NoisyMix: Boosting Model Robustness to Common Corruptions

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Abstract

The robustness of neural networks has become increasingly important in real-world applications where stable and reliable performance is valued over simply achieving high predictive accuracy. To address this, data augmentation techniques have been shown to improve robustness against input perturbations and domain shifts. In this paper, we propose a new training scheme called NoisyMix that leverages noisy augmentations in both input and feature space to improve model robustness and in-domain accuracy. We demonstrate the effectiveness of NoisyMix on several benchmark datasets, including ImageNet-C, ImageNet-R, and ImageNet-P. Additionally, we provide theoretical analysis to better understand the implicit regularization and robustness properties of NoisyMix.

1 INTRODUCTION

While deep learning has achieved remarkable results in computer vision and natural language processing, much of the success is driven by improving test accuracy. However, it is well-known that deep learning models are typically brittle and sensitive in noisy and adversarial environments. This limits their application in many real-world settings which require, at a minimum, models that are stable to small perturbations of the input data. Structural stability is particularly important when compression techniques (e.g., quantization) or different hardware systems introduce small errors to the model’s weights. Input stability is important when models (e.g., a vision model in a self-driving car) should operate reliably in noisy environments. Overall, robustness is an important topic within machine learning and deep learning, but the many facets of robustness prohibit the use of a single metric for measuring it (Hendrycks et al., 2021).

In this work, we are specifically interested in studying input stability (robustness) with respect to common data corruptions and domain shifts that naturally occur in many real-world settings. To do so, we use datasets such as ImageNet-C (Hendrycks and Dietterich, 2018) and ImageNet-R (Hendrycks et al., 2021). ImageNet-C provides examples to test the robustness of a model with respect to common corruptions, including noise corruptions like Gaussian noise, motion blur, and brightness changes. Figure 1 shows the test accuracy of different data augmentation techniques on ImageNet-C and ImageNet-R datasets.

Figure 1: ResNet-50 models trained with data augmentations. NoisyMix improves the test accuracy on ImageNet-C and ImageNet-R, indicating improved robustness to common corruptions and domain shifts.

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Data augmentation methods such as Mixup (Zhang et al., 2018), AutoAugment (Cubuk et al., 2019), training on stylized ImageNet (Geirhos et al., 2018), and AugMix (Hendrycks et al., 2020) have the ability to improve the resilience to various data perturbations and domain shifts. Figure 1 shows the test accuracies on ImageNet-C, and ImageNet-R for ResNet-50 models trained with different augmentation schemes on ImageNet (see Sec. 4). Clearly, data augmentation helps to achieve better test accuracies on these corrupted and domain-shifted test sets.

Among these, our proposed method, NoisyMix, is the most effective in improving robustness. In contrast, adversarial training (Shafahi et al., 2019a) decreases robustness to common corruptions. The reason for this is that while adversarial training helps to improve robustness to perturbations that are manifested in the high-frequency domain, it reduces robustness to common corruptions that are often manifested in the low-frequency domain (Yin et al., 2019).

Contributions. Our main contributions in this paper are summarized as follows.

- We formulate and study a novel, theoretically justified training scheme, NoisyMix, that substantially outperforms standard stability training and data augmentation schemes in improving robustness to common corruptions and in-domain test accuracy.
- We study NoisyMix via the lens of implicit regularization and provide theoretical results that identify the effects (see Theorem 1-2 in App. B). Importantly, we show that, under assumptions similar to those made in Lamb et al. (2019); Lim et al. (2022), minimizing the NoisyMix loss corresponds to minimizing an upper bound on the sum of an adversarial loss, a stability objective, and data-dependent regularizers (see Theorem 3 in App. C). This suggests that NoisyMix provides an inductive bias towards improving model robustness with respect to common corruptions, when compared to standard training.
- We provide empirical results to show that NoisyMix improves the robust accuracy on ImageNet-C and ImageNet-R by over 4% compared to AugMix, while achieving comparable accuracy on the clean ImageNet validation set. Moreover, NoisyMix has comparable training costs to AugMix. We also show that NoisyMix achieves state-of-the-art performance on CIFAR-10-C and CIFAR-100-C, using preactivated ResNet-18 and Wide-ResNet-28x2 architectures. Our empirical results further suggest that the improved effects of NoisyMix come from implicit regularization. To understand this better, we illustrate these effects via toy experiments (see App. A).

Notation. \([K] := \{1, \ldots, K\}\), \(\odot\) denotes Hadamard product, \(\mathds{1}\) is the vector with all elements equal one. For a vector \(v\), \(\|v\|_p\) denotes its \(l_p\) norm for \(p > 0\).\(M_\lambda(a, b) := \lambda a + (1 - \lambda)b\), for random variables \(a, b, \lambda\).

2 RELATED WORK

The stability of modeling methods for real-world phenomena has a long been studied and foundational to statistics, numerical analysis, and dynamical systems (Yu, 2013). The field of robust statistics is concerned with perturbations to model parameters (Huber, 2004), while other branches of statistics are concerned with the perturbation at the input level (Efron, 1992). The importance of developing robust methods for scientific discovery is at hand, and thus it is no surprise that robustness is an emerging research area within the deep learning community as well. In this work, we are mainly concerned with improving input stability of deep neural networks for computer vision tasks. In particular, there are two types of input perturbations that have recently led to a surge of interest: (i) adversarial perturbations, and (ii) common corruptions. The class of adversarial perturbations consists of artificially crafted perturbations specifically designed to fool neural networks while being imperceptible for the human eye (Szegedy et al., 2013; Goodfellow et al., 2014). There has been a tremendous effort on designing attack and defense methods (see Chakraborty et al. (2018) for a recent overview), and an interesting result of this line of work is that adversarially-trained models (Madry et al., 2018) learn features that are more aligned with human perception (Ilyas et al., 2019). However, it can be argued that adversarial perturbation are a hypothetical thread in real-world applications. Moreover, adversarial robust models turn out to be less accurate on clean data as well as on common corruptions (Yin et al., 2019; Rusak et al., 2020). Common corruptions are a class of perturbations that naturally occur in real-world applications (Hendrycks and Dietterich, 2018), and are often more relevant in real-world science and engineering applications than adversarial perturbations (Rusak et al., 2020).

Early work by Dodge and Karam (2016) has revealed...
how sensitive neural networks (trained on high-quality images) are to slight degradation of image quality and common corruptions. Hendrycks and Dietterich (2018) introduced a more extensive benchmark dataset for testing robustness on a comprehensive set of common corruptions with varying intensity levels. Subsequently, it has been shown that data augmentation and noise injection schemes can help to improve robustness (Yin et al., 2019) to common corruptions. In contrast, Taori et al. (2020) criticize that most work on robustness focuses on synthetic image perturbations and they instead suggest that it is required to train models on larger and more diverse datasets in order to improve robustness to distribution shifts that are arising from natural variations in datasets. As an alternative, Hendrycks et al. (2019) and Xie et al. (2020) showed that pretraining on larger and more diverse datasets (e.g., JFT-300M (Hinton et al., 2015)) can help to improve robustness, but Hendrycks et al. (2021) showed that these strategies do not consistently do better if a broader range of distribution shifts is considered. The disadvantages of pretraining on larger datasets are the increased computational costs and the limited availability of massive computer vision datasets. Hence, we limit our discussion in this paper to data augmentation and noise injection strategies that can be applied to standard architectures trained on common computer vision benchmark datasets.

Most relevant for our work are data augmentation methods that aim to reduce the generalization error and improve robustness by introducing implicit regularization (Kukavčka et al., 2017) into the training process. By data augmentation, we refer to schemes that do not introduce new data but train a model with virtual training examples (e.g., proper transformations of the original data) in the vicinity of a given training dataset. Basic examples are random cropping and horizontal flipping of input images (Krizhevsky et al., 2012).

Mixup (Zhang et al., 2018) is a popular data augmentation method, which creates new training examples by forming linear interpolations between random pairs of examples and their corresponding labels. It is a simple but extremely efficient scheme that can improve the generalization and robustness of models with minimal computational overhead. There are added advantages in not introducing additional overhead during training or requiring modifications on the network architecture. Various other proposed methods have since been inspired by Mixup. Manifold Mixup (Verma et al., 2019) extends the idea of interpolating between data points in input space to hidden representations, leading to models that have smoother decision boundaries. However, our experiments show that the advantages compared to Mixup are mixed. Noisy Feature Mixup (NFM) (Lim et al., 2022) generalizes the idea of both Mixup and Manifold Mixup by taking linear combinations of pairs of perturbed input and hidden representations. Another approach involves zeroing out parts of an input image (DeVries and Taylor, 2017) to prevent a model from focusing on a limited image region. CutMix (Yun et al., 2019) extends this idea by replacing the removed parts with a patch from another image. This improves the test accuracy, but it does not help much to improve the robustness to common corruptions. PuzzleMix (Kim et al., 2020) is a mixup method that further improves CutMix by leveraging saliency information and local statistics of the original input data to decide how to mix two data points.

Geirhos et al. (2018) proposed to train models on a stylized version of ImageNet, which helps to increase shape bias and in turn improves robustness to common corruptions. To do so, they use a style transfer network that applies artwork styles to images that are then used to train the model. Similar good results are achieved by AutoAugment (Cubuk et al., 2019), a method that automatically searches for improved data augmentation policies. AugMix (Hendrycks et al., 2020) is designed to improve robustness to common corruptions, which is achieved by leveraging a diverse set of basic data augmentations (e.g., translate, posterize, solarize) and applying consistency regularization (using the Jensen-Shannon divergence loss). This approach improves robustness when compared to the previously discussed data augmentation methods. Recently, Wang et al. (2021) proposed AugMax, a strong data augmentation scheme, that learns an adversarial mixture of several sampled augmentation operators.

Noise injections are another form of data augmentations, which introduce regularization into the training process (Bishop, 1995). Most commonly, the models are trained with noise perturbed inputs (An, 1996). Noise can also be injected into the activation functions (Gulcehre et al., 2016), or the hidden layers of a neural network (Camuto et al., 2020; Lim et al., 2022). Recently, Rusak et al. (2020) demonstrated that training a model with various noise types helps to improve robustness to common corruptions.

3 METHOD

At a high level, NoisyMix implements feature mixup and noise injections (Lim et al., 2022) on top of a stability training scheme (Zheng et al., 2016) on an augmented data set (Hendrycks et al., 2020) in a single setup to boost both accuracy and robustness. While components underlying NoisyMix exist in the literature, the devil is in the details to get the method right, so we motivate and describe the procedures of NoisyMix.
for multi-class classification in the following.

First, we augment the clean data with appropriately transformed data, obtained by applying the operations used in AugMix to the clean data. The transformed data are diverse because the operations are sampled stochastically and then layered on the same input image. Such diverse transformations are crucial for enabling model robustness by discouraging the model from memorizing fixed augmentations (Hendrycks et al., 2020).

To improve the robustness further, we judiciously combine the stability training scheme with mixup and noise injections at the level of input and hidden layers. The latter approach, NFM (Lim et al., 2022), generalizes input mixup (Zhang et al., 2018) and Manifold Mixup (Verma et al., 2019) in the sense that noise is injected when taking convex combinations of pairs of input and hidden layer features. It has been shown that this makes the decision boundary smoother and helps improving robustness when predicting on out-of-distribution data (Lim et al., 2022).

Applying the procedures of NFM on top of AugMix, and using stability training, leads to our proposed method, NoisyMix, which we describe in detail next.

3.1 Detailed Formulation of NoisyMix

We consider multi-class classification with $K$ labels. Denote the input space by $\mathcal{X} \subset \mathbb{R}^d$ and the output space by $\mathcal{Y} := \Delta^{K-1}$, where $\Delta^{K-1}$ is the probability simplex, i.e., a categorical distribution over $K$ classes. The classifier, $g$, is constructed from a learnable deep neural network map $p : \mathcal{X} \rightarrow \Delta^{K-1}$, mapping an input $x$ to its label, $g(x) = \text{arg max}_k p_k(x) \in [K]$. We are given a training set, $Z_n := \{(x_i, y_i)\}_{i=1}^n \cup \{(A(x_i), y_i)\}_{i=1}^n$, consisting of $2n$ pairs of input and one-hot label, with each training pair $z_i := (x_i, y_i), z_i' := (A(x_i), y_i) \in \mathcal{X} \times \mathcal{Y}$. The training pairs $(x_i, y_i)$ are drawn independently from a ground-truth distribution $D := D_x \times D_y$. Here, $A$ denotes the random perturbation of an input, i.e., the AugmentAndMix function in Hendrycks et al. (2020). In the sequel, we denote $A(x)$ as the underlying conditional (on $x$) distribution from which $A(x)$ is sampled.

More precisely, for an (original) input $x$, we construct a virtual data point $x_{am}$ as

$$x_{am} = A(x) := mx + (1-m) \sum_{i=1}^3 w_i C(x) \sim A(x),$$

where $m \sim \text{Beta}(\alpha, \alpha)$, the $w_i \sim \text{Dirichlet}(\alpha, \ldots, \alpha)$ (with $\alpha := 1$), and $C$ uniformly drawn from a set of operations whose elements including compositions of transformations such as rotating, translating, auto-contrast, equalizing, posterizing, solarizing, and shearing. They can be viewed as randomly perturbed input data, i.e., $A(x) = x + \delta x$, where the random perturbation $\delta x := (1-m) \left( \sum_{i=1}^3 w_i C(x) - x \right)$ is additive and data-dependent. The transformed data are highly diverse because the operations are sampled stochastically and then layered on the same input image. Such diverse transformations are crucial for enabling model robustness by discouraging the model from memorizing fixed augmentations (Hendrycks et al., 2020).

Within this setting, we propose the NoisyMix loss:

$$L_{\text{NoisyMix}} = L_{\text{NFM}} + \gamma L_{\text{stability}},$$

where $\gamma \geq 0$. This loss is a sum of two components: the expected NFM loss (Lim et al., 2022) and an expected stability loss (Zheng et al., 2016). As in (Lim et al., 2022), the expected NFM loss is given by:

$$L_{\text{NFM}} = E_{(x,y),(x',y') \sim \mathcal{D}} E_{\xi \sim \text{Beta}(\alpha,\beta)} [l(p(M_{\lambda,\xi}(x, x')), M_{\lambda}(y, y'))],$$

for some loss function (e.g., cross-entropy) $l : \Delta^{K-1} \times \Delta^{K-1} \rightarrow [0, \infty)$ (note the dependence of both $l$ and $p$ on the learnable parameter $\theta$ is suppressed in the notation), $\xi := (\xi_{\text{mult}}, \xi_{\text{add}})$ are drawn from some probability distribution $\mathcal{Q}$ with finite first two moments, and

$$M_{\lambda,\xi}(x, x') = (1 + \sigma_1 \xi_{\text{mult}}) \odot M_{\lambda}(x, x') + \sigma_2 \xi_{\text{add}},$$

where $\sigma_1$ and $\sigma_2$ are tuning parameters. In other words, this approach combines stability training with mixup and noise injections at the level of input and hidden layers. In App. A we illustrate that the combination of these techniques introduce implicit regularization that makes the decision boundary smoother. In turn, this helps improving robustness when predicting on out-of-distribution data.

We choose the distance measure $D$ in the expected stability loss to be the Jensen-Shannon divergence (JSD) and minimize the deviation between the distribution on clean input data and that on the transformed input data, i.e., we minimize $L_{\text{stability}} := L_{\text{JSD}}$, given by:

$$E_{x,x' \sim D_x} E_{x_{am} \sim A(x), x'_{am} \sim A(x')} E_{\xi \sim \text{Beta}(\alpha,\beta)} [JSD(p(M_{\lambda,\xi}(x, x')), p(M_{\lambda,\xi}(x_{am}, x'_{am})))],$$

where $JSD$ denotes Jensen-Shannon divergence with the weights $\pi$, defined as follows: for $p_1, p_2 \in \Delta^{K-1}$ with the corresponding weights $\pi := (\pi_1, \pi_2) \in \Delta$, the JSD (Lin, 1991) between $p_1$ and $p_2$ is

$$JSD(p_1, p_2) := H(\pi_1 p_1 + \pi_2 p_2) - \pi_1 H(p_1) - \pi_2 H(p_2),$$

with $H$ the Shannon entropy, defined as $H(p) := - \sum_{k=1}^K p_k \ln(p_k)$ for $p \in \Delta^{K-1}$, Alternatively, $JSD(p_1, p_2) = \pi_1 KL(p_1 || p_1 + p_2) +$
\[ \pi_2 KL(p_2 \| \pi_1 p_1 + \pi_2 p_2) \], where \( KL \) denotes the Kullback–Leibler divergence.

JSD measures the similarity between two probability distributions, and its square root is a true metric between distributions. Unlike the KL divergence and cross-entropy, it is symmetric, bounded, and does not require absolute continuity of the involved probability distributions. Similar to the KL divergence, \( JSD(p_1, p_2) \geq 0 \), with equality if and only if \( p_1 = p_2 \). This follows from applying Jensen’s inequality to the concave Shannon entropy. Moreover, JSD can interpolate between cross-entropy and mean absolute error for \( \pi_1 \in (0, 1) \) (see Proposition 1 in Englesson and Azizpour (2021)). In GANs, JSD is used as a measure to quantify the similarity between the generative data distribution and the real data distribution (Goodfellow et al., 2020; Weng, 2019). We refer to Lin (1991); Englesson and Azizpour (2021) for more properties of JSD. Note that, unlike \( L^{NFM} \), \( L^{JSD} \) is computed without the use of the labels.

### A Note on Stability Training

We consider a stability training scheme on the augmented training data (Zheng et al., 2016). Stability training stabilizes the output of a model against small perturbations to input data, thereby enforcing robustness. Since the classifiers are probabilistic in nature, we consider a probabilistic notion of robustness, which amounts to making them locally Lipschitz with respect to some distance on the input and output space. This ensures that a small perturbation in the input will not lead to large changes (as measured by some statistical distance) in the output.

We now formalize a notion of robustness. Let \( p > 0 \). We say that a model \( f : \mathcal{X} \to \mathcal{Y} \) is \( \alpha_p \)-robust if for any \( (x, y) \sim \mathcal{D} \) such that \( f(x) = y \), one has, for any data perturbation \( \delta x \in \mathcal{X} \),

\[ \|\delta x\|_p \leq \alpha_p \Rightarrow f(x) = f(x + \delta x). \]

An analogous definition can be given for distribution-valued outputs, which leads to a notion of probabilistic robustness. Let \( D \) be a metric or divergence between two probability distributions. We say that a model \( F : \mathcal{X} \to P(\mathcal{Y}) \), i.e., the space of probability distributions on \( \mathcal{Y} \), is \( (\alpha_p, \epsilon) \)-robust with respect to \( D \) if, for any \( x, \delta x \in \mathcal{X} \),

\[ \|\delta x\|_p \leq \alpha_p \Rightarrow D(F(x), F(x + \delta x)) \leq \epsilon. \]

In practice, stability training involves formulating a loss that aims to flatten the model output in a small neighborhood of any input data, forcing the output to be similar between the original and perturbed copy. For instance, AugMix incorporates such stability training in the following manner. In contrast to the standard training scheme, one does not simply train with the cross-entropy loss, but instead train with a sum of the cross-entropy (on the clean data) and a suitable stability objective. This encourages model prediction to be more constant around the input data while mitigating underfitting. This is done by combining the cross-entropy loss with a suitable stability objective, encouraging model prediction to be more constant around the input data while mitigating underfitting. However, stability training increases the computational costs when compared to training with simply the standard cross-entropy loss.

#### 3.2 Theoretical Results

**NoisyMix** seeks to minimize a stochastic approximation of \( L^{NoisyMix}_n \) by sampling a finite number of \( \lambda, \xi, x_{am}, x'_{am} \) values and using minibatch gradient descent to minimize this loss approximation. The empirical loss to be minimized during the training is

\[ L^{NoisyMix}_n = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \mathbb{E}_{\xi \sim \text{Beta}(\alpha, \beta)} \mathbb{E}_{\xi' \sim Q} [a_{i,j} + \gamma b_{i,j}], \]

where

\[ a_{i,j} := l(p(M_{\lambda, \xi}(x_i, x_j)), M_{\lambda}(y_i, y_j)), \]

\[ b_{i,j} := JS_\pi(p(M_{\lambda, \xi}(x_i, x_j)), p(M_{\lambda, A}(x_i, A(x_j)))). \]

In App. A, we illustrate the effectiveness of **NoisyMix** in smoothing the decision boundary for a binary classification task. Overall, we observe that **NoisyMix** leads to the smoothest decision boundary when compared to other methods (see Fig. 3), and thus more robust models, since the predicted label stays the same even if the data are perturbed.

To understand these regularizing effects of **NoisyMix** better, we provide some mathematical analysis via the lens of implicit regularization (Mahoney, 2012) in App. B (see Theorem 1 and Theorem 2). Such a lens was adopted to understand stochastic optimization (Ali et al., 2020; Smith et al., 2021) and regularizing effects of noise injections (Camuto et al., 2020; Lim et al., 2021; Gong et al., 2021). One common approach to study implicit regularization is to approximate it by an appropriate explicit regularizer, which we follow here for the regime of small mixing coefficient and noise levels. In this regime, we shall show that **NoisyMix** can lead to models with enhanced robustness and stability.

Following Lamb et al. (2019); Lim et al. (2022), we consider the binary cross-entropy loss, setting \( h(z) = \log(1 + e^z) \), with the labels \( y \) taking value in \( \{0, 1\} \) and the classifier model \( f: \mathbb{R}^d \to \mathbb{R} \). In the following, we assume that the model parameter \( \theta \in \Theta := \{\theta : y_i f(x_i) + (y_i - 1) f(x_i) \geq 0 \text{ for all } i \in [n]\} \). We remark
that this set contains the set of all parameters with correct classifications of training samples (before applying NoisyMix), since \( \{ \theta : 1_{f(x_i) \geq 0} = y_i \text{ for all } i \in [n] \} \subset \Theta \). Therefore, the condition of \( \theta \in \Theta \) is fulfilled when the model classifies all labels correctly for the training data before applying NoisyMix. Since the training error often becomes zero in finite time in practice, we shall study the effect of NoisyMix on model robustness in the regime of \( \theta \in \Theta \).

Working in the data-dependent parameter space \( \Theta \), we obtain the following result, which is an informal version of our main theoretical result (see Theorem 3 in App. C for the precise statements).

**Theorem** (Informal). Let \( \epsilon > 0 \) be a small parameter, and rescale \( 1 - \lambda \rightarrow \epsilon(1 - \lambda) \), \( \sigma_{\text{add}} \rightarrow \epsilon\sigma_{\text{add}} \), \( \sigma_{\text{mult}} \rightarrow \epsilon\sigma_{\text{mult}} \). Then, under additional reasonable assumptions,

\[
I_n^{\text{NoisyMix}} \geq \frac{1}{n} \sum_{i=1}^{n} \max_{\|\delta_i\| \leq \epsilon_{\text{mix}}} l(f(x_i + \delta_i), y_i) + \gamma \hat{L}_n^{\text{JSD}} + c^2 L_n^{\text{reg}} + c^2 \phi(\epsilon),
\]

for some data-dependent radii \( \epsilon_{\text{mix}} > 0 \), stability objective \( \hat{L}_n^{\text{JSD}} \), data-dependent regularizer \( L_n^{\text{reg}} \), and function \( \phi \) such that \( \lim_{\epsilon \to 0} \phi(\epsilon) = 0 \).

This theorem implies that minimizing the NoisyMix loss results in a small regularized adversarial loss and a stable model. Therefore, training with NoisyMix not only enhances robustness with respect to input perturbations, but it also imposes additional smoothness.

Lastly, we remark that one could generalize the JSD (designed for only two distributions) to provide a similarity measure for any finite number of distributions. This leads to the following definition (Lin, 1991):

\[
JS_{\pi}(p_1, \ldots, p_K) := \sum_{k=1}^{K} \pi_k KL\left(p_k \left\| \sum_{j=1}^{K} \pi_j p_j \right\right)
\]

where \( p_1, \ldots, p_K \) are \( K \) probability distributions with weights \( \pi := (\pi_1, \ldots, \pi_K) \). We note that such a measure, with \( K := 3 \) and the \( \pi_i := 1/3 \), will be used in our experiments and was also used in Hendrycks et al. (2020) for obtaining optimal results.

### 4 EXPERIMENTAL RESULTS

In this section, we compare NoisyMix to other data augmentation schemes, studying in particular the robustness with respect to various common corruptions and data shifts. We will first discuss the experimental details, and then present results for ImageNet, followed by results for smaller architectures trained on CIFAR-10 and CIFAR-100. Finally, we will present a detailed ablation study, and investigate whether robustness transfers to downstream tasks.

Research code is provided on our GitHub repository: https://github.com/erichson/NoisyMix.

#### 4.1 Experiment Details

**Datasets.** We center our experiments around several datasets to benchmark the robustness of neural networks to common corruptions and perturbations.

The ImageNet-C, CIFAR-10-C, and CIFAR-100-C datasets (Hendrycks and Dietterich, 2018) provide corrupted versions of the original ImageNet (Deng et al., 2009) and CIFAR (Krizhevsky, 2009) test sets. Specifically, each dataset emulates noise, blur, weather, and digital distortions. In total, there are 15 different perturbation and corruption types, and for each type, there are sets with 5 severity levels which allows us to study the performance with respect to increasing data shifts. Figure 5 in the App. illustrates the different corruption types. The average accuracy across all corruption types and severity levels provides a score for robustness.

We also consider the ImageNet-R dataset (Hendrycks et al., 2021), which is designed to measure the robustness of models to various abstract visual renditions (e.g., graffiti, origami, cartoons). This dataset provides 30,000 image renditions for a subset of 200 ImageNet object classes. The collected renditions have textures and local image statistics that are distinct from the standard ImageNet examples. This dataset provides a test for robustness to naturally occurring domain shifts, and it is complementary to ImageNet-C, which is used to test robustness to synthetic domain shifts.

In addition, we consider the ImageNet-P dataset (Hendrycks and Dietterich, 2018), which provides 10 different perturbed sequences for each ImageNet validation image. Each sequence contains 30 or more frames, where each subsequent frame is a perturbed version of the previous frame. The applied perturbations are relatively subtle, ensuring that the frames do not go too far out-of-distribution. This dataset is specifically designed to evaluate the prediction stability of a model \( f \), using the flip probability as a metric to evaluate stability. Specifically, the flip probability is computed as \( F_{\text{Bp}} = \frac{1}{m(n-1)} \sum_{i=1}^{m} \sum_{j=2}^{n} I(f(x_i) \neq f(x_j^{(i)})) \), for given \( m \) sequences \( \{(x_i^{(i)}, \ldots, x_n^{(i)})\}_{i=1}^{m} \) of length \( n \) for a corruption type \( p \). Here \( I \) denotes an indicator function. Unstable models are characterized by a high flip probability, i.e., the prediction behavior across consecutive frames is erratic, whereas consistent predictions indicate stable statistical predictions. For sequences that are constructed with noise
Table 1: Clean test accuracy of ResNet-50 models on ImageNet, and average robust accuracy on ImageNet-C and ImageNet-R (higher values are better). In addition, we show the mean flip rate for ImageNet-P (lower values are better) and the robustness to adversarial examples constructed with FGSM (higher values are better). The values in parenthesis are the average robust accuracy for ImageNet-C and mean flip probability for ImageNet-P excluding noise perturbations.

|                    | ImageNet (%) | ImageNet-C (%) | ImageNet-R (%) | ImageNet-P (%) | FGSM (%) |
|--------------------|--------------|----------------|----------------|----------------|----------|
| Baseline           | 76.1         | 39.2 (42.3)    | 36.2           | 58.0 (57.8)    | 6.6      |
| Adversarial Trained| 63.9         | 32.1 (35.4)    | 38.9           | 33.3 (33.4)    | 43.1     |
| Stylized ImageNet  | 74.9         | 45.2 (46.6)    | 41.5           | 54.4 (55.2)    | 7.8      |
| AutoAugment        | 77.6         | 45.7 (47.3)    | 39.0           | 56.5 (57.7)    | 9.9      |
| Mixup              | 77.5         | 46.2 (48.4)    | 39.6           | 56.4 (58.7)    | 23.5     |
| Manifold Mixup     | 76.7         | 43.9 (46.5)    | 39.7           | 56.0 (58.2)    | 27.7     |
| Puzzle Mix         | 78.7         | 44.6 (46.4)    | 39.5           | 55.5 (57.0)    | 28.4     |
| AugMix             | 77.5         | 48.3 (50.5)    | 41.0           | 37.6 (37.2)    | 9.9      |
| NoisyMix (ours)    | 77.6         | **52.3 (52.4)**| **45.7**       | **28.5 (29.7)**| 29.6     |

The advantage of NoisyMix is also pronounced when clean data as compared to the baseline model. However, it is less accurate than models trained with augmentation schemes such as PuzzleMix, and CutMix. In contrast, adversarial training and style transfer reduce the in-distribution test accuracy.

On ImageNet-C, NoisyMix gains about 13% compared to the baseline model and about 4% compared to AugMix. Since NoisyMix uses white noise during training, we also show the average robust accuracy in parentheses, excluding any noise perturbations (i.e., excluding white, shot and impulse noise). Table 6 in the App. provides results for each perturbation type. Further, Figure 2 shows the robust accuracy as a function of the severity level of the corruptions, and it can be seen that the advantage of our model is more pronounced with increasing severity levels.

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Table 2: Results for ResNet-18 models trained on CIFAR-10/100 and evaluated on CIFAR-10/100-C. We train all models with 5 different seeds and show the mean and (minimum/maximum) test accuracy. (Note, the results for AugMax, indicated by a *, are adapted from Wang et al. (2021.).)

| Method            | CIFAR-10 (%) | CIFAR-10-C (%) | CIFAR-100 (%) | CIFAR-100-C (%) |
|-------------------|--------------|----------------|--------------|----------------|
| Baseline          | 95.3 (95.2 / 95.4) | 73.8 (73.5 / 74.2) | 77.7 (77.5 / 78.1) | 48.4 (48.0 / 48.8) |
| Mixup             | 95.8 (95.3 / 96.1) | 80.4 (79.7 / 80.9) | 79.7 (79.5 / 79.9) | 54.2 (53.9 / 54.8) |
| Manifold Mixup    | 95.9 (95.7 / 96.1) | 77.7 (77.3 / 78.2) | 80.3 (79.8 / 80.7) | 53.4 (52.9 / 53.6) |
| CutMix            | **96.5 (96.4 / 96.6)** | 73.0 (71.8 / 73.9) | 79.3 (78.8 / 80.0) | 47.9 (47.6 / 48.3) |
| Puzzle Mix        | **96.5 (96.3 / 96.6)** | 84.1 (83.7 / 84.3) | 79.4 (79.1 / 79.6) | 57.4 (56.9 / 57.8) |
| NFM               | 95.4 (95.2 / 95.6) | 83.3 (82.7 / 83.9) | 79.4 (78.9 / 79.8) | 59.7 (59.2 / 60.2) |
| AugMix            | 95.5 (95.0 / 95.8) | 88.8 (88.1 / 89.3) | 77.0 (76.6 / 77.3) | 64.6 (64.4 / 64.8) |
| AugMax*           | - | 90.36 | - | 65.75 |
| NoisyMix (ours)   | 95.3 (95.2 / 95.3) | **91.2 (91.1 / 91.3)** | 79.7 (79.6 / 79.9) | **71.0 (70.3 / 71.3)** |

Table 3: Results for Wide-ResNet-28x2 trained on CIFAR-10/100 and evaluated on CIFAR-10/100-C. We train all models with 5 different seeds and show the mean and (minimum/maximum) test accuracy.

| Method            | CIFAR-10 (%) | CIFAR-10-C (%) | CIFAR-100 (%) | CIFAR-100-C (%) |
|-------------------|--------------|----------------|--------------|----------------|
| Baseline          | 96.0 (95.9 / 96.1) | 74.9 (74.5 / 75.4) | 79.0 (78.8 / 79.1) | 47.4 (47.0 / 48.0) |
| Mixup             | 96.7 (96.6 / 96.8) | 80.8 (80.7 / 81.0) | 80.9 (80.8 / 81.0) | 53.9 (53.1 / 54.3) |
| Manifold Mixup    | 96.6 (96.5 / 96.7) | 77.7 (76.3 / 78.4) | **81.8 (81.5 / 82.0)** | 53.2 (52.4 / 53.7) |
| CutMix            | 96.7 (96.6 / 96.8) | 71.6 (71.1 / 71.9) | 80.3 (80.0 / 80.6) | 48.5 (48.1 / 48.9) |
| Puzzle Mix        | **97.0 (96.9 / 97.2)** | 82.6 (82.1 / 83.0) | 81.2 (81.0 / 81.6) | 56.3 (55.8 / 56.7) |
| NFM               | 96.2 (96.1 / 96.3) | 82.3 (81.8 / 82.8) | 80.2 (79.6 / 80.6) | 58.7 (58.2 / 59.2) |
| AugMix            | 96.5 (96.4 / 96.6) | 90.2 (89.7 / 90.7) | 80.5 (79.9 / 80.8) | 66.7 (65.9 / 67.2) |
| NoisyMix (ours)   | 96.3 (96.2 / 96.4) | **91.8 (91.2 / 92.7)** | 81.3 (81.0 / 81.6) | **71.9 (71.7 / 72.3)** |

evaluated on real-world examples as provided by the ImageNet-R dataset. This shows that NoisyMix not only improves the robustness with respect to synthetic distribution shifts but also to natural distribution shifts. It can also be seen that the stylized ImageNet model performs well on this task. We also use the ImageNet-R dataset to show that NoisyMix yields well-calibrated predictions, see Table 11 in the Appendix.

NoisyMix also achieves state-of-the-art result on ImageNet-P, reducing the mFP from 58% down to 28.5%. Again, we also show the mFP computed for a subset that excludes noise perturbations (i.e., white and shot noise). Table 6 in the App. shows detailed results for all perturbation types.

To provide another metric for robustness, we evaluate the resilience of ImageNet models with respect to adversarial examples crafted by the simple fast gradient sign method (FGSM) (Goodfellow et al., 2014). NoisyMix helps to improve adversarial robustness with respect to these weak adversarial examples crafted with FGSM ($\epsilon = 4/255$). In contrast, AugMix is less resilient.

4.3 CIFAR-10 and CIFAR-100 Results

We train ResNet-18 and Wide-ResNet-28x2 models (with 5 seeds) on both CIFAR-10 and CIFAR-100, using different data augmentation schemes. Table 2 and 3 provide the results, showing the average, minimum, and maximum test accuracy for the various test settings. Similar to the previous results, PuzzleMix achieves the best test accuracy on clean test sets. Further, we can see that the advantage of NoisyMix is pronounced on CIFAR-100-C, where we improve the robust accuracy by more than 5% as compared to AugMix.

4.4 Ablation Study

At a high-level, we can decompose NoisyMix into 3 building blocks: (i) noisy feature mixup, (ii) stability training, and (iii) data augmentations. Further, noisy feature mixup can be decomposed into feature mixup and noise injections. We provide a detailed ablation to show that all components are required to achieve state-of-the-art performance.

We train models on CIFAR-100 and evaluate the in-domain test accuracy and robust accuracy on CIFAR-100-C. Table 4 shows the results, and it can be seen that data augmentations have a significant impact on robustness, i.e., robust accuracy is improved by 17%. The combination of feature mixup and noise injections helps to improve the in-domain test accuracy, while the robust accuracy is improved by about 11%. The combination of both considerably helps to improve in-domain test accuracy and robustness. The performance is further improved when stability training is
Table 4: Ablation study using a Wide-ResNet-28x2 trained on CIFAR-100. The combination of feature mixing and noise injections with stability training, using augmented data, helps to boost clean and robust accuracy.

| Augmentations | Feature Mixing | Noise Injections | JSD Loss | CIFAR-100 (%) | CIFAR-100-C (%) | Robustness Gain (%) |
|---------------|----------------|------------------|----------|--------------|----------------|---------------------|
| ✗ ✗ ✗ ✗       |     ✗          |                ✗  |          | 79.0         | 47.4           | -                   |
| ✗ ✗ ✓ ✗       |     ✗          |                ✗  |          | 79.1         | 49.5           | +2.1                |
| ✗ ✓ ✓ ✗       |     ✗          |                ✗  |          | 81.8         | 53.2           | +5.8                |
| ✓ ✓ ✓ ✗       |     ✗          |                ✗  |          | 80.2         | 58.7           | +11.3               |
| ✓ ✓ ✗ ✓       |     ✗          |                ✗  |          | 79.1         | 64.0           | +16.6               |
| ✓ ✓ ✗ ✗       |     ✗          |                ✗  |          | 80.9         | 66.7           | +19.3               |
| ✓ ✓ ✓ ✓       |     ✗          |                ✗  |          | 80.5         | 66.9           | +19.5               |
| ✓ ✓ ✗ ✓       |     ✗          |                ✗  |          | 80.4         | 68.6           | +21.2               |
| ✓ ✓ ✗ ✗       |     ✗          |                ✗  |          | 82.1         | 69.3           | +21.9               |
| ✓ ✓ ✗ ✓       |     ✗          |                ✗  |          | 81.3         | 69.3           | +21.9               |

Table 5: Transfer learning results for ResNet-50 models trained on ImageNet and transferred to CIFAR-10/100. Then robustness is evaluated on CIFAR-10-C/100-C. We show the average robust accuracy across all common corruptions / across noise corruptions / across whether, blur and digital corruptions. (Higher values are better.)

| Source Model | CIFAR-10 (%) | CIFAR-10-C (%) | CIFAR-100 (%) | CIFAR-100-C (%) |
|--------------|-------------|----------------|---------------|----------------|
| Baseline     | 97.0        | 77.1 / 48.8 / 86.5 | 84.3          | 57.1 / 28.0 / 66.8 |
| PuzzleMix    | 97.1        | 73.7 / 47.7 / 87.2 | 84.5          | 57.2 / 26.1 / 67.6 |
| Mixup        | 97.1        | 78.6 / 51.7 / 87.6 | 84.5          | 57.3 / 26.4 / 67.6 |
| AutoAugment  | 97.2        | 78.7 / 54.6 / 86.7 | 84.9          | 57.6 / 27.3 / 67.6 |
| AugMix       | 97.7        | 79.9 / 53.9 / 88.6 | **86.5**      | 58.8 / 24.2 / 66.8 |
| NoisyMix (ours) | **97.7** | **81.8 / 60.3 / 88.9** | 85.9          | **60.3 / 34.5 / 68.8** |

used in addition, yielding our NoisyMix scheme. Indeed, NoisyMix (i.e., the favorable combination of all components) is outperforming all other variants.

4.5 **Does Robustness Transfer?**

Motivated by Shafahi et al. (2019b), who showed that adversarial robustness transfers, we study whether robustness to common corruptions transfers from a source model to a target task. To do so, we fine-tune ImageNet models on CIFAR-10 and CIFAR-100, and then use the corrupted test datasets to evaluate robustness.

Table 5 shows that target models inherit some robustness from the source models. More robust ImageNet models, i.e., models trained with NoisyMix, lead to more robust target models. In particular, robustness to noise perturbations is transferred. For example, the parent model trained with NoisyMix yields target models that are about 12% more robust to noise perturbations on CIFAR-10-C, and about 6% more robust on CIFAR-100-C. Moreover, a robust parent model can help to improve the in-domain accuracy on the target task, which is in alignment with the results by Utrera et al. (2020); Salman et al. (2020).

5 **CONCLUSION**

Desirable properties of deep classifiers for computer vision are (i) in-domain accuracy, (ii) robustness to domain shifts, and (iii) well-calibrated estimation of class membership probabilities. Motivated by the challenge to achieve these, we propose NoisyMix, a novel and theoretically justified training scheme. Our empirical results demonstrate the advantage, i.e., improvement of (i), (ii), and (iii), when compared to standard training and other data augmentation methods. These findings are supported by theoretical results that show that NoisyMix can improve model robustness.

A limitation of NoisyMix is that it is tailored towards computer vision tasks and not directly applicable to natural language processing tasks, or time series tasks. Thus, future work will explore how NoisyMix can be extended to a broader range of tasks. In particular, we anticipate that NoisyMix will be effective in improving the robustness of models used in Scientific Machine Learning (e.g., models used for climate predictions). Since this paper studies a new training scheme to improve model robustness there are no potential negative societal impacts of our work.
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Checklist

1. For all models and algorithms presented, check if you include:
   (a) A clear description of the mathematical setting, assumptions, algorithm, and/or model. [Yes]
   (b) An analysis of the properties and complexity (time, space, sample size) of any algorithm. [No]
   (c) (Optional) Anonymized source code, with specification of all dependencies, including external libraries. [Yes]

2. For any theoretical claim, check if you include:
   (a) Statements of the full set of assumptions of all theoretical results. [Yes]
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3. For all figures and tables that present empirical results, check if you include:
   (a) The code, data, and instructions needed to reproduce the main experimental results (either in the supplemental material or as a URL). [Yes]
   (b) All the training details (e.g., data splits, hyperparameters, how they were chosen). [Yes]
   (c) A clear definition of the specific measure or statistics and error bars (e.g., with respect to the random seed after running experiments multiple times). [Yes]
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APPENDIX FOR “NoisyMix”

In this Appendix, we provide additional materials to support our results in the main paper. In particular, we provide mathematical analysis of NoisyMix to shed light on its regularizing and robustness properties, as well as additional empirical results and the details.

Organizational Details. This Appendix is organized as follows.

• In Section A, we illustrate the regularizing effects of stability training with the JSD, Manifold Mixup, and noise injection on a toy dataset.

• In Section B, we study NoisyMix via the lens of implicit regularization. Our contributions there are Theorem 1 and Theorem 2, which show that minimizing the NoisyMix loss function is approximately equivalent to minimizing a sum of the original loss and data-dependent regularizers, amplifying the regularizing effects of AugMix according to the mixing and noise levels.

• In Section C, we show that NoisyMix training can improve model robustness when compared to standard training. Our main contribution there is Theorem 3, which shows that, under appropriate assumptions, NoisyMix training approximately minimizes an upper bound on the sum of an adversarial loss, a stability objective and data-dependent regularizers.

• In Section D, we provide additional experimental results and their details.

We recall the notation that we use in the main paper as well as this Appendix.

Notation. I denotes identity matrix, \([K] := \{1, \ldots, K\}\), the superscript \(T\) denotes transposition, \(\circ\) denotes Hadamard product, \(\mathbb{1}\) denotes the vector with all components equal one. For a vector \(v, v^i\) denotes its \(i\)th component and \(\|v\|_p\) denotes its \(l_p\) norm for \(p > 0\). \(M_\lambda(a, b) := \lambda a + (1 - \lambda)b\), for random variables \(a, b, \lambda\). For \(\alpha, \beta > 0\), \(\tilde{D}_\lambda := \frac{\alpha}{\alpha + \beta}\text{Beta}(\alpha + 1, \beta) + \frac{\beta}{\alpha + \beta}\text{Beta}(\beta + 1, \alpha)\) is a uniform mixture of two Beta distributions. For the two vectors \(a, b\), \(\cos(a, b) := \langle a, b \rangle / \|a\|_2 \|b\|_2\) denotes their cosine similarity.

To simplify mathematical analysis and enable clearer intuition, we shall restrict to the special case when the mixing is done only at the input level. Moreover, we consider the setting described in the main paper for the case of \(K = 2\) and \(\pi_0 = \pi_1 = 1/2\) for our analysis. The results for the general case can be derived analogously using our techniques.

A Illustration of the Regularizing Effects of Stability Training, Manifold Mixup and Noise Injection on a Toy Dataset

We consider a binary classification task for the noise corrupted 2D dataset whose points are separated by a concave polygon-shaped buffer band. Inner and outer points that disjoint the band directly correspond to two label classes, while those falling onto the band are randomly assigned to one of the classes. We generate 500 samples by setting the scale factor to be 0.5 and adding Gaussian noise with zero mean and standard deviation of 0.2 to the shape. Fig. 3 shows the data points.

We train a fully connected feedforward shallow neural network (with 32 neurons) with the ReLU activation function on these data, using 250 points for training and 250 for testing. All models are trained with Adam, learning rate of 0.1, and batch size of 50 for 100 epochs. The seed is fixed across all experiments.

We focus on the effects of stability training (with the JSD), Manifold Mixup, noise injection, and their combinations on the test performance. Note that we do not explore AugMix’s data augmentation here, since the operations used there are not meaningful for the toy dataset here.

Fig. 4 illustrates how different training schemes affect the decision boundaries of the neural network classifier. First, we can see that NoisyMix (which combines feature mixup, noise injections, and stability training) is most effective at smoothing the decision boundary of the trained classifiers, imposing the strongest smoothness on the dataset. Here it also yields the best test accuracy among all considered training schemes. We can also see that stability training helps to boost the effectiveness of using Manifold Mixup. In contrast, NFM without stability training performs better than the baseline and Manifold Mixup, but not as well as NoisyMix and Manifold Mixup with stability training.
Figure 3: The dataset in $\mathbb{R}^2$ that we use for binary classification.

Figure 4: Decision boundaries and test accuracy (in parenthesis) for different training schemes on a 2D binary classification problem. NoisyMix is most effective at smoothing the decision boundary, and it also yields the best test accuracy among all considered training schemes.

B Implicit Regularization of NoisyMix

In this section, we are going to identify the implicit regularization effects of NoisyMix by analyzing the NFM loss and the stability loss.

We emphasize that while the analysis for the NFM loss can be adapted from that in Lim et al. (2022) in a straightforward manner, the analysis and results for the stability loss (see Theorem 2) are novel and needs to be derived independently. Importantly, Theorem 2 is a crucial ingredient for the proof of our main theoretical result, Theorem 3 (stated informally in the main paper). The analysis for the stability loss is rather tedious, and we are going to provide the details of the derivation for completeness (see Subsection B.2).
B.1 Implicit Regularization for the NFM Loss

Recall that
\[
L_{n}^{NFM} = \mathbb{E}_{(x,y),(x',y') \sim D_n} \mathbb{E}_{\lambda \sim \text{Beta}(\alpha, \beta)} \mathbb{E}_{\xi \sim \mathcal{Q}} l(p(M_{\lambda, \xi}(x, y')), M_{\lambda}(y, y')).
\]  
(4)

We consider loss functions of the form \( l(p(x), y) := h(p(x)) - yp(x) \), which includes standard choices such as the logistic loss and the cross-entropy loss. Denote \( L_{n}^{std} := \frac{1}{n} \sum_{i=1}^{n} l(p(x_i), y_i) \) and let \( D_x \) be the empirical distribution of training samples \( \{x_i\}_{i \in [n]} \). We shall show that NFM exhibits a natural form of implicit regularization, i.e., regularization imposed implicitly by the stochastic learning strategy or approximation algorithm, without explicitly modifying the loss.

Recall from the main paper that the NFM loss function to be minimized is
\[
L_{n}^{NFM} = \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} \mathbb{E}_{\lambda \sim \mathcal{D}_{\lambda}} \mathbb{E}_{x_i \sim \mathcal{D}_x} \mathbb{E}_{\xi \sim \mathcal{Q}} l(p(M_{\lambda, \xi}(x_i, x_j)), M_{\lambda}(y_i, y_j)),
\]  
(5)

where \( l \) is a loss function, \( \xi := (\xi^{\text{add}}, \xi^{\text{mult}}) \) are drawn from some probability distribution \( \mathcal{Q} \) with finite first two moments (with zero mean), and
\[
M_{\lambda, \xi}(x, x') := (I + \sigma_{\text{mult}} \xi^{\text{mult}}) \circ M_{\lambda}(x, x') + \sigma_{\text{add}} \xi^{\text{add}}.
\]  
(6)

We emphasize that this loss is computed using only the clean data.

Let \( \epsilon > 0 \) be a small parameter. In the sequel, we assume that \( l(p(x), y) = h(p(x)) - yp(x) \), and rescale \( 1 - \lambda \mapsto \epsilon (1 - \lambda) \), \( \sigma_{\text{add}} \mapsto \epsilon \sigma_{\text{add}} \), \( \sigma_{\text{mult}} \mapsto \epsilon \sigma_{\text{mult}} \). We recall Lemma 2 in Lim et al. (2022) (adapted to our setting), which will be useful later, in the following.

Lemma 1. The NFM loss \( (5) \) can be equivalently written as
\[
L_{n}^{NFM} = \frac{1}{n^2} \sum_{i=1}^{n} \mathbb{E}_{\lambda \sim \mathcal{D}_{\lambda}} \mathbb{E}_{x_i \sim \mathcal{D}_x} \mathbb{E}_{\xi \sim \mathcal{Q}} [h(p(x_i + \epsilon e_{i}^{NFM})) - y_i p(x_i + \epsilon e_{i}^{NFM})],
\]  
(7)

with
\[
e_{i}^{NFM} = (I + \epsilon \sigma_{\text{mult}} \xi^{\text{mult}}) \circ e_{i}^{\text{mixup}} + e_{i}^{\text{noise}}.
\]  
(8)

Here \( e_{i}^{\text{mixup}} = (1 - \lambda)(x_r - x_i) \) and \( e_{i}^{\text{noise}} = \sigma_{\text{mult}} \xi^{\text{mult}} \circ x_i + \sigma_{\text{add}} \xi^{\text{add}} \), with \( x_i, x_r \in \mathcal{X} \) and \( \lambda \sim \text{Beta}(\alpha, \beta) \).

Note that the explicit mixing of the labels is absent in the formulation of Lemma 1. This gives a more natural interpretation of \( (5) \), in the sense that the inputs are sampled from the vicinal distribution induced by certain (data-dependent) random perturbations and the labels are kept the same.

Denote \( \nabla p \) and \( \nabla^2 p \) as the first and second directional derivative of \( p \) with respect to \( x \) respectively. By working in the small parameter regime, we can relate the NFM empirical loss \( L_{n}^{NFM} \) to the original loss \( L_{n}^{std} \) and identify the regularizing effects of NFM. More precisely, we perform a second-order Taylor expansion of the terms in bracket in Eq. (7) and arrive at the following result.

Theorem 1. Let \( \epsilon > 0 \) be a small parameter, and assume that \( h \) and \( p \) are twice differentiable. Then,
\[
L_{n}^{NFM} = L_{n}^{std} + \epsilon R_1 + \epsilon^2 \frac{3}{2} R_2 + \frac{1}{2} \epsilon^2 \frac{3}{2} R_3 + \epsilon^2 \phi(\epsilon),
\]  
(9)

with
\[
\begin{align*}
R_2 &= R_2 + \sigma_{\text{add}}^2 R_{2} + \sigma_{\text{mult}}^2 R_{2} \\
R_3 &= R_3 + \sigma_{\text{add}}^2 R_{3} + \sigma_{\text{mult}}^2 R_{3}
\end{align*}
\]  
(10)
where
\begin{align}
R_1 &= \frac{E_{\lambda \sim \tilde{D}_\lambda} [1 - \lambda]}{n} \sum_{i=1}^{n} h'(p(x_i) - y_i) \nabla p(x_i)^T E_{x_i \sim D_x} [x_r - x_i], \\
R_2 &= \frac{E_{\lambda \sim \tilde{D}_\lambda} [(1 - \lambda)^2]}{2n} \sum_{i=1}^{n} h''(p(x_i)) \nabla p(x_i)^T \\
&\quad \times E_{x_i \sim D_x} [(x_r - x_i)(x_r - x_i)^T] \nabla p(x_i), \\
R_3 &= \frac{E_{\lambda \sim \tilde{D}_\lambda} [(1 - \lambda)^2]}{2n} \sum_{i=1}^{n} (h'(p(x_i)) - y_i) \\
&\quad \times E_{x_i \sim D_x} [(x_r - x_i)^T \nabla^2 p(x_i)(x_r - x_i)], \\
R_{2d}^{add} &= \frac{1}{2n} \sum_{i=1}^{n} h''(p(x_i)) \nabla p(x_i)^T E_{\xi \sim Q}(\xi^{add}(\xi^{add})^T) \nabla p(x_i), \\
R_2^{mult} &= \frac{1}{2n} \sum_{i=1}^{n} h''(p(x_i)) \nabla p(x_i)^T (E_{\xi}[(\xi^{mult}(\xi^{mult})^T) \circ x_i x_i^T]) \nabla p(x_i), \\
R_{3d}^{add} &= \frac{1}{2n} \sum_{i=1}^{n} (h'(p(x_i)) - y_i) E_{\xi}[(\xi^{add})^T \nabla^2 p(x_i) \xi^{add}], \\
R_3^{mult} &= \frac{1}{2n} \sum_{i=1}^{n} (h'(p(x_i)) - y_i) E_{\xi}[(\xi^{mult} \circ x_i)^T \nabla^2 p(x_i)(\xi^{mult} \circ x_i)],
\end{align}
and \(\phi(\epsilon) = E_{\lambda \sim \tilde{D}_\lambda} E_{x, x' \sim D_x} E_{\xi \sim Q}[\phi(\epsilon)]\), with \(\phi\) some function such that \(\lim_{\epsilon \to 0} \phi(\epsilon) = 0\).

Theorem 1 implies that, when compared to Manifold Mixup, NFM introduces additional smoothness, regularizing the directional derivatives, \(\nabla p(x_i)\) and \(\nabla^2 p(x_i)\), with respect to \(x_r\), according to the noise levels \(\sigma_{add}\) and \(\sigma_{mult}\), and thus amplifying the regularizing effects of Manifold Mixup and noise injection. In particular, making \(\nabla^2 p(x_i)\) small can lead to smooth decision boundaries (at the input level), while reducing the confidence of model predictions. On the other hand, making the \(\nabla p(x_i)\) small can lead to improvement in model robustness.

### B.2 Implicit Regularization for the Stability Objective

Recall that \(L^{JS\text{SD}}\) is given by:
\begin{align}
E_{x, x' \sim D_x} E_{x_{am} \sim A(x), x'_{am} \sim A(x')} E_{\lambda \sim Beta(\alpha, \beta)} E_{\xi \sim Q} JS_\pi(p(M_{\lambda, \xi}(x, x')), p(M_{\lambda, \xi}(x_{am}, x'_{am}))).
\end{align}

The stability objective to be minimized is
\begin{align}
L_n^{JS\text{SD}} &= \frac{1}{n^2} \sum_{i=1}^{n} \sum_{j=1}^{n} E_{\xi \sim Q} JS_\pi(p(M_{\lambda, \xi}(x_i, x_j)), p(M_{\lambda, \xi}(A(x_i), A(x_j)))),
\end{align}
where \(\xi := (\xi^{add}, \xi^{mult})\) are drawn from some probability distribution \(Q\) with finite first two moments (with zero mean), and
\begin{align}
M_{\lambda, \xi}(x, x') := (I + \sigma_{mult} \xi^{mult}) \circ M_{\lambda}(x, x') + \sigma_{add} \xi^{add}.
\end{align}

Note that this loss is computed using both the clean data and the diversely transformed data.

Denote
\begin{align}
\hat{L}_n^{std} := \frac{1}{n} \sum_{i=1}^{n} JS_\pi(p(x_i), p(A(x_i)));
\end{align}
and let \(D_x\) be the empirical distribution of training samples \(\{x_i\}_{i \in [n]}\). We shall show that the stability objective exhibits a natural form of implicit regularization.

Let \(\epsilon > 0\) be a small parameter, and rescale \(1 - \lambda \mapsto \epsilon(1 - \lambda)\), \(\sigma_{add} \mapsto \epsilon\sigma_{add}\), \(\sigma_{mult} \mapsto \epsilon\sigma_{mult}\) as before. For this stability objective, we have the following result, derived analogously to that in Lemma 1.
Lemma 2. The JSD loss (20) can be equivalently written as

$$L_n^{JSD} = \frac{1}{n} \sum_{i=1}^{n} \mathbb{E}_{\lambda \sim \mathcal{D}, \mathcal{E}_r \sim \mathcal{D}_r} \mathbb{E}_{\xi \sim \mathcal{Q}} [JS_\pi(p(x_i + \epsilon_i^{\text{NFM}}), p(A(x_i) + \epsilon_i^{JSD}))],$$

with

$$\epsilon_i^{\text{NFM}} = (1 + \epsilon \sigma_{\text{mult}} \xi_{\text{mult}}) \circ \epsilon_i^{\text{mixup}} + \epsilon_i^{\text{noise}},$$

and

$$\epsilon_i^{JSD} = (1 + \epsilon \sigma_{\text{mult}} \xi_{\text{mult}}) \circ \epsilon_i^{\text{mixup}} + \epsilon_i^{\text{noise}}.$$  

Here \( \epsilon_i^{\text{mixup}} = (1 - \lambda)(x_r - x_i), \epsilon_i^{\text{noise}} = \sigma_{\text{mult}} \xi_{\text{mult}} \circ x_i + \sigma_{\text{add}} \xi_{\text{add}}, \) and \( \epsilon_i^{\text{mixup}} = (1 - \lambda)(A(x_r) - A(x_i)) \) and \( \epsilon_i^{\text{noise}} = \sigma_{\text{mult}} \xi_{\text{mult}} \circ A(x_i) + \sigma_{\text{add}} \xi_{\text{add}}, \) with \( x_i, x_r \in \mathcal{X}, A \sim \mathcal{A} \) and \( \lambda \sim \text{Beta}(\alpha, \beta). \)

The following proposition will be useful later.

Proposition 1. Let \( \epsilon > 0 \) be a small parameter. For \( i = 0, 1, \ldots, M, \) denote \( \tilde{p}_i := p(z_i + R_i(\epsilon)), p_i := p(z_i), \) \( \nabla p_i := \nabla p(z_i) \) and \( \nabla^2 p_i := \nabla^2 p(z_i), \) with the \( z_i, \delta z_i \in \mathcal{X}, \) and \( R_i \) some twice differentiable function such that \( R_i(0) = 0, R_i'(0) = \delta z_i, \) and \( R_i''(0) = \delta^2 z_i. \) Suppose that \( p : \mathcal{X} \to \Delta^{K-1} \) is twice differentiable. Then, we have

$$JS_\pi(\tilde{p}_0, \ldots, \tilde{p}_M) = JS_\pi(p_0, \ldots, p_M) + \epsilon \sum_{k=1}^{K} \sum_{l=0}^{M} \pi_l \left( \frac{\nabla^2 p_i^k}{p_i^k} \right) \ln \left( \frac{p_i^k}{p_j^k} \right) \right) \right) - \frac{\epsilon^2}{2} \sum_{k=1}^{K} \sum_{l=0}^{M} \pi_l \left( (\nabla^2 p_i^k)^T \delta z_i \right) \ln \left( \frac{p_i^k}{p_j^k} \right) + O(\epsilon^3),$$

as \( \epsilon \to 0. \)

Proof. By definition,

$$JS_\pi(\tilde{p}_0, \ldots, \tilde{p}_M) := H \left( \sum_{l=0}^{M} \pi_l \tilde{p}_l \right) - \sum_{l=0}^{M} \pi_l H(\tilde{p}_l)$$

$$= - \sum_{k=1}^{K} \left( \sum_{l=0}^{M} \pi_l \tilde{p}_l^k \right) \ln \left( \sum_{l=0}^{M} \pi_l \tilde{p}_l^k \right) + \sum_{l=0}^{M} \pi_l \sum_{k=1}^{K} \tilde{p}_l^k \ln(\tilde{p}_l^k)$$

$$= \sum_{l=0}^{M} \sum_{k=1}^{K} \pi_l A_t^k(\epsilon),$$

where

$$A_t^k(\epsilon) := p^k(z_l + R_l(\epsilon)) \ln \left( \frac{p^k(z_l + R_l(\epsilon))}{\sum_j \pi_j p^k(z_j + R_j(\epsilon))} \right).$$

Then, using twice-differentiability of \( p, \) we perform a second-order Taylor expansion on (31) in the small parameter \( \epsilon: \)

$$A_t^k(\epsilon) = A_t^k(0) + \epsilon (A_t^k)'(0) + \frac{1}{2} \epsilon^2 (A_t^k)''(0) + o(\epsilon^2).$$
as $\epsilon \to 0$.

It remains to compute $(A^k_t)'(0)$ and $(A^k_t)''(0)$.

Using chain rule, we obtain:

$$
(A^k_t)'(\epsilon) = (\nabla p^k(z_l + R_t(\epsilon)))^T R'_t(\epsilon) \left(1 + \ln \left( \frac{p^k(z_l + R_t(\epsilon))}{\sum_j \pi_j p^k(z_j + R_j(\epsilon))} \right) \right) 
- p^k(z_l + R_t(\epsilon)) \sum_j \pi_j \nabla p^k(z_l + R_j(\epsilon))^T R'_t(\epsilon) \sum_l \pi_l p^k(z_l + R_l(\epsilon)).
$$

(33)

Therefore, using the fact that $R_t(0) = 0$ and $R'_t(0) = \delta z_l$, we have:

$$
(A^k_t)'(0) = (\nabla p^k(z_l))^T \delta z_l \left(1 + \ln \left( \frac{p^k(z_l)}{\sum_j \pi_j p^k(z_j)} \right) \right) 
- p^k(z_l) \sum_j \pi_j (\nabla p^k(z_l))^T \delta z_j \sum_l \pi_l p^k(z_l).
$$

(34)

Similarly, applying chain rule, we obtain:

$$
(A^k_t)''(\epsilon) 
= [R'_t(\epsilon)^T \nabla^2 p^k(z_l + R_t(\epsilon)) R'_t(\epsilon) 
+ (\nabla p^k(z_l + R_t(\epsilon)))^T R''_t(\epsilon)] 
\left(1 + \ln \left( \frac{p^k(z_l + R_t(\epsilon))}{\sum_j \pi_j p^k(z_j + R_j(\epsilon))} \right) \right) 
+ \frac{(\nabla p^k(z_l + R_t(\epsilon)))^T R'_t(\epsilon))^2}{p^k(z_l + R_t(\epsilon))} 
- 2\nabla p^k(z_l + R_t(\epsilon))^T R'_t(\epsilon) \left(\frac{\sum_j \pi_j \nabla p^k(z_l + R_j(\epsilon))^T R'_t(\epsilon)}{\sum_l \pi_l p^k(z_l + R_l(\epsilon))} \right) 
- p^k(z_l + R_t(\epsilon)) \sum_j \pi_j (R''_t(\epsilon))^T \nabla^2 p^k(z_l + R_j(\epsilon)) R'_t(\epsilon) + \nabla p^k(z_l + R_j(\epsilon))^T R''_t(\epsilon) 
\sum_l \pi_l p^k(z_l + R_l(\epsilon)) 
+ p^k(z_l + R_t(\epsilon)) \left(\frac{\sum_l \pi_l \nabla p^k(z_l + R_t(\epsilon))^T R'_t(\epsilon)}{\sum_j \pi_j p^k(z_j + R_j(\epsilon))} \right)^2.
$$

(35)

Therefore, using the fact that $R_t(0) = 0$, $R'_t(0) = \delta z_l$, and $R''_t(0) = \delta^2 z_l$, we have:

$$
(A^k_t)''(0) = [\delta z_l]^T \nabla^2 p^k(z_l) \delta z_l + (\nabla p^k(z_l))^T \delta^2 z_l \left(1 + \ln \left( \frac{p^k(z_l)}{\sum_j \pi_j p^k(z_j)} \right) \right) 
+ \frac{(\nabla p^k(z_l))^T \delta z_l)^2}{p^k(z_l)} 
- 2\nabla p^k(z_l)^T \delta z_l \left(\frac{\sum_j \pi_j \nabla p^k(z_l)^T \delta z_j}{\sum_l \pi_l p^k(z_l)} \right) 
- p^k(z_l) \sum_j \pi_j ((\delta z_l)^T \nabla^2 p^k(z_l) \delta z_l + \nabla p^k(z_l)^T \delta^2 z_l) 
\sum_l \pi_l p^k(z_l) 
+ p^k(z_l) \left(\frac{\sum_l \pi_l \nabla p^k(z_l)^T \delta z_l}{\sum_j \pi_j p^k(z_j)} \right)^2.
$$

(36)

Combining the above results with the Taylor expansion (32) and simplifying the resulting expression lead to (26).

\[\square\]

Specializing Proposition 1 to our setting, we arrive at the following result.

**Theorem 2.** Let $\epsilon > 0$ be a small parameter, and assume that $p$ is twice differentiable. Define, for $k \in [K]$, 

$$
q^k(x, y) := \nabla^2 p^k(x) \ln \left( \frac{2p^k(x)}{p^k(x) + p^k(y)} \right) + \nabla p^k(x) (\nabla p^k(x))^T \frac{p^k(y)}{p^k(x)(p^k(x) + p^k(y))}.
$$

(37)
Then,
\[ L_n^{SD} = \tilde{L}_n^{std} + \epsilon S_1 + \epsilon^2 \tilde{S}_2 + \epsilon^2 \phi(\epsilon), \tag{38} \]
with
\[ \tilde{S}_2 = S_2 + \sigma_{\text{add}}^2 S_{2}^{\text{add}} + \sigma_{\text{mult}}^2 S_{2}^{\text{mult}}, \tag{39} \]
where
\[
S_1 = \frac{E_\lambda[1 - \lambda]}{2n} \sum_{i=1}^{n} \sum_{k=1}^{K} \left( (\nabla p^k(x_i))^T (E_{x_i}[x_r] - x_i) \cdot \ln \left( \frac{2p^k(x_i)}{p^k(x_i) + p^k(A(x_i))} \right) \right) + \left( (\nabla p^k(A(x_i)))^T (E_{x_i}[A(x_r)] - A(x_i)) \cdot \ln \left( \frac{2p^k(A(x_i))}{p^k(x_i) + p^k(A(x_i))} \right) \right), \tag{40} \]
\[
S_2^{\text{add}} = \frac{1}{4n} \sum_{i=1}^{n} \sum_{k=1}^{K} \left( E_\xi[(\xi^{\text{add}})^T q^k(x_i, A(x_i)) \xi^{\text{add}}] + E_\xi[(\xi^{\text{add}})^T q^k(A(x_i), x_i) \xi^{\text{add}}] - 2(\nabla p^k(x_i))^T E_\xi[\xi^{\text{add}}(\xi^{\text{add}})^T] \nabla p^k(A(x_i)) \right), \tag{41} \]
\[
S_2^{\text{mult}} = \frac{1}{4n} \sum_{i=1}^{n} \sum_{k=1}^{K} \left( E_\xi[(\xi^{\text{mult}} \odot x_i)^T q^k(x_i, A(x_i)) (\xi^{\text{mult}} \odot x_i)] + E_\xi[(\xi^{\text{mult}} \odot A(x_i))^T q^k(A(x_i), x_i) (\xi^{\text{mult}} \odot A(x_i))] - 2(\nabla p^k(x_i))^T E_\xi[(\xi^{\text{mult}} \odot x_i)(\xi^{\text{mult}} \odot A(x_i))^T] \nabla p^k(A(x_i)) \right), \tag{42} \]
and \( \phi(\epsilon) = E_\lambda \sim D_\lambda E_{x_i} \sim D_{x_i} E_\xi \sim \phi[\varphi(\epsilon)] \), with \( \varphi \) some function such that \( \lim_{\epsilon \to 0} \varphi(\epsilon) = 0 \).

**Proof.** By assumption, \( p \) is twice differentiable, so we apply Proposition 1 (with \( M = 1, K = 2, \pi_0 = \pi_1 = 1/2 \), \( z_0 = x_i, z_1 = A(x_i), R_0(\epsilon) = \epsilon (e_i^{\text{mixup}} + e_i^{\text{noise}}) + \epsilon^2 \sigma_{\text{mult}} \xi^{\text{mult}} \odot e_i^{\text{mixup}}, R_1(\epsilon) = \epsilon (e_i^{\text{mixup}} + e_i^{\text{noise}}) + \epsilon^2 \sigma_{\text{mult}} \xi^{\text{mult}} \odot e_i^{\text{mixup}} \) to Eq. (7). Simplifying (using linearity of expectation, and the expression for the \( e_i^{\text{mixup}}, e_i^{\text{noise}} \) and \( \tilde{e}_i^{\text{noise}} \) given in Lemma 2) and rearranging the resulting expression then lead to the expression given in the theorem.

Theorem 2 implies that, when compared to the stability training in AugMix, the stability training in NoisyMix introduces additional smoothness, regularizing the directional derivatives, \( \nabla p(x_i), \nabla p(A(x_i)), \nabla^2 p(x_i) \) and \( \nabla^2 p(A(x_i)) \), with respect to both the clean data \( x_i \) and augmented data \( A(x_i) \), according to the mixing coefficient and the noise levels.

Combining this with the analogous interpretation of Theorem 1 and recalling that \( L_n^{\text{NoisyMix}} = L_n^{\text{NFM}} + \gamma L_n^{\text{stability}} \), we see that NoisyMix amplifies the regularizing effects of AugMix, and can lead to smooth decision boundaries and improvement in model robustness. Since NoisyMix implicitly reduces the \( \nabla p(x_i) \) and \( \nabla p(A(x_i)) \), following the argument in Section C of Lim et al. (2022), we see that NoisyMix implicitly increases the classification margin, thereby making the model more robust to input perturbations. We shall study another lens in which NoisyMix can improve robustness in the next section.

## C Robustness of NoisyMix

We now demonstrate how NoisyMix training can lead to a model that is both adversarially robust and stable.
We consider the binary cross-entropy loss, setting $h(z) = \log(1 + e^z)$, with the labels $y$ taking value in $\{0, 1\}$ and the classifier model $f : \mathbb{R}^d \to \mathbb{R}$. In the following, we assume that the model parameter $\theta \in \Theta := \{\theta : y_i f(x_i) + (y_i - 1)f(x_i) \geq 0 \text{ for all } i \in [n]\}$. We remark that this set contains the set of all parameters with correct classifications of training samples (before applying NoisyMix), since $\{\theta : 1_{f(x_i)\geq 0} = y_i \text{ for all } i \in [n]\} \subset \Theta$. Therefore, the condition of $\theta \in \Theta$ is fulfilled when the model classifies all labels correctly for the training data before applying NoisyMix. Since the training error often becomes zero in finite time in practice, we shall study the effect of NoisyMix on model robustness in the regime of $\theta \in \Theta$.

Working in the data-dependent parameter space $\Theta$, we obtain the following result.

**Theorem 3.** Let $\theta \in \Theta := \{\theta : y_i f(x_i) + (y_i - 1)f(x_i) \geq 0 \text{ for all } i \in [n]\}$ be a point such that $\nabla f_x \in \mathbb{R}^d$, $\nabla f(A(x_i))$, $\nabla^2 f(A(x_i))$ and $\nabla^2 f(A(x_i))$ exist for all $i \in [n]$. Assume that $f(x_i) = \nabla f_x(x_i) f_{x_i}^T x_i, \nabla^2 f(x_i) = 0$ for all $i \in [n]$. In addition, suppose that $E_{r \sim D}[r] = E_{r \sim D}[A(r)] = 0$ and $\|x_i\|_2 \geq c_r \sqrt{d}$ for all $i \in [n]$. Then,

$$L_n^{\text{NoisyMix}} \geq \frac{1}{n} \sum_{i=1}^n \max_{\|\delta_i\|_2 \leq \epsilon_i^{\text{mix}}} \ell(f(x_i + \delta_i), y_i) + \gamma L_n^{\text{JS}} + \epsilon^2 L_n^{\text{reg}} + \epsilon^2 \phi(\epsilon),$$

where

$$\epsilon_i^{\text{mix}} = \epsilon E_{r \sim \mathcal{D}}[1 - \lambda] \cdot c_x \sqrt{d} \cdot \cos(\nabla f(x_i), x_i),$$

$$\tilde{L}_n^{\text{JS}} = \frac{1 - \epsilon E_{r \sim \mathcal{D}}[1 - \lambda]}{n} \sum_{i=1}^n J_{S_n}(f(x_i), f(A(x_i))),$$

$$L_n^{\text{reg}} = \frac{1}{2n} \sum_{i=1}^n |h''(f(x_i))| (\epsilon_i^{\text{reg}})^2 + \gamma \tilde{S}_2,$$

with $\tilde{S}_2$ given in (39) and

$$(\epsilon_i^{\text{reg}})^2 = \|\nabla f(x_i)\|_2^2 \left( E_{r \sim \mathcal{D}}[1 - \lambda] \cdot \nabla f_x \cdot \|x_r\|_2^2 \cos(\nabla f(x_i), x_r)^2 \right)$$

$$+ \sigma_{add}^2 E_{r \sim \mathcal{D}}[\|x_{add}\|_2^2 \cos(\nabla f(x_i), x_{add})^2]$$

$$+ \sigma_{mul}^2 E_{r \sim \mathcal{D}}[\|x_{mul} \cap x_i\|_2^2 \cos(\nabla f(x_i), x_{mul} \cap x_i)^2],$$

and $\phi$ is some function such that $\lim_{\epsilon \to 0} \phi(\epsilon) = 0$.

**Proof of Theorem 3.** The inequality in the theorem follows upon applying Theorem 2 in Lim et al. (2022) together with Theorem 2. \hfill \Box

We remark that the assumption made in Theorem 3 is similar to the one made in Lamb et al. (2019); Zhang et al. (2020), and is satisfied by linear models and deep neural networks with ReLU activation function and max-pooling (for a proof of this, we refer to Section B.2 in Zhang et al. (2020)).

Theorem 3 says that $L_n^{\text{NoisyMix}}$ is approximately an upper bound of sum of an adversarial loss with $l_2$-attack of size $\epsilon^{\text{mix}} = \min_i \epsilon_i^{\text{mix}}$, a stability objective, and a data-dependent regularizer. Therefore, **minimizing the NoisyMix loss would result in a small regularized adversarial loss, while promoting stability for the model**. This not only amplifies the robustness benefits of Manifold Mixup, but also imposes additional smoothness, due to the noise injection in NFM and stability training on the noise-perturbed transformed data. The latter can also help reduce robust overfitting and improve test performance (Rice et al., 2020; Rebuffi et al., 2021).
D Additional Experiments and Details

In this section we show additional results to support and complement the findings in Section 4.

D.1 Additional ImageNet-C, CIFAR-10-C and CIFAR-100-C Results

ImageNet-C, CIFAR-10-C, and CIFAR-100-C consist of 15 different corruption types that are used to evaluate the robustness of a given model. These corruption types are illustrated in Figure 5. Here we show examples that corresponds to severity level 5, i.e., the most severe perturbation level. The considered corruptions cover a wide verity of perturbations that can potentially occur in real-world situations (i.e., a self-driving car might need to navigate in a snow storm).

![Figure 5: The ImageNet-C example shows the 15 different corruption types at severity level 5.](image)

**ImageNet-C Results.** Table 6 shows the robust accuracy with respect to each perturbation type for the different ResNet-50 models trained on ImageNet-1k. NoisyMix performs best on 9 out of the 15 corruption types. The advantage is particularly pronounced for noise perturbations and weather corruptions. To illustrate this further, we compute the average robustness accuracy for the 4 meta categories (i.e., noise, weather, blur, and digital corruptions). The results are shown in Figure 6, where it can be seen that NoisyMix is the most robust model with respect to noise, weather, and digital corruptions, while AugMix is slightly more robust to blur corruptions.

**CIFAR-C Results.** Table 7 shows the detailed robustness properties of ResNet-18 models trained with different data augmentation schemes and evaluated on CIFAR-10-C. Table 8 shows similar results for ResNet-18 models that are evaluated on CIFAR-100-C. Overall, NoisyMix performs best across most of the different input perturbations and data corruptions as compared to various baselines in the data augmentation space. In general, the combination of a noisy training scheme paired with stability training appears to prescribe our model a bias towards stochastic noise due to the nature of fitting a reconstruction over a stochastic representation. This can be seen by the good robust accuracy on the noise, weather and digital perturbations being higher for NoisyMix and lower in the other methods that place less encompassing emphasis on stochastic perturbations. Overall, the advantage of training models with NoisyMix is more pronounced for the CIFAR-100 task, where NoisyMix dominates AugMix.

The limitations of certain neural network architectures in terms of their abilities to learn high-frequency features in low-dimensional domains (Tancik et al., 2020) could also imply that our model will tend towards learning features similar in frequency to the in-domain task, and hence perform most accurately in specific out-of-domain perturbations on the same dataset containing similar frequencies. We leave this to future work.
Table 6: Detailed results for ResNet-50s trained with various data augmentation schemes and evaluated on ImageNet-C. All values indicate the robust accuracy (in %) as a function of the considered 15 perturbation types (higher values are better).

| Scheme                     | Noise White | Noise Shot | Noise Impulse | Blur Defocus | Blur Glass | Blur Motion | Blur Zoom | Weather Snow | Weather Frost | Weather Fog | Weather Bright | Digital Contrast | Digital Elastic | Digital Pixel | Digital JPEG |
|----------------------------|-------------|------------|---------------|--------------|------------|-------------|------------|--------------|---------------|-------------|----------------|------------------|----------------|--------------|-------------|
| Baseline                   | 29.3        | 27.0       | 23.8          | 38.8         | 26.8       | 38.7         | 36.2       | 32.5         | 38.1          | 45.8        | 68.0           | 39.1             | 45.3           | 44.8         | 53.4         |
| Adversarial Trained        | 22.4        | 21.0       | 13.1          | 24.6         | 34.8       | 33.7         | 35.2       | 30.6         | 29.6          | 6.2         | 54.4           | 8.5              | 49.4           | 57.3         | 60.5         |
| Stylized ImageNet          | 41.3        | 40.3       | 37.2          | 42.9         | 32.3       | 45.5         | 35.8       | 41.0         | 41.6          | 47.0        | 67.4           | 43.3             | 49.5           | 55.6         | 57.7         |
| Fast AutoAugment           | 40.9        | 40.3       | 36.3          | 42.0         | 35.1       | 41.2         | 36.3       | 40.2         | 43.0          | 52.9        | **72.0**       | 51.7             | 47.5           | 48.5         | 57.7         |
| Mixup                      | 40.5        | 36.9       | 34.2          | 41.9         | 29.1       | 43.9         | 41.1       | 42.0         | 45.5          | **57.5**    | 71.4           | 51.0             | 47.1           | 51.8         | 58.6         |
| Manifold Mixup             | 36.3        | 34.2       | 31.0          | 39.7         | 27.6       | 42.0         | 40.2       | 38.6         | 49.0          | 54.6        | 69.1           | 51.3             | 45.5           | 45.7         | 54.2         |
| CutMix                     | 36.0        | 34.1       | 28.4          | 40.2         | 25.4       | 40.6         | 36.9       | 34.1         | 39.6          | 50.3        | 70.3           | 46.8             | 45.2           | 36.4         | 51.2         |
| Puzzle Mix                 | 22.4        | 21.0       | 13.1          | 24.6         | 34.8       | 33.7         | 35.2       | 30.6         | 29.6          | 6.2         | 54.4           | 8.5              | 49.4           | 57.3         | 60.5         |
| AugMix                     | 40.6        | 41.1       | 37.7          | **47.7**     | 34.9       | **53.5**     | **49.0**   | 39.9         | 43.8          | 47.1        | 69.5           | 51.1             | 52.0           | 57.0         | 60.3         |
| *NoisyMix* (ours)          | **53.1**    | **52.5**   | **51.7**      | 47.1         | **37.5**   | 52.2         | 47.2       | **45.0**     | **52.6**      | 52.9        | 71.1           | **52.6**        | **52.6**       | 53.2         | **63.8**     |
Table 7: Detailed results for ResNet-18s trained with various data augmentation schemes and evaluated on CIFAR-10-C. All values indicate the robust accuracy (in %) as a function of the considered 15 perturbation types (higher values are better).

| Scheme                  | Noise | Blur | Weather | Digital | White | Shot | Impulse | Defocus | Glass | Motion | Zoom | Snow | Frost | Fog | Bright |
|-------------------------|-------|------|---------|---------|-------|------|---------|---------|-------|--------|------|------|-------|-----|--------|
| Baseline                | 81.0% | 76.8%| 56.3%   | 69.8%   | 11.1% | 41.2%| 69.0%   | 11.1%   | 41.2% | 69.0% | 11.1%| 41.2%| 69.0% | 11.1%| 41.2%  |
| Mixup                   | 85.2% | 80.3%| 60.0%   | 71.2%   | 13.8% | 51.6%| 75.2%   | 13.8%   | 51.6% | 75.2% | 13.8%| 51.6%| 75.2% | 13.8%| 51.6%  |
| Manifold Mixup          | 85.5% | 76.4%| 52.2%   | 68.5%   | 15.6% | 48.2%| 71.3%   | 15.6%   | 48.2% | 71.3% | 15.6%| 48.2%| 71.3% | 15.6%| 48.2%  |
| CutMix                  | 82.8% | 78.2%| 58.5%   | 73.7%   | 17.1% | 54.8%| 79.1%   | 17.1%   | 54.8% | 79.1% | 17.1%| 54.8%| 79.1% | 17.1%| 54.8%  |
| Puzzle Mix              | 81.7% | 76.9%| 53.9%   | 69.4%   | 16.3% | 47.9%| 74.2%   | 16.3%   | 47.9% | 74.2% | 16.3%| 47.9%| 74.2% | 16.3%| 47.9%  |
| NFM                     | 87.8% | 83.5%| 73.4%   | 80.3%   | 20.0% | 64.1%| 84.0%   | 20.0%   | 64.1% | 84.0% | 20.0%| 64.1%| 84.0% | 20.0%| 64.1%  |
| AugMix                  | 88.5% | 87.9%| 88.4%   | 93.8%   | 22.1% | 78.8%| 91.5%   | 22.1%   | 78.8% | 91.5% | 22.1%| 78.8%| 91.5% | 22.1%| 78.8%  |
| NoisyMix (ours)         | 89.9% | 91.8%| 94.6%   | 93.6%   | 24.7% | 85.9%| 92.7%   | 24.7%   | 85.9% | 92.7% | 24.7%| 85.9%| 92.7% | 24.7%| 85.9%  |

Note: Detailed results for ResNet-18 trained with various data augmentation schemes and evaluated on CIFAR-10-C.
Table 8: Detailed results for ResNet-18s trained with various data augmentation schemes and evaluated on CIFAR-100-C. All values indicate the robust accuracy (in %) as a function of the considered 15 perturbation types (higher values are better).

| Noise     | Baseline | Mixup   | Manifold Mixup | CutMix | NFM | AugMix | \( N_{\text{geogMix}} \) (ours) |
|-----------|----------|----------|----------------|--------|-----|--------|-------------------------------|
| White Shot | 22.1 27.2 | 24.7 23.1 | 13.1 47.3 | 50.2 46.8 | 50.2 46.8 | 65.3 69.2 77.8 76.8 68.6 73.7 75.4 76.8 77.8 |
| Shot Impulse | 30.3 36.0 | 33.6 33.6 | 20.6 54.3 | 57.5 54.9 | 57.5 54.9 | 69.2 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Impulse | 24.6 60.2 | 21.7 65.1 | 29.8 64.1 | 42.1 64.8 | 42.1 64.8 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Defocus | 63.7 65.9 | 65.9 62.4 | 64.1 64.1 | 64.8 64.8 | 64.8 64.8 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Glass | 57.4 63.8 | 57.4 63.8 | 33.1 33.1 | 33.1 33.1 | 33.1 33.1 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Motion | 54.8 63.8 | 54.8 63.8 | 59.6 59.6 | 59.6 59.6 | 59.6 59.6 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Zoom | 48.9 60.3 | 48.9 60.3 | 50.0 50.0 | 50.0 50.0 | 50.0 50.0 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Snow | 74.9 61.8 | 74.9 61.8 | 61.0 61.0 | 61.0 61.0 | 61.0 61.0 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Frost | 73.0 69.3 | 73.0 69.3 | 73.0 73.0 | 73.0 73.0 | 73.0 73.0 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Bright | 75.4 75.4 | 75.4 75.4 | 75.4 75.4 | 75.4 75.4 | 75.4 75.4 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Contrast | 76.8 76.8 | 76.8 76.8 | 76.8 76.8 | 76.8 76.8 | 76.8 76.8 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Elastic | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Pixel | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| JPEG | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 77.8 77.8 | 71.1 77.8 77.8 76.8 73.7 75.4 76.8 77.8 |
| Digital | 58.6 68.0 | 58.6 68.0 | 60.7 60.7 | 60.7 60.7 | 60.7 60.7 | 68.0 72.2 70.5 | 68.9 |
Table 9: Detailed results for Wide-ResNet models trained with data augmentation schemes and evaluated on CIFAR-10-C. All values indicate the robust accuracy (in %) as a function of the considered 15 perturbation types (higher values are better).

| Scheme       | Baseline | CutMix | Manifold Mixup | Mixup | NoisyMix (ours) |
|--------------|----------|--------|----------------|-------|----------------|
| Noise        |          |        |                |       |                |
| White Shot   | 85.1     | 82.2   | 87.7           | 90.4  | 89.8           |
| Impulse      | 81.1     | 85.7   | 84.9           | 87.9  | 88.2           |
| Blur         | 59.6     | 73.4   | 76.4           | 78.8  | 78.8           |
| Defocus      | 84.7     | 73.4   | 73.4           | 78.8  | 78.8           |
| Glass        | 85.6     | 84.7   | 84.7           | 87.9  | 88.2           |
| Motion       | 83.0     | 83.0   | 83.0           | 87.9  | 88.2           |
| Zoom         | 78.4     | 78.4   | 78.4           | 78.8  | 78.8           |
| Weather      | 88.0     | 88.0   | 88.0           | 87.9  | 88.2           |
| Snow Frost   | 81.0     | 81.0   | 81.0           | 87.9  | 88.2           |
| Fog          | 79.4     | 79.4   | 79.4           | 78.8  | 78.8           |
| Bright       | 78.4     | 78.4   | 78.4           | 78.8  | 78.8           |
| Contrast     | 77.4     | 77.4   | 77.4           | 78.8  | 78.8           |
| Elastic      | 85.4     | 85.4   | 85.4           | 87.9  | 88.2           |
| Pixel JPEG   | 83.2     | 83.2   | 83.2           | 87.9  | 88.2           |
| JPEG         | 83.0     | 83.0   | 83.0           | 87.9  | 88.2           |
| NFM          | 74.8     | 74.8   | 74.8           | 81.2  | 81.2           |

Note: Detailed results for Wide-ResNet models trained with data augmentation schemes and evaluated on CIFAR-10-C. All values indicate the robust accuracy (in %) as a function of the considered 15 perturbation types (higher values are better).
Table 10: Detailed results for Wide-ResNet models trained with data augmentation schemes and evaluated on CIFAR-100-C. All values indicate the robust accuracy (in %) as a function of the considered 15 perturbation types (higher values are better).

|               | Noise       | Blur       | Weather  | Digital  |
|---------------|-------------|------------|----------|----------|
|               | White Shot | Impulse   | Defocus  | Glass    | Motion   | Zoom      | Snow | Frost | Fog | Bright |
| Baseline      | 18.4 27.0  | 19.1       | 60.6     | 18.4     | 54.0     | 54.5      | 55.7 | 48.1  | 64.4| 74.9    |
| Mixup         | 25.3 34.3  | 21.0       | 66.5     | 23.6     | 61.8     | 61.3      | 64.5 | 56.1  | 71.1| 77.1    |
| Manifold Mixup| 20.5 29.7  | 24.2       | 65.7     | 23.9     | 60.9     | 60.9      | 64.8 | 56.8  | 70.6| **78.2**|
| CutMix        | 13.1 20.8  | 31.2       | 62.2     | 22.6     | 57.0     | 55.7      | 61.2 | 52.4  | 67.8| 75.4    |
| Puzzle Mix    | 38.6 47.7  | 33.0       | 64.4     | 29.4     | 60.5     | 58.7      | 66.3 | 61.2  | 71.2| 77.5    |
| NFM           | 49.4 57.2  | 39.4       | 62.9     | 35.8     | 57.3     | 58.3      | 66.5 | 65.5  | 66.0| 76.7    |
| AugMix        | 46.6 55.5  | 65.2       | **78.2** | 49.3     | **74.8** | **76.1**  | 69.0 | 66.1  | 71.5| 77.6    |
| NoisyMix (ours)| **68.4** | **71.9**   | **80.0** | 77.2     | 59.5     | 73.8      | 71.7 | 70.7  | 72.2| 77.7    |

All values are in percentage and indicate robust accuracy as a function of the considered 15 perturbation types. Higher values are better.
Figure 6: Average robust accuracy of ResNet-50 models for the 4 meta categories noise, blur, weather, and digital distortions. *NoisyMix* dominates in 2 out of the 4 categories, while the robustness to digital corruptions is nearly on par with AugMix.

Figure 7: Average robust accuracy of Wide-ResNet-28x2 models, trained on CIFAR-10, for the 4 categories noise, blur, weather, and digital distortions. *NoisyMix* dominates in 1 out of the 4 categories, while the robustness to blur, weather, and digital corruptions is nearly on par with AugMix.

Figure 8: Average robust accuracy of Wide-ResNet-28x2 models, trained on CIFAR-100, for the 4 categories noise, blur, weather, and digital distortions. *NoisyMix* dominates in 4 out of the 4 categories.

Table 9 shows the detailed robustness properties of Wide-ResNet-28x2 models evaluated on CIFAR-10-C, and
Table 10 results for CIFAR-100-C, respectively. NoisyMix benefits from the wide architecture and can improve the robustness as compared to the ResNet-18 models. Again, it can be seen that NoisyMix significantly improves the robustness to noise perturbations as well as to weather corruptions. Figure 7, and Figure 8 show the average robustness accuracy for the 4 meta categories (i.e., noise, weather, blur, and digital corruptions) for CIFAR-10-C and CIFAR-100-C, respectively.

D.2 Calibration Results for ImageNet-R

State-of-the-art neural networks tend to be biased towards high accuracy prediction, while being poorly calibrated. In turn, these models tend to be less reliable and often also show to lack fairness. This can lead to poor decision making in mission critical applications, e.g., medical imaging. In such applications it might be not sufficient to just compute a score $s(x)$ that can then be used to rank the input from the most probable member to the least probable member of a class $c$. Rather it is desirable to obtain an reliable estimate for the class membership probability that can be assigned an example-dependent misclassifications cost (Zadrozny and Elkan, 2001).

Typically, these probability estimates are obtained by squashing the activations of the output layer through a normalized exponential function that is known as the softmax function.

One hypothesis that explains why DNNs (such as ResNets) are poorly calibrated is their high capacity, i.e., these models have the ability to learn highly-specific features that overfit to the source dataset. This phenomenon is shown by Guo et al. (2017), where calibration error increases as the number of filters per layer increases. We show that data augmentation can mitigate this effect and produce better calibrated models. Table 11 shows the RMS calibration error and the area under the response rate accuracy curve (AURRA), evaluated on ImageNet-R, as metrics to measure how well a model is calibrated. NoisyMix considerably improves the calibration of the ImageNet model, as compared to standard training or training with other data augmentation methods. We believe that this is explained by the fact that NoisyMix “wash-out” features that are highly task specific.

D.3 Additional ImageNet-P and CIFAR-10-P Results

ImageNet-C Results. Table 12 shows the flip rates with respect to each perturbation type for the different ResNet-50 models trained on ImageNet-1k. NoisyMix dominates on 6 out of the 10 corruption types, resulting in the lowest mean flip rate overall. While NoisyMix shows a clear advantage compared to the model trained with AugMix, the adversarial trained model is performing nearly as good as NoisyMix on this task. This is surprising, since the adversarial trained model shows a poor performance on the ImageNet-C task. However, this shows the many different facets of robustness and supports the claim that there is no single metric for measuring robustness.

Table 13 and 14 show results for ResNet-18 and Wide-ResNet-28x2 models trained on CIFAR-10 and evaluated on CIFAR-10-P. Here, the models trained with NoisyMix clearly dominate. Note, that we do not normalize the flip rates here.

Table 11: Calibration errors measured by the RMS calibration error (lower values are better), and the area under the response rate accuracy curve (higher values are better).

| Method                      | RMS Calibration Error (↓%) | AURRA (↑%) |
|-----------------------------|-----------------------------|------------|
| Baseline ResNet-50          | 19.7                        | 64.6       |
| Adversarial Trained (Salman et al., 2020) | 8.9 | 68.8 |
| Stylized ImageNet (Geirhos et al., 2018) | 16.2 | 69.7 |
| AutoAugment (Cubuk et al., 2019) | 19.9 | 67.6 |
| Mixup (Zhang et al., 2018)  | 17.5                        | 68.6       |
| Manifold Mixup (Verma et al., 2019) | 5.0 | 68.7 |
| CutMix (Yun et al., 2019)   | 13.6                        | 63.2       |
| Puzzle Mix (Kim et al., 2020) | 15.8 | 68.8 |
| AugMix (Hendrycks et al., 2020) | 14.5 | 70.1 |
| NoisyMix (ours)             | **3.9**                     | **74.8**   |
Table 12: Detailed results for ResNet-50s trained with various data augmentation schemes and evaluated on ImageNet-P. The values indicate the flipping rates as a function of the 10 perturbation types (lower values are better). The mean flip rate (mFR) summarizes the results and shows that NoisyMix improves perturbation stability by approximately 5%.

| Noise   | Blur  | Weather  | Digital  | mFR  |
|---------|-------|----------|----------|------|
|          | White | Motion   | Snow     | Translate | Rotate | Tilt | Scale |       |
| Baseline | 75.4  | 37.8     | 64.5     | 72.1     | 63.2   | 61.9 | 44.2   | 31.9   | 56.9   | 48.1  | 58.9  |
| Adversarial Trained | 25.6 | 40.1     | 26.9     | 33.1     | 18.3   | 60.6 | 25.7   | 31.3   | 30.5   | 40.4  | 33.3  |
| Stylized ImageNet | 31.1 | 28.8     | 41.8     | 58.1     | 43.1   | 50.3 | 36.7   | 41.2   | 42.9   | 42.1  | 54.4  |
| Fast AutoAugment | 53.7 | 49.9     | 63.8     | 79.3     | 66.5   | 58.2 | 43.2   | 49.0   | 57.7   | 43.6  | 56.6  |
| Mixup | 47.6  | 47.2     | 60.8     | 73.9     | 58.9   | 62.1 | 51.2   | 55.0   | 61.1   | 46.3  | 56.4  |
| Manifold Mixup | 49.1 | 46.2     | 66.1     | 72.4     | 62.6   | 59.9 | 46.5   | 53.7   | 58.8   | 45.2  | 56.0  |
| CutMix | 54.7  | 52.7     | 69.4     | 77.4     | 73.1   | 59.2 | 43.2   | 51.6   | 59.1   | 46.1  | 58.6  |
| Puzzle Mix | 50.1 | 48.3     | 64.6     | 72.5     | 63.3   | 56.7 | 44.3   | 50.4   | 57.9   | 46.6  | 55.5  |
| AugMix | 40.9  | 37.8     | 30.0     | 54.8     | 37.3   | 46.1 | 26.9   | 32.3   | 36.4   | 33.8  | 37.6  |
| NoisyMix (ours) | 25.1 | 22.8     | 30.2     | 41.5     | 32.4   | 34.1 | 20.5   | 24.5   | 27.7   | 26.3  | 28.5  |

Table 13: Detailed results for ResNet-18s trained with various data augmentation schemes and evaluated on CIFAR-10-P. The values indicate the flipping rates as a function of the 10 perturbation types (lower values are better). The mean flip rate (mFR) summarizes the results and shows that NoisyMix improves perturbation stability by approximately 1%.

| Noise   | Blur  | Weather  | Digital  | mFR  |
|---------|-------|----------|----------|------|
|          | White | Motion   | Snow     | Translate | Rotate | Tilt | Scale |       |
| Baseline | 44.6  | 27.6     | 9.2      | 0.4      | 2.5    | 0.5  | 2.1    | 2.8    | 0.8    | 3.1   | 9.4   |
| Mixup | 26.3  | 17.2     | 6.1      | 0.4      | 1.8    | 0.6  | 2.2    | 2.4    | 0.8    | 2.5   | 6.0   |
| Manifold Mixup | 33.4 | 21.6     | 7.8      | 0.4      | 2.1    | 0.6  | 2.2    | 2.6    | 0.8    | 2.9   | 7.4   |
| NFM | 11.5  | 7.5      | 6.8      | 0.3      | 1.5    | 0.4  | 1.8    | 2.0    | 0.6    | 2.2   | 3.5   |
| CutMix | 72.3  | 50.8     | 9.1      | 0.5      | 1.9    | 0.6  | 1.9    | 2.3    | 0.8    | 3.1   | 14.3  |
| Puzzle Mix | 16.1 | 11.2     | 6.3      | 0.3      | 1.4    | 0.4  | 2.3    | 1.8    | 0.6    | 2.5   | 4.3   |
| AugMix | 11.4  | 7.1      | 2.4      | 0.2      | 1.0    | 0.4  | 1.9    | 1.2    | 0.5    | 1.8   | 2.8   |
| NoisyMix (ours) | 6.6  | 4.5      | 2.3      | 0.1      | 0.9    | 0.4  | 1.6    | 1.0    | 0.4    | 1.4   | 1.9   |

Table 14: Detailed results for Wide-ResNet-28x2 trained with various data augmentation schemes and evaluated on CIFAR-10-P. The values indicate the flipping rates as a function of the 10 perturbation types (lower values are better). The mean flip rate (mFR) summarizes the results and shows that NoisyMix improves perturbation stability by approximately 5%.

| Noise   | Blur  | Weather  | Digital  | mFR  |
|---------|-------|----------|----------|------|
|          | White | Motion   | Snow     | Translate | Rotate | Tilt | Scale |       |
| Baseline | 42.7  | 27.6     | 8.2      | 0.3      | 2.1    | 0.4  | 1.4    | 2.4    | 0.6    | 2.6   | 8.8   |
| Mixup | 30.9  | 18.5     | 5.7      | 0.3      | 1.5    | 0.5  | 1.6    | 1.8    | 0.7    | 2.1   | 6.4   |
| Manifold Mixup | 43.8 | 26.6     | 7.0      | 0.3      | 1.6    | 0.4  | 1.3    | 1.9    | 0.6    | 2.2   | 8.6   |
| NFM | 13.1  | 7.8      | 8.0      | 0.3      | 1.4    | 0.4  | 1.4    | 2.0    | 0.6    | 2.1   | 3.7   |
| CutMix | 78.0  | 60.1     | 9.6      | 0.5      | 1.9    | 0.6  | 1.5    | 2.4    | 0.8    | 3.2   | 15.9  |
| Puzzle Mix | 14.6 | 9.5      | 7.1      | 0.4      | 1.3    | 0.5  | 1.8    | 1.9    | 0.7    | 2.5   | 4.0   |
| AugMix | 11.2  | 6.6      | 2.1      | 0.2      | 0.9    | 0.3  | 1.3    | 1.0    | 0.4    | 1.5   | 2.5   |
| NoisyMix (ours) | 6.3  | 4.2      | 2.0      | 0.1      | 0.7    | 0.3  | 1.0    | 0.8    | 0.3    | 1.1   | 1.7   |