A control allocation method based on equivalent virtual control surfaces

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Abstract

Most existing allocation methods need the control effectiveness matrix to solve the problem, and their allocation accuracy directly depends on the identification precision of the control effectiveness matrix. If the matrix is poorly identified, the designed allocator may not work well. This paper presents an allocation method based on equivalent virtual control surface whose aerodynamic effect is equivalent to the real effectors. The method divides the design of control allocation system into two parts: the longitudinal and lateral, solve the problem and find the allocation law by choosing reasonable equivalent virtual effectors. Simulation results show that the proposed method can make the responses of roll angle and yaw angle follow the command signals well.

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Introduction

In a conventional aircraft, there are three major control effectors: aileron, elevator and rudder. Flight control systems are usually designed utilizing one control effector for each rotational degree of freedom. Essentially, the aileron is used differentially to produce a rolling moment, the elevator is to generate a pitching moment, and the rudder controls the yawing moment. With three desired moments and three independent control effectors to generate these moments, a unique solution can be found. However, with the improvement of the aircraft performance in reliability and maneuverability, the number of the control effectors is increased. So control modes and combination modes of the control surfaces are no longer unique. How to distribute the three-axis moments into the control surfaces reasonably becomes a complex problem to be solved by control allocation techniques [1].

In order to distribute the control surfaces effectively, scholars have done a lot of research work. Many existing methods [2] to the control allocation problem involve inverting non-square matrix using
generalized inverses, such as: pseudo-inverse, generalized inverse and daisy chain method. This non-
square matrix is called control effectiveness matrix. It is a very important concept. Most allocation
methods can not solve the allocation problem without the matrix. Besides above methods, the matrix is
also used in the calculations of other methods, such as direction allocation method [3] and quadratic
programming based allocation methods [4]. Direction allocation method need the matrix to solve the
attainable moment subset. Allocation methods based on quadratic programming usually involve an
objective function which contains the matrix.

Most allocation methods need the control effectiveness matrix to solve the problem, and their
allocation accuracy directly depends on the identification precision of the control effectiveness matrix. If
the matrix is poorly identified, the designed allocator may not work well against model errors.
Furthermore, most methods consider the longitudinal allocation problem and lateral allocation problem
together. It’s complicated for engineering application. Focus on these questions, this paper presents an
allocation method based on equivalent virtual control surface whose aerodynamic effect is equivalent to
the real effectors. The proposed method divides the allocator design into two parts: the longitudinal and
lateral. It can provide allocation solutions without the directly using of control effectiveness matrix.

1. Description of the Problem

Consider an m-dimensional control space \( u(t) \in \mathbb{R}^m \). The controls are constrained to minimum and
maximum values, defined by the constrained control subset:
\[
\Omega = \{ u \in \mathbb{R}^m | u_i \leq u \leq u_i \} \subset \mathbb{R}^m
\]
These controls generate moments \( v(t) \) through a linear mapping \( B : \mathbb{R}^m \rightarrow \mathbb{R}^3 (m > 3) \) onto a 3-
dimensional moment space \( v(t) \in \mathbb{R}^3 \). If \( v(t) \) is specified, control allocation is the solving of the constrained equation:

\[
Bu(t) = v(t)
\]

Here \( u(t) \) is the sought which should meet the desired system performance. With the introduction of
an allocator, the state equation of the aircraft can be expressed as:

\[
\dot{x} = Ax + Bu
\]

The structure diagram of flight control system is shown as follows:

When FCS is designed, it is not only necessary to design controller to satisfy the request of the system
performance, but also design the allocator to distribute the moments to control surfaces reasonably.

Design of the flight control system is a very complicated job. Engineers usually divide the work into
two parts: longitudinal control law design and lateral control law design. Consider the habits of the
engineer, this paper solve the longitudinal allocation problem and lateral allocation problem respectively.

2. Longitudinal control allocation design

In order to discuss the problem convenient, here we use a generic dynamic model of an aircraft with
canard configuration. Its longitudinal state equation is expressed as equation 3.Here \( \alpha \) is the attack angle;
\( q \) is the pitch rate. The control \( \delta_c \) is symmetric canard deflection; \( \delta_e \) is symmetric elevons deflection;
\( \delta_f \) is symmetric trailing edge flap deflection.
In a conventional aircraft, the elevator is used to generate a pitching moment to control pitch movement. It’s hoped the experiences and control methods used in conventional aircraft can also be used in advanced aircrafts. So we give an equivalent transformation and convert the state equation of aircraft with multiple effectors to that of conventional aircraft. Suppose the aerodynamic effect of above three control surfaces is equivalent to a virtual elevator ($E_\delta$). The state equation 3 is transformed to:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} Z_\alpha & Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_\alpha} & Z_{\delta_q} \\ M_{\delta_\alpha} & M_{\delta_q} \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}$$

(3)

Here, $Z_{\delta_\alpha}$ and $M_{\delta_\alpha}$ are the effectiveness values of virtual control surface. According to the equivalency of aerodynamic effect, following equation should be satisfied:

$$\begin{bmatrix} \alpha \\ q \end{bmatrix} = \begin{bmatrix} Z_\alpha & Z_q \\ M_\alpha & M_q \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} Z_{\delta_\alpha} & Z_{\delta_q} \\ M_{\delta_\alpha} & M_{\delta_q} \end{bmatrix} \begin{bmatrix} \delta_x \\ \delta_y \end{bmatrix}$$

(4)

Our main idea is to design its control law by the methods and experiences used in conventional aircraft. Then keep the control law unchanged, substitute the virtual effector with real effectors and solve the allocation problem. According to formula 5, in order to solve the allocation problem, the value $Z_{\delta_\alpha}$ and $M_{\delta_\alpha}$ should be obtained first. These two values depend on the physical features of virtual effector. It is very important to give a reasonable physical description for the virtual effector. The position limits and rate limits are two major performance indexes. Usually the controller parameters are scheduled with small input angles, so the position limit of virtual effector can be ignored at first. Check it after design.

The choice of the rate limits has great influence on the dynamic response of the system. If the chosen value is too big, the real effectors will not reach the same aerodynamic effect. It is safe and efficient to choose the values close to the rate limits of the real effectors. Such a choice ensures the aerodynamic effect of virtual effector is no more than that of real effectors. When the design is finished with virtual effector substituted by the real effectors, the choice will make the system still follow the command well.

Although the value $Z_{\delta_\alpha}$ and $M_{\delta_\alpha}$ can not be estimated directly, their ranges can be decided by analyzing the relationship between equation 3 and equation 4:

$$Z_{\delta_\alpha} \in \max(|Z_{\delta_\alpha}|, |Z_{\delta_q}|, |Z_{\delta_x}|, |Z_{\delta_y}|, |Z_{\delta_\alpha}| + |Z_{\delta_q}| + |Z_{\delta_x}| + |Z_{\delta_y}|), \quad M_{\delta_\alpha} \in \max(|M_{\delta_\alpha}|, |M_{\delta_q}|, |M_{\delta_x}|, |M_{\delta_y}|, |M_{\delta_\alpha}| + |M_{\delta_q}| + |M_{\delta_x}|)$$

If all the rate limits of effectors are the same, the values of $Z_{\delta_\alpha}$ and $M_{\delta_\alpha}$ can be simply selected by:

$$Z_{\delta_\alpha} = |Z_{\delta_\alpha}| + |Z_{\delta_q}| + |Z_{\delta_x}| + |Z_{\delta_y}| \quad \text{and} \quad M_{\delta_\alpha} = |M_{\delta_\alpha}| + |M_{\delta_q}| + |M_{\delta_x}| + |M_{\delta_y}|.$$ With the effectiveness value ranges and the rate limit of the virtual effector decided, we can use the genetic algorithm to solve the longitudinal control allocation problem. Here PID control method is used and the control law is:

$$\delta_\epsilon = K_\theta \Delta \theta + K_q \Delta q + K_\alpha \Delta \alpha.$$ 

Choose $J = \int |\theta| dt$ ($\theta$ is the pitch angel) as the fitness function and use the genetic algorithm to optimize the control system. All these controller parameters can be obtained. Then keep the control law unchanged and substitute the virtual effector with real effectors. Choose formula 6 as the allocation law and optimize values $Z_{\delta_\alpha}$ and $M_{\delta_\alpha}$ again. So the longitudinal control allocation system is designed.
3. Lateral control allocation design

The lateral state equation of the aircraft mentioned above is:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{p} \\
\dot{\gamma}
\end{bmatrix} =
\begin{bmatrix}
Y_{\beta} & \sin\alpha_{0} & -\cos\alpha_{0} \\
L_{\beta} & L_{p} & l_{r} \\
N_{\beta} & N_{p} & N_{r}
\end{bmatrix}
\begin{bmatrix}
\beta \\
p \\
r
\end{bmatrix} +
\begin{bmatrix}
Y_{\delta_{c}} & Y_{\delta_{e}} & Y_{\delta_{r}} \\
L_{\delta_{c}} & L_{\delta_{e}} & L_{\delta_{r}} \\
N_{\delta_{c}} & N_{\delta_{e}} & N_{\delta_{r}}
\end{bmatrix}
\begin{bmatrix}
\delta_{c} \\
\delta_{e} \\
\delta_{r}
\end{bmatrix}
\]

(6)

Here \( \beta \) is sideslip angle; \( p \) is roll rate; \( r \) is yaw rate; control \( \delta_{c} \) is differential canard deflection; \( \delta_{c} \) is differential elevons deflection; \( \delta_{\text{f}} \) is differential trailing edge flap deflection; \( \delta_{r} \) is rudder. Similar with the design of the longitudinal control allocation system, the equivalent lateral state equation of aircraft with multiple aircraft is:

\[
\begin{bmatrix}
\dot{\beta} \\
\dot{p} \\
\dot{\gamma}
\end{bmatrix} =
\begin{bmatrix}
Y_{\beta} & \sin\alpha_{0} & -\cos\alpha_{0} \\
L_{\beta} & L_{p} & l_{r} \\
N_{\beta} & N_{p} & N_{r}
\end{bmatrix}
\begin{bmatrix}
\beta \\
p \\
r
\end{bmatrix} +
\begin{bmatrix}
Y_{\delta_{c}} & Y_{\delta_{e}} \\
L_{\delta_{c}} & L_{\delta_{e}} \\
N_{\delta_{c}} & N_{\delta_{e}}
\end{bmatrix}
\begin{bmatrix}
\delta_{c} \\
\delta_{e} \\
\delta_{r}
\end{bmatrix}
\]

(7)

Here \( \delta_{c} \) is the virtual aileron and \( \delta_{r} \) is the virtual rudder. \( Y_{\delta_{c}}, Y_{\delta_{e}}, L_{\delta_{c}}, L_{\delta_{e}}, N_{\delta_{c}}, N_{\delta_{e}} \) are effectiveness values. According to the equivalency of aerodynamic effect, follow equation is satisfied:

\[
\begin{bmatrix}
\delta_{\text{Dc}} \\
\delta_{\text{Da}} \\
\delta_{\text{DF}}
\end{bmatrix} =
\begin{bmatrix}
Y_{\delta_{c}} & Y_{\delta_{c}} & Y_{\delta_{c}} & Y_{\delta_{c}} & Y_{\delta_{c}} & Y_{\delta_{c}} \\
L_{\delta_{c}} & L_{\delta_{c}} & L_{\delta_{c}} & L_{\delta_{c}} & L_{\delta_{c}} & L_{\delta_{c}} \\
N_{\delta_{c}} & N_{\delta_{c}} & N_{\delta_{c}} & N_{\delta_{c}} & N_{\delta_{c}} & N_{\delta_{c}}
\end{bmatrix}
\begin{bmatrix}
\delta_{D} \\
\delta_{D} \\
\delta_{D}
\end{bmatrix}
\]

(8)

For lateral movement, it’s desired that the roll control and yaw control can be designed decoupled. So we make the virtual rudder only produced by real rudder. It means the aerodynamic effect of control surfaces \( \delta_{c}, \delta_{a}, \delta_{f} \) is equivalent to a virtual aileron and the effectiveness values of virtual rudder can be directly selected by \( Y_{\delta_{c}}, L_{\delta_{c}}, N_{\delta_{c}} \). The choice of the rate limits and the decision of \( Y_{\delta_{c}}, L_{\delta_{c}}, N_{\delta_{c}} \) are the same with the longitudinal. Use the PID control method to design the controller. The control law is:

\[
\begin{align*}
\delta_{D} &= K_{p}\Delta \phi + K_{i}\Delta \phi + K_{d}\Delta \beta, \\
\delta_{r} &= K_{p}\Delta \phi + K_{i}\Delta \phi + K_{d}\Delta \beta
\end{align*}
\]

(9)

\( K_{p}, K_{i}, K_{d}, K_{p}, K_{i}, K_{d}, K'_{p}, K'_{d} \) are the parameters to be optimized. Choose \( J = \int \Delta \phi dt + \int \Phi \Delta \beta dt \) as the fitness function and use the genetic algorithm to optimize the control system. All the parameters of the controller are obtained. Keep the control law unchanged and substitute the virtual effectors with real effectors. Choose formula 8 as the allocation law and optimize values \( Y_{\delta_{c}}, L_{\delta_{c}}, N_{\delta_{c}} \) again. So the longitudinal control allocation system is designed.

4. Simulations

The following example applies above techniques to design the control allocation system. Equation 10 is a state-equation of an aircraft. The data in matrices are based on a linear approximation model which is linearized at a flight condition of 3000m, Mach 0.3. The control surfaces are canards, ailerons, trailing edge flaps and rudder: whose deflection limits are shown in Table 1.
\[
\begin{bmatrix}
\dot{\beta} \\
\dot{\rho} \\
\dot{r}
\end{bmatrix} =
\begin{bmatrix}
-0.168 & 0.1303 & -0.9816 \\
-14.5955 & -1.3423 & 0.5268 \\
0.8291 & -0.0889 & -0.2992
\end{bmatrix}
\begin{bmatrix}
\beta \\
\rho \\
r
\end{bmatrix}
+ 
\begin{bmatrix}
-0.012 & 0.0064 & 0.027 & 0.0395 \\
1.5968 & -9.1574 & -7.8826 & 2.6919 \\
-0.794 & -0.4028 & -0.8512 & -1.6265
\end{bmatrix}
\begin{bmatrix}
\delta_Dc \\
\delta_Da \\
\delta_Df
\end{bmatrix}
\]

(10)

First convert the state equation according to formula 7. Because the rate limits of all the control surfaces are the same, effectiveness values of virtual effectors can be simply selected:

\[
\begin{bmatrix}
Y_{\delta_c} & L_{\delta_c} & N_{\delta_c}
\end{bmatrix} =
\begin{bmatrix}
0.0395 & 2.6919 & -1.6265
\end{bmatrix}
\]

\[
\begin{bmatrix}
Y_{\delta_a} & L_{\delta_a} & N_{\delta_a}
\end{bmatrix} =
\begin{bmatrix}
0.0454 & -18.6368 & -2.048
\end{bmatrix}
\]

Choose the rate ±50°/s as the rate limits of the virtual effectors. Use the genetic algorithm to optimize the control law given in formula 9. The optimization result is:

\[
K_\phi = -5.27 \\
K_\rho = -1.66 \\
K_\beta = 0.58 \\
K_\varphi = -15.9 \\
K_\psi = -12 \\
K_\beta' = -0.638
\]

Keep the control law unchanged, substitute the virtual effectors with real effectors and choose formula 8 as the allocation law. Give the command \(\phi = 5^\circ, \varphi = 5^\circ\). The simulation results are shown in figure 2. It can be seen that the control allocation system can make the roll angle and yaw follow the command well.

![Fig 3. (a) Response of the roll angle](image1)

![Fig 3. (b) Response of the yaw angle](image2)

5. Conclusions

A control allocation method based on equivalent virtual control surfaces is proposed. The method divides the design of allocator into two parts: the longitudinal and lateral, solve the problem and find the corresponding allocation law by choosing reasonable equivalent virtual effectors. Simulation results show that the proposed method can make the responses of roll angle and yaw angle follow the command signals well.

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