Estimation of water level and steam temperature using ensemble Kalman filter square root (EnKF-SR)

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Abstract. The equipment unit which has the most vital role in the steam-powered electric power plant is boiler. Steam drum boiler is a tank functioning to separate fluida into has phase and liquid phase. The existence in boiler system has a vital role. The controlled variables in the steam drum boiler are water level and the steam temperature. If the water level is higher than the determined level, then the gas phase resulted will contain steam endangering the following process and making the resulted steam going to turbine get less, and the by causing damages to pipes in the boiler. On the contrary, if less than the height of determined water level, the resulted height will result in dry steam likely to endanger steam drum. Thus an error was observed between the determined. This paper studied the implementation of the Ensemble Kalman Filter Square Root (EnKF-SR) method in nonlinear model of the steam drum boiler equation. The computation to estimate the height of water level and the temperature of steam was by simulation using Matlab software. Thus an error was observed between the determined water level and the steam temperature, and that of estimated water level and steam temperature. The result of simulation by Ensemble Kalman Filter Square Root (EnKF-SR) on the nonlinear model of steam drum boiler showed that the error was less than 2%. The implementation of EnKF-SR on the steam drum boiler r model comprises of three simulations, each of which generates 200, 300 and 400 ensembles. The best simulation exhibited the error between the real condition and the estimated result, by generating 400 ensemble. The simulation in water level in order of 0.00002145 m, whereas in the steam temperature was some 0.00002121 kelvin.

1. Introduction

In Indonesia, electric power generation is carried out by various types of complex power plant installations. One of them is PLTU (Steam Power Plant). PLTU is one of the producers of electric power that provides electricity to meet the needs of all people who are expected to work optimally, therefore it is intended that the electricity distribution process is not hampered [1]. To support achieving that goal would require supporting components or equipments that always operate in good condition. Among the supporting equipments one unit that plays an important role is the boiler. Steam drum boiler is a tank that serves to separate the fluid into gas phase and liquid phase [2]. Its presence in the system plays a very important role. Measurement of water level and steam temperature in steam drum boilers is of great importance for the safety and operational efficiency of the boiler. It can be said that steam drum boilers are the heart of a boiler. This is where the steam used to turn turbines was first produced [3]. Than an estimator is required to predict and correct the water level and steam temperature of the steam drum boiler. Estimation is made for problem solving that requires previous information so that the next step can be determined in solving the problem. Estimation methods are often used in robotics such as for estimation of AUV trajectory [4,5,6] and Missile Trajectory [7]. Maglev ball position estimation [8]. Kalman filter is a method of estimating state variables of a discrete linear dynamic system that minimizes the estimation error covariance.

Kalman filter was first introduced by Rudolph E. Kalman in 1960. It was about problem solving on linear discrete data filtering. In the nonlinear model another approach is required as an extension of the Kalman filter called Ensemble Kalman Filter (EnKF). In the EnKF method, the algorithm is
executed by generating a certain number of ensembles to calculate the mean value and the error covariance of the state variable [5]. The development of EnKF method is through modification of algorithm by appending a square root scheme at EnKF correction stage which can produce Ensemble Kalman Filter Square Root (EnKF-SR) method.

The contribution of this paper is that estimation of both water level and steam temperature in the steam drum boiler can be an effective way to predict the increase and the decrease of water level and steam temperature in the steam boiler in future. In this paper, the implementation of square root scheme on Ensemble Kalman Filter (EnKF) method on Steam Drum Boiler equation was done, then it was applied to estimate the water level and the vapor temperature by simulation using Matlab software so that the error was observed in water level and vapor temperature determined with water level estimation and steam temperature.

2. Steam Drum Boiler

Steam drum is a container to store water in a large volume and to separate steam and water after heating takes place inside the boiler. The mathematical model of the boiler steam drum is formulated with two variables: the water level and the steam temperature.

![Figure 1 steam drum boiler system](image)

The mathematical model of the above figure is [2]:

\[
\begin{align*}
A \frac{dh}{dt} &= F_{in} - F_{out} \\
Ah \frac{dT}{dt} &= F_{in}(T_{in} - T) + \frac{Q}{\rho C_p} \\
\text{with } F_{out} &= kw\sqrt{h}
\end{align*}
\]

whereas:

- \( F_{in} \): Flow of in-coming water (kg/hour)
- \( F_{out} \): Flow of out-going water (kg/hour)
- \( T \): Temperature of vapor (K)
- \( T_{in} \): Temperature of in-coming water (K)
- \( Q \): Flow of vapor (kg/hour)
- \( V \): Volume of water (m\(^3\))
- \( A \): Area of steam drum boiler (m\(^2\))
- \( h \): Height of water level (m)
- \( \rho \): Mass of water (kg/m\(^3\))
- \( C_p \): Capacity of heat in steam drum (J/kg K)
- \( kw \): control valve of water flow (m)
- \( k \): coefficient of control valve (m\(^{3/2}\)/hour)

Because the system requires discretization, so the steam drum boiler model in equation (1) – (2) must be discreted using the finite difference method.

Equation (1) and (2), If \( h_k \) water level and steam temperature \( T \) are
The change of state variables respect to the time are approximated by forward scheme of finite difference. Thus we will get

\[ \dot{h} = \frac{dh}{dt} \approx \frac{h_{k+1} - h_k}{\Delta t} \]

\[ \dot{T} = \frac{dT}{dt} \approx \frac{T_{k+1} - T_k}{\Delta t} \]

from equation (4) and (5) will be gotten the modified steam drum boiler model in (6) below

\[
\begin{bmatrix}
T_{k+1} \\
h_{k+1}
\end{bmatrix} = 
\begin{bmatrix}
F_{in} - kw\sqrt{h_k} \\
F_{in}(T_{in} - T_k) + \frac{Q}{\rho c_p}
\end{bmatrix}
\]

3. **Ensemble Kalman Filter Square Root (EnKF-SR)**

This section present EnKF-SR algorithm to estimated nonlinear or linear dynamic system and measurement model, the algorithm *Ensemble Kalman Filter Square Root (EnKF-SR)* can be seen [7]:

Model system and measurement model

\[ x_{k+1} = f(x_k, u_k) + w_k \]

\[ z_k = Hx_k + v_k \]

\[ w_k \sim N(0, Q_k), \quad v_k \sim N(0, R_k) \]

1. **Initialization**

   Generate *N ensemble* as the first guess \( \tilde{x}_0 \)

   \[ x_{0,i} = [x_{0,1}, x_{0,2}, \ldots, x_{0,N}] \]

   The first Mean Ensemble: \( \bar{x}_{0,i} = x_{0,i} \)

   The first Ensemble error: \( \tilde{x}_{0,i} = x_{0,i} - \bar{x}_{0,i} = x_{k,i}(1 - I_N) \)

2. **Time Update**

   \( \tilde{x}_{k,i} = f(\tilde{x}_{k-1,i}, u_{k-1,i}) + w_{k,i} \)

   where \( w_{k,i} \sim N(0, Q_k) \)

   Mean Ensemble: \( \bar{x}_{k,i} = \tilde{x}_{k,i} \)

   Error Ensemble : \( \tilde{x}_{k,i} = \tilde{x}_{k,i} - \bar{x}_{k,i} = \tilde{x}_{k,i}(I - 1_N) \)

3. **Measurement Update**

   \( z_{k,i} = Hx_{k,i} + v_{k,i} \)

   where \( v_{k,i} \sim N(0, R_k) \)

   \[ S_k = H\tilde{x}_{k,i}, \quad E_k = (v_1, v_2, ..., v_N) \]

   \[ C_k = S_k S_k^T + E_k E_k^T \]

   Mean Ensemble :

   \( \tilde{x}_{k,i} = \tilde{x}_{k-1,i} + \tilde{x}_{k-1,i}^T C_k^{-1}(\tilde{x}_{k,i} - H\tilde{x}_{k-1,i}) \)

   **Square Root Scheme:**

   - eigenvalue decomposition from \( C_k = U_k \Lambda_k U_k^T \)

   - determine matrix \( M_k = \frac{1}{2} U_k^T S_k \)

   - determine SVD from \( M_k = Y_k \Lambda_k V_k^T \)

   Ensemble Error: \( \tilde{x}_{k,i} = \tilde{x}_{k,i}^T V_k (I - L_k^{-1} L_k^T)^{\frac{1}{2}} \)

   Ensemble Estimation: \( \hat{x}_{k,i} = \tilde{x}_{k,i} + \tilde{x}_{k,i} \)

To evaluate of estimation result accuracy from EnKF-SR algorithm, can be show with calculate Root Mean Square Error (RMSE) [5].
\[ RMSE = \sqrt{\frac{\sum_{i=1}^{n}(X_{\text{obs},i}(k)-X_{\text{model},i}(k))^2}{n}} \]  

With  
\(X_{\text{obs},i}(k)\) = observation data  
\(X_{\text{model},i}(k)\) = model data  
\(n\) = iteration

4. Computational Result

This simulation was made by applying the EnKF-SR algorithm to the boiler steam drum model. The simulation results were evaluated and compared with real conditions. This simulation consisted of three simulations of 200, 300 and 400 ensembles applied for estimation of the water level and the steam temperature. The value of \(\Delta t\) used was \(\Delta t = 0.1\), and the initial condition used was \(h = 0.765\) m and \(T = 786\) K.

Figures 2 and 3 show the estimation of the water level and the vapor temperature of steam drum boilers by generating 400 ensembles, while figure 3 and 4 show the result of estimation by generating 300 ensembles. Figure 2 shows very accurate estimation results with an error of 0.0021 meters and accuracy of 98.2%. Figure 3 shows a very accurate estimation result with an error of 0.004 Kelvin and accuracy of 99.1%. Figures 2 and 3 illustrate that the estimation by EnKF-SR with 400 ensembles and iteration of 200 showed significantly accurate result which was very close to the real condition of the height of the water level.

Figure 2. Estimation of water level using 400 Ensemble
**Figure 3.** Estimation of Steam Temperature using 400 Ensemble

**Figure 4.** Estimation of water level using 300 Ensemble
Further comparison of estimation results with 200, 300 and 400 ensembles in Table 1 shows that with the 400 ensembles the most accurate results were in water level and vapor estimates with errors of \(2.145 \times 10^{-5}\) meter dan \(2.121 \times 10^{-5}\) Kelvin. In terms of simulation time, with 400 ensembles it took a longer time than it did with 200 and 300 ensembles because the generation of numbers of ensemble greatly affected the computing time.

In general as seen in the table 1, the result of the three simulations were highly accurate. The first simulation by generate 200 ensemble with error of water level 0.0003704 meter or accuracy of 99\% and error of steam temperature 0.00003655 kelvin or accuracy of 99.1\%. the second simulation by generate 300 ensemble with error of water level 0.00003028 m or accuracy of 99.2\% and accuracy of steam temperature 99.3\%.

![Figure 5](image_url)  
Figure 5. Estimation of Steam Temperature using 300 Ensemble

| Table 1. Comparison of the values of RMSE by EnKF-SR method by generating 200, 300 and 400 ensembles |
|---------------------------------------------|
|                                    | 200 Ensemble | 300 Ensemble | 400 Ensemble |
|---------------------------------------------|
| Water Level                               | 0.00003704 m | 0.00003028 m | 0.00002145 m |
| Steam Temperature                         | 0.00003655 kelvin | 0.00003015 kelvin | 0.00002121 kelvin |
| Time Simulation                           | 3.3438 s     | 5.2156 s     | 6.974 s       |

5. Conclusion  
Based on the results of the simulation analysis by generating 200, 300 and 400 ensembles, the EnKF-SR method could be applied to estimate water levels and steam temperatures with a high degree of accuracy. Water level errors and vapor temperatures were of less than 2\%. Square Root Scheme could affect estimation result because it could be implemented in the correction phase of the EnKF method.

Open problem. How to implemented Unscented Kalman Filter (UKF) and Unscented Kalman filter Square Root (UKF-SR) for estimation of water level and steam temperature in steam drum boiler.
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