SCALAR-EMITTING MODES IN DOUBLE-BETA DECAY

C.P. BURGESS

Physics Department, McGill University
3600 University St., Montréal, Québec, Canada, H3A 2T8.

ABSTRACT

The sum-energy spectrum of electrons emitted in double-beta decay is a well-known diagnostic for the nature of the physics which is responsible for the decay. Three types of spectra are usually considered when these experiments are analysed: one each for the standard two-neutrino ($\beta\beta_{2\nu}$) decay, neutrinoless ($\beta\beta_{0\nu}$) and a Majoron-emitting ($\beta\beta_{\phi}$) decay. It has recently been shown that two other electron spectra can be possible for scalar-emitting modes, in addition to these traditional three. One of these is softer than the Standard-Model $\beta\beta_{2\nu}$ decay, while the other is intermediate between the $\beta\beta_{2\nu}$ and the usual $\beta\beta_{\phi}$ spectra. The models which predict these new spectra are generically more natural than those which predict the traditional $\beta\beta_{\phi}$ spectrum, in that they can accommodate the constraints following from the steadily improving limits on $\beta\beta_{0\nu}$ decay without requiring the fine-tuning that is endemic to the usual models. This article reviews the properties of the physics which can produce the new kinds of electron spectra.

1. Introduction

Double-beta ($\beta\beta$) decay is an extremely rare process in which two nuclear neutrons simultaneously convert into two protons and two electrons. Within the Standard Electroweak Model (SM) this decay occurs at second order in the charged-current weak interactions, and is accompanied by the emission of two antineutrinos, giving rise to a characteristic ($\beta\beta_{2\nu}$) electron spectrum. Despite the extremely long half-lives involved — typically $10^{20}$ yr or more — heroic efforts over the past ten years have been rewarded by its experimental detection.

Because it is such a rare process, $\beta\beta$ decay experiments also furnish a unique window onto whatever new physics may replace the SM at energies very much higher than those that are directly accessible in today’s accelerators. This is because the effects for these experiments

*Invited talk presented to the Workshop on Double Beta Decay and Related Topics, Trento Italy, April 1995.
of new interactions can in some circumstances compete with those of
run-of-the-mill SM decays.

In order to be detectable in $\beta\beta$ experiments, new physics must
have either or both of the following properties:

1. It must violate a selection rule — *e.g.*: electron-number ($L_e$) con-
servation — which is satisfied by the SM contribution;

2. It must contain new particle states that are light enough to be
produced in $\beta\beta$ decay. Since $Q \sim 1$ MeV is typical of the energy
release in these decays, any such new particles must be much
lighter than this scale.

The purpose of this article is to outline the features of types of
new physics which satisfy the second of these properties, produc-
ing $\beta\beta$ decays in which new light particles are emitted. The title refers
to ‘scalar-emitting’ decay modes because, with the occasional excep-
tion [2] this new light particle is usually taken to be spinless. A spinless
particle can be particularly well-motivated if it is a (possibly pseudo-)
Nambu-Goldstone boson (NGB), since in that case its small mass is
naturally understood. In particular, the focus is on comparatively re-
cent work [3–5] which shows that models of this sort generally have very
different features — including qualitatively different experimental sig-
natures, such as electron spectra — than are usually assumed in the
analyses of current $\beta\beta$ experiments.

2. The Electron Energy Spectrum

The experimental quantity that is used to distinguish exotic de-
cays from the ordinary SM events in $\beta\beta$ experiments is the shape of the
decay rate as a function of the energies, $\varepsilon_1$ and $\varepsilon_2$, of the two emitted
electrons. For instance, if no new light particles are produced, then $L_e$-
violating new physics can be identified if it produces decays which are
‘neutrinoless’ ($\beta\beta_{0\nu}$) in the sense that only electrons emerge from the
decaying nucleus. In this case the decay rate vanishes unless the sum
of the two electron energies, $\varepsilon = \varepsilon_1 + \varepsilon_2$, equals the released energy, $Q$.

Decays into electrons plus light scalars, on the other hand, predict
a continuous decay distribution throughout the entire kinematically
allowed interval, $2m_e \leq \varepsilon \leq Q$, that is distinguishable from both the
SM $\beta\beta_{2\nu}$ distribution, and the neutrinoless $\beta\beta_{0\nu}$ contribution at the
endpoint, $\varepsilon = Q$. Indeed, the prediction of the scalar-mediated ($\beta\beta_\phi$)
decay in models like the Gelmini-Roncadelli (GR) model of lepton-
number breaking [6,7] helped to motivate the original $\beta\beta$ experiments.
2.1. The Spectral Index

The different spectra that are possible in the various decays can be characterized (except for \(\beta\beta^0\nu\)) in terms of a single integer, or ‘spectral index’, \(n\). This is because the decay distribution quite generally can be written in the following form:

\[
\frac{d\Gamma}{d\varepsilon_1 d\varepsilon_2} = \Gamma_0 (Q - \varepsilon_1 - \varepsilon_2)^n [p_1 \varepsilon_1 F(\varepsilon_1)] [p_2 \varepsilon_2 F(\varepsilon_2)],
\]

where \(F(\varepsilon_i)\) is the Fermi function which describes the distortion of the decay distribution due to the electrostatic field of the nucleus, and \(p_i = |p_i|\), for \(i = 1, 2\), represent the magnitudes of the three-momenta of the electrons. We neglect the mass, \(\mu\), of the light scalar (or scalars) that are emitted in writing eq. (1). Should \(\mu\) not be negligible in comparison with \(Q\), then the factor \((Q - \varepsilon_1 - \varepsilon_2)^n\) should be replaced by \([(Q - \varepsilon_1 - \varepsilon_2)^2 - \mu^2]^{n/2}\). Of course, if there are several types of possible decay, the total spectrum is a sum of terms like eq. (1).

A plot of the spectral shape which follows from eq. (1) for various choices for \(n\), is given in Fig. 1. The spectral shape is determined by \(n\) because the normalization, \(\Gamma_0\), does not depend, to a very good approximation, on the two electron energies, \(\varepsilon_1\) and \(\varepsilon_2\). This is because the most important quantity which sets the scale for contributions to \(\Gamma_0\) is the typical momenta, \(p_N \sim 60\) MeV, of the decaying neutrons. Since the sizes of the light particle momenta are set by the net energy release, \(Q \sim 1\) MeV, they are negligible in their contribution to \(\Gamma_0\), and in this approximation the spectral shape becomes determined purely by phase space and the Fermi functions.

For example, the phase space of the two emitted neutrinos (plus the electrons) of \(\beta\beta^2\nu\nu\) in the SM implies the corresponding spectral index is \(n_{SM} = 5\). Using the phase space of a single scalar instead of two neutrinos similarly gives the result \(n_{GR} = 1\) for the spectrum predicted for \(\beta\beta^\phi\) decay by the GR model. With one early exception,\(\dagger\) this same spectrum is also predicted by all of the alternative models for scalar-emitting decays which have been proposed ever since the original GR paper.

Of course, Nature need not be limited to the two choices \(n = 5\) and \(n = 1\), and in general other values for \(n\) might be expected to be possible signals for \(\beta\beta\) experiments. It turns out that this naive expectation is true, and that models exist which predict both \(n = 3\) and \(n = 7\)\(\dagger\). Furthermore, some of these models — particularly those

\(\dagger\) As will become clear later, the cases \(n = 5, 9, \ldots\) can also be obtained by combining the features which produce the \(n = 1, 3\) and \(7\) decays.
for which \( n = 3 \) — can produce observable signals in \( \beta \beta \) experiments while preserving agreement with all other constraints, such as those arising from precision electroweak measurements on the \( Z \) resonance.

![Figure 1](image)

The \( \beta \beta \) spectrum as a function of the two electrons’ total kinetic energy for various choices of the ‘spectral index’ \( n \). \( n = 1 \) corresponds to the dotted line, \( n = 3 \) is the dashed line, \( n = 5 \) is the solid line and \( n = 7 \) is the dash-dotted line. All four curves have been arbitrarily assigned the same maximal value for purposes of comparison.

2.2. How to Generate \( n \neq 1 \)

The remainder of this article is intended to briefly outline the properties of models with each of the values \( n = 1, 3 \) and 7. Before doing so in detail, however, it is worth identifying the two general features which go into the prediction of the index \( n \) for any model. These are:

1. Phase Space: As was noted above, the phase space that is associated with a decay into two electrons and a scalar in \( \beta \beta_\phi \) decay implies an index \( n = 1 \), as for the GR model. The phase space of each additional scalar that appears in the final state similarly increases \( n \) by 2, so that in the absence of other contributions to \( n \), a two-scalar decay (\( \beta \beta_\phi \phi \)) should have: \( n = 3 \), a three-scalar decay: \( n = 5 \), and so on.
2. **Nambu-Goldstone Bosons:** As is well known, if a scalar is a NGB, then its couplings all must vanish in the limit of zero energy and momentum. This generally implies a suppression of the contribution of such a scalar in any low-energy process, and in particular for $\beta\beta$ decay. Since this suppression implies that the emission amplitude for each NGB is proportional to a factor of the NGB momentum, every such particle in the $\beta\beta$ final state should also increase $n$ by 2, in addition to the requirements of phase space. \[\dagger\]

It is immediately clear how to generate scalar-emitting $\beta\beta$ decays for which $n \neq 1$. If two scalars whose emission amplitude is not derivatively suppressed are produced in $\beta\beta_\varphi$ decay, then the index for the decay should be $n = 3$. If $N$ such scalars are emitted then $n = 2N - 1$. Alternatively, if a single scalar is emitted in a $\beta\beta_\varphi$ event, but this scalar has the derivatively-suppressed couplings of a NGB, then we again expect $n = 3$. Similarly, if two such derivatively-suppressed scalars are emitted, then the index for the corresponding decay is $n = 7$. These arguments are borne out by detailed calculations.\[\dagger\]

### 2.3. A Puzzle with the GR Model

A puzzle remains as to how the original GR model itself, and its many successors over the years, fit into the above counting scheme. After all, the light scalar which is emitted in $\beta\beta_\varphi$ decay in the GR model is a NGB: It is the NGB for the spontaneous breaking of lepton number. Yet even so, the spectral index which is predicted for this decay is $n_{GR} = 1$, rather than $n = 3$ as the previous counting would have predicted. The resolution of this puzzle is instructive because it reveals an additional criterion for increasing the spectral index of a model.

The resolution to this puzzle\[\dagger\] goes as follows. It is useful to think in terms of variables for which the NGB of the GR model is explicitly derivatively coupled. (These variables may be obtained from the standard ones by performing a field-dependent lepton-number rotation.) In these new variables the NGB couples directly to the electron, as well as to the various neutrinos of the model. The graphs which then dominate $\beta\beta_\varphi$ decay at low energies turn out to be those for which the NGB of the final state is emitted from the external electron lines, rather than from the neutrino lines.

But the emission of a massless boson with a vector coupling by an external electron line introduces a potential singularity into the amplitude at low momenta. Indeed if the emitted particle had been a photon

---

\[\dagger\] See, however, section 2.3 below for a qualification to this statement.
rather than a NGB, such graphs would really be infrared divergent. For NGB emission, however, the infrared singularity of these graphs is compensated by the derivative coupling of the massless scalar, giving a nonzero and finite result in the zero-momentum limit.

In practice, then, in order to obtain a real suppression of $\beta\beta_\varphi$ or $\beta\beta_{\varphi\varphi}$ decay because of the NGB nature of the emitted scalar, it is also necessary to forbid the emission of the scalar from the external electrons in the decay. As is shown below, this is an automatic feature of many models once they are required to not produce experimentally unacceptable rates for $\beta\beta_0\nu$ decay in a natural way.

3. Models: Preliminaries

The remainder of the article is devoted to outlining the properties of the various kinds of scalar-emitting decays which the $\beta\beta$ experiments can see. The present section sketches those features which are required as preliminaries to model building, and the properties of some explicit models are briefly discussed in the section immediately following. Some of the more general conclusions that can be drawn from a comparison of these models are finally summarized in section 5.

3.1. The Naturalness Problem

The purpose of this section is to argue that a serious fine-tuning problem exists for virtually all of the models that have been proposed to date which predict $n = 1$ for the single-scalar $\beta\beta_\varphi$ decays, or which predict $n = 3$ for the two-scalar $\beta\beta_{\varphi\varphi}$ decays. This fine tuning arises from a naturalness issue which has strong implications for any model which purports to predict a detectably large scalar-emitting $\beta\beta$ decay rate.

This naturalness issue hinges on the following two questions which any such model must address:

1. Masses: The first question is: How can an elementary scalar have a mass that is smaller than $Q \sim 1\,\text{MeV}$, and so be light enough to permit its production in $\beta\beta$ decay? Being some five orders of magnitude lighter than the electroweak scale, such a small scalar mass introduces a fine-tuning, or ‘heirarchy’, problem unless there is a mechanism which can protect it from virtual effects at the weak scale.

2. $\beta\beta_0\nu$ Decay: The second question concerns the size that is predicted for $\beta\beta_0\nu$ decay. A model which breaks $L_e$ generically also predicts a nonzero $\beta\beta_0\nu$ decay rate, and this rate must not be larger than the current experimental limit. Since this rate is often related to the rate for the scalar-emitting $\beta\beta$ decay, which
is by assumption detectable, it can be difficult to suppress the one reaction without suppressing both. This makes the resulting constraint quite powerful.

There are several ways in which the various existing models handle these issues. Two mechanisms have been proposed which can permit such a naturally small scalar mass. Although (somewhat surprisingly) supersymmetry can be used to do the job, the resulting models are quite complicated and fairly contrived. The much simpler alternative is to simply follow the original workers and to require that the light scalar be the NGB for an exact or approximate global symmetry.

The second question becomes important if the symmetry for which the scalar is the NGB is electron number itself, as is the case for virtually all proposed models which predict \( n = 1 \) as the index for the scalar-emitting \( \beta\beta \) decays. This includes essentially all models that have been considered until very recently, and is one of the main motivations for taking the models with \( n \neq 1 \) as important alternatives.

The problem arises because the same vacuum expectation value (v.e.v.), \( v \), which breaks \( L_e \) typically also generates a Majorana mass for the electron neutrino whose size is \( m_{\nu_e}\nu_e \sim g_{\text{eff}} v \), where \( g_{\text{eff}} \) is the relevant scalar-neutrino Yukawa coupling constant. Such a Majorana mass gives rise to \( \beta\beta \nu_e \nu_e \) decays which would have been seen if they exceed the current experimental limit, which is \( |m_{\nu_e}\nu_e| \ll 1 \) eV. But this limit cannot be satisfied simply by making \( g_{\text{eff}} \) very small, since if the \( \beta\beta \nu_e \nu_e \) decay itself is to be observably large, then \( g_{\text{eff}} \) cannot be made smaller than \( 10^{-4} \). From this lower limit for \( g_{\text{eff}} \) we learn that the \( L_e \)-breaking v.e.v. must satisfy \( v \lesssim 10 \) keV.

But this upper bound requires the appearance in the scalar potential of a scale that is more than 5 orders of magnitude smaller than the electroweak scale. As such, it poses precisely the same type of fine-tuning problem that would have occurred if it were the scalar mass itself that was to be fine tuned. In either case we are led to a mass scale in the scalar potential that is at largest several hundred keV or so.

It is tempting to argue that a hierarchy of the size of order \( v/M_W \sim 100 \text{keV}/100 \text{GeV} \sim 10^{-6} \) is not so small if the \( \varphi - \nu \) coupling is really \( g_{\text{eff}} \sim 10^{-4} \), since in this case radiative corrections would be \( \delta v \sim (g_{\text{eff}}/4\pi) \sim 10^{-5} \). Although seductive, this argument turns out to be wrong. The coupling, \( g_{\text{eff}} \), which appears in \( \beta\beta \nu_e \nu_e \) decay is not simply a yukawa coupling, \( g \), of the lagrangian, but is really an effective coupling in which a yukawa coupling is multiplied by a small mixing angle: \( g_{\text{eff}} \sim g \sin^2 \theta \). The mixing angle arises because the precision data at LEP precludes the direct coupling of any new light scalar to the electroweak sector. Scalar-emitting \( \beta\beta \) decay must therefore be
induced through the mixing of an electroweak eigenstate with another state which couples to the light scalar. This mixing can occur in either the neutrino sector, giving sterile-neutrino mediated decays, or, for example, in the scalar sector. In either case, the mixing angles typically cannot be too big without running into conflict with other experiments. In sterile-neutrino models, for example, weak interaction bounds imply that $\sin \theta$ cannot be larger than 10% or so.

Now comes the main point. Because of its suppression by powers of $\sin \theta$, $g_{\text{eff}}$ can only be as large as $10^{-4}$ if the underlying Yukawa coupling is considerably bigger. For example $\sin \theta \lesssim 10\%$ implies $g \gtrsim 10^{-2}$. But it is $g$ and not $g_{\text{eff}}$ which controls the radiative corrections to the scalar potential, so $\delta v$ is of order $(g/4\pi)$ rather than $(g_{\text{eff}}/4\pi)$ as was argued above. As a result the estimate using $g_{\text{eff}}$ underestimates the correction to $v$ by several orders of magnitude. More realistic estimates show that these corrections can only be sufficiently small in restrictive corners of parameters space, for which new degrees of freedom (like sterile neutrinos) are quite light, and so are strongly constrained phenomenologically.

To be sure, the $\beta\beta_{0\nu}$ decay rate could be made naturally small if $L_e$ were conserved, and so if $v$ were to vanish. But in this case the corresponding NGB disappears and the problem with the scalar mass must be solved in some other way.

These considerations point to a natural way out of this dilemma. Both the scalar mass and the $\beta\beta_{0\nu}$ decay rate can be naturally zero if: (a) the scalar is a NGB, and (b) $L_e$ is unbroken. It follows that the light scalar must then be a NGB for some symmetry other than that responsible for electron-number conservation. These two conditions are sufficient in themselves to imply a spectral index for the associated scalar-emitting $\beta\beta$ processes which is respectively $n = 3$ for $\beta\beta_{\varphi}$, and $n = 7$ for $\beta\beta_{\varphi\varphi}$ decay. This is easily seen since these decays can only proceed if the emitted scalar itself carries nonzero electron number which, together with $L_e$ and electric-charge conservation, precludes the possibility of scalar emission from the external electron lines.

3.2. $\Gamma_0$ and Nuclear Form Factors

Before turning to representative models for each type of decay, it is useful to pause to record expressions for the normalization of the various $\beta\beta$ decay rates in a way which facilitates the comparison of different models. These expressions are required in order to determine the kinds of couplings which would be necessary in order to obtain observable scalar-emitting $\beta\beta$ decay rates.

It is useful to use the $\beta\beta$ decay rate in the form given in eq. (1),
with the normalization given by

\[ \Gamma_0(\beta\beta_i) = \frac{(G_F \cos \theta_C)^4}{8(2\pi)^5} |\mathcal{A}(\beta\beta_i)|^2, \]  

(2)

where \(G_F\) is the Fermi constant, \(\theta_C\) the Cabibbo angle, and \(\mathcal{A}\) an amplitude which depends on the decay process being computed (\(\beta\beta_i\), with \(i = 2\nu, \varphi\) or \(\varphi\varphi\)), on the couplings of the model, and on some soon-to-be-identified nuclear matrix elements.

\(\mathcal{A}\) can be written, for \(0^+ \rightarrow 0^+\) transitions, as a convolution:

\[ \mathcal{A} = \int \frac{d^4\ell}{(2\pi)^4} L^{\mu\nu}(\ell) W_{\mu\nu}(\ell), \]  

(3)

where \(L^{\mu\nu}\) depends on the detailed properties of the leptons in the model, and \(W_{\mu\nu}(\ell)\) contains the nuclear matrix elements that are relevant to the decay:

\[ W_{\mu\nu}(\ell) \equiv (2\pi)^3 \sqrt{EE'} \int d^4x \langle N' | T^* [J_\mu(x)J_\nu(0)] | N \rangle e^{i\ell x}. \]  

(4)

Here \(J_\mu = \bar{u} \gamma_\mu (1 + \gamma_5) d\) is the weak charged current that causes transitions from neutrons to protons, and \(|N\rangle\) and \(|N'\rangle\) represent the initial and final \(0^+\) nuclei in the decay. \(E\) and \(M\) are the energy and mass of the initial nucleus, \(N\), while \(E'\) and \(M'\) are the corresponding properties for the final nucleus, \(N'\). The symmetries of the problem ensure that the most general possible form for \(W_{\mu\nu}\) is:

\[ W_{\mu\nu}(\ell) = w_1 \eta_{\mu\nu} + w_2 u_\mu u_\nu + w_3 \ell_\mu \ell_\nu + w_4 (\ell_\mu u_\nu + \ell_\nu u_\mu) + w_5 (\ell_\mu u_\nu - \ell_\nu u_\mu) + iw_6 \epsilon_{\mu\nu\sigma\rho} u^\sigma \ell^\rho, \]  

(5)

where \(u_\mu\) is the four-velocity of the initial and final nucleus, and the six Lorentz-invariant form factors, \(w_a = w_a(u \cdot \ell, \ell^2), a = 1, \ldots, 6\), are functions of the two independent invariants that can be constructed from \(\ell_\mu\) and \(u_\mu\).

These form factors can be related to the nuclear matrix elements as they are quoted in the literature. For example, in many situations (such as \(\beta\beta_{2\nu}\), \(\beta\beta_{0\nu}\) and some kinds of \(\beta\beta_\varphi\) and \(\beta\beta_{\varphi\varphi}\) decays) \(L^{\mu\nu} \propto \eta^{\mu\nu}\) and so the decay rate only depends on the combination \(W^\mu_{\mu}\). This is simply the difference between the Fermi and Gamow-Teller form factors, which are defined by \(w_F \equiv W_{00}\) and \(w_{\text{GT}} \equiv \sum_{k=1}^3 W_{kk}\). These form factors, when computed using the closure and nonrelativistic impulse
approximations in a model of the nucleus, become

\[
w_F = \frac{2i\epsilon g_V^2}{\ell_0^2 - \ell^2 + i\epsilon} \langle \langle N' | \sum_{nm} e^{-i\mathbf{r}_{nm} \cdot \mathbf{r}} \tau_n^+ \tau_m^+ | N \rangle \rangle; \\
w_{\text{GT}} = \frac{2i\epsilon g_A^2}{\ell_0^2 - \ell^2 + i\epsilon} \langle \langle N' | \sum_{nm} e^{-i\mathbf{r}_{nm} \cdot \mathbf{r}} \tau_n^+ \tau_m^+ \sigma_n^+ \sigma_m | N \rangle \rangle. \tag{6}
\]

Here \( \epsilon \equiv \overline{E} - M \) is the average excitation energy of the intermediate nuclear state, \( \mathbf{r}_{nm} \) is the separation in position between the two decaying nucleons, \( \ell \) is the spatial component of \( \ell' \) in the nuclear rest frame, and \( g_V \simeq 1 \) and \( g_A \simeq 1.25 \) are the vector and axial couplings of the nucleon to the weak currents. Finally, \( \langle \langle \mathbf{N}' | \mathbf{O} | \mathbf{N} \rangle \rangle \) represents a reduced matrix element from which the nuclear centre-of-mass motion has been extracted, and \( \tau_n^+ \) (or \( \sigma_n \)) are the raising operators for nuclear isospin (or the nuclear spin operators) acting on the \( n \)’th nucleon.

### 4. Models: Specific Examples

We now turn to the details of representative models for which \( n = 1, 3 \) and \( 7 \). Although much of what follows applies more generally we choose, for simplicity and for the purposes of comparison, models in which the corresponding \( \beta \beta \) decay proceeds due to the mixing of the electron neutrino, \( \nu_e \), with a collection of sterile neutrinos, \( N_i \). The reader who is not interested in the specific properties of these models, can skip to the next, concluding, section in which their general features are compared.

#### 4.1. The Case \( n = 1 \)

The simplest models to build are those for which the spectral index takes its traditional value, \( n = 1 \). Theories of this sort, when constructed using sterile neutrinos, are similar to the original singlet-majoron models. They, as well as other variants with \( n = 1 \) which do not rely on sterile neutrinos to produce \( \beta \beta e \) decays, have recently received renewed attention.

Suppose the neutrino mass eigenstates are denoted generically by \( \nu_i \), and their overlap with the electron-neutrino flavour eigenstate is called \( V_{ei} \). Suppose also that \( \varphi \) represents the light scalar of the model. Then, taking the general Yukawa coupling lagrangian between these particles to be

\[
\mathcal{L}_{\varphi \nu \nu} = -\frac{1}{2} \bar{\nu}_i (a_{ij} \gamma_L + b_{ij} \gamma_R) \nu_j \varphi^* + \text{c.c.}, \tag{7}
\]
and evaluating the Feynman graph of Fig. 2, gives an amplitude $\mathcal{A}$ of the form of eq. (3), with

$$L^{\mu\nu}(\ell) = 4\sqrt{2} \sum_{ij} V_{ei} V_{ej} \left[ \frac{(a_{ij} m_i m_j - \ell^2 b_{ij}) \eta^{\mu\nu}}{(\ell^2 + m_i^2 - i\epsilon)(\ell^2 + m_j^2 - i\epsilon)} \right].$$

(8)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{Figure 2}
\end{figure}

The Feynman graph which is responsible for $\beta\beta\varphi$ decay in models for which $\beta\beta$ decay can arise because of sterile-neutrino exchange.

The remaining question is whether an observably large $\beta\beta\varphi$ decay rate can be obtained from these expressions without becoming in conflict with any existing phenomenological bounds. This can be addressed only by constructing an explicit model in which all of the relevant observables can be computed as a function of a common set of underlying parameters. We therefore next present an representative model for this kind of theory.

An explicit sterile-neutrino model which produces $\beta\beta\varphi$ decays with $n = 1$ is simple to construct. Add to the SM two electroweak-singlet, left-handed neutrinos $s_{\pm}$, which carry $\pm 1$ unit of an unbroken global lepton number symmetry. Add also a singlet scalar field with lepton number $-2$. The most general renormalizable couplings of these

\footnote{Our conventions here use $\eta^{\mu\nu} = \text{diag}(-+++)$.}
new particles, consistent with lepton number conservation and the SM
gauge symmetries, are

\[ \mathcal{L} = -\lambda LH\gamma_{\mu} s_- - M s_+ \gamma_{\mu} s_+ - \frac{g_+}{2} s_+ \gamma_{\mu} s_+ \varphi - \frac{g_-}{2} s_- \gamma_{\mu} s_- \varphi^* + c.c. \] (9)

Here \( L \) and \( H \) respectively denote the usual SM lepton- and Higgs
doublets, and \( \gamma_L \) and \( \gamma_R \) denote the usual projections onto left- and
right-handed spinors. Majorana spinors are used throughout to rep-
resent the neutrinos, so the spinor conjugate used above employs the
charge-conjugation matrix, \( C \), according to \( \nu_i^c = \nu_i^T C^{-1} \).

As discussed previously, the experimental absence of \( \beta\beta_{0}\nu \) decay
requires the expectation value of the scalar field, \( \langle \varphi \rangle \), not to be larger
than \( \sim 100 \text{ keV} \). Rather than fine-tuning in such a small scale, it is sim-
pler to take \( \langle \varphi \rangle = 0 \), so that lepton number is not spontaneously broken.
Then the fine-tuning simply becomes the requirement that the scalar
mass be \( \lesssim 10 \text{ keV} \). In this case, the spectrum contains three massless
neutrinos, \( \nu^e, \nu^\mu \) and \( \nu^\tau \), together with a massive Dirac neutrino, \( \nu_h \),
which mixes with the electron-type charged-current weak interactions.

The model can have a detectable \( \beta\beta_{\nu \varphi} \) decay rate, provided that
the masses and mixings are chosen judiciously. Because the scale of
the factor \( W_{\mu \nu} \) in eq. (3) is set by the nucleon momentum, \( p_N \sim 60 \text{ GeV} \), \( \beta\beta \) decay rate tends to become suppressed if all neutrino masses
are much larger than, or much smaller than this scale. As a result,
a successful choice of parameters, which can also avoid bounds from
other laboratory experiments, puts the heavy neutrino mass eigenstate
at several hundred MeV, with a fairly large (somewhat less than 10\%) mixing with \( \nu_e \). More of the qualitative features and problems that arise
with these models are outlined in section 5, below. Those interested in
the details of the analysis are referred to the recent literature.

4.2. The Case \( n = 3 \): Two-scalar Decays

There are two possibilities for producing \( n = 3 \) decays: two light
scalars could be emitted without a NGB suppression of the emission
amplitude, or one light NGB-suppressed scalar could be emitted. An
example of the two-scalar decay is given here, even though the two-
scalar emission models involve the same fine-tuning problems as do the
\( n = 1 \) models just described. The alternative models, for which \( n = 3 \)
arises in single-scalar decays, are the subject of the next subsection.

Consider in this case a theory of two types of left-handed neutrinos,
\( \nu_i \) and \( N_a \), which respectively carry lepton number \( L_e(\nu_i) = +1 \)
and \( L_e(N_a) = 0 \), and which are coupled to the light scalar, \( \varphi \), which
has lepton number \( L_e(\varphi) = +1 \). The most general renormalizable and
$L_e$-conserving Yukawa couplings involving these fields are:

$$\mathcal{L}_{yuk} = - \bar{\nu}_i (A_{ia} \gamma_L + B_{ia} \gamma_R) N_a \varphi + h.c.,$$  \hspace{1cm} (10)

where $A_{ia}$ and $B_{ia}$ represent arbitrary Yukawa-coupling matrices. These neutrinos are endowed with a set of generic lepton-number-conserving masses, $m_{\nu_i}$ and $m_{N_a}$. For the $L_e = 0$ neutrinos, $N_a$, this is accomplished by simply introducing a general majorana-mass term. For the $L_e = 1$ neutrinos, however, an additional collection of $L_e = 1$ right-handed neutrinos are required, with which $L_e$-invariant Dirac masses may be formed.

This type of theory produces $\beta\beta\varphi\varphi$ decay due to the Feynman graph of Fig. 3. Two scalars must be emitted in this decay because of $L_e$ conservation. Evaluating this graph gives a result of the form of eqs. 1, 2 and 3, with 5:

$$L_{\mu \nu}(\ell) = \frac{2}{3\pi^2} \sum_{ija} \frac{V_{e\nu_i} V_{e\nu_j} N_{ija}}{(\ell^2 + m_{\nu_i}^2 - i\epsilon)(\ell^2 + m_{\nu_j}^2 - i\epsilon)(\ell^2 + m_{N_a}^2 - i\epsilon)},$$  \hspace{1cm} (11)

where the factor, $N_{ija}$, denotes:

$$N_{ija} \equiv (-\ell^2)[A_{ia} B_{ja} m_{\nu_i} + A_{ja} B_{ia} m_{\nu_j} + B_{ia} B_{ja} m_{N_a}] + A_{ia} A_{ja} m_{\nu_i} m_{\nu_j} m_{N_a}.$$  \hspace{1cm} (12)

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure3.png}
\caption{The Feynman graph which is responsible for $\beta\beta\varphi\varphi$ decay in models for which $\beta\beta$ decay arises because of sterile-neutrino exchange.}
\end{figure}
It is just possible to obtain a detectably large decay rate in this kind of model without running into conflict with other laboratory bounds. Because of the comparatively soft electron spectrum (since \( n = 3 \)), the total integrated decay rate tends to be suppressed compared to \( n = 1 \) models by an additional two powers of the small ratio \( Q/p_N \sim 10^{-1} \). The additional phase space also introduces additional suppression due to dimensionless factors of \( 1/2\pi \). As a result the total rate tends to be much smaller than in a comparable model for which \( n = 1 \). This makes it more difficult to obtain observable decays, and a sufficiently large rate generally requires sterile neutrinos to lie in the mass range in the vicinity of 100 MeV which optimizes the \( \beta\beta \) decay rate.

4.3. The Case \( n = 3 \): Single-scalar Decays

We next turn to models which predict only single-scalar \( \beta\beta \phi \), but for which the spectrum nevertheless has \( n = 3 \). This may be ensured by constructing the model to ensure that the emitted scalar is a NGB which carries two units of conserved electron number.

The rate for \( \beta\beta \phi \) decay is given by evaluating the result for the Feynman graph of Fig. 2. using the generic Yukawa coupling of eq. (7). By virtue of the light scalar being a NGB carrying unbroken electron number, one finds that the leading result in powers of the lepton momenta, as given by expression (8), vanishes identically. It is therefore necessary to work to next order in these momenta, which raises the resulting spectral index from \( n = 1 \) to \( n = 3 \). The resulting amplitude then satisfies \( L^{0\nu} = -L^{\nu0} \), with

\[
L^{0m} = -4\sqrt{2} \sum_{ij} V_{ei} V_{ej} b_{ij} \left[ \frac{\ell^m}{(\ell^2 + m_i^2 - i\epsilon)(\ell^2 - m_j^2 + i\epsilon)} \right],
\]

\[
L^{mn} = -4\sqrt{2} \sum_{ij} V_{ei} V_{ej} b_{ij} \left[ \frac{\epsilon^{mnt} \ell_r}{(\ell^2 + m_i^2 - i\epsilon)(\ell^2 - m_j^2 + i\epsilon)} \right].
\] (13)

An example of a renormalizable model for which this is the decay formula is given by requiring the theory to have a nonabelian flavour symmetry, \( G = SU_F(2) \times U_L'(1) \) which gets broken down to the unbroken electron number. To implement this symmetry-breaking pattern, introduce an electroweak-singlet scalar field, \( \Phi_i \), which transforms under \( G \) like \( (2, -1) \). Also introduce the electroweak-singlet left-handed neutrino fields, \( \tilde{N} \sim (2, 0) \) and \( s_\pm \sim (1, \pm 1) \).

The most general renormalizable lagrangian involving the new fields which respects all of the symmetries is

\[
\mathcal{L} = -\lambda \bar{L} H \gamma_5 \gamma_R s_- - M \bar{s}_+ \gamma_R s_- - g_+ (\bar{N} \gamma_L s_+) \Phi - g_- (\bar{N} \gamma_L s_-) \bar{\Phi} + c.c.
\] (14)
\( \Phi = i\tau_2 \Phi^* \) represents the conjugate \( SU_F(2) \) doublet, where \( \tau_2 \) is the second Pauli matrix acting on flavour indices. The scalar potential is then chosen to ensure that \( \Phi \) gets a VEV, which we assume to take the form \( \langle \Phi \rangle = \left( \frac{0}{v} \right) \).

The soft \( n = 3 \) spectrum suppresses the integrated decay rate, giving a detectable spectrum only when all couplings and masses are optimal. This once again places the sterile neutrinos in the mass range of several hundred MeV, with significant mixing with the electron neutrino.

### 4.4. The Case \( n = 7 \)

As a final example, consider the case with spectral index \( n = 7 \). This decay is produced by ensuring that the light scalar is a NGB, and that it carries only one unit of electron number so that the decay rate is derivatively suppressed, and two scalars must be emitted in the decay.

The \( \beta \beta_{\phi \phi} \) decay rate is found by evaluating the Feynman graph of Fig. 3. Once again, the symmetry properties make it necessary to work to higher order in the lepton momenta than was necessary for the \( n = 3 \) \( \beta \beta_{\phi \phi} \) decay. It turns out that the expression for the resulting amplitude is quite cumbersome when given in terms of the couplings of eq. (10), and so we use instead variables for which the derivative coupling nature of the Goldstone bosons is manifest from the outset.

The trilinear coupling to neutrinos of a Goldstone boson carrying \( L_e = 1 \), then becomes:

\[
L_{gb} = -i \, \mathcal{D}_i \gamma^\mu (X_{ia} \gamma_L + Y_{ia} \gamma_R) N_a \, \partial_\mu \phi + h.c.,
\]

where the coefficients \( X_{ia} \) and \( Y_{ia} \) are coupling matrices that can be computed in any specific model. Using these interactions to evaluate the amplitude given by the diagram of Fig. 3 gives, for the special case \( Y_{ia} = 0 \) :

\[
L^{\mu \nu}(\ell) = \left( \frac{4}{105 \pi^2} \right) \sum_{ija} \frac{V_{ie} \, V_{ej} \, \tilde{N}_{ija} \gamma^{\mu
u}}{(\ell^2 + m^2_{\nu_i} - i\epsilon) \, (\ell^2 + m^2_{\nu_j} - i\epsilon) \, (\ell^2 + m^2_{N_a} - i\epsilon)},
\]

\( \tilde{N}_{ija} \) represents the following expression:

\[
\tilde{N}_{ija} \equiv (-\ell^2) X_{ia} X_{ja} m_{N_a}.
\]
for detecting this kind of decay. Firstly, since the electron spectrum is softer than that of the SM $\beta\beta_{2\nu}$ decay, the decay electrons tend to come out with low energies, and so would be difficult to distinguish from background.

Secondly, the soft spectrum greatly suppresses the integrated total decay rate, making it very difficult to get a detectable decay in a model which also satisfies all other laboratory bounds. Even though models which produce this spectrum have been constructed, none have been found which are both phenomenologically viable and which predict a detectable $\beta\beta_{\phi\phi}$ decay rate.

5. Models: General Features and Conclusions

A comparison of models in these four classes leads to a number of reasonably robust conclusions.

1. The models for which $n = 1$ and those which predict $n = 3$ $\beta\beta_{\phi\phi}$ decays illustrate in detail the general fine-tuning problem that was argued in section 2 to be endemic to these kinds of $\beta\beta_{\phi}$ decay. The requirement that a light scalar should exist, and that $\beta\beta_{0\nu}$ should not be predicted at an unacceptably large rate, taken together require a fine tuning of the scalar potential to ensure either an extremely small scalar mass, or an equally small $L_\phi$-breaking v.e.v. Although supersymmetric models along these lines have been constructed which are technically natural, they are also quite contrived and complicated.

2. Models with softer electron spectra give smaller integrated decay rates, since the total decay becomes suppressed by higher powers of the small endpoint energy $Q$. This implies that theories predicting $n = 1$ decays have an easier time producing observably large decay rates than do models for which $n$ is larger. As a result these models tend to offer the largest latitude to accommodate other laboratory limits and phenomenological constraints.

To produce acceptably large scalar-emitting $\beta\beta$ decay rates, models for which $n = 3$ must typically have all of the relevant dimensionless couplings be $O(1)$, have the mixings of the relevant sterile neutrino be as large as are phenomenologically allowed ($\lesssim 10\%$), and have the participating new neutrino states have masses in the optimal mass range of a few hundred MeV. This mass range is preferred since it is comparable to the typical momenta, $p_N$ of the decaying nucleons within the nucleus, and so does not lead to suppressions of the form of $M/p_N$ or $p_N/M$. The resulting models therefore can work, but do not leave a great deal of freedom to
accomodate other limits. Of the \( n = 3 \) models, those which emit only a single scalar tend to predict larger decay rates than the two-scalar decays because they are not suppressed by additional small phase-space factors.

Models for which \( n = 7 \) are the worst case, and have decay rates that are sufficiently suppressed by powers of \( Q/p_N \) that they are unlikely to be experimentally detectable for the foreseeable future.

3. Models with electron spectra as soft as the \( n = 7 \) decays are also harder to detect for another reason, besides the size of their total decay rate. Most of the background in \( \beta\beta \) decay experiments occurs for electron energies which are comparatively soft, and so it is the soft electrons which are the ones which are hard to dig out of this background. This makes it all the more unlikely that these decays will turn up in the experimental data.

4. Because the contribution of sterile neutrinos to scalar-emitting \( \beta\beta \) decays become suppressed when their masses are much larger or much smaller than around 100 MeV, and since couplings and mixings tend to have to be large in order to produce an observable decay, the experimental discovery of \( \beta\beta_\varphi \) or \( \beta\beta_{\varphi\varphi} \) decay would strongly suggest the existence of new particles in this mass range. The signals for such particles could be: anomalous bumps in \( K \rightarrow e\nu \) or \( \pi \rightarrow e\nu \) decays; violations of weak universality in leptonic \( \pi \) decays, possible Zenlike monojet events at LEP, etc. Signals in beam dump experiments would not necessarily be expected, since the heavy neutrinos would dominantly decay invisibly into ordinary neutrinos and the light scalars.

5. All models which produce scalar-emitting \( \beta\beta \) decays necessarily have at least one very light scalar which is significantly coupled to the electron neutrino. As a result, all of these models generically run into trouble with Big Bang nucleosynthesis. The scalars tend not to decouple from the ordinary neutrinos, and so tend to violate the constraints on the gravitating degrees of freedom which can exist at the epoch around \( T \approx 1 \text{ MeV} \). This bound can be evaded for comparatively special values of the masses and couplings of the particles involved. The loophole arises since there can be an interval during which the neutrinos are no longer in chemical equilibrium with the protons and neutrons, but where the \( n/p \) ratio has still not frozen out. During this interval the annihilation of sterile neutrinos can raise the electron neutrino abundance, which in turn acts to deplete the neutron abundance.
A sterile neutrino which annihilates in this way effectively counts as a negative number of neutrino species, and so can counteract the positive contribution of the light scalars.

Of course, the details of the loophole are less interesting than the simple fact that the loophole exists. Although nucleosynthesis considerations generically disfavour models with additional light scalars, nucleosynthesis should and would be re-evaluated should a scalar-emitting $\beta\beta$ decay mode be observed.

6. The nuclear matrix elements that are required to evaluate the $\beta\beta$ decay rates in essentially all of the models are the same as those which have long been studied within the context of $\beta\beta_0\nu$ and $\beta\beta_\varphi$ decay in the GR model. This can be seen since $L^{\mu\nu} \propto \eta^{\mu\nu}$, which implies that the decay rate depends only on the nuclear form factor combination: $W^\mu_\mu = w_{GT} - w_{F}$. This is precisely the same combination as appears in $\beta\beta_2\nu$ and $\beta\beta_0\nu$ decays, and so it is well studied in the literature.

The exception to this rule are those models which predict $n = 3$ single-scalar $\beta\beta_\varphi$ decay, which instead depend on the antisymmetric part of $W^\mu_\mu$. The corresponding matrix elements are less well studied, with a corresponding greater uncertainty in the predicted decay rates. (Explicit formulae for these matrix elements in terms of nucleon operators are given in the literature.)

7. All of the models given here predict the same decay distribution as a function of the opening angle of the two electrons, so this observable cannot be used to distinguish one from another (as it can be used to distinguish decays mediated through right-handed currents). The only exception to this statement among the models considered are those for which the final electrons are emitted from an $L_e = 2$ scalar having electric charge $Q = -2$. Unfortunately the bounds on the masses of such scalars make the resulting $\beta\beta$ decay rate undetectable.

To summarize, $\beta\beta$ experiments can legitimately expect to see scalar emitting decays even though the original models which proposed this decay mode have since been ruled out by constraints such as those coming from LEP. If such decays are seen, they are most likely to point to new physics with properties which are quite different than what would have been expected from these traditional models. Although decay spectra with $n = 1, 3, 5, 7, \ldots$ are all possible, the basic combinations are $n = 1, 3$ and 7. The $n = 7$ spectrum is very unlikely to seen, however, due to the strong suppression of this decay rate by powers of the small
energy release, $Q$. Two-scalar $n = 3$ decays also tend to be suppressed compared to single-scalar decays having the same spectrum due to the presence of additional small phase-space factors. Furthermore, naturalness considerations (driven by the strong bound on the occurrence of the $\beta\beta_{0\nu}$ decay mode) disfavour those models which predict $n = 1$ $\beta\beta_1\phi$ decays, or $n = 3$ $\beta\beta_3\phi\phi$ decays. Thus, from a theoretical perspective the $n = 3$ $\beta\beta_3\phi$ decays are the most natural. If such decays are seen we may also probably expect some new developments in nucleosynthesis and in precision experiments such as those which constrain lepton universality.

May we live to see such exciting times!

6. Acknowledgements

I would like to thank the workshop’s organizers for providing such a splendid setting for the workshop, and for their kind invitation to speak. It is also a pleasure to acknowledge my collaborators in the research described here: Peter Bamert, Jim Cline and Rabi Mohapatra. Our funds have been provided by linear combinations of N.S.E.R.C. of Canada, les Fonds F.C.A.R. du Qu´ebeq, the Swiss National Foundation and the U.S. National Science Foundation.

References
1. Recent reviews of the experimental situation may be found in: M. Moe, Int. J. Mod. Phys. E2 (1993) 507; M. Moe and P. Vogel, Ann. Rev. of Nucl. and Part. Sc. (to appear).
2. C.D. Carone, Phys. Lett. B308 (1993) 85.
3. C.P. Burgess and J.M. Cline, Phys. Lett. B298 (1993) 141; Phys. Rev. D49 (1994) 5925.
4. C.P. Burgess and J.M. Cline, in the proceedings of The 1st International Conference on Nonaccelerator Physics, Bangalore, January 1994, (World Scientific, Singapore).
5. P. Bamert, C.P. Burgess and R. Mohapatra, preprint McGill-94/18, NEIP-94-010, UMD-PP-95-78 [hep-ph/9412365].
6. G.B. Gelmini and M. Roncadelli, Phys. Lett. B99 (1981) 411.
7. H.M. Georgi, S.L. Glashow and S. Nussinov, Nuc. Phys. B193 (1981) 297.
8. R. Mohapatra and E. Takasugi, Phys. Lett. B211 (1988) 192.
9. C.P. Burgess and O. Hernández, Phys. Rev. D48 (1993) 4326.
10. Y. Chikashige, R.N. Mohapatra and R.D. Peccei, Phys. Rev. Lett. 45 (1980) 1926.
11. See e.g. M. Beck et.al., Phys. Rev. Lett. 70 (1993) 2853 and references therein, or J.-L. Vuilleumier et.al., Nucl. Phys. (Proc. Suppl.) B31 (1993).
12. Kai Zuber, in Relativistic Astrophysics and Particle Cosmology, ed. by C.W. Akerlof and M.A. Srednicki, Annals of the New York Academy of Sciences, Vol. 688 (New York, 1993).

13. A. Balysh, et al., Heidelberg preprint, [hep-ex/9502007] (July 1994).

14. M. Doi, T. Kotani and E. Takasugi, Prog. Theor. Phys. Suppl. 83 (1985) 1; T. Tomoda, Rept. Prog. Phys. 54 (1991) 53.

15. See also: W.C. Haxton and G.J. Stevenson, Progress in Particle and Nuc. Phys. B12 (1984) 409; T. Tomoda, A. Faessler and K.W. Schmid, Nucl. Phys. A452 (1986) 591.

16. A. Halprin, P. Minkowski, H. Primakoff and P. Rosen, Phys. Rev. D13 (1976) 2567; S.P. Rosen, Arlington preprint UTAPHY-HEP-4 (1992); and references therein.

17. H. Klapdor-Kleingrothaus, K. Muto and A. Staudt, Europhys. Lett. 13, 31 (1990); M. Hirsch et. al. Zeit. Phys. A 345, 163 (1994); H. Klapdor-Kleingrothaus, Prog. Part. Nucl. Phys. 32, 261 (1994) for a review.

18. Z.G. Berezhiani, A.Yu. Smirnov and J.W.F. Valle, Phys. Lett. B291 (1992) 99.

19. K. Enqvist, K. Kainulainen and M. Thomson, Phys. Rev. Lett. 68 (1992) 744.