Probing Turbulence in the Interstellar Medium of Galaxies

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Abstract. The power spectrum of HI intensity fluctuation of the interstellar medium carries information of the turbulence dynamics therein. We present a method to estimate the power spectrum of HI intensity fluctuation using radio interferometric observations. The method involves correlating the visibilities in the $u-v$ plane at different baselines. This method is particularly useful for evaluating the power spectrum for the faint dwarf galaxies. We apply this method to 3 spiral galaxies and 5 dwarf galaxies. The measured power spectrum seem to follow a power law $P_{HI}(U) = AU^α$, suggesting turbulence to be operational. Further, depending on the slope of the power spectrum, we expect the presence of 2D and 3D turbulence in those galaxies.

1. Introduction

Evidence has been mounting in recent years that turbulence plays an important role in the physics of the ISM as well as in governing star formation. It is believed that turbulence is responsible for generating the hierarchy of structures present across a range of spatial scales in the ISM (e.g. Elmegren & Scalo 2004a; Elmegren & Scalo 2004b). In such models the ISM has a fractal structure and the power spectrum of intensity fluctuations is a power law, indicating that there is no preferred “cloud” size.

On the observational front, power spectrum analysis of HI intensity fluctuations is an important technique to probe the structure of the neutral ISM in galaxies. The power spectra of the HI intensity fluctuations in our own galaxy, the LMC and the SMC all show power law behaviour (Crovisier & Dickey 1983; Green 1993; Deshpande et al. 2000; Elmegreen et al. 2001; Stanimirovic et al. 1999) which is a characteristic of a turbulent medium.

Recently Begum et al. (2006) have presented a visibility based formalism for determining the power spectrum of HI intensity fluctuations in galaxies with extremely weak emission. This formalism was applied to a dwarf galaxy, DDO 210 and a spiral galaxy NGC 628 (Dutta et al. 2008). Interestingly, the HI power spectrum of both of these galaxies were found to be power law. In this report we present a elaborate description of this method. We also report the result of estimation of the power spectrum of 3 spiral galaxies and 5 dwarf galaxies.
2. A visibility based power spectrum estimator

The specific intensity of the HI emission from a galaxy may be modelled as

\[ I_\nu(\vec{\theta}) = W_\nu(\vec{\theta}) \left[ \bar{I}_\nu + \delta I_\nu(\vec{\theta}) \right]. \] (1)

Here we have assumed that this is the sum of a smooth component and a fluctuating component. We express the fluctuating component of the specific intensity as \( W_\nu(\vec{\theta}) \delta I_\nu(\vec{\theta}) \), where \( \delta I_\nu(\vec{\theta}) \) is assumed to be a statistically homogeneous and isotropic stochastic fluctuation and \( W_\nu(\vec{\theta}) \) is the window function which quantify the overall large scale HI distribution of the galaxy. Since, the angular extent of the galaxies that we consider here is much smaller than the primary beam, we can write the visibility, the quantity directly measured by the radio interferometers, as

\[ V_\nu(\vec{U}) = \tilde{W}(\vec{U}) \bar{I}_\nu + \tilde{W}(\vec{U}) \otimes \tilde{\delta I}_\nu(\vec{U}) + N_\nu(\vec{U}) \] (2)

where the tilde \( \tilde{\cdot} \) denotes the Fourier transform of the corresponding quantity and \( \otimes \) denotes a convolution. In addition to the signal, each visibility also contains a system noise contribution \( N_\nu(\vec{U}) \) which we have introduced in eq. (2). The noise in each visibility is a Gaussian random variable and the noise in the visibilities at two different baselines \( \vec{U} \) and \( \vec{U}' \) is uncorrelated. At larger baselines, where the effect of the window function can be neglected, we can write,

\[ V_\nu(\vec{U}) = \tilde{W}(\vec{U}) \tilde{\delta I}_\nu(\vec{U}) + N_\nu(\vec{U}) \] (3)

We use the power spectrum of HI intensity fluctuations \( P_{HI}(U) \) defined as

\[ \langle \delta \tilde{I}_\nu(\vec{U}) \delta \tilde{I}_\nu^*(\vec{U}') \rangle = \delta^2(\vec{U} - \vec{U}') P_{HI}(U) \] (4)

to quantify the statistical properties of the intensity fluctuations. We use angular averaging in place of the ensemble averaging denoted by the angular brackets. The square of the visibilities can, in principle, be used to estimate \( P_{HI}(U) \)

\[ \langle V_\nu(\vec{U})V_\nu^*(\vec{U}) \rangle = \left| \tilde{W}_\nu(\vec{U}) \right|^2 \otimes P_{HI}(\vec{U}) + \left| \langle N_\nu(\vec{U}) \rangle \right|^2 \] (5)

The last term \( \left| \langle N_\nu(\vec{U}) \rangle \right|^2 \), which is the noise variance, introduces a positive bias in estimating the power spectrum. The noise bias can be orders of magnitude larger than the power spectrum for the faint external galaxies considered here. The problem of noise bias can be avoided by correlating visibilities at two different baselines for which the noise is expected to be uncorrelated. We define the power spectrum estimator

\[ \hat{P}_{HI}(\vec{U}, \Delta \vec{U}) = \langle V_\nu(\vec{U})V_\nu^*(\vec{U} + \Delta \vec{U}) \rangle \]

\[ = \int d^2U' W_\nu(\vec{U} - \vec{U}') W_\nu^*(\vec{U} + \Delta \vec{U} - \vec{U}') P_{HI}(\vec{U}') \] (6)

In our analysis we consider two different models for the window function of a galaxy. We present them here in the Fourier transformed form,

\[ \tilde{W}_\nu(\vec{U}) = \frac{2}{2\pi \theta_0 U} J_1(2\pi \theta_0 U) \] (7)
and
\[ \tilde{W}_\nu(\vec{U}) = \exp(-\pi^2 \theta_0^2 U^2/2) \] (8)

It is to note that the visibilities at two different baselines will be correlated only if \( |\Delta \vec{U}| < (\pi \theta_0)^{-1} \), and not beyond. In our analysis we restrict the difference in baselines to \( |\Delta \vec{U}| \ll (\pi \theta_0)^{-1} \) so that \( \tilde{W}_\nu(\vec{U} + \Delta \vec{U} - \vec{U}') \approx \tilde{W}_\nu(\vec{U} - \vec{U}') \) and the estimator \( \hat{P}_{HI}(\vec{U}, \Delta \vec{U}) \) no longer depends on \( \Delta \vec{U} \). We then use the visibility correlation estimator

\[
\hat{P}_{HI}(\vec{U}) = \langle \nu_\nu(\vec{U}) \nu_\nu^*(\vec{U} + \Delta \vec{U}) \rangle = \int d^2 U' | W_\nu(\vec{U} - \vec{U}')|^2 P_{HI}(\vec{U}').
\] (9)

We use the real part of the estimator \( \hat{P}_{HI}(\vec{U}) \) to estimate power spectrum \( P_{HI}(U) \). The real part is the power spectrum of HI intensity fluctuations convolved with the square of the window function. At large baselines, the effect of the window function can be neglected and we then have

\[
\hat{P}_{HI}(\vec{U}) = C P_{HI}(\vec{U})
\] (10)

where \( C = \int | \tilde{W}_\nu(U) |^2 d^2 U \) is a constant. The estimator \( \hat{P}_{HI}(\vec{U}) \) also has a small imaginary part that arises from the HI power spectrum because the assumption that \( \tilde{W}_\nu(\vec{U} + \Delta \vec{U} - \vec{U}') \approx \tilde{W}_\nu(\vec{U} - \vec{U}') \) is not strictly valid. We use the requirement that the imaginary part of \( \hat{P}_{HI}(\vec{U}) \) should be small compared to the real part as a self-consistency check to determine the range of validity of our formalism.

The real and imaginary parts of the measured value of the estimator \( \hat{P}_{HI}(\vec{U}) \) both have uncertainties arising from (1.) the sample variance and (2.) the system noise. We add both these contributions to determine the \( 1 - \sigma \) error-bars. The reader is referred to Section 3.2 of Ali et al. (2008) for further details of the error estimation.

3. Result and Discussion

The dwarf galaxy data used here is from Giant Metrewave Radio Telescope (GMRT) observations. We use Very Large Array (VLA) archival data, for the spiral galaxies NGC 628, NGC 2403 and NGC 1058. Details can be found in Dutta et al. 2009 (in preparation). All the data are reduced in the usual way using standard tasks in classic AIPS. For each galaxy, after calibration the frequency channels with HI emission were identified and a continuum image was made by combining the line free channels. The continuum was hence subtracted from the data in the \( u - v \) plane using the AIPS task UVSUB. We correlate the visibilities in each channel as in eq. (7) and then average over channels to get \( \hat{P}_{HI}(U) \). We then fit a power law \( A U^\alpha \) to the estimator \( \hat{P}_{HI}(U) \). The best

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1 NRAO Astrophysical Image Processing System, a commonly used software for radio data processing.
fit $A$ and $\alpha$ were determined through a $\chi^2$ minimization. To test whether the impact of the window function is actually small we have convolved the best fit power spectrum with $|\tilde{W}(U)|^2$. We estimate the goodness of fit ($\chi^2$) to the data for the convolved power spectrum. The fit is accepted only after ensuring that the effect of the convolution can actually be ignored. The results are listed in table 1.

The galaxies in our sample have dichotomy in the slope $\alpha$ ranging from $-2.6$ to $-1.1$. We have proposed a possible explanation for this in Dutta et al. (2008). This was based on the fact that DDO 210, where the power spectrum was measured across length-scales $100 - 500$ pc, had a slope of $-2.6$ while NGC 628, a nearly face-on galaxy where the power spectrum was measured across length-scales $0.8 - 8$ kpc had a slope of $-1.6$. We have interpreted the former as three dimensional (3D) turbulence operational at small scales whereas the latter was interpreted as two dimensional (2D) turbulence in the plane of the galactic disk. For a nearly face-on disk galaxy we expect the transition from 2D to 3D turbulence to be seen at a length-scale corresponding to the scale height of the galaxy. Continuing with this interpretation implies that we have also measured 3D turbulence in NGC 3741, and 2D turbulence in NGC 2403, UGC 4459, GR 8 and AND IV.

The slope $\alpha = -1.0 \pm 0.2$ gives a good fit to the power spectrum measured at the length-scales $1.5 - 10.0$ kpc for NGC 1058, where for the length scales $600$ pc$-1.5$ kpc the best fit slope is $\alpha = -2.5 \pm 0.6$. This, following the argument of dimensionality, is a transition from 2D to 3D turbulence. Since the length-scale $10.0$ kpc is definitely larger than the scale height of any spiral galaxy, we interpret the wave-length $-1.5 \pm 0.3$ kpc at this transition as the scale-height of the HI disk of NGC 1058. To our knowledge this is the first observational
Table 1. The results for the 5 dwarf and 3 spiral galaxies in our sample. Rows 1-3 gives (1) the best fit slope $\alpha$, (2) length-scales over which the power law fit is valid and (3) the goodness of fit $\chi^2$ per degree of freedom.

| Dwarf          | DDO 210 | NGC 3741 | UGC 4459 | GR 8  | AND IV |
|----------------|---------|----------|----------|-------|--------|
| (1) $\alpha$   | $-2.6 \pm 0.6$ | $-2.2 \pm 0.4$ | $-1.8 \pm 0.6$ | $-1.1 \pm 0.4$ | $-1.3 \pm 0.3$ |
| (2) range (kpc) | $0.10 - 0.50$ | $0.15 - 3.75$ | $0.16 - 1.78$ | $0.1 - 1.5$ | $0.56 - 6.2$ |
| (3) $\chi^2/\nu$ | 0.6     | 0.4      | 0.8      | 0.8   | 0.4    |
| Spiral         | NGC 628 | NGC 2403 | NGC 1058 | NGC 1058 |
| (1) $\alpha$   | $-1.7 \pm 0.2$ | $-1.8 \pm 0.2$ | $-1.0 \pm 0.2$ | $-2.5 \pm 0.6$ | - |
| (2) range (kpc) | $1.0 - 10.0$ | $0.5 - 4.4$ | $1.5 - 10.0$ | $0.6 - 1.5$ | - |
| (3) $\chi^2/\nu$ | 0.2     | 0.9      | 0.9      | 0.5   | -      |

determination of the scale-height of a nearly face on spiral galaxy through its HI power spectrum.

We have plan to use the same estimator to estimate the power spectrum of a large sample of spiral galaxy and to find the correlation of the slope of the power spectrum of those galaxies with the other measurable dynamical parameters like scale height, total HI mass etc. of the galaxies.

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