Critical analysis of modified grouping efficacy measure; new weighted modified grouping efficiency measure

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\textbf{ABSTRACT}

Different grouping efficiency measures have been used to evaluate block-diagonal forms in group technology. One of the well-known measures is grouping efficacy. Many grouping efficiency measures were developed in the literature are based on or derived from grouping efficacy. One of these measures is called modified grouping efficacy (MGE). The limitation of MGE measure is that the case of zero efficiency can’t be determined. Moreover, the efficiency of the system is close to zero or very low compared with other measures in the case, where the number of voids is greater than number of exceptions inside the cell. To overcome these limitations, a new measure called weighted modified grouping efficacy is developed in this paper. This measure is tested against some problems from the literature and the results demonstrate the ability of this measure to be used in the evaluation of all types of Structure data.

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\section{1. Introduction}

In recent years many firm operating in a small-to-medium-size batch manufacturing environment have moved towards cellular manufacturing (CM), and the use of group technology (GT), to improve the design and operation of their production system, so as to become more competitive and cost-efficient (Srivastava & Chen, 1995). Group technology (GT) is a manufacturing approach, which organizes and uses the information about an item’s similarity (parts and/or machines) to enhance the efficiency and effectiveness of batch manufacturing systems (Chen & Guerrero, 1994). The basic idea of GT is to group together parts that require similar operations and machines corresponding to these operations (Kusiak & Cho, 1992). The input to the GT problem is a zero-one in matrix $A$ where $a_{ij} = 1$ indicates the visit of component $j$ to machine $i$, and $a_{ij} = 0$ otherwise.

Grouping of components into families and machines into cells results in a transformed matrix with diagonal blocks where ones occupy the diagonal blocks and zeros occupy the
off-diagonal blocks. The resulting diagonal blocks represent the manufacturing cells (Nair & Narendran, 1998). The case where all the ones occupy the diagonal blocks and all the zeros, occupy the off-diagonal blocks is called perfect diagonal blocks. But this case is rarely accomplished in practice. For that the most desirable solution of cellular manufacturing systems is that which gives minimum number of zero entries inside a diagonal block (known as voids) and minimum number of one entries outside the diagonal blocks (known as the exceptional elements). A void indicates that a machine assigned to a cell is not required for the processing of a part in the cell. An exceptional element is created when a part requires processing on a machine that is not available in the allocated cell of the part. Voids and exceptional elements have adverse implications in terms of system operations for more details see Adil, Rajamani, and Strong (1996). One of the difficulties in GT is the objective evaluation of solutions to the part family – machine cell formation problem (Sandbothe, 1998). Evaluating the structure of the part-machine solution matrix was first explored with creation of a grouping efficiency criterion (Chandrasekharan & Rajagopalan, 1986). A second measure is called grouping efficacy (Kumar & Chandrasekharan, 1990). The structure of the final machine-component matrix significantly affects the effectiveness of the corresponding cellular manufacturing system (Seifoddini & Djassemi, 1996). For this reason, the choice of grouping methodology must be based on criteria that can indicate the goodness of a grouping solution. Hence, a number of grouping measures have been developed to evaluate the efficiency of block-diagonal forms. For other measures that are available in the literature see Sarker and Mondal (1999), Sarker and Khan (2001), Mukattash el al. (2018), Kellie, Evelyn, and Tabitha (2007) and Lee and Ahn (2013).

The following definitions will be used in this paper:

**Block**: A sub-matrix of the machine-component incidence matrix formed by the intersection of columns representing a component family and rows representing a machine cell. 

**Voids**: A zero element appearing in a diagonal block.

**Exceptional element** (or exception): A one appearing in the off – diagonal blocks.

**Perfect block-diagonal form**: A block-diagonal form in which all diagonal blocks contain ones and all off-diagonal blocks contain zeros (Kumar & Chandrasekharan, 1990).

**Sparsity** (**Block-diagonal** space): Total number of elements within the diagonal blocks of the solved matrix (Sarker & Khan, 2001).

**Optimal solution**: A system that contains minimum sum of voids and exceptions in the solved matrix.

This paper introduces a modified measure called Weighted Modified Grouping Efficacy (WMGE), which is more accurate to determine the efficiency of a block-diagonal form for developing cellular manufacturing systems. Moreover, WMGE measure is considered to be more effective since it can be used to evaluate all types of structure data. WMGE measure can distinguish between two or more alternative manufacturing systems having the same sum of voids and exceptions, because sparsity of the manufacturing system is taken into consideration.

### 2. Overview of performance measures

The most available used measures for goodness of cells are shown and discussed in the following sub-section. All these efficiency measures are based or derived from grouping efficacy.
• Grouping Efficacy ($\tau$), (Kumar & Chandrasekharan, 1990).

The main limitations of grouping efficacy have been exposed by Sandbothe (1998), Sarker and Mondal (1999), Sarker and Khan (2001), Ng (1993) and Nair and Narendran (1996).

Grouping efficacy ($\tau$) is defined as:

$$\tau = \frac{1 - \Psi}{1 + \phi} \quad (1)$$

where

$$\Psi = \frac{Number \ of \ exceptional \ elements}{Total \ number \ of \ operations \ in \ the \ MP \ matrix}$$

$$\phi = \frac{Number \ of \ voids \ in \ the \ diagonal \ blocks}{Total \ number \ of \ operations \ in \ the \ MP \ matrix}$$

Also $\tau$ can be written as:

$$\tau = \frac{k}{k + e_0 + v} \quad (2)$$

And $k + e$: total number of operations in the MP matrix; $k$: number of operations in the diagonal block; $e$: number of exceptions; $v$: number of voids

• Modified grouping efficacy ($\tau_2$), (Nair & Narendran, 1996):

$$\tau_2 = \frac{B - qe_v + (1 - q)e_0}{B + qe_v + (1 - q)e_0} \quad (3)$$

where $B$: is the sparsity of the solved matrix; $e_0$: number of exceptions; $e_v$: number of voids; $q$: weighted factor

• Grouping Index ($\gamma$), (Nair & Narendran, 1996):

where $\gamma$ is derived from the modified grouping efficacy by introducing a correction factor.

$$\gamma = \frac{1 - \frac{qe_v + (1 - q)(e_0 - A)}{B}}{1 + \frac{qe_v + (1 - q)(e_0 - A)}{B}} \quad (4)$$

where $A = 0$ for $e_0 \leq B$ and $A = e_0 - B$ for $e_0$ greater than $B$

$\tau$ can be written as:

$$\tau = \frac{1 - \alpha}{1 + \alpha} \quad (5)$$

where

$$\alpha = \frac{qe_v + (1 - q)(e_0 - A)}{B} \quad (6)$$
where \( A \) is a correction factor and \( B \) is the sparsity of the matrix.

- Weighted grouping efficacy (\( \omega \)), (Ng, 1993):

\[
\omega = \frac{q(e - e_0)}{q(e + e_v - e_0) + (1 - q)e_0}
\]  

where \( e \): total number of operations in the MP matrix; \( e_0 \): number of exceptions; \( e_v \): number of voids; \( q \): weighted factor

- Comprehensive Grouping Efficacy (CGE), (Mukattash et al., 2018):

\[
\text{CGE} = \frac{B_1}{B} \left( \frac{k_1}{k_1 + v_1 + e_1} \right) + \frac{B_2}{B} \left( \frac{k_2}{k_2 + v_2 + e_2} \right) + \ldots + \frac{B_p}{B} \left( \frac{k_p}{k_p + v_p + e_p} \right)
\]

where \( B_1, B_2, B_p \), and \( B \) represent the sparsity of the first, the second and the \( p \)th cell in the solved matrix, respectively. Also, \( B \) represents the sparsity of the solved matrix, which is defined as the total number of elements within diagonal blocks of the solve matrix. Here, \( B \) represents the sparsity of the solved matrix and \( B_1 = n_1 \times m_1, B_2 = n_2 \times m_2 \) and \( B_p = n_p \times m_p \).

where \( m \) = total number of parts in the matrix; \( n \) = total number of machines in the matrix; \( m_j \) = number of parts in the \( j \)th diagonal block [\( j \)th cell]; \( n_j \) = number of machines in the \( j \)th diagonal block [\( j \)th cell]; \( v_j \) = number of voids in the \( j \)th diagonal block; \( e_j \) = number of exceptional elements in the \( j \)th off-diagonal block; \( k_j \) = number of operations in the \( j \)th diagonal block; \( p \) = total number of diagonal blocks [total number of cells in the matrix].

### 3. Critical analysis of the modified grouping efficacy measure

In this section, a mathematical and critical analysis will be dressed for the Modified Grouping Efficacy measure (MGE), (Rajesh, Chalapathi, Chaitanya, Sairam, & Anildeep, 2016). MGE can be written as:

\[
\text{MGE} = \frac{(1 - \lambda)}{(1 + \xi)}
\]

\[
\lambda = \frac{\text{Number of voids within the cell}}{\text{Total number of operations}}
\]

\[
\xi = \frac{\text{Number of operations outside bdf}}{\text{Total number of operations}}
\]

then

\[
\lambda = \frac{v}{k + e}
\]

and
where $k + e$: total number of operations in the MP matrix; $k$: number of operations in the diagonal block; $e$: number of exceptions; $v$: number of voids.

Then, MGE can be written as:

$$\xi = \frac{e}{k + e}$$  \hspace{1cm} (13)

or can be written as:

$$\text{MGE} = 1 - \frac{\frac{v}{k+e}}{1 + \frac{\frac{e}{k+e}}{k+2e}}$$  \hspace{1cm} (14)

or can be written as:

$$\text{MGE} = \frac{k + e - v}{k + 2e}$$  \hspace{1cm} (15)

### 3.1. Mathematical properties of modified grouping efficacy function

1. Physical meaning of extremes:
   - When all the ones in the perfect diagonal-block are outside the diagonal block (condition of zero efficiency where $k$ is equal to zero).

   $$\text{MGE} = \frac{e - v}{2e}$$  \hspace{1cm} (16)

   It is clear that when $k = 0$, MGE is not equal to zero. This limitation will lead to have inaccurate results. The numerator of MGE is equal to $e - v$, which will equal to zero if and only if when $e = v$. But the case of zero efficiency doesn't mean that number of voids should be equal to number of exceptions.

   - For perfect diagonal block [condition of 100% efficiency], then MGE = 1 because $e = v = 0$

2. Non negativity: The case of zero efficiency may lead to a negative or very small value of efficiency, if number of voids is greater than number exceptions.

3. From property 1 and property 2 it is found that MGE efficiency is less than or equal to 1, but it may be less than zero, while in all grouping efficiency measures should be greater than or equal to zero and less than or equal to 1.

### 3.2. Testing the modified grouping efficacy measure

In order to analyze MGE measure a computer programming model has been used to test different case studies from literature. One of these cases containing 7 machines and 11 parts and its machine–part matrix is given below in Figure 1. This case study is provided by Boctor (1991) and solved by Mukattash et al. (2017). The two alternative optimal solutions (two-cells) for this case study are shown below in Figures 2 and 3. The two alternative optimal solutions are considered to be the best formation scheme for cell formation of the case study shown in Figure 1. From Table 1, it is clear that the results of using MGE measure
are not the same when using other well-known measures. The results of using MGE measure are inaccurate. The optimal solution is considered to be the best scheme, which gives minimum sum of voids and exceptions.
## Table 1. Evaluation of different measures for Figures 2 and 3.

|                | # of cells \((p)\) | # of voids \((v)\) | # of exceptions \((e)\) | \(v + e\) | Sparsity \((B)\) | Total number of operations in the MP matrix \((K + e)\) | # of operations inside the cells \((k)\) | Grouping index \((y)\) | Grouping efficacy \((r)\) | Modified grouping efficacy \((r_2)\) | Weighted grouping efficacy \((ω)\) | Comprehensive grouping efficacy \((CGE)\) | Modified grouping efficacy \((MGE)\) |
|----------------|-------------------|-------------------|-----------------|--------|----------------|-------------------------------|-------------------|-----------------|----------------|---------------------|-----------------|-------------------------------|---------------------|
| 2-cell 1st solution | 20                | 2                 | 22              | 39     | 21             | 19                           | .56               | .45             | .6              | .463                | .463            | .463                           | .043                |
| 2-cell 2nd solution | 19                | 3                 | 22              | 37     | 21             | 18                           | .542              | .45             | .6              | .45                 | .456            | .456                           | .083                |
To study the effectiveness of MGE measure the optimal solutions (Figures 2 and 3) are analyzed and the results are compared with different efficiency measures as shown below in Table 1.

From Table 1, we can notice that the results of MGE measure are inaccurate compared with other well-known efficiency measures. For the first solution the efficiency is around 4.3%, while the other measures it is more than 45%. The reason behind that, the effect of number of voids is more than the effect of number exceptions on MGE measure as discussed above in Section 3.1. In this case, the number of voids is greater than the number of exceptions which will lead to illogical result.

4. Proposed measure

4.1. Weighted modified grouping efficacy

In this section, a new grouping measure called Weighted Modified Grouping Efficacy (WMGE) is proposed to overcome the limitations of MGE measure. The new grouping measure can be expressed as:

\[
WMGE = \frac{1}{2} \left[ \frac{k + e - v}{k + 2e} \right] + \frac{1}{2} \left[ \frac{k - e + v}{k + 2e} \right]
\]  

Equation (17) can be written as shown below:

\[
WMGE = \frac{1}{2} \left[ \frac{k + e - v}{k + 2e} \right] + \frac{1}{2} \left[ \frac{B - e}{k + 2e} \right] = WMGE = \left[ \frac{B - v}{k + 2e} \right], \text{ where } B = k + v
\]

For that, ½ has been used just to write the equation in different form. It is not a weight. The best ratio (if exist) is depending on the equation (model) itself, if it is used as a weighted factor for the model or in order to make the model satisfy the mathematical properties of the grouping measure.

The mathematical properties of the proposed function showed that the efficiency is within the range between zero and one, (0 ≤ WMGE ≤ 1).

where \(k + e\): total number of operations in the MP matrix; \(k\): number of operations in the diagonal block; \(e\): number of exceptions; \(v\): number of voids; \(B\): Sparsity: Total number of elements within the diagonal blocks of the solved matrix, where \(B = k + v\).

4.2. Mathematical properties of weighted modified grouping efficacy function

WMGE results and other well-known measures results are not different in evaluating the manufacturing systems, and WMGE and other well measures satisfy the following properties:

(1) Non negativity: All the elements of weighted modified grouping efficacy measure are positive.

(2) Physical meaning of extremes:
• When all the ones in the perfect diagonal-block are outside the diagonal block [condition of zero efficiency], WMGE = 0 because $k = 0$.
• For perfect diagonal block [condition of 100% efficiency], WMGE = 1 because $v = e = 0$.

(3) From property 1 and property 2 it is found that $0 \leq WMGE \leq 1$.

MGE failed to satisfy all the above properties.

From the above discussion it is clear that, why MGE measure has inaccurate results (see Table 1), compared to WMGE and other well-known measures.

4.3. Superiority of weighted modified grouping efficacy measure

WMGE measure can distinguish between two or more alternative manufacturing systems having the same sum of voids and exceptions, because sparsity of the manufacturing system is taken into consideration. Moreover, WMGE can be used to find the efficiency of all types of structure data with logical results. From Table 3, it is clear that some measures can’t distinguish between the two solutions shown in Figures 2 and 3.

4.4. Testing the weighted modified grouping efficacy measure

To analyze and study the behavior of WMGE measure the optimal solutions (Figures 2 and 3) are analyzed and the results are summarized in Table 2.

Moreover, in order to evaluate the performance of the proposed measure, a comparison will be done with different case studies taken from literature. The following table (Table 3) summarizes the results.

From Table 3 it is clear that MGE has an inaccurate result in the first case study compared with other efficiency measures. In this case the efficiency is around 32.2% while for the other measures it is more than 49%. This result shows the deficiency in the MGE measure.

The following case study (small cell size, $8 \times 10$) will be used to test WMGE measure and compare the results with other well known measures. Consider the solution matrix of Figure 4, taken from the literature (Kusiak & Chow, 1987).

The results shown in Tables 3 and 4, illustrate the validity of WMGE measure to evaluate block-diagonal forms in group technology with different cell sizes, compared with other measures. Moreover, the superiority of WMGE measure is shown in Table 2, through distinguishing between the two optimal alternative solutions.

Table 2. Evaluation of WMGE measure for Figures 2 and 3.

| # of cells     | Voids (v) | Exceptions (e) | $v + e$ | Sparsity (B) | Total number of operations in the MP matrix | # of operations inside the cells | Weighted modified grouping efficacy (WMGE) |
|----------------|-----------|----------------|--------|--------------|--------------------------------------------|--------------------------------|----------------------------------------|
| 2-cell 1st solution | 20        | 2              | 22     | 39           | 21                                         | 19                             | .826                                   |
| 2-cell 2nd solution  | 19        | 3              | 22     | 37           | 21                                         | 18                             | .75                                    |
Table 3. Comparison of WMGE with some commonly known measures, ($q = .5$).

| Source | Problem size ($n \times m$) | # of voids ($v$) | # of exceptions ($e$) | $v + e$ | Sparsity ($B$) | Total number of operations in the MP matrix ($k + e$) | # of operations inside the cells ($k$) | Grouping Index ($\gamma$) | Grouping efficacy ($\tau$) | Modified grouping efficacy ($\tau_2$) | Weighted grouping efficacy ($\omega$) | Weighted modified grouping efficacy (WMGE) | Modified grouping efficacy (MGE) |
|--------|-------------------------------|------------------|----------------------|--------|----------------|-----------------------------------------------|--------------------------------------|-------------------------------|------------------|-------------------------------|-----------------------------|--------------------------------------|-----------------------------|
| Burbidge (1973), solved by Boctor (1991) | 16 $\times$ 43 | 77 | 26 | 103 | 177 | 126 | 100 | .549 | .493 | .663 | .493 | .6577 | .322 |
| Solved by Rajesh et al. (2016) | 16 $\times$ 24 | 16 | 33 | 49 | 69 | 86 | 53 | .4759 | .5196 | .828 | .5196 | .445 | .588 |
5. Summary and concluding remarks

A mathematical and sensitivity analysis has been done to the modified grouping efficacy measure (MGE). It is found that the first limitation of this measure is that the condition of zero efficiency can’t be reached using MGE measure. While the second limitation is that, inaccurate results will be obtained in the case when number of voids is greater than number of exceptions. To avoid these limitations a new measure called weighted modified grouping efficacy (WMGE) is proposed in this paper. The result of using the new measure showed that it can be used to evaluate all types of structure data. Moreover, WMGE measure can distinguish between two or more alternative manufacturing systems having the same sum of voids and exceptions with logical results. The weakness of the MGE measure is shown in the mathematical formula. This function failed to satisfy the mathematical properties which should be inherent inside the formula, as any other well-known measure. Moreover, to prove the invalidity of this measure, different case studies have been used and the results were illogical and inaccurate. While the strengths of WMGE measure is clear in satisfying these properties, the validity to be used with different cell sizes and the ability to distinguish between different optimal solutions.

Figure 4. Optimal solution for problem Y.
Table 4. Validity of WMGE measure.

| Figure | # machines in 1st cell | # machines in 2nd cell | # machines in 3rd cell | # parts in 1st cell | # parts in 2nd cell | # parts in 3rd cell | $e + \nu$ | $\tau$ | $\gamma$ | $\omega$ | $\tau_x$ | CGE | WMGE |
|--------|------------------------|------------------------|------------------------|---------------------|---------------------|---------------------|--------|-------|-------|-------|-------|-----|-------|
| 4      | 4                      | 3                      | 3                      | 3                   | 3                   | 2                   | 5      | 82.76 | 83.10 | 82.7  | 89.8  | .835 | .857   |
Disclosure statement

No potential conflict of interest was reported by the authors.

References

Adil, G. K., Rajamani, D., & Strong, D. (1996). Cell formation considering alternate routeings. *International Journal of Production Research, 34,* 1361–1380.

Boctor, F. F. (1991). A linear formulation of the machine-part cell formation problem. *International Journal of Production Research, 29,* 334–356.

Burbidge, J. L. (1973, November). *Production flow analysis on a computer.* Third Annual Conference of the Institute of Production Engineers, Sheffield.

Chandrasekharan, M. P., & Rajagopalan, R. (1986). An ideal seed non-hierarchical clustering algorithm for cellular manufacturing. *International Journal of Production Research, 24,* 451–464.

Chen, H. G., & Guerrero, H. H. (1994). A general search algorithm for cell formation in group technology. *International Journal of Production Research, 32,* 2711–2724.

Kellie, B., Evelyn, C. B., & Tabitha, L. J. (2007). Grouping efficiency measures and their impact on factory measures for the machine-part cell formation problem; A simulation study. *Engineering Applications of Artificial Intelligence, 20,* 63–78.

Kumar, C. S., & Chandrasekharan, M. P. (1990). Grouping efficacy; a quantitative criterion for goodness of block diagonal forms of binary matrices in group technology. *International Journal of Production Research, 28,* 233–243.

Kusiak, A., & Cho, M. (1992). Similarity coefficient algorithms for solving the group technology problem. *International Journal of Production Research, 30,* 2633–2646.

Kusiak, A., & Chow, W. S. (1987). Efficient solving of the group technology problem. *Journal of Manufacturing Systems, 6*(2), 117–124.

Lee, K., & Ahn, K.-I. (2013). GT efficacy: A performance measure for cell formation with sequence data. *International Journal of Production Research, 51,* 6070–6081.

Mukattash, A., Dahmani, N., Al-Bashir, A., & Qamar, A. (2018). Comprehensive grouping efficacy: A new measure for evaluating block-diagonal forms in group technology. *International Journal of Industrial Engineering Computations, 9*(1), 155–172.

Nair, G. J., & Narendran, T. T. (1996). Grouping index: A new quantitative criterion for goodness of block-diagonal forms in group technology. *International Journal of Production Research, 34,* 2767–2782.

Nair, G. J., & Narendran, T. T. (1998). A clustering algorithm for cell formation with sequence data. *International Journal of Production Research, 36,* 157–179.

Ng, S. M. (1993). Worst-case analysis of an algorithm for cellular manufacturing. *European Journal of Operational Research, 69,* 384–398.

Rajesh, K. V. D., Chalapathi, P. V., Chaitanya, A. B. K., Sairam, V., & Anildeep, N. (2016). Modified grouping efficacy and new average measure of flexibility: Performance measuring parameters for cell formation applications. *ARPN Journal of Engineering and Applied Sciences, 11,* 9212–9215.

Sandbothe, R. A. (1998). Two observations on the grouping efficacy measure for goodness of block diagonal forms. *International Journal of Production Research, 36,* 3217–3222.

Sarker, B. R., & Khan, M. (2001). A comparison of existing grouping efficiency measures and a new weighted grouping efficiency measure. *IIE Transactions, 33,* 11–27.

Sarker, B. R., & Mondal, S. (1999). Grouping efficiency measures in cellular manufacturing: A survey and critical review. *International Journal of Production Research, 37,* 285–314.

Seifoddini, H., & Djassemi, M. (1996). A new grouping measure for evaluation of machine-component matrices. *International Journal of Production Research, 34,* 1179–1193.

Srivastava, B., & Chen, W. H. (1995). Efficient solution for machine cell formation in group technology. *International Journal of Computer Integrated Manufacturing, 8,* 255–264.