Spin Contents of Nucleons with SU(3) Breaking

Jechiel LICHTENSTADT\textsuperscript{1}\textsuperscript{*} and Harry J. LIPKIN\textsuperscript{1,2}\textsuperscript{†}
\textsuperscript{1}School of Physics and Astronomy
The Raymond and Beverly Sackler Faculty of Exact Sciences
Tel Aviv University, 69978 Tel Aviv, Israel

and

\textsuperscript{2}Department of Nuclear Physics
The Weizmann Institute of Science, 76100 Rehovot, Israel

Abstract

We apply a model for SU(3) breaking to the analysis of the spin contents of the nucleon from the latest data on first moments of the spin dependent structure functions and include higher order QCD corrections. The results show that the value of the total quark spin contribution to the nucleon spin $\Delta \Sigma \equiv \Delta u + \Delta d + \Delta s$ remains about 0.3 and is very insensitive to SU(3) breaking, while the result for the strange quark contribution varies considerably with SU(3) breaking.

\textsuperscript{*}Supported by the Basic Research Fund of the Israeli Academy of Sciences

\textsuperscript{†}Supported in part by grant No. I-0304-120-.07/93 from The German-Israeli Foundation for Scientific Research and Development
Recent experiments of polarized deep inelastic scattering (DIS) provided us with high quality data for the spin structure functions of the proton, deuteron and neutron [1, 2, 3, 4]. These measurements are used to evaluate the first moments of the spin dependent structure functions which can be interpreted in terms of the contributions of the quark spins ($\Delta \Sigma = \Delta u + \Delta d + \Delta s$) to the total spin of the nucleon. The first results from the measurements of the proton spin structure function by the EM Collaboration [5] were very surprising, implying that $\Delta \Sigma$ is rather small (about 10%) and that the strange sea is strongly polarized. The new measurements [3, 4], together with the new values of $F$ and $D$ [6, 7], suggest that $\Delta \Sigma$ is significantly larger than what was inferred from the EMC experiment, concluding that $\Delta \Sigma \approx 0.27 \pm 0.14$ and $\Delta s = -0.10 \pm 0.05$. Incorporating also the recent SLAC E143 data for the proton with all other proton data, has not changed these results, and the combined analysis yields [8] for the first moment $\Gamma_p^1 (Q^2_0 = 5 GeV^2) = 0.137 \pm 0.010$ and $\Delta \Sigma = 0.28 \pm 0.09$ and $\Delta s = -0.10 \pm 0.03$.

With $\int_0^1 g(x) dx = \frac{1}{2} \sum_{\text{flavors}} e^2_f \Delta q_f$ the underlying hypothesis in these analyses is the quark parton model (QPM) interpretation of the neutron weak decay constant $g_A = \Delta u - \Delta d$ and the SU(3) flavor symmetry interpretation of $3F - D = \Delta u + \Delta d - 2\Delta s$.

It is usually implied that the experimental evaluation of $\Delta \Sigma$ and $\Delta s$ is based on exact SU(3) flavor symmetry. However, the extracted values are always quoted with errors which are derived from the experimental errors on the measured quantities ($\Gamma_1, g_A$ and $3F - D$). Little attention has been given to evaluating the effect of SU(3) breaking on these results.

We first examine the information obtainable from experimental data to separate results which do not assume SU(3) from those which do. We will see that the value of $\Delta \Sigma$ deduced from the measured first moment of the spin dependent structure function is not sensitive to the assumption of exact SU(3), or to the values of the SU(3) parameters $F$ and $D$.

In the quark parton model, $\Gamma_p^1$ can be expressed as

$$\Gamma_p^1 = \int_0^1 g_1^p(x) dx = C_{ns}(\frac{\alpha_s}{\pi}) \frac{1}{12} \left[ \frac{4\Delta u - 2\Delta d - 2\Delta s}{3} \right] + C_s(\frac{\alpha_s}{\pi}) \frac{1}{9} \Delta \Sigma \quad (1)$$

where $C_{ns}(\frac{\alpha_s}{\pi})$ and $C_s(\frac{\alpha_s}{\pi})$ include the higher order QCD corrections which are taken from Ref. [3] for the linear combinations of the matrix elements of the axial-vector current which transform under SU(3) like members of an octet and singlet respectively. Here, anomalous contributions to $\Delta \Sigma$ in the singlet term were neglected, and $\Delta u, \Delta d$ and $\Delta s$ are assumed to be the same in the singlet and non-singlet term. At this stage there is no assumption of SU(3) symmetry for the nucleon wave function. There is therefore no relation between the values of the contributions to the quark spins $\Delta u, \Delta d$ and $\Delta s$ and other parameters determined from weak decays.
When SU(3) symmetry is assumed for the octet baryon wave function, the expression \(4\Delta u - 2\Delta d - 2\Delta s\) can be expressed entirely in terms of the quantities measured in weak decays \((g_A\) and \(3F - D)\). Substituting

\[3F - D = \Delta u + \Delta d - 2\Delta s = \Delta \Sigma - 3\Delta s\]  

(2)

gives the conventional expression,

\[
\Gamma_1^{p(n)} = \int_0^1 g_1^{p(n)}(x) dx = C_{ns}(\frac{\alpha_s}{\pi}) \frac{1}{12} \left[ (+(-)g_A + \frac{3F - D}{3}) + C_s(\frac{\alpha_s}{\pi}) \frac{1}{9} \Delta \Sigma \right]  
\]

(3)

(where the (-) refers to the neutron case). A similar expression can be written for the deuteron first moment which can be expressed as a sum of those of the proton and neutron:

\[
\Gamma_1^d = (\Gamma_1^p + \Gamma_1^n)(\frac{1 - 1.5\omega_D}{2})  
\]

(4)

where corrections for the D-state contributions in the deuteron ground state were considered.

It is interesting to examine the case when only isospin symmetry is assumed for the nucleon wave function. The expression \(4\Delta u - 2\Delta d - 2\Delta s\) can be written in terms of the quantity \(g_A\) measured with high precision in the neutron decay, \(\Delta \Sigma\) and \(\Delta s\):

\[
\Gamma_1^{p(n)} = \int_0^1 g_1^{p(n)}(x) dx = C_{ns}(\frac{\alpha_s}{\pi}) \frac{1}{12} \left[ (+(-)g_A + \frac{\Delta \Sigma}{3} - \Delta s) + C_s(\frac{\alpha_s}{\pi}) \frac{1}{9} \Delta \Sigma \right]  
\]

(5)

We can then express \(\Delta \Sigma\) as a function of \(\Delta s\) for given values of \(g_A\) and \(\Gamma_1^N\). Taking the result for the “world” value for the proton [8]: \(\Gamma_1^p(Q_0^2 = 5 \text{ GeV}^2) = 0.137 \pm 0.010\) and the recent results of SLAC E143 for the deuteron [10] \(\Gamma_1^d(Q_0^2 = 3 \text{ GeV}^2) = 0.042 \pm 0.005\) we show in Fig. [1] the dependence of \(\Delta \Sigma\) with the variation of \(\Delta s\) from -0.2 to +0.15. We can conclude that \(\Delta \Sigma\) obtained from the spin structure function first moment is well established, and variations of \(\Delta s\) by some model (like using input from hyperon decays with SU(3) breaking) will have a relatively small effect on \(\Delta \Sigma\).

Introducing SU(3) breaking into the analysis of hyperon decays is nontrivial and model dependent. The quantity denoted by \(g_A\) is really a ratio of axial-vector and vector matrix elements. Both matrix elements can be changed by SU(3) symmetry-breaking, but it is only the axial matrix element that is relevant to the spin structure. The information from hyperon decays used in conventional treatments of spin structure is expressed in terms of \(D\) and \(F\) parameters which characterize the axial couplings only under the assumption that the vector coupling is pure \(F\) and normalized by the conserved vector current, where the whole SU(3) octet currents are conserved. Thus any attempts to parameterize SU(3) breaking in fitting hyperon data by defining “effective” \(D\) and \(F\) parameters immediately encounter the difficulty of how much of the breaking comes from the
axial couplings and how much comes from the vector and the breakdown of the conserved vector currents for strangeness changing currents. The vector matrix element is uniquely determined by Cabibbo theory in the SU(3) symmetry limit. The known agreement of experimental vector matrix elements with Cabibbo theory places serious constraints on possible SU(3) breaking in the baryon wave functions. On the other hand the strange quark contribution to the proton sea is already known from experiment be reduced roughly by a factor of two from that of a flavor-symmetric sea [11].

To assess the the uncertainty on $\Delta \Sigma$ and $\Delta s$ due to flavor symmetry breaking we use a model consistent with the above constraints [12] suggested by one of us (H. J. L.), to incorporate the weak decay data with the polarized DIS results. The good results of Cabibbo theory for the weak vector decays are kept by assuming that the weak current strangeness changing matrix elements are due to a strangeness change in a valence quark, while the sea wave functions have a nearly 100% overlap and that the valence quark wave functions satisfy SU(3). In this picture it is convenient to express $\Delta q$ as the sum of its valence and sea contributions, $\Delta q = \Delta q^v + \Delta q^s$. We immediately obtain the result that the SU(3) fits to hyperon decay data give information only about the wave functions of the valence quarks and the values of the $D$ and $F$ parameters give information about the spin contributions $\Delta u^v$, $\Delta d^v$ and $\Delta s^v$ from the valence quarks.

In this model the flavor SU(3) symmetry breaking relevant for the $\Sigma^- \to n$ decay is realized by a suppression of the strange pair production in both neutron and $\Sigma^-$ seas, and described by a parameter $\epsilon$. Noting that the neutron has no valence strange quarks, and that $\Delta u^s$ is the same for both neutron and $\Sigma^-$ seas the following results were obtained [12]:

$$\Delta s^v(n) = 0; \quad \Delta u^s(\Sigma^-) \approx \Delta u^s(n) = (1 + \epsilon)\Delta s(n)$$

(6)

$$\frac{G_A}{G_V}(\Sigma^- \to n) = F - D = \Delta u^v(n) - \Delta s^v(n) =$$

$$= \Delta u(n) - \Delta s(n) - [\Delta u^s(n) - \Delta s^s(n)] = \Delta u(n) - (1 + \epsilon)\Delta s(n)$$

(7)

Combining this result with the corresponding relation for the neutron weak decay constant (and SU(2) isospin symmetry):

$$g_A = F + D = \Delta u(p) - \Delta d(p) = \Delta d(n) - \Delta u(n) = 1.257 \pm 0.003$$

(8)

we obtain for the proton or neutron:

$$3F - D = 2 \left[ \frac{G_A}{G_V}(\Sigma^-) \right] + g_A = \Delta u(N) + \Delta d(N) - 2(1 + \epsilon)\Delta s(N) = 0.575 \pm 0.016$$

(9)
or
\[ 3F - D = \frac{3 + 2\epsilon}{3} (\Delta u + \Delta d - 2\Delta s) - \frac{2\epsilon}{3} \Delta \Sigma \]  
(10)

with the values for \( g_A \) and \( 3F - D \) taken from [3, 7].

With the new QPM interpretation of \( 3F - D \) under the flavor symmetry breaking we can substitute \( 3F - D \) in Eq. 1 with

\[ \Delta u + \Delta d - 2\Delta s = \frac{3(3F - D)}{3 + 2\epsilon} + \frac{2\epsilon}{3 + 2\epsilon} \Delta \Sigma \]  
(11)

which then reads:

\[ \Gamma_p^1 = \frac{C_{ns}(\frac{3F - D}{3 + 2\epsilon})}{12} \left[ g_A + \frac{3F - D}{3 + 2\epsilon} \right] + \left[ \frac{C_s(\frac{3F - D}{3 + 2\epsilon})}{9} + \frac{C_{ns}(\frac{3F - D}{3 + 2\epsilon})}{12} \right] \frac{2\epsilon}{3(3 + 2\epsilon)} \Delta \Sigma \]  
(12)

and

\[ \Delta s = \frac{\Delta \Sigma - (3F - D)}{3 + 2\epsilon}. \]  
(13)

Taking for experimental value for \( \Gamma_p^1 \), we show in Fig. 2 the variations of \( \Delta \Sigma \) and \( \Delta s \) as a function of the asymmetry parameter \( \epsilon \). There is hardly a noticeable change in \( \Delta \Sigma \) which moves from 0.28 at \( \epsilon = 0 \) (which is the exact SU(3) symmetry limit), to 0.31 for \( \epsilon = 2 \). On the other hand \( \Delta s \) varies from -0.10 to -0.04, which implies no significant contribution due to the strange sea if flavor symmetry is significantly broken. The same conclusions can be drawn from the neutron results or from the deuteron measurement of \( \Gamma_d^1 \) as shown in Fig. 3.

In conclusion, the value of the quark spin contribution to the nucleon spin extracted from the first moments of the spin dependent structure functions is relatively independent of the assumption exact SU(3) flavor symmetry. The latter affects the value obtained for the “strange sea” contribution \( \Delta s \). It is interesting to assess the anomalous contributions in the singlet term, and study the variations of the extracted spin contents with different models for the gluon contribution in the spin structure functions.
References

[1] SMC, B. Adeva et al., Phys. Lett. B302 (1993) 533.
[2] E-142, P.L. Anthony et al., Phys. Rev. Lett. 71 (1993) 959.
[3] SMC, D. Adams et al., Phys. Lett. B329 (1994) 399.
[4] E-143, K. Abe et al., Phys. Rev. Lett. 74 (1995) 346.
[5] EMC, J. Ashman et al., Phys. Lett. B206, (1988) 364; Nucl. Phys. B328 (1989) 1.
[6] F. E. Close and R. G. Roberts, Phys. Lett. B 316 (1993) 165
[7] S. Y. Hsueh et al., Phys. Rev. D38, (1988) 2056.
[8] The SM Collaboration, to be published.
[9] S.A. Larin, Phys. Lett. B 334 (1994) 192.
[10] E-143, K. Abe et al., SLAC-PUB-95-6734, submitted to Phys. Rev. Lett.
[11] A. O. Bazarko et al (CCFR Collaboration), Nevis Preprint R1502 (June 30, 1994), submitted to Physics Letters B
[12] Harry J. Lipkin, Phys. Lett. B 337 (1994) 157.
Figure 1: $\Delta \Sigma$ vs $\Delta s$ for the proton (a) and for the deuteron (b).
Figure 2: $\Delta \Sigma$ (a) and $\Delta s$ (b) for the proton as a function of the SU(3) symmetry breaking parameter $\epsilon$. 

\[ \Gamma^p = 0.137 \pm 0.009 \]
Figure 3: $\Delta \Sigma$ (a) and $\Delta s$ (b) for the deuteron as a function of the SU(3) symmetry breaking parameter $\epsilon$. 

$\Gamma^d_1 = 0.042 \pm 0.005$