Extra $W$-Boson Mass from a D3-Brane

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Abstract

Motivated by the recent CDF measurement of the $W$-boson mass, we study string-based particle physics models which can accommodate this deviation from the Standard Model. We consider an F-theory GUT in which the visible sector is realized on intersecting 7-branes, and extra sector states arise from a probe D3-brane near an E-type Yukawa point. The D3-brane worldvolume naturally realizes a strongly coupled sector which mixes with the Higgs. In the limit where some extra sector states get their mass solely from Higgs vevs, this leads to a contribution to the $\rho$ parameter which is too large, but as the D3-brane is separated from the 7-brane stack, this effect is suppressed, leading to O(1) - O(10) TeV scale extra sector states and a correction to $\rho$ which would be in accord with the CDF result. We also estimate the contribution to the oblique electroweak parameter $S$, and find that it is compatible with existing constraints. This also leads to additional signatures, including diphoton resonances (as well as other diboson final states) in the O(1) - O(10) TeV range.

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1 Introduction

Recently, CDF announced an updated measurement of the $W$-boson mass [1] which is in tension with expectations from global Standard Model (SM) fits (see e.g. [2, 3]) as well as previous measurements, e.g., from ATLAS [4] and LHCb [5]:

CDF II: $M_W = 80,433.5 \pm 9.4$ MeV (1.1)
ATLAS: $M_W = 80,370 \pm 19$. MeV (1.2)
LHCb: $M_W = 80,354 \pm 32$. MeV (1.3)
SM Global Fit: $M_W = 80,357 \pm 6.0$ MeV. (1.4)

Assuming the CDF II measurement is correct, and that all other quantities such as $c_W$ and $M_Z$ remain in accord with the SM, a shift in the $\rho$ parameter $M_W^2/c_W^2M_Z^2$ (see [6]) would need to occur at the level of $\delta\rho = 2\rho_{SM}(\delta M_W/M_W)$, i.e., a deviation of order $10^{-3}$. For recent theoretical discussions of the $W$-boson mass anomaly and its implications for physics beyond the Standard Model, see references [7–16].

This would signal the presence of additional states sensitive to the physics of electroweak symmetry breaking. In terms of the higher dimension operators contributing to the precision electroweak observables [17,18] such as $|H^\dagger D_\mu H|^2/(4\pi f_{\text{nat}}^2)$, this suggests the appearance of new states with a “natural mass” on the order of $f_{\text{nat}} \sim 400$ GeV. Such a low mass scale is presumably already ruled out by LHC direct searches. However, in the limit where the new states also have a vector-like contribution to the mass whilst still coupling to the Higgs, one can presumably evade LHC direct searches and still accommodate a non-zero shift in the $\rho$ parameter. Note that even in this limit, sizable Yukawa couplings between the Higgs and extra states would appear to be necessary in order to fit the observed data, indicating a preference for strong coupling effects.

Of course, it could happen that the announced result is incorrect.

But it could also be true! Indeed, it is important to ask whether models written down with possibly other motivations in mind might provide a natural explanation. In this short note, we consider a particular class of models motivated by top-down string constructions. The entire construction can be defined in purely field theoretic terms, although the motivation for the particular structures we present here which are quite natural from a stringy perspective might appear “contrived” from a bottom-up perspective. Moreover, the model comes with additional signatures which can likely be tested in the coming years.

2 D3-Branes Near a Yukawa Point

The scenario we wish to consider is based on F-theory GUT models [19–22] (see [23, 24] for reviews, and [25] for a recent “landscape scan” of SM-like F-theory models). These are string-based models in which the Standard Model (visible sector) is realized on a configuration of
intersecting 7-branes. A remarkable feature of this stringy particle physics scenario is that in a parametric limit where gravity decouples, getting the correct matter content, gauge interactions and flavor structure [26–29] leaves little room for anything else [30]. Extra sectors can still be included, but they typically arise either from other stacks of 7-branes far from the visible sector, or from mobile D3-branes which can probe the visible sector. See figure 1 for a depiction of the different sectors of an F-theory GUT model.

We would like to understand how these models can generate a shift in the $\rho$ parameter. In what follows, we state our model in terms of a supersymmetric theory, in part because this is the case where we can actually calculate the quantities of interest. While there is of course no evidence for superpartners in the visible sector, it could still happen that extra sector states are approximately supersymmetric. The main role of supersymmetry breaking is to set an overall mass scale and effective potential for the modulus controlling the separation of the D3-brane from the visible sector [31].

Now, in models with a mobile D3-brane, the same physics which generates hierarchical Yukawa couplings also attracts D3-branes to an E-type Yukawa point [32–35]. The resulting worldvolume theory is a strongly coupled supersymmetric conformal field theory (SCFT), the formal properties and resulting phenomenology of which were determined in a series of papers [34, 36–41, 31, 42–44]. Treating, for the moment, all states of the Standard Model as non-dynamical, the probe D3-brane theory is specified by a particular relevant deformation of the $\mathcal{N} = 2$ Minahan-Nemeschansky theory with $E_8$ flavor symmetry [45,46]. The resulting CFT$_{\text{intermediate}}$ comes with a collection of operators transforming (descending from the adjoint of $E_8$) in various representations of the SM gauge group. In particular, there are couplings to the Higgs doublets of the MSSM, as well as the heaviest generation of quarks and leptons. Our interest here is in the Higgs / D3-brane couplings, and these can be viewed as an additional relevant deformation of CFT$_{\text{intermediate}}$ by superpotential couplings of the form:

$$\int d^2 \theta \lambda_u H_u O_u + \lambda_d H_d O_u.$$  \hspace{1cm} (2.1)

In a weakly coupled model, these terms could be interpreted as Yukawa couplings to a vector-like fourth generation. In the case at hand where the extra sector is strongly coupled, these terms drive CFT$_{\text{intermediate}}$ to a new fixed point CFT$_{\text{IR}}$ in which the scaling dimension of the Higgs fields changes to $\Delta_u = 1 + \delta_u$ and $\Delta_d = 1 + \delta_d$. This modifies the Kähler potential for the Higgs fields, which in turn distorts the resulting masses for the $W$- and $Z$-bosons, as well as the spectrum of excitations in the two Higgs doublet model [39]. See figure 2 for a depiction of the different fixed points.

In particular, it is possible to calculate the contribution to the $\rho$ parameter from such
Figure 1: Depiction of D3-brane probe separated from the stack of intersecting 7-brane configuration used to generate the Standard Model (i.e., visible sector). Visible sector states arise from massless $7 - 7'$ strings, while the states of the D3-brane sector arise from $7 - 3$ and $3 - 3$ strings. Separating the D3-brane from the visible sector results in a mass for the $7 - 3$ strings and triggers a vector-like mass term for the states which mix with the Higgs fields.

couplings. Precisely because we are at strong coupling, the end result is of the general form:\(^{1}\)

$$\delta \rho_{\text{CFT}} \simeq f_u(\beta)\delta_u + f_d(\beta)\delta_d \simeq \delta_{\text{CFT}},$$

(2.2)

where $f_u(\beta)$ and $f_d(\beta)$ are order one trigonometric functions which sum to one, and $\tan \beta \equiv v_u^{1/\Delta_u} / v_d^{1/\Delta_d}$ is a ratio of scaled Higgs vevs. In the last approximation of line (2.2), we have made the estimate $\delta_{\text{CFT}} = (\delta_u + \delta_d)/2$ to get a “typical” sense of the size of the deviation. In the most phenomenologically successful scenarios such as the “Dih$^{(2)}_4$” model of reference [28], the resulting value of $\delta_{\text{CFT}}$ is of order 0.01, although larger values can be generated in other scenarios. Returning to the Higgs / extra sector couplings of line (2.1), these extra sector states can be interpreted as weakly coupled states with large Yukawa couplings, with corresponding masses on the order of $f_{\text{nat}} \sim 400 \text{ GeV}$, so we take this as a reference value in what follows. Taken at face value, getting such a large value of $\delta_{\text{CFT}}$ would result in a contribution to $\delta \rho$ which is too large by a factor of 10 to accommodate the CDF result, let alone the other measurements which are in accord with the Standard Model.

Of course, for various reasons, we must consider a deformation of the CFT$_{\text{IR}}$ theory. For one thing, the presence of a large number of additional low mass states charged under the Standard Model gauge group would have been detected by now. The simplest, well-motivated possibility is to assume that while the D3-brane is nearby the Standard Model

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\(^{1}\)As explained in [39, 41], to extract the $W$- and $Z$-boson masses, we use the fact that a modification in the scaling dimension for the Higgs fields corrects its Kähler potential, e.g., $(H_u^\dagger H_u + H_d^\dagger H_d)^{1/\Delta}$ in a case where a custodial $SU(2)$ is retained to leading order. Indeed, for a supersymmetric theory with superfields $\Phi^i$, the corresponding kinetic term $g_{ij}D^\mu \Phi^i D^\nu \Phi^j$ then generates a shift in the $\rho$ parameter.
Figure 2: Depiction of the different deformations of the D3-brane probe superconformal field theory (SCFT). An F-theory GUT is generated by an intricate pattern of intersecting 7-branes. In the limit with no tilting, the D3-brane SCFT realizes the $E_8$ Minahan-Nemeschansky theory (left). Switching on 7-brane tilting with “T-brane deformations” (see [47, 36]) generates an $\mathcal{N}=1$ deformation to a new conformal fixed point (middle). Coupling to dynamical Standard Model fields, and in particular mixing with the Higgs sector results in another $\mathcal{N}=1$ fixed point (right) in which contributions to the $\rho$ parameter are calculable.

stack, it is actually somewhat removed from it, i.e., separated from it in the extra dimensions by a characteristic scale $f_{D3}$ (see figure 1). As explained in [31], $f_{D3}$ is expected to be the same order of magnitude as the mass scales of the extra sector. Assuming present LHC limits, we take this to be on the order of 1 TeV. In this case, the net contribution to higher-dimension operators such as $|H^\dagger D_\mu H|^2$ will receive some additional suppression relative to the purely conformal case. We can model such effects by introducing an effective $\delta_{\text{eff}} \equiv \delta_{\text{CFT}} \times (4\pi f_{\text{nat}}/4\pi f_{D3})^2$, with $f_{\text{nat}} \sim 400$ GeV. All told, then, we expect the contribution to the $\rho$ parameter from the D3-brane sector to be:

$$\delta \rho_{D3} \approx 10^{-3} \times \left(\frac{\delta_{\text{CFT}}}{0.01}\right) \times \left(\frac{1.3 \text{ TeV}}{f_{D3}}\right)^2.$$  \hspace{1cm} (2.3)

Surveying the list of F-theory GUT models with different spectral cover monodromy [28], the values of $\delta_{\text{CFT}}$ for the different models were collected in line (3.64) of [38]. This results
in the D3-brane mass scales:

| One D3-Brane | $\mathbb{Z}_2^{(1)}$ | $\mathbb{Z}_2^{(2)}$ | $\mathbb{Z}_2 \times \mathbb{Z}_2$ | $S_3$ | Dih$_4^{(1)}$ | Dih$_4^{(2)}$ |
|--------------|-------------------|-------------------|-------------------------------|------|-------------|-------------|
| $\delta_{\text{CFT}}$ | 0.065             | 0                 | 0.178                         | 0.08 | 0.145       | 0.01        |
| $f_{\text{D3}}$      | 3.2 TeV           | X                 | 5.3 TeV                       | 3.6 TeV | 4.8 TeV    | 1.3 TeV     |

where an “X” indicates that we cannot fit the shift in the $\rho$ parameter mass for any value of $f_{\text{D3}}$. We stress that these are order of magnitude estimates since we have imperfect knowledge of the microscopic details of the strongly coupled CFT.

One can repeat this analysis in the case of multiple D3-branes, but beyond two D3-branes leads to a loss of asymptotic freedom and perturbative gauge coupling unification [38]. The values of $\delta_{\text{CFT}}$ for the different models were collected in line (3.67) of [38]. This results in the D3-brane mass scales:

| Two D3-Branes | $\mathbb{Z}_2^{(1)}$ | $\mathbb{Z}_2^{(2)}$ | $\mathbb{Z}_2 \times \mathbb{Z}_2$ | $S_3$ | Dih$_4^{(1)}$ | Dih$_4^{(2)}$ |
|---------------|-------------------|-------------------|-------------------------------|------|-------------|-------------|
| $\delta_{\text{CFT}}$ | 0.11              | 0                 | 0.225                         | 0.21 | 0.29        | 0.08        |
| $f_{\text{D3}}$      | 4.2 TeV           | X                 | 6.0 TeV                       | 5.8 TeV | 6.8 TeV    | 3.6 TeV     |

The main point is that the CFT$_{\text{IR}}$ fixed point would generate a too large contribution to the $\rho$ parameter, but we can decouple the effects by introducing vector-like masses, which in some cases need to be rather large to sufficiently suppress the resulting contributions.

It is also important to ask about the contribution from these extra sector states to the oblique electroweak parameter $S$ [17, 18]. We can estimate this contribution using the methods of [41]. In many of the D3-brane scenarios considered here, the leading order contribution is a shift of order:

$$\delta S \simeq \frac{\delta b_{SU(5)}}{2\pi} \left( \frac{f_{\text{nat}}}{f_{\text{D3}}} \right)^2,$$

where $\delta b_{SU(5)} \simeq 2 - 4$ is the size of the threshold correction to the unified $SU(5)$ gauge coupling. By inspection, this is typically on the order of $10^{-2}$ to $10^{-3}$, which is within the tolerance of recently updated precision electroweak fits [9,10,12,14].

A subtle feature of many electroweak global fits is possible mixing with other higher dimension operators. We expect the dominant contributions to be associated with the $S$ and $T$ (and thus implicitly $\rho$) parameters, which has been our main focus here, but of course it would be important to explore this more systematically in future work, perhaps along the lines of [12].

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2 We thank C. Kilic for comments on this point.
3 Further Signatures

The requirement that we separate the D3-brane from the visible stack leads to additional signatures. For example, in this class of scenarios the vev of a dynamical modulus controls the separation between the D3-brane and the visible sector. The mass of this state is roughly of the same order of magnitude as $f_{D3}$, and will produce some additional resonances which can in principle be detected in future LHC searches. This modulus couples to the SM vector bosons via the higher dimension operator:

$$\frac{\delta b_G}{32\pi^2} \left( \frac{s}{f_{D3}} \right) \text{Tr}_G F_{\mu\nu}^G F_{\mu\nu}^G$$

with $F_{\mu\nu}^G$ the corresponding field strength for a gauge group factor $G$. The order one coefficient $\delta b_G$ in front of this dimension five operator is controlled by the contribution from the D3-brane sector to the beta function for each gauge group factor. In fact, precisely this mechanism was explored in [31] as a way to generate a 750 GeV diphoton resonance via gluon fusion, i.e., $gg \rightarrow s \rightarrow \gamma\gamma$. To get a 5 fb production cross section, this required a somewhat lower value of $4\pi f_{D3}$ of order 1 TeV.\(^3\) Scaling up to the list of $f$’s given in lines (2.4) and (2.5), we expect a suppressed production cross section of order $\sim 5$ fb $\times (1 \text{ TeV} / 4\pi f_{D3})^2$. This should be interpreted as a crude order of magnitude estimate, i.e., there could be a stray order one factor multiplying $f_{D3}$.\(^4\)

While the evidence for the 750 GeV diphoton resonance has since collapsed, the generic expectation that such resonant diphoton signatures will appear at some scale remains a feature of all of these D3-brane scenarios. Additional signatures, as well as their rates include the corresponding contribution from $s$ and its axionic partner $a$ include (see [31] for further discussion):

- $pp \rightarrow s/a \rightarrow \gamma\gamma$
- $pp \rightarrow s/a \rightarrow gg$
- $pp \rightarrow s/a \rightarrow ZZ$
- $pp \rightarrow s/a \rightarrow WW$
- $pp \rightarrow s/a \rightarrow Z\gamma$

\(^3\)One might ask whether the model of [31] was in fact compatible with the precision electroweak constraints known at that time. In the model considered there, the mixing from the Higgs sector was basically neglected, and this can be arranged by tuning the $\lambda_u H_u O_u$ and $\lambda_d H_d O_d$ couplings to small values. One can of course reintroduce this tuning and in so doing suppress any shift to the $\rho$ parameter.

\(^4\)For example, the mass of the modulus controlling the separation of the D3-brane from the visible sector will in general differ from the suppression scale $f_{D3}$ entering in the dimension five operator $s \text{Tr}_G F^2$. 
See, e.g., [48] for a recent update on resonant diphoton searches. From figure 5 of [48], we observe that the typical production rate is below current limits in all of our scenarios, but with increased luminosity, such scenarios may eventually be probed. For example, the current bound on a 1 TeV diphoton scalar resonance is around $10^{-1}$ fb, while the Dih$_4^{(2)}$ scenario would indicate (roughly) a value of order $10^{-2}$ fb.

Of course, the overall cross section depends on various details of the model and in particular the mass of the modes $s$ and $a$. As the mass increases, the overall cross section will of course decrease further. From the perspective of a UV complete model, the mass depends on the local potential for the D3-brane probe in the F-theory model. Following [31] (see equation (2.2) of [31]), we note that non-perturbative corrections to the D3-brane superpotential induce a mass for this modulus of order $M_{GUT}(M_{IR}/M_{GUT})^{\Delta-1}$, with $M_{IR} \sim$ TeV an infrared scale of conformal symmetry breaking, and $\Delta \sim 2$ the scaling dimension of the operator responsible for the motion of the D3-brane. This in turn leads to a mass scale in the TeV range. Supersymmetry breaking can induce an additional model dependent mass splitting between $s$ and $a$.

Another natural question is whether the presence of such a D3-brane might introduce contributions to other beyond the Standard Model signatures such as proton decay. This question was actually considered in reference [37]. It was found there that the coupling to a conformal extra sector has a large conformal suppression in the resulting higher-dimension operators. An additional comment here is that the D3-brane probe theory enjoys approximate global $U(1)$ symmetries which further suppress such contributions to proton decay.

4 Conclusions

The reported $W$-boson mass measurement by CDF is quite exciting. Though it will need to be confirmed, it is already worth asking how well various scenarios for physics beyond the Standard Model can accommodate this measurement. While there are clearly many possible models which can do the job (see e.g., [7–16]), in this note we have explained how top-down motivated F-theory GUT models with an extra sector generated by a probe D3-brane provide a natural candidate scenario which might otherwise appear “contrived” from purely bottom-up considerations. Thankfully, much of the required formal analysis of Higgs / D3-brane sector mixing was already performed in previous work. For suitable choices of mass scales and parameters, this class of models can indeed produce shifts in the $\rho$ parameter of the correct size, whilst remaining compatible with constraints on the $S$-parameter and direct searches at the LHC. The model also predicts additional signatures, especially with respect to vector boson production. This holds out the prospect of vaulting the gap from the electroweak scale to the Planck scale. We anticipate that the coming years will provide additional clarity.
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