that we confirm the findings of [25, 26] concerning the importance of quark initiated corrections. They are – in the case of CC production of charm – not suppressed with respect to gluon initiated corrections for all reasonable values of the factorization scale. Only for small choices of $\mu^2 \sim Q^2 + m_c^2$ can the quark initiated correction be neglected. In this region of $\mu^2$ also gluon initiated corrections are moderate and Born approximation holds within $\sim 10\%$ [15]. For reasons explained in section 3.1 the absolute value of both corrections – gluon and quark initiated – become very significant when large factorization scales like $p_T^{\text{max}}$ are chosen. This can be inferred by looking at the region indicated by the arrow in Fig. 6 which marks the scale $\mu = 2p_T^{\text{max}}$ which was used in [7]. Analyses which use ACOT with a high factorization scale and neglect quark initiated corrections therefore undershoot the complete $O(\alpha_s^3)$ result by the difference between the solid and the dot-dashed curve, which can be easily as large as $\sim 20\%$. For reasons explained in the introduction to this section we have used the radiative strange sea of GRV94. When larger strange seas like CTEQ4 are used the inclusion of the quark initiated contributions is even more important.

4 Conclusions

In this article we have calculated and analysed DIS on massive quarks at $O(\alpha_s^3)$ within the ACOT scheme for heavy quarks. For NC DIS this contribution differs significantly from its massless $\overline{\text{MS}}$ analogue for $\mu^2 = Q^2$. Both give, however, a very small contribution to the total charm structure function such that the large relative difference is phenomenologically immaterial. At higher values of the factorization scale $\mu^2 \sim (p_T^{\text{max}})^2$ the contributions become significant and their relative difference vanishes. The $Q S^{(1)}$ contribution of section
2.2 can therefore be safely approximated by its much simpler $\overline{\text{MS}}$ analogue at any scale. For CC DIS quark initiated corrections should always be taken into account on the same level as gluon initiated corrections. Due to the smallness of the strange quark mass ACOT gives results which are almost identical to $\overline{\text{MS}}$.

**Acknowledgements**

We thank E. Reya for advice, useful discussions and a careful reading of the manuscript. This work has been supported in part by the 'Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie’, Bonn.
Appendix A: Real Gluon Emission

We define a partonic tensor
\[ \hat{\omega}^{\mu\nu} \equiv \sum_{\text{color}} \sum_{\text{spin}} \left\langle Q_2(p_2), g(k) \right| \bar{Q}_2 \gamma^\mu (V - A_3) Q_1 \left| Q_1(p_1) \right\rangle \times \left( \mu \rightarrow \nu \right)^* \] \hspace{1cm} (A1)

which can be decomposed into its different tensor components as usual
\[ \hat{\omega}^{\mu\nu} = -\hat{\omega}_1^Q g^{\mu\nu} + \hat{\omega}_2^Q p_1^\mu p_1^\nu + i\hat{\omega}_3^Q \varepsilon_{\alpha\beta}^{\mu\nu} p_1^\alpha q^\beta + \hat{\omega}_4^Q q^\mu q^\nu + \hat{\omega}_5^Q (q^\mu p_1^\nu + q^\nu p_1^\mu) \] \hspace{1cm} (A2)

\( \hat{\omega}_{\mu\nu} \) can be easily calculated from the general Feynman rules for invariant matrix elements which are customarily expressed as functions of the mandelstam variables \( \hat{s} \equiv (p_1 + q)^2 \) and \( \hat{t} \equiv (p_1 - k)^2 \) to which we will refer in the following. Projection onto the individual \( \hat{\omega}_{i=1,2,3}^Q \) in Eq. (A2) is performed for nonzero masses and in \( n = 4 + 2\varepsilon \) dimensions with the following operators
\[ P_1^{\mu\nu} = \frac{-1}{2(1 + \varepsilon)} \left\{ g^{\mu\nu} + \left[ m_1^2 q^\mu q^\nu - Q^2 p_1^\mu p_1^\nu - (p_1 \cdot q)(q^\mu p_1^\nu + p_1^\mu q^\nu) \right] \right\} \]
\[ \times 4\Delta^{-2}[m_1^2, \hat{s}, -Q^2] \]
\[ P_2^{\mu\nu} = 2 \left[ -g^{\mu\nu} Q^2 + 4 q^\mu q^\nu \frac{2(1 + \varepsilon)(p_1 \cdot q)^2 - m_1^2 Q^2}{\Delta^2[m_1^2, \hat{s}, -Q^2]} \right. 
\[ + 4(3 + 2\varepsilon) Q^2 \frac{Q^2 p_1^\mu p_1^\nu + (p_1 \cdot q)(q^\mu p_1^\nu + p_1^\mu q^\nu)}{\Delta^2[m_1^2, \hat{s}, -Q^2]} \left\{ (1 + \varepsilon) \Delta^2[m_1^2, \hat{s}, -Q^2] \right\}^{-1} \]
\[ P_3^{\mu\nu} = \frac{-2i}{\Delta^2[m_1^2, \hat{s}, -Q^2]} \varepsilon^{\mu\nu\lambda\kappa} p_1^\lambda q^\kappa \] \hspace{1cm} (A3)

such that \( P_i \cdot \hat{\omega} = \hat{\omega}_i^Q \). The normalization in Eqs. (A1), (A2) is such that real gluon emission contributes \( F_i^R \) to the hadronic structure functions via
\[ F_1^R = \frac{1}{8\pi} \int \frac{d\xi}{\xi} Q_1(\xi) \int \hat{\Delta} \hat{\omega}_1^Q \]
\[ F_2^R = \frac{2\varepsilon}{16\pi} \int \frac{d\xi}{\xi} \frac{\Delta^2[m_1^2, \hat{s}, -Q^2]}{2Q^2} Q_1(\xi) \int \hat{\Delta} \hat{\omega}_2^Q \]
\[ F_3^R = \frac{1}{8\pi} \int \frac{d\xi}{\xi} \Delta[m_1^2, \hat{s}, -Q^2] Q_1(\xi) \int \hat{\Delta} \hat{\omega}_3^Q \] \hspace{1cm} (A4)

where \[ \int \hat{\Delta} \hat{\omega} = \frac{1}{8\pi} \frac{\hat{s} - m_2^2}{\hat{s}} \frac{1}{\Gamma(1 + \varepsilon)} \left[ \frac{(\hat{s} - m_2^2)^2}{4\pi\hat{s}} \right]^{\xi} \int_0^1 \left( y(1 - y) \right)^{\varepsilon} dy \] \hspace{1cm} (A5)
is the partonic phase space. In Eq. (A5) \( y \) is related to the partonic centre of mass scattering angle \( \theta^* \) and the partonic mandelstam variable \( \hat{t} \) via

\[
y \equiv \frac{1}{2} \left( 1 + \cos \theta^* \right)
\]

\[
= \frac{1}{2\Delta[m_1^2, \hat{s}, -Q^2]} \left[ Q^2 + m_1^2 + \hat{s} + \Delta[m_1^2, \hat{s}, -Q^2] + \frac{2\hat{s}(\hat{t} - m_1^2)}{\hat{s} - m_2^2} \right] . \tag{A6}
\]

We have chosen dimensional regularization for the soft gluon poles stemming from \( \hat{s} \to m_2^2 \) which arise from propagators in the \( \hat{\omega}_i \) times phase space factors in \( d\hat{\Omega}S \). In Eq. (A4) we use

\[
(\hat{s} - m_2^2)^{2\varepsilon-1} \sim \left( 1 - \frac{\chi}{\xi} \right)^{2\varepsilon-1} = \frac{1}{2\varepsilon} \delta(1 - \chi/\xi) + \frac{1}{(1 - \chi/\xi)^+} + \mathcal{O}(\varepsilon) \tag{A7}
\]

which separates hard gluon emission (\( \sim \hat{j}_i^Q \)) from soft gluon \( (S_i) \) contributions in Eq. (A4). Note that in Eqs. (3), (4) the integration variable \( \xi \), which is implicitly defined in Eq. (I), has been changed to \( \xi' \equiv \chi/\xi \) for an easier handling of the distributions. For the relation between \( \hat{s} \) and \( \xi' \) see Eq. (C3) below.

Since all quark masses are kept nonzero, no poles in \( y \) (collinear singularities) are contained in the integration volume. The \( \hat{j}_i^Q \) which occur in Eq. (3) and which are given below in Eq. (C4) are therefore straightforward integrals of the \( \hat{\omega}_i^Q \)

\[
\hat{j}_i^Q = (g_s^2 C_F)^{-1} \int_0^1 dy \, \hat{\omega}_i^Q \tag{A8}
\]

and the \( S_i \) in Eqs. (3), (C1) pick up the pole in Eq. (A7)

\[
S_i \sim \frac{1}{\varepsilon} \int_0^1 dy \, [y(1-y)]^\varepsilon \left[ \hat{\omega}_i^Q(\hat{s} - m_2^2)^2 \right]_{\xi=\chi} , \tag{A9}
\]

where the proportionality is given by kinematical and phase space factors which must be kept up to \( \mathcal{O}(\varepsilon) \).

The normalization of our hadronic structure functions in Eq. (A4) can be clearly inferred from the corresponding LO results in Eq. (2). Nevertheless, for definiteness we also give the hadronic differential cross section to which it corresponds

\[
\frac{d^2\sigma^{l,l}}{dx dy} = \frac{1}{n_l} \frac{(G_{l}^{B,B'})^2}{2\pi} \frac{(G_{q}^{B,B'})^2}{2M_N E_l} \left[ S_{l,+} (1-y)F_2 + S_{l,+} y^2xF_1 \pm R_{l,+} 2y(1-\frac{y}{2})xF_3 \right] , \tag{A10}
\]

17
where \( (G_{t,q}^{B,B'})^2 = \frac{g_{t,q}^2(Q^2 + M_B^2)}{g_{t,q}^2(Q^2 + M_{B'}^2)} \) is the effective squared gauge coupling – including the gauge boson propagator – of the \( \gamma_\mu (V - A_\gamma_5) \) lepton and quark current, respectively, and \( n_l \) counts the spin degrees of freedom of the lepton, e.g. \( n_l = 1, 2 \) for \( l = \nu, e^- \). The leptonic couplings \( S_{l,+}, R_{l,+} \) are defined analogous to the quark couplings in Eq. (5). As noted below Eq. (5), \( B = B' \) for non-interference (pure \( B \) scattering).

### Appendix B: Vertex Correction

We have calculated the vertex correction in \( n = 4 + 2 \varepsilon \) dimensions at \( \mathcal{O}(\alpha_s^1) \) for general masses and couplings using the Feynman gauge. The unrenormalized vertex \( \Lambda_6^\mu \) [Fig. 1(c.1)] has the structure

\[
\Lambda_6^\mu = C_F \frac{\alpha_s}{4\pi} \Gamma(1-\varepsilon) \left( \begin{array}{c} \frac{Q^2}{4\pi^2} \\ \frac{m_1^2 - m_2^2}{4\pi^2} \end{array} \right) \epsilon \left\{ C_{0,-} \gamma^\mu L_5 + C_{+} \gamma^\mu R_5 + \ C_{1,-} m_2 p_1^\mu \ L_5 + C_{1,+} m_1 p_1^\mu \ R_5 + \ C_{q,-} m_2 q^\mu \ L_5 + C_{q,+} m_1 q^\mu \ R_5 \right\}
\]

(B1)

with \( L_5 = (V - A_\gamma_5) \), \( R_5 = (V + A_\gamma_5) \). The coefficients read

\[
C_{0,-} = \frac{1}{\varepsilon} (-1 - \Sigma_{++} I_1) + \left[ \frac{\Delta^2}{2Q^2} + \Sigma_{++} \left( 1 + \ln \left( \frac{Q^2}{\Delta} \right) \right) \right] I_1
\]

\[
+ \frac{1}{2} \ln \left( \frac{Q^2}{m_1^2} \right) + \frac{1}{2} \ln \left( \frac{Q^2}{m_2^2} \right) + \frac{m_2 - m_1}{2Q^2} \ln \left( \frac{m_2^2}{m_1^2} \right) + \frac{\Sigma_{++}}{\Delta}
\]

\[
\times \left\{ \frac{1}{2} \ln^2 \left( \frac{\Delta - \Sigma_{++}}{2Q^2} \right) + \frac{1}{2} \ln^2 \left( \frac{\Delta - \Sigma_{+-}}{2Q^2} \right) - \frac{1}{2} \ln^2 \left( \frac{\Delta + \Sigma_{+-}}{2Q^2} \right) - \frac{1}{2} \ln^2 \left( \frac{\Delta + \Sigma_{++}}{2Q^2} \right) \right\}
\]

\[
- \left\{ \frac{1}{2} \ln^2 \left( \frac{\Delta - \Sigma_{++}}{2\Delta} \right) - \frac{1}{2} \ln^2 \left( \frac{\Delta - \Sigma_{+-}}{2\Delta} \right) + \frac{1}{2} \ln^2 \left( \frac{\Delta + \Sigma_{+-}}{2\Delta} \right) + \frac{1}{2} \ln^2 \left( \frac{\Delta + \Sigma_{++}}{2\Delta} \right) \right\}
\]

(B2)

\[
C_{+} = 2m_1 m_2 I_1
\]

\[
C_{1,-} = \frac{-1}{Q^2} \left[ \Sigma_{+-} I_1 + \ln \left( \frac{m_1^2}{m_2^2} \right) \right]
\]

\[
C_{1,+} = \frac{-1}{Q^2} \left[ \Sigma_{+-} I_1 - \ln \left( \frac{m_1^2}{m_2^2} \right) \right]
\]

\[
C_{q,-} = \frac{1}{Q^4} \left[ (\Delta^2 - 2m_2^2 Q^2) I_1 - 2Q^2 + \Sigma_{+-} \ln \left( \frac{m_1^2}{m_2^2} \right) \right]
\]

\[
C_{q,+} = \frac{1}{Q^4} \left[ (\Delta^2 - 2m_2^2 Q^2 - \Sigma_{+-} Q^2) I_1 + 2Q^2 + (\Sigma_{+-} + Q^2) \ln \left( \frac{m_1^2}{m_2^2} \right) \right]
\]

(B3)
with
\[ I_1 = \frac{1}{\Delta} \ln \left[ \frac{\Sigma_{++} + \Delta}{\Sigma_{++} - \Delta} \right] \]

\[ I_2 = I_1 \ln \Delta - \frac{1}{\Delta} \]
\[ \times \left\{ \frac{1}{2} \ln^2 \left| \frac{\Delta - \Sigma_{+-}}{2\Delta^2} \right| + 2 \ln^2 \left| \frac{\Delta - \Sigma_{+-}}{2\Delta^2} \right| - \frac{1}{2} \ln^2 \left| \frac{\Delta + \Sigma_{+-}}{2\Delta^2} \right| - \frac{1}{2} \ln^2 \left| \frac{\Delta + \Sigma_{+-}}{2\Delta^2} \right| \right\} \]
\[ - \left( L_{i_2} \left( \frac{\Delta - \Sigma_{+-}}{2\Delta} \right) - L_{i_2} \left( \frac{\Delta - \Sigma_{+-}}{2\Delta} \right) + L_{i_2} \left( \frac{\Delta + \Sigma_{+-}}{2\Delta} \right) + L_{i_2} \left( \frac{\Delta + \Sigma_{+-}}{2\Delta} \right) \right) \} \].

The renormalized vertex [Fig. 1 (c.1)-(c.3)] is obtained by wave function renormalization:
\[ \Lambda_{R} = \gamma^\mu L_5 (Z_1 - 1) + \Lambda^\mu_0 + O(\alpha_s^2) \]  \hspace{1cm} (B5)
where \( Z_1 = \sqrt{Z_2(p_1)Z_2(p_2)} \). The fermion wave function renormalization constants are defined on mass shell:
\[ Z_2(m_i) = 1 + C_F \frac{\alpha_s}{4\pi} \Gamma(1 - \varepsilon) \left( \frac{m_i^2}{4\pi\mu^2} \right)^\varepsilon \frac{1}{\varepsilon} \left[ 3 - 4 \varepsilon + O(\varepsilon^2) \right] \]  \hspace{1cm} (B6)
such that
\[ \Sigma_{++} = 1 + C_F \frac{\alpha_s}{4\pi} \Gamma(1 - \varepsilon) \left( \frac{Q^2}{4\pi\mu^2} \right)^\varepsilon \left[ 3 - 2 \ln \left( \frac{Q^2}{m_1^2} \right) - 2 \ln \left( \frac{Q^2}{m_2^2} \right) - 4 \right] \].  \hspace{1cm} (B7)

The final result for the renormalized vertex \( \Lambda_{R}^\mu \) reads
\[ \Lambda_{R}^\mu = C_F \frac{\alpha_s}{4\pi} \Gamma(1 - \varepsilon) \left( \frac{Q^2}{4\pi\mu^2} \right)^\varepsilon \left\{ C_{R,-} \gamma^\mu L_5 + C_{+} \gamma^\mu R_5 \right. \]
\[ + C_{1,-} m_2 p_{1\mu} L_5 + C_{1,+} m_1 p_{1\mu} R_5 + C_{q,-} m_2 q^\mu L_5 + C_{q,+} m_1 q^\mu R_5 \left\} \]  \hspace{1cm} (B8)
with \( C_{+}, C_{1,\pm}, C_{q,\pm} \) as given above and
\[ C_{R,-} = \frac{1}{\varepsilon} (2 - \Sigma_{++} I_1) + \left[ \frac{\Delta^2}{2Q^2} + \Sigma_{++} \left( 1 + \ln \left( \frac{Q^2}{\Delta} \right) \right) \right] I_1 \]
\[ + \frac{m_2^2 - m_1^2}{2Q^2} \ln \left( \frac{m_1^2}{m_2^2} \right) - \ln \left( \frac{Q^2}{m_1^2} \right) - \ln \left( \frac{Q^2}{m_2^2} \right) - 4 + \frac{\Sigma_{++}}{\Delta} \]
\[ \times \left\{ \frac{1}{2} \ln^2 \left| \frac{\Delta - \Sigma_{+-}}{2Q^2} \right| + \frac{1}{2} \ln^2 \left| \frac{\Delta - \Sigma_{+-}}{2Q^2} \right| + \frac{1}{2} \ln^2 \left| \frac{\Delta + \Sigma_{+-}}{2Q^2} \right| - \frac{1}{2} \ln^2 \left| \frac{\Delta + \Sigma_{+-}}{2Q^2} \right| \right\} \]
\[ - L_{i_2} \left( \frac{\Delta - \Sigma_{+-}}{2\Delta} \right) - L_{i_2} \left( \frac{\Delta - \Sigma_{+-}}{2\Delta} \right) + L_{i_2} \left( \frac{\Delta + \Sigma_{+-}}{2\Delta} \right) + L_{i_2} \left( \frac{\Delta + \Sigma_{+-}}{2\Delta} \right) \} \].
Appendix C: Real and Virtual Contributions to Structure Functions

The soft real contributions \( S_i \) to the coefficient functions in Eq. (3) are given by

\[
S_1 = \frac{1}{\varepsilon}(-2 + \Sigma_+ I_1) + 2 + \frac{\Sigma_{++}}{\Delta} \left[ \Delta I_1 + \text{Li}_2 \left( \frac{2\Delta}{\Delta - \Sigma_{++}} \right) - \text{Li}_2 \left( \frac{2\Delta}{\Delta + \Sigma_{++}} \right) \right] \\
+ \ln \frac{\Delta^2}{m_2^2 Q^2} (-2 + \Sigma_+ I_1)
\]

\[
S_{2,3} = S_1,
\]

with \( I_1 \) given in Appendix B and where \( \chi \) is given in Eq. (6). The virtual contributions are derived from the renormalized vertex in Eq. (B8) by using the projectors in Eq. (A3):

\[
V_1 = C_{R,-} + S_{-\Sigma_+} - 2S_{m_1^2 m_2} C_+
\]

\[
V_2 = C_{R,-} + \frac{1}{2} \left( m_2^2 C_{1,+} + m_2^2 C_{1,-} \right) + \frac{S_{-\Sigma_+}}{S_+} \left[ C_+ + \frac{m_1 m_2}{2} (C_{1,+} + C_{1,-}) \right]
\]

\[
V_3 = C_{R,-} + \frac{R_-}{R_+} C_+
\]

where the \( C \)'s are given in Appendix B. Note that the soft poles \( (1/\varepsilon) \) of \( S_i, V_i \) cancel in the sum \( S_i + V_i \) in Eq. (3) as must be.

The massive matrix elements \( \hat{f}_i^Q(\xi') \) are most conveniently given as functions of the mandelstam variable \( \hat{s}_1(\xi') \equiv (p_1 + q)^2 - m_2^2 \), i.e. \( \hat{f}_i^Q(\xi') \equiv \hat{f}_i^Q[\hat{s}_1(\xi')] \) with

\[
\hat{s}_1(\xi') \equiv \hat{s} - m_2^2 = \frac{1 - \xi'}{2\xi'}[(\Delta - \Sigma_{+-})\xi' + \Delta + \Sigma_{+-}].
\]

From the real graphs of Fig. 1 (b) one obtains

\[
\hat{f}_1^Q(\hat{s}_1) = \frac{8}{\Delta^2} \left\{ -\Delta^2 (S_+ \Sigma_{++} - 2m_1 m_2 S_-) I_{\xi'} + 2m_1 m_2 S_- \left( \frac{1}{\hat{s}_1}[\Delta^2 + 4m_2^2 \Sigma_{+-}] \right) \right\} \\
+ \Delta \hat{s}_1 \left[ \frac{\Delta^2 + 2\Sigma_{+-} \Sigma_{++} + (m_2^2 + Q^2) \hat{s}_1}{\hat{s}_1^2} \right] L_{\xi'} \\
+ \left[ \frac{-m_2^2 \Sigma_{++}}{(\hat{s}_1 + m_2^2)^2} (\Delta^2 + 4m_2^2 \Sigma_{+-}) - \frac{1}{4(\hat{s}_1 + m_2^2)} \left[ 3\Sigma_{++} \Sigma_{+-} + 4m_2^2 (10 \Sigma_{++} \Sigma_{+-} - \Sigma_{++} \Sigma_{+-} - m_1^2 \Sigma_{++} + \hat{s}_1 [-7 \Sigma_{+-} \Sigma_{+-} + 18 \Delta^2 - 4m_1^2 (7Q^2 - 4m_2^2 + 7m_2^2)] \right) \right]
\]
\[
\frac{\Delta^2}{\Delta'} \left\{ -2 \Delta^4 S_1 I_{\xi'} + 2 m_1 m_2 S_1 \left( \frac{\hat{s}_1 + m_2^2}{\Delta'} \right) (\Delta'^2 - 6 m_1^2 Q^2) L_{\xi'} \right. \\
- \frac{\Delta^2}{2(\hat{s}_1 + m_2^2)} + \left( 2 \Delta'^2 - 3 Q^2 (\hat{s}_1 + \Sigma_{++}) \right) + S_+ \left( -2(\Delta'^2 - 6 m_1^2 Q^2)(\hat{s}_1 + m_2^2) \right. \\
- 2 (m_1^2 + m_2^2) \hat{s}_1^2 - 9 m_2^2 \Sigma_{++}^2 + \Delta^2 (2 \Sigma_{++} - m_2^2) + 2 \hat{s}_1 (2 \Delta'^2 + (m_1^2 - 5 m_2^2) \Sigma_{++}) \\
+ \frac{(\Delta'^2 - 6 Q^2 (m_2^2 + \hat{s}_1)) \Sigma_{++} (\hat{s}_1 + \Sigma_{++})}{\hat{s}_1} - \frac{2 \Delta^2}{\hat{s}_1} (\Delta'^2 + 2(2 m_2^2 + \hat{s}_1) \Sigma_{++}) \\
+ \frac{(\hat{s}_1 + m_2^2)}{\Delta'} \left[ -2 \frac{\Delta^2(\Delta^2 + 2 \Sigma_{++} \Sigma_{++}) - 2 \hat{s}_1 (\Delta^2 - 6 m_1^2 Q^2)}{\hat{s}_1} \right] \\
- (\Delta'^2 - 18 m_1^2 Q^2) \Sigma_{++} - 2 \Delta^2 (\Sigma_{++} + 2 \Sigma_{++}) [L_{\xi'}] \left. \right\} \\
\frac{\Delta^2}{\Delta'} \left\{ -2 \Delta^2 R_1 I_{\xi'} + 2 m_1 m_2 R_+ \left( 1 - \frac{\Sigma_{++}}{\hat{s}_1} + \frac{(\hat{s}_1 + m_2^2)(\hat{s}_1 + \Sigma_{++})}{\Delta' \hat{s}_1} \right) L_{\xi'} \right. \\
+ R_+ \left( \Sigma_{++} - 3 \Sigma_{++} - \frac{2}{\hat{s}_1} (\Delta^2 + 2 m_2^2 \Sigma_{++}) - \frac{2(\hat{s}_1 - \Sigma_{++})(\hat{s}_1 + \Sigma_{++})}{2(\hat{s}_1 + m_2^2)} \right) \\
+ \frac{\hat{s}_1 + m_2^2}{\Delta' \hat{s}_1} \left[ -\hat{s}_1^2 + 4 (m_1^2 \Sigma_{++} - \Delta^2) - 3 \hat{s}_1 \Sigma_{++} \right] L_{\xi'} \right\} \\
\right\} 
\]

with

\[ L_{\xi'} \equiv \ln \left( \frac{\Sigma_{++} + \hat{s}_1 - \Delta'}{\Sigma_{++} + \hat{s}_1 + \Delta'} \right) \]

and

\[ I_{\xi'} = \left( \frac{\hat{s}_1 + m_2^2}{\hat{s}_1} + \frac{\hat{s}_1 + m_2^2}{\Delta' \hat{s}_1} \Sigma_{++} \right) L_{\xi'} \].

\[ \Delta \] is given below Eq. (3) and \[ \Delta' \equiv \Delta[m_1^2, \hat{s}, -Q^2] \].

Finally, the normalization factors in Eq. (1) are

\[ N_1 = \frac{S_+ \Sigma_{++} - 2 m_1 m_2 S_-}{2 \Delta} \], \[ N_2 = \frac{2 S_+ \Delta}{(\Delta')^2} \], \[ N_3 = \frac{2 R_+}{\Delta'} \].
References

[1] C. Adloff et al., H1 collab., Z. Phys. C72, 593 (1996).

[2] J. Breitweg et al., ZEUS collab., Phys. Lett. B407, 402 (1997).

[3] W. G. Seligman et al., Phys. Rev. Lett. 79, 1213 (1997);
   W. G. Seligman, Ph. D. thesis, Columbia University, Nevis-292 (1997).

[4] A. Vogt, proceedings of the International Workshop on Deep Inelastic Scattering and
   Related Phenomena (DIS 96), Rome, Italy, April 1996; Edited by G. D’Agostini and
   A. Nigro, World Scientific, 1997.

[5] H. Abramowicz et al., CDHSW collab., C. Phys. C15, 19 (1982).

[6] S. A. Rabinowitz et al., CCFR collab., Phys. Rev. Lett. 70, 134 (1993).

[7] A. O. Bazarko et al., CCFR collab., Z. Phys. C65, 189 (1995);
   A. O. Bazarko, Ph. D. thesis, Columbia University, Nevis-285 (1994).

[8] M. Glück, E. Reya and M. Stratmann, Nucl. Phys. B422, 37 (1994).

[9] M. A. G. Aivazis, F. I. Olness and W.- K. Tung, Phys. Rev. D50, 3085 (1994).

[10] M. A. G. Aivazis, J. C. Collins, F. I. Olness and W.- K. Tung, Phys. Rev. D50, 3102
    (1994);
    J. C. Collins PSU-TH/198, hep-ph 9806259.

[11] R. S. Thorne and R. G. Roberts, RAL-TR-97-049, hep-ph 9709442;
    R. S. Thorne and R. G. Roberts, RAL-TR-97-061, hep-ph 9711223.

[12] A. D. Martin, R. G. Roberts, M. G. Ryskin and W. J. Stirling, Eur. Phys. J. C2, 287 (1998).

[13] M. Buza, Y. Matiounine, J. Smith, W. L. van Neerven, Eur. Phys. J. C1, 301 (1998).

[14] J. C. Collins and W.- K. Tung, Nucl. Phys. B278, 934 (1986).
[15] M. Glück, S. Kretzer and E. Reya, Phys. Lett. \textbf{B380}, 171 (1996); Erratum \textbf{B405}, 391 (1996).

[16] B. Mele and P. Nason, Nucl. Phys. \textbf{B361}, 626 (1991).

[17] J. J. van der Bij and G. J. Oldenborgh, Z. Phys. \textbf{C51}, 477 (1991).

[18] G. Altarelli, R. K. Ellis and G. Martinelli, Nucl. Phys. \textbf{B157}, 461 (1979).

[19] W. Furmanski and R. Petronzio, Z. Phys. \textbf{C11}, 293 (1982).

[20] E. Hoffmann and R. Moore, Z. Phys. \textbf{C20}, 71 (1983).

[21] S. J. Brodsky, P. Hoyer, C. Peterson and N. Sakai, Phys. Lett. \textbf{B93}, 451 (1980);
    S. J. Brodsky, C. Peterson and N. Sakai, Phys. Rev. \textbf{D23}, 2745 (1981).

[22] B. W. Harris, J. Smith, R. Vogt, Nucl. Phys. \textbf{B461}, 181 (1996);
    J. F. Gunion and R. Vogt, UCD-97-14, LBNL-40399, \texttt{hep-ph 9706252}.

[23] M. Glück, R. M. Godbole and E. Reya, Z. Phys. \textbf{C38}, 441 (1988); Erratum \textbf{C39}, 590 (1988).

[24] G. Schuler, Nucl. Phys. \textbf{B299}, 21 (1988);
    U. Baur and J. J. van der Bij, Nucl. Phys. \textbf{B304}, 451 (1988).

[25] M. Glück, S. Kretzer and E. Reya, Phys. Lett. \textbf{B398}, 381 (1997); Erratum \textbf{B405}, 392 (1997).

[26] S. Kretzer and I. Schienbein, Phys. Rev. \textbf{D56}, 1804 (1997).

[27] S. Kretzer and I. Schienbein, (in preparation).

[28] H. L. Lai \textit{et al.}, CTEQ collab., Phys. Rev. \textbf{D55}, 1280 (1997).

[29] M. Glück, E. Reya and A. Vogt, Z. Phys. \textbf{C53}, 127 (1992).

[30] M. Glück, E. Reya and A. Vogt, Z. Phys. \textbf{C67}, 433 (1995).
[31] M. A. G. Aivazis, F. I. Olness and W.-K. Tung, Phys. Rev. Lett. 65, 2339 (1990).

[32] J. C. Collins, Proceedings of the XXVth Rencontre de Moriond: High Energy Hadronic Interactions, ed. J. Tran Than Van, Gif-sur-Yvette: Editions Frontières, 1990, page 123.

[33] C. R. Schmidt, proceedings of the 5th International Workshop on Deep Inelastic Scattering and QCD (DIS 97), Chicago, IL, USA, April 1997.

[34] U. D’Alesio, Ph. D. thesis (in Italian), Turin University (1996).

[35] T. Gottschalk, Phys. Rev. D23, 56 (1981).
Figure Captions

Fig. 1 Feynman diagrams for the $QS^{(0)}$ [Fig. 1 (a)] and $QS^{(1)}$ [Fig. 1 (b), (c)] contributions to ACOT structure functions in Eqs. (2) and (8), respectively.

Fig. 2 $x$-dependence of the subtracted $QS^{(1)}$ contribution to the NC charm structure function $F_{2c}^{\sigma}$ (solid line). $Q^2 = \mu^2 = 10$ GeV$^2$ is fixed. For comparison the $\overline{\text{MS}}$ analogue in Eq. (19) is shown (dashed line). The GRV92 parton distributions have been used.

Fig. 3 The same as Fig. 2 but varying $Q^2(= \mu^2)$ for fixed $x$.

Fig. 4 The complete $\mathcal{O}(\alpha_s^1)$ neutral current structure function $F_{2c}^{\sigma}$ and all individual contributions over a wide range of $Q^2$, calculated from the CTEQ4M distributions. Details of the distinct contributions are given in the text.

Fig. 5 $\mu^2$ dependence of the complete $\mathcal{O}(\alpha_s^1)$ NC structure function (CTEQ4M) in Eq. (17) (solid line) and of the structure function in Eq. (18) (dot-dashed line) where the subtracted $QS^{(1)}$ contribution is neglected. Also shown are the different subtracted $\mathcal{O}(\alpha_s^1)$ contributions $GF$ and $QS^{(1)}$.

Fig. 6 (a), (b) The charm production contribution to the charged current structure function $F_2$ for a wide range of the factorization scale $\mu^2$ using GRV94. The curves are as for the neutral current case in Fig. 5. In Fig. 6 (a) the thicker curves have been obtained with a (purely regularizing) strange mass of 10 MeV which according to Eq. (13) (and to the analogous limit for the subtracted $GF$ term) numerically reproduces $\overline{\text{MS}}$. For the thinner curves a strange mass of 500 MeV has been assumed. In Fig. 6 (b) all curves correspond to $m_s = 10$ MeV ($\equiv \overline{\text{MS}}$).
Fig. 1
Fig. 2

$Q^2 = 10 \text{ GeV}^2$

$F_2: QS^{(1)} - \text{SUB}_q$

$\overline{\text{MS}}$

Fig. 3

$F_2: QS^{(1)} - \text{SUB}_q$

$\overline{\text{MS}}$

$x = 0.05$
Fig. 4

\[ F_2(x,Q^2) \]

\[ x = 0.005 \]

\[ Q^2 = 25 \text{ GeV}^2 \]

Fig. 5

\[ F_2(x,Q^2) \]

\[ x = 0.005 \]

\[ Q^2 = 25 \text{ GeV}^2 \]
Fig. 6 (a)

\[ F_2^c(x, Q^2) = \frac{\mu^2}{(Q^2 + m_c^2)} \]

- **x = 0.015**
- **Q^2 = 2.4 GeV^2**

Fig. 6 (b)

\[ F_2^c(x, Q^2) = \frac{\mu^2}{(Q^2 + m_c^2)} \]

- **x = 0.125**
- **Q^2 = 17.9 GeV^2**