Convex-Optimization-Based Power-Flow Calculation Method for Offshore Wind Systems

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Abstract: Offshore wind farms have boomed worldwide due to the sustainability of wind power and ocean resources. Power grid companies should consider the wind power consumption problem with more power generated. Power-flow calculation is the most fundamental tool in energy management. This paper proposes the convex-relaxation-based method for offshore wind farms’ power flow. In this method, the traditional equations’ problem solving is transferred into standard convex optimization, which can be solved efficiently with unique optimum. Second-order cone relaxations are imposed to describe the quadratic relationship. The exactness of the relaxation is guaranteed with the special definition of the objective function. The superiority of the proposed method is tested on the case study, for which a computational efficiency improvement is shown. Moreover, the reliability of the power-flow results is verified within the precision tolerance.

Keywords: convex optimization; power flow; wind farms; exact convex relaxations

1. Introduction

Power-flow analysis is to obtain the voltage and power information when the system’s topology is fixed. With this information, the power system’s planning, operation, and system-stability management states can be guided and decided [1]. Offshore wind energy resources are abundant and stable, and the global wind power elaboration shows a trend of developing from onshore to offshore [2].

Calculating the power flow of offshore wind systems helps to regulate and dispatch the energy. Since offshore wind systems are mostly in radial topology or weak-meshed topology, it is necessary to evaluate the methods of solving power flow. The main difficulty of the power-flow problem is solving a series of non-linear equations that describe the physical connection of the offshore wind system. The history of power flow can be traced back to the mid-20th century. In the early 1950s, power-flow calculation was carried out through the physical simulation of the DC computing table and the AC computing table. With the development of computer science, power-flow calculation transferred from analog technology to digital technology [3]. From then on, power-flow analysis developed with memory requirements, calculation speed and precision, and convergence. Thus, various kinds of power-flow calculation algorithms are mushrooming around the world [4].

Initially, the matrix-based Gauss–Seidel method [5] was used in power-flow calculation. This method only requires limited storage but shows a slow convergence rate. Later, the impedance matrix-based iterative method has been developed [6,7]. In comparison, the computational memory’s consumption of this method limits the system’s scale. As the power system becomes more and more complicated, this method cannot be applied anymore. At the same time, the Newton–Raphson method [8] has been created for power-flow analysis. Moreover, the PQ decomposition method [9], the artificial intelligence-based method [10], the backward/forward sweep method [11], and the approximated linear method [12] have also been applied in power-flow calculation. The stability of considered
power flow has been proposed in [13]. The power-flow problem considering renewable energy has been proposed in [14]. However, the heuristic methods require much storage and computational time in the calculation [15]. The approximated linear equations will result in non-negligible errors [16], which is regarded as DC power flow. The Newton-series method is most sensitive to the initial value [17] since the essence of power-flow calculation is to solve a series of non-linear equations [18]. Sometimes, the solution of a specific power flow is not unique. Therefore, secondary processing is also required [19].

Another way to analyze power flow is to solve this through an optimization process since optimal power flow is the power flow solution with specific objectives. Among the optimization formats, convex optimization has strong points considering the character of the global optimum [20]. Therefore, the combination of convex optimization and power flow can guarantee a unique solution. Moreover, the convex optimization shows high efficiency in solving [21]. While the power flow constraints are non-linear, many relaxation methods should be implemented to relax the non-convex equations into convex ones, such as convex hull relaxations [22], convex-concave procedure technique [23], and semi-definite relaxation [24], second-order-cone relaxation [25] and so on. With the convex relaxation implemented, it is meaningful to check the relaxation gap. The large gaps will result in an infeasible optimal solution of the optimization before relaxations [26]. Therefore, to take advantage of convex optimization, exact relaxations should be guaranteed.

In this paper, the power-flow analysis of offshore wind farms is implemented. Convex optimization is combined with power-flow analysis, which helps to enhance the solving efficiency and guarantee a unique solution. The exactness of the relaxation is proved with the specific configuration of the objective function. The real wind farm system is tested in the case study to evaluate the performance of the proposed method with other methods, in which the superiority of the proposed method is demonstrated.

The main technical contributions made in this work can be summarized as follows:

(1) A convex-relaxation-based method for power-flow calculation of offshore wind farms has been proposed.
(2) The exactness of the relaxation can be guaranteed due to the design of the optimization objectives.
(3) The unique solution of the power flow can be guaranteed with the strong points of the convex optimization.

This paper is organized as follows: In Section 2, the formulation of the power-flow problem is described. In Section 3, convex relaxations are imposed to convexify the power-flow equations, the exactness of which is proved mathematically. In Section 4, the proposed method is applied in a real case study. In Section 5, the main findings of the paper are summarized.

2. Power-Flow Calculation Problem

In this section, the original power flow will be demonstrated from the aspects of the notations, formulations, and descriptions.

2.1. Notations

The offshore wind system is a typical power system. In this paper, we make the following definitions.

- **Variables**
  Define the voltage of node \( i \) with the complex variable \( V_i = U_i e^{j\theta_i} \). \( U_i \) and \( \theta_i \) are the real variables, which denote the amplitude and the phase angle of node \( i \), respectively. The active power and reactive power of node \( i \) are defined as the real variables \( P_i \) and \( Q_i \), respectively. The apparent power of node \( i \) is defined as the complex variable \( S_i \). Similarly, the corresponding voltage and power variables are defined for node \( j \) as \( V_j = U_j e^{j\theta_j} \), \( S_j = P_j + Q_j i \). The injected current of node \( i \) is set as \( I_i \), where \( I_i \) is the complex variable. For the transmission line \( i \sim j \), the transmitted
active power and reactive power are shown with the real variables $P_{ij}$ and $Q_{ij}$. For the apparent power of $i \sim j$, the complex variables are $S_{ij}$, where $S_{ij} = P_{ij} + iQ_{ij}$.

- Constants
  For the transmission line $i \sim j$, the conductance and susceptance of the transmission line are defined as the real constants $G_{ij}$ and $B_{ij}$, where the complex constant admittance is $Y_{ij} = G_{ij} + iB_{ij}$. It is important to mention that the transmission line in this paper is set as the in-line equivalent circuit, where the susceptance to the ground is neglected.

- Operator
  $*$ is the conjugate operator of complex variables and constants.

The above notations are summarized in Figure 1.

![Figure 1. The notations of power system.](image)

A summary table of the notations has been added in the end of the paper.

### 2.2. Power Flow Formulation

For each node $i$ in the power system, the relationship between the injected current and node’s voltage can be represented as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \quad (i, j = 1, 2, \ldots, n), \quad (1)$$

where $n$ is the total number of nodes in the system. This is the basic principle in the power system. The injected current of each node equals the total current of each connected node.

To solve the power-flow problem, the apparent power of the node and corresponding current should be used. Due to the physical principle, the relationship is shown as

$$I_i = \frac{P_i - iQ_i}{V_i^*} \quad (j = 1, 2, \ldots, n), \quad (2)$$

in mathematics. By substituting (2) into (1), the basic power flow formulation is obtained. It will be

$$\frac{P_i - iQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j \quad (j = 1, 2, \ldots, n). \quad (3)$$

In (3), there are $n$ complex non-linear equations. In the power system, by constructing the equation with each node’s current, we can connect the branch’s information with the nodes'. The different ways to deal with this equation constructs various methods to solve power flow equations.

### 2.3. Description of Power Flow Analysis

In the power system, the operation state of each node $i$ can be represented with a set of four variables: $U_i, \theta_i, P_i, Q_i$. Therefore, for the power system with $n$ nodes, there are $4n$ variables in total. While not all of them are unknown, some of them is known before calculation.

To simplify the power-flow equations, the real parts and imaginary parts are decoupled individually. In the power-flow calculation (3), the $n$ complex equations can be rewritten as $2n$ real equations by decoupling the active power and reactive power, respectively. So, only $2n$ variables can be obtained by solving power-flow equations with the other $2n$ variables.
fixed. The nodes in the power system can be classified into three types: PQ node, PV node and slack node, according to the operation state. Whether the nodes’ information is known or unknown depends on the type of the nodes. More specifically, three types of nodes are defined as follows:

- **PQ node**
  PQ nodes that characterize the known active power and the reactive power’s value. Additionally, the power-flow calculation aims to solve the node’s voltage amplitude and phase angle. The nodes connected with generators are always regarded as the PQ node in the power system. In offshore wind systems, the node of wind turbines is PQ nodes.

- **PV node**
  The PV node characterizes the known active power and the phase angle’s value. Additionally, the power-flow calculation aims to solve the node’s voltage amplitude and reactive power. This node is embedded with reactive power sources to set the specific voltage amplitude in operation. The power plant buses with certain reactive power reserves are the PV nodes. In the offshore wind system, there are few nodes connected with loads. Sometimes, the storage units are hidden by the wind system. Thus, these nodes can be regarded as PV nodes. To guarantee the electricity’s quality of wind farms, there are regulations for the voltage limit of each node.

- **Slack node**
  Only one node can be set as the slack node in power-flow calculation. This node regulates the reference phase angle of the voltage in the system. The node’s active and reactive power should be solved for the slack node. The slack node always poises the power balance of the whole system. Mostly, the frequency adjustment bus of the power plant is set as the slack node. In an offshore wind system, the grid node that transfers the energy between the power system and the offshore wind system can be regarded as the slack node.

To sum up, the power-flow analysis is used to solve Equation (3). In this equation, considering the features of the nodes PQ node, PV node, and slack node, there are $2n$ known variables and $2n$ unknown variables. The target of the power-flow analysis for the offshore wind system is to solve the unknown variables efficiently and accurately.

### 3. Convex Optimization Based Method

Since Equation (3) is non-convex, if the convex relaxation is implemented, the relaxations should be taken as precautions. In this section, the variables’ relaxation is defined firstly. Thereafter, the convex relaxation is imposed for the power-flow equations, which are used to convert the power-flow problem into a convex optimization.

#### 3.1. Variables Relaxations

The first step to relaxing the non-linear equations is by variables’ relaxations. In the relationship between the injected power of each node and the voltage, there will be square variables of voltages. In the variables’ relaxation, we will focus on them. For two arbitrary connecting nodes $i\sim j$ in the offshore wind system, the variables $x_i$, $x_j$, $x_{ij1}$, and $x_{ij2}$ are defined as

\[
x_i = U_i^2, \quad (4)
\]

\[
x_j = U_j^2, \quad (5)
\]

\[
x_{ij1} = U_i U_j \sin(\theta_i - \theta_j), \quad (6)
\]

\[
x_{ij2} = U_i U_j \cos(\theta_i - \theta_j). \quad (7)
\]

The defined variables are the quadratic format of voltage variables, which describes the squared voltages of nodes, the real and imaginary of $V_i V_j$. 


According to the definition above, we reconstruct some indirect variables. We define three variables $x_{ij}, y_{ij},$ and $z_{ij}$, referring to the value of $x_i, x_j, x_{ij1},$ and $x_{ij2}$, that is

$$x_{ij} = x_{ij1},$$ (8)

$$y_{ij} = x_i - x_j,$$ (9)

$$z_{ij} = x_i + x_j - 2x_{ij2}.$$ (10)

From the above equations, it can be known that the variables $x_i, x_j, x_{ij1},$ and $x_{ij2}$ can be replaced by the linear representations of variables $x_{ij}, y_{ij},$ and $z_{ij}$.

When comparing (4)–(7) with (8)–(10), it can be known that with the value of $x_{ij}, y_{ij},$ and $z_{ij}$ we can have endless sets of value $x_i, x_j, x_{ij1},$ and $x_{ij2}$ as

$$
\begin{bmatrix}
1 & -1 & 0 \\
1 & 1 & 2
\end{bmatrix}
\begin{bmatrix}
x_i \\
x_j \\
x_{ij2}
\end{bmatrix} =
\begin{bmatrix}
y_{ij} \\
z_{ij}
\end{bmatrix}
$$ (11)

When solving (11), we can see that there is an infinite set of solutions. By inducing an arbitrary variable $\xi$, we can obtain a set of solutions, for example,

$$
\begin{cases}
x_i = \frac{z_{ij}}{\pi} + \frac{y_{ij}}{\pi} + \xi_{ij} \\
x_j = \frac{y_{ij}}{\pi} - \frac{z_{ij}}{\pi} + \xi_{ij} \\
x_{ij1} = x_{ij} \\
x_{ij2} = -\frac{z_{ij}}{\pi} + \xi_{ij}
\end{cases}
$$ (12)

With different $\xi$, there will be various relationship representations between the variables $x_i, x_j, x_{ij1}, x_{ij2}$ and $x_{ij}, y_{ij},$ and $z_{ij}$.

### 3.2. Convex Relaxation

The power-flow equations can be rewritten with the relaxed variables $x_{ij}, y_{ij}$ and $z_{ij}$. In the power system, considering the relationship of the transmission line’s active power and reactive power with $x_{ij}, y_{ij}$, and $z_{ij}$, we have

$$
\begin{bmatrix}
P_{ij} \\
P_{ji} \\
Q_{ij} \\
Q_{ji}
\end{bmatrix} =
\begin{bmatrix}
B_{ij} & G_{ij} & G_{ij} & -G_{ij} \\
-G_{ij} & B_{ij} & G_{ij} & G_{ij} \\
-G_{ij} & -G_{ij} & B_{ij} & G_{ij} \\
G_{ij} & G_{ij} & -G_{ij} & B_{ij}
\end{bmatrix}
\begin{bmatrix}
x_{ij} \\
y_{ij} \\
z_{ij}
\end{bmatrix}.
$$ (13)

From Equation (13), the non-linear power-flow equations are converted into linear ones with the relaxed variables. This equation explains the power-flow equations from the aspects of the active power and the reactive power. The decoupling of this step converts the complex variables into real variables.

For the voltage constraints of the real operation state, we have the corresponding constraints for $x_{ij}, y_{ij},$ and $z_{ij}$. In the offshore wind systems, we have some limitations for each node’s voltages. That is, to ensure the effectiveness of the solution, both the voltage’s amplitude and phase angle are limited. That is,

$$
U_i^{\text{min}} \leq U_i \leq U_i^{\text{max}},
$$ (14)

$$
U_j^{\text{min}} \leq U_j \leq U_j^{\text{max}},
$$ (15)

$$
\theta_{ij}^{\text{min}} \leq \theta_i - \theta_j \leq \theta_{ij}^{\text{max}}.
$$ (16)
The superscript min and max are the lower and upper bounds of the variables, which are fixed as the constants due to the stable operation of the system. With the ranges in (14)–(16), the value of variables $x_{ij}$, $y_{ij}$, and $z_{ij}$ can be limited as well.

$$\min(U_i U_j \sin(\theta_i - \theta_j)) \leq x_{ij} \leq \max(U_i U_j \sin(\theta_i - \theta_j)), \quad (17)$$

$$\min(U_i^2 - U_j^2) \leq y_{ij} \leq \max(U_i^2 - U_j^2), \quad (18)$$

$$\min(U_i^2 + U_j^2 - 2U_i U_j \cos(\theta_i - \theta_j)) \leq z_{ij} \leq \max(U_i^2 + U_j^2 - 2U_i U_j \cos(\theta_i - \theta_j)). \quad (19)$$

The min and max symbol mentioned above are the minimize and maximize function. With the fixed value ranges in (14)–(16), the minimization and maximization will be fixed with constants accordingly.

Moreover, due to the power balance law, the active power and reactive power of the node should be equal to the sum of the total power from connected branches.

$$P_i = \sum_{ij \in E} P_{ij}, \quad (20)$$

$$Q_i = \sum_{ij \in E} Q_{ij}. \quad (21)$$

where $E$ denotes the set of all the branches in the power system.

From (4)–(7), we can know that there is the following relationship among variables $x_i$, $x_j$, $x_{ij1}$, $x_{ij2}$.

$$x_i x_j = x_{ij1}^2 + x_{ij2}^2 \quad (22)$$

This is a quadratic constraint with squared terms and the product term. This equation constructs the non-linearity and non-convexity of the power-flow problem. Thus, convex relaxations are imposed as follows.

To make this optimization problem a convex optimization problem, we relax the equality constraints into inequality constraints as a second-order cone format. By substituting (12) into convex-relaxed (22), (22) can be represented as

$$x_{ij1}^2 + \frac{1}{4} y_{ij}^2 + \frac{1}{12} z_{ij}^2 \leq z_{ij} y_{ij} \quad (23)$$

A schematic diagram of the second-order-cone relaxations is shown in Figure 2. The original constraints in (22) can be described as a surface of cones, as shown in the left picture in Figure 2. With relaxations, the constraints’ region is enlarged, as shown in the whole space contained in the cone in the right picture in Figure 2. Obviously, when the optimal solution is obtained on the surface of the cone, we can consider that the relaxation is exact. That is to say, although the solution is obtained by relaxation, the optimal solution is the same as the one obtained before relaxation.

![Figure 2. The second-order-cone relaxation.](image-url)
Thus, the power-flow problem can be described as

$$\min \sum_{ij \in E} \xi_{ij}$$

subject to (13), (17)–(21), (23) \hspace{1cm} (24)$$

with convex relaxations.

This optimization problem becomes a problem relating to the variables $x_{ij}, y_{ij}, z_{ij}$ and $\xi_{ij}$. Since $\xi_{ij}$ is a variable with a random possible value, this relaxation always tends to be exact. The exactness of the relaxation will be discussed in the next section.

In Equation (24), both the equality and inequality constraints are in a convex format. The objective function is a linear representation of the variable $\xi_{ij}$, which is also a convex function. Thus, optimization (24) is a standard convex optimization problem. Due to the strong points of convex optimization, (24) can be solved efficiently with one unique optimum. That is, only the power flow solution, which meets the power system’s physical requirements, can be obtained with (24). Compared with the method in [23], the (24) shows the method for power-flow calculation, in which the second-order cone relaxation is concluded, while in [23], the method for power flow optimization is proposed, whose aim is to optimize the power loss. To apply the method in [23] for power-flow calculation, the objective function should be changed as a constant. Moreover, in [23], the convex relaxation is based on the convex-concave procedure technique.

3.3. Exactness of Relaxations

In Section 3.2, it can be shown that the power-flow problem can be transferred into a convex optimization with relaxations. Since the convex relaxations have been implemented, it is essential to discuss the exactness. The exactness refers to the fact that the optimal solution is solved when (23) has an equality operator. That is the same as the one before convex relaxations. If the relaxation is not exact, the optimal value of the objective function could not be obtained.

So in this section, we try to prove the exactness of the relaxation mathematically. The following theorem is given.

**Theorem 1.** For $\xi_{ij}$ with any value, the second-order-cone relaxation (23) will always be exact.

**Proof of Theorem 1.** The proof is carried out by the contradiction.

Assume after convex optimization (24), a $\xi_{ij}^*$ is obtained on the transmission line \(i\sim j\) when

$$x_{ij}^2 + \frac{1}{4}y_{ij}^2 + \frac{1}{12}z_{ij}^2 < z_{ij}\xi_{ij}^*. \hspace{1cm} (25)$$

Therefore, the gap of (25) can be recorded as $\epsilon_{ij}^*$, where

$$\epsilon_{ij}^* = z_{ij}\xi_{ij}^* - x_{ij}^2 - \frac{1}{4}y_{ij}^2 - \frac{1}{12}z_{ij}^2. \hspace{1cm} (26)$$

Due to (25), it can be known that

$$\epsilon_{ij}^* > 0. \hspace{1cm} (27)$$

So there must be an arbitrary constant $\epsilon'_{ij}$, where

$$\epsilon'_{ij} > \epsilon_{ij}^* \geq 0. \hspace{1cm} (28)$$

In the power-flow calculation, $\epsilon_{ij}$ is a defined variables with no real physical meaning. As shown in the optimization, there are no specific limitations for this. Therefore, both of $\epsilon_{ij}^*$ and $\epsilon'_{ij}$ can be the optimized solution. Since in convex optimization (24), the local optimum
will be the global one. Thus, there will be only one unique solution, and the optimum of $\epsilon^*_ij$ is refuted.

That implies that, unless the exactness of the relaxation is obtained, the optimal solution of convex optimization will not be obtained. This finishes the proof. □

Since the exactness of the relaxations can be ensured, with the value of $\xi, x_{ij}, y_{ij}$, and $z_{ij}$ from the optimal solution in (24), the value of voltage variables $x_i, x_j, x_{ij1}$, and $x_{ij2}$ can be recovered with (12).

3.4. Solving Process

The whole solving process of the proposed convex-relaxation-based method for power flow can be summarized in the following Algorithm 1.

**Algorithm 1:** Solve power-flow problem by the proposed method

| Input: | (P_i, Q_i) of the PQ nodes; (U_i, P_i) of the PV nodes |
| Output: | (U_i, \theta_i) of the PQ nodes; (Q_i, \theta_i) of the PV nodes |
| 1. Define variables $x_i, x_j, x_{ij1}, x_{ij2}$; define variables $x_{ij}, y_{ij}, z_{ij}$; |
| 2. Introduce the variable $\xi_{ij}$; |
| represent $x_i, x_j, x_{ij1}, x_{ij2}$ with $x_{ij}, y_{ij}, z_{ij}$ and $\xi_{ij}$; |
| 3. Construct the optimization problem with limited ranges and set the objective function; |
| 4. Relax the non-convex equality constraints with second-order-cone relaxation; |
| 5. Solve the optimization problem and recover the electrical variables from $x_i, x_j, x_{ij1}, x_{ij2}$, $x_{ij}, y_{ij}, z_{ij}$ and $\xi_{ij}$; |

From the above, the whole process can be divided into five steps.

- **Step 1:** The first step focuses on the variables’ relaxations. As stated in (4)–(7), the variables $x_i, x_j, x_{ij1}$, and $x_{ij2}$ are defined. Afterwards, the variables $x_{ij}, y_{ij}$, and $z_{ij}$ are defined according to $x_i, x_j, x_{ij1}$, and $x_{ij2}$ in (8)–(10).
- **Step 2:** The slack variable $\xi_{ij}$ is introduced in (12). Thus, the variables $x_i, x_j, x_{ij1}$, and $x_{ij2}$ can be represented with $x_{ij}, y_{ij}, z_{ij}$, and $\xi_{ij}$.
- **Step 3:** With the newly-defined variables, the power-flow problem can be represented in an optimization format, where the defined variables should be limited according to the ranges of voltage. Moreover, the power flow equation should be satisfied as an linear constraint. The objective function is set with the slack variable $\xi_{ij}$.
- **Step 4:** Relax the non-convex constraint (22) with the second-order-cone relaxation as (23).
- **Step 5:** Solve the convex optimization problem (24) and obtain the value of $x_{ij}, y_{ij}, z_{ij}$ and $\xi_{ij}$. Then recover $x_i, x_j, x_{ij1}$, and $x_{ij2}$ with (12). It needs to be mentioned that the active power and reactive power can be obtained from the optimization (24). Additionally, there is no need to recover them.

4. Case Study

In this section, a real offshore wind farm will be implemented as the case study. In this wind farm, there are 103 wind turbines in total. The topology of the wind farm is shown in Figure 3, which is radial.

The Arabic numerals shown in Figure 3 denote the number of wind turbines. The differences between cables come from the distance to the offshore boosters. The nearer to the booster, the more over-current should be considered. The information on the connected wind turbines and their offshore submarine cables is shown in Table 1.
Figure 3. The topology of the offshore wind farm.

Table 1. The information of the connected wind turbines and their offshore submarine cables.

| Loop No. | Wind Turbine No. | Length of Cables (km) | Cross-Section of Cables (mm) |
|----------|------------------|-----------------------|-----------------------------|
| 1        | 18               | 12,784                | 3 × 300                     |
| 2        | 12               | 16,982                | 3 × 300                     |
| 3        | 6                | 17,896                | 3 × 300                     |
| 4        | 31               | 4010                  | 3 × 500                     |
| 5        | 46               | 5625                  | 3 × 500                     |
| 6        | 72               | 16,987                | 3 × 300                     |
| 7        | 77               | 2057                  | 3 × 500                     |
| 8        | 24               | 10,247                | 3 × 300                     |
| 9        | 38               | 1987                  | 3 × 500                     |
| 10       | 39               | 2367                  | 3 × 500                     |
| 11       | 53               | 10,782                | 3 × 500                     |
| 12       | 66               | 16,442                | 3 × 300                     |
| 13       | 84               | 12,783                | 3 × 500                     |
| 14       | 60               | 10,257                | 3 × 300                     |
| 15       | 97               | 13,578                | 3 × 300                     |
| 16       | 103              | 11,264                | 3 × 300                     |
To test the effect of the proposed method, we evaluate the results from these aspects: 1. the exactness of the relaxations; 2. a comparison of the solving efficiency and solving accuracy with other methods.

The simulation is implemented by the Matlab program in the computer with an Intel 7th gen CPU. By constructing the optimization problem (24) in Yalmip [27], the power-flow problem can be solved by the optimizer toolbox Cplex [28].

Firstly, to test the exactness of the relaxation, for each offshore cable, we define the relaxation gap as
\[
\epsilon_{ij} = |x_{ij}^2 + \frac{1}{4}y_{ij}^2 + \frac{1}{12}z_{ij}^2 - z_{ij}\xi_{ij}|. \tag{29}
\]

By performing sampling calculation, we randomly select ten branches each time and calculate the average gap value of their gaps by
\[
\overline{\epsilon}_{ij} = \frac{\sum |x_{ij}^2 + \frac{1}{4}y_{ij}^2 + \frac{1}{12}z_{ij}^2 - z_{ij}\xi_{ij}|}{10}. \tag{30}
\]

The values of \(\overline{\epsilon}_{ij}\) changing in each sample are shown in the Figure 4.

![Average Gap](image)

Figure 4. The value of \(\overline{\epsilon}_{ij}\) changing in each sample.

From this figure, we can know that by setting the calculation accuracy to \(10^{-6}\), each sample of the test meets the requirement. Although the gaps vary in the test, all of the gaps are very small. By carefully checking the gap of each branch, it can be found that all of them are less than \(10^{-6}\). The accuracy standard is set as \(10^{-6}\) in the solver considering the computational efficiency.

Secondly, to test the solving efficiency, we compare the proposed method with the Gauss–Seidel Method. The test results are shown in Table 2. The iteration times of the algorithm and the computational time are compared.

| Method          | Computational Time | Iteration Times |
|-----------------|--------------------|-----------------|
| Gauss–Seidel    | 1.7 s              | 486             |
| Proposed Method | 0.68 s             | 4               |

From the comparison, it can be found that the proposed method requires fewer iterations and thus has higher computational efficiency. The solution results of each node and branch’s power flow have been compared as well, which demonstrates the same solution.

Moreover, we extend the case study into a bigger wind farm, which is a combination of three proposed wind farms whose topology is shown in Figure 5.
In the case study, we calculate the power flow of this big system with the Gauss–Seidel Method and the proposed method. To quantify the solution differences of these two methods, we define the variable $\tau$ as

$$\tau = \sum_{i=1,2,3...k} |P_{i}^{\text{proposed method}} - P_{i}^{\text{Gauss–Seidel method}}| + |Q_{i}^{\text{proposed method}} - Q_{i}^{\text{Gauss–Seidel method}}|. \quad (31)$$

In this equation, $k$ indicates the total nodes of the whole system. The result of the case study is shown in the Table 3.

**Table 3. Power flow results’ comparison.**

| Method               | Computational Time | Iteration Times | $\tau$ |
|----------------------|--------------------|-----------------|--------|
| Gauss–Seidel         | 1.27 s             | 384             |        |
| Proposed Method      | 0.82 s             | 21              | 0      |

From the Table 3, it can be found that when the system becomes bigger, the solving efficiency is improved. While the proposed method is still faster than the Gauss–Seidel method, the value of $\tau$ equals zero under the tolerance of $10^{-6}$, which demonstrates the same electrical solution of the two methods. In conclusion, the proposed convex relaxation shows exact relaxations in the power-flow calculation, which guarantees the correctness of the solution. Moreover, the solving efficiency is relatively high compared with the existing method.

5. Conclusions

In this paper, a convex-optimization-based method for power-flow calculation of offshore wind systems has been proposed. The convex relaxation is implemented with an arbitrary defined variable, of which the exactness is proved. The method is applied in a real offshore wind farm. In the case study, both the solving efficiency and the correctness of the solution are verified. Additionally the superiority of the method is performed. The main findings of this manuscript can be summarized as
• The computational time of the proposed method is significantly improved compared with the Gauss–Seidel method.
• The average gap of the proposed convex relaxation is within the set accuracy standard.

In our future work, the proposed method will be compared with more existing methods, such as the fast decoupled load-flow method, the direct-current load-flow method, and the deep-learning-based artificial intelligence method. Moreover, technical improvements should be implemented to boost the computational efficiency further.

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Notations
The following notations are used in this manuscript:

\[ V_i = U_i \angle \theta_i \] Voltage of node \( i \)
\[ U_i \] Amplitude of \( V_i \)
\[ \angle \theta_i \] Phase angle of the \( V_i \)
\[ P_i \] Active power of node \( i \)
\[ Q_i \] Reactive power of node \( i \)
\[ S_i = P_i + i Q_i \] Apparent power of node \( i \)
\[ I_i \] Injected current of node \( i \)
\[ P_{ij} \] Transmitted active power of the transmission line \( i \sim j \)
\[ Q_{ij} \] Transmitted reactive power of the transmission line \( i \sim j \)
\[ S_{ij} = P_{ij} + i Q_{ij} \] Transmitted apparent power of the transmission line \( i \sim j \)
\[ G_{ij} \] Conductance of the transmission line \( i \sim j \)
\[ B_{ij} \] Susceptance of the transmission line \( i \sim j \)
\[ Y_{ij} = G_{ij} + i B_{ij} \] Admittance of the transmission line \( i \sim j \)

* The conjugate operator

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