Computer-aided analysis of system dynamics models

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Abstract. The system dynamics methodology originally developed by J. W. Forrester for modelling and simulation of decision-making processes at enterprises is nowadays applied to many different fields, such as transportation, ecology, healthcare, biology, economics, etc. Instead of traditional flow diagrams and equations in the DYNAMO language, a block-textual specification is proposed. This form is considered to be more convenient for engineers familiar with control engineering. A model of transportation of kimberlite ore created in the modelling and simulation environment ISMA is given in order to illustrate the approach.

1. Introduction

System dynamics is an approach to modelling of complex dynamical systems with interacting material and information flows, delays, feedbacks, nonlinear loops from a variety of fields [1, 2, 3]. Models in J.W. Forrester’s system dynamics are continuous, and the author steadily follows the ideology of studying systems of this kind [3]. For example, to model a 10% increase in the retail sales $RRR(t)$ of the production-distribution system in [4], in lieu of the steady-state value $RRR(t) = 1000$, Forrester uses the piecewise-continuous function of sales:

$$RRR(t) = \begin{cases} 
1000 + kt, & t \leq t^*, \\
1100, & t > t^*. 
\end{cases}$$

The time instant $t^*$ can be calculated as $t^* = 100/k$. Taking into account that $k \gg 1$, we conclude that $[0, t^*) \ll [0, T]$, where $T$ is the simulation run length. Hence, using the terminology of hybrid systems [5, 6], we can consider the interval $[0, t^*)$ as a time gap. Then we have an instant change in the retail sales in the production-distribution model and can analyze it in the methodology of hybrid systems.

System dynamics models are described in a textual-visual form. The visual part consists of a set of flow diagrams representing the processes and information flows in the system. The textual component is a description of the model equations in a special language called DYNAMO. In contrast to Forrester’s textual-visual specification, the computer model specification developed for a modeling and simulation environment named ISMA will be used below [7]. Block diagrams traditional for engineers are used as the visual part of models in this environment. The textual elements of models in ISMA are described in a language called LISMA in a form close to the natural mathematical notation for differential-algebraic systems of equations. Such models do not lack in visual clarity as well as flow diagrams, although using known control engineering elements allows engineers to read the model easier and faster.

Level equations in the DYNAMO language, which was developed for simulating system dynamics models, are difference equations, which are the result of using the explicit Euler method of the first order of accuracy, which is the least accurate numerical method. The models are simulated with a constant
step size without error control, which aggravates the situation even more. Moreover, the explicit Euler method is practically inapplicable in the case of so-called stiff problems, when the solution has both slowly and rapidly changing components. Although J.W. Forrester considered only smooth systems, many real system models are stiff in actual fact. Besides, as the dimension of a system increases, the stiffness ratio inevitably grows as well. Piecewise functions employed in systems dynamics models can be effectively and correctly described in the modern theory of hybrid systems.

Although a block-textual model can be created in any modeling environment including a library of control engineering elements and allowing to add textual blocks, ISMA will be used later as it does not have the aforementioned drawbacks.

2. Features of ISMA
A Cauchy problem is a system of differential equations with initial conditions:
\[
y' = f(y,t),
\]
\[
y(t_0) = y_0,
\]
\[
t \in [t_0, t_e],
\]
where \( y \in R^N \), \( f : R^N \times R \rightarrow R^N \), \( t \in R, y_0 \in R^N, t_e \) is the simulation end time.

If a dynamical system has several modes, for instance, an electrical network with a circuit breaker interrupting the circuit once the current exceeds a preset threshold, a hybrid system is used instead of (1). A hybrid system is a set of systems (1), each along with:
\[
pr : g(y,t) < 0,
\]
where \( g : R^N \times R \rightarrow R \).

The constraint on the so-called event function \( g(y,t) < 0 \) means that the phase space trajectory of the system in the current mode must not ever cross the event surface \( g(y,t) = 0 \). As long as \( pr = true \), the system stays in the current mode. The global behavior of a hybrid system is a sequence of modes activated one after another.

ISMA (“Computer-Aided Analysis Tools” in Russian) is an environment for modeling and simulation of complex dynamical systems, which is developed by the Automated Control Systems department of Novosibirsk State Technical University (Russia). One of its key features is the support of several modeling languages. The general purpose ones are the textual language LISMA reflecting the notation of mathematics and a visual language of block diagrams. The software suite also includes editors for modeling problems from specific domains, such as chemical kinetics and power engineering.

The presence of discontinuities of the first kind in the first derivatives does not allow solving the Cauchy problems with symbolic computation methods. Explicit numerical methods have limited stability domains; that is why implicit numerical methods are traditionally used to solve stiff problems. From the other hand, they demand decomposing the Jacobian matrix. It is a very laborious process in terms of computational resources. Moreover, implicit numerical methods should not be used to integrate hybrid systems for they might try to compute the right-hand side of system (1) in a singular region, where the function is not defined [6]. Original explicit one-step schemes with improved stability domains and with error and stability control are, if possible, used to simulate stiff problems (1) and (2) in ISMA [8]. ISMA’s library contains an algorithm implementing the Runge-Kutta-Menson scheme with stability control named STEKS, DP78ST based on the Dormand-Prince method with stability control, implicit (m, k)-methods with freezing the Jacobian matrix (MK22, MK11), adaptive algorithm of variable structure and step DISPF1_RADAU5 [9], etc.

Apart from the continuous behavior described by system (1) and constraint (2), a hybrid system in general exhibits discrete behavior. Determining the exact moment of a transition is a considerably complicated problem, which can be approached in different ways. Using a simple algorithm, such as the bisection method, does not always guarantee detecting the event, if the event function crosses the event surface several times or has acute angles. That is why ISMA employs an original event detection algorithm originally justified by Professor J. Esposito in [10]. The algorithm adjusts the step size so that the phase space trajectory asymptotically approaches the event surface never crossing it. Thus, the
integration step size in ISMA is controlled with regard to the error and stability of the solution and, which is especially important, with regard to the behavior of the event function of the current mode.

3. Kimberlite ore transportation model

Let us consider a model of the transportation of kimberlite ore from the Inter mine to Concentration Plant 3 of the Mirny mining and processing complex in the Town of Mirny, Sakha Republic, Russia. A schematic diagram of the workflow is depicted in figure 1. It includes the maximum capacities of the warehouses, the distances between the objects, and the processing rate of the plant. The ore is transported by haul trucks BelAZ 7547, whose capacity is 45 tonnes. After the extraction from the mine Inter, ore is moved to the intermediate warehouse, from which it is then transported to the main warehouse. Ore is supplied from the main warehouse to Concentration Plant 3 according to its demand. There are five haul trucks transporting the ore from Inter. The trucks are loaded one after another by one loader at every site. Ore is extracted and transported round the clock in three shifts. The demand of Concentration Plant 3 is considered to be constant.

Figure 1. Schematic diagram of the kimberlite ore transportation.

Figure 2 shows the top level of the hierarchical block-textual model of the process in the ISMA environment. The model consists of the four blocks “Concentration Plant 3”, “Warehouse”, “Intermediate Warehouse”, “Mine” connected via third-order delays modeling ore transportation.

Figure 2. Top level of the model.

Figure 3 depicts the contents of the third-order exponential delay D3(WOR), which consists of three first-order exponential delays connected one to another. The block «Main Warehouse» consists of two closely intertwined blocks: «Current State» and «Ordering». The contents of the former is demonstrated in figure 4. The level of unfilled orders WUO is formed by the ordering rate of the plant.
WRR and the warehouse output rate WOR. The WOR rate is bounded from above by the maximal loading rate. The inventory level at the main warehouse, WWL, is determined by the unloading rate of ore from the intermediate warehouse WUR and WOR. WIR is the rate at which ore is received for unloading from the intermediate warehouse, which becomes WUR after going through the delay. WWU is the amount of ore being unloaded right now. The block «Intermediate Warehouse» is analogous to the «Warehouse» block. The only difference is the additional outport TUO, which stands for the unfilled orders.

4. Simulations

4.1. Instant increase in the factory production rate.

At $t' = 1$ day, it is decided to increase the production capacity at Concentration Plant 3 by 10%, which results in an instant increase in the ore ordering rate to the main warehouse by 10%. A change in the rate of this kind can be modeled by a piecewise function $WRR(t) = \begin{cases} \frac{WRR_1}{t < t'}, \\ \frac{WRR_2}{t \geq t'} \end{cases} \text{ (t/day)}$, which can be implemented with a «Nonlinear function» block of ISMA. The simulation results are depicted in figure 5.

In 1 day after the beginning of the experiment, the ordering rate to the intermediate warehouse TRR increases by 105% and gradually decreases then. In 1.5 days after the increase of the ordering rate of the plant, the ordering rate to the mine MRR increases by 900%. From $t = 4.5$ day to $t = 10.7$ day, the ordering rate to the mine drops to 0 t/day, which compensates for the abnormally high amount of orders in the previous phase.
From day 5 to day 8, the mine is working at the limit of its capacity, which is 3 000 t/day, and by the middle of day 12, the production rate MOR drops by 74% below the initial one. The inventory at the main warehouse does not change considerably, whereas the minimum inventory at the intermediate warehouse TWL is 1% below the initial one at the middle of day 3, and the maximal inventory is 28% higher by the beginning of day 10. It takes 20 days for the system to reach a steady state.

4.2. Accident at the intermediate warehouse.
With \( t' = 1 \) days there is an accident at the intermediate warehouse, for instance the building collapses, which results in its complete uselessness and removal from the workflow of transportation. The simulation results are depicted in figure 6.

In 5 hours after the accident, the ordering rate of the mine drops by 46%, and by the middle of day 3, the mining rate is 30% below that of \( t = 0 \). The system reaches a steady state in two weeks with the normal inventory 3.4% lower than the original one.
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