Precession of Isolated Neutron Stars

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Abstract.
I summarize the evidence for precession of isolated neutron stars and theoretical effort to understand the observations. I discuss factors that might set the precession period, describe constraints on the material properties of the crust, and conclude with a brief discussion of possible sources of stress that would deform a neutron star to the extent required.

1. Introduction

Observations of precession of isolated neutron stars afford a variety of interesting new probes of the physics of neutron stars. The precession period is determined by the deformation of the star, and hence by stresses in the crust and core. Observation of a clear signature of damping of precession would constrain the dissipative processes that enforce corotation between the crust and liquid core. As a neutron star wobbles, the external torque that spins it down could vary, giving a distinct timing signature. Precession could therefore provide a direct probe, perhaps the only probe, of the dependence of the spin-down torque on the angle between the magnetic axis and the angular velocity. Also, as the neutron star wobbles, the observer looks down the beam at different angles through the precession cycle, allowing mapping of the beam morphology. A rapidly-spinning precessing neutron star could be a strong source of gravitational waves which might be detectable by LIGO or LISA.

2. Observational Evidence for Precession

The two isolated pulsars that provide the most compelling evidence for precession are PSR 1642-03 (Cordes 1993; Shabanova, Lyne, & Urama 2001) and PSR B1828-11 (Stairs, Lyne, & Shemar 2000). PSR B1828-11 is particularly convincing as it displays variations in pulse duration and shape that are highly correlated with the timing residuals (see Fig. 1), as one would expect for a wobbling neutron star. The pulse width variations are about 5°, suggesting a wobble angle (the angle between the star’s symmetry axis and the angular momentum) of similar magnitude. The timing data are highly periodic, but non-sinusoidal, indicating harmonic structure. Over 13 years, the timing residuals show Fourier power at $\sim 1000$ d, $\sim 500$ d and $\sim 250$ d. The 1000-d component, however, does not appear clearly in the Fourier analyses of the period, period derivative and beam shape and so may not represent the precession period.
Figure 1. The left panel shows the phase, period and period derivative residuals for about 2000 days of data from PSR B1828-11. The data are highly periodic but non-sinusoidal. The bottom graph shows the correlated changes in beam width (the beam shape parameter is small when the beam is wide, large when the beam is narrow). The solid curves in the timing plots are fits from the model described in the text. The right panel shows templates for the pulse profile in the wide and narrow states. Data provided courtesy of I. Stairs.

3. Theoretical Description of Precession

3.1. Free Precession

Precession is the motion that results when a non-spherical rigid body is set into rotation about any axis other than a principal axis. Consider an oblate, biaxial, rigid object with principal moments of inertia $I_3 > I_2 = I_1$. (The effects of triaxiality will be discussed later). Let $I_3 = I_1(1 + \epsilon)$ where $\epsilon \ll 1$ is the oblateness. Set the body rotating with angular velocity $\omega$ about an axis other than the symmetry axis $\hat{s}$, as shown in Fig. 2. At any instant, the conserved angular momentum $L$, $\omega$ and $\hat{s}$ span a plane. The wobble angle $\theta_w$ and $|\omega|$ are both constants of the motion. Free precession consists of a superposition of two rotations: 1) a fast wobble about $L$ at approximately the spin rate, and, 2) a slow, retrograde rotation about the symmetry axis at frequency $\omega_p \simeq \epsilon \omega$ (for $\theta_w \ll 1$). With respect to a coordinate system fixed in the body, $\omega$ takes a circular path in a right-handed sense about $\hat{s}$, completing the circle in a precession period $p_p = 2\pi/\omega_p$.

If a beam in direction $\hat{b}$, taking an angle $\chi$ with respect to $\hat{s}$, is affixed to the body, a distant observer will see a pulse when the beam passes through the plane defined by the observer and $L$. The observer will see modulation of the pulse arrival times at frequency $\omega_p$, accompanied by variations in pulse duration as she looks into the beam at different angles. The phase variations have magnitude $\delta \phi = \theta_w \cot \chi \sin \omega_p t$ (see, e.g., Nelson, Finn, & Wasserman 1990), corresponding to period variations of

$$\frac{\delta p}{p} = \frac{p}{p_p} \theta_w \cot \chi \cos \omega_p t,$$

where $p$ is the spin period. Assuming the precession period of PSR B1828-11 is 500 d, and a wobble angle of $5^\circ$, the observed amplitude of the period
variations (about 1 ns) implies $\chi \simeq 20^\circ$. These period variations, however, have no harmonics. I next describe how the star’s spin-down torque could introduce the harmonic structure observed in PSR B1828-11 and PSR 1642-03.

3.2. Precession Under Spin-down Torque

The precession of an isolated neutron star is not truly torque-free; the star is being spun down by electromagnetic torque. If the torque depends on the angle between the star’s magnetic moment and the angular velocity, then the spin-down torque will vary over the precession period (Cordes 1993; Link & Epstein 2001). The variable torque will introduce variations in the star’s spin rate (with respect to the secular spin-down) which will add to the timing effects described above. These variations are given by:

$$\frac{1}{2} I_1 \frac{d\omega^2}{dt} \simeq \omega \cdot N,$$

where $N$ is the torque on the crust. Though the full spin-down torque and its dependence on $\omega$ are unknown, let us consider the vacuum dipole torque (Davis & Goldstein 1970) as an example:

$$N = \frac{2\omega^2}{3c^3} (\omega \times m) \times m,$$

where $m$ is the star’s magnetic dipole moment.\(^1\) The quantity $\omega \cdot N$ has non-linear dependence on the components of $\omega$ which is particularly strong for $\chi \simeq \pi/2$. Neglecting internal dissipation and treating the precessing star as (effectively) a rigid body, this model has three free parameters: the wobble angle $\theta_w$, the dipole angle $\chi$ and the oblateness $\epsilon$. The solid curves shown in the

\(^1\)The near-field contribution to the torque, which does not affect the star’s spin rate, has been ignored.
Figure 3. A schematic representation of the beam shape inferred for PSR B1828-11.

timing plots of Fig. 1 are fits for $\theta_w = 3.2^\circ$, $\chi = 89^\circ$ and $\epsilon = 9.1 \times 10^{-9}$ (Link & Epstein 2001). The precession period is 511 d, with a harmonic at 256 d arising from non-linearity in the torque. The derived wobble angle is consistent with the observed pulse width variations. A simple model of these variations allows crude mapping of the beam morphology, and gives the hour-glass beam depicted in Fig. 3. This beam shape, though non-standard, is similar to that found by Weisberg & Taylor (2002) for the binary pulsar B1913+16.

4. The Precession Period

For a neutron star (or any object) to precess, it must have a deformation axis which cannot follow the instantaneous spin vector. If the moment of inertia corresponding to this deformation is $\Delta I_d$, the precession period is

$$p_p = p \left( \frac{I_c}{\Delta I_d} \right),$$

where $I_c$ is the moment of inertia of the crust plus any component of the star that corotates with the crust over timescales less than $p$. If the precession period of PSR B1828-11 is $\approx 500$ d, then $\Delta I_d/I_c \approx 10^{-8}$. This deformation is far less than the rotational deformation, which follows the instantaneous spin axis and therefore does not affect the precession period. What might sustain a deformation of $\Delta I_d/I_c \approx 10^{-8}$?

4.1. Precession of an Elastic, Relaxed Body

An elastic, relaxed (i.e., unstressed) body can precess, since rigidity will prevent a portion of the spin-induced bulge from following the instantaneous rotation vector once precession is excited. Consider an unprecessing, spinning star with some rigidity. If the star is relaxed, it will have an excess moment of inertia $\Delta I_o$ about its rotation axis which is the same as that for a self-gravitating fluid at the same spin rate. If the star is carefully spun down to zero angular velocity without cracking or otherwise relaxing, the star will not become spherical, but will remain oblate by some amount $\Delta I_d$ under the stresses that have developed from the spin down. The principal axis of the star is aligned with the original
rotation vector. Now suppose the star is spun up to its original spin rate but about a different axis. The star will precess, because the built-in deformation is not aligned with the new spin axis. The star is also relaxed (except for the stresses associated with the precession itself). The built-in deformation is some fraction of the original spin deformation:

$$\Delta I_d = b \Delta I_\omega,$$

where $b$ is a *rigidity parameter* in the range $0 \leq b \leq 1$. For a fluid $b = 0$, while for an infinitely rigid solid, $b = 1$. The precession period of a relaxed, rigid object is then

$$p_p = \frac{p}{b \epsilon_{rot}},$$

where $\epsilon_{rot}$ is the rotational oblateness of a self-gravitating fluid with spin period $p$. For the Earth, $b$ is about 0.7; that is, the Earth behaves more like a solid than a liquid. For the Earth, the above expression gives $p_p = 440$ d, which is the period of the famous Chandler Wobble.

Might the precession of PSR B1828-11 be similar to the Chandler Wobble? A calculation of $b$ for a neutron star, using a realistic model for the internal structure, gives $b \approx 2 \times 10^{-7}$ for reasonable equations of state (Cutler, Ushomirsky, & Link 2002). This value is a factor of $\sim 30$ smaller than the previous estimate of Baym & Pines (1971). The smallness of $b$ is because the gravitational energy density of a neutron star far exceeds the crust’s shear modulus; consequently, the crust behaves much more like a liquid – although a slightly “rigid liquid” – than a solid. The implied precession period of PSR B1828-11, *if its crust is relaxed*, is about 100 years (Cutler et al. 2002). Hence, if crust rigidity is responsible for the observed precession, the crust must be significantly strained.

### 4.2. The Precession Period of a Stressed Crust

Strain in the crust will arise naturally as the star spins down. As stress builds, it can be partially relieved by crustquakes or plastic flow. Terrestrial solids fail (i.e., crack or flow) when the strain reaches a critical value $\theta_c$, typically in the range $10^{-5} < \theta_c < 0.1$. At present, the critical strain for a given solid cannot be calculated from first principles, and so must be determined empirically. The critical strain for the stellar crust can be constrained by calculating the strain field of a spinning-down neutron star and determining how much strain is needed to sustain a given amount of deformation. To account for the precession period of PSR B1828-11, the average critical strain of the crust must satisfy (Cutler et al. 2002)

$$\theta_c \geq 5 \times 10^{-5} \left( \frac{p_p}{511 \text{d}} \right)^{-1} \left( \frac{I_c/I}{0.01} \right),$$

where $I$ is the total moment of inertia of the star and a fiducial value of $I_c$ comparable to crust moment of inertia has been chosen. Even if $I_c$ is comparable to $I$, the inferred lower limit on $\theta_c$ is not unreasonable (by terrestrial standards), suggesting that crust rigidity is sufficient to sustain the required deformation.
5. Effects of Vortex Pinning

The lattice of the inner crust is expected to coexist with superfluid neutrons. A rotating superfluid, such as liquid helium, is threaded by quantized vortex lines. In a neutron star, an attractive interaction between nuclei and vortices exists which might pin the vortices to the lattice (Anderson & Itoh 1975). As originally pointed out by Shaham (1975), pinning is disastrous for long-period precession; the vortex array acts as a gyroscope which drives the star to precess with a period that is at most $\simeq 100$ spin periods if most of the vortices in the crust are pinned. Under these circumstances, damping of the precession is also very quick (Sedrakian, Wasserman, & Cordes 1999). However, if PSR B1828-11 is precessing with a wobble angle of $\simeq 3^\circ$, pinning is most likely unstable, because the forces exerted on the pinned vortex lattice in a precessing star are sufficient to cause global unpinning (Link & Cutler 2002).

6. Effects of Triaxiality

So far I have focused on precession of a biaxial star. However, given that the evolution and relaxation of the crust is probably quite complex, and that magnetic stresses might also significantly deform the star (Wasserman 2002), a neutron star is almost certainly a triaxial object. For a triaxial, precessing object, the wobble angle and $|\omega|$ are not conserved quantities. Some aspects of the precessional dynamics are then determined by the dimensionless parameter

$$k^2 = \frac{\epsilon'}{\epsilon(\epsilon - \epsilon')} \tan^2 \theta_0,$$

where $\epsilon' (< \epsilon)$ is defined by $I_2 = I_1(1 + \epsilon')$ and $\theta_0$ is the minimum value of the wobble angle. For $k$ of order unity, which is possible for sufficiently large $\epsilon'$, the arrival time residuals have strong harmonics of $\omega_p$ even for free precession. However, the angle between the beam and the observer at the time of the pulse varies enormously during the precession cycle – by $50^\circ$ or more. Hence, $k$ of order unity cannot be tolerated as an explanation of PSR B1828-11’s timing behavior, as we would lose sight of the beam.

For small $\theta_0$, $k$ can be $\ll 1$ even for extreme triaxiality ($\epsilon \simeq \epsilon'$). In this limit, the biaxial solution presented in Section 3.2 changes only slightly, and so is robust for significant triaxiality.

7. Summary and Discussion

The interpretation that PSR B1828-11 is a precessing neutron star is supported by a simple model of torque-assisted precession with a wobble angle of $\simeq 3^\circ$. The harmonic structure seen in the timing data could be produced by a torque that depends on the angle between the rotation vector and the magnetic moment, as in the vacuum dipole model. These results are essentially unchanged if the star is significantly triaxial. This model, if correct, provides evidence that the spin-down torque does actually depend on the relative orientation of the spin and dipole axes, as is usually assumed. The vacuum dipole model for the torque
(though alternatives should certainly be considered) requires a rather extreme
dipole angle of $\chi \simeq 89^\circ$. Perhaps this conclusion is telling us something about
the preferred rotational state of a precessing star.

A precession period of $\simeq 500$ d in PSR B1828-11 cannot be explained if the
star is relaxed. Rather, the star must be under stresses that sustain deformation.
Crustal stresses alone could account for the deformation, though Wasserman
(2002) has proposed a different model of precession in which both magnetic
stresses and crustal stresses deform the star.

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