Density Perturbations in the Universe from Massive Vector Fields

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Abstract. I discuss the possibility of using a massive vector field to generate the density perturbation in the Universe. I find that a scale-invariant superhorizon spectrum of vector field perturbations is possible to generate during inflation. The associated curvature perturbation is imprinted onto the Universe following the curvaton scenario. The mechanism does not generate a long-range anisotropy because an oscillating massive vector field behaves as a pressureless isotropic fluid.

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INTRODUCTION

The use of scalar fields in theoretical cosmology is extensive. For example, the dynamics of inflation is taken to be controlled by a scalar field (the inflaton) [1]. In fact, in many inflation models several scalar fields are employed (e.g. hybrid inflation, assisted inflation). Alternatives to minimal inflation, such as the curvaton paradigm, also use scalar fields [2]. Scalar fields have been used in other aspects of cosmology such as Dark Energy (quintessence) or baryogenesis (Affleck-Dine mechanism). In fact one might wonder whether we could do any cosmology without scalar fields.

Considering that scalar fields play an important role in the Early Universe is well motivated by the theory. Indeed, scalar fields are ubiquitous in theories beyond the standard model such as Supersymmetry (scalar partners, flat directions) and String Theory (moduli fields). However, no scalar field has ever been observed. This means that, designing models using unobserved scalar fields undermines their predictability and falsifiability, despite the recent precision of the observational data.

Given the above addiction of cosmologists to scalar fields and the approaching collider experiments which might disprove some of our favourite theories, it is necessary to explore other means of tackling the cosmological problems. Here we discuss the possibility of generating the curvature perturbation in the Universe using a massive vector field.

Despite the fact that we do have evidence of the existence of both massive and massless vector fields their use in cosmology is limited. This is so because of the following fundamental prejudice against them: Were a vector field to dominate the Universe it would generate a large-scale anisotropy, in conflict with the CMB observations. A further prejudice applies to particle production of vector fields during inflation: Particle production of a light vector field is negligible because a massless vector field is conformally invariant and is not gravitationally generated during inflation. We will show that both these prejudices can be evaded. Moreover, there is evidence that some weak large-scale anisotropy might be present in the CMB ("Axis of Evil" [3]).
MASSIVE ABELIAN VECTOR FIELDS

The equations of motion

Consider a massive Abelian vector field with Lagrangian density

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} m^2 A_\mu A^\mu, \]  

(1)

where, the field strength tensor is \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \). The equations of motion are:

\[ \partial_\mu + \left( \partial_\mu \ln \sqrt{-g} \right) \left( \partial_\mu A_\nu - \partial_\nu A_\mu \right) + m^2 A_\mu = 0, \]

where \( g \) is the determinant of the metric tensor \( g_{\mu\nu} \). We assume that inflation lasts long enough to homogenise the Universe and inflate away its spatial curvature before the cosmological scales exit the horizon. Hence, we can use the flat-FRW metric:

\[ ds^2 = dt^2 - a(t)^2 dx^i dx^i. \]

Similarly, we expect inflation to homogenise the vector field too, so that \( A_\mu = A_\mu(t) \). Using the above, the equations of motion of the spatial components of the homogeneous vector field are [4]

\[ \ddot{A}_i + H \dot{A}_i + m^2 A_i = 0, \]

(2)

while for the temporal component we have \( A_t(t) = 0 \), where the dot denotes time derivative. Perturbing the field equations and switching to momentum space we obtain the equations of motion of the Fourier components of the vector field perturbations [4]

\[ \partial_i^2 + H \partial_i + m^2 \left( k^2 + \frac{(k/a)^2}{k^2 + (am)^2} \right) \delta \mathcal{A}_i^\perp = 0, \]

(3)

\[ \partial_i^2 + \left( 1 + \frac{2k^2}{k^2 + (am)^2} \right) H \partial_i + m^2 \left( k^2 + \frac{(k/a)^2}{k^2 + (am)^2} \right) \delta \mathcal{A}_i^\parallel = 0, \]

(4)

where \( \delta \mathcal{A}_i^\parallel \equiv k_i (k_j \delta \mathcal{A}_j) / k^2 \) is the component parallel to \( k_i \) and \( \delta \mathcal{A}_i^\perp \equiv \delta \mathcal{A}_i - \delta \mathcal{A}_i^\parallel \), with \( k^2 \equiv k_i k_j \). Using the above equations we can investigate particle production of \( A_\mu \) during inflation. Our aim is to generate a scale invariant spectrum of vector field perturbations. We will ignore hereafter the longitudinal component, since \( \delta \mathcal{A}_i^\parallel \) can generate such a spectrum only under the condition \( m \ll e^{-N_* H_*} \) [4], where \( N_* \approx 40 - 60 \) is the number of e-folds before the end of inflation corresponding to the cosmological scales and \( H_* \) is the inflationary Hubble scale. It turns out that this condition is impossible to meet if the field is to generate the curvature perturbation in the Universe [4]. Hence, in the following we focus on the transverse component \( \delta \mathcal{A}_i^\perp \) and we drop the \( \perp \) symbol.

Particle production

Solving Eq. (3) with boundary condition \( \lim_{k/aH \to \infty} \delta \mathcal{A}_k = e^{ik/aH} / \sqrt{2k} \), (i.e. matching to the Bunch-Davies vacuum at early times so that the perturbations start out as vacuum fluctuations) we get [4]

\[ \delta \mathcal{A}_k = \sqrt{\frac{\pi}{aH}} \frac{e^{i\pi(v-\frac{1}{2})/2}}{1 - e^{i2\pi v}} \left[ J_v(k/aH) - e^{i\pi v} J_{-v}(k/aH) \right], \]

(5)
where with $J_\nu$ we denote Bessel functions of the first kind and $v \equiv \sqrt{\frac{1}{2} - (m/H)^2}$. The power spectrum of the generated perturbations is obtained in the limit of late times, when the perturbations have grown to superhorizon scales. Using the above we find \( (4) \)

\[
\mathcal{P}_\delta \equiv \frac{k^3}{2\pi^2} \lim_{k/aH \to 0} |\delta \mathcal{A}_k|^2 \approx \frac{8\pi \Gamma(1 - v)|\Im v|^2}{(1 - \cos 2\pi v)} (\frac{aH}{2\pi})^2 \left( \frac{k}{2aH} \right)^{3 - 2v} \tag{6}
\]

For a light field $v \approx \frac{1}{2}$ and so $\mathcal{P}_\delta \text{ vac} \approx (k/2\pi)^2$. This is the vacuum spectrum, that can be readily obtained using the Bunch-Davies vacuum expression for $\delta \mathcal{A}_k$. Thus, in the massless limit, we recover the conformal invariant result where there is no particle production. However, what we are after is a scale invariant spectrum. From Eq. (6) it is evident that scale invariance requires $v \approx \frac{3}{2}$, which is equivalent with: $m^2 \approx -2H^2$.

This condition can be understood by noting that $A_\mu$ is the *comoving* vector field. Indeed, the mass term in Eq. (1), when using the flat-FRW metric, becomes $\delta \mathcal{L}_m = \frac{1}{2} m^2 A_\mu A^\mu = \frac{1}{2} m^2 (A_i^2 - a^{-2} A_i A_i)$. Since the Lagrangian is a physical quantity we find that the components of the physical vector field are $W_i \equiv A_i / a$. Introducing these into Eq. (2) during inflation we obtain: $\ddot{W}_i + 3\dot{W}_i + (2H^2 + m^2)W_i = 0$ \( (4) \). Hence, the equation of motion is identical to the one of a scalar field with mass $\bar{m}^2 = 2H^2 + m^2$. Thus, a scale invariant spectrum is generated when the *physical* vector field becomes light $\bar{m} \to 0$, i.e. its Compton wavelength exceeds the horizon. The power spectrum, in this case, is given by the Hawking temperature $\mathcal{P}_\delta = \mathcal{P}_\delta / a^2 \approx (H/2\pi)^2$.

**VECTOR CURVATON**

We now employ the scale invariant superhorizon spectrum of vector field perturbations obtained above to generate the curvature perturbation. To retain isotropy, the vector field cannot be the inflaton. The vector field can act as a *curvaton* provided it safely dominates the Universe after inflation. Hence, studying its post-inflationary evolution is necessary.

The stress-energy tensor is: $T_{\mu\nu} = \frac{1}{4\sqrt{-g}} F_{\mu\rho} F^{\rho\sigma} - F_{\mu\rho} F^{\rho\nu} + m^2 (A_\mu A_\nu - \frac{1}{4} g_{\mu\nu} A_{\rho} A^{\rho})$ \( (4) \). This can be written as $T_{\mu\nu} = \text{diag}(\rho_A, -p_\perp, -p_\perp, +p_\perp)$, which is similar to a perfect fluid with the crucial difference that the pressure along the longitudinal direction is of opposite sign to the transverse pressure $p_\perp$, where $\rho_A \equiv \rho_{\text{kin}} + V$ and $p_\perp \equiv \rho_{\text{kin}} - V$, with $\rho_{\text{kin}} \equiv -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ and $V \equiv -\frac{1}{2} m^2 A_{\mu} A^{\mu}$. If $p_\perp \neq 0$ then the fluid is anisotropic and cannot be allowed to dominate the Universe. This is why $A_\mu$ cannot be the inflaton.

However, suppose that, after the end of inflation a phase transition renders the mass $m^2$ positive for the vector field. This not only restores Lorentz invariance in the vacuum, but allows the field to begin coherent oscillations after inflation, when $m > H(t)$, as evident from Eq. 2. During these quasi-harmonic oscillations we have on average $\bar{\rho}_{\text{kin}} \approx V / m^2 \approx 0$. Hence, the coherently oscillating homogeneous massive Abelian vector field behaves as pressureless *isotropic* matter. Consequently, its density decreases as $\rho_A \propto a^{-3}$ \( (4) \), so that it can dominate the radiation background without introducing significant anisotropy. Therefore, the vector field can act as a curvaton and impose, upon domination, its own curvature perturbation onto the Universe. Hence, the observed
curvature perturbation $\zeta = 5 \times 10^{-5}$ can be attributed to the vector curvaton field, i.e.

$$\zeta = \zeta_A \equiv -H \frac{\delta \rho_A}{\dot{\rho}_\Lambda} = \frac{1}{3} \frac{\delta \rho_A}{\rho_A} \left|_{\text{dec}} \right. \simeq \frac{2}{3} \frac{\delta A}{A} \left|_{\text{dec}} \right. = \frac{2}{3} \frac{\delta A}{A} \left|_{\text{osc}} \right. \approx \frac{2}{3} \frac{a_{\text{osc}}}{a_s} \frac{\delta A}{A} \left|_{s} \right. \simeq \frac{H_*}{3\pi W_{\text{osc}}}, \quad (7)$$

where we defined $\zeta_A$ on a foliage of spacetime along spatially flat hypersurfaces and we used that: $\rho_A \propto a^{-3}$ at the decay time of the field (denoted by ‘dec’); $\rho_A^{\text{dec}} \approx 2V \propto A^2$; $\delta A/A \simeq$ const. during oscillations but, before their onset (denoted by ‘osc’), $A$ is frozen ($A_{\text{osc}} \approx A_s$) and $\delta A/A \approx a$; $\delta A_s \simeq a_s H_* / 2\pi$ and $W_{\text{osc}} = (A/a)_{\text{osc}} \approx A_s / a_{\text{osc}}$ with $^{\text{**}}$ denoting the epoch when the cosmological scales exit the horizon.

As shown in Eq. (7), $\zeta_A \propto \delta \rho_A / \rho_A$. Since $\rho_A$ is a scalar quantity, the generated perturbations are scalar and not vector in nature, despite originating from a vector field.

**CONCLUSIONS**

A vector field can generate the curvature perturbation in the Universe. To obtain a super-horizon spectrum of perturbations, its mass should satisfy the condition: $m^2 \approx -2H^2$. After inflation, the vector field can act as a curvaton provided $m > H$ (i.e. the vacuum is Lorentz invariant). In this case the vector field undergoes quasi-harmonic oscillations, during which it acts as a pressureless isotropic fluid. Hence, it can dominate the Universe and impose its own, scalar curvature perturbation without introducing any anisotropy.

The challenge is to realise the above conditions in theories beyond the standard model. In supergravity, one may dispense with the requirement that $m^2 < 0$ during inflation. Indeed, the gauge kinetic function $f$ of vector fields is expected to vary significantly during inflation. This is because supergravity corrections to the potential give the scalar fields masses of order $H_*$, which means that they are fast-rolling down the potential slopes resulting in $\dot{f} / f \sim H$. It can be shown that, if $f \propto a^2$, then a light vector field can obtain a scale invariant spectrum of perturbations without the need of $m^2 < 0$.

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