**Minisurf** – A minimal surface generator for finite element modeling and additive manufacturing

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**A R T I C L E I N F O**

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- Finite element modeling (FEM)
- Additive manufacturing (AM)
- Computer-aided design (CAD) files
- 3D printing
- Architected materials

**A B S T R A C T**

Triply periodic minimal surfaces (TPMSs) have long been studied by mathematicians but have recently garnered significant interest from the engineering community as ideal topologies for shell-based architected materials with both mechanical and functional applications. Here, we present a TPMS generator, **Minisurf**. It combines surface visualization and CAD file generation (for both finite element modeling and additive manufacturing) within one single GUI. **Minisurf** presently can generate 19 built-in and one user-defined triply periodic minimal surfaces based on their level-set surface approximations. Users can fully control the periodicity and precision of the generated surfaces. We show that **Minisurf** can potentially be a very useful tool in designing and fabricating architected materials.

**1. Introduction**

For decades, scientists and engineers have been striving to design and fabricate new multiphase materials with controlled phase topologies – often termed “architected materials” or “metamaterials” – with unprecedented and tunable combinations of properties; architected cellular materials, where one phase is void, are the most notable examples. In terms of mechanical behavior, significant efforts have focused on designing architected materials that are stiff, strong and...
tough at very low density, by optimizing the topology of the material phases. Traditionally, topologies have largely been limited to beam-based structures, such as honeycombs in 2D [1–7] and octet lattices in 3D [8–14]. More recently, interest has shifted to shell-based topologies with minimal surface characteristics, such as triply periodic minimal surfaces (TPMS) [15–20] and isotropic stochastic spinodal minimal surfaces [21–23]; while more challenging to fabricate, these topologies are devoid of nodes and other stress intensification regions, which results in improved strength and toughness [21,24–26] as well as efficient fluid transport at low pressure drops [27–29]. Many studies of these minimal surface topologies have been motivated by the development of superior additive manufacturing (AM) technologies that enable their fabrication, and generally employ finite element modeling (FEM) for calculation of their mechanical and functional response; as a consequence, there is an increasing need for quick and accurate generation of computer-aided design (CAD) files for periodic cellular materials based on TPMS topologies, to be employed both for numerical analysis and additive manufacturing.

In this article, we present an efficient software application called "MiniSurf", which combines surface visualization and CAD file generation (for both FEM and AM) within one single graphical user interface (GUI). We briefly describe and illustrate the main software features. In addition, we highlight the impact of this package on current and potential applications in the field of architected materials design. Finally, we discuss the software limitations and future improvements.

2. Description and features

MiniSurf is a software package that runs on Matlab Runtime (a freely accessible Matlab compiler) for visualization and generation of triply periodic minimal surface CAD files (with .inp extension for FEM through Simulia Abaqus and/or .stl extension for AM). The software package has a sleek and simple GUI consisting of two panels: a control panel (left) and a visualization panel (right), as shown in Fig. 1. The control panel allows users to select from the built-in library of minimal surfaces, as well as to type in the custom level-set equation of any desired surface. To facilitate generation of periodic architected materials, users can adjust the number of unit cells $N_i$ along the x, y, and z-directions, to produce specimens of different aspect ratios and number of unit cells, as shown in Fig. 2(a) and (b). In addition, the precision of the generated surfaces, governed by number of composing facets, can be fine-tuned by changing the number of mesh grid points $P_i$ along the x, y, and z-directions. The generated minimal surfaces will be shown in the visualization panel in either non-mesh or mesh mode, as illustrated in Fig. 3(a) and (b).

MiniSurf currently has 19 built-in minimal surfaces. All these minimal surfaces are generated by meshing their implicit level-set approximations $f(x, y, z) = c$, where $c$ is a constant and $x$, $y$, and $z$ represent the location of $P_i \times P_i \times P_i$ grid points in a 3D volume of size $N_x \times N_y \times N_z$; equations for all built-in surfaces are reported in Table 1. The meshing is executed via the Matlab built-in function isosurface, that discretizes the minimal surfaces into many triangular facets, thus...
These studies are recent and the interest of the mechanics community in truss-based lattices in terms of specific strength and toughness \[41–43\] for both AM and FEM. TPMS shell-based architected materials have additional ones frequently added) and automatically creates CAD files complete library of equations for the most interesting TPMSs (with the connectivity is then subsequently used to write CAD files in .inp .stl formats. The idea of using level-set surface equations to approximate TPMSs is currently used in two ongoing projects that includes a nearly.

Table 1
The level-set surface equations in the form of \( f(x, y, z) = c \) for the 19 built-in triply periodic minimal surfaces. For a single unit cell, \( x, y, \) and \( z \) are bounded by \([0, 2\pi]\).

| TPMS            | Level-set equation for the TPMS \( f(x, y, z) = c \) |
|-----------------|-----------------------------------------------------|
| Schwarz P \[30,31\] | \( \cos(x) + \cos(y) + \cos(z) = 0 \) |
| Double Primitive \[32\] | \( 0.5 \cos(x) \cos(y) + \cos(z) (\cos(x) + \cos(y) + \cos(2z)) = 0 \) |
| Schwarz D \[31,33\] | \( \sin(x) \sin(y) \sin(z) + \sin(x) \cos(y) \cos(z) + \cos(x) \sin(y) \cos(z) + \cos(x) \cos(y) \sin(z) = 0 \) |
| Complementary D \[33\] | \( \cos(3x + y) \cos(z) - \sin(3x - y) \sin(z) + \cos(x + 3y) \cos(z) + \cos(x - 3y) \sin(z) + \cos(x - y) \cos(2z) - \sin(x + y) \sin(3z) = 0 \) |
| Double Diamond \[30\] | \( 0.5 \sin(x) \sin(y) + \sin(y) \sin(z) + \sin(x) \sin(z) + \sin(2x) \sin(2y) + \sin(2y) \sin(2z) + \sin(2z) \sin(2x) = 0 \) |
| Gyroid \[30,33\] | \( \cos(x) \sin(y) + \cos(y) \sin(z) + \cos(z) \sin(x) = 0 \) |
| Gyroid' \[30\] | \( \sin(2x) \cos(y) \sin(z) + \sin(2y) \cos(z) \sin(x) = -0.32 \) |
| Double gyroid \[32\] | \( 2.75 \sin(2x) \sin(2y) \sin(z) + \sin(2y) \sin(z) \sin(x) + \sin(2z) \sin(y) \cos(x) - (\cos(2x) \cos(2y) \cos(2z) + \cos(2z) \cos(2y) \cos(2x)) = 0.95 \) |
| Karcher K \[30\] | \( 0.3 \cos(x) + \cos(y) + \cos(z) + 0.3 \cos(x) \cos(y) + \cos(3z) \cos(x) + \cos(z) \cos(x) + \cos(z) \cos(x) = -0.4 \cos(2x) \cos(2y) + \cos(2z) = -0.25 \) |
| O, CT-O \[30\] | \( 0.6 \cos(x) \cos(y) + \cos(x) \cos(z) + \cos(x) \cos(z) + \cos(x) \cos(y) = 0 \) |
| Lidinoid \[33,34\] | \( 0.5 \sin(2x) \cos(y) \sin(z) + \sin(2y) \cos(z) \sin(x) + \sin(2z) \cos(x) \sin(y) - 0.5 \cos(2x) \cos(2y) + \cos(2z) \cos(2x) + \cos(2z) \cos(2x) = -0.15 \) |
| Neovius \[30,31\] | \( 3 \cos(x) \cos(y) + \cos(z) + \cos(z) + 4 \cos(x) \cos(y) \cos(z) + \cos(2z) \cos(x) = 0 \) |
| I-WP \[31,33\] | \( 2 \cos(x) \cos(y) + \cos(y) \cos(z) + \cos(z) \cos(x) - \cos(2x) + \cos(2y) + \cos(2z) = 0 \) |
| Fisher-Roch \( S \) \[32,33\] | \( \cos(2x) \cos(2y) \cos(z) + \cos(2y) \cos(z) \sin(z) + \sin(x) \cos(2y) \cos(z) = 0 \) |
| Fisher-Roch \( C(S) \) \[33\] | \( \cos(2x) \cos(2y) \cos(2z) + \cos(2z) + 2 \sin(3x) \sin(2y) \cos(z) + \cos(z) \sin(2z) \sin(3x) \sin(2y) \cos(z) + \cos(x) \cos(2y) \cos(z) + \cos(x) \cos(2y) \cos(z) = 0 \) |
| Fisher-Roch \( Y \) \[33\] | \( \cos(x) \cos(z) \cos(x) \cos(z) \cos(z) \cos(x) \cos(z) \cos(z) \cos(x) \cos(z) = 0 \) |
| Fisher-Roch \( C(Y) \) \[33\] | \( -\sin(x) \sin(y) \sin(z) + \sin(2x) \sin(y) \sin(z) + \sin(2y) \sin(z) \sin(x) - \cos(x) \cos(2y) \cos(z) \cos(2z) \cos(2y) \cos(z) = 0 \) |
| P-RD \[30,32,33\] | \( 4 \cos(x) \cos(y) \cos(z) - \cos(2x) \cos(2y) \cos(2z) + \cos(2y) \cos(2z) \cos(2y) \cos(2z) = 0 \) |

providing information on the facet-vertex connectivity. Information on the connectivity is then subsequently used to write CAD files in .inp and .stl formats.

3. Impact overview

The idea of using level-set surface equations to approximate TPMSs has been explored extensively in various multidisciplinary research projects for years \[35–40\]; however, to the best of our knowledge, there is no available software package like MiniSurf that includes a nearly complete library of equations for the most interesting TPMSs (with additional ones frequently added) and automatically creates CAD files for both AM and FEM. TPMS shell-based architected materials have remarkable mechanical properties, which makes them superior to classic truss-based lattices in terms of specific strength and toughness \[41–43\]. These studies are recent and the interest of the mechanics community in the structural performance of TPMS-based materials is only expected to grow. MiniSurf will certainly support a number of future projects in this field. As examples, MiniSurf is currently used in two ongoing projects in our research group: (i) Mechanical properties of 3D printed interpenetrating phase composites with shell-based reinforcements \[44\]. MiniSurf is used to generate CAD files for Schwarz P surface shell-based reinforcements for interpenetrating phase composites. These composites can be readily fabricated by multi-material jetting in VeroWhite (a hard polymeric material for reinforcement) and Agilus (a soft elastomeric material for the matrix) using a Connex 3D printer. The effect of the matrix/reinforcement interpenetration on the mechanical properties of the composites are subsequently investigated both experimentally and numerically (for the numerical studies, MiniSurf-generated meshes are used in finite elements analyses of deformation and damage of the composites). (b) Architected materials designs for long bone implants \[45\]. In this effort, we are investigating the performance of minimal surface-based porous materials as implants for long bone repair. Schwarz P CAD files are generated using MiniSurf for the purpose of surface area calculations and finite element modeling. The results are then used to draw comparisons among different topological designs and identify optimal topologies.

At the same time, we expect MiniSurf to have a broad impact on multidisciplinary studies far beyond the solid mechanics field. The
interest of the engineering community in TPMS shell-based materi-
als is documented in several recent studies where TPMS-based ar-
chitected materials are manufactured and investigated for their mul-
tifunctionality, including (1) thermal properties (e.g., thermal con-
ductivity [46,47], coefficient of thermal expansion [48] and heat ex-
change [49–51]), (2) acoustic properties (e.g., sound absorption and
acoustic bandgaps [52,53] and audible coloration [54]) and (3) elec-
trochemical properties (e.g., electrical conductivity [55,56]).

4. Limitations

Despite being user friendly and freely accessible to all researchers
and engineers, MinSurf has three main limitations:

(1) Suboptimal mesh

In general, meshing in Matlab is done through the Delaunay
triangulation algorithm [57,58], which connects a given set of
discrete points. Although such algorithm tends to avoid triangu-
lar facets with acute angles, meshing of highly curved minimal
surfaces – based on the initial user-defined 3D uniform grid
points – still results in many triangular facets with bad aspect
ratios (thin and long).

(2) Zero-thickness surface

MiniSurf generates minimal surfaces composed of many facets
without any physical thickness. Postprocessing to thicken these
surfaces is often required. Fortunately, many commercial finite
element packages (for example, Simulia Abaqus) or additive
manufacturing software (for example, Geomagic Design X) have
such postprocessing ability.

(3) Nonparallel computing

Currently, MiniSurf can only execute calculations with one
single-core processor, although it can still efficiently generate
highly meshed surfaces (300 × 300 × 300 initial mesh grid
points) under one minute.

5. Conclusion and future improvements

In this paper, we presented a software package, MiniSurf, that
efficiently produces CAD files of shell-based architectural materials con-
sisting of periodic arrays of minimal surface unit cells, for additive
manufacturing and finite element modeling. The surface description
is provided via implicit level-set equations. Currently, the software
library has 19 built-in minimal surfaces, but any user-defined level-set
surface is also allowed. Despite the limitations discussed in Section 4,
we expect the software package to be impactful, given the profound
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