\gamma\text{-Ray} Annihilation in Milky Way Satellite Galaxies: An Analysis with Particle Dark Matter Models for 45 Dwarf Spheroidals

\textbf{Ashadul Halder},\textsuperscript{1} \textbf{Shibaji Banerjee},\textsuperscript{1} \textbf{Madhurima Pandey},\textsuperscript{2} \textbf{and Debasish Majumdar}\textsuperscript{2}

\textsuperscript{1}Department of Physics, St. Xavier's College
30, Mother Teresa Sarani, Kolkata - 700016, India

\textsuperscript{2}Astroparticle Physics and Cosmology Division, Saha Institute of Nuclear Physics, HBNI
1/AF Bidhannagar, Kolkata 700064, India

(Received; Revised; Accepted)

\textbf{ABSTRACT}

This has been suggested that the dwarf satellite galaxies in the Milky Way may contain substantial amount of dark matter in them. These dark matters may undergo self-annihilation to produce \gamma\text{-rays. The satellite borne \gamma\text{-rays telescope such as Fermi-LAT reported the detection of \gamma\text{-rays from around 45 Dwarf Spheroidals (dSphs) of Milky Way. In this work, we consider a particle dark matter model and after studying its phenomenology, we calculate the \gamma\text{-ray fluxes of each of these 45 dSphs and compare our results with the upper bounds of mass vs annihilation cross-sections of dark matter provided by the Fermi-LAT collaboration. We calculate these fluxes by considering different dark matter density profiles and make a comparison of the results. We also repeat our analysis with another dark matter candidate namely Kaluza-Klein dark matter inspired by extra-dimensional models. We make a critical comparison between the results obtained for each of these models vis-a-vis the observational results.

\textit{Keywords: dwarf galaxy — \gamma\text{-ray astronomy — dark matter}

1. INTRODUCTION

Although the existence of dark matter in the Universe is now well established, but any direct signature of the dark matter is still eluding the worldwide endeavors at different direct dark matter search experiments. The indirect search for dark matter involves in detecting the known standard model (SM) particles that can be produced by possible dark matter annihilation (or decay). Although the cosmic relics, the dark matter can undergo self annihilation if accumulated in considerable magnitude by being captured inside massive astrophysical bodies under the influence of gravity. While the emission of excess \gamma\text{-rays are observed from Galactic Centre region, the dwarf spheroidal galaxies could also be rich in dark matter. The dwarf spheroidal galaxies are the satellite galaxies to the Milky Way and they fail to grow as matured galaxies. These dwarf spheroidal galaxies (dSphs) are generally of low luminosities and contain population older stars with little dust. The dwarf spheroidal could be very rich in dark matter. These galaxies would have been tidally disrupted but for the presence of dark matter that provides the necessary gravitational pull. The presence of dark matter in dwarf spheroidals can also be realised by studying their mass to luminosity ratios. From several observations the estimated mass to luminosity ratios (M/L) are found to be much more than the same for the sun (|\frac{M}{L}|). The dark matter at dSphs can undergo self annihilation and produce \gamma\text{-rays.

The Fermi-LAT satellite borne observations and the collaborative observations of Fermi-LAT and Dark Energy Survey (DES) has reported the upper bounds of the \gamma\text{-ray spectra from several dwarf galaxies. Here in this work, we consider a particle dark matter model and analyse the \gamma\text{-ray results for all the 45 dSphs with our model. Under the framework of the model the \gamma\text{-ray fluxes as the annihilation of the dark matter are calculated for all the fortyfive dSphs and compare our results with observational upperbound of the same. We repeat our analysis for the dark matter from another model namely the Kaluza-Klein dark matter inspired by the theories of extra dimensions.

Corresponding author: Ashadul Halder
ashadul.halder@gmail.com
For the particle dark matter, we consider a two component dark matter model where one component is the Feebly Interacting Massive particle or FIMP and the other component is the Weakly Interacting Massive Particle or WIMP. The phenomenology of the model is elaborately given in an earlier work involving two of the present authors (Ref. Dutta Banik et al. (2017)). While the FImP component of the two component dark matter model could explain the phenomena such as dark matter self interactions, the WIMP component is useful in explaining the excess $\gamma$-rays from dSphs when they annihilate into $qq$ to finally produce gamma. The model is constructed by minimal extension of Standard Model with a Dirac fermion $\chi$, a real scalar $S$ and a pseudoscalar $\phi$. While the fermion $\chi$ and the scalar $S$ are singlets under SM gauge group, the fermion has an additional U(1)$_{DM}$ charge. This prevents the fermion $\chi$ to interact with SM fermions ensuring stability. A $Z_2$ symmetry is imposed on the scalar $S$. The Lagrangian is CP invariant but the CP invariance is broken when the pseudoscalar $\phi$ acquires a vev. On the other hand, the scalars develops a vev when the $Z_2$ symmetry is spontaneously broken. Thus after the spontaneous symmetry breaking of the symmetries (SU(2)$_L \times$ U(1), Z$_2$, CP), the scalars in the theory namely the Higgs $H$, S and $\phi$ acquire vev s and their real components will mix together. The lightest mass eigenstate after diagonalisation with small mixings with other scalars is taken to be the FImP candidate. But in this work, in order to calculate the $\gamma$-rays from the annihilation of dark matter in each of the chosen 45 dwarf galaxies, the WIMP component which is the Dirac singlet fermion in this model will be useful. The WIMP candidate in this model interacts with SM sector through Higgs portal. All the channels of the annihilation cross sections are derived and given in Ref. Dutta Banik et al. (2017). The $\gamma$-ray fluxes for each of the 45 dwarf galaxies are then computed with the $J$-factor values obtained from different observational groups for the dwarf galaxies.

The analysis is also repeated for the Kaluza-Klein (KK) dark matter inspired by the theories of extra dimensionsCheng et al. (2002); Hooper et al. (2008); Majumdar (2003). If only one spatial extra dimension is considered and this extra dimension is compactified over a circle of compactification radius $R$, say, then the effective four dimensional theory as obtained by integrating the extra spatial dimension over the periodic coordinate ($(y \rightarrow y + 2\pi R)$, compactification over a circle), gives rise to a tower of Kaluza-Klein modes with mass of each mode given by $m_k = k/R$, where $k$ is called the Kaluza-Klein number or KK number. As KK number is associated with the quantized momentum in compactified dimension ($E^2 = p^2 + m_k^2$), the KK number is conserved and hence the Lightest Kaluza-Klein particle or LKP is stable and can be a candidate for dark matter. We have taken a range of masses for KK dark matter and demonstrate how well the $\gamma$-rays produced from the annihilation of such a dark matter candidate agrees with the observational results for all the dwarf galaxies considered. The computation of annihilation and the $\gamma$-ray spectrum are made by using the MICROMEGAS (Belanger et al. (2011)) computer code.

We then extend our analyses for extragalactic $\gamma$-rays also. The observed extragalactic $\gamma$-ray signal may contain the component of $\gamma$-ray from dark matter annihilations at extragalactic sources (Ullio et al. (2002); Bergström et al. (2001); Gao et al. (1991); Stecker (1978); Taylor & Silk (2003); Ng et al. (2014)). The extragalactic $\gamma$-rays can have many components other than those possibly from dark matter annihilations. There are attempts to extract dark matter annihilation signals from the extragalactic gamma background or EGB (Bringmann et al. 2014; Cholis et al. 2014; Tavakoli et al. 2014a; Sefusatti et al. 2014; Ajello et al. 2015; Di Mauro & Donato 2015; Di Mauro 2015; Ackermann et al. 2015). The possible contribution to the EGB may come from BL Lac objects, millisecond pulsars, radio galaxies etc. More detailed knowledge and their possible contribution to the EGB not only helps to look for any such dark matter annihilation signals beyond the EGB but also is useful to put stringent bound on dark matter annihilation cross-sections. With the two particle dark matter models considered here, we have made an attempt in this work to estimate whether any significant signal from the dark matter annihilation can be obtained from extragalactic sources.

In an earlier work (Modak & Majumdar (2015)), by one of the present authors, similar analyses have been performed. But in that analyses only 18 dSphs were considered with inert doublet dark matter but in the present analyses we take into account as many as 45 dSphs. Also, the analyses are performed for two particle dark matter models one of which is the Higgs portal component of a two component DM (Dutta Banik et al. (2017)) and the other is the dark matter obtained from extra dimensional model (Cheng et al. (2002); Hooper et al. (2008); Majumdar (2003)).

The paper is organised as follows. In Sect. 2 we give the formalism to calculate $\gamma$-ray flux. Section 3 deals with the observational data, the calculations and results for the dwarf galaxies. The calculational procedures for the estimation of extra galactic $\gamma$-ray background and the contribution from possible dark matter annihilation are given in Sect. 4. Finally in Sect. 5 we conclude with a summary and some remarks.

2. FORMALISM FOR $\gamma$-RAY FLUX CALCULATIONS
The observed flux from cosmic dark matter source depends significantly on the dark matter annihilation cross-section \(\langle \sigma v \rangle\) (Ackermann et al. 2015) as well as total DM contained within the solid angle subtended by the the source (i.e. \(J\)-factor). The empirical expression of \(J\)-factor is defined as

\[
J = \int_{l.o.s} \rho(r)^2 ds = r_\odot \rho_\odot^2 J,
\]

In the above equation \(J\) represents the dimensionless form of \(J\)-factor given by,

\[
J = \int_{l.o.s} \frac{1}{r_\odot} \left( \frac{\rho(r)}{\rho_\odot} \right)^2 ds,
\]

where \(\rho(r)\) be the DM density at radial distance \(r\) from the galactic centre. Radial distance \(r\) can be expressed in terms of line of sight \(s\) as,

\[
r = \sqrt{s^2 + r_\odot^2 - 2sr_\odot \cos l \cos b} \quad l, b \text{ coordinate},
\]

\[
r = \sqrt{s^2 + r_\odot^2 - 2sr_\odot \cos \theta} \quad r, \theta \text{ coordinate}.
\]

There are several choices of DM halo profiles which are discussed in Appendix A. The differential \(\gamma\)-ray flux observed due to dark matter annihilation od mass \(M_\chi\) is given by (Cirelli et al. 2011),

\[
\frac{d\phi}{d\Omega dE_\gamma} = \frac{1}{8\pi\alpha} \sum_f \frac{\langle \sigma v \rangle_f}{M_\chi^2} \frac{dN_{f}^\gamma}{dE_\gamma} J,
\]

where \(\rho_\odot\) (0.3 GeV/cm\(^3\)) be the dark matter density at the distance \(r_\odot\) (8.33 kpc) from the galactic centre (at the solar system) and \(\alpha = 1\).

As mentioned earlier, we have considered two particle dark matter models namely a two component WIMP-FImP model where the WIMP component contributes to the self annihilation leading to \(\gamma\)-rays and the other is a KK dark matter. The annihilation cross-sections for the former for all possible channels are all derived and given in Appendix of Ref. Dutta Banik et al. (2017) whereas for the latter we have used microMEGAS for computing the \(\gamma\)-rays.

3. THE \(\gamma\)-RAY FLUX CALCULATIONS FOR THE DWARF GALAXIES AND THEIR COMPARISON WITH THE OBSERVATIONS

DM rich dwarf spheroidal galaxies (dSphs) have turned out to be an important topic to understand the nature of dark matter and its astrophysical implications. The satellite observations by Fermi Lat (Atwood et al. 2009) and later Dark Energy Survey (DES) and Fermi-LAT collaboration reveal a sum of 45 dwarf spheroidal galaxies in the energy range 0.5 \(\sim\) 500 GeV (Ackermann et al. 2015). The details of these dSphs are furnished in Table 1. The observational results are given in Fig. 1. In Fig. 1, the upper bound of the \(\gamma\)-ray flux for each of the 45 galaxies are shown in green arrows.

From the observational data we have constructed a heatmap. From to the heatmap diagram for the observed upper bound of \(\gamma\)-ray spectrum (Fig. 1, one observes that in the higher energy range (50 \(\sim\) 400 GeV) upper bounds are very localized (\(J\)-factors are different for different dSphs (see Table 2)). This implies that flux is less dependent of Dark Matter in this range of energy. But in the range of 0.5 \(\sim\) 50 GeV, upper bounds are not so localized and average slope is also quite different. Consequently, one can justify that, the DM annihilation dominates the lower energy region (0.5 \(\sim\) 50 GeV) whereas in the higher energy range (50 \(\sim\) 400 GeV) any exotic non-DM power spectra is maximized.

| Table 1. Benchmark points for both model. |
|------------------------------------------|
| Model | \(M_\chi\) in GeV |
|------|----------------|
| Model I (Dutta Banik et al. (2017)) | 50 |
| Model II (Cheng et al. (2002)) | \(500 \sim 800\) |

The observed flux from cosmic dark matter source depends significantly on the dark matter annihilation cross-section \(\langle \sigma v \rangle\) (Ackermann et al. 2015) as well as total DM contained within the solid angle subtended by the the source (i.e. \(J\)-factor). The empirical expression of \(J\)-factor is defined as

\[
J = \int_{l.o.s} \rho(r)^2 ds = r_\odot \rho_\odot^2 J,
\]

In the above equation \(J\) represents the dimensionless form of \(J\)-factor given by,

\[
J = \int_{l.o.s} \frac{1}{r_\odot} \left( \frac{\rho(r)}{\rho_\odot} \right)^2 ds,
\]

where \(\rho(r)\) be the DM density at radial distance \(r\) from the galactic centre. Radial distance \(r\) can be expressed in terms of line of sight \(s\) as,

\[
r = \sqrt{s^2 + r_\odot^2 - 2sr_\odot \cos l \cos b} \quad l, b \text{ coordinate},
\]

\[
r = \sqrt{s^2 + r_\odot^2 - 2sr_\odot \cos \theta} \quad r, \theta \text{ coordinate}.
\]

There are several choices of DM halo profiles which are discussed in Appendix A. The differential \(\gamma\)-ray flux observed due to dark matter annihilation od mass \(M_\chi\) is given by (Cirelli et al. 2011),

\[
\frac{d\phi}{d\Omega dE_\gamma} = \frac{1}{8\pi\alpha} \sum_f \frac{\langle \sigma v \rangle_f}{M_\chi^2} \frac{dN_{f}^\gamma}{dE_\gamma} J,
\]

where \(\rho_\odot\) (0.3 GeV/cm\(^3\)) be the dark matter density at the distance \(r_\odot\) (8.33 kpc) from the galactic centre (at the solar system) and \(\alpha = 1\).

As mentioned earlier, we have considered two particle dark matter models namely a two component WIMP-FImP model where the WIMP component contributes to the self annihilation leading to \(\gamma\)-rays and the other is a KK dark matter. The annihilation cross-sections for the former for all possible channels are all derived and given in Appendix of Ref. Dutta Banik et al. (2017) whereas for the latter we have used microMEGAS for computing the \(\gamma\)-rays.

3. THE \(\gamma\)-RAY FLUX CALCULATIONS FOR THE DWARF GALAXIES AND THEIR COMPARISON WITH THE OBSERVATIONS

DM rich dwarf spheroidal galaxies (dSphs) have turned out to be an important topic to understand the nature of dark matter and its astrophysical implications. The satellite observations by Fermi Lat (Atwood et al. 2009) and later Dark Energy Survey (DES) and Fermi-LAT collaboration reveal a sum of 45 dwarf spheroidal galaxies in the energy range 0.5 \(\sim\) 500 GeV (Ackermann et al. 2015). The details of these dSphs are furnished in Table 1. The observational results are given in Fig. 1. In Fig. 1, the upper bound of the \(\gamma\)-ray flux for each of the 45 galaxies are shown in green arrows.

From the observational data we have constructed a heatmap. From to the heatmap diagram for the observed upper bound of \(\gamma\)-ray spectrum (Fig. 1, one observes that in the higher energy range (50 \(\sim\) 400 GeV) upper bounds are very localized (\(J\)-factors are different for different dSphs (see Table 2)). This implies that flux is less dependent of Dark Matter in this range of energy. But in the range of 0.5 \(\sim\) 50 GeV, upper bounds are not so localized and average slope is also quite different. Consequently, one can justify that, the DM annihilation dominates the lower energy region (0.5 \(\sim\) 50 GeV) whereas in the higher energy range (50 \(\sim\) 400 GeV) any exotic non-DM power spectra is maximized.
Figure 1. Heatmap for the observed upper bound of γ-ray spectrum for 45 dSphs

Figure 2. Upper limit of the DM annihilation cross section for $b\bar{b}$ channel ($\langle \sigma v \rangle_{b\bar{b}}$) as function of dark matter mass ($M_\chi$) for individual dSphs. The black dashed line represents the working area of model I i.e. 50GeV and the region within the black rectangle shows the same for model II.

In the present work we have estimated γ-flux for all of those 45 dSphs from DM annihilation for the benchmark points for dark matter masses (or the mass range) for two chosen particle dark matter models as described earlier. In Table 1 the benchmark mass points are given. We first choose these two masses (or mass range) from Table 1 and without considering any dark matter model, calculated the limiting value of $\langle \sigma v \rangle$ for each of the dwarf galaxies for which the γ-ray fluxes do not cross the upper limit of the observational results. The upper limits for $\langle \sigma v \rangle$ for the chosen dark matter mass as in Table 1 for each of the dwarf galaxies are plotted in colour coded Fig. 2. In Fig. 2, the dashed line corresponds to the dark matter mass of 50 GeV and the solid rectangular block is for the dark matter mass range given in Table 1. The colour code corresponding to each line describes the upper limit of the value of $\langle \sigma v \rangle$ for a particular dwarf galaxy corresponding the chosen dark matter mass.

Figs. 3 and 4 show observed γ-ray flux along with DM annihilation flux and corresponding uncertainty for the benchmark points for two models chosen. The red and blue lines represent the flux for Model I and Model II respectively. The yellow shaded area indicate the uncertainty corresponding to Model I. In the calculation of γ-ray flux for individual dSphs, $dN/dE$ is calculated for given DM mass $M_\chi$. Integrated $J$-factor over a solid angle of $\Delta\Omega = 2.4 \times 10^{-4}$ sr (field of view of Fermi-LAT $\sim 0.5$) are measured from stellar kinematics data. The numerical values of $J$-factor for all dSphs are tabulated in Table 2. For Model I, the calculations are performed for three dark matter halo profiles namely NFW,
Einasto and Burkart profiles (see Appendix). It appears from Figs. 3 and 4 that for Model I (the WIMP component of a FImP-WIMP model) and for all the three density profiles the fluxes are below the observational upper limits of all the 45 dSphs. Similar results corresponding to the mass range chosen for Model II (the Kauza-Klein model) are shown in Figs. 3 and 4 in colour codes. The band structure in Figs. 3 and 4 arises because of the spread in $\langle \sigma v \rangle$ for each mass in the mass range (designated by a particular colour).  

4. EXTRAGALACTIC $\gamma$-RAY BACKGROUND AND EXTRAGALACTIC $\gamma$-RAYS FROM DARK MATTER ANNIHILATIONS

The $\gamma$-rays from dark matter annihilations could have extragalactic origin too. But whether such a gamma signal can be identified by terrestrial telescopes depend on the background $\gamma$-rays from different types of extragalactic sources. Therefore, to study the gamma rays from extragalactic dark matter annihilations, one needs to estimate the flux from other possible sources.

The extragalactic $\gamma$-ray when detected at terrestrial detectors, may also contain the galactic $\gamma$-ray component. It is therefore essential to understand such components and subtract from the extragalactic signal. The resulting signal after subtraction is the diffuse $\gamma$-ray background which is to be estimated to extract the possible signal from possible extragalactic dark matter annihilation.

The rate of photons emitted from volume element $dV$ having energy ranges $E + dE$ and observed by detector having effective area $dA$ during the time interval $dt_0$ in the redshifted $(z)$ energy $dE_0$ is given by,

$$
\frac{dN_{\gamma}}{dE} = e^{-\tau} \left[ (1 + z)^3 \int dM \frac{dn}{dM}(M, z) \times \frac{dN_{\gamma}}{dE}(E, M, z) \right] \frac{dV dA}{4\pi (R_0 S_k(r))^2} dE_0 dt_0.
$$

In the above, the volume element $dV$ is given by

$$
dV = (R_0 S_k(r))^2 R_0 \frac{dV}{(1 + z)^3} d\Omega_{\text{detector}},
$$

where $S_k(r)$ is the spatial curvature given by Robertson-Walker metric. The quantity $\frac{dn}{dM}(M, z)$ is the halo mass function and $\frac{dN_{\gamma}}{dE}(E, M, z)$ is the photon energy flux $^1$. The $\tau(z, E_0)$ represents optical depth of extragalactic $\gamma$-rays and $e^{-\tau(z, E_0)}$ be the corresponding attenuation factor varies with $z$ and $E_0$ as described in Fig. 5 (Cirelli et al. 2011). For ultraviolet background we have adopted the model as described in the work of Domínguez et al. (2011); Franceschini et al. (2008). Now the diffuse extragalactic $\gamma$-ray flux due to DM annihilation, can be written as,

$$
\frac{d\phi_{\gamma}}{dE} = \frac{dN_{\gamma}}{dM dtdE_0} \frac{dE_0}{h(z)} \int dM \frac{dn}{dM}(M, z) \frac{dN_{\gamma}}{dE}(E_0 (1+z), M, z),
$$

where $c$ is the speed of light in vacuum, $H_0$ is the Hubble constant at the present epoch and $h(z) = \sqrt{\Omega_M(1+z)^3 + \Omega_\Lambda}$ for spatially flat universe ($\Omega_k = 0$). $\Omega_M$ and $\Omega_\Lambda$ respectively the matter and dark energy density parameter respectively. The cosmological dark matter halo function $\frac{dn}{dM}(M, z)$ can be written in the form (Press & Schechter 1974),

$$
\frac{dn}{dM} = \frac{\rho_{0,m}}{M^2} \nu f(\nu) \frac{d\log \nu}{d\log M},
$$

where $\rho_{0,m}$ is the comoving background matter density and $\nu = \delta_c / \sigma(M)$. $\delta_c$ ($\simeq 1.686$) be the critical overdensity for spherical collapse and $\sigma(M)$ denotes the variance or the root mean square density fluctuations of the linear density field in sphere having mean mass $M \simeq (4/3)\pi R^3 \rho_c(z)$; here the subscript $c$ denotes for the collapsing of halos. The term $\sigma^2(M)$ can be expressed as (Sheth & Tormen 1999),

$$
\sigma^2(M) = \int d^3k \tilde{W}^2(kR)P(k),
$$

where $P(k)$ is the linear power spectrum $\tilde{W}(kR)$ is the Fourier transform of the top hat window function and $R$ is the comoving length scale. The power spectrum $P(k) \propto k^3T^2(k)$ where $n$ is the spectral index and $T$ is the transfer

$^1 dt_0 = \frac{dz}{1+z}(1+z) dt_0$, where $t_0$ and $E_0$ are the time and energy respectively at $z = 0$
Figure 3. Comparison of γ ray upper limit of energy flux along with computed γ flux produced by DM annihilation process (Blue represents the flux according to the Model I and red represents the same for Model II). See text for detail.
Table 2. Upper limit of dark matter annihilation cross section for both of the benchmark DM mass and J-factor for individual dSphs.

| dSphs name   | Logitude $l$ (deg) | Latitude $b$ (deg) | Distance (kpc) | $\log_{10} \mathcal{J}_{NFW}^{\nu}$ | Upper limit of $\langle \sigma v \rangle$ (cm$^3$ s$^{-1}$) (Model I) |
|-------------|--------------------|--------------------|----------------|--------------------------------------|-------------------------------------------------------------|
| Bootes I    | 358.1              | 69.6               | 66             | 18.17 ± 0.30                         | $2.95 \times 10^{-25}$                                     |
| Bootes II   | 353.7              | 68.9               | 42             | 18.90 ± 0.60                         | $3.00 \times 10^{-25}$                                     |
| Bootes III  | 35.4               | 75.4               | 47             | 18.80 ± 0.60                         | $1.08 \times 10^{-25}$                                     |
| Canes Venatici I | 74.3           | 79.8               | 218            | 17.42 ± 0.16                         | $7.77 \times 10^{-25}$                                     |
| Canes Venatici II | 113.6          | 82.7               | 160            | 17.82 ± 0.47                         | $6.24 \times 10^{-25}$                                     |
| Carina      | 260.1              | −22.2              | 105            | 17.83 ± 0.10                         | $6.67 \times 10^{-25}$                                     |
| Cetus II    | 156.47             | −78.53             | 30             | 19.10 ± 0.60                         | $1.30 \times 10^{-25}$                                     |
| Columba I   | 231.62             | −28.88             | 182            | 17.60 ± 0.60                         | $8.24 \times 10^{-24}$                                     |
| Coma Berenices | 241.9              | 83.6               | 44             | 19.00 ± 0.36                         | $3.51 \times 10^{-26}$                                     |
| Draco       | 86.4               | 34.7               | 76             | 18.83 ± 0.12                         | $2.23 \times 10^{-26}$                                     |
| Draco II    | 98.29              | 42.88              | 24             | 19.30 ± 0.60                         | $9.78 \times 10^{-26}$                                     |
| Eridanus II | 249.78             | −51.65             | 330            | 17.28 ± 0.34                         | $5.88 \times 10^{-24}$                                     |
| Eridanus III| 274.95             | −59.6              | 95             | 18.30 ± 0.40                         | $3.06 \times 10^{-25}$                                     |
| Fornax      | 237.1              | −65.7              | 147            | 18.09 ± 0.10                         | $1.19 \times 10^{-24}$                                     |
| Grus I      | 338.68             | −58.25             | 120            | 17.90 ± 0.60                         | $1.77 \times 10^{-24}$                                     |
| Grus II     | 351.14             | −51.94             | 53             | 18.70 ± 0.60                         | $7.74 \times 10^{-25}$                                     |
| Hercules    | 28.7               | 36.9               | 132            | 17.37 ± 0.53                         | $5.41 \times 10^{-23}$                                     |
| Horologium I| 271.38             | −54.74             | 87             | 18.40 ± 0.40                         | $7.95 \times 10^{-25}$                                     |
| Horologium II| 262.48            | −54.14             | 78             | 18.30 ± 0.60                         | $1.54 \times 10^{-24}$                                     |
| Hydra II    | 295.62             | 30.46              | 134            | 17.80 ± 0.60                         | $1.13 \times 10^{-24}$                                     |
| Indus II    | 354                | −37.4              | 214            | 17.40 ± 0.60                         | $2.88 \times 10^{-23}$                                     |
| Kim 2       | 347.2              | −42.1              | 69             | 18.60 ± 0.40                         | $1.99 \times 10^{-24}$                                     |
| Leo I       | 226                | 49.1               | 254            | 17.64 ± 0.14                         | $1.23 \times 10^{-24}$                                     |
| Leo II      | 220.2              | 67.2               | 233            | 17.76 ± 0.2                          | $1.67 \times 10^{-25}$                                     |
| Leo IV      | 265.4              | 56.5               | 154            | 16.40 ± 1.15                         | $2.77 \times 10^{-22}$                                     |
| Leo V       | 261.86             | 58.54              | 178            | 17.65 ± 0.97                         | $3.93 \times 10^{-22}$                                     |
| Pegasus III | 69.85              | −41.81             | 205            | 18.30 ± 0.94                         | $1.64 \times 10^{-24}$                                     |
| Phoenix II  | 323.69             | −59.74             | 95             | 18.30 ± 0.40                         | $3.99 \times 10^{-25}$                                     |
| Pictor I    | 257.29             | −40.64             | 126            | 18.10 ± 0.40                         | $7.63 \times 10^{-25}$                                     |
| Piscis II   | 79.21              | −47.11             | 182            | 17.60 ± 0.40                         | $1.90 \times 10^{-24}$                                     |
| Reticulum II| 266.3              | −49.74             | 32             | 18.68 ± 0.35                         | $5.78 \times 10^{-25}$                                     |
| Reticulum III| 273.88            | −45.65             | 92             | 18.20 ± 0.60                         | $1.92 \times 10^{-24}$                                     |
| Sagittarius II | 18.94            | −22.9              | 67             | 18.40 ± 0.60                         | $1.70 \times 10^{-24}$                                     |
| Sculptor    | 287.5              | −83.2              | 86             | 18.58 ± 0.05                         | $2.48 \times 10^{-25}$                                     |
| Segue 1     | 220.5              | 50.4               | 23             | 19.12 ± 0.54                         | $4.97 \times 10^{-26}$                                     |
| Sextans     | 243.5              | 42.3               | 86             | 17.73 ± 0.13                         | $9.53 \times 10^{-25}$                                     |
| Triangulum II| 140.9             | −23.82             | 30             | 19.10 ± 0.60                         | $1.20 \times 10^{-25}$                                     |
| Tucana II   | 328.04             | −52.35             | 58             | 18.80 ± 0.40                         | $6.98 \times 10^{-25}$                                     |
| Tucana III  | 315.38             | −56.18             | 25             | 19.30 ± 0.60                         | $2.97 \times 10^{-25}$                                     |
| Tucana IV   | 313.29             | −55.29             | 48             | 18.70 ± 0.60                         | $6.64 \times 10^{-25}$                                     |
| Tucana V    | 316.31             | −51.89             | 55             | 18.60 ± 0.60                         | $8.78 \times 10^{-26}$                                     |
| Ursa Major I| 159.4              | 54.4               | 97             | 18.26 ± 0.28                         | $8.73 \times 10^{-25}$                                     |
| Ursa Major II| 152.5            | 37.4               | 32             | 19.44 ± 0.40                         | $1.67 \times 10^{-26}$                                     |
| Ursa Minor  | 105                | 44.8               | 76             | 18.75 ± 0.12                         | $1.56 \times 10^{-26}$                                     |
| Willman 1   | 158.6              | 56.8               | 38             | 18.90 ± 0.60                         | $4.49 \times 10^{-25}$                                     |
function which depends on the nature of DM and baryon density in the universe. Thus the transfer function can be calculated from the cosmic microwave background data. The variation of the power spectrum $P(k)$ with wavenumber $k$ for a range of redshifts is shown in Fig. 8d. Fig. 8c describes the variation of variance $\sigma$ with halo mass $M$ for those particular values of $z$. The multiplicity function $f(\nu)$ can be modelled in the ellipsoidal collapse model (Sheth...
\[ \nu f(\nu) = 2A \left( 1 + \frac{1}{\nu' a} \right) \frac{\nu'^2}{2\pi} \exp \left( -\frac{\nu'^2}{2} \right) \] (10)

where \( \nu' = \sqrt{a}\nu \). Fitting the Eq. 8 with \( N \)-body simulation of Virgo consortium (Jenkins et al. 1998) the numerical values of \( a(= 0.707) \) and \( p = 0.3 \) can be obtained.

The function \( f(\sigma) \) can be formulated by plugging \( \nu = \delta_{\text{vir}}/\sigma M \) in Eq. 10. In Fig. 8a the variations of the mass collapse fraction \( f(\sigma) \) in the ellipsoidal collapse model is demonstrated for several values of redshift \( z \) (0 ~ 10) as well as the halo mass \( M \) are demonstrated. Fig. 8b describes the variations of the considered halo mass function \( dn/dM \) of Sheth-Torman model (Sheth & Tormen 1999) with redshift \( z \) and the halo mass \( M \). All the numerical calculations related to Fig. 8 have been performed using \texttt{HMFCalc} (Murray et al. 2013) code. For the halo profile we have chosen NFW halo profile (Navarro et al. 1996; Navarro et al. 1997) \( \rho(r) = \rho_s g(r/r_s) \) (see Appendix A) Any DM halo of mass \( M_h \) enclosed at a radius \( r_h \) is,

\[ M_h = 4\pi\rho_s r_h^3 f(r_s/r_h) = \frac{4\pi}{3} \Delta_{\text{vir}} \bar{\rho}(z) R_{\text{vir}}^3, \] (11)

where \( f(x) = x^3 \left( \ln(1+x^{-1}) - (1 + x^{-1}) \right) \), \( \bar{\rho}(z) \) be the mean background density and \( R_{\text{vir}} \) be the virial radius defined as a sphere having the same radius contains total halo mass \( M \) with mean halo density \( \Delta_{\text{vir}} \bar{\rho}(z) \). The term \( \Delta_{\text{vir}} \) is the virial overdensity with respect to the mean matter density which may depend on the cosmological parameters. For the flat universe \( (\Omega_k = 0) \), \( \Delta_{\text{vir}}(z) \) takes the form \((O'Shea & Norman 2007)\),

\[ \Delta_{\text{vir}} \simeq (18\pi^2 + 82d - 39d^2), \] (12)

with \( d \equiv d(z) = \frac{\Omega_m(1+z)^3}{(\Omega_m(1+z)^3 + \Omega_X)} - 1 \).

Now the \( \gamma \)-ray energy spectrum \( \frac{dN_{\gamma}}{dE}(E_0 (1 + z), M, z) \) for the \( \gamma \)-ray (Eq. 7) emitted from a halo of mass \( M \) at redshift \( z \) can be written in the form,

\[ \frac{dN_{\gamma}}{dE}(E, M, z) = \frac{\langle \sigma v \rangle}{2} \frac{dN_s(E)}{dE} \int d^3 r P(c_{\text{vir}}) \left( \frac{\bar{\rho}(z)}{M_X} \right)^2 \int d^3 r g^2 (r/a). \] (13)

In the above equation \( \langle \sigma v \rangle \) is the thermally averaged value of DM annihilation cross-section times the relative velocity, \( \frac{dN_{\gamma}}{dE} \) is the differential \( \gamma \)-ray energy spectrum. The log-normal distribution \( P(c_{\text{vir}}) \) of the concentration parameter \( c_{\text{vir}} \) around the mean value is chosen within 1\( \sigma \) deviation (Sheth & Tormen 1999), for halos with mass \( M \). Finally one can write,

\[ \frac{dN_{\gamma}}{dE}(E, M, z) = \frac{\sigma v}{2} \frac{dN_s(E)}{dE} \frac{M}{M_X^2} \Delta_{\text{vir}} \bar{\rho}(z) \int d^3 r P(c_{\text{vir}}) \left( \frac{c_{\text{vir}} r_2 - 2}{I_1(c_{\text{vir}} r_2)} \right)^3 I_2(x_{\text{min}}, c_{\text{vir}} r_2). \] (14)
In the above $r_{-2}$ is the ratio between $r^{(-2)}_s$ and $r_s$ where $r^{(-2)}_s$ is the radius at which the effective logarithmic slope $-2$ that follows from the relation, $d/dr \left( r^2 g(r) \right) |_{r=r^{(-2)}_s} = 0$. For NFW profile, $r^{(-2)}_s = r_s$. Hence $c_{\text{vir}} r_{-2} = R_{\text{vir}}/r$ and the form of integration $I_n(x_{\text{min}}, x_{\text{max}})$ is given be $I_n(x_{\text{min}}, x_{\text{max}}) = \int_{x_{\text{min}}}^{x_{\text{max}}} dx x^2 g^\nu(x)$. Now putting the above equation in Eq. 7, the analytic form of extragalactic $\gamma$-ray flux from DM annihilation can be obtained as (Ullio et al. 2002)

$$\frac{d\phi_{\gamma}}{dE_0} = \frac{\sigma v}{8\pi H_0 M_{\chi}^2} \int dz (1+z)^3 \frac{\Delta^2(z) dN_{\gamma}(E_0(1+z))}{h(z)} \frac{dE}{dE_0} e^{-\tau(z,E_0)}, \tag{15}$$

where the expression for $\Delta^2(z)$ can be given by,

$$\Delta^2(z) = \int dM \frac{\nu(z,M) f(\nu(z,M))}{\sigma(M)} \left| \frac{d\sigma}{dM} \right| \Delta^2_M(z,M), \tag{16}$$

with

$$\Delta^2_M(z,M) \equiv \frac{\Delta_{\text{vir}}(z,M)}{3} \int dc_{\text{vir}} \mathcal{P}(c_{\text{vir}}) \frac{I_2(x_{\text{min}}, c_{\text{vir}}(z,M) r_{-2})}{I_1(x_{\text{min}}, c_{\text{vir}}(z,M) r_{-2})} (c_{\text{vir}}(z,M) r_{-2})^3. \tag{17}$$

In all of the above the concentration parameter, $c_{\text{vir}}$ is defined as

$$c_{\text{vir}} = \frac{R_{\text{vir}}}{r^{(-2)}_s}, \tag{18}$$

In this work, two forms for the concentration parameter $c_{\text{vir}}$ are chosen following Macciò et al. (2008) and power law model (Neto et al. 2007; Macciò et al. 2008). For the first choice (Macciò et al.) $c_{\text{vir}}(M,z) = k_{200} (\mathcal{H}(z_f(M))/\mathcal{H}(z))^{2/3}$, where $k_{200} \simeq 3.9$, $\mathcal{H}(z) = H(z)/H_0$ and $z_e(M)$ is the effective redshift during the formation of a halo with mass $M$. The second choice is adopted (in the power law model) such that $c_{\text{vir}}(M,z) = 6.5 \mathcal{H}(z)^{-2/3} (M/M_\star)^{-0.1}$, $M_\star = 3.37 \times 10^{12} h^{-1} M_\odot$.

The substructures of dark matter within halo form bound objects. The minimum subhalo mass, $M_{\text{min}}$ is determined from the decoupling (kinematically) temperature of dark matter from the cosmic background.

The present analyses are performed for two typical values of $M_{\text{min}}$ namely $M_{\text{min}} = 10^{-6} M_\odot$ and $10^{-9} M_\odot$ (Martinez et al. 2009; Bringmann 2009).

It is found that the extragalactic $\gamma$-ray spectrum is reduced to almost a power law spectrum for the energy range few hundred MeV $\sim$ few hundred GeV ($\frac{dN_{\gamma}}{dE} \propto E^n$). The diffuse $\gamma$-ray background may include contributions from BL Lacertea objects (BL Lacs), flat spectrum radio quasars (FSRQs), millisecond pulsars (MSPs), star forming galaxy.
Figure 7. Observed extragalactic $\gamma$-ray fluxes by EGRET and Fermi-LAT compared with the sum total $\gamma$-ray fluxes obtained from the DM annihilation for Model-I DM and other possible non-DM $\gamma$-rays extragalactic sources.

All non-DM fluxes are plotted in fig. 6 and 7 along with the observed extragalactic $\gamma$-ray fluxes by EGRET and Fermi-LAT. The red and black dashed line in fig. 6 demonstrate the contribution of DM annihilation in the background flux for $M_{\text{min}} = 10^{-6} M_\odot$ and $10^{-9} M_\odot$ respectively according to the Maccio' et al. (2008). Fig. 7 represents the same for powerlaw model. It appears from fig. 6 and 7 that while the $\gamma$-ray from DM annihilation may not be detected (below the extragalactic background) when powerlaw spectrum is considered (Fig. 7), for the formal case fig. 7 the $\gamma$-ray spectrum from DM annihilation (black solid line of fig. 6, for $M_{\text{min}} = 10^{-9} M_\odot$) appears to lie a shade above the extra galactic background.
5. SUMMARY AND DISCUSSIONS

In this work, we explore the observational upper limits of \( \gamma \)-ray flux from 45 dwarf spheroidal galaxies and relate this to the \( \gamma \)-rays that could be produced from annihilation of dark matter in dSphs. From the mass to luminosity ratios, dSphs could be rich in dark matter and the dark matter can undergo annihilation to produce \( \gamma \)-rays. For our analysis, we consider two particle dark matter models. One is a two component WIMP-FImP model of which the WIMP component undergoes annihilation to produce the \( \gamma \)-ray flux. The WIMP component is a Dirac singlet fermion and its additional \( U(1)_{DM} \) charge prevents its interaction with SM fermions. But this sector (the fermion WIMP) interacts with SM sector via Higgs portal. The benchmark mass for this dark matter is chosen to be 50 GeV for the present analysis. The other particle dark matter chosen for the analysis is Kaluza-Klein (KK) dark matter inspired by models of extra dimensions. For the KK dark matter, which is stable since the KK number of the Kaluza-Klein tower is conserved, the chosen mass range is \( \sim 500 - 800 \) GeV, much higher than the Higgs portal fermionic dark matter in Model I. It appears from the analysis that for both the Higgs portal model and KK model, the dark matter annihilation to \( \gamma \)-rays at 45 dwarf galaxies are well within the observational upper bounds of the \( \gamma \)-ray flux for all the 45 galaxies considered. While the Higgs portal dark matter (Model I) covers a shorter range, the Kaluza-Klein dark matter with higher mass range can probe the \( \gamma \)-ray flux at higher energy range.

We have also extended our analysis for the case of possible extragalactic signature of \( \gamma \)-rays from dark matter annihilations. For this purpose, one need to estimate \( \gamma \)-rays from other extragalactic sources. Also the galactic halo part is to be subtracted to obtain the the diffuse extragalactic \( \gamma \)-ray background. Our calculation shows that for
certain circumstances, the γ-rays from the annihilation of dark matter in the present dark matter models may barely give signals just above the extragalactic gamma background. Thus such analyses not only throw light on the indirect signatures of dark matter, but also help constraining the particle dark model and the theory.

ACKNOWLEDGMENTS

Two of the authors (S.B. and A.H.) wish to acknowledge the support received from St.Xaviers College, Kolkata. One of the authors (A.H.) also acknowledges the University Grant Commission (UGC) of the Government of India, for providing financial support, in the form of UGC-CSIR NET-JRF. One of the authors (MP) thanks the DST-INSPIRE fellowship grant by DST, Govt. of India.

APPENDIX

A. DARK MATTER HALO PROFILE

\begin{align*}
\text{NFW} & \quad \rho_{\text{NFW}} = \rho_s \left( \frac{r_s}{r} \right)^2 \\
\text{Einasto} & \quad \rho_{\text{Ein}} = \rho_s \exp \left[ -\frac{2}{\alpha} \left( \left( \frac{r}{r_s} \right)^{\alpha} - 1 \right) \right] \\
\text{Isothermal} & \quad \rho_{\text{Iso}} = \frac{\rho_s}{1+\left( r/r_s \right)^2} \\
\text{Burkert} & \quad \rho_{\text{Bur}} = \frac{\rho_s}{\left( 1+r/r_s \right)^{2/3}} \\
\text{Moore} & \quad \rho_{\text{NFW}} = \rho_s \left( \frac{r_s}{r} \right)^{1.16} \left( 1+\frac{r}{r_s} \right)^{-1.84}
\end{align*}

REFERENCES

Ackermann, M., et al. 2015, JCAP, 1509, 008, doi: 10.1088/1475-7516/2015/09/008
Ackermann, M., Albert, A., Anderson, B., et al. 2015, Physical Review Letters, 115, 231301, doi: 10.1103/PhysRevLett.115.231301
Ajello, M., Gasparinni, D., Sánchez-Conde, M., et al. 2015, ApJL, 800, L27, doi: 10.1088/2041-8205/800/2/L27
Atwood, W. B., Abdo, A. A., Ackermann, M., et al. 2009, ApJ, 697, 1071, doi: 10.1088/0004-637X/697/2/1071
Belanger, G., Boudjema, F., Brun, P., et al. 2011, Comput. Phys. Commun., 182, 842, doi: 10.1016/j.cpc.2010.11.033
Bergström, L., Edsjö, J., & Ullio, P. 2001, Physical Review Letters, 87, 251301
Bringmann, T. 2009, New Journal of Physics, 11, 105027, doi: 10.1088/1367-2630/11/10/105027
Bringmann, T., Calore, F., Di Mauro, M., & Donato, F. 2014, Phys. Rev. D, 89, 023012, doi: 10.1103/PhysRevD.89.023012
Cheng, H.-C., Feng, J. L., & Matchev, K. T. 2002, Phys. Rev. Lett., 89, 211301, doi: 10.1103/PhysRevLett.89.211301
Cholis, I., Hooper, D., & McDermott, S. D. 2014, Journal of Cosmology and Astroparticle Physics, 2014, 014, doi: 10.1088/1475-7516/2014/02/014
Cirelli, M., Corcella, G., Hektor, A., et al. 2011, Journal of Cosmology and Astroparticle Physics, 2011, 051, doi: 10.1088/1475-7516/2011/03/051
Di Mauro, M. 2015, in 5th International Fermi Symposium Nagoya, Japan, October 20-24, 2014. https://arxiv.org/abs/1502.02566
Di Mauro, M., & Donato, F. 2015, Phys. Rev., D91, 123001, doi: 10.1103/PhysRevD.91.123001
Dominguez, A., Primack, J. R., Rosario, D. J., et al. 2011, MNRAS, 410, 2556, doi: 10.1111/j.1365-2966.2010.17631.x
Dutta Banik, A., Pandey, M., Majumdar, D., & Biswas, A. 2017, The European Physical Journal C, 77, 657, doi: 10.1140/epjc/s10052-017-5221-y
Franceschini, A., Rodighiero, G., & Vaccari, M. 2008, A&A, 487, 837, doi: 10.1051/0004-6361:200809691
Gao, Y.-T., Stecker, F. W., & Cline, D. B. 1991, A&A, 249, 1
