HOURLY VARIABILITY IN Q0957 + 561

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Received 1999 October 27; accepted 2000 April 5

ABSTRACT

We have continued our effort to re-reduce archival Q0957 + 561 brightness monitoring data and present results for 1629 R-band images using the methods of galaxy subtraction and seeing correction that we reported previously. The new data set comes from four observing runs, several nights apiece, with sampling of typically 5 minutes, which allows the first measurement of the structure function for variations in the R band from timescales of hours to years. Comparison of our reductions to previous reductions of the same data and to r-band photometry produced at Apache Point Observatory shows good overall agreement. Two of the data runs, separated by 417 days, allow us for the first time to use intranight variations to estimate the time delay. However, our sharpened value of 417.4 days is valid only if the time delay is close to the now-fashionable 417 day value, and worse, it is highly subject to diurnal windowing effects: our statistical estimators have little discriminatory power where there is no overlap of data, namely, at half-day offsets. Results herein show no unambiguous signature of daily microlensing, though a suggestive feature is found in the data. Because both time delay measurement and microlensing searches suffer from the lack of sampling at half-day offsets, inevitable at a single observatory, we discuss the need for round-the-clock monitoring with participation by multiple observatories.

Subject headings: dark matter — galaxies: halo — gravitational lensing — methods: data analysis — quasars: individual (0957 + 561) — techniques: photometric

1. INTRODUCTION

Previous studies of the Q0957 + 561A,B brightness fluctuations have indicated the existence of daily brightness fluctuations. If these are intrinsic to the quasar, then the time delay can be determined to a precision of a fraction of a day; however, if such fluctuations are at least in part due to microlensing, they would probably signal the nature of a baryonic dark matter component.

A recent report by Colley & Schild (1999, hereafter Paper I) demonstrated precision aperture photometry close to the photon limit. The procedure subtracts out the lens galaxy’s light according to the galaxy’s profile determined recently from a Hubble Space Telescope (HST) image and removes cross talk light from the A and B image apertures. We now apply this reduction to 1629 frames collected in four runs, with continuous monitoring for 10 hr per night over, typically, five nights.

Previous analysis of these data showed that small brightness fluctuations were observed during these runs, but because the effects of seeing were known to limit conclusions about such low-amplitude fluctuations, only nightly averages were published. With our new photometric methods, we are able to present here new reductions with sampling of hours. Sampling at this frequency should clarify conclusions about time delay and microlensing.

In the first several sections to follow, new results unravel the systematic effects that seeing has on the aperture photometry. Next, the measured brightness fluctuations give a structure function that describes the amplitude of variations observed in this lensed/microlensed system on various timescales. Final sections present time delay calculations and possible microlensing results from two observing runs separated by 417 days.

2. THE PHOTOMETRY

For two decades, R. E. S. has monitored Q0957 and amassed a “master data set” of the variations of images A and B (Schild & Thomson 1995 and references therein). The data presented here come from all-night monitoring in one of two similar observational configurations. About half of the data, as in Paper I, derive from unbinned 1k × 1k CCD images with pixel scales of one-third of an arcsecond, exposure times of 450 s, read noise of 8e−, and FWHM seeing of 1″5-2″. Some of the data come from otherwise similar circumstances but derive from a 2k × 2k CCD binned down to 1k × 1k; here shorter exposure times are used, and the pixel scale is twice as great. Both of these setups typically allow for five unsaturated comparison stars in the R band on each data frame. As with Schild’s previous work, we stay with 6″ diameter apertures to minimize effects of seeing. This paper presents the photometry from more than 1600 such frames.

To reduce 1600 frames, one requires a highly automated photometry reduction code (one free of mouse clicks and manual entry of parameters). In Paper I, we detailed such a method, which is not only automated, but includes two basic improvements upon the basic aperture photometry scheme historically employed to reduce Q0957 + 561 data images: (1) use of HST imaging to subtract the galaxy and (2) correction for “cross talk” between the narrowly separated (6″1) A and B images. Typically, the galaxy, whose core is inside the B aperture, contributes 17% to the B aperture and 2% to the A aperture. The cross talk contributes typically 2% to each image. One could calibrate those out if the seeing were steady, but poor seeing obviously introduces more cross talk contamination and has the curious effect of spreading much more galaxy light into the A image. These variable effects confuse the search for both intrinsic quasi-stellar object (QSO) variation and microlensing. Worse still, while the seeing introduces a correlation in the A and B photometry (spreading more cross talk light into both images from each other), it also introduces an anticorrelation due to the galaxy, whose light is
spread out from the B aperture and into the A aperture. To disentangle these effects, a complete galaxy subtraction and measurement of cross talk is necessary.

Slight modifications to the photometry scheme in Paper I have been implemented, principally for sake of speed and increased automation. As before, the astrometry is accomplished via point-spread function (PSF) fitting, which requires a model PSF. The model is empirical, built by stacking several standards. The first pass utilizes methods described by Alard & Lupton (1998), but only even terms in the polynomial expansion are used. An explicitly even model PSF for each standard can be fitted to the standard itself to determine the astrometry of that standard. Shifting and stacking (by median) all of the standards forms the model PSF. This method has the advantages of being quite fast and of eliminating cosmic rays automatically from the model PSF. Other modifications include increased sensitivity to erratic photometry in the standards (by simply censoring 5 σ outliers) and more careful measurement of cross talk (again by filtering out gross outliers). Each of these methods increases the stability of the photometry appreciably.

3. THE EFFECTS OF THE LENS GALAXY

Paper I described our method for subtracting the lens galaxy’s contribution to the apertures for images A and B. We used a profile for the galaxy determined from HST imaging (Bernstein et al. 1997) and convolved it with the point-spread function produced by co-adding the standard stars on the frame. Figure 1 shows the contribution of the lens galaxy’s light to the A and B measurement apertures as a function of seeing, as measured from all the image frames of this data set. Plotted is the percentage of the signal from the lens galaxy in the A and B apertures as a function of FWHM seeing. Because seeing effects originate in different levels of the terrestrial atmosphere and because the seeing is not known to produce a unique profile, adoption of a simple FWHM parameter is only a first-order statistic and cannot fully describe the variety of seeing profiles supplied by nature.

In the top panel of Figure 1 the typical percentage contribution to the light of image B is 17.7% for average seeing. The average brightness of the B image during this time was 16.51 mag. The observed contribution is equivalent to an 18.39 mag source in the measurement aperture. This is in excellent agreement with the $R = 18.34$ mag correction to the lens galaxy magnitude that has been historically adopted throughout the 20 yr Q0957 brightness monitoring project (Schild & Weekes 1984). The historical correction was made from a compilation of aperture photometry at optical and infrared wavelengths and the known colors of an elliptical galaxy at moderate redshift. Published data tables of Schild and colleagues usually had a correction for an $R = 18.34$ mag galaxy applied but no correction for the galaxy contribution to the A aperture and no aperture cross talk corrections.

The bottom panel in Figure 1 shows the brightness contribution from the lens galaxy to the aperture of the A image. For average seeing, this correction is 3%. Because this contribution originates in the previously unknown outer profile of the lens galaxy, no correction for it has been made in the historical brightness records (where bad seeing images were removed). This has little effect on the historical record, since it causes just a slight reduction in the measured brightness fluctuations of image A and a slight 3% correction to the inferred A/B continuum brightness ratio.

Best-fit parabolic curves have been overplotted on the Figure 1 data; we give those fits below. With seeing expressed as a FWHM (represented below as $x$) in units of arcseconds, the fitted curves express the percent contribution of the lens galaxy to the $R$ magnitude in a 6” diameter A aperture $\delta R(A)$ and B aperture $\delta R(B)$ as

$$
\delta R(A) = (3.154 - 0.6775x + 0.3542x^2)\% ,
\delta R(B) = (18.725 - 0.5962x)\% .
$$

We found that the best-fit quadratic term for B was very nearly zero, so we omit that here. These empirical low-order fits apply only in the range shown on the plots and should not be used for seeing with FWHM less than 1”. In such cases, the minimum value shown for the A contamination is probably quite a good approximation, since the curve is obviously flattening by 1” anyway. It seems, however, that the B contamination is still growing as the seeing improves toward 1”, and perhaps one could extrapolate somewhat, but the B contamination would obviously level below some FWHM; hence, correction for the galaxy in the B aperture during very good seeing remains somewhat unresolved.

4. THE APERTURE CORRECTIONS

As the seeing gets worse, relatively more light will spill over from quasar image A into the adjacent B aperture (and vice versa) and cause systematically the cross talk effects noted by Schild & Cholfin (1986). This effect can be corrected for by measuring the amplitude of the effect in nearby

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**Fig. 1.—** Galaxy contamination as a function of seeing. Plotted is the percentage of light in the A and B apertures due to the lens galaxy. Contamination by the galaxy not only alters the average photometry of the QSO images, but adds a component that is variable as a function of seeing. This variable component has an amplitude of up to 3% from good (1”) to bad (2.5”) seeing, large enough to corrupt searches for few percent brightness variations. As seeing deteriorates, light of the lens galaxy is systematically scattered out of the image B aperture, which contains the galaxy’s nucleus, but scattered into the A aperture. The fitted curves are discussed in § 3.
Figure 2 demonstrates to be important since seeing effects can produce photometric errors of several percent.

5. Results: The Brightness Record

The final brightness data after corrections for the lens galaxy and aperture cross talk appear in Figure 3. Plotted points are the result from each data frame, so the amplitude of the "swarm" for each night relates to the accuracy of the photometry. The accuracy is comparable to our results in Paper I, where it was found that the photometry is within 15% of the limit imposed by Poisson statistics. Note that the swarm for the first observing run in 1994 December covering Julian Dates 2,449,702–08 has a larger amplitude than for later runs; this is because the pixel size was larger by a factor of 2 on the first observing run, so exposure times are 4 times shorter and the Poisson noise greater. But although the noise per image frame was larger, there are more of them and the overall precision of the photometry is about the same as for subsequent observing runs.

Correlations in the A and B photometry could arise because of coherent observational errors, such as cross talk (Schild & Cholón 1986), though no significant correlations appear (verified by the Pearson's correlation coefficient applied to raw, galaxy-subtracted, and cross talk-corrected photometry). Although it might at first appear that correlated fluctuations were seen in the 1995 February run, with both images brightening with time, note that during this time the A image was unusually faint and the B image was unusually bright. So we conclude that by chance, both images were seen brightening at the same time.

These brightness records give unambiguous evidence of daily brightness fluctuations. The best is the record for image B on night 2,449,704, which we have highlighted with an arrow in Figure 3 (the same feature is just as obvious in the uncorrected photometry). Here image B increased in brightness by more than 2% from the night before and returned to its normal level the night after. We conclude that the quasar lens system is capable of producing 2% brightness fluctuations on a timescale of a day. In § 7 below we will compute the structure function for Q0957 brightness fluctuations on long and short timescales.

6. Comparison of Old and New Reductions

The new reductions allow a comparison with the original CCD data record published by Schild and colleagues over the years. The data have been published in Schild & Thomson (1995, and references therein). Here we are concerned with the question of how well the new reductions agree with the original published and tabulated data. Recall that the basic procedure in both the old and the new reductions is aperture photometry and that the new reduction differs primarily in the treatment of subtraction of the lens galaxy G1 and in the correction for aperture cross talk.

Plotted in Figures 4 and 5 are the photometry from this work in filled circles. In open circles are plotted the previous reductions of these data from Schild's master data set. In

\[ \delta R = (0.7087 + 0.6320x - 0.7387x^2 + 0.2788x^3)\% . \]
FIG. 3.—Brightness record after corrections for the lens galaxy and aperture cross talk. Each data point is for a single image frame, and the "swarms" give an idea of the spread of photometry image to image. Despite the photon noise scatter, variations night-to-night are visible. One particularly interesting feature is in the B data from 1994 December, pointed out by the arrow, where for a single night the B image jumps by 2%. Nightly averages for these data are shown in Fig. 5, and the hourly binned data for 1994 December and 1996 February are shown as Fig. 7.

In Figure 4, no correction or shift has been applied to either the old or the new photometry for image A. The overall agreement is obviously quite good (almost always within 2%). Meanwhile, for image B, we have simply added back in the galaxy light subtracted in Schild’s master data set (his 18.34 mag galaxy correction), and there is quite good agreement (rarely exceeding more than 1% error). The reason the observation times appear slightly different for these two data sets is that, in the Schild data set, filtering for seeing removed large fractions of data from each run and hence shifted the mean time of observation for each night. If one linearly interpolates the Schild data and compares them directly with the photometry of this work, one finds that the image A photometry averages just 3 mmag brighter than Schild’s, while the standard deviation of the comparisons is 8 mmag (statistically consistent with no offset at all). If one does the same for image B, one finds an average discrepancy of 184 mmag with a standard deviation of 6 mmag. This 184 mmag means there is an additional 18.31 mag source in our raw B aperture, very close to the 18.34 mag correction Schild made for the lens galaxy.

The (A, B) rms differences between the photometries of only (8, 6) mmag show that the nightly averages published by Schild and colleagues are confirmed, since the published errors average about the same amount, which adds credence to many microlensing conclusions reported based upon these data. Note particularly from Figure 4 that the discrepancies between the old and the new reductions are on a timescale of individual nights, which should not affect the microlensing trends on larger timescales proposed from wavelet analysis by Schild (1999).

Figure 5 gives the same data from Schild but plots our corrected photometry. In previous sections, we discussed...
the contributions of the galaxy and cross talk during typical seeing. For aperture A the lens galaxy contributes approximately 3% during typical seeing, while the cross talk between apertures contributes another 1.2%. Our corrected photometry is accordingly 44 mmag fainter than the Schild photometry with an rms of 7 mmag (a slight improvement over the comparison with raw photometry). The image B photometry shows a 40 mmag deficit to the Schild photometry with an rms of 5 mmag (also a slight improvement over the raw photometry). The improvement in rms is perhaps expected because of the filtering for good seeing done by Schild.

Schild & Thomson (1995) report an internal photometric error of 10 and 12 mmag for A and B, respectively, during this epoch. Therefore, the agreement of the Schild photometry and current photometry at the 5 mmag level is about as good as one could expect. It is perhaps noteworthy that the agreement exists despite completely different software for the aperture photometry (IRAF dophot vs. Interactive Data Language [IDL] programming from scratch) and completely different methods for the relative photometry (pencil and calculator vs. Honeycutt 1992 matrix reduction).

Because we expect to continue re-reducing archival data frames with our improved data reductions, it is worthwhile to summarize the differences between the new and old reductions. Relative to the old reductions, the new results for image (A, B) are (44, 40) mmag fainter than the tabulated Schild photometry, primarily because of the new corrections for aperture cross talk but also partly because of improved knowledge of the effects of the lens galaxy. Thus, to correct our old photometry to the new system, one would add (.044, .040) mag to the table entries in the Schild master data set. The true errors of the photometry for the dates we have compared are (7, 5) mmag.
For reference, the open triangles in Figures 4 and 5 give the Apache Point Observatory (APO) $r$-band photometry where applicable (W. Colley et al. 2000, in preparation), with arbitrary offsets. While the gross features of the photometry are reproduced, the agreement is not as good, but this might be expected because (1) $r$ is a different filter, (2) there is no correction or censoring for seeing effects, and (3) the APO photometry derives from only a handful of frames on a given night (hence the larger error bars), compared with several tens of frames per night at Mount Hopkins.

7. THE STRUCTURE FUNCTION FOR Q0957 BRIGHTNESS FLUCTUATIONS

For many purposes related to the discussion of the Q0957 time delay and for the physics of active galactic nuclei in general, it is useful to know the structure function for the brightness fluctuations. The structure function, which is the expected excess variance as a function of temporal separation, can be approximated as a power law $V = 10^{a t^b}$. For our corrected photometry, we find $a = -4.5; b = 0.54$, in rough agreement with values found by W. Colley et al. (2000, in preparation) for $r$-band photometry of $a = -4.1; b = 0.47$, and by Press, Rybicki, & Hewitt (1992, hereafter PRH) for $R$-band photometry of $a_A \approx -0.34, b_A \approx 0.54$ and $a_B \approx -0.22, b_B \approx 0.68$ for the A and B images, respectively. The difference in the A and B power laws in PRH shows that measurement of the structure function is perhaps not the most stable process (even perhaps not the best way to describe QSO variation) and suggests that we should not be troubled by the smallish discrepancies between each of these estimates. Fortunately, within reason, the details of the structure function have little bearing on the PRH statistic itself.

Figure 6 contains a plot of the structure function for the fluctuations averaged for images A and B. Note that Schild (1996) has found from wavelet analysis that the A and B images have equal brightness fluctuations on timescales from days to 2 months, and Pelt et al. (1998) have shown that on timescales of decades, the fluctuations are larger in B, presumably because of its larger optical depth to micro-
lensing. Note that the systematic errors due to seeing, which we have discussed previously, would tend to produce larger fluctuations in A than in B.

We have shown, for the first time, the structure function for timescales of less than a day. Note that there is no obvious departure from the power law from the smallest to the longest scales. For timescales of a day, the results are consistent with the frequent remark by Schild (1996) that this quasar’s daily brightness fluctuations are typically of order a percent (0.6% according to this plot).

8. TIME DELAY ESTIMATES

Because two of the observing runs, 1994 December and 1996 February were separated by 417 days, the presently favored value of the time delay (W. Colley et al. 2000, in preparation; Pelt et al. 1998), it should be possible to determine the gravitational lens time delay to a fraction of a night. We have completed two time delay calculations for the new data. For both of the calculations, to be discussed below, the data have been binned by 1 hr.

Our first calculation, shown as a dotted line in Figure 7, is based upon simple linear interpolation of the data and is equivalent to the calculation by Kundić et al. (1997), in which errors and values are linearly interpolated between the nearest points, and a simple $\chi^2$ statistic is computed. This autocorrelation calculation produces a network of minima separated by 1.0 days, with the deepest minimum occurring for 417.3 days.

A time delay calculation using the PRH calculation is shown as the solid line in Figure 7. This calculation uses the Figure 6 structure function to estimate the permitted range of brightness values in place of linear interpolation. It also shows a network of minima separated by 1.0 days with the deepest minimum at 417.5 days.

These minima at $n + \frac{1}{2}$ days can be understood easily in terms of the statistics used to compute the best-fit time delay, both of which prefer not to overlap data. The linear method prefers no overlap for a simple reason. Because the data are binned by 1 hr, the actual number of data in the first and last bins varies according to chance. When a small number of data occurs in those bins, the error bars are, of course, larger by root-$n$ than those of typical bins. Hence, the linearly interpolated errors are larger by the same factor and are thus far more tolerant in the no-overlap regions. The unusually large error bars at the end points also explain why the absolute $\chi^2$ minimum for the linear method is less than one.

The PRH method prefers no overlap for a different reason but with similar results. The PRH method, of course, finds that the structure of variations of A and B independently match the structure function (Fig. 6) measured from the A and B variations. It tries to keep A and B independent rather than fighting to find the correct overlap. This propensity to find the gaps was recognized as a possible weakness of PRH by its authors in the original paper.

While both of these methods have proved excellent for determining the time delay for well-sampled data on different timescales (W. Colley et al. 2000, in preparation), both show weakness for intermittently sampled data, inevitable at a single observatory at middle latitudes. Because of this complication, we conclude that our program of intensive monitoring over many nights separated by the time delay does not allow us to sharpen the time delay significantly, and we do not claim that our overall best-fit value 417.4 days is an improvement over other recent determinations. This problem motivates the need for round-the-clock monitoring of the QSO.
9. AGREEMENT OF THE TIME-SHIFTED BRIGHTNESS CURVES

Figure 8 contains the 1994 December A brightness record (filled circles) and the shifted B brightness record (open circles). At bottom, the image B photometry has been shifted by the best-fit time delay from the linear interpolation \( \chi^2 \) method; at top, the PRH fit is shown. We show both, because of the qualitatively different appearance of the fits and because we have no predisposition to favor either method particularly. For reasons discussed in the previous section, we might be inclined to place more faith in the PRH method for producing the best overall fit, because it is less subject to the edge effects due to binning.

In both panels of Figure 8, there does seem to be a qualitative agreement between the behavior in A and B. Particularly, there seems to be a coherent wavy behavior in both A and B. Some encouragement that the signal is real lies in the fact that night to night neither A nor B shows systematic behavior that is obviously spurious. Namely, neither A nor B is always increasing, decreasing or inflecting in the same way on each night. Such behavior would lead to suspicion that the waviness was an observational artifact. Without “filling in the gaps,” however, the actual validity of these fits is moot, and we are left with only one certainty, that the QSO often varies by about a percent within a given night. This variation means that with round-the-clock observations, highly precise measurement of the time delay would be possible, and any daily microlensing residing in the signal should become apparent.

Looking at Figure 8, it is hard to find any definitive microlensing signal, but most suggestive is the apparent gap at JD 2,449,705.8. While the rest of the curve (in the top panel) seems to obey a coherent pattern, this last interface looks slightly off. This is hardly irrefutable evidence for microlensing, but again, with round-the-clock monitoring, we would be able to be much more definitive if such an event arose. Furthermore, a null result would be of some interest: quasar microlensing searches for the missing mass have been producing exclusion diagrams for possible MACHO masses (Schmidt & Wambsganss 1998; Refsdal et al. 2000). The intermittence of microlensing and clumping of caustics (Wambsganss, Witt, & Schneider 1992) are complications, however, and a detection such as the 1994 December–1996 March event (Schild & Gibson 1999) is challenging, not so much to observe but to confirm, because of the low amplitudes of the events.

10. SUMMARY AND CONCLUSIONS

Using improved photometric methods as described in Paper I, we have begun our program of re-reduction of archival Q0957 CCD images acquired over the last 20 years. We report herein results for four data runs when the quasar was continuously observed with a 1.2 m telescope for typically six consecutive nights, including observing runs in December 1994 and February 1996, which are separated by 417 days, the currently favored time delay between images A and B (Kundic et al. 1997).

For the new data we show how the contribution of the lens galaxy varies with the seeing. During good seeing, the lens galaxy contribution to the A aperture is nearly constant at 2.5% and appears to maximize at about 18.5% in the B aperture. As seeing deteriorates beyond FWHM = 2"5, the A aperture contribution increases by 1% while the B aperture contribution decreases by 1%.

For the same data we evaluate the contribution of A-B aperture cross talk and find that significant deterioration occurs in both apertures for seeing with FWHM greater than 1"8. The deviation from average due to this error source reaches 2% during FWHM = 3" seeing. Thus, we find that for deteriorating seeing, galaxy contamination adds to aperture cross talk in aperture A and compensates for half the cross talk in aperture B. These effects can very significantly affect aperture photometry for the system, but historically they have been only qualitatively understood.

For the fully corrected data we present hourly binned brightness curves that show significantly detected variations on timescales of hours. The quasar can produce 2% brightness fluctuations on the timescales of 24 hr and 1% brightness fluctuations within individual nights of continuous observation.

Comparison of our new photometry with the previous reductions in the master data set (Schild & Thomson 1995) yields excellent agreement. The new raw photometry agrees with the historical data within an rms of about 7 mmag. The new corrected photometry agrees within slightly better rms limits after an offset of about +42 mmag is applied to the historical photometry.

With two data runs separated by 417 days, the currently favored time delay, one can search for a precise delay and

\[ \chi^2 \]

\[ 2 \]

\[ \text{The data set is also available at http://cfa-www.harvard.edu/~rschild.} \]
for any daily microlensing residual. Two different time delay estimation procedures favor values near half-day values. These estimators, as would most, tend to find gaps where there is no data overlap between the two runs. The best microlensing candidate in these runs is a significant discrepancy on day 2,449,705.8 (Fig. 8), which is unfortunately straddling a half-day gap. The tendency for time delay estimators to find the half-day gaps where either A or B could not be observed because of daylight, and the lack of conclusiveness available for microlensing due to such gaps, necessitate round-the-clock monitoring of Q0957 + 561.

REFERENCES

Alard, C., & Lupton, R. H. 1998, ApJ, 503, 325
Bernstein, G., Fischer, P., Tyson, J. A., & Rhee, G. 1997, ApJ, 483, L79
Colley, W., & Schild, R. 1999, ApJ, 518, 153 (Paper I)
Honeycutt, R. K. 1992, PASP, 104, 435
Kundic, T., et al. 1997, ApJ, 482, 75
Pelt, J., Schild, R.,Refsdal, S., & Stabell, R. 1998, A&A, 336, 829
Press, W. H., Rybicki, G. B., & Hewitt, J. N. 1992, ApJ, 385, 404 (PRH)
Refsdal, S., et al. 1999, Boston Conf. on Gravitational Lensing, ed. T. Brainerd & C. Kochanek, in press
Schild, R. E. 1999, ApJ, 514, 598
Schild, R. E., & Cholin, B. 1986, ApJ, 300, 209
Schild, R. E., & Gibson, C. 1999, preprint (astro-ph/9904366)
Schild, R. E., & Thomson, D. J. 1995, AJ, 109, 1970
Schmidt, R., & Wambsganss, J. 1998, A&A, 335, 379
Wambsganss, J., Witt, H., & Schneider, P. 1992, A&A, 258, 591