Confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions

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Abstract: This paper proposes new confidence intervals for the weighted coefficients of variation (CV) of two-parameter exponential distributions based on the adjusted method of variance estimates recovery method (adjusted MOVER). This is then compared with the generalized confidence interval method (GCI) and the large sample method. The performance of these confidence intervals in terms of coverage probabilities and average lengths were evaluated via a Monte Carlo simulation. Simulation studies showed that the GCI should be considered as an alternative to the confidence interval estimation for the weighted CV of two-parameter exponential distributions. However, the adjusted MOVER confidence interval (CIAM) can be used to estimate the weighted CV when the coefficient of variation is a positive value. The proposed confidence intervals are illustrated using a real example.

Subjects: Science; Mathematics & Statistics; Statistics & Probability; Statistics

Keywords: two-parameter exponential distribution; generalized confidence interval; large sample confidence interval; coefficient of variation; adjusted MOVER confidence interval

1. Introduction
In probability and statistics, the two-parameter exponential distribution is used to represent the time to failure in many applications, such as lifetime data, survival, and reliability analysis (Hahn & Meeker, 1991). This distribution is widely used in many fields and confidence interval estimation of its parameter is importance. The confidence interval provides information respecting the population

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PUBLIC INTEREST STATEMENT
The problem of estimating the parameter of two-parameter exponential distribution has been studied continuously. In practice, the data are collected at different settings. This study has provided two methods and proposed a novel method for confidence interval estimation for the weighted coefficients of variation of two-parameter exponential distributions based on the adjusted method of variance estimates recovery method (adjusted MOVER), then compared with the generalized confidence interval method (GCI), and the large sample method. It is concluded that the GCI should be considered as an alternative to the confidence interval estimation for the weighted coefficients of variation, whereas the adjusted MOVER confidence interval CIAM can be used when the coefficient of variation is a positive.
value of the quantity much more than the point estimate (Casella & Berger, 2002). Therefore, many studies have examined confidence interval estimation for the parameters in this distribution. For example, Chiou (1997) presented a method for confidence interval estimation of scale parameters following a pre-test for two exponential distributions. Roy and Mathew (2005) proposed the generalized confidence interval approach to construct an exact lower confidence limit for the reliability function of a two-parameter exponential distribution. Li and Zhang (2010) considered the problem of estimation of asymptotic confidence interval for the ratio of means of two-parameter exponential distributions. Kharrati-Kopaei, Malekzadeh, and Sadooghi-Alvandi (2013) presented simultaneous fiducial generalized confidence intervals for the successive differences of two-parameter exponential location parameters in three populations when both scale parameters and sample sizes are possibly unequal. Singh and Singh (2013) proposed simultaneous confidence intervals of ordered pairwise differences of exponential location parameters under heteroscedasticity. Li, Song, and Shi (2015) proposed a parametric bootstrap method to construct simultaneous confidence intervals for all pairwise differences of means from several two-parameter exponential distributions.

Coefficient of variation is the ratio of the standard deviation to the mean (Kelley, 2007). It is a measure of relative variability. The coefficient of variation is often used to compare two distributions measured on different units. The coefficient of variation has been used in many fields, such as science, medicine, economics, and life insurance. For example, it is used to analyze the cycle variation in hydrogen-fueled engines with direct injection (Kim, Lee, & Choi, 2005). It is also used to measure the variation in socioeconomic status and the prevalence of smoking in tobacco control environments (Gulhar, Kibria, Albatineh, & Ahmed, 2012). In medical study, the coefficient of variation is used to measure precision within and between laboratories (Tian, 2005). In diet study, the coefficient of variation is used to compare the variability in the ratio of total/HDL cholesterol with the variability in vessel diameter change because the ratio of total/HDL cholesterol and the vessel diameter change are measured in different units (Tian, 2005). Several researchers have studied focusing on confidence interval for the coefficient of variation, for example, Vangel (1996) presented confidence intervals for a normal coefficient of variation. Tian (2005) developed the procedures for confidence interval estimation and hypothesis testing for the common coefficient of variation of normal distributions. Wong and Wu (2002) proposed confidence intervals for the coefficient of variation of normal and nonnormal models. Mahmoudvand and Hassani (2009) proposed two new confidence intervals for the coefficient of variation in a normal distribution. Curto and Pinto (2009) proposed statistical tests for two coefficients of variation comparison in non-iid case. Banik and Kibria (2011) reviewed several interval estimators proposed by different researcher at different times for estimating the population coefficient of variation and compared with bootstrap interval estimators when data are generated from various distributions. Niwitpong (2013) presented the confidence intervals for coefficient of variation of log-normal distribution with restricted parameter space.

In practical applications, the independent samples are collected from different settings. Therefore, inference procedures regarding several coefficients of variation are of interest. Several researchers proposed statistical tests to test the equality of two or more coefficients of variation, i.e. see Ahmed (1995), Gupta and Ma (1996), Fung and Tsang (1998), Curto and Pinto (2009), and Gokpinar and Gokpinar (2015). Sangnawakij and Niwitpong (2016) proposed confidence intervals for the single coefficient of variation and the difference of coefficients of variation in the two-parameter exponential distributions. Sangnawakij, Niwitpong, and Niwitpong (2016) presented confidence intervals for the ratio of coefficients of variation in the two-parameter exponential distributions. Tian (2005) noted that confidence interval estimation or hypothesis testing about the common population coefficient of variation from several sample is need in many circumstances. Combine the data of several independent samples is used in clinical trials and social and behavioral sciences. Collecting independent sample from different populations with the common coefficient of variation but possibly with different variances, then the problem of interest is to estimate or construct a confidence interval for the common coefficient of variation. This problem arises in situations where different instruments, different methods, or different laboratories are used to measure the products to estimate the average quality. Tian (2005) proposed the generalized variable approach to make inferences...
about the common coefficient of variation based on several independent normal samples. Ng (2014) presented the generalized variable approach for making inferences about the common coefficient of variation based on several independent log-normal samples. To our knowledge, no paper exists for weighted coefficients of variation in \( k \) two-parameter exponential populations. Then, inference procedures are of practical and theoretical importance, to develop procedures for confidence interval estimation for the weighted coefficients of variation in several two-parameter distributions. Hence, this paper will fill this gap by developing novel methods for confidence interval for the weighted coefficients of variation in several two-parameter exponential populations. We applied the results of Sangnawakij and Niwitpong (2016) for the weighted coefficients of variation in two or more populations. Moreover, this paper searches for a confidence interval for weighted coefficients of variation of several two-parameter exponential populations that is easy to use in practice.

The samples are collected from several independent two-parameter exponential populations which are of interest. Therefore, the goal of this paper is to provide two methods and propose a novel method for confidence interval estimation of weighted coefficients of variation of two-parameter exponential derived from several independent samples. The first method was constructed based on the concept of generalized confidence interval method (GCI). Weerahandi (1993) introduced the GCI which could successfully construct the confidence interval for common parameters; for example, see Krishnamoorthy and Lu (2003), Tian (2005), Tian and Wu (2007), and Ye, Ma, and Wang (2010). The second method was constructed according to the large sample method which was based on central limit theorem (CLT). Tian and Wu (2007) presented the GCI and the large sample method to construct confidence intervals for the common mean of several log-normal populations. The third method, the proposed method, was constructed based on the adjusted method of variance estimates recovery method (adjusted MOVER). The adjusted MOVER method was motivated and extended based on the method of variance estimates recovery method (MOVER), for example see, Zou and Donner (2008) and Zou, Taleban, and Hao (2009), and was inspired by the score interval method proposed by Bartlett (1953). Several researchers have successfully used the MOVER method to construct the confidence interval; see i.e. Zou and Donner (2008), Zou et al. (2009), Donner and Zou (2010), and Sangnawakij et al. (2016). From our knowledge, there are no proposed methods for the weighted coefficients of variation of several two-parameter exponential populations.

The organization of this paper is as follows. Section 2 describes the theory and computational procedures to construct the confidence intervals. Section 3 demonstrates the simulation results and illustrates the proposed methods with a real example. Section 4 summarizes this paper.

2. Method

2.1. The generalized confidence interval method (GCI)

Let \( X = (X_1, X_2, \ldots, X_n) \) be a random variable follows a two-parameter exponential distribution with probability density function

\[
f_X(x; \lambda, \beta) = \begin{cases} 
\frac{1}{\lambda} \exp \left( -\frac{x - \beta}{\lambda} \right), & x > \beta, \beta \in \mathbb{R}, \lambda > 0,
\end{cases}
\]

where \( \lambda \) is a scale parameter and \( \beta \) is a location parameter.

The mean and variance of \( X \) are

\[
E(X) = \lambda + \beta,
\]

\[
\text{Var}(X) = \lambda^2.
\]

The maximum likelihood estimators of parameters \( \beta \) and \( \lambda \) are

\[
\hat{\beta} = X_{(1)} = \min (X_1, X_2, \ldots, X_n),
\]

\[
\hat{\lambda} = \bar{X} - X_{(1)},
\]
where ̄X = \frac{1}{n} \sum_{j=1}^{n} X_j.

Let X_i, i = 1, 2, \ldots, k be random samples from two-parameter exponential distributions. Let θ_i = \frac{λ_i}{λ_i + β_i}

be the coefficient of variation of X_i. This paper is interested in constructing confidence intervals for the weighted coefficients of variation, based on Graybill and Deal (1959), defined as follows:

$$\bar{θ} = \frac{\sum_{i=1}^{k} \hat{θ}_i}{\sum_{i=1}^{k} 1/\text{Var}(\hat{θ}_i)},$$

where \( \hat{θ}_i \) is an unbiased estimator based on the \( i \)-th sample; see Lin and Lee (2005).

Let \( X_{ip}, i = 1, 2, \ldots, k; j = 1, 2, \ldots, n_i \) be random samples from the \( X_i \).

Let \( X_i \) and \( X_{i1} \) denote the sample mean and the smallest sample for data \( X_{ip}, i = 1, 2, \ldots, k; j = 1, 2, \ldots, n_i \). And let \( \bar{x}_i \) and \( x_{i1} \) denote the observed sample mean and the smallest observed sample, respectively.

The maximum likelihood estimator of \( \hat{θ}_i \) is

$$\hat{θ}_i = \frac{\hat{λ}_i}{\hat{λ}_i + \hat{β}_i} = \frac{\bar{x}_i - x_{i1}}{\bar{x}_i - x_{i1} + x_{i1}}.$$

(1)

where \( i = 1, 2, \ldots, k \).

According to Sangnawakij and Niwitpong (2016), the expectation and variance of \( \hat{θ}_i \) are defined as follows:

$$E(\hat{θ}_i) = \frac{(n_i - 1)λ_i}{n_i(λ_i + β_i)}$$

and

$$\text{Var}(\hat{θ}_i) = \frac{λ_i^2(n_i - 1)(λ_i^2 + n_iβ_i^2 + 2λ_iβ_i)}{n_i^3(λ_i + β_i)^4}.$$  

Thus, \( \hat{θ}_i \) is a biased estimator of \( θ_i \). To use the pooled estimators of Graybill and Deal (1959), an unbiased estimator of \( θ_i \) is needed. The unbiased estimator is defined by

$$\bar{θ}_i = \frac{n_i}{n_i - 1} \hat{θ}_i.$$

(2)
The expectation of \( \hat{\theta}_i \) is

\[
E(\hat{\theta}_i) = E\left( \frac{n_i}{n_i - 1} \hat{\theta}_i \right) = \frac{n_i}{n_i - 1} E(\hat{\theta}_i)
\]

\[
= \frac{n_i}{n_i - 1} \frac{(n_i - 1) \lambda_i}{n_i (\lambda_i + \beta_i)} = \frac{\lambda_i}{\lambda_i + \beta_i}.
\]

The variance of \( \hat{\theta}_i \) is

\[
\text{Var}(\hat{\theta}_i) = \text{Var}\left( \frac{n_i}{n_i - 1} \hat{\theta}_i \right) = \left( \frac{n_i}{n_i - 1} \right)^2 \text{Var}(\hat{\theta}_i)
\]

\[
= \left( \frac{n_i}{n_i - 1} \right)^2 \frac{\lambda_i^2 (n_i - 1) (\lambda_i^2 + n_i \beta_i^2 + 2 \lambda_i \beta_i)}{n_i^2 (\lambda_i + \beta_i)^4}
\]

\[
= \frac{\lambda_i^2 (\lambda_i^2 + n_i \beta_i^2 + 2 \lambda_i \beta_i)}{(n_i - 1) n_i (\lambda_i + \beta_i)^4}.
\]

Following Weerahandi (1993): let \( X = (X_1, X_2, \ldots, X_n) \) be a random sample from a distribution \( F_x(x; \theta, \delta) \), where \( \theta \) is a scalar parameter of interest and \( \delta \) is a nuisance parameter. Let \( X = (X_1, X_2, \ldots, X_n) \) be an observed sample. A generalized confidence interval for \( \theta \) is computed using the percentiles of a generalized pivotal quantity \( R(X; x, \theta, \delta) \) which is a function of \( X, x, \theta \) and \( \delta \) if the following two conditions are satisfied:

(i) For a given \( x \), the distribution of \( R(X; x, \theta, \delta) \) is free of all unknown parameters.

(ii) The observed value of \( R(X; x, \theta, \delta) \), \( X = x \), is the parameter of interest.

When the conditions (i) and (ii) are hold, the quantiles of \( R(X; x, \theta, \delta) \) form a \( 1 - \alpha \) confidence interval for \( \theta \). Let \( R(\alpha) \) be the \( \alpha \)-th quantile of \( R(X; x, \theta, \delta) \). Then, \( (R(\alpha/2), R(1 - \alpha/2)) \) becomes \( 100(1 - \alpha)% \) two-sided generalized confidence interval for parameter of interest \( \theta \).

As in Lawless (1982), then

\[
\frac{2n_i \lambda_i}{\lambda_i} = V_i \sim \chi^2_{2n_i - 2}
\]

and

\[
\frac{2n_i (\hat{\beta}_i - \beta_i)}{\lambda_i} = U_i \sim \chi^2.
\]
where $\chi^2_{2n_i-2}$ denotes a chi-square distribution with degrees of freedom $2n_i - 2$ and $\chi^2_2$ denotes a chi-square distribution with degrees of freedom 2. Then

$$\lambda_i = \frac{2n_i \hat{\lambda}_i}{V_i}$$

and

$$\beta_i = \hat{\beta}_i - \frac{U_i \hat{\lambda}_i}{2n_i}.$$ 

The generalized pivotal quantity for $\lambda_i$ is defined as follows:

$$R_{\lambda_i} = \frac{2n_i \hat{\lambda}_i}{V_i} = \frac{2n_i (\bar{x}_i - x_{i1i})}{V_i}.$$ (7)

The generalized pivotal quantity for $\beta_i$ is defined as follows:

$$R_{\beta_i} = \hat{\beta}_i - \frac{U_i R_{\lambda_i}}{2n_i} = x_{i1i} - \frac{U_i R_{\lambda_i}}{2n_i}.$$ (8)

According to Sangnawakij and Niwitpong (2016), the generalized pivotal quantity for $\theta_i$ is

$$R_{\theta_i} = \left(1 + \frac{1}{2n_i} \left( \frac{x_{i1i} V_i}{\bar{x}_i - x_{i1i}} - U_i \right) \right)^{-1}.$$ (9)

According to Ye et al. (2010), the generalized pivotal quantity for the weighted coefficients of variation $\theta_i$ is a weighted average of the generalized pivot $R_{\theta_i}$ based on $k$ individual samples defined as follows:

$$R_{\theta} = \frac{\sum_{i=1}^{k} R_{\theta_i}}{\sum_{i=1}^{k} R_{\text{Var}(\theta_i)}},$$ (10)

where (from Equation (4))

$$R_{\text{Var}(\theta_i)} = \frac{R_{\lambda_i}^2 (R_{\lambda_i}^2 + n_i R_{\beta_i}^2 + 2 R_{\lambda_i} R_{\beta_i})}{(n_i - 1) n_i (R_{\lambda_i} + R_{\beta_i})}.$$ (11)

The generalized confidence interval for the weighted coefficients of variation $\theta$ can be constructed from $R_{\theta}$. Therefore, the $100(1 - \alpha)%$ two-sided confidence interval for the weighted coefficients of variation $\theta$ based on GCI is

$$\text{CI}_{GCI} = (l_{GCI}, u_{GCI}) = (R_{\theta}(\alpha/2), R_{\theta}(1 - \alpha/2)),$$ (12)

where $R_{\theta}(\alpha/2)$ and $R_{\theta}(1 - \alpha/2)$ denote the $\alpha/2$-th and $1 - \alpha/2$-th quantiles of $R_{\theta}$, respectively.
The following algorithm can be used for estimating the $R_{\alpha}(a/2)$ and $R_{\alpha}(1-a/2)$.

Algorithm 1

input : $x_i$, $i = 1, 2, \ldots, k$

output: $R_{\alpha}(a/2)$ and $R_{\alpha}(1-a/2)$

begin

for $g = 1$ to $m$
do

Generate $V_i$ from chi-square distribution with degrees of freedom $2n_i - 2$; 
Generate $U_i$ from chi-square distribution with degrees of freedom $2$; 
Compute $R_{\lambda_i}$ from equation (7); 
Compute $R_{\phi_i}$ from equation (8); 
Compute $R_{\vartheta_i}$ from equation (9); 
Compute $R_{\text{var}(\phi_i)}$ from equation (11); 
Compute $R_{\theta_i}$ from equation (10); 

end

Compute the $\alpha/2$-th quantile of $R_{\phi_i}$; 
Compute the $1 - \alpha/2$-th quantile of $R_{\theta_i}$; 

end

2.2. The large sample method

The large sample estimate of the coefficient of variation is a pooled estimated unbiased estimator of the coefficient of variation, based on Graybill and Deal (1959), defined as follows:

$$\bar{\theta} = \frac{\sum_{i=1}^{k} \bar{\theta}_i}{\sum_{i=1}^{k} \frac{1}{\text{Var}(\bar{\theta}_i)}},$$

(13)

where

$$\bar{\theta}_i = \frac{n_i}{n_i - 1} \hat{\theta}_i$$

$$= \frac{n_i}{n_i - 1} \left( \frac{\bar{\lambda}_i}{\hat{\lambda}_i + \hat{\beta}_i} \right)$$

$$= \frac{n_i}{n_i - 1} \left( \frac{\bar{x}_i - x_{1i\bar{\theta}}}{\bar{x}_i - x_{1i\bar{\theta}} + x_{1i\bar{\theta}}} \right)$$

$$= \frac{n_i}{n_i - 1} \left( \frac{\bar{x}_i - x_{1i\bar{\theta}}}{\bar{x}_i} \right)$$

$$= \frac{n_i}{n_i - 1} \left( 1 - \frac{x_{1i\bar{\theta}}}{\bar{x}_i} \right)$$

and it is easy to see that $\text{Var}(\bar{\theta}_i)$ is

$$\text{Var}(\bar{\theta}_i) = \frac{(\bar{x}_i - x_{1i\bar{\theta}})^2 \left( (\bar{x}_i - x_{1i\bar{\theta}})^2 + n_i(x_{1i\bar{\theta}})^2 + 2(\bar{x}_i - x_{1i\bar{\theta}})x_{1i\bar{\theta}} \right)}{(n_i - 1)n_i(x_{1i\bar{\theta}})^4}$$

$$= \frac{\left( \frac{\bar{x}_i - x_{1i\bar{\theta}}}{\bar{x}_i - x_{1i\bar{\theta}} + x_{1i\bar{\theta}}} \right)^2 \left( (\bar{x}_i - x_{1i\bar{\theta}})^2 + n_i(x_{1i\bar{\theta}})^2 + 2(\bar{x}_i - x_{1i\bar{\theta}})x_{1i\bar{\theta}} \right)}{(n_i - 1)n_i(x_{1i\bar{\theta}})^4}.$$
\[ CI_{LS} = (l_{LS}, u_{LS}) \]
\[ = \left( \hat{\theta} - z_{1-\alpha/2} \sqrt{\frac{1}{\sum_{i=1}^{k} \frac{1}{\text{Var}(\hat{\theta}_i)}}, \hat{\theta} + z_{1-\alpha/2} \sqrt{\frac{1}{\sum_{i=1}^{k} \frac{1}{\text{Var}(\hat{\theta}_i)}}} \right), \]

where \( z_{1-\alpha/2} \) denotes the \( 1 - \alpha/2 \)-th quantile of the standard normal distribution.

### 2.3. The adjusted method of variance estimates recovery method (adjusted MOVER)

For two parameters case, the method of variance estimates recovery method (MOVER) was introduced by Zou and Donner (2008) and Zou et al. (2009). Let \( \theta_1 \) and \( \theta_2 \) be the parameters of interest. The MOVER method is used to construct a \( 100(1 - \alpha)\% \) two-sided confidence interval of \( \theta_1 + \theta_2 \).

Using the central limit theorem and the assumption of independence between the point estimates \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) the lower limit \( L \) is defined as follows:

\[ L = \hat{\theta}_1 + \hat{\theta}_2 - z_{\alpha/2} \sqrt{\text{Var}(\hat{\theta}_1) + \text{Var}(\hat{\theta}_2)}, \]

where \( z_{\alpha/2} \) denotes the \( \alpha/2 \)-th quantile of the standard normal distribution.

The \( (l_1, u_1) \) and \( (l_2, u_2) \) contain the parameter values for \( \theta_1 \) and \( \theta_2 \), respectively. The lower limit \( L \) must be closer to \( l_1 + l_2 \) than to \( \theta_1 + \theta_2 \). The variance estimate for \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) at \( \theta_j = l_j \) is

\[ \text{Var}(\hat{\theta}_i) = \frac{(\hat{\theta}_i - l_i)^2}{z_{\alpha/2}^2}; \quad i = 1, 2. \]

Substituting back into Equation (15), then

\[ L = \hat{\theta}_1 + \hat{\theta}_2 - \sqrt{(\hat{\theta}_1 - l_1)^2 + (\hat{\theta}_2 - l_2)^2}. \]

By performing similar steps with this idea, the upper limit \( U \) must be closer to \( u_1 + u_2 \), which gives

\[ U = \hat{\theta}_1 + \hat{\theta}_2 + \sqrt{(u_1 - \hat{\theta}_1)^2 + (u_2 - \hat{\theta}_2)^2}. \]

Consider \( k \) parameters case, let \( \theta_1, \theta_2, \ldots, \theta_k \) be the parameters of interest. The MOVER method is motivated to construct a \( 100(1 - \alpha)\% \) two-sided confidence interval for \( \theta_1 + \theta_2 + \cdots + \theta_k \). Using the central limit theorem and the assumption of independence between the point estimates \( \hat{\theta}_1, \hat{\theta}_2, \ldots, \hat{\theta}_k \), the lower limit \( L \) is defined as follows:

\[ L = \hat{\theta}_1 + \cdots + \hat{\theta}_k - z_{\alpha/2} \sqrt{\text{Var}(\hat{\theta}_1) + \cdots + \text{Var}(\hat{\theta}_k)}, \]

where \( z_{\alpha/2} \) denotes the \( \alpha/2 \)-th quantile of the standard normal distribution.

The \( (l_1, u_1), (l_2, u_2), \ldots, (l_k, u_k) \) contain the parameter values for \( \theta_1, \theta_2, \ldots, \theta_k \), respectively. The lower limit \( L \) must be closer to \( l_1 + l_2 + \cdots + l_k \) than to \( \theta_1 + \theta_2 + \cdots + \theta_k \). The lower limit \( L \) for \( \theta_1 + \theta_2 + \cdots + \theta_k \) is

\[ L = \hat{\theta}_1 + \cdots + \hat{\theta}_k - \sqrt{(\hat{\theta}_1 - l_1)^2 + \cdots + (\hat{\theta}_k - l_k)^2} \]

and similarly, the upper limit

\[ U = \hat{\theta}_1 + \cdots + \hat{\theta}_k + \sqrt{(u_1 - \hat{\theta}_1)^2 + \cdots + (u_k - \hat{\theta}_k)^2}. \]
The adjusted MOVER method was motivated based on concepts of the large sample method in Equations (13)–(14) and MOVER method in Equations (15)–(19). According to Graybill and Deal (1959), the weighted coefficients of variation \( \theta \) is weighted average of the coefficient of variation \( \hat{\theta}_i \) based on \( k \) individual samples is defined as

\[
\hat{\theta} = \sum_{i=1}^{k} \frac{\hat{\theta}_i}{\text{Var}(\hat{\theta}_i)} / \sum_{i=1}^{k} \frac{1}{\text{Var}(\hat{\theta}_i)},
\]

where the variance estimate for \( \hat{\theta}_i \) at \( \theta_i = l_i \) and \( \theta_i = u_i \) is the average variance between these two variances and given by

\[
\text{Var}(\hat{\theta}_i) = \frac{1}{2} \left( \frac{(\hat{\theta}_i - l_i)^2}{z_{a/2}^2} + \frac{(u_i - \hat{\theta}_i)^2}{z_{a/2}^2} \right); \quad i = 1, 2, \ldots, k.
\]

The lower limit \( L \) and upper limit \( U \) for the weighted coefficients of variation \( \theta \) are

\[
L = \hat{\theta} - z_{1-a/2} \sqrt{\frac{1}{\sum_{i=1}^{k} \frac{z_{a/2}^2}{(\hat{\theta}_i - l_i)^2}}},
\]

and

\[
U = \hat{\theta} + z_{1-a/2} \sqrt{\frac{1}{\sum_{i=1}^{k} \frac{z_{a/2}^2}{(u_i - \hat{\theta}_i)^2}}}.
\]

Therefore, the \( 100(1 - \alpha)\% \) two-sided confidence interval for the weighted coefficients of variation \( \theta \) based on adjusted MOVER method is

\[
\text{CI}_{AM} = (L_{AM}, U_{AM}) = (\hat{\theta} - z_{1-a/2} \sqrt{\frac{1}{\sum_{i=1}^{k} \frac{z_{a/2}^2}{(\hat{\theta}_i - l_i)^2}}}, \hat{\theta} + z_{1-a/2} \sqrt{\frac{1}{\sum_{i=1}^{k} \frac{z_{a/2}^2}{(u_i - \hat{\theta}_i)^2}}})
\]

where \( z_{a/2} \) and \( z_{1-a/2} \) denote the \( a/2 \)-th and \( 1 - a/2 \)-th quantiles of the standard normal distribution, respectively.

The lower limit \( l_{ij} \) and upper limit \( u_{ij} \) for coefficient of variation \( \theta_i = \frac{\hat{\theta}_i}{z_{0.1}} = \frac{\hat{\theta}_i}{z_{0.9}} \) were obtained by substituting \( \hat{\theta}_i = x_i - x_{1.0} \) and \( \hat{\theta}_i = \bar{x}_i \) according to Sangnawakij and Niwitpong (2016), are given by

\[
l_{ij} = \frac{\hat{\theta}_i \hat{\theta}_j - \sqrt{(\hat{\theta}_i \hat{\theta}_j)^2 - l_{01}^2 u_{02}^2 (2\hat{\theta}_j - l_{01})(2\hat{\theta}_j - u_{02})}}{u_{02}^2 (2\hat{\theta}_j - u_{02})}
\]

\[
u_{ij} = \frac{\hat{\theta}_i \hat{\theta}_j + \sqrt{(\hat{\theta}_i \hat{\theta}_j)^2 - u_{01}^2 l_{02}^2 (2\hat{\theta}_j - l_{01})(2\hat{\theta}_j - l_{02})}}{l_{02}^2 (2\hat{\theta}_j - l_{02})}
\]

where
Therefore, the confidence interval for coefficient of variation \( \theta \) is given by

\[
(l_{2i}, u_{2i}) = \left( \frac{n_i}{n_i - 1} \left(1 - \frac{x_{1i}}{\bar{x}_i}\right) - z_{a/2} \hat{\epsilon}_i, \frac{n_i}{n_i - 1} \left(1 - \frac{x_{1i}}{\bar{x}_i}\right) + z_{a/2} \hat{\epsilon}_i \right)
\]  

(27)

where

\[
\hat{\epsilon}_i = \frac{(\bar{x}_i - x_{1i})}{\bar{x}_i^2} \sqrt{\frac{\hat{\sigma}^2 + n_i \hat{\sigma}^2 + 2 \lambda \beta}{(n_i - 1) n_i}}.
\]

Therefore, the 100(1 - \( \alpha \))% two-sided confidence intervals for the weighted coefficients of variation \( \theta \) based on adjusted MOVER method are

\[
CI_{AM1} = (l_{AM1}, u_{AM1}) = \left( \hat{\theta} - z_{1-a/2} \left( \frac{1}{\sum_{i=1}^{k} \left( \hat{\theta}_i - l_{1i} \right)^2} \right)^{1/2}, \hat{\theta} + z_{1-a/2} \left( \frac{1}{\sum_{i=1}^{k} \left( \hat{\theta}_i - u_{1i} \right)^2} \right)^{1/2} \right)
\]

(28)

and

\[
CI_{AM2} = (l_{AM2}, u_{AM2}) = \left( \hat{\theta} - z_{1-a/2} \left( \frac{1}{\sum_{i=1}^{k} \left( \hat{\theta}_i - l_{2i} \right)^2} \right) \hat{\theta} + z_{1-a/2} \left( \frac{1}{\sum_{i=1}^{k} \left( \hat{\theta}_i - u_{2i} \right)^2} \right) \right)
\]

(29)
3. Results

3.1. Comparative analysis

In this section, the performance of four confidence intervals in terms of coverage probabilities and average lengths are compared via a Monte Carlo simulation. The generalized confidence interval is defined as \( C_{GCI} \), the large sample confidence interval is defined as \( C_{LS} \), and two adjusted MOVER confidence intervals are defined as \( C_{AM1} \) and \( C_{AM2} \). A confidence interval, with the values of coverage probability at least or close to the nominal confidence level \( 1 - \alpha \) and also has the shortest average length, was chosen.

In this simulation study, the numbers of population were assigned \( k = 2, 4, \) and \( k = 6 \) with the sample sizes \( n_1 = n_2 = \cdots = n_k = n = 30, 50, 100, \) and \( 200 \). In two-parameter exponential distribution, the scale parameters were \( \lambda_1 = \lambda_2 = \cdots = \lambda_k = \lambda = 1.0 \) and the location parameters were computed by \( \beta_i = \lambda_i (1 - \theta_i) / \theta_i \), where \( i = 1, 2, \ldots, k \). The coefficients of variation were \( \theta_1 = \theta_2 = \cdots = \theta_k = \theta = -1.0, -0.5, -0.3, 0.3, 0.5, \) and \( 1.0 \). For each parameter and sample size setting, 5,000 random samples were generated. For the GCI, for each of the 5000 random samples, \( 2,500 \) \( R_i \)'s were obtained.

Tables 1–6 presents the coverage probabilities and average lengths of confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions for 2, 4, and 6 sample cases, respectively. The results show that the adjusted MOVER confidence interval \( C_{AM2} \) performs as well as the generalized confidence interval \( C_{GCI} \) for large sample size, i.e. \( n \geq 100 \) and \( \theta \) is a positive value. For a negative value of \( \theta \), the generalized confidence interval \( C_{GCI} \) performs the best confidence interval compared with the other confidence intervals. Hence, the generalized confidence interval \( C_{GCI} \) and the adjusted MOVER confidence interval \( C_{AM2} \) can be used for estimating the weighted coefficients of variation of two-parameter exponential distributions when \( \theta \) is a positive value only, otherwise the generalized confidence interval \( C_{GCI} \) will be chosen. Note that, from simulation results, the large sample confidence interval \( C_{LS} \) performs as well as the adjusted MOVER confidence interval \( C_{AM2} \) for every case.

The following algorithm can be used to estimate the coverage probability and average length.

**Algorithm 2**

**input**: \( M, m, k, n_1, n_2, \ldots, n_k, \lambda_1, \lambda_2, \ldots, \lambda_k, \beta_1, \beta_2, \ldots, \beta_k, \theta \)

**output**: The coverage probability and the average length

**begin**

for \( h = 1 \) to \( M \) do

\[ \text{Generate } x_{ij} \text{ from } \exp (\lambda_i, \beta_i), \text{ for } i = 1, 2, \ldots, k, j = 1, 2, \ldots, n_i; \]

\[ \text{Compute } z_i; \]

\[ \text{Use Algorithm 1 to construct } (L_{GCI(h)}, U_{GCI(h)}); \]

\[ \text{Use equation 14 to construct } (L_{LS(h)}, U_{LS(h)}); \]

\[ \text{Use equation 28 to construct } (L_{AM1(h)}, U_{AM1(h)}); \]

\[ \text{Use equation 29 to construct } (L_{AM2(h)}, U_{AM2(h)}); \]

if \( (L_{(h)} \leq \theta \leq U_{(h)}) \) then

\[ p_{(h)} = 1; \]

else

\[ p_{(h)} = 0; \]

end

\[ \text{Compute } U_{(h)} - L_{(h)}; \]

end

Compute mean of \( p_{(h)} \) defined by the coverage probability;

Compute mean of \( U_{(h)} - L_{(h)} \) defined by the average length;

**end**
Table 1. The coverage probabilities of 95% two-sided confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions: 2 sample cases

| n    | λ   | θ  | CIGCI | CILS | CIAM | CIAM2 |
|------|-----|----|-------|------|------|-------|
| 30   | 1.0 | −1.0| 0.9198| 0.8018| 0.8226| 0.8018|
|      |     | −0.5| 0.9342| 0.8522| 0.8878| 0.8522|
|      |     | −0.3| 0.9392| 0.8696| 0.9122| 0.8696|
|      |     | 0.3 | 0.9504| 0.9264| 0.9250| 0.9264|
|      |     | 0.5 | 0.9512| 0.9374| 0.9296| 0.9374|
|      |     | 1.0 | 0.9292| 0.9626| 1.0000| 0.9626|
| 50   | 1.0 | −1.0| 0.9324| 0.8500| 0.8172| 0.8500|
|      |     | −0.5| 0.9378| 0.8770| 0.8640| 0.8770|
|      |     | −0.3| 0.9410| 0.8956| 0.8964| 0.8956|
|      |     | 0.3 | 0.9486| 0.9354| 0.9498| 0.9354|
|      |     | 0.5 | 0.9492| 0.9416| 0.9980| 0.9416|
|      |     | 1.0 | 0.9424| 0.9646| 1.0000| 0.9646|
| 100  | 1.0 | −1.0| 0.9378| 0.8866| 0.8132| 0.8866|
|      |     | −0.5| 0.9402| 0.9102| 0.8570| 0.9102|
|      |     | −0.3| 0.9416| 0.9200| 0.8864| 0.9200|
|      |     | 0.3 | 0.9444| 0.9360| 0.9728| 0.9360|
|      |     | 0.5 | 0.9504| 0.9504| 0.9992| 0.9504|
|      |     | 1.0 | 0.9446| 0.9560| 1.0000| 0.9560|
| 200  | 1.0 | −1.0| 0.9392| 0.9146| 0.8228| 0.9146|
|      |     | −0.5| 0.9492| 0.9342| 0.8528| 0.9342|
|      |     | −0.3| 0.9452| 0.9060| 0.8836| 0.9060|
|      |     | 0.3 | 0.9564| 0.9532| 0.9866| 0.9532|
|      |     | 0.5 | 0.9472| 0.9470| 0.9990| 0.9470|
|      |     | 1.0 | 0.9506| 0.9550| 1.0000| 0.9550|

Table 2. The average lengths of 95% two-sided confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions: 2 sample cases

| n    | λ   | θ  | CIGCI | CILS | CIAM | CIAM2 |
|------|-----|----|-------|------|------|-------|
| 30   | 1.0 | −1.0| 1.1703| 0.9379| 0.7914| 0.9379|
|      |     | −0.5| 0.4162| 0.3332| 0.3239| 0.3332|
|      |     | −0.3| 0.2131| 0.1849| 0.1817| 0.1849|
|      |     | 0.3 | 0.1129| 0.1060| 0.1951| 0.1060|
|      |     | 0.5 | 0.1383| 0.1331| 0.3870| 0.1331|
|      |     | 1.0 | 0.0809| 0.0933| 1.5634| 0.0933|
| 50   | 1.0 | −1.0| 0.8674| 0.7456| 0.5844| 0.7456|
|      |     | −0.5| 0.3169| 0.2815| 0.2372| 0.2815|
|      |     | −0.3| 0.1625| 0.1472| 0.1330| 0.1472|
|      |     | 0.3 | 0.0854| 0.0822| 0.1385| 0.0822|
|      |     | 0.5 | 0.1034| 0.1012| 0.2619| 0.1012|
|      |     | 1.0 | 0.0497| 0.0558| 0.8247| 0.0558|
| 100  | 1.0 | −1.0| 0.5913| 0.5347| 0.3980| 0.5347|
|      |     | −0.5| 0.2180| 0.2031| 0.1609| 0.2031|
|      |     | −0.3| 0.1125| 0.1062| 0.0903| 0.1062|
|      |     | 0.3 | 0.0593| 0.0582| 0.0920| 0.0582|
|      |     | 0.5 | 0.0712| 0.0705| 0.1693| 0.0705|
|      |     | 1.0 | 0.0258| 0.0278| 0.4706| 0.0278|
| 200  | 1.0 | −1.0| 0.4113| 0.3854| 0.2796| 0.3854|
|      |     | −0.5| 0.1516| 0.1454| 0.1117| 0.1454|
|      |     | −0.3| 0.0781| 0.0756| 0.0625| 0.0756|
|      |     | 0.3 | 0.0415| 0.0411| 0.0632| 0.0411|
|      |     | 0.5 | 0.0497| 0.0494| 0.1144| 0.0494|
|      |     | 1.0 | 0.0131| 0.0139| 0.3027| 0.0139|
Table 3. The coverage probabilities of 95% two-sided confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions: 4 sample cases

| n   | λ   | θ  | CI_GCI | CI_LS  | CI_AM1 | CI_AM2 |
|-----|-----|-----|--------|--------|--------|--------|
| 30  | 1.0 | −1.0 | 0.7468 | 0.6598 | 0.7028 | 0.6598 |
|     |     | −0.5 | 0.8242 | 0.7548 | 0.8420 | 0.7548 |
|     |     | −0.3 | 0.8592 | 0.7982 | 0.8858 | 0.7982 |
|     |     | 0.3  | 0.9358 | 0.9030 | 0.8960 | 0.9030 |
|     |     | 0.5  | 0.9480 | 0.9288 | 0.9934 | 0.9288 |
| 50  | 1.0 | −1.0 | 0.9284 | 0.9702 | 1.0000 | 0.9702 |
|     |     | −0.5 | 0.8042 | 0.7550 | 0.7284 | 0.7550 |
|     |     | −0.3 | 0.8640 | 0.8202 | 0.8394 | 0.8202 |
|     |     | 0.3  | 0.9448 | 0.9072 | 0.9414 | 0.9072 |
|     |     | 0.5  | 0.9492 | 0.9396 | 0.9970 | 0.9396 |
| 100 | 1.0 | −1.0 | 0.8476 | 0.8224 | 0.7464 | 0.8224 |
|     |     | −0.5 | 0.9000 | 0.8820 | 0.8432 | 0.8820 |
|     |     | −0.3 | 0.9024 | 0.8874 | 0.8724 | 0.8874 |
|     |     | 0.3  | 0.9404 | 0.9330 | 0.9624 | 0.9330 |
|     |     | 0.5  | 0.9438 | 0.9426 | 0.9994 | 0.9426 |
| 200 | 1.0 | −1.0 | 0.9396 | 0.9674 | 1.0000 | 0.9674 |
|     |     | −0.5 | 0.8924 | 0.8818 | 0.7792 | 0.8818 |
|     |     | −0.3 | 0.9198 | 0.9112 | 0.8498 | 0.9112 |
|     |     | 0.3  | 0.9360 | 0.9300 | 0.8906 | 0.9300 |
|     |     | 0.5  | 0.9488 | 0.9450 | 0.9450 | 0.9450 |
|     |     | 1.0  | 0.9540 | 0.9622 | 1.0000 | 0.9622 |

Table 4. The average lengths of 95% two-sided confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions: 4 sample cases

| n   | λ   | θ  | CI_GCI | CI_LS  | CI_AM1 | CI_AM2 |
|-----|-----|-----|--------|--------|--------|--------|
| 30  | 1.0 | −1.0 | 0.6814 | 0.5942 | 0.5067 | 0.5942 |
|     |     | −0.5 | 0.2786 | 0.2385 | 0.2207 | 0.2385 |
|     |     | −0.3 | 0.1478 | 0.1270 | 0.1256 | 0.1270 |
|     |     | 0.3  | 0.0809 | 0.0747 | 0.1366 | 0.0747 |
|     |     | 0.5  | 0.0996 | 0.0941 | 0.2713 | 0.0941 |
|     |     | 1.0  | 0.0549 | 0.0660 | 1.0995 | 0.0660 |
| 50  | 1.0 | −1.0 | 0.5596 | 0.4934 | 0.3888 | 0.4934 |
|     |     | −0.5 | 0.2180 | 0.1928 | 0.1634 | 0.1928 |
|     |     | −0.3 | 0.1137 | 0.1019 | 0.0925 | 0.1019 |
|     |     | 0.3  | 0.0609 | 0.0581 | 0.0974 | 0.0581 |
|     |     | 0.5  | 0.0740 | 0.0716 | 0.1850 | 0.0716 |
|     |     | 1.0  | 0.0344 | 0.0395 | 0.5821 | 0.0395 |
| 100 | 1.0 | −1.0 | 0.4081 | 0.3668 | 0.2735 | 0.3668 |
|     |     | −0.5 | 0.1538 | 0.1416 | 0.1125 | 0.1416 |
|     |     | −0.3 | 0.0792 | 0.0741 | 0.0632 | 0.0741 |
|     |     | 0.3  | 0.0421 | 0.0411 | 0.0649 | 0.0411 |
|     |     | 0.5  | 0.0506 | 0.0498 | 0.1195 | 0.0498 |
|     |     | 1.0  | 0.0180 | 0.0197 | 0.3327 | 0.0197 |
| 200 | 1.0 | −1.0 | 0.2890 | 0.2683 | 0.1947 | 0.2683 |
|     |     | −0.5 | 0.1070 | 0.1021 | 0.0785 | 0.1021 |
|     |     | −0.3 | 0.0553 | 0.0533 | 0.0441 | 0.0533 |
|     |     | 0.3  | 0.0294 | 0.0291 | 0.0446 | 0.0291 |
|     |     | 0.5  | 0.0352 | 0.0349 | 0.0808 | 0.0349 |
|     |     | 1.0  | 0.0093 | 0.0098 | 0.2141 | 0.0098 |
### Table 5. The coverage probabilities of 95% two-sided confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions: 6 sample cases

| n   | λ   | θ     | CI_{GCI} | CI_{LS} | CI_{AM1} | CI_{AM2} |
|-----|-----|-------|----------|---------|----------|----------|
| 30  | 1.0 | −1.0  | 0.5634   | 0.5572  | 0.6172   | 0.5572   |
|     |     | −0.5  | 0.6936   | 0.6726  | 0.7910   | 0.6726   |
|     |     | −0.3  | 0.7478   | 0.7156  | 0.8578   | 0.7156   |
|     |     | 0.3   | 0.9300   | 0.8904  | 0.8672   | 0.8904   |
|     |     | 0.5   | 0.9514   | 0.9286  | 0.9936   | 0.9286   |
|     |     | 1.0   | 0.9316   | 0.9728  | 1.0000   | 0.9728   |
| 50  | 1.0 | −1.0  | 0.6414   | 0.6548  | 0.6302   | 0.6548   |
|     |     | −0.5  | 0.7674   | 0.7706  | 0.7990   | 0.7706   |
|     |     | −0.3  | 0.8084   | 0.8088  | 0.8584   | 0.8088   |
|     |     | 0.3   | 0.9342   | 0.9160  | 0.9234   | 0.9160   |
|     |     | 0.5   | 0.9512   | 0.9406  | 0.9964   | 0.9406   |
|     |     | 1.0   | 0.9422   | 0.9732  | 1.0000   | 0.9732   |
| 100 | 1.0 | −1.0  | 0.7508   | 0.7688  | 0.6910   | 0.7688   |
|     |     | −0.5  | 0.8410   | 0.8530  | 0.8214   | 0.8530   |
|     |     | −0.3  | 0.8752   | 0.8762  | 0.8724   | 0.8762   |
|     |     | 0.3   | 0.9428   | 0.9334  | 0.9612   | 0.9334   |
|     |     | 0.5   | 0.9492   | 0.9454  | 0.9984   | 0.9454   |
|     |     | 1.0   | 0.9410   | 0.9658  | 1.0000   | 0.9658   |
| 200 | 1.0 | −1.0  | 0.8290   | 0.8560  | 0.7510   | 0.8560   |
|     |     | −0.5  | 0.8788   | 0.8962  | 0.8270   | 0.8962   |
|     |     | −0.3  | 0.9072   | 0.9124  | 0.8806   | 0.9124   |
|     |     | 0.3   | 0.9488   | 0.9456  | 0.9824   | 0.9456   |
|     |     | 0.5   | 0.9510   | 0.9482  | 0.9994   | 0.9482   |
|     |     | 1.0   | 0.9462   | 0.9594  | 1.0000   | 0.9594   |

### Table 6. The average lengths of 95% two-sided confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions: 6 sample cases

| n   | λ   | θ     | CI_{GCI} | CI_{LS} | CI_{AM1} | CI_{AM2} |
|-----|-----|-------|----------|---------|----------|----------|
| 30  | 1.0 | −1.0  | 0.5406   | 0.4698  | 0.4026   | 0.4698   |
|     |     | −0.5  | 0.2258   | 0.1909  | 0.1773   | 0.1909   |
|     |     | −0.3  | 0.1203   | 0.1021  | 0.1012   | 0.1021   |
|     |     | 0.3   | 0.0664   | 0.0610  | 0.1112   | 0.0610   |
|     |     | 0.5   | 0.0818   | 0.0768  | 0.2203   | 0.0768   |
|     |     | 1.0   | 0.0441   | 0.0539  | 0.8975   | 0.0539   |
| 50  | 1.0 | −1.0  | 0.4509   | 0.3931  | 0.3105   | 0.3931   |
|     |     | −0.5  | 0.1786   | 0.1562  | 0.1326   | 0.1562   |
|     |     | −0.3  | 0.0931   | 0.0827  | 0.0751   | 0.0827   |
|     |     | 0.3   | 0.0498   | 0.0474  | 0.0793   | 0.0474   |
|     |     | 0.5   | 0.0606   | 0.0584  | 0.1506   | 0.0584   |
|     |     | 1.0   | 0.0279   | 0.0372  | 0.4751   | 0.0372   |
| 100 | 1.0 | −1.0  | 0.3337   | 0.2971  | 0.2217   | 0.2971   |
|     |     | −0.5  | 0.1255   | 0.1150  | 0.0914   | 0.1150   |
|     |     | −0.3  | 0.0649   | 0.0603  | 0.0515   | 0.0603   |
|     |     | 0.3   | 0.0344   | 0.0336  | 0.0529   | 0.0336   |
|     |     | 0.5   | 0.0414   | 0.0407  | 0.0975   | 0.0407   |
|     |     | 1.0   | 0.0147   | 0.0161  | 0.2715   | 0.0161   |
| 200 | 1.0 | −1.0  | 0.2360   | 0.2176  | 0.1580   | 0.2176   |
|     |     | −0.5  | 0.0874   | 0.0830  | 0.0639   | 0.0830   |
|     |     | −0.3  | 0.0452   | 0.0434  | 0.0359   | 0.0434   |
|     |     | 0.3   | 0.0240   | 0.0238  | 0.0364   | 0.0238   |
|     |     | 0.5   | 0.0288   | 0.0285  | 0.0660   | 0.0285   |
|     |     | 1.0   | 0.0076   | 0.0080  | 0.1748   | 0.0080   |
3.2. Example

To illustrate the computation of confidence intervals proposed in this paper, we use the data from a clinical trial provided by Freireich et al. (1963) and Sangnawakij and Niwitpong (2016). The data showed the time (weeks) to relapse of patients after treated by a drug 6-mercaptopurine (6-MP) and a placebo. In sampling, 42 children with acute leukemia were selected into two groups for treatments, 21 patients for 6-MP and another for a placebo. The histograms of these data are presented by Sangnawakij and Niwitpong (2016). The summary statistics are $X_1 = 17.10$, $X_2 = 8.67$, $X_{1/2} = 6.00$, $X_{2/2} = 1.00$, $\theta_1 = 0.65$, $\theta_2 = 0.88$, $n_1 = 21$, and $n_2 = 21$. Sangnawakij and Niwitpong (2016) showed that the data comes from two-parameter exponential distribution. The generalized confidence interval was $CI_{SCI} = (0.7690, 0.8992)$ with the length of interval 0.1302. The large sample confidence interval was $CI_{ILS} = (0.7554, 0.9023)$ with the length of interval 0.1469. In comparison, the adjusted MOVER confidence intervals were $CI_{AM1} = (0.2236, 1.5606)$ with the length of interval 1.3370 and $CI_{AM2} = (0.7554, 0.9023)$ with the length of interval 0.1469. The numerical results show that the confidence interval $CI_{AM1}$ contains the true coefficients of variation in all cases. Hence, this confirms our simulation study in the previous section for $k = 2$.

4. Discussion and conclusions

This paper proposes confidence intervals for the weighted coefficients of variation of two-parameter exponential distributions. The confidence intervals were constructed based on the adjusted MOVER method, then compared with the GCI and the large sample method. The coverage probabilities and average lengths of the proposed confidence intervals were evaluated through Monte Carlo simulations. The adjusted MOVER confidence interval $CI_{AM1}$ is a conservative confidence interval. Therefore, the adjusted MOVER confidence interval $CI_{AM1}$ is not recommended for confidence interval for the weighted coefficients of variation of two-parameter exponential distributions. However, the generalized confidence interval $CI_{SCI}$ should be chosen to estimate the weighted coefficients of variation of two-parameter exponential distributions. Moreover, the adjusted MOVER confidence interval $CI_{AM2}$ can be used when the coefficient of variation is a positive value.

As a final note, Sangnawakij and Niwitpong (2016) showed that the generalized confidence interval for single coefficient of variation of two-parameter exponential distribution is an exact confidence interval, i.e. the coverage probability is at least the nominal confidence level of 0.95. This paper has indicated that the generalized confidence interval for the weighted coefficients of variation of two-parameter exponential distributions is not an exact confidence interval for $k \geq 2$.

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