Strangeness Enhancement in the Parton Model

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I. INTRODUCTION

The production of strange particles in heavy-ion collisions has been a subject of intense study in the past twenty years, ever since the proposal that it may reveal a signal of quark-gluon plasma formation \[\require{AMSsymbols}Q^2 = 1 \text{(GeV/c)}^2\]. Various approaches to the problem have been adopted, ranging from statistical thermal model \[\text{[3,4]}\] to simple quark coalescence model \[\text{[5,6]}\] to dual parton model \[\text{[7]}\]. Despite differences in diverse viewpoints, the major theme is to explain the phenomenon of strangeness enhancement in nuclear collisions \[\text{[8]}\]. Although the experimental definition of strangeness enhancement is the increase of the strangeness content of the produced hadrons with increasing number of participants from \(pp\) to \(AA\) collisions, a more appropriate theoretical description of strangeness enhancement is in terms of the increase of strange quarks before hadronization. In this paper we present a quantitative treatment of the enhancement factor in the framework of the parton model, and obtain a numerical measure of Pauli blocking.

The point stressed in the original explanation for strangeness enhancement \[\text{[9]}\] is that when the quark degrees of freedom are liberated, it is easier to create strange quark pairs than strange hadrons because the \(\bar{s}s\) threshold is lower. Deconfinement then leads naturally to the possibility of plasma formation. In our view the quarks have always been the basis for understanding hadron production even in \(pp\) collisions for \(\sqrt{s} > 10\) GeV. The recombination model has been able to reproduce the low-\(p_T\) inclusive distributions in the fragmentation region by treating the hadronization processes at the parton level \[\text{[9]}\]. Thus the relevance of the quark degrees of freedom in heavy-ion collisions at SPS is nothing new. In collisions at such energies the nucleons are broken up and thus deconfined, but it does not mean that there is thermalized quark-gluon plasma, which no one would associate with \(pp\) collisions.

Once one descends to the parton level, the notion of strangeness enhancement (SE) can take on a quantitative description in terms of the strange quark population. The baseline for the unenhanced \(s\) quark distribution in the quiescent sea should be pinned down by the parton distribution functions of the nucleon studied exhaustively by several groups \[\text{[10]}\]. We shall use the distribution functions of the CTEQ global analysis \[\text{[11]}\] that fits some 1300 data points obtained for many reactions in 16 experiments. Their extrapolations to \(Q^2 = 1\) (GeV/c)\(^2\) are presented in the form of graphs available on the web \[\text{[12]}\]. Since gluons do not hadronize directly, there being no glueballs found, they are converted to quark pairs which subsequently hadronize by recombination. How much the conversion goes into the strange sector gives us a measure of SE.

Gluon conversion is not a new process that we must consider for heavy-ion collisions. Even in hadronic collisions gluons must convert in order to hadronize. Such conversion has been included in the study of inclusive distributions of hadrons produced in the fragmentation region in the framework of the valon-recombination model \[\text{[10]}\], and more recently using the CTEQ parton distribution functions \[\text{[13]}\] to reproduce various hadronic spectra \[\text{[12]}\]. Our attention in this paper is shifted from the fragmentation region in hadronic collisions, where the \(x\) dependence is an issue, to the central region in nuclear collisions, where the relative yields of particles produced are the focus.

Clearly, there is no way to study SE from first principles. Our investigation here is phenomenological. Our goal is very modest. It is not possible to compute particle ratios from the parton model alone. We shall use a large set of particle ratios as experimental inputs to guide us in the determination of the SE factor. The effect of Pauli blocking in the non-strange sector is included in the those inputs, and are not amenable to first-principle calculations. Our theoretical input is essentially the counting of quarks and antiquarks in their partition into the
II. QUARK COUNTING

We begin by drawing the boundary of our concern here. Since the yields on multi-strange hyperons are low compared to $K$ and $\Lambda$, we shall in first approximation ignore the production of $\Xi$ and $\Omega$, and aim at results with accuracies not better than 90%. With such simplification we can better exhibit the spirit of our approach to the problem and make more transparent the issues involved in SE. Improvements that include the $\Xi$ and $\Omega$ particles can be considered later. We shall also consider isosymmetric dense medium at mid-rapidity so that we need not distinguish $u$ and $d$ quarks. Proton and neutron will be equal in number, as do $\pi^+$ and $\pi^-$. The strange quarks $s$ and $\bar{s}$ are produced in equal numbers, but $K^+$ and $K^-$ will not be produced in equal numbers because of associated production.

Let us use the following notation to denote the numbers of hadrons and non-strange quarks, e.g., $N$ is the number of nucleons, and $q$ is the number of light quarks.

$$N = p + n,$$

$$\bar{N} = \bar{p} + \bar{n},$$

$$\Pi = \pi^+ + \pi^- + \pi^0,$$

$$Y = \Lambda + \Sigma^0 + \Sigma^+ + \Sigma^-,$$

$$K = K^+ + K^0,$$

$$\bar{K} = K^- + \bar{K}^0,$$

$$q = u + d.$$ (1-5)

Then there are linear relations among these numbers based on counting the number of valence quarks in the various hadrons

$$3N + \Pi + 2Y + K = q,$$ (6)

$$3\bar{N} + \Pi + 2\bar{Y} + \bar{K} = \bar{q},$$ (7)

$$Y + \bar{K} = s,$$ (8)

$$\bar{Y} + K = \bar{s}.$$ (9)

The right-hand sides of the above equations all refer to the numbers of quarks after enhancement from gluon conversion. Let $\kappa$ be the fraction of $s$ quarks that recombine with non-strange antiquarks to form anti-kaons, and similarly $\bar{\kappa}$ be the fraction of $\bar{s}$ to form kaons. That is, we define

$$K = \kappa s, \quad \bar{K} = \bar{\kappa} \bar{s},$$ (10)

Then on account of Eqs. (8) and (9), we have

$$Y = (1 - \kappa) s, \quad \bar{Y} = (1 - \bar{\kappa}) \bar{s}.$$ (11)

Define the hadronic ratios

$$r = K/\bar{K}, \quad R = \bar{Y}/Y.$$ (12)

It then follows that

$$\kappa = \frac{1 - r}{r - R}, \quad \bar{\kappa} = r \kappa,$$ (13)

where $s = \bar{s}$ has been used. For experimental values of $r$ and $R$, we assume that $K/\bar{K} \approx K^+ / K^-$ and $\bar{Y}/Y \approx \bar{\Delta}/\Delta$. For $Pb-Pb$ collisions at SPS the values are

$$r = 1.8, \quad R = 0.13,$$ (14)

so we obtain

$$\kappa = 0.52, \quad \bar{\kappa} = 0.94.$$ (15)

With these values of $\kappa$ and $\bar{\kappa}$ we can proceed to consider the non-strange sector. Define

$$\rho = \bar{N}/N$$ (16)

so that we can obtain from Eqs. (6) and (7)

$$N = \frac{1}{3(1 - \rho)} \left[ q - \bar{q} + 3 s (\kappa - \bar{\kappa}) \right],$$ (17)

$$\Pi = \frac{1}{1 - \rho} \left[ -\rho q + \bar{q} - s \left[ (1 + 2 \rho) \kappa - (2 + \rho) \bar{\kappa} + 2(1 - \rho) \right] \right].$$ (18)

The particle ratios involving abundant strange and non-strange hadrons are

$$\frac{N}{K} = \frac{1}{1 - \rho} \left[ \frac{q_v}{3s\kappa} + \frac{1}{r - 1} \right],$$ (19)

$$\frac{\Pi}{K} = \frac{1}{(1 - \rho)\bar{\kappa}} \left[ (1 - \rho) \frac{q}{s} - \frac{q_v}{s} - (1 + 2 \rho) \kappa + (2 + \rho) \bar{\kappa} - 2(1 - \rho) \right].$$ (20)

Where $q_v$ denotes the number of valence quarks, i.e., $q_v = q - \bar{q}$. The RHS can be determined from parton distributions, assuming that the parameters $\kappa$, $\bar{\kappa}$ and $\rho$ are known from experiments. The LHS can be related approximately to $p/K^+$ and $\pi^+/K^+$:

$$\frac{p}{K^+} = \frac{N}{K}, \quad \frac{\pi^+}{K^+} = \frac{2 \Pi}{3 K}.$$ (21)

The experimental values of these ratios at SPS are

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by computing the ratios of the corresponding quark dis-

$\frac{p^+}{K^+} = 1.0, \quad \frac{\pi^+}{K^+} = 4.76. \quad (22)$

The values of $\rho$ is $^{[21]}$

$\rho = 0.07. \quad (23)$

With these experimental inputs there should be no difficulty in satisfying Eq. (19) and (20) by varying the quark numbers.

However, in our approach the quark numbers must fit into our scheme of quark enhancement via gluon conversion. Moreover, there is the issue of what precisely is the central region where the experimental numbers of the hadron ratios are measured. Clearly, the valence to sea quark ratio depends on the region of small $x$ considered. The experiments do not have a common and unique definition of the central rapidity region. For our analysis in the following we define the central rapidity region to correspond to a value $x_0$, which depends on $\sqrt{s}$. In fact, when we make prediction later on for RHIC energies we shall use the relation

$x_0 = (s_0/s)^{1/2}. \quad (24)$

For now at SPS we use $x_0$ as an adjustable parameter. The important physics input is that for $x \leq x_0$, which is what Feynman called the “wee” $x$ region, we assume that all partons distributions are constant (consistent with the notion of saturation) so that the quark number ratios in the wee region, whatever the flavor, can be determined by computing the ratios of the corresponding quark distributions at $x = x_0$.

In the calculation of the quark distributions after gluon conversion, we shall do it in a simplified way for the central region, different from how it has been treated in the fragmentation region $^{[13]}$. The reasons are because firstly we need not distinguish $u$ and $d$ types quarks in the central region and secondly we need not track the $x$ dependences. It is important to first refer all quark numbers to those given by CTEQ at $Q^2 = 1$ GeV and $x = x_0$. That is our baseline, from which we discuss enhancement in the following. Now, CTEQ gives distributions, not number of partons. For example, $u(x_0)$ is the probability of having a $u$ quark at $x = x_0$. In the following we use $q_0$ to denote the number of $u$ and $d$ quarks in $x \leq x_0$, and equate it to $u(x_0) + d(x_0)$, multiplied by a factor that is proportional to the relevant phase space volume. Such a factor will cancel later upon taking the ratio of quark numbers, so it will not appear explicitly in any of the expressions for parton numbers below. Similarly, we use $s_0$ and $g_0$ to denote the number of $s$ quark and gluons in the region $x \leq x_0$, but identified with $s(x_0)$ and $g(x_0)$, respectively, of the CTEQ distributions.

III. GLUON CONVERSION

Before gluon conversion we have $q_v$ valence quarks, $2q_0$ non-strange sea quarks, and $s_0 + \bar{s}_0$ strange quarks. In the case of hadronic collisions, it has been shown that the inclusive distributions of produced hadrons can be reproduced without any free parameters, if the gluons are completely converted to non-strange sea quarks before hadronization through recombination $^{[4]}$. Now, in the case of $AA$ collisions we must consider the conversion of gluons to strange quarks in addition to the non-strange quarks because of Pauli blocking in the light sector. We use $\gamma$ to denote the fraction in the strange sector. That is, the number of converted strange and non-strange quarks, labeled with subscript $c$, are

$s_c = \gamma g_0, \quad q_c = (1 - \gamma) g_0, \quad (25)$

with the corresponding antiquarks $\bar{s}_c$ and $\bar{q}_c$ being equal in number, respectively. Thus after conversion we have

$q = q_v + q_0 + q_c, \quad (26)$

$s = s_0 + s_c. \quad (27)$

The quark and gluon distributions at $x_0$ can be either obtained from the graphs posted by CTEQ4LQ $^{[4]}$, or determined numerically from the analytic formulas given in Ref. $^{[22]}$. We use the latter to fix $q_v, q_0, s_0$ and $g_0$ for every $x_0$, while $q_c$ and $s_c$ depend on $\gamma$. Hence, we have two free parameters, $x_0$ and $\gamma$, to fit the data through the use of Eqs. $^{[19]-[23]}$.

From Eq. (19) one gets $q_v/s = 3.86$. Using that in (20) yields $\bar{q}/s = 7.53$, whereupon one obtains

$\frac{\bar{q}}{q} = \frac{1}{1 + q_v/\bar{q}} = 0.66. \quad (28)$

From (23) and (27) we have

$s = s_0 + \gamma g_0 = q_v / 3.86, \quad (29)$

$q_0 + (1 - \gamma) g_0 = 7.53 s; \quad (30)$

together they give

$q_0 + s_0 + g_0 = 2.21 q_v. \quad (31)$

This is an equation that depends on CTEQ distributions only, so we can solve for the value of $x_0$. The result then determines also the values of $q_v, s_0$ and $g_0$, which, when used in Eq. (29), fix $\gamma$. The process yields

$x_0 = 0.135, \quad (32)$

$\gamma = 0.08. \quad (33)$

The value of $x_0$ is reasonable, but the value of $\gamma$ seems surprisingly low, since 8% conversion from the gluons seems insufficient to justify the notion of SE.

IV. STRANGENESS ENHANCEMENT

To appreciate the value of $\gamma$ found above, let us examine the quark distributions at $x_0$ before gluon conversion. Our solution of Eq. (31) gives

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The document is a scientific text discussing quark and gluon distributions, quark enhancement, and strangeness enhancement in hadronic collisions. It references experiments and calculations to determine fractions of quarks in different regions and uses the CTEQ distributions to model these fractions. The text is focused on the process of converting gluons to strange quarks and analyzing the impact of this conversion on quark distributions at different scales.
\[ x_0 q_e = 0.462, \quad x_0 \bar{q}_e = 0.118, \]
\[ x_0 s_0 = 0.052, \quad x_0 g_0 = 0.85. \quad (34) \]

Thus from Eq. (24) we have \( x_0 s_c = 0.068 \). Comparing \( s_c \) with \( s_0 \), we see that the strangeness enhancement factor \( E_s \) at the quark level is
\[ E_s = \frac{s}{s_0} = 1 + \frac{s_c}{s_0} = 2.3. \quad (35) \]

This indicates quite an appreciable amount of increase of the strange quarks, qualitatively consistent with the hyperon enhancement. The point is that there are so many gluons that an 8\% conversion significantly enhances the strangeness content. The remaining 92\% conversion to \( q_c \) should be compared to 100\% conversion in the case of hadronic collisions [23]. Let us call the light quark population in the sea after 100\% conversion \( \bar{q}_1 \), i.e.,
\[ \bar{q}_1 = \bar{q}_0 + g_0. \quad (36) \]

Then the change in the sea from \( pp \) to \( AA \) collisions can be characterized by the ratio \( B \):
\[ B = \frac{\bar{q}}{\bar{q}_1} = \frac{\bar{q}_0 + (1 - \gamma) g_0}{\bar{q}_0 + g_0} = 0.94. \quad (37) \]

This may be regarded as a numerical factor quantifying Pauli blocking in the light quark sector. Note that it is less than one by only a small amount, but enough to boost \( E_s \) from one by more than a factor of two.

The extension of this consideration to RHIC energies is straightforward. We first use Eq. (24) to determine \( s_0 \) from the values of \( x_0 \) at SPS. Setting \( \sqrt{s} = 17 \) GeV, we obtain \( \sqrt{s_0} = 2.3 \) GeV. Now, holding \( s_0 \) fixed, we have the corresponding \( x_0 \) value (call it \( x'_0 \)) at \( \sqrt{s} = 130 \) GeV to be
\[ x'_0 = 0.0177. \quad (38) \]

The values of \( q_c, \bar{q}_0, s_0 \) and \( g_0 \) at \( x'_0 \) are (from CTEQ)
\[ x'_0 q'_e = 0.149, \quad x'_0 \bar{q}'_e = 0.184, \]
\[ x'_0 s'_0 = 0.089, \quad x'_0 g'_0 = 1.229. \quad (39) \]

Note that \( q'_e \) is much smaller than \( q_e \), as expected, so that \( q'_0/\bar{q}_0 = 0.55 \), even before gluon conversion. Thus we expect antiparticle/particle ratios to be much closer to one.

As before, we need the experimental inputs at RHIC. From Refs. [23, 24] we have at \( \sqrt{s} = 130 \) GeV
\[ r = 1.136, \quad R = 0.77, \quad \rho = 0.64. \quad (40) \]

So we get from Eq. (13)
\[ \kappa = 0.628, \quad \bar{\kappa} = 0.713. \quad (41) \]

Assuming that \( \gamma \) remains constant, we now can calculate the quark ratios
\[ \frac{q'_e}{s'} = 0.8, \quad \frac{\bar{q}'_e}{s'} = 7.06, \quad (42) \]

which, when used in (19) and (20), enable us to calculate the hadron ratios. As a consequence, we obtain
\[ \frac{p}{K} = 0.71, \quad \frac{K^+}{\pi^+} = 0.18. \quad (43) \]

The latter, compared to the value, 0.21, at SPS, is a 14\% decrease and agrees well with the data at RHIC [24], which shows \( K^+/\pi^+ = 0.176 \pm 0.004 \). For the former we find indirect confirmation from the following ratios reported by STAR [23, 27] for \( \sqrt{s} = 130 \) GeV:
\[ \frac{p}{K^-} = 0.65 \pm 0.07, \quad \frac{p}{\pi^-} = 0.08 \pm 0.01, \]
\[ \frac{K^-}{\pi^-} = 0.149 \pm 0.02, \quad \frac{K^-}{K^+} = 0.88 \pm 0.05. \]

These numbers can be used to imply
\[ \frac{p}{K^-} = 0.73 \pm 0.1, \quad (44) \]

which agrees well with the calculated number in (43). These results give support to our assumption that \( \gamma \) is constant when the energy is increased and to our procedure of treating the energy dependence.

The SE factor becomes at RHIC \( (\sqrt{s} = 130 \) GeV\)
\[ E_s = \frac{s'}{s_0} = 1 + \frac{s'_c}{s_0} = 2.1. \quad (45) \]

Although the gluon density increases by 45\% as \( x_0 \) decreases to \( x'_0 \), the s quark density in the quiescent sea increases by even more, so the net enhancement factor \( E_s \) decreases slightly. This small decrease is in agreement with that of the statistical model [24], although the physics is totally different. The Pauli blocking factor becomes
\[ B = 0.93, \quad (46) \]

which is essentially unchanged from Eq. (27).

V. CONCLUSION

Since we have left out the multi-strange hyperons from our consideration, we cannot expect the numbers calculated to be accurate. Moreover, the necessity to use such experimental inputs as \( r, R, \rho \) to determine \( \kappa \) and \( \gamma \) renders the approach highly phenomenological, far from first principles. However, the basic attributes of this line of study are to use the parton model (and the distributions of CTEQ) as the basis for the investigation of particle ratios in nuclear collisions at the quark level, and to use simple linear relations, Eqs. (3)-(4), based on quark counting as the only constraints among the strange and
non-strange hadrons. We have found consistency within this simple approach, and can successfully describe the energy dependence. We have not assumed thermal equilibrium, nor relied on the statistical model. We have also deliberately avoided treating mesons and baryons as products of quark densities, as have been attempted in Refs. [5,6], since they lead to either $s \neq \bar{s}$ or undetermined constants.

As we have stated at the outset, it is not our aim to predict particle ratios. We have used the experimental values of the ratios to lead us to the determination of the SE factor, $E_s$, and the Pauli blocking factor, $B$, defined at the quark level. In so doing we have gained some insight into how the enhancement mechanism works through the process of gluon conversion. We have further learned that a slight suppression of the conversion into the non-strange sector gives rise to a substantial increase in the strange sector. Such a small change from hadronic to nuclear collisions makes strangeness enhancement an unreliable signature for the formation of quark-gluon plasma.

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