Lightness of Higgs Boson and Spontaneous CP Violation in Lee Model

Ying-nan Mao $^1$ and Shou-hua Zhu $^{1,2,3}$

1 Institute of Theoretical Physics & State Key Laboratory of Nuclear Physics and Technology, Peking University, Beijing 100871, China

2 Collaborative Innovation Center of Quantum Matter, Beijing, China

3 Center for High Energy Physics, Peking University, Beijing 100871, China

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We propose a mechanism in which the lightness of Higgs boson and the smallness of CP-violation are correlated based on the Lee model, namely the spontaneous CP-violation two-Higgs-doublet-model. In this model, the mass of the lightest Higgs boson $m_h$ as well as the quantities $K$ and $J$ are $\propto t_\beta s_\xi$ in the limit $t_\beta s_\xi \to 0$ (see text for definitions of $t_\beta$ and $s_\xi$), namely the CP conservation limit. Here $K$ and $J$ are the measures for CP-violation effects in scalar and Yukawa sectors respectively. It is a new way to understand why the Higgs boson discovered at the LHC is light. We investigated the important constraints from both high energy LHC data and numerous low energy experiments, especially the measurements of EDMs of electron and neutron as well as the quantities of B-meson and kaon. Confronting all data, we found that this model is still viable. It should be emphasized that there is no standard-model limit for this scenario, thus it is always testable for future experiments. In order to pin down Lee model, it is important to discover the extra neutral and charged Higgs bosons and measure their CP properties and the flavor-changing decays. At the LHC with $\sqrt{s} = 14$TeV, this scenario is favored if there is significant suppression in the $b\bar{b}$ decay channel or any vector boson fusion (VBF), $V+H$ production channels. On the contrary, it will be disfavored if the signal strengths are standard-model-like more and more. It can be easily excluded at $(3 \sim 5)\sigma$ level with several fb$^{-1}$ at future $e^+e^-$ colliders, via the accurately measuring the Higgs boson production cross sections.
I. INTRODUCTION

How to realize the electro-weak gauge symmetry breaking and CP violation are important topics in the standard model (SM) and beyond the SM (BSM) in particle physics. In order to induce the spontaneous gauge symmetry breaking, the Higgs mechanism was proposed in 1964 [1]. Meanwhile in the SM the CP violation is put in by hand via the complex Yukawa couplings among Higgs field and fermions, namely Kobayashi and Maskawa (KM) mechanism [2]. In 1973, Kobayashi and Maskawa [2] proposed that if there are three generations of fermions, there would be a nontrivial phase which leads to CP violation in the fermion mixing matrix (CKM matrix [2][3]). In a word, one single scalar field plays the two-fold roles. In the SM, only one doublet Higgs field is introduced. After symmetry spontaneous breaking, there exists one physical scalar, the Higgs boson. It is essential to discover and measure the properties of the Higgs boson, in order to test the SM or discover the BSM.

A. Status of experimental measurements on new scalar boson

Experimentally, in July 2012, both CMS [4] and ATLAS [5] discovered a new boson with the mass around 125.7 GeV in $\gamma\gamma$ and $ZZ^*$ final states with the luminosity of about $10^{fb^{-1}}$. At the LHC, the SM Higgs boson can be produced through the following three processes: (1)gluon-gluon fusion (ggF); (2)vector boson fusion (VBF); (3) associated production with a vector boson (VH). It can also be produced associated with a pair of top quarks due to the large $m_t$, but the cross section is suppressed by its phase space and parton distribution function (PDF) of proton. A SM Higgs boson would mainly decay to fermion pairs ($b\bar{b}, \tau^+\tau^-$, or $t\bar{t}$ if heavier than $2m_t$), massive gauge boson pair ($W^+W^-, Z^0Z^0$), massless gauge boson pair ($gg, \gamma\gamma$), etc. The decay properties for a 125.7 GeV SM higgs boson are listed in Table 1 for the production and decay properties, see also the reviews [6] and [7].

The updated searches by CMS [8-11] and ATLAS [12-15] with the luminosity of about $25 fb^{-1}$ till the end of 2012 gave the significance $s$ and signal strengths $\mu$ (defined as the ratios between observed $\sigma \cdot Br$ and the corresponding SM prediction) for some channels. Because the measurements will be utilized to constrain the new model in this paper, we list

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1 Some new analysis updated in 2014 are used as well which modify the old results a little bit.
TABLE I: Table for the SM prediction for the decay branching ratios of a 125.7GeV Higgs boson, the numbers are from [7].

| Decay Channel | Branching Ratio (%) | Relative Uncertainty (%) |
|---------------|---------------------|--------------------------|
| $b\bar{b}$    | 56.6                | $\pm 3.3$                |
| $c\bar{c}$    | 2.85                | $\pm 12.2$               |
| $\tau^+\tau^-$| 6.21                | $\pm 5.6$                |
| $gg$          | 8.51                | $^{+10.2}_{-9.9}$        |
| $WW^*$        | 22.6                | $^{+4.2}_{-4.1}$         |
| $ZZ^*$        | 2.81                | $^{+4.2}_{-4.1}$         |
| $\gamma\gamma$| 0.228               | $\pm 4.9$                |
| $Z\gamma$     | 0.16                | $^{+8.9}_{-8.8}$         |
| Total Width   | 4.17MeV             | $\pm 3.9$                |

TABLE II: Signal strengths for some production and decay channels of the new boson at CMS (with combined significance over 3σ).

|           | $\mu$(VBF/VH) | $\mu$(ggF) | $\mu$(combined) | significance |
|-----------|---------------|------------|-----------------|--------------|
| $\gamma\gamma$| $1.58^{+0.77}_{-0.68}$(VBF) | $1.12^{+0.37}_{-0.32}$ | $1.14^{+0.26}_{-0.23}$ | 5.7σ         |
| $ZZ^*$    | $1.7^{+2.2}_{-2.1}$(VBF)     | $0.80^{+0.46}_{-0.36}$ | $0.93^{+0.29}_{-0.25}$ | 6.8σ         |
| $WW^*$    | $0.60^{+0.57}_{-0.46}$(VBF)  | $0.74^{+0.22}_{-0.20}$ | $0.72^{+0.20}_{-0.18}$ | 4.3σ         |
| $\tau^+\tau^-$| $0.94 \pm 0.41$           |             | $0.78 \pm 0.27$ | 3.2σ         |

The results in Table II for CMS and Table III for ATLAS. The new boson has a combined mass 125.7GeV and it is also favored as a 0+ particle in spin and parity by the data [8, 16, 17] if we assume that there is no CP violation induced by this boson.

The experimental measurements of the new particle are in agreement with the SM predictions within the current accuracy. In the SM the electro-weak fitting results [18] also favors a light one. It allows a SM Higgs boson lighter than 145GeV at 95% C.L. inferred from the oblique parameters [19] with fixed $U = 0$. However there are still spacious room for the BSM. For example, if we assume that the new particle is a CP-mixing state, the

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2 The VBF events are usually easy to tag with two jets which have large invariant mass, while sometimes it is difficult to tag a gluon fusion event.
TABLE III: Signal strengths for some production and decay channels of the new boson at ATLAS (with combined significance over 3σ).

|       | μ(VBF/VH)       | μ(ggF)        | μ(combined) | significance |
|-------|-----------------|---------------|-------------|--------------|
| γγ    | 0.8 ± 0.7 (VBF) | 1.32 ± 0.38   | 1.17 ± 0.27 | 5.2σ         |
| ZZ*   | 0.26^{+1.64}_{−0.94} | 1.66^{+0.51}_{−0.44} | 1.43^{+0.40}_{−0.33} | 8.1σ |
| WW*   | 1.7 ± 0.8       | 0.82^{+0.33}_{−0.32} | 0.99^{+0.31}_{−0.28} | 3.8σ |
| τ⁺τ⁻  | 1.6^{+0.8}_{−0.7} | 1.1^{+1.3}_{−1.0} | 1.4^{+0.5}_{−0.4} | 4.1σ |
typical scale $\Lambda \sim 4\pi f_\pi \sim \mathcal{O}(1)\text{GeV}$. Thus we can argue that the new boson with mass 125.7\text{GeV} is rather light compared with the strong interaction. As a side remark, compared with $\sigma$ the pion mass $m_\pi \sim \mathcal{O}(f_\pi)$ is light due to the approximate chiral symmetry. This has motivated the idea that new scalar boson may be the pseudo-Nambu-Goldstone boson for certain unknown symmetry breaking.

Theoretically, in some BSM models there exists a light scalar naturally. For example, (1) in the minimal super-symmetric model, the lightest Higgs boson should be lighter than 140\text{GeV} including higher-order corrections \[22\] (at tree level it should be lighter than the mass of $Z_0$ boson); (2) in the little higgs model, a Higgs boson which is treated as a pseudo-Nambu-Goldstone boson must be light due to classical global symmetry and it acquires mass through quantum effects only \[23\]; (3) similarly, anomalous in scale invariance can also generate a light Higgs boson as well \[24\]; (4) the lightness of Higgs boson can intimately connect with the spontaneous CP violation \[25\]. While the first three approaches base on the conjectured symmetry, the last one utilizes the observed approximate CP symmetry. Historically Lee proposed the spontaneous CP violation in 1973 \[26\] as an alternative way to induce CP violation. For the fourth approach, Lee’s idea is extended to account for the lightness of the observed Higgs boson.

\section*{C. The lightness of new scalar boson and spontaneous CP violation}

CP violation was first discovered in neutral K-meson in 1964 \[27\]. Experimentally people have already measured several kinds of CP violated effects in neutral K- and B-meson, and charged B meson systems \[28\]. These CP violation can be successfully accounted for by the CKM matrix, which is usually parameterized as the Wolfenstein formalism \[29\]

\[V_{\text{CKM}} = \begin{pmatrix}
1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\
-\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\
A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1
\end{pmatrix} + \mathcal{O}(\lambda^4) \quad (4)\]

The Jarlskog invariant \[28\] \[30\]

\[J = A^2\lambda^6\eta = (2.96^{+0.20}_{-0.16}) \times 10^{-5} \quad (5)\]

measures the CP violation in flavor sector. The smallness of $J$ means the smallness of CP-violation in the real world in SM. Another possible explicit CP-violation comes from the $\theta$
term

\[ \mathcal{L}_{\text{str}} = \frac{\theta \alpha_s}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \]  

(6)
in the QCD lagrangian \[31\][32]. The parameter \( \theta \) is strongly constrained by the neutron electric dipole moment (EDM) measurement \[33\][34], namely \( |\theta| \lesssim 10^{-10} \). Why \( \theta \) is extremely small is known as the strong CP problem. It is often interesting, necessary and useful to search for other sources of CP violation beyond the KM-mechanism. As a common reason, for example, CP-violation is one of the conditions to produce the matter-antimatter asymmetry in the universe today \[35\], but SM itself cannot provide strong enough first order electro-weak phase transition and large enough CP-violated effects to get the right asymmetry between matter and anti-matter \[28, 36–38\].

In 1973, Lee proposed a 2HDM (Lee model) \[26\] in which all parameters in the scalar potential are real but it is possible to leave a nontrivial phase \( \xi \) between the vacuum expectation values (VEV) of the two Higgs doublets. CP can be spontaneously broken in this model. Chen et. al. \[39\] proposed the possibility that the complex vacuum could lead to a correct CKM matrix, which means that we can set all Yukawa couplings real thus the complex vacuum would become the only source of CP-violation. It is also a possible way to solve strong CP problem that, in spontaneous CP-violation scenarios, \( \theta \) arises only from the determinant of quark mass matrix. Keeping \( \theta \equiv 0 \) at tree level, the loop corrections can generate naturally small \( \theta \) \[40\][41], the so-called ”calculable \( \theta \)“ \[31\]. Without imposing symmetry \[42\], the Yukawa couplings are arbitrary which will generate the flavor changing neutral currents (FCNC) at tree level. FCNC is severely constrained by experiments. Cheng and Sher proposed an ansatz \[43\] that the flavor changing couplings should be \( \propto \sqrt{m_i m_j} \) for two fermions with mass \( m_i \) and \( m_j \). One of the authors of this paper had proposed a mechanism \[25\] to understand the lightness of Higgs boson in the \( \xi \to 0 \) limit. In this paper, we will explore the relation between the smallness of CP-violation and the lightness of Higgs boson in a similar way in Lee model further. Specifically we will study the full phenomenology of the Lee model and to see whether this model is still viable confronting LHC data and numerous low energy measurements.

We should mention that there are also cosmological implication for Lee model. In this model, CP is a spontaneously broken discrete symmetry thus it may face the domain wall problem \[44\] during the electro-weak phase transition. It is argued that if there is a small initial bias thus one of the vacuum states is favored, the domain walls would disappear soon...
[44][45], for example, if there is small explicit CP-violation [46]. In the soft CP breaking model, the electro-weak baryogenesis effects is estimated by Cohen et. al. [38] at early time, and was estimated again by Shu and Zhang [47] after including LHC data. They found that the observed matter-anti-matter asymmetry can be explained. It is also discussed numerically that an inflation during the symmetry breaking would forbid the domain wall production [48].

This paper is organized as following. Section II presents the Lee model and the scenario that lightness of Higgs boson and smallness of CP violation are correlated. Section III and IV contain the constraints on Lee model from high energy and low energy data respectively. Section V studies the perspectives for Lee model for future experiments. The last section collects our conclusions and discussions.

II. THE LEE MODEL: MASS SPECTRUM AND COUPLINGS

We begin with the description of Lee model [26] assuming that in the whole lagrangian there are no explicit CP-violated terms, which means all the CP-violated effects come from a complex vacuum $^3$.

For the Lee model, the interactions of scalar fields read [26]

$$\mathcal{L} = (D_\mu \phi_1)(D^\mu \phi_1) + (D_\mu \phi_2)(D^\mu \phi_2) - V(\phi_1, \phi_2).$$

(7)

Here

$$\phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{v_1 + R_1 + iI_1}{\sqrt{2}} \end{pmatrix}, \quad \phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{v_2 e^{i\xi} + R_2 + iI_2}{\sqrt{2}} \end{pmatrix}$$

(8)

are the two higgs doublets. We can get the masses of gauge bosons

$$m_W = g \sqrt{\frac{v_1^2 + v_2^2}{2}}, \quad m_Z = \frac{\sqrt{(g^2 + g'^2)(v_1^2 + v_2^2)}}{2}$$

(9)

by setting $v = \sqrt{v_1^2 + v_2^2} = 246\text{GeV}$. Defining $R(I)_{ij}$ as the real( imaginary) part of $\phi_i^+ \phi_j$, we

$^3$ For a review on two Higgs doublet models (2HDM), the interested reader can read Ref. [49]
can write a general potential as

\[ V = V_2 + V_4 \]
\[ = \mu_1^2 R_{11} + \mu_2^2 R_{22} \]
\[ + \lambda_1 R_{11}^2 + \lambda_2 R_{11} R_{12} + \lambda_3 R_{11} R_{22} \]
\[ + \lambda_4 R_{12}^2 + \lambda_5 R_{12} R_{22} + \lambda_6 R_{22}^2 + \lambda_7 I_{12}^2; \]  

(10)
in which we can always perform a rotation between \( \phi_1 \) and \( \phi_2 \) to keep the coefficient of \( R_{12} \) term zero in \( V_2 \). We can also write the general Yukawa couplings as

\[ L_y = -\bar{Q}_i (Y_1 d \phi_1 + Y_2 d \phi_2)_{ij} D_R j - \bar{Q}_i (Y_1 u \tilde{\phi}_1 + Y_2 u \tilde{\phi}_2)_{ij} U_R j, \]

(11)
in which \( \tilde{\phi}_i = i\sigma_2 \phi_i^* \) and all Yukawa couplings are real.

Minimizing the higgs potential, and for some parameter choices, we can get a nonzero phase difference \( \xi \) between two higgs VEVs, which would induce spontaneous CP violation. We can always perform a gauge transformation to get at least one of the VEVs real like in (8). When \( v_1, v_2, \xi \neq 0 \), we can express

\[ \mu_1^2 = -\lambda_1 v_1^2 - \frac{\lambda_3 + \lambda_7}{2} v_2^2 - \frac{\lambda_2}{2} v_1 v_2 \cos \xi; \]  

(12)
\[ \mu_2^2 = -\frac{\lambda_3 + \lambda_7}{2} v_1^2 - \frac{\lambda_5}{2} v_2^2 - \frac{\lambda_6}{2} v_1 v_2 \cos \xi. \]  

(13)

Define \( \tan \beta \equiv v_2 / v_1 \) as usual, where \( \beta \) is the rotational angel which we should use to get the Goldstones. We also have an equation about \( \xi \)

\[ \frac{\lambda_2}{2} v_1^2 + \frac{\lambda_5}{2} v_2^2 + (\lambda_4 - \lambda_7) v_1 v_2 \cos \xi = 0, \]

(14)
which requires \( \lambda_2 v_1^2 + \lambda_5 v_2^2 < 2|\lambda_4 - \lambda_7| v_1 v_2 \). Of course, the couplings \( \lambda_i \) must keep the vacuum stable, for the conditions see section A for details.

All the CP-violated effects in the real world are small (see the data in [28]) corresponding to the smallness of the off-diagonal elements in the CKM-matrix which leads to the smallness of the Jarlskog invariant. As a limit, when \( t_\beta \equiv \tan \beta \rightarrow 0 \), or we may write \( t_\beta s_\xi \rightarrow 0 \) instead since \( |s_\xi| < 1 \) always holds, there would be no CP-violation in the scalar sector. The CKM-matrix would be real thus there would be no CP-violation in flavor sector as well. In

\footnote{We write \( s_\alpha \equiv \sin \alpha, c_\alpha \equiv \cos \alpha, t_\alpha \equiv \tan \alpha \) for short in this paper.}
In this paper we will consider the small $t_\beta$ limit, in which all CP-violated effects tend to zero as $t_\beta \to 0$. We treat the whole world as an expansion around the point without CP-violation.

The two higgs doublets contain 8 degrees of freedom, 3 of which should be eaten by massive gauge bosons as Goldstones. So there are 5 physical scalars left, 2 of which are charged and 3 of which are neutral. If CP is a good symmetry, there will be 2 CP even and 1 CP odd scalars among the 3 neutral ones, however when CP is spontaneously breaking, the CP eigenstates will mix with each other thus the neutral scalars have no certain CP charge. We have the Goldstones as

$$G^\pm = c_\beta \phi_1^\pm + e^{\mp i \xi} \phi_2^\pm, \quad \text{(15)}$$

$$G^0 = c_\beta I_1 + s_\beta c_\xi I_2 - s_\beta s_\xi R_2. \quad \text{(16)}$$

The charged Higgs boson is the orthogonal state of the charged Goldstone as

$$H^\pm = -e^{\mp i \xi} s_\beta \phi_1^\pm + c_\beta \phi_2^\pm, \quad \text{(17)}$$

and its mass square should be

$$m_{H^\pm}^2 = -\frac{\lambda_7 v^2}{2}. \quad \text{(18)}$$

While for the neutral part, we write the mass square matrix as $\tilde{m}n^2/2$ in the basis $(-s_\beta I_1 + c_\beta c_\xi I_2 - c_\beta s_\xi R_2, R_1, s_\xi I_2 + c_\xi R_2)^T$. The symmetric matrix $\tilde{m}$ is

$$\begin{pmatrix}
(\lambda_4 - \lambda_7)s_\xi^2 & -(\lambda_4 - \lambda_7)s_\beta c_\xi & -(\lambda_4 - \lambda_7)c_\beta c_\xi \\
+\lambda_2 c_\beta)s_\xi & +\lambda_5 s_\beta)s_\xi & 4\lambda_1 c_\beta^2 + 2\lambda_2 c_\beta s_\beta c_\xi \\
+(\lambda_4 - \lambda_7)s_\beta^2 c_\xi^2 & +(\lambda_4 - \lambda_7)c_\beta^2 & (2(\lambda_3 + \lambda_7) + (\lambda_4 - \lambda_7)c_\beta^2) s_\beta c_\xi \\
+2\lambda_5 s_\beta c_\xi c_\xi & +\lambda_5 c_\beta c_\xi + \lambda_5 s_\beta^2 c_\xi & (\lambda_4 - \lambda_7)c_\beta^2 c_\xi^2 \\
+2\lambda_5 s_\beta c_\xi + 4\lambda_6 s_\beta^2 c_\xi & & \end{pmatrix} \quad \text{(19)}$$

and its three eigenvalues correspond to the masses of three neutral bosons.

We expand the matrix $\tilde{m}$ in series of $t_\beta(s_\xi)$ as

$$\tilde{m} = \tilde{m}_0 + (t_\beta s_\xi)\tilde{m}_1 + (t_\beta s_\xi)^2\tilde{m}_2 + \cdots \quad \text{(20)}$$
to get the approximate analytical behavior of its eigenvalues and eigenstates. Certainly we have

$$\lim_{t_βs_ξ \to 0} \det(\tilde{m}) = \det(\tilde{m}_0) = 0$$

which means a zero eigenvalue of $\tilde{m}_0$ thus there must be a light neutral scalar when $t_βs_ξ$ is small. To the leading order of $t_βs_ξ$, for the lightest scalar $h$, we have

$$m_h^2 = \frac{v^2t_βs_ξ^2}{2} \left( \frac{(\tilde{m}_1)^2}{(m_0)^{22}} + \frac{(\tilde{m}_1)^2}{(m_0)^{33}} + (\tilde{m}_2)_{11} \right)$$

$$= \frac{v^2t_βs_ξ^2}{2} \left[ 4(\lambda_3 + \lambda_7)^2 \left( \frac{c_θ^2}{(m_0)^{22}} + \frac{s_θ^2}{(m_0)^{33}} \right) + \lambda_5^2 \left( \frac{s_θ^2}{(m_0)^{22}} \right) \left( \frac{c_θ^2}{(m_0)^{33}} \right) - 2\lambda_5(\lambda_3 + \lambda_7)s_θ^2 \left( \frac{1}{(m_0)^{22}} - \frac{1}{(m_0)^{33}} \right) + 4\lambda_6 \right];$$

$$h = I_2 + t_βs_ξ \left( \frac{(\tilde{m}_1)_2}{(m_0)^{22}}(c_θR_1 + s_θR_2) + \frac{(\tilde{m}_1)_3}{(m_0)^{33}}(c_θR_2 - s_θR_1) - \frac{I_1}{t_ξ} \right)$$

$$= I_2 + t_βs_ξ \left[ \frac{2(\lambda_3 + \lambda_7)}{(m_0)^{22}} \left( \frac{c_θ^2}{(m_0)^{22}} + \frac{s_θ^2}{(m_0)^{33}} \right) + \frac{\lambda_5s_θ^2}{2} \left( \frac{1}{(m_0)^{22}} \right) \right]$$

$$+ \frac{1}{(m_0)^{22}} \left( \frac{\lambda_3 + \lambda_7}{s_θ} \right)^2 \left( \frac{1}{(m_0)^{22}} - \frac{1}{(m_0)^{33}} \right)$$

$$+ \lambda_5 \left( \frac{s_θ^2}{(m_0)^{22}} + \frac{c_θ^2}{(m_0)^{33}} \right) \left( \begin{array}{c} \beta R_1 - \frac{I_1}{t_ξ} \\ \beta R_2 - \frac{I_1}{t_ξ} \end{array} \right).$$

While for the two heavier neutral Higgs, we have

$$m_{2(3)}^2 = \frac{v^2}{2} \left( (m_0)^{22(33)} + O(t_βs_ξ) \right),$$

in which $(m_0)^{22(33)}$ are the other two eigenvalues of $\tilde{m}_0$ and

$$(m_0)^{22(33)} = \frac{4\lambda_1 + \lambda_4 - \lambda_7}{2} \pm \left( \frac{4\lambda_1 - (\lambda_4 - \lambda_7)}{2} c_θ + \lambda_2s_θ \right)$$

where $θ = (1/2) \tan^{-1}(2λ_2/(4λ_1 - λ_4 + λ_7))$. The physical states are

$$\left( \begin{array}{c} h_2 \\ h_3 \end{array} \right) = \left( \begin{array}{cc} c_θ & s_θ \\ -s_θ & c_θ \end{array} \right) \left( \begin{array}{c} R_1 \\ R_2 \end{array} \right) + O(t_βs_ξ).$$

For all the details about scalar spectra and its small $t_βs_ξ$ expansion series, the interesting reader can see section B.

From the Yukawa couplings we will get the mass matrixes for fermions as

$$(M_U)_{ij} = \frac{v}{\sqrt{2}}(Y_{1i}c_β + Y_{2i}s_βe^{-iξ})_{ij},$$

$$(M_D)_{ij} = \frac{v}{\sqrt{2}}(Y_{1d}c_β + Y_{2d}s_βe^{iξ})_{ij}.$$
We can always perform the diagonalization for $M_{U(D)}$ with matrixes $U(D)_L$ and $U(D)_R$ as

$$U_L M_U U_R^\dagger = \begin{pmatrix} m_u & m_c \\ m_c & m_t \end{pmatrix}, \quad D_L M_D D_R^\dagger = \begin{pmatrix} m_d & m_s \\ m_s & m_b \end{pmatrix}. \quad (29)$$

And $V_{CKM} = U_L D_L^\dagger$ is the CKM matrix.

In this scenario, the couplings for the discovered light Higgs boson should be modified from SM by a factor as

$$\mathcal{L}_{h,\text{eff}} = c_V \left( \frac{2m_W^2}{v} W_\mu^+ W_\mu^- + \frac{m_Z^2}{v} Z_\mu Z_\mu \right) h - c_\pm v H^+ H^- h$$

$$- \sum_i \left( c_{U_i} U_{Li} U_{Ri} + c_{D_i} D_{Li} D_{Ri} + \text{h.c.} \right) h, \quad (30)$$

where the factors $c_\pm$ and $c_V$ must be real, but $c_{U_i}$ and $c_{D_i}$ may be complex. According to (C.1)-(C.4) in section C, to the leading order of $t_\beta s_\xi$, we straightforwardly have

$$c_V = t_\beta s_\xi (1 + \eta_1); \quad (31)$$

$$c_{D_i} = \frac{t_\beta s_\xi}{\sqrt{2}} \left( \eta_1 (Y_{1d}^{*})_{ii} + \eta_2 (Y_{2d}^{*})_{ii} \right) + \frac{i (Y_{2d}^{*})_{ii}}{\sqrt{2}}; \quad (32)$$

$$c_{U_i} = \frac{t_\beta s_\xi}{\sqrt{2}} \left( \eta_1 (Y_{1u}^{*})_{ii} + \eta_2 (Y_{2u}^{*})_{ii} \right) - \frac{i (Y_{2u}^{*})_{ii}}{\sqrt{2}}; \quad (33)$$

and the coupling including charged higgs should be

$$c_\pm = t_\beta s_\xi \left( 2\lambda_6 - \lambda_7 \right) + \lambda_3 \eta_1 + \frac{\lambda_5 \eta_2}{2}; \quad (34)$$

where

$$\eta_1 = c_\theta (\bar{m}_1)_{12} - s_\theta (\bar{m}_1)_{13} (\bar{m}_0)_{33} \quad = 2(\lambda_3 + \lambda_7) \left( \frac{c_\theta^2}{(\bar{m}_0)_{22}} + \frac{s_\theta^2}{(\bar{m}_0)_{33}} \right) + \lambda_5 s_{2\theta} \left( \frac{1}{(\bar{m}_0)_{22}} - \frac{1}{(\bar{m}_0)_{33}} \right); \quad (35)$$

$$\eta_2 = s_\theta (\bar{m}_1)_{12} + c_\theta (\bar{m}_1)_{13} (\bar{m}_0)_{33} \quad = (\lambda_3 + \lambda_7) s_{2\theta} \left( \frac{1}{(\bar{m}_0)_{22}} - \frac{1}{(\bar{m}_0)_{33}} \right) + \lambda_5 \left( \frac{c_\theta^2}{(\bar{m}_0)_{22}} + \frac{s_\theta^2}{(\bar{m}_0)_{33}} \right). \quad (36)$$

We choose all the nine free parameters as nine observables in Higgs sector: masses of four scalars $m_h, m_2, m_3$ and $m_{H^\pm}$; vacuum expected values $v_1, v_2, \xi$ and two mixing angles for neutral bosons. We choose them $c_1$ and $c_2$ defined as

$$\mathcal{L}_{hVV} = c_i h_i \left( \frac{2m_W^2}{v} W_\mu^+ W_\mu^- + \frac{m_Z^2}{v} Z_\mu Z_\mu \right) \quad (37)$$
which just means the $hVV$ vertex strength comparing with that in SM\textsuperscript{5}. In the scalar sector, for non-degenerate neutral Higgs bosons, a quantity $K = c_1c_2c_3$ measures the CP violation effects \[49\] \[50\], while in Yukawa sector, the Jarlskog invariant $J$ \[30\] measures that. In this scenario, to the leading order of $t_\beta s_\xi$, we have

$$K = c_1c_2c_3 = -s_\beta c_\delta (1 + \eta_1)t_\beta s_\xi \propto t_\beta s_\xi$$ \hspace{1cm} (38)

In order to calculate $J$, we define matrix $\hat{C}$ as

$$\hat{C} \equiv [M_U M_U^\dagger, M_D M_D^\dagger].$$ \hspace{1cm} (39)

We can always choose a basis in which the diagonal elements of $\hat{C}$ are zero. Thus

$$\hat{C} = \begin{pmatrix} 0 & C_3 & -C_2 \\ -C_3 & 0 & C_1 \\ C_2 & -C_1 & 0 \end{pmatrix} + i \begin{pmatrix} 0 & C_3^* & C_2^* \\ C_3^* & 0 & C_1^* \\ C_2^* & C_1^* & 0 \end{pmatrix} = \left( \text{Re} \hat{C} + i \text{Im} \hat{C} \right)$$ \hspace{1cm} (40)

in which using equations (27) and (28), to the leading order of $t_\beta s_\xi$, we have

$$\text{Re} \hat{C} = \frac{v^4 c_\beta^4}{4} \left[ Y_{u1} Y_{u1}^\dagger, Y_{d1} Y_{d1}^\dagger \right];$$ \hspace{1cm} (41)

$$\text{Im} \hat{C} = \frac{v^4 c_\beta^4}{4} \left( \left[ Y_{u1} Y_{u2}^\dagger - Y_{u2} Y_{u1}^\dagger, Y_{d1} Y_{d1}^\dagger \right] \right.$$

$$+ \left. \left[ Y_{u1} Y_{u1}^\dagger, Y_{d2} Y_{d1}^\dagger - Y_{d1} Y_{d2}^\dagger \right] \right) t_\beta s_\xi \propto t_\beta s_\xi.$$ \hspace{1cm} (42)

To the leading order of $t_\beta s_\xi$, the determinant

$$\det \left( i \hat{C} \right) = 2J \prod_{i<j} \left( m^2_{U_i} - m^2_{U_j} \right) \prod_{i<j} \left( m^2_{D_i} - m^2_{D_j} \right)$$

$$= C_1 C_2 C_3 \left( \frac{C_1^*}{C_1} + \frac{C_2^*}{C_2} + \frac{C_3^*}{C_3} \right),$$ \hspace{1cm} (43)

where $C_i^* \propto t_\beta s_\xi$, thus

$$J = \frac{\prod_i C_i \sum (C_i^*/C_i)}{\prod_{i<j} \left( m^2_{U_i} - m^2_{U_j} \right) \prod_{i<j} \left( m^2_{D_i} - m^2_{D_j} \right)} \propto t_\beta s_\xi.$$ \hspace{1cm} (44)

\textsuperscript{5} There is a sum rule $c_1^2 + c_2^2 + c_3^2 = 1$ due to spontaneous electro-weak symmetry broken, thus only two of the $c_i$ are free, and $c_1$ here is just the $c_V$ in \[31\].

\textsuperscript{6} If at least two of the neutral bosons have degenerate mass, we can always perform a rotation among the neutral fields to keep $K = 0$. 
According to the equations (38), (44), and (22), we propose that the lightness of the Higgs boson and the smallness of CP-violation effects could be correlated through small $t_\beta s_\xi$ since both the Higgs mass $m_h$ and the quantities $K$ and $J$ to measure CP-violation effects are proportional to $t_\beta s_\xi$ at the small $t_\beta s_\xi$ limit.

In the following two sections, we will study whether the Lee model is still viable confronting the current numerous high and low energy measurements. From Eq. (31), it is quite clear that couplings of discovered scalar boson differ from those in the SM, namely Lee model does not have SM limit. Provided that LHC obtained only a small portion of its designed integrated luminosity, there would be spacious room for Lee model. In the long run, LHC and future facilities have the great potential to discover/exclude Lee model. We will discuss this part in Section V.

### III. CONSTRAINTS FROM HIGH ENERGY PHENOMENA

In this model there are two more neutral bosons and one more charged boson pair comparing with SM, these degree of freedoms may affect on the physics at electro-weak scale, and they could also be constrained by direct searches at the LHC. For the discovered boson, SM predicts the decay branching ratios for a Higgs boson with mass 125.7GeV in Table I. However in Lee model, the modified couplings will change the total width and branching ratios due to equations (31)-(34), together with the production cross sections modified by (33) for gluon fusion and (31) for vector boson fusion and the associated production with vector bosons. Of course, this model may also affect top physics because the couplings between Higgs boson and top quark are not suppressed and it may also change the flavor changing couplings especially for top quark. Thus it is necessary to discuss the constraints to this model from high energy phenomena.

#### A. Constraints on heavy neutral bosons

A heavy Higgs boson may decay to $W^+W^-$, $2Z_0$, $2h$, $t\bar{t}$ (for neutral bosons heavier than $2m_t \approx 346$GeV), or $H^+H^-$ (for light charged Higgs and a neutral boson heavier than $2m_{H^\pm}$). Based on the searches for the SM Higgs boson using diboson final state [51], masses and couplings of the other two heavier neutral Higgs bosons should be constrained by the data.
For a neutral Higgs boson heavier than 350GeV, the $t\bar{t}$ resonance search [52] may also give some constraints.

In our scenario, the total width of a heavy boson can be expressed as

$$\Gamma_i = \Gamma_{i,VV} + \Gamma_{i,\pm} + \Gamma_{i,2h} + \Gamma_{i,t\bar{t}}, \quad (45)$$

where $\Gamma_{i,VV}$, $\Gamma_{i,\pm}$, $\Gamma_{i,2h}$, $\Gamma_{i,t\bar{t}}$ correspond to massive gauge boson pairs, charged Higgs pair, neutral Higgs pair and top quark pair final states respectively. The partial decay width for a heavy neutral Higgs with mass $m_i$ are

$$\frac{\Gamma_{i,VV}}{m_i} = \frac{3c_i^2}{16\pi} \left(\frac{m_i}{v}\right)^2 \left( m_i \gg m_V \right); \quad (46)$$

$$\frac{\Gamma_{i,t\bar{t}}}{m_i} = \frac{3|c_{t,i}|^2}{8\pi} \left(\frac{m_t}{v}\right)^2 \sqrt{1 - \frac{4m_t^2}{m_i^2}} \left( 1 - \frac{4m_t^2\cos^2(\arg(c_{t,i}))}{m_i^2} \right); \quad (47)$$

$$\frac{\Gamma_{i,\pm}}{m_i} = \frac{\lambda_{i,\pm}^2}{16\pi} \left(\frac{v}{M}\right)^2 \sqrt{1 - \frac{4m_{h,\pm}^2}{m_i^2}}; \quad (48)$$

$$\frac{\Gamma_{i,hh}}{m_i} = \frac{\lambda_{i,hh}^2}{32\pi} \left(\frac{v}{M}\right)^2 \sqrt{1 - \frac{4m_h^2}{m_i^2}}. \quad (49)$$

in unit of its mass. Here we have the vertices

$$\lambda_{i,\pm} = \frac{1}{v} \frac{\partial^3 V}{\partial h_i \partial H^+ \partial H}, \quad \lambda_{i,hh} = \frac{1}{v} \frac{\partial^3 V}{\partial h_i \partial h \partial h}. \quad (50)$$

The couplings $\lambda_{i,\pm}, \lambda_{i,hh} \sim O(1)$.

The signal strength is defined as

$$\mu = \frac{\sigma}{\sigma_{SM}} \cdot \frac{\Gamma_{i,VV}}{\Gamma_i} \cdot \frac{1}{Br_{SM(VV)}} \cdot (51)$$

for a production channel. The $\sigma/\sigma_{SM} \lesssim O(1)$ for different channels. For a heavy Higgs with $m_i \leq 2m_t$, $Br_{SM}(VV)$ is very close to 1; while for $m_i > 2m_t$, $Br_{SM}(VV)$ has a minimal value of about 0.8 when $m_i \sim 500$GeV. According to (46) - (49), we can estimate that for both $m_i \sim v$ and $m_i \gg v$, $\mu \sim O(0.1 - 1)$.

Thus according to the figures in [51], we have three types of typical choices for the mass of two heavy neutral higgs particles in Table IV. (Here we write the mass of the lighter boson $m_2$ and the heavier one $m_3$.)
### TABLE IV: Typical choices for the masses of the two heavy neutral scalars.

| Case | Allowed $m_2$(GeV) | Allowed $m_3$(GeV) |
|------|-------------------|-------------------|
| I    | $\lesssim 300$    | $\lesssim 300$    |
| II   | $\lesssim 300$    | $\gtrsim 700$    |
| III  | $\gtrsim 700$    | $\gtrsim 700$    |

#### B. Constraints due to Oblique Parameters

After the discovery of the new boson, there are new electro-weak fit for the standard model \[18\]. Choosing $m_{t,\text{ref}} = 173\text{GeV}$ and $m_{h,\text{ref}} = 126\text{GeV}$, the oblique parameters \[19\] are

\[
S = 0.03 \pm 0.10, \quad T = 0.05 \pm 0.12, \quad U = 0.03 \pm 0.10,
\]

\[
R_{ST} = +0.89, \quad R_{SU} = -0.54, \quad R_{TU} = -0.83,
\]

(52)

with $R$ the correlation coefficient between two quantities; or

\[
S = 0.05 \pm 0.09, \quad T = 0.08 \pm 0.07, \quad R = +0.91,
\]

(53)

with fixed $U = 0$, where R is the correlation coefficient between S and T. The basic mathematica code to draw the S-T ellipse can be found on the webpage \[53\].

The contribution to S and T parameters due to multi-higgs doublets were calculated in \[54\] (see the formulae in \[49\]).

\[
\Delta T = \frac{1}{16\pi s_W^2 m_W^2} \left[ \sum_{i=1}^{3} (1 - c_i^2) F(m_{H^\pm}^2, m_i^2) - c_i^2 F(m_2^2, m_3^2) - c_2^2 F(m_3^2, m_1^2) 
- c_3^2 F(m_1^2, m_2^2) + 3 \sum_{i=1}^{3} c_i^2 (F(m_2^2, m_i^2) - F(m_W^2, m_i^2)) 
- 3(\sum_{i=1}^{3} c_i^2 H(z_i) + \ln \left( \frac{m_i^2}{m_{H^\pm}^2} \right)) - H \left( \frac{m_{h,\text{ref}}^2}{m_Z^2} \right) - \ln \left( \frac{m_{h,\text{ref}}^2}{m_{H^\pm}^2} \right) \right];
\]

(54)

\[
\Delta S = \frac{1}{24\pi} \left[ (1 - 2s_W^2)^2 G(z_\pm, z_\pm) + c_2^2 G(z_2, z_3) + c_2^2 G(z_3, z_1) + c_3^2 G(z_1, z_2) 
+ \sum_{i=1}^{3} \left( c_i^2 H(z_i) + \ln \left( \frac{m_i^2}{m_{H^\pm}^2} \right) \right) - H \left( \frac{m_{h,\text{ref}}^2}{m_Z^2} \right) - \ln \left( \frac{m_{h,\text{ref}}^2}{m_{H^\pm}^2} \right) \right];
\]

(55)

---

7 Assuming Gaussian distribution, the second $\Delta \chi^2$ should be 6.0 instead of 6.8 in the code. See the 36th chapter (statistics) of the reviews in PDG \[28\].
FIG. 1: S-T ellipse for case I, $m_2 = 280\text{GeV}$ and $m_3 = 300\text{GeV}$.

where $c_i$ is the rate of the $h_i V_\mu V^\mu$ coupling to that in SM ($c_1$ represents above-mentioned $c_V$) and $\sum c_i^2 = 1$. $m_{h,\text{ref}} = m_1 = 126\text{GeV}$ is the reference point for Higgs Boson, $z_{\pm} = (m_{H^{\pm}}/m_Z)^2$ and $z_i = (m_i/m_Z)^2$. The functions $F, G, H$ read (following the formulae in [49])

$$F(x, y) = \frac{x+y}{2} - \frac{xy}{x-y} \ln \left(\frac{x}{y}\right); \quad (56)$$

$$G(x, y) = -\frac{16}{3} + 5(x+y) - 2(x-y)^2$$

$$+ 3 \left(\frac{x^2+y^2}{x-y} + y^2 - x^2 + \frac{(x-y)^3}{3}\right) \ln \left(\frac{x}{y}\right) + (1-2(x+y))$$

$$+ (x-y)^2 f(x+y-1, 1-2(x+y)+(x-y)^2); \quad (57)$$

$$H(x) = -\frac{79}{3} + 9x - 2x^2 + \left(-10 + 18x - 6x^2 + x^3 - 9 \frac{x^2 + 1}{x^2 - 1}\right) \ln x$$

$$+ (12 - 4x + x^2) f(x, x^2 - 4x); \quad (58)$$

where

$$f(x, y) = \begin{cases} 
\sqrt{y} \ln \left|\frac{x-\sqrt{y}}{x+\sqrt{y}}\right|, & y \geq 0; \\
2 \sqrt{-y} \arctan \left(\frac{\sqrt{-y}}{x}\right), & y < 0. 
\end{cases} \quad (59)$$

At removable singularities the functions are defined as the limit.

The parameter $U$ is usually small so that we fix $U = 0$ from now on. We take the benchmark points according to the cases in Table IV. We show the contours in Figure 1, Figure 3 for the cases listed in Table IV in last section. Throughout the paper, the region outside the green area is excluded at 68% C.L. and the region outside the yellow area is excluded at 95% C.L. Firstly, for case I, we take $m_2 = 280\text{GeV}$ and $m_3 = 300\text{GeV}$. The
FIG. 2: S-T ellipse for case II, $m_2 = 300\text{GeV}$ and $m_3 = 700\text{GeV}$.

![S-T ellipse for case II](image)

FIG. 3: S-T ellipse for case III, $m_2 = 700\text{GeV}$ and $m_3 = 750\text{GeV}$.

![S-T ellipse for case III](image)

typical values for the left diagram in Figure 1 are $c_1^2 = 0.2$, $c_2^2 = c_3^2 = 0.4$. Here the blue and red lines refer to $86\text{GeV} < m_{H^\pm} < 126\text{GeV}$ and $312\text{GeV} < m_{H^\pm} < 350\text{GeV}$ respectively. For the right diagram, $c_1^2 = 0.25$, $c_2^2 = 0.4$, $c_3^2 = 0.35$, and the blue and red lines refer to $94\text{GeV} < m_{H^\pm} < 136\text{GeV}$ and $312\text{GeV} < m_{H^\pm} < 351\text{GeV}$ respectively.

Secondly, for case II, we take $m_2 = 300\text{GeV}$ and $m_3 = 700\text{GeV}$. The typical values for the left diagram in Figure 2 are $c_1^2 = 0.2$, $c_2^2 = 0.5$, $c_3^2 = 0.3$. Here the blue and red lines refer to $127\text{GeV} < m_{H^\pm} < 149\text{GeV}$ and $580\text{GeV} < m_{H^\pm} < 600\text{GeV}$ respectively. For the right diagram, $c_1^2 = c_3^2 = 0.25$, $c_2^2 = 0.5$, and the blue and red lines refer to $141\text{GeV} < m_{H^\pm} < 163\text{GeV}$ and $598\text{GeV} < m_{H^\pm} < 618\text{GeV}$ respectively.
Thirdly, for case III, we take $m_2 = 700\text{GeV}$ and $m_3 = 750\text{GeV}$. The typical values for the left diagram in Figure 3 are $c_1^2 = 0.2$, $c_2^2 = c_3^2 = 0.4$. Here the blue and red lines refer to $218\text{GeV} < m_{H^\pm} < 235\text{GeV}$ and $748\text{GeV} < m_{H^\pm} < 765\text{GeV}$ respectively. For the right diagram, $c_1^2 = 0.25$, $c_2^2 = 0.4$, $c_3^2 = 0.35$, and the blue and red lines refer to $250\text{GeV} < m_{H^\pm} < 269\text{GeV}$ and $749\text{GeV} < m_{H^\pm} < 767\text{GeV}$ respectively.

In type II 2HDM the charged Higgs should be heavier than 360GeV \cite{55} \cite{56} mainly due to the constraint from inclusive $b \rightarrow s\gamma$ process. However in other models, there is no such strict constraints. Direct searches by LEP told us that the charged Higgs boson should be heavier than 78.6GeV \cite{57}. In case I and II above, a light (around 100-200GeV) charged Higgs boson is allowed, while in case III the charged Higgs boson cannot be lighter than about 250GeV. In case I and III, a charged higgs boson with the mass near the heavy neutral bosons is allowed, while in case II a heavy charged higgs boson must be lighter than the heaviest neutral scalar.

C. Constraints due to Signal Strengths

In Table II and Table III for a certain channel, the signal strength is defined as

$$
\mu_f = \frac{\sigma \cdot \text{Br}_f}{(\sigma \cdot \text{Br}_{f,\text{SM}})} = \frac{\sigma}{\sigma_{\text{SM}}} \cdot \frac{\Gamma_f}{\Gamma_{f,\text{SM}}} \cdot \frac{\Gamma_{\text{tot,SM}}}{\Gamma_{\text{tot}}},
$$

(60)

in which $\sigma/\sigma_{\text{SM}} = |c'_f|^2$ for gluon fusion processes and $\sigma/\sigma_{\text{SM}} = c_V^2$ for vector boson fusion (VBF) processes and associated productions with a gauge boson. For decays without interference, we simply have $\Gamma_f/\Gamma_{f,\text{SM}} = |c_f|^2$ such as for $f = V, b, \tau$. While for the two photons final state, we have \cite{6} \cite{22}

$$
\frac{\Gamma_{\gamma\gamma}}{\Gamma_{\gamma\gamma,\text{SM}}} = \left| \frac{(4/3)c'_t A_{1/2}(x_t) + c_V A_1(x_W) + (c_{\pm} v^2/2m_{H^\pm}^2) A_0(x_{\pm})}{(4/3)A_{1/2}(x_t) + A_1(x_W)} \right|^2,
$$

(61)

in which $x_i = m_i^2/4m^2$ for $i = t, W, H^\pm$. The loop integration functions are

$$
A_0(x) = \frac{1}{x^2} (x - f(x))
$$

(62)

$$
A_{1/2}(x) = -\frac{2}{x^2} (x + (x - 1)f(x))
$$

(63)

$$
A_1(x) = \frac{1}{x^2} (2x^2 + 3x + 3(2x - 1)f(x))
$$

(64)
for scalar, fermion and vector boson loop respectively and

\[ f(x) = \begin{cases} 
\arcsin^2 \sqrt{x}, & x \leq 1; \\
-\frac{1}{4} \left( \ln \left( \frac{1+\sqrt{1-x}}{1-\sqrt{1-x}} \right) - \pi i \right)^2, & x > 1.
\end{cases} \]  

(65)

In a spontaneous CP-violated model, \( c'_t \) (together with other \( c_f \) for fermions) can be complex while \( c_V \) and \( c_\pm \) must be real. Notice all the \( c_V, c_\pm, c_b \) and \( c_\tau \) are the same with those in (31)-(34), but \( c_t \) should be modified to \( c'_t \) as

\[ c'_t = \text{Re}(c_t) + i \frac{B_{1/2}(m_h^2/4m_t^2)}{A_{1/2}(m_h^2/4m_t^2)} \text{Im}(c_t) \]  

(66)

in which the function

\[ B_{1/2}(x) = -\frac{2f(x)}{x}. \]  

(67)

Thus defining \( \alpha_t \equiv \arg(c_t) \) and \( \alpha'_t \equiv \arg(c'_t) \), numerically we have

\[ \alpha'_t = \arctan(1.52\alpha_t), \quad |c'_t| = |c_t| \sqrt{1 + 1.31 \sin^2 \alpha_t}. \]  

(68)

Assuming there is no unknown decay channel which contributes several percentages or more to the total width, we can estimate that

\[ \frac{\Gamma_{\text{tot}}}{\Gamma_{\text{tot,SM}}} = 0.57|c_b|^2 + 0.25c_V^2 + 0.06|c_\tau|^2 + 0.03|c_c|^2 + 0.09|c'_t|^2 \]  

(69)

according to [Table 1].

Define the \( \chi^2 \)

\[ \chi^2 = \sum_{i,f} \left( \frac{\mu_{i,f,\text{obs}} - \mu_{i,f,\text{pre}}}{\sigma_{i,f}} \right)^2 \]  

(70)

where \( i = \text{VBF, } \text{ggF, } \text{VH} \) and \( f = \gamma\gamma, WW^*, ZZ^*, \tau^+\tau^- \) at a detector (CMS or ATLAS). The \( \mu_{i,f,\text{obs(pre)}} \) are the observed (predicted) signal strength for the production channel \( i \) and final state \( f \). We ignored all correlation coefficients between channels since they are small.

Numerically we find that the minimal \( \chi^2 \) is not sensitive to the charged Higgs mass since the scalar loop contributes less than the top and \( W \) loop in \( \gamma\gamma \) decay channel. Thus we take the benchmark point as \( m_{H^\pm} = 150\text{GeV} \). For six degrees of freedom, parameter space with \( \chi^2 \leq 7.0 \) is allowed at 68%C.L. and \( \chi^2 \leq 12.6 \) is allowed at 95%C.L. For both CMS and ATLAS data, the minimal \( \chi^2 \) is very sensitive to \( c_V \) and \( c'_t \), since they give dominant contributions to most production cross sections and partial decay widths; it is sensitive to \( c_b \) as well since the total width is sensitive to \( |c_b| \). With the CMS data, we have \( c_V \geq 0.22 \);
FIG. 4: Allowed $|c'_t| - \alpha'_t$ contour when taking $c_V = 0.4$, $c_\pm = 0.2$ and $|c_\tau| = 0.8$ for CMS data. $|c_b| = 0.1$ for the left figure and $|c_b| = 0.4$ for the right figure.

and with the ATLAS data, we have $c_V \geq 0.31$, both at 95\% C.L. So $c_V = 0.5$ is a good benchmark point as we have chosen in the last section, and it will also be taken around this point in later analysis. The $\chi^2$ is not very sensitive to $|c_\tau|$ and $c_\pm$, as both of them contribute to only one channel, and the charged Higgs loop contributes less in the $\gamma\gamma$ decay channel. Thus for most analysis we don’t discuss these two parameters carefully.

For the CMS data, when $c_V \sim 0.5$, the $\chi^2_{\text{min}} \approx 2$. The data favors smaller $|c_b|$ but the minimal value of $\chi^2$ changes little as $c_b$ varies, since the points are far away from the 95\% allowed boundary $c_V = 0.22$. Figure 4 and Figure 6 show CMS allowed $|c'_t|$ and $\alpha'_t \equiv \arg(c'_t)$ for some benchmark points $^8$.

In Figure 4 and Figure 5, we choose $c_V = 0.4$. Fixing $c_\pm = 0.2$ and $|c_\tau| = 0.8$, and taking $|c_b| = 0.1, 0.4, 0.7$, we have the three figures in Figure 4 and Figure 5. The best fit point for $|c_t|$ has positive correlation with $|c_b|$. For larger $|c_b|$, the best fit point for $|c_\tau|$ increases as well, thus in the right figure in Figure 5 we set $|c_\tau| = 1.3$ and get better fitting result.

In Figure 6, we have $c_V = 0.5$. Fixing $c_\pm = 0.2$ and $|c_\tau| = 0.9$, and taking $|c_b| = 0.1, 0.3, 0.5, 0.7$, we get the four figures. The fitting results are less sensitive to $|c_\tau|$ than in Figure 4 and Figure 5 and the best fit point for $|c'_t|$ has positive correlation with $|c_b|$ as well.

$^8$ In this paper, the benchmark points are close to the best fit points for a certain case thus the allowed regions are typical enough.
FIG. 5: Allowed $|c'_t| - \alpha'_t$ contour when taking $c_V = 0.4$, $c_\pm = 0.2$ and $|c_b| = 0.7$ for CMS data. $|c_\tau| = 0.8$ for the left figure and $|c_\tau| = 1.3$ for the right figure.

Usually $\alpha'_t \sim 0$ is disfavored while for smaller $|c_b|$ and larger $c_V$ any $\alpha'_t$ is allowed. For each case, the best fit point is about $|\alpha'_t| \sim 1.2$.

For the ATLAS data, when $c_V \sim 0.5$, the $\chi^2_{\text{min}} \approx 7$ which is near the $1\sigma$ allowed boundary. The data favor smaller $|c_b|$ as well just like the CMS case. Figure 7 and Figure 8 show ATLAS allowed $|c'_t|$ and $\alpha'_t \equiv \arg(c'_t)$ for some benchmark points.

In Figure 7 we show the allowed regions for $c'_t$. Fixing $c_V = 0.5$, $c_\pm = 0.4$, and $|c_\tau| = 0.7$, choosing $|c_b| = 0.2, 0.4$, we have the two figures. In Figure 8 fixing $c_V = 0.6$, $c_\pm = 0.4$, and $|c_\tau| = 0.8$, and taking $|c_b| = 0.1, 0.3, 0.5, 0.7$, we have the four figures. Usually $\alpha'_t \sim 0$ is disfavored while for smaller $|c_b|$ and larger $c_V$ any $\alpha'_t$ is allowed. The best fit points for $\alpha'_t$ are around $|\alpha'_t| \sim 1.2$, all these behaviors are similar as the results from CMS data.

For both CMS and ATLAS data, smaller $|c_b|$ is favored. In most case, the best fit points for $|c'_t|, |c_\tau|$ are around 1, and $c_\pm \sim O(0.1)$. The fitting results for $\alpha'_t$ favor smaller $|\alpha'_t|(\sim 1.2)$ by both data for most $|c_b|$ input. We also have the $\chi^2$ for SM as

$$\chi^2_{\text{SM,CMS}} = 2.4, \quad \chi^2_{\text{SM,ATLAS}} = 4.4$$

(71)

close to the minimal $\chi^2$ for Lee model we discussed in this paper. So the Lee model can fit the current data as well as that in the SM.
FIG. 6: Allowed $|c'_t| - \alpha'_t$ contour when taking $c_V = 0.5$, $c_{\pm} = 0.2$ and $|c_T| = 1$ for CMS data. The four figures correspond to $|c_b| = 0.1, 0.3, 0.5, 0.7$ respectively.

D. Same Sign Top Production

We put no additional symmetries in the Yukawa sector to avoid tree-level FCNC, thus the model must be constrained by processes including flavor-changing interactions. The tree-level FCNC for up type quarks will lead to same sign top quarks production at the LHC. An upper limit at 95%C.L. was given as [58]

$$\sigma_{tt} < 0.37\text{pb}$$

by the CMS group with an integrated luminosity 19.5fb$^{-1}$ at $\sqrt{s} = 8$TeV.
FIG. 7: Allowed $|c_t'| - \alpha_t'$ contour when taking $c_V = 0.5$, $c_\pm = 0.4$ and $|c_t| = 0.7$ for ATLAS data. The left and right figures correspond to $|c_b| = 0.2$ and $|c_b| = 0.4$ respectively.

In this model, we can write the interaction which can induce same sign top quark production at the LHC as

$$\mathcal{L}_{I,tuh} = -\frac{1}{\sqrt{2}}(\xi_{1tu} + \xi_{2tu} \gamma^5)uh + \text{h.c.}$$

(73)

The lightest neutral boson gives the dominant contribution when the effect couplings are similar. A direct calculation gives

$$\sigma_{tt} = \int dx_1 dx_2 f_u(x_1) f_u(x_2)\sigma_0$$

(74)

in which

$$\sigma_0 = \frac{|\xi_{tu}|^4 \beta_t}{64\pi s_0} \int_{-1}^1 dc_\theta \left[ \left( \frac{1 - \beta_t c_\theta}{1 + \beta_t^2 + 4m_h^2/s_0 - 2\beta_t c_\theta} \right)^2 + \frac{1 + \beta_t^2 c_\theta^2}{(1 + \beta_t^2 + 4m_h^2/s_0)^2 - 4\beta_t^2 c_\theta^2} \right]$$

(75)

if $\xi_{1tu}\xi_{2tu} + \xi_{2tu}\xi_{1tu} = 0$ where $s_0$ is the square of energy in the frame of momentum center of two partons (both $u$ quarks). $\beta_t = \sqrt{1 - 4m_t^2/s_0}$ is the velocity of a top quark and $\theta$ is the radiative angel in the same frame and $|\xi_{tu}| = \sqrt{|\xi_{1tu}|^2 + |\xi_{2tu}|^2}$. Using the MSTW2008 PDF [59] and comparing with (72), we can estimate that $|\xi_{tu}| \lesssim 0.4$. 
FIG. 8: Allowed \(|c'_t| - \alpha'_t\) contour when taking \(c_V = 0.6, c_\pm = 0.4\) and \(|c_\tau| = 0.8\) for ATLAS data.

The four figures correspond to \(|c_b| = 0.1, 0.3, 0.5, 0.7\) respectively.

E. Top Rare Decays

In this model, the FCNC interactions including up type quarks will induce rare decay processes of top quark, such as \(t \to ch\) and \(t \to uh\), usually with a larger rate than that in the SM. When the charged Higgs boson is lighter than the top quark, there will be a new decay channel \(t \to H^+b\) as well. Direct search results at \(\sqrt{s} = 8\) TeV by CMS at LHC gave the top pair production cross section\(^{[60]}\) \(\sigma_{tt} = (237 \pm 13)\)pb assuming \(m_t = 173\) GeV and \(Br(t \to bW) = 1\), while theoretical calculation predicts that\(^{[61]}\) \(\sigma_{tt,\text{pre}} = (246^{+9}_{-11})\)pb. Assuming there is no effects beyond SM during the production of top pair, these results can
constrain the top rare decay (all channels except $bW$) branching ratio

$$Br_{t,\text{rare}} = 1 - Br(t \to bW) < 7.4\%$$  \hspace{1cm} (76)

at 95% C.L.

For the rare decay processes above, the interactions can be written as

$$\mathcal{L}_{I,tch} = -\frac{1}{\sqrt{2}} \bar{t}(\xi_{1tc} + \xi_{2tc}\gamma^5)ch + \text{h.c.} \hspace{1cm} (77)$$

$$\mathcal{L}_{I,tbH^+} = -\bar{t}(\xi_{1tb} + \xi_{2tb}\gamma^5)bH^+ + \text{h.c.} \hspace{1cm} (78)$$

together with (73). Direct calculations give the decay rates

$$\Gamma_{hu(hc)} = \frac{\mid\xi_{tu(tc)}\rangle^2m_t}{32\pi} \left(1 - \frac{m_h^2}{m_t^2}\right)^2$$  \hspace{1cm} (79)

$$\Gamma_{H^+b} = \frac{\mid\xi_{tb}\rangle^2m_t}{16\pi} \left(1 - \frac{m_{H^+}^2}{m_t^2}\right)^2$$  \hspace{1cm} (80)

where $\mid\xi_{ti}\rangle = \sqrt{\mid\xi_{1ti}\rangle^2 + \mid\xi_{2ti}\rangle^2}$.

Direct search for $t \to c(u)h \to c(u)\gamma\gamma$ decays \cite{62} at ATLAS gives the bound for branching ratios

$$Br(t \to ch) + Br(t \to uh) < 0.79\% \cdot \left(\frac{Br(h \to \gamma\gamma)_{\text{SM}}}{Br(h \to \gamma\gamma)}\right)$$  \hspace{1cm} (81)

at 95% C.L. which leads to

$$\sqrt{\mid\xi_{tu}\rangle^2 + \mid\xi_{tc}\rangle^2} < 0.16\kappa, \hspace{0.5cm} \text{where} \hspace{0.5cm} \kappa = \sqrt{\frac{Br(h \to \gamma\gamma)_{\text{SM}}}{Br(h \to \gamma\gamma)}} \sim \mathcal{O}(1)$$  \hspace{1cm} (82)

and $\kappa = 1$ in the SM. For most cases it is a stronger constraint on $\mid\xi_{tu}\rangle$ than that in the same sign top production process, but they are of the same order. A similar measurement by CMS \cite{63} gives a 95% upper limit $Br(t \to ch) < 0.56\%$ hence $\mid\xi_{tc}\rangle < 0.14$ with the combination of Higgs decaying to diphoton or multileptons assuming the SM decay branching ratios of Higgs boson. If we allow different branching ratios to the SM, the constraints on this coupling is still of that order. Adopting the Cheng-Sher ansatz \cite{43}, we have

$$\frac{\mid\xi_{tc}\rangle v}{\sqrt{2m_tm_c}} \lesssim 1.5, \hspace{1cm} \text{and} \hspace{1cm} \frac{\mid\xi_{tu}\rangle v}{\sqrt{2m_tm_u}} \lesssim 44$$  \hspace{1cm} (83)

assuming SM branching ratios of Higgs. For other branching ratio, the constraints are of the same order.
TABLE V: Constraints on the $t \rightarrow bH^+ \rightarrow b\tau^+ \nu (c\bar{s})$ from direct searches for light charged Higgs boson (lighter than top quark).

| Process $(H^+ \rightarrow f)$ | Charged Higgs mass (GeV) | $Br(t \rightarrow bH^+ \rightarrow b\tau)$ (95\%C.L.) |
|-----------------------------|-------------------------|-------------------------------------------|
| $H^+ \rightarrow c\bar{s}$ (ATLAS) | $90 - 150$ | $< (1.2\% - 5.1\%)$ |
| $H^+ \rightarrow \tau^+ \nu$ (ATLAS) | $90 - 160$ | $< (0.8\% - 3.4\%)$ |
| $H^+ \rightarrow \tau^+ \nu$ (CMS) | $80 - 160$ | $< (1.9\% - 4.1\%)$ |
| $H^+ \rightarrow c\bar{s}$ (CMS) | $90 - 160$ | $< (1.7\% - 7.0\%)$ |

TABLE VI: Constraints on the $tbH^+$ vertex coupling $|\xi_{tb}|$ for some typical mass of the charged Higgs boson.

| Mass(GeV) | 100 | 120 | 150 |
|-----------|-----|-----|-----|
| CMS($\tau\nu$) | $0.17/\sqrt{Br(\tau\nu)}$ | $0.20/\sqrt{Br(\tau\nu)}$ | $0.38/\sqrt{Br(\tau\nu)}$ |
| CMS($c\bar{s}$) | $0.15/\sqrt{Br(c\bar{s})}$ | $0.16/\sqrt{Br(c\bar{s})}$ | $0.43/\sqrt{Br(c\bar{s})}$ |
| ATLAS($\tau\nu$) | $0.16/\sqrt{Br(\tau\nu)}$ | $0.12/\sqrt{Br(\tau\nu)}$ | $0.25/\sqrt{Br(\tau\nu)}$ |
| ATLAS($c\bar{s}$) | $0.17/\sqrt{Br(c\bar{s})}$ | $0.16/\sqrt{Br(c\bar{s})}$ | $0.27/\sqrt{Br(c\bar{s})}$ |

Direct searches for $t \rightarrow bH^+ \rightarrow b\tau^+ \nu (c\bar{s})$ at ATLAS [64] for $90\text{GeV} < m_{H^\pm} < 160(150)\text{GeV}$ and at CMS [65] for $80\text{GeV} < m_{H^\pm} < 160\text{GeV}$ gave the results in Table V. These results lead to the upper limits region on $|\xi_{tb}|$ at 95\%C.L. as

$$|\xi_{tb}| < \begin{cases} 
(0.15 - 0.59)/\sqrt{Br(\tau\nu)}, & \text{(CMS, 80GeV < m}_{H^\pm}< 160\text{GeV)}; \\
(0.15 - 1.12)/\sqrt{Br(c\bar{s})}, & \text{(CMS, 90GeV < m}_{H^\pm}< 160\text{GeV)}; \\
(0.13 - 0.45)/\sqrt{Br(\tau\nu)}, & \text{(ATLAS, 90GeV < m}_{H^\pm}< 160\text{GeV)}; \\
(0.19 - 0.27)/\sqrt{Br(c\bar{s})}, & \text{(ATLAS, 90GeV < m}_{H^\pm}< 150\text{GeV)}.
\end{cases}$$

(84)

For some typical mass of charged Higgs boson (which are allowed for some cases in the S-T ellipse tests) we have the upper limits of $|\xi_{tb}|$ in Table VI. From all the direct searches for top decays, we must have an relation

$$Br(t \rightarrow hc) + Br(t \rightarrow hu) + Br(t \rightarrow bH^+) < 7.4\%$$

(85)

according to [76] at 95\% C.L. as well.
IV. CONSTRAINTS FROM LOW ENERGY PHENOMENA

The Lee model we discussed in this paper contains additional sources of CP violation and tree-level FCNC interactions, therefore they will affect many kinds of low energy phenomena, especially for the CP violation observables and the FCNC processes. For the CP violation observables, we will focus on the constraints from the electric dipole moments (EDM) of electron and neutron [66]. For the constraints on FCNC interactions, we will focus on the mesonic measurements.

A. Constraints due to EDM and Strong CP Phase

Direct searches of the electric dipole moment (EDM) for electron(d_e) and neutron(d_n) are given as [34] [67]

\[ d_e = (-2.1 \pm 4.5) \times 10^{-29} e \cdot cm, \quad d_n = (0.2 \pm 1.7) \times 10^{-26} e \cdot cm \] (86)

which will constrain the corresponding CP-violated interactions.

The effective interaction for electron can be written as [66]

\[ \mathcal{L}_{\text{e, EDM}} = -\frac{ie}{2} d_e \bar{e} \sigma^{\mu\nu} \gamma^5 e F_{\mu\nu} \] (87)

where \( d_e \) is the EDM for electron. In our scenario, the dominant contribution to electron EDM should be due to the two-loop Barr-Zee type diagrams [68] involving the lightest scalar as follows

\[
\frac{d_e}{e} = \left( \frac{d_e}{e} \right)_{W^\pm} + \left( \frac{d_e}{e} \right)_t + \left( \frac{d_e}{e} \right)_{H^\pm} \\
= \frac{2\sqrt{2}\alpha_{em}G_F m_e}{(4\pi)^3} \left( -c_V\text{Im}(c_e)J_1(m_W,m_h) + \frac{8}{3}\text{Re}(c_e)\text{Im}(c_t)J_{1/2}(m_t,m_h) \right.
\]

\[ + \frac{8}{3}\text{Im}(c_e)\text{Re}(c_t)J_{1/2}(m_t,m_h) - c_\pm\text{Im}(c_e)J_0(m_{H^\pm},m_h) \right) \] (88)

in which the loop integration functions \( J_1 \) comes from the \( W \) loop, \( J_{1/2} \) comes from
the top loop and \( J_0 \) comes from the charged scalar loop. The analytical expressions are

\[
J_1(m_W, m_h) = -\frac{m_W^2}{m_h^2} \left( 5 - \frac{m_h^2}{2 m_W^2} \right) I_1(m_W, m_h) + \left( 3 + \frac{m_h^2}{2 m_W^2} \right) I_2(m_W, m_h); \tag{89}
\]

\[
J_{1/2}(m_t, m_h) = -\frac{m_t^2}{m_h^2} I_1(m_t, m_h); \tag{90}
\]

\[
J'_{1/2}(m_t, m_h) = -\frac{m_t^2}{m_h^2} I_2(m_t, m_h); \tag{91}
\]

\[
J_0(m_{H^\pm}, m_h) = -\frac{v^2}{2m_h^2} (I_1(m_{H^\pm}, m_h) - I_2(m_{H^\pm}, m_h)); \tag{92}
\]

where

\[
I_1(m_1, m_2) = \int_0^1 dz \frac{m_2^2}{m_1^2 - m_2^2 z (1-z)} \ln \left( \frac{m_2^2 z (1-z)}{m_1^2} \right); \tag{93}
\]

\[
I_2(m_1, m_2) = \int_0^1 dz \frac{m_2^2 (1 - 2z (1-z))}{m_1^2 - m_2^2 z (1-z)} \ln \left( \frac{m_2^2 z (1-z)}{m_1^2} \right). \tag{93}
\]

Numerically, the contribution from charged Higgs loop is usually small comparing with the \( W \) and top loop, especially for heavy charged Higgs. As a benchmark point, take \( m_{H^\pm} = 150 \text{GeV} \), we have

\[
d_e = [- (14.0 c_V + 1.28 c_\pm) \text{Im}(c_e) + 6.53 \text{Re}(c_t) \text{Im}(c_e)] \times 10^{-27} e \cdot \text{cm}. \tag{94}
\]

As benchmark points, take \( c_V = c_\pm = 0.5, |c_t| = 1 \). For both CMS and ATLAS data, small \( \alpha(< \pi/2) \) is favored. Take \( \alpha' = 1.2 \) around the best fit point thus \( \alpha_t \approx 1.0 \), the EDM data strongly constrains the coupling \( c_e \). For most \( \alpha_e \equiv \text{arg}(c_e), \) the coupling strength \( |c_e| \) is constrained to be as small as \( \mathcal{O}(10^{-2} - 10^{-1}) \). But for some special angles, as \( \alpha_e \approx -2.04 \) and \( \alpha_e \approx 1.09, |c_e| \) may be as large as \( \mathcal{O}(1) \). But the windows are very narrow, in Figure 9 we show the constraints close to the special angles.

If adding the contributions from heavy neutral Higgs, the constraints on \( c_e \) would be shifted. Since both heavy scalars are CP-even dominant, we can estimate that

\[
\text{arg}(c_{e, 2}) \simeq \text{arg}(c_{e, 3}) \simeq \text{arg}(c_{t, 2}) \simeq \text{arg}(c_{t, 3}) \sim \mathcal{O}(0.1) \tag{95}
\]

and for the two \( |c_{e, i}|, \) at least one of them is of \( \mathcal{O}(1) \) because of its mass; which is the same for \( |c_{t, i}|. \) For the couplings to gauge bosons, we can estimate

\[
c_2^2 + c_3^2 = 1 - c_1^2 \simeq 0.7 \tag{96}
\]
thus at least one of them must be large enough to be close to $O(1)$. For a neutral Higgs with mass $m_2 \sim 300$GeV or $m_2 \sim 700$GeV, the contributions can be estimated as

\[
d_{e,2} \simeq (1 \sim 5) \times 10^{-28} e \cdot cm;
\]

\[
d_{e,3} \simeq (0.5 \sim 3) \times 10^{-28} e \cdot cm.
\]

As an example, if the heavy scalars contribute a $d_e = 2 \times 10^{-28} e \cdot cm$, Figure 9 would be changed to Figure 10. It still imposes strict constraints on $c_e$ but the behaviors are different from that without including the contributions from the heavy scalars.
For neutron, the effective interaction can be written as \[ L_{n,\text{EDM}} = -\frac{i}{2} \sum_q (d_q \bar{q} \gamma^5 q F_{\mu \nu} + \bar{d}_q g_s \bar{q} \gamma^5 q G_{\mu \nu}^a) \]

\[ -\frac{w}{3} f^{abc} G_{\mu \nu}^a G_{\rho \sigma}^{b,c} + \frac{\theta \alpha_s}{8\pi} G_{\mu \nu} \tilde{G}^{\mu \nu}. \] (99)

The first two operators correspond to the EDM($d_q$) and color EDM($\bar{d}_q$) of light quarks; the third operator is the Weinberg operator; and the last operator, in which $\theta = \arg(\det(M_u \cdot M_d))$ is the strong CP phase. The EDM of neutron \[66, 70, 71\] is

\[ \frac{d_n}{e} \simeq 1.4 \left( \frac{d_d}{e} - 0.25 \frac{d_u}{e} \right) + 1.1 \left( \bar{d}_d + 0.5 \bar{d}_u \right) + (2.5 \times 10^{-16} \theta + 4.3 \times 10^{-16} w(\text{GeV}^{-2})) \text{cm} \] (100)

at the hadron scale with a theoretical uncertainty of about 50%. At weak scale the EDM and CEDM for quarks are given as \[70, 71\]

\[ \frac{d_q}{e} = \frac{2\sqrt{2} \alpha_{em} Q_q G_F m_q}{(4\pi)^3} \left( c_Y \text{Im}(c_q) J_1(m_W, m_h) + c_{\pm} \text{Im}(c_q) J_0(m_{H^\pm}, m_h) \right. \]

\[ -\frac{8}{3} \left( \text{Re}(c_q) \text{Im}(c_t) J_{1/2}(m_t, m_h) + \text{Im}(c_q) \text{Re}(c_t) J_{1/2}'(m_t, m_h) \right) \right); \] (101)

\[ \frac{\bar{d}_q}{e} = -\frac{2\sqrt{2} \alpha_s G_F m_q}{(4\pi)^3} \left( \text{Re}(c_q) \text{Im}(c_t) J_{1/2}(m_t, m_h) \right. \]

\[ +\text{Im}(c_q) \text{Re}(c_t) J_{1/2}'(m_t, m_h) \left. \right); \] (102)

and the Weinberg operator

\[ w = \frac{\sqrt{2} G_F g_s \alpha_s}{4 \cdot (4\pi)^3} \text{Re}(c_t) \text{Im}(c_t) g \left( \frac{m_t^2}{m_h^2} \right) \] (103)

with

\[ g(x) = 4x^2 \int_0^1 dv \int_0^1 du \frac{u^3 v^3(1-v)}{(x v (1-u v) + (1-u)(1-v))^2}. \] (104)

Following the appendix in \[70\], with the input $m_u = 2.3$MeV, $m_d = 4.8$MeV and $\alpha_s(m_t) = 0.11$ \[28\], numerically the EDM for neutron is

\[ d_n \simeq (0.5 \sim 1.5) \times \left( - (7.0 \text{Re}(c_u) \text{Im}(c_t) + 4.9 \text{Im}(c_u) \text{Re}(c_t)) \right. \]

\[ - (29 \text{Re}(c_d) \text{Im}(c_t) + 20 \text{Im}(c_d) \text{Re}(c_t)) \]

\[ - (2.8 c_Y + 0.25 c_{\pm}) \text{Im}(c_d) - (0.66 c_Y + 0.06 c_{\pm}) \text{Im}(c_u) \]

\[ + 2.5 \times 10^{10} \theta + 2.3 |c_t|^2 \sin(2\alpha_t') \right) \times 10^{-26} e \cdot \text{cm} \] (105)
FIG. 11: Plots on the allowed $\alpha_d - \alpha_u$. For the left figure, $\alpha_t = 0.7$ favored by CMS data and for the right one, $\alpha_t = 2.2$ favored by ATLAS data.

Take benchmark points as usual, and fix $c_V = c_\pm = 0.5$ and $|c_t| = 1$, $\alpha_t = 1.0$ as usual. For $|c_u| \simeq |c_d| \sim O(0.1)$, there is almost no constraints on $\alpha_u \equiv \arg(c_u)$ and $\alpha_d \equiv \arg(c_d)$. For $|c_u| \simeq |c_d| \sim O(1)$, constraints on $\alpha_d$ and $\alpha_u$ are shown in Figure 11. Ignoring the $\theta$ term, for $|c_u| = |c_d| = 1$, $\alpha_d$ is constrained in two bands with a width of $\Delta \alpha_d \simeq (0.2 \sim 1)$ from the uncertainties in calculating $d_n$. And the width are more sensitive to $c_d$, for example, if $|c_d| = 0.5$, $\Delta \alpha_d \simeq (0.5 \sim 2)$. The constraints by neutron EDM are less strict comparing with those by electron EDM in this model. Contributions from heavy neutral Higgs bosons and nonzero $\theta(\lesssim 10^{-10})$ would also change the location of the bands.

B. Meson Mixing and CP Violation

In SM the neutral mesons $K^0, D^0, B^0_d$ and $B^0_s$ mix with their corresponding anti-particles through weak interactions. Usually BSM will give additional contributions to the mixing matrix elements $\langle \bar{M}^0 | H_{\Delta F=2} | M^0 \rangle$ thus they will modify the mass splitting and mixing induced CP-violated observables. We can parameterize the new physics effects as [72]

$$M_{12,M} \equiv \frac{1}{2m_M} \langle \bar{M}^0 | H_{\Delta F=2} | M^0 \rangle = M_{12,M,SM}(1 + \Delta_M e^{i\delta_M}).$$  \hspace{1cm} (106)

For mass splitting, we list the world averaging results [28, 73, 74] and SM predictions [75, 77] for $\Delta m$ in Table VIII in section D. The useful decay constants and bag parameters are from
the lattice results [78]. Only for the $D^0 - \bar{D}^0$ system it is difficult to predict $\Delta m_D$ since the long-distance effects are the dominant contributions. Nonzero $\delta_M$ from new physics will modify the CP violating effects from those in the SM, thus it will be constrained by CP-violated observable, as $\epsilon_K$ in $K^0 - \bar{K}^0$ mixing and $\sin(2\beta_{d(s)})$ in $B^0_{d(s)} - \bar{B}^0_{d(s)}$ mixing et. al. They are defined as

$$\epsilon_K = \frac{1}{3} \left( \frac{\mathcal{M}(K_L \to 2\pi^0)}{\mathcal{M}(K_S \to 2\pi^0)} \right) + \frac{2}{3} \left( \frac{\mathcal{M}(K_L \to \pi^+\pi^-)}{\mathcal{M}(K_S \to \pi^+\pi^-)} \right)$$

(107)

where $K_{L(S)}$ is the long(short) lived neutral kaon and $\mathcal{M}$ is the amplitude for the process and

$$\beta = \arg \left( -\frac{V_{tb} V_{td}^*}{V_{cb} V_{cd}^*} \right), \quad \beta_s = \arg \left( \frac{V_{tb} V_{ts}^*}{V_{cb} V_{cs}^*} \right)$$

(108)

where $V_{ij}$ are CKM matrix elements.

First assuming the charged Higgs is heavy and considering the contribution only from the 126GeV Higgs Boson, we can write the flavor-changing effective interaction as

$$\mathcal{L}_{ij} = \bar{f}_i (\xi_{1ij} + \xi_{2ij} \gamma^5) f_j h + \text{h.c.}$$

(109)

The lightest neutral Higgs boson contribution to matrix elements for meson mixing is [79] [80]

$$M_{12,M,SM} \Delta_M e^{i\delta_M} = \frac{f_M^2 B_M m_M}{6m_h^2} \left( \xi_{1ij}^2 - \xi_{2ij}^2 + \frac{(\xi_{11ij}^2 - 11\xi_{22ij}^2)m_M^2}{(m_i + m_j)^2} \right).$$

(110)

The parameters $f_M$, $B_M$ and $m_M$ are the decay constant, bag parameter and mass for meson $M^0$, and $m_{i(j)}$ are masses for the quark $f_{i(j)}$. For $B^0_{d(s)} - \bar{B}^0_{d(s)}$ mixing, according to fitting results [81] (see the plots in [82] for details), for different $\delta_{B_d(B_s)}$,

$$\Delta_{B_d} \lesssim (0.1 \sim 0.4) \quad \text{and} \quad \Delta_{B_s} \lesssim (0.1 \sim 0.3).$$

(111)

For $\delta_{B_d(B_s)} = 0$, the upper limit on $\Delta_{B_d(B_s)}$ is about 0.2. Comparing with (106), (110) and adopting the Cheng-Sher ansatz [43], the typical upper limit on $\xi_{bs(bd)}$ have the order

$$\frac{|\xi_{bs}|v}{\sqrt{2m_b m_s}} \lesssim 2 \times 10^{-2} \quad \text{and} \quad \frac{|\xi_{bd}|v}{\sqrt{2m_b m_d}} \lesssim 6 \times 10^{-2}$$

(112)

both of $\mathcal{O}(10^{-2} \sim 10^{-1})$. For $D^0 - \bar{D}^0$ mixing, we have the upper limit

$$\frac{|\xi_{cu}|v}{\sqrt{2m_c m_u}} \lesssim 0.1$$

(113)

For $K^0 - \bar{K}^0$ mixing, when $\delta_K \approx 0$ or $\pi$, we have $\Delta_K \lesssim 0.25$ which leads to

$$\frac{|\xi_{sd}|v}{\sqrt{2m_s m_d}} \lesssim 2 \times 10^{-2}.$$  

(114)
While for a general $\delta_K$, $\Delta_K$ is strongly constrained to be less than $\mathcal{O}(10^{-3})$ because of the smallness of $\epsilon_K$. New CP-violated effects must be very small in neutral $K$ system while they are allowed or even favored \[81\] for other meson.

Next, consider the contribution to $B_{d(s)}^0 - \bar{B}_{d(s)}^0$ mixing from charged Higgs boson. Box diagrams with one or two charged Higgs boson instead of $W$ boson will contribute to $\Delta_{B_d(B_s)} \exp(i \delta_{B_d(B_s)})$ as \[83\] \[84\]

$$
\Delta_{B_d(B_s)} e^{i \delta_{B_d(B_s)}} = \frac{\mathcal{F}_1(x_{tW}, x_{tH}, x_{HW}) + \mathcal{F}_2(x_{tH})}{\mathcal{F}_0(x_{tW})} \tag{115}
$$

where

$$
\mathcal{F}_0(x_{tW}) = 1 + \frac{9}{1 - x_{tW}} - \frac{6}{(1 - x_{tW})^2} - \frac{6x_{tW}^2 \ln x_{tW}}{(1 - x_{tW})^3} \tag{116}
$$

$$
\mathcal{F}_1(x_{tW}, x_{tH}, x_{HW}) = \eta_{d(s)}^2 \frac{x_{tH}}{1 - x_{HW}} \left( \frac{8 - 2x_{tW}}{1 - x_{tH}} + \frac{(2x_{HW} - 8) \ln x_{tH}}{(1 - x_{tH})^2} + \frac{6x_{HW} \ln x_{tW}}{(1 - x_{tW})^2} \right) \tag{117}
$$

$$
\mathcal{F}_2(x_{tH}) = \eta_{d(s)}^4 (1 - x_{tH}^2 + 2x_{tH} \ln x_{tH}) \tag{118}
$$

at leading order in which $\eta_{d(s)} \approx (\xi_{1d} \xi_{1s}/2V_{tb}V_{ts}^*)^{1/2} v/m_t$ and $x_{ij} = (m_i/m_j)^2$. We can parameterize the interactions \[80\] as

$$
\mathcal{L}_{1,4D,H^+} = - \frac{V_{td}}{v} i (X_t m_t P_L + X_{D_1} m_{D_1} P_R) D_i + h.c. \tag{119}
$$

in which $P_{L(R)} = (1 \mp \gamma^5)/2$. Thus $\eta_{d(s)} \approx |X_t| v/\sqrt{2} m_t$ and it is not sensitive to $X_{D_1}$ if they are of the same order as $X_t$. According to the constraints in section III E for light charged Higgs $m_{H^\pm} < m_t$, with a typical coupling $|X_t| \lesssim 0.5$, $\Delta_{B_d(B_s)} \lesssim 0.2$ holds for $m_{H^\pm} \geq 100\text{GeV}$ and additional CP-violated effects induced by charged Higgs mediated loop are negligible. Thus take a benchmark point $m_{H^\pm} = 150\text{GeV}$ as usual, it is allowed by $B$ meson mixing data. While for heavy charged Higgs $m_{H^\pm} > m_t$, the coupling $X_t$ is not constrained by $t \to bH^+ \ell^-$ decay process. We can give an upper limit $|X_t| \lesssim (0.6 \sim 1)$ when $200\text{GeV} < m_{H^\pm} < 600\text{GeV}$.

In the $D^0 - \bar{D}^0$ mixing, another useful constraint comes from the neutral Higgs mediated box diagram. Its contribution to $\Delta m_D$ is \[85\]

$$
\Delta m_D^* = \frac{G_F^2 v^4 |\xi_{tu} \xi_{tc}|^2}{12 \pi^2 m_t^2} f_D m_D B_{Br} \mathcal{F}_2(x_{th}) \approx 4 \times 10^{-9} |\xi_{tu} \xi_{tc}|^2 \tag{120}
$$
where \( r = (\alpha_s(m_t)/\alpha_s(m_b))^{6/23}(\alpha_s(m_b)/\alpha_s(m_c))^{6/25} \approx 0.8 \) and loop function \( F_2 \) is the same as that in (118). For \( \Delta m_D^* \) contributes less than the order of measured \( \Delta m_D \), we have \(|\xi_{tu}\xi_{tc}| \lesssim 1.5 \times 10^{-3}\) and hence we can put a stronger constraint than (83) on the flavor changing interactions including top as
\[
\frac{|\xi_{tu}\xi_{tc}|v^2}{2m_t\sqrt{m_u m_c}} \lesssim 5 \quad \text{which is of } \mathcal{O}(1).
\]

\[\text{C. The B Leptonic Decays}\]

The rare decay process \( B_{s,d} \to \mu^+\mu^- \) has been measured by LHCb [86] and CMS [87] Collaborations respectively with the results
\[
\overline{Br}(B_s \to \mu^+\mu^-) = \begin{cases} 2.9^{+1.1}_{-1.0} \times 10^{-9}, & \text{(LHCb), 4.0σ significance),} \\ 3.0^{+1.0}_{-0.9} \times 10^{-9}, & \text{(CMS), 4.3σ significance)} \end{cases},
\]
and
\[
\overline{Br}(B_d \to \mu^+\mu^-) = \begin{cases} 3.7^{+2.5}_{-2.1} \times 10^{-10}, & \text{(LHCb),} \\ 3.5^{+2.5}_{-1.5} \times 10^{-10}, & \text{(CMS)} \end{cases}.
\]
A combination result is \( \overline{Br}(B_d \to \mu^+\mu^-) = (2.9 \pm 0.7) \times 10^{-9} \) by CMS and LHCb Collaborations[88]. There is no evidence for the process \( B_d \to \mu^+\mu^- \). The results correspond to the SM prediction[89] (and updated results[90] in 2014)
\[
\overline{Br}(B_s \to \mu^+\mu^-)_{SM} = (3.65 \pm 0.23) \times 10^{-9},
\]
\[
\overline{Br}(B_d \to \mu^+\mu^-)_{SM} = (1.06 \pm 0.09) \times 10^{-10}.
\]
Where the modified branching ratio \( \overline{Br} \) means the averaged time-integrated branching ratio and it has the relation with the branching ratio \( Br \) as[91][92]
\[
Br(B_s \to \mu^+\mu^-) = \overline{Br}(B_s \to \mu^+\mu^-) \left(1 + \mathcal{O}\left(\frac{\Delta \Gamma}{\Gamma}\right)\right).
\]
See section D for details.

Consider the neutral Higgs mediated flavor changing process first. Using the constraints in (112), we can estimate the contributions to \( Br(B_{s(d)} \to \mu^+\mu^-) \) as
\[
\delta Br(B_s \to \mu^+\mu^-) = \frac{m_{B_s}|c_\mu|^2}{8\pi \Gamma_{B_s,tot}} \left( \frac{f_{B_s} m_{B_s}^2 m_\mu}{(m_b + m_s) v m_h^2} \right)^2 \lesssim 4 \times 10^{-12} |c_\mu|^2; \quad (127)
\]
\[
\delta Br(B_d \to \mu^+\mu^-) = \frac{m_{B_d}|c_\mu|^2}{8\pi \Gamma_{B_d,tot}} \left( \frac{f_{B_d} m_{B_d}^2 m_\mu}{(m_b + m_d) v m_h^2} \right)^2 \lesssim 1 \times 10^{-12} |c_\mu|^2. \quad (128)
\]
We cannot get stronger constraints through these processes on $|c_\mu|$ than direct search\cite{93} which gives $|c_\mu| \lesssim 7$.

Next consider the charged Higgs contribution. For $|X_t| \sim |X_{b,s,\mu}| \sim \mathcal{O}(1)$, the charged Higgs loop is sensitive to $X_t$ and $m_{H^\pm}$ only\cite{94}. According to \cite{94}, it is estimated that

$$\frac{\delta Br(B_s \to \mu^+\mu^-)}{Br(B_s \to \mu^+\mu^-)} \approx \left(1 - \frac{|X_t^2| Y_{2\text{HDM}}}{\eta Y_{\text{SM}}} \right)^2$$

(129)

where $\eta = 0.987$ is the electro-weak and QCD correction factor and

$$Y_{\text{SM}} = \frac{x_{tW}}{8} \left( \frac{x_{tW} - 4}{x_{tW} - 1} + \frac{3x_{tW}}{(x_{tW} - 1)^2} \ln x_{tW} \right);$$

(130)

$$Y_{2\text{HDM}} = \frac{x_{tW}^2}{8} \left( \frac{1}{x_{HW} - x_{tW}} + \frac{x_{HW}}{(x_{HW} - x_{tW})^2} \ln \left( \frac{x_{tW}}{x_{HW}} \right) \right).$$

(131)

If the charged Higgs is light ($m_{H^\pm} < m_t$), $|X_t| = 0.5$ is allowed at 95\%C.L. While for a heavy charged Higgs, when $200\text{GeV} < m_{H^\pm} < 600\text{GeV}$, we have the 95\%C.L. upper limit on $|X_t|$ as $|X_t| \lesssim (0.6 \sim 1.1)$ with the combined experimental results or $|X_t| \lesssim (0.8 \sim 1.4)$ with single experimental result.

D. The B Radiative Decays

The inclusive radiative decays branching ratio of $\bar{B}$ meson $\bar{B} \to X_s \gamma$ (or we say $b \to s\gamma$ at parton level) has the averaged value \cite{73}

$$Br(\bar{B} \to X_s \gamma) = (3.43 \pm 0.22) \times 10^{-4}$$

(132)

with the photon energy $E_\gamma > 1.6\text{GeV}$. The SM prediction for that value is $(3.15\pm0.23)\times10^{-4}$ to $\mathcal{O}(\alpha_s^2)$ \cite{56}. In a 2HDM, the dominant contribution to modify this decay rate is from a loop containing a charged Higgs instead of the $W$ boson in SM. The neutral Higgs loop contribution is negligible because of the suppression in $\xi_{bs}$ and $m_{b(s)}/v$.

The charged Higgs loop is sensitive to both $X_t$ and $X_b$ that we should take some benchmark points. Define $\alpha_{bt} \equiv \arg(X_b/X_t)$, for a light charged Higgs boson, take $|X_t| = 0.5$ and $m_{H^\pm} = 150\text{GeV}$ as before; while for a heavy charged Higgs boson, take $|X_t| = 0.8$ and $m_{H^\pm} = 500\text{GeV}$. We show the allowed region for $\alpha_{bt} - |X_b|$ in Figure 12 utilizing the calculations in \cite{56}. From the figures, we can see that for most $\alpha_{bt}$ the coupling $|X_b|$ is constrained to be $\lesssim \mathcal{O}(1)$; while for some angles it can be larger$^9$.

$^9$ That’s because with merely the decay rate, we can only determine the absolute value for the $b \to s\gamma$
FIG. 12: Plots on allowed $\alpha_{bt} - |X_b|$. For the left figure, $|X_t| = 0.5$ and $m_{H^\pm} = 150\text{GeV}$; for the right figure, $|X_t| = 0.8$ and $m_{H^\pm} = 500\text{GeV}$.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure12}
\caption{Plots on allowed $\alpha_{bt} - |X_b|$.}
\end{figure}

V. FEATURES OF LEE MODEL AND ITS FUTURE PERSPECTIVES

One of the main goals of this paper is the thorough phenomenological studies on the Lee model with spontaneous CP-violation [26]. We can see from last two sections that Lee model is still viable confronting the high and low energy experiments. The next natural question is how to confirm/exclude this model at future facilities.

In the scalar sector there are nine free parameters $\mu^2_1, \mu^2_2$, and $\lambda_{1,2,...,7}$, corresponding to nine observables:

- Four masses $m_h, m_2, m_3$ and $m_{H^\pm}$;
- VEVs and a physical phase $v_1, v_2, \xi$ (or equivalently $v, \tan \beta, \xi$);
- Two neutral scalar mixing angles, equivalently we choose the ratios $c_1$ and $c_2$ of the couplings to gauge boson compared to the corresponding ones in the SM.

We treat the discovered scalar with mass 126GeV as the lightest neutral Higgs boson. If Lee model is true, the extra neutral and charged Higgs bosons should be discovered at high energy colliders. As the general rules, the lighter the extra Higgs bosons, the easier they can

amplitude. The largest allowed $X_b$ can reach 14 for the left figure and 28 for the right figure, in which case the new physics contributes twice as large as the SM but with the opposite sign.
be produced. In order to confirm the Lee model, another possible signal can be the FCNC
decay of the neural Higgs bosons which are unobservable small in the SM. Furthermore the
CP properties of the Higgs boson are essential measurements, though it is a very challenging
task.

As we have pointed out that there is no SM limit in this scenario, thus it is always
testable at the future colliders, such as LHC with $\sqrt{s} = 14$TeV, CEPC, ILC, or TLEP with
$\sqrt{s} = (240 \sim 250)$TeV, even before the discovery of other neutral Higgs bosons and charged
Higgs boson. The coupling between the lightest Higgs boson and other particle (especially
for massive gauge bosons $W^\pm$ and $Z^0$) are usually suppressed by the factor of $O(t_\beta s_\xi)$. In
the $b\bar{b}$ decay channel or any VBF, VH production channel, a significant suppression can be
the first sign of this scenario. On the contrary if the signals become even more SM-like, this
scenario will be disfavored.

For future LHC with $\sqrt{s} = 14$TeV, the signal strengths will be measured with an uncer-
tainty of about 10% at the luminosity $300\text{fb}^{-1}$\cite{97,98}. Perform the same $\chi^2$ fit as in (70),
and add the $b\bar{b}$ decay mode in. The value of $\chi^2$ is sensitive to $c_V$ and $c_b$, and the magnitude
of $c_V$ is a criterion for this model. A Higgs boson with $c_V \gtrsim (0.6 \sim 0.7)$ is hardly to be
pseudoscalar dominant thus if $c_V \lesssim (0.6 \sim 0.7)$ is excluded, we can say this scenario is
excluded. So we can test this scenario by fitting the signal strengths. We list the estimating
results in Table VII

| TABLE VII: Abilities to test the scenario at $\sqrt{s} = 14$TeV LHC. Lower limit for the allowed $c_V$ at
| $2\sigma$ and $3\sigma$ level are listed in the tables. For the left/right tables we assume all signal strengths are
| consist with SM at $1\sigma/2\sigma$ level respectively. |
|---|---|---|---|---|
| Excluded level | $2\sigma$ | $3\sigma$ | Excluded level | $2\sigma$ | $3\sigma$
| $300\text{fb}^{-1}$ | 0.62 | 0.55 | $300\text{fb}^{-1}$ | 0.53 | 0.45
| $3000\text{fb}^{-1}$ | 0.77 | 0.72 | $3000\text{fb}^{-1}$ | 0.7 | 0.65

If all signal strengths and the overall $\chi^2$ are consist with SM at $1\sigma$ level, For the integrated
luminosity $300\text{fb}^{-1}$, all $c_V \lesssim 0.62$ can be excluded at 95\%C.L.$(2\sigma)$ while all $c_V \lesssim 0.55$ can
be excluded at 99.7\%C.L.$(3\sigma)$; For the integrated luminosity $3000\text{fb}^{-1}$, all $c_V \lesssim 0.77$ can
be excluded at 95\%C.L.$(2\sigma)$ while all $c_V \lesssim 0.72$ can be excluded at 99.7\%C.L.$(3\sigma)$. If all
signal strengths and the overall $\chi^2$ are consistent with SM at $2\sigma$ level, For the integrated
luminosity $300\text{fb}^{-1}$, all $c_V \lesssim 0.53$ can be excluded at 95\%C.L.($2\sigma$) while all $c_V \lesssim 0.45$ can be excluded at 99.7\%C.L.($3\sigma$); For the integrated luminosity $3000\text{fb}^{-1}$, all $c_V \lesssim 0.7$ can be excluded at 95\%C.L.($2\sigma$) while all $c_V \lesssim 0.65$ can be excluded at 99.7\%C.L.($3\sigma$). All the results are for the largest parameter space in this scenario because the true ability to test this scenario by $\chi^2$ depends strongly on the real signal strengths from future experiments.

Another useful observable is $f_{a_3}$ defined in\cite{2}. For $\sqrt{s} = 14\text{TeV}$, the 95\%C.L. upper limit on $f_{a_3}$ will reach about 0.14(0.04) for the luminosity $300(3000)\text{fb}^{-1}$\cite{97, 98} which leads to the constrains $|a_3/a_1| < 1.0(0.5) \sim \mathcal{O}(1)$ separately. For $|c_t| \sim \mathcal{O}(1)$, it is still too large to give direct constrains on $\alpha_t \equiv \text{arg}(c_t)$.

At a Higgs factory with the $e^+e^-$ initial state at $\sqrt{s} = (240 \sim 250)\text{GeV}$, the dominant production process for a Higgs boson is associated with a $Z^0$ boson. Another important production process is through VBF. In this scenario it is suppressed by a factor $c_1^2$ thus we can exclude this scenario if the total cross section favors SM. For the total cross section, a measurement with $\mathcal{O}(10\%)$ uncertainty is accurate enough to distinguish the scenario we discussed in this paper and SM at 3$\sigma$ or even 5$\sigma$ significance. Such accuracy can be achieved at CEPC/ILC/TLEP. At $\sqrt{s} = 240\text{GeV}$ TLEP, the total cross section can be measured with an uncertainty $0.4\%$ for the integrated luminosity $500\text{fb}^{-1}$\cite{99, 100}, while that value is about $3\%$ for the integrated luminosity $250\text{fb}^{-1}$ ILC at $\sqrt{s} = 500\text{GeV}$\cite{101}.

VI. CONCLUSIONS AND DISCUSSIONS

In this paper we proposed a scenario in which the smallness of CP-violation and the lightness of Higgs boson are correlated through small $t_\beta s_\xi$, based on the Lee model, namely the spontaneous CP-violated 2HDM. The basic assumption is that CP, which spontaneously broken by the complex vacuum, is an approximate symmetry. We found that $m_h$ as well as the quantities $K$ and $J$ are $\propto t_\beta s_\xi$ in the limit $t_\beta s_\xi \to 0$. Here $K$ and $J$ are the measures for CP-violation effects in scalar and Yukawa sectors respectively. It is a new way to understand why the Higgs boson discovered at LHC is light. In this scenario, all the three neutral physical degrees of freedom mix with each other thus none of them is a CP eigenstate.

\footnote{Almost the same for CMS and ATLAS detector, with $300(3000)\text{fb}^{-1}$ luminosity, the upper limit can reach 0.15(0.037) for ATLAS and 0.13(0.04) for CMS, see details in the references.}
We then investigated the phenomenological constraints from both high energy and low energy experiments and found the scenario still alive. The lightest Higgs boson usually couples with SM gauge and fermion particles with a smaller strength than in the SM, thus the total width must be narrower than that in SM. Such choice of the parameters makes Lee model still allowed by the CMS or ATLAS data. The LHC search for heavy neutral bosons implies the masses of other two neutral bosons should be away from the region $300 - 700 \text{GeV}$. The S-T ellipse also strictly constrains the mass relation between the charged and neutral bosons as can be seen in Figure 1 - Figure 3. We also fitted the CMS and ATLAS data respectively, for example, see Figure 4 - Figure 8. We found that this scenario is still allowed for either data. It does not sensitive to the charged Higgs contribution. After considering all the data, a light charged Higgs with the mass about 100 GeV is still allowed. Small $h\bar{b}b$ vertex is favored for both CMS and ATLAS data. The minimal $\chi^2$ is close to the $\chi^2$ in SM, thus we cannot conclude that SM is better than Lee model.

We forbid the explicit CP-violation in the whole lagrangian including the Yukawa sector, thus we must tolerate the tree-level FCNC. The flavor-changed couplings including top quark are constrained by same sign top production process and the top quark rare decay, besides the constraints by B physics processes. The tree-level FCNC vertices including five light quarks are strongly constrained to be less than $O(10^{-2} \sim 10^{-1}) \sqrt{2m_im_j/v}$ while for the vertices including top quark it should be less than $O(1) \sqrt{2m_tm_q/v}$. The coupling $X_t$ for $tbH^+$ vertex are constrained to be less than $O(0.1 \sim 1)$ for different $m_{H^\pm}$, while $X_b \sim O(1)$ are usually allowed by $b \to s\gamma$ data.

The constraints by EDMs are usually very important in discussing a CP-violated model, because new sources of CP-violation may modify the theoretical prediction of EDMs from the SM by several orders of magnitude, and maybe testable by the experiments now. The EDM for electron gave very strict constraints on the $h\bar{e}e$ vertex as shown in Figure 9 - Figure 10. While the EDM for neutron gave weaker constraints on $h\bar{d}d$ and $h\bar{u}u$ vertices, see Figure 11.

There is no SM limit for the lightest Higgs boson in this scenario, thus it is testable at future colliders. At $\sqrt{s} = 14 \text{TeV}$, besides discovering the extra neutral and charged Higgs bosons, the ability to test this scenario depends on how far the signal strengths for the 126 GeV Higgs boson differ from the SM predictions, as listed in Table VII. From the discovery point of view, if any suppression in the VBF, VH production channel or $b\bar{b}$ decay channel are confirmed, this scenario would be favored. On the contrary, if all signal are SM-like
more and more at future colliders, this scenario would be disfavored by data. For most cases \(300\text{fb}^{-1}\) luminosity is not enough to exclude this scenario, while \(3000\text{fb}^{-1}\) luminosity is better. At \(\sqrt{s} = (240 \sim 250)\text{GeV}\) \(e^+e^-\) colliders, several fb\(^{-1}\) luminosity is enough to distinguish this scenario and SM at \((3 \sim 5)\sigma\) level by accurately measuring the total cross section. We emphasize that measuring the CP properties and the flavor-changing decay of the Higgs bosons are essential to pin down Lee model.

We did not build the model for flavor sector in details thus we did not solve the natural FCNC and strong CP problems. It is possible to solve the FCNC and strong CP problems together, for example, see the model proposed by Liao [102]. We also did not discuss the constraints from flavor changing processes in lepton sector. As a CP-violated model, there may also some new CP-violation effects, especially in top, \(\tau\) and neutral \(D\) sector where no CP-violation has been discovered. We did not study the cosmological effects in this paper, like the domain wall and electro-weak baryogenesis in this model. All these consequences will be further scrutinized in the future.

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\[\begin{align*}
[1] & \text{F. Englert and R. Brout, Phys. Rev. Lett. 13, 321 (1964); P. W. Higgs, Phys. Rev. Lett. 13, 508 (1964); G. S. Guralnik, C. R. Hagen and T. W. B. Kibble, Phys. Rev. Lett. 13, 585 (1964)} \\
[2] & \text{M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973)} \\
[3] & \text{N. Cabbibo, Phys. Rev. Lett. 10, 531 (1963)} \\
[4] & \text{CMS Collaboration, Phys. Lett. B 716, 30 (2012), arXiv:1207.7235} \\
[5] & \text{ATLAS Collaboration, Phys. Lett. B 716, 1 (2012), arXiv:1207.7214} \\
[6] & \text{A. Djouadi, Phys. Rept. 457 (2008)} \\
[7] & \text{The LHC Higgs Cross Section Working Group, CERN-2013-004, arXiv:1307.1347}
\end{align*}\]
[8] CMS Collaboration, arXiv:1407.0558, CMS-PAS-HIG-14-009
[9] S. Chatrchyan et. al. (CMS Collaboration), Phy. Rev. D 89, 092007 (2014)
[10] P. Govoni (on behalf of the CMS Collaboration), https://indico.ific.uv.es/indico/getFile.py/access?contribId=258&sessionId=23&resId=0&materialId=slides&confId=2025
[11] CMS Collaboration, arXiv:1401.5041
[12] ATLAS Collaboration, Phys. Lett. B 726, 88 (2013), arXiv:1307.1427; ATLAS Collaboration, arXiv:1408.7084
[13] ATLAS Collaboration, arXiv:1408.5191
[14] ATLAS Collaboration, ATLAS-CONF-2013-030
[15] ATLAS Collaboration, ATLAS-CONF-2013-108
[16] CMS Collaboration, CMS-PAS-HIG-13-002, CMS-PAS-HIG-13-005
[17] ATLAS Collaboration, ATLAS-CONF-2013-013
[18] The GFitter Group, Eur. Phys. J. C72, 2205 (2012), arXiv:1209.2716
[19] M. E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65, 964 (1990); M. E. Peskin and T. Takeuchi, Phys. Rev. D 46, 381 (1992)
[20] Y. Gao et. al. Phys. Rev. D 81, 075022 (2010)
[21] Q.-H. Cao, C. B. Jackson, W.-Y. Keung, Ian Low, and Jing Shu, Phys. Rev. D. 81, 015010 (2010)
[22] A. Djouadi, Phys. Rept. 459 (2008)
[23] M. Perelstein, Prog. Part. Nucl. Phys. 58: 247 (2007), arXiv:hep-ph/0512128,
[24] A. Farzinnia, H. J. He, and J. Ren, Phys. Lett. B 727, 141 (2013), arXiv:1308.0295
[25] S. H. Zhu, arXiv:1211.2370; Y. Hu, Y. K. Wang, P. F. Yin and S. H. Zhu, Front. Phys. 8, 516 (2013).
[26] T. D. Lee, Phys. Rev. D 8, 1226 (1973)
[27] J.H. Christenson et. al., Phys. Rev. Lett. 13, 138 (1964)
[28] J. Beringer et. al. (Particle Data Group), Phys. Rev. D 86, 010001 (2012); K. A. Olive et al. (Particle Data Group), Chin. Phys. C 38, 090001 (2014 updated)
[29] L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983)
[30] C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985)
[31] J. E. Kim and G. Garosi, Rev. Mod. Phys. 82, 557 (2010)
[32] G. 't Hooft, Phys. Rev. Lett. 37, 8 (1976)
[33] V. Baluni, Phys. Rev. D 19, 2227 (1979)
[34] C. Baker et. al. Phys. Rev. Lett. 97, 131801 (2006)
[35] A.D. Sakharov, Pisma Zh. Eksp. Theor. Fiz. 5, 32 (1967); or JETP Lett. 5, 24 (1967)
[36] Planck Collaboration, arXiv:1303.5076
[37] D. E. Morrissey and M. J. Ramsey-Musolf, arXiv:1206.2942
[38] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Phys. Lett. B 263, 86 (1991); A. G. Cohen, D. B. Kaplan, and A. E. Nelson, Ann. Rev. Nucl. Part. Sci. 43, 27 (1993)
[39] S. L. Chen, N. G. Deshpande, X. G. He, J. Jiang, and L. H. Tsai, Eur. Phys. J. C 53, 607 (2008)
[40] G. Segrè and H. A. Weldon, Phy. Rev. Lett. 42, 1191 (1979)
[41] S. M. Barr, Phys. Rev. Lett. 53, 329 (1984)
[42] S. L. Glashow and S. Weinberg, Phys. Rev. D 15, 1958 (1977)
[43] T. P. Cheng and M. Sher, Phys. Rev. D 35, 3484 (1987)
[44] Y. B. Zel’dovich, I. Y. Kobzarev and L. B. Okun, Zh. Eksp. Teor. Fiz. 67, 3 (1974); or Sov. Phy. JEPT. 40, 1 (1975)
[45] T. W. B. Kibble, J. Phys. A: Math. Gen. 9 1387 (1976)
[46] L. M. Krauss and S.-J. Rey, Phys. Rev. Lett. 69, 1308 (1992)
[47] J. Shu and Y. Zhang, Phys. Rev. Lett. 111, 091801 (2013)
[48] M. F. Parry and A. T. Sornborger, Phys. Rev. D 60, 103504 (1999)
[49] G. C. Branco et. al. Phys. Rept. 516 (2012)
[50] A. Méndez and A. Pomaral, Phys. Lett. B 272, 313 (1991)
[51] CMS Collaboration, CMS-PAS-HIG-13-014
[52] ATLAS Collaboration, ATLAS-CONF-2013-052
[53] http://people.bridgewater.edu/~doneil/STellipseModule.nb
[54] W. Grimus, L. Lavoura, O. M. Ogreid, and P. Osland, J. Phys. G 35: 075001 (2008), arXiv:0711.4022; W. Grimus, L. Lavoura, O. M. Ogreid, and P. Osland, Nucl. Phys. B 801: 81 (2008), arXiv:0802.4353
[55] O. Deschamps et. al. Phys. Rev. D 82, 073012 (2010)
[56] T. Hermann, M. Misiak, and M. Steinhauser, JHEP 1211, 036 (2012); arXiv:1208.2788
[57] ALEPH, DELPHI, L3 and OPAL Collaborations (the LEP Higgs Working Group), LHWG
[58] CMS Collaboration, CMS-PAS-SUS-13-013

[59] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Eur. Phys. J. C63, 189 (2009), arXiv:0901.0002; see also this webpage http://mstwpdf.hepforge.org/

[60] CMS Collaboration, JHEP 02, 024 (2014), arXiv:1312.7582

[61] Michal Czakon, Paul Fiedler, and Alexander Mitov, Phys. Rev. Lett. 110 (2013) 252004, arXiv:1303.6254

[62] ATLAS Collaboration, arXiv:1403.6293

[63] CMS Collaboration, CMS-PAS-HIG-13-034

[64] ATLAS Collaboration, Eur. Phys. J. C, 73 6, 2465 (2013) arXiv:1302.3694; ATLAS Collaboration, JHEP 03, 076 (2013), arXiv:1212.3572

[65] CMS Collaboration, JHEP 07, 143 (2012), arXiv:1205.5736; CMS Collaboration, CMS PAS HIG-13-035

[66] M. Pospelov and A. Ritz, Ann. Phys. 318, 169 (2005)

[67] The ACME Collaboration, Science 343 6168, 269(2014), arXiv:1310.7534

[68] S. M. Barr and A. Zee, Phys. Rev. Lett. 65, 21 (1990) and Phy. Rev. Lett. 65, 2920 (1990, errata added)

[69] T. Abe, J. Hisano, T. Kitahara, and K. Tobioka, JHEP 1401, 106 (2014), arXiv:1311.4704

[70] J. Brod, U. Haisch and J. Zupan, JHEP 1311, 180 (2013), arXiv:1310.1385

[71] K. Cheung, J. S. Lee, E. Senaha and P.-Y. Tseng, CNU-HEP-14-02, arXiv:1403.4775

[72] A. H"ocker and Z. Ligeti, Ann. Rev. Nucl. Part. Sci. 56, 501 (2006)

[73] http://www.slac.stanford.edu/xorg/hfag/

[74] Heavy Flavor Averaging Group, arXiv:1207.1158

[75] J. Yu, PoS (LATTICE 2013) 398, arXiv:1312.0306

[76] A. Lenz and U. Nierste, TTP11-03, TUM-HEP-792/11, arXiv:1102.4274

[77] A. Lenz et. al. (CKMfitter Group), Phys. Rev. D 83, 036004 (2011)

[78] J. Laiho, E. Lunghi and R. S. Van de Water, Phy. Rev. D 81, 034503 (2010); see also this webpage http://www.latticeaverages.org/

[79] R. S. Gupta and J. D. Wells, Phys. Rev. D 81, 055012 (2010)

[80] B. McWilliams and O. Shanker, Phys. Rev. D 22, 2853 (1980)

[81] A. Lenz et. al. Phys. Rev. D 86, 033008 (2012)
Appendix A: Vacuum Stability Conditions

For the potential (10), when \( |\phi_i| \equiv \sqrt{\phi_i^\dagger \phi_i} \to \infty, V \geq 0 \) must hold to keep the vacuum stable. Write \( |\phi_1| = r_1, |\phi_2| = r_2 \) and \( \phi_1^\dagger \phi_2 = r \exp(i\alpha) \), we have

\[
|\phi_1^\dagger \phi_2| = r \leq |\phi_1| \cdot |\phi_2| = r_1 r_2.
\]  

(A.1)
The $R_{ij}$ and $I_{ij}$ can be expressed as

\begin{align*}
R_{11} &= r_1^2, \quad R_{22} = r_2^2, \quad R_{12} = r \cos \alpha, \quad I_{12} = r \sin \alpha. \tag{A.2}
\end{align*}

Thus we have the equation

\begin{align*}
V &= \lambda_1 r_1^4 + \lambda_3 r_1^2 r_2^2 + \lambda_6 r_2^4 \\
&\quad + r c_\alpha (\lambda_2 r_1^2 + \lambda_5 r_2^2) + r^2 (\lambda_4 c_\alpha^2 + \lambda_7 s_\alpha^2). \tag{A.3}
\end{align*}

holds for any $\alpha$ and $0 \leq r \leq r_1 r_2$.

Another type of conditions is that the potential should be minimized when

\begin{align*}
\langle \phi_1 \rangle &= \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad \langle \phi_2 \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 e^{i \xi} \end{pmatrix}. \tag{A.4}
\end{align*}

It equivalents to the conditions that the mass matrix for neutral Higgs, $\tilde{m}$ in [19], must be positive definite. We write the conditions as

\begin{align*}
\text{tr} \tilde{m} > 0, \quad (\text{tr} \tilde{m})^2 - \text{tr}(\tilde{m})^2 > 0, \quad \det \tilde{m} > 0. \tag{A.5}
\end{align*}

**Appendix B: Scalar Spectra and Small $t_\beta s_\xi$ Expansion**

In the unitary gauge the mass square matrix for charged scalars reads

\begin{align*}
M^2_{\pm} = -\frac{\lambda_7}{2} \begin{pmatrix} v_2^2 & -v_1 v_2 e^{-i \xi} \\ -v_1 v_2 e^{i \xi} & v_1^2 \end{pmatrix}. \tag{B.1}
\end{align*}

The eigenvalues are

\begin{align*}
m^2_{G^\pm} &= 0; \quad m^2_{H^\pm} = -\frac{\lambda_7 v^2}{2}; \tag{B.2}
\end{align*}

where the zero eigenvalue corresponds to the charged goldstones which will be eaten by the longitudinal part of W bosons. Diagonalize (B.1) by performing a rotation

\begin{align*}
\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} &= \begin{pmatrix} \cos \beta & e^{-i \xi} \sin \beta \\ -e^{i \xi} \sin \beta & \cos \beta \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \phi_2^+ \end{pmatrix}. \tag{B.3}
\end{align*}
For the neutral parts, in the basis \((I_1, I_2, R_1, R_2)^T\), for any angel \(\alpha\) to appear below, we have the mass square matrix \(M_0^2 = \frac{v^2}{2} m_{ij}\), where

\[
\begin{align*}
  m_{11} &= (\lambda_4 - \lambda_7) s_\beta^2 s_\xi^2; \\
  m_{12} &= \lambda_5 s_\beta^2 s_\xi^2; \\
  m_{13} &= (\lambda_2 c_\beta + (\lambda_4 - \lambda_7) s_\beta c_\xi) s_\beta s_\xi; \\
  m_{14} &= ((\lambda_4 - \lambda_7) c_\beta + \lambda_5 s_\beta c_\xi) s_\beta s_\xi; \\
  m_{22} &= 4\lambda_6 s_\beta^2 s_\xi^2; \\
  m_{23} &= 2((\lambda_3 + \lambda_7) c_\beta + \lambda_5 s_\beta c_\xi) s_\beta s_\xi; \\
  m_{24} &= (\lambda_5 c_\beta + 4\lambda_6 s_\beta c_\xi) s_\beta s_\xi; \\
  m_{33} &= 4\lambda_1 c_\beta^2 + 2\lambda_2 c_\beta s_\beta c_\xi + (\lambda_4 - \lambda_7) c_\xi^2 s_\beta^2; \\
  m_{34} &= \lambda_2 c_\beta^2 + (2\lambda_3 + \lambda_4 + \lambda_7) s_\beta c_\beta c_\xi + \lambda_5 s_\beta^2 c_\xi^2; \\
  m_{44} &= (\lambda_4 - \lambda_7) c_\beta^2 + 2\lambda_5 c_\beta s_\beta c_\xi + 4\lambda_6 s_\beta^2 c_\xi^2.
\end{align*}
\]

Perform the same rotation as \((B.3)\) between \(\phi_1\) and \(\phi_2\), which in the basis above can be written as

\[
R = R_1 R_2 = \begin{pmatrix}
  c_\beta & s_\beta & 0 & 0 \\
- s_\beta & c_\beta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
  1 & 0 & 0 & 0 \\
0 & c_\xi & 0 & -s_\xi \\
0 & 0 & 1 & 0 \\
0 & s_\xi & 0 & c_\xi
\end{pmatrix}
\]

(B.4)

we have

\[
\tilde{M}_0^2 = RM_0^2 R^{-1} = \frac{v^2}{2} \begin{pmatrix}
  0 \\
 0 & \tilde{m}
\end{pmatrix}_{3 \times 3}
\]

(B.5)

where the zero eigenvalue corresponds to the neutral Goldstone,

\[
G^0 = c_\beta I_1 + s_\beta c_\xi I_2 - s_\beta s_\xi R_2,
\]

(B.6)
which will be eaten by the longitudinal part of Z boson. The matrix elements for \( \tilde{m} \) in the basis \((-s_\beta I_1 + c_\beta c_\xi I_2 - c_\beta s_\xi R_2, R_1, s_\xi I_2 + c_\xi R_2)^T\) should be

\[
\begin{align*}
\tilde{m}_{11} &= (\lambda_4 - \lambda_7)s_\xi^2; \\
\tilde{m}_{12} &= -(\lambda_2 c_\beta + (\lambda_4 - \lambda_7)s_\beta c_\xi)s_\xi; \\
\tilde{m}_{13} &= -(\lambda_5 s_\beta + (\lambda_4 - \lambda_7)c_\beta c_\xi)s_\xi; \\
\tilde{m}_{22} &= 4\lambda_1 c_\beta^2 + 2\lambda_2 c_\beta s_\beta c_\xi + (\lambda_4 - \lambda_7)s_\beta^2 c_\xi^2; \\
\tilde{m}_{23} &= \lambda_2 c_\beta^2 c_\xi + (2\lambda_3 + \lambda_7) + (\lambda_4 - \lambda_7)c_\xi^2) s_\beta c_\beta + \lambda_5 s_\beta^2 c_\xi; \\
\tilde{m}_{33} &= (\lambda_4 - \lambda_7)c_\beta^2 c_\xi^2 + 2\lambda_5 s_\beta c_\beta c_\xi + 4\lambda_6 s_\beta^2. 
\end{align*}
\] (B.7)

We can expand \( \tilde{m} \) in powers of \( t_\beta s_\xi \) as follows,

\[
\tilde{m} = \tilde{m}_0 + (t_\beta s_\xi)\tilde{m}_1 + (t_\beta s_\xi)^2\tilde{m}_2 + \cdots \quad (B.8)
\]

In the basis \((-s_\beta I_1 + c_\beta c_\xi I_2 - c_\beta s_\xi R_2, R_1, s_\xi I_2 + c_\xi R_2)^T\) the matrix \( \tilde{m}_0 \) can be written as

\[
\tilde{m}_0 = \begin{pmatrix}
(\lambda_4 - \lambda_7)s_\xi^2 & -\lambda_2 s_\xi & -(\lambda_4 - \lambda_7)s_\xi c_\xi \\
-\lambda_2 s_\xi & 4\lambda_1 & \lambda_2 c_\xi \\
-(\lambda_4 - \lambda_7)s_\xi c_\xi & \lambda_2 c_\xi & (\lambda_4 - \lambda_7)c_\xi^2
\end{pmatrix}
\] (B.9)

Diagonalize it with a \( 3 \times 3 \) matrix

\[
r = r_1 r_2 = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_\theta & s_\theta \\
0 & -s_\theta & c_\theta
\end{pmatrix} \begin{pmatrix}
c_\xi & 0 & s_\xi \\
0 & 1 & 0 \\
-s_\xi & 0 & c_\xi
\end{pmatrix}
\] (B.10)

we have

\[
rr\tilde{m}_0 r^{-1} = \begin{pmatrix}
0 & (\tilde{m}_0)_{22} \\
(\tilde{m}_0)_{32} & (\tilde{m}_0)_{33}
\end{pmatrix},
\] (B.11)

in which

\[
(\tilde{m}_0)_{22(33)} = \frac{4\lambda_1 + \lambda_4 - \lambda_7}{2} \pm \left( \frac{4\lambda_1 - (\lambda_4 - \lambda_7)^2}{4} c_{2\theta} + \lambda_2 s_{2\theta} \right); \quad (B.12)
\]

\[
\theta = \frac{1}{2} \arctan \left( \frac{2\lambda_2}{4\lambda_1 - (\lambda_4 - \lambda_7)} \right). \quad (B.13)
\]

The two heavy scalars have their masses

\[
m^2_{2(3)} = \frac{v^2}{2}((\tilde{m}_0)_{22(33)} + \mathcal{O}(t_\beta s_\xi)). \quad (B.14)
\]
The new basis is then
\[
  r \begin{pmatrix} c_\xi I_2 - s_\xi R_2 \\ R_1 \\ s_\xi I_2 + c_\xi R_2 \end{pmatrix} = \begin{pmatrix} I_2 \\ c_\theta R_1 + s_\theta R_2 \\ -s_\theta R_1 + c_\theta R_2 \end{pmatrix}
\]
(B.15)
in which the useful matrix elements for \( \tilde{m} \) are
\[
  (\tilde{m}_1)_{11} = 0; \quad (B.16)
\]
\[
  (\tilde{m}_1)_{12} = (2(\lambda_3 + \lambda_7)c_\theta + \lambda_5 s_\theta); \quad (B.17)
\]
\[
  (\tilde{m}_1)_{13} = (\lambda_5 c_\theta - 2(\lambda_3 + \lambda_7)s_\theta); \quad (B.18)
\]
\[
  (\tilde{m}_2)_{11} = 4\lambda_6. \quad (B.19)
\]

Thus to the leading order of \( t_\beta s_\xi \), for the lightest scalar \( h \) we have
\[
  m_h^2 = \frac{v^2 t_\beta^2 s_\xi^2}{2} \left( \frac{(\tilde{m}_1)_{12}^2}{(\tilde{m}_0)_{22}} + \frac{(\tilde{m}_1)_{13}^2}{(\tilde{m}_0)_{33}} + (\tilde{m}_2)_{11} \right)
  = \frac{v^2 t_\beta^2 s_\xi^2}{2} \left[ 4(\lambda_3 + \lambda_7)^2 \left( \frac{c^2_\theta}{(\tilde{m}_0)_{22}} + \frac{s^2_\theta}{(\tilde{m}_0)_{33}} \right) + \lambda_5^2 \left( \frac{s^2_\theta}{(\tilde{m}_0)_{22}} \right. \right.
  \left. + \frac{c^2_\theta}{(\tilde{m}_0)_{33}} \right) - 2\lambda_5 (\lambda_3 + \lambda_7) s_\theta \left( \frac{1}{(\tilde{m}_0)_{22}} - \frac{1}{(\tilde{m}_0)_{33}} \right) + 4\lambda_6 \right];
\]
(B.20)
\[
  h = I_2 + t_\beta s_\xi \left( \frac{(\tilde{m}_1)_{12}}{(\tilde{m}_0)_{22}} (c_\theta R_1 + s_\theta R_2) + \frac{(\tilde{m}_1)_{13}}{(\tilde{m}_0)_{33}} (c_\theta R_2 - s_\theta R_1) - \frac{I_1}{t_\xi} \right)
  = I_2 + t_\beta s_\xi \left[ 2(\lambda_3 + \lambda_7) \left( \frac{c^2_\theta}{(\tilde{m}_0)_{22}} + \frac{s^2_\theta}{(\tilde{m}_0)_{33}} \right) + \lambda_5 s_\theta \frac{1}{2} \left( \frac{1}{(\tilde{m}_0)_{22}} \right. \right. \left. \left. - \frac{1}{(\tilde{m}_0)_{33}} \right) \right] R_1 + \left( \lambda_3 + \lambda_7 \right) s_\theta \left( \frac{1}{(\tilde{m}_0)_{22}} - \frac{1}{(\tilde{m}_0)_{33}} \right)
  + \lambda_5 \left( \frac{s^2_\theta}{(\tilde{m}_0)_{22}} + \frac{c^2_\theta}{(\tilde{m}_0)_{33}} \right) R_2 - \frac{I_1}{t_\xi}. \quad (B.21)
\]
Appendix C: Some Useful Feynman-Rules in this Model

From the lagrangian we have some useful coupling vertexes directly,

\[ \mathcal{L}_{hVV} = \left( \frac{2m_{W}^{2}}{v} W_{\mu}^{+} W_{\mu}^{-} + \frac{m_{Z}^{2}}{v} Z_{\mu}Z_{\mu} \right) \]
\[ \left( c_{\beta} R_{1} + s_{\beta} c_{\xi} R_{2} + s_{\beta} s_{\xi} I_{2} \right); \] \hspace{1cm} (C.1)

\[ \mathcal{L}_{hH^{+}H^{-}} = -v H^{+} H^{-} \left[ \frac{\lambda_{2} + \lambda_{5}}{2} s_{\beta} s_{\xi} I_{1} \right. \]
\[ \left. + \left( \lambda_{3} c_{\beta}^{2} + \left( -\lambda_{2} + \frac{\lambda_{5}}{2} \right) c_{\beta}^{2} s_{\beta} c_{\xi} \right) \right] \]
\[ + \left( 2\lambda_{1} - \lambda_{4} c_{\xi}^{2} - \lambda_{7} s_{\xi}^{2} \right) c_{\beta} s_{\beta} c_{\xi} + \frac{\lambda_{2}}{2} s_{\beta}^{3} c_{\xi} \right) R_{1} \]
\[ + \left( (2\lambda_{6} - \lambda_{7}) c_{\xi}^{2} - \lambda_{5} c_{\beta} s_{\beta} c_{\xi} + \lambda_{3} s_{\beta}^{2} \right) I_{2} \]
\[ + \left( \frac{\lambda_{5}}{2} c_{\beta}^{2} - (\lambda_{4} - 2\lambda_{6}) c_{\beta}^{2} s_{\beta} c_{\xi} \right) \]
\[ + \left( \frac{\lambda_{2}}{2} - \lambda_{5} c_{\xi}^{2} \right) c_{\beta} s_{\beta}^{2} + \lambda_{3} s_{\beta}^{3} c_{\xi} \right) R_{2} \] \hspace{1cm} (C.2)

\[ \mathcal{L}_{hDD} = -\frac{1}{\sqrt{2}} \bar{D}_{Li} (Y'_{1d}(R_{1} + iI_{1}) + Y'_{2d}(R_{2} + iI_{2}))_{ij}D_{Rj} + \text{h.c.}; \] \hspace{1cm} (C.3)

\[ \mathcal{L}_{hUU} = -\frac{1}{\sqrt{2}} \bar{U}_{Li} (Y'_{1u}(R_{1} - iI_{1}) + Y'_{2u}(R_{2} - iI_{2}))_{ij}U_{Rj} + \text{h.c.}; \] \hspace{1cm} (C.4)

\[ \mathcal{L}_{Ch} = -\frac{1}{\sqrt{2}} \bar{U}_{Li} (V_{CKM})_{ij} (-Y'_{1d} s_{\beta} e^{-i\xi} + Y'_{2d} c_{\beta})_{jk}D_{Rk}H^{+} \]
\[ -\frac{1}{\sqrt{2}} \bar{D}_{Li} (V_{CKM}^{\dagger})_{ij} (Y'_{1u} s_{\beta} e^{i\xi} - Y'_{2u} c_{\beta})_{ji}U_{Rk}H^{-} + \text{h.c.} \] \hspace{1cm} (C.5)

The \( Y' \) in Yukawa couplings means the couplings in the mass eigenstates. For neutral Higgs triple vertex, the Feynman rules are all from

\[ -i\lambda_{ijk} = -\frac{i\partial^{3}V}{\partial h_{i}\partial h_{j}\partial h_{k}} \] \hspace{1cm} (C.6)

Appendix D: Formalism for Neutral Meson

The for meson \( K^{0}, D^{0}, B^{0}_{d} \) and \( B^{0}_{s} \) can mix with their charged conjugate particles, through weak interaction in SM. We begin with the Schrödinger equation

\[ i \frac{\partial}{\partial t} \begin{pmatrix} |M_{0}\rangle \\ |\bar{M}_{0}\rangle \end{pmatrix} = \left( \mathbf{m} - \frac{i}{2} \mathbf{\Gamma} \right) \begin{pmatrix} |M_{0}\rangle \\ |\bar{M}_{0}\rangle \end{pmatrix} \] \hspace{1cm} (D.1)

where \( \mathbf{m} \) and \( \mathbf{\Gamma} \) are \( 2 \times 2 \) matrix. Write the hamiltonian as

\[ \mathcal{H} = \mathcal{H}_{0} + \mathcal{H}_{\Delta F=1} + \mathcal{H}_{\Delta F=2}, \] \hspace{1cm} (D.2)
we have the matrix elements

\[
\left( m - \frac{i}{2} \Gamma \right)_{ij} = m_M \delta_{ij} + \frac{1}{2m_M} \langle \psi_i | \mathcal{H}_{\Delta F=2} | \psi_j \rangle
\]

\[
+ \frac{1}{2m_M} \int d\Pi_f \frac{\langle \psi_i | \mathcal{H}_{\Delta F=1} | f \rangle \langle f | \mathcal{H}_{\Delta F=1} | \psi_j \rangle}{m_M - E(f) + i\epsilon}
\]

(D.3)

with the normalized condition \( \langle \psi_i | \psi_j \rangle = 2m_M \delta_{ij} \) where \( \psi_{i,j} = |M^0 \rangle \) or \( \bar{M}^0 \). The second and third terms come from short-distance and long-distance effects separately and according to (D.3)

\[
\Gamma_{ij} = \frac{1}{2m_M} \int d\Pi_f \langle \psi_i | \mathcal{H}_{\Delta F=1} | f \rangle \langle f | \mathcal{H}_{\Delta F=1} | \psi_j \rangle 2\pi \delta(E(f) - m_M)
\]

(D.4)

The solutions for the eigenvalues are

\[
m_{H(L)} = m_M \pm \text{Re} \left( \sqrt{\left( m_{12} - \frac{i}{2} \Gamma_{12} \right) \left( m_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \right); \quad \text{(D.5)}
\]

\[
\Gamma_{H(L)} = \Gamma \mp \text{Im} \left( \sqrt{\left( m_{12} - \frac{i}{2} \Gamma_{12} \right) \left( m_{12}^* - \frac{i}{2} \Gamma_{12}^* \right)} \right). \quad \text{(D.6)}
\]

The \( H(L) \) means the heavy(light) mass eigenstate

\[
|M_{H(L)}\rangle = p|M^0 \rangle \mp q|\bar{M}^0 \rangle \quad \text{(D.7)}
\]

where

\[
|p|^2 + |q|^2 = 1; \quad \text{and} \quad \left( \frac{p}{q} \right)^2 = \frac{m_{12} - i\Gamma_{12}/2}{m_{12}^* - i\Gamma_{12}^*/2}. \quad \text{(D.8)}
\]

The time-dependent solution

\[
\begin{pmatrix}
|M_0(t)\rangle \\
|\bar{M}_0(t)\rangle
\end{pmatrix} = \begin{pmatrix}
g_+(t) & -(q/p)g(t) \\
g_+(t) & -(p/q)g(t)
\end{pmatrix} \begin{pmatrix}
|M_0(0)\rangle \\
|\bar{M}_0(0)\rangle
\end{pmatrix}
\]

(D.9)

where

\[
g_{\pm}(t) = \frac{1}{2} \left( e^{-im_H t - \frac{\Gamma_H t}{2}} \pm e^{-im_L t - \frac{\Gamma_L t}{2}} \right). \quad \text{(D.10)}
\]

For \( \Gamma_{12} \sim m_{12} \) and \( m_{12} \) is almost real like \( K^0 \) system, \( \Delta m \approx 2\text{Re}m_{12} \); while for \( \Gamma_{12} \ll m_{12} \) like \( D^0_{d(s)} \) system, \( \Delta m \approx 2|m_{12}| \). All the measurements and SM predictions are listed here. It is difficult to estimate the long-distance effects which give the dominant contribution in \( D^0 \) system.
TABLE VIII: SM predictions and Experimental values for mass difference in meson mixing.

| Meson      | $\Delta m_{\text{exp}}$(GeV)                  | $\Delta m_{\text{SM}}$(GeV)          |
|------------|-----------------------------------------------|--------------------------------------|
| $K^0(d\bar{s})$ | $(3.474 \pm 0.006) \times 10^{-15}$                | $(3.30 \pm 0.34) \times 10^{-15}$       |
| $D^0(c\bar{u})$   | $(1.0 \pm 0.3) \times 10^{-14}$                  | $-$                                     |
| $B^0_d(d\bar{b})$ | $(3.33 \pm 0.03) \times 10^{-13}$                | $(3.3 \pm 0.4) \times 10^{-13}$       |
| $B^0_s(s\bar{b})$ | $(1.1663 \pm 0.0015) \times 10^{-11}$            | $(1.14 \pm 0.17) \times 10^{-11}$    |

For decay processes to CP eigenstate $f$, for example, $B^0 \to \mu^+\mu^-$, the direct observable is time integrated averaged branching ratio which has an relation

\[
\overline{Br}(M \to f) \equiv \frac{1}{2} \int_0^\infty (\Gamma(M(t) \to f) + \Gamma(\bar{M}(t) \to f))
\]

which leads to

\[
\overline{Br}(M \to f) = \frac{1 + A\Delta\Gamma/\Gamma}{1 - (\Delta\Gamma/\Gamma)^2} Br(M \to f)
\]

where $-1 \leq A \leq 1$ and in SM $A = 1$. 

