Lorentz-violating gaugeon formalism for rank-2 tensor theory

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We develop a BRST symmetric gaugeon formalism for the Abelian rank-2 antisymmetric tensor field in the Lorentz breaking framework. The Lorentz breaking is achieved here by considering a proper subgroup of Lorentz group together with translation. In this scenario, the gaugeon fields together with the standard fields of the Abelian rank-2 antisymmetric tensor theory get mass. In order to develop the gaugeon formulation for this theory in VSR, we first introduce a set of dipole vector fields as a quantum gauge freedom to the action. In order to quantize the dipole vector fields, the VSR-modified gauge-fixing and corresponding ghost action are constructed as the classical action is invariant under a VSR-modified gauge transformation. Further, we present a Type I gaugeon formalism for the Abelian rank-2 antisymmetric tensor field theory in VSR. The gauge structures of Fock space constructed with the help of BRST charges are also discussed.

Keywords: Very Special Relativity, 2-form gauge theory, Gaugeon formalism.

I. OVERVIEW AND MOTIVATIONS

It is well-known that Lorentz symmetry is an essential ingredient of a highly successful model of particle physics known as Standard Model (with gauge group $SU(3) \times SU(2) \times U(1)$). The evidences of Lorentz symmetry violation at high energy are still there in the context of string/M-theory \cite{1,2} and loop quantum gravity \cite{3}. Remarkably, the violation of Lorentz symmetry at high energy helps us to renormalize the non-renormalizable interactions, for instance, two fermion-two scalar vertices and four fermion vertices \cite{4}. Two interesting theories have been proposed in search of Lorentz symmetry violation. One of them considers Planck-scale effects into account by introducing an invariant Planckian parameter into the theory of special relativity and known as Doubly Special Relativity \cite{5-9}. Another is the Very Special Relativity (VSR) which suggests that the laws of physics need not be invariant under the full Lorentz group but rather under a proper subgroup of it \cite{10}. Since energy and momentum should be conserved for theory, therefore it is mandatory to include spacetime translation together with the $SIM(2)$ group. The semi-direct product of $SIM(2)$ group with spacetime translation group is known as $ISIM(2)$ group, which an 8-dimensional subgroup of the Poincaré group. The terms of action that are invariant only under this subgroup necessarily break discrete symmetries, however the CPT symmetry is preserved and many empirical successes of special relativity are still functioned. This subgroup when supplemented by any of the discrete symmetries $P$, $T$ or $CP$ enlarges to the full Poincaré group.

The idea of breaking the Lorentz symmetry under VSR framework has been considered widely in the literature. For instance, the generalization of VSR to the physical situation (de Sitter spacetime), in which a cosmological constant is present, has been made \cite{11}. There, it is shown that if there is in addition a breaking of de Sitter symmetry, there are corrections near the endpoint of the beta decay spectrum proportional to the ratio of the square of the mass, divided by the product of its momentum times its energy. The gaugeon formalism and Sakoda’s extension of the gauge freedom of the vector field are investigated in the context of VSR and it is found that the gaugeon modes together with gauge modes become massive \cite{12}. A possibility is discussed that the symmetries underlying the standard model matter...
and gauge fields are those of Lorentz, while the event space underlying the dark matter and the dark gauge fields supports the algebraic structure underlying VSR \[13\]. The VSR is generalized to curved spacetimes and have been found that gauging the $\text{SIM}(2)$ symmetry does not, in general, provide the coupling to the gravitational background \[14\]. The generalization of VSR to noncommutative spacetimes is also studied where noncommutative parameter $\theta_{\mu\nu}$ has light-like character \[15, 16\]. The algebra followed by VSR gauge transformations is found as closed algebra and the actions coupling the gauge field to various matter fields is also constructed within VSR framework \[17\]. One possible way to use the spontaneous symmetry-breaking mechanism to give a flavor-dependent VSR mass to the gauge bosons is also discussed \[17, 18\]. An interesting VSR based description is proposed that VSR plays the same role for the field theoretic structure of dark matter as special relativity plays for standard model \[19\]. The VSR provides a new mechanism for neutrino mass which conserves lepton number without introducing additional sterile states \[20\]. The Super-Yang-Mills theory in $\text{SIM}(1)$ superspace and three dimensional Chern-Simons theory \[21\] are discussed in such schemes \[22–24\]. The superspace description of Lorentz violating $p$-form gauge theories is also presented \[25\]. The VSR generalization to axion electrodynamics is studied and it is found that the VSR effects give a health departure from the usual axion field theory \[26\]. In spite of such investigations, spontaneous breaking of Lorentz symmetry due to ghost condensation is also explored \[27–30\]. In fact, there are various models in literature which discuss BRST symmetry \[31–35\] and Lorentz breaking scheme \[36, 37\].

On the other hand, the gaugeon formulation was developed originally for the quantum electrodynamics to settle the issues of renormalization of gauge parameter \[38\]. The main idea behind the gaugeon formalism was to introduce the gaugeon fields to the action which describe the quantum gauge freedom. Since the gauge freedom at the quantum level doesn’t exist as the quantum action is defined only after fixing the gauge. By introducing the gaugeon fields, it is shown that there exists a gauge freedom even after fixing the gauge. The underlying gauge transformation is called as the $q$-number gauge transformation. In this mechanism, the occurrence of shift in gauge parameter during renormalization was addressed naturally by connecting theories in two different gauges within the same family by a $q$-number gauge transformation. The gaugeon formalism has been applied, and well studied for various gauge fields, such as, Abelian gauge fields \[39–44\], non-Abelian gauge fields \[45–48\], string theories \[49, 50\], and gravitational fields \[51, 52\]. Recently, The gaugeon formulation of the Lorentz invariant gauge-fixed and quantized dipole vector field is studied \[53\]. The field-dependent BRST transformation \[54–60\] and $q$-number gauge transformation are constructed within gaugeon formalism which help in connection of different gauge-fixing terms of the action \[61, 62\]. The VSR description of the gauge-fixed and quantized dipole vector (gaugeon) field is not studied yet. This provides us an opportunity to bridge this gap by analyzing the VSR effects on the gauge-fixed and quantized vector gaugeon field.

In this paper, we present the BRST quantization of Abelian rank-2 tensor theory in VSR framework and define physical Hilbert space under Kugo-Ojima condition. This is achieved by adding an appropriate Lorentz breaking non-local terms to the standard action. In this regard, we find that the Kalb-Ramond fields together with ghost and ghost of ghost fields get the mass but this can not be an alternative to Higgs mechanism as all the fields get same tiny mass. Even the fields become massive, the theory admits a VSR-type gauge transformation and needs quantization which is done via Faddeev-Popov trick. In order to assign different mass for different fields there must occur a spontaneous symmetry breaking. Recently, in Ref. \[53\], the massless gaugeon dipole vector model is studied for Abelian rank-2 tensor theory. However, in Lorentz violating framework, we find that in order to discuss the quantum gauge freedom for Abelian rank-2 tensor theory a dipole vector field becomes massive under VSR framework. This classical dipole vector theory also admits a VSR-modified gauge invariance. To remove the superfluous degrees of freedom, we fix the gauge and introduce corresponding ghost terms. The (non-local) Faddeev-Popov action for dipole vector field is invariant under a non-local BRST symmetry. The non-local generator for this BRST symmetry is calculated. Furthermore, we construct the gaugeon action for the Abelian rank-2 tensor theory in VSR framework, where dipole vector fields play the role of quantum (gaugeon) fields. In order to have a BRST symmetric gaugeon formalism, we introduced a massive ghost fields corresponding to gaugeon fields. The non-local BRST symmetric gaugeon action admits a non-local $q$-number gauge transformation. The form-invariance of action requires a shift in gauge parameter automatically. The gaugeon action in VSR admits various sets of BRST transformation and consequently various BRST
charges exist. The Fock spaces constructed with the help of these charges are embedded in the physical Hilbert space.

The plan of the paper is as following. In Sec. II we discuss the standard BRST quantization of Abelian rank-2 tensor theory in the VSR framework. In Sec. III we construct a classical theory for dipole vector in VSR scenario and discuss its dynamics. The BRST quantization of dipole vector field theory is provided in section IV. In Sec. V, we develop BRST symmetric gaugeon formalism for 2-form gauge theory in VSR which admits $q$-number gauge transformation. We discuss the gauge structure of Fock space for such theory in VSR. Finally, we summarize this work in section VI.

II. BRST QUANTIZATION OF ABELIAN RANK-2 TENSOR THEORY IN VSR

The main feature of VSR is violation of the Lorentz symmetry. We briefly review the relevant subgroups involved in VSR. The generators $T_1 = K_1 + L_2$ and $T_2 = K_2 - L_1$ form a group, isomorphic to the group of translations in the plane and satisfy following algebra:

$$[T_1, T_2] = 0, \quad [L_3, T_1] = T_2, \quad [L_3, T_2] = -T_2.$$  

The generator $K_3$ together with $T_1$ and $T_2$ form a group known as $HOM(2)$ and satisfy following algebra:

$$[K_3, T_1] = T_1, \quad [K_3, T_2] = T_2.$$  

The generators $K_3$ and $L_3$ together with $T_1$ and $T_2$ form $SIM(2)$ group, isomorphic to the four parameter similitude group. We study the Abelian antisymmetric rank-2 tensor gauge theory under the Lorentz breaking but $SIM(2)$-invariant setting.

Let us begin with the $D$-dimensional classical action describing Abelian antisymmetric rank-2 tensor gauge field $B_{\mu\nu}$ in VSR framework as [63]

$$S_0 = \frac{1}{12} \int d^Dx \tilde{H}^{\mu\nu\lambda} \tilde{H}_{\mu\nu\lambda},$$  

where the 3-form wiggle field strength tensor $\tilde{H}_{\lambda\mu\nu}$ is defined as

$$\tilde{H}_{\lambda\mu\nu} = \partial_\lambda B_{\nu\lambda} + \partial_\nu B_{\lambda\mu} + \partial_\mu B_{\lambda\nu} - \frac{1}{2} m^2 n^\lambda n_\mu B_\nu - \frac{1}{2} m^2 n_\nu B_\mu - \frac{1}{2} m^2 n_\lambda n_\mu B_{\nu\lambda}. $$  

Here null vector $n_\mu = (1, 0, 0, -1)$ changes only by a constant factor under boosts in the $z$-direction. Therefore, the presence of equal number of null vector in the numerator and the denominator of quotient leads to the invariance of quotient under $HOM(2)$ and $SIM(2)$. The Wiggle field strength $H_{\lambda\mu\nu}$ and, hence, action (3) remain invariant under the following VSR modified gauge transformation:

$$\delta B_{\mu\nu} = \partial_\mu \theta_\nu - \partial_\nu \theta_\mu - \frac{1}{2} m^2 n_\mu \theta_\nu + \frac{1}{2} m^2 n_\nu \theta_\mu,$$  

where $\theta_\mu$ is a vector transformation parameter. The choice of gauge parameter, $\theta_\mu = \partial_\mu \zeta - \frac{1}{2} m^2 n_\mu \zeta$, leads to $\delta B_{\mu\nu} = 0$, this implies that the gauge transformation (5) is reducible. For this theory, the further reducibility identity does not exist. In order to do BRST quantization, we need Faddev-Popov ghosts ($\bar{\rho}_\mu$ and $\rho_\mu$), ghost of ghost fields ($\phi, \bar{\phi}, d, \bar{d}$ and $\eta$) as the theory is reducible. Now, we define the
Faddeev-Popov action for Abelian rank-2 tensor field in VSR as following:

\[
S_{FP} = \int d^D x \left[ \frac{1}{12} \tilde{H}^{\mu\nu\lambda} \tilde{H}_{\mu\nu\lambda} - \partial^\mu B^\nu B_{\mu\nu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B^\nu B_{\mu\nu} - \frac{\alpha_1}{2} B^\mu B_\mu + B^\mu \partial_\mu \eta \right]
\]

where \(\alpha_1\) and \(\alpha_2\) are gauge parameters and \(B_\mu\) is a Nakanishi-Lautrup type auxiliary multiplier field. This action is invariant under the following \(SIM(2)\)-invariant off-shell nilpotent BRST transformation \((s_b^2 = 0)\):

\[
s_b B_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \rho_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \rho_\mu,
\]

\[
s_b \rho_\mu = -i \partial_\mu \phi + i \frac{m^2}{2 n \cdot \partial} n_\nu \phi, \quad s_b \phi = 0,
\]

\[
s_b \rho_\mu = i B_\mu, \quad s_b B_\mu = 0, \quad s_b \phi = 0, \quad s_b \phi = 0.
\]

This BRST transformation is important to prove renormalizability of Lorentz-violating \(SIM(2)\)-invariant tensor field theory. According to Noether’s theorem, it is easy to calculate the conserved charge corresponding to the above BRST transformation, which is given by

\[
Q_{FP} = \int d^{D-1} x \left[ B^\lambda \tilde{Q}_0 \rho_\lambda + \tilde{d} \tilde{Q}_0 \phi + (1 - \alpha_2) B_0 d \right],
\]

where \(\tilde{Q}_0 = \tilde{Q}_0 - \tilde{Q}_0\) with \(\tilde{Q}_0 = \partial_0 - \frac{1}{4} \frac{m^2}{n_0} n_0\). From the above expression, it is evident that this operator (BRST charge), which implements the BRST symmetry in the Hilbert space, is nilpotent. In order to have probabilistic interpretation, all the physical states must be projected in the positive definite Hilbert space. Now, this charge helps in defining the physical states in total Hilbert space of the theory by annihilating the physical states of the total Hilbert space as following:

\[
Q_{FP}|_{\text{phys}} = 0.
\]

Now, Euler-Lagrange equations of motion for \(B_{\mu\nu}, B_\mu\) and \(\eta\) are given, respectively, by

\[
\partial^\lambda H_{\lambda\mu\nu} + \partial_\mu B_{\nu} - \partial_\nu B_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B_{\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu B_\mu = 0,
\]

\[
\partial^\lambda B_{\lambda\mu} + \partial_\mu \eta - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\lambda B_{\lambda\mu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \eta - \alpha_1 B_\mu = 0,
\]

\[
\partial^\mu B_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B_\mu = 0.
\]

These field equations further lead to

\[
(\Box - m^2) B_\mu = 0,
\]

\[
(\Box - m^2) \eta = 0.
\]

These equations suggest that both the fields \(B_\mu\) and \(\eta\) are massive. The equation can be recognized as \(SIM(2)\)-invariant the Lorentz-like gauge condition.
III. A CLASSICAL DIPOLE VECTOR FIELD THEORY IN VSR

In this section, we discuss the classical theory of a dipole vector field $Y_{\mu}$, which can be considered as a gaugeon field, in the VSR framework. It explores a possibility of changing the gauge-fixing parameter within family under the so-called $q$-number gauge transformation, given by

$$B_{\mu\nu} \rightarrow \tilde{B}_{\mu\nu} = B_{\mu\nu} + \tau \left( \partial_\nu Y_\mu - \partial_\mu Y_\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu Y_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu Y_\mu \right). \quad (14)$$

Here $\tau$ is a bosonic transformation parameter. The gauge condition in VSR (11) changes under such a $q$-number gauge transformation as following:

$$\partial^\mu B_{\mu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu B_{\mu\nu} + \tau \left[ \Box Y_\nu - \partial^\mu \partial_\nu Y_\mu - m^2 Y_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu (\partial \cdot Y) - \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial_\nu (n \cdot Y) \right. \left. + \frac{1}{4} \frac{m^4}{(n \cdot \partial)^2} n^\mu n_\nu Y_\mu \right] - (\alpha_1 + \tau) B_{\mu\nu} + \partial_\mu \eta - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \eta = 0. \quad (15)$$

It should be noted that the gauge-fixing parameter changes to $\alpha_1 + \tau$. In order to write the classical action for the dipole vector (gaugeon) field in VSR, we generalize the framework of Froissart model for dipole scalar field [64]. Thus, the classical action for the dipole vector field theory in VSR reads

$$S_D = \int d^Dx \left[ (\Box - m^2) Y_\nu Y_\nu + \partial^\mu Y_\nu \partial_\mu Y_\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\mu Y_\nu n_\nu Y_\mu \right. \left. - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu Y_\nu \partial_\mu Y_\nu + \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 (n \cdot Y_\mu)(n \cdot Y) - \frac{\varepsilon}{2} Y_\mu Y_\mu \right], \quad (16)$$

where the sign factor $\varepsilon = \pm 1$ and $Y_\mu$ is an auxiliary vector field. This action is invariant under the VSR-modified gauge transformation

$$Y_\mu \rightarrow Y_\mu + \partial_\mu \theta - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \theta, \quad (17)$$

where $\theta$ represents an arbitrary scalar function.

The Euler-Lagrange field equations for the dipole vector fields $Y_{\nu\nu}$ and $Y_\nu$, respectively, can be calculated from the above action (16) as follows,

$$\Box Y_{\nu\nu} - \partial^\nu \partial_\nu Y_\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\mu n_\nu Y_\mu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \partial_\nu Y_\mu \left. - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n^\mu n_\nu Y_\mu - \varepsilon Y_{\nu\nu} = 0, \quad (18)\right.$$ \n
$$\Box Y_{\nu\nu} - \partial^\mu \partial_\mu Y_{\nu\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\mu n_\nu Y_{\nu\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \partial_\nu Y_{\nu\mu} \left. - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n^\mu n_\nu Y_{\nu\mu} = 0. \quad (19)\right.$$ \n
These two field equations reflect the following conditions:

$$\partial^\nu Y_{\nu\nu} - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\nu Y_{\nu\nu} = 0, \quad (\Box - m^2) Y_{\nu\nu} = 0. \quad (20)$$

Exploiting equations (18) and (20), we get the equation of motion for field $Y_\mu$ as follows,

$$\Box Y_\nu - \partial^\mu \partial_\mu Y_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\mu n_\mu Y_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \partial_\nu Y_\mu \left. - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n^\mu n_\nu Y_\mu \right.$$

$$+ m^2 \partial^\mu \partial_\mu Y_\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\mu n_\mu Y_\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n^\mu \partial_\nu Y_\mu + \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n^\mu n_\nu Y_\mu = 0. \quad (21)$$

The equations of motion (20) and (21) suggest that the gaugeon fields for the antisymmetric tensor fields would be a massive dipole fields. Thus, the gaugeon fields for Abelian rank-2 tensor field theory get mass $m$ in VSR framework.
IV. BRST QUANTIZATION OF DIPOLE VECTOR FIELD IN VSR

Since action (16) is gauge invariant, so possesses some superficial degrees of freedom. In order to quantize it correctly, we need to impose gauge-fixing condition which removes the redundancy in gauge degrees of freedom. The essential requirements for gauge-fixing condition are (i) it must fix the gauge completely, i.e., there must not be any residual gauge freedom, and (ii) using the transformations it must be possible to bring any configuration specified by gauge fields into one satisfying the gauge condition. The gauge-fixing can be achieved by adding the appropriate gauge-symmetry breaking terms to the classical action (16) as follows,

\[
S_{\text{DGF}} = S_D + \int d^Dx \left[ Y_\mu^\nu \partial_\mu Y + \partial^\mu Y_\nu \partial_\nu Y_\mu - \frac{1}{2} m^2 n_\mu n_\nu Y - \frac{1}{2} m^2 n_\nu Y_\mu(n \cdot Y) + \alpha_3 Y_\mu Y \right],
\]

where \(\alpha_3\) is the gauge-fixing parameter. Here, \(Y_\mu\) and \(Y\) represent scalar multiplier fields.

Since the Fock space corresponding above gauge-fixed action (22) is not positive definite. In order to make it positive definite, we add the Faddeev-Popov ghosts \(K_\mu\) and \(K_{s\mu}\) along with scalar FP ghosts \(K\) and \(K_s\) to the action, which compensate the determinant due to the gauge-fixing term within functional integral, as follows

\[
S_{\text{DFP}} = \int d^Dx \left[ (\square - m^2)Y_\mu^\nu Y_\nu + \partial^\mu Y_\nu \partial_\nu Y_\mu + \frac{1}{2} m^2 n_\mu n_\nu Y + \frac{1}{2} m^2 n_\nu Y_\mu(n \cdot Y) + \frac{1}{2} m^2(n \cdot Y)(n \cdot Y) \right. \\
+ \left. \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 (n \cdot Y)(n \cdot Y) - \frac{\epsilon}{2} m^2 n_\mu Y_\nu \partial_\nu Y_\mu - \frac{1}{2} m^2(n \cdot Y_\nu \partial_\nu Y_\mu) - \frac{1}{2} m^2 Y_\nu \partial_\nu Y_\mu \right. \\
- \left. \frac{i m^2}{2 n \cdot \partial}(n \cdot Y_\mu) + \alpha_3 Y_\mu Y + i(m - m^2)K_\mu^\nu K_\nu + i\partial^\mu K_\mu^\nu \partial_\nu K_\mu + \frac{i m^2}{2 n \cdot \partial}(n \cdot K_\mu)(n \cdot K) \right. \\
+ \left. \frac{i}{2} \partial^\mu K_\mu^\nu K_\nu(n \cdot K) + i\partial^\mu K_\mu^\nu K_\nu - \frac{i m^2}{2 n \cdot \partial}(n \cdot K_\mu)K_\nu + i\alpha_3 K_\mu K_\nu \right].
\]

The quantum action (23) is invariant under following nilpotent BRST transformation:

\[
s_b Y_\mu = K_\mu, \quad s_b K_\mu = 0, \quad s_b K_{s\mu} = i Y_{s\mu}, \quad s_b Y_{s\mu} = 0, \\
s_b Y = K, \quad s_b K_s = 0, \quad s_b K_s = i Y_s, \quad s_b Y_s = 0,
\]

Here we note that the gauge-fixed parts of the action is unphysical as it is BRST-exact and so do not contribute to the physical Hilbert space and can be written in terms of gauge-fixing fermion \(\Psi\) as follows

\[
S_{\text{DFP}} = s_b \Psi,
\]

where the expression of \(\psi\) is given by

\[
\Psi = \int d^Dx \left[ \partial^\mu K_\mu^\nu(\partial_\mu Y - \partial_\mu Y_\mu) + m^2 K_\mu^\nu Y_\nu + \frac{1}{2} m^2 n_\mu n_\nu Y + \frac{1}{2} m^2 n_\nu Y_\mu(n \cdot Y) \right. \\
- \left. \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 (n \cdot K_\mu)(n \cdot Y) + \frac{\epsilon}{2} K_\mu^\nu Y_\nu - \partial^\mu K_\mu^\nu Y_\nu - \frac{1}{2} m^2 K_\mu^\nu Y_\mu(n \cdot Y) \right. \\
+ \left. \frac{1}{2} \partial^\mu K_\mu^\nu K_\nu(n \cdot Y) - \alpha_3 K_\mu K_\nu \right].
\]

Now, using Noether’s theorem, we calculate the conserved charge \(Q_{\text{DFP}}\) corresponding to the BRST transformation (24). This is given by

\[
Q_{\text{DFP}} = \int d^{D-1}x \left[ Y_\mu \tilde{\partial}_0 K_\mu + (1 - \alpha_3)(Y_\mu K_\nu - Y_\nu K_\mu) \right].
\]
One can check that this charge is nilpotent. This charge helps to define the physical Hilbert space out of total Hilbert space under Kugo-Ojima condition.

V. GAUGEON FORMALISM OF 2-FORM THEORY IN VSR

In this section, we discuss the gaugeon formalism for the Abelian antisymmetric tensor field $B_{\mu\nu}$ in VSR. It is important to study the gaugeon formalism as the renormalized gauge parameter appears naturally and also this formalism connects two different gauges within the same family by the quantum gauge transformation. The gaugeon action for the Abelian 2-form gauge theory in VSR framework is given by

$$S = S_{FP}(\alpha_1 = 0, \alpha_2) + S_{DFP}(\alpha_3 = \alpha_2) + \int d^Dx \left[ \frac{\epsilon}{2} Y^\mu Y_{\mu} - \frac{\epsilon}{2} (Y^\mu + aB^\mu)(Y_{\mu} + aB_{\mu}) \right],$$

$$= \int d^Dx \left[ \frac{1}{12} \hat{H}_{\mu\nu\lambda} \hat{H}^{\mu\nu\lambda} - \partial^\mu B^\nu B_{\mu\nu} + \frac{m^2}{2n \cdot \partial} n^\mu B^\nu B_{\mu\nu} + B^\mu \delta_\mu \eta \frac{1}{2n \cdot \partial} (n \cdot B) \eta + \partial^\mu \phi \partial_\mu \phi 
+ m^2 \delta_\phi \phi - i \partial^\mu \phi^o \partial_\mu \rho_{\nu} - i m^2 \phi^o \partial_\mu \rho_{\mu} + i \phi^o \partial_\mu \partial_\nu \rho_{\mu} + \frac{i m^2}{2n \cdot \partial} (n \cdot \partial) (\partial \cdot \rho) + \frac{i m^2}{2n \cdot \partial} (\partial \cdot \rho) (n \cdot \partial) 
+ \frac{1}{4} \frac{m^4}{(n \cdot \partial)^2} (n \cdot \partial) (n \cdot \partial) \frac{\epsilon}{2} (Y^\mu + aB^\mu)(Y_{\mu} + aB_{\mu}) - \frac{1}{2n \cdot \partial} Y_{\mu} (n \cdot Y_{\mu}) + Y^\mu \partial_\mu Y 
+ \frac{1}{2n \cdot \partial} Y_{\mu} (n \cdot Y_{\mu}) + Y^\mu \partial_\mu Y 
- \frac{1}{2n \cdot \partial} Y_{\mu} (n \cdot Y_{\mu}) + Y^\mu \partial_\mu Y 
+ \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 (n \cdot Y_{\mu}) (n \cdot Y_{\mu}) - \frac{\epsilon}{2} (Y^\mu + aB^\mu)(Y_{\mu} + aB_{\mu}) - \frac{1}{2n \cdot \partial} Y_{\mu} (n \cdot Y_{\mu}) + Y^\mu \partial_\mu Y 
+ \frac{1}{2n \cdot \partial} (n \cdot K^o) (n \cdot K^o) + \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 (n \cdot K^o) (n \cdot K^o) + i \phi^o K^o \partial_\mu K^\mu - \frac{i m^2}{2n \cdot \partial} K^o (n \cdot K^o), \right]$$

(28)

where $a$ denotes the group vector valued gauge-fixing parameter. It should be noted that the gauge-fixing parameter $\alpha_1$ mentioned in Faddeev-Popov action for Abelian rank-2 tensor field can be recognized through the parameter $a$ by

$$\alpha_1 = \alpha a^2.$$

Remarkably, the action (28) possesses an extra symmetry, so-called $q$-number gauge transformation, which leaves the action form-invariant. Under $q$-number gauge transformation fields transform as

$$\delta_q B_{\mu\nu} = \hat{B}_{\mu\nu} - B_{\mu\nu} = \tau \left( \partial_\mu Y_{\nu} - \partial_\nu Y_{\mu} - \frac{m^2}{2n \cdot \partial} \nu_{\mu} Y_{\nu} + \frac{m^2}{2n \cdot \partial} \nu_{\nu} Y_{\mu} \right),$$

$$\delta_q \rho_{\mu} = \hat{\rho}_{\mu} - \rho_{\mu} = \tau K_{\mu}, \quad \delta_q Y_{\mu} = \hat{Y}_{\mu} - Y_{\mu} = -\tau B_{\mu}, \quad \delta_q B_{\mu} = \hat{B}_{\mu} - B_{\mu} = 0,$$

$$\delta_q Y_{\mu} = \hat{Y}_{\mu} - Y_{\mu} = 0, \quad \delta_q K_{\mu} = \hat{K}_{\mu} - K_{\mu} = -\tau \rho_{\mu}, \quad \delta_q \rho_{\mu} = \hat{\rho}_{\mu} - \rho_{\mu} = 0,$$

$$\delta_q K_{\mu} = \hat{K}_{\mu} - K_{\mu} = 0, \quad \delta_q d = \hat{d} - d = 0, \quad \delta_q \eta = \hat{\eta} - \eta = \tau Y,$$

$$\delta_q Y = \hat{Y} - Y = 0, \quad \delta_q K = \hat{K} - K = 0, \quad \delta_q \phi = \hat{\phi} - \phi = 0, \quad \delta_q \phi = \hat{\phi} - \phi = 0,$$

(30)

where $\tau$ is an infinitesimal transformation parameter having bosonic nature. The form-invariance of the gaugeon action (28) under quantum gauge transformation (30) leads to the following shift in the
The anti-ghost and ghost fields satisfy the following equations of motions:

\[ \Box \bar{\phi} + \frac{m^2}{2 n \cdot \partial} \bar{\phi} = 0, \]  
\[ \Box \phi - \frac{m^2}{2 n \cdot \partial} \phi = 0, \]

then the transformed fields satisfy the same field equations as the original fields do.

The equations of motion for the \( B_{\mu\nu}, B_{\nu}, \eta, \bar{\phi} \) and \( \phi \) fields corresponding to action (28) are given by

\[ \Box \bar{B}_{\mu\nu} - \frac{m^2}{2 n \cdot \partial} \bar{B}_{\mu\nu} + \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} - \frac{1}{2 n \cdot \partial} n_{\mu} B_{\nu} + \frac{1}{2 n \cdot \partial} n_{\nu} B_{\mu} = 0, \]  
\[ \partial^\mu B_{\mu\nu} - \frac{m^2}{2 n \cdot \partial} n_{\mu} B_{\mu\nu} + \partial_{\nu} \eta - \frac{1}{2 n \cdot \partial} n_{\nu} \eta = \varepsilon \sigma(Y_{\sigma\nu} + a B_{\nu}) = 0, \]  
\[ \partial^\nu B_{\mu} - \frac{m^2}{2 n \cdot \partial} (n \cdot B) = 0, \]  
\[ (\Box - m^2) \phi = 0, \]  
\[ (\Box - m^2) \bar{\phi} = 0. \]

It is obvious here that both the fields \( \phi \) and \( \bar{\phi} \) are massive. The gaugeon fields satisfy the following field equations:

\[ \Box Y_{\nu} - \partial^\mu \partial_{\nu} Y_{\mu} + \frac{1}{2 n \cdot \partial} n_{\nu} (\partial \cdot Y) + \frac{1}{2 n \cdot \partial} \partial_{\nu} (n \cdot Y) - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_{\nu} (n \cdot Y) \]
\[ - \varepsilon (Y_{\sigma\nu} + a B_{\nu}) + \partial_{\nu} Y - \frac{1}{2 n \cdot \partial} n_{\nu} Y = 0, \]
\[ (\Box - m^2) Y_{\nu} = \frac{1}{2 n \cdot \partial} n_{\nu} Y_{\nu} = 0. \]

The anti-ghost and ghost fields satisfy the following equations of motions:

\[ (\Box - m^2) \rho_{\nu} - \partial^\mu \partial_{\nu} \rho_{\mu} + \frac{m^2}{2 n \cdot \partial} n_{\nu} (\partial \cdot \rho) + \frac{m^2}{2 n \cdot \partial} \partial_{\nu} (n \cdot \rho) - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_{\nu} (n \cdot \rho) \]
\[ + \partial_{\nu} d - \frac{1}{2 n \cdot \partial} n_{\nu} d = 0, \]
\[ (\Box - m^2) \bar{\rho}_{\nu} - \partial^\mu \partial_{\nu} \bar{\rho}_{\mu} + \frac{m^2}{2 n \cdot \partial} n_{\nu} (\partial \cdot \bar{\rho}) + \frac{m^2}{2 n \cdot \partial} \partial_{\nu} (n \cdot \bar{\rho}) - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_{\nu} (n \cdot \bar{\rho}) \]
\[ + \partial_{\nu} \bar{d} - \frac{1}{2 n \cdot \partial} n_{\nu} \bar{d} = 0, \]

The equations of motion for anti-ghost and ghost fields corresponding to gaugeon fields, respectively, are

\[ (\Box - m^2) K_{\nu} - \partial^\mu \partial_{\nu} K_{\mu} + \frac{m^2}{2 n \cdot \partial} n_{\nu} (\partial \cdot K) + \frac{m^2}{2 n \cdot \partial} \partial_{\nu} (n \cdot K) - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_{\nu} (n \cdot K) \]
\[ + \partial_{\nu} K - \frac{1}{2 n \cdot \partial} n_{\nu} K = 0, \]
\[ (\Box - m^2) \bar{K}_{\nu} - \partial^\mu \partial_{\nu} \bar{K}_{\mu} + \frac{m^2}{2 n \cdot \partial} n_{\nu} (\partial \cdot \bar{K}) + \frac{m^2}{2 n \cdot \partial} \partial_{\nu} (n \cdot \bar{K}) - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 n_{\nu} (n \cdot \bar{K}) \]
\[ + \partial_{\nu} \bar{K} - \frac{1}{2 n \cdot \partial} n_{\nu} \bar{K} = 0. \]
The effective gaugeon action for 2-form theory in VSR [28] is invariant under the following nilpotent BRST transformations:

\[ s_b B_{\mu\nu} = \partial_\mu \rho_\nu - \partial_\nu \rho_\nu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \rho_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu \rho_\mu \]

\[ s_b \rho_\mu = -i \partial_\mu \phi + \frac{i}{2} \frac{m^2}{n \cdot \partial} n_\nu \rho_\mu \phi, \quad s_b \phi = 0, \quad s_b B_\mu = 0, \]

\[ s_b \bar{\phi} = i B_\mu, \quad s_b \bar{\eta} = \bar{d}, \quad s_b \eta = d, \quad s_b d = 0, \]

\[ s_b Y_\mu = K_\mu, \quad s_b K_\mu = 0, \quad s_b K_{*\mu} = i Y_{*\mu}, \quad s_b Y_{*\mu} = 0, \]

\[ s_b Y = K, \quad s_b K = 0, \quad s_b K_{*} = i Y_{*}, \quad s_b Y_{*} = 0. \]

(43)

Here we see that the fields of gaugeon sector form the BRST quartet. The unphysical parts of the action [28] is BRST exact and can be written in terms of gauge-fixing fermion \( \Psi \) as

\[ S = \int d^D x \left[ \frac{1}{12} \Lambda \mu \nu \lambda \Lambda_{\mu \nu \lambda} + s_b \Psi_g, \right] \]

(44)

where

\[ \Psi_g = i \int d^D x \left[ \partial^\mu \bar{\rho}^\nu (\partial_\mu B_\nu - \partial_\nu B_\mu) + m^2 \bar{\rho}^\nu B_\nu - \alpha_2 K_{*\mu} Y + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\mu \bar{\rho}^\nu n_\nu B_\mu - \alpha_2 \bar{\eta} \right] \]

\[ + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu \partial_\nu B_\nu + \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 (n \cdot \bar{\rho})(n \cdot B) - \bar{\rho}^\nu \partial_\mu \eta + \frac{1}{2} \frac{m^2}{n \cdot \partial} (n \cdot \bar{\rho}) \eta + \partial^\mu \bar{\rho} \partial_\mu \eta \]

\[ - \frac{1}{2} \frac{m^2}{n \cdot \partial} \phi (n \cdot \rho) + \frac{\varepsilon}{2} (K_{*\mu} + a \bar{\rho}^\nu) (Y_{*\mu} + a B_\mu) + \partial^\mu K_{2\nu} (\partial_\nu Y_\mu - \partial_\nu Y_{*\mu}) - m^2 K_{2\nu} Y_\mu \]

\[ + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu K_{2\nu} \partial_\nu Y_{*\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} \partial^\mu K_{2\nu} n_\nu Y_{*\nu} - \frac{1}{4} \left( \frac{m^2}{n \cdot \partial} \right)^2 (n \cdot Y)(K_{*\mu} - K_{*\nu}) \]

\[ + \partial^\mu K_{*\mu} Y_{*\mu} + \frac{1}{2} \frac{m^2}{n \cdot \partial} (K_{*\mu} - K_{*\nu}) Y_{*\nu} - \frac{m^2}{2} K_{*\mu} (n \cdot Y). \]

(45)

The invariance of the action (44) under BRST transformation is obvious due to nilpotency of the BRST transformation. We calculate the conserved BRST charge \( Q_B \) with the help of Noether’s theorem as follows,

\[ Q_B = \int d^{D-1} x \left[ B^\lambda \partial_0 \rho_\lambda + \bar{d} \partial_0 \bar{\rho} + (1 - \alpha_2) B_0 d + Y_{*\lambda} \partial_0 K_{*\lambda} \right] \]

\[ + (1 - \alpha_2) (Y_{*0} K_{*} - Y_{*} K_{*0})]. \]

(46)

The above BRST charge helps to define the Fock space of the system (which is Kernel of BRST charge) in following manner:

\[ \mathcal{V}_{\text{phys}} = \{ |\Phi\rangle; Q_B |\Phi\rangle = 0 \}. \]

(47)

The BRST transformations [28] commute with the \( q \)-number gauge transformation [30]. This suggests that the the BRST charge is invariant under the \( q \)-number gauge transformations, i.e.,

\[ \delta_q Q_B = 0. \]

(48)

Consequently, the fock space (the physical subspace of states) \( \mathcal{V}_{\text{phys}} \) is invariant under the \( q \)-number gauge transformation, i.e.,

\[ \delta_q \mathcal{V}_{\text{phys}} = 0. \]

(49)

As a result, the physical Hilbert space of 2-form gauge theory in VSR \( \mathcal{H}_{\text{phys}} = \mathcal{V}_{\text{phys}} / \text{Im} Q \) is also invariant under both the BRST and the \( q \)-number gauge transformations.
There exist many nilpotent symmetry and corresponding charges for the action (28) in addition to charge (46). For instance, these charges are

\[ Q_{FP} = \int d^{D-1}x \left[ B^\mu \partial_0 \phi + d \partial_0 \phi + (1 - \alpha_2)B_0 d \right], \]

(50)

\[ Q_{DFP} = \int d^{D-1}x \left[ Y^\mu \partial_0 K_\mu + (1 - \alpha_3)(Y_0 K - Y_0 K_0) \right], \]

(51)

\[ Q'_B = \int d^{D-1}x \left[ B^\mu \partial_0 K_\mu + (1 - \alpha_3)B_0 K \right]. \]

(52)

The charge \( Q_{FP} \) is the generator of the BRST transformation (7) while charge \( Q_{DFP} \) is the generator of the BRST transformation (43). The charge \( Q'_B \) generates following BRST transformation \( s'_b \):

\[ s'_b B_{\mu\nu} = \partial_\mu K_\nu - \partial_\nu K_\mu - \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\mu K_\nu + \frac{1}{2} \frac{m^2}{n \cdot \partial} n_\nu K_\mu, \]

\[ s'_b K_{\star\mu} = i B_\mu, \quad s'_b \eta = K, \quad s'_b (\text{other fields}) = 0. \]

(53)

These charges satisfy following anticommutation relation among themselves:

\[ \{ Q_{FP}, Q_{DFP} \} = \{ Q_{DFP}, Q'_B \} = \{ Q'_B, Q_{FP} \} = 0. \]

(54)

The BRST charge \( Q_{FP} \) acts separately for fields of standard formalism sector and the BRST charge \( Q_{DFP} \) act separately for fields of gaugeon sector. The net BRST charge (46), which acts on the fields of both the standard formalism sector and gaugeon sector, is given by

\[ Q_B = Q_{FP} + Q_{DFP}. \]

(55)

This charge is the generator of BRST transformation (43).

In VSR framework also, we can define a subspace of states in total Hilbert space \( \mathcal{V}_{\text{phys}}^{(a)} \) as

\[ \mathcal{V}_{\text{phys}}^{(a)} = \ker Q_{FP} \cap \ker Q_{DFP} = \{ |\Phi\rangle; Q_{FP}|\Phi\rangle = Q_{DFP}|\Phi\rangle = 0 \}. \]

(56)

Here, the index \((a)\) in \( \mathcal{V}_{\text{phys}}^{(a)} \) signifies the dependence of its definition on the gauge-fixing parameter \( a \).

For such space, the Kugo-Ojima condition, \( Q_{FP}|\Phi\rangle = 0 \), removes the superfluous modes of the standard formalism sector. However, the Kugo-Ojima condition, \( Q_{DFP}|\Phi\rangle = 0 \), removes the superfluous modes of the gaugeon sector. It is very obvious now that the space \( \mathcal{V}_{\text{phys}}^{(a)} \subset \mathcal{V}_{\text{phys}} \).

Also, the BRST charges \( Q_{FP} \) and \( Q_{DFP} \) transform under the \( q \)-number gauge transformation (30) as follows,

\[ \delta_q Q_{FP} = \tau Q'_B, \]

\[ \delta_q Q_{DFP} = -\tau Q'_B, \]

(57)

which insures that the sum of the charges \((Q_B)\) is invariant under \( q \)-number gauge transformation.

We also define the subspace \( \mathcal{V}^{(a)} \) of the total Fock space as

\[ \mathcal{V}^{(a)} = \ker Q_{DFP} = \{ |\Phi\rangle; Q_{DFP}|\Phi\rangle = 0 \}. \]

(58)

This space coincides with the space of physical dipole vector field for \( \alpha_1 = \varepsilon a^2 \) gauge. This implies that the action (28) in \( \alpha_1 = \varepsilon a^2 \) gauge can be written as

\[ S = S_{FP}(\alpha_1 = \varepsilon a^2) + i \int dt \{ Q_B, \Theta \}, \]

(59)
Our motivation here is to discuss the SIM and their generator are constructed for the resulting effective action. The nilpotent BRST transformation it contains superfluous degrees of freedom. In order to remove these extra degrees of freedom, we have fixed the gauge which introduced the Faddeev-popov ghost terms. The same arguments hold for the quantum gauge transformed BRST charges also as for the original BRST charges. If we define the subspaces $\mathcal{V}^{(a+)}$ and $\mathcal{V}^{(a+)}_{\text{phys}}$ annihilated by the $q$-number gauge transformed BRST charges as

$$\mathcal{V}^{(a+)} = \ker(Q_{\text{DFP}} + \delta q Q_{\text{DFP}}),$$

$$\mathcal{V}^{(a+)}_{\text{phys}} = \ker(Q_{\text{FP}} + \delta q Q_{\text{FP}}) \cap \ker(Q_{\text{DFP}} + \delta q Q_{\text{DFP}}).$$

where $\alpha_1 = \varepsilon(a + \tau)^2$, and the corresponding physical subspace is $\mathcal{V}^{(a+)}_{\text{phys}}$. Consequently, in VSR scenario also, the single Fock space corresponding to BRST invariant gaugon action of 2-form gauge theory embeds the Fock spaces of the 2-form gauge theory in family of linear gauges.

We would like to comment here that the Type II gaugeon formalism for the present theory can also be developed in VSR framework. For Type II theory, the standard gauge-fixing parameter $\alpha_1$ is expressed as $\alpha_1 = a$ and the sign of $\alpha_1$ can also be changed in Type II theory, however, for Type I theory $\alpha_1 = \varepsilon a^2$. In both cases, the gauge-fixing parameter $a$ can be shifted as $\hat{a} = a + \tau$ by the $q$-number gauge transformation.

For Type II, the action in VSR is given by

$$S_{\text{II}} = S_{\text{FP}}(\alpha_1 = a; \alpha_2) + S_{\text{DFP}}(\alpha_3 = \alpha_2) + \int d^D x \left[ \frac{\varepsilon}{2} Y^\mu Y_{\ast \mu} - \frac{1}{2} Y^\mu B_\mu \right].$$

Here, the standard gauge-fixing parameter $\alpha_1$ can be identified with $\alpha_1 = a$. The action (62) remains invariant under the BRST transformations generated by the BRST charges (44), (49), (51), and (52) too. Therefore, the single Fock space corresponding to Type II gaugeon action of 2-form gauge theory embeds the Fock spaces of the 2-form gauge theory in family of linear gauges.

**VI. CONCLUDING REMARKS**

In this paper, we have analysed the BRST quantization of SIM(2) invariant Abelian rank-2 tensor theory framework and define physical Hilbert space under Kugo-Ojima condition. The rotational symmetry of full Lorentz group is broken by fixing the null direction. This is achieved by adding the appropriate Lorentz breaking non-local terms to the standard action. In this regard, we find that the Abelian rank-2 tensor fields together with ghost and ghost of ghost fields satisfy the Proca-type equations and get the mass but this can not be an alternative to Higgs mechanism as all the fields get same value of mass. The spontaneous symmetry breaking should take place in order to assign different mass for different fields. We have found that the VSR modified action is not invariant under usual gauge transformation rather this is invariant under a VSR-modified gauge transformation. As the theory possesses the gauge invariance, it contains superfluous degrees of freedom. In order to remove these extra degrees of freedom, we have fixed the gauge which introduced the Faddeev-popov ghost terms. The nilpotent BRST transformation and their generator are constructed for the resulting effective action.

Recently, the massless gaugeon dipole vector model is studied for Abelian rank-2 tensor theory satisfying full Lorentz group $\mathfrak{so}(2,1)$. Our motivation here is to discuss the SIM(2)-invariant generalization of the
gaugeon dipole vector theory. We have observed that the dipole vector field gets mass under the VSR framework, still this classical dipole vector action also admits a VSR-modified gauge invariance. In order to remove the redundant degrees of freedom due to gauge symmetry, we have chosen a VSR-modified gauge. This is implemented in the action by adding the suitable non-local gauge-fixing and ghost terms. The BRST transformation and their generator are constructed for the (non-local) effective action for dipole vector field. Furthermore, in order to discuss the quantum gauge freedom for Abelian rank-2 tensor theory in VSR, we have constructed the gaugeon action for the Abelian rank-2 tensor theory, where dipole vector fields play the role of quantum (gaugeon) fields. In order to replace the Gupta-Bleuler type subsidiary condition which removes the unphysical gaugeon mode to Kugo-Ojima type subsidiary condition, we have developed a BRST symmetric gaugeon formalism by introducing the massive ghost fields corresponding to gaugeon fields to the action. We have shown that the $SIM(2)$-invariant BRST symmetric gaugeon action admits a (non-local) $q$-number gauge transformation. The form-invariance of action requires a shift in gauge parameter which can be identified as the renormalized gauge parameter. We noted here that the BRST transformation commutes with the $q$-number gauge transformation in VSR also, therefore, the physical Hilbert space of 2-form gauge theory $H_{phys} = V_{phys} / \text{Im}Q$ is also invariant under both the BRST and the $q$-number gauge transformations. We have found that there exist various sets of BRST transformation for the gaugeon action in VSR and therefore various BRST charges exist. We have shown that the Fock spaces constructed with the help of these charges are embedded into a single physical Hilbert space.

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