Primary Energy Spectra and Elemental Composition.
GAMMA Experiment

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On the basis of the Extensive Air Shower (EAS) data observed by the GAMMA experiment the energy spectra and elemental composition of the primary cosmic rays have been derived in the $10^3 \div 10^5$ TeV energy range. Reconstruction of the primary energy spectra is carried out in the framework of the SIBYLL and QGSJET interaction models and the hypothesis of the power-law steepening primary energy spectra. The obtained energy spectra of primary $H, He, O, Fe$ nuclei along with the SIBYLL interaction model agree with the corresponding extrapolations of known balloon and satellite data at the $\sim 10^3$ TeV energies. The energy spectra obtained from the QGSJET model, show predominant proton composition of cosmic rays in the knee region. The evident rigidity-dependent behavior of the primary energy spectra for both interaction models are displayed at the following rigidities: $E_R \approx 2400 \div 3000$ TV (SIBYLL) and $E_R \approx 3400 \pm 200$ TV (QGSJET).

Using parametric event-by-event method of the primary energy evaluation by measured $N_{ch}, N_\mu (E_\mu > 5$ GeV, $R < 50m$) and age ($s$) shower parameters, the all-particle energy spectra were obtained. All presented results are derived taking into account the detector response, reconstruction uncertainties of EAS parameters and fluctuation of EAS development.

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I. INTRODUCTION

The investigation of the energy spectra and elemental composition of primary cosmic rays in the knee region ($10^3 \div 10^5$ TeV) remains to be one of the intriguing problems of the modern high energy cosmic-ray physics. Despite the fact that these investigations have been carried out for more than half a century, the data on the elemental primary energy spectra at energies of $E > 10^3$ TeV need improvement. However, a bend of the all-particle energy spectra at around $3 \cdot 10^3$ TeV (called the "knee") at overall spectrum $\sim E^{-2.7}$ until the knee and $\sim E^{-3.1}$ beyond the knee, may be considered as an avowed fact. Moreover, assuming that the supernova explosion is the main source of the cosmic rays, different theoretical models of the high energy cosmic-ray origin and propagation through the Galaxy, predict the rigidity-dependent steepening primary energy spectra in the knee region [1, 2, 3].

High statistical accuracies of the last EAS experiments [4, 5, 6] already allowed us to infer that the rigidity-dependent steepening energy spectra of primary nuclei can approximately describe the observed EAS size spectra in the knee region in the framework of conventional interaction models. However, the accuracies of the obtained elemental primary energy spectra are still insufficient due to both the uncertainty of interaction model and the accuracy of the solutions of the EAS inverse problem.

The Gamma facility (Fig. 1) was designed at the beginning of 90’s in the framework of the ANI experiment [7] and the preliminary results of EAS investigations presented in [8, 9, 10, 11]. The main characteristic features of the GAMMA experiment are the mountain disposition, symmetric location of the EAS detectors and underground muon scintillation carpet that detects EAS muon components at $E_\mu > 5$ GeV energies.

Here, the description of GAMMA facility, EAS inverse approach determining the primary energy spectra using the observed EAS data, and main results of investigation...
during 2002-2004 [10, 11] are presented in comparison with the corresponding MC-simulated data in the framework of the SIBYLL [12] and QGSJET [13] interaction models.

II. GAMMA EXPERIMENT

The GAMMA installation is a ground based array of 33 surface particle detection stations and 150 underground muon detectors located on the south side of Mount Aragats, Armenia. Elevation of the GAMMA facility is 3200 m above sea level, which corresponds to 700 g/cm² of atmospheric depth. The diagrammatic layout is shown in Fig. 1.

The surface stations of the EAS array are located on 5 concentric circles of radii: 20, 28, 50, 70, 100 m and each station contains 3 square plastic scintillation detectors with the following dimensions: 1x1x0.05 m³. Each of the central 9 stations contains an additional (4-th) small scintillator with dimensions 0.3x0.3x0.05 m³ (Fig. 1) for high particle density (> 10² particles/m²) measurements.

The photomultiplier tube is positioned on the top of the aluminum casing covering the scintillator. One of the three station’s detectors is examined by two photomultipliers, one of which is designed for fast-time measurements.

150 underground muon detectors (muon carpet) are compactly arranged in the underground hall under 2.3 Kg/cm² of concrete and rock. The dimensions, casing and applied photomultipliers are the same as in the EAS surface detectors.

A. Detector system and triggering

The output voltage of each photomultiplier is converted into the pulse burst by logarithmic ADC and transmitted to the CAMAC array where the corresponding electronic counters produce a digital number (”code”) of pulses in the burst. Four inner (”trigger”) stations are monitored by a coincidence circuit. If each of at least two scintillators of each trigger station detects more than 3 particles, the information from all detectors are then recorded along with the time between the master trigger pulse and the pulses from all fast-timing detectors. The given trigger condition provides EAS detection with the EAS size threshold Nch > (0.5 ÷ 1) · 10⁵ at the location of the EAS core within the R < 25 m circle.

Before being placed on the scintillation casing, all photomultipliers are tested by a test bench using a luminodiode method where the corresponding parameters of logarithmic ADC and the upper limits ((0.5 ÷ 1) · 10⁴) of the measurement ranges are determined. The number of charged particles (nᵢ) passing through the i-th scintillator is computed using a logarithmic transformation: ln nᵢ = (C - C₀)/d, where the scale parameter d ≃ (9 ÷ 10) ± 0.35 is preliminarily determined by the test bench, C = (0 ± 2⁷ − 1) is an output digital code from the CAMAC array corresponding to the energy deposit of n charged particles into the scintillator, C₀ ≃ (5 ÷ 6) ± 0.25 is determined for each hour of run and is equal to the mode of the background single particle digital code spectra (Fig. 2).

The time delay ∆tᵢ = tᵢ − t₁ of each i-th (i = 2, . . . , 33) fast timing detector is estimated by the pair-delay method [14] at the resolution time about 4 ÷ 5 ns.

B. Reconstruction of EAS parameters

EAS zenith angle (θ) is estimated on the basis of measured shower front arrival times by 33 fast-timing surface detectors, applying the maximum likelihood method and flat-front approach [14, 15]. The corresponding uncertainties are tested by MC simulations and is equal to: σ(θ) ≃ 1.5⁰.

The reconstruction of the EAS size (Nch), shower age (s) and core coordinates (x₀, y₀) are performed based on the NKG approximation of measured charged particle densities {nᵢ, i = 1, . . . , m} using the χ² minimization to estimate x₀, y₀ and the maximum likelihood method to estimate the Nch taking into account the measurement errors. The logarithmic transformation L(nᵢ) = ln nᵢ − (1/m) ∑ ln nᵢ at nᵢ ≠ 0, allows to obtain the analytical solution for the EAS age parameter (s) at the χ² minimization [14, 15]. Nearby (< 5%) estimations of Nch, s, x₀, y₀ shower parameters are obtained at Nch > 5 · 10⁵, 0.3 < s < 1.6, and R < 25 m from the shower core to the center of the EAS array distances.

Corresponding accuracies are derived from MC simulations by the CORSIKA(EGS) [17] and are equal to: ∆Nch/Nch ≃ 0.1, ∆s ≃ 0.05, ∆x ≃ ∆y ≃ ± 0.5 ± 1 m. The reconstruction of the total number of EAS muons (Nµ) by the detected muon densities {nµ,j, j = 1, . . . , 150} from the underground muon hall is carried out by restricting the distance Rₘ < 50 m from the shower core (so-called ”truncated” EAS muon size [20]) and Greisen approximation of the muon lateral distribution function: ρµ(r) = cNµ(R < 50m) exp (−r/r₀)/(r/r₀)⁰.⁷, where r₀ = 80 m, c = 1/2π ∫₀⁻⁵₀ ρ(r)rdr. The truncated muon size Nµ(R < 50m) is estimated at known (from the EAS surface array) shower core coordinates in the underground muon hall. The unbiased estimations for muon size are obtained at Nµ > 10⁵ using the maximum likelihood method and assuming Poisson fluctuations of detected muon numbers. The reconstruction accuracies of the truncated muon size are equal to ∆Nµ/Nµ ≃ 0.2 ÷ 0.35 at Nµ ≃ 10⁵ ÷ 10⁶ respectively.

Notice, that the detected muons in the underground hall are always accompanied by the electron-positron equilibrium spectrum which is produced when muons pass through the matter (2300 g/cm²) over the scintillation carpet. Since this spectrum depends on the muon energy (∼ ln Eµ), overestimations (∼ 25 %) of the reconstructed muon size have to depend on the primary energy.
III. EAS SIMULATION

A. Key assumptions

All observed quantities \( \frac{\Delta F}{\Delta \hat{q}_u} \) in the high energy EAS physics are obtained via convolutions of the energy spectra \( dI_A/dE \) of primary nuclei \( A \equiv H, He, O, Fe \) at least up to \( \sim 5 \cdot 10^5 \) TeV energy range. The spectral index \(-1.5\) was chosen to provide high statistical accuracies of the simulated data beyond the knee.

The resulting energy deposit in the scintillator is converted to the ADC code and inverse decoded into a number of "detected" charge particles taking into account all uncertainties of the ADC parameters \( C_0, d \) and the fluctuation of the light collected by the photomultiplier \( \sigma_t \simeq 0.25/\sqrt{n} \).

Using the simulation scenario above, 100 EAS events were simultaneously simulated by the CORSIKA routine at the EGS and NKG modes for \( A \equiv H, He, O, Fe \) primary nuclei at log-uniform energy spectra in the \( 5 \cdot 10^2 \div 5 \cdot 10^5 \) TeV energy range. The computations of the charged particle densities in the surface detectors at NKG mode of the CORSIKA were performed by applying the two-dimensional interpolations of the corresponding particle density matrix from the CORSIKA routine.

The agreement \( \sim 5\% \) of the EGS and NKG simulated data was attained at the \( E_e \simeq 1 \pm 1 \) MeV kinetic energy threshold of the EAS electrons (positrons) at NKG mode (input parameter of CORSIKA code). However,
the energy threshold for the detection of a vertical single minimal ionizing background particle by scintillation counters is about 8 ± 9 MeV and differences obtained by the CORSIKA NKG prediction are completely explained by contribution of EAS γ-quanta.

Thus, the EAS simulations by the CORSIKA with fast computation NKG mode at \( E_\gamma > 1 \text{ MeV} \) is adequate to the EAS simulation by the EGS mode taking into account the EAS γ-quanta and peculiarity of the GAMMA surface array.

All EAS muons with energies of \( E_\mu > 4 \text{ GeV} \) on the GAMMA observation level have passed through the 2.3 Kg/cm\(^2\) of rock to the muon scintillation carpet (the underground muon hall). Fluctuations of the muon ionization losses and electron (positron) accompaniment due to the muon bremsstrahlung, direct pair production, knock on and photo-nuclear interactions are taken into account. The transformation of the energy deposit to the number of detected muons is performed the same way as for the surface detectors.

The EAS simulations were performed at \( 4.5 \times 10^4 \) primary \( H, 4.3 \times 10^4 \) \( He, 2.4 \times 10^4 \) \( O, 2.4 \times 10^4 \) \( Fe \) nuclei using the CORSIKA NKG routine at the SIBYLL interaction model. Corresponding statistics at the QGSJET interaction model were: \( 4.1 \times 10^4, 4.2 \times 10^4, 2.1 \times 10^4, 2.1 \times 10^4 \). The energy thresholds of primary nuclei were the same for both interaction models and were set 0.5, 0.7, 1, 1.2 PeV respectively at \( 5 \times 10^3 \) PeV upper energy limit.

**IV. MEASUREMENT ERRORS AND DENSITY SPECTRA**

The close disposition of \( k = 1, 2, 3 \) scintillators in each of the \( i \)-th detector station of the GAMMA surface array allows to auto-calibrate the measurement error by detected EAS data. The measured and simulated particle density divergences \( (n_k - \rho)/\rho \) versus average value \( \rho = (1/3) \sum n_k \) at \( R_i > 10 \text{ m} \) distances from shower core are shown in Fig. 2 (circle symbols). The obtained dependences are completely determined by Poisson fluctuations (at \( R_i \gg 1 \text{ m} \) ) and measurement errors.

The agreement of the measured and simulated dependences allowed to extract the real measurement errors of the GAMMA detectors. In Fig. 2 the corresponding results are shown (square symbols).

The background single particle spectra (in the units of ADC code) detected by GAMMA surface scintillators for 78 sec operation time are shown in Fig. 3 (dotted lines). The background single particle spectra detected by underground muon scintillators have the same shape at about 10 times less intensities.

These spectra are used for the operative (each hour) determination of ADC parameters \( C_0 \) during an experiment. The symbols and solid lines in Fig. 3 display the corresponding expected spectra obtained by MC-simulation taking into account the measurement errors (symbols) and without errors (line) respectively. The minimal primary energy in simulation of the background particle spectra was confined to the 7.6 GV primary particle's geomagnetic rigidity.

Because the effective primary energies responsible for the single particle spectra at observation level 700 g/cm\(^2\) are about \( 100 \text{ GeV} \) and the energy range is studied by direct measurements in the balloon and satellite experiments, the primary energy spectra and elemental composition at MC-simulation were taken from approximations [21]. Notice, that the expected single particle spectra at these energies are practically the same for QGSJET and SIBYLL interaction models.

Fig. 4a,b (symbols) display the charged particle density
spectra detected by the corresponding surface detectors (a) and underground muon detectors (b) at $R_i < 50$ m and different EAS size thresholds: $N_{ch} > 5 \cdot 10^5$, $N_{ch} > 10^7$ (and additionally $N_{ch} > 2 \cdot 10^6$ for muon density spectra).

The showers were selected at $\theta < 30^\circ$ and the shower core location in the $R < 25$ m range from center of the GAMMA facility (Fig. 1). The corresponding expected spectra (lines) at different interaction models are also shown in Fig. 4. The primary energy spectra and elemental composition at MC-simulations were taken from EAS inverse problem solution (see below). There is a good agreement of the expected and observed data for the surface array in the measurement range (about four orders of magnitude). However, the agreement of the detected muon density spectra with expected ones is attained only in the $N_{ch} < 10^7$ range. The observed discrepancies for the muon density spectra at $N_{ch} > 10^7$ are unaccounted for the present and demand subsequent investigations.

V. EAS DATA

The main EAS data of the GAMMA experiment are shown in Fig. 5-10 (symbols). These results were obtained at the $6.19 \cdot 10^7$ sec operation time and following selection criteria: $N_{ch} > 5 \cdot 10^5$, $R < 25$ m, $\theta < 30^\circ$, $0.3 < s < 1.6$. All the lines and shaded areas in Fig. 5-10 correspond to the expected spectra according to the QGSJET and SIBYLL interaction models.

The EAS size spectra $(N_{ch}^{2.5} \cdot dF(\theta)/dN_{ch})$ at 3 zenith angular intervals are shown in Fig. 6. These spectra normalized to the EAS intensity at $N_{ch} > 5 \cdot 10^5$ and $\theta < 30^\circ$. The EAS size spectra at $\theta < 30^\circ$ and different thresholds of the truncated EAS muon size are shown in Fig. 7. The normalized EAS truncated muon size spectra at different EAS size thresholds are shown in Fig. 8. Fig. 9 displays the average EAS age parameter dependence on EAS size. The lines are the expected dependences according to QGSJET (dotted line) and SIBYLL (solid line) models. The obtained $N_{mu}(N_{ch})$ dependences and corresponding expected values at the primary Hydrogen, Iron and mixed compositions computed in the frame of the SIBYLL and QGSJET interaction models are plotted on Fig. 10.

VI. EAS INVERSE PROBLEM AND PRIMARY ENERGY SPECTRA

A. Combined approximations of EAS data

Direct computations of the expected EAS spectra using the integral expression (1) is possible only in the framework of a given interaction model and known primary energy spectra. Moreover, the Gamma data shown in Fig. 4-10 may only formally compare with the same data obtained by other EAS experiments performed at both similar atmospheric depths and depths close to the sea level. The correct comparison is possible only at known primary energy spectra and known interaction model because both transformation of the detected EAS spectra to the spectra at a given observation level and the extrapolation of the obtained spectra to another atmosphere depth in a general case are folded by the integral expressions similar to (1).

In such case the more reliable way to interpret the experimental data is to unfold the integral expression (1) at a given interaction model. As a criterion of the validity of the solutions, the $\chi^2$ test of the detected and expected data may be performed. The agreement between the obtained energy spectra at different primary nuclei and the corresponding extrapolations of known balloon and satellite data to the given measurement range will also validate the solutions.

Evidently, the accuracies of the unfolding of expression (1) depend not only on number of measurement points (bins) and different measured spectra but also on the wealth of information about the primary energy spectra and the interaction model involved in the given measured EAS spectra. The amount of information contained in the expression (1) reveals itself via stability and uncertainties of the solutions.

It is shown in Ref. [24], that the EAS size spectra and EAS truncated muon size spectra at three zenith angular intervals allow to reliably unfold expression (1) at a given interaction model for not more than 2 kinds of primary nuclei. The unreliability of solutions of (1) for 4 kinds of primary nuclei was shown in Ref. [27], as well.

Taking into account the above, we used the parameterization of the integral equation (1) similar to Ref. [22]. The solutions for the primary energy spectra in (1) were sought based on the theoretically known power-law function with the “knee” at the rigidity-dependent energies $E_k(A) = E_R \cdot Z$ and the same indices $(-\gamma_1)$ and $(-\gamma_2)$ before and after the knee respectively, for all kinds of primary nuclei ($A$):

$$\frac{dI_A}{dE} \equiv \Phi_A E_{-\gamma_1} \left( \frac{E}{E_k} \right)^{-\gamma}$$

(2)

where $\gamma = \gamma_1 \equiv 2.65$ at $E \leq E_k(A)$, $\gamma = \gamma_2$ at $E > E_k(A)$, $E_R$ is particle’s rigidity and $Z$ is a charge of a nucleus.

Thus, the integral equation (1) is transformed into a parametric equation with unknown spectral parameters: $\Phi(A), E_k(A), \gamma_2$, which are determined by minimization of $\chi^2$ function:

$$\chi^2 = \frac{1}{U} \sum \sum \frac{(\zeta_{u,v} - \xi_{u,v})^2}{\sigma^2(\zeta_{u,v}) + \sigma^2(\xi_{u,v})}$$

(3)

where $U$ is the number of examined functions $\zeta_{u,v} \equiv \Delta F_u/\Delta q_{u,v}$ (Fig. 5-10, symbols) obtained from the experiment with statistical accuracies $\sigma(\zeta_{u,v})$...
FIG. 4: Detected (symbols) and expected (lines) particle density spectra of surface scintillators (left panel, a) and underground muon scintillators (right panel, b).

FIG. 5: EAS size spectra at 3 zenith angular intervals (symbols) and corresponding expected spectra according to the SIBYLL (shaded area) and QGSJET interaction models (lines).

FIG. 6: Normalized EAS truncated muon size spectra at 3 zenith angular intervals (symbols). The lines correspond to the expected spectra at the SIBYLL (solid) and QGSJET (dashed) interaction models.

at \( v = 1, \ldots, V_u \) measured points (bins), and \( \xi_{u,v} \) and \( \sigma(\xi_{u,v}) \) are the corresponding expected values of the examined data set.

Using the aforementioned formalism and \( U = 6 \) 2-dimensional examined functions from Fig. 5-8 (symbols) and 1-dimensional functions from Fig. 9,10, the unknown spectral parameters \( \Phi(A), E_k(A), \gamma_2 \) were derived by the minimization of \( \chi^2 \) function (3) at \( \gamma_1 = 2.65 \) and the degree of freedom \( \sum V_u \simeq 350 \).

The values of spectral parameters (2) obtained by the solution of the parameterized equation (1) are presented in Table 1 at the QGSJET and SIBYLL interaction models. The derived primary energy spectra for \( p, He, O, Fe \) nuclei are shown in Fig. 11 (shaded areas) in comparison with the KASCADE data (symbols) from [25]. The expected spectra conforming the examined data set according to the solutions above are shown in Fig. 5-10 (lines and shaded area) for the QGSJET and SIBYLL interaction models. It is necessary to note, that the obtained results in the framework of the SIBYLL interaction model are more consistent and slightly dependent on a number of examined functions.
B. 4-Dimensional approach

The combination of 1,2-dimensional approximations of EAS data above does not take into account all the information about primary energy spectra folded in the detected EAS data. In general, the EAS inverse problem can be formulated in the multidimensional space of EAS parameters. In case of the 4-parametric ($N_{ch}, N_{\mu}, s, \theta$) analysis, the expression (1) is written as:

$$\Delta F = \frac{1}{C} \sum_A \int_E \frac{dI_A}{dE} \int_Q \int_D G_A(E, \theta) \times R(\theta)d\theta dN_{\mu}dN_{ch}d\Delta \Omega,$$

where $G_A(E, \theta) = \partial^3 W_A(E, \theta)/\partial N_{ch} \partial N_{\mu} \partial s$, are the multidimensional differential EAS spectra at given $A, E, \theta$ parameters of the primary nucleus, $R(\theta) = \partial^3 R(\theta)/\partial N_{ch} \partial N_{\mu} \partial s$ are the error functions of the experiment. The parameters with a tilde symbol are the
reconstructed values of corresponding EAS parameters. Evidently, the amount of information about primary energy spectra contained in the detected multidimensional spectrum ∆F is always greater than the cumulative amount of information contained in the 1,2-dimensional spectra ∆F_u/∆q_u, q_u = N_{ch}, N_μ, s of the expression (1). The difference is determined by the inter-correlations of EAS parameters that are taken into account in the expression (4).

On the basis of the EAS data set of the GAMMA experiment, the simulated EAS database (section III) and parameterization (2), the equations (4) were resolved by the χ²-minimization method. The computations were performed at the following bin dimensions: ∆ ln N_{ch} = 0.15, ∆ ln N_μ = 0.25, ∆ sec θ = 0.05 and ∆ s = 0.15 on the left and right hand side of s* = 0.85 and ∆ s = 0.3 in other cases. The total number of the degree of freedom at 4-dimensional χ²-minimization was equal to 1560. The values of spectral parameters (2) obtained by the solution of the parameterized equation (4) are presented in Table 2 at the QGSJET and SIBYLL interaction models. As it is seen from Fig. 11 and Tables 1,2, the derived expected primary energy spectra significantly depend on interaction model. The expected abundance of primary nuclei at energy E = 10^3 TeV in the framework of SIBYLL model agrees well with corresponding extrapolations of the balloon and satellite data [21], whereas the predictions according to the QGSJET model point out to a predominantly proton primary composition in the 10^3 ÷ 10^5 TeV energy range.

VII. EVENT-BY-EVENT ANALYSIS

The mountain location of the GAMMA experiment and the agreements of observed and simulated data in the measurement range 5 ÷ 10^5 ≤ N_{ch} < 10^7 (Fig. 4-10) allowed, apart from above, to obtain the all-particle energy spectra with high reliability. The method is based on an event-by-event evaluation of primary energy using the reconstructed parameters N_{ch}, N_μ, s, θ of detected

| Parameters | SIBYLL | QGSJET |
|------------|--------|--------|
| Φ_H       | 0.081 ± 0.004 | 0.164 ± 0.004 |
| Φ_{He}    | 0.072 ± 0.008 | 0.005 ± 0.008 |
| Φ_O       | 0.028 ± 0.008 | 0.005 ± 0.006 |
| Φ_{Fe}    | 0.028 ± 0.003 | 0.018 ± 0.003 |
| E_R       | 2560 ± 200  | 3400 ± 150  |
| a_γ_1     | 2.65      | 2.65      |
| γ_2       | 3.21 ± 0.04 | 3.10 ± 0.03 |
| χ²        | 2.5       | 2.6       |

*Parameter was fixed.

FIG. 11: Energy spectra and abundance of the primary nuclei (shaded areas) at the SIBYLL (left panel) and QGSJET (right panel) interaction models. The symbols are the KASCADE data from [25].
EAS. Such possibilities have been studying for a long time in different papers \[10, 29, 30\] and the main difficulty was to obtain an unbiased energy estimation at an existent abundance of the primary nuclei taking into account the fluctuations of shower development and detector response.

Using the simulated database, \( J = 1.5 \cdot 10^4 \) EAS events were taken for each of \( k = 1, \ldots, 4 \) kinds \((H, He, O, Fe)\) of primary nuclei and each interaction model \((\text{SIBYLL, QGSJET})\). The reconstructed \( N_{ch}, \tilde{N}_\mu, \tilde{s} \) shower parameters, known zenith angle \( \theta \) and primary energy \( E_0 \) were used at minimization

\[
\chi^2(a_1, \ldots, a_p) = \frac{1}{4J} \sum_{k=1}^4 \sum_{j=1}^J \frac{(\ln E_{1,k,j} - \ln E_{0,k,j})^2}{\sigma_E^2} \tag{5}
\]

where \( E_1 = f(a_1, \ldots, a_p; \tilde{N}_{ch}, \tilde{N}_\mu, \tilde{s}, \theta) \) is the investigated parametric function with \( a_1, \ldots, a_p \) parameters, \( \sigma_E \) is expected accuracy of the \( E_1 \) evaluated energy. The best estimations were achieved at 7-parametric \( (p = 7) \) fit:

\[
\ln E_1 = a_1 x + \frac{a_2 y}{c} + a_3 + a_4 c + a_5 e^y + \frac{a_6}{(x - a_7 y)}, \tag{6}
\]

where \( x = \ln \tilde{N}_{ch}, y = \ln \tilde{N}_\mu(R < 50m), c = \cos \theta \) and energy \( E_1 \) has units of GeV. The values of the \( a_1, \ldots, a_7 \) parameters for both interaction models and the corresponding \( \chi^2 \) obtained from (5) at \( \sigma_E = 0.15 \) are displayed in Table 3. The root mean square deviations of the energy estimation by 7-parametric fit (4) in the framework of the SIBYLL model is shown in Fig. 12. The corresponding results at three (only \( x, \tilde{s} \) variables) and 4-parametric \((x, \tilde{s}, \cos \theta)\) fit are shown in Fig. 12 as well.

The obtained error distributions estimating primary energy by 7-parametric approximation (6), are shown in Fig. 13 for \( H, He, O, Fe \) nuclei. The red line corresponds to the Gaussian distribution at the same parameters as cumulative distribution (black solid line).

The all-particle energy spectrum derived on the basis of the GAMMA 2002-2004 EAS data set and fit (6), at the QGSJET (filled red square symbols) and SIBYLL (filled blue circle symbols) interaction models are shown in Fig. 14.

Notice, that the energy spectrum obtained by event-by-event method claims additional corrections, because the errors \( \sigma_E = \sigma(\Delta E/E) \) and power-law energy spectra \((\sim E^{-\gamma})\) lead to an overestimation of the spectrum \( \eta = \exp((\gamma - 1)\sigma_E)^2/2) \) times. Moreover, the inevitable biases of energy estimations \( \epsilon(A) = \langle E_1/E_0 \rangle > 1 \) impossible to take into account without information about abundance of primary nuclei.

The observed biases of 7-parametric fit (6) are distributed from \( \epsilon(p) \approx 1.02\% \) up to \( \epsilon(Fe) \approx 0.96\% \) (Fig. 13) and here are neglected. In the results shown in Fig. 14, the corrections of \( \eta(E) \) are taken into account using the expected accuracies from Fig. 12.

\[\begin{array}{cccccccc}
\text{Model} & a_1 & a_2 & a_3 & a_4 & a_5 & a_6 & a_7 & \chi^2 \\
\hline
\text{SIBYLL} & 1.03 & -3.98 & -4.3 & 2.01 & -1.2 & 11.5 & 0.94 & 0.85 \\
\text{QGSJET} & 1.03 & 4.38 & -4.6 & 2.35 & -1.3 & 11.5 & 0.96 & 0.94
\end{array}\]

FIG. 12: Accuracy (RMSD) of the primary energy evaluations at different number of approximation parameters.

FIG. 13: Distribution of errors of the primary energy estimation by event-by-event 7-parametric fit at different primary nuclei.
event-by-event analysis of the GAMMA data at the QGSJET interaction model using \( \alpha \)-parametric method also shown in Fig. 14 (asterisk symbols). The dotted line in Fig. 14 represents the parametrized solutions of the EAS inverse problem for the KASCADE EAS data at rigidity-dependent steepening primary energy spectra. The results of KASCADE02 in Fig. 14 obtained by the non-parametric event-by-event analysis was taken from review. The KASCADE01,05 data obtained by the iterative method of unfolding of the EAS inverse problem were taken from respectively.

VIII. CONCLUSION

The self-consistency of results (Fig. 2-10) obtained by GAMMA experiment at least up to \( N_{ch} \approx 10^7 \) and corresponding predictions in the framework of hypothesis of the rigidity-dependent steepening primary energy spectra and validity of the SIBYLL or QGSJET interaction models point towards:

- The anomalous behavior of the EAS muon spectra (overestimation, Fig. 4b,8,10) and EAS age parameter at EAS size \( N_{ch} > 10^7 \). The same behavior of the EAS age parameter had been observed also in.

- The obtained abundances and energy spectra of primary \( p, He, O, Fe \) nuclei depend on interaction models. The SIBYLL interaction model is more preferable in terms of the extrapolation of the derived expected primary spectra (Fig. 11) to the energy range of the direct measurements.

- The rigidity-dependent steepening energy spectra of primary nuclei describe the EAS data of the GAMMA experiment at least up to \( N_{ch} \approx 10^7 \) with average accuracy < 10% at particle’s magnetic rigidity \( E_R \approx 2400 \div 3000 \text{ TV (SIBYLL)} \) and \( E_R \approx 3300 \pm 200 \text{ TV (QGSJET)} \).

- The 4-dimensional approach at the EAS inverse problem solution is more preferable in terms of the stability and accuracies of solutions.

- The obtained all-particle energy spectra slightly depend on interaction model and are practically the same at both the event-by-event reconstruction method and the EAS inverse approach.

The obtained energy spectra of primary nuclei \( A \equiv p, He, O, Fe \) in the energy range \( 10^6 < E_A < 5 \cdot 10^7 \text{ GeV} \) disagree (see Fig. 11) with the same KASCADE data obtained by the iterative method. However, the discrepancies of all-particle energy spectra (see Fig. 14) obtained by the GAMMA and KASCADE experiments are sufficiently small (~ 20%).

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