On the Benefits of Waiting in Status Update Systems

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Abstract—This paper explores the potential of waiting before packet transmission in improving the Age of Information (AoI) in status update systems. We consider a non-preemptive queue with Poisson arrivals and independent general service distribution, which we term $M/GI/1^\ast$. There is a single unit buffer that captures the latest arriving status update packet and discards the previous one. As a means to improve AoI performance, we allow “server waiting” in this queuing model. Depending on idle or busy system state, the server waits for a deterministic time before starting service of the packet so as to capture a potential newer arrival. Different from most existing works, we analyze AoI evolution by indexing the incoming packets, which is enabled by an alternative method of partitioning the area under the evolution of instantaneous AoI to calculate its time average. We obtain average AoI expressions for the $M/GI/1^\ast$ discipline with waiting for two distributions. Our numerical results demonstrate that waiting before service can bring significant improvement in average age – more than 75% in some cases – particularly, for heavy-tailed service distributions.

I. INTRODUCTION

Age of Information (AoI) defines the staleness of available information at the receiver of a system that monitors a physical phenomenon of interest and updates the status. Since its early treatments in [1], [2] for queuing models motivated from vehicular status update systems, the AoI metric has been found useful in and related to numerous applications that require timely availability of information at the receiving end of a communication system. In particular, [3], [4] investigates the role of packet management with the possibility of packet deadlines to improve the AoI at the monitoring node. [5] provides a general treatment of stationary probability analysis of AoI in various preemptive and non-preemptive queuing disciplines; see also [6], [7] for more specialized studies. [8] provides an information-theoretic treatment of the tradeoff between AoI and throughput in an energy harvesting timing channel. We also refer to [9]–[13] for AoI in energy harvesting communication systems. [14] considers AoI under link capacity constraints and [15] considers non-linear age dimension into the problem. Evolutions of AoI through multiple hops in networks have been characterized in [16]–[19]. References [20]–[22] consider AoI optimization over broadcast and multi-access scenarios.

In this paper, we consider an abstraction of a point-to-point communication system where the transmitting node sends status updates to the receiving node. The status update is a message whose time of generation is critical and is the only relevant quantity in this abstraction. In this setting, the status update age is the time elapsed since the last received sample was generated. We investigate the average age where randomly generated samples arrive according to a Poisson process and the time it takes for a packet to be transmitted has a general probability distribution. At this point, we bring the seminal paper [23] into attention. In this reference, general insights and analysis are provided as to when “waiting before updating” is useful to improve AoI performance in a point-to-point status update system. In the setting of [23], the samples are generated one at a time at the source in the presence of perfect knowledge of the server state. It has been shown analytically and numerically that heavy-tailed service distributions are especially amenable to provide cases of boosted AoI when a deliberate waiting period is introduced in the status update generation process. Our current paper explores the benefits of waiting further. However, our system is different from [23] in that status update packets are generated at random times one after the other independently in our model. We assume no feedback of the server state, and packet generation is oblivious to the transmitter state. Additionally, we allow the transmitter to manage packet transmissions by discarding earlier updates when a later update arrives at the transmitter, and introducing a delay before transmitting an available status update. Through decoupling the update generation process from the communication process, we aim to capture a natural characteristic of various types of applications in which sensors generate updates oblivious to server state and the service of the status update packets is separately handled.

In this paper, we consider a modified version of the $M/GI/1^\ast$ queuing discipline compatible with Kendall notation, reminiscent of the one used in [3] (see also [5] where this discipline is called $M/GI/1$ with last come first serve and discarding). In this model, the updates arrive at the transmitter according to a Poisson process, and the time it takes for a packet to be transmitted is a random variable that has a general distribution, is independent over time, and is also independent of other events in the system. Additionally, a single buffer is available in the queue, so the system can store one packet while the server is busy. The server is not equipped with the option to preempt service for a new arrival. Instead, the server waits an additional time before continuing to serve the latest arriving packet. Finally, we assume that any packet in the buffer is discarded if a new update arrives while the
server is busy or waiting before service. The potential benefit expected from waiting is to capture newer packets at the expense of longer wait times for packets in service. We allow deterministic amounts of waiting after idle and busy periods of the server, and perform stationary distribution analysis to obtain expressions for average AoI.

We combine waiting and packet management together in a status update system and determine closed form expressions for average AoI. We provide numerical results that highlight the benefits of waiting in AoI minimization. Our numerical results show that waiting is helpful for heavy-tailed service distributions such as inverse Gaussian distribution while the improvement is limited for light-tailed ones such as exponential and Erlang distributions.

II. THE MODEL

We consider a point-to-point link with a single transmitter (or server) and a single receiver. The status updates arrive to the transmitter in a packet form according to a Poisson process of rate $\lambda$. The transmitter node transmits the status update packets one at a time. The time for a packet to be served is independent of other system variables and independent for each packet with a general service time density function $f_s(s)$, $s \geq 0$. We use $\text{MGF}_s^{(S)}$ to denote the moment generating function of the service distribution evaluated at $-\gamma$:

$$\text{MGF}_s^{(S)} = \mathbb{E}[e^{-\gamma S}]$$

where we are interested in $\gamma \geq 0$. To proceed in the ensuing analysis, we also define the following:

$$\text{MGF}_\lambda^{(S,1)} \triangleq \mathbb{E}[Se^{-\lambda S}], \quad \text{MGF}_\lambda^{(S,2)} \triangleq \mathbb{E}[S^2e^{-\lambda S}]$$

where $\text{MGF}_\lambda^{(S,1)}$ and $\text{MGF}_\lambda^{(S,2)}$ are the first and second derivative of the moment generating function of $S$ at $-\lambda$.

We use $t_i$ to denote the time stamp of the event that packet $i$ enters the queue, and $t'_i$ to denote the time stamp of the event that the service of packet $i$ (if selected for service) is completed and it is delivered to the receiver.

We consider a packet management scheme similar to the one in [3, 5], which we term M/GI/1/2* or equivalently non-preemptive last come first serve with discarding. In this scheme, we assume that a single packet may be kept in queue. The transmitter chooses to send the latest arriving update and discards the previous updates. With Poisson arrivals and general service time, a single server, and one space in the buffer, this model of packet management is in the form of an M/GI/1/2* queue, referring to the usual M/GI/1/2 with the additional modification due to packet discarding.

A. Deterministic Waiting Policy

In our model, the server waits before starting service in the same spirit as [23]. When a packet arrives to an idle system, the server waits a deterministic time $\epsilon_t$ instead of taking the packet right away into service. If during this $\epsilon_t$ duration another packet arrives, then the newer packet is taken into service at the end of the ongoing waiting period, while the existing packet is discarded. Similarly, if a service ends and the queue is not empty, we introduce an additional waiting, which we term waiting after a busy period of a deterministic amount $\epsilon_B$. If during this $\epsilon_B$ period a new packet arrives, the new arrival is served and the existing one is discarded at the end of the ongoing waiting period. We assume that the waiting times are decided beforehand and are applied invariantly throughout the process. With M/GI/1/2* scheme and waiting, the instantaneous Age of Information (AoI) is measured by the difference of the current time and the time stamp of the latest delivered packet at the receiver:

$$\Delta(t) = t - u(t)$$

where $u(t)$ is the time stamp of the latest received packet at time $t$.

B. Equivalent Queuing Model

We now present another queuing model that yields an identical AoI pattern to our system’s, and one we will use to analyze the AoI in our system. In this model, the data buffer capacity is unlimited. Each arriving packet is stored in the queue and no packet is discarded. We allow multiple packets to be served at the same time. An arriving packet may find the system in four different states: i) Idle (I), ii) Busy (B), iii) Waiting after an Idle Period (WaI) and iv) Waiting after a Busy Period (WaB). If a packet finds the system in state (I), then that packet’s service starts after $\epsilon_I$ units of time together with all other packets that arrive during this waiting period finding the system in state (WaI). If an arriving packet finds the system in state (B), its service starts after the end of the current service period and the additional waiting period $\epsilon_B$. The packets arriving to the system in state (B) are served together with all other packets that arrive during the same busy period and the following (WaB) period.

This equivalent queue model is not physically the same as the original model in that the queue can hold at most one packet and the server cannot serve multiple packets simultaneously in the original model. Nevertheless, this queuing model yields an AoI evolution over time that is identical to the original non-preemptive M/GI/1/2* model with last come first serve and discarding the existing packets in the queue upon a new arrival. We essentially allow the discarding of the existing packets to happen at the end of the ensuing service time and this has no influence on the AoI evolution. Hence, in this equivalent model, no packet is discarded and this enables us to index the arriving packets.

We provide an instantiation of the AoI evolution under this equivalent queuing model in Fig. 1. We assume packet 1 finds the server idle and the server waits $\epsilon_I$ time units to capture an incoming packet. No packet arrives in this interval. The queue is not empty at $t'_1$ and so the server waits $\epsilon_B$ time units as the system state is busy at $[t'_1]^-$. In this waiting period, packet 4 is captured and the packets 2, 3 and 4 are served simultaneously at the end of the waiting period. Note that the end of service times $t'_2$, $t'_3$, $t'_4$ coincide as shown in Fig. 1. In the original model, only packet 4 is served and packets 2, 3 are discarded. At time $t'_4$, the system enters idle state, and packet 5 finds
between

we have the following stationary probabilities for each state:

In view of the renewal structure, these definitions are identical to those in [2].

We define the areas $Q_i$ under the triangular regions of the AoI curve in the same order as the arriving packet indices as shown in Fig. 1. These definitions are identical to those in [3].

where first come first serve (FCFS) queuing is assumed. Note

that the equivalent model in our current work is in the category of FCFS in that the packets are never discarded and they are served in the same order as they arrive. We now define $X_1$ as the length of time interval between the arrivals of packets $i-1$ and $i$ and $T_i$ as the system time for packet $i$ in the equivalent queuing model. These definitions are identical to those in [2].

We, therefore, have the average AoI as:\footnote{More generally, the $k$th moment of AoI is as follows [3]}

$$E[Δ] = λ \left( E[XT] + \frac{E[X^2]}{2} \right) = λE[XT] + \frac{1}{λ} \quad (5)$$

Since the system is ergodic, we are able to work with the generic variables for inter-arrival time $X$ and system time $T$.

III. AVERAGE AoI FOR M/GI/1/2* WITH WAITING

To calculate $E[Δ]$ in (5), it suffices to find the correlation between $X$ and $T$. As outlined in Section II-B, the system can be in four different states. In view of the renewal structure, we have the following stationary probabilities for each state:

$$p_I = \frac{1}{T_{cycle}}, \quad p_B = \frac{E[S]}{T_{cycle}MGF_λ^{(S)}}$$

$$p_{WaB} = \frac{ε_B}{T_{cycle}} \left( \frac{1}{MGF_λ^{(S)}} - 1 \right), \quad p_{WaI} = \frac{ε_I}{T_{cycle}} \quad (7)$$

$$E[Δ^k] = λ \left( E[X + T]^{k+1} - E[T^{k+1}] \right)$$

(Note this equal to the expected time spent in system state $s$ in one renewal cycle divided by the expected cycle length.

$$T_{cycle} = \frac{1}{λ} + ε_I + ε_B \left( \frac{1}{MGF_λ^{(S)}} - 1 \right) + \frac{E[S]}{MGF_λ^{(S)}} \quad (8)$$

These expressions are obtained by a standard application of Renewal Reward Theorem (see, e.g., [24], [25]). In one renewal cycle in our queuing model, the system first starts in (I) state and shifts to (WaI) when an arrival occurs. The expected length of staying in (I) is $\frac{1}{λ}$. After staying in (WaI) for a deterministic $ε_I$ time units, the system state switches to (B) and stays there for a service time. The system may repeatedly switch between (WaB) and (B) in a single cycle. If an arrival occurs during service time, the system state switches to (WaB) and after a deterministic $ε_B$ units, the system state goes back to (B). If no arrival occurs in one service time, then system state shifts to (I) and this completes one renewal cycle.

The expected length of back and forth between (B) and (WaB) is:

$$E[∑_{n=1}^{N} S_n + ∑_{n=2}^{N} ε_B]$$

We next evaluate $E[X_i | T_i | (s)]$ for $s ∈ S$ and conditioning is on the system state observed by packet $i-1$, denoted as $P_{i-1}$. Due to PASTA property, $Pr[P_{i-1} = (s)] = p_s$ where $p_s$, $s ∈ S$ are as in (6-7). We denote $S_{i-1}$, $S_i$ as independent random variables representing the service times of packets $i-1$ and $i$. Once packet $i-1$ arrives (shown in Fig. 2 as a big green rectangle), the next inter-arrival time $X_1$ determines the next state (the interval the small green rectangle falls in in Fig. 2) and how long packet $i$ spends in the system (which is $T_i$). We defer the evaluation of $E[X_i | T_i | (I)]$ to Appendix A and start with conditioning on (WaI).
A. $\mathbb{E}[X_i T_i]_{(\text{WaI})}$

Since waiting time is deterministic and the arrivals are Poisson, any packet that arrives in (Wal) state of the system could arrive at any point in this deterministic interval with uniform probability. Hence, the residual waiting time in this case is uniformly distributed: $R_i \sim U[0, \epsilon_i]$. By replacing the initial deterministic waiting time $\epsilon_i$ in the calculation of $\mathbb{E}[X_i T_i|\{I\}]$ with uniformly distributed $R_i \sim U[0, \epsilon_i]$, we can determine expressions for $\mathbb{E}[X_i T_i|\{WaI\}]$. To this end, we define the function $g(r)$ in Appendix E: $g(r)$ defines $\mathbb{E}[X_i T_i|R_i = r]$ where $R_i$ denotes residual time for packet $i - 1$ to start service. In particular, we have $g(\epsilon_i) = \mathbb{E}[X_i T_i|\{I\}]$. With this definition, we can express the desired expectation as

$$\mathbb{E}[X_i T_i|\{WaI\}] = \frac{1}{\epsilon_i} \int_{0}^{\epsilon_i} g(r)dr$$

We obtain closed form expressions for the integral in the RHS of (9) in Appendix C.

B. $\mathbb{E}[X_i T_i]_{(\text{WaB})}$

Due to identical reasoning to the (Wal) case, the residual waiting time in (Wal) state is uniformly distributed: $R_i \sim U[0, \epsilon_B]$. We thus have the following expression for the expectation:

$$\mathbb{E}[X_i T_i|\{Wal\}] = \frac{1}{\epsilon_B} \int_{0}^{\epsilon_B} g(r)dr$$

Closed form expressions for (10) are obtained in Appendix C.

C. $\mathbb{E}[X_i T_i|\{B\}]$

In this case, we first note the following lemma.

**Lemma 1** The residual service time for a packet arriving to the queue in (B) state has the following density function:

$$f_R(r) = \mathbb{P}[S > r] = \frac{\mathbb{E}[S]}{\mathbb{E}[S]}$$

where $S$ represents the general service time. Additionally,

$$MGF^\gamma_R = \frac{1 - MGF^\gamma}{{\gamma}\mathbb{E}[S]}$$

In particular, $\mathbb{E}[R_i|\{B\}] = \frac{\mathbb{E}[S]}{\mathbb{E}[S]}$. This lemma can be proved by using PASTA property; see also [5, Eq. (6)]. We use the same notation as in (2) to define the first and second derivative as $MGF^{(R,1)}_\gamma$ and $MGF^{(R,2)}_\gamma$. We use $R_i$ to denote the residual time of packet $i - 1$ if it arrives at system state (B) and separately consider the cases $R_i < X_i$ and $R_i \geq X_i$, to evaluate the desired expectation $\mathbb{E}[X_i T_i|\{B\}]$.

1) $R_i < X_i$: We have $\mathbb{P}[R_i < X_i] = MGF^\gamma_R$. In this case, $X_i - R_i$ is distributed exponentially with the same rate $\lambda$ conditioned on $R_i < X_i$. Hence, we get the same form of expressions by replacing $X_i - R_i$ with $X_i$ and $\epsilon_i$ with $\epsilon_B$. Decomposing $X_i$ as $X_i - R_i + R_i$, we have

$$\mathbb{E}[(X_i - R_i - 1)T_i|\{B\}, E_4] = g(\epsilon_B)$$

where the event $E_4$ denotes $R_i < X_i$. Also let $E_5$ be the complement of $E_4$. We evaluate $\mathbb{E}[R_i T_i|\{B\}, E_4]$ and $\mathbb{E}[X_i T_i|\{B\}, E_5]$ in Appendix D.

Having evaluated all cases, we can now use the law of total expectation to obtain

$$\mathbb{E}[X_i T_i|\{B\}] = \sum_{i=4}^{5} \mathbb{E}[X_i T_i|\{B\}, E_i]Pr[E_i]$$

Finally, we have

$$\mathbb{E}[X_i T_i] = \sum_{s \in S} \mathbb{E}[X_i T_i|\{s\}]p_s$$

and $\mathbb{E}[\Delta] = \lambda \mathbb{E}[X_i T_i] + \frac{1}{\lambda}$.

**IV. Numerical Results**

In this section, we provide numerical comparisons for AoI under Inverse Gaussian and Gamma service distributions. We use the analytical expressions derived in our paper to obtain the plots in this section. Additionally, we verified these plots using packet-based simulations using random number generators in MATLAB where we use a minimum of $10^6$ packets (around $10^8$ for longer tailed cases) for convergence. We observe in each case that these expressions yield accurate results.

**A. Inverse Gaussian Service Distribution**

Inverse Gaussian distribution is defined as $f_{S}(s) = \sqrt{\frac{\alpha}{2\pi s^3}}e^{-\frac{\alpha(s-1/\mu)^2}{2s\mu^2}}$ for $s \geq 0$. For this distribution, $\mathbb{E}[S] = \frac{1}{\alpha}$ and $\alpha$ is the shape parameter that determines the variance and tail behavior. As $\alpha$ gets smaller, the tail gets heavier. We have the following closed-form expressions for Inverse Gaussian distribution:

$$MGF^{(S)}_{\lambda} = e^{\alpha\mu(1 - \sqrt{1 + 2\lambda/(\alpha\mu^2)})}$$

$$MGF^{(S,1)}_{\lambda} = \frac{MGF^{(S)}}{\lambda \sqrt{1 + 2\lambda/(\alpha\mu^2)}}$$

$$MGF^{(S,2)}_{\lambda} = \frac{MGF^{(S,1)}_\lambda}{\lambda \sqrt{1 + 2\lambda/(\alpha\mu^2)}} + \frac{MGF^{(S)}}{\alpha\mu^3 \left(1 + 2\lambda/(\alpha\mu^2)\right)^2}$$

In Fig. 3, we plot the average AoI versus $\alpha$ for different mean service rates when $\epsilon_I = 0$, $\epsilon_B = 0$ for the Inverse Gaussian service distribution with shape parameter $\alpha = 0.1$. We observe that average AoI is monotonic with $\mathbb{E}[S]$, and attains its minima at smaller values of $\lambda$ as the service rate becomes smaller.

In Fig. 4, we show average AoI versus $\alpha$ under inverse Gaussian service distribution with $\mu = 0.1$ and for $\lambda = 0.1$ and $\lambda = 1$. We provide comparisons of AoI performances for zero-waiting and optimal deterministic waiting in Appendix E. In here, we determine optimal deterministic waiting through exhaustive search over all $(\epsilon_I, \epsilon_B)$ pairs. We observe that the improvement...
Fig. 3. Average AoI versus λ for different mean service rate values when \( \epsilon_I = 0, \epsilon_B = 0 \) for inverse Gaussian service distribution with \( \alpha = 0.1 \).

Fig. 4. Average AoI versus \( \alpha \) with inverse Gaussian service distribution with \( \mu = 0.1 \) and different system loads λ.

Fig. 5. Average AoI versus \( \epsilon_I \) for Gamma distributed service time with \( \mu = 0.1, \epsilon_B = 0, k = 0.1 \) and various λ values.

Closed form expressions for Gamma distribution:

\[
MGF^{(S)}_\lambda = \left( 1 + \frac{\lambda}{k\mu} \right)^{-k} \\
MGF^{(S,1)}_\lambda = \frac{1}{\mu} \left( 1 + \frac{\lambda}{k\mu} \right)^{-k-1} \\
MGF^{(S,2)}_\lambda = \frac{k+1}{k\mu} \left( 1 + \frac{\lambda}{k\mu} \right)^{-k-2}
\]

In Fig. 5, we plot average AoI versus \( \epsilon_I \) for Gamma distributed service time with \( \mu = 0.1, \epsilon_B = 0, k = 0.1 \) and various λ values. In this particular case, \( \epsilon_B = 0 \) coincides with the jointly optimal selection of the pair \((\epsilon_I, \epsilon_B)\). We observe that longer waiting periods are more useful for larger arrival rates. This is directly related to the fact that longer waiting periods enable capturing newer arrivals and available service rate is made more efficient use with waiting. In this particular case, optimal deterministic waiting enables a significant drop in average AoI, especially for larger \( \lambda \) values.

Note that in Fig. 5, the service distribution has a much larger variance with respect to the exponential and Erlang distributions \((k \geq 1)\), i.e., for fixed mean service rate smaller \( k \) means larger variance for the service distribution. In all of our numerical experiments, we observe invariantly that average AoI increases and the improvement brought by waiting is more significant for larger variances with fixed mean. In Fig. 6, we directly address this point.

In Fig. 6, we compare the average AoI with zero-waiting and optimal deterministic waiting (over all pairs \((\epsilon_I, \epsilon_B)\)) with respect to the parameter \( k \) where we set \( \lambda = 1 \) and \( \mu = 0.1 \).

In general, as the variance increases, the tail of the service distribution gets heavier and hence this observation supports [23] in our queueing system. We also observe that exponential and Erlang type service distributions promise little improvement in average AoI.

B. Gamma Service Distribution

Gamma distribution is defined as \( f_S(s) = \frac{k^k}{\Gamma(k)} s^{k-1} e^{-k\mu s} \) for \( s \geq 0 \). For this distribution, \( E[S] = \frac{1}{\mu} \) and \( k > 0 \) is the shape parameter that determines the variance and tail behavior. As \( k \) gets smaller, the tail gets heavier. We have the following
waiting. Service time is Gamma distributed with $\mu$ and various $\lambda$ states in the same renewal theory-based spirit of reference [23].

includes optimization of the waiting times according to system $M/GI/1$ partitioning the area under AoI evolution curve to calculate $W$. We highlight the benefits of waiting in the average AoI under $M/GI/1$ on the system state, the server waits a deterministic time and no feedback of the server state is available. Depending when preemption is not an option. In this paper, we apply 1 $M/GI/1$ $A$. A useful byproduct of our analysis is a new method of $V.C.O.N.C.L.U.S.I.O.N.S$ $A.P.P.E.N.D.I.X$

V. CONCLUSIONS

In this paper, we investigate Age of Information (AoI) for $M/GI/1/2^*$ discipline with deterministic waiting deliberately introduced before service starts. Waiting is known to be useful in terms of minimizing Age of Information especially when preemption is not an option. In this paper, we apply waiting when packets arrive randomly from an outside source and no feedback of the server state is available. Depending on the system state, the server waits a deterministic time before starting service of the packet in the queue. We determine average AoI expressions for $M/GI/1/2^*$ discipline. We highlight the benefits of waiting in the average AoI under inverse Gaussian and Gamma (exponential and Erlang as special cases) distributed service times.

A useful byproduct of our analysis is a new method of partitioning the area under AoI evolution curve to calculate time average AoI, yielding a direct connection of the average AoI expression for $M/GI/1/2^*$ packet management to the expressions for FCFS queuing discipline [2]. Future work includes optimization of the waiting times according to system states in the same renewal theory-based spirit of reference [23].

APPENDIX

A. $E[X_i T_i | (I)]$

1) $\epsilon_i - X_i \leq 0, \epsilon_i - X_i + S_{i-1} > 0$: In this case, $T_i = \epsilon_i - X_i + S_{i-1} + \epsilon_B + S_i$. Let the event $E_1$ denote $\epsilon_i - X_i \leq 0, \epsilon_i - X_i + S_{i-1} > 0$. We have

$$E[X_i T_i | (I), E_1 | Pr[E_1] = E[X_i (\epsilon_i - X_i + S_{i-1} + \epsilon_B + S_i)] Pr[E_1]$$

$$= (2E[S] + \epsilon_i + \epsilon_B) e^{-\lambda \epsilon_i} \frac{1 + \lambda \epsilon_i}{\lambda}$$

$$- \frac{e^{-\lambda \epsilon_i}}{\lambda^2} (2 + 2\lambda \epsilon_i + \lambda^2 \epsilon_i^2)$$

$$- (E[S] + \epsilon_B + \epsilon_i) \frac{e^{-\lambda \epsilon_i}}{\lambda}$$

$$((1 + \lambda \epsilon_i) MGF^{(S)}_\lambda + \lambda MGF^{(S,1)}_\lambda)$$

$$- \frac{e^{-\lambda \epsilon_i}}{\lambda^2} ((1 + \lambda \epsilon_i) MGF^{(S,1)}_\lambda + \lambda MGF^{(S,2)}_\lambda)$$

$$+ \frac{e^{-\lambda \epsilon_i}}{\lambda^2} ((2 + 2\lambda \epsilon_i + \lambda^2 \epsilon_i^2) MGF^{(S)}_\lambda$$

$$+ (2 + 2\lambda \epsilon_i + \lambda^2 \epsilon_i^2) MGF^{(S,1)}_\lambda$$

$$+ (2 + 2\lambda \epsilon_i + \lambda^2 \epsilon_i^2) MGF^{(S,2)}_\lambda$$

2) $\epsilon_i > X_i$: In this case, $T_i = \epsilon_i - X_i + S_i$. Let the event $E_2$ denote $\epsilon_i > X_i$. We have

$$E[X_i T_i | (I), E_2] Pr[E_2] = \int_0^{\epsilon_i} x (\epsilon_i - x + E[S]) \lambda e^{-\lambda x} dx$$

$$= \frac{\epsilon_i + E[S]}{\lambda} (1 - e^{-\lambda \epsilon_i} (1 + \lambda \epsilon_i))$$

$$- \frac{1}{\lambda^2} (2 - e^{-\lambda \epsilon_i} (2 + 2\lambda \epsilon_i + \lambda^2 \epsilon_i^2))$$

3) $\epsilon_i + S_{i-1} \leq X_i$: In this case, $T_i = \epsilon_i + S_i$. Let the event $E_3$ denote $\epsilon_i + S_{i-1} \leq X_i$. We have

$$E[X_i T_i | (I), E_3] Pr[E_3] = E[X_i (\epsilon_i + S_i)] Pr[E_3]$$

$$= (\epsilon_i + E[S]) \frac{e^{-\lambda \epsilon_i}}{\lambda} ((1 + \lambda) MGF^{(S)}_\lambda + \lambda MGF^{(S,1)}_\lambda)$$

We finally sum the three expressions to get $E[X_i T_i | (I)]$:

$$E[X_i T_i | (I)] = \sum_{i=1}^3 E[X_i T_i | (I), E_i] Pr[E_i]$$

(14)

B. Definition of $g(r)$

$g(r)$ defines $E[X_i T_i | R_{i-1} = r]$ where $R_{i-1}$ denotes residual time for packet $i$ to start service. It is expressed as:

$$g(r) = \frac{r + E[S]}{\lambda} (1 - e^{-\lambda r} (1 + \lambda r))$$

$$- \frac{1}{\lambda^2} (2 - e^{-\lambda r} (2 + 2\lambda r + \lambda^2 r^2))$$

$$+ (2E[S] + r + \epsilon_B) e^{-\lambda r} \frac{1 + \lambda r}{\lambda} - \frac{e^{-\lambda r}}{\lambda^2} (2 + 2\lambda r + \lambda^2 r^2)$$

$$- (E[S] + \epsilon_B + r) e^{-\lambda r} ((1 + \lambda r) MGF^{(S)}_\lambda + \lambda MGF^{(S,1)}_\lambda)$$

$$- \frac{e^{-\lambda r}}{\lambda} ((1 + \lambda r) MGF^{(S,1)}_\lambda + \lambda MGF^{(S,2)}_\lambda)$$

6
Then, we have
\[ h(x) = \frac{1}{x^2} \left( e^{-\lambda x} (2E[S] - 3) + (E[S] - \lambda - 2)x + 0.5\lambda x^2 + xe^{-\lambda x} (E[S] - 1) \right) - e^{-\lambda x} (2\epsilon_B + 2E[S] - 3x) - xe^{-\lambda x} (\epsilon_B + E[S] + x) + \lambda e^{-\lambda x} (MGF^{(S)}_\lambda (2 + x) - E[S] (x + 2)) - \lambda e^{-\lambda x} (MGF^{(S)}_\lambda (2x^2 + 2MGF^{(S)}_\lambda x + MGF^{(S)}_\lambda)) + 4MGF^{(S)}_\lambda \lambda x + 4MGF^{(S)}_\lambda (2 + MGF^{(S)}_\lambda) - e^{-\lambda x} (\epsilon_I + E[S]) (MGF^{(S)}_\lambda (2 + x) + MGF^{(S)}_\lambda)) \]

Then, we have
\[ \mathbb{E}[X, T_i (W aI)] = \frac{h(\epsilon_I) - h(0)}{\epsilon_I} \]
\[ \mathbb{E}[X, T_i (W aB)] = \frac{h(\epsilon_B) - h(0)}{\epsilon_B} \]

Let the event \( E_3 \) denote \( R_{i-1} \geq X_i \). Let the event \( E_3 \) denote \( R_{i-1} \geq X_i \). We have
\[ \mathbb{E}[X, T_i (W aB)] = \mathbb{E}[X_i (R_{i-1} - X_i + \epsilon_B + S_i)] e^{\lambda(\epsilon_B + E[S] + E[R])} - 2 + \frac{MGF^{(R,1)}_\lambda (2 - \lambda(\epsilon_B + E[S]))}{\lambda^2} \]

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