The Letter presents an exact expression for the non-adiabatic non-cyclic geometric phase of photons propagating inside a noncoplanarly curved optical fiber by using the Lewis-Riesenfeld invariant theory. It is shown that the helicity inversion of photons arises in the curved fiber. Since we have exactly solved the time-dependent Schrödinger equation that governs the propagation of photons in a curved fiber and, moreover, the chronological product is not involved in this exact solution, our formulation therefore has several advantages over other treatments based on the classical Maxwell’s theory and the Berry’s adiabatic quantum theory. The potential application of helicity inversion of photons to information science is briefly suggested.

PACS: 03.65.Vf; 42.50.Ct; 03.67.-a

Keywords: Geometric phase; Helicity inversion; Curved optical fiber; Time evolution

I. INTRODUCTION

In 1984, Berry found the existence of the Berry’s topological phase (i.e., the adiabatic geometric phase) of wave function in adiabatic quantum process where the cyclic evolution of wave function yields the original state plus a phase shift, which is a sum of a dynamical phase and a geometric phase shift [1]. Berry’s discovery opens up new opportunities for investigating the global and topological properties of quantum evolution [2–6]. It is now well known that geometric phase arises in systems with the time-dependent Hamiltonian, or in systems whose Hamiltonian possesses some evolution parameters [7,8]. Differing from dynamical phase that depends on dynamical quantities of systems such as energy, frequency, velocity as well as coupling coefficients, geometric phase is independent of these dynamical quantities. Instead, it is only related to the geometric nature of the pathway along which quantum systems evolve. This, therefore, implies that geometric phase presents the topological and global properties of quantum systems in time-evolution process, and that it possesses the physical significance and can thus be applied to various fields of physics [9–11]. Geometric phases attract attentions of many physicists in considerable fields such as gravity theory [2,3], differential geometry [12], atomic and molecular physics [4,13,14], nuclear physics [5], quantum optics [9], condensed matter physics [10,11], molecular systems and chemical reaction [4] as well.

The first physical realization of the Berry’s geometric phase is that of the polarized photon propagating inside the noncoplanar optical fiber, of which whose theory is proposed by Chiao and Wu [15]. Based on the experimental work of Chiao et al. [16], many researchers further studied this problem in both experiments and theories [17–21] afterwards, where they tried to consider this geometric phase by using the classical Maxwell’s theory, differential geometry or quantum mechanics. Although Chiao-Wu theory [15] concerning the propagation of photons inside the optical fiber was very successful in predicting this geometric phase, and in satisfactorily investigating the polarization and propagation of photons in the cyclic adiabatic process in the fiber experiment performed by Kwiat and Chiao et al. [15,17], we still argue that there exist at least three points which should be further discussed. They are given as follows:

(i) since it is based on Berry’s formula of adiabatic geometric phase [1], Chiao-Wu formulation can be applicable only to the adiabatic and cyclic quantum process, i.e., this method is not appropriate to deal with the general non-adiabatic non-cyclic geometric quantum phase;

(ii) Chiao-Wu theory is first-quantized. However, we suggest the second-quantized treatment of the photon in the fiber, so that this second-quantized formulation enables us to consider the geometric phase in the fiber at quantum
level, and thus provides us with a theoretical tool to deal with the hot arguments as to whether this geometric phase inside the fiber belongs to the classical or the quantum level [17];

(iii) Chiao-Wu theory has no expression for the Hamiltonian describing the interaction of photons field with fiber medium. Instead, it investigated the geometric phase merely by taking into consideration the eigenvalue equation of the helicity of photons in curved fiber. Note, however, that this eigenvalue equation cannot always be regarded as the Schrödinger equation governing the propagation of photons in the noncoplanarly curved fiber, although in the sense of adiabatic process, it can be considered the Schrödinger equation indeed. This, therefore, means that the Chiao-Wu formulation should be generalized to a more complete one that can satisfactorily study the non-adiabatic evolution of wave function of the photon in curved fiber.

In the present paper, we resolve these problems by making use of the Lewis-Riesenfeld invariant theory and the invariant-related unitary transformation formulation [22,23]. By constructing an effective second-quantized Hamiltonian describing the interaction between photons field and fiber medium, we obtain the exact solution of the time-dependent Schrödinger equation and, based on this result, we obtain the non-adiabatic non-cyclic geometric phase of photons, rather than the phase in the sense of Berry’s adiabatic quantum process. Note that the exact solution presented here does not contain the chronological product operator, namely, it may be considered the explicit solution. On the basis of the above work, we consider the helicity inversion of photons in the optical fiber, to which most of the methods based on the Maxwell’s theory, differential geometry (parallel transport) [12] or Berry’s phase theory [1] is not applicable since these methods are not in connection with the time-evolution equation (i.e., the time-dependent Schrödinger equation) of the photon in the fiber.

Time-dependent system is governed by the time-dependent Schrödinger equation. The invariant theory [23] suggested by Lewis and Riesenfeld can solve the time-dependent Schrödinger equation. Gao et al. proposed a generalized invariant theory [23,24], by introducing the basic invariants which enable one to find the complete set of commuting invariants for some time-dependent multi-dimensional systems [24]. Since the exact solution to quantum systems with time-dependent Hamiltonian obtained by the invariant-related unitary transformation formulation contains both the geometric phase and the dynamical phase and, fortunately, all these results are explicit rather than formal (i.e., there exists no chronological product operator in wave function), the Lewis-Riesenfeld theory thus developed into a powerful tool for treating the time-dependent Schrödinger equation and the geometric phase factor.

II. TIME EVOLUTION AND NONCYCLIC GEOMETRIC PHASE OF PHOTONS IN A CURVED FIBER

Consider a noncoplanarly curved optical fiber that is wound smoothly on a large enough diameter [16], an equation of motion of the photon propagating along the fiber is of the form

\[
\dot{k} + k \times \left( \frac{k \times \dot{k}}{k^2} \right) = 0
\]

(1)

with \(k(t)\) being the wave vector of the photon, \(k \times \left( \frac{k \times \dot{k}}{k^2} \right)\) and \(\frac{k \times \dot{k}}{k^2}\) may be considered the generalized Lorentz magnetic force (Coriolis force) and the generalized magnetic field strength, respectively. The reason for \(k \times \left( \frac{k \times \dot{k}}{k^2} \right)\) being called generalized Lorentz force is that Eq. (1) is somewhat analogous to the equation of motion of a photon moving in the gravitomagnetic fields [25]. Dot in the equation of motion (1) denotes the time rate of change of \(k(t)\). Further analysis shows that the infinitesimal rotation operator of motion of a photon in the fiber is given as \(U_R = 1 - i \theta \cdot J\) with \(J\) being the total angular momentum operator of photons field and \(\theta = \frac{\kappa(t) \times \dot{k(t)}}{k^2} dt\). It follows from the evolution operator, \(U_R\), that the photon state \(|\sigma, k(t)\rangle\) (\(\sigma\) denotes the eigenvalue of \(I(t) = \frac{k(t)}{\hbar} \cdot J\) that is proved to be an invariant in what follows) satisfies the following time-evolution equation (in the unit \(c = \hbar = 1\))

\[
\frac{d}{dt} |\sigma, k(t)\rangle = \frac{k(t) \times \dot{k(t)}}{k^2} \cdot J |\sigma, k(t)\rangle
\]

(2)

with \(H_{eff}(t) = \frac{k(t) \times \dot{k(t)}}{k^2} \cdot J\) being the effective Hamiltonian of photon inside the fiber. Apparently, \(I(t)\) and the effective Hamiltonian \(H_{eff}(t)\) agree with the invariant equation (Liouville-Von Neumann equation) [22]

\[
\frac{d}{dt} I(t) + \frac{1}{i} [I(t), H_{eff}(t)] = 0.
\]

(3)

Substitution of the expressions for \(I(t)\) and \(H_{eff}(t)\) into this invariant equation yields the equation, \(\dot{k} + k \times \left( \frac{k \times \dot{k}}{k^2} \right) = 0\), of motion of the photon in the fiber. It follows that the eigenvalue of \(\frac{k(t)}{\hbar} \cdot J\) of the photon is conserved in motion and
it is therefore an invariant in terms of the invariant equation (3). It can be seen from the form of $H_{eff}(t)$ that the problem of the rotation of polarization plane is actually in analogy with that of the time-dependent quantum spin model [8]. Set the components of momentum of a photon

$$\frac{k(t)}{k} = (\sin \lambda(t) \cos \gamma(t), \sin \lambda(t) \sin \gamma(t), \cos \lambda(t)), \quad (4)$$

where the time-dependent parameters $\lambda(t)$ and $\gamma(t)$ denote the angle displacements of $k(t)$ in the spherical polar coordinate system, and the invariant $I(t) = \frac{k(t)}{k} \cdot J$ may therefore be rewritten

$$I(t) = \frac{1}{2} \sin \lambda(t) \exp[-i \gamma(t)]J_+ + \frac{1}{2} \sin \lambda(t) \exp[i \gamma(t)]J_- + \cos \lambda(t) J_3 \quad (5)$$

with $J_\pm = J_1 \pm i J_2$, $[J_3, J_\pm] = \pm J_\pm$, $[J_+, J_-] = 2J_3$.

In order to obtain the analytical solution of the time-dependent Schrödinger equation (2), we introduce an invariant-related unitary transformation operator $V(t)$ that is of the form

$$V(t) = \exp[\beta(t)J_+ - \beta^*(t)J_-], \quad (6)$$

where the time-dependent parameters $\beta(t) = -\frac{\lambda(t)}{2} \exp[-i \gamma(t)]$, $\beta^*(t) = -\frac{\lambda(t)}{2} \exp[i \gamma(t)]$. $V(t)$ can be easily shown to transform the time-dependent invariant $I(t)$ to $I_V \equiv V(t)I(t)V(t)^\dagger = J_3$, which is time-independent. The eigenstate of $I_V = J_3$ corresponding to the eigenvalue $\sigma$ is denoted by $\ket{\sigma}$. By making use of $V(t)$ in expression (6) and the Baker-Campbell-Hausdorff formula [26], one can obtain $H_V(t)$ from $H(t)$, i.e.,

$$H_V(t) = V(t)^\dagger H(t)V(t) - V(t)^\dagger \frac{\partial V(t)}{\partial t} = \left\{ [\cos \lambda \cos \theta + \sin \lambda \sin \theta \cos(\gamma - \varphi)] + \dot{\gamma}(1 - \cos \lambda) \right\} J_3, \quad (7)$$

where the time-dependent parameters $\theta$ and $\varphi$ represent the angle displacements of $\frac{k(t) \times k(t)}{k^2}$ in the spherical polar coordinate system, i.e., $\frac{k(t) \times k(t)}{k^2} = (\sin \theta \cos \varphi, \sin \theta \sin \varphi, \cos \theta)$. It follows from Eq. (7) and the expression $I_V = J_3$ that $H_V(t)$ differs from $I_V$ only by a time-dependent c-number factor $\exp \left[ \frac{1}{2} \int_0^t \dot{\gamma}(t')[\sigma H_V(t')][\sigma] \, dt' \right]$. By using the invariant equation (3), the two auxiliary equations, which are used to determine the time-dependent parameters $\theta$ and $\varphi$, can be derived

$$\dot{\gamma}\sin^2 \lambda = \cos \theta, \quad \dot{\lambda} \cos \gamma - \dot{\gamma} \cos \lambda \sin \sin \gamma = \sin \theta \sin \varphi. \quad (8)$$

Fortunately, it is shown from the auxiliary equations that

$$\cos \lambda \cos \theta + \sin \lambda \sin \theta \cos(\gamma - \varphi) = 0, \quad (9)$$

the expression (7) for $H_V(t)$ can thus be simplified into $H_V(t) = \dot{\gamma}(t)[1 - \cos \lambda(t)] J_3$.

Based on the invariant theory, the geometric phase of a photon whose initial eigenvalue of helicity is $\sigma$ can be expressed by

$$\phi_\sigma(t) = \left\{ \int_0^t \dot{\gamma}(t') [1 - \cos \lambda(t')] \, dt' \right\} \langle \sigma | J_3 | \sigma \rangle. \quad (10)$$

Since it is easy to obtain the eigenvalues and eigenstates of $I_V(t) = J_3$, with the help of (6) and (7), one can arrive at the general solution of the time-dependent Schrödinger equation, which governs the motion of photons in the fiber experiment. The result is given as follows

$$|\Psi(t)\rangle_s = \sum_\sigma C_\sigma \exp \left[ \frac{1}{i} \phi_\sigma(t) \right] |H_V(t)\rangle \langle \sigma | \quad (11)$$

with the coefficients $C_\sigma = \langle \sigma, t = 0 | \Psi(0) \rangle_s$. It is noted that the exact solution presented here does not contain the chronological product and may be considered the explicit solution.

In the noncoplanar optical fiber, according to the expression for $H_V(t)$, the dynamical phase

$$\phi_\sigma^{(d)}(t) = \left\{ \int_0^t \left[ \cos \lambda(t') \cos \theta(t') + \sin \lambda(t') \sin \theta(t') \cos(\gamma(t') - \varphi(t')) \right] \, dt' \right\} \langle \sigma | J_3 | \sigma \rangle \quad (12)$$
of the photon due to the effective Hamiltonian vanishes (But note that the Hamiltonian of free photons, $H_0 = \frac{1}{(2\pi)^3} \int d\mathbf{k} \omega_k (a_R(\mathbf{k}) \dagger a_R(\mathbf{k}) + a_L(\mathbf{k}) \dagger a_L(\mathbf{k}) + 1)$ with $a_R(\mathbf{k})$ and $a_L(\mathbf{k})$ being respectively the creation (annihilation) operators for the right- and left- polarized photons, which is not involved in the effective Hamiltonian, may contribute a dynamical phase to the wave function of photons in the curved fiber), and its geometric phase is expressed by the expression (10). The time evolution operator of wave function is $U(t) = \exp[\frac{i}{\hbar} \phi_{\sigma}(t)] V(t)$. For the sake of $U(t = 0) = 1$ and consequently $\lambda(t = 0) = 0$, the initial conditions are taken to be $k_1 = 0, k_2 = 0, k_3 = k$. Thus the expectation value of the third component of the orbital angular momentum $\mathbf{L}$ of the linear polarized photon vanishes, namely, $\langle \sigma | L_3 | \sigma = \frac{k}{i} \mathbf{k} \rangle, \langle \sigma | \mathbf{L} | \sigma = 0$. This means that we should only analyze the expectation value of the third component of spin of the photon. It follows from the spin operator of non-normal product

$$S = \frac{1}{2} \int d\mathbf{k} k \left\{ [a_R(\mathbf{k}) \dagger a_R(\mathbf{k}) + a_R(\mathbf{k}) a_R(\mathbf{k}) \dagger] - [a_L(\mathbf{k}) \dagger a_L(\mathbf{k}) + a_L(\mathbf{k}) a_L(\mathbf{k}) \dagger] \right\}$$

$$= \int d\mathbf{k} k \left\{ [a_R(\mathbf{k}) \dagger a_R(\mathbf{k}) + \frac{1}{2}] - [a_L(\mathbf{k}) \dagger a_L(\mathbf{k}) + \frac{1}{2}] \right\}$$

(13)

that the total spin of the photons field comprises the contributions of both the right- and left- rotation photons. It can be found that the zero-point energy of both the right- and left- rotation photons exists in the effective Hamiltonian. This time-dependent zero-point energy possesses the physical meaning and also contributes to the geometric phase factor. Thus, it follows that there exists the quantum-vacuum geometric phase

$$\phi_{\pm}^{(g)} = \pm \frac{1}{2} \int_{0}^{t} \hat{\gamma}(t')[1 - \cos \lambda(t')] dt'$$

(14)

($\pm$ corresponding to the right- and left- polarized photon, respectively), in addition to the classical geometric phase $\phi_{\pm}^{(c)} = \pm \int_{0}^{t} \gamma(t')[1 - \cos \lambda(t')] dt'$ that has been measured in the fiber experiment performed by Tomita and Chiao. However, it should be pointed out that, even at the quantum level, the observable quantum-vacuum geometric phase is absent in the fiber experiment, since the sign of the quantum-vacuum geometric phase of the left- and right- rotation photons is opposite, and their quantum-vacuum geometric phase are therefore counteracted, hence the observable geometric phase is merely the classical geometric phase, which, arising as the adiabatic cyclic Berry’s phase in the fiber experiment, has been proved existing by Tomita and Chiao [16].

In the adiabatic and cyclic case, it is of physical interest to discuss the topological property of the geometric phase arising in the fiber experiment. Note that in the adiabatic and cyclic case the expressions for geometric phase presented in this Letter is reduced to that in the Chiao-Wu theory [15], namely, in this case our calculation is consistent with the result obtained by Chiao and Wu. The adiabatic geometric phase ( Berry’s phase ) in a cycle associated with $\gamma(t)$ in the parameter space of the invariant $\frac{k}{i} \mathbf{J}$ is $\phi_{\pm}(T) = 2\pi \sigma (1 - \cos \lambda)$, where $\lambda$ is taken to be time-independent just as what has been done in the Chiao-Wu theory and the subsequent experiment performed by Tomita et al. [16]. It is apparent that in the adiabatic and cyclic geometric phase, $2\pi (1 - \cos \lambda)$ is the solid angle over the parameter space of the invariant, which are said to present the geometric meanings of the phase. This, therefore, implies that geometric phase reflects the global and topological properties of time-dependent evolution of quantum systems. It is worthwhile to point out that although the adiabatic solution describing the time evolution of photons in the curved fiber has sufficient features to indicate the global and topological properties of geometric phase of photons, the exact solution obtained via the Lewis-Riesenfeld invariant theory possesses more rich characters and can therefore treat more general time-dependent evolution of wavefunctions in Quantum Mechanics regimes in addition to the narrower class of the quantum adiabatic process.

III. HELICITY INVERSION OF PHOTONS IN A CURVED FIBER

In the optical fiber that is wound smoothly on a large enough diameter, the variation of the helicity of the photon cannot be easily observed. However, in the sharply curved fiber, it is apparent that the helicity depends strongly upon the geometric shape of the fiber. In order to see whether or not this is the true case, in what follows we deal with the helicity inversion of photons inside the curved fiber. The principal reasons for the importance of helicity inversion lie in that: (i) it is believed that neither the adiabatic quantum theory nor the classical electrodynamics can satisfactorily deal with this problem. Only the explicit wave function of photons, which does not involve the chronological product, in the optical fiber is obtained can we deal with the time evolution of wave function and the helicity inversion of photons moving along the curved fiber; (ii) helicity inversion may be employed to the field of
information science, since the left-polarized laser (representing 1) may be inverted to the right-polarized laser (representing 0) by the mirror reflection and the optical helix. The Hamiltonian of free photons field, $\mathbf{p} \cdot \mathbf{v}(t)$ with $\mathbf{v}(t) = (\sin \lambda(t) \cos \gamma(t), \sin \lambda(t) \sin \gamma(t), \cos \lambda(t))$, possesses the same eigenvalue of $\mathbf{k} \cdot \mathbf{J}$ divided by $\sigma$. The particular solution of Eq. (2) corresponding to the eigenvalue, $\sigma$, is

$$|\sigma, \mathbf{k}(t)\rangle = V(t) \exp \left[ \frac{i}{\hbar} \int_{t_0}^{t} H_V(t') dt' \right] |\sigma\rangle$$

(15)

that can be rewritten in the interaction representation as

$$|\sigma, \mathbf{k}(t)\rangle = V(t) \exp \left\{ \frac{1}{\hbar} \int_{t_0}^{t} \left[ H_V(t') - \mathbf{k}(t') \cdot \mathbf{J} \right] dt' \right\} \exp \left\{ \frac{1}{2} \int_{t_0}^{t} dt' \int_{t_0}^{t} dt'' \left[ H_V(t'), \mathbf{k}(t'') \cdot \mathbf{J} \right] \right\} |\sigma, t\rangle_I$$

(16)

with $|\sigma, t\rangle_I = \exp \left[ \frac{i}{\hbar} \int_{t_0}^{t} \mathbf{p} \cdot \mathbf{v}(t') dt' \right] |\sigma\rangle$. It follows that $H_V(t) - \mathbf{k}(t) \cdot \mathbf{J} = -\mathbf{K} \cdot \mathbf{J}$, where $\mathbf{K}$ is defined to be

$$\mathbf{K} = k(\sin \lambda \cos \gamma, \sin \lambda \sin \gamma, -\frac{1}{k}(1 - \cos \lambda) + \cos \lambda),$$

(17)

and consequently the module of $\mathbf{K}$ is $K = \varsigma k$ with

$$\varsigma = \sqrt{1 - \frac{2\gamma}{k}(1 - \cos \lambda) \cos \lambda + \left( \frac{\Omega}{k} \right)^2 (1 - \cos \lambda)^2}.$$  

(18)

From the expression for $|\sigma, \mathbf{k}(t)\rangle$ in the interaction picture the helicity operator of the photon is of the form $\frac{1}{k} \mathbf{K} \cdot \mathbf{J}$. Since we have the time-evolution operator of wave function of photons, which does not involve the chronological product, we can in principle investigate the evolution and inversion of the helicity states of photons propagating inside the curved fiber.

It is significant to apply the formulation presented here to the Chiao-Wu case (i.e., the case of photon moving along a helical fiber) in order to unfold the physical meanings of our results. Consider the Chiao-Wu adiabatic cyclic case in which the rotating frequency of photon moving on the helicoid reads $\dot{\gamma} = \Omega$ with $\Omega = \frac{2\pi}{\sqrt{d^2 + (4\pi a)^2}}$ where $d$ and $a$ respectively denote the pitch length and the radius of the helix, and $c$ is the speed of light. When the initial condition is taken to be $k_1 = 0, k_2 = 0, k_3 = \kappa$, i.e., $\cos \lambda = 1$, it is readily verified that the initial expectation value

$$\left\langle \frac{1}{k} \mathbf{K} \cdot \mathbf{J} \right\rangle = \frac{\sigma \cos \lambda - \frac{\Omega}{k}(1 - \cos \lambda)}{\sqrt{1 - \frac{2\Omega}{k}(1 - \cos \lambda) \cos \lambda + \left( \frac{\Omega}{k} \right)^2 (1 - \cos \lambda)^2}}$$

(19)

of the helicity of photon is equal to $\sigma$. If, for example, the length scales, $d$ and $a$, of the optical fiber are taken to be $d \to 0$, $a \to 0$, which means that the fiber is curved extremely on a small enough diameter, and in consequence the rotating frequency $\Omega$ of photon moving along the curved fiber tends to $\infty$ (and $\lambda$ is negligibly small), then the expectation value, $\left\langle \frac{1}{k} \mathbf{K} \cdot \mathbf{J} \right\rangle$, of the helicity of photon in the evolution process may approach $-\sigma$ with $\sigma = \pm 1$ corresponding to the right- and left-polarized photon, respectively. The detailed calculation is given as follows: since in this case the rotating frequency $\Omega$ tends to be infinite and $\lambda$ is small, the fraction on the right-handed side of Eq. (19)

$$\left\langle \frac{1}{k} \mathbf{K} \cdot \mathbf{J} \right\rangle \to \frac{\sigma - \Omega \left( \frac{\gamma}{k} \right)}{\sqrt{\left( \frac{\Omega}{k} \right)^2 \left( \frac{\gamma}{k} \right)^2}} = -\sigma.$$  

(20)

Note that taking the Chiao-Wu case as an illustrative example here does not mean that no attempt is made to achieve theoretical rigor, since this case is a special limit (i.e., an adiabatic limit in which the optical fiber is helically curved) in our treatment of the propagation of photons moving in a noncoplanarly curved Fiber. Hence, from what has been discussed above, we can conclude without exaggeration that the formulation based on the Lewis-Riesenfeld invariant theory, rather than on the Berry’s adiabatic phase theory, can satisfactorily deal with the helicity inversion of photons in the arbitrarily curved optical fiber.

As is presented previously, in the case of curved fiber the operator $\frac{1}{k} \mathbf{K} \cdot \mathbf{J}$ together with the effective Hamiltonian $H_{eff}(t)$, can describe the motion of photons in the fiber, for the reason that it is the conserved operator with the time-independent eigenvalue $\sigma$, and can therefore be considered a Lewis-Riesenfeld invariant. But here it is worth
noticing that in a curved optical fiber the helicity of a propagating photon is defined to be \( \frac{1}{\hbar} \mathbf{k} \cdot \mathbf{J} \), rather than the conventional definition \( \frac{1}{\hbar} \mathbf{k} \cdot \mathbf{J} \) which has well described the state of photon in free space. For the extremely curved fiber, the expectation value of photon helicity may varies (even inverses) [27] in the propagation inside the fiber, and therefore the invariant \( \frac{1}{\hbar} \mathbf{k} \cdot \mathbf{J} \) may no longer considered the definition of photon helicity. It follows from (17) that in smoothly curved fiber (i.e., the helix diameter is large enough and consequently the angle \( \lambda \) in the momentum space is small), or in the case where the rotating frequency, \( \dot{\gamma} \), vanishes, it is apparently seen that the operator \( \frac{1}{\hbar} \mathbf{k} \cdot \mathbf{J} \) tends to the conventional definition of photon helicity, \( \frac{1}{\hbar} \mathbf{k} \cdot \mathbf{J} \).

IV. POTENTIAL APPLICATIONS OF HELICITY INVERSION TO INFORMATION SCIENCE

What interaction results in the helicity inversion of photons in the curved optical fiber? It follows from both the equation of motion (1) and the time-evolution equation (2) of photons that the generalized Lorentz force (Coriolis force) [8,25] may lead to the reversal of photon helicity. The effective magnetic field \( \frac{\hbar}{\mathbf{k}} \mathbf{k} \times \dot{\mathbf{k}} \) causes the variations of the direction of \( \mathbf{k} \) (the wave vector of photon) and can therefore be coupled to the generalized magnetic moment (proportional to the total angular momentum, \( \mathbf{J} \), of photons). It should be noted that the interaction Hamiltonian (effective Hamiltonian \( H_{\text{eff}}(t) \)) in the present Letter is obtained via the infinitesimal rotation operator of motion of photon in the fiber, rather than through analyzing the electromagnetic interaction between the photons and the medium of the optical fiber. This means that the effective Hamiltonian is a phenomenological Hamiltonian, which is based on the previous assumption that the wave vector \( \mathbf{k} \) of photon is always along the tangent of the fiber at each point at arbitrary time [15]. This assumption enables us to treat the propagation of photons in the fiber by using the purely geometric method (containing the infinitesimal rotation operator in 3-dimensional space). Since Chiao-Wu theory was consistent with the fiber experiment [16], we think that our generalized approach to this problem may also be appropriate to consider the more general cases, e.g., the non-adiabatic non-cyclic cases. Perhaps someone may argue that the investigation of photon-atom interaction in the curved fiber based on the purely physical mechanism is also essentially significant. This may be the case, but from the point of view of us, when the theory is compared with the experimental results, the previous assumption is still necessary, so long as the length scale is larger than the wavelength. Only in the extremely curved case (in which the length scales, \( d \to 0 \) and \( a \to 0 \), are less than the wavelength) should we take into account the detailed physical mechanism on the basis of the electromagnetic interaction between photons and media of the fiber. In the smoothly curved case, however, the geometric description of the electromagnetic interaction between them as presented in this Letter is convenient. It is readily verified that the helicity inversion of photons results from this interaction.

For the present, physicists’ control over the behavior of photons has spread to include the photon number, phase [28] and polarization [29] of light wave. If we could engineer all the degrees of freedom of photons, our technology would benefit. Already, the fiber communication, which simply guides light, has revolutionized the telecommunications industry. It is now possible for physicists to carry the laser communication and quantum computation into practice, with the development of the modern optics, fiber technology, quantum information and so on. For instance, in the last decades a new frontier (photonic crystal [30]) has emerged with the goal to control the behavior (amplitude or photon number) of light; Muller et al. [29] applied the polarization or spin of photons to the optical communication, and Duan et al. [28] suggested the application of the adiabatic Berry’s phase driven by the laser field to quantum computation. Apparently, it is of essential significance to control and utilize the degrees of freedom of photons (photon number, polarization, helicity, geometric phase, etc.) in information science and technology. It is with these goals in mind that the helicity inversion of photons is taken into account in the present Letter. In the curved optical fiber, the interaction of the photon spin with the wave vector causes the helicity inversion of the photon, which is in exact analogy with the transition operation from 0 to 1 in digital circuit and may be said to possess the potential applications in information technology. This inversion of helicity is controllable by manipulating the spatial shape and helix radius of the curved optical fiber. Furthermore, since geometric phase contains the global properties of the varying geometric shape of the fiber, and the measurement of the time-dependence of geometric phase may obtain the information about the varying geometric shape of the fiber, we think that the possible application of geometric phase of photons to the optical communication deserves considerations.

V. CONCLUDING REMARKS

The present approach to the geometric phase, time evolution, polarization as well as the helicity inversion of photons propagating along the optical fiber has several advantages over other treatments based on the classical Maxwell’s theory...
and the Berry’s adiabatic quantum theory. In this Letter, we are concerned with the non-adiabatic non-cyclic case. Furthermore, the adiabatic cyclic geometric phase $2\pi \sigma (1 - \cos \lambda)$ reduced from (10) is consistent with the result in Chiao-Wu theory, and the Chiao-Wu case is therefore considered the adiabatic limit of our case. Additionally, We show that when it is propagating in the curved (or noncoplanarly curved) optical fiber, the helicity of the photon varies with time, and the method in the present Letter is appropriate to treat the time-dependence of helicity and the helicity inversion. If the fiber is extremely curved, then the helicity inversion will take place; i.e., the change of the helicity of the photon is dependent strongly on the spatial appearance and the helix radius of the noncoplanarly curved fiber. Most of the treatments regarding the propagation of photons in the fiber had no Hamiltonian describing the motion of photons in the fiber, but the Hamiltonian is indeed very necessary in investigating the time evolution of photons in the fiber, so, in this Letter we construct an effective Hamiltonian and then transform the problem of motion of photons in the optical fiber into that of the time-dependent quantum spin model [31,32]. The invariant-related unitary transformation formulation replaces eigenstates of the time-dependent invariants with that of the time-independent invariants through the unitary transformation [33,34] and then obtain the explicit expression for the time-evolution operator, instead of the formal solution associated with the chronological product operator. It is apparently of essential significance to obtain the exact solution of Eq. (2), since in the treatment of helicity inversion of photons in arbitrarily noncoplanarly curved optical fiber, the exact solution, rather than the adiabatic solution obtained based on the Berry’s phase theory, is of real importance. All these advantages enable us to study the helicity inversion of photons in the fiber in more detail. We hold that not only this research would be of physical interest and essential significance both experimentally and theoretically, but also the applications of both geometric phase and helicity inversion of photons inside the fiber to optical communication, quantum information and related fields deserved further investigations.

Acknowledgements This project is supported in part by the National Natural Science Foundation of China under the project No. 90101024. Great thanks are due to Xiao-Chun Gao who led us to this subject and gave many helpful suggestions.

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