Phase-Space Coalescence for heavy and light quarks at RHIC

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Abstract. We review the application and successes of a phase-space coalescence plus fragmentation model that has been applied for hadronization at RHIC. The physical concept is discussed together with the practical implementation. The robustness of main predictions is reviewed together with several open issues like relevance of three dimensional calculation, finite width of the wave functions, effects of quark masses, energy-entropy conservation, space-momentum correlation. Eventually the relevance of coalescence also for the study of the microscopic interaction of heavy quarks is highlighted.

1 Introduction

The goal of ultra-relativistic heavy-ion collisions is to create a new state of nuclear matter with quarks and gluons as degrees of freedom. Such a state of matter is predicted to be one of the Quantum Chromodynamics (QCD) phases, usually referred to as quark-gluon plasma (QGP), and it is expected to occur at energy densities above $\epsilon_c \simeq 0.7 \text{ GeV/fm}^3$ and temperatures above $T_c \sim 180 \text{ MeV}$ [1]. These conditions are believed to be reached at the Relativistic Heavy Ion Collider (RHIC) as confirmed by the success of ideal hydrodynamics [2,3] (at low $p_T < 1.5 \text{ GeV}$) and the observed strong nuclear suppression $R_{AA}$ in agreement with jet-quenching predictions [4]. In fact both entail an energy density $\epsilon \geq 10 \text{ GeV/fm}^3 \gg \epsilon_c$.

Such a hot and dense medium is therefore expected to be in a deconfined state, i.e. a state where quarks and gluons act directly as degrees of freedom. The success of hydrodynamics based on a QGP equation state in reproducing the observed large elliptic flow [23], together with the failure of hadronic transport model [5], constitutes a first indirect proof of the partonic phase. However in the search for signatures of the QGP and in the study of its properties an important role should be played by the understanding of hadronization dynamics, as persistently stressed by J. Zimanyi [6]. However because of its intrinsic non-perturbative nature there has been a general skeptical view on the real possibility to have hints of quark degrees of freedom. Surprisingly the wide availability of exclusive data from RHIC experiments and the power of the phenomenological approach have made possible to spot signatures of hadronization via coalescence of massive quarks [7,8,9,10,11,12].

Coalescence is a quite general concept that has been applied to nuclear physics for light cluster production for more than two decades [13]. First application to ultra-relativistic heavy-ion collisions is due again to J. Zimanyi and his group in Budapest who implemented an ALgebraic COalescence Rehadronization (ALCOR) model that studies the chemical composition of the hadronic matter produced in heavy ion collisions at SPS and RHIC [14]. However it was only in 2002-03 that some puzzling behavior of the experimental data in Au+Au collisions at RHIC showed features specific of valence quark coalescence: enhancement of baryon/meson...
ratio at intermediate $p_T$, absence of $R_{AA}$ suppression for baryons, and the scaling of elliptic flow according to the constituent quark number [8,9,11,12,13].

In this talk, we will review the general idea of coalescence and its phase-space implementation for the hadronization. Then we will show the comparison with the experimental data for the particle spectra, their ratios and elliptic flows. Finally we will discuss the robustness of the basic features of a naïve coalescence approach together with some open issues. Quark coalescence was initially applied to light quarks, but I will also draw the attention to its role in the investigation of the in-medium interaction of heavy quarks and quarkonia in the QGP [10,16,17].

2 Coalescence and Fragmentation

A lot of efforts has been done to work on the hadronization process, even if a fully satisfying theoretical description is still missing. Because of its highly non-perturbative nature, hadronization is treated through different phenomenological approaches that have shown their validity in different momentum scale regions: string fragmentation, independent fragmentation, dual parton model, coalescence. We will discuss mainly a coalescence plus fragmentation model that has the advantage to describe in an economical way different features of the hadronization and the collective behavior of the hadronic matter created at RHIC.

We briefly remind the main features of independent fragmentation (IF) used as standard hadronization mechanism for enough large $p_T$. IF relies on the QCD factorization theorem that allows to write the hadron production as a convolution $f_H(p_T) = \sum_p f_p(p_T/\mu) \otimes D_{p \rightarrow h}(z, \mu)$, where $f_p$ is the parton distribution in the system, $D_{p \rightarrow h}$ is the fragmentation function, with $z$ fraction of the momentum of the parton carried by the hadron and $\mu \sim p_T^2$ the perturbative scale. In $e^+ e^-$ and $hh'$ collisions, $f_p$ can be determined by the pQCD cross section folded with the parton distribution functions (PDF’s), while in heavy-ion collisions they are calculated as superposition of $pp$ collision plus nuclear shadowing and, moreover, radiative energy loss due to in-medium gluon-radiation [4]. In such an approach non-perturbative features are encoded in the fragmentation function $D_{p \rightarrow h}(z, \mu)$, that gives the probability to produce from a parton $p$ a hadron with momentum $p_h = zp_T$. Such a function is considered to be universal and is extracted from the available data in $e^+ e^-$ collisions and then applied to study also hadron-hadron collisions. This approach has been successful in reproducing the $\pi^0$ transverse momentum distribution down to $p_T \sim 2$ GeV [24] for $pp$ collisions at RHIC energy.

Despite the success of IF for high energy collisions and hadrons at large $p_T$, at lower energy or large rapidity there have been evidences of hadronization mechanism that proceeds through a coalescence mechanism. For example the $D^-/D^+$ ratio in hadron-nucleus reaction at Fermilab [20,21] or the particle production in the fragmentation region (large rapidity) that cannot be explained by IF while coalescence seems a more suitable approach [19]. In such physical conditions it seems necessary to postulate that hadrons come from a convolution of two parton distribution folded by a wave function $f_H(p) = f_p(p_1) \otimes f_p(p_2) \otimes \Phi_M(p_1 - p_2)$, where $p_1$ and $p_2$ are the momenta of coalescing partons. One naively expects that when the phase space is enough dense and the process involves a low virtuality, the production of quarks from vacuum is not likely and the recombination of quarks becomes the dominant mechanism of hadronization. In this perspective HIC’s provide the ideal environment much denser than $hp$ collisions in a wider kinematical region (respect to $hh'$ collisions), for which is natural to think that recombination processes may play a dominant role, at least at not much high $p_T$.

On the other hand it is quite natural to think that nature has a smooth transition between the two processes as a function of phase space and virtuality, hence IF or, at lower $p_T$, string fragmentation could always be present together with coalescence.

3 Phase Space Formulation and Implementation

We describe our implementation of coalescence here, while the fragmentation part is described in the next subsection. Our approach is based on the Wigner formalism [14] that allows a more
direct connection with the dynamical phase-space description of heavy-ion collisions (HIC’s). In this formalism the transverse momentum spectrum of hadrons that consist of $n$ (anti-) quarks is given by the overlap between the hadron wave function and the $n$ quark phase-space distribution function $f_q(x_i, p_i)$:

$$
\frac{dN_H}{d^2P_T} = g_H \int \prod_{i=1}^n \frac{d^3p_i}{(2\pi)^3} E_i \cdot p_i \cdot d\sigma_i f_q(x_i, p_i) f_H(x_1 \ldots x_n; p_1 \ldots p_n) \delta^{(2)} \left(P_T - \sum_{i=1}^n p_{T,i} \right)
$$

(1)

where $d\sigma$ denotes an element of a space-like hypersurface, $f_H(x_1 \ldots x_n; p_1 \ldots p_n)$ is the Wigner distribution function of the hadron, $g_H$ is the probability of forming from $n$ colored quarks a color neutral object with the spin of the hadron considered. In Eq.(1) it is already assumed that the $n$ quark phase-space distribution is approximated by the product of the single quark distribution function

$$
f_q(x_1 \ldots x_n; p_1 \ldots p_n) = \prod_{i=1}^n f(x_i, p_i)
$$

(2)

and therefore no quark-quark correlations are included. The formalism can be formally extended to include such correlations [27], but then it is necessary a dynamical study of the in-medium correlations that asks for transport approaches.

As light hadrons wave function we have used a sphere in both space and momentum, with radii $\Delta_\tau$ and $\Delta_p$, respectively, which in the Wigner formalism are related by $\Delta_\tau \cdot \Delta_p = 1$. A good description of pion, kaon, proton, antiproton spectra can be obtained with a radius parameter $\Delta_p = 0.24 \text{ GeV}$ for mesons and $0.36 \text{ GeV}$ for baryons, which in terms of mean square radius corresponds to take a slightly larger radius for baryons respect to mesons. The multidimensional integral Eq.(1) is evaluated in the full 6D phase space by the Monte Carlo method via test particle method [9].

3.1 Bulk properties

For partons in the quark-gluon plasma we take a thermal distribution for transverse momenta up to $p_T = 2$ GeV in agreement with the hydrodynamical behavior observed at low $p_T$. For their longitudinal momentum distribution we assume boost-invariance, i.e. a uniform rapidity distribution in the rapidity range $y \in (-0.5, +0.5)$. To take into account collective flow of quark-gluon plasma, these partons are boosted by a flow velocity $v_T = \beta_0 (r_T/R)$, depending on their transverse radial position $r_T$, $R$ being the transverse size of the quark-gluon plasma at hadronization, and $\beta_0$ is the maximum collective flow velocity of the quark-gluon plasma. Therefore for light quarks and antiquarks transverse momentum spectra are given by

$$
\frac{dN_{q,\bar{q}}}{d^2p_T d^2p_T} = g_{q,\bar{q}} \tau n_T \left(\frac{m_T}{2\pi}\right)^3 \times \exp \left(-\frac{\gamma_T (m_T - p_T \cdot v_T + \mu_q)}{T}\right),
$$

(3)

where $g_q = g_{\bar{q}} = 6$ is the spin-color degeneracy of light quarks and antiquarks, and the minus/plus signs of chemical potentials are for quarks and antiquarks, respectively. The slope parameter $T$ is taken to be $T = 170 \text{ MeV}$, consistent with the phase transition temperature from lattice QCD calculations [1].

The masses of thermal quarks are taken to be those of constituent quarks, $m_{u,d} = 300 \text{ MeV}$, $m_s = 475 \text{ MeV}$, $m_c = 1.4 \text{ GeV}$ and $m_b = 4.8 \text{ GeV}$ (effect of masses on the typical observed scaling is discussed in Section 4). We notice that in such an approach non perturbative effect are also encoded in the quark masses, in fact a large part of the interaction can be accounted for by quark thermal masses [23,24,25]; however the relation of this masses to the masses associated to scalar condensates remains still an open challenging theoretical question. For the quark chemical potential $\mu_q$, we use a value of $\mu_q = 10 \text{ MeV}$ to give a light antiquark to quark ratio of 0.89, which would then lead to an antiproton to proton ratio of about $(0.89)^2 = 0.7$, consistent with the observed ratio at midrapidity in heavy ion collisions at RHIC.
Above $p_0$, partons are from the quenched pQCD minijets \cite{18}, and their masses are those of current quarks (which schematically resembles the $p-$dependence of the constituent quark masses). Both soft thermal and hard minijet partons are assumed to be distributed uniformly in a fireball of volume of $950 \text{ fm}^3$ (which implies a transverse radius $R = 8.1 \text{ fm}$ and longitudinal length of $\tau = 4.4 \text{ fm}$ for a Bjorken plus transverse expansion). Such a volume is fixed to reproduce the measured transverse energy of $750 \text{ GeV}$ \cite{34}, together with radial flow parameter $\beta_0 = 0.5$ which is consistent with both experimental data and hydrodynamical calculations \cite{3}. In other words the reconstruction of the fireball tells us that the system used for coalescence hadronizes at an energy density of $0.8 \text{ GeV/fm}^3$, which is essentially the energy density at which the phase transition is expected to occur from lattice QCD calculations \cite{1}. In addition the entropy of such a system is $dS/dy \cong 4800$ in agreement with the value inferred from experimental data by S. Pratt and S. Pal \cite{45}. Therefore even if employed mainly at intermediate $p_T$ coalescence model relies on a bulk distribution that is consistent with what can be inferred from hydrodynamics and experimental data.

### 3.2 Minijet Distribution

The transverse momentum distribution for $p_T > 2 \text{ GeV}$ is taken to be that of minijet partons in the midrapidity. In particular we use the one that can be obtained from an improved perturbative QCD calculation \cite{30}. It is given by $dN_{\text{jet}}/d^2p_T = 1/\sigma_{\text{tot}}^{0-10}d\sigma_{\text{jet}}/d^2p_T$ in terms of $\sigma_{\text{tot}}^{0-10}$ corresponding to the total cross section at central 10% of the collisions and the jet production cross section from nucleus-nucleus collisions,

\[
\frac{d\sigma_{\text{jet}}}{d^2p_T} = \int d^2b \int d^2r \ t_{\text{Au}}(r) t_{\text{Au}}(b-r) \sum_{ab} \int dx_a dx_b d^2k_a d^2k_b g(k_a) g(k_b) \\
\times f_{a/AU}(x_a, Q^2) f_{b/AU}(x_b, Q^2) \frac{\delta}{\pi} \delta(s + \hat{t} + \hat{u}) d\sigma^{ab}/dt. \quad (4)
\]

In the above, $t_{\text{A}}(r)$ is the thickness function of Au at transverse radius r. The parton distribution function in a nucleon in the nucleus Au is denoted by $f_{a/AU}(x, Q^2)$ plus a transverse smearing $g(k_T)$. The cross section $d\sigma^{ab}/dt$ is the parton scattering cross section. Kinematic details and a systematic analysis of $pp$ collisions can be found in Ref. \cite{30}. Using the GRV94 LO result for the PDF \cite{31} and the KKP fragmentation function from Ref. \cite{32}, measured data in the reaction $pp \rightarrow \pi^0 X$ at $\sqrt{s} = 200 \text{ GeV}$ can be reproduced with $Q = 0.75p_T$ and $\langle k_T^2 \rangle = 2 \text{ GeV}^2$.

In heavy ion collisions at RHIC, minijet partons are expected to lose energy by radiating soft partons as they traverse through the quark-gluon plasma \cite{4}. This effect is taken into account by lowering their transverse momenta by the energy loss $\Delta E$, which depends on both the parton energy $E$ and an effective opacity parameter $L/\lambda$ according to the GLV model \cite{18}. An effective opacity $L/\lambda = 3.5$ is used, as extracted from a fit to the spectrum of high transverse momentum pions measured at RHIC \cite{18} \footnote{Such an assumption is in principle too drastic for minijets, however it does not affect our results because jet-jet coalescence is never taken into account.}

We note that our approach is indeed quite schematic in separating the thermal spectrum from the one of minijets. In reality there is only one spectrum with possible correlations also at $p_T < p_0$. Our simplification does not affect the one-body observables described in the following, but can be essential for studying two or three particle angular correlations \cite{70,75}.

### 4 Coalescence for light quarks

In the coalescence model, hadrons are formed from quarks that are close in phase space if a hadron wave function with a small width is considered. As a result, baryons with momentum

\footnote{In this calculation is not included the QCD analog of the Ter-Mikayelian effect. When included a $L/\lambda = 5$ has to be used to have the same amount of quenching \cite{61}.}
$p_T$ are produced from quarks with momenta $\sim p_T/3$, while mesons with same momentum are from quarks with momenta $\sim p_T/2$. Since the transverse momentum spectra of quarks decrease with $p_T$, production of high momentum baryons from quark coalescence is favored respect to the fragmentation where baryons are penalized with respect to mesons as more quarks are needed from the vacuum. In the independent fragmentation the estimated value of $p/\pi$ ratio is of $\sim 0.2 \div 0.3$ in approximate agreement with the ratio measured in $pp$ collisions. For Au+Au collisions results for $\bar{p}$ and $\pi^-$ spectra are shown (solid lines) together with the data from PHENIX [35] in Fig. 1(left). The resulting $\bar{p}/\pi$ and the $K^0_s/\Lambda$ ratios based on Eq. (1) are shown in Fig. 1(right) together with data points [26,38]. Calculations shown for $\pi$, $K$ include the contribution from independent fragmentation that is essential to reproduce the meson spectra already at $p_T \sim 3$ GeV (especially for pions). Although different models have been used, they all lead to enhanced $\bar{p}/\pi$ ratio and $K^0_s/\Lambda$ ratio of similar magnitude [7,11]. As shown in Fig. 1(left), our approach [8,9], which includes resonance decays and avoids the collinear approximation by using the Monte Carlo method to evaluate the multi-dimensional coalescence integral, gives a good description of spectra, $\bar{p}/\pi$ and $K^0_s/\Lambda$ ratios also at $p_T < 2$ GeV, even if in this momentum region the assumptions used are less under control (for a more extended discussion see next Section). At lower energy, Au+Au at 62 AGeV, a direct extrapolation with the same model predicts an enhancement of the $p/\pi$ ratio and a decrease of $\bar{p}/\pi$ respect to 200 AGeV. Such a trend is again in reasonable agreement with experimental data [28,29].

In HICs the anisotropy of particle momentum distributions with respect to the azimuthal angle offers the possibility to get information on the dynamics of the collisions and the properties of the created hot and dense matter [40]. At RHIC energies it has been shown to be an important probe for the equation of state of the quark-gluon plasma [3] and of the parton cross section [36,37], and for hadrons at high transverse momentum it is a probe of the initial energy density that causes the jet quenching [4]. Moreover it has been shown that it is built-in very early in the dynamical evolution being thus a probe of the interaction during the partonic stage [38], a picture recently confirmed by the measurement of $\phi$ meson $v_2$ [51].

Thanks to coalescence models it was realized that elliptic flow provides also a way to understand the hadronization mechanism in itself. In fact if hadronization goes through coalescence of constituent quarks the anisotropy at partonic level propagates at hadronic level according to [12]:

$$v_{2,M}(p_T) \approx 2v_{2,q}(p_T/2), \quad v_{2,B}(p_T) \approx 3v_{2,q}(p_T/3).$$

This effect together with the shape of rise and saturation at parton level predicted by the parton cascade [37] for the partonic flow allows to see the characteristic feature of coalescence that is a larger elliptic flow for baryons, by a factor 3/2, and a similar shift of the $p_T$ at which
the $v_2$ reaches the maximum value. These features are clearly seen in the experimental data, as anticipated by Voloshin [13], see Fig.2. We notice that the coalescence mechanism translates the hydrodynamical behavior into higher $p_T$ at hadronic level with the effect being larger for baryons. This is seen also from the agreement of hydrodynamics $v_2(p_T)$ for mesons up to $\sim 1.5$ GeV and for baryons up to $p_T \sim 2.5$ GeV. As a general remark we like to point out that coalescence is the only known hadronization mechanism that can lead to an enhancement of the partonic elliptic flow. Someone may think that hydrodynamics gives the larger $v_2$, but at fixed elliptic flow in the partonic stage coalescence can still enhance the elliptic flow. However, Eq. (5) is valid under the assumption of a uniform phase space density [46] and the approximation of a one dimensional space in which only collinear and equal momentum partons recombine. Relaxation of these assumptions can in principle break “quark number scaling” of the elliptic flow. For a discussion about effects beyond the simplest (naive) coalescence, see the next Section.

In Fig. 2 (left) we show the scaled elliptic flow for pions and protons obtained with our model once the quark $v_2^q$ is fitted to reproduce $K$ elliptic flow, then elliptic flow of $p, \bar{p}, K, \Lambda$ is well reproduced [9,68]. The calculations include the effect of resonance decays that is another possible source of scaling breaking [50], as discussed more in detail in the next Section. The contribution to hadronic elliptic flow from minijet fragmentation is not included in this calculation. Its inclusion would lead to a universal hadron elliptic flow at momentum above $p_T \sim 6$ GeV [11].

In Fig. 2 (right), taken from Ref. [65], the scaling of $v_2$ for various identified hadrons is shown together with the relative baryon to meson $((B-M)/(B+M))$ deviation from the naive scaling Eq. (5). The circles are the experimental data, the dotted line is the prediction of our model [50], and the solid line is the effect coming from higher Fock state in the hadron wave function discussed in Ref. [78]. The agreement of our model with the data comes from a combination of the finite width of the hadron wave function and the effects of resonance decays.

In Fig. 3 (left) it is shown the effect of quark mass on the scaling of $v_2$ at fixed wave function width ($\Delta p = 0.24$ GeV). In panel (a) it is shown an ideal case in which the quark $v_{2q}(p_T)$ is flat, it is evident a strong violation of the scaling at low $p_T$. The source of such a violation of the scaling is the full 6D phase space that allows coalescence of non-collinear momentum at low $p_T$ where the boost effect is small. In such a case the effect is nearly independent on the quark mass. In the lower panel (b) a more realistic case is shown, i.e. a $v_2(p_T)$ that rise and saturates at $v_2 = 0.1$. In such a case the violation of the scaling at low $p_T$ is drastically reduced, while at higher $p_T$ it is significantly mass dependent. We clearly see a violation of $\sim 25 - 30\%$ if a mass $m_q = 0.03$ GeV is used. This is due to the fact that when the mass is small, even if the wave function width is small in the rest frame of the hadron, particles with large difference in the relative momenta can still coalesce due to boost effects. In such a case particles at higher
$p_T$ ($v_2 \sim 0.1$) can coalesce with particles at small $p_T$ ($v_2 \sim 0$) and this effect is of course larger for smaller quark mass.

\[
\begin{align*}
\text{Fig. 3. Left: Scaled elliptic flows, the solid black line is the quark } v_{2q}, \text{ the dashed and dot-dashed line are the } v_2 \text{ of mesons from coalescence for two different values of the quark mass } m_q, \text{ see text for details. Right: } v_4 \text{ in Au+Au collisions at 200 AGeV for charged (solid line), pion (dashed line) and proton (dot-dashed line) evaluate from coalescence; circles are data taken from Ref.\cite{STAR}.}
\end{align*}
\]

It will be important to see experimentally where the scaling breaks down as a function of the beam energy but also as a function of the rapidity. In principle one would expect that at very forward (backward) rapidity one reaches conditions that are comparable to the midrapidity one at lower energy, therefore it could be that a QGP phase is no longer dominant at large rapidity. Indeed we have seen that in the AMPT model the magnitude of $v_1$ and $v_2$ at $y \geq 3$ are consistent with a pure hadronic matter dynamics\cite{AMPT,55}.

The study of azimuthal anisotropy has been mainly limited to the elliptic flow (second harmonic of a Fourier expansion in the azimuthal angle) but a sizeable amount of the fourth order momentum anisotropy has been predicted\cite{54} and confirmed experimentally\cite{52}. This provides the possibility to further test the coalescence model, in fact also for the $v_4$ precise relations between mesonic and baryon $v_4$ are expected\cite{39}, as for example:

\[
\frac{v_{AB}}{v_{2B}}(3p_T) \approx \frac{2}{3} \frac{v_{1M}}{v_{2M}}(2p_T) + \frac{1}{6}.
\]

Finite momentum spread and resonance decay do not change significantly such a prediction, especially at $p_T > 1$ GeV\cite{68,39}. For the details on the derivation of this and other scaling relations also for odd harmonics we remand to Ref.\cite{55}. In Fig.3 (left) predictions for pion and proton $v_4$ are shown, the calculation is done fitting $v_{4q}$ to the charged hadron $v_4$. Recent data\cite{56} again confirm the trend predicted by coalescence\cite{39,68}.

5 Beyond the simplest implementation

The RHIC program has provided a remarkable evidence that coalescence of massive quarks supplies a simple and succesful model for hadronization from a deconfined plasma. Nonetheless there are several aspects that are still problematic and act as a stimolous to a deeper and advanced formulation of the coalescence process in the context of hadronization. Some of these problems are more serious if hadronization via coalescence is applied to bulk production of hadrons. Nevertheless, once coalescence is recognized to be the dominant mechanism at intermediate $p_T$, it is appealing to investigate its behavior at low momenta where the phase space is
denser and coalescence probability increases. Till now we have simply extended the approach at quite low momenta noticing that no striking contradiction with the experimental data are observed which validates also the good agreement at intermediate $p_T$. Even if one should be aware of the accompanying problems, that we discuss in the following, also the success of ALCOR in reproducing the total yields \[15\] has to be considered. Furthermore the reasonable description of the experimental data on particle spectra also at low $p_T$ in our approach is found also in a more recent based on ALCOR/MICOR approach \[14\]. We want also to point out that an important probe for the formation of QGP and the hadronization mechanism is given by the charge fluctuations \[72\]. First data \[74\] agrees better with predictions from coalescence \[73\]. Furthermore charge fluctuation of the bulk are reproduced by a coalescence hadronization mechanism if the number of quarks and antiquarks is of the order of $dN/dy \approx 1300$ \[81\] (for most central collisions) which is similar to the number used in the model discussed here at intermediate $p_T$ and to what is extracted by ALCOR model that is instead dedicated to the study of the multiplicities \[80\]. This shows again an internal consistency of coalescence models.

The simplest approach to hadronization by coalescence considers a uniform distribution of particles that combine if have the same $p_T$ forming directly the stable hadrons. Of course this seems too naive for a number of reasons that I try to list here together with some investigations beyond this simple picture that have been or could be done in the next future:

1. **Resonances** - The baryon/meson anomaly and moreover the scaling of elliptic flow are directly applicable to stable hadrons, on the other hand it is well-known that especially for pions there is a large feed-down from resonance decays. In the model we have presented here and published originally in Ref. \[9\] resonances were already included. As discussed more in detail in Ref. \[50\], at intermediate $p_T$ their role is quite reduced and therefore their discard by other groups is justified when looking at $p_T \approx 3$ GeV. However our calculation shows that when included both $p_T$ spectra and $v_2(p_T)$ have a better agreement with data at lower $p_T$. In particular, we have studied how the elliptic flows of pions and other stable hadrons are affected by decays of resonances, such as $\rho \to 2 \pi$, $\omega \to 3 \pi$, $K^* \to K \pi$, and $\Delta \to p \pi$ \[50\], see Fig.4. It turns out that particles like $p$, $\Lambda$, and $K$ from resonance decays have elliptic flows that are very similar to the directly produced ones. Therefore, the inclusion of resonances does not destroy the coalescence scaling of these stable hadrons. On the other hand, pions from the decay of $\omega$, $K^*$, $\Delta$ show a significant enhancement of their elliptic flow at $p_T < 2$ GeV. The breaking of coalescence scaling due to resonance decays together with that due to finite quark momentum spread (see next point) lead to a better agreement with available data as shown in Fig.2 where the decay of resonances is included.

   However pions are not expected to be well described, but one has also to consider that at intermediate $p_T$ the mass mismatch is less relevant and at low momenta most of the pions...
are coming from resonance decays. This can be the reason behind the approximate good description of pions in our model. [9,50].

2. Wave function - Our formalism described in Section 3 contains the full 3D phase space and a wave function with a finite width $\Delta_p$ of the order of the Fermi momentum [50]. The effect $\Delta_p$ is shown in Fig. 4 (right) where we can see a significant increase of the violation of the scaling between partons and mesons as a function of the width of the wave function. However with the same width used to reproduce the $p_T$ spectra the inclusion of a full 3D phase space and wave function does not destroy but even improve the agreement with experimental data, see Fig 2 and the relative discussion [4]. An extreme ansatz for the wave function could be considered, but if the wave function is flat in term of the relative momentum, the naive formulas Eq.(5) (right) are strongly modified leading to a reduced difference between baryons and mesons [48] not observed experimentally.

3. Energy and Entropy Conservation - Another issue related to Eq.(1) is the energy conservation. In the present approach on shell massive quarks are recombined into one particle respecting momentum conservation, but not energy conservation. Of course this is a drawback of the simplest implementation while recombination processes always happen in nature involving a third particle or off-shell recombining particles.

In our model pions come mainly from $\rho$ decay hence the energy violation is less serious even at low $p_T$, at the same time both the $p_T$ distribution and the elliptic flow get closer to data thanks to a shift to lower $p_T$. For example the $v_2$ of pions gets closer to a scaling with $m_T - m_0$ as found experimentally [70], which can be seen as an indirect effect of energy conservation. More recently energy conservation in the coalescence process has been investigated by Ravagli and Rapp [69] using a formulation of coalescence from the Boltzmann collision integral. They find that if energy conservation is implemented then the $v_2$ scaling as a function of $p_T$ moves toward a scaling versus $m_T - m_0$. Another important development is the inclusion of a mass distribution for the quarks as an effective way for including in-medium interaction [6], this is another way to allow for both momentum and energy conservation and leads to a fairly good agreement with the data, for more details see the contribution of T. Biró to these Proceedings.

An issue related also to energy conservation is the entropy conservation. Coalescence reduces the number of particles by about a factor two (if resonance production is not taken into account), this rises suspes of a strong decrease of entropy. Of course entropy is not the number of particle, but depends also on the degeneracy of the two phases and on the mass of the particles. For example converting one gluon into one pion with the same mass leads to a huge entropy violation. In other words a large chemical potential is enforced by number

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3. The $v_2(p_T)$ used here is only schematic to show the effect of the violation that for a more realistic $v_2(p_T)$ is however reduced to about a 10% for $\Delta_p = 0.24$ GeV.
conservation causing an entropy decrease unless at the same time there is a large volume expansion.

In Fig. 5 it is shown the evolution of $dS/dy$ (for $|y| < 0.5$ as a function of time if one assumes that coalescence takes place during a mixed phase of 4 fm (and 8 fm upper line), during which the volume expands with $\beta = 0.5$ and quarks are gradually converted into hadrons which are allowed to decay according to their width. For the full calculation (expansion plus decays) a decrease of about 15\% is found, but in our coalescence model there is also a decrease in the energy by about 15\% ($S = (E + \ln Z)/T$, where $Z$ is the partition function). Therefore if coalescence is implemented taking care of the energy conservation probably entropy can also be conserved. For the multiplicity of various hadrons the calculation is based on the results from our coalescence model (but only $u,d$ matter is considered). The important role of the decays and the moderate role of the expansion are shown by the different cases considered, see Fig. 5. We also see that assuming a longer mixed phase ($t_f = 8$ fm), and therefore a larger volume at the end entropy can be conserved even in such a simple approach. In addition one should also take into account the interaction among quarks; it has been shown using the IQCD equation of state for an isentropically expanding fireball that the evolution of the effective number of particles reduces significantly around the crossover temperature [71]. This of course helps to solve the entropy problem inherent to quark coalescence, as pointed out also by Nonaka et al. [81].

4. **Space-momentum correlation and $v_2$ scaling** - The constituent quark number scaling of the elliptic flow was derived by Voloshin and Molnar assuming that the x-space can be decoupled from p-space and integrated out. This means that the scaling has been explicitly demonstrated only if the coalescence probability is homogeneous in space and if space and momentum are not correlated. A detailed discussion on effects coming from space and momentum correlation as a source of scaling violation can be found in Ref.[46] together with classes of distribution that lead to an approximate scaling. Another study on the effect of phase space distribution can be found in Ref.[66].

Results presented here with our coalescence model do not assume the coalescence scaling and show that if radial flow space-momentum correlation (but still in a uniform fireball) is included the scaling is still recovered [9]. An important tool of investigation in this context is supplied by parton cascade studies that calculate the time evolution of the phase space. A first investigation by Molnar finds that the scaling between baryons and mesons still persists even if a strong violation is found respect to the $v_2$ at quark level [63]. However it is not clear the dependence on the freeze-out criteria, on the wave function width and on the interplay with indipendent fragmentation mechanism as sources of scaling breaking, see also D. Molnar contribution to these Proceedings.

Finally it is clear that a better understanding of HBT measurements can supply fundamental information on this issue that is the potential main source of scaling violation and therefore should be investigated in deeper detail. At LHC the different dynamical evolution may lead to an even stronger scaling violation.

5. **Jet-like correlation** - The last but not the least issue is related to the correlation among hadrons in the hadronization process. At variance with the other issues this is the one more relevant at intermediate $p_T$. At RHIC it has been possible to measure the correlation between hadrons at momentum $p_T^{\text{trig}} > 4-6$ GeV and the associated particles with momenta $p_T < p_T^{\text{trig}}$ as a function of the azimuthal angle respect to the trigger particle [75,76]. Such a kind of measurement has shown that hadrons at intermediate $p_T$ come with associated particles at lower momentum in the opposite direction. In the seminal papers [59,70] we have taken into account the possibility of coalescence between thermal partons and minijets and this was a primitive way to propose the idea that at intermediate $p_T$ particle from coalescence could follow the di-jet correlation. A formal framework for the effect of quark correlations has been studied by R. Fries et al. [27] and R. Hwa and C.B. Yang [62]. They have shown that coalescence goes along with particle angular correlation, even if more stringent tests are currently under investigation. Anyway the main problem remains the study of the origin of correlation at parton level which asks for a transport approach, a first explorative study has been performed by Molnar [64].
In the future it is desirable to have a deeper theoretical investigation of coalescence, in fact it is easy to predict that such a kind of processes will be even more dominant at the Heavy-Ion experiments at the Large-Hadron-Collider (LHC) at CERN, see also P. Levai contribution to these Proceedings. In fact hard processes will be largely dominant together with a larger jet quenching and a thermal source with larger radial flow which should lead to an overwhelming production of hadrons from coalescence in a wider momentum range [79,80]. Therefore the features of more sophisticated models will have wide chances to undergo many experimental tests at both the LHC and RHIC programs.

6 Phase Space Coalescence for heavy quarks

The coalescence model has been also applied to study open and hidden charm and bottom production. For the evaluation of charmonia yield the idea of coalescence [58] or recombination at phase boundary [59] has been investigated by several authors as a competing effect to charmonia suppression. We have suggested instead that the dominance of coalescence mechanism in a wide range of momenta can be applied also for the study of the D and B mesons [10,17,89,90] that are the principal tool for investigating heavy quark interaction and thermalization in the QGP. In a first paper we have shown that such an approach allows to estimate both spectra that are the principal tool for investigating heavy quark interaction and thermalization in the QGP. In a first paper we have shown that such an approach allows to estimate both spectra that are the principal tool for investigating heavy quark interaction and thermalization in the QGP. In a first paper we have shown that such an approach allows to estimate both spectra that are the principal tool for investigating heavy quark interaction and thermalization in the QGP. In a first paper we have shown that such an approach allows to estimate both spectra that are the principal tool for investigating heavy quark interaction and thermalization in the QGP.

Surprisingly, data from the Relativistic Heavy-Ion Collider (RHIC) for single electrons \(e^\pm\) associated with semileptonic B and D decays in semi-central Au-Au collisions exhibited a \(v_2\) of up to 10% [82,84], indicating substantial collective behavior of charm \((c)\) quarks consistent with the assumption of a c-quark \(v_2\) similar to the one for light quarks, apart from a \(p_T\) shift due to radial-flow effects [10]. In addition, the nuclear suppression factor was found to be comparable to the pion one, \(R_{AA}\approx0.3\) [83,84].

On the other hand for heavy quarks (HQ) the predicted amount of energy-loss has been shown to be insufficient (at variance with the light quark case) to account for the observed nonphotonic single electrons nuclear suppression (small \(R_{AA}\)) and for a strong degree of collectivity (large \(v_2\) [88]. Therefore for HQ the challenge is mainly the understanding of the quark medium interaction, even if the acquired knowledge on hadronization mechanism from light quarks plays a significant role.

Based on lattice QCD (lQCD) which suggests a resonance structure in the meson-correlation function at moderate temperatures [77], an effective model for heavy-light quark scattering via \(D\) and B resonances has been suggested. In Ref. [10] it was found that resonances cause a reduction of about a factor of three in the thermalization time respect to pQCD estimates. To relate such a microscopic description to the single electron data, we employ a Fokker-Planck approach for \(c\) and \(b\) quarks in the QGP based on elastic scattering with light quarks via \(D\)- and \(B\)-meson resonances with a width \(T=400\text{--}750\text{ MeV}\) (supplemented by perturbative interactions in color singlet channel) [10]. Heavy-quark (HQ) kinetics in the QGP is then treated as a relativistic Langevin process [17]:

\[
\frac{\partial f_Q}{\partial t} = \gamma \frac{\partial (p f_Q)}{\partial p} + D_p \frac{\partial^2 f_Q}{\partial p^2} ,
\]

where \(f_Q\) is the HQ phase space distribution, \(\gamma\) and \(D_p\) are the corresponding drag and (momentum) diffusion constants which determine the approach to equilibrium and satisfy the Einstein relation, \(T = D_p/\gamma M_Q\). The medium is modeled by a spatially homogeneous elliptic thermal fireball which expands isentropically. Finally the hadronization is treated by a coalescence (see Eq. 1) plus fragmentation approach with the distribution of HQ that undergoes a fragmentation process evaluated as \(f_c,\Lambda_0(p_T) \ast [1 - P_{c,b\rightarrow(D,\Lambda_0),(B,\Lambda_0)}(p_T)]\), where \(P_{c,b\rightarrow(D,\Lambda_0),(B,\Lambda_0)}\) is the probability for a HQ to coalesce according to Eq. 1. Results for Au+Au at 200 AGeV from the Langevin simulation including hadronization by coalescence+fragmentation (left) and fragmentation only (right) are shown in Fig. together with experimental data [83,86]. It is clear that elastic scattering in a pQCD scheme is insufficient to account for the small \(R_{AA}\) and...
large $v_2$ (see Fig. 7) independently on the hadronization scheme applied. The red band shows the full calculation with $c,b$ quarks that scatter in the presence of hadronic-like resonances with a width $\Gamma \sim 0.4 - 0.75$ GeV. We notice that contamination of single-electrons from B decay is significant already at $p_T \sim 2$ GeV (corresponding to a cross-point between $c$ and $b$ spectra around 4 – 5 GeV in agreement with FNNLO calculation [57]). Therefore it is necessary to include the $B$ mesons (despite the inherent uncertainties in the $b/c$ ratio) to draw any conclusion on the interaction processes behind the experimental results.

Comparing the band in the left and right panel of Figs. 6 and 7, we can appreciate the effect of coalescence in the increasing of $R_{AA}$ at $p_T \sim 1 - 4$ GeV and the simultaneous increase of the elliptic flow $v_2$, see Fig. 7 [9,10]. Therefore coalescence mechanism reverts the usual correlations between $R_{AA}$ and $v_2$, and allows for a reasonable agreement with the experimental data of both $R_{AA}$ and $v_2$.

Very recently within a Brueckner many-body scheme in medium T-matrices for heavy-quarks scattering off light quarks [89] have been evaluated starting from lQCD potential. Therefore the existence and width of $D$ and $B$-like resonances are no longer assumed like in the effective lagrangian but extracted from lQCD. It is found even a better agreement with the data [89]. Even if inherent uncertainties in the extrapolation of the potential have to be evaluated in the next future, this approach constitutes a promising tool to connect the observables to the information from lQCD. Furthermore we note that if resonant scattering with increasing strength at decreasing temperature is the dominant interaction channel this would lead to a natural merg-
7 Summary and Conclusions

The first stage of RHIC program has shown clear signs of modification of the hadronization mechanism respect to pp collisions in the light quark sector. There are several evidences that hadronization proceeds through coalescence of massive quarks close in phase space. The baryon-to-meson ratio enhancement observed at intermediate \( p_T \) and the scaling of the elliptic flow with the number of valence quarks, are robust effects against improvements of the naive coalescence picture: inclusion of resonances, finite width of the wave function, effective quark mass distribution, gluons in the higher Fock states of the wave function, energy conservation ... All these studies that improve the original coalescence picture from a theoretical point of view lead generally to an even better agreement with the data. The main open question remains the development of a dynamical coalescence model that can shed light on the issue of space-momentum correlation and jet-like correlations that could be significant sources of deviation from naive coalescence. 

The knowledge of hadronization for light quarks seems to play a role also in the new challenges posed by heavy-quark probes. Here the main issue is the dominant interaction mechanism and its relation to the microscopic structure of the QGP. The role of coalescence is mainly a modification of the correlation between \( R_{AA} \) and \( v_2 \). Finally, coalescence in the heavy quark sector can provide an inherent consistency between the in-medium interaction of HQ and the subsequent hadronization considering that a pole in the HQ propagator above \( T_c \) can be viewed as a precursor of coalescence.

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