Promoting cooperation by reputation-driven group formation

Han-Xin Yang\textsuperscript{1} and Zhen Wang\textsuperscript{2,3}

\textsuperscript{1} Department of Physics, Fuzhou University, Fuzhou 350116, People’s Republic of China
\textsuperscript{2} Qingdao University, Qingdao, Shandong 266071, People’s Republic of China
\textsuperscript{3} Interdisciplinary Graduate School of Engineering Sciences, Kyushu University, Kasuga-koen, Kasuga-shi, Fukuoka 816-8580, Japan

E-mail: yanghanxin001@163.com

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Abstract. In previous studies of the spatial public goods game, each player is able to establish a group. However, in real life, some players cannot successfully organize groups for various reasons. In this paper, we propose a mechanism of reputation-driven group formation, in which groups can only be organized by players whose reputation reaches or exceeds a threshold. We define a player’s reputation as the frequency of cooperation in the last $T$ time steps. We find that the highest cooperation level can be obtained when groups are only established by pure cooperators who always cooperate in the last $T$ time steps. Effects of the memory length $T$ on cooperation are also studied.

Keywords: evolutionary game theory
1. Introduction

Cooperation exists widely in human society and the animal world [1]. Understanding the emergence of cooperation among selfish individuals remains an interesting problem. So far, evolutionary game theory has provided a powerful mathematical framework to address this problem [2]. Researchers have proposed various game models, among which the public goods game (PGG) has been a prevailing paradigm [3].

Due to the rapid development of complex networks [4], the PGG and other evolutionary game models have been extensively studied in various kinds of structured populations [5–7], including regular lattices [8–12], random graphs [13], scale-free networks [14–17] and dynamic networks [18–23]. Network structure helps cooperators survive through the formation of clusters. Within clusters, cooperators can assist each other and benefits of mutual cooperation outweigh losses against the defector.

Apart from network reciprocity, a number of mechanisms have been discovered that facilitate cooperation. Szabó and Hauert have studied the voluntary participation and found that the presence of loners leads to a cyclic dominance of the strategies [24]. Szolnoki and Szabó have found that inhomogeneous activity can promote cooperation [25]. Perc and Szolnoki have shown that social diversity is an efficient promoter of cooperation [26]. Szolnoki and Perc have considered that the collective benefits of group membership can only be harvested if the fraction of cooperators within the group, i.e. their critical mass, exceeds a threshold value [9]. Perc and Wang have found that heterogeneous aspirations promote cooperation [27]. Xia et al have revealed the dynamic instability of cooperation due to diverse activity patterns [28]. Szolnoki and Perc have found that conformity enhances network reciprocity in evolutionary social dilemmas [29]. Chen and Szolnoki have found that individual wealth-based selection supports cooperation in spatial public goods games [30].

In the spatial PGG, each group is composed of a focal player and all its nearest neighbors. Thus, for a given network, the number of different PGG groups is equal to
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the network size. In the previous studies, each PGG group is assumed to be existent all the time and each player is always involved in several independent groups which are determined by the interaction graph. However, in real life, not all groups can be successfully organized and players are reluctant to participate in some groups for various reasons. Very recently, Szolnoki and Chen proposed a model where only those players whose previous payoff exceeds a threshold level can establish a PGG group [31]. They demonstrated that a carefully chosen threshold to establish a PGG group could efficiently improve the cooperation level.

Motivated by the pioneering work of Szolnoki and Chen, in this paper we propose a reputation-driven group formation mechanism where PGG groups are organized by players whose reputation reaches or exceeds a threshold. A player’s reputation is defined as the frequency of cooperation in the past few time steps. We find that cooperation can be greatly promoted when PGG groups are established by high-reputation players.

2. Model

Our model is described as follows.

Players are located on a $L \times L$ square lattice with periodic boundary conditions. Each PGG group is composed of a focal player and its four neighbors. Thus the size of each PGG group is five. A player $i$ may participate in five different PGG groups organized by $i$ and its four neighbors respectively.

At each time step, every cooperator contributes a unit cost to each involved PGG group. Defectors invest nothing. The total cost of a group is multiplied by a factor, and is then redistributed uniformly to all the five players in this group. We denote $i$’s strategy at time $t$ as $s_i(t) = 1$ for cooperation and $s_i(t) = 0$ for defection. At time $t$, the payoff that player $i$ gains from the group organized by player $j$ is

$$\Pi_i^j(t) = -s_i(t) + \frac{r}{5} \sum_{x=0}^{4} s_x(t),$$

where $x = 0$ stands for player $j$, $x > 0$ represent the neighbors of $j$ and $r$ is the multiplication factor. The total payoff of the player $i$ at $t$ is calculated by

$$R_i(t) = \sum_{j \in \Omega_i} \Delta_j(t) \Pi_i^j(t),$$

where $\Omega_i$ denotes the community of neighbors of $i$ and itself, $\Delta_j(t) = 1$ if player $j$’s reputation $R_j(t)$ reaches or exceeds a threshold $H$ otherwise $\Delta_j(t) = 0$. The reputation of a player $i$ at time $t$ is defined as the frequency of cooperation in the last $T$ time steps, that is

$$R_i(t) = \frac{\sum_{m=1}^{T} s_i(t-m)}{T}.$$

We set $R_i(0) = 1$ so that initially ($t = 0$) all players can organize PGG groups. When the reputation threshold $H = 0$, our model is reverted to the original model in which
players can organize PGG groups all the time. Initially, cooperators and defectors are randomly distributed with the equal probability 0.5. After each time step, all individuals synchronously update their strategies as follows. Each individual $i$ randomly chooses a neighbor $j$ and adopts the neighbor $j$’s strategy with the probability:

$$W[s_i(t + 1) \leftarrow s_j(t)] = \frac{1}{1 + e^{(P_j(t) - P_i(t))I/K}},$$

(4)

where $K$ characterizes the noise introduced to permit irrational choices [32].

3. Main results and analysis

We assume that players occupy nodes on a $100 \times 100$ square lattice and the noise $K = 0.5$. Players can be divided into four types: $C_s$ ($D_s$) denotes cooperators (defectors) who successfully organize PGG groups, and $C_f$ ($D_f$) denotes cooperators (defectors) who fail to organize PGG groups. The key quantity for characterizing the cooperative behavior of the system is the fraction of cooperators (including $C_s$ and $C_f$) $\rho_c$ in the steady state. In all simulations below, $\rho_c$ is obtained by averaging over the last 5000 time steps of the entire 50 000 time steps. Each data is obtained by averaging over 50 different realizations.

Figure 1 shows the fraction of cooperators $\rho_c$ as a function of the multiplication factor $r$ for different values of the reputation threshold $H$. From figure 1, we can see that for any given value of $H$, $\rho_c$ increases from 0 to 1 as $r$ increases. In figure 2, we plot the full $r - H$ phase diagram for the memory length $T = 5$. There are regions: full cooperators ($C$), full defectors ($D$), and the coexistence of cooperators and defectors ($C + D$). One can find that the region of $C + D$ phase becomes very narrow as the reputation threshold $H$ increases. This phenomenon indicates that the phase transition from full $D$ to the coexistence of $C + D$ is discontinuous when $H$ and $T$ is very large.

Figure 3 shows the fraction of cooperators $\rho_c$ as a function of the reputation threshold $H$ for different values of the memory length $T$ when the multiplication factor $r = 3$. For fixed values of the multiplication factor $r$ and the memory length $T$, $\rho_c$ increases with $H$, indicating that cooperation is best promoted when PGG groups can only be organized by pure cooperators who always cooperate in the last $T$ time steps. Since some groups may not be organized, it is necessary for us to investigate whether higher cooperation level brings higher payoff. Figure 4 shows the average payoff of players $\langle P \rangle$ in the steady state as a function of the reputation threshold $H$ for different values of the multiplication factor $r$. One can see that for each value of $r$, $\langle P \rangle$ increases with $H$, manifesting that the average payoff is positively related with the cooperation level in our model.

To intuitively understand the mechanism of cooperation enhancement through reputation-driven group formation, we plot the spatial distribution of players at different time steps for the reputation threshold $H = 0.6$ (see figure 5) and $H = 0.2$ (see figure 6) respectively.

For the high threshold value (e.g. $H = 0.6$), most of the players become $D_f$ players in the early evolution (see figure 5(a)). As time evolves, $C_s$ players form some compact
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Figure 1. The fraction of cooperators $\rho_c$ as a function of the multiplication factor $r$ for different values of the reputation threshold $H$. The memory length $T$ is 2, 3, 5 for (a)–(c) respectively. For each value of $T$, $\rho_c$ reaches the highest when $H = 1$, indicating that cooperation is best promoted when PGG groups can only be organized by pure cooperators who always cooperate in the last $T$ time steps.

Figure 2. Full $r - H$ phase diagram for the memory length $T = 5$. There are three phases: full cooperators ($C$), full defectors ($D$), and the coexistence of cooperators and defectors ($C + D$). As $H$ increases, the region for $C + D$ phase becomes narrower.

clusters (see figure 5(b)) and $C_s$ clusters continually expand (see figures 5(c) and (d)). Finally, $C_s$ players occupy the whole system (results are not shown here). Because of the high reputation threshold, only a very small fraction of $D_s$ players can dispersedly survive during the whole evolution. It is noted that during the evolution, $C_s$ clusters are surrounded by $C_f$ players. These $C_f$ players act as protective layers which can effectively prevent the invasion of defectors. On the one hand, defectors outside the
protective layers cannot gain payoffs from \textit{Cf} players who fail to organize PGG groups. One the other hand, \textit{Cf} players can receive aid from \textit{Cs} players inside the protective layers by participating in PGG groups organized by \textit{Cs} players.

For the low reputation threshold (e.g. $H = 0.2$), in the beginning most of the players become $D_s$ or $D_f$ players while $C_s$ players can only form tiny clusters (see figure 6(a)). As time evolves, $C_s$ clusters expand (see figure 6(b)). After a long time, $C_s$ clusters stop expanding and are surrounded by the three other types of players (see figures 6(c) and 6(d)).

**Figure 3.** The fraction of cooperators $\rho_c$ as a function of the reputation threshold $H$ for different values of the memory length $T$. The multiplication factor $r = 3$. For each value of $T$, $\rho_c$ increases with $H$.

**Figure 4.** The average payoff of players $\langle P \rangle$ in the steady state as a function of the reputation threshold $H$ for different values of the multiplication factor $r$. The memory length $T = 10$. For each value of $r$, $\langle P \rangle$ increases with $H$. 
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For the low reputation threshold, \( C_f \) players cannot form protective layers around \( C_s \) clusters and \( C_s \) players are exploit by the surrounding defectors. Thus, it becomes difficult to reach the full cooperation in the case of low reputation threshold.

Next, we study the stationary density of the four types of players as a function of the multiplication factor \( r \) for \( H = 0.2 \) and \( H = 0.6 \) respectively. From figure 7, one can see that the stationary density of \( D_f \) players decreases as \( r \) increases. In contrast, the stationary density of \( C_s \) players increases with \( r \). Both of the stationary density of \( C_f \) and \( D_s \) players are not a monotonic function of \( r \). In fact, the stationary density of \( C_f \) and \( D_s \) players are maximized at moderate values of \( r \). For the low reputation threshold (\( H = 0.2 \)), the number of \( D_s \) players is much larger than that of \( C_f \) players (see figure 7(a)). For the high reputation threshold (\( H = 0.6 \)), the number of \( D_s \) players is almost the same as that of \( C_f \) players (see figure 7(b)).

Figure 5. Snapshots of typical distributions of four types of players at different time steps \( t \) when the reputation threshold \( H = 0.6 \), the multiplication factor \( r = 3.5 \) and the memory length \( T = 5 \). Successful cooperators (\( C_s \)) are marked by blue, whereas failed cooperators (\( C_f \)) are denoted by green. Successful defectors (\( D_s \)) are marked by red and failed defectors (\( D_f \)) are denoted by gray. The time step is \( t = 5, 15, 25 \) and 50 for (a)–(d) respectively. In the case of high threshold value, \( C_f \) players form a protective layer around \( C_s \) clusters, which reduces payoffs of external defectors.

Figure 6. Snapshots of typical distributions of four types of players at different time steps \( t \) when the reputation threshold \( H = 0.2 \), the multiplication factor \( r = 3.5 \) and the memory length \( T = 5 \). Successful cooperators (\( C_s \)) are marked by blue, whereas failed cooperators (\( C_f \)) are denoted by green. Successful defectors (\( D_s \)) are marked by red and failed defectors (\( D_f \)) are denoted by gray. The time step is \( t = 5, 25, 100 \) and 1000 for (a)–(d) respectively. In the case of low threshold value, \( C_s \) clusters are surrounded by \( D_s \) and \( D_f \) clusters.

(d)). For the low reputation threshold, \( C_f \) players cannot form protective layers around \( C_s \) clusters and \( C_s \) players are exploit by the surrounding defectors. Thus, it becomes difficult to reach the full cooperation in the case of low reputation threshold.
Figure 7. Stationary density of the four types of players as a function of the multiplication factor $r$ for (a) $H = 0.2$ and (b) $H = 0.6$. The memory length $T = 5$. $C_s$ ($D_s$) denotes cooperators (defectors) who successfully organize PGG groups, and $C_f$ ($D_f$) denotes cooperators (defectors) who fail to organize PGG groups. The stationary density of $C_s$ ($D_f$) players increases (decreases) as $r$ increases. The stationary density of $C_f$ and $D_s$ players are maximized at moderate values of $r$.

Figure 8. The fraction of cooperators $\rho_c$ as a function of the multiplication factor $r$ for different values of the memory length $T$. The reputation threshold $H$ is 0.2, 0.6, 1 for (a)–(c) respectively. For fixed values of $H$ and $r$, the highest cooperation level can be reached at an optimal value of $T$ (hereafter denoted by $T_{opt}$). For $H = 0.2$, $T_{opt} = 1$ (see figure 8(a)). For $H = 0.6$, $T_{opt} = 2$ (see figure 8(b)). For $H = 1$, the cooperation level increases as $T$ increases, indicating that $T_{opt} = \infty$ (see figure 8(c)). The value of $T_{opt}$ can be determined by the following rule. For $i(i + 1) < H \leq (i + 1)(i + 2)$, $T_{opt} = i + 1$ ($i = 0, 1, 2...$). Note that for a given value of $H$ and the corresponding $T_{opt}$, only pure cooperators who always cooperate in the last $T_{opt}$ time steps can organize PGG groups.
Finally, we study the effects of the memory length $T$ on cooperation. Figure 8 shows the fraction of cooperators $p_c$ as a function of the multiplication factor $r$ for different values of the memory length $T$. From figure 8(a), we can see that for a given value of $r$, $p_c$ decreases as $T$ increases when the reputation threshold $H = 0.2$. However, for $H = 1$, the cooperation level increases with $T$ (see figure 8(c)). For $H = 0.6$, the highest cooperation level is obtained when $T = 2$ (see figure 8(b)). In fact, we can determine the optimal value of $T$ which leads to the highest cooperation level as follows. For $i((i+1)) < H \leq ((i+1)(i+2)) (i = 0,1,2,...)$, the optimal value of $T$ (hereafter denoted by $T_{opt}$) is $T_{opt} = i + 1$. For examples, $T_{opt} = 2$ when $1/2 < H \leq 2/3$ and $T_{opt} = 3$ when $2/3 < H \leq 3/4$ (the results for $T_{opt} = 3$ are not shown here). Note that for a given value of $H$ and the corresponding $T_{opt}$, only pure cooperators who always cooperate in the last $T_{opt}$ time steps can organize PGG groups. Taking $H = 0.6$ as an example, defectors with reputation $R = 2/3$ still can organize groups if $T = 3$. However, for $T = 1$ or $T = 2$, only pure cooperators who always cooperate in the last $T$ time steps can organized groups. According to the results in figure 1, cooperation can be best enhanced if PGG groups can only be organized by pure cooperators. The criteria of pure cooperators becomes stricter as the memory length $T$ increases. Thus, the highest cooperation level is obtained when $T = 2$.

4. Conclusions

In conclusion, we have studied the impact of reputation-driven group formation on the evolution of cooperation. We define the reputation of a player as the frequency of cooperation in the last $T$ time steps. Here $T$ represents a memory length. A player can organize a group only when his/her reputation reaches or exceeds a threshold $H$. We find that both of the cooperation level and the average payoff increase with the threshold $H$, manifesting that cooperation can be best promoted when groups are only organized by pure cooperators who never change strategy during the memory length. For the high threshold $H$, failed cooperators who cannot successfully organize groups form a protective layer around those successful cooperators. The dependence of the memory length on cooperation is found to be non-monotonic.

Our results are useful for understanding the role of reputation in modern society. Leaders who want to organize a group should be of high reputation. Low reputation will destroy the stability of a group, leading to a lower cooperation level. Since individuals with high reputation usually gain high payoff, our results are consistent with that in [31]. The full-cooperator state disappears if the threshold level of payoff is too high in [31]. While in our model, the full-cooperator phase still exists when the reputation threshold is very high. Together [31] and our work provide a deeper understanding of the impact of group formation on the evolution of cooperation.

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