Complete orbital angular momentum Bell-state measurement and superdense coding

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Quantum protocols require access to large-scale entangled quantum states, due to the requirement of channel capacity. As a promising candidate, the high-dimensional orbital angular momentum (OAM) entangled states have been implemented, but only one of four OAM Bell states in each individual subspace can be distinguished. Here we demonstrate the first realization of complete OAM Bell-state measurement (OAM-BSM) in an individual subspace, by seeking the suitable unitary matrix performable using only linear optics and breaking the degeneracy of four OAM Bell states in ancillary polarization dimension. We further realize the superdense coding via our complete OAM-BSM with the average success probability of ∼82% and the channel capacity of ∼1.1(4) bits. This work opens the window for increasing the channel capacity and extending the applications of OAM quantum states in quantum information in future.

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Quantum protocols require access to large-scale entangled quantum states, due to the requirement of channel capacity [1]. As a promising candidate, the high-dimensional orbital angular momentum (OAM) entangled states have been implemented [2–11], but only one of four OAM Bell states in each individual subspace can be distinguished. A significant challenge is to resolve a complete Bell-state measurement, due to the requirement of channel capacity. The Bell-state measurement is required for quantum dense coding [12, 13], quantum teleportation [14, 15], quantum key distribution [16, 17], and entanglement swapping [18, 19]. For polarization entangled Bell states, the complete Bell-state measurement using only linear optics and the classical communication with a 100% efficiency is impossible [20, 21]. Although the complete polarization Bell-state measurement has realized by using nonlinear optics [22] or ancillary photons and linear optics [23], they are inefficient and impractical in practical application. The polarization Bell-states with only linear optics have been distinguished completely in ancillary degree of freedom (DOF) [24, 25], where the message is carried by the polarization and while the ancillary DOF is used to expand only the measurement space.

However, due to the limit of two dimensions, quantum protocols based on the polarization-entangled Bell states is limited to be 2 bits for a single qubit. Many quantum protocols require large-scale entangled quantum states [1]. The orbital angular momentum (OAM) carrying by the helical phase spatial mode [26] can generate a higher dimension Herbst space. Since the OAM entanglement was firstly realized [2], it has attracted considerable attention in quantum optics [3–9]. The OAM quantum states should be a most promising candidate to provide access to the higher-dimension quantum states [10, 11] and then to increase the storage and processing potential of quantum information. However, only one of the four OAM Bell states can be distinguished from others in each individual subspace so far [11], due to that the measurement results of three among four OAM Bell states are degenerate with the Hong-Ou-Mandel (HOM) interference. Therefore, the degeneracy of four OAM Bell states in each individual subspace must be broken to distinguish completely them and then to increase the channel capacity.

Here we propose an idea for the first realization of complete OAM-BSM in each individual subspace with only linear optics and without auxiliary photons. The key of our idea is to break the degeneracy of four OAM Bell states in ancillary polarization dimension and seek the suitable unitary matrix performable using linear optics only. To confirm our idea, we finish the experimental verification for the complete OAM-BSM in an individual subspace with the quantum number of $m = 1$. We secondly realize the superdense coding by utilizing our complete OAM-BSM with the average success probability of ∼82% and the channel capacity of ∼1.1(4).

Using the traditional method, we firstly generate the polarization-entangled photon states, and further produce the polarization-OAM entangled photon states similar to the method in Ref. [24], so-called hyperentanglement. The introduction of polarization DOF is just used to expand the measurement space for breaking the degeneracy of OAM Bell states. When a fundamental Gaussian mode pumps a nonlinear $\beta$-BaBO$_3$ (BBO) crystal, such a kind of hyperentangled photon-pairs generated via the spontaneous parametric down-conversion (SPDC) can be described as

$$|\Psi\rangle = \sum_{m=0}^{\infty} c_m |\Psi^{-m}\rangle \otimes |\Psi^{+}\rangle.$$  (1)
Here $c_m$ is a complex coefficient, $|\Psi^{s+}\rangle = (|H\rangle_A|V\rangle_B + |V\rangle_A|H\rangle_B)/\sqrt{2}$ is one of four polarization-entangled Bell states, $H$ ($V$) represents the horizontal (vertical) polarization, and the subscripts $A$ and $B$ label the photon path. $|\Psi^{m+}\rangle$ is one of four OAM Bell states in the $m$th-order subspace, which are written as

$$|\Psi^{m\pm}\rangle = \frac{1}{\sqrt{2}} (|+m\rangle_A|-m\rangle_B \pm |-m\rangle_A|+m\rangle_B),$$
$$|\Phi^{m\pm}\rangle = \frac{1}{\sqrt{2}} (|+m\rangle_A|+m\rangle_B \pm |-m\rangle_A|-m\rangle_B),$$

where $|-m\rangle (|+m\rangle)$ represents a photon state with an OAM of $-mh (+mh)$, while the case of $m = 0$ is ignored.

In an individual subspace, by applying one of four unitary operations on the OAM of one photon state $|\Psi^{m+}\rangle \otimes |\Psi^{s+}\rangle$, the other hyperentangled states can be obtained as follows. (1) with $|+m\rangle_A \rightarrow |\exp(\pm j\pi/2)|\pm m\rangle_A$, which can be realized with a pair of Dove prisms oriented at an angle $\pi/(4m)$ with respect to each other [3, 27], $|\Psi^{m+}\rangle \otimes |\Psi^{s+}\rangle \rightarrow |\Psi^{m-}\rangle \otimes |\Psi^{s+}\rangle$; (2) with $|+m\rangle_A \leftrightarrow |-m\rangle_A$, which can be realized by using a Dove prism, $|\Psi^{m+}\rangle \otimes |\Psi^{s+}\rangle \rightarrow |\Phi^{m+}\rangle \otimes |\Psi^{s+}\rangle$; (3) with $|+m\rangle_A \rightarrow |\exp(\pm j\pi/2)|\pm m\rangle_A$ at the same time, $|\Psi^{m+}\rangle \otimes |\Psi^{s+}\rangle \rightarrow |\Phi^{m-}\rangle \otimes |\Psi^{s+}\rangle$ (see Fig. 4a for details). It should be noted that the Dove prism has no influence on the spin state of photons.

Here one of our two goals is to distinguish four OAM Bell states in an individual subspace with the aid of polarization entanglement. Combining Eqs. (1) and (2), the four hyperentangled states $|\Psi^{m+}\rangle \otimes |\Psi^{s+}\rangle$, $|\Psi^{m-}\rangle \otimes |\Psi^{s+}\rangle$, $|\Phi^{m+}\rangle \otimes |\Psi^{s+}\rangle$, and $|\Phi^{m-}\rangle \otimes |\Psi^{s+}\rangle$ can be written as a superposition of the basis $\{|B^i_A|B^j_B\}$ (see Supplementary Information for details), where $k, l \in \{1, 2, 3, 4\}$ and $|B^i_1\rangle = |-m\rangle_H$, $|B^3_2\rangle = |+m\rangle_V$, $|B^1_3\rangle = |+m\rangle_V$, and $|B^4_4\rangle = |+m\rangle_H$. After projecting the four photon-pair entangled states into the basis $\{|B^i_4\}$ with $k = \{1, 2, 3, 4\}$ and performing the coincidence measurement between photons $A$ and $B$, the results are the same for $|\Psi^{m+}\rangle \otimes |\Psi^{s+}\rangle$ and $|\Psi^{m-}\rangle \otimes |\Psi^{s+}\rangle$ ($|\Phi^{m+}\rangle \otimes |\Psi^{s+}\rangle$ and $|\Phi^{m-}\rangle \otimes |\Psi^{s+}\rangle$) (Supplementary Fig. S1). That is to say, although there has the assistance of polarization DOF $|\Psi^{s+}\rangle$, four OAM Bell states $|\Psi^{m+}\rangle$, $|\Psi^{m-}\rangle$, $|\Phi^{m+}\rangle$, and $|\Phi^{m-}\rangle$ cannot still be distinguished completely under the basis $\{|B^i_4\}$ measurement. Therefore, a key step is to transform the initial basis $\{|B^i_4\}$ with $k = \{1, 2, 3, 4\}$ into a suitable project basis $\{|B^i_4\}$ with $k = \{1, 2, 3, 4\}$, which need to find out a suitable unitary transformation matrix $U(4)$ (Fig. 1a for idea). This suitable project basis is called the target basis here.

Inspired by Krenn et al. [28], we construct a theoretical model to seek a target basis vectors (Fig. 1). Our calculation results demonstrate that there are a series of unitary matrices $U(4)$, in other words, the unitary matrices $U(4)$ satisfying our criterion are not the only one.

We use one of them, as shown in Eq. (3) below, to guide our experiment

$$U(4) = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}.$$  (3)

Then the corresponding target basis is

$$\begin{pmatrix} |B^1_1\rangle \\ |B^2_1\rangle \\ |B^3_1\rangle \\ |B^4_1\rangle \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} |+m\rangle_H + |+m\rangle_V \\ |+m\rangle_H - |+m\rangle_V \\ |+m\rangle_V + |+m\rangle_H \\ |+m\rangle_V - |+m\rangle_H \end{pmatrix}.  \quad (4)$$

With this target basis vector, the four hyperentangled states become into

$$|\Psi^{m+}\rangle \otimes |\Psi^{s+}\rangle = \frac{1}{2} (|B^1_1\rangle_A|B^1_1\rangle_B - |B^2_1\rangle_A|B^2_1\rangle_B + |B^3_1\rangle_A|B^3_1\rangle_B - |B^4_1\rangle_A|B^4_1\rangle_B),$$
$$|\Psi^{m-}\rangle \otimes |\Psi^{s+}\rangle = \frac{1}{2} (|B^3_1\rangle_A|B^3_1\rangle_B + |B^1_1\rangle_A|B^1_1\rangle_B - |B^2_1\rangle_A|B^2_1\rangle_B),$$
$$|\Phi^{m+}\rangle \otimes |\Psi^{s+}\rangle = \frac{1}{2} (|B^1_3\rangle_A|B^1_3\rangle_B + |B^3_3\rangle_A|B^3_3\rangle_B + |B^2_3\rangle_A|B^2_3\rangle_B + |B^4_3\rangle_A|B^4_3\rangle_B),$$
$$|\Phi^{m-}\rangle \otimes |\Psi^{s+}\rangle = \frac{1}{2} (|B^1_3\rangle_A|B^1_3\rangle_B - |B^3_3\rangle_A|B^3_3\rangle_B - |B^2_3\rangle_A|B^2_3\rangle_B).$$

In fact, there are sixteen combination basis states of photons $A$ and $B$ ($\{|B^i_4\}_A|B^j_4\}_B$ with $k, l \in \{1, 2, 3, 4\}$). We can see from Eq. (5) that each hyperentangled state is a unique superposition of four among the sixteen possible combination target basis states. By carrying out the local Bell-state measurement, the four OAM Bell states in an individual subspace can be distinguished completely.

In experiment, the hyperentangled photon-pairs source is generated by pumping a 2-mm-thick BBO nonlinear crystal (see Fig. 2a). The pump light is a femtosecond pulsed laser with a power of 230 mW, a pulse duration of $\sim 140$ fs, a repetition rate of $\sim 80$ MHz, and a central wavelength of 405 nm. Under the type-II phase matching, the down-converted photons at a degenerate wavelength of 810 nm are emitted into two cones. Only the photons located at the two overlapping areas of two cones (inset of Fig. 2a) are useful. Three Dove prisms (DP1, DP2, and DP3) are used to transform the OAM Bell states with each other. The pump light is a fundamen tal Gaussian beam with a beam waist of $w_0 \sim 0.9$ mm. The beam waist locates at the center plane of the BBO crystal.

The photons in each path will meet a polarization beam splitter (PBS), which makes the photons of different polarization enter into the first beam splitter (BS)
The coincidence measurement for \( m = 1 \) has been carried out to verify our idea and experimental scheme (Fig. 3). For any one of the four OAM Bell states, only a group of unique four combinations give the coincidence measurement signals. Therefore, the four OAM Bell states can be distinguished completely. If one of the combination states \( |B_i^k\rangle_A |B_j^k\rangle_B \) with \( k = \{1, 2, 3, 4\} \) is fired, the input OAM Bell state must be \( |\Psi^{++}\rangle \). Similarly, for \( |\Psi^{--}\rangle \), the combination \( |B_2^0\rangle_A |B_3^1\rangle_B \), \( |B_0^0\rangle_A |B_2^1\rangle_B \), \( |B_0^0\rangle_A |B_3^1\rangle_B \) or \( |B_2^0\rangle_A |B_3^1\rangle_B \) should give the coincidence signals. And \( |B_2^1\rangle_A |B_3^0\rangle_B \), \( |B_1^1\rangle_A |B_3^0\rangle_B \), \( |B_1^1\rangle_A |B_2^0\rangle_B \), or \( |B_0^1\rangle_A |B_2^0\rangle_B \) has coincidence for the state \( |\Phi^{++}\rangle \) (\( |\Phi^{--}\rangle \)).

In our experiment, the signal-noise ratio (SNR) of the state, which is defined as the ratio between the sum of four ratios of the actual states and the sum of the other twelve ratios, are \( \text{SNR}_{\Psi^{++}} = 6.78 \), \( \text{SNR}_{\Psi^{--}} = 4.6 \), \( \text{SNR}_{\Phi^{++}} = 5.09 \), and \( \text{SNR}_{\Phi^{--}} = 3.12 \).

Based on our OAM-BSM, a superdense-coding protocol can be realized (Fig. 4a). The hyperentanglement source is produced via the SPDC in a nonlinear BBO crystal (as described above). Photon pairs are entangled in OAM, polarization, and emission time synchronously 

Alice encodes two-bits message carried by the OAM Bell state, by operating her photon with three Dove prisms properly (Fig. 4a). Then Bob performs the OAM-BSM and decodes the two-bits message sent by Alice with...
FIG. 2: Preparation and analysis of the OAM Bell states. a, The preparation of the source of the hyperentanglement state. A half wave plate (HWP) and a compensate crystal in each path are used to compensate the work-off effects. b, the experimental setup of the OAM Bell-state analyzer. Photon A (B) of an OAM Bell state is separated by a PBS according to its polarization. Then a modified Mach-Zehnder interferometer (MZI) makes the photon in the state $|−m⟩|H⟩$ or $|+m⟩|V⟩$ ($|−m⟩|V⟩$ or $|+m⟩|H⟩$) exit on the top (bottom) output port of the second BS. Actually, the PBS and the MZI play the part of the unitary matrix $U(4)$ in Fig. 1(a). The role of the q-plate (sandwiched by two quarter wave plates) and the PBS@45° together is the unitary matrix $U(2)$ in Fig. 1(a). An interference filter (not be shown) with a 3-nm bandwidth centred at 810 nm is used to remove the unwanted photons in front of each single mode fiber (SMF).

FIG. 3: Experimental results of OAM-BSM for $m = 1$. The vertical axis represents the coincidence counts in 10 seconds.

the OAM-BSM. Our superdense-coding implementation has been characterized, by switching between the four states and measuring the output states. As the coincidence counts for each input state (Fig. 4b), due to the imperfections in the optical elements, alignment, and input states, the average success probability is $\sim 82\%$. The channel capacity is calculated by

$$CC = \max_{p(x)} \sum_{x,y=1}^{4} p(y|x)p(x) \log \frac{p(y|x)}{\sum_{x'=1}^{4} p(y|x')p(x')}.$$  \hspace{1cm} (6)$$

Here $p(y|x)$ represents the conditional probabilities when
the state x is sent by Alice and the state y is decoded by Bob, $p(x) = \{p(\Psi^{1+}), p(\Psi^{1-}), p(\Phi^{1+}), p(\Phi^{1-})\}$ maximizes the capacity. In our experiment, for a uniform probability of transmission, $CC = 1.1(4)$.

We firstly resolved an important problem—the complete OAM-BSM, which is the requirement of channel capacity in quantum information. Since the limit of the two dimensions, the channel capacity of the polarization entanglement can never be higher than two bits. To break the limit of two-bits channel capacity, the OAM entanglement is a good candidate, because the OAM states can construct an infinite dimension Hermitian space. So far, however, only one of the four OAM Bell states in an individual subspace can be distinguished from others with the HOM interference. The realization of a complete OAM-BSM is still a crucial challenge. The key of our solution is to break completely the degeneracy of four OAM Bell states, by introducing the polarization-OAM hyperentanglement and performing the suitable unitary transform with only linear optics. We propose a theoretical model to seek the suitable unitary matrix $U(4)$, which can transform the initial basis $\{|B^i\rangle\}$ into the target basis $\{|B'^i\rangle\}$. In particular, the unitary transformation can be performed experimentally using only linear optics. We verified experimentally the efficient and practical realization of the complete OAM-BSM in the individual OAM subspace of $m = 1$. We secondly realized the superdense coding by utilizing our complete OAM-BSM, and the results show that the average success probability is $\sim 82\%$ and the channel capacity is $\sim 1.1(4)$. Since the OAM states can generate an infinitely dimensional discrete Hilbert space, the channel capacity can be higher than 2 by encoding the message. Although our experimental results does not reach this goal at present, our research opens the window for increase of the channel capacity and extend the applications of photon OAM states in quantum information science in future.

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