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Study on Scanning Forming Methods of Machining Work-piece Surface

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Abstract: In the process of machine tool cutting, there are strict geometric relations among the cutting edge curve / tool surface, machine tool movement and workpiece surface, and the machine tool movement is also related to the type of tool. Firstly, the forming methods of cutting workpiece surface are analyzed and summarized from the geometric point of view, and the scanning forming method and its geometric expression are studied, and the research technical route of forming turning scanning forming is put forward. Then, the mathematical modeling and Simulation of forming turning are carried out according to the proposed technical route. Finally, taking the groove of the inner ring of the formed turning ball bearing as an example, the mathematical modeling of the design surface of the workpiece and the machined surface of the workpiece is carried out. The radial dimension changes of the workpiece caused by the cutting force and tool wear are analyzed, and the simulation of the machined surface of the workpiece is carried out.

Key words: Cutting Edge Curve/Tool Surface; Machine Tools Motion; Work-Piece Surface; Geometry; Scanning Forming

1. Introduction

Cutting process is an important method to manufacture mechanical parts by forming the surface of the workpiece by the movement of the tool relative to the workpiece.

The cutting forming method of mechanical parts and the required machine tool motion belong to the research category of machine tool kinematics. Machine tool kinematics studies the forming method of the workpiece surface, the various motions required and the relationship between the motions during the machining of the machine tool. The basic theory was put forward by the former Soviet Union scholar Professor Golovin, who believed that the large number of existing various types of machine tools are just the transformation results of some basic primitive mechanisms [1,2]. If the components of these original mechanisms are regarded as tools and workpieces, and given motions separately, they will be transformed into a machine tool [1,3]. When cutting the surface of a workpiece,
there is a rigorous geometric relationship among the tool cutting edge curve/tool surface, the machine motion and the workpiece surface.

With the rapid development of society and economy, in order to meet the needs of product functions, more and more mechanical parts with complex curved surfaces are used on the surface, and the cutting and forming movements of machine tools are also becoming more complicated. In 1952, CNC machine tools [3,4] came into being. The cutting and forming process of CNC machine tool processing and ordinary machine tool processing mechanical parts is the same, only in the control method is different, there are also problems with the theory and method of part cutting and forming. Yuan-Liu Chen et al.[5] presented a self-evaluation method for submicron precision measurement of the cutting edge contour of a micro diamond tool with a force sensor integrated fast tool servo (FS-FTS) on an ultra-precision lathe. Zhihuang Shen et al.[6] presented a digital graphic scanning (DGS) method based on computer scanning images to generate grinding profiles, avoiding the difficulties caused by complex contact line equations. Zhiyong Chang[7] formulated effective cutting edge equations to accurately calculate the dimensional errors and the surface roughness, providing a new approach for high precision CNC turning programming. Cheng Yajun et al.[8] innovated the processing technology of ball bearing and summarized the grooves of ball bearing turned by forming turning tool. M.L.Wu and Zhang KF et al. [9,10] proposed an evaluation method of interrupt shaping planning based on radial raceway contour change (monitoring point). S.J.Wang et al.[11] designed a three-axis computer numerical control (CNC) grinding machine for grinding micro-V-shaped groove arrays on hard and brittle materials. Material removal and machining strategy were studied for the grinding of V-grooves array. J-F Hsieh et al.[12] presented a simple yet comprehensive method for the design and machining of a Geneva indexing mechanism with curved slots. The kinematics model of grooved wheel mechanism was established by using homogeneous coordinate transformation and conjugate surface theory. Jui-Tang Tseng[13] presented the generative motion of machining curved-tooth cylindrical gear with CNC hobbing machine. Based on the cutting mechanism and the gear theory, the surface equation of this kind of gear is established, which is the function of the design parameters of the hob. Based on the mathematical model of the gear, the computer graph of the curved gear is given, and the tooth surface deviation caused by setting the nominal radius of the circular arc tooth line is studied. Yan He et al.[14,15] established a mathematical model and an analytical model based on tangential motion conditions, and deduced the profile of the rotary cutter according to the machining parameters and the required geometrical shape of the workpiece. The accuracy, calculation time and geometric flexibility are compared. Yingxue Yao et al.[16] analyzed and modeled the forming process of the workpiece in turning and the error sources affecting the machining accuracy of the workpiece, and presented to represent the geometric errors of the workpiece with the representation of the tool path and attitude. Hu Gongp et al.[17] presented a new spiral tool path generation method for quasi-rotating diamond turning optical free-form surfaces based on spatial Archimedes spiral. Kun He et al. [18] used the forming tool to grind the spiral surface, calculated the point vector envelope method of the spiral surface forming tool contour, and verified the effectiveness of the method.

Based on the consideration of cutting tools classification, this paper analyzes and
summarizes the forming methods of workpiece surface in cutting process from the Angle of geometry, and studies the scanning forming method among them. The mathematical modeling and simulation of forming turning were carried out by taking forming turning groove of ball bearing inner ring as an example.

2.Cutting shape of the workpiece surface

2.1Formation of the workpiece surface

Based on the geometric angle, the cutting tools used in machining are divided into two categories. ①Non-rotating tools, such as turning tools, planers, etc., are geometrically embodied as cutting edge curves; ②Rotating tools, such as milling cutters, grinding wheels, etc., are geometrically embodied as the tool surface where the cutting edge of the tool is located.

Based on the geometric point of view, through the analysis and induction of the machining methods of the machine tool, there are the following two methods for the surface cutting and forming of the workpiece.

Use the first type of tool, that is, a non-rotating tool. The tool is embodied as a cutting edge curve, and the surface of the workpiece is formed by the motion scanning of the cutting edge curve of the tool.

Use the second type of tool, that is, the rotary tool, the tool is embodied as the tool surface where the cutting edge of the tool is located, and the surface of the workpiece is the envelope of the surface family formed by the movement of the tool surface.

When forming cutting, the cutting edge of the tool is in line contact with the surface of the workpiece. It belongs to the method of using the first type of tool to scan the surface of the formed workpiece with one pass. The machine tool only needs a single parameter movement, and the cutting efficiency is high.

2.2Formation of the workpiece surface

When using a shaped planer for planing processing, the curved translational movement of the cutting edge of the shaped planer forms the surface of the workpiece, as shown in Fig.1. Transform the cutting edge curve to form the surface of the workpiece.

![Fig. 1 Shaped and planed curved surface](image)

In order to ensure the tool angle during forming turning, the relative posture of the forming turning tool and the workpiece must be kept unchanged. Therefore, the workpiece revolving surface is formed by the circular trajectory of the cutting edge curve of the forming turning tool around the axis of the workpiece spindle, that is, the normal motion scanning of the cutting edge curve of the forming turning tool forms the surface of the workpiece [3].

As shown in Fig.2, the cutting edge curve \( L_t \) is rotated around the workpiece axis to obtain the workpiece surface \( S_p \).
2.3 The Research Technology Route of Forming Turning Scanning Forming

1. Mathematical modeling of tool cutting edge curve

Establish the tool coordinate system consolidated with the tool and the workpiece coordinate system consolidated with the workpiece, establish the mathematical model of the tool cutting edge curve in the tool coordinate system, and use the translation transformation matrix from the tool coordinate system to the workpiece coordinate system to convert the tool cutting edge curve move to the workpiece coordinate system.

2. Mathematical modeling of workpiece design surface

Using the rotation transformation matrix, a mathematical model of the workpiece design surface is formed from the cutting edge curve of the tool.

3. Mathematical modeling and simulation of the surface of the formed turning workpiece considering the relative displacement

When cutting, factors such as cutting force and tool wear will cause the relative displacement between the tool and the workpiece, and then produce machining errors between the surface of the workpiece and the design surface. Consider the relative displacement between the tool and the workpiece, and use coordinate transformation to establish a mathematical model of the workpiece surface.

4. Research on influencing factors

Study the influence of factors such as cutting force and tool wear on the relative displacement between the tool and the workpiece.

3. Mathematical Modeling of Workpiece Design Surface for Forming Turning

As shown in Fig. 3, establish a tool coordinate system $\sigma_c(O_c - X_c Y_c Z_c)$ that is consolidated with the tool. Select the point on the cutting edge curve of the forming turning tool that is closest to the axis of the machine tool spindle as the tool coordinate system origin $O_b$, and the $Z_0O_0X_0$ plane is coplanar with the workpiece rotation axis.
The parameter Eq.(1) for establishing the cutting edge curve \( L_t \) is as follows:

\[
L_t = \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix} = \begin{bmatrix}
x(u) \\
y(u) \\
z(u) \\
1
\end{bmatrix}
\]

(1)

### 3.2 Mathematical modeling and solution of design surface

As shown in Fig.4, establish a workpiece coordinate system \( \sigma_i(O_i-X_i Y_i Z_i) \) that is consolidated with the workpiece. The workpiece coordinate system is parallel to the tool coordinate system, and the Z axis coincides with the machine tool spindle. The X axis of the workpiece coordinate system coincides with the X axis of the tool coordinate system. The distance between the origins of the two coordinate systems on the X axis is \( r_0 \), and \( r_0 \) is the minimum radius of gyration of the workpiece surface.

Transform the tool cutting edge curve \( L_t \) in the tool coordinate system to the workpiece coordinate system, the Homogeneous coordinate translation transformation matrix \( T \) is determined as Eq.(2).

\[
T = \begin{bmatrix}
1 & 0 & 0 & T_x \\
0 & 1 & 0 & T_y \\
0 & 0 & 1 & T_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

(2)

In the formula, \( T_x \) is the X-axis coordinate of the tool coordinate system origin \( O_0 \) in the
workpiece coordinate system, \( T_x = -r_0 \). After translation transformation, the parameter Eq.(3) of the cutting edge curve in the workpiece coordinate system is obtained.

\[
L_e = \begin{bmatrix}
  x' \\
  y' \\
  z' \\
  1
\end{bmatrix} = \begin{bmatrix}
  1 & 0 & 0 & T_x & x(u) \\
  0 & 1 & 0 & 0 & y(u) \\
  0 & 0 & 1 & 0 & z(u) \\
  0 & 0 & 0 & 1 & 1
\end{bmatrix} = \begin{bmatrix}
  x(u)+T_x \\
  y(u) \\
  z(u)
\end{bmatrix}
\]

(3)

The homogeneous coordinate rotation transformation matrix \( R_Z \) is Eq.(4).

\[
R_Z = \begin{bmatrix}
  \cos \theta & -\sin \theta & 0 & 0 \\
  \sin \theta & \cos \theta & 0 & 0 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}
\]

(4)

After the rotation transformation, the parameter Eq.(5) of the workpiece design surface \( S_p \) in the workpiece coordinate system is obtained.

\[
S_p = \begin{bmatrix}
  \cos \theta & -\sin \theta & 0 & 0 & x(u)+T_x \\
  \sin \theta & \cos \theta & 0 & 0 & y(u) \\
  0 & 0 & 1 & 0 & z(u)
\end{bmatrix}
\]

(5)

In the tool coordinate system, the coordinate of any point \( P_e \) on the cutting edge curve is \((x, y, z) = (C_e \cos \gamma_a, C_e \sin \gamma_a, z)\).

The size \( C_e \) on the rake face of the tool corresponding to the point \( P_e \) is determined as Eq.(6).[4]

\[
C_e = \sqrt{r_e^2 - (r_e \sin \gamma_a)^2} - r_e \cos \gamma_a
\]

(6)

In the formula, \( C_e \) is the distance from each point on the cutting edge of the tool to the Z axis, \( r_e \) is the distance from each point of the generatrix on the \( X_0O_0Z_0 \) surface to the centerline of the rotation axis, and \( r_e = x + r_e \), \( x = x(z) \) is the generatrix equation on the \( X_0O_0Z_0 \) surface.

The parameter equation of the cutting edge curve in the tool coordinate system is determined as Eq.(7).

\[
L_e = \begin{bmatrix}
  x \\
  y \\
  z \\
  1
\end{bmatrix} = \begin{bmatrix}
  \sqrt{(x(z)+r_e)^2-(r_e \sin \gamma_a)^2} - r_e \cos \gamma_a \times \cos \gamma_a \\
  \sqrt{(x(z)+r_e)^2-(r_e \sin \gamma_a)^2} - r_e \cos \gamma_a \times \sin \gamma_a \\
  z
\end{bmatrix}
\]

(7)

After translation transformation and rotation transformation, the parameter Eq.(8) of the workpiece design surface \( S_p \) in the workpiece coordinate system can be established.

\[
S_p = \begin{bmatrix}
  \cos \theta & -\sin \theta & 0 & 0 & \sqrt{(x(z)+r_e)^2-(r_e \sin \gamma_a)^2} - r_e \cos \gamma_a \times \cos \gamma_a + T_x \\
  \sin \theta & \cos \theta & 0 & 0 & \sqrt{(x(z)+r_e)^2-(r_e \sin \gamma_a)^2} - r_e \cos \gamma_a \times \sin \gamma_a \\
  0 & 0 & 1 & 0 & z \\
  0 & 0 & 0 & 1 & 1
\end{bmatrix}
\]

(8)

Taking forming turning tool turning standard GB/T 276-2013 6202 deep groove ball bearing inner ring groove as an example, the design surface of the workpiece is solved.

Fig.5 shows the inner ring dimensions of 6202 deep groove ball bearings.
Fig. 5 6202 deep groove ball bearing inner ring dimensions

Fig. 6 Formed turning ball bearing inner ring raceway

As shown in Fig.6, The arc equation on the X0O0Z0 plane is determined as Eq.(9).

\[ x = -\sqrt{r_1^2 - z^2} + r_1 \quad (-d_2 / 2 \leq z \leq d_1 / 2) \] (9)

Substituting Eq.(9) into Eq.(8) and solving with Matlab, the design surface shown in Fig.7 is obtained.

Fig. 7 Application of Matlab to solve the design surface

4.Mathematical Modeling of the Surface of Scanning Formed Workpiece in Forming Turning

4.1Mathematical modeling of workpiece surface

In the cutting process, factors such as cutting force and tool wear will cause relative...
displacement between the tool and the workpiece, which will result in machining errors. In the X and Y directions, the relative displacement between the tool and the workpiece is expressed as $\delta_X$ and $\delta_Y$, respectively. In the tool coordinate system, the parameter Eq.(10) of the cutting edge curve considering the relative displacement between the tool and the workpiece is established.

$$L_{\text{eq}} = \begin{bmatrix} x_x \\ y_x \\ z_x \\ 1 \end{bmatrix} = \begin{bmatrix} x(u) + \delta_x \\ y(u) + \delta_y \\ z(u) \\ 1 \end{bmatrix}$$  \hspace{1cm} (10)

After translation transformation, the parameter Eq.(11) of the cutting edge curve in the workpiece coordinate system considering the relative displacement between the tool and the workpiece is established

$$L_{\text{eq}}' = \begin{bmatrix} x_x' \\ y_x' \\ z_x' \\ 1 \end{bmatrix} = \begin{bmatrix} x(u) + \delta_x + T_x \\ y(u) + \delta_y \\ z(u) \\ 1 \end{bmatrix}$$  \hspace{1cm} (11)

After rotating transformation, the workpiece surface Eq.(12) in the workpiece coordinate system considering the relative displacement between the tool and the workpiece is obtained:

$$S_{X} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 \\ \sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x(u) + \delta_x + T_x \\ y(u) + \delta_y \\ z(u) \\ 1 \end{bmatrix}$$  \hspace{1cm} (12)

![Fig. 8 The influence of X-direction cutting force](image)

The surface of the workpiece considering the influence of the X-direction cutting force is shown in Fig. 8. The relative displacement between the tool and the workpiece under the action of the X-direction cutting force is expressed as $\delta_{X}$, and the parameter Eq.(13) of the cutting edge curve in the tool coordinate system is established as follows:

$$L_{\text{eq}} = \begin{bmatrix} x_{\text{eq}} \\ y_{\text{eq}} \\ z_{\text{eq}} \\ 1 \end{bmatrix} = \begin{bmatrix} x(u) + \delta_{v_x} \\ y(u) \\ z(u) \\ 1 \end{bmatrix}$$  \hspace{1cm} (13)

After translation transformation and rotation transformation, the workpiece surface
parameter Eq.(14) in the workpiece coordinate system considering the influence of the X-direction cutting force can be obtained.

\[
S_{\text{px}} = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 & 0 \\
\sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x(u) + \delta_x + T_x \\
y(u) \\
z(u) \\
1
\end{bmatrix}
\]  

(14)

**Fig. 9** The influence of Y-direction cutting force

Fig.9 shows the workpiece surface considering the influence of Y-direction cutting force. The relative displacement between the tool and the workpiece under the action of the Y-direction cutting force is expressed as \( \delta_{\text{y}} \), and the parameter Eq.(15) of the cutting edge curve in the tool coordinate system is established as follows:

\[
L_{\text{y}} = \begin{bmatrix}
x_{\text{y}} \\
y_{\text{y}} \\
z_{\text{y}} \\
1
\end{bmatrix} = \begin{bmatrix}
x(u) \\
y(u) + \delta_{\text{y}} \\
z(u) \\
1
\end{bmatrix}
\]  

(15)

After translation transformation and rotation transformation, the workpiece surface parameter Eq.(16) in the workpiece coordinate system considering the influence of the Y-direction cutting force can be obtained.

\[
S_{\text{py}} = \begin{bmatrix}
\cos \theta & -\sin \theta & 0 & 0 \\
\sin \theta & \cos \theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
x(u) + T_x \\
y(u) + \delta_{\text{y}} \\
z(u) \\
1
\end{bmatrix}
\]  

(16)

3. Workpiece surface modeling considering the comprehensive influence of X and Y cutting forces
The surface of the workpiece considering the comprehensive influence of the X-direction Y cutting force is shown in Fig.10. The parameter Eq.(17) of the cutting edge curve in the tool coordinate system is established.

$$L_{xy} = \begin{bmatrix} x(u) + \delta_y \\ y(u) + \delta_x \\ z(u) \\ 1 \end{bmatrix}$$  \hspace{2cm} (17)

After translation transformation and rotation transformation, the workpiece surface parameter Eq.(18) in the workpiece coordinate system considering the comprehensive influence of X and Y cutting forces can be obtained.

$$S_{xy} = \begin{bmatrix} \cos \theta & -\sin \theta & 0 & 0 & x(u) + \delta_x + T_x \\ \sin \theta & \cos \theta & 0 & 0 & y(u) + \delta_y \\ 0 & 0 & 1 & 0 & z(u) \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$  \hspace{2cm} (18)

As shown in Fig.11, from the perspective of machining accuracy, the tool wear NB along the radial direction of the workpiece is generally used as a measure of bluntness during turning.

The relative displacement relationship Eq.(19) between the tool and the workpiece considering tool wear is as follows:
The parameter Eq.(20) of the cutting edge curve in the tool coordinate system considering the influence of tool wear is established as follows:

\[
\begin{bmatrix}
\delta_x = NB \\
\delta_y = NB \tan \gamma_0
\end{bmatrix}
\]  

(19)

After translation transformation and rotation transformation, the workpiece surface parameter Eq.(21) in the workpiece coordinate system considering the influence of tool wear can be obtained.

\[
L_u = \begin{bmatrix}
x_u \\
y_u \\
z_u \\
1
\end{bmatrix} = \begin{bmatrix}
x(u) + \delta_{ax} \\
y(u) + \delta_{ay} \\
z(u)
\end{bmatrix}
\]  

(20)

4.2 Surface analysis of forming turning workpiece

Under the action of X-direction cutting force, the radius of the workpiece is given by, and \(\delta_f = 0.005\text{mm}\) is selected as this paper.

Using Matlab software to solve the groove surface of Eq.(5) and Eq.(14), the deviation between the workpiece surface and the design surface produced by the X-direction cutting force is shown in Fig.12.

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Fig. 12 The influence of X-direction cutting force
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Under the action of Y-direction cutting force, the radius of the workpiece is given by \(r \rightarrow r_f, r_f = \sqrt{r^2 + \delta_{fy}}\), and \(\delta_{fy}\) is selected as \(0.01\text{mm}\) in this paper.

Using Matlab software to solve the groove surface of Eq.(5) and Eq.(16), the deviation between the workpiece surface and the design surface produced by the Y-direction cutting force is shown in Fig.13.
Fig. 13 The influence of Y-direction cutting force

Under the combined influence of X and Y cutting forces, the radius of the workpiece is determined by

\[ r_{r} \rightarrow r_{r} + \delta_{r} = \sqrt{(r + \delta_{r}x)^2 + \delta_{r}^2} \cdot \]

Using Matlab software to solve the groove surface of Eq.(5) and Eq.(18), the deviation between the workpiece surface and the design surface produced by the comprehensive influence of the X and Y cutting forces is shown in Fig.14.

Fig. 14 Comprehensive influence of X and Y cutting force

Because the linear velocity of each point on the cutting edge is different, the amount of tool wear at each point on the cutting edge is also different. In this paper, the relationship between tool wear and cutting speed is simplified as a proportional relationship.

Due to tool wear, the radius of the workpiece is changed by

\[ r_{r} \rightarrow r_{r} \cdot \delta_{r} = \sqrt{(r + \delta_{r}x)^2 + \delta_{r}^2} \cdot \]

Using Matlab to solve the groove surface of Eq.(5) and Eq.(21), the deviation between the workpiece surface and the design surface caused by tool wear is shown in Fig.15.
5. Conclusion

(1) From a geometric point of view, cutting tools are divided into two categories: non-rotating tools and rotating tools. Geometrically, the first type of tool is embodied as the cutting edge curve, and the second type of tool is embodied as the tool surface where the cutting edge of the tool is located. Based on the classification of tools, there are two methods for cutting and forming the surface of the workpiece: ①Using the first type of tool, the surface of the workpiece is scanned by the motion of the cutting edge curve. ②Using the second type of tool, the surface of the workpiece is the envelope of the curved surface family formed by the curved surface movement of the tool.

(2) The research technical route of forming turning scanning forming is proposed. Taking the forming and turning of the groove of the inner ring of a ball bearing as an example, a mathematical model of the cutting edge curve and the design surface of the workpiece is established.

(3) In view of the relative displacement between the tool and the workpiece caused by the cutting force and tool wear factors, a mathematical model of the surface of the formed turning workpiece was established, and the deviation between the workpiece surface and the design surface was simulated by Matlab software.

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- The authors have no conflicts of interest to declare that are relevant to the content of this article.
- All authors certify that they have no affiliations with or involvement in any organization or entity with any financial interest or non-financial interest in the subject...
matter or materials discussed in this manuscript.

- The authors have no financial or proprietary interests in any material discussed in this article.

  - c. Availability of data and material (data transparency)
    The datasets used or analysed during the current study are available from the corresponding author on reasonable request.

  - d. Code availability (software application or custom code)
    The code in MATLAB is feasible

  - e. Ethics approval (include appropriate approvals or waivers)
    - No conflict of interest
    - All the authors participated in the discussion of the paper
    - Informed consent

  - f. Consent to participate (include appropriate statements)
    All authors agree to participate in the paper and agree to the feasibility of the paper

  - g. Consent for publication (include appropriate statements)
    All authors read and approved the final manuscript and agreed to publish it

  - h. Authors’ contributions (optional: please review the submission guidelines from the journal whether statements are mandatory)
    Xinghua Niu and Xueteng Wang developed the idea of the study, participated in its design and coordination and helped to draft the manuscript. Zizhao Yu provided critical review and substantially revised the manuscript. All authors read and approved the final manuscript.

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Figures

Figure 1
Shaped and planed curved surface

Figure 2
Forming turning surface
Figure 3

Tool coordinate system
Figure 4

Workpiece coordinate system
6202 deep groove ball bearing inner ring dimensions

\[ d_1 = 4.2958, \quad d_2 = 3.3521, \quad r_2 = 3.07 \]
Formed turning ball bearing inner ring raceway

Figure 7

Application of Matlab to solve the design surface
Figure 8

The influence of X-direction cutting force
Figure 9

The influence of Y-direction cutting force

Figure 10

Comprehensive influence of X and Y cutting force
Figure 11

Tool wear
Figure 12
The influence of X-direction cutting force

Figure 13
The influence of Y-direction cutting force

Figure 14
Comprehensive influence of X and Y cutting force
Figure 15

The effect of tool wear