Complex Network for Solar Active Regions

Farhad Daei, Hossein Safari ©, and Neda Dadashi

Department of Physics, Faculty of Science, University of Zanjan, P.O. Box 45195-313, Zanjan, Iran; safari@znu.ac.ir

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Abstract

In this paper we developed a complex network of solar active regions (ARs) to study various local and global properties of the network. The values of the Hurst exponent (0.8–0.9) were evaluated by both the detrended fluctuation analysis and the rescaled range analysis applied on the time series of the AR numbers. The findings suggest that ARs can be considered as a system of self-organized criticality (SOC). We constructed a growing network based on locations, occurrence times, and the lifetimes of 4227 ARs recorded from 1999 January 1 to 2017 April 14. The behavior of the clustering coefficient shows that the AR network is not a random network. The logarithmic behavior of the length scale has the characteristics of a so-called small-world network. It is found that the probability distribution of the node degrees for undirected networks follows the power law with exponents of about 3.7–4.2. This indicates the scale-free nature of the AR network. The scale-free and small-world properties of the AR network confirm that the system of ARs forms a system of SOC. Our results show that the occurrence probability of flares (classified by GOES class C > 5, M, and X flares) in the position of the AR network hubs takes values greater than that obtained for other nodes.

Key words: Sun: activity – Sun: flares

Supporting material: machine-readable table

1. Introduction

Solar active regions (ARs) are the origin of various energetic phenomena. It is believed that solar flares and coronal mass ejections are direct results of the changes in the topology and structure of the ARs’ magnetic field (Priest & Forbes 2002; Aschwanden 2005). Because of the important role of ARs in solar activity, numerous attempts have been made to study the statistical properties of ARs (Künzel 1959; Howard 1989, 2000; Sammis et al. 2000; Leka & Barnes 2003; Georgoulis & Rust 2007; Schrijver 2007; Falconer et al. 2008; Mason & Hoeksema 2010; Falconer et al. 2011; Georgoulis et al. 2012; Abramenko 2015; Arish et al. 2016; Barnes et al. 2016; Raboonik et al. 2017). Zhang et al. (2010) studied both the statistical (e.g., the frequency distribution of size and magnetic flux) and physical properties of 1730 ARs detected from 1996 to 2008. The information (occurrence times and locations on solar disk) about ARs and sunspots are recorded by the Royal Observatory of Belgium (ROB), the Solar Influences Data Analysis Center (SIDC), and the National Oceanic and Atmospheric Administration (NOAA). In the NOAA catalog, each AR has a unique identification number. There is important evidence to suggest that systems of solar events such as flares, bright points, and so on are in a self-organized criticality (SOC; Zhang et al. 2001; Alipour & Safari 2015; Aschwanden et al. 2016). Aschwanden (2013) provided an extended review on SOC in solar physics and astrophysics. De Arcangelis et al. (2006) studied the universality properties of both solar flares and earthquakes. They showed that both phenomena follow the same law for the size distribution and interoccurrence times. Also, they interpreted the temporal power-law dependency of the afterflare sequence as the Omori law for earthquakes.

The complex network is an approach to study effectively large complex systems (e.g., earthquakes, bulk electrical power systems, computers, the brain, and social systems). This approach explains significant characteristics of complex systems using network representations that have their roots in mathematical studies and are known as the graph theory. In most studies, a network or its equivalent graph is considered as a collection of some nodes together with edges while modeling a network (Newman 2003; Humphries & Gurney 2008; Rubinov & Sporns 2010; Lotfi & Darooneh 2012, 2013; Rezaei et al. 2017). Probability theory is a useful tool for interpreting the details of the complexity of the networks. The scale-free and small-world networks are the two main complex networks that have been widely used to study large complex systems (e.g., Watts & Strogatz 1998; Amaral et al. 2000; Barabási & Bonabeau 2003; Kim & Wilhelm 2008; Buldyrev et al. 2010). Small-world networks are basically intersections of the random and regular networks. In these kinds of networks, the length scale behavior is the same as in the random networks, but with a high average in the clustering coefficient (Watts & Strogatz 1998). The quantitative measurement of the small-world property can be computed by different methods—all comparing the network’s parameters with the parameters of an equivalent random or regular network (Humphries & Gurney 2008; Telesford et al. 2011). Humphries & Gurney (2008) presented a measure for the small-world property of a network by comparing both its length scale and clustering coefficient with an equivalent random network. This factor is employed in many articles, especially in the field of neuroscience (Bullmore & Sporns 2009). However, Telesford et al. (2011) explained that the Humphries method cannot correctly recognize the small-world networks. They introduced a new factor, using the length scale of a random network and the clustering coefficient of a regular network to evaluate the small-world property of a network.

In this paper, we construct a network for the ARs using their locations, occurrence times, and lifetimes. We compute the length scale, clustering coefficient, and degree distribution of nodes for the network. We describe important properties of the AR network in the category of scale-free and small-world networks.
The details are discussed as follows: Section 2 introduces the data set. In Section 3, the application of both detrended fluctuation and rescaled range (R/S) analysis on the time series of ARs is explained. Sections 4 and 5 describe the construction of AR networks and their properties. Sections 6 and 7 present the results and conclusions, respectively.

2. Data Processing

The solar monitor (www.solarmonitor.org) records the solar data observed by several solar space observatories and missions (e.g., GOES, GONG, ACE, STEREO, SDO, etc.).

SolarMonitor.org provides daily tables for ARs that occur on the main side of the Sun. The daily tables include the unique identification NOAA numbers, sunspot areas, Hale and McIntosh classifications, produced flares, and number of spots (Gallagher et al. 2002). The AR positions are tabulated daily in both the heliographic (in latitude and longitude) and heliocentric (in arcseconds) coordinates. The needed information of 4227 ARs during 1999 January 1 to 2017 April 14 for building the AR network are the NOAA numbers, rotated positions, occurrence times, and lifetimes. By tracking the NOAA numbers in daily tables, the lifetimes and positions of the ARs are extracted. The Appendix of this paper contains a table (Table 1) that includes the NOAA numbers, dates, and positions in the heliographic coordinates (longitudes and latitudes) for 4227 ARs.

Using the SunPy software (SunPy Community et al. 2015), the locations of the ARs are rotated with respect to the first occurrence time of the reference AR (NOAA 8419). For more simplicity, we used the location of the ARs at their first occurrence time during their lifetimes (on the main side). In Figure 1, the rotated positions at the first occurrence time of 4227 ARs in the front and far hemispheres are presented. The rotated positions of ARs (longitudes $\varphi$ and latitudes $\theta$) are restricted to the range of $0^\circ$–$360^\circ$ and $0^\circ$–$180^\circ$, respectively.

3. Hurst Exponent

The Hurst exponent is a key parameter to measure the autocorrelation (self-dependency) of the time series. Both the detrended fluctuation analysis (DFA; Mandelbrot 1975; Peng et al. 1994; Weron 2002; Alipour & Safari 2015) and R/S analysis (Buldyrev et al. 1995; Weron 2011) are useful tools to study the self-affinity (temporal dependency) in the time series of ARs (Figure 2). In order to compute the Hurst exponent, the time series of ARs’ $x(t)$ with length $T$ is divided into adjacent
sub-time series of length \( P \) while \( T = mP \). Each sub-time series is labeled by \( x_i^j (t_i) \) \( (i = 1, \ldots, P, j = 1, \ldots, m) \). For each sub-time series, the rescaled range \((R/S)\) is computed by

\[
(R/S)_j = \frac{\max(y^i_j, \ldots, y^n_j) - \min(y^i_j, \ldots, y^n_j)}{\sqrt{\frac{1}{P} \sum_{i=1}^P (x^i_j - \bar{x}^j)^2}},
\]

where \( \bar{x}^j = \frac{1}{P} \sum_{i=1}^P x^i_j \) and \( y^j_k = \frac{1}{P} \sum_{i=1}^P (x^i_j - \bar{x}^j) \), \( k = 1, \ldots, P \). The average of the normalized range for the \( P \) sub-time series is given by \((R/S)_p = \frac{1}{P} \sum_{j=1}^P (R/S)_j\). Mandelbrot (1975) and Weron (2002) have shown that the R/S analysis shows asymptotic behavior as \((R/S)_p \propto (H^2)\), in which \( H \) is the Hurst exponent, which can be obtained from a linear fit in the log-log scale of \((R/S)_p\) versus \( P \).

In the DFA for each cumulative time series \( \sum_{i=1}^k x^i_j - \bar{x}^j \), a straight line \( \alpha_j t_j + \beta_j \) is fitted. The mean value of the mean square fluctuation for all sub-time series is computed using

\[
F(P) = \frac{1}{m} \frac{1}{P} \left( \frac{1}{P} \sum_{j=1}^P \left( \sum_{i=1}^k (x^i_j - \bar{x}^j) - \alpha_j t_j - \beta_j \right)^2 \right)^{1/2} .
\]

The Hurst exponent \((H)\) can be obtained by a linear fit (in a log-log scale) for \( F(P) \) versus \( P \).

DFA and R/S analyses are used to extract the value of the Hurst exponent \((H)\) to investigate the behavior of the time series. In the analysis of time series based on the value of the Hurst exponent \((H)\), if \( 0 < H < 0.5 \) and \( 0.5 < H < 1 \), we can say that the time series has long temporal negative and positive correlations, respectively. If \( H \) equals 0.5, the time series is uncorrelated (white noise; Buldyrev et al. 1995). By applying the DFA and R/S on the time series of ARs, the values of the Hurst exponent are obtained as 0.8 and 0.94, respectively. These values show that the time series of ARs has a long temporal dependency and suggest that the system of ARs is one of SOC (Carreras et al. 2001; Dobson et al. 2007; Alipour & Safari 2015). SOC is an important way to describe the complex nature of a high degree of freedom and nonlinearity in many physical and astrophysical phenomena (e.g., Bak et al. 1987; Tang & Bak 1988; Wang & Dai 2013; Aschwanden et al. 2016).

In the remainder of this paper, we study the behavior of the SOC of the AR system in the framework of the complex network.

### 4. AR Network

The information of 4227 recorded ARs is used to construct a complex network concerning the following steps:

1. The solar spherical surface is divided into \( N \times N \) cells with equal areas considering the spherical coordinates \((\theta, \varphi)\) as \( A_{ij} = 4\pi R^2 \), \( i, j = 1, \ldots, N \), where \( R \) is the solar radius. The angles \( \theta \) and \( \varphi \) for each equal area cell are given by

   \[
   \varphi_{i+1} = \varphi_i + \frac{2\pi}{N}, \quad \varphi_1 = 0, \\
   \cos(\theta_{i+1}) = \cos(\theta_i) - \frac{1}{N}, \quad \theta_1 = 0.
   \]

2. The ARs are assigned to cells according to their locations at their first occurrence times rotated with respect to the first occurrence time of the reference AR (NOAA 8419) (Figure 1). To avoid more complexity in our analysis, the variations on the latitudes and longitudes of ARs during their lifetimes are ignored. Cells with no ARs are removed from our network analysis. However, the remaining ones are used as the nodes of the network, indicated by \( n_i (\ell = 1 \ldots L) \).

3. An edge connects the node \( n_i \) to \( n_j \) if an AR appears at node (cell) \( n_j \) during the lifetime of another AR at node \( n_i \). A loop (connecting node \( n_i \) to itself) is formed when an AR appears during the lifetime of another one within the same cell. In Figure 3, a small part of the AR network with six nodes, 15 edges, and four loops is shown.

Figure 3. Small part of the AR network, comprising six nodes. Each arrow represents the edge between the two nodes. For example, the AR (NOAA 10081) emerges in node \( n_{600} \) during the lifetime of the AR (NOAA 10063) presented in node \( n_{557} \), so that a directed edge is drawn from \( n_{557} \) to \( n_{600} \). Each self-loop shows the occurrence of two coinciding ARs in the same node. For example, the AR (NOAA 10205) appearing at the same node of NOAA 10221 formed the self-loop.

Figure 4. Clustering coefficient of AR networks and their equivalent random networks (with the same number of nodes and edges). The size of the network ranges from 50 to 2700 (nodes).
4. We organized a directed and weighted network as an \( L \times L \) adjacency matrix \( (A') \). In order to analyze the properties of the AR network, \( A' \) is converted to a symmetric matrix with all diagonal elements equal to zero, which is representative of the undirected, unweighted, and self-loop-free graph.

5. Network Parameters

A graph that consists of nodes and edges is a mathematical representation of a network. In general, a graph can be categorized into a directed or an undirected and a weighted or an unweighted graph depending on its edges. A directed network or its equivalent graph is defined by a set of nodes including directed connections (edges). A graph with bidirectional edges is called an undirected graph. A graph with different real numbers assigned to its edges is named a weighted network. The unweighted ones are those for which all the weights are set to be equal to 0 or the edges have no number assigned to them. In the complex network approach, using the adjacency matrix made for a graph, the topological properties in both the local and global scales were studied (Steen 2010). In the present section, we briefly review some network parameters.

In order to characterize a graph, one can use an adjacency matrix \( (A') \), including information about edges and nodes. The adjacency matrix is an \( L \times L \) matrix in which the element \( a'_{ij} \) is the number of edges (i.e., weight of edge) connecting node \( n_i \) to node \( n_j \). In this way, the sum of \( i \)th row elements represents the outbound edges from the node \( n_i \) (outbound degree), and the sum of the \( i \)th column elements represents the number of inbound edges to the node \( n_i \) (inbound degree). The diagonal elements represent self-loop(s) of nodes (i.e., a node connecting to itself by an edge). For a simple undirected and unweighted graph, the adjacency matrix is a symmetric matrix with all elements equal to 0 or 1. Undirected and unweighted adjacency matrices are used to determine the characteristics of the AR network, including the average of local clustering coefficients, mean of shortest path length, diameter, and the nodes’ degree of probability distribution function (PDF).

To convert the network to an unweighted, undirected, and loop-free network, the diagonal elements of the adjacency matrix are set to 0, and every nonzero element is replaced by 1. Then, the new adjacency matrix \( (A) \) is symmetrized.

The local clustering coefficients \( (c_i) \) and clustering coefficient \( (C) \) are computed using the adjacency matrix \( A \):

\[
c_i = \frac{\sum_{j=1,j\neq i}^{L} a_{ij} a_{ji} a_{jj}}{k_i (k_i - 1)},
\]

\[
C = \frac{1}{L} \sum_{i=1}^{L} c_i,
\]

where \( k_i \) and \( L \) are the number of edges connected to the \( i \)th node and the total number of nodes, respectively.

The average of the shortest path between all pairs of nodes is another interesting parameter of the network, expressed as

\[
ls = \frac{1}{L(L-1)} \sum_{i,j=1,i\neq j}^{L} d_{ij},
\]

where \( d_{ij} \) is the shortest path length between \( i \) and \( j \) nodes.

The parameter \( d_{ij} \) is computed using the Floyd–Warshall algorithm (Floyd 1962), which is an efficient method to find the length of the shortest path between all nodes in a graph. The largest value of the shortest paths is called the diameter of a graph. Another interesting property of a network is the PDF of the nodes’ degree, which determines the probability of finding a node with a certain degree.

In this study, we refer to some properties of such well-known networks as regular, random, small-world, and scale-free networks.

In regular graphs with all nodes having the same degree, typically the clustering coefficient and shortest path length possess large values. A network is called random if all pairs of nodes are connected with the same probability (Erdos & Rényi 1960). A random graph, in comparison with a regular one, has smaller values for both the clustering coefficient and the shortest path length. For a random network, the average values of the shortest path length \( (ls_{\text{rand}}) \) and clustering coefficient \( (C_{\text{rand}}) \) can be obtained by the following equations:

\[
ls_{\text{rand}} = \frac{\ln(L_{\text{rand}}) - \gamma}{\ln \langle k \rangle} + \frac{1}{2},
\]

\[
C_{\text{rand}} \approx \frac{E_{\text{rand}}}{L_{\text{rand}}^2},
\]

where \( E_{\text{rand}}, L_{\text{rand}}, \langle k \rangle, \) and \( \gamma = 0.5772 \) are the number of nodes, number of edges, average degree of nodes, and Euler constant, respectively (Fronczak et al. 2004). One can build a random graph using a model proposed by Erdos & Rényi (1960). The intersection of the random and regular graphs is called a small-world graph (Watts & Strogatz 1998). A small-world graph has a small average shortest path. However, the clustering coefficient takes the larger value. We can find enough examples of such graphs in nature—for instance, in the brainstem reticular network (Humphries et al. 2005), human protein network (Stelzl et al. 2005), and power grid network (Mei et al. 2011). Bullmore & Sporns (2009) reviewed the small-world networks in neurosciences. In some small-world networks (e.g., the brainstem reticular network), the PDF of the nodes’ degree \( (P(k)) \) follows a power law, \( P(k) \propto k^{-\gamma} \). This kind of network is representative of scale-free events.
6. Results

In order to construct a complex network for the solar ARs, the locations and dates of the first occurrence times and the lifetimes of 4227 ARs were used (Section 4). The nodes of the network were created by dividing the solar surface into equal-area cells. The adjacency matrix was prepared for both the directed and undirected AR networks. The weighted adjacency matrix was determined using directed connections (edges) between the pair of ARs. Loops, the connections between two successive ARs occurring at the same cell (node), were considered as the diagonal elements of the adjacency matrix ($a_{ii}$). The empty cells of the directed and weighted adjacency matrix were removed from our network analysis. The effect of the network size (the number of nodes, $L$) on the shortest path length, clustering coefficient, and probability distribution of the nodes’ degree are studied. An equivalent random network was constructed corresponding to the AR network with the same size and edges.

The clustering coefficients of the AR networks and their equivalent random networks (with the same number of nodes and edges) are shown in Figure 4. The size of the networks varies from 54 to 2693 (nodes). The clustering coefficient for the AR and random networks are computed by Equations (4) and (7), respectively. It is found that the clustering coefficient of the AR network is noticeably greater than the equivalent random network by about a factor of two or more.

In Figure 5, the length scale of the AR network versus the size (logarithm) of the network is plotted. For the small size of the AR networks ($L < 500$), most of the nodes are connected together, and the networks tend to the complete graphs. For the networks with size 500–2700, a linear fitting was applied and the slope of the fit was obtained ($\ell_s = 0.79 \log_{10} L$). According to the small values of the length scale and its logarithmic dependency on the size, we conclude that the AR network is classified as a small-world network.

Figure 6 presents the PDF of the nodes’ degree for the undirected networks with the size varying from 1687 to 2693. The lower ends of the degree distributions ($k < k_{\text{trunc}}$) suffer from some truncation effects due to incomplete detection of small ARs and far-side ARs and therefore should not be considered in the power-law fitting. The truncated power-law ($p(k) \sim k^{-\gamma}$) fitting was carried out on the probability distribution of the nodes’ degree (Aschwanden 2015). The values of the power-law exponent range from 3.7 to 4.2. The power-law nature of the PDF indicates that a few nodes have high connectivity values. In the context of the complex network, nodes with a large number of connections are named hubs of the network (Albert & Barabási 2002). One may ask, what is the criterion for selecting a node as a hub? In a network, nodes with degrees higher than a threshold are considered as hubs. The threshold is defined as the maximum degree of nodes in the equivalent random network.

We found that for the AR network with 1986 nodes, there are 53 hubs with degrees larger than 80. On increasing the network size to 2693 nodes, 78 hubs with connections larger than 61 are found. In Figure 7, the relation between the average number of large energetic flares ($C > 5$, M, and X classes) appearing at the position of the AR network nodes is presented. The average number of flares increases with an increase in the degree of the nodes. The average number of large flares appearing at the position of the AR network hubs is larger than the other nodes by at least a factor of two. In Figure 8, the dependency of the averaged local clustering coefficient ($\bar{C}$) for the nodes with the same degree is presented. The thresholded...
power law \( (C \sim (k + k_{th})^{-\gamma}) \) is fitted to the average local clustering coefficient, in which \( k_{th} \) is the threshold value. Aschwanden (2015) modeled the thresholded power law for size distributions of some natural phenomena (e.g., solar and stellar flares). They show that in most size distributions of detected data, the thresholded power laws are significantly well fitted. We found that by increasing the size of the network, the power-law exponent of the average clustering coefficient slightly exceeds a value of 1.

7. Conclusion

Typically, 220 ARs can be observed on the solar disk each year. Within some of the ARs, large-scale magnetic phenomena such as flares and coronal mass ejections are stochastically emerged. The exact physical mechanism underlying these phenomena remains unknown. The results of applying both the detrended fluctuation and R/S analysis with the Hurst exponent (0.8–0.9) suggest that the AR system is categorized into SOC, which motivated us to confirm such a characteristic by using the complex network approach. In the present work, we designed the AR network based on their locations at the first occurrence times and lifetimes. In our complex network, all ARs that occurred in the lifetime of a specific AR are linked with each other. The unweighted adjacency matrix was used to calculate the length scale and clustering coefficient of the network. The degree of nodes was computed for both the directed and undirected AR networks. The main results of this study are organized as follows:

1. A comparison between both the values and behavior of the clustering coefficient of the AR network and the equivalent random network (with the same number of edges and nodes) indicates that the AR network is not a random network.
2. The small values of the length scale and its logarithmic behavior \( (l = 0.74 \log_{10} L) \) related to the network size demonstrate that the AR network is a small-world network. The obtained results for the large values of the clustering coefficient confirm that the AR network can be classified as a category of the small-world network.
3. The truncated power-law distribution was fitted to the PDF of the nodes’ degree. The power-law nature of the

Figure 7. Average number of flares (circles) and connected mean values of nonuniform binning (dashed line) observed within the position of the AR network nodes vs. the degree of nodes for the network size \( (L) \) (a) 1552, (b) 1986, (c) 2382, and (d) 2693. The nodes with high degrees (hubs) are presented on the right side of the dotted lines.
nodes’ degree demonstrates the scale-free feature of the AR network. Recent observations have shown that some of the ARs produce energetic phenomena (e.g., flares, CMEs) and some others do not. Such characteristics of the AR network are representative of the prescription of the heterogeneous networks (e.g., Abe & Suzuki 2009).

4. We observed an increase in flare occurrence in each cell corresponding to the hubs. In other words, the nodes with higher degrees in the AR network have a higher likelihood to trigger flares (Figure 7). This behavior of the AR network raises an important question about the prediction capability of flares based on the AR network. More statistical studies are required to examine the flare prediction with the ARs’ complex network.

In the present study, we used the AR information appearing on the main side of the solar surface. By employing the tracking algorithms on the reconstructed helioseismic maps (Lindsey & Braun 2000; Hernández et al. 2013), the returning far-side ARs can be identified. By increasing the lifetimes of some ARs from 2 weeks to 1 month or more, the number of connections (edges and loops) for some nodes of the AR network may be increased. Additionally, some of the nodes may be empty and, therefore, will be removed from the analysis. Expectedly, by decreasing the number of nodes in the presence of the ARs with the large lifetimes, the diameter and the mean path length of the network may also decrease, while the clustering coefficient may increase. More quantitative studies are required to address the effects of the far-side AR information on the properties of the network.

In the next step, we attempt to include more characteristics of ARs (e.g., magnetic class, Hall class, McIntosh class, sunspots) and study the effects of changing solar latitudinal and longitudinal displacements of the ARs during their lifetimes in order to investigate the prediction capability for energetic solar events.

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Figure 8. Average of the clustering coefficients of nodes with the same degree (triangles) in the undirected AR networks and the fitted thresholded power laws ($\mathcal{C} \sim (k + k_{tr})^{-\gamma}$) presented for network sizes (a) 1552, (b) 1986, (c) 2382, and (d) 2693. The value of the threshold ($k_{th}$) is selected to be 10, and the fitted power-law exponents are obtained to be in the range of 1.02–1.06.
Table 1
Solar Active Region Data

| ID   | Year | Month | Day | Lat. | Long. |
|------|------|-------|-----|------|-------|
| 8419 | 1999 | 01    | 01  | 28   | 91    |
| 8419 | 1999 | 01    | 02  | 28   | 77    |
| 8420 | 1999 | 01    | 01  | 20   | 43    |
| 8420 | 1999 | 01    | 02  | 20   | 40    |
| 8420 | 1999 | 01    | 03  | 20   | 39    |
| 8421 | 1999 | 01    | 01  | 27   | 40    |

(This table is available in its entirety in machine-readable form.)

Appendix

A sample information of six ARs that includes the NOAA numbers, dates, and positions in the heliographic coordinates (longitudes and latitudes) are presented in Table 1.

ORCID iDs

Hossein Safari © https://orcid.org/0000-0003-2326-3201

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