Harmonic quarks and their precise masses

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Abstract

An examination of charged two-quark meson masses hinted that the mass ratio of neighboring quarks could simply be a constant. The concept of a harmonic quark oscillator based on a quark-antiquark pair is introduced. Unstable symmetric state of the harmonic quark oscillator can be broken to an asymmetric state with the formation the neighboring quark of lesser mass due to a weak reaction. A new recurrent equation of quark masses is obtained and the model of harmonic quark family is developed. The quark masses in the model are bound together into one rigid chain. With the electromagnetic contribution taken into account the quark masses are calculated with the 0.03 percent inaccuracy. It follows from the model that the muon is a single u-quark mass state fixed as a lepton. The neutral pion is probably a stable harmonic oscillator based on a quark-antiquark u-pair.

1 Introduction

Let us briefly review the present state of the quark mass problem, considering that, modern physics has enough experimental data to be certain in existence of quarks. Quarks acquire their masses in a process which is related to the Higgs sector and goes back to the original Higgs’s work of 1964 [1]. Quarks are interacting with the Higgs boson (bosons) proportionally to masses of quarks. The extensive review of the Higgs boson search and theories presented in [2]. The Higgs boson is believed to have the mass 115 GeV [3] or perhaps even up to 200 GeV [4]. In view of indeterminacy of a theoretical position, both the number of bosons and their masses remain unknown and therefore the concrete mechanism of quark mass formation is also unclear. As twenty years ago [5] and now [6] practically the same opinion is stated:

“... we do not at all understand the lepton and quark mass spectrum ...”

“The fact, that the mass spectrum of quarks and leptons envelops 5 orders of magnitude, is a mystery from any point of view... Even with universal recognition of standard model, these problems seem very far from resolution” [6].

Theory urges us to employ the concept of a quark mass with a great deal of discretion, because only bound quark systems (i.e. particles) are observable. Depending on the concrete character of quark interaction, we can expect either
positive or negative binding energy. The negative binding energy is analogous to mass defect in nucleus when the mass sum of the component quarks is more than the mass of the relevant hadron. The positive binding energy signifies that in addition to quark masses there is an additional energy in a hadron, for example, the energy of color field, which provides the quark confinement. The notion of quark mass and its value depends upon the theoretical model used in a given experimental or theoretical work. With respect to quark mass modern physics developed a number of notions: there exist bare quark mass, physical quark mass, constituent quark mass and effective quark mass. Effective quark mass is inherently related to the impulse used to calculate it \[6\] and we won’t consider it here. An interacting field theory has a well-defined notion of the rest mass of a single particle state, which is called physical mass. There can also be discrete bound states of the particle-antiparticle pairs.

The mass parameter in the QCD Lagrangian is considered to be a bare quark mass \[6\]. The various estimations within the QCD theory using the \(\overline{MS}\)-schemes and lattice simulations show that the bare masses of the light \(u\)- and \(d\)-quarks are less than 10 MeV \[7\]. The estimation of the bare mass ratios for the \(u\)-, \(d\)- and \(s\)-quarks is 1 : 2 : 40 respectively. Therefore, it is accepted, that the fundamental Lagrangian of strong interaction does not contain any hint on a symmetry with respect to quark masses \[6\].

Approximate constituent quark masses can be obtained from experimental data of the hadron masses \[8\]. Constituent quark is deemed to be formed from the bare quark and its “coat”, i.e. surrounding gluon field energy, the sea quarks and all other unsolved hadron structure problems. First of all, the “coat” notion makes the concept of a constituent quark mass fuzzy, because the quark’s incapability of solitary existence makes the mass of its “coat” dependent on a particular environment, i.e. from the quark structure of hadron and the quark-gluon dynamics.

On the other hand, the mass discreteness of the light hadrons with equal quantum numbers is several times greater than pion masses. It is difficult to understand why a light constituent quarks of pions have to dress in additional heavy “coat”. At the same time significant discreteness of the hadron mass spectrum makes it possible to hope that the quark-gluon field energy is also subject to quantization.

On the whole, after the forty years the birth of the quark model, the solution of the quark mass problem seems to be far still. It is also apparent that both the hadron internal energetic structure and quark-quark interaction dynamics is impossible to fully understand without the knowledge of quark mass quantization and the precise values of their spectrum. These reasons and the author’s conviction in the existence of the analytic relation for quark masses gave rise to the present work. Hereinafter under a quark mass we shall understand a physical rest mass of the single particle state of some interacting quantum field.
2 Quark properties and experimental data

As a first stage of the study we shall ascertain the fact that there is both theoretical reason and experimental data in favor of certain relation between quark masses. However, let’s begin with a few words about flavor as it is the only known quantum number, which is obviously connected with quark mass. All other quantum numbers is strictly quantized and are either constant or periodic by quark generations. Let’s ask ourselves, is quark mass quantized? It evidently is, for all available energetic information about matter indicates that quantum effects and energy discrete levels became dominant as an object dimensions decrease. The mass spectrum of the hadrons with different flavors support the idea of quark mass quantization. Therefore another question looms: is quark mass strictly quantized? According to field theory, the answer is yes. The flavor quantum number is essentially a reflection of quark’s internal energy—its physical mass. Let’s see how the flavor changes in the basic weak interactions of quarks.

\[ Q_n + W^\pm \leftrightarrow Q_{n+1} \]  
\[ Q_n + e^\pm \leftrightarrow Q_{n+1} + \nu \]

These are transition reactions between quarks with neighboring flavors. It is irrelevant whether real or virtual bosons the quarks are interacting with. The point is that both the quark’s mass and charge are changed. Therefore, two successive transformations will result in double mass transformation with the charge remaining constant—that is, flavor will be the only quantum number that will change.

We shall now examine theoretical computations of bare quark masses and certain mesons \[7\]. The values of bare quark masses are known with one digit precision at best. Let’s select for the analysis the charged mesons consisting of two neighboring quarks \( Q_n \) and \( Q_{n+1} \) with minimal mass in their ground state. In the table 1 these masses are given together with their relations 1.

| quark | quark mass MeV | quark mass ratio | meson | meson mass MeV | meson mass ratio |
|-------|----------------|------------------|-------|----------------|-----------------|
| \( d \) | 3–9            | -                | -     | -              | -               |
| \( u \) | 1–5            | \( \approx 0.5 \) (u/d) | \( \pi^\pm \) (du) | 139.57         | -               |
| \( s \) | 75–170         | \( \approx 40 \) (s/u) | \( K^\pm \) (us) | 493.64         | 3.54 (\( K^\pm /\pi^\pm \)) |
| \( c \) | 1150–1350      | \( \approx 10 \) (c/s) | \( D^\pm \) (sc) | 1968.5         | 3.99 (\( D^\pm /K^\pm \)) |
| \( b \) | 4000–4400      | \( \approx 3.4 \) (b/c) | \( B^\pm \) (cb) | 6400           | 3.25 (\( B^\pm /D^\pm \)) |
| \( t \) | 174300         | \( \approx 42 \) (t/b) | -     | -              | -               |

The bare quark mass ratios do not show any regularity but the meson mass ratios definitely present certain degree of uniformity. The mean value of their

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1The table 1 data are taken from the Particle Data Group listing \[7\]
mass ratios is 3.6. Assuming that the neighboring quark mass ratio is actually a constant of 3.6 we can obtain the following quark masses (starting from the mean table value for the b-quark): 4200(b), 1167(c), 324(s), 90(u) and 25(d). The sum of the neighboring quark masses will be about 17% less than the meson masses for all considered mesons except $D_s^+$ ($\approx 25\%$). This sounds promising, for after a proportional increase of the quark masses we’ll have a match for three of the four examined mesons. Thus, taking the b-quark mass value equal to 5000 MeV we’ll have for the mesons $\pi^\pm$, $K^\pm$ and $B_s^\pm$ the mass values of 137, 493 and 6390 MeV respectively.

Other examples that meson mass ratio is roughly constant are given in the table 2.

| meson | meson mass, MeV | meson mass ratio |
|-------|-----------------|------------------|
| $\omega$ | 782             | -                |
| $J$    | 3097            | 3.96 ($J/\omega$) |
| $\Upsilon(1S)$ | 9460          | 3.05 ($\Upsilon/J$) |
| $\varphi$ | 1019.4         | -                |
| $\Psi(2S)$ | 3686          | 3.61 ($\Psi/\varphi$) |
| $\Upsilon(2S)$ | 10023         | 2.72 ($\Upsilon(2S)/\Psi$) |

In contrast to table 1 the mesons given here have a hidden flavor (that is, a quark-antiquark pair). Thus, we can claim that this experimental data set also supports the idea of approximate constancy of the neighboring quarks mass ratio ($\approx 3.6$).

So then, there is certain multiplicative pattern in mass transformation between quark flavors. This probably means that quarks are in a similar state in these mesons with various masses. This is quite hard to explain considering that quark and meson masses differ by two orders of magnitude. The light quarks of a pion should be relativistic whereas the heavy quarks of a heavy meson are not very relativistic even when taken at their bare masses. There would be no such contradiction if the quark masses themselves have the scale similarity with the same factor that charged mesons do. Then the quarks should not be very relativistic in all these mesons.

### 3 Construction of the quark mass model

Now we will attempt to build a quark transformation model and to calculate the quark mass transformation factor. We should investigate the quark interaction process that is essentially to guess the new key moments of the strong interaction. A couple of simple question to start things off: first of all—what is the character of quark-antiquark interaction, and second of all—what is the result of that interaction? It is rather easy to answer the second question.

This interaction produces either a meson based on the quark-antiquark pair (e.g. a vector meson) or the complete annihilation of the pair with the birth
of photons or smaller mass quarks and other particles. The formation of vector meson has only deferred the quark-antiquark annihilation. There are seemingly two extremes—either the pair is present (meson) or it disappears (annihilation). But this “yes or no” situation can be avoided by introducing the concept of a discrete annihilation spectrum or discrete coupling of a quark-antiquark pair. It is not much of a novelty actually. A good old discrete bound states of an interacting field theory, which are lower than the doubled physical mass of the quark. Bound quark-antiquark states are well known (e.g., their paired states are presented in table 2). We are approaching the main question now—what is the binding energy value of quark-antiquark bound state and what quantum laws operate by the levels of binding energy? It is still an open question now as we poorly know even the quark masses. It is similar a movement on the closed circle and our chance only to guess. Then new variant of the answer is below stated.

**Proposition #1**

The quark-antiquark pair of the same flavor $n$ can, instead of the complete annihilation, annihilate partly to the bound state of a harmonic oscillator with the mass equal to $m_n \cdot \frac{4}{\pi}$.

The partial annihilation means that the quark pair with total mass $2m_n$ are involved in such an interaction with the following equation for their bound state with total energy $m_{sum}$:

$$m_{sum} = 2m_n \int_0^{\pi} \frac{\sin(x)}{\pi} dx = \frac{4}{\pi} m_n$$  \(3\)

This process (let us name it the harmonic quark oscillation) can be also interpreted as a harmonic oscillation of the quark-antiquark pair over space-time from complete separation of quarks to their complete annihilation. So for example, the time-dependent quadratic mass term can be noted as the equation:

$$m^2(\tau) = (2m_n)^2 \cos^2(\omega \tau)$$  \(4\)

The time-averaged mass will also be equal to $m_n \cdot \frac{4}{\pi}$. In eq. 4 we are not considering the transition of the mass into other energy forms and these oscillations should be forbidden by the law of energy conservation. Nonetheless in the virtual meaning with the help of uncertainty relation we have a right to consider them to a certain boundary frequency $\omega$.

The equation 3 is also corresponds to a stable state of the quark pair with a certain mass defect value ($\approx 36\%$). This state could be similar to quarkonium solution of the wave equations with quarks mutually revolving on some stable orbit, except that it has the very high binding energy.

At this stage, it’s important to examine the stability of the harmonic oscillation. The process defined by 4 would not be stable for exactly the same reason which has allowed not to forbid it. Namely, that we are already at the energy fluctuation level comparable to the quark mass in the context of uncertainty relation, which means that the other fluctuations would immediately destroy the harmonic oscillator. But what will follow the destruction? There are at
least two ways to overcome this instability. The first one is that an unstable annihilating oscillator is fast damped and then quark pair goes to mentioned stationary state, which corresponds to equation (3). The second way is to accept the proposition two.

**Proposition #2**

*The labile symmetric state of the harmonic annihilating oscillator based on a quark-antiquark pair of the same flavor can be broken in one or another way and would subsequently pass into the asymmetric state based on quarks of different kinds, i.e. consisting of a quark-antiquark pair with neighboring flavors.*

To clarify this proposition let us write one of the possible reaction of $u$-antiquark birth.

\[
(s\bar{s})_{ho} + e^- = s + \bar{u} + \nu
\]  

(5)

where $(s\bar{s})_{ho}$ is a harmonic oscillator based on a strange quark-antiquark pair and $\nu$ is a neutrino. Reactions similar to (5) can be written down for any quark harmonic oscillator, both with actual and virtual electrons. Doubling the number of participants we can write the reactions with quarks only:

\[
2(s\bar{s})_{ho} = s + \bar{s} + u + \bar{u}
\]  

(6)

This reaction demonstrates the birth of the quark-antiquark $u$-pair from two harmonic oscillators based on strange quark-antiquark pairs. The main point here is the possibility of such reactions with neither extra participants nor energy addition. Note that, it is weak interaction with participation of other particles that is responsible for the symmetry breaking in one harmonic oscillator with the subsequent transition into the asymmetric state.

Now we can write down a recurrent relation between quarks of neighboring flavors, considering that a harmonic state with the energy value of $\frac{4}{\pi}m_n$ generates two quarks with masses $m_n$ and $m_{n-1}$:

\[
m_{n-1} = \frac{4}{\pi} m_n - m_n = m_n \left( \frac{4}{\pi} - 1 \right) = 0.27323954 m_n
\]

\[
\frac{m_n}{m_{n-1}} = \frac{\pi}{(4 - \pi)} = 3.6597924 \overset{\text{def}}{=} MQ
\]  

(7)

Thus, having introduced two propositions we managed to obtain an equation, which links together the masses of neighboring quarks. Hereafter using the word quarks, we shall mean the harmonic quarks, i.e. the quarks, which subject to the equation (7).

Let’s summarize the implications of the equation (7):

- Quarks form a rigid chain and their masses are all bound together
- The quarks have a logarithmically equidistant mass spectrum
• The multiplication factor $MQ$ between neighboring quark masses is equal to 3.66 (which is practically equal to the mean meson mass ratio (3.6) mentioned above).

• A quark flavor uniquely determines the transformation number.

To stress this last point, the (7) can be written as

$$m_n = m_0 \left[ \frac{\pi}{4 - \pi} \right]^n$$

where $m_0$ is a hypothetical initial quark mass, and the quark order number $n$ is essentially its flavor. Thus we have:

| n  | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|----|--|--|--|--|--|--|--|
| quark | d | u | s | c | b | t | b' |

It’s worthwhile to note that the total mass of any neighboring quark pair is precisely equal to the mass of the harmonic oscillator based on the quark with greater mass. Moreover, with respect to a given quark its two neighbors can be considered as an upward excitation (the quark with greater mass) and a downward excitation (the quark with lesser mass), with the equal electric charges of both excitations.

Finally, we have the chance to calculate actually the genuine quark masses and afterwards to verify the validity of the original propositions using ample experimental data.

4 Harmonic quark masses

4.1 Quark masses estimation

We can estimate the harmonic quark masses assuming that the charged meson mass is a total mass of the valence quarks composing it. Table 3 presents these estimations for each of the four stable mesons from the table 1.

Table 3. Harmonic quark masses, MeV (zero approximation)

| meson     | d  | u  | s  | c  | b  | t  |
|-----------|----|----|----|----|----|----|
| $\pi^\pm(ud)$ | 29.9 | 109.6 | 401 | 1468 | 5373 | 19665 |
| $K^\pm(su)$  | 28.9 | 105.9 | 387.7 | 1419 | 5193 | 19005 |
| $D^\pm(cs)$  | 31.5 | 115.4 | 422.4 | 1546 | 5658 | 20708 |
| $B^\pm(bc)$  | 28.0 | 102.5 | 375.3 | 1373 | 5026 | 18396 |

The dispersion of these estimations does not exceed 12%. It is worth mentioning that $B_c^\pm$ meson mass value is measured with 8% inaccuracy and the estimation for meson $D_c^\pm$ supposedly produce an overestimation. The pion-

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2The reasons for this overestimation are beyond the scope of the present paper and will be explained in further publications.
and kaon-estimated masses differ by 3.5%. This difference has a natural explanation if the sizes $\pi^\pm$ and $K^\pm$ are approximately equal. The only difference of their structure is in $d$ and $s$-quarks and the localization of the $d$-quark, which is different by an order of mass magnitude from the $s$-quark will demand an additional kinetic energy. That’s why the pion-estimated quark masses will be greater.

### 4.2 Bounds of harmonic quark masses

Keeping in mind that quark masses in the model are rigidly bound, the heavier mesons we use for the quark mass determination, the more accurate it will be. Therefore we would start with the heavy charged two-quark meson with the mass measured exact enough, for example, $b^\pm$ with the mass $5279.0 \pm 0.5$ MeV. The kaon-estimated (table 3) total mass of $b$ and $u$-quarks is 5299 MeV. Provided that binding energy in the meson is not greater than the $u$-quark mass, in other words not greater than 50% of its mass per single quark (a binding energy in harmonic oscillator is only 36% from mass of the everyone quark) we have that $b^\pm$ mass is an upper bound for the harmonic $b$-quark mass. The lower bound can be determined as follows. Let’s assume that the value of binding energy in a meson is positive but less than a pion producing threshold (about 140 MeV). Hence the lower bound of a harmonic $b$-quark mass is equal to $5279 - 140 - 109 = 5030$ MeV. Now we can calculate bounds for all other quarks using equation (8). The calculated data are given in the table 4.

|        | $d$   | $u$   | $s$   | $c$   | $b$   | $t$   |
|--------|-------|-------|-------|-------|-------|-------|
| lower bound | 28.0  | 102.6 | 375   | 1374  | 5030  | 18409 |
| upper bound | 29.4  | 107.7 | 394   | 1442  | 5279  | 19320 |
| mean value | 28.7  | 105.2 | 384   | 1408  | 5154  | 18864 |

We may see that the bound values for quark masses situate inside same data of tables 3 and the mean values are almost equal to kaon-calculated values. The fact that $\pi^\pm$ mass exceeds the sum of the upper mass bounds of the $u$ and $d$ by $\approx 2.5$ MeV means that pion binding energy is positive. It means that quarks are completely separated in $\pi^\pm$ and there is an additional energy of several MeVs. What kind of energy it is? The $u$ and $d$-quarks of a pion charged alike and will repel each other. The repulsion force will be the more effective the less distance is between the quarks. On the other hand, the color field would act on the longer distances. Thus the additional kinetic energy would be maximal at the color and Coulomb forces equality and would transform to Coulomb energy on the small distances and to color-field energy on large distances. The value of this additional energy is naturally corresponds to the electromagnetic split in hadronic duplets and triplets. The complete separation of the quarks in $\pi^\pm$ and the structural similarity of the stable charged mesons discovered earlier makes it possible to suppose that the quarks in other charged mesons (in particular $K^\pm$ and $b^\pm$) are also completely separated.
4.3 Electromagnetic split is taken into account

To calculate the mean value of electromagnetic split the author used well-known mass differences of the charged and neutral particles up to 1350 MeV, i.e. of pions, kaons (with \( J = 0, 1 \) and basic baryon octet. The seven differences altogether. The mean split value is 4.13 ± 1.47 MeV. Decreasing by this the mass values of \( \pi^\pm \), \( K^\pm \) and \( b^\pm \) and using equation 7 we obtain the quark masses to a first approximation (table 5). The inaccuracy value given depends only on electromagnetic split measurement inaccuracy and the meson mass. (For the \( b^\pm \) its mass measurement inaccuracy was also considered).

Table 5. Harmonic quark masses (MeV) determined with electromagnetic split consideration (first approximation)

| meson | \( d \) | \( u \) | \( s \) | \( c \) | \( b \) | \( t \) | error,\% |
|-------|-------|-------|-------|-------|-------|-------|--------|
| \( \pi^\pm \) | 29.066 | 106.374 | 389.31 | 1424.8 | 5214.4 | 19083.7 | ±1.08 |
| \( K^\pm \) | 28.706 | 105.058 | 384.49 | 1407.2 | 5149.9 | 18847.5 | ±0.30 |
| \( b^\pm \) | 28.815 | 105.456 | 385.95 | 1412.5 | 5169.4 | 18919.0 | ±0.030 |

There are two positive moments to emphasize here. First of all, pion- and kaon-calculated quark masses differ only by 1.2%, which is three times less than the same difference of masses to a zero approximation and, moreover, is in accordance with the inaccuracy calculated. And second of all, the most accurate values of the harmonic quark masses, calculated with the \( b^\pm \) meson (\( \approx 0.03\% \) inaccuracy), fall inside the range of pion- and kaon-calculated values. Hereafter the phrase “harmonic quark masses” would relate to these most accurate values.

5 First results

We will now some demonstrate the ability of harmonic model to solve real-world issues. Even the most superficial examination of the harmonic quark masses produce two impressive results:

- The total mass of \( u \) and \( d \) quarks is extremely close to the mass of neutral pion.
- \( u \)-quark mass is extraordinarily close to the muon mass.

Recollecting that neutral pion is the lightest existing hadron and the truly neutral particle it seems that **neutral pion with great share of probability is a stationary harmonic oscillator based on quark-antiquark \( u \)-pair.** The pion mass (134.97 MeV) differs from the oscillator mass (134.27 MeV by 0.5% only. It’s likely that the relativistic correction would account for that.

What is to \( u \)-quark and muon, the difference between them is only 0.2 MeV, with the \( u \)-quark mass value inaccuracy of 0.03 MeV which is seven times less than the difference. The obvious conclusion is that **muon is a successful attempt of Nature to explicitly fix the single \( u \)-quark mass state as**
a lepton suppressing color and fractional charge. As muon charge is
greater by $\frac{1}{3}e$ than the $u$-quark charge, the Coulomb energy of muon should also
be greater. If electron mass is practically Coulomb energy then the charge in $\frac{1}{3}e$
add to the quark mass $\approx 1/3$ from the electron mass. Hence, the difference in
0.2 MeV has an easy explanation. Thus the mystery of muon mass is naturally
solved. It is strong result and good argument for harmonic model. Furthermore,
the $MQ$ factor works well not only with quarks but with pions too, thereby
supporting the idea of similar structural formation of the mesons with different
flavors:

- $m_{\pi^0} \cdot MQ = 493.99$ MeV, which is only 0.34 MeV greater than the mass
  of the first meson containing strangeness ($K^\pm$)
- $m_{\pi^\pm} \cdot MQ^2 = 1869.4$ (MeV), which is precisely equal to the mass value of
  the first meson containing charm ($D^\pm$)

In the last case, the additional energy of the pion quarks is also transformed by
the factor $MQ^2$, which leads to the effectively increase of the $D^\pm$ meson mass.
This could be one of the above-mentioned reasons of the harmonic quark masses
overestimation when calculated with $D^\pm$ and $D_s^\pm$ mesons.

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