A near-NHEK/CFT correspondence

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Abstract

We consider excitations around the recently introduced near-NHEK metric describing the near-horizon geometry of the near-extremal four-dimensional Kerr black hole. This geometry has a $U(1)_L \times U(1)_R$ isometry group which can be enhanced to a pair of commuting Virasoro algebras. We present boundary conditions for which the conserved charges of the corresponding asymptotic symmetries are well defined and non-vanishing and find the central charges $c_L = 12J/h$ and $c_R = 0$ where $J$ is the angular momentum of the black hole. Applying the Cardy formula reproduces the Bekenstein-Hawking entropy of the black hole. This suggests that the near-extremal Kerr black hole is holographically dual to a non-chiral two-dimensional conformal field theory.
1 Introduction

The near-horizon geometry of an extremal four-dimensional Kerr black hole [1] is described by the so-called NHEK metric. According to the recently conjectured Kerr/CFT correspondence [2], the corresponding quantum theory is holographically dual to a chiral Conformal Field Theory (CFT) in two dimensions. In the spirit of [3] and using the formalism of [4], it was found that certain boundary conditions enhance the $U(1)_L$ symmetry of the $U(1)_L \times SL(2,\mathbb{R})_R$ isometry group to a Virasoro algebra. Strong evidence [2] for the correspondence is found in the exact agreement between the macroscopic Bekenstein-Hawking entropy [5] of the black hole and the Cardy formula for the CFT entropy. This analysis has been successfully generalized and applied to a variety of extremal black holes [6, 7, 8, 9, 10, 11, 12, 13]. Extending the Kerr/CFT correspondence to the near-extremal Kerr black hole, however, has presented some serious challenges. The dual two-dimensional CFT should be non-chiral. It is the main objective of the present work to offer a possible resolution to one of these obstacles.

Boundary conditions enhancing the $SL(2,\mathbb{R})_R$ isometries to a Virasoro algebra were examined in [14, 15]. Since no self-consistent set of boundary conditions was found which enhances the $U(1)_L$ isometry at the same time, these works do not capture the full non-chiral CFT in the dual picture. Several results on near-extremal black holes have since been obtained [16] and have provided significant insight. Extending the approach of [17], the work [18] on black-hole superradiance and the subsequent generalizations thereof [19] have provided further and highly non-trivial support for the Kerr/CFT correspondence. These results on the near-extremal Kerr black hole are all based on deviations from the geometrical approach of [2] since consistent boundary conditions which allow for both left- and right-moving sectors have not yet been identified. Another very recent development is the analysis of a so-called hidden conformal symmetry [20] which is not a symmetry of the spacetime geometry.

Here we reapply the original approach of [2] to the near-extremal Kerr black hole. Its near-horizon geometry is described by the so-called near-NHEK metric derived in [18]. This geometry has a $U(1)_L \times U(1)_R$ isometry group which we find can be enhanced to a pair of commuting Virasoro algebras. We present the corresponding boundary conditions and argue that the conserved charges of the corresponding asymptotic symmetries are well defined and non-vanishing. The central charges are $c_L = 12J/\hbar$ and $c_R = 0$ where $J$ is the angular momentum of the black hole. Applying the Cardy formula reproduces the Bekenstein-Hawking entropy of the black hole. This supports the assertion that the near-extremal Kerr black hole is holographically dual to a non-chiral two-dimensional CFT. We refer to the corresponding duality as the near-NHEK/CFT correspondence since the original Kerr/CFT correspondence [2] actually concerns a duality of the form NHEK/CFT.

The Virasoro algebra has been observed in other black-hole contexts as well. Extending the work [21] on the entropy of three-dimensional black holes such as the BTZ black hole [22], it was found in [23, 24] that a copy of the Virasoro algebra appears in the near-horizon region of any black hole and that it reproduces the black-hole entropy using the Cardy formula. Free-field approaches to the dual description of black holes are discussed in [24, 25], while black-hole solutions of gravity theories with higher-derivative corrections are considered in [26] and references therein.
2 Kerr/CFT correspondence

2.1 NHEK geometry

The Near-Horizon Extremal Kerr (NHEK) geometry [1, 2] is described by

\[ d\bar{s}^2 = 2GJ\Gamma \left( -r^2 dt^2 + \frac{dr^2}{r^2} + d\theta^2 + \Lambda^2 (d\phi + rdt)^2 \right) \]

(2.1)

where

\[ \Gamma = \Gamma(\theta) = \frac{1 + \cos^2 \theta}{2}, \quad \Lambda = \Lambda(\theta) = \frac{2 \sin \theta}{1 + \cos^2 \theta} \]

(2.2)

and where \( \theta \in [0, \pi] \) and \( \phi \sim \phi + 2\pi \). The corresponding isometry group \( U(1)_L \times SL(2, \mathbb{R})_R \) is generated by

\[ \{ \partial_\phi \} \cup \{ \partial_t, t\partial_t - r\partial_r, (t^2 + \frac{1}{r^2})\partial_t - 2tr\partial_r - \frac{2}{r}\partial_\phi \} \]

(2.3)

while the ADM mass \( M \) and angular momentum \( J \) of the associated extremal Kerr black hole are related through

\[ J = GM^2 \]

(2.4)

2.2 Conserved charges

The interest here is in perturbations \( h_{\mu\nu} \) of the near-horizon geometry of the extremal black hole whose background metric \( \bar{g}_{\mu\nu} \) is defined in (2.1). Asymptotic symmetries are generated by the diffeomorphisms whose action on the metric generates metric fluctuations compatible with the chosen boundary conditions. We are thus looking for contravariant vector fields \( \eta \) along which the Lie derivative of the metric is of the form

\[ \mathcal{L}_\eta \bar{g}_{\mu\nu} \sim h_{\mu\nu} \]

(2.5)

To such an asymptotic symmetry generator \( \eta \), one associates [4] the conserved charge

\[ Q_\eta = \frac{1}{8\pi G} \int_{\partial \Sigma} \sqrt{-\bar{g}} k_\eta [h; \bar{g}] = \frac{1}{8\pi G} \int_{\partial \Sigma} \sqrt{-\bar{g}} \frac{\epsilon_{\alpha\beta\mu\nu} d\eta^{\mu\nu}[h; \bar{g}]}{4} dx^\alpha \wedge dx^\beta \]

(2.6)

where

\[ d\eta^{\mu\nu}[h; \bar{g}] = \eta^\nu \bar{D}^\mu h - \eta^\mu \bar{D}_\sigma h^{\mu\sigma} + \eta_\sigma \bar{D}^\mu h^{\mu\sigma} - h^{\nu\sigma} \bar{D}_\eta h^{\mu\sigma} + \frac{1}{2} h \bar{D}^{\nu} \eta^{\mu} + \frac{1}{2} h^{\sigma\nu} (\bar{D}^{\mu} \eta_{\sigma} + \bar{D}_{\sigma} \eta^{\mu}) \]

(2.7)

and where \( \partial \Sigma \) is the boundary of a three-dimensional spatial volume, ultimately near spatial infinity. Here, indices are lowered and raised using the background metric \( \bar{g}_{\mu\nu} \) and its inverse, \( \bar{D}_\mu \) denotes a background covariant derivative, while \( h \) is defined as \( h = \bar{g}^{\mu\nu} h_{\mu\nu} \). To be a well-defined charge in the asymptotic limit, the underlying integral must be finite as \( r \to \infty \). If the charge vanishes, the asymptotic symmetry is rendered trivial. The asymptotic symmetry group is generated by the diffeomorphisms whose charges are well-defined and non-vanishing. The algebra generated by the set of well-defined charges is governed by the Dirac brackets computed [4] as

\[ \{ Q_\eta, Q_\hat{\eta} \} = Q_{[\eta, \hat{\eta}]} + \frac{1}{8\pi G} \int_{\partial \Sigma} \sqrt{-\bar{g}} k_{[\eta, \hat{\eta}]} [\mathcal{L}_\eta \bar{g}; \bar{g}] \]

(2.8)

where the integral yields the eventual central extension.
2.3 Boundary conditions

Written in the ordered basis \( \{ t, r, \phi, \theta \} \), the boundary conditions considered in [2] are the fall-off conditions

\[
h_{\mu\nu} = O \begin{pmatrix} r^2 & r^{-2} & 1 & r^{-1} \\ r^{-3} & r^{-1} & r^{-2} & 1 \\ 1 & r^{-1} & r^{-1} & r^{-1} \end{pmatrix}, \quad h_{\mu\nu} = h_{\nu\mu} \tag{2.9}
\]

and the zero-energy condition

\[ Q_{\partial_t} = 0 \tag{2.10} \]

Consistency of these conditions was confirmed in [11]. The generators of the corresponding asymptotic symmetry group read

\[
\xi = -\epsilon'(\phi) r \partial_r + \epsilon(\phi) \partial_\phi \tag{2.11}
\]

and form the centreless Virasoro algebra

\[
[\xi_\epsilon, \xi_{\hat{\epsilon}}] = \xi_{\epsilon\hat{\epsilon}' - \hat{\epsilon}' \epsilon} \tag{2.12}
\]

This symmetry is an enhancement of the exact \( U(1)_L \) isometry generated by the Killing vector \( \partial_\phi \) of (2.1) as the latter is recovered by setting \( \epsilon(\phi) = 1 \). The usual form of the Virasoro algebra is obtained by choosing an appropriate basis for the functions \( \epsilon(\phi) \) and \( \hat{\epsilon}(\phi) \), where we recall the periodicity \( \phi \sim \phi + 2\pi \). With respect to the basis \( \xi_n(\phi) \), where

\[
\epsilon_n(\phi) = -e^{-in\phi} \tag{2.13}
\]

one introduces the dimensionless quantum versions

\[
L_n = \frac{1}{\hbar} \left( Q_{\xi_n} + \frac{3J}{2} \delta_{n,0} \right) \tag{2.14}
\]

of the conserved charges. After the usual substitution \{.,.\} \( \rightarrow -\frac{i}{\hbar} [.,.] \) of Dirac brackets by quantum commutators, the quantum charge algebra is recognized [2] as the centrally-extended Virasoro algebra

\[
[L_n, L_m] = (n - m)L_{n+m} + \frac{c}{12} n(n^2 - 1) \delta_{n+m,0}, \quad c_L = \frac{12J}{\hbar} \tag{2.15}
\]

This quantum charge algebra also arises when considering boundary conditions sufficiently similar to those in (2.9). A partial classification of such alternatives can be found in [15].

3 Near-NHEK/CFT correspondence

3.1 Near-NHEK geometry

We are interested in infinitesimal excitations above extremality of the Kerr black hole. To describe this near-extremal Kerr black hole, we follow [18] and consider a generalization of the NHEK geometry in which the temperature of the near-horizon geometry is fixed and non-zero. This temperature is denoted by \( T_R \) and the corresponding near-NHEK geometry is described by

\[
ds^2 = 2J \Gamma \left( -r(r + 2\alpha) dt^2 + \frac{dr^2}{r(r + 2\alpha)} + d\theta^2 + \Lambda^2 (d\phi + (r + \alpha) dt)^2 \right) \tag{3.1}
\]
where
\[ \alpha = 2\pi T_R \] (3.2)
while \( \Gamma \) and \( \Lambda \) are given in (2.2). Here and in the following, we use the unit convention of [18] where \( G = \hbar = c = 1 \). We denote the background metric (3.1) by \( \bar{g} \), as we did with the NHEK geometry (2.1), and hope that no confusion will arise from this abuse of notation. The NHEK geometry (2.1) follows immediately from the near-NHEK geometry (3.1) by setting \( T_R = 0 \). We also note that the determinant of the near-NHEK metric \( \bar{g} \) only depends on \( \theta \) as
\[ \sqrt{-\bar{g}} = 4J^2 \Gamma^2(\theta) \Lambda(\theta) = 2J^2 \sin \theta(1 + \cos^2 \theta) \] (3.3)

It is readily verified that
\[ \{ \partial_\phi \} \cup \{ \partial_t \} \] (3.4)
generate exact \( U(1)_L \times U(1)_R \) isometries of the near-NHEK geometry. The \( U(1)_R \) subgroup of the \( SL(2, \mathbb{R})_R \) isometries of the NHEK geometry is thus generated by the lowering (or raising) operator of \( SL(2, \mathbb{R})_R \), not by the Cartan generator of the corresponding Lie algebra.

### 3.2 Boundary conditions and conserved charges

First, we modify the fall-off conditions (2.9) by introducing
\[ \hat{h}_{\mu\nu} = \mathcal{O} \begin{pmatrix} r^2 & r^{-3} & r^{-2} \\ r^{-4} & r^{-1} & r^{-3} \\ 1 & r^{-2} & r^{-2} \end{pmatrix}, \quad \hat{h}_{\mu\nu} = \hat{h}_{\nu\mu} \] (3.5)
and find the asymptotic Killing vectors
\[ K_\epsilon = [\mathcal{O}(r^{-4})] \partial_t + [-(r + \alpha) \epsilon'(\phi) + \mathcal{O}(r^{-1})] \partial_r + [\epsilon(\phi) + \mathcal{O}(r^{-3})] \partial_\phi + [\mathcal{O}(r^{-2})] \partial_\theta \]
\[ K_\epsilon = [\epsilon(t) + \mathcal{O}(r^{-4})] \partial_t + [\mathcal{O}(r^{-1})] \partial_r + [\mathcal{O}(r^{-3})] \partial_\phi + [\mathcal{O}(r^{-2})] \partial_\theta \] (3.6)

where \( \epsilon(\phi) \) and \( \epsilon(t) \) are smooth functions. The generators of the corresponding asymptotic symmetries read
\[ \xi = -(r + \alpha) \epsilon'(\phi) \partial_r + \epsilon(\phi) \partial_\phi, \quad \zeta = \epsilon(t) \partial_t \] (3.7)
and form a commuting pair of centreless Virasoro algebras
\[ [\xi_\epsilon, \xi_\epsilon'] = \xi_{\epsilon\epsilon'} - \epsilon_\epsilon', \quad [\zeta_\epsilon, \xi_\epsilon] = \zeta_{\epsilon\epsilon'}, \quad [\xi_\epsilon, \zeta_\epsilon] = 0 \] (3.8)

Along \( \xi \) and \( \zeta \), the Lie derivatives of the near-NHEK metric are worked out to be
\[ \mathcal{L}_\xi \bar{g}_{\mu\nu} = 2J \bar{g}_{\mu\nu} \begin{pmatrix} -2(\Lambda^2 - 1)(r + \alpha)^2 \epsilon'(\phi) & 0 & 0 \\ 0 & \frac{2\alpha^2 \epsilon'(\phi)}{r^2(r + 2\alpha)^2} & -\frac{(r + \alpha) \epsilon''(\phi)}{r(r + 2\alpha)} \Lambda^2 \epsilon'(\phi) \\ 0 & -\frac{(r + \alpha) \epsilon''(\phi)}{r(r + 2\alpha)} & 2\Lambda^2 \epsilon'(\phi) \end{pmatrix} \]
\[ \mathcal{L}_\zeta \bar{g}_{\mu\nu} = 2J \bar{g}_{\mu\nu} \begin{pmatrix} 2(\Lambda^2 - 1)r(r + 2\alpha) + 2\alpha^2 \Lambda^2 & \Lambda^2(r + \alpha) & 0 \\ 0 & 0 & 0 \\ \Lambda^2(r + \alpha) & 0 & 0 \end{pmatrix} \] (3.9)
Second, we supplement the fall-off conditions (3.5) by a weakened zero-energy condition (2.10) where instead of requiring \(Q_{\partial_t} = 0\), we allow this conserved charge to be proportional to \(\alpha\). An inspection of the asymptotic \(r\)-expansion of \(k_{\partial_t}[\hat{h}; \hat{g}]\) reveals that the divergent term (linear in \(r\)) is independent of \(\alpha\) while the constant term (independent of \(r\)) is at most linear in \(\alpha\). Subleading terms in \(r\) can here be ignored as \(r \to \infty\). We thus impose the condition

\[
(1 - \alpha \partial_\alpha)Q_{\partial_t} = 0
\]  

(3.10)

Under this constraint, only perturbations \(\hat{h}\) preserving it and only background metrics \(g\) which can be reached from the near-NHEK geometry via a path of such perturbations are considered. As in the similar situation [2] arising when imposing (2.10) in addition to (2.9), this is presumably a complicated nonlinear submanifold of the geometries allowed by the linear boundary conditions (3.5).

Third, \(Q_\zeta\) must be well-defined for all \(\zeta\), not just for \(\varepsilon(t) = 1\) as in (3.10), so we should study the consequences of the constraint (3.10) on the asymptotic expansion of \(Q_\zeta\). To this end, we examine the integrands of the corresponding charges (2.6) and obtain the remarkably simple expression

\[
\sqrt{-g} (k_{\zeta}[\hat{h}; \hat{g}] - \varepsilon(t) k_{\partial_t}[\hat{h}; \hat{g}]) \bigg|_{\partial_\phi \wedge d\theta} = -\frac{1}{4} \Lambda \varepsilon'(t)(r + \alpha) \hat{h}_{r\phi}
\]  

(3.11)

valid for all \(r\). It follows, in particular, that the divergent part of the asymptotic \(r\)-expansion of \(k_{\zeta}[\hat{h}; \hat{g}]\) matches the divergent part of \(\varepsilon(t) k_{\partial_t}[\hat{h}; \hat{g}]\) where

\[
\sqrt{-g} k_{\partial_t}[\hat{h}; \hat{g}] \bigg|_{\partial_\phi \wedge d\theta} = \frac{1}{4\Lambda} \left[ -\Lambda^4 r^{-1} \hat{h}_{tt} + 2\Lambda^2 (\Lambda^2 - r \partial_r) \hat{h}_{t\phi} + 2(1 - \Lambda^2)r^2 \partial_\phi \hat{h}_{r\phi} \right. \\
\left. - (\Lambda^4 + \Lambda^2 - 2) r \hat{h}_{\phi\phi} \right] + \mathcal{O}(r^0)
\]  

(3.12)

Here we have used that \(\partial_\phi \hat{h}_{\phi\phi} = \mathcal{O}(r^{-2})\). This matching ensures that the conserved charge \(Q_\zeta\) is finite when (3.5) and (3.10) are satisfied.

Fourth, and despite the results above, the expression (3.11) actually indicates that (3.5) and (3.10) provide an inconsistent set of boundary conditions. The problem is that, while contravariant vector fields of the form \(\zeta\) in (3.7) satisfy the Jacobi identity for commutators, the corresponding conserved charges \(Q_\zeta\) may not satisfy the Jacobi identity for Dirac brackets. This is illustrated by \(\varepsilon_\gamma(t) = t^{n_\gamma}\), \(j = 1, 2, 3\), if \(n_2 = 1 - n_1\) with \(n_1\) and \(n_3\) generic, since then \(Q_{[\zeta_1, \zeta_2]} = (1 - 2n_1)Q_{\partial_t} = 0\). We should therefore prevent the term (3.11) from contributing to the surface integral (2.6) in the definition of \(Q_\zeta\). However, \(L_\zeta \bar{g}_{\phi\phi}\) goes like \(r^{-1}\) asymptotically so we cannot strengthen the fall-off condition \(\hat{h}_{r\phi}\) without affecting the conserved charge \(Q_\zeta\). As a remedy, one may require that \(\hat{h}_{r\phi}\) is a total \(\phi\)-derivative on the boundary. Such a requirement will presumably also reduce possible back-reaction effects, see [11] on the extremal Kerr/CFT correspondence. We therefore suggest to impose the condition

\[
\hat{h}_{r\phi} \bigg|_{\partial \Sigma} = \partial_\phi \hat{H}_{r\phi}
\]  

(3.13)

and note that it is satisfied by the perturbations (3.9) generated by \(\xi\) and \(\zeta\). The combined set of boundary conditions given in (3.5), (3.10) and (3.13) should then ensure that \(Q_\xi\) and \(Q_\zeta\) are well defined.

Now, taking the constraint (3.10) explicitly into account, \(Q_\zeta\) can be written as

\[
Q_\zeta = \frac{1}{8\pi} \int_{\partial \Sigma} \sqrt{-g} k_{\zeta}[\hat{h}; \hat{g}]
\]  

(3.14)
where
\[
\sqrt{-g} k_\xi [\hat{h}; \bar{g}] = \frac{\alpha \varepsilon(t)}{4\Lambda} \left( \Lambda^4 r^{-2} \hat{h}_{tt} - 2\Lambda^2 \partial_r \hat{h}_{t\phi} - (\Lambda^4 + \Lambda^2 - 2) \hat{h}_{\phi\phi} \right) d\phi \wedge d\theta + \ldots
\] (3.15)

The dots indicate that terms not contributing to the charge (3.14) have been omitted. In particular, total \(\phi\)-derivatives can be ignored. We also find that the integrand in (2.6) for the conserved charge \(Q_\xi\) is given by
\[
\sqrt{-g} k_\xi [\hat{h}; \bar{g}] = \Lambda^2 \left( \varepsilon(t) \left[ -\Lambda^2 \hat{h}_{tt} + \frac{(\Lambda^4 - r \partial_r) \hat{h}_{t\phi}}{r} - \Lambda^2 \frac{\varepsilon' (t)}{\Lambda^2 r} \right] + \varepsilon''(t) \left[ \frac{\hat{h}_{t\phi}}{2\Lambda^2 r} \right] \right) d\phi \wedge d\theta + \ldots
\] (3.16)

We finally argue that the conserved charge \(Q_\xi\) is non-vanishing. Separating the boundary condition (3.10) into a vanishing condition on the divergent part of \(Q_\partial_t\) (computed using (3.12)) and a vanishing condition on the non-divergent part could render \(Q_\xi\) trivial. We find that this is not the case for general perturbations satisfying the fall-off conditions (3.5). Instead, we find that a simple re-scaling of the divergent part by a factor of \(\alpha/r\) can be used to simplify the integrand of (3.14)
\[
\sqrt{-g} k_\xi [\hat{h}; \bar{g}] \rightarrow \frac{\alpha \Lambda^3 \varepsilon(t)}{2} \left( \frac{\hat{h}_{tt}}{r^2} - \frac{\hat{h}_{t\phi}}{r} \right) d\phi \wedge d\theta + \ldots
\] (3.17)

It is recalled that the NHEK geometry is obtained by setting \(\alpha = 0\) in which case the conserved charge \(Q_\xi\) is seen to vanish. The conserved charge \(Q_\xi\), on the other hand, is unaffected by setting \(\alpha = 0\).

### 3.3 Central charges

First, we note that the left-moving sector obtained from the conserved charges \(Q_\xi\) is generated by the same Virasoro modes (2.14) forming the same Virasoro algebra (2.15) as in the NHEK case. The corresponding central charge is thus given by
\[
c_L = \frac{12 J}{\hbar}
\] (3.18)

To obtain a quantum algebra in the right-moving sector, one can perform an analytic continuation of \(t\) and introduce \(14\)
\[
L_n = \oint \frac{dt}{2\pi i} Q_\zeta_n, \quad \zeta_n = \varepsilon_n(t) \partial_t, \quad \varepsilon_n(t) = -t^{n+1}
\] (3.19)

Since \(\sqrt{-g} k_\xi [L_\zeta \bar{g}; \bar{g}]\) is linear in \(\varepsilon(t) \varepsilon'(t)\) and therefore only involves a single derivative, the eventual central extension of the corresponding Virasoro algebra can be absorbed in a redefinition of the quantum generator \(L_{-2}\), much akin to the redefinition of \(L_0\) in (2.14). This implies that the central charge of the right-moving sector vanishes
\[
c_R = 0
\] (3.20)

By evaluating \(\sqrt{-g} k_\xi [L_\zeta \bar{g}; \bar{g}]\) and \(\sqrt{-g} k_\xi [L_\xi \bar{g}; \bar{g}]\) explicitly, it is verified that there is no central charge mixing the two conformal sectors. This establishes that the two Virasoro algebras are indeed mutually commutative.
3.4 Entropy

The quantum theory in the Frolov-Thorne vacuum [27] restricted to the extremal Kerr black hole has the left-moving temperature [2]

\[ T_L = \frac{1}{2\pi} \] (3.21)

Applying the Cardy formula to the dual CFT yields the entropy

\[ S = \frac{\pi^2}{3}(c_LT_L + c_RT_R) \] (3.22)

For NHEK, the dual CFT has \( c_L = \frac{12J}{\hbar} \) and is chiral (implying \( c_R = T_R = 0 \)) thereby reproducing the Bekenstein-Hawking entropy [5]

\[ S_{BH} = \frac{2\pi J}{\hbar} \] (3.23)

In the case of the near-extremal Kerr black hole, the near-horizon geometry is described by the near-NHEK metric. By construction, the right-moving sector of the dual non-chiral CFT is excited with the non-zero temperature \( T_R = \frac{\alpha}{2\pi} \) while we have found that the corresponding central charge \( c_R \) vanishes (3.20). The left-moving sector is characterized by the same \( c_L \) and \( T_L \) as in the extremal case. The entropy is therefore given by the same expression (3.23) as in the extremal case.

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