Numerical Study of 1D Wave Propagation through Double Negative Photonic Crystal

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Abstract. Photonic crystals are having many applications in different fields like wave guides, high efficiency antenna etc. The conventional photonic crystal contains alternating layers of Double Positive media having high and low positive refractive index values. Recently another kind of unconventional photonic crystal is proposed which consists of a Double Negative meta-material and a Double Positive medium. While in conventional photonic crystals the band gaps are formed due to Bragg scattering of fields, in unconventional photonic crystals the band gaps are formed due to both Bragg scattering and volume averaged zero refractive index of the unit cell. Here we propose a new kind of photonic crystal which consists of two Double Negative meta-materials having different refractive indices. The computational study of the propagation of electromagnetic wave through one dimensional Double Negative photonic crystal is reported. One dimensional Finite Difference Time Domain method is used for the study. Drude model is used in this paper to define the dispersion property of Double Negative media.

1. Introduction
A conventional photonic crystal consists of alternate layers of materials having different dielectric constants [1,2]. These periodic arrangements of material affect the propagation of electromagnetic wave by blocking or reflecting a particular range of frequencies. The frequencies which are forbidden by this structure is called photonic band gap. Here the refractive indices of the constituent materials are positive. Another kind of photonic crystal which consists of a Double Negative Material (DNG) and a Double Positive medium (DPS) is proposed by Jensen Li et.al [3]. In this DNG-DPS photonic crystal, volume averaged zero refractive index of the unit cell is responsible for the band gap [3,4]. A lot of research has been done on different combination of materials to form photonic crystals for different applications [5-12]. In this report we propose a photonic crystal in which both the layers are made up of Double Negative Materials. The values of permittivity and permeability of a DNG medium are negative [13-15]. The region of interest of our study lies in Mid Wave Infrared (MWIR) frequency range which is mainly used in guided missile technology applications. The possibility of enhancing the width of band gap in MWIR region using DNG-DNG photonic crystal is studied. If it is possible to increase the width of band gap, it will be useful for masking the object which is being traced with MWIR frequency for a wide range of frequencies. One dimensional Finite Difference Time Domain (FDTD) method is used in this paper to study the band gap.

2. Theory
For a DNG medium the energy density can be defined as [16]
\[ W = \frac{\partial (\varepsilon \omega)}{\partial \omega} E^2 + \frac{\partial (\mu \omega)}{\partial \omega} H^2 \]  

where \( E \) and \( H \) are the amplitudes of electric field and magnetic field, \( \omega \) is the angular frequency and \( \varepsilon \) and \( \mu \) are frequency dependent permittivity and permeability.

For modeling the frequency dependent permittivity and permeability, Drude dispersion model is used in this paper. For an x-polarized electric field and y-polarized magnetic field propagating in z direction, the one dimensional Maxwell’s equation \([16]\) can be written as

\[ \partial_t E_x = \frac{1}{\varepsilon_0} (- \partial_y H_y - J_x) \]  
\[ \partial_t H_y = -\frac{1}{\mu_0} (\partial_x E_x + K_y) \]  

where \( J_x \) and \( K_y \) are electric current density and magnetic current density.

We use Drude model to define the current densities as \([16]\)

\[ \partial_t J_x + \Gamma J_x = \varepsilon_0 \omega_{pm}^2 E_x \]  
\[ \partial_t K_y + \Gamma K_y = \mu_0 \omega_{pm}^2 H_y \]  

where \( \omega_{pe} \) and \( \omega_{pm} \) are electric and magnetic plasma frequencies and \( \Gamma \) is the dissipation factor.

FDTD method \([17,18]\) is used to solve these equations. The electric field, magnetic field and current densities are discretized in time and space using central difference formula. Standard Yee algorithm is used to implement FDTD method where electric field and magnetic fields are located in leap frog arrangement. The electric field is located at the cell edges for integer time step and the magnetic field is located at the cell center for half integer time step. Electric and magnetic current densities are located at the cell centers \([16]\). In this paper total field-scattered field (TF-SF) formalism is used to introduce a Gaussian pulse in the total field region \([17,18]\). One dimensional Mur Absorbing Boundary condition is used to terminate the FDTD lattice \([17,18]\).

### 3. Observations and Calculations

A structure consists of two DNG slabs arranged in periodic order with 15 periods is modeled. The thicknesses of the slabs are chosen as \( \lambda_0/4n \), where \( \lambda_0 \) is the target wavelength and \( n \) is the refractive index of each slab at the target wavelength. We fix the region of interest of our study in MWIR region (3µm - 8µm). A Gaussian pulse of bandwidth 100 THz is allowed to be incident on the periodic structure at normal incidence. The frequency dependent permittivity and permeability according to Drude model is \([16]\)

\[ \varepsilon(\omega) = \varepsilon_0 \left[ 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\Gamma)} \right] \]  
\[ \mu(\omega) = \mu_0 \left[ 1 - \frac{\omega_{pm}^2}{\omega(\omega + i\Gamma)} \right] \]  

The electric plasma frequency and magnetic plasma frequency are properly chosen such that for the entire frequency range of interest the permittivity and permeability remain negative. For \( \omega_{pe} = 17.36e14 \text{ rad/s} \) and \( \omega_{pm} = 6.14e14 \text{ rad/s} \), \( \varepsilon = -15 \) and \( \mu = -2 \) and for \( \omega_{pe} = 6.14e14 \text{ rad/s} \) and \( \omega_{pm} = 9.7e14 \text{ rad/s} \), \( \varepsilon = -1 \) and \( \mu = -3 \) at the target frequency. With these values of permittivity and permeability two DNG slabs of refractive indices -3.873 and -2 are arranged periodically. The periodic structure starts at the 21st cell in the FDTD lattice. First 20 cells and last 20 cells in the FDTD lattice are filled with air. The reflection spectrum is recorded at the entrance of periodic structure and transmission spectrum is recorded at the exit of periodic structure. The reflection and transmission coefficients of DNG-DNG photonic crystal are calculated and plotted against frequency as shown in figure 1. The result is compared with DPS-DPS photonic crystal having refractive index +3.873 and +2 which is shown in figure 2. It is observed that the band gap is a little wider in the case of DNG-DNG photonic crystal.
Here in figure 1, the band gap is formed between 53.02THz and 98.65THz (45.63THz wide) in MWIR region. From the band gap diagram of dielectric photonic crystal for identical geometry having the magnitude of refractive indices same as the DNG-DNG photonic crystal, it is seen that the band gap is smaller (28.81THz wide from 54.45THz to 83.26THz).

In dielectric photonic crystal, the band gap is formed due to Bragg scattering of electromagnetic waves incident on the periodic structure. In the same way, in DNG-DNG photonic crystal also, the band gap is formed due to Bragg scattering. Here the plasma frequencies are tuned to different values.

Figure 1. Band gap formed in DNG-DNG photonic crystal. Band gap is from 53.02THz to 98.65THz

Figure 2. Band gap formed in DPS-DPS photonic crystal. Band gap is from 54.45THz to 83.26THz.

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Figure 3. Transmission spectra of DNG-DNG photonic crystal for different combination of plasma frequencies. (a) $\omega_{pe1}=8.68e14$rad/s, $\omega_{pm1}=10.63e14$rad/s and $\omega_{pe2}=6.14e14$rad/s, $\omega_{pm2}=9.7e14$rad/s. (b) $\omega_{pe1}=8.68e14$rad/s, $\omega_{pm1}=10.63e14$rad/s and $\omega_{pe2}=\omega_{pm2}=7.52e14$rad/s

for the same refractive index and the band gaps are studied. The band gap study for different combination of electric and magnetic plasma frequencies are shown in figure 3a and figure 3b. It is found that when the difference between electric plasma frequency and magnetic plasma frequency increases, width of the band gap for the desired frequency range also increases.

4. Conclusion
The propagation of electromagnetic wave in the MWIR region through DNG-DNG photonic crystal structure is studied using a home built FDTD code. In a dielectric photonic crystal, band gaps are formed when electromagnetic wave passes through it due to Bragg scattering. In a similar way, it is found in DNG-DNG photonic crystal also a photonic band gap is formed in MWIR region. By varying the plasma frequencies which is used to define the permittivity and permeability of DNG material using Drude model, the band gaps are studied. It is found that as the separation increases the width of
band gap increases. For maximum separation of plasma frequencies, the band width is wider than that in the case of DPS-DPS photonic crystal having same refractive index.

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References
[1] Eli Yablonovitch 1987 Phys. Rev. Lett. 20 58
[2] Sajeev John 1987 Phys. Rev. Lett. 23 58
[3] Jensen Li, Lei Zhou, Chan C T and Sheng P 2003 Phys. Rev. Lett. 8 90
[4] Jiang HT, Chen H, Li H, Zhang Y and Zhu S 2003 Appl. Phys. Lett. 83 26
[5] Makhan M, Ramchurn S K, 2007 J. Opt. Soc. Am. B 24 12 3040-47
[6] Tao Bo, Fu-Li Li and Yu Feng 2009 Modern Physics Letters B 23 25 2943-53
[7] Li-Gang Wang, Hong Chen and Shi-Yao Zhu 2004 Phys. Rev. B 70 245102
[8] Xiang Y, Dai X, Wen S and Fan D 2007 J. Opt. Soc. Am. A 24 10
[9] Tang T, Chen F and Sun B 2011 Optics & Laser Technology 43 237-41
[10] Zhang L, Wang J, Li He, Qiao W, Chen L, Wang Q and Zhao Y 2013 Physica B 431 127-31
[11] Joseph S and Hafiz A K 2014 Optik 125 2734-38
[12] Upadhyay M, Mehta A, Aswathi S K, Srivastava S K, Shukla S N and Ojha S P 2014 J. Intense Pulsed Lasers and Applications in Advanced Physics 4 3 45-54
[13] Veselago V G 1968 Soviet Physics. Uspekhi 10 4 509-14
[14] Anantha Ramakrishna S 2005 Reports on Progress in physics 68 2
[15] John B Pendry and David R Smith 2004 Physics Today 57 6
[16] Richard W Ziolkowski and Ehud Heyman 2001 Physical Review E 64 056625
[17] Dennis M Sullivan 2000 Electromagnetic Simulation using the FDTD method (NewYork: IEEE Press)
[18] Allen Taflove and Susan C Hagness 2005 Computational Electrodynamics Finite Difference Time Domain Method (London: Artech House, INC).