Models and Methods of Aggregating Linguistic Information in Multi-criteria Hierarchical Quality Assessment Systems

T V Azarnova¹, I A Titova¹ and S A Barkalov²

¹ Department of Applied Mathematics, Informatics and Mechanics, Voronezh State University, Voronezh, Russia
² Department of Economics, Management and IT, Voronezh State Technical University, Voronezh, Russia

E-mail: ivdas92@mail.ru, tia1404@yandex.ru

Abstract. The article presents an algorithm for obtaining an integral assessment of the quality of an organization from the perspective of customers, based on the method of aggregating linguistic information on a multilevel hierarchical system of quality assessment. The algorithm is of a constructive nature, it provides not only the possibility of obtaining an integral evaluation, but also the development of a quality improvement strategy based on the method of linguistic decomposition, which forms the minimum set of areas of work with clients whose quality change will allow obtaining the required level of integrated quality assessment.

1. Introduction
Client capital is an important factor in the competitiveness of modern companies working in the service sector. Manage client portfolio [1] and building long-term relationships with customers [2] are a priority in the work of each organization. To effectively manage customer capital, tools are needed to measure service quality from the perspective of customers, implementing feedback and assessing the degree to which the service meets the requirements and expectations of customers. The problem of assessing the quality of service is weakly structured, there are no established generally accepted approaches to its solution. Widely used methods are: secret client (Mystery Shopping); SCI (Customer Satisfaction Index), push-button surveys of customers and SERVQUAL [3]. The SERVQUAL method, on which the algorithms proposed in this article are based, is based on the assumption that the quality of service for any evaluation characteristic (service criterion) is measured by the discrepancy between the "expectation" and "perception" of the service level for a given characteristic. For the practical implementation of this approach to quality assessment, special tools are needed to measure the degree of discrepancy between "expectation" and "perception".

The article presents a hierarchical system of quality assessment, reflecting the process of aggregating evaluation information, structured by client groups. Depending on the specific activities of the organization’s customers are divided into groups, each of which in turn is divided into sub-groups. The criteria for this segmentation can be such factors as socio-economic (income level, education level, etc.), geographical (region of residence, climate, etc.), demographic (sex, age, etc.), etc. Based on the proposed model and methodology, SERVQUAL...
developed a linguistic algorithm for assessing quality and a linguistic algorithm for shaping a quality improvement strategy.

2. Algorithms for the Formation of an Integral Linguistic Quality Assessment on a Multicriteria Hierarchical System

As the information base of the research, the results of the customer questioning for each selected segment are presented. The questionnaires correspond to the structure of the hierarchical model for each client category and provide a list of criteria and corresponding subcriteria reflecting the full range of services of the organization in question. For example, for a restaurant business [4], the criterion can be "Serving a waiter", and the sub criteria of this criterion is "Service speed", "Courtesy", etc. The evaluation criteria are divided according to their orientation (ordering of alternatives) into two groups, the criteria of the first group (symbol \( \nearrow \)) satisfy the condition — the higher the value of the evaluation, the higher the client’s satisfaction, and the criteria of the second group (symbol \( \searrow \)) to the condition — the lower the estimate value, the higher the customer satisfaction. In the process of questioning, the respondent for each of the selected items (subcriteria) should indicate: an assessment of importance; assessment of quality expectations; assessment of the perception of quality. The estimation is carried out in a linguistic scale [5] with the term set \( S = \{ S_1 = VL, S_2 = L, S_3 = M, S_4 = H, S_5 = VH \} \) \((VL — very low, L — low, M — average, H — high, VH — very high). The multicriteria hierarchical system for assessing the quality of the generalized (not tied to a particular problem) is shown in Figure 1. The above structure contains the following notation:

- \( K^V \) — final integrated customer service quality assessment;
- \( K^{IV} = (k^{IV}_1, \ldots, k^{IV}_p, \ldots, k^{IV}_n) \) — vector integral group quality estimates;
- \( p = \overline{1, \overline{p}} \) — group number, \( \overline{p} \) — number of groups;
- \( W^{IV} = (w^{IV}_1, \ldots, w^{IV}_p, \ldots, w^{IV}_n) \) — vector of assessments of the importance of the groups provided by the experts of the enterprise in question;
- \( K^{III} = (k^{III}_1, \ldots, k^{III}_p, \ldots, k^{III}_n) \) — vector of summary quality estimates of subgroups of the \( p \)-th group;
- \( t_p = \overline{1, \overline{t}_p} \) — subgroup number of the \( p \)-th group, \( \overline{t}_p \) — number of subgroups of the \( p \)-th group;
- \( W^{III} = (w^{III}_1, \ldots, w^{III}_p, \ldots, w^{III}_n) \) — vector of assessments of the importance of the subgroups provided by the experts of the enterprise in question;
- \( K^{II} = (k^{II}_1, \ldots, k^{II}_p, \ldots, k^{II}_n) \) — vector of integral quality estimates of the criteria of the \( p \)-th group of the \( t_p \)-th subgroup;
- \( m_{tp} = \overline{1, \overline{m}_{tp}} \) — number of the criterion of the \( p \)-th group of the \( t_p \)-th subgroup, \( \overline{m}_{tp} \) — number of criteria for the \( p \)-th group of the \( t_p \)-th subgroup;
- \( k^{I} = (k^{I}_1, \ldots, k^{I}_p, \ldots, k^{I}_n) \) — vector of integral quality estimates of subcriteria of the \( m \)-th criterion of the \( p \)-th group of the \( t_p \)-th subgroup;
- \( n_{mtp} = \overline{1, \overline{n}_{mtp}} \) — sub-criterion number of the \( m \)-th criterion of the \( p \)-th group of the \( t_p \)-th subgroup, \( \overline{n}_{mtp} \) — number of subcriteria of the \( m \)-th criterion of the \( p \)-th group of the \( t_p \)-th subgroup;
- \( K^{is}_{i=1, \overline{i}} \) — expectations of service quality, issued by the \( i \)-th client;
- \( K^{ex}_{i=1, \overline{i}} \) — assessment of service quality perception, exposed by the \( i \)-th client;
- \( X \) — variable indicating the number of estimates for this sub-criterion;
- \( \omega_{i=1, \overline{X}} \) — assessment of the importance of subcriteria, exposed by the \( i \)-th client;
- \( \overline{K}_{i=1, \overline{X}} \) — estimates aggregated for each \( i \)-th client for each sub-criterion on the basis of estimates \( K^{is}_{i=1, \overline{i}} \) and \( K^{ex}_{i=1, \overline{i}} \).
Algorithm 1. Computing an integral evaluation of the quality of customer service for each element of a multicriteria hierarchical system [6].

Step 1. The results of the questionnaire are structured in accordance with the main groups/subgroups of clients.

Step 2. For each respondent, linguistic assessments of quality are calculated based on the questionnaires that were filled in during questioning on the basis of linguistic assessments of expectations and perceptions of quality.

Let $K_{is}^m = S_i$ — the linguistic estimate of the expected quality from the $m$-th sub-criterion, $K_{ex}^m = S_j$ — the linguistic estimate of the perceived quality from the $m$-th sub-criterion, then the linguistic quality of service estimate for the $m$-th sub-criterion with orientation (↗) will be calculated according to the formula:

$$K_m = \overline{S}_{\text{max}\{\min\{2j-i;n\};1\}}, \quad (1)$$

where $n$ — number of terms of the linguistic scale $S = \{S_i\}_{i=1,n}$. For sub-criteria with orientation (↙), the quality estimate is computed similarly after converting the estimates $K_{is}^m$ and $K_{ex}^m$ in to estimates of type (↗) through the negation operation for linguistic variables.

For each subcriter, many pairs are formed $L = \{(w_l, k_l) : l = 1, \ldots, n\}$, where $n$ — the number of respondents who rated the relevant item, $w_l$ — assessment of the importance for $l$-th of respondent, $k_l$ — quality assessment for $l$-th of respondent. The pairs are structured into two vectors: the importance vector $W = (w_1, \ldots, w_n)$ and quality vector $K = (k_1, \ldots, k_n)$.
The importance estimation vector \( W = (w_1, \ldots, w_n) \) containing duplicate values generates a decomposition of the quality estimation vector into non-empty groups \( K_1, \ldots, K_5 \). Each nonempty group \( K_i, i = 1, \ldots, 5 \) is constructed from a certain value of importance and consists of the components of the vector \( K = (k_1, \ldots, k_n) \) for which the corresponding components of the vector \( W \) are equal to the selected value of importance. Values of importance are viewed in ascending order \( S_1 = VL, S_2 = L, S_3 = M, S_4 = H, S_5 = VH \). We denote by \( n_i \) the number of elements in the group \( K_j \). For each of the groups \( K_1, \ldots, K_5 \), a generalized quality estimate is constructed by convolving their components with the help of the linguistic LOWA-operator \( \Phi_U(K) \), which is defined as follows [7–9].

Let \( U \) — vector of weights, \( (u_i \in [0, 1], \sum_{i=1}^{n} u_i = 1) \), \( K = \{k_1, k_2, \ldots, k_n\} \) — components of some non-empty group, then

\[
\Phi_U(K) = C^n\{\{(k, b_k), \; k = \frac{1}{m}, n\} = u_1 \otimes b_1 \oplus C^{n-1}\{\{(\lambda_k, b_k), \; k = \frac{2}{m}, n\}, \quad (2)
\]

where \( B = (b_1, b_2, \ldots, b_n) \) — is the vector obtained from \( K = \{k_1, k_2, \ldots, k_n\} \) ordering by non-increase of linguistic terms, \( \lambda_k = \sum_{m=2}^{n} \omega_m (k = \frac{1}{m}, n) \) — the normalized vector of weights obtained after removing the maximum weight \( u_1, \sum_{m=2}^{n} \lambda_m = 1; C^n, C^{n-1} \) — convex combinations, \( n \) and \((n-1)\) terms respectively.

For \( n = 2 \), the convex combination of linguistic terms \( b_1 = S_j \) and \( b_2 = S_i \) \((j \geq i)\) is determined by the rule:

\[
C^2\{(u_1, b_1), (u_2, b_2)\} = u_1 \otimes b_1 \oplus u_2 \otimes b_2 = S_k, \quad (3)
\]

where \( k = \min\{T, i + \text{round}(u_1 \cdot (j - i))\} \), \text{round} corresponds to the rounding operation.

Vector of weights \( U = (u_1, \ldots, u_n) \) for the aggregation operator is calculated through special functions of quantification \( Q(x) \):

\[
u_i = Q\left(\frac{i}{n}\right) \quad \forall \quad \left(i \in \mathbb{Z}, n\right) \left(u_i = Q\left(\frac{i}{n}\right) - Q\left(\frac{i-1}{n}\right)\right).
\]

As a function of quantification \( Q(x) \) the function is used \( Q(x) = x^\alpha \) with different values of \( \alpha \) for different groups \( K_1, \ldots, K_5 \) with higher importance values, the aggregation strategy is closer to the conjunctive, with lower ones to disjunctive): for the group \( K_1 \) (importance \( S_1 = VL \) — \( Q_1(x) = x^{1/3} \); for the group \( K_2 \) (importance \( S_2 = L \) — \( Q_2(x) = x^{1/2} \); for the group \( K_3 \) (importance \( S_3 = M \) — \( Q_3(x) = x \); for the group \( K_4 \) (importance \( S_4 = H \) — \( Q_4(x) = x^2 \); for the group \( K_5 \) (importance \( S_5 = VH \) — \( Q_5(x) = x^3 \).

For the chosen quantification functions we calculate the weights \( U = (u_1, \ldots, u_n) \) operator of aggregation and generalized estimates for groups \( K_1, \ldots, K_5 \).

As a result of the performed operations, three vectors are formed: the importance vector \( W = (w_1, \ldots, w_n) \) the vector of generalized quality estimates \( K = (k_1, \ldots, k_n) \), and the frequency vector \( V = (v_1, \ldots, v_n) \), where \( v_i = \frac{n_i}{n} \), \( n \) — the number of respondents evaluating the criterion in question.

Using the pairs \((w_i, v_i)\), obtained, linguistic confidence coefficients \( q_i \), are constructed that characterize the degree of confidence in the estimates of \( k_i \). Confidence coefficients are calculated using linguistic scales, constructed individually for each value \( s = 2, 3, 4, 5 \) (the number of pairs \((w_i, v_i)\)) and each valuation of \( VL, L, M, H, VH \).

Algorithm 2. The construction of a scale for a specific value \( w = S_j \).

Step 1. On the OX axis, the interval \([0, 1]\) is the range of possible frequencies.

Step 2. At the point \( \frac{1}{s} \), the membership function for a term equal to the value of the considered importance of \( S_j \), is assumed to be 1:

\[
\mu_{S_j}\left(\frac{1}{n}\right) = 1.
\]
Step 3. The remaining terms are completed as follows: for terms with indices \( j = 1, \ldots, j^* \) the membership functions at the points \( \frac{j - 1}{n \cdot (j^* - 1)} \) are assumed to be 1:

\[
\mu_{S_j} \left( \frac{j - 1}{n \cdot (j^* - 1)} \right) = 1.
\]

For terms with indices \( j = j^*, \ldots, 5 \), the membership functions at the points \( \left( \frac{1}{n} + \frac{n - 1}{n \cdot (5 - j^*)} \cdot (j - j^*) \right) \) are assumed to be 1:

\[
\mu_{S_j} \left( \frac{1}{n} + \frac{n - 1}{n \cdot (5 - j^*)} \cdot (j - j^*) \right) = 1.
\]

The graphs mark the points obtained and complete the triangular numbers, connecting the right end of the \( i \)-th fuzzy number to the left end \((i + 1)\)-th in the marked values.

For \( w = VL \), a fuzzy trapezoidal number with coordinates: \( \left( 0; \frac{1}{n}; 0; \frac{n - 1}{4n} \right) \) corresponds to the term \( VL \) in the scale being formed. For \( w = VH \), a fuzzy trapezoidal number with coordinates \( \left( \frac{1}{n}; 1; \frac{1}{4n}; 0 \right) \) corresponds to the \( VL \) term in the scale being formed.

Based on the vector of linguistic quality estimates \( K = (k_1, k_2, \ldots, k_s) \) and the vector of linguistic confidence coefficients \( Q = (q_1, q_2, \ldots, q_s) \) \((s \leq 5)\), a generalized quality estimate for each criterion \( \bar{K} \) is constructed.

It is assumed that \( s \geq 2 \), the case \( s = 1 \) is trivial and does not require convolution construction. For \( s = 2 \), the number of possible different combinations \((K, Q)\), where \( Q = (q_1, q_2) \) and \( K = (k_1, k_2) \) is 325, which allows us to consider and specify all possible rules for fuzzy inference:

- If \( k_1 = VL \) and \( q_1 = VL \), \( k_2 = VL \) and \( q_2 = VL \), then \( K = VL \)
- If \( k_1 = L \) and \( q_1 = VL \), \( k_2 = VL \) and \( q_2 = VL \), then \( K = VL \)
- If \( k_1 = L \) and \( q_1 = VL \), \( k_2 = L \) and \( q_2 = VL \), then \( K = L \)

These rules can be presented in the form of a table, where for estimates \((k_i, q_i)\) and \((k_j, q_j)\) the generalized estimate \( \bar{K} \) stands at the intersection of the \( i \)-th row and the \( j \)-th column.

For the three-dimensional case, the number of different combinations increases already to 7875 and the application of expert judgment on the values of generalized estimates becomes too time-consuming. Therefore, for \( s \geq 3 \), there is a need for a mechanism that would not involve experts. Let’s describe a variant of such a mechanism.

**Algorithm 3. Construction of generalized estimates \( \bar{K} \) for the case \( s \geq 3 \).**

Let there be 2 vectors: \( K = (k_1, \ldots, k_s) \) and \( Q = (q_1, \ldots, q_s) \). We denote by \( l \) the indices of the estimates in the vectors \( K \) and \( Q \), \( l = 1, \ldots, s \), and by \( i_l \) and \( j_l \) are the indices of the terms corresponding to these estimates to the scale \( S \).

Step 1. Each pair of estimates \((k_l, q_l) = (S_{i_l}, S_{j_l})\) is translated into the numerical estimate \( c_{i_lj_l} \) in accordance with Table 1.

Step 2. The values are calculated:

\[
H_i = \sum_{l=1}^{s} c_{i_lj_l}, \quad i = 1, \ldots, 5, \quad H^* = \sum_{l=1}^{s} c_{i_lj_l}.
\]  

Step 3. An output linguistic scale is constructed, containing 5 terms that correspond to the possible final estimates \((VL, L, M, H, VH)\), as follows: on the OX axis the \( H_i \) points are marked.
Table 1. The mechanism of forming a numerical estimate.

|   | q | VL | L | M | H | VH |
|---|---|----|---|---|---|----|
| VL| 55| 45 | 20| 13| 4 |
| L | 66| 55 | 40| 27| 9 |
| M | 79| 67 | 60| 42| 30|
| H | 90| 85 | 80| 65| 64|
| VH|100|100 |100|100|100|

In these values, the membership functions of the corresponding evaluation terms will be equal to 1. Triangular numbers with the following coordinates are completed: \((H_1, H_1, H_2)\) — for the term \(S_1 = VL\); \((H_{i-1}, H_i, H_{i+1})\) — for the terms \(S_i\), \(i = 2, 3, 4\); \((H_4, H_5, H_5)\) for the term \(S_5 = VH\).

Step 4. The scale indicates the value of \(H^*\). The value of the term \(S_i\), for which the membership function for the given point takes the greatest value, is the final generalized estimate \(K = S_i\).

The algorithm described above allows us to form a vector of partial estimates of the subcriteria \(K = (k_1, k_2, \ldots, k_m)\). For each term \(S_i\) from the scale \(S\), the number of occurrences of this term in the vector of partial estimates \(K\) is counted. Let: \(r_1\) — the number of estimates of \(VL\) in the vector \(K\); \(r_2\) is the number of estimates of \(L\) in the vector \(K\); \(r_3\) is the number of estimates of \(M\) in the vector \(K\); \(r_4\) is the number of estimates of \(H\) in the vector \(K\); \(r_5\) is the number of \(VH\) estimates in the vector \(K\). Denote the required integral estimate for a certain stage of aggregation by \(S_t\), then the index \(t\) of the final estimate will be calculated by the formula:

\[
t = \text{round} \left( \frac{1}{m} \sum_{i=1}^{5} (r_i \ast i) \right),
\]

where \(\text{round}\) is the rounding operation. In this case, you can use 3 types of policies for calculating the final rating: hard, neutral, soft. With a hard policy, the rounding down method (the nearest integer not exceeding the value obtained) is used to round the result. When neutral, the index \(t\) is rounded in the traditional way. If you choose a soft policy when rounding, the round-up method is used (the nearest integer is larger than the value obtained).

3. Algorithm for forming a strategy for improving quality

The algorithm described in the previous section is of a constructive nature, its structure and the calculation technologies used allow us to develop tools for reverse decomposition and the development of a strategy for increasing the integral quality assessment [10].

Consider the algorithm for determining the minimum set of criteria, changing the integral linguistic quality assessment by which for some (if possible minimum) number of items up the linguistic scale, allow to increase the overall integral assessment of the quality of the organization as a whole.

Algorithm 4. The solution of the problem of reverse decomposition according to the multicriteria hierarchical system of quality assessment:

Step 1. Set the desired upper (fifth) level \(K^V\).

Step 2. Determine all possible variants of the vector \(K^IV_s = (k^IV_1, \ldots, k^IV_p)\), where \(K^IV_s\) is the vector of quality grades by groups, vector \(K^IV\) and is the input vector for obtaining the desired estimate \(K^V_s\).
Step 3. Determine for each vector $K_{III}^{*i} = (k_{III}^{*1}, \ldots, k_{III}^{*p})$, where $K_{III}^{*i}$ is the vector of quality estimates by subgroups, vector $K_{III}$ and $K_{IV}^{*i}$ are the input vector for obtaining the desired estimates of the vectors $K_{IV}^{*i}$.

Step 4. For each vector $K_{III}^{*i}$ find a minimum set of criteria, the assessment of which must be shifted one point up the linguistic scale.

Step 5. Construct a tree of options for the task of improving the quality assessment.

Step 6. Evaluate all the sets of criteria obtained in step 3, select expertly the best one.

Step 7. For the set of criteria selected in step 5, consider the sub-criteria, identify those that require increased attention from the enterprise.

Let us dwell in more detail on the implementation of steps 2, 3 of the algorithm.

The method of calculating the inverse problem will depend on the technique used in direct calculation, which in turn is determined by the number of given groups:

- for $p = 2$, the convolution was performed using a knowledge base represented in the form of a table,
- at $3 \leq p \leq 5$ — the convolution was performed using linguistic scales. Consider the formation of changes for both cases:

  1. for $p = 2$ we modify the used table:
     - at the intersection of rows and columns corresponding to the given estimates of the quality and importance of the groups, instead of the resulting estimate, we insert the value of the indexes of the first and second group quality ratings.
     - the resulting estimates with the corresponding terms will be denoted by shading the areas in the table with the defined colors for each term as follows: $VL$ — blue, $L$ — light blue, $M$ — yellow, $H$ — light red, $VH$ – dark red (Table 2).

**Algorithm 5. Formation of possible variants of changes for $p = 2$.**

- Step 1. Find the cell of the resulting estimate $K_{V}^{*i} = S_{j}$ in the table and determine the tabular value corresponding to the given estimate. We denote it by $z$.

- Step 2. Find the domain of the term for a given desired estimate $K_{V}^{*i} = S_{j}$.

- Step 3. Find in the given area all the values of $z^{*}$, which are strictly right and/or lower than the value of $z$ and for which the difference $(z^{*} - z)$ is minimal.

- Step 4. Determine all possible quality estimation vectors $K_{IV}^{*i} = (k_{IV}^{*1}, k_{IV}^{*2})$, where $k_{IV}^{*1}$ and $k_{IV}^{*2}$ correspond to the values found $z^{*}$ possible estimates of the groups.

If the number of groups $p$: $3 \leq p \leq 5$, then the quality estimates were no longer calculated using a table, but based on linguistic scales.

**Algorithm 6. Formation of possible variants of changes for $3 \leq p \leq 5$.**

| $\omega_{1}$ | $L$ | $L$ | $L$ | $L$ | $L$ |
|-------------|----|----|----|----|----|
| $\omega_{2}$ | $k_{1}^{V}$ | $VL$ | $L$ | $M$ | $H$ | $VH$ |
| $H$ | $VL$ | 2 | 3 | 4 | 5 | 6 |
| $H$ | $L$ | 3 | 4 | 5 | 6 | 7 |
| $H$ | $M$ | 4 | 5 | 6 | 7 | 8 |
| $H$ | $H$ | 5 | 6 | 7 | 8 | 9 |
| $H$ | $VH$ | 6 | 7 | 8 | 9 | 10 |
In order to shift the estimate \( k^I_{IV} \) to the value \( k^I_{IV*} \), it is necessary to shift the numerical value obtained on the basis of the table to the right according to the linguistic scale until the desired result is obtained. The minimum value on the linguistic scale at which the desired estimate \( k^I_{IV*} \) is obtained is the intersection of the scales of the terms of the estimate \( k^I_{IV} \) and the estimate with the index is one less. The intersection point is the midpoint of the segment connecting the vertices of the scales of these terms.

We denote by \( l \) the indices of the estimates in the vectors \( k^I_{IV*}, W^I_{IV} \) and \( K^I_{IV*} \), \( l = 1, 2, 3 \), and by \( i_l, j_l \) and \( i_l^* \) are the indices of the terms of the scale \( S \).

Step 1. Translate each pair of estimates \( (k^I_{IV}, w^I_{IV}) = (S_{i_l}, S_{j_l}) \) into a numerical estimate of \( c_{i_l,j_l} \) in accordance with Table 1.

Step 2. Calculate the quantities:

\[
H_i = \prod_{l=1}^{5} c_{i_l,j_l}, \quad i = i_l^* - 1, i_l^*.
\]  

(7)

Step 3. Calculate the value of \( \overline{H} \) — the middle of the segment \( [H_i_{l-1}; H_i_l] \):

\[
\overline{H} = \frac{H_i_{l-1} + H_i_l}{2}.
\]  

(8)

Step 4. Introduce the vector \( K^I_{a_1...a_\overline{p}} \) with the coordinates \( (S_{1+a_1}, \ldots, S_{\overline{p}+a_\overline{p}}) \) \( (a_i \text{ — integers that can take the values } 0, 1, 2, 3 \text{ or } 4, \overline{p} \text{ is the number of groups, for example, for } \overline{p} = 3, \text{ the vector } K^I_{000} = K^I_{IV} = (k^I_{11}, k^I_{22}, k^I_{33}) ). \) Put \( k' = a_1 + \cdots + a_\overline{p} = 1, \) and for all \( a_i \text{ satisfying the above requirement, calculate the quantities:} \)

\[
H_{a_1...a_\overline{p}} = \prod_{l=1}^{3} c_{(i_l+m_l),j_l} = c_{(i_1+m_1),j_1} * c_{(i_2+m_2),j_2} * \cdots * c_{(i_\overline{p}+m_\overline{p}),j_\overline{p}}
\]  

(9)

Step 5. If one of the quantities \( H_{a_1...a_\overline{p}} > \overline{H} \), then we put \( K^I_{IV*} = (k^I_{IV*}, \ldots, k^I_{IV*}) = (S_{i_1+a_1}, \ldots, S_{\overline{p}+a_\overline{p}}) \), otherwise put \( k' = k' + 1 \) and go to Step 4.

Step 3 of Algorithm 4 assumes the presence of vectors of the third level and is performed in the same way as step 2, with the inverse problem being solved for the desired estimates of each group separately.

Let’s consider in more detail step 4 of Algorithm 4.

All criteria are ranked by the value of the following value:

\[
R_i = s_i * (u_i * d_i + v_i + g_i),
\]  

(10)

where

\[
s_i = \begin{cases} 
1, & \text{if enough answers are received}, \\
0, & \text{otherwise}, 
\end{cases}
\]

\[
u_i = \begin{cases} 
1, & \text{if you can increase all the estimates } f \text{ for the groups under consideration}, \\
0, & \text{otherwise}, 
\end{cases}
\]

\[
v_i = \begin{cases} 
1, & \text{if the evaluation of the criterion for a non-considered group changes}, \\
0, & \text{otherwise}, 
\end{cases}
\]

\[
g_i = \begin{cases} 
1, & \text{if in the groups under consideration there is an estimate } VL, \\
0.5, & \text{if in the groups under consideration there is an estimate } L, \\
0, & \text{otherwise} \end{cases}
\]
\[d_i = k \ast 5 - \sum j_i;\]  
(11)

\[k \text{ — number of groups, } \sum j_i \text{ — sum of the indexes of the evaluation terms in these groups.}\]

In the future, the selection criteria will be in accordance with the list obtained in the ranking.

Let \(K^{III} = (k_1^{III}, k_2^{III}, k_3^{III}), k_j^{III} = S_j \) — the vector of subgroup quality assessments obtained in the direct aggregation process, \(K^{III*} = (k_1^{III*}, k_2^{III*}, k_3^{III*}), k_j^{III*} = S_j*\) — the vector of necessary estimates of subgroups found in the reverse process. Then for each of the subgroups we calculate the quantity:

\[
\Delta = \begin{cases} 
\tilde{j} \ast m - \sum_{i=1}^{5} (r_i \ast i), & \text{if } k_j^{III*} > k_j^{III} \\
\tilde{j}^*, & \text{if } k_j^{III*} = k_j^{III},
\end{cases}
\]

(12)

where

\[
\tilde{j} = \begin{cases} 
(j^* - 0.95) & \text{with a loyal policy of rounding} \\
(j^* - 0.5) & \text{with a neutral rounding policy} \\
j^* & \text{with a strict rounding policy}
\end{cases}
\]

The minimum number of criteria for which you need to shift the estimate to achieve the desired result will be calculated using the formula:

\[k = \max\{\Delta_1, \ldots, \Delta_t\},\]

(13)

where \(\Delta_1, \ldots, \Delta_t\) — rounded up values of \(\Delta\) for each of the subgroups of clients.

In connection with the possibility of obtaining the same values of \(rang_i\) in the ranking, it becomes possible to obtain more than one solution at the end of step 4. The previous steps (with the exception of the first one) are also characterized by many variants of the solution of the problem. In this connection, it becomes necessary to construct a tree of all possible solutions and to choose the optimal one among them. After determining the optimal set of criteria that should be analyzed by the organization in order to improve the evaluation of the quality of customer service, it is suggested to define as recommendations, those sub-criteria of the found criteria that have the greatest value and the lowest satisfaction rating from the point of view of consumers. To obtain an aggregated estimate of the importance of the subcriteria, a linguistic LOWA operator with a quantification function \(Q(x) = x\) is used.

4. Conclusion

The algorithms proposed in the work allow to obtain an integral linguistic estimation of the quality of services from the perspective of clients, and also to implement the reverse process for finding weak positions in the service, the strengthening of which will increase the integral quality assessment. Algorithms are implemented in software, tested on test data and ready for practical use.

References

[1] Chinarian R A 2013 The clients and managing client portfolio 2 82–101
[2] Reicheld F and Stamps R 2013 The Sincere loyalty. The key to winning customers for life (Moscow: Mann, Ivanov and Ferber)
[3] Parasuraman A, Berry L and Zeithaml V 1988 Journal of Retailing 69 197–199
[4] Mill R 2009 To Manage a restaurant: a textbook (Moscow: YUNITI-DATE)
[5] Borisov A N, Alekseyev A V and Krumberg O A 1982 Models of Decision-making on the Basis of a Linguistic Variable (Riga: Zinatne)
[6] Azarnova T V and Titova I A 2012 Modern Economy: Problems and Solutions 9 151–157
[7] Ledeneva T M 2006 *Processing fuzzy information* (Voronezh: Voronezh State University)
[8] Beliakov G, Pradera A and Calvo T 2007 *Aggregation Functions: A Guide for Practitioners* (Berlin: Springer)
[9] Yager R R 1994 *Fuzzy Sets and Systems* 67 129–145
[10] Proskurin D K, Asnina N G, Azarnova T V and Titova I A 2017 *Economics and management control systems* 2.1(24) 172–182