The $g_T(x)$ contribution to single spin asymmetry in SIDIS

Abhiram Kaushik
University of Zagreb

Benić, Hatta, Li, AK Phys. Rev. D 104 (2021) 9, 094027

DIS 2022, Santiago de Compostela, Spain, May 2-6, 2022
Single-Spin Asymmetries

- Collisions involving transversely polarised hadrons - left-right asymmetry in particle production
- Observed in various \( pp \) and \( ep \) processes since the 70s

\[
A_N = \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow}
\]
Single-Spin Asymmetries

- QCD is time reversal invariant
- SSAs are T-odd:

\[ A_N \propto S \cdot (P \times k) \]

An phase (interference) is required!

- Quest for theoretical description of SSAs is a quest for sources of a complex phase
Theoretical description of SSAs

- In a collinear factorization framework, SSAs have been described in terms of three-parton correlation functions, i.e. twist-3 parton distribution functions (ETQS) functions and twist-3 fragmentation functions.
  
  Efremov, Teryaev, Sov. J. Nucl. Phys. 36, 140 (1982)
  Qiu, Sterman, Phys. Rev. D 59, 014004 (1999)
  Yuan, Zhou Phys. Rev. Lett. 103 (2009) 052001
  Kang, Yuan, Zhou Phys. Lett. B 691 (2010) 243
  Kanazawa, Koike Phys. Rev. D 88 (2013) 074022

- Eg., above hard part can be convolved with ETQS distribution. Complex phase arises when internal propagator goes on-shell. $1/(k^2 + i\epsilon) = \text{PV}(1/k^2) + i\pi\delta(k^2)$

- Twist-3 distributions not very well known.
Theoretical description of SSAs

SSA from $g_T(x)$:

- In an earlier work, my collaborators and D.J. Yang had shown that the twist-3 quark distribution $g_T(x)$ can lead to SSA in SIDIS at the two-loop level.

  Benić, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027

- Imaginary phase arises from when certain internal propagators are cut.

- $g_T$ is a chiral-even transverse-spin dependent contribution to the quark-quark correlator

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS | \bar{\psi}(0) \psi(\lambda n) | PS \rangle = \frac{M_N}{2} \gamma_5 S_T g_T(x) + \ldots$$
SSAs at two loops

• Schematically,

\[ d\sigma^\uparrow - d\sigma^\downarrow \propto \frac{d\Delta\sigma}{dP_{hT}} \sim g_T(x) \otimes H^{(2)} \otimes D_1(z) \]

• Wandzura-Wilczek relation:

\[ g_T(x) = \int_x^1 \frac{dx'}{x'} \Delta q(x') + \text{(genuine twist-3)} \]

• In effect,

\[ \frac{d\Delta\sigma}{dP_{hT}} \sim g_T(x) \otimes H^{(2)} \otimes D_1(z) \sim \Delta q(x) \otimes H^{(2)} \otimes D_1(z) \]

• SSA completely determined in terms of well understood twist-2 distributions!
In this work we,

1. extend the analysis to include gluon-initiated contribution from the twist-3 distribution $\mathcal{G}_3 T$, which is the gluonic counterpart of $g_T$.
   - In analogy with the quark case,
     \[
     \frac{d\Delta \sigma}{dP_{hT}} \sim \mathcal{G}_3 T(x) \otimes H_g^{(2)} \otimes D_1(z) \sim \Delta G(x) \otimes H_g^{(2)} \otimes D_1(z)
     \]

2. present numerical estimates for asymmetry in SIDIS through these mechanisms at COMPASS and EIC.
Why $g_T$ at two loops?

- $g_T$ appears in the hadronic tensor through the collinear expansion of the two-parton correlator $M^{(0)}$.

$$M^{(0)} \sim \hat{\rho} f(x) + M_N \hat{\rho} \gamma_5 (S \cdot n) \Delta q(x) + M_N S_T \gamma_5 g_T(x) + \ldots$$

- $g_T$ appears in correlator with $\gamma_5$. Traces involving $\gamma_5$ produce a factor of $i \implies g_T$ receives no contributions from hard part $S^{(0)}$ at the Born level

Eguchi, Koike, Tanaka Nucl. Phys. B 763, 198, (2007)

- Can receive non-zero contributions beyond Born level.

Benić, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
Gluon initiated contribution $G_{3T}$

Two-gluon correlator in a polarised proton,

$$M^{(0)}_{\alpha \beta} \sim \langle PS_T | F^{n \alpha} W F^{n \beta} | PS_T \rangle \sim xG(x)g_T^{\alpha \beta} + iM_N x \Delta G(x) (S \cdot n) \epsilon^{n r \alpha \beta} + iM_N x G_{3T}(x) \epsilon^{n \alpha \beta S \perp}$$

Ji, Phys. Lett. B 289, 137 (1992)
Hatta, Tanaka, Yoshida, JHEP 02, 003 (2013)

- $G_{3T}$ - transverse spin dependent contribution to the two-gluon correlator.
- WW approximation

$$G_{3T}(x) = \frac{1}{2} \int_x^1 \frac{dx'}{x'} \Delta G(x') + \text{(genuine twist three)}$$
Calculation of hard part

Asymmetry can be written schematically as,

\[ A_{UT}^{\sin(\alpha \phi_h + \beta \phi_S)} \sim \frac{\alpha_s^2}{\alpha_s} \left( xg_T(x) \text{ or } xG_{3T}(x) \right) \otimes H^{(2)} \otimes D_1(z) \sim \alpha_s \frac{x \Delta q \text{ or } x \Delta G}{q(x)} \]

All contributions to \( H^{(2)} \) have a generic 'AMA' structure:

- Each blob represents 2-2 scattering
- Phase arises from cutting internal lines, i.e., regions of loop momentum \( l_2 \) where the two lines go on-shell.
- Potential divergence when \( l_2 \) gluon is collinear to proton cancels out between \( S_{\mu \nu}^{(0)} \) and \( \frac{dS_{\mu \nu}^{(0)}}{dk_T^\alpha} \)
Calculation of hard part

\[
\frac{d^6 \Delta \sigma}{dx_B dQ^2 dz_f dq_T^2 d\phi d\chi} = \frac{\alpha_{em}^2 \alpha_s^2 M_N}{16\pi^2 x_B^2 S_{ep} Q^2} \sum_k A_k S_k \int \frac{dx}{x} \int \frac{dz}{z} \times \delta \left( \frac{q_T^2}{Q^2} - \left(1 - \frac{1}{\hat{x}}\right) \left(1 - \frac{1}{\hat{z}}\right) \right)
\]

\[
\times \sum_f e_f^2 \left[ D_f(z) x^2 \frac{\partial g_{TF}(x)}{\partial x} \Delta \hat{\sigma}_{qq} + D_f(z) x g_{TF}(x) \Delta \hat{\sigma}_{k} + (qg \text{ channel}) + (gq \text{ channel}) \right]
\]
Numerical results

\[
A_{UT}^{\sin(\alpha \phi_h + \beta \phi_S)} = \frac{2 \int_0^{2\pi} d\phi_h d\phi_S \sin(\alpha \phi_h + \beta \phi_S) \left[ d\sigma(\phi_h, \phi_S) - d\sigma(\phi_h, \phi_S + \pi) \right]}{\int_0^{2\pi} d\phi_h d\phi_S \left[ d\sigma(\phi_h, \phi_S) + d\sigma(\phi_h, \phi_S + \pi) \right]},
\]

- Five independent moments including Sivers \(A_{UT}^{\sin(\phi_h - \phi_S)}\) and Collins \(A_{UT}^{\sin(\phi_h + \phi_S)}\)
- Sivers and Collins asymmetry NOT from Sivers and Collins functions.
- \(g_T(x)\) and \(G_3^T\) from helicity distributions using the WW approximation.

\[
g_T(x) = \int_x^1 \frac{dx'}{x'} \Delta q(x'), \quad G_3^T(x) = \int_x^1 \frac{dx'}{x'} \Delta G(x')
\]

- We used the latest fits of helicity distributions from NNPDF and JAM.

Nocera, Ball, Forte, Ridolfi, and Rojo, Nucl. Phys. B 887, 276 (2014)
Ethier, Sato, and Melnitchouk, Phys. Rev. Lett. 119, 132001 (2017)
• Sivers asymmetry up to 2% with JAM at moderate-to-large $x$ and low $Q^2$.
• Decreases at low-$x$. 
• cancellation between \(qq\) and \(qg\) channels
• \(qg\) channel kinematically suppressed at large \(z_f\) — sign change in asymmetry with NNPDF
• JAM has smaller \(g \rightarrow \pi^+\) FF \(\Rightarrow\) less cancellations
• Negligible contribution from gluon-initiated (gq) channel (\(x^2\) suppression).
All $A_{UT}$ moments at EIC

- $\sin(\phi_h - \phi_S)$ (Sivers), $\sin(\phi_S)$ and $\sin(2\phi_h - \phi_S)$ moments are at percent level. Collins negligible.
Sivers and Collins at COMPASS

- Sivers at percent level for $\pi^+$ with JAM. Collins negligible.
- Only available datapoint from COMPASS at $P_{hT} \approx 1.5$ shows positive Sivers asymmetry ($\sim 2.5\%$) but with large errors.

Adolph et al., Phys. Lett. B744, 250 (2015)
Comparison with the KPR estimate

Kane, Pumplin and Repko (1978) presented the first parametric estimate of SSA in pQCD as

\[ A_N \sim \alpha_s \frac{m_q}{P_{hT}} \]

expected to vanish since \( m_q \rightarrow 0 \)

but in this mechanism,

\[ A_N \sim \alpha_s \frac{xM_N}{P_{hT}} \]
Conclusions

Presented numerical results for $g_T$ contribution to SSA at two loops.

- Asymmetry can be calculated entirely in terms of well-known twist-2 distributions.
- Upto 2% Sivers asymmetry at EIC at low-$Q^2$ with $P_{hT} > 1$.
- Asymmetry suppressed at small-$x$.
- Two-loop mechanism needs to be accounted for when constraining other mechanisms (ETQS, twist-3 fragmentation etc.).

Work in progress...

- Calculating similar contributions for SSA in $pp$
- Open-charm in SIDIS - isolates the $G_3T$ contribution.
Appendix
\( g_T \) contribution to SSA

Hadronic tensor in SIDIS

\[ W_{\mu \nu} = \int_z \frac{dz}{z^2} D(z) w_{\mu \nu}, \]

\[ w_{\mu \nu} = \int_k M^{(0)}(k) S^{(0)}_{\mu \nu}(k) + \int_{k_1} \int_{k_2} M^{(1)}_{\sigma}(k_1, k_2) S^{(1)}_{\mu \nu \sigma}(k_1, k_2) \]

- Need to include twist-3 distributions \( \Rightarrow \) consider hadronic tensor upto three parton correlator.

- \( M^{(0)}_{ij} \sim \langle PS_T | \bar{\psi}_j \psi_i | PS_T \rangle \)

- \( M^{(1)}_{ij} \sim \langle PS_T | \bar{\psi}_j g A^\sigma \psi_i | PS_T \rangle \)
**$g_T$ contribution to SSA**

- **Hard part** - expand in collinear limit: $k^\mu = xP^\mu + k_T^\mu$

  $$S^{(0)}_{\mu\nu}(k) = S^{(0)}_{\mu\nu}(xP) + k_T^\alpha \frac{dS^{(0)}_{\mu\nu}}{dk_T^\alpha}(xP)$$

- **Soft part**

  $$M^{(0)} \sim \not{p}f(x) + M_N \not{p}\gamma_5 (S \cdot n) \Delta q(x) + M_N S_T \gamma_5 g_T(x)$$

  + transversity and higher twist terms...

- $g_T$ appears in correlator with $\gamma_5$. Traces involving $\gamma_5$ produce a factor of $i$.

- Hence $g_T$ receives no contributions from $S^{(0)}$ at the Born level.

  
  Eguchi, Koike, Tanaka Nucl. Phys. B 763, 198, (2007)

- Can receive non-zero contributions beyond Born level.

  Benić, Hatta, Li, Yang Phys. Rev. D 100 (2019) 9, 094027
\( g_T \) contribution to SSA

All order and gauge invariant result:

\[ w_{\mu\nu} = \frac{M_N}{2} \int dx g_T(x) \text{Tr} \left[ \gamma_5 S_T^{(0)}(xP) \right] \]

\[ - \frac{M_N}{4} \int dx \tilde{g}(x) \text{Tr} \left[ \gamma_5 \not{\partial} S_T^{(0)}(k) \right] \bigg|_{k=xP} \]

\[ + \frac{iM_N}{4} \int dx_1 dx_2 \text{Tr} \left[ \left( \not{\epsilon} \not{P} S_T^{(0)} \frac{\not{G}_F(x_1, x_2)}{x_1 - x_2} + i\gamma_5 \not{\partial} S_T^{(0)} \frac{\tilde{G}_F(x_1, x_2)}{x_1 - x_2} \right) S_T^{(1)}(x_1 P, x_2 P) \right] \]

- \( \tilde{g}(x) \) is another "kinematic" twist-3 distribution - first moment of worm-gear TMD \( g_1 T \) (talk by Shohini Bhattacharya)

\[ g_T(x) + \frac{\tilde{g}(x)}{2x} = \int dx' \frac{G_F(x, x') + \tilde{G}_F(x, x')}{x - x'} \]

Eguchi, Koike, Tanaka, Nucl. Phys. B 763, 198 (2007)

Neglecting explicit twist-3 contributions (WW approximation),

\[ w_{\mu\nu} \approx \frac{M_N}{2} \int dx g_T(x) S_T^{(0)} \left( \frac{\partial}{\partial k_T^{\alpha}} \text{Tr} \left[ \gamma_5 k S_T^{(0)}(k) \right] \right) \bigg|_{k=xP} \]
Energy dependence

- Percent level Sivers contribution (JAM) at highest EIC energy.