Parametric Sequential Method for MRI-based Wall Shear Stress Quantification

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Abstract—Wall shear stress (WSS) has been suggested as a potential biomarker in various cardiovascular diseases and it can be estimated from phase-contrast Magnetic Resonance Imaging (PC-MRI) velocity measurements. We present a new parametric sequential method for MRI-based WSS quantification consisting of a geometry identification and a subsequent approximation of the velocity field. The new method was validated on well controlled, high resolution in vitro measurements of turbulent stationary flows and physiological pulsatile flows in phantoms. Basic applicability to in vivo was demonstrated using 2D PC-MRI data of the ascending aorta of three volunteers.

Index Terms—Flow MRI, MR velocimetry, phase-contrast MRI, wall shear stress, blood flow

I. INTRODUCTION

PATHOLOGICAL mechanisms leading to cardiovascular diseases may be discovered by studying blood flow patterns in the cardiovascular system [1]. Various parameters, e.g. volume flow rate, peak velocity, or wall shear stress, are examined with respect to cardiovascular diseases [2]. Wall shear stress (WSS) is the tangential force per unit surface area exerted by flowing blood on the vessel wall [3]. WSS is sensed by endothelial cells located at the blood-vessel wall interface [4] and may serve as a potential biomarker for atherosclerosis, aortic stenosis, aneurysms, bicuspid aortic valves [5]–[7]. See also a review [8] and references therein.

Blood – a shear-thinning non-Newtonian fluid [9] – behaves as an incompressible Newtonian fluid with constant viscosity in larger arteries at high shear rates [10]. In 2D case the WSS at the wall \( \partial \Omega \) is defined as

\[
\tau_w = -\mu \frac{\partial u}{\partial n},
\]

where \( \mu \) is the dynamic viscosity, \( u \) is the velocity component parallel to \( \partial \Omega \), \( n \) is the outer normal to \( \partial \Omega \) [3]. Phase-contrast Magnetic Resonance Imaging (PC-MRI) [11] is a non-invasive method for measuring blood flow velocity \( u \). MRI-based WSS quantification typically requires two steps: 1) the identification of the boundary \( \partial \Omega \), 2) the computation of \( \partial u / \partial n \) at \( \partial \Omega \).

Previous WSS quantification methods locally fitted linear [12]–[14] or parabolic [14] polynomials to the velocity data near the vessel boundary. This approach was generalized in the sectored paraboloid method [15] to the 3D case and using linear interpolation to handle 4D MRI data [16]. These methods relied on sufficiently high spatial resolution and moderate noise near the boundary.

More recently global velocity approximation methods have been developed. An important milestone was the introduction of a method based on B-spline interpolation of the Gaussian filtered measurement data [17]. This method [17] was used in [18]–[20], where the aortic vessel was manually delineated for each 2D analysis plane and WSS estimation was based on a direct interpolation of the local velocity derivative on the segmented vessel contour. In [21] peak-systolic WSS vectors were calculated by fitting the 3D velocity data with B-splines surfaces and computing velocity derivatives on the vessel lumen segmented using the method [17].

In [22] a finite element mesh was introduced, nodal velocity values were derived by cubic interpolation and WSS was computed using the derivative of the finite element function. In [23] the velocity reconstruction using smoothing splines was performed in interior voxels only and the resulting gradient was

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extrapolated to the boundary. A similar approach [24] used Sobel filters [25] (a combination of numerical differentiation and orthogonal smoothing) to estimate the velocity gradient in the domain interior, afterwards the extrapolated gradient was evaluated at the boundary. See [8] and references therein on MRI-based WSS quantification methods.

Further research has been dedicated to pre- and post-processing techniques. Pre-processing involves methods for reducing systematic noise and background phase distortions in MRI data. Post-processing becomes increasingly important due to an increasing trend to process 4D flow MRI data. For instance, centerline [26] and – for more complicated flow geometries - Laplacian based [27] techniques were developed to define an axial direction and analyze axial and circumferential WSS components.

The assessment of the accuracy of MRI-based WSS estimators is difficult due to the lack of reference data for \textit{in vivo} [2]. WSS values obtained from the 4D flow \textit{in vivo} data with the highest available spatio-temporal resolution might serve as reference [28]. For validation, \textit{in vitro} measurements of controlled flow regimes, Computational Fluid Dynamics (CFD) simulations and Laser Doppler Velocimetry (LDV) experiments [29], [30] can be used. For some flow regimes, analytical solutions or known formulas exist, e.g. the Womersley analytical solution for laminar pulsatile pipe flows [29] and friction factor formula for fully developed turbulent stationary flows in straight pipes [8], [31]. The generic nonlinear regression method for MRI-based WSS quantification [8] automatized the graphical Clauer plot method for fully developed turbulent stationary pipe flows [31], [32].

The accuracy of MR-based WSS estimators depends on the PC-MR acquisition parameters, the boundary identification accuracy, and the accuracy of the velocity reconstruction [2], [14], [23], [28], [33]. PC-MRI-based WSS quantification methods tend to underestimate the WSS values [14], [17], [28], [34]. See [8] and references therein on accuracy and validation of MR-based WSS quantification methods.

The flow velocity reconstruction from MRI data and, subsequent estimation of WSS has three major difficulties.

A first difficulty arises from the flow velocity character. Since the blood velocity is zero outside the vessel and the shear rate at the wall does not vanish, the velocity exhibits a kink at the vessel boundary. This makes an accurate reconstruction of the velocity near the boundary, and in particular its derivatives, difficult by standard methods. For instance, a spline interpolation of such a velocity profile with a kink [17] exhibits the Gibb’s phenomenon [35]: the approximation of the velocity derivative that features a jump at the kink of the velocity profile, contains a constant error, even in the case of noise-free and infinitely highly resolved data. Therefore, the field of view should be decomposed into a flow region and an exterior domain. The velocity is smooth in the flow domain, in principle allowing an accurate representation using standard approximation methods as it is done in the polynomial fitting methods [12]–[16] as well as in novel approaches [23], [24], [36]. Therefore, a reliable identification of the flow domain from MRI data is crucial.

The second difficulty is the observed dependence on spatial resolution of the underlying velocity data. The limited spatial resolution is the main factor for the variability of MR-derived WSS [2], [14], [17], [23], [28], [33], [37]. The correct near-wall behavior must be appropriately resolved in the data, especially in the case where the velocity profile changes its character in a thin boundary layer. The measurement process incorporates a spatial and temporal local averaging. Most approaches interpret the velocity data as nodal values at voxel centers. However, the voxel value is substantially affected by the flow velocity in a local neighborhood around the voxel center. Furthermore, partial volume effects impact the data in the critical near-wall region. Finally, MRI is not able to capture instantaneous velocity fluctuations or oscillations of the vessel geometry. An accurate modeling of the data acquisition process would minimize the data error and improves stability of the estimates against considerably low data resolution. The temporal resolution may introduce a significant underestimation of WSS in special situations like the emerging of vortex rings in an aneurysm [30]. In the setups investigated in this work temporal resolution has only a minor impact on the WSS estimates [28], [33].

The third difficulty is the evaluation of velocity derivatives at the boundary. The data noise on the boundary is relatively high due to several reasons. The no-slip boundary condition and the continuity of the velocity field lead to low velocities near the boundary. Therefore, the fluid near the boundary is magnetically saturated and its signal-to-noise ratio (SNR) is reduced. Moreover, the velocity encoding (VENC) parameter, that is typically chosen according to the expected maximum velocity to avoid phase aliasing [11], introduces relatively high phase noise near the boundary. The data noise becomes further amplified by numerical differentiation [38]. Therefore, velocity derivatives must be evaluated in a stable manner. In most approaches, regularization by discretization is applied, where the amount of regularization depends on the data resolution only [12]–[16], [24]. This is problematic, since the amount of regularization should be chosen respective to the current flow situation and data fidelity [38] as it was done with the controlled regularization by Gaussian filtering in [17] or smoothing splines in [23].

The proposed method offers an approach to handle all three difficulties appropriately in order to achieve considerable improvements in WSS quantification.

II. METHODS

Key ideas
Our method uses 2D-PC-MRI data given as magnitude and axial velocity values on a regular grid consisting of \( N \) voxels to reconstruct both the flow domain \( \Omega \), in particular its outer normal \( \mathbf{n} \), as well as the velocity \( \mathbf{u} \).

The method has been designed to address the three main challenges:
• The global velocity is non-smooth, exhibiting a kink at the vessel wall.
• Dependence on data resolution.
• Difficulty in evaluation of velocity derivatives at the boundary.

We use regularization methods [39] to deal with the ill-posedness of the problem indicated by the last two points. To overcome the difficulty with the non-smooth global velocity, we implement a modular sequential approach: first the magnitude data is used to reconstruct the flow geometry, afterwards the identified geometry is used to reconstruct the flow velocity inside the domain. Finally, a WSS estimate is computed. A mathematical analysis of the method is given in [40] and provides error estimates in terms of the measurement error.

Preprocessing of the magnitude data
The geometry registration method requires normalized magnitude data \( \tilde{m}_i = |V_i \cap \Omega|/|V_i| \), which is equal to the fraction of the voxel \( V_i \) covered by the flow domain \( \Omega \). To extract these values from the original data, we model the magnitude \( m_i \) by

\[
m_i = \int_{V_i} \rho(x) dx,
\]

where \( \rho \) is the signal amplitude depending amongst other things on the proton density and the relaxation times. Assuming the signal amplitude in the flow domain \( \Omega \) and the exterior domain to be locally nearly constant, we arrive at

\[
m_i = \tilde{m}_i m_{f,i} + (1 - \tilde{m}_i) m_{e,i},
\]

with magnitude level \( m_{f,i} \) in the fluid and \( m_{e,i} \) in the exterior domain around \( V_i \). Thus, after selecting the characteristic magnitude levels, the normalized data is obtained as

\[
\tilde{m}_i = \min \left\{ 1, \max \left\{ 0, \frac{m_i - m_{e,i}}{m_{f,i} - m_{e,i}} \right\} \right\}. \tag{1}
\]

In the investigated *in vitro* cases, \( m_{f,i} \) and \( m_{e,i} \) can be chosen independently of \( V_i \) by the histogram of the magnitude [40]. For *in vivo* cases, in a first step, a region growing algorithm is applied. For every voxel a surrounding patch is investigated: if all voxels in the patch are picked by the region growing algorithm, the former voxel is designated an interior voxel. If some voxels in the patch are picked, the voxel is called a boundary voxel. Otherwise it is an exterior voxel. For interior and exterior voxels, \( \tilde{m}_i \) is set to 1 respective 0. For boundary voxels, the normalization (1) is applied with the characteristic magnitude levels \( m_{f,i} \) and \( m_{e,i} \), selected by the statistics of the nearest interior respective exterior voxel magnitude data. The local magnitude contrast \( c_i = m_{f,i} - m_{e,i} \) is used as an indicator for data fidelity.

**Geometry registration**
Given its center of gravity \( x_0 \) computed from the normalized magnitude \( \tilde{m}_i \), the flow domain can be parametrically represented using a radius function \( R \) as

\[
\Omega(R) = \{ x \in \mathbb{R}^2 : |x - x_0| < R(\varphi(x - x_0)) \},
\]

where \( \varphi(x) \) is the angle between first axis and the vector \( x \). To determine the radius function \( R \), such that \( \Omega(R) \) is the flow domain, we introduce the virtual measurement operator

\[
F_{geo}(R)_i = |V_i \cap \Omega(R)|/|V_i|
\]

and the Tikhonov functional

\[
J_{geo}(R) := \frac{1}{N} \sum_{i=1}^{N} \tilde{c}_i |F_{geo}(R)_i - \tilde{m}_i|^2 + \alpha \|R''\|_L^2(0,2\pi)^2. \tag{2}
\]

The first term forces accordance to the data, where the data fidelity indicator \( \tilde{c}_i = c_i/\max(c_i) \) is the normalized magnitude contrast \( c_i = m_{f,i} - m_{e,i} \) (\( \tilde{c}_i = 1 \) for exterior and boundary voxels and *in vitro* cases). The second term ensures the smoothness of \( \Omega(R) \). \( R \) is computed by minimizing \( J_{geo} \) over the space of functions of type

\[
R(\varphi) = b_0 + \sum_{k=1}^{n} a_k \sin(k\varphi) + b_k \cos(k\varphi).
\]

In practice, the forward operator has to be smoothed to enable gradient methods for the minimization. The regularization parameter \( \alpha \) (2) to be chosen carefully (here \( \alpha = 0.1 \) was selected) and \( n \) was chosen to be sufficiently large avoiding discretization errors. For details we refer to [40].

**Velocity approximation**
Contrary to the global velocity field, the velocity field restricted to the flow domain \( \Omega \) is smooth and can be approximated by standard techniques. The parametrization \( R \) and \( x_0 \) of the identified geometry defines a diffeomorphism \( \phi \), mapping the unit disk \( B = \{ x \in \mathbb{R}^2 : \|x\| < 1 \} \) to the flow domain \( \Omega \). With \( R_0 = \min_{\theta \in [0,2\pi]} R(\varphi) \) the transformation \( \phi \) is given by

\[
\phi : \begin{pmatrix} r \cos(\varphi) \\ r \sin(\varphi) \end{pmatrix} \mapsto x_0 + (R_0 r^2 + (R(\varphi) - R_0) r^4) \begin{pmatrix} \cos(\varphi) \\ \sin(\varphi) \end{pmatrix}.
\]

Every arbitrary function \( v : B \to \mathbb{R}^n \) defined on the unit disk \( B \) can be associated with a function \( u : \Omega \to \mathbb{R}^n \) on the flow domain by \( u(x) = v(\phi^{-1}(x)) \). Modelling the velocity measurement by the averaging
\[ u_i = \frac{1}{|V_i \cap \Omega|} \int_{V_i \cap \Omega} u(x) \, dx \]

leads to the forward operator

\[ F_{\text{flow}}(v)_i = \frac{1}{|V_i \cap \Omega|} \int_{V_i \cap \Omega} v(\phi^{-1}(x)) \, dx. \]

The associated Tikhonov functional reads

\[ J_u(v) = \frac{1}{|\Omega|} \sum_{|V_i \cap \Omega| > 0} |V_i \cap \Omega| \left\| F_{\text{flow}}(v)_i - u_i \right\|^2 + \beta \| \Delta v \|^2_{L^2(B)} \]

(3)

\( v \), and therefore, the velocity approximation \( u = v \circ \phi^{-1} \), is obtained by minimizing the Tikhonov functional over the truncated spectral space

\[ V_M = \text{span} \left\{ v \in L^2(B) \mid \begin{array}{l} -\Delta v = \lambda v \quad \text{in} \ B \\ v = 0 \quad \text{on} \ \partial B \\ \lambda \leq M \end{array} \right\}. \]

(4)

Note, that the approximation incorporates the no-slip condition. A typical velocity exhibiting a boundary layer shows high derivatives in normal direction and small derivatives in the angular direction. The former requires a low regularization parameter (here \( \beta = 10^{-6} \)) not to oversmooth the boundary layers. A larger smoothing in the angular direction is applied by discretization: using polar coordinates the basis functions in (4) are of the type \( v(r, \phi) = \Re(e^{i \phi} r^l) \) for \( l \leq N_\phi \) and high-oscillatory for \( k > N_\phi \). Low-oscillatory basis functions are included up to an eigenvalue of \( \lambda \leq M_1 \), high-oscillatory basis functions only up to \( \lambda \leq M_2 \). In the in vivo cases the domain is a circle and the flow is nearly symmetric, therefore we choose \( N_\phi = 0 \), \( M_1 = 200 \) and \( M_2 = 20 \), the latter to compensate for small deviations from the symmetric velocity. This idealization does not hold true in the in vivo situation, therefore we choose \( N_\phi = 4 \), \( M_1 = 200 \) and \( M_2 = 0 \).

### Evaluation of wall shear stress

The unit outer normal \( n \) and the gradient \( \nabla u \) of the velocity field are analytically computable at any point from the known geometry parametrization \( x_0 \) and \( R \), the domain transformation \( \phi^{-1} \) and the velocity approximation \( \psi \) respective their derivatives in a stable manner [40]. At the boundary point \( x \) with angle \( \phi = \phi(x - x_0) \) the WSS estimate is given by

\[ \tau_w(\phi) = -\mu n(\phi)^T (\nabla u)(x). \]

For nonstationary flows we also estimate the oscillatory shear index (OSI) [42] – a spatially distributed quantity measuring oscillations of the WSS over time at the boundary parametrized by the angle \( \phi \) in our approach:

\[
\text{OSI}(\phi) = \frac{1}{2} \left( 1 - \frac{\int_0^T \tau_w(\phi, t) \, dt}{\int_0^T |\tau_w(\phi, t)| \, dt} \right).
\]

The absolute physical position of boundary may slightly move over time due to temporal variations of the geometry. Note, that only measurements of the axial velocity were conducted and, therefore, incorporated into the OSI computation, which may lead to a distorted OSI especially in vivo, were the circumferential component of the WSS is not negligible.

### III. Results

#### Validation in vitro

For validation, two in vitro flow setups were used. The first setup was a turbulent stationary flow in a straight circular pipe. The WSS values computed using the friction factor formula [8], [31] and the generic nonlinear regression method [31] served as reference values. The second setup – two laminar physiological pulsatile flows in a straight circular pipe – mimicked flow conditions in the human aorta [29]. The WSS values computed from the LDV experiments [29] and the Womersley analytical solution [29] served here as a reference.

In addition, the WSS values were also quantified with a MATLAB-based software flow-tool developed in Medical Center – University of Freiburg (Freiburg, Germany). Flow-tool is an implementation of the B-spline interpolation method [17], which is widely used as the current standard. Since in the studied flow regimes the velocity gradients are large near the boundary and the velocity has a kink at the boundary, Gaussian smoothing – even with a small radius – leads to severe underestimation of WSS [17]. Therefore, flow-tool was used without Gaussian smoothing. It is important to note, that as we quantify mean WSS values, smoothing is implicitly applied in the tangential direction. Instead of the flow-tool embedded manual geometry identification, our new automatic geometry identification was used. Without Gaussian filtering in flow-tool it is important to ensure that the velocity derivative is evaluated inside the flow domain. Therefore, prior to using with flow-tool, the automatically identified geometry was shrunk by an empirically defined value equal to 7/12 of the in-plane voxel size.

2D PC-MRI measurements were done using a 3T whole-body scanner (MAGNETOM Prisma, Siemens Healthineers, Erlangen, Germany). The detailed description of the experimental setup is given in [29]. The flow setups were carefully controlled to ensure the reproducibility of the experiments.

A conventional phase-contrast MRI sequence was used, based on a spoiled 2D gradient echo sequence with bipolar velocity encoding along the slice selection direction. 2D PC-MR images were acquired in a plane oriented perpendicular to the pipe axis (i.e. axial slice orientation) for different in-plane resolutions with velocity encoding along the through-plane direction.
**Turbulent stationary flow**

A turbulent stationary flow in a straight pipe was provided at Reynolds numbers $Re = 8060, 5370, 3000$. VENC = 0.25 m/s was chosen for all $Re$ in order to better resolve the velocities close to the vessel boundary. The images were acquired for nine different in-plane resolutions (range: 0.30 mm x 0.30 mm – 1.00 mm x 1.00 mm). Other scan parameters were TR/TE = 17.8-18.6/5.66-5.68 ms, FOV was 96 mm x 96 mm, slice thickness was 3 mm, flip angle was $5^\circ$.

The geometry was identified accurately (relative approximation errors of 0.01-0.08% for turbulent stationary flow with $Re = 5370$), i.e. sub-voxel resolution is achieved. As observed from Fig. 1, in contrast to *flow-tool* the estimates of the parametric sequential method deviate only slightly from the reference values and reveal no dependence on the data resolution. The relative deviation between the values obtained by the parametric sequential method and the friction factor formula was 4.2%±2.5%. The deviation between the two reference values was 2.1%±1.0% and between *flow-tool* and the friction factor formula 26.3%±17.3%.

![Fig. 1. MR-derived WSS estimates by the parametric sequential method, *flow-tool*, the friction factor formula [8], [31] and the generic nonlinear regression method [31] for turbulent stationary flows with (a) $Re = 8060$, (b) $Re = 5370$, (c) $Re = 3000$ and different in-plane resolutions.](image.png)

**Laminar nonstationary (physiological pulsatile) flows**

Two physiological pulsatile flows in a straight pipe with maximal Reynolds numbers $Re_{\text{max}} = 3952$ (resting state) and $Re_{\text{max}} = 7651$ (exercising state) mimicked realistic flow conditions of the human aorta [29]. A Womersley number $Wo = 20.3$ was taken as characteristic for the ascending aorta [43]. The images were acquired for four different in-plane resolutions (range: 0.30 mm x 0.30 mm – 1.56 mm x 1.56 mm). Other scan parameters for resting state were VENC = 0.15 m/s, TR/TE = 70.4-72.0/5.7795-5.68 ms, FOV was 32 mm x 32 mm – 96 mm x 96 mm, slice thickness was 3 mm, flip angle was $10^\circ$. The scan parameters for exercising state were VENC = 0.25 m/s, TR/TE = 64.4-68.0/5.24-5.25 ms, FOV was 32 mm x 32 mm – 160 mm x 160 mm, slice thickness was 3 mm, flip angle was $10^\circ$.

Again, the method shows low dependence on the data resolution in contrast to *flow-tool* (Fig. 2). In general, the estimates are in good agreement with the reference (Fig. 3). However, the method reveals a phase difference to the reference values during the phase around the second positive WSS peak. One possible reason is that the inertia driven velocities in the middle of the pipe influence the WSS estimate. These velocities lag behind the velocities in the boundary layer, driven by viscous forces and follow the changing pressure gradient faster. Nevertheless, this phase difference does not seriously affect important quantities such as peak WSS and OSI, which is in good agreement with the reference (see Table 1).

![Fig. 2. MR-derived WSS estimates by the parametric sequential method and *flow-tool* for the laminar physiological pulsatile flows with (a) $Re_{\text{max}} = 3952$, (b) $Re_{\text{max}} = 7651$; resolutions: black lines – 0.30 mm x 0.30 mm, blue lines – 0.52 mm x 0.52 mm, green lines – 0.78 mm x 0.78 mm, red lines – 1.56 mm x 1.56 mm.](image.png)

![Fig. 3. MR-derived WSS estimates by the parametric sequential method and *flow-tool* for the in-plane resolution 0.3 mm x 0.3 mm, the WSS values obtained from LDV [29], the Womersley analytical solution [29] for the laminar physiological pulsatile flows with a) $Re_{\text{max}} = 3952$ and b) $Re_{\text{max}} = 7651$.](image.png)
**Influence of regularization parameters**

As expected, the method is sensitive to the choice of the regularization parameter $\beta$ (3). The discretization of the velocity space affects the estimates only for coarse resolutions ($M_t = 20$), since the results with the different discretizations of the velocity space by $M_t = 100$, $M_t = 200$, and $M_t = 300$ nearly coincide (Fig. 4).

**Application in vivo**

The parametric sequential method was applied to *in vivo* 2D PC-MRI data of the ascending aorta acquired for three healthy volunteers (men, 42, 29 and 20 years old) using ECG-triggering and breathing navigator gating. A scan plane was placed axially in the ascending aorta (Fig. 5) and the images were acquired for four different in-plane resolutions (range: $0.30 \text{ mm} \times 0.30 \text{ mm} – 1.56 \text{ mm} \times 1.56 \text{ mm}$). The VENC was $1.5 \text{ m/s}$, TR/TE were $21.6\sim22.8/2.9 \text{ ms}$, FOV was $500 \text{ mm} \times 250 \text{ mm}$, slice thickness was $5 \text{ mm}$, flip angle was $15^\circ$.

**Table I. Oscillatory shear index (OSI)**

| in-plane resolution | $Re_{\text{max}} = 3952$ | $Re_{\text{max}} = 7651$ |
|---------------------|-----------------|-----------------|
| $0.30 \text{ mm} \times 0.30 \text{ mm}$ | 0.4614 | 0.4580 |
| $0.52 \text{ mm} \times 0.52 \text{ mm}$ | 0.4535 | 0.4686 |
| $0.78 \text{ mm} \times 0.78 \text{ mm}$ | 0.4655 | 0.4649 |
| $1.56 \text{ mm} \times 1.56 \text{ mm}$ | 0.4638 | 0.4636 |

Volunteers 1, 2 and 3 (a)–(c) were scanned using $\beta = 100$ (a), $\beta = 200$ (b) and $\beta = 300$ (c) and different values of $M_t$ (d), discretization parameter $M_t$.

Fig. 4. MR-derived WSS estimates by the parametric sequential method for physiological pulsatile flow with $Re_{\text{max}} = 7651$, in-plane resolution $0.30 \text{ mm} \times 0.30 \text{ mm}$ and different values of (a) Tikhonov regularization parameter $\beta$, (b) discretization parameter $M_t$.

In vivo, the vessel boundary is optically well identified for various image contrasts and SNR that are present during the cardiac cycle (Fig. 6).

![Fig. 6. Magnitude images of the ascending aorta with the vessel boundary identified by the parametric sequential method for: (a) time step 6, (b) time step 17, (c) time step 23, (d) time step 40.](image)

The WSS estimates of the parametric sequential method show little dependence on the spatial resolution and are substantially larger than the values provided by flow-too (Fig. 7). The order of magnitude of the former estimates is in accordance with the values reported in the literature: $3.106 \pm 0.479/3.385 \pm 1.553 \text{ Pa}$ [44] and $2.23\pm1.04 \text{ Pa}$ [37].

![Fig. 7. MR-derived WSS estimates by the parametric sequential method and flow-too for in vivo MRI measurements for three volunteers; resolutions: black lines – $0.30 \text{ mm} \times 0.30 \text{ mm}$, blue lines – $0.52 \text{ mm} \times 0.52 \text{ mm}$, green lines – $0.78 \text{ mm} \times 0.78 \text{ mm}$, red lines – $1.56 \text{ mm} \times 1.56 \text{ mm}$.](image)
IV. Discussion

In this work, a new parametric sequential method for MRI-based wall shear stress quantification was presented. The method was validated on well controlled in vitro measurements and applied to in vivo data of the ascending aorta of three volunteers.

Due to the presence of boundary layers, both investigated in vitro setups are challenging for generic WSS estimators and relevant for the in vivo situation. The parametric sequential method copes well with both in vitro cases. The resulting WSS estimates are robust against low data resolution and show low deviation from the reference values. For the physiological flows, the important quantities peak WSS and OSI are estimated especially well. Although the results reveal high sensitivity on the regularization parameter, the good performance of the parametric sequential method with fixed regularization parameter $\beta = 10^{-6}$ in all setups validates it as a promising tool to estimate WSS in vivo.

Based on comparison to the literature, the method in fact provides reliable estimates of WSS in the ascending aorta.

During our investigations the accurate identification of the flow domain turned out to be crucial for reasonable WSS estimates, as it is expected [40]. This does not affect the estimates in vitro, due to the highly accurate geometry identification, but requires careful adjustments of the region growing parameters of the magnitude pre-processing in vivo. Furthermore, the accuracy of the velocity reconstruction may be further enhanced by including fluid dynamical models in the smoothing process [45].

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REFERENCES

[1] V. P. Kamphuis, J. J. M. Westenberg, R. L. F. van der Palen, N. A. Blom, A. de Roos, R. van der Geest, M. S. M. Elbaz, and A. A. W. Roest, “Unravelling cardiovascular disease using four dimensional flow cardiovascular magnetic resonance,” Int. J. Cardiovasc. Imaging, vol. 33, no. 7, pp. 1069–1078, 2017.

[2] P. Dyverfeldt, M. Bissell, A. J. Barker, A. F. Bolger, C.-J. Carlhäll, T. Ebbers, C. J. Francios, A. Frydrychowicz, J. Geiger, D. Giese, M. D. Hope, P. J. Kilner, S. Kozerke, S. Myerson, S. Neubaeyer, O. Wieben, and M. Markl, “4D flow cardiovascular magnetic resonance consensus statement,” J. Cardiovasc. Magn. Reson., vol. 17, no. 1, pp. 72–91, Aug. 2015.

[3] Z. U. A. Warsi, Fluid dynamics: theoretical and computational approaches, 3rd ed. CRC Press, 2005.

[4] S. Lehoux and A. Tedgui, “Cellular mechanics and gene expression in blood vessels,” Cardiovasc. Biomech., vol. 36, no. 5, pp. 631–643, May 2003.
interpolation and for solving the Poisson equation in a disk: Zernike

J. P. Boyd and F. Yu

and wall shear stress from magnetic resonance images,” Inverse Probl.,

H. Egger and G. Teschner, “On the stable estimation of flow geometry

45.

H. W. Engl, M. Hanke, and A. Neub

and wall shear stress measured using four

F. M. Callaghan and S. M. Grieve, “Normal patterns of thoracic aortic

Magn. Reson. Med., vol. 80, no. 2, pp. 748–755, Jul. 2018.

[22]  J. Sotelo, J. Urbina, J. Valverde, C. Tejos, P. Irrázaval, M. E. Andia, S.

Urine, and D. E. Hurtado, “3D quantification of wall shear stress and

oscillatory shear index using a finite element method in 3D CINE PC-

MRI data of the thoracic aorta,” IEEE Trans. Med. Imaging, vol. 35, no. 6,

pp. 1475–1487, Jun. 2016.

[23]  W. V. Potters, P. Ooij, H. Marquering, E. van Bavel, and A. J. Nederveen,

“Volumetric arterial wall shear stress calculation based on cine phase

contrast MRI,” J. Magn. Reson. Imaging, vol. 41, no. 2, pp. 505–516, Feb.

2015.

[24]  F. Piatti, S. Pirola, M. Bissell, I. Nesteruk, F. Sturla, A. Della Corte, A.

Redaelli, and E. Votta, “Towards the improved quantification of in vivo

abnormal wall shear stresses in BAV-affected patients from 4D-flow imaging:

Benchmarking and application to real data,” J. Biomechanics, vol. 50, pp. 93–101, Jan. 2017.

[25]  F. Y. Shih, Image processing and pattern recognition: fundamentals and
techniques. Wiley-IEEE Press, 2010.

[26]  W. V. Potters, H. A. Marquering, E. VanBavel, and A. J. Nederveen,

“Measuring wall shear stress using velocity-encoded MRI,” Curr.

Cardiovasc. Imaging Rep., vol. 7, no. 4, p. 9257, Feb. 2014.

[27]  J. Sotelo, L. Dux-Santoy, A. Guala, J. Rodríguez-Palomares, A.

Evangelista, C. Sing-Long, J. Urbina, J. Mura, D. E. Hurtado, and S.

Urine, “3D axial and circumferential wall shear stress from 4D flow MRI
data using a finite element method and a laplacian approach,” Magn.

Reson. Med., vol. 79, no. 5, pp. 2816–2823, May 2018.

[28]  C. Montalba, J. Urbina, J. Sotelo, M. E. Andia, C. Tejos, P. Irrázaval,

D. E. Hurtado, I. Valverde, and S. Uribe, “Variability of 4D flow

parameters when subjected to changes in MRI acquisition parameters

using a realistic thoracic aortic phantom,” Magn. Reson. Med., vol. 79,

no. 4, pp. 1882–1892, Jul. 2017.

[29]  A. Bauer, S. Wegt, M. Bopp, S. Jakirlic, C. Tropea, A. J. Krafft, N.

Shokina, J. Hennig, G. Teschner, and H. Egger, “Comparison of wall

shear stress estimates obtained by laser Doppler velocimetry, magnetic

resonance imaging and numerical simulations,” Exp. Fluids, vol. 60, p.

112, 2019.

[30]  A. Bauer, M. Bopp, S. Jakirlic, C. Tropea, A. J. Krafft, N. Shokina, and J.

Hennig, “Analysis of the wall shear stress in a generic aneurysm under

pulsating and transitional flow conditions,” Exp. Fluids, vol. 61, no. 2, p.

59, Feb. 2020.

[31]  N. Shokina, A. Bauer, G. Teschner, W. B. Buchenberg, C. Tropea, H.

Egger, J. Hennig, and A. J. Krafft, “MR-based wall shear stress

measurements in fully developed turbulent flow using the Clauser plot

method,” J. Magn. Reson., vol. 305, pp. 16–33, Jan. 2019.

[32]  F. H. Clauser, “The turbulent boundary layer...” in Advances in Applied

Mechanics, vol. 4, H. L. Dryden and Th. von Kármán, Eds. Elsevier, 1956,

pp.–51.

[33]  J. Zimmermann, D. Demedts, H. Mirzaee, P. Ewert, H. Stern, C.

Meierhofer, B. Menze, and A. Hennebuth, “Wall shear stress estimation

in the aorta: Impact of wall motion, spatiotemporal resolution, and phase

noise,” J. Magn. Reson. Imaging, vol. 48, no. 3, pp. 718–728, Apr. 2018.

[34]  P. van Ooij, A. L. Powell, W. V. Potters, J. C. Carr, M. Markl, and A. J.

Barker, “Reproducibility and interobserver variability of systolic blood

flow velocity and 3D wall shear stress derived from 4D flow MRI in the

healthy aorta,” J. Magn. Reson. Imaging, vol. 43, no. 1, pp. 236–248, Jan.

2016.

[35]  D. Gottlieb and C.-W. Shu, “On the Gibbs Phenomenon and Its

Resolution,” SIAM Rev., vol. 39, no. 4, pp. 644–668, Jan. 1997.

[36]  E. M. Masutani, F. Contijoch, E. Kyubwa, J. Cheng, M. T. Alley, S.

Vasanawala, and A. Hsiao, “Volumetric segmentation-free method for

rapid visualization of vascular wall shear stress using 4D flow MRI,”

Magn. Reson. Med., vol. 80, no. 2, pp. 748–755, Aug. 2018.

[37]  F. M. Callaghan and S. M. Grieve, “Normal patterns of thoracic aortic

wall shear stress measured using four-dimensional flow MRI in a large

population,” Am. J. Physiol.-Heart Circ. Physiol., vol. 315, no. 5, pp.

H1174–H1181, Jul. 2018.

[38]  W. Hackbusch, The concept of stability in numerical mathematics, vol.

45. Springer-Verlag Berlin Heidelberg, 2014.

[39]  H. W. Engl, M. Hanke, and A. Neubauer, Regularization of inverse

problems, vol. 375. Springer Netherlands, 2000.

[40]  H. Egger and G. Teschner, “On the stable estimation of flow geometry

and wall shear stress from magnetic resonance images,” Inverse Probl.,

2019.

[41]  J. P. Boyd and F. Yu, “Comparing seven spectral methods for

interpolation and for solving the Poisson equation in a disk: Zernike

polynomials, Logan–Shepp ridge polynomials, Chebyshev–Fourier

Series, cylindrical Robert functions, Bessel–Fourier expansions, square-
to-disk conformal mapping and radial basis functions,” J. Comput. Phys.,

vol. 230, no. 4, pp. 1408–1438, Feb. 2011.

[42]  X. He and D. N. Ku, “Palsatilie flow in the human left coronary artery

bifurcation: average conditions...” J. Biomech. Eng., vol. 118, no. 1, pp.

74–82, Feb. 1996.

[43]  C. G. Caro, T. J. Pedley, R. C. Schrotter, and W. A. Seed, The Mechanics

of the Circulation, 2nd ed. Cambridge: Cambridge University Press, 2011.

[44]  R. L. F. van der Palen, A. A. W. Roest, P. J. van den Boogaard, A. de

Roos, N. A. Blom, and J. J. M. Westenberg, “Scan-rescan reproducibility

of segmental aortic wall shear stress as assessed by phase-specific

segmentation with 4D flow MRI in healthy volunteers,” MAGMA, vol.

31, no. 5, pp. 653–663, 2018.

[45]  H. Egger, T. Seitz, and C. Tropea, “Enhancement of flow measurements

using fluid-dynamic constraints,” J. Comput. Phys., vol. 344, pp. 558–574,

Sep. 2017.