Testing dynamical vacuum models with CMB power spectrum from Planck

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The comprehensive analysis of a large family of observational data indicate that around ∼ 95% of the Universe content corresponds to unknown sectors, usually called dark matter (around ∼ 25%) and dark energy (around ∼ 70%). Dynamically, the latter component plays a key role in cosmic expansion because it is responsible for the outstanding phenomenon of cosmic acceleration [1,2]. It is worth mentioning that the nature of dark energy (hereafter DE) has been a topic of interest ever since its first appearance in Einstein’s equations as a cosmological constant. Even though the term was briefly discarded as unnecessary, the cosmological constant is to date the most favored candidate as a dark energy which coexists with cold dark matter (CDM) and ordinary baryonic matter (see [3] for review). These components form the so-called ΛCDM model which fits accurately the current cosmological data.

Although the ΛCDM model is considered as a successful cosmic scenario, it is not without its problems [9–12]. Examples of theoretical impairments of the model include the fine tuning and coincidence problems. The fine tuning problem reflects the gap between the expected (Planck natural unit) vacuum energy density ρvac, which, using quantum field theory (QFT), is calculated at a remarkable ∼ 120 orders of magnitude larger than the observed value of ρΛ at the present time. On the other hand, the coincidence problem source from the approximate equality of ρm and ρΛ prior to the present time, even though the former is a dynamical quantity and the latter is a constant.

These problems have given rise to a large body of cosmological models which mainly extends the traditional Einstein-Hilbert action of general relativity using either a new field ([13–16], or a modified gravity theory that increases the number of degrees of freedom ([17,22], see also [23]). Among the class of DE models, the introduction of a dynamical vacuum, Λ(t), is perhaps the simplest modification of the Einstein-Hilbert action towards alleviating the aforementioned theoretical issues (see [24–26]). Here, the time dependence is not introduced via the equation of state (EoS) parameter wΛ which, like the ΛCDM, is strictly set to wΛ = −1, but is inherited to the pressure through pΛ(t) = −wΛ(t)ρΛ(t). Notice, that the idea to deal with a vacuum which varies with cosmic time (or redshift) has a long history in cosmology and it is perfectly allowed by the cosmological principle ([27,45]).

Usually, the dynamical vacuum energy density ρΛ evolves slowly as a power series of the Hubble rate (for a review see [46,47,48]). In this scenario, the decaying vacuum energy density has an interesting feature, namely it predicts that the spacetime emerges from a non-singular initial de Sitter vacuum stage, hence the phase of the universe changes in a smooth way from inflation to a radiation epoch ("graceful exit"). Then the universe enters into the dark-matter and vacuum-dominated phases, before finally, entering to a late-time de Sitter phase [25,26]. From the observational view point, recently Sola et al. [49–51] tested the performance of the running vacuum models against the latest cosmological data and they found that the Λ(H) models are favored over the usual ΛCDM model at ∼ 3 − 4σ statistical level (see also [52]). These results have led to growing interest in Λ(H) cosmological models.

In this work we attempt to study the performance of various dynamical vacuum models at the expansion level. A likelihood analysis, involving the Planck CMB power spectrum [8], is implemented in order to constrain a large family of dynamical vacuum models. Then we combine CMB in an overall likelihood analysis with other cosmological probes (SNe type Ia, BAOs, H0) in order to extract the probability distribution (via Monte Carlo MCMC method) of the solutions for a large set of cosmological parameters, as well as, to search for deviations from the concordance ΛCDM model for which Λ(H) = const.
The structure of our paper is as follows: in section II we provide a brief introduction of the running vacuum cosmology and we present the most popular Λ(H) models that have appeared in the literature. In section III we discuss the methodology and the cosmological data that we utilize and we perform a detailed statistical analysis aiming to provide the corresponding best fit values and contour plots for the current Λ(H) models. Finally, we discuss our results in the conclusions.

II. BACKGROUND EXPANSION IN RUNNING VACUUM MODELS

In this section we briefly describe the main features of the dynamical vacuum models for which Λ is not constant but evolves with cosmic time. This is perfectly allowed by the cosmological principle embedded in the FLRW metric \( \text{II} \). In general, if we model the expanding universe as a mixture of perfect fluids \( N = 1, 2, \ldots \) then the total energy momentum tensor is given by

\[
T^\mu_\nu = \sum_N T^N_\mu\nu = \sum_N \left[ -p_N \delta^\mu_\nu + \left( \rho_N + p_N \right) U^N_\mu U^N_\nu \right],
\]

where \( U^N_\mu \) is the 4-velocity field. In this case the components of \( T^\mu_\nu \) are written as

\[
T^0_0 = \sum_N \rho_N \equiv \rho_T, \quad T^i_j = -\sum_N p_N \delta^i_j \equiv -p_T \delta^i_j, \quad (2)
\]

where the quantities \( \rho_T \) and \( p_T \) are the total pressure and energy density in the comoving frame \( (U^0_N, U^i_N) = (1, 0) \), respectively. The next step is to apply the covariant local conservation law for the mixture, namely \( \nabla_\mu T^\mu_\nu = 0 \). Inserting Eq. (1) into the latter expression and with the aid of the relation \( U^N_\mu \nabla_\mu U^N_\nu = 0 \) (which comes from the fact that for any four-velocity vector, we have \( U^N_\mu U^N_\nu = 1 \) we arrive at

\[
\sum_N \left[ U^N_\mu \nabla_\mu \rho_N + \left( \rho_N + p_N \right) \nabla_\mu U^N_\mu \right] = 0. \quad (3)
\]

For a Friedmann-Lemaître-Robertson-Walker (FLRW) metric, it is easy to check that for a comoving frame \( (U^0_N = \delta^0_\mu) \), one obtains:

\[
\nabla_\mu U^0_N = 3H \quad (N = 1, 2, \ldots),
\]

and thus Eq. (3) reduces to

\[
\sum_N \left[ \dot{\rho}_N + 3H (\rho_N + p_N) \right] = 0, \quad (5)
\]

where \( H = \dot{a}/a \) is the Hubble parameter and \( a(t) \) is the scale factor of the universe normalized to unity at the present epoch. For the rest of the paper we focus on the spatially flat FLRW metric.

In the aforementioned discussion we did not address the physics of the fluids involved. The total density \( \rho_T \) receives contributions from non-relativistic matter (cold dark matter and baryons) \( \rho_m = \rho_b + \rho_{dm} \) (\( \rho_m = 0 \)), radiation \( \rho_r \) (\( \rho_r = \rho_r/3 \)) and vacuum \( \rho_{\Lambda} \) (\( \rho_{\Lambda} = -\rho_{\Lambda} \)). In this context the index \( N \) specifies the specific components of the cosmic fluid, namely \( \{\text{dm}, b, r, \Lambda\} \). Assuming that baryons and radiation are self-conserved, namely the corresponding densities evolve in the nominal way \( \rho_b = \rho_{b0} a^{-3} \) and \( \rho_r = \rho_{r0} a^{-4} \) the overall conservation law \( \dot{\rho}_T \) becomes

\[
\dot{\rho}_T + 3H \rho_T = 0 \quad (6)
\]

or

\[
\dot{\rho}_{dm} + 3H \rho_{dm} = Q \quad (7)
\]

\[
\dot{\rho}_{\Lambda} = -Q. \quad (8)
\]

Notice that \( Q \) is the interaction term between dark matter and running vacuum, such that a small amount of vacuum decays into dark matter or vice versa. It is worth mentioning that the above expression is the outcome of imposing the covariant conservation of the total energy density of the combined system of matter and vacuum, hence is a direct consequence of the Bianchi identity in the context of general relativity. Of course in the case of the concordance ΛCDM model (\( \rho_{\Lambda} = \text{const.}, Q = 0 \), we recover the standard dark matter conservation law \( \dot{\rho}_{dm} + 3H \rho_{dm} = 0 \).

Within this framework, the Friedmann equations of the system formed by the above fluid components are given by (see \[49\] and references therein):

\[
H^2 = \frac{8\pi G}{3} \left( \rho_m + \rho_r + \rho_{\Lambda} \right) \quad (9)
\]

\[
2H^2 + 3H^2 = -8\pi G \left( \frac{1}{3} \rho_r - \rho_{\Lambda} \right). \quad (10)
\]

A. Specific Λ(H) models

Now let us briefly present the running vacuum models studied in this article. For each one of these models we provide the term of interaction \( Q \) and thus we calculate the evolution of main cosmological quantities, namely \( \rho_{\Lambda}(a), \rho_{dm}(a) \) and \( H(a) \). Notice that the baryon and the radiation densities obey the standard laws, namely \( \rho_b(a) \propto a^{-3} \) and \( \rho_r(a) \propto a^{-4} \) respectively. Specifically, the current Λ(H) models read as follows.

The first model under consideration is the running vacuum model as described in \[24,26\] (hereafter Λ(H)CDM). In this case we have \( Q = \nu H (3\rho_m + 4\rho_r) \), hence solving the system of equations \( (7), (8) \) one finds an expression for the evolution of both densities:

\[
\rho_{dm} = \rho_{dm0} a^{-3(1-\nu)} + \rho_{b0} (a^{3(1-\nu)} - a^{-3}) + \frac{4\nu}{1 + 3\nu} \rho_{r0} \left( a^{3(1-\nu)} - a^{-4} \right) \quad (11)
\]
mological quantities become

\[ \rho_\Lambda = \rho_{\Lambda,0} + \frac{\nu \rho_{m,0}}{1 - \nu} (a^{3(1-\nu)} - 1) + \frac{\nu \rho_{r,0}}{1 - \nu} \left( \frac{1 - \nu}{1 + 3\nu} a^{-4} - 4\nu \frac{1}{1 + 3\nu} a^{-3(1-\nu)} - 1 \right) \],

(12)

while the normalized Hubble parameter \( E(a) = H(a)/H_0 \) is given by

\[ E^2(a) = 1 + \frac{\Omega_m}{1 - \nu} (a^{3(1-\nu)} - 1) + \frac{\Omega_r}{1 - \nu} \left( \frac{1 - \nu}{1 + 3\nu} a^{-4} - 4\nu \frac{1}{1 + 3\nu} a^{-3(1-\nu)} - 1 \right). \]

(13)

The second phenomenological model that we take into account is that with \( Q = 3\nu H \rho_{dm} \) (hereafter \( \Lambda(H)\)CDM). In this context, we have

\[ \rho_{dm} = \rho_{dm,0} a^{-3(1-\nu)} \]

(14)

\[ \rho_\Lambda = \rho_{\Lambda,0} + \frac{\nu \rho_{dm,0}}{1 - \nu} (a^{-3(1-\nu)} - 1) \]

(15)

and

\[ E^2 = 1 + \Omega_b (a^{-3-1}) + \frac{\Omega_{dm}}{1 - \nu} (a^{-3(1-\nu)} - 1) + \Omega_r (a^{-4-1}). \]

(16)

The final vacuum model consists of \( Q = 3\nu H \rho_\Lambda \) (hereafter \( \Lambda(H)\)CDM). Within this framework the basic cosmological quantities become

\[ \rho_{dm} = \rho_{dm,0} + \frac{\nu \rho_{dm,0}}{1 - \nu} (a^{-3\nu} - a^{-3}) \]

(17)

\[ \rho_\Lambda = \rho_{\Lambda,0} a^{-3\nu} \]

(18)

\[ E^2 = \frac{a^{-3\nu} - \nu a^{-3}}{1 - \nu} + \frac{\Omega_m}{1 - \nu} (a^{-3} - a^{-3\nu}) + \Omega_r \left( a^{-4} + \frac{\nu}{1 - \nu} a^{-3} - 1 \right). \]

(19)

It is important to note that for \( \nu = 0 \) the aforementioned equations boil down to those of \( \Lambda\)CDM, as they should.

### III. FITTING DYNAMICAL VACUUM MODELS TO THE PLANCK CMB SPECTRUM

For this analysis, the publicly available CAMB code was modified to admit dynamical vacuum models, and used in combination with the MCMC package to restrain the usual set of cosmological parameters of the standard model for Cosmology, with the addition of the dynamical vacuum parameter \( \nu \). Notice that Gomez-Valent & Sola [54] provided a detailed analysis regarding the behavior of dynamical vacuum model at the perturbation level (for similar studies see [48, 55]). There authors found that both Newtonian and synchronous gauges provide the same differential equation of the matter density contrast which is different with respect to that of \( \Lambda\)CDM. Also, in the comoving frame Ref. [54] showed that vacuum perturbations are negligible with respect to matter fluctuations at subhorizon scales.

We use the Planck satellite, 2015 release which includes the CMB power spectrum, TT, TE, EE + lowP. For completeness we also give the constrains of the ACMD model. Notice that the parameter space is \( \{\Omega_m, \sigma_8, h, \nu, n_s, \tau\} \), where \( \sigma_8 \) is the mass variance at \( 8h^{-1}\)Mpc, \( h = H_0/100 \), \( n_s \) spectral index, \( \tau \) is the optical depth. Notice that in CAMB, the density of photons is calculated through \( r_\tau = \frac{4\pi a T_{\text{cmb}}}{c} \), with \( T_{\text{cmb}} \) being the Stefan - Boltzmann constant, and \( T_{\text{cmb}} \) the temperature of the CMB.

![FIG. 1: 2D likelihood contours for the \( \Lambda(H)\)CDM1 vacuum model. We present 1\& and 2\& likelihood contours of all the sampled parameters when testing against the CMB full spectrum. The straight line corresponds to \( \nu = 0 \).](image)

| Model | \( \Lambda\)CDM | \( \Lambda(H)\)CDM1 | \( \Lambda(H)\)CDM2 | \( \Lambda(H)\)CDM3 |
|-------|-------------|----------------|----------------|----------------|
| \( \Omega_m \) | \( 0.306 \pm 0.018 \) | \( 0.309 \pm 0.03 \) | \( 0.330 \pm 0.03 \) | \( 0.342 \pm 0.026 \) |
| \( \sigma_8 \) | \( 0.83 \pm 0.03 \) | \( 0.84 \pm 0.04 \) | \( 0.822 \pm 0.03 \) | \( 0.832 \pm 0.035 \) |
| \( h \) | \( 0.679 \pm 0.012 \) | \( 0.678 \pm 0.02 \) | \( 0.662 \pm 0.02 \) | \( 0.653 \pm 0.018 \) |
| \( \nu \) | \( 0.04 \pm 0.07 \) | \( -0.60 \pm 0.59 \) | \( -0.9 \pm 0.8 \) | \( -1.0 \pm 0.9 \) |
| \( n_\tau \) | \( 0.964 \pm 0.008 \) | \( 0.956 \pm 0.008 \) | \( 0.959 \pm 0.008 \) | \( 0.958 \pm 0.009 \) |
| \( \tau \) | \( 0.066 \pm 0.037 \) | \( 0.071 \pm 0.038 \) | \( 0.064 \pm 0.028 \) | \( 0.068 \pm 0.030 \) |

| TABLE I: The analysis results for models \( \Lambda(H)\)CDM1, \( \Lambda(H)\)CDM2 and \( \Lambda(H)\)CDM3 using the data from Planck (TE,TT,EE+lowP), quoting the 68% CL for the parameters. |

In Table I we show an overall presentation of the current observational constraints imposed by the CMB spectrum, while in Figs. 1, 2, and 3 we plot the 1\& and 2\& contours in various planes for the explored \( \Lambda(H)\)CDM models.

In particular, we find:
We observe that the results from CAMB suggest that the best fit parameters of the dynamical vacuum models are in agreement with those of \( \Lambda \)CDM case. Moreover, the current observational constraints are compatible with those of TT, TE, EE + lowE Planck 2018 data provided by [2] (see also [6]), namely \( \Omega_m = 0.3166 \pm 0.0084, \sigma_8 = 0.812 \pm 0.0073, h = 0.6727 \pm 0.006, n_s = 0.9649 \pm 0.0044 \) and \( \tau = 0.0544_{-0.0081}^{+0.0070} \). Concerning the Hubble constant problem, namely the observed Hubble constant \( H_0 = 73.48 \pm 1.66 \text{ Km/s/Mpc} \) found by [29] is in \( \sim 3.7\sigma \) tension with that of Planck \( H_0 = 67.36 \pm 0.54 \text{ Km/s/Mpc} \) [4], we find that the \( H_0 \) values extracted from the \( \Lambda(H) \) models are closer to the latter case (see also [51]). Moreover our \( H_0 \) results are in excellent agreement with those of Shanks et al. [57] who found \( H_0 = 67.6 \pm 1.52 \text{ Km/s/Mpc} \) using the GAIA parallax distances of Milky Way Cepheids.

Moreover, we would also like to compare our results with those of Yang et al. [60] who have tested three scalar field DE models against various datasets. Particularly, for the Planck data the constraints of Yang et al. [60] (see Tables 1,2,4) stand as follows. For the potential \( V(\phi) \propto \cosh(\beta \phi) \) they found \( \Omega_m = 0.312 \pm 0.009, \sigma_8 = 0.83 \pm 0.014, h = 0.6748 \pm 0.006, \beta = 4.32_{-3.32}^{+1.5}, n_s = 0.9661 \pm 0.0045 \) and \( \tau = 0.081 \pm 0.017 \). In the case of \( V(\phi) \propto 1 + \text{sech}(\alpha \phi) \) they obtained \( \Omega_m = 0.313 \pm 0.009, \sigma_8 = 0.829 \pm 0.013, h = 0.6746 \pm 0.0065, \alpha = 4.77_{-3.77}^{+5.23}, n_s = 0.966 \pm 0.0046 \) and \( \tau = 0.079 \pm 0.017 \). Lastly, for the potential \( V(\phi) \propto [1 + \delta(\phi/M_P)]^2 \), where \( M_P \) is the Planck mass, Yang et al. [60] found \( \Omega_m = 0.312 \pm 0.009, \sigma_8 = 0.829 \pm 0.013, h = 0.6747 \pm 0.0064, \delta = 4.843_{-4.843}^{+5.157}, n_s = 0.966 \pm 0.0044 \) and \( \tau = 0.079 \pm 0.017 \). Obviously the constraints of the \( \Lambda(H) \) models (see Table I) are compatible within 1\( \sigma \) with those of scalar field DE models.

A. Combining with other probes

In this section we implement a joint likelihood analysis using SNIa from JLA sample [55], Baryonic Acoustic Oscillations (BAOs) [24] and measurements of \( H_0 \) [60], in order to place tight constraints on the corresponding parameter space of the models. The best fit parameters are listed in Table II. In order to visualize the solution space in Figs.4, 5 and 6 we show the 1\( \sigma \) and 2\( \sigma \) contours for various planes. Specifically, the joint likelihood function peaks at (with the corresponding 68% errors)

- For the \( \Lambda(H) \)CDM model: \( \Omega_m = 0.325 \pm 0.008, \sigma_8 = 0.808 \pm 0.03, h = 0.661 \pm 0.008, \nu \times 10^3 = 1.2_{-0.5}^{+0.6}, n_s = 0.959 \pm 0.006 \) and \( \tau = 0.103_{-0.053}^{+0.029} \).
- For the \( \Lambda(H) \)CDM model: \( \Omega_m = 0.291 \pm 0.009, \sigma_8 = 0.815 \pm 0.02, h = 0.691 \pm 0.008, \nu \times 10^3 = 1.0_{-0.9}^{+0.8}, n_s = 0.958 \pm 0.009 \) and \( \tau = 0.086_{-0.039}^{+0.030} \).
- For the \( \Lambda(H) \)CDM model we find \( \Omega_m = 0.306 \pm 0.018, \sigma_8 = 0.83 \pm 0.03, h = 0.679 \pm 0.012, n_s = 0.964 \pm 0.008 \) and \( \tau = 0.065_{-0.026}^{+0.037} \).

where we have quoted 1\( \sigma \) error bars.

\[ \nu = 0. \]

\[ \text{FIG. 2: The CMB spectrum analysis, repeated for the } \Lambda(H) \text{CDM}_2 \text{ model. Again, the straight line corresponds to } \nu = 0. \]

\[ \text{FIG. 3: 2D likelihood contours for the pairs of sampled parameters for the } \Lambda(H) \text{CDM}_3 \text{ model under the scope of the CMB data. The red line corresponds to } \nu = 0. \]
$10^3 = -0.08^{\pm 0.72}, n_s = 0.967 \pm 0.008$, and $\tau = 0.059^{+0.008}_{-0.008}$.

- For the $\Lambda (H)CDM_3$ model: $\Omega_m = 0.295 \pm 0.013$, $\sigma_8 = 0.797 \pm 0.04$, $h = 0.684 \pm 0.01$, $\nu \times 10^3 = 0.85^{+1.2}_{-0.9}$, $n_s = 0.962 \pm 0.008$, and $\tau = 0.073^{+0.019}_{-0.024}$.

- Lastly, for the usual $\Lambda CDM$ we find $\Omega_m = 0.298 \pm 0.014$, $\sigma_8 = 0.81 \pm 0.01$, $h = 0.686 \pm 0.010$, $n_s = 0.967 \pm 0.008$ and $\tau = 0.047^{+0.011}_{-0.006}$.

We find that the incorporation of more data sets via joint analyses improves the fitting for all models, hence the current running vacuum models are very efficient and in very good agreement with observations. Among the three $\Lambda (H)$ models the $\Lambda (H)CDM_1$ is the one with a small but non-zero deviation from the concordance $\Lambda CDM$ cosmology. Indeed we find that the deviation parameter $\nu$ is different from zero at $\sim 2.4 \sigma$ level.

First we verified that the combined constraints of the $\Lambda (H)$ models (see Table II) are in agreement within $1 \sigma$ errors with those Yang et al. [60] who considered the case of scalar field DE (see also [61]), where these authors found: $\Omega_m = 0.306 \pm 0.006$, $\sigma_8 = 0.83 \pm 0.013$, $h = 0.6794 \pm 0.0046$, $\beta = 4.522^{+1.677}_{-4.522}$, $n_s = 0.9688 \pm 0.0038$ and $\tau = 0.085 \pm 0.016$ [model $V(\phi) \propto \cosh(\beta \phi)$], $\Omega_m = 0.306 \pm 0.006$, $\sigma_8 = 0.83 \pm 0.014$, $h = 0.6797 \pm 0.0049$, $\alpha = 3.845^{+1.361}_{-3.845}$, $n_s = 0.9691 \pm 0.0038$ and $\tau = 0.086 \pm 0.018$ [model $V(\phi) \propto 1 + \sech(\alpha \phi)$] and finally $\Omega_m = 0.306 \pm 0.006$, $\sigma_8 = 0.83 \pm 0.013$, $h = 0.6794 \pm 0.0046$, $\delta = 4.764^{+5.236}_{-4.764}$, $n_s = 0.9688 \pm 0.0039$ and $\tau = 0.085 \pm 0.017$ for the potential $V(\phi) \propto [1 + \delta(\phi/M_P)]^2$.

Concerning the $\Lambda (H)CDM_1$ model our results can be compared with those of Wang [62] who combined SNIa/H(z)/BAO/CMB data. Notice that regarding the cosmological parameters $\{\Omega_m, \nu, h\}$ our CMBshift/SNIa/BAO/H0 constraints are similar (within $1 \sigma$) with those of Sola et al. [51] (see also [48, 50] and references therein) who found, combing cosmic chronometer, SNIa (JLA), CMB shift parameters and BAO data, $\{\Omega_m, \nu, h\} = \{0.304 \pm 0.005, 0.00014 \pm 0.00103, 0.684 \pm 0.007\}$, $\{\Omega_m, \nu, h\} = \{0.304 \pm 0.005, 0.00019 \pm 0.00126, 0.685 \pm 0.007\}$, and $\{\Omega_m, \nu, h\} = \{0.304 \pm 0.005, 0.00009 \pm 0.0033, 0.686 \pm 0.004\}$ for the $\Lambda (H)CDM_1$, $\Lambda (H)CDM_2$ and $\Lambda (H)CDM_3$ models, respectively.

Although our observational constraints are in qualitative agreement with previous studies [18, 51] we would like to spell out clearly the main reason why the results of the present work are novel. Indeed, to our knowledge, our analysis includes the Planck CMB power spectrum in a large family of dynamical vacuum models and thus we can trace the Hubble expansion of the $\Lambda (H)$ models in the recombination era.

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1 For the $\Lambda (H)CDM_3$ model we refer the reader the work of [68].
TABLE II: The analysis results for models $\Lambda(H)CDM_1$, $\Lambda(H)CDM_2$ and $\Lambda(H)CDM_3$ using a joint analysis of CMB data from Planck, BAO’s, SNIa and $H_0$. In this table we quote the 68% limits for the parameters.

| Model       | $\Omega_{m0}$ | $\sigma_8$ | $h_0$ | $\nu \cdot 10^3$ | $n_s$ | $\tau$ |
|-------------|----------------|------------|--------|-----------------|-------|--------|
| $\Lambda_{CDM}$ | $0.298 \pm 0.014$ | $0.325 \pm 0.008$ | $0.291 \pm 0.009$ | $0.295 \pm 0.013$ | $-1.2^{+0.5}_{-0.7}$ | $0.85^{+1.2}_{-0.9}$ |
| $\Lambda(H)CDM_1$ | $0.81 \pm 0.01$ | $0.808 \pm 0.03$ | $0.815 \pm 0.02$ | $0.797 \pm 0.04$ | $0.967 \pm 0.008$ | $0.967 \pm 0.008$ |
| $\Lambda(H)CDM_2$ | $0.686 \pm 0.010$ | $0.661 \pm 0.008$ | $0.691 \pm 0.008$ | $0.684 \pm 0.010$ | $0.047^{+0.006}_{-0.006}$ | $0.073^{+0.009}_{-0.009}$ |

IV. CONCLUSIONS

We extracted observational constraints on various dynamical vacuum models, using the CMB power spectrum from Planck. We used the most popular $\Lambda(H)$ models and in all of them we studied their deviation from the usual $\Lambda$CDM model through a sole parameter. Modifying CAMB we found that the best fit parameters of the explored running vacuum models are in agreement with those of $\Lambda$CDM. For completeness we combined the CMB spectrum in a joint analysis with other cosmological probes (SNe type Ia, BAOs, $H_0$) in order to place tight constraints on the cosmological parameters of the dynamical vacuum models. We find that $\Lambda(H)CDM_2$ and $\Lambda(H)CDM_3$ do not show deviations from the $\Lambda$CDM case. However, for the $\Lambda(H)CDM_1$ vacuum model we found a small but non-zero deviation from $\Lambda$CDM, where the confidence level is close to $\sim 2.4\sigma$. This is an indication that dark energy could be dynamical.

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