Pair-breaking due to orbital magnetism in iron-based superconductors

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We consider superconductivity in the presence of impurities in a two-band model suited for the description of iron-based superconductors. We analyze the effect of interband scattering processes on superconductivity, allowing for orbital, i.e., non-spin-magnetic time-reversal-symmetry-breaking impurities. Pair-breaking in such systems is described by a nontrivial phase in an interband-scattering matrix element. We find that the transition temperature of conventional superconductors can be suppressed due to interband scattering, whereas unconventional superconductors may be unaffected. As an example, we consider impurities associated with orbital density waves that are of interest for iron-based superconductors.

I. INTRODUCTION

Conventional superconductivity is astonishingly robust against impurity scattering. The transition temperature $T_c$ remains approximately constant in the presence of nonmagnetic impurities as follows from the Anderson theorem. Magnetic impurities, on the other hand, are pair-breaking and suppress the transition temperature that vanishes at a critical value of the scattering rate. Unconventional superconductors, in contrast, are already sensitive to nonmagnetic impurities, and again, superconductivity vanishes at a critical scattering rate. The suppression of $T_c$ with increasing scattering rate due to nonmagnetic impurities is therefore considered as a signature of unconventional superconductivity.

In iron-based superconductors, there is strong evidence supporting an $s^+^-$ scenario for superconductivity in these materials, where the pairing gap changes sign between different bands without breaking a point group symmetry. However, the pairing state is still under debate and in particular the relatively weak suppression of the superconducting transition temperature with increasing concentration of nonmagnetic impurities has been used as an argument in favor of a conventional pairing state. One explanation for this behavior is that intraband and interband scattering are not equally strong in iron pnictides, and transport properties are mainly determined by intraband scattering effects whereas the suppression of $T_c$ is due to interband scattering. Moreover, in this paper, we show that the discrimination of $s^+$ and $s^−$ pairing state based on their response to the presence of apparently nonmagnetic impurities is not always possible.

Iron pnictides are multiband superconductors in which electrons from different orbitals contribute to superconductivity and/or magnetic order. Furthermore, competing states of order are a characteristic of iron pnictides. Model calculations show that orbital density waves are expected to compete with antiferromagnetism and superconductivity in these materials. Thus, the detailed impact of orbital magnetism on pairing in these multiband systems is an interesting open topic.

In this paper, we consider a two-band model for iron-based superconductors with impurities causing intraband and interband scattering processes, and investigate how the interplay between pairing and orbital magnetism takes place. In particular, we find that impurities associated with orbital magnetism can lead to the suppression of $T_c$ in conventional superconductors. In addition, we will see that the transition temperature in unconventional superconductors may remain unaffected, i.e., there exists an Anderson theorem for the $s^+^-$ pairing state which is protected against time-reversal-symmetry-breaking interband scattering. As we will show, this effect can be due to impurities that nucleate local orbital magnetic states. Therefore, it is important for our theory that we allow for spatially extended impurity potentials.

II. DISORDERED TWO-BAND MODEL

We consider a two-band superconductor with impurities, described by the Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}} + \hat{H}_{\text{dis}}$. The noninteracting part is given by

$$\hat{H}_0 = \sum_{\mathbf{k}, \sigma} \sum_{\alpha} \xi_{\alpha, \mathbf{k}} \hat{\psi}^\dagger_{\alpha, \mathbf{k}, \sigma} \hat{\psi}_{\alpha, \mathbf{k}, \sigma},$$

where $\alpha$ labels the two bands, $\sigma$ denotes spin, and $\xi_{\alpha, \mathbf{k}} = \varepsilon_{\alpha, \mathbf{k}} - \mu$ is the dispersion of band $\alpha$, measured from the chemical potential. We assume that the quasiparticles in band 1 have small momenta near the center of the Brillouin zone (Γ point), while the momenta of quasiparticles in band 2 are close to $2\mathbf{Q}$, where $2\mathbf{Q}$ is a reciprocal primitive vector, as it is suitable for iron pnictide superconductors. The concrete form of the dispersion relation is not important for our calculations. For simplicity, we assume the density of states near the Fermi level to have the same value $\rho_F$ in both bands. The generalization to different densities of states in the two bands is straightforward.
Furthermore, we consider superconductivity (SC) due to interband pairing, described in a BCS-like model,

\begin{equation}
\hat{H}_{\text{int}} = \sum_{k,k'} \sum_{\alpha,\alpha'} V^\alpha_{\alpha',k} \hat{\psi}_{\alpha,k}^\dagger \hat{\psi}_{\alpha',-k',\dagger} \hat{\psi}_{\alpha,-k} \hat{\psi}_{\alpha',k'},
\end{equation}

where \( \alpha^{\prime} \) labels the band other than \( \alpha \).

The most generic Hamiltonian of disorder in such a system reads

\begin{equation}
\hat{H}_{\text{dis}} = \sum_{s,s'} \sum_{\alpha,\alpha'} \hat{\psi}_{\alpha,s}^\dagger (R_s) W_{\alpha,\alpha'}(R_s, R_{s'}) \hat{\psi}_{\alpha',s'}(R_{s'}),
\end{equation}

where the indices \( s \) and \( s' \) label lattice sites \( R_s \) and \( R_{s'} \). Here, the \( \hat{\psi}_{\alpha}(R_s) \) and \( W_{\alpha,\alpha'}(R_s, R_{s'}) \) are vectors and matrices in spin space, respectively, i.e., \( \hat{\psi}_{\alpha}(R_s) = (\hat{\psi}_{\alpha,\uparrow}(R_s), \hat{\psi}_{\alpha,\downarrow}(R_s))^T \), where by \( \hat{\psi}_{\alpha,\sigma}(R) \) we denote field operators in position space which have to be understood as convolution with momenta in band \( \alpha \) only.

This disorder is typically represented by identical impurities with random locations \( R_i \),

\begin{equation}
\hat{H}_{\text{dis}} = \sum_{i=1}^N \hat{U}_i R_i,
\end{equation}

\begin{equation}
\hat{U}_i = \sum_{\alpha,\beta} \sum_{s,s'} \hat{\psi}_{\alpha,s}^\dagger (R_s + R_i) J_{ss'}^{\alpha\beta} \hat{\psi}_{\beta,s'}(R_{s'} + R_i).
\end{equation}

These two formulations of the impurity Hamiltonian, Eqs. (3) and (4), are connected by

\begin{equation}
W_{\alpha,\alpha'}(R_s, R_{s'}) = \sum_i J^{\alpha\beta}_{ss'} \delta_{\alpha,\alpha'}.
\end{equation}

The matrix element \( J^{\alpha\beta}_{ss'} \) can account for intraband \( \alpha = \beta \) as well as interband \( \alpha \neq \beta \) scattering processes. In general, \( J^{\alpha\beta}_{ss'} \) are the matrix elements of a non-diagonal matrix in position space, allowing us to describe spatially extended scattering centers, which is essential, e.g., to account for orbital-magnetic impurities.

At the same time, in what follows, we assume for simplicity that the disorder is short-correlated on the scale \( k_F^{-1} \), where \( k_F \) is the largest of the Fermi wavevectors in the two bands.

### III. Symmetry Considerations

Before we explicitly calculate the effect of impurities on the SC transition temperature of \( s^{++} \) and \( s^- \) superconductors, we will provide an extension of Anderson’s theorem[34] for two-band superconductors. Specifically, it will be demonstrated that the \( s^{++} \) pairing state is robust against time-reversal-symmetric (TRS) scattering while for the \( s^- \) pairing state, the gap is unchanged by time-reversal-antisymmetric (TRA) interband disorder.

We consider the two-band \( s \)-wave superconductor as defined in Eqs. (1) and (2). The corresponding mean-field Hamiltonian is given by

\begin{equation}
\hat{H}_\text{MF} = \sum_{k,\alpha} \hat{\psi}_{\alpha,k}^\dagger \tilde{\xi}_{\alpha,k} \hat{\psi}_{\alpha,k}
\end{equation}

\begin{equation}
+ \sum_{k,\alpha} \frac{\Delta_\alpha}{2} \left[ \hat{\psi}_{\alpha,k}^\dagger \hat{\sigma}_2 (\hat{\psi}_{\alpha,-k}^T + \hat{\psi}_{\alpha,-k}^\dagger \hat{\sigma}_2) \right],
\end{equation}

where \( \Delta_\alpha \in \mathbb{R} \) denotes the pairing in band \( \alpha \) which is taken to be momentum independent (s-wave), as in Eq. (2). We introduce Nambu spinors \( \hat{\Psi}_{\alpha}(k) = (\hat{\psi}_{\alpha,k}, i\hat{\sigma}_2 (\hat{\psi}_{\alpha,-k}^\dagger))^T \) and \( \hat{\Psi}_{\alpha}^\dagger(k) = (\hat{\psi}_{\alpha,k}^\dagger, \hat{\psi}_{\alpha,-k}^\dagger) \) to write the mean-field Hamiltonian in the quadratic form

\begin{equation}
\hat{H}_\text{MF} = \frac{1}{2} \sum_{k,\alpha} \hat{\Psi}_{\alpha}^\dagger(k) \left[ \Delta_{\alpha,\beta} \tilde{\xi}_{\alpha,k} \hat{\Psi}_{\beta}(k) \right].
\end{equation}

It’s convenient to consider a given disorder realization[35], as described by the general quadratic term \( \hat{H}_\text{dis} \), in momentum space, where it reads

\begin{equation}
\hat{H}_\text{dis} = \frac{1}{2} \sum_{k,\alpha,k',\alpha'} \hat{\Psi}_{\alpha,k}^\dagger(k) \begin{pmatrix} W_{\alpha,\alpha'}(k, k') & 0 \\ 0 & -i\sigma_y W_{\alpha',\alpha}(k', -k) \end{pmatrix} \hat{\Psi}_{\alpha'}(k').
\end{equation}

The only constraint on \( W_{\alpha,\alpha'}(k, k') \) is \( W_{\alpha',\alpha}(k', k) = W_{\alpha,\alpha'}(k, k') \) due to Hermiticity. For the following analysis of time-reversal symmetry, it is convenient to split \( W_{\alpha,\alpha'}(k, k') \) according to

\begin{equation}
W_{\alpha,\alpha'}(k, k') = W_{\alpha,\alpha'}^+(k, k') + W_{\alpha,\alpha'}^-(k, k').
\end{equation}

into parts that are symmetric and antisymmetric under time reversal,

\begin{equation}
W_{\alpha,\alpha'}(k, k') = \frac{1}{2} \left[ W_{\alpha,\alpha'}(k, k') \pm \hat{T} W_{\alpha,\alpha'}(-k, -k') \hat{T}^{-1} \right],
\end{equation}

where \( \hat{T} = \sigma_y \sigma_3 \).
where $\hat{T} = i\sigma_3 \hat{K}$ denotes the time-reversal operator for spin-$\frac{1}{2}$, with $\hat{K}$ representing complex conjugation. Introducing Pauli matrices $\hat{\tau}_i$ acting in band space, and defining $\Delta_{\pm} = \frac{1}{\sqrt{2}}(\Delta_1 \pm \Delta_2)$, the Hamiltonian can be written compactly as $\hat{H}_{MF} + \hat{H}_{\text{dis}} = \frac{1}{2} \sum_{k,k'} \sum_{\alpha\alpha'} \Psi_\alpha(k) h_{\alpha\alpha'}(k,k') \Psi_{\alpha'}(k')$ with
\[
\hat{h} = \left( \frac{\hat{\xi} + \hat{W}^+ + \hat{W}^-}{\sqrt{2}} (\Delta_+ \hat{\tau}_0 + \Delta_- \hat{\tau}_3) - \left( \frac{\hat{\xi} + \hat{W}^+ + \epsilon \hat{1}}{\sqrt{2}} \right) \right), \tag{11}
\]
where $\hat{\xi}$ is the diagonal matrix of band energies $\xi_\alpha(k)$.

The spectrum of $\hat{h}$ is found by solving $\det(\hat{h} - \epsilon \hat{1}) = 0$ for $\epsilon$, where we can use that
\[
\det \begin{pmatrix} A & B \\ C & D \end{pmatrix} = \det(AD - CB) \tag{12}
\]
holds for arbitrary square matrices $A, B, C,$ and $D$, if $[A,C] = 0$ is satisfied.

### A. Nonmagnetic disorder

We start by considering TRS disorder, i.e., we assume $\hat{W}_- = 0$ but $\hat{W}_+ \neq 0$ in Eq. (11). From the Anderson theorem, we expect the $s^{++}$ state to be robust against such nonmagnetic impurities. The spectrum of $\hat{h}$ can straightforwardly be found from the condition
\[
\det \left( \frac{\hat{\xi} + \hat{W}^+ + \epsilon \hat{1}}{\sqrt{2}} (\Delta_+ \hat{\tau}_0 + \Delta_- \hat{\tau}_3) - \left( \frac{\hat{\xi} + \hat{W}^+ + \epsilon \hat{1}}{\sqrt{2}} \right) \right) = 0 , \tag{13}
\]
where for a pure $s^{++}$ pairing state, $\Delta_- = 0$ and $\Delta_+ \neq 0$ holds in addition. Then the commutator
\[
\left[ \frac{\hat{\xi} + \hat{W}^+ + \epsilon \hat{1}}{\sqrt{2}} (\Delta_+ \hat{\tau}_0 + \Delta_- \hat{\tau}_3) - \left( \frac{\hat{\xi} + \hat{W}^+ + \epsilon \hat{1}}{\sqrt{2}} \right) , \right] \frac{1}{\sqrt{2}} (\Delta_+ \hat{\tau}_0 + \Delta_- \hat{\tau}_3) = \frac{1}{\sqrt{2}} \Delta_- \left[ \hat{W}^+, \hat{\tau}_3 \right] \tag{14}
\]
vanishes, and we can use Eq. (12) for the evaluation of the determinant. We obtain the eigenvalues of $\hat{h}$ in the case of TRS disorder in an $s^{++}$ superconductor,
\[
\pm \sqrt{(\xi_i + W_i)^2 + \Delta_+^2/2} , \tag{15}
\]
where $\xi_i + W_i$ denote the different real eigenvalues of the Hermitian matrix $\hat{\xi} + \hat{W}^+$. Consequently, the gap of the disordered system is larger than or equal to $|\Delta_+|/\sqrt{2}$, the gap of the clean system. We have hereby shown that the gap will be unaffected by the presence of the disorder potential which indicates the stability of the $s^{++}$ superconducting state against TRS impurity scattering, and thus obtained the Anderson theorem for $s^{++}$ superconductors.

We note that the commutator (14) also vanishes for $\Delta_- \neq 0$ if the disorder potential is purely band diagonal, i.e., no interband scattering processes occur. Therefore, from similar reasoning, we obtain that the $s^{+-}$ pairing state is protected against nonmagnetic intraband scattering.

### B. Anderson theorem for $s^{+-}$ superconductors

The same approach can be used to motivate an analog of the Anderson theorem for the $s^{+-}$ pairing state. We rewrite the determinant by performing a unimodular transformation in bandspace,
\[
\det \left( \hat{h} - \epsilon \hat{1} \right) = \det \left( \begin{pmatrix} \tau_0 & 0 \\ 0 & \tau_3 \end{pmatrix} \left( \hat{h} - \epsilon \hat{1} \right) \begin{pmatrix} \tau_0 & 0 \\ 0 & \tau_3 \end{pmatrix} \right) . \tag{16}
\]
For the specific microscopic scattering mechanism to be discussed below, that is, for a purely band diagonal TRS component and a purely band off-diagonal TRA component of the disorder potential, it follows
\[
\hat{\tau}_3 \hat{W}^+ \hat{\tau}_3 = \hat{W}^+, \quad \hat{\tau}_3 \hat{W}^- \hat{\tau}_3 = -\hat{W}^- , \tag{17}
\]
and, hence, we have to solve
\[
\det \left( \frac{\hat{\xi} + \hat{W} - \epsilon \hat{1}}{\sqrt{2}} (\Delta_+ \hat{\tau}_3 + \Delta_- \hat{\tau}_0) - \left( \frac{\hat{\xi} + \hat{W} - \epsilon \hat{1}}{\sqrt{2}} \right) \right) = 0 . \tag{18}
\]
From the analysis of section IIIA, we know that the relevant quantity for the sensitivity to disorder is the commutator
\[
\left[ \frac{\hat{\xi} + \hat{W} - \epsilon \hat{1}}{\sqrt{2}} (\Delta_+ \hat{\tau}_3 + \Delta_- \hat{\tau}_0) - \left( \frac{\hat{\xi} + \hat{W} - \epsilon \hat{1}}{\sqrt{2}} \right) , \right] \frac{1}{\sqrt{2}} (\Delta_+ \hat{\tau}_0 + \Delta_- \hat{\tau}_3) = \frac{1}{\sqrt{2}} \Delta_+ \left[ \hat{W}, \hat{\tau}_3 \right] . \tag{19}
\]
It vanishes in case of $s^{+-}$ SC where $\Delta_+ = 0$ and $\Delta_- \neq 0$, but assumes finite values for the $s^{+-}$ superconductor when TRA interband scattering is present. This is the algebraic reason for why the $s^{+-}$ superconductor is in general prone to TRA scattering, while the $s^{+-}$ state is stable against TRA interband disorder and TRS intraband disorder. Let us finally emphasize that this conclusion holds irrespective of the form of the bands (as long as $\xi_{\alpha,-k} = \xi_{\alpha,k}$ holds) and the detailed momentum dependence of $W_{\alpha,\alpha'}(k,k')$. In particular, the disorder potential does not have to be momentum independent within each band for the $s^{+-}$ Anderson theorem to hold. Furthermore, it does not rely on the disorder potential breaking time-reversal symmetry due to spin or orbital magnetism. It is only important that $\hat{\tau}_3 \hat{W}^{\pm} \hat{\tau}_3 = \pm \hat{W}^{\pm}$ holds. Here, the insensitivity to spin results from the investigation of singlet pairing.

### IV. DISORDER AVERAGING

In the following sections, we do not consider spin-magnetic impurities. To evaluate physical observables, we use the disorder-averaging diagrammatic technique [13]. A basic element of this technique is the impurity line
Here \( U_{\alpha\beta}^{kk'} \) is the matrix element of the perturbation due to a single impurity at site \( R = 0 \), \( (...)_R = \Omega^{-1} \int dR \ldots \) is the averaging with respect to the position \( R_i \) of impurity \( i \), \( n_{\text{imp}} = N/\Omega \) denotes the impurity concentration, and \( \Omega \) is the \( d \)-dimensional volume. It holds that \( K = 0 \) if all or two of the momenta \( k_1, k_2, k'_1, k'_2 \) belong to the same band, and \( K = Q \) if one momentum belongs to one band, and three other to the other band. [In Eq. (20) we have taken into account that \( 2Q \) is a reciprocal vector].

The impurity line, Eq. (20), describes the elastic scattering of two momentum states \( k_1 \) and \( k_2 \) into another two momentum states \( k'_1 \) and \( k'_2 \). The scattering can occur within the same band or involve interband processes, as shown in Fig. 1. The \( \delta \)-function in Eq. (20) represents the conservation of quasimomentum, and the quantity \( \Gamma_{\alpha\beta\gamma\delta}(k_1, k'_1, k_2, k'_2) \), defined in Eq. (21), is hereinafter referred to as the rate of elastic scattering between the pair of momentum states \( k_1, k_2 \) and \( k'_1, k'_2 \), respectively.

\[
\langle k_1, \alpha | \hat{U}_R | k_2, \beta \rangle \langle k_2, \gamma | \hat{U}_R | k'_2, \delta \rangle = (2\pi)^d \Gamma_{\alpha\beta\gamma\delta}(k_1, k'_1, k_2, k'_2) \delta(k_1 + k_2 - k'_1 - k'_2 + K),
\]

where \( \Gamma_{\alpha\beta\gamma\delta}(k_1, k'_1, k_2, k'_2) = n_{\text{imp}} U_{\alpha\beta}^{k_k'} U_{\gamma\delta}^{k_k'} \).

**FIG. 1.** Scattering processes that can occur in a two-band system. (a) Intraband scattering process in band \( \alpha \). (b)–(e) Interband scattering processes.

The **intraband scattering** process within band \( \alpha \) is depicted in Fig. 1(a), and we abbreviate the corresponding scattering rate by \( \Gamma_{\alpha} \approx \Gamma_{\alpha\alpha\alpha\alpha} \). For sufficiently short-correlated disorder considered in this paper, the rates

\[ \Gamma_{\alpha} \approx n_{\text{imp}} |U_{\alpha\alpha}^{kk'}|^2 \]

are independent of the momenta \( k_1, k_2, k'_1, \) and \( k'_2 \). Such intraband scattering processes are pair-breaking neither for conventional nor for unconventional superconducting states. We emphasize that in general \( \Gamma_1 \neq \Gamma_2 \), because the momenta states in the two bands may have different structure, e.g., in terms of sublattices or atomic orbital degrees of freedom, and thus may be scattered differently by impurities.

Processes involving **interband scattering** are shown in Figs. 1(b)–(e). The process in Fig. 1(b) requires a momentum transfer of \( K = Q \) which is not a reciprocal lattice vector and thus this scattering process is forbidden due to the conservation of quasimomentum. The process in Fig. 1(c) effects neither the quasiparticle self-energy part nor the superconductive properties but can be important, e.g., for the magnetic properties of the material. The process depicted in Fig. 1(d) affects the quasiparticle self-energy part, as we discuss in section IV A. In what follows, we assume that the respective rate \( \Gamma_{\alpha\alpha\alpha\alpha} \) is independent of the momenta \( k_1, k_2, k'_1 \), and \( k'_2 \) and, generally speaking, is different from \( \Gamma_1 \) and \( \Gamma_2 \). Such assumption is rather generic and may be justified, e.g., if the disorder (perturbation \( \hat{U}_R \)) has components varying both on length scales significantly smaller than \( 1/Q \) and on scales \( \lambda \) : \( 1/Q \ll \lambda \ll 1/k_F \). The former will contribute to the intraband scattering rates as well as to the interband scattering rates whereas the latter contributes significantly only to the interband scattering rates. The rate of the process in Fig. 1(d) is

\[ \Gamma_{\alpha\alpha\alpha\alpha} \approx n_{\text{imp}} |U_{\alpha\alpha}^{kk'}|^2 \]

We note that the rate given in (23) is real, \( \Gamma_{\alpha\alpha\alpha\alpha} \in \mathbb{R} \). On the contrary, the scattering process shown in Fig. 1(e) in general comes with a phase

\[ \Gamma_{\alpha\alpha\alpha\alpha} \approx n_{\text{imp}} |U_{\alpha\alpha}^{kk'}|^2 e^{i\phi} \]

where \( \phi \neq 0 \) (modulo \( 2\pi \)) if \( \text{Im} U_{\alpha\alpha}^{kk'} \neq 0 \). This process describes the scattering of a pair of momentum states in one band into a pair of momentum states in the other band. Since \( \Gamma_{\alpha\alpha\alpha\alpha} = \Gamma_{\alpha\alpha\alpha\alpha} \), we introduce the notation

\[ \Gamma_{12} \equiv \Gamma_{\alpha\alpha\alpha\alpha} = \Gamma_{\alpha\alpha\alpha\alpha} e^{-i\phi} \]

In principle, the phase \( \phi \) is defined relative to a similar phase of the BCS coupling matrix element \( V_{kk'}^{\alpha\alpha} \), that is contained in Eq. (2), which couples pairs of momentum states in different bands. Thus, the interplay of the scattering process in Fig. 1(e) and the superconductive coupling may affect the superconductive properties of the system. The fact that \( \phi \) must be understood as a relative phase becomes more evident in our discussion in section V.
A. Self energy and Cooperons

For the remainder of this paper, we assume the scattering to be sufficiently weak such that the mean free path $l = v_F \tau$ satisfies $k_F l \gg 1$. This allows us to neglect single-particle interference effects, i.e., diagrams with crossed impurity lines, since they are suppressed by a factor $1/k_F l$.

Because the process in Fig. 1(b) is forbidden by quasi-momentum conservation, only the processes in Fig. 1(a) and Fig. 1(d) contribute to the electron self-energy part in the Born approximation,

$$\Sigma_{\alpha} = \alpha + \frac{1}{\nu_n - \xi_{\alpha} k + \frac{i}{\tau} \text{sgn} \nu_n},$$

and therefore, in the disorder-averaged electron propagator,

$$G_{\alpha, k}(\nu_n) = \frac{1}{\nu_n - \xi_{\alpha} k + \frac{i}{\tau} \text{sgn} \nu_n},$$

$$\tau = \left[2\pi \rho_F (\Gamma_\alpha + \Gamma_{12})\right]^{-1},$$

the full scattering rate that determines the elastic scattering time $\tau$ is a sum of the intraband $\Gamma_\alpha$ and the interband $\Gamma_{12}$ rates.

Further corrections due to impurity scattering can be conveniently summarized into vertex corrections, as they appear in the diagrams contributing to the SC transition temperature shown in Fig. 2. In the presence of intraband as well as interband scattering processes, most contributions can be accounted for by a generalized form $C_\alpha$ of the Cooperon ladder of the impurity line. This generalized Cooperon is indicated in dark gray in the diagrams in Fig. 2 and accounts for all combinations of scattering processes starting and ending in band $\alpha$, including (pairwise) interband scattering processes and intermediate scattering processes in band $\bar{\alpha}$. To calculate this generalized Cooperon, a single rung of the Cooperon ladder in band $\alpha$ as known from one-band models has to be modified as

$$C_\alpha(\nu_n) = \frac{(\pi \rho_F \Gamma_{12} + |\nu_n|) (\pi \rho_F (\Gamma_\alpha + \Gamma_{12}) + |\nu_n|)}{|\nu_n| (2\pi \rho_F \Gamma_{12} + |\nu_n|)}$$

at vertices associated with the order parameter $\Delta_\alpha$. We note that the vertex corrections $C_\alpha(\nu_n)$ associated with $\Delta_\alpha$ only depend on the intraband scattering rate in the respective band $\alpha$, and on the interband scattering rate $\Gamma_{12}$. The vertex corrections are independent of the other band to which electrons are scattered in intermediate processes and within which they can also be scattered.

In addition, to avoid double-counting in the interband diagrams $d_{12}$ and $d_{21}$, we need the usual single-band Cooperon ladder in band $\alpha$, $C_{\alpha}^0$, which is indicated in light gray in the diagrams in Fig. 2 and given by

$$C_{\alpha}^0(\nu_n) = \frac{|\nu_n| + \pi \rho_F (\Gamma_\alpha + \Gamma_{12})}{|\nu_n| + \pi \rho_F \Gamma_{12}}.$$
the most prominent example for superconductors being the superconducting transition temperature \( T_c \), as established in the following section \( \square \).

V. TRANSITION TEMPERATURE IN THE PRESENCE OF IMPURITY SCATTERING

The action associated with the interacting Hamiltonian given in Eq. \( \mathbb{3} \) can be decoupled by introduction of the auxiliary fields \( \Delta_{\pm} = \frac{1}{\sqrt{2}} (\Delta_1 \pm \Delta_2) \), where \( \Delta_1 \) and \( \Delta_2 \) are the values of the order parameter on the respective sheets of the Fermi surface. For attractive interaction, we use

\[ e^{-V b_{\uparrow}^* b_{\downarrow}} = \int \mathcal{D}\Delta_+^* \mathcal{D}\Delta_- e^{i \Delta_+^* \Delta_+ + i \Delta_-^* \Delta_- + i \Delta_+ b_{\uparrow} + i \Delta_- b_{\downarrow}} , \]

\[ e^{V b_{\downarrow}^* b_{\uparrow}} = \int \mathcal{D}\Delta_+^* \mathcal{D}\Delta_- e^{i \Delta_-^* \Delta_- + i \Delta_+^* \Delta_+ + i \Delta_- b_{\downarrow} + i \Delta_+ b_{\uparrow}} , \]  

where \( b_{\pm} = \frac{1}{\sqrt{2}} (b_{\uparrow} \pm b_{\downarrow}) \) which are linked to the original fermionic fields by \( b_{\nu} = \sum_k \psi_{\nu, k+} \psi_{\nu, -k} \). The respective decoupling for repulsive interaction has the same structure, but the factor \( i \) is then associated with the \( \Delta_+ \) mode rather than the \( \Delta_- \) mode, ensuring the convergence of the integral.

Then the SC transition temperature can be extracted from the quadratic part of an expansion of the free energy in terms of the order parameters \( \Delta_+ \) and \( \Delta_- \), which can be written in matrix form as

\[ \Delta F = (\Delta_+^* \Delta_+ - \Delta_-^* \Delta_-) \left( \begin{array}{cc} a_{++} & a_{+-} \\ a_{-+} & a_{--} \end{array} \right) \left( \begin{array}{c} \Delta_+ \\ \Delta_- \end{array} \right) . \]  

The sign change of the lower eigenvalue of this quadratic form,

\[ \lambda_{1,2} = \frac{1}{2} (a_{++} + a_{--}) \pm \frac{1}{2} \sqrt{(a_{++} + a_{--})^2 + 4a_{+-}a_{-+}}, \]

determines the transition temperature. The coefficients in this expansion of the free energy in the presence of disorder can be obtained from our microscopic model, and the intraband and interband diagrams \( d_{ij} \) contributing to the quadratic coefficients are depicted in Fig. 2.

The quadratic coefficients in terms of these diagrams read

\[ a_{++} = \frac{1}{|V|} + \frac{1}{2} \text{sgn} \, V \left[ d_{11} + d_{22} + d_{12} + d_{21} \right] , \]

\[ a_{--} = \frac{1}{|V|} - \frac{1}{2} \text{sgn} \, V \left[ d_{11} + d_{22} - d_{12} - d_{21} \right] , \]

\[ a_{+-} = -\frac{1}{2} \left[ d_{11} - d_{22} + d_{12} - d_{21} \right] , \]

\[ a_{-+} = -\frac{1}{2} \left[ d_{11} - d_{22} - d_{12} + d_{21} \right] . \]

Since for equal density of states in the two bands, \( d_{11} = d_{22} \) and \( d_{12} = d_{21} \), the eigenvalues reduce to

\[ \lambda_{1,2} = \frac{1}{|V|} \text{sgn} \, V \, \text{Re} d_{12} \pm \sqrt{d_{11}^2 - (\text{Im} \, d_{12})^2} , \]

and the sign change of the lower one determines the transition temperature. The respective diagrams can be evaluated analytically, and expressed in terms of digamma functions \( \psi_0 \),

\[ d_{11} = d_{22} = \frac{\rho_F}{2} \left[ \psi_0 \left( \frac{1}{2} + \frac{\Delta}{2\pi T} \right) - \psi_0 \left( \frac{1}{2} \right) \right] + \psi_0 \left( \frac{1}{2} + \frac{\Delta}{2\pi T} + \frac{\rho_F \Gamma_{12}}{2} \right) - \psi_0 \left( \frac{1}{2} + \frac{\rho_F \Gamma_{12}}{2} \right) \]

\[ = \frac{\rho_F}{2} \left\{ \ln \left( \frac{2 \rho_F \Gamma_{12}}{2\pi T} \right) - \psi_0 \left( \frac{1}{2} \right) \right\} \]

\[ = \frac{\rho_F}{2} \left\{ \ln \left( \frac{2 \rho_F \Gamma_{12}}{2\pi T} \right) - \psi_0 \left( \frac{1}{2} \right) \right\} \]

\[ \approx \frac{\rho_F}{2} \left\{ \left( \frac{2 \rho_F \Gamma_{12}}{2\pi T} \right) - \psi_0 \left( \frac{1}{2} \right) \right\} , \]

\[ \approx \frac{\rho_F}{2} \left\{ \ln \left( \frac{2 \rho_F \Gamma_{12}}{2\pi T} \right) - \psi_0 \left( \frac{1}{2} \right) \right\} , \]

where we also gave the results in the limiting cases of a clean system and strong interband scattering (but in the sense that \( 1/k_F l \ll 1 \) still holds). The transition temperature can be determined numerically from these diagrams for arbitrary phases of \( \phi \), but it is most instructive to highlight three important limits, namely \( \phi = 0 \), \( \phi = \frac{\pi}{2} \), and \( \phi = \pi \). Our results for the SC transition temperature as a function of the interband scattering rate are shown in Fig. 3 for attractive and repulsive interaction.

In the clean case and for \( \phi = 0 \), we reproduce well-known results, namely that, depending on the sign of the coupling constant \( V \), one of the two modes condenses. In case of attractive interaction, \( s^{++} \) superconductivity, characterized by the order parameter \( \Delta_+ \), is realized, whereas for repulsive interaction, it is \( s^{-+} \) superconductivity characterized by \( \Delta_- \). The SC transition occurs at the critical temperature \( T_{c,0} \), as known from BCS theory,

\[ T_{c,0} = \frac{2 e^\gamma}{\pi} \rho_F \Gamma_{12} \]

where \( \gamma \) denotes the Euler constant. Furthermore, the consideration of \( \phi = 0 \) in a dirty superconductor is also consistent with previous work. In case of attractive interaction, the \( \Delta_+ \) mode condenses, and the transition temperature is unaffected by the presence of impurities, \( T_c \approx T_{c,0} \). This result for \( s^{++} \) SC is known as the Anderson theorem. For repulsive interaction, we find the \( \Delta_- \) mode to be the one that condenses, and now (unconventional) SC is affected by the presence of impurities, and the suppression of the transition temperature is given by the usual Abrikosov-Gorkov law. Particularly, at a critical scattering rate

\[ \Gamma_c = \frac{T_{c,0}}{4 e^\gamma \rho_F} \]

\( s^{-+} \) superconductivity vanishes completely.

However, for a phase of \( \pi \), we find the reversed situation: Conventional superconductivity is now harmed by impurities, and even suppressed at a critical scattering rate, whereas for \( s^{+-} \) SC, there exists an analog of the
interaction. For $\phi = 0$ (green dotted line), $\phi = \frac{\pi}{2}$ (red lines), and $\phi = \pi$ (blue dashed line) in case of (a) attractive and (b) repulsive results for $\rho_F$.

In the case of $V$ of the BCS coupling matrix element $V_{s,s}$, this result can also be understood in terms of a redefinition of the electron operators in order to absorb the phase of the impurity line associated with the scattering process with rate $\Gamma_{1212}$,

$$
\hat{\psi}_{1,k,\sigma} \rightarrow \hat{\psi}'_{1,k,\sigma} = e^{i\frac{\phi}{2}} \hat{\psi}_{1,k,\sigma},
$$

$$
\hat{\psi}_{2,k,\sigma} \rightarrow \hat{\psi}'_{2,k,\sigma} = \hat{\psi}_{2,k,\sigma}.
$$

This leaves the intraband scattering processes as well as the interband scattering process associated with rate $\Gamma_{1221}$ unaffected, but entails a simultaneous rescaling of the BCS coupling matrix element $V \rightarrow V' = e^{-i\phi}V$.

In the case of $\phi = \pi$ this corresponds to $V \rightarrow V' = -V$, and thus, an attractive interaction in this description effectively becoming repulsive, and vice versa. Therefore, for a phase of $\phi = \pi$, we find an Anderson theorem for the $s^{+\pi}$ pairing state, whereas the transition temperature of the $s^{+\pi}$ pairing state is suppressed according to the Abrikosov-Gorkov law.

In the case of $\phi = \frac{\pi}{2}$, we find that the transition temperature is suppressed for attractive as well as repulsive interaction. However, in neither case, a critical scattering rate at which superconductivity vanishes is found,

$$
T_c = \begin{cases} 
T_{c,0} - \frac{\pi^2}{16} \rho_F \Gamma_{12} & , T \gg \rho_F \Gamma_{12} , \\
\Lambda \frac{2\pi^2}{16} e^{-\frac{\rho_F \Gamma_{12}}{2}} \gamma \pi & , T \ll \rho_F \Gamma_{12} ,
\end{cases}
$$

where $V_{\text{eff}}(\Gamma_{12}) = \rho_F |V|^2 \ln(\frac{\Lambda}{2\pi\rho_F \Gamma_{12}})$. Furthermore, the pairing state in case of such an intermediate phase is a superposition of the $\Delta_+$ and $\Delta_-$ mode.

VI. APPLICATION TO IRON PNICTIDES

We showed that $s^{++}$ SC can be destroyed by impurities which cause certain interband scattering processes characterized by a nontrivial phase in the impurity line, whereas the $s^{+\pi}$ pairing state remains robust under certain conditions. In this section, we establish the connection of our preceding observations to the situation in iron-based superconductors. We reveal the necessity of time-reversal-symmetry-breaking for the occurrence of the effect in iron pnictides and discuss the nucleation of orbital density wave order around impurities as a possible origin of time-reversal-symmetry-breaking associated with orbital magnetism in these materials.

A. Role of time-reversal symmetry

As anticipated in section III B, the consideration of time-reversal-symmetry-breaking interband scattering allows to formulate an analog of the Anderson theorem for $s^{+\pi}$ superconductivity. This was formalized in section II V by the introduction of a nontrivial phase in the interband scattering rate $\Gamma_{1212}$.

In this section, we elucidate the role of time-reversal-symmetry-breaking impurities in iron pnictides, where electrons from $d$-orbitals are forming the superconducting condensate.

Since $\Gamma_{a\alpha a\alpha} \propto (U_{\alpha a})^2$, in order to have a nontrivial phase in the impurity line, we need $U_{\alpha a} = \sum_s R_s e^{-i\mathbf{R}_s \cdot \mathbf{Q}} J_{ss'}^{\alpha \alpha} \Phi_{a}^{\alpha}$ to have a nonzero imaginary part. Since $2\mathbf{Q}$ is a reciprocal lattice vector, and thus $\exp(-i\mathbf{R} \cdot \mathbf{Q}) = \pm 1$ for any lattice vector $\mathbf{R}$, this requirement can only be met if the matrix element

$$
J_{ss'}^{\alpha \alpha} = \int \mathbf{d}r \langle \Phi_{\alpha}^{\alpha}(\mathbf{R})^\ast |U_{\mathbf{R}=0} \Phi_{\alpha}^{\alpha}(\mathbf{R}) \rangle
$$

itself has a nonzero imaginary part. Here, $\Phi_{\alpha}^{\alpha}(\mathbf{R})$ denotes the Wannier function of band $\alpha$ centered around site $\mathbf{R}_a$. The Wannier functions in band space are related to the tight-binding wave functions in orbital space by an orthogonal, that is, real, transformation matrix, since the dispersion in band space is symmetric.
The wavefunctions of electrons on \( d \)-orbitals with which we are concerned in the iron pnictides, can be chosen real, so \( J^{ss}_{\alpha \beta} \) can have an imaginary part only due to the phases in the impurity Hamiltonian.

In the absence of spin-orbital coupling, the Hamiltonian can be split into an orbital and a spin part, \( \hat{H}_{\text{imp}} = \hat{H}_{\text{orb}} \otimes \hat{H}_{\text{spin}}^{\text{imp}} \). We consider the transformation properties under time reversal, described by the operator

\[
\hat{T} = (\hat{T}^{\text{orb}} \otimes \hat{T}^{\text{spin}}) \hat{\mathcal{C}},
\]

where \( \hat{\mathcal{C}} \) denotes complex conjugation. For spin-\( \frac{1}{2} \), the spin part \( \hat{T}^{\text{spin}} \) is given by the Pauli matrix \( \hat{\sigma}_2 \). In real space, the orbital part \( \hat{T}^{\text{orb}} \) is just the identity, \( \hat{T}^{\text{orb}} = \hat{1}^{\text{orb}} \).

We consider the most generic time-reversal symmetric impurity Hamiltonian \( \hat{H}_{\text{imp}} = \hat{T} \hat{H}_{\text{imp}} \hat{T}^{-1} \), and if \( \hat{H}_{\text{imp}} \) is invariant under time reversal, the matrix element \( J^{ss}_{\alpha \beta} \) is invariant as well. If we do not consider scattering processes involving spin flips, that is, if \( \hat{H}^{\text{spin}} \propto \hat{\sigma}_0 \), then the spin part is also invariant under time reversal, and as a consequence, the orbital part of the Hamiltonian is real, yielding \( J^{ss}_{\alpha \beta} \in \mathbb{R} \).

In conclusion, impurities that are invariant under time-reversal symmetry are not able to generate nontrivial phases in the scattering matrix elements such that a nontrivial phase can arise. Since we are not concentrating on spin magnetism, this implies that a nontrivial phase is caused by orbital magnetism in multiband superconductors.

### B. ODW impurities

The renormalization group analysis\(^{[13]}\) of the two-band Hubbard model with particle-hole symmetry, as suited for the description of iron pnictides, revealed the existence of a fixed point where the Hamiltonian exhibits an SO(6) symmetry, and three different states of order compete\(^{[13]}\) spin-density wave (SDW), superconductivity (SC), and orbital-density waves (ODW). Thus, at this fixed point, the free energy \( F \) is a function of a combined order parameter, \( F = F(M^2 + |\Delta|^2 + \rho^2) \).

Since iron pnictides are only close to this SO(6)-symmetric fixed point, the SDW instability occurs first, and ODW order has not been observed in any iron pnictide superconductor so far, although being close in energy. It is, however, a conceivable scenario that such order could nucleate around impurities in these materials, similar to SDW order\(^{[13],[14]}\). Such ODW-type impurities break time-reversal symmetry and thereby are responsible for orbital magnetism. Thus we consider such ODW impurities as an example to demonstrate the emergence of a nontrivial phase in the impurity line in iron-based superconductors.

An ODW-type impurity at site \( \mathbf{R}_i \) is described by

\[
\hat{U}_{\mathbf{R}_i} = -\frac{i}{2} \sum_{s,\sigma} \sum_{s',\sigma'} e^{i(\mathbf{k}_s - \mathbf{k}_{s'} + \mathbf{Q}) \cdot \mathbf{R}_{s'}} \psi_{s',i+s',\sigma}^\dagger \psi_{s,i+s,\sigma} \psi_{s',i+s,\sigma'}^\dagger \psi_{s,i+s',\sigma'} - \psi_{s,i+s',\sigma'}^\dagger \psi_{s',i+s,\sigma} \psi_{s',i+s',\sigma'}^\dagger \psi_{s,i+s,\sigma},
\]

where for a short-ranged impurity, the sum over lattice sites \( s \) can, for example, be restricted to nearest neighbors (NN). In momentum space, the corresponding matrix element is given by

\[
\langle \mathbf{k}, \alpha | \hat{U}_{\mathbf{R}_i} | \mathbf{k}', \beta \rangle = -\frac{i}{2} \sum_{s} e^{i(\mathbf{k}_s - \mathbf{k}_{s'} + \mathbf{Q}) \cdot \mathbf{R}_{s'}} \sum_{s} e^{i(\mathbf{k}_s - \mathbf{k}_{s'} + \mathbf{Q}) \cdot \mathbf{R}_{s}} \delta_{ss'} \langle \delta_{s,1} \delta_{s',2} - \delta_{s,2} \delta_{s',1} \rangle,
\]

and thus the scattering rate is given by

\[
\Gamma_{\alpha \beta \gamma \delta}(\mathbf{k}_1, \mathbf{k}_1', \mathbf{k}_2, \mathbf{k}_2') = -\frac{\hbar}{4} \sum_{s} \sum_{s} e^{i(\mathbf{k}_s - \mathbf{k}_{s'} + \mathbf{Q}) \cdot \mathbf{R}_{s}} \sum_{s} \sum_{s} e^{i(\mathbf{k}_s - \mathbf{k}_{s'} + \mathbf{Q}) \cdot \mathbf{R}_{s}} \delta_{\text{band}}.
\]

For the interband scattering process corresponding to the exchange of two electrons between the bands, \( \alpha = \delta \neq \gamma = \beta \), it holds that \( \delta_{\text{band}} = -1 \). The interband scattering process from which a phase in the impurity line might arise is associated with \( \delta_{\text{band}} = +1 \) and corresponds to a Cooper pair being scattered to the other band, that is, \( \alpha = \gamma \neq \beta = \delta \). All other combinations of band indices yield \( \delta_{\text{band}} = 0 \), reflecting that this particular type of impurities can only cause certain interband scattering processes.

Keeping in mind that a global prefactor of \(-1\) corresponds to a phase of \( \pi \), we evaluate the imaginary part of the impurity line that might yield arbitrary phases. It is determined from the phase factors,

\[
\text{Im} \Gamma_{\alpha \beta \gamma \delta}(\mathbf{k}_1, \mathbf{k}_1', \mathbf{k}_2, \mathbf{k}_2') \propto \text{Im} \left[ \sum_{s} e^{i(\mathbf{k}_s - \mathbf{k}_{s'} + \mathbf{Q}) \cdot \mathbf{R}_{s}} \sum_{l} e^{i(\mathbf{k}_l - \mathbf{k}_{l'} + \mathbf{Q}) \cdot \mathbf{R}_{l}} \right] = \sum_{s,t} \sin((\mathbf{k}_s - \mathbf{k}_{s'} + \mathbf{Q}) \cdot \mathbf{R}_{s} + (\mathbf{k}_l - \mathbf{k}_{l'} + \mathbf{Q}) \cdot \mathbf{R}_{l})
\]

which can be evaluated assuming a lattice possessing certain symmetries and a finite range of the impurity. As long as inversion symmetry is present in the crystal, the imaginary part of the impurity line is zero. However,
phases of 0 and π are possible even in case of an inversion-
symmetric lattice. For example, in case of zero incom-
ing momenta, \(k_1 = k_2 = 0\), and outgoing momenta \(Q\),
\(k'_1 = k'_2 = Q\), an inversion-symmetric lattice, and short-
 ranged impurities that only affect neighboring sites, we
find a phase of \(\pi\) since

\[
\Gamma_{1212} = -\frac{n_{\text{imp}}}{4} (N_{\text{NN}})^2,
\]

where \(N_{\text{NN}}\) is the number of nearest-neighbor sites.
When, additionally, the lattice breaks inversion symme-
try, even arbitrary phases are conceivable, also leading to
suppression of \(T_c\), but with a different functional behav-
ior.

VII. CONCLUSION

We consider a two-band superconductor in the pres-
ence of impurities. Depending on the interaction leading
to superconductivity, this model describes conventional
or unconventional superconductivity which is known to
react differently to the presence of impurities, also de-
pending on whether the impurities are sensitive to the
spin of the scattered electrons or not. Extended potential
impurities, although insensitive to spin, can still break
time-reversal symmetry, and in this paper, we consider
the effect of such impurities associated with orbital mag-
netism on the transition temperature. One example for
the occurrence of this effect could be a competing state
of order nucleated by the impurity. Such a scenario is
conceivable in the case of iron pnictides, where orbital
density waves are a hidden state of order competing with
superconductivity.

Orbital magnetism, that as competing ordered state
nucleates near impurities, manifests itself in a nontrivial
phase in the impurity line of one interband scattering pro-
cess, and we classify different limits by this phase. Our
results for the transition temperature are summarized in
Fig. 3. The trivial phase \(\phi = 0\) corresponds to the well-
known situation: The transition temperature \(T_c\) of con-
ventional superconductors remains unaffected by impu-
rieties, whereas for unconventional superconductors, \(T_c\)
is suppressed with increasing interband scattering rate, and
even vanishes completely at a critical scattering rate. The
functional behavior of \(T_c\) on the interband scattering rate
corresponds to the functional behavior originally only as-
sociated with paramagnetic impurities by Abrikosov and
Gorkov. For a phase of \(\phi = \pi\), however, we find the
reversed situation. Then, nonmagnetic impurities are
pair-breaking for conventional superconductors with the
same functional behavior, and there exists an analog of
the Anderson theorem for unconventional superconduc-
tors. This scenario is indeed realized in case of the orbital
density wave state discussed in Refs. [12] and [13]. In the
intermediate regime, impurities are pair-breaking for both
pairing states, but there is no critical interband scattering
rate at which superconductivity is suppressed completely.
As an example, we consider \(\phi = \frac{\pi}{2}\), and find linear sup-
pression of \(T_c\) for small interband scattering rates, and
exponential suppression of \(T_c\) in the dirty limit.

In conclusion, in the presence of impurities associated
with orbital magnetism, pair-breaking due to interband
scattering does not only occur in unconventional super-
conductors, and the robustness of \(T_c\) against impurities
does not necessarily imply conventional superconductiv-
ity.

We note that the effect of spin-magnetic impurities
(not considered here microscopically) on the supercon-
ductive transition has been addressed recently in Ref. [20].
Their results are consistent with our general symmetry
analysis of section III while our diagrammatic calcula-
tion of sections IV and V focusses on the other case of
orbital-magnetic impurities and a possible microscopic
mechanism for such impurities.

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Appendix A: Calculation of the generalized
Cooperon ladder

This appendix provides details on the calculation of
the generalized form of the Cooperon ladder, denoted by
\(C_\alpha\). A single rung of the ladder \(C_\alpha\) is given by
where the second term appears in addition to the usual Cooperon ladder for scattering in single-band models or in models with intraband scattering only. In Eq. (A1), the propagators drawn in light gray are only shown for clarification of the respective scattering processes and not part of the calculation. The last line has been obtained by performing the energy integration.

\[
\int_{\nu} G_{\alpha,k}(\nu_n)G_{\alpha,-k}(-\nu_n) \left[ 1 + \frac{\Gamma_1}{\Gamma_2} \int_k^{k'} G_{\alpha,k'}(\nu_n)G_{\alpha,-k'}(-\nu_n) \sum_{m=0}^{\infty} \left( \frac{\Gamma_1}{\Gamma_2} \int_{k''}^{k'} G_{\alpha,k''}(\nu_n)G_{\alpha,-k''}(-\nu_n) \right)^m \right]
\]

(A1)

In order to obtain the full generalized Cooperon ladder, the result for a single rung, Eq. (A1), is summed, yielding

\[
C_{\alpha}(\nu_n) = \sum_{m=0}^{\infty} \left( \frac{\pi \rho_F \Gamma_1 |\nu_n| + (\pi \rho_F)^2 \Gamma_2 (\Gamma_1 + \Gamma_2)}{(\pi \rho_F)^2 \Gamma_2 + |\nu_n| |\rho_F (\Gamma_1 + \Gamma_2) + |\nu_n|)} \right)^m
\]

(A2)

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