Motivation for the sampling rate of the excitation model

To determine the sampling rate of our excitation model we note that many body quantum dynamics undergoes decoherence due to the elastic scattering of Rydberg electrons with the ground state atoms. Here we show the estimation of this rate for different Rydberg states and deduce a constant rate of 5 MHz for our parameters. To estimate the rate of this process we use the semiclassical approximation for the radial part of kinetic energy (which for \( l = 0 \) states becomes the total kinetic energy) of the Rydberg electron at a distance \( r \) from the ion:

\[
E_{\text{kin}}(r) = -\frac{e^2}{2a_0} \frac{1}{(n - \delta(l))^2} - \frac{\hbar^2 l(l+1)}{2m_e r^2} + \frac{e^2}{r},
\]

where \( a_0 \) is the Bohr radius, \( n \) is the principal quantum number and \( \delta(l) \) denotes the quantum defect for the states with orbital quantum number \( l \) [1]. We estimate the scattering rate using the formula:

\[
\Gamma_{\text{scat}} = \int I \sigma(\Omega) \rho d^3r d\Omega,
\]

where \( I \) is the density of flux corresponding to the Rydberg electron, \( \sigma(\Omega) \) is the differential cross section for the electron-atom scattering and \( \rho \) is the density of condensed atoms. The flux density is \( I = v |\Psi_{\text{Ryd}}(\vec{r})|^2 \), where \( v = \sqrt{2E_{\text{kin}}(r)/m_e} \) is the electron velocity. In the case of the \( s \)-states the above formula is reduced to:

\[
\Gamma_{\text{scat}} = \int 4\pi a^2 \sqrt{2E_{\text{kin}}(r)/m_e} |\psi_r(r)|^2 r^2 dr \rho,
\]

where \( \psi_r(r) \) is the normalized radial part of the electron Rydberg wavefunction. The result for a momentum-dependent scattering length \( a(k) \) and a constant scattering length of \( a = -16.1a_0 \) for \( s \)-states is shown in Fig. S1. Based on this we assume that the time difference between two scattering events is approximately 200 ns.

Rydberg electron interaction time

The interaction between the Rydberg electron and surrounding ground state atoms decreases exponentially with time (Fig. S2). The measured interaction time \( \sim 10 \mu s \) is much shorter than the Rydberg atom lifetime in a thermal cloud [2]. A full explanation of this effect is not yet complete.
The interaction between a Rydberg electron and surrounding ground state atoms [2] is strong enough to create a significant density change in a Rb BEC. This enables direct visualization of the Rydberg electron orbit. As soon as the Rydberg atom is excited, the condensate responds to the newly imprinted potential and atoms start to flow into regions where a high Rydberg electron probability density can be found. To determine the density change due to a Rydberg electron an image of the BEC must be taken which will disturb the quantum nature of the object.

Every spatial measurement technique will project the atom probability distribution of the BEC into a number state in a certain region [4]. The integrated column density results in an integrated atom number over the cloud thickness and will show a shot to shot fluctuation of the atom number \( N \) as \( \sqrt{N} \) since the atoms are Poissonianly distributed in a BEC [5]. This noise source will be dominant since even in the center region (\( 1 \mu m^2 \)) a typical mean atom number of around 300 can be found resulting in a minimum atom number shot noise background (Fig. S3) over the image of \( \sim 6 \% \). By averaging over many images this noise source will eventually average to zero. Other noise sources, for example, photon shot noise can be overcome by choosing an appropriate light intensity. In phase contrast imaging this can be set well above the photon shot noise without disturbing a single image.

The main parameters and properties which have to be taken into account to image the orbital in a single shot experiment are: the quantum number of the Rydberg state (size, potential depth, lifetime of the Rydberg state), the density of the cloud (size, lifetime, atom number fluctuations), the trap frequencies (size, aspect ratio), the evolution time after which the image is taken and the imaging resolution.

The latter will always give a spatial limit of what can be observed. All structures below the resolution limit, which is in a high resolution setup around 1 \( \mu m \) (\( 1/e^2 \) width), will be smoothed out, so the size of the electron wavefunction itself has to be larger than this to observe the different orbitals. As a second effect the depth of view will also affect the image. For a resolution of 1 \( \mu m \) and an imaging laser with a wavelength of \( \lambda = 780 \) nm the Rayleigh range is about 1 \( \mu m \), which causes an additional smoothing of the observed density distribution. However, this can be neglected in the following since most of the signal arises from the center area, which lies within the depth of focus of 2 \( \mu m \) and only the longitudinal imaging direction is significantly affected by the depth of view.

As a second parameter the trap frequencies have to be chosen such that the radial cloud size matches the size of the Rydberg orbital. If the cloud is too big the light has to travel through dense volumes which are not affected by the Rydberg electron. This would cause a loss of contrast. In addition a very elongated cloud is preferable so that a reference picture shows no structure in the \( x \) direction on the scale of the Rydberg orbital.

The cloud density in the area where the Rydberg atom is excited should be between the following two limits: a too dense cloud reduces the interaction time, whereas a too dilute cloud would result in higher atom number fluctuations (noise) and less scattered signal.

Finally, also the quantum state has to be chosen to match the following conditions. On the one hand the structure of the state has to be bigger than the imaging resolution and a higher quantum number also increases the interaction time slightly [2]. On the other hand the state has to be low enough such that it still causes a reasonable Rydberg electron ground state atom interaction.

One suggested parameter set, taking into account the explained conditions, consists of an atom number of \( 5 \cdot 10^4 \) atoms with trap frequencies \( \omega_x = 2\pi \cdot 10 \) Hz, \( \omega_y = 2\pi \cdot 200 \) Hz, \( \omega_z = 2\pi \cdot 200 \) Hz, which results in a peak density of \( \rho \sim 10^{14} \) \( \text{cm}^{-3} \). The optimal time when a picture should be taken can be determined by looking at the response of the condensate to the potential of the Rydberg atom (Fig. S4). After the Rydberg electron is gone the remaining motion increases the contrast until it is washing out the structure again at longer times. Fig. S5 shows the final results, in which the different orbitals are clearly visible.

**Imaging the Rydberg electron orbital**

**Optimal parameters for a single shot experiment**

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![Image](https://via.placeholder.com/150)

**Figure S3.** Simulation of the atom distribution of a BEC in equilibrium. The top panel shows the smooth absolute square of the wavefunction with a finite imaging resolution of 1 \( \mu m \) in contrast to the image below which shows a projected number state distribution convolved with the same finite resolution. The visible noise is only due to spatial number fluctuations of a BEC. The peak density in the center is \( \rho \sim 10^{14} \) \( \text{cm}^{-3} \) which results in 300 ± 17 atoms in the central column \( a = 1\mu m^2 \).
Figure S.4. Time evolution of the condensate (140D m=0) for a single Rydberg excitation. The top graph shows the visibility defined as $v = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}}) - v_{t=0}$ in the center area of the picture ($\Delta x = 6 \mu m$, $\Delta y = 1.4 \mu m$). The vertical lines show the specific times of the simulated images (bottom part). At the time when the imprint of the Rydberg atom ends ($t = 30 \mu s$) almost no signal can be observed. Since the atoms are still in motion the signal increases for the given simulation until ($t = 210 \mu s$) and washes out afterwards. To point out this effect the atom number shot noise is not taken into account in this simulation.

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Figure S5. Resulting simulated phase contrast images including a finite resolution (1 μm) and atom number shot noise. Both images are simulated for $n = 140$ and show the corresponding density distribution. The colorscale is chosen such that the reference peak intensity at $t = 0$ equals to 1. For both simulations the Rydberg imprint was assumed to be $30 \mu s$ which is the case in every twentieth shot if the average lifetime of a Rydberg electron is $10 \mu s$. The left image ($m_l = 0$) was simulated for $t = 210 \mu s$ where the visibility reaches a maximum. The right image ($m_l = 2$) is taken at a shorter time $t = 150 \mu s$ since it has smaller structures which wash out afterwards.