Pulsar Timing Array Constraints on Primordial Black Holes with NANOGrav
11-Year Data Set

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The detection of binary black hole coalescences by LIGO/Virgo has aroused the interest in primordial black holes (PBHs), because they could be both the progenitors of these black holes and a compelling candidate of dark matter (DM). PBHs are formed soon after the enhanced scalar perturbations re-enter horizon during radiation dominated era, which would inevitably induce gravitational waves as well. Searching for such scalar induced gravitational waves (SIGWs) provides an elegant way to probe PBHs. We perform the first direct search for the signals of SIGWs accompanying the formation of PBHs in North American Nanohertz Observatory for Gravitational waves (NANOGrav) 11-year data set. No statistically significant detection has been made, and hence we place a stringent upper limit on the abundance of PBHs in DM at 95% confidence level. Our results rule out the existence of PBHs in the mass range of $[1.7 \times 10^{-3}, 4.2 \times 10^{-1}] M_\odot$ (the abundance of PBHs in this mass range is less than one part in million).

Introduction. Over the past few years, the great achievement of detecting gravitational waves (GWs) from binary black holes (BBHs) [1–7] and a binary neutron star (BNS) [8] by LIGO/Virgo has led us to the era of GW astronomy, as well as the era of multi-messenger astronomy. Various models have been proposed to account for the formation and evolution of these LIGO/Virgo BBHs, among which the PBH scenario [9–11] has attracted a lot of attention recently. PBHs are predicted to undergo gravitational collapse from overdense regions in the infant universe [12, 13] when the corresponding wavelength of enhanced scalar curvature perturbations re-enter the horizon [14–18].

The PBH scenario is appealing because it can not only account for the event rate of LIGO/Virgo BBHs, but also be a promising candidate for the long elusive missing part of our Universe – dark matter (DM). It is inconclusive that whether PBH can represent all DM or not, yet the abundance of PBHs in the form of DM ($f_{\text{PBH}}$) has been constrained by a variety of observations, such as extra-galactic $\gamma$-ray bursts [19], femtolensing of $\gamma$-ray bursts [20], existence of white dwarfs in our local galaxy [21], Subaru/HSC microlensing [22], Kepler milli/microlensing [23], OGLE microlensing [24], EROS/MACHO microlensing [25], dynamical heating of ultra-faint dwarf galaxies [26], X-ray/radio constraints [27], accretion constraints by cosmic microwave background (CMB) spectra [28–31] and GWs either through the null detection of sub-solar mass BBHs [32–35] or the null detection of stochastic GW background (SGWB) from BBHs [34, 36]. See revisiting constraints in [37] as well. But PBHs in a substantial window in the approximate mass range $[10^{-16}, 10^{-14}] \cup [10^{-13}, 10^{-12}] M_\odot$ are still allowed to account for all of the DM. We refer to [34] for a recent summary.

There exists another way to probe the PBH DM scenario, namely through the scalar induced GWs (SIGWs) which would be inevitably generated in conjunction with the formation of PBHs [38, 39]. The feature for distinguishing SIGW from other sources was sketched out in [40] recently. Since PBHs are supposed to form from the tail of the probability density function of the curvature perturbations, the possibility to form a single PBH is quite sensitive to the amplitude of curvature perturbation power spectrum [41]. Consequently the abundance of PBHs is extremely sensitive to the amplitude of the corresponding SIGW. Therefore a detection of SIGW will provide evidence for PBHs, while the null detection of SIGW will put a stringent constraint on the abundance of PBHs.

The peak frequency of the SIGW ($f_*$) is determined by the peak wave-mode of the comoving curvature power spectrum, and thus is related to the mass of PBHs by $f_* \sim 3 \text{Hz} \left( m_{\text{pbh}}/10^{-18} M_\odot \right)^{-1/2}$ [39]. The mass of PBHs constituting DM should be heavier than $10^{-18} M_\odot$, otherwise they would have evaporated due to Hawking radiation. As a result, the corresponding peak frequency of the SIGW should be lower than 3Hz, and then it is difficult for the ground-based detectors like LIGO/Virgo to detect the corresponding SIGWs. On the other hand, the GW observatories hunting for low frequency signals are especially suitable to explore the PBH DM hypothesis, and the prospective constraints on the abundance of...
PBHs by LISA [42] and pulsar timing observations such as IPTA [43], FAST [44] and SKA [45] have been investigated in [46]. See some other related works in [47–53].

Despite the data of current pulsar timing array (PTA) has been used to constrain the amplitude of SGWBs, those results strongly depend on the assumption of some special power-law form which is quite different from SIGWs [46]. Therefore, in this article, we perform the first search in the public available PTA data set for the signal of SIGWs in order to test the PBH DM hypothesis. In particular, the null detection of SIGWs in the current NANOGrav 11-year data set [54] provides a constraint on the abundance of PBHs through SIGWs in the mass range of $[3 \times 10^{-4}, 1] M_\odot$.

**PBH DM and SIGW.** In this article, we consider the monochromatic formation of PBHs, corresponding to a $\delta$-power spectrum of the scalar curvature perturbation, i.e.,

\[
P_\zeta(k) = A f \delta(f - f_\ast),
\]

where $A$ is the dimensionless amplitude of the power spectrum. In this case, the mass of the PBHs is related to the peak frequency $f_\ast$ by [12, 13],

\[
m_{\mathrm{pbh}} M_\odot \simeq 2.3 \times 10^{18} \left(\frac{H_0}{f_\ast}\right)^2,
\]

where $H_0$ is the Hubble constant. The formation of PBH is a threshold process which is described by three-dimensional statistics of Gaussian random fields, also known as peak theory [55], and the abundance of PBH in DM, $f_{\mathrm{pbh}} \equiv \Omega_{\mathrm{pbh}}/\Omega_{\mathrm{DM}}$, is given by [56],

\[
f_{\mathrm{pbh}} \simeq 1.9 \times 10^{7} \left(\frac{\xi_c}{A} - 1\right) e^{-\frac{c^2}{2}} \left(\frac{m_{\mathrm{pbh}}}{M_\odot}\right)^{-\frac{1}{2}},
\]

where $\xi_c \simeq 1$ [57–62] is the threshold value for the formation of PBHs.

In [63] the energy density of a GW background $\rho_{\mathrm{GW}}$ takes the form

\[
\rho_{\mathrm{GW}} = \int \rho_{\mathrm{GW}}(f, \eta) \, \mathrm{d} \ln f = \frac{M_p^2}{16 \pi^2} \left(\frac{h_{ij}}{\sqrt{g}}\right),
\]

where $M_p$ is the Planck mass and the overline stands for time average and $a$ is the scale factor. It is useful to introduce the dimensionless GW energy density parameter per logarithm frequency $\Omega_{\mathrm{GW}}(\eta, k)$ defined by

\[
\Omega_{\mathrm{GW}}(\eta, f) \equiv \frac{\rho_{\mathrm{GW}}(f, \eta)}{\rho_{\mathrm{cr}}},
\]

where $\rho_{\mathrm{cr}}$ is the critical energy of the present Universe. For a monochromatic formation of PBHs, the present $\Omega_{\mathrm{GW}}(f)$ of the SIGW in radiation domination can be estimated as [46]

\[
\Omega_{\mathrm{GW}}(f) = \Omega_{\mathrm{GW}}^{(2)}(f) + \Omega_{\mathrm{GW}}^{(3)}(f).
\]

Here, the leading order contribution $\Omega_{\mathrm{GW}}^{(2)}(f)$ is given by [64, 65],

\[
\Omega_{\mathrm{GW}}^{(2)}(f) = \frac{3 \tilde{f}^2 A^2}{512} \bar{\Omega}_r (4 - \tilde{f}^2)^2 (3 \tilde{f}^2 - 2)^2 \Theta(2 - \tilde{f}) \times 
\]

\[
\left[\pi^2 (3 \tilde{f}^2 - 2)^2 \Theta(2\sqrt{3} - 3 \tilde{f})
\right.
\]

\[
+ \left. \left(4 + (3 \tilde{f}^2 - 2) \log \left|1 - \frac{4}{3 \tilde{f}^2}\right| \right)^2 \right],
\]

where $\tilde{f} \equiv f / f_\ast$ is the dimensionless frequency, $\bar{\Omega}_r$ is the current density parameter of radiation, and $\Theta$ is the Heaviside theta function. In addition, the third-order correction $\Omega_{\mathrm{GW}}^{(3)}(f)$ reads, [46],

\[
\Omega_{\mathrm{GW}}^{(3)}(f) = \frac{A^3}{192 \tilde{f}^2} \bar{\Omega}_r \left(M_2 \tilde{f}_I^2 + M_1 \tilde{f}_I \tilde{f}_E\right).
\]

The definitions of $M_1$, $M_2$, $I_2$, $I_3$, and $I_4$ are complicated and can be found in [46].

**PTA data analysis.** Null detection of certain GW backgrounds has been reported by the current PTAs such as NANOGrav\(^1\), PPTA\(^2\) and EPTA\(^3\), and the upper bounds on the amplitude of those GW backgrounds have also been continuing improved. For instance, NANOGrav constrained on the SGWB produced by supermassive black holes [66] and other spectra [67] such as power-law, broken-power-law, free and Gaussian-process ones. Similar studies were also performed by the PPTA collaboration [68] and the EPTA collaboration [69]. In this article we search for the signal of SIGW using the NANOGrav 11-year data set which consists of time of arrival (TOA) data and pulsar timing models presented in [54]. Similar to [70], we choose six pulsars which have relatively good TOA precision and long observation time. A summary of the basic properties of these pulsars is presented in Table I.

| Pulsar name | RMS [\mu s] | $N_{\text{epoch}}$ | $N_{\text{TOA}}$ | $T_{\text{obs}}$ [yr] |
|-------------|-------------|-------------------|-----------------|------------------|
| J0613−0200  | 0.422       | 324               | 11,566          | 10.8             |
| J1012+5307  | 1.07        | 493               | 16,782          | 11.4             |
| J1600−3053  | 0.23        | 275               | 12,433          | 8.1              |
| J1713+0747  | 0.108       | 789               | 27,571          | 10.9             |
| J1744−1134  | 0.842       | 322               | 11,550          | 11.4             |
| J1909−3744  | 0.148       | 451               | 17,373          | 11.2             |

\(^1\) http://nanograv.org  
\(^2\) https://www.atnf.csiro.au/research/pulsar/ppta  
\(^3\) http://www.epta.eu.org
The presence of a GW background will manifest as the unexplained residuals in the TOAs of pulsar signals after subtracting a deterministic timing model that accounts for the pulsar spin behavior and the geometric effects due to the motion of the pulsar and the Earth [71, 72]. It is therefore feasible to separate GW-induced residuals, which have distinctive correlations among different pulsars [73], from other systematic effects, such as clock errors or delays due to light propagation through interstellar medium, by regularly monitoring TOAs of pulsars from an array of the most rotational stable millisecond pulsars [74]. An $N_{\text{TOA}}$ length vector $\mathbf{\delta t}$ representing the timing residuals for a single pulsar can be modeled as follows [67, 75, 76]

$$\mathbf{\delta t} = M \boldsymbol{\epsilon} + \mathbf{n},$$

where $M$ is the timing model design matrix, $\boldsymbol{\epsilon}$ is a vector denoting small offsets for the timing model parameters, and $M \boldsymbol{\epsilon}$ is the residual due to inaccuracies of the timing model. The timing model design matrix is obtained through 1ibstempo\(^4\) package which is a python interface to TEMPO2 \(^5\) [77, 78] timing software. The measurement noise $\mathbf{n}$ in Eq. (9) is assumed to be Gaussian and can be categorized as

$$\mathbf{n} = \mathbf{n}_{\text{RN}} + \mathbf{n}_{\text{WN}} + \mathbf{n}_{\text{SSE}} + \mathbf{n}_{\text{SIGW}}.$$  \hspace{1cm} (10)

The first term on the right hand side of Eq. (10), $\mathbf{n}_{\text{RN}}$, represents the red noise via a Fourier decomposition,

$$\mathbf{n}_{\text{RN}} = \sum_{j=1}^{N_{\text{mode}}} \left[ a_j \sin \left( \frac{2\pi j t}{T} \right) + b_j \cos \left( \frac{2\pi j t}{T} \right) \right] = \mathbf{F} \mathbf{a},$$

where $N_{\text{mode}}$ is the number of frequency modes included in the sum, $T$ is the total observation time span, $\mathbf{F}$ is the Fourier design matrix with components of alternating sine and cosine functions for frequencies in the range $[1/T, N_{\text{mode}}/T]$, and $\mathbf{a}$ is a vector giving the amplitude of the Fourier basis functions. In the analysis, we choose $N_{\text{mode}} = 30$. The covariant matrix of the red noise coefficients $a_i$ at frequency modes $i$ and $j$ will be diagonal, namely

$$\langle a_i a_j \rangle = P(f_i) \delta_{ij},$$

where the power spectrum $P(f)$ is usually well described by a power-law model,

$$P(f) = \frac{A_{\text{RN}}^2}{12\pi^2} \left( \frac{f}{\gamma_{\text{RN}} f_T} \right)^{3-\gamma_{\text{RN}}} f^{-3},$$

with $A_{\text{RN}}$ and $\gamma_{\text{RN}}$ the amplitude and spectral index of the power-law, respectively. Note that in Eq. (12), $f_i$ is defined by $i/T$ if $i$ is odd, and $(i-1)/T$ if $i$ is even.

The second term, $\mathbf{n}_{\text{WN}}$, accounts for the influence of white noise on the timing residuals, including a scale parameter on the TOA uncertainties (EFAC), an added variance (EQUAD) and a per-epoch variance (ECORR) for each backend/receiver system. This white noise is assumed to follow Gaussian distribution and can be characterized by a covariance matrix as

$$\mathbf{C}_{\text{WN}} = \mathbf{C}_{\text{EFAC}} + \mathbf{C}_{\text{EQUAD}} + \mathbf{C}_{\text{ECORR}},$$

where $\mathbf{C}_{\text{EFAC}}$, $\mathbf{C}_{\text{EQUAD}}$ and $\mathbf{C}_{\text{ECORR}}$ are the correlation functions for EFAC, EQUAD and ECORR parameters, respectively. Explicit expressions for these correlation functions can be found in [70].

The third term, $\mathbf{n}_{\text{SSE}}$, is a noise due to inaccuracies of a solar system ephemeris (SSE) which is used to convert observatory TOAs to an inertial frame centered at the solar system barycenter. The SSE noise can seriously affect the upper limits and Bayes factors when searching for stochastic gravitational-wave backgrounds [67]. To account for the SSE errors, we employ the physical model BAYSESEPHEM introduced in [67] and implemented in NANOGrav’s flagship package enterprise\(^6\). The BAYSESEPHEM model has eleven parameters, including four parameters correspond to perturbations in the masses of the outer planets, one parameter describes a rotation rate about the ecliptic pole, and six parameters characterize the corrections to Earth’s orbit generated by perturbing Jupiter’s average orbital elements [67].

The last term, $\mathbf{n}_{\text{SIGW}}$, is the observed timing residuals due to the SIGW, which are described by the cross-power spectral density [79]

$$S_{IJ}(f) = \frac{H_0^2}{16\pi^2 f_5^7} \Gamma_{IJ}(f) \Omega_{\text{GW}}(f),$$

where $\Gamma_{IJ}$ is the Hellings & Downs coefficients [73] measuring the correlation of the pulsars $I$ and $J$ in the array. The expression for $\Omega_{\text{GW}}(f)$ is given by Eq. (6). The free parameters for the SIGW are the amplitude $A$ and the peak frequency $f_*$. For a fixed $f_*$, the mass of PBH is given by Eq. (2). In this sense, the free parameter $A$ is directly related to the abundance of PBHs $f_{pbh}$.

For the timing model parameters and TOAs, we use the publicly available data files from NANOGrav 11-year data set [54]. To extract information from the data, we perform a Bayesian inference by closely following the procedure in [67]. The parameters of our model and their prior distributions are presented in Table II. In order to reduce the computational costs, a common strategy is to

\(^4\)https://vallis.github.io/1ibstempo
\(^5\)https://bitbucket.org/psrsoft/tempo2.git
\(^6\)https://github.com/nanograv/enterprise
fix the white noise parameters to their max likelihood values determined from independent single-pulsar analysis, in which only the white and red noises are considered. Fixing white noise parameters can greatly reduce the number of free parameters.

Assuming the noise is Gaussian and stationary, for a PTA with M pulsars, the likelihood function can be evaluated as, [80],

\[
\mathcal{L} = \frac{1}{\sqrt{\det(2\pi\Sigma)}} \exp \left( -\frac{1}{2} \mathbf{N}^T \mathbf{\Sigma}^{-1} \mathbf{N} \right),
\]

(16)

where \( \mathbf{N} \equiv [\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_M]^T \) is a vector of noise time series for all pulsars, and \( \mathbf{\Sigma} \equiv \langle \mathbf{NN}^T \rangle \) is the covariance matrix. Following the common practice in [81–83], we marginalize over the timing model parameter \( \mathbf{\epsilon} \) when evaluating the likelihood. The likelihood is calculated by using the pulsar timing package enterprise. To achieve parallel tempering, we use PTMCMCSampler package to do the Markov chain Monte Carlo sampling.

Given the observational data \( \mathcal{D} \), one needs to distinguish two exclusive models: a noise-only model \( \mathcal{H}_0 \) and a noise-plus-signal model \( \mathcal{H}_1 \). The model selection is quantified by the Bayes factor

\[
B_{10} = \frac{\text{evidence}[\mathcal{H}_1]}{\text{evidence}[\mathcal{H}_0]} = \frac{p(A = 0|\mathcal{H}_1)}{p(A = 0|\mathcal{D}, \mathcal{H}_1)},
\]

(17)

where the numerator and denominator are the prior and posterior probability density of \( A = 0 \) in the model \( \mathcal{H}_1 \), respectively. We have used the Savage-Dickey formula [84] to estimate the Bayes factor in Eq. (17).

### Results and conclusion.

The upper limits and the Bayes factor for the power spectrum amplitude \( A \) as a function of the peak frequency \( f_* \) from the NANOGrav 11-year data set are showed in Fig. 1 at the 95% confidence level. Since the Bayes factor \( B_{10} \) for each peak frequency is less than 3, it indicates that the data is consistent with containing noise only. The upper limits on the abundance of PBHs in DM \( f_{\text{pbh}} \), as a function of the PBH mass \( m_{\text{pbh}} \), are given in Fig. 2 at the 95% confidence level. Note that \( m_{\text{pbh}} \) is related to \( f_* \) by Eq. (2), and \( f_{\text{pbh}} \) is related to \( A \) and \( m_{\text{pbh}} \) by Eq. (3). Our results imply that the current PTA data set has already been able to place a stringent constraint on the abundance of PBHs through the SIGWs. According to Fig. 2, the abundance of PBHs is less than \( 10^{-6} \) in the mass range of \([1.7 \times 10^{-3}, 4.2 \times 10^{-1}] M_\odot\).

In this article, we give the first search for the signal of SIGWs inevitably accompanying the formation of PBHs in the NANOGrav 11-year data set. Since no significant signal is found, we place a 95% upper limit on the abundance of PBHs in the mass range of \([3 \times 10^{-4}, 1] M_\odot\). In particular, the abundance of PBHs in the mass range of \([1.7 \times 10^{-3}, 4.2 \times 10^{-1}] M_\odot\) is less than \( 10^{-6} \) which is much better than any other observational constraints in this mass range in literature. Although we assume the mass function of PBH to be monochromatic, our result may be robust since the extended mass distribution will not dramatically change the amplitude of the SIGW (see e.g., Fig.2 in [85]).

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**Table II. Parameters and their prior distributions used in the analyses.**

| Parameter | Description | Prior | Comments |
|-----------|-------------|-------|----------|
| \( A \)  | GWB strain amplitude | Uniform \([10^{-5}, 10^0]\) (upper limits) | one parameter for PTA |
| \( f_* \) | peak frequency | delta function | fixed |
| \( E_k \) | EFAC per backend/receiver system | Uniform \([0, 10]\) | single-pulsar analysis only |
| \( Q_k[s] \) | EQUAD per backend/receiver system | log-Uniform \([-8.5, -5]\) | single-pulsar analysis only |
| \( J_k[s] \) | ECORR per backend/receiver system | log-Uniform \([-8.5, -5]\) | single-pulsar analysis only |
| \( A_{RN} \) | red-noise power-law amplitude | Uniform \([10^{-20}, 10^{-11}]\) (upper limits) | one parameter per pulsar |
| \( \gamma_{RN} \) | red-noise power-law spectral index | log-Uniform \([-20, -11]\) (model comparison) | one parameter per pulsar |
| \( z_{\text{drift}} \) | drift-rate of Earth’s orbit about ecliptic z-axis | Uniform \([-10^{-9}, 10^{-9}]\) | one parameter for PTA |
| \( \Delta M_{\text{Jupiter}} [M_\odot] \) | perturbation to Jupiter’s mass | \( N(0, 1.55 \times 10^{-11}) \) | one parameter for PTA |
| \( \Delta M_{\text{Saturn}} [M_\odot] \) | perturbation to Saturn’s mass | \( N(0, 8.17 \times 10^{-12}) \) | one parameter for PTA |
| \( \Delta M_{\text{Uranus}} [M_\odot] \) | perturbation to Uranus’ mass | \( N(0, 5.72 \times 10^{-11}) \) | one parameter for PTA |
| \( \Delta M_{\text{Neptune}} [M_\odot] \) | perturbation to Neptune’s mass | \( N(0, 7.96 \times 10^{-11}) \) | one parameter for PTA |
| PCA \( i \) | principal components of Jupiter’s orbit | Uniform \([-0.05, 0.05]\) | six parameters for PTA |

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*https://github.com/jellis18/PTMCMCSampler*
95% upper limit for $f_{pbh}$ from the NANOGrav 11-year data set. The horizontal dotted line corresponds to $10^{-6}$. 

FIG. 2. The 95% upper limits on the abundance of PBHs in DM $f_{pbh}$ as a function of the PBH mass $m_{pbh}$ from the NANOGrav 11-year data set. Results from OGLE microlensing (OGLE) [24], EROS/MACHO microlensing (EROS) [25], and SGWB [34] are also shown. The horizontal dotted line corresponds to $10^{-6}$.

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