Self-similar structure of the magnetized radiation-dominated accretion disks

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ABSTRACT
We investigate the effects of a large-scale magnetic field with open field lines on the steady-state structure of a radiation-dominated accretion disk, using self-similarity technique. The disk is supposed to be turbulent and possesses an effective viscosity and an effective magnetic diffusivity. We consider the extreme case in which the generated energy due to viscous and magnetic dissipation is balanced by the advection cooling. While the magnetic field outside of the disk is treated in a phenomenological way, the internal field is determined self-consistently. Magnetized and nonmagnetized solutions have the same radial dependence, irrespective of the values of the input parameters. Generally, our self-similar solutions are very sensitive to the viscosity or diffusivity coefficients. For example, the density and the rotation velocity increase when the viscosity coefficient decreases. The gas rotates with sub-Keplerian angular velocity with a factor less than unity which depends on the magnetic field configuration. Magnetic field significantly reduce disk thickness, however, tends to increase the radial velocity comparing to the nonmagnetic self-similar solutions.

Key words: accretion, accretion disks - black hole physics - MHD

1 INTRODUCTION
Advection-dominated accretion flows (ADAFs) have been studied by many authors during recent years (e.g., Abramowicz et al. 1988; Narayan & Yi 1994; Chen 1995; Narayan, Kato & Honma 1997). A key feature of radiatively inefficient accretion flows is that radiative energy losses are small so that most of the energy is advected with the gas. However, advection-dominated accretion flows can occur in two regimes, depending on the accretion rate and the optical depth. When the accretion rate is high, the optical depth becomes very high and the radiation is trapped in the gas. These kinds of solutions which are known under the name 'slim accretion disk' have been studied in detail by Abramowicz et al. (1988). On other hand, we may have optically thin accretion flows with very low mass accretion rate (e.g., Rees et al. 1982; Narayan & Yi 1994; Abramowitz et al. 1995; Chen 1995).

Because of the complexity of the equations, similarity technique can help us to explore the relevant physics of radiatively inefficient accretion flows. As long as we are not interested in the boundaries of the problem, such solutions that describe the behavior of the flow in an intermediate region far from the radial boundaries. Originally, Narayan & Yi (1994) studied optically thin advection-dominated accretion disk using their self-similar solutions. They speculated, on the basis of some numerical calculations, that the self-similar solution is the natural state for an advection-dominated flow. Subsequent analysis (e.g., Narayan, Kato & Honma 1997) showed that the global solutions achieve approximate self-similar behavior within a short distance from the outer boundary and the approach to self-similarity is quite impressive. Considering these achievements, Wang & Zhou (1999) constructed self-similar solutions for optically thick advection dominated accretion flow, in which photon
trapping and advection dominate over surface diffusion cooling and used these to explore its different properties from optically thin self-similar solutions. However, both solutions have the same power index of radius.

A remarkable problem arises when the accretion disk is threaded by magnetic field. There are good reasons for believing that magnetic fields are important to the physics of accretion disk. Schwartzman (1971) was the first to point out the importance of the magnetic field in an accretion process. He proposed a hypothesis of equipartition between the magnetic and kinetic energy densities and this picture is usually accepted in the modern picture of viscous ADF models. Bisnovatyi-Kogan & Lovelace (2000) suggested that recent papers discussing ADF as a possible solution for astrophysical accretion should be treated with caution, particularly because of ignorance of the magnetic field. While they obtained a solution for a time-averaged magnetic field in a quasispherical accretion flow, an analysis of energy dissipation and equipartition between magnetic and flow energies has been presented (Bisnovatyi-Kogan & Lovelace 2000). Numerical simulations of magnetized, radiatively inefficient flows have been done recently by several authors (e.g., Machida, Matsumoto & Mineshige 2001; Igumenshchev, Narayan & Abramowicz 2003). However, in most of these the resistive terms in the MHD equations have been neglected, or the resistivity has been considered only in the induction equation without accounting the corresponding dissipation in the energy equation.

Some authors tried to study magnetized accretion flow analytically. For example, Kaburaki (2000) presented a set of analytical solutions for a fully advective accretion flow in a global magnetic field and the conductivity is assumed to be constant for simplicity. Shadmehri (2004) extended this analysis by considering non-constant resistivity. He obtained a set of self-similar solutions in spherical coordinates that describes quasi-spherical magnetized accretion flow. Lai (1998) and Lee (1999) calculated transonic disk accretion flows around a weekly magnetized neutron star, where it was assumed the disk is fully advective. While Lee (1999) considered both the thermal and the radiation pressures, Lai (1998) assumed the radiation pressure dominates over the gas pressure.

In this study, we present self-similar solutions of an idealized height-integrated set of equations that describe magnetized radiation-dominated accretion flow. In fact, this analysis extends self-similar solutions of Wang & Zhou (1999) for optically thick advection-dominated accretion flow to the magnetized case. For a steady state disk, there can be a final, steady configuration of magnetic field, in which the inward dragging of field lines by the disk is balanced everywhere by the outward movement of filed lines due to magnetic diffusivity. We show that the radial structure of a magnetized radiation-dominated accretion flow does not differ from the non-magnetic solutions. But we can see significantly different behaviors, because of the effects of the magnetic fields. The equations of the model are presented in the second section. We obtain and solve the set of self-similar equations analytically in the third section. For a set of illustrative parameters the solutions will be discussed in this section.

2 GENERAL FORMULATION

We employ a cylindrical coordinate system \((R, \varphi, z)\) centered on a central object (e.g., a black hole) with mass \(M_\ast\) which accretes matter at a steady state \(\dot{M}\) from a geometrically thin axisymmetric accretion disk in steady state threaded by an ordered magnetic field. Our model generalize the usual slim disks around black holes (e.g., Muchotrzeb & Paczyński 1982; Matsumoto et al. 1984; Abramowicz et al. 1988) by including the effect of magnetic fields. General relativistic effects are neglected and outside of the disk, dissipative effects are assumed to be negligible. For the magnetic field geometry, we are following a general approach presented by Lovelace, Romanova & Newman 1994 (hereafter LRN), in which even filed symmetry is assumed so that \(B_R(R, z) = -B_R(R, -z)\), \(B_\varphi(R, z) = -B_\varphi(R, -z)\) and \(B_z(R, z) = +B_z(R, -z)\). However, we note that the solution for the magnetic field outside of disk should match to the field solution inside the disk at its surface. But our model does not present a self-consistent model for the magnetic field outside of the disk. In analogy to LRN, we parameterize the magnetic field outside of the disk. Also, it is assumed that the accreting matter is confined to a thin disk, and we do not formally introduce a magnetosphere into our model.

The basic equations are integrated over the vertical thickness of the disk. The mass continuity equation takes the form

\[ -2\pi R \Sigma v_R = \dot{M}, \tag{1} \]

where \(v_R\) is the radial velocity and \(\Sigma = \int dz \rho \simeq 2h \rho\) is surface density of the disk. The half-thickness is denoted by \(h\), where we consider the magnetic field effect on the disk thickness. We will see that the disk can be compressed or flattened depending on the field configuration. Also, we note that since the radial velocity is negative for accretion (i.e., \(v_R < 0\)), the accretion rate \(\dot{M}\) as an input parameter of our model is positive.

The radial momentum equation reads

\[ \Sigma v_R \frac{dv_R}{dR} - \frac{\Sigma v_R^2}{R} = -\frac{dP}{dR} - \frac{\Sigma GM_\ast}{R^2} + \int F_{\text{mag}}^\text{in} \, dz, \tag{2} \]

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where \( P = \int dz P \) is the integrated disk pressure. We consider an extreme case in which the radiation pressure dominates over the gas pressure and the generated energy is balanced by the advection cooling. The first assumption implies \( P \approx 2aT^4/3 \), where \( a \) is black body constant and \( T \) denotes the midplane temperature of the disk. The energy equation is written based on the second assumption. The last term of equation (2) represents the height integrated radial magnetic force which can be written as (LRN)

\[
\int \frac{F_{Ma}^{\text{mag}}}{dR} dz = \frac{1}{2\pi} (B_R B_\phi)_h - \frac{1}{4\pi R^2} \frac{d}{dR}[h R^2 < B_\phi^2 - B_R^2 >]
- \frac{1}{4\pi} \frac{d}{dR}[h < B_\phi^2 >] + \frac{1}{4\pi \eta R} (B_\phi^2 + B_R^2 - B_R^2)_h,
\]

where \( < \ldots > \equiv \int dz (\ldots) / (2h) \), and the \( h \) subscript denotes that the quantity is evaluated at the upper disk plane, i.e. \( z = h \). Similarity, integration over \( z \) of the azimuthal equation of motion gives

\[
R \Sigma \nu_R \frac{d}{dR} (R v_\phi = \frac{d}{dR} \left[ R^2 \Sigma \frac{d}{dR} (v_\phi R) \right] + \int F_{Ma}^{\text{mag}} dz,
\]

where

\[
\int \frac{F_{Ma}^{\text{mag}}}{dR} dz = \frac{1}{2\pi} (R^2 B_R B_\phi)_h - \frac{1}{2\pi} \frac{d}{dR} (R^2 B_R B_\phi)_h
+ \frac{1}{2\pi} \frac{d}{dR}[h R^2 < B_R B_\phi >].
\]

Here, the last term of equation (4) represents the height integrated toroidal component of magnetic force. Note that this form of azimuthal equation of motion is not exactly similar to the original slim disk model (e.g., Abramowicz et al. 1988), in which the well-known \( \alpha \) prescription of Shakura & Sunyaev (1973) has been used as a general approximate form for the \( \alpha \) component of the viscous stress tensor \( (\tau_{\alpha r} = -\alpha \rho v) \). Here, we replaced \( \alpha \rho v \) prescription by a diffusive viscosity prescription, i.e.

\[
\tau_{\alpha r} = \rho \nu_r \frac{\partial \Omega}{\partial R},
\]

where \( \rho \) is the density, \( \nu_r \) is a kinematic viscosity coefficient, and \( \Omega \) is the angular velocity of matter in the disk. The above prescription leads to equation (4). In our model, we also assume

\[
\nu = \alpha_c l \eta,
\]

where \( \alpha_c \) is the local sound speed and \( \alpha \) is a constant less than unity.

The \( z \) component of equation of motion gives the condition for vertical hydrostatic balance, which can be written as (Lovelace et al. 1987)

\[
\frac{h}{R} \frac{d}{dR} (\frac{h}{R})^2 + q(\frac{h}{R}) - \frac{2P}{4\pi \Sigma v_K^3} = 0,
\]

where \( v_K = \sqrt{GM_* / R} \) is the Keplerian velocity and

\[
q = \frac{R (B_\phi)^2 + (R B_R)_h^2}{4\pi \Sigma v_K^3}.
\]

Now, we can treat the internal magnetic field using the induction equation. LRN showed that the variation of \( B_\phi \) with \( z \) within the disk is negligible for even field symmetry. Moreover, \( B_R \) and \( B_\phi \) are odd functions of \( z \) and consequently \( \partial B_R / \partial z \approx (B_R)_h / h \) and \( \partial B_\phi / \partial z \approx (B_\phi)_h / h \). Thus,

\[
B_R(R, z) = \frac{z}{h} (B_R)_h, \quad B_\phi(R, z) = \frac{z}{h} (B_\phi)_h,
\]

and the induction equation reads

\[
- \frac{R B_\phi v_R}{h} - \frac{\eta R}{h} (B_R)_h + \eta R \frac{d B_R}{d R} = 0,
\]

where the magnetic diffusivity \( \eta \) has the same units as kinematic viscosity. We assume that the magnitude of \( \eta \) is comparable to that of the turbulent viscosity \( \nu \) (e.g., Bisnovatyi-Kogan & Ruzmaikin 1976; Shadmeht 2004). Exactly in analogy to \( \alpha \) prescription for \( \nu \), we are using a similar form for the magnetic diffusivity \( \eta \),

\[
\eta = \eta_0 c_s h,
\]

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where $\eta_0$ is a constant. Note that $\eta$ is not constant and depends on the physical variables of the flow, and in our self-similar solutions, as we will show, $\eta$ scales with radius as a power law. This form of scaling for diffusivity has been widely used by many authors (e.g., Lovelace, Wang & Sulkanen 1987; LRN; Ogilvie & Livio 2001; Rüdiger & Shalybkov 2002).

While equation (11) describes transport of a large-scale magnetic field (here, $B_0(R)$), the values of $(B_{0,R})_h$ and $(B_{0,\phi})_h$ are determined by the field solutions external to the disk. Instead, we are following approach of LNR, in which the external field solutions obey the relations

$$(B_{0,\phi})_h = \beta_\phi B_z, \quad (B_{0,R})_h = \beta_R B_z,$$

where $\beta_\phi$ and $\beta_R$ are constants of order unity ($\beta_\phi < 0$). Thus, one can simply show that $< B_{0,R}^2 >= \beta_R^2 B_z^2/3$, $< B_{0,\phi}^2 >= \beta_\phi^2 B_z^2/3$ and $< B_0 B_\phi >= \beta_R \beta_\phi B_z^2/3$.

To close the equations of our model, we can write the energy equation describing the thermal state of the flow as

$$\rho T v_n \frac{dS}{dR} = Q_{\text{vis}} + Q_{\text{Joule}},$$

where $S$ is the specific entropy (per unit mass) and $T$ is midplane temperature of the disk. For the heating term, we may have two sources of dissipation: the viscous and resistive dissipations due to a turbulence cascade. So, $Q_{\text{vis}}$ and $Q_{\text{Joule}}$ represent viscous dissipation due to the radial motion and the Joule heating rate, respectively.

$$Q_{\text{vis}} = \rho \nu R^2 \left( \frac{d\Omega}{dR} \right)^2,$$

and

$$Q_{\text{Joule}} = \frac{\eta}{4\pi R^2} [2(B_{0,R})_h^2 + \frac{3}{5}(B_{0,\phi})_h^2]$$

Now we have constructed our model and the main equations of the model are equations (1), (2), (4), (8), (11) and (14). In the next section, we will present self-similar solutions of these equations.

### 3 SELF-SIMILAR SOLUTIONS

The equations of our model are reduced to standard equations of the slim disk, if we set all the magnetic terms equal to zero. As we discussed, we are considering a radiation-dominated disk, in which the gas pressure has been neglected comparing to the radiation pressure. After some algebraic manipulations we get to the following set of self-similar solutions:

$$\Sigma(R) = a \Sigma_0 \left( \frac{R}{R_0} \right)^{-1/2},$$

$$v_R(R) = b \left( \frac{GM_*}{R_0} \right) \left( \frac{R}{R_0} \right)^{-1/2},$$

$$v_\phi(R) = -c \left( \frac{GM_*}{R_0} \right) \left( \frac{R}{R_0} \right)^{-1/2},$$

$$P(R) = d \frac{\Sigma_0 GM_*}{R_0} \left( \frac{R}{R_0} \right)^{-3/2},$$

$$B_\phi(R) = e \sqrt{4 \pi \Sigma_0 \frac{GM_*}{R_0^2}} \left( \frac{R}{R_0} \right)^{-5/4},$$

$$h(R) = f R_0 \left( \frac{R}{R_0} \right),$$

where $\Sigma_0$ and $R_0$ provide convenient units with which the equations can be written in non-dimensional form. Thus, we obtain the following system of dimensionless equations, to be solved for $a$, $b$, $c$, $d$, $e$ and $f$:

$$ac = \dot{m},$$

$$-\frac{1}{2} ab^2 - ab^2 = \frac{3}{2} d - a + [2\beta_\phi + \left( \frac{5 + \beta_\phi^2 - \beta_R^2}{2} \right) f] c^2,$$

$$-\frac{1}{2} abc = -\frac{3a}{4} \sqrt{d} \frac{ab \beta_\phi (2 - \beta_\phi f) c^2},$$

$$af^2 + (\beta_\phi^2 + \beta_R^2) f c^2 - 2d = 0,$$
where $\dot{m} = M/(2\pi \Sigma_0 \sqrt{G M/R_0})$ is nondimensional mass accretion rate. Thus, our input parameters are $\alpha$, $\eta_0$, $\beta_r$, $\beta_\varphi$, and $\dot{m}$. We will present values of the physical quantities in non-dimensional form.

In the limit of nonmagnetic case, the above equations can be solved analytically:

\begin{align}
a &= \frac{3 \sqrt{2} \dot{m} \alpha}{\sqrt{25 + 36 \alpha^2} - 5} \approx \frac{5 \sqrt{2} \dot{m}}{6 \alpha}, \\
b &= \frac{1}{3 \alpha} \sqrt{\frac{25 + 36 \alpha^2}{25} - 5} \approx \frac{2}{\sqrt{5}}, \\
c &= \frac{25 + 36 \alpha^2}{3 \sqrt{2} \alpha} - 5 \approx \frac{6 \alpha}{5 \sqrt{2}}, \\
d &= \frac{\sqrt{3} \dot{m}}{3 \alpha}, \\
f &= \frac{\sqrt{3}}{3 \alpha} \sqrt{\frac{25 + 36 \alpha^2}{25} - 5} \approx \frac{2}{\sqrt{5}}.
\end{align}

The second relation in each equation refers to the limit $\alpha \ll 1$. The scaling of our solutions are different from Wang & Zhou (1999) who analyzed self-similar solution of optically thick advection-dominated flows. Because they applied $\omega p$ prescription for viscous stress tensor, but a diffusive prescription has been used in our model, i.e. equation (7).

The above nonmagnetic solutions show that the surface density increases with accretion rate, and decreases inversely with $\alpha$. However, the radial velocity is directly proportional to the viscosity coefficient $\alpha$. The gas rotates with sub-Keplerian angular velocity, i.e. $\Omega \approx \sqrt{2} \Omega K$. Note that except for the surface density and the pressure, the other physical quantities are independent of the accretion rate but depend only on the viscosity coefficient $\alpha$. An interesting feature is that the opening angle of the disk is fixed $h/R \approx 2/\sqrt{5}$, independent of $\alpha$ and of the mass accretion rate $\dot{m}$.

Although the radial scaling of the solutions are similar to the nonmagnetic case, we can see significant differences because of the magnetic field effect. The first important effect of the magnetic field on the disk structure is a squeezing effect, where the scale height $h$ is reduced compared to the nonmagnetic case. In fact, the squeezing effect of the large-scale magnetic field counterbalances the thickening of the disk generated by advection. Figure 1 shows the behaviour of physical disk quantities as a function of $\beta_\varphi$, for three different values of $\beta_r = -0.8, -0.9, -1.1$. The other input parameters are assumed as $\dot{m} = 1$, $\eta_0 = 0.01$, $\alpha = 0.01$. Generally, $\beta_r$ and $\beta_\varphi$ are parameters of order unity, however, we consider $\beta_\varphi$ around unity but changing $\beta_r$ from 0.5 to 1.2. For these input parameters, the surface density significantly reduces from $\alpha \approx 117$ to a value between 25 and 45 depending on the magnetic field configuration. However, as $\beta_\varphi$ increases, the surface density slightly increases for a fixed $\beta_r$. The rotation velocity is below Keplerian and as $\beta_\varphi$ increases, the rotation rate reaches to a maximum and then decreases. Also, the radial velocity increases with $\beta_r$ or $\beta_\varphi$, however, is significantly below free-fall velocity. But comparing to the nonmagnetic solution, the magnetic field tends to increase the radial velocity. For example, the above input parameters gives $c = 8.48 \times 10^{-3}$ for nonmagnetic flow, but the field causes $c$ increases to a value between 0.02 to 0.04. Magnetic field causes the opening angle of the disk decreases.

Figure 2 is the same as Figure 1, but with lower viscosity coefficient, i.e. $\alpha = 0.001$. We see this decrease of $\alpha$ causes the surface density increases. While the radial velocity decreases with $\alpha$, the rotation velocity becomes closer to the Keplerian rate. We see that the opening angle of disk for $\alpha = 0.001$ is smaller than $\alpha = 0.01$. Generally, the physical quantities are sensitive to the viscosity coefficient $\alpha$.

We repeated the above calculations for $\alpha = \eta_0 = 0.01$ and found that the solutions weakly change because of the variations of $\beta_r$ and $\beta_\varphi$. In this case, the solutions are qualitatively similar to Figures 1 and 2, but the scaling is somewhat different. For example, for these input parameters, we find the ratio of $v_\varphi/v_K$ between 0.66 and 0.69 depending on $\beta_r$ and $\beta_\varphi$. Regarding to Figure 1 which is for $\eta_0 = 0.1$, we can say as the resistivity coefficient $\eta_0$ decreases, the dependence of solutions on the outside field solutions becomes weaker. The nonmagnetic solutions show that the rotational and the radial velocity and the opening angle are independent of the mass accretion rate. This result is valid even in magnetic case, as we found by changing the accretion rate and keeping other input parameters constant.
Figure 1. Profiles of some height-averaged physical quantities for the disk, as a function of $\beta_r$, for $\dot{m} = 1$, $\eta_0 = 0.1$, $\alpha = 0.01$ and $\beta_\varphi = -0.8$ (solid line), $\beta_\varphi = -0.9$ (dashed line) and $\beta_\varphi = -1.1$ (dotted line). The surface density $\Sigma$, the radial velocity $v_R$, the rotational velocity $v_\varphi$, the pressure $P$, half-thickness $h$ are drawn in dimensionless form. The ratio of the magnetic pressure at the surface of the disk to the pressure is presented by $\beta_m$. 
Figure 2. Same as Figure 1, but in the case of $\alpha = 0.001$, $\dot{m} = 1$, $\eta_0 = 0.1$. Solid, dashed, and dotted lines refer to $\beta_\varphi = -0.8$, $\beta_\varphi = -0.9$ and $\beta_\varphi = -1.1$. 

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4 CONCLUSION

Although investigating the behavior of the magnetic field and associated currents within the disk was not the main purpose of our study, we studied the effect of a large-scale magnetic field with open field lines on the structure of an optically thick accretion disk. While the field outside of the disk treated in a phenomenological way, we solved the height-averaged MHD equations self-consistently using similarity technique in analogy to the original study of optically thin ADAF by Narayan & Yi (1994). Our self-similar solutions reduce to the nonmagnetic solutions of Wang & Zhou (1999) for optically thick advection-dominated accretion flow. The disk structure and the field geometry are closely linked. The magnetic field is dragged by the accreting flow, however, the field tends to squeeze the disk and to increase the radial velocity. The angular velocity of the flow is less than the local Keplerian angular velocity by a factor which depends on the magnetic field configuration.

Our simple self-similar solutions show that the effect of the magnetic field can not be ignored in a realistic accretion model. The present solutions may be applied to the X-ray galactic and extragalactic sources, when the accretion rate is high and radiation dominated regime takes place. The emergent spectrum of such a disk can be calculated using our magnetized self-similar solutions and considering the energy transfer in the vertical direction. We will discuss this problem in future.

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