Azimuthal asymmetry in electro-production of neutral pions in semi-inclusive DIS

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Abstract

Recently HERMES has observed an azimuthal asymmetry \(A_{UL}\) in electro-production of neutral pions in semi-inclusive deep-inelastic scattering of unpolarized positrons off longitudinally polarized protons. This asymmetry (like those observed in the production of charged pions) is well reproduced theoretically by using the non-perturbative calculation of the proton transversity distribution \(h_1^a\) in the effective chiral quark-soliton model combined with experimental DELPHI-data on the new T-odd Collins fragmentation function \(H_{\perp}^a(z_h)\). There are no free, adjustable parameters in the analysis. Using the \(z_h\)-dependence of the HERMES azimuthal asymmetry and the calculated transversity distributions the \(z_h\)-dependence of the Collins fragmentation function is obtained. The value obtained from HERMES data is consistent with the DELPHI result, even though these results refer to different scales.

1 Introduction

Recently a large azimuthal asymmetry has been observed by HERMES in the electro-production of neutral pions in semi-inclusive deep-inelastic scattering (SIDIS) of unpolarized positrons off longitudinally polarized protons \cite{1}. A similarly large azimuthal asymmetry in the production of \(\pi^+\) has been observed before, while no such azimuthal asymmetry was found in the production of \(\pi^-\) \cite{2}. Azimuthal asymmetries were also observed in SIDIS off transversely polarized protons at SMC \cite{3}. These asymmetries contain information on the proton transversity distributions \(h_1^a(x)\) and on the Collins fragmentation function \(H_{1}^{a}(z_h)\) \cite{4}. The transversity distribution function \(h_1^a(x)\) describes the distribution of transversely polarized quarks of flavour \(a\) in the nucleon \cite{5}. The T-odd fragmentation function \(H_{1}^{a}(z_h)\) describes the left-right asymmetry in fragmentation of transversely polarized quarks of flavour \(a\) into a hadron \cite{4,6,7,8,9} (the so-called ”Collins asymmetry”). Both \(H_{1}^{a}(z_h)\) and \(h_1^a(x)\) are twist-2, chirally odd, and not known experimentally. Only in the last years experimental indications to the T-odd fragmentation function \(H_{1}^{a}(z_h)\) in \(e^+e^-\)-annihilation have appeared \cite{10,11}, while the HERMES and SMC experiments \cite{1,2,3} can be viewed as the very first experimental indications to \(h_1^a(x)\).

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\textsuperscript{1} We use the notation of the Ref. \cite{4,6,7}.
Here we will explain the azimuthal asymmetry in \( \pi^0 \) production \cite{1} by using information on \( H_1^+ \) from DELPHI \cite{7, 11} and the predictions from the chiral quark-soliton model (\( \chi \)QSM) for the transversity distribution \( h_1^a(x) \) \cite{12}. Our analysis is free of any adjustable parameters. In this way the azimuthal asymmetries for \( \pi^\pm \) \cite{2, 3} have been explained in Ref. \cite{13}. We recalculate them using a bit more exact experimental cuts. In Ref. \cite{0} the data is compared to results of a similar analysis, which is based on the approach of Ref. \cite{14} and which, however, makes use of adjustable parameters and certain assumptions about \( h_1^a(x) \).

In order to use information from DELPHI on \( H_1^+ \), we have to assume that \( \langle H_1^+ \rangle / \langle D_1 \rangle \), the ratio of the T-odd to the usual fragmentation function (averaged over \( z_h \) and over flavours), varies little with scale. We will investigate whether this assumption is justified. For that we will use the prediction of \( h_1^a(x) \) from \( \chi \)QSM to extract \( H_1^+(z_h) \) from \( z_h \)-dependence of HERMES data. We will show that the results for \( \langle H_1^+ \rangle / \langle D_1 \rangle \) from HERMES \cite{0, 2, 3}, SMC \cite{4} and DELPHI \cite{10, 11} are consistent with each other.

## 2 Ingredients for analysis: \( h_1 \) and \( H_1^+ \)

### Transversity distribution function \( h_1 \). 

We will take the predictions of the chiral quark-soliton model (\( \chi \)QSM) as input for \( h_1^a(x) \) \cite{12}.

The \( \chi \)QSM is a quantum field-theoretical relativistic model with explicit quark and antiquark degrees of freedom. This allows an unambiguous identification of quark as well as antiquark distributions in the nucleon. Due to its field-theoretical nature the quark and antiquark distribution functions obtained in this model satisfy all general QCD requirements: positivity, sum rules, inequalities, etc \cite{15}. The model results for the unpolarized quark and antiquark distribution function \( f_1^a(x) \) and for the helicity quark distribution function \( g_1^a(x) \) agree within (10 - 20\%) with phenomenological parametrizations. This encourages confidence in the model predictions for \( h_1^a(x) \). In Fig. 1 the results of the model are shown at the average \( Q^2 = 4 \text{ GeV}^2 \) close to the HERMES experiment.

The application of the model results has yet another advantage. When using the model results for twist-2 parton distributions it is consequent to neglect systematically twist-3 distributions for the following reason. The \( \chi \)QSM has been derived from the instanton model of the QCD vacuum, and in the latter nucleon matrix elements of twist-3 operators are suppressed with respect to the leading twist-2 \cite{14}. In the case of the twist-3 distribution \( h_2^a(x) \) this has been shown explicitly in Ref. \cite{17}.

### The T-odd fragmentation function \( H_1^+ \). 

The Collins fragmentation function \( H_1^+(z_h, k_{2}^2) \) describes a left–right asymmetry in the fragmentation of a transversely polarized quark with spin \( \sigma \) and momentum \( k = (k_L, k_\perp) \) into a hadron with momentum \( P_h = -z_h k \): the relevant structure is \( H_1^+(z_h, k_\perp^2) \sigma(k \times P_{1h})/(|k| \langle P_{1h} \rangle) \). Here \( \langle P_{1h} \rangle \) is the average transverse...
momentum of the final hadron.

This fragmentation function is responsible for a specific azimuthal asymmetry of a hadron in a jet around the axis in direction of the second hadron in the opposite jet. This asymmetry was measured using the DELPHI data collection. For the leading particles in each jet of two-jet events, averaged over quark flavours (assuming $H_1^\perp = \sum_h H_1^\perp q/h$ is flavour independent), the most reliable value of the analyzing power is given by

$$\left\langle \frac{H_1^\perp}{D_1} \right\rangle = (6.3 \pm 2.0)\%$$

with presumably large systematic errors. The result Eq. (1) refers to the scale $M_Z^2$ and to an average over $k_\perp$ and over $z_h$ with $\langle z_h \rangle \approx 0.4$.

3 The HERMES experiment for $A_{UL}$

In the HERMES experiment the cross section for $lp \rightarrow l'\pi^0X$ was measured in dependence of the azimuthal angle $\phi$, which is the angle between lepton scattering plane and the plane defined by momentum $q$ of virtual photon and momentum $P_h$ of produced pion, see Fig. 2.

Denoting momentum of the target proton by $P$, momentum of the incoming lepton by $l$ and momentum of the outgoing lepton by $l'$, the relevant kinematical variables – center of mass energy square $s$, four momentum transfer $q$, invariant mass of the photon-proton system $W$, $x$, $y$ and $z_h$ – are defined as

$$s := (P + l)^2, \quad q := l - l', \quad Q^2 := -q^2,$$

$$W^2 := (P + Q)^2 = s(1 - x)y + M_N^2,$$

$$x := \frac{Q^2}{2Pq}, \quad y := \frac{2Pq}{s} \quad \text{and} \quad z_h := \frac{P_P h_P h}{P q}.$$ (2)

In this notation the azimuthal asymmetry $A_{UL}^{\sin \phi}(x)$ measured by HERMES reads

$$A_{UL}^{\sin \phi}(x) = \frac{1}{2} \int dy dz_h d\phi \sin \phi \left( \frac{1}{S^+} \frac{d^4\sigma^+}{dx dy dz_h d\phi} - \frac{1}{S^-} \frac{d^4\sigma^-}{dx dy dz_h d\phi} \right).$$ (3)

The subscript “U” reminds of the unpolarized beam, and “L” reminds of the longitudinally (with respect to the beam direction) polarized proton target. $S^\pm$ denotes the proton spin, where “+” means polarization opposite to the beam direction. When integrating over $y$ and $z_h$ one has to consider the experimental cuts

$$W^2 > W_{min}^2 = 4 \text{GeV}^2, \quad Q^2 > Q_{min}^2 = 1 \text{GeV}^2, \quad 0.2 < y < 0.85, \quad 0.2 < z_h < 0.7.$$

Notice the different normalization factor compared to $\langle P_h \rangle$, $\langle P_h \perp \rangle$ instead of $M_h$.

A similar value was also obtained from the pion asymmetry in inclusive $pp$-scattering.
The azimuthal asymmetry. The cross sections entering the asymmetry $A_{UL}^\sin \phi$ Eq.\(\text{(3)}\) have been computed in Ref.\(\text{[3]}\) at tree-level up to order $1/Q$. The denominator in Eq.\(\text{(3)}\) is the cross section for pion production from scattering of unpolarized positrons on unpolarized target protons

$$\frac{1}{2} \left( \frac{d^3 \sigma^+}{dx \ dy \ dz_h d\phi} + \frac{d^3 \sigma^-}{dx \ dy \ dz_h d\phi} \right) = \frac{d^3 \sigma_{UL}}{dx \ dy \ dz_h d\phi}. \quad (5)$$

The numerator in Eq.\(\text{(3)}\) consists of two parts — a longitudinal and a transverse part with respect to the photon momentum $q$

$$\frac{1}{S^+} \frac{d^3 \sigma^+}{dx \ dy \ dz_h d\phi} - \frac{1}{S^-} \frac{d^3 \sigma^-}{dx \ dy \ dz_h d\phi} = \frac{2}{S} \frac{d^3 \sigma_{UL}}{dx \ dy \ dz_h d\phi} + \frac{2}{S} \frac{d^3 \sigma_{UT}}{dx \ dy \ dz_h d\phi}. \quad (6)$$

The cross sections are given by

$$\frac{d^4 \sigma_{UU}}{dx \ dy \ dz_h d\phi} = \frac{\alpha^2 s}{Q^4} \left( 1 + (1 - y)^2 \right) \sum \frac{e_a^2}{x} f_1^a(x) \ D_1^a(z_h)$$

$$\frac{d^4 \sigma_{UL}}{dx \ dy \ dz_h d\phi} = \sin \phi S_L \frac{\alpha^2 s}{Q^4} \frac{M_{\chi}}{Q} \frac{8(2 - y)\sqrt{1 - y}}{\langle z_h \rangle = \sqrt{1 + \langle P_{2L}^2 \rangle/\langle k_L^2 \rangle}} \sum \frac{e_a^2}{x} \int \frac{d\xi}{\xi^2} h_1^a(\xi) \ H_1^1(z_h)$$

$$\frac{d^4 \sigma_{UT}}{dx \ dy \ dz_h d\phi} = \sin \phi S_T \frac{\alpha^2 s}{Q^4} \frac{2(1 - y)}{\langle z_h \rangle = \sqrt{1 + \langle P_{2L}^2 \rangle/\langle k_L^2 \rangle}} \sum \frac{e_a^2}{x} h_1^a(x) \ H_1^1(z_h). \quad (7)$$

In Eq.\(\text{(7)}\) terms have been omitted which vanish after the (weighted) integration over $\phi$, and pure twist-3 contributions have been systematically neglected for reasons mentioned above so that for $h_L$ entering $\sigma_{UL}$ the Wandzura-Wilczek type relation $h_L(x) = 2x f_1^a d\xi (h_1^a(\xi)/\xi^2)$ is hold (see Ref.\(\text{[3]}\) and Appendix). A term proportional to $\overline{H}_1^1(z_h)$ is also neglected, even though it contains a twist two contribution due to $\overline{H}_1^1(z_h) = z_h (d/dz_h) H_1^1(z_h) + \text{twist-3}$\(\text{[3]}\). However the contribution of this term to $\sigma_{UL}$ is very small, see the Appendix. $\langle P_{2L}^2 \rangle$ and $\langle k_L^2 \rangle = \langle P_{2L}^2 \rangle/\langle z_h \rangle$ are the mean square transverse momenta of quarks in the distribution and fragmentation functions, respectively. $S_L$ is the longitudinal, $S_T$ is the transverse component of target spin $S$ with respect to the 3-momentum of the virtual photon

$$S_L = S \cos \theta_\gamma \simeq S \left( 1 - \frac{2M_N^2 x (1 - y)}{m_y} \right), \quad S_T = S \sin \theta_\gamma \simeq S \sqrt{\frac{4M_N^2 x (1 - y)}{m_y}} \quad (8)$$

where $\theta_\gamma$ is the angle of virtual photon with respect to incoming beam.

Assuming isospin symmetry and favoured fragmentation the following relations hold

$$D_1^{d/\pi^+} = D_1^{d/\pi^+} = D_1^{d/\pi^-} = D_1^{d/\pi^-} \quad D_1^{d/\pi^+} = D_1^{d/\pi^+} = D_1^{d/\pi^-} = D_1^{d/\pi^-} \simeq 0 \quad (9)$$

$$D_1^{s/o} = D_1^{s/o} = D_1^{s/o} = D_1^{s/o} \quad \text{and} \quad D_1^{d/\pi^+} = D_1^{d/\pi^-} = \frac{1}{2} D_1^{d/\pi^+} \text{ def} \ D_1,$$

where the arguments $z_h$ are omitted. The same relations hold for $H_1^1$. Inserting Eq.\(\text{(3)}\) and \(\text{(7)}\) into Eq.\(\text{(3)}\) for the azimuthal asymmetry $A_{UL}^\sin \phi$ yields for the production of the pion

$$A_{UL}^\sin \phi (x, \pi) = \frac{1}{\langle z_h \rangle \sqrt{1 + \langle P_{2L}^2 \rangle/\langle k_L^2 \rangle}} \left( \frac{H_1^1}{D_1} \right)$$

$$\times \left( B_L(x) \sum_{z_h} \frac{e_a^2 x^2 \int_1^0 d\xi h_1^a(\xi)/\xi^2}{\sum_{a'} e_a^2 f_1^{a'}(x)} + B_T(x) \frac{\sum_{a'} e_a^2 h_1^a(x)}{\sum_{a'} e_a^2 f_1^{a'}(x)} \right). \quad (10)$$
where $\sum_a^p$ means summation only over those flavours which contribute to the favoured fragmentation into the specific pion asymmetry, i.e. in the $\pi^0$ case e.g.

$$\frac{\sum_a^p e_a^2 h_1^o(x)}{\sum_a^o e_a^2 f_1^o(x)} = \frac{(4h_1^u + 4h_1^u + h_1^t + h_1^t)}{(4f_1^u + 4f_1^u + f_1^t + f_1^t(x))}.$$

The prefactors $B_L(x), B_T(x)$ introduced in Eq.(10) are given by

$$B_L(x) = \frac{\int dy 2(2-y) \sqrt{1-y} \cos \theta_d M_s/Q^2}{\int dy (1-(1-y)^2)/Q^4} \quad \text{and} \quad B_T(x) = \frac{\int dy 2(1-y) \sin \theta_d/Q^4}{\int dy (1-(1-y)^2)/Q^4}. \quad (11)$$

When integrating over $y \in [y_1(x), y_2(x)]$ one has to keep in mind that $Q, \sin \theta_d$, and $\cos \theta_d$ are functions of $x$ and $y$, according to Eq.(2) and Eq.(8). The $x$-dependent integration range of variable $y$ is due to the experimental cuts Eq.(4)

$$y_1(x) := \max \left(0.2, \frac{Q_{\min}^2}{s_x}, \frac{W_{\min}^2 - M_5^2}{s(1-x)} \right) \leq y \leq y_2(x) := 0.85. \quad (12)$$

The implicit dependence of $h_1^u, f_1^u, H_1, D_1$ on $y$ through $Q$ is neglected. The distributions will be taken at the average value $Q_{av}^2 = 4 \text{GeV}^2$ close to the HERMES experiment.

**Results.** In the HERMES experiment $\langle P_{\perp N}^2 \rangle \simeq \langle P_{\perp L}^2 \rangle = \langle z_h^2 \rangle \langle k_l^2 \rangle$ and $\langle z_h \rangle = 0.41$. Approximating $\langle z_h^2 \rangle \simeq \langle z_h \rangle^2$ and using the result Eq.(11), the overall prefactor in Eq.(10) is

$$\frac{1}{\langle z_h \rangle \sqrt{1 + \langle P_{\perp N}^2 \rangle / \langle k_l^2 \rangle}} \langle H_1^+ \rangle / \langle D_1 \rangle = 0.12 \pm 0.04. \quad (13)$$

The error is due to the experimental error of the analyzing power $\langle H_1^+ \rangle / \langle D_1 \rangle$ Eq.(11), of which only the modulus is known. Here we have chosen the positive sign, for which the analysis of azimuthal asymmetries for $\pi^\pm$ gave evidence for $[13]$. When using the DELPHI result Eq.(11) to explain the HERMES experiment, we assume a weak scale dependence of the analyzing power. For $h_1^u(x)$ we take the results of the chiral quark-soliton model $[12]$ and for $f_1^u(x)$ the parametrization from Ref. $[20]$, both LO-evolved to the average scale $Q_{av}^2 = 4 \text{GeV}^2$.

It is instructive to investigate how much the longitudinal spin (twist-3) part and the transverse spin (twist-2) part contribute to the total azimuthal asymmetry $A_{UL}^{a \phi}(x)$. Comparing the $x$-dependent prefactors $B_L(x)$ and $B_T(x)$ Eq.(11), we note that $B_L(x) \gg B_T(x)$, see Fig. 3a. This is due to the fact that $\cos \theta_d \simeq 1$ appears in $B_L(x)$, while in $B_T(x)$ we have $\sin \theta_d = O(M_s/\sqrt{s})$ which is very small. However this effect is partially canceled by the fact that $x^2 \int_x^1 dy h_1^u(y)/y^2$, which contributes to the longitudinal (twist-3) part, is much smaller than $h_1^t(x)$, which contributes to the transverse (twist-2) part. The results of the chiral quark-soliton model for $h_1^u(x)$ satisfy $|x^2 \int_x^1 dy h_1^u(y)/y^2| < 0.1 |h_1^t(x)|$ in the whole $x$ region. In Fig. 3b this is demonstrated for the $u$ quark. As a result the longitudinal and the transverse part give – with increasing $x$ – comparably large contributions to the total $A_{UL}^{a \phi}(x)$. However, the longitudinal part gives always the major contribution, see Fig. 3c. The results shown in Fig. 3c correspond to the central value of the numerical prefactor, Eq.(13). For comparison data from Ref. $[14]$ are included in Fig. 3c.

Repeating the same steps for charged pions, we obtain the results shown in Fig. 4. In this figure we compare the HERMES data on $A_{UL}^{a \phi}(x)$ and $A_{UL}^{a 2 \phi}(x)$ for $\pi^0$, $\pi^+$ and $\pi^- \[4, 2\]
Clearly \( B_L(x) \gg B_T(x) \) for HERMES kinematics.

One observes that \( x h_1^u(x) \gg x^2 \int_0^1 d\xi h_1^u(\xi)/\xi^2 \). The situation is similar for other flavours.

The contribution of longitudinal (L, dashed) and transverse (T, dotted) spin part to the total (tot, solid line) azimuthal \( x^0 \) asymmetry \( A_{UL}^{sin \phi} \) and data from Ref. [3] vs. \( x \).

Azimuthal asymmetries \( A_{UL}^{W(\phi)}(x, \pi) \) weighted by \( W(\phi) = \sin \phi \) and \( \sin 2\phi \), respectively, for \( \pi^0 \) (a), \( \pi^+ \) (b) and \( \pi^- \) (c) as function of \( x \). The rhombuses denote data on \( A_{UL}^{sin \phi}(x, \pi) \), the squares data on \( A_{UL}^{2sin \phi}(x, \pi) \) from Ref. [4][5]. The enclosed areas correspond to the azimuthal asymmetries evaluated using the prediction of the chiral quark-soliton model for \( h_1^u(x) \) and the DELPHI result for the analyzing power \( \langle H_1^+ \rangle / \langle D_1 \rangle = (6.3 \pm 2.0)\% \) [7], and take into account the statistical error of the analyzing power.

with the results which follow from our analysis. The results shown here differ slightly from those obtained previously in Ref. [12] since there the lower \( y \)-cut was taken to be \( y > 0 \), instead of \( y > 0.2 \), see Eq.[1].

Finally, integrating the azimuthal asymmetries (numerator and denominator separately) over the \( x \)-region covered by the HERMES experiment, \( 0.023 \leq x \leq 0.4 \), we obtain the results for \( A_{UL}^{sin \phi} \) and \( A_{UL}^{2sin \phi} \) for \( \pi^0 \) and \( \pi^\pm \) production which are summarized in Table 1. We conclude that the azimuthal asymmetries obtained with the chiral quark-soliton model prediction for \( h_1^u(x) \) [12] combined with the DELPHI result for the analyzing power [14] are
Asymmetry | $\chi_{\text{QSM}}$ \[12\] + DELPHI \[10\] | HERMES exp. \[1, 2\] |
|-----------------|-----------------|-----------------|
| $A_{UL}^{\sin\phi}$ | | |
| $\pi^0$ | $0.017 \pm 0.005$ | $0.019 \pm 0.007 \pm 0.003$ |
| | $0.024 \pm 0.008$ | $0.022 \pm 0.005 \pm 0.003$ |
| | $-0.0046 \pm 0.0015$ | $-0.002 \pm 0.006 \pm 0.004$ |
| $\pi^-$ | | |
| $A_{UL}^{\sin 2\phi}$ | | |
| $\pi^0$ | $0.0044 \pm 0.0014$ | $0.006 \pm 0.007 \pm 0.003$ |
| | $0.0063 \pm 0.0020$ | $-0.002 \pm 0.005 \pm 0.010$ |
| | $-0.0011 \pm 0.0003$ | $-0.005 \pm 0.006 \pm 0.005$ |
| $\pi^-$ | | |

Table 1: The integrated azimuthal asymmetries $A_{UL}^{\sin\phi}$ and $A_{UL}^{\sin 2\phi}$ for $\pi^+$, $\pi^0$ and $\pi^-$. 2nd column: Results obtained with the chiral quark-soliton model prediction for proton transversity distribution $h_1^a(x)$ \[12\] and the DELPHI result for $H_1^\perp$ \[10\]. The error is due to the statistical error of the DELPHI result, Eq.(1). 3rd column: Experimental data from HERMES \[1, 2\].

consistent with experiment.

4 Determining $H_1^\perp(z_h)$

We used the DELPHI result for the analyzing power $\langle H_1^\perp / \langle D_1 \rangle$, Eq.(1), in order to explain the HERMES experiment. When doing so we presumed that the analyzing power varies weakly with scale. This assumption can be questioned. Therefore let us reverse the logic here, and use the HERMES results for the $\pi^0$ and $\pi^+$ azimuthal asymmetries to estimate $H_1^\perp(z_h)/D_1(z_h)$. For that we will use the chiral-quark soliton model prediction for $h_1^a(x)$, and this will introduce a model dependence. However, since the results of the model for known distribution functions agree within (10 – 20)% with parametrizations, we expect a similar “accuracy” for the model prediction for $h_1^a(x)$. With this in mind, the model dependence can be viewed as an additional systematic error, which however is “under control” and of order (10 - 20)%.

From the HERMES data on $A_{UL}^{\sin\phi}(z_h)$ for $\pi^0$ and $\pi^+$ we obtain the results shown in Fig. 5. The data can be described by the fit

$$H_1^\perp(z_h) = a z_h D_1(z_h) \quad \text{with} \quad a = \text{const} = 0.15 \pm 0.03 . \quad (14)$$

The error is the statistical error of the HERMES data. One should keep in mind that there is also a systematical error of the HERMES data (which varies with $z_h$), and a systematical error due to the uncertainty of the theoretical calculation of $h_1^a(x)$.

Averaging over $z_h$ we obtain

$$\frac{\langle H_1^\perp \rangle}{\langle D_1 \rangle} = \begin{cases} (5.8 \pm 1.3 \pm 0.8)\% & \text{from HERMES } \pi^+ \text{ data} \\ (7.1 \pm 2.6 \pm 0.8)\% & \text{from HERMES } \pi^0 \text{ data} \\ (6.1 \pm 0.9 \pm 0.8)\% & \text{combined HERMES result.} \end{cases} \quad (15)$$
Figure 5: a. $H^+_1(z_h)/D_1(z_h)$ vs. $z_h$, as extracted from HERMES data \cite{1, 2} on the azimuthal asymmetries $A_{UL}^{\sin \phi}(z_h)$ for $\pi^+$ and $\pi^0$ production using the prediction of the chiral quark-soliton model for $h_1^q(x)$ \cite{12}. The error-bars are due to the statistical error of the data.

Figure 5: b. The same as Fig. 5a with data points from $\pi^+$ and $\pi^0$ combined. The line plotted in both figures is the best fit to the form $H^+_1(z_h)/D_1(z_h) = a z_h$ with $a = 0.15$.

Here the statistical and the systematical errors of the HERMES data are considered. Again one should keep in mind an additional error of (10 - 20)% due to the uncertainty of theoretical prediction for $h_1^q(x)$.

Note that from the SMC data for the azimuthal asymmetry in the production of charged hadrons in SIDIS off transversely polarized protons \cite{3}, we obtain in this way the value

$$\langle H^+_1 \rangle / \langle D_1 \rangle = (10 \pm 5)\%$$

from SMC data. (16)

The results for the analyzing power from HERMES Eq.(15), SMC Eq.(16) and DELPHI Eq.(1) are all consistent with each other. This indicates that the scale dependence of $\langle H^+_1 \rangle / \langle D_1 \rangle$ might be indeed weak.

The observation that $H^+_1(z_h) \propto z_h D_1(z_h)$ – if it will be confirmed by future and more accurate data – is physically very appealing. The smaller the momentum fraction transferred from the parent parton to the hadron, the less the produced hadron knows about the polarization of the parton. One should notice that such behaviour differs from those obtained in simplest model calculation \cite{8} and used by other authors for explanation of HERMES asymmetries \cite{14}.

**Conclusions**

Using the result for $\langle H^+_1 \rangle / \langle D_1 \rangle$ from DELPHI \cite{10} and the chiral quark-soliton model prediction for the nucleon transversity distribution $h_1^q(x)$ \cite{12}, we obtain a good description of the azimuthal asymmetries measured in the semi-inclusive $\pi^0$, $\pi^+$ and $\pi^-$ production by HERMES \cite{1, 2}. The HERMES data suggest a strong flavour dependence of the transversity...
distribution, a feature the chiral quark-soliton model results for \( h_1^q(x) \) successfully account for. We stress that our description has no free adjustable parameters.

From the \( z_h \)-dependence of the HERMES data \cite{1, 2} and the chiral quark-soliton model prediction for \( h_1^q(x) \) \cite{3} we extracted \( H_1^+(z_h)/D_1(z_h) \) as function of \( z_h \). We find the Collins fragmentation function \( H_1^+(z_h) \) proportional to \( z_h D_1(z_h) \) within (the large) error-bars in the \( z_h \) region covered in the experiment.

After averaging over \( z_h \) we obtain a value for \( \langle H_1^+ \rangle /\langle D_1 \rangle \) very close to the DELPHI measurement. This suggests that the scale dependence of the analyzing power \( \langle H_1^+ \rangle /\langle D_1 \rangle \) might be rather weak. As an example of a similar behaviour of an asymmetry could serve the ratio \( A_1 \propto G_1/F_2 \) for which weak \( Q^2 \) dependence agrees with the QCD evolution equations \cite{21}. Recently the evolution equation for \( H_1^+ \) in the large \( N_c \) limit was obtained \cite{22}. This allows to make a similar investigation for \( H_1^+/D_1 \) which is under current study.

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\section{Azimuthal asymmetries}

\textbf{Unpolarized cross section} \( \sigma_{UU} \). The unpolarized differential cross section follows from Eq.(113) of Ref. 6

\[
\frac{d^5 \sigma_{UU}}{dxdydz_hd^2p_{\perp h}} = \frac{4\pi \alpha_s^2}{Q^4} \sum_{a,\bar{a}} e_a^2 \left( 1 + (1 - y)^2 \right) \frac{1}{2} x f_1^a(x) D_1^a(z_h) + \ldots \right) \cdot \frac{G(Q_{\perp}, R)}{z_h^2} . \tag{A.1}
\]

The dots denote terms which cancel out after the integration over \( \phi \). \( Q_{\perp} = |q_{\perp}| \) and \( q_{\perp} = -(p_{\perp h}/z_h) \). The dependence of the distribution and fragmentation functions on transverse quark momenta is assumed to be

\[
G(Q_{\perp}, R) = \frac{R^2}{\pi} \exp \left( -Q_{\perp}^2 R^2 \right) \quad \text{with} \quad \int d^2p_{\perp h} \frac{G(Q_{\perp}, R)}{z_h^2} = 1 . \tag{A.2}
\]

After the integration over transverse momenta \( d|p_{\perp h}|/|p_{\perp h}| \), we obtain the spin averaged cross section Eq.(6)

\[
\frac{d^4 \sigma_{UU}}{dxdydz_hd\phi} = \frac{\alpha_s^2}{Q^4} \left( 1 + (1 - y)^2 \right) \sum_{a,\bar{a}} e_a^2 x f_1^a(x) D_1^a(z_h) .
\]

\textbf{Longitudinal part} \( \sigma_{UL} \). The part of \( \sigma_{UL} \) which is proportional to \( \sin \phi \) is given by Eq.(115) of Ref. 6

\[
\frac{d^5 \sigma_{UL}^{\sin \phi}}{dxdydz_hd^2p_{\perp h}} = \sin \phi \cdot S_L \cdot \frac{4\pi \alpha_s^2}{Q^4} \left( 2(2 - y)\sqrt{1 - y} \right) \cdot \frac{Q_{\perp}}{Q} \sum_{a,\bar{a}} e_a^2 \left( \frac{R_6^2}{M_N \langle p_{\perp h} \rangle R_N^2 R_h^4} \left[ \frac{R_h^2 - R_N^2}{R^2} - Q_{\perp}^2 \right] x h_1^L(x) H_1^{L\bar{a}}(z_h) \right)
\[
\times \frac{M_N^2 R_2^2}{\langle p_{\perp h} \rangle R_N^2} x h_1^L(x) H_1^{L\bar{a}}(z_h) \right)
\[
- \frac{M_N^2 R_2^2}{M_N \langle p_{\perp h} \rangle R_N^2} x h_1^L(x) H_1^{L\bar{a}}(z_h) \right) \cdot \frac{G(Q_{\perp}, R)}{z_h^2} \right) . \tag{A.3}
\]
According to Eq.(D7) in Ref. [6] the term containing $\sigma_{UL}^{\sin \phi} = \sigma_{UL}^{\sin \phi} [H_1^\perp] + \sigma_{UL}^{\sin \phi} [H] = \sigma_{UL}^{\sin \phi}$ and compute first the part $\sigma_{UL}^{\sin \phi} [H_1^\perp] \propto H_1^\perp$ in Eq.(A.3). Let us decompose $\sigma_{UL}^{\sin \phi} = \sigma_{UL}^{\sin \phi} [H_1^\perp]$ due to Eq.(D5) in Ref. [6]. The result Eq.(A.7) we insert into the cross section Eq.(A.3) and observe that the contribution of $\tilde{h}_L^a(x)$ in Eq.(A.3) arises from the convolution Eq.(A.6) in Ref. [6].

The Wandzura–Wilczek type relation, Eq.(C11) in Ref. [6] when performing the integral $I$, observe that the contribution of $\tilde{h}_L^a(x)$ cancels out exactly

$$\frac{d^4 \sigma_{UL}^{\sin \phi}[H_1^\perp]}{dx dy dz_h d^2 P_{\perp h}} = \sin \phi \frac{4 \pi \alpha^2 S_L}{Q^4} 2(2-y)\sqrt{1-y} \sum a c_a^2 x^2 h_L^a(z_h)$$

$$\times \frac{Q L}{Q} \frac{M_h R^2}{z_h^2} \frac{G(Q^2; R)}{z_h^2}.$$  \hspace{1cm} (A.8)

$$\frac{d^4 \sigma_{UL}^{\sin \phi}[H_1^\perp]}{dx dy dz_h d\phi} = \sin \phi \frac{4 \pi \alpha^2 S_L M_h}{Q^5} 2(2-y)\sqrt{1-y} I_1 \sum a c_a^2 x^2 h_L^a(z_h) H_1^\perp(z_h).$$  \hspace{1cm} (A.9)

where

$$I_1 = \int d|P_{\perp h}| |P_{\perp h}| \frac{Q L}{Q} \frac{M_h R^2}{z_h^2} \frac{G(Q^2; R)}{z_h^2} = \frac{1}{2\pi \langle z_h \rangle} \frac{1}{\sqrt{1 + (P_{\perp h}^2)/P_{\perp h}^{1/2} z_h^2}}.$$  \hspace{1cm} (A.10)

When performing the integral $I_1$ we made use of the definitions

$$\langle Q \rangle = \int d^2 Q_{\perp} |Q_{\perp}| \mathcal{G}(Q_{\perp}; R_h) = \frac{\sqrt{\pi}}{2 R_h}, \quad \langle Q^2 \rangle = \int d^2 Q_{\perp} \frac{Q_{\perp}^2}{Q_{\perp}^2} \mathcal{G}(Q_{\perp}; R_h) = \frac{1}{R_h^2}.$$  \hspace{1cm} (A.11)
and analog definitions for $\langle P_N^\perp \rangle$ and $\langle P_N^\perp \rangle^2$. By means of Eq.(19) in Ref. [1] – where we neglect systematically current quark mass terms and the twist-3 contribution $\tilde{h}_L$ –

$$h_L(x) = 2x \int \frac{1}{x} d\xi \frac{h_1(\xi)}{\xi^2} + O(\tilde{h}_L) + O(m_q/M_N), \quad (A.12)$$

we finally arrive at the result quoted in Eq.[3]

$$\frac{d^4 \sigma_{UL}(H)}{dx dy dz_h d\phi} = \sin \phi S_L \frac{\alpha^2 s}{Q^4} M_N \frac{8(2-y)\sqrt{1-y}}{(z_h)\sqrt{1+(\langle P_{2\perp}^2 \rangle/\langle P_{h}^2 \rangle^2)}} \sum_a \epsilon_a^2 x^3 \int \frac{1}{x} d\xi \frac{h_1(\xi)}{\xi^2} H_1^{+a}(z_h).$$

Next we turn to the contribution $\sigma_{UL}(H) \propto \bar{H}$, and show that we can neglect it. Note that $\bar{H}$ is normalized analogously to Eq.(A.4). After integration over $|P_{1h}| d|P_{1h}|$ we obtain

$$\frac{d^4 \sigma_{UL}(\bar{H})}{dx dy dz_h d\phi} = -\sin \phi S_L \frac{4\pi \alpha^2 s}{Q^4} 2(2-y)\sqrt{1-y} \frac{M_N^2 R_{h^2}}{Q} I_1 \sum_a \epsilon_a^2 x h_1^{+a}(z_h) \frac{H_1^{+a}(z_h)}{z_h}, \quad (A.13)$$

with $I_1$ as defined in Eq.(A.11). From Ref. [3], Eq.(C.15) and (C.19), we obtain the relation

$$h_1^{+a}(z_h) = -\frac{M_N^2}{\langle P_{2\perp}^2 \rangle} \frac{2}{x} \int d\xi \frac{h_1^{+a}(\xi)}{\xi^2} + \ldots, \quad (A.14)$$

where the dots denote twist-3 terms and contributions proportional to current quark masses, which we neglect. We also use the relation [13]

$$\frac{d}{dz_h} \left( z_h H_1^{+a}(z_h) \right) + \ldots,$$

where we neglect consistently a twist-3 contribution, and obtain

$$\frac{d^4 \sigma_{UL}(\bar{H})}{dx dy dz_h d\phi} = \sin \phi S_L \frac{4\pi \alpha^2 s}{Q^4} 2(2-y)\sqrt{1-y} \frac{M_N^2 R_{h^2}}{Q} I_1 \sum_a \epsilon_a^2 x^3 \int \frac{1}{x} d\xi \frac{h_1^{+a}(\xi)}{\xi^2} \frac{d}{dz_h} \left( z_h H_1^{+a}(z_h) \right). \quad (A.16)$$

Thus, using the relations Eq.[3] and the definition Eq.(A.11), we see that

$$\frac{d^4 \sigma_{UL}(\bar{H})}{dx dy dz_h d\phi} = \left( \pi \langle z_h \rangle^2 M_N^2 / 4 \langle P_{1h}^2 \rangle^2 \right) \frac{d}{dz_h} \left( z_h H_1^{+a}(z_h) \right) \cdot \frac{d^4 \sigma_{UL}(\bar{H})}{dx dy dz_h d\phi} \propto \frac{d^4 \sigma_{UL}(\bar{H})}{dx dy dz_h d\phi}, \quad (A.17)$$

not only due to $\pi \langle z_h \rangle^2 M_N^2 / 4 \langle P_{1h}^2 \rangle^2 \approx 0.1$ in the HERMES experiment. Also $\frac{d}{dz_h} \left( z_h H_1^{+a}(z_h) \right) = z_h \frac{d}{dz_h} (\ln z_h H_1^{+a}(z_h))$ is small provided $H_1^{+a}(z_h)$ is a smooth function for $0.2 \leq z_h \leq 0.7$.

**Transverse part $\sigma_{UT}$.** According to Eq.(116) in Ref. [3] the only term which is non-zero after the (sin-φ-weighted) integration over φ reads

$$\frac{d^3 \sigma_{UT}(\phi)}{dx dy dz_h m^2 P_{1h}} = -\sin(\phi + \phi_s) S_T \frac{4\pi \alpha^2 s}{Q^4} \frac{1}{\langle P_{1h} \rangle R_h^2} \sum_a \epsilon_a^2 x h_1^{+a}(x) H_1^{+a}(z_h) \frac{G(Q_{1\perp} R)}{z_h^2}, \quad (A.18)$$

with $\phi_s = -\pi$ for the longitudinally polarized target in the HERMES experiment. After the integration over transverse momenta we obtain the result quoted in Eq.[7]

$$\frac{d^4 \sigma_{UT}}{dx dy dz_h d\phi} = \sin \phi S_T \frac{\alpha^2 s}{Q^4} \frac{2(1-y)}{(z_h)\sqrt{1+(z_h^2)\langle P_{1h}^2 \rangle/\langle P_{1h}^2 \rangle}} \sum_a \epsilon_a^2 x h_1^{+a}(x) H_1^{+a}(z_h).$$
References

[1] A. Airapetian et al., Phys. Rev. D64 (2001) (in press), hep-ex/0104005.
[2] A. Airapetian et al., Phys. Rev. Lett. 84 (2000) 4047, hep-ex/9910062.
H. Avakian, Nucl. Phys. (Proc. Suppl.) B79 (1999) 523.
[3] A. Bravar, Nucl. Phys. (Proc. Suppl.) B79 (1999) 521.
[4] D. Boer and P. J. Mulders, Phys. Rev. D57 (1998) 5780.
[5] D. Boer, R. Jakob and P. J. Mulders, Phys. Lett. B424 (1998) 143.
[6] P. J. Mulders and R. D. Tangerman, Nucl. Phys. B461 (1996) 197 [Erratum-ibid. B484 (1996) 538], hep-ph/9510301.
D. Boer and R. D. Tangerman, Phys. Lett. B381 (1996) 305.
[7] J. Ralston and D. E. Soper, Nucl. Phys. B152 (1979) 109.
J. L. Cortes, B. Pire and J. P. Ralston, Z. Phys. C55 (1992) 409.
R. L. Jaffe and X. Ji, Phys. Rev. Lett. 67 (1991) 552; Nucl. Phys. B375 (1992) 527.
[8] J. C. Collins, Nucl. Phys. B396 (1993) 161.
X. Artru and J. C. Collins, Z. Phys. C69 (1996) 277.
[9] A. V. Efremov, L. Mankiewicz and N. Törnqvist, Phys. Lett. B284 (1992) 394.
[10] A. V. Efremov, O. G. Smirnova and L. G. Tkachev, Nucl. Phys. (Proc. Suppl.) B74 (1999) 49 and B79 (1999) 554; hep-ph/9812522.
[11] A. V. Efremov et al., Czech. J. Phys. 49 (1999) 75, Suppl. S2; hep-ph/9901216.
[12] P. V. Pobylitsa and M. V. Polyakov, Phys. Lett. B389 (1996) 350.
P. Schweitzer, D. Urbano, M. V. Polyakov, C. Weiss, P. V. Pobylitsa, K. Goeke, Phys. Rev. D64, 034013 (2001), hep-ph/0101300.
[13] A. V. Efremov, K. Goeke, M. V. Polyakov and D. Urbano, Phys. Lett. B 478 (2000) 94, hep-ph/0001119.
[14] K. A. Oganesyan, H. R. Avakian, N. Bianchi and A. M. Kotzinian, hep-ph/9808368.
E. De Sanctis, W.-D. Nowak, K. A. Oganesyan, Phys. Lett. B483, 69 (2000).
[15] D. I. Diakonov, V. Yu. Petrov, P. V. Pobylitsa, M. V. Polyakov and C. Weiss, Nucl. Phys. B480 (1996) 341.
[16] D. I. Diakonov, M. V. Polyakov and C. Weiss, Nucl. Phys. B461 (1996) 539, hep-ph/9510232.
[17] B. Dressler and M. V. Polyakov, Phys. Rev. D61 (2000) 097501, hep-ph/9912376.
[18] M. Anselmino, M. Boglione and F. Murgia, Phys. Rev. D60 (1999) 054027, hep-ph/9901442.
M. Boglione and P. J. Mulders, Phys. Rev. D60 (1999) 054007, hep-ph/9903354.
[19] M. Boglione and P. J. Mulders, Phys. Lett. B478 (2000) 114, hep-ph/0001196.
[20] M. Glück, E. Reya and A. Vogt, Z. Phys. C67 (1995) 433.
[21] A. V. Kotikov, D. V. Pesheknonov, Phys. Atom. Nucl. 60 (1997) 653 [Yad. Fiz. 60 (1997) 736]; Eur. Phys. J. C9 (1999) 55, hep-ph/9810224.
[22] A. A. Henneman, D. Boer and P. J. Mulders, hep-ph/0104271.