Developing an Efficient Waste Management System: An Approach by Using Some Methods of Mathematical Modelling

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Abstract: Human community has outgrown breaking the laws of the nature. This has caused serious threat to the nature and hence to the human community itself. Day to day activities of the man creates different kind of waste materials. These materials are thrown in the surroundings, which creates unpredictable damage to the environment. The matter of management of waste has prime priority in sustainable development. In this paper we mainly discuss various characteristics of waste management systems such as efficiency, sufficiency and consistency. A waste management system is developmental if it generates employment and income to the country. We develop some mathematical models which to ensure the efficiency and developmental characteristics of the waste management systems

1. INTRODUCTION

Management of waste is a multifaceted problem. It contains mainly three components. Waste Reuse Management (WReuM), Waste Recycle Management (WRecM) and Waste Disposal Management (WDM). An object which is of no use for some people will be valuable for some others. Finding the people who need a product which has reuse value comes under waste reuse management. Separation of reusable and recyclable materials from the waste and processing it to convert into usable materials are the functions of waste recyclable management. Some portions of waste can always be classified as useless. Such materials must be disposed permanently. It is the role of waste disposal management. A waste management system can be effective only if the three components of WMS function properly. Modelling and Planning of each component must be done separately to ensure its effectiveness and systems developed can be integrated to obtain a unified waste management system [3].

Mathematical modelling is an integral part of effective planning of an efficient waste management system. A number of models were developed during the period 1970 – 90 in which mathematical modelling techniques are used. A survey of the models is given by Morrissey et al. [1]. In the paper the authors mainly review the types of models that are currently being used in the area of municipal waste management and highlight some major shortcomings the models. They have commented that most of the municipal waste models identified in the literature are decision support models. For simplifying the research, they divided the models into three categories. The first type is based on cost benefit analysis, the second models are based on life cycle assessment and the third models are based on multicriteria decision making. Two typical examples of mathematical modeling applied in the study of solid waste management are available in the papers [4] and [5].
All developed counties have their own plans of waste management and well developed system for implementing the plans. In a developing country like India, it is only in the initial stage. In India it is treated as a responsibility of local authorities. We have a clearly defined and properly decentralized system of government. Municipality and Panchayath come as the micro level of local self government. State and Central government system which form the macro level do not take direct initiative to solve the problem of waste management. But the management of waste usually dement huge investment and less return. So it cannot be properly managed by local authorities of Municipalaities or Panchayath. When ideas are planed and implemented by many people it lack uniformity and perfection. A system can be efficient only if it is planned and implemented in perfect way. For example, the postal network system and railway network system in India run effectively because it is administered by the central government. Educational planning and implementation is done by both central and state government. Water management is the responsibility of central, state and local authorities. Central as well as state governments should assume major role and responsibilities in the matter of planning and implementing Waste Management Systems (WMS). But the problem of waste generation is a local issue, which is very well understood by local authorities. So in an efficient WMS which is suitable for India the role of local authorities is at the apex level. We suggest a network based mathematical model to represent the whole WMS in the country. It is a macro network, in which due consideration given to LWMS, state level WMS (SWMS) and National level WMS (NWMS). In the following section we discuss the nature a National level WMS network and give one example.

2. NETWORK OF NATIONAL WMS

In India we have very poor waste management system. We have to approach to the problem of waste management in a way which is entirely different from that of developed other countries. Because the majority of Indian population are illiterate and financially very backward. Sunil Kumar et al. in [2] describe the challenges opportunities associated with the waste management systems in India. This paper is a discussion on the pros and cons of waste generation and management in India. It points out that the main drawbacks of waste management system in India are inadequate waste collection, transport, treatment, and disposal. Fast growth in population, rapid urbanization and indifferent behavior of citizens etc. has caused unparalleled increase in the quantity of waste. The diverse culture of India is making the situation even worst.

Waste management is a matter of the greatest concern in our country. We do not have an efficient waste management system. Developing a comprehensive network of waste management is the biggest challenge that we are facing today. We intend to develop a national network of WMS which contains local communities representing the activities of local waste management systems. State level waste management system is the most crucial point in the network. Middle level management is extremely important in WMS and it is the responsibility of each state government. The role of central government can be limited mainly in the area of policy making, planning, monitoring, evaluation, revision and correction of the whole system. The Figure 1 depicts a model of national network of WMS.

The proposed network is a hierarchical structure in which the Central Government (node C) occupy the highest position. State Governments are represented by the nodes $S_1$, $S_2$, $S_3$ etc., which come at the middle level management. The bottom level in the network comprise of district and local authorities such as Corporation, Municipality and Panchayaths. Each local unit may have to depend on other units in the network for improving managerial efficiency and improving the areas of weakness.
3. COMMUNITIES IN THE WMS NETWORK

The network representing WMS must have community structure. A network is said to have community structure if the nodes of the network divide into groups (possibly overlapping groups) so that the nodes in each group are interconnected by edges. In the Figure the nodes $S_1$, $D_{11}$, $L_1$, $L_2$, $L_3$ and $L_4$ make a community. Similarly $S_1$, $D_{12}$, $L_5$, $L_6$ and $L_7$ make another community. Nodes in these communities are interlinked because they collaborate or interact with each other to build an effi-

![Network model of National WMS](image)

client system of WMS. Each local area WMS itself is a big network of which contains sources of waste materials, sorting centres, recycling and processing centres, landfills etc. Structure of these LWMS (micro level networks) need be studied separately, because the functions in such networks are different from that of the network shown in the Figure. A model for LWMS is studied in the article [3] and an example is given which represent the LWMS of Thiruvananthapuram city, the capital of Kerala. It is found that the LWMS of the city is not efficient and some suggestions to make the system efficient is also given.

4. MATHEMATICAL MODELLING OF LWMS

When we think about waste management at the local level, we have to be concerned about the type of waste generated, quantity, its collection, separation, and processing. The collection and separation according to its type require enormous manpower and huge investment. But the income generated from the WMS is at present extremely low. This difference in expense and earning naturally withdraw the attention of government from developing a WMS. The matter is not of any financial benefit but that of social responsibilities and sustainable development. So we need to invest huge amount in the area of WMS and simultaneously we must seek the ways to make the system cost effective.

We initially assume that all quantity of waste materials are properly identified and separated at the source or else it is done at the collection centre, which are denoted by $A_i$. The methods, effort and cost for it are matter of a separate study. So it does not come under the purview of the current research. Also the collection of materials from its source needs attention separately. In this model we assume that all items are properly collected and transported to the collection centres. But we give due consideration to the manpower needed to collect and spate the materials and the job opportunities generated by it. For a country like India finding new employment opportunities is very essential for social security and development. The money spend for this purpose is a national asset. In this sense we try to maximize the job opportunities that may arise from the waste management system.
Furthermore employing more people will make the system more efficient. The collected materials are transported to various processing centres and landfills. The processing centres are denoted by $P_j$ and the landfills are by $L_j$. Landfills are continuously numbered after all the processing centres. We denote the quantity of $n$ items collected at the centre $Ai$ by a vector $(Q_{i1}, Q_{i2}, Q_{i3}, ..., Q_{in})$, where $\sum_i Q_{il} = Q_l$ for $i = 1, 2, 3, ..., n$. This is the total quantity of item $l$ collected in all collection centers. Assuming that the collection cost of item $l$ is $CC_l$, the total collection cost of the particular item is $\sum_i Q_{il} \times CC_l = TCC_l$. We use the simplifying assumption that the collection cost of the item $l$ is same in all collection centers. $Q_{il}^l$ is the portion of the waste material $Q_{il}$ which is transported from the $i^{th}$ collection center to the $j^{th}$ processing center. So $\sum_i Q_{il}^l = Q_{il}$, the total quantity of the item $l$ collected at the center $i$. We assuming that $TC_{il}^j$ is the transportation cost of unit quantity of item $l$ from $i^{th}$ collection center to the $j^{th}$ processing center. So the total transportation cost at the center $j$ of the item $l$ is given by $\sum_i TC_{il}^j \times Q_{il}^l$. The total transportation cost of the item $l$ in the whole system is given by $\sum_j \sum_i TC_{il}^j \times Q_{il}^l$. Processing cost per unit of the item $l$ at the $j^{th}$ processing center is given by $PC_i^j$ and hence the total processing cost at the $j^{th}$ center is given by $\sum_i \sum_j Q_{il}^j \times PC_i^j$. Let the income generated by one unit of the item $l$ at the center $j$ be $I_l^j$. Then the total income generated at the center $j$ by the item $l$ is $\sum_i \sum_j Q_{il}^j \times I_l^j$. Hence the net cost at the $j^{th}$ processing center is the sum of the transportation cost, processing cost and the income generated. We use negative signs before each term representing cost and positive signs before the income. So the net cost at the $j^{th}$ processing center is given by $\sum_j \sum_i Q_{il}^j \times (I_l^j - PC_i^j - \sum_i TC_{il}^j)$.

So the net cost of the whole WMS (NCWMS) is represented by the equation

$$NCWMS = \sum_l \sum_i Q_{il} \times CC_l + \sum_j \sum_i \sum_l Q_{il}^l \times (I_l^j - PC_i^j - \sum_i TC_{il}^j)$$

Let $(L_j, L_{j2}, L_{j3}, ..., L_{jn})$, be the maximum possible capacity of the $j^{th}$ processing center to handle various types to waste materials. Then we have $\sum_i Q_{il}^l \leq L_{jl}$ for each $j = 1, 2, 3, ...$

We also have $Q_{il}^l \geq 0$ for each $i, j$, and $l$. $Q_{il}^l \geq 0$ for each $i$ and $l$. The equation (1) together with the constraints make the mathematical model which represent a local waste management system.

The three characteristics of a WMS (efficient, sufficient and consistent) are defined in [3]. A waste management system is efficient if it can handle all kind and quantity of waste in a meaningful, useful effective way during a particular time period. If a system can handle all waste materials in some way is sufficient. So a system which can process only a portion of waste is not sufficient. A system is consistent if it can manage the waste generated constantly for a long time. A WMS, which is sufficient may not be efficient. But a efficient system must be sufficient also. Consistency checks whether a system shows steady performance for a long time. This measure evaluates the performance of the system based on time. An efficient system must satisfy all the constraints of the model and optimize the cost function. A LWMS is sufficient if the model satisfies the constraint $\sum_l Q_{il}^l \leq L_{jl}$ for each $j = 1, 2, 3, ...$ A waste management system is progressive if it meets the standards of green initiatives and sustainable development. It must also generate income for the country and create job opportunities.

5. PROBLEM

In a local WMS there are three collection centers, which collect four items plastic (Q1), metal (Q2), paper (Q3) and garbage (Q4). The quantity of items collected during a period of time is given in the Table 1.
Table 1. Quantity of items collected in Ton.

|     | Q1  | Q2  | Q3  | Q4  |
|-----|-----|-----|-----|-----|
| A1  | 124 | 215 | 280 | 590 |
| A2  | 250 | 285 |  60 | 790 |
| A3  | 400 | 450 | 600 | 430 |
| Total| 774 | 950 | 940 |1810 |

The processing charge per ton of the four items is given in the Table 2 in thousand rupees. Transportation charges of the items Q1, Q2, Q3 and Q4 from the collection centers to the processing centers are assumed equal and are given in the Table 3. Material collection cost at the collection centers are 20, 30, 8 and 2 (in thousands per ton) for the items Q1, Q2, Q3 and Q4 respectively. The income that can be generated out of the processing of the materials are respectively 80, 250, 20 and 25 (in thousands per ton). Also suppose the processing limit of items in the processing centers is as given in the Table 4.

Table 2. Processing charge of items per ton in the processing centers.

|     | Q1  | Q2  | Q3  | Q4  |
|-----|-----|-----|-----|-----|
| P1  | 24  | 30  |  8  | 10  |
| P2  | 20  | 25  | 10  | 15  |
| L3  | 15  | 15  |  5  |  3  |

Table 3. Transportation charge per ton of Quantity Q1, Q2, Q3 and Q4 from the collection centers to the Processing center or Land fill in thousand rupees.

|     | P1 | P2 | L3 |
|-----|----|----|----|
| A1  | 2  | 1  | 2  |
| A2  | 3  | 2  | 5  |
| A3  | 3  | 4  | 3  |

Table 4. Maximum capacity to Process the items in ton in the processing centers.

|     | Q1  | Q2  | Q3  | Q4  |
|-----|-----|-----|-----|-----|
| P1  | 400 | 300 | 350 | 600 |
| P2  | 400 | 500 | 300 | 700 |
| L3  |  0  |  0  | 300 | 750 |
| Total| 800 | 800 | 950 |2050 |

The problem can be formulated as follows. Let \( Q_{ij}^l \) for \( i = 1, 2, 3 \) and \( j = 1, 2, 3 \) be the quantity of item \( l \) transported from item \( i \) to the item \( j \). Also \( l = 1, 2, 3, 4 \).

\[
NCWMS = -20[Q_{11} + Q_{21} + Q_{31}] - 30[Q_{12} + Q_{22} + Q_{32}] - 8[Q_{13} + Q_{23} + Q_{33}] - 2[Q_{14} + Q_{24} + Q_{34}] + \sum_i \sum_j \sum_l \sum_\ell Q_{ij} \times (I_{ij}^\ell - PC_{ij}^\ell - \sum_\ell TC_{ij}^\ell)
\]

Total transportation charges is \( TC = 2[Q_{11}^1 + Q_{11}^2 + Q_{11}^3 + Q_{11}^4] + 1[Q_{12}^1 + Q_{12}^2 + Q_{12}^3 + Q_{12}^4] + 2[Q_{13}^1 + Q_{13}^2 + Q_{13}^3 + Q_{13}^4] + 3[Q_{14}^1 + Q_{14}^2 + Q_{14}^3 + Q_{14}^4] + 3[Q_{21}^1 + Q_{21}^2 + Q_{21}^3 + Q_{21}^4] + 2[Q_{22}^1 + Q_{22}^2 + Q_{22}^3 + Q_{22}^4] + 5[Q_{23}^1 + Q_{23}^2 + Q_{23}^3 + Q_{23}^4] + 3[Q_{24}^1 + Q_{24}^2 + Q_{24}^3 + Q_{24}^4] + 3[Q_{31}^1 + Q_{31}^2 + Q_{31}^3 + Q_{31}^4] + 4[Q_{32}^1 + Q_{32}^2 + Q_{32}^3 + Q_{32}^4] + 3[Q_{33}^1 + Q_{33}^2 + Q_{33}^3 + Q_{33}^4] + 3[Q_{34}^1 + Q_{34}^2 + Q_{34}^3 + Q_{34}^4]
\]
Total Processing cost at the centers is given by is \( PC = 24[Q_{11} + Q_{21} + Q_{31}] + 30[Q_{12} + Q_{22} + Q_{32}] + 8[Q_{13} + Q_{23} + Q_{33}] + 10[Q_{14} + Q_{24} + Q_{34}] + 20[Q_{15} + Q_{25} + Q_{35}] + 25[Q_{16} + Q_{26} + Q_{36}] + 10[Q_{17} + Q_{27} + Q_{37}] + 15[Q_{18} + Q_{28} + Q_{38}] + 15[Q_{19} + Q_{29} + Q_{39}] + Q_{22} + Q_{33}] + 3[Q_{14} + Q_{24} + Q_{34}] \)

Total income from processing of waste is given by \( TI = 80[Q_{11} + Q_{21} + Q_{31}] + 250[Q_{12} + Q_{22} + Q_{32}] + 20[Q_{13} + Q_{23} + Q_{33}] + 25[Q_{14} + Q_{24} + Q_{34}] + 80[Q_{15} + Q_{25} + Q_{35}] + 250[Q_{16} + Q_{26} + Q_{36}] + 20[Q_{17} + Q_{27} + Q_{37}] + 25[Q_{18} + Q_{28} + Q_{38}] + 250[Q_{19} + Q_{29} + Q_{39}] + 20[Q_{13} + Q_{23} + Q_{33}] + 25[Q_{14} + Q_{24} + Q_{34}] + 250[Q_{15} + Q_{25} + Q_{35}] + 20[Q_{17} + Q_{27} + Q_{37}] + 25[Q_{18} + Q_{28} + Q_{38}] + 250[Q_{19} + Q_{29} + Q_{39}] \)

\( NCWMS = TI - TC - PC - 20[Q_{11} + Q_{21} + Q_{31}] - 30[Q_{12} + Q_{22} + Q_{32}] - 8[Q_{13} + Q_{23} + Q_{33}] - 2[Q_{14} + Q_{24} + Q_{34}] \)

Our objective is to maximize the \( NCWMS \) subject to the following conditions.

First item must satisfy the following constraints.
\[
Q_{11} + Q_{21} + Q_{31} \leq 400 \\
Q_{12} + Q_{22} + Q_{31} \leq 400 \\
Q_{13} + Q_{23} + Q_{31} \leq 0
\]

Similarly the second item must satisfy,
\[
Q_{11} + Q_{21} + Q_{32} \leq 300 \\
Q_{12} + Q_{22} + Q_{32} \leq 500 \\
Q_{13} + Q_{23} + Q_{32} \leq 0
\]

The third item must satisfy
\[
Q_{11} + Q_{21} + Q_{33} \leq 350 \\
Q_{12} + Q_{22} + Q_{33} \leq 300 \\
Q_{13} + Q_{23} + Q_{33} \leq 300
\]

And the fourth item must satisfy
\[
Q_{11} + Q_{21} + Q_{34} \leq 600 \\
Q_{12} + Q_{22} + Q_{34} \leq 700 \\
Q_{13} + Q_{23} + Q_{34} \leq 750
\]

The quantity of items transported from the collection centers to the processing centers must satisfy the following set of equations also.
\[
Q_{11} + Q_{12} + Q_{13} = 124 \\
Q_{21} + Q_{22} + Q_{23} = 250 \\
Q_{31} + Q_{32} + Q_{33} = 400 \\
Q_{11} + Q_{12} + Q_{13} = 215 \\
Q_{21} + Q_{22} + Q_{23} = 285 \\
Q_{31} + Q_{32} + Q_{33} = 450 \\
Q_{13} + Q_{12} + Q_{13} = 280 \\
Q_{23} + Q_{22} + Q_{23} = 60 \\
Q_{31} + Q_{32} + Q_{33} = 600 \\
Q_{14} + Q_{13} + Q_{14} = 590 \\
Q_{24} + Q_{23} + Q_{24} = 790
\]
\[ Q_{34}^{31} + Q_{34}^{32} + Q_{34}^{33} = 430 \]

\[ Q_{ll}^i \geq 0 \text{ for each } i, j, \text{ and } l \]

The system given in the example does not have the facility to process all materials of the second type. Total quantity of type two collected is 950 tons. But the facility allows us to process only 800 tons of waste. So the system is not sufficient and hence it is not efficient.

6. CONCLUSION

In this paper first we discussed the structure of a Network of NWMS. Its components at three level of hierarchy are given along with its relative importance. Cluster of nodes at the root level is most important as they belong to the WMS of local area. It is followed by a discussion of the modelling of the local area WMS. Before concluding the paper we have seen a method of classifying the WMS based on their characteristics. This paper is only a preliminary study on the requirements of a waste management system in India. Each aspect of the system needs detailed attention and separate treatment. Mathematical modelling techniques which are suitable for each level of the system development must be identified and used to expand the currently existing models to improve the efficiency of the entire system.

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