Scattering of $^7$Be and $^8$B and the astrophysical $S_{17}$ factor

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Measurements of scattering of $^7$Be at 87 MeV on a melamine (C$_3$N$_6$H$_6$) target and of $^8$B at 95 MeV on C were performed. For $^7$Be the angular range was extended over previous measurements and monitoring of the intensity of the radioactive beam was improved. The measurements allowed us to check and improve the optical model potentials used in the incoming and outgoing channels for the analysis of existing data on the proton transfer reaction $^{14}$N($^7$Be,$^8$B)$^{13}$C. The results lead to an updated determination of the asymptotic normalization coefficient for the virtual decay $^8$B $\rightarrow$ $^7$Be + p. We find a slightly larger value, $C^0_{\text{tot}}(^8B) = 0.466 \pm 0.047$ fm$^{-1}$, for the melamine target. This implies an astrophysical factor, $S_{17}(0) = 18.0 \pm 1.8$ eV-b, for the solar neutrino generating reaction $^7$Be($p$,$\gamma$)$^8$B.

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I. INTRODUCTION

Measurements of the energetic neutrinos produced in $^8$B beta decay have played a prominent role in our new understanding of neutrino properties (see [1, 2, 3, 4, 5] and references therein). $^8$B is produced in the sun by the $^7$Be($p$,$\gamma$)$^8$B reaction. A good understanding of this reaction rate is needed in order to calculate the expected neutrino flux in the standard solar model [6]. The determination of the astrophysical $S_{17}$ factor has, therefore, been the subject of intense experimental and theoretical effort over the past decade. This work has been summarized in several recent publications [7, 8]. In spite of these efforts, there is no clear consensus on the value of $S_{17}(0)$ at the desired 5% precision. Consequently, several new experiments are under way or planned.

We previously reported measurements of the asymptotic normalization coefficients (ANC) for $^8$B using the proton transfer reactions $^{10}$B($^7$Be,$^8$B)$^9$Be [9] and $^{14}$N($^7$Be,$^8$B)$^{13}$C [10]. The ANCs determine the amplitude of the tail of the overlap integral of the ground state wave function of $^8$B onto the two-body channel $^7$Be + p. To find the ANCs with the $^{14}$N($^7$Be,$^8$B)$^{13}$C transfer reaction, a distorted wave Born approximation (DWBA) calculation is carried out and compared to the measured data. The DWBA calculation needs optical model parameters (OMP) for both the incoming $^7$Be+$^{14}$N and outgoing $^8$B+$^{13}$C channels. Here we report a measurement of $^7$Be elastic scattering on a melamine (C$_3$N$_6$H$_6$) target, where we doubled the angular range and improved the monitoring of the intensity of the $^7$Be radioactive beam relative to our previous measurement [10]. The extension of the angular range was done to obtain a better determination of the optical potential in the incoming channel. We also measured the elastic scattering of a $^8$B beam on a C target with the aim of checking, for the first time, the OMP that were used in the outgoing channel $^8$B+$^{13}$C.

Below we describe the radioactive beam production, the experimental setups, and the procedure for the data reduction. We then give results of calculations for optical potentials in both the incoming and outgoing channels based on a double folding procedure with an effective nucleon-nucleon interaction. We discuss the consequences of the improved secondary beam normalization, and compare revised DWBA calculations to the $^{14}$N($^7$Be,$^8$B)$^{13}$C data from Ref. [10] in order to extract a new value for the ANCs and the corresponding astrophysical factor $S_{17}$.

II. THE EXPERIMENTS

The $^7$Be radioactive beam was produced and separated using the Momentum Achromatic Recoil Spectrometer (MARS) [11]. The primary beam was $^7$Li at 18.6 MeV/A delivered by the K500 superconducting cyclotron at Texas A&M University. It bombarded a liquid nitrogen cooled H$_2$ gas target at a pressure of 2 atm, with entrance and exit windows of 12 $\mu$m thick Havar. A secondary beam of $^7$Be at 12.5 MeV/A was filtered from other reaction products by MARS. The characteristics of the beam spot, which were measured with a 900 $\mu$m thick two-dimensional Position Sensitive Silicon Detector (PSSD) placed at the MARS focal plane (the target detector), were a spot size of 2.5 mm $\times$ 3.6 mm FWHM (horizontal $\times$ vertical) and an angular spread of 1.8$^\circ$ $\times$ 0.6$^\circ$. The purity of the $^7$Be beam was 99% at an average rate of $\sim$80 kHz. Alpha particles were
the primary contaminant. For a detailed description of radioactive beam production with MARS, see Ref. 12.

Following beam tuning, the secondary target, a melamine foil with a thickness of 1.5 mg/cm², was moved into the beam spot. Four 5 × 5 cm² PSSDs were placed symmetrically around the target on an aluminum plate, as shown in Fig. 1. Detectors 1 and 2 (110 µm thick) covered a laboratory angular range from 4° to 19°, and detectors 3 and 4 (65 µm thick) covered 16° to 30°. All four PSSDs were backed by 500 µm thick silicon detectors providing particle identification spectra (ΔE, E). Each PSSD was position calibrated using a mask with 6 slots that were 0.8 mm wide and were spaced 8 mm apart. The detectors were cooled to approximately −10°C with two electric thermocoolers fixed on the aluminum plate in order to decrease the inverse current in the detectors and minimize their noise. The assembly was placed on a XYZ optical table for precise positioning.

In our previous experiments with a 7Be beam 2,13, the number of secondary beam particles was determined indirectly by measuring the intensity of the 7Li primary beam in a Faraday cup, and normalizing the yield at low primary beam intensities by counting the 7Be with the target detector. Periodically (typically once a day) the calibration procedure was repeated to check for any rate variations due to drifts in MARS power supplies. The primary 7Li beam intensity was substantially higher for the experiment on the melamine target 11 than the experiment on the 10B 8 target.

Following these two measurements, we modified the experimental setup by adding a monitor detector to count the radioactive beam particles directly. The beam monitoring system, which is shown in Fig. 1, used a wire mesh screen to reduce the secondary beam intensity and a plastic scintillator coupled with a photomultiplier tube to count the radioactive beam particles that passed through the target. In parallel, we ran the old monitoring system with a Faraday cup for the primary beam, and compared the results. In subsequent measurements with a high-intensity 11B primary beam, we observed a difference between the two normalization procedures. Beam heating reduced the density of the gas in the production target, causing a drop in the isotope production rate per nA of primary beam current and increasing the central beam energy.

By scaling the heat deposition of the beam in the gas target, we concluded that this effect may have produced a small but non-negligible shift in the beam normalization during the previous 14N(7Be,8B)13C experiment. The effect was to over estimate the number of secondary beam particles and hence reduce the cross section and the resulting ANC. In contrast, the primary beam intensity for the 10B(7Be,8B)9Be measurement was sufficiently low to have a negligible effect. For the present experiment, the monitor detector was a NE102A plastic scintillator coupled to a photomultiplier tube. To minimize rate-dependent effects in the photomultiplier tube, two screens with a transparency of 9% each were added to attenuate the beam intensity. The yield in the monitor detector was calibrated using the procedure described in Ref. 13.

For the 8B elastic scattering measurement, the radioactive beam was produced via the 14N(10B,8B)7Be reaction using a 27 MeV/A 10B primary beam on the same LN₂-cooled gas cell. The cell contained H₂ gas at 3 atm pressure, corresponding to a target thickness of ≈ 10.8 mg/cm². Entrance and exit windows were made of 50 µm (42 mg/cm²) Havar. A 137 mg/cm² Al degrader was placed behind the gas cell to reduce the secondary beam energy. A 95 MeV radioactive beam of 4Be was focused at the end of MARS with a rate of about 5 kHz. The beam purity was better than 95%, with α particles being the primary contaminant. The full-width energy spread was limited to 1.6 MeV using momentum defining slits. Beam emittance was optimized using a pair of slits after the last quadrupole in MARS. The plastic scintillator behind the target was used for direct counting of the secondary beam particles, in this case without any wire mesh screen. Two telescopes, each consisting of a 110 µm thick PSSD backed by a 500 µm thick Si detector, observed the secondary reaction products. The telescopes covered the angular range θlab = 4° − 19°. A 1.9 mg/cm² C target was used for the elastic scattering measurement. In both experiments, target properties such as thickness and uniformity were verified using the radioactive 7Be and 8B beams directly by detecting beam particles at 0° with and without the target. The resultant energy loss measurements were compared to calculations with the computer code SRIM 14 to extract the thicknesses.

III. RESULTS

A. 7Be elastic scattering

There were two motivations for measuring elastic scattering of 7Be from melamine. The new detector geometry allowed us to extend the angular region for elastics which, in turn, helps to define optical model parameters. Further, it allowed us to normalize the elastic scattering yield directly by counting 7Be particles after the secondary target.

The kinematic reconstruction of the elastically scattered reaction products was performed using the energy and position information from the four detector telescopes. The events selected corresponded to elastic scattering of 7Be on 14N and 12C since the two contributions could not be separated. First, we identified all the events with (ΔE, E) corresponding to 7Be, then we utilized the correlation of 7Be energy vs. scattering angle to select those that were consistent with elastic scattering off either 14N or 12C. This discriminated against scattering on H and inelastic scattering populating excited states in either 14N or 12C. However, it was not possible to separate the elastically scattered events from inelastic scattering leading to the first excited state in 7Be at Eex = 0.429
FIG. 1: A three-dimensional view of the detector assembly.

MeV. We estimated this contribution using data obtained from inelastic scattering of $^7$Li on $^{13}$C at 63 MeV [15] to the analog state at $E_{ex} = 0.477$ MeV. Assuming that the deformation lengths are equal for the analog states, we calculated the inelastic cross section for $^7$Be on $^{14}$N and $^{12}$C at the current energy and subtracted it from the data. The correction was negligible at all but the largest angles, where it amounted to a few percent. The solid angle calculation as a function of scattering angle was done using a Monte Carlo simulation that took the measured properties of the beam spot and the geometrical specifications of the detector assembly as input data. The procedure has been described in previous publications (e.g. see Ref. [12]). Figure 2 shows the resulting angular distribution corresponding to elastically scattered $^7$Be on $^{14}$N and $^{12}$C.

The angular distribution predicted from the optical model parameters used in Ref. [10] is compared to the data in Fig. 2. The Monte Carlo calculation was used to provide the proper angular distribution that takes into account the finite angular binning in the data.

The results of our measurements are compatible with those reported in Ref. [10] at small angles and with the predictions of the optical model calculations done at that time (dotted curve). However, the new experimental data fall above the predictions at larger angles, suggesting a smaller absorption than was assumed in Ref. [10]. In order to obtain a better description of the elastic scattering, calculations were carried out with a range of new parameters. These were also used for the entrance channel to generate DWBA predictions for the $^{14}$N($^7$Be,$^8$B)$^{13}$C proton transfer reaction.

The optical parameters used in Ref. [10] were based on results from an analysis of elastic scattering of loosely bound p-shell nuclei [16], which demonstrated that the data can be described with double-folded potentials. The potentials quoted in Ref. [10] were obtained from calculated nuclear matter densities folded with an effective nucleon-nucleon interaction (JLM, [17]), smeared (two range parameters, $t_V$ and $t_W$) and renormalized (two strength parameters, $N_V$ and $N_W$) to produce:

$$U_{DF}(r) = N_V V(r, t_V) + iN_W W(r, t_W).$$  (1)

The calculations for previous $^7$Be studies were done using the JLM1 effective interaction with standard range parameters: $t_V = 1.20$ fm, $t_W = 1.75$ fm, and average renormalizations $N_V = 0.37$, $N_W = 1.00$ (for details see Ref. [16] and references therein). These parameters served as the starting point for the new calculations. Elastic scattering of $^7$Be at 87.7 MeV on $^{14}$N and $^{12}$C were calculated in the center of mass frame, then transformed into the lab frame and added with weights 1.0 and 0.5, respectively, equal to the ratio of $^{14}$N to $^{12}$C nuclei in the melamine. The resulting curve was “smoothed” using the Monte Carlo code described above. The parameters for the folding potential were varied simultaneously and identically for both target nuclei. This approach is supported by the fact that both target nuclei are well bound and have similar densities in the surface region and by experiments we have carried out with melamine and C targets using other radioactive beams, such as $^{13}$N [18], $^8$B (present experiment) and $^{17}$F [19]. The extended angular coverage of the present data was still not sufficient to attempt an optical model fit with free parameters. Rather, the two normalization and two range parameters were varied. The parameters for various calculations and the reduced $\chi^2$
values obtained by comparison to the data are presented in Table I.

Four entries in the Table (A, B, C and H) were obtained by adjusting the renormalization of the real and imaginary parts of the potential. The smearing ranges of the interaction, $t_V$ and $t_W$, were adjusted for three cases (D, E, F), and the density dependence was adjusted in one case (G) where the JLM2 interaction was used. The best results were obtained for cases D, E, G and H. The small differences between the $\chi^2$ values show that it is difficult to choose a "best solution". Rather, we did DWBA calculations for the $^{14}$N($^{7}$Be,$^{8}$B)$^{13}$C transfer reaction for the four most promising potentials. The results are compared to the previous calculations in Table I.

A far/near decomposition of the scattering amplitudes shows that the observed angular range covers the region of Fraunhofer oscillations generated by the interference of the two components (see Fig. 3). Their crossover is around 20°, and at larger angles the far component becomes dominant. But in the region included in our measurements, the interference is still important. The calculations show that after about 60° the angular distributions develop a rainbow type pattern, typical for the cases found recently in our $^6$,$^7$Li elastic scattering data [20].

A similar analysis was used for the $^{8}$B elastic scattering on a natural C target. The resulting angular distribution is shown in Fig. 3 where it is compared to calculations made with the folded potentials using the average parameters $t_V = 1.20$ fm, $t_W = 1.75$ fm, $N_V = 0.37$ and $N_W = 1.00$. The solid (dashed) line shows the results after (before) smoothing with the Monte Carlo calculation. The $^8$B density used in the folding procedure was that calculated in [16] using the correct ANC for the last proton. Due to the limited angular range of the data, we did not attempt to produce a better fit by adjusting parameters. Based on the similar densities for $^{12}$C and $^{13}$C and on results found in cases where scattering on both $^{12}$C and $^{13}$C were measured, we assume that the parameters ($t_V$, $t_W$, $N_V$, $N_W$) extracted for the natural C target are valid for the $^8$B+$^{13}$C channel in DWBA calculations of the transfer reaction.
The results of the calculations are given in Table I where the data. This quantity contains the DWBA cross sections weighted with the ANCs for $^{14}$N, the single particle ANCs calculated for the appropriate Woods-Saxon proton binding potentials in $^8$B and $^{14}$N and the mixing ratio in the ground state of $^8$B, $\delta^2$. Since the reaction is peripheral, the results do not depend on the geometry assumed for the proton binding potentials, which are chosen to be Woods-Saxon shape with depths adjusted to reproduce the experimental proton binding energies in $^8$B and $^{14}$N, respectively. The results shown were calculated using the reduced radius $r_0 = 1.20$ fm and the diffuseness $a = 0.60$ fm, and the same spin-orbit term as in Ref. [10]. The exit channel parameters were fixed to the previous values. Calculations were done at $E_{lab} = 83.5$ MeV, the energy of the previous experiment with the four optical model sets in Table I for the entrance channel that have the lowest $\chi^2$, B, D, E, G, and H. In column 8 we give the ratio of the present calculations to the same quantity calculated in Ref. [10]. The average of the four results, weighted by the $\chi^2$’s, gives the ratio $\langle R \rangle = 0.968 \pm 0.047$. The new ANC, calculated with the relation

$$C^2_{sB,p_{3/2}}(new) = (1.055/\langle R \rangle)C^2_{sB,p_{3/2}}(old),$$  

where the $\sigma$’s are the calculated DWBA differential cross sections for proton transfer from the $p_{3/2}$ and $p_{1/2}$ orbitals in $^{14}$N to the $p_{3/2}$ and $p_{1/2}$ orbitals in $^8$B, the $b_{ij}$’s are the ANCs for the single particle orbitals used in the DWBA calculation, and the $C_{14N,p_{3/2}}$’s and $C_{sB,p_{3/2}}$’s are the ANCs for $^{14}$N $\rightarrow$ $^{13}$C $+ p$ and $^8$B $\rightarrow$ $^7$Be $+ p$, respectively. The ANCs, $C^2_{14N,p_{1/2}} = 18.6(12)$ fm$^{-1}$ and $C^2_{14N,p_{3/2}} = 0.93(14)$ fm$^{-1}$, were measured in [21, 22]. The calculations were done with the code PTOLEMY [23]. The results of the calculations are given in Table I where the value calculated and shown in column 7 is the quantity in the last square bracket in Eq. (2). $\sigma_{DW}$, integrated over the angular region $\theta_{c.m.} = 4^\circ$ - $25^\circ$ to match the data. This quantity contains the DWBA cross sections weighted with the ANCs for $^{14}$N, the single particle ANCs calculated for the appropriate Woods-Saxon proton binding potentials in $^8$B and $^{14}$N and the mixing ratio in the ground state of $^8$B, $\delta^2$. Since the reaction is peripheral, the results do not depend on the geometry assumed for the proton binding potentials, which are chosen to be Woods-Saxon shape with depths adjusted to reproduce the experimental proton binding energies in $^8$B and $^{14}$N, respectively. The results shown were calculated using the reduced radius $r_0 = 1.20$ fm and the diffuseness $a = 0.60$ fm, and the same spin-orbit term as in Ref. [10]. The exit channel parameters were fixed to the previous values. Calculations were done at $E_{lab} = 83.5$ MeV, the energy of the previous experiment with the four optical model sets in Table I for the entrance channel that have the lowest $\chi^2$, B, D, E, G, and H. In column 8 we give the ratio of the present calculations to the same quantity calculated in Ref. [10]. The average of the four results, weighted by the $\chi^2$’s, gives the ratio $\langle R \rangle = 0.968 \pm 0.047$. The new ANC, calculated with the relation

$$C^2_{sB,p_{3/2}}(new) = (1.055/\langle R \rangle)C^2_{sB,p_{3/2}}(old),$$
is $C_{2B,p/2}^2\text{(new)} = 0.414 \pm 0.041 \text{ fm}^{-1}$. The overall uncertainty contains contributions from statistics (2.6%), absolute normalization of the cross section (5%), input parameters in the Monte Carlo simulation of the experiment (1.4%), and uncertainties in the ANC for the $^{14}\text{N}$ vertex (6.4%). The contribution of each of these factors remains the same as in the original analysis. The uncertainty due to the optical model parameters was taken from the standard deviation of the calculated cross sections (column 7 or 8 in Table II) and is 5% compared with the previous value of 8.1%.

The relation between the ANCs and the astrophysical factor $S_{17}(0)$, in eV·b, is

$$S_{17}(0) = 38.6 \left( C_{p3/2}^2 + C_{p1/2}^2 \right) = 38.6 C_{p3/2}^2 (1 + \delta^2) \quad (4)$$

Using the new value of the ANC we find $S_{17}(0) = 18.0 \pm 1.8$ eV·b. This value is very close to the value obtained from the reaction $^{10}\text{Be}(^{7}\text{Be},^8\text{Be})^9\text{Be}$ in Ref. [6] where we found $S_{17}(0) = 18.4 \pm 2.5$ eV·b. The weighted average of the two results is $S_{17}(0) = 18.2 \pm 1.7$ eV·b.

V. CONCLUSION

Elastic scattering of $^7\text{Be}$ at about 12 MeV/A has been measured over an extended angular range on a melamine target. The results provide a better determination of the optical model parameters used for the entrance channel of the $^{12}\text{N}(^7\text{Be},^8\text{Be})^{13}\text{C}$ reaction. For the first time elastic scattering of $^8\text{Be}$ was measured on a C target, thus allowing for a check the optical model parameters used for the exit channel in the DWBA calculation. In the measurement of the $^7\text{Be}$ elastic scattering, we directly counted the secondary beam particles. This resulted in a 5.5% increase of the transfer reaction cross section from Ref. [10], which used an indirect method to obtain the secondary beam intensity. We also used the mixing ratio between the $1p_{1/2}$ and $1p_{3/2}$ components from a ($^7\text{Li},^8\text{Li}$) measurement [13], rather than a theoretical prediction. These improvements lead to the revised value of the ANC for $^8\text{Be} \rightarrow ^7\text{Be} + p$ from the $^{14}\text{N}(^7\text{Be},^8\text{Be})^{13}\text{C}$ reaction of $C_{2B,p/2}^2\text{(new)} = 0.414 \pm 0.041 \text{ fm}^{-1}$, resulting in $C_{tot}^2(8B; new) = C_{p3/2}^2 + C_{p1/2}^2 = 0.466 \pm 0.047 \text{ fm}^{-1}$. This, in turn leads to a larger value for the astrophysical S factor for the $^{7}\text{Be}(p,\gamma)^8\text{Be}$ reaction, $S_{17}(0) = 18.0 \pm 1.8$ eV·b. This new value is very close to the one from the same reaction on the $^{14}\text{B}$ target. Averaging the two results, we obtain $S_{17}(0) = 18.2 \pm 1.7$ eV·b from the proton transfer reactions.

Our result for $S_{17}(0)$ is a bit over 2σ lower than the extrapolation of the most recent and precise direct measurement of $^{7}\text{Be}(p,\gamma)^8\text{Be}$ by Junghans et al. [10]. Our central value is about 1.5σ lower than the average central value obtained by Cyburt et al. [8] in a recent analysis that uses all of the best available capture data, under the assumption that they are independent. Including the uncertainty quoted by Cyburt et al. our results are consistent at the 1σ level. We do not understand the reason for the discrepancy between our ANC result and the extrapolated value from Junghans et al. However, we note that direct measurements with both radioactive beams and targets and indirect measurements continue to be carried out on this important proton capture reaction.

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TABLE I: The optical model parameters, the corresponding $\chi^2$ per degree of freedom for the $^7$Be elastic scattering fits, the calculated DWBA cross section for the $^{14}$N($^7$Be,$^8$B)$^{13}$C reaction (see text), and the ratio of the calculated DWBA calculation to that in Ref. [10].

| Calculation | $N_V$ | $N_W$ | $t_V$ [fm] | $t_W$ [fm] | $\chi^2/N$ | $\bar{\sigma}_{DW}$ | Ratio $R = \frac{\sigma_{DW}(\text{new})}{\sigma_{DW}(\text{orig})}$ |
|------------|-------|-------|------------|------------|-------------|-----------------|-----------------|
| Ref. [10]  | 0.37  | 1.00  | 1.20       | 1.75       | 35.19       | 2.469           | 1.00            |
| A          | 0.45  | 0.90  | 1.20       | 1.75       | 10.72       |                 |                 |
| B          | 0.40  | 0.92  | 1.20       | 1.75       | 15.02       | 2.385           | 0.966           |
| C          | 0.42  | 0.92  | 1.20       | 1.75       | 12.84       |                 |                 |
| D          | 0.42  | 0.90  | 0.80       | 1.75       | 10.31       | 2.408           | 0.975           |
| E          | 0.42  | 0.90  | 0.80       | 1.55       | 7.97        | 2.538           | 1.028           |
| F          | 0.52  | 0.78  | 0.12       | 2.59       | 25.72       |                 |                 |
| G          | 0.40  | 0.85  | 1.20       | 1.75       | 7.49        | 2.375           | 0.962           |
| H          | 0.40  | 0.85  | 1.20       | 1.75       | 9.22        | 2.137           | 0.900           |

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