Primordial black holes in a dimensionally reduced universe

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Received December 4, 2018
Accepted December 9, 2018
Published December 19, 2018

Abstract. We investigate the spontaneous creation of primordial black holes in a lower-dimensional expanding early universe. We use the no-boundary proposal to construct instanton solutions for both the background and a black hole nucleated inside this background. The resulting creation rate could lead to a significant population of primordial black holes during the lower dimensional phase. We also consider the subsequent evaporation of these dimensionally reduced black holes and find that their temperature increases with mass, whereas it decreases with mass for 4-dimensional black holes. This means that they could leave stable sub-Planckian relics, which might in principle provide the dark matter.

Keywords: primordial black holes, quantum black holes, quantum gravity phenomenology

ArXiv ePrint: 1811.09518
1 Introduction

Gravity is the most familiar and least understood fundamental interaction. We experience its influence in everyday life but progress towards its full understanding has been slow. On the experimental side, the accuracy of the value of the Newton’s constant $G_N \approx 6.67384(80) \times 10^{-11}$ N/(m/kg)$^2$ [1] has barely improved since the time of Cavendish [2]. On the theoretical side, the situation is no more promising. General relativity suffers from bad short-distance behavior and may need to be modified at large distances unless one invokes ad hoc dark sectors. Black holes are plagued with paradoxes connected to their information content [3].

The quantization of gravity as a standard quantum field theory has been frustrated by its non-remormalizable character, as indicated by the dimensionality of its coupling constant, $G_N = M_P^{-2}$ in natural units. Alternative quantization schemes, such as superstring theory, loop quantum gravity and asymptotically safe gravity, have made major breakthroughs but there is no consensus on which approach is correct.

Against this background, there has been a radical proposal to circumvent the difficulties in reconciling gravity and quantum mechanics. Following the seminal paper by ’t Hooft [4], one can conjecture that the Universe behaves two-dimensionally at the Planck scale, as a consequence of spontaneous dimensional reduction. From this perspective, the non-renormalizabity of gravity would be an apparent low energy feature that shows up only when the Universe “oxidates” to the standard, infrared $(3 + 1)$-dimensional manifold. In other words, general relativity would be just an effective description of a fundamental theory of quantum gravity governed by a dimensionless coupling constant $G_{(2)}$.

To understand the meaning of a Universe behaving two-dimensionally, one has to invoke an alternative definition of dimension. The study of quantum fluctuations at the Planck scale suggests that fractal dimensionality is the most appropriate concept in the early Universe. The spectral dimension is an indicator well suited for this purpose and its computation has been the focus of several investigations based on a variety of quantum spacetime models. For example, causal dynamical triangulation [5], loop quantum gravity [6], asymptotically safe gravity [7, 8], noncommutative geometry [9], multi-fractal geometry [10], modified dispersion
Figure 1. Illustrating the Universe’s dimensional oxidation. During the Planck era it behaves two-dimensionally and below some critical temperature, $T_{\text{crit}}$, it has the conventional four-dimensional form. From a string theory perspective, there may exist an intermediate three-dimensional phase with $T_{\text{crit}} < T_{\text{Hag}} \ll T_P \approx 1.42 \times 10^{32}$ K during which it is filled with a gas of strings [14].

relations [11] all suggest a continuous dimensional flow to two dimensions. It has also been shown that, due to their scaling dimension, unparticles — a sector of massive particles beyond the Standard Model (SM), conjectured to play a fundamental role in the discretization of the Universe at the Planck scale — can naturally arise in a dimensionally-reduced, fractalized Universe [12]; see [13] for a recent review.

The interpretation of the above results is as follows: while the number of spacetime dimension remains the usual topological one, i.e. $(3 + 1)$, matter fields and gravity itself may not be able to perceive all of them due to the local loss of resolution expected at Planckian energies. These results have led to a resurgence of interest in $(1 + 1)$-dimensional black holes in relation to such quantum gravitational characteristics as singularity avoidance [15], gravitational ultraviolet self-completeness [16, 17] and the recently proposed black hole chemistry [18].

In light of these developments, this paper addresses the issue of primordial black hole (PBH) production in a dimensionally reduced Universe. One usually expects gravitational collapse into PBHs to have occurred as a consequence of large density fluctuations in the early Universe [19]. After their evaporation some part of the initial PBH mass might survive as a cold remnant, thereby contributing to the dark matter [20]. However, in the lower-dimensional case, such a scenario needs to be revised [21]. First, in $(2 + 1)$-dimensions, black hole solutions of the Einstein equations exist only in Anti-de Sitter space, suggesting that the aforementioned oxidation from two to four dimensions might have been a non-analytic phase transition, as indicated in figure 1. Second, $(1 + 1)$-dimensional dilaton gravity black holes radiate with power proportional to their mass squared. Therefore, in marked contrast with their $(3 + 1)$-dimensional counterparts, lighter black holes would be more stable than heavier ones. This idea has been expanded upon in the context of the Bose-Einstein condensate corpuscular picture, wherein the end stage of black hole evaporation obeys a dimensionally-reduced behavior [22]. The probability of lower-dimensional black hole formation was also estimated using the horizon wavefunction approach [23], a method which was recently applied to production of PBHs in $(3 + 1)$-dimensions [24].
In order to estimate the PBH nuclear rate, we follow the gravitational instanton approach proposed by Mann & Ross [25] and Bousso & Hawking [26]. Within the Euclidean quantum gravity formulation [27], the Hartle-Hawking no-boundary proposal [28] implies that a spacetime can be represented quantum-mechanically by a wavefunction $\Psi$, defined by a path integral over positive-definite Euclidean metric configurations. In a semiclassical approach, the dominant contribution comes from instantons at the saddle points of the Euclidean gravitational action and $\Psi$ is determined by the instanton action [29]. Squaring this then gives the relative probability ($\Gamma$) of two universes [30], one representing an empty background and the other a black hole inside such a background. For $\Gamma > 1$, de Sitter space is quantum mechanically unstable, despite being classically stable. $\Gamma$ depends exponentially on the inverse of the cosmological constant [31, 32], so PBH production is strongly suppressed for the presently observed value of this constant ($\Lambda_c \sim 10^{-120}$) but non-negligible for $\Lambda_c \sim 1$.

In this paper, we use the instanton formalism to calculate the PBH production rate in a dimensionally-reduced Universe. However, this raises several important issues, none of which is fully resolved at this stage. One needs to distinguish these issues, since they involve different conceptual problems. The first issue concerns the nature of the transition from the $1+1$ to $3+1$ phase. One possibility is that there was an intermediate $2+1$ phase. For example, Atick and Witten have shown that a gas of strings, heated up to a critical temperature, $T_{\text{crit}}$, below the Hagedorn temperature, $T_{\text{Hag}}$, undergoes a phase transition to an effective $2+1$ phase [14]. This is illustrated in figure 1 but does not address the question of how to describe the black hole during this intermediate stage. The important point is that this scenario allows the dimensional transition to occur below the Planck temperature.

The second issue concerns the behavior of the cosmological constant during the transition. One must distinguish between the 2D cosmological constant $\Lambda$ and the current 4D cosmological constant $\Lambda_c$. In this context, one might expect the $1+1$ phase to be inflationary, since one can always choose a spacelike slicing in which the metric has an exponentially expanding form [33]. However, in this case, is the $1+1$ phase complete before the conventional $3+1$ inflationary phase or does it replace it? One might be reluctant to replace it because then one could lose some of the attractions of the standard scenario (e.g. the form of the density fluctuations). On the other hand, if the $1+1$ phase ends before $3+1$ inflation, there would be no observational consequences of $1+1$ PBH production.

The third issue concerns the black hole solution itself. One must understand how a $1+1$ black hole turns into a $3+1$ one and this is non-trivial. Possibly the region close to the black hole remains $(1+1)$-dimensional after the cosmological transition. Indeed, this is implicit in our discussion of the evaporation of a $1+1$ black hole in a $3+1$ cosmological background. This would entail the introduction of a spatial inhomogeneity but that is implicit in PBH production anyway.

The paper is organized as follows. In section 2 we review dilaton/Liouville gravity as a consistent candidate to replace Einstein gravity in $(1+1)$-dimensions and we calculate the instanton for the de Sitter background. In section 3 we derive two instantons for the $(1+1)$-dimensional Schwarzschild-de Sitter universe, one for a lukewarm black hole with a generic horizon structure and the other for a Nariai black hole with a degenerate horizon. In section 4 we estimate the black hole production rate inside such a two-dimensional universe and in section 5 we consider observational consequences of this. In section 6 we draw some conclusions.
2 Dimensionally reduced de Sitter instanton

First, we recall the gravitational instanton approach of Mann & Ross [25] and Bousso & Hawking [26]. Within the Euclidean quantum gravity formulation [27], the Hartle-Hawking no-boundary proposal [28] states that a spacetime can be treated quantum-mechanically by means of a wavefunction $\Psi$, which is defined by a path integral over positive-definite Euclidean metric configurations $g_{\mu\nu}$:

$$\Psi = \int \mathcal{D}[g_{\mu\nu}] \, e^{-S_E[g_{\mu\nu}]},$$

(2.1)

where $S_E$ is the Euclidean version of the gravitational action. This integral does not always converge because $S_E$ need not be positive. For this reason, one uses a semiclassical approach, according to which the dominant contribution comes from instantons at the saddle points of $S_E$ [29]. In this case, apart from a prefactor, the wavefunction becomes

$$\Psi \approx e^{-\mathcal{I}},$$

(2.2)

where $\mathcal{I}$ is the instanton action. Squaring this gives a probability, so the relative probability of two universes [30], one representing an empty background ($\mathcal{I}_{bg}$) and the other a black hole inside such a background ($\mathcal{I}_{bh}$), is given by

$$\Gamma = \exp \left[ -2(\mathcal{I}_{bh} - \mathcal{I}_{bg}) \right].$$

(2.3)

This indicates how suppressed ($\Gamma < 1$) or favored ($\Gamma > 1$) black hole formation is in such a universe. In the latter case, one can infer that de Sitter space is quantum mechanically unstable. Although $\Gamma$ is sometimes described as a “rate”, we note that it is just a dimensionless probability. For instance, in $(3 + 1)$-dimensions the Schwarzschild-de Sitter-Nariai creation rate becomes [31]

$$\Gamma = e^{-\pi/\Lambda_c},$$

(2.4)

in geometrical units with $G_N = 1$. The rate is strongly suppressed for the presently observed value of the cosmological constant ($\Lambda_c \sim 10^{-120}$) but becomes non-negligible for $\Lambda_c \sim 1$. This means that Planckian PBHs might have been produced prolifically from the decay of de Sitter space.

We now extend this approach to the lower dimensional case. Einstein gravity in $(1 + 1)$-dimensions is trivial because the Einstein tensor vanishes for every metric [34]. For this reason, dilaton gravity has been the focus of a vast research activity — see for instance [33, 35–42] — and offers a valuable alternative to describe systems in a dimensionally reduced spacetime. The dilaton is a scalar field that describes gravitational degrees of freedom alongside the graviton. It is present in higher dimensional gravitational theories, most notably in Kaluza-Klein and string theory. More importantly, it has been shown that a dilaton action can be derived by taking the continuous limit, $D \to 2$, of the $D$-dimensional Einstein-Hilbert action. This guarantees a high degree of correspondence between the original fully dimensional Universe and its reduced counterpart. Specifically, the corresponding dilaton action becomes [43]

$$S[g_{\mu\nu}, \psi] = \int d^2x \sqrt{-g} \left[ -\frac{1}{16\pi G_{(2)}} \left( \psi R + \frac{1}{2}(\nabla \psi)^2 - 2\Lambda \right) + \mathcal{L}_m \right],$$

(2.5)
where $R$ is the Ricci scalar, $\psi$ is the dilaton, $\Lambda$ is the cosmological constant, $\mathcal{L}_m$ is the matter Lagrangian and $G_{(2)}$ is Newton’s constant in $(1 + 1)$-dimensions. In natural units with $\hbar = c = k_B = 1$, the constant $G_{(2)}$ is dimensionless and can be set to 1 by convention — see [16] for reference.

The above action depends on both the metric and the dilaton field, so its variation generates two equations of motion. The variation in $\delta \psi$ gives

$$\nabla^2 \psi = R,$$

while the variation in $\delta g^{\mu \nu}$ gives

$$\frac{1}{2} \nabla_\mu \psi \nabla_\nu \psi - \frac{1}{4} g_{\mu \nu} (\nabla \psi)^2 + g_{\mu \nu} \nabla_\lambda \nabla_\nu \psi - \nabla_\mu \nabla_\nu \psi + \Lambda g_{\mu \nu} = 8\pi G_{(2)} T_{\mu \nu},$$

where the stress-energy tensor is defined as

$$T_{\mu \nu} = \frac{-2}{\sqrt{-g}} \frac{\delta (\sqrt{-g} \mathcal{L}_m)}{\delta g^{\mu \nu}}.$$ 

By combining eq. (2.6) with the trace of (2.7) one gets

$$R + 2\Lambda = 8\pi G_{(2)} T,$$

where $T = T^\mu_{\mu}$. This is called the Liouville field equation and governs gravity in $(1 + 1)$-dimensions. Equation (2.9) is usually regarded as the best analogue of the Einstein field equations.

In order to construct an instanton, we must first solve the Liouville field equation for a specific spacetime. In analogy with the known static and stationary solutions of the $(3 + 1)$-dimensional case, we make the following ansatz for the line element:

$$ds^2 = -V(x) dt^2 + dx^2 / V(x).$$

For vacuum solutions ($T = 0$), eq. (2.9) takes the simple form

$$-\frac{d^2 V(x)}{dx^2} + 2\Lambda = 0,$$

with solution

$$V(x) = \Lambda x^2 + Dx + C,$$

where $C$ and $D$ are integration constants. Because of its dimensionality, the constant $D$ may be identified with a mass parameter in $(1 + 1)$-dimensions [44]. Thus we can set it equal to zero ($D = 0$) for an empty background. The value of $C$ is arbitrary but its sign plays an important role for the existence of horizons. For this spacetime there are no physical singularities, while conical singularities may be found by taking $V(x) = 0$.

The de Sitter space has a cosmological horizon at

$$|x_c| = \sqrt{-\frac{C}{\Lambda}},$$

so $C$ and $\Lambda$ must have opposite signs for real horizons to exist. As can be checked from the metric potential (2.12), a $(1 + 1)$-dimensional de Sitter universe is characterized by a negative
cosmological constant \((\Lambda < 0)\) [18], so we can replace \(\Lambda\) with \(-|\Lambda|\). Only by choosing \(C > 0\) and \(\Lambda < 0\) can one get a compact and expanding \((1+1)\)-dimensional Universe, the shape of whose potential \(V\) is in analogy with the \((3+1)\)-dimensional de Sitter case. Since the value of \(C\) is arbitrary and positive, we can set it to unity without loss of generality. Thus expression (2.13) for the cosmological horizon becomes \(|x_c| = 1/\sqrt{|\Lambda|}\) and one finds

\[
V_{\text{dS}}(x) = 1 - |\Lambda|x^2. \tag{2.14}
\]

The shape of (2.14) is shown in figure 2. For the regularity of the instanton, we analytically continue the time, \(\tau = it\). Hawking radiation is associated with the periodicity of \(\tau\), the black hole temperature being the inverse of the period. This is \(\beta = 2\pi/\kappa\), where \(\kappa\) is the surface gravity, given by

\[
\kappa = \frac{1}{2} \left. \frac{dV}{dx} \right|_{x=x_c} = |\Lambda x_c| = \sqrt{|\Lambda|}, \tag{2.15}
\]

using eqs. (2.13) and (2.14). So in our case, the period is

\[
\beta = \frac{2\pi}{\sqrt{|\Lambda|}} \tag{2.16}
\]

and this removes the conical singularity.

The regularity of the instanton can be seen through an appropriate coordinate transformation via variables \(\xi\) and \(\chi\), with periods \(2\pi\) and \(\pi\), respectively:

\[
\tau = \frac{1}{\sqrt{|\Lambda|}} \xi, \tag{2.17}
\]

\[
x = \frac{1}{\sqrt{|\Lambda|}} \cos \chi. \tag{2.18}
\]

Under the above transformation, the de Sitter instanton can be written as

\[
ds^2 = \frac{1}{|\Lambda|} \left( d\chi^2 + \sin^2 \chi \; d\xi^2 \right). \tag{2.19}
\]
It can be seen that $\chi = 0$ ($x = x_c$) is the axis of the polar coordinates $(\chi, \xi)$ and the manifold is perfectly regular there. As shown in appendix A, the above line element can be cast into the two-dimensional Robertson-Walker form,

$$\text{d}s^2 = -\text{d}t^2 + a^2(t)\text{d}x^2, \quad a(t) = e^{\sqrt{|\Lambda|}t},$$

(2.20)

where the exponential scale factor corresponds to a $1 + 1$ inflationary model.

The Euclidean version of the dilaton action (2.5) for this compact spacetime is

$$\mathcal{I} = -\int d^2x \sqrt{g} \left[ \frac{1}{16\pi G(2)} \left( \psi R + \frac{1}{2} (\nabla \psi)^2 - 2\Lambda \right) + \mathcal{L}_m \right],$$

(2.21)

so, after integrating by parts and using (2.6), the ($1 + 1$)-dimensional de Sitter instanton-action is

$$\mathcal{I}_{\text{dS}} = -\frac{|\Lambda|}{8\pi G(2)} \int d^2x \sqrt{g} \left( \frac{1}{2} \psi + 1 \right).$$

(2.22)

By solving the equations of motion (2.6), the dilaton becomes

$$\psi = \psi_{0}^{(\text{dS})} - \text{arctanh}(x\sqrt{|\Lambda|}) - \ln \left( 1 - |\Lambda| x^2 \right)$$

(2.23)

and by inserting this into the instanton action (2.22) we finally obtain

$$\mathcal{I}_{\text{dS}} = -\psi_{0}^{(\text{dS})} + 4 - \ln 4 \cdot \frac{4G(2)}{4G(2)}.$$

(2.24)

Interestingly, this result does not depend on the cosmological constant or any specific energy scale, $G(2)$ being dimensionless. This means that, in contrast to the $(3 + 1)$-dimensional case, the de Sitter instability occurs irrespective of the value of the cosmological constant. Indeed, the production rate depends only on the ratio of the mass of the tunneled object and the cosmological constant, so the de Sitter background will not affect the production rate. We therefore normalize the background contribution to unity by choosing the integration constant $\psi_{0}^{(\text{dS})}$ so that $\mathcal{I}_{\text{dS}} = 0$.

### 3 Schwarzschild-de Sitter instanton

For the Schwarzshild-de Sitter spacetime, the solution of the field equations (2.9) can be derived after imposing reflection symmetry ($x \to |x|$) around the origin. The line element then becomes [44]

$$\text{d}s^2 = -(C + 2M|x| - |\Lambda|x^2)\text{d}t^2 + \frac{\text{d}x^2}{C + 2M|x| - |\Lambda|x^2}.$$  

(3.1)

The signs of $M$ and $C$ determine several distinct classes of solutions and their associated causal structures [15]. Here we require $M > 0$ because a negative mass implies a naked singularity [44]. Moreover, the existence of a compact spacetime, having both an event horizon $x_h$ and a cosmological horizon $x_c$, is possible only if $M^2 + C|\Lambda| \geq 0$ with $C < 0$. Since the value of $C$ is arbitrary, we can put $C = -1$. On symmetry grounds, we consider the spacetime region with $x > 0$ and drop the absolute value in the expression for the instanton.
The general form of the potential for the (1+1)-dimensional Schwarzschild-de Sitter instanton is therefore
\[ V(x) = -1 + 2Mx - |\Lambda|x^2 \quad \text{for} \quad x > 0, \tag{3.2} \]
with the corresponding horizons
\[ x_h = \frac{1}{|\Lambda|} \left( M - \sqrt{M^2 - |\Lambda|} \right) \equiv a \tag{3.3} \]
\[ x_c = \frac{1}{|\Lambda|} \left( M + \sqrt{M^2 - |\Lambda|} \right) \equiv b. \tag{3.4} \]

One gets two distinct horizons for \( M^2 > |\Lambda| \) (the lukewarm case) but a degenerate horizon for \( M^2 = |\Lambda| \) (the Nariai case). These two cases lead to smooth and regular Euclidean manifolds.

Let us start with the lukewarm case \([45, 46]\). The event horizon is given by (3.3) and the cosmological horizon by (3.4). We analytically continue to the Euclidean section by putting \( \tau = it \) and then choose the region \( a \leq x \leq b \) where the metric is positive-definite. Remarkably the two horizons have the same temperature. Therefore we can remove the conical singularities by demanding \( \tau \) to be periodic with the period being the inverse of the temperature
\[ \beta = T^{-1} = \left( \frac{1}{2\pi} \sqrt{M^2 - |\Lambda|} \right)^{-1}. \tag{3.5} \]

As expected, the temperature in (1+1)-dimensions is proportional to the black hole mass \([21]\) for \( M \gg \sqrt{\Lambda} \), so heavier black holes are hotter. This means they have a positive heat capacity and relax towards smaller, colder configurations.

One can write the metric coefficient in the lukewarm case as
\[ V_L(x) = \frac{(x - a)(b - x)}{ab}, \tag{3.6} \]
whose form is plotted in figure 3. The instanton action for the Schwarzschild-de Sitter spacetime is
\[ I_{\text{SdS}} = -\frac{|\Lambda|}{8\pi G(2)} \int d^2x \sqrt{g} \left( \frac{1}{2} \psi + 1 \right). \tag{3.7} \]
and from (2.6) one obtains
\[ \psi = \psi_0^{(L)} - \ln \left( -1 + 2Mx - |\Lambda|x^2 \right), \quad (3.8) \]
where \( \psi_0^{(L)} \) is an integration constant. The lukewarm instanton action is therefore
\[ \mathcal{I}_L = -\frac{1}{8G(2)} \left[ 2\psi_0^{(L)} + 8 + \ln \left( \frac{\Lambda^2}{16(|\Lambda| - M^2)^2} \right) \right] \quad (3.9) \]
and this becomes
\[ \mathcal{I}_L = -\frac{1}{8G(2)} \ln \left( \frac{\Lambda^2}{(|\Lambda| - M^2)^2} \right) \quad \text{for} \quad M > \sqrt{|\Lambda|} \quad (3.10) \]
providing we choose
\[ \psi_0^{(L)} = \psi_0^{(dS)} = -4 + \ln 4. \quad (3.11) \]

For the \((1 + 1)\)-dimensional Nariai instanton, \( M^2 = |\Lambda| \) and so there is a degenerate horizon at
\[ x_h = x_c = \rho = \frac{M}{|\Lambda|}. \quad (3.12) \]
The double root implies that the proper distance from any point to the degenerate horizon is infinite [25]. The surface gravity on the horizon is therefore zero, corresponding to the extremal case, and the mass and cosmological constant are
\[ M = \frac{1}{\rho}, \quad |\Lambda| = \frac{1}{\rho^2}. \quad (3.13) \]
By substituting (3.13) into (3.2), one therefore gets the form of the potential for the \((1 + 1)\)-dimensional Nariai instanton,
\[ V_N(x) = -\left( \frac{x - \rho}{\rho^2} \right)^2, \quad (3.14) \]
and this is illustrated in figure 4.
By Wick-rotating ($\tau = it$) we get the regular instanton metric

$$ds^2 = -\frac{(x-\rho)^2}{\rho^2} d\tau^2 - \frac{\rho^2}{(x-\rho)^2} dx^2.$$  \hfill (3.15)

To demonstrate its regularity, we can apply the Nariai transformation \[47, 48\]:

$$\tau = \frac{\xi}{|\Lambda|\epsilon}, \quad x = \rho - \epsilon \cos \chi, \quad x_h = \rho - \epsilon, \quad x_c = \rho + \epsilon,$$  \hfill (3.16)

where the two horizons coincide in the limit $\epsilon \to 0$. Under this transformation, the instanton (3.15) can be written in the form of eq. (2.19), so this can again be transformed into the inflating form (2.20). One might wonder why this does not represent a black hole in an expanding background; the reason is that the black hole and cosmological event horizons coincide in this case, so — in some sense — the background universe is itself a black hole.

For the Nariai instanton ($M = \sqrt{|\Lambda|}$), the action turns to be

$$I_N = -\psi^{(N)}_0 + \frac{4 - \ln \left(\frac{4\epsilon^2|\Lambda|}{|\Lambda|/\mu_0^2}\right)}{4G(2)},$$  \hfill (3.17)

where $\psi^{(N)}_0$ is an integration constant. The action then becomes

$$I_N = \frac{\ln \left(|\Lambda|/\mu_0^2\right)}{4G(2)}$$  \hfill (3.18)

providing we choose

$$\psi^{(N)}_0 = -4 + 2 \ln(2\epsilon\mu_0).$$  \hfill (3.19)

Here $\mu_0$ is an arbitrary mass scale, at least in principle unconnected to $M$ or $M_P$.

4 Creation rate of PBHs in (1 + 1)-dimensions

We now have all the elements needed to calculate the rate — or, more strictly, probability — of PBH production in (1 + 1)-dimensions. Before going into the details, it is worth recalling what has been obtained so far. First, the “rate” depends only on the instanton action of the nucleated object,

$$\Gamma = \exp \left[-2I_{bh}\right],$$  \hfill (4.1)

so the de Sitter background does not contribute to the rate, which is a great simplification. Second, the two-dimensional topology allows for a richer instanton structure than in four dimensions (eg. the lukewarm case). Third, the absence of a Planck mass in two dimensions has the important consequence that the de Sitter space is unstable, irrespective of the values of the cosmological constant and the black hole mass. Only their ratio affects the decay rates. This extends the (3 + 1)-dimensional scenario and allows for sub-Planckian PBH nucleation.
We now focus on the (1 + 1)-dimensional lukewarm spacetime, for which the rate is
\[ \Gamma_L = \left[ \frac{\Lambda^2}{(M^2 - |\Lambda|)^2} \right]^{\frac{1}{3\pi(2)}}. \]  
\[ (4.2) \]

We plot this as a function of mass for a fixed $|\Lambda|$ in figure 5. We see that the rate increases as the mass of the black hole decreases. To summarise, one has:

- For $M \gg \sqrt{|\Lambda|}$, the rate is highly suppressed ($\Gamma_L \ll 1$).
- For $M = \sqrt{2|\Lambda|}$, $\Gamma_L = 1$ and the two universes have equal probability.
- For $\sqrt{|\Lambda|} < M < \sqrt{2|\Lambda|}$, the rate $\Gamma_L$ exceeds 1, corresponding to a highly unstable de Sitter space.
- For $M \approx \sqrt{|\Lambda|}$, the rate diverges ($\Gamma_L \gg 1$).

Next we consider the (1 + 1)-dimensional Nariai instanton. After choosing suitable boundary conditions, one obtains
\[ \Gamma_N = \left( \frac{\mu_0^2}{|\Lambda|} \right)^{\frac{1}{3\pi(2)}}. \]  
\[ (4.3) \]

Here $\mu_0$ is not set a priori and PBHs can be produced prolifically for any value of $\Lambda$. As shown in figure 6, the Nariai instanton allows comparison with the four-dimensional case.

We now consider models in which the 2D and 4D cosmological constants are equal (i.e. $|\Lambda| = \Lambda_c$). From eqs. (2.4) and (4.3), the 2D PBH production rate then dominates the 4D one for any value of the (joint) cosmological constant smaller than a critical value:
\[ \Lambda_c < \Lambda_{\text{crit}} \approx 4.31 \mu_0^2. \]  
\[ (4.4) \]

In general, the cosmological terms $\Lambda$ and $\Lambda_c$, entering the two-dimensional and four-dimensional gravity actions, are not related. Even if action (2.5) is the dimensionally reduced...
Einstein-Hilbert action, there is no reference scale to set $\Lambda$, since $G_{(2)}$ is dimensionless in two dimensions. We have already noted such a scale freedom, since the production rate depends on either $M^2/\Lambda$ or $\mu_0^2/\Lambda$ but not $\Lambda$ itself. As a result, due to the arbitrariness of $\Lambda$, we can have two additional regimes: $|\Lambda|/\Lambda_c \ll 1$ and $|\Lambda|/\Lambda_c \gg 1$.

The first regime implies a prolific production of light two-dimensional Nariai black holes with $M = \sqrt{|\Lambda|}$. For ease of discussion, one can describe these as black holes with sub-Planckian masses. To this end, we indirectly introduce Planck units in two dimensions via the relation between $\Lambda$ and $\Lambda_c$, e.g. $\Lambda \ll \Lambda_c \sim 1$ for $G_N = 1$. Such sub-Planckian black holes have large radii and are thermodynamically stable due to their positive heat capacity. Eventually they oxidate, i.e., undergo a phase transition to the four-dimensional form. Unfortunately, the Schwarzschild metric cannot be used to connect a $1+1$ model with a $3+1$ one. The major obstacle is the presence of a $2M/r$ term, which contrasts with the $Mr$ term which appears in two dimensions. Rather one should employ the recently proposed holographic metric \[ds^2 = -\left(1 - \frac{2M\ell_P^2}{r^2 + L^2}\right)dt^2 + \left(1 - \frac{2M\ell_P^2}{r^2 + L^2}\right)^{-1}dr^2 + r^2d\Omega^2,\] with $L \sim \ell_P = 1/M_P$. For $r \gg L$, this becomes the Schwarzschild spacetime, while for $r \ll L$ it becomes the effectively two-dimensional metric \[ds^2 \longrightarrow -\left(1 - 2Mr\ell_P^2/L^2\right)dt^2 + \left(1 - 2Mr\ell_P^2/L^2\right)^{-1}dr^2 + \mathcal{O}\left(r^2/L^2\right).\]

The second regime, $|\Lambda|/\Lambda_c \gg 1$, corresponds to a two-dimensionally driven inflationary scenario that dominates over the standard four-dimensional one. The expansion is now controlled by the scale factor appearing in (2.20). Chan and Mann argue that a pure-radiation model is static and that one needs some exotic matter content to drive inflation [33]. However, their framework is different from ours, since they do not include a cosmological term in the gravity action. In our model, the presence of a negative $\Lambda$ suffices.
5 Observational consequences

In order to understand the observational consequences of this scenario, we need to consider the implications of the 1 + 1 phase for the standard 3 + 1 inflationary model. If we follow the Bousso-Hawking and Mann-Ross approaches, which assume 3 + 1 dimensions, the quantum mechanical decay of the universe into a spacetime with a black hole occurs only when the cosmological constant has the Planckian value of around $10^{133}$ eV m$^{-3}$. In the usual scenario, $\Lambda_c$ has an energy density of $10^{117}$ eV m$^{-3}$ at the start of inflation, which is 10$^{16}$ times smaller, so there should be no black hole production by this mechanism in 3 + 1 dimensions. However, in 1 + 1 dimensions, there is no Planck scale, since Newton’s constant is dimensionless, so black holes can be produced even without Planckian values of the cosmological constant. Indeed, dimensional reduction in quantum gravity may occur at an energy far below the Planck scale.

If the 1 + 1 phase ends before inflation is complete, there are no observational consequences because the black hole number density is exponentially diluted. However, we stress that we are only considering “effective” 1 + 1 dimensionality. This is because ’t Hooft’s argument for the self-renormalizability of gravity requires that gravity perceives just 1 + 1 dimensions but the Universe itself must remain 3 + 1 dimensional in order to describe the ultraviolet (Planckian) regime. Otherwise there would be no Planck scale at all. For present purposes, we therefore adopt a compromise model, in which the 1+1 phase persists throughout inflation, while the ambient cosmological space evolves 3 + 1 dimensionally, as in the standard model. In this way, we avoid modifying the standard inflation scenario but still expect observational consequences from PBH production during the 1 + 1 phase.

In summary we have:

- PBHs produced before inflation have no observational consequence, irrespective of the number of dimensions.
- 3+1 PBHs cannot be produced during the inflation since the rate depends on the 3+1 Newtonian constant and is small unless $\Lambda$ is Planckian.
- The 1 + 1 dimensional phase is only “effective”, the Universe itself remaining 3 + 1 dimensional.
- If the 1+1 dimensional phase persists through inflation, PBHs are produced prolifically since the rates do not depend on the Planck mass, $G_{(2)}$ being dimensionless. However, their number density is exponentially diluted unless they are produced at the end of inflation.

We conclude that the decay of de Sitter universe has observational consequences only if the (1 + 1)-dimensional phase persisted until the end of inflation. We do not necessarily regard this as a plausible situation, since one does not wish to risk sacrificing the attractions of the standard scenario. Nevertheless, we will confine attention to this possibility in what follows.

In order to study the cosmological consequences of PBHs produced through this mechanism, we first recall the Hawking temperature $T(M)$ and production rate $\Gamma(M)$ for PBHs of mass $M$. For the ease of the presentation we set $G_{(2)} = 1$. For $M \ll \sqrt{|\Lambda|}$, which includes the sub-Planckian case for $\sqrt{|\Lambda|} \ll M_P$, these are

$$T \approx M/(2\pi), \quad \Gamma_L \approx (\Lambda/M^2)^{1/2} \ll 1. \quad (5.1)$$
For $M = \sqrt{2|\Lambda|}$, they are
\[ T = \sqrt{|\Lambda|}/(\sqrt{2\pi}), \quad \Gamma_L = 1. \] (5.2)

For $\sqrt{|\Lambda|} < M < \sqrt{2|\Lambda|}$, they are
\[ T \approx \sqrt{|\Lambda|}/(2\pi), \quad \Gamma_L \approx 1. \] (5.3)

For $M = \sqrt{|\Lambda|}(1 + \epsilon)$ with $\epsilon \ll 1$, they are
\[ T \approx \sqrt{\epsilon|\Lambda|}/(2\pi) \ll \sqrt{|\Lambda|}, \quad \Gamma_L \approx \epsilon^{-1/2} \gg 1. \] (5.4)

For $M = \sqrt{|\Lambda|}$, corresponding to the Nariai black hole, they are
\[ T = 0, \quad \Gamma_N = (\mu_0/\sqrt{|\Lambda|}). \] (5.5)

For $M < \sqrt{|\Lambda|}$, there is no black hole solution but a naked singularity. So $\Gamma(M)$ cuts off below the peak at $M = \sqrt{|\Lambda|}$ and has a power-law decline for $M \gg \sqrt{|\Lambda|}$.

The initial collapse fraction of PBHs of mass $M$ is roughly
\[ \beta(M) \sim \Gamma(M), \] (5.6)

with no $H(M)$ factor because $\Gamma$ is a probability rather than a rate. Since the scale factor in the de Sitter background scales as $\exp(t\sqrt{|\Lambda|})$, $H(M)$ is taken to be $\sqrt{|\Lambda|}$. After the PBHs have formed, the density of the de Sitter background is constant, whereas the PBH density decreases as $a^{-1}$ in a $1 + 1$ model, so the fraction of the Universe’s total (radiation) density in PBHs decreases as
\[ \rho_{\text{PBH}}/\rho_R \propto a^{-1} \propto \exp(-t\sqrt{|\Lambda|}), \] (5.7)

where we have neglected evaporation. So the fraction decreases exponentially but not as fast as in the subsequent $3 + 1$ inflationary phase, where it decreases as $a^{-3}$. The current PBH mass function should be
\[ \frac{dn}{dM} \sim \frac{\beta(M)}{M^2} \sim \frac{\Gamma(M)}{M^2} \] (5.8)

and this is shown in figure 7. The density of PBHs of mass $M$, denoted by $\rho(M)$, is just $M^2$ times this and therefore comparable to $\Gamma(M)$. Both functions cut off below the peak at

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure7}
\caption{Initial PBH mass function $dn/dM$, without the contribution from Nariai black holes. At the present epoch the mass function may have collapsed down to a delta-function at $M \sim M_{\text{CMB}}$ due to evaporation.}
\end{figure}
$M = \sqrt{|\Lambda|}$ and have a power-law decline for $M \gg \sqrt{|\Lambda|}$. The delta-function contribution from the Nariai black holes depends on $\mu_o$ and is therefore not included in figure 7. There would be an exponential reduction given by eq. (50) if the black holes formed before the end of inflation and in this case the PBHs would have no observational consequences at all.

We now consider two possible cosmological consequences of these black holes: (1) those with $M \gg \sqrt{|\Lambda|}$ have a temperature $T \propto M$, so their evaporation consequences are very different from those of $3 + 1$ black holes; (2) Nariai black holes with $M = \sqrt{|\Lambda|}$ have zero temperature and are therefore stable, possibly contributing to the dark matter (cf. the Planck mass relics of $3 + 1$ black holes if their evaporation stops at the Planck scale).

### 5.1 Evaporating sub-Planckian black holes

In this section, we discuss the evolution of a $1 + 1$ PBH of a specific mass $M$, leaving to the next section the issues of what value of $M$ might be expected and the effect of an extended mass function. We initially neglect the effect of the cosmic background radiation, which would suppress evaporation if it were hotter than the black hole, since this might not exist at early times, but we return to this point at the end. We also neglect the effect of accretion, which could also suppress evaporation if it were large, since this is expected to be small [19].

From eqs. (3.3) and (3.5), the black hole radius and temperature are

$$R_S \approx \frac{1}{2M}, \quad T \approx \frac{M}{2\pi},$$

(5.9)

for $M \gg \sqrt{|\Lambda|}$, where $R_S$ corresponds to what was previously called $x_h$. For $\sqrt{|\Lambda|} \ll M_P$, the black holes can have less than the Planck mass and they would then resemble the sub-Planckian ones considered in ref. [17] in the $3 + 1$ context. In that work it was unclear whether black holes could form with sub-Planckian mass but here we have proposed a specific mechanism. Note that the condition $M \gg \sqrt{|\Lambda|}$ also allows $M > M_P$, so eq. (5.9) should even apply in the super-Planckian regime. Indeed, for $\sqrt{|\Lambda|} \gg M_P$, which may be unphysical, it could only apply in that regime.

In ref. [17] the luminosity of the black hole was written as

$$L \sim R_S^2 T^4 \sim \gamma M^2,$$

(5.10)

with $\gamma \equiv c^2/h$ and no dependence on $G_N$. This formula may seem suspect in the present context since it assumes the black hole is $3$-dimensional, whereas one expects the area to scale as $R_S^{n-1}$ and the black-body emission to scale as $T^{1+n}$ with $n$ spatial dimensions. Curiously, however, this gives $L \propto M^2$ for $n = 1$, so eq. (5.10) still applies [21]. Indeed, since there is no $G_N$ dependence in eq. (5.10), the scaling $L \propto M^2$ is required on purely dimensional grounds.

If the black hole forms at time $t_i$ with mass $M_i$, then eq. (5.10) implies its mass subsequently evolves according to

$$t - t_i \sim \frac{1}{\gamma} \left( \frac{1}{M} - \frac{1}{M_i} \right).$$

(5.11)

Although it decreases on a ‘Compton’ timescale,

$$\tau \sim \frac{1}{\gamma M} \sim h/(Mc^2),$$

(5.12)

the black hole never evaporates entirely because eq. (5.11) shows that it takes an infinite time for $M$ to reach zero. However, we note that there is a value of $M$ for which $\tau$ is comparable to the age of the Universe ($t_0 \sim 10^{17}$s) and this is

$$M_* \sim \frac{1}{\gamma t_0} \sim h/(c^2 t_0) \sim 10^{-65} \text{g}.$$
Figure 8. Current black hole mass $M(t_0)$ as a function of initial mass $M_i$, showing mass $M_*$ for which evaporation timescale equals age of Universe and mass $M_{CMB}$ for which black hole has current CMB temperature. Both mass-scales decrease with time but the latter does so more slowly.

From eq. (5.11), the mass of the black hole at the present epoch is then

$$M = \frac{M_i}{1 + \gamma M_i(t_0 - t_i)} \approx \frac{M_i}{1 + M_i/M_*}. \quad (5.14)$$

Hence $M \approx M_i$ for $M_i \ll M_*$ (i.e. the mass is unchanged) but $M \approx M_*$ for $M_i \gg M_*$. This is indicated by the dotted line in figure 8.

This mass-scale $M_*$ is associated with a radius of $10^{27}$ cm (the current cosmological horizon size) and a temperature of $10^{-28}$ K (the Hawking temperature for a black hole with the mass of the Universe). It might seem implausibly small but this mass-scale arises naturally in some estimates for the photon or graviton mass (e.g. in the work of Mureika and Mann [51]). It might also have observational consequences associated with gravitational effects on the scale of clusters and the Dvali-Gabadadze-Porrati (DPG) effect.

The above analysis neglects the effect of the cosmic microwave background (CMB). This may be appropriate until the end of inflation, since there may be no background radiation then. However, the assumption fails after reheating and evaporation will be suppressed whenever the black hole temperature is less than the CMB temperature ($T_{CMB}$). This means that the PBH mass may never actually reach the tiny value $M_*$. Indeed, the CMB should prevent PBH evaporation below an epoch-dependent mass

$$M_{CMB} = 10^{-36}(T_{CMB}/3K) \text{ g.} \quad (5.15)$$

Since accretion is expected to be unimportant [19], the PBH mass should effectively freeze at this value, i.e., its value can be approximated as $M_i$ for $M_i < M_{CMB}$ and $M_{CMB}$ for $M_i > M_{CMB}$. This is indicated by the solid line in figure 8. Note that $M_*$ and $M_{CMB}$ both decrease with time but as $t^{-1}$ and $t^{-2/3}$, respectively, so $M_*$ falls faster.

5.2 Lower dimensional PBHs as dark matter

The above scenario leads to stable relics which might in principle provide the dark matter. The relic PBH mass decreases with time but the current value is around $10^{-4}$ eV, which is close to the mass-scale associated with the dark energy. This coincidence reflects the fact that the dark energy density ($\rho_{DE} \sim \Lambda_c$) and CMB density ($\rho_{CMB} \sim T_{CMB}^4 \sim M_{CMB}^4$) are not so different at the present epoch, corresponding to just a factor of 10 in $M_{CMB}$. 

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We have seen that the PBHs formed in this scenario can only have an appreciable density if they form at the end of inflation. In this case, eq. (5.8) should still apply but with the value of the cosmological constant at the end of inflation ($\Lambda_i$). The dominant contribution to the density would then come from PBHs with initial mass $\sqrt{\Lambda_i}$. Providing this exceeds the mass $M_{\text{CMB}}$, the analysis of section 5.1 suggests that all these PBHs will have shrunk to $M_{\text{CMB}}$, with the mass function shown in figure 7 turning into a delta-function at around that mass. Otherwise they will still have the mass $\sqrt{\Lambda_i}$.

We must also consider the effects of the Nariai black holes, these necessarily having a current mass $\sqrt{\Lambda_i}$ since they do not evaporate at all. From eq. (5.8), their density at formation is

$$\rho_N(\Lambda_i) \sim M^2 \frac{dn_N}{dM} \sim \Gamma_N(\Lambda_i) \sim \frac{\mu_0}{\sqrt{\Lambda_i}}.$$  \hspace{1cm} (5.16)

(Strictly, one should integrate over $\Lambda$ but the dominant effect clearly comes from $\Lambda_i$ due to the exponential dilution prior to reheating.) The current PBH mass function should therefore comprise two delta-functions, one at $M \sim \sqrt{\Lambda_i}$ and the other at $M \sim M_{\text{CMB}}$. This raises the question of which component could most plausibly provide the dark matter. The Nariai contribution depends on the value of $\mu_0$ in eq. (5.5), this being essentially a free parameter. Of course, it requires very fine-tuning of $\Lambda_i$, $t_i$ and $\mu_0$ to explain the dark matter but such fine-tuning is a feature of all PBH scenarios. Having the dark matter in $(1 + 1)$-dimensional objects with zero temperature might seem rather radical but one does expect the PBH mass function to peak at this mass.

6 Summary and future work

In this paper we have studied the spontaneous production of PBHs if the universe was effectively $(1+1)$-D before it became $(3+1)$-D. We have investigated this quantum nucleation process semi-classically by using the instanton method and constructing instantons which represent an early $(1+1)$-D universe. Our estimate of the PBH creation rate suggests that they could be very abundant and have an initial mass $\sim \sqrt{\Lambda}$ and size $\sim 1/\sqrt{\Lambda}$, where $\Lambda$ is the value of the cosmological constant in the (inflating) $1+1$ background. The ones which form at the end of the inflationary phase will dominate the current density. Providing $\sqrt{\Lambda_i}$ exceeds the mass of the CMB photons, the PBHs will decay down to that mass, leaving stable relics which could provide the dark matter. Alternatively, the dark matter could be associated with Nariai black holes which retain the mass $\sqrt{\Lambda_i}$ because they never evaporate at all. If one identifies $|\Lambda_i|$ with the current 4D cosmological constant, this naturally explains why the dark energy and dark matter have comparable densities. For this model to be viable, the $1+1$ phase must persist until the end of inflation and we suggest a scenario in which this happens plausibly in an accompanying paper.

Acknowledgments

The work of P.N. has been supported by the project “Evaporation of the microscopic black holes” of the German Research Foundation (DFG) under the grant NI 1282/2-2 and by the Helmholtz International Center for FAIR within the framework of the LOEWE program (Landesoffensive zur Entwicklung Wissenschaftlich-Ökonomischer Exzellenz) launched by the State of Hesse. A.G.T, P.N. and J.M. are grateful to the Max Planck Institute for Radioastronomy, Bonn, for hospitality during the early stages of this work.
A Robertson-Walker metric in two dimensions

To map the two-dimensional de Sitter solution,
\[
ds^2 = -(1 - |\Lambda| x^2) dt^2 + \frac{dx^2}{(1 - |\Lambda| x^2)}, \tag{A.1}
\]
into the two-dimensional Robertson-Walker metric,
\[
ds^2 = -dt^2 + a^2(t) dx^2, \tag{A.2}
\]
one performs the following coordinate transformation:
\[
t = T - \frac{1}{2 \sqrt{|\Lambda|}} \ln \left( -\frac{1}{|\Lambda|} + r^2 e^{2 \sqrt{|\Lambda|} T} \right), \tag{A.3}
\]
\[
x = r e^{\sqrt{|\Lambda|} T}. \tag{A.4}
\]
After differentiation, one gets
\[
dt = \sqrt{|\Lambda|} \frac{dT}{1 - |\Lambda| r^2 e^{2 \sqrt{|\Lambda|} T}} + r e^{\sqrt{|\Lambda|} T} dr, \tag{A.5}
\]
\[
dx = e^{\sqrt{|\Lambda|} T} dr + r \sqrt{|\Lambda|} e^{\sqrt{|\Lambda|} T} dT. \tag{A.6}
\]
Putting (A.4), (A.5) and (A.6) into (A.1) gives
\[
ds^2 = -dT^2 + e^{2 \sqrt{|\Lambda|} T} dr^2. \tag{A.7}
\]
With the relabelling \( T \to t \) and \( r \to x \), this becomes
\[
ds^2 = -dt^2 + e^{2 \sqrt{|\Lambda|} t} dx^2, \tag{A.8}
\]
allowing us to identify the scale factor in (A.2) as
\[
a(t) = e^{\sqrt{|\Lambda|} t}. \tag{A.9}
\]

We can also map the Nariai metric into the 1 + 1 FRW metric by introducing the coordinates
\[
\xi = i \sqrt{|\Lambda|} \left( T - \frac{1}{2 \sqrt{|\Lambda|}} \ln \left( -\frac{1}{|\Lambda|} + r^2 e^{2 \sqrt{|\Lambda|} T} \right) \right), \tag{A.10}
\]
\[
\chi = \arccos \left( \sqrt{|\Lambda|} r e^{\sqrt{|\Lambda|} T} \right). \tag{A.11}
\]
Differentiating then gives
\[
d\xi = i \sqrt{|\Lambda|} \frac{dT + |\Lambda| r e^{2 \sqrt{|\Lambda|} T} dr}{1 - e^{2 \sqrt{|\Lambda|} T} |\Lambda| r^2}, \tag{A.12}
\]
\[
d\chi = -\frac{\sqrt{|\Lambda|} e^{\sqrt{|\Lambda|} T}}{\sqrt{1 - e^{2 \sqrt{|\Lambda|} T} |\Lambda| r^2}} \left( dr + r \sqrt{|\Lambda|} dT \right). \tag{A.13}
\]
Substituting these relations into (2.19) again yields metric (A.8). Note that the cosmological background can itself be regarded as a black hole in the Nariai solution.
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