Deterministic Selection of Phase Sequences in Low Complexity SLM Scheme

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Abstract—Selected mapping (SLM) is a suitable scheme, which can solve the peak-to-average power ratio (PAPR) problem. Recently, many researchers have concentrated on reducing the computational complexity of the SLM schemes. One of the low complexity SLM schemes is the Class III SLM scheme which uses only one inverse fast fourier transform (IFFT) operation for generating one orthogonal frequency division multiplexing (OFDM) signal sequence. By selecting rotations and cyclic shifts randomly, it can generate $\frac{N}{3}$ alternative OFDM signal sequences, where $N$ is the FFT size. But this selection can not guarantee the optimal PAPR reduction performances. Therefore, in this paper, we propose a simple deterministic cyclic shifts selection method which is optimal in case of having low variance of correlation coefficient between two alternative OFDM signal sequences. And we show that cyclic shifts are highly dependent on the PAPR reduction performance than rotations. For small FFT size and the number of alternative signal sequences is close to $N/S$, simulations results show that the proposed scheme can achieve better PAPR reduction performance than the Class III SLM scheme.

Index Terms—Correlation coefficient, peak-to-average power ratio (PAPR), selected mapping (SLM), variance of correlation.

I. INTRODUCTION

Orthogonal frequency division multiplexing (OFDM) is one of the most popular multicarrier modulation technique. Because of the orthogonality of subcarriers, the receiver can recover the transmitted data without interferences. Due to this robustness against the multipath channel, OFDM has been adopted as the standards for various wireless communication systems such as IEEE 802.11 a (WLAN), IEEE 802.16 (WiMAX), and long term evolution (LTE). But, it also has the serious drawback which is high peak-to-power ratio (PAPR). When OFDM signals pass through the high power amplifier (HPA), it has in-band distortion and out-of-band radiation. Thus, to reduce the PAPR, several schemes are proposed.

Clipping is the simplest technique, tone reservation reserves the tones for peak canceling signal, and probabilistic method, selected mapping (SLM) and partial transmit sequence (PTS), generate alternative signal sequences and choose the one with the lowest PAPR.

This paper is organized as follows. Section II introduces the Class III SLM scheme. Optimal condition and deterministic cyclic shifts generation method are proposed in Section III. In Section IV, simulation results are shown and Section V concludes this paper.

II. CLASS III SLM SCHEME

The Class III SLM scheme is introduced by Li et al. [1] and it only uses one inverse fast fourier transform (IFFT) operation for generating one orthogonal frequency division multiplexing (OFDM) signal sequence. Fig. 1 shows the block diagram of Class III SLM scheme. Quadrature phase-shift keying (QPSK) or $M$-ary quadrature amplitude modulation ($M$-QAM) modulated input symbol sequence, $X = [X_0, X_1, X_2, \ldots, X_{N-1}]$, is the input of IFFT operation, where $N$ is the FFT size. Then, OFDM symbol sequence, $x = [x_0, x_1, x_2, \ldots, x_{N-1}]$, performs $N$-point circular convolution with 4 base vectors. The 4 base vectors are defined as

\[
\begin{align*}
\mathbf{p}_1 &= \begin{bmatrix} 1,0,\ldots,0,1,0,\ldots,0,1,0,\ldots,0,1,0,\ldots,0 \end{bmatrix} \\
\mathbf{p}_2 &= \begin{bmatrix} 1,0,\ldots,0,0,\ldots,0,0,\ldots,0,0,\ldots,0 \end{bmatrix} \\
\mathbf{p}_3 &= \begin{bmatrix} 1,0,\ldots,0,1,0,\ldots,0,0,\ldots,0,0,\ldots,0 \end{bmatrix} \\
\mathbf{p}_4 &= \begin{bmatrix} 0,\ldots,0,1,0,\ldots,0,1,0,\ldots,0,1,0,\ldots,0 \end{bmatrix}
\end{align*}
\]

(1)

Then, each sequence is right cyclic shifted, $\tau_i$, and rotated, $c_i$, to generate alternative signal sequences, $x^{(a)}$. Finally, select the one with the lowest PAPR. The conversion vector for
Because circular convolution in time domain is identical to

denotes complex conjugate operation, and

where

is equivalent Class III SLM scheme which is expressed using

and can also be applied to other partitioning vectors. After all, the

Thus, the Class III SLM can generate

alternative signal sequences. Fig. 2 shows the

th alternative signal sequence has the general form

where

and

By (2), the Class III SLM can be interpreted as an interleaved

scheme can be expressed as

where \( P_i^{(u)} \) denotes the linear phase in frequency domain

denoted as in time domain. And then, we can

consider \( \sum_{i=1}^{4} c_i^{(u)} P_i^{(u)} \) in (5) as a phase sequence in

SLM scheme. Thus, \( u \) th phase sequence in the Class III SLM

scheme can be expressed as

For \( i = 1 \) in (6), it can be computed as

With this result, we can compute the \( u \) th phase sequence as

where the elements are

Finally, we can substitute (8) to (3) and then consider only inner parts of the magnitude |·|. We can replace it as \( A(\tau) \)

and then we can get 4 terms. We can also replace each term with \( A_i(\tau) \), \( i = 1, 2, 3, \) and 4, as follows

Because circular convolution in time domain is identical to

case of wise multiplication in frequency domain, \( x \oplus_N P_i = x \oplus P_i \), input symbol sequence is multiplied by

\( P_i \), \( i = 1, 2, 3, \) and 4. It means that the 4 vectors, \( P_i \), are partitioning

vectors same as the partitioning in the PTS. For example, \( P_1 \) has all zero elements except when its indices are 0 mod 4.

Thus, \( X \oplus P_1 = [X_0, 0, 0, 0, X_4, 0, 0, 0, \cdots, X_{N-4}, 0, 0, 0] \). It can also be applied to other partitioning vectors. After all, the

Class III SLM scheme can be interpreted as an interleaved partitioned PTS scheme [5].

A(\tau) = \sum_{k=0}^{N-1} P_i^{(k)} P_j^{(k)} e^{-j^{\frac{2\pi k}{N} \tau}}

= \sum_{k=0}^{N-1} c_i^{(u)} c_j^{(u)} e^{-j^{\frac{2\pi i k}{N} \tau}} (\tau + r_i^{(u)} - r_j^{(u)})

= \sum_{k=0}^{N-1} c_i^{(u)} c_j^{(u)} e^{-j^{\frac{2\pi k}{N} \tau}} (\tau + r_i^{(u)} - r_j^{(u)})

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= \sum_{k=0}^{N-1} c_i^{(u)} c_j^{(u)} e^{-j^{\frac{2\pi k}{N} \tau}} (\tau + r_i^{(u)} - r_j^{(u)})

= A(\tau) + A(2)(\tau) + A(3)(\tau) + A(4)(\tau). (10)
Then, we can calculate all possible values of $\bar{A}$.

Let us consider one $\bar{A}(\tau)$. And take the rotation $c_1^{(i)} c_1^{(j)*}$ out of summation, then we can get $\hat{A}(\tau)$.

$$
\hat{A}(\tau) = \sum_{v=0}^{N-1} c_1^{(i)} c_1^{(j)*} e^{-j \frac{2\pi v (\tau + \tau_1^{(i)} - \tau_1^{(j)})}{N}}
$$

We can make the variance of correlation coefficient low.

Optimal condition: $d_{\tau_1} \neq d_{\tau_2} \neq d_{\tau_3} \neq d_{\tau_4} \mod \frac{N}{4}$. (13)

That is, to achieve optimal PAPR reduction performance, we have to choose the cyclic shifts with different $d_{\tau_1}$, between any $\{U\}$ alternative signal sequence pairs, where $U$ is the total number of alternative signal sequences. Suppose 2 or more $\bar{A}(\tau)$ have the values in Table 1 at same $\tau$. By Parseval's Theorem, the magnitude should be larger than a case which only have one signal at $\tau$. Therefore, sharing same $\tau$ might increase the variance of correlation and it is not desirable [3].

C. Deterministic Cyclis Shifts Generation Method

In this subsection, we propose a simple cyclic shifts generation method. Table 3 shows the cyclic shifts for each $\tau_i$, where $u$ denotes the Alternative signal sequence index. $\tau_2$ has the values which are multiple of 1 at an increasing order. Similarly, $\tau_3$ has multiple of 2 and $\tau_4$ has multiple of 3. For any $u_1, u_2$, where $u_1 \neq u_2$ and $u_1 < u_2$, the corresponding differences are $d_{\tau_1} = 0$, $d_{\tau_2} = u_2 - u_1$, $d_{\tau_3} = 2(u_2 - u_1)$, and $d_{\tau_4} = 3(u_2 - u_1) = 2(u_2 - u_1) + (u_2 - u_1)$, respectively.

If $u_2 - u_1 = \frac{N}{3}$, $d_{\tau_4} = d_{\tau_4} \mod \frac{N}{3}$. Therefore, $u_2 - u_1 < \frac{N}{3}$.

Now, we have show that $d_{\tau_1} - d_{\tau_1} = k(u_2 - u_1) \neq 0 \mod \frac{N}{4}$, where $i, j \in \{1, 2, 3, 4\}, i < j$, and $k = 1, 2, 3$. When $k = 1$ and 2, this condition holds by $u_1 < u_2$ and $u_2 - u_1 < \frac{N}{3}$. And when $k = 3$, because of $N = 2m$, where $m$ is natural number, $3(u_2 - u_1) \neq 0 \mod \frac{N}{3} = 2m-2$. Thus, when $u_2 - u_1 = \frac{N}{3}$, it always satisfy the optimal condition and the maximum number of alternative signal sequences is $\frac{N}{3}$.

### IV. Simulation Result

In this section, we compare the PAPR reduction performance between Class III SLM (C3-SLM) scheme and the proposed deterministic selection SLM scheme (DS-SLM). PAPR reduction performance is evaluated by complementary cumulative distribution function (CCDF) which shows a probability that is larger than a certain threshold level, $\gamma$. In C3-SLM scheme, cyclic shift and rotation values are randomly
While in DS-SLM scheme, cyclic shift values are generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, generated. While in DS-SLM scheme, cyclic shift values are taken from Table II and rotation values are all 1, that is, generated. When $N$ is large, DS-SLM (OPT) can achieve better PAPR reduction performance than C3-SLM (RANDOM).

**V. CONCLUSION**

In this paper, we propose a simple deterministic cyclic shifts generation method which satisfy the optimal condition and show that cyclic shifts are highly dependent on the PAPR reduction performance than rotations. Simulation results show that the proposed DS-SLM scheme can achieve better PAPR reduction performance than the C3-SLM scheme for small FFT size and $U$ is close to $N/8$.

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