Abstract

We calculate $\sin^2 \theta(W(M_Z))$ in the MSSM in terms of $\alpha_{EM}$, $G_F$, $m_t$, $\tan \beta$ and SUSY mass parameters with the same accuracy as the present calculations of $\sin^2 \theta(M_Z)$ in the SM. We compare the results with the standard leading logarithmic approximation used for SUSY threshold corrections and find important differences in the case of light sparticles. We give approximate formulae connecting coupling constants in the SM and in the MSSM and comment on process dependence of such formulae. The obtained values of the MSSM couplings $\alpha_i(M_Z)$ are used to investigate gauge coupling unification in the minimal SUSY $SU(5)$ model. Our non-logarithmic corrections lower the predicted value of the Higgs triplet mass. The interplay between the supersymmetric and GUT thresholds in achieving unification for the coupling constants in the range of the experimentally acceptable values is quantified.

*Supported in part by the Polish Committee for Scientific Research.
1. INTRODUCTION.

The successful gauge coupling unification in the Minimal Supersymmetric Standard Model (MSSM) \[1\] has spurred recently a lot of interest in the SUSY GUT scenario. The standard approach to the coupling unification is based on the fact that physics at a given scale can be described in terms of an effective renormalizable theory, with the effects of the larger scales decoupled. For SUSY GUTs, one naturally considers the three energy regimes described by the Standard Model (SM), the MSSM and the full GUT. The first two models are effective renormalizable theories which approximate the full theory up to higher dimension operators suppressed by the scales of new physics.

The standard approach to the gauge coupling unification consists of the following steps:

i) Measurements at energy \( E \leq M_Z \) are used to extract the couplings \( \alpha_i(M_Z) \) \( i = y, 2, 3 \) using the Dimensional Reduction regularization (DR) \[2\] and the Modified Minimal Subtraction Scheme \( (\overline{MS}[3]) \) in the framework of the SM. In addition to \( \alpha_3(M_Z) \) and \( \alpha_{EM}(M_Z) \) it is convenient to use \( \sin^2\theta_W(M_Z) \). It can be calculated e.g. in terms of \( G_F = 1.16639(2) \times 10^{-5} \) GeV\(^{-2} \), \( \alpha_{EM}^{OS} = 1/137.0359895(61) \), \( M_Z = 91.1888(44) \) GeV \[4\], \( m_t \) and \( M_{h^0} \) (the top quark and SM Higgs boson masses respectively). The first three parameters are known with very high accuracy and the latter two enter through radiative corrections. In practice, one takes \( \alpha_{EM}(M_Z) \), \( \sin^2\theta_W(M_Z) \) and \( \alpha_3(M_Z) \) determined in the ordinary Dimensional Regularization (DIMR) and \( \overline{MS} \) and subsequently converts them to the DR regularization using the prescription \[5\]

\[
\frac{1}{\alpha_i(M_Z)} = \frac{1}{\alpha_i^{DIMR}(M_Z)} - \frac{C_i}{12\pi} \tag{1}
\]

with \( C_y = 0, C_2 = 2, C_3 = 3 \). The Weinberg angle can also be extracted (or crosschecked) from other observables: e.g. \( G_F \) can be replaced by \( \sin^2\theta_W^{eff} \) measured at LEP and SLD \[4\]. However, at present the most precise determination of the couplings in the SM follows from the measured values of \( G_F, M_Z \) and \( \alpha_{EM}^{OS} \).

ii) The coupling constants are evolved to higher scales by means of the 2–loop RGE of the SM, up to some scale \( M_{SUSY} \), and then using the RGE of the MSSM. The threshold corrections to the running of \( \alpha_i(Q) \) due to the splitting in sparticle masses are included at the 1-loop level and result in the contribution \[1, 7, 8\]

\[
\frac{1}{\alpha_i^{MSSM}(M_{SUSY})} - \frac{1}{\alpha_i^{SM}(M_{SUSY})} \equiv 4\pi\Delta_i(M_{SUSY}) = \sum_k \Delta b_{ik} \log \frac{m_k}{M_{SUSY}} \tag{2}
\]

We call \( \alpha_y \) the coupling related to the ordinary hypercharge reserving the symbol \( \alpha_1 \) for the rescaled coupling appropriate for discussion of unification; \( \alpha_1 = (5/3)\alpha_y \).
where the coefficients $\Delta b_{ik}$ are the contributions to the $\beta_i$ function of the coupling $\alpha_i$ from the sparticle $k$. It has to be stressed, that the effects of the VEV contribution to sparticle masses as well as the effects of the left-right mixing between sfermions cannot be treated consistently in this way and must be neglected. Thus the masses $m_k$ appearing in the formula are the lagrangian $SU(2) \times U(1)$ invariant mass parameters and not the physical masses.

The one loop formula for $\alpha_i^{MSSM}(Q)$ at some scale $Q > \text{max}(m_k)$:

\[
\frac{1}{\alpha_i^{MSSM}(Q)} = \frac{1}{\alpha_i^{SM}(M_Z)} + 2b_i^{SM} \log \frac{M_{SUSY}}{M_Z} + 4\pi \Delta_i(M_{SUSY}) + 2b_i^{MSSM} \log \frac{Q}{M_{SUSY}}
\] (3)

can be rewritten as

\[
\frac{1}{\alpha_i^{MSSM}(Q)} = \frac{1}{\alpha_i^{MSSM}(M_Z)} + 2b_i^{MSSM} \log \frac{Q}{M_Z}
\] (4)

with

\[
\frac{1}{\alpha_i^{MSSM}(M_Z)} = \frac{1}{\alpha_i^{SM}(M_Z)} + 4\pi \Delta_i(M_Z)
\] (5)

having the obvious interpretation of the couplings $\alpha_i(M_Z)$ extracted at the scale $M_Z$ directly in the MSSM. This is possible, because the threshold corrections are given in eq.(2) in the form in which the 1-loop logarithms are resummed. Therefore the dependence on $M_{SUSY}$ in eq.(3) drops out and one can take $M_{SUSY} = M_Z$. One should stress that this is true only in the leading logarithmic approximation for the threshold corrections. In general the results would depend on the scale $M_{SUSY}$ and this dependence is always a next order correction.

iii) The MSSM equations are used for the running up to some scale $M_{GUT}$ where the unification is achieved (or not) without or with the GUT scale threshold effects which are taken into account also in the leading logarithmic approximation. In the latter case the final result depends on the specific GUT. In addition it depends on our criterion of an “acceptable” GUT spectrum.

The standard approach is justified in the presence of well separate scales ($M_Z \ll M_{SUSY} \ll M_{GUT}$). However, it often happens that the scales of “new physics” are not large enough to neglect its non–logarithmic contributions. This may well be the case with the Standard Model and its supersymmetric extension: it is quite plausible that the superpartner masses vary from $O(M_Z)$ (some of them can even be lighter than $Z^0$ boson) to, say, $O(1 \text{ TeV})$. In this case the (renormalizable) SM is not a consistent
approximation at the scale $M_Z$ to the full MSSM and higher dimension (non-renormalizable) operators cannot be neglected in step $i$, i.e. in the procedure of extracting the SM couplings from the data.

An equivalent, more straightforward and more convenient approach is to work at the scale $M_Z$ directly with the MSSM. This point has been emphasized by Faraggi and Grinstein [10]. This means that the couplings at $M_Z$ extracted from experimental data are the MSSM (and not the SM) couplings. All SUSY threshold effects are already included at this step and the RG running is based on 2-loop MSSM equations from $M_Z$ up to the GUT scale. From this point of view formula (5) is a formula for $\alpha^{\text{MSSM}}_i(M_Z)$ calculated e.g. in terms of $G_F$, $\alpha^{\text{OS}}_E(M_Z)$, $M_Z$, $m_t$ and other SUSY parameters with only 1-loop logarithmic SUSY contributions retained, and resummed to all orders. We call it Leading Logarithmic Threshold (LLT) Approximation.

The main part of this paper is the calculation in Sec. 2 of the $\sin^2 \theta_W(M_Z)$ (and also, as a byproduct, of $M_W$) in terms of $\alpha^{\text{OS}}_E$, $G_F$, $M_Z$, $m_t$, $\tan \beta$ and other SUSY parameters in the $\overline{\text{MS}}$ Scheme directly in the MSSM with the same accuracy as the present calculations of $\sin^2 \theta_W(M_Z)$ in the SM. In Sec. 3 we compare the results with the standard LLT approximation and find important differences in the case of light sparticles. Also in Sec. 3 we comment on the Farragi-Grinstein (FG) [11] approach and its ambiguity related to the process dependence. In an approximation analogous to theirs we give the formulae connecting the coupling constants in the SM and in the MSSM, if calculated in terms of $G_F$, $M_Z$ and $\alpha^{\text{OS}}_E$. In Sec.4 we use the obtained values of the MSSM couplings $\alpha_i(M_Z)$ to investigate the gauge coupling unification in the minimal SUSY $SU(5)$ model. In particular, we find that our non-logarithmic corrections lower the predicted values of the Higgs triplet mass. Also, we study in some detail the interplay between the supersymmetric and GUT thresholds in achieving unification for the coupling constants in the range of the experimentally acceptable values.

2. CALCULATION OF $\sin^2 \theta_W(M_Z)$ IN THE MSSM.

The calculation of $\sin^2 \theta_W(M_Z)$ proceeds through the calculation of the Fermi constant measured in the $\mu \rightarrow e\bar{\nu}_e \nu_\mu$ decay. We follow closely the analogous calculation in the SM whose details are analyzed in [11]. The equation for $\hat{s}^2 \equiv \sin^2 \theta_W(M_Z)$ reads (for the running coupling constants in the MSSM taken at the scale $Q = M_Z$ we use the abbreviations: $\hat{\alpha} \equiv \alpha_E(M_Z)$ and $\hat{\alpha}_i \equiv \alpha_i(M_Z)$ for $i=y,2,3$; analogous couplings in the SM will be distinguished always by the superscript “SM”):
where
\[ \Delta \hat{r}_W = \frac{\hat{N}_{WW}(0)}{M_W^2} + \frac{\hat{N}_{WW}(M_W^2)}{M_W^2} + \delta_{VB} \tag{7} \]
and \( M_W (M_Z) \) is the physical mass of the \( W^\pm (Z^0) \) boson. The \( \hat{N}_{V,V}(q^2) \)'s are the vector boson self energies calculated in DR and renormalized by the Modified Minimal Subtraction \( \overline{MS} \); the expressions for \( WW, ZZ, \gamma\gamma \) and \( Z\gamma \) self energies in DR can be found in \([12]\) and \( \delta_{VB} = \delta_{V_B}^{SM} + \delta_{V_B}^{SUSY} \) is the contribution of “non-oblique” vertex, box and external fermion wave function renormalization factors. In the \( \overline{MS} \) scheme, the SM contribution \( \delta_{V_B}^{SM} \) is given by \([11]\):
\[ \delta_{V_B}^{SM} = \frac{\alpha}{4\pi s^2} \left[ 6 + \frac{7 - 5s_W^2 + \hat{s}^2 (3c_W^2/c_W^2 - 10)}{2s_W^2} \log c_W^2 \right] \tag{8} \]
with \( c_W^2 = 1 - s_W^2 \) used as an abbreviation for \( M_W^2/M_Z^2 \). It has the same form in the DIMR and DR schemes. The \( \delta_{V_B}^{SUSY} \) has exactly the same form as in the on-shell calculation (with the obvious replacement of \( s_W, c_W \) and \( \alpha_{EM} \) by \( \hat{s}, \hat{c} \) and \( \hat{\alpha} \)) and can be taken over from the Appendix A of ref. \([13]\): the counterterms to the vertices and neutrino self energies have in the \( \overline{MS} \) calculation the interpretation of the electron and muon wave function renormalization factors.

One way to organize the calculation with \( G_F, M_Z \) and \( \alpha_{EM}^{OS} \) taken as the input parameters is \([11]\) to express the physical mass of the \( W^\pm \) boson in terms of \( M_Z \):
\[ M_W^2 = c^2 \hat{\rho} M_Z^2 \tag{9} \]
with
\[ \frac{1}{\hat{\rho}} = \frac{1 - \hat{N}_{WW}(M_W^2)}{\hat{N}_{ZZ}(M_Z^2) - \hat{N}_{ZZ}(M_Z^2)} + \delta \hat{\rho} \tag{10} \]
where \( \hat{N}_{ZZ}(M_Z^2) \) is the contribution of the \( Z - \gamma \) mixing to the relation between the physical and bare \( Z^0 \) boson masses:
\[ \delta \hat{N}_{ZZ}(M_Z^2) = \frac{(\hat{N}_{Z\gamma}(M_Z^2))^2}{M_Z^2 - \hat{N}_{\gamma\gamma}(M_Z^2)} \tag{11} \]
and \( \delta \hat{\rho} = \delta \hat{\rho}^{QCD} + \delta \hat{\rho}^{HIGGS} \) stands for the leading irreducible 2-loop corrections to \( \hat{\rho} \) of order \( O(\alpha_3 G_F m_t^2) \) (QCD) and \( O(G_F^2 m_t^4) \) (HIGGS). \( \delta \hat{\rho}^{QCD} \) in the SM and in the limit of a heavy top quark reads \([14]\):
\[ \delta \hat{\rho}^{QCD} = \frac{2\alpha_3 (m_t^2)}{3\pi} \left( 1 + \frac{\pi^2}{3} \right) N_c \left( \frac{G_F m_t^2}{8\sqrt{2}\pi^2} \right) \tag{12} \]
where \( m_t \) is the top quark pole mass. This expression is expected to account for the dominant part of QCD corrections also in the MSSM since corrections coming from gluons attached to squark loops, which themselves contribute a small piece of \( \hat{\rho} \), should be very small. In the SM, \( \delta \hat{\rho}^{\text{HIGGS}} \) was calculated in ref \[15\] in the limit \( M_{\phi^0} = 0 \). Recently this calculation has been extended to \( M_{\phi^0} \neq 0 \) by Barbieri et al. \[16\]. In the absence of explicit calculation of \( \delta \hat{\rho}^{\text{HIGGS}} \) in the MSSM we use the following interpolating formula:

\[
\delta \hat{\rho}^{\text{HIGGS}} = -N_c \left( \frac{G_F m_t^2}{8\sqrt{2}\pi^2} \right)^2 \left[ \left( \frac{\cos \alpha}{\sin \beta} \right)^2 \rho^{(2)} \left( \frac{M_{H^0}}{m_t} \right) + \left( \frac{\sin \alpha}{\sin \beta} \right)^2 \rho^{(2)} \left( \frac{M_{H^0}}{m_t} \right) \right]
\]

(13)

where \( \rho^{(2)}(x) \) is given in \[16\] (see also \[17\]), \( M_{h^0} \) and \( M_{H^0} \) are masses of the lighter and heavier MSSM scalar Higgs bosons, \( \tan \beta \equiv v_2/v_1 \) and \( \alpha \) is the mixing angle between scalar Higgs bosons.

Finally, the running coupling constant \( \hat{\alpha} \equiv \alpha(M_Z) \) is obtained from the formula \[17\]:

\[
\hat{\alpha} = \frac{\alpha}{1 - \Delta \hat{\alpha}}
\]

(14)

with \( \alpha \equiv \alpha^{\text{OS}}_{\text{EM}} \) and

\[
\Delta \hat{\alpha} = 0.0684 \pm 0.0009 + \frac{7\alpha}{4\pi} \log \frac{M_W}{M_Z} - \frac{8\alpha}{9\pi} \log \frac{m_t}{M_Z} - 4\pi\alpha \Delta e
\]

(15)

where

\[
8\pi^2 \Delta e = \sum_{i=1}^{6} \frac{4}{3} \log \frac{m_{C_i}}{M_Z} + \frac{1}{3} \log \frac{M_{H^\pm}}{M_Z} + \sum_{i=1}^{6} \frac{1}{9} \log \frac{M_{E_i}}{M_Z} + \sum_{i=1}^{6} \frac{4}{9} \log \frac{M_{U_i}}{M_Z}
\]

(16)

contains logarithms of the physical masses of all charged supersymmetric particles. Notice, that as compared to the DIMR (see e.g. \[17\]), the quantity \( \Delta \hat{\alpha} \) in DR does not contain any constant term coming from integrating out \( W^\pm \) and \( Z^0 \) bosons \[\text{\textsuperscript{2}}\].

The term \( 0.0684 \pm 0.0009 \) accounts for the contribution of light fermions and contains a nonperturbative part. It is extracted from the experimental data on \( e^+e^- \to \gamma^* \to \text{hadrons} \) \[18\]. The error 0.0009 is the most important theoretical uncertainty in the determination of \( \hat{s}^2 \) from eq. (6).

\[\text{\textsuperscript{2}}\]This is the content of the theorem \[5\] stating that in the DR scheme there are no threshold corrections when gauge bosons are integrated out.
The fine structure constant $\alpha_{EM}^{OS}$ is obtained from the Thomson scattering in pure QED, which is the effective theory at the scale of the electron mass. At this scale, all higher dimensional operators which remain after integrating out all heavier particles are totally negligible and so are the corrections to eq. (14).

Eq. (6) is solved iteratively by calculating at each step of iteration the physical $W$ boson mass from eq. (9) and $\hat{\alpha}$ from eq. (14).

In the limit of very heavy sparticles SUSY corrections to self energies of the vector bosons are dominated by large logarithms of the soft SUSY breaking masses. In this limit our approach should give the same results as given in eq. (6) in the LLT approximation, where the logarithms read explicitly:

\[
8\pi^2 \Delta_y = \frac{1}{3} \sum_{i=1}^{3} \log \frac{m_{E_i}}{M_Z} + \frac{1}{6} \sum_{i=1}^{3} \log \frac{m_{L_i}}{M_Z} + \frac{1}{9} \sum_{i=1}^{3} \log \frac{m_{D_i}}{M_Z} + \frac{4}{9} \sum_{i=1}^{3} \log \frac{m_{U_i}}{M_Z} + \frac{1}{18} \sum_{i=1}^{3} \log \frac{m_{Q_i}}{M_Z}
\]

\[
+ \frac{2}{3} \log \frac{|\mu|}{M_Z} + \frac{1}{6} \log \frac{M_{A_0}}{M_Z}
\]

(17)

\[
8\pi^2 \Delta_2 = \frac{1}{6} \sum_{i=1}^{3} \log \frac{m_{E_i}}{M_Z} + \frac{1}{2} \sum_{i=1}^{3} \log \frac{m_{L_i}}{M_Z} + \frac{2}{3} \log \frac{|\mu|}{M_Z} + \frac{1}{6} \log \frac{M_{A_0}}{M_Z} + \frac{4}{3} \log \frac{M_{\tilde{g}^2}}{M_Z}
\]

(18)

$\Delta_{y}^{\Delta}$ replaced by $\alpha_{SM}^{\Delta} \Delta_{y}$ holds for $\hat{c}^2$. Since in the limit of heavy sparticles the physical masses are dominated by large soft SUSY breaking masses we have $\Delta_e = \Delta_y + \Delta_2 + O(\frac{M_{\tilde{g}}}{M_{SUSY}})$ and it is easy to see that indeed $\hat{\alpha}$ calculated from eq. (14) approaches $\hat{\alpha}$ determined from eq. (19). Next, it is important to notice that in the limit considered, by
using the asymptotic form of the SUSY contributions to self energies:

\[
\hat{\Pi}_{WW}(q^2) - \hat{\Pi}_{WW}(0) \sim 4\pi \frac{\hat{\alpha}}{s^2} q^2 \Delta_2,
\]

\[
\hat{\Pi}_{ZZ}(q^2) - \hat{\Pi}_{ZZ}(0) \sim 4\pi \frac{\hat{\alpha}}{s^2 c^2} q^2 (\hat{c}^4 \Delta_2 + \hat{s}^4 \Delta_y),
\]

\[
\hat{\Pi}_{Z\gamma}(q^2) \sim 4\pi \frac{\hat{\alpha}}{s c} q^2 (\hat{c}^2 \Delta_2 - \hat{s}^2 \Delta_y),
\]

\[
\hat{\Pi}_{\gamma\gamma}(q^2) \sim 4\pi \hat{\alpha} q^2 (\Delta_2 + \Delta_y).
\]

(21)

eq(6) can be rewritten in the following form:

\[
\hat{s}^2 = \hat{s}^2_{SM} \frac{1 + 4\pi \hat{\alpha}^2_{SM} \Delta_2}{1 + 4\pi \hat{\alpha}^2_{SM} (\Delta_y + \Delta_2)} \frac{1}{1 - \Delta \hat{r}_{SM} 4\pi \hat{\alpha}^2_{SM} \Delta_2}
\]

(22)

which is the same as eq.(19) (up to higher order terms). Thus, we reproduce correctly the leading logarithmic threshold corrections in the limit of heavy sparticles.

With regard to eqs.(9) and (10) we see that they connect with each other two observables \(M_W\) and \(M_Z\). Therefore, according to the Appelquist–Carazzone decoupling theorem, heavy SUSY particles have no effect on this relation; one can explicitly check this cancellation in the limit of heavy sparticles with the help of eqs.(21) and (22).

Before presenting our results for \(\sin^2 \theta(M_Z)\) (see Sec. 3) we comment on various consistency checks of our calculation. The higher order correction in eq.(19) coming from neglected (non–top exchange) 2–loop diagrams can be estimated as in the SM \cite{17} to be of order \(\delta \hat{s}^2 < 3 \times 10^{-5}\); the uncertainty due to the difference between eqs.(19) and (22) turns out to be \(\leq 1 \times 10^{-4}\) for \(M_{\text{SUSY}} \leq 1.5\) TeV. The \(W^\pm\) mass calculated in the MSSM in \(\overline{MS}\) scheme (eqs.(9) and (10)) and in the on-shell scheme \cite{13} agree to better than \(\mathcal{O}(10)\) MeV for a wide range of sparticle masses. The \(W^\pm\) mass calculated in the MSSM (by using eqs.(9) and (10)) with sparticle masses greater than 1.5 TeV agrees to better than 10 MeV with the \(W^\pm\) mass calculated in the SM (in the \(\overline{MS}\) scheme or in the on–shell scheme) with the appropriate SM Higgs boson mass \(M_{\phi^0}\).

\[3\] Subtraction of \(\hat{\Pi}(0)\) for \(W^\pm\) and \(Z^0\) self energies is required because of the chargino/neutralino contribution which unlike scalars do not decouple in that limit from renormalized self–energies at \(q^2 = 0\); however, \(\hat{\Pi}_{WW}(0)/M_W^2 - \hat{\Pi}_{ZZ}(0)/M_Z^2\) vanishes in the limit of heavy charginos and neutralinos as it should.

\[4\] In the present version, our code used in \cite{13} has been improved to incorporate \(\delta \rho_{higgs}\) as in eq. (13) and the leading top dependent corrections to the Higgs boson masses using the effective potential approach \cite{19} as described in \cite{20}.

8
3. COMPARISON WITH THE LEADING LOGARITHMIC AND FARAGGI-GRINSTEIN APPROXIMATIONS.

We begin the presentation of our results with a brief summary on $\hat{s}_{SM}^2$ ($\sin^2\theta_W(M_Z)$ in the Standard Model) calculated with our code in terms of $G_F$, $M_Z$, $\alpha \equiv \alpha_{EM}^{OS}$, $m_t$ and $M_{\phi^0}$. A comparison with earlier results [11, 17] is a useful check of our calculation and remembering several numerical values provides a convenient reference frame for comparison with the MSSM results. In Table 1 we collect a sample of values for $\hat{s}_{SM}^2$ (in DIMR) and $M_W$:

| $m_t$ | 160 | 160 | 170 | 170 | 180 | 180 |
|-------|-----|-----|-----|-----|-----|-----|
| $M_{\phi^0}$ | 60 | 130 | 60 | 130 | 60 | 130 |
| $\hat{s}_{SM}^2$ | 0.23155 | 0.23195 | 0.23125 | 0.23164 | 0.23093 | 0.23132 |
| $M_W$ | 80.344 | 80.301 | 80.406 | 80.362 | 80.471 | 80.426 |

We reproduce the results of ref. [11] within ±0.00008 accuracy in $\hat{s}_{SM}^2$ for a wide range of $m_t$ and $M_{\phi^0}$ values. Our input parameters (with which the Table 1 has been obtained) are: $M_Z = 91.1888$ GeV, $G_F = 1.166739 \times 10^{-5}$ GeV$^{-2}$, $\alpha \equiv \alpha_{EM}^{OS} = 1/137.0369895$ and $\alpha_3(M_Z) = 0.118$. The running electromagnetic coupling $\hat{\alpha}_{SM} \equiv \alpha_{EM}(M_Z)$ in the $\overline{MS}$ scheme reads:

$$\frac{1}{\hat{\alpha}_{SM}} = 127.87 + \frac{8}{9\pi} \log \frac{m_t}{M_Z}$$

and is calculated from eq.(14) by taking

$$\Delta \hat{\alpha} = 0.0684 - \frac{\alpha}{6\pi} + \frac{7\alpha}{4\pi} \log \frac{M_W}{M_Z} - \frac{8\alpha}{9\pi} \log \frac{m_t}{M_Z}$$

appropriate for the calculation in DIMR (the conversion to DR can always be performed with the help of eq.(1)). The uncertainty in the value of $\hat{\alpha}_{SM}$ related to the hadronic contribution to $\Delta \hat{\alpha}$ ($\delta(\Delta \hat{\alpha}) = \pm 0.0009$ [18]) is a source of uncertainties: $\delta \hat{s}^2 = \pm 0.0003$ and $\delta M_W = \pm 17$ MeV.

Similarly, our values of the physical mass $M_W$ obtained in the Standard Model from eqs.(9) and (10) agree very well with the previous calculation [11, 17] (up to a few MeV). As mentioned earlier, the present calculation in the $\overline{MS}$ scheme also agrees with the calculation in the on shell renormalization scheme: for $90 < m_t < 240$ GeV and $60 < M_{\phi^0} < 1000$ GeV the difference is less than 5 MeV and for $m_t < 210$ GeV and $M_{\phi^0} < 500$ GeV the agreement is typically within 2 MeV.

Provided as in ref. [11] we take $M_Z = 91.17$ GeV, $\delta \rho^{QCD} = 0$, $\delta \rho^{Higgs} = 2\pi^2 - 19$, and adjust the hadronic vacuum polarization to their value.
For later use it is very convenient to have simple interpolating formulae for \( \hat{s}_{SM}^2 \) and \( M_W \) in the SM. For the central values of the input parameters we find that the following formulae:

\[
\hat{s}_{SM}^2 = 0.23166 + 5.4 \times 10^{-6} h - 2.4 \times 10^{-8} h^2 \\
- 3.03 \times 10^{-5} t - 8.4 \times 10^{-8} t^2 
\]  \( (25) \)

\[
(M_W)^{SM} = 80.347 - 6.4 \times 10^{-4} h + 2.5 \times 10^{-6} h^2 \\
+ 6.2 \times 10^{-3} t + 1.25 \times 10^{-5} t^2 ,
\]  \( (26) \)

where \( h \equiv M_{\phi^0} - 100 \) and \( t \equiv m_t - 165 \) (both masses in GeVs), reproduce the results for \( \hat{s}_{SM}^2 \) and \( M_W \) in the SM with the accuracy 0.00001 and 2 MeV respectively (for the range \( 130 < m_t < 200 \) GeV and \( 60 < M_{\phi^0} < 140 \) GeV which is relevant for comparison with MSSM). These formulae are therefore used as input in eq.(5) in our further study whenever we refer to the values of \( \hat{s}_{SM}^2 \) and \( M_W \) in the Standard Model.

Turning now to our results for \( \hat{s}^2 \) in the MSSM, there are two points to be discussed first: the departure from the SM result (with the same values of the input parameters \( G_F, M_Z, \alpha_{EM}^{OS}, m_t \) and \( M_{h^0} \)) as a function of the SUSY particle masses and comparison with the LLT approximation\([6, 21]\). Both are illustrated in Fig.1 a-d where we present the results of our complete calculation and of the LLT approximation for a scan over the sparticle masses in a wide range of values given in the Appendix. In our scan we include only those values of the parameters which are consistent with experimental constraints on the sparticle masses, the mass of the lighter MSSM scalar Higgs boson\([20]\) and give \( M_W \) within \( 2\sigma \) of the measured value\([22]\). In Fig.1, plots for \( m_t = 180 \) GeV contain smaller number of points because most of the cases with light sparticles are eliminated by the mentioned above experimental cuts\([13, 20]\). All points for \( m_t = 160 \) (180) GeV in Fig.1 correspond to the values \( \hat{s}_{SM}^2 = 0.23155 - 0.23195 \) \( (0.23093 - 0.23132) \), with the ranges reflecting the (weak) dependence of the \( \hat{s}_{SM}^2 \) on \( M_{\phi^0} \) in the range \( 60 - 130 \) GeV (see Table 1)\([6]\) and show the dependence of the running couplings in the MSSM on SUSY thresholds.

Comparing the complete calculation with the LLT approximation we see that the difference in \( \hat{s}^2 \) is up to \( \mathcal{O}(0.002) \) for light supersymmetric spectrum and therefore very relevant for the discussion of unification of couplings. There is also a nonnegligible dependence of \( \hat{\alpha} \) on the contribution from the electroweak symmetry breaking to the charged sparticle masses (see Figs.2-4).

The origin of the differences between the full and approximate calculations can be understood from formulae quoted in the Appendix. Qualitatively, the

\[\hat{s}_{SM}^2 = 0.2326 - 1.03 \times \times 10^{-7}(m_t^2 - 138^2) \]

This dependence is neglected in the frequently used fit \( \hat{s}_{SM}^2 = 0.2326 - 1.03 \times 10^{-7}(m_t^2 - 138^2) \)\([6]\).
largest deviations from the LLT approximation come from the contribution of sfermion and, to the smaller extent, chargino/neutralino sectors. This is further illustrated in Figs.2-4 where we show the dependence of our results on the chosen sparticle masses keeping other SUSY particles heavy ($\mathcal{O}(1 \text{ TeV})$).

Finally we discuss the approximate method proposed by Faraggi and Grinstein [10] of translating the values of the couplings in the SM into the values of the couplings in the MSSM, without performing the complete calculation in the MSSM, as in Sec. 2.

In this approach one is instructed to choose a set of observables $O_i$ which can be calculated in the SM and MSSM as

$$O_i = F_i(g_{k}^{SM})$$ (27)

and

$$O_i = \tilde{F}_i(g_{k}^{MSSM}, e_n)$$ (28)

respectively, where $e_n$ stand for SUSY parameters not present in the SM. Writing next $g_{k}^{MSSM} = g_{k}^{SM} + \delta g_{k}$,

$$F_i(g_{k}^{MSSM}, e_n) = F_i(g_{k}^{MSSM}) + \Delta F_i(g_{k}^{MSSM}, e_n)$$

and equating the RHS of the above two equations one arrives (assuming that $\delta g_{k}$ are small enough) at the set of equations

$$\sum_l \delta g_l \frac{\partial F_i(g_{k}^{SM})}{\partial g_l^{SM}} + \Delta F_i(g_{k}^{SM}, e_n) = 0$$ (29)

which allow for the determination of the MSSM couplings provided the SM couplings $g_{k}^{SM}$ have been determined from the chosen set of observables $O_i$.

This prescription can be easily applied to our set of observables: $\alpha_{EM}^O$, $G_F$ and $M_Z$ leading to the following approximate formulae (which can be also derived directly from eqs.(6-10) without any reference to the FG method):

$$\frac{\delta g_2^2}{g_2^{SM}} = \frac{1}{c_2^{SM} - s_2^{SM}} \left[ 4\pi\hat{\alpha}^{SM} s_2^{SM} \Delta_e + c_2^{SM} R e \left( \frac{\hat{\Pi}_{WW}(0)}{M_W^2} - \frac{\hat{\Pi}_{ZZ}(M_Z^2)}{M_Z^2} \right) \right]^{SUSY}$$

$$\frac{\delta g_y^2}{g_y^{SM}} = \frac{1}{s_2^{SM} - c_2^{SM}} \left[ 4\pi\hat{\alpha}^{SM} c_2^{SM} \Delta_e + s_2^{SM} R e \left( \frac{\hat{\Pi}_{WW}(0)}{M_W^2} - \frac{\hat{\Pi}_{ZZ}(M_Z^2)}{M_Z^2} \right) \right]^{SUSY}$$ (30)

or, equivalently

$$\frac{\delta \hat{\alpha}}{\hat{\alpha}^{SM}} = -4\pi\hat{\alpha}^{SM} \Delta_e$$ (31)
\[
\delta \hat{s}_\text{SM}^2 = \frac{\hat{c}_\text{SM}^2}{\hat{s}_\text{SM}^2 - \hat{c}_\text{SM}^2} \left[ 4\pi \hat{\alpha}_\text{SM} \Delta_e + \text{Re} \left( \frac{\hat{\Pi}_{WW}(0)}{M_W^2} - \frac{\hat{\Pi}_{ZZ}(M_Z^2)}{M_Z^2} \right) \right] \tag{32}
\]

Similarly, one can easily find the formula for correction to the $W^\pm$ mass ($\delta M_W^2 \equiv (M_W^2)_{\text{MSSM}} - (M_W^2)_{\text{SM}}$):

\[
\frac{\delta M_W^2}{(M_W^2)_{\text{SM}}} = -\frac{\hat{s}_\text{SM}^2}{\hat{c}_\text{SM}^2} \left( \frac{\delta \hat{s}_\text{SM}^2}{\hat{s}_\text{SM}^2} \right) + \text{Re} \left( \frac{\hat{\Pi}_{WW}(M_W^2)}{M_W^2} - \frac{\hat{\Pi}_{ZZ}(M_Z^2)}{M_Z^2} \right) \tag{33}
\]

In the above formulae, $(...)_{\text{SUSY}}$ stands for superparticle contributions to the vector boson self energies and includes also the difference between the Higgs sector contributions to these self energies in MSSM and SM. The formulae for $\hat{\Pi}_{WW}(0)_{\text{SUSY}}$ and $\hat{\Pi}_{ZZ}(M_Z^2)_{\text{SUSY}}$ are collected for the reader’s convenience in the Appendix.

Using the interpolating formulae (25, 26) for $\hat{s}_\text{SM}^2$ and $(M_W)_{\text{SM}}$ we can check that equations (31, 32) give an excellent approximation to our complete results for the couplings. This is illustrated in Figs. 2-4. The approximation for $(M_W)_{\text{MSSM}}$ given by eq. (33) is typically within 20 MeV as compared with the full calculation in the $\overline{\text{MS}}$ scheme.

One should stress that the FG approximation is consistent only if the SM couplings are determined from the same set of observables $\mathcal{O}_i$ as used to calculate the couplings in the MSSM: $g_k^\text{MSSM} = g_k^{\text{SM}} + \delta g_k$. This is because the corrections $\delta g_k$ are process dependent and correlated with the values of $g_k^{\text{SM}}$ extracted from the same set of observables: since we are concerned with the situation when the SM (without higher dimension non–renormalizable operators) is not a sufficiently accurate effective theory at $M_Z$ scale, the values of the couplings $g_k^{\text{SM}}$ extracted from different experiments (a different set of observables $\mathcal{O}_i$) are also process dependent and $g_k^{\text{SM}}(\mathcal{O}_i) - g_k^{\text{SM}}(\mathcal{O}_j)$ can be of the order of $\delta g_k$ itself. It is therefore inconsistent to apply the FG approach to $\hat{s}_\text{SM}^2$ obtained from an overall fit to the data in the SM; such a fit gives meaningless numbers if non-logarithmic SUSY corrections are important. It is also inconsistent to apply FG method to correct the SM couplings obtained from a set $\mathcal{O}_i$ by using the equations for $\delta g_k$ derived by referring to a set $\mathcal{O}_j$. This is shown in Figs. 2-4 where the results obtained from the two variants of the FG formulae [10] derived by referring to the scattering processes and applied to the SM couplings obtained from our set of observables $M_Z$, $G_F$ and $\alpha_{\text{EM}}^{\text{Qs}}$ are marked by dotted lines.

It is easy to check, using formulae (21) that all the approximations give the same asymptotic behaviour of the corrections to $\hat{\alpha}$ and $\hat{s}^2$ when the SUSY particles are heavy. Nevertheless, as is clear from Figs. 2-4, for light SUSY particles they give very different results from the complete calculation.

4. UNIFICATION OF COUPLINGS.
Our calculation of the running Weinberg angle in the MSSM can be used to investigate unification of the gauge couplings, with the supersymmetric threshold effects taken exactly into account at the 1–loop level. The main uncertainty in our value of \(\sin^2 \theta_W(M_Z)\) (for fixed \(m_t, M_{h^0}\) and superpartner masses) follows from the uncertainty in \(\hat{\alpha}\) which gives \(\delta \hat{s}^2 = \pm 3 \times 10^{-4}\) (theoretical uncertainty of the calculation like higher order corrections etc. is smaller, \(\mathcal{O}(1 \times 10^{-4})\)). This can be compared with the precision \(\pm 6 \times 10^{-4}\) of \(\sin^2 \theta_W(M_Z)\) in the SM obtained from the global fits to the LEP and the low energy data \[23, 7\]. Note, however, that as explained earlier, from the point of view of the MSSM the value of \(\sin^2 \theta_W(M_Z)\) in the SM obtained from global fits is meaningless as soon as we want to go beyond the LLT approximation. To increase the statistical significance of the value of \(\hat{s}^2\) in the MSSM, i.e. to obtain it from a global fit to more observables than just \(G_F, M_Z\) and \(\alpha_{EM}^{OS}\) one has to perform similar calculation to ours for other observables and to have an independent fit for each chosen set of values for \(m_t\) and \(M_{h^0}\) and for superpartner masses. In particular it is important to remember about the correlation between the value of \(\sin^2 \theta_W(M_Z)\) and the values of the top quark and Higgs boson masses which is already present in the Standard Model results (see e.g. ref. \[17\] and eq. (25)).

Given the high accuracy of our approximate formulae \[31, 32, 33\] which translate \(\hat{s}_{SM}^2\) into \(\hat{s}^2\) in the MSSM (for any chosen superpartner spectrum) and hoping for similar accuracy of the formula connecting \(\alpha_3^{SM}\) with \(\hat{\alpha}_3\) in the MSSM \[10\]:

\[
\delta \hat{\alpha}_3 \over \alpha_3^{SM} = -R e \left( \frac{\hat{\Pi}_{gg}(M_Z)^{SUSY}}{M_Z^2} \right) \tag{34}
\]

where \(\hat{\Pi}_{gg}^{SUSY}\) is the SUSY contribution to the gluon self energy (provided we apply this formula to \(\hat{\alpha}_3^{SM}\) extracted from processes where the gluon four-momentum \(q^2\) is of order of \(M_Z^2\), it is convenient to use the values of \(\hat{s}_{SM}^2\) and \(\hat{\alpha}_3^{SM}\) as a label for the set of values of \(\hat{s}^2\) and \(\hat{\alpha}_3\) in the MSSM which correspond to the same values of \(\hat{s}_{SM}^2\) and \(\hat{\alpha}_{SM}^3\) in the approximation \[31, 32, 33, 34\]. The values used in the following are as in Table 1:

\[
m_t = 160 \text{ GeV} \quad \hat{s}_{SM}^2 = 0.23155 - 0.23195
\]
\[
m_t = 180 \text{ GeV} \quad \hat{s}_{SM}^2 = 0.23093 - 0.23132
\]

For the strong coupling constant \(\hat{\alpha}_3^{SM}\) we shall be using values 0.115, 0.125 and 0.13, for the purpose of illustration of various effects. Thus, in this paper, by “experimental data” we mean the above values of \(\hat{s}_{SM}^2\), \(\hat{\alpha}_{SM}^3\) and the corresponding sets of values of \(\hat{s}^2\), \(\hat{\alpha}_3\) in the MSSM (now dependent on SUSY parameters). Exactly analogous remarks apply to \(\hat{s}^3\) and \(\hat{\alpha}_3\). The “experimental data” for the MSSM will be compared with the
relation between $\hat{s}^2$, $\hat{\alpha}$ and $\hat{\alpha}_3$ predicted after imposing the unification conditions in the MSSM. Note that since we work directly in the MSSM, the predictions which follow from unification can be obtained, as a relation between $\hat{s}^2$, $\hat{\alpha}_3$ and $\hat{\alpha}$ without any reference to the “experimental data”.

The final point we have to specify before the actual discussion of unification is the choice of the supersymmetric spectrum. The very general scan over the sparticle masses performed in Section 3 is not very illuminating from the point of view of unification: by stretching the parameters and correlating them in some specific way one has very little constraint on unification scenarios. It is much more interesting to study unification of the gauge couplings under the additional assumption that the supersymmetric spectrum is obtained from the requirement of radiative electroweak symmetry breaking. In this paper we use the spectra of ref. [24] obtained with universal boundary conditions at the GUT scale and with one-loop corrections to the effective potential included. The spectra of ref. [24] correspond to a complete scanning over the available parameter space with cut–off on the physical squark masses $M_\tilde{q} < 2$ TeV. The requirement of radiative breaking results in very strong correlation between masses of different sparticles.

From the point of view of the gauge couplings unification a useful parameter which characterizes the spectrum is $T_{SUSY}$ defined so that in the LLT approximation (neglecting GUT threshold corrections) the SUSY threshold correction to the value of $\hat{\alpha}_{SM}^3$ predicted from the gauge coupling unification reads [7, 8]:

$$\Delta(1/\hat{\alpha}_3) = \frac{1}{2\pi} \frac{19}{14} \log \frac{T_{SUSY}}{M_Z}$$

(35)

Assuming that soft SUSY breaking mass terms are the same for all generations we get (see the Appendix for notation):

$$T_{SUSY} = \mu \left( \frac{M_\mu^2}{M_\mu^2} \right)^{\frac{11}{20}} \left( \frac{m_L^2}{m_Q^2} \right)^{\frac{1}{20}} \left( \frac{M_\mu^2}{\mu^2} \right)^{\frac{1}{38}} \left( \frac{M_\mu^2}{\mu^2} \right)^{\frac{2}{9}}$$

(36)

One of the consequences of the correlations in the sparticle spectra obtained in the model with radiative electroweak breaking is that, generally, the effective parameter $T_{SUSY} < 300$ GeV for $M_\tilde{q} < 2$ TeV, and it is strongly correlated with the supersymmetric parameter $\mu$ (only in the fixed point regime, $T_{SUSY}$ extends to 600 GeV for $M_\tilde{q} < 2$ TeV and this is due to large values of $\mu$, which in this case necessary for proper radiative electroweak symmetry breaking) [8, 25]. This is shown in Fig.5 for several values of $m_t$ and $\tan\beta$. It is important to realize that, in the context

---

7 The mass $m_t$ is the pole mass which is related to the running mass as follows:

$$m_t = m_t(m_t) \left( 1 + \frac{4\alpha_3}{3\pi} + 11.4 \left( \frac{\alpha_3}{\pi} \right)^2 \right)$$
of radiative electroweak symmetry breaking with universal boundary conditions at the GUT scale, the often referred to value of $T_{SUSY} = 1$ TeV would correspond to very heavy squark spectra.

We are now equipped to study unification of the gauge couplings. First, we address the question of unification with no GUT threshold and higher dimension operator GUT corrections.

This requirement, as discussed before, determines a relation between $\hat{s}^2$, $\hat{\alpha}_3$ and $\hat{\alpha}$ (all three are MSSM couplings). Fortunately the “experimental value” of $\hat{\alpha}$ changes only weakly for the spectra of ref. [24]: $(\hat{\alpha}_{\min(\max)})^{-1} = 131.8$ (129.0) for heavy (light) charged sparticles. Therefore we present the unification prediction (obtained from 2-loop running with the MSSM RGEs from $M_Z$ to the GUT scale) as a narrow band in the $\hat{s}^2 - \hat{\alpha}_3$ plane obtained for $\hat{\alpha}$ in its “experimental range”. This band can be compared with “experimental data” for $\hat{s}^2$ and $\hat{\alpha}_3$ obtained with the use of the chosen SUSY spectra, bearing in mind that $(\hat{\alpha}_{\min(\max)})^{-1}$ is more appropriate for heavy (light) spectra. Labelling the “experimental data” by the corresponding Standard Model values of $\hat{s}^2_{SM}$ and $\hat{\alpha}_3^{SM}$ we see in Fig. 6 a clear trend: unification with no GUT threshold corrections occurs only for sufficiently large $\hat{\alpha}_3^{SM}$ and sufficiently heavy supersymmetric spectrum (the actual numbers depend on $m_t$ and $\tan \beta$ values). The dependence on $m_t$ has two sources: firstly it enters into the 1-loop correction to $\hat{s}^2$ and also to $\hat{\alpha}_3$ and $\hat{\alpha}$ as already in the SM (see the values (25)). Secondly the features of the SUSY spectrum depend on $m_t$ and $\tan \beta$ because of the tight constraints imposed by universality of the soft terms and by radiative symmetry breaking. In particular, as discussed above and seen in Fig. 5 for the same cut – off for $M_\tilde{q}$ the maximal value of $T_{SUSY}$ is $m_t$ and $\tan \beta$ dependent. In Fig. 7 we plot the predicted $\hat{\alpha}_3^{SM}$ in the approximation (31,32,34) (from unification with no GUT thresholds) as a function of $T_{SUSY}$ for several values of $m_t$ and $\tan \beta$ and with $\hat{s}^2_{SM}$ and $\hat{\alpha}_{SM}$ in their experimental ranges specified earlier. The uncertainty $\delta \hat{s}^2 = \pm 3 \times 10^{-4}$ (correlated with uncertainty in $\hat{\alpha}$, eq. (17)) translates itself into $\delta \hat{\alpha}_3^{SM} = \pm 1.5 \times 10^{-3}$ in this plot. The dependence of $\hat{s}^2_{SM}$ on $m_t$ (eq. (25)) is of course reflected in the predicted value of $\hat{\alpha}_3^{SM}$. Changing $m_t$ from 160 GeV to 160 $\pm$ 20 GeV (and leaving the sparticle spectrum unaffected) results in a shift $\delta \hat{\alpha}_3^{SM} = \pm (2 - 2.5) \times 10^{-3}$.

In the LLT approximation and neglecting the dependence of $\hat{s}^2_{SM}$ on the Higgs boson mass, the predicted $\hat{\alpha}_3^{SM}$ is a function of $T_{SUSY}$ only. The scatter plot obtained in the LLT approximation reflects the dependence of the input $\hat{s}^2_{SM}$ on the Higgs boson mass which is varied from 60 to 130 GeV. In the full calculation there is stronger departure from a universal

In this paper, as in ref. [24], we neglect other contributions to the pole mass.

The coupling constant usually quoted in the literature which we call $\hat{\alpha}_3^{SM}$ throughout the paper is actually $\hat{\alpha}_3^{QCD+QED}$ in the theory with the top quark decoupled.
dependence of $\hat{\alpha}_3^{SM}$ on $T_{SUSY}$ which is due to the non-logarithmic threshold corrections. Going beyond the LLT approximation results in larger values of $\hat{\alpha}_3^{SM}$ for small $T_{SUSY}$. Moreover, in the whole range of $T_{SUSY}$ reachable with our spectra, the corrections to the LLT approximation are non-negligible. This happens because the spectra always contain some light sparticle(s) which is responsible for the difference. In the absence of the GUT threshold corrections the unification scale can be unambiguously defined as the point of crossover of the couplings. It depends only weakly on the SUSY spectra and varies between $1.3 \times 10^{16}$ and $3 \times 10^{16}$.

Next we study unification with GUT thresholds included. We restrict ourselves to the minimal supersymmetric $SU(5)$ with three different mass scales: $M_{H_3}$, $M_{\Sigma}$ and $M_V$ corresponding to the Higgs triplet, Higgs bosons originating from the 24 representation and heavy vector bosons (leptoquarks) respectively. We have then [21, 26]:

$$\frac{1}{\alpha_3(Q)} = \frac{1}{\alpha_5(Q)} + \frac{1}{2\pi} \left( 4 \log \frac{M_V}{Q} - 3 \log \frac{M_{\Sigma}}{Q} - \log \frac{M_{H_3}}{Q} \right)$$

$$\frac{1}{\alpha_2(Q)} = \frac{1}{\alpha_5(Q)} + \frac{1}{2\pi} \left( 6 \log \frac{M_V}{Q} - 2 \log \frac{M_{\Sigma}}{Q} \right)$$

$$\frac{1}{\alpha_1(Q)} = \frac{1}{\alpha_5(Q)} + \frac{1}{2\pi} \left( 10 \log \frac{M_V}{Q} - \frac{2}{5} \log \frac{M_{H_3}}{Q} \right)$$

(37)

for any scale $Q$ close to $M_{V,\Sigma,H_3}$ i.e. a scale at which the higher order (logarithmic) threshold corrections to the RHS can be neglected. It follows from eqs. (37) that $M_{H_3}$, and a combination of $M_{\Sigma}$ and $M_V$ are uniquely related to the values of the coupling constants:

$$\left( \frac{3}{\alpha_2} - \frac{2}{\alpha_3} - \frac{1}{\alpha_1} \right)(Q) = \frac{6}{5\pi} \log \frac{M_{H_3}}{Q}$$

$$\left( \frac{5}{\alpha_1} - \frac{3}{\alpha_2} - \frac{2}{\alpha_3} \right)(Q) = \frac{6}{\pi} \log \frac{M_V^2 M_{\Sigma}}{Q^3}$$

(38)

and $\alpha_i(Q)$ are related to $\hat{\alpha}_i \equiv \alpha_i(M_Z)$ by the RGE in the MSSM.

The curves of fixed $M_{H_3}$ in the $(\hat{s}_2^2, \hat{s}_3)$ plane are given by eq. (38) (for fixed $\hat{\alpha}_{EM}$). They are shown in Fig. 6 for $M_{H_3} = 5 \times 10^{15}$, $2 \times 10^{16}$, $1 \times 10^{17}$ (for $\hat{\alpha}_{EM} = 1/131.8$). Thus, each point in the $(\hat{s}_2^2, \hat{s}_3)$ plane corresponds to fixed values of $M_{H_3}$ and of the product $M_V^2 M_{\Sigma}$ (modulo the uncertainty in $\hat{\alpha}_{EM}$). Running with 2-loop equations from $M_Z$ to Q such that LHS of the second of eqs. (38) vanishes we obtain $(M_V^2 M_{\Sigma})^{1/3} (= Q)$ and $M_{H_3}$. With GUT threshold corrections, unification of couplings can be always achieved at the expense of large enough splitting.
between $M_{H_3}$ and the combination $(M_V^2 M_\Sigma)^{1/3}$. There are, however, certain natural requirements which constrain these three masses and their splitting. First of all, $M_{H_3}$, $M_\Sigma$ and $M_V$ should be smaller than the Planck mass. Other requirements follow from the superpotential which breaks $SU(5)$ and the condition that its couplings remain perturbative up to the Planck scale. This gives \[21\]:

$$M_{H_3} < 2 M_V, \quad M_\Sigma < 1.8 M_V$$

(39)

Another requirement is more technical: the discussion of unification based on the 1–loop threshold corrections eqs. \[37\] is meaningful only if there exist a common scale $Q$ such that all logarithms $\log(M_{H_3}/Q)$, $\log(M_\Sigma/Q)$ and $\log(M_V/Q)$, are small enough to make higher order corrections (next to leading logarithms) negligible. Those arguments still leave a lot of freedom for GUT corrections. It is interesting to know how much, say, the predicted $\hat{\alpha}^{SM}_3$ is modified by the splitting $(M_V^2 M_\Sigma)^{1/3}/M_{H_3} >> 1$. Using the formulae \[38\] with 1-loop running to $Q = M_Z$ we obtain

$$\delta_{GUT} \left( \frac{1}{\hat{\alpha}_3} \right) = - \frac{3}{14\pi} \log \frac{M_{H_3}^2}{M_V^2 M_\Sigma}$$

(40)

which yields $\delta_{GUT}\hat{\alpha}^{SM}_3 = -0.01 (-0.02)$ for $(M_V^2 M_\Sigma)^{1/3}/M_{H_3} = 30 (1000)$ (which is not strongly modified by 2–loop effects) respectively. We demonstrate the effect of GUT scale threshold correction in Fig. 6 where we show the bounds in the $(s^2, \hat{\alpha}_3)$ plane in the MSSM which are consistent with unification with

$$\max(M_{H_3}, M_\Sigma, M_V) < \min(M_{H_3}, M_\Sigma, M_V) < 30 \text{ or } 1000$$

(41)

respectively. The only (weak) dependence of those bounds on the supersymmetric spectrum is through the value of $\hat{\alpha}_{EM}$. The asymmetry of the bounds with respects to the unification curve with no GUT threshold corrections, seen in Fig.6, is due to condition \[39\]. Below this curve $M_V = M_\Sigma = 30 \text{ (1000)}$ $M_{H_3}$ and above $M_{H_3} = 2M_V = 30 \text{ (1000)}$ $M_\Sigma$ which results in a much smaller splitting between $M_{H_3}$ and $(M_V^2 M_\Sigma)^{1/3}$.

As stressed in ref. \[26\], the uncertainty in the unification prediction for $\hat{\alpha}_3$ due to the unknown GUT thresholds makes direct tests of the unification idea impossible, even if we knew the low energy superparticle spectrum and no matter how precise the experimental data for the couplings at $M_Z$ are. Only the reversed programme is realizable: the more precise the experimental data for the couplings are the more accurate predictions we can obtain for $M_{H_3}$ and for the splitting between $M_{H_3}$ and $(M_V^2 M_\Sigma)^{1/3}$. Comparing the bounds with our “experimental data” it is clear that for light SUSY spectrum GUT threshold corrections, corresponding to large splitting $M_V(\Sigma)/M_{H_3} \sim$
30 are needed for unification with $\hat{\alpha}_3^{SM} = 0.125$. If one takes $\hat{\alpha}_3^{SM} = 0.115$, close to its experimental lower bound, the same GUT splitting is needed even for $T_{SU3} = 300$ GeV and it has to reach $M_{V(\Sigma)}/M_{H_3} \sim 1000$ for light spectra. For the same value of $\hat{\alpha}_3^{SM}$, the non-logarithmic corrections calculated in this paper tend to increase GUT corrections needed to achieve unification.

It is also very interesting to show the predictions for $M_{H_3}$ which follow from our “experimental data” points. The Higgs triplet mass is directly related to the proton lifetime [27]. There have been claims in the literature that even $M_{H_3} \sim 10^{17}$ GeV can be reconciled with the gauge coupling unification and at the same time it is large enough to make the proton lifetime consistent with experimental limits [28] also for large values of $\tan \beta$ [24]. Our predictions for $M_{H_3}$ are plotted in Fig. 8 as a function of $T_{SU3}$ for $\hat{\alpha}_3^{SM} = 0.125$. The value of $M_{H_3}$ is rather sensitive to changes of the MSSM couplings at $M_Z$. For example, a change $\delta s^2 = 3 \times 10^{-4}$ increases $M_{H_3}$ by a factor of $\sim 1.5$ whereas a change $\delta \hat{\alpha}_3 = -2 \times 10^{-3}$ decreases it by a factor $\sim 2.2$.

We stress again that those results depend on the SUSY spectrum which is taken from the minimal supergravity model (radiative electroweak symmetry breaking with universal boundary conditions at the GUT scale) and with cut–off on the physical squark masses, $M_{\tilde{q}} < 2$ TeV. We obtain an upper bound on $M_{H_3}$ which is lower than the one in ref. [21]. For the same value of $\mu = 1$ TeV, $\hat{\alpha}_3^{SM} = 0.125$, $m_t = 160$ GeV and including the uncertainty $\delta s^2 = \pm 3 \times 10^{-4}$ we obtain $M_{H_3}^{max} = 3 \times 10^{16}$ GeV. The discrepancy with the upper bound given in ref. [21] is due to a different $s^2_{SM}$ value used in that paper. Our bounds on $M_{H_3}$ are in agreement with those given in ref. [23]. We also observe that our non–logarithmic SUSY threshold corrections decrease the predicted value of $M_{H_3}$.

Finally we would like to discuss the impact on the unification of gauge couplings of the condition that quark Yukawa couplings remain in the perturbative regime. In particular we impose

$$Y_t^2 < 4\pi$$

This condition is relevant only for a heavy top quark and small $\tan \beta$ (i.e. close to the quasi IR–fixed point [3, 8]). As it is clear from the structure of the RGE for the $Y_t$, for given values of $m_t$ and $\tan \beta$, the condition (42) gives a lower bound on $\hat{\alpha}_3^{MSSM}$ [9]. Superimposed on the unification plots in Fig. 6, it favours light superpartner spectrum: for heavy spectrum the

\[\text{The bound depends (weakly) on } \sin^2 \theta_W \text{ because a change in } \sin^2 \theta_W \text{ induces a change in the final point of the renormalization evolution where } Y_{top} \text{ grows very quickly. In this discussion we neglect the contribution of SUSY and GUT threshold corrections to the Yukawa couplings and in this sense our conclusions here should be taken as only qualitative.} \]
values of $\hat{\alpha}_3^{MSSM}$ corresponding to reasonable values of $\hat{\alpha}_3^{SM}$ (say, below 0.13) are too small to be consistent with perturbative top quark Yukawa coupling. It is interesting to observe that, for small $\tan \beta$, large $m_t$, and $\hat{\alpha}_3^{SM} < 0.125(0.115)$, perturbativity of $Y_t$ puts strong upper bounds on $T_{SUSY}$: $T_{SUSY} < 200(600) \text{ GeV}$ for $m_t = 160(180) \text{ GeV}$ and $\tan \beta = 1.25(2.0)$ respectively.

5. CONCLUSIONS.

In this paper we have calculated the running Weinberg angle at $M_Z$ directly in the MSSM in terms of $G_F$, $M_Z$, $\alpha_{EM}^{OS}$, $m_t$ and supersymmetric parameters. The accuracy of this calculation is similar to the analogous calculation in the Standard Model. Using the language of supersymmetric threshold corrections to the Standard Model our calculation is equivalent to a complete calculation of those corrections at the one-loop level. A detailed comparison with LLT approximation is presented. Also highly accurate approximate formulae are given which translate the SM couplings into the MSSM couplings.

Those results are subsequently used to study the gauge coupling unification in the minimal supersymmetric $SU(5)$ model, with the superpartner spectra taken from the minimal supergravity model (i.e. with radiative electroweak symmetry breaking and universal boundary conditions for soft supersymmetry breaking parameters at the unification scale).

For unification with no GUT thresholds, the value of the strong coupling constant is calculated as a function of supersymmetric spectra. The non-logarithmic threshold corrections increase the predicted value of $\hat{\alpha}_3^{SM}$ for light spectra. For the spectra obtained in the minimal supergravity model with radiative electroweak breaking, with cut–off $M_0 < 2 \text{ TeV}$, one generically obtains $\hat{\alpha}_3^{SM}$ above the experimental range $\hat{\alpha}_3^{SM} = 0.118 \pm 0.007$ (with the exception of the low $\tan \beta$ values and $m_t$ close to its quasi–IR fixed point). In the presence of GUT thresholds, a direct precise test of the unification idea in the minimal $SU(5)$ is no longer possible [26]. Only the reversed programme is realizable: with precise information on the couplings at $M_Z$ we can calculate the Higgs triplet mass and the combination $(M_H^2 M_\Sigma)^{1/3}$ as a function of the supersymmetric spectrum. For the same values of the couplings at $M_Z$, the non-logarithmic corrections calculated in this paper decrease the value of $M_{H_3}$ and increase the splitting between $M_{H_3}$ and $(M_H^2 M_\Sigma)^{1/3}$ needed to achieve unification. Information from the proton life time (sensitive to $M_{H_3}$) and from $b - \tau$ Yukawa coupling unification (sensitive to the splitting) gives additional constraints on the model. It will be interesting to use the results of the present paper for improving the precision of those constraints.
Acknowledgments. P.H. Ch. would like to thank Professor W.F.L. Hollik for his kind hospitality extended to him during his stay at the University of Karlsruhe and many discussions. His work was partly supported by European Union under contract CHRX-CT92-0004. Z.P. Is grateful to M. Olechowski for providing him with programs computing MSSM spectra and many discussions. S.P. is grateful to the Aspen Center for Physics for its hospitality during the completion of this work.
APPENDIX.

In this Appendix we make explicit our convention and notation for particle masses. In order to avoid confusion with signs, often present in the literature, we will be explicit here. We follow closely the convention (but not the notation) of [30].

From the superpotential of the model

\[ W = \epsilon_{ij} Y_l \hat{H}_1^i \hat{L}^j + \epsilon_{ij} Y_d \hat{H}_1^i \hat{Q}^j \hat{D}^+_R + \epsilon_{ij} Y_u \hat{H}_2^i \hat{Q}^j \hat{U}^-_R + \epsilon_{ij} \mu \hat{H}_1^i \hat{H}_2^j \]  

(43)

(where \( Y_a \) are the Yukawa couplings and \( \epsilon_{12} = -\epsilon_{21} = -1 \)) and the relevant soft SUSY breaking part of the lagrangian

\[ \mathcal{L}_{soft} = -m_{H_1}^2 |H_1|^2 - m_{H_2}^2 |H_2|^2 + m_\epsilon^2 \epsilon_{ij} (H_1^i H_2^j + c.c.) - m_\nu^2 |\nu_L|^2 - m_\nu^2 |\nu_R|^2 - m_D^2 |D_R^\tau|^2 + \epsilon_{ij} Y_t A_t \hat{H}_1^i \hat{Q}^j \hat{D}^+_R + \epsilon_{ij} Y_u A_u \hat{H}_1^i \hat{Q}^j \hat{U}^-_R \]  

(44)

where \( H_1, H_2 \) and \( L, Q \) are the \( SU(2) \) doublets and adding contributions of the D–terms one gets the mass matrices for up and down squarks as well as charged sleptons (because we neglect the intergenerational mixing we write them for one generation only; therefore \( U (u) \) represents generically up–type squarks (quarks) and the same is understood for dow–type ones and sleptons (leptons)):

\[ M_U^2 = \begin{pmatrix} M_{UL}^2 & M_{ULR}^2 \\ M_{ULR}^2 & M_{UR}^2 \end{pmatrix} \]  

(45)

with

\[ M_{UL}^2 = m_Q^2 + m_u^2 + \frac{1}{6} t(M_Z^2 - 4M_W^2) \]

\[ M_{UR}^2 = m_Q^2 + m_u^2 - \frac{2}{3} t(M_Z^2 - M_W^2) \]

\[ M_{ULR}^2 = -m_u (A_u + \mu \cot \beta) \]

\[ M_{DL}^2 = m_Q^2 + m_u^2 - \frac{1}{6} t(M_Z^2 + 2M_W^2) \]

\[ M_{DR}^2 = m_Q^2 + m_u^2 + \frac{1}{3} t(M_Z^2 - M_W^2) \]

\[ M_{DLR}^2 = -m_d (A_d + \mu \tan \beta) \]

\[ M_{EL}^2 = m_L^2 + m_e^2 - \frac{1}{2} t(M_Z^2 - 2M_W^2) \]

\[ M_{ER}^2 = m_L^2 + m_e^2 + t(M_Z^2 - M_W^2) \]

\[ M_{ELR}^2 = -m_e (A_e + \mu \tan \beta) \]  

(48)
Here \( t \equiv (\tan^2 \beta - 1)/(\tan^2 \beta + 1) \) and \( m_{u,d,e} \) stand for ordinary fermion masses. These matrices are diagonalized by the appropriate rotations. The sneutrinos have masses given by:

\[
M_{\tilde{\nu}}^2 = m_L^2 + m_e^2 + \frac{t}{2} M_Z^2
\]  

(49)

Part of the lagrangian relevant for the chargino/neutralino sector reads:

\[
\mathcal{L} = \frac{1}{2} M_{g2} \psi^a \psi^a + \frac{1}{2} M_g \chi \chi - \mu \epsilon_{ij} h_1^i h_2^j + i \sqrt{2} g_2 H_1^a T^a \chi h_1 \psi^a
\]  

(50)

\[
+ \ i \sqrt{2} g_2 H_2^a T^a h_2 \psi^a - \frac{i}{\sqrt{2}} g_y H_1^a h_1 \chi + \frac{i}{\sqrt{2}} g_y H_2^a h_2 \chi + \text{h.c.}
\]

where \( \psi^a \) (a=1,2,3) and \( \chi \) are the \( SU(2) \) and \( U(1) \) gauginos respectively, \( h_1 \) and \( h_2 \) are the two higgsino doublets and \( T^a \) are the \( SU(2) \) generators. Defining \( \sqrt{2} \psi^\pm = \psi^1 \mp i \psi^2 \) we get in the basis \((-i \psi^+, h_1^1, -i \psi^-, h_2^2) \equiv (\chi^+, \chi^-)\) the chargino mass matrix:

\[
\mathcal{L}_{\text{mass}} = -\frac{1}{2} (\chi^+, \chi^-) \left( \begin{array}{cc} 0 & X^T \\ X & 0 \end{array} \right) (\chi^+) + \text{h.c.}
\]

(51)

with

\[
X = \left( \begin{array}{cc} M_{g2} & \sqrt{2} M_W \sin \beta \\ \sqrt{2} M_W \cos \beta & \mu \end{array} \right)
\]

(52)

Diagonalization requires two unitary mixing matrices \( Z_+ \) and \( Z_- \) which rotate \( \chi^\pm \) fields to mass eigenstates \( \lambda^\pm \): \( \lambda^\pm = Z^\pm \chi^\pm \). The two physical chargino masses are denoted as \( m_{C_i}, i = 1, 2 \).

The neutralino mass matrix in the basis \( \chi^0 = (-i \chi, -i \psi^3, h_1^1, h_2^2) \) has the well known form:

\[
\mathcal{M}_N = \begin{pmatrix}
M_{g1} & 0 & -M_Z \hat{s} \cos \beta & M_Z \hat{s} \sin \beta \\
0 & M_{g2} & M_Z \hat{c} \cos \beta & -M_Z \hat{c} \cos \beta \\
-M_Z \hat{s} \cos \beta & M_Z \hat{c} \cos \beta & 0 & -\mu \\
M_Z \hat{s} \sin \beta & -M_Z \hat{c} \cos \beta & -\mu & 0
\end{pmatrix}
\]

(53)

and is diagonalized by rotation \( \chi^0 = Z_N^{-1} \chi^0 \). Physical neutralino masses are denoted as \( m_{N_i}, i = 1, \ldots, 4 \). Four component chargino (Dirac) and neutralino (Majorana) fields are then built as

\[
\psi^C_{C_i} = \left( \begin{array}{c} \lambda^+_i \\ \lambda^-_i \\ \chi_i \\ \bar{\chi}_i \end{array} \right) \quad \psi_{N_i} = \left( \begin{array}{c} \lambda^0_i \\ \chi_i \end{array} \right)
\]

(54)

Finally, gluino, which does not mix with anything, has a mass denoted as \( \tilde{M}_g \).
We do not describe here the Higgs sector referring the reader to refs. \[3, 12\]. We recall only that at the tree level, the Higgs boson masses and couplings can be conveniently parametrized by the ratio of the vacuum expectation values of the two Higgs doublets $\tan \beta \equiv v_2/v_1$ and the $CP$–odd Higgs boson mass $M_{A^0}$.

In terms of the parameters defined above, the general scan used in Sec. 3 can be described as follows. $50 < M_{A^0} < 500$ GeV, $50 < m_{C_L}$, $m_N < 1500$ GeV, $120 < M_{B_L} < 1000$ GeV, $0.4M_{B_L} < M_{T_R} < 2.5M_{T_L}$, $0 < A_t < M_{B_L}$, $0.4M_{B_L} < M_{\tilde{t}} < 0.8M_{B_L}$ keeping $M_{Q_L} = M_{Q_R} = M_{B_L}$ and $A_q = 0$ for the first two generations as well as $M_{E_R} = M_{\tilde{e}}$ with $A_t = 0$ (for all the three generations).

Next, we display the formulae for $(\Pi_{WW}(q^2))^{SUSY}$ and $(\Pi_{ZZ}(q^2))^{SUSY}$ which allow easy use of our approximation given by eqs. (25,26,31,32). As explained in Sec. 3, besides the genuine sparticle contributions, $(\Pi_{WW}(q^2))^{SUSY}$ and $(\Pi_{ZZ}(q^2))^{SUSY}$ include also the difference of contributions coming from the Higgs sectors of MSSM and SM and therefore depend also on the SM Higgs boson mass $M_{\phi^0}$. Obviously, calculating the corrections to $s^2$ one has to take the same $M_{\phi^0}$ in formulae below and in the fit (25). It is convenient to choose $M_{\phi^0} = M_{h^0}$.

Let us first consider $W^\pm$ self energy. The difference of the MSSM and SM Higgs boson contributions gives:

$$4\pi(\Pi_{WW}(q^2))^h = \frac{\hat{\alpha}}{8\pi}M_W^2 \left[ b_0(q^2, M_W, M_{\phi^0}) - c_{\beta\alpha}^2 b_0(q^2, M_W, M_{H^0}) - s_{\beta\alpha}^2 b_0(q^2, M_W, M_{H^0}) \right]$$

$$+ \frac{\hat{\alpha}}{8\pi} \left[ -\tilde{b}_{22}(q^2, M_W, M_{\phi^0}) + \tilde{b}_{22}(q^2, M_{H^0}, M_{A^0}) \right]$$

$$+ s_{\beta\alpha}^2 \tilde{b}_{22}(q^2, M_{H^0}, M_{H^0}) + c_{\beta\alpha}^2 \tilde{b}_{22}(q^2, M_{H^0}, M_{H^0})$$

where the functions $b_0$ and $\tilde{b}_{22}$ are defined in eqs. (56,57), $s_{\beta\alpha}^2 \equiv \sin^2(\beta - \alpha)$ and $\alpha$ is the mixing angle between the two MSSM scalar Higgs bosons defined e.g. as in \[31, 30\]. We recall that we use $M_{h^0}$, $M_{H^0}$ and $\alpha$ with 1–loop corrections included via the effective potential approach of ref. \[19\].

Each generation of sleptons contributes

$$4\pi(\Pi_{WW}(q^2))^{sd} = 2\frac{\hat{\alpha}}{8\pi} \left[ c_L^2 \tilde{b}_{22}(q^2, M_{E_1}, M_{\tilde{e}}) + s_L^2 \tilde{b}_{22}(q^2, M_{E_2}, M_{\tilde{e}}) \right]$$

where $c_L \equiv \cos \phi_L$ and the mass eigenstates of charged sleptons are defined by their relations to the left and right handed states:

$$E_L^- = c_L E_1^- - s_L E_2^- \quad E_R^- = s_L E_1^- + c_L E_2^-$$

(57)
Similarly, one generation of sleptons gives

\[ 4\pi(\tilde{\Pi}_{WW}(q^2))^{sq} = 6\frac{\hat{\alpha}}{s^2} \left[ c_U^2 s_D^2 \tilde{b}_{22}(q^2, M_{U_1}, M_{D_1}) + s_U^2 c_D^2 \tilde{b}_{22}(q^2, M_{U_2}, M_{D_1}) \right. \\
\left. + c_U^2 s_D^2 \tilde{b}_{22}(q^2, M_{U_1}, M_{D_2}) + s_U^2 s_D^2 \tilde{b}_{22}(q^2, M_{U_2}, M_{D_2}) \right] \]

with \( c_{U,D} \equiv \cos \phi_{U,D} \) defined by

\[ U_L^+ = c_U U_1^+ - s_U U_2^+ \quad U_R^+ = s_U U_1^+ + c_U U_2^+ \]
\[ D_L^- = c_D D_1^- - s_D D_2^- \quad D_R^- = s_D D_1^- + c_D D_2^- \]

Finally, contribution of two charginos and four neutralinos reads:

\[ 4\pi(\tilde{\Pi}_{WW}(q^2))^{CN} = 2\frac{\hat{\alpha}}{s^2} \sum_{i=1}^{4} \sum_{j=1}^{4} \left[ |c_{L,C/N}^{ji}|^2 + |c_{R,C/N}^{ji}|^2 \right] \left[ -4\tilde{b}_{22}(q^2, m_{C_j}, m_{N_i}) \right. \\
\left. - (q^2 - m_{C_j}^2 - m_{N_i}^2) b_0(q^2, m_{C_j}, m_{N_i}) \right] \]

The explicit formulae for the left and right couplings \( c_{L,R,C/N}^{ji} \) of charginos and neutralinos to \( W^\pm \) will be given shortly.

For the \( \tilde{\Pi}_{ZZ} \) the difference of the MSSM and SM Higgs sectors reads:

\[ 4\pi(\tilde{\Pi}_{ZZ}(q^2))^{h} = \frac{\hat{\alpha}}{s^2 c^2} M_Z^2 \left[ b_0(q^2, M_Z, M_{\phi^0}) \right. \\
\left. - c_{\beta\alpha}^2 b_0(q^2, M_Z, M_{H^0}) - s_{\beta\alpha}^2 b_0(q^2, M_Z, M_{H^0}) \right] \\
\left. + \frac{\hat{\alpha}}{s^2 c^2} \left[ s_{\beta\alpha}^2 \tilde{b}_{22}(q^2, M_Z, M_{H^0}) + c_{\beta\alpha}^2 \tilde{b}_{22}(q^2, M_Z, M_{H^0}) \right] \right) \]

One generation of sleptons yields:

\[ 4\pi(\tilde{\Pi}_{ZZ}(q^2))^{sl} = 3\frac{\hat{\alpha}}{s^2 c^2} \left[ 2c_U^2 s_D^2 \tilde{b}_{22}(q^2, M_{U_1}, M_{U_2}) + c_U^2 s_D^2 \tilde{b}_{22}(q^2, M_{D_1}, M_{D_2}) \right. \\
\left. + (c_U^2 - \frac{4}{3} s_U^2) \tilde{b}_{22}(q^2, M_{U_1}, M_{U_1}) + (s_U^2 - \frac{4}{3} s_U^2) \tilde{b}_{22}(q^2, M_{U_2}, M_{U_2}) \right. \\
\left. + (c_D^2 - \frac{2}{3} s_D^2) \tilde{b}_{22}(q^2, M_{D_1}, M_{D_1}) + (s_D^2 - \frac{2}{3} s_D^2) \tilde{b}_{22}(q^2, M_{D_2}, M_{D_2}) \right] \]
Two charginos and four neutralinos give

\[4\pi(\Pi_{ZZ}(q^2))^{CN} = \frac{1}{4} \frac{\hat{\alpha}}{s^2 c^2} \sum_{j=1}^{2} \sum_{i=1}^{2} \left(|c_{L,C}^{ji}|^2 + |c_{R,C}^{ji}|^2\right) \left[-4\tilde{b}_{22}(q^2, m_{C_j}, m_{C_i})ight]
\]

\[-(q^2 - m_{C_j}^2 - m_{C_i}^2)b_0(q^2, m_{C_j}, m_{C_i})\]

\[-\frac{\hat{\alpha}}{s^2} \sum_{j=1}^{2} \sum_{i=1}^{2} \text{Re}(c_{L,C}^{ji}c_{R,C}^{ji*})m_{C_j}m_{C_i}b_0(q^2, m_{C_j}, m_{C_i})\]

\[+ \frac{1}{8} \frac{\hat{\alpha}}{s^2 c^2} \sum_{j=1}^{4} \sum_{i=1}^{4} \left(|c_{L,N}^{ji}|^2 + |c_{R,N}^{ji}|^2\right) \left[-4\tilde{b}_{22}(q^2, m_{N_j}, m_{N_i})\right]
\]

\[-(q^2 - m_{N_j}^2 - m_{N_i}^2)b_0(q^2, m_{N_j}, m_{N_i})\]  \hspace{1cm} (64)

\[-\frac{\hat{\alpha}}{s^2} \sum_{j=1}^{4} \sum_{i=1}^{4} \text{Re}(c_{L,N}^{ji}c_{R,N}^{ji*})m_{N_j}m_{N_i}b_0(q^2, m_{N_j}, m_{N_i})\]

The function \( \tilde{b}_{22} \) can be easily expressed in terms of standard \( a \) and \( b_0 \) functions [32, 12] as:

\[\tilde{b}_{22}(q^2, m_1, m_2) = \frac{1}{6} a(m_1) + \frac{1}{6} a(m_2) + \frac{1}{3} m_2^2 b_0(q^2, m_1, m_2)\]

\[-\frac{(q^2 - m_1^2 + m_2^2)^2}{12q^2} b_0(q^2, m_1, m_2)\]  \hspace{1cm} (65)

\[-\frac{m_1^2 - m_2^2}{12q^2} [a(m_1) - a(m_2)] + \frac{1}{6} (m_1^2 + m_2^2 - \frac{1}{3} q^2)\]

Here \( a \) and \( b_0 \) are the “renormalized in \( \overline{MS} \) scheme” functions:

\[a(m) = m^2(-1 + \log \frac{m^2}{M_Z^2})\]  \hspace{1cm} (66)

\[b_0(q^2, m_1, m_2) = -\int_0^1 dx \log \frac{x(x-1)q^2 + (1-x)m_1^2 + xm_2^2}{M_Z^2}\]  \hspace{1cm} (67)

In terms of the mixing matrices \( Z_{\pm} \) and \( Z_{N} \) the couplings needed in eqs.(60,64) take the form [30, 31]: (the fermion \( i \) is incoming and \( j \) is outgoing)

\[c_{L,C/N}^{ji} = Z_{+}^{1j*} Z_{N}^{2i} - \frac{1}{\sqrt{2}} Z_{+}^{2j*} Z_{N}^{4i}\]

\[c_{R,C/N}^{ji} = Z_{-}^{1j} Z_{N}^{2i*} + \frac{1}{\sqrt{2}} Z_{+}^{2j} Z_{N}^{3i*}\]  \hspace{1cm} (68)

\[\text{Factors } c_{L,N}^{ji} \text{ and } c_{R,N}^{ji} \text{ given in [30] are not properly symmetrized.}\]
\[ c_{L,C}^{ji} = Z_{1j}^{1i} Z_{1i}^{1j} + (\hat{c}^2 - \hat{s}^2) \delta^{ji} \]
\[ c_{R,C}^{ji} = Z_{3j}^{1i} Z_{3i}^{1j} + (\hat{c}^2 - \hat{s}^2) \delta^{ji} \]  
(69)

and

\[ c_{L,N}^{ji} = Z_{Nj}^{4i} Z_{Ni}^{4j} - Z_{Nj}^{3i} Z_{Ni}^{3j} \]
\[ c_{R,N}^{ji} = Z_{Nj}^{3i} Z_{Ni}^{3j} - Z_{Nj}^{4i} Z_{Ni}^{4j} \]  
(70)

We end this Appendix with two comments. First, as long as neutralinos appear only in closed loops it is not necessary to do anything with negative eigenvalues of their mass matrix (53). Using \( \frac{(p + m)}{(p^2 - m^2)} \) for \( m < 0 \) as a propagator gives the same physical results as other more complicated prescriptions which require modification of couplings (see eg. [31]). It is possible, however, to choose the \( Z_N \) matrix such that all mass eigenvalues are positive.

Second, the transition from Weyl spinors \( \lambda_i^{0,\pm} \) to four component fields \( \psi_{N_i} \) and \( \psi_{C_i}^{+} \) is not unique. One can work for example with

\[ \psi_{C_i}^{-} = \begin{pmatrix} \lambda_i^- \\ \lambda_i^+ \end{pmatrix} \]  
(71)

instead of \( \psi_{C_i}^{+} \). This freedom in building four component fields from two component ones is in fact the basis of the recently developed technique [33] for handling Majorana particles which often produce "clashing arrows" in Feynman diagrams. However realizing that (at the level of lagrangian) e.g.

\[ a^{ji} \bar{\psi}_{C_i}^{+} (1 - \gamma^5) \psi_{N_i} \phi^+ = a^{ji} \bar{\psi}_{N_i} (1 - \gamma^5) \psi_{C_i}^{-} \phi^+ \]  
(72)

and that in different vertices of a given Feynman diagram different forms of the same interaction can be used allow avoiding essentially all problems with Majorana particles.
References

[1] J. Ellis, S. Kelley, D.V. Nanopoulos Phys. Lett. 260B (1991) 131, 
P. Langacker, M. Luo Phys. Rev. D44 (1991) 817, 
F. Anselmo, L. Cifarelli, A. Peterman, A. Zichichi Nuovo Cimento 104A (1991) 1817, 105A (1992) 581, 
U. Amaldi, W. de Boer, H. Fürstenau Phys. Lett. B (1991).

[2] W. Siegel Phys. Lett. 94B (1980) 37, 
D.M. Capper, D.R.T Jones, P. van Nieuwenhuizen Nucl. Phys. B167 (1980) 479.

[3] W.A. Bardeen, A.J. Buras, D.W. Duke, T. Muta Phys. Rev. D18 (1978) 3998.

[4] D. Schaile talk at XXVII Int. Conf. on High Energy Physics, Glasgow, July 1994.

[5] I. Antoniadis, C. Kounnas, K. Tamvakis Phys. Lett. 119B (1982) 377, 
I. Antoniadis, C. Kounnas, R. Lacaze Nucl. Phys. B211 (1983) 216.

[6] G.G. Ross, R.G. Roberts Nucl. Phys. B377 (1992) 368.

[7] P. Langacker, N. Polonsky Phys. Rev. D47 (1993) 4028.

[8] M. Carena, S. Pokorski C.E.M. Wagner Nucl. Phys. B406 (1993) 59.

[9] M. Carena, T.E. Clark, C.E.M. Wagner, W.A. Bardeen, K. Sasaki, Nucl. Phys. B369 (1992) 33, 
C.T. Hill, Phys. Rev. D24 (1981) 691, 
C.T. Hill, C.N. Leung, S. Rao, Nucl. Phys. B262 (1985) 517.

[10] A.E. Faraggi, B. Grinstein Nucl. Phys. B422 (1994) 3.

[11] G. Degrassi, S. Fanchiotti, A. Sirlin Nucl. Phys. B351 (1991), 49.

[12] P.H. Chankowski, S. Pokorski, J. Rosiek Nucl. Phys. B423 (1994) 437.

[13] P.H. Chankowski et al. Nucl. Phys. B417 (1994), 101.

[14] A. Djouadi, C. Verzegnassi Phys. Lett. 195B (1987) 265, 
A. Djouadi Nuovo Cimento 100A (1988) 357, 
D.Yu. Bardin, V.A. Chizov Dubna preprint E2-89-525 (1989), 
B.A. Khniel, J.H. Kühn, R.G. Stuart Phys. Lett. 214B (1988) 621, 
B.A. Khniel Nucl. Phys. B347 (1990) 86, 
F.A. Halzen, B.A. Khniel Nucl. Phys. B353 (1991) 567, 
S. Fanchiotti, B.A. Khniel, A. Sirlin preprint CERN-TH 6749/92 and NYU-TH 92/12/05.
[15] J.J. van der Bij, F. Hoogeveen Nucl. Phys. B283 (1987) 477.

[16] R. Barbieri, M. Beccaria, P. Ciafaloni, G. Curci, A. Viceré Phys. Lett. 288B (1992) 95.

[17] W. Hollik Munich preprint MPI-Ph/93-21, April 1993

[18] H. Burkhard, F. Jegerlehner, G. Penso, C. Verzegnassi Z. Phys. C43 (1989) 497,
F. Jegerlehner preprint PSI-PR-91-16 (1991) in: Progress in Particle and Nuclear Physics, ed. A. Fassler, Pergamon Press, Oxford, U.K.

[19] J. Ellis, G. Ridolfi, F. Zwirner Phys. Lett. 262B (1991) 477.

[20] P.H. Chankowski Phys. Lett. 336B (1994) 423.

[21] J. Hisano, H. Murayama, T. Yanagida Phys. Rev. Lett. 69 (1992) 1014,
Nucl. Phys. B402 (1993) 46.

[22] M. Demarteau et al. Combining W Mass Measurements, CDF/PHYS/CDF/PUBLIC/2215 and D0NOTE 2115.

[23] P. Langacker, M. Luo Phys. Rev. D44 (1991) 817.

[24] M. Olechowski, S. Pokorski Nucl. Phys. B404 (1993) 590.

[25] M. Carena, M. Olechowski, S. Pokorski, C. Wagner Nucl. Phys. B419 (1994) 213.

[26] R. Barbieri, L.J. Hall Phys. Rev. Lett. 68 (1992) 752.

[27] N. Sakai, T. Yanagida Nucl. Phys. B197 (1982) 533,
S. Weinberg Phys. Rev. D26 (1982) 287.

[28] Particle Data Group, K. Hikasa et al. Phys. Rev. D45 (1992) 1.

[29] R. Arnowitt, P. Nath Phys. Rev. Lett. 69 (1992) 725,
P. Nath, R. Arnowitt Phys. Lett. 289B (1992) 368.

[30] J. Rosiek Phys. Rev. D41 (1990) 3464.

[31] J.F. Gunion, H.E. Haber, G. Kane, S. Dawson The Higgs Hunter’s Guide, Addison-Wesley (1990).

[32] G. Passarino, M Veltman Nucl. Phys. B160 (1978) 151,
G. ’t Hooft, M. Veltman Nucl. Phys. B153 (1973) 365,
A. Axelrod Nucl. Phys. B209 (1982) 349.

[33] A. Denner, H. Eck, O. Hahn, J. Küblbeck Nucl. Phys. B387 (1992) 467.
FIGURE CAPTIONS

Figure 1.
Comparison of $\sin^2 \theta(M_Z)^{\text{FULL}}$ with $\sin^2 \theta(M_Z)^{\text{LLT}}$ in the MSSM for sparticle spectra corresponding to the general scan described in the text in four different cases:

a) $m_t = 160 \text{ GeV}$ $\tan \beta = 2$,  
b) $m_t = 160 \text{ GeV}$ $\tan \beta = 50$,  
c) $m_t = 180 \text{ GeV}$ $\tan \beta = 2$,  
d) $m_t = 180 \text{ GeV}$ $\tan \beta = 50$.

Circles denote spectra with all SUSY particles heavier than 500 GeV and $50 < M_{A^0} < 500$ GeV. Stars correspond to spectra with $M_{A^0} \leq 250$ GeV, $m_{C_{ij}}$, $m_{N_j} \leq 250$ GeV and heavy sfermions, $\geq 500$ GeV. Squares correspond to spectra with all sparticles light, $< 250$ GeV and $50 < M_{A^0} < 250$ GeV.

Figure 2.

a) $\sin^2 \theta(M_Z)$ and b) $\alpha(M_Z)^{\text{EM}}$ in the MSSM as a function of the (common for all the three generations) sneutrino mass. All other SUSY particles (including the $A^0$ Higgs boson) have masses 2 TeV, with no Left – Right mixing. Solid lines correspond to the full calculation, dashed ones to the LLT approximation, dot–dashed lines show our approximation (eqs. (31,32) combined with the fit (25)). Two dotted lines show the results of application of the Faraggi Grinstein formulae (see [10], eqs. (5.10)) to the SM couplings obtained from our fit (25).

Figure 3.

a) $\sin^2 \theta(M_Z)$ and b) $\alpha(M_Z)^{\text{EM}}$ in the MSSM as a function of the (purely) left handed sbottom mass. All other SUSY particles (including the $A^0$ Higgs boson) have masses 2 TeV, with no Left – Right mixing. Solid lines correspond to the full calculation, dashed ones to the LLT approximation, dot–dashed lines show our approximation (eqs. (31,32) combined with the fit (25)). Two dotted lines show the results of application of the Faraggi Grinstein formulae (see [10], eqs. (5.10)) to the SM couplings obtained from our fit (25).

Figure 4.

a) $\sin^2 \theta(M_Z)$ and b) $\alpha(M_Z)^{\text{EM}}$ in the MSSM as a function of the pseudoscalar mass $M_{A^0}$ mass. All SUSY particles have masses 2 TeV, with no Left – Right mixing. Solid lines correspond to the full calculation, dashed ones to the LLT approximation, dot–dashed lines show our approximation (eqs. (31,32) combined with the fit (25)). Three dotted lines show the results of application of the Faraggi Grinstein formulae (see [10], eqs. (5.10)) to the SM couplings obtained from our fit (25).
Figure 5.
Parameter $\mu$ as a function of $T_{SUSY}$ for spectra of ref. [24, 25] obtained for different values of $m_t$ and $\tan \beta$ and subject to the conditions: i) Squark masses below 2 TeV, ii) Sparticle masses above the experimental limits.

Figure 6.
Values of $\hat{\alpha}_3$ and $\hat{s}^2$ (vertical and horizontal axis respectively) calculated with the use of sparticle spectra characterized in Fig.5 and for the same $m_t$ and $\tan \beta$ values. Stars (diamonds) denote LLT (full) calculation of the MSSM couplings. The upper (lower) group of points in Figs. a), c) and d) corresponds to $\hat{\alpha}_3^{SM} = 0.125(0.115)$. The three groups of points in Fig. b) correspond to $\hat{\alpha}_3^{SM} = 0.13, 0.125, 0.115$. Contours in the plot denote bounds imposed by SUSY SU(5) unification conditions with $\hat{\alpha}_{em}$ (in the MSSM) allowed to vary in the range: $1/129.0 - 1/131.8$: Solid lines - no GUT scale thresholds; dashed (dash-dotted) lines $- M_{GUT}^{max}/M_{GUT}^{min} = 30 (1000)$ (for details see text). The three dotted lines correspond to fixed values of $M_{H_3} = 1 \times 10^{17}$ (uppermost), $2 \times 10^{16}, 5 \times 10^{15}$ (for $\hat{\alpha}_{EM} = 1/131.8$). The region below the long-dashed lines correspond to violation of perturbativity condition $Y_t^2(M_{GUT})/4\pi < 1$.

Figure 7.
Values of $\hat{\alpha}_3^{SM}$ predicted by SUSY unification without GUT scale thresholds as a function of $T_{SUSY}$ calculated with the use of sparticle spectra characterized in Fig.5 and for the same $m_t$ and $\tan \beta$ values. Stars (diamonds) denote LLT (full) calculation of the MSSM couplings. The region below the long-dashed line corresponds to violation of perturbativity condition $Y_t^2(M_{GUT})/4\pi < 1$.

Figure 8.
$M_{H_3}$ as a function of $T_{SUSY}$ for $\hat{\alpha}_3^{SM}(M_Z) = 0.125$ and sparticle spectra characterized in Fig.5. Stars (diamonds) denote LLT (full) calculation of the MSSM couplings.
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411233v1
This figure "fig2-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411233v1
This figure "fig1-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411233v1
This figure "fig2-2.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411233v1
This figure "fig1-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411233v1
This figure "fig2-3.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411233v1
This figure "fig1-4.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411233v1
This figure "fig1-5.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9411233v1