On the dust abundance gradients in late-type galaxies: I. Effects of destruction and growth of dust in the interstellar medium

Lars Mattsson1*, Anja C. Andersen1 & Joakim D. Munkhammar2
1 Dark Cosmology Centre, Niels Bohr Institute, University of Copenhagen, Juliane Maries Vej 30, DK-2100, Copenhagen Ø, Denmark
2 Dept. of Engineering Sciences, Solid State Physics, Uppsala University, Box 534, S-751 21 Uppsala, Sweden

ABSTRACT
We present basic theoretical constraints on the effects of destruction by supernovae (SNe) and growth of dust grains in the interstellar medium (ISM) on the radial distribution of dust in late-type galaxies. The radial gradient of the dust-to-metals ratio is shown to be essentially flat (zero) if interstellar dust is not destroyed by SN shock waves and all dust is produced in stars. If there is net dust destruction by SN shock waves, the dust-to-metals gradient is flatter than or equal to the metallicity gradient (assuming the gradients have the same sign). Similarly, if there is net dust growth in the ISM, then the dust-to-metals gradient is steeper than or equal to the metallicity gradient. The latter result implies that if dust gradients are steeper than metallicity gradients, i.e., the dust-to-metals gradients are not flat, then it is unlikely dust destruction by SN shock waves is an efficient process, while dust growth must be a significant mechanism for dust production. Moreover, we conclude that dust-to-metals gradients can be used as a diagnostic for interstellar dust growth in galaxy discs, where a negative slope indicates dust growth.

Key words: Galaxies: evolution, ISM; ISM: clouds, dust, extinction, evolution, supernova remnants;

1 INTRODUCTION
The lifetime of dust grains in the interstellar medium (ISM) is a critical parameter for the evolution of the dust component in a galaxy. Shock-waves originating from supernovae (SNe) arguably contain enough energy to destroy (or at least shatter) dust grains as these waves propagate through the ISM. The time scale for such dust destruction depends on several physical conditions, where the supernova rate (SNR) and efficiency of dust destruction in a SN-shock are the most important (McKee 1989; Draine 1990). Shock destruction of dust grains has been considered quite efficient in many studies, e.g., Jones, Tielens & Hollenbach (1996); Jones (2004); Serra Díaz-Cano & Jones (2008), suggest a grain lifetime of a few times 108 yr for many dust species, but note that a recent re-evaluation of dust lifetimes by Jones & Nuth (2011) showed that the lifetimes of silicate grains may be comparable to the injection time scale of such grains.

While this high dust-destruction efficiency seems consistent with the Milky Way (solar neighbourhood), it has been shown by several authors that very efficient dust destruction is unlikely in high-z objects (Dwek et al. 2007; Gall, Andersen & Hjorth 2011; Mattsson 2011). It may of course be that dust destruction by SNe is less efficient in high-z galaxies, but also modelling of nearby late-type galaxies seems to work nicely without significant net destruction of dust (Inoue 2003; Hirashita 1999). In fact, no or little net dust destruction makes it easier to explain the dust-to-gas ratios, since stellar dust production is likely not sufficient for neither the Milky Way, nor any of the other late-type local group galaxies (Hirashita 1999; Zhukovska, Gail & Truloff 2008).

Observational constraints imply dust production in SNe is rather inefficient (Kotak et al. 2006, 2009), which suggest the high dust masses detected in some, relatively old, SN remnants (see, e.g., Morgan & Edmunds 2003; Morgan et al. 2003; Dunne et al. 2009; Gall, Hjorth & Andersen 2011) could be the result of subsequent dust growth, and/or heating of pre-existing dust, rather than dust production in the actual SN. However, since SN remnants with large dust masses typically contain vast amounts of cold dust, some degree of growth appear to be necessary even if there is a component of heated swept-up dust. This picture is consistent with theoretical results which suggest 90% of the dust produced in SNe is destroyed by the reverse shock before it reaches the ISM (Bianchi & Schneider 2007). Hence AGB stars could be the most important source of stellar dust as a significant fraction of the metals expelled by these stars is expected to be in the form of dust (Edmunds & Eales 1998; Ferrarotti & Gail 2006), which is supported by observational detections of dust (see, e.g., the recent results by Ramstedt et al. 2011). It should be noted, however, that this picture may need to be revised due to the recent discovery of a large...
amount of cold dust associated with SN 1987A (Matsuura et al. 2011).

In models of dust evolution for the solar neighbourhood by Dwek (1998) and Zhukovska, Gail & Trieloff (2008), the limited dust production in stars and possible dust destruction by SN-shock waves are more than well compensated by an efficient dust growth in the ISM, which is supported by observations indicating the existence of large, micrometer-sized dust grains in dense molecular clouds (Pagani et al. 2010). There is further evidence from dust-evolution modelling (see, e.g. Mattsson 2011; Pipino et al. 2011; Valiante et al. 2011) along with some observational constraints (see, e.g. Michalowski et al. 2010) which suggests a need for significant dust growth in the early Universe as well. Dust growth appears to dominate over dust destruction also in the local, present-day Universe (Hirashita 1999; Inoue 2003; Hirashita & Kuo 2011; Asano et al. 2012). It is difficult to separate one scenario where dust growth is totally dominating over dust destruction from another where there is very little dust destruction and less dust growth. But whether there is net growth/destruction it should have observable consequences, however.

We propose here a diagnostic tool for determining whether there is net dust growth or net dust destruction in the ISM of a late-type galaxy for which dust-to-gas as well as metallicity gradients can be derived. As we will show in this paper, the change (gradient) of the dust-to-metals ratio along a galactic disc is closely connected to growth and destruction of dust in the ISM. In an associated paper (Mattsson & Andersen 2011, hereafter cited as Paper II), we investigate the implications of observed dust-to-metals profiles in a selection of late-type galaxies from the SINGS (Kennicutt et al. 2003) sample.

2 BASIC EQUATIONS

In order to obtain analytical solutions and be able to manipulate the basic equations of the dust-enrichment problem in such a way that we can derive some basic constraints, we use the instantaneous recycling approximation (IRA), which essentially means all stars are assumed to have negligible lifetimes with respect to the overall time scale for the build-up of metals and dust, see Page (1997) throughout this paper. No delayed element production due to stellar lifetimes is considered.

For convenience, we also define the dust destruction rate relative to the growth rate of the stellar component $\Sigma_\text{ISM}$ as

$$ D(r,t) \equiv \Sigma_\text{ISM}(r,t) \left( \frac{d \Sigma}{dt} \right)^{-1}, $$

where $\Sigma_\text{ISM}$ is the dust destruction rate due to SNe and the variables $r, t$ are the galactocentric distance and time/age respectively (a notation that we will assume is understood in the following). Similarly, we also define

$$ G(r,t) \equiv \Sigma_g(r,t) \left( \frac{d \Sigma}{dt} \right)^{-1}, $$

where $\Sigma_g$ is the rate of grain growth (in mass units) in the ISM.

Assuming a 'closed box', where dust destruction in the ISM is from SN-shocks, the equations for the metallicity $Z$ and the dust-to-gas ratio $Z_d$ becomes

$$ \frac{dZ}{dt} = y_Z \frac{d\Sigma}{dt} = -y_d \frac{d\Sigma_g}{dt}, $$

$$ \frac{dZ_d}{dt} = y_d \frac{d\Sigma_g}{dt} + Z_d(r,t) [G(r,t) - D(r,t)] \frac{d\Sigma}{dt}, $$

where $\Sigma_g$ is the gas density, $\Sigma_d$ is the dust density, and the yield $y_i$ is defined as

$$ y_i = \frac{1}{\alpha} \int_{m_{\text{lo}}}^{m_{\text{up}}} p_i(m) \phi(m) \, dm, $$

for both metals ($i = Z$) and stellar dust ($i = d$). In equation (5) above, $p_i$ is the fraction of the initial mass $m$ of a star ejected in the form of newly produced metals or dust, $\alpha$ is the stellar lock-up fraction (i.e., the fraction of the baryon mass being locked up in long lived stars) and $\phi(m)$ is the mass-normalised IMF, with $m_{\text{lo}}, m_{\text{up}}$ being the lower and upper mass cuts, respectively. Combining equations (3) and (4), we have

$$ \frac{d\Sigma}{dt} = \frac{\partial D}{\partial Z} \frac{dZ}{dt} + \frac{\partial D}{\partial Z_d} \frac{dZ_d}{dt} = \frac{\partial G}{\partial Z} \frac{dZ}{dt} + \frac{\partial G}{\partial Z_d} \frac{dZ_d}{dt}. $$

which thus have no explicit dependence on the gas mass density $\Sigma$ or the stellar mass density $\Sigma_\text{ISM}$, although $G, D$ and $Z_d$ of course may have implicit dependencies on the amount of gas and stars being present in a certain galactic environment.

3 CONSTRAINTS ON DUST-TO-METALS GRADIENTS

We will now prove some basic properties of dust-to-metals ($\zeta$) gradients relative to the metallicity ($Z$) gradient. For 'logarithmic' dust-to-metals and metallicity gradients we use the following notations,

$$ \Delta_{\zeta} \equiv \frac{\partial \ln \Sigma_\zeta}{\partial \ln Z} = \frac{\partial Z_\zeta}{Z_\zeta} \frac{\partial Z_\zeta}{\partial Z} = \frac{1}{\partial Z} \frac{\partial Z_\zeta}{\partial Z}, $$

$$ \Delta_Z \equiv \frac{\partial \ln Z}{\partial \ln Z} = \frac{1}{Z} \frac{\partial Z}{\partial Z} = \frac{1}{Z} \frac{\partial Z}{\partial Z} = \frac{1}{Z} \frac{\partial Z}{\partial Z}, $$

which are used since they both have the same unit (length$^{-1}$). The two gradients $\Delta_{\zeta}$ and $\Delta_Z$ can be regarded as coupled through a function $f$ which may be seen as a function of a number of physical parameters, but in general we may say it is a function of time $t$ and radial position (galactocentric distance) $r$ along the galaxy. Hence, we consider a relation of the form $\Delta_{\zeta}(r,t) = f(r,t) \Delta_Z(r,t), t$. In the following we will implicitly assume all quantities except $y_d, y_Z$ are functions of $r$ and $t$. We will also refer to the case of a zero derivative with respect to $r$ as a 'flat' gradient, which of course could be seen as the case of no gradient. However, we prefer to describe the $\zeta$- and $Z$-gradient as being either positive, flat or negative, where 'negative' refers to a gradient (derivative) which decreases with galactocentric radius and vice versa for 'positive' gradients. Below we also use the sign function $\text{sgn}(x) \equiv x/|x|$ to denote the sign of $\Delta_{\zeta}$ and $\Delta_Z$.

THEOREM. For a closed-box model, without any pre-enrichment, and where the IRA and constant yields $y_Z, y_d$ have been adopted, the following always hold:

(i) A flat (no slope) dust-to-metals gradient can only be obtained if there is neither net growth, nor any net destruction of dust in the ISM ($G = D$) or if the metallicity gradient is flat.

(ii) If the dust-to-metals and metallicity gradients have the same sign, there has to be net growth ($G > D$) of dust in the ISM.

(iii) If the dust-to-metals and metallicity gradients have opposite signs, there has to be net destruction ($G < D$) of dust in the ISM.
Proof. From the basic equations of dust evolution (see section 2) one finds
\[
\frac{\partial c}{\partial Z} = \frac{y_0}{yz} \left[ (G - D) \frac{Z}{yz} - 1 \right] \xi.
\]
By use of the chain rule we get
\[
\frac{dc}{dr} = \frac{y_0}{yz} \left[ \frac{y_0}{z^2} + \xi \left( \frac{G - D}{yz} - 1 \right) \frac{dZ}{dr} \right],
\]
which in terms of \( \Delta_c \) and \( \Delta_z \), can be written as
\[
\Delta_c = \frac{y_0}{yz} \xi + \frac{Z}{yz} \left( G - D \right) - 1 \Delta_z.
\]
The function \( f \) (see definition above) is then generally expressed
\[
f = \frac{y_0}{yz} \xi + \frac{Z}{yz} \left( G - D \right) - 1.
\]
(i) If \( \Delta_z = 0 \), then obviously \( \Delta_c = 0 \) as a consequence of Equation (11). In case there is neither net growth, nor any net destruction of dust in the ISM \((G = D)\), we have
\[
f = \frac{y_0}{yz} \xi - 1.
\]
Equation (6) gives
\[
\frac{dZ_a}{dZ} = \frac{y_0}{yz},
\]
and again by the chain rule,
\[
\left( \frac{dZ_a}{dr} \right)_{G,D} = \frac{y_0}{yz} \frac{dZ}{dr}.
\]
Integrating equation (15), together with the natural initial conditions \( Z(r,0) = Z_0(r,0) = 0 \) (no pre-enrichment), one obtains \( \xi = y_0/yz \), or
\[
\frac{y_0}{yz} \xi = 1.
\]
Hence, according to Equation (13), we must have \( \Delta_c = 0 \), since \( f = 0 \), which proves part (i).

(ii) First, we note that if \( \text{sgn}(\Delta_c) = \text{sgn}(\Delta_z) \), then \( f > 0 \). In case \( G > D \), Equation (6) gives
\[
\left( \frac{dZ_a}{dr} \right)_{G,D} > \left( \frac{dZ_a}{dr} \right)_{G,D}.
\]
Then, by Equation (8) and the fact that \( f = 0 \) if \( G = D \), we have \( \Delta_c,G,D > \Delta_c,G,D = 0 \), which implies \( f > 0 \). In case \( G < D \), Equation (6) gives
\[
\left( \frac{dZ_a}{dr} \right)_{G,D} < \left( \frac{dZ_a}{dr} \right)_{G,D}.
\]
Again, using Equation (8) and if \( f = 0 \) if \( G = D \), we have \( \Delta_c,G,D < \Delta_c,G,D = 0 \), which implies \( f < 0 \). Hence,
\[\text{sgn}(\Delta_c) = \text{sgn}(\Delta_z) \text{ is possible if (and only if) } G > D, \text{ which proves part (ii)}.\]

(iii) In this case, if \( \text{sgn}(\Delta_c) \neq \text{sgn}(\Delta_z) \), then \( f < 0 \). If \( G > D \), Equation (6) gives
\[
\left( \frac{dZ_a}{dr} \right)_{G,D} > \left( \frac{dZ_a}{dr} \right)_{G,D}.
\]
Analogous to case (ii) we have \( \Delta_c,G,D > \Delta_c,G,D = 0 \), which implies \( f > 0 \). In case \( G < D \), Equation (6) gives
\[
\left( \frac{dZ_a}{dr} \right)_{G,D} < \left( \frac{dZ_a}{dr} \right)_{G,D}.
\]
Thus, we have \( \Delta_c,G,D < \Delta_c,G,D = 0 \), which implies \( f < 0 \) and therefore \( \text{sgn}(\Delta_c) \neq \text{sgn}(\Delta_z) \) is possible if (and only if) \( G < D \), which proves part (iii).

\[\square\]

4 SIMPLE MODELS OF DUST GROWTH AND DUST DESTRUCTION

4.1 Dust growth in the ISM

The most likely dominant type of \textquoteleft secondary\textquoteright dust production is that by accretion of atoms (or small molecules) onto pre-existing interstellar dust grains. Dust grains can in principle also grow by coagulation, but this process will not affect the total dust mass very much since it is mostly smaller dust grains being joined together into larger grains. Hence, we will here only discuss dust growth by accretion.

We define the rate per unit volume at which the number of atoms \( N_A \) in dust grains grows by accretion of metals onto these dust grains in a similar way as (see, e.g., Dwek 1998)
\[
d\frac{N_A}{dt} = f_i \pi a^2 n_g \langle v_f \rangle,
\]
where \( n_g \) and \( n_g \) are the total atomic metals and dust-grain number densities in the ISM, respectively, \( a \) is the typical grain radius and \( f_i \) is the sticking coefficient (i.e., the probability that an atom will stick to the grain). \( \langle v_f \rangle \) is the mean thermal speed of the gas particles (including metals), which is defined as
\[
\langle v_f \rangle \equiv \int_0^\infty v f(v) dv = \frac{8 kT}{\pi m_A},
\]
where \( f(v) \) is the Maxwell distribution, \( k \) is the Boltzmann constant, \( T \) is the kinetic temperature of the gas and \( m_A \) is the atomic weight of the gas particles. In terms of surface densities in the molecular gas clouds where the dust may grow, we can write
\[
d\frac{\Sigma_d}{dt} = f_i \pi a^2 \Sigma_g \langle v_f \rangle \langle m_g \rangle d_i, \tag{23}
\]
where \( \Sigma_g \) is the surface density of free (atomic) metals, \( \langle m_g \rangle \) is the mean mass of the dust grains in the ISM and \( d_i \) is the size of the molecular cloud in which the dust is growing. The timescale of grain growth can then be expressed as
\[
\tau_{gr} = \tau_0 \left( \frac{Z_i}{Z} \right)^{-1}, \tag{24}
\]
where
\[
\tau_0 = \frac{\langle m_g \rangle d_i}{f_i \pi a^2 \Sigma_g \langle v_f \rangle} \approx \frac{\langle m_g \rangle d_i}{f_i \pi a^2 \Sigma_{mol} \langle v_f \rangle}, \tag{25}
\]
in which $\Sigma_{\text{mol}}$ is the surface density of molecular gas, and $Z$ the metallicity.

For simplicity we will assume $\Sigma_{\text{mol}} \approx \Sigma_{\text{H$_2$}}$, since most of the gas in the molecular gas clouds is in the form of molecular hydrogen. We also assume $\Sigma_{\text{H$_2$}}$ traces the star-formation rate, i.e.,

$$\Sigma_* = \tau \Sigma_{\text{H$_2$}},$$

(26)

as indicated by several observational studies (e.g. Rownd & Young 1999, Wong & Blitz 2002, Bigiel et al. 2008; Leroy et al. 2008; Bigiel et al. 2011; Feldmann, Gnedin & Kravtsov 2011; Schruba et al. 2011). Such a relation is also supported by theory and recent numerical experiments (see, e.g. Krumholz, McKee & Tumlison 2009, Krumholz, Leroy & McKee 2011). Moreover, the mean thermal speed $(v_\text{t})$ is roughly constant in the considered ISM environment and the typical grain radius does not vary much. Hence, the timescale $\tau_0$ is essentially just a simple function of the metallicity, the gas abundance and the growth rate of the stellar component,

$$\tau_0^{-1} \approx \frac{\epsilon Z d\Sigma_\epsilon}{\Sigma dt},$$

(27)

the constant $\epsilon$ will, in the following, be treated as an essentially free (but not unconstrained) parameter of the model. The expected value is on the order of a few hundred, which is required to obtain $\tau_0 \approx 10^7$ yr, suggested above. We will here adopt

$$\left(\frac{d\Sigma_\epsilon}{dt}\right)_\epsilon = \left(1 - \frac{Z_0}{Z}\right) \frac{\Sigma_\epsilon}{\tau_0} = \frac{Z_0}{\tau_\epsilon},$$

(28)

as the rate of change of the dust-to-gas ratio $Z_\epsilon$ due to accretion of metals onto pre-existing dust grains in the ISM. Note that this formulation of ‘secondary’ dust production differs from that used by Edmunds (2001) and Mattsson (2011) in that it also depends on the dust abundance in the ISM and the depletion of metals in atomic state.

4.2 Dust destruction

The dominant mechanism for dust destruction is by sputtering in the high-velocity interstellar shocks driven by SNe, which can be directly related to the energy of the SNe (Nozawa & Kozasa 2006). Following McKee (1989), Dwek et al. (2007) the dust destruction time-scale is

$$\tau_d = \frac{\Sigma_\epsilon}{\langle m_{\text{ISM}} \rangle R_{\text{SN}}},$$

(29)

where $\Sigma_\epsilon$ is the gas mass density, $\langle m_{\text{ISM}} \rangle$ is the effective gas mass cleared of dust by each SN event, and $R_{\text{SN}}$ is the SN rate, which may be approximated as

$$R_{\text{SN}}(t) \approx \Sigma_\epsilon (r, t) \int_{\text{SN}} \phi(m) dm.$$  

(30)

The integral in equation (30) is a constant with respect to time, and is not likely to vary much over the disc either, hence the time scale $\tau_d$ may be expressed as

$$\tau_d^{-1} \approx \frac{\delta}{\Sigma_\epsilon} \frac{d\Sigma_\epsilon}{dt},$$

(31)

where $\delta$ will be referred to as the dust destruction parameter. This parameter is dimensionless, and as such it can be seen as a measure of the efficiency of dust destruction. More precisely, however, the efficiency is set by the fraction $f_d$ of interstellar dust destroyed in an encounter with a SN shock wave, which occurs in the definition of $\langle m_{\text{ISM}} \rangle$ McKee (1989), Dwek et al. (2007).

$$\langle m_{\text{ISM}} \rangle \equiv \int_{v_\text{SN}}^{v_\text{SN}_{\text{max}}} f_d(v) \left| \frac{dM_{\text{sw}}}{dv_s} \right| dv_s,$$

(32)

where $M_{\text{sw}}$ is the swept-up gas mass (during Sedov-Taylor expansion), $v_s$ is the shock velocity, and $v_\text{SN}$, $v_{\text{SN}_{\text{max}}}$ are the initial and the final velocity, respectively. Note that in this way $\delta$ is similar to the $\epsilon$-parameter (average grain-destruction efficiency) used by McKee (1989), which should not be confused with the $\epsilon$ (dust-growth parameter) introduced in the previous section. It should also be stressed that $f_d$ is not a constant, but a function of the shock velocity $v_s$.

A Larson (1998) IMF and $\langle m_{\text{ISM}} \rangle \sim 1000M_\odot$ (Dwek et al. 2007) suggests $\delta \sim 10$, which is likely close to an upper limit for $\delta$. Just as in the case of $\epsilon$ above, it is not absolutely clear, however, that $\delta$ can be treated as a parameter that does not vary during the course of evolution of the ISM in a galaxy, but it seems in a given environment a fair approximation.

5 ANALYTIC SOLUTIONS

For simplicity we have assumed a closed box (see section 3), i.e., no in- or outflows to/from the disc. This is not in agreement with the widely accepted ideas about galaxy-disc formation, where the baryons (in the form of essentially pristine gas) are assumed to be accreted over an extended period of time. But as shown by Edmunds (1990), the only major effect of unenriched infall is to make the effective yield smaller, i.e., to dilute the gas so that the metallicity builds up more slowly. As we in this study uses the present-day metallicity as input, the overall effects of assuming a closed box are rather small, and in general only accretion of metal-enriched gas can affect the dust-to-metals ratio significantly (see Appendix B).

5.1 General solution

Adopting the closed-box scenario, the dust destruction and dust growth models as described above, results in an equation for dust evolution,

$$\Sigma_\epsilon \frac{dZ_\epsilon}{dt} = \left[ y_\epsilon + Z_\epsilon \left( \frac{1}{Z} - \frac{Z_\epsilon}{Z} \right) \right] \frac{d\Sigma_\epsilon}{dt}.$$  

(33)

which combined with the metallicity $Z$ gives

$$\frac{dZ_\epsilon}{dZ} = \frac{1}{y_\epsilon} \left[ y_\epsilon + Z_\epsilon \left( \frac{1}{Z} - \frac{Z_\epsilon}{Z} \right) \right],$$

(34)

where $y_\epsilon$ is the metal yield. Provided $Y_\epsilon < y_\epsilon$, the general closed-box solution (of equation (33) for the dust-to-gas ratio $Z_\epsilon$ in terms of the metallicity $Z$ is (see Appendix C) for a sketchy derivation),

$$Z_\epsilon = \frac{Y_\epsilon}{y_\epsilon} \left( Z - \frac{\delta}{\epsilon} \right) \left[ \varphi_1 + \frac{1}{y_\epsilon} \left( \frac{\theta_1}{\gamma_1} - \frac{\gamma_1}{\theta_1} \left( 1 - \frac{\theta_1}{\gamma_1} \right) \right) \right]$$

(35)

for

$$\varphi_i(k, x, y) = M \left[ i + \frac{k}{2}, i + \frac{1}{2}, \frac{x}{2}, \frac{y}{2} \right] U \left[ i + \frac{k}{2}, i + \frac{1}{2}, \frac{y}{2}, \frac{y}{2} \right].$$

(36)

The functions $M$ and $U$ are the confluent hypergeometric Kummer-Tricomi functions of the first and second kind, respectively, Kummer (1837), Tricomi (1947), see also Appendix D.
In the equations above, \( y_d \) and \( y_z \) are the stellar dust and metal yields, respectively, \( \delta \) is the 'dust destruction parameter' (see section 4.2) and \( \epsilon \) is the 'grain-growth parameter' (see section 4.1). Equation (35) is singular at \( Z = \delta / \epsilon \), which means this general solution must be used with care. It is relatively straightforward to implement the Kummer-Tricomi functions numerically (see Appendix D), but there is a regular singularity at the origin in Kummer’s equation (to which \( M \) and \( U \) are linearly independent solutions) which can cause potential problems in the vicinity of \( Z = \delta / \epsilon \).

5.2 Special cases

The general solution presented above is obviously not simple to use in practice, not the least because of the singularity at \( Z = \delta / \epsilon \). However, in the special case \( y_d \rightarrow 0 \) (negligible net contribution of dust from stars) the singularity can be removed and the solution expressed as (see Appendix C)

\[
\frac{Z_d}{Z_{d,0}} = \exp\left(\sqrt{\frac{1}{2}} \left(\frac{\epsilon Z - \delta^2}{\epsilon y_z}\right)^2\right) \exp\left(\sqrt{\frac{1}{2}} \left(\frac{\epsilon Z_{0} - \delta^2}{\epsilon y_z}\right)^2\right) \exp\left(\eta_0 Z_d,0 \left[\text{erf}(\xi) - \text{erf}(\xi_0)\right]\right)^{-1}
\]  

(37)

with \( Z_{d,0} \) being the initial dust-to-metals ratio,

\[
\xi \equiv \frac{1}{\sqrt{2}} \left(\frac{\epsilon Z - \delta^2}{\epsilon y_z}\right)^2, \quad \xi_0 \equiv \frac{1}{\sqrt{2}} \left(\frac{\epsilon Z_{0} - \delta^2}{\epsilon y_z}\right)^2, \quad \eta_0 \equiv \frac{\sqrt{\pi \epsilon}}{2 y_z},
\]  

(38)

and \( Z_0 \) the initial metallicity. In the solution above, \( \text{erf}(z) \) is the imaginary error function, related to the ordinary error function \( \text{erf}(z) \) as \( \text{erf}(z) = -i\text{erf}(iz) \), where \( \text{erf}(z) \) is defined as

\[
\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.
\]  

(39)

Physically, one may interpret this solution as describing the subsequent evolution (where the dust contribution from stars may be considered negligible) after an initial phase of metal enrichment and dust formation leading up to the point where \( Z = Z_0 \) and \( Z_d, Z_{d,0} \). It may not be entirely realistic, but it demonstrates the interstellar “battle” between growth and destruction of dust grains in a very nice way (see Section 5.3 and Figure 5).

Even when there is a significant net contribution from stars, we can still find simpler solutions for special cases. In case there is no dust destruction by SNe (\( \delta = 0 \)) the solution reduces to

\[
Z_d = \frac{y_d}{y_z} \frac{M\left(1 + \frac{1}{2} + \frac{1}{2} \frac{\epsilon Z}{y_z} + \frac{1}{2} \frac{\delta^2}{y_z^2}\right)}{M\left(1 + \frac{1}{2} + \frac{1}{2} \frac{\epsilon Z_0}{y_z} + \frac{1}{2} \frac{\delta^2}{y_z^2}\right)} Z,
\]  

(40)

and if there is dust destruction, but no grain growth in the ISM (\( \epsilon = 0 \)), then

\[
Z_d = \frac{y_d}{\sigma} \left[1 - \exp\left(-\delta \frac{Z}{y_z}\right)\right].
\]  

(41)

If there is neither growth, nor destruction of dust in the ISM (\( \epsilon = \delta = 0 \)), we have the trivial case

\[
Z_d = \frac{y_d}{y_z} Z,
\]  

(42)

corresponding to pure stellar dust production and obviously a flat dust-to-metals gradient. All the special cases above evade the inconvenient singularity at \( Z = \delta / \epsilon \).

6 GRAPHIC ANALYSIS

Using the numerical implementation of \( M \) and \( U \) described in Appendix D we will here demonstrate the general behaviour of the dust-to-metals ratio \( \zeta = Z_d/Z \) using contour plots. Unless anything else is stated, we assume \( y_d = 0.02 \) is a good typical metal yield (which is consistent with the results of Paper II, but note that, e.g., Garnett et al. 2002 finds a lower value) and \( y_d = \frac{1}{2} y_z \) for simplicity.

6.1 General effects of growth destruction of dust in the ISM

In case of no dust destruction (\( \delta = 0 \)) the dust-to-metals ratio builds up to a maximum (where \( \zeta \sim 1 \)) as \( \epsilon \) and the metallicity \( Z \) increases (see Figure 6 left panel). At low metallicities (half of solar or less, in the present case) the effect of increasing \( \epsilon \) is relatively small once we get beyond a certain \( \epsilon \), while at higher metallicities \( \delta \) grows rapidly until the metals reservoir is exhausted and \( \zeta \) approaches unity (as also found in the models by, e.g., Hirashita & Kuc 2011; Asano et al. 2012). In case of no dust growth (\( \epsilon = 0 \)) the dust-to-metals ratio is on a steep "downhill slope" (approaching \( \zeta = 0 \)) for essentially all metallicities and \( \delta \)-values on the considered interval (see figure 2 left panel). Note that \( \delta \) is very small at high metallicity if there is significant dust destruction.

6.2 Dust-to-metals gradients

As shown by the theorem in section 4 the effect of dust destruction and dust growth on the dust-to-metals gradient in a galaxy disc is to make it steeper or flatter. The effect of growth and destruction of dust in the ISM can be illustrated in a more intuitive fashion if we consider the specific effects on a given metallicity profile. We here assume that metals in a disc follows an exponential distribution,

\[
Z(R) = Z_d \exp\left(-\frac{R}{R_0}\right),
\]  

(43)

where we set the central metallicity to \( Z_0 = 0.055 \) and the e-folding scale length \( R_0 \) is set to be the unit for the galactocentric distance. The right panel of figure 6 shows how dust growth creates a dust-to-metals gradient that falls of with galactocentric distance and becomes increasingly steeper as \( \epsilon \) increases (for \( \epsilon = 0 \) the gradient is flat). Similarly, the right panel of figure 6 shows how dust destruction creates an inwards gradient, starting from a flat gradient for \( \delta = 0 \).

In the context of dust-to-metals gradients as signs of either net dust growth or net dust destruction, one should as well note that if the metallicity gradient and the dust-to-gas gradients are essentially flat, it is more or less impossible to distinguish between pure stellar dust production (albeit with a high stellar dust yield) and scenario including dust growth and/or dust destruction in the ISM.

6.3 Dust growths vs. destruction

Growth and destruction of dust must likely occur together. As we describe in Appendix C the general solution with both \( \epsilon \) and \( \delta \) non-zero, has a singularity at \( Z = \delta / \epsilon \), which makes the analysis of how growth and destruction compete somewhat complicated and not least limited. Hence, we will here consider the special (and not entirely realistic) case where stellar dust production is considered negligible (see equation 37 starting from a point in time when the metallicity \( Z = 1.0 \cdot 10^{-3} \) and the dust-to-metals ratio \( \zeta = 0.5 \). In
6 Mattsson, Andersen & Munkhammar

figure 3, we show ħ as a function of e and δ for a fixed present-day metallicity Z = 0.02. Increasing the efficiency of dust destruction counteracts the dust growth, which is shown by the "downhill slope" towards high ħ and low e values. Clearly, a high efficiency of dust destruction is not likely if there is to be a significant net production of dust without invoking a ridiculously short time scale for the dust growth in the ISM. More precisely, it is required that eZ ≫ δ, which in case Z = 0.02 and δ = 10, would imply e ≫ 500. With such a large e the typical growth-time scale is down to ~10^6 yr or less. As we mentioned in section 4.1, ħ should not exceed values of a few hundred if the dust-growth time scale τd is to be consistent with the suggested numbers for the local ISM of the Milky Way. It is quite possible that τd can be significantly shorter in, e.g., a denser environment, but a deeper analysis of this goes beyond the scope of this paper.

6.4 "Critical" metallicity for dust growth

Just as Zhukovska, Gail & Trieloff (2008), Hirashita & Kud (2011) and Asano et al. (2012), we find that there exist a "critical metallicity" Zcrit where the dust-mass contribution from grain growth increases rapidly. But this rapid increase over orders of magnitude occurs only if the stellar dust yield Yd is significantly lower than the metal yield (see figure 3, left upper panel). Moreover, Zcrit depends somewhat on the dust-growth time scale (or e), which can be seen in figure 4 (right upper panel). Hence, Zcrit should not be viewed as a universal constant. In fact, a reasonable definition of Zcrit would be the metallicity at which stellar dust production and the net dust growth in the ISM contribute equally to build-up of the interstellar dust component. In such a case, adopting the model used above,

\[ Z_{\text{crit}} = Z_d + \frac{Y_d}{eZ_d} \frac{\delta}{\epsilon} \]  

(44)

If dust growth dominates over stellar dust production and dust destruction in the ISM is negligible, i.e., if \( Y_d/e \approx 1 \) and \( \delta/\epsilon \approx 1 \), then \( Z_{\text{crit}} \approx Z_d \), which suggests \( Z_{\text{crit}} \sim Y_d/Y_d \). At metallicities below this value, the dust evolution (as function of metallicity) should be essentially identical to the case of pure stellar dust production - without any growth or destruction of dust grains in the ISM. In the right upper panel of figure 4, Zcrit = Yd/Yd is marked by a vertical dashed line. At lower metallicities all model curves are indeed the same.

This critical metallicity Zcrit has an interesting implication for dust-to-metals/gas gradients, as it predicts the existence of bends also in logarithmic slopes and the existence of a critical galactocentric distance in between an inner and an outer "plateau" where \( \xi \) is constant (see Fig 3, lower panels). This non-linear feature is the consequence of the interstellar dust-growth rate (see equation 39) having a non-linear \( (Z_d)^2 \) term. Although equation 33 and equation 35 together represent one specific model, all models of interstellar dust growth will be non-linear as long as they depend on the amount of dust and "free" metals (not locked-up in dust) available.

7 DISCUSSION AND CONCLUSIONS

We have shown that dust destruction by shock waves from exploding SNe and interstellar dust growth acts in opposite ways on the dust-to-metals gradient over a galaxy disc (see the theorem proved in section 3). This is hardly surprising, but starting from an exactly flat gradient (or no gradient, more precisely) dust destruction will over time create an inwards slope, while dust growth will create an outwards slope, provided the dust-to-gas ratio as well as the metallicity have negative gradients, i.e., decreases with galactocentric distance. Hence, we expect dust-to-metals gradients to have positive (inwards) gradients if dust destruction is more important than dust growth, and if dust growth is the more important process we expect them to be negative in general. The dust-to-metals gradient thus appears to be a useful diagnostic for the existence of interstellar dust growth.

Our simple model of dust growth has just one adjustable parameter. This parameter (e) can have a rather wide range of numerical values depending on what one assumes about the physical properties of the dust grains as well as the gas in ISM. In principle e is proportional to the gas mass density \( \Sigma_g \), if all other quantities remain constant, but the star formation efficiency \( \eta \) is likely proportional to \( \Sigma_g \) raised to some power (Krumholz & McKee 2009) and since the cloud size \( d_c \) is also related to \( \Sigma_g \), e may not be much dependent on \( \Sigma_g \) after all. More precisely, the star-formation efficiency (or time scale) is expected to correlate with the free-fall time scale, i.e., \( \Sigma_g \propto \Sigma_g / t_{\text{ff}} \), where \( t_{\text{ff}} \propto \Sigma_g^{-1/2} \) (Krumholz, McKee & Tumlinson 2009), assuming \( \Sigma_g \propto \rho_v \) (Elmegreen 2003), and the scale of the cloud size \( d_c \) is given by Jeans length \( L_j \propto \Sigma_g^{-1/2} \). As \( e \propto \tau_d \), \( \Sigma_g \) and \( (\tau_d) \) is roughly constant (isothermal conditions), the effective dependence on \( \Sigma_g \) is expected to be weak, if not negligible. Thus, it is fair to assume that e is (effectively) only very weakly dependent on \( \Sigma_g \) within a galaxy, although from one galaxy to another e may vary significantly, however (see Paper II). Below we analyse the range of possible e values considering just mean/characteristic values of \( \Sigma_g, \eta \) and \( (\tau_d) \).

In terms of the included physical parameters (see section 4.1), we find

\[ e \approx \frac{f_3 \pi^2 \langle v_g \rangle}{\alpha (\eta) (d_c) (m_{\text{gr}}) \langle \Sigma_g \rangle} \]  

(45)

The lock-up fraction \( \alpha \) is 0.6 - 0.8 for a normal IMF (we use here \( \alpha = 0.7 \), see Mattsson 2011 figure 1), \( \langle \eta \rangle \sim 1 \text{ Gyr}^{-1} \) and since the typical size of a molecular cloud \( d_c \) is 10 - 100 pc, we adopt \( (d_c) = 50 \text{ pc} \). The average grain mass \( (m_{\text{gr}}) \) of course depends on the typical grain size \( a \), where the latter ranges between 0.001 μm for the smallest seed particles and ~ 1 μm for large full-grown dust grains. Hence, it is more convenient to introduce the characteristic grain density \( \rho_{\text{gr}} = (m_{\text{gr}}) / (V_g) \), where \( V_g \) is the volume of a dust grain. The grain density \( \rho_{\text{gr}} \) is typically 3.3 g cm⁻³ (Draine & Lee 2007) for silicates and 1.85 g cm⁻³ for amorphous carbon (Rouleau & Martin 1991), but other values can also be found in the literature. Taking \( \rho_{\text{gr}} = 2.5 \text{ g cm}^{-3} \) as representative figure for cosmic dust in general, we arrive at

\[ e \approx 4.2 \times f_3 \left( \frac{a}{\mu m} \right)^{-1} \left( \frac{\langle \Sigma_g \rangle}{M_c \text{ pc}^{-2}} \right) \]  

(46)

Assuming that all metals that come in contact with a dust grain will stick to that dust grain (\( f_s = 1 \)), a small characteristic grain size \( a = 0.01 \mu m \) and a relatively high average gas density of \( \Sigma_g = 50 M_c \text{ pc}^{-2} \), will result in an \( e \) of roughly 2 × 10^6 corresponding to a typical grain-growth time scale of \( (\tau_d) \sim 10^6 \text{ yr} \) if the gas consumption rate is similar to that of the solar neighbourhood. Such high values of \( e \) may be expected in young star-forming systems (e.g., late-type dwarf galaxies) where one has reasons to believe that gas densities are quite high and the grain-size distribution is biased towards small grains. The latter is due to insufficient time for extensive grain growth, and grain shattering owing to an elevated SN rate and strong UV radiation as consequences of recent star formation. If \( f_s = 0.1 \) (which is more consistent with silicate growth),
Figure 1. Left: Dust-to-metals ratio $\zeta = Z_d/Z$ as a function of the metallicity $Z$ and the dust-growth parameter $\epsilon$ for the case where there is no dust destruction due to SNe ($\delta = 0$). Right: Same as the left panel, but as a function of the galactocentric distance in a galaxy disc assuming an exponential distribution of metals.

Figure 2. Left: Dust-to-metals ratio $\zeta = Z_d/Z$ as a function of the metallicity $Z$ and the dust-destruction parameter $\delta$ for the case where there is no dust growth in the ISM ($\epsilon = 0$). Right: Same as the left panel, but as a function of the galactocentric distance in a galaxy disc assuming an exponential distribution of metals.

The model of dust destruction due to SN shock waves has effectively only one parameter as well. This dust destruction parameter $\delta$ can be expressed as

$$\delta = \frac{(m_{\text{ISM}})}{\alpha} \int_{m_{\odot}}^{1000 M_{\odot}} \phi(m) \, dm,$$

(47)

where $\phi$ is the IMF and $\alpha$ is the lock-up fraction, as previously defined. With $\alpha = 0.7$ and a normal IMF (see, e.g., Larson 1998), we find

$$\delta \approx 0.018 \left( \frac{(m_{\text{ISM}})}{M_{\odot}} \right),$$

(48)

which suggest $\delta$ is of order ten, if $(m_{\text{ISM}}) \sim 1000 M_{\odot}$. The actual
efficiency of dust destruction, and thus the effective interstellar gas mass cleared of dust, is not very well known. Therefore, it is reasonable to treat $\delta$ as an essentially free parameter. In order to have net growth of dust in the ISM, the value of $\delta$ needs to be $\delta < \epsilon (Z - Z_d)$. This means $\delta \sim 10$ is likely at the upper end of possible values for such a scenario, assuming $Z - Z_d \approx 0.01$, which suggest $\delta = 10$ would require $\epsilon > 1000$ or $\tau_{gr} \lesssim 10^7$ yr. High values of $\delta$ may be found in starburst environments, where high SN rates and possibly also top-heavy IMFs are expected. However, in general $\delta$ is likely small, since high rates of dust destruction are somewhat inconsistent with the fact that dust is ubiquitous throughout the Universe.

Although simplifying assumptions have been made in this study in order to obtain a reasonably simple parametric model in terms of $\epsilon$ and $\delta$, a clear outwards slope is unlikely to be the result of any other mechanism than dust growth in the ISM. Other mechanisms, which however appear less effective:

- Accretion of dust free material onto the galactic disc may affect the dust-to-metals ratio if the infalling gas contains some fraction of atomic metals (see Appendix B for further details and worked out examples). The metallicity of the accreted gas is likely much less than that of the ISM, so the effect cannot be very large and it would also mimic the effect of dust destruction rather than dust growth.
- Secondary dust production in stars, i.e., a stellar dust yield which increases as the metallicity of stars increases, may in principle create a dust-to-metals gradient along a galaxy disc. However, the relative increase of the stellar dust yield along the disc cannot be arbitrarily large. In particular, the dust-to-metals gradient can never become steeper than the metallicity gradient only owing to secondary dust production in stars (for further details and a more quantitative analysis, see Appendix A).
- The lifetime of stars may also play a role, but since the very same stars that are producing the metals are also responsible for the stellar production of dust, this effect cannot be dominant. In fact, it should be negligible.

Thus, we conclude that dust-to-metals gradients can be used as a diagnostic for interstellar dust growth in galaxy discs, where a negative slope indicates dust growth.

Dust growth has a non-linear nature as the time scale for it must depend on both the metallicity and the amount of available seed grains. As a consequence there is a “critical” metallicity (which depends on the dust-growth and dust-destruction time scales as well as the dust-to-gas ratio) at which the dust production by interstellar grain growth exceeds stellar dust production and the dust-to-gas ratio diverges from the steady increase obtained in case the dust mass is owing to stars only. This allows for bends in the logarithmic slopes of the dust-to-metals profile even if the metallicity follows an exponential fall-off with galactocentric distance. Dust destruction in the ISM due to SNe may also affect the shape of the dust-to-metals profile, creating a central depression as the dust-to-gas ratio, the metallicity and the integrated number of SNe typically increases in the central parts of a galaxy disc compared...
Figure 4. Effects of the critical metallicity for dust growth domination and its dependence on the stellar dust yield and dust growth parameter $\epsilon$. The upper panels show the evolution of the dust mass as a function of metallicity for various values of the stellar dust yield $y_d$ with a fixed $\epsilon = 200$ (left panel) and various values of $\epsilon$ with a fixed stellar dust yield $y_d = 5.0 \cdot 10^{-3} y_Z$ (right panel). The lower panels show the corresponding plots of the dust-to-metals ratio $\zeta$ as a function of galactocentric distance assuming an $\epsilon$-folding decay of the metallicity along the disc (see equation 43).

to the outer disc. However, since dust growth increases as well, the expected net effect is an increased dust-to-metals ratio in any case.

Finally, we note that combining recent observational results (Munoz-Mateos et al. 2009; Moustakas et al. 2010) one finds that dust-to-metals gradients in late-type galaxy discs appear relatively steep (and negative), i.e., show a clear fall-off with galactocentric distance, which suggest interstellar dust growth is more important than stellar dust production. In Paper II of this series, where we compare theoretical models and observational results in more detail, we return to this fact and look for more quantitative evidence of interstellar dust growth being the dominant dust production mechanism in late-type galaxies.

ACKNOWLEDGMENTS

The authors thank the reviewer, Anthony Jones, for constructive and helpful comments and criticism that greatly helped to improve the presentation. L.M. acknowledges support from the Swedish Re-
search Council (Vetenskapsrådet). The Dark Cosmology Centre is funded by the Danish National Research Foundation.

REFERENCES

Asano R.S., Takeuchi T.T., Hirashita H. & Inoue A.K., 2012, Earth, Planets & Space, submitted
Bianchi S. & Schneider R., 2007, MNRAS 378, 973
Bigiel F., Leroy, A., Walter, F., Bigel, E., de Blok, W. J. G., Madore, B., & Thornley, M. D. 2008, AJ, 136, 2846
Bigiel F., Leroy A. K., Walter F., Brinks E., de Blok W. J. G., Kramer C., Rix H. W., Schruba A. et al, 2011, ApJ, 730, L13+Clayton, D. D. 1987, ApJ, 315, 451
Davidge T. J., 2008, PASP, 120, 1145
Draine B., 1990, in Blitz L., ed., ASP Conf. Ser. Vol. 12, The Evolution of the Interstellar Medium. Astron. Soc. Pac., San Francisco , p. 193
Draine B. T. & Li A. 2007, ApJ, 657, 810
Draine B. T. & Salpeter E. E., 1979, ApJ, 231, 438
Draine B. T., Dale D. A., Bendo G., et al., 2007, ApJ, 663, 866
Dunne L., Maddox S. J., Ivison R. J., et al. 2009, MNRAS, 394, 1307
Dwek E., 1998, ApJ, 501, 643
Dwek E., Galliano F. & Jones A.P., 2007, ApJ, 662, 927
Edmunds M.G., 1990, MNRAS, 246, 678
Edmunds M.G. & Eales S.A., 1998, MNRAS, 299, L29
Edmunds M.G., 2001, MNRAS, 328, 223
Falgarotti A.S. & Gail H.-P., 2006, A&A, 447, 553
Gall C., Andersen A. C. & Hjorth J., 2011, A&A, 528, A13
Gall C., Hjorth J. & Andersen A. C., 2011, A&AR, 19, 43
Garnett, D.R., 2002, ApJ, 581, 1019
Garnett D.R., Shields G. A., Skillman E. D., Sagan S. P. & Dufour R. J., 1997, ApJ, 489, 63
Hirashita H., 1999, ApJ, 510, L99
Hirashita H. & Kuo T.-M., 2011, MNRAS, accepted
Ince E. L., 1956, "Ordinary Differential Equations", New York: Dover Publications
Inoue A. K., 2003, PASJ, 55, 901
Issa M.R., MacLaren I. & Wolfendale A.W., 1990, A&A, 236, 237
Jones, A. P. 2004, in ASP Conf. Ser. 309, Astrophysics of Dust, ed. A. N. Witt, G. C. Clayton, & B. T. Draine (San Francisco: ASP), 347
Jones A. P., Duley W. W. & Williams D. A., 1990, QJRAS, 31, 567
Jones A. P. & Nuth J. A., 2011, A&A, 530, A44
Jones A. P., Tielens A. G. G. M. & Hollenbach D. J., 1996, ApJ, 469, 740
Kennicutt, R. C. Jr., et al., 2003, PASP, 115, 928
Kotak R., Meikle P., Pozzo M., et al., 2006, ApJ, 651, L117
Kotak R., Meikle W.P.S., Farrah D., et al., 2009, ApJ, 704, 306
Krumholz M. R. & McKee C. F., 2005, ApJ, 530, 250
Krumholz M. R., McKee C. F. & Tumlinson J., 2009, ApJ, 699, 850
Krumholz M. R., Leroy A. K. & McKee C. F., 2011, ApJ, 731, 25
Kummer E. E., 1837, Journal für die reine und angewandte Mathematik, 17, 228
Larson R. B. 1998, MNRAS, 301, 569
Leroy A. K., Walter F., Brinks E., Bigiel F., de Blok W. J. G., Madore B., Thornley M.D. & Adam Leroy, 2008, AJ, 136, 2782
Magrini L., Bianchi S., Corbelli E., et al., 2011, A&A, accepted, arXiv:1106.0618
Matsuura M., Dwek E., Meixner M., et al., 2011, Science, 333, 1258
Mattsson L., 2011, MNRAS, 414, 781
Mattsson L. & Andersen A. C., 2011, MNRAS, submitted (Paper II)
McKee C. F., 1989, IAUS, 135, 431
Michalowski M., Murphy E.J., Hjorth J., Watson D., Gall C. & Dunlop J.S., 2010, A&A, 522, 15
Morgan H.L. & Edmunds M.G., 2003, MNRAS, 343, 427
Morgan H.L., Dunne L., Eales S.A., Ivison R.J., Edmunds M.G., 2003, ApJ, 597, L33
Moustakas J., Kennicutt R. C., Tremonti C. A., et al. 2010, ApJS, 190, 233
Muñoz-Mateos J. C., et al. 2009, ApJ, 701, 1965
Nozawa T. & Kozasa T., 2006, ApJ, 648, 435
Pagani L., Steinacker J., Baczmann A., Stutz A. & Henning T., 2010, Science, 329, 1622
Pagel B.E., 1997, "Nucleosynthesis and Chemical Evolution of Galaxies", Cambridge Univ. Press
Pilyugin L. S., Vilchez J. M. & Contini T., 2004, A&A, 425, 849
Pipino A., Fan X. L., Matteucci F. et al., 2011, A&A, 525, 61
Ramstedt S., Maercker M., Olofsson G., Olofsson H. & Schier F. L., 2011, A&A, 531, A148
Recchi S., Spitoni E., Matteucci F. & Lanfranchi G. A., 2008, A&A, 489, 555
Rouleau F. & Martin P.G., 1991, ApJ, 377, 526
Rownd B. K., & Young J. S., 1999, AJ, 118, 670
Schruba A., Leroy A. K., Walter F., et al., 2011, AJ, 142, 37
Serra D´ıez-Cano L. & Jones A. P., 2008, A&A, 492, 127
Taylor S. D. & Williams D. A., 1993, MNRAS, 260, 280
Tricomi F. G., 1947, Annali di Matematica Pura ed Applicata: Serie Quarta, 26,141
Valiante R., Schneider R., Salvadori, S. & Bianchi, S., 2011, MNRAS, 416, 1916
Vollmer C., Hoppe P., Brencher F. E., & Holzapfel C., 2007, ApJ, 666, L49
Walter F., Brinks E., de Blok W. J. G., Bigiel F., Kennicutt Jr., R.C., Thornley M.D. & Leroy A., 2008, AJ, 136, 2563
Whittet D.C.B., 1991, Dust in the Galactic Environment, IOP Publishing, Bristol
Wong T. & Blitz L. 2002, ApJ, 569, 157
Zhukovska S., Gail H.-P. & Trieloff M., 2008, A&A, 479, 453

APPENDIX A: EFFECTS OF SECONDARY STELLAR DUST PRODUCTION

It is important to remember that interstellar dust growth is not the only mechanism that can give rise to a dust-to-metals gradient. Secondary dust production in stars, i.e., a metallicity-dependent yield, could in principle have similar effects. Splitting the stellar dust yield into two components, the constant primary yield $\gamma_p$ and the metallicity-dependent secondary yield $\gamma_s = y_s d\phi \times Z/Z_\odot$, and assuming there is no growth, nor destruction of dust in the ISM, we obtain

$$\frac{dZ}{dZ} = \frac{1}{\gamma_p} \left[ y_p + y_s \phi \times Z/Z_\odot \right], \tag{A1}$$

which, with the initial condition $Z(0) = 0$, has the solution...
On the dust abundance gradients in late-type galaxies I

Given the initial condition no metals in any form, we have the equation

\[ Z_d = \frac{Z}{y_Z} \left( \frac{y_d}{y_Z} \cdot \frac{Z}{Z_0} \right). \]  
(A2)

Using the notation introduced in section 3, we can also write

\[ \Delta_\ell = \left( 1 + \frac{2y'_d}{y_d} \cdot \frac{1}{Z_a} \right)^{-1} \Delta_Z. \]  
(A3)

In the limit where \( y'_d \ll y_d, Z \) we then have \( \Delta_\ell \approx \Delta_Z \), which is the steepest dust-to-metals gradient obtainable for a given metallicity gradient. Hence, if the dust-to-metals gradient is steeper than the metallicity gradient, then there must be dust growth in the ISM to account for that steepness.

It is quite unlikely that \( y'_d \ll y_d, Z \) as dust production in stars is likely primary to almost the same extent as the metals production is. Since most of the metals are primary, the secondary dust component cannot be dominant, which implies \( \Delta_\ell < \Delta_Z \). In fact, modelling of stellar dust production suggests yield curves can be relatively high even at \( Z = 0 \) (see Gall, Hjorth & Andersen 2011, and references therein). It is actually reasonable to assume the secondary yield is no more (likely less) than 50% of the primary yield at solar metallicity \( Z_0 \). If \( y'_d = y_d/2 \), then

\[ \Delta_\ell = \left( 1 + \frac{4}{Z} \right)^{-1} \Delta_Z, \]  
(A4)

where we note that \( 4/Z_0 \approx 300 \). More precisely, this implies \( \Delta_\ell \) is at least about two orders of magnitude smaller than \( \Delta_Z \) for all reasonable metallicities along a galaxy disc. Thus, we conclude that although the dust-to-metals gradient \( \Delta_c \) is technically non-zero in this case, it is still consistent with a flat dust-to-metals profile as metallicity gradients are rarely very steep (Piluvug, Vilchez & Contini 2004). Secondary dust production in stars cannot be responsible for a significant dust-to-metals gradient.

**APPENDIX B: EFFECTS OF INFALL**

Throughout this paper, we have treated the dust evolution in late-type galaxies assuming they are "closed boxes", i.e., that there is neither any inflow, nor any outflow of gas and metals to/from the disc. In reality, accretion of gas and minor mergers with smaller galaxies are important for the chemical evolution of a galaxy disc and thus also important for the shaping of the dust component. Hence, an inflow component in equation (34) would have been in its place, but we omitted it for simplicity. However, as we will show here, the effect of infall is not such that it can qualitatively change any of our results. In fact, a theorem similar to that presented in section 3 could likely be formulated, but it would be less transparent as regards the effects of growth and destruction of dust in the ISM.

In case of no growth or destruction of dust in the ISM and accretion of pristine (unenriched) gas at a rate \( \Sigma_{inf} \), which contains no metals in any form, we have the equation

\[ \frac{dZ_d}{dZ} = \frac{y_d - Z_d A}{y_Z - Z}. \]  
(B1)

If \( A \) is constant, the solution to the equation above is

\[ Z_d = \frac{y_d}{y_Z} Z, \]  
(B2)

given the initial condition \( Z_d(0) = Z(0) = 0 \). Hence, pristine inflow likely does not affect the dust-to-metals ratio much.

If the accreted gas contains metals the situation is quite different. Including metal-enriched inflow, equation (B1) becomes

\[ \frac{dZ_d}{dZ} = \frac{y_d - Z_d A}{y_Z - (Z - Z_{inf}) A}. \]  
(B3)

Assuming an outflow of interstellar gas where some of the metals in that gas are accreted back onto the disc, we can write \( Z_{inf} = v Z \) (usually referred to as a "galactic fountain" model, see Recchi et al. 2008) the solution is

\[ Z_d = \frac{y_d}{A} \left( 1 - \left[ 1 + (v - 1) Z \right]^{-1} \right), \]  
(B4)

where \( y'_d \) is a reduced metal yield to account for the metals lost in the outflow. For the specific case \( v = 1/2 \), we obtain the solution

\[ Z_d = \frac{y_d}{y_Z} \left( 1 - \frac{A Z}{4 y_Z} \right), \]  
(B5)

from which it is easy to see that the effect of infall is reminiscent of the effect of dust destruction in the ISM due to SNe, i.e., that the dust-to-metals ratio decreases with metallicity (cf. equation 41). Moreover, if the dust destruction term is included in equation (B1), we have

\[ Z_d = \frac{y_d}{A + \delta} \left( 1 - \left[ 1 + (v - 1) Z \right]^{-1} \right), \]  
(B6)

which is a solution of the same mathematical form as above. Hence, it is quite clear that metal-enriched inflow has an effect which is very similar to that of dust destruction by SNe, which means that inflow alone cannot create a dust-to-metals gradient with the same sign as the metallicity gradient.

**APPENDIX C: GENERAL SOLUTION OF EQUATION (34)**

The general solution to equation (34) presented in section 3 is expressed in terms of a product of the confluent hypergeometric functions \( Kummer \) and \( Tricomi \) functions \( M \) and \( U \) (Kummer 1874; Tricomi 1944). This solution exist because equation (34) is related to Kummer’s equation (also known as the confluent hypergeometric equation), i.e.,

\[ z \frac{d^2w}{dz^2} + (b - z) \frac{dw}{dz} - aw = 0, \]  
(C1)

which has the solution \( w(z) = c_1 M(a, b; z) + c_2 U(a, b; z) \), where \( c_1 \) and \( c_2 \) are arbitrary constants. With the variable change

\[ \xi \equiv \frac{1}{2} \left( \frac{1}{e^2} \right) \]  
(C2)

equation (34) can be rewritten as an equation of the form

\[ \frac{dZ_d}{d\xi} = \frac{y_d}{\sqrt{e y_Z \xi}} + Z_d = \frac{e}{\sqrt{e y_Z \xi}} Z_d. \]  
(C3)

This is a Riccati equation, which has the general form

\[ \frac{dy}{dx} = q_0(x) + q_1(x) y(x) + q_2(x) y^2(x). \]  
(C4)

Such non-linear equations can be reduced to a second order linear ordinary differential equation (Ince 1956) of the form

\[ \frac{d^2u}{dx^2} + R(x) \frac{du}{dx} + S(x) u(x) = 0, \]  
(C5)

where

\[ R(x) = q_1(x) + \frac{1}{q_2(x)} \frac{dq_2}{dx}, \quad S(x) = q_0(x) q_2(x). \]  
(C6)

A solution to equation (C5) provides a solution to equation (C4) as
Identifying the Riccati coefficients as

\[ q_0(\xi) = \frac{y_0}{\sqrt{2\epsilon y_0}} \quad q_1(\xi) = 1 \quad q_2(\xi) = -\frac{1}{\sqrt{2\epsilon y_0}} \]

we find the associated second order linear ordinary differential equation,

\[
\xi^2 \frac{d^2 u}{d\xi^2} + \left( 1 - \frac{1}{2 - \xi} \right) \frac{du}{d\xi} - \frac{1}{2} y_0^2 u(\xi) = 0. \tag{C9}
\]

This is the Kummer equation for \( b = 1/2 \) and \( a = y_0/2y_z \). Reverse Riccati reduction and back-substitution, together with the natural initial condition \( Z(0) = 0 \), will provide the general solution given in section 5 after some algebra.

The Kummer equation has an awkward property: it has a regular (order one) singularity at the origin (at \( \xi = 0 \) in the case above). This means that no solution exists at \( \xi = 0 \) (or \( Z = \delta/\epsilon \)) and that the region near this point must be avoided when applying this solution. In particular, when the Kummer-Tricomi functions are implemented numerically, the algorithm for computing them will be unstable in the vicinity of this singularity point. We have therefore considered the special (and not entirely realistic) case where stellar dust production is negligible from a point in time when the metallicity has a certain value \( Z_0 \) and the dust-to-metals ratio has a value \( \xi_0 \). Assuming \( y_0 \rightarrow 0 \) equation (34) reduces to

\[ y_x \frac{dZ_x}{dZ} = (\epsilon Z - \delta) Z_x - \epsilon Z_x^2. \tag{C10}
\]

Riccati reduction as above yields

\[
\frac{d^2 w}{d\xi^2} \left( \frac{1}{2 - \xi} \right) \frac{dw}{d\xi} = 0, \tag{C11}
\]

where \( \xi \) is as previously defined (equation C2). This equation is non-singular, but requires that \( Z > Z_0 \), where \( Z_0 \) is some finite initial value. The general solution is

\[ u(\xi) = C_0 + C_1 \sqrt{\xi} \operatorname{erfi} \left( \sqrt{\xi} \right), \tag{C12}
\]

where \( C_0, C_1 \) are constants to be fixed by initial conditions as we do reverse the Riccati reduction and back-substitute. With \( Z_0 = Z(t_0) \neq 0 \) and \( Z_{0,0} = Z_0(0) \neq 0 \) as the initial conditions it is then possible to obtain the solution given as equation (37) in section 5.2.

**APPENDIX D: NUMERICAL IMPLEMENTATION OF THE KUMMER-TRICOMI FUNCTIONS**

The Kummer-Tricomi functions, used in section 5 and Appendix C above, can be defined in terms of integral quantities (Kummer 1837; Tricomi 1947),

\[ M(a, b; z) = \sum_{n=0}^{\infty} \frac{(\theta^n z)^n}{(a)_{n}} = \, _{1}F_{1}(a, b; z) \tag{D3} \]

where

\[ \theta^n = a(a+1)(a+2)\cdots(a+n-1) = \frac{(a+n)}{(a)}. \tag{D4} \]

is the Pochhammer symbol and \( \Gamma \) is the Gamma function,

\[ \Gamma(z) = \int_{0}^{\infty} t^{z-1} e^{-t} \, dt. \tag{D5} \]

The function \( M \) can thus be implemented numerically by computing the above series until some arbitrary precision is obtained. The typical number of terms needed to reach the precision limit of a standard Intel processor is at most a few hundred. This may still cause problems when computing the Pochhammer symbol, since this will have to be done using some limited implementation of \( \Gamma \) to obtain reasonable computation speed. The basic issue is the fact that \( \Gamma \), as well as the factorial, is usually not implemented for large arguments. For example, in IDL and MATLAB the argument \( z \) cannot exceed \( \sim 170 \). However, this situation rarely occurs.

The function \( U \) can be defined in terms of the function \( M \) (Tricomi 1947) by

\[ U(a, b; z) = \frac{\Gamma(1-b)}{\Gamma(a-b+1)} M(a, b; z) + \frac{\Gamma(b-1)}{\Gamma(a)} z^{1-b} M(a-b+1, 2-b; z), \tag{D6} \]

which is straightforward to implement, except for integer \( b \) (where \( U \) is not defined). The general solution to equation (34) does not give rise to any integer values for the \( b \), so we will not consider how to implement the analytical extension of \( U \) for integer \( b \). This can be done, but goes beyond the scope of this study.