Friedmann cosmology with bulk viscosity: a concrete model for dark energy

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The universe content is considered as a non-perfect fluid with bulk viscosity and can be described by a general equation of state (endowed some deviation from the conventionally assumed cosmic perfect fluid model). An explicitly bulk viscosity dark energy model is proposed to confront consistently with the current observational data sets by statistical analysis and is shown consistent with (not deviated away much from) the concordant Λ Cold Dark Matter (CDM) model by comparing the decelerating parameter. Also we compare our relatively simple viscosity dark energy model with a more complicated one by contrast with the concordant ACDM model and find our model improves for the viscosity dark energy model building. Finally we discuss the perspectives of dark energy probes for the coming years with observations.

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I. INTRODUCTION

The cosmological observations indicate that the expansion of our universe accelerates [1]. Recently lots of research work on its possible mechanism, such as extended gravity [2], decaying Λ Cold Dark Matter (CDM) model [3], modifying equation of state (hereafter EOS) or by introducing kinds of the so called dark energy models are to explain the cosmic acceleration expansion observed. To consider causality and the hydrodynamical instability, an interesting barotropic dark energy model is proposed [4] that includes a linear EOS of with a general form, \( p = \alpha (\rho - p_0) \) [5], which incorporated into cosmological model can describe the hydrodynamically stable dark energy behaviors.

The astrophysical observations also indicate that the universe media is not a perfect fluid [6] and the viscosity is concerned in the evolution of the universe [7, 8, 9]. On the other hand, in the standard cosmological model, if the EOS parameter \( \omega \) is less than \(-1\), the universe shows the future finite singularity called Big Rip [10, 11]. Several ideas are proposed to prevent the big rip singularity as thought it un-physical, like by introducing quantum effects terms in the action [12], or by including universe viscosity media for the Universe evolution [13]. So an interesting question naturally arises: what kind of role the cosmic viscosity element can play for helping the above two facets, dark energy and cosmic dark energy model singularity? Considering some deviation from the ideal fluid model is also very helpful to nowadays cosmology probes advancement.

In Refs. [14, 15, 16, 17, 18, 19, 20], the bulk viscosity in cosmology has been studied in various aspects. Dissipative processes are thought to be present in any realistic theory of the evolution of the universe. In the early universe, the thermodynamics is far from equilibrium, the viscosity should be concerned in the studies of the early stage for cosmological evolution. But even in the later cosmic evolution stage, for example, the temperature for the intergalactic medium (IGM), baryonic gas, generally is about 10000K to 1000000K and the complicated IGM is rather non-trivial. The sound speed \( c_s \) in the baryonic gas is only a few \( \text{km s}^{-1} \) to a few tens \( \text{km s}^{-1} \) and the Jeans length \( \lambda \) yields a term as an effective viscosity \( c_s \lambda \). On the other hand, the bulk velocity of the baryonic gas is of the order of hundreds \( \text{km s}^{-1} \) [21]. So it is helpful to consider the viscosity element in the later cosmic evolution. It is well known that in the framework of Friedmann Robertson Walker (FRW) metric, the shear viscosity has no contribution in the energy momentum tensor, and the bulk viscosity behaves like an effective pressure. At the late times, since we do not know the nature of the universe content (dark matter and dark energy components) very clearly, concerning the bulk viscosity is reasonable and practical. Moreover, the cosmic viscosity here can also be regarded as an effective quantity as caused by complicated astrophysics mechanisms and may play a role as a dark energy candidate [22, 23, 24] or a possible unification scheme for the two mysterious dark components (dark matter and dark energy) [24, 25, 26] as they may be facets of the same problem, and then evidence for dark matter is also evidence for dark energy.

The EOS \( p = (\gamma - 1)\rho \) and the bulk viscosity \( \zeta = \alpha \rho^s \) is studied in the full causal theory of bulk viscosity, and the case \( s = 1/2 \) has possessed exact solutions as shown in [16, 17]. However, both the pressure and the bulk viscosity coefficient may have constant components. We argue that the non-causal approximation is reasonable in the late times of the universe evolution. In our previous papers, we have shown that the Friedmann equations can be solved with both a more general EOS and bulk viscosity detailed as follows. (Note that the EOS with a constant pressure gets degenerate with ACDM model.)

\[
p = (\gamma - 1)\rho + p_0,
\]

where \( p_0 \) and \( \gamma \) are two parameters. The bulk viscosity
where \( \zeta_0 \) and \( \zeta_1 \) are two constants conventionally, and the overhead dot stands for derivative with respect to time. The motivation of considering this bulk viscosity is that by fluid mechanics we know the transport/viscosity phenomenon is involved with the "velocity" \( \dot{a} \), which is related to the scalar expansion \( \theta = 3\dot{a}/a \). Both \( \zeta = \zeta_0 \) (constant) and \( \zeta \propto \theta \) are considered in the previous papers \([6, 25]\). Then the Friedmann equations give:

\[
8\pi G \rho = 3 \left( \frac{\dot{a}}{a} \right)^2 = 3H^2
\]

where the expansion parameter \( \theta = U_{\mu}^\mu = 3\dot{a}/a \).

If we assume that the cosmic fluid possesses a bulk viscosity as shown explicitly in the pressure as \( \Pi = -\xi \theta \) (we take another Greek character to represent the bulk viscosity which is different from the before papers), the energy-momentum tensor could be written fully as:

\[
T_{\mu\nu} = \rho U_\mu U_\nu + (p - \xi \theta) H_{\mu\nu}
\]

and assume that the cosmic fluid possesses a bulk viscosity, and conventionally \( \zeta_0 \) and \( \zeta_1 \) are regarded as positive. To choose the parameters properly, it can prevent the Big Rip problem or some kind of singularity for the cosmology model, like in the phantom energy phase, as demonstrated before. Additionally, the sound speed in this model can also keep the causality condition.

This present paper is a continuous work following our previous efforts, which is organized as follows. In Sec. II we present a relatively simple cosmology model with extremely non-relativistic dark matter and viscosity dark energy by an explicit form. With these we give out the exact solution and discuss the acceleration phase in this model. In the last section (Sec.III) we discuss and summarize our conclusions.

## II. MODEL AND CALCULATIONS

We consider the Friedmann-Roberson-Walker metric in the flat space geometry \((k=0)\) as the case favored by WMAP satellite mission on cosmic background radiation (CMB) data

\[
ds^2 = -dt^2 + a(t)^2(d^2 + r^2d\Omega^2),
\]

and assume that the cosmic fluid possesses a bulk viscosity. The energy-momentum tensor can be written as

\[
T_{\mu\nu} = \rho U_\mu U_\nu + (p + \Pi) H_{\mu\nu},
\]

where in the co-moving coordinates \( U^\mu = (1, 0) \), and \( H_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu \). By defining the effective pressure as \( \bar{p} = p + \Pi \) and from the Einstein equation \( R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G T_{\mu\nu} \), we obtain the Friedmann equations

\[
\begin{align}
\dot{a}^2 &= 8\pi G \rho, \\
\dot{a}/a &= -\frac{4\pi G}{3}(\rho + 3\bar{p}).
\end{align}
\]

The covariant conservation equation for energy \( T^{00}_{\mu\nu} \) yields

\[
\dot{\rho} + (\rho + \bar{p})\theta = 0, \quad (7)
\]

where the expansion parameter \( \theta = U_{\mu}^\mu = 3\dot{a}/a \).

For comparison with the observational data, we should derive \( h \) from Eq.\((9)\) by writing it as the equation of redshift \( z \) as:

\[
-2(1+z)\frac{dh}{dz} + 3h = 9\lambda
\]

where \( \lambda = \xi H_0/\rho_{cr} \) is the critical density. Using this definition, we could rewrite equation \((8)\) as:

\[
\frac{dh^2}{dt} + 3h^3 = 9\lambda h^2
\]

and assume that the cosmic fluid possesses a bulk viscosity as shown explicitly in the pressure as \( \Pi = -\xi \theta \) (we take another Greek character to represent the bulk viscosity which is different from the before papers), the energy-momentum tensor could be written fully as:

\[
T_{\mu\nu} = \rho U_\mu U_\nu + (p - \xi \theta) H_{\mu\nu}
\]

where \( U_\mu \) denotes 4 - velocity, as before defined \( \theta = 3H \), and \( \xi \) is the bulk viscosity with its explicit form to be present below.

The cold dark matter has been assumed to be extremely non-relativistic so that we can take its pressure \( p = 0 \), and we now suppose that the effect of dark energy on cosmos evolution is included in the viscous term \(-\xi \theta \) which has the dimension of pressure. In this present work we treat it as the effective pressure of dark energy. Then the covariant conservation of \( T_{\mu\nu} \) (Eq.(7)) yields:

\[
\dot{\rho} + (\rho - \xi \theta) \theta = 0
\]

so,

\[
\dot{\rho} + \rho \theta = \xi \theta^2
\]

Here, it will be convenient to define a dimensionless parameter below:

\[
h = \frac{H^2}{H_0^2} = \frac{\rho}{\rho_{cr}}
\]

where \( \rho_{cr} \) is the critical density. Using this definition, we could rewrite equation \((8)\) as:

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we propose an explicit red-shift dependent bulk viscosity, which gets a constant limit (an effective cosmological constant) today as \( z = 0 \). And we will show that this relatively simple form is better than the complex one in ref.\[32\] when compared with ΛCDM model. The explicit red-shift dependent viscosity is

\[
9\lambda = \lambda_0 + \lambda_1(1 + z)^n
\]

(11)

where \( n \) is an arbitrary integer, \( \lambda_0 \), and \( \lambda_1 \) are two arbitrary constants, which could all be best fitted from the observational data sets.

With this new bulk viscosity, we could write Eq. (10) as:

\[
-2(1 + z)\frac{dh}{dz} + 3h = \lambda_0 + \lambda_1(1 + z)^n
\]

(12)

which is a first-order differential equation of \( h \) and the exact solution of \( h \) from this equation is:

\[
h = \lambda_2(1 + z)^{1.5} - \frac{\lambda_1}{2n - 3}(1 + z)^n + \frac{\lambda_0}{3}
\]

(13)

where \( \lambda_2 \) is an integration constant. The obtained relation above could be regarded that the cosmic expansion rate is from a combined result of the viscosity term: the matter component effect plus the effective dark energy (including the cosmological constant like term \( \frac{\lambda}{a^2} \)) contributions. Because of the consistent requirement \( h = 1 \) for the spatial flat universe at present, from the above we have

\[
\frac{\lambda_0}{3} = 1 - \lambda_2 + \frac{\lambda_1}{2n - 3}
\]

With the analysis of dimension of term \( \lambda_2 \) and the fact that matter density of our cosmos \( \rho_m \propto a(t)^{-3} \), where \( a(t) \) is the scalar scale factor we will show that the value of \( \lambda_2 \) from data fitting below does consist with the result of matter component given by WMAP5 data sets\[28\]. To make the fitting result compared with ΛCDM model, we make an assumption that \( \lambda_2 = \sqrt[5]{\Omega_m} \), where \( \Omega_m \equiv 8\pi G \rho_m / (3H_0^2) \). So in our viscosity dark energy model, the first term as labelled by \( \lambda_2 \) could be looked as matter contribution and the other terms of \( h(z) \) is due to the viscosity dark energy, which with limit case (\( z = 0 \) today) as the concordant ΛCDM model. The consistent requirement relation also gets clear meaning, that is, the relationship between the matter and dark energy (including the cosmological constant contribution) components.

The observations of the SNe Ia have provided the first direct evidence of the accelerating expansion for our current universe. so any model attempting to explain the acceleration mechanism should be consistent with the SNe Ia data implying results, as a basic requirement. As we know the observations of supernovas measure essentially the apparent magnitude \( m \), which is related to the luminosity distance \( D_L \) by

\[
m = M + 5 \log_{10} D_L(z)
\]

(14)

where the distance \( D_L(z) \equiv (H_0) d_L(z) \) is the dimensionless luminosity and

\[
d_L = (1 + z)d_M(z),
\]

(15)

where \( d_M \) is the co-moving distance given by

\[
d_M = \int_0^z \frac{1}{H(z)} dz'
\]

(16)

Also,

\[
M = M + 5 \log_{10} \left( \frac{1/H_0}{1 Mpc} \right) + 25,
\]

(17)

where \( M \) is the absolute magnitude which is believed to be constant for all supernovas of type Ia. In this paper, we use the 307 Union SNe Ia data sets compiled in reference\[26\]. The data points in these samples are given in terms of the distance modulus

\[
\mu_{obs} \equiv m(z) - M_{obs}(z)
\]

(18)

We employ it for doing the standard statistic analysis. So the \( \chi^2 \) is calculated from

\[
\chi^2 = \sum_{i=1}^{n} \frac{[\mu_{obs}(z_i) - M' - 5 \log_{10} D_{Lth}(z_i; c_\alpha)]^2}{\sigma_{obs}(z_i)}
\]

(19)

where \( M' = M - M_{obs} \) is a free parameter and \( D_{Lth}(z_i; c_\alpha) \) is the theoretical prediction for the dimensionless luminosity distance of a supernovae at a particular distance, for a given model with parameter \( c_\alpha \).

On the other hand, the shift parameter \( R \) and the distance parameter \( A \) are considered to give contributions to data fitting. The shift parameter \( R \) is defined in refs.\[29\] and \[30\] as

\[
R \equiv \sqrt{\Omega_m} \int_0^{z_e} \frac{dz'}{h(z')}
\]

(20)

and WMAP5 results\[28\] have updated the redshift of recombination to be \( z_e = 1090 \). Its detail meaning can be found in reference\[31\]. The distance parameter \( A \) is defined as

\[
A \equiv \sqrt{\Omega_m} h(z_b)^{-\frac{1}{2}} \left( \frac{1}{z_b} \int_0^{z_b} \frac{dz'}{h(z')} \right)^{\frac{3}{2}}
\]

(21)

where \( z_b = 0.35 \).

Considering \( R \) and \( A \), we use the total \( \chi^2 \) to make the standard statistic analysis, and data fitting:

\[
\chi^2_{total} = \chi^2 + \left( \frac{R - R_{obs}}{\sigma_R} \right)^2 + \left( \frac{A - A_{obs}}{\sigma_A} \right)^2
\]

(22)

The result of data fitting is listed in TABLE 1.

In Fig. 2, we plot the deceleration parameter \( q \) relation with the redshift as model comparisons:

\[
q = \frac{1 + z}{H} \frac{dH}{dz} - 1
\]

(23)
FIG. 1: The $1\sigma$, $2\sigma$, $3\sigma$ C.L. contours of $\lambda_1$ and $\lambda_2$ in the best fitting condition $n = -1$. The black dot corresponds to the best fitting values.

FIG. 2: $q-z$ relation. Red line (the lowest one) represents $\Lambda$CDM, blue thick line and black line are our models with $n = -1$ and $n = 1$ respectively. By including the more complex viscosity form in [32], we get another result (dashed line) for illustration.

At the same time, we plot the deceleration parameter relations of $q(z)$ in $\Lambda$CDM model and the more complex bulk viscosity in ref. [32] for contrast in Fig. 2, too.

To demonstrate clearly how and when the viscosity dark energy components catch up of the matter contribution, and overpass it we also plot the two components evolution in Fig. 3. With our best fitting result of the $n = -1$ case, we could calculate the EOS of viscosity dark energy. The pressure of dark energy is $-\xi\theta$, so state parameter:

$$\omega = \frac{\xi\theta}{\rho_X}$$

where $\rho_X = \rho(1 - \Omega_m)$ is the density of dark energy. Then we have,

$$\omega = -\frac{3H\xi}{\rho(1 - \Omega_m)}$$

With the relation of $\xi$ and $\lambda$ defined above, we could get $\omega$ today:

$$\omega_0 = -\frac{\lambda_0 + \lambda_1}{3(1 - \Omega_m)}$$

By the best fitting data we give an approximated value of EOS at present ($h = 1$) $\omega \simeq -1.04$. There is a little deviation from $\Lambda$CDM model by a value of -0.04.

Recently, ref. [27] has proposed $Om$ diagnose method to differentiate a new model from the $\Lambda$CDM model with the constant equation of state parameter exactly as -1.

The diagnostic parameter $Om$ is defined as:

$$Om = \frac{h^2 - 1}{x^3 - 1}$$

where $x = 1 + z$. The dimensionless expansion parameter $b(z)$ for a dark energy model with constant equation of state can be written clearly as:

$$h(x)^2 = \Omega_{m0}x^3 + (1 - \Omega_{m0})x^{3(1+\omega)}$$

So, for the $\Lambda$CDM model with its EOS parameter $\omega = -1$, $Om = Om_0$. This result also gives us a null - test of cosmology constant. For model we proposed (here we use the best fitting results with $n = -1$, which gives a better $q(z)$ evolution compared with the $\Lambda$CDM model), its $h^2$ could be written as:

$$h^2 = (\lambda_2 x^{1.5} + \frac{\lambda_1}{5} x^{-1} + \frac{\lambda_0}{3})^2$$

so the $Om$ evolution for this viscosity dark energy model is shown in Fig. 4 to compare with the $\Lambda$CDM model which is a horizontal line in the plot.

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**TABLE I: Fitting results for model parameters**

| $n$  | $\lambda_0$ | $\lambda_1$ | $\lambda_2$ | $\chi^2$ |
|------|-------------|-------------|-------------|-----------|
| -1   | 0.18027     | 2.0923      | 0.52145     | 312.8     |
| -0.8 | -0.04672    | 2.2784      | 0.52027     | 313.0747  |
| -2   | 0.63824     | 1.8331      | 0.52538     | 312.2193  |
| 1    | 1.0464      | -0.15136    | 0.50016     | 323.3296  |

---

FIG. 3: The bulk viscosity dark energy part (the slow changing almost horizontal line) evolutions vs matter component parameter:

$$\omega = -\frac{3H\xi}{\rho(1 - \Omega_m)}$$

In the best fitting condition $n = -1$. The black dot corresponds to the best fitting values.

---

FIG. 4: The $Om$ evolution for this viscosity dark energy model is shown to compare with the $\Lambda$CDM model which is a horizontal line in the plot.
FIG. 4: $\Omega_m - z$ relation as clearly shown the viscosity dark energy model deviations from ΛCDM model. The dashed line corresponds ΛCDM model while the thick curve corresponds to our viscosity dark energy model contribution.

FIG. 5: $q(z) - z$ relations for three different models. The nearby two lines are ours with the explicit viscosity form and the concordant ΛCDM model($\Omega = 0.3$). The dashed line corresponds the complicated one, which can degenerate the previous two models for certain redshift values

With this diagnostic method, we could also see that this model has different properties from ΛCDM model with the exact constant EOS parameter $\omega = -1$.

With the above discussions we can see our relative simple viscosity model fits the concordant ΛCDM model better. It is due to the simpler viscosity form. As a matter of fact so far we still have not obtained a systematic way to express the cosmic viscosity effects. In view of the astrophysical reality any progresses in this field is very helpful. To show the similarities and differences among the concordant ΛCDM model, our model and the complicated one we present a much clear figure below with the hope there will be much relative works to appear soon. Hope the study for viscosity dark energy can be stimulated further.

III. SUMMARY AND DISCUSSIONS

In this letter we present an explicit viscosity form to mimic dark energy behaviors and confront it with current observational data sets. Though the viscosity dark energy form is simple, the results are better. The possible Universe evolution fate, like possible future singularity types from this viscosity model can be also discussed and we will work it out elsewhere. We emphasize that perfect fluid is just a limit case of a general viscosity media that is more practical in the astrophysics sense.

Discovery of dark energy is about ten years old, but its nature and origin have been still puzzling. Fundamental as it has promised to physics foundations there are several possibilities to develop with great expectations. One of these is the unification scenario of dark matter and dark energy, that is, they are two facets of one secret. If we finally discover the dark matter either by accelerators such as LHC or future ILC or by satellite missions, like PAMELA and GLAST now on the running, we may infer the existence of dark energy therefore. In this aspect viscosity dark energy has already shown its unification efforts, to write uniformly all the possible cosmic components in a formula with distinct scaling law with respect to scalar factor. It is worthy further studying.

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