Non-gaussian spatial correlations dramatically weaken localization

H. Javan Mard,1 E. C. Andrade,2 E. Miranda,3 and V. Dobrosavljević1

1Department of Physics and National High Magnetic Field Laboratory, Florida State University, Tallahassee, FL 32306
2Institut für Theoretische Physik, Technische Universität Dresden, 01062 Dresden, Germany
3Instituto de Física Gleb Wataghin, Unicamp, R. Sérgio Buarque de Holanda, 777, Campinas, SP 13083-859, Brazil

(Dated: May 11, 2014)

We perform variational studies of the interaction-localization problem to describe the interaction-induced renormalizations of the effective (screened) random potential seen by quasiparticles. Here we present results of careful finite-size scaling studies for the conductance of disordered Hubbard chains at half-filling and zero temperature. While our results indicate that quasiparticle wave functions remain exponentially localized even in the presence of moderate to strong repulsive interactions, we show that interactions produce a strong decrease of the characteristic conductance scale \( g^* \) signaling the crossover to strong localization. This effect, which cannot be captured by a simple renormalization of the disorder strength, instead reflects a peculiar non-Gaussian form of the spatial correlations of the screened disordered potential, a hitherto neglected mechanism to dramatically reduce the impact of Anderson localization (interference) effects.

PACS numbers: 71.10.Fd, 71.23.An, 71.30.+h, 72.15.Rn

According to the scaling theory of localization [1], any amount of disorder suffices to localize all (non-interacting) electrons at \( T = 0 \) in dimension \( d \leq 2 \). In the presence of electron-electron interactions, however, no such general statement exists, and the transport behavior of disordered interacting electrons remains an outstanding open problem [2]. Since the relevant analytical results are available only in some limiting cases [3–5], complementary computational methods play a crucial role in providing insight and information. Several numerical approaches have been recently utilized to investigate transport properties of these systems, including variational Hartree-Fock (HF) [6–8] and slave boson (Gutzwiller approximation) [9] methods, as well as (numerically exact) quantum Monte Carlo techniques [10–12].

These studies provided evidence that repulsive electron-electron interactions generally increase the conductance in small systems, with the suppression of electronic localization being tracked down to partial screening of the disorder potential. In principle, interactions could modify either the amplitude or the form of spatial correlations [6] of the renormalized disorder potential. The former mechanism is known to be significantly enhanced by strong correlation effects [9] and to survive even in high dimensions, the latter is more pronounced [13] in the weak-coupling regime and in low dimensions [4].

Despite this progress, several important questions remained unanswered: (1) What is the dominant physical mechanism for disorder screening, and can it qualitatively modify the noninteracting picture? (2) Can the interaction effects overcome Anderson localization and stabilize the metallic phase in low dimensions? The task to carefully and precisely answer these important questions in a model calculation is the the main goal of this Letter. To do this, we utilize two different variational methods to describe the statistics of the renormalized disorder potential in an idealized dirty Fermi liquid. In contrast to most previous attempts, here we perform a careful finite-size scaling analysis of the conductance, which allows us to reach conclusive results for the transport properties of the model we consider.

**Model and method.** We study the paramagnetic phase of a disordered Hubbard model

\[
H = -t \sum_{i,j,\sigma} (c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}) + \sum_{i,\sigma} \varepsilon_i n_{i\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow},
\]

where \( t \) is the hopping amplitude between nearest-neighbor sites, \( c_{i\sigma}^\dagger, c_{i\sigma} \) are the creation (annihilation) operators of an electron with spin \( \sigma = \uparrow, \downarrow \), \( U \) is the on-site Hubbard repulsion, and \( n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma} \). The spatially uncorrelated random site energies \( \varepsilon_i \) are drawn from a uniform distribution of zero mean and width \( W \). We work at half-filling, and units such that \( t = a = e^2/h = 1 \), where \( a \) is the lattice spacing, \( h \) is the Planck constant, and \( e \) is the electron charge. To be able to carry out large scale computations needed for conclusive finite-size scaling of the conductance, we focus our attention on a one-dimensional model; within the variational description of a dirty Fermi liquid we consider, we expect the main trends to persist in higher dimensions.

Our starting point is the non-magnetic HF scheme [6], where the renormalized site energies \( v_i \) are given by

\[
v_i = \langle n_i \rangle = \varepsilon_i + \frac{U}{2} \langle n_i \rangle.
\]

Here, \( \langle n_i \rangle = \sum_\sigma \langle n_{i\sigma} \rangle \), the average site occupation, is determined self-consistently in the ground state, for each disorder realization. To cross-check our HF predictions within a theory that is able to capture strong correlation effects, we repeated the same calculations using the
slave boson (SB) mean-field theory (i.e. the Gutzwiller approximation) of Kotliar and Ruckenstein [9, 14], generalized to disordered systems [13]. The SB theory features two local variational parameters: the renormalized site energies $v_i$ and the quasiparticle weight $Z_i$ ($Z_i = 1$ within HF) [15–17]. We found that, for moderate interaction strength (not too close to the Mott transition) and the low-dimensional situation we consider, both methods produce qualitatively the same behavior (see Fig. 2 below), dominated by a peculiar type of spatial correlation of the screened disorder potential. The strong correlation effects (corresponding to $Z_i \ll 1$) do not appear to play a significant role in this regime (in contrast to the situation explored in Ref. [13, 16]); this makes it possible to search for the relevant screening mechanism within the simpler and physically very transparent HF scheme, which we focus on in presenting most of our results.

To study the nature of the ground state we focus on the dimensionless conductance $g$, which we obtain applying the standard Landauer approach to our quasiparticle Hamiltonian [17, 18]. We numerically calculate $g$ in a setup where we attach our system to two non-interacting metallic leads at its ends [19]. For simplicity, we consider the wide band limit, where the leads’ self-energies are simply given by $\Sigma_{i(L)} = -i\eta/2$ [17], and in all our results we consider $\eta = 1.0t$ (we carefully checked that all our conclusions are independent of $\eta$ [17]). Since we are working in one dimension, the conductance displays wide sample to sample fluctuations; we therefore focus on its typical value, as given by the geometrical average $g = g_{typ} = \exp\ln g_s$ [20, 21]. In every case, we averaged our results over 2,000 disorder realizations, which was sufficient to obtain very accurate results.

Conductance scaling. — In the non-interacting limit, the dependence of the conductance on disorder and the system size can be expressed in a simple scaling function $g_0(x)$, with $x = L/\xi$, where $\xi = \xi(W)$ is the localization length [1, 22]. Specifically, $g_0 \propto L^{d-1}$ for $g \gg g^*$ (ohmic regime, $x \ll 1$) and $g_0 \propto \exp(-L/\xi)$ for $g \ll g^*$ (localized regime, $x \gg 1$), where $g^*$ is the characteristic dimensionless conductance which marks the crossover between these two regimes. In particular, we use the expected exponential decay of the conductance to determine $\xi$ for fixed values of $W$.

Using this scaling Ansatz, we can collapse the system size dependence of the conductance onto a scaling curve $g(L/\xi)$ even in the presence of interactions, as shown in Fig. 1. We find that the localization length increases considerably with $U$ (see [17] for more details). This enhancement of the localization length with interactions has been often observed in studies of disordered interacting systems [7, 12, 23, 24]. We should nevertheless stress that, despite the huge enhancement of $\xi$ with $U$, there is always an exponential decrease for large $L$ and we do not see any evidence of extended states.

Interestingly, all the curves $g(L/\xi)$ for different interaction strengths can be made to collapse onto a single universal curve by a proper interaction-dependent rescaling of the conductance, see Fig. 2. We call the conductance rescaling factor $g^*(U)$, and stress that this characteristic conductance $g^*$ is a function of the Hubbard $U$ only. Its $U$-dependence for both HF and SB approaches is shown in the inset of Fig. 2, where an exponential decrease with $U$ fits well the data in both cases.

Since the collapsed curves encompass the non-interacting case, the above scaling implies that the full disorder and interaction dependence of the conductance can be written as

$$g = g^*(U) g_0[L/\xi(W, U)],$$

where $g_0(x)$ is the non-interacting scaling function, $g^*(U)$ sets the crossover conductance which separates the weak localization regime ($g \gg 1$) from the strongly localized one ($g \ll 1$), and we have explicitly shown all the $W$ and $U$ dependences.

The scaling function in Eq. (3) can then be used to generate the beta function $\beta(g) = d\ln g/d\ln L$. It follows immediately that the only effect introduced by interactions on $\beta(g)$, as compared to its non-interacting counterpart, is the rescaling of $g$ by the characteristic conductance $g^*(U)$

$$\beta(g) = \beta_0[g/g^*(U)],$$

where $\beta_0(g)$ is non-interacting beta function. In particular, if we assume the form of $\beta_0(g)$ proposed in ref. [20] we obtain

![Figure 1](image_url)
The validity of Eq. (5) can be double-checked through a direct examination of the behavior of the beta function for different values of $U$, as shown in Fig. 3. We stress that interaction-induced renormalizations of the localization length alone are not capable of describing the results of Fig. 3, as they drop out of the beta function. Finally, using (5), we are able to give an operational definition of the characteristic conductance: $g = g^* (U)$ at $L = L^* = (\ln 2) \xi (W, U)$ [17].

Disorder screening and non-Gaussian spatial correlations.— A commonly invoked explanation for this conductance enhancement is the fact that interactions act to “screen” the one-body potential [6, 9, 16]. Within a mean-field description, an electron moving in the one-body potential $v_i$ “sees” site energies renormalized by the average interaction with the other electrons, as in Eq. (2). In the inset of Fig. 4, we compare the conductance in the full HF calculation for $W = 0.5t$ and $U = 1t$ with the one obtained in the non-interacting case with an effective disorder $W_{\text{eff}}$ obtained from the width of the $v_i$ distribution [17]. It is clear that the screening effect by itself is not enough to reproduce the conductance enhancement of the full HF calculation. This is further confirmed when, after obtaining the fully converged self-consistent HF values of $v_i$’s, we then calculate the conductance of a non-interacting system whose site energies are a random permutation (RP) of the same $v_i$’s. Not surprisingly, the conductance of the randomized system is essentially the same as the one for the non-interacting system with uncorrelated site energies distributed uniformly with strength $W_{\text{eff}}$ (inset of Fig. 4). In main panel of Fig. 4, we also show the beta function obtained from the RP of the HF results. As can be seen, it reduces to the non-interacting one. We stress that the effect of a RP of the renormalized site energies is to eliminate the spatial correlations between them. In the following we argue that it is precisely these correlations which shift the crossover scale $g^* (U)$ to much smaller values as compared to the $U = 0$ case.

To further elucidate the pivotal role of spatial correlations, we start by looking at the limit of weak disorder $W \rightarrow 0$. A perturbative calculation shows that the correlations among the $v_i$’s are given by $\langle v_i v_j \rangle / W^2 \sim r_{ij}^{-1}$, for $r_{ij} \gg 1$, where $r_{ij} = |\vec{r}_i - \vec{r}_j|$ [13, 17]. These long-ranged correlations of the effective disorder potential come from the usual Friedel oscillations. When properly tailored, a correlated disorder potential may drive a metal-insulator transition in $d = 1$ [25–28]. In order to go beyond weak disorder, we first generate numerically the two point correlation function $B_{ij} = \langle v_i v_j \rangle / W_{\text{eff}}^2$, from our HF results [17]. We then implement a standard procedure to generate random $v_i$’s with gaussian correlations given by $B_{ij}$ (note that the generated data have no correlations beyond gaussian). Finally, we calculate the conductance of a non-interacting system with the latter site energies. Essentially, we want to know if the gaussian correlations contained in $B_{ij}$ are sufficient to account for the $g^*$ renormalization. Fig. 5 displays the results of this numerical procedure (which we dubbed gaussian spatial correlations (GSC)). Although the conductance is enhanced as in the case of $W_{\text{eff}}$ (see the inset of Fig. 5), the scaling curve
Figure 4. Beta function for $U = 1t$ obtained after randomizing the self-consistently determined HF renormalized energies $v_i$’s (red circles; see also the text for an explanation of the procedure). Black and green lines are plotted using Eq. (5). In the inset, the conductance $g$ is shown as a function of $L$ for: (a) a system with $W = 0.5t$ and $U = 1t$ in the HF approximation (black dots), (b) a non-interacting system whose site energies are a random permutation of the renormalized site energies $v_i$ of the HF approximation (red squares), and (c) a non-interacting system with $U = 0$ and $W = W_{\text{eff}} = 0.41t$ (gold diamonds).

Figure 5. Comparison of the conductance scaling function of a disordered chain with $U = 2t$ in the HF approximation and the corresponding non-interacting system with the same gaussian correlated site energies (GSC) (the inset shows the unscaled data). Solid lines are drawn by using $g = g^*/(\exp(x) - 1)$, as in Fig. (1).

The renormalization $g \to g^*/g^*$ should dramatically reduce the amplitude of the weak-localization correction; precisely such an effect was observed in $d = 2$ magnetoresistance experiments [30]. In practice, this would open the possibility that competing (e.g. Mott or Wigner-Mott) mechanisms for localization [31, 32] could become dominant well before Anderson localization effects set in.

**Conclusions.**— Adding interactions to a disordered system gives rise to new effects that assist transport even if the single particle states are all Anderson localized. Our careful numerical studies show that the typical value of the scaled conductance follows the same non-interacting behavior but with a large decrease of the conductance scale $g^* (U)$ signaling the crossover to the strongly localized regime. Consistent with scaling, this shift is independent of the system size or the strength of disorder, and only depends on the interaction strength $U$. Surprisingly, we find that the form of the spatial two-point correlation function $B_{ij}$ characterizing the renormalized disorder potential is not enough to explain this shift. This indicates that non-gaussian (higher order) correlations play a crucial role in decreasing localization effects, a mechanism overlooked in previous works. Clarifying the precise origin and role of these higher-order correlations is not an easy task, but it opens an exciting new door to understanding the effects of interactions in disordered systems.

We acknowledge support by DFG through grants FOR 960 and GRK 1621 (ECA), CNPq through grant 304311/2010-3 (EM), FAPESP through grant 07/57630-5 (EM) and NSF through grant DMR-1005751 (VD and HJM).
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Supplemental notes for: “Non-gaussian spatial correlations dramatically weaken localization”

H. Javan Mard,1 E. C. Andrade,2 E. Miranda,3 and V. Dobrosavljević1

1Department of Physics and National High Magnetic Field Laboratory, Florida State University, Tallahassee, FL 32306
2Institut für Theoretische Physik, Technische Universität Dresden, 01062 Dresden, Germany
3Instituto de Física Gleb Wataghin, Unicamp, R. Särgio Buarrque de Holanda, 777, Campinas, SP 13083-859, Brazil
(Dated: May 11, 2014)

PACS numbers: 71.10.Fd, 71.23.An, 71.30.+h, 72.15.Rn

I. COMPUTATION OF THE CONDUCTANCE

The dimensionless conductance computed in this work is defined as \( g = G/ (2e^2 / h) \). The sample conductance \( g_s \), with the effects of the contacts already removed, is given by the Landauer relation [1]

\[
g_s = \frac{<T_s>_{typ}}{1 - <T_s>_{typ}},
\]

where \( T_s \) is the total sample transmittance, and ‘typ’ refers to the geometrical average \(<T_s>_{typ} = \exp \ln T_s\) (the overbar denoting an average over realizations). The transmission function is simply given by [2]

\[
T(\omega) = \frac{1}{2} \text{Tr} \left[ G_S^\dagger (\omega) \Gamma_R (\omega) G_S (\omega) \Gamma_L (\omega) \right].
\] (2)

Here, all bold-face quantities are matrices in the lattice site basis. \( \Gamma_{L(R)} (\omega) \) are the coupling matrices, describing the coupling of the system to non-interacting left (L) and right (R) leads, and are determined by the leads’ self-energies as

\[
\Gamma_{L(R)} (\omega) = i \left( \Sigma_{L(R)} - \Sigma_{L(R)}^\dagger \right).
\] (3)

For the sake of simplicity, we assume

\[
\Sigma_{L(R)} = -i \eta B_{L(R)},
\] (4)

where the matrix \( B_{L(R)} \) is equal to 1 for a site \( i \) connected to the left (right) lead and 0 otherwise. This constant lead self-energy is equivalent to the assumption of wide lead bands. \( G_S \) is the sample Green’s function, given by

\[
G_S = \left[ (\omega + \mu) 1 - H_S^0 - \Sigma_S - \Sigma_L - \Sigma_R \right]^{-1},
\] (5)

where \( \mu \) is the chemical potential. \( H_S^0 \) is the non-interacting sample Hamiltonian and \( \Sigma_S \) is the sample self-energy, which accounts for the electronic correlations. As we employ a Hartree-Fock approach, the sample self-energy is given by

\[
\Sigma_S = \frac{U}{2} n + \mu 1.
\] (6)

\( n \) is a diagonal matrix in the site basis whose elements \( n_i \) correspond to the occupation of the site \( i \). It can be seen from Eqs. (5), (6) and (4) that the sample Green’s function can be obtained through a single diagonalization of an effective Hamiltonian, as opposed to a matrix inversion for every frequency value. Even with this simplification, we are still left with a non-Hermitian matrix. We have implemented this diagonalization step using standard LAPACK routines. A particular efficient implementation can be achieved using the OpenMP version of Intel’s MKL library, which allowed us to perform these calculations on a desktop computer. Furthermore, using Eqs. (3) and (4), we can rewrite Eq. (2) as

\[
T(\omega) = 2 \eta^2 \text{Tr} \left[ G_{1L}(\omega) G_{L1}^\dagger (\omega) \right],
\] (7)

where \( G_{1L}(\omega) \) is the sample Green’s function from the first site \((i = 1)\) (connected to the left lead) to the last one \((i = L)\) (connected to the right lead).

II. THE LOCALIZATION LENGTH

The localization length \( \xi \) is calculated using the exponential behavior of the conductance \( e^{-x} \) at very large \( x = \frac{\xi}{\Delta} \) (for this, either \( L \) or \( W \) should be very large). In other words, it can be computed from the slope of the conductance as a function of \( x \) on a semi-log scale (assuming all states are localized, which we always find). Fig. 1 shows that adding interactions considerably increases the localization length, even though all states remain localized.
III. OPERATIONAL DEFINITION OF THE CHARACTERISTIC CONDUCTANCE

We now show that the characteristic conductance \( g^* \) can be determined in practice as the value at the point \( L = L^* = (\ln 2) \xi \), or \( x = \ln 2 \). Given the beta function for 1D localization, we can write

\[
\ln \frac{L}{L_0} = \int_{g_0}^{g^*} \frac{d \ln g}{\beta(g)} = F(g) - F(g_0),
\]

where we define the function

\[
F(g) = \int_{g_0}^{g} \frac{d \ln g}{\beta(g)},
\]

and \( g_0 \) is a reference conductance at scale \( L_0 \). From Eq. (8) we get

\[
F[g(L)] = \ln \left( \frac{L e^{F(g_0)}}{L_0} \right),
\]

which implicitly gives \( g(L) \). Defining the reference conductance \( g = g^* \) at \( L = L^* \)

\[
F[g(L^*)] = \ln \left( \frac{L e^{F(g^*)}}{L^*} \right).
\]

Using the form proposed in the main text \( \beta(g) = \beta_0 \left( g/g^* \right) \) and the non-interacting beta function of ref. 3, we find

\[
F(g) = - \int_{g_0}^{g} \frac{d \ln g}{\left( 1 + \frac{g^*}{g} \right) \ln \left( 1 + \frac{g^*}{g} \right)} = \ln \left( B \ln \left( 1 + \frac{g^*}{g} \right) \right),
\]

where \( B \) is an arbitrary constant. It then follows from Eq. (11) that

\[
F(g) = \ln \left( \frac{L}{L^*} B \ln 2 \right).
\]

On the other hand, the scaling function proposed in ref. 3 is

\[
\frac{g^*}{g} = e^{L/\xi} - 1,
\]

which, when plugged into Eq. (12), leads to

\[
F(g) = \ln \left( B \frac{L}{\xi} \right).
\]

Finally, comparing Eqs. (13) and (15) gives us the anticipated result \( L^* = (\ln 2) \xi \approx 0.69 \xi \).

IV. SPATIAL CORRELATIONS OF THE RENORMALIZED DISORDER POTENTIAL

A. Weak disorder

An important question is how to characterize the spatial correlations between the renormalized site energies \( v_i \). For weak disorder, this problem is solvable analytically. Expanding the average on the right-hand side of Eq. (2) of the main text to first order in the bare disorder, we find the spatial Fourier component [4? , 5]

\[
v_q = \frac{\varepsilon_q}{1 - U \Pi_q} + O(\varepsilon_q^2),
\]

where \( \Pi_q \) is the usual static Lindhard polarization function of the clean, non-interacting system, which in the case of a one-dimensional tight-binding chain is given by

\[
\Pi_q = -\rho(\varepsilon_F) \frac{1}{\sin(q/2)} \ln \left( \frac{\cos(q/2)}{1 - \sin(q/2)} \right),
\]

where \( \rho(\varepsilon_F) \) is the density of states at the Fermi level.

Using Eqs. (16) and (17) we can generate a renormalized potential \( v_i \) in this linear response approach (LRA). The result of this procedure is shown in Fig. 2. As can be seen from Fig. 2, although the LRA potential exhibits both disorder screening and spatial correlations, it does not capture the renormalization of \( g^* \): the scaling function (and its mathematical equivalent, the beta function) coincides with the one from the non-interacting calculation. This result strongly suggests that the \( g^* \) renormalization is a non-perturbative effect of disorder and a fully self-consistent solution is needed in order to capture it.
B. Gaussian correlations

To go beyond the weak disorder limit, we calculate the fully self-consistent HF two-point correlation function $B_{ij} = \langle v_i v_j \rangle / W_{\text{eff}}^2$ as defined in the main text with $B_{ij} = B(r_i - r_j)$. Here $\langle \cdots \rangle$ denotes an average over both pairs and disorder realizations. $W_{\text{eff}}$ is the effective disorder width, which is simply given by the standard deviation of $v_i$. Defined in this fashion, $B(U = 0) = 1$. In Fig. 3 we show $B(r)$ as a function of the distance $r$ between two sites. It displays Friedel-like oscillations which are enhanced as $U$ increases. For comparison, we also show $B(r)$ obtained within LRA at $U = t$.

Since a Gaussian distribution is fully determined by its two-point correlation function, we can use $B_{ij}$ to generate Gaussian correlated variables. To implement this procedure, we first calculate the Fourier transform of $B_{ij}$, $B(q)$, which is shown in Fig. 4. As expected, its derivative diverges at $q = \pm 2k_F$, giving rise to long-ranged Friedel-like oscillations, as shown in Fig. 3. Once we have $B(q)$, we generate Gaussian distributed random numbers $v_q$ using the Box-Muller method with a $q$-dependent variance $B(q)$. We can then obtain their real space values $v_i$ after a numerical Fourier transform.

The result of this procedure is shown in the main text, where it is shown to be insufficient to capture both the
renormalization of the crossover scale $g^*$ and the enhancement of the localization length (see Fig. 5). Therefore, we conclude that this phenomenon is not only non-perturbative in disorder but also determined by higher order disorder correlations beyond the Gaussian level.

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