Production of axial-vector $D_{sJ}$ in $e^+e^-$ annihilation

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Abstract

If one of the recently discovered charmed-strange mesons ($D_{sJ}(2317)$) is the $0^+$ state of $c\bar{s}$, the other ($D_{sJ}(2460)$) is most likely the $1^+$ state with $j = \frac{3}{2}$. They could be produced in $e^+e^-$ annihilation at $E_{cm} = m_{\Upsilon(4S)}$ either by fragmentation from $c\bar{s}$ jets or as decay products of the $B$ mesons from $\Upsilon(4S) \rightarrow B\bar{B}$. If one analyzes the $c\bar{s}$ jet events and the $\Upsilon(4S)$ decay events separately, one will have a direct test as to whether $D_{sJ}(2460)$ is the $j = \frac{1}{2}$ state or not, how much $D_{sJ}(2460)$ is mixed with the $j = \frac{3}{2}$ state, and also whether the four-quark interpretation is viable.

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I. INTRODUCTION

The BaBar Collaboration [1] discovered a narrow peak at 2317 MeV in the invariant mass of $D_s\pi^0$. The decay into $D_s\pi^0$ suggests $J^P = 0^+$ for this state $D_{sJ}(2317)$ since good candidates already exist for $1^-$ and $2^+$. The observation of this peak was subsequently confirmed by the CLEO [2] and the Belle Collaboration.

The BaBar data show another sharp peak at 2460 MeV in the invariant mass of $D_s^*\pi^0$ ($D_s\gamma\pi^0$). Although BaBar was initially cautious in calling it as a resonance, CLEO concluded that it is another narrow resonance. Absence of the decay $D_{sJ}(2460) \not\rightarrow D_s\pi^0$ leads us to speculate that $D_{sJ}(2460)$ is one of $J^P = 1^+$ states and forms with $D_{sJ}(2317)$ a $j = \frac{1}{2}$ multiplet in terms of total momentum $j = 1 + s$ of the $\bar{s}$ quark in the heavy-light limit of $c\bar{s}$. Then the existing $1^+$ resonance $D_{s1}(2536)$ [3] should be assigned to a $j = 3/2$ multiplet with the $2^+$ candidate $D_{sJ}(2573)$.

Theorists have worked on spectroscopy of the heavy-light quark states for long time. Schnitzer [4] argued on the basis of the $l$-$s$ coupling in a naive two-body potential model that for heavy-light mesons, the $j = \frac{3}{2}$ multiplet should be lighter than the $j = \frac{1}{2}$ multiplet. It was pointed out many years later that $K$ meson resonances seemed to show this “inversion” of the spin-orbit coupling sign despite the relative lightness of the $s$-quark [5]. Then the spin-orbit inversion in the heavy-light system was studied systematically by Isgur [6]. More recently a detailed computation was presented in the potential model [7]. However, the potential picture has an uncertainty of long-distance physics since the energy scale of the heavy-light potential is the reduced mass, i.e., the light mass, not by the heavy mass. Upon the discovery of $D_{sJ}(2317)$, Cahn and Jackson [8] re-examined the $c\bar{s}$ states with the potential model. It is fair to say that the potential model is inconclusive about which of the $j$-multiplets is heavier than the other. By keeping the inversion scenario back in mind, many theorists proposed the exotic possibility that $D_{sJ}(2317)$ is a four-quark state or its mixing [9,10]. A cursory examination by lattice QCD was also reported [11].

Another approach to the heavy-light mesons is to treat the light component as a chiral-symmetric cloud instead of a constituent quark [12]. Bardeen et al made a strong case for noninversion of $j = \frac{1}{2}$ and $j = \frac{3}{2}$ with detailed calculations [13]. Among others the observed $D_{sJ}$ mass difference, $m(2460) - m(2317) \simeq m(1^-) - m(0^-) = 143$ MeV, is a successful consequence of chiral symmetry. As for magnitude of the splitting between $j = \frac{1}{2}$ and $\frac{3}{2}$, however, an earlier work [14] had found much larger uncertainty than Bardeen et al. did.

While the case for $j = \frac{1}{2}$ appears strong for $D_{sJ}(2317)$ and $D_{sJ}(2460)$, an independent experimental confirmation is desirable. At $E^{\text{cm}}_{\gamma\pi}$ = the $\Upsilon(4S)$ mass, $D_{sJ}$ can be produced through $c\bar{s}$ jets or $B$ decays. Two types of processes occur with roughly the same rate and can be separated without difficulty by event topology and by $B$ decay vertices. We point out here that production rates of the axial-vector $D_{sJ}$ in $c\bar{s} \rightarrow D_{sJ}X$ and in $\Upsilon(4S) \rightarrow B\overline{B} \rightarrow D_{sJ}X$ are sensitive to $j = \frac{1}{2}$ vs $\frac{3}{2}$ and therefore that analyzing the $D_{sJ}(2460)$ production in $c\bar{s}$ and in $B\overline{B}$ separately will give us an additional clue about as to whether $D_{sJ}(2460)$ is $j = \frac{1}{2}$ or

$^{1}$Since then, however, the measured branching fractions of $\tau \rightarrow K_1\nu_\tau$ had shifted so that the inversion argument is no longer supported by $\tau$ decay data. Only the $s$-$d$ ratio of $K_1 \rightarrow K^*\pi/\rho K$ may favor the inversion, if at all.
\( \frac{3}{2} \) of \( c\bar{s} \), or else a four-quark state \( c\bar{s}q\bar{q} \). The two types of production occur with roughly the same rate and separable by event topology and \( B \) decay vertices.

II. B DECAY

Production of charmed strange mesons is one of the dominant nonleptonic \( B \) decay processes. It occurs mainly through the effective decay operators of tree type:

\[
\mathcal{O}_1 = (\tau s)_{V-A} (\tau c)_{V-A}, \\
\mathcal{O}_2 = (\bar{c} s)_{V-A} (\bar{b} s)_{V-A},
\]

where colors are contracted within each bracket. Although detailed comparison between theory and experiment has not been available for \( B \to D_s \), the similar decays \( B \to D\pi \), \( D^*\pi \), and \( D\rho \) and so forth that occur through \( (ud)_{V-A} (bc)_{V-A} \) and \( (uc)_{V-A} (bd)_{V-A} \) have been measured with good accuracy \([3]\). These color-favored two-body decays agree well with the theoretical values computed in the factorization approximation. The factorization is even simpler for the inclusive color-favored decays since they are free from the quark distribution involving the spectator quark. We therefore proceed by assuming that the color-favored decay \( B \to D_{sJ} X \) is described by the factorization.\(^2\) In the factorization limit only \( D_{sJ} \) of 0\(^-\), 1\(^-\), and 1\(^+\) can be produced. As for the 1\(^+\) states, we shall see below that the 1\(^+\) state of \( j = \frac{1}{2} \) would be produced preferentially for \( m_s/m_c \ll 1 \).

In the simple factorization the inclusive decay \( B \to D_{sJ} X \) is determined by the short-distance quark decay of \( b \to c \). While inclusive production of 0\(^-\) and 1\(^-\) in raw data contains the contribution from strong and electromagnetic cascade decays of higher \( D_{sJ} \) states, production of 1\(^+\) is very likely free of such contamination. The reason is that only the radially excited 0\(^-\) and 1\(^\pm\) states below the \( DK \) threshold are possible sources of cascade decays down to the 1\(^+\) states in the factorization. Such excited states have not been seen in experiment. They are expected to be above the \( DK \) threshold. Therefore the \( D_{s1}(2460) \) reconstructed in \( B \) decay may be counted entirely as primary decay products of \( B \) meson.

The production amplitude for \( b \to 1^+\tau \) is given by

\[
A(b \to D_{s1}\tau) = (G_\mu/\sqrt{2}) V^*_{cb} V_{cs} (C_1 + C_2/3) f_{A_j} m_{D_{s1}} e^\mu (\bar{\tau} b \gamma_\mu \gamma_5 v_c), \quad (j = 1/2, 3/2)
\]

where the axial-vector decay constant \( f_{A_j} \) is defined with the normalization \( \langle p|p' \rangle = (2\pi)^3 2E_p \delta(p - p') \) by

\[
\langle D_{s1}(p, \epsilon) |(\tau \gamma^\mu \gamma_5 s) |0 \rangle = f_{A_j} m_{D_{s1}} e^\mu.
\]

We have introduced the additional subscript \( j \) for \( D_{s1} \) to distinguish between two eigenstates of \( j^2 \). If one evaluates Eq. (3) in the rest frame of \( D_{s1} \) by treating \( D_{s1} \) as being made of the \( c \)-quark and the remainder carrying the \( \bar{s} \)-quark quantum numbers (still denoted by \( \bar{s} \)), the left-hand side is written in the Pauli spinors as

\[2\]This is not true for the color-suppressed decays such as \( B^0 \to D^0 \pi^0 \) and charmonium production. The factorization-forbidden charmonia of \( J^{PC} = 0^{++} \) and \( 2^{++} \) are abundantly produced. \([15]\)
\[
\langle D_{s1j}(0, \epsilon, j)|(\bar{\psi} \gamma_5 \sigma s)|0 \rangle = N \left[ \left( \frac{1}{E_s + m_s} + \frac{1}{E_c + m_c} \right) i \chi_c(p_s \times \sigma) \chi_\pi^\dagger \right. \\
+ \left. \left( \frac{1}{E_s + m_s} - \frac{1}{E_c + m_c} \right) \chi_\pi^\dagger p_s \chi_\pi \right],
\]
where \( N \) is an normalization factor. The spinors form \( ^3P_1 \) and \( ^1P_1 \) in the first and the second term, respectively, in the right-hand side of Eq. (4). In the limit of \( m_s/m_c \to 0 \), the right-hand side approaches \( \chi^j \sigma (\sigma \cdot p_s) \chi^\pi \) up to an overall constant, which is exactly the \( j = \frac{1}{2} \) combination of \( ^3P_1 \) and \( ^1P_1 \). Therefore, the weak axial-vector current can produce only the \( j = \frac{1}{2} \) state of \( D_{s1} \) in the large \( m_c \) limit [16]. For \( m_s/m_c \neq 0 \), the weak current produces the combination of
\[
|D_{s1\frac{1}{2}}\rangle \cos \alpha - |D_{s1\frac{3}{2}}\rangle \sin \alpha,
\]
where
\[
\tan \alpha = \frac{2\sqrt{2}}{3(E_c + m_c)/(E_s + m_s) + 1}
\]
in the phase convention of \( |j = \frac{1}{2}\rangle = \sqrt{\frac{3}{2}} |^1P_1\rangle - \sqrt{\frac{3}{2}} |^3P_1\rangle \) and \( |j = \frac{3}{2}\rangle = \sqrt{\frac{3}{2}} |^1P_1\rangle + \sqrt{\frac{3}{2}} |^3P_1\rangle \). For a nonrelativistic binding with \( m_s/m_c \simeq \frac{1}{3} \), the mixture of \( |j = \frac{3}{2}\rangle \) is small (\( \tan^2 \alpha \simeq 0.09 \) in probability) and production of \( D_{s1\frac{3}{2}} \) is almost negligible. To obtain the production rates of the mass eigenstates, one needs to know about a small mixing between \( j = \frac{1}{2} \) and \( \frac{3}{2} \) in the mass eigenstates:
\[
|D_{s1}(2460)\rangle = \cos \theta |D_{s1\frac{1}{2}}\rangle - \sin \theta |D_{s1\frac{3}{2}}\rangle,
|D_{s1}(2536)\rangle = \sin \theta |D_{s1\frac{1}{2}}\rangle + \cos \theta |D_{s1\frac{3}{2}}\rangle.
\]
Then the ratio of the branching fractions for two mass eigenstates is
\[
\frac{B(B \to D_{s1}(2536)X)}{B(B \to D_{s1}(2460)X)} = \tan^2(\alpha - \theta).
\]
In the case that \( D_{s1}(2460) \) consists mostly of \( j = \frac{1}{2} \), the mixing angle \( \theta \) is \( O(m_s/m_c) \) so that the production rate of \( D_{s1}(2536) \) is one order of magnitude smaller than that of \( D_{s1}(2460) \). Theoretical estimate of the value of \( \theta \) is not possible because of unknown long-distance effects. We should use Eq. (8) to determine the mixing angle \( \theta \) albeit Eq. (6) has some model dependence.

If the value of the decay constant \( f_{A_j} \) is given for \( j = \frac{1}{2} \), we can compute the branching fraction of the inclusive \( B \to D_{s1\frac{1}{2}}X \) decay. The authors in [16] gave one estimate, which corresponds to \( f_A(= f_{A\frac{1}{2}}) \simeq 200\text{MeV} \) for \( m_c \simeq 1.35\text{GeV} \) in our definition Eq. (3). Using this value in the amplitude of Eq. (2), we obtain with a straightforward computation the branching fraction,
\[
\frac{B(B \to D_{s1\frac{1}{2}}X)}{\Gamma(B \to Xl^+\nu_l)_{\text{th}}} \times \Gamma(B \to D_{s1\frac{1}{2}}X),
\simeq (1.7 \times 10^{-2}) \times (f_A/200\text{MeV})^2,
\]

where $\mathcal{B}(B \rightarrow Xl^+\nu_l)_{\text{exp}} \simeq 0.104$ [3], $C_1 + \frac{1}{3}C_2 \simeq 1.02$ [17], and the short-distance corrected value of $\Gamma(B \rightarrow Xl^+\nu_l)_{\text{th}}$ have been used to obtain the numerical result. This number is subject to uncertainty of the values for $m_b(\simeq 4.5\text{GeV})$ and $m_c(\simeq 1.35\text{GeV})$. The branching fraction is quite large since it is one of the dominant factorizable processes. If we compare two-body decays $B \rightarrow D_{s1\frac{1}{2}}\overline{D}$ and $D_{s}\overline{D}$, for example, we find

$$\mathcal{B}(B \rightarrow D_{s1\frac{1}{2}}\overline{D})/\mathcal{B}(B \rightarrow D_{s}\overline{D}) \simeq 1.5 \times (f_A/f_{D_s})^2, \tag{10}$$

where $f_{D_s}$ is the decay constants of $D_s$.

III. FRAGMENTATION FROM CHARM-ANTICHARM JET

Fragmentation of a heavy meson from a heavy quark was studied by many theorists in perturbative pictures [18]. In the heavy limit of a heavy quark, the fragmentation functions for $^1P_1$ and $^3P_1$ have a similar dependence on $z = 2E/m_b$ and ratio of the integrated fragmentation probabilities is $\simeq 2/3$ in the perturbative calculation [19] when the $^3P_1 - ^1P_1$ mass splitting is ignored. In terms of the ratio of $j = \frac{1}{2}$ to $\frac{3}{2}$, this number corresponds to $\simeq 0.88$. After the $O(m_s/m_c)$ corrections are included, the ratio shifts a little but stays close to unity: $\int D_{j=\frac{1}{2}}(z)dz/\int D_{j=\frac{3}{2}}(z)dz \simeq 1$. Actually, physics of fragmentation of a heavy-light meson is not entirely a short-distance process even if one takes the heavy quark limit. We do not know of how to estimate nonperturbative effects reliably. One can understand complexity of long-distance effects if one thinks of cascade feeding from higher resonance states.

For two $1^+$ states, the orbital wavefunctions are the same in nonrelativistic models. Cascade contributions are unimportant since higher resonances decay into $DK$ channels. If dominant long-distance effects are spin independent like the confining force, the ratio of the fragmentation probabilities would not change much with long-distance effects. In the absence of a compelling reason for otherwise, it is not unreasonable to expect that nonperturbative effects do not upset the perturbative prediction on the fragmentation ratio:

$$\mathcal{B}(e^+e^- \rightarrow c\overline{c} \rightarrow D_{s1\frac{1}{2}}X) \simeq \mathcal{B}(e^+e^- \rightarrow c\overline{c} \rightarrow D_{s1\frac{3}{2}}X). \tag{11}$$

This is markedly different from $B$ decay in which $D_{s1\frac{1}{2}}$ is dominantly produced.

IV. FOUR-QUARK STATE

Many theorists proposed [9] that $D_{sJ}(2317)$ may be an $0^+$ state of $c\overline{c}q\overline{q}$. Although it has been speculated that some of light scalar mesons might be four-quark states [20], we have not yet had a resonance that is proven to be a four-quark meson. Consequently, we do not have much knowledge of dynamical properties of four-quark states such as production and decay.

We consider four-quark states, which we denote them generically by $D_{s}^{(4)}$. First in $B$ decay. The production amplitude for $D_{s}^{(4)}$ is obtained by superposition of a four-quark production amplitude in momentum space:
\[ A_{B \to D_s^{(4)} X}(p_X) = \prod_{i=1,2,3} \int \frac{d^3q_i}{(2\pi)^3} \tilde{\Psi}(q_i) A_{\overline{b} \to (\pi q q)\pi}(p_X, q_i), \]  

where \( \tilde{\Psi}(q_i) \) is the four-quark wavefunction in momentum space, \( q_i \) \( (i = 1, 2, 3) \) are the relative quark momenta inside \( D_s^{(4)} \), and \( p_X \) is momentum of \( D_s^{(4)} \). In the loose-binding approximation, the decay amplitude turns into a simple form,

\[ A_{B \to D_s^{(4)} X}(p_X) \approx \tilde{\Psi}(0) A_{\overline{b} \to (\pi q q)\pi}(p_X, 0), \]

where \( \tilde{\Psi}(0) \) is the four-quark wavefunction in coordinate space with all three relative coordinates set equal to zero. \( |\tilde{\Psi}(r_j)|^2 \) has dimension of the ninth power of energy. For a loosely bound molecular \( DK \) state,

\[ |\tilde{\Psi}(0)|^2 = O(\Lambda_{QCD}^{-6} \Delta^3) \]

where \( \Delta \) is the binding energy. For intrinsic four-quark states in which no \( q \overline{q} \) pair is in a color-singlet, \( |\tilde{\Psi}(0)|^2 \) would be comparable with or smaller than that of the molecular state since the binding is loose. As for the production amplitude, the relevant effective interactions are six-quark operators. The dominant interaction is of the form

\[ \mathcal{L}_{\text{int}} \sim (G_\mu/\sqrt{2}) V_{cb}^* V_{cs}(\pi \alpha_s / E^3)(\bar{b} \gamma\mu(1 - \gamma_5)c)(\bar{q} q)=(s)\mu, \]

where \( (\bar{q} q)=(s)\mu \) is a Lorentz vector made of four quark fields such as \( (\bar{q} \gamma\mu \partial_\nu q)(\bar{q} \gamma^\nu s) \), and \( E \) is determined by the energy scale involved in creation of \( q \) and \( \overline{q} \). Then simple dimension counting gives us

\[ \Gamma(B \to D_s^{(4)} X) \sim G^2 |V_{cb}^* V_{cs}|^2 (\pi \alpha_s)^2 \left| \frac{\tilde{\Psi}(0)}{m_{D_s^{(4)}}^2} \right|^2 \frac{m_{B}^3}{E^0}. \]

Using Eq. (14), we can estimate the branching fraction. If we express in the ratio to the branching to \( B \to D_s X \) for comparison,

\[ \mathcal{B}(B \to D_s^{(4)} X) \sim (\pi \alpha_s)^2 \left( \frac{\Lambda_{QCD}}{E} \right)^6 \left( \frac{\Delta^3}{m_{D_s^{(4)}}^2 f_{D_s}^2} \right) \times \mathcal{B}(B \to D_s X). \]

Since the four-quark binding energy \( \Delta \) is much smaller than \( m_{D_s}^{(4)} \) and \( f_{D_s} \), \( \mathcal{B}(B \to D_s^{(4)} X) \) is minuscule as compared with \( \mathcal{B}(B \to D_s X) \). The same estimate as Eq. (17) holds for perturbative fragmentation of \( D_s^{(4)} \) from \( c \overline{c} \) jets. We thus conclude that short-distance production of \( D_s^{(4)} \) is negligibly small both in \( B \) decay and in jet fragmentation. In fact, suppression of a loosely bound multiparticle state is a general rule at high energies. The factor \( \Delta^3 / m_{D_s}^2 f_{D_s}^2 \) in Eq. (17) comes from the ratio of the wavefunctions of \( D_s^{(4)} \) and \( D_s \). However, there is a chance of large long-distance enhancement in the final state if \( D_s^{(4)} \) and a \( c \overline{c} \) meson state of the same quantum numbers happen to be almost degenerate in mass. In this case the transition of \( D_{sJ}(c \overline{c}) \leftrightarrow D_s^{(4)} \) is enhanced by the factor \( |\tilde{\Psi}(0)|^2 / (M_{D_s^{(4)}}^2 - M_{D_{sJ}}^2) \). This enhancement factor is square of the \( D_{sJ} - D_s^{(4)} \) mixing itself. We may state therefore that production of \( D_s^{(4)} \) is possible only if it has a large mixing to a quark-antiquark meson state [10].
How much could $D_s(2317)$ be produced if it is the $0^+$ state of $D_s^{(4)}$ mixed with a $c\bar{s}$ state? Since $D_s(c\bar{s})$ of $J^P = 0^+$ cannot be produced in the factorization, a mixing to it does not help production of $D_s^{(4)}$ in $B$ decay. On the other hand $D_s^{(4)}$ production through fragmentation would be realized if it is mixed substantially with $D_{sJ}(c\bar{s})$ of $J^P = 0^+$. Since the ratio of fragmentation functions of $0^+$ and $1^+$ is approximately 0.36 for $m_s/m_c \simeq 1/3$ in the perturbative calculation [19], even a 50-50 mixing to $D_{s0}(c\bar{s})$ would allow fragmentation of $D_s^{(4)}$ only at the level of one fifth of $D_{s1\frac{1}{2}}$. If $D_s^{(4)}$ of $J^P = 1^+$ mixes with $D_{s1\frac{1}{2}}$ strongly, we may be able to see it in $B$ decay. In fragmentation, mixing to $1^+$ of either $j = \frac{1}{2}$ or $j = \frac{3}{2}$ helps $D_s^{(4)}$ production.

V. SUMMARY

The BaBar, CLEO, and Belle Collaborations should be able to sort out $\Upsilon(4S)$ events and $c\bar{s}$ jet events by event topology and by $B$ meson decay vertices. By analyzing the two types of events separately, we shall obtain useful information as to which of $D_{s1}(2460)$ and $D_{s1}(2536)$ is $j = \frac{1}{2}$ or $j = \frac{3}{2}$. The $1^+$ meson produced abundantly in $B$ decay is $D_{s1\frac{1}{2}}$. The $D_{s1\frac{1}{2}}$ meson should be looked for in the $c\bar{s}$ jet events. If both $D_{s1j}$ states should happen to be produced significantly in $B$ decay, or if only one of two $D_{s1j}$ states is produced from the $c\bar{s}$ jet, it would be an indication of a large mixing between $j = \frac{1}{2}$ and $\frac{3}{2}$, that is, failure of the heavy quark approximation to the $c$ quark.

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