The nuclear configurational entropy impact parameter dependence in the Color-Glass Condensate

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The impact parameter (b) dependence on the saturation scale, in the framework of the Color Glass Condensate (b-CGC) dipole model, is investigated from the configurational point of view. During the calculations and analysis of the quantum nuclear states, the critical points of stability in the configurational entropy setup are computed, matching the experimental parameters that define the onset of the quantum regime in the b-CGC in the literature with very good accuracy. This new approach is crucial and important for understanding the stability of quantum systems in study of deep inelastic scattering processes.

PACS numbers: 89.70.Cf,24.85.+p

I. INTRODUCTION

The nuclear configurational entropy [1] takes into the cross-section as a natural localized, square-integrable functions, driving the configurational entropy setup in nuclear physics. This concept has been already applied to gauge dualities involving quantum chromodynamics (QCD) models in Refs. [2, 3] for studying the stability of mesons and scalar glueballs. The nuclear configurational entropy is based upon the information entropy, implemented by the recently introduced configurational entropy [6–9], for spatially-localized physical systems. As the the cross section of any nuclear reaction is spatially-localized and employed to characterize the probability that any reaction occurs, the analysis of the shape complexity of classical field configurations can be implemented in the context of cross sections. In nuclear physics one measures the cross section values for the systems with a finite spatial extent. In order to describe the system, thus, one needs as much information, as the higher the informational entropy is. A brief analysis between the configurational entropy lattice approach and statistical mechanics has been paved in Ref. [2]. In a similar context, the Hawking-Page transition was analysed [5] and Bose-Einstein condensates have been studied [4] with interesting results.

The strong interaction at high energy regimes has been studied in both experiment and theory. QCD is a theory of strong interactions that is able to explain most of their underlying experimental results to remarkable accuracy. Another important source of information is numerical lattice calculation, which produces correct solution to the QCD field equations. However, a lot of questions still is open and the lattice data still need to be interpreted in order to find the mechanisms and principles that lead to these numerical results. In this case, the Shannon based informational entropy can shed some light onto the principal results based on the lattice QCD setup and introduces a quantitative theoretical apparatus for studying the instability of high-excited nuclear system. In this work, the cross sections will be analysed with regard to some principal points of the associated informational entropy. Our focus in this letter is applying the Color Glass Condensate to nuclear collisions.

Colliding heavy nuclei with each other makes QCD to predict that a new state of matter, called the quark-gluon plasma (QGP), is then formed [10]. The most precise measurements of the quark and gluon structure of the proton come from the HERA particle accelerator, which collided electrons and positrons with protons. In these experiments, the proton can look very differently, depending on at which scale it is measured. When the proton structure is probed with a photon that has a long wavelength compared with the proton size, a charged particle with electric charge +e is seen. The inner structure of the proton becomes visible when the photon wavelength is decreased to the order of the proton radius. First, one observes three valence quarks having a fractional electric charge and carrying a fraction ∼ 1/3 of the proton longitudinal momentum, viewed in the frame where the proton energy is very large. When the wavelength of the photon decreases more, a richer structure becomes visible. The photon starts to see a large number of sea quarks and antiquarks that carry a small fraction of the proton longitudinal momentum, denoted by Bjorken-
a new vision. In the KKT model [11], it was assumed that that the gluons inside particles are seen to other particles
as a gluon wall, that describes the Color Glass Condensate itself. In other words, gluons from the inside have a high
density distribution. Increasing the energy increases the
momentum states that are occupied by the gluons, forcing a weaker coupling among the gluons [11–13]. This
leads to the gluon saturation effect, which corresponds to a
multiparticle Bose condensate state.

QCD at high energies can be described as a many-body
theory of partons which are weakly coupled albeit non-
perturbative due to the large number of partons. Such a
system is called a Color Glass Condensate (CGC) and de-
scribes an effective perturbative weak-coupling field the-
ory approach in the small-$x$ regime of QCD. The initial
conditions for high energy collisions are determined by
the free partons in the wave functions of the colliding
cnuclei.

Partons represent the localized-energy systems, they
can be considered as the spatially-coherent field config-
urations in an informational entropic context, together
with the Shannon entropy of information theory [8]. The
critical points of the configurational entropy correspond
to the onset of instability of a spatially-bound configura-
tion. In a considerable amount of studies, the configura-
tional entropy has been applied to analyze aspects in a
variety of models, which comprise the higher spin mesons
and glueballs stability [2, 3] as well as Bose-Einstein con-
densates of long-wavelength gravitons, which describes
black holes. [4]. The physical systems, that were studied
in above mentioned works, had configurations of classical
fields and have been successfully analyzed from the view
of critical points, including the derivation of the Higgs
mass [15, 16].

It is interesting to analyze the configurational entropy
from the point of view of spatially-localized reaction cross
sections as [1]

$$\sigma(\tilde{k}) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \sigma(r) e^{i\tilde{k}\cdot \tilde{r}} d\tilde{r}. \quad (1)$$

Then the modal fraction can be expressed as [6–9]:

$$f_\sigma(\tilde{k}) = \frac{|\sigma(\tilde{k})|^2}{\int_{-\infty}^{\infty} |\sigma(\tilde{k})|^2 d\tilde{k}}. \quad (2)$$

Finally, one can define the configurational entropy anal-
ogous to what has been defined for the energy density in
Ref. [8], but this time for the cross section, as

$$S_\sigma[f] = -\int_{-\infty}^{\infty} f_\sigma(\tilde{k}) \log f_\sigma(\tilde{k}) d\tilde{k}, \quad (3)$$

where $f(\tilde{k}) = f(\tilde{k})/f_{\max}(\tilde{k})$. Critical points of the con-
figurational entropy imply that the system has informa-
tional entropy that is critical with respect to the maximal
entropy $f_{\max}(\tilde{k})$, corresponding to more dominant states
[2, 4, 5].

II. THE CONFIGURATIONAL ENTROPY AND
KKT MODEL

Study of deep inelastic scattering (DIS) reactions
and exclusive diffractive processes of leptons on protons
and/or nuclei, such as, for instance, exclusive vector
mesons production and virtual Compton scattering at
small-$x$, is turned toward understanding the mechanism
of QCD. Many interesting questions which have arisen
since the beginning of the investigation of strong inter-
actions, such as the behavior of cross sections at high
energies, the universality of hadronic interactions at high
ergies, have acquired fresh vigor. In the CGC approxi-
mation, one of the important ingredients for particle pro-
duction is the universal dipole amplitude, represented by
the imaginary part of the quark-antiquark scattering am-
plitude on a target.

The impact parameter ($b$) dependence of the dipole
amplitude is essential for understanding exclusive diffrac-
tive processes in the CGC or the color dipole approach.
Therefore, in this work we will briefly represent a simple
dipole model, b-CGC, that incorporates all known prop-
erties of the gluon saturation, and the impact parameter
dependence of dipole amplitude [17]. The b-CGC model
has been applied to various reactions, as deep inelas-
tic scattering and diffractive processes [17] and proton-
nucleus [18] collisions at RHIC and the LHC.

At small $x$, the deep inelastic scattering is character-
ized by the fluctuation of the virtual photon $\gamma^*$ into a
quark-antiquark pair $qq$ with size $r$, which then scatters
off the hadronic or nuclear target via gluon exchanges.
Then, the total deep inelastic cross section, $\gamma^*p$, for a
given Bjorken $x$ and virtuality $Q^2$ will be expressed as
[19]:

$$\sigma_{L,T}^{\gamma^*p}(Q^2, xQ, \Psi \sum_f \int d^2 \vec{r} \int d^2 \vec{b} \int_{0}^{1} dz |\Psi_{L,T}^{(f)}(r, z, m_f; Q^2)|^2 \times \mathcal{N}(x, r, b), \quad (4)$$

where $z$ is the fraction of the light-front momentum of the
virtual photon carried by the quark, $m_f$ is the quark
mass, and $\mathcal{N}(x, r, b)$ is the imaginary part of the forward
$q\bar{q}$ dipole-proton scattering amplitude with dipole size
$r$ and impact parameter $b$. The form of light front wave
function $\Psi_{L,T}^{(f)}$, for $\gamma^*$ fluctuations into $qq$, was taken
from Ref. [20]. The subscript $L,T$ in Eq. (5) denotes the lon-
gitudinal and transverse polarizations of the virtual pho-
on. In Ref. [21] it was suggested a simple dipole model,
which links two limiting behavior of Eq. (5), namely in
the vicinity of the saturation line for small dipole sizes,
$r \ll 1/Q_s$, and for the case of deep inside the saturation
region for larger dipoles, $r \gg 1/Q_s$. Such a model is
historically called CGC dipole model, in which the color
dipole-proton amplitude is given by:

$$N(x, r, b) = \begin{cases} N_0 \left(\frac{Q_s}{Q}\right)^{2\gamma-1}, & rQ_s \leq 2, \\ 1 - \exp(-A\log^2(BrQ_s)), & rQ_s > 2, \end{cases} \quad (5)$$
where the effective anomalous dimension is expressed by formula
\[ \gamma_{eff} = \gamma_s + \frac{1}{\kappa Y} \log \left( \frac{2}{rQ_s} \right), \]  
(6)
where \( Y = \log(1/x) \) and \( \kappa = \chi''(\gamma_s)/\chi'(\gamma_s) \), with \( \chi \) being the characteristic function. The scale \( Q_s \) in Eqs. (5, 6), is generally called the saturation scale. In the CGC dipole model, the scale \( Q_s \) is given by following expression:
\[ Q_s \rightarrow Q_s(x) = \left( \frac{x_0}{x} \right)^{2} \text{GeV}. \]  
(7)
The parameters \( A \) and \( B \) in Eq.(5), are determined from the matching of the dipole amplitude and its logarithmic derivatives at \( rQ_s = 2 \):
\[ A = - \frac{N_s^2}{(1-N_0)^2} \log(1-N_0), \quad B = \frac{1}{2} (1-N_0)^{-1} = N_0 \]  
(8)
The amplitude is considered to be independent from the impact parameter in the Color Glass Condensate model. Thus, the integral over impact parameter in Eq. (4) can be considered as a normalization factor \( \sigma_0 = 2 \int d^2b \), and is determined by a fit to data. Therefore, the total dipole cross section will be estimated as \( \sigma_{CGC} = \sigma_0 N(x,r) \). The parameters \( \kappa = 9.9 \) and \( N_0 = 0.7 \) are fixed [17, 21], and the other four parameters, namely \( \gamma_s, x_0, \lambda, \sigma_0 \), are obtained by a fit to the HERA data via a \( \chi^2 \) minimization procedure.
In the so-called b-CGC model [17] the authors have been extended the CGC dipole model by involving the dependence of an amplitude of an impact parameter. Therefore, the dependence of the saturation scale on impact parameter in expression (8) can be changed by:
\[ Q_s \rightarrow Q_s(x,b) = \left( \frac{x_0}{x} \right)^{2} \exp \left( - \frac{b^2}{4\gamma_s B_{CGC}} \right) \text{GeV}. \]  
(9)
In Eq. (9), instead of \( \sigma_0 \) in the CGC dipole model, the free parameter is \( B_{CGC} \). It was determined by other reactions, for instance, the \( t \)-distribution of the exclusive diffractive processes at HERA.
In order to compute the informational entropy associated to the Color Glass Condensate, let us start by calculating the spatial Fourier transform of the total deep inelastic cross section, and thus the color dipole-proton amplitude, is defined in b-CGC model [17]. For such a purpose, Eqs. (1 - 3) are used in the 2-dimensional case. First of all, the Fourier transform of the total deep inelastic cross section (4) together with Eq. (5) can be computed using Eq. (1). After it, the result of the calculations is used to estimate the modal fraction, Eq. (2). At last, using an Eq. (3) one can calculate the informational entropy utilizing the modal fraction.
The numerical results follows, after awkward algebraic manipulations. The results were obtained for three given values of impact parameter, equal to \( b = 0; b = 0.404 \), and \( b = 0.693 \), for \( x = 10^{-4} \) (Fig. 1) and \( x = 10^{-5} \) (Fig. 2), respectively. As one can see from Fig. 1, the black dotted curve that depicts the minimum of the nuclear configurational entropy for \( b = 0 \) corresponds to the value of parameter \( c = 4.100 \). In the case of impact parameter \( b = 0.404 \) (gray dotted curve), the nuclear configurational entropy minimum occurs at \( c = 4.050 \) and in the more peripheral collision when \( b = 0.693 \), which is shown by light gray curve on Fig.1, the minimum of the informational entropy falls at \( c = 4.120 \), establishing the onset of quantum regime in the CGC.
In the case of the Bjorken variable given by \( x = 10^{-6} \), which is shown by Fig. 2, the minimum in the black dotted curve for impact parameter value \( b = 0 \) corresponds to \( c = 4.080 \). When value of the impact parameter turn to the \( b = 0.404 \) (grey curve), the minimum of the nuclear configurational entropy corresponds to \( c = 4.040 \) and in the more peripheral collision when \( b = 0.693 \), which is shown by light gray curve on Fig.2, the minimum of the nuclear configurational entropy falls at \( c = 4.120 \). Therefore, one can conclude the best results of the impact parameter \( b = 0.404 \), that corresponds to the fitted parameter \( c \), and was assumed to be \( c = 4.01 \), in Ref. [1], to derive the results therein presented. These results match the one in Ref. [12].
The calculation shows that such it is a natural choice provided by the analysis of the minima of the nuclear configurational entropy, which derives the universal dipole amplitude with impact parameter of the collision in the Color Glass Condensate model setup. The uncertainties of the calculation are upper bound by \( \sim 1.23\% \).

![FIG. 1. Configurational entropy as a function of the onset of the gluon anomalous dimension, in the Color-Glass Condensate regime, for distinct values of the Bjorken variable \( x = 10^{-4} \). Each curve was generated for intervals of \( c = 0.01 \). The black curve is depicted for \( b = 0 \text{ GeV}^{-1} \), the light grey curve is plot for \( b = 0.7 \text{ GeV}^{-1} \) and the grey curve represents \( b = 0.4 \text{ GeV}^{-1} \).](image-url)

**A. Outlook**

The nuclear configurational entropy was used to derive the onset of the CGC, with impact parameter dependence, matching results in the literature with a very good accuracy [1, 12], of around \( \sim 1\% \). Figs. 1 and 2, and the analysis that respectively follow, illustrate the critical...
points of the nuclear configurational entropy as a way to derive the onset of the quantum regime in the CGC, for different values of the impact parameter \( b \). A direction to be further studied encompasses quantum mechanics fluctuations. The wave function used in those equations can be explored in the context of topological defects proposed, e. g., in Refs. [22–28], in the configurational entropy setup. In order to improve the understanding of the nature of deep inelastic scattering, as well as the diffractive processes, further systematically investigation is needed concerning the gluon saturation in the proton wave function.

**ACKNOWLEDGMENTS**

GK thanks to FAPESP (grant No. 2016/18902-9), for partial financial support.

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