On the long-term tidal evolution of GJ 436b in the presence of a resonant companion

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ABSTRACT

In order to explain the significant orbital eccentricity of the short-period transiting Neptune-mass planet GJ 436b and at the same time satisfy various observational constraints and anomalies, Ribas, Font-Ribera and Beaulieu have proposed the existence of an eccentric low-mass companion planet at the position of the outer 2:1 resonance. The authors demonstrate the viability of their proposal using point-mass three-body integrations, arguing that as long as the system appears to be dynamically stable, the short-term secular variations ought to dominate the long-term dissipative evolution. Here we demonstrate that if one includes tidal dissipation, both orbits circularize after a few times the circulation timescale of the inner planet. We conclude that with or without a nearby companion planet, in or out of the 2:1 resonance, the $Q$-value of GJ 436b must be near the upper bound estimate for Neptune if the system is as young as 1 Gyr, and an order of magnitude higher if the system is as old as 10 Gyr. We show detail of passage through resonance and conclude that even out of resonance, a companion planet should still be detectable through transit timing variations.

Key words: planetary systems – celestial mechanics – methods: analytical – planetary systems: formation

1 INTRODUCTION

GJ 436b was discovered in a radial velocity survey by Butler et al. (2004), and has since been observed in transit by Gillon et al. (2007) and Alonso et al. (2008) as the only transiting hot Neptune to date. Its system parameters are listed in Table 1 together with those for the hypothetical companion presented here (see Section 3.1). Note that the mass

particular which works against the intuition that GJ 436b ought to be circularized, and that is that the planet’s distance from the star in units of its radius is quite large at 160 compared to, for example, HD 209458b at 76. Since the circularization timescale depends on the fifth power of this ratio, it turns out to be relatively long if one uses estimates for the $Q$-value of Neptune. The latter has been estimated by Banfield & Murray (1992) to be in the range \(1.2 \times 10^4 < Q_N < 3.3 \times 10^5\), while Tittemore & Wisdom (1989) estimate the $Q$-value of Uranus, whose mass is 0.85 times that of Neptune, to be less than \(3.9 \times 10^4\). Using the expression

\[
\tau_{\text{circ}} = \frac{e_b}{\dot{e}_b} \approx \frac{2}{42\pi} \left( \frac{Q_b}{k_b} \right) \left( \frac{m_b}{m_*} \right) \left( \frac{a_b}{R_b} \right)^5 P_b
\]

for the circulation timescale of a synchronous system \(\text{Goldreich \\ Soter} (1966)\) where the subscript $b$ refers to quantities associated with GJ 436b, $m_*$ is the stellar mass and $P_b = 2.64 d$ is the orbital period, one obtains the range \(5.3 \times 10^7 \text{yr} < \tau_{\text{circ}} < 1.5 \times 10^9 \text{yr}\), where we have used the \(\text{Banfield \\ Murray} (1992)\) estimates for Neptune

\[1\text{ Note that Goldreich \\ Soter (1966) use a modified } Q\text{-value},\]

\[Q'_b, \text{ which absorbs the Love number such that } Q'_b = 3Q_b/2k_b.\]
This simple analysis suggests that the circularization time which is longer than the age of the system, the non-zero eccentricity to be simply a result of a circularization timescale, but is entirely inadequate for understanding the past evolution. In particular, the second-order secular theory of the system cannot be older than 0.38(\(Q_b/10^5\)) Gyr.

In this Letter we focus on the proposal of Ribas et al. (2008) that, given the circularization timescale is considerably less than the age of the system, the eccentricity is sustained by the presence of a low-mass companion planet positioned at the location of the outer 2:1 resonance, in an orbit which is inclined at around 10° to that of GJ 436b. The authors claim to have found a strong peak at 5.2 days in a periodogram analysis of the RV data, consistent with a body in the 2:1 resonance and with a false alarm probability of 20%. This is supported by a fit to the residuals of the two-body fit to the inner orbit. Moreover, the authors claim that the planet in the 2:1 resonance allowed them to produce the observed eccentricity of GJ 436b with a planet whose mass is low enough not to have previously been detected in the RV data. Their scenario was further strengthened by the fact that it provides a natural explanation for the apparent change in the inclination to the line of sight of the orbit of GJ 436b, \(i_b\), which would bring it into transit between the time of the null result of Butler et al. (2004) and the positive detection of Gillon et al. (2007). The authors estimated that a rate of change of \(i_b\) of around 0.1° yr\(^{-1}\) was needed to be consistent with these observations (note that this generally includes a combination of modulation and precession of the orbital plane). However, Alonso et al. (2008) have more recently reported an upper limit for the current rate of change of \(i_b\) of 0.03 ± 0.05° yr\(^{-1}\). Since 0 < \(\dot{i}_b/\Omega_b < 0.03°\) yr\(^{-1}\) for about 20% of the precession cycle (see Figure 1), this in itself does not rule out a Ribas et al.-type system.

In the following section we briefly consider the tidal evolution of a single-planet system, putting a more accurate lower bound on \(Q_b\) for that case. In Section 3 we study the proposed orbital solution of Ribas et al. (2008) using a direct integration scheme for three bodies which includes perturbing accelerations due to tidal dissipation and spin-orbit coupling in both the star and the innermost planet, and the post-Newtonian relativistic contribution to the potential of the star (Mardling & Lin 2002). We demonstrate that long-term evolution tends towards the doubly-circular state, independent of the tidal circularization timescale of the outer planet, with the orbits inclined to each other by a fixed angle. Section 4 discusses stability while Section 5 presents a conclusion.

### Table 1. Observed and hypothetical orbital and structural data for a GJ 436 two-planet system

|          | \(m\) (M\(\odot\)) | \(R\) (R\(\odot\)) | \(a\) (AU) | \(e\) | \(\omega\) | \(\Omega\) | \(i\) | \(M\) | \(Q\) | \(k\) | \(R_b/R\) |
|----------|-----------------|-----------------|--------|----|--------|--------|----|------|------|-----|--------|
| star     | 0.452 M\(\odot\) | 0.452 R\(\odot\) |        |    |        |        |    |      |      |     |        |
| GJ 436b  | 23.2 M\(\odot\)  | 27,600 km       | 0.0287 | 0.15 | 343    | 86.54  | 38 | [10\(^5\)] | [0.028] | [0.276] |
| GJ 436c  | (4.7 M\(\odot\)) | -               | (0.0295) | [0.2] | [343] | [90] | [86.54] | (0) | ? | [0.346] | [0.511] |

\(Q = 10^5\) Gyr.

Figure 1. Past and Future Evolution of GJ 436b Without a Companion. The system cannot be older than 0.38(\(Q_b/10^5\)) Gyr.

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\(^2\) Note that we find a maximum value of only 0.1 for \(e_b\) over a modulation cycle using their initial conditions, independent of initial phases and longitudes.
Tidal evolution of GJ 436b

3 LONG-TERM TIDAL EVOLUTION OF TWO-PLANET SYSTEMS

The second simplest explanation for a non-zero eccentricity in a system like GJ 436 is the presence of a companion planet or star. In general the induced eccentricity of a stable system will vary quasi-periodically, with periods of variation dominated by the rate of change of the angle between the apsidal lines, the rate of change of the resonance angle(s) if the system is in a resonance, and the rate of change of the argument of periastron if the system is significantly non-coplanar (Murray & Dermott 2000). The amplitude of variation depends on the ratios of semimajor axes and planet masses as well as the initial eccentricities and the angle between the apsidal lines, and when relativistic effects are important, on the ratio of the inner planet mass to the stellar mass. Formulae for the amplitude and period of variation of the eccentricities are given in Mardling (2007) for coplanar non-resonant systems with moderate inner eccentricity ($e_b \lesssim 0.2$). The expression for the amplitude is also accurate for moderately non-coplanar and/or resonant systems, while that for the modulation period severely overestimates the true value for a system in or near the 2:1 resonance because for such a close system, higher order and resonant terms should be included.

A mistake often made is that secular variations of the orbital elements persist for the lifetime of the system, even when dissipative forces are significant (e.g. Maness et al. 2004). In fact such variations are damped out after a few circularization timescales, with the system evolving to a pseudo-equilibrium configuration which itself evolves on a generally longer timescale. A secular theory for two-planet coplanar systems with dissipation has recently been developed (Mardling 2007), in which it is shown that the eccentricity of the innermost planet together with the angle between the apsidal lines, $\eta$, evolve towards a fixed point in ($e_b, \eta$) space on a timescale of three times the circularization timescale of the inner planet, after which the system evolves to the doubly circular state on timescale given by equation (60) in Mardling (2007). Note that this timescale is independent of the Q-value of the outermost planet. When resonant terms are important and/or the system is moderately inclined, the behaviour is only slightly modified (as long as the system is stable). The fixed point or equilibrium eccentricity, a quantity which is independent of the initial values of $e_b$, phases and longitudes, is given by

$$e_b^{(eq)} = \frac{(5/4)(a_b/a_c)e_c e_r^2}{1 - \sqrt{a_b/a_c} (m_b/m_c) e_r e_c^{-1} + \gamma e_r^2},$$

where $\gamma = \sqrt{1 - e_r^2}$ and $\gamma = 4(n_i a_i/c)(m_i/m_c)(a_i/a_c)^3$ with $n_i$ the mean motion of the inner planet and $c$ the speed of light. Note, however, that (2) was derived assuming that the average value of the outer eccentricity doesn’t vary much on the circularization timescale. When it does (as happens for the hypothetical GJ 436 system because the outer mass is so low), the estimate (2) tends to overestimate the equilibrium eccentricity. Thus (2) can be regarded as an upper bound for $e_b^{(eq)}$.

3.1 GJ 436b with a resonant companion

If the circularization timescale of GJ 436b is less than a third of the age of the system, it will already have reached and evolved past the equilibrium eccentricity. Thus we begin by calculating the range of companion masses and eccentricities capable of producing an equilibrium eccentricity of 0.15, that is, the observed eccentricity of GJ 436b. As in the previous Section, this will allow us to put a lower bound on its $Q$-value.
eccentricity corresponding to the hypothetical companion of Ribas et al. (2008) (with $e_c = 0.2$) is 0.06.

One can conclude from this that if GJ 436b does have a single nearby low-mass companion in or near the 2:1 resonance, the system cannot yet have evolved to its pseudo-equilibrium state. Its circularization timescale must therefore be longer than one third of the age of the system, thereby providing the weaker constraint on the $Q$-value of GJ 436b than in the single-planet case that $Q_b > 7.7 \times 10^4 (t_{age}/\text{Gyr})$. In fact, we can do better than this, and argue that after only one circularization timescale the system will be in apsidal libration with zero minimum $e_b$ and a maximum which depends on $e_c$ (Mardling 2007). From stability considerations $e_c$ can’t be much more than around 0.3 (given it needs to be able to move safely through the resonance as it tidally evolves; see Figure 5). Since this corresponds to a $maximum$ value of $e_b$ in a libration cycle of 0.12, we must have that $\tau_{circ} > \tau_{age}$, putting the same lower bound on $Q_b$ as in the single-planet case.

We finish this section by demonstrating the behaviour discussed above. Using the data in Table 1 we performed a direct integration using the code described in the Introduction (Mardling & Lin 2002). The star’s $Q$-value is an estimate, while the Love numbers $k$ (twice the apsidal motion constant) and radii of gyration $R_g$ for the star and planet correspond to $n = 3$ and $n = 1$ polytropes respectively (Sterne 1941), the latter often taken to approximate the structure of Jupiter. The spin period of the star was taken to be 20 days while the spin of the planet was taken to be synchronous with the orbital motion. Both were taken to be aligned with the orbit normal.

The secular analysis in (Mardling 2007) demonstrates that varying the $Q$-value of the inner-most planet merely changes the timescale on which the system evolves towards the final state while not affecting the local secular oscillation period. Since tidal dissipation in the planet dominates the tidal evolution while the inner orbit’s eccentricity is non-zero, we took $Q_b = 0.1$ in order to show the detail of various stages of evolution, and confirmed that the evolutionary timescale scales linearly with $Q_b$ for $Q_b = 1$ and 10. We also confirmed that the system remains stable for $2 \times 10^5 \, \text{yr}$ with $Q_b = 10^4$, consistent with the results of Section 3.2.

Figure 3 shows 4000 years of evolution, equivalent to $4(Q_b/10^3) \, \text{Gyr}$ for general $Q_b$, except that $Q_b/0.1$ as many secular oscillations will have occurred in that time. It passes through its present (hypothetical) configuration after $1600 (Q_b/0.1) \, \text{yr}$. Panel (a) shows the evolution of $e_b$ (bottom curve) and $e_c$, with the system entering resonance at around $200 (Q_b/0.1) \, \text{yr}$, and entering apsidal libration (panel (b)) when $e_b$ (temporarily) hits zero at $1000 (Q_b/0.1) \, \text{yr}$. The latter occurs on a timescale of $1 \, \tau_{circ}$ after which the system evolves towards the quasi-equilibrium phase on a timescale of $2 \tau_{circ}$. The subsequent approach of both eccentricities to zero is well approximated by the secular theory and occurs on a timescale of around $6 \tau_{circ}$ in this case (equation (60) Mardling 2007). Panel (c) shows the approach to a fixed value of around $10^{-3}$ of the relative inclination.

Figure 4 shows detail of the passage through resonance. While the amplitude of variation of the eccentricity is only slightly enhanced (so that estimates provided by the secular theory are reasonably accurate), the amplitude of variation of the orbital period ratio $P_c/P_b$ suddenly increases as the system crosses the separatrix and enters the 2:1 resonance, with a width of around 0.035. The corresponding variation of the orbital period of GJ 436b is around 15 mins, a significant increase on the variation outside the resonance, whose effect on the transit timing would be easily measured (note that the libration period for this system is around 200 days). For
Figure 5. A stability map of the region surrounding the 2:1 resonance for a range of values of \( \varepsilon_c \).

coplanar systems in the 2:1 resonance with \( m_b, m_c \ll m_\ast \), this variation is given by (Mardling, in preparation)

\[
\delta P_b / P_b = 2 \left( 1 + (m_b / m_\ast) \sigma^{2/3} \right)^{-1} \delta \sigma / \sigma \approx 2^{-2/3} (m_c / m_b) \delta \sigma,
\]

where \( \sigma = P_c / P_b \), \( \delta \sigma = \sigma - 2 \), that is, the “distance” from resonance, and the approximation holds for \( m_c \ll m_b \).

However, the GJ 436 system passes through the resonance in around 300 \((Q_b / 0.1) \) yr = 0.3 \((Q_b / 10^5) \) Gyr so unless \( Q_b \) is at least equal to the upper estimate of Neptune’s \( Q \)-value, we would not expect to see the system in resonance now (given that it was deposited into the resonance around the time of formation). Alonso et al. (2008) find no obvious departure from linear ephemeris and conclude that the proposed resonant solution of Ribas et al. (2008) is unlikely. Once the system leaves the resonance, the amplitude of variation of \( P_b \) is given by (3) with \( \delta \sigma \) replaced by the quantity \( \sigma - \sqrt{\Delta \sigma^2 - \Delta \sigma^2} \), where \( \Delta \sigma \) is the width of the resonance (Mardling, in preparation). \( \delta P_b \) reduces quickly as the system crosses the resonance, however, even when the system becomes doubly circularized \( \delta P_b \) is still significant at around one minute. Thus we conclude that even out of resonance, the companion planet proposed by Ribas et al. (2008) would be detectable through transit timing variations given current accuracies (Alonso et al. 2008).

Panel (c) shows the variation of the inclination of GJ 436b to the line of sight, \( i_b \). The resonance has very little effect on the period and amplitude of variation, with the average rate of change of \( i_b \) equal to approximately 0.06° \( \) yr\(^{-1} \). This compares to the observational upper bound of 0.03 ± 0.05° \( \) yr\(^{-1} \) by Alonso et al. (2008).

3.2 Stability

Figure 5 shows a stability map for a system composed of GJ 436b and a companion planet with a mass of 4.7 \( M_\oplus \), aligned periastra, a mutual inclination of 10°, and with initial companion eccentricity \( \varepsilon_c \) and period \( P_c = \sigma P_b \) indicated by the position in the plot. Three-body integrations were performed, with stability being determined using the procedure described in Mardling (2008). Initial conditions corresponding to unstable systems are indicated in the figure by red dots; stable systems are left blank. A clearly defined boundary is evident, indicating that a system with a sufficiently low initial value of \( \varepsilon_c \) would be free to tidally evolve through the 2:1 resonance without the danger of instability. Other resonances are also evident, for example, the 3:2 and the 5:2.

4 CONCLUSION

The main conclusion from this study is that with or without a single nearby companion planet, in or out of the 2:1 resonance, the \( Q \)-value of GJ 436b must be greater than the upper bound estimate for Neptune if the age of the system is around 1 Gyr, and up to an order of magnitude greater for an age of up to 10 Gyr. Passage through resonance of a Ribas et al. two-planet system occurs on a timescale of 0.3 \((Q_b / 10^5) \) Gyr, remaining stable throughout and beyond. However, even if it were now no longer in resonance, the companion planet should still be detectable through transit timing variations.

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\footnote{See also Holman & Murray (2005) and Agol et al. (2005) but note that our expression is independent of the eccentricities, and is an approximation to an expression valid for any masses.}