A study of the surface current distribution on the microwave transmission line using Green’s function

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Abstract

A method for estimating the currents on the microwave transmission line is proposed. In this method, the substrate is divided into a wire-grid model. Then the magnetic near field strength above the microwave transmission line is calculated and measured. Based on the magnetic near field strength, the magnetic field integral equation is established by applying the Green’s function and is used to estimate the currents flowing on the microwave transmission line. In this paper, the current distribution which is calculated by FDTD method with the interpolation (first order) and estimated current distribution by applying the Green’s function on the microstrip line and the coplanar line is evaluated by using the calculated magnetic field components. As a result, a good agreement between both results is observed and the validity of this method is confirmed.

Furthermore, the validity of the current estimation by measuring the magnetic near fields is confirmed by comparing the current obtained from the measured magnetic fields and the calculated magnetic fields. Comparing between the measured and calculated results, a good agreement is observed between both results on the strip line area. However, differences are also observed on the dielectric substrate area.

Keywords: Microstrip line; Coplanar line; Magnetic near field; Current; Green’s function; Conjugate gradient method; L-curve method

1. Introduction

With the innovative miniaturization and the high speed operation of the recent electric devices, the operating frequency of the recent digital computers and the communication equipments reached to the incredibly high frequency range. Hence, the leakage electromagnetic field radiation from these devices also gets to the high frequency range and leads to the serious electromagnetic interference (EMI) problems, e.g. interference between the different devices and also with the printed circuit board (PCB) itself [1,2].

To solve these EMI problems, the partial shielding technique is effective [5], as an example, which arranges the shielding material on the specific part on the PCB that radiates large strength of fields, (i.e. where impediment currents flow). For this purpose, it is necessary to estimate the current distributions on the PCB. However, the current distributions on the PCB cannot be measured directly using the probe without the electromagnetic coupling problems [3,4]. Moreover, the complex structure of the board makes it difficult to model the PCB, (e.g. motherboard) numerically on the electromagnetic calculation tools.

Owing to these backgrounds, the estimation method for the current distributions on the PCB, which uses the measured electromagnetic near field data is applied in this study [4]. However, to estimate the current distributions on the PCB by measuring the electromagnetic near field is inherently ill-posed inverse problem and leads to an ill-conditioned system of linear equations. To solve this ill-posed inverse problem, the conjugate gradient method is used and the L-curve method is applied to find the right value of the regularization parameter.

In this paper, a method for estimating the currents on the microwave transmission line is proposed. The microstrip line and the coplanar line are chosen as the example. At first, the microstrip line and the coplanar line are divided into a wire-grid model. Then the magnetic near field strength above these line is calculated by FDTD method. Based on the magnetic near field strength, the magnetic field integral equation is established by applying the Green’s function and is used to estimate the currents flowing on the microstrip line and the coplanar line. This inverse transformation (inverse problem) of the currents is calculated numerically by applying the conjugate gradient method and the L-curve method. The validity of this method is demonstrated by comparing
the current distribution which is calculated by FDTD method with the interpolation (first order) [6], with the estimated current distribution by applying the Green’s function. Furthermore, the validity of estimation current based on the measurement is also demonstrated by comparing the current obtained with the measured magnetic fields and the calculated magnetic fields on the microstrip line by applying the Green’s function.

2. Theory

2.1. Modeling of substrate [7]

Fig. 1 shows the model of the substrate. The substrate is modeled with a thin wire grid, which radiates the electromagnetic field. At first, the substrate is divided into \( n \times n \) and the number of the measuring points above the substrate is \( m \times m \). The currents are the unknown coefficient flowing on the thin wire grid. Each segment is assumed to carry a constant currents. And the charges are concentrated at the intersections of the grid. The charges can be obtained as follows

\[
Q_{\mu \nu \xi} = \frac{1}{j\omega} \left( I_{x,\mu-1,\nu,\xi} - I_{x,\mu,\nu-1,\xi} + I_{y,\mu,\nu,\xi} - I_{y,\mu,\nu-1,\xi} \right)
+ I_{z,\mu,\nu,\xi-1} - I_{z,\mu,\nu,\xi} \tag{1}
\]

where \( \mu, \nu, \xi \) are the arbitrary point of free space in the direction of \( x, y, z \).

Moreover, the measuring point of magnetic near field is \( h \) from the substrate to the measuring plane. Based on the magnetic near field strength, the magnetic field integral equation can be obtained by applying the Maxwell’s equations as follows

\[
\mathbf{H}(x,y,z) = \sum_{\mu=1}^{k-1} \sum_{\nu=1}^{l-1} \sum_{\xi=1}^{m-1} I_{x,\mu,\nu,\xi} \delta(x - \mu, y - \nu, z + b \xi)
+ \sum_{\mu=1}^{k-1} \sum_{\nu=1}^{l-1} \sum_{\xi=1}^{m-1} I_{y,\mu,\nu,\xi} \delta(y - \mu, x - \nu, z + b \xi)
+ \sum_{\mu=1}^{k-1} \sum_{\nu=1}^{l-1} \sum_{\xi=1}^{m-1} I_{z,\mu,\nu,\xi} \delta(z - \mu, y - \nu, x + b \xi) \tag{2}
\]

where \( n = \begin{cases} m-1 & \text{ground present} \\ m & \text{ground absent} \end{cases} \) (3)

where the coordinates are normalized to the grid constant \( a \) and to the height \( b \). And \( \bar{x}, \bar{y}, \bar{z} \) and \( \bar{b} \) are defined as follows:

\[
\bar{x} = \frac{x}{a}, \quad \bar{y} = \frac{y}{a}, \quad \bar{z} = \frac{z}{a}, \quad \bar{b} = \frac{b}{a} \tag{4}
\]

2.2. Computation of Green’s function

The inverse problem for the calculation of the current distribution above the microwave transmission line is illustrated in Fig. 2. Here, we restrict the theory to the substrate with in-plane excitation without \( z \) component. The substrate is a homogeneous isotropic loss dielectric of thickness \( d \) and complex permittivity. The ground plane is considered to be a perfect conductor which is located at \( z = -d \). We consider an \( x \)-directed horizontal electric dipole moment \( I_d \) located in the air–dielectric interface, which is chosen as the origin of the coordinate system [8,9]. The structure is assumed to be of infinite length. The electromagnetic fields in such a structure can be derived from a scalar and a vector potential by using Eqs. (5) and (6).

\[
H_x = G_{\text{II}} I_y \tag{5}
\]

\[
H_y = G_{\text{II}} I_x \tag{6}
\]

As stated above, the magnetic filed \( H \) is given by integral of the product of the current distribution \( I \) with the dyadic magnetic Green’s function \( G_{\text{II}} \). The electromagnetic field is derived by special dyadic magnetic Green’s function \( G_{\text{II}} \) at the air–dielectric interface as Eqs. (7) and (8)

\[
H_x(\rho, \varphi, z) = \frac{I}{2\pi} \int_{-1}^{1} \sin 2\varphi \int_{0}^{\infty} \frac{k_{\rho}^2}{D_{TM} D_{TE}} e^{-k_{\rho} z} \times \left( I_0(k_{\rho} \rho) - 2 I_1(k_{\rho} \rho) \right) d\rho \tag{7}
\]
2.3. Inverse problems

Eqs. (7) and (8) are discrete two-dimensional Fredholm integral equations. In order to obtain a meaningful solution of such a problem, regularization has to be applied. For a system of linear equation

\[ b = Ax, \]  

where \( b \) (the left-hand side) and matrix \( A \) (the kernel) are known and \( x \) is a function to be calculated. For this matrix calculation, the conjugate gradient method is applied in this study. The term of the left-hand side \( b \) is generally representing the measured values, which often contain the inevitable errors induced by the measurement. However, the small deviations (measurement error) may cause large changes in the solution \( x \). Therefore, this kind of equation belongs to the group of ill-posed problems. In order to solve such a problem, we have applied L-curve method developed by Hansen [10]. The L-curve is shown in Fig. 3. The regularization method is based on finding the solution for

\[
\min \{ \| Ax - b \|^2 + \lambda^2 \| L(x - x_0) \|^2 \} \]  

where \( x_0 \) is an estimated solution and \( L \) is a suitably chosen matrix. \( \lambda \) is called a regularization parameter. The L-curve is a simple plot of residual norm \( \| Ax - b \| \) against the (semi) norm \( \| L(x - x_0) \| \) in log–log scale for a set of admissible regularization parameters. In this way, the L-curve displays a compromise between the minimization of these two quantities. When the curve appears as L-shaped, the examination of the current distribution can be obtained.

3. Analysis

3.1. Analysis model

Fig. 4 shows the analysis model of the microstrip line and coplanar line. In this section, the magnetic near field strength above the microstrip line and coplanar line is calculated by using FDTD method. Based on the magnetic near field components \( H_x \) and \( H_y \), the magnetic field integral equation is established by applying the Green’s function and is used to estimate the currents flowing on the microstrip line and coplanar line. The analysis dimension \((x \times y \times z)\) by the FDTD calculation is \(100 \times 4.8 \times 21\) mm. To decrease the influence from the substrate, the thickness of the air layer (cells) is increased to twice that of the substrate. To achieve the infinite line, the Mur 1 boundary condition is used for the \(yz\)-plane. In this analysis, the CW wave (900 MHz) is used for the source excitation.

Moreover, the dimensions of both substrates for the microstrip line and the coplanar line are \(100 \times 21\) mm, with thickness \( d = 1.6\) mm and relative dielectric constant \( \varepsilon_r = 4.3\). The dimensions of strip are \(100 \times 3\) mm. In this case, the characteristic impedance of the line becomes 50 \( \Omega \).

3.2. Estimation results by changing distance

Fig. 5 shows the estimated current distributions on the microstrip line by changing the estimation distance heightwise \((h = 0, 0.025, 0.05, 0.075\) mm). The cell size in each \( x, y \) and \( z \) directions is set as an uneven cell size, considering the calculation time and the memory size to determine the unevenness. As shown in this result, the high current values are observed above the strip line area as expected, which is caused by the concentration of electric conduct currents. Furthermore, the current values are shifted when the estimation distance increases. This could be caused by the effects of the decreasing of propagation of the magnetic field.
Hence, the estimation point is arranged above the surface of the microstrip line and coplanar line with the constant distance of 0 mm for the future study.

3.3. Comparison with FDTD method

In this section, the currents on the microstrip line and the coplanar line are evaluated by using the calculated magnetic field components $H_x$ and $H_y$ above both lines. Figs. 6 and 7 show the results, which are calculated with the interpolated (first order) current distribution (FDTD method) and the estimated current distribution with the Green’s function. For the validation, the results of the interpolated current is considered as the criterion value.

As shown in these results, the high current values are observed above the center strip line area for both microstrip line and coplanar line, and also by the side strip of the coplanar line. Furthermore, a good agreement is observed between the results of the Green’s function and the FDTD method. Due to these agreement, the validity of Green’s function method is confirmed.

4. Measurement

In this section, the microstrip line is chosen as an example. The current distribution is estimated with the measured magnetic near fields on the microstrip line by applying the Green’s function. The validation of the current estimation by measuring the magnetic near fields which include the influence of measurement environment and measurement probe is conducted by comparing the current obtained with the measured magnetic fields (actual microstrip line) and the calculated magnetic fields (used the numerical microstrip line model and calculated the fields with the FDTD method).
4.1. Measurement system

The magnetic near fields above the microstrip line are measured with the measurement system of the near magnetic field. Figs. 8–10 show the experimental setting of the measurement system. As shown in Fig. 8, the magnetic field probe (NEC: CP-2S) is arranged vertically above the surface of the microstrip line with the constant distance of 1.0 mm. The sensor is mounted on the scanning rack-arm, whose scanning positions are controlled with the personal computer and driven by the electric motor (field mapping system). This field mapping system scans the field for two-dimensionally (x and y directions) on the surface plane of the microstrip line (Figs. 9 and 10), with the spatial resolution of 0.25 mm. The scanning area for the magnetic field measurement is same as the substrate area. Furthermore, the loop aperture of the magnetic field probe is allocated parallel to the x-axis to measure the $H_x$ field component, on the other hand, parallel to the y-axis for the $H_y$ field component.

The measure and aspect of the applied microstrip line is shown in Fig. 10. The size of the dielectric substrate is $21 \times 100$ mm and 1.6 mm of thickness, whereas the line is $3 \times 100$ mm. For the material of the dielectric substrate, the glass epoxy (FR-4) is used to fabricate the microstrip line. Furthermore, as shown in Fig. 8, the signal generator (SG) is connected on the left hand side of the microstrip line with the SMA connector, and on the other side, the 50 $\Omega$ terminator is connected. During the actual measurement, the absorbers are arranged around the microstrip line and measurement system to maintain the ideal measurement environment.

4.2. Results

The surface currents on the microstrip line are estimated with the calculated (FDTD) results, and also with the measured results. In both results, the magnetic field components of $H_x$ and $H_y$ are considered, and the Green’s function method...
(described in Section 2) is applied for the current estimation. As shown in Fig. 11, high current values are observed on the strip line area as expected, which is caused by the concentration of electric conduct currents. Furthermore, comparing between the measured and calculated results, a good agreement is observed between both results on the strip line area. However, differences are also observed on the dielectric substrate area (beside the strip line), e.g., the shifting of the zero point and the difference of the levels. These could be caused by the measurement error, induced from the electromagnetic field coupling between the magnetic field sensor and the strip line.

5. Conclusion

In this paper, a method for estimating the surface currents on the microwave transmission line using Green’s function is proposed. The microstrip line and coplanar line are chosen as an example. The currents on the surface of the microstrip line and the coplanar line are evaluated by using the calculated magnetic field components. Based on the magnetic near-field strength, the magnetic field integral equation is established and used to estimate the currents. As a result, the validation of this method is conducted by comparing the results between the Green’s function and the interpolated results. A good agreement between both results are observed, and the validity is confirmed.

Furthermore, the validation of the current estimation by measuring the magnetic fields, which include the influence of measurement environment and measurement probe is conducted by comparing the current obtained with the measured magnetic fields and the calculated magnetic fields. The estimated current distribution on the strip line area shows a good agreement between the calculated (FDTD) and the measured results. However, differences are observed on the dielectric substrate area. These differences could be caused by the electromagnetic field coupling between the magnetic field sensor and strip line. Hence, it is necessary to measure it accurately. For the future study, the increase of the current accuracy should be in consideration, by taking into account the field component of $E_z$ in the estimation.

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