Lepton flavor violating decays of the SM-like Higgs boson

$h \to e_i e_j$, and $e_i \to e_j \gamma$ in a flipped 3-3-1 model

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Abstract

In the framework of the flipped 3-3-1 model introduced recently [1], the lepton-flavor-violating (LFV) decay $\mu \to 3e$ was predicted to have a large branching ratio (Br) close to the recent experimental limit. We will show that the Br of LFV decays of the standard-model-like (SM-like) Higgs boson decays (LFVHD) $Br(h \to e_a e_b)$ may also be large. Namely, the $Br(h \to \mu \tau, e\tau)$ can reach values of $O(10^{-4}) - O(10^{-5})$, which will be reach the upcoming experimental sensitivities. On the other hand, for LFV decays of charged leptons (cLFV) ($e_b \to e_a \gamma$), the branching ratios are well below experimental bounds.

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I. INTRODUCTION

Since the SM-like Higgs boson was discovered in 2012 [2–4], the LFV decays of this Higgs boson have been sought for by experiments at the Large Hadron Collider (LHC) [5–10]. Recent experimental lower bounds on the LFV decays of the standard-model-like (SM-like) Higgs decays (LFVHD) $h \to e_b e_a$ are

$$
\text{Br}(h \to \tau\mu), \text{Br}(h \to \tau e) \leq \mathcal{O}(10^{-3}),
$$

$$
\text{Br}(h \to \mu e) < 3.5 \times 10^{-4}.
$$

An updated lower bound $\text{Br}(h \to \mu e) < 6.1 \times 10^{-5}$ has been reported recently by the ATLAS Collaboration [11]. Recent studies have predicted that lower bounds from experiments for $\text{Br}(h \to \mu\tau, e\tau)$ could reach the orders of $\mathcal{O}(10^{-4}) - \mathcal{O}(10^{-5})$ [12–15].

The LFVHD has been studied in many models beyond the SM, from seesaw and inverse seesaw models [16–20] to more complicated ones [21–54], including the suppersymmetric versions [55–67]. Many of these models predict very large $\text{Br}(h \to \tau\mu, \tau e)$ with the order of $\mathcal{O}(10^{-5})$, implying that LFVHD decays will be signals as new physics that will be tested experimentally in the near future.

The models beyond the SM constructed by extending the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ into the group $SU(3)_C \times SU(3)_L \times U(1)_X$ (3-3-1) models may predict large LFV decay branching ratios. This can be explained based on the common property of the popular 3-3-1 models [68–72] that left-handed fermions are usually arranged into $SU(3)_L$ (anti)triplets. Hence, there will appear couplings of new heavy leptons in the third components of these lepton representations with normal charged leptons and gauge or Higgs bosons. The mixing of these heavy leptons is an important source of LFV mediation at the one-loop level. Therefore, LFV decays of charged leptons in the framework of 3-3-1 models have been widely investigated [73–80]. Many of the 3-3-1 models can explain the recent lower bounds on the decays $\text{Br}(e_b \to e_a \gamma)$ [81–82]

$$
\text{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8},
$$

$$
\text{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8},
$$

$$
\text{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}.
$$

In future projects, new sensitivities for these decay channels will be $\text{Br}(\mu \to e\gamma) \sim$
\( \mathcal{O}(10^{-14}) \) \cite{83} and \( \text{Br}(\tau \rightarrow \mu \gamma, e\gamma) \sim \mathcal{O}(10^{-9}) \) \cite{84}. They will be used to determine allowed regions of the parameter spaces of the 3-3-1 models for further studying other LFV decays such as those of the SM-like Higgs boson \( h \rightarrow e^+_b e^-_a \). They just have been investigated in just a few specific 3-3-1 models \cite{45, 52}, where the LFV sources come from the mixing of heavy neutrinos. In particular, the 3-3-1 model with inverse seesaw neutrinos \cite{52} predicts very small regions of parameter space that give large \( \text{Br}(h \rightarrow \tau \mu, \tau e) \sim O(10^{-5}) \) and also satisfy the current bounds of \( \text{Br}(\mu \rightarrow e\gamma) \). Recently, an interesting flipped 3-3-1 model has been constructed \cite{1}, where the left-handed lepton is arranged in a lepton sextet, while the left-handed \( \tau \) and \( \mu \) are still the same as those known previously. In addition, all left-handed quarks are also arranged in the same \( SU(3)_L \) triplets so that the model is anomaly free. The treel-level flavor neutral changing currents caused by the heavy neutral boson \( Z' \) do not appear; hence \( m_{Z'} \) is not constrained by the corresponding experimental data. The active neutrino and electron masses can be produced consistent with experiments through loop corrections \cite{85}. The effect of the Higgs sextet on fermion and Higgs boson couplings was discussed in ref. \cite{86}. The Higgs potentials relating to the Higgs sextets were studied in refs. \cite{87, 88}. Based on these ingredients, our aim in this work is to investigate the LFV decays of charged leptons \( e_b \rightarrow e_a \gamma \) and the SM-like Higgs boson \( h \rightarrow e_b e_a \) in the framework of the flipped 3-3-1 model.

Our work is arranged as follows. In Sects \text{\textit{II}} and \text{\textit{III}}, we will collect the main content of the flipped 3-3-1 model, where masses, physical states, and needed couplings for calculating branching ratios of the LFV decays are presented. The analytic formulas of LFV branching ratios and the corresponding numerical investigations will be shown in section \text{\textit{IV}}. We will summary main results in Sect \text{\textit{V}}. Finally, there are two appendices showing the details of the one loop formulas contributing to the LFV decays of charged lepton (cLFV) amplitudes of the decays \( e_b \rightarrow e_a \gamma \) and the equations for minimal conditions of the Higgs potential considered in this work.
II. THE FLIPPED 3-3-1 MODEL

A. The model review

We follow the model introduced in Ref. [1], where the particle content is presented in Table I. All fermions are written in terms of Dirac spinors.

| Name  | 331 rep. | SM group decomposition | Components | # flavors |
|-------|----------|------------------------|------------|----------|
| $L_e$ | $\left(1, 6, -\frac{1}{3}\right)$ | $\left(1, \bar{3}, 0\right)$ + $\left(1, \bar{2}, -\frac{1}{2}\right)$ + $\left(1, \bar{1}, -1\right)$ | $\left(\Sigma_3^-\right)$, $\frac{1}{\sqrt{2}}\Sigma_3^0 L$, $\frac{1}{\sqrt{2}}\nu e L$ | 1 |
| $L_{\alpha=\mu,\tau}$ | $\left(1, 3, -\frac{2}{3}\right)$ | $\left(1, \bar{2}, -\frac{1}{2}\right)$ + $\left(1, \bar{1}, -1\right)$ | $\left(v_\alpha, e_\alpha, E_\alpha\right)_L^T$ | 2 |
| $e_{\alpha R}$ | $\left(1, 1, -1\right)$ | $\left(\nu_{\alpha}, e_\alpha\right)$ | $\nu_\alpha$ | 6 |
| $Q_\alpha$ | $\left(3, 3, \frac{1}{3}\right)$ | $\left(3, \bar{2}, \frac{1}{2}\right)$ + $\left(3, \bar{1}, \frac{2}{3}\right)$ | $\left(d_{\alpha}, -u_\alpha, U_{\alpha}\right)_L^T$ | 3 |
| $u_{\alpha R}$ | $\left(3, 1, \frac{2}{3}\right)$ | $\left(3, \bar{1}, \frac{2}{3}\right)$ | $u_{\alpha R}$ | 6 |
| $d_{\alpha R}$ | $\left(3, 1, -\frac{2}{3}\right)$ | $\left(3, \bar{1}, -\frac{2}{3}\right)$ | $d_{\alpha R}$ | 3 |
| $\phi_{i=1,2}$ | $\left(1, 3, \frac{1}{3}\right)$ | $\left(1, \bar{2}, \frac{1}{2}\right)$ + $\left(1, \bar{1}, 0\right)$ | $\left(H_1^0, H_0^0, \sigma_i^0\right)^T$ | 2 |
| $\phi_3$ | $\left(1, 3, -\frac{2}{3}\right)$ | $\left(1, \bar{2}, -\frac{1}{2}\right)$ + $\left(1, \bar{1}, -1\right)$ | $\left(H_3^0, H_0^0, \sigma_3^0\right)^T$ | 1 |
| $S$ | $\left(1, 6, \frac{2}{3}\right)$ | $\left(1, \bar{3}, 1\right)$ + $\left(1, \bar{2}, \frac{1}{2}\right)$ + $\left(1, \bar{1}, 0\right)$ | $\left(\Delta^{++}, \frac{1}{\sqrt{2}}\Delta^{+}, \frac{1}{\sqrt{2}}H_3^0\right)$ | 1 |

TABLE I: Representations for the flipped 3-3-1 model, taken from Ref. [1], the notations of fermions are Dirac spinors.

The electric charge operator is:

$$Q = T^3 + \frac{1}{\sqrt{3}} T^8 + X,$$

where $T^3, T^8$ are diagonal generators of the $SU(3)$ group.

These Higgs bosons develop vacuum expectation values (VEV) defined as

$$\sigma_i^0 = n_i + \frac{1}{\sqrt{2}} (R_{\sigma_i} + iI_{\sigma_i}), \quad \langle \sigma_i^0 \rangle = n_i, \quad i = 1, 2, S,$$

$$H_\alpha^0 = k_\alpha + \frac{1}{\sqrt{2}} (R_\alpha + iI_\alpha), \quad \langle H_\alpha^0 \rangle = k_\alpha, \quad \alpha = 1, 2, 3, S,$$
\[ \Delta^0 = \epsilon_S + \frac{1}{\sqrt{2}} \left( + R_\Delta + i I_\Delta \right), \quad \langle \Delta^0 \rangle = \epsilon_S, \]  

(4)

where \( \epsilon_S \ll k_{1,2,3}, S \ll n_{1,2,3} \) in general [1]. In addition, it was shown that \( \epsilon_S \) and \( k_S \) should be small to successfully generate neutrino mass consistent with experimental data. Hence, we can take \( k_s = \epsilon_S \simeq 0 \) when solving the masses and physical states of Higgs and gauge bosons.

The Yukawa Lagrangian for the lepton sector is

\[ - \mathcal{L}^Y_{\text{lepton}} = \sum_{i=1}^{2} \sum_{\alpha=\mu,\tau} \sum_{\beta=1}^{6} y_{\alpha\beta}^{(i)} \overline{e_{\beta R} L_\alpha} \phi_i^* + \sum_{\beta=1}^{6} y_{\beta R} \overline{e_{\beta R} L_\alpha S^*} + y^{\nu n} (\overline{L_e} c) L_e S + \text{h.c.}, \]  

(5)

where the invariant term of the tensor product of the three sextets is expanded as \( (\overline{L_e})^c L_e S = \epsilon_{abc} \epsilon_{ijk} (\overline{L_e})_a^c e_{bi} (L_e)_j S_{ck} \) [73, 89], \( (L_e)_a^c \equiv C (\overline{L_e})_a^c \). Note that \( \phi_3 \) only appears in the Yukawa part of the quark.

The fermions are presented as two-component spinors in the original version; see table I in Ref. [1]. In this work, we will use the Dirac (four-component) spinor notation, based on the equivalence given in detail in Ref. [90]. In particular, a Dirac spinor \( f = (f_L, f_R)^T \), where \( f_{L,R} \) is the respective left (right) component of a Dirac fermion, namely \( f_L = P_L f \) and \( f_R = P_R f \). The Dirac conjugation is \( \overline{f} = f^\dagger \gamma^0 = (\overline{f}_R, \overline{f}_L) \). The charge conjugation is \( f^C \equiv C f^T = ((f_R)^c, (f_L)^c)^T \), implying that \( (f_{R,L})^c = P_{L,R} f^C \). A Majorana fermion satisfying \( f^C = f \) results in \( f_{L,R} = (f_{R,L})^c \). The mass term of all fermions at tree level is

\[ - \mathcal{L}^\text{mass}_{\text{lepton}} = \sum_{i=1}^{2} \sum_{\alpha=\mu,\tau} \sum_{\beta=1}^{6} y_{\alpha\beta}^{(i)} \overline{e_{\beta R} (e_{\alpha L} k_i + E_{\alpha L} n_i)} + \sum_{\beta=1}^{6} y_{\beta R} \overline{e_{\beta R} (\Sigma^- e_S + E_L k_s + E_e n_s)} \]

\[ + y^{\nu n} \left[ 2 \epsilon_S \left( \overline{\Sigma^-} E_{e L} - (\nu_e L)^c \nu_e L \right) + 2 k_s \left( \overline{\Sigma^-} e_L + \frac{1}{\sqrt{2}} (\nu_e L)^c \nu_e L \right) \right] + n_s \left( 2 \overline{\Sigma^-} \Sigma^- - (\Sigma^- L)^c \Sigma^- L \right) + \text{h.c.}, \]  

(6)

where we have used the identity \( \overline{\psi^e_a} \psi^e_b = \overline{\psi_b} \psi_a \) for leptons.

According to the discussion on Ref. [1], in the basis \( \Psi_{L,R}^T = (e_\alpha^, \phantom{.}, E_\alpha, \phantom{.}, E_e, \phantom{.}, e, \phantom{.}, \Sigma^-)^T \) the mass matrix of charged leptons always has one massless eigenstate at tree level, corresponding to the normal electron mass \( m_e = 0 \). This is also the case for active neutrinos. However, when the loop corrections are included, the consistent masses of electrons and active neutrinos are obtained. The one-loop Feynman diagrams corresponding to these corrections are given in Fig. [1] and were pointed out in Ref. [1], along with a very detailed discussion
FIG. 1: Feynman diagrams giving one-loop corrections to the masses of electron (left panel) and active neutrinos (right panel) [1], where $\ell^c$ means $\ell^c_\alpha \equiv (e_\alpha R)^c$.

on this property of the flipped 3-3-1 model. Accordingly, using the minimal Higgs sector given in Table II, the experimental data of an inverse hierarchy for active neutrinos can be fitted. Adding more scalar fields to the model will be another way to solve the problem of the neutrino oscillations that can be fitted with recent experimental data. As we will show, this problem does not affect significantly our discussion on LFV decays.

Because loop corrections are needed to generate masses of only very light leptons, namely electrons and active neutrinos, the other corrections to the lepton mass matrices are also smaller than other heavy masses appearing in the model. This is also because of another reason that one-loop corrections are suppressed by the two factors $1/(16\pi^2)$ and $1/M^2$ relating respectively to the one-loop integral and new heavy masses $M$ of a new particle running in the loop. In conclusion, loop corrections make tiny contributions to the lepton mass matrices. Hence, we will ignore loop corrections to the masses of heavy particles from now on.

For simplicity, in this work we will assume that only exotic charged leptons $E_e, E_\mu, E_\tau$ mix with each other to guarantee the existence of LFV couplings that contribute to one-loop amplitudes of the LFV decays. On the other hand, all of the original states of the SM charged leptons and $\Sigma^-$ are physical. This corresponds to the condition that $\epsilon, k_S, n_1, k_2 \simeq 0$. The large Yukawa couplings of the physical states $\mu, \tau$ and $\Sigma^-$ are

$$y^{(1)}_{11} = \frac{m_\mu}{k_1}, \quad y^{(1)}_{22} = \frac{m_\tau}{k_1}, \quad y^{\Sigma^-} = \frac{m_{\Sigma^-}}{2n_S}.$$  \hspace{1cm} (7)

Note that the masses of electrons and active neutrinos come from loop corrections.

The original basis $(E_\mu, E_\tau, E_e)$ corresponds to the following mass term:

$$-L^E_{mass} = \left( E_\mu R \ E_\tau R \ E_e R \right) M_{E,\mu,\tau,e} \left( E_{\tau L} \ E_{\mu L} \ E_{e L} \right)^T + h.c.,$$

\hspace{1cm} 6
\[ \mathcal{M}_{E_{\mu,\tau,e}} = n_2 \begin{pmatrix} y_{13} & y_{14} & y_{15} \\ y_{23} & y_{24} & y_{25} \\ y_3 & y_4 & y_5 \end{pmatrix} \equiv n_2 Y^\ell, \]

where we have used the assumption that some of the Yukawa couplings in the Lagrangian \[^5\] are zeros. The lepton mass matrix in Eq. (8) is arbitrary; hence it is diagonalized by the following transformation:

\[ V_R^{E^\dagger} \mathcal{M}_{E_{\mu,\tau,e}} V_L^E = \text{diag}(m_{E_1}, m_{E_2}, m_{E_3}), \]

\[ \begin{pmatrix} E_\mu \\ E_\tau \\ E_e \end{pmatrix}_{R,L} = V_{R,L} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \end{pmatrix}_{R,L}, \]

where \( m_{E_i} \) are masses of the physical states \( E_{iL(R)} \), \( i = 1, 2, 3 \). For simplicity, in this work we will choose \( V_R^E = I_3 \), while \( V_L^E \) is parameterized in terms of three free mixing angles \( \theta_{ij}^E \), \( i,j = 1, 2, 3 \) \((i < j)\), namely

\[ V_L^E \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23}^E & s_{23}^E \\ 0 & -s_{23}^E & c_{23}^E \end{pmatrix} \begin{pmatrix} c_{13}^E & 0 & s_{13}^E \\ 0 & 1 & 0 \\ -s_{13}^E & 0 & c_{13}^E \end{pmatrix} \begin{pmatrix} c_{12}^E & s_{12}^E & 0 \\ -s_{12}^E & c_{12}^E & 0 \\ 0 & 0 & 1 \end{pmatrix}, \]

\[ \equiv \begin{pmatrix} c_{23} c_{13}^E & c_{12}^E s_{13}^E & s_{12}^E \\ c_{13}^E & c_{12}^E & s_{12}^E \\ c_{13}^E & s_{13}^E & c_{12}^E \end{pmatrix}, \]

where \( s_{ij} \equiv \sin \theta_{ij}^E, c_{ij}^E \equiv \cos \theta_{ij}^E \), and all Dirac and Majorana phases are set to be zeros. This matrix exactly satisfies the unitary property. We will use \( s_{ij}^E \) as free parameters.

Other Yukawa couplings are non-zero for generating active neutrino masses and mixing consistent with experiments (see discussions in ref. \[^1\]), but they are assumed to be suppressed in this work. We also note that the conditions in Eq. (7) still allow right SM quark masses and mixing consistent with experimental data. Similarly, there is one heavy Majorana neutrino \( \Sigma_M = (\Sigma^0, \Sigma^0)^T \) with the mass term \(-1/2(-2y^\ell n_s)\Sigma^0\Sigma^0 + h.c.\). Three other active neutrinos get consistent masses and mixing from loop corrections, which prefer the inverted order of active neutrino data oscillation. Their physical states are denoted as \( n_1, n_2, n_3 \) \[^1\]. The masses and mass eigenstates of heavy neutral leptons are

\[ n_4 = i\Sigma_M, \quad m_{n_4} = m_{\Sigma^-} = 2n_S y^\ell. \]
Yukawa coupling terms in the Lagrangian (5) containing normal charged leptons are

\[ \mathcal{L}_Y^\ell = -\frac{m_\mu}{k_1} \left[ H_1^0 \mu_R \mu_L + \sigma_1^{0*} \mu_R E_\mu L + H_1^- \mu_R \nu_{\mu L} \right] \\
- \frac{H_2^{*}}{\sqrt{2}} \left[ E_{\mu R} y_{13}^{(2)} + E_{\tau R} y_{14}^{(2)} + E_{e R} y_{15}^{(2)} \right] \mu_L \\
- \frac{m_\tau}{k_1} \left[ H_1^0 \tau_R \tau_L + \sigma_1^{0*} \tau_R E_{\tau L} + H_1^- \tau_R \nu_{\tau L} \right] \\
- \frac{H_2^{*}}{\sqrt{2}} \left[ E_{\mu R} y_{23}^{(2)} + E_{\tau R} y_{24}^{(2)} + E_{e R} y_{25}^{(2)} \right] \tau_L \\
- \frac{H_3^{*}}{\sqrt{2}} \left[ E_{e R} y_3^{(2)} + E_{\tau R} y_4^{(2)} + E_{e R} y_5^{(2)} \right] e_L + \frac{m_{\Sigma^-}}{n_S} H_S^{0*} \Sigma_R e_L \\
- \frac{m_{\Sigma^-}}{\sqrt{2} n_S} \Delta^{+} \nu_{e L} e_L + \frac{m_{\Sigma^-}}{n_S} \Delta^{+} (\bar{e}_L) e_L + \frac{i m_{\Sigma^-}}{\sqrt{2} n_S} H_S^{+} n_{4L} e_L + h.c. \quad (12) \]

Corresponding to the above assumption that all charged leptons are diagonal, Yukawa couplings relating to one-loop corrections must guarantee that new Higgs bosons should couple to different SM charged leptons. As we will show later, the SM-like Higgs bosons will be \( h \simeq R_3 \) when we assume that \( k_1 \ll k_3 \). Combined with Lagrangian (12), we can see that tree-level couplings of the SM-like Higgs boson \( h e_i e_j \) do not appear. The heavy neutral lepton \( n_4 \) does not couple with normal charged leptons. The couplings \( h e_i e_i \) appear from the small mixing of \( R_3 \) and \( R_1 \) for \( e_i = \mu, \tau \) and loop corrections for the electron. These couplings have small effects on the LFV decays so we omit them from now on.

After breaking, the masses and physical states of all gauge bosons are determined as follows.

### III. HIGGS AND GAUGE BOSONS

#### A. Gauge boson

The covariant derivative of the \( SU(3)_L \times U(1)_X \) is defined as

\[ D_\mu \equiv \partial_\mu - ig W_\mu^a T^a - ig X T^9 X \mu, \quad (13) \]

where \( T^a (a = 1, 2, .., 8) \) is the \( SU(3) \) generator with respective gauge boson \( W_\mu^a \), \( T^9 = \frac{1}{\sqrt{6}} \) is the \( U(1)_X \) generator with the gauge boson \( X_\mu \), and \( X \) is the \( U(1)_X \) charge of the field acted by the covariant derivative. The particular forms of the generators are:

- For an \( SU(3)_L \) singlet: \( T^a = 0 \ \forall a = 1, 2, .., 8 \), \( T^9 = \frac{1}{\sqrt{6}} \).
• For an $SU(3)_L$ triplet: $T^a = \frac{1}{2} \lambda_a \ \forall a = 1, 2, \ldots, 8$, $T^9 = \frac{1}{\sqrt{6}} I_3$, where $\lambda_a$ are Gell-Mann matrices. The covariant part can be written as:

$$W_\mu \equiv W^a T^a = \frac{1}{2} \begin{pmatrix}
W^3_\mu + \frac{1}{\sqrt{3}} W^8_\mu & \sqrt{2} W^\prime_\mu & \sqrt{2} Y^\prime_\mu \\
\sqrt{2} W^{-}_\mu & -W^3_\mu + \frac{1}{\sqrt{3}} W^8_\mu & \sqrt{2} Y^0_\mu \\
\sqrt{2} Y^{-}_\mu & \sqrt{2} Y^0_\mu & -\frac{2}{\sqrt{3}} W^8_\mu
\end{pmatrix}, \quad (14)$$

where we have defined the mass eigenstates of the charged gauge bosons as

$$W^\prime_\mu = \frac{1}{\sqrt{2}} (W^1_\mu \mp i W^2_\mu), \quad Y^\prime_\mu = \frac{1}{\sqrt{2}} (W^4_\mu \mp i W^5_\mu), \quad Y^0_\mu = \frac{1}{\sqrt{2}} (W^6_\mu - i W^7_\mu). \quad (15)$$

• For an $SU(3)_L$ antitriplet: $T^a = -\frac{1}{2} \lambda^*_a = -\frac{1}{2} \lambda^T_a \ \forall a = 1, 2, \ldots, 8$, $T^9 = \frac{1}{\sqrt{6}} I_3$.

• For an $SU(3)_L$ sextet denoted as $S \sim (6, 2/3)$, given in table [1], the action of an $SU(3)_L$ generator can be written in terms of the Gell-Mann matrix, $T^a S = S \lambda_a/2 + \lambda_a/2 S^T$ [91]. Hence, the corresponding covariant derivative can be written in terms of the generators of the $SU(3)$ triplet [91, 92], namely

$$D_\mu S = \partial_\mu S - ig \left[ SW_\mu + SW^T_\mu \right] - ig X \frac{X}{\sqrt{6}} X_\mu S. \quad (16)$$

The symmetry-breaking pattern is $SU(3)_L \times U(1)_X \xrightarrow{(\sigma^0)} SU(2)_L \times U(1)_Y \xrightarrow{(H^0)} U(1)_Q$, where $i = 1, 2, S$ and $\alpha = 1, 2, 3, S$.

The covariant kinetic terms of the Higgs bosons are

$$L^H_{kin} = \sum_{i=1}^3 (D_\mu \phi_i)^\dagger (D^\mu \phi_i) + (D_\mu S)^\dagger (D^\mu S). \quad (17)$$

From this, the squared mass matrix of the charged gauge bosons in the basis $(W^\prime_\mu, Y^\prime_\mu)$ is given by

$$M^2_{V^\pm} = \frac{g^2}{2} \begin{pmatrix}
k_1^2 + k_2^2 + k_3^2 + k_s^2 + 2 \varepsilon_S^2 & k_1 n_1 + k_2 n_2 + \sqrt{2} k_S n_S + \sqrt{2} k_s \varepsilon_S \\
k_1 n_1 + k_2 n_2 + \sqrt{2} k_S n_S + \sqrt{2} k_s \varepsilon_S & k_3^2 + k_s^2 + n_1^2 + n_2^2 + 2 n_S^2
\end{pmatrix}. \quad (18)$$

It is enough to assume that $k_i/n_i \ll 1$ for $i = 2, S$ so that the non-diagonal term in the squared mass matrix (18) can be ignored. In this work we will accept that

$$n_1 = 0, \quad \frac{k_2}{n_2} = \frac{k_s}{n_S} \ll 1. \quad (19)$$

9
In particular, we will choose $k_{1,2,S} \sim \mathcal{O}(10)$ GeV and $n_{2,s} \sim \mathcal{O}(10^3)$ GeV, leading to the consequence that $k_1 n_i \text{GeV}^2/(246 \text{GeV})^2 \ll 1$. The non-zero values of $k_1$ still allow the reasonable Yukawa couplings of normal charged leptons given in Lagrangian (12). We note that this choice of VEV values are still allowed for generating consistent quark masses, as discussed previously [1]. The masses and physical states $\{W^\pm, Y^\pm\}$ of charged gauge bosons are determined as

$W^\pm \simeq W'^\pm$, $m_W^2 = \frac{g^2}{2} v^2$, $v^2 \equiv (k_1^2 + k_2^2 + k_3^2 + k_S^2)$,

$Y^\pm \simeq Y'^\pm$, $m_Y^2 = \frac{g^2}{2} u^2$, $u^2 \equiv (k_3^2 + k_S^2 + n_1^2 + n_2^2 + n_S^2)$.

Identifying the $W^\pm$ with the SM one, we have $v \simeq 174 \text{ GeV}$. If $k_{1,2,S} = \mathcal{O}(10)$ GeV, we have $k_3 \simeq v$. Using the assumption in Eq. (19) the neutral gauge boson mass can be determined as follows.

The non-Hermitian gauge bosons $V^0$ and $V^{0*}$ do not mix with the Hermitian ones. The masses and physical states are

$V^0 \simeq V'^0$, $m_V^2 = \frac{g^2}{2} (u^2 + n_S^2)$.

For simplicity in calculating the masses and mass eigenstates of the Hermitian neutral gauge bosons, we will safely use the limit that $k_1, k_2, k_S, \epsilon_S \ll k_3$. Accordingly, these neutral gauge bosons will decouple with the Re$V^0$. In the basis $(X_\mu, W_3^\mu, W_8^\mu)$, the squared mass matrix is

$\mathcal{M}_{X38}^2 = \frac{g^2}{2} \begin{pmatrix}
\frac{2}{3} t^2 (3 n_S^2 + u^2 + 4 v^2) & -\frac{2}{3} \sqrt{\frac{2}{3}} t v^2 & -\frac{2}{3} \sqrt{\frac{2}{3}} t (3 n_S^2 + u^2 + v^2) \\
-\frac{2}{3} \sqrt{\frac{2}{3}} t v^2 & v^2 & \frac{v^2}{\sqrt{3}} \\
-\frac{2}{3} \sqrt{\frac{2}{3}} t (3 n_S^2 + u^2 + v^2) & \frac{v^2}{\sqrt{3}} & \frac{1}{3} (12 n_S^2 + 4 u^2 + v^2)
\end{pmatrix}$,

where $t = g_X/g$. This matrix will be diagonalized by a mixing matrix $C$ defined by

$M_d^2 = C^T \mathcal{M}_{X38}^2 C = M_d^2 = \text{diag}(0, M_{Z_1}^2, M_{Z_2}^2)$.

This mixing matrix $C$ can be summarized in the three breaking steps as follows: $X_\mu, W_3^\mu, W_8^\mu \rightarrow B_\mu, W_3^\mu, Z'_\mu \rightarrow A_\mu, Z'_\mu \rightarrow A_\mu, Z_{1,2}^\mu, Z_{2,2}^\mu$ corresponding to three physical gauge bosons. Two of them are identified with the massless photon $A_\mu$ and the SM-like neutral gauge boson $Z_1$ found experimentally. After the first breaking step, the gauge couplings and $U(1)_Y$ charges are identified with the SM, leading to the following consequences:

$Y = \frac{1}{\sqrt{3}} T^8 + X, \quad t = \frac{g_X}{g} = \frac{3 \sqrt{2} s_W}{\sqrt{3 - 4 s_W^2}}$. 

(24)
where $g$ and $s_W$ are the well-known parameters defined in the SM, i.e., the $SU(2)_L$ gauge couplings and the sine of the Weinberg angle. In the first step, the two neutral gauge bosons $W_\mu^s$ and $X_\mu$ mix, giving rise to the two bosons $B_\mu$ and $Z'_\mu$. The mixing angle is denoted by $\theta_{331}$ and is given by \cite{91}

$$s_{331} \equiv \sin \theta_{331} = \frac{\sqrt{6g}}{\sqrt{6g^2 + \frac{g^2}{3}}} = \sqrt{1 - t_W^2/3}, \quad c_{331} \equiv \cos \theta_{331} = \frac{t_W}{\sqrt{3}}.$$ \hspace{1cm} (25)

The relation between the original and physical basis of the neutral gauge bosons are

$$\begin{pmatrix} X_\mu \\ W_\mu^3 \\ W_\mu^8 \end{pmatrix} = \begin{pmatrix} s_{331} & 0 & c_{331} \\ 0 & 1 & 0 \\ c_{331} & 0 & -s_{331} \end{pmatrix} \begin{pmatrix} c_W & -s_W & 0 \\ s_W & c_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_\theta & -s_\theta \\ 0 & s_\theta & c_\theta \end{pmatrix} \begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix} = C \begin{pmatrix} A_\mu \\ Z_{1\mu} \\ Z_{2\mu} \end{pmatrix},$$

$$C = \begin{pmatrix} s_{331}c_W, & (-s_{331}s_Wc_\theta + c_{331}s_\theta), & (s_{331}s_Ws_\theta + c_{331}c_\theta) \\ s_W, & c_Wc_\theta, & -s_\theta c_w \\ c_{331}c_W, & -(c_{331}s_Wc_\theta + s_{331}s_\theta), & (c_{331}s_Ws_\theta - s_{331}c_\theta) \end{pmatrix},$$ \hspace{1cm} (26)

Using the limit $\epsilon_S^2 \ll k_\alpha^2 \ll n_{2,S}^2$, the mixing angle $\theta$ is determined as \cite{93}

$$s_\theta \equiv \sin \theta \simeq \frac{\sqrt{3 - 4s_W^2v^2}}{4c_W^4(u^2 + 3n_S^2) + 2(2s_W^2 - 1)v^2}. \hspace{1cm} (27)$$

The masses for the neutral gauge bosons in this limit are

$$m_A^2 = 0, \quad m_{Z_1}^2 \simeq m_Z^2 \simeq \frac{g^2v^2}{2c_W^2}, \quad m_{Z_2}^2 \simeq m_{Z'}^2 = \frac{4g^2c_W^2(u^2 + 3n_S^2)}{3 - 4s_W^2}. \hspace{1cm} (28)$$

As usual for 3-3-1 models with non-zero $Z - Z'$ mixing, in the limit $m_{Z'}^2 \gg m_Z^2$ the tree-level contribution to the $\rho$ parameter defined by $\rho \equiv m_W^2/(m_Z^2c_W^2)$ is estimated approximately by the following formula \cite{93}

$$\Delta \rho \simeq \left(\frac{m_{Z'}^2}{m_Z^2}\right) s_\theta \simeq \left(\frac{m_{Z}^2}{m_{Z'}^2}\right) \times \frac{2}{\sqrt{3 - 4s_W^2}}.$$ \hspace{1cm} (29)

where $s_\theta$ is given in Eq. (27). The recent experimental lower bound of $m_{Z'} \geq 4$ TeV \cite{96} results in that $\Delta \rho \leq 7 \times 10^{-4}$, which still satisfies $3\sigma$ allowed range of experimental data \cite{94}. Previous studies of one-loop contributions from heavy gauge and Higgs bosons to the $\rho$ parameter in some particular 3-3-1 models \cite{79,93,99} suggest that these contributions from the heavy gauge bosons are very suppressed with $m_{Z'} \geq 4$ TeV, while those from Higgs bosons can be negative and have the order of $\mathcal{O}(10^{-4})$. Hence the total contributions to
Δρ may satisfy the experimental constraint even with $m_{Z'}$ smaller than 4 TeV, which was reported from the ATLAS experiment at LHC [96]. We will use this lower bound of $m_{Z'}$ in the numerical investigation.

To determine the SM-like Higgs from its couplings to the gauge bosons $W^\pm$ and $Z$, the relevant terms are

$$
\mathcal{L}_{VS} = \frac{g^2}{2} (W^+.W^-) \left[ \sum_{i=1}^{3} 2k_i R_i + 2k_S R_S + 4\epsilon_S R_\Delta + \sum_{i=1}^{3} R_i^2 + R_S^2 + 2R_\Delta^2 \right] + \frac{g^2}{4c_W^2} Z^2 \left[ \sum_{i=1}^{3} 2k_i R_i + 2k_S R_S + 8\epsilon_S R_\Delta + \sum_{i=1}^{3} R_i^2 + R_S^2 + 4R_\Delta^2 \right].
$$

(30)

In the limit $k_{1,2,S}, \epsilon_S \ll k_3$, we have $k_3 \simeq v = \sqrt{2}m_W/g$. Then we can see that $R_3$ should be identified with the SM-like Higgs boson because it has the same couplings with the SM gauge bosons as those predicted by the SM.

As noted in previous works, $m_{Z_2'}^2 \gg m_{Z_2}^2$, so we get $s_\theta \ll 1$ based on Eq. (27), hence the $Z - Z'$ mixing will be ignored in one-loop formulas involving with LFV decays. An interesting property of the heavy gauge bosons is that they get masses from two large vev $n_2$ and $n_S$. Hence, in principle, $n_2$ can get low values of 1 TeV, even when $m_{Z'}$ are constrained to be very heavy from recent experiments.

### B. Higgs boson

The Higgs potential is $^1$:

$$
V_h = V(\phi_1, \phi_2, \phi_3) + V(S) + V(S, \phi),
$$

$$
V(\phi_1, \phi_2, \phi_3) = \sum_{i=1}^{3} \left[ \mu_i^2 \phi_i^\dagger \phi_i + \lambda_i \left( \phi_i^\dagger \phi_i \right)^2 \right] + \left( \mu_{12} \phi_1^\dagger \phi_2 + \text{h.c.} \right) + \sum_{i<j,i,j=1}^{3} \left[ \lambda_{ij} \left( \phi_i^\dagger \phi_i \right) \left( \phi_j^\dagger \phi_j \right) + \lambda_{ij}^{\phi} \left( \phi_i^\dagger \phi_j \right) \left( \phi_j^\dagger \phi_i \right) \right] + \left[ \lambda_{12}^{\phi} \left( \phi_1^\dagger \phi_2 \right)^2 + \text{h.c.} \right] - \sum_{i<j<k,i,j,k=1}^{3} \sqrt{2} f^\phi (\epsilon_{ijk} \phi_i \phi_j \phi_k + \text{h.c.}),
$$

$$
V(S) = \text{Tr} \left[ \mu_S^2 (S^\dagger S) + \lambda_1^S (S^\dagger S)^2 \right] + \lambda_2^S \left[ \text{Tr}(S^\dagger S) \right]^2,
$$

$^1$ We thank the referee for pointing out a missing term of this Higgs potential in the previous version.
\[
V(S, \phi) = \text{Tr}(S^\dagger S) \sum_{i=1}^{3} \lambda_i^\phi \phi_i^\dagger \phi_i + \text{Tr}(S^\dagger S) \left( \lambda_1^\phi \phi_1^\dagger \phi_1 + \text{h.c.} \right) \\
+ \sum_{i=1}^{3} \lambda_i^\phi \left[ \phi_i^\dagger S S^* \phi_i \right] + \left( \lambda_1^\phi \phi_1^\dagger S S^* \phi_2 + \text{H.c.} \right) \\
+ \sum_{i<j,i,j=1}^{2} f_{ij}^\phi \left( \phi_i^T S^* \phi_j + \text{H.c.} \right) + \lambda^\phi S \left[ (\phi_2^*)_i S_{li}(\phi_1)_j(\phi_3)_k \epsilon_{ijk} + \text{H.c.} \right],
\]

where the invariant terms containing Higgs sextets were derived based on ref. [88], \( \epsilon_{ijk} \) is the total antisymmetric tensor.

For one-loop contributions of Higgs bosons to LFV decays of the SM-like Higgs boson and charged leptons, we pay attention to Higgs components appearing in the Yukawa terms given in Eq. (12). Furthermore, if \( \Sigma^- \) or \( E_e \) does not mix with \( E_\mu \), the LFV decays containing \( e \) as a final state are suppressed; these get Higgs contributions from light active neutrinos and singly charged Higgs exchanges. Here, the simple case of \( k_1 \ll k_3 \) allows us to take \( k_1 \simeq 0 \) in the squared mass matrices of all Higgs bosons. We note that \( k_1 \neq 0 \) is still necessary for generating right quark masses as well as couplings of the SM-like Higgs boson with normal charged leptons.

For simplicity in finding physical states and masses of neutral Higgs bosons, we use the following limit:

\[
\lambda^\phi S \rightarrow 0, \lambda^\phi_{23} \rightarrow 0, \lambda^\phi_{3} \rightarrow 0, \lambda^\phi_{12} \rightarrow -2 \lambda^\phi_{22}, \lambda^\phi_{2} \rightarrow -\frac{f_{22}^\phi}{n_S} - \lambda^\phi_{2}. \quad (32)
\]

We remind the reader of the other assumptions that we mentioned above that can be applied for finding physical states of the Higgs bosons: \( k_2, k_s, k_1 \simeq 0 \) and \( n_1 = 0 \). There are eight neutral Higgs components in the Higgs sector, corresponding to eight equations of the minimum conditions of the Higgs potential. The minimal equations are listed in Appendix B. Inserting them into the Higgs potential (31), we will find the masses and mixing matrices of all physical Higgs bosons as follows.

There are six physical states of CP-even neutral Higgs bosons that are the original states themselves, namely

\[
R_1 \equiv h_1^0, \ R_3 \equiv h, \ R_{\sigma_1} \equiv h_2^0, \ R_{\sigma_2} \equiv h_3^0, \ R_{\sigma_3} \equiv h_4^0, \ R_\Delta \equiv h_5^0 \quad (33)
\]

with corresponding masses as follows:

\[
m_{R_1}^2 = \mu_1^2 = 2k_1^2 \lambda_1^\phi + k_3^2 \lambda_{13}^\phi + n_2^2 \lambda_{12}^\phi + n_S^2 \lambda_2^\phi.
\]
\[ m_{R_1}^2 = 4\lambda_3 k_3^2, \quad m_{R_{s_1}}^2 = \mu_1^2 + (\lambda_{12} + 2\lambda_{12}) n_2^2 + \tilde{\lambda}_{12}^2 n_S^2, \quad m_{R_{s_2}}^2 = 4\lambda_2 n_2^2, \]
\[ m_{s_s}^2 = 4n_S^2(\lambda_1^s + \lambda_2^s) - n_2^2 f_{22}^\phi, \quad m_R^2 = -\frac{n_2^2(n_S\tilde{\lambda}_2^\phi + f_{22}^\phi)}{n_S} - 2n_S^2\lambda_1^s. \tag{34} \]

The squared matrix of the two states \((R_2, R_S)\) is
\[
M_{2s}^2 = \begin{pmatrix}
-n_S(n_S\tilde{\lambda}_2^\phi + 2f_{22}^\phi) & \frac{n_2(n_S\tilde{\lambda}_2^\phi + 2f_{22}^\phi)}{\sqrt{2n_S}} \\
\frac{n_2(n_S\tilde{\lambda}_2^\phi + 2f_{22}^\phi)}{\sqrt{2n_S}} & -\frac{n_2^2(n_S\tilde{\lambda}_2^\phi + 2f_{22}^\phi)}{2n_S}
\end{pmatrix}, \tag{35}
\]
which gives two mass eigenstates corresponding to one Goldstone boson of \(V^0\) and one physical state, which are denoted as \(G_V\) and \(h_6^0\). Their masses and relations to the original states are
\[
m_{G_V}^2 = 0, \quad m_{h_6}^2 = -(n_2^2 + 2n_S^2) \left( \frac{f_{22}^\phi}{n_S} + \frac{\tilde{\lambda}_2^\phi}{2} \right),
\]
\[
\begin{pmatrix}
R_2 \\
R_S
\end{pmatrix} = \begin{pmatrix}
c_{2s} & -s_{2s} \\
s_{2s} & c_{2s}
\end{pmatrix} \begin{pmatrix}
G_V \\
h_6^0
\end{pmatrix}, \quad c_{2s} = \frac{n_2}{\sqrt{n_2^2 + 2n_S^2}}, \quad s_{2s} = \frac{\sqrt{2}n_S}{\sqrt{n_2^2 + 2n_S^2}}. \tag{36}
\]

We can see that the above assumptions of the VEV and Higgs self-couplings gives one Goldstone boson \(G_V\) of the non-Hermitian gauge boson \(V\) and a light CP-even neutral Higgs boson \(h \equiv R_3\). It will be identified with the SM-like Higgs boson found by LHC through its couplings with fermions and gauge bosons, as we will show later.

The model contains only one pair of doubly charged Higgs bosons \(\Delta^{\pm\pm}\) with mass
\[
m_{\Delta^{++}}^2 = k_3^2\tilde{\lambda}_3^\phi + n_2^2 \left( \frac{f_{22}^\phi}{n_S} - \tilde{\lambda}_2^\phi \right) - 2n_S^2\lambda_1^s. \tag{37}\]

Regarding singly charged scalars, we have found two zero mass eigenvalues corresponding to two Goldstone bosons of \(W^\pm\) and \(Y^\pm\). There are three original states that are also the mass eigenstates,
\[ G_W^\pm \equiv H_3^\pm, \quad m_{G_W} = 0, \]
\[ m_{H_3^+}^2 = \mu_1^2 + k_3^2\tilde{\lambda}_{13}^\phi, \quad m_{\Delta^+}^2 = \frac{1}{2} \left( k_3^2\tilde{\lambda}_3^\phi - \frac{2n_2^2(n_S\tilde{\lambda}_2^\phi + f_{22}^\phi)}{n_S} - 4n_S^2\lambda_1^s \right). \tag{38}\]

Corresponding to three other singly charged Higgs states \((H_3^+, \sigma^\pm, H_S^\pm)\), the squared mass matrix is
\[
M_{3sS}^2 = \begin{pmatrix}
k_3^2\tilde{\lambda}_3^\phi - n_S(2f_{22}^\phi + n_S\tilde{\lambda}_2^\phi) & k_3n_2\tilde{\lambda}_{23}^\phi & \frac{n_2(2f_{22}^\phi + n_S\tilde{\lambda}_2^\phi)}{\sqrt{2}} \\
k_3n_2\tilde{\lambda}_{23}^\phi & \frac{k_3n_2\tilde{\lambda}_3^\phi}{n_S\sqrt{2}} + n_S^2\tilde{\lambda}_3^\phi & \frac{k_3n_2\tilde{\lambda}_3^\phi}{\sqrt{2}} \\
k_3n_2\tilde{\lambda}_{23}^\phi & \frac{k_3n_2\tilde{\lambda}_3^\phi}{n_S\sqrt{2}} + n_S^2\tilde{\lambda}_3^\phi & \frac{1}{2} \left( k_3^2\tilde{\lambda}_3^\phi - \frac{n_2^2(2f_{22}^\phi + n_S\tilde{\lambda}_2^\phi)}{n_S} \right)
\end{pmatrix}. \tag{39}\]
It is easily seen that $\text{Det}[M^2_{IσS}] = 0$, leading to a massless eigenstate that can be identified with the Goldstone boson of $V^\pm$.

In the CP-odd neutral Higgs spectrum, there are three massless eigenstates corresponding to three Goldstone bosons of gauge bosons $Z, Z'$ and $V^0$. In particular, the three mass eigenstates and two Goldstone bosons are

$$m^2_{I_1} = \mu_1^2, m^2_{I_{σ_1}} = m^2_{I_Δ} = -\frac{n_2^2(n_S\tilde{\phi}^S_2 + f_{22}^φ)}{n_S} - 2n_S^2\lambda_1^S, \quad m^2_{G_Z} = 0, \quad G_Z \equiv I_3,$$

where $G_Z$ is the Goldstone boson absorbed by the gauge boson $Z$. Five remaining states divide into two sub-matrices of squared masses, corresponding to bases $(I_2, I_S)$ and $(I_{σ_1}, I_{σ_2}, I_{σS})$, namely

$$M^2_{I_{2S}} = \begin{pmatrix}
\frac{n_2^2(2f_{22}^φ + n_S\tilde{\phi}^S_2)}{\sqrt{2}} & \frac{n_2^2(2f_{22}^φ + n_S\tilde{\phi}^S_2)}{n_S} \\
\frac{n_2^2(2f_{22}^φ + n_S\tilde{\phi}^S_2)}{\sqrt{2}} & \frac{n_2^2(2f_{22}^φ + n_S\tilde{\phi}^S_2)}{n_S}
\end{pmatrix},$$

$$M^2_{I_{σ_{1,2}Δ}} = \begin{pmatrix}
\tilde{\phi}^{φ}_{12} - 2\tilde{\phi}^{φ}_{12} n_2^2 + n_S^2\tilde{\phi}^{φ}_{12} + \mu_1^2 - 4n_S^2\lambda_1^{φS} & 2n_2n_S\lambda_1^{φS} \\
4n_S^2\lambda_1^{φS} & 2n_2n_S\lambda_1^{φS}
\end{pmatrix}. \quad (40)$$

The first $2 \times 2$ matrix gives one Goldstone boson of $V^0$ denoted as $G'_V$, $m_{G'_V} = 0$, and a physical CP-odd neutral Higgs $a_6$. Their mass and mixing matrix is

$$m_{G'_V} = 0, \quad m_{a_6}^2 = (-n_2^2 - 2n_S^2)f_{22}^φ \left(\frac{n_2^2\tilde{\phi}^φ_2}{n_S} + \frac{\tilde{\phi}^{φS}_2}{2}\right),$$

$$\begin{pmatrix}
I_2 \\
I_S
\end{pmatrix} = \begin{pmatrix}
c_{2σ} & -s_{2σ} \\
s_{2σ} & c_{2σ}
\end{pmatrix} \begin{pmatrix}
G'_V \\
a_6
\end{pmatrix}. \quad (42)$$

Regarding to the second matrix in Eq. (41), it is easy to check that $\text{Det}[M^2_{I_{σ_{1,2}Δ}}] = 0$; equivalently, there exists one massless state that can be identified with the Goldstone boson of $Z'$. Because $I_{σ_2}$ and $I_Δ$ are irrelevant with the couplings in Eq. (12), which contribute to the one-loop amplitude of LFV decays, we choose a simple case that $\lambda_2^{φS} = 0$ so that $I_{σ_1}$ is itself physical. The CP-odd neutral Higgs bosons relating to the one-loop contributions to LFV decays are $I_{σ_1}$ and $a_6$.

According to the above discussion on the Higgs sector, we can see that $R_{σ_1}$ and $I_{σ_1}$ are the real and imaginary parts of a physical Higgs boson $σ_1$ with mass $m_{σ_1}^2 = (\tilde{\phi}_{12}^{φ} - 2\tilde{\phi}_{12}^{φS})n_2^2 +$.
Similarly, there is another neutral complex Higgs boson denoted as \( h_6 = (h_6^0 + i a_6)/\sqrt{2} \) with mass \( m_{h_6}^2 = m_{a_6}^2 = m_{h_6}^0 \) given in Eqs. (36) and (42).

According to the above discussion on the Higgs sector, we can see that \( h_6^0 \) and \( a_6 \) can be considered as real and imaginary parts of a physical neutral complex Higgs boson denoted as \( h_6 \equiv (h_6^0 + i a_6)/\sqrt{2} \sigma_1 \) with mass \( m_{h_6}^2 = m_{a_6}^2 = m_{h_6}^0 \) given in Eqs. (36) and (42). Similarly, in the limit of the unknown parameter \( \tilde{\lambda}_{12} \sigma_1 = 0 \), \( R_{\sigma_1} \) and \( I_{\sigma_1} \) can be considered as the real and imaginary parts of a physical Higgs boson \( \sigma_1 \) with mass \( m_{\sigma_1}^2 = \tilde{\lambda}_{12} \sigma_1 \tilde{\lambda}_{12} \sigma_1 n_2^2 + \tilde{\lambda}_{1} \sigma_1 n_2^2 + \mu_{1}^2 \). More interesting, \( R_{\sigma_1} \) and \( I_{\sigma_1} \) give the same qualitative contributions to the amplitudes of the LFV decays. Therefore, we will use this limit for our numerical investigation to avoid unnecessary and lengthy private one-loop contributions of \( R_{\sigma_1} \) and \( I_{\sigma_1} \) to LFV decay amplitudes.

From the simple Higgs potential shown above, the Feynman rules for Higgs self-couplings of the SM-like Higgs boson that contribute to the LFVHD are shown in Table II. Note that the coupling \( hh_6 h_6 \) is zero. After determining the masses and mixing matrices of all leptons,

| Coupling | Vertex | Coupling | Vertex |
|----------|--------|----------|--------|
| \( h\sigma_1^0 \sigma_1^{0*} \) | \(-i\lambda_{13} m_W / g \) | \( h\sigma_1^0 h_6 \) | \( if^\phi s_{2s}/2 \) |

TABLE II: Feynman rules for Higgs self-couplings that contribute to LFVHD decays.

gauge and Higgs bosons, the branching ratios of LFV decays \( h \to e_b e_a \) and \( e_b \to e_a \gamma \) can be computed in the next section.

IV. LFV DECAYS \( e_b \to e_a \gamma \) AND \( h \to e_a e_b \)

A. Analytic formulas of branching ratios

In this section, we only pay attention to couplings that contribute to the LFV decay amplitudes \( h \to e_b e_a \) and \( e_b \to e_a \gamma \) at the one-loop level. We also apply the results introduced in Ref. [45] to calculate the amplitudes of the decays \( h \to e_a e_b \). In this model, couplings of charged leptons with active neutrinos result in suppressed contributions to the LFV decay, similar to the case of the SM with very light neutrinos. Hence the non-trivial LFV couplings with normal charged leptons that give large LFV effects relate to only heavy charged leptons \( E_i \), leading to that the LFV couplings that we consider here being only
V^0E_ie_a or s^0E_ie_a, and their Dirac conjugations.

The $ffV$ couplings are contained in the covariant kinetic terms of leptons,

\[ \mathcal{L}_{ffV} = \sum_{i=1}^{7} i\bar{e}_{iR}\gamma^\mu D_{\mu} e_{iR} + \sum_{i=e,\mu,\tau} i\bar{L}_i\gamma^\mu D_{\mu} L_i, \]  

(43)

see the detailed explanation of the relations between these notations in Ref. [90]. The following terms are involved with LFV couplings:

\[ \mathcal{L}_{sff}^{\text{LFV}} = g \left[ \bar{E}_{eL}\gamma^\mu e_L + \frac{1}{\sqrt{2}} (\bar{E}_{\mu L}\gamma^\mu \mu_L + \bar{E}_{\tau L}\gamma^\mu \tau_L) \right] \ V_\mu^0 + \text{h.c.} \]

\[ = g \left[ (V_L^{\text{E}^*})_{3i} \bar{E}_i\gamma^\mu P_L e + \frac{1}{\sqrt{2}} \left[ (V_L^{\text{E}^*})_{1j} \bar{E}_j\gamma^\mu P_L \mu + (V_L^{\text{E}^*})_{2i} \bar{E}_i\gamma^\mu P_L \tau \right] \right] \ V_\mu^0 + \text{h.c.} \]  

(44)

Based on the general Feynman rules for one-loop contributions to the decay amplitude $h \to e_a e_b$, the diagrams need vertices with non-zero couplings $hV_0^0 V_0^0$, or $hs^0 V_0^0$, where $s^0$ is a neutral Higgs boson. In the model under consideration these kinds of couplings do not appear in the model. In contrast, the couplings given in Eq. (44) do contribute to the decay amplitudes $e_b \to e_a \gamma$.

The $ffs^0$ couplings come from the Yukawa Lagrangian [12]. In the physical basis, the Yukawa couplings involved to LFVHD are

\[ \mathcal{L}_{sff} = -\frac{H_1^{0*}}{k_1} \left[ m_{\mu} \bar{P}_{\mu L} + m_{\tau} \bar{P}_{\tau L} \right] - \sigma_1^0 \sum_{i=1}^{3} \left[ \frac{m_{\mu}}{k_1} (V_L^{E^*})_{1i} \bar{E}_i P_{\mu L} + \frac{m_{\tau}}{k_1} (V_L^{E^*})_{2i} \bar{E}_i P_{\tau L} \right] \]

\[ - h_6^* \sum_{i=1}^{3} \left[ s_{2s} (Y_{1i}^{E^*}) \bar{E}_i P_{\mu L} + (Y_{2i}^{E^*}) \bar{E}_i P_{\tau L} \right] + \frac{c_{2s} s_n}{n_2} Y_{3i}^{E^*} \bar{E}_i P_L e \]  

(45)

where the matrix $Y^\ell$ is given in Eq. (8), which can be written in terms of heavy charged lepton masses and mixing parameters based on Eq. (9):

\[ Y^\ell = \frac{1}{n_2} \text{diag}(m_{E_1}, m_{E_2}, m_{E_3}) V_L^{E^*}. \]  

(46)

For convenience in calculating the one-loop contributions of Higgs mediation to the LFV amplitudes, Lagrangian (45) is written in the following form:

\[ \mathcal{L}_{sff} = -\frac{H_1^{0*}}{k_1} \left[ m_{\mu} \bar{P}_{\mu L} + m_{\tau} \bar{P}_{\tau L} \right] - \sigma_1^0 \sum_{i=1}^{3} \sum_{j=1}^{2} Y_{ji}^{0*} \bar{E}_i P_{R e_{(j+1)}} \]

\[ - h_6^* \sum_{i=1}^{3} \left[ 2 \sum_{j=1}^{2} Y_{ji}^{h_6} \bar{E}_i P_{L e_{(j+1)}} + Y_{3i}^{h_6} \bar{E}_i P_L e \right] + \text{h.c.} \]  

(47)
where the coupling $Y_{ji}^s$, $i, j = 1, 2, 3$, is defined as follows:

$$
Y_{ji}^s = \begin{cases} 
\frac{m_{e(j+1)}}{k_1} (V_L^E s_{ji}), & j = 1, 2, \\
0, & j = 3
\end{cases}, \quad Y_{ji}^{h\alpha} = \begin{cases} 
s_{2s} Y_{ji}^\ell, & j = 1, 2, \\
\frac{s_{2s}}{\sqrt{2}} Y_{ji}^\ell, & j = 3
\end{cases},
$$

(48)

where we have used $s_{2s} = e^{2\gamma E_{\text{ms}}} / m_{\nu_2}^2$.

The corresponding one-loop Feynman diagrams that contribute to the LFVHD amplitude are shown in Fig. 2. Although the model under consideration contains charged Higgs bosons, their one-loop contributions to the LFV decay are tiny. The LFV couplings of the doubly and singly charged Higgs bosons $\Delta_{\pm\pm}$ and $H^\pm_S$ do not appear because they only couple with electron; see Eq. (12). The other singly charged Higgs bosons only couple with active neutrinos having tiny masses; hence one-loop contributions involving with them to LFV decay amplitudes are proportional to the deviations between the squared masses of the active neutrinos $\Delta m^2_{ij} \equiv m^2_i - m^2_j$, with $i \neq j$ and $i, j = 1, 2, 3$. This result can be derived using Taylor expansion in terms of the squared masses of the active neutrinos and applying the Glashow-Iliopoulos-Maiani (GIM) mechanism $\sum_i V_{ia}^* V_{ib} = 0$ to cancel large contributions independent of $m_i$, see previous discusion on LFV decays [46, 100]. Hence these contributions from singly charged Higgs bosons are very suppressed so we then safely ignore them.

The partial decay width of the decays $h \to e_a e_b$ is defined as follows:

$$
\Gamma(h \to e_a e_b) \equiv \Gamma(h \to e^-_a e^+_b) + \Gamma(h \to e^+_a e^-_b) = \frac{m_h}{8\pi} \left( |\Delta_{(ba)L}|^2 + |\Delta_{(ba)R}|^2 \right),
$$

(49)

with the condition $m_h \gg m_{a,b}$ and $m_{a,b}$ charged lepton, $a, b = 1, 2, 3$ corresponding to $e, \mu, \tau$. The on-shell conditions for external particles are $p_{a,b}^2 = m_{a,b}^2$ and $p_h^2 \equiv (p_a + p_b)^2 = m_h^2$. The
LFVHD decay rate is $\text{Br}(h \to e_a e_b) = \Gamma(h \to e_a e_b)/\Gamma_h^{\text{total}}$ where $\Gamma_h^{\text{total}} = 4.1 \times 10^{-3}$ GeV. In the notations constructed in Ref. [45], the $\Delta_{(ba) L,R}$ can be written as

$$
\Delta_{(ba) L,R} = \sum_{i=1}^{5} \Delta_{(ba) L,R}^{(i)},
$$

where detailed calculations to derive analytic formulas of $\Delta_{(ba) L,R}^{(i)}$ are given in Ref. [45]. In previous works [19, 45], we can see that $\Delta_{(ba) L,R}^{(2+3)}$ and $\Delta_{(ba) L,R}^{(4+5)}$ are very suppressed, hence we focus only to $\Delta_{(ba) L,R}^{(1)} = \Delta_{(ba) L,R}^{(1)}$ with the following analytic forms for non-zero contributions:

$$
\Delta_{(32) L,R} = \Delta_{(32) L,R}^{(0) \sigma_1} \sigma_1^0 \sigma_1^0 + \Delta_{(32) L,R}^{(0) h_6} h_6 + \Delta_{(32) L,R}^{(0) \sigma_1 h_6} h_6,
$$

$$
\Delta_{(b1) L,R} = \Delta_{(b1) L,R}^{(0) \sigma_1 h_6} h_6,
$$

where $b = 2, 3,$ and

$$
\Delta_{(32) L}^{(0) \sigma_1} \sigma_1^0 = \frac{m_\tau \lambda_{13} m_W}{16 \pi^2 g} \sum_{i=1}^{3} Y_{1i}^{(0) \sigma_1} Y_{2i}^{(0) \sigma_1} \left[ -C_2(0,0; m_{E_i}, m_{\sigma_1}, m_{\sigma_1}^2) \right],
$$

$$
\Delta_{(32) R}^{(0) \sigma_1} \sigma_1^0 = \frac{m_\mu \lambda_{13} m_W}{16 \pi^2 g} \sum_{i=1}^{3} Y_{1i}^{(0) \sigma_1} Y_{2i}^{(0) \sigma_1} \left[ C_1(0,0; m_{E_i}, m_{\sigma_1}, m_{\sigma_1}^2) \right],
$$

$$
\Delta_{(32) L}^{(0) h_6} h_6 = - \frac{f_5 s_{2s}}{32 \pi^2} \sum_{i=1}^{3} Y_{1i}^{(0) h_6} Y_{2i}^{(0) h_6} \left[ m_{E_i} C_0(0,0; m_{E_i}, m_{h_6}, m_{h_6}^2) \right],
$$

$$
\Delta_{(32) R}^{(0) h_6} h_6 = 0,
$$

$$
\Delta_{(32) L}^{(0) \sigma_1 h_6} h_6 = 0,
$$

$$
\Delta_{(b1) L}^{(0) \sigma_1 h_6} h_6 = 0,
$$

$$
\Delta_{(b1) R}^{(0) \sigma_1 h_6} h_6 = - \frac{f_5 s_{2s}}{32 \pi^2} \sum_{i=1}^{3} Y_{3i}^{(0) h_6} Y_{(b-1)i}^{(0) h_6} \left[ m_{E_i} C_0(0,0; m_{E_i}^2, m_{h_6}, m_{h_6}^2) \right].
$$

The functions $C_{1,2}(0,0; m_{E_i}, m_{\sigma_1}, m_{\sigma_1}^2) \equiv C_{0,1,2}(m_{E_i}, m_{\sigma_1}, m_{\sigma_1}^2)$ are one-loop three-point Passarino-Veltman (PV) functions introduced in Ref. [15].

The $\Delta_{(32) L}^{(0) h_6}$ arises from the chirality flip in the Yukawa couplings of heavy fermions with $\sigma_1^0$ and $h_6^0$ given in Eq. (47), similar to the cases mentioned in Refs. [64, 98], which relates to the Yukawa couplings with chirality flip. In our work, the $\Delta_{(32) L,R}^{(0) \sigma_1 h_6} h_6$ arises from the chirality flip in the Yukawa couplings of heavy fermions with $\sigma_1^0$ and $h_6^0$ given in Eq. (47). This may
give an interesting result that $\text{Br}(h \to e_b e_a)$ may be large with large Yukawa couplings of $E_i$ in the perturbative limit.

In the unitary gauge, the one-loop three-point Feynman diagrams contributing to the decay amplitudes $e_b \to e_a \gamma$ ($a < b$) are shown in Fig. 3.

For low energy, the branching ratios of the cLFV decays can be written in a more convenient form as follows:

$$
\text{Br}(e_b \to e_a \gamma) = \left(1 - \frac{m_a^2}{m_b^2}\right)^3 \times \frac{3\alpha_e}{2\pi} \left(|F_{(ba)\ell\ell}|^2 + |F_{(ba)\ell R}|^2\right) \times \text{Br}(e_b \to e_a \bar{\nu}_a \nu_b),
$$

where $\alpha_e \approx 1/137$, $F_{(ba)\ell\ell} = \frac{C_{(ba)\ell\ell}}{m_b} \times \left(\frac{g^2 e}{32\pi^2 m_W^2}\right)^{-1}$, and $C_{(ba)\ell\ell}$ are the one-loop contributions originating from the diagrams shown in Fig. 3. The well-known experimental values of $\text{Br}(e_b \to e_a \bar{\nu}_a \nu_b)$ are $\text{Br}(\tau \to \mu \bar{\nu}_\mu \nu_\tau) \approx 17.41\%$, $\text{Br}(\tau \to e \bar{\nu}_e \nu_\tau) \approx 17.83\%$, and $\text{Br}(\mu \to e \bar{\nu}_e \nu_\mu) \approx 100\%$. The analytical forms of $C_{(ba)\ell\ell}$ are derived based on previous results [77, 94]. Accordingly, we can use the limit $m_a^2, m_b^2 \approx 0$, where the results are as follows,

$$
F_{(ba)\ell L, R} = F^{(1)}_{(ba)\ell L, R} + F^{(2)}_{(ba)\ell L, R},
$$

$$
F^{(1)}_{(32) L} = \sum_{i=1}^{3} \frac{2m_W^2 Y_{ij}^2 \gamma_{ij} \gamma_i^2}{g^2 m_i^2} g_s(t_{a_i^o j}), + \sum_{i=1}^{3} \frac{2m_W^2 Y_{ij}^2 \gamma_{ij} \gamma_i^2}{m_r g^2 m_h^2} g_s(t_{a_i^o j}),
$$

$$
F^{(1)}_{(32) R} = \sum_{i=1}^{3} \frac{2m_W^2 Y_{ij}^2 \gamma_{ij} \gamma_i^2}{m_r g^2 m_i^2} g_s(t_{a_i^o j}), + \sum_{i=1}^{3} \frac{2m_W^2 Y_{ij}^2 \gamma_{ij} \gamma_i^2}{g^2 m_h^2} g_s(t_{a_i^o j}),
$$

$$
F^{(1)}_{(b1) L} = \frac{m_e}{m_b} F^{(1)}_{(b1) R} = \sum_{i=1}^{3} \frac{2m_e m_W^2 Y_{3i}^2 \gamma_{3i} \gamma_i^2}{m_b g^2 m_h^2} g_s(t_{a_i^o j}),
$$

$$
F^{(2)}_{(32) L} = \frac{m_\mu}{m_\tau} F^{(2)}_{(32) R} = \frac{2m_W^2}{m_{V^0}} \sum_{i=1}^{3} V_{i1}^* V_{2i}^* g_\ell(t_{v, i}),
$$

20
\[ F^{(2)}_{(b)R} = \frac{m_e}{m_b} F^{(2)}_{(b)R} = \frac{2m_W^2}{m_{V^0}} \sum_{i=1}^{3} V_{3i}^E V_{(b-1)i}^E \mathcal{g}_v(t_{v,i}), \]  

where \( t_{x,i} = m_{E_i}^2/m_X^2 \) \( (x = \sigma_1^0, h_6, V^0) \),

\[ V_{ai}^E = \begin{cases} (V_L^E)_{ai}, & a = 3 \\ \frac{1}{\sqrt{2}}(V_L^E)_{ai}, & a = 1, 2 \end{cases}, \]

and the functions \( g_s(t_{s,i}), g_v(t_{v,i}) \) are derived in Appendix A.

We note that \( \sigma_1^0 \) only contributes to LFV decays \( t \rightarrow \mu \gamma \) and \( h \rightarrow \mu \tau \). Because of the \( \sigma_1^0 \) couplings with only \( \mu \) and \( \tau \). This is the proper property of the flipped 3-3-1 model, where left-handed electron is a component of a sextet, while the \( \tau \) and \( \mu \) are arranged in triplets as other usual 3-3-1 models. Consequently, the amplitudes of the two decays \( h \rightarrow \mu \tau \) and \( \tau \rightarrow \mu \gamma \) receive more one-loop contributions than the remaining decay amplitudes, hence we expect that the \( \text{Br}(h \rightarrow \tau \mu) \) and \( \text{Br}(\tau \rightarrow \mu \gamma) \) will be large.

### B. Numerical discussions

In this numerical discussion, the unknown input parameters are: the masses and mixing parameters of the heavy leptons \( s_{ij}^E \) and \( m_{E_i} \); heavy neutral Higgs masses and mixing \( m_{\sigma_1^0}, m_{h_6} \) and \( s_{2s} \). In addition, the unknown VEVs in the model are \( k_1 \) and \( n_2 \). From Eqs. (36) and (28), we have

\[ n_S = \frac{s_{2s} n_2}{c_{2s} \sqrt{2}}, \quad n_2^2 (1 + 2t_{2s}^2) = \frac{(3 - 4s_W^2) m_{Z'}^2}{4g^2 c^2_W}, \]

where \( t_{2s} = s_{2s}/c_{2s} \). This means that \( n_2^2 + 4n_S^2 \simeq (2.15 m_{Z'})^2 \). For the latest lower bound of \( m_{Z'}^2 \geq 4 \text{ TeV} \) reported from experiment [96], we have \( \sqrt{n_2^2 + 4n_S^2} \geq 8.3 \text{ TeV} \). For our numerical investigation in this work, we will fix \( \sqrt{n_2^2 + 4n_S^2} = 8.3 \text{ TeV} \), \( n_2 = 1 \text{ TeV} \), \( n_S \geq 4 \text{ TeV} \), leading to \( t_{2s} = \sqrt{2} n_S/n_2 = 4\sqrt{2} \); equivalently \( s_{2s} \simeq 0.985 \). The large \( s_{2s} \) corresponds to the large Yukawa coupling \( Y_{h_6} \) given in Eq. (48). Because \( k_1 \) generates masses for the lepton \( \tau \) at the tree level, it should not be too small. In addition, \( \mu^2_{12} \) given in Eq. (B1) is too large if \( k_1 \) is too small. Hence we will choose that \( 10 \text{ GeV} \leq k_1 \leq 50 \text{ GeV} \). The above particular choice of \( m_{E_i} \) is an illustration for a general consideration where large \( \text{Br}(h \rightarrow e_b e_a) \) needs \( m_{E_i} - m_{E_j} = \mathcal{O}(10^2) \text{ GeV} \) when \( m_{E_i} = \mathcal{O}(1) \text{ TeV} \) is applied in our discussion.
In the first numerical investigation, the default values of the inputs are $k_1 = 20$ GeV, $\lambda_{13} = 1$, $f^\phi = 2$ TeV, $m_{E_1} = 1$ TeV, $m_{E_k} = m_{E_1} - k \times 100$ GeV, $n_2 = 1$ TeV, $s_{2s} = 0.985$, $m_{\sigma^0_1} = m_{h_6} = 1$ TeV. The perturbative limit of the Yukawa couplings relating to heavy lepton masses gives $m_{E_1} \leq n_2\sqrt{4\pi} = 3.5$ TeV for $n_2 = 1$ TeV. Values of $m_{E_{2,3}}$ are chosen to avoid the degenerate masses of the three charged heavy leptons which result in $\text{Br}(\epsilon_b \rightarrow e_a\gamma) = 0$.

All other well-known parameters are taken from Ref. [94], namely the Higgs boson mass and its total decay width $m_h = 125.01$ GeV and $\Gamma_h = 4.07 \times 10^{-3}$ GeV; the mass of the $W$ boson, the masses of normal leptons $m_e$, $m_\mu$, $m_\tau$, the gauge couplings and $\alpha_e$.

Regarding the mixing matrix $V^E_L$, we first consider three cases of only one of $s^E_j = 1/\sqrt{2}$, which correspond to the maximal mixing of only two heavy charged leptons. Hence, these result in large branching ratios of some of the LFV decays while the remaining ones vanish. 

This help us to estimate the largest branching ratios of LFV decays. In the case of $s_{12} = 1/\sqrt{2}$ and $s_{13} = s_{23} = 0$, we always have $\text{Br}(h \rightarrow \mu e) = \text{Br}(h \rightarrow \tau e) = \text{Br}(\mu \rightarrow e\gamma) = \text{Br}(\tau \rightarrow e\gamma) = 0$. In contrast, the $\text{Br}(h \rightarrow \tau\mu)$ and $\text{Br}(\tau \rightarrow \mu\gamma)$ as functions of $m_{E_1}$ with different fixed $k_1$ are shown in Fig. 4. It can be seen that $\text{Br}(\tau \rightarrow \mu\gamma)$ is much smaller than the current experimental bound given in Eq. [2]. Although the $\text{Br}(h \rightarrow \tau\mu) \sim \mathcal{O}(10^{-3})$ is close to the current experimental bound in Eq. [2], the lower bounds obtained from near-future experiments can be used to constrain the parameter space. The two parameters $k_1$ and $m_{E_1}$ strongly affect on $\text{Br}(h \rightarrow \tau\mu)$ but $\text{Br}(\tau \rightarrow \mu\gamma)$ depends weakly on them. This property can be explained as follows. The dominant contribution to the $h \rightarrow \tau\mu$ decay amplitude is $\Delta^{h_0\sigma^0_1}_{32}$, which is proportional to $f^\phi m_\tau m^2_{E_1}/k_1$ and $C_0 \sim 1/m^2_{E_1}$ for $m^2_{E_1} \gg m^2_{h_6}, m^2_{\sigma^0_1}$. For the decay amplitude $\tau \rightarrow \mu\gamma$ the contribution relating to $\sigma^0_1$ is much smaller than that relating

![FIG. 4: Br(h → τμ) and Br(τ → μγ) as functions of m_{E_1} in the cases s^{E}_{12} = \frac{1}{\sqrt{2}} and s^{E}_{13} = s^{E}_{23} = 0.](image-url)
Similarly, with \( s_{12}^E = s_{23}^E = 0 \) and \( s_{13}^E = \frac{1}{\sqrt{2}} \), we have only two non-zero \( \text{Br}(h \rightarrow \mu e) \) and \( \text{Br}(\mu \rightarrow e\gamma) \). Illustrations of these branching ratios as functions of \( m_{E_1} \) with different fixed \( k_1 \) are shown in Fig. 5. Accordingly, \( \text{Br}(\mu \rightarrow e\gamma) \leq \mathcal{O}(10^{-15}) \), which still satisfies the lower bound in Eq. (2). It is noted that although \( \text{Br}(h \rightarrow \mu e) \) is sensitive to \( k_1 \), the \( \text{Br}(\mu \rightarrow e\gamma) \) is not, because it does not receive contribution from Yukawa coupling of \( \sigma_1^0 \).

The case of \( s_{12}^E = s_{23}^E = 0 \) and \( s_{13}^E = \frac{1}{\sqrt{2}} \) correspond to the two non-zero \( \text{Br}(h \rightarrow \tau e) \) and \( \text{Br}(\tau \rightarrow e\gamma) \). Illustrations of these branching ratios as functions of \( m_{E_1} \) with different fixed \( k_1 \) are shown in Fig. 6. In this case, \( \text{Br}(h \rightarrow \tau e) \) has the same order as \( \text{Br}(h \rightarrow \tau\mu) \) because both of them get dominant contributions from \( \Delta_{ba}^{\Delta R} \). Other contributions to \( \Delta_{(ba)} \) have been checked numerically and shown to be very suppressed. Similarly, the case of \( \text{Br}(\tau \rightarrow \mu\gamma) \), \( \text{Br}(\tau \rightarrow e\gamma) \) is much smaller than the current and upcoming experimental

![FIG. 5: \( \text{Br}(h \rightarrow \mu e) \) and \( \text{Br}(\mu \rightarrow e\gamma) \) as functions of \( m_{E_1} \) in the case \( s_{12}^E = \frac{1}{\sqrt{2}} \) and \( s_{12}^E = s_{13}^E = 0 \).]

![FIG. 6: \( \text{Br}(h \rightarrow \tau e) \) and \( \text{Br}(\tau \rightarrow e\gamma) \) as functions of \( m_{E_1} \) in the case \( s_{23}^E = \frac{1}{\sqrt{2}} \) and \( s_{12}^E = s_{13}^E = 0 \).]
sensitivities.

In order to illustrate the effects of heavy lepton masses on the magnitude of different LFV decays, we consider the case of all equal non-zero \( s_{ij}^E = \frac{1}{\sqrt{2}} \). The branching ratios of all LFV decays are functions of \( m_{E_1} \), numerical illustrations of which are shown in Fig. 7.

We consider a region with large \( \text{Br}(\mu \rightarrow e\gamma) \), where the necessary conditions are large \( s_{13}^E \), small values of \( n_2 \) and small \( m_{h_6} \). The illustration is shown in Fig. 8, where we fix \( m_{Z'} = 4 \text{ TeV} \) and \( m_{h_6} = 500 \text{ GeV} \), then plot branching ratios of LFV decays as functions of \( n_2 \) with different \( m_{E_1}/n_2 \leq \sqrt{4\pi} \) satisfying the perurbative limit. We can see again that

\[ \text{Br}(\mu \rightarrow e\gamma) \leq \mathcal{O}(10^{-15}). \]

The large \( \text{Br}(\mu \rightarrow e\gamma) \) corresponds to the regions of small \( n_2 \) and small \( m_{h_6} \).
Similarly, the $\text{Br}(h \to \tau\mu)$ and $\text{Br}(h \to \tau e)$ as functions of $n_2$ are shown in Fig. 9. The $\text{Br}(\tau \to \mu \gamma, e\gamma)$ are much smaller than current experimental constraints so we do not show them again. We just mention here a property that all $\text{Br}(h \to e_b e_a)$ are enhanced with increasing $m_{E_1}$, which has an upper bound originating from the perturbative limit of the Yukawa couplings. Hence the upper bounds of $\text{Br}(h \to e_b e_a)$ correspond to the largest values of the Yukawa couplings. In contrast, all $\text{Br}(e_b \to e_a \gamma)$ decrease with increasing $m_{E_1}$ when $n_2$ is large enough.

To estimate how large the LFV branching ratios can become when $m_{Z'}$ is large, we fix $n_2 = m_{Z'}/4 \geq 1 \text{ TeV}$, then $t_{2s}$ and $n_S$ are determined from the relations given in Eq. [55]. The Br of LFV decays as functions of $m_{Z'}$ are illustrated in Fig. 10. In this case we can see that all LFV branching ratios decrease with larger $m_{Z'}$, but $\text{Br}(h \to \tau \mu)$ and $\text{Br}(h \to \tau e)$ are
still close to the order of $O(10^{-5})$ or larger. Hence these decay channels are still interesting for experiments. On the other hand, all $\text{Br}(e_b \rightarrow e_a\gamma)$ decrease rapidly with increasing $m_{Z'}$. They will not be detected by upcoming experiments.

Apart from the LFV decay $\text{Br}(\mu \rightarrow e\gamma)$, the LFV decay $\mu \rightarrow e\bar{e}e$ is also highly constrained from experimental data, $\text{Br}(\mu \rightarrow e\bar{e}e) < O(10^{-12})$ [101]. A discussion in ref. [1] showed that there exists a tree-level contribution from the heavy gauge boson $Z'$ to this decay amplitude; see the first Feynman diagram in Fig. [1] . Accordingly, the experimental upper bound of $\text{Br}(\mu \rightarrow e\bar{e}e) < O(10^{-12})$ was shown to give a constraint of $m_{Z'} \geq 3$ TeV, which is less strict than that obtained from LHC. In addition, there appear one-loop contributions to this decay because of the same LFV couplings as those result in the LFV decay $\mu \rightarrow e\gamma$, see the second and third diagrams in Fig. [1] . From previous works [76, 102], it can be seen that the one-loop contributions to the two mentioned LFV decays are of same orders. Therefore, the numerical investigations on the $\text{Br}(\mu \rightarrow e\gamma)$ show that the tree-level contribution of $Z'$ to $\mu \rightarrow e\bar{e}e$ is still dominant, and can be used to constrain the $m_{Z'}$.

V. CONCLUSIONS

We have investigated LFV decays of the SM-like Higgs boson $h \rightarrow e_b e_a$ and charged leptons $e_b \rightarrow e_a\gamma$ in the framework of the flipped 3-3-1 model. The Higgs potential was considered in a simple case, where we have shown that the model contains an SM-like Higgs boson that can be identified as the one found experimentally. The main LFV sources originate from the heavy charged leptons. Because electron is arranged in a sextet, which is different from the two other charged leptons $\tau$ and $\mu$, one-loop contributions to the LFV amplitudes of the decays $h \rightarrow \mu\tau$ and $\tau \rightarrow \mu\gamma$ are larger than the remaining $h \rightarrow \tau e, \mu e$ and $\tau, \mu \rightarrow e\gamma$, respectively. Assuming that all new heavy particles are in the TeV scale, the $\text{Br}(h \rightarrow \tau\mu, \tau e)$ and $\text{Br}(h \rightarrow \mu e)$ can reach the orders of $O(10^{-3} - 10^{-4})$, and $O(10^{-6})$, respectively.
respectively. These values are very close to the recent lower bounds reported by experiments, and they should be considered for constraining the parameter space of the model if improved lower bounds on these decay rates are published. The large values of \( \text{Br} \) for LFVHD still appear even with heavy \( m_{Z'} \sim \mathcal{O}(10) \) TeV. On the other hand, the \( \text{Br}(e_b \rightarrow e_a \gamma) \) always satisfies the current experimental constraints. In addition, our numerical investigation shows that \( \text{Br}(\tau \rightarrow \mu \gamma, e \gamma) \leq \mathcal{O}(10^{-14}) \), which is much smaller than the planned sensitivities of upcoming experiments. Similarly, \( \text{Br}(\mu \rightarrow e \gamma) \) can reach the order of \( \mathcal{O}(10^{-15}) \) which is more promising for searching by experiments.

**Acknowledgments**

This research is funded by the An Giang University under Grant No. 19.02.TB.

**Appendix A: One loop contribution to the decay amplitudes \( e_b \rightarrow e_a \gamma \)**

The one-loop contributions to the decays \( e_b \rightarrow e_a \gamma \) is calculated based on the notations of the PV functions defined in ref. [77].

\[
C^{\sigma^0}_{(32)L} = \sum_{i=1}^{3} \frac{-m_\tau Q_E Y_{i1}^\sigma Y_{2i}^\sigma}{16\pi^2} \left[ C_1([p_i^2]; m_{\sigma_1}^2, m_{E_1}, m_{E_i}^2) + C_{11}(\ldots) + C_{12}(\ldots) \right],
\]

\[
C^{\sigma^0}_{(32)R} = \sum_{i=1}^{3} \frac{-m_\mu Q_E Y_{i1}^\sigma Y_{2i}^\sigma}{16\pi^2} \left[ C_2([p_i^2]; m_{\sigma_1}^2, m_{E_1}, m_{E_i}^2) + C_{12}(\ldots) + C_{22}(\ldots) \right],
\]

\[
C^{h_{6}}_{(32)L} = \sum_{i=1}^{3} \frac{-m_\tau Q_E Y_{i1}^{h_{6}} Y_{2i}^{h_{6}}}{16\pi^2} \left[ C_1([p_i^2]; m_{\sigma_1}^2, m_{E_1}, m_{E_i}^2) + C_{11}(\ldots) + C_{12}(\ldots) \right],
\]

\[
C^{h_{6}}_{(32)R} = \sum_{i=1}^{3} \frac{-m_\mu Q_E Y_{i1}^{h_{6}} Y_{2i}^{h_{6}}}{16\pi^2} \left[ C_2([p_i^2]; m_{\sigma_1}^2, m_{E_1}, m_{E_i}^2) + C_{12}(\ldots) + C_{22}(\ldots) \right],
\]

\[
C^{h_{6}}_{(b1)R} = \sum_{i=1}^{3} \frac{-m_\tau Q_E Y_{i1}^{h_{6}} Y_{2i}^{h_{6}}}{16\pi^2} \left[ C_1([p_i^2]; m_{\sigma_1}^2, m_{E_1}, m_{E_i}^2) + C_{11}(\ldots) + C_{12}(\ldots) \right],
\]

\[
C^{h_{6}}_{(b1)L} = \sum_{i=1}^{3} \frac{-m_\mu Q_E Y_{i1}^{h_{6}} Y_{2i}^{h_{6}}}{16\pi^2} \left[ C_2([p_i^2]; m_{\sigma_1}^2, m_{E_1}, m_{E_i}^2) + C_{12}(\ldots) + C_{22}(\ldots) \right],
\]

\[
C^{V}_{(32)L} = \frac{e g^2 m_\mu Q_E}{16\pi^2 m_{V_0}^2} \sum_{i=1}^{3} V_{L1i} V_{E2i}^* \left[ m_{V_0}^2 (C_0([p_i^2]; m_{E_0}^2, m_{E_1}^2, m_{E_i}^2) + C_1(\ldots) + 2C_2(\ldots) + C_{12}(\ldots) + C_{22}(\ldots)) \right]
\]
These results are consistent with the formulas introduced in ref. \[97\], used to discuss on the

\[ C_{(32)R} = \frac{e g^2 m_e Q_E}{16 \pi^2 m_{V_0}^2} \sum_{i=1}^{3} V_{11i}^* V_{2i}^* V_{2i}^* V_{b}^* \]

\[ \times \left[ 2m_{V_0}^2 \left( C_0(p_i^2); m_{V_0}, m_{E_i}, m_{E_2}\right) + 2C_1(...) + 2C_2(...) + C_{11}(...) + C_{12}(...) \right] \]

\[ + m_{E_1}^2 \left( -C_2(...) + C_{11}(...) + C_{12}(...) \right) + m_{a}^2 \left( C_2(...) + C_{12}(...) + C_{22}(...) \right) \] ,

\[ C_{(b1)L} = \frac{e g^2 m_0 Q_E}{16 \pi^2 m_{V_0}^2} \sum_{i=1}^{3} V_{3i}^* V_{(b-1)i}^* V_{(b-1)i}^* \]

\[ \times \left[ 2m_{V_0}^2 \left( C_0(p_i^2); m_{V_0}, m_{E_i}, m_{E_2}\right) + C_1(...) + 2C_2(...) + C_{12}(...) + C_{22}(...) \right] \]

\[ + m_{E_1}^2 \left( -C_1(...) + C_{12}(...) + C_{22}(...) \right) + m_{b}^2 \left( C_1(...) + C_{11}(...) + C_{12}(...) \right) \] ,

\[ C_{(b1)R} = \frac{e g^2 m_0 Q_E}{16 \pi^2 m_{V_0}^2} \sum_{i=1}^{3} V_{3i}^* V_{(b-1)i}^* V_{(b-1)i}^* \]

\[ \times \left[ 2m_{V_0}^2 \left( C_0(p_i^2); m_{V_0}, m_{E_i}, m_{E_2}\right) + 2C_1(...) + C_{11}(...) + C_{12}(...) \right] \]

\[ + m_{E_1}^2 \left( -C_2(...) + C_{11}(...) + C_{12}(...) \right) + m_{a}^2 \left( C_2(...) + C_{12}(...) + C_{22}(...) \right) \] ,

(A1)

where \( p_i^2 = m_b^2, 0, m_a^2 \) relate to external momenta and the symbols (...) stands for the list of arguments shown in the first terms. In the limit \( m_a^2, m_b^2 \simeq 0 \), the PV functions
\( C_{0,i,ij}(0,0,0; m_B^2, m_F^2, m_F^2) \) are written as follows \[95\]

\[ C_0 = \frac{1 - t + \ln(t)}{m_B^2(t - 1)^2}, \quad C_1 = C_2 = \frac{3 - 4t + t^2 + 2 \ln(t)}{4m_B^2(t - 1)^3}, \]

\[ C_{11} = C_{22} = 2C_{12} = \frac{11 - 18t + 9t^2 - 2t^3 + 6 \ln(t)}{18m_B^2(t - 1)^4}, \] (A2)

where \( t = m_F^2/m_B^2 \). Using these approximations we have

\[ g_s(t) \equiv [C_1 + C_{11} + C_{12}] m_B^2 = \frac{t^3 - 6t^2 + 3t + 6t \ln(t) + 2}{12(t - 1)^4}, \]

\[ g_v(t) = 2m_{V_0}^2 \left( C_0 + 2C_1 + C_2 + C_{11} + C_{12}\right) + m_{E_1}^2 \left( -C_2 + C_{11} + C_{12} \right) \]

\[ = \frac{-5t^4 + 14t^3 - 39t^2 + 18t^2 \ln(t) + 38t - 8}{12(t - 1)^4}. \] (A3)

These results are consistent with the formulas introduced in ref. \[97\], used to discuss on the
muon anomalous magnetic moments.
Appendix B: Equations for minimal conditions of the Higgs potential

We have eight independent equations corresponding to eight neutral Higgs bosons \( \{H_0^1, H_0^2, H_0^3, H_0^S, \sigma_0^1, \sigma_0^2, \sigma_0^S, \Delta_0\} \). In the limit of \( \epsilon, k_2, k_S, n_1 = 0 \) and the conditions in Eq. (32) being applied, there are seven independent equations that result in the following functions:

\[
\begin{align*}
\mu_1^2 &= -2k_1^2 \lambda_1^\phi + \frac{\sqrt{2}k_3n_2 f^\phi}{k_1} - k_3^2 \lambda_{13} - n_2^2 \lambda_{12} - n_S^2 \lambda_{2}^S, \\
\mu_{12}^2 &= 0, \\
\mu_2^2 &= k_1^2(-\lambda_{13}^\phi) + \frac{\sqrt{2}k_1n_2 f^\phi}{k_3} - 2k_3^2 \lambda_2^\phi, \\
f_{12}^S &= 0, \\
\mu_2^2 &= k_1^2(-\lambda_{12}^\phi) + \frac{\sqrt{2}k_1k_3 f^\phi}{n_2} - 2n_2^2 \lambda_2^\phi - n_S f_{22}^S, \\
\mu_S^2 &= k_1^2(-\lambda_{2}^S) - 2n_S^2 \lambda_{1}^S - 2n_{S}^2 \lambda_{2}^S, \\
f_{11}^S &= 0.
\end{align*}
\]

(B1)

Inserting them into the Higgs potential to cancel the dependent parameters, we can find the physical states and masses of the Higgs bosons as we discussed above.

[1] R. M. Fonseca and M. Hirsch, JHEP 1608, 003 (2016) [arXiv:1606.01109 [hep-ph]].
[2] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1 [arXiv:1207.7214 [hep-ex]].
[3] S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30 [arXiv:1207.7235 [hep-ex]].
[4] S. Chatrchyan et al. [CMS Collaboration], JHEP 1306 (2013) 081 [arXiv:1303.4571 [hep-ex]].
[5] V. Khachatryan et al. [CMS Collaboration], Phys. Lett. B 749 (2015) 337 [arXiv:1502.07400 [hep-ex]].
[6] G. Aad et al. [ATLAS Collaboration], JHEP 1511 (2015) 211 [arXiv:1508.03372 [hep-ex]].
[7] V. Khachatryan et al. [CMS Collaboration], Phys. Lett. B 763 (2016) 472 [arXiv:1607.03561 [hep-ex]].
[8] G. Aad et al. [ATLAS Collaboration], Eur. Phys. J. C 77 (2017) no.2, 70 [arXiv:1604.07730 [hep-ex]].
[9] A. M. Sirunyan et al. [CMS Collaboration], JHEP 1806 (2018) 001 [arXiv:1712.07173 [hep-ex]].
[10] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 800, 135069 (2020) [arXiv:1907.06131 [hep-ex]].
[11] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 801, 135148 (2020) [arXiv:1909.10235 [hep-ex]].
[12] S. Banerjee, B. Bhattacherjee, M. Mitra and M. Spannowsky, JHEP 1607 (2016) 059 [arXiv:1603.05952 [hep-ph]].
[13] I. Chakraborty, A. Datta and A. Kundu, J. Phys. G 43 (2016) no.12, 125001 [arXiv:1603.06681 [hep-ph]].
[14] I. Chakraborty, S. Mondal and B. Mukhopadhyaya, Phys. Rev. D 96 (2017) no.11, 115020 [arXiv:1709.08112 [hep-ph]].
[15] Q. Qin, Q. Li, C. D. L, F. S. Yu and S. H. Zhou, Eur. Phys. J. C 78 (2018) no.10, 835 [arXiv:1711.07243 [hep-ph]].
[16] A. Pilaftsis, Phys. Lett. B 285 (1992) 68.
[17] J. G. Korner, A. Pilaftsis and K. Schilcher, Phys. Rev. D 47 (1993) 1080 [hep-ph/9301289].
[18] E. Arganda, M. J. Herrero, X. Marcano and C. Weiland, Phys. Rev. D 91 (2015) no.1, 015001 [arXiv:1405.4300 [hep-ph]].
[19] N. H. Thao, L. T. Hue, H. T. Hung and N. T. Xuan, Nucl. Phys. B 921 (2017) 159 [arXiv:1703.00896 [hep-ph]].
[20] X. Marcano and R. A. Morales, Front. in Phys. 7, 228 (2020) [arXiv:1909.05888 [hep-ph]].
[21] J. L. Diaz-Cruz and J. J. Toscano, Phys. Rev. D 62 (2000) 116005 [hep-ph/9910233].
[22] R. Harnik, J. Kopp and J. Zupan, JHEP 1303 (2013) 026 [arXiv:1209.1397 [hep-ph]].
[23] A. Falkowski, D. M. Straub and A. Vicente, JHEP 1405 (2014) 092 [arXiv:1312.5329 [hep-ph]].
[24] A. Celis, V. Cirigliano and E. Passemar, Phys. Rev. D 89 (2014) 013008 [arXiv:1309.3564 [hep-ph]].
[25] A. Dery, A. Efrati, Y. Nir, Y. Soreq and V. Susi, Phys. Rev. D 90 (2014) 115022 [arXiv:1408.1371 [hep-ph]].
[26] X. G. He, J. Tandean and Y. J. Zheng, JHEP 1509 (2015) 093 [arXiv:1507.02673 [hep-ph]].
[27] I. Dorner, S. Fajfer, A. Greljo, J. F. Kamenik, N. Konik and I. Niandic, JHEP 1506 (2015)
[28] J. Heeck, M. Holthausen, W. Rodejohann and Y. Shimizu, Nucl. Phys. B 896 (2015) 281 [arXiv:1412.3671 [hep-ph]].

[29] A. Crivellin, G. D'Ambrosio and J. Heeck, Phys. Rev. D 91 (2015) no.7, 075006 [arXiv:1503.03477 [hep-ph]].

[30] L. de Lima, C. S. Machado, R. D. Matheus and L. A. F. do Prado, JHEP 1511 (2015) 074 [arXiv:1501.06923 [hep-ph]].

[31] Y. Omura, E. Senaha and K. Tobe, JHEP 1505 (2015) 028 [arXiv:1502.07824 [hep-ph]].

[32] M. D. Campos, A. E. Crcamo Hernandez, H. Ps and E. Schumacher, Phys. Rev. D 91 (2015) no.11, 116011 [arXiv:1408.1652 [hep-ph]].

[33] A. Crivellin, G. D'Ambrosio and J. Heeck, Phys. Rev. Lett. 114 (2015) 151801 [arXiv:1501.00993 [hep-ph]].

[34] D. Das and A. Kundu, Phys. Rev. D 92 (2015) no.1, 015009 [arXiv:1504.01125 [hep-ph]].

[35] A. Lami and P. Roig, Phys. Rev. D 94 (2016) no.5, 056001 [arXiv:1603.09663 [hep-ph]].

[36] Y. Omura, E. Senaha and K. Tobe, Phys. Rev. D 94 (2016) no.5, 055019 [arXiv:1511.08880 [hep-ph]].

[37] W. Altmannshofer, S. Gori, A. L. Kagan, L. Silvestrini and J. Zupan, Phys. Rev. D 93 (2016) no.3, 031301 [arXiv:1507.07927 [hep-ph]].

[38] C. F. Chang, C. H. V. Chang, C. S. Nugroho and T. C. Yuan, Nucl. Phys. B 910 (2016) 293 [arXiv:1602.00680 [hep-ph]].

[39] C. H. Chen and T. Nomura, Eur. Phys. J. C 76 (2016) no.6, 353 [arXiv:1602.07519 [hep-ph]].

[40] K. Huitu, V. Keus, N. Koivunen and O. Lebedev, JHEP 1605 (2016) 026 [arXiv:1603.06614 [hep-ph]].

[41] K. Cheung, W. Y. Keung and P. Y. Tseng, Phys. Rev. D 93 (2016) no.1, 015010 [arXiv:1508.01897 [hep-ph]].

[42] N. Bizot, S. Davidson, M. Frigerio and J.-L. Kneur, JHEP 1603 (2016) 073 [arXiv:1512.08508 [hep-ph]].

[43] M. Aoki, S. Kanemura, K. Sakurai and H. Sugiyama, Phys. Lett. B 763 (2016) 352 [arXiv:1607.08548 [hep-ph]].

[44] H. K. Guo, Y. Y. Li, T. Liu, M. Ramsey-Musolf and J. Shu, Phys. Rev. D 96 (2017) no.11, 115034 [arXiv:1609.09849 [hep-ph]].
[45] L. T. Hue, H. N. Long, T. T. Thuc and T. Phong Nguyen, Nucl. Phys. B 907 (2016) 37 [arXiv:1512.03266 [hep-ph]].

[46] T. T. Thuc, L. T. Hue, H. N. Long and T. P. Nguyen, Phys. Rev. D 93 (2016) no.11, 115026 [arXiv:1604.03285 [hep-ph]].

[47] K. H. Phan, H. T. Hung and L. T. Hue, PTEP 2016 (2016) no.11, 113B03 [arXiv:1605.07164 [hep-ph]].

[48] J. Herrero-Garca, T. Ohlsson, S. Riad and J. Wirn, JHEP 1704 (2017) 130 [arXiv:1701.05345 [hep-ph]].

[49] Y. Cai, J. Herrero-Garca, M. A. Schmidt, A. Vicente and R. R. Volkas, Front. in Phys. 5 (2017) 63 [arXiv:1706.08524 [hep-ph]].

[50] B. Yang, J. Han and N. Liu, Phys. Rev. D 95 (2017) no.3, 035010 [arXiv:1605.09248 [hep-ph]].

[51] E. Arganda, M. J. Herrero, X. Marcano, R. Morales and A. Szynkman, Phys. Rev. D 95, no. 9, 095029 (2017) [arXiv:1612.09290 [hep-ph]].

[52] T. P. Nguyen, T. T. Le, T. T. Hong and L. T. Hue, Phys. Rev. D 97 (2018) no.7, 073003 [arXiv:1802.00429 [hep-ph]].

[53] S. Chamorro-Solano, A. Moyotl and M. A. Prez, J. Phys. G 45 (2018) no.7, 075003 [arXiv:1707.00100 [hep-ph]].

[54] A. Vicente, Front. in Phys. 7 (2019) 174 [arXiv:1908.07759 [hep-ph]].

[55] A. Brignole and A. Rossi, Phys. Lett. B 566 (2003) 217 [hep-ph/0304081].

[56] J. L. Diaz-Cruz, JHEP 0305 (2003) 036 [hep-ph/0207030].

[57] A. Brignole and A. Rossi, Nucl. Phys. B 701 (2004) 3 [hep-ph/0404211].

[58] E. Arganda, A. M. Curiel, M. J. Herrero and D. Temes, Phys. Rev. D 71 (2005) 035011 [hep-ph/0407302].

[59] P. T. Giang, L. T. Hue, D. T. Huong and H. N. Long, Nucl. Phys. B 864 (2012) 85 [arXiv:1204.2902 [hep-ph]].

[60] M. Arana-Catania, E. Arganda and M. J. Herrero, JHEP 1309 (2013) 160 Erratum: [JHEP 1510 (2015) 192] [arXiv:1304.3371 [hep-ph]].

[61] D. T. Binh, L. T. Hue, D. T. Huong and H. N. Long, Eur. Phys. J. C 74 (2014) no.5, 2851 [arXiv:1308.3085 [hep-ph]].

[62] E. Arganda, M. J. Herrero, X. Marcano and C. Weiland, Phys. Rev. D 93 (2016) no.5, 055010 [arXiv:1508.04623 [hep-ph]].
[63] E. Arganda, M. J. Herrero, R. Morales and A. Szynkman, JHEP 1603, 055 (2016) [arXiv:1510.04685 [hep-ph]].
[64] S. Baek and Z. F. Kang, JHEP 1603 (2016) 106 [arXiv:1510.00100 [hep-ph]].
[65] S. Baek and K. Nishiwaki, Phys. Rev. D 93 (2016) no.1, 015002 [arXiv:1509.07410 [hep-ph]].
[66] H. B. Zhang, T. F. Feng, S. M. Zhao, Y. L. Yan and F. Sun, Chin. Phys. C 41 (2017) no.4, 043106 [arXiv:1511.08979 [hep-ph]].
[67] U. Chattopadhyay, D. Das and S. Mukherjee, arXiv:1911.05543 [hep-ph].
[68] M. Singer, J. W. F. Valle and J. Schechter, Phys. Rev. D 22 (1980) 738.
[69] F. Pisano and V. Pleitez, Phys. Rev. D 46 (1992) 410 doi:10.1103/PhysRevD.46.410 [hep-ph/9206242].
[70] P. H. Frampton, Phys. Rev. Lett. 69 (1992) 2889. doi:10.1103/PhysRevLett.69.2889
[71] R. Foot, H. N. Long and T. A. Tran, Phys. Rev. D 50 (1994) no.1, R34 [hep-ph/9402243].
[72] J. C. Montero, F. Pisano and V. Pleitez, Phys. Rev. D 47 (1993) 2918 [hep-ph/9212271].
[73] J. T. Liu and D. Ng, Phys. Rev. D 50 (1994) 548 [hep-ph/9401228].
[74] S. M. Boucenna, J. W. F. Valle and A. Vicente, Phys. Rev. D 92 (2015) no.5, 053001 [arXiv:1502.07546 [hep-ph]].
[75] G. Arcadi, C. P. Ferreira, F. Goertz, M. M. Guzzo, F. S. Queiroz and A. C. O. Santos, Phys. Rev. D 97 (2018) no.7, 075022 doi:10.1103/PhysRevD.97.075022 [arXiv:1712.02373 [hep-ph]].
[76] M. Lindner, M. Platscher and F. S. Queiroz, Phys. Rept. 731 (2018) 1 [arXiv:1610.06587 [hep-ph]].
[77] L. T. Hue, L. D. Ninh, T. T. Thuc and N. T. T. Dat, Eur. Phys. J. C 78 (2018) no.2, 128 [arXiv:1708.09723 [hep-ph]].
[78] L. T. Hue, D. T. Huong and H. N. Long, Nucl. Phys. B 873 (2013) 207 [arXiv:1301.4652 [hep-ph]].
[79] H. N. Long, N. V. Hop, L. T. Hue, N. H. Thao and A. E. Crcamo Hernndez, Phys. Rev. D 100 (2019) no.1, 015004 [arXiv:1810.00605 [hep-ph]].
[80] A. E. Crcamo Hernndez, Y. Hidalgo Velsquez and N. A. Prez-Julve, Eur. Phys. J. C 79 (2019) no.10, 828 [arXiv:1905.02323 [hep-ph]].
[81] A. M. Baldini et al. [MEG Collaboration], Eur. Phys. J. C 76 (2016) no.8, 434 [arXiv:1605.05081 [hep-ex]].
[82] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 104 (2010) 021802 [arXiv:0908.2381 [hep-ex]].
[83] A. M. Baldini et al., “MEG Upgrade Proposal,” arXiv:1301.7225 [physics.ins-det].
[84] T. Aushev et al., “Physics at Super B Factory,” arXiv:1002.5012 [hep-ex].
[85] R. M. Fonseca and M. Hirsch, Phys. Rev. D 94 (2016) no.11, 115003 [arXiv:1607.06328 [hep-ph]].
[86] A. G. Dias, J. C. Montero and V. Pleitez, Phys. Rev. D 73 (2006) 113004 [hep-ph/0605051].
[87] R. A. Diaz, R. Martinez and F. Ochoa, Phys. Rev. D 72, 035018 (2005) [arXiv:hep-ph/0411263, hep-ph/0411263].
[88] R. A. Diaz, R. Martinez and F. Ochoa, Phys. Rev. D 69 (2004) 095009 [hep-ph/0309280].
[89] G. De Conto, A. C. B. Machado and V. Pleitez, Phys. Rev. D 92 (2015) no.7, 075031 [arXiv:1505.01343 [hep-ph]].
[90] H. K. Dreiner, H. E. Haber and S. P. Martin, Phys. Rept. 494 (2010) 1 [arXiv:0812.1594 [hep-ph]].
[91] A. J. Buras, F. De Fazio, J. Girrbach and M. V. Carlucci, JHEP 1302 (2013) 023 [arXiv:1211.1237 [hep-ph]].
[92] V. Pleitez and M. D. Tonasse, Phys. Lett. B 430 (1998) 174 [hep-ph/9707298].
[93] A. J. Buras, F. De Fazio and J. Girrbach-Noe, JHEP 1408 (2014) 039 [arXiv:1405.3850 [hep-ph]].
[94] M. Tanabashi et al. [Particle Data Group], Phys. Rev. D 98 (2018) no.3, 030001.
[95] L. Lavoura, Eur. Phys. J. C 29 (2003) 191 [hep-ph/0302221].
[96] M. Aaboud et al. [ATLAS Collaboration], JHEP 1801 (2018) 055 [arXiv:1709.07242 [hep-ex]].
[97] A. Freitas, J. Lykken, S. Kell and S. Westhoff, JHEP 1405 (2014) 145 Erratum: [JHEP 1409 (2014) 155] [arXiv:1402.7065 [hep-ph]].
[98] J. Herrero-Garcia, N. Rius and A. Santamaria, JHEP 1611, 084 (2016) [arXiv:1605.06091 [hep-ph]].
[99] H. N. Long and T. Inami, Phys. Rev. D 61, 075002 (2000) [hep-ph/9902475].
[100] T. P. Cheng and L. F. Li, “Gauge Theory Of Elementary Particle Physics,” Oxford, Uk: Clarendon (1984) 536 P. (Oxford Science Publications)
[101] U. Bellgardt et al. [SINDRUM Collaboration], Nucl. Phys. B 299, 1 (1988).

34
[102] A. Abada, M. E. Krauss, W. Porod, F. Staub, A. Vicente and C. Weiland, JHEP 1411, 048 (2014) [arXiv:1408.0138 [hep-ph]].