The recent discovery of unstable travelling waves (TWs) in pipe flow has been hailed as a significant breakthrough with the hope that they populate the turbulent attractor. We confirm the existence of coherent states with internal fast and slow streaks commensurate in both structure and energy with known TWs using numerical simulations in a long pipe. These only occur, however, within less energetic regions of (localized) ‘puff’ turbulence at low Reynolds numbers ($Re = 2000 - 2400$), and not at all in (homogeneous) ‘slug’ turbulence at $Re = 2800$. This strongly suggests that all currently known TWs sit in an intermediate region of phase space between the laminar and turbulent states rather than being embedded within the turbulent attractor itself. New coherent fast streak states with strongly decelerated cores appear to populate the turbulent attractor instead.

The transition to turbulence in wall-bounded shear flows is a classical problem that has challenged physicists for over a century. Some flows, such as that between differentially-heated parallel plates or between rotating concentric cylinders, exhibit a smooth progression to increasingly complicated flows via an initial linear instability. Plane Couette flow and pipe flow, however, abruptly adopt a turbulent state. The problem is further complicated by changes in the spatio-temporal character of the observed flows at different flow rates. Pipe flow exhibits a quasi-stable localised turbulent ‘puff’ state as well as a globally turbulent ‘slug’ flow \cite{1}. A finite amplitude disturbance is required to trigger turbulence, the amplitude of which has been shown to depend critically on its ‘shape’ \cite{2}. Similarly, plane Couette flow exhibits localised spots of turbulence which may be either short lived transients or survive to arbitrarily long times \cite{3}. Plane Poiseuille flow, on the other hand, exhibits a linear instability but at flow rates well beyond those at which turbulence is typically observed.

The discovery of exact travelling wave (TW) solutions in wall-bounded shear flows \cite{5,6,7,8,9,10} has spurred a flurry of excitement within the community and prompted the speculation that the states lie inside the turbulent attractor. We confirm the existence of coherent states with internal fast and slow streaks commensurate in both structure and energy with known TWs using numerical simulations in a long pipe. These only occur, however, within less energetic regions of (localized) ‘puff’ turbulence at low Reynolds numbers ($Re = 2000 - 2400$), and not at all in (homogeneous) ‘slug’ turbulence at $Re = 2800$. This strongly suggests that all currently known TWs sit in an intermediate region of phase space between the laminar and turbulent states rather than being embedded within the turbulent attractor itself. New coherent fast streak states with strongly decelerated cores appear to populate the turbulent attractor instead.

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for structures indicated that a ‘good’ correlation is achieved. The projection function \( z \) was calculated for each \( u, z \).

FIG. 2: Axial component of vorticity in \((r, z)\)-plane, 25 \(D\) shown of 50 \(D\) computational domain; (a) slug turbulence at \(Re = 2800\); (b) inhomogeneous turbulence at \(Re = 2400\) (c) puff turbulence at \(Re = 2000\). Cross-sections in \((r, \theta)\) show the axial flow relative to the laminar profile with fast streaks (light/white) and slow streaks (dark/red), contour lines each 0.2 \(U\); (d) \(m = 4 \) and \(m = 3\) structures seen upstream and downstream of the trailing edge \(z_{TE}\) (flow is left to right); (e) sections from a puff where no clear structures are observed; (f) energetic section at \(Re = 2800\), but resembling \(m = 5\) structure; (g) exact solution with 3-fold rotational symmetry.

The skin friction is a useful indicator of the flow response — see Fig. 1. A localised ‘puff’ structure of apparently stable length \(\approx 20D\) is observed for \(Re \lesssim 2250\). At \(2250 \lesssim Re \lesssim 2500\) the puff gradually expands while translating along the pipe, possibly dividing into multiple puffs, giving rise to an uneven patch of turbulence in which the turbulent intensity is spatially inhomogeneous (see Fig. 2a-c). By \(Re \approx 2800\), the turbulent intensity has become much more spatially homogenized indicating ‘slug’-like turbulence [1].

As a puff is spatially inhomogeneous, the search for coherent structures was conducted at fixed relative positions, up- and downstream, of the puff’s steadily translating trailing edge, \(z_{TE}(t)\), which is itself characterized by a sharp jump in the streamwise velocity \(u_z\) on the pipe axis [22]. The search focused upon the appearance of fast streaks near the pipe wall, thus correlations in the streamwise velocity were examined using the function

\[
C(\theta, z') = \frac{2 \langle u'_z(\theta + \phi, z') u'_z(\phi, z') \rangle_{\phi}}{(\max_{\phi,z}(u'_z)^2)} \Bigg|_{r=0.4D} \tag{2}
\]

where \(\langle \cdot \rangle_{\phi}\) indicates averaging over the subscripted variable, and \(u'_z\) is the deviation from the time-averaged profile calculated for each \(z' = z - z_{TE}\) position in the puff. The projection function \(C_m(z') = 2 \langle C(\theta, z') \cos(m\theta) \rangle_{\theta}\) was used to extract the signature of structures of azimuthal wavenumber \(m\). Experience of examining flow structures indicated that a ‘good’ correlation is achieved for \(C_m(z')\) larger than 0.1, as indicated by Fig. 3 which shows the correlation results for the puff snapshot of Fig. 2. The magnitude of the correlation measures are relatively large at the positions indicated in Fig. 3 despite not being located at \(z_{TE}\), where the turbulent intensity is greatest (\(u'_z\) is largest) for the puff. Cross-sections in \((r, \theta)\) of the flow field are shown in Fig. 2 with lines indicating their position. For comparison purposes, cross-sections for another puff snapshot are reproduced in Fig. 2 where \(C_m(z')\) is less than 0.1 for all \(z'\). Particularly for the plots upstream of the trailing edge, the similarity to known TWs is remarkable where individual slow streaks are also reproduced in the interior (compare, for example, with Figs. 9a and 13(lower left) in [9]).

The probabilities of finding a correlation greater than 0.1 at different parts of the puff are plotted in Fig. 4. From around \(z' = -D\) to \(z' = +5D\) fast streak structures of \(m = 3\) and \(m = 4\) are seen approximately 10–15\% of the time which is in good agreement with the frequencies observed in [11, 14, 18]. The appearance of coherent fast-streaks, however, does not necessarily imply an observation of a TW. Of the coherent fast-streaks found, those which look most like TWs (in terms of fast and slow streaks) are found away from the most energetic regions in puff turbulence. The disturbance energy at the trailing edge itself (Fig. 5) is far too high to be compatible with any known TW at the same \(Re\) with the roll energy, in particular, an order of magnitude too large. Cross-sections at \(z_{TE}\) also exhibit small-scale structure uncharacteristic of TWs (see
There are two coherent structures simultaneously present in the 2D/3D instant correlation episode in split puffs at \(Re = 2400\), where we find well formed coherent structures, the magnitudes of streak and roll energies are both consistent with TWs which all have a characteristically small roll-to-streak energy ratio.

At larger Reynolds numbers, \(Re > 2800\), the turbulent intensity in slug turbulence is uniformly high everywhere, being comparable to that at the trailing edge of a puff (in units of \(\rho u^2\) at each cross-section), and is never as low as that of the TWs (see Fig. 5). Although fast streak structures are still observed, they are of too high energy to be associated with the turbulent part of phase space as \(t\) increases, to reach the fully turbulent region near the trailing edge. On leaving here as time increases, it passes back through the TW region to the origin as it relaminarises far downstream (\(t \to +\infty\)). Consequently, TWs are only visited just upstream and downstream of the trailing edge: Fig 5 is a good 2-D representation of this process (where \(z\) plays the role of \(t\) and a phase space norm based on the streak and roll energies is implied).

Our results confirm emerging evidence that lower branch TWs lie strictly between the laminar and turbulent states in phase space \([11, 13, 20, 21]\). However, the fact that upper branches of the known TWs also have too low energy to be associated with the turbulent part of phase space is a surprise. It is quite plausible that the coherent structures observed so far for \(Re > 2800\) and characterised by an outer ring of fast streaks together with a strongly decelerated core represent a more-energetic branch of TWs which is embedded in the turbulent attractor. The fact that their roll-to-streak energy ratio is so much larger than for currently known TWs suggests that they may have a different sustaining mechanism.

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FIG. 4: Probability at different parts of a puff of a ‘good’ correlation $C_m(z') \gtrsim 0.1$ (see Fig. 2) at $Re = 2000$ (left). Probability of a correlation $C_m > C$ at any given point in the flow for $Re = 2400$ (middle) and 2800 (right).

FIG. 5: Roll and streak energies at different parts of the puff of Fig. 2 (Re = 2000). Expanding in Fourier modes $m$ in $\theta$, the total streak and roll energies are defined as $E_{\text{streak}}(z') := Re^2 \pi \sum_{m \neq 0} |u_m'|^2 dr$ and $E_{\text{roll}}(z') := Re^2 \pi \sum_{m \neq 0} (|u_m'|^2 + |u_m|)^2 r dr$ in units $\rho \nu^2$. Here $u' = (u_r, u_\theta, u_z)$ is the deviation from the laminar profile, for easier comparison with the TWs. Energies for 2-, 3- and 4-fold rotationally-symmetric TWs [12] are shown using lower thick, middle thin and upper thick red solid lines respectively (the closed loops are produced by the finite continuum of TW axial wavenumbers which can exist at $Re = 2000$). The pink short-dotted line ‘cloud’ in the top right hand corner corresponds to a slug at $Re = 2800$.

FIG. 6: Correlations in inhomogeneous turbulence ($Re = 2400$) at time intervals of $4D/U$, in a fixed window of $25D$ of the domain. The middle snapshot corresponds to that of Fig. 3. Same scales as Fig. 3. The black lines mark positions of waves at the different times indicating the translation of the TWs. The phase speeds appear $\approx (1 \pm 0.1)U$.

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[23] Plotting the flow relative to a different profile may change Fig. 2 slightly, and may reduce the relative streak energy of the perturbation, but cannot resolve the disparity in roll energies.