Resource Allocation for Selection-Based Cooperative OFDM Networks

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Abstract

This paper considers resource allocation to achieve max-min fairness in a selection-based orthogonal frequency division multiplexing network wherein source nodes are assisted by fixed decode-and-forward relays. The joint problem of transmission strategy selection, relay assignment, and power allocation is a combinatorial problem with exponential complexity. To develop effective solutions to these questions, we approach these problems in two stages. The first set of problems assume ideal source-relay channels; this simplification helps illustrate our general methodology and also why our solutions provide tight bounds. We then formulate the general problem of transmission strategy selection, relay assignment, and power allocation at the sources and relays considering all communication channels, i.e., finite power source-relay channels. In both sets of problems mentioned so far, transmissions over subcarriers are assumed to be independent. However, given the attendant problems of synchronization and the implementation using a FFT/IFFT pair, resource allocation at the subcarrier level appears impractical. We, therefore, consider resource allocation at the level of an entire OFDM block. While optimal resource management requires an exhaustive search, we develop tight bounds with lower complexity. Finally, we propose a decentralized block-based relaying scheme. Simulation results using the COST-231 channel model show that this scheme yields close-to-optimal performance while offering many computational benefits.

Index Terms

Cooperative communication, orthogonal frequency division multiplex (OFDM), resource allocation.
I. INTRODUCTION

In a cooperative wireless network, geographically distributed nodes share the available resources to achieve the benefits of multiple-input multiple-output systems and combat the impact of fading via relaying. The initial work in [1]–[3] sparked much research activity in this area. Of specific interest here is the decode-and-forward (DF) protocol where the relay node decodes and re-encodes the source’s data [2]. If multiple relays are available, selection, wherein sources choose one “best” relay, has been shown to provide almost all the benefits of the cooperative diversity while minimizing overhead. Most importantly, selection avoids issues of synchronization across relays; selection-based cooperation has now been studied in various contexts [4]–[8]. However, relay selection becomes more crucial in multi-source networks when simultaneous data flows are allowed. Since each relay must split its available power amongst all the source nodes it supports, the individually optimal relay allocation scheme may not be globally optimal. Optimal relay assignment is a combinatorial optimization problem with exponential complexity. Without addressing power allocation at the relays, in [5] the authors present low complexity relay selection schemes for multi-source networks. In [9], the authors extend this to include power allocation in a single-carrier cellular network, but assuming an ideal source-relay channel.

In a separate track, orthogonal frequency division multiplexing (OFDM/OFDMA) is an increasingly popular technique to mitigate the impact of multipath fading and enables high data rates for current and emerging wireless communication technologies. Furthermore, because each subcarrier experiences a different channel realization, resource allocation can significantly enhance performance [10]–[13]. OFDM benefits from the crucial implementation advantage that the transmitted signal can be obtained from an Inverse Fast Fourier Transform (IFFT) of the data. This IFFT is paired with a FFT at the receiver. However, as we will see, this pairing also restricts how nodes can cooperate.

In the context of cooperative OFDM networks, optimal relay assignment and dynamic subcarrier and power allocation have received significant attention. In [14] Li et al. developed a graph-theoretical approach to maximize the sum rate under fairness constraints; here fairness is imposed by limiting the number of sources a single relay can help. The work in [15] maximizes the minimum rate in a two-hop cooperative network while allowing for subcarrier permutation. In [16], Ng and Yu constructed a utility maximization framework to solve for relay selection,
relaying strategies, and power allocation in cellular OFDMA access-based networks. Using the decomposition method and assuming a finite discrete set of rates, they use an exhaustive search to solve the optimization problem. The same approach is employed in [17] in order to minimize the required power subject to data rate constraints on each flow. The authors of [18] proposed a resource allocation scheme for a two-hop clustered-based cellular network with relays chosen \textit{a priori}.

A key concern is that all these works deal with the OFDM transmission on a per-subcarrier basis, i.e., as if each were an \textit{independent} transmission. In OFDM-based networks, the raw data is encoded for error control before data modulation and the IFFT; the data is therefore spread over all subcarriers. In turn, this implies that DF requires decoding \textit{all subcarriers}. Most of the subcarrier-based resource allocation is, therefore, theoretically optimal, but impractical. Hence, relay selection and resource allocation must happen over the entire OFDM block of subcarriers.

In this paper, we consider a selection-based cooperative OFDM mesh network of access points (AP) where relays use the DF protocol. We begin with the assumption that all relay nodes can always decode each individual data streams, i.e., we assume that the relay can always decode the source transmissions without error. While this may be valid in a few practical scenarios, e.g., when relays are installed close enough to the source nodes, this is clearly not a universally valid assumption. However, as we will see, the solution based on this simplifying assumption provides useful insights into finding a near-optimal solution for a subcarrier-based selection scheme.

Building on the work in [9], here we prove that subcarrier selection at the relays is the optimal assignment for \textit{most, not all}, subcarriers. This contradicts the results in [20] which claims that subcarrier based selection is the optimal strategy. Furthermore, using the Karush-Kuhn-Tucker (KKT) conditions, we characterize an upper bound to the original subcarrier-based problem which leads to joint relay and power allocation for each subcarrier. We also derive a simple tight lower bound on the solution of the original problem. We then deal with selection for an entire OFDM block and propose a simple selection scheme, but with performance close to using an exhaustive search and not much different from the per-subcarrier relaying scheme. To the best of our knowledge, there has been no published work on selection and resource allocation at the level of an entire OFDM block.

In the next section, we solve the general form of the relay selection and resource allocation problem for OFDM-based networks irrespective of the positions of the relays, i.e., unlike previous
works, we take source-to-relay (S-R), source-to-destination (S-D), and relay-to-destination (R-D) channels into account. Furthermore, our scheme allows for direct transmission (no cooperation) if it is optimal. By introducing time-sharing coefficients, we transform the original combinatorial problem into a standard convex optimization problem resulting in an upper bound on performance. In addition, using the same approach, we formulate block-level selection for multi-source networks and develop an upper bound to the achievable rate. A tight lower bound for subcarrier-based (respectively block-based) schemes can be achieved by enforcing the selection constraint, i.e., each subcarrier (resp. block) is transmitted either via a single relay or directly to the destination. Finally, we propose a distributed, decentralized, selection scheme which offers large computational advantages, but with close-to-optimal performance. We emphasize that unlike most previous works, neither the transmission strategy nor the relay nodes are chosen a priori.

The remainder of this paper is organized as follows. Section II presents our system model in some detail. Section III investigates node selection and resource allocation under the assumption that all relays can decode. Section IV deals with the optimization problem for both subcarrier-based and block-based schemes by considering all communication channels while taking both selection and per-node power constraints into account. Section V presents simulation results that quantify the performance of different relaying and resource allocation algorithms. Finally, we wrap up this paper in Section VI with some conclusions.

II. SYSTEM MODEL

This paper considers an OFDM-based static mesh network of access points (APs) as shown in Fig. I. The network comprises $K$ sources assisted by $J$ dedicated relays. Each source node has its own destination which is not within the set of sources and relays. Let $K = \{1, 2, ..., K\}$, $J = \{1, 2, ..., J\}$, and $N_k = \{1, 2, ..., N\}$ be the set of source nodes, relay nodes, and subcarriers of source $k$, respectively. Note that the available bandwidth is divided into $K \times N$ orthogonal subcarriers and each source node is assigned a block of OFDM and communicates over its corresponding band, i.e., simultaneous transmissions do not interfere. Here, for the sake of simplicity, we assume that all sources are allocated the same number of subcarriers. However, the presented results can be easily generalized to the case of an unequal number of subcarriers per source node or to the case of OFDMA where multiple sources share an OFDM block. Furthermore, upon the admission of any new source node in the network, the subcarrier allocation
needs to be updated. This, however, is beyond the scope of this paper.

All sources and relays are attached to the power supply and transmit with constant and maximum total power of $P$. We consider DF relaying wherein each relay receives, decodes, and re-encodes the information with the same codebook as the transmitter, and forwards it to the destination. Nodes meet a half-duplex constraint. The inter-node wireless channels are modeled as frequency-selective. Since individual subcarriers of each source node experiences different channel realizations, adaptive transmission strategy and implementing power allocation at the sources and relays can enhance the system performance. Finally, we assume that all inter-node channels vary slowly enough for the channel state information (CSI) to be fed back to a centralized unit with limited overhead, making resource allocation possible.

The half-duplex constraint imposes the need for a two-stage version of the DF protocol. In Fig. 1, the solid arrows indicate the first, time-sharing, stage wherein each source broadcasts its data using $N$ subcarriers and each relay receives the OFDM block from all source nodes on orthogonal channels. During the second stage, represented by dashed arrows, only those relays that can fully decode the received information are nominated to participate. Finally, the destination node combines messages received in the two phases to decode the original information. This paper considers two cooperative scenarios; treating each subcarrier as an independent transmission and, more practically, cooperation at the level of an entire OFDM block.

The focus of this paper is to achieve max-min fairness in a multi-source mesh network, i.e., to maximize the minimum rate across all source nodes. In the next section, we present resource allocation schemes in such networks under the assumption that all relay nodes can successfully decode received symbols.

III. RESOURCE ALLOCATION WITH IDEAL S-R CHANNELS

This section develops optimal relay selection and power allocation in a subcarrier-based fashion to achieve max-min fairness. In particular, two block-based relaying schemes with different complexities are proposed. The assumption of an ideal S-R channel, wherein a relay can always decode the sources’ transmission, is valid when the relays are close to the sources or the S-R channels have a line-of-sight component. Note that this is a stepping stone to the next section where we drop this assumption. Further, we assume that a source divides its power equally
across all subcarriers.

A. Subcarrier-Based Resource Allocation with Ideal S-R Channels

In this section, each subcarrier is treated as an independent transmission. In practice, the total number of subcarriers in a network, \( KN \), is much higher than the number of relay nodes, \( J \). Hence, a relay is most probably required to support multiple subcarriers. In order to meet its power constraint, a relay must distribute its available power amongst all subcarriers that it supports. Under these conditions, the achievable rate of source \( s_k \) over its \( n^{th} \) subcarrier is

\[
R^{(n)}_k = \max_j \min \left\{ I^{(n)}_{skr_j}, I^{(n)}_{skr_j d_k} \right\},
\]

(1)

\[
I^{(n)}_{skr_j} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{kj} \alpha^{(n)}_{kj} |h^{(n)}_{kj}|^2 \right),
\]

(2)

\[
I^{(n)}_{skr_j d_k} = \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{0k} \alpha^{(n)}_{0k} |h^{(n)}_{0k}|^2 + \text{SNR}_{jk} \alpha^{(n)}_{jk} |h^{(n)}_{jk}|^2 \right),
\]

(3)

where \( h^{(n)}_{0k}, h^{(n)}_{kj}, \) and \( h^{(n)}_{jk} \) denote the complex S-D, S-R, and R-D channels over the \( n^{th} \) subcarrier of \( s_k \). \( \text{SNR}_{0k}, \text{SNR}_{kj}, \) and \( \text{SNR}_{jk} \) are the ratios of the total transmitted power to the power of noise; \( \alpha^{(n)}_{0k} \) and \( \alpha^{(n)}_{jk} \) are, respectively, the fraction of the allocated power to the \( n^{th} \) subcarrier of source \( k \) at the source node and relay \( j \). Eqn. (1) declares that the rate of each source node over its individual subcarriers is the minimum of the S-R rate (Eqn. (2)) and the compound S-R-D rate (Eqn. (3)), i.e., the cooperative rate requires that both the relay and destination fully decode the received data. The total rate then is \( R_k = \sum_{N_k} R^{(n)}_k \). Moreover, we assume that

- all S-R channels are ideal in that for any potential source rate

\[
I^{(n)}_{skr_j} \geq I^{(n)}_{skr_j d_k} \Rightarrow R^{(n)}_k = I^{(n)}_{skr_j d_k}, \quad \forall k, n.
\]

- source nodes distribute their available power equally amongst their subcarriers, i.e., \( \alpha^{(n)}_{0k} = 1/N \). The power allocation problem is, therefore, relevant at the relays only.

In keeping with its many benefits described earlier, we impose a selection constraint in the second, relaying, phase, i.e., each subcarrier of a source node is relayed through at most one of
the relays in the network. Therefore, the optimization problem we wish to solve is

$$\max_\alpha \min_k R_k$$

s.t. $C_1 : \alpha_{j_1k}^{(n)} \times \alpha_{j_2k}^{(n)} = 0$, \( \forall k, n, \text{ and } j_1 \neq j_2 \),

$$C_2 : \alpha_{jk}^{(n)} \geq 0, \quad \forall j, k, n,$$

$$C_3 : \sum_k \sum_{N_k} \alpha_{jk}^{(n)} = 1, \quad \forall j,$$

where $R_k = \sum_{N_k} \frac{1}{2} \log_2 \left( 1 + \frac{1}{N} \text{SNR}_{0k} |h_{0k}^{(n)}|^2 + \sum_J \text{SNR}_{jk} \alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2 \right)$.

Constraint $C_1$ enforces selection by allowing only one node to devote power to each subcarrier. Constraints $C_2$ and $C_3$ state that the amount of allocated power must be non-negative and that the total available power of all relays is limited. Due to the selection constraint, (4)-(7) is an, essentially intractable, mixed-integer programming optimization problem. One proposed solution \[11\], \[13\] separates the power allocation and selection problems. First, subcarriers are selected assuming equal power allocation; then, power is distributed based on this selection. However, with $K$ sources and $J$ relays, there are $J^KN$ relay assignments to be checked. Therefore, even this scheme is infeasible for realistic values of $K$, $J$, and $N$. We build on an alternative approach developed in \[9\] to form an approximate solution that is also an upper bound.

1) An Approximate Solution and Upper Bound: The objective function of the optimization problem is increasing and concave in $\alpha_{jk}^{(n)}$. Other than the integer constraint of (5), the constraints in the original problem of (4)-(7) are convex. In order to find an approximate, tractable solution we first ignore the selection constraint; hence, the solution to this modified optimization problem will be an upper bound to the original subcarrier-based (UBSB) resource allocation problem. The revised formulation, stated here in epigraph form, is a concave maximization problem solvable

\[\text{It is worth emphasizing that while the solution methodology here is similar to that of \[9\], both our problem formulation and solution are significantly different. The development here, using the epigraph form, leads to effective solutions to OFDM-based relaying and allows us to show that selection is a sub-optimal solution to the resource allocation problem.}\]
in polynomial time using available efficient solvers [21].

\[
\max_{\{t, \alpha\}} t \\
\text{s.t. } C_1 : \sum_{N_k} \frac{1}{2} \log_2 \left( 1 + \frac{1}{N} \text{SNR}_{0k} |h_{0k}^{(n)}|^2 + \sum_{J} \text{SNR}_{jk} \alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2 \right) - t \geq 0, \quad \forall k, \\
C_2 : \alpha_{jk}^{(n)} \geq 0, \quad \forall j, k, n, \\
C_3 : \sum_{k} \sum_{N_k} \alpha_{jk}^{(n)} = 1, \quad \forall j.
\]

The solution to this convex optimization problem is characterized by the KKT conditions [21]. The Lagrangian is given by

\[
\mathcal{L} (\alpha_{jk}^{(n)}, \gamma_k, \mu_j, \lambda_{jkn}) = t + \sum_{k} \gamma_k \left( \sum_{N_k} \frac{1}{2} \log_2 \left( 1 + \frac{1}{N} \text{SNR}_{0k} |h_{0k}^{(n)}|^2 + \sum_{J} \text{SNR}_{jk} \alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2 \right) - t \right) \\
+ \sum_{J} \mu_j \left( 1 - \sum_{k} \sum_{N_k} \alpha_{jk}^{(n)} \right) + \sum_{J} \sum_{k} \sum_{N_k} \lambda_{jkn} \alpha_{jk}^{(n)},
\]

wherein \(\gamma_k, \mu_j,\) and \(\lambda_{jkn}\) are Lagrangian multipliers.

For the sake of clarity, let us assume that \(J = 2\), i.e., a cooperative network comprising two relays. Since the problem is a standard convex optimization problem and satisfies Slater’s conditions, any solution for the amount of power that relays \(r_1\) and \(r_2\) allocate to the \(n^{th}\) subcarrier of source \(k\) satisfies the KKT conditions. Suppose that both relays allocate some power to the \(n^{th}\) subcarrier of \(s_k\). Therefore, the convex problem satisfies the complementary slackness condition, i.e., \(\lambda_{r_1 kn} = \lambda_{r_2 kn} = 0\). Now, we can conclude that \(\frac{\mu_2}{\mu_2} = \frac{|h_{r_1k}^{(n)}|^2}{|h_{r_2k}^{(n)}|^2}\). Similarly, if the same two nodes contribute to relaying the \(i^{th}\) subcarrier of source \(k\), using the same KKT conditions, \(\frac{\mu_2}{\mu_2} = \frac{|h_{r_1k}^{(i)}|^2}{|h_{r_2k}^{(i)}|^2}\). These two equations cannot be simultaneously satisfied since channel gains are continuous random variables. Thus, \emph{at most one subcarrier} of each source can be helped by more than one relay.

Now, let us evaluate all the possible relay selections in a network with \(J = 3\), assuming the \(n^{th}\) subcarrier of source \(k\) is being helped by all relay nodes. The KKT conditions state that \(\frac{\mu_1}{|h_{r_1k}^{(n)}|^2} = \frac{\mu_2}{|h_{r_2k}^{(n)}|^2} = \frac{\mu_3}{|h_{r_3k}^{(n)}|^2}\). Now suppose that the \(i^{th}\) subcarrier of the same source is relayed through \(r_1\) and \(r_2\), i.e., \(\frac{\mu_1}{|h_{r_1k}^{(i)}|^2} = \frac{\mu_2}{|h_{r_2k}^{(i)}|^2}\). Thus, we have \(|h_{r_1k}^{(n)}|^2/|h_{r_2k}^{(n)}|^2| = |h_{r_1k}^{(i)}|^2/|h_{r_2k}^{(i)}|^2|\), which is a zero-probability event.

Consider, again, the scenario in which none of the subcarriers can be helped with all three relays. As an example, consider the case where the \(n^{th}\) subcarrier is relayed via nodes \(r_1\) and \(r_2\).
and the $i^{th}$ subcarrier is helped by node $r_1$ and $r_3$. Applying the same KKT conditions, it follows that $\frac{\mu_1}{|h_{r_1k}|^2} = \frac{\mu_2}{|h_{r_2k}|^2}$ and $\frac{\mu_1}{|h_{r_1k}|^2} = \frac{\mu_3}{|h_{r_3k}|^2}$. Now, the $m^{th}$ subcarrier can be helped by node $r_2$ and $r_3$ only if $\frac{|h_{r_2k}|^2}{|h_{r_1k}|^2} = \frac{|h_{r_3k}|^2}{|h_{r_1k}|^2}$, which happens with zero probability. Therefore, when two subcarriers are relayed by two nodes, all others can be helped by at most one node.

Generalizing this to the network with $K$ sources and $J$ relays, one concludes that at most $J - 1$ subcarriers of each source can be helped by more than one relay and selection is imposed on $(N - J + 1)$ subcarriers. In practice, $N \gg J$ which means that a large fraction of subcarriers meet the selection criterion, i.e., selection is an approximate, though not optimal solution, to the relaxed optimization problem. Note that this contradicts the work in [20] which suggests that selection is optimal.

2) A Heuristic Algorithm and a Lower Bound: By neglecting the selection constraint, the solution to the modified problem provides an upper bound to that of the original optimization problem in (4)-(7). Here, we use this to develop a heuristic solution to the original problem. We force the subcarriers that do not meet the constraint (a maximum of $J - 1$ of them) to receive power only from the single relay that achieves a higher data rate. Mathematically speaking

$$r_k^{(n)} = r_m^{(n)}; \quad m = \arg \max_j \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{1}{N} \frac{\mathrm{SNR}_{n} |h_{nk}^{(n)}|^2 + \mathrm{SNR}_{jk} \alpha_{nk}^{(n)} |h_{jk}^{(n)}|^2}{\alpha_{nk}^{(n)} |h_{nk}^{(n)}|^2 + \alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2} \right) \right\},$$

where $r_k^{(n)}$ is the relay node which contributes to the transmission of source $k$ on the $n^{th}$ subcarrier. Since this solution meets all the constraints of the original problem, this is also a lower bound to the solution of the subcarrier based (LBSB) optimization problem. In Section V we will show that the upper and lower bounds are indistinguishable. As a result, this heuristic approach provides almost the exact solution to the original mixed-integer optimization problem with significantly reduced solution complexity.

B. Block-Based Resource Allocation with Ideal S-R Channels

The optimization problem and solution detailed so far is in keeping with existing literature. It allows different subcarriers within an OFDM block to be helped by different relays. This is problematic for two reasons. One, while not explicitly stated, most of the previous work assumes a relay can treat each subcarrier as an independent transmission. In DF-based relaying, the decoding constraint is at the level of a subcarrier, e.g., [1]. However, in OFDM, the data is first protected by a channel code, modulated and then a block of $N$ subcarriers is formed. It is not
possible to decode information without receiving and decoding an entire OFDM block. Second, practical OFDM systems depend heavily on accurate time and frequency synchronization. This would be extremely difficult in a distributed mesh network.

In a multi-source network, as long as each relay has to divide its available power amongst all allocated sources, the solution to the relay assignment problem is not immediate. Here, we separate the problem into selection followed by power allocation (via waterfilling) across subcarriers. As in [5], two selection schemes with different levels of complexity are proposed and results will be compared in terms of the max-min rate in Section V.

1) Optimal Relay Selection: In a network with $K$ sources and $J$ relays, there are $J^K$ different possible relay assignments. The optimal scheme is exhaustive search over all possible relay selections and pick the one which provides the maximal minimum rate. This is clearly impossible for any reasonable $K$ and $J$.

2) Decentralized Relay Selection: The decentralized or simple relay selection scheme ignores all other sources. Each source chooses its best relay with the assumption that the corresponding relay distributes its power equally over all subcarriers of only that source. In particular

$$r_k = r_m, \quad m = \arg \max_j \left( \sum_{N_k} \log_2 \left( 1 + \frac{1}{N} \text{SNR}_{jk} |h_{jk}^{(n)}| \right) \right).$$

With each source having selected the relays, power is allocated via waterfilling, to the assigned sources. Note that since each source-destination pair only needs local CSI and selection is performed independently of all other sources, this scheme can be implemented in a decentralized manner. In a network with $J$ dedicated relays, only $J$ water-filling problems need to be solved.

IV. RESOURCE ALLOCATION WITH FINITE-POWER S-R CHANNELS

The previous section developed solutions under the assumption of an ideal S-R channel. In this section we consider the general case of resource allocation across the S-D, S-R, and R-D channels. The solution to this optimization problem also chooses the best transmission strategy for each source, i.e., direct transmission is a valid solution if that is optimal. Our approach also allows us to move beyond heuristics for block-based relaying.
A. Subcarrier-Based Resource Allocation with Finite-Power S-R Channels

Given the fact that each source is allowed to switch between DF and direct transmission, one concludes that in a network with \( K \) sources and \( J \) relays, with subcarrier based relaying

\[
R_k^{(n)} = \max \left\{ I_{s_kd_k}^{(n)}, \max_j \min_i \left\{ I_{s_kr_j}^{(n)}, I_{s_kr_jd_k}^{(n)} \right\} \right\},
\]

wherein \( I_{s_kd_k}^{(n)} = \log_2 \left( 1 + \text{SNR}_{0_k}^{(n)} \alpha_{0k}^{(n)} |h_{0k}^{(n)}|^2 \right) \). Eqn. (8) declares that the rate of each source node over its individual subcarriers is the maximum of the direct and cooperative transmission rates; in turn, the cooperative rate requires that both the relay and destination fully decode the received data. The total rate is then the sum of achievable rates of all subcarriers. In addition, by taking the S-R channel into account, optimal power allocation at the source nodes may alter relay selection and further enhance the performance of the network.

Let \( J_+ = \{0, 1, 2, \ldots, J\} \) be an extended set of relays; the ‘0’ index indicates direct transmission. Therefore, the formal optimization problem is

\[
\max_{\alpha} \min_k R_k
\]

s.t. \( C_1 : \alpha_{j_1k}^{(n)} \times \alpha_{j_2k}^{(n)} = 0, \forall k, n, \{j_1, j_2\} \in J, \)

\[
C_2 : \alpha_{jk}^{(n)} \geq 0, \forall k, n, j \in J_+,
\]

\[
C_3 : \sum_k \sum_{n} \alpha_{jk}^{(n)} \leq 1, \forall j \in J,
\]

\[
C_4 : \sum_{n} \alpha_{0k}^{(n)} = 1, \forall k,
\]

Eqns. (10)-(11) are equivalent to the constraints (5)-(7) of the original optimization problem of the previous section. Unlike the previous optimization problem, since source nodes are allowed to transmit directly, some relays might stay silent during the second time-slot. Hence, as stated in Eqn. (12), the power constraint can be satisfied by inequality. Eqn. (13) limits the available power of each source node. Similar to the previous scenario, since each relay must split its available power amongst all source nodes which it supports, transmission strategy selection, relay assignment, and power allocation problem is combinatorial and needs to be solved jointly.

1) An Approximate Solution and Upper Bound: To make the problem mathematically tractable, we introduce \( KN(J + 1) \) indicator variables to the objective function. Therefore, the new
optimization problem can be expressed as

$$\max_{\{\alpha, \rho \in \{0,1\}\}} \min_k R_k$$

s.t. $C_1 : \alpha_{jk}^{(n)} \geq 0, \rho_{jk}^{(n)} \geq 0, \forall k, n, j \in J_+$, $C_2 : \sum_k \sum_{N_k} \rho_{jk}^{(n)} \alpha_{jk}^{(n)} \leq 1, \forall j \in J$, $C_3 : \sum_{J_+} \sum_{N_k} \rho_{jk}^{(n)} \alpha_{0k}^{(n)} = 1, \forall k$, $C_4 : \sum_{J_+} \rho_{jk}^{(n)} = 1, \forall k, n.$

If source $k$ allocates a fraction of its available power to the $n$th subcarrier, i.e, $\alpha_{0k}^{(n)} \neq 0$, for any set of $\alpha_{jk}^{(n)}$ satisfying (10)-(12), the following equation is true

$$\rho_{jk}^{(n)} = \begin{cases} 1, & \alpha_{jk}^{(n)} \neq 0, \\ 0, & \alpha_{jk}^{(n)} = 0. \end{cases}$$ (14)

Eqn. (14), along with the fact that only one indicator variable of each source can be non-zero at a time, enforces the selection constraint of the original problem. The total rate of $s_k$ is

$$R_k = \sum_{N_k} \rho_{0k}^{(n)} \log_2 \left( 1 + \text{SNR}_{0k} \alpha_{0k}^{(n)} |h_{0k}^{(n)}|^2 \right) + \sum_{J_+} \sum_{N_k} \rho_{jk}^{(n)} \min \left\{ \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{kj} \alpha_{0k}^{(n)} |h_{kj}^{(n)}|^2 \right), \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{0k} \alpha_{0k}^{(n)} |h_{0k}^{(n)}|^2 + \text{SNR}_{jk} \alpha_{jk}^{(n)} |h_{jk}^{(n)}|^2 \right) \right\}.$$

Note that, since indicator variables can only take integer values, the problem is still a combinatorial optimization problem. As in the previous section, our solution methodology is to relax the corresponding constraint and allow each stream to be transmitted both directly as well as cooperatively through multiple relays. Thus, indicator variables of each individual subcarriers can take any rational value on the convex hull of the original discrete set. Consequently, the resulting solution is an upper bound to the min-rate of the original subcarrier-based problem (UBSB) formulated in (9)-(13). Furthermore, $\rho_{jk}^{(n)}$ can now be interpreted as a fraction of time that $s_k$ transmits over its $n$th subcarrier directly ($j = 0$) and cooperatively ($j \in J$).

The rate $R_k$ comprises three different terms: S-D, S-R, and S-R-D rates. One can easily show that none of them is jointly concave in the set of variables. By a change of variables as in [10], we set

$$\rho_{jk}^{(n)} \alpha_{jk}^{(n)} = r_{jk}^{(n)}, j \in J_+, \quad \rho_{jk}^{(n)} \alpha_{0k}^{(n)} = p_{jk}^{(n)}, j \in J.$$

It is worth noting that this is a key difference from the work in [15], [22] which did not take the coupling constraint between time-sharing coefficients and power allocation into account.
The new optimization problem in terms of \((\rho, r, p)\) can be formulated as

\[
\max_{\{\rho, r, p\}} \min_k R_k \tag{15}
\]

s.t. \(C_1: \rho_{jk}^{(n)} \geq r_{jk}^{(n)} \geq 0, \forall k, n, j \in J_+\), \(C_2: \rho_{jk}^{(n)} \geq p_{jk}^{(n)} \geq 0, \forall k, n, j \in J\), \(C_3: \sum_{k} \sum_{n} \rho_{jk}^{(n)} \leq 1, \forall j \in J\), \(C_4: \sum_{j} \sum_{n} r_{jk}^{(n)} = 1, \forall k\), \(C_5: \sum_{j} \rho_{jk}^{(n)} = 1, \forall k, n\). \(\tag{16}\)

Hence, the achievable rate of source \(k\) is expressed as

\[
R_k = \sum_{N_k} \rho_{0k}^{(n)} \log_2 \left( 1 + \frac{\text{SNR}_{0k} \rho_{0k}^{(n)} |h_{0k}^{(n)}|^2}{\rho_{0k}^{(n)}} \right) + \sum_{J} \sum_{N_k} \rho_{jk}^{(n)} \min \left\{ \frac{1}{2} \log_2 \left( 1 + \frac{\text{SNR}_{jk} \rho_{jk}^{(n)} |h_{jk}^{(n)}|^2}{\rho_{jk}^{(n)}} \right), \frac{1}{2} \log_2 \left( 1 + \frac{\text{SNR}_{0k} \rho_{0k}^{(n)} |h_{0k}^{(n)}|^2}{\rho_{0k}^{(n)}} + \frac{\text{SNR}_{jk} \rho_{jk}^{(n)} |h_{jk}^{(n)}|^2}{\rho_{jk}^{(n)}} \right) \right\}. \tag{17}
\]

**Theorem 1:** The objective function in (15) is jointly concave in \(\rho, r, \) and \(p\).

**Proof:** The S-D and S-R rates are in the form of \(f(x, y) = x \log (1 + y/x)\) and the rate of the compound S-R-D channel is in the form of \(g(x, y, z) = x \log (1 + y/x + z/x)\). In addition, \(x, y, \) and \(z\) are non-negative variables. One can show that the Hessian of \(f\) is

\[
\nabla^2 f = \frac{1}{(1 + y/x)^2} \begin{bmatrix} -y^2/x^3 & y/x^2 \\ y/x^2 & -1/x \end{bmatrix}. \tag{18}
\]

The determinant of \(\nabla^2 f\), the product of the eigenvalues, is zero. The trace of the \(\nabla^2 f\), the sum of the eigenvalues, is a negative value, which certifies that the \(\nabla^2 f \preceq 0\), i.e., the Hessian evaluated within the optimization region is a negative semi-definite matrix. Now, let us follow the same strategy to show that the third term is also jointly concave in the set of consisting variables. Therefore

\[
\nabla^2 g = \frac{1}{(1 + y/x + z/x)^2} \begin{bmatrix} -(y + z)^2/x^3 & (y + z)/x^2 & (y + z)/x^2 \\ (y + z)/x^2 & -1/x & -1/x \\ (y + z)/x^2 & -1/x & -1/x \end{bmatrix}. \tag{19}
\]

Similar to the previous case, the determinant and trace of the \(\nabla^2 g\) are, respectively, zero and negative. Moreover, \(\nabla^2 g\) is a rank one matrix, i.e., it has one non-positive eigenvalue and two zero ones. Thus, \(\nabla^2 g\) is a negative semi-definite matrix which proves that the rate of the S-R-D
channel is jointly concave in \((\rho, r, p)\). It is also known that a point-wise minimum and the non-negative summation of a set of concave functions are also concave functions \([21]\). Hence, the underlying objective function is jointly concave in \((\rho, r, p)\).

Although the objective function is jointly concave, it is not differentiable. By rewriting it in the epigraph form, the final optimization problem can be stated as follows

\[
\max_{\{t, \zeta, \rho, r, p\}} t
\]
\[s.t. \ C_1 : (16) - (18)\]
\[C_2 : \sum_{J_k} \sum_{N_k} \zeta^{(n)}_{jk} \geq t, \forall k,\]
\[C_3 : \rho^{(n)}_{0k} C \left( \frac{\text{SNR}_{0k} r^{(n)}_{0k} |h^{(n)}_{0k}|^2}{\rho^{(n)}_{0k}} \right) \geq \zeta^{(n)}_{0k}, \forall k, n,\]
\[C_4 : \frac{\rho^{(n)}_{jk}}{2} C \left( \frac{\text{SNR}_{kj} r^{(n)}_{jk} |h^{(n)}_{kj}|^2}{\rho^{(n)}_{jk}} \right) \geq \zeta^{(n)}_{jk}, \forall k, n, j \in J,\]
\[C_5 : \frac{\rho^{(n)}_{jk}}{2} C \left( \frac{\text{SNR}_{0k} r^{(n)}_{0k} |h^{(n)}_{0k}|^2 + \text{SNR}_{kj} r^{(n)}_{kj} |h^{(n)}_{kj}|^2}{\rho^{(n)}_{jk}} \right) \geq \zeta^{(n)}_{jk}, \forall k, n, j \in J,\]

where \(C(x) = \log_2(1 + x)\). This modified version is a standard convex optimization problem which, again, can be solved using well established and efficient iterative algorithms \([21]\).

2) A Heuristic Algorithm and a Lower Bound: The upper bound derived in the previous section approximates, but does not meet the selection constraint. As in Section III-A2 our approach to imposing selection is to assign to each subcarrier the transmission strategy and the relay that provides the maximum achievable rate. Thus, the selection constraint is enforced as

\[
R^{(n)}_k = \max \left\{ I^{(n)}_{s_k d_k}, \min \left\{ I^{(n)}_{s_k r_j m}, I^{(n)}_{s_k r_j m d_k} \right\} \right\}, \quad m = \arg \max \min \left\{ I^{(n)}_{s_k r_j}, I^{(n)}_{s_k r_j d_k} \right\}.
\]

Since this solution satisfies all constraints of the original problem in (9)-(13), this heuristic scheme provides a lower bound (LBSB). Moreover, the power freed up by the selection step can be reused by waterfilling over other source nodes which are helped by each individual relays. However, as we show in Section VII the performance gap is not noticeable; thus, there is no need to apply a second round of waterfilling. Finally, it is worth emphasizing that if direct transmission were optimal, the power allocated at all relays would be zero, i.e., the approach is adaptive across relay strategies.
B. Block-Based Resource Allocation with Finite-Power S-R Channels

This section deals with the selection and power allocation at the level of an entire OFDM block in the general case of finite-power S-D, S-R and R-D channels. As in the previous section, the solution to this problem optimizes the transmission strategy for each individual source node. The achievable rate of each source node across the entire OFDM block is

\[ R_k = \max \left\{ \sum_{n \in N_k} I_s^{(n)}(n), \max_j \min_{\alpha_j} \left\{ \sum_{n \in N_k} I_s^{(n)}(n), \sum_{n \in N_k} I_s^{(n)}(n) \right\} \right\}, \]

which states that each OFDM block can be transmitted either directly or via the relay node which supports a higher data rate. Here, unlike in (8), the individual terms in the rate expression include a sum over all subcarriers; in block-based selection, all subcarriers are relayed by the same relay node. The formal optimization problem is therefore

\[
\max_{\alpha} \min_k R_k \\
\text{s.t. } C_1 : \sum_{n \in N_k} \alpha^{(n)}_{j1k} \times \sum_{n \in N_k} \alpha^{(n)}_{j2k} = 0, \forall k, \{j_1, j_2\} \in J, \\
C_2 : \alpha^{(n)}_{jk} \geq 0, \forall k, n, j \in J_+, \\
C_3 : \sum_{k} \sum_{n \in N_k} \alpha^{(n)}_{jk} \leq 1, \forall j \in J, \\
C_4 : \sum_{n \in N_k} \alpha^{(n)}_{0k} = 1, \forall k.
\]

C_1 states that each OFDM block can be helped by at most one relay node. Other constraints are similar to those of the original subcarrier-based scheme formulated in Section IV-A.

1) An Approximate Solution and Upper Bound: The block-level optimization problem is an integer programming with exponential complexity. Thus, again, we introduce time-sharing coefficients to the objective function and rewrite the achievable rate of source \( k \) as

\[ R_k = \rho_{0k} \sum_{n \in N_k} \log_2 \left( 1 + \text{SNR}_{0k} \alpha^{(n)}_{0k} |h_0^{(n)}|^2 \right) + \]

\[ \sum_{j \in J} \rho_{jk} \min \left\{ \frac{1}{2} \sum_{n \in N_k} \log_2 \left( 1 + \text{SNR}_{kj} \alpha^{(n)}_{0k} |h_k^{(n)}|^2 \right), \sum_{n \in N_k} \frac{1}{2} \log_2 \left( 1 + \text{SNR}_{0k} \alpha^{(n)}_{0k} |h_0^{(n)}|^2 + \text{SNR}_{jk} \alpha^{(n)}_{jk} |h_j^{(n)}|^2 \right) \right\}. \]

Note that, since relaying is block-based, only \( K(J+1) \) time-sharing coefficients are required. Following the same approach as the previous section, we relax the selection constraint and set

\[ \rho_{jk} \alpha^{(n)}_{0k} = r^{(n)}_{jk}, j \in J_+, \quad \rho_{jk} \alpha^{(n)}_{jk} = p^{(n)}_{jk}, j \in J. \]

Using Theorem 1, it is straightforward to prove that the resulting optimization problem is jointly concave in \((\rho, r, p)\). Finally, by rewriting the objective function in epigraph form, a standard
convex optimization problem can be formulated. Since we relaxed the selection constraint, this solution provides an upper bound to the minimum rate of the original block-based relaying (UBBB). The approach developed in the Section IV-A therefore provides the basis for block-based optimization as well.

2) A Heuristic Algorithm and a Lower Bound: Having generalized our approach in Section IV-A2 to the block-based relaying, the lower bound to the block-based scheme (LBBB) can be achieved by choosing the best relay for individual source nodes.

3) Decentralized Resource Allocation: The solution to the optimization problem detailed so far requires joint selection of the transmission strategy, the relay node, and power allocation to each source in the network. This solution requires a central resource allocation unit which has the full CSI of all channels. This requires significant transmission and coordination overhead, potentially making the solution impractical. In this section, we develop a simplified decentralized scheme, wherein, similar to III-B2 at the first stage each source selects its best relay independently

\[ r_k = r_m, \quad m = \arg \max_j \min \left\{ \sum_{n_k} I_{s_k r_j}, \sum_{n_k} I_{s_k r_j d_k} \right\}. \]

Second, the transmission strategy is chosen by comparing the rates of the direct and relaying transmissions. Given that each individual source node has already selected its transmission strategy, at most \( J \) waterfilling problems need to be solved to maximize the minimum rate across source nodes. Furthermore, if a source node has decided to transmit directly, power is distributed based on the S-D channel state. In Section V we will show that, in fact, the performance of the distributed scheme closely tracks that of the UBBB algorithm.

V. SIMULATION RESULTS AND DISCUSSION

This section presents simulation results for the proposed relay selection and resource allocation schemes described in Sections III and IV. We consider two different network geometries. In the first scenario, all inter-node channels are modeled as independent and identically distributed (i.i.d.) random variables. The second network setup is more realistic; in that nodes are randomly distributed. The communication channels are modeled using the COST-231 channel model recommended by the IEEE 802.16j working group [23]. The chosen parameters for the COST-231 are given in Table I.
A. Resource Allocation with Infinite-Power S-R Channels in I.I.D. Scenario

Our first example implements relay selection and resource allocation for a mesh network with $K = 3$ and $K = 4$ APs with $N = 32$. The S-R channels are assumed ideal; the average SNR of all S-D channels is set to 5dB. Fig. 2 plots the minimum achievable rate across the $K$ source nodes for different values of the R-D SNRs. As can be seen from the figure, the upper and lower bounds are indistinguishable. In this setup, at most one of the subcarriers of each source node can be helped by both relays. Since the number of subcarriers, $N$, is generally much larger than the number of relays, $J$, selection is close-to-optimal.

Given the additional flexibility of subcarrier-based cooperation schemes, both UBSB and LBSB outperform block-based schemes. Moreover, although the optimal block-based relaying scheme is computationally much more complex than the decentralized scheme, the performance benefit is negligible. Enforcing direct transmission has the worst performance, validating the fact that cooperation transmission can boost network performance under the max-min metric.

B. Resource Allocation with Infinite-Power S-R Channels in Distributed Scenario

In this example, nodes are geographically distributed and inter-node channels are modeled using the COST-231 channel model. We generate the random node locations over an square area of $200 \times 200$ m. Source and destination nodes are located on the edges of the square area while relays are randomly distributed within the area. The variance of the log-normal fading is set to 10.6dB. In this experiment, for each set of locations, independent channel realizations are simulated and results averaged over both node locations and channel states.

Fig. 3 plots the max-min achievable rate across all APs and compares the performance of various resource allocation schemes. From the figure, the performance gap between UBSB and LBSB is, again, negligible; the heuristic method to find the solution of the original convex optimization problem is almost exact. However, it is worth emphasizing that in both Figs. 2 and 3 there is a difference, albeit minuscule, between the UBSB and LBSB performance. This proves the fact that selection, is an approximate, not optimal solution.

Fig. 3 also compares the performance of block-based schemes. Simple relay selection closely tracks the optimal relay selection method, but with significantly less complexity. This result indicates that the simple relay selection scheme can be implemented in a decentralized manner without significant performance loss.
Fig. 4 illustrates the importance of node locations on the performance of different resource allocation algorithms. This example simulates a single source-destination pair with two relay nodes. The S-D distance is fixed to $0.2\sqrt{2}$ km. Relays are located on both sides of the S-D path. Clearly one wants to use the relay close to the destination; however, note that this may impact the assumption that relays can always decode. Simulation results show that relaying schemes outperform direct transmission whenever relays are located between the source and destination nodes. While the upper bound on subcarrier-based selection outperforms block-based selection, the performance loss for this more practical approach is surprisingly small.

C. Resource Allocation with Finite-Power S-R Channels in I.I.D. Scenario

With finite-power S-R channels, we now use the comprehensive resource allocation and relay assignment schemes developed in Section IV. Fig. 5 ($K = 3$) and 6 ($K = 4$) plot the achievable minimum rate across source nodes for various values of R-D SNRs. The SNR of the S-R and S-D channels are, respectively, set to 10dB and 5dB. Both figures show that at high SNRs, subcarrier-based methods outperform other resource allocation schemes. This is expected since subcarrier-based methods exploit the frequency diversity across relays provided by the assumption that individual subcarriers can be transmitted independently. However, at low SNRs, the UBBB outperforms the LBSB scheme and the decentralized selection scheme outperforms the centralized LBBB. This can be explained by recognizing the fact that our heuristic method to impose selection on individual flows does not use all available power at the relay nodes. If we apply a second round of power allocation at the relays, power freed up from enforcing selection can be distributed amongst all other source nodes which are assigned to those relays and a tighter lower bound will result.

D. Resource Allocation with Finite-Power S-R Channels in Distributed Scenario

Fig. 7 plots the minimum rate across users versus the maximum available power of the sources and relays when the nodes are geographically distributed and channels are simulated using the COST-231 model. Although the decentralized scheme uses only local CSI, it has a close-to-optimal performance. This method also decreases the computation and coordination burden of the network. Again, since LBBB does not use the total available power, it is probable that its achievable rate is less than that of the decentralized scheme.
VI. Conclusion

This paper developed subcarrier and block-based relaying methods for the selection-based OFDM networks to maximize the minimum rate across sources. Considering subcarriers as independent transmissions and assuming that relays are capable of decoding all received signals, we proved that selection is violated in a maximum of $J - 1$ out of $N$ subcarriers. Furthermore, by relaxing the ideal S-R channel assumption, we proposed a generalized subcarrier-based relaying scheme which jointly selects the transmission strategy, assigns relays, and allocates power to source nodes. We also characterized tight lower bounds on the minimum achievable rates of both subcarrier-based algorithms.

However, given the implementation restrictions that IFFT/FFT pair imposed on the transmitted OFDM signals, considering individual subcarriers as independent transmissions is invalid. To hurdle this issue, we then considered block-based relaying for multi-source networks. While heuristic algorithms with different computation complexities are proposed for the ideal S-R scenario, we formulate the optimization problem for the comprehensive case which simultaneously solves transmission strategy selection, relay assignment, and power allocation problem. Similar to the subcarrier-based method, tight lower bounds are also developed.

We also proposed a simple, decentralized, relaying algorithm and the required guidelines to allocate network-wide resources amongst source nodes. Compared to the previous schemes, this method significantly decreases the required computational complexity while it has a close-to-optimal performance.

References

[1] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity, part I. system description,” IEEE Trans. Commun., vol. 51, no. 11, pp. 1927–1938, Nov. 2003.
[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” IEEE Trans. Inform. Theory, vol. 50, no. 12, pp. 3062–3080, Dec. 2004.
[3] J. N. Laneman and G. W. Wornell, “Distributed space-time-coded protocols for exploiting cooperative diversity in wireless networks,” IEEE Trans. Inform. Theory, vol. 49, no. 10, pp. 2415–2425, Oct. 2003.
[4] A. Bletsas, A. Khisti, D. P. Reed, and A. Lippman, “A simple cooperative diversity method based on network path selection,” IEEE J. Select. Areas Commun., vol. 24, no. 3, pp. 659–672, March 2006.
[5] E. Beres and R. S. Adve, “Selection cooperation in multi-source cooperative networks,” IEEE Trans. Wireless Commun., vol. 7, no. 1, pp. 118–127, Jan. 2008.
[6] Y. Zhao, R. S. Adve, and T. J. Lim, “Improving amplify-and-forward relay networks: optimal power allocation versus selection,” IEEE Trans. Wireless Commun., vol. 6, no. 8, pp. 3114–3123, August 2007.
[7] D. Michalopoulos and G. Karagiannidis, “Performance analysis of single relay selection in rayleigh fading,” IEEE Trans. Wireless Commun., vol. 7, no. 10, pp. 3718–3724, October 2008.

[8] J. Chu, R. Adve, and A. Eckford, “Relay selection for low-complexity coded cooperation,” in IEEE Global Telecommunications Conference, nov. 2007, pp. 3932–3936.

[9] S. Kadloor and R. S. Adve, “Optimal relay assignment and power allocation in selection based cooperative cellular networks,” in IEEE ICC, June 2009, pp. 1–5.

[10] C. Y. Wong, R. S. Cheng, K. B. Lataief, and R. D. Murch, “Multiuser OFDM with adaptive subcarrier, bit, and power allocation,” IEEE J. Select. Areas Commun., vol. 17, no. 10, pp. 1747–1758, Oct 1999.

[11] W. Rhee and J. M. Ciofﬁ, “Increase in capacity of multiuser OFDM system using dynamic subchannel allocation,” in VTC Spring, 2000, pp. 1085–1089 vol.2.

[12] J. Jang and K. B. Lee, “Transmit power adaptation for multiuser OFDM systems,” IEEE J. Select. Areas Commun., vol. 21, no. 2, pp. 171–178, Feb 2003.

[13] Z. Shen, J. G. Andrews, and B. L. Evans, “Adaptive resource allocation in multiuser OFDM systems with proportional rate constraints,” IEEE Trans. Wireless Commun., vol. 4, no. 8, pp. 2726–2737, Nov 2005.

[14] G. Li and H. Liu, “Resource allocation for OFDMA relay networks with fairness constraints,” IEEE J. Select. Areas Commun., vol. 24, no. 11, pp. 2061–2069, Nov. 2006.

[15] H. Li, H. Luo, X. Wang, C. Lin, and C. Li, “Fairness-aware resource allocation in OFDMA cooperative relaying network,” in IEEE ICC, June 2009, pp. 1–5.

[16] T. C. Y. Ng and W. Yu, “Joint optimization of relay strategies and resource allocations in cooperative cellular networks,” IEEE J. Select. Areas Commun., vol. 25, no. 2, pp. 328–339, February 2007.

[17] L. Weng and R. D. Murch, “Cooperation strategies and resource allocations in multiuser OFDMA systems,” IEEE Trans. Veh. Technol., vol. 58, no. 5, pp. 2331–2342, Jun 2009.

[18] Y. Cui, V. K. N. Lau, and R. Wang, “Distributive subband allocation, power and rate control for relay-assisted OFDMA cellular system with imperfect system state knowledge,” IEEE Trans. Wireless Commun., vol. 8, no. 10, pp. 5096–5102, October 2009.

[19] A. R. Bahaei, B. R. Saltzberg, and M. Ergen, Multi Carrier Digital Communications: Theory and Applications of OFDM. Springer NewYork, 2004.

[20] Y. Pan, A. Nix, and M. Beach, “Resource allocation techniques for OFDMA-Based decode-and-forward relaying networks,” in VTC Spring, 2008, pp. 1717–1721.

[21] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge University Press, 2004.

[22] H.-X. Li, H. Yu, H.-W. Luo, J. Guo, and C. Li, “Dynamic subchannel and power allocation in OFDMA-based DF cooperative relay networks,” in IEEE Global Telecommunications Conference, Dec. 2008, pp. 1–5.

[23] “Multi-hop Relay System Evaluation Methodology,” available online at http://ieee802.org/16/relay/docs/80216j-06_013r3.pdf, Tech. Rep.
Fig. 1. Cooperative OFDM-based multi-source multi-destination mesh network with $K = 3$ and $J = 2$.

**TABLE I**

**PARAMETER VALUES IN COST-231**

| Parameter       | Value  | Parameter       | Value  |
|-----------------|--------|-----------------|--------|
| AP Height       | 15m    | Frequency       | 3.5 GHz|
| Building Spacing| 50m    | Rooftop Height  | 30m    |
| Destination Height | 15m    | Road Orientation| 90 deg.|
| Street Width    | 12m    | Noise PSD       | -174 dBm|

Fig. 2. Achievable min rate of different resource allocation strategies in “i.i.d channel” scenario with $J = 2$ and $N = 32$. 
Fig. 3. Achievable min rate of different resource allocation strategies in “distributive” scenario with $J = 2$ and $N = 16$.

Fig. 4. Source transmission rate in a single source-destination pair network with $J = 2$ and $N = 16$. 
Fig. 5. Max-min rate across all source nodes of different resource allocation strategies in the “i.i.d. channel” scenario with $K = 3$, $J = 2$, and $N = 8$.

Fig. 6. Max-min rate across all source nodes of different resource allocation strategies in the “i.i.d. channel” scenario with $K = 4$, $J = 2$, and $N = 8$. 
Fig. 7. Max-min rate across all source nodes of different resource allocation strategies in the “distributive” scenario with $K = 3$, $J = 2$, and $N = 8$. 