A mixed-frequency approach for exchange rates predictions

Raffaele Mattera\textsuperscript{b}, Michelangelo Misuraca\textsuperscript{*a}, Germana Scepi\textsuperscript{b}, and Maria Spano\textsuperscript{b}

\textsuperscript{a}University of Calabria - Department of Business Administration and Law, address
\textsuperscript{b}University of Naples "Federico II" - Department of Economics and Statistics, address

June 2, 2021

Selecting an appropriate statistical model to forecast exchange rates is still today a relevant issue for policymakers and central bankers. The so-called Meese and Rogoff puzzle assesses that exchange rate fluctuations are unpredictable. In the literature, a lot of studies tried to solve the puzzle finding alternative predictors and statistical models based on temporal aggregation. In this paper, we propose an approach based on mixed frequency models to overcome the lack of information caused by temporal aggregation. We show the effectiveness of our approach in comparison with other proposed methods by performing CAD/USD exchange rate predictions.

\textbf{keywords:} MIDAS, linear regression, frequency alignment, forecasting

1 Introduction

According to a popular quote attributed to Alan Greenspan, the former U.S. Federal Reserve Chairman: "\textit{implicit in any monetary policy action or inaction is an expectation of how the future will unfold, that is, a forecast}" (Carlstrom and Fuerst, 1999). Exchange rate forecasting is an essential issue for policymakers and central bankers because these predictions are used to project in the future the potential consequences of given monetary policies. Central bank policies in the U.S. and EU are described by interest rate rules, where interest rates respond to forecasts of inflation and economic activities rather than just outcomes (Wieland and Wolters, 2013). Equally important, exchange rate predictions result extremely decisive for heavy importer/exporter countries’ central banks.

\textsuperscript{*}Corresponding author: michelangelo.misuraca@unical.it
Several choices have to be made to forecast the exchange rates. First of all, a set of significant predictors has to be defined. Economic theory (e.g. Fisher 1896, Frenkel 1976, Choi and Oh 2003) provides a powerful guide. Among classical theoretical frameworks, it can be mentioned the interest rate differential (uncovered interest rate parity theory), the price levels differential (purchasing power parity theory) and the money supply (monetary theory). A subsequent but equally relevant aspect in forecasting is the time-frequency. Some studies on exchange rates focused on monthly predictions (e.g. Molodtsova and Papell 2009), whereas other studies aimed at forecasting exchange rates with quarterly predictions (e.g. Cheung et al. 2005, 2019). Time horizon is an important choice since there is an interest in obtaining either short-run and long-run forecasts. A third aspect concerns the selection of a statistical model. Models could either be based on single or multiple relations, with linear or nonlinear specifications, allowing or not allowing co-integration by considering an error correction term (see Rossi 2013).

The so-called Meese and Rogoff puzzle (Meese and Rogoff 1983, 1988) assesses that, differently from what is claimed by the economic theory, exchange rate fluctuations are challenging to predict in practice. As a main result, the simple random walk model provides more accurate forecasts than the most competing models based on classical predictors. Previous studies tried to solve the puzzle by finding new competing predictors and statistical models to forecast exchange rates better than the random walk model. Concerning the choice of the variables, Meese and Rogoff (1988) tested classical predictors’ forecasting accuracy against the random walk hypothesis. A similar approach was proposed by Mark (1995), Chinn and Meese (1995), Cheung et al. (2005, 2019). Molodtsova and Papell (2009) showed that the Taylor rule-based variables are, to some extent, able to forecast the exchange rates. Similarly, Ferraro et al. (2015) showed that oil price fluctuations play an important role at this aim. As regards the choice of statistical models, instead, Meese and Rogoff used the classical linear regression to obtain predictions, whereas Mark proposed long-run relationships among predictors and exchange rates with error correction models (ECM). Nevertheless, this approach showed several drawbacks in forecasting exchange rates. The puzzle is still not properly solved, and some questions remain open. For example, it is not clear why with monthly data some authors as Molodtsova and Papell and Ferraro et al. had evidence in favour of predictability, while other authors Cheung et al. obtained a favourable result with quarterly data. Among the others, Meese and Rogoff (1988) tried to explain the puzzle through sampling errors or model misspecification.

Exchange rate data are daily available, and all the related forecasting studies used a data aggregation step. This operation induces the so-called temporal aggregation bias (Marcellino 1999), consisting of a considerable loss of information once data aggregation is used. In this paper, we aim at exploring the role of temporal aggregation bias in the Meese and Rogoff puzzle. The challenge is how to handle the mixture of sampling frequencies in exchange rates’ predictions. For this purpose, we implemented a strategy based on the so-called Mixed Data Sampling regression (MIDAS: Foroni et al., 2015), which allows analysing data with different time frequency.

The paper is structured as follows. In Section 2, we briefly reviewed the predictors used to forecast exchange rates by previous studies. Then, in Section 3, we described the
implemented statistical methodology. In Section 4, we provided empirical evidence of
the forecasting ability of mixed frequency models, presenting a case study related with
CAD/USD exchange rate prediction. We closed the paper with some remarks.

2 Classical predictors for exchange rates

In exchange rates’ forecasting, the class of theoretical models that have been tested over
time against the random walk hypothesis is vast. The selected benchmark, the random
walk without drift, could be written as:

\[ \Delta s_t = \Delta s_{t-1} + \epsilon_t \] (1)

where \( \Delta s_t \) is the exchange rate differential and \( \epsilon_t \) an error term. In this section, we
briefly examine the most relevant models used into the reference literature.

2.1 Uncovered Interest Rate Parity

According to the uncovered interest rate parity (UIRP) theory (Fisher, 1896), the interest
rate differentials between two countries should explain fluctuations in the exchange rates.
However, many previous studies showed that more accurate forecasts can be obtained
by using the random walk. Several authors found good results using monthly frequency
data (Clark and West, 2006; Molodtsova and Papell, 2009). The UIRP model could be
specified by estimating the following equation:

\[ \Delta s_t = \alpha + \beta(i_t - i_t^*) + \epsilon_t \] (2)

where \( \alpha \) is the intercept term of the model, \( i_t \) is the domestic short-term interest rate, \( i_t^* \)
the foreign short-term interest rate, \( \beta \) is the relation between the interest rate differentials
and the exchange rate fluctuations and \( \epsilon_t \) is the error term. Some studies restricted \( \alpha = 0 \).
A positive value of \( \beta \) produces a forecast of the interest rate depreciation.

2.2 Purchasing Power Parity

Another classical predictor could be find in the purchasing power parity (PPP) theory (PPP)
considering the price level differentials of two countries. In particular, to test the validity
of the PPP theory for exchange rate forecasting, the following equation is estimated:

\[ \Delta s_t = \alpha + \beta(p_t - p_t^*) + \epsilon_t \] (3)

where \( p_t \) is the domestic price level, \( p_t^* \) the foreign price level, \( \alpha \) and \( \beta \) the parameters to
be estimated and \( \epsilon_t \) the error term. As above, \( \alpha \) could either be restricted to zero or not.
Previous studies showed that the out-of-sample performances are not good for the PPP
model. In particular, Cheung et al. (2005) found that predictors based on PPP produce
more accurate forecasts than random walk within a long timescale but their performance are never significantly better. Molodtsova and Papell (2009) showed instead that the PPP model is significantly worse than a random walk in shorter time horizons. Similar results can be found in Cheung et al. (2019).

2.3 Monetary models

The monetary models (e.g. Frenkel, 1976) assess that the exchange rates are determined by the movements in countries’ relative money supply, outputs, interest rates and prices. Assuming that UIRP and PPP hold, the following equation is estimated:

\[ \Delta s_t = \alpha + \beta_1 (i_t - i_t^*) + \beta_2 (y_t - y_t^*) + \beta_3 (m_t - m_t^*) + \epsilon_t \]  

(4)

where \( y_t \) and \( m_t \) are the output and the money supply, respectively. The \( \beta_3 \) coefficient on money differentials is usually restricted to 1, whereas \( \beta_2 \) is assumed to be negative since \( (y_t - y_t^*) < 0 \) implies a domestic currency depreciation with an increasing (ceteris paribus) foreign country output. Equation 4 has been defined flexible price version of the monetary model by Meese and Rogoff (1988).

Another monetary model is the so called sticky price, where it is supposed that the PPP holds only in the long run. The main difference with respect to the specification in Eq. 4 is that the functional relation is enriched by the price levels’ differentials:

\[ \Delta s_t = \alpha + \beta_1 (i_t - i_t^*) + \beta_2 (y_t - y_t^*) + \beta_3 (m_t - m_t^*) + \beta_4 (p_t - p_t^*) + \epsilon_t \]  

(5)

Even if Mark (1995) found strong and statistically significant evidence in favour of these predictors in a very long time horizon (three to four years), these results have been later argued by several scholars (e.g. Chinn and Meese, 1995; Cheung et al., 2005; Molodtsova and Papell, 2009; Cheung et al., 2019). Meese and Rogoff (1983) demonstrated that the random walk is better than any monetary models in forecasting exchange rates. These findings have been confirmed by Chinn and Meese for a short timescale, by Cheung et al. for very long horizon time (five years) and by Molodtsova and Papell which found good evidence just for few countries.

2.4 Taylor rule fundamentals

Some authors proposed to use predictors based on the Taylor rule of monetary policy (Taylor, 1993) to forecaste the exchange rates. Taylor theorised that monetary authorities set the real interest rate as a function of how inflation differs from a given target. According to this claim, the central banks’ response function can be expressed as:

\[ \hat{i}_t = \pi_t + \phi (\pi_t - \bar{\pi}) + \gamma y_t^{gap} + \bar{r} \]  

(6)

where \( \hat{i}_t \) is the target short-term interest rate, \( \pi_t \) is the inflation rate at current time,
\((\pi_t - \bar{\pi})\) is the deviation of the current inflation rate from its target level \(\bar{\pi}\), \(y_t^{gap}\) is the output gap and \(\bar{r}\) is the equilibrium level of real interest rate. The parameters \(\phi\) and \(\gamma\) define how the inflation rate and the output gap affect the target interest rate. Following \textbf{Molodtsova and Papell}, we can combine \(\pi_t\) and \(\bar{r}\) into a constant term such that:

\[
\hat{i}_t = \mu_t + \phi \pi_t + \gamma y_t^{gap}
\] (7)

where \(\mu_t = \bar{r} - \phi \bar{\pi}\). The same relation hold for a foreign central bank:

\[
\hat{i}_t^* = \mu_t^* + \phi \pi_t^* + \gamma y_t^{*gap}
\] (8)

Assuming that the interest rate \(i_t\) immediately reaches the target \(\hat{i}_t\), and that both central banks set the interest rates according to a Taylor rule, if the UIRP holds we get:

\[
\Delta s_t = \alpha + \beta_1 (\pi_t - \pi_t^*) + \beta_2 (y_t^{gap} - y_t^{*gap}) + \epsilon_t
\] (9)

The above specification is known as instantaneous Taylor rule. However, we can suppose that the interest rate \(i_t\) slowly adjusts to the target. An example of this adjustment process can be found in \textbf{Molodtsova and Papell}, where:

\[
i_t = (1 - \rho) \hat{i}_t + \rho i_{t-1} + \epsilon_t
\] (10)

Supposing that the Eq. 10 is applied to the data of a foreign country, we estimate:

\[
\Delta s_t = \alpha + \beta_1 (\pi_t - \pi_t^*) + \beta_2 (y_t^{gap} - y_t^{*gap}) + \beta_3 (i_t - i_{t-1}) + \epsilon_t
\] (11)

that is defined as a Taylor rule with smoothing. The presence of smoothing reflects the assumption made about the adjustment mechanism to the interest rate target. Using Taylor rule fundamentals as predictors, \textbf{Molodtsova and Papell} found that the out-of-sample exchange rates forecasts are significantly better than the random walk model for several countries. Other studies (e.g. \textbf{Molodtsova et al., 2011} \textbf{Giacomini and Rossi, 2010} \textbf{Rossi and Inoue, 2012}) also found evidence in favour of Taylor rule fundamentals. On the other hand, \textbf{Rogoff and Stavrakeva} (2008) found that the empirical evidence in favour of this fundamentals is not robust, assessing that the Taylor rule framework is a good description of monetary policies only for the past three decades. Nowadays, after the financial crises, monetary policies have been changed. It is interesting to highlight that \textbf{Rogoff and Stavrakeva} analysed quarterly data instead of monthly data, as well as \textbf{Molodtsova and Papell}, \textbf{Molodtsova et al.} and \textbf{Rossi and Inoue}.\n
3 Statistical methodology

Traditional literature of exchange rate forecasting implemented standard statistical models that incorporate economic predictors (Meese and Rogoff, 1983, 1988). These statistical models are mainly based on single equations within a linear regression framework, where the estimation of the relationships showed in Section 2 are done by ordinary least squares (OLS). This approach has been followed, for example, by Cheung et al. (2005); Bacchetta et al. (2009); Ferraro et al. (2015). Alternatively, some authors proposed to include some lags, fitting a distributed lag model (e.g. Wright, 2008; Molodtsova and Papell, 2009) Molodtsova et al. (2011). Moreover, in the class of single-equation models, another widely used alternative is the error correction model (ECM), which assumes a long-run relationship between exchange rate levels and predictor levels.

The co-integration vector parameter can be either calibrated (e.g. Mark, 1995; Chinn and Meese, 1995; Abhyankar et al., 2005; Berkowitz and Giorgianni, 2001; Kilian, 1999) or estimated (e.g. Alquist and Chinn, 2008; Chinn, 2012; Cheung et al., 2005, 2019). Positive evidence favouring the ECM model within a long time horizon has been found by Mark, whereas most of the other authors find no predictive ability. More interestingly – using exactly the same ECM specification of Mark Kilian, Groen (1999) and Groen (2002) find no predictive ability for monetary models. In other words, single-equation models without a co-integrating relation provide better out-of-sample forecasts for exchange rates. For this reason, in the following, we focused on single-equation models involving mixed-frequencies (Ghysels et al., 2004, 2007).

The main problem related to the usual single-equation approaches is that mixed frequencies are not allowed, even if we know that the sampling frequency affects forecasting accuracy (Silva et al., 2018). As discussed by Rossi (2013) about estimating the equations with quarterly data, the majority of previous papers did not find favourable evidence for the classical economic predictors, differently to the studies that used monthly data (e.g. Molodtsova and Papell). As we stated initially, the Meese and Rogoff puzzle could be potentially explained by temporal aggregation biases that rise in aggregating monthly data to a quarterly frequency. Because of temporal aggregation, much information is lost (Marcellino, 1999). To avoid the consequences of the bias, a statistical model that allows incorporating all the monthly information available in the data should be preferred. In exchange rate forecasting, temporal aggregation is a choice related to the dependent variable, which is available weekly or daily or at higher frequencies. Instead, macroeconomic fundamentals like the interest rates, the price levels or the monetary aggregates are all monthly available at the highest frequency. Thus, differently from the dependent variable, temporal aggregation for the predictors is not a researcher choice.

In the class of single-equation models, mixed data sampling (MIDAS; Foroni et al., 2015) regression seems to be very promising in facing this kind of problems. MIDAS shares some features with distributed lag models, and from several point of views they are very similar. The basic single equation with high-frequency regressor and low frequency

\[1\text{In some cases there are good high-frequency proxies for low-frequency variables. An example is quarterly GDP that could be replaced, under certain circumstances, by the monthly Industrial Production (see Carannante et al., 2020).}\]
dependent variable is:

\[ y_t = \beta_0 + \beta_1 B(L^{1/m}; \theta) x_t^{(m)} + \epsilon_t^{(m)} \]  

(12)

where \( (L^{1/m}; \theta) = \sum_{k=0}^{K} B(k; \theta)L^{k/m} \) and \( L^{1/m} x_t^{(m)} = x_{t-1}^{(m)} \). (for \( t = 1, ..., T \)). We suppose that \( y_t \) is observed at low frequency (e.g. quarterly) and \( x_t^{(m)} \) is observed \( m \) times in the same period (suppose \( x_t^{(m)} \) is monthly or \( m = 4 \)). Therefore, it is clear that we are projecting \( y_t \) onto a history of lagged observation of the high-frequency variable \( x_{t-k}^{(m)} \). The parameterisation of the lagged coefficients of \( B(k; \theta) \) in a parsimonious way is one of the MIDAS key features that avoid parameter proliferation. Diverse are the choices for \( B(k; \theta) \) but the most common are the exponential Almon lag and Beta function (Ghysels et al., 2007).

Foroni et al. introduced also the so-called unrestricted MIDAS (U-MIDAS) which has very appealing features. As the authors showed in their study, when the difference in sampling frequencies between the dependent variable and the regressors is not so large (as often happen with macroeconomic applications), it might not be necessary to employ distributed lag functions \( B(k; \theta) \). The essential operation made in estimating equations within the MIDAS framework is the so-called temporal alignment. The frequency alignment is used to transform an high-frequency vector \( x \) with \( mT \) elements into a low-frequency matrix \( X \) with \( T \) rows and \( m \) columns known as stacked vectors:

\[
\begin{bmatrix}
  x_1 \\
  \vdots \\
  x_{(mT)}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  x_m & x_{m-1} & \cdots & x_1 \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{(mT)} & x_{(mT-1)} & \cdots & x_{(mT-(m-1))}
\end{bmatrix} = X
\]

The MIDAS mapping follows a simple time-ordering aggregation scheme. Suppose that \( y_t \) is observed quarterly and the aim is to explain its relationship with the monthly-observed variable \( x_t \). Stated that each quarter has three months, a value of \( m = 3 \) has to be used. Let consider that only the monthly data in the current quarter have explanatory power (i.e. we are estimating a single equation without lags). Assuming that the exchange rates are quarterly observed, it is possible to transform a high-frequency predictor \( x \) in a low-frequency matrix \( X \) with \( m = 3 \) stacked vectors:

\[
\begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_{(3T)}
\end{bmatrix}
\rightarrow
\begin{bmatrix}
  x_3 & x_2 & x_1 \\
  x_6 & x_5 & x_4 \\
  \vdots & \vdots & \vdots \\
  x_{(3T)} & x_{(3T-1)} & x_{(3T-2)}
\end{bmatrix} = X
\]
The previous formalisation indicates that for the quarter \( t \) we want to model \( y_t \) as a linear combination of the monthly predictors observed within each quarter \( t \). Alternatively, we can write:

\[
y_t = \alpha + \beta_1 x_{3t} + \beta_2 x_{3t-1} + \beta_3 x_{3t-2} + \epsilon_t
\]  

(15)

The frequency alignment procedure turns a MIDAS regression into a classical time series regression where all the variables are observed at the same frequency. Moreover, this operation makes the single equation model to be estimated by OLS. An important advantage is that with MIDAS techniques we use all the available information for predicting the subsequent quarter. Moreover, MIDAS regression is very promising in explaining the role of temporal aggregation bias in exchange rate predictions. Therefore, we propose the mixed-frequency extensions of the models presented in the Section 2. For example, we can estimate the mixed frequency UIRP model as in the following:

\[
\Delta s_t = \alpha + \beta_1 (i_{3t} - i_{3t}^*) + \beta_2 (i_{3t-1} - i_{3t-1}^*) + \beta_3 (i_{3t-2} - i_{3t-2}^*) + \epsilon_t
\]  

(16)

where \((i_{3t} - i_{3t}^*)\), \((i_{3t-1} - i_{3t-1}^*)\) and \((i_{3t-2} - i_{3t-2}^*)\) are the inter-quarterly interest rates differences. In a similar fashion, we can extend the PPP model using mixed frequencies:

\[
\Delta s_t = \alpha + \beta_1 (p_{3t} - p_{3t}^*) + \beta_2 (p_{3t-1} - p_{3t-1}^*) + \beta_3 (p_{3t-2} - p_{3t-2}^*) + \epsilon_t
\]  

(17)

where \((p_{3t} - p_{3t}^*)\), \((p_{3t-1} - p_{3t-1}^*)\) and \((p_{3t-2} - p_{3t-2}^*)\) are the inter-quarterly price levels’ differences. The same specification for the monetary models (4) and (5) produces:

\[
\Delta s_t = \alpha + \beta_1 (i_{3t} - i_{3t}^*) + \beta_2 (i_{3t-1} - i_{3t-1}^*) + \beta_3 (i_{3t-2} - i_{3t-2}^*) + \\
+ \gamma_1 (y_{3t} - y_{3t}^*) + \gamma_2 (y_{3t-1} - y_{3t-1}^*) + \gamma_3 (y_{3t-2} - y_{3t-2}^*) + \\
+ \delta_1 (m_{3t} - m_{3t}^*) + \delta_2 (m_{3t-1} - m_{3t-1}^*) + \delta_3 (m_{3t-2} - m_{3t-2}^*) + \epsilon_t
\]  

(18)

\[
\Delta s_t = \alpha + \beta_1 (i_{3t} - i_{3t}^*) + \beta_2 (i_{3t-1} - i_{3t-1}^*) + \beta_3 (i_{3t-2} - i_{3t-2}^*) + \\
+ \gamma_1 (y_{3t} - y_{3t}^*) + \gamma_2 (y_{3t-1} - y_{3t-1}^*) + \gamma_3 (y_{3t-2} - y_{3t-2}^*) + \\
+ \delta_1 (m_{3t} - m_{3t}^*) + \delta_2 (m_{3t-1} - m_{3t-1}^*) + \delta_3 (m_{3t-2} - m_{3t-2}^*) + \\
+ \zeta_1 (p_{3t} - p_{3t}^*) + \zeta_2 (p_{3t-1} - p_{3t-1}^*) + \zeta_3 (p_{3t-2} - p_{3t-2}^*) + \epsilon_t
\]  

(19)

In the case of instantaneous Taylor rule, it easily follows that:

\[
\Delta s_t = \alpha + \beta_1 (\pi_{3t} - \pi_{3t}^*) + \beta_2 (\pi_{3t-1} - \pi_{3t-1}^*) + \beta_3 (\pi_{3t-2} - \pi_{3t-2}^*) + \gamma (y_{t}^{gap} - y_{t}^{gap}) + \epsilon_t
\]  

(20)
An inter-quarterly adjustment mechanism for the interest rate can be supposed to be:

\[ i_{3t} = (1 - \rho_1 - \rho_2 - \rho_3)\hat{i}_t + \rho_1 i_{3t-1} + \rho_2 i_{3t-2} + \rho_3 i_{3t-3} + \epsilon_t \]  (21)

where \( \hat{i}_t \) is the quarterly target level of the interest rate, \( i_{3t} \) is the interest rate in the end of the quarter, \( i_{3t-2} \) the second month of the quarter and \( i_{3t-1} \) the first one. To test the validity of the Taylor rule with inter-quarterly smoothing mechanism of the interest rate, we can finally estimate the following relation:

\[ \Delta s_t = \alpha + \beta_1 (\pi_{3t} - \pi_{3t}^*) + \beta_2 (\pi_{3t-1} - \pi_{3t-1}^*) + \beta_3 (\pi_{3t-2} - \pi_{3t-2}^*) + \gamma (y_{gap}^{3t} - y_{gap}^t) + \delta_1 (i_{3t} - i_{3t}^*) + \delta_2 (i_{3t-1} - i_{3t-1}^*) + \delta_3 (i_{3t-2} - i_{3t-2}^*) + \epsilon_t \]  (22)

4 An application to CAD/USD exchange rate

To show the forecasting ability of the proposed approach, we considered the quarterly data of the Canadian Dollar (CAD) / U.S. Dollar (USD) exchange rate. We collected data from FRED database\(^2\) and computed the logarithm of the nominal monthly CAD/USD exchange rate from 01/01/1985 to 01/01/2019. More recent data about 2020 were not considered because of the COVID-19 pandemic. We then aggregated the monthly data into quarterly data and calculated the returns of exchange rates (Fig. 1).

To empirically test the performances of the UIRP-based model, we used data related to the short-term interest rate collected by the OECD database\(^3\) as in Molodtsova and Papell (2009). The price levels, necessary to make forecasts with PPP-based predictors, are captured by the monthly logarithmic Consumer Price Index (CPI). For monetary models, we downloaded data of money supply index from the OECD database according to M3 monetary stock definition and compute the logarithm of this variable. As output variable, we considered the quarterly GDP measured in logarithmic levels. All the variables (Fig. 2) are expressed as the differences between the domestic (Canada) and foreign (U.S.) countries’ values.

Moreover, we need the output gap for the implementation of Taylor rule-based models. Following the literature, we computed the output gap as the GDP deviation from its long-run trend, obtained by applying the Hodrick and Prescott (1997) filter. The country’s inflation rate are computed as the first difference of the price levels logarithm (the predictors are shown in the Fig. 3).

We evaluated the performances of the standard models as well as of the proposed mixed frequency approach. Following Ramzan et al. (2012) and Chung and Zhang (2017), we compared model performances with an out-of-sample analysis. We considered both a recursive approach, where the sample has an increasing size, and a rolling-window

\(^2\)https://fred.stlouisfed.org/tags/series
\(^3\)https://data.oecd.org/
Figure 1: Exchange rate CAD/USD: levels vs returns

Figure 2: Relevant predictors for classical models
approach with a fixed sample size. To evaluate the forecasting accuracy, we used the Mean Square Forecast Error (MSFE) defined as:

$$\text{MSFE} = \frac{\sum_n (\hat{\Delta}s_t - \Delta s_t)^2}{n}$$ \hspace{1cm} (23)

where, in general, the quantity $\hat{\Delta}s_t - \Delta s_t$ represents the forecast error. The model with the lowest value of the associated loss function is the best one.

Since assessing the accuracy’s improvement is not enough, it is necessary to test that the forecasts obtained with alternative models are statistically different. There are several possible approaches for this purpose. The primary predictive accuracy test in the forecasting literature is the Diebold-Mariano test (Diebold and Mariano, 2002). Given two alternative forecasting models $i$ and $j$, and considering a generic loss function $g(\epsilon_{i,t})$, the loss difference is computed as in the following:

$$d_t = g(\epsilon_{i,t}) - g(\epsilon_{j,t})$$ \hspace{1cm} (24)

The null hypothesis of the test is:

$$H_0 : E(d_t) = 0$$
where \( d_t \) follows a \( N(0, 1) \) distribution. However, the main drawback of this approach – as pointed out by Diebold (2015) – is that the test compares forecasts but does not compare models. Therefore, following the exchange rate forecasting literature, we also used the Clark and West (2006) test to compare the performances of the proposed complex models.

5 Empirical results and discussion

According to the unit root tests of Said and Dickey (1984) (ADF) and Kwiatkowski et al. (1992) (KPSS), we obtained evidence of stationarity for all the considered variables (see Table 1). For the ADF test, we considered a null hypothesis of not stationarity, while for KPSS we considered a null hypothesis of stationarity. To get consistent estimates, stationarity of all the involved variables is required. The integration order represents the number of differentiations that the time series need in order to be covariance-stationarity according to both tests.

| Table 1: Unit root test results |
|--------------------------------|
| ADF                      | KPSS | Integration order |
| CAD/US exchange rate     | -4.2404*** | 0.1205 | \( I(0) \) |
| Interest rate differential | -2.4043 | 0.9066*** | \( I(1) \) |
| Price level differential  | -3.1993*  | 0.2240 | \( I(0) \) |
| Money supply differential | -2.8085 | 2.1546*** | \( I(1) \) |
| Output differentials      | -2.0744 | 1.1409*** | \( I(1) \) |
| Canada output gap         | -4.7458*** | 0.0337 | \( I(0) \) |
| U.S. output gap           | -4.1525*** | 0.0362 | \( I(0) \) |
| Canada inflation rate     | -5.5314*** | 0.0324 | \( I(0) \) |
| U.S. inflation rate       | -5.2739*** | 0.0221 | \( I(0) \) |

*** significance at 1%, ** at 5% and * at 10%

To make the out-of-sample analysis, the first step is to split the sample into a training set and a testing set. We obtained the forecasts according to both a recursive and a rolling window schemes. As training set we selected the period from 1985 to 1994, while as testing set we selected the time window between 1995 and 2019. Table 2 contains the list of the estimated models in our empirical analysis.

The results of the forecasting accuracy obtained by the recursive scheme and the associated predictive accuracy are in Table 3. The majority of the forecasting models (the only exception is the MM_2 model) provided more accurate forecasts than the benchmark in Eq. 1. We considered a random walk model without drift. Forecasts are one step ahead (\( h = 1 \)). Under the DM column are reported the value associated with the
Diebold and Mariano test, computed assuming MSFE loss function. Under the CW column, instead, is reported the value associated with the Clark and West test.

We observed a better performance of the Taylor rule-based models. In particular, we achieved the most accurate forecasts with the instantaneous Taylor rule model (TYLR₁), whereas the Taylor rule model with smoothing (TYLR₂) performed poorer. The worst model seemed to be the sticky price version of the monetary model (MM₂). The Taylor rule with inter-quarterly adjustment (MF-TYLR₂), proposed in this paper, provided the most accurate forecasts within the class of mixed-frequency models.

In conclusion, with a recursive approach, the mixed frequency based extensions improved the forecasting accuracy in comparison with the classical models. In particular, the MF-UIRP was the 2.3% more accurate than UIRP, the MF-PPP was the 8.2% more accurate than PPP, and the MF-TYLR₂ was the 18% more accurate than TYLR₂. The highest benefit of considering a mixed-frequency model was reached, in terms of accuracy, with the mixed-frequency sticky price version of the monetary model (MF-MM₂), that was the 53.6% more accurate than the classical MM₂ model.

In a similar fashion, we evaluated the forecasting accuracy of the proposed models according to a rolling window scheme (Table 4).

The results were completely different from those reported in Table 3. First of all, the presence of the Meese and Rogoff (1983) puzzle is here more evident. For example, the classical UIRP model is not able to predict the exchange rate, as well as the monetary models. This result was confirmed both by the Diebold and Mariano and the Clark and West tests. As in Molodtsova and Papell (2009) and Molodtsova et al. (2011), the Taylor rule-based forecasts provided good results, since both the instantaneous rule (TYLR₁) and the smoothing rule (TYLR₂) provided better forecasts than the benchmark.

Table 2: List of estimated models

| Acronym | Model description |
|---------|-------------------|
| UIRP    | Uncovered Interest Rate Parity estimated by the equation (2) |
| PPP     | Purchasing Power Parity estimated by the equation (3) |
| MM₁     | Flexible price monetary model estimated by the equation (4) |
| MM₂     | Sticky price monetary model estimated by the equation (5) |
| TYLR₁   | Instantaneous Taylor rule estimated by the equation (9) |
| TYLR₂   | Taylor rule with smoothing estimated by the equation (11) |
| MF-UIRP | Mixed frequency version of (2) estimated by (16) |
| MF-PPP  | Mixed frequency version of (3) estimated by (17) |
| MF-MM₁ | Mixed frequency version of (4) estimated by (18) |
| MF-MM₂ | Mixed frequency version of (5) estimated by (19) |
| MF-TYLR₁| Mixed frequency version of (9) estimated by (20) |
| MF-TYLR₂| Mixed frequency version of (11) estimated by (22) |
Table 3: Out-of-sample analysis: recursive approach

|           | MSFE   | DM   | CW         |
|-----------|--------|------|------------|
| Random Walk | 0.001984 | -    | -          |
| UIRP      | 0.001649 | 7.7800*** | 1.53e-18*** |
| PPP       | 0.001776 | 6.6951*** | 5.87e-13*** |
| MM1       | 0.001500 | 7.3806*** | 8.98e-19*** |
| MM2       | 0.003241 | -7.6012*** | 1.01e-16*** |
| TYLR1     | 0.001155 | 3.2853*** | 9.71e-17*** |
| TYLR2     | 0.001547 | 7.3827*** | 1.12e-18*** |
| MF-UIRP   | 0.001612 | 7.7164*** | 8.21e-19*** |
| MF-PPP    | 0.001630 | 7.5522*** | 3.28e-16*** |
| MF-MM1    | 0.001439 | 6.9542*** | 3.15e-18*** |
| MF-MM2    | 0.001504 | 7.1243*** | 5.87e-18*** |
| MF-TYLR1  | 0.001318 | 6.0342*** | 3.39e-16*** |
| MF-TYLR2  | 0.001266 | 5.8580*** | 1.40e-18*** |

*** significance at 1%, ** at 5% and * at 10%

Table 4: Out-of-sample analysis: rolling w. approach

|           | MSFE   | DM   | CW         |
|-----------|--------|------|------------|
| Random Walk | 0.001984 | -    | -          |
| UIRP      | 0.002137 | -1.2658 | 0.1498     |
| PPP       | 0.001596 | 7.1651*** | 2.73e-15*** |
| MM1       | 0.002355 | -2.0977*** | 0.1238     |
| MM2       | 0.005823 | -7.1600*** | 0.9999     |
| TYLR1     | 0.001189 | 4.2468*** | 1.25e-17*** |
| TYLR2     | 0.001550 | 5.2236*** | 8.15e-11*** |
| MF-UIRP   | 0.001923 | 0.5376 | 0.001346*** |
| MF-PPP    | 0.001482 | 7.1987*** | 9.84e-18*** |
| MF-MM1    | 0.005702 | -3.2726*** | 0.08108*   |
| MF-MM2    | 0.007596 | -3.5049*** | 0.1251     |
| MF-TYLR1  | 0.001447 | 6.6033*** | 4.21e-18*** |
| MF-TYLR2  | 0.001844 | 2.9112*** | 1.43e-04*** |

Note: *** significance at 1%, ** at 5% and * at 10%
As in the recursive approach, with the mixed-frequency extensions we obtained better results than the classical models for the majority of the cases. The MF-UIRP model provided better results than the benchmark in terms of MSFE (the classical UIRP specification based on temporal aggregation provided less accurate forecasts). The gain in accuracy of the MF-UIRP with respect the classical UIRP was exactly equal to 10%. While the classical UIRP provided the same forecasts of the random walk without drift, with the MF-UIRP specification we obtained a significant over performance. In other words, the unpredictability of UIRP model was explained by temporal aggregation. Similar conclusions can be drawn for the MF-PPP model, since it provided more accurate forecasts than the classical PPP model with an accuracy gain close to 7.2%.

The overall evidence of the presented study suggests different findings with respect to other recent studies (e.g. [Cheung et al. (2019)]), in which there are evidence against predictability for CAD/USD exchange rate using quarterly data. An important result is that by incorporating mixed frequency we were able to improve forecasting accuracy.

6 Conclusions

According to the economic theory, several variables can be used to explain the exchange rate fluctuations. From an empirical viewpoint, a vast literature argued instead that the variables used in the most popular frameworks are not able to forecast the exchange rate better than a simple random walk model. This result is called the Meese and Rogoff puzzle. The most shared explanation of the puzzle is that, stated the validity of the economic theory, the unpredictability should derive by the presence of sampling errors or to statistical models’ misspecification.

The most common statistical approach to exchange rate forecasting is based on the classical linear regression model. Starting from the work of [Mark (1995)], several authors tried to incorporate the long-run relationships among the predictors as additional variables, with poorer results than standard linear regression model. In this paper, we claimed that a possible explanation of the Meese and Rogoff puzzle can be found in the so called temporal aggregation biases. These biases, caused by the information lost induced by temporal aggregation, can be seen as a source of misspecification when some important (high-frequency) variables are omitted from the model. This intuition lies on the fact that the results presented in the literature are clearly affected by the frequency at which exchange rates are sampled. Even thought exchange rate data are daily available, many studies focused on monthly or quarterly frequencies [Rossi 2013].

The mixed-frequency regression model is a well known technique able to overcome this issue. Here we proposed for the first time, to the best of our knowledge, to use the monthly-sampled predictors to forecast the (long-run) quarterly exchange rates by means of a Mixed Data Sampling (MIDAS) regression. The main empirical finding of the paper – on the basis of a case study concerning the CAD/USD exchange rate – is that the mixed-frequency regression model improves the predictive ability in comparison with the classical models, that are instead affected by the temporal aggregation bias. Therefore, the contribution of this paper is two-fold. First of all, we showed the implementation of
the MIDAS regression to predict quarterly exchange rates with very promising results, offering a new applicative domain for this approach. Second of all, we provided a possible explanation of the Meese and Rogoff puzzle. These findings can be interesting for a varied audience, including both scholars and practitioners.

References

Abhyankar, A., Sarno, L., and Valente, G. (2005). Exchange rates and fundamentals: evidence on the economic value of predictability. *Journal of International Economics*, 66(2):325–348.

Alquist, R. and Chinn, M. D. (2008). Conventional and unconventional approaches to exchange rate modelling and assessment. *International Journal of Finance & Economics*, 13(1):2–13.

Bacchetta, P., van Wincoop, E., and Beutler, T. (2009). Can parameter instability explain the meese-rogoff puzzle? *NBER International Seminar on Macroeconomics*, 6(1):125–173.

Berkowitz, J. and Giorgianni, L. (2001). Long-horizon exchange rate predictability? *Review of Economics and Statistics*, 83(1):81–91.

Cheung, Y.-W., Chinn, M. D., and Pascual, A. G. (2005). Empirical exchange rate models of the nineties: Are any fit to survive? *Journal of International Money and Finance*, 24(7):1150–1175.

Cheung, Y.-W., Chinn, M. D., Pascual, A. G., and Zhang, Y. (2019). Exchange rate prediction redux: new models, new data, new currencies. *Journal of International Money and Finance*, 95:332–362.

Chinn, M. (2012). Macro approaches to foreign exchange determination. *Handbook of Exchange Rates*, pages 45–71.

Chinn, M. D. and Meese, R. A. (1995). Banking on currency forecasts: how predictable is change in money? *Journal of International Economics*, 38(1-2):161–178.

Choi, W. G. and Oh, S. (2003). A money demand function with output uncertainty, monetary uncertainty, and financial innovations. *Journal of Money, Credit, and Banking*, 35(5):685–709.

Chung, S. S. and Zhang, S. (2017). Volatility estimation using support vector machine: Applications to major foreign exchange rates. *Electronic Journal of Applied Statistical Analysis*, 10(2):499–511.

Clark, T. E. and West, K. D. (2006). Using out-of-sample mean squared prediction
errors to test the martingale difference hypothesis. *Journal of Econometrics*, 135(1-2):155–186.

Diebold, F. X. (2015). Comparing predictive accuracy, twenty years later: A personal perspective on the use and abuse of diebold–mariano tests. *Journal of Business & Economic Statistics*, 33(1):1–1.

Diebold, F. X. and Mariano, R. S. (2002). Comparing predictive accuracy. *Journal of Business & Economic Statistics*, 20(1):134–144.

Ferraro, D., Rogoff, K., and Rossi, B. (2015). Can oil prices forecast exchange rates? an empirical analysis of the relationship between commodity prices and exchange rates. *Journal of International Money and Finance*, 54:116–141.

Fisher, I. (1896). *Appreciation and Interest: A Study of the Influence of Monetary Appreciation and Depreciation on the Rate of Interest with Applications to the Bimetallic Controversy and the Theory of Interest*. Macmillan Company.

Foroni, C., Marcellino, M., and Schumacher, C. (2015). Unrestricted mixed data sampling (midas): Midas regressions with unrestricted lag polynomials. *Journal of the Royal Statistical Society: Series A (Statistics in Society)*, 178(1):57–82.

Frenkel, J. A. (1976). A monetary approach to the exchange rate: doctrinal aspects and empirical evidence. *The Scandinavian Journal of Economics*, pages 200–224.

Ghysels, E., Santa-Clara, P., and Valkanov, R. (2004). The midas touch: Mixed data sampling regression models.

Ghysels, E., Sinko, A., and Valkanov, R. (2007). Midas regressions: Further results and new directions. *Econometric Reviews*, 26(1):53–90.

Giacomini, R. and Rossi, B. (2010). Forecast comparisons in unstable environments. *Journal of Applied Econometrics*, 25(4):595–620.

Groen, J. J. (1999). Long horizon predictability of exchange rates: Is it for real? *Empirical Economics*, 24(3):451–469.

Groen, J. J. (2002). Cointegration and the monetary exchange rate model revisited. *Oxford Bulletin of Economics and Statistics*, 64(4):361–380.

Hodrick, R. J. and Prescott, E. C. (1997). Postwar us business cycles: an empirical investigation. *Journal of Money, Credit, and Banking*, pages 1–16.

Kilian, L. (1999). Exchange rates and monetary fundamentals: what do we learn from long-horizon regressions? *Journal of Applied Econometrics*, 14(5):491–510.

Kwiatkowski, D., Phillips, P. C., Schmidt, P., and Shin, Y. (1992). Testing the null hypothesis of stationarity against the alternative of a unit root: How sure are we that economic time series have a unit root? *Journal of Econometrics*, 54(1-3):159–178.

Marcellino, M. (1999). Some consequences of temporal aggregation in empirical analysis. *Journal of Business & Economic Statistics*, 17(1):129–136.

Mark, N. C. (1995). Exchange rates and fundamentals: Evidence on long-horizon predictability. *The American Economic Review*, pages 201–218.

Meese, R. A. and Rogoff, K. (1983). Empirical exchange rate models of the seventies: Do they fit out of sample? *Journal of International Economics*, 14(1-2):3–24.
Meese, R. A. and Rogoff, K. S. (1988). Was it real? the exchange rate-interest differential relation over the modern floating-rate period. *The Journal of Finance*, 43(4):933–948.

Molodtsova, T., Nikolsko-Rzhevskyy, A., and Papell, D. H. (2011). Taylor rules and the euro. *Journal of Money, Credit and Banking*, 43(2-3):535–552.

Molodtsova, T. and Papell, D. H. (2009). Out-of-sample exchange rate predictability with taylor rule fundamentals. *Journal of International Economics*, 77(2):167–180.

Ramzan, S., Ramzan, S., and Zahid, F. M. (2012). Modeling and forecasting exchange rate dynamics in pakistan using arch family of models. *Electronic Journal of Applied Statistical Analysis*, 5(1):15–29.

Rogoff, K. S. and Stavrakeva, V. (2008). The continuing puzzle of short horizon exchange rate forecasting. Working paper 14071, National Bureau of Economic Research.

Rossi, B. (2013). Exchange rate predictability. *Journal of economic literature*, 51(4):1063–1119.

Rossi, B. and Inoue, A. (2012). Out-of-sample forecast tests robust to the choice of window size. *Journal of Business & Economic Statistics*, 30(3):432–453.

Said, S. E. and Dickey, D. A. (1984). Testing for unit roots in autoregressive-moving average models of unknown order. *Biometrika*, 71(3):599–607.

Silva, E. S., Hassani, H., and Otero, J. (2018). Forecasting inflation under varying frequencies. *Electronic Journal of Applied Statistical Analysis*, 11(1):307–339.

Taylor, J. B. (1993). Discretion versus policy rules in practice. *Carnegie-Rochester conference series on public policy*, 39:195–214.

Wieland, V. and Wolters, M. (2013). Forecasting and policy making. In Elliott, G. and Timmermann, A., editors, *Handbook of economic forecasting*, volume 2A, pages 239–325. North Holland.

Wright, J. H. (2008). Bayesian model averaging and exchange rate forecasts. *Journal of Econometrics*, 146(2):329–341.