Stability of Scheduled Multi-access Communication over Quasi-static Flat Fading Channels with Random Coding and Independent Decoding

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Abstract—The stability of scheduled multi-access communication with random coding and independent decoding of messages is investigated. The number of messages that may be scheduled for simultaneous transmission is limited to a given maximum value, and the channels from transmitters to receiver are quasi-static, flat, and have independent fades. Requests for message transmissions are assumed to arrive according to an i.i.d. arrival process. Then, we show the following: (1) in the limit of large message alphabet size, the stability region has an interference limited information-theoretic capacity interpretation, (2) state-independent scheduling policies achieve this asymptotic stability region, and (3) in the asymptotic limit corresponding to immediate access, the stability region for non-idling scheduling policies is shown to be identical irrespective of received signal powers.

I. INTRODUCTION

The random-coding upper bound on mean message error probability for maximum likelihood decoding in the presence of noise is exponentially tight in the code length for rates sufficiently close to capacity. Multi-access random-coded communication with independent decoding, of messages that arrive in a Poisson process to an infinite transmitter population, and that achieves any desired value for the upper bound by determining message signal durations appropriately, has been considered in [1] and [2]. In the present work the number of messages that may be scheduled for simultaneous transmission is limited to a given maximum value, and the channels from transmitters to receiver are quasi-static, flat, and have independent fades. Requests for message transmissions are assumed to arrive according to an i.i.d. arrival process. Then, we show the following: (1) in the limit of large message alphabet size, the stability region has an interference limited information-theoretic capacity interpretation, (2) state-independent scheduling policies achieve this asymptotic stability region, and (3) in the asymptotic limit corresponding to immediate access, the stability region for non-idling scheduling policies is shown to be identical irrespective of received signal powers.

First, we describe the model of [1] and [2] in brief. Consider a multi-access message communication system. Requests for message transmissions over a flat bandpass additive white Gaussian noise (AWGN) channel arrive according to Poisson process. Messages are selected from a finite message alphabet of size $M$. Each message has to be transmitted reliably with reliability quantified by the tolerable message decoding error probability, $P_e$. Upon each message arrival the receiver assigns a codebook of Gaussian signals with zero mean, equal power $P$ and uniform power spectral density over a narrow frequency band of width $W$, following random coding. The receiver uses the codebook of a transmitter in independent maximum-likelihood decoding of the message of the transmitter. Each transmitter transmits its signal, starting at its message arrival time, for a random duration determined by the receiver. This signalling duration of a message is chosen so that the random coding bound on the expected probability of message decoding error equals a pre-set value. A noiseless feedback channel from the receiver to the transmitters, and mechanism by which transmitters inform the receiver of their intention to transmit are assumed. In [1] and [2] this random-coded multi-access system is then modelled as a processor-sharing queue in which the transmitters are “customers” that are “served” by the decoder. The processor-sharing model is then analyzed to determine the stability condition and the mean delays experienced by the incoming messages, by determining steady-state probabilities.

In the present work, we generalize this model. First, we limit the maximum number of simultaneous message transmissions on the channel to a finite positive integer $K$. This assumption has the effect of limiting the interference as seen by any message transmission, i.e., at most $K - 1$ transmissions can interfere. Second, we assume independent quasi-static flat fading from the transmitters to the receiver in the channels. With this assumption, there is an i.i.d. multiplicative gain in the channel from each transmitter to the receiver. Thus, for a transmitted signal power $P_m$ for message $m$, the received signal power is equal to $|h_m|^2 P_m$, where the multiplicative gain $h_m$, which is assumed to be known at the receiver, is a random variable that has a finite number of finite possible magnitudes. We assume that each message is assigned a message class upon arrival, that is uniquely identified by the power level it chooses from a finite set of power levels and by its service requirement.

Further, we assume that the receiver schedules messages for transmission in each Nyquist-sample-time-slot and for each system state based on their message classes. We consider two classes of scheduling policies (defined in sections III and IV): ...
(1) non-idling policies, defined in section III and denoted by \( \Omega_K \), which are required to schedule as many as possible up to \( K \) simultaneous message transmissions, and (2) state-independent scheduling policies, defined in section IV and denoted by \( \Omega^K \), which are required to schedule not more than certain numbers of transmissions of the various classes and a total of not more than \( K \) transmissions.

We derive inner and outer bounds to the stability region achievable by scheduling policies in the classes \( \Omega_K \) and \( \Omega^K \) policies. We show that in the limit of transmissions of the various classes and a total of \( S \), where \( S = -\ln P_e + \rho \ln M \) can be considered to be the service requirement of message \( m \). The corresponding signal duration for message \( m \), measured in terms of number of scalar channel uses or slots, is \( d \).

Requests for message transmissions are assumed to arrive at slot boundaries in batches. Each message, upon arrival, is assigned a class \((l,j)\) based on its service requirement and fading value, where \( 1 \leq l \leq L \) and \( 1 \leq j \leq J \), assuming that service requirement takes \( L \) different values and fading takes \( J \) different values. Let \( P_{e,l} \) and \( M_l \) be the tolerable error probability and message alphabet size for class-\((l,j)\) messages respectively. A class-\((l,j)\) message then has a service requirement equal to \( S_l = -\ln P_{e,l} + \rho \ln M_l < \infty \) and received power \( P_J > 0 \). Throughout this paper, the suffix \( l \) will identify service requirement class and suffix \( j \) the received power class.

Let the random variable \( A_l \), with finite moments \( E A_l \) and \( E A_l^2 \), represent the number of messages of class-\((l,j)\) that arrive in any slot, with the pmf \( \Pr(A_l = k) = p_k(l), k \geq 0 \). We assume that \( \{A_l\} \) are independent random variables. Let \( EA = (E A_1, E A_2, \ldots, E A_l, \ldots, E A_L) \in \mathbb{R}_+^L \). Let \( \lambda_l \) denote the arrival rate of class-\((l,j)\) messages. Since each slot is of duration \( 1/P_e \), we have \( \lambda_l = WP_eA_l \). Let \( \Gamma_l = \frac{\rho}{N_{MW}} \) be the received-message-signal to noise ratio (SNR) of a message received at power \( P_J \).

We construct a discrete-time countable state space Markov-chain model and then analyze for the stability (c-regularity) [3] of the model. These stability results are derived by obtaining appropriate drift conditions for suitably defined Lyapunov functions of the state of the Markov chain. In particular, we prove that the Markov chain is c-regular by applying Theorem 10.3 from [3], and then show finiteness of the stationary mean number of messages in the system.

Thus, in the following sections we obtain inner and outer bounds to the achievable stability region for scheduling policies in \( \Omega^K \) and \( \Omega_K \). In some particular cases we derive exact characterization of the stability region.

Let \( s = (s_{i1}, s_{i2}, \ldots, s_{ij}, \ldots, s_{iL}) \in \mathbb{Z}_+^L \) be a vector of non-negative integers and define two sets \( \mathcal{S}_K = \{ s : 0 \leq \sum_{l=1}^L \sum_{j=1}^L s_{lj} \leq K \} \), and \( \mathcal{S}_K^j = \{ s : \sum_{l=1}^L \sum_{j=1}^L s_{lj} = K \} \subset \mathcal{S}_K \) where \( \mathcal{S}_K \) denotes the set of all schedules that schedule exactly \( K \) messages simultaneously for transmission and \( \mathcal{S}_K^j \) denotes the set of all schedules that do not schedule more than \( K \) transmissions. We also define the following subset of \( \mathcal{S}_K^j \): \( \mathcal{S}_K^j(s) = \{ s : \sum_{l=1}^L s_{lj} > 0 \} \) denotes the set of schedules in \( \mathcal{S}_K \) that schedule at least one class-\((..,j)\) message. Let \( \phi_j(s) > 0 \) be the available service quantum per class-\((..,j)\) message under the schedule \( s \in \mathcal{S}_K^j(s) \), and \( \phi_j(s) = 0 \) otherwise. Let \( \phi = \max_{s \in \mathcal{S}_K^j(s)} \phi_j(s) \), \( \bar{\phi} = \min_{s \in \mathcal{S}_K^j(s)} \phi_j(s) \). The following notation will be used in the rest of the paper: for any \( x > 0 \) and \( q > 0 \), \([x]_q = \min(n \geq 1 : x \leq nq)q\).
III. STABILITY FOR NON-IDLING SCHEDULING POLICIES

Formally, a non-idling deterministic scheduling policy in the class $\Omega_K$ of policies, denoted by $\omega$, is defined by the mapping $\{\omega(\alpha) : \mathcal{X} \to \mathcal{S}_K\}$. We also define a non-idling random scheduling policy $\omega$ in the class $\Omega_K$ of policies as a collection of random variables $\mathcal{U}_\alpha$, $\alpha \in \mathcal{X}$. Here, $\mathcal{U}_\alpha$ takes values in $\varphi(\alpha)$ with probability measure $p(\varphi(\alpha))$, where $\varphi(\alpha)$ denotes the set of schedules implementable in state $\alpha$. Throughout this paper we use the same notation $\omega$ to denote both deterministic as well as random policy. Formally, this class of scheduling policies is represented as $\{\mathcal{U}_\alpha, p(\varphi(\alpha)), \alpha \in \mathcal{X}\}$. Thus the set $\Omega_K$ is composed of both deterministic and random non-idling scheduling policies $\omega$.

Let $x$ denote the residual service requirement of a message of any class. Define state as

$$
\alpha = \left\{ (x_1,l_1,j_1), (x_2,l_2,j_2), \ldots, (x_n(\alpha),l_n(\alpha),j_n(\alpha)) \right\}
$$

(1)

where $n(\alpha)$ is the number of messages in state $\alpha$, and $(x_k,l_k,j_k)$ gives the residual service requirement, service requirement class, and received power class, respectively for the $k$th message in state $\alpha$. Let $\mathcal{X}$ be the countable set of state vectors $\alpha$ defined in (1) over which the discrete-time Markov chain $\{X_n, n \geq 0\}$ is defined. Countability of the state space $\mathcal{X}$ follows from the fact that the number of service quantum $x_k$ that a message of class-$k$ is composed of both deterministic and random non-idling scheduling policies $\omega$.

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Then we prove the following two lemmas and two theorems.

**Lemma 3.1:** Let $K \geq 1$. For $\alpha \in \mathcal{X}$, let $c(\alpha) = \sum_{k=1}^{n(\alpha)} \left[ \frac{x_k}{2\rho_k} \right] + 1$ and

$$
V(\alpha) = \frac{c^2(\alpha)}{2 \left( K - \sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j \left[ \frac{S_l}{\rho} \right] \right)}.
$$

Then for $\omega \in \Omega_K$ the Markov chain is $\zeta$-regular if

$$
\sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j \left[ \frac{S_l}{\rho} \right] < K.
$$

**Lemma 3.2:** Let $K \geq 2$. For $\alpha \in \mathcal{X}$, let $c(\alpha) = \sum_{k=1}^{n(\alpha)} (x_k + \rho_{j_k}) + 1$ and

$$
V(\alpha) = \frac{c^2(\alpha)}{2 \left( \phi - \sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j \left( S_l + \rho_{j_k} \right) \right)}.
$$

Then for $\omega \in \Omega_K$ the Markov chain is $\zeta$-regular if

$$
\sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j \left( S_l + \rho_{j_k} \right) < \phi.
$$

**Theorem 3.1:** Let $K = 1$. Then for $\omega \in \Omega_K$ the Markov chain is

(a) positive recurrent and yields finite stationary mean for the number of messages of each class if

$$
\sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j \left[ \frac{S_l}{\rho} \right] < 1,
$$

(b) transient if

$$
\sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j \left[ \frac{S_l}{\rho} \right] > 1.
$$

**Theorem 3.2:** Let $K \geq 2$.

(a) For $\omega \in \Omega_K$, an arrival rate vector $E_A$ satisfying inequality (2) or inequality (3) below belongs to $\mathcal{R}_{in}(\Omega_K)$.

$$
\sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j \left[ \frac{S_l}{\rho} \right] < K \quad (2)
$$

$$
\sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j \left( S_l + \rho_{j_k} \right) < \phi \quad (3)
$$

(b) For $\omega \in \Omega_K$, for every non-empty subset $B$ of $\{1,2,\ldots,J\}$, an arrival rate vector $E_A$ satisfying inequality (4) below belongs to $\mathcal{R}_{out}(\Omega_K)$.

$$
\sum_{l=1}^{L} \sum_{j=1}^{J} S_l E_A t_j \geq \max_{s \in \mathcal{S}_K} \sum_{l=1}^{L} \sum_{j \in B} s_{lj} \rho_{j_k} \quad (4)
$$

Part (a) of Theorem 3.1 follows from Lemma 3.1. To prove Part (b) we show that for the Lyapunov function $V(\alpha) = 1 - \theta c(\alpha)$, where $c(\alpha)$ is as defined in Lemma 3.1 there exists a value for $\theta$, $0 < \theta < 1$, for which $V(\alpha)$ satisfies the conditions for the theorem for transience [4]. Inequalities (2) and (3) of Theorem 3.2 follow from Lemmas 3.1 and 3.2 respectively. To prove inequality (4) of Theorem 3.2 we proceed as follows.

For a non-empty subset $B$ of $\{1,2,\ldots,J\}$, define $r_B(\alpha) = \sum_{k=1}^{n(\alpha)} x_k I_{\{l_k \in B\}}$, where $I_{\{l_k \}}$ is the indicator function, and $V(\alpha) = 1 - \theta^r c(\alpha)$. We then show that for this Lyapunov function there exists a value for $\theta$, $0 < \theta < 1$, for which $V(\alpha)$ satisfies the conditions for the theorem for transience.

**Corollary 3.1:** In the limit $K \to \infty$ the Markov chain $\{X_n, n \geq 0\}$ is

(a) positive recurrent and yields finite stationary mean for the number of messages if $\sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j S_l < \frac{1}{\rho}$, and

(b) transient if

$$
\sum_{l=1}^{L} \sum_{j=1}^{J} E_A t_j S_l > \frac{1}{\rho}.
$$

Corollary 3.1 follows from inequality (3) of Theorem 3.2 and inequality (4) of the same theorem specialized for the set $B = \{1,2,\ldots,J\}$. The corollary says that, in the limit $K \to \infty$, the stability result is independent of message SNR-s and their distribution.

The stability results for the continuous-time models in [1] and [5] coincide with the corresponding results, stated in Corollary 3.1 for the discrete-time model in the limit of large number of simultaneous transmissions.

In the remainder of this section we consider the special case $J = 1$ (corresponding to no fading) for which exact characterization of the stability region $\mathcal{R}_{in}(\Omega_K)$ can be given as follows.
Corollary 3.2: Let \( J = 1 \) and \( K \geq 1 \). For \( \omega \in \Omega_K \) the Markov chain \( \{X_n, n \geq 0\} \) is

(a) positive recurrent and yields finite stationary mean for the number of messages of each class if

\[
K \phi_1 < \sum_{l=1}^{L} E A_{11} [S_l] \phi_l \quad \text{in the limit } K \to \infty, \quad \sum_{l=1}^{L} E A_{11} S_l < \Gamma + 1, \quad \text{and}
\]

and the limit \( K \to \infty, \sum_{l=1}^{L} E A_{11} S_l > \Gamma + 1 \).

(b) transient if

\[
K \phi_1 > \sum_{l=1}^{L} E A_{11} [S_l] \phi_l \quad \text{in the limit } K \to \infty, \quad \sum_{l=1}^{L} E A_{11} S_l < \Gamma + 1, \quad \text{and}
\]

and the limit \( K \to \infty, \sum_{l=1}^{L} E A_{11} S_l > \Gamma + 1 \).

Part (a) of Corollary 3.2 follows from inequality (4) of Theorem 3.2. To prove Part (b) we proceed as follows. Define

\[
r(\alpha) = \sum_{k=1}^{n(\alpha)} [x_k] \phi_k, \quad V(\alpha) = 1 - \theta^r(\alpha),
\]

We then show that for this Lyapunov function there exists a value for \( \theta \), \( 0 < \theta < 1 \), for which the conditions for the theorem for transience are satisfied.

Figure 1 shows plots of message arrival rate stability limit versus \( K \), for the special case \( J = L = 1 \) and for different values of \( \Gamma \) with parameters \( \rho, M_1 \) and \( P_{o,1} \) fixed. From these plots we see that, for sufficiently small transmit powers, as many simultaneous message transmissions as possible should be scheduled, i.e., immediate access should be granted to messages to increase the throughput of the system. For large transmit power, scheduling many transmissions hurts the system throughput. This behaviour can be explained as follows. For small transmit powers, the effective noise seen by a transmission comes mainly from thermal noise, rather than from interference caused by other ongoing message transmissions. Thus, interference from other signal transmissions has insignificant effect on any given transmission, and scheduling as many transmissions as possible is advantageous from the stability viewpoint. For large transmit powers, interference dominates the effective noise seen by any message transmission. Hence, limiting the number of simultaneous transmissions is desirable.

Let \( f_1(M_1) = \frac{K \phi_1 \ln M_1}{[S_1] \phi_1} \) be the maximum stable nat arrival rate in the special case \( L = 1 \). Then the following corollary can be proved.

**Corollary 3.3 (Capacity Interpretation):** Let \( L = J = 1 \).

For a given \( \Gamma, K \), and \( \rho \), the threshold on nat arrival rate, \( f_1(M_1) \), increases with the message alphabet size \( M_1 \) (in the sense that for any given positive integer \( M_1^2 \) there exists a positive integer \( M_1^2 > M_1^2 \) such that \( f_1(M_1^2) > f_1(M_1^2) \)), and approaches the limit \( K W \ln \left( 1 + \frac{\rho}{(1+\rho)[(K-1)\rho+M_0 W]} \right) \).

As \( \rho \to 0 \), this limit has its supremum that is equal to \( K \) times the information-theoretic capacity of an AWGN channel with SNR \( \Gamma \).

**IV. STABILITY FOR STATE-INDEPENDENT SCHEDULING POLICIES**

In this section we consider the class \( \Omega^K \) of state-independent scheduling policies. Formally, a policy in this class is defined by a probability measure \( p(s) \), where \( s \in S_\alpha \). To implement a scheduling policy \( \omega \in \Omega^K \) we first classify incoming messages based on the particular schedule \( s \) to be assigned to them.

For each message arrival class \( (-l, j) \), a schedule \( s \in \{ s \in S_\alpha : s_lj > 0 \} \) is chosen randomly with some fixed probability measure and the message is further classified by assigning the subclass \( (-l, j, s) \) to it. With this classification a message of subclass \( (-l, j, s) \) will be scheduled to transmit only when the schedule \( s \) gets chosen. We first fix a scheduling policy \( \omega = p(s) \) and then, in each time slot, a schedule \( s \) is chosen from the set \( S_\alpha \), independent of the state \( \alpha \), with probability \( p(s) \). Let \( \bar{x} \) denote the residual service requirement of a message of any class. Define state as

\[
\alpha = (x_1, l_1, j_1, z_1), (x_2, l_2, j_2, z_2), \ldots (x_n(\alpha), l_n(\alpha), j_n(\alpha), z_n(\alpha))
\]

where \( n(\alpha) \) is the number of messages in state \( \alpha \) and \( x_k, l_k, j_k, z_k \) gives the residual service requirement, service requirement class, received power class, and the assigned schedule, respectively, for the \( k \)th message in state \( \alpha \). When trying to implement a schedule \( s \) the following two possibilities can occur:

1) there are enough messages of each subclass in state \( \alpha \) to implement the chosen schedule, i.e., \( n_{ij} s(\alpha) \geq s_lj(\alpha) \), where \( n_{ij} s(\alpha) \) denotes the number of messages of subclass \( (-l, j, s) \), in state \( \alpha \). Then we are able to completely implement the schedule.

2) for at least one message subclass, the number of messages is less than that required by the schedule; then we transmit as many messages of such subclasses as are present in the system, and exactly as are required by the schedule in case of other subclasses.

Let \( \phi_2(s) \) denote the available service quantum per class \((-l, j, s)\) message when schedule \( s \) is completely implemented. Let \( \mathcal{X} \) be the countable set of state vectors \( \alpha \) defined in 3.5 over which the discrete-time Markov chain \( \{X_n, n \geq 0\} \) is defined. Countability of the state space \( \mathcal{X} \) follows from the fact that the number of service quantum messages that a message of subclass \( (-l, j, s) \) needs for its service completion is bounded above by \( \lfloor S_l/\phi_2(s) \rfloor \). Then the following lemma and theorem are proved.
Lemma 4.1: Let $K \geq 1$. For $\alpha \in \mathcal{A}$, and for each subclass-$(l, j, s)$ let

$$c_{lj} (\alpha) = \sum_{k: (l_k, j_k, s_k) = (l, j, s)} |x_k| \phi_j(s)$$

Define

$$c(\alpha) = 1 + \sum_{lj s} c_{lj} (\alpha), \quad \text{and}$$

$$V(\alpha) = \sum_{lj s} 2 (p(s) s_{lj} \phi_j(s) - |S_l| \phi_j(s) EA_{lj s})$$

Then for $\omega \in \Omega^K$ the Markov chain is $c$-regular if, for each subclass-$(l, j, s)$,

$$EA_{lj s} < \frac{p(s) s_{lj} \phi_j(s)}{|S_l| \phi_j(s)}.$$ 

where $EA_{lj s}$ is the mean number of subclass-$(l, j, s)$ messages that arrive in any slot.

Define

$$\psi_{lj} = \sum_{\{s \in S_K: s_{lj} > 0\}} p(s) s_{lj} \phi_j(s)$$

and the set

$$R_{in} (\Omega^K) = \bigcup_{\{p(s) \in \Omega^K\}} \{ \beta \in \mathbb{R}_+^{LJ} : \beta_{lj} \leq \psi_{lj} \}$$

Corollary 4.1: For any given message arrival rate vector $EA \in R_{in} (\Omega^K)$ there exists a scheduling policy $p(s) \in \Omega^K$ such that the Markov chain is positive recurrent and yields finite stationary mean for the number of messages of each subclass.

In the following corollary we give information-theoretic capacity interpretation of nat arrival rate stability region.

Corollary 4.2 (Capacity Interpretation): For given $\Gamma$, $K$, $\rho$, and $p(s) \in \Omega^K$, the threshold on nat arrival rate of class-$(l, j)$ messages increases with the message alphabet size $M_l$ and approaches the limit

$$\sum_{\{s \in S_K: s_{lj} > 0\}} p(s) s_{lj} \phi_j(s) \rho.$$ 

As $\rho \to 0$, this limit has its supremum that is equal to

$$\sum_{s \in S_K: s_{lj} > 0} p(s) s_{lj} W \ln \left( 1 + \frac{P_j}{\sum_{l=1}^{L} \sum_{j=1}^{J} s_{lj} P_j - P_j + N_0 W} \right).$$

V. A GENERAL OUTER BOUND TO THE STABILITY REGION

Consider message arrival processes $\{A_{lj}, 1 \leq l \leq L; 1 \leq j \leq J\}$ and a stationary scheduling policy $\omega$ that does not schedule more than $K$ simultaneous transmissions. Let $\pi(s)$ be a probability measure on $S_K$. Define

$$\Psi_{lj} = \sum_{\{s \in S_K: s_{lj} > 0\}} \pi(s) \frac{s_{lj} \phi_j(s)}{S_l}$$

and the set

$$R_{out} = \bigcup_{\pi(s)} \{ \beta \in \mathbb{R}_+^{LJ} : \beta_{lj} \leq \Psi_{lj} \}$$

Theorem 5.1: Let the Markov chain $\{X_n, n \geq 0\}$ be positive recurrent and yield finite stationary mean for the number of messages in the system for the message arrival processes $\{A_{lj}\}$ and the stationary scheduling policy $\omega$. Then $EA \in R_{out}$.

Define

$$R_{in} (\Omega^K) = \bigcup_{\{p(s) \in \Omega^K\}} \{ \beta \in \mathbb{R}_+^{LJ} : \beta_{lj} \leq \psi_{lj} \},$$

where $\psi_{lj} = \psi_{lj} \ln M_l$, and

$$R_{out} = \bigcup_{\pi(s)} \{ \beta \in \mathbb{R}_+^{LJ} : \beta_{lj} \leq \Psi_{lj} \},$$

where $\Psi_{lj} = \psi_{lj} \ln M_l$. Then the following corollary is proved.

Corollary 5.1: In the limit $\min_{1 \leq l \leq L} M_l \to \infty$ we have $R_{in} (\Omega^K) = R_{out}$.

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