The impact of weak lensing on Type Ia supernovae luminosity distances

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ABSTRACT
When Type Ia supernovae are used to infer cosmological parameters, their luminosities are compared to those from a homogeneous cosmology. In this note we propose a test to examine to what degree SN Ia have been observed on lines of sight where the average matter density is not representative of the homogeneous background. We apply our test to the Pantheon SN Ia compilation, and find two redshift bins which indicate a moderate bias to over-density at \( z \approx 2 \). We modify the Tripp estimator to explicitly de-lens SN Ia magnitudes, and show that this reduces scatter of Hubble diagram residuals. Using our revised Tripp estimator, the effect on cosmological parameters from Pantheon in \( \Lambda \)CDM is however small with a change in mean value from \( \Omega_{m} = 0.317 \pm 0.027 \) (baseline) to \( \Omega_{m} = 0.312 \pm 0.025 \) (de-lensed). For the Flat \( \Lambda \)CDM case it is \( \Omega_{m} = 0.332 \pm 0.049 \) and \( w = -1.16 \pm 0.16 \) (baseline) versus \( \Omega_{m} = 0.316 \pm 0.048 \) and \( w = -1.12 \pm 0.15 \) (de-lensed). We note that the effect of lensing on cosmological parameters may be larger for future high-z surveys.

Key words: gravitational lensing: weak – supernovae: general – cosmology: observations – cosmology: dark matter – cosmology: cosmological parameters

1 INTRODUCTION
Type Ia supernovae are used extensively in cosmology as, once standardised, their absolute magnitudes have a low and well-characterised scatter. As they are observed from our cosmic neighbourhood out to redshift \( z \approx 2 \), they may be used to build a Hubble diagram of their apparent magnitudes and redshift which connects the expansion history of the universe from when it was matter-dominated, through to the current epoch of dark-energy domination (Riess et al. 1998; Perlmutter et al. 1999). This Hubble diagram may be compared to the theoretical prediction of a given homogeneous model, in order to determine \( \Omega_{m} \) and the equation of state of dark energy in simple extensions of \( \Lambda \)CDM (Scolnic et al. 2018). If the absolute magnitudes of SN Ia are calibrated then the Hubble constant \( H_{0} \) is also measured (see for example Shah et al. (2021) for a review). A standard data set often used in cosmological analyses is Pantheon (Scolnic et al. 2018), which combines observations from multiple surveys, and we use Pantheon in this note.

The accuracy of cosmological parameters determined from SN Ia depends on reliable standardisation and bias corrections, which is done in Pantheon via a distance modulus estimator (Tripp & Branch 1999):

\[
\mu = m_{B} - M_{B} + a x_{1} - \beta c + \Delta_{M} + \Delta_{B},
\]

where the observables are \( m_{B} \), the log of the flux normalisation of the SN Ia lightcurve; \( x_{1} \), a stretch parameter determined by the duration of the light curve; and \( c \), the deviation of the \( B - V \) colour from the mean. \( M_{B} \) is the absolute magnitude of a fiducial mean SN Ia light curve. \( \Delta_{M} \) accounts for environmental effects in the host galaxy, and \( \Delta_{B} \) is a Malmquist bias correction.

Weak gravitational lensing, which causes non-Gaussian fluctuations in the magnitudes of SN Ia, is usually treated as a source of scatter. SN Ia seen on over-dense lines of sight (compared to a homogeneous universe of the same average matter density) will be magnified, and those on under-dense ones de-magnified. Although the distribution of lensing is not Gaussian, a rough guide to its size is the r.m.s. scatter \( \sigma_{\text{lens}} = (0.06 \pm 0.017)(d_{C}(z)/d_{C}(z = 1))^{3/2} \) mag, where \( d_{C}(z) \) is the comoving distance to redshift \( z \) (Shah et al. (2022), hereafter S22).

There are two principal effects of lensing on cosmological parameter estimation. If treated as a source of noise, the accuracy of high-z SN Ia data is degraded as \( \sigma_{\text{lens}} \) approaches the intrinsic scatter of SN Ia (Holz & Linder 2005). Secondly, if SN Ia are preferentially selected depending on whether they are magnified or not, a bias is introduced. While this bias may in principle be simulated and incorporated into \( \Delta_{B} \) in Equation 1 (see Kessler & Scolnic (2017) for details), it requires an estimation of the lensing probability distribution which was not available to Pantheon at the time. A further assumption is that the actual selection process of SN Ia is modelled by the simulation and that the numbers of SN Ia suffice to converge to the mean.

In this note, we have two goals. Firstly, we compare values of the weak lensing convergence estimator proposed in S22 between the lines of sight to Pantheon SN Ia and randomly chosen ones. This is to statistically test if residual bias exists, and to what equivalent magnitude. Secondly, we use de-lensed magnitudes to evaluate the effect this may have on cosmological parameters.

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2 MODIFYING THE TRIPP ESTIMATOR FOR WEAK LENSING

Working to first order in weak lensing convergence $\kappa$, the change in magnitude $m$ is $\Delta m = -(5/\log 10) \kappa + O(\kappa^2, \gamma^2)$ where $\gamma$ is the shear. We may write the magnification of supernovae $i$ as the sum of the contribution from $N_i$ individual lenses as
\[
\Delta m_i = \sum_{j=1}^{N_i} \delta m_{ij}(\lambda) .
\]
\[(2)\]
$\Delta m_i$ is defined as the magnification relative to the “empty beam” value, so $\Delta m_i < 0$ always. As detailed in S22, by taking lensing as due to dark matter haloes surrounding foreground galaxies, the parameters determining $\delta m_{ij}$ are $\lambda = \{M_r, z_s, z_d, \theta, \Gamma, \beta\}$ are the $r$-band absolute magnitude of the galaxy, the spectroscopic redshift of the source, photometric redshift of the lens and its angular distance to the line of sight, mass-to-light ratio $\Gamma$ and halo radial matter density slope $\beta$ respectively. Again to first order in the convergence, flux conservation may be used to express the estimated magnification of a given SN Ia as
\[
\Delta \text{lens} = \Delta m_i - (\Delta m'(z_s)) .
\]
\[(3)\]
The average in the second term is over random lines of sight to the same redshift as the source. The distribution of $\Delta \text{lens}$ is skewed : large numbers of slightly de-magnified SN Ia will be balanced by smaller numbers of magnified SN Ia (for a quantification of skew, see Kaimulainen & Marra (2009)). To first order, averaging over sources is equivalent to averaging over area (Kaiser & Peacock 2016).

By correlating the lensing estimator $\Delta \text{lens}$ to the Hubble diagram residual $\mu_{\text{res}} = \mu - \mu_{\text{model}}$ where $\mu_{\text{model}}(z)$ is the distance modulus of a best-fit homogeneous cosmology, Bayesian estimates for the mass-to-light ratio $\Gamma$ and radial slope $\beta$ may be found. S22 found $\beta = 1.8 \pm 0.3$ (slightly shallower than the NFW profile (Navarro et al. 1996) which has $\beta = 2$), and $\Gamma = 197^{+58}_{-80} h M_\odot/L_{r,\odot}$ where 65% credibility intervals are indicated. Strictly speaking we should marginalise over the full posterior pdf $p(\Gamma, \beta)$, but for the purposes of this note it suffices to use the mean values. These were determined using Pantheon and foreground galaxy data from the Sloan Digital Sky Survey (SDSS) (Eisenstein et al. 2011). We therefore modify the Tripp estimator as
\[
\mu = m_B - M_B + \alpha x_1 - \beta c + \Delta M + \Delta \text{b} - \eta \Delta \text{lens} ,
\]
\[(4)\]
where $\eta$ is a parameter to adjust for the magnitude limitation of the SN Ia and galaxy data ($\eta = 1$ for volume-limited surveys). It was found in S22 that $\eta = 1$ is adequate for the Pantheon and SDSS combination. Here $\Delta \text{lens}$ is a line-of-sight “environmental” variable akin to $\Delta M$. The minus sign is for de-lensing, analogous to the de-redening term $\beta c$.

For the second term in Equation (3) we generate 10,000 lines of sight at random within the SDSS footprint, terminating in a redshift that has randomly selected from Pantheon (that is, the random lines of sight shuffle only the sky positions of SN Ia and not their redshift distribution). We use the same selection criteria as S22 to remove heavily masked fields (but do not select fields on whether a host has been identified or not; most of our random LOS will intentionally not lie near any putative host galaxy).

For Pantheon, we calculate $\Delta \text{lens}$ for the 727 SN Ia lying within the SDSS footprint between 0.05 $< z < 1.0$ (SDSS is not deep enough to reliably estimate lensing for SN Ia with $z > 1$). $\Delta \text{lens}$ varies between $-0.19$ and 0.09 and has a skew of $-3.0$. The standard deviation increases with the comoving distance to the source per the formula given above.

3 ARE SN IA LINES OF SIGHT SPECIAL?

SN Ia candidates far outnumber those confirmed, with the main constraint being the observing time on spectroscopic platforms. Selection occurs both with the detection of candidates (detection efficiency) and spectroscopic confirmation (selection efficiency). It is argued in Scolnic et al. (2018) that both selections are well-characterised by an estimated probability $f(m_r) (0,1)$ which depends solely on the r-band magnitude $m_r$ of the SN Ia (see section 3.3 of Scolnic et al. (2018)). The observed population does indeed show a drift with redshift : supernovae from the high-z SNLS subset are bluer and more stretched than the Low-z subset, amounting to $\sim 0.2$ magnitudes brighter before standardisation. The $\Delta \text{b}$ term in Equation (1) corrects the light curve parameters for the consequent Malmquist bias. It is calculated by simulations convolving a model for the scatter of SN Ia parameters with $f(m_r)$ according to the method outlined in Kessler & Scolnic (2017). For Pantheon, a $\sigma_{\text{lens}}$ term was included in the covariance matrix (see Eqn. 5 below) but not in the bias correction calculation.$^2$

Can magnitude be the sole selection criteria for SN Ia? Weinberg (1976) noted that if background SN Ia are obscured by foreground galaxies, sources are de-magnified on average towards the “empty beam” value. In our model this would be the luminosity distance in a homogeneous universe where all matter associated with virialised halos has been removed. He argued a “radius of avoidance” of $R \sim 10\text{kpc}$ around a foreground galaxy is sufficient to trigger this effect (a quantitative prescription in terms of survival probability of a light ray is given in Kaimulainen & Marra (2011)). SN Ia may not need to be strictly obscured : a desire to avoid blending or fibre collisions with foreground galaxies in spectroscopic selection would create an equivalent effect.

One could then argue for bias in either direction. Also, Pantheon combines multiple surveys each of which has its own selection characteristics. It is therefore worthwhile to compare the distribution of $\Delta m'$ for the lines of sight in Pantheon to the random set, which are the two terms in Equation (3).

We perform a two-tailed Kolmogorov-Smirnov test in broad redshift bins (chosen to have ~ 70 Pantheon SN Ia each). The results are summarised in Table 1. While there is some statistical inconsistency in the two low redshift buckets, indicating a preference to find SN Ia in clusters, these bins will not be significant for cosmological fits. In general, Pantheon lines of sight are somewhat over-dense compared to the random set, but not at large significance. The exceptions are the two bins $0.25 < z < 0.3$ and $0.6 < z < 0.8$, which interestingly are close to the magnitude limits of the main surveys in Pantheon (SDSS, SNLS and Pan-Starrs; see Figure 10 of Scolnic et al. (2018)). These are over-dense at a significance of around $2\sigma$; supernovae in these bins seem not to be on typical lines of sight. Figure 1 illustrates the distribution for two bins.

$^2$ More recent datasets such as the Dark Energy Survey (Smith et al. 2020) and PantheonPlus (Scolnic et al. 2022) do incorporate lensing probability distributions derived from N-body simulations into their bias calculations (Kessler et al. 2019; Broust et al. 2022). In this case, $\Delta \text{b}$ should be re-calculated before de-lensing the distances as done in Eqn. 4.
Random 37
-0.0003 -0.0008
-0.0012 -0.0031
0.9685
48
-0.0269 -0.0336
0.036
-0.073 -0.0825
0.6921
0.0059
108
-0.0143 -0.0136
52
0.3429
48
90
114
-0.0028 -0.0039
-0.0939 -0.0905
-0.0081 -0.0098
80
0.8977
c

\(\mathcal{C}\) is a Gaussian random scatter term of size given by \(\sigma_{\text{lens}} = 0.055\xi\) (Jönnson et al. 2010). The Pantheon likelihood is \(\mathcal{L} = \exp(-\chi^2/2)\)

\[\chi^2 = \mu_{\text{res}}^T \mathcal{C}^{-1} \mu_{\text{res}}.\] (6)

Here \(\mu_{\text{res}} = \mu - \mu_{\text{model}}\) where \(\mu\) is as published in Scolnic et al. (2018) (baseline), or modified by Equation (4) (de-lensed). For the de-lensed case, we adjust the Pantheon covariance diagonal elements to remove the stated lensing variance: \(C' = C - (0.055)^2\). The additional variance due to the uncertainty in our lensing estimator is small compared to other variance terms and we have found adding it does not affect the results. The standard FRLW cosmological formulae are used for \(\mu_{\text{model}}\). Our baseline values are consistent with, but somewhat different to Scolnic et al. (2018) due to the smaller numbers of SN Ia used.

Cosmological parameters are often fit using the minimum-\(\chi^2\) method (see for example Section 6 of Scolnic et al. (2018)). Seen from a Bayesian perspective, this is equivalent to a maximum-likelihood if the likelihood is Gaussian, of known covariance and a uniform prior is taken for the cosmological parameters. In other words, the mode of the posterior distribution is found. However as lensing is a skewed distribution, a likelihood incorporating its effect will be non-Gaussian. It is then possible that de-lensing may result in larger differences in maximum likelihood values compared to the mean values. One can understand the effect on the dark energy equation of state parameter \(w\) in the \(\Lambda\)CDM model as follows. If \(w < -1\), SN Ia appear dimmer compared to \(\Lambda\)CDM. However, as the maximum likelihood value for lensing also results in a dimmer SN Ia, de-lensing may be expected to increase the maximum likelihood of \(w\). Amendola et al. (2010) and Holz & Linder (2005) investigated this using simulations, and state \(w\) is biased lower by 0.1 if the likelihood is falsely assumed to be Gaussian. However a similar analysis by Sarkar et al. (2008) found minimal bias. Unmodelled systematics in the observing strategy as described in Section 3 may also contribute to skew effects.

We are therefore motivated to compare both the modes and mean of the posterior between the baseline and de-lensed cases given uniform priors of \(\Omega_m \in (0.2, 0.4)\) and \(w \in (-0.5, -1.5)\). Our cosmological parameter shifts are unaffected by whether we fix the nuisance parameter \(M = -19.425\) (which is equivalent to \(H_0 = 67.4\ km\ s^{-1}\ Mpc^{-1}\)) or marginalise over it. We also assume that the nuisance parameters \(\alpha\) and \(\beta\) are unaffected by lensing. The PolyChord package (Handley et al. 2015) was used to generate the chains.

**Table 1.** Comparison between the empty-beam average (\(\Delta m'\)) computed on random lines of sight within the SDSS footprint, and the lines of sight to Pantheon SN Ia. There is a general trend for Pantheon SN Ia to appear more magnified than the random sample. The probability-to-exceed of a two-tailed Kolmogorov-Smirnov test is given in the last column. While the distributions are compatible for many buckets, in particular for \(z \in (0.25, 0.3)\) and \(z \in (0.6, 0.8)\) the distributions are distinct at moderate significance. These buckets roughly correspond to the magnitude limits of the SDSS, Pan-Starrs and SNLS SN Ia surveys respectively.

\[
\begin{array}{cccccc}
\bar{z}_\text{min} & \bar{z}_\text{max} & N & \text{Random }\langle \Delta m'\rangle & \text{Pantheon }\langle \Delta m'\rangle & p\text{-value} \\
0.05 & 0.1 & 48 & -0.0003 & -0.0008 & 0.0022 \\
0.1 & 0.15 & 80 & -0.0012 & -0.0031 & 0.0059 \\
0.15 & 0.2 & 110 & -0.0028 & -0.0039 & 0.3429 \\
0.2 & 0.25 & 108 & -0.0053 & -0.0056 & 0.6921 \\
0.25 & 0.3 & 90 & -0.0081 & -0.0098 & 0.036 \\
0.3 & 0.4 & 114 & -0.0143 & -0.0136 & 0.9685 \\
0.4 & 0.5 & 52 & -0.0269 & -0.0336 & 0.4433 \\
0.5 & 0.6 & 37 & -0.0479 & -0.0482 & 0.9989 \\
0.6 & 0.8 & 48 & -0.073 & -0.0825 & 0.0211 \\
0.8 & 1.0 & 39 & -0.0939 & -0.0905 & 0.8977 \\
\end{array}
\]
5 RESULTS

For $\Lambda$CDM, we find the mean $\Omega_m = 0.317 \pm 0.027$ (baseline) and $\Omega_m = 0.312 \pm 0.025$ (de-lensed). For Flat $w$CDM, we find $\Omega_m = 0.332 \pm 0.049$ and $w = -1.16 \pm 0.16$ (baseline), whereas $\Omega_m = 0.316 \pm 0.048$ and $w = -1.12 \pm 0.15$ for the de-lensed case. The posteriors for $\Lambda$CDM for are shown in Figure 2. $\Delta\chi^2 \sim -15$ in both $\Lambda$CDM and $w$CDM for the de-lensed data.

The differences of $\Delta\Omega_m = -0.005 \pm 0.002$ ($\Lambda$CDM) and $(\Delta\Omega_m, \Delta w) = (-0.016 \pm 0.008, +0.04 \pm 0.02)$ ($w$CDM) are low compared to the cosmological uncertainty (of the overall Pantheon sample). We have computed the errors on the shifts by sampling from the posteriors for the model parameters and photometric redshifts. Our statistical test of the preceding paragraph isolated the significance of the lensing correction independent of the background cosmological uncertainty.

The maximum likelihood value changes for $\Lambda$CDM by $\Delta\Omega_m = -0.006$, and for $w$CDM the change is $(\Delta\Omega_m, \Delta w) = (-0.017, +0.04)$. We emphasize we have computed the change for realised Pantheon sample, rather than the average of a simulated ensemble.

6 CONCLUSION

Our answer to the question “are SN Ia biased probes of cosmological parameters?” is then a qualified “no” for Pantheon. We have found moderate evidence of an uncorrected bias to select magnified SN Ia in two buckets around the magnitude limits of the main contributing surveys. However, the change in cosmological parameters caused by de-lensing SN Ia magnitudes is not significant compared to their uncertainty. We find no evidence that a “zone of avoidance” around foreground galaxies has selected SN Ia on under-dense lines of sight, although it may be that some offset between the two effects is present.

Our test can be performed on any future survey to prospectively or retrospectively check for bias, and uses only observational data with no need for simulations of the lensing pdf. It will be particularly well-suited to high-z surveys such as the Rubin LSST and the Roman Space Telescope, due to the larger effect of lensing and also by being paired with a galaxy catalog assembled from the same platform. It may also be necessary to consider $O(\kappa^2)$ effects at higher redshifts, which can introduce further bias (Kaiser & Peacock 2016).

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DATA AVAILABILITY

Upon publication, a file containing computed $\Delta l_{\text{lens}}$ for Pantheon will be made available at https://github.com/paulshah/SNLensing.

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