Analysis of Chatter and Turning Errors of Basalt Fiber Reinforced Polymer Concrete Lathe

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Abstract: In order to improve the turning accuracy of the lathe, a basalt fiber reinforced polymer concrete lathe bed was designed. In the light of theory, the turning error model of the lathe, the vibration of the lathe-workpiece system and the average amplitude calculation model of turning error are established. The Runge-Kutta method is used to calculate the vibration of the lathe-workpiece. The influence of the workpiece radius and turning parameters on the average amplitude of the turning error is studied. The results shows that average amplitude of the turning error is significantly influenced by feed rate, workpiece radius, back engagement and spindle speed from large to small.

1. Introduction

At present, most of traditional lathe beds are made of cast iron material. The cast iron material is widely used because of their ideal mechanical properties, such as high strength, good stiffness and easy processing, but their damping and thermal stability is not as per the standards of the modern machine tool manufacturing. The advantages of basalt fiber reinforced polymer concrete is high damping property, high specific strength and stiffness, better thermal stability\textsuperscript{[1,2]}. Therefore, a basalt fiber reinforced polymer concrete lathe bed was designed and manufactured. The study aims to improve the turning accuracy of the lathe by using these advantages of basalt fiber reinforced polymer concrete. In the existing literature on the vibration of traditional lathes, most of the discussion is about the linear vibration of lathe-workpiece system. Generally, these studies failed to reflect the actual turning vibration effectively because of ignoring the non-linear factors in the system\textsuperscript{[3,4]}. It is no doubt that these studies have limitations, especially for the fast feed rate of turning. This paper studies the non-linear vibration reaction of basalt fiber reinforced polymer concrete lathe under the integrative action of all the nonlinear factors, such as the stiffness of the lathe-workpiece system, the tool-post and guide rail system, the turning force.

2. Structure Design of Basalt Fiber Reinforced Polymer Concrete Bed

The basalt fiber reinforced polymer concrete lathe bed is designed, based on a type of traditional lathe bed, as shown in Fig.1. The bed is mainly made up of basalt fiber reinforced polymer concrete foundation and steel guide rail. The two guide rails are connected by bolts embedded in the basalt fiber reinforced polymer concrete lathe bed.
3. Error Analysis of Turning process

When the cylindrical turning, the bed and the tool and the workpiece position relationship is shown in Fig.2. The coordinate system $O_{xyz}$ is built at the center of mass of the bed, the vertical distance of the spindle center of the lathe to the upper surface of the guide rail is $H$, the distance between the upper surface of the guide rail and the central mass of the bed is $h = 125$. The workpiece speed is $n$, the initial radius is $r_0$, the desired radius of turning is $r$. After cutting the basic back engagement is $a_p$, the ideal position of the turning tool is horizontal position, as shown in the figure. However, the central mass of the bed and the rotation of the workpiece center position both have changed slightly, due to turning force in turning process, so the actual radius of the workpiece is $r_1$. In order to work out the $r_1$ of the workpiece, the displacement of the center of mass of the bed and the center of rotation of the workpiece are separated as shown in Fig.2.

(1) The deflection of the lathe bed around its center of mass is $\theta_1(t)$, and the displacements along the $y$ and $z$ axes are $y_1(t)$ and $z_1(t)$, respectively, as the procedure $\overset{1}{\rightarrow}\overset{2}{\rightarrow}\overset{3}{\rightarrow}$ in Fig.2.

The coordinate of the cutter tip in the horizontal position is $(y, z) = (r, H + h)$, and its angle with the $z$ axis is $\theta = \arctan[r/(h + H)]$, the distance between the cutter tip and the $O$ point is

$$ l = \sqrt{(H + h)^2 + r^2} \tag{1} $$

Through the process $\overset{1}{\rightarrow}$, the coordinate of the cutter tip is

$$(y, z) = [l \cdot \sin(\theta + \theta_1(t)), l \cdot \cos(\theta + \theta_1(t))] \tag{2}$$

Through the process $\overset{2}{\rightarrow}$, the coordinate of the cutter tip is

$$(y, z) = [l \cdot \sin(\theta + \theta_1(t)), l \cdot \cos(\theta + \theta_1(t)) - z_1(t)] \tag{3}$$

Through the process $\overset{3}{\rightarrow}$, the coordinate of the cutter tip is

$$(y, z) = [l \cdot \sin(\theta + \theta_1(t)) + y_1(t), l \cdot \cos(\theta + \theta_1(t)) - z_1(t)] \tag{4}$$

(2) The displacements of the workpiece along the $y$ and $z$ axes are $u_1(t)$ and $v_1(t)$, respectively, through step $\overset{4}{\rightarrow}\overset{5}{\rightarrow}$, shown in Fig.2.

The initial coordinate of the rotation center of the workpiece is $(y, z) = (0, H + h)$; through the process $\overset{4}{\rightarrow}$, the coordinate of the rotation center of the workpiece is

$$(y, z) = [-u_1(t), H + h] \tag{5}$$

Through the process $\overset{5}{\rightarrow}$, the coordinate of the rotation center of the workpiece is

$$(y, z) = [-u_1(t), H + h + v_1(t)] \tag{6}$$

After all of the above processes, the actual radius of the workpiece can be expressed as

$$r_1(t) = \sqrt{[l \cdot \sin(\theta + \theta_1(t)) + y_1(t) + u_1(t)]^2 + [l \cdot \cos(\theta + \theta_1(t)) - z_1(t) - (H + h + v_1(t))]^2} \tag{7}$$

The basic back engagement of $a_p$ can be expressed as

$$a_p = r_0 - r \tag{8}$$

The turning error of the workpiece can be expressed as

$$\Delta(t) = r_1(t) - r \tag{9}$$

The actual back engagement can be expressed as
4. Vibration Analysis of Turning Process

4.1 Analysis of turning force

The surface of the workpiece is corrugated\cite{5,6}, as shown in Fig.3. The turning error $\Delta(t)$, analyzed above, is generated when the lathe vibrates. In addition, taking into account the error $\Delta(t-T)$ formed by the previous pass, the actual size of the back engagement can be expressed as $[a_p+\mu\Delta(t-T)-\Delta(t)]$, during two adjacent cutting. Where $\mu$ is the overlap coefficient and $T$ is the time interval between adjacent two cutting.

The force of the turning tool is decomposed along the $y$-axis, $z$-axis and $x$-axis, exponential form of the cutting force calculation formula is modified, due to the actual effect of the size of the back engagement. The three component forces can be expressed as

$\begin{align*}
F_c[a_p, f, v_c, \Delta(t), \Delta(t-T)] &= C_{F_c}[a_p + \mu\Delta(t-T) - \Delta(t)]^\nu_c f^{\nu_{f_c}} v_c^{n_{f_c}} K_{F_c} \\
F_f[a_p, f, v_c, \Delta(t), \Delta(t-T)] &= C_{F_f}[a_p + \mu\Delta(t-T) - \Delta(t)]^\nu_f f^{\nu_{f_c}} v_c^{n_{f_c}} K_{F_f} \\
F_p[a_p, f, v_c, \Delta(t), \Delta(t-T)] &= C_{F_p}[a_p + \mu\Delta(t-T) - \Delta(t)]^\nu_p f^{\nu_{f_c}} v_c^{n_{f_c}} K_{F_p}
\end{align*}$

Where $F_c, F_f, F_p$ are the main cutting force, the feeding force, and the radial thrust force; $C_{F_c}, C_{F_f}, C_{F_p}$ are the processing condition coefficients; $x_F, y_F, n_F$ are the influencing factors of each factor on the cutting force; $K_{F_c}, K_{F_f}, K_{F_p}$ are the correction coefficients; $v_c, f$ are the cutting speed and feed size.

According to Fig.3, it can be seen that the size of the bed by the torque in the $x$-axis direction can be expressed as

$M_s[a_p, f, v_c, \Delta(t), \Delta(t-T)] = [l \cdot \sin(\theta + \theta_1(t)) + y_1(t)] \times F_c + [l \cdot \cos(\theta + \theta_1(t)) - z_1(t)] \times F_p$

4.2 Lathe chatter model

The lathe-workpiece chatter system is shown in Fig.4, the equivalent mass of the lathe bed and the tool-post and the spindle box is $m_1$, the equivalent moment of inertia is $J$, the equivalent mass of the workpiece is $m_2$. The equivalent spring stiffness and damping of the bed along the $y_1$-axis, $z_1$-axis are $k_1, c_1$ and $k_2, c_2$. The equivalent torsional spring stiffness and damping of the bed around the $x$-axis are $k_c$ and $c_c$, which are not shown in the Fig.4. The equivalent stiffness and damping of the tool-post and guide rails system are $k_3$ and $c_3$. The equivalent spring stiffness and damping of the workpiece along the $u_1$-axis, $v_1$-axis are $k_4, c_4$ and $k_5, c_5$ respectively. $l_1$ is the vertical distance between the center of mass of the tool-post and the center of mass of the bed. The displacement and rotation of the bed center of mass are $y_1, z_1$ and $\theta_1$. The displacement of the workpiece center of mass are $u_1, v_1$. The vibration equation of the system is established by the Lagrange equation, which can be expressed as
4.3 Spring stiffness of lathe-workpiece system

4.3.1 Spring stiffness of bed

The error along the x-axis is ignored, because the spring stiffness of the x-axis direction of the bed is large, so the deformation of this direction is small. The spring stiffness of bed can be expressed as

\[
k_1 = 3EI_y a_1(t)^3 [a_o - a(t)]^2; k_2 = 3EI_z a_1(t)^3 [a_o - a(t)]^2, 0 < a(t) < a_o
\]

Where \(k_1, k_2\) are the spring stiffness of the y-axis and z-axis of the bed; \(EI_y, EI_z\) are the equivalent deflection rigidity of the y-axis and z-axis of the bed; the effective span of the bed is \(a_o\). The distance between the position of the cutting force and the fixed end of the bed is \(a(t)\) which can be expressed as \(a(t) = a_1 + f \cdot n \cdot t\), when the time of is \(t\). The initial distance between the position of the cutting force and the fixed end is \(a_1\).

The torsion spring stiffness of the bed can be expressed as

\[
k_c = GI_p / [a_o - a(t)] + GI_p / a(t), 0 < a(t) < a_o
\]

Where \(GI_p\) is the equivalent torsional rigidity.

4.3.2 Spring stiffness of tool-post and guide rail system

The spring stiffness of system is the segmentation function during the loading process, due to the gaps between the components of the system. The spring stiffness of the system is measured by the octagonal loop resistance turning dynamometer, and the spring stiffness of the system can be expressed as

\[
k_3 = \begin{cases} 
1.187 \times 10^5 N/m, F_{lim} < 432 N, \text{Loading stage} \\
2.507 \times 10^7 N/m, F_{lim} \geq 432 N, \text{Loading stage} \\
4.979 \times 10^7 N/m, \text{Unloading stage}
\end{cases}
\]

4.3.3 Spring stiffness of workpiece

The cross section shape of the workpiece is circular and in a rotating state, so the workpiece’s spring stiffness of the y-axis and z-axis directions are observed. The workpiece is fixed between the chuck and the live center, and the spring stiffness of the workpiece is deduced by the deflection formula of the simply supported beam, which can be expressed as

\[
k_4 = k_5 = 3EI_{wp}a_{wp}a(t)^3 [a_{wp} - a(t)]^2, 0 < a(t) < a_{wp}
\]

Where \(k_4\) and \(k_5\) are the spring stiffness of the workpiece in the y-axis and z-axis directions. \(I_{wp}\) is the moment of inertia of the workpiece cross section, \(I_{wp} = \pi r^4/4\). \(E_{wp}\) is the elastic modulus of the workpiece. \(a_{wp}\) is the length of the workpiece.

4.4 Damping of lathe-workpiece system

The specific damping of each part is calculated according to formula (13), which can be expressed as

\[
c_i = 2\zeta_c \sqrt{k_i m_j}, c_c = 2\zeta_c \sqrt{k_c J}
\]

In the formula (18), when \(i = 1,2,3, j = 1\); when \(i = 4,5, j = 2\).

5. A Case

The calculation case is turning round parts, which turning tools and workpiece materials are carbide
and structural steel. The strength limit of structural steel materials is $\sigma_b = 650\text{MPa}$. The length of the workpiece is the span of the lathe, and the initial turning position is the middle of the workpiece. The turning parameters are $ap = 4\text{mm}$, $f = 0.4\text{mm/r}$ and spindle speed $n = 250\text{r/min}$. The parameters of the tool are $\kappa_r = 45^\circ$, $\gamma_o = 10^\circ$, $\lambda_o = 0^\circ$ and other parameters are shown in Tab.1.

**Tab.1** Main parameters\(^7\)

| Code name | Numerical value | Code name | Numerical value | Code name | Numerical value |
|-----------|----------------|-----------|----------------|-----------|----------------|
| $H$       | 0.205m         | $m_2$     | 207.9759kg     | $x_{Fc}$ | 1.0            |
| $h$       | 0.12445m       | $J$       | 7.758kg m\(^2\) | $x_{Ff}$ | 1.0            |
| $r$       | 0.075m         | $EI_p$   | 5.969E6Pa m\(^4\) | $x_{Fp}$ | 0.9            |
| $\mu$     | 1              | $EI_c$   | 2.926E7Pa m\(^4\) | $y_{Fc}$ | 0.75           |
| $l_c$     | 0.223m         | $GI_p$   | 1.399E7Pa m\(^4\) | $y_{Ff}$ | 0.5            |
| $a_o$     | 1.5m           | $E_{Fc}$ | 2.09E11Pa       | $y_{Ff}$ | 0.6            |
| $a_1$     | 0.75m          | $C_{Fc}$ | 2795            | $n_{Fc}$ | 0.15           |
| $a_{ap}$  | 1.5m           | $C_{Ff}$ | 2880            | $n_{Ff}$ | -0.4           |
| $m_1$     | 175.5053kg     | $C_{Fp}$ | 1940            | $n_{Fp}$ | -0.3           |

we introduce new variables $Y_1$, $Y_2$, $Y_3$, $Y_4$, $Y_5$, $Y_6$, $Y_7$, $Y_8$, $Y_9$ and $Y_{10}$ as $Y_1 = y_{Fc}$, $Y_2 = y_{Ff}$, $Y_3 = z_{Fc}$, $Y_4 = z_{Ff}$, $Y_5 = \dot{y}_{Fc}$, $Y_6 = \dot{y}_{Ff}$, $Y_7 = \ddot{u}_t$, $Y_8 = \ddot{u}_c$, $Y_9 = v_{Fc}$, $Y_{10} = v_{Ff}$ and vibration equation (13) as

$$
Y_z = \frac{1}{m_1} \left[ F_r - (c_1 + c_2) Y_z - l_1 c_1 Y_6 - (k_1 + k_2) Y_7 - l_2 k_2 Y_8 \right], \ Y_c = \frac{1}{m_1} \left[ F_r - c_2 Y_6 - k_2 Y_7 \right].
$$

(19)

6. Factors Affecting Turning Error Analysis

We use the method of analog roughness to describe the average amplitude of turning error. The vibration curve is divided into $l_{i1}$, $l_{i2}$, $l_{i3}$, …, which is the sampling curves, as shown in Fig.5. The red line in each sample curve is the mid line of amplitude. When the sampling length is small, it is assumed that the size of mid line is constant which can be expressed as

$$
A_j = \sum_{i=1}^{\Delta t_j} \Delta \omega_j(t) / n_j \quad i_j = 1, 2, 3, \ldots, n_j; \quad j = 1, 2, \ldots, p
$$

(20)

Where $A_j$ is the size of the $j$-th mid line. $n_j$ is the number of samples included in the $j$-th segment. $[\Delta \omega_j(t)]$ is the size of the $i$-th error for the $j$-th segment. $p$ is the total number of samples.

The average amplitude $\bar{A}$ of the turning error can be expressed as

$$
\bar{A} = \sum_{j=1}^{p} \left( \sum_{i=1}^{\Delta t_j} \Delta \omega_j(t) - A_j \right) / n_j / p \quad i_j = 1, 2, \ldots, n_j; \quad j = 1, 2, \ldots, p
$$

(21)
The effect of the workpiece radius and the turning parameters on the average amplitude of the turning error is noted, where the size of back engagement is 1~7 mm, workpiece radius is 25~100 mm, feed rate is 0.05~1.55 mm and spindle speed is from 100~400 r/min.

According to equations (19)~(21), the results are shown in Fig.7~Fig.10. According to Fig.7, it can be seen that the turning error increases linearly from 0.9942 μm to 2.14 μm nearly, when the back engagement is from 1 mm to 7 mm. Furthermore, it can be seen that the turning error quickly reduced from 242.2 μm to 0.2324 μm, and stabilized eventually, when the workpiece radius is from 25 mm to 100 mm, according to Fig.8. Furthermore, it can be seen that the turning error is rapidly increasing from 0.9262 μm to 22.36 μm, once the feed rate is from 0.05 mm to 1.55 mm, and the change appears slow firstly, and quick later, according to Fig.9. It is observed that the turning error is reduced from 1.802 μm to 1.382 μm with spindle speed is from 100 r/min to 400 r/min, explained by Fig.10. The order of influence of turning error is: feed rate, workpiece radius, back engagement and spindle speed, according to the calculation results of each average sensitivity.

7. Conclusions

The chatter model of basalt fiber reinforced concrete lathe bed-workpiece system is studied by theoretical analysis. The vibration response is analyzed in the case that the workpiece radius is 0.075 m, the length is 1.5 m, the initial turning position in the middle of the workpiece, and the back engagement $a_{p}=4$ mm, the feed rate $f=0.4$mm/r, the spindle speed $n=250$ r/min. The effect of the average amplitude of the turning error is from large to small: feed rate, workpiece radius, back engagement and spindle speed.

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