Quasinormal mode expansion of optical far-field quantities

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Quasinormal mode (QNM) expansion is a popular tool to analyze light-matter interaction in nanoresonators. However, expanding far-field quantities such as the energy flux is an open problem because QNMs diverge with an increasing distance to the resonant systems. We introduce a theory to compute modal expansions of far-field quantities rigorously. The presented approach is based on the complex eigenfrequencies of QNMs. The divergence problem is circumvented by using contour integration with an analytical continuation of the far-field quantity into the complex frequency plane. We demonstrate the approach by computing the angular resolved modal energy flux in the far-field of a nanophotonic device.

Introduction.— Modern nanotechnology allows for exploring new regimes in tailoring light-matter interaction [1]. Applications comprise the design of nanophotonic devices for quantum information technology [2], tuning photochemistry applications with nanoresonators [3], using plasmonic nanoparticles for biosensing [4], and miniaturization of optical components using dielectric metasurfaces [5]. Most approaches are based on resonance phenomena. Optical resonances are characterized by their wavelength-dependent localized and radiated field energies. They may appear as, e.g., plasmonic resonances in metals [6] or resonances in dielectric materials, such as Mie resonances [7] or bound states in the continuum [8].

The theoretical description of the resonances is essential for understanding the physical properties of the systems and for designing and optimizing related devices. A popular approach is the modeling with QNMs, which are the eigen-solutions of resonant systems [9, 10]. In typical nanophotonic setups, outgoing radiation conditions have to be fulfilled yielding complex eigenfrequencies and an exponential decay of the QNMs in time. This means that the QNMs diverge exponentially with an increasing distance to the resonators [9–12]. Nevertheless, QNM-based expansion approaches, where electromagnetic fields are expanded into weighted sums of QNMs, have been derived to describe light-matter interaction in various applications [13–18]. These approaches are based on the expansion of electromagnetic fields inside and in the close vicinity of the resonators. In this way, modal near-field quantities, such as the modal Purcell enhancement [19–21], can be computed. For time-dependent problems, methods have been proposed to overcome the divergence problem [22–24].

In many applications, time-averaged far-field quantities are of special interest [1, 2, 5]. However, the divergence of QNMs is a key issue for modal expansion of such quantities [9, 10]. From a physics perspective, for time-harmonic sources, the excited electromagnetic near- and far-field distributions are clearly nondiverging. This has motivated a discussion about the general applicability of QNMs and alternative approaches [25–27]. Also, the question of how to normalize QNMs is related to their exponential divergence [9, 10, 28–30].

In this work, we present a general approach for modal analysis which allows for expansions of physical observables in the far-field region. The approach is based on the complex eigenfrequencies of the resonant systems, however, the diverging behavior of the corresponding QNMs is circumvented by using contour integration of the relevant far-field quantities. The method is validated by comparing the modal expansion to a direct solution of the corresponding scattering problem. The approach is applied to compute the modal expansion of the angular resolved energy flux radiated to the far-field by a localized source in a resonant nanostructure.

Modal expansion of far-field quantities.— The QNMs of a resonant system are diverging outgoing waves. Figure 1(a) illustrates the electric field corresponding to a QNM in a one-dimensional resonator defined by layers with different refractive indices. In nano-optics, in the steady-state regime, electric fields \(\mathbf{E}(\omega_0)\) ∈ \(\mathbb{C}^3\) are solutions to the time-harmonic Maxwell’s equations in second-order form,

\[
\nabla \times \mu(\omega_0)^{-1} \nabla \times \mathbf{E}(\omega_0) - \omega_0^2 \epsilon(\omega_0) \mathbf{E}(\omega_0) = i\omega_0 \mathbf{J},
\]

where \(\omega_0 \in \mathbb{R}\) is the angular frequency and \(\mathbf{J} \in \mathbb{C}^3\) is the source field. For a simpler notation, we omit the spatial dependence of the quantities and write, e.g., \(\mathbf{E}(\omega_0)\) instead of \(\mathbf{E}(\mathbf{r}, \omega_0)\), where \(\mathbf{r} \in \mathbb{R}^3\) is the position. The permittivity tensor and the permeability tensor are defined by \(\epsilon(\omega_0)\) and \(\mu(\omega_0)\), respectively. For optical frequencies, \(\mu(\omega_0)\) is typically equal to the vacuum permeability \(\mu_0\). QNMs are solutions to Eq. (1) equipped with outgoing radiation conditions and without a source, i.e., \(\mathbf{J} = 0\). The eigenfrequencies \(\omega_k \in \mathbb{C}\) have negative imaginary parts and are given by the complex resonance poles of the analytical continuation \(\mathbf{E}(\omega)\) of the electric field \(\mathbf{E}(\omega_0)\) into the complex plane \(\omega \in \mathbb{C}\).

We use the Riesz projection expansion (RPE) [17, 31] for modal expansion of the energy flux in the far-field, which can be expressed as a quadratic form with a
where \( E \) is the electric field and \( \mathbf{n} \) is the normal on the corresponding far-field sphere. The RPE is based on contour integration in the complex frequency plane. Since the complex conjugation of the electric field makes \( s(\mathbf{E}(\omega_0), \mathbf{E}^*(\omega_0)) \) nonholomorphic, the evaluation of this function for complex frequencies is problematic. This challenge can be addressed by exploiting the relation \( \mathbf{E}^*(\omega_0) = \mathbf{E}(\omega_0) \) for \( \omega_0 \in \mathbb{R} \). The field \( \mathbf{E}(\omega_0) \) is a solution to Eq. (1) as well. For the harmonic time dependency \( e^{-i\omega_0 t} \) with a negative frequency, the radiation conditions are sign-inverted. The field \( \mathbf{E}(\omega_0) \) has an analytical continuation into the complex plane \( \omega \in \mathbb{C} \), which we denote by \( \mathbf{E}^0(\omega) \). This yields the required analytical continuation of \( s(\mathbf{E}(\omega_0), \mathbf{E}^*(\omega_0)) \), which is given by \( s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \). To expand \( s(\mathbf{E}(\omega_0), \mathbf{E}^*(\omega_0)) = s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \) into modal contributions, Cauchy’s integral formula,

\[
s(\mathbf{E}(\omega_0), \mathbf{E}^0(\omega_0)) = \frac{1}{2\pi i} \oint_{C_0} s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \frac{d\omega}{\omega - \omega_0},
\]

is then exploited. The contour \( C_0 \) is a closed integration path around \( \omega_0 \) so that \( s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \) is holomorphic inside of \( C_0 \). Deforming the integration path and applying Cauchy’s residue theorem yield

\[
s(\mathbf{E}(\omega_0), \mathbf{E}^0(\omega_0)) = -\sum_{k=1}^{K} \frac{1}{2\pi i} \oint_{C_k} s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \frac{d\omega}{\omega - \omega_0} - \frac{1}{2\pi i} \oint_{C^*} s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \frac{d\omega}{\omega - \omega_0},
\]

where \( C_1, \ldots, C_K \) are contours around the resonance poles of \( \mathbf{E}(\omega) \), given by \( \tilde{\omega}_1, \ldots, \tilde{\omega}_K \), and \( C_1^*, \ldots, C_K^* \) are contours around the resonance poles of \( \mathbf{E}^0(\omega) \), given by \( \tilde{\omega}_1^*, \ldots, \tilde{\omega}_K^* \). The outer contour \( C_t \) includes \( \omega_0 \), the resonance poles \( \tilde{\omega}_1, \ldots, \tilde{\omega}_K \) and \( \tilde{\omega}_1^*, \ldots, \tilde{\omega}_K^* \), and no further poles, as sketched in Fig. 1(b). The Riesz projections

\[
s_k(\mathbf{E}(\omega_0), \mathbf{E}^0(\omega_0)) = -\frac{1}{2\pi i} \oint_{C_k} s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \frac{d\omega}{\omega - \omega_0} - \frac{1}{2\pi i} \oint_{C_k^*} s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \frac{d\omega}{\omega - \omega_0},
\]

are modal contributions for the energy flux. The Riesz projections \( s_k(\mathbf{E}(\omega_0), \mathbf{E}^0(\omega_0)) \) are associated with the eigenfrequencies \( \tilde{\omega}_k \) as the integration is performed along the contours \( C_k \) and \( C_k^* \). The contribution

\[
s_r(\mathbf{E}(\omega_0), \mathbf{E}^0(\omega_0)) = \frac{1}{2\pi i} \oint_{C_r} s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \frac{d\omega}{\omega - \omega_0}
\]

FIG. 1. One-dimensional resonator defined by layers with different refractive indices, where \( n_2 > n_1 \). Electric field solutions, \( \mathbf{E}(x, \omega) \) and \( \mathbf{E}^0(x, \omega) \), are obtained by solving the Helmholtz equation with a source term corresponding to incoming plane waves with unit amplitude. Only scattered fields (a.u.) outside the resonator are shown. (a) Diverging field \( \mathbf{E}(x, \tilde{\omega}_k, \Delta) = A e^{i(n_1 \tilde{\omega}_k, \Delta/|x|)} \), where \( \tilde{\omega}_k \) is a resonance pole of \( \mathbf{E}(x, \omega) \) and \( \tilde{\omega}_k, \Delta = \tilde{\omega}_k + \Delta \tilde{\omega}_k \) is a frequency close to \( \tilde{\omega}_k \). (b) Illustration of resonance poles and integration contours corresponding to the RPE for the energy flux given by Eq. (2). The analytical continuation of the energy flux has resonance poles with negative and with positive imaginary parts. (c) Nondiverging field \( \mathbf{E}^0(x, \tilde{\omega}_k, \Delta) = B e^{-i(n_1 \tilde{\omega}_k, \Delta/|x|)} \). (d) Constant product \( \mathbf{E}(x, \tilde{\omega}_k, \Delta) \cdot \mathbf{E}^0(x, \tilde{\omega}_k, \Delta) \), which relates to the energy flux.
is the remainder of the expansion containing nonresonant components as well as components corresponding to eigenfrequencies outside of the contour \( C_t \).

The RPE is based on evaluating \( s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \) by solving Eq. (1) for the frequencies \( \omega \) and \( -\omega \). Consequently, the quadratic form \( s(\mathbf{E}(\omega), \mathbf{E}^0(\omega)) \), where a product of \( \mathbf{E}(\omega) \) and \( \mathbf{E}^0(\omega) \) is involved, does not diverge. This is due to the cancellation of the factors \( e^{i(n\omega/c)r} \) and \( e^{-i(n\omega/c)r} \) of the fields in the far-field region, where \( r = |r| \). In this way, it becomes possible to compute modal expansions of far-field quantities with nondiverging expansion terms. To illustrate this, we consider a one-dimensional resonator and compute electric fields, \( \mathbf{E}(x, \omega) \) and \( \mathbf{E}^0(x, \omega) \), fulfilling the corresponding Helmholtz equation. Figures 1(a) and 1(c) sketch the diverging field \( \mathbf{E}(x, \omega_k) \) and the nondiverging field \( \mathbf{E}^0(x, \omega_k) \) outside of the resonator, respectively. The frequency \( \omega_k = \omega + \Delta \omega_k \) represents an evaluation point on an integration contour \( C_t \). Figure 1(d) shows the nondiverging product \( \mathbf{E}(x, \omega_k) \cdot \mathbf{E}^0(x, \omega_k) \), which relates to the energy flux. The approach also applies to arbitrary three-dimensional problems, where, in the far-field region, \( \mathbf{E}(r, \omega) \sim e^{i(n\omega/c)r}/r \) and \( \mathbf{E}^0(r, \omega) \sim e^{-i(n\omega/c)r}/r \).

**Application.—** The presented approach is used for modal analysis of a quantum technology device. We revisit an example from the literature [33], where a quantum dot acts as single-photon source. For a specific far-field region, the photon collection efficiency (PCE) has been enhanced by using a numerically optimized circular Bragg grating nanoresonator. Such devices can be realized experimentally by using deterministic fabrication technologies [34]. For more details on the specific device and material properties, the reader is referred to [33]. The geometry is sketched in Fig. 2(a). To numerically analyze the light source, we spatially discretize the system with the finite element method (FEM) using the solver JCMsuite [35].

The quantity of interest is the energy flux in the far-field \( s(\omega_0, \theta) = s(\mathbf{E}(\omega_0, \theta), \mathbf{E}^0(\omega_0, \theta)) \), see Eq. (2), where \( \theta \) is the inclination angle as shown in Fig. 2(a). For the modal expansion of \( s(\omega_0, \theta) \), the outer contour \( C_t \) is chosen to enclose the wavelength range of interest, 1280 nm \( \leq \lambda_0 \leq 1400 \) nm, where \( \lambda_0 = 2\pi c/\omega_0 \). We compute all eigenfrequencies inside of the contour, which are listed in Tab. 1. Note that only those rotationally symmetric QNMs which can couple to the dipole source are computed. Figure 2(a) sketches the electric field intensity of the QNM corresponding to \( \omega_d \) in the near-field of the structure. The QNM exhibits a maximum of the field intensity at the center of the resonator and it diverges in the far-field region.

For a fixed dipole frequency, the radiation diagrams for the total modal expansion \( s_{\text{tot}}(\omega_0, \theta) = \sum_{k=1}^N s_k(\omega_0, \theta) + s_\text{tot}(\omega_0, \theta) \) and for the quasiexact solution \( s(\omega_0, \theta) \) are depicted in Fig. 2(b). The quasiexact solution is computed by solving scattering problems given by Eq. (1) directly. The total modal expansion coincides with the quasiexact solution with an absolute error of \( s(\theta)/s_{\text{max}} < 5 \times 10^{-3} \).
and, for the angle region $-60^\circ < \theta < 60^\circ$, with a relative error smaller than $3 \times 10^{-5}$. The differences in these solutions are related to numerical discretization errors and would decrease further by refining the numerical parameters. The agreement demonstrates that, although the associated QNMs diverge in the far-field, the RPE of the energy flux gives correct results with nondiverging expansion terms. Figure 2(c) shows the modal energy fluxes $s_2(\omega_0, \theta)$, $s_3(\omega_0, \theta)$, and $s_4(\omega_0, \theta)$. These are the significant contributions for the total energy flux and they have different directivities corresponding to the different diffraction intensities of the Bragg grating. The contributions $s_3(\omega_0, \theta)$ and $s_4(\omega_0, \theta)$ also have negative values. A negative modal energy flux can be understood as suppression of light emission into specific directions arising from the interference of various modes excited by the source at the frequency $\omega_0$. Negative modal contributions have been reported also for QNM expansions of near-field quantities [19]. Note that, as physically expected, the total modal expansion of the energy flux, $s_{\text{tot}}(\omega_0, \theta)$, is positive for all angles $\theta$.

Next, the RPE is used to obtain insight into the properties of the device for the wavelength range $1280 \text{ nm} \leq \lambda_0 \leq 1400 \text{ nm}$. Figure 3(a) shows the normalized decay rate, also termed Purcell enhancement,

$$\Gamma(\omega_0) = -\frac{1}{2} \text{Re}(\mathbf{E}(\omega_0) \cdot \mathbf{j}^*) / \Gamma_b,$$

where $\mathbf{j} = -i\omega \mathbf{p}$ with the dipole moment $\mathbf{p}$ and $\Gamma_b$ is the dipole emission in homogeneous background material [17]. It can be observed that, in the wavelength range of interest, the three resonances corresponding to the eigenfrequencies $\tilde{\omega}_2$, $\tilde{\omega}_3$, and $\tilde{\omega}_4$ are significant for the Purcell enhancement. The resonance with the eigenfrequency $\tilde{\omega}_1$ has a very small influence. The nonresonant contributions and the contributions associated with other eigenfrequencies are negligible. Figure 3(b) shows the PCE,

$$\eta(\omega_0) = \frac{1}{P_{\text{DE}}} \int_{\Omega} \frac{1}{2} \text{Re} \left( \mathbf{E}^*(\omega_0) \times \frac{1}{i\omega_0 \mu_0} \nabla \times \mathbf{E}(\omega_0) \right) \cdot d\mathbf{S},$$

where $\delta\Omega$ is the far-field region corresponding to $\text{NA} = 0.8$ and $P_{\text{DE}}$ is the emitted power of the dipole emitter into the upper hemisphere. In the case of the PCE, the resonances corresponding to $\tilde{\omega}_1$, $\tilde{\omega}_2$, $\tilde{\omega}_3$, and $\tilde{\omega}_4$ play an important role. In contrast to the Purcell enhancement, the modal contribution $\tilde{\eta}_1(\omega_0)$ is significant for the PCE. It contributes to $\eta_{\text{tot}}(\omega_0)$ for the wavelength region near to its maximum. Note that the behavior of the remaining contributions, $\sum_{k=5}^{9} \tilde{\eta}_k(\lambda_0) + \eta_k(\lambda_0)$, is partially based on resonances with eigenfrequencies outside the integration contour $C_r$.

**Conclusions.**—A theoretical approach to investigate modal quantities in the far-field of resonant systems was presented. Although the QNMs decay exponentially in time and thus represent diverging outgoing waves, modal expansions can be computed rigorously. The approach was applied to expand the energy flux in the far-field of

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**TABLE I.** Eigenfrequencies of the resonator shown in Fig. 2(a). The eigenfrequencies $\tilde{\omega}_k$ are contained in the circular contour $C_r$, which is centered at $1.41 \times 10^{15} \text{ s}^{-1}$ and has a radius of $6.8 \times 10^{13} \text{ s}^{-1}$.

| $k$ | $\text{Re}(\tilde{\omega}_k) \times 10^{15} \text{ s}^{-1}$ | $\text{Im}(\tilde{\omega}_k) \times 10^{13} \text{ s}^{-1}$ |
|-----|-----------------|-----------------|
| 1   | 1.441           | -0.109          |
| 2   | 1.428           | -0.182          |
| 3   | 1.399           | -0.232          |
| 4   | 1.372           | -0.568          |
| 5   | 1.370           | -1.025          |
| 6   | 1.398           | -2.475          |
| 7   | 1.406           | -0.470          |
| 8   | 1.422           | -0.875          |
| 9   | 1.435           | -1.942          |

**FIG. 3.** Modal expansions of Purcell enhancement and PCE for the resonator with a localized light source shown in Fig. 2(a). Eigenfrequencies $\tilde{\omega}_1, \ldots, \tilde{\omega}_9$ are considered, see Tab I. (a) Modal expansion of the Purcell enhancement. The contributions $\tilde{\Gamma}_1(\lambda_0), \ldots, \tilde{\Gamma}_4(\lambda_0)$ correspond to the eigenfrequencies $\tilde{\omega}_1, \ldots, \tilde{\omega}_4$, respectively. The remaining modal contributions are added to the remainder of the expansion, $\sum_{k=5}^{9} \tilde{\Gamma}_k(\lambda_0) + \tilde{\Gamma}_1(\lambda_0)$. The term $\tilde{\Gamma}_1(\lambda_0)$ includes also modal contributions corresponding to eigenfrequencies outside the integration contour $C_r$. (b) Modal expansion of the PCE. Total modal expansion, $\tilde{\eta}_{\text{tot}}(\lambda_0) = \sum_{k=1}^{9} \tilde{\eta}_k(\lambda_0) + \eta_k(\lambda_0)$, single modal contributions, $\tilde{\eta}_1(\lambda_0), \ldots, \tilde{\eta}_4(\lambda_0)$, and the sum of other contributions, $\sum_{k=5}^{9} \tilde{\eta}_k(\lambda_0) + \eta_k(\lambda_0)$. It contributes to $\tilde{\eta}_{\text{tot}}(\omega_0)$ for the wavelength region near to its maximum.
a nanoresonator with an embedded point source. It was demonstrated that, by computing modal far-field patterns, those resonances which contribute significantly to the scattering response of the nanophotonic device can be identified. Thus, deeper physical insights into the system are gained.

The method cannot only be used to efficiently compute the scattering response and to compare to experimental results, but also for an optimization of devices for a tailored functionality. It can be applied to any quadratic form with a sesquilinear map, related to far-field as well as to near-field quantities. Examples are quantities involving the electromagnetic energy flux or the electromagnetic absorption. We expect that with resolving the key issue of the far-field treatment in QNM modeling, the presented approach will enable usage of QNMs in various fields. Applications include systems in nano-optics with any material dispersion and any resonant system in general, e.g., in acoustics or quantum mechanics.

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[1] L. Novotny and N. van Hulst, Nat. Photonics 5, 83 (2011).
[2] X. Ding, Y. He, Z.-C. Duan, N. Gregersen, M.-C. Chen, S. Unsleber, S. A. Maier, C. Schneider, M. Kamp, S. Höfling, C.-Y. Lu, and J.-W. Pan, Phys. Rev. Lett. 116, 020401 (2016).
[3] Y. Zhang, S. He, W. Guo, Y. Hu, J. Huang, J. R. Mulcahy, and W. D. Wei, Chem. Rev. 118, 2927 (2018).
[4] J. N. Anker, W. P. Hall, O. Lyandres, N. C. Shah, J. Zhao, and R. P. Van Duyne, Nat. Mater. 7, 442 (2008).
[5] N. Yu and F. Capasso, Nat. Mater. 13, 139 (2014).
[6] M. S. Tame, K. R. McEnery, S. K. Özdemir, J. Lee, S. A. Maier, and M. S. Kim, Nature Phys. 9, 329 (2013).
[7] A. I. Kuznetsov, A. E. Miroshnichenko, M. L. Brongersma, Y. S. Kivshar, and B. Luk'yanchuk, Science 354, aag2472 (2016).
[8] C. W. Hsu, B. Zhen, A. D. Stone, J. D. Johannopoulos, and M. Soljačić, Nat. Rev. Mater. 1, 16048 (2016).

[9] P. Lalanne, W. Yan, K. Vynck, C. Sauvan, and J.-P. Hugonin, Laser Photonics Rev. 12, 1700113 (2018).
[10] P. T. Kristensen, K. Hermann, F. Intravaia, and K. Busch, Adv. Opt. Photonics, 10.1364/AOP.377940, to be published.
[11] H. Lamb, Proc. London Math. Soc. s1-32, 208 (1900).
[12] G. Beck and H. M. Nussenzveig, Il Nuovo Cimento (1955-1965) 16, 416 (1960).
[13] E. S. C. Ching, P. T. Leung, A. Maassen van den Brink, W. M. Suen, S. S. Tong, and K. Young, Rev. Mod. Phys. 70, 1545 (1998).
[14] E. A. Muljarov, W. Langbein, and R. Zimmermann, EPL 92, 50010 (2010).
[15] B. Vial, F. Zolla, A. Nicolet, and M. Commandré, Phys. Rev. A 89, 023829 (2014).
[16] F. Zolla, A. Nicolet, and G. Demévy, Opt. Lett. 43, 5813 (2018).
[17] L. Zschiedrich, F. Binkowski, N. Nikolay, O. Benson, G. Kewes, and S. Burger, Phys. Rev. A 98, 043806 (2018).
[18] W. Yan, R. Faggiani, and P. Lalanne, Phys. Rev. B 97, 205422 (2018).
[19] C. Sauvan, J.-P. Hugonin, I. S. Maksymov, and P. Lalanne, Phys. Rev. Lett. 110, 237401 (2013).
[20] X. Zambrana-Puyalto and N. Bonod, Phys. Rev. B 91, 195422 (2015).
[21] E. A. Muljarov and W. Langbein, Phys. Rev. B 94, 235438 (2016).
[22] R. Colom, R. McPhedran, B. Stout, and N. Bonod, Phys. Rev. B 98, 085418 (2018).
[23] M. I. Abdelrahman and B. Gralak, OSA Continuum 1, 340 (2018).
[24] M. Kamandar Dezfooli and S. Hughes, Phys. Rev. B 97, 115302 (2018).
[25] D. J. Bergman, Phys. Rev. B 19, 2359 (1979).
[26] H. E. Türeci, A. D. Stone, and B. Collier, Phys. Rev. A 74, 043822 (2006).
[27] P. Y. Chen, D. J. Bergman, and Y. Sivan, Phys. Rev. Appl. 11, 044018 (2019).
[28] P. T. Kristensen, R.-C. Ge, and S. Hughes, Phys. Rev. A 92, 053810 (2015).
[29] E. A. Muljarov and W. Langbein, Phys. Rev. A 96, 017801 (2017).
[30] P. T. Kristensen, R.-C. Ge, and S. Hughes, Phys. Rev. A 96, 017802 (2017).
[31] F. Binkowski, L. Zschiedrich, M. Hammerschmidt, and S. Burger, Phys. Rev. B 100, 155406 (2019).
[32] J. D. Jackson, Classical Electrodynamics, 3rd ed. (Wiley, New York, 1998).
[33] L. Rickert, T. Kupko, S. Rodt, S. Reitzenstein, and T. Heinzel, Opt. Express 27, 36824 (2019).
[34] P. Senellart, G. Solomon, and A. White, Nat. Nanotechnol. 12, 1026 (2017).
[35] J. Pomplun, S. Burger, L. Zschiedrich, and F. Schmidt, Phys. Status Solidi B 244, 3419 (2007).