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Authors: Ekrem Çengelci
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Full Order Symbolic Small Signal Analysis of Peak-Current-Controlled SEPIC by PWM-Switch Model

Ekrem Çengelci

Abstract

Full order small signal analysis of peak-current controlled non-isolated Single Ended Primary Inductor Converter (SEPIC) is presented by utilizing pwm-switch model in Continuous Conduction Mode (CCM). The analysis provides control to output voltage transfer function together with its zeros and poles in symbolic form taking into account parasitic resistances of all four reactive components in the SEPIC topology. The resultant transfer function is with 4th order numerator and 5th order denominator, which necessitates approximation in deriving formulas of zeros and poles in symbolic form. Symbolically derived transfer function of the SEPIC is validated on two different numerical examples, one with operating in step down mode and another operating in step up mode, by frequency domain PSpice simulations on average circuit models and by time domain LTspice simulations on switching models. The mathematical analysis, PSpice and LTspice simulations, and measurement results of control to output voltage transfer function of SEPIC agree very well proving that the symbolic control to output voltage transfer function of SEPIC together with its zeros and poles are successfully derived.

Keywords: SEPIC, peak current-mode control, pwm-switch, small signal analysis, continuous conduction mode.

1. INTRODUCTION

The SEPIC topology first introduced in [1] as shown in Fig. 1 is used in wide range of applications, such as wind, solar, and fuel cell power conversions, lighting, aerospace and battery. It is one of the dual inductor dc-dc converter topologies that can step up and step down the input voltage. Major advantages of SEPIC are the output voltage polarity being same as that of the input, non-pulsating input current thanks to presence of inductor $L_1$ at its input, driving the switch $S_1$ being easier since it is a low-side switch.

Current-mode control [2] is a control method of switching dc/dc converters with several advantages [3] such as eliminating phase-lag from control voltage to the switch/inductor current, inherent pulse-by-pulse current limiting protecting the converter against overloads, ease of paralleling converter outputs, ease of applying output current feed-forward minimizing output voltage deviations under load transients, inherent sensitivity to static and dynamic variations of input voltage.

A major disadvantage of dc/dc converters with dual inductors, like SEPIC, is the complexity of their small signal transfer functions due to presence of four reactive components in the topology (inductors $L_1$, $L_2$ and capacitors $C_1$, $C_{out}$ in Fig. 1). The pwm-switch model presented in [4] is convenient in analyzing small signal behavior of switching dc/dc converters in voltage mode control in CCM while [5,6] present pwm-switch model with peak current-

1 Türkiye Bilimsel ve Teknolojik Araştırma Kurumu, MAM, Türkiye, ORCID: 0000-0002-4938-8961. ekrem.cengelci@tubitak.gov.tr
mode control in CCM, both of which facilitate fully circuit oriented small signal analysis of switching dc/dc converters as opposed the state space averaging technique involving matrix operations [7]. The model in [5,6] is capable of predicting the subharmonic instability of the current loop causing 180° phase reversal at half the switching frequency.

[8] performs small signal analysis of SEPIC with peak current-mode control in CCM using the pwm-switch model only on a numerical example without providing equations of the control to output voltage transfer function and its zeros and poles in symbolic form.

[9] provides small signal control to output voltage transfer function of SEPIC in symbolic form by state space averaging technique including the parasitic resistances of the reactive elements in the topology. However, it neither provides fully expanded control to output voltage transfer function, nor symbolic equations of zeros and poles of the transfer function.

Control to output voltage transfer function of SEPIC in symbolic form is not reported in the literature with constant switching frequency peak current-mode control based on the pwm-switch model including the parasitic resistances of all four reactive elements in the topology and will be presented in this paper. Equations for zeros and poles of the transfer function will be derived in symbolic form with full complexity including the parasitic resistances of the reactive components in SEPIC topology. Symbolic equations provided in the paper enable researchers and engineers to understand effect of each parasitic resistance on the transfer function of the converter. The resultant transfer function and the equations of zeros and poles are too complex for hand manipulations. However, present mathematics software packages, such as Mathcad, Mathematica, Maple, have symbolic arithmetic capabilities that facilitate manipulation of symbolic equations with thousands of parameters with no difficulty. Therefore, once the symbolic transfer function provided in the paper is transferred into a mathematics software, it can be manipulated in its symbolic form by either using it with full complexity or reducing it to a simpler form setting desired parameters to zero by the choice of designer.

Since the derived transfer function is with 4th order numerator and 5th order denominator, equations of the zeros and poles in symbolic form are be derived with approximation assuming locations of zeros and poles are well separated.

2. PWM SWITCH MODEL WITH PEAK CURRENT-MODE CONTROL IN CCM

Pwm-switch models presented in [4], [11]-[14] enable circuit-oriented small signal analysis of switching dc/dc converters. It represents the switch pair in a dc/dc converter with three terminals, which are active terminal “a”, passive terminal “p” and common terminal “c”. The pwm-switch model has dc, large signal and small signal models. Fig. 2 shows significant control signal waveforms, the dc and small signal ac circuit diagrams of the pwm-switch model with peak current-mode control in CCM. In Fig. 2b, upper case voltages and currents represent dc operating point quantities of the converter while lower case ones with tilde accent marks represent small signal ac quantities of the converter. The dc model is utilized to solve for dc operating point quantities while the ac model facilitates to determine the desired small signal transfer function of the dc/dc converter to be analyzed, such as control to output voltage or input to output voltage transfer functions, input or output impedances.

Equations of the pwm-switch model parameters in Fig. 2 are shown in (1a) to (1f).

\[ k_o = \frac{1}{R_i} \]  
\[ g_o = \frac{T_s}{L} \left( D \cdot \frac{S_e}{S_n} + 0.5 - D \right) \]  
\[ g_f = D \cdot g_o - \frac{D \cdot D' \cdot T_s}{2 \cdot L} \]  
\[ g_i = -\frac{i_a}{V_{ap}} \]  
\[ g_r = \frac{i_c}{V_{ap}} \]  
\[ C_s = \frac{4}{L \cdot \left( \frac{1}{T_o} \right)} \]

where,
$R_i$ is a scaling constant that transforms the “c” terminal current to voltage signal (Ω).

$D$ is the duty cycle in steady state and $D' = 1 - D$.

$T_s$ is the switching period (s),

$$L = \frac{L_1 + L_2}{L_1 + L_2} (H) \tag{2a}$$

$S_n = \frac{V_{ac}}{L_i} R_i \text{ (V/s)} \tag{2b}$

$S_f = \frac{V_{cp}}{L_i} R_i \text{ (V/s)} \tag{2c}$

$S_e$ is slope of the external compensation signal (V/s).

Note that $S_n$ is slope of the current coming out of terminal “c” as reflected to the control voltage by the scaling constant $R_i$. The capacitor $C_s$ in Fig. 2c is to model the subharmonic instability of the current loop [5,6].

The steady state value of control voltage $V_c$ (Fig. 2a) can be solved from (3) given below (see [5,6])

$$I_c = \frac{V_c - V_{cp}}{R_i} - \frac{V_{cp}}{V_{ap}} \cdot \frac{T_s S_e}{R_i} - V_{cp} \cdot \left(1 - \frac{V_{cp}}{V_{ap}}\right) \cdot \frac{T_s}{2L} \tag{3}$$

**3. Control to Output Voltage Transfer Function of SEPIC with Peak Current-Mode Control in CCM**

Switches $S1$ and $D1$ in Fig. 1 are not connected at a common terminal. To carry out the small signal analysis of SEPIC by the pwm-switch model, we need to represent the switch pair $S1$ and $D1$ with three terminals by connecting one of their terminals together. This can be achieved by either moving $D1$ from top to bottom (Fig. 3a) or $C1$ from top to bottom (Fig. 3b). Both works and for the purpose of the analysis of SEPIC in this paper Fig. 3a is chosen for no specific reason.

To solve for the pwm-switch model parameters in (1a) to (1f), we need to obtain dc equivalent circuit of SEPIC by substituting the dc pwm-switch model in Fig. 2b into the switch pair shown within dotted box in Fig. 3a. After shorting capacitors $C1$ and $Cout$ and opening inductors $L1$ and $L2$, the circuit parameters $D, Ia, Ic, Vap$ and $Vac$ in (1b-1e) and (2b, 2c) are solved using the resultant dc equivalent circuit, which is shown in Fig. 4.

If $D, Ia, Ic, Vap$ and $Vac$ are solved from Fig. 4 and substituted into (1b-1c) and (2b, 2c), equations in (4a-4k) can be obtained.

$$\frac{v_o}{V_i} = \frac{D}{1 - D} \tag{4a}$$

$$D = \frac{V_o}{V_i} \tag{4b}$$

$$Vap = \frac{V_o}{D} \tag{4c}$$

$$Ic = \frac{V_o}{R(1 - D)} \tag{4d}$$

$$Vac = Vi \tag{4e}$$

$$Ia = \frac{V_o}{R(1 - D)} \tag{4f}$$

$$Vcp = V_o \tag{4g}$$

$$gi = -\frac{D^2}{R(1 - D)} \tag{4h}$$

$$gr = -\frac{D}{R(1 - D)} \tag{4i}$$

$$S_n = Vi \cdot \frac{R_i}{L_1 + L_2} \tag{4j}$$

$$S_f = \frac{V_o}{2L} \cdot \frac{R_i}{L_1 + L_2} \tag{4k}$$

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Using (3) and (4a-4k) the control voltage $V_c$ can be solved for SEPIC as given in (5).

$$V_c = \frac{R_i I_0 + (1 - D) D s e}{2 L f s} + \frac{R_i I_0}{(1 - D)} + \frac{D s e}{f s} \quad (5)$$

Now that we have solved for the pwm-switch model parameters for SEPIC as shown in (4a-4k) and (5), next step is to obtain its ac pwm-switch model to derive the transfer function of $\tilde{V}_o/\tilde{V}_c$ by substituting the ac pwm-switch model in Fig. 2c into the switch pair shown within dotted box in Fig. 3a and shorting the dc input voltage source, which results in an equivalent circuit shown in Fig. 5.

Equations in (6a-6g) can be written by applying Kirchhoff’s current and voltage laws to the ac equivalent circuit in Fig. 5.

$$\dot{i}_a = D \cdot i_c + \tilde{v}_{ap} \cdot g_i + \imath r_c \cdot \tilde{v}_{cp} \quad (6a)$$

$$\dot{i}_c = \tilde{v}_{ap} \cdot g_f + \imath o \cdot \tilde{v}_c - \tilde{v}_{cp} \cdot (g_o + s \cdot C_s) \quad (6b)$$

$$\tilde{v}_{cp} = \frac{(iL1 + iC) \cdot (s \cdot L2 + rL2) + (iL1 + iC) \cdot \frac{R (s C_o + 1)}{s C_o}}{iL1 + iC + iC1} \quad (6c)$$

$$\dot{i}C1 = \frac{(\tilde{v}_{ap} - \tilde{v}_o)}{rC1 + s C1} \quad (6d)$$

$$\frac{\tilde{v}_{ap}}{\tilde{v}_c} = \frac{\dot{i}L1}{iL1 + iC + iC1} \quad (6e)$$

$$\dot{i}L1 = - \left( i_0 + \frac{\tilde{v}_{ap} - \tilde{v}_o}{rC1 + s C1} \right) \quad (6f)$$

$$\frac{\tilde{v}_o}{\tilde{v}_c} = \frac{(iL1 + iC + \frac{\tilde{v}_{ap} - \tilde{v}_o}{rC1 + s C1}) \cdot \frac{R (s C_o + 1)}{s C_o}}{R_+ (s C_o + 1)} \quad (6g)$$

If the equations in (6a-6g) are simultaneously solved, $\tilde{v}_o/\tilde{v}_c$ can be derived and organized as shown in (7).

$$\frac{\tilde{v}_o}{\tilde{v}_c} = -R \cdot k_o \cdot \frac{N0 + N1 s + N2 s^2 + N3 s^3 + N4 s^4}{D0 + D1 s + D2 s^2 + D3 s^3 + D4 s^4 + D5 s^5} \quad (7)$$

The numerator coefficients $N0$ to $N4$ and denominator coefficients $D0$ to $D5$ in (7) are given in Table 4 in Appendix. The pwm-switch model parameters $k_o$, $g_o$, $g_f$, $g_i$, $g_r$, $C_s$ in Table 4 are given in (1) and (4). The denominator of (7) is 5th order because there are five reactive components of $C1$, $Cout$, $L1$, $L2$, and $Cs$ in the ac equivalent circuit of SEPIC in Fig. 5.

Reorganizing the numerator of (7) yields (8) as follows:

$$\frac{\tilde{v}_o}{\tilde{v}_c} = -R \cdot k_o \cdot \frac{1 + \frac{s}{\omega z1}}{D0 + D1 s + D2 s^2 + D3 s^3 + D4 s^4 + D5 s^5} \quad (8)$$

where,

$$\omega z1 = \frac{1}{Cout \cdot rCout} \quad (9a)$$

$$M0 = 1 - D + g_i \cdot rL1 \quad (9b)$$
The 3rd order polynomial in the numerator of (8) can be assumed to have a real and a complex pole pair as shown in (10).

\[
\begin{align*}
M1 &= gi \cdot (L1 + C1 \cdot rC1 \cdot rL1) + \\
&\quad C1 \cdot \left[ (rC1 + rL2) \cdot (1 - D) + \right. \\
&\quad \left. rL1 \cdot (1 - D - gr \cdot rL2) \right] \\
M2 &= C1 \cdot \left[ L2 \cdot (1 - D - gr \cdot rL1) + \\
&\quad L1 \cdot (1 - D + gi \cdot rC1 - gr \cdot rL2) \right] \\
M3 &= -C1 \cdot gr \cdot rL1 \cdot L2
\end{align*}
\]

Exact numeric solutions exist for \( \omega z, \omega o, z, \) and \( Qz \) in (10) but exact symbolic solutions don’t. However, approximate symbolic solutions for \( \omega z, \omega o, z, \) and \( Qz \) can only be found by assuming that \( \omega z, \omega o, z, \) and \( Qz \) are well-separated [10].

If \( \omega z \gg \omega o, z, \) then approximate solution of \( \omega z, \omega o, z, \) and \( Qz \) are,

\[
\omega z \approx \frac{M2}{M3} = \frac{\omega o, z \approx \frac{M0}{M2} = \frac{L2 \cdot (1 - D - gr \cdot rL1) + L1 \cdot (1 - D + gi \cdot rC1 - gr \cdot rL2)}{gr \cdot rL1 \cdot L2}}{\frac{1 - D + gi \cdot rL1}{L2 \cdot (1 - D - gr \cdot rL1) + L1 \cdot (1 - D + gi \cdot rC1 - gr \cdot rL2)}}
\]

If \( \omega z \ll \omega o, z, \) then approximate solution of \( \omega z, \omega o, z, \) and \( Qz \) are,

\[
\omega z \approx \frac{M0}{M1} = \frac{\frac{1 - D + gi \cdot rL1}{L2 \cdot (1 - D - gr \cdot rL1) + L1 \cdot (1 - D + gi \cdot rC1 - gr \cdot rL2)}}{\frac{1 - D + gi \cdot rL1}{L2 \cdot (1 - D - gr \cdot rL1) + L1 \cdot (1 - D + gi \cdot rC1 - gr \cdot rL2)}}
\]

The denominator of (7) is 5th order yielding five poles. Subharmonic instability of the current loop is included in the pwm-switch model through \( C_5, \)

\[
M1 = gi \cdot (L1 + C1 \cdot rC1 \cdot rL1) + \\
\frac{1}{M1 \cdot s + M2 \cdot s^2 + M3 \cdot s^3} = \left( 1 + \frac{s}{\omega z z^2} \right) (1 + \frac{s}{\omega z z^2}) + \frac{s}{\omega z z^2}
\]

whose equation is given in (1f). It can be deduced from (1f) that \( C_5 \) and the inductors \( L1 \) and \( L2 \) place a double-pole at half the switching frequency. The remaining 3 poles of (7) are a low frequency real pole (\( \omega p \)) and a complex pole pair (with resonant frequency \( \omega o, p1 \) and quality factor \( Qp1 \)) as shown in (13).

\[
\frac{\omega p}{\omega o} = K \cdot \frac{(1 + \frac{s}{\omega o z}) + (1 + \frac{s}{\omega o z Qp1}) + (1 + \frac{s^2}{(\omega o z Qp1)^2})}{(1 + \frac{s}{\omega o z Qp1}) + (1 + \frac{s^2}{(\omega o z Qp1)^2})}
\]

Exact solutions to poles of the 5th order denominator of (13) can be found numerically but exact symbolic solutions do not exist. Approximate symbolic solutions for \( \omega p, \omega o, p1, Qp1, \omega o, p2, Qp2 \) can only be found by assuming \( \omega p, \omega o, p1 \) and \( \omega o, p2 \) are well-separated [10].

If \( \omega o, p2 \gg \omega o, p1 \gg \omega p, \) then approximate solutions of \( \omega p, \omega o, p1 \) and \( Qp1, \omega o, p2 \) and \( Qp2 \) are,

\[
\omega p \approx \frac{\omega o}{\omega o} \approx \frac{\omega o p1}{\omega o p1} \approx \frac{Qp1}{D1} \approx \frac{D2}{D2} \approx \frac{Qp2}{D4}
\]

The terms from \( D0 \) to \( D5 \) in (14a-14e) and (15a-15e) are given in Table 4 in Appendix. Although the high frequency resonant pole of \( \omega o, p2 \) is expressed in terms of \( D0 \) to \( D5 \) in (14a-14e) and (15a-15e) yielding lengthy expression, \( \omega o, p2 \) is set to half the angular switching frequency by
definition of the PWM-switch model. Therefore, it can be simplified with loss of negligible accuracy as,

\[ \omega_0 p^2 \cong \frac{D_5}{D_5} \cong \frac{\omega_s}{2} = \pi \cdot f_s \]  

(16)

where \( f_s \) is the switching frequency of the converter.

The dc gain \( K \) in (13) can be found by setting \( s = 0 \) in (7) as follows:

\[ K = -k_0 \cdot R \cdot \frac{N_0}{D_0} \]  

(17)

The equations of poles in (14a-14e) and (15a-15e) are too complex to manipulate manually in symbolic form due to lengthy expressions with the denominator coefficients \( D_0 \) to \( D_5 \) in Table 4. However, present mathematics software packages have capabilities of performing symbolic arithmetic operations. Once the coefficients of (7) in Table 4 are entered into a mathematics software with symbolic arithmetic capabilities, equations of poles in (14a-14e) and (15a-15e) can be manipulated easily and effectively keeping them in full complexity or simplifying them to certain degrees by setting some parameters to zero. This facilitates to analyze the effect of converter parameters on the transfer function in symbolic manner at any complexity level desired.

4. SIMULATION RESULTS

In this section, two different SEPIC’s are analyzed, one of which operate in step down mode in Example I and the other in step up mode in Example II. In both examples, control to output voltage transfer functions of the converters are numerically computed using (7) and determined by frequency domain PSpice simulations on average circuit models and by time domain LTspice simulations on switching models. Results of the mathematical model by (7), and LTspice and PSpice simulations are plotted on the same graph for comparison. Since PSpice and mathematical models are both based on pwm-switch model with circuit averaging theory, both are supposed to yield the same transfer function at all frequencies even well above the switching frequency. Therefore, the fact that PSpice simulations of the transfer functions produce the same results as of the mathematical model prove that the symbolic mathematical model parameters in (7) in Table 4 are correctly derived. The transfer function obtained through switching LTspice time domain simulations are not based on circuit averaging. Therefore, the fact that LTspice simulations match the circuit-averaging-based symbolic mathematical model and PSpice simulations proves validity of the circuit averaging based mathematical model by a switching simulation model.

4.1. Example I: A SEPIC operating in step down mode

Consider a SEPIC operating in step down mode with the parameters below.

\[ V_i = 12V \quad V_o = 5V \quad I_o = 5A \]
\[ R = 1\Omega \quad f_s = 300kHz \quad \omega_s = 1885krad/s \]
\[ R_i = 40m\Omega \quad rCout = 3.5m\Omega \quad Cout = 330\mu F \]
\[ C1 = 82\mu F \quad rC1 = 20m\Omega \quad Se = 40kV/s \]
\[ L1 = 18\mu H \quad L2 = 8.2\mu H \quad rL1 = rL2 = 8m\Omega \]

The parameters above are chosen such that the ripple voltages across capacitors \( Cout \) and \( C_1 \) are low and ratios of peak-to-peak ripple to average currents of inductors \( L_1 \) and \( L_2 \) are about 30%. Some external slope compensation is assumed \( (Se) \) although it is not mandatory for stability of the current loop because the duty cycle is lower than 50%.

Fig. 6 shows the LTspice circuit schematic of the SEPIC with the parameters above with peak-current-mode control. The LTspice simulation model is open loop with the dc voltage source \( V_c \) controlling the duty cycle hence output voltage of the converter. Control voltage \( V_c \) is calculated as 0.364V using (5) and determined by LTspice simulation as 0.368V (dc voltage source \( "V_c" \) in Fig. 6 is varied until output voltage is set to 5V).

Fig. 7 shows the time domain LTspice simulation
and load current, current through inductor L1, external slope signal, positive input of the comparator, and logic gate drive signal of the MOSFET.

From (1a-1f), (2a-2c), (4a-4k) and (17), the following parameters can be calculated.

\[ D = 0.294 \quad g_o = 0.318 \text{mho} \quad L = 5.634 \mu\text{H} \]
\[ K = 11.76 \quad g_i = -0.123 \text{mho} \quad S_f = 35.5 \text{kV/s} \]
\[ C_s = 200 \text{nF} \quad g_r = 0.417 \text{mho} \quad S_n = 85.2 \text{kV/s} \]
\[ k_o = 25 \text{mho} \quad g_f = 0.032 \text{mho} \]

With the parameters of this example SEPIC, the transfer function of \( \frac{v_o}{v_c} \) in (7) calculates out as shown in (18).

\[ \frac{v_o}{v_c} = 25 \cdot \left( \frac{0.705 + 6.883 \times 10^{-7} s + 1.506 \times 10^{-9} s^2}{-3.304 \times 10^{-1} s^2 + 5.825 \times 10^{-21} s^4 + 1.499 + 3.389 \times 10^{-9} s + 4.871 \times 10^{-9} s^2 + 7.881 \times 10^{-21} s^4} \right) \]  \( \text{(18)} \)

Solving the roots of the numerator and denominator of (18) yields (19) as follows:

\[ \frac{v_o}{v_c} = 11.76 \cdot \left( \frac{1 + \frac{s}{865.8 \times 10^6}}{1 + \frac{s}{795.1 \times 10^6 - 507.48 \times 10^6 s}} \right) \]  \( \text{(19)} \)

Angular frequencies of real zeros and poles, resonant angular frequencies and their quality factors of the complex zeros and poles are calculated from (19) and summarized in column “Exact Solutions” in Table 1.

Note that poles and zeros in ‘Exact Solutions’ column in Table 1 are well separated so that the approximate equations derived in (11a-11c), (12a-12c), (14a-14e) and (15a-15e) are expected to yield values close to those of exact solutions. In
“Approximate Solutions” column of Table 1, (11a-11c) and (14a-14e) are used instead of (12a-12c) and (15a-15e) since \( \omega z \gg \omega_o, z \) and \( \omega, p2 \gg \omega_o, p1 \gg \omega \). It is It is observed in Table 1 that approximate equations in (11a-11c) and (14a-14e) calculate the parameters with negligible errors except \( Qz \) and \( Qp1 \) thanks to zeros and poles of the converter in this example being well-separated.

It is noticed from Table 1 that the zero \( \omega z \) is on the Right Half Plane (RHP).

For the SEPIC example operating in step down mode, frequency domain simulation of the transfer function \( \tilde{v}_o/\tilde{v}_c \) can be performed by PSpice using CMLSCCM model in its library, which is a large signal peak current-mode pwm-switch model in CCM. The PSpice circuit model of the example SEPIC is shown in Fig. 8 for frequency domain analysis.

A flaw has been discovered in the netlist of PSpice model, CMLSCCM. The capacitor \( C_s \) has been connected across the nodes “c” and “p” in its netlist resulting in miscalculation of the phase at frequencies nearly half the switching frequency and above. The correct modeling requires connecting the capacitor \( C_s \) across the nodes “ca” and “p” instead. In the PSpice simulations presented in this paper, CMLSCCM model has been corrected accordingly. Frequency domain analysis of the example SEPIC is carried out through PSpice using the circuit model in Fig. 8 and through LTspice using the circuit model in Fig. 6. Fig. 9 compares Bode plots obtained through LTspice and PSpice simulations as well as the transfer function in (18). Gain and Phase plots of PSpice simulation and the transfer function in (18), labelled as “Analysis”, match perfectly in Fig. 9 even well above the switching frequency expectedly because both CMLSCCM PSpice model and the transfer function in (18) are based on average peak current-mode pwm-switch model in CCM. This proves that the symbolic transfer function in (7) and its coefficients in Table 4 are derived successfully.

The LTspice Bode plots in Fig. 9 are very well in agreement with those of the PSpice simulation and transfer function in (18) but deviate starting at nearly half the switching frequency and above due to the effect of switching with the LTspice circuit model.

As observed in “Exact Solutions” column in Table 1, the angular resonant frequencies of the double zeros and double poles (\( \omega_o,z \) and \( \omega_o,p1 \)) and their quality factors (\( Qz \) and \( Qp1 \)) are nearly equal. Therefore, they cancel each other in the Bode plots in Fig. 9 yielding smooth gain and phase plots around the angular frequency of \( \omega_o,z \). The parameters in the “Approximate Solutions”

![Figure 9. Comparison of Bode plots of \( \tilde{v}_o/\tilde{v}_c \) as simulated by LTspice and PSpice and as calculated by the transfer function in (18).](image)

![Figure 10. Deviation with gain and phase plots of \( \tilde{v}_o/\tilde{v}_c \) around \( \omega_o,z \) because approximated values of \( Qz \) and \( Qp1 \) differ from their exact values.](image)

4.1. Example II: A SEPIC operating in step up mode

Consider a SEPIC operating in step up mode with the following parameters:

\[
\begin{align*}
Vi &= 12V \\
Vo &= 24V \\
Se &= 150kV/s \\
R &= 8\Omega \\
f_s &= 400kHz \\
\omega_s &= 2.512M\text{rad/s} \\
Ri &= 50m\Omega \\
Io &= 3A \\
C_{out} &= C1 = 100\mu F \\
L1 &= 15\mu H \\
rL1 &= 12m\Omega \\
rCout &= rC1 = 20m\Omega \\
L2 &= 10\mu H \\
rL2 &= 10m\Omega
\end{align*}
\]
Parameters of LTspice circuit schematic in Fig. 6 have been updated with the parameters of the SEPIC operating in step up mode. Control voltage \( V_c \) is calculated as 0.783V using (5) and determined by LTspice simulation as 0.79V.

Fig. 11 shows the time domain LTspice simulation waveforms of the SEPIC operating in the step up mode. Like in the case of the previous SEPIC example operating in step down mode, poles and zeros in “Exact Solutions” column in Table 2 are well separated with the SEPIC example operating in step up mode so that the parameter values in the “Approximate Solutions” column calculated using (11a-11c) and (14a-14e) are very close to those of Table 2. Zeros And Poles of The Transfer Function In (21).

\[
\begin{align*}
\frac{\theta_o}{\theta_c} &= 27.73 \cdot \frac{(1 + \frac{s}{500 \cdot 10^7})}{(1 + \frac{s}{244.1 - 20.06 \cdot 10^{-5} j})} \\
&\quad \cdot \frac{(1 + \frac{s}{495.6 - 20.01 \cdot 10^{-5} j})}{(1 + \frac{s}{659.12 \cdot 10^{-5} - 1.75 \cdot 10^{-7} j})} \\
&\quad \cdot \frac{(1 + \frac{s}{659.12 \cdot 10^{-5} + 1.075 \cdot 10^{-7} j})}{(1 + \frac{s}{2381})}
\end{align*}
\] (21)

Angular frequencies of real zeros and poles, and resonant angular frequencies and their quality factors of the complex zeros and poles are calculated from (21) and summarized in column “Exact Solutions” in Table 2.

Figure 12. Comparison of Bode plots of \( \theta_o/\theta_c \) as simulated by LTspice and PSpice and as calculated by the transfer function \( \frac{\theta_o}{\theta_c} \).
function in (20). The frequency range on the bottom graph is narrower to show details of the gain and phase plots with sharp changes around \( \omega_0, z \equiv \omega_0, p1 \cong 20 \text{krad/s} \).

the parameters in “Exact Solutions” column except \( Q_z \) and \( Q_p1 \). (see “Exact Solutions” column in Table 2).

Similar to the SEPIC operating in step down mode, \( \omega_z^2 \) in Table 2 is on the RHP, too.

The LTspice simulation model in Fig. 6 and the PSpice simulation model in Fig. 8 are updated with the parameters of the SEPIC example operating in step up mode and frequency domain simulations are carried out with both simulators. Fig. 12 compares the LTspice and PSpice simulation results with the transfer function in (20), labelled as “Analysis”. Similar to the plots in Fig. 9, the PSpice simulations and the transfer function in (20) match perfectly at all frequencies even well above switching frequency as expected. LTspice simulation results in Fig. 12 agree very well with the PSpice simulations and the transfer function in (20) up to nearly half the switching frequency and deviate above that due to the switching effect with the LTspice simulation model. There is sudden phase change around the frequency of \( \omega_0, z \equiv \omega_0, p1 \) this is because \( Q_z \) and \( Q_p1 \) differ in value (see “Exact Solutions” column in Table 2).

Because of the fact that exact and approximate values do not match in Table 2 for \( Q_z \) and \( Q_p1 \), the Bode plots generated using the values in exact and approximate values differ in the vicinity of \( \omega_0, z \equiv \omega_0, p1 \). This mismatch is shown in Fig. 13.

As seen in both examples of SEPIC operating in step up and step down modes, the LTspice and PSpice simulations and the symbolic transfer function in (7) agree very well validating the mathematical analysis of SEPIC with current-mode control in CCM presented in this paper.

5. EXPERIMENTAL RESULTS

LT3759 controller by Analog Devices Inc. has been used for experimental verification. A demo board of LT3759 (part number: DC1787A) has been modified as shown in Table 3. Transfer function \( \tilde{\eta}_o/\tilde{\eta}_i \) has been measured using frequency response analyzer PSM1735 by Newtons4th Ltd. See Fig. 14 for the picture of the experimental set up.

The output voltage, output current and switching frequency of the converter are set to 12V, 1A and 300kHz, respectively. Experimental tests are carried out with the converter operating in step up mode with \( V_i = 9V \) and in step down mode with \( V_i = 18V \).

As seen in Fig. 15, the calculated transfer function in (7) and measured transfer function match well.

![Figure 13. Bode plots of \( \tilde{\eta}_o/\tilde{\eta}_i \) with exact and approximate solutions in Table 2. The mismatch in Bode plots around \( \omega_0, z \equiv \omega_0, p1 \cong 20 \text{krad/s} \) is due to difference between exact and approximate values of \( Q_z \) and \( Q_p1 \) in Table 2.](image)

![Figure 14. Picture of the experimental set up.](image)

**Table 3 Components Used On The Demo Board Of LT3759**

| Reference Designator | Component Value | Manufacturer and Part Number |
|----------------------|-----------------|------------------------------|
| \( C, C_{out} \)     | \( C = 100 \mu F, esr = 0.25 \Omega \) | Panasonic Corp., EEU-EB1V221B |
| \( L1, L2 \)         | \( L = 33 \mu H + 47 \mu H \) \( DCR = 67 \Omega + 68 \Omega \) | Viking Tech Corp. PCS127-T330 and PCS127-T470 connected in series |
| \( D1 \)             | 3A Schottky Diode | Diodes Inc., PDS360-13 |
| \( S1 \)             | 60V, 7m\( \Omega \), N-Ch. MOSFET | Renesas Elec. Corp., RJK0652DPB-00-J5 |
| \( Ri \)             | 5m\( \Omega \), 1/2W, 1% | Vishay Inc., WSL20105L000FEA |
Since the symbolic equations provided in the paper are derived including the parasitic resistances of all four reactive components in SEPIC, they enable researchers and engineers to understand effect of all these parasitic resistances on the transfer function of the converter. On the other hand, they make the resultant transfer function and the equations of zeros and poles too complex for hand manipulations. Present mathematics software packages have symbolic arithmetic capabilities that facilitate manipulation of symbolic equations with thousands of parameters with no difficulty. Therefore, once the symbolic transfer function provided in the paper is transferred into a mathematics software, it can be manipulated in its symbolic form by either using it with full complexity or reducing it to a simpler form setting desired parameters to zero by the choice of designer.

7. APPENDIX

Table 4 Coefficients of The Transfer Function in (7).

| N0  | 1 − D + gi · rL1 |
|-----|------------------|
| N1  | gi · (L1 + C1 · rC1 · rL1 + Cout · rCout · rL1) + (1 − D) · (Cout · rCout + C1 · (rC1 + rL1)) + C1 · (1 − D − gr · rL1) · rL2 |
| N2  | Cout · rCout + C1 · L2 − gr · rL2 · rL1 · L1 · (1 − D + gi · rC1 − gr · rL1) + Cout · rCout · (rC1 + rL1 + gi · rC1 · rL1 + rL2 − gr · rL1 − D · (L2 + Cout · rCout · (rC1 + rL1 + rL2))) |
| N3  | C1 · [Cout · rCout · ((1 − D) · (L1 + L2 + gi · L1 · rC1) − gr · (L2 + Cout · rCout · rL1 + L1 · L2 + Cout · rCout · rL2))]
| N4  | −C1 · Cout · gr · L1 · L2 · rCout |
| D0  | −1 − [go · (1 − D) + gr · (rL1 + rL2 − gi · (rL1 + go · D · (rL1 + rL2 − gi · (rL1 + go · rL1 − rL1 + Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 − gi · gr · rL2 − rL1 + gi · gO · rL1 − gi · go · rL1 − C1 · go · rC1 − C1 · go · rC1 − Cout · gO · gr · rCout − C1 · rL1 − gi · go · rL2 - |
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