Synthesis of the CNC machine control algorithms optimized with respect to the contour error using the genetic algorithm

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Abstract. Traditionally control systems of CNC machines are built using the positional system structure, e.g. the uncoupled system (US) tuned on the symmetrical optimum. These systems have a relatively small contouring quality factor – the ratio of the contour speed to the contour error. To increase the quality factor of such systems, we proposed two new algorithms: the US tuned on the contour error optimum (US-C) and the neural coordinated control algorithm (NCCA). Both algorithms were tuned on a non-convex criterion using the genetic algorithm which allowed us to reduce the tuning time and acquire better optimal values of the criterion in comparison to the gradient descent method. The experiments were conducted using a mathematical model of a CNC machine based on real servo motors and lead screws. The results of the experiments showed that the usage of the proposed algorithms led to a significant increase in the quality factor, which has a positive effect on the overall productivity of the CNC machine. Furthermore, the NCCA use available resources better than the other tested algorithms which can be seen in the experiments with a lower initial contour error, a lower radius of the circle, and a higher contour speed. Besides, the US-C can maintain the contour speed better than other tested algorithms.

1. Introduction

CNC control systems traditionally built using the uncoupled system (US) structure regardless of the operations they conduct [1-6]. The US is a positional system [5-6] because it uses the positional error – the difference between the desired and the real positions of the end effector – or the axes reference errors to form the control signals. In positional systems we are not interested in the trajectory of the end effector – as long as it does not collide with an obstacle – what is important is the final position accuracy [5-6].

Another way of implementing a control system of a CNC machine is the contour system structure [5-6] otherwise known as the coupled structure [1-3], which uses the contour error – the minimal distance between the desired trajectory and the position of the end effector – to form the control vector. The relations of the contour error, the position error, and the reference errors are shown in figure 1.
In contour systems, on the contrary to positional systems, the accuracy of the contouring of the trajectory is crucial, since it determines the quality of the technical operations executed by such systems, like welding, cutting, milling and other [1-3, 5, 6]. Examples of systems with the coupled structure are the path precompensation method [7-9], methods based on passive velocity field control [10, 11], cross-coupled systems [12, 13] and others [14, 15]. Although the US can deal with pure positional operations – like drilling and riveting – well, operations which require movement along the desired trajectory are better handled by contour systems [1-6] for the following reasons: (1) the US minimizes the contour error indirectly through minimization of the positional error, whereas it is possible to have the contour error equal to zero with a none zero position error by coordinating the axes of the CNC machine [1-6]; (2) it is hard to set control priorities of the US – like the ratio between the contour error and the contour speed error – due to the small amount of tuning parameters [1-2].

The goal of the paper is to increase the quality factor of tracking the circular arc by a CNC machine. The goal is achieved by developing two new algorithms: the US optimized with respect to the contour error (US-C) and the neural coordinated control algorithm (NCCA) also optimized with respect to the contour error. Both algorithms are tuned using the genetic algorithm [16].

The idea of the first proposed algorithm is the US-C, although it is impossible to make the US a coupled system without changing its structure, it might be possible to increase its quality factor by adjusting the PID regulators parameters with the genetic algorithm in order to minimize the contour error. Moreover, the transition from the US to the US-C requires changing only the PID parameters and can be easily done on most modern CNC machines and manipulators programmatically, without changing its software and its hardware.

The NCCA [1] is the second proposed algorithm it is a variation of the path precompensation method (PPM) [3, 7-9] first introduced by Huan J. in 1982 [17]. At present, a variety of the modifications of the PPM were developed for a wide range of mechanisms including CNC machines [1, 9] and different robots [3, 7, 8]. However, most PPMs are different only in the regulator structure, it can have PD regulator, fuzzy logic regulator, PID regulator, and relay regulator. Whereas, the NCCA is the first modification of the PPM which uses the neural network regulator [18-21].

There are many approaches in mathematical optimization which can be used for tuning the neural network, like gradient methods and coordinate descent, in the previous paper [1] we already discussed the batch gradient descent method [22] which required a great amount of retuning for a non-convex score function in order to achieve a sufficient optimum. In this paper, we decided to tune the neural network using the genetic algorithm which can escape from less optimal minimums to more optimal minimums using mutation and crossing. It allows the algorithm to be tuned in fewer steps on a better optimum [21]. The genetic algorithm is not the only method that can be used for tuning non-convex

![Figure 1. The relations of the contour error, the position error, and the reference errors. $e_c$ – the contour error vector, $e_p$ – the position error vector, $e_x$, $e_y$ – the reference errors.](image)
functions, another method that can escape less optimal minimums is the stochastic gradient descent method, which is not discussed in this paper.

2. Algorithms synthesis
In this chapter, we develop the proposed control algorithms figure 2: the US-C with the uncoupled system structure and the NCCA with the coupled system structure.

The proposed algorithms are synthesized for the two-axial CNC machine model. Axes consist of DC or synchronous [23] motors and lead screws, figure 3.

The US-C is a positional system, figure 2a; it uses the positional error (eₚ) to form the control signal vector (r*) and has the uncoupled structure which is poorly suitable for contour tasks. However, by tuning the coefficients of the PID regulator on the contour error optimum it should be able to achieve a better quality factor than the classical US, which is optimized with respect to positional error.

The NCCA figure 2b is also tuned for optimum contour error. The main idea of the NCCA is to form the control signal as the sum of two components: the tangential speed (Vₜ) that sets the movement along the trajectory and the returning speed (Vₑ) which minimizes the contour error. The geometrical interpretation of this idea is shown in figure 4.
The NCCA structure of the neural network figure 5 is chosen to have two hidden layers with six neurons each. Since the optimization of the proposed algorithms is not the subject of this paper, the structure choice is done without sufficient reason, a better way to do that would be to use some hyperparameters optimization method, e.g. grid search.

The transition between the neural network layers is represented in equation (1), where \( x_i \) – the signals from the i-th layer; \( W_i \) – the weights of the i-th layer; \( b_i \) – the biases of the i-th layer, \( \sigma \) – the logistic sigmoid function:

\[
x_n = \sigma(W_{n-1} \cdot x_{n-1} + b_{n-1}).
\] (1)

As the main goal of the NCCA is the elimination of the contour error, the neural network input is the contour error vector projections and the neural network regulator output is the returning speed vector (\( \mathbf{V}_e \)). Whereas the tangential speed (\( \mathbf{V}_\tau \)) (2) is formed using the actual position of the end effector (\( x, y \)) and the equation of the desired path which is a circular arc (3):

\[
\mathbf{V}_\tau = \begin{bmatrix} V_k \cdot \cos(\frac{\pi}{2} - \tan^{-1}\frac{y}{x}) \\ -V_k \cdot \sin(\frac{\pi}{2} - \tan^{-1}\frac{y}{x}) \end{bmatrix}
\] (2)

\[
x^2 + y^2 = R^2.
\] (3)

Using the desired trajectory equation (3), it is possible to analytically calculate the contour error. So, we get the following equations for the contour error module (4) and the contour error vector (5):
The contour error is the minimization criterion which has a variety of sources that can be broadly grouped into two categories: quasi-static errors and dynamic contour errors. Quasi-static errors are those that are slowly varying in time and arise due to geometrical inaccuracies, temperature expansions [24], and errors in the speed transducers. Whereas dynamic errors are fast-changing positioning errors, they arise due to deformation, vibration, parametric disturbances (for example disturbances in the electrical network [25-28]), coordinate disturbances, and imperfections in control algorithms. Several methods can be used to avoid or compensate for the contour error [5], like building a well-designed, accurate, and stiff CNC machine, using look-up tables, building an error model, or designing advanced control systems as we do in this paper.

The proposed algorithms are tuned using the genetic algorithm with the contour error score function. The genetic algorithm [18, 29] is a heuristic optimization algorithm, it uses processes similar to natural selection figure 6, such as replication, elitism, mutation, and crossing. Each member of the generation is characterized by a genome and a reproduction probability. The probability of an individual to reproduce is in inverse proportion to its score value. In the case of the US, the genome is the PID regulator coefficients and in the case of the NCCA, the genome is the parameters of the neural network and the PID regulator coefficients. The genetic algorithm can also be used to tune fuzzy logic systems [30].

\[ e_k = \sqrt{x^2 + y^2} - R^2 \]  
\[ e_k = \left[ e_k \cdot \frac{x}{\sqrt{x^2+y^2}} \right] + \left[ e_k \cdot \frac{y}{\sqrt{x^2+y^2}} \right]^T. \]  

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\[ \langle e_p \rangle = \frac{1}{T} \int_0^T e_p^2 dt. \]  

During the experiments, we measured the following criteria: the mean square of the quality factor (7) which corresponds to the path accuracy and the mean absolute square contour speed error (8) which shows how well the chosen contour speed is maintained. To evaluate these errors, we also calculated the mean square contour error (9) and the module of the speed error (10):

3. Experiment

To compare synthesized algorithms with the reference algorithms we used simulation modeling. We built a mathematic model of a CNC machine using BALDOR’s MT-3353-B as the CNC machine motors and lead screws with a speed ratio equal to 63 mm/ratio.

The reference control systems were the classical US tuned on symmetrical optimum and the US optimized with respect to the position error (US-P) using the genetic algorithm and the mean square positional error as the score function (6):

\[ \langle e_p \rangle = \frac{1}{T} \int_0^T e_p^2 dt. \]  

During the experiments, we measured the following criteria: the mean square of the quality factor (7) which corresponds to the path accuracy and the mean absolute square contour speed error (8) which shows how well the chosen contour speed is maintained. To evaluate these errors, we also calculated the mean square contour error (9) and the module of the speed error (10):

\[ \langle e_p \rangle = \frac{1}{T} \int_0^T e_p^2 dt. \]
The optimization of the proposed algorithms is not a subject of this paper, hence the proposed algorithms were tuned only once using the parameters from the experiment N 1 in table 1.

We chose 10 different conditions for the experiments (see table 1) to explore how different initial contour errors, different trajectory radiiuses, different contour speeds, and asymmetry of the axes influence the criteria.

4. Results
The results of the experiments are shown in table 1. Using the experimental data, we can plot the comparison of the quality factors of the algorithms in figure 7 and the comparison of the absolute contour speed errors in figure 8.

| N | $e_{k0}$ | R | $V_k$ | $T_{e1}$ | $\langle \nu \rangle$ | $e_{\nu k}$ | $\langle \nu \rangle$ | $e_{\nu k}$ | $\langle \nu \rangle$ | $e_{\nu k}$ | $\langle \nu \rangle$ | $e_{\nu k}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.2 | 1 | 0.1 | 1.0 | 195 | 11.8 | 223 | 6.8 | 356 | 2.0 | 333 | 3.2 |
| 2 | 0.3 | 1 | 0.1 | 1.0 | 90 | 37.4 | 95 | 15.4 | 176 | 4.3 | 131 | 5.0 |
| 3 | 0.1 | 1 | 0.1 | 1.0 | 390 | 3.2 | 406 | 1.7 | 489 | 0.6 | 827 | 1.5 |
| 4 | -0.2 | 1 | 0.1 | 1.0 | 195 | 11.9 | 218 | 6.8 | 350 | 2.0 | 332 | 3.3 |
| 5 | 0.2 | 2 | 0.1 | 1.0 | 195 | 11.8 | 229 | 6.8 | 420 | 2.0 | 333 | 3.2 |
| 6 | 0.2 | 0.5 | 0.1 | 1.0 | 195 | 11.8 | 202 | 6.8 | 244 | 2.0 | 330 | 3.2 |
| 7 | 0.2 | 0.05 | 1.0 | 198 | 41.3 | 115 | 25.3 | 221 | 7.6 | 166 | 12.3 |
| 8 | 0.2 | 0.2 | 1.0 | 390 | 3.2 | 305 | 1.6 | 276 | 0.6 | 622 | 0.8 |
| 9 | 0.2 | 0.1 | 1.1 | 195 | 11.9 | 223 | 6.8 | 356 | 2.0 | 332 | 3.2 |
| 10 | 0.2 | 0.1 | 0.9 | 184 | 13.0 | 219 | 6.8 | 352 | 2.0 | 326 | 3.3 |

Figure 7. The comparison of the quality factors of the classical US, the US-P, the US-C, and the NCCA for each experiment.
Figure 7 shows that the proposed algorithms have the overall quality factor higher than the reference algorithms, with the only exception being experiment N 8 with higher contour speed, where the US-C has the quality factor lower than the reference systems. Furthermore, the NCCA in some cases achieved even greater quality factors than the US-C, e.g. in experiments N 3 and N 8, where the NCCA quality factor was 1.7 and 2.3 times higher than the US-C quality factor.

The proposed algorithms apart from having generally a better quality factor than the reference systems also have lower contour speed errors, which can be seen in figure 8, with the overall absolute contour speed error less than or equal to 12.3% for the NCCA and 7.6% for the US-C.

5. Discussion
The results show that using the proposed algorithms indeed leads to an increase in the quality factor in comparison with the classical US and the US-P without any loss in contour speed maintenance quality. However, the US-C showed a worse tracking accuracy than the NCCA which corresponds to their structures the US-C has an uncoupled structure whereas the NCCA has a coupled structure that is more suitable for contouring tasks [1-6]. That conclusion can also be drawn from other studies about the path precompensation method and its variations [1-3, 7-9, 17]. Moreover, these studies show that the path precompensation methods are more suitable for controlling robots with complex kinematics than the positional methods [2, 3, 7-9], hence it might be another possible application for the NCCA.

The neural network structure used in the paper is not symmetrical, meaning it uses different functions to form x and y component of $U_c$, moreover, using this structure leads to the creation of cross-coupled bounds which effect on the quality factor and the contour error is not yet studied. Furthermore, control vector components $V_\tau$ and $V_e$ in systems with non-symmetrical neural networks are not perpendicular which means that they might impede each other from achieving their control goals. So, the next logical step would be to synthesize the NCCA with a symmetrical neural network regulator.

Additionally, it could be possible to set control priorities for the proposed algorithms, which cannot be done for the reference algorithms, by choosing the score function, for example, the score function might include the contour speed error in order to reduce this error, or it might include control signals, so the control system would be more power-efficient. This needs additional research.

6. Conclusion
In this study, we proposed the NCCA and the US-C algorithms in order to increase the quality factor of tracking a circular arc trajectory by a CNC machine. These algorithms were optimized using the genetic algorithm on the contour error criterion.

The comparison of the proposed algorithms and the reference systems on the simulation model showed that these algorithms indeed increased the quality factor of the system, moreover the
algorithms can be implemented on many existing CNC machines without modifying their structure because they are using information only from speed and position sensors.

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