Arbitrarily High Super-Resolving Phase Measurements at Telecommunication Wavelengths

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We present two experiments that achieve phase super-resolution at telecommunication wavelengths. One of the experiments is realized in the space domain and the other in the time domain. Both experiments show high visibilities and are performed with standard lasers and single-photon detectors. The first experiment uses six-photon coincidences, whereas the latter needs no coincidence measurements, is easy to perform, and achieves, in principle, arbitrarily high phase super-resolution. Here, we demonstrate a 30-fold increase of the resolution. We stress that neither entanglement nor joint detection is needed in these experiments, demonstrating that neither is necessary to achieve phase super-resolution.

I. INTRODUCTION

Interference plays a crucial role in many physical measurements, such as detection of gravitational waves [1–3], metrology [4–5], interferometry and atomic spectroscopy [6–8], imaging [9], and lithography [10]. Improvements of these schemes can be achieved with the help of phase super-resolution, where n oscillations (fringes) appear in the interference pattern over a range which usually would have given only one oscillation [11,12]. In a similar vein, phase super-sensitivity decreases the phase uncertainty in such experiments, so that the measurement sensitivity would surpass the classical limit, i.e., beating the standard quantum limit [13,14].

It was believed that entangled states are needed to achieve phase super-resolution [13]. One such state is a path-entangled number-state, the so-called NOON-state [16],

$$|\Psi(0)\rangle = \frac{1}{\sqrt{2}} (|N\rangle |0\rangle + |0\rangle |N\rangle) , \tag{1}$$

where N denotes the number of particles (most often photons). This state is a superposition of N particles in one path and no particles in the other path, and vice versa. The production of such states at satisfying rates gets extremely difficult already for small N. So far, only experiments with N up to 4 have been reported [11,17]. Recently, however, schemes were proposed and shown, where phase super-resolution could be achieved by unentangled, coherent light [13]. In that paper an experiment was reported, where n = N = 6 oscillations occur over the period of one classical oscillation. For their experiment, Resch et. al. used 6-photon coincidence, but got rather limited counting rates ($\leq$ 27 counts/10 s) and moderate visibility (50-90 %). Other experiments with unentangled states and phase super-resolution with n > 4 have been reported [18]. All of those experiments used light at wavelengths around 800 nm.

In this letter, we report on two super-resolution experiments. One is in the space domain like all the experiments hitherto reported, but using an innovative approach resulting in higher counting rates and very high visibility. The other experiment is performed in the time domain and shows very promising counting rates and visibility. Both of the experiments could in principle be scaled to high numbers of n > 100. Whereas the first experiment requires more components for higher numbers of n, the second experiment only requires a longer measurement time. Our space- and time-domain experiments were performed up to n = 6 and n = 30, respectively. Furthermore, these experiments are, to the best of our knowledge, the first ones performed at telecommunication wavelengths which allows efficient transmission of the photons over long distance via optical fiber and thereby allow more opportunities for applications of phase super-resolution.

II. PHASE SUPER RESOLUTION - “DEQUANTIFIED”

We briefly introduce the theory of phase super-resolution.

Consider the state given in Eq. (1). Imposing a relative phase-shift of $\phi$ between the two modes transforms the state into

$$|\Psi(\phi)\rangle = \frac{1}{2} (|N\rangle |0\rangle + e^{iN\phi} |0\rangle |N\rangle) , \tag{2}$$

since energy (difference) is the generator of (relative) phase. The phase $N\phi$ will therefore grow linearly with the number of particles N. Treating the two modes for example as spatial modes and combining them via a 50/50 beamsplitter would result in a detection probability $P \propto 1 \pm \cos (N\phi)$ in the two outputs of the beamsplitter when measuring N-fold coincidence. $P$ will exhibit phase super-resolution, since it oscillates n = N times when $\phi$ varies from 0 to 2$\pi$. The same effect, however, can also be reached without entanglement by a so-called time-reversal measurement [13]. In the cited
paper the effect is explained by using the inherent time-reversal symmetry of quantum mechanics and measurement of entanglement. A different view of the experiment is the following, based on the mathematical relation
\[
\frac{\sin(n\phi)}{2n-1} = \sin(\phi) \sin\left(\frac{\phi + \pi}{n}\right) \sin\left(\frac{\phi + 2\pi}{n}\right) \cdots \\
\cdots \sin\left(\frac{\phi + (n-1)\pi}{n}\right). \tag{3}
\]

A phase super-resolving measurement can hence be implemented as a multiplication (e.g., coincidence detection) of \(n\) ordinary phase measurements, each shifted by \(k\pi/n\), where \(k = 0, 1, \ldots, n-1\).

A coherent state can be split into several modes, where each of these will be a coherent state. Since the ensuing multi-mode state is separable, there are no quantum correlations between the states. E.g., a coherent-state in an optical beam can be split in two halves by a mirror, as in Fig. 1. If the two “half beams” come from the same or from different, identical sources makes no difference. It also does not matter if the beam is split after the interaction by a totally reflecting mirror inserted halfway into the beam, or by a semitransparent beam splitter, insofar the measured object characteristic does not vary over the width of the beam. Hence, we may as well split the beam already before the object, as in Fig. 2. It also does not matter if the coherent state mode is split spatially, temporally or in frequency insofar the object does not vary over the corresponding space-, time-, or frequency range. In previous experiments the state has been split spatially, by beam splitters, after the interaction. We find it simpler to split the beam before the interaction, or in time, and this and the observation delineated in Eq. (3) are the basic ingredients of our experiments and the interpretation thereof.

III. EXPERIMENT - SPACE DOMAIN

In our experiment, to measure phase super-resolution in the space domain we built the setup illustrated in Fig. 2. Except for the fact that we split the coherent state before the half-wave plate (HWP), the setup is essentially identical to that in Ref. 13. The coherent state is generated by a \(\lambda = 1550\) nm laser, which is pulsed at a rate of \(2.5\) MHz. Every pulse has a duration of around \(500\) ps and the average power of the laser is \(1\) mW, which, directly after the laser, is attenuated by \(50 - 55\) dB (not shown in the figure) to avoid “overexposure” of our single-photon detectors (SPD). A polarised beam-splitter (PBS) in front of the laser assures that the light is horizontally (H) polarised. Two 50/50 beam-splitters (BS) divide the beam into three paths, which are superimposed on a HWP. The HWP can be rotated to any angle \(\phi\) by a motor. One has to assure that the angle \(\alpha\) between the beams is small so that the pathlengths of the beams in the HWP (the object) are essentially the same. In our case, we use an angle of \(\alpha = 2.9^\circ\). Since \(\varphi = 45^\circ\) turns the polarisation from H to vertical (V) one can see one oscillation by turning the HWP by \(90^\circ\) (corresponding to a relative-phase shift of \(360^\circ\) in our figures) and detect, for example, only H-polarised photons. Two
of the beams pass another HWP rotated by the angles 15° and 30°, respectively, to rotate the polarization further in such a way that they fulfill the relation in Eq. (3) after detection. Each beam then pass another PBS, carefully oriented so that the H-polarised light goes into one arm and the V-polarised light into the other arm. These 6 beams are then coupled into single-mode fibres which lead to SPDs. The SPDs are gated and open only for 1 ns when a coherent-state pulse is coming, and the outputs of the SPDs are led to a six-channel coincidence counter, and subsequently to a computer where the coincidences are registered and stored.

The results can be seen in Fig. 3. Every SPD detected one oscillation while turning the HWP by 90° (corresponding to a phase shift of 360 degrees in the figure), but from one SPD to the next the peak of the oscillation was shifted by 60 degrees. Multiplying the stored, individual counts from all the six SPDs in the computer gives the black dots in the figure, where the scale is in arbitrary units. In the case of the 6-fold coincidences (grey dots) the scale to the left applies. One can clearly distinguish 6 peaks as expected, so a sixfold increase of the phase resolution is achieved. At every angle we were measuring for 10 seconds and got counting rates of more than 1500 counts/s for the 6-fold coincidences and about 10⁶ counts/s for the individual SPDs. The visibility is high, between 98.6 % and 96.7 % in the case of the multiplication of the single detections and between 98.6 % and 97.0 % in the case of the coincidence detection. For calculating the visibility we took the lowest of the to minima around a peak. The reason that two of the minima are higher than the other ones is due to imperfections in one of the PBSs, which does not perfectly split the beam into H- and V-polarised components. Having better PBSs would probably result in a more uniform visibility >98 %. Another improvement would be to replace the first 50/50 BS with a 33/66 BS so that all the beams have the same intensity. This would neither change the visibility nor the resolution, but would increase the coincidence rate. In principle, the setup can be extended to detect phase super-resolution with a factor \( n > 6 \) by adding more arms. The drawbacks are that this requires more components and that the coincidence counting-rate gets exponentially lower so that one would have to increase the measurement time correspondingly.

**IV. EXPERIMENT - TIME DOMAIN**

To obtain phase super-resolution with \( n > 6 \) it therefore is more practical to do the measurement in the time domain. To this end we built the setup in Fig. 4. The parameters of the laser are the same as in the setup in the space domain, except a slightly higher attenuation. Instead of having several physical arms in the setup we used a computer to store the data sequentially as to have \( n \) “arms after each other” in time. In this setup we counted and stored the clicks in each SPD as a function of \( \varphi_1 \), turned the second HWP by an angle \( \Delta \varphi_2 = 90°/n \), repeated the process \( n \) times and numerically multiplied the results according to equation (3). For \( n = 6 \) we set the second HWP at the angles \( \varphi_2 = 0°, 15°, 30°, \ldots, 75° \) when using only one SPD.

The results of our measurements can be seen in Fig. 5. We were rotating the first HWP by small increments over
the range $0 \leq \varphi_1 \leq 90^\circ$ to introduce a differential phase-shift. Then we change the angle of the second HWP after each scan of $\varphi_1$ by $\Delta \varphi_2 = 9^\circ$ to get phase super-resolution by a factor of $n = 10$. At each combination of angles we measured for one second. The upper panel of Fig. 5 shows the results when we used both SPDs followed by a coincidence counter, and the lower panel shows the results when we used only one SPD, requiring twice as many settings of the angle $\varphi_2$. As one can see, the first method gives higher visibility, 99.6 %, and takes only half the time, and is therefore more efficient. On the other hand does the second method need neither the second SPD nor the coincidence detector in Fig. 4. This clearly shows that phase super-resolution can be achieved with neither entanglement nor joint detection.

Further results can be seen in Fig. 6, where we achieved phase super-resolution by a factor of $n = 30$. (Note that the $x$-axis in this case only spans the range $0^\circ$ to $45^\circ$.) In this case too, every combination of angles $\varphi_1, \varphi_2$ was measured for one second. The reason that the visibility is degraded is most certainly due to the insufficient stability of our laser’s intensity over long times. Since higher values of $n$ in our experiments require smaller measurement increments of $\varphi_1$ to resolve one oscillation period, we had to measure for a long time to get phase super-resolution with $n = 30$ over the whole range of $\varphi_1$ between 0 and 360 degrees. (1800 settings of $\varphi_1$, 30 settings of $\varphi_2$ and 1 s measurement time per setting plus time for rotation gives around 20 hours.) Under practical conditions, however, this should not pose any substantial problem, since one usually is not interested in scanning the whole range between 0 and 360 degrees with phase super-resolution, but rather make some rough scans to start with, and then limit oneself to a narrow range of angles $\varphi_1$ or phases. Furthermore, in most cases, measurement times of less than one second per setting should give sufficient visibility.

An advantage of the time-domain method is that one can get phase super-resolution with high factors $n$. A technical limitation of our setup is the stepsize resolution $\Delta \varphi_1$ and $\Delta \varphi_2$ in turning the HWP. Stepsizes of $\Delta \varphi = 0.1^\circ$ posed no problem in our setup, theoretically the rotator specification allows a resolution of around one minute of arc. If $m$ denotes the number of points in each fringe this gives the possibility to achieve $n = 90/(m \Delta \varphi)$, for example $m = 3$ and one minute of arc as the stepsize of the rotator would allow $n = 1800$. A more fundamental problem is the measurement time. The object under investigation should not undergo any changes in phase during the measurement time. Additionally, the intensity of the laser should be stable during the whole measurement time because the SPDs have no way to tell whether a change in the counting rate is due to a phase change or on due to an intensity variation of the laser. Since our laser was not sufficiently stable over long times we could not use the full advantage of the small stepsizes achievable by our rotators, but “only” achieved $n = 30$.

We stress that our experiments did not manifest any phase super-sensitivity since we used coherent states, subject to shot noise. For phase super-sensitivity it seems that entanglement is required [17].

V. INTERPRETATION

Phase super-resolution turns out to be quite a simple and mostly classical phenomenon. We have shown that neither entanglement nor joint detection is needed to achieve it (note that the coincidence unit and the second detector are not necessary in Fig. 4 as explained in the previous section). There is neither a need for time-reversal symmetry as introduced in [13] nor, e.g., measurement induced, post-selection entanglement. Basically everything one has to do is to shift sine-functions as in Eq. (3) and multiplying them. This task can be done with quite ordinary standard optical components and multiplication, either by coincidence detection or by numerical multiplication of the acquired data in a computer.

In principle these findings were known since Glauber’s pioneering work on quantum optics [19]. The correlation functions $\langle g^{(n)}(x_1 \cdots x_{2n}) \rangle$ of any order $n$, where $x$ denotes the time- and space-coordinates are symmetric in time and space. This, together with Glauber’s finding that $n$-fold delayed coincidences which are “detected by ideal photon counters reduce to a product of the detection rates of the individual counter” leads to the fact that our experiments in time- and space-domain give the same kind of physics as all the other hitherto reported experiments, yet with much better results. We have capitalized on this knowledge for the design and explanation of our setup, that outperforms the experiments shown so far, and we hope that it will clarify the requirements for achieving super-resolving phase measurements.
VI. CONCLUSION

In summary, we showed that super-resolving phase measurements with simultaneously high $n$ (denoting the relative increase in the number of fringes) and high visibility can be achieved. Whereas there are clear limitations to the increase of $n$ in the space domain, it is less demanding to reach high values of $n$ in the time domain. We showed the principle up to $n = 30$ and explained how one could reach higher values without requiring any extra components by just using a stabilized laser and high quality optics. We furthermore performed the experiment at telecommunications wavelengths, enabling possible remote applications since the phase-shifting (including the polarization analyzers) and the detectors, coincidence unit and control computer can be stationed at different, remote locations.

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