Performance analysis of FSO using relays and spatial diversity under log-normal fading channel

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Abstract—The performance analysis of free space optical communication (FSO) systems using relays and spatial diversity at the source is studied in this paper. The effect of atmospheric turbulence and attenuation has also been considered caused by different weather conditions and geometric losses. The exact closed form expressions are presented for bit-error rate (BER) for M-ary quadrature amplitude modulation (M-QAM) for different types of FSO systems under log-normal channel model. The performance of the link of FSO system is compared for different system as on-off keying, M-ary pulse amplitude modulation. Results shows that M-QAM outperforms other systems with same spectral efficiency for each system with the help of relays.

Keywords—Free Space Optical Communications, Log-Normal Fading Channel, Quadrature Amplitude Modulation, Spatial Diversity, Bit Error Rate, Decode and Forward Relays

I. INTRODUCTION

Free space optical communications (FSO) is a rapidly evolving technology to handle high data rate and has very huge information handling capacity. FSO systems are proved to be an alternative to the fiber optics technology. In many cases, only the free space transmission is possible to establish a connection between the source and destination over a line-of-sight (LOS) path. Optical signal transmission in free space has explored the unexplored areas in wireless communication since decades and appearing as the basic part of many systems and varieties of applications for example radio frequency (RF) wireless transmission, satellite communications, long-haul connections and optical fiber back up. FSO systems have to face many challenges like atmospheric turbulence, beam wandering, beam attenuation, weather attenuation, geometric losses and scintillation and many more [1]. In literature, varieties of channel models are proposed and used to exactly model the atmospheric turbulence which matches well with the experimental results. Log-normal, gamma-gamma and negative exponential channel models are used for weak, strong-moderate and strong turbulence conditions, respectively [2][4]. Fading and attenuation has affected the performance of FSO systems in large extent. Turbulence and weather conditions like snow, fog and haze also affecting the FSO transmission. Furthermore, geometric losses also lessen the performance of FSO systems. Varieties of spatial diversity techniques are proposed and used to mitigate the effect of atmospheric turbulence in FSO [5][6] where three main spatial diversity techniques are most popular among the available diversity schemes which are Orthogonal Space Time Block Codes (OSTBCs), Transmit Laser Selection (TLS) and Repetition Codes (RCs) where TLS is the best with channel state information (CSI) at the source with increased system complexity [7][8].

In this paper, M-ary quadrature amplitude modulation (M-QAM) is used against a log-normal channel model to analyze the link performance of the FSO system under different weather circumstances. Bit-error-rate (BER) is considered as a metric to measure the performance of FSO systems. For analysis purpose, effect of turbulence, path losses, scattering and scintillation has also been considered. The BER analysis for FSO multiple-input single-output (MISO) system is analyzed. Link reliability is also improved with the help of relays. With decode and forward (DF) relays for intensity modulation and direct detection (IM/DD), multi-hop MISO system are studied over log-normal channel model for short distance communication links. Among the transmitters, considering the correlation effects, the performance of the multi-hop MISO system for M-QAM is compared with IM/DD SISO multi-hop system using M-PAM, SISO with on-off keying (OOK) and MISO with RCs and OOK [9]. Spectral efficiency is considered same for all systems and ML decoding technique is used for receiving the QAM signals at the receiver.

II. SYSTEM DESCRIPTION

A. DF-Relay MISO Multi-hop System

For transmission of signals, a MISO system is employed with M transmitters in form of lasers and a photo detector as receiver which is shown in Fig. 1. At the source, transmission of signals are in the form of binary data which is modulated by a $M_t \times M_Q$ electrical-QAM modulator and using the Gray coding, mapping of data into QAM symbols is done for the transmission using M paths. In addition, sufficient channel distance is considered to mitigate the spatial correlation effect.

The probability of error of QAM signals can be easily calculated from the probability of error of PAM signals as the phase-quadrature signal components can be perfectly extracted at the QAM demodulator. For the M-ary QAM signals, the...
probability of a correct decision is written as \[ P_C = (1 - P_{\sqrt{\gamma}})^2, \] \[ (1) \]
where \( P_{\sqrt{\gamma}} \) is the probability of error for \( \sqrt{M} \)-ary PAM with half the average power in each quadrature component of the signal of the corresponding QAM system. After some appropriate modifications, the probability of error of M-ary PAM can be written as
\[ P_M = 2 \left( 1 - \frac{1}{\sqrt{M}} \right) Q \left( \frac{3\log_2 M \gamma}{M - 1} \right), \] \[ (2) \]
where \( \gamma \) represents the average SNR per symbol. So, the probability of a symbol error for M-ary QAM signals can be written as
\[ P_M = 1 - (1 - P_{\sqrt{\gamma}})^2, \] \[ (3) \]
which is valid for \( M = 2^k \) and even \( k \). By using the optimum detector, the tightly upper bounded symbol error probability can be written as
\[ P_M \leq 1 - \left[ 1 - 2Q \left( \frac{\sqrt{\gamma}}{M - 1} \right) \right], \] \[ (4) \]

Here, introduction of relays creates some effect in FSO system for SISO and MISO configurations with the help of DF relays which helps to achieve significant improvement in the link performance. \((K - 1)\) DF relays are set up in between the source and destination for \( K \) hops. The received signal at the \( k \)th hop can be written as
\[ r_k(t) = x(t)\eta I_k + n_k(t), \] \[ (5) \]
In this scheme, \( K \) time slots are required for transmission of the signal from source to destination which is not the case for MISO and SISO systems. For the benefit of achieving similar spectral efficiency, \( 2^K \)-ary M-QAM modulation techniques are considered which is finally compared for analysis purpose at the end as discussed earlier. DF relays used in between the transceiver pair create shorter communication links which helps to reduce the effects of turbulence and path losses significantly. The upper bound BER for multi-hop DF relay system can be written as \[ BER \leq 1 - \prod_{k=1}^{K} (1 - BER_k). \] \[ (6) \]
As the identical statistical properties is considered for all hops, \( (6) \) can be approximated by
\[ BER \approx \frac{1}{2} \left[ 1 - (1 - 2BER_k)^K \right]. \] \[ (7) \]

The conditional BEP of M-QAM can be written as \[ 11 \] as
\[ P(e|\gamma) \approx \frac{2 \left( 1 - \frac{1}{\sqrt{\gamma}} \right) \log_2 M}{\log_2 M} \left[ Q \left( \frac{\sqrt{3\log_2 M \gamma}}{2(M - 1)} \right) \right], \] \[ (8) \]
The instantaneous SNR \( \gamma \) is given by
\[ \gamma = \frac{2P^2}{\sigma_n^2 R}, \] \[ (9) \]
where \( P \) implies the signal power and \( R \) signifies the bit rate. Now, by applying the approximate \( Q \) function on \( (8) \), the conditional BEP of M-QAM can be written as
\[ P(e|\gamma) \approx \frac{2 \left( 1 - \frac{1}{\sqrt{\gamma}} \right) \log_2 M}{\log_2 M} \times \left[ \frac{1}{12} \exp \left( - \frac{3\log_2 M \gamma}{4(M - 1)} \right) + \frac{1}{4} \exp \left( - \frac{\log_2 M \gamma}{(M - 1)} \right) \right], \] \[ (10) \]

where the block diagram representing this system is presented in Fig. 2. \((K - 1)\) relays are employed to receive the signal from \( N_t \) transmitters and re-transmits it simultaneously with the help of RCs. For this system, the number of hops are considered to be identical with the transmitters per relay. The signal received at each hop can be given as
\[ r_k(t) = x(t)\eta \sum_{i=1}^{N_t} I_{ki} + n_k(t), \] \[ (12) \]

Similarly, using the conditional BEP expression for M-
QAM, the BER expression for $k$th hop can be written as

$$\text{BER}_k \approx \sum_{n_i=1}^{N} \cdots \sum_{n_i=1}^{N} \prod_{i=1}^{N_t} \frac{w_{n_i}}{\sqrt{\pi}} \times F \frac{12}{\exp} \left( - \frac{3\log_2(M)\beta_{kn_i}^2}{4(M-1)N_t} \sum_{i=1}^{N_i} \exp \left( \frac{32}{\sqrt{\pi}} \sum_{j=1}^{N_i} c'_{ij}x_{n_ij} - 4\sigma_k^2 \right) \right) $$

$$+ \sum_{n_i=1}^{N} \cdots \sum_{n_i=1}^{N} \prod_{i=1}^{N_t} \frac{w_{n_i}}{\sqrt{\pi}} \times F \frac{4}{\exp} \left( - \frac{\log_2(M)\beta_{kn_i}^2}{(M-1)N_t} \sum_{i=1}^{N_i} \exp \left( \frac{32}{\sqrt{\pi}} \sum_{j=1}^{N_i} c'_{ij}x_{n_ij} - 4\sigma_k^2 \right) \right),$$

(13)

where $F = 2(1 - \frac{1}{\sqrt{M}})/\log_2(M)$ for $\beta_{kn_i}^2$ and $c'_{ij}$ represents the $(i,j)$th coefficients of $\Gamma_{ij} = \Gamma^{1/2}$. By substituting these results in (9) and (7), expressions of the upper bound BER and the closed form average BER of the multi-hop MISO system can be determined, respectively. The BER expression of M-PAM multi-hop FSO system is shown in (9) as Eq. (28).

III. Numerical Analysis and Discussions

Numerical results are presented to verify the derived results. All the parameters are defined and tabulated in Table I to obtain the results to achieve a target BER of $10^{-9}$. Fig. 3 presents the BER for multi-hop FSO system using two relays using one or three transmit lasers with 8-QAM, 8-PAM, MISO with RCs and OOK and SISO using OOK modulation techniques and demonstrates the effect due to the increment in transmit diversity and relays in a MISO system and multi-hop system, respectively under clear weather conditions. At the target BER, with three transmitters, SNR gains of 19.5 dB and 26 dB is achieved for multi-hop MISO and multi-hop SISO systems as compared to MISO with OOK and RCs and SISO with OOK, respectively, for 8-PAM. But with 8-QAM modulation, SNR gains of 13 dB and 14 dB is achieved for multi-hop MISO and multi-hop SISO as compared to multi-hop MISO and multi-hop SISO, respectively, again with three transmitters using 8-PAM. Multi-hop MISO system outperforms the multi-hop SISO system by 2.8 dB and 2.6 dB, respectively, for M-PAM and M-QAM.

In Fig. 4, BER for multi-hop FSO systems using two relays with one or three transmit lasers for 8-QAM, 8-PAM, MISO with RCs and OOK with three transmitters and SISO with OOK has been shown under light fog conditions. For multi-hop system using two relays, 8-QAM helps to achieve an equal spectral efficiency as SISO and MISO systems. Analytic results shows that multi-hop systems are still outperforms other systems and the increment in the performance of the systems is 10 dB which is significant. When number of transmit lasers are increased to three, a increment of 2 dB and 1.8 dB has been noticed for M-PAM and M-QAM, respectively, at the target BER.

42.8 dB and 47.5 dB is achieved at the target BER for multi-hop MISO and multi-hop SISO systems in comparison with MISO with RCs and OOK and SISO with OOK with 8-PAM whereas, using 8-QAM technique, SNR gains of 13 dB and 12.5 dB is achieved for multi-hop MISO and multi-hop SISO systems with three transmit lasers as compared to

**TABLE I**

| Simulation Parameters | Numerical Values |
|-----------------------|------------------|
| Relay spacing         | 400 m            |
| Wavelength ($\lambda$)| 1550 nm          |
| Link distance (l)     | 1200 m           |
| Beam divergence angle ($\theta_{p}$) | 2 mrad |
| Correlation coefficient ($\rho$) | 0.3 |
| Transmitter and receiver aperture diameter ($D_{t}$ and $D_{r}$) | 20 cm |
| Attenuation constant ($\alpha$) | 0.43 dB/km (clear weather) |
| 20 dB/km (light fog) |
| Refractive index constant ($C_{D}^2$) | $5 \times 10^{-14} \text{m}^{-2/3}$ (clear weather) |
| 1.7 \times 10^{-14} \text{m}^{-2/3}$ (light fog) |
multi-hop MISO and multi-hop SISO with PAM using single transmit lasers. Multi-hop MISO system outperforms the multi-hop SISO system by 1.2 dB and 1.4 dB in case of M-QAM and M-PAM, respectively.

IV. CONCLUSIONS

The effects of moderate turbulence under clear weather and weak turbulence with light fog on the the performance of different types of FSO systems are studied. Diverse conditions in atmosphere have large effect on the performance of MISO multi-hop FSO system with DF relay where derived analytical results show that multi-hop MISO system with M-QAM modulation outperforms other systems such as SISO with OOK and MISO with RCs and OOK and multi-hop SISO system with M-PAM considering the same spectral efficiency. Results also demonstrates that Multi-hop systems are capable of mitigate the effects of turbulence and path losses caused by geometric loss and attenuation whereas, MISO systems can counteract turbulence effects only. Consequently, results demonstrate that the overall performance of the FSO system can be improved by increasing the number of relays, significantly.

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