Testing Quantised Inertia on Galactic Scales.

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Abstract

Galaxies and galaxy clusters have rotational velocities (v) apparently too fast to allow them to be gravitationally bound by their visible matter (M). This has been attributed to the presence of invisible (dark) matter, but so far this has not been directly detected. Here, it is shown that a new model that modifies inertial mass by assuming it is caused by Unruh radiation, which is subject to a Hubble-scale (Θ) Casimir effect predicts the rotational velocity to be: \( v^4 = \frac{2GMc^2}{\Theta} \) (the Tully-Fisher relation) where G is the gravitational constant, M is the baryonic mass and c is the speed of light. The model predicts the outer rotational velocity of dwarf and disk galaxies, and galaxy clusters, within error bars, without dark matter or adjustable parameters, and makes a prediction that local accelerations should remain above \( 2c^2/\Theta \) at a galaxy’s edge.

1 Introduction

Zwicky (1933) first noticed that galaxies in galaxy clusters were moving too fast to be held together gravitationally by their visible matter, and proposed the existence of an invisible (dark) matter that provides the extra required gravitational pull. A similar problem in disc galaxy rotation was proven by the accurate rotation curves of Rubin et al. (1980). Dark matter is still the most popular explanation for these problems, but, after decades of searching, it has not been directly detected, though many efforts are ongoing, such as CDMS-II (2009) and XENON10 (2009).

Milgrom (1983) proposed an alternative explanation for galaxy rotation. He speculated that either 1) the force of gravity may increase or 2) the inertial mass \( m_i \) may decrease for the low accelerations at a galaxy’s edge. His empirical scheme, called Modified Newtonian Dynamics (MoND), can fit disc galaxy rotation curves, and has the advantage of being less tunable than dark matter. However, it does require one arbitrary parameter, the acceleration \( a_0 \), and it

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does not predict the dynamics of galaxy clusters (Aguirre et al., 2001, Sanders, 2002). The model proposed in this paper has no adjustable parameters, and fits these clusters more closely (it fits spirals less well, but is still within error bars).

Haisch et al. (1994) proposed that inertia might be due to a reaction to the magnetic component of Unruh radiation, which is seen only by accelerating bodies (the work of Haisch et al. was an initial inspiration in the development of the model of inertia presented in this paper, although the actual mechanism that produces inertia is not specified here, and need not be that of Haisch et al.).

The wavelength of Unruh radiation lengthens as acceleration reduces, and Milgrom (1994) noted that as galactic radius increases, the rotational acceleration reduces, the Unruh waves lengthen, and, at the radius where galactic dynamics start to become non-Newtonian, the Unruh waves reach the Hubble scale. He speculated, without assigning a specific cause, that this event may abruptly reduce inertia, affecting dynamics and perhaps explaining MoND. However, an abrupt loss of inertia at a particular galactic radius is not what is seen. The observations show a more gradual deviation from Newtonian behaviour.

McCulloch (2007) proposed a model for inertia that could be called a Modification of inertia resulting from a Hubble-scale Casimir effect (MiHsC) or Quantised Inertia. MiHsC assumes that the inertial mass of an object is caused by Unruh radiation resulting from its acceleration with respect to surrounding matter, and that this radiation is subject to a Hubble-scale Casimir effect. This means that only Unruh waves that fit exactly into twice the Hubble diameter are allowed, so that an increasingly greater proportion of the Unruh waves are disallowed as accelerations decrease and these waves get longer, leading to a new gradual loss of inertia as acceleration reduces. This loss of inertia is far more gradual than Milgrom’s proposal, discussed above. In MiHsC the inertial mass becomes

\[ m_I = m_g \left( 1 - \frac{\beta \pi^2 c^2}{\Theta} \right) \sim m_g \left( 1 - \frac{2 c^2 \Theta}{|a|} \right) \]  

(1)

where \( m_g \) is the gravitational mass, \( \beta = 0.2 \) (part of Wien’s displacement law), \( c \) is the speed of light, and \( \Theta \) is the Hubble diameter (2.7 × 10^{26} \text{m}, from Freedman, 2001). For the derivation of Eq. 1 see McCulloch (2007) and for a justification for the use of the modulus of the acceleration see McCulloch (2008) and McCulloch (2011). MiHsC has now been tested quite successfully on several anomalies that have been observed in environments where accelerations are small (see McCulloch, 2007, 2008, 2010, 2011). MiHsC violates the equivalence principle, but not in a way that could have been detected in a torsion balance experiment (McCulloch, 2011).

McGaugh et al. (2009) studied the baryonic mass of disc galaxies and showed that there were none with a baryonic mass of less than 10^9 M_⊙. This minimum
mass is also predicted by MiHsC (McCulloch, 2010) since in MiHsC mutual accelerations must always be above $2c^2/\Theta$ (close to the acceleration attributed to dark energy). Using this prediction of a minimum acceleration, in this paper MiHsC is applied to the rotation of a wider range of cosmic structures.

2 Method and Results

Starting with Newton’s second law, and his gravity law for a star of mass $m$ orbiting in a galaxy of mass $M$

$$F = m_i a = \frac{GMm}{r^2}$$

(2)

where $m_i$ is the inertial mass, $a$ is the rotational acceleration ($v^2/r$) of the star towards the galactic centre, $G$ is the gravity constant, and $M$ is the mass within a radius $r$, and $m$ is the gravitational mass of a star. Replacing the inertial mass following McCulloch (2007, 2008) (see Eq. 1, above)

$$m \left( 1 - \frac{2c^2}{|A|\Theta} \right) a = \frac{GMm}{r^2}$$

(3)

where $|A|$ is the total of all the accelerations of the star relative to nearby matter. This acceleration $A$ can be separated into a mean part ($a$) that is constant in time, and a variable part ($a'$) that is allowed to vary due to local accelerations. Therefore

$$\left( 1 - \frac{2c^2}{(|a| + |a'|)\Theta} \right) a = \frac{GM}{r^2}$$

(4)

Multiplying through by $|a| + |a'|$ gives

$$\left(|a| + |a'| - \frac{2c^2}{\Theta} \right) a = \frac{GM(|a| + |a'|)}{r^2}$$

(5)

MiHsC predicts a minimum allowed acceleration of $2c^2/\Theta$. This occurs because as acceleration reduces, the wavelength of the Unruh radiation increases and a greater proportion of the waves are disallowed by the Hubble-scale Casimir effect in MiHsC, so the inertia decreases and it becomes easier for the object to be accelerated again by the same external force. A balance is predicted to occur at a minimum acceleration of $2c^2/\Theta$ (see McCulloch, 2007, 2010). At a galaxy’s edge the rotational acceleration $a$ is smaller than this, so it is assumed here that this residual minimum acceleration is found in smaller scale motions.
of the stars: in \( a' \), so at the galaxy’s edge \( a' = 2c^2/\Theta \) and terms 2 and 3 on the left hand side of Eq. 5 cancel, leaving

\[
a^2 = \frac{GM(|a| + |a'|)}{r^2}
\]  

(6)

At the galaxy’s edge the radius is large, therefore the rotational acceleration is tiny, so \( a \ll a' \) and so

\[
a^2 = \frac{GM|a'|}{r^2}
\]  

(7)

Since \( a = v^2/r \)

\[
v^4 = GM |a'|
\]  

(8)

Therefore MiHsC predicts a Tully-Fisher relation (Tully & Fisher, 1977) with a constant of \( a' = 2c^2/\Theta = 6.7 \pm 0.6 \times 10^{-10} \text{m/s}^2 \) (the uncertainty of 0.6 arises from uncertainties in the Hubble constant of 9%, taken from Freedman, 2001), so

\[
v^4 = \frac{2GMc^2}{\Theta}
\]  

(9)

Figure 1 shows the observed rotation velocity of galaxies and galaxy clusters binned into different baryonic mass ranges as compiled by McGaugh et al. (2009). The x axis is the baryonic mass (gas plus stellar) in Solar masses. The y axis (and the solid line with black circles) shows the observed rotation (circular) speed (Vc) in km/s with an error of 30% caused by extrapolating Vc to larger radii (McGaugh et al., 2009).

The dashed line shows the predictions of MiHsC from Eq. 9 which has a 20% error because of: 1) a 9% uncertainty in the Hubble constant and therefore the value of \( \Theta \) used, and 2) a factor of 2 uncertainty in the stellar mass to light ratio (McGaugh et al., 2009) which has a lesser impact for the darker low mass dwarf systems.

For dwarf galaxies (the three cases on the left) the velocity predicted by MiHsC is higher than that observed by up to 44%, but MiHsC agrees with the observations, given the 30% uncertainty in the observed velocity and the 20% uncertainty in the prediction. For the spiral galaxies the predicted velocity is higher than that observed by between 30-54%. This is also in agreement given the errors. For the galaxy clusters, MiHsC overpredicts the velocity by 3-38% but is still in agreement with the observations.
To summarise: MiHsC, which forbids accelerations below $2c^2/\Theta$, can predict the rotation velocity of dwarf galaxies, spiral galaxies and galaxy clusters within error bars, from the baryonic mass only, and with no adjustable parameters (see Eq. 9), but tends to overpredict the velocities by between a third and a half.

3 Discussion

The prediction of MoND is shown in Fig. 1 with the two dotted lines, using the formula $v^4 = \sqrt{GMa_0}$, and using two values of $a_0$ to represent the range of values used in MoND: $a_0 = 1.2 \times 10^{-10}m/s^2$ (the darker dotted line) and $a_0 = 2 \times 10^{-10}m/s^2$ (the lighter dotted line). MoND underestimates galaxy cluster velocities and dark matter must be added to the clusters to fit MoND to the observations (Sanders, 2002). If we assume error bars of 30% on the observations then both MoND and MiHsC agree with the data. MoND performs better for spirals and MiHsC performs better for galaxy clusters. However, MoND requires an unexplained adjustable parameter $a_0$ to fit it to the data. With MiHsC no adjustable parameters are needed (see Eq. 9).

When the mean acceleration $a$ is large (close to the galactic centre) then the assumptions used here are not valid and the prediction of MiHsC is Newtonian. As the galactic radius increases the mean acceleration $(v^2/r)$ decreases, but MiHsC predicts that the acceleration cannot fall below $6.7 \times 10^{-10}m/s^2$ so a testable prediction of MiHsC is that in the outer edges of a galaxy smaller scale accelerations must increase to offset the decrease in rotational accelerations with radius and keep accelerations above $6.7 \times 10^{-10}m/s^2$.

This analysis assumes a balance between gravity and the inertial force so is only valid for rotationally-supported disc galaxies, and not elliptical galaxies and the galaxy clusters (although MiHsC seems to predict those well). A similar derivation could be tried for pressure-supported systems.

4 Conclusion

A relation between the velocity dispersion of a disc galaxy or galaxy cluster and its visible mass (a Tully-Fisher relation) can be derived by assuming that inertia is due to Unruh radiation which is subject to a Hubble-scale Casimir effect (MiHsC).

The derived relation is, $v^4 = 2GMc^2/\Theta$, and without adjustable parameters or dark matter, MiHsC predicts the dispersion velocity of dwarf galaxies, spiral galaxies and galaxy clusters within the error bars in these values (overestimating the observed velocities typically by one third to one half) and predicts that local accelerations should remain above $2c^2/\Theta$ at a galaxy’s edge.
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Figures

Figure 1. The observed outer rotational velocity for bins of galaxies and galaxy clusters of various masses from McGaugh et al. (2009) (black circles, solid line) and the prediction of MoND (dotted line) and MiHsC (dashed line). MiHsC overestimates the velocity for spirals by 30-54%, but outperforms MoND for galaxy clusters.