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Abstract Using inclusive decays of the J/ψ, a precise determination of the number of J/ψ events collected with the BESIII detector is performed. For the two data sets taken in 2009 and 2012, the numbers of J/ψ events are recalculated to be $(224.0\pm1.3)\times10^6$ and $(1088.5\pm4.4)\times10^6$ respectively, which are in good agreement with the previous measurements. For the J/ψ sample taken in 2017–2019, the number of events is determined to be $(8774.3\pm6)\times10^6$. The total number of J/ψ events collected with the BESIII detector is determined to be $(10087\pm44)\times10^6$, where the uncertainty is dominated by systematic effects and the statistical uncertainty is negligible.

Key words number of J/ψ events, BESIII detector, inclusive J/ψ decays

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1 Introduction

As a charmonium ground state, the \( J/\psi \) offers a unique laboratory for studying light hadron spectroscopy. In particular, \( J/\psi \) decays can be used to search for exotic hadrons composed of light quarks and gluons, which are key to a fuller understanding of the nature of the strong interaction.

Many important results in light hadron spectroscopy have been reported based on \( (1310.6\pm7.0)\times10^6 \) \( J/\psi \) events collected by the BESIII experiment in 2009 and 2012. An additional large sample of \( J/\psi \) events was collected by BESIII during 2017–2019 to improve the precision of the measurements and search for new processes. The three data samples of \( J/\psi \) events collected at BESIII are summarized in Table 1.

| Data set | \( \sqrt{s} \) | \( \mathcal{L}_{\text{online}} \) | Date (duration) |
|----------|----------------|--------------------------|----------------|
| \( J/\psi \) | 3.097 GeV | 2678 pb\(^{-1}\) | 2017-08-12 – 2019-06-02 |
| QED1 | 3.08 GeV | 48 pb\(^{-1}\) | 2018-04-12 – 2018-04-14 |
| QED2 | 3.08 GeV | 88 pb\(^{-1}\) | 2019-02-07 – 2019-02-11 |
| \( \psi(3686) \) | 3.686 GeV | 25 pb\(^{-1}\) | 2018-05-20 |
| \( J/\psi \) | 3.097 GeV | 323 pb\(^{-1}\) | 2012-04-10 – 2012-05-22 |
| QED1 | 3.08 GeV | 13 pb\(^{-1}\) | 2012-04-08 |
| QED2 | 3.08 GeV | 17 pb\(^{-1}\) | 2012-05-23 – 2012-05-24 |
| \( \psi(3686) \) | 3.686 GeV | 7.5 pb\(^{-1}\) | 2012-05-26 |
| \( J/\psi \) | 3.097 GeV | 82 pb\(^{-1}\) | 2009-06-12 – 2009-07-28 |
| QED | 3.08 GeV | 0.3 pb\(^{-1}\) | 2009-06-19 |
| \( \psi(3686) \) | 3.686 GeV | 150 pb\(^{-1}\) | 2009-03-07 – 2009-04-14 |

This paper reports a precise determination of the total number of \( J/\psi \) events, which is an important quantity for many analyses using these data samples. The number of \( J/\psi \) events for the new samples collected in 2017-2019 is determined with the same method as the one used in the previous measurements. In addition, in this analysis we also recalculate the number of \( J/\psi \) events for the two data samples taken in 2009 and 2012, reconstructed using the latest BESIII software. The number of \( J/\psi \) events, \( N_{\psi(3686)} \), is calculated as

\[
N_{\psi(3686)} = \frac{N_{\text{mc}} - N_{\text{bg}}}{\epsilon_{\text{trig}} \times \epsilon_{\text{data}}^{\psi(3686)} \times f_{\text{cor}}} \tag{1}
\]

where \( N_{\text{mc}} \) is the number of inclusive \( J/\psi \) decays selected from the \( J/\psi \) data; \( N_{\text{bg}} \) is the number of background events estimated with continuum data taken at \( \sqrt{s} = 3.08 \) GeV; \( \epsilon_{\text{trig}} \) is the trigger efficiency; \( \epsilon_{\text{data}}^{\psi(3686)} \) is the inclusive \( J/\psi \) detection efficiency determined experimentally using the \( J/\psi \) sample from the reaction \( \psi(3686) \rightarrow \pi^+\pi^-J/\psi \). \( f_{\text{cor}} \) is a correction factor that accounts for the difference in the detection efficiency between the \( J/\psi \) events produced at rest and those produced in the reaction \( \psi(3686) \rightarrow \pi^+\pi^-J/\psi \). \( f_{\text{cor}} \) is expected to be approximately unity, and is determined by the Monte Carlo (MC) simulation sample with

\[
f_{\text{cor}} = \frac{\epsilon_{\text{MC}}^{\psi(3686)}}{\epsilon_{\text{MC}}^{\psi(3686)}}, \tag{2}
\]

where \( \epsilon_{\text{MC}}^{\psi(3686)} \) is the detection efficiency of inclusive \( J/\psi \) events determined from the MC sample of \( J/\psi \) events produced directly in electron-positron collisions, and \( \epsilon_{\text{MC}}^{\psi(3686)} \) is that from the MC sample of \( \psi(3686) \rightarrow \pi^+\pi^-J/\psi \). For the number of \( J/\psi \) events determined with Eq. 1 only \( f_{\text{cor}} \) depends on MC simulation. According to Eq. 2, the uncertainties related to MC simulation (including generator, detector response etc.) almost cancel since they impact both the numerator and denominator, which improves the precision of the number of \( J/\psi \) events. In the MC simulation, the production of \( J/\psi \) and \( \psi(3686) \) resonances is simulated with a GEANT4-based MC software, which includes the geometric description of the BESIII detector and the detector response. The simulation models the beam energy spread and initial state radiation (ISR) in the \( e^+e^- \) annihilations with the generator KKMC. The known decay modes are modeled with EVTGEN using branching fractions taken from the Particle Data Group, and the remaining unknown charmonium decays are modeled with LUNDCHARM, PETRA and FIRE.

2 Inclusive \( J/\psi \) selection criteria

Candidate events must contain two or more charged tracks which are required to have a momentum less than \( 2.0 \) GeV/c and to be within a polar angle \( \theta \) range of \( |\cos \theta| < 0.93 \), where \( \theta \) is defined with respect to the axis of the Main Drift Chamber (MDC). The distance of closest approach to the interaction point (IP) must be less than \( 15 \) cm along the \( z \)-axis, \( |V_z| \), and less than \( 1 \) cm in the transverse plane, \( V_r \). Photon candidates are identified using isolated showers in the electromagnetic calorimeter (EMC). The deposited energy of each shower must be more than \( 25 \) MeV in the barrel region, \( |\cos \theta| < 0.83 \), and more than \( 50 \) MeV in the end cap region, \( 0.86 < |\cos \theta| < 0.93 \). To suppress electronic noise and showers unrelated to the event, the difference between the EMC time and the event start time is required to be within \( [0, 700] \) ns.

To suppress events from Quantum Electrodynamics (QED) processes (i.e. Bhabha and dimuon events), from cosmic rays, beam-induced backgrounds and electronic noise, a series of selection criteria are applied to the candidate events.

The sum of charged particle energies computed from the track momenta assuming a pion mass and the neutral shower energies deposited in the EMC, \( E_{\text{vis}} \), is required to be greater than \( 1.0 \) GeV. Figure 1 shows a comparison of the \( E_{\text{vis}} \) distribution between the \( J/\psi \) data, the data taken at \( \sqrt{s} = 3.08 \) GeV, and the inclusive \( J/\psi \) MC sample. The requirement \( E_{\text{vis}} > 1.0 \) GeV removes one third of the background events while retaining \( 99.5\% \) of...
the signal events.

For events with only two charged tracks, the momentum of each track is required to be less than 1.5 GeV/c to exclude Bhabha and dimuon events. Figure 2 shows a scatter plot of the momenta of the two charged tracks, and the solid lines depict the momentum requirement. Figure 3 displays the distribution of energy deposited in the EMC for the charged tracks of 2-prong events for J/ψ data (dots with error bars) and for the combined, normalized MC simulations of e⁺e⁻ → e⁺e⁻ (γ) and J/ψ → e⁺e⁻ (γ) (histogram). After the above requirements, N_sel = (6912.03±0.08)×10⁶ candidate events are selected from the J/ψ data taken in 2017–2019. The distributions of the track parameters of closest approach along the beam line and in radial direction (Vz and Vr), the polar angle (cosθ), and the total energy deposited in the EMC (EEMC) after subtracting background events estimated with the continuum data taken at √s = 3.08 GeV (see Sec. 3 for details) are illustrated in Fig. 4. The multiplicity of charged tracks (N_good) is shown in Fig. 5 where the MC sample generated according to the standard MC model agrees very well with the data while the MC sample generated with an ‘incomplete’ MC model deviates from the data. The standard MC model includes the known decay processes listed in the PDG and the unknown ones modeled with LUNDCHARM, while the incomplete MC model only includes the known decay modes listed in the PDG. The effect of this discrepancy on the determination of the number of J/ψ events is small, as described in Sec. 6.
3 Background analysis

In this analysis, the potential background sources include QED processes, beam-induced background, cosmic rays, and electronic noise. The continuum data samples at √s = 3.08 GeV are taken in close chronological order to each J/ψ sample to estimate these backgrounds.

The integrated luminosity is determined using the process e+e− → γγ. The candidate events are selected by requiring at least two showers in the EMC within |cosθ| < 0.8 and with the energy of the second most energetic shower between 1.2 and 1.6 GeV. The number of signal events is determined from the number of events in the signal region |Δφ| < 2.5°, and the background is estimated from those in the side-band region 2.5° < |Δφ| < 5°, where Δφ = |φ+ − φ−|−180° and φ± are the azimuthal angles of the two photon candidates. Taking into account the detector efficiency obtained from the MC simulation and the cross section of the QED process e+e− → γγ, the integrated luminosities of the J/ψ data sample and the sample taken at √s = 3.08 GeV in 2017–2019 are determined to be (2568.07 ± 0.40) pb−1 and (136.22 ± 0.09) pb−1, respectively, where the errors are statistical only.

After applying the same selection criteria as for the J/ψ data, N3.08 = 6,363.941 ± 2,523 events are selected from the continuum data taken at √s = 3.08 GeV. Assuming the same detection efficiency at √s = 3.08 GeV as for the J/ψ peak and taking into account the energy-dependent cross section of the QED processes, the number of background events for the J/ψ sample, Nbg, is estimated to be

\[
N_{bg} = N_{3.08} \times \frac{L_{J/\psi}}{L_{3.08}} \times \frac{s_{3.08}}{s_{J/\psi}} = (118.66 \pm 0.05) \times 10^6, \quad (3)
\]

where \( L_{J/\psi} \) and \( L_{3.08} \) are the integrated luminosities for the J/ψ data sample and the data sample taken at √s = 3.08 GeV, respectively, and \( s_{3.08} \) and \( s_{J/\psi} \) are the corresponding squares of the center-of-mass energies. The background is calculated to be (1.717±0.002)% of the number of selected inclusive J/ψ events taken in 2017–2019.

4 Determination of the detection efficiency and correction factor

In this analysis, the detection efficiency is determined experimentally using a sample of J/ψ events from the reaction Ψ(3686) → π+π−J/ψ to reduce the uncertainty related to any discrepancies between the MC simulation and the data. To ensure that the beam conditions and detector status are similar to those of the sample collected at the J/ψ peak, a dedicated Ψ(3686) sample taken on May 20, 2018 is used for this study.

For a candidate Ψ(3686) → π+π−J/ψ event, there must be at least two soft pions with opposite charge detected in the MDC with |cosθ| < 0.93. Each candidate pion is required to have a momentum less than 0.4 GeV/c,
and the distance of closest approach to the IP must satisfy $|V_z| < 15$ cm and $V_z < 1$ cm. No further selection criteria on the remaining charged tracks or showers are required. The distribution of the invariant mass recoiling against all possible soft $\pi^+\pi^-$ pairs is shown in Fig. 6. A prominent peak around 3.1 GeV/c², corresponding to the decay of $\psi(3686) \rightarrow \pi^+\pi^-J/\psi$, is observed over a smooth background. The number of inclusive J/ψ events, $N_{\text{inc}} = (3538.5\pm3.6)\times10^3$, is obtained by fitting a double-Gaussian function for the J/ψ signal plus a second-order Chebychev polynomial for the background to the $\pi^+\pi^-$ recoil mass spectrum.

To measure the detection efficiency of inclusive J/ψ events, the same selection criteria as described in Sec. 2 are applied to the remaining charged tracks and showers. The number of selected inclusive J/ψ events, $N_{\text{sel}}$, is determined to be $(2717.6\pm3.4)\times10^3$ using a fit to the recoil mass distribution of the selected events with the same function as described above. The detection efficiency of inclusive J/ψ events, $\epsilon_{\psi(3686)}^{\text{inc}}/(76.80\pm0.05)\%$, is calculated from the ratio of the number of inclusive J/ψ events with and without the inclusive J/ψ event selection criteria applied.

To account for the efficiency difference between the J/ψ produced at rest and the J/ψ from the decay $\psi(3686) \rightarrow \pi^+\pi^-J/\psi$, a correction factor, defined in Eq. 2, is used. Two large statistics MC samples, inclusive $\psi(3686)$ and inclusive J/ψ events, are produced and are subjected to the same selection criteria as the data samples. The detection efficiencies of inclusive J/ψ events are determined to be $\epsilon_{\psi(3686)}^{\text{MC}} = (76.93\pm0.02)\%$, and $\epsilon_{\psi(3686)}^{\text{MC}} = (77.56\pm0.01)\%$ for the two inclusive MC samples, respectively. The correction factor $f_{\text{cor}}$ for the detection efficiency is therefore taken as

$$f_{\text{cor}} = \frac{\epsilon_{\psi(3686)}^{\text{MC}}}{\epsilon_{\psi(3686)}^{\text{MC}}} = 1.0082\pm0.0007, \quad \text{(4)}$$

where the error is statistical only.

5 Number of J/ψ events

With Eq. 1 and the corresponding parameter values summarized in Table 2, the number of J/ψ events collected in 2017–2019 is determined to be $(8774.0\pm40.2)\times10^3$. The trigger efficiency of the BESIII detector is taken to be 100%, based on a study of various reactions [11]. With the same procedure, the numbers of J/ψ events taken in 2012 and 2009 are recalculated to be $(1088.5\pm0.1)\times10^6$ and $(224.0\pm0.1)\times10^6$, respectively, where the uncertainties are statistical only. The statistical uncertainties of $N_{\psi}$ are taken into account as part of the systematic uncertainty (see Sec. 6.4). The systematic uncertainties from different sources are discussed in detail in Sec. 6.

6 Systematic uncertainty

The sources of systematic uncertainties, including the MC model, track reconstruction efficiency, fit to the J/ψ peak, background estimation, random trigger mixing and the efficiency of selecting the two soft pions recoiling against J/ψ, are investigated in detail below, and the corresponding contributions are summarized in Table 3.

6.1 MC model uncertainty

In the measurement of the number of J/ψ events, only the efficiency correction factor, $f_{\text{cor}}$, depends on the MC simulation. To evaluate the uncertainty due to the MC model, MC samples are generated with the incomplete MC model, and the correction factor based on these samples is compared to its nominal value. As shown in Fig. 6, the charged track multiplicity distribution of the incomplete MC sample deviates much from the experimental data, which means this method will overestimate the systematic uncertainty. To be conservative, the change in the correction factor, 0.18%, is taken as the systematic uncertainty due to the MC model on the number of J/ψ events collected in 2017–2019 (0.18% for 2012 and 0.27% for 2009).

6.2 Track reconstruction efficiency

The charged track reconstruction efficiencies in MC simulation and experimental data are studied, and the disagreement between them is less than 1% for each charged track. In the analysis, the detection efficiency
for inclusive $J/\psi$ decays is obtained using the $\psi(3686)$ data sample. The consistency of charged track reconstruction efficiency between the MC and data samples in $\psi(3686)$ decays is assumed to be the same as that in $J/\psi$ decays since the $\psi(3686)$ data is taken in close chronological proximity to the $J/\psi$ sample. To evaluate the effect of a possible difference, the track reconstruction efficiencies in both $J/\psi$ and $\psi(3686)$ MC samples are varied by $-1\%$ to determine the uncertainty due to the MDC tracking. As expected, the change in the correction factor is very small, 0.02%, and this value is taken as a systematic uncertainty (0.03% for 2012).

In 2009, the $J/\psi$ and $\psi(3686)$ data samples were collected in different time periods, and there may be slight differences in the tracking efficiencies between the two data sets. Here, the difference between the MC/data consistencies in the $J/\psi$ and $\psi(3686)$ samples is assumed to be 0.5%, half of the data/MC inconsistency, 1%. To estimate the corresponding systematic uncertainty, we modify the track reconstruction efficiency by $-0.5\%$ in the $J/\psi$ MC sample, keeping it unchanged for the $\psi(3686)$ MC sample. The resulting change in the correction factor, 0.31%, is taken as a systematic uncertainty on the number of $J/\psi$ events in 2009.

### 6.3 Fit to the $J/\psi$ peak

The $\psi(3686)$ data sample is used to measure the selection efficiency of inclusive $J/\psi$ events. The yield of $J/\psi$ events in $\psi(3686)$ decays is determined by fitting the $J/\psi$ peak in the mass spectrum recoiling against $\pi^+\pi^-$. The uncertainties due to the fit are investigated: (a) the fit: we propagate the statistical uncertainties of the $J/\psi$ signal yield from the fit to the selection efficiency, and the resulting uncertainties, 0.07% and 0.03% for $\epsilon_{\text{data}}^{\psi(3686)}$ and $\epsilon_{\text{MC}}^{\psi(3686)}$, respectively, are considered to be the uncertainties from the fit itself. (b) the fit range: the fit range on the $\pi^+\pi^-$ recoil mass is changed from [3.07, 3.13] GeV/$c^2$ to [3.08, 3.12] GeV/$c^2$, and the change of the result, 0.07%, is taken as the corresponding systematic uncertainty. (c) the signal description: we perform an alternative fit by describing the $J/\psi$ signal with a histogram (convolved with a Gaussian function) obtained from the recoil mass spectrum of $\pi^+\pi^-$ in $\psi(3686) \rightarrow \pi^+\pi^- J/\psi$, $J/\psi \rightarrow \mu^+\mu^-$, and the resulting change, 0.01%, is considered to be the associated systematic uncertainty. (d) the background shape: the uncertainty due to the background shape, 0.03%, is estimated by replacing the second-order Chebychev polynomial with a first-order or third-order Chebychev polynomial. By assuming that all of the sources of systematic uncertainty are independent, the fit uncertainty for the 2017–2019 $J/\psi$ sample, 0.10%, is obtained by adding all of the above effects in quadrature.

The same sources of systematic uncertainty are considered for the $J/\psi$ sample taken in 2012 (2009). The fit has an uncertainty of 0.07% (0.02%) for $\epsilon_{\text{data}}^{\psi(3686)}$ and 0.03% (0.03%) for $\epsilon_{\text{MC}}^{\psi(3686)}$. The uncertainties from the fit range, signal function and background shape are 0.08% (0.03%), 0.15% (0.06%) and 0.10% (0.04%), respectively. The total uncertainty from the fit for the 2012 (2009) data is 0.21% (0.09%).

### 6.4 Background uncertainty

In this analysis, the events selected from the experimental data sample include the $J/\psi$ events and background: QED processes, cosmic rays, beam-induced backgrounds, and electronic noise. The contribution of the background is estimated by normalizing the number of events in the continuum data sample taken at $\sqrt{s} = 3.08$ GeV according to Eq. (3). The uncertainty due to the background estimation mainly comes from the normalization method, the statistics of the continuum sample, the statistical uncertainty of the integrated luminosity and the uncertainty due to beam associated backgrounds.

The cosmic ray background, beam associated backgrounds and electronic noise can not be normalized properly with Eq. (3), since the number of cosmic rays is proportional to the time of data taking, while beam associated backgrounds depend on the vacuum status and beam currents during data taking in addition to the time of data taking, and the electronic noise also depends on the detector status. To estimate the associated systematic uncertainty, the difference in the estimated number of background events with and without the energy-dependent factor in Eq. (3) is used.

During 2017–2019, two data samples at $\sqrt{s} = 3.08$ GeV were taken at different times during the $J/\psi$ data taking. They are compared to each other to estimate the uncertainty of the background related with the stability of the beam and vacuum status. Each of the two continuum data samples is used to estimate the background with Eq. (3), and the maximum difference to the nominal result, 0.03%, is taken as the related systematic uncertainty. The corresponding systematic uncertainty for the 2012 sample is 0.09%. Only one continuum data sample was taken for $J/\psi$ data in 2009. The selected background events from the continuum sample are compared to those from the $J/\psi$ data to estimate the corresponding uncertainty as described in detail in Ref. [12].

Assuming that all the above effects are independent, their contributions are added quadratically. The resulting uncertainties on the number of $J/\psi$ events due to the background estimation are determined to be 0.04%, 0.10% and 0.14% for the data taken in 2017–2019, 2012 and 2009, respectively.

### 6.5 Random Trigger mixing

In the MC simulation, events recorded by a random trigger are mixed into the MC events to simulate the electronic noise and beam-induced background. In the $\psi(3686)$ MC sample the random trigger events from the $\psi(3686)$ data taking were replaced by the random trigger events from the $J/\psi$ data taking to estimate the effect of
the different background levels. The change of the correction factor for the detection efficiency, 0.06%, is taken as the systematic uncertainty due to random trigger mixing for the number of J/ψ events taken in 2017–2019. The corresponding uncertainties for the 2012 and 2009 samples are 0.02% and 0.12%, respectively.

### 6.6 Uncertainty of selection efficiency of two soft pions

Study of the MC sample shows that the selection efficiency of soft pions, \( \epsilon_{\pi^+\pi^-} \), recoiling against the J/ψ in \( \psi(3686) \rightarrow \pi^+\pi^-J/\psi \) depends on the multiplicity of charged tracks in the J/ψ decays. Differences between the data and MC samples may lead to a change in the number of J/ψ events. The dependence of \( \epsilon_{\pi^+\pi^-} \) in the data is obtained by comparing the multiplicity distribution of J/ψ/ in the data to that of the J/ψ/MC data at rest. Then the efficiency of J/ψ in the \( \psi(3686) \rightarrow \pi^+\pi^-J/\psi(J/\psi \rightarrow \text{inclusive}) \) MC sample, \( \epsilon_{MC}^{\psi(3686)} \) in Eq. (2), can be reweighted with the dependence of \( \epsilon_{\pi^+\pi^-} \) from the data sample. The resulting changes in the number of J/ψ events, 0.40%, 0.27%, 0.32% are taken as the uncertainties for the data taken in 2017–2019, 2012 and 2009, respectively.

### 6.7 Summary of systematic uncertainties

The systematic uncertainties from the different sources studied above are summarized in Table 3. The total systematic uncertainty for the number of J/ψ events in 2017–2019, 0.45%, is the quadratic sum of the individual uncertainties. Correspondingly, the uncertainties for 2012 and 2009 are 0.40% and 0.56%, respectively.

| Sources                      | 2017–2019(%) | 2012 (%) | 2009(%) |
|------------------------------|--------------|----------|---------|
| MC model uncertainty         | 0.18*        | 0.18*    | 0.27*   |
| Tracking efficiency          | 0.02*        | 0.03*    | 0.31    |
| Fit to J/ψ peak              | 0.10         | 0.21     | 0.09    |
| Background uncertainty       | 0.04         | 0.10     | 0.14    |
| Noise mixing                 | 0.06         | 0.02     | 0.12    |
| \( \epsilon_{\pi^+\pi^-} \) uncertainty | 0.40*        | 0.27*    | 0.32*   |
| Total                        | 0.45         | 0.40     | 0.56    |

### 7 Summary

Using the inclusive J/ψ decays, the number of J/ψ events collected with the BESIII detector in 2017–2019 is determined to be \((8774.0 \pm 39.4) \times 10^4\), where the uncertainty is completely dominated by systematics and the statistical uncertainty is negligible. The numbers of J/ψ events taken in 2009 and 2012 are recalculated to be \((224.0 \pm 1.3) \times 10^6\) and \((1088.5 \pm 4.4) \times 10^6\), which are consistent with the previous measurements [3], but with improved precision.

The total number of J/ψ events taken with BESIII detector is determined to be \( N_{J/\psi} = (10087 \pm 44) \times 10^6\). Here, the total uncertainty is determined by adding the common uncertainties linearly and the independent ones in quadrature.

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