Interplay of disorder and interactions in an optical lattice

Hubbard model

S. S. Kondov, W. R. McGehee, B. DeMarco

1Department of Physics, University of Illinois, 1110 W Green St., Urbana, IL 61801

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Abstract

Open questions abound regarding how the interplay between disorder and inter-particle interactions affects essential properties of electronic solids. For example, a complete understanding of how competing phases and disorder conspire to create inhomogeneity in the high-temperature superconducting cuprates and the role of disorder in materials that exhibit colossal magnetoresistance, such as the manganites, has been elusive despite decades of active research [1–3]. Fundamental transport issues, such as whether interactions enhance or inhibit localization of metallic phases, also remain unresolved [1, 4, 5]. Here we use ultracold atoms trapped in a disordered optical lattice to realize a minimal model for strongly correlated, disordered electronic solids [5]—the disordered Fermi Hubbard model (DFHM) [6]—and to probe the impact of disorder on the metallic and Mott insulator (MI) phases. We measure a disorder-driven metal–insulator transition, and we observe that interactions raise the critical disorder energy, thereby stabilizing the metallic phase. Introducing disorder in the MI regime creates density fluctuations that are consistent with disruption of the Hubbard gap. Because we utilize fully known and controllable disorder, our work supports new and more stringent tests of theoretical and numerical approaches to understanding the influence of disorder on strongly correlated systems (see Refs. [6, 7] for reviews and Refs. [8, 9] for recent work, for example). If lower temperatures can be reached in optical lattices [10], our methods may enable measurements of the effect of disorder on the analogue of high-temperature superconductivity.
In our experiment, fermionic $^{40}$K atoms cooled below the Fermi temperature $T_F$ and trapped in a cubic optical lattice potential formed from three pairs of counter-propagating laser beams play the role of the electrons in a solid [11]. As shown in Fig. 1, the atoms are harmonically confined by an additional dipole trap. An approximately equal mixture of two atomic hyperfine states (\$|F = 9/2, m_F = 9/2\$ and $|F = 9/2, m_F = 7/2\$) are used to mimic the spin of the electrons. In the lattice, the atoms can tunnel between adjacent sites and two atoms on the same site (in different hyperfine states) interact through a low-energy collision, thereby realizing the Hubbard model [11, 12]. The equivalent of material parameters, such as the ratio of interaction to tunneling energy, are precisely known and tunable over orders of magnitude by adjusting the power of the $\lambda = 782.2$ nm lattice laser, which controls the lattice potential depth $s$.

By disordering the lattice potential using optical speckle [13 14], we explore the DFHM model with ultracold atoms for the first time. In contrast with experiments on solids, the disorder is precisely known (via optical microscopy) and continuously tunable, from being absent to the largest energy scale present. Our work builds on the achievement of the MI phase [15 16] and transport measurements [17 18] for fermionic atoms trapped in clean lattices, and the realization of 3D Anderson localization (AL) [19 20] and a disorder-induced bosonic superfluid–insulator transition [14] for gases disordered by optical speckle.

The optical speckle field is produced by passing a 532 nm laser beam through a holographic diffuser and is focused onto the atoms, as in Refs. [13 14 19]. The atoms experience a potential proportional to the optical speckle intensity, which varies randomly in space. The strength of this disorder, which we characterize by the average disorder potential energy $\Delta$, can be continuously adjusted by tuning the 532 nm laser power. The disorder causes the “clean” Hubbard model occupation $\epsilon$, interaction $U$, and tunneling $t$ energies to vary from site to site in the lattice, and thus the atoms realize a single-band DFHM described by the Hamiltonian $H = \sum_i U \hat{n}_{i\uparrow} \hat{n}_{i\downarrow} - \sum_{\langle ij \rangle, \sigma} t_{ij} \left( \hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + h.c. \right) + \sum_i \left( \epsilon_i + m \omega^2 r_i^2 / 2 \right) \hat{n}_i$, where $i$ indexes the lattice sites, $\hat{c}_{i\sigma}^\dagger$ is the operator that creates an atom on site $i$ in spin state $\sigma = \uparrow, \downarrow$, \$\langle ij \rangle\$ indicates a sum over adjacent sites, $m$ is the atomic mass, $\omega$ is the geometric mean of the trap frequencies, $r_i$ is the distance from the trap centre to site $i$, and $\hat{n}_{i\sigma} = \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma}$ is the number operator. We work at sufficiently low temperature such that the atoms occupy only the lowest energy band. The statistical distributions of Hubbard parameters are given in Ref. [13]: the standard deviation of $\epsilon_i$ distribution is approximately equal to $\Delta$. We measure the
Hubbard energies and $\Delta$ in units of the atomic recoil energy $E_R = \frac{h^2}{8md^2} = 390 \ k_B \cdot \text{nK}$, where $d = \lambda/2$ is the lattice spacing, and $h$ and $k_B$ are Planck’s and Boltzmann’s constants.

We employ two experimental probes—transport and double occupancy—to capture the impact of disorder on the defining characteristics of the metallic and MI phases. We access these phases by adjusting the number of atoms $N$, $\omega$, and $s$ to control the characteristic density $\tilde{\rho} = N \left( \frac{m\omega^2 d^2}{12t} \right)^{\frac{3}{2}}$ [21]. For $s$ such that $U$ is less than the maximum kinetic energy $12t$ and sufficiently low $\tilde{\rho}$, the gas is in a metallic phase with a smoothly varying density profile, and for $U > 12t$ (corresponding to $s \gtrsim 8 \ E_R$) and $\tilde{\rho} \gtrsim 5$, a MI region of unit filling emerges in the centre of the gas (Fig. 1). We adjust $N$ and $\omega$ to maintain approximately a central unit filling in the clean lattice, resulting in $\tilde{\rho} = 2$–5 for the metallic state and $\tilde{\rho} \approx 25$ for the MI regime (with unit filling across approximately 50% of the gas at zero temperature).

We measure transport as the response of the atomic quasimomentum distribution $n(q)$ to an applied impulse [14]. In the metallic phase, applying an external force induces a centre-of-mass (COM) velocity, which is manifest as an asymmetry in $n(q)$. We observe that the introduction of disorder obstructs transport, leading to an insulating phase. In contrast, in a clean, uniform system at zero temperature, transport does not occur in the MI phase since the band is filled and all available quasimomentum states lie beyond an energy gap. The interaction induced localization causing this insulating behavior leads to suppressed density fluctuations and fewer doubly occupied states compared with a gas at lower interaction strength and the same $\tilde{\rho}$ [15, 22]. We measure that the introduction of disorder to the MI phase produces increased density fluctuations, evident as an increase in the fraction of doubly occupied sites. Throughout this work, we use an atomic limit calculation [15] to interpret measurements (see Supplementary Information).

To measure transport, we apply an external force to the gas by turning on a magnetic field gradient for 2 ms, which is short compared with the confining trap period. The magnitude of the impulse is chosen so that the maximum COM quasimomentum is less than $\hbar \pi/2d$, and thus the sign of the dispersion is positive for the COM velocity. Relative displacement between the spin components during the impulse resulting from a slight magnetic-moment mismatch is less than 0.7 $\mu$m, which is small compared with the typical 5 $\mu$m root mean square (RMS) radius of the gas along the impulse direction. Immediately following the impulse, the lattice is turned off in 200 $\mu$s, and we measure $n(q)$ by bandmapping and absorption imaging after 10 ms time-of-flight (TOF) [23]. The COM velocity $v_{\text{COM}}$ of $n(q)$
is determined by measuring the displacement of the centroid of the imaged density profile from the case without an impulse.

Sample images and data showing \( v_{\text{COM}} \) for a range \( s = 4-7 \ E_R \) (corresponding to \( U/12t \approx 0.20-0.75 \)) and \( \Delta \approx 0-1.5 \ E_R \) are displayed in Fig. 2. At all lattice potential depths, increasing \( \Delta \) causes \( v_{\text{COM}} \) to decrease. Sufficient \( \Delta \) to completely arrest motion, signifying a metal–insulator transition, is achieved for all \( s \). As revealed in the images shown in Fig. 3, disorder has a minor impact on \( n(q) \) before the impulse. The insulating state we observe emerge is therefore qualitatively distinct from a band or Mott insulator—the quasimomentum distribution is narrow and states proximate in quasimomentum to the band edge are unfilled. To quantitatively identify the transition to the insulating state, we measure the characteristic disorder strength \( \Delta_c \), which is the average disorder potential energy necessary to eliminate \( v_{\text{COM}} \) within the experimental resolution \( v_{\text{res}} = 0.05 \ \text{mm/sec} \). The grey band in Fig. 2 shows \( v_{\text{res}} \), which is the standard error of the mean in \( v_{\text{COM}} \) when an impulse is not applied. \( \Delta_c \) is determined from a heuristic fit to the data at fixed \( s \) to

\[
v_{\text{COM}} = Ae^{-\Delta/\Delta_c \log(A/v_{\text{res}})}
\]

with \( A \) and \( \Delta_c \) as free parameters.

The universal behavior for the metal-to-insulator transition is shown in Fig. 3 in terms of the dimensionless quantities \( \Delta_c/12t \) and \( U/12t \). In the metallic regime, \( \Delta_c/12t \) increases with \( U/12t \), implying that interactions shift the metal–insulator transition to higher disorder, effectively stabilizing the metallic phase. This behavior is qualitatively consistent with a statistical dynamical mean-field theory (SDMFT) prediction for this system \[9\] and with a scenario in which interactions create stationary many-particle states that can be Anderson localized in the same fashion as in a non-interacting system \[4, 24, 25\]. By transferring population to higher energy states (evident in the images shown in Fig. 3 for \( s = 4 \) and \( 7 \ E_R \)), interactions may cause \( \Delta_c \) to increase relative to \( 12t \), since more disorder is required to increase the mobility edge and localize all states. The \( \Delta_c/12t \) we measure is approximately a factor of 3–4 smaller compared with the SDMFT prediction for a metal–Anderson-MI transition \[9\]. This discrepancy may be explained by the Bethe lattice geometry used in the SDMFT and the presence of the trap in the experiment. To exclude the observed metal–insulator transition as a percolation transition, in which particles are confined to finite spatial region of energetically accessible sites, we follow the procedure from Ref. \[26\] to estimate the percolation threshold, which is shown in Fig. 3 as a dashed line and exceeds the measured \( \Delta_c/12t \) by an order of magnitude at high \( U/12t \).
To determine the impact of disorder on the MI regime, we measure the fraction of doubly occupied sites $f$ as $\Delta$ is varied at $s = 10 \, E_R$ (corresponding to $U/12t = 2.25$). The density distribution is fixed before measuring $f$ by increasing the lattice potential depth to $s = 32 \, E_R$ over 200 $\mu$s to reduce the tunneling time $h/t$ to 360 ms. Atoms on doubly occupied sites are then expelled by exposing the gas to light 22 MHz detuned from the transition between the $4S_{1/2}, F = 9/2$ and $4P_{3/2}$ electronic states. This light induces binary, light-assisted collisions between atoms that impart kinetic energy much greater than the lattice potential depth, so that pairs of atoms occupying the same lattice site are rapidly lost \[^{27}\] (Supplementary Figures 1 and 2). We infer the fraction of atoms on doubly occupied sites $f$ by measuring the fraction of atoms removed from the lattice after a 0.5 ms exposure to this light (see Supplementary Information).

We observe an increase in the fraction of doubly occupied sites as disorder is added to the clean lattice (Fig. 4). The doubly occupied fraction rises from $f \approx 0.02$ over a range $\Delta = 0.5 \, E_R$, saturating to $f \approx 0.13$ at high $\Delta$. This scale over which $f$ increases is approximately equal to $U$, which suggests that the mechanism behind the increase in double occupancy is disorder in the on-site energies disrupting the Hubbard gap—when $\Delta$ is comparable to $U$, the energetic barrier to double occupancy is removed. As a quantitative check of this model, we compare double occupancy to the prediction from the atomic limit calculation with disorder in $\epsilon$ included. We calculate $f$ at temperatures in the lattice corresponding to a low entropy limit $S/N = (2.0 \pm 0.2) \, k_B$ determined by measuring $T/T_F$ before turning on the lattice and a high entropy limit $S/N = (2.8 \pm 0.3) \, k_B$ inferred from temperature measured after turning the lattice on and slowly off (over 150 ms). We find that the upper limit is constant within 0.4$k_B$ over the range of disorder sampled here. As shown in Fig. 4, the measured fraction agrees quantitatively with the atomic limit calculation. Density profiles generated from the atomic limit calculation (Fig. 4) illustrate how disorder leads to doubly occupied sites emerging in the centre of the gas.

Although disorder seemingly transforms the finite-temperature MI phase into a more compressible state, $f$ at high $\Delta$ does not reach that of a gas at comparable $\tilde{\rho}$ and lower interaction strength. For comparison, a measurement in the metallic regime at $s = 4 \, E_R$ (corresponding to $U = 0.1 \, E_R$) and similar $\tilde{\rho}$ yields $f = 0.30 \pm 0.02$ (Fig. 4). The higher doubly occupied fraction at this lower interaction strength is due to the formation of a band insulator in the centre of the trap. The failure of the disordered MI to reach an $f$ similar
to that achieved in the metallic regime may be related to the disorder-induced appearance of empty sites that possess high occupation energies, which are apparent in the calculated density profiles shown in Fig. 4.

The complete nature of the disordered MI state remains unresolved. SDMFT for this system predicts that disorder transforms the MI phase into a disordered correlated metal [9]. Probing such changes in transport properties using the impulse method is complicated in the MI regime by the coexisting metallic shell and localized single-particle states [23, 28] that are occupied when $k_B T_F > 12t$. Alternatively, methods that use a chemical potential imbalance imposed across a channel may be employed in the future to explore transport in the MI regime [29] and to probe if disorder leads to non-Fermi-liquid behavior in the metallic regime [1].

**Methods Summary**

We create ultracold gases of $^{40}$K atoms cooled to temperatures below $T_F$ in a crossed-beam 1064 nm optical dipole trap using standard techniques. For the transport measurement, we used $(44 \pm 1) \times 10^3$ atoms cooled to $(0.16 \pm 0.01) T_F$ in the dipole trap. For the double occupancy data at $s = 10 E_R$, $(80 \pm 10) \times 10^3$ atoms were cooled to $(0.22 \pm 0.03) T_F$, and for the double occupancy data at $s = 4 E_R$, $(123 \pm 8) \times 10^3$ atoms were cooled to $(0.16 \pm 0.02) T_F$. Imaging artifacts such as interference fringes are removed using a background image basis decomposition [30] for all absorption images. The geometric mean $\omega$ of the harmonic trap frequencies is varied across $2\pi \times (95–120)$ Hz and is determined by the dipole trap optical power and a contribution from the profile of the lattice laser beams. The ratio of the overall vertical to horizontal trap frequencies is approximately fixed at 1.6.

The lattice and optical speckle light is superimposed on the atoms over 150 ms, keeping the ratio $\Delta/s$ fixed. The average speckle field intensity is modulated by a Gaussian envelope with a $170 \mu$m $1/e^2$ radius and is uniform to within a few percent over the gas. The lattice potential depth is calibrated to within 10% using lattice modulation, and $\Delta$ is calibrated to within 10% statistical uncertainty by measuring the dipole force from the speckle envelope. There is a systematic uncertainty in $\Delta$ related to measuring the speckle envelope that is less than 40%. Since the speckle field propagates along a non-lattice direction, resulting in a speckle correlation length along all lattice axes comparable to $d$, the Hubbard energies are weakly correlated between adjacent sites and are nearly isotropic in space [13]. The intensity of the speckle is approximately Gaussian correlated, with 310 nm and 1600 nm...
RMS autocorrelation lengths along the transverse and propagation directions, respectively.

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FIG. 1. **Experimental geometry and lattice phases.** The atoms are cooled in a magnetic trap (copper) and an optical dipole trap formed from 1064 nm laser beams (grey lines). Optical lattice laser beams (red) superimposed on the trap form a cubic lattice potential. In the combined parabolic–lattice potential, a metallic phase is present at low lattice potential depth, and a MI phase emerges in the centre of the trap (at zero temperature) as the lattice laser intensity is increased. A 532 nm optical speckle field (green) is focused onto the atoms using a 0.9 f-number lens (blue hemisphere). Atoms in two hyperfine states (red and blue spheres) are trapped in the disordered lattice potential (false colour) formed at the intersection of all the laser beams.
FIG. 2. Impulse data. The COM velocity of the atom gas measured after an applied impulse is shown for $s = 4$ (blue squares), 5 (red circles), 6 (green triangles), and 7 $E_R$ (orange diamonds). Sample images used to determine $v_{\text{COM}}$ are shown in false colour for $s = 4$ $E_R$ at $\Delta = 0$ $E_R$ (a) and $\Delta = 1.46$ $E_R$ (b). The field-of-view for all absorption images used in this work is 0.54 mm. The projection of the Brillouin zone onto the imaging plane is indicated using solid black lines. The blue dotted line is the exponential fit used to determine $\Delta_c$ for $s = 4$ $E_R$; the arrow indicates $\Delta_c$ for $s = 4$ $E_R$. The error bars are the standard error in the mean for the 7–9 experimental runs that are averaged for each data point.
FIG. 3. Critical disorder for metal–insulator transition. The critical disorder strength $\Delta_c/12t$ is shown for varying interaction strength $U/12t$, which is controlled by tuning the lattice potential depth $s$. The data are inconsistent with the predicted percolation threshold, which is shown as a dashed line. Representative images at $s = 4\ E_R$, $\Delta = 0\ E_R$ (a) and $\Delta = 1.46\ E_R$ (b), and at $s = 7\ E_R$, $\Delta = 0\ E_R$ (c) and $\Delta = 1.46\ E_R$ (d) illustrate that disorder does not strongly affect the quasimomentum distribution, which broadens at higher interaction strengths. The error bars show the uncertainty in the fit to the data in Fig. 2 used to determine $\Delta_c$. 
**FIG. 4. Double occupancy.** Double occupancy $f$ measured for varied $\Delta$ at $s = 10 \, E_R$ (black circles) and for $\Delta = 0$ and $\tilde{\rho} = 34$ at $s = 4 \, E_R$ (red square). Atomic limit predictions are shown for the measured low (dashed line) and high (dotted line) entropy limits. The grey band is the standard error in the mean for $f$ for a spin polarized gas, for which double occupancy is disallowed. The error bars show the standard error in the mean for the 11–15 experimental runs averaged for each data point. The false-colour images are a two-dimensional slice (with a 63 $\mu$m field-of-view) through the in-trap density profile predicted by the atomic limit calculation for $S/N = 2.4 \, k_B$ and $\Delta = 0 \, E_R$ (a), $0.5 \, E_R$ (b), and $1 \, E_R$ (c). Empty sites are shown in white, and sites with unit and greater than unit filling are shown in red and black, respectively.