MATHEMATICAL PROBLEM SOLVING ABILITY IN APOS MODIFIED LEARNING MODEL (M-APOS)

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Abstract. The purpose of this research is to analyze and describe student’s mathematical problem solving abilities in the implementation of APOS modified learning model (M-APOS). This learning model adopts from APOS theory which apply activity, class discussion and exercise cycle. The type of this research is an explorative descriptive research with the subject is 37 respondents of UPY Mathematic Education Study Program’s students who joint the course of Group Theory. The data collection procedure is by observation, test, and interview. Respondents were given a written test of three questions related to problem solving abilities. From the results of the analysis it was found that the indicator of the ability to understand the problem, plan for completion, and solve the problem according to plan, the highest percentage was in high ability students with percentages of 76.12%, 60.05%, and 44.68%. This means that the ability of students to understand problems, plan for completion, and implement the plan in the implementation of the M-APOS learning model is in good category. While the indicator of re-examine for all steps reviews, the highest percentage is the moderate ability category with percentage is 39.28%. The implementation of APOS learning in this study is in very good category.

1. Introduction

Mathematics is a science that has an important role in effort to mastery of science, technology and also in everyday of our life [1]. It is not only learned in class, but it is very close to our activities. In this era, who understand and can do mathematics well, will have better opportunities for shaping their future. Productive future will be opened if we have a good mathematical competence, and vice versa [2]. Every students must have opportunity and support necessary to learn mathematics with depth and understanding. Cockroft [in 1] argued that "Mathematics should be taught to the students because of (1) is always used in all facets of life; (2) all areas of life requires appropriate mathematical skills; (3) is a powerful means of communication, concise, and clear; (4) can be used to present information in a variety of ways; and (5) give satisfaction to attempt to solve a challenging problem.

National Council of Teachers of Mathematics (NCTM) recognized standard process of learning mathematics is problem solving, reasoning and proof, communication, connections and the representation [2]. Problem solving is a standard part of the mathematical process which very important because in the learning process and completion, students are allowed to use the skills and experience they have to be applied in the resolution of problems that are not routine because after studying, the students will go into the community that full of social problems [1]. The ultimate goal of teaching is students able to conduct mathematical investigations by themselves, and able to identify where the
mathematics they have learned is applicable in the real world situations [3]. In the phrase of the mathematician [in 3], problem solving is “the heart of mathematics”.

Problem solving is very important that it becomes a general purpose math teaching math even as his heart [4]. The process of thinking in problem solving strategy requires the ability to organize. It will make student to think critically, logically, creatively indispensable in dealing with the development of society. It was often found the students are only concerned with the final answer without understanding how the process to answer, and they feel difficult to determine what concepts are used to solve the problem [5].

However, many students assume that mathematics is difficult to understand, they feel bored and do not respond well. They are less interested and less motivated. Methods of learning which taken by teacher often not varied. Problem solving is one of the most fundamental goals of teaching mathematics, but also one of the most elusive for student. One of issue of mathematical problem solving in Indonesia, it has not yielded a satisfactory result although it has been becoming one of curriculum contents of school mathematics since year 1968 [6]. Student’s ability to solve mathematical problem is still weak, it was pointed out by TIMSS (Trends in International Mathematics and Science Study) and PISA (Programme Internationale for Student Assesment), international survey on education. Result of TIMSS 2011, Indonesia ranked 38th out of 42 countries [7]. Therefore in this study an analysis of students' problem solving abilities will be conducted.

Polya say that in the problem solving, there are four steps to be taken: “(1) to understand the problem, (2) plan for completion, (3) do the problem according to plan, and (4) to re-examine for all steps are done” [8]. In the PISA mathematics literature, the problem-solving stage is to formulate, to form into the mathematical model, and finally to get the solution plan [9].

Chang state that strategy and model of learning that encourage the development of problem-solving skills is very useful for students on aspects of cognitive, psychomotor, and affective. The accuracy of the selection of learning models will have an impact on the learning success of students and the achievement of learning goals [10]. The selection of the right learning model is something that needs to be done, so that it matches the characteristics of students and the material to be taught. M-APOS is a modification of the APOS learning model. The M-APOS and APOS learning models are learning models based on the APOS theory (Action, Process, Object, Schema). Stated that APOS is a learning theory which fulfilled six characteristics of learning theory [11]. These six characteristics are as follows: 1) Supporting the prediction, 2) Being able used to describe, 3) Being able to be applied for a large phenomenon, 4) Helping to organize thoughts about learning phenomenon, 5) As the data analysis tools and 6) Giving a term for communication in learning. Learning using APOS theory emphasizes the acquisition of knowledge through mental construction [12]. There are two things that are considered as learning characteristics based on APOS theory, namely learning includes: (i) mental constructions in understanding a mathematical concept and (ii) using the ADL cycle (Activities, class discussions, practice questions) [13]. In the learning activity phase with the APOS model is a learning activity carried out in a computer laboratory. While the M-APOS learning model uses the Student Activity Sheet as a substitute for activities in the computer laboratory in the activity phase. As for the class discussion phase and practice phase the problem is the same as the APOS learning model, which uses the activity, class discussion, and problem exercise phase.

In this study researchers will analyze problem solving abilities in the implementation of the APOS Modification learning model (M-APOS).

2. Method

This research was carried out in the Mathematics Education Study Program of PGRI Yogyakarta University academic year 2017/2018. The research subjects were students who attended the Algebra Structure course in a number of 37 students. This type of research is explorative descriptive. The researcher will describe the problem solving ability of students in problems solving related to the Group. The description of the ability of students to solve Group questions is based on four indicators of mathematical problem solving abilities, namely the ability to understand problems, the ability to plan for completion, the ability to do the problems according to plan, and the ability to re-examine for all steps. Data is collected through observation, tests, and interviews. Observation data is used to check the
implementation of learning with the M-APOS learning model. The research subjects were given a written test of 3 questions.

Before the test questions are used in the research class, the questions are analyzed for validity and reliability first. A test is said to be valid if the test is able to measure what you want to measure [14]. The formula used to determine the validity of the item is the product moment correlation formula as follows:

\[
r = \frac{N \sum_{i=1}^{n} X_i Y_i - (\sum_{i=1}^{n} X_i)(\sum_{i=1}^{n} Y_i)}{\sqrt{N \sum_{i=1}^{n} X_i^2 - (\sum_{i=1}^{n} X_i)^2}(N \sum_{i=1}^{n} Y_i^2 - (\sum_{i=1}^{n} Y_i)^2)}
\]

Information:
- \( r \): validity test coefficient
- \( N \): the number of students
- \( X \): score item
- \( Y \): total score

The interpretation of the correlation coefficient above uses the following criteria [14]:

| \( r \) | Interpretation |
|--------|----------------|
| 0,80 \leq r \leq 1,00 | Very High |
| 0,60 \leq r \leq 0,80 | High |
| 0,40 \leq r \leq 0,60 | Medium |
| 0,20 \leq r \leq 0,40 | Low |
| 0,00 \leq r \leq 0,20 | Very Low |

A test is said to be reliable or trustworthy if it provides a fixed result even though it is used repeatedly on the same subject [13]. [14] states that the reliability coefficient of a test form description can be estimated using the Alpha formula as follows:

\[
\alpha = \frac{K}{K-1} \left(1 - \frac{\sum S_i^2}{S_2^2}\right)
\]

Information:
- \( \alpha \) = test reliability coefficient
- \( \sum S_i^2 \) = the amount of variance score of each item
- \( K \) = the number of test items
- \( S_2^2 \) = total variance

| \( r \) | Interpretation |
|--------|----------------|
| 0,80 < \alpha \leq 1,00 | Very High |
| 0,60 < \alpha \leq 0,80 | High |
| 0,40 < \alpha \leq 0,60 | Medium |
| 0,20 < \alpha \leq 0,40 | Low |
| 0,00 < \alpha \leq 0,20 | Very Low |

Regarding the analysis of the level of students' problem solving abilities, the values obtained in each problem for each indicator stage are as follows [15]:
\[ N_i = \frac{S_i \times 100}{T_i}, \quad i = 1, 2, 3, 4 \]

Information:
- \( i \) = indicator stage of problem solving ability
- \( N_i \) = Student grades for each stage
- \( S_i \) = Student score for each stage
- \( T_i \) = Maximum score for each indicator

While the calculation of the final value of the three problems in each indicator is as follows:

\[ NA_i = \frac{Q_i \times 100}{E_i}, \quad i = 1, 2, 3, 4 \]

Information:
- \( i \) = indicator stage of problem solving ability
- \( NA_i \) = Student grades for each stage
- \( Q_i \) = Total student scores for each indicator
- \( E_i \) = Total Maximum Score for each indicator

The score obtained are categorized according to the level of student ability determined as follows:

| Student Ability Score | Category |
|-----------------------|----------|
| \( 0 \leq \text{Score} \leq 60 \) | Low      |
| \( 60 < \text{Score} \leq 75 \) | Medium   |
| \( 75 < \text{Score} \leq 100 \) | High     |

Based on table 3, the ability category of students is divided into 3 three parts, namely low, medium and high. Then this category is used to measure problem-solving abilities.

3. Results and Discussions

The learning applied in this study is APOS Modification learning model (M-APOS) with activity, discussions, and exercises cycles. The activity phase includes learning material both with lectures, and working on individual and group worksheets. When working in groups this is a discussion activity running. While the training phase aims to establish and apply concepts that have been constructed in the form of solving problems. The implementation of learning with the M-APOS learning model is in a very good category.

The material tested in this study is about proving the Group. The number of questions tested was three questions. The results of the analysis of the validity of the items can be seen in the following table:

| Number of Question | \( r \) Score | Interpretation |
|--------------------|---------------|----------------|
| 1                  | 0.76          | High           |
| 2                  | 0.72          | High           |
| 3                  | 0.78          | High           |

Table 4 shows that all the questions in the test are valid to measure what is being measured, namely the students' problem solving abilities in the subject matter of the Group Theory based on the Problem Solving learning model. For the level of reliability of the test, obtained a value = 0.68 with a high interpretation. This means that a given test is reliable or reliable even though it is used repeatedly on the same subject. The percentage of students in the level of problem solving skills is as follows:
Table 5. Percentage of Students in Problem Solving Ability

| Problem Solving Ability Indicator                  | Student Ability Level (%) |
|---------------------------------------------------|---------------------------|
|                                                   | High          | Medium        | Low           |
| Understand the problem                            | 76.12         | 18.24         | 5.64          |
| Plan for completion                               | 60.05         | 26.27         | 13.68         |
| Solve the problem according to plan               | 44.68         | 34.06         | 21.26         |
| re-examine for all steps reviews                  | 30.11         | 39.28         | 30.61         |

At the stage of understanding the problem, the highest percentage is in students in a relatively high level of ability that is equal to 76.12%. This shows that students ability to understand problems is very good. Likewise in the stage of making plan for completion and the stage of implementing the completion plan, the ability of students in this case is quite high, it can be seen from the highest percentage in this indicator is for students in a relatively high level of ability, although at the stage of implementing percentage completion plans for higher abilities little if compared to the ability to understand problems and compile a settlement plan. While the stage of re-examine for all steps the highest percentage is in the medium ability category. This shows that the ability of students of this stage is still in good condition, even though the percentage for high and low ability levels is almost the same. In general, it is seen from the percentage at each level of problem-solving ability, that the percentage of high-category abilities decreases from indicators of understanding the problem, plan for completion, implementing a resolution plan, to re-examine for all steps it again.

From the results of interviews with students it was found that the second and third stages were more difficult when compared to the first stage. Students convey that they understand the problem given, but sometimes in operating how to solve it, they still feel confused. Students sometimes forget to review the results of the problem. For students with high average ability, from the four stages of ability that are most decisive are the stages of preparing a completion plan. If in this step alone is wrong, the next step will also be incorrect. From the information of students with an average level of moderate ability, it was found that they had a picture in solving the problem, they had a resolution plan, but in applying the plan still experienced confusion and in general the work was still wrong. Whereas from the information of students the average ability is low that they sometimes still have difficulty understanding the problem, so the next stage will become more difficult for them.

4. Conclusion

Problem solving ability is one of the main competencies in mastering mathematics, even there is an opinion that problem solving ability is the heart of mathematics. In this study, exploratory descriptive analysis was carried out on problem solving ability in the implementation of APOS modification learning model (M-APOS). This learning model is carried out in a cycle of activities, discussions and exercises. The implementation of learning in this study is in a very good category. From the results of the problem solving ability test, it was found that for the indicator of the ability to understand the problem, plan for completion, and solve the problem according to plan, the highest percentage was for students with high abilities. This means that for these three indicators the ability of students is in good condition. Nevertheless, an indicator ability to re-examine the highest percentage is in the medium category.

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