Fault Detection and Isolation Tools (FDITOOLS)

User's Guide

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Abstract

The Fault Detection and Isolation Tools (FDITOOLS) is a collection of MATLAB functions for the analysis and solution of fault detection and model detection problems. The implemented functions are based on the computational procedures described in the Chapters 5, 6 and 7 of the book: "A. Varga, Solving Fault Diagnosis Problems – Linear Synthesis Techniques, Springer, 2017". This document is the User's Guide for the version V0.4 of FDITOOLS. First, we present the mathematical background for solving several basic exact synthesis problems of fault detection filters and model detection filters. Then, we give in-depth information on the command syntax of the main analysis and synthesis functions. Several examples illustrate the use of the main functions of FDITOOLS.

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Notations and Symbols

General notations

\( \mathbb{C} \) field of complex numbers
\( \mathbb{R} \) field of real numbers
\( \mathbb{C}_s \) stability domain (i.e., open left complex half-plane in continuous-time or open unit disk centered in the origin in discrete-time)
\( \partial \mathbb{C}_s \) boundary of stability domain (i.e., extended imaginary axis with infinity included in continuous-time, or unit circle centered in the origin in discrete-time)
\( \overline{\mathbb{C}}_s \) closure of \( \mathbb{C}_s \): \( \overline{\mathbb{C}}_s = \mathbb{C}_s \cup \partial \mathbb{C}_s \)
\( \mathbb{C}_u \) open instability domain: \( \mathbb{C}_u := \mathbb{C} \setminus \overline{\mathbb{C}}_s \)
\( \overline{\mathbb{C}}_u \) closure of \( \mathbb{C}_u \): \( \overline{\mathbb{C}}_u := \mathbb{C}_u \cup \partial \mathbb{C}_s \)
\( \mathbb{C}_g \) "good" domain of \( \mathbb{C} \)
\( \mathbb{C}_b \) "bad" domain of \( \mathbb{C} \): \( \mathbb{C}_b = \mathbb{C} \setminus \mathbb{C}_g \)
\( s \) complex frequency variable in the Laplace transform: \( s = \sigma + i \omega \)
\( z \) complex frequency variable in the Z-transform: \( z = e^{sT}, T \) – sampling time
\( \lambda \) complex frequency variable: \( \lambda = s \) in continuous-time or \( \lambda = z \) in discrete-time
\( \overline{\lambda} \) complex conjugate of the complex number \( \lambda \)
\( \mathbb{R}(\lambda) \) set of rational matrices in indeterminate \( \lambda \) with real coefficients and unspecified dimensions
\( \mathbb{R}(\lambda)^{p \times m} \) set of \( p \times m \) rational matrices in indeterminate \( \lambda \) with real coefficients
\( \delta(G(\lambda)) \) McMillan degree of the rational matrix \( G(\lambda) \)
\( G^\sim(\lambda) \) Conjugate of \( G(\lambda) \in \mathbb{R}(\lambda) \): \( G^\sim(s) = G^T(-s) \) in continuous-time and \( G^\sim(z) = G^T(1/z) \) in discrete-time
\( \ell_2 \) Banach-space of square-summable sequences
\( \mathcal{L}_2 \) Lebesgue-space of square-integrable functions
\( \mathcal{L}_\infty \) Space of complex-valued functions bounded and analytic in \( \partial \mathbb{C}_s \)
\( \mathcal{H}_\infty \) Hardy-space of complex-valued functions bounded and analytic in \( \mathbb{C}_u \)
\( \|G\|_\infty \) \( \mathcal{H}_\infty \)- or \( \mathcal{L}_\infty \)-norm of the transfer function matrix \( G(\lambda) \)
\( M^T \) transpose of the matrix \( M \)
\( M^{-1} \) inverse of the matrix \( M \)
\( \sigma(M) \) largest singular value of the matrix \( M \)
\( \sigma(M) \) least singular value of the matrix \( M \)
\( N(M) \) kernel (or right nullspace) of the matrix \( M \)
\( N_L(G(\lambda)) \) left kernel (or left nullspace) of \( G(\lambda) \in \mathbb{R}(\lambda) \)
\( N_R(G(\lambda)) \) right kernel (or right nullspace) of \( G(\lambda) \in \mathbb{R}(\lambda) \)
\( \mathbb{R}(M) \) range (or image space) of the matrix \( M \)
\( I_n \) or \( I \) identity matrix of order \( n \) or of an order resulting from context
\( e_i \) the \( i \)-th column of the (known size) identity matrix
\( 0_{m \times n} \) or \( 0 \) zero matrix of size \( m \times n \) or of a size resulting from context
Fault diagnosis related notations

\( y(t) \) measured output vector: \( y(t) \in \mathbb{R}^p \)

\( y(\lambda) \) Laplace- or Z-transformed measured output vector

\( u(t) \) control input vector: \( u(t) \in \mathbb{R}^{m_u} \)

\( u(\lambda) \) Laplace- or Z-transformed control input vector

\( d(t) \) disturbance input vector: \( d(t) \in \mathbb{R}^{m_d} \)

\( d(\lambda) \) Laplace- or Z-transformed disturbance input vector

\( w(t) \) noise input vector: \( w(t) \in \mathbb{R}^{m_w} \)

\( w(\lambda) \) Laplace- or Z-transformed noise input vector

\( f(t) \) fault input vector: \( f(t) \in \mathbb{R}^{m_f} \)

\( f(\lambda) \) Laplace- or Z-transformed fault input vector

\( x(t) \) state vector: \( x(t) \in \mathbb{R}^n \)

\( G_u(\lambda) \) transfer function matrix from \( u \) to \( y \)

\( G_d(\lambda) \) transfer function matrix from \( d \) to \( y \)

\( G_w(\lambda) \) transfer function matrix from \( w \) to \( y \)

\( G_f(\lambda) \) transfer function matrix from \( f \) to \( y \)

\( G_{f_j}(\lambda) \) transfer function matrix from the \( j \)-th fault input \( f_j \) to \( y \)

\( A \) system state matrix

\( E \) system descriptor matrix

\( B_u, B_d, B_w, B_f \) system input matrices from \( u, d, w, f \)

\( C \) system output matrix

\( D_u, D_d, D_w, D_f \) system feedthrough matrices from \( u, d, w, f \)

\( r(t) \) residual vector: \( r(t) \in \mathbb{R}^q \)

\( r(\lambda) \) Laplace- or Z-transformed residual vector

\( n_r \) number of components of residual vector \( r \)

\( r^{(i)}(t) \) \( i \)-th residual vector component: \( r^{(i)}(t) \in \mathbb{R}^{q_i} \)

\( r^{(i)}(\lambda) \) Laplace- or Z-transformed \( i \)-th residual vector component

\( Q(\lambda) \) transfer function matrix of the implementation form of the residual generator from \( y \) and \( u \) to \( r \)

\( Q_y(\lambda) \) transfer function matrix of residual generator from \( y \) to \( r \)

\( Q_u(\lambda) \) transfer function matrix of residual generator from \( u \) to \( r \)

\( Q^{(i)}(\lambda) \) transfer function matrix of the implementation form of the \( i \)-th residual generator from \( y \) and \( u \) to \( r^{(i)} \)

\( R(\lambda) \) transfer function matrix of the internal form of the residual generator from \( u, d, w \) and \( f \) to \( r \)

\( R_u(\lambda) \) transfer function matrix from \( u \) to \( r \)

\( R_d(\lambda) \) transfer function matrix from \( d \) to \( r \)

\( R_w(\lambda) \) transfer function matrix from \( w \) to \( r \)

\( R_f(\lambda) \) transfer function matrix from \( f \) to \( r \)

\( R_{f_j}(\lambda) \) transfer function matrix from the \( j \)-th fault input \( f_j \) to \( r \)

\( R^{(i)}_{f_j}(\lambda) \) transfer function matrix from the \( j \)-th fault input \( f_j \) to \( r^{(i)} \)

\( S \) binary structure matrix

\( S_{R_f} \) binary structure matrix corresponding to \( R_f(\lambda) \)
Model detection related notations

- $M_r(\lambda)$: transfer function matrix of a reference model from $f$ to $r$
- $\theta(t)$: residual evaluation vector
- $i(t)$: binary decision vector
- $\tau, \tau_i$: decision thresholds

For an $i$-th residual vector component:

- $y^{(i)}(t)$: measured output vector: $y(t) \in \mathbb{R}^p$
- $y^{(j)}(\lambda)$: Laplace- or $\mathcal{Z}$-transformed measured output vector
- $u^{(i)}(t)$: control input vector: $u(t) \in \mathbb{R}^m$
- $u^{(j)}(\lambda)$: Laplace- or $\mathcal{Z}$-transformed control input vector
- $u^{(j)}(t)$: control input vector of $j$-th model: $u^{(j)}(t) := u(t) \in \mathbb{R}^m$
- $u^{(j)}(\lambda)$: Laplace- or $\mathcal{Z}$-transformed control input vector of $j$-th model
- $d^{(i)}(t)$: disturbance input vector of $j$-th model: $d^{(j)}(t) \in \mathbb{R}^{m_d}$
- $d^{(j)}(\lambda)$: Laplace- or $\mathcal{Z}$-transformed disturbance input vector of $j$-th model
- $w^{(i)}(t)$: noise input vector of $j$-th model: $w^{(j)}(t) \in \mathbb{R}^{m_w}$
- $w^{(j)}(\lambda)$: Laplace- or $\mathcal{Z}$-transformed noise input vector of $j$-th model
- $y^{(j)}(\lambda)$: output vector of $j$-th model: $y^{(j)}(t) \in \mathbb{R}^p$
- $y^{(j)}(t)$: Laplace- or $\mathcal{Z}$-transformed output vector of $j$-th model
- $x^{(j)}(t)$: state vector of $j$-th model: $x^{(j)}(t) \in \mathbb{R}^{n_i}$
- $G_u^{(j)}(\lambda)$: transfer function matrix of $j$-th model from $u^{(j)}$ to $y^{(j)}$
- $G_d^{(j)}(\lambda)$: transfer function matrix of $j$-th model from $d^{(j)}$ to $y^{(j)}$
- $G_w^{(j)}(\lambda)$: transfer function matrix of $j$-th model from $w^{(j)}$ to $y^{(j)}$
- $A^{(j)}$: system state matrix of $j$-th model
- $E^{(j)}$: system descriptor matrix of $j$-th model
- $B_u^{(j)}$, $B_d^{(j)}$, $B_w^{(j)}$: system input matrices of $j$-th model from $u^{(j)}$, $d^{(j)}$, $w^{(j)}$
- $C^{(j)}$: system output matrix of $j$-th model
- $D_u^{(j)}$, $D_d^{(j)}$, $D_w^{(j)}$: system feedthrough matrices of $j$-th model from $u^{(j)}$, $d^{(j)}$, $w^{(j)}$
- $r^{(i)}(t)$: $i$-th residual vector component: $r^{(i)}(t) \in \mathbb{R}^q$
- $r^{(i)}(\lambda)$: Laplace- or $\mathcal{Z}$-transformed $i$-th residual vector component
- $r(t)$: overall residual vector: $r(t) \in \mathbb{R}^q$, $q = \sum_{i=1}^{N} q_i$
- $r(\lambda)$: Laplace- or $\mathcal{Z}$-transformed overall residual vector
- $Q^{(i)}(\lambda)$: transfer function matrix of the implementation form of the $i$-th residual generator from $y$ and $u$ to $r^{(i)}$
- $Q_y^{(i)}(\lambda)$: transfer function matrix of residual generator from $y$ to $r^{(i)}$
- $Q_u^{(i)}(\lambda)$: transfer function matrix of residual generator from $u$ to $r^{(i)}$
- $Q(\lambda)$: transfer function matrix of the implementation form of the overall residual generator from $y$ and $u$ to $r$
- $R^{(i,j)}(\lambda)$: the transfer function matrix of the internal form of the overall residual generator from $(u^{(j)}, d^{(j)}, w^{(j)})$ to $r^{(i)}$
$R_u^{(i,j)}(\lambda)$ the transfer function matrix of the internal form of the overall residual generator from $u^{(j)}$ to $r^{(i)}$

$R_d^{(i,j)}(\lambda)$ the transfer function matrix of the internal form of the overall residual generator from $d^{(j)}$ to $r^{(i)}$

$R_w^{(i,j)}(\lambda)$ the transfer function matrix of the internal form of the overall residual generator from $w^{(j)}$ to $r^{(i)}$

$\theta(t)$ $N$-dimensional residual evaluation vector

$\iota(t)$ $N$-dimensional binary decision vector

$\tau_i$ decision threshold for $i$-th component of the residual vector
Acronyms

EFDP Exact fault detection problem
EFDIP Exact fault detection and isolation problem
EMDP Exact model detection problem
EMMP Exact model matching problem
FDD Fault detection and diagnosis
FDI Fault detection and isolation
LTI Linear time-invariant
LFT Linear fractional transformation
LPV Linear parameter-varying
MIMO Multiple-input multiple-output
MMP Model-matching problem
TFM Transfer function matrix
1 Introduction

The Fault Detection and Isolation Tools (FDITOOLS) is a collection of MATLAB functions for the analysis and solution of fault detection problems. FDITOOLS supports various synthesis approaches of linear residual generation filters for continuous- or discrete-time linear systems. The underlying synthesis techniques rely on reliable numerical algorithms developed by the author and described in the Chapters 5, 6 and 7 of the author’s book [11]:

Andreas Varga, Solving Fault Diagnosis Problems - Linear Synthesis Techniques, vol. 84 of Studies in Systems, Decision and Control, Springer International Publishing, xxviii+394, 2017.

The functions of the FDITOOLS collection rely on the Control System Toolbox [1] and the Descriptor System Tools (DSTOOLS) V0.6 [2]. The current release of FDITOOLS is version V0.4, dated August 31, 2017. FDITOOLS is distributed as a free software via the Bitbucket repository.\(^1\) The codes have been developed under MATLAB 2015b and have been also tested with MATLAB 2016a, 2016b and 2017a. To use the functions of FDITOOLS, the Control System Toolbox and the DSTOOLS collection must be installed in MATLAB running under 64-bit Windows 7, 8, 8.1 or 10.

This document describes version V0.4 of the FDITOOLS collection. It will be continuously extended in parallel with the implementation of new functions. The book [11] represents an important complementary documentation for the FDITOOLS collection: it describes the mathematical background of solving synthesis problems of fault detection and model detection filters and gives detailed descriptions of the underlying synthesis procedures. Additionally, the M-files of the functions are self-documenting and a detailed documentation can be obtained online by typing help with the M-file name. Please cite FDITOOLS as follows:

A. Varga. FDITOOLS – The Fault Detection and Isolation Tools for MATLAB.
https://sites.google.com/site/andreasvargaccontact/home/software/fditools.

\(^1\)https://bitbucket.org/DSVarga/fditools
2 Fault Detection Basics

In this section we describe first the basic fault monitoring tasks, such as fault detection and fault isolation, and then introduce and characterize the concepts of fault detectability and fault isolability. Six “canonical” fault detection problems are formulated in the book [11] for the class of linear time-invariant (LTI) systems with additive faults. Of the formulated six problems, three involve the exact synthesis and three involve the approximate synthesis of fault detection filters. The current release of FDITOOLS covers the three exact synthesis techniques described in [11]. The approximate synthesis methods will be progressively implemented and included in the future releases of FDITOOLS. Jointly with the formulation of the fault detection problems, general solvability conditions are given for each problem in terms of ranks of certain transfer-function matrices. More details and the proofs of the results are available in Chapters 2 and 3 of [11].

2.1 Basic Fault Monitoring Tasks

A fault represents a deviation from the normal behaviour of a system due to an unexpected event (e.g., physical component failure or supply breakdown). The occurrence of faults must be detected as early as possible to prevent any serious consequence. For this purpose, fault diagnosis techniques are used to allow the detection of occurrence of faults (fault detection) and the localization of detected faults (fault isolation). The term fault detection and diagnosis (FDD) includes the requirements for fault detection and isolation (FDI).

A FDD system is a device (usually based on a collection of real-time processing algorithms) suitably set-up to fulfill the above tasks. The minimal functionality of any FDD system is illustrated in Fig. 1.

![Figure 1: Basic fault diagnosis setup.](image)

The main plant variables are the control inputs $u$, the unknown disturbance inputs $d$, the noise inputs $w$, and the output measurements $y$. The output $y$ and control input $u$ are the only measurable signals which can be used for fault monitoring purposes. The disturbance inputs $d$ and noise inputs $w$ are non-measurable “unknown” input signals, which act adversely on the system performance. For example, the unknown disturbance inputs $d$ may represent physical disturbance inputs, as for example, wind turbulence acting on an aircraft or external loads acting...
on a plant. Typical noise inputs are sensor noise signals as well as process input noise. However, fictive noise inputs can also account for the cumulative effects of unmodelled system dynamics or for the effects of parametric uncertainties. In general, there is no clear-cut separation between disturbances and noise, and therefore, the appropriate definition of the disturbance and noise inputs is a challenging aspect when modelling systems for solving fault detection problems. A fault is any unexpected variation of some physical parameters or variables of a plant causing an unacceptable violation of certain specification limits for normal operation. Frequently, a fault input \( f \) is defined to account for any anomalous behaviour of the plant.

The main component of any FDD system (as that in Fig. 1) is the residual generator (or fault detection filter, or simply fault detector), which produces residual signals grouped in a \( q \)-dimensional vector \( r \) by processing the available measurements \( y \) and the known values of control inputs \( u \). The role of the residual signals is to indicate the presence or absence of faults, and therefore the residual \( r \) must be equal (or close) to zero in the absence of faults and significantly different from zero after a fault occurs. For decision-making, suitable measures of the residual magnitudes (e.g., signal norms) are generated in a vector \( \theta \), which is then used to produce the corresponding decision vector \( \iota \). In what follows, two basic fault monitoring tasks are formulated and discussed.

**Fault detection** is simply a binary decision on the presence of any fault \((f \neq 0)\) or the absence of all faults \((f = 0)\). Typically, \( \theta(t) \) is scalar evaluation signal, which approximates \( ||r||_2 \), the \( \mathcal{L}_2 \)- or \( \ell_2 \)-norms of signal \( r \), while \( \iota(t) \) is a scalar decision making signal defined as \( \iota(t) = 1 \) if \( \theta(t) > \tau \) (fault occurrence) or \( \iota(t) = 0 \) if \( \theta(t) \leq \tau \) (no fault), where \( \tau \) is a suitable threshold quantifying the gap between the “small” and “large” magnitudes of the residual. The decision on the occurrence or absence of faults must be done in the presence of arbitrary control inputs \( u \), disturbance inputs \( d \), and noise inputs \( w \) acting simultaneously on the system. The effects of the control inputs on the residual can be always decoupled by a suitable choice of the residual generation filter. In the ideal case, when no noise inputs are present \((w \equiv 0)\), the residual generation filter must additionally be able to **exactly** decouple the effects of the disturbances inputs in the residual and ensure, simultaneously, the sensitivity of the residual to all faults (i.e., complete fault detectability, see Section 2.4). In this case, \( \tau = 0 \) can be (ideally) used. However, in the general case when \( w \neq 0 \), only an approximate decoupling of \( w \) can be achieved (at best) and a sufficient gap must exist between the magnitudes of residuals in fault-free and faulty situations. Therefore, an appropriate choice of \( \tau > 0 \) must avoid false alarms and missed detections.

**Fault isolation** concerns with the exact localization of occurred faults and involves for each component \( f_j \) of the fault vector \( f \) the decision on the presence of \( j \)-th fault \((f_j \neq 0)\) or its absence \((f_j = 0)\). Ideally, this must be achieved regardless the faults occur one at a time or several faults occur simultaneously. Therefore, the fault isolation task is significantly more difficult than the simpler fault detection. For fault isolation purposes, we will assume a partitioning of the \( q \)-dimensional residual vector \( r \) in \( n_b \) stacked \( q_i \)-dimensional subvectors \( r^{(i)} \), \( i = 1, \ldots, n_b \), in the form

\[
rg = \begin{bmatrix}
  r^{(1)} \\
  \vdots \\
  r^{(n_b)}
\end{bmatrix}, \tag{1}
\]
where \( q = \sum_{i=1}^{n_b} q_i \). A typical fault evaluation setup used for fault isolation is to define \( \theta_i(t) \), the \( i \)-th component of \( \theta(t) \), as a real-time computable approximation of \( \| r^{(i)} \|_2 \). The \( i \)-th component of \( \iota(t) \) is set to \( \iota_i(t) = 1 \) if \( \theta_i(t) > \tau_i \) (\( i \)-th residual fired) or \( \iota_i(t) = 0 \) if \( \theta_i(t) \leq \tau_i \) (\( i \)-th residual not fired), where \( \tau_i \) is a suitable threshold for the \( i \)-th subvector \( r^{(i)}(t) \). If a sufficiently large number of measurements are available, then it can be aimed that \( r^{(i)} \) is influenced only by the \( i \)-th fault signal \( f_i \). This setting, with \( n_b \) chosen equal to the actual number of fault components, allows strong fault isolation, where an arbitrary number of simultaneous faults can be isolated. The isolation of the \( i \)-th fault is achieved if \( \iota_i(t) = 1 \), while for \( \iota_i(t) = 0 \) the \( i \)-th fault is not present. In many practical applications, the lack of a sufficiently large number of measurements impedes strong isolation of simultaneous faults. Therefore, often only weak fault isolation can be performed under simplifying assumptions as, for example, that the faults occur one at a time or no more than two faults may occur simultaneously. The fault isolation schemes providing weak fault isolation compare the resulting \( n_b \)-dimensional binary decision vector \( \iota(t) \), with a predefined set of binary fault signatures. If each individual fault \( f_j \) has associated a distinct signature \( s_j \), then the \( j \)-th fault can be isolated by simply checking that \( \iota(t) \) matches the associated signature \( s_j \). Similarly to fault detection, besides the decoupling of the control inputs \( u \) from the residual \( r \) (always possible), the exact decoupling of the disturbance inputs \( d \) from \( r \) can be strived in the case when \( w \equiv 0 \). However, in the general case when \( w \neq 0 \), only approximate decoupling of \( w \) can be achieved (at best) and a careful selection of tolerances \( \tau_i \) is necessary to perform fault isolation without false alarms and missed detections.

### 2.2 Plant Models with Additive Faults

The following input-output representation is used to describe LTI systems with additive faults

\[
y(\lambda) = G_u(\lambda)u(\lambda) + G_d(\lambda)d(\lambda) + G_f(\lambda)f(\lambda) + G_w(\lambda)w(\lambda),
\]

where \( y(\lambda), u(\lambda), d(\lambda), f(\lambda), \) and \( w(\lambda) \), with boldface notation, denote the Laplace-transformed (in the continuous-time case) or Z-transformed (in the discrete-time case) time-dependent vectors, namely, the \( p \)-dimensional system output vector \( y(t) \), \( m_u \)-dimensional control input vector \( u(t) \), \( m_d \)-dimensional disturbance input vector \( d(t) \), \( m_f \)-dimensional fault vector \( f(t) \), and \( m_w \)-dimensional noise input vector \( w(t) \) respectively. \( G_u(\lambda), G_d(\lambda), G_f(\lambda) \) and \( G_w(\lambda) \) are the transfer-function matrices (TFMs) from the control inputs \( u \), disturbance inputs \( d \), fault inputs \( f \), and noise inputs \( w \) to the outputs \( y \), respectively. According to the system type, \( \lambda = s \), the complex variable in the Laplace-transform in the case of a continuous-time system or \( \lambda = z \), the complex variable in the Z-transform in the case of a discrete-time system. For most of practical applications, the TFMs \( G_u(\lambda), G_d(\lambda), G_f(\lambda) \) and \( G_w(\lambda) \) are proper rational matrices. However, for complete generality of our problem settings, we will allow that these TFMs are general improper rational matrices for which we will not a priori assume any further properties (e.g., stability, full rank, etc.).

The main difference between the disturbance input \( d(t) \) and noise input \( w(t) \) arises from the formulation of the fault monitoring goals. In this respect, when synthesizing devices to serve for fault diagnosis purposes, we will generally target the exact decoupling of the effects of disturbance inputs. Since generally the exact decoupling of effects of noise inputs is not achievable, we will simultaneously try to attenuate their effects, to achieve an approximate decoupling.
sequently, we will try to solve synthesis problems exactly or approximately, in accordance with the absence or presence of noise inputs in the underlying plant model, respectively.

An equivalent descriptor state-space realization of the input-output model (2) has the form

\[
E \lambda x(t) = Ax(t) + B_d d(t) + B_f f(t) + B_w w(t),
\]

\[
y(t) = C x(t) + D_d u(t) + D_f f(t) + D_w w(t),
\]  

(3)

with the \( n \)-dimensional state vector \( x(t) \), where \( \lambda x(t) = \dot{x}(t) \) or \( \lambda x(t) = x(t+1) \) depending on the type of the system, continuous- or discrete-time, respectively. In general, the square matrix \( E \) can be singular, but we will assume that the linear pencil \( A - \lambda E \) is regular. For systems with proper TFMs in (2), we can always choose a standard state-space realization where \( E = I \). In general, it is advantageous to choose the representation (3) minimal, with the pair \( (A - \lambda E, C) \) observable and the pair \( (A - \lambda E, [B_u B_d B_f B_w]) \) controllable. The corresponding TFMs of the model in (2) are

\[
G_u(\lambda) = C(\lambda E - A)^{-1}B_u + D_u,
\]

\[
G_d(\lambda) = C(\lambda E - A)^{-1}B_d + D_d,
\]

\[
G_f(\lambda) = C(\lambda E - A)^{-1}B_f + D_f,
\]

\[
G_w(\lambda) = C(\lambda E - A)^{-1}B_w + D_w
\]

(4)
or in an equivalent notation

\[
\begin{bmatrix}
G_u(\lambda) & G_d(\lambda) & G_f(\lambda) & G_w(\lambda)
\end{bmatrix} := \begin{bmatrix}
A - \lambda E & B_u & B_d & B_f & B_w \\
C & D_u & D_d & D_f & D_w
\end{bmatrix}.
\]

2.3 Residual Generation

A linear residual generator (or fault detection filter) processes the measurable system outputs \( y(t) \) and known control inputs \( u(t) \) and generates the residual signals \( r(t) \) which serve for decision-making on the presence or absence of faults. The input-output form of this filter is

\[
r(\lambda) = Q(\lambda) \begin{bmatrix}
y(\lambda) \\
u(\lambda)
\end{bmatrix} = Q_y(\lambda)y(\lambda) + Q_u(\lambda)u(\lambda),
\]

(5)

with \( Q(\lambda) = [Q_y(\lambda) \quad Q_u(\lambda)] \), and is called the implementation form. The TFM \( Q(\lambda) \) for a physically realizable filter must be proper (i.e., only with finite poles) and stable (i.e., only with poles having negative real parts for a continuous-time system or magnitudes less than one for a discrete-time system). The dimension \( q \) of the residual vector \( r(t) \) depends on the fault detection problem to be addressed.

The residual signal \( r(t) \) in (5) generally depends on all system inputs \( u(t), d(t), f(t) \) and \( w(t) \) via the system output \( y(t) \). The internal form of the filter is obtained by replacing in (5) \( y(\lambda) \) by its expression in (2), and is given by

\[
r(\lambda) = R(\lambda) \begin{bmatrix}
u(\lambda) \\
d(\lambda) \\
f(\lambda) \\
w(\lambda)
\end{bmatrix} = R_u(\lambda)u(\lambda) + R_d(\lambda)d(\lambda) + R_f(\lambda)f(\lambda) + R_w(\lambda)w(\lambda),
\]

(6)
with \( R(\lambda) = [R_u(\lambda) \ R_d(\lambda) \ R_f(\lambda) \ R_w(\lambda)] \) defined as

\[
\begin{bmatrix}
R_u(\lambda) & R_d(\lambda) & R_f(\lambda) & R_w(\lambda)
\end{bmatrix} := Q(\lambda)
\begin{bmatrix}
G_u(\lambda) & G_d(\lambda) & G_f(\lambda) & G_w(\lambda) \\
I_{m_u} & 0 & 0 & 0
\end{bmatrix}.
\] (7)

For a properly designed filter \( Q(\lambda) \), the corresponding internal representation \( R(\lambda) \) is also a proper and stable system, and additionally fulfills specific fault detection and isolation requirements.

### 2.4 Fault Detectability

The concepts of fault detectability and complete fault detectability deal with the sensitivity of the residual to an individual fault and to all faults, respectively. For the discussion of these concepts we will assume that no noise input is present in the system model (2) \((w \equiv 0)\).

**Definition 1.** For the system (2), the \(j\)-th fault \(f_j\) is **detectable** if there exists a fault detection filter \(Q(\lambda)\) such that for all control inputs \(u\) and all disturbance inputs \(d\), the residual \(r \neq 0\) if \(f_j \neq 0\) and \(f_k = 0\) for all \(k \neq j\).

**Definition 2.** The system (2) is **completely fault detectable** if there exists a fault detection filter \(Q(\lambda)\) such that for each \(j\), \(j = 1, \ldots, m_f\), all control inputs \(u\) and all disturbance inputs \(d\), the residual \(r \neq 0\) if \(f_j \neq 0\) and \(f_k = 0\) for all \(k \neq j\).

We have the following results, proven in [11], which characterize the fault detectability and the complete fault detectability properties.

**Proposition 1.** For the system (2) the \(j\)-th fault is detectable if and only if

\[
\text{rank} \begin{bmatrix} G_d(\lambda) & G_f(j, \lambda) \end{bmatrix} > \text{rank} G_d(\lambda),
\] (8)

where \(G_f(j, \lambda)\) is the \(j\)-th column of \(G_f(\lambda)\) and \(\text{rank}(\cdot)\) is the normal rank (i.e., over rational functions) of a rational matrix.

**Theorem 1.** The system (2) is completely fault detectable if and only if

\[
\text{rank} \begin{bmatrix} G_d(\lambda) & G_f(j, \lambda) \end{bmatrix} > \text{rank} G_d(\lambda), \quad j = 1, \ldots, m_f.
\] (9)

Strong fault detectability is a concept related to the reliability and easiness of performing fault detection. The main idea behind this concept is the ability of the residual generators to produce persistent residual signals in the case of persistent fault excitation. For example, for reliable fault detection it is advantageous to have an asymptotically non-vanishing residual signal in the case of persistent faults as step or sinusoidal signals. On the contrary, the lack of strong fault detectability may make the detection of these type of faults more difficult, because their effects manifest in the residual only during possibly short transients, thus the effect disappears in the residual after an enough long time although the fault itself still persists.

The definitions of strong fault detectability and complete strong fault detectability cover several classes of persistent fault signals. Let \(\partial \mathcal{C}_s\) denote the boundary of the stability domain,
which, in the case of a continuous-time system, is the extended imaginary axis (including also the infinity), while in the case of a discrete-time system, is the unit circle centered in the origin. Let \( \Omega \subset \partial \mathbb{C}_s \) be a set of complex frequencies, which characterize the classes of persistent fault signals in question. Common choices in a continuous-time setting are \( \Omega = \{0\} \) for a step signal or \( \Omega = \{i\omega\} \) for a sinusoidal signal of frequency \( \omega \). However, \( \Omega \) may contain several such frequency values or even a whole interval of frequency values, such as \( \Omega = \{i\omega \mid \omega \in [\omega_1, \omega_2]\} \). We denote by \( F_\Omega \) the class of persistent fault signals characterized by \( \Omega \).

**Definition 3.** For the system (2) and a given set of frequencies \( \Omega \subset \partial \mathbb{C}_s \), the \( j \)-th fault \( f_j \) is **strong fault detectable** with respect to \( \Omega \) if there exists a stable fault detection filter \( Q(\lambda) \) such that for all control inputs \( u \) and all disturbance inputs \( d \), the residual \( r(t) \neq 0 \) for \( t \to \infty \) if \( f_j \in F_\Omega \) and \( f_k = 0 \) for all \( k \neq j \).

**Definition 4.** The system (2) is **completely strong fault detectable** with respect to a given set of frequencies \( \Omega \subset \partial \mathbb{C}_s \), if there exists a stable fault detection filter \( Q(\lambda) \) such that for each \( j = 1, \ldots, m_f \), all control inputs \( u \) and all disturbance inputs \( d \), the residual \( r(t) \neq 0 \) for \( t \to \infty \) if \( f_j \in F_\Omega \) and \( f_k = 0 \) for all \( k \neq j \).

For a given stable filter \( Q(\lambda) \) checking the strong detection property of the filter for the \( j \)-th fault \( f_j \) involves to check that \( R_{f_j}(\lambda) \) has no zeros in \( \Omega \). A characterization of strong detectability as a system property is given in what follows.

**Theorem 2.** Let \( \Omega \subset \partial \mathbb{C}_s \) be a given set of frequencies. For the system (2), \( f_j \) is strong fault detectable with respect to \( \Omega \) if and only if \( f_j \) is fault detectable and the rational matrices \( G_{e,j}(\lambda) \) and \( \begin{bmatrix} G_{e,j}(\lambda) \\ F_{e}(\lambda) \end{bmatrix} \) have the same zero structure for each \( \lambda_z \in \Omega \), where

\[
G_{e,j}(\lambda) := \begin{bmatrix} G_{f_j}(\lambda) & G_u(\lambda) & G_d(\lambda) \\ 0 & I_{m_u} & 0 \end{bmatrix}, \quad F_{e}(\lambda) := \begin{bmatrix} 1 & 0_{1 \times m_u} & 0_{1 \times m_d} \end{bmatrix}.
\]

**Remark 1.** Strong fault detectability implies fault detectability, which can be thus assimilated with a kind of weak fault detectability property. For the characterization of the strong fault detectability, we can impose a weaker condition, involving only the existence of a filter \( Q(\lambda) \) without poles in \( \Omega \) (instead imposing stability). For such a filter \( Q(\lambda) \), the stability can always be achieved by replacing \( Q(\lambda) \) by \( M(\lambda)Q(\lambda) \), where \( M(\lambda) \) is a stable and invertible TFM without zeros in \( \Omega \). Such an \( M(\lambda) \) can be determined from a left coprime factorization with least order denominator of \( [Q(\lambda) \ R_f(\lambda)] \).

For complete strong fault detectability the strong fault detectability of each individual fault is necessary, however, it is not a sufficient condition. The following theorem gives a general characterization of the complete strong fault detectability as a system property.

**Theorem 3.** Let \( \Omega \) be the set of frequencies which characterize the persistent fault signals. The system (2) with \( w \equiv 0 \) is completely strong fault detectable with respect to \( \Omega \) if and only if each fault \( f_j \), for \( j = 1, \ldots, m_f \), is strong fault detectable with respect to \( \Omega \) and all \( G_{f_j}(\lambda) \), for \( j = 1, \ldots, m_f \), have the same pole structure in \( \lambda_p \) for all \( \lambda_p \in \Omega \).
2.5 Fault Isolability

While the detectability of a fault can be individually defined and checked, for the definition of fault isolability, we need to deal with the interactions among all fault inputs. Therefore for fault isolation, we assume a structuring of the residual vector $r$ into $n_b$ subvectors as in (1), where each individual $q_i$-dimensional subvector $r^{(i)}$ is differently sensitive to faults. We assume that each fault $f_j$ is characterized by a distinct pattern of zeros and ones in a $n_b$-dimensional vector $s_j$ called the signature of the $j$-th fault. Then, fault isolation consists of recognizing which signature matches the resulting decision vector $\iota$ generated by the FDD system in Fig. 1 according to the partitioning of $r$ in (1).

For the discussion of fault isolability, we will assume that no noise input is present in the model (2) ($w \equiv 0$). The structure of the residual vector in (1) corresponds to a $q \times m_f$ TFM $Q(\lambda)$ ($q = \sum_{i=1}^{n_b} q_i$) of the residual generation filter, built by stacking a bank of $n_b$ filters $Q^{(1)}(\lambda), \ldots, Q^{(n_b)}(\lambda)$ as

$$Q(\lambda) = \begin{bmatrix} Q^{(1)}(\lambda) \\ \vdots \\ Q^{(n_b)}(\lambda) \end{bmatrix}. \quad (11)$$

Thus, the $i$-th subvector $r^{(i)}$ is the output of the $i$-th filter with the $q_i \times m_f$ TFM $Q^{(i)}(\lambda)$

$$r^{(i)}(\lambda) = Q^{(i)}(\lambda) \begin{bmatrix} y(\lambda) \\ u(\lambda) \end{bmatrix}. \quad (12)$$

Let $R_f(\lambda)$ be the corresponding $q \times m_f$ fault-to-residual TFM in (6) and we denote $R^{(i)}_{f_j}(\lambda) := Q^{(i)}(\lambda) \begin{bmatrix} G_{f_j}(\lambda) \\ 0 \end{bmatrix}$, the $q_i \times 1$ $(i,j)$-th block of $R_f(\lambda)$ which describes how the $j$-th fault $f_j$ influences the $i$-th residual subvector $r^{(i)}$. Thus, $R_f(\lambda)$ is an $n_b \times m_f$ block-structured TFM of the form

$$R_f(\lambda) = \begin{bmatrix} R^{(1)}_{f_1}(\lambda) & \cdots & R^{(1)}_{f_{m_f}}(\lambda) \\ \vdots & \ddots & \vdots \\ R^{(n_b)}_{f_1}(\lambda) & \cdots & R^{(n_b)}_{f_{m_f}}(\lambda) \end{bmatrix}. \quad (13)$$

We associate to such a structured $R_f(\lambda)$ the $n_b \times m_f$ structure matrix $S_{R_f}$ whose $(i,j)$-th element is defined as

$$S_{R_f}(i,j) = \begin{cases} 1 & \text{if } R^{(i)}_{f_j}(\lambda) \neq 0, \\ 0 & \text{if } R^{(i)}_{f_j}(\lambda) = 0. \end{cases} \quad (14)$$

If $S_{R_f}(i,j) = 1$ then we say that the residual component $r^{(i)}$ is sensitive to the $j$-th fault $f_j$, while if $S_{R_f}(i,j) = 0$ then the $j$-th fault $f_j$ is decoupled from $r^{(i)}$.

Fault isolability is a property which involves all faults and this is reflected in the following definition, which relates the fault isolability property to a certain structure matrix $S$. For a given structure matrix $S$, we refer to the $i$-th row of $S$ as the specification associated with the $i$-th residual component $r^{(i)}$, while the $j$-th column of $S$ is called the signature (or code) associated with the $j$-th fault $f_j$. 

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Definition 5. For a given $n_b \times m_f$ structure matrix $S$, the model (2) is $S$-fault isolable if there exists a fault detection filter $Q(\lambda)$ such that $S R_f = S$.

When solving fault isolation problems, the choice of a suitable structure matrix $S$ is an important aspect. This choice is, in general, not unique and several choices may lead to satisfactory synthesis results. In this context, the availability of the maximally achievable structure matrix is of paramount importance, because it allows to construct any $S$ by simply selecting a (minimal) number of achievable specifications (i.e., rows of this matrix). The M-function `genspec`, allows to compute the maximally achievable structure matrix for a given system.

The choice of $S$ should usually reflect the fact that complete fault detectability must be a necessary condition for the $S$-fault isolability. This requirement is fulfilled if $S$ is chosen without zero columns. Also, for the unequivocal isolation of the $j$-th fault, the corresponding $j$-th column of $S$ must be different from all other columns. Structure matrices having all columns pairwise distinct are called weakly isolating. Fault signatures which results as (logical OR) combinations of two or more columns of the structure matrix, can be occasionally employed to isolate simultaneous faults, provided they are distinct from all columns of $S$. In this sense, a structure matrix $S$ which allows the isolation of an arbitrary number of simultaneously occurring faults is called strongly isolating. It is important to mention in this context that a system which is not fault isolable for a given $S$ may still be fault isolable for another choice of the structure matrix.

To characterize the fault isolability property, we observe that each block row $Q(i)(\lambda)$ of the TFM $Q(\lambda)$ is itself a fault detection filter which must achieve the specification contained in the $i$-th row of $S$. Thus, the isolability conditions will consist of a set of $n_b$ independent conditions, each of them characterizing the complete detectability of particular subsets of faults. We have the following straightforward characterization of fault isolability.

Theorem 4. For a given $n_b \times m_f$ structure matrix $S$, the model (2) is $S$-fault isolable if and only if for $i = 1, \ldots, n_b$

$$\operatorname{rank} \left[ G_d(\lambda) \hat{G}_d^{(i)}(\lambda) G_f(\lambda) \right] > \operatorname{rank} \left[ G_d(\lambda) \hat{G}_d^{(i)}(\lambda) \right], \quad \forall j, \ S_{ij} \neq 0,$$

where $\hat{G}_d^{(i)}(\lambda)$ is formed from the columns $G_f(\lambda)$ of $G_f(\lambda)$ for which $S_{ij} = 0$.

The conditions (15) of Theorem 4 give a very general characterization of isolability of faults. An important particular case is strong fault isolability, in which case $S = I_{m_f}$, and thus diagonal. The following result characterizes the strong isolability.

Theorem 5. The model (2) is strongly fault isolable if and only if

$$\operatorname{rank} \left[ G_d(\lambda) G_f(\lambda) \right] = \operatorname{rank} G_d(\lambda) + m_f.$$

Remark 2. In the case $m_d = 0$, the strong fault isolability condition reduces to the left invertibility condition

$$\operatorname{rank} G_f(\lambda) = m_f.$$

This condition is a necessary condition even in the case $m_d \neq 0$ (otherwise $R_f(\lambda)$ would not have full column rank). □
Remark 3. The definition of the structure matrix $S_{Rf}$ associated with a given TFM $R_f(\lambda)$ can be extended to cover the strong fault detectability requirement defined by $\Omega \subset \partial \mathcal{C}_s$, where $\Omega$ is the set of relevant frequencies. For each $\lambda_z \in \Omega$, we can define the strong structure matrix at the complex frequency $\lambda_z$ as

$$S_{Rf}(i,j) = 1 \text{ if } R_f^{(i,j)}(\lambda_z) \neq 0,$$

$$S_{Rf}(i,j) = 0 \text{ if } R_f^{(i,j)}(\lambda_z) = 0. \quad (18)$$

2.6 Fault Detection and Isolation Problems

In this section we formulate three basic exact synthesis problems of fault detection and isolation filters for LTI systems. For the solution of these problems we seek linear residual generators (or fault detection filters) of the form (5), which process the measurable system outputs $y(t)$ and known control inputs $u(t)$ and generate the residual signals $r(t)$, which serve for decision-making on the presence or absence of faults. The standard requirements on all TFM\'s appearing in the implementation form (5) and internal form (6) of the fault detection filter are properness and stability, to ensure physical realizability of the filter $Q(\lambda)$ and to guarantee a stable behaviour of the FDD system. The order of the filter $Q(\lambda)$ is its McMillan degree, that is, the dimension of the state vector of a minimal state-space realization of $Q(\lambda)$. For practical purposes, lower order filters are preferable to larger order ones, and therefore, determining least order residual generators is also a desirable synthesis goal. Finally, while the dimension $q$ of the residual vector $r(t)$ depends on the fault detection problem to be solved, filters with the least number of outputs, are always of interest for practical usage.

For the solution of fault detection and isolation problems it is always possible to completely decouple the control input $u(t)$ from the residual $r(t)$ by requiring $R_u(\lambda) = 0$. Regarding the disturbance input $d(t)$ and noise input $w(t)$ we aim to impose a similar condition on the disturbance input $d(t)$ by requiring $R_d(\lambda) = 0$, while considering the noise input $w(t)$ as an auxiliary input via a stable TFM $G_w(\lambda)$ whose effects on the residual is described thus by a stable $R_w(\lambda)$.

In all fault detection problems formulated in what follows, we require that by a suitable choice of a stable fault detection filter $Q(\lambda)$, we achieve that the residual signal $r(t)$ is fully decoupled from the control input $u(t)$ and disturbance input $d(t)$. Thus, the following decoupling conditions must be fulfilled for the filter synthesis

$$\begin{align*}
(i) & \quad R_u(\lambda) = 0, \\
(ii) & \quad R_d(\lambda) = 0. \quad (19)
\end{align*}$$

In the case when condition (ii) can not be fulfilled (e.g., due to lack of sufficient number of measurements), we can redefine some (or even all) components of $d(t)$ as noise inputs and include them in $w(t)$.

For each fault detection problem formulated in what follows, specific requirements have to be fulfilled, which are formulated as additional synthesis conditions. For all formulated problems we also give the existence conditions of the solutions of these problems. For the proofs of the results consult [11].
2.6.1 EFDP – Exact Fault Detection Problem

For the exact fault detection problem (EFDP) the basic additional requirement is simply to achieve by a suitable choice of a stable and proper fault detection filter $Q(\lambda)$ that, in the absence of noise input (i.e., $w \equiv 0$), the residual $r(t)$ is sensitive to all fault components $f_j(t), j = 1, \ldots, m_f$. If a noise input $w(t)$ is present, then we assume the TFM $G_w(s)$ is stable (thus $R_w(\lambda)$ is stable too). Thus, the following detection condition has to be fulfilled:

$$\text{(iii) } R_{f_j}(\lambda) \neq 0, \quad j = 1, \ldots, m_f \text{ with } R_f(\lambda) \text{ stable.}$$  \hspace{1cm} (20)

This is precisely the complete fault detectability requirement (without the stability condition) and leads to the following solvability condition:

**Theorem 6.** For the system (2), the EFDP is solvable if and only if the system (2) is completely fault detectable.

Let $\Omega \subset \partial C_s$ be a given set of frequencies which characterize the relevant persistent faults. We can give a similar result in the case when the EFDP is solved with a strong detection condition:

$$\text{(iii)' } R_{f_j}(\lambda_z) \neq 0, \quad \forall \lambda_z \in \Omega, \quad j = 1, \ldots, m_f \text{ with } R_f(\lambda) \text{ stable.}$$  \hspace{1cm} (21)

The solvability condition of the EFDP with the strong detection condition above is precisely the complete strong fault detectability requirement as stated by the following theorem.

**Theorem 7.** Let $\Omega$ be the set of frequencies which characterize the persistent fault signals. For the system (2), the EFDP with the strong detection condition (21) is solvable if and only if the system (2) is completely strong fault detectable with respect to $\Omega$.

2.6.2 EFDIP – Exact Fault Detection and Isolation Problem

For a row-block structured fault detection filter $Q(\lambda)$ as in (11), let $R_f(\lambda)$ be the corresponding block-structured fault-to-residual TFM as defined in (13) with $n_b \times m_f$ blocks, and let $S_{R_f}$ be the corresponding $n_b \times m_f$ structure matrix defined in (14) (see Section 2.5). Let $s_j, j = 1, \ldots, m_f$ be a set of $n_b$-dimensional binary signature vectors associated to the faults $f_j, j = 1, \ldots, m_f$, which form the desired structure matrix $S := [s_1 \ldots s_{m_f}]$. The exact fault detection and isolation problem (EFDIP) requires to determine for a given $n_b \times m_f$ structure matrix $S$, a stable and proper filter $Q(\lambda)$ of the form (11) such that the following condition is additionally fulfilled:

$$\text{(iii) } S_{R_f} = S, \quad \text{with } R_f(\lambda) \text{ stable.}$$  \hspace{1cm} (22)

We have the following straightforward solvability condition:

**Theorem 8.** For the system (2) with $w \equiv 0$ and a given structure matrix $S$, the EFDIP is solvable if and only if the system (2) is $S$-fault isolable.

A similar result can be established for the case when $S$ is the $m_f$-th order identity matrix $S = I_{m_f}$. We call the associated synthesis problem the strong EFDIP. The proof is similar to that of Theorem 8.

**Theorem 9.** For the system (2) with $w \equiv 0$ and $S = I_{m_f}$, the EFDIP is solvable if and only if the system (2) is strongly fault isolable.
2.6.3 EMMP – Exact Model-Matching Problem

Let $M_r(\lambda)$ be a given $q \times m_f$ TFM of a stable and proper reference model specifying the desired input-output behaviour from the faults to residuals as $r(\lambda) = M_r(\lambda)f(\lambda)$. Thus, we want to achieve by a suitable choice of a stable and proper $Q(\lambda)$ satisfying (i) and (ii) in (19), that we have additionally $R_f(\lambda) = M_r(\lambda)$. For example, a typical choice for $M_r(\lambda)$ is an $m_f \times m_f$ diagonal and invertible TFM, which ensures that each residual $r_i(t)$ is influenced only by the fault $f_i(t)$. The choice $M_r(\lambda) = I_m$ targets the solution of an exact fault estimation problem (EFEP).

To determine $Q(\lambda)$, we have to solve the linear rational equation (7), with the settings $R_u(\lambda) = 0$, $R_d(\lambda) = 0$, and $R_f(\lambda) = M_r(\lambda)$ ($R_u(\lambda)$ and $G_w(\lambda)$ are assumed empty matrices). The choice of $M_r(\lambda)$ may lead to a solution $Q(\lambda)$ which is not proper or is unstable or has both these undesirable properties. Therefore, besides determining $Q(\lambda)$, we also consider the determination of a suitable updating factor $Q(\lambda)$ of $M_r(\lambda)$ to ensure the stability and properness of the solution $Q(\lambda)$ for $R_f(\lambda) = M(\lambda)M_r(\lambda)$. Obviously, $M(\lambda)$ must be chosen a proper, stable and invertible TFM. Additionally, by choosing $M(\lambda)$ diagonal, the zero and nonzero entries of $M_r(\lambda)$ can be also preserved in $R_f(\lambda)$ (see also Section 2.6.2).

The exact model-matching problem (EMMP) can be formulated as follows: given a stable and proper $M_r(\lambda)$, it is required to determine a stable and proper filter $Q(\lambda)$ and a diagonal, proper, stable and invertible TFM $M(\lambda)$ such that, additionally to (19), the following condition is fulfilled:

$$ (iii) \quad R_f(\lambda) = M(\lambda)M_r(\lambda). \quad (23) $$

The solvability condition of the EMMP is the standard solvability condition of systems of linear equations:

**Theorem 10.** For the system (2) with $w \equiv 0$ and a given $M_r(\lambda)$, the EMMP is solvable if and only if the following condition is fulfilled

$$ \text{rank} \begin{bmatrix} G_d(\lambda) & G_f(\lambda) \end{bmatrix} = \text{rank} \begin{bmatrix} G_d(\lambda) & G_f(\lambda) \\ 0 & M_r(\lambda) \end{bmatrix}. \quad (24) $$

**Remark 4.** When $M_r(\lambda)$ has full column rank $m_f$, the solvability condition (24) of the EMMP reduces to the strong isolability condition (16) (see also Theorem 9).

**Remark 5.** It is possible to solve a slightly more general EMMP, to determine $Q(\lambda)$ and $M(\lambda)$ as before, such that, for given $R(\lambda) = [R_u(\lambda) \quad R_d(\lambda) \quad R_f(\lambda) \quad R_w(\lambda)]$, they satisfy

$$ Q(\lambda) \begin{bmatrix} G_u(\lambda) \\ G_d(\lambda) \\ 0 \\ G_f(\lambda) \\ 0 \\ 0 \end{bmatrix} = M(\lambda) \begin{bmatrix} R_u(\lambda) \quad R_d(\lambda) \quad R_f(\lambda) \quad R_w(\lambda) \end{bmatrix}. \quad (25) $$

This formulation may arise, for example, if $R(\lambda)$ is the internal form resulted from an approximate synthesis, for which $R_u(\lambda) \approx 0$, $R_d(\lambda) \approx 0$ and $R_w(\lambda) \approx 0$.

The solvability condition is simply that for solving the linear system (25) for $M(\lambda) = I$

$$ \text{rank} \begin{bmatrix} G_d(\lambda) & G_f(\lambda) & G_w(\lambda) \end{bmatrix} = \text{rank} \begin{bmatrix} G_d(\lambda) & G_f(\lambda) & G_w(\lambda) \\ R_d(\lambda) & R_f(\lambda) & R_w(\lambda) \end{bmatrix}. \quad (26) $$

□
The solvability conditions (see Theorem 10) become more involved if we strive for a stable proper solution \( Q(\lambda) \) for a given reference model \( M_r(\lambda) \) without allowing its updating. For example, this is the case when solving the EFEP for \( M_r(\lambda) = I_{mf} \). For a slightly more general case, we have the following result.

**Theorem 11.** For the system (2) with \( w \equiv 0 \) and a given stable and minimum-phase \( M_r(\lambda) \) of full column rank, the EMMP is solvable with \( M(\lambda) = I \) if and only if the system is strongly fault isolable and \( G_f(\lambda) \) is minimum phase.

**Remark 6.** If \( G_f(\lambda) \) has unstable or infinite zeros, the solvability of the EMMP with \( M(\lambda) = I \) is possible provided \( M_r(\lambda) \) is chosen such that

\[
\begin{bmatrix}
G_f(\lambda) & G_d(\lambda)
\end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix}
G_f(\lambda) & G_d(\lambda) \\
M_r(\lambda) & 0
\end{bmatrix}
\]

have the same unstable zero structure. For this it is necessary that \( M_r(\lambda) \) has the same unstable and infinity zeros structure as \( G_f(\lambda) \). \( \square \)
3 Model Detection Basics

In this section we describe first the basic model detection task and introduce and characterize the concept of model detectability. Two model detection problems are formulated in the book [11] relying on LTI multiple models. The formulated synthesis problems, involve the exact synthesis and the approximate synthesis of model detection filters. The current release of FDITOOLS covers the exact synthesis technique described in [11]. The approximate synthesis method will be addressed in a future release of FDITOOLS. Jointly with the formulation of the exact model detection problem, general solvability conditions are given in terms of ranks of certain transfer function matrices. More details and the proofs of the results are available in Chapters 2 and 4 of [11].

3.1 Basic Model Detection Task

Multiple models which describe various fault situations have been frequently used for fault detection purposes. In such applications, the detection of the occurrence of a fault comes down to identifying, using the available measurements from the measurable outputs and control inputs, that model (from a collection of models) which best matches the dynamical behaviour of the faulty plant. The term model detection describes the model identification task consisting of the selection of a model from a collection of $N$ models, which best matches the current dynamical behaviour of a plant.

A typical model detection setting is shown in Fig. 2. A bank of $N$ residual generation filters (or residual generators) is used, with $r^{(i)}(t)$ being the output of the $i$-th residual generator. The $i$-th component $\theta_i$ of the $N$-dimensional evaluation vector $\theta$ usually represents an approximation...

![Figure 2: Basic model detection setup.](image-url)
of \( \|r^{(i)}\|_2 \), the \( L_2 \)- or \( \ell_2 \)-norm of \( r^{(i)} \). The \( i \)-th component of the \( N \)-dimensional decision vector \( \iota \) is set to 0 if \( \theta_i \leq \tau_i \) and 1 otherwise, where \( \tau_i \) is a suitable threshold. The \( j \)-th model is “detected” if \( \iota_j = 0 \) and \( \iota_i = 1 \) for all \( i \neq j \). It follows that model detection can be interpreted as a particular type of week fault isolation with \( N \) signature vectors, where the \( N \)-dimensional \( j \)-th signature vector has all elements set to one, excepting the \( j \)-th entry which is set to zero. An alternative decision scheme can also be devised if \( \theta_i \) can be associated with a distance function from the current model to the \( i \)-th model. In this case, \( \iota \) is a scalar, set to \( \iota = j \), where \( j \) is the index for which \( \theta_j = \min_{i=1:N} \theta_i \). Thus, the decision scheme selects that model \( j \) which best fits with the current model characterized by the measured input and output data.

The underlying synthesis techniques of model detection systems rely on multiple-model descriptions of physical fault cases. Since different degrees of performance degradations can be easily described via multiple models, model detection techniques have potentially the capability to address certain fault identification aspects too.

### 3.2 Multiple Physical Fault Models

For physically modelled faults, each fault mode leads to a distinct model. Assume that we have \( N \) LTI models describing the fault-free and faulty systems, and for \( j = 1, \ldots, N \) the \( j \)-th model is specified in the input-output form

\[
y^{(j)}(\lambda) = G_u^{(j)}(\lambda)u^{(j)}(\lambda) + G_d^{(j)}(\lambda)d^{(j)}(\lambda) + G_w^{(j)}(\lambda)w^{(j)}(\lambda),
\]

where \( y^{(j)}(t) \in \mathbb{R}^p \) is the output vector of the \( i \)-th system with control input \( u^{(j)}(t) \in \mathbb{R}^{m_u} \), disturbance input \( d^{(j)}(t) \in \mathbb{R}^{m_d} \) and noise input \( w^{(j)}(t) \in \mathbb{R}^{m_w} \), and where \( G_u^{(j)}(\lambda) \), \( G_d^{(j)}(\lambda) \) and \( G_w^{(j)}(\lambda) \) are the TFMs from the corresponding plant inputs to outputs. The significance of disturbance and noise inputs, and the basic difference between them, have already been discussed in Section 2.2. The state-space realizations corresponding to the multiple model (28) are for \( j = 1, \ldots, N \) of the form

\[
E^{(j)}x^{(j)}(t) = A^{(j)}x^{(j)}(t) + B_u^{(j)}u^{(j)}(t) + B_d^{(j)}d^{(j)}(t) + B_w^{(j)}w^{(j)}(t),
\]

\[
y^{(j)}(t) = C^{(j)}x^{(j)}(t) + D_u^{(j)}u^{(j)}(t) + D_d^{(j)}d^{(j)}(t) + D_w^{(j)}w^{(j)}(t),
\]

where \( x^{(j)}(t) \in \mathbb{R}^{n^{(j)}} \) is the state vector of the \( j \)-th system and, generally, can have different dimensions for different systems.

The multiple-model description represents a very general way to describe plant models with various faults. For example, extreme variations of parameters representing the so-called parametric faults, can be easily described by multiple models.

### 3.3 Residual Generation

Assume we have \( N \) LTI models of the form (28), for \( j = 1, \ldots, N \), but the \( N \) models originate from a common underlying system with \( y(t) \in \mathbb{R}^p \), the measurable output vector, and \( u(t) \in \mathbb{R}^{m_u} \), the known control input. Therefore, \( y^{(j)}(t) \in \mathbb{R}^p \) is the output vector of the \( j \)-th system with the control input \( u^{(j)}(t) \in \mathbb{R}^{m_u} \), disturbance input \( d^{(j)}(t) \in \mathbb{R}^{m_d} \) and noise input \( w^{(j)}(t) \in \mathbb{R}^{m_w} \),
respectively, and $G_u^{(j)}(\lambda)$, $G_d^{(j)}(\lambda)$ and $G_w^{(j)}(\lambda)$ are the TFMs from the corresponding plant inputs to outputs. We assume that all models are controlled with the same control inputs $u^{(j)}(t) := u(t)$, but the disturbance and noise inputs $d^{(j)}(t)$ and $w^{(j)}(t)$, respectively, may differ for each component model. For complete generality of our problem formulations, we will allow that these TFMs are general rational matrices (proper or improper) for which we will not a priori assume any further properties.

Residual generation for model detection is performed using $N$ linear residual generators, which process the measurable system outputs $y(t)$ and known control inputs $u(t)$ and generate $N$ residual signals $r^{(j)}(t)$, $i = 1, \ldots, N$, which serve for decision making on which model best matches the current input-output measurement data. As already mentioned, model detection can be interpreted as a week fault isolation problem with an $N \times N$ structure matrix $S$ having all its elements equal to one, excepting those on its diagonal which are zero. The task of model detection is thus to find out the model which best matches the measurements of outputs and inputs, by comparing the resulting decision vector $\iota$ with the set of signatures associated to each model and coded in the columns of $S$. The $N$ residual generation filters in their implementation form are described for $i = 1, \ldots, N$, by the input-output relations

$$r^{(i)}(\lambda) = Q^{(i)}(\lambda) \begin{bmatrix} y(\lambda) \\ u(\lambda) \end{bmatrix}, \quad (30)$$

where $y$ and $u$ is the actual measured system output and control input, respectively. The TFMs $Q^{(i)}(\lambda)$, for $i = 1, \ldots, N$, must be proper and stable. The dimension $q_i$ of the residual vector component $r^{(i)}(t)$ can be chosen always one, but occasionally values $q_i > 1$ may provide better sensitivity to model mismatches.

Assuming $y(t) = y^{(j)}(t)$, the residual signal component $r^{(i)}(t)$ in (30) generally depends on all system inputs $u^{(j)}(t)$, $d^{(j)}(t)$ and $u^{(j)}(t)$ via the system output $y^{(j)}(t)$. The internal form of the $i$-th filter driven by the $j$-th model is obtained by replacing in (30) $y(\lambda)$ with $y^{(j)}(\lambda)$ from (28) and $u(\lambda)$ with $u^{(j)}(\lambda)$. To make explicit the dependence of $r^{(i)}$ on the $j$-th model, we will use $\tilde{r}^{(i,j)}$, to denote the $i$-th residual output for the $j$-th model. After replacing in (30), $y(\lambda)$ with $y^{(j)}(\lambda)$ from (28), and $u(\lambda)$ with $u^{(j)}(\lambda)$, we obtain

$$\tilde{r}^{(i,j)}(\lambda) := R^{(i,j)}(\lambda) \begin{bmatrix} u^{(j)}(\lambda) \\ d^{(j)}(\lambda) \\ w^{(j)}(\lambda) \end{bmatrix}, \quad (31)$$

with $R^{(i,j)}(\lambda) := \begin{bmatrix} R_u^{(i,j)}(\lambda) & R_d^{(i,j)}(\lambda) & R_w^{(i,j)}(\lambda) \end{bmatrix}$ defined as

$$\begin{bmatrix} R_u^{(i,j)}(\lambda) & R_d^{(i,j)}(\lambda) & R_w^{(i,j)}(\lambda) \end{bmatrix} := Q^{(i)}(\lambda) \begin{bmatrix} G_u^{(j)}(\lambda) & G_d^{(j)}(\lambda) & G_w^{(j)}(\lambda) \\ I_{m_u} & 0 & 0 \end{bmatrix}. \quad (32)$$

For a successfully designed set of filters $Q^{(i)}(\lambda)$, $i = 1, \ldots, N$, the corresponding internal representations $R^{(i,j)}(\lambda)$ in (31) are also a proper and stable.
3.4 Model Detectability

The concept of model detectability concerns with the sensitivity of the components of the residual vector to individual models from a given collection of models. Assume that we have \( N \) models, with the \( j \)-th model specified in the input-output form (28). For the discussion of the model detectability concept we will assume that no noise inputs are present in the models (28) (i.e., \( w^{(j)} \equiv 0 \) for \( j = 1, \ldots, N \)). For model detection purposes, \( N \) filters of the form (30) are employed. It follows from (31) that the \( i \)-th component \( r^{(i)} \) of the residual \( r \) is sensitive to the \( j \)-th model provided

\[
R^{(i,j)}(\lambda) := \begin{bmatrix} R^{(i,j)}(\lambda) \\ R^{(i,j)}(\lambda) \end{bmatrix} \neq 0. \tag{33}
\]

We can associate to the \( N \times N \) blocks \( R^{(i,j)}(\lambda) \) defined in (33), the \( N \times N \) structure matrix \( S_R \) with the \((i,j)\)-th element set to 1 if \( R^{(i,j)}(\lambda) \neq 0 \) and set to 0 if \( R^{(i,j)}(\lambda) = 0 \). As already mentioned, model detection can be interpreted as a week fault isolation problem with an \( N \times N \) structure matrix \( S \) having all its elements equal to one, excepting those on its diagonal which are zero. Having this analogy in mind, we introduce the following concept of model detectability.

**Definition 6.** The multiple model defined by the \( N \) component systems (28) with \( w^{(j)} \equiv 0 \) for \( j = 1, \ldots, N \), is model detectable if there exist \( N \) filters of the form (30), such that \( R^{(i,j)}(\lambda) \) defined in (33) fulfills \( R^{(i,i)}(\lambda) = 0 \) for \( i = 1, \ldots, N \) and \( R^{(i,j)}(\lambda) \neq 0 \) for all \( i, j = 1, \ldots, N \) such that \( i \neq j \).

The following result, proven in [11], characterizes the introduced model detectability property.

**Theorem 12.** The multiple model defined by the \( N \) component systems (28) with \( w^{(j)} \equiv 0 \) for \( j = 1, \ldots, N \), is model detectable if and only if for \( i = 1, \ldots, N \)

\[
\text{rank} \left[ G^{(i)}_a(\lambda) \ G^{(j)}_a(\lambda) \ G^{(i)}_d(\lambda) - G^{(j)}_d(\lambda) \right] > \text{rank} G^{(i)}_d(\lambda) \ \forall j \neq i. \tag{34}
\]

We can define the concept of strong model detectability with respect to classes of persistent input and disturbance signals characterized by a set of complex frequencies \( \Omega \subset \mathbb{C}_s \), in analogy with the definition of the strong fault detectability with respect to \( \Omega \).

**Definition 7.** The multiple model defined by the \( N \) component systems (28) with \( w^{(j)} \equiv 0 \) for \( j = 1, \ldots, N \), is strong model detectable with respect to \( \Omega \) if there exist \( N \) stable filters of the form (30), such that \( R^{(i,j)}(\lambda) \) defined in (33) fulfills \( R^{(i,i)}(\lambda) = 0 \) for \( i = 1, \ldots, N \) and \( R^{(i,j)}(\lambda_z) \neq 0 \) for all \( i, j = 1, \ldots, N \) such that \( i \neq j \) and for all \( \lambda_z \in \Omega \).

The following result characterizes the strong model detectability property in an important practice relevant situation.

**Theorem 13.** Let \( \Omega \subset \partial \mathbb{C}_s \) be a given set of frequencies, such that none of \( \lambda_z \in \Omega \) is a pole of any of the component system (28), for \( j = 1, \ldots, N \). Then, the multiple model (28) with \( w^{(j)} \equiv 0 \) for \( j = 1, \ldots, N \), is strong model detectable with respect to \( \Omega \) if and only if for \( i = 1, \ldots, N \)

\[
\text{rank} \left[ G^{(i)}_a(\lambda_z) \ G^{(j)}_a(\lambda_z) \ G^{(i)}_d(\lambda_z) - G^{(j)}_d(\lambda_z) \right] > \text{rank} G^{(i)}_d(\lambda_z) \ \forall \lambda_z \in \Omega \ 	ext{and} \ \forall j \neq i. \tag{35}
\]
3.5 Model Detection Problems

In this section we formulate the exact synthesis problems of model detection filters for the collection of $N$ LTI systems (28) (the approximate synthesis problem is not yet addressed in FDITOOLS). As in the case of the EFDIP, we seek $N$ linear residual generators (or model detection filters) of the form (30), which process the measurable system outputs $y(t)$ and known control inputs $u(t)$ and generate the $N$ residual signals $r^{(i)}(t)$ for $i = 1, \ldots, N$. These signals serve for decision-making by comparing the pattern of fired and not fired residuals with the signatures coded in the columns of the associated standard $N \times N$ structure matrix $S$ with zeros on the diagonal and ones elsewhere. The standard requirements for the TFM of the filters $Q^{(i)}(\lambda)$ in (30) are properness and stability. For practical purposes, the orders of the filter $Q^{(i)}(\lambda)$ must be as small as possible. Least order filters $Q^{(i)}(\lambda)$ can be usually achieved by employing scalar output least order filters.

In analogy to the formulations of the EFDIP, we use the internal form of the $i$-th residual generator (31) to formulate the basic model detection requirements. Independently of the presence of the noise inputs $w^{(j)}$, we will target that the $i$-th residual is exactly decoupled from the $i$-th model if $w^{(i)} \equiv 0$ and is sensitive to the $j$-th model, for all $j \neq i$. These requirements can be easily translated into algebraic conditions using the internal form (31) of the $i$-th residual generator:

\begin{align}
(i) \quad [R^{(i,i)}_u(\lambda) \quad R^{(i,i)}_d(\lambda)] &= 0, \quad i = 1, \ldots, N, \\
(ii) \quad [R^{(i,j)}_u(\lambda) \quad R^{(i,j)}_d(\lambda)] &\neq 0, \quad \forall j \neq i, \quad \text{with} \quad [R^{(i,j)}_u(\lambda) \quad R^{(i,j)}_d(\lambda)] \text{ stable}. \quad (36)
\end{align}

Here, $(i)$ is the model decoupling condition for the $i$-th model in the $i$-th residual component, while $(ii)$ is the model sensitivity condition of the $i$-th residual component to all models, excepting the $i$-th model. In the case when condition $(i)$ cannot be fulfilled (e.g., due to lack of sufficient measurements), some (or even all) components of $d^{(i)}(t)$ can be redefined as noise inputs and included in $w^{(i)}(t)$.

In what follows, we formulate the exact model detection problem, for which we give the existence conditions of the solution. For the proof of the results consult [11].

3.5.1 EMDP – Exact Model Detection Problem

The standard requirement for solving the exact model detection problem (EMDP) is to determine for the multiple model (28), in the absence of noise input (i.e., $w^{(j)} \equiv 0$ for $j = 1, \ldots, N$), a set of $N$ proper and stable filters $Q^{(i)}(\lambda)$ such that, for $i = 1, \ldots, N$, the conditions (36) are fulfilled. These conditions are similar to the model detectability requirement and lead to the following solvability condition:

**Theorem 14.** For the multiple model (28) with $w^{(j)} \equiv 0$ for $j = 1, \ldots, N$, the EMDP is solvable if and only if the multiple model (28) is model detectable.

Let $\Omega \subset \partial C_s$ be a given set of frequencies which characterize the relevant persistent input and disturbance signals. We can give a similar result in the case when the EMDP is solved, by replacing the condition $(ii)$ in (36), with the strong model detection condition:

\begin{align}
(ii)' \quad [R^{(i,j)}_u(\lambda_z) \quad R^{(i,j)}_d(\lambda_z)] &\neq 0, \quad \forall \lambda_z \in \Omega \text{ and} \forall j \neq i, \quad \text{with} \quad [R^{(i,j)}_u(\lambda) \quad R^{(i,j)}_d(\lambda)] \text{ stable}. \quad (37)
\end{align}
The solvability condition of the EMDP with the strong model detection condition above is precisely the strong model detectability requirement as stated by the following theorem.

**Theorem 15.** Let $\Omega$ be the set of frequencies which characterize the persistent input and disturbance signals. For the multiple model (28) with $w^{(j)} \equiv 0$ for $j = 1, \ldots, N$, the EMDP with the strong model detectability condition (37) is solvable if and only if the multiple model (28) is strong model detectable.
4 Description of FDITOOLS

This user’s guide is intended to provide users basic information on the FDITOOLS collection to solve the fault detection and isolation problems formulated in Section 2.6 and the model detection problem formulated in Section 3.5. The notations and terminology used throughout this guide have been introduced and extensively discussed in the accompanying book [11], which also represents the main reference for the implemented computational methods underlying the analysis and synthesis functions of FDITOOLS. Information on the requirements for installing FDITOOLS are given in Appendix A.

In this section, we present first a short overview of the existing functions of FDITOOLS and then, we illustrate a typical work flow by solving an EFDIP. In-depth information on the command syntax of the functions of the FDITOOLS collection is given is Sections 4.3 and 4.4. To execute the examples presented in this guide, simply paste the presented code sequences into the MATLAB command window. More involved examples are given in several case studies presented in [11].

4.1 Quick Reference Tables

The current release of FDITOOLS is version V0.3, dated April 7, 2017. The corresponding Contents.m file is listed in Appendix B. This section contains quick reference tables for the functions of the FDITOOLS collection. All M-files available in the current version of FDITOOLS are listed below by category, with short descriptions.

| Demonstration                  |
|-------------------------------|
| FDIToolsdemo                  |
| Demonstration of Fault Detection and Isolation Tools | |

| Analysis                      |
|-------------------------------|
| fditspec                      |
| Computation of the structure matrix of a system. |
| fdispec                       |
| Computation of the strong structure matrix of a system. |
| genspec                       |
| Generation of achievable fault detection specifications. |

| Synthesis of fault detection filters |
|--------------------------------------|
| efdsyn                               |
| Exact synthesis of fault detection filters. |
| efdisyn                              |
| Exact synthesis of fault detection and isolation filters. |
| emmsyn                               |
| Exact model matching based synthesis of FDI filters. |

| Synthesis of model detection filters |
|--------------------------------------|
| emdsyn                               |
| Exact synthesis of model detection filters. |

| Miscellaneous                     |
|-----------------------------------|
| efdbasesel                        |
| Selection of admissible basis vectors to solve the EFDP. |

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2 Use [https://sites.google.com/site/andreasvargacontact/home/book/matlab](https://sites.google.com/site/andreasvargacontact/home/book/matlab) to download the case study examples presented in [11].
4.2 Getting Started

In this section we shortly illustrate the typical steps of solving a fault detection and isolation problem, starting with building an adequate fault model, performing preliminary analysis, selecting the suitable synthesis approach, and evaluating the computed results.

4.2.1 Building Models with Additive Faults

In-depth information on how to create and manipulate LTI system models and arrays of LTI system models are available in the online documentation of the Control System Toolbox and in its User’ Guide [1]. These types of models are the basis of the data objects used in the FDITOOLS collection.

The input plant models with additive faults used by all synthesis functions of the FDITOOLS collection are LTI models of the form (2), given via their equivalent descriptor system state-space realizations of the form (3). The object-oriented framework employed in the Control System Toolbox has been used to define the LTI plant models, by defining several input groups corresponding to various input signal. The employed standard definitions of input groups are: 'controls' for the control inputs $u(t)$, 'disturbances' for the disturbance inputs $d(t)$, 'faults' for the fault inputs $f(t)$, and 'noise' for the noise inputs $w(t)$. There are several ways to define input groups, as described in the documentation of the Control System Toolbox [1]. In what follows we illustrate one of these possibilities.

Once you have a plant model for a system without faults, you can construct models with faults using simple commands in the Control System Toolbox. For example, consider a plant model $sys$ with 3 inputs, 3 outputs and 3 state components. Assume that the first two inputs are control inputs which are susceptible to actuator faults and the third input is a disturbance input, which is not measurable and therefore is considered as an unknown input. All outputs are measurable, and assume that the first output is susceptible to sensor fault. Using the following standard Control System Toolbox commands, the plant model with additive faults can be setup as follows:

```matlab
rand('seed',16); randn('seed',16);
sys = rss(3,3,3);
% extract system matrices
[A,B,C,D]=ssdata(sys);
% build input and feedthrough matrices extended with fault inputs
Be = [B(:,1:3), zeros(size(B,1),1), B(:,1:2)]; % Be = [Bu Bd Bf]
De = [D(:,1:3), eye(size(D,1),1), D(:,1:2)]; % De = [Du Dd Df]
% define dimensions of control, disturbance and fault input vectors
mu = 2; md = 1; mf = 3;
% build system with fault inputs
sysf = ss(A,Be,C,De);
```
% define the input groups for control, disturbance and fault inputs
sysf.InputGroup.controls = 1:mu;
sysf.InputGroup.disturbances = mu+(1:md);
sysf.InputGroup.faults = mu+md+(1:mf);

4.2.2 Determining the Achievable Fault Signature

To determine the achievable strong fault signatures with respect to constant faults, the function

genspec

can be used as follows:

\[ S = \text{genspec}(\text{sysf},\text{struct('FDFreq',0)}); \]

For the above example, the possible fault signatures are contained in the generic structure matrix

\[
S = \begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0 \\
1 & 1 & 1
\end{bmatrix},
\]

which indicates that the EFDP is solvable (last row of \( S \)) and the EFDIP is also solvable using

the specifications contained in the rows of

\[
S_{\text{FDI}} = \begin{bmatrix}
0 & 1 & 1 \\
1 & 0 & 1 \\
1 & 1 & 0
\end{bmatrix}.
\]

4.2.3 Designing an FDI Filter Using efdisyn

To solve the EFDIP, an option structure is used to specify various options for the synthesis

function efdisyn. Frequently used options are the desired stability degree for the poles of the

fault detection filter, the requirement for performing least order synthesis, or values for the
tolerances used for rank computations or fault detectability tests. The user options can be

specified by setting appropriately the respective fields in a MATLAB structure \( \text{options} \). For

example, the desired stability degree of \(-2\) of the filter, the targeted fault signature specification

\( S_{\text{FDI}} \), and the frequency 0 for strong synthesis (for constant faults), can be set using

\[
\text{options} = \text{struct('sdeg',-2,'SFDI',S(1:3,:),'FDFreq',0)};
\]

A solution of the EFDIP, for the selected structure matrix \( S_{\text{FDI}} \), can be computed using the

function \( \text{efdisyn} \) as given below

\[
[Q,R,\text{info}] = \text{efdisyn}(:\text{sysf},\text{options});
\]

The resulting bank of scalar output fault detection filters, in implementation form, is contained
in \( Q \) (stored as an one-dimensional cell array of systems), while the corresponding internal forms
of the bank of filters are contained in the cell array \( R \). The information structure \( \text{info} \) contains
further information on the resulting designs.
4.2.4 Assessing the Residual Generator

For assessment purposes, often simulations performed using the resulting internal form provide sufficient qualitative information to verify the obtained results. The example below illustrates how to simulate step inputs from faults using the computed cell array \( R \) containing the internal form of the filter.

\[
Rf = [R{1};R{2};R{3}]; \quad \% \text{build the global internal form of the filter}
\]

\[
Rf.\text{OutputName} = \text{strcat(strseq('r_{',1:3},'}');}
\]

\[
Rf.\text{InputName} = \text{strcat(strseq('f_{',1:mf},'}');}
\]

\[
\text{step(Rf)};
\]

A typical output of this computation can be used to assess the achieved fault signatures as shown in Fig. 3. As it can be observed, the diagonal entries of the transfer-function matrix of \( Rf \) are zero, while all off-diagonal entries are nonzero. The precise information on the resulting strong structure matrix can be obtained using

\[
\text{S\_strong} = \text{fdisspec(Rf)}
\]

![Step Response](image)

**Figure 3:** Step responses from the fault inputs.

4.3 Functions for Analysis

The analysis functions cover the evaluation of the weak and strong structure matrices of a system and the generation of achievable weak and strong fault detection specifications.
4.3.1 fditspec

Syntax

\[
\text{SMAT} = \text{fditspec}(\text{SYS}) \\
\text{SMAT} = \text{fditspec}(\text{SYS}, \text{TOL}) \\
\text{SMAT} = \text{fditspec}(\text{SYS}, \text{TOL}, \text{FDTOL}) \\
\text{SMAT} = \text{fditspec}(\text{SYS}, \text{TOL}, \text{FDTOL}, \text{FREQ})
\]

Description

\text{fditspec} \text{ determines the binary structure matrix } \text{SMAT} \text{ corresponding to the zero and nonzero elements of the transfer-function matrix of the LTI system } \text{SYS}.

Input data

\text{SYS} \text{ is a LTI system in a descriptor system state-space form}

\[
E\lambda x(t) = Ax(t) + Bu(t), \\
y(t) = Cx(t) + Du(t).
\]

(38)

If \text{SYS} \text{ is given in an input-output form}

\[
y(\lambda) = G(\lambda)u(\lambda),
\]

(39)

then it is automatically converted to an equivalent minimal order state-space form (38).

\text{TOL} \text{ is a relative tolerance used for observability tests. A default value is internally computed if } \text{TOL} \leq 0 \text{ or is not specified at input.}

\text{FDTOL} \text{ is an absolute threshold for the magnitudes of the nonzero elements in the system matrices } B, C \text{ and } D. \text{ Any element of these matrices whose magnitude does not exceed } \text{FDTOL} \text{ is considered zero. Additionally, if } \text{FREQ} \text{ is nonempty, } \text{FDTOL} \text{ is also used for the singular-value-based rank tests performed on the system matrix } [A \lambda E B C D]. \text{ If } \text{FDTOL} \leq 0 \text{ or not specified at input, the default value } \text{FDTOL} = 10^{-4} \text{ max}(1, \|B\|_1, \|C\|_{\infty}, \|D\|_1) \text{ is used for the magnitude of nonzero elements in } B, C \text{ and } D. \text{ If } \text{FDTOL} \leq 0 \text{ and if } \text{FREQ} \text{ is nonempty, the value } \text{FDTOL} = 10^{-4} \text{ max}(1, \| [A B C D] \|_1, \|E\|_1) \text{ is used for the rank tests on the system matrix.}

\text{FREQ} \text{ is a real vector, which contains the frequency values to be used to check for zeros of the elements of the transfer-function matrix } G(\lambda). \text{ By default, } \text{FREQ} \text{ is empty if not specified. Note that, if } \omega_k \text{ is a real frequency contained in } \text{FREQ}, \text{ then the corresponding complex frequency } \lambda_k \text{ used to check the elements of } G(\lambda) \text{ to have } \lambda_k \text{ as zero is } \lambda_k = i\omega_k, \text{ in the continuous-time case, and } \lambda_k = \exp(i\omega_k T), \text{ in the discrete-time case, where } T \text{ is the sampling time of the system.}
Output data

SMAT is a logical array which contains the structure matrix $S_G$ corresponding to the zero and nonzero elements of the transfer-function matrix $G(\lambda)$ of the LTI system SYS. If FREQ is empty or not specified at input, then SMAT is a two-dimensional logical array, which contains the weak structure matrix corresponding to the zero and nonzero elements of the transfer function matrix $G(\lambda)$ of the LTI system SYS (see (14) for the definition of the weak structure matrix). Accordingly, $\text{SMAT}(i, j) = \text{true}$, if the $(i, j)$-th element of $G(\lambda)$ is nonzero. Otherwise, $\text{SMAT}(i, j) = \text{false}$.

If FREQ is nonempty, then SMAT is a three-dimensional logical array which contains in the $k$-th page $\text{SMAT}(\cdot, \cdot, k)$, the strong structure matrix corresponding to the zero and nonzero elements of the transfer function matrix $G(\lambda)$ of the LTI system SYS evaluated for the $k$-th frequency $\omega_k$ contained in FREQ (i.e., for $\lambda = \lambda_k$, see description of FREQ) (see (18) for the definition of the strong structure matrix at a complex frequency $\lambda_k$). Accordingly, $\text{SMAT}(i, j, k) = \text{true}$, if the $(i, j)$-th element of $G(\lambda)$ has no zero in $\lambda_k$. Otherwise, $\text{SMAT}(i, j, k) = \text{false}$.

Method

For the definition of the weak and strong structure matrices, see Section 2.5. For the determination of the weak structure matrix, observable realizations are determined for each row of $G(\lambda)$ and input observability tests are performed to identify nonzero elements (see [11, Corollary 7.1]). For the determination of the strong structure matrix, minimal realizations are determined for each element of $G(\lambda)$ and the absence of zeros are assessed by checking the full rank of the corresponding system matrix for all frequencies specified in FREQ (see [11, Corollary 7.2]).

4.3.2 fdisspec

Syntax

$$[\text{SMAT}, \text{GAINS}] = \text{fdisspec}(\text{SYS})$$
$$[\text{SMAT}, \text{GAINS}] = \text{fdisspec}(\text{SYS}, \text{FDGAINTOL})$$
$$[\text{SMAT}, \text{GAINS}] = \text{fdisspec}(\text{SYS}, \text{FDGAINTOL}, \text{FREQ})$$

Description

fdisspec determines the strong structure matrix SMAT corresponding to the zero and nonzero elements of the transfer-function matrix of the LTI system SYS.

Input data

SYS is a LTI system in a state-space form (38) or in an input-output form (39).

FDGAINTOL is a threshold for the magnitudes of the frequency-response gains of the transfer-function matrix $G(\lambda)$ of the LTI system SYS. If FDGAINTOL = 0 or not specified at input, the default value FDGAINTOL = 0.01 is used.
FREQ is a real vector, which contains the frequency values to be used to check for zeros of the elements of the transfer-function matrix \( G(\lambda) \). By default, \( \text{FREQ} = 0 \), if not specified at input. The complex frequencies corresponding to the real frequencies specified in \( \text{FREQ} \) must be disjoint from the set of poles of \( G(\lambda) \). Note that, if \( \omega_k \) is a real frequency contained in \( \text{FREQ} \), then the corresponding complex frequency \( \lambda_k \) used to evaluate \( G(\lambda_k) \) is \( \lambda_k = i\omega_k \) in the continuous-time case and \( \lambda_k = \exp(i\omega_k T) \) in the discrete-time case, where \( T \) is the sampling time of the system.

Output data

\( \text{SMAT} \) is a three-dimensional logical array which contains in the \( k \)-th page \( \text{SMAT}(; ; k) \), the strong structure matrix corresponding to the zero and nonzero elements of the transfer function matrix \( G(\lambda) \) of the LTI system \( \text{SYS} \) evaluated for the \( k \)-th frequency \( \omega_k \) contained in \( \text{FREQ} \) (i.e., for \( \lambda = \lambda_k \), see description of \( \text{FREQ} \)) (see (18) for the definition of the strong structure matrix at a complex frequency \( \lambda_k \)). Accordingly, \( \text{SMAT}(i,j,k) = \text{true} \), if the magnitude of the \( (i,j) \)-th element of \( G(\lambda_k) \) is greater than or equal to \( \text{FDGAINTOL} \). Otherwise, \( \text{SMAT}(i,j,k) = \text{false} \).

\( \text{GAINS} \) is a real nonnegative array, whose \( (i,j) \)-th element contains the minimum value of the frequency-response gains of the \( (i,j) \)-th element of \( G(\lambda) \) evaluated over all complex frequencies corresponding to \( \text{FREQ} \) (see above). This value is a particular instance of the \( \mathcal{H}_{\infty} \) index of the \( (i,j) \)-th element.

Method

For the definition of the strong structure matrix at a given frequency, see Remark 3 of Section 2.5. The value contained in \( \text{GAINS} \) corresponds to an element-wise evaluation of the \( \mathcal{H}_{\infty} \) index on a discrete set of frequency values (see [11, Section 5.3]).

4.3.3 genspec

Syntax

\[
\text{S} = \text{genspec}(\text{SYSF}, \text{OPTIONS})
\]

Description

genspec determines all achievable fault detection specifications for the LTI state-space system \( \text{SYSF} \) with additive faults.

Input data

\( \text{SYSF} \) is a LTI system in the state-space form

\[
E \lambda x(t) = Ax(t) + Bu(t) + Bd(t) + Bf(t),
\]

\[
y(t) = Cx(t) + Du(t) + Dd(t) + Df(t),
\]

\( (40) \)
where any of the inputs components \( u(t), d(t), \) and \( f(t) \) can be void. For the system \( \text{SYSF} \), the input groups for \( u(t), d(t), \) and \( f(t) \), have the standard names ‘controls’, ‘disturbances’, and ‘faults’, respectively. Any additionally defined input groups are ignored.

If no standard input groups are explicitly defined, then \( \text{SYSF} \) is assumed to be a partitioned LTI system
\[
\text{SYSF} = [\text{SYS1} \ \text{SYS2}]
\]

in a state-space form
\[
\begin{align*}
E\lambda x(t) &= Ax(t) + B_d d(t) + B_f f(t), \\
y(t) &= Cx(t) + D_d d(t) + D_f f(t),
\end{align*}
\]

where the inputs components \( d(t) \in \mathbb{R}^{m_1} \), and \( f(t) \in \mathbb{R}^{m_2} \) (both input components can be void). \( \text{SYS1} \) has \( d(t) \) as input vector and the corresponding state-space realization is \((A-\lambda E, B_d, C, D_d)\), while \( \text{SYS2} \) has \( f(t) \) as input vector and the corresponding realization is \((A-\lambda E, B_f, C, D_f)\). The dimension \( m_1 \) of the input vector \( d(t) \) is specified by the \text{OPTIONS} field \text{OPTIONS.m1} (see below). For compatibility with the previous version, if \text{OPTIONS.m1} is specified, then the form (41) is assumed, even if the standard input groups have been explicitly defined.

\text{OPTIONS} is a MATLAB structure used to specify various synthesis options and has the following fields:

| Option fields | Description |
|---------------|-------------|
| tol           | tolerance for rank determinations (Default: internally computed) |
| FDTol         | threshold for assessing weak specifications (see function \text{fditspec}) (Default: 0.0001) |
| FDGainTol     | threshold for assessing strong specifications (see function \text{fdisspec}) (Default: 0.01) |
| m1            | the number \( m_1 \) of the inputs of \( \text{SYS1} \) (Default: 0); if \text{OPTIONS.m1} is explicitly specified, then \( \text{SYSF} \) is assumed to be partitioned as \( \text{SYSF} = [\text{SYS1} \ \text{SYS2}] \) with a state-space realization of the form (41) and the definitions of input groups are ignored. |
| FDFreq        | real frequency value to assess strong fault specifications (Default: \([ \ ]\)) |
| sdeg          | prescribed stability degree for the poles of the internally generated filters (see Method): in the continuous-time case, the real parts of filters poles must be less than or equal to \text{OPTIONS.sdeg}, while in discrete-time case, the magnitudes of filter poles must be less than or equal to \text{OPTIONS.sdeg}; (Default: if \text{OPTIONS.FDFreq} is empty, then \text{OPTIONS.sdeg} = \([ \ ]\), i.e., no stabilization is performed; if \text{OPTIONS.FDFreq} is nonempty, then \text{OPTIONS.sdeg} = -0.05 in the continuous-time case and \text{OPTIONS.sdeg} = 0.9 in the discrete-time case). |

35
Output data

$S$ is a logical array, whose rows contains the achievable fault detection specifications. Specifically, the $i$-th row of $S$ contains the $i$-th achievable specification, obtainable by using a certain (e.g., scalar output) fault detection filter $Q^{(i)}(\lambda)$, whose internal form is $R_f^{(i)}(\lambda)$, with $R_f^{(i)}(\lambda) \neq 0$ (see Method). Thus, the row $S(i,:) = S(i,:)$ is the structure matrix of $R_f^{(i)}(\lambda)$, such that $S(i,j) = \text{true}$ if $R_f^{(i)}(\lambda) \neq 0$ and $S(i,j) = \text{false}$ if $R_f^{(i)}(\lambda) = 0$. If a real frequency value $\omega$ for strong specifications is provided in $\text{OPTIONS.FDFreq}$, then $S(i,j) = \text{true}$ if $\|R_f^{(i)}(\omega)\| \geq \text{OPTIONS.FDGainTol}$ for the complex frequency $\lambda_\omega$ corresponding to $\omega$, and $S(i,j) = \text{false}$ otherwise. Note that $\lambda_\omega = i\omega$ in the continuous-time case and $\lambda_\omega = \exp(i\omega T)$ in the discrete-time case, where $T$ is the sampling time of the system.

Method

The implementation of $\text{genspec}$ is based on the Procedure $\text{GENSPEC}$ from [11, Sect. 5.4]. The nullspace method of [6] is recursively employed to generate the complete set of achievable specifications, obtainable using suitable fault detection filters. The method is also described in [8]. In what follows we give some details of this approach.

Assume the system $\text{SYSF}$ in (40) has the input-output form

$$y(\lambda) = G_u(\lambda)u(\lambda) + G_d(\lambda)d(\lambda) + G_f(\lambda)f(\lambda).$$

(42)

If $\text{SYSF}$ has the form (41), then we simply assume that $u(t)$ is void in (42). To determine the $i$-th row of $S$, which contains the $i$-th achievable specification, a certain (e.g., scalar output) fault detection filter is employed, with the input-output implementation form

$$r^{(i)}(\lambda) = Q^{(i)}(\lambda) \left[ \begin{array}{c} y(\lambda) \\ u(\lambda) \end{array} \right].$$

(43)

and its internal form

$$r^{(i)}(\lambda) = R_f^{(i)}(\lambda)f(\lambda),$$

(44)

with $R_f^{(i)}(\lambda)$ defined as

$$R_f^{(i)}(\lambda) := Q^{(i)}(\lambda) \left[ egin{array}{c} G_f(\lambda) \\ 0 \end{array} \right].$$

(45)

The resulting $i$-th row of $S$ is the structure matrix (weak or strong) of $R_f^{(i)}(\lambda)$. Recursive filter updating based on nullspace techniques is employed to systematically generate particular filters which are sensitive to certain fault inputs and insensitive to the rest of inputs.

The check for nonzero elements of $R_f^{(i)}(\lambda)$ is performed by using the function $\text{fditspec}$ to evaluate the corresponding weak specifications. The corresponding threshold is specified via $\text{OPTIONS.FDTol}$. If a frequency value for tests are provided in $\text{OPTIONS.FDFreq}$, then the magnitudes of the elements of $R_f^{(i)}(\lambda)$ must be above a certain threshold for the complex frequencies corresponding to the specified real frequency values in $\text{OPTIONS.FDFreq}$. For this purpose, the
function `fdisspec` is used to evaluate the corresponding strong specifications. The corresponding threshold is specified via `OPTIONS.FDGainTol`. The call of `fdisspec` requires that the set of poles of $R_f^{(i)}(\lambda)$ and the complex frequency corresponding to the real frequency specified in `OPTIONS.FDFreq` are disjoint. This condition is fulfilled by ensuring a certain stability degree for the poles of $R_f^{(i)}(\lambda)$, specified via `OPTIONS.sdeg`.

**Example**

*Example 1.* This is the example of [12] of a continuous-time state-space model of the form (40) with $E = I_4$,

\[
A = \begin{bmatrix}
-1 & 1 & 0 & 0 \\
1 & -2 & 1 & 0 \\
0 & 1 & -2 & 1 \\
0 & 0 & 1 & -2 \\
\end{bmatrix},
B_u = \begin{bmatrix}
1 \\
0 \\
0 \\
0 \\
\end{bmatrix},
B_d = 0,
B_f = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & -1 \\
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix},
D_u = 0,
D_d = 0,
D_f = 0.
\]

The achievable 18 weak fault specifications and 12 strong fault specifications, computed with the following script, are:

\[
S_{weak} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 & 0 \\
1 & 1 & 1 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 1 & 1 & 0 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

\[
S_{strong} = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
\end{bmatrix}
\]

Observe that there are 6 weak specifications, which are not strong specifications.

---

3If $\omega$ is a real frequency, then the corresponding complex frequency is $\lambda_\omega = i\omega$ in the continuous-time case, and $\lambda_\omega = \exp(i\omega T)$ in the discrete-time case, where $T$ is the sampling time of the system.
% Example of Yuan et al. IJC (1997)
p = 3; mu = 1; mf = 8;

\[ A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 1 & -2 & 1 \\ 0 & 0 & 1 & -2 \end{bmatrix}; \quad Bu = [1 \ 0 \ 0 \ 0]^T; \]
\[ Bf = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}; \quad Du = zeros(p,mu); \quad Df = zeros(p,mf); \]

% setup the model with additive faults
sysf = ss(A,[Bu Bf],C,[Du Df]);

% set input groups
set(sysf,'InputGroup',struct('controls',1:mu,'faults',mu+(1:mf)));

% compute the achievable weak specifications
opt = struct('tol',1.e-7,'FDTol',1.e-5);
S_weak = genspec(sysf,opt), size(S_weak)

% compute the achievable strong specifications for constant faults
opt = struct('tol',1.e-7,'FDTol',0.0001,'FDGainTol','.001',...\n       'FDFreq',0,'sdeg',-0.05);
S_strong = genspec(sysf,opt), size(S_strong)

4.4 Functions for the Synthesis of Residual Generator Filters

4.4.1 efdsyn

Syntax

\[ [Q,R,INFO] = \text{efdsyn}(\text{SYSF},\text{OPTIONS}) \]

Description

efdsyn solves the exact fault detection problem (EFDP) (see Section 2.6.1), for a given LTI system SYSF with additive faults. Two stable and proper filters, Q and R, are computed, where Q contains the fault detection filter representing the solution of the EFDP, and R contains its internal form.

Input data

SYSF is a LTI system in the state-space form

\[ \begin{align*}
E \dot{x}(t) &= Ax(t) + Bu(t) + Bd(t) + Bf f(t) + Bw w(t) + Bv v(t), \\
y(t) &= Cx(t) + Du u(t) + Dd d(t) + Df f(t) + Dw w(t) + Dv v(t),
\end{align*} \tag{46} \]

where any of the inputs components \( u(t), d(t), f(t), w(t) \) or \( v(t) \) can be void. The auxiliary input signal \( v(t) \) can be used for convenience. For the system SYSF, the input groups for \( u(t), d(t), f(t), \) and \( w(t) \) have the standard names 'controls', 'disturbances', 'faults', and 'noise', respectively. For the auxiliary input \( v(t) \) the standard input group 'aux' can be used.
OPTIONS is a MATLAB structure used to specify various synthesis options and has the following fields:

| OPTIONS fields | Description |
|----------------|-------------|
| **tol**        | relative tolerance for rank computations (Default: internally computed) |
| **tolmin**     | absolute tolerance for observability tests (Default: internally computed) |
| **FDTol**      | threshold for fault detectability checks (Default: $10^{-4}$) |
| **FDGainTol**  | threshold for strong fault detectability checks (Default: $10^{-2}$) |
| **rdim**       | desired number $q$ of residual outputs for $Q$ and $R$ (Default: [], in which case $q = 1$, if OPTIONS.minimal = true, or $q = p - r_d$, if OPTIONS.minimal = false; see Method) |
| **FDFreq**     | vector of real frequency values for strong detectability checks (Default: []) |
| **smarg**      | stability margin for the poles of the filters $Q$ and $R$ (Default: $-\sqrt{\text{eps}}$ for a continuous-time system SYSF; $1-\sqrt{\text{eps}}$ for a discrete-time system SYSF). |
| **sdeg**       | prescribed stability degree for the poles of the filters $Q$ and $R$ (Default: $-0.05$ for a continuous-time system SYSF; $0.95$ for a discrete-time system SYSF). |
| **poles**      | complex vector containing a complex conjugate set of desired poles (within the stability domain) to be assigned for the filters $Q$ and $R$ (Default: []) |
| **simple**     | option to employ a simple proper basis for filter synthesis true – use a simple basis; false – use a non-simple basis (default) |
| **minimal**    | option to perform a least order filter synthesis true – perform least order synthesis (default); false – perform full order synthesis. |
| **tcond**      | maximum allowed condition number of the employed non-orthogonal transformations (Default: $10^4$). |
| **HDesign**    | full row rank design matrix $H$ to build OPTIONS.rdim linear combinations of the left nullspace basis vectors (see Method) (Default: []) |

Output data

$Q$ is the resulting fault detection filter in a standard state-space form

$$
\lambda x_Q(t) = A_Q x_Q(t) + B_Q y(t) + B_Q u(t),
$$

$$
r(t) = C_Q x_Q(t) + D_Q y(t) + D_Q u(t),
$$

where the residual signal $r(t)$ is a $q$-dimensional vector. The resulting value of $q$ depends on the selected options OPTIONS.rdim and OPTIONS.minimal (see Method).
system object $Q$, two input groups 'outputs' and 'controls' are defined for $y(t)$ and $u(t)$, respectively, and the output group 'residuals' is defined for the residual signal $r(t)$.

$R$ is the resulting internal form of the fault detection filter has a standard state-space representation

$$\lambda x_R(t) = A_Q x_R(t) + B_{R_f} f(t) + B_{R_w} w(t) + B_{R_v} v(t),$$

$$r(t) = C_Q x_R(t) + D_{R_f} f(t) + D_{R_w} w(t) + D_{R_v} v(t)$$

and the input groups 'faults', 'noise' and 'aux' are defined for $f(t)$, $w(t)$, and $v(t)$, respectively, and the output group 'residuals' is defined for the residual signal $r(t)$. Note that the realizations of $Q$ and $R$ share the matrices $A_Q$ and $C_Q$.

INFO is a MATLAB structure containing additional information as follows:

| INFO fields | Description |
|-------------|-------------|
| tcond       | maximum of the condition numbers of the employed non-orthogonal transformation matrices; a warning is issued if INFO.tcond $\geq$ OPTIONS.tcond. |
| degs        | if OPTIONS.simple = true, the orders of the basis vectors of the employed simple nullspace basis; if OPTIONS.simple = false, the degrees of the basis vectors of an equivalent polynomial nullspace basis |
| S           | binary structure matrix corresponding to $HG_f(\lambda)$ as computed (see Method) |
| HDesign     | design matrix $H$ employed for the synthesis of the fault detection filter (see Method) |

**Method**

The function `efdsyn` implements an extension of the Procedure EFD from [11, Sect. 5.2], which relies on the nullspace-based synthesis method proposed in [3]. In what follows, we succinctly present this extended procedure, in terms of the input-output descriptions. Full details of the employed state-space based computational algorithms are given in [11][Chapter 7].

Let assume the system SYSF in (46) has the equivalent input-output form

$$y(\lambda) = G_u(\lambda) u(\lambda) + G_d(\lambda) d(\lambda) + G_f(\lambda) f(\lambda) + G_w(\lambda) w(\lambda) + G_v(\lambda) v(\lambda),$$

where the vectors $y$, $u$, $d$, $f$, $w$ and $v$ have dimensions $p$, $m_u$, $m_d$, $m_f$, $m_w$ and $m_v$, respectively.

The resulting fault detection filter in (47) has the input-output form

$$r(\lambda) = Q(\lambda) \begin{bmatrix} y(\lambda) \\ u(\lambda) \end{bmatrix},$$

where the resulting dimension $q$ of the residual vector $r$ is $q = \min(\text{OPTIONS.rdim}, p-r_d)$, where $r_d = \text{rank} G_d(\lambda)$.

The synthesis method which underlies Procedure EFD, essentially determines the filter $Q(\lambda)$ as a stable rational left annihilator of

$$G(\lambda) := \begin{bmatrix} G_u(\lambda) & G_d(\lambda) \\ I_{m_u} & 0 \end{bmatrix},$$

(51)
The solvability condition of the EFDP is checked by verifying that

\[ R(\lambda) := \begin{bmatrix} R_f(\lambda) & R_w(\lambda) & R_v(\lambda) \end{bmatrix} := Q(\lambda) \begin{bmatrix} G_f(\lambda) & G_w(\lambda) & G_v(\lambda) \end{bmatrix}. \]  

(52)

The resulting internal form of the fault detection filter (50) is

\[ r(\lambda) = R(\lambda) \begin{bmatrix} f(\lambda) \\ w(\lambda) \\ v(\lambda) \end{bmatrix} = R_f(\lambda)f(\lambda) + R_w(\lambda)w(\lambda) + R_v(\lambda)v(\lambda), \]  

(53)

with \( R(\lambda) \), defined in (52), stable.

If \texttt{OPTIONS.minimal = false}, then \( Q(\lambda) \) is determined in the form

\[ Q(\lambda) = M(\lambda)HN_i(\lambda), \]

where: \( N_i(\lambda) \) is a \((p-r_d) \times (p+m_w)\) rational left nullspace basis satisfying \( N_i(\lambda)G(\lambda) = 0 \), with \( r_d := \text{rank} G_d(\lambda); H \) is a suitable design matrix used to build \( q \) (e.g., \( q = \text{OPTIONS.rdim} \)) linear combinations of the \( p-r_d \) left nullspace basis vectors; and \( M(\lambda) \) is a stable invertible transfer function matrix determined such that \( Q(\lambda) \) and the corresponding \( R(\lambda) \) have a desired dynamics (specified via \texttt{OPTIONS.sdeg} and \texttt{OPTIONS.poles}). The internal form of the filter \( Q(\lambda) \) is obtained as

\[ R(\lambda) = M(\lambda)H\overline{G}(\lambda), \]

where

\[ \overline{G}(\lambda) := \begin{bmatrix} \overline{G}_f(\lambda) & \overline{G}_w(\lambda) & \overline{G}_v(\lambda) \end{bmatrix} = N_i(\lambda) \begin{bmatrix} G_f(\lambda) & G_w(\lambda) & G_v(\lambda) \end{bmatrix}. \]

The solvability condition of the EFDP is checked by verifying that

\[ H\overline{G}_f(\lambda) \neq 0, \quad j = 1, \ldots, m_f. \]

The design parameter matrix \( H \) is set as follows: if \texttt{OPTIONS.HDesign} is nonempty, then \( H = \text{OPTIONS.HDesign} \); if \texttt{OPTIONS.HDesign} = [ ], then \( H = I_{p-r_d} \), if \texttt{OPTIONS.rdim} = [ ], or \( H \) is a randomly generated \texttt{OPTIONS.rdim} \( \times \) \((p-r_d)\) real matrix, if \texttt{OPTIONS.rdim} is empty. If \texttt{OPTIONS.simple = true}, then \( N_i(\lambda) \) is determined as a simple rational basis. The orders of the basis vectors are provided in \texttt{INFO.degs}. These are also the degrees of the basis vectors of an equivalent polynomial nullspace basis.

If \texttt{OPTIONS.minimal = true}, then \([Q(\lambda) \ R(\lambda)]\) has the least McMillan degree, with \( Q(\lambda) \) a left annihilator of \( G(\lambda) \) in (51) with \texttt{OPTIONS.rdim} outputs. \( Q(\lambda) \) and \( R(\lambda) \) are determined in the form

\[ [Q(\lambda) \ R(\lambda)] = M(\lambda)[\bar{Q}(\lambda) \ \bar{R}(\lambda)], \]

where

\[ [\bar{Q}(\lambda) \ \bar{R}(\lambda)] = H [N_i(\lambda) \overline{G}(\lambda)] + Y(\lambda)[N_i(\lambda) \overline{G}(\lambda)] \]

with \([\bar{Q}(\lambda) \ \bar{R}(\lambda)]\) and \( Y(\lambda) \) the least order solution of a left minimal cover problem [10], and \( M(\lambda) \) is a stable invertible transfer function matrix determined such that \([Q(\lambda) \ R(\lambda)]\) has
a desired dynamics. The global least order for $[Q(\lambda) \ R(\lambda)]$ can be achieved by choosing OPTIONS.rdim = 1. If OPTIONS.HDesign is nonempty, then $H = $ OPTIONS.HDesign, and if OPTIONS.HDesign = [ ], then a suitable randomly generated $H$ is employed, which fulfills the fault detectability conditions

$$R_{f_j}(\lambda) \neq 0, \ j = 1, \ldots, m_f.$$ 

The structure field INFO.HDesign contains the employed value of the design matrix $H$.

**Example**

**Example 2.** This is Example 5.4 from the book [11]. Consider an unstable continuous-time system with the TFMs

$$G_u(s) = \begin{bmatrix} \frac{s + 1}{s - 2} \\ \frac{s + 2}{s - 3} \end{bmatrix}, \quad G_d(s) = \begin{bmatrix} \frac{s - 1}{s + 2} \\ 0 \end{bmatrix}, \quad G_f(s) = \begin{bmatrix} \frac{s + 1}{s - 2} & 0 \\ \frac{s + 2}{s - 3} & 1 \end{bmatrix}, \quad G_w(s) = 0, \quad G_v(s) = 0,$$

where the fault input $f_1$ corresponds to an additive actuator fault, while the fault input $f_2$ describes an additive sensor fault in the second output $y_2$. The TFM $G_d(s)$ is non-minimum phase, having an unstable zero at 1. We want to design a fault detection filter $Q(s)$ with scalar output and a stability degree of $-3$ for the poles. The results computed with the following script are

$$Q(s) = \begin{bmatrix} 0 & \frac{s - 3}{s + 3} & -\frac{s + 2}{s + 3} \end{bmatrix}, \quad R_f(s) = \begin{bmatrix} \frac{s + 2}{s + 3} & \frac{s - 3}{s + 3} \end{bmatrix}.$$
% resulting fault detection filter
tf(Q), tf(Rf)

% check synthesis conditions: Q[Gu Gd;I 0] = 0 and Q[Gf; 0] = Rf
syse = [sysf;eye(mu,mu+md+mf)];  % form Ge = [Gu Gd Gf;I 0 0];
norm_Ru_Rd = norm(Q*syse(:,{'controls','disturbances'}),inf)
norm_rez = norm(Q*syse(:, 'faults')-Rf,inf)

% check weak and strong fault detectability
S_weak = fditspec(Rf)
[S_strong,abs_dcgains] = fdisspec(Rf)

% evaluate step responses
set(Rf, 'InputName', {'f_1', 'f_2'}, 'OutputName', 'r');
step(Rf);
title('Step responses from the fault inputs'), ylabel('')

4.4.2 efdisyn

Syntax

[Q,R,INFO] = efdisyn(SYSF,OPTIONS)

Description

efdisyn solves the exact fault detection and isolation problem (EFDIP) (see Section 2.6.2), for a given LTI system SYSF with additive faults and a given structure matrix S_{FDI} (specified via the OPTIONS structure). Two banks of stable and proper filters are computed in the nb-dimensional cell arrays Q and R, where nb is the number of specifications contained in S_{FDI} (i.e., the number of rows of the structure matrix S_{FDI}). Q{i} contains the i-th fault detection filter (12) in the overall solution (11) of the EFDIP and R{i} contains its internal form.

Input data

SYSF is a LTI system in the state-space form

\[
\begin{align*}
E\lambda x(t) &= Ax(t) + Bu(t) + Bd(t) + Bf(t) + Bw(t), \\
y(t) &= Cx(t) + Du(t) + Dd(t) + Df(t) + Dw(t),
\end{align*}
\]

(54)

where any of the inputs components u(t), d(t), f(t), or w(t) can be void. For the system SYSF, the input groups for u(t), d(t), f(t), and w(t) have the standard names 'controls', 'disturbances', 'faults', and 'noise', respectively.

OPTIONS is a MATLAB structure used to specify various synthesis options and has the following fields:
| OPTIONS fields | Description |
|----------------|-------------|
| SFDI           | the desired structure matrix $S_{FDI}$ to solve the EFDIP (Default: $[1 \ldots 1]$, i.e., solve an exact fault detection problem) |
| tol            | relative tolerance for rank computations (Default: internally computed) |
| tolmin         | absolute tolerance for observability tests (Default: internally computed) |
| FDTol          | threshold for fault detectability checks (Default: 0.0001) |
| FDGainTol      | threshold for strong fault detectability checks (Default: 0.01) |
| rdim           | desired number $q$ of residual outputs for the component filters $Q\{i\}$ and $R\{i\}$ (Default: [], in which case $q = 1$, if OPTIONS.minimal = true, or is internally computed for each filter, if OPTIONS.minimal = false; see Method) |
| FDFreq         | vector of real frequency values for strong detectability checks (Default: []) |
| smarg          | stability margin for the poles of the component filters $Q\{i\}$ and $R\{i\}$ (Default: $-\sqrt{\text{eps}}$ for a continuous-time system SYSF; $1-\sqrt{\text{eps}}$ for a discrete-time system SYSF). |
| sdeg           | prescribed stability degree for the poles of the component filters $Q\{i\}$ and $R\{i\}$ (Default: $-0.05$ for a continuous-time system SYSF; $0.95$ for a discrete-time system SYSF). |
| poles          | complex vector containing a complex conjugate set of desired poles (within the stability domain) to be assigned for the component filters $Q\{i\}$ and $R\{i\}$ (Default: []) |
| simple         | option to employ simple proper bases for the synthesis of the component filters $Q\{i\}$ and $R\{i\}$ (Default: [ ]): true – use simple bases; false – use non-simple bases (default) |
| minimal        | option to perform least order synthesis of the component filters $Q\{i\}$ and $R\{i\}$ (Default: [ ]): true – perform least order synthesis (default); false – perform full order synthesis |
| tcond          | maximum allowed condition number of the employed non-orthogonal transformations (Default: $10^4$). |
| FDSelect       | integer vector with increasing elements containing the indices of the desired filters to be designed (Default: $[1, \ldots, n_b]$) |
| HDesign        | $n_b$-dimensional cell array; OPTIONS.HDesign$\{i\}$, if not empty, is a full row rank design matrix employed for the synthesis of the $i$-th fault detection filter (Default: [ ]) |
Output data

Q is a cell array, where \( Q[i] \) for \( i = 1, \ldots, n_b \), contains the resulting \( i \)-th filter in a standard state-space representation

\[
\lambda x_Q^{(i)}(t) = A_Q^{(i)} x_Q^{(i)}(t) + B_Q^{(i)} y(t) + B_{Q_u}^{(i)} u(t),
\]

where the residual signal \( r^{(i)}(t) \) is a \( q_i \)-dimensional vector. For each system object \( Q[i] \), two input groups ‘outputs’ and ‘controls’ are defined for \( y(t) \) and \( u(t) \), respectively, and the output group ‘residuals’ is defined for the residual signal \( r^{(i)}(t) \). \( Q[i] \) is empty if the index \( i \) is not selected in OPTIONS.FDSelect.

R is a cell array, where \( R[i] \) contains the resulting internal form of the \( i \)-th filter in a standard state-space representation

\[
\lambda x_R^{(i)}(t) = A_R^{(i)} x_R^{(i)}(t) + B_R^{(i)} f(t) + B_R^{(i)} w(t),
\]

The input groups ‘faults’ and ‘noise’ are defined for \( f(t) \), and \( w(t) \), respectively, and the output group ‘residuals’ is defined for the residual signal \( r^{(i)}(t) \). Note that the realizations of \( Q[i] \) and \( R[i] \) share the matrices \( A_Q^{(i)} \) and \( C_Q^{(i)} \). \( R[i] \) is empty if the index \( i \) is not selected in OPTIONS.FDSelect.

INFO is a MATLAB structure containing additional information as follows:

| INFO fields | Description |
|-------------|-------------|
| tcond       | \( n_b \)-dimensional vector; \( \text{INFO.tcond}(i) \) contains the maximum of the condition numbers of the employed non-orthogonal transformation matrices to determine the \( i \)-th filter component \( Q[i] \); a warning is issued if any \( \text{INFO.tcond}(i) \geq \text{OPTIONS.tcond} \). |
| degs        | \( n_b \)-dimensional cell array; if \( \text{OPTIONS.simple} = \text{true} \), \( \text{INFO.degs}(i) \) contains the orders of the basis vectors of the employed simple nullspace basis for the synthesis of the \( i \)-th filter component \( Q[i] \); if \( \text{OPTIONS.simple} = \text{false} \), \( \text{INFO.degs}(i) \) contains the degrees of the basis vectors of an equivalent polynomial nullspace basis |
| HDesign     | \( n_b \)-dimensional cell array; \( \text{INFO.HDesign}(i) \) is the \( i \)-th design matrix actually employed for the synthesis of the \( i \)-th fault detection filter \( Q[i] \). \( \text{INFO.HDesign}(i) \) is empty if the index \( i \) is not selected in OPTIONS.FDSelect. |

Method

The Procedure EFDI from [11, Sect. 5.4] is implemented, which relies on the nullspace-based synthesis method proposed in [5]. This method essentially determines each filter \( Q^{(i)}(\lambda) \) and
its internal form $R^{(i)}(\lambda)$, by solving a suitably formulated EFDP for a reduced system without control inputs, and with redefined disturbance and fault inputs. For this purpose, the function efdisyn calls internally the function efdsyn to solve a suitably formulated EFDP for each specification (i.e., row) contained in the structure matrix $S_{FDI}$.

If the faulty system SYSF has the input-output form

$$y(\lambda) = G_u(\lambda)u(\lambda) + G_d(\lambda)d(\lambda) + G_f(\lambda)f(\lambda) + G_w(\lambda)w(\lambda)$$

and the $i$-th fault detection filter $Q^{(i)}(\lambda)$ contained in $Q\{i\}$ has the input-output form

$$r^{(i)}(\lambda) = Q^{(i)}(\lambda) \begin{bmatrix} y(\lambda) \\ u(\lambda) \end{bmatrix},$$

then, taking into account the decoupling conditions (19), the resulting internal form of the $i$-th fault detection filter $R^{(i)}(\lambda)$, contained in $R\{i\}$, is

$$r^{(i)}(\lambda) = R^{(i)}(\lambda) \begin{bmatrix} f(\lambda) \\ w(\lambda) \end{bmatrix} = R_f^{(i)}(\lambda)f(\lambda) + R_w^{(i)}(\lambda)w(\lambda),$$

with $R^{(i)}(\lambda) = [R_f^{(i)}(\lambda) \ R_w^{(i)}(\lambda)]$ defined as

$$\begin{bmatrix} R_f^{(i)}(\lambda) \\ R_w^{(i)}(\lambda) \end{bmatrix} := Q^{(i)}(\lambda) \begin{bmatrix} G_f(\lambda) \\ 0 \\ G_w(\lambda) \\ 0 \end{bmatrix}.$$

According to (22), the structure matrix of $R^{(i)}(\lambda)$ is equal to the $i$-th row of the specified $S_{FDI}$.

To design the $i$-th filter $Q^{(i)}(\lambda)$, the function efdisyn is called, which internally uses a design matrix $H^{(i)}$, which can be specified in OPTIONS.HDesign\{i\} (see Method for efdisyn). The actually employed design matrix is returned in INFO.HDesign\{i\}.

Examples

Example 3. This is Example 5.10 from the book [11], which considers a continuous-time system with triplex sensor redundancy on its measured scalar output, which we denote, respectively, by $y_1$, $y_2$ and $y_3$. Each output is related to the control and disturbance inputs by the input-output relation

$$y_i(s) = G_u(s)u(s) + G_d(s)d(s), \quad i = 1, 2, 3,$$

where $G_u(s)$ and $G_d(s)$ are $1 \times m_u$ and $1 \times m_d$ TFMs, respectively. We assume all three outputs are susceptible to additive sensor faults. Thus, the input-output model of the system with additive faults has the form

$$y(s) := \begin{bmatrix} y_1(s) \\ y_2(s) \\ y_3(s) \end{bmatrix} = \begin{bmatrix} G_u(s) \\ G_u(s) \\ G_u(s) \end{bmatrix} u(s) + \begin{bmatrix} G_d(s) \\ G_d(s) \\ G_d(s) \end{bmatrix} d(s) + \begin{bmatrix} f_1(s) \\ f_2(s) \\ f_3(s) \end{bmatrix}.$$
The maximal achievable structure matrix is

\[ S_{\text{max}} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \]

We assume that no simultaneous sensor failures occur, and thus we can target to solve an EFDIP for the structure matrix

\[ S_{\text{FDI}} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \]

where the columns of \( S_{\text{FDI}} \) codify the desired fault signatures.

The resulting least order overall FDI filter has the generic form (i.e, independent of the numbers of control and disturbance inputs)

\[
Q(s) = \begin{bmatrix} Q^{(1)}(s) \\ Q^{(2)}(s) \\ Q^{(3)}(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & 0 & \cdots & 0 \\ 1 & -1 & 0 & 0 & \cdots & 0 \end{bmatrix}
\]

and the corresponding overall internal form is

\[
R_{f}(s) = \begin{bmatrix} R^{(1)}_{f}(s) \\ R^{(2)}_{f}(s) \\ R^{(3)}_{f}(s) \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}
\]

\% Example - Solution of an EFDIP

\p = 3; \text{mf} = 3; \% enter output and fault vector dimensions
\% generate random dimensions for system order and input vectors
rand('seed',16); randn('seed',16);
u = floor(1+4*rand); \text{mu} = floor(1+4*rand);
nd = floor(1+4*rand); \text{md} = floor(1+4*rand);
\% define random Gu(s) and Gd(s) with triplex sensor redundancy
\% and Gf(s) for sensor faults
\text{Gu} = ones(3,1)*rss(u,1,\text{mu}); \% enter Gu(s) in state-space form
\text{Gd} = ones(3,1)*rss(nd,1,\text{md}); \% enter Gd(s) in state-space form
\text{Gf} = eye(3); \% enter Gf(s) for sensor faults
\% build model with faults
\text{sysf} = [\text{Gu} \text{Gd} \text{Gf}];
\% set input groups
\text{sysf}.InputGroup.controls = 1:u; \% controls
\text{sysf}.InputGroup.disturbances = u+(1:md); \% disturbances
\text{sysf}.InputGroup.faults = u+md+(1:mf); \% faults
SFDI = [ 0 1 1; 1 0 1; 1 1 0];  % enter structure matrix

% set options for least order synthesis with EFDISYN
options = struct('tol',1.e-7,'sdeg',-1,'rdim',1,'SFDI',SFDI);
[Qt,Rft] = efdisyn(sysf,options);

% normalize Q and Rf to match example
scale = sign([ Rft{1}.d(1,2) Rft{2}.d(1,3) Rft{3}.d(1,1)]);
for i = 1:3, Qt{i} = scale(i)*Qt{i}; Rft{i} = scale(i)*Rft{i}; end
Q = [Qt{1};Qt{2};Qt{3}]; Rf = [Rft{1};Rft{2};Rft{3}];
Q = set(Q,'InputName',['y1';'y2';'y3';'u1';'u2'], 'OutputName', ['r1';'r2';'r3']);
Rf = set(Rf,'InputName', ['f1';'f2';'f3'], 'OutputName', ['r1';'r2';'r3']);

% check synthesis conditions: Q[Gu Gd;I 0] = 0 and Q[Gf; 0] = Rf
syse = [sysf;eye(mu,mu+md+mf)];  % form Ge = [Gu Gd Gf;I 0 0];
norm_Ru_Rd = norm(Q*syse(:,{'controls','disturbances'}),inf)
norm_rez = norm(Q*syse(:,'faults')-Rf,inf)

% check strong fault detectability
[S_strong,abs_dcgains] = fdisspec(Rf)

% evaluate step responses
set(Rf,'InputName',strseq('f_',1:mf), 'OutputName',strseq('r_',1:size(SFDI,1)));
step(Rf); title('Step responses from the fault inputs'), ylabel('Residuals')
Example 4. This is the example of [12], already considered in Example 1. Using efdisyn, we can easily determine a bank of least order fault detection filters, which achieve the computed maximal weak structure matrix $S_{\text{weak}}$. With the default least order synthesis option, we obtain a bank of 18 filters, each one of order one or two. The overall filters $Q(s)$ and $R_f(s)$ obtained by stacking the 18 component filters have state-space realizations of order 32, which are usually non-minimal. Typically, minimal realizations of orders about 20 can be computed for each of these filters. The bank of 12 component filters ensuring strong fault detection can easily be picked-out from the computed filters.

In this example, we show that using the pole assignment feature, the overall filters $Q(s)$ and $R_f(s)$ can be determined with minimal realizations of order 6, which is probably the least achievable global order. To arrive to this order, we enforce the same dynamics for all component filters by assigning, for example, all poles of the component filters to lie in the set $\{-1, -2\}$. The resulting least order of the overall filter $Q(s)$ can be easily read-out from the plot of its Hankel-singular values shown in Fig. 4. The synthesis procedure also ensures that $R_f(s)$, and even of the joint overall filter $[Q(s) \ R_f(s)]$ have minimal realizations of order 6!. It is also straightforward to check that the resulting weak structure matrix of $R_f(s)$ and $S_{\text{weak}}$ coincide.

% Example of Yuan et al. IJC (1997)

\begin{verbatim}
% p = 3; mu = 1; mf = 8;
A = [-1 1 0 0; 1 -2 1 0; 0 1 -2 1; 0 0 1 -2 ];
Bu = [1 0 0 0];
Bf = [1 0 0 1 0 0 0; 0 1 0 0 -1 1 0 0; 0 0 1 0 0 -1 1 0; 0 0 0 1 0 0 -1 1];
C = [1 0 0 0; 0 0 1 0; 0 0 0 1];
Du = zeros(p,mu); Df = zeros(p,mf);
% setup the model with additive faults
sysf = ss(A,[Bu Bf],C,[Du Df]);
% set input groups
set(sysf,'InputGroup',struct('controls',1:mu,'faults',mu+(1:mf)));

% compute the achievable weak specifications
opt = struct('tol',1.e-7,'FDTol',1.e-5);
S_weak = genspec(sysf,opt);

% set options for least order synthesis with pole assignment
options = struct('tol',1.e-7,'sdeg',-5,'smarg',-5,'poles',[-1 -2],... 
                 'FDTol',0.0001,'rdim',1,'simple',false,'SFDI',S_weak);

\end{verbatim}
% check that minimal order = 6
hsvd(Qtot)

% check achieved structure matrix
isequal(S_weak,fdspec(Rtot))

4.4.3 emmsyn

Syntax

\[ [Q,R,INFO] = \text{emmsyn}(\text{SYSF,SYSR,OPTIONS}) \]

Description

emmsyn solves the exact model matching problem (EMMP) (as formulated in the more general form (25) in Remark 5; see Section 2.6.3), for a given LTI system SYSF with additive faults and a given stable reference filter SYSR. Two stable and proper filters, Q and R, are computed, where Q contains the fault detection and isolation filter representing the solution of the EMMP, and R contains its internal form.
Input data

SYSF is a LTI system in the state-space form

\[
E_\lambda x(t) = A x(t) + B_u u(t) + B_d d(t) + B_f f(t) + B_w w(t),
\]

\[
y(t) = C x(t) + D_u u(t) + D_d d(t) + D_f f(t) + D_w w(t),
\]

(61)

where any of the inputs components \(u(t), d(t), f(t),\) or \(w(t)\) can be void. For the system SYSF, the input groups for \(u(t), d(t), f(t),\) and \(w(t)\) have the standard names ‘controls’, ‘disturbances’, ‘faults’, and ‘noise’, respectively.

SYSR is a proper and stable LTI system in the state-space form

\[
\lambda x_R(t) = A_R x_R(t) + B_{R_u} u(t) + B_{R_d} d(t) + B_{R_f} f(t) + B_{R_w} w(t),
\]

\[
r(t) = C_R x_R(t) + D_{R_u} u(t) + D_{R_d} d(t) + D_{R_f} f(t) + D_{R_w} w(t),
\]

(62)

where the residual signal \(r(t)\) is a \(q\)-dimensional vector and any of the inputs components \(u(t), d(t), f(t),\) or \(w(t)\) can be void. For the system SYSR, the input groups for \(u(t), d(t), f(t),\) and \(w(t)\) have the standard names ‘controls’, ‘disturbances’, ‘faults’, and ‘noise’, respectively.

OPTIONS is a MATLAB structure used to specify various synthesis options and has the following fields:

| OPTIONS fields | Description |
|----------------|-------------|
| tol            | relative tolerance for rank computations (Default: internally computed) |
| tolmin         | absolute tolerance for observability tests (Default: internally computed) |
| smarg          | stability margin for the poles of the filters Q and R (Default: \(-\sqrt{\text{eps}}\) for a continuous-time system SYSF; \(1-\sqrt{\text{eps}}\) for a discrete-time system SYSF). |
| sdeg           | prescribed stability degree for the poles of the filters Q and R (Default: \(-0.05\) for a continuous-time system SYSF; \(0.95\) for a discrete-time system SYSF). |
| poles          | complex vector containing a complex conjugate set of desired poles (within the stability domain) to be assigned for the filters Q and R (Default: [ ] ) |
| simple         | option to employ simple proper bases for synthesis true – use simple bases; false – use non-simple bases (default) |
| minimal        | option to perform least order synthesis of the filter Q true – perform least order synthesis (default); false – perform full order synthesis |
| tcond          | maximum allowed condition number of the employed non-orthogonal transformations (Default: \(10^4\)). |
**normalize** | option for the normalization of the diagonal elements of the updating matrix $M(\lambda)$:

- `'gain'` – scale with the gains of the zero-pole-gain representation (default)
- `'dcgain'` – scale with the DC-gains
- `'infnorm'` – scale with the values of infinity-norms

**freq** | complex frequency value to be employed to check the left-invertibility-based solvability condition (see **Method**)
(Default: [], i.e., a randomly generated frequency).

**HDesign** | full row rank design matrix $H$ employed for the synthesis of the filter $Q$ (see **Method**)
(Default: [])

### Output data

$Q$ is the resulting fault detection filter in a standard state-space representation

$$
\lambda x_Q(t) = A_Q x_Q(t) + B_{Q_y} y(t) + B_{Q_u} u(t),
\quad r(t) = C_Q x_Q(t) + D_{Q_y} y(t) + D_{Q_u} u(t),
$$

(63)

where the residual signal $r(t)$ is a $q$-dimensional vector. For the system object $Q$, two input groups 'outputs' and 'controls' are defined for $y(t)$ and $u(t)$, respectively, and the output group 'residuals' is defined for the residual signal $r(t)$.

$R$ is the resulting internal form of the fault detection filter in a standard state-space representation

$$
\lambda \tilde{x}_R(t) = \tilde{A}_R x_R(t) + \tilde{B}_{R_u} u(t) + \tilde{B}_R d(t) + \tilde{B}_R f(t) + \tilde{B}_R w(t),
\quad r(t) = \tilde{C}_R x_R(t) + \tilde{D}_{R_u} u(t) + \tilde{D}_R d(t) + \tilde{D}_R f(t) + \tilde{D}_R w(t)
$$

(64)

and the same input groups are defined as for $SYSR$ and the output group 'residuals' is defined for the residual signal $r(t)$.

INFO is a MATLAB structure containing additional information as follows:
### INFO fields

| Field  | Description |
|--------|-------------|
| **tcond** | the maximum of the condition numbers of the employed non-orthogonal transformation matrices; a warning is issued if \( \text{INFO.tcond} \geq \text{OPTIONS.tcond} \). |
| **degs** | the left Kronecker indices of \( G(\lambda) := \begin{bmatrix} G_u(\lambda) & G_w(\lambda) \\ I & 0 \end{bmatrix} \) (see Method); also the increasingly ordered degrees of a left minimal polynomial nullspace basis of \( G(\lambda) \); \( \text{INFO.degs} = [ \] if no explicit left nullspace basis is computed) |
| **M** | state-space realization of the employed updating matrix \( M(\lambda) \) (see Method) |
| **freq** | complex frequency value employed to check the left invertibility condition; \( \text{INFO.freq} = [ \] if no frequency-based left invertibility check was performed. |
| **HDesign** | design matrix \( H \) employed for the synthesis of the fault detection filter; \( \text{INFO.HDesignq} = [ \] if no design matrix was explicitly involved in the filter synthesis. |

### Method

Extensions of the Procedure EMM and Procedure EMMS from [11, Sect. 5.6] are implemented in the function `emmsyn`. The Procedure EMM relies on the model-matching synthesis method proposed in [4], while Procedure EMMS uses the inversion-based method proposed in [9] in conjunction with the nullspace method. The Procedure EMM is employed to solve the general EMMP (see below), while Procedure EMMS is employed to solve the more particular (but practically more relevant) strong exact fault detection and isolation problem (strong EFDIP).

Assume that the system \( \text{SYSF} \) has the input-output form

\[
y(\lambda) = G_u(\lambda)u(\lambda) + G_d(\lambda)d(\lambda) + G_f(\lambda)f(\lambda) + G_w(\lambda)w(\lambda)
\]

and the reference model \( \text{SYSR} \) has the input-output form

\[
r(\lambda) = R_u(\lambda)u(\lambda) + R_d(\lambda)d(\lambda) + R_f(\lambda)f(\lambda) + R_w(\lambda)w(\lambda),
\]

where the vectors \( y, u, d, f, w \) and \( r \) have dimensions \( p, m_u, m_d, m_f, m_w \) and \( q \), respectively. The resulting fault detection filter in (63) has the input-output form

\[
r(\lambda) = Q(\lambda) \begin{bmatrix} y(\lambda) \\ u(\lambda) \end{bmatrix},
\]

where the resulting dimension of the residual vector \( r \) is \( q \).

The function `emmsyn` determines \( Q(\lambda) \) by solving the general exact model-matching problem

\[
Q(\lambda) \begin{bmatrix} G_u(\lambda) & G_d(\lambda) & G_f(\lambda) & G_w(\lambda) \\ I_{m_u} & 0 & 0 & 0 \end{bmatrix} = M(\lambda) \begin{bmatrix} R_u(\lambda) \\ R_d(\lambda) \\ R_f(\lambda) \\ R_w(\lambda) \end{bmatrix},
\]
Two cases are separately addressed, depending on the presence or absence of $R_w(\lambda)$ in the reference model \((66)\).

In the first case, when $R_w(\lambda)$ is present, the general EMMP \((68)\) is solved, with $R_u(\lambda)$, or $R_d(\lambda)$, or both of them, explicitly set to zero if not present in the reference model. In the second case, when $R_w(\lambda)$ is not present, then $Q(\lambda)$ is determined by solving the EMMP

$$Q(\lambda) \begin{bmatrix} G_u(\lambda) & G_d(\lambda) & G_f(\lambda) \\ 0 & 0 & 0 \end{bmatrix} = M(\lambda) \begin{bmatrix} R_u(\lambda) & R_d(\lambda) & R_f(\lambda) \end{bmatrix}$$ \hspace{1cm} (70)$$

with $R_u(\lambda)$, or $R_d(\lambda)$, or both of them, explicitly set to zero if not present in the reference model. In this case, if $G_w(\lambda)$ is present in the plant model \((65)\), then $\tilde{R}_w(\lambda)$ is explicitly computed as

$$\tilde{R}_w(\lambda) = Q(\lambda) \begin{bmatrix} G_w(\lambda) \\ 0 \end{bmatrix}.$$  

The particular EMMP formulated in Section 2.6.3 corresponds to solve the EMMP \((70)\) with $R_u(\lambda) = 0$, $R_d(\lambda) = 0$.

If both $R_u(\lambda)$ and $R_d(\lambda)$ are not present in reference model \((66)\), then the nullspace method is employed as the first computational step of solving the EMMP (see Procedure EMM in [11]). The strong EFDIP arises if additionally $R_f(\lambda)$ is diagonal and invertible, in which case, an extension of the Procedure EMMS in [11] is employed. In fact, this procedure works for arbitrary invertible $R_f(\lambda)$ and this case was considered for the implementation of emmsyn. The solution of a fault estimation problem can be targeted by choosing $R_f(\lambda) = I_m$, and checking that the resulting $M(\lambda) = I_m$. Recall that $M(\lambda)$ is provided in INFO.M. In what follows, we give some details of the implemented synthesis approach employed if $R_u(\lambda)$, $R_d(\lambda)$ and $R_w(\lambda)$ are not present in reference model.

If OPTIONS.minimal = false, then $Q(\lambda)$ is determined in the form

$$Q(\lambda) = M(\lambda)Q_2(\lambda)HN_l(\lambda),$$

where: $N_l(\lambda)$ is a $(p - r_d) \times (p + m_u)$ rational left nullspace basis satisfying

$$N_l(\lambda) \begin{bmatrix} G_u(\lambda) & G_d(\lambda) \\ I_m & 0 \end{bmatrix} = 0,$$

with $r_d := \text{rank} G_d(\lambda)$; $H$ is a suitable full row rank design matrix used to build $q$ linear combinations of the $p - r_d$ left nullspace basis vectors ($q$ is the number of outputs of SYSR); $Q_2(\lambda)$ is the solution of $Q_2(\lambda)HG_f(\lambda) = R_f(\lambda)$, where

$$G_f(\lambda) = N_l(\lambda) \begin{bmatrix} G_f(\lambda) \\ 0 \end{bmatrix}.$$
and \( M(\lambda) \) is a stable invertible transfer function matrix determined such that \( Q(\lambda) \) and the corresponding \( \tilde{R}(\lambda) \) in (69) have desired dynamics (specified via OPTIONS.sdeg and OPTIONS.poles). The internal form of the filter \( Q(\lambda) \) is obtained as
\[
\tilde{R}(\lambda) = M(\lambda)Q_2(\lambda)H\mathcal{G}(\lambda),
\]
where
\[
\mathcal{G}(\lambda) := \begin{bmatrix} \mathcal{G}_f(\lambda) & \mathcal{G}_w(\lambda) \end{bmatrix} = N_l(\lambda) \begin{bmatrix} G_f(\lambda) & G_w(\lambda) \end{bmatrix}.
\]
The solvability condition of the strong EFDIP is verified by checking the left invertibility condition
\[
\text{rank} H\mathcal{G}_f(\lambda_s) = m_f,
\]
where \( \lambda_s \) is a suitable frequency value, which can be specified via the OPTIONS.freq. The design parameter matrix \( H \) is set as follows: if OPTIONS.HDesign is nonempty, then \( H = \text{OPTIONS.HDesign} \); if OPTIONS.HDesign = [ ], then \( H = I_{p-r_d} \), if \( q = p - r_d \), or \( H \) is a randomly generated \( q \times (p-r_d) \) real matrix, if \( q < p - r_d \). If OPTIONS.simple = true, then \( N_l(\lambda) \) is determined as a simple rational basis. The orders of the basis vectors are provided in INFO.degs. These are also the degrees of the basis vectors of an equivalent polynomial nullspace basis.

If OPTIONS.minimal = true, then \( [Q(\lambda) \mid \tilde{R}_f(\lambda)] \) has the least McMillan degree, with \( Q(\lambda) \) having \( q \) outputs. \( Q(\lambda) \) and \( \tilde{R}_f(\lambda) \) are determined in the form
\[
\begin{bmatrix} Q(\lambda) & \tilde{R}_f(\lambda) \end{bmatrix} = M(\lambda)Q_2(\lambda)\begin{bmatrix} \mathcal{G}(\lambda) & \tilde{R}_f(\lambda) \end{bmatrix},
\]
where
\[
\begin{bmatrix} \mathcal{G}(\lambda) & \tilde{R}_f(\lambda) \end{bmatrix} = H\begin{bmatrix} N_l(\lambda) & \mathcal{G}_f(\lambda) \end{bmatrix} + \begin{bmatrix} Y(\lambda) & N_l(\lambda) & \mathcal{G}_f(\lambda) \end{bmatrix}
\]
with \( [\mathcal{G}(\lambda) \mid \tilde{R}_f(\lambda)] \) and \( Y(\lambda) \) the least order solution of a left minimal cover problem [10]; \( Q_2(\lambda) \) is the solution of \( Q_2(\lambda)\tilde{R}_f(\lambda) = R_f(\lambda) \); and \( M(\lambda) \) is a stable invertible transfer function matrix determined such that \( [Q(\lambda) \mid \tilde{R}(\lambda)] \) has a desired dynamics. If OPTIONS.HDesign is nonempty, then \( H = \text{OPTIONS.HDesign} \), and if OPTIONS.HDesign = [ ], then a suitable randomly generated \( H \) is employed, which fulfills the left invertibility condition (71).

The actually employed design matrix \( H \) is provided in INFO.HDesign, and INFO.HDesign = [ ] if the solution of the EMMP is obtained by directly solving (68).

**Examples**

Example 5. This is Example 5.12 from the book [11] and was used in Example 3, to solve an EFDIP for a system with triplex sensor redundancy. To solve the same problem by solving an EMMP, we use the resulting \( \tilde{R}_f(s) \) to define the reference model
\[
M_r(s) := \tilde{R}_f(s) = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}.
\]
The resulting least order filter $Q(s)$, determined by employing \texttt{emmsyn} with Procedure EMM, has the generic form

$$Q(s) = \begin{bmatrix} 0 & 1 & -1 & 0 & \cdots & 0 \\ -1 & 0 & 1 & 0 & \cdots & 0 \\ 1 & -1 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$ 

% Example - Solution of an EMMP

\begin{verbatim}
p = 3; mf = 3;  \% enter output and fault vector dimensions
% generate random dimensions for system order and input vectors
rand('seed',16); randn('seed',16);
nu = floor(1+4*rand); mu = floor(1+4*rand);
nd = floor(1+4*rand); md = floor(1+4*rand);
% define random Gu(s) and Gd(s) with triplex sensor redundancy
% and Gf(s) for triplex sensor faults
Gu = ones(3,1)*rss(nu,1,mu); \% enter Gu(s) in state-space form
Gd = ones(3,1)*rss(nd,1,md); \% enter Gd(s) in state-space form
Gf = eye(3); \% enter Gf(s) for sensor faults

% build model with faults
sysf = [Gu Gd Gf];
% set input groups
set(sysf,'InputGroup',struct('controls',1:mu,'disturbances',mu+(1:md),...'
faults',mu+md+(1:mf)));

% enter reference model for the TFM from faults to residual
Mr = ss([ 0 1 -1; -1 0 1; 1 -1 0]);
set(Mr,'InputGroup',struct('faults',1:mf));

% solve an exact model-matching problem using EMMSYN
[Q,R,info] = emmsyn(sysf,Mr);

% check the synthesis: Q*Ge = M*Me and R = M*Mr, where
% Ge = [Gu Gd Gf; I 0 0] and Me = [0 0 Mr ].
Ge = [Gu Gd Gf; eye(mu+md+mf)]; Me = [zeros(p,mu+md) Mr];
norm(gminreal(Q*Ge)-info.M*Me,inf)
norm(R-info.M*Mr,inf)
\end{verbatim}
Example 6. This is Example 5.13 from the book [11], with a continuous-time system with additive actuator faults, having the transfer-function matrices

\[
G_u(s) = \begin{bmatrix}
\frac{s}{s+1} & 0 \\
\frac{s^2 + 3s + 2}{s+2} & \frac{1}{s+2} \\
0 & \frac{1}{s+2}
\end{bmatrix}, \quad G_d(s) = 0, \quad G_f(s) = G_u(s), \quad G_w(s) = 0.
\]

We want to solve an EMMP with the reference model

\[
M_r(s) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},
\]

which is equivalent to solve a strong EFDIP with the structure matrix

\[
S_{FDI} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\]

A least order stable filter \(Q(s)\), with poles assigned to \(-1\), has been determined by employing \texttt{emmsyn} with \textbf{Procedure EMMS}. The resulting \(Q(s)\) is

\[
Q(s) = \begin{bmatrix}
0 & 1 & 0 & -\frac{s}{s+1} & 0 \\
0 & 0 & \frac{s+2}{s+1} & \frac{s}{s+1} & 0 \\
0 & 0 & \frac{1}{s+1} & \frac{1}{s+1} & 0
\end{bmatrix}
\]

and has the McMillan degree equal to 2. The resulting updating factor is

\[
M(s) = \begin{bmatrix}
\frac{s}{s+1} & 0 \\
0 & \frac{1}{s+1}
\end{bmatrix}
\]

and has McMillan degree 2. The presence of the zero at \(s = 0\) in \(R(s) := M(s)M_r(s)\) is unavoidable for the existence of a stable solution. It follows, that while a constant actuator fault \(f_2\) is strongly detectable, a constant actuator fault \(f_1\) is only detectable during transients.

% Example - Solution of a strong EFDIP as an EMMP

% define s as an improper transfer function
s = tf('s');
% enter Gu(s), Gf(s) and Mr(s)
Gu = [s/(s^2+3*s+2) 1/(s+2); 
     0; 
     0 1/(s+2)];
Gf = Gu;
Mr = ss(eye(2)); % enter Mr(s)
[p,mf] = size(Gf); mu = size(Gu,2);

tol = 1.e-7; % set tolerance
sdeg = -1; % set stability degree

% build model with faults
sysf = ss([Gu Gf]);
% set input groups
set(sysf,'InputGroup',struct('controls',1:mu,'faults',mu+(1:mf)));

% setup reference model
sysr = Mr;
set(sysr,'InputGroup',struct('faults',1:mf));

% solve a strong EFDIP using EMMSYN (for an invertible reference model)
opts_emmsyn = struct('tol',tol,'sdeg',sdeg);
[Q,R,info] = emmsyn(sysf,sysr,opts_emmsyn);

% check solution
G = [Gu Gf;eye(mu,mu+mf)];
norm(Q*G-info.M*[zeros(mf,mu) Mr],inf)

% display results
minreal(tf(Q)), tf(info.M)

4.4.4 emdsyn

Syntax

[Q,R,INFO] = emdsyn(SYSM,OPTIONS)

Description

emdsyn solves the exact model detection problem (EMDP) (see Section 3.5.1), for a given stable LTI multiple model SYSM containing N models. A bank of N stable and proper residual generation filters $Q^{(i)}(\lambda)$, for $i = 1, \ldots, N$, is determined, in the form (30). For each filter $Q^{(i)}(\lambda)$, its associated internal forms $R^{(i,j)}(\lambda)$, for $j = 1, \ldots, N$, are determined in accordance with (32).

Input data

SYSM is multiple model which contains $N$ stable LTI systems in the state-space form

$$
E^{(j)} \dot{x}^{(j)}(t) = A^{(j)} x^{(j)}(t) + B_u^{(j)} u^{(j)}(t) + B_d^{(j)} d^{(j)}(t) + B_w^{(j)} w^{(j)}(t),
$$

$$
y^{(j)}(t) = C^{(j)} x^{(j)}(t) + D_u^{(j)} u^{(j)}(t) + D_d^{(j)} d^{(j)}(t) + D_w^{(j)} w^{(j)}(t),
$$

(72)
where \( x^{(j)}(t) \in \mathbb{R}^{n^{(j)}} \) is the state vector of the \( j \)-th system with control input \( u^{(j)}(t) \in \mathbb{R}^{m_u} \), disturbance input \( d^{(j)}(t) \in \mathbb{R}^{m_d} \), and noise input \( w^{(j)}(t) \in \mathbb{R}^{m_w} \), and where any of the inputs components \( u^{(j)}(t), d^{(j)}(t), \) or \( w^{(j)}(t) \) can be void. The multiple model \( \text{SYSM} \) is either an array of \( N \) LTI systems of the form \( (72) \), in which case \( m^{(j)}_d = m_d \) and \( m^{(j)}_w = m_w \) for \( j = 1, \ldots, N \), or is a \( 1 \times N \) cell array, with \( \text{SYSM}\{j\} \) containing the \( j \)-th component system in the form \( (72) \). The input groups for \( u^{(j)}(t), d^{(j)}(t), \) and \( w^{(j)}(t) \) have the standard names ‘controls’, ‘disturbances’, and ‘noise’, respectively.

\( \text{OPTIONS} \) is a MATLAB structure used to specify various synthesis options and has the following fields:

| OPTIONS fields | Description |
|----------------|-------------|
| tol            | relative tolerance for rank computations (Default: internally computed) |
| tolmin         | absolute tolerance for observability tests (Default: internally computed) |
| MDTol          | threshold for model detectability checks (Default: \( 10^{-4} \)) |
| MDGainTol      | threshold for strong model detectability checks (Default: \( 10^{-2} \)) |
| simple         | option to compute a simple proper basis: |
|                | true – compute a simple basis; the orders of the basis vectors are provided in \text{INFO.degs}; |
|                | false – no simple basis computed (default) |
| tcond          | maximum allowed value for the condition numbers of the employed non-orthogonal transformation matrices (Default: \( 10^8 \)) |
|                | (only used if \text{OPTIONS.simple = true}) |
| smarg          | prescribed stability margin for the resulting filters \( Q\{i\} \) |
|                | (Default: \(-\sqrt{\text{eps}}\) for continuous-time component systems; \(1-\sqrt{\text{eps}}\) for discrete-time component systems. |
| sdeg           | prescribed stability degree for the resulting filters \( Q\{i\} \) |
|                | (Default: \(-0.05\) for continuous-time component systems; \(0.95\) for discrete-time component systems. |
| poles          | complex vector containing a complex conjugate set of desired poles (within the stability margin) to be assigned for the resulting filters \( Q\{i\} \) |
|                | (Default: \([\,\,]\) |
| rdim           | desired number \( q \) of residual outputs for the filters \( Q\{i\} \) and \( R\{i,j\} \) |
|                | (Default: \([\,\,]\), in which case \( q = 1 \), if \text{OPTIONS.minimal = true}, or \( q = n_b \), the number of the nullspace basis vectors used for the synthesis (see Method), if \text{OPTIONS.minimal = false}) |
| minimal        | option to perform least order filter syntheses: |
|                | true – perform least order syntheses (default); |
|                | false – perform full order syntheses. |
| MDSelect       | integer vector with increasing elements containing the indices of the desired filters to be designed (Default: \([1,\ldots, N]\)) |
| HDesign        | \( N \)-dimensional cell array; \text{OPTIONS.HDesign}\{i\}, if not empty, is a full row rank design matrix employed for the synthesis of the \( i \)-th filter (Default: \([\,\,]\)) |
Output data

Q is a $N \times 1$ cell array of filters, where $Q\{i\}$ contains the resulting $i$-th filter in a standard state-space representation

$$\lambda x_Q^{(i)}(t) = A_Q^{(i)} x_Q^{(i)}(t) + B_Q^{(i)} y(t) + B_Q^{(i)} u(t),$$

$$r^{(i)}(t) = C_Q^{(i)} x_Q^{(i)}(t) + D_Q^{(i)} y(t) + D_Q^{(i)} u(t),$$

where the residual signal $r^{(i)}(t)$ is a $q_i$-dimensional vector, with $q_i \leq q$. For each system object $Q\{i\}$, two input groups 'outputs' and 'controls' are defined for $y(t)$ and $u(t)$, respectively, and the output group 'residuals' is defined for the residual signal $r^{(i)}(t)$. $Q\{i\}$ is empty for all $i$ which do not belong to the index set specified by \texttt{OPTIONS.MDSelect}.

R is an $N \times N$ cell array of filters, where the $(i,j)$-th filter $R\{i,j\}$, is the internal form of $Q\{i\}$ acting on the $j$-th model. The resulting $R\{i,j\}$ has a standard state-space representation

$$\lambda x_R^{(i,j)}(t) = A_Q^{(i,j)} x_R^{(i,j)}(t) + B_R^{(i,j)} y(t) + B_R^{(i,j)} d^{(j)}(t) + B_R^{(i,j)} w^{(j)}(t),$$

$$r^{(i,j)}(t) = C_Q^{(i,j)} x_R^{(i,j)}(t) + D_R^{(i,j)} y(t) + D_R^{(i,j)} d^{(j)}(t) + D_R^{(i,j)} w^{(j)}(t),$$

and the input groups 'controls', 'disturbances' and 'noise' are defined for $u^{(j)}(t)$, $d^{(j)}(t)$, and $w^{(j)}(t)$, respectively, and the output group 'residuals' is defined for the residual signal $r^{(i,j)}(t)$. $R\{i,j\}$, $j = 1, \ldots, N$ are empty for all $i$ which do not belong to the index set specified by \texttt{OPTIONS.MDSelect}.

INFO is a MATLAB structure containing additional information, as follows:

| INFO fields | Description |
|-------------|-------------|
| tcond       | $N$-dimensional vector; \texttt{INFO.tcond}(i) contains the maximum of the condition numbers of the non-orthogonal transformation matrices used to determine the $i$-th filter $Q\{i\}$; a warning is issued if any \texttt{INFO.tcond}(i) $\geq$ \texttt{OPTIONS.tcond}. |
| degs        | $N$-dimensional cell array; if \texttt{OPTIONS.simple} = true, \texttt{INFO.degs}\{i\} contains the orders of the basis vectors of the employed simple nullspace basis for the synthesis of the $i$-th filter component $Q\{i\}$; \texttt{INFO.degs}\{i\} = [ ] for all $i$ which do not belong to the index set specified by \texttt{OPTIONS.MDSelect}. If \texttt{OPTIONS.simple} = false and $m_d^{(i)} > 0$, then \texttt{INFO.degs}\{i\} contains the degrees of the basis vectors of an equivalent polynomial nullspace basis. If \texttt{OPTIONS.simple} = false and $m_d^{(i)} = 0$, then \texttt{INFO.degs}\{i\} = [ ]. |
$N \times N$-dimensional array containing the resulting model detection performance measure, given as the gains associated with the internal representations: if $\text{OPTIONS.MDFreq} = \{\}$, then $\text{INFO.MDperf}(i,j)$ is the $\mathcal{H}_\infty$-norm of $R\{i,j\}$; if $\text{OPTIONS.MDFreq}$ is non-empty, then $\text{INFO.MDperf}(i,j)$ is the minimum of the $\infty$-norm of the frequency-response gains of the transfer function matrix of $R\{i,j\}$ evaluated over all frequencies in $\text{OPTIONS.MDFreq}$. Ideally, $\text{INFO.MDperf}(i,j)$ should reflect the distance between the $i$-th and $j$-th models (e.g., as provided by the $\nu$-gap metric).

$\text{INFO.MDperf}(i,j) = -1$, for $j = 1, \ldots, N$ and for all $i$ which do not belong to the index set specified by $\text{OPTIONS.MDSelect}$.

$H\text{Design}$ $N$-dimensional cell array, where $\text{INFO.HDesign}\{i\}$ contains the $i$-th design matrix $H(\lambda)$ employed for the synthesis of the $i$-th filter (see Method).

**Method**

An extension of the Procedure EMD from [11, Sect. 6.2] is implemented, which relies on the nullspace-based synthesis method proposed in [7]. Assume that the $j$-th model has the input-output form

$$y(j)(\lambda) = G_u^{(j)}(\lambda)u(j)(\lambda) + G_d^{(j)}(\lambda)d(j)(\lambda) + G_w^{(j)}(\lambda)w(j)(\lambda)$$

(73)

and the resulting $i$-th filter $Q(i)(\lambda)$ has the input-output form

$$r(i)(\lambda) = Q(i)(\lambda) \begin{bmatrix} y(\lambda) \\ u(\lambda) \end{bmatrix}.$$  

(74)

The synthesis method, which underlies Procedure EMD, essentially determines each filter $Q(i)(\lambda)$ as a stable rational left annihilator of

$$G(i)(\lambda) := \begin{bmatrix} G_u^{(i)}(\lambda) & G_d^{(i)}(\lambda) \\ I_{m_u} & 0 \end{bmatrix},$$

such that for $i \neq j$ we have $[R_u^{(i,j)}(\lambda) R_d^{(i,j)}(\lambda)] \neq 0$, where

$$R^{(i,j)}(\lambda) := \begin{bmatrix} R_u^{(i,j)}(\lambda) & R_d^{(i,j)}(\lambda) \end{bmatrix} R^{(i,j)}(\lambda) = Q(i)(\lambda) \begin{bmatrix} G_u^{(i)}(\lambda) & G_d^{(i)}(\lambda) & G_w^{(i)}(\lambda) \\ I_{m_u} & 0 & 0 \end{bmatrix}$$

is the internal form of $Q(i)(\lambda)$ with respect to the $j$-th model.

If $\text{OPTIONS.minimal} = \text{false}$, then $Q(i)(\lambda)$ is determined in the form

$$Q(i)(\lambda) = M(i)(\lambda)H(i)N_l(i)(\lambda),$$

where: $N_l(i)(\lambda)$ is a $(p - r_d(i)) \times (p + m_u)$ rational left nullspace basis satisfying $N_l(i)(\lambda)G(i)(\lambda) = 0$, with $r_d(i) := \text{rank } G_d(i)(\lambda)$; $H(i)$ is a suitable full row rank design matrix used to build $q_i$ (e.g.,
is \( q_i = \text{OPTIONS.rdim} \) linear combinations of the \( p - r_d^{(i)} \) left nullspace basis vectors; and \( M^{(i)}(\lambda) \) is a stable invertible transfer function matrix determined such that \( Q^{(i)}(\lambda) \) has a desired dynamics (specified via \text{OPTIONS.sdeg} and \text{OPTIONS.poles}). \( H^{(i)} \) is set as follows: if \text{OPTIONS.HDesign} is nonempty, then \( H^{(i)} = \text{OPTIONS.HDesign} \); if \text{OPTIONS.HDesign} = \( [ ] \), then \( H^{(i)} = I_{p - r_d^{(i)}} \), if \text{OPTIONS.rdim} = \( [ ] \), or \( H^{(i)} \) is a randomly generated \text{OPTIONS.rdim} \times (p - r_d^{(i)}) real matrix, if \text{OPTIONS.rdim} is nonempty. If \text{OPTIONS.simple} = \text{true}, then \( Q^{(i)}(\lambda) \) is determined as a simple rational basis. The orders of the basis vectors are provided in \text{INFO.degs}. These are also the degrees of the basis vectors of an equivalent polynomial nullspace basis. If \( m_d^{(i)} = 0 \), then the simple choice \( N_1^{(i)}(\lambda) = [I - G_d^{(i)}(\lambda)] \) is employed.

If \text{OPTIONS.minimal} = \text{true}, then \( Q^{(i)}(\lambda) \) is a least McMillan degree left annihilator with \text{OPTIONS.rdim} outputs. \( Q^{(i)}(\lambda) \) is determined in the form

\[
Q^{(i)}(\lambda) = M^{(i)}(\lambda)\tilde{Q}^{(i)}(\lambda),
\]

where \( \tilde{Q}^{(i)}(\lambda) := H^{(i)}N_1^{(i)}(\lambda) + Y^{(i)}(\lambda)N_1^{(i)}(\lambda), \) and \( \tilde{Q}^{(i)}(\lambda) \) and \( Y^{(i)}(\lambda) \) are the least order solution of a left minimal cover problem [10], and \( M^{(i)}(\lambda) \) is a stable invertible transfer function matrix determined such that \( Q^{(i)}(\lambda) \) has a desired dynamics. The global least order for \( Q^{(i)}(\lambda) \) can be achieved by choosing \text{OPTIONS.rdim} = 1. If \text{OPTIONS.HDesign} is nonempty, then \( H^{(i)} = \text{OPTIONS.HDesign} \), and if \text{OPTIONS.HDesign} = \( [ ] \), then a suitable randomly generated \( H^{(i)} \) is employed.

The resulting \( N \times N \) matrix contained in \text{INFO.MDperf} can be used to assess the achieved overall model detection performance. If \text{MDFreq} = \( [ ] \), then the \((i, j)\)-th element of \text{INFO.MDperf} is \( \| [ R_u^{(i,j)}(\lambda) \quad R_d^{(i,j)}(\lambda) ] \|_{\infty} \), and, ideally, represents a measure of the distance between the \( i \)-th and \( j \)-th component systems (e.g., \( \| [ G_u^{(i)}(\lambda) - G_u^{(j)}(\lambda) \quad G_d^{(i)}(\lambda) - G_d^{(j)}(\lambda) ] \|_{\infty} \) or similar measures). If \text{MDFreq} is nonempty, then the \((i, j)\)-th element of \text{INFO.MDperf} is the minimum of the \( \infty \)-norm of the frequency-response gains of \( [ R_u^{(i,j)}(\lambda) \quad R_d^{(i,j)}(\lambda) ] \), evaluated for all frequencies specified in \text{MDFreq}. In this case, \text{INFO.MDperf} ideally represents a measure of the distance between the frequency responses of the \( i \)-th and \( j \)-th component systems, evaluated in a selected set of frequency values.

**Example**

*Example 7.* This is Example 6.1 from the book [11], which deals with a continuous-time state-space model, describing, in the fault-free case, the lateral dynamics of an F-16 aircraft with the matrices

\[
A^{(1)} = \begin{bmatrix}
-0.4492 & 0.046 & 0.0053 & -0.9926 \\
0 & 0 & 1.0000 & 0.0067 \\
-50.8436 & 0 & -5.2184 & 0.7220 \\
16.4148 & 0 & 0.0026 & -0.6627
\end{bmatrix}, \quad B_u^{(1)} = \begin{bmatrix}
0.0004 & 0.0011 \\
0 & 0 \\
-1.4161 & 0.2621 \\
-0.0633 & -0.1205
\end{bmatrix},
\]

\( C^{(1)} = I_4, \quad D_u^{(1)} = 0_{4 \times 2} \).

The four state variables are the sideslip angle, roll angle, roll rate and yaw rate, and the two input variables are the aileron deflection and rudder deflection. The model detection problem addresses
the synthesis of model detection filters for the detection and identification of loss of efficiency of the two flight actuators, which control the deflections of the aileron and rudder. The individual fault models correspond to different degrees of surface efficiency degradation. A multiple model with $N = 9$ component models is used, which correspond to a two-dimensional parameter grid for $N$ values of the parameter vector $\rho := [\rho_1, \rho_2]^T$. For each component of $\rho$, we employ the three grid points $\{0, 0.5, 1\}$. The component system matrices in (72) are defined for $i = 1, 2, \ldots, N$ as:

$$
E^{(i)} = I_4, \ A^{(i)} = A^{(1)}, \ C^{(i)} = C^{(1)}, \text{ and } \ B^{(i)}_u = B^{(1)}_u \Gamma^{(i)},
$$

where $\Gamma^{(i)} = \text{diag}(1 - \rho_1^{(i)}, 1 - \rho_2^{(i)})$ and $(\rho_1^{(i)}, \rho_2^{(i)})$ are the values of parameters $(\rho_1, \rho_2)$ on the chosen grid:

$$
\begin{array}{c|cccccccc}
\rho_1 & 0 & 0 & 0 & 0.5 & 0.5 & 0.5 & 1 & 1 & 1 \\
\rho_2 & 0 & 0.5 & 1 & 0 & 0.5 & 1 & 0 & 0.5 & 1 \\
\end{array}
$$

For example, $(\rho_1^{(1)}, \rho_2^{(1)}) = (0, 0)$ corresponds to the fault-free situation, while $(\rho_1^{(9)}, \rho_2^{(9)}) = (1, 1)$ corresponds to complete failure of both control surfaces. It follows, that the TFM $G^{(i)}_u(s)$ of the $i$-th system can be expressed as

$$
G^{(i)}_u(s) = G^{(1)}_u(s)\Gamma^{(i)},
$$

(75)

where

$$
G^{(1)}_u(s) = C^{(1)}(sI - A^{(1)})^{-1}B^{(1)}_u
$$

is the TFM of the fault-free system. Note that $G^{(N)}_u(s) = 0$ describes the case of complete failure.

The distances between the $i$-th and $j$-th models can be evaluated—for example, as the $\mathcal{H}_\infty$-norm of $G^{(i)}_u(s) - G^{(j)}_u(s)$, for $i, j = 1, \ldots, N$ and are plotted in Fig. 5.

![Figure 5: Distances between component models in terms of $\|G^{(i)}_u(s) - G^{(j)}_u(s)\|_\infty$](image)

For the design of the model detection system, we aim to determine the $N$ filters $Q^{(i)}_u(s)$, $i = 1, \ldots, N$ with scalar outputs, having least McMillan degrees and satisfactory dynamic responses.
Additionally, the resulting model detection performance measures \(\| R_u^{(i,j)}(s) \|_\infty \) should (ideally) reproduce the shape of distances plotted in Fig. 5. For the design of scalar filters, we used the same \(1 \times p\) design matrix \(H\) for the synthesis of all filters, which has been chosen, after some trials with randomly generated values, as

\[
H = \begin{bmatrix} 0.7645 & 0.8848 & 0.5778 & 0.9026 \end{bmatrix}.
\]

The filter synthesis, performed by employing \texttt{emdsyn}, led to first order stable filters, which, as can be observed in Fig. 6, produces similar shapes of the model detection performance measure as those in Fig. 5.

Figure 6: Model detection performance in terms of \(\| R_u^{(i,j)}(s) \|_\infty\)
In Fig. 7 the step responses from $u_1$ (aileron) and $u_2$ (rudder) are presented for the $9 \times 9$ block array, whose entries are the computed TFMs $R^{(i,j)}(s)$. Each column corresponds to a specific model for which the step responses of the $N$ residuals are computed.

Figure 7: Step responses of $R^{(i,j)}(s)$ from $u_1$ (blue) and $u_2$ (red) for least order syntheses.

The following script implements the model building, synthesis and analysis steps.

% Example - Solution of an exact model detection problem (EMDP)

% Define a lateral aircraft dynamics model (without faults) with
% n = 4 states
% mu = 2 control inputs
% p = 4 measurable outputs

A = [-.4492 0.046 .0053 -.9926;
     0 0 1 0.0067;
     -50.8436 0 -5.2184 .722;
     16.4148 0 .0026 -.6627];
Bu = [0.0004 0.0011; 0 0; -1.4161 .2621; -0.0633 -0.1205];
C = eye(4); p = size(C,1); mu = size(Bu,2);
% define the loss of efficiency (LOE) faults as input scaling gains
% \Gamma(i,:) = \begin{bmatrix} 1-\rho_1(i) & 1-\rho_2(i) \end{bmatrix}
\Gamma = 1 - \begin{bmatrix} 0 & 0 & 0.5 & 0.5 & 1 & 1 & 1; & 0.5 & 1 & 0 & 0.5 & 1 \end{bmatrix}';
N = size(\Gamma,1); % number of LOE cases

% define a multiple physical fault model \( G_{ui} = G_u*\text{diag}(\Gamma(i,:)) \)
sysu = ss(zeros(p,mu,N,1));
for i=1:N
  sysu(:,:,i,1) = ss(A,Bu*diag(\Gamma(i,:)),C,0);
end

% set input groups
set(sysu,'InputGroup',struct('controls',1:mu));

% distance plots
distinf = zeros(N);
for i = 1:N
  for j = 1:N
    distinf(i,j) = norm(sysu(:,:,i)-sysu(:,:,j),\text{inf});
  end
end
figure, mesh(distinf)
colormap hsv
title('Distances between component models')
ylabel('Model numbers')
xlabel('Model numbers')

% call of EMDSYN with the options for stability degree -1 and pole -1 for
% the filters, tolerance and a design matrix \( H \) to form a linear combination
% of the left nullspace basis vectors
[H{1:N}] = deal([ 0.7645 0.8848 0.5778 0.9026 ]);
emdsyn_options = struct('sdeg',-1,'poles',-1,'HDesign',{H});
[Q,R,info] = emdsyn(sysu,emdsyn_options);

% inspect achieved performance
figure, mesh(info.MDperf)
colormap hsv
title('Model detection performance')
ylabel('Residual numbers')
xlabel('Model numbers')
% plot the step responses for the internal filter representations
figure
k1 = 0;
for j = 1:N,
    k1 = k1+1;
    k = k1;
    for i=1:N,
        subplot(N,N,k),
        [r,t] = step(R{j,i},4);
        plot(t,r(:,:,1),t,r(:,:,2)),
        if i == 1, title(['Model ',num2str(j)]), end
        if i == j, ylim([-1 1]), end
        if j == 1, ylabel(['r^(^', num2str(i),'^)'],'FontWeight','bold'), end
        if i == N && j == 5, xlabel('Time (seconds)','FontWeight','bold'), end
        k = k+N;
    end
end
References

[1] MathWorks. Control System Toolbox (R2015b), User’s Guide. The MathWorks Inc., Natick, MA, 2015.

[2] A. Varga. DSTOOLS – The Descriptor System Tools for MATLAB. https://sites.google.com/site/andreasvargacontact/home/software/dstools.

[3] A. Varga. On computing least order fault detectors using rational nullspace bases. In Proceedings of the IFAC Symposium SAFEPROCESS, Washington D.C., USA, 2003.

[4] A. Varga. New computational approach for the design of fault detection and isolation filters. In Advances in Automatic Control, M. Voicu, editor, volume 754 of The Kluwer International Series in Engineering and Computer Science, Kluwer Academic Publishers, Dordrecht, 2004, pages 367–381.

[5] A. Varga. On designing least order residual generators for fault detection and isolation. In Proceedings of the 16th International Conference on Control Systems and Computer Science, Bucharest, Romania, 2007, pages 323–330.

[6] A. Varga. On computing nullspace bases – a fault detection perspective. In Proceedings of the IFAC World Congress, Seoul, Korea, 2008, pages 6295–6300.

[7] A. Varga. Least order fault and model detection using multi-models. In Proceedings of the Conference on Decision and Control, Shanghai, China, 2009, pages 1014–1019.

[8] A. Varga. On computing achievable fault signatures. In Proceedings of the IFAC Symposium SAFEPROCESS, Barcelona, Spain, 2009, pages 935–940.

[9] A. Varga. New computational paradigms in solving fault detection and isolation problems. Annu. Rev. Control, 37:25–42, 2013.

[10] A. Varga. Descriptor System Tools (DSTOOLS) User’s Guide. 2017. https://arxiv.org/abs/1707.07140.

[11] A. Varga. Solving Fault Diagnosis Problems – Linear Synthesis Techniques, volume 84 of Studies in Systems, Decision and Control. Springer International Publishing, 2017.

[12] Z. Yuan, G. C. Vansteenkiste, and C. Y. Wen. Improving the observer-based FDI design for efficient fault isolation. Int. J. Control, 68(1):197–218, 1997.
A Installing FDITOOLS

FDITOOLS runs with MATLAB R2015b (or later versions) under 64-bit Windows 7 (or later). Additionally, the Control System Toolbox (Version 9.10 or later) and the Descriptor Systems Tools (DSTOOLS) collection (Version 0.6 or later) are necessary to be installed. To install FDITOOLS, perform the following steps:

- download FDITOOLS and DSTOOLS as zip files from Bitbucket\(^4\)
- create on your computer the directories fditools and dstools
- extract, using any unzip utility, the functions of the FDITOOLS and DSTOOLS collections in the corresponding directories fditools and dstools, respectively
- start MATLAB and put the directories fditools and dstools on the MATLAB path, by using the `pathtool` command; for repeated use, save the new MATLAB search path, or alternatively, use the `addpath` command to set new path entries in `startup.m`
- try out the installation by running the demonstration script `FDIToolsdemo.m`

Note: The software accompanying the book [11] can be also downloaded as a zip file,\(^5\) which also includes FDITOOLS V0.2 and DSTOOLS V0.5. To install and execute the example and case-study scripts listed in the book, follow the steps indicated in the web page. An updated collection of MATLAB scripts with examples is also available from Bitbucket.\(^6\)

\(^4\)Download FDITOOLS from https://bitbucket.org/DSVarga/fditools, and DSTOOLS from https://bitbucket.org/DSVarga/dstools
\(^5\)https://sites.google.com/site/andreasvargacontact/home/book/matlab
\(^6\)https://bitbucket.org/DSVarga/fdibook_examples
B  Current Contents.m File

The M-functions available in the current version of FDITOOLS are listed in the current version of the Contents.m file, given below:

% FDITOOLS - Fault detection and isolation filter synthesis tools.
% Version 0.4 31-August-2017
% Copyright 2016-2017 A. Varga
%
% Demonstration.
% FDIToolsdemo - Demonstration of FDITOOLS.
%
% Analysis functions.
% fditspec - Computation of the structure matrix of a system.
% fdisspec - Computation of the strong structure matrix of a system.
% genspec - Generation of achievable fault detection specifications.
%
% Synthesis functions of fault detection filters.
% efdsyn - Exact synthesis of fault detection filters.
% efdisyn - Exact synthesis of fault detection and isolation filters.
% emmsyn - Exact model matching based synthesis of FDI filters.
%
% Synthesis functions of model detection filters.
% emdsyn - Exact synthesis of model detection filters.
%
% Miscellaneous.
% efdbasesel - Selection of admissible basis vectors to solve the EFDP.
% emmbasesel - Selection of admissible basis vectors to solve the strong EFDIP.
% emdbasesel - Selection of admissible basis vectors to solve the EMDP.
%
C   FDITOOLS Release Notes

The FDITOOLS Release Notes describe the changes introduced in the successive versions of the FDITOOLS collection, as new features, enhancements to functions, or major bug fixes.

C.1  Release Notes V0.2

This is the initial version of the FDITOOLS collection of M-functions, which accompanies the book [11]. All numerical results presented in this book have been obtained using this version of FDITOOLS.

C.1.1  New Features

The M-functions available in the Version 0.2 of FDITOOLS are listed below:

```
% FDITools - Fault detection and isolation filter synthesis tools.
% Version 0.2 31-Dec-2016
% Copyright 2017 A. Varga
%
% Demonstration.
% FDIToolsdemo - Demonstration of FDITools.
%
% Analysis functions.
% fditspec - Computation of the structure matrix of a system.
% fdisspec - Computation of the strong structure matrix of a system.
% genspec - Generation of achievable fault detection specifications.
%
% Synthesis functions.
% efdsyn - Exact synthesis of fault detection filters.
% efdisyn - Exact synthesis of fault detection and isolation filters.
%
% Miscellaneous.
% efdbasesel - Selection of admissible basis vectors to solve the EFDP.
```  

C.2  Release Notes V0.21

Version 0.21, dated February 15, 2017, is a minor improvement over version 0.2 of FDITOOLS.

C.2.1  New Features

A new version of the function genspec is provided, with an enhanced user interface. The new calling syntax, which also covers the previously used calling syntax, is similar to that used by the synthesis functions and allows the direct handling of systems having control, disturbance and additive fault inputs.
C.3 Release Notes V0.3

Version 0.3, dated April 7, 2017, provides a complete set of functions implementing the exact synthesis approaches of fault detection and isolation filters.

C.3.1 New Features

A new function \texttt{emmsyn} is provided for the exact synthesis of fault detection and isolation filters, by using an exact model-matching approach.

A new function \texttt{emmbasesel} is provided for the selection of admissible left nullspace basis vectors to solve the strong exact fault detection and isolation problem, using an exact model-matching approach.

C.4 Release Notes V0.4

Version 0.4, dated August 31, 2017, is a major new release, including substantial revisions of most functions, by adding exhaustive input parameter checks, new user options and several enhancements and simplifications of the implemented codes. Besides these modifications, two new functions have been implemented for the exact synthesis of model detection filters.

C.4.1 New Features

Several new features have been implemented:

- The functionality of \texttt{fditspec} has been enhanced, by generating a three-dimensional structure matrix in the case of several frequency values specified at input. In this case, the pages (along the third dimension) of this array contain the strong structure matrices at different frequency values.

- The functionality of \texttt{fdisspec} has been enhanced, by generating a three-dimensional structure matrix in the case of several frequency values, whose pages (along the third dimension) contain the strong structure matrices at different frequency values. An error message is issued if any of the specified frequencies is a system pole.

- The functionality of \texttt{genspec} has been restricted, by only allowing a single frequency value to generate strong specifications.

- In the function \texttt{efdsyn} a new option to specify a design parameter has been implemented and the internally employed/generated value of this parameter is returned in the \texttt{INFO} structure. This allows, among others, the reproducibility of the computed results. This feature also can serve for optimization purposes (e.g., minimizing the sensitivity conditions; see [11, Remark 5.6]).

- In the function \texttt{efdisyn} a new option has been implemented to specify design parameters for the synthesis of individual filters. The internally employed/generated values of these
design parameters are returned in the INFO structure. This allows, among others, the reproducibility of the computed results. This feature also can serve for optimization purposes (e.g., minimizing the sensitivity conditions of the individual filters; see [11, Remark 5.6]).

- In the function efdisyn a new option has been implemented to specify a subset of indices of the filters to be designed. This allows, for example, to design separately individual filters of the overall bank of filters.

- In the function emmsyn several enhancements of the algorithm implementation have been performed, new options have been implemented for the normalization of the diagonal elements of the updating factor, for the specification of a design parameter and for the specification of a complex frequency value to be used for solvability checks. The actually employed design matrix and frequency are returned in two fields of the INFO structure.

- A new function emdsyn is provided for the exact synthesis of model detection filters, by using a nullspace-based synthesis approach.

- The implementation of efdbasesel has been simplified and its functionality has been enhanced, by allowing as input, a three-dimensional structure matrix as computed by fdisspec. Also, the selection of admissible vectors is performed, regardless the degree information for the basis vectors is provided or not.

- The implementation of emmbasesel has been simplified and the selection of admissible vectors is performed, regardless the degree information for the basis vectors is provided or not.

- A new function emdbasesel is provided for the selection of admissible left nullspace basis vectors for solving the exact model detection problem.

C.4.2 Bug Fixes

The function efdsyn has been updated by fixing a bug in the strong fault detectability test, in the case of more than one frequency values, or in the case when one of the specified frequency values coincides with a system pole.

C.4.3 Compatibility Issues

The function emmsyn has been updated to comply with the version V0.6 of DSTOOLS.
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