FIELD REDEFINITIONS IN STRING THEORY AS
A SOLUTION GENERATING TECHNIQUE

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Abstract
The purpose of this work is to show that there exists an additional invariance of the $\beta$-function equations of string theory on $d + 1$-dimensional targets with $d$ toroidal isometries. It corresponds to a shift of the dilaton field and a scaling of the lapse function, and is reminiscent of string field redefinitions. While it preserves the form of the $\beta$-function equations, it changes the effective action and the solutions. Thus it can be used as a solution generating technique. It is particularly interesting to note that there are field redefinitions which map solutions with non-zero string cosmological constant to those with zero cosmological constant. Several simple examples involving two- and three-dimensional black holes and black strings are provided to illustrate the role of such field redefinitions.

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Vigorous research in string theory in non-trivial backgrounds in the past few years has yielded a number of novel approaches for the construction and classification of string vacua. One such method is known as string duality, or twisting procedure, or \( O(d, d+n) \) boosting (in presence of \( n \) independent gauge fields) \[1\]-[7]. It employs the fact that many string solutions posses toroidal symmetries associated with the appearance of commuting translational Killing vectors spanning the tangent-space basis. At the level of the effective field theory on target space, the symmetry takes a very simple form. One can rewrite the effective action in terms of the independent degrees of freedom, after dimensional reduction \( a \ la \) Kaluza-Klein, and extract the \( O(d, d+n) \) group as a part of the full invariance group of the theory. Although the action and the equations of motion (the \( \beta \)-function equations) are invariant under this group, the initial conditions are not. Thus under an \( O(d, d+n) \) transformation a solution to the equations of motion transforms, in principle, to a different solution. (One has to remember that \( O(d, d+n) \) transformations contain diffeomorphisms and gauge transformations which have to be factored out, since they don’t change solutions [4].)

In this letter, I will show that there exists another transformation which leaves the \( \beta \)-function equations form-invariant, although the action and the solutions are changed. It is realized by shifting the dilaton field and scaling the lapse function, and in its appearance is similar to a string field redefinition, which does not change the physics of the fundamental string [8]. For this reason, and for the sake of brevity, I will tentatively refer to it as a ”field redefinition”, dropping the term ”string” to indicate that in general they are different. Therefore, in what follows, by field redefinition it is meant a transformation in the above sense, while the label string field redefinition refers to the conventional ambiguity of string theory. This field redefinition is not completely arbitrary, for one must require that the transformation preserve the form of the lowest order \( \beta \)-function equations while changing solutions. Viewed as a consistency condition, this imposes a constraint on the field redefinition parameters. The constraint admits non-trivial solutions for all string configurations from the class considered here, namely with all but one toroidal isometries. The field redefinitions
which solve the constraint are sufficiently general to produce new solutions. It is particularly interesting to note that among them there are field redefinitions which map solutions with string cosmological constant to those without. The role of such field redefinitions is illustrated with several simple examples involving two- and three-dimensional black holes and black strings.

The effective action of string theory describing dynamics of the massless bosonic background fields to the lowest order in the inverse string tension $\alpha'$ expansion is, in the world-sheet frame

$$S = \int d^{d+1}x \sqrt{g} e^{-\Phi} \left( R + \partial_{\mu} \Phi \partial^{\mu} \Phi - \frac{1}{12} H_{\mu\nu\lambda} H^{\mu\nu\lambda} - \frac{\alpha'}{4} F_{\mu\nu} F^{\mu\nu} + 2\Lambda \right)$$

(1)

The action above is written in Planck units $\kappa^2 = 1$. Here $F_{\mu\nu} = \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu}$ are field strengths of $n$ Abelian gauge fields $A^j_{\mu}$, $H_{\mu\nu\lambda} = \partial_\lambda B_{\mu\nu} + \text{cyclic permutations} - (\alpha'/2) \Omega_{\mu\nu\lambda}$ is the field strength associated with the Kalb-Ramond field $B_{\mu\nu}$ and $\Phi$ is the dilaton field. The $n$ Abelian gauge fields should be thought of as the components of a non-Abelian gauge field $A$ residing in the Cartan subalgebra of the gauge group, while the rest have been set equal to zero. In this letter I will set $\alpha' = 1$. The Maxwell Chern-Simons form $\Omega_{\mu\nu\lambda} = \sum_N A_{\mu}^N F_{\nu\lambda}^N + \text{cyclic permutations}$ appears in the definition of the axion field strength due to the Green-Schwarz anomaly cancellation mechanism, and can be understood as a model-independent residue after dimensional reduction from ten-dimensional superstring theory. In fact, this term is a necessary ingredient of the theory if one wants to ensure the $O(d, d+n)$ invariance, as shown by Maharana and Schwarz [7].

Under the assumption that all but one coordinates are toroidal, one can dimensionally reduce the action (1) to its effective form describing dynamics of those stationary points with $d$ commuting isometries. The reduction is a generalization of the Kaluza-Klein dimensional reduction in presence of matter fields, and is straightforward, if tedious [1]-[7]. Below I will briefly review some of the main ingredients, to the extent necessary for the purpose of this letter. I will work with the assumption that there are no cross terms of the form $dr dx^k$ in the metric and the axion. In fact, it is easy to show that when all but one coordinates are toroidal, this is the most general ansatz, by employing gauge transformations and diffeomorphisms. The cross terms $dr dx^k$ in the metric corresponding
to the "shift" functions can be removed by coordinate transformations $x^k \rightarrow x^k + F^k(r)$. The remaining cross terms in the axion field and the $r$ component of the gauge fields can be removed by gauge transformations. To see that this is consistent with the equations of motion for these modes, one just needs to recall that they are homogeneous for gauge degrees and diffeomorphism and gauge invariant, and hence admit trivial solutions. Therefore, the backgrounds studied here will be of the form

$$
\begin{align*}
    ds^2 &= \Gamma(r) \, dr^2 + G_{jk}(r) \, dx^j dx^k \\
    B &= \frac{1}{2} B_{jk}(r) dx^j \wedge dx^k \\
    A^N &= A^N_j(r) dx^j \\
    \Phi &= F(r)
\end{align*}
$$

where the $d \times d$ matrix $G_{jk}(r)$ is either of signature $d$ (in which case $\Gamma > 0$ and $r$ is the time coordinate) or of signature $d-2$ (when $\Gamma < 0$ and $r$ is a space-like coordinate). The lapse function $\Gamma$ is kept arbitrary as its variation in (1) yields the constraint equation.

With this ansatz, the action (1) can be rewritten in the manifestly $O(d, d+n)$ invariant form:

$$
S_{\text{eff}} = \int dr \sqrt{\Gamma} e^{-\phi} \left( \frac{1}{\Gamma} \phi'^2 + \frac{1}{8\Gamma} Tr(\mathcal{M}'\mathcal{L})^2 + 2\Lambda \right)
$$

where the prime denotes the derivative with respect to $r$. One has to distinguish between the true dilaton $\Phi$ and the effective dilaton after dimensional reduction $\phi = \Phi - (1/2) \ln |\det G|$. The two matrices $\mathcal{M}$ and $\mathcal{L}$ which appear in the action (3) are defined by

$$
\mathcal{M} = \begin{pmatrix}
g^{-1} & -g^{-1}C & -g^{-1}A \\
-C^T g^{-1} & g + a + C^T g^{-1}C & A + C^T g^{-1}A \\
-g^{-1}A & A + C^T g^{-1}A & 1 + A^T g^{-1}A
\end{pmatrix}
$$

$$
\mathcal{L} = \begin{pmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{pmatrix}
$$
Here \( g \) and \( b \) are \( d \times d \) matrices defined by the dynamical degrees of freedom of the metric and the axion: \( g = (G_{jk}) \) and \( b = (B_{jk}) \). The matrix \( A \) is a \( d \times n \) matrix built out of the gauge fields: \( A_{kN} = A_N^k \). The matrix \( a \) is defined by \( a = AA^T \), and \( C \) by \( C = (1/2)a + b \). The first two blocks \( 1 \) in \( L \) are \( d \times d \) matrices, and the last is \( n \times n \). Note that \( M^T = M \) and \( M^{-1} = LML \). Therefore, \( M \) is a symmetric element of \( O(d, d + n) \). An invariance of the theory then is obviously a chiral \( O(d, d + n) \) rotation \( M \rightarrow \Omega M \Omega^T \), which preserves the symmetry property of \( M \). The complete set of \( \beta \)-function equations can be obtained from (3) by the standard variational procedure [2]:

\[
\frac{\phi'^2}{\Gamma} + \frac{1}{8\Gamma} Tr(M'L)^2 = 2\Lambda
\]

\[
\left(\frac{e^{-\phi}}{\sqrt{\Gamma}}\phi'\right)' + 2\Lambda \sqrt{\Gamma} e^{-\phi} = 0
\]

\[
\left(\frac{e^{-\phi}}{\sqrt{\Gamma}} M'M^{-1}\right)' = 0
\]

(5)

The \( M \) equation is readily integrable. Obviously [2],

\[
M' = \sqrt{\Gamma} e^\phi J M
\]

(6)

We recognize \( J \) to be a restriction on \( M \) of the Maurer-Cartan form for the group \( O(d, d + n) \). As a consequence, \( J \) is a constant element of the Lie algebra of \( O(d, d + n) \), and satisfies two relations which in terms of \( K = JL \), can be written as \( K^T = -K \) and \( KLML = -MKL \). Now one can compute the trace of \((M'L)^2\): \( Tr(M'L)^2 = -\Gamma \exp(2\phi)Tr J^2 = -8\lambda \Gamma \exp(2\phi) \). I have introduced \( \lambda = (1/8)Tr J^2 = \text{const.} \) as a short-hand notation for this trace. The remaining equations of (5) can be rewritten as

\[
\frac{\phi'^2}{\Gamma} = 2\Lambda + \lambda e^{2\phi}
\]

\[
\left(\frac{e^{-\phi}}{\sqrt{\Gamma}}\phi'\right)' = -2\Lambda \sqrt{\Gamma} e^{-\phi}
\]

(7)
They and the first integral (6) comprise the full set of equations equivalent to the \( \beta \)-function system (5). Actually, the first of Eq. (7) is the first integral of the second, so in principle the second equation can be ignored. However, it will be kept here as it will turn out to be useful later.

Before solving these equations, it is advisable to inspect if they admit any symmetries which could be of interest. Indeed, the equations (6)-(7) are of fairly general nature, and knowing any symmetries in addition to the \( O(d, d + n) \) invariance could provide us with further insight into the structure of the solution space of the theory. The requirement of symmetry can be somewhat relaxed. One can ask if there are invariances of the \( \beta \)-function equations (i.e., their first integrals (6)-(7)) which leave the form of the \( \beta \)-functions the same but change the action (3) and the solutions. In this case, all one needs to require is that the realization of this invariance must map solutions on solutions. That there is an additional invariance of the system (6)-(7) of this type can be seen as follows. Let the dilaton and the lapse simultaneously undergo a shift and scaling according to

\[
\phi \to \tilde{\phi} = \phi + \chi \quad \Gamma \to \tilde{\Gamma} = \Gamma \exp(-2\chi)
\]

while the "matter" \( \mathcal{M} \) is left unchanged; obviously, the equation (6) is invariant under this transformation. Note that this implies that the metric, axion and gauge fields in the subspace of the original \( d + 1 \)-dimensional manifold spanned by the Killing vectors also remain unchanged. To assure that \( \tilde{\chi}, \tilde{\Gamma} \) are solutions one must impose the form-invariance of (7) under the transformation (8). This will be true if the field redefinition parameter is constrained to satisfy

\[
\chi^2 + 2\phi'\chi' \left. \left. - 2\Gamma \left( \Lambda e^{-2\chi} - \Lambda \right) \right| \right. = 0
\]

\[
\left( \frac{e^{-\phi}}{\sqrt{\Gamma}} \chi' \right)' + 2\sqrt{\Gamma} e^{-\phi} \left( \Lambda e^{-2\chi} - \Lambda \right) = 0
\]

Here \( \Lambda \) appears since (6)-(7) was required to be form-invariant, which is more relaxed. This shows that \( \tilde{\chi}, \tilde{\Gamma} \) solve (6)-(7) with a different value of the cosmological constant. As a consequence, if there exist solutions of (6) with \( \Lambda \neq \Lambda \), the transformations (8) generated by them will represent
maps between solutions with different cosmological constant. Moreover, using (9) it is easy to see
that the action changes by just a boundary term: \( \delta S = \tilde{S}_{eff} - S_{eff} = 2 \int dr \left[ \exp(-\phi) / \sqrt{\Gamma} \right] \chi' \).
Therefore, the field redefinition (8) represents a symmetry of the action in the generalized sense,
since the variation is reduced to the boundary.

At this point one needs to solve the constraint (9), and see if it admits solutions other then the trivial, \( \chi = 0 \). Recalling that the lapse \( \Gamma \) is a gauge parameter, one can fix it to be \( \Gamma = \exp 2\phi \). Then Eq. (9) reduce to

\[
\chi'' + \frac{\Gamma'}{\Gamma} \chi' = 2\Gamma \left( \tilde{\Lambda} e^{-2\chi} - \Lambda \right)
\]

The simplicity of the second equation has been hinted at earlier by retaining the second order
differential equation in (7). The general non-trivial solution is \( \chi = \chi_0 + \ln(r + \omega) \) with the two integration constants \( \chi_0 \) and \( \omega \) constrained by the first of Eq. (10). In order to see if there are any allowed values of the parameters, one needs to find \( \Gamma \). In the gauge chosen for it, the system (7) also simplifies greatly, and the most general solution for \( \Gamma \) is

\[
\Gamma = \left( 2\Lambda r^2 + \omega r + \frac{\omega^2}{8\Lambda} - \frac{\lambda}{2\Lambda} \right)^{-1}
\]

which is correct even when \( \Lambda \to 0 \) (as can be seen from (7), \( \omega^2 \to 4\lambda + \Lambda \eta_0 \), so there exists a non-singular limit). Substituting this in the first of Eq. (10) gives the algebraic form of the constraint (9). It is

\[
4\Lambda \omega = \omega \pm \sqrt{4\lambda + 16\tilde{\Lambda} e^{-2\chi_0}}
\]

After determining those parameters which solve (12), the new solution can be obtained from (2)
and (8), keeping in mind that the metric, axion and gauge fields in the subspace spanned by the
Killing vectors do not change (and therefore the physical dilaton \( \Phi \) changes in the same way as the
effective dilaton \( \phi \) because \( | \det G | \) is invariant):
\[ ds^2 = e^{-2\chi(r)} \Gamma(r) \, dr^2 + G_{jk}(r) \, dx^j dx^k \]

\[ B = \frac{1}{2} B_{jk}(r) dx^j \wedge dx^k \]  

\[ A^N = A^N_j(r) dx^j \]

\[ \Phi = F(r) + \chi(r) \]  

(13)

It is obvious that in general (13) will represent a solution different from (2) if \( \chi \) exists. There still remains the question if this solution will also correspond to a different physical situation. That there might be such an ambiguity can be noted from the similarity between the field redefinition (8) and a special type of the string field redefinition, defined by (for simplicity, in the absence of gauge fields):

\[ G_{\mu\nu} \rightarrow \tilde{G}_{\mu\nu} = G_{\mu\nu} + S(R, (\partial \Phi)^2) \partial_\mu \Phi \partial_\nu \Phi \]

\[ \Phi \rightarrow \tilde{\Phi} = \Phi + T(R, (\partial \Phi)^2) \]  

(14)

with the axion unchanged. In the case studied here, since all the fields depend only on one variable \( r \), the equations (8) would be of the form (14), if \( \chi = T = -(1/2) \ln[1 + S(\partial \Phi)^2] \). To determine whether the field redefinition (8) is precisely the same as (14), one needs to restore the \( \alpha' \) dependence in the action and the solutions and then evaluate \( \chi \). However, since \( \chi \) relates solutions with different cosmological constants, it is hard to see how in general it would be possible to write it as an analytic function of \( R \) and \( (\partial \Phi)^2 \) only. I hope to return to this issue elsewhere.

In order to illustrate the arguments above, I will present several examples involving two- and three-dimensional black holes, and investigate the effect of the field redefinition (8) on these solutions. The two-dimensional solutions are the simplest possible. There are several apparently different two-dimensional stringy solutions, related to the Witten two-dimensional black hole [9] either by an \( O(1, 2) \) duality or by extending the Witten solution to include all the higher order \( \alpha' \)
corrections. Actually, as noted recently by Tseytlin [10], the extension is related to the original solution by a string field redefinition, and therefore to distinguish between them we need to investigate dynamics of other fields in these backgrounds (e.g., tachyon [10]). On the other hand, it is well known that the 2d black hole extended to all orders in $\alpha'$ is non-singular [11]. This indicates that the singularity of the Witten solution is not as dangerous as are the singularities in conventional General Relativity, because it disappears from one version of the two solutions. It is introduced in the metric by the particular form of the string field redefinition, or in other words, it is related to the specific subtraction scheme adopted in string field theory. In its semiclassical singular form, the Witten 2d black hole is given by

$$
 ds^2 = \frac{1}{2\Lambda} \frac{dr^2}{r(r-\mu)} - (1 - \frac{\mu}{r})dt^2
$$

$$
 \Phi = - \ln \sqrt{2\Lambda r}
$$

(15)

Since $\phi = \Phi - (1/2)|\det G|$, and so $\exp(2\phi) = 2\Lambda r(r-\mu)$, this is precisely the gauge in which the detailed form of the field redefinition (8) is worked out. For this solution, $\omega = -2\Lambda\mu$ and $\lambda = \Lambda^2\mu^2$.

The constraint (12) then reads

$$
 \varpi = \frac{1}{2}\mu \pm \frac{1}{2} \sqrt{\mu^2 + 4\frac{\Lambda}{\Lambda} e^{-2\chi_0}}
$$

(16)

In the case when $\tilde{\Lambda} \neq 0$, the values of cosmological constants and $\chi_0$ combine into a single parameter, and (16) can be inverted to give $\tilde{\Lambda} = \Lambda \varpi (\varpi + \mu) \exp(2\chi_0)$. I will skip the details and just state that the redefined solution actually turns out to be identical to (13), after a coordinate transformation, as can be verified by some straightforward algebra. The only interesting cases then are when $\tilde{\Lambda} = 0$. Then either $\varpi = -\mu$ or $\varpi = 0$. The field redefinition by $\chi = \chi_0 + \ln(r-\mu)$ gives, after some manipulation, the dual of the Rindler solution in two dimensions

$$
 ds^2 = dz^2 - \frac{dt^2}{z^2}
$$

$$
 \Phi = \Phi_0 - 2 \ln z
$$

(17)
which is related by Wick rotation to the singular 2d cosmology discovered by Veneziano \[1\]. The other field redefinition, $\chi = \chi_0 + \ln r$, gives, as one should guess, the dual of \((17)\), i.e. the 2d Rindler solution:

$$
    ds^2 = dz^2 - z^2 dt^2 \\
    \Phi = \Phi_0
$$

(18)

One may wonder why these field redefinitions have not reproduced the result of Tseytlin, i.e. gave the exact form of \((15)\) to all orders in $\alpha'$. This is because the constraint \((9)\) was enforced on the field redefinition, so it does not change the form of \((3)\)-(\(5\)), whereas the string field redefinition used in \[10\] changes the equations of motion by introducing corrections of higher order in $\alpha'$. Thus the two solutions do not solve the same equations although they represent the same physical situation.

In three dimensions, it is most convenient to investigate the action of \((8)\) on the black string solution of Horne and Horowitz \[12\]. It is not difficult to see that all field redefinitions with $\varpi$ different from both $-\mu$ and $-q^2/\mu$ and with $\tilde{\Lambda} \neq 0$ do not change the form of the black string, much the same way as in the two-dimensional black hole example above. For brevity, I will consider in detail only the two special field redefinitions which change the solution. The black string solution is, in the gauge adopted here, given by

$$
    ds^2 = \frac{1}{2\Lambda} \frac{dr^2}{(r-(q^2/\mu))(r-\mu)} + (1 - \frac{q^2}{\mu r})dx^2 - (1 - \frac{\mu}{r})dt^2 \\
    B = \frac{q}{r} dx \wedge dt \\
    \Phi = -\ln \sqrt{2\Lambda r}
$$

(19)

Here, $\omega = -2\Lambda \mu (1 + (q/\mu)^2)$ and $\lambda = \Lambda^2 \mu^2 (1 - (q/\mu)^2)^2$. The constraint \((12)\) becomes

$$
    \varpi = -\frac{1}{2} \mu (1 + \frac{q^2}{\mu^2}) \pm \frac{1}{2} \sqrt{\mu^2 (1 - \frac{q^2}{\mu^2})^2 + \frac{4}{\Lambda} e^{-2\chi_0} \tilde{\Lambda}}
$$

(20)
Let $\varpi = 0$ and $\tilde{\Lambda} = \Lambda$ (then, $\exp(-2\chi_0) = q^2$). The transformed solution is, in terms of the new coordinates $z = (\mu - r)/\mu r$, $t = \vartheta/\sqrt{\mu}$ and $x = \sqrt{\mu} \tau/q$, with $m = (\mu^2 - q^2)/q^2 \mu$ and after a gauge shift of the axion $B \rightarrow B - d(\vartheta d\vartheta)/\mu$, found to be exactly the stringy version \[13\] of the static three-dimensional black hole \[14\]:

$$
\begin{align*}
 ds^2 &= \frac{1}{2\Lambda} \frac{dz^2}{z(z - m)} + zd\vartheta^2 - (z - m)d\tau^2 \\
 B &= z d\tau \wedge d\vartheta \\
 \Phi &= \Phi_0
\end{align*}
$$

(21)

It has been demonstrated before that the three-dimensional black string was related to the three-dimensional spinning black hole by a duality transformation. With the field redefinition approach one can relate the black string to the static three-dimensional black hole. Note that this is the only way to field-redefine the solution so that the dilaton is constant when the axion is present. When $q = 0$, the solution reduces to the Witten black hole extended with a flat coordinate, and then it is possible to map it to a flat solution in three dimensions, as illustrated by the argument leading to \[18\].

In the last example, let the black string \[19\] be extremal: $q = \mu$. Then, if $\tilde{\Lambda} = 0$, the constraint \[20\] admits the unique solution $\varpi = -\mu$. After some manipulation, the new solution can be put in the form of a singular configuration of the type studied in \[15\]:

$$
\begin{align*}
 ds^2 &= dz^2 + \frac{d\vartheta^2 - d\tau^2}{z} \\
 B &= \frac{1}{z} d\tau \wedge d\vartheta \\
 \Phi &= \Phi_0 - \ln z
\end{align*}
$$

(22)

To summarize, it is clear that the field redefinition method presented in this letter can be useful in the study of string solutions. It is a concise and practical tool for the construction of semiclassical string vacua. While not a substitute for the powerful method of string duality, in the
simplest situations exhibited here, their usefulness is perhaps compatible. In fact, its value and versatility are enhanced when used in conjunction with duality. The method provides a supplement to duality because it can be used to relate solutions with different values of the string cosmological constant. This offers further means to distinguish between the roles played by what we perceive as the cosmological constant in General Relativity and in string theory. It has been proposed recently in the first reference of [13] and in [16] that the status of the string cosmological constant must be fundamentally different from the status of the cosmological constant in General Relativity. The arguments were based on the observation that string duality in some cases relates asymptotically flat solutions to those which are not asymptotically flat, although both solve the $\beta$-function equations with the string cosmological term (e.g. the 3$d$ black string - 3$d$ black hole duality). The field redefinitions described here support this observation, as they provide direct means to connect solutions with different string cosmological term. Furthermore, the field redefinitions could also be useful when combined with the singular limits approach, another unorthodox method for generating string solutions [17].

It would be interesting to see how this procedure generalizes on backgrounds which have less isometries than what was assumed here, i.e. with two or more non-toroidal coordinates. In fact, it is not very hard to conclude that the procedure will admit rather straightforward generalization to the problems with three or more non-toroidal coordinates, on the basis of the form of the dimensionally reduced action in those cases. For the problems with two non-toroidal coordinates (which encompass such physically interesting situations as four-dimensional stationary spherically symmetric backgrounds) the issue is more subtle and needs to be addressed with more care. If it turns out to be possible to adapt the method described here to such cases, then we may be able to use it with the string version of the Geroch group-extended duality and thus enhance both approaches [18].
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