Measure of fuzzy \((i, j)-s\)-compactness

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Abstract
In this paper, the notion of fuzzy \((i, j)-s\)-compactness degrees is introduced in \(L\)-fuzzy topological spaces by means of the implication operation of \(L\). Characterizations of fuzzy \((i, j)-s\)-compactness degrees in \(L\)-fuzzy topological spaces are obtained, and some properties of fuzzy \((i, j)-s\)-compactness degrees are researched.

Keywords
\(L\)-bitopological spaces, fuzzy \((i, j)-s\)-compactness, Fuzzy \((i, j)-s\)-compactness degree.

AMS Subject Classification
54A40, 54D30, 03E72.

1 Introduction
It is known that compactness and its stronger and weaker forms play very important roles in topology. Based on fuzzy topological spaces introduced by Chang [4], various kinds of fuzzy compactness [4, 7] have been established. However, these concepts of fuzzy compactness rely on the structure of \(L\) and \(L\) is required to be completely distributive. In [10], for a complete De Morgan algebra \(L\), author introduced a new definition of fuzzy compactness in \(L\)-topological spaces using open \(L\)-sets and their inequality. This new definition does not depend on the structure of \(L\). In this paper, the notion of fuzzy \((i, j)-s\)-compactness degrees is introduced in \(L\)-fuzzy topological spaces by means of the implication operation of \(L\). Characterizations of fuzzy \((i, j)-s\)-compactness degrees in \(L\)-fuzzy topological spaces are obtained, and some properties of fuzzy \((i, j)-s\)-compactness degrees are researched.

2 preliminaries
Throughout this paper, \((L, \lor, \land, ^{'}\) is a complete De Morgan algebra, \(X\) a nonempty set and \(L^{X}\) the set of all \(L\)-fuzzy sets (or \(L\)-sets for short) on \(X\). The smallest element and the largest element in \(L\) are denoted by \(0\) and \(1\). The smallest element and the largest element in \(L^{X}\) are denoted by \(0\) and \(1\). An element \(a\) in \(L\) is called a prime element if \(b \land c \leq a\) implies that \(b \leq a\) or \(c \leq a\). An element \(a\) in \(L\) is called a co-prime element if \(a^{'}\) is a prime element [6]. The set of nonunit prime elements in \(L\) is denoted by \(P(L)\) and the set of nonzero co-prime elements in \(L\) by \(M(L)\). The binary relation \(<\) in \(L\) is defined as follows: for \(a, b \in L, a < b\) if and only if for every subset \(D \subseteq L\), the relation \(b \leq \sup D\) always implies the existence of \(d \in D\) with \(a \leq d\) [5]. In a completely distributive De Morgan algebra \(L\), each element \(b\) is a sup of \(\{a \in L|a < b\}\). The set \(s(b) = \{a \in L|a < b\}\) is called the greatest minimal family of \(b\) in the sense of [7, 13]. Now, for \(b \in L\), we define \(s^{'}(b) = s(b) \cap M(L), \alpha(b) = \{a \in L|a^{'} < b^{'}\}\) and \(\alpha^{'}(b) = \alpha(b) \cap P(L)\). In a complete De Morgan frame \(L\), there exists a binary operation \(\rightarrow\). Explicitly the implication is given by \(a \rightarrow b = \forall c \in L.a \land c \leq b\). We interpret \([a \leq b]\) as the degree to which \(a \leq b\), then \([a \leq b] = a \rightarrow b\).

Definition 2.1. [15] An \(L\)-topology on a set \(X\) is a mapping \(\tau: L^{\rightarrow}L\) which satisfies the following conditions:

1. \(\tau(1) = \tau(0) = 1\);
2. for any \(A, B, \tau(A \cap B) \geq \tau(A) \land \tau(B)\);
3. for any \(A_{\lambda} \in L^{X}, \lambda \in \Delta, \tau(\bigvee_{\lambda \in \Delta} A_{\lambda}) \geq \bigwedge_{\lambda \in \Delta} \tau(A_{\lambda})\).

The pair \((X, \tau)\) is called an \(L\)-fuzzy topological space. \(\tau(U)\) is called the degree of openness of \(U\), \(\tau^{'}(U) = \tau(U')\)
is called the degree of closedness of $U$, where $U'$ is the $L$-complement of $U$. For any family $\mathcal{W} \subset L^X$, $\tau(\mathcal{W}) = \bigwedge_{A \in \mathcal{W}} \tau(A)$ is called the degree of openness of $U$.

For a subfamily $\Phi \subset L^X$, $2(\Phi)$ denotes the set of all finite subfamilies of $\Phi$. For any $a \in L$, $\alpha$ denotes a constant value mapping from $X$ to $L$, its value is $a$.

**Definition 2.2.** An $L$-bitopological space (or $L$-bts for short) is an ordered triple $(X, \tau_1, \tau_2)$, where $\tau_1$ and $\tau_2$ are subfamilies of $L^X$ which contains $\emptyset, X$ and is closed for any suprema and finite infima.

**Definition 2.3.** [24] An $L$-fuzzy inclusion on $X$ is a mapping $\tilde{\tau} : L^X \times L^X \rightarrow L$ defined by the equality $\tilde{\tau}(A, B) = \bigwedge_{x \in X} (A'(x) \vee B(x))$.

In this paper, we will write $[A \tilde{\tau} B]$ instead of $\tilde{\tau}(A, B)$.

**Definition 2.4.** [9] Let $(X, \tau)$ be an $L$-ts, $a \in L \setminus \{1\}$, and $A \in L^X$. A family $\mu \subseteq L^X$ is called

1. an $a$-shading of $A$ if for any $x \in X$, $A'(x) \vee \bigwedge_{B \in \mu} B(x) \not\subseteq a$.

2. a strong $a$-shading of $A$ if $\bigwedge_{x \in X} (A'(x) \vee \bigwedge_{B \in \mu} B(x)) \not\subseteq a$.

**Definition 2.5.** [9] Let $(X, \tau)$ be an $L$-ts, $a \in L \setminus \{0\}$ and $A \in L^X$. A family $\mu \subseteq L^X$ is called

1. an $a$-remote neighborhood family of $A$ if for any $x \in X$, $(A(x) \wedge \bigwedge_{B \in \mu} B(x)) \not\supseteq a$.

2. a strong $a$-remote neighborhood family of $A$ if $\bigwedge_{x \in X} (A(x) \wedge \bigwedge_{B \in \mu} B(x)) \not\supseteq a$.

3. a $s_a$-cover of $A$ if for any $x \in X$, it follows that $a \in s(A'(x) \bigwedge_{B \in \mu} B(x))$.

4. a strong $s_a$-cover of $A$ if for any $x \in X$, it follows that $a \in s\left(\bigwedge_{x \in X} (A'(x) \bigwedge_{B \in \mu} B(x))\right)$.

5. A $Q_a$-cover of $A$ if for any $x \in X$, it follows that $\bigwedge_{B \in \mu} B(x) \not\subseteq a$.

**Definition 2.6.** [11] Let $(X, \tau_1, \tau_2)$ be an $L$-bts, $A \in L^X$. Then $A$ is called an $(i, j)$-semi-open set if $A \leq j \text{Cl}(i \text{Int}(A))$. The complement of an $(i, j)$-semi-open set is called an $(i, j)$-semi-closed set. Also, $(i, j)$-$SO(L^X)$ and $(i, j)$-$SC(L^X)$ will always denote the family of all $(i, j)$-semi-open sets and $(i, j)$-semi-closed sets respectively. Obviously, $A \in (i, j)$-$SO(L^X)$ if and only if $A' \in (i, j)$-$SC(L^X)$.

**Definition 2.7.** [11] Let $(L^X, \tau_1, \tau_2)$ be an $L$-bitopological space, $A, B \in L^X$. Let $(i, j)$-$\text{Int}(A) = \bigvee \{B \in L^X | B \leq A, B \in (i, j)$-$SO(L^X)\}$, $(i, j)$-$\text{Cl}(A) = \bigwedge \{B \in L^X | A \leq B, B \in (i, j)$-$SC(L^X)\}$. Then $(i, j)$-$\text{Int}(A)$ and $(i, j)$-$\text{Cl}(A)$ are called the $(i, j)$-semi-interior and $(i, j)$-semi-closure of $A$, respectively.

**Definition 2.8.** Let $(X, \tau_1, \tau_2)$ be an $L$-fuzzy bitopological space on $X$. For any $A \in L^X$, define a mapping $\tau_{(i,j)} : L^X \rightarrow L$ by $\tau_{(i,j)}(A) = \bigwedge_{B \in A} (\tau_i(B)) \wedge \bigwedge_{x \in X} (\tau_j(D'))$. Then $\tau_{(i,j)}$ is called the $L$-fuzzy $(i, j)$-semi-open operator induced by $\tau_1$ and $\tau_2$, where $\tau_{(i,j)}(A)$ can be regarded as the degree to which $A$ is $(i, j)$-semi-open and $(i, j)$-$\text{Int}(A) = \tau_{(i,j)}(A' \bigwedge \{B \in L^X \mid A \subseteq B, B \in (i, j)$-$SC(L^X)\})$ can be regarded as the degree to which $B$ is $(i, j)$-semi-closed. For any family $\mathcal{W} \subset L^X$, $\tau_{(i,j)}(\mathcal{W}) = \bigwedge_{A \in \mathcal{W}} \tau_{(i,j)}(A)$ is called the degree of $(i, j)$-semi-openness of $\mathcal{W}$.

**Definition 2.9.** Let $(X, \tau_1, \tau_2)$ be an $L$-fuzzy bitopological space on $X$ and let $\tau_{(i,j)}$ be the $L$-fuzzy $(i, j)$-semi-open operator induced by $\tau_1$ and $\tau_2$. Then $\tau_{(i,j)}(A) \leq \tau_{(i,j)}(A)$ for any $A \in L^X$.

**Definition 2.10.** Let $(X, \tau_1, \tau_2)$ be an $L$-fuzzy bitopological space. $G \in L^X$ is said to be $L$-fuzzy $(i, j)$-s-compact if for every family $\mathcal{W} \subset L^X$, it follows that $\bigwedge_{A \in \mathcal{W}} \tau_{(i,j)}(A) \wedge \bigwedge_{x \in X} (G'(x) \vee \bigwedge_{Y \in \mathcal{W}} A(x)) \leq \bigwedge_{Y \in \mathcal{W}} G'(x) \vee \bigwedge_{X \in \mathcal{W}} A(x)$.

**3. Measures of fuzzy $(i, j)$-s-compactness**

Let $(X, \tau_1, \tau_2)$ be an $L$-bitopological space and $G \in L^X$. Then $G$ is fuzzy $(i, j)$-s-compact if and only if for every family $\mathcal{W}$ of $(i, j)$-semi-open $L$-sets, it follows that $\bigwedge_{A \in \mathcal{W}} (G'(x) \vee \bigwedge_{x \in X} A(x)) \leq \bigwedge_{Y \in \mathcal{W}} G'(x) \vee \bigwedge_{X \in \mathcal{W}} A(x)$.

So for every family $\mathcal{W}$ of $(i, j)$-semi-open $L$-sets, $[G \tilde{\tau} \mathcal{W}] = \bigwedge_{Y \in \mathcal{W}} G'(x) \vee \bigwedge_{X \in \mathcal{W}} A(x) = 1$. We know that an $L$-topology $\tau$ can be looked as a special $L$-fuzzy topology. Therefore, $A \in L^X$ is an $(i, j)$-semi-open set if and only if $\tau_{(i,j)}(A) = 1$. Thus $G$ is fuzzy $(i, j)$-s-compact if and only if for every family $\mathcal{W} \subset L^X$, it follows that $\tau_{(i,j)}(U) \leq [G \tilde{\tau} \mathcal{W}] \leq [G \tilde{\tau} \mathcal{W}] = 1$.

Therefore, we can naturally generalize the notion of fuzzy $(i, j)$-s-compactness degrees to $L$-fuzzy bitopological spaces as follows:

**Definition 3.1.** Let $(X, \tau_1, \tau_2)$ be an $L$-fuzzy bitopological space and $G \in L^X$. The fuzzy $(i, j)$-s-compactness degree $cd_{(i,j)}(G)$ of $G$ is defined as $cd_{(i,j)}(G) = \bigwedge_{\mathcal{W} \subset L^X} (\tau_{(i,j)}(\mathcal{W}) \rightarrow (G \tilde{\tau} \mathcal{W}) \rightarrow G'(x) \vee A(x)) \rightarrow \bigwedge_{X \in \mathcal{W}} A(x)) $.

**Theorem 3.2.** Let $(X, \tau_1, \tau_2)$ be an $L$-fuzzy bitopological space and $G \in L^X$. Then $G$ is fuzzy $(i, j)$-s-compactness in $(X, \tau_1, \tau_2)$ if and only if $cd_{(i,j)}(G) = 1$.

**Proof.** Straightforward.

**Theorem 3.3.** Let $(X, \tau_1, \tau_2)$ be an $L$-fuzzy bitopological space and $G \in L^X$. Then $G$ is fuzzy $(i, j)$-s-compactness in $(X, \tau_1, \tau_2)$ if and only if $cd_{(i,j)}(G) = 1$. 

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Lemma 3.5.\textit{ And only if } \forall(G) \ni \alpha \leq 1. By the definition of } cd(\chi_{\tau_{1}}), \text{ the conclusion is hold.}

Theorem 3.4.\textit{ Let } (X, \tau_{1}, \tau_{2}) \text{ be an } L\text{-fuzzy bitopological space and } G \subseteq L^{X}. \textit{G is } L\text{-fuzzy } (i, j)-s\text{-compactness in } (X, \tau_{1}, \tau_{2}) \text{ if and only if } \forall(G) \ni \alpha \leq 1. By the definition of } cd(\chi_{\tau_{1}}), \text{ the conclusion is hold.}

Lemma 3.5.\textit{ Let } (X, \tau_{1}, \tau_{2}) \text{ be an } L\text{-fuzzy bitopological space and } G \subseteq L^{X}. \textit{Then } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the property (6) of } \rightarrow, \textit{we obtain that } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the definition of } cd(\chi_{\tau_{1}}), \text{ the conclusion is hold.}

Theorem 3.6.\textit{ Let } (X, \tau_{1}, \tau_{2}) \text{ be an } L\text{-fuzzy bitopological space and } G \subseteq L^{X}. \textit{Then } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the property (6) of } \rightarrow, \textit{we obtain that } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the definition of } cd(\chi_{\tau_{1}}), \text{ the conclusion is hold.}

Theorem 3.7.\textit{ Let } (X, \tau_{1}, \tau_{2}) \text{ be an } L\text{-fuzzy bitopological space and } G \subseteq L^{X}. \textit{Then } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the property (6) of } \rightarrow, \textit{we obtain that } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the definition of } cd(\chi_{\tau_{1}}), \text{ the conclusion is hold.}

Theorem 3.8.\textit{ Let } (X, \tau_{1}, \tau_{2}) \text{ be an } L\text{-fuzzy bitopological space and } G \subseteq L^{X}, a \in L\{0\}. \textit{Then the following conditions are equivalent:}

1. \textit{cd}(\chi_{\tau_{1}})(G) \ni \alpha \ni a.

2. \textit{For any } b \in P(L), b \ni a, \textit{each strong } b\text{-shading } \forall(G) \ni \alpha \leq 1. \textit{By the property (6) of } \rightarrow, \textit{we obtain that } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the definition of } cd(\chi_{\tau_{1}}), \text{ the conclusion is hold.}

3. \textit{For any } b \in P(L), b \ni a, \textit{each strong } b\text{-shading } \forall(G) \ni \alpha \leq 1. \textit{By the property (6) of } \rightarrow, \textit{we obtain that } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the definition of } cd(\chi_{\tau_{1}}), \text{ the conclusion is hold.}

4. \textit{For any } b \in P(L), b \ni a, \textit{each strong } b\text{-shading } \forall(G) \ni \alpha \leq 1. \textit{By the property (6) of } \rightarrow, \textit{we obtain that } cd(\chi_{\tau_{1}})(G) \ni \alpha \leq 1. \textit{By the definition of } cd(\chi_{\tau_{1}}), \text{ the conclusion is hold.}
6. For any \( b \in M(L), b \not\leq a \), each strong \( b \)-remote family \( \mathcal{P} \) of \( G \) with \( \tau^s_{(i,j)b}(\mathcal{P}) \not\leq b' \), there exists a finite subfamily \( \mathcal{H} \) of \( \mathcal{P} \) and \( r \in s^*(b) \) such that \( \mathcal{H} \) is an \( r \)-remote family of \( G \).

7. For any \( b \in M(L), b \not\leq a, \) each strong \( b \)-remote family \( \mathcal{P} \) of \( G \) with \( \tau^s_{(i,j)b}(\mathcal{P}) \not\leq b' \), there exists a finite subfamily \( \mathcal{H} \) of \( \mathcal{P} \) and \( r \in s^*(b) \) such that \( \mathcal{H} \) is a strong \( r \)-remote family of \( G \).

8. For any \( b \leq a, r \in s(b), b, r \neq 0 \), each \( Q_b \)-cover \( \mathcal{U} \subset (\tau^s_{(i,j)b})_b \) of \( G \) has a finite subfamily \( \mathcal{V} \) which is a \( Q_r \)-cover of \( G \).

9. For any \( b \leq a, r \in s(b), b, r \neq 0 \), each \( Q_b \)-cover \( \mathcal{U} \subset (\tau^s_{(i,j)b})_b \) of \( G \) has a finite subfamily \( \mathcal{V} \) which is a strong \( s_r \)-cover of \( G \).

10. For any \( b \leq a, r \in s(b), b, r \neq 0 \), each \( Q_b \)-cover \( \mathcal{U} \subset (\tau^s_{(i,j)b})_b \) of \( G \) has a finite subfamily \( \mathcal{V} \) which is a \( s_r \)-cover of \( G \).

11. For any \( b \leq a, r \in s(b), b, r \neq 0 \), each strong \( s_b \)-cover \( \mathcal{U} \subset (\tau^s_{(i,j)b})_b \) of \( G \) has a finite subfamily \( \mathcal{V} \) which is a \( Q_r \)-cover of \( G \).

12. For any \( b \leq a, r \in s(b), b, r \neq 0 \), each strong \( s_b \)-cover \( \mathcal{U} \subset (\tau^s_{(i,j)b})_b \) of \( G \) has a finite subfamily \( \mathcal{V} \) which is a strong \( s_r \)-cover of \( G \).

13. For any \( b \leq a, r \in s(b), b, r \neq 0 \), each strong \( s_b \)-cover \( \mathcal{U} \subset (\tau^s_{(i,j)b})_b \) of \( G \) has a finite subfamily \( \mathcal{V} \) which is a \( s_r \)-cover of \( G \).

In Theorem 3.8 (8)-(13), if we replace \( b, r \neq 0 \) and \( r \in s(b) \) with \( b \in M(L) \) and \( r \in s^*(b) \), then the conclusions are still right.

**Theorem 3.9.** Let \((X, \tau_1, \tau_2)\) be an \( L \)-fuzzy bitopological space and \( G \in L^X, a \in L \setminus \{0\} \). If for any \( c, d \in L, s(c \land d) = s(c) \land s(d) \). Then the following conditions are equivalent:

1. \( \text{cd}_{(i,j)a}(G) \geq a \).

2. For any \( b \in s(a), b \neq 0 \), each strong \( s_b \)-cover \( \mathcal{U} \) of \( G \) with \( b \in s(\tau^s_{(i,j)b}(\mathcal{U})) \) has a finite subfamily \( \mathcal{V} \) which is a \( Q_b \)-cover of \( G \).

3. For any \( b \in s(a), b \neq 0 \), each strong \( s_b \)-cover \( \mathcal{U} \) of \( G \) with \( b \in s(\tau^s_{(i,j)b}(\mathcal{U})) \) has a finite subfamily \( \mathcal{V} \) which is a \( s_b \)-cover of \( G \).

4. For any \( b \in s(a), b \neq 0 \), each strong \( s_b \)-cover \( \mathcal{U} \) of \( G \) with \( b \in s(\tau^s_{(i,j)b}(\mathcal{U})) \) has a finite subfamily \( \mathcal{V} \) which is a strong \( s_b \)-cover of \( G \).

**Theorem 3.10.** Let \((X, \tau_1, \tau_2)\) be an \( L \)-fuzzy bitopological space and \( G, H \in L^X \). Then \( \text{cd}_{(i,j)a}(G \land H) = \text{cd}_{(i,j)a}(G) \land \text{cd}_{(i,j)a}(H) \).

**Proof.** By Theorem 3.7, we have \( \text{cd}_{(i,j)a}(G \land H) = \bigvee \{ a \in L : \tau_{(i,j)a}(\mathcal{U}) \land [(G \land H) \land \mathcal{U}] \leq a \leq \bigwedge \{ (G \land H) \land \mathcal{U} \} \} \).

**Theorem 3.11.** Let \((X, \tau_1, \tau_2)\) be an \( L \)-fuzzy bitopological space and \( G, H \in L^X \). Then \( \text{cd}_{(i,j)a}(G \land H) = \text{cd}_{(i,j)a}(G) \land \text{cd}_{(i,j)a}(H) \).

**Proof.** By Theorem 3.7, \( \text{cd}_{(i,j)a}(G \land H) = \bigvee \{ a \in L : \tau_{(i,j)a}(\mathcal{U}) \land [(G \land H) \land \mathcal{U}] \leq a \leq \bigwedge \{ (G \land H) \land \mathcal{U} \} \} \).

**Corollary 3.12.** Let \((X, \tau_1, \tau_2)\) be an \( L \)-fuzzy bitopological space and \( G \in L^X \). Then \( \text{cd}_{(i,j)a}(G) = \text{cd}_{(i,j)a}(\mathbb{1}) \land \text{cd}_{(i,j)a}(H) \).

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