Alcock–Paczynsky Test with the Evolution of Redshift-space Galaxy Clustering Anisotropy

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Received 2019 April 6; revised 2019 June 8; accepted 2019 June 26; published 2019 August 22

Abstract

We develop an improved Alcock–Paczynsky (AP) test method that uses the redshift-space two-point correlation function (2pCF) of galaxies. Cosmological constraints can be obtained by examining the redshift dependence of the normalized 2pCF, which should not change apart from the expected small nonlinear evolution. An incorrect choice of cosmology used to convert redshift to comoving distance will manifest itself as redshift-dependent 2pCF. Our method decomposes the redshift difference of the two-dimensional correlation function into the Legendre polynomials whose amplitudes are modeled by radial fitting functions. Our likelihood analysis with this 2D fitting scheme tightens the constraints on $\Omega_m$ and $\Lambda$ by $\sim$40% compared to the method of Li et al. that uses one-dimensional angular dependence only. We also find that the correction for the nonlinear evolution in the 2pCF has a non-negligible cosmology dependence, which has been neglected in previous similar studies by Li et al. With an accurate accounting for the nonlinear systematics and use of full two-dimensional shape information of the 2pCF down to scales as small as 5 h$^{-1}$ Mpc it is expected that the AP test with redshift-space galaxy clustering anisotropy can be a powerful method to constraining the expansion history of the universe.

Key words: cosmological parameters – cosmology: theory – dark energy – large-scale structure of universe

1. Introduction

The accelerated expansion of the universe remains a deep mystery that remains unsolved by contemporary cosmology. The most popular cosmological model, $\Lambda$CDM, exquisitely fits the peaks of the cosmic microwave background (Hinshaw et al. 2013; Planck Collaboration et al. 2018), the distance–redshift relation of distant SNeIa (Perlmutter et al. 1999), and the distribution of galaxies (Li et al. 2016). All the more reassuring is the small set of parameters required and the simplicity in the underlying assumptions of homogeneity, Gaussianity, and near scale invariance of the initial perturbation spectrum.

However, we are still left with the unsavory prospect that if we are to believe $\Lambda$CDM then we are forced to include within our cosmic inventory a vacuum energy that is 120 orders of magnitude smaller than theoretical predictions (e.g., Weinberg 1989) and a large component of that is not contained within the standard model of particle physics (e.g., Trimble 1987).

This has led many theorists to consider alternative models that include a scalar field remnant from the big bang (see Li et al. 2011 for review) and modifications to Einstein’s general relativity (see Koyama 2016 for review). Thus, the endeavor of cosmology today does not lack a rich variety of theoretical models; rather, we lack precise observations of the expansion of the universe, which would allow us discern among the proposed models.

A redshift survey is one of the most reliable ways to obtain data for uncovering the underlying cosmology of the universe. Many statistical tools have been developed to extract information on the initial conditions of the cosmic structures from the observed spatial distribution of galaxies. The size and depth of surveys have been improved thanks to technological advances. Upcoming and ongoing surveys such as DESI (Dark Energy Spectroscopic Instrument; Flaugher & Bebek 2014), eBOSS (Zhao et al. 2016), and PFS (Subaru Prime Focus Spectroscopy; Takada et al. 2014) will measure the locations of millions of galaxies up to a redshift of 2 or greater, together with many of those at redshifts below 1.

In the linear regime, clusters of galaxies on very large scales retain information about the early universe, and therefore can be directly compared with the predictions of theoretical models. If there are statistical measures of galaxy clustering that suffer from little nonlinear evolution effects either because the scale under study is safely in the linear regime or because they are intrinsically insensitive to nonlinear effects, they will be very useful for uncovering the physics of the early universe. In addition, they can be used for reconstructing the expansion history of the universe. This is because the conversion of observed redshift to comoving distance can create artificial systematic distortion of galaxy clustering when the cosmology adopted for the conversion is incorrect. Park & Kim (2010) adopted this idea and proposed using the shapes of the 2pCF, power spectrum, or the genus topology of large-scale structures in the universe as cosmic invariants for constraining the cosmological parameters governing the expansion history of the universe (see also Appleby et al. 2017, 2018b).

The Alcock–Paczynsky (AP) test is one of the statistical means of extracting cosmological parameters from galaxy redshift survey data (Alcock & Paczyński 1979; see Figure 1). It is a test of the geometry of cosmic objects or galaxy
distribution, which should appear isotropic at all redshifts if the objects and galaxy clusters are spherical or isotropic, and the redshift of the galaxy is converted to distance from correct objects and galaxy clusters are spherical or isotropic, and the distribution, which should appear isotropic at all redshifts if the assumption is correct.

For the test, the baryonic acoustic oscillation (BAO) feature in the galaxy 2pCF is often used due to its distinct excess at $r \sim 100 \ h^{-1} \text{ Mpc}$. The AP test with BAO has been proven to put a moderately powerful constraint on parameters like the matter density parameter $\Omega_m$, dark energy equation of state $w$, and so on. A weakness of the BAO method is that it uses clustering information on very large scales where the statistics of a given sample is relatively weak (Li et al. 2016).

An extension of the AP test came from the observation that even though the observed galaxy clustering in the redshift space is quite anisotropic due to the redshift-space distortion effects, its redshift evolution is almost conserved, with only small nonlinear effects (Li et al. 2015). The redshift evolution of the shape of the 2pCF turned out to be a powerful tool for uncovering the expansion history of the universe, as the signal of the 2pCF is much stronger near the $10 \ h^{-1} \text{ Mpc}$ scale than at the BAO scale of $\sim 100 \ h^{-1} \text{ Mpc}$ due to a much larger number of galaxy pairs. It has been shown that the shape of the redshift-space 2pCF down to the $6 \ h^{-1} \text{ Mpc}$ scale does not evolve as much as the incorrect cosmology assumption would distort it, making it possible to separate the systematic effects due to adopting an incorrect cosmology (Li et al. 2015, 2016, 2018; Appleby et al. 2017, 2018a, 2018b).

There is another stream of efforts by Ramanah et al. (2019) that tackles this problem using a Bayesian inference framework. Their work also aims to improve the cosmological constraint by utilizing two-point statistics of galaxies on the entire range of scales.

This work is a continuation of our effort to improve the AP test with galaxy clustering anisotropy. Previous works by Li et al. (2016, 2017, 2018) left room to improve the method, and we attempt to accommodate those past studies here. The main contribution of this paper is as follows.

(1) Li et al. (2016, 2017, 2018) used the angular dependence of the radially integrated 2pCF, which potentially dilutes the constraints from the radial shape of the 2pCF. We shall attempt to use both angular and radial shapes of the 2pCF to place the constraints and see how much the constraints improve.

(2) A potential caveat in the previous work is that the systematic correction due to intrinsic evolution of the redshift-space 2pCF was assumed to be cosmology-independent without proof. Now that we have a data set for multiple cosmologies (see Section 2.2 for detail), we can verify whether or not the assumption is correct.

The rest of the paper is as follows. In Section 2, we list and explain the N-body simulations and mock galaxy catalogs used for our analysis. We propose our new AP test method in detail in Section 3. We present the results in Section 4. Finally, we summarize and conclude in Section 5.

2. Data

2.1. Horizon Run 4

The Horizon Run 4 (HR4) simulation (Kim et al. 2015) is a massive cosmological simulation that evolved $N_p = 6300^3$ particles in a cubic box of a side length of $L_{\text{box}} = 3150/h^{-1}\text{ Mpc}$. It uses a flat $\Lambda$CDM cosmological model in concordance with a Wilkinson Microwave Anisotropy Probe (WMAP) 5 yr observation (Dunkley et al. 2009), where the matter density fraction, dark energy density fraction, and dark energy equation of state at $z = 0$ are $(\Omega_m, \Omega_{\Lambda}, w) = (0.26, 0.74, -1)$. The volume of the HR4 is big enough to simulate the formation of large-scale structures, and at the same time its force and mass resolutions are high enough to simulate the formation of individual galaxies down to a relatively small mass scale. Thanks to these unique features, the HR4 has been extensively used for cosmological model tests and study of galaxy formation under the influence of large-scale structures in the universe (Kim et al. 2015; Hwang et al. 2016; Li et al. 2016, 2017; Appleby et al. 2017, 2018b; Einasto et al. 2018; Uhlmann et al. 2018a, 2018b).

Rich information on structure formation is contained in the merger trees of dark matter (DM) halos forming in the big simulation box of HR4, constructed at 75 time steps between $z = 12$ and 0 with the time interval of $\sim 0.1 \text{ Gyr}$. In each snapshot, DM halos are found with the Friends-of-friends
(FoF) algorithm with the linking length of $l_{\text{FoF}} = 0.1 \, h^{-1} \text{Mpc}$. The minimum number of DM particles constructing DM halos is set to 30, which corresponds to the minimum DM halo mass of $M_{\text{min}} = 2.7 \times 10^{11} h^{-1} M_*$. The mock galaxy catalogs of HR4 were modeled by applying the most bound halo particle (MBP)-galaxy abundance matching to its DM halo merger tree (Hong et al. 2016). For each DM halo at each snapshot, we found the most gravitationally bound member particle (MBP). The particle is marked as the center of a “galaxy” if the given halo is isolated or if it is the most massive member halo (namely the central halo) in the merger events. On the other hand, for less massive member halos (satellites), we trace their “galaxies” from the time when they were isolated ones just before merger until they are completely disrupted. The time between the infall and the complete disruption of satellite galaxies is estimated by adopting the modified merger timescale model of Jiang et al. (2008):

$$t_{\text{merge}} = \frac{t_{\text{dyn}}}{\ln[1 + (M_{\text{host}}/M_{\text{sat}})]} \left( \frac{M_{\text{host}}}{M_{\text{sat}}} \right)^\epsilon,$$

where $\epsilon$, $M_{\text{host}}, M_{\text{sat}}$, $t_{\text{dy}}$ are the circularity of the satellite’s orbit, mass of central and satellite halos, and the orbital period of virialized objects, respectively. We set $\epsilon = 1.5$, which makes the $2p$CF of our mock galaxies match that of the SDSS Main galaxies down to scales below 1 $h^{-1}$ Mpc (Zehavi et al. 2011).

For our analysis, we divide the HR4 simulation box into 6 pieces in each dimension, thereby creating 216 sub-cube mock samples that are 525 $h^{-1}$ Mpc long on a side. This choice is made to have a sufficient number of samples for likelihood analysis. Galaxy surveys like the SDSS cover a larger volume of virialized objects, respectively. We set $a = 1.5$, which the $2p$CF of our mock galaxies match that of the SDSS Main galaxies down to scales below 1 $h^{-1}$ Mpc (Zehavi et al. 2011).

We adopt $10^{-3}$ galaxy per $(h^{-1} \text{Mpc})^3$ for the galaxy number density in the mock sample, which corresponds to 0.145 million in each sub-cube mock. This number density roughly correspond to the $r$-band magnitude $M_r = 5 \log h < -20.3$ at $z = 0$ (Choi et al. 2010) and it is also similar to the expected number density galaxies to be observed by the PFS survey. We will also show some results with 10 times more galaxies for comparison. We note that these mass cuts are rather arbitrary. The actual value to be used in the analysis of a given observational data should be determined by the survey data.

2.2. Multiverse Simulations

The multiverse simulations are a set of cosmological $N$-body simulations designed to see illustrate the effects of cosmological parameters on the clustering and evolution of cosmic structures. We changed the cosmological parameters around those of the standard concordance model with $\Omega_m = 0.26$, $\Omega_{\text{de}} = 0.74$, and $w = -1$. We used exactly the same set of random numbers to generate the initial density fluctuations of all the simulations, which allow us to make a proper comparison between the models with the effects of the cosmic variance compensated.

Five multiverse simulations we use in this paper are listed in Table 1. Two models have the matter density parameter shifted by 0.05 from the fiducial model, while the dark energy equation of state is fixed to $w = -1$. The other two quintessence models (Sefusatti & Vernizzi 2011) have $w$ shifted by 0.5 from the fiducial value of $-1$, while $\Omega_m$ is fixed to 0.26. These parameters are chosen so that they are reasonably large enough to cover the area in the $\Omega_{\text{m}}-w$ space constrained by many existing studies at the time WMAP 5 yr results have been announced (Spergel et al. 2003).

The power spectrum of each model is normalized in such a way that the rms of the matter fluctuation linearly evolved to $z = 0$ has $\sigma_8 = 0.794$ when smoothed with a spherical top hat with $R = 8 h^{-1}$ Mpc.

The number of particles evolved is $N_p = 2048^3$ and the comoving size of the simulation box is 1024 $h^{-1}$ Mpc. The starting redshift is $z_{\text{init}} = 99$ and the number of global time steps is 1980 with equal step size in the expansion parameter, $a$. We have used the CAMB package to calculate the power spectrum at $z_{\text{init}}$. We have extended the original GOTPM code (Dubinski et al. 2004) to gravitationally evolve particles according to the modified Poisson equation of

$$\nabla^2 \phi = 4\pi G a^2 \rho_m \delta_m \left[ 1 + \frac{D_{\text{de}}}{D_m} \frac{\Omega_{\text{de}}(a)}{\Omega_m(a)} \right],$$

where $D_{\text{de}}$ and $D_m$ are the linear growth factors of the dark energy and matter, respectively (see Sefusatti & Vernizzi 2011 for details).

3. Methodology

3.1. Shape of the Two-Point Correlation Function

In our AP test we use the two-dimensional shape of the galaxy 2PCF in the plane of the line of sight and across the line-of-sight directions. However, we exclude the region $r_s < r_{\text{cut}} = 3 h^{-1}$ Mpc of the plane from our analysis to minimize the impact of the highly nonlinear Fingers-of-God effects. This leaves us with only a mildly nonlinear contribution to $\xi$ in our analysis. Because we use only the shape of $\xi$ and not its amplitude in the AP test, we shall normalize $\xi$ by its volume integral up to the radial separation of $r_{\text{max}} = 45 h^{-1}$ Mpc. Namely, the normalization factor is

$$\xi_s(z) \equiv (2\pi) \int_0^1 d\mu \int_0^{r_{\text{max}}} r^2 dr \xi(r, \mu, z),$$

where $r$ is radial separation of galaxy pair and $\mu$ is the cosine of the angle between the line of sight and pair-separation direction. Then, the normalized 2pCF is

$$\hat{\xi}(r, \mu, z) \equiv \frac{\xi(r, \mu, z)}{\xi_s(z)}.$$

Regarding the choice of $r_{\text{max}} = 45 h^{-1}$ Mpc, we picked a value that is large enough to make the normalization insensitive to the highly nonlinear small-scale physics of the Fingers-of-God effect. We find our results are generally insensitive to the
choice of \( r_{\text{max}} \). The left panel of Figure 2 shows \( \hat{\xi} \) from one of the sub-cube mock galaxy samples.

In this work, we use the Legendre polynomials \( P_0 = 1 \), \( P_2 = (3 \mu^2 - 1)/2 \) and \( P_4 = (35 \mu^4 - 30 \mu^2 + 3)/8 \) to approximate the angular dependence of \( \hat{\xi} \) at each \( r \):

\[
\hat{\xi}(r, \mu, z) = \hat{\xi}_0(r, z) P_0(\mu) + \hat{\xi}_2(r, z) P_2(\mu) + \hat{\xi}_4(r, z) P_4(\mu).
\]

(5)

Here, \( \hat{\xi}_0 \), \( \hat{\xi}_2 \), and \( \hat{\xi}_4 \) are similar to the monopole, quadrupole, and hexadecapole moments at a given \( r \), except we exclude \( r_{\perp} < 3 \, h^{-1} \, \text{Mpc} \) in the fitting. In this case, we cannot use a decomposition formula like \( \hat{\xi} = \int \hat{\xi}(\mu) P(\mu) d\mu \) and have to make a \( \chi^2 \) fitting instead. We write \( \hat{\xi}_{0,2,4} \) when we refer to \( \hat{\xi}_0 \), \( \hat{\xi}_2 \), and \( \hat{\xi}_4 \) altogether in the rest of the paper. Thanks to this exclusion of the highly nonlinear part of the 2pCF, the fits by these three moments are highly accurate (Figure 2). In principle, \( \hat{\xi} \) can be decomposed into Legendre polynomials of arbitrary order, but we find that higher-order moments do not help much with tightening the constraint.

The cosmic variance in finite survey volume would give an intrinsic scatter to \( \hat{\xi}_{0,2,4} \). The uncertainty ranges of those moments are computed from the 216 sub-cube mock samples of HR4 and are shown as shaded regions in Figure 3. For our main analysis, we shall use the differences \( \Delta \hat{\xi} \) across different redshifts, which measure the shape change of 2pCF between redshift. We shall introduce the difference, \( \Delta \hat{\xi} \), in Section 3.4, along with our motivation for adopting it.

3.2. Geometrical Distortion Effects due to Choice of Incorrect Cosmology

If an incorrect cosmology is adopted when converting redshift to distance, then the apparent spatial distribution of galaxies will be distorted. Recalling that the comoving displacements are \( \Delta r \parallel = \Delta z [c/H(z)] \) and \( \Delta r_{\perp} = (1 + z) D_h(z) \Delta \theta \), for an object subtending \( \Delta z \) and \( \Delta \theta \) in the parallel and perpendicular to the line-of-sight direction, respectively, the distortion in each direction can be parameterized by

\[
\alpha_J(z) = \frac{H_{\text{adopted}}(z)}{H_{\text{true}}(z)}
\]

\[
\alpha_L(z) = \left[ \frac{D_{\Lambda,\text{adopted}}(z)}{D_{\Lambda,\text{true}}(z)} \right]^{-1}.
\]

(6)

In the case of our fiducial cosmology (\( \Omega_m, w = (0.26, -1) \)), if we adopt “incorrectly” that \( \Omega_m, w = (0.41, -0.5) \) then \( \alpha_0 = 0.735 \) and \( \alpha_L = 0.841 \) at \( z = 1 \). Relative to the reference point this is a \(-26.5\%\) and \(-15.9\%\) change in \( r_\parallel \) and \( r_{\perp} \), respectively (see Figure 1), which makes the apparent shape of the galaxy distribution compressed relatively more along the line of sight.

The distortion effect in the 2pCF due to adopting incorrect cosmology is described by Li et al. (2016). The effect is described by the coordinate transformation (Li et al. 2018)

\[
\xi'(r'_\parallel, r'_\perp) = \xi(r_\parallel/\alpha_0, r_{\perp}/\alpha_L).
\]

(7)

In polar coordinates, the transformation is

\[
\xi'(r', \mu') = \xi(r, \mu),
\]

where

\[
r = r' \sqrt{\alpha_0^{-2} \mu'^2 + \alpha_L^{-2} (1 - \mu'^2)}
\]

\[
\mu = \frac{\mu'}{\sqrt{\alpha_0^{-2} \mu'^2 + \alpha_L^{-2} (1 - \mu'^2)}}.
\]

(8)

We normalize the 2pCF after the transformation, as in Equations (3) and (4).

Using the transformation, we show how \( \hat{\xi}_{0,2,4} \) is affected by choosing incorrect cosmology. In the left panel of Figure 3, the black solid lines are \( \hat{\xi}_{0,2,4} \) for the fiducial case of \( \Omega_m, w = (0.26, -1) \) and the other lines are those with the distortion effect applied for four incorrect cosmologies (\( \Omega_m, w \) = (0.26, -0.5), (0.26, -1.5), (0.21, -1), and (0.31, -1)). The significance of the distortion effect appears strongest for \( \hat{\xi}_2 \) at separations roughly between 7 and 15 \( h^{-1} \) Mpc. In the incorrect cosmologies, \( \hat{\xi}_2 \) fall nearly outside the 2\sigma uncertainty in that range. The distortion effect is smaller for \( \hat{\xi}_0 \) and \( \hat{\xi}_4 \), but it does seem to be significant for certain separations.

![Figure 2](image-url)
3.3. Cosmology Dependence in the Shape of a Two-point Correlation Function

If the amount of redshift distortion of $\hat{\xi}_{0,2,4}$ remained the same for different cosmologies, we would be able to use $\hat{\xi}_{0,2,4}$ to constrain the cosmology using the corrections for nonlinear evolution effects found for just one cosmology. However, $\hat{\xi}_{0,2,4}$ does have some cosmology dependence, as we shall show in this section.

In the right panel of Figure 3, we show $\hat{\xi}_{0,2,4}$ for $(\Omega_m, w) = (0.26, -1), (0.26, -0.5), (0.26, -1.5), (0.21, -1)$, and $(0.31, -1)$ at $z = 1$, which are calculated from the multiverse simulation set. In comparison to the left panel, where we plotted the effect of an incorrect cosmology choice, we describe the intrinsic cosmology dependence of $\hat{\xi}_{0,2,4}$ in the right panel.

The cosmology dependence is tiny for $\hat{\xi}_{0}$, but it is significantly large for $\hat{\xi}_{2}$ and $\hat{\xi}_{2}$ reaches nearly the 2$\sigma$ level for certain cases. In the next section it is shown that the redshift evolution of $\hat{\xi}_{0,2,4}$ also depends on cosmology, and this is what needs to be corrected.

3.4. Evolution of the Shape of the Two-point Correlation Function

Our AP method does not care whether or not the amplitude or shape of the correlation function depends on cosmology as it cares only about whether or not the function changes across redshifts. We shall show that the redshift evolution of 2pCF, namely $\Delta \hat{\xi}_{0,2,4}(z_i, z_j) = \hat{\xi}_{0,2,4}(z_i) - \hat{\xi}_{0,2,4}(z_j)$ between two different redshifts for example, is not sensitive to the underlying cosmology in this section.

We compute $\Delta \hat{\xi}_{0,2,4}(z_i, z_j)$ for $z_i = 1$ and $z_j = 0$ for every possible pair of mocks of the total 216 HR4 mocks, excluding the cases that use the same mock for both $\hat{\xi}_{0,2,4}(z_i = 1)$ and $\hat{\xi}_{0,2,4}(z_j = 0)$. Using the same volume for $z = 1$ and 0 would underestimate the scatter in $\Delta \hat{\xi}$ because the cosmic variance will be mostly canceled out. We thus have 215$^3$ cases of $\Delta \hat{\xi}_{0,2,4}(z_i, z_j)$, which we plot in Figure 4 with the effect of incorrect cosmology choice in the left panel and intrinsic cosmology dependence in the right panel.

$\Delta \hat{\xi}$ is very small compared to $\hat{\xi}$ (see Figure 3). This was also addressed in Li et al. (2016). Therefore, even though the shape of the 2pCF itself is significantly distorted due to the redshift-space distortion effect, it is roughly a cosmic invariant and can be used for the AP test. In reality, the shape of the redshift-space 2pCF mildly evolves due to nonlinear gravitational evolution and the change of the type of galaxies. Due to the nonlinear effect, $\Delta \hat{\xi}$ shows a nonzero residual that decreases with $r$. However, we can see from the left panel of Figure 4 that this change is smaller compared to that produced by the geometrical distortion caused by adopting an incorrect cosmology (see also Li et al. 2016).

The cosmology dependence of $\Delta \hat{\xi}$ appears significantly smaller than that of $\hat{\xi}$. There is more than a 2$\sigma$ level of change in $\hat{\xi}_{0}$ when changing $\Omega_m$ from 0.26 to 0.21 or 0.31 (Figure 3), but that in $\Delta \hat{\xi}_{0}$ is well below the 1$\sigma$ uncertainty at most.
separations. The change in $x^2$ ranges from the 1-$\sigma$ to 2-$\sigma$ level, but that in $x^D$ is mostly below 1-$\sigma$. Both $x^4$ and $x^D4$ do not seem to be affected by background cosmology to a significant level. However, the residual cosmology dependence in $x^D$ is not negligibly small everywhere and it has to be taken into account. We expect the 1-$\sigma$ level change in $x^D2$ at separations $5 \sim 10 \, h^{-1}$ Mpc to significantly affect the results (see the middle right panel of Figure 4). That is where we expect to have the strongest constraint from the geometrical distortion effect. We describe how we subtract this effect in Section 3.6.

3.5. Parameterization of the Redshift Evolution of 2pCF

In order to use $\Delta \hat{\xi}$ for the AP test, we need to compress the information in $\Delta \hat{\xi}$ into a small number of parameters. If the number of parameters is comparable to that of the samples, the covariance matrix will be significantly biased and the error will propagate to the likelihood evaluation (Hartlap et al. 2007; Percival et al. 2014).

We fit the $r$-dependence of the moments, $\Delta \hat{\xi}_{0,2,4}$, with second-order polynomials as follows:

$$
\Delta \hat{\xi}_{0,f}(r, z_i, z_j) = r^{-2}(a_1 + a_2[\log(r)] + a_3[\log(r)]^2)
$$

$$
\Delta \hat{\xi}_{2,f}(r, z_i, z_j) = r^{-2}(a_4 + a_5[\log(r)] + a_6[\log(r)]^2)
$$

$$
\Delta \hat{\xi}_{4,f}(r, z_i, z_j) = r^{-2}(a_7 + a_8[\log(r)] + a_9[\log(r)]^2).
$$

(9)

This fitting results in nine parameters $a \equiv (a_1, a_2, \ldots, a_9)$ that describe $\Delta \hat{\xi}$. Namely

$$
\Delta \hat{\xi} \approx \sum_{\ell=0,2,4} \Delta \hat{\xi}_{\ell,f}(r)P_{\ell}(\mu).
$$

(10)

We shall use this nine-element vector $a$ to describe $\Delta \hat{\xi}$ to calculate the likelihood for each cosmology. We find that the constraint is strongest when we fit between $r = 5$ and $15 \, h^{-1}$ Mpc. At $r > 15 \, h^{-1}$Mpc $\hat{\xi}$ does not contribute to the constraint of the cosmology and we exclude it from our analysis. We thus use that range to generate $a$.

3.6. Likelihood Analysis

The redshift difference in the shape of 2pCF, $\Delta \hat{\xi}$, is much smaller than the shape itself ($\hat{\xi}$), but it does have a nonzero residual, as can be seen from the right panel of Figure 4. We shall refer to the value as the systematics and use the superscript “sys” to denote it. Our goal is to make an accurate subtraction of $a^{\text{sys}}$ to the observed value of $a$ in our likelihood analysis.

$\Delta \hat{\xi}_{0,2,4}(z_i, z_j)$ is cosmology-dependent, as shown in the right panel of Figure 4. Li et al. (2016, 2017, 2018) assumed it is cosmology-independent, which is a potential caveat of their studies.

In principle, $a^{\text{sys}}$ should be computed for every cosmology under consideration, which would be too expensive. In this work we linearly interpolate and extrapolate five cases of $a^{\text{sys}}$.
from the multiverse simulations with \((\Omega_m, w) = (0.21, -1), (0.26, -0.5), (0.26, -1), (0.26, -1.5), \) and \((0.31, -1)\). For example, when we compute the likelihood for a given cosmology of \((\Omega_m, w) = (0.33, -1.1)\), our systematics correction is
\[
a^{\text{sys}}(\Omega_m, w) = a^{\text{sys}}_{\text{fid}} + (\Omega_m - 0.26) a^{\text{sys}}_{\text{High-}\Omega_m - \Omega_{\text{fid}}} - 0.31 - 0.26
+ (w - (-1)) a^{\text{sys}}_{\text{Low-}w - \Omega_{\text{fid}}} - 1.5 - (-1).
\]
The regions of the parameter space with \(\Omega_m < 0.21, \Omega_m > 0.31, w > -0.5, \) or \(w > -1.5\) require extrapolation in the systematics, which might be less reliable than interpolation. However, we expect the likelihood for that part of the parameter space to be fairly low and have minimal impact on the high-likelihood region. We use the systematics-corrected fitting parameters
\[
p(\Omega_m, w) \equiv a(\Omega_m, w) - a^{\text{sys}}(\Omega_m, w)
\]
for the likelihood calculation.

Then, we calculate the covariance matrix, \(C_{ij}\), using \(p\) from 216² combinations of HR4 sub-cube mocks at \(z = 0\) and 1 for the fiducial cosmology \((\Omega_m, w) = (0.26, -1)\). This matrix describes the uncertainty range of \(\Delta \xi\) in the fiducial cosmology.

Next, we compute \(p\) for arbitrary cosmology. For an adopted cosmology, we transform \(\xi^{\text{sys}}\) using Equation (8) and compute \(a\) from it. This would put the center of our constraint at the fiducial cosmology \(\Omega_m = 0.26\) and \(w = -1\). Then, \(p = a - a^{\text{sys}}\) contains the distortion effect from incorrect cosmology choice, but not the cosmic variance. Finally, the likelihood for any adopted cosmology is \(\mathcal{L} = \exp(-\chi^2/2)\), where
\[
\chi^2(\Omega_m, w) \equiv \sum_i \sum_j p_i(\Omega_m, w) \cdot C_{ij} \cdot p_j(\Omega_m, w).
\]

We shall show the likelihood results from the above equation in the next section.

4. Results: Cosmological Constraints

4.1. Constraints from Different Redshift Intervals

The likelihood contour for \(\Omega_m - w\) from our AP test is shown in Figure 5. The size of the contour is our prediction for the constraining power of our method. As mentioned above, the center of the constraint is designed to be at the fiducial cosmology. In an actual analysis of observational data from the fiducial cosmology, the center of constraint will be located randomly within the range of uncertainty (i.e., the area enclosed by the contour).

The constraint from the evolution between \(z = 1\) and 0 \((\Delta \xi(z_i = 1, z_j = 0); \) top left panel\) forms a stretched region. The direction of the stretch is similar to the line that satisfies \(\alpha_{||} = \alpha_{\perp}\) in the parameter space. This is because the AP test loses the constraining power when the ratio of distortions parallel and perpendicular to the line-of-sight direction, \(\alpha_{||}/\alpha_{\perp}\), remains unchanged.

The likelihood contour for \(\Delta \xi(z_i = 0.5, z_j = 0)\) (upper right panel of Figure 5) has a similar stretched shape, but is more tilted toward the horizontal direction. This is because the distortion factors \(\alpha_{||}\) and \(\alpha_{\perp}\) have a different dependence on \(\Omega_m\) and \(w\) at different redshift. At low redshifts \(w\) plays an increasingly important role in determining the expansion history, thus the AP method becomes more sensitive to \(w\) with low-redshift data.

Due to the difference in the slopes of constraint from \(\Delta \xi(0.5, z_j = 0)\) and \(\Delta \xi(z_i = 0.5, z_j = 0)\), combining the two data set tightens the constraint significantly. We compute the combined constraint using
\[
p^* \equiv (p_1^{z_i=0.5}, ..., p_6^{z_i=1}, p_1^{z_j=0.5}, ..., p_6^{z_j=1}),
\]
where we simply concatenated \(p\)’s from the two redshift pairs, \((z_i, z_j) = (1, 0)\) and \((0.5, 0)\). We show \(\mathcal{L}(\Omega_m, w)\) from \(p^*\) in the lower panel of Figure 5. Combining extra redshift data tightens the constraint, confining the parameters within a 1σ uncertainty of \(\Delta \Omega_m \approx 0.04\) or \(\Delta w \approx 0.2\) when marginalized over \(w\) or \(\Omega_m\), respectively.

4.2. Impact of Cosmology Dependence in the Systematics

In previous works by Li et al. (2016, 2017, 2018), the systematics was modeled from a single background cosmology and the same correction is made for the entire range of cosmology considered. Here, we assess the impact of accounting for the cosmology dependence of the systematics correction, \(a^{\text{sys}}\). In Figure 5 we show the cosmological constraints when a fixed systematics correction of \(a^{\text{sys}}\) is used regardless of adopted cosmology (yellow and magenta contours).

The likelihood contour is much more stretched for fixed systematics correction cases, showing stronger degeneracy along the line of \(\alpha_{||} = \alpha_{\perp}\). For the combined constraint, the shape is more elongated for fixed systematics, but the area of the contour is not very affected: ignoring the cosmology dependence of the systematics correction underestimated the uncertainty in the parameter estimation only slightly (20%). However, we note that if the cosmology dependence is not taken into account, the central value of the constraint is likely to shift in the analysis of real observational data by more than 1σ, as shown in \(\Delta \xi\) across different cosmologies in the right panel of Figure 4. Our likelihood results are designed to be centered at the fiducial cosmology.

The cosmology dependence in the systematics may seem to help break the degeneracy for parameter sets that are far from our fiducial choice \((\Omega_m = 0.26\) and \(w = -1)\). However, the systematics correction outside the coverage of the multiverse simulations (see the diamond in Figure 5) involves less reliable extrapolation and we cannot draw conclusions for those parameters in this work. More simulations are needed to extend the coverage. In practice, the result is unlikely to change significantly due to the extra simulations, as other cosmological probes like the CMB show that the constrained area will be within the range of multiverse simulations.

4.3. Dependence of Number Density and Type of Galaxies

The constraining power of our AP test is determined by the size of the uncertainty in \(\Delta \xi_{0.24}\), which is quantified by the covariance matrix in Equation (12). The uncertainty range is also described as the shaded areas in Figure 4. The smaller the shades are, the stronger the constraint is. This uncertainty is for our fiducial sample that is \((525\ Mpc/h)^3\) in volume with \(10^{-3}\) galaxies per \((\text{Mpc}/h)^3\).
The uncertainty is presumably from cosmic variance in finite volume and Poisson noise due to a limited number of pairs. In that case, one can reduce the uncertainty by increasing either the survey volume or the number density of sample galaxies. In this section, we show the impact of increasing the sample density by 10 times. The denser sample contains about 1.45 million galaxies and includes relatively less massive galaxies. We note that having such a dense sample from near future surveys is not practical for redshifts of our interest \((z \lessapprox 0.5)\), while we will likely have a larger volume than our mock from those surveys. We shall explore the impact of enlarged survey volume in future works with larger simulations.

In Figure 6, we compare the uncertainty range of our fiducial case \(n = 10^{-3} (h^{-1} \text{Mpc})^{-3}\) with another sample with 10 times higher galaxy number density \(n = 10^{-2} (h^{-1} \text{Mpc})^{-3}\). In the case with higher number density, the uncertainty is substantially reduced for \(\Delta \tilde{\xi}_{2}^{c}\) and \(\Delta \tilde{\xi}_{4}\) at \(\lessapprox 10 h^{-1} \text{Mpc}\), where most of the constraint comes from. As a result, the predicted cosmological constraint in the high number density case turns out be substantially tighter. It gives a nearly five times smaller 1\(\sigma\) area and more than two times tighter constraint for each parameter (see Figure 7), giving marginalized constraints of \(\Delta \Omega_{m} \approx 0.017\) or \(\Delta w \approx 0.09\) with 1\(\sigma\) uncertainty. Note that this is more than a factor of two improvement compared to the result of the fiducial case, \(\Delta \Omega_{m} \approx 0.04\) and \(\Delta w \approx 0.2\).

Note that the average distance between galaxies is \(10 (h^{-1} \text{Mpc})\) for \(n = 10^{-3} (h^{-1} \text{Mpc})^{-3}\), while it is \(\sim 4.6 (h^{-1} \text{Mpc})\) for \(n = 10^{-2} (h^{-1} \text{Mpc})^{-3}\). The uncertainty is significantly reduced at \(\lessapprox 10 h^{-1} \text{Mpc}\) as we increase the number density, while it stays nearly the same at larger scales. The uncertainty seems to be dominated by the cosmic variance on scales larger than the mean galaxy separation, while the shot noise seems to dominate on scales shorter than the mean separation. Also, it can be seen in Figure 6 that the size of systematics is larger for the less massive galaxies, particularly for \(\Delta \tilde{\xi}_{2}^{c}\). Therefore, it is necessary to estimate the systematics and have the abundance of galaxies matched with observations.

4.4. Comparison with the Previous Method

Li et al. (2016, 2017, 2018) used the binned values of the radially integrated 2pCF, which is similar to

\[
\Delta \tilde{\xi}_{\Delta r}(\mu, z_i, z_j) \equiv \int_{r_{\text{min}}}^{r_{\text{max}}} \tilde{\xi}(r, \mu, z_i) dr - \int_{r_{\text{min}}}^{r_{\text{max}}} \tilde{\xi}(r, \mu, z_j) dr,
\]

where \(r_{\text{min}} = 5 h^{-1} \text{Mpc}\) and \(r_{\text{max}} = 45 h^{-1} \text{Mpc}\) are chosen in their AP test. Their method potentially suffers from loss of information in the radial shape of the 2pCF. Note that the radial
integrals on the right side of Equation (14) are practically dominated by $\xi$ at $r = r_{\text{min}}$ because of the $r^{-2}$-like scaling of $\xi$.

We expect the constraining power of the AP test to improve with our method, which uses both the radial and angular dependence of $\xi$. To compare the constraining power of the two methods, we reproduce their AP test with nine $\mu$-bins of $\Delta\hat{\xi}_{\Delta\mu}(\mu, z_i = 1, z_j = 0)$ and $r_{\text{min}} = 5 \, h^{-1} \text{Mpc}$ and $r_{\text{max}} = 45 \, h^{-1} \text{Mpc}$. We set the number of $\mu$-bins to be the same as the number of parameters we use to fit $\Delta\hat{\xi}_{m, 2.4}$. The systematics correction is assumed to be cosmology-independent in those works. That is, $a^{\mu\nu} = a_{\text{sys}}$ for all cosmologies.

The resulting likelihood for $\Omega_m - w$ is shown in Figure 8. The constraint from the method of Li et al. (2016, 2017, 2018) gives about a 40% larger uncertainty in the parameter estimation (black and gray areas). Clearly, using the full shape improves the constraint by a significant amount.

The amount of improvement in constraint, however, may not look very impressive considering that we added an extra dimension in the analysis. This is because the geometric distortion effect in $\Delta\hat{\xi}_{m, 2.4}$ over different $r$ is correlated to some extent. As we see in the left panel of Figure 4, incorrect cosmology choice shifts or tilts $\Delta\hat{\xi}_{m, 2.4}$ more or less uniformly over $r$. In this case, combining constraints over different $r$ does not add up perfectly. Nevertheless, using the full shape does improve the constraint by a significant amount.

5. Summary and Conclusions

The AP test that uses the evolution of redshift-space 2PCF as proposed by Li et al. (2015, 2016) is a powerful method for constraining the cosmological parameters governing the expansion of the universe. We presented an improved method for the AP test that utilizes the two-dimensional shape of the anisotropic galaxy clustering down to a scale as small as
Our method decomposes the two-dimensional galaxy 2pCF into Legendre polynomials whose amplitudes are modeled by radial fitting functions (Equations (5) and (9)). This allows us to describe the 2D shape of the 2pCF with a reasonably small number of parameters. Our likelihood analysis with this fitting scheme tightens the constraint on $\Omega_m$ and $w$ by 40% compared to the previous method of Li et al. (2016, 2017, 2018) that uses one-dimensional angular dependence only.

We found that the systematic effects in the shape of 2pCF have a non-negligible amount of cosmology dependence over $\Omega_m = 0.21-0.31$ and $w = -0.5-1.5$, which can result in changes in the shape of the constraint. The cosmology dependence is likely to change the center of constraint in the case of observational data. Therefore, future works should account for cosmology dependence with more simulations from different background cosmologies.

The constraint on $\Omega_m$ and $w$ from a single pair of redshifts has a degeneracy for the parameter sets that give the same $\alpha_r/\alpha_l$. This degeneracy can be broken by adding extra pairs of redshifts in the analysis. Most of the constraint comes from the smallest scales we consider, between $r = 5$ and $10 h^{-1}$ Mpc. Reducing the uncertainty in the shape of 2pCF at those scales is the key to tightening the constraint; this can be achieved by increasing the number of galaxy pairs or enlarging the survey volume. When we increased the mock galaxy number density from $n = 10^{-3}$ to $10^{-2} (h^{-1} \text{Mpc})^{-3}$ while the physical size of the sample was fixed to $(525 h^{-1} \text{Mpc})^3$, the constraint tightened by nearly two times. However, enlarging the sample volume is more relevant to the expected outcome of upcoming surveys, which we shall explore in future works.

The authors thank C. Pichon, H. S. Hwang, E. Komatsu, D. Jeong, and T. Sunayama for the helpful comments on this work. X.D.L. acknowledges the support from an NSFC grant (No. 11803094). C.G.S. acknowledges financial support from the National Research Foundation (NRF; #2017R1D1A1B03034900, #2017R1A2B2004644 and #2017R1A4A1015178). S.E.H. was supported by Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education(2018R1A6A1A06024977). The authors acknowledge the Korea Institute for Advanced Study for providing computing resources (KIAS Center for Advanced Computation Linux Cluster System). This work was supported by the Supercomputing Center/Korea Institute of Science and Technology Information, with supercomputing resources including technical support (KSC-2016-C3-0071), and the simulation data were transferred through a high-speed network provided by KREONET/GLORIAD.

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