Remarks on Screening in a Gauge-Invariant Formalism

Patricio Gaete *, and Iván Schmidt †

Departamento de Física, Universidad Técnica F. Santa María, Valparaíso, Chile

Abstract

In this paper we display a direct and physically attractive derivation of the screening contribution to the interaction potential in the Chiral Schwinger model and generalized Maxwell-Chern-Simons gauge theory. It is shown that these results emerge naturally when a correct separation between gauge-invariant and gauge degrees of freedom is made. Explicit expressions for gauge-invariant fields are found.

PACS number(s): 12.20.Ds, 11.15.Tk

I. INTRODUCTION

The binding energy of an infinitely heavy quark-antiquark pair represents a fundamental concept which is expected to play an important role in the understanding of non-Abelian theories and especially of quark confinement. In fact, a linearly increasing quark-antiquark pair static potential provides the simplest criterion for confinement, although unfortunately there is up to now no known way to analytically derive the confining potential from first principles. However, as it is well known, in certain theories the rising potential can be

*E-mail: pgaete@fis.utfsm.cl

†E-mail: ischmidt@fis.utfsm.cl
screened at large distances by dynamical charges. An illustrative example arises when one considers two dimensions of spacetime, where it was shown in Ref. [1,2] that matter fields can screen any Abelian fractional charge. In three dimensions massive fermions screen any charge [3]. In this context it may be recalled the confining and screening nature of the potential in QED3 [4,5]. As the three dimensional fermions induce a topological mass for the photon, the logarithmically rising potential is transformed into an exponentially decreasing one. We further note that recently the stability of the above potential in the presence of a self-interaction among fermions has been studied [6]. In particular it was considered the Thirring interaction. As a result it was argued that when the Thirring coupling $g$ is positive, the new model displays a marked departure of a qualitative nature from the results of Ref. [4] at short distances. More precisely, the resulting potential obtained has the form of a well, which may be contrasted with the result of [4,5] where for small separation the potential tends to a logarithmic Coulomb potential. In this way, the author of Ref. [6] is lead to the conclusion that the effect of adding the Thirring term is to stabilize the charge system. Despite their relevance, this study was carried out in a gauge fixed scheme, and we think that their results should be confirmed by a gauge independent analysis.

Meanwhile, in a previous paper [5] we have proposed a general framework for studying the confining and screening nature of the potential in QED3 in terms of gauge-invariant but path-dependent field variables. According to this formalism, the interaction potential between two static charges is obtained once a judicious identification of the physical degrees of freedom is made. This procedure leads to the physical phenomena of electrostatic and dressed electrons, where we refer to the cloud made out of the vector potentials around the fermions as dressing. In this sense, our formalism has provided a method for the determination of the potential between charges which, in our view, is of interest both for its simplicity and physical content. We also point out that a similar analysis has been developed for the Schwinger model [6].

In this Brief Report we will continue our program [5,7] to study the structure of the interaction energy between charged fermions. In the next section we develop further the discussion that was begun in [6] by considering how the charged fermions behave in the
Chiral Schwinger model [8]. Section III is concerned with the calculation of the interaction energy in QED3 with a Thirring interaction term among fermions. Particular care is paid to reexamine the consequences of including this term in the confining and screening nature of the potential.

II. PRELIMINARY: GAUGE-INvariant VARIABLES FORMALISM

First, let us briefly review the framework of the gauge-invariant but path-dependent field variables formalism as described in Ref. [9,5,7]. Accordingly, we consider the gauge-invariant field

$$A_\mu(y) = A_\mu(y) + \partial_\mu \Lambda(y). \quad (1)$$

The function $\Lambda(y)$ is defined by

$$\Lambda(y) = - \int_{C_{\xi y}} dz^\nu A_\nu(z), \quad (2)$$

where the path integral is to be evaluated along some contour $C_{\xi y}$ connecting $\xi$ and $y$. Here $A_\mu$ is the usual electromagnetic potential and, in principle, it is taken in an arbitrary gauge. It can easily be verified that $A_\mu(y)$ is invariant with respect to gauge transformations $A_\mu(y) \mapsto A_\mu(y) + \partial_\mu \Theta(y)$. It is now important to notice that the gauge invariant field (1) depends not only on the points $\xi$ and $y$ but also on the path. Furthermore, by choosing a spacelike path from the point $\xi^k$ to $y^k$, on a fixed time slice, it is possible to express the gauge-invariant field in terms of the magnetic ($B$) and electric ($E$) fields as:

$$A_0 (t, y) = - y \cdot \int_0^1 d\alpha E (t, \alpha y), \quad (3)$$

$$A (t, y) = - y \wedge \int_0^1 d\alpha B (t, \alpha y), \quad (4)$$

where $\alpha$ $(0 \leq \alpha \leq 1)$ is the parameter describing the contour $y^k = \xi^k + \alpha (y - \xi)^k$ with $k = 1, 2, 3$. For simplicity we have assumed the reference point $\xi$ at the origin. The above
expressions coincide with the Poincaré gauge conditions \[9\] for the path-dependent fields \(A_\mu\), while other contour choices coincide with other gauge conditions for these fields. For reasons that will become evident later, we now focus our attention on the fermion field. In the context of our formalism, the charged matter field together with the electromagnetic cloud (dressing) which surrounds it, is given by \[9,17\],

\[
\Psi(y) = \exp \left( -ie \int_{C_{\xi y}} dz^\mu A_\mu(z) \right) \psi(y). \tag{5}
\]

Thanks to our path choice, the physical fermion \[5\] then becomes

\[
\Psi(y) = \exp \left( -ie \int_0^y dz^k A_k(z) \right) \psi(y). \tag{6}
\]

The expressions \[3\], \[4\] and \[6\] will form the basis of our subsequent considerations, where the gauge invariance of the formalism guarantees that the relevant physical information will be preserved. We conclude this brief introduction to gauge invariant variables by pointing out that the breaking of the gauge invariance of the fields in the standard formalism is transformed into breaking of the translational invariance in the path-dependent formalism. This drawback is avoided by letting the reference point \(\xi^k\) go to infinity.

As already mentioned, we now want to study the interaction energy between external probe sources in the Chiral Schwinger model (CSM) \[8\], which consists of a U(1) gauge field coupled to chiral fermions in two-dimensional spacetime. As was shown by Jackiw-Rajaraman \[10\] this theory can be consistently quantized and the quantum theory is unitary, in spite of its gauge anomaly. Faddeev and Shatashvili \[11\] have suggested a modification of the canonical quantization by addition new degrees of freedom through a Wess-Zumino action. At present two formalisms of the Chiral Schwinger model are available: the gauge noninvariant one \[11\] and the gauge invariant one \[11–13\].

We begin by recalling the bosonized form of the gauge-invariant version under consideration \[8\]:

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 + \frac{1}{2} (\partial_\mu \varphi)^2 - \frac{g}{\sqrt{\pi}} (g^{\mu\nu} - \varepsilon^{\mu\nu}) \partial_\mu A_\nu \varphi + \frac{ag^2}{2\pi} A_\mu A^\mu + \frac{1}{2} (a - 1) (\partial_\mu \theta)^2 - \frac{g}{\sqrt{\pi}} [(a - 1) g^{\mu\nu} + \varepsilon^{\mu\nu}] \partial_\mu A_\nu \theta - A_0 J_0, \tag{7}
\]
where \( J^0 \) is the external current. The parameter \( a \) reflects the ambiguity of fermionic radiative corrections or may be considered as a bosonization ambiguity parameter. Here we focus our attention to the \( a > 1 \) case. As we have already indicated in [7], to compute the interaction energy we need to carry out the integration over \( \varphi \) and \( \theta \) in Eq.(7). Once this is done, we arrive at the following effective Lagrangian for the gauge fields

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} \left( 1 + \frac{g^2}{\pi} \frac{a}{a-1} \frac{1}{\partial^2} \right) F^{\mu\nu} - A_0 J^0.
\]

One immediately sees that this expression is very similar to the effective Lagrangian for the Schwinger model. Notwithstanding, in order to put our discussion into context it is useful to summarize the relevant aspects of the analysis described previously [7]. Thus, our first undertaking is to calculate the expectation value of the Hamiltonian in the physical state \( |\Omega\rangle \), which we will denote by \( \langle H \rangle_\Omega \).

We now proceed to obtain the Hamiltonian. For this we restrict our attention to the Hamiltonian framework of this theory. The canonical momenta read \( \Pi^\mu = \left( 1 + \frac{g^2}{\pi} \frac{a}{a-1} \frac{1}{\partial^2} \right) F^{\mu\alpha} \), and one immediately identifies the sole primary constraint \( \Pi^0 = 0 \). The canonical Hamiltonian is given by

\[
H_C = \int dx \left( -\frac{1}{2} \Pi_1 \left( 1 + \frac{g^2}{\pi} \frac{a}{a-1} \frac{1}{\partial^2} \right)^{-1} \Pi^1 + \Pi^1 \partial_1 A_0 + A_0 J^0 \right).
\]

With this at hand, the consistency condition \( \dot{\Pi}_0 = 0 \) lead to the secondary constraint

\[
\Omega_1 (x) = \partial_1 \Pi^1 - J^0,
\]

and the time stability of the secondary constraint does not induce further constraints. Therefore, in this case there are two constraints, which are first class. The extended Hamiltonian that generates translations in time then reads

\[
H = H_C + \int dx (c_0(x) \Pi_0 (x) + c_1(x) \Omega_1 (x)),
\]

where \( c_0(x) \) and \( c_1(x) \) are the Lagrange multiplier fields. Moreover, it follows from this Hamiltonian that \( \dot{A}_0 (x) = [A_0 (x), H] = c_0 (x) \), which is an arbitrary function. Since
$\Pi^0 = 0$ always, neither $A^0$ nor $\Pi^0$ are of interest in describing the system and may be discarded from the theory. Thus the Hamiltonian takes the form

$$H = \int dx \left\{ -\frac{1}{2} \Pi_1 \left( 1 + \frac{g^2}{\pi} \frac{a}{a - 1} \right)^{-1} \Pi_1 + c'(x) \left( \partial_1 \Pi_1 - J^0 \right) \right\}, \quad (12)$$

where $c'(x) = c_1(x) - A_0(x)$.

Now the presence of the arbitrary quantity $c'(x)$ is undesirable since we have no way of giving it a meaning in a quantum theory. To avoid this trouble we introduce a supplementary condition on the vector potential such that the full set of constraints becomes second class. A particularly convenient condition is

$$\Omega_2(x) = \int_0^1 d\alpha x^1 A_1(\alpha x) = 0, \quad (13)$$

where, as before, $\alpha$ is the parameter describing a spacelike straight line of integration. In this case, the basic Dirac brackets between the canonical variables have the following form:

$$\left\{ A_1(x), A^1(y) \right\}^* = 0 = \left\{ \Pi_1(x), \Pi^1(y) \right\}^*, \quad (14)$$

$$\left\{ A_1(x), \Pi^1(y) \right\}^* = \delta^{(1)}(x - y) - \partial_1 \int_0^1 d\alpha x^1 \delta^{(1)}(\alpha x - y). \quad (15)$$

Next, as remarked by Dirac [17], the physical states $|\Omega\rangle$ correspond to the gauge invariant ones. In this way, we consider the stringy gauge-invariant $|\overline{\Psi}(y)\Psi(y')\rangle$ state,

$$|\Omega\rangle \equiv |\overline{\Psi}(y)\Psi(y')\rangle = \overline{\psi}(y) \exp \left( -iq \int_y^{y'} dz^1 A_1(z) \right) \psi(y) |0\rangle, \quad (16)$$

where $|0\rangle$ is the physical vacuum state.

We have finally assembled the tools to determine the interaction energy. Recalling again that the fermions are taken to be static, we can therefore substitute $\partial^2$ by $-\partial_1^2$ in the Hamiltonian. As a consequence, the expectation value $\langle H \rangle_\Omega$ simplifies to

$$\langle H \rangle_\Omega = \langle \Omega | \int dx_1 \left( -\frac{1}{2} \Pi_1 \left( 1 - \frac{g^2}{\pi} \frac{a}{a - 1} \right)^{-1} \Pi_1 \right) |\Omega\rangle. \quad (17)$$

Taking into account the above Hamiltonian structure, the resulting interaction energy of the dressed fermion-antifermion system takes the form
\begin{align}
\langle H \rangle_\Omega &= \langle H \rangle_0 + \frac{g^2 \sqrt{\pi}}{2g} \sqrt{\frac{a-1}{a}} \left( 1 - e^{-\frac{g}{\sqrt{a-1}} |y-y'|} \right), \quad (18)
\end{align}

where \( \langle H \rangle_0 = \langle 0 | H | 0 \rangle \). The second term on the right-hand side of Eq. (18) is clearly dependent on the distance between the external static fields. Therefore the potential for two opposite charges located at \( y \) and \( y' \) is given by

\begin{align}
V &= \frac{g^2 \sqrt{\pi}}{2g} \sqrt{\frac{a-1}{a}} \left( 1 - e^{-\frac{g}{\sqrt{a-1}} |y-y'|} \right). \quad (19)
\end{align}

This result agrees with that of Ref. [14], and finds here an independent derivation. An immediate consequence of the Eq. (19) is that it saturates at \( \frac{g^2 \sqrt{\pi}}{2g} \sqrt{\frac{a-1}{a}} \) for large \( |y - y'| \). In other terms, the charged fermions behave in the same way as in the Schwinger model, that is, the probe charges are screened. The point we wish to emphasize, however, is that within this framework one circumvents completely the need to introduce the Wilson loop, where subtleties related to the correct calculation must be considered [15,16]. Thus, in the context of our present formalism, the potential energy between fermions can be directly obtained once the structure of the electromagnetic cloud around static fermions is known.

We also observe at this stage that the gauge-invariant variables \( A_\mu \) commute with the sole first class constraint (Gauss law), corroborating the fact that these fields are physical variables [17]. This last point enables us to arrive at the result (19) by an alternative but equivalent way. To this end we first note that the physical electron (i.e. an electron together with the electric field surrounding it), Eq. (5), may be rewritten as

\begin{align}
\Psi (y) &= \exp \left( -ie \int_0^1 dz A_1^L (z) \right) \psi (z), \quad (20)
\end{align}

where \( A_1^L \) refers to the longitudinal part of \( A_1 \). It is worth noting here that the above expression uses a modified form for the electromagnetic cloud in the Poincaré gauge Eq. (3), which is equivalent to the Coulomb gauge [18]. Having made this observation and from the previous Hamiltonian analysis, we can write immediately the following physical scalar potential

\begin{align}
A_0 (t, x) = \int_0^1 d\alpha x \int_0^1 d\alpha x \left( 1 - \frac{g^2}{\pi} \frac{a}{a-1} \right)^{-1} \frac{1}{\theta_1^{\alpha x}} (-J^0 (\alpha x)) \frac{1}{\theta_2^{\alpha x}}, \quad (21)
\end{align}
where \( J^0 \) is the external current. The static current describing two opposites charges \( q \) and \(-q\) located at \( y \) and \( y' \) is then described by

\[
J^0(t, x) = q \{ \delta(x - y) - \delta(x - y') \}. \tag{22}
\]

Substituting this back into the Eq. (21), we obtain

\[
V = q (A_0(y) - A_0(y')) = \frac{q^2 \sqrt{\pi}}{2g} \sqrt{\frac{q - 1}{a}} \left( 1 - e^{-\frac{q}{\sqrt{\pi a}} |y - y'|} \right). \tag{23}
\]

It is clear from this discussion that a correct identification of physical degrees of freedom is a key feature for understanding the physics hidden in gauge theories. According to this viewpoint, once that identification is made, the computation of the potential is achieved by means of Gauss law [15].

III. GENERALIZED MAXWELL-CHERN-SIMONS GAUGE THEORY

We now extend the analysis of the previous section to a (2+1)-dimensional topologically massive gauge theory, which includes a self-interaction among fermions, the so-called generalized Maxwell-Chern-Simons gauge theory [6]. As mentioned above, in this section we concentrate on the effect of including the self-interaction term in the confinement and screening nature of the potential.

Before going to the derivation of the interaction energy, we will describe very briefly the model under consideration. It is described by the following Lagrangian:

\[
\mathcal{L} = -\frac{pe^2}{4} F_{\mu\nu} F^{\mu\nu} + \frac{qe^2}{2} \varepsilon_{\mu\nu\lambda} A^\mu F^{\nu\lambda} + \bar{\psi} i \gamma^\mu (\partial_\mu - ieA_\mu) \psi - m \bar{\psi} \psi + \frac{g}{2} |\bar{\psi} \gamma^\mu \psi|^2 + J^0 A^0, \tag{24}
\]

where \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), \( J^0 \) is an external current and \( p = \frac{1}{e^2}, \ q = \frac{\chi}{2e^2} \).

Next, in order to linearize this theory, we introduce the auxiliary field \( B_\mu \). It follows that the expression (24) can be rewritten as

\[
\mathcal{L} = -\frac{pe^2}{4} F_{\mu\nu}^2 + \frac{qe^2}{2} \varepsilon_{\mu\nu\lambda} A^\mu F^{\nu\lambda} + \bar{\psi} i \gamma^\mu (\partial_\mu - ieA_\mu - iB_\mu) \psi - \frac{1}{2g} B_\mu^2 - m \bar{\psi} \psi + J^0 A^0. \tag{25}
\]
By using bosonization methods, it can be shown that in the large fermion mass limit, the
Lagrangian (25) then becomes $O(\frac{1}{m})$:

$$L = \frac{-a}{4} W_{\mu\nu} W^{\mu\nu} + \frac{\alpha}{2} \varepsilon_{\mu\nu\lambda} W^\mu W^\nu - \frac{1}{2 g} B^2 - \frac{pe}{4} F_{\mu\nu} F^{\mu\nu} + \frac{qe^2}{4} \varepsilon_{\mu\nu\lambda} A^\mu F^{\nu\lambda},$$  \hspace{1cm} (26)

where $W_\mu = B_\mu + e A_\mu$, $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$, $\alpha = -\frac{1}{8\pi}$ and $a = -\frac{1}{8\pi m}$.

Following our earlier discussion, this expression allows us to derive an effective Lagrangian. Thus, after the $B_\mu$ fields are integrated away, one gets up to $O(\frac{1}{m})$ a generalized Maxwell-Chern-Simons gauge theory:

$$L = -\frac{1}{4} F_{\mu\nu} \left( 1 + e^2 a \left[ 1 - 12\alpha^2 g^2 \partial^2 \right] \right) F^{\mu\nu} + \frac{\chi}{2} \varepsilon_{\mu\nu\lambda} A^\mu \partial^\nu A^\lambda.$$  \hspace{1cm} (27)

It is now once again straightforward to apply the gauge-invariant formalism discussed in the preceding section. For this purpose, we start by observing that the canonical momenta read $\Pi^\mu = -(1 + e^2 a \left[ 1 - 12\alpha^2 g^2 \partial^2 \right]) F^{0\mu} + \frac{\chi}{2} \varepsilon^{0\mu\nu} A_\nu$. As we can see there is one primary constraint $\Pi^0 = 0$, and $\Pi^i = (1 + e^2 a \left[ 1 - 12\alpha^2 g^2 \partial^2 \right]) E^i + \frac{\chi}{2} \varepsilon^{ij} A_j$ $(i, j = 1, 2)$. The canonical Hamiltonian for this system is in this case

$$H_C = \int d^2 x \left\{ -\frac{1}{2} F_{\mu0} \left( 1 + e^2 a \left[ 1 - 12\alpha^2 g^2 \partial^2 \right] \right) F^{\mu0} + \frac{1}{2} F_{ij} \left( 1 + e^2 a \left[ 1 - 12\alpha^2 g^2 \partial^2 \right] \right) F^{ij} \right\} + \Pi^i \partial_i A_0 - \frac{\chi}{2} \varepsilon^{ij} A_0 \partial_i A_j + A_0 J^0 \right\}.$$  \hspace{1cm} (28)

The conservation in time of the constraint $\Pi^0$ leads to the secondary constraint (Gauss law)

$$\Omega_1 (x) = \partial_i \Pi^i + \frac{\chi}{2} \varepsilon_{ij} \partial^i A^j - J^0 = 0.$$  \hspace{1cm} (29)

There are no more constraints in the theory and the two we have found are first class. It follows, therefore, that the total Hamiltonian (first class) is given by

$$H = H_C + \int d^2 x \left( c_0 (x) \Pi_0 (x) + c_1 (x) \Pi_1 (x) \right),$$  \hspace{1cm} (30)

where we recall from Eq. (\text{11}) that $c_0(x)$ and $c_1(x)$ are arbitrary functions. We also recall that $A_0$ is not a dynamical variable. Thus the total Hamiltonian $H$ is given as
\[ H = \int d^2x \left\{ -\frac{1}{2} F_{i0} F^{i0} + \frac{1}{4} F_{ij} F^{ij} + c'(x) \left[ \partial_i \Pi^i + \frac{\chi}{2} \varepsilon^{ij} \partial_i A_j - J^0 \right] \right\}, \]  

(31)

where \( c'(x) = c_1(x) - A_0(x) \).

As before, we now proceed to impose a supplementary condition on the vector potential such that the full set of constraints becomes second class. Therefore, we once again write the supplementary condition as

\[ \Omega_2(x) = \int_0^1 d\alpha x^i A_i(\alpha x) = 0, \]  

(32)

where \( \alpha \) is the parameter describing a spacelike straight line of integration. Correspondingly, the fundamental Dirac brackets are given by

\[ \{ A_i(x), A^j(y) \}^* = 0 = \{ \Pi_i(x), \Pi^j(y) \}^*, \]  

(33)

\[ \{ A_i(x), \Pi^j(y) \}^* = \delta_i^j \delta^{(2)}(x-y) - \partial_i \int_0^1 d\alpha x^j \delta^{(2)}(\alpha x - y). \]  

(34)

We now have all the information required to compute the potential energy for this theory. As we pointed above, this calculation is straightforward by using

\[ \mathcal{A}_0(t, x) = \int_0^1 d\alpha x^i E^L_i(t, \alpha x), \]  

(35)

which may be rewritten as

\[ \mathcal{A}_0(t, x) = \int_0^1 d\alpha \frac{1}{1 + e^2 a (1 - 2\alpha^2 g^2 \partial^2)} \frac{x^i \partial^x - J^0 (\alpha x)}{\nabla_{\alpha x}^2 - \chi^2}. \]  

(36)

Accordingly, for \( J^0(t, x) = q \delta^{(2)}(x, a) \), expression (36) takes the form

\[ \mathcal{A}_0(t, x) = q \frac{1}{2\pi} \frac{1}{1 + e^2 a (1 - 2\alpha^2 g^2 \partial^2)} (K_0 (\chi|x - a|) - K_0 (\chi|a|)). \]  

(37)

By means of Eq. (37) we evaluate the potential energy for a pair of static pointlike opposite charges at \( y \) and \( y' \), that is,

\[ V = q(\mathcal{A}_0(y) - \mathcal{A}_0(y')) = -\frac{q^2}{\pi} \frac{1}{1 + e^2 a (1 - 2\alpha^2 g^2 \partial^2)} K_0 (\chi|y - y'|). \]  

(38)
Considering again that the fermions are taken to be static, we can write immediately the potential energy as

\[ V = -Q^2 K_0 (\chi|y - y'|) - \frac{\beta}{\eta} Q^2 \nabla^2 K_0 (\chi|y - y'|), \quad (39) \]

where \( Q \equiv \frac{a^2}{\pi a}, \ \beta = -\frac{1}{2} e^2 a \alpha^2 g^2, \ \eta = 1 + e^2 a \) and \( K_0 \) is a modified Bessel function. This result explicitly shows the effect of including the self-interaction term, in the form of the second term on the right-hand side of Eq.(39). It is straightforward to see that when \( g = 0 \) the potential (39) reduces to the one in [4,5], as indeed it should. Now we recall that the limiting form of \( K_0 \) for small \( \chi r \) \( (r \equiv |y - y'|) \) is \( K_0 \sim -\ln (\chi r) \), and hence expression (39) reduces to

\[ V = Q^2 \left( \ln z - \gamma \frac{1}{z^2} \right), \quad (40) \]

where \( z \equiv \chi r \) and \( \gamma \equiv \frac{12 e^2 a \alpha^2 g^2}{1 + e^2 a} \), which is a rather small number since \( a \sim \frac{1}{m} \). In this way one obtains that in the short distances regime the potential has the form of a well (Fig.1), as was noted in Ref. [6]. Figure 1 represents the potential energy Eq.(40) for the case of \( \gamma = 0.01 \), plotted with respect to \( z \). It is important to notice that the presence of the second term on the right-hand side of Eq.(40), which predominates for small \( z \) values, causes \( V \) to be attractive at short distances. One is thus lead to the interesting conclusion that the effect of adding the self-interaction term is to generate stable bound states of quark-antiquark pairs at short distances. However, the central difference between the above analysis and that of Ref. [6] lies on the fact that the potential Eq.(39) is present at the classical level too. In this context, the present gauge-invariant investigation supplements the earlier analysis done in Ref. [5], as well as it reveals the general applicability of the gauge-invariant approach.
FIG. 1. Shape of the potential, Eq.(40), at short distances.

IV. ACKNOWLEDGMENTS

Work supported in part by Fondecyt (Chile) grant 1980149 and grant 1000710, and by a Cátedra Presidencial (Chile).
REFERENCES

[1] D. J. Gross, I. R. Klebanov, A. V. Matytsin and A. V. Smilga, Nucl. Phys. B461, 109 (1996).

[2] E. Abdalla, R. Mohayaee and A. Zadra, Int. J. Mod. Phys. A12, 4539 (1997).

[3] E. Abdalla, R. Banerjee and C. Molina, Eur. Phys. J. C17, 467 (2000).

[4] E. Abdalla and R. Banerjee, Phys. Rev. Lett. 80, 238 (1998).

[5] P. Gaete, Phys. Rev. D59, 127702 (1999).

[6] S. Ghosh, J. Phys. A: Math. Gen. 33, 1915 (2000).

[7] P. Gaete and I. Schmidt, Phys. Rev. D61, 125002 (2000).

[8] D. Boyanovsky, I. Schmidt and M. Golterman, Ann. Phys. (N.Y.) 185, 111 (1988).

[9] P. Gaete, Z. Phys. C76, 355 (1997).

[10] R. Jackiw and R. Rajaraman, Phys. Rev. Lett. 54, 1219 (1985).

[11] L. D. Faddeev and S.S. Shatashvili, Phys. Lett. B167, 225 (1986).

[12] N. F. Falck and G. Kramer, Ann. Phys. (N.Y.) 176, 330 (1987).

[13] H. O. Girotti and H. J. Rothe, Intern. J. Mod. Phys. A4, 3041 (1989).

[14] T. Berger, N. K. Falck and G. Kramer, Int. J. Mod. Phys. A4, 427 (1989).

[15] P. E. Haagensen and K. Johnson, eprint [hep-th/9702204].

[16] H. A. Falomir, R. E. Gamboa Saravi and F. A. Schaposnik, Phys. Rev. D25, 547 (1982).

[17] P. A. M. Dirac, The Principles of Quantum Mechanics (Oxford University Press, Oxford, 1958), 4th ed.; Can. J. Phys. 33, 650 (1955).

[18] P. Gaete, C. A. P. Galvão and B. Pimentel, Int. J. Mod. Phys. A6, 5373 (1991).