Massless Scalar QED
with Non-Minimal Chern-Simons Coupling

by

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ABSTRACT: 2+1 dimensional massless scalar QED with \((\phi^* \phi)^3\) scalar self-coupling is modified by the addition of a non-minimal Chern-Simons term that couples the dual of the electromagnetic field strength to the covariant current of the complex scalar field. The theory is shown to be fully one-loop renormalizable. The one loop effective potential for the scalar field gives rise to spontaneous symmetry breaking which induces masses for both the scalar and vector fields. At high temperature there is a symmetry restoring phase transition.

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1 Introduction

Lower dimensional field theories with Chern-Simons terms are of interest for a variety of reasons. They possess interesting theoretical properties[1] and have been proposed to describe the physics of planar systems relevant to the fractional Hall effect [2] and high temperature superconductivity [3]. Any theory that can describe superconductivity must contain a phase transition or, in the language of quantum field theory, such a theory must exhibit spontaneous symmetry breaking. Previous work has attempted to study the superconducting phase transition by considering scalar quantum electrodynamics (QED) minimally coupled to a Chern-Simons (CS) term (see Burgess et al [4] and references therein). This model is a natural place to start since scalar QED in 3 + 1 dimensions exhibits spontaneous symmetry breaking and, in 2 + 1 dimensions, the coupling to the CS term gives fractional statistics to the scalar fields. The Lagrangian has the form:

$$\mathcal{L} = \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) - \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{\tau}{4!} (\phi^* \phi)^2 - \frac{\lambda}{6!} (\phi^* \phi)^3 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{m_{CS}}{4} A^\mu \tilde{F}_\mu$$  \hspace{1cm} (1)

$\phi = (\chi + i\eta)$ is a complex-valued scalar field, $\{\mu, \nu,... = 0, 1, 2\}$ are Lorentz indices and the covariant derivative is:

$$D_\mu \phi = \partial_\mu \phi + ie A_\mu \phi \hspace{1cm} (2)$$

$\tilde{F}_\mu[A] = \epsilon^{\mu\nu\alpha} F_{\nu\alpha}$ is the dual of the electromagnetic field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ with $\epsilon^{\mu\nu\alpha}$ the totally antisymmetric tensor density, $\epsilon^{012} = 1$. A detailed analysis of the dynamics of the phase transition in the above model[4] has shown that the CS term has essentially no effect other than that of shifting the zero-point energy. If the phase transition is to be interpreted as a superconducting phase transition it seems surprising that its physical properties are independent of the term that is responsible for generating fractional statistics.

In the present paper we will examine a model in which the scalar field is non-minimally coupled to a manifestly gauge invariant CS term. In particular, we will replace the standard CS term in the Lagrangian Eq.(1) by a term of the form,

$$\mathcal{L}_{CS} = -\frac{i}{4} \gamma[|\phi|] \tilde{F}_\mu[A] J_\mu[A, \phi] \hspace{1cm} (3)$$

The covariant electromagnetic current associated with the scalar field in our model is,

$$J_\mu[\phi, A] = \frac{1}{2} [\phi^* (D_\mu \phi) - (D_\mu \phi)^* \phi] \hspace{1cm} (4)$$
and $\gamma[|\phi|]$ is an arbitrary function of the magnitude of the scalar field. This term is the only gauge invariant expression that can be added to the Lagrangian without giving up one-loop renormalizability. We will show below that the standard CS term is, in fact, contained in this more general expression. As in standard Abelian CS theory, $L_{CS}$ is odd under parity reversal, and is topological, in the sense that it does not depend on the space time metric. In the present case, it is also manifestly gauge invariant. Its connection with the standard CS term is most easily seen by going to the parametrization:

$$\phi = |\phi|e^{i\alpha}. \quad (5)$$

In this case,

$$L_{CS} = \frac{1}{4}e \gamma[|\phi|]|\phi|^2 \phi^*(A_\mu + \frac{1}{e} \partial_\mu \alpha)\tilde{F}^\mu. \quad (6)$$

If one chooses $\gamma = m/(e\phi^*\phi)$, then $L_{CS}$ is a manifestly gauge invariant form of the Abelian CS term with topological mass $m$. The second term, which is necessary for gauge invariance, is a total divergence and can be dropped, leaving the standard CS Lagrangian.

For $\gamma = k/|\phi|$, the theory is equivalent to one considered by Paul and Khare who started with a kinetic term for the scalar field written in terms of a non-minimal covariant derivative:

$$D_\mu \phi = D_\mu \phi + ig \tilde{F}_\mu \phi \quad (7)$$

The Lagrangian they used was:

$$\bar{\mathcal{L}} = \frac{1}{2}(D^\mu \phi)^*(D_\mu \phi) - V(\phi) - \frac{1}{4}(1 + g^2)F^{\mu\nu}F_{\mu\nu}$$

$$= \frac{1}{2}(D^\mu \phi)^*(D_\mu \phi) - V(\phi) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{ig}{|\phi|} \tilde{F}^\mu J_\mu \quad (8)$$

which is clearly equivalent to the one above with $\gamma = 4g/|\phi|$.

In this paper we will examine in detail the quantum effects in this theory at the one loop level. One of the more remarkable results is that renormalizability of the effective potential puts severe restrictions on the form of $\gamma$. In fact, if we restrict $\gamma$ to be a polynomial function of the magnitude of the scalar field, then there are only two distinct possibilities: One possibility is

$$\gamma = \gamma_0 + m_{CS}/(e\phi^*\phi) \quad (9)$$
In this case, the CS part of the Lagrangian Eq. (3) has the simple form:

\[ \mathcal{L}_{CS} = \frac{1}{4} m_{CS} A^{\mu} \tilde{F}_{\mu} + \frac{1}{4} \gamma_0 J^{\mu} \tilde{F}_{\mu} \]  

(10)

Note that the first term in Eq. (10) is just the usual CS term. Moreover, as will be shown in section 3, Eq. (3) is the most general expression, that includes the standard CS term, that can be added to the Lagrangian without destroying one-loop renormalizability. The second possibility is essentially the Lagrangian of Paul and Khare. However, this choice is somewhat unnatural in the sense that it introduces an explicit dependence on \(|\phi|\) into a Lagrangian with other terms that depend only on \(\phi^* \phi\).

In the following, we will therefore restrict consideration to \(\gamma\) of the form Eq. (9) given above. Since our purpose is to consider a theory in which all masses are radiatively induced we will also set the bare masses \(\mu\) and \(m_{CS}\) to zero in Eq. (1) and Eq. (10). We will show that a finite value for \(m_{CS}\) would not effect the renormalizability of the theory, or the effective potential except for a shift of the zero point energy. The paper is organized as follows: In section 2 we give the equations of motion, the conserved current, and the Feynman rules for this theory. In section 3 we show that the symmetric vacuum is unstable against radiative corrections. The one loop effective potential for the scalar field has a symmetry breaking minimum which induces masses for both the scalar and vector modes in the theory. As in the standard Coleman-Weinberg mechanism, the resulting mass ratios can be expressed in terms of the only dimensionless coupling constants in the theory, namely \(\lambda\) and \(\delta \equiv \gamma_0 e\). In section 4 we derive the necessary one-loop counter terms and show that they are all of the form of terms found in Eq. (1). In section 5 we derive the finite temperature contribution to the one loop effective potential and show that there is a symmetry restoring phase transition at a high temperature. Finally, in section 6 we close with a discussion of the results and prospects for future work.

2 Feynman Rules

We start with the bare Lagrangian,

\[ \mathcal{L} = \frac{1}{2} (D_\mu \phi)^* (D^\mu \phi) - \frac{\lambda}{6!} (\phi^* \phi)^3 - \frac{1}{4} F_{\mu \nu}^{\mu} F_{\nu}^{\mu} - \frac{i}{4} \gamma_0 \tilde{F}^{\mu} J_{\mu} \]  

(11)

The conserved current associated with the gauge invariance is

\[ \mathcal{J}_\mu = J_\mu + \frac{i}{4} \gamma_0 \phi^* \phi \tilde{F}_\mu \]  

(12)
The equations of motion for the gauge boson field and the scalar field are,

\[ \partial^\tau F_{\tau\mu} = ie J_\mu + \frac{i}{2} \gamma_0 \epsilon_{\mu\tau\lambda} \partial^\tau J^\lambda \]  
(13)

\[ D^\mu D_\mu \phi + \frac{\lambda}{5!} (\phi^* \phi)^2 \phi + i \frac{\gamma_0}{2} \tilde{F}_\mu D^\mu \phi = 0 \]  
(14)

Since we are interested in generating masses for the fields by radiative corrections, we have set the bare masses \( \mu \) and \( m_{CS} \) to zero. Mass counter-terms will appear, however, because there is no symmetry of the theory that ensures that the bare parameters will vanish in the limit that the renormalized parameters are zero. We have verified that none of our results would change if we had used finite bare masses, other than the form of the mass counter-terms which are necessary to give zero renormalized masses at \( v = 0 \), where \( v \) is the vacuum expectation value of the real part of the scalar field. Thus, the values of the bare masses have no physical significance. We will obtain a renormalized theory with radiatively induced spontaneous symmetry breaking by setting the physical renormalized masses to zero at \( v = 0 \), and using this equation as a renormalization condition. For simplicity, we have also chosen to set the bare quartic coupling to zero. We will derive the quartic counter-term that is necessary to give a renormalized quartic coupling of zero at \( v = 0 \).

To quantize the theory we must add a gauge fixing term to the Lagrangian in Eq.(11). We use the \( R_\xi \) gauge where power counting arguments are possible. The gauge fixing term is,

\[ \mathcal{L}_{GF} = -\frac{1}{2\xi} (\partial_\mu A^\mu - \xi e v \eta)^2 \]  
(15)

To study spontaneous symmetry breaking, we shift the real part of the scalar field:

\[ \chi \rightarrow \chi + v. \]  

The Lagrangian becomes:

\[
\mathcal{L} = \frac{1}{2} (\partial_\mu \chi)^2 + \frac{1}{2} (\partial_\mu \eta)^2 - \frac{1}{2} \xi e^2 v^2 \eta^2 - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu)^2 \\
+ \frac{1}{2} e^2 g_{\mu\nu} A^\mu A^\nu + \frac{1}{2} \delta \epsilon^{\mu
u\alpha\beta} v^2 A_\mu \partial_\nu A_\alpha + e A^\mu (\chi \partial_\mu \eta - \eta \partial_\mu \chi) \\
+ \frac{1}{2} e^2 g_{\mu\nu} A^\mu A^\nu (\eta^2 + \chi^2) + \frac{\gamma_0}{2} (\chi \partial_\mu \eta - \eta \partial_\mu \chi) \epsilon^{\mu\alpha\nu} \partial_\alpha A_\nu + \frac{\delta}{2} \epsilon^{\mu\alpha\nu} (\chi^2 + \eta^2) A_\mu \partial_\alpha A_\nu \\
+ \delta v \chi \epsilon^{\mu\alpha\nu} A_\mu \partial_\alpha A_\nu + e^2 v \chi g_{\mu\nu} A^\mu A^\nu \\
- \frac{\lambda}{6!} [\chi^6 + \eta^6 + 3 \chi^4 \eta^2 + 3 \chi^2 \eta^4 + v(6 \chi^5 + 12 \chi^3 \eta^2 + 6 \chi \eta^4) + v^2 (15 \chi^4 + 3 \eta^4 + 18 \eta^2 \chi^2) \\
+ v^3 (20 \chi^3 + 12 \chi \eta^2) + v^4 (15 \chi^2 + 3 \eta^2)]
\]  
(16)
We obtain the propagators from the terms in the Lagrangian that are quadratic in the fields by inverting the appropriate operators. We use the Landau value for the gauge fixing parameter $\xi = 0$ so that there are no contributions from ghost degrees of freedom. This procedure gives,

$$iS_\chi(P) = \frac{i}{P^2 - m_\chi^2 + i\epsilon}$$

$$iS_\eta(P) = \frac{i}{P^2 - m_\eta^2 + i\epsilon}$$

$$-iD_{\mu\nu}(K) = -i\frac{(K^2 - e^2v^2)(g_{\mu\nu}K^2 - K_\mu K_\nu)}{K^2(K^2 - m_1^2)(K^2 - m_2^2)} + \frac{\delta v^2 K^\alpha \epsilon_{\mu\nu\alpha}}{(K^2 - m_1^2)(K^2 - m_2^2)}$$

where $m_\chi^2 = \frac{1}{4!}v^4$, $m_\eta^2 = \frac{1}{5!}v^4$ and,

$$m_{1,2}^2 = e^2v^2 \left[ 1 + \frac{1}{2}\gamma_0^2 v^2 \pm \frac{1}{2}\gamma_0 v\sqrt{4 + \gamma_0^2 v^2} \right]$$

When the vev of the scalar field is non-zero, the particle spectrum contains four massive modes. All the masses are real as long as $\gamma_0$ is real. In the absence of the CS term ($\gamma_0 = 0$), the gauge boson masses become degenerate: $m_{1,2} = ev$. Note that the Feynman rules would in general be more complicated if we had used arbitrary $\gamma = \gamma[v]$ instead of the constant value $\gamma = \gamma_0$. However, the photon propagator is valid for arbitrary $\gamma = \gamma[v]$, and one can immediately obtain masses for standard CS theory by simply substituting $\gamma_0 = m_{CS}/ev$ into Eq. (19) so that:

$$m_{1,2}^2 = \frac{1}{2}m_{CS}^2 + e^2v^2 \pm \frac{1}{2}m_{CS}^2\sqrt{1 + 4e^2v^2/m_{CS}^2}$$

In this case when $v = 0$, the gauge boson contains one massive and one massless mode as expected for Maxwell-CS theory without symmetry breaking.

The vertices are obtained from the interaction terms in the Lagrangian. They are shown in Fig. 1. We use the following notation: The dotted lines are gauge bosons and all momenta are ingoing. Scalar lines are labelled $\chi$ or $\eta$ and the notation $\chi/\eta$ indicates two separate diagrams. The results are,

$$V_{1a} = -i\lambda; \quad V_{1b} = -i\lambda/5$$

$$V_{2a} = -i\lambda v; \quad V_{2b} = V_{2c} = -i\lambda v/5;$$

$$V_{3a} = -i\lambda v^2/2; \quad V_{3b} = V_{3c} = -i\lambda v^2/10;$$
\[ V_{4a} = -i\lambda v^3/6; \quad V_{4b} = -i\lambda v^3/30 \]
\[ V_5 = e(P - Q)_\lambda \]
\[ V_6 = i\gamma_0 \epsilon^{\alpha\lambda} K_\tau P_\alpha \]
\[ V_7 = 2ie^2 g_{\alpha\beta} \]
\[ V_8 = -\delta e^{\alpha\lambda\beta} (P - Q)_\lambda \]
\[ V_9 = -\delta e^{\mu\alpha\nu}(K - Q)_\alpha \]
\[ V_{10} = 2ie^2 v g_{\mu\nu} \]

\section{One Loop Effective Potential}

In this section we outline the calculation of the zero temperature renormalized one loop effective potential. When performing this calculation, we want to consider the general expression \( \gamma = \gamma[v] \) in order to show that renormalizability requires us to choose \( \gamma = \gamma_0 \), a constant. To obtain the one loop effective potential, we only need to know the propagators and, as discussed earlier, the result for the photon propagator given in the previous section is valid for arbitrary \( \gamma = \gamma[v] \). A straightforward calculation yields the Euclidean momentum space integral for the one loop effective potential:

\[ V^{(1)}(v) = \frac{\hbar}{4\pi^2} \int dkk^2 \left( \ln(k^2 + m_\chi^2) + \ln(k^2 + m_\eta^2) + \ln(k^2 + m_1^2) + \ln(k^2 + m_2^2) \right) \]

(22)

Introducing a cut-off \( \Lambda \), the basic zero temperature integral that needs to be considered is (for \( m^2 > 0 \)):

\[ \int_0^\Lambda dkk^2 \ln(k^2 + m^2) = \frac{1}{3} \Lambda^3 \ln \Lambda^2 - \frac{2}{3} \Lambda^3 + \frac{2}{3} m^2 \Lambda - \frac{1}{3} \pi |m|^3 \]

(23)

Using this result in Eq.(22) we find an expression for the complete one loop effective potential. Since Eq.(19) is valid arbitrary \( \gamma = \gamma[v] \), we obtain an expression for the effective potential as a function of arbitrary \( \gamma[v] \). All masses are assumed positive and from now on we drop the absolute value signs. We obtain,

\[ V(v) = \frac{\lambda}{6!} v^6 + \frac{\hbar}{6\pi^2} (2e^2 v^2 + e^2 \gamma^2 v^4 + \lambda v^4/4! + \lambda v^4/5!) \Lambda \]
\[ + \frac{\gamma}{2} v^2 + \frac{\Omega}{4!} v^4 + \frac{\Xi}{6!} v^6 \]
where we have dropped field independent terms and added the counter-terms appropriate for scalar QED in 2+1 dimensions. Clearly the theory will be one loop renormalizable only if

$$\gamma^2 v^4 = a + bv^2 + cv^4 + dv^6$$  \hspace{1cm} (25)$$

for some constants \(\{a, b, c, d\}\). However, by inspection of the last line in Eq.(24), we see that unless \(d = 0\), the effective potential will be unbounded below for large \(v\). Hence there is at most a three parameter family of allowed Lagrangians. If we require \(\gamma\) to be a sum of rational functions of \(|\phi|\), then there are only two distinct possible solutions. The first solution is obtained if \(c, a = 0\) which gives \(\gamma[v] = \sqrt{b/v}\). This solution is used by Paul and Khare [5]. We note, however, that a CS term of this form introduces an explicit dependence on \(|\phi|\) into a Lagrangian with other terms that depend only on \(\phi^* \phi\) and therefore we will not consider this solution. The second solution is obtained if the right hand side of Eq.(23) is a perfect square which gives a solution of the form \(\gamma[v] = m_{CS}/ev^2 + \gamma_0\). This solution is the only one that preserves the structure of the original Lagrangian. As was discussed in the Introduction, the first term in this expression gives the usual CS term. Thus we have the remarkable result that \(\gamma[|\phi|] = \gamma_0 = \text{constant}\) is the unique non-minimal CS term that can be added to the usual CS term while preserving one loop renormalizability of the effective potential.

For \(\gamma_0 = \text{constant}\), the mass and quartic coupling can be renormalized to zero at \(v = 0\) by choosing:

$$\Upsilon = -\frac{2\hbar e^2 \Lambda}{3\pi^2}$$ \hspace{1cm} (26)$$

$$\Omega = -\frac{\hbar}{6\pi^2}(4!e^2\gamma_0^2 + 6\lambda/5)\Lambda$$

All the derivatives of the finite part of the effective potential to sixth order are zero when \(v\) approaches zero from the right. Therefore, in contrast to the Coleman and Weinberg problem, all the renormalizations can be done at \(v\) equal to zero, and do not receive contributions from the derivatives of the finite part of the effective potential.

As noted by Coleman and Weinberg, self-consistency of the loop expansion demands that the one loop contributions proportional to \(m_\chi^3 \sim m_\eta^3 \sim \lambda^{3/2} v^6\) be neglected compared to the tree level \(\lambda\). In this case, since the renormalized mass and quartic
couplings are chosen to be zero, the remaining one loop effective potential is completely gauge and parametrization independent. The same result is obtained in the unitary gauge in [8]. One way to understand this is to note that all such problems occur in the scalar sector of the theory, which we are neglecting at the one loop level. Equivalently, we note that the effective potential is manifestly gauge invariant because it is evaluated “on shell” [9]: when \( \lambda \approx 0, \phi = \text{constant} \) solves the classical field equations. Consequently, the Vilkovisky-DeWitt corrections used in [4] to ensure manifest gauge fixing independence, for example, are identically zero.

The gauge invariant renormalized one loop effective potential is:

\[
V = \frac{\lambda v^6}{6!} - \frac{\hbar}{12\pi} e^3 v^3 f(x)
\]

\[
= \frac{\lambda}{6!\gamma_0} (x^6 - Ax^3 f(x))
\]

where we have defined the dimensionless quantities \( x = \gamma_0 v \) and

\[
A = \frac{6!\hbar}{12\pi} \left( \frac{\delta^3}{\lambda} \right).
\]

In the above:

\[
f(x) = [1 + \frac{1}{2} x^2 + \frac{1}{2} x \sqrt{4 + x^2}]^{(3/2)} + [1 + \frac{1}{2} x^2 - \frac{1}{2} x \sqrt{4 + x^2}]^{(3/2)}
\]

The potential in Eq.(27) exhibits symmetry breaking as long as the constant \( A < 1 \). This is easily seen by noting first that as \( x \to 0, V \sim -x^3 \), while for large \( x, V \sim (1-A)x^6 \). Thus, the requirement that the potential be bounded below guarantees the presence of symmetry breaking, and puts the following restriction on the ratio of dimensionless couplings in the theory:

\[
\frac{\delta^3}{\lambda} < \frac{12\pi}{6!\hbar}
\]

Since consistency of the one loop approximation requires \( \lambda < 1 \), this also puts a corresponding bound on \( \delta \) of approximately \( \delta < (1/20)^{1/3} \). In Fig. 2, the effective potential is plotted (up to an overall constant) for \( A = 1/4 \). The zero temperature potential is given by the curve labeled \( g = \gamma_0 T/e = 0 \). The location of the symmetry
breaking minimum $\tau = \gamma_0 \tau$ is easily found in terms of the couplings. The implicit equation is:

$$A = \frac{6\tau^3}{(3f(\tau) + \tau f'(\tau))}$$

(31)

A more useful form of the above is:

$$\frac{\lambda}{5!} = \frac{e^3}{v^3} \frac{\hbar}{12\pi} (3f(\tau) + \tau f'(\tau))$$

(32)

In contrast to 4-d scalar QED, the scalar self-coupling cannot be expressed solely in terms of the electromagnetic coupling constant, since now the latter has dimensions of $L^{(-\frac{1}{2})}$.

We can evaluate the mass ratios of the gauge bosons and scalar particles. The $\eta$ mass is $m_\eta^2(\tau) = \lambda \tau^4/5!$ and the masses of the gauge bosons are given by $m_1^2(\tau)$ and $m_2^2(\tau)$, where $m_1^2$ and $m_2^2$ are defined in Eq.(19). The mass of the $\chi$ field is,

$$m_\chi^2 = \frac{\partial^2 V}{\partial v^2} \bigg|_\tau$$

$$= \frac{\hbar e^3 \tau}{12\pi} (9f(\tau) - \tau f'(\tau) - \tau^2 f''(\tau))$$

(33)

Using $e/\tau = \delta/\tau$, we obtain,

$$\frac{m_\chi^2}{m_{1,2}^2} = \frac{\hbar \delta}{12\pi} \left( \frac{9f(\tau) - \tau f'(\tau) - \tau^2 f''(\tau)}{\tau (1 + \frac{1}{2}\tau^2 \pm \frac{1}{2}\tau \sqrt{4 + \tau^2})} \right)$$

$$\frac{m_\eta^2}{m_{1,2}^2} = \frac{\lambda}{5\delta^2} \left( \frac{\tau^2}{1 + \frac{1}{2}\tau^2 \pm \frac{1}{2}\tau \sqrt{4 + \tau^2}} \right)$$

(34)

Using $e/\tau = \delta/\tau$ in Eq.(32), we have $\tau$ as a function of $\lambda$ and $\delta$. Thus, Eq.(34) effectively expresses the mass ratios as functions of these two dimensionless couplings, as required. By expanding Eq.(34) in $\delta$, we obtain an expression that gives the effect of the non-minimal CS term on the physical parameters. We obtain,

$$\frac{m_\chi^2}{m_{1,2}^2} = \frac{3}{5!} \left( \frac{6!h}{12\pi} \right)^{2/3} \left( 1 \mp \delta \left( \frac{6!h}{12\pi \lambda} \right)^{1/3} \right)$$

(35)

The contribution from the non-minimal CS term has an appreciable effect on the mass ratios when

$$\frac{\delta^3}{\lambda} \sim \frac{12\pi}{6!h}$$

(36)
We note that the contributions in Eq.(35) become equal at precisely the point at which the potential becomes unbounded when $x$ approaches infinity Eq.(30). Therefore, the non-minimal CS term that we have introduced can never dominate the mass ratios. From the lowest curve in Fig. 2 we obtain $x = .87$, when $T = 0$ and $A = 1/4$. This gives $m^2_\chi = 0.7e^3v$, $m^2_1 = 2.33e^2v^2$ and $m^2_2 = 0.43e^2v^2$. The mass ratios are, $m^2_\chi/m^2_1 \sim 0.34\delta$ and $m^2_\chi/m^2_2 \sim 1.8\delta$.

4 One Loop Renormalizability

In this section we discuss the one loop renormalizability of the Lagrangian Eq.(11), Eq.(15). Power counting tells us that the superficial degree of divergence of any diagram is given by

$$\Delta = 3 - \frac{1}{2}(E_B + E_S) - \frac{1}{2}V_2 - V_3 - \frac{3}{2}V_4 - \frac{1}{2}V_5 - \frac{1}{2}V_6 - V_7 - \frac{1}{2}V_9 - \frac{3}{2}V_{10}$$

(37)

where $E_B$ and $E_S$ are the number of external gauge boson and scalar lines, and $V_i$ indicates the number of vertices of the $i$th type, as defined in Fig. 1. We note that this expression is only valid at one loop: There are two derivatives in the term in the Lagrangian that corresponds to $V_6$ but, in any given one loop diagram, a vertex of the type $V_6$ can only contribute one power of internal momentum to the loop integral because of the presence of the epsilon tensor. This argument is not valid for higher loop diagrams and thus our proof of renormalizability is only valid to one loop order. We use the standard BPHZ formulation of the renormalization procedure [10]. We define the renormalized quantities in terms of the bare quantities:

$$\chi_R = Z^\chi_\chi^{-\frac{1}{2}} \chi$$

$$\eta_R = Z^\eta_\eta^{-\frac{1}{2}} \eta$$

$$A^\mu_R = Z^{A^\mu}_A^{-\frac{1}{2}} A^\mu$$

$$\epsilon_R = Z_\epsilon \epsilon$$

(38)

These definitions allow us to rewrite the original Lagrangian:

$$\mathcal{L} = \mathcal{L}_R + \mathcal{L}_{CT}$$

(39)
\( \mathcal{L}_R \) is of the same form as the original Lagrangian with the bare quantities replaced by the renormalized ones. The counter-term Lagrangian \( \mathcal{L}_{CT} \) is obtained by calculating the contributions from divergent diagrams using the renormalized Lagrangian and constructing \( \mathcal{L}_{CT} \), also as a function of renormalized quantities, to cancel these divergences. We use Pauli-Villiars regularization to avoid the difficulties associated with the use of dimensional regularization for the calculation of diagrams containing epsilon tensors. From now on, all quantities are the renormalized ones, and the subscripts \( R \) are omitted.

To identify divergent diagrams we require \( \Delta \geq 1 \) since, in 2+1 dimensions, all diagrams with \( \Delta = 0 \) give zero by symmetric integration. In addition, there are many diagrams with \( \Delta \geq 1 \) which are, in fact, finite. This effect occurs when a dependence on external momenta reduces the actual degree of divergence. The identification of these diagrams is straightforward except for the diagrams in Fig. 3. For these diagrams, in order to preserve gauge invariance, the Pauli-Villiars regulator must be applied to the closed scalar loop and not to the individual propagators. The resulting expression is finite. The technique is exactly analogous to that used for the closed fermion loop contribution to the photon polarization tensor in ordinary spinor electrodynamics.

The divergent diagrams are shown in Figs. 4-15. On each diagram, the vertices are labeled as in Fig. 1. The dotted lines are gauge bosons, and all momenta are ingoing. The scalar lines are labeled \( \chi \) or \( \eta \) and the notation \( \chi/\eta \) indicates two separate diagrams. The results are given below, where the variable \( M \) is the Pauli-Villiars mass.

Fig. 4a \( \rightarrow i e^2 v M/2\pi \)
Fig. 4b \( \rightarrow i \lambda v^3 M/40\pi \)
Fig. 4c \( \rightarrow i \delta^2 v^3 M/2\pi \)
Fig. 5a \( \rightarrow i e^2 M/2\pi \)
Fig. 5b \( \rightarrow 3 i \lambda v^2 M/40\pi \)
Fig. 5c \( \rightarrow i \lambda v^2 M/40\pi \)
Fig. 5d \( \rightarrow i \delta^2 v^2 M/2\pi \)
Fig. 5e \( \rightarrow i \delta^2 v^2 M/\pi \)
Fig. 6a \( \rightarrow 3 i \lambda v M/20\pi \)
Fig. 6b \( \rightarrow 3 i \delta^2 v M/\pi \)
Fig. 7a → \( i\lambda vM/20\pi \)
Fig. 7b → \( i\delta^2 vM/\pi \)
Fig. 8a → \( 3i\lambda M/20\pi \)
Fig. 8b → \( 3i\delta^2 M/\pi \)
Fig. 9a → \( i\lambda M/20\pi \)
Fig. 9b → \( i\delta^2 M/\pi \)
Fig. 10a → \( 5M\delta \epsilon_{\alpha\beta}P^\lambda/12\pi \)
Fig. 10b → \( i\gamma_0^2 M(g_{\alpha\beta}P^2 - P_\alpha P_\beta)/12\pi \)
Fig. 10c → \( -i\delta^2 v^2 M g_{\alpha\beta}/4\pi \)
Fig. 11 → \( -\delta\gamma_0 MP_\mu/4\pi \)
Fig. 12 → \( -i\delta^2 M g_{\mu\nu}/2\pi \)
Fig. 13 → \( -i\gamma_0^2 M/4\pi \)
Fig. 14 → \( -\delta\gamma_0 vMP_\mu/4\pi \)
Fig. 15 → \( -i\delta^2 v^2 M g_{\mu\nu}/2\pi \)

To cancel these divergences, the counter-term Lagrangian must have the form

\[
\mathcal{L}_{CT} = \frac{1}{2} \alpha_1 (\partial_\mu \chi)^2 + \frac{1}{2} \alpha_2 (\partial_\mu \eta)^2 + \frac{1}{2} \beta_1 e^2 g^{\mu\nu} A_\mu A_\nu \chi^2 + \frac{1}{2} \beta_2 e^2 g^{\mu\nu} A_\mu A_\nu \eta^2
+ \nu e A^\mu (\chi \partial_\mu \eta - \eta \partial_\mu \chi) + \frac{1}{2} \rho \chi g^{\mu\nu} A_\mu A_\nu + \tilde{\gamma} e A^\mu \partial_\mu \eta
+ \frac{1}{2} \gamma_1 A^\mu (\Box g_{\mu\nu} - \partial_\mu \partial_\nu) A^\nu + \frac{1}{2} \gamma_2 e^2 v^2 g^{\mu\nu} A_\mu A_\nu + \frac{1}{2} \gamma_3 \delta v^2 \epsilon^{\mu\nu\lambda\sigma} A_\mu \partial_\alpha A_\nu
+ a \chi - \frac{1}{2} \mu^2 \chi^2 - \frac{1}{2} \mu^2 \eta^2 - \frac{\tau_1}{4!} \chi^4 - \frac{\tau_2}{4!} \eta^4 - \frac{2\tau_3}{3!} \chi^2 \eta^2 - \frac{\theta_1}{3!} \chi^3 - \frac{\theta_2}{2} \chi \eta^2
\]

The values of the coefficients in this expression are determined from the divergences as given in Eq. (40). We have,

\[
\alpha_1 = \alpha_2 = \beta_1 = \beta_2 = \nu = z_2 = \frac{\gamma_0^2 M}{4\pi}
\]

\[
\rho = \frac{\delta^2 v M}{2\pi}
\]

\[
\tilde{\gamma} = \frac{\gamma_0 v M}{4\pi}
\]

13
\[
\begin{align*}
 a & = -\frac{M}{2\pi} \left[ e^2 v + \frac{\lambda}{20} v^3 + \delta^2 v^3 \right] \\
 \mu_1^2 & = \frac{M}{2\pi} \left[ e^2 + \frac{3\lambda}{20} v^2 + 3\delta^2 v^2 \right] \\
 \mu_2^2 & = \frac{M}{2\pi} \left[ e^2 + \frac{\lambda}{20} v^2 + \delta^2 v^2 \right] \\
 \tau_1 & = \tau_2 = \tau_3 = \frac{M}{2\pi} \left[ \frac{3\lambda}{10} + 6\delta^2 \right] \\
 \theta_1 & = \frac{M}{2\pi} \left[ \frac{3\lambda}{10} v + 6\delta^2 v \right] \\
 \theta_2 & = \frac{M}{2\pi} \left[ v + 2\delta^2 v \right] 
\end{align*}
\]

To verify that the addition of the counter-term Lagrangian does not introduce any terms that were not present in the original Lagrangian, we shift the real part of the scalar field back to its original position: \( \chi \rightarrow \chi - v \). After making this shift, the counter-term Lagrangian is,

\[
\mathcal{L}_{CT} = \frac{1}{2} \alpha_1 (\partial_\mu \chi)^2 + \frac{1}{2} \alpha_2 (\partial_\mu \eta)^2 + \frac{1}{2} \beta_1 e^2 g^{\mu\nu} A_\mu A_\nu \chi^2 + \frac{1}{2} \beta_2 e^2 g^{\mu\nu} A_\mu A_\nu \eta^2 + \nu e A^\mu (\chi \partial_\mu \eta - \eta \partial_\mu \chi) + \frac{1}{2} z_1 A^\mu (\Box g_{\mu\nu} - \partial_\mu \partial_\nu) A^\nu + \frac{1}{2} z_3 \delta^2 e^2 e^{\mu\nu\alpha} A_\mu \partial_\alpha A_\nu + \frac{1}{2} \gamma g^{\mu\nu} A_\mu A_\nu [z_2 e^2 v^2 + \beta_1 e^2 v^2 - \rho v] + \chi g^{\mu\nu} A_\mu A_\nu [\frac{1}{2} \rho - \beta_1 e^2 v] + A_\mu \partial^\mu \eta [ev^2 + e \tilde{\gamma}] + \chi [a + \nu \mu_1^2 + \frac{\tau_1}{3!} v^3 - \frac{1}{2} \theta_1 v^2] + \chi^2 [\frac{1}{2} \mu_1^2 - \frac{\tau_1}{4} v^2 + \frac{1}{2} \theta_2 v] + e^2 \frac{\tau_3}{12} v^2 + \eta^2 \frac{1}{2} \mu_2^2 + \frac{1}{2} \theta_1 v + \chi \eta^2 \frac{1}{2} \mu_2^2 + \frac{1}{2} \theta_1 v + \chi \eta^2 [\frac{\tau_1}{3!} v - \frac{1}{3!} \theta_1] + \chi \eta^2 \frac{1}{2} \theta_2 v - \frac{\tau_1}{4} \chi^4 - \frac{\tau_2}{4!} \eta^4 - \frac{2\tau_3}{4!} \chi^2 \eta^2
\]

Substituting in the values for the coefficients as given by Eq. (42) we obtain,

\[
\mathcal{L}_{CT} = (Z^{-1}_\phi - 1) \frac{1}{2} |D_\mu \phi|^2 - (Z^{-1}_A - 1) \frac{1}{4} F_{\mu\nu} F^{\mu\nu}
- \frac{5M\delta}{24\pi} \epsilon_{\mu\nu\rho} A^\mu \partial^\rho A^\nu - \frac{e^2 M}{4\pi} \phi^* \phi - \frac{M}{2\pi 4!} \left[ \frac{3\lambda}{10} + 6\delta^2 \right] (\phi^* \phi)^2
\]
where,

\[ Z_{\phi}^{-1} = 1 + \frac{\gamma_0^2 M}{4\pi}; \quad Z_{A}^{-1} = 1 + \frac{\gamma_0^2 M}{12\pi}; \quad Z_e = Z_A^{1/2} \tag{45} \]

We note that it is the non-minimal CS term that we have introduced that generates the counter-term with the form of the standard CS term. The last three terms in Eq.(44) do not appear in the original Lagrangian because we explicitly set them to zero at the tree level. They are analogous to the mass renormalization term that appears in the Coleman-Weinberg computation of radiatively induced spontaneous symmetry breaking for the $\phi^4$ theory with zero bare mass [6]. These terms appear in our counter-term Lagrangian because the theory does not possess a symmetry which guarantees the vanishing of the bare scalar mass, quartic scalar coupling and standard CS coupling in the limit where the renormalized values of these parameters are zero. It is straightforward to see that the results would remain unchanged if we had included non-zero bare scalar and CS mass terms at the tree level: The theory does not contain any diagrams with a degree of divergence greater than one and therefore, the divergence structure of the theory would not change if we had non-zero bare masses. The only difference in the calculation would be a shift in the values of the mass counter-terms which are necessary to give zero renormalized masses at $v = 0$. We obtain a renormalized theory with radiatively induced spontaneous symmetry breaking by setting the physical renormalized masses to zero at $v = 0$ and using this equation as a renormalization condition.

5 Finite Temperature Effective Potential

At finite temperature, $T$, the calculation of the effective potential in the Matsubara formalism proceeds by analytically continuing to imaginary time $t_E$ and imposing periodic boundary conditions at $t_E = 0, \beta = 1/(kT)$. In Eq.(22), the integral over $k_0$ is replaced by a sum over discrete $k_0 = 2\pi n T$ in the usual way [11]. A straightforward calculation gives the temperature dependent part of the effective potential:

\[ V_T(v) = \frac{\hbar T}{4\pi^2} \int d^2k \left[ \ln(1 - e^{-\beta\omega_1}) + \ln(1 - e^{-\beta\omega_2}) \right] \tag{46} \]

where $\omega_{1,2} = \vec{k}^2 + m_{1,2}^2$. We define the dimensionless variables $\theta = |\vec{k}|/T$, $g = \gamma_0 T/e$ and

\[ y_{1,2}^2 = \frac{m_{1,2}^2}{T^2} = \frac{x^2}{g^2} \left( 1 + \frac{x^2}{2} \pm \frac{x}{2}\sqrt{4 + x^2} \right) \tag{47} \]
Then,
\[ V_T(v) = \frac{hT^3}{2\pi} \int d\theta \sum_{i=1,2} \ln(1 - e^{-\sqrt{\theta^2 + y_i^2}}) \]  
(48)

Combining with Eq.(27), Eq.(28) we have,
\[ V(v) = \frac{\lambda}{6! \gamma_0} \left( x^6 - A g^3 \sum_{i=1,2} y_i^3 + 6 A g^3 \int d\theta \sum_{i=1,2} \ln(1 - e^{-\sqrt{\theta^2 + y_i^2}}) \right) \]  
(49)

Note that as in the zero temperature case we drop the contribution from the scalar self coupling.

The integral can be evaluated numerically, or we can find an analytic expression in the high temperature limit. Integrating by parts we can write,
\[ V_T(v) = -\frac{3hT^3}{2\pi} \sum_{i=1,2} h_4[y_i] \]  
(50)

where,
\[ h_4(y) = \frac{1}{\Gamma(4)} \int_0^\infty \frac{x^3}{(x^2 + y^2)^{3/2}} \left[ \frac{1}{e^{x^2 + y^2} - 1} \right] dx. \]  
(51)

When \( y < 1 \) we can expand
\[ h_4(y) - h_4(0) = \frac{1}{12} \left[ y^2 \ln y^2 - y^2 \right] + O(y^4) \]  
(52)

where we have dropped the terms \( \sim y^3 \) because they are temperature independent. This gives,
\[ V_T(v) = -\frac{hT^3}{8\pi} \sum_{i=1,2} y_i^2 (\ln y_i^2 - 1). \]  
(53)

Combining Eq.(48), Eq.(49), Eq.(53) we obtain,
\[ V(v) = \frac{\lambda}{6! \gamma_0} \left( x^6 - A g^3 \sum_{i=1,2} y_i^3 - \frac{3}{2} A g^3 \sum_{i=1,2} y_i^2 (\ln y_i^2 - 1) \right) \]  
(54)

When \( y < 1 \) this expression agrees with the result obtained from Eq.(45). In Fig. 1 we show the numerical results for two different values of \( g \). A first order phase transition is indicated.
6 Conclusions

We have introduced a non-minimal CS term to massless scalar QED. Renormalizability of the one loop effective potential puts severe restrictions on the form of the non-minimal coupling. In particular, it was shown that only the term in Eq.(3) with \( \gamma = \gamma_0 = constant \) can be added to the usual Chern-Simons Lagrangian without destroying the renormalizability of the effective potential at one loop. It was shown that this theory is fully one loop renormalizable. The purely massless theory is unstable due to radiative corrections which generate masses for all the particles through spontaneous symmetry breaking. At one loop, the symmetry is restored at high temperatures via what appears to be a first order phase transition. This phase transition is qualitatively the same in the minimally coupled CS theory however, unlike the standard CS term which gives only a shift in the zero-point energy, the non-minimal CS term introduced in this paper does affect the physics of the phase transition.

Although the theory with \( \gamma = constant \) appears to be quite sensible at the one loop level, more work is required to discover whether or not higher loops affect the results significantly. As discussed in section 4, the power counting analysis will clearly not work at two loops in the same way that it does at one loop. However, it is not impossible that the theory is renormalizable beyond one loop. Since the naive power counting argument indicates that the theory is not renormalizable, even at one loop, it is clear that power counting arguments can be misleading. In addition, it is possible that the theory is a valid effective theory in some restricted energy regime, and that in this regime the question of renormalizability is not relevant. Given the fact that the symmetry is broken radiatively in this (and other CS couplings) it would be interesting to look for vortex solutions \(^{12}\) with the given induced potential and CS coupling. This work is in progress.

After the completion of this work, some additional references \(^{13},^{14}\) were brought to our attention which motivate the physical interpretation of the coupling \( \gamma \) as a scalar magnetic moment. This idea is discussed in detail in \(^{15}\).

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Figure Captions

1) Feynman rules for vertices
2) The one-loop effective action for various values of $g = \gamma_0 T/\epsilon$
3) A finite contribution to the scalar polarization tensor
4) Some divergent contributions to the scalar one-point function
5) Some divergent contributions to the scalar two-point function
6) Some divergent contributions to the scalar three-point function
7) Some divergent contributions to the scalar three-point function
8) Some divergent contributions to the scalar four-point function
9) Some divergent contributions to the scalar four-point function
10) Some divergent contributions to the gauge boson two-point function
11) A divergent three-point function
12) A divergent four-point function
13) A divergent two-point function
14) A divergent two-point function
15) A divergent three-point function
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