Corrections to Bino Annihilation I: Sfermion Mixing

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Abstract

We consider corrections to bino annihilation due to sfermion mixing.
Within the standard model of electroweak interactions, the only plausible dark matter candidate is the neutrino. A neutrino ($\nu_\tau$, for example) with a mass of 10-30 eV would make a significant contribution to the overall mass density of the Universe. A light neutrino, however, is a hot dark matter candidate and as such cannot account for the observed structure on small (galactic) scales. Cold dark matter alone also has its difficulties. It is difficult to make large scale structure and conflicts with observation of the microwave background anisotropies. At present, the best mix of dark matter and baryons seems to be 60-70% cold, about 30% hot, and the remainder in baryons (see [1] and references therein). Cold dark matter still appears to be a necessity. Before their demise, heavy (masses $\sim$ a few GeV) neutrinos were by far the simplest cold dark matter candidates. The minimal supersymmetric standard model (MSSM) is perhaps the simplest logical extension which offers a cold dark matter candidate. Because of an unbroken discrete symmetry, $R$-parity, the lightest supersymmetric partner (LSP) is expected to be stable. This makes the LSP a natural candidate for role of the non-baryonic dark matter.

The identity of the LSP is concealed by the relatively large number of unknown parameters of the MSSM. Nevertheless, by a combination of i) theoretical presumptions (eg that the MSSM will eventually be contained in a grand unified theory), ii) limits from accelerator experiments and iii) cosmological bounds, it is possible to identify the more plausible regions of the parameter space. In previous studies, the lightest neutralino, a linear combination of the supersymmetric partners of the neutral $SU(2)$ gauge boson (neutral wino), $U(1)$ hypercharge gauge boson (bino), and the two neutral components of the Higgs doublets (higgsinos) has been suggested as the most likely candidate for the LSP [2]. The mass matrix for these states contains three unknown parameters: (1) $M_2$, the supersymmetry breaking $SU(2)_L$ gaugino mass; (2) $\epsilon$, the higgsino mixing mass; and (3) $\tan \beta = v_1/v_2$, the ratio of the vacuum expectation values of the Higgs doublets. In a large portion of this parameter space, the least massive eigenstate is a nearly pure bino [3].

Restricting our attention to the bino $\tilde{B}$ as the candidate LSP, it is straightforward to use its couplings to the other fields in the MSSM to compute the relic abundance. These calculations were previously performed under a variety of assumptions: degenerate squark and
slepton masses with \( m_{\tilde{q}} = m_{\tilde{f}} > m_{\tilde{B}} \) taken as a relatively free parameter [3, 4, 5]; unequal squark and slepton masses with values run down from the GUT scale using renormalization group equations with supergravity-inspired boundary conditions, and sfermion mixing neglected [3]; the same with sfermion mixing included [7, 8]. Note there is also a great variety in the level of detail of these calculations. Only in [3, 8] have all LSP–LSP annihilation channels been included. In [8] the annihilations of the LSP and the next lightest neutralino (co-annihilations [9]) were also included.

In general, the sfermion mass matrix contains off diagonal terms connecting the left and right sfermions. These are proportional to the fermion-Higgs coupling and hence to the fermion mass. Sfermion mixing is also dependent on the higgsino mixing mass \( \epsilon \). Since the region of the parameter space containing the almost pure bino is in the range of large \( \epsilon \), for \( m_{\tilde{B}} > m_t \) we expect the off-diagonal terms to have a large effect on bino annihilation by exchange of a top squark.

Though mixing was included in [8], all parameters were set according to a particular GUT renormalization (albeit a good one). The purpose of this paper is to examine in more detail the full effect of including sfermion mixing. A subsequent paper will deal with another correction to the bino annihilation process, s-channel annihilation to fermions via a \( Z^0 \) at the one loop level.

We begin by deriving the annihilation cross-section for binos, keeping the explicit dependence on sfermion mixing. The general form of the sfermion mass matrix is [10]

\[
\left( \begin{array}{c}
\tilde{f}_L^* \\
\tilde{f}_R^*
\end{array} \right) \left( \begin{array}{cc}
M_L^2 & m^2 \\
m^2 & M_R^2
\end{array} \right) \left( \begin{array}{c}
\tilde{f}_L \\
\tilde{f}_R
\end{array} \right),
\]

where \( m^2 = m_f (A_f - \epsilon \cot \beta) \) for weak isospin \( +1/2 \) fermions and \( m_f (A_f - \epsilon \tan \beta) \) for weak isospin \( -1/2 \). \( A_f \) is the soft supersymmetry breaking trilinear mass term arising from superpotential terms such as \( H_1 Q(u^c, d^c) \) or \( H_2 L e^c \), and \( \epsilon \) is the Higgs mixing parameter, often written as \( -\mu \); see [11] for an overview of couplings in the MSSM. This mass matrix is easily diagonalized by writing the diagonal sfermion eigenstates as

\[
\tilde{f}_1 = \tilde{f}_L \cos \theta_f + \tilde{f}_R \sin \theta_f , \\
\tilde{f}_2 = -\tilde{f}_L \sin \theta_f + \tilde{f}_R \cos \theta_f .
\]
With these conventions we have the diagonalizing angle and mass eigenvalues
\[
\theta_f = \text{sign}\left[-m^2\right] \begin{cases} 
\frac{\pi}{2} - \frac{1}{2} \arctan \frac{2m^2}{(M^2_L - M^2_R)}), & M^2_L > M^2_R, \\
\frac{1}{2} \arctan \frac{2m^2}{(M^2_L - M^2_R)}, & M^2_L < M^2_R,
\end{cases}
\]
\[
m^2_{1,2} = \frac{1}{2} \left[(M^2_R + M^2_L) \mp \sqrt{(M^2_R - M^2_L)^2 + 4m^4}\right].
\]

Here \(\theta_f\) is chosen so that \(m_1\) is always lighter than \(m_2\). Note that in the special case \(M_L = M_R = M\), we have \(\theta_f = \text{sign}\left[-m^2\right](\pi/4)\) and \(m^2_{1,2} = M^2 \mp |m^2|\).

From here it is straightforward (if tedious) to enumerate the Feynman diagrams contributing to the annihilation process. The dominant contribution is due to sfermion exchange and is derived from bino-fermion-sfermion interaction lagrangian,
\[
\mathcal{L}_{ff\tilde{B}} = \frac{1}{\sqrt{2}} \bar{f} \left( Y_R \bar{f} P_L \tilde{B} \tilde{f}_R + Y_L \bar{f} P_R \tilde{B} \tilde{f}_L \right) + \text{h.c.} = \frac{1}{\sqrt{2}} \bar{f} \left( \tilde{f}_1 x P_L + \tilde{f}_2 w P_L + \tilde{f}_1 y P_R - \tilde{f}_2 z P_R \right) \tilde{B} + \text{h.c.}
\]
where \(x = Y_R \sin \theta_f, y = Y_L \cos \theta_f, w = Y_R \cos \theta_f, z = Y_L \sin \theta_f, P_{R,L} = (1 \pm \gamma_5)/2, Y_R = 2Q_f\) and \(Y_L = 2(Q_f - T^3)\) where \(Q_f\) is the fermion charge and \(T^3\) is the fermion weak isospin.

To derive a thermally averaged cross section, we make use of the technique of ref. [12].

We expand \(\langle \sigma v_{rel} \rangle\) in a Taylor expansion in powers of \(x = T/m_{\tilde{B}}\)
\[
\langle \sigma v_{rel} \rangle = a + bx + O(x^2)
\]
The coefficients \(a\) and \(b\) are given by
\[
a = \sum_f v_f \tilde{a}_f
\]
\[
b = \sum_f v_f \left[ \tilde{b}_f + \left( -3 + \frac{3m^2_f}{4v^2_fm^2_{\tilde{B}}} \right) \tilde{a}_f \right]
\]
where \(\tilde{a}_f\) and \(\tilde{b}_f\) are computed from the expansion of the matrix element squared in powers of \(p\), the incoming bino momentum, and \(v_f = (1 - m^2_f/m^2_{\tilde{B}})^{1/2}\) is a factor from the phase space integrals.
We summarize the result by quoting the computed \( \tilde{a}_f \):

\[
\tilde{a}_f = \frac{g'^4}{128\pi} \left[ \frac{\Delta_1 (m_f w^2 - 2m_B w z + m_f z^2) + \Delta_2 (m_f x^2 + 2m_B x y + m_f y^2)}{\Delta_1 \Delta_2} \right]^2
\]

(8)

where \( \Delta_i = m_{\tilde{f}_i}^2 + m_{\tilde{B}}^2 - m_f^2 \). The result for \( \tilde{b}_f \) is too lengthy for presentation here, but was computed and used in the numerical integrations to obtain the relic densities. The results reduce to the results quoted in [3] in the limit of no mixing and equal left and right sfermion masses.

We next discuss the effect of considering this mixing interaction on the relic density and detectability of binos. In all of the following cases we have chosen a value of \( \tan \beta = 2 \) and \( \epsilon = \pm 3000 \text{ GeV} \) and assume a top quark mass of 120 GeV. This choice of \( \epsilon \) places us in the region of parameter space where the LSP is a nearly pure bino for a wide range of gaugino masses, \( M_2 \simeq 50 - 5000 \text{ GeV} \). We also take the diagonal part of the sfermion mass matrix to have a constant value for all squarks and sleptons, and denote the common value of \( M_L \) and \( M_R \) by \( M \). This assumption works best when the sfermion masses are large compared with \( m_t, m_{\tilde{B}}, \) and \( M_Z \). For simplicity, we take this condition to apply in all cases. Previous results can be viewed as a special case of this more general form. Indeed, when mixing is neglected one must make a particular choice of \( A_f = \epsilon \cot \beta \) and \( A_f = \epsilon \tan \beta \) for \( T_3 = \pm 1/2 \) fermions. Thus results for the relic bino density when mixing is neglected implicitly assume that a specific (and different) value of \( A_f \) is chosen for each value of \( \tan \beta \) and \( \epsilon \). In the mixed case we will also take a common value for all \( A_f \) and denote it by \( A \).

Sfermion mixing has several effects on the cross-section that can be readily understood. First, the cross-section is no longer p-wave suppressed. In (8), we see that the zero temperature cross-section now has a piece proportional to \( m_{\tilde{B}}^2 / \Delta_2^2 \) rather than \( m_f^2 / \Delta_2^2 \). (Without mixing, when \( m_1 = m_2 \), this piece disappears.) For \( m_{\tilde{B}} > m_f \), this serves to increase the cross-section. However this aspect of mixing produces little effect on \( b \), which already includes terms proportional to \( m_{\tilde{B}}^2 / \Delta_2^2 \). On the other hand, the splitting of the sfermion masses makes \( \Delta_2 > \Delta_1 \). The cross-section (relative to the unmixed case) may be either larger or smaller depending on which mass term is held fixed. When the diagonal elements are fixed, the presence of the lighter sfermion in the mixed case tends to increase the cross-
section; however, since the bino couples with different strengths to the two sfermion mass eigenstates, the cross-section can actually decrease [see fig. (1)]. When the lightest mass eigenvalue \( m_1 \) is fixed, the cross-section goes down as half the sfermions get heavier.

In fig. (1), we show the effect of mixing on the coefficient \( a \) given by (8) as a function of the bino mass (which for our chosen values of \( \epsilon \) is directly proportional to the gaugino mass \( M_2 \)). In the upper and lower curve where mixing is included, we have taken \( A_f = 0 \) and chosen \( M \) so that \( m_1 = m_\tilde{B} \), the smallest mass consistent with the identification of the bino as the LSP. In the upper curve \( \epsilon = 3000 \text{ GeV} \), while in the lower curve \( \epsilon = -3000 \text{ GeV} \). The middle curve, where mixing is neglected, has the same value of \( M \) as the other curves but with off diagonal mass entries set to zero by adjusting the \( A_f \)'s to the appropriate values. In these extreme cases, mixing may either increase or decrease the cross-section depending on the sign of \( \epsilon \). The increase in cross-section is primarily due to the presence of a lighter sfermion in the propagator. When \( \epsilon = -3000 \text{ GeV} \), there is a cancellation which then decreases the cross-section at larger \( m_\tilde{B} \). (However, in this case the relic abundance turns out to be too high, as we will discuss below.) Notice also the large effect of crossing the top quark threshold.

In fig. (2), we fix the relic density \( \Omega_\tilde{B} h^2 \) and consider the effects of mixing on the zero temperature cross-section. \( \Omega_\tilde{B} \) is the mass density of the binos in units of the critical density, and \( h \) is the Hubble parameter in units of 100 km s\(^{-1}\) Mpc\(^{-1}\). In the mixed case, we set \( A_f = 0 \), and for each value of \( m_\tilde{B} \), vary \( M \) until the value of \( \Omega_\tilde{B} h^2 \) equals 1/4, corresponding to a critical mass density of binos with a value of \( h = 1/2 \). Here \( \epsilon = 3000 \text{ GeV} \) only, as there are no solutions to \( \Omega_\tilde{B} h^2 = 1/4 \) with \( \epsilon = -3000 \text{ GeV} \). In the unmixed case, we set the \( A_f \)'s appropriately and again adjust \( M \) so that \( \Omega_\tilde{B} h^2 = 1/4 \). The ratio of the \( a \) values in the mixed and unmixed cases is plotted in fig. (2). Notice here that \( a \) is approximately twice as large in the mixed case.

The enhanced value for the zero-temperature cross-section has implications for the prospects of detecting neutralino dark matter. Among the proposed effects signaling the presence of the particles comprising dark matter are (i) gamma ray line flux from annihilations in the galactic halo \( \text{[13]} \) (ii) high energy neutrino flux from dark matter particles annihilating after being trapped in the Sun or the Earth \( \text{[14]} \) and (iii) direct cryogenic detection of DM particles
scattering off nuclei [15] (for a review, see [16]). Referring again to fig. (2) we see that the inclusion of mixing allows the relic abundance of binos to be the same as in the nonmixing case in spite of the increased cross section $a$, due to a large decrease in the p-wave part of the cross-section given by $b$. The increased value of $a$ will directly increase the expected gamma flux in (i) as the flux is proportional to $a$. Bino annihilation in the sun is not affected, as the rate there is limited by the sun’s ability to trap binos. In cryogenic detectors, the effect will also be important, as pointed out in [17], due to the effect of mixing on the elastic cross-section (see also [18, 19] for more on the elastic scattering of binos).

In fig. (3), we show the region of the $m_B - A$ parameter space excluded by the conditions 1) $\Omega_B h^2 < 1/4$, 2) the lightest sfermion is heavier than the bino, and 3) the lightest sfermion is heavier than 74 GeV. Here we have set $\epsilon = 3000$ GeV, but note that for other choices of $\epsilon$ the only substantial change in the picture is a shift in the values of $A_f$ (since the stop is dominant over much of the space, the relic abundance depends largely only on the off-diagonal matrix elements for the stops). The upper limit on the bino mass has the same physical origin as in the unmixed case. For no mixing, the smallest value of $\Omega_B$ occurs when $m_B = m_{\tilde{\tau}}$; as both $m_B$ and $m_{\tilde{\tau}}$ are increased, $\Omega_B$ also increases. When we include mixing, the upper limit depends on the trilinear mass term $A$, as shown in fig. (3). Again, the smallest value of $\Omega_B$ for a fixed $A_f$ and $m_B$ occurs when the bino is as heavy as the lightest sfermion.

The lower limit on $m_B$ seems to be a general feature of including mixing. The stop mixing effect becomes large outside of a region a few hundred GeV wide near $A_f = 1500$ GeV and the condition that $m_B > m_{\tilde{\tau}}$ forces a large value for the diagonal entries, $M$. The large sfermion masses suppress the annihilation rate below the top threshold. Near $A = 1500$ GeV, there is little stop mixing, $M$ can be much smaller, and binos can efficiently annihilate into $\tau$’s and $b$’s. In this region, the condition $m_{\tilde{\tau}} > 74$ GeV limits the annihilation rate and determines the lower bound on $m_B$.

Finally, in fig. (4) we show the effect of $A$ in the off-diagonal mass term. The bino mass is fixed at 150 GeV, $\epsilon = 3000$ GeV and we plot $\Omega_B h^2$ as a function of $A$ for several values of $M$. We effectively pass through the nonmixing case at a value of $A = 1500$ GeV (where the stops become unmixed). Again the points plotted are the largest range of $A$ consistent with
the identification of the bino as the LSP. In the absence of mixing, as the sfermion mass was increased the cross section decreased leading to increased \( \Omega_{\widetilde{B}h^2} \). So choosing a desired value for \( \Omega_{\widetilde{B}h^2} \) fixed the value of the sfermion mass in relation to the bino mass. Now with the addition of the parameter \( A \) it is possible to achieve a range of possible sfermion masses consistent with a given relic density as is evidenced in fig. 4.

To summarize, we have calculated the effect of right and left sfermion mixing on the annihilation cross-section for binos. For a fixed galactic halo density of binos, we find mixing enhances the zero-temperature cross-section; this may improve prospects of detecting neutralino dark matter through gamma ray flux from annihilations in the halo or cryogenic detection of binos scattering from nuclei. When the mixing effect for stops becomes pronounced we also find stronger upper and lower bounds on \( m_{\widetilde{B}} \) coming from the requirement that \( \Omega_{\widetilde{B}h^2} < 1/4 \) and the consistency condition that the bino be the LSP. Finally, given a halo density \( \Omega_{\widetilde{B}h^2} \), bino mass \( m_{\widetilde{B}} \) and choice of \( \epsilon \) we note that in the mixing case we are allowed a range of sfermion masses through variation of the trilinear mass terms \( A_f \), whereas the sfermion mass was uniquely determined in the unmixed case.

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Figure Captions

**Figure 1:** The zero-temperature cross-section, $a$, as a function of the bino mass when the diagonal entries of the sfermion mass matrix are held fixed. The upper curve ($\epsilon = 3000 \text{ GeV}$) and lower curve ($\epsilon = -3000 \text{ GeV}$) assume sfermion mixing. In the middle dotted curve the values of $A_f$ are taken in such a way as to create no mixing, but with the same diagonal mass entries as in the mixed cases.

**Figure 2:** The ratio of the zero-temperature cross-sections, $a_{\text{mix}}/a_{\text{nomix}}$ as a function of the bino mass when the relic density $\Omega^{\tilde{B}}h^2 = \frac{1}{4}$ is fixed. $\epsilon = 3000 \text{ GeV}$ and the diagonal mass choices are made as in Figure 1.

**Figure 3:** Allowed regions ($\Omega^{\tilde{B}}h^2 \leq \frac{1}{4}$) in the $m_{\tilde{B}} - A_f$ plane for $\epsilon = 3000 \text{ GeV}$.

**Figure 4:** The relic bino density $\Omega^{\tilde{B}}h^2$ at $\epsilon = 3000 \text{ GeV}$ as a function of $A_f$ for several values (as labeled) of the diagonal entry of the sfermion mass matrix, $M$. 

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