The running coupling in lattice Landau gauge with unquenched Wilson fermion and KS fermion

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The running coupling of the Wilson fermion (JLQCD/CP-PACS) and that of Kogut-Susskind (KS) fermion (MILC) are measured in the lattice Landau gauge QCD in \( \tilde{MOM} \) scheme. The quark propagator of the KS fermion is also measured and we find that it is infrared suppressed. The renormalization factor of the running coupling and the tadpole renormalization define the scale of the quark wave function. Effects of the \( A_\mu^2 \) condensates of a few GeV\(^2\) are observed in the running coupling and also in the quark propagator.

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1. Introduction

The mechanism of dynamical chiral symmetry breaking and confinement is one of the most fundamental problems of hadron physics. The propagator of dynamical quarks in the infrared region provides information on dynamical chiral symmetry breaking and confinement. In the previous paper\cite{1}, we measured gluon propagators and ghost propagators of unquenched gauge configurations obtained with quark actions of Wilson fermions (ILQCD/CP-PACS) and those of Kogut-Susskind (KS) fermions (MILC)\cite{2,3} in Landau gauge and observed that the configurations of the KS fermion are closer to the chiral limit than those of Wilson fermions.

In the analysis of running coupling obtained from the gluon propagator and the ghost propagator, with use of the operator product expansion of the Green function, we observed possible contribution of the quark condensates and $A^2$ condensates in the configurations of the KS fermion\cite{1}. The quark propagator of quenched KS fermion was already measured in \cite{4}, and possible contribution of these condensates are reported. Unquenched KS fermion propagator of $20^3 \times 64$ lattice (MILC$_c$) was measured in \cite{5}, but to distinguish the gluon condensates and the quark condensates, it is desirable to measure the quark propagator of larger lattice (MILC$_f$) and to compare with data of MILC$_c$. We measure quark propagator of gauge configuration of 1) MILC$_c$ $20^3 \times 64$, $\beta = 6.76$ and 6.83 and 2) MILC$_f$ $28^3 \times 96$, $\beta = 7.09$ and 7.11, using the Staple+Naik action\cite{6}.

2. The running coupling in \(\widetilde{\text{MOM}}\) scheme

The running coupling in \(\widetilde{\text{MOM}}\) scheme is given with use of the vertex renormalization factor $\tilde{Z}_1$ as

$$\alpha_s(q) = \alpha_R(\mu^2)Z_R(q^2,\mu^2)G_R(q^2,\mu^2)^2 = \frac{\alpha_0(\Lambda_{\text{UV}})}{\tilde{Z}_1(\beta,\mu)^2}Z(q^2,\beta)G(q^2,\beta)^2$$

where $Z$ and $G$ are the gluon and the ghost dressing function, respectively, and $\tilde{Z}_1$ is the vertex renormalization factor.

**Figure 1:** The $\log G(qa)$ as a function of $\log qa$ ($a$ is the lattice spacing of MILC$_f$) of MILC$_f$ $\beta_{\text{imp}} = 7.09$ (stars) and MILC$_c$ $\beta_{\text{imp}} = 6.76$ (triangles).

**Figure 2:** The running coupling $\alpha_s(q)$ as a function of $\log_{10} q(\text{GeV})$ of MILC$_f$ $\beta_{\text{imp}} = 7.09$ (stars) and 7.11 (diamonds).
Our data suggests that the gluon propagator is infrared finite as in Dyson-Schwinger equation\cite{7}. The running coupling in the infrared is suppressed as shown in Figure 2, but the main origin is the suppression of the ghost propagator in the infrared.

We parametrize the difference of the lattice data and the pQCD 4-loop result\cite{10,11} in the $1\text{GeV} < q < 6\text{GeV}$ region in the form, with a minor correction term $d$ as

$$
\Delta \alpha_s(q) = \alpha_{s,latt}(q) - \alpha_{s,\text{pert}}(q) = \frac{c_1}{q^2} + \frac{c_2}{q^4} + d, \quad (2.2)
$$

where the $A^2$ condensates gives $c_1$ and the gluon condensates and/or quark condensates gives $c_2$. Although statistics is not large, running coupling of CP-PACS suggests $c_1 \sim 2\text{GeV}$. The MILC data suggests $c_1 \sim 4\text{GeV}$ and $c_2 \sim -2\text{GeV}$. There is an analytical calculation that suggests correlation between the $A_2$ condensates and the the horizon function parameter\cite{12}.

### 3. The quark wave function renormalization

We renormalize the quark field as $\psi_{\text{bare}} = \sqrt{Z_2} \psi_R$, and define the colorless vector current vertex\cite{7}

$$
\Gamma_\mu(q,p) = S^{-1}(q)G_\mu(q,p)S^{-1}(q+p), \quad (3.1)
$$

where

$$
G_\mu(p,q) = \int d^4xd^4y e^{i(p \cdot y + q \cdot x)} \langle q(y) \bar{q}(x) \gamma_\mu q(x) \bar{q}(0) \rangle \quad (3.2)
$$

and $S(q)$ is the quark propagator.

The vertex of the vector current with $p = 0$ is written as

$$
\Gamma_\mu(q) = \delta_{a,b}\{g_1(q^2)\gamma_\mu + ig_2(q^2)p_\mu + g_3(q^2)q_\mu \bar{q} + ig_4(q^2)[\gamma_\mu, \bar{q}]\} \quad (3.3)
$$

The Ward identity implies

$$
Z_{\psi}^{\text{MOM}} \Gamma_\mu(q) = -i\frac{\partial}{\partial q^\mu} S^{-1}(q) \quad (3.4)
$$

where $Z_{\psi}^{\text{MOM}} g_1(q^2) = Z_\psi(q^2)$ and $Z_{\psi}^{\text{MOM}} = 1$ when there is no lattice artefact.

The running coupling $g$ of the ghost, anti-ghost, gluon coupling

$$
g(q) = \tilde{Z}_1^{-1}Z_3^{1/2}(\mu^2, q^2)\tilde{Z}_3(\mu^2, q^2) \quad (3.5)
$$

and that of quark, gluon coupling

$$
g(q) = Z_1^{-1}Z_3^{1/2}(\mu^2, q^2)Z_2(\mu^2, q^2) \quad (3.6)
$$

are identical due to the Slavnov-Taylor identity. At the renormalization point $q = \mu$, we fix $Z_2(\mu^2, \mu^2) = 1$ and $Z_3(\mu^2, \mu^2) = 1$, and so $\tilde{Z}_1^{-1}Z_3^{1/2}\tilde{Z}_3 = Z_1^{-1}Z_3^{1/2}Z_2$ implies $\tilde{Z}_1 = Z_1$. On the other hand $Z_{\psi}(q^2) = Z_{\psi}^{\text{MOM}} g_1(q^2)$ where for 16 tastes

$$
g_1(q^2) = \frac{1}{48N_c} \text{Tr}[\Gamma_\mu(q, p = 0)(\gamma_\mu - q_\mu \bar{q}/q^2)] \quad (3.7)
$$

When $Z_{\psi}^{\text{MOM}} = 1$, $g_1(\mu^2)$ is identical to $\tilde{Z}_1$ which is defined by the renormalization of the running coupling on the lattice defined at $\mu \sim 6\text{GeV}$ and summarized in Table \[\text{\cite{12}}\].
The running coupling in lattice Landau gauge

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The MILC configurations are produced by the Asqtad action with the bare masses 13.6MeV, 27.2MeV and 68.0MeV in the MILC, and 11.5MeV, 65.7MeV and 82.2MeV in the MILC. The gluon wave function renormalization $Z_3$ of the Asqtad action is renormalized by $1/u_0^2$, where $u_0$ is the fourth root of the plaquette value. The tadpole renormalization on quark wave function is $u_0$ and we normalize $S(q)$ by multiplying the average of $u_0 Z_3^i$ i.e. 1/1.36 for MILC and 1/1.38 for MILC. The quark propagators $Z_2(q)$ after this renormalization are infrared suppressed as shown in Figures 3 and 4. The apparent difference in the formulae of [5] and our work are only in the expression and in fact they are equivalent and the results agree with [5].

Using the pQCD result of the inverse quark propagator [6, 7] and $(A^2)$ and $\bar{c}_2$ (the contribution from the mixed condensate $\langle \bar{q} A q \rangle$ [8]) as fitting parameters, we calculate

$$Z_\psi(q) = \frac{1}{Z_2(q)} = Z_\psi^{pert}(q^2) + \frac{\frac{\alpha(q)}{\mu(q)}}{q^2} - \frac{\gamma_0 + \gamma_2}{q^2} A^2(\mu) \frac{\langle A^2(\mu) \rangle}{4(N_c^2 - 1)} Z_\psi^{pert}(\mu^2) + \frac{\bar{c}_2}{q^4} \tag{3.8}$$

| $\beta_1/K_{sea}$ | $\beta_2/K_{sea}$ | average | configurations |
|------------------|------------------|---------|----------------|----------------------------------|
| CP-PACS          | 1.07(8)          | 1.21(10)| 1.14           | $K_{sea} = 0.1357, 0.1382$      |
| MILC             | 1.49(11)         | 1.43(10)| 1.46           | $\beta_1, 2 = 6.83, 6.76$       |
| MILC             | 1.37(9)          | 1.41(12)| 1.40           | $\beta_1, 2 = 7.11, 7.09$       |

Table 1: The $1/Z_1^2$ factor of the unquenched SU(3).

Figure 3: The renormalization factor $Z_2(q)$ of MILC with bare mass $m_0 = 13.6$MeV stars, 27.2MeV diamonds and 68.0MeV triangles.

Figure 4: Same as Fig. 3 but quark of MILC with bare mass $m_0 = 11.5$MeV stars, 65.7MeV diamonds and 82.2MeV triangles. The last two almost overlap.

Figure 5: The quark wave function renormalization factor $Z_\psi(q)$ of MILC with bare mass $m_0 = 27.2$MeV. Dashed line is the pQCD result and solid line is the result including the $A^2$ condensates. The points below 0.6GeV are $g_1(q^2)$ approximated by the running coupling obtained by the ghost anti-ghost gluon vertex.
where $\alpha(q)$ are data calculated in the MOM scheme using the same MILC$_f$ gauge configurations. We estimate that $\bar{c}_2$ is small. In Figure 5, we show a fit of the MILC$_f$ $m_0 = 13.6$MeV data, by taking the renormalization point at $\mu \sim 3.8$GeV with use of $\langle \Lambda^2(\mu) \rangle \sim 1.6(3)$GeV$^2$ and $\bar{c}_2 = 0$. These parameters are consistent with $[7]$. In $q > 1.5$GeV region, dynamical mass of a quark in pQCD is expressed as $[4]$

$$M(q) = -\frac{4\pi^2 d_M \langle \bar{q}q \rangle_\mu}{3q^2 \log(\mu^2/\Lambda_{QCD}^2)} + \frac{m(\mu^2) \log(\mu^2/\Lambda_{QCD}^2)}{\log(q^2/\Lambda_{QCD}^2)} d_M,$$  

where $d_M = \frac{12}{33-2N_f}$. The second term is the contribution of the massive quark. In the analysis of the lattice data, we observe that the quark condensates $-\langle \bar{q}q \rangle_\mu$ and $\Lambda_{QCD}$ roughly satisfy $-\langle \bar{q}q \rangle_\mu = (0.70\Lambda_{QCD})^3 [8]$, with $\Lambda_{QCD} = 0.69$GeV.

For the global fit of $M(q)$, we try the phenomenological monopole type $[13]$

$$M(q) = \frac{c\Lambda^3}{q^2 + \Lambda^2} + m_0$$

where $m_0$ is the bare quark mass.

Figures 6 and 7 show the mass function $M(0)$ of MILC$_f$ and MILC$_c$, respectively.

**Figure 6**: The dynamical mass of the MILC$_f$ quark with bare mass $m_0 = 13.6$MeV (stars), 27.2MeV (diamonds) and 68.0MeV (triangles) and the phenomenological fits.

**Figure 7**: The dynamical mass of the MILC$_c$ quark with bare mass $m_0 = 11.5$MeV (stars), 65.7MeV (diamonds) and 82.2MeV (triangles) and the phenomenological fits.

We observe that the product $c\Lambda$ becomes larger as the bare quark mass becomes heavy and it depends on $\beta$ in the case of MILC$_f$ but not in the case of MILC$_c$. In the case of MILC$_f$ $m_0 = 13.6$MeV, the lowest three momentum points of $M(q)$ are systematically smaller than the other points. Ignoring these points we find $c\Lambda$ of $\beta = 7.09$ is smaller than that of 7.11 and that of MILC$_c$ as shown in Figure 8. In the chiral limit $m_0 \to 0$, we obtain $M(0) = 0.35 \sim 0.37$GeV, which is larger than $[5]$ and that of the Wilson fermion $[13]$ by about 20%.

**4. Discussion and conclusion**

We measured running coupling of unquenched Wilson fermion and KS fermion and the quark wave function renormalization factor and mass function of the KS fermion. The renormalization

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factor $Z_\psi$ obtained as $1/Z_2$ is infrared finite. The Kugo-Ojima confinement criterion favours infrared vanishing of $Z_\psi$ and $g_1(q^2)$ approximated by the running coupling suggests this behavior, but it could be a lattice artefact. With infrared finite $Z_2$, infrared vanishing of $Z_1\psi$ is necessary for the confinement criterion to be satisfied.

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