MAGNETIC POLARIZABILITY OF DIQUARKS IN BARYONS

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Abstract

We study the response of diquark wave function in Λ-type baryons to strong magnetic fields. It is found that quantum state of $J=0$ diquark ($ud$) in the magnetic field changes due to magnetic polarizability, and constituent quarks in ($ud$) diquark become polarized. The phenomenon influences polarized quark distribution functions $\Delta u(x)$ and $\Delta d(x)$, which therefore may be sensitive to the internal electromagnetic fields in hypernuclei. We also speculate, that strange quark polarization in nucleon may originate from the interaction of virtual $s\bar{s}$ quark pairs with the intrinsic magnetic field of nucleon $B \approx 10^{13} T$.

1 Introduction

It has been suggested many years ago [1], that baryons and mesons contain fractionally charged fermions - constituent quarks. According to Dirac equation, magnetic moment of charged particles with spin $s = 1/2$ is $\mu = eQ/2m^*$, and therefore, constituent quarks should have magnetic moments. For baryons this concept works surprisingly well, and measured magnetic moments of hyperons $\Omega^-$, $\Xi^0$, $\Xi^-$, $\Sigma^-$, $\Lambda^0$, proton and neutron, can be understood as originating from the magnetic moments $\mu_u=1.85\mu_N$, $\mu_d=-0.97\mu_N$, $\mu_s=-0.61\mu_N$ of quarks with constituent masses $m_u^*, m_d^* \approx 330$MeV and $m_s^* \approx 510$MeV.

Consequently, open-flavor vector mesons should also have magnetic moments. For example, $K^{*+}$ meson (bound state of $u, s$ quarks with parallel spins) may be expected to have magnetic moment $\mu_{K^{*+}} = |\mu_u| + |\mu_s| = 2.5\mu_N$ (here $\mu_N=3.1\cdot10^{-14}$MeV/T).

The response of pseudoscalar mesons and scalar diquarks to external magnetic fields can be understood using the analogy of $(q\bar{q})'$ bound states with muonium $(e^-\mu^+)$ and positronium $(e^-e^+)$. Similarly to singlet $(J=0)$ ground state of positronium or muonium, mesons $\eta_c, \eta_b, \eta', \pi, K, D, B$ should have zero magnetic moment [2]. In the magnetic field however, due to magnetic polarizability of pseudoscalar mesons, induced magnetic moment $\tilde{\mu}[B]$ is expected to appear [3], due to partial polarization of $q\bar{q}$ pair in $J=0$ quantum state. If the analogy with positronium behavior [4] is indeed correct, wave function $(\uparrow\downarrow + \downarrow\uparrow)/\sqrt{2}$ of $(m_z=0)$ substate of vector mesons can acquire the admixture of pseudoscalar state $(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$ in the magnetic field, and quenching [5] of $\Psi(c\bar{c})$, $\Upsilon(b\bar{b})$ and $\varphi(s\bar{s})$ meson decays may occur [6] in static external fields $B \approx 10^{14} - 10^{15}$T.

Internal spin structure of scalar diquarks [7] in Λ-type baryons resembles quantum state of pseudoscalar mesons: $(\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$. In strong magnetic field, a superposition of $(J=0)$ diquark with its excited state $(J=1, m_z=0)$ can take place. In this contribution we discuss the magnetic polarizability of diquarks in baryons due to fields $B \approx 10^{13} - 10^{14}$T.
2 Spin structure of Λ baryons

Internal spin structure of Λ^{0}_{1/2}(1116) baryon differs from that of proton, neutron and other spin 1/2 hyperons [8]. Typical s = 1/2 baryon contains two quarks (diquark) in (J=1) triplet state accompanied with the third quark, as described by naive SU(6) function Ψ_{1/2}^{T} in Eq.(1). One may directly guess that ground state wave function of proton (uud) is similar to Σ^{+} (uus), since both they contain quarks (uu) accompanied by third d or s quark. Almost equal masses of Σ^{0}(1193), Σ^{−}, Σ^{0} hyperons then suggest, that their constituent quantum spin structure is similar, (given by Ψ_{1/2}^{T}) as in the case of proton and neutron.

However, constituent quarks (uds) of Σ^{0}_{1/2} hyperon can enter a lower-energy quantum state Ψ_{1/2}^{S}, with different configuration of quark spins. Such state is observed experimentally as Λ^{0}_{1/2}(1116). Mass difference (δM=77MeV) between Σ^{0}(1193) and Λ^{0} baryon comes from different interaction energy of constituent quark color-magnetic moments.

Quantum structure of Λ^{0} hyperon thus contains scalar (J=0) diquark accompanied with the third quark, which is then responsible for the spin of such baryon. A question, which quarks enter the scalar (J=0) diquark state in flavor-degenerate baryons of type Λ^{0}(uds), Ξ^{+}(usc) or Ω^{0}_{c}(scb) has been discussed already by Franklin et al. [10]. The conclusion was that two quarks with similar masses form a scalar diquark state, with small admixture of other diquark flavor configurations. For Λ^{0}(uds), Λ^{+}(udc), Λ^{0}_{b}(udb) this means that scalar diquark (ud) is accompanied with heavier s, c or b quark.

If all three constituent quark spins are oriented in parallel, baryon has spin s=3/2, which corresponds to experimentally observed Ω hyperon and Δ, Σ^{*} and Ξ^{*} resonances. Spin wave function Ψ_{3/2} of such baryons is shown Eq.(1).

3 Internal hyperfine magnetic fields in baryons

Within the framework of MIT bag model [9], constituent quarks are bound together in a small (R ≈ 1fm) volume, which contains strong gluon fields and also virtual partons. Constituent quarks are the source of magnetic dipole and electric fields, which are not screened by the external vacuum. In a simplified picture of Λ^{0} hyperon as purely (ud)-s state, the measured magnetic moment µ_{Λ} = −0.613µ_{N} is to be generated by s-quark: µ_{Λ} = µ_{s}, because diquarks in quantum state Ψ^{S} = (↑↓ − ↓↑)/√2 (as well as pseudoscalar mesons) should have zero magnetic moment.

However, the above said is not completely true. Magnetic dipole field lines, which constitute the dipole field of Λ^{0} hyperon are contained in (penetrating) the "bag" volume of baryon (see Fig.1). Therefore, scalar (ud) diquark state, described by the spin wave function Ψ^{S} can be altered in the magnetic field, and achieve (due to its magnetic polarizability) an induced magnetic moment µ[B], as discussed for the η-meson case in [8].
In such picture, virtual quark-antiquark pairs and scalar diquarks are swimming in a deconfined QCD medium (the "bag") containing also gluons and strong magnetic field.

Let us estimate the strength of hyperfine magnetic field inside baryons: Since the source of the magnetic dipole field is localized inside the hadronic "bag" volume, we shall assume, that dipole magnetic moment \( \mu = c_1 \mu_N \) of baryon comes from the fictious current loop of radius \( R_B = r_o[10^{-15}\text{m}] \) (for proton \( c_1 = 2.79 \), for \( \Lambda \) hyperon \( c_1 = -0.61 \)). One has

\[
\mu = I \cdot S = I \cdot \pi R_B^2 \quad \rightarrow \quad I = (c_1/\pi r_o^2) \mu_N 10^{30} \approx 5 (c_1/\pi r_o^2) 10^3 \text{A}
\]  

(2)

using \( \mu_N = 5 \cdot 10^{-27} \text{J/T} \). Magnetic field \( B_{\text{int}} \) at the center of such current loop is

\[
B_{\text{int}} = \mu_o I/2R_B \quad \rightarrow \quad B_{\text{int}} \approx (2c_1/\pi r_o^3) 10^{12} \text{T},
\]  

(3)

if magnetic permeability \( \mu_o = 4\pi \cdot 10^{-7} \text{NA}^{-2} \) of vacuum is used. For \( \Lambda^0 \) hyperon we then obtain internal magnetic field \( B_{\text{int}}^0 \approx 4 \cdot 10^{12} \text{T} \) (assuming \( r_o = 0.67 \text{[fm]} \)), and for proton \( B_{\text{int}}^p \approx 10^{13} \text{Tesla} \), assuming fictious current loop radius \( r_o = 0.82 \text{[fm]} \).

## 4 Scalar diquarks in the magnetic field

External and intrinsic magnetic field of baryons can influence quantum state of scalar diquarks via interaction term: \( H_{\text{int}} = -\vec{\mu}_q \cdot \vec{B} \). Similarly to the case of Positronium and Muonium, spin-singlet state \( \Psi^S[B] \) becomes a quantum superposition of triplet and singlet states [4], and induced magnetic moment [6] of scalar (ud) diquark appears

\[
\Psi^S[B] = \frac{c_a - s_a}{\sqrt{2}} | \uparrow\downarrow \rangle - \frac{c_a + s_a}{\sqrt{2}} | \downarrow\uparrow \rangle; \quad \langle \Psi^S|\hat{\mu}|\Psi^S \rangle = (|\mu_u| + |\mu_d|) \sin(2\alpha) = \Delta \mu
\]  

(4)

where \( s_a = \sin(\alpha) = y/\sqrt{1 + y^2} \), \( c_a = \cos(\alpha) = \sqrt{1 - s_a^2} \), and \( y = x/(1 + \sqrt{1 + x^2}) \) depends on magnetic field \( B \) via parameter \( x = 2(|\mu_u| + |\mu_d|)B/\Delta E_{hf} \). Hyperfine splitting \( \Delta E_{hf} \) is \( (M_{\Lambda} - M_{\Sigma}) = 77\text{MeV} \) for \( \Lambda^0 \) hyperon, and 166MeV and 194MeV for \( \Lambda^+_c \) and \( \Lambda_b \) hyperons. In the limit \( B \rightarrow \infty \), \( \alpha = 45^\circ \), and scalar diquark becomes fully polarized \( \Psi^S = -|\downarrow\uparrow \rangle \) in its \( (J=0) \) state: quark magnetic moments become oriented along field \( \vec{B} \) direction, while their spins are anti-parallel. In such extreme case, polarized quark distribution functions \( \Delta u(x) \) and \( \Delta d(x) \) of \( \Lambda^0 \) baryon are substantially affected.

Induced magnetic moment \( \Delta \mu \) of scalar diquark should contribute to the magnetic moment of \( \Lambda^0 \) hyperon, as pointed out already by Franklin et al. [10]. In the limiting case \( \Delta \mu \rightarrow |\mu_u| + |\mu_d| = 2.8\mu_N \). For our intrinsic magnetic field \( B = 4 \cdot 10^{12} \text{T} \) in \( \Lambda^0 \) hyperon: \( \sin(2\alpha) \approx x = 2(|\mu_u| + |\mu_d|)B/\Delta E_{hf} = 0.0091 \) and \( \Delta \mu = 0.026\mu_N \), which is 4% of \( \mu_A \). Here, we did not take into account the full wave function \( \Psi^{T/2}_I \) of \( \Sigma \) baryon (see Eq.11), which contains term \( (|\uparrow\downarrow + \downarrow\uparrow \rangle) \sqrt{2} \) with probability \( (1/\sqrt{3})^2 \). Magnetic polarizability of scalar (us) and (ds) diquarks in \( \Sigma_c \) hyperons originates from the same mechanism: the superposition of \( \Psi^S \) with \( \Psi^T \) triplet state of \( \Sigma_c \) hyperons (they correspond to \( \Sigma^0 \)). Due to different quark magnetic moment orientation relative to quark spin in (us) and (ds) diquarks, magnetic polarizability \( \beta_0 = 2\langle \Psi^S|\hat{\mu}_{ds}|\Psi^T \rangle^2/\Delta E_{hf} \) of (ds) diquark is expected to be much smaller compared to (us) and (ud) diquarks (see Eq.10 and Eq.11 in [2]).

The interaction of color-magnetic dipole moments of quarks induces additional hyperfine mixing [8][10] of wave functions \( \Psi^T \) and \( \Psi^S \), which is independent from purely electromagnetic effects we study here.
5 Virtual $s\bar{s}$ pairs polarization in nucleon

Similarly to virtual $e^+e^-$ pairs, which contribute to anomalous magnetic moments of electron and muon, virtual ($s\bar{s}$) pairs can influence nucleon properties. Various experimental results suggest, that ($s\bar{s}$) quark pairs in nucleon are polarized: $\Delta s = -0.1 \pm 0.02$.

Let us assume here, that intrinsic magnetic field $B_{int} \approx 10^{13}$T in nucleon affects quantum state of virtual ($s\bar{s}$) pairs. Inside the hadronic bag, without any external fields, virtual $s\bar{s}$ pairs would appear in pure $J=0^{+-}$ singlet state $\Psi^S = (\uparrow\downarrow - \downarrow\uparrow)/\sqrt{2}$, or in $J=1^{-+}$ triplet state. Due to its smaller energy, pseudoscalar configuration $\Psi^S$ should be more probable. If internal magnetic field $B_{int} \approx 10^{13}$T in nucleon modifies the wave function $\tilde{\Psi}^S[B]$ of scalar $s\bar{s}$ pairs as described by Eq.(4), induced magnetic moment of $J=0$ ($s\bar{s}$) pairs appears: $\langle \hat{\mu}\rangle_{s\bar{s}} = 2|\mu_s|\sin(2\alpha)$, which may contribute to the nucleon magnetic moment. At the same time, net polarization of virtual $s$ quarks occurs.

6 Conclusions

We have discussed that quantum state of scalar diquarks in $\Lambda^0$-type hyperons can be influenced by internal and external magnetic fields. Our estimate of the intrinsic (hyperfine) magnetic field for $\Lambda^0$ hyperon is $B_{int} = 4 \cdot 10^{12}$T. We suggest, that polarized quark distribution functions $\Delta q(x)$ of $\Lambda^0$-type hyperons can be modified due to polarization of scalar ($ud$) diquark in strong electromagnetic field, which may be remotely related to EMC effect. We also suggest, that virtual $s\bar{s}$ pairs in nucleon are effectively polarized due to the intrinsic magnetic field of nucleon $B_{int} \approx 10^{13}$T.

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