How important is the three-nucleon force?

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Abstract

By calculating the contribution of the $\pi - \pi$ three-body force to the three-nucleon binding energy in terms of the $\pi N$ amplitude using perturbation theory, we are able to determine the contribution of the different $\pi N$ partial waves to the three-nucleon force. The division of the $\pi N$ amplitude into a pole and nonpole gives a unique procedure for the determination of the $\pi NN$ form factor in the model. The total contribution of the three-body force to the binding energy of the triton is found to be very small.
The discrepancy between the results of the exact calculation of the binding energy of the triton using a number of realistic nucleon-nucleon potentials, and the experimental value of 8.48 MeV, has been an outstanding problem in nuclear physics for a number of years. A commonly accepted solution has been the introduction of a three-nucleon force that will bridge the gap between the calculated binding energy, based on two-body interaction, and the experimental binding energy. The origin of such a three-body force lies in the fact that the nucleons are treated as point particles interacting via a two-body potential that is often assumed to be local. This approximation neglects the contribution of the mechanisms whereby one of the nucleons emits a meson that scatters off a second nucleon and then gets absorbed onto the third nucleon; see Fig. 1. Thus in a theory where the meson degrees of freedom are suppressed, one needs to include the contribution from the three-body force given in Fig. 1.

Over the past ten years there have developed two main approaches to the determination of this three-nucleon interaction from an underlying meson-nucleon theory. One of the first such three-body forces was developed by the Tucson-Melbourne (TM) group. They determined the \( \pi N \) amplitude that goes into the calculation of the three-body force by emphasizing the role of symmetry, and in particular, stressed the importance of using current algebra and soft pion theorems, to fix the behavior of the \( \pi N \) t-matrix, off-mass shell and near the \( \pi N \) threshold. This amplitude was then used to calculate the three-body force given in Fig. 1. The second approach emphasized the fact that the \( \pi N \) amplitude is dominated at medium energies by the \( \Delta(1230) \) resonance, which is considered as the first excited state of the nucleon. This suggests that one should introduce coupling between the \( NN, N\Delta, \) and \( \Delta\Delta \) channels, and solve the coupled channel problem for the \( BBB \) system, where the baryon \( B = N, \Delta \). In this way the \( \Delta \) component of the three-nucleon force is automatically included in the three baryon system.

To motivate the need to reexamine the above approaches to the three-nucleon force, we should recall that the \( NN \) amplitude, needed to calculate the binding energy of the three-nucleon system using the Faddeev equations, is \( t^{NN}_\alpha(k, k'; E_{NN}) \), where \( \alpha \) labels the partial
wave, and the energy of the $NN$ system, $E_{NN}$, is restricted to the domain $-\infty < E_{NN} < -E_T$, with $E_T$ being the binding energy of the three-nucleon system. The fact that we need this amplitude over the above specified energy domain is a result of the fact that in the three-nucleon system, the total energy is fixed at $E = -E_T$, and the spectator particle can have any energy from zero to $\infty$. If we now examine, in the same spirit, the contribution of the $\pi N$ amplitude to the three-body force, then the off-energy-shell partial wave amplitudes we require are $t^\pi_N(k, k'; E_{\pi N})$ for all energies in the domain $-\infty < E_{\pi N} < (m_N - E_T)$, where in this case we have included the rest mass of the nucleon and pion in the $\pi N$ energy $E_{\pi N}$. Here, the need for the $\pi N$ amplitude in this energy domain is a result of the fact that the total energy is still $-E_T$, and the two spectator nucleons have their full range of kinetic energy. As a result, we have an integral of the $\pi N$ amplitude over this energy domain when we calculate the contribution of the three-nucleon force to the binding energy of the triton. The second question that needs to be examined is that of the relative contribution of the different $\pi N$ partial waves to the binding energy. In the $NN - N\Delta$ coupled channel approach, it is assumed that the $P_{33}$ is the only partial wave contributing to the three-body force, while in the TM approach it is not clear how much of the $P$-wave contribution is included, since the $\pi N$ amplitude is fixed by the low energy experimental data and soft pion theorems. Finally, we need to consider the $\pi NN$ form factor used in Fig. 1 for the initial emission and final absorption of the pion. Traditionally, the $\pi NN$ form factor is taken to be a function of the pion momentum only. This is a result of assuming that the nucleons are on-mass-shell while the pion is off-mass-shell. At the $NN$ level, the cutoff mass in the $\pi NN$ form factor is determined by fitting the $NN$ data, while for the three-nucleon force this cutoff mass is treated either as a parameter [1,2], or adjusted [3] to fit the Goldberger-Treiman relation [14].

For the present analysis we have taken the approach used in the formulation of the $NN - \pi NN$ problem [15,16] in which the off-energy-shell $\pi N$ amplitude is divided into a pole and nonpole contribution. The nonpole contribution gives the off-energy-shell $\pi N$ amplitude that describes the $\pi N$ scattering in Fig. 1. The removal of the nucleon pole contribution
from the $\pi N$ amplitudes avoids the double counting of the one-pion exchange. The residue of the off-energy-shell $\pi N$ amplitude at the nucleon pole defines the pion absorption and production vertex in Fig. 1. The corresponding dressed $\pi NN$ form factor is a function of the total $\pi N$ energy and the relative $\pi N$ momentum. To maintain consistency with the three-nucleon wave function (i.e., point nucleons), and avoid the problem of undercounting due to incomplete nucleon dressing, we have used the $\pi NN$ form factor extracted from the $\pi N$ amplitude rather than that extracted from the $NNN - \pi NNN$ equations.

To simplify our model for the $\pi N$ amplitude, which is a solution of the Lippmann-Schwinger equation, we can choose the $\pi N$ potential in the $P_{11}$ channel to be the sum of an $s$-channel pole diagram and a background term that is represented by a separable potential. For all other $S$- and $P$-wave channels we have used the separable potential of Thomas. This potential, constructed for use in $\pi - d$ elastic scattering, was adjusted to fit the $\pi N$ phase shifts and scattering lengths in all $S$ and $P$ waves except the $P_{11}$. This potential gives a partial wave amplitude that is of the form

$$t_\alpha(k, k'; E) = g_\alpha(k) \tau_\alpha(E) g_\alpha(k').$$

(1)

For the $P_{11}$ partial wave, where the $s$-channel nucleon pole plays an important role, we have used the $\pi N$ potentials of McLeod and Afnan. These potentials were developed for, and used in $\pi - d$ scattering and pion absorption on the deuteron. These potentials give a $\pi N$ scattering amplitude that is a sum of a pole term and a nonpole term, i.e.,

$$t_\alpha(k, k'; E) = f^{R}(k, E) d^{R}(E) f^{R}(k', E)$$

$$+ g_\alpha(k) \tau_\alpha(E) g_\alpha(k')$$

for $\alpha = P_{11}$

(2)

where the second term in Eq. (2) is the nonpole amplitude. The parameters of these potentials have been adjusted to fit the experimental data, which consist of the scattering volume, the phase shifts up to the pion production threshold, and the position of the nucleon pole with the residue at the pole being the $\pi NN$ coupling constant when all legs in the vertex
are on-mass-shell. The renormalized $\pi NN$ form factor $f^R(k, E)$, which is energy dependent, is given by [23]

$$f^R(k, E) = Z_2^{1/2}(E) \left[ f_0(k) + g_\alpha(k) \tau_\alpha(E) \langle g_\alpha | G_0(E) | f_0 \rangle \right], \quad (3)$$

with $\alpha = P_{11}$ [26]. Here, $G_0(E)$ is the free $\pi N$ Green’s function, $Z_2(E)$ is the wave function renormalization, and $f_0(k)$ is the bare $\pi NN$ form factor. The nonpole part of the $P_{11} \pi N$ amplitude gives, in this case, the dressing to the $\pi NN$ form factor. The dressed nucleon propagator $d^R(E)$ has a pole with unit residue at $E = m_N$, i.e., $d^R(E) = (E - m_N)^{-1}$. The important feature of this $\pi N P_{11}$ amplitude is that the background term $g_\alpha(k) \tau_\alpha(E) g_\alpha(k')$ is part of the amplitude that contributes to $\pi N$ scattering in the three-nucleon force, while the pole term gives the $\pi NN$ form factor for the pion production and absorption vertices. In this case the dressed $\pi NN$ form factor $f^R(k, E)$ is not free, but is constrained by the experimental $\pi N$ data in the $P_{11}$ channel. To that extent the parameters of this three-nucleon force are completely determined by the $\pi N$ data. The off-shell behavior of this $\pi N$ amplitude is completely determined by the choice of the bare $\pi NN$ form factor $f_0(k)$ and the form factors of the separable potential $g_\alpha(k)$. This completely defines the three-body force corresponding to the diagram in Fig. [1].

For the present calculation we have taken the triton wave function to be the solution to the 18 channel Faddeev equation for the Paris-Ernst-Shakin-Thaler (PEST) potential [27,28] which is a separable expansion to the Paris potential [29]. We have taken sufficient terms in the separable expansion to get the binding energy, $S_-, S_-', S_-'$, and $D$-state probability for the triton to be 7.318 MeV, 90.111%, 1.430%, and 8.393%, respectively. These are in very good agreement with the corresponding results from the exact coordinate space Faddeev calculations [2] which give 7.388 MeV, 90.130%, 1.395%, and 8.409%. This establishes that our three-nucleon wave function is a good approximation to the exact wave function extracted from the coordinate space Faddeev equations.

In Table [1], we present the contribution of the three-body force, in keV, to the binding
energy of the triton from the different $\pi N$ partial waves and for different choices of the interaction in the $P_{11}$ channel. The potentials $PJ$ and $M1$ are those of McLeod and Afnan \cite{23}. The potential $PJ$ has a dipole bare form factor $f_0(k) \propto k/(k^2 + \alpha^2)^2$ with $\alpha = 3.82 \text{ fm}^{-1}$ ($\approx 754 \text{ MeV}$), while potential $M1$ has a monopole bare form factor $f_0(k) \propto k/(k^2 + \alpha^2)$ with $\alpha = 2.77 \text{ fm}^{-1}$ ($\approx 547 \text{ MeV}$). For comparison, we have also included results for two potentials $A$ and $D$ \cite{30}, which have been adjusted to fit the same $\pi N$ data as the potentials $PJ$ and $M1$. These have a bare form factor which is a monopole with a range or cutoff mass of $\alpha = 9.32 \text{ fm}^{-1}$ ($\approx 1840 \text{ MeV}$) and $\alpha = 1.64 \text{ fm}^{-1}$ ($\approx 323 \text{ MeV}$) respectively. From these results we may conclude that: (i) The total contribution from all $\pi N$ partial waves is very small, less than 50 keV, and not of any significance in explaining the discrepancy between the results of Faddeev calculations based on a two-body $NN$ interaction and the experimental binding energy. (ii) The largest contributions are from the $P_{11}$, $P_{33}$, and $S_{31}$ partial waves. This justifies the fact that the $\Delta(1230)$ plays a large role in the determination of the three-body force. But then the $S_{31}$ and $P_{11} \pi N$ amplitude are equally important. Here we should point out that the $S$-wave $\pi N$ interaction used has scattering lengths such that $a_1 + 2a_3 = -0.011 m_\pi^{-1}$, consistent with the requirement of current algebra. (iii) There is a cancellation between the $P$-wave and $S$-wave contributions, making the overall contribution of the three-body force even smaller. (iv) The results are sensitive to the choice of the bare $\pi NN$ form factor, to the extent that potential $A$ gives a much larger contribution than potential $D$. However, this variation is not very large considering the range of cutoff masses. This is a result of the fact that the corresponding dressed form factors are not drastically different, see Fig. 2. This is an indication that the requirement to fit the experimental data puts considerable constraint on the dressed form factor \cite{31}, and that in turn constrains the variation in the contribution of the three-body force to the binding energy of the three-nucleon system. (v) Finally, there is a correlation between the sign of the contribution of the different partial waves and the corresponding phase shift, which is expected considering the fact that the sign of the phase shift tells us if the interaction in that partial wave is attractive or repulsive. Here we should note that the $P_{11}$ contribution, which is due to the nonpole
part of the amplitude in this channel, is attractive, as is the case with the \( P_{33} \). This is due to the fact that the nonpole \( P_{11} \) phases are similar in behavior to the \( P_{33} \) phases \[21,22\].

To examine the possible reasons for this small three-body force contribution to the triton binding energy, and in particular, the role of the energy dependence of the \( \pi N \) amplitude and the cutoff mass in the \( \pi NN \) form factor, we need to consider different approximations to the above results. We first consider the energy dependence of the dressed \( \pi NN \) form factor. We could take the energy in this form factor to be the nucleon mass, i.e.,

\[
f^R(k, E_{\pi N}) \rightarrow f^R(k, m_N) .
\] (4)

The results of this approximation for potential \( PJ \), labeled (i) in Table[4], give us some additional binding, but nothing substantial. In this case both the \( S \)-wave and \( P \)-wave contributions have increased in magnitude, but the cancellation between the \( S_{31} \) and the \( P_{33} \) reduces the overall contribution. If in addition we take the energy in the \( \pi N \) amplitude to be fixed at the nucleon mass, i.e.,

\[
\tau_{\alpha}(E_{\pi N}) \rightarrow \tau_{\alpha}(m_N) ,
\] (5)

then we get a substantial increase in the overall contribution to the binding, see Table[4] line labeled (ii). This is partly due to the fact that the \( S_{31} \) contribution has been reduced, while the \( P_{33} \) and \( P_{11} \) contributions have increased. As a result, the total contribution to the binding energy has increased by a factor of ten, when the energy in both the \( \pi N \) scattering amplitude and the dressed \( \pi NN \) form factors are fixed to be \( E_{\pi N} = m_N \). To understand this change in cancellation as we change the \( \pi N \) energy, we should recall that 

\[
\tau_{\alpha}(E_{\pi N}) = \left[ \lambda^{-1} - \langle g_{\alpha}|G_0(E_{\pi N})|g_{\alpha} \rangle \right]^{-1},
\]

where \( \lambda \), the strength of the potential, is \(-1\) for attractive potentials such as is the case for the \( P_{33} \) channel, and \(+1\) for repulsive potentials as is the case for the \( S_{31} \). Since \( \langle g_{\alpha}|G_0(E_{\pi N})|g_{\alpha} \rangle \) is negative for \( E_{\pi N} < (m_\pi + m_N) \), then the two terms in the denominator of \( \tau_{\alpha}(E_{\pi N}) \) add for repulsive potentials and subtract for attractive potentials. Furthermore, since the magnitude of \( \langle g_{\alpha}|G_0(E_{\pi N})|g_{\alpha} \rangle \) increases as we approach the \( \pi N \) threshold from below, the contribution of the repulsive channels is
reduced while that of the attractive channels increases. In fact if we take the energy in the 
\( \pi N \) amplitude to be the \( \pi N \) threshold, i.e., \( E_{\pi N} = (m_\pi + m_N) \), then the contribution of the 
three-body force to the binding energy of the triton increases by another factor of two as 
illustrated in Table I line labeled (iii). This establishes the fact that the inclusion of the 
energy dependence of the \( \pi N \) amplitude and dressed \( \pi NN \) form factor into the calculation 
has reduced the total contribution to the binding energy by a factor of \( \approx 40 \).

We now turn to the role of the \( \pi NN \) form factor in determining the magnitude of the 
three-nucleon force contribution to the binding energy of the triton. Here we observe that 
there is a bare form factor \( f_0(k) \) that can be chosen to be a monopole or a dipole form factor 
with some fixed cutoff mass. However, the \textit{dressed} \( \pi NN \) form factor given in Eq. (3) and 
used for the pion absorption and production vertex is determined by fitting the experimental 
data. In Fig. 2 we compare the dressed \( \pi NN \) form factors for the potentials considered in 
Table I with a monopole form factor, i.e., \( F_0(k) = \frac{\Lambda^2}{k^2 + \Lambda^2} \), having a cutoff mass of 400 
and 800 MeV. Here we observe that the dressed form factors are almost identical despite 
the fact that the different potentials have either a monopole form factor (potentials \( M1, A, \) 
and \( D \)) with drastically different cutoff masses, or a dipole form factor (potential \( PJ \)). This 
strongly suggests that the dressing and the requirement of fitting the experimental data has 
the effect of constraining the dressed \( \pi NN \) form factors irrespective of the choice for the 
cutoff mass in the bare form factor \[31\]. Since in previous determinations of the contribution 
of the three-nucleon potential to the binding energy of the triton, no experimental constraint 
was imposed on the \( \pi NN \) form factor other than the Goldberger-Treiman relation \[14\] for 
the TM potential, we now examine the result of replacing the dressed \( \pi NN \) form factors 
in our model with the potential \( PJ \), by the monopole form factor \( F_0(k) \). At the same time 
we fix the energy of the \( \pi N \) amplitude at the nucleon pole and take the form factor for the 
separable potentials, \( g_\alpha(k) \), to be also the monopole form factor \( F_0(k) \). This can be achieved 
by the substitutions

\[
\frac{f^R(k, E_{\pi N})}{k} \rightarrow \left. \frac{f^R(k, m_N)}{k} \right|_{k=0} F_0(k),
\]
\[
\tau_\alpha(E_{\pi N}) \rightarrow \tau_\alpha(m_N) ,
\]
\[
g_\alpha(k) \rightarrow \frac{g_\alpha(k)}{k^\ell} \bigg|_{k=0} \frac{k^\ell}{F_0(k)} ,
\]
where \( \ell \) is zero for \( S \) waves and one for \( P \) waves. This procedure is similar to that implemented in the TM potential \([32]\). This approximation involves the assumption that the form factor in the separable potentials is the same in all partial waves, and of the monopole type \([34]\). In Table II we present the results of such a substitution for \( \Lambda = 400 \) and 800 MeV. The substitution in Eq. (6) has the dramatic effect of increasing the contribution of the three-nucleon force by one to two orders of magnitude, with the final result being very sensitive to the choice of \( \Lambda \). Note that the difference between the results with \( \Lambda = 400 \) and 800 MeV should be compared with the results for, say, the potentials \( M1 \) and \( A \) in Table I, which have cutoff masses for the bare form factors of 547 and 1822 MeV. To understand this large change, we recall from Fig. 2 that for \( k \approx 3 \), the monopole form factor for \( \Lambda = 800 \) MeV is about three times greater than that of the potential \( PJ \), and this form factor is raised to the fourth power, since we have four factors of \( F_0(k) \), two from the \( \pi NN \) form factors, and two from the separable potential form factors. This should give us an increase by roughly two orders of magnitude, which is consistent with the result of comparing the fourth and last line in Table II. On the other hand, by comparing the results for the potentials \( M1, A, \) and \( B \) in Table I, we observe some sensitivity to the dressed \( \pi NN \) form factor. This sensitivity is not as dramatic partly because the energy dependence of the \( \pi N \) amplitude and the \( \pi NN \) form factor result in a cancellation between the repulsive \( S_{31} \) and the attractive \( P_{11} \) and \( P_{33} \) contribution, and partly because the dressed form factors for the different potentials have similar momentum dependence. The role of the cancellation between the contribution of the different partial waves can be illustrated by comparing the results for the potential \( A \) in Table II with those for \( \Lambda = 400 \) MeV in Table I, both of which have comparable \( \pi NN \) form factors, see Fig. 2.

Although this analysis is based on perturbation theory, the fact that the results are so small suggests that a more exact treatment of the three-body force based on the present
formalism is not warranted since higher order contributions from the three-nucleon force cannot change the overall magnitude of the three-body force substantially \cite{33}. Within the framework of $\pi NN$ dynamics there should be a component of the three-nucleon system that corresponds to $\pi NNN$. This comes in at higher order if the perturbation expansion is about three point nucleons. We have chosen not to include this effect on the grounds that it would most likely give a small contribution. The inclusion of the other time order $\pi - \pi$ three-body force can at most increase the contribution by a factor of 4, if all contributions add coherently and are of the same magnitude, which is still too small to bridge the gap between the experimental result of 8.48 MeV and the value for the binding energy based on the Paris two-body interaction of 7.39 MeV. In the $NN - \Delta N$ coupled channel approach there is a cancellation between the three-body force contribution and the dispersive effect. If this cancellation persists in channels other than the $P_{33}$, we would expect a further suppression of the three-body contribution to the binding energy. The present results are based on the use of separable potential for the nonpole interaction, and as a result only the $P_{11}$ data put a constraint on the dressed $\pi NN$ form factor. For chiral Lagrangians, e.g., \cite{35}, the total $\pi N$ data are used to constrain the dressed $\pi NN$ form factor. We need to establish if this limited variation in the $\pi NN$ form factor is a special feature of the present model, or is more generally valid.

In summary, we have found two factors that suppress the contribution of the three-nucleon force to the binding energy of the triton. These are the following: (i) The energy dependence of the $\pi N$ amplitude and the dressed $\pi NN$ form factor reduce this contribution by $\approx 40$. Part of this effect is due to the fact that there is a substantial cancellation between the repulsive $S_{31}$ contribution and the attractive $P_{11}$ and $P_{33}$ contributions. (ii) The use of a monopole form factor with a large cutoff mass can enhance the contribution of the three-nucleon force by as much as two orders of magnitude provided the energy dependence of the $\pi N$ amplitude and the $\pi NN$ form factor are neglected.

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FIGURES

FIG. 1. The contribution to the three-nucleon force.

FIG. 2. Comparison of the dressed $\pi NN$ form factors for the potentials $P J$, $M 1$, $A$, and $D$ with a monopole form factor having a cutoff mass of $\Lambda = 400$ and 800 MeV.
TABLE I. Comparison of the results for the contribution of the three-body force to the triton binding energy in keV for two of the $P_{11}$ $\pi N$ potentials of Ref. [8]. For comparison, we have also included the results for potential $A$ and $D$ which have monopole bare form factors with a cutoff mass of $\approx 1822$ MeV and $\approx 323$ MeV, respectively.

| $P_{11}$ potential | $S_{11}$ | $S_{31}$ | $P_{11}$ | $P_{31}$ | $P_{13}$ | $P_{33}$ | Total  |
|-------------------|---------|---------|---------|---------|---------|---------|--------|
| $P_{J}$           | -4.8    | 26.4    | -8.8    | -3.6    | 4.5     | -16.0   | -2.3   |
| $M1$              | -5.0    | 28.9    | -15.3   | -2.1    | 6.2     | -22.1   | -9.4   |
| $A$               | -9.2    | 51.4    | -38.9   | -4.3    | 10.0    | -38.3   | -29.3  |
| $D$               | -3.8    | 19.3    | -4.9    | -5.2    | 1.4     | -6.7    | 1.0    |

TABLE II. The effect of removing the energy dependence in the $\pi NN$ form factor (i), and the $\pi N$ amplitude (ii) and (iii). Also included are the results of replacing the dressed $\pi NN$ form factor and separable potential form factors by a monopole with a cutoff mass of 400 and 800 MeV, while fixing the energy of the $\pi N$ amplitude to $m_N$. The total includes the contribution from all $S$- and $P$-wave $\pi N$ amplitudes. The results in this table are for the $P_{11}$ potential $P_{J}$, and all energies are in keV.

| Approx. | $S_{31}$ | $P_{11}$ | $P_{33}$ | Total  |
|---------|---------|---------|---------|--------|
| exact   | 26.4    | -8.8    | -16.0   | -2.3   |
| (i)     | 31.7    | -13.0   | -22.3   | -6.8   |
| (ii)    | 28.1    | -20.1   | -39.8   | -36.0  |
| (iii)   | 23.4    | -28.7   | -70.3   | -80.9  |
| 400     | 18.7    | -58.8   | -311.5  | -231.1 |
| 800     | 113.2   | -1146.7 | -3172.5 | -3897.3|