Exploring timelike exclusive processes in the light-front approach

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We discuss a necessary nonvalence contribution in timelike exclusive processes. Utilizing a Schwinger-Dyson type of approach, we relate the nonvalence contribution to an ordinary light-front wave function that has been extensively tested in the spacelike exclusive processes. An application to $K_{\ell 3}$ decays provides encouraging results.

Not only the new and upgraded B-meson factories but also the lower-lying meson facilities such as the $\tau$-Charm factories at Cornell and the recent PEP-N project at SLAC demand intensive theoretical analyses of exclusive meson decays and form factors. Unlike the leading twist structure functions measured in deep inelastic scattering, such exclusive channels are sensitive to the structure of the hadrons at the amplitude level and to the coherence between the contributions of the various quark currents and multi-parton amplitudes. The central unknown required for reliable calculations of weak decay amplitudes are thus the hadronic matrix elements.

Perhaps, one of the most popular formulations for the analysis of exclusive processes involving hadrons may be provided in the framework of light-front (LF) quantization \cite{1}. In particular, the Drell-Yan-West ($q^{+} = q^{0} + q^{3} = 0$) frame has been extensively used in the calculation of various electroweak form factors and decay processes \cite{2,3}. As an example, only the parton-number-conserving (valence) Fock state contribution is needed in $q^{+} = 0$ frame when the “good” component of the current, $J^{+}$ or $J_{\perp} = (J_{x}, J_{y})$, is used for the spacelike electromagnetic form factor calculation of pseudoscalar mesons \cite{4}. On the other hand, the analysis of timelike exclusive processes has remained as a rather significant challenge in the LF approach. In principle, the $q^{+} \neq 0$ frame can be used to compute the timelike processes but then it is inevitable to encounter the particle-number-nonconserving Fock state (or nonvalence) contribution. The main source of difficulty in
constituent quark model (CQM) phenomenology is the lack of information on the non-wave-function vertex (black blob in Fig. 1(a)) in the nonvalence diagram arising from the quark-antiquark pair creation/annihilation.

In this talk, we thus present a way of handling the nonvalence contribution. Our aim of new treatment is to make the program more suitable for the CQM phenomenology specific to the low momentum transfer processes. More details of our effective treatment can be found in Ref. [5].

The crux of our method [5] is the link between the non-wave-function vertex (black blob) and the ordinary LF wave function (white blob) as shown in Fig. 2:

\[
(M^2 - M_0^{'2})\Psi'(x_i, k_{\perp i}) = \int [dy][d^2l_{\perp}]K(x_i, k_{\perp i}; y_j, l_{\perp j})\Psi(y_j, l_{\perp j}), \tag{1}
\]

where \( M \) is the mass of outgoing meson and \( M_0^{'2} = (m_1^2 + k_{\perp 1}^2)/x_1 - (m_2^2 + k_{\perp 2}^2)/(-x_2) \) with \( x_1 = 1 - x_2 > 1 \) due to the kinematics of the non-wave-function vertex. With this link made by a Schwinger-Dyson (SD) type equation (Eq. (1)), we can now get the nonvalence contribution (Fig. 1(a)) as a sum of LF time-ordered amplitudes (Figs. 1(b) and (c)) and moreover find that the four-body energy denominator \( D_4 \) is exactly cancelled in summing the LF time-ordered amplitudes; i.e., \( 1/D_4D_2^g + 1/D_4D_2^h = 1/D_2^gD_2^h \). We thus obtain the amplitude identical to the nonvalence contribution in terms of ordinary LF wave functions.
of gauge boson (W) and hadron (white blob) as drawn in Fig.1(d). This method, however, requires to have some relevant operator depicted as the black square (K) in Fig. 2 (See also Fig.1(d)), that is in general dependent on the involved momenta connecting one-body to three-body sector. While the relevant operator K is in general dependent on all internal momenta (x, k⊥, y, l⊥), a sort of average on K over y and l⊥ depends only on x and k⊥ (See Eq. (1)). In the semileptonic decay processes involving small momentum transfers such as the Kℓ3 decays, we can kinematically justify [5] that the r.h.s. of Eq. (1) may be approximated as a constant.

If the initial and final mesons are pseudoscalar (0−), the relevant matrix element for the semileptonic decay of the initial meson (Q1¯q bound state) with four-momentum P1μ and mass M1 into the final meson (Q2¯q bound state) with P2μ and M2 is given by

\[ J^\mu(0) = \langle P_2 | \bar{Q}_2 \gamma^\mu Q_1 | P_1 \rangle = f_+(q^2)(P_1 + P_2)\mu + f_-(q^2)q^\mu, \]

where qμ = (P1 − P2)μ is the four-momentum transfer to the lepton pair (ℓν) and mℓ2 ≤ q2 ≤ (M1 − M2)2. We compute [6] the matrix element in a purely longitudinal momentum frame where q+ > 0 and P1⊥ = P2⊥ = 0 so that q2 = q+q− > 0. For the check of frame-independence, we also compute the “+” component of the current J^µ_D in the Drell-Yan-West (q+ = 0) frame where only valence contribution exists. Since the form factor f_+(q^2) obtained from J^µ_D in q+ = 0 frame is immune to the zero-mode contribution [5], the comparison of f_+(q^2) in the two completely different frames (i.e. q+ = 0 and q+ ≠ 0) would reveal the validity of existing model with respect to a covariance (or frame-independence).

The comparison of f_−(q^2), however, cannot give a meaningful test of covariance because of the zero-mode complication as noted in Ref. [5]. Indeed, the difference between the two (q+ = 0 and q+ ≠ 0) results of f_−(q^2) amounts to the zero-mode contribution.

In our numerical calculation for the processes of Kℓ3 decays, we use the linear potential parameters presented in Refs. [2,3]. In Table 1, we summarize the experimental observables for the Kℓ3 decays, where \( \lambda_i = M_{\pi}^2f'_i(0)/f_i(0) (i = +, 0) \) and \( \xi_A = f_−(0)/f_+(0) \). Incidentally, the Kℓ3(ℓ = e, µ) decays involving rather low momentum transfers bear a substantial contribution from the nonvalence part and their experimental data are better known than other semileptonic processes with large momentum transfers. As one can see in Table 1, our new results (column 2) are now much improved and comparable with the data. More results including heavier mesons are discussed in Ref. [6].

In summary, we presented an effective treatment of the LF nonvalence contributions
Table 1
Model predictions for the parameters of $K^0_{e3}$ decays. The decay width is in units of $10^6 \text{s}^{-1}$. The used CKM matrix is $|V_{us}| = 0.2196 \pm 0.0023$ from the Particle Data Group, D. E. Groom et al., Eur. Phys. J. C 15, 1 (2000).

| $f_+(0)$ | $q^+ = 0$ | Experiment |
|---------|---------|-----------|
| 0.962 [0.962] | 0.962 [0.962] | $0.962 \pm 0.0015 [K^0_{e3}]$ |
| $\lambda_+$ | 0.026 [0.083] | 0.026 [0.026] | $0.025 \pm 0.006 [K^0_{\mu3}]$ |
| $\lambda_0$ | 0.025 $[-0.017]$ | 0.001 $[-0.009]$ | $-0.11 \pm 0.09 [K^0_{\mu3}]$ |
| $\xi_A$ | $-0.013 [-1.10]$ | $-0.29 [-0.41]$ | $-0.11 \pm 0.09 [K^0_{\mu3}]$ |
| $\Gamma(K^0_{e3})$ | $7.3 \pm 0.15$ | $7.3 \pm 0.15$ | $7.5 \pm 0.08$ |
| $\Gamma(K^0_{\mu3})$ | $4.92 \pm 0.10$ | $4.66 \pm 0.10$ | $5.25 \pm 0.07$ |

crucial in the timelike exclusive processes. Using a SD-type approach and summing the LF time-ordered amplitudes, we obtained the nonvalence contributions in terms of ordinary LF wave functions of gauge boson and hadron that have been extensively tested in the spacelike exclusive processes [2,3]. Including the nonvalence contribution, our results on $K_{e3}$ not only show a definite improvement in comparison with experimental data but also exhibit a covariance (i.e frame-independence) of our approach.

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