On the Dearth of Ultra-faint Extremely Metal-poor Galaxies

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Abstract

Local extremely metal-poor galaxies (XMPs) are of particular astrophysical interest since they allow us to look into physical processes characteristic of the early universe, from the assembly of galaxy disks to the formation of stars in conditions of low metallicity. Given the luminosity–metallicity relationship, all galaxies fainter than $M_r \approx -13$ are expected to be XMPs. Therefore, XMPs should be common in galaxy surveys. However, they are not common, because several observational biases hamper their detection. This work compares the number of faint XMPs in the SDSS-DR7 spectroscopic survey with the expected number, given the known biases and the observed galaxy luminosity function (LF). The faint end of the LF is poorly constrained observationally, but it determines the expected number of XMPs. Surprisingly, the number of observed faint XMPs ($\sim$10) is overpredicted by our calculation, unless the upturn in the faint end of the LF is not present in the model. The lack of an upturn can be naturally understood if most XMPs are central galaxies in their low-mass dark matter halos, which are highly depleted in baryons due to interaction with the cosmic ultraviolet background and to other physical processes. Our result also suggests that the upturn toward low luminosity of the observed galaxy LF is due to satellite galaxies.

Key words: galaxies: abundances – galaxies: dwarf – galaxies: formation – galaxies: luminosity function, mass function – galaxies: statistics – intergalactic medium

1. Motivation

Galaxies having a gas-phase metallicity smaller than a tenth of the solar metallicity are often known to be extremely metal poor (e.g., Kunth & Östlin 2000). They are of astrophysical interest for a number of reasons, among which include determining the primordial He abundance produced during the Big Bang (Peimbert et al. 2010; Cyburt et al. 2016), studying star formation in conditions of low metallicity (Shi et al. 2014; Elmegreen & Hunter 2015; Rubin et al. 2015; Filho et al. 2016), understanding the formation of the universe (Fisher et al. 2014), analyzing primitive interstellar media (Izotov & Thuan 2007), constraining the properties of the first stars (Kehrig et al. 2015; Thuan & Izotov 2005), following the assembly of primitive disks (Elmegreen et al. 2012, 2013; Sánchez Almeida et al. 2015; Ceverino et al. 2016), and studying the intergalactic gas (Sánchez Almeida et al. 2014a, 2014b).

Unfortunately, the number of known extremely metal-poor galaxies (XMPs) remains small. The review paper by Kunth & Östlin (2000) contained only 31 XMPs; Kniazev et al. (2003) added eight new targets from the early data release of the Sloan Digital Sky Survey (SDSS), the exploration in SDSS-DR6 by Guseva et al. (2009) yielded 44 sources, and the systematic bibliographic search for all XMPs in literature and the SDSS-DR7 carried out by Morales-Luis et al. (2011) rendered 140 sources. Morales-Luis et al. included targets found by Kniazev et al. (2004), Izotov et al. (2004, 2006), and Izotov & Thuan (2007). Although new local metal-poor objects have been discovered since 2011 (e.g., Izotov et al. 2012; Skillman et al. 2013; Guseva et al. 2015; James et al. 2015; Hirschauer et al. 2016; Sánchez Almeida et al. 2016), XMPs remain uncommon.

The scarcity of known XMPs is in sharp contrast with the expectation that most galaxies are actually XMPs. This problem has been put forward by several authors (McQuinn et al. 2013; Skillman et al. 2013; James et al. 2015; Sánchez Almeida et al. 2016), with the following argument: there is a well-known relation between absolute luminosity or stellar mass and gas-phase metallicity (e.g., Skillman et al. 1989; Sánchez Almeida et al. 2008); consequently all faint or low-mass galaxies should be XMPs. Using as the metallicity threshold

$$12 + \log(O/H) \leq 7.65,$$

the metallicity versus absolute magnitude relationship by Berg et al. (2012) implies that galaxies with absolute $B$-band magnitude

$$M_B \geq -12.5$$

are XMPs. Similarly, the metallicity–stellar-mass relationship in the paper by Berg et al. (2012) implies that galaxies with stellar masses

$$M_* \leq 1.1 \times 10^7 M_\odot,$$

correspond to XMPs. Equation (2) can be rewritten for the $r$-band magnitude using the transformation from $B$ to $r$ in Jester et al. (2005) for typical colors of gas-rich galaxies ($g - r \approx 0.4$; Blanton & Moustakas 2009), leading to

$$M_r \geq -13.3.$$
and/or masses below the thresholds in Equations (2)–(4), will be designated here as quiescent XMPs, or QXMPs. The remaining XMPs are often denoted as active XMPs. Note that, by definition, active XMPs are low-metallicity outliers of the luminosity–metallicity relationship.

Hence, why are XMPs so unusual among the observed galaxies? It is argued that most QXMPs also have low surface brightness, so low as to be below the detection threshold of the largest surveys (see Skillman et al. 2013; James et al. 2015).

As far as we are aware, this qualitative argument has never been quantified. In other words, (1) what is the number of QXMPs to be expected in current surveys, and (2) is this prediction consistent with the number of known QXMPs? Our paper addresses these two questions, finding that observational biases alone cannot account for the scarcity of observed XMPs. The luminosity function (LF) of XMPs must decline for objects fainter than the limits in Equations (2) and (3). We show that such a drop is expected from cosmological numerical simulations, provided XMPs are central, rather than, satellite galaxies. The ultraviolet (UV) background is then expected to prevent the formation of low-mass galaxies (e.g., Efstathiou 1992; Thoul & Weinberg 1996; Kravtsov et al. 2004; Wyithe & Loeb 2006; Okamoto et al. 2008). Several physical processes may suppress gas accretion and star formation in low-mass dark matter halos. The cosmological UV background heats the intergalactic gas and establishes a minimum mass for halos that can accrete gas. The gas in low-mass halos may also be photo-evaporated by the UV background after re-ionization. In addition, the ionizing radiation dissociates molecular hydrogen, which is the main coolant for low-metallicity gas, thus preventing star formation even before the gas is completely stripped from the halos by other quenching processes. Stellar feedback processes, such as supernova explosions and stellar winds, are also able to remove gas from galaxy disks, reducing the star formation efficiency in low-mass halos, where the gravitational binding energy is particularly low (e.g., White & Frenk 1991; Dalla Vecchia & Schaye 2008; Governato et al. 2010; Silk & Mamon 2012; Vogelsberger et al. 2014; Schaye et al. 2015). Thus, when the Galaxy has a stellar mass smaller than several $10^8 M_{\odot}$, most of the gas that could be used to form stars returns unused to the intergalactic medium (e.g., Davé et al. 2012; Shen et al. 2012; Sánchez Almeida et al. 2014a; Christensen et al. 2016).

The paper is organized as follows. Section 2 describes the number of QXMPs observed in the spectroscopic sample of the SDSS-DR7, which is chosen because this survey provides most of the known XMPs. Section 3 is devoted to estimating the expected number of QXMPs from the Galaxy LF, first by extrapolating the observed LF to faint objects, and then including the effect of the baryon fraction changing with the dark matter halo mass (Section 3.3). The results are discussed in Section 4, including an analysis of the factors that limit the number of observed QXMPs and how they can be overcome in future searches (Section 4.2). The number of observed QXMPs to be expected in other existing and forthcoming surveys is determined in Section 4.3. The results are collected and summarized in Section 5. Throughout the paper, the Hubble constant $H_0$ is taken to be 70 km s$^{-1}$ Mpc$^{-1}$.

### 2. Number of Observed QXMPs in the SDSS-DR7

The search for XMPs during the last decade has been very much focused on the spectroscopic sample of the SDSS-DR7 (Abazajian et al. 2009). The purpose of this section is to evaluate how many QXMPs have been found as part of these SDSS-DR7-based searches. A comprehensive list is needed to compare the number of observed and expected QXMPs.

We have built the list by searching all recent papers that may have XMPs from the SDSS-DR7. The galaxies in this paper were filtered so as to keep only those from the SDSS-DR7 with metallicity and luminosity below the thresholds in Equations (1) and (4), respectively.

The samples that were analyzed are:

1. The Morales-Luis et al. (2011, ML+11) XMP sample, which contains a compilation of all low-metallicity (12 + log(O/H) $\lesssim$ 7.65) sources from the literature until the date of publication, plus several new targets from the SDSS-DR7 spectroscopic sample. It uses photometry from the SDSS.
2. The Sánchez Almeida et al. (2016, SA+16) XMP sample from the SDSS-DR7, with metallicity from SDSS spectra, and photometry also from the SDSS.
3. The Karachentsev et al. (2013, Kara+13) sample, which is the latest version of the nearby galaxy reference catalog. Only sources with $M_B \geq -12.5$ mag are considered here, where both the metallicity and photometry are compiled from the literature.
4. The Berg et al. (2012, Berg+12) sample, a subsample of low-luminosity galaxies within 11 Mpc, with metallicities and photometry from Multiple Mirror Telescope (MMT) observations.
5. The Izotov et al. (2012, Izo+12) sample, a study of metal-poor emission-line SDSS-selected galaxies, with metallicities from Apache Point Observatory (APO) 3.5 m and/or MMT, and photometry from the SDSS.
6. The James et al. (2015, James+15) sample, a set of SDSS-selected blue diffuse dwarf galaxies, with metallicities from MMT observations and photometry from the SDSS.

From these six samples, we selected galaxies fulfilling the following criteria: (a) appearance in the SDSS-DR7 spectroscopic catalog, (b) B-band absolute magnitudes larger than $-12.5$, and (c) metallicities $12 + \log(O/H) \lesssim 7.65$. For the selected sources, we checked whether the photometry was reliable, particularly in those cases where only SDSS photometry is available. For the SDSS photometry, we first visually inspected the SDSS images of the sources and registered the source size. The visual sizes were then compared with the SDSS Petrosian g-band radius at 90% of the light, obtained at the position of the SDSS spectroscopic target. If the sizes were well matched, the SDSS photometry was deemed reliable, and the source was retained as a QXMP. If the sizes were not well matched, we then searched the literature for a value for the photometry. Most of them happen to have alternative photometry, which allowed us to exclude 90% of them as QXMPs. Only three objects had no alternative photometry, and they were discarded using the argument that their chance of being a QXMP is the same as the chance for objects with photometry. This chance is only 10%, which amounts to 0.3 sources.

Figures 1(a) and (b) summarize the selection procedure and its outcome. They show oxygen abundance versus absolute B magnitude for the galaxies in the previous references. In order to avoid overcrowding, the samples are split into two plots, and
only XMPs with reliable magnitudes are shown for \( M_B > -12.5 \). Figure 1(a) contains the compilation ML+11, plus the XMPs from the SDSS-DR7 recently identified in SA+16. There are only five targets with \( M_B > -12.5 \) that seem to be bona fide XMPs in the SDSS-DR7 spectroscopic database. Those are encircled with black outlines in Figure 1(a). One of these appears in the two samples with a slightly different magnitude and metallicity; the two corresponding points are encircled together.

Figure 1(b) contains the low-luminosity objects with measured metallicity from the four remaining samples: Kara+13 (green symbols), Izo+12 (brown symbols), James+15 (purple symbols), and Berg+12 (magenta symbols). The galaxies in Berg+12 are those used to derive the luminosity–metallicity relationship leading to the QXMP magnitude limit in Equation (2); the relationship is shown as a slanted black line in Figures 1(a) and (b). The prototype QXMP Leo P (Skillman et al. 2013) and the recently discovered AGC 198691 (Hirschauer et al. 2016) are also included in the figure for reference, even though they are not part of the SDSS-DR7 spectroscopic sample. Four new QXMPs have been identified, and are marked with a solid outline. One of the four (J105b+36) has three H II regions having slightly different abundances. The objects with the dashed outlines are QXMPs already included in (a)—two of these have two different abundance estimates, hence the dashed outline is elongated.

in the figure. They are already in the samples by ML+11 or SA+16, and so they do not contribute to the total number of QXMPs.

At the end of our selection process, we are left with nine QXMPs, i.e.,

\[
N_{QXMP}^{\text{obs}} = 9 \pm 3, \tag{5}
\]

where the error considers only the Poissonian statistical error associated with the process of counting (e.g., Martin 1971). It is important to realize that Equation (5) probably represents an upper limit. Since QXMPs are so uncommon, the chances of having false positives (non-QXMPs misidentified as QXMPs) are expected to be much greater than the chances of having false negatives (QXMPs excluded by mistake). The coordinates and main properties of the nine QXMPs are listed in Table 1, while their SDSS images are shown in Figure 2. Many of them are contained transversely in more than one of the analyzed samples.

As we argue in Section 1, QXMPs are expected to have low surface brightness (SB). Figure 3 shows the SB versus absolute magnitude in the \( r \)-band for the QXMPs with reliable SDSS photometry. All SBs referred to in the paper are half-light surface brightnesses. Despite their low SB (22–24 mag arcsec\(^{-1}\); see Figure 3 and Table 1), they tend to be brighter than expected from extrapolating the relationship between magnitude and SB found for brighter galaxies (the black solid line in Figure 3, from Blanton et al. 2005). Only two of the targets appear to follow the relationship (J0959+46
and J1231+42; compare their images with Leo P in Figure 2). The origin of this unexpectedly large SB can be pinned down to the bias of the observation toward high-SB objects, and the intrinsic scatter in the relationship between SB and magnitude. Given an absolute magnitude, observations preferentially pick up the objects with largest SB. This scatter is important for estimating the expected number of QXMPs, and so it is discussed and treated in Section 3.2 and the Appendix.

3. Number of Faint XMPs to be Expected in the SDSS-DR7

3.1. Calculation of the Expected Number of QXMPs

The galaxy LF, $\Phi(M)$, is defined as the number of galaxies with absolute magnitude $M$, per unit volume and unit magnitude. Figure 4(a) shows the LF in the $r$-band determined by Blanton et al. (2005) from the SDSS-DR2 data. The number of galaxies in a survey fainter than a given limiting magnitude, $M_{\text{lim}}$, is

$$N(M > M_{\text{lim}}) = \int_{M_{\text{lim}}}^{M_l} S(M)\Phi(M)dM,$$

(6)

with $S(M)$ the selection function that provides the effective volume that is sampled by the survey. Equation (6) assumes $S$ to depend only on the absolute magnitude of the Galaxy, which is reasonable in our case, and simplifies the treatment. This assumption will be relaxed later on. Equation (6) also assumes the existence of a limit for the magnitude of the faintest galaxy, $M_l$.

In the case of a volume-limited sample, $S$ is just the volume of the sample, and so independent of the absolute magnitude. In the case of an apparent-magnitude-limited survey, where all galaxies brighter than the apparent magnitude, $m_{\text{lim}}$, are included, $S(M)$ is the volume where galaxies of magnitude $M$ have apparent magnitude $m_{\text{lim}}$ or brighter, i.e.,

$$S(M) = V(M) = \frac{d^3}{3}\Omega,$$

(7)

with $\Omega$ the solid angle covered by the survey and $d$ the maximum distance at which a galaxy of magnitude $M$ can be observed, i.e.,

$$\log(d) = \frac{1}{5}(m_{\text{lim}} - M) - 5,$$

(8)

with $d$ in Mpc. The SDSS spectroscopic survey was designed to be apparent-magnitude-limited, with a limit in the $r$-band given by

$$m_{\text{lim}} = 17.77.$$

(9)

In practice, all surveys have a limit in surface brightness. Blanton et al. (2005) work it out for the SDSS, showing that the completeness of the spectroscopic survey decreases drastically for an average surface brightness, $SB_\lambda$, fainter than 23 mag arcsec$^{-2}$, reaching only 10% completeness at 24 mag arcsec$^{-2}$. This bias against low-SB objects is particularly severe for QXMPs. Galaxies fainter than the limit in Equation (2) may, in principle, have any surface brightness. However, faint galaxies tend to have a low SB as well (e.g., Kormendy 1985; Skillman 1999). Blanton et al. (2005) give the following relationship between surface brightness and absolute magnitude in the $r$-band,

$$SB_\lambda = 23.8 + 0.45(M_r + 13.3).$$

(10)

This relationship is in close agreement with others found in the literature (e.g., Kormendy 1985; Geller et al. 2012), and appears to be valid down to very low magnitudes (even for $M_B > -8$; e.g., Karachentsev et al. 2013). Equation (10) implies that galaxies fainter than the QXMP limit (Equation 4) are fainter than 23.8 mag arcsec$^{-2}$, and so potentially subject to severe incompleteness in the SDSS. Completeness is quantified using the completeness function, which gives the fraction of galaxies with a given SB that are detected in the survey, $C(SB)$. Combining the SDSS completeness function by Blanton et al. (2005, Figure 3) with Equation (10), one finds the completeness function, $C(M)$, in Figure 5 (the symbols joined by a solid line). Including completeness, the selection function turns out to be

$$S(M) = V(M)C(M),$$

(11)

which remains a function of the absolute magnitude only. As we show in the Appendix, Equation (11) remains formally valid when the scatter of the relationship between $SB$ and $M$ is

Table 1

Bona Fide QXMPs in the SDSS-DR7 Spectroscopic Survey

| Name# | $12 + \log(O/H)$ | $M_B$ | $M_r$ | $SB_\lambda$ (mag arcsec$^{-2}$) | $D$ (Mpc) | Reference | Also in
|-------|-----------------|-------|-------|-------------------------------|-----------|-----------|---------|
| J084338.0 + 402547.1 | 7.57 ± 0.06 | −12.3 | −12.8 | 22.1 | 10.2 | Izo+12 | ...
| J091159.4 + 313534.4 | 7.51 ± 0.14 | −12.4 | −12.7 | 21.9 | 11.6 | ML+11 | ...
| J095905.7 + 462650.5 | 7.50 ± 0.05 | −12.3 | −11.9 | 24.0 | 8.0 | Izo+12 | ...
| J105604.3 + 360827.9 | 7.26 ± 0.09 | −12.1 | ... | ... | 9.2 | Kara+13 | (M$_B$) Izo+12 (12 + log(O/H))
| J115754.2 | 7.62 ± 0.11 | −11.7 | −11.6 | 22.3 | 5.8 | SA+16 | Kara+13, James+15
| J121546.6 + 522338.8 | 7.39 ± 0.15 | −11.4 | −11.6 | 22.5 | 2.1 | SA+16 | ML+11, Berg+12
| J123109.1 + 420533.9 | 7.62 ± 0.04 | −12.2 | −12.1 | 23.8 | 8.2 | Izo+12 | Kara+13
| J123839.1 + 324555.9 | 7.28 ± 0.07 | −11.4 | ... | ... | 3.1 | ML+11 | Kara+13, Berg+12
| J125840.1 + 141308.1 | 7.65 ± 0.06 | −12.1 | ... | ... | 2.2 | ML+11 | Kara+13, Berg+12

Notes.

# The name includes the coordinates R.A. and decl.

Distance adopted in the respective reference to determine absolute magnitudes.

Morales-Luis et al. (2011, ML+11), Izotov et al. (2012, Izo+12), Berg et al. (2012, Berg+12), Karachentsev et al. (2013, Kara+13), James et al. (2015, James+15), and Sánchez Almeida et al. (2016, SA+16).
taken into account. This scatter is bound to be important in the analysis, since most observed QXMPs are high-SB outliers of the $SB_r - M_r$ relationship (Figure 3). In this case, the completeness, $C(M)$, in Equation (11) must be replaced with an effective completeness,

$$S(M) = V(M)C_{\text{eff}}(M),$$

where

$$C_{\text{eff}}(M) = \int_{SB} P(SB|M)C'(SB)dSB.$$  

$P(SB|M)$ stands for the conditional probability function of having a surface brightness $SB$ when the magnitude of the Galaxy is $M$. $C'(SB)$ represents the completeness function in terms of $SB$. $P(SB|M)$ is also provided by Blanton et al. (2005) (see Appendix A), and the resulting $C_{\text{eff}}(M)$ is represented in Figure 5 by the red solid line. In order to evaluate the integral in Equation (13), we use an erf function fit to the actual discrete completeness measured by Blanton et al. (2005) (i.e., we use the smooth thick solid line in Figure 5, meant to reproduce the symbols). As can be appreciated in Figure 5, the scatter increases the effective completeness of the survey at low SB, and this occurs because many targets happen to have an SB larger than that assigned by Equation (10), and those are the ones that are preferentially selected.

The integrand of Equation (6) gives the number of galaxies of a given magnitude to be expected in a survey. Figure 4(b) shows this integrand for the SDSS-DR7 spectroscopic survey, which implies employing the $m_{\text{limit}}$ in Equation (9) and $\Omega = 2.45$ sr (Abazajian et al. 2009). We use the LF for extremely low-luminosity galaxies determined by Blanton et al. (2005). The actual LF is shown by the red solid line in Figure 4(a), although we use the double Schechter function fitted to the observations by Blanton et al. (2005) for the calculations, i.e.,

$$\Phi(M) = \left(0.4 \ln 10 \ h^3 \ \text{Mpc}^{-3}\right) \exp \left[-10^{-0.4(M-M_*)}\right]$$

$$\times \left[\phi_{*,1} 10^{-0.4(M-M_*)^{(1+\alpha_1)}} + \phi_{*,2} 10^{-0.4(M-M_*)^{(1+\alpha_2)}}\right]$$

with $\phi_{*,1} = 0.0134$, $\phi_{*,2} = 0.0086$, $\alpha_1 = 0.33$, $\alpha_2 = -1.40$, $M_* = -19.99 - 5 \log h$, and $h$ the Hubble constant normalized to 100 km s$^{-1}$ Mpc$^{-1}$. This double Schechter function is shown by the black solid line in Figure 4(a). Thus, our

\[\text{Equation} 11\]
estimates are based on the extrapolation to low luminosity of the observed LF, which yields a function that continues growing toward the region occupied by the QXMPs (to the left of the vertical black line in Figure 4(a)). Assuming the Malmquist bias alone, i.e., using Equation (7) for the selection function, the number of expected QXMPs, \( N_{QXMP} \), is 149 (see Table 2). This figure results from integrating the solid line in Figure 4(b) up to the QXMP threshold. When the full selection function is considered (Equation (12)), the total number of expected QXMP galaxies turns out to be

\[ N_{QXMP} \approx 42. \]  

We integrate from \( M_r = -13.3 \) to \(-8 \). The upper limit is taken from Karachentsev et al. (2013, Figure 10) as the faintest magnitude of the late-type galaxies in the local universe. This upper limit, however, does not affect \( N_{QXMP} \) since the selection function is very small at low luminosities. Even though the number in Equation (15) represents a minuscule fraction of the galaxies to be expected in the full survey \((1.1 \times 10^6)\); Table 2), the actual number exceeds the \( 9 \pm 3 \) QXMP galaxies that are observed (Section 2). The predicted \( N_{QXMP} \) depends on the completeness function, and it falls off toward low SB, which is uncertain. We work out the error budget in Section 3.2, yielding the number of expected QXMPs to be between 12 and 73, with the high end of range strongly favored. Thus, the discrepancy between observations and predictions remains even when uncertainties are taken into account.

Our detailed description of differences between the number of observed and predicted QXMPs should not override the fact that these two numbers are always very small. QXMPs are, at most, a few tens in a survey such as the SDSS-DR7, which contains almost one million galaxies with spectra. QXMPs outnumber any other type of galaxy, but only a tiny fraction of these are detected; in the prediction described above, 87% of the galaxies are QXMPs, but they constitute only 0.004% of the detected galaxies (Table 2). The actual percentages depend on the specific assumptions (see Table 2), but the vast disproportion between the true number of QXMPs and their paucity in surveys always holds true.

### 3.2. Error Budget for the Expected Number of QXMPs

The number of QXMPs depends on several assumptions, as explained in the previous section. These assumptions are...
modified here to determine their impact on the estimated number of QXMPs.

1. *Neglecting the scatter in the SB_−M_ relationship.* The scatter increases the effective completeness quite substantially. If the scatter is neglected, then one is left with the completeness function, C, either the actual completeness determined by Blanton et al. (2005, the symbols in Figure 5) or the erf function fit to this completeness (the smooth solid black line in Figure 5). If the completeness is used, then N_QXMP ≥ 12. If the erf fit is used, then N_{QXMP} ≥ 19. These estimates are closer to (but still larger than) the number of observed QXMPs (Equation 5). However, they must be regarded as lower limits to the predicted N_{QXMP}. The scatter in the SB_−M_ relationship is a key ingredient of the detection process, as proven by the fact that the observed QXMPs are high-SB outliers of the extrapolated SB_−M_ relation (Figure 5). Consequently, the scatter must be included, and N_{QXMP} ≥ 12.

2. *The completeness function.* We have used the erf fit to the completeness function by Blanton et al. (2005) to evaluate the effective completeness leading to the limit in Equation (15). If rather than the fit the actual completeness is used (i.e., the symbols in Figure 5), then N_{QXMP} ≥ 38. The difference between the erf and the completeness function by Blanton et al. (2005) is about ±0.5 mag, in the sense that if the erf fit is shifted by ±0.5 mag then it encompasses all of the points. If the effective completeness is evaluated using the completeness shifted by ±0.5 mag, one obtains the black dashed and dotted lines in Figure 5. They yield an N_{QXMP} between 36 and 50.

3. *The absolute magnitude limit.* The absolute magnitude limit to be a QXMP, given in Equation (4), depends on a number of factors. The value of this limit is relevant because the majority of the QXMPs are within one magnitude of the cutoff (i.e., the number of QXMPs is dominated by objects with metallicity just below the metallicity cutoff and its corresponding absolute magnitude). If we use exactly one-tenth of the solar abundance by Asplund et al. (2009), then the limit in Equation (1) becomes 7.69, so that the mass–metallicity relationship by Berg et al. (2012) predicts M_B ≥ −12.9, and the limit in the r-band becomes M_r ≥ −13.7. This brighter limit allows for more QXMPs, specifically, N_{QXMP} ≃ 73. The conversion from Equation (3) to Equation (4) employs both the magnitude transformation by Jester et al. (2005) and a single color g − r ≈ 0.4 for all galaxies. If one uses the full range of colors for galaxies in the blue cloud, 0.2 ≤ g − r ≤ 0.6 (e.g., Blanton & Moustakas 2009), then M_r goes from −13.0 to −13.5, which increases N_{QXMP} from 28 to 55. Finally, if the uncertainties in the relation derived by Berg et al. (2012) are propagated into the magnitude cutoff, then M_B ≥ −12.5 ± 0.3, and so M_r ≥ −13.3 ± 0.3, leading to values of N_{QXMP} between 28 to 64.

4. *The relationship between absolute magnitude and surface brightness.* The relationship between the absolute magnitude and surface brightness in Equation (10) is given by

### Table 2

| Description | Number or Percentage | Comment |
|-------------|----------------------|---------|
| **Total number**, LF with C = 1 | 1.1 × 10^6 | dashed line in Figure 4(b) |
| QXMP, C = 1 | 149 | solid line in Figure 4(b) |
| QXMP, C = 1 | 42 | dashed line in Figure 4(b) |
| % QXMP in the survey | 0.004 | uncertainties in Section 3.2 |
| % QXMP in a volume | 87 | |
| **Total number**, varying f_n, C = 1 | 1.1 × 10^6 | red dashed line in Figure 6(b) |
| QXMP, C = 1 | 20 | red solid line in Figure 6(b) |
| QXMP, C = 1 | 6 | red dashed line in Figure 6(b) |
| % QXMP in the survey | 0.0005 | uncertainties like in Section 3.2 |
| % QXMP in a volume | 45 | |
| **Total number**, exponential f_n, C = 1 | 1.1 × 10^6 | green dashed line in Figure 6(b) |
| QXMP, C = 1 | 24 | green solid line in Figure 6(b) |
| QXMP, C = 1 | 7 | green dashed line in Figure 6(b) |
| % QXMP in the survey | 0.0006 | uncertainties like in Section 3.2 |
| % QXMP in a volume | 42 | |
| **Total number**, LF for centrals | 0.8 × 10^6 | solid line in Figure 8(b) |
| QXMP, C = 1 | 15 | solid line in Figure 8(b) |
| QXMP, C = 1 | 4 | dashed line in Figure 8(b) |
| % QXMP in the survey | 0.0005 | uncertainties like in Section 3.2 |
| % QXMP in a volume | 52 | |

**Notes.**

* The real SDSS-DR7 spectroscopic survey has 0.93 × 10^6 galaxies (Abazajian et al. 2009).
* In an unbiased, purely volume-limited survey.
* From Yang et al. (2009).
Blanton et al. (2005). In order to test the uncertainty introduced by the use of this relationship, we also employed the law by Geller et al. (2012) for blue objects, $SB_B = 29.9 + 0.46 M$, and by Kormendy (1985), $SB_B = 30.3 + 0.47 M$. The latter was digitized from Figure 3 in Kormendy’s paper. The magnitudes were then transformed from V and B to r (Jester et al. 2005), and the central surface brightness was converted to the half-light surface brightness assuming an exponential light profile. The use of these two alternative relationships renders an $N_{QXMP}$ always around 42.

5. A combination of the previous assumptions. In the previous items, the ingredients that determine $N_{QXMP}$ are analyzed independently. We have also checked the combined effect of all of them operating simultaneously. We carried out a Monte Carlo simulation where $N_{QXMP}$ was estimated, simultaneously varying the completeness function, the magnitude limit, and the mapping between $SB_B$ and $M_c$. Explicitly, the center of the completeness function and the absolute magnitude limit were randomly changed following Gaussian distributions with standard deviations of 0.5 mag (item #2) and 0.3 mag (item #3), respectively. In addition, the three relations between $SB_B$ and $M_c$, discussed in item #4 were assumed to be equally probable. As a result of 1000 trials, we obtain $N_{QXMP} = 46 \pm 20$, which is similar to the range of values inferred separately from the individual factors (Table 2).

6. The adopted LF. We separate the properties of the LF into two parts: the shape and the normalization. Changes in the shape are analyzed in Section 3.3, and produce significant changes in the number of expected QXMPs. The normalization, however, is fairly well constrained by the total number of galaxies in the SDSS-DR7 spectroscopic sample, which amount to $0.93 \times 10^9$ galaxies. By scaling the LF to reproduce the actual number of sources in the DR7 (i.e., to go from 1.1 million to 0.93 million; see Table 2), one finds $N_{QXMP} \approx 35$.

3.3. Including Galaxy Formation Quenching Induced by the UV Background

As we have shown above, the number of observed QXMPs (around nine; Equation (5)) is not consistent with the number of QXMPs expected from extrapolating the observed LF to low luminosities (from 12 to 73, with the best value around 42; Table 2). Such an extrapolation of the LF implicitly neglects the quenching of galaxy formation in low-mass halos expected from numerical models of galaxy formation (see Section 1). These predict a rapid fall-off of the baryon fraction, $f_b$, in low-mass halos due to various physical processes, such as heating of the intergalactic medium by the UV background or stellar feedback (see Section 1). The drop in $f_b$ induces a drop in the gas that fuels star formation, and hence a drop in the number of the low-mass, low-luminosity galaxies affected by the decrease of baryons.

The effect of the baryon fraction on the LF can be modeled by considering that the magnitude of a galaxy is related to the baryon fraction as follows:

$$M - M^c = -2.5 \log [f_b(1 - f_c) f_b / \mu].$$

where $M^c, f_c,$ and $f_b$ stand for the absolute solar magnitude, the light-to-stellar mass ratio (in solar units), and the gas fraction, respectively. The symbol $\mu$ in Equation (16) stands for the total mass, including dark matter, gas, and stars. Equation (16) allows us to express $\Phi$ in terms of the LF obtained by assuming the baryon fraction to be constant, $\Phi_0$. In this case, the mapping between $M$ and the magnitude $M_0$ when $f_b$ is a constant equal to $f_{b0}$ turns out to be

$$M - M_0 = -2.5 \log [f_b / f_{b0}].$$

so that the LFs for $\Phi(M)$ and $\Phi_0(M_0)$ are linked according to (e.g., Martin 1971)

$$\Phi(M) = \Phi_0(M_0) \frac{dM_0}{dM}.$$

The ratio between the two LFs at the same magnitude, $X(M)$, quantifies the drop in LF induced by the drop in the baryon fraction, i.e.,

$$X(M) = \frac{\Phi(M)}{\Phi_0(M_0)} = \frac{\Phi_0(M_0)}{\Phi_0(M)} \frac{dM_0}{dM}.$$

Neglecting variations of the mass-to-light ratio and the gas fraction with halo mass, then

$$\frac{dM_0}{dM} = \frac{dM_0 / d\mu}{dM / d\mu} = \frac{1}{1 + d \ln f_b / d \ln \mu}.$$

The baryon fraction is usually expressed in terms of the half-mass fraction, so that at mass $\mu = \mu_c$, the baryon fraction, $f_b$, is half the cosmic baryon fraction ($f_b$). In the parametrization determined by Gnedin (2000), and subsequently adopted by many others,

$$f_b = \langle f_b \rangle [1 + (2^{a/3} - 1)(\mu/\mu_c)^{1-a}]^{3/a},$$

with $a \approx 2$, as constrained by numerical simulations (Okamoto et al. 2008). Numerical simulations also give $\mu_c \approx 9.3 \times 10^9 M_\odot$ in the local universe at redshift zero (Okamoto et al. 2008, Figure 3). If the baryon fraction is given by Equation (21), then the transformation between $\Phi_0$ and $\Phi$ can be computed analytically, since

$$\frac{d \ln f_b}{d \ln \mu} = \frac{3(2^{a/3} - 1)}{1 + (2^{a/3} - 1)(\mu_c/\mu)^{1-a}} \left(\frac{\mu_c}{\mu}\right)^{a-1}.$$

It is important to realize that, in the limit of very low luminosities, $\mu \ll \mu_c$, so that Equation (22) predicts

$$\frac{d \ln f_b}{d \ln \mu} \approx 3.$$

Therefore, there is a drop in the LF associated with the vanishing baryon fraction, but it is not very large,

$$\Phi(M) / \Phi_0(M_0) \approx 0.25.$$

In fact, $\Phi(M)$ flattens for very low masses because for a given variation of $M$, $M_0$ changes very little, so that $\Phi_0(M_0)$ in Equation (24) is approximately constant, and $\Phi(M)$ also becomes independent of $M$. The LFs in Figure 6(a) show this behavior—see the red solid line, which is computed from Equations (18), (17), (20), and (21) with $\langle f_b \rangle = 0.158$ (Planck Collaboration et al. 2016), $f_{b0} = \langle f_b \rangle$, $f_L = 1$, and $f_c = 0.9$.

The damping of the LF caused by the baryon fraction in Equation (21) is never very large. In order to make the drop
more pronounced, we also tried a negative exponential parametrization of the baryon fraction,

\[ f_b = \langle f_b \rangle \exp \left( -\frac{\mu}{\mu_c} \ln 2 \right), \]

which is hardly distinguishable from Equation (21) in the representation used to compare with numerical simulations (see Figure 7(a)), but which produces a linear drop of the LF (Figure 7(b)), since

\[ \frac{d \ln f_b}{d \ln \mu} = \frac{\mu}{\mu_c} \ln 2 / \mu, \]

so that at low luminosity, where \( \mu \ll \mu_c \),

\[ \Phi(M)/\Phi_0(M_0) \simeq \mu/\langle \mu \rangle \ln 2 \to 0. \]

The LF resulting from the exponential fall-off of the baryon fraction is shown as a green solid line in Figure 6(a).

Figure 6(a) is similar to Figure 4(a), except that it reproduces the LFs when including the decrease of baryon fraction toward low-mass halos. The red and the green lines correspond to Equations (21) and (25), respectively, whereas the black solid line is the same as in Figure 4(a), and has been included for reference. Figure 6(b) shows the number of galaxies expected in the SDSS-DR7 spectroscopic survey, considering only the apparent-magnitude threshold (solid lines), and both the apparent-magnitude threshold and the incompleteness (black dashed lines). In the case of the baryon fraction given in Equation (21), and considering the apparent-magnitude limit and incompleteness,

\[ N_{\text{QXMP}} \simeq 6, \]

which is significantly smaller than the estimate for the LF with the upturn at low luminosity (Equation (15)), and consistent with the observed number of QXMPs (Equation (5)). The agreement with observations is enhanced even further after considering the error budget expounded in the last paragraph. The exponential baryon fraction (Equation (25)) gives \( N_{\text{QXMP}} \simeq 7 \), as is reflected in Table 2.

Similar to the estimate in Equation (15), the number in Equation (28) is quite uncertain. We have repeated the exercise in Section 3.2 for the case of LFs with varying baryon fraction. The result is an \( N_{\text{QXMP}} \) in the range between 3 and 12 objects. Another source of uncertainty, which we do not treat in Section 3.2, is the mapping between masses and magnitudes. According to Equations (15) and (21), the free parameters of this mapping, \( f_b \), \( 1 - f_b \), and \( \langle f_b \rangle \), appear in the equations as a single parameter, corresponding to their product. If this product is two times larger or smaller, \( N_{\text{QXMP}} \) changes from 5 to 8, respectively. If, on the other hand, the half-baryonic mass is varied from \( 2 \times 10^9 M_\odot \) to \( 2 \times 10^{10} M_\odot \), then \( N_{\text{QXMP}} \) goes from 12 to 4. (The nominal value we use is \( 9.3 \times 10^9 M_\odot \).) These uncertainties in the estimate of \( N_{\text{QXMP}} \) are summarized in Table 2.

### 4. Discussion

#### 4.1. Are QXMPs Central or Satellite Galaxies?

The number of QXMPs in the SDSS-DR7 is not consistent with the extrapolation to low luminosities of the observed LF. Observations and theory agree much better if a varying baryon fraction is included to flatten the upturn of the observed LF at low luminosities, which implicitly assumes the QXMPs to be central galaxies of their dark matter halos. The baryon fraction of satellite galaxies is determined by interactions with nearby galaxies (tidal stripping and harassment) and with the circumgalactic medium of the central galaxy (ram pressure stripping and starvation; e.g., Combes 2004; Benson 2010). Hence, the baryon fraction depends not only on the halo mass but on many other factors, and expressions for \( f_b \) like
Equation (21) are no longer valid. Consequently, the consistency of the estimated $N_{QXMP}$ with observations suggests that the upturn in the observed LF is caused by the presence of satellites. This result is in agreement with the conclusion reached by Lan et al. (2016). They model the LF by Blanton et al. (2005) as the sum of an LF for centrals (i.e., the most massive galaxy in its dark matter halo) plus an LF for satellites (i.e., galaxies sharing a dark matter halo with other more massive galaxies). This decomposition reveals that the LF of field galaxies is dominated by satellite galaxies at $M_r > -17$, and that only halos more massive than $10^{10} M_\odot$ contribute to the LF at $M_r < -12$.

The conjecture that QXMPs are central galaxies rather than satellite galaxies is also qualitatively consistent with the observed $N_{QXMP}$. If one uses the empirical LF for central galaxies determined by Yang et al. (2009), and simply extrapolates it to low luminosity, then $N_{QXMP} \approx 3$–9; see Figures 8(a) and (b), and Table 2. This LF for centrals does not show the upturn and stays below the LF by Blanton et al. (2005) (see Figure 8). The lack of an upturn produces a major drop in the number of QXMPs. The LF for centrals also predicts 30% less galaxies than the reference LF by Blanton et al. (2005). This difference is due to low-luminosity ($M_r > -17$) satellite galaxies, included by Blanton et al. (2005) but separated out by Lan et al. (2016).

4.2. Factors Limiting the Number of Observed QXMPs

Assuming that our model for the selection function provides a good representation of the SDSS properties, one can investigate which, among the parameters of the survey, are responsible for the limited number of observed QXMPs. In principle, there are three main parameters that control such a number, namely, (1) the completeness, (2) the area coverage of the survey, and (3) the apparent-magnitude limit.

The number of objects of a particular type in the survey linearly scales with the area of the survey. Since SDSS already covers a significant part of the sky (around 20%; Abazajian et al. 2009), this is not a limiting factor in the present case. In other words, no dramatic (tenfold) increase in the number of QXMPs will follow from increasing the area covered by the SDSS-DR7.

Even though completeness is an important factor, it is not the key factor that determines $N_{QXMP}$. At the absolute magnitude limit that characterizes the QXMPs, the completeness is around 30% (see Figure 5). Although the completeness can be increased, it cannot exceed the value of one, which in turn implies a moderate increase in $N_{QXMP}$. This dependence can be apprised in Figure 9(a), where the expected number of QXMPs is represented for different SB cutoffs of the completeness function. To compute the number, we have used the same completeness of the SDSS-DR7 (Figure 5, the red solid line), shifted in $SB$, with the shift parameterized as one-half of the $SB$ completeness. As the SB of the drop increases, $N_{QXMP}$ also increases. However, it saturates at a value that is only 10 times larger than the value corresponding to the SDSS-DR7 spectroscopic survey (the point of lower SB in Figure 9(b)). The curves in Figures 9(a) and (b) assume the LF shown as a solid line in Figure 6(a).

The apparent-magnitude limit turns out to be the critical parameter that limits the number of QXMPs. It determines the volume sampled by the survey, which, in principle, can be increased indefinitely as $m_{lim}$ increases. The behavior is shown in Figure 9(b). Just to provide an idea of the increase, if one considers the apparent-magnitude limit of the SDSS-DR7 photometric survey ($m_{lim} = 22.2$ in the $r$-band), then Figure 9(b) predicts the presence of approximately 2800 QXMPs, which should be compared to the prediction of 6 QXMPs in the SDSS-DR7 spectroscopic sample ($m_{lim} = 17.77$; see Table 2 and Figure 9(b)).

4.3. Number of QXMPs in Other Surveys

The formalism developed in Section 3 allows us to estimate the number of expected QXMPs in any magnitude-limited survey, given their magnitude limit, half-completeness $SB_h$ and coverage area. If the completeness function is assumed to have the functional form of the SDSS completeness, then the half-completeness $SB_h$ fully describes it. We carried out the exercise of estimating $N_{QXMP}$ for a number of ongoing and forthcoming
large area surveys, specifically, for the the Dark Energy Survey (DES; The Dark Energy Survey Collaboration 2005), the Galaxy and Mass Assembly survey (GAMA; Liske et al. 2015), the Kilo-Degree Survey (KIDS; de Jong et al. 2015), and the Large Synoptic Survey Telescope (LSST; Ivezic et al. 2008). The parameters that define the surveys are given in Table 3. LSST, DES, and KIDS do not explicitly give the half-completeness SB. In these cases, we infer it from the depth for point sources as described in Appendix B.

As shown in Table 3, the number of QXMPs expected in the next generation of wide area surveys is very large, reaching up to $10^7$ in the 10 year average LSST. The actual number changes by one order of magnitude, depending on whether the LF increases toward low luminosity (the black solid line in Figure 6(a)) or if it flattens out (the red solid line in Figure 6(a)). Therefore, it should be easy to discriminate both trends using these new surveys. For example, the ongoing GAMA survey predicts 10 or 100 QXMPs, depending on the LF faint end. In agreement with the conclusion in the previous section, the key factor determining $N_{QXMP}$ is not so much the SB cutoff but the limiting magnitude, $m_{\text{lim}}$. Note, however, that even if the surveys contain all these new QXMPs, it will be impossible to confirm the XMP nature of many of the faint objects. The spectroscopic follow-up required to determine abundances will be possible only in those QXMPs where star-forming regions are bright enough.

### Table 3

| Survey             | $r$-band $m_{\text{lim}}$ (mag) | Half-completeness SB (mag arcsec$^{-2}$) | Area (deg$^2$) | $N_{QXMP}$ | $N_{QXMP}$ |
|--------------------|---------------------------------|------------------------------------------|---------------|-----------|-----------|
| SDSS$^a$ spectroscopic | 17.8                            | 23.4                                     | 8032         | 42        | 6         |
| GAMA$^d$           | 19.8                            | 26.0                                     | 286          | 83        | 14        |
| SDSS$^b$ photometric | 22.2                            | 23.4                                     | 8423         | 2.0 \times 10^4 | 2.8 \times 10^5 |
| KIDS$^c$           | 24.0                            | 27.2                                     | 1500         | 1.5 \times 10^5 | 2.0 \times 10^5 |
| DES$^e$            | 24.1                            | 27.1                                     | 5000         | 5.8 \times 10^5 | 7.6 \times 10^5 |
| LSST$^f$, single visit | 24.7                           | 25.9                                     | 18000        | 4.3 \times 10^6 | 6.0 \times 10^6 |
| LSST$^f$, co-added 10 year | 27.5                           | 28.7                                     | 18000        | 2.3 \times 10^8 | 3.0 \times 10^7 |

Notes.

$^a$ Assuming an LF function growing with increasing magnitude (the black solid line in Figure 6(a)).

$^b$ Assuming an LF function decreasing with increasing magnitude (the red solid line in Figure 6(a)).

$^c$ For DR7. Parameters from the Web page http://classic.sdss.org/dr7/.

$^d$ Galaxy and Mass Assembly. Parameters from Liske et al. (2015).

$^e$ Kilo-Degree Survey. Parameters from de Jong et al. (2015).

$^f$ Dark Energy Survey, after five years. Parameters from The Dark Energy Survey Collaboration (2005).

$^g$ Large Synoptic Survey Telescope. Parameters from Ivezic et al. (2008).

5. Conclusions

Galaxies follow a relationship between luminosity and gas-phase metallicity, so that faint galaxies tend to be metal-poor galaxies as well. Since the LF increases steeply toward low luminosity, one would naively expect that most observed galaxies are metal poor. This is not the case. This apparent inconsistency is usually attributed to the low luminosity of the metal-poor objects, which are underrepresented in galaxy surveys. First, this occurs due to the Malmquist bias: surveys are apparent-magnitude-limited, so that the sampled volume drops down dramatically for faint sources. Second, low-luminosity galaxies are also low-SB galaxies, and the surveys tend to miss extended, low-SB objects.

This dearth of metal-poor galaxies is particularly severe for the so-called XMPs, with a gas-phase metallicity smaller than a tenth of the solar metallicity. They are of astrophysical interest for a number of reasons highlighted in Section 1, but they represent only a tiny fraction of the galaxies in the most popular surveys (e.g., 0.02% in the recent SDSS-DR7 search by Sánchez Almeida et al. 2016). Moreover, most of the observed XMPs are outliers in the luminosity–metallicity relationship. Therefore, they are not part of the predicted sea of faint XMPs. (We denote the faint XMPs as quiescent XMPs or QXMPs.) The question arises as to whether the actual number of observed XMPs is quantitatively consistent with the expected number. We address this question in the present paper. Most known XMPs come from the SDSS-DR7 spectroscopic survey, so we compare the number of observed QXMPs in this survey with the expected number. The main conclusion of our work is that they disagree, unless the LF for QXMPs is considerably shallower than the extrapolation to low luminosity of the observed LF.

The number of QXMPs in the SDSS-DR7 spectroscopic survey turns out to be $9 \pm 3$, with the error bar representing the Poissonian fluctuation (Section 2, Table 1, and Figures 1 and 2). Extrapolating to $M_r > -13.3$ the LF for faint galaxies observed by Blanton et al. (2005, shown in Figure 4(a)), the expected number is 42 (Section 3). This estimate takes into account the Malmquist bias plus the finite completeness of SDSS for low-SB objects. In addition, it includes the scatter in the SB–magnitude relationship, which effectively increases the completeness of the survey at low luminosities (Appendix A). Once the various uncertainties involved in the estimate are considered (Section 3.2), the expected number of QXMPs is in the range between 12 and 73, with the low value highly disfavored.

On the other hand, if the previous LF is modified to include the decrease of baryon fraction in low-mass dark matter halos, then the upturn at low luminosity disappears (Figure 6(a)), rendering an expectation of only six QXMPs. (The number is between 3 and 12 when uncertainties are taken into account; see Section 3.3.) The tension with observation automatically disappears. Including the varying baryon fraction implicitly assumes the QXMPs to be central galaxies in their dark matter halos. In fact, the LF for centrals determined by Yang et al. (2009; see Figure 8(a)) does not show the upturn, and it predicts between three and nine QXMPs (Section 4.1 and
Table 2). The agreement with observations has several implications. First, QXMPs seem to be central, rather than satellite, galaxies. QXMPs become tracers of low-mass halos not gravitationally bound to massive halos, and so they can be used to trace these halos observationally. These low-mass dark matter halos are of clear astrophysical interest in the context of characterizing the building blocks in the hierarchical formation of galaxies and in particular, the effect of the cosmic UV background in their baryonic content. The fact that QXMPs seem to be isolated and in low-density regions of the universe (e.g., Filho et al. 2015; Sánchez Almeida et al. 2016), second, the upturn in the faint end of the observed LF appears to be due to satellite galaxies, and this should be taken into account when comparing observations and numerical models. Third, the baryon fraction predicted by the numerical models is consistent with observations. First, QXMPs seem to be central, rather than satellite, galaxies. QXMPs become tracers of low-mass halos not gravitationally bound to more massive halos, and so they can be used to trace these halos observationally. These low-mass dark matter halos are of clear astrophysical interest in the context of characterizing the building blocks in the hierarchical formation of galaxies and in particular, the effect of the cosmic UV background in their baryonic content. The fact that QXMPs seem to be isolated and in low-density regions of the universe (e.g., Filho et al. 2015; Sánchez Almeida et al. 2016), second, the upturn in the faint end of the observed LF appears to be due to satellite galaxies, and this should be taken into account when comparing observations and numerical models. Third, the baryon fraction predicted by the numerical models is consistent with observations.

Assuming that our modeling of the SDSS biases is correct, we have studied which among the parameters defining the survey restricts the number of QXMPs the most (Section 4.2). It turns out to be the apparent-magnitude limit, which is more relevant than the incompleteness. Thus, the photometric SDSS-DR7 survey, which is 4 magnitudes deeper than the spectroscopic one, should contain as many as 2800 QXMPs. The expected numbers in various other surveys are determined in Section 4.3, and a summary is presented in Table 3. Future surveys are predicted to detect QXMPs in large quantities.

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Appendix A

Scatter in the $SB$–$M$ Relationship

Equation (11) assumes a one-to-one correspondence between the SB and the absolute magnitude of the source. The fact that most observed QXMPs are high-SB outliers of the $SB$–$M$ relationship indicates that the scatter in this relation may be of importance in our estimate. In simple terms, an object is preferentially picked out by SDSS if it is of high SB for its absolute magnitude.

In order to treat this case in our formalism, we assume the LF to be the marginal probability density function (PDF), $P(M, SB)$, which quantifies the number of galaxies with absolute magnitude $M$ and surface brightness $SB$, i.e.,

$$
\Phi(M) = \int_{SB} P(M, SB) dSB.
$$

Then computing the number is equivalent to Equation (6), but it involves a double integral over $M$ and $SB$,

$$
N(M > M_{lim}) = \int_{M_{lim}}^{M_{lim}} \int_{SB} P(M, SB) V(M) C'(SB) dSB dM,
$$

with the symbol $C'$ standing for the completeness function expressed in terms of the surface brightness. The bivariate distribution function can be expressed in terms of the conditional probability function, $P(SB|M)$ (the probability of having a surface brightness $SB$, given that the absolute magnitude is $M$),

$$
P(M, SB) = P(SB|M) \Phi(M).
$$

Inserting expression (31) into Equation (30), one recovers Equations (6) and (11), provided that $C(M)$ is replaced with an effective completeness function, $C_{eff}$, given by

$$
C_{eff}(M) = \int_{SB} P(SB|M) C'(SB) dSB.
$$

With minimal assumptions, one can write down the conditional PDF as

$$
P(SB|M) = \frac{1}{\Delta_0} G\left(\frac{SB - SB_0}{\Delta_0}\right),
$$

with

$$
SB_0 = SB_0(M),
$$

$$
\Delta_0 = \Delta_0(M),
$$

and $G$ any positive function properly normalized,

$$
\int_{-\infty}^{+\infty} G(x) dx = 1.
$$

The case without scatter in the $SB$–$M$ relationship corresponds to an infinitely narrow conditional probability function, which can easily treat in the limit $\Delta_0 \rightarrow 0$, so that

$$
\frac{1}{\Delta_0} G\left(\frac{SB - SB_0}{\Delta_0}\right) \rightarrow \delta(SB - SB_0),
$$

with $\delta$ a Dirac delta function. Inserting the previous expression into Equation (32) yields

$$
C_{eff}(M) = C(M) = C'(SB_0(M)),
$$

with $SB_0(M)$ given by expression (10). In general, the computation of $N_{QXMP}$ must make use of the full expression, where the effective completeness function is a type of convolution of the completeness function with the conditional PDF. Following Blanton et al. (2005), we will assume that the conditional PDF is a Gaussian,

$$
G(x) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right),
$$

with the mean, $SB_0$, and the variance, $\Delta^2_0(M)$, measured to be

$$
SB_0(M) = 23.8 + 0.45(M + 13.3),
$$

$$
\Delta^2_0(M) = 1.16 + 0.081(M + 13.3).
$$
Equation (38) is identical to Equation (10), and has been included here for comprehensiveness. The effective completeness function resulting from $C$ in Blanton et al. (2005) and the above parametrization of the conditional function is shown by the red solid line in Figure 5. The net effect is a significant enhancement of the completeness at low luminosity.

We note that in the limiting case when (1) $C'(SB)$ is a Heaviside step function (zero for $SB$ larger than a given surface brightness, and one elsewhere), and when (2) $G$ is a Gaussian of constant width $\Delta_0$, then $C_G$ in Equation (32) has the shape of an erf function. This approximation is used in the main text.

Appendix B

Estimate of the SB Limit from the Depth of the Survey

In order to predict the number of XQMPs expected in various surveys, one needs to assign an SB limit to them. The requirements of the survey are often set in terms of the depth for point sources, i.e., the flux that a point source must have to grant detection with a signal-to-noise ratio a number of times above the noise level. In order to transform this parameter to the corresponding SB limit, we proceed as follows: the depth for point sources, $n_{point}$, is defined as

$$m_{point} = -2.5 \log (\xi \sqrt{n_{point}}) + \kappa,$$

(39)

where $\sigma$ stands for the noise per pixel, $\xi$ for the level above noise to grant detection, and $n_{point}$ for the number of pixels covered by a point source. $\kappa$ sets the zero of the magnitude scale. Equation (39) assumes the noise of adjacent pixels to be independent, so that the noise of their sum adds up quadratically. We will define the SB limit as the SB of an extended source exceeding the noise in one arcsecond, i.e.,

$$m_{ext} = -2.5 \log (\sigma \sqrt{n_{arc}}) + \kappa,$$

(40)

with $n_{arc}$ the number of pixels in one arcsecond. Since

$$n_{point}/n_{arc} = \pi (\text{FWHM}/2)^2,$$

(41)

with FWHM the full width at half-maximum of the point-spread function in arcseconds, then Equations (39) and (40) lead to

$$m_{ext} = m_{point} + 2.5 \log \left(\frac{\sqrt{\pi}}{2} \text{FWHM}\right).$$

(42)

Equation (42) links the depth, $m_{point}$, with the SB limit, $m_{ext}$.

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