MORE ON THE NONEXISTENCE OF ODD PERFECT NUMBERS OF CERTAIN FORMS

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Abstract. Euler showed that if an odd perfect number exists, it must be of the form

\[ N = p^\alpha q_1^{2\beta_1} \cdots q_k^{2\beta_k}, \]

where \( p, q_1, \ldots, q_k \) are distinct odd primes, \( \alpha, \beta_1, \ldots, \beta_k \in \mathbb{N} \), with \( p \equiv \alpha \equiv 1 \pmod{4} \). In 2005, Evans and Pearlman showed that \( N \) is not perfect, if \( 3|N \) or \( 7|N \) and each \( \beta_i \equiv 2 \pmod{5} \). We improve on this result by removing the hypothesis that \( 3|N \) or \( 7|N \) and show that \( N \) is not perfect, simply, if each \( \beta_i \equiv 2 \pmod{5} \).

1. Introduction

We define \( \sigma(N) \) to be the sum of the positive divisors of \( N \). We say \( N \) is perfect when \( \sigma(N) = 2N \). For example, \( \sigma(6) = 1 + 2 + 3 + 6 = 12 \) and \( \sigma(28) = 1 + 2 + 4 + 7 + 14 + 28 = 56 \), making both 6 and 28 perfect. It is still an open question as to whether or not there exists an infinite number of even perfect numbers or even a single example of an odd perfect number. Nevertheless, we let \( \mathcal{O} \) be the set of all odd perfect numbers.

Euler showed that if an odd perfect number exists, it must be of the form:

\[ N = p^\alpha q_1^{2\beta_1} \cdots q_k^{2\beta_k} \tag{1.1} \]

where \( p, q_1, \ldots, q_k \) are distinct odd primes, \( \alpha, \beta_1, \ldots, \beta_k \in \mathbb{N} = \{0, 1, 2, \ldots\} \), with \( p \equiv \alpha \equiv 1 \pmod{4} \). The prime \( p \) is often referred to as the special prime of \( N \), and \( p^\alpha \) as the Eulerian component of \( N \). Throughout this paper, when we say \( N \in \mathcal{O} \), unless otherwise stated, we assume \( N \) has the form given in (1.1).

Assuming \( \beta_1 = \cdots = \beta_k = \beta \), it has been shown for all fixed \( \beta \leq 14 \), except \( \beta = 9 \), that \( N \) cannot be odd perfect. Additional results include infinite congruence classes for \( \beta \). For example, McDaniel proved in [7] that \( N \) is not perfect if each \( \beta_i \equiv 1 \pmod{3} \). Iannucci and Sorli, in [5], showed \( N \) is not perfect if \( 3|N \) and each \( \beta_i \equiv 1 \pmod{3} \) or \( \beta_i \equiv 2 \pmod{5} \). See Evans and Pearlman, [3] for a more detailed account of these types of results. In that same paper, Evans and Pearlman show

**Theorem 1.** Suppose \( N \in \mathcal{O} \) and each \( \beta_i \equiv 2 \pmod{5} \), then \( \gcd(N, 21) = 1 \) and \( p \equiv 1 \pmod{12} \).

Using this result, and applying a different method of proof, we show

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Theorem 2. Suppose N is as described in (1.1) and each \( \beta_i \equiv 2 \pmod{5} \), then N is not perfect.

Theorem 2 subsumes a number of theorems like Theorem 1.

2. Preliminary Results

We start with some elementary properties of \( \sigma \):

1. if \( q \) is prime, \( k \in \mathbb{N} \), then \( \sigma(q^k) = 1 + q + q^2 + \ldots + q^k \).
2. \( \sigma \) is multiplicative. That is, if \( \gcd(m, n) = 1 \), then \( \sigma(m \ast n) = \sigma(m) \ast \sigma(n) \).

For \( N \in \mathbb{O} \):

\[
\sigma(N) = \sigma(p^\alpha)\sigma(q_1^{2\beta_1}) \ldots \sigma(q_k^{2\beta_k}) = 2N
\]

(2.1)

Which makes clear that any prime \( r \) dividing \( \sigma(q_i^{2\beta_i}) \) for some \( i \), must also divide \( N \). Additionally, \( \alpha \) odd implies that for \( \alpha = 2a + 1 \)

\[
\sigma(p^\alpha) = 1 + p + \ldots + p^{2k} + p^{2a + 1} = (1 + p) + \ldots + (1 + p)p^{2a} = (1 + p)(1 + p^2 + \ldots + p^{2a})
\]

(2.2)

Thus, \( (p + 1)|\sigma(p^\alpha) \) and so any odd prime dividing \( (p + 1) \) also divides \( N \).

We will make use of the following lemmas. The proof of the first can be found in [8] and the second in [6].

Lemma 1. \( \sigma(s^f)|\sigma(s^{f + (f + 1)m}) \) for all primes \( s \) and all \( m, f \in \mathbb{N} \).

Let \( N \in \mathbb{O} \) be written as in (1.1). Define \( \gamma_i = 2\beta_i + 1 \) for \( 1 \leq i \leq k \).

Lemma 2. Suppose \( d|\gamma_i \) for each \( i \), then \( d^a|N \).

Remark 1. Observe the assumption in Theorem 2 that each \( \beta_i \equiv 2 \pmod{5} \) is equivalent to assuming \( d = 5 \) in Lemma 2.

Further note, with \( f = 4 \) and \( 2\beta_i \equiv 4 \pmod{5} \) for some \( m \in \mathbb{N} \), Lemma 1 implies

\[
\sigma(s^4)|\sigma(s^{5m + 4}) = \sigma(s^{2\beta_i}).
\]

(2.3)

Though Lemma 1 is often discussed in the context of cyclotomic polynomials, we won’t need results any more powerful than the above. These two ideas lie at the foundation of every result similar to Theorem 1 and Theorem 2.

3. \( \sigma \)-Chains

Under the hypotheses of Theorem 2, suppose \( r_0 \) is a prime that divides \( \sigma(q_i^4) \). It follows immediately that

\[
r_0|\sigma(q_i^4)|\sigma(q_i^{2\beta_i})|N \Rightarrow r_0|N.
\]

(3.1)

Thus, if \( r_0 \not\equiv 1 \pmod{4} \), then \( r_0 \) cannot be the special prime in the prime factorization of \( N \), so there must be some \( q_i \) where \( r_0 = q_i \).

Remark 2. Applying Theorem 1, we can reach this same conclusion so long as \( r_0 \not\equiv 1 \pmod{12} \). Additionally, since we know \( 7 \nmid N \), (2.2) implies \( r_0 \not\equiv 6 \pmod{7} \).
For the rest of this paper, we assume our special prime $p$ satisfies these two conditions.

Applying the same reasoning to a prime, say $r_1$, such that $r_1|\sigma(r_1^4)$, so long as $r_1$ does not satisfy the conditions of Remark 2, then we may conclude $r_1|N$. Continuing this process, we may construct a chain $\{r_0, r_1, r_2, \ldots\}$ of primes satisfying $r_{i+1}|\sigma(r_i^4)$ whereby each $r_{i+1}$ must divide $N$.

Traditionally, these chains have been used to accumulate enough primes to show that $\sigma(N) > 2$, thus contradicting $\sigma(N) = 2$. We will be using them to create a contradiction on a different internal structure of $N$. For the moment, we will use them to show the following

**Proposition 1.** If $N \in \mathbb{O}$ satisfies the hypotheses of Theorem 2, each prime in the collection $T = \{5, 11, 31, 41, 71, 101, 131, 151, 181, 191, 211\}$ divides $N$.

**Proof.** By Remark 1, we know $5|N$, which we use to construct our first two chains:

\[
\{5, 11\}
\]

\[
\{5, 71, 211, 1361, 11831, 17249741, 41\}
\]

Once we know $211|N$, we may construct the chain:

\[
\{211, 292661, 191, 13001, 32491, 34031, 101, 31\}
\]

and after $191|N$,

\[
\{191, 1871, 151\}
\]

\[
\{191, 13001, 17981, 613680341, 1478611, 520392931, 336491, 4231, 216211, 131\}
\]

\[
\{191, 13001, 32491, 34031, 350411, 47791, 26561, 181\}
\]

A quick scan through each chain shows that none of the primes are congruent to $1 \pmod{12}$, except 181 which is also $6 \pmod{7}$. Thus, none of the primes can be the special prime, and since each element of $T$ appears, we are done. \(\square\)

**Proposition 2.** For the same $N$, each prime in $T$ divides $N$ at least 5 times.

**Proof.** We skip most of the details and simply point out that

- $11$ divides $\sigma(5^4), \sigma(31^4), \sigma(71^4), \sigma(191^4)$, and $\sigma(311^4)$
- $31$ divides $\sigma(101^4), \sigma(281^4), \sigma(1031^4), \sigma(1151^4)$, and $\sigma(1721^4)$

and similarly, every element of $T$ can be shown to appear in at least 5 $\sigma$-chains of various primes under 10000, except 181, which requires us to go to as high as 18521. \(\square\)

For the rest of the paper we expand the definition of $T$ to include all of the primes demonstrated through $\sigma$-chains to divide $N$. Suppose now that it has been demonstrated $\{821, 55001\} \subset T$. Consider the following two chains:

\[
\{821, 241, 61\}
\]

\[
\{55001, 2521, 61\}
\]

Observe that 61, 241, and 2521 each satisfy the conditions of Remark 2 and are candidates for being the special prime. Neither chain has any primes in common except for the last element, 61. At most, one of 241 or 2521 may be the special prime. By virtue of this fact, one of the two chains shows $61 \in T$. 

4. A Closer Look at the Form of $N$

In 1980, Ewell [4] demonstrated the following:

**Theorem 3.** Let $N = p^\alpha q_1^{2\beta_1} \cdots q_k^{2\beta_k}$ be odd perfect. Set $\alpha = 4\epsilon+1$ and $q_i = 2\pi_i - 1$. Assume $3 \nmid N$, then

1. $p \equiv 1 \pmod{12}$,
2. $\epsilon \equiv 0$ or $-1 \pmod{3}$,
3. $\pi_i \beta_i \equiv 0$ or $-1 \pmod{3}$ for each $i$,
4. The number of elements in the set $\{\pi_1 \beta_1, \ldots, \pi_k \beta_k\}$ for which $\pi_i \beta_i \equiv -1 \pmod{3}$ is even when $\epsilon \equiv 0 \pmod{3}$ and odd when $\epsilon \equiv -1 \pmod{3}$.

Applying our assumption $\beta_i \equiv 2 \pmod{5}$ to this result, simple algebra shows that $N$, after some relabeling, may be written as

$$N = p^\alpha \prod_{i=1}^{s} q_i^{10\beta_i + 4} \prod_{j=1}^{t} r_j^{30\gamma_j + 4} \prod_{k=1}^{u} s_k^{30\delta_k + 24}$$

(4.1)

Where $p, q_i, r_j, s_k$ are primes with $p \equiv 1 \pmod{12}$, each $q_i \equiv -1 \pmod{6}$, and each $r_j, s_k \equiv 1 \pmod{6}$ and, for their respective subscripts, each $\beta, \gamma, \delta$, and $\alpha \in N$ with $\alpha \equiv 1$ or $9 \pmod{12}$.

This form of $N$ is a bit cumbersome, but it does clearly demonstrate one useful piece of information.

**Remark 3.** From Proposition 2, it is immediate that $5|N$ at least 14 times, $11|N$ at least 14 times, $31|N$ at least 24 times, and so on, according to each prime’s residue (mod 6).

5. The Sum of the Reciprocal of the Primes of $N$

We now consider [1]. Cohen demonstrates the following for odd perfect numbers such that $5|N$ and $3 \nmid N$:

$$\sum_{q \mid N} \frac{1}{q} < .677637$$

To this author’s knowledge, the best upper and lower bounds of the types appear in [2], and we include them here.

| Divisors | LowerBound | UpperBound |
|----------|-------------|------------|
| $3 \mid N, 5 \nmid N$ | .647649 | .677637 |
| $3 \nmid N, 5 \mid N$ | .667472 | .693148 |
| $3 \mid N, 5 \mid N$ | .596063 | .673634 |
| $3 \nmid N, 5 \mid N$ | .604707 | .657304 |

Given that we have additional knowledge regarding the structure of $N$, it is natural we try to improve on this bound for our particular case. Cohen starts his proof off by demonstrating a stronger form of the following inequality, for $0 < x \leq \frac{1}{3}$

$$1 + x + x^2 > \exp(x)$$

(5.1)
We assume (5.1) and let \( N \in O \). Thus,

\[
2N = \sigma(N) = (1 + p + p^2 + \ldots + p^\alpha) \prod_{i=1}^{k} (1 + q_i + q_i^2 + \ldots + q_i^{2^{s_i}})
\]

Dividing through by \( N \) and utilizing \( \alpha \geq 1 \) yields,

\[
2 \geq (1 + \frac{1}{p}) \prod_{i=1}^{k} (1 + \frac{1}{q_i} + \frac{1}{q_i^2} + \ldots + \frac{1}{q_i^{2^{s_i}}})
\]

We separate the \( q_i \) based on whether or not they appear in \( T \). As per Remark 3, we let \( \gamma_i \) be 14 or 24 depending on whether \( q_i \equiv -1 \pmod{6} \) or \( q_i \equiv 1 \pmod{6} \), respectively for \( q_i \in T \). And finally, we truncate for the primes not in \( T \).

\[
2 \geq (1 + \frac{1}{p}) \prod_{q_i \in T} (1 + \frac{1}{q_i} + \frac{1}{q_i^2} + \ldots + \frac{1}{q_i^{2^{s_i}}}) \prod_{q_i \notin T} (1 + \frac{1}{q_i} + \frac{1}{q_i^2})
\]

Apply (5.1) to the components of \( q_i \notin T \) and take the log,

\[
\ln 2 > \ln (1 + \frac{1}{p}) + \sum_{q_i \in T} \ln (1 + \frac{1}{q_i} + \frac{1}{q_i^2} + \ldots + \frac{1}{q_i^{2^{s_i}}}) + \sum_{q_i \notin T} \frac{1}{q_i}
\]

Let

\[
\Delta = \sum_{q_i \in T} \ln (1 + \frac{1}{q_i} + \frac{1}{q_i^2} + \ldots + \frac{1}{q_i^{2^{s_i}}}) - \frac{1}{q_i}
\]

We substitute \( \Delta \) into (5.2), add \( \frac{1}{p} \) to both sides, and rearrange,

\[
\ln 2 - \Delta - \ln (1 + \frac{1}{p}) + \frac{1}{p} > \frac{1}{p} + \sum_{i=1}^{k} \frac{1}{q_i}
\]

We now observe the right hand side of the inequality is the sum of the reciprocal of the primes that divide \( N \). Because \( \Delta \) is a straight forward calculation, it would seem our upper bound is singularly dependent upon \( p \), however, we can still do a little better by adding a few more primes to \( T \). Remark (2) implies \( p = 37, 61, 73 \), or \( p \geq 109 \) are the only options for the special prime. It will be convenient for us to refer to \( \gamma_q \) for the exponent of a particular prime \( q \in T \) as we consider each case in turn.

1. \( p = 37 \), then \( 19 \mid (p+1) \) (and thus \( N \)). Additionally, we know 61 is not the special prime and since we have seen two chains (with appropriate assumptions) that demonstrate 61|\( N \) we can put \( 19, 61 \in T \) with \( \gamma_{19} = \gamma_{61} = 4 \).

2. \( p = 61 \), then \( 31 \mid (p+1) \), which is already in \( T \).

3. \( p = 73 \), then \( 37 \mid (p+1) \), and again, we can put \( 37, 61 \in T \) with \( \gamma_{37} = \gamma_{61} = 4 \).

4. \( p \geq 109 \). As before, we can put \( 61 \in T \) with \( \gamma_{61} = 4 \).

When \( p \geq 109 \) we must change our bound slightly. By virtue of the Maclaurin series of \( \ln (1 + x) \), \( \ln (1 + x) > x - \frac{x^2}{2} \) for small \( x \). Now, (5.3) may be written as

\[
\ln 2 - \Delta + \frac{1}{2p^2} > \frac{1}{p} + \sum_{i=1}^{k} \frac{1}{q_i}
\]

After computing the different incarnations of \( \Delta \) for each case, we get the following upper bounds for the right hand side of (5.3):

\[
\ln 2 - \Delta + \frac{1}{2p^2} > \frac{1}{p} + \sum_{i=1}^{k} \frac{1}{q_i}
\]
As can be seen, .6646602 is the greatest of these bounds, so we take this to be, and will refer to it as, the upper bound for our \( N \).

This may seem like a lot of work to lower the previous upper bound by less than .013. However, when one considers optimal solutions, by which we mean, every allowable prime in \( T \) without concern for special primes. It turns out that \( T \) must contain 100,369 primes: 5 and every 1 (mod 5) prime less than 5,826,451. This improved bound requires 5 and every 1 (mod 5) prime less than 2,647,111 or a mere 48,250 primes. Of course, the primes that appear in \( \sigma \)-chains is far from optimal and about 20% of them are special primes, which as we have seen, require extra attention.

6. Programming Methodology

It should be noted that \( \sigma \)-chains as a term is a bit misleading. A more appropriate term would be \( \sigma \)-trees. Of course, presenting such information in tree form becomes unwieldy. Though as chains get progressively longer, they too become unwieldy.

Considering we had to construct chains to include over 960,000 primes, even after taking great pains to reduce the upper bound in Section 5, the approximately 10,000 pages of data is likely why Theorem 2 has not been demonstrated sooner.

In an effort to simplify the process of constructing chains, we started with chains of length 2:

\[
\{ \text{Known Seed Prime} \} \rightarrow \{ \text{New Prime} \}
\]

Chains resulting in a potential special prime or a composite too large for Mathematica to conveniently factor were set aside\(^1\). The new primes were put into a “seed” list. The next ordered pair, or 2-chain, was created using the smallest number from the seed list. This is not a new idea. The advantage is always working with the smallest numbers from the full \( \sigma \)-tree, thus avoiding the need to factor large numbers to continue a chain. More importantly, keeping the data as uniform 2-chains made programming chain manipulations much easier to automate and separate between two computers when convenient.

Breaking the upper bound without including candidates for the special prime, though technically feasible, would probably take years. The 2-chains with special primes, previously set aside, were extended to 3-chains where the 3rd element was also a special prime, as follows

\[
\{ \text{Seed Prime} \} \rightarrow \{ \text{Special Prime} \} \rightarrow \{ \text{Special Prime} \}
\]

This collection of 3-chains was then sorted by the last element. The first two appearances of a special prime in the 3rd element were paired together and used to confirm that prime was an element of \( T \). After removing chains with duplicate

\(^1\)The chains involving large composites were never used. We were able to generate enough primes to prove Theorem 2 without factoring any of these numbers.
terminus (keeping one chain from the confirmed primes), the 3-chains were extended
to 4-chains, again, only extending in the cases where a special prime appeared as
the 4th element (that had not already been confirmed in the previous step). The
last element of each 4-chain was then compared to the 3rd and 4th element of each
of the other 4-chains. Any matches were used to confirm the corresponding special
prime was in $T$. This process is not optimal. Among other considerations, we are
allowed to have non-special primes appear in these extended chains, however, a
broader approach turned out to be unnecessary.

The calculations needed to create the primes included in our set $T$, required 2
laptops a little over 3 weeks to perform. The previous month had been spent trying
to work with chains of arbitrary length. As aforementioned this becomes quite
cumbersome. Once this 2-chain method was implemented, the previous month’s
work (in some sense) was replicated in about 4 days.

7. Proof Methodology

As was previously mentioned, the typical method of proof for theorems like
Theorem 1 and 2 is to accumulate enough primes to show $\sigma(N) > 2N$. In the
context of optimal solutions, this bound can be easier to surpass than the sum of
the reciprocal of primes. Taking every allowable prime to be in $T$ without concern
for special primes, and assuming each prime divides $N$ four times, we achieve
$\sigma(N) > 2N$ after 47335 primes; 5 and every 1 (mod 5) prime less than 2592521. A
marginal improvement compared to our methodology. Though without any special
knowledge regarding the number of times an arbitrary prime divides $N$, the sum
of the reciprocal of primes may be an easier bound to surpass. Regardless, when
nearly a million primes are needed for either method of proof, the choice largely
becomes a matter of taste.

With regards to section 4. If one compares the improvement to the upper bound
from increasing the number of times a prime divides $N$ from 4 to 14 (or 24), the
improvements are minor. This begs the question, “Why use this in our proof?”
The answer is simple. Since we are showcasing a different methodology for these
types of proofs, we hope something here may spur innovation in someone else by
broadening the view of the problem.

8. Data Summary

Occasionally, Mathematica would hang trying to show a number to be provably
prime. This is why Module 17 has about half as much data as the other modules.
The first time this happened, we started the next module, but after this instance,
we skipped the seed prime, and restarted the module with the next number in the
seed list.

Despite setting the recursion level to infinity, Mathematica did not seem to like
using our algorithm after about 24,000 new seeds cycled through. To remedy this,
we stopped after 20,000 iterations and called it a data module. We have 30 data
modules for the non-special primes using this algorithm. A proper remedy, namely
taking the local variables within the program and making them global, solved this
problem and allowed us to use as many as 50,000 seeds without issue in Modules
32-34.

Due to a lapse in programming, the primes that were 1 (mod 12) and also 6
(mod 7) were originally collected with the special primes. After the oversight was
noticed, we separated them out and called them Module 31. The new primes were
used as seeds for chains in Modules 32-34.

The filters resulting from Modules 1-18 were used to filter Module 20. As a
result, 6305 primes were duplicated between Modules 19 and 20. The two data sets
were combined for simplicity of filtering out duplicates.

Data Summary

| Module | # Primes | Total Primes | Module Sum  | Total Sum   |
|--------|----------|--------------|-------------|-------------|
| 1      | 29253    | 29253        | 0.5430147   | 0.5430147   |
| 2      | 27967    | 57220        | 0.0105602   | 0.5535749   |
| 3      | 27451    | 84671        | 0.0060902   | 0.5596651   |
| 4      | 27040    | 111711       | 0.0040347   | 0.5636998   |
| 5      | 27071    | 138782       | 0.0032882   | 0.5669880   |
| 6      | 26680    | 165462       | 0.0025726   | 0.5695606   |
| 7      | 26386    | 191848       | 0.0020041   | 0.5715647   |
| 8      | 26154    | 218002       | 0.0017367   | 0.5733014   |
| 9      | 26386    | 244388       | 0.0016088   | 0.5749102   |
| 10     | 26008    | 270396       | 0.0014174   | 0.5763276   |
| 11     | 25915    | 296311       | 0.0012916   | 0.5776192   |
| 12     | 25933    | 322244       | 0.0012246   | 0.5788438   |
| 13     | 25837    | 348081       | 0.0010565   | 0.5799003   |
| 14     | 25562    | 373643       | 0.0009254   | 0.5808257   |
| 15     | 25825    | 399468       | 0.0008905   | 0.5817163   |
| 16     | 25543    | 425011       | 0.0008013   | 0.5825176   |
| 17     | 12262    | 437273       | 0.0003992   | 0.5829168   |
| 18     | 25497    | 462770       | 0.0007536   | 0.5836704   |
| 19-20  | 44626    | 507396       | 0.0011789   | 0.5848493   |
| 21     | 25447    | 532843       | 0.0006398   | 0.5854891   |
| 22     | 25055    | 557898       | 0.0005856   | 0.5860747   |
| 23     | 25086    | 582984       | 0.0005580   | 0.5866328   |
| 24     | 25278    | 608262       | 0.0005657   | 0.5871985   |
| 25     | 25292    | 633554       | 0.0005443   | 0.5877428   |
| 26     | 25100    | 658654       | 0.0004953   | 0.5882381   |
| 27     | 25094    | 683748       | 0.0004879   | 0.5887260   |
| 28     | 25150    | 708898       | 0.0004763   | 0.5892022   |
| 29     | 25034    | 733932       | 0.0004517   | 0.5896539   |
| 30     | 25069    | 759001       | 0.0004470   | 0.5901010   |
| 31     | 41963    | 800964       | 0.0160788   | 0.6061797   |
| 32     | 36037    | 837901       | 0.005961    | 0.6067758   |
| 33     | 54933    | 892834       | 0.008298    | 0.6076056   |
| 34     | 66071    | 958905       | 0.009650    | 0.6085706   |
| Special Primes | 2511    | 961416       | 0.0567245   | 0.6652951   |

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