Change in Soil Porosity under Load

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Abstract. The theoretical basis for the process of soil compaction under various loading paths is considered in the article, the theoretical assumptions are compared with the results of the tests of clay soil on a stabilometer. The variant of the critical state model of the sealing plastic-rigid environment is also considered the strength characteristics of which depend on the porosity coefficient. The loading surface is determined by the results of compression and stabilometrical tests. In order to clarify the results of this task, it is necessary to carry out stabilometric tests under conditions of simple loading, i.e. where the vertical pressure would be proportional to the compression pressure \( \sigma_3 = k \sigma_1 \). Within the study the attempts were made to confirm the model given in the beginning of the article by laboratory tests. After the analysis of the results, the provided theoretical assumptions were confirmed.

1. Introduction
In the studies [1-4] a certain value of the porosity corresponds to the equation of loading surface

\[ \sigma_3 = C + A \sigma_1 + b(p) \sigma_1^2 \]  

where \( p \) is such a parameter that when

\[ \sigma_1 = \sigma_3 = -p \]

the point on the loading surface (9) determines the hydrostatic stress condition. Therefore, the \( b(p) \) coefficient is determined by the following formula

\[ b(p) = \frac{C}{p^2} + \frac{A - 1}{p} \]  

As the \( p \) parameter increases, the \( b(p) \) coefficient tends to zero, and the loading surface tends to the limiting yield surface. For variables

\[ S_\sigma = \frac{\sigma_1 + \sigma_3}{2} \quad and \quad \tau_{max} = \frac{\sigma_1 - \sigma_3}{2} \]

The loading surfaces corresponding to the condition (9) are shown in Figure 1. According to [4-20], the \( p \) parameter can be written as a porosity function:
After the formula is analyzed (3), the construction of a soil plasticity model can be considered, the parameters of which are determined in standard tests.

$$p(e) = \frac{2C}{1 - A + \sqrt{(A - 1)^2 + \frac{A^2}{f(e)/C} - 1}}$$

After the formula is analyzed (3), the construction of a soil plasticity model can be considered, the parameters of which are determined in standard tests.

Figure 1. Traces of loading surfaces: 1 - limiting yield surface; 2,3,4 - loading surfaces at \( p = 0.2, \) 0.4, 0.6 MPa, respectively.

The loading surfaces obtained from various sources are not very different, we can expect that the corresponding solutions will be similar.

If the load applied to the plastic-rigid body does not exceed the carrying capacity corresponding to the given porosity, the body remains in a rigid (elastic) state. Otherwise, a velocity field appears in the body, a slow yielding flow begins. If the volumetric strain rate turns out to be positive, i.e. the porosity increases, it will mean that the ultimate resistance is exceeded. If all the volumetric strain rates turn out to be negative, then the porosity decreases, the yielding flow will necessarily be limited and damped, if necessary. The body goes into a new state of equilibrium. Hardening takes place, the applied load can be increased.

2. Planar deformation
Referring to the equations of equilibrium in the following form:

$$\begin{align*}
\frac{\partial \sigma_1}{\partial u} &= (\sigma_3 - \sigma_1) \frac{\partial \theta}{\partial v} \\
\frac{\partial \sigma_3}{\partial v} &= (\sigma_3 - \sigma_1) \frac{\partial \theta}{\partial u}
\end{align*}$$

(4)
where $\theta$ is the angle between the first principal direction and the x axis, $\frac{\partial \theta}{\partial u}, \frac{\partial \theta}{\partial v}$ - derivatives of the first and second main directions.

The equation of the loading surface is as follows:

$$\sigma_3 = -C + A\sigma_1 + b(e)\sigma_1^2$$  \hspace{1cm} (5)

and the yield function, respectively

$$f = -\sigma_3 - C + A\sigma_1 + b(e)\sigma_1^2$$  \hspace{1cm} (6)

In the formulas (5) and (6), $e$ is the porosity coefficient.

The normal yield rule

$$\dot{\varepsilon}_i = \lambda \frac{\partial f}{\partial \sigma_i}$$

where $i = 1,2,3$, for the yield function (6) is written as follows:

$$\dot{\varepsilon}_1 = \lambda (A + 2b\sigma_1), \ \dot{\varepsilon}_2 = 0, \ \dot{\varepsilon}_2 = -\lambda$$  \hspace{1cm} (7)

The results of the compression tests are represented by the logarithmic Terzaghi law:

$$e = \Gamma - 1 - \mu \ln \frac{P_i}{P_{i0}}$$

where $P_i$ is the compression pressure, $\Gamma, \mu$ - constants, and $P_{i0} = 0.1$ MPa, if the compression pressure is measured in MPa.

In this model, according to the formulas, the compression pressure is related to the $b$ parameter by the dependence

$$P_i = \frac{A^2}{4b} + C$$  \hspace{1cm} (8)

Taking into account (8) the Terzaghi law will be rewritten as follows:

$$e = \Gamma - 1 - \mu \ln \frac{\frac{A^2}{4b} + C}{P_{i0}}$$  \hspace{1cm} (9)

The equation (9) determines the dependence of the $b$ parameter on the porosity coefficient $e$.

The system of equations (1) and (9) connects the loading surfaces with the soil porosity coefficient.

Figure 1 shows the loading surfaces based on the above-mentioned conclusions within the framework of our plastic-rigid. At each point on the loading surface the soil is in the same physical state, i.e. bears the same porosity coefficient.

In order to clarify the dependencies between the stress state of the soil and the porosity coefficient, soil samples were tested using triaxial compression. This is facilitated by the following advantages:

- loading can be carried out on different trajectories;
- it is possible to load the sample with various combinations of $\sigma_1 \sigma_3$;
- at each moment of time the radial and vertical deformations of the sample are tracked and, hence, changes in the volume of the sample are tracked as well.

3. Results of experimental studies
The studies were carried out for sandy and clay-bearing soils. The authors developed a theory for soils that transfer to the limiting state with a decreasing volume. During the experiments with sandy soil while increasing the load, sections with an increase in the porosity coefficient were observed, so a detailed analysis for the sandy soil was not performed.

While carrying out the experiments, we used a loamy, soft, unbranched structure with the following characteristics (Table 1).

| №  | Description                                  | Value  |
|----|---------------------------------------------|--------|
| 1  | Density of soil particles $\rho_s$, kN/m³  | 27     |
| 2  | Soil density in the natural state $\rho_0$, kN/m³ | 19.27 |
| 3  | Density of dry soil $\rho_d$, kN/m³         | 15.3   |
| 4  | Soil moisture content in the natural state W | 0.257  |
| 5  | Soil moisture at the limit of plasticity W_p | 0.213  |
| 6  | Soil moisture content at the yield point W_l  | 0.371  |
| 7  | The plasticity number I_p                   | 0.158  |
| 8  | Flow index I_l                              | 0.278  |
| 9  | Porosity coefficient e                      | 0.765  |

As the load in the soil sample increases, the porosity coefficient must decrease to a certain, critical value after which the sample breaks down, it goes into a critical state, a state of unlimited yielding flow.

So, for each simple loading path the porosity coefficient will decrease from the initial to the critical. It should be noted that a simple loading trajectory will cross all loading surfaces once.

The following solution for the purpose at hand:
- perform soil tests on a triaxial compression device under different loading paths;
- construct loading trajectories in the axes $\sigma$ - $\tau$;
- mark the points with the same porosity coefficient on the loading paths;
- build loading surfaces. Compare them with the theoretical ones.

The software of the device contains the GOST algorithm for sample loading, loading takes place in the following sequence:
- compression of the sample;
- load shedding;
- compression of the sample;
- with constant lateral compression, the vertical load increases in steps as a percentage of lateral reduction.

This loading path is shown in the graph of figure 2.

![Figure 2. Loading path.](image-url)
In figure 2 the section 0-1 corresponds to the stage of compression of the sample by uniform pressure. At the point 1 the soil is in the state of hydrostatic reduction when $\sigma_1 = \sigma_3$. The point 2 corresponds to the state of failure of the sample, its transition to the state of yielding flow.

Complex loading is implemented in the device. At the point 2 near the ground the porosity coefficient may significantly change (depending on the value of the lateral compression pressure).

A series of experiments with lateral pressures of 100, 200, 300, 400, 500 kPa were performed. For each stage of application of the vertical load in the section 0-2 the porosity coefficients were calculated. According to the tests, it was possible to construct loading surfaces that approach the yield surface as the load increases (figure 3).

4. Conclusion

It should be noted that, while loading the sample from a certain moment, the porosity coefficient ceased to change, that is, only the change in the shape of the sample occurred, and this moment can be considered as the transition of the soil into a state of yielding flow.

In order to clarify the results of this task, it is necessary to carry out stabilometric tests under the conditions of simple loading, i.e. where the vertical pressure would be proportional to the compression pressure $\sigma_3 = k\sigma_1$.

Within the study the attempts were made to confirm the model given in the beginning of the article by laboratory tests. After the analysis of the results, theoretical assumptions were confirmed.

![Figure 3. Results of stabilometric tests.](image)

Lines 7 - isolines with the same porosity coefficient. Lines 1,2,3,4,5 - loading paths. Line 6 is the limit line.

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