Possible Frictionless States at Room-Temperature Regime for Many Fermions in Confined Domain

Zotin K.-H. Chu
P.O. Box 39, Toudiban, Road Xihong, Urumqi 830000, China

Abstract
We investigate the possible frictionless transport of many composite (condensed) fermions at room temperature regime along an annular tube with transversely wavy-corrugations by using the verified transition-rate model and boundary perturbation approach. We found that for certain activation volume and energy there exist possible frictionless states at room temperature regime.

Keywords: Activation energy, boundary perturbation

1 Introduction
Understanding the frictionless transport of system of (interacting or noninteracting) fermions is crucial to our knowledge of the nature considering the neutron star, the superconductivity, the superfluidity, etc. One relevant study is the existence and characteristics of the Fermi surface for a system of interacting fermions. Usually such a surface is introduced into momentum space only for a system of noninteracting fermions.

To be specific, this surface represents the limit of occupation of the different single-particle momentum states in the ground state of the system. All the states with momentum contained within this surface are occupied, all those with momentum outside this surface are unoccupied. However, Migdal [1] noticed that, under some circumstances, the mean occupation number of different single-particle momentum states in the true ground state still possessed a discontinuity for a system of interacting fermions. Meanwhile, as Luttinger pointed out, the existence of the Fermi surface depends on the nature of forces between the fermions [2].

Other relevant study is, e.g., thermodynamical properties of trapped noninteracting Fermi gases in gravitational fields [3] which was originated in view of the successful experiments with the trapping and cooling of weakly interacting fermionic isotopes [4]. There is no doubt that it is important to theoretically investigate the thermodynamical and statistical properties of system of fermions even there are many approaches [5-11]. Here we shall investigate the possible frictionless transport of many fermions in a confined domain via a different approach considering the shearing response of many-fermion system. Note that researchers have been interested in the question of how matter responds to an external mechanical load. External loads cause transport, in Newtonian or various types of non-Newtonian ways. Amorphous matter, composed of poly-
mers, metals, or ceramics, can deform under mechanical loads, and the nature of the response to loads often dictates the choice of matter in various applications. To the best knowledge of the author, the simplest model that makes a prediction for the rate and temperature dependence of shear yielding is the rate-state model of stress-biased thermal activation [12-14]. Structural rearrangement is associated with a single energy barrier $E$ that is lowered or raised linearly by an applied stress $\sigma$: 

$$R_{\pm} = \nu_0 \exp\left[-E/(k_BT)\right] \exp[\pm \sigma V^*/(k_BT)]$$

where $k_B$ is the Boltzmann constant, $\nu_0$ is an attempt frequency and $V^*$ is a constant called the 'activation volume'. In amorphous matter, the transition rates are negligible at zero stress. Thus, at finite stress one needs to consider only the rate $R_+$ of transitions in the direction aided by stress.

The linear dependence will always correctly describe small changes in the barrier height, since it is simply the first term in the Taylor expansion of the barrier height as a function of load. It is thus appropriate when the barrier height changes only slightly before the system escapes the local energy minimum. This situation occurs at higher temperatures; for example, Newtonian transport is obtained in the rate-state model in the limit where the system experiences only small changes in the barrier height before thermally escaping the energy minimum. As the temperature decreases, larger changes in the barrier height occur before the system escapes the energy minimum (giving rise to, for example, non-Newtonian transport). In this regime, the linear dependence is not necessarily appropriate, and can lead to inaccurate modeling. To be precise, at low shear rates ($\dot{\gamma} \leq \dot{\gamma}_c$), the system behaves as a power law shear-thinning material while, at high shear rates, the stress varies affinely with the shear rate. These two regimes correspond to two stable branches of stationary states, for which data obtained by imposing either $\sigma$ or $\dot{\gamma}$ exactly superpose.

In this paper, considering the general forcing (either gravity [3] or electric field [6]) we shall adopt the verified transition-rate-state model [12-14] to study the frictionless transport of many fermions within a corrugated annular tube. To obtain the law of shear-thinning matter for explaining the too rapid annealing at the earliest time, because the relaxation at the beginning was steeper than could be explained by the bimolecular law, a hyperbolic sine law between the shear (strain) rate $\dot{\gamma}$ and shear stress $\tau$ was proposed and the close agreement with experimental data was obtained. This model has sound physical foundation from the thermal activation process [12-14] (a kind of (quantum) tunneling which relates to the matter rearranging by surmounting a potential energy barrier was discussed therein). With this model we can associate the (shear-thinning) fluid with the momentum transfer between neighboring atomic clusters on the microscopic scale and reveals the atomic interaction in the relaxation of flow with dissipation (the momentum transfer depends on the activation (shear) volume $V^* \equiv V_h$ which is associated with the center distance between atoms and is equal to $k_BT/\tau_0$ (T is temperature in Kelvin, and $\tau_0$ a constant with the dimension of stress).

To consider the more realistic but complicated boundary conditions in the walls of the annular tube, however, we will use the boundary perturbation technique [15-16] to handle the presumed
wavy-roughness along the walls of the annular tube. To obtain the analytical and approximate solutions, here, the roughness is only introduced in the radial or transverse direction. The relevant boundary conditions along the wavy-rough surfaces will be prescribed below. We shall describe our approach after this section: Introduction with the focus upon the transition-rate approach and boundary perturbation method. The approximate expression of the transport is then demonstrated at the end. Finally, we will illustrate our results and give discussions therein.

2 Formulations

We firstly take into account the condensed system of many fermions subjected to random thermal fluctuations (under external forcing). In thermally-activated motion, mobile fermions in a many-fermion system may interact with other fermions even they are already in a preferred motion. The rate of deformation (strain) is controlled by the rate at which thermal energy can help the composite systems overcome their energy barriers, allowing the rest of other fermions to spread. The attractive interactions involving pairs of moving fermions lower the energy as the mechanisms that enable them to form a composite moving subsystem cost energy. Repulsive interactions, on the other hand, require less work to overcome, and do not usually transform the composite subsystem or leave residual subsystem after the interaction. Parts of condensed fermions easily pull away after having been forced to pass the repulsive obstacle by an external stress. Accordingly, attractive interactions require thermal activation and are said to be temperature-dependent, while repulsive ones are not.

In fact, thermal energy is supplied by random thermal fluctuations, and motion of composite (condensed) fermions depends on the number of fluctuations that supply the interacting subsystems the energy they need. The number of such successful outcomes is \( N_s = NP(\text{success}) \), where \( N \) is the number of attempts in the complete many-fermion system. The probability of success is the probability that the thermal jump \( U_j \) is greater than \( \Delta E \), the energy required to surmount the barrier and assumed to follow an Arrhenius law [12-14] which is \( P(\text{success}) = P(U_j > \Delta E) = \exp(-\Delta E/k_BT) \). Considering the strain gained at each successful attempt leads to the thermal-activation controlled expression for the strain rate \( \dot{\gamma}_p = \dot{\gamma}_{p_0} \exp(-\Delta E(\tau_s)/k_BT) \). Here \( \dot{\gamma}_{p_0} \) is an intrinsic value that has units of strain rate and depends on the average waiting time at the intersection point, the strain released after the events, and the frequency of thermal fluctuations, which is some fraction of the Debye frequency \( (10^{13} \text{ s}^{-1}) \). We remind the readers that \( \Delta E(\tau_s) \) is a function of \( \tau_s \) which is a concentrated shear stress due to the short-range interaction with another subsystem (or obstacle), emphasizing the localized nature of thermally-activated events. It is possible to attribute \( \Delta E \) in above expression to one type of thermally-activated process. This simplification applies when one process has a much smaller \( \Delta E \) than the rest. Thermally controlled deformation, however, is a complex collective phe-
nomenon of many thermally-activated processes. To continue to use this theory, $\Delta E$ must be treated as an effective energy covering all possible types of such processes.

We shall consider a steady transport of many fermions in a wavy-rough annular tube of $r_1$ (mean-averaged inner radius) with the inner interface being a fixed wavy-rough surface: $r = r_1 + \epsilon \sin(k\theta + \beta)$ and $r_2$ (mean-averaged outer radius) with the outer interface being a fixed wavy-rough surface: $r = r_2 + \epsilon \sin(k\theta)$, where $\epsilon$ is the amplitude of the (wavy) roughness, $\beta$ is the phase shift between two walls, and the roughness wave number: $k = 2\pi/L$ ($L$ is the wavelength of the surface modulation in transverse direction).

Firstly, this matter (composed of many condensed (composite) fermions) can be expressed as

$$\dot{\gamma} = \dot{\gamma}_0 \sinh(\tau/\tau_0),$$

where $\dot{\gamma}$ is the shear rate, $\tau$ is the shear stress, $\tau_0 = 2k_BT/V_h$, and $\dot{\gamma}_0 = C_k k_B T \exp(-\Delta E/k_B T)/h$ is a constant relating rate of strain to the jump frequency ($V_h = \lambda_2 \lambda_3 \lambda$, $V_m = \lambda_2 \lambda_3 \lambda_1$, $\lambda_2 \lambda_3$ is the cross-section of the transport unit on which the shear stress acts, $\lambda$ is the distance jumped on each relaxation, $\lambda_1$ is the perpendicular distance between two neighboring layers of particles sliding past each other), accounting for the interchain co-operation required, $h$ is the Planck constant, $\Delta E$ is the activation energy.

In fact, the force balance gives the shear stress at a radius $r$ as $\tau = -(r \delta \mathcal{G})/2$ [15]. $\delta \mathcal{G}$ is the net effective external (gravity or electric field) forcing along the transport (or tube-axis: $z$-axis) direction (considering $dz$ element). Introducing the forcing parameter $\phi = -(r_2/2\tau_0)\delta \mathcal{G}$ then we have $\dot{\gamma} = \dot{\gamma}_0 \sinh(\phi r/r_2)$. As $\dot{\gamma} = -du/dr$ ($u$ is the velocity of the transport in the longitudinal ($z$)-direction of the annular (cosmic) string), after integration, we obtain

$$u = u_s + \frac{\dot{\gamma}_0 r_2}{\phi} [\cosh \phi - \cosh(\frac{\phi r}{r_2})],$$

(1)

here, $u_s (\equiv u_{slip})$ is the velocity over the (inner or outer) surface of the annular (cosmic) string, which is determined by the boundary condition. We noticed that a general boundary condition for transport over an interface [15] was

$$\delta u = L_s^0 \dot{\gamma}_s (1 - \frac{\dot{\gamma}}{\dot{\gamma}_c})^{-1/2},$$

(2)

where $\delta u$ is the velocity jump over the interface, $L_s^0$ is a constant slip length, $\dot{\gamma}_c$ is the critical shear rate at which the slip length diverges. The slip (velocity) boundary condition above (related to the slip length) is closely linked to the mean free path of the particles together with a geometry-dependent factor (it is the quantum-mechanical scattering of Bogoliubov quasiparticles which is responsible for the loss of transverse momentum transfer to the confined surfaces [17]). The value of $\dot{\gamma}_c$ is a function of the corrugation of interfacial energy.

With the slip boundary condition [15], we can derive the velocity fields and transport rates along the wavy-rough annular tube below using the verified boundary perturbation technique [15-16] and dimensionless analysis. We firstly select $L_s^0$ to be the characteristic length scale and set $r' = r/L_s^0$, $R_1 = r_1/L_s^0$, $R_2 = r_2/L_s^0$, $\epsilon' = \epsilon/L_s^0$. After this, for simplicity, we drop all the
primes. It means, now, \( r, R_1, R_2 \) and \( \epsilon \) become dimensionless (\( \phi \) and \( \dot{\gamma} \) also follow). The wavy boundaries are prescribed as \( r = R_2 + \epsilon \sin(k\theta) \) and \( r = R_1 + \epsilon \sin(k\theta + \beta) \) and the presumed steady transport is along the \( z \)-direction (annulus-axis direction).

### 2.1 Boundary Perturbation

Along the outer boundary (the same treatment below could also be applied to the inner boundary), we have \( \dot{\gamma} = (du)/(dn) \) on interfaces. Here, \( n \) means the normal. Let \( u \) be expanded in \( \epsilon \):

\[
u = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \cdots,
\]

and on the boundary, we expand \( u(r_0 + \epsilon dr, \theta(= \theta_0)) \) into

\[
u(r, \theta)|_{(r_0 + \epsilon dr, \theta_0)} = u(r_0, \theta) + \epsilon [du_1 + \epsilon [du_2 + \cdots]]dr, \theta.
\]

Considering \( L \approx R_1, R_2 \gg \epsilon \) case, we can derive the velocity field \( u \) up to the second order:

\[
u(r, \theta) = -(R_2\gamma_0/\phi)\{\cosh(\phi r/R_2) - \cosh \phi [1 + \epsilon^2 \phi^2 \sin^2(k\theta)/(2R_2^2)]\} + u_{slip}|_{r=R_2+\epsilon r \sin(k\theta)}.
\]

The key point is to firstly obtain the slip velocity along the boundaries or surfaces. After lengthy mathematical manipulations, we obtain the velocity fields (up to the second order) and then we can integrate them with respect to the cross-section to get the transport (volume flow) rate \( Q \), also up to the second order:

\[
Q = \int_0^{\theta_0} \int_{R_2 + \epsilon \sin(k\theta)}^{R_2+\epsilon \sin(k\theta)} u(r, \theta) r d\theta = Q_0 + \epsilon Q_{p1} + \epsilon^2 Q_{p2}.
\]
In fact, the approximate (up to the second order) net transport (volume flow) rate reads:

\[
Q = \pi \gamma_0 \left(L_s^0(R_2^2 - R_1^2) \sinh \phi \left(1 - \frac{\sinh \phi}{\gamma_c/\gamma_0}\right)^{-1/2} + \frac{R_2}{\phi} \left[(R_2^2 - R_1^2) \cosh \phi - \frac{2}{\phi}(R_2^2 \sinh \phi - R_1 R_2 \sinh(\phi R_1/R_2)) + \frac{2 R_2^2}{\phi^2} (\cosh \phi - \cosh(\phi R_1/R_2))\right]\right] + \epsilon^2 \left\{ \frac{\pi}{2} u_{\text{slip}_0}(R_2^2 - R_1^2) + L_s^0 \frac{\pi}{4} \gamma_0 \sinh \phi \left(1 + \frac{\sinh \phi}{\gamma_c/\gamma_0}\right) \left(-k^2 + \phi^2\right) [1 - \left(\frac{R_1}{R_2}\right)^2] + \frac{\pi}{2} \gamma_0 R_1 \sinh(\frac{R_1}{R_2} \phi) - R_2 \sinh \phi\right\} + \frac{\pi^2}{2} \gamma_0 \frac{R_2}{\phi} \cosh \phi + L_s^0 \frac{\pi}{4} \gamma_0 \frac{\cosh \phi}{\gamma_c/\gamma_0} \left[\sinh \phi + L_s^0 \cosh(1 + \frac{\sinh \phi}{\gamma_c/\gamma_0})(R_2 - R_1 \cos \beta)\right] + \frac{\pi}{2} \gamma_0 \frac{R_2}{\phi} \cosh \phi.
\]

Here,
\[
u_{\text{slip}_0} = L_s^0 \gamma_0 \left[\sinh \phi \left(1 - \frac{\sinh \phi}{\gamma_c/\gamma_0}\right)^{-1/2}\right].
\]

3 Results and Discussions

With above detailed derivations, now, we firstly check the presumed wavy-roughness effect (or combination of curvature and confinement effects) upon the possible frictionless transport of many condensed (composite) fermions because there are no available experimental data and numerical simulations for the same geometric configuration (annular tube with wavy corrugations in transverse direction). With a series of forcings (due to externally imposed gravity or electric field forcings) : \(\phi \equiv -R_2(\delta G)/(2\gamma_0)\), we can determine the enhanced shear rates \((d\gamma/dt)\) due to these forcings. From equation (5), we have (up to the first order)

\[
\frac{d\gamma}{dt} = \frac{d\gamma_0}{dt} [\sinh \phi + \epsilon \sin(k\theta) \frac{\phi}{R_2} \cosh \phi].
\]

The parameters are fixed below (the orientation effect : \(\sin(k\theta)\) is fixed here). \(r_2\) (the mean outer radius) is selected as the same as the slip length \(L_s^0\). The amplitude of wavy roughness can be tuned easily. The effect of wavy-roughness is significant once the forcing \((\phi)\) is rather large (the maximum is of the order of magnitude of \(\epsilon[\phi \tan(h)/(R_2)]\)).

If we select a (fixed) temperature, then from the expression of \(\tau_0\), we can obtain the shear stress \(\tau\) corresponding to above gravity forcings \((\phi)\):

\[
\tau = \tau_0 \sinh^{-1} [\sinh(\phi) + \epsilon \sin(k\theta) \frac{\phi}{R_2} \cosh(\phi)].
\]

There is no doubt that the orientation effect \((\theta)\) is also present for the condensed many-fermion system. For illustration below, we only consider the maximum case : \(|\sin(k\theta)| = 1\). The wave number of roughness in transverse direction is fixed to be a constant.
As the primary interest of present study is related to the possible frictionless transport or formation of superfluidity (presumed to be relevant to the many-fermion system as mentioned in Introduction) due to strong shearing, we shall present our main results in the following. Note that, based on the absolute-reaction-rate Eyring model (of stress-biased thermal activation), structural rearrangement is associated with a single energy barrier (height) $\Delta E$ that is lowered or raised linearly by a (shear) yield stress $\tau$. If the transition rate is proportional to the plastic (shear) strain rate (with a constant ratio $: C_0; \dot{\gamma} = C_0 R_t$, $R_t$ is the transition rate in the direction aided by stress), we have

$$
\tau = 2\left[ \frac{\Delta E}{V_h} + \frac{k_B T}{V_h} \ln\left( \frac{\dot{\gamma}}{C_0 \nu_0} \right) \right] \quad \text{if} \quad \frac{V_h \tau}{k_B T} \gg 1
$$

(10)

where $\nu_0$ is an attempt frequency or transition rate, $C_0 \nu_0 \sim \dot{\gamma}_0 \exp(\Delta E/k_B T)$, or

$$
\tau = 2\frac{k_B T}{V_h} \frac{\dot{\gamma}}{C_0 \nu_0} \exp(\Delta E/k_B T) \quad \text{if} \quad \frac{V_h \tau}{k_B T} \ll 1.
$$

(11)

It is possible that the frictional resistance (or shear stress) can be almost zero (existence of $\tau \sim 0$) from above equations (say, equation (10) considering a sudden jump of the resistance).

The nonlinear character only manifests itself when the magnitude of the applied stress times the activation volume becomes comparable or greater in magnitude than the thermal vibrational energy.

Normally, the value of $V_h$ is associated with a typical volume required for a microscopic shear rearrangement. Thus, the nonzero transport rate (of the condensed many-fermion system) as forcing is absent could be related to a barrier-overcoming or tunneling for shear-thinning matter along the wavy-roughness (geometric valley and peak served as atomic potential surfaces) in annular tubes when the wavy-roughness is present. Once the geometry-tuned potentials (energy) overcome this barrier, then the tunneling (spontaneous transport) inside wavy-rough annular tubes occurs.

Finally, We demonstrate in Fig. 1 that if we select the activation energy to be $4 \times 10^{-19}$ J we can then observe a sudden drop of the resistance (frictional or shear stress) around 3 order of magnitude at $T = 300.5^\circ K$ ($V_h \approx 3.12 \times 10^{-21} \text{m}^3$). It means there is a rather low resistance below the temperature : $T \sim 300^\circ K$ for the material parameters selected. As $\tau \sim 0$ (below $T \sim 300$ K), from $|\tau| = r_2 \delta G/2 \ (r_2 \neq 0)$, we can understand that there is no need for any external (gravity or electric field) forcing ($\delta G \sim 0$) once the persistent current occurs.

The possible reasoning for this frictionless transport of many condensed (composite) fermions can be stressed again as a brief summary. It could be due to the strong shearing driven by larger external (say, gravity or electric field) forcings along a confined wavy-rough tube. The shear-thinning (the viscosity diminishes with increasing shear rate) reduces the viscosity for the transport of this condensed many-fermion system significantly. One possible outcome for almost vanishing viscosity is the nearly frictionless transport. We shall investigate other relevant issues [18-20] in the future.
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Fig. 1 Calculated (shear) stresses or resistance using an activation energy $4 \times 10^{-19}$ J. There is a sharp decrease of shear stress around $T \sim 300.5^\circ K$. Below around 300 K ($V_h \approx 3.12 \times 10^{-21} m^3$), the transport of many composite (condensed) fermions is nearly frictionless.