Stochastic System with Colored Noise and Absorbing States:
Path Integral Solution

D. O. Kharchenko∗
Sumy State University, 40007 Sumy, Ukraine

Abstract

The behavior of the most probable values of the order parameter \(x\) and the amplitude \(\phi\) of conjugate force fluctuations is studied for a stochastic system with a colored multiplicative noise with absorbing states. The phase diagrams introduced as dependencies the noise self-correlation time vs temperature and noise growth velocity are defined. It is shown that phase half-plane \((x, \phi)\) can be split into isolated domains of large, intermediate, and small values of \(x\). System behavior in these domains is studied by the probability represented as path integral. In the region \(x \ll 1\), the trajectories converge to the point \(x = \phi = 0\) for \(0 < a < 1/2\) and to \(x = 0, \phi \to \infty\) for \(1/2 < a \leq 1\). In the former case, the probability of realization of trajectories is finite, while in the latter case it is vanishingly small, and an absorbing state can be formed.

A wide class of stochastic systems with multiplicative noise whose intensity is a function of the stochastic variable \(x\) determining the state of the subsystem can exhibit absorbing states. An absorbing configuration is one in which a system can get trapped, from which it can not escape. The most popular example of this kind is the Verchilst model proposed for explaining the population kinetics; the analysis of this model remains a topical question. Another example is the percolation model in which a linear dependence on \(x\) is observed for noise intensity. Indicated models can be generalized by representing the noise intensity as a stochastic variable power dependence with an arbitrary exponent. A kinetic features of stochastic system with that kind noise was studied in the white noise approximation and with accounting inhomogeneity of the space. An evolution of averages in that model was discussed in. A distinguishing feature of our approach is that we shall study the dependencies of most probable values vs system parameters using supposition of the noise correlation. This is realized on the basis of a field scheme in which the probability functional has an exponential form with the exponent that can be reduced (correct to the sign) to the standard action. Consequently, a description of phase transition in terms of the most probable values corresponds to the application of the least action principle. Such an approach makes it possible to extend the self-consistent field method to a description of systems with multiplicative noise. This reveals the following nontrivial peculiarity: the most probable values

∗dikh@ssu.sumy.ua
can tend either to free energy minima or to points corresponding to unstable thermodynamic states depending on the system parameters. The singular nature of noise indicating the tendency of its intensity to zero for the value \( x = 0 \) of the order parameter (more probably magnitude of \( x \)) is responsible for the emergence of a region on the phase portrait in which all trajectories converge to the axis \( x = 0 \) over a finite time. In other words, an absorbing state is formed, in which the system becomes closed [1].

We consider following model. The time dependence of the hydrodynamic mode amplitude \( x(t) \) is determined by the Langevin equation

\[
\dot{x} = f_0(x) + g_0(x)\zeta(t).
\]  

(1)

Here the dot indicates the differentiation with respect to time \( t \), deterministic force \( f_0(x) \) is defined according to the thermodynamic potential \( V_0(x) \) as \( f_0 = -\partial V_0(x)/\partial x \), \( \zeta(t) \) is the colored multiplicative noise with amplitude defined by function \( g_0(x) \). A specific form for the noise \( \zeta(t) \) we choose in kind of Ornstein–Ulenbeck process, i.e.

\[
\tau \dot{\zeta} = -\zeta + \xi(t),
\]  

(2)

where \( \tau \) is a self–correlation time, \( \zeta \) is the Gaussian distributed variable with zero mean and exponentially decaying correlations:

\[
\langle \zeta(t)\zeta(t') \rangle = (1/2\tau) \exp(-|t - t'|/\tau),
\]  

(3)

\( \xi(t) \) is a white noise—namely, Gaussian stochastic variable with zero mean and \( \delta \)–correlated: \( \langle \xi(t)\xi(t') \rangle = \delta(t - t') \). The amplitude \( g_0(x) \) of the multiplicative noise in Eq.(1) is chosen in the simple form

\[
g(x) = |x|^a,
\]  

(4)

where \( a \) is a positive exponent varying from 0 to 1 [3]. We shall describe our system by master parameter, self–correlation time and velocity of noise growth defined by exponent \( a \).

If we take time derivative of Eq.(1), replace first \( \dot{\zeta} \) in terms of \( \zeta \) and \( \xi \) from Eq.(2) and then \( \zeta \) in terms of \( \dot{x} \) and \( x \) from Eq.(1), we obtain the following non–Markovian stochastic differential equation (SDE) (see Ref.[4]):

\[
\tau \left( \ddot{x} - \dot{x}^2 \partial_x g_0(x)/g_0(x) \right) = -\sigma(x)\dot{x} + f_0(x) + g_0(x)\zeta(t),
\]  

(5)

where

\[
\sigma(x) = 1 - \tau f_0(x)\partial_x \ln \left[ \frac{f_0(x)}{g_0(x)} \right]
\]  

(6)

(throughout the paper, the Itô interpretation for the SDE will be meant). In the limit \( \dot{x}^2 = 0 \) a system behavior was described in [10, 11]. Here, following “unified colored noise approximation” Ref.[12] we can recover a Markovian SDE. It needs to use adiabatic elimination (neglecting \( \ddot{x} \)) and to neglect \( \dot{x}^2 \) so that the system’s dynamics be governed by a Fokker–Planck equation. In order to make correct transition to an ordinary, linear in \( \dot{x} \), SDE we have to use Itô’s differential rule [13]. In above mentioned suppositions SDE (5) takes form

\[
\sigma(x)\dot{x} = f_0(x) + g_0(x)\zeta(t).
\]  

(7)
Expression in lhs in Eq.(7) defines time derivative for variable $z$. The relation between $z$ and $x$ is set by expression $dz = \sigma(x)dx$. An equivalent equation to Eq.(7) can be write down as

$$\dot{x} = f(x) + h(x) + g(x)\xi(t),$$

with

$$f(x) = \frac{f_0(x)}{\sigma(x)}, \quad h(x) = -\frac{1}{2} \left( \frac{g_0(x)}{\sigma(x)} \right)^2 \partial_x \ln \sigma(x), \quad g(x) = \frac{g_0(x)}{\sigma(x)}.$$  

Here we stress that drift term $h(x)$ appears only by accounting the noise self-correlation time.

Let us now go over to a new field $y(t)$ connected with the initial field $x(t)$ through the relation

$$dy = g(x(y)) dt.$$  

The new stochastic field taking into account Eq.(10) satisfies the Langevin equation

$$\dot{y} = \tilde{h}(x(y)) + \xi,$$

with additive noise and effective force

$$\tilde{h} \equiv \frac{(f + h)}{g} - \frac{\partial_x g}{2}.\quad (12)$$

The obtained equation (11) allows us to use the standard field scheme [7] based on analysis of the generating functional. The latter has the form of the functional Laplace transform

$$Z \{u(t)\} = \int Z \{y(t)\} \exp \left( \int u(y) dt \right) Dy(t)$$  

for the partition function

$$Z \{y(t)\} = \left\langle \prod_t \delta \left\{ \dot{y} - \tilde{h} - \xi \right\} \det \left| \frac{\delta \xi}{\delta y} \right|_\xi \right\rangle,$$  

where $Dy$ denotes integration over all paths starting at $y(0)$ for $t = 0$ and ending at $y(t_f)$ for $t = t_f$. It is defined as

$$Dy = \lim_{N \to \infty} \frac{N^{N-1}}{\epsilon} \prod_{i=1}^{N-1} dy(t_i),$$

where $y(t_i)$ is a field at time $t_i = i\epsilon$, having sliced the interval 0 to $t_f$ in $N$ parts of size $\epsilon = t_f/N$. The argument of the $\delta$-function in Eq.(14) can be reduced to the Langevin equation (13), and the determinant ensuring a transition from continual integration with respect to $\xi(t)$ to $y(t)$ is equal to unity in Itô calculus.

In the approach developed by Zinn-Justin [7], $n$-fold variation of functional (13) over the auxiliary field $u(t)$ makes it possible to find the $n$-th order correlator for the hydrodynamic mode amplitude $y(t)$ and to construct the perturbation theory. We shall proceed, however, from expression (14) for the conjugate functional $Z\{y(t)\}$ whose variation leads to the most probable realization of the stochastic field $y(t)$.

Going over to an analysis of functional (14), we write the $\delta$-function in integral form

$$\delta \{y(t)\} = \int_{-i\infty}^{i\infty} \exp \left( - \int qy dt \right) Dq.$$  

3
Averaging over the noise $\xi$ with the help of the Gaussian distribution

$$P_0\{\xi\} \propto \exp\left\{-\frac{1}{2} \int \xi^2(t)dt\right\},$$

(17)
corresponding to (3) and taking into account Eq.(16), we reduce functional Eq.(14) to the standard form

$$Z\{y(t)\} = \int P\{y(t),q(t)\}Dq, \quad P \equiv e^{-S}.$$ 

(18)

Here the probability distribution $P\{y,q\}$ is specified by the action $S = \int Ldt$, where the Lagrangian is given by

$$L(y,q) = q\left(\dot{y} - \tilde{h}\right) - q^2/2.$$ 

(19)

We shall use in subsequent analysis the Euler equations

$$\frac{\partial L}{\partial z} - \frac{d}{dt}\frac{\partial L}{\partial \dot{z}} = \frac{\partial R}{\partial \dot{z}}, \quad z \equiv \{y,q\},$$

(20)
in which the dissipative function is defined as

$$R(y) = \dot{y}^2/2.$$ 

(21)

As a result, the equations for the most probable realizations of the stochastic fields $y(t)$ and $q(t)$ assume the form

$$\dot{y} = \tilde{h} + q,$$ 

(22)

$$\dot{q} = -q\left(1 + \partial_x\tilde{h}\right) - \tilde{h}.$$ 

(23)

A comparison of Eq.(22) with the stochastic equation (11) having the same form shows that the fields $y(t)$ and $q(t)$ are the most probable values of amplitudes of the auxiliary hydrodynamic mode defined by relation (10) as well as by fluctuation of the conjugate force. Obviously, the latter can be reduced to the conjugate momentum $q = \partial L/\partial \dot{y}.$

In order to return to the initial stochastic field $x(t)$, we shall use the relation (10) and definition (12). As a result, the Lagrangian (19) and the dissipative function (21) assume the form

$$L(x,\phi) = \phi(\dot{x} - f - h + g\partial_x g/2) - g^2\phi^2/2,$$ 

(24)

$$R(x) = \dot{x}^2/2g^2,$$ 

(25)

where we have used the definition of the conjugate momentum $\phi = \partial L/\partial \dot{x}$ leading to the relation

$$\phi = q/g.$$ 

(26)

In this case, the Euler equations (20) assume the form

$$\dot{x} = f + h - g\partial_x g/2 + g^2\phi,$$ 

(27)

$$\dot{\phi} = -\phi\left[1 + \partial_x f + \partial_x h - \partial_x(g\partial_x g)/2 + \phi g\partial_x g\right] - (f + h)/g^2 + \partial_x g/2g.$$ 

(28)
Similar equations can be obtained directly from the system (22), (23) using the relations (10) and (26) as well as definition (12).

In order to calculate noise self–correlation time influence on the system behavior we shall use the $x^4$-model for potential

$$V_0(x) = \frac{\varepsilon}{2} x^2 + \frac{1}{4} x^4, \quad \varepsilon \in [-1, 1]$$  \hspace{1cm} (29)

and the definition (3) of multiplicative function. Since the Lagrangian (24) does not change its form upon simultaneous reversal of the signs of $x$ and $\phi$, the phase portraits will possess central symmetry relative to the origin $x = \phi = 0$. On the other hand, the axis $x = 0$ on which the noise intensity assumes zero value is singular in accordance with (4). For this reason, we can confine our analysis only to the upper part of the phase plane corresponding to the value of $x > 0$.

First of all let us investigate steady states of the system. In the stationary case $\dot{x} = \dot{\phi} = 0$, we have two equations

$$\phi = -\frac{\sigma f_0}{g_0} + \frac{1}{2} \partial_x \ln g_0,$$  \hspace{1cm} (30)

$$\phi \left\{ \frac{\sigma f_0}{g_0} \partial_x \ln \left[ \frac{f_0}{g_0} \right] + \frac{1}{2} (\partial_x \ln \sigma) \partial_x \ln g_0 - \frac{\partial_x g_0}{2g_0} \right\} = 0.$$  \hspace{1cm} (31)

From Eq.(30) it follows that at noise self–correlation time not exceeding the value $\tau_0$ (Fig.1) the form of the phase portrait is characterized by the presence of a single saddle point $S$ whose position is defined by the solution of Eq.(31) and corresponds to $\phi \neq 0$. Figure 2 shows that bifurcation takes place at $\phi = 0$ at noise self–correlation time $\tau_0$ (Fig.2a, 2b). This is accompanied by the emergence of an additional saddle point $S$ and an attractive node $C$ whose positions are determined by the condition $\phi = 0$ and specified by the coordinates $x \pm$.

In Fig.3 we show critical magnitude of the noise self–correlation time $\tau_c$ vs temperature $\varepsilon$ (as maximum of the dependence $\tau_0(a)$). As shown in Fig.1, above $\tau_c$ the system is always ordered for any velocity of the noise growth, defined by exponent $a$. Obviously, the saddle point $S$ and the node $C$ merge at the point corresponding to the correlation time $\tau_c$.

The above analysis shows that in systems with $\tau < \tau_0$ the stationary state corresponds to the point $S$ whose coordinates are defined as

$$\phi = -\frac{f + h}{g^2} + \frac{1}{2} \partial_x g,$$  \hspace{1cm} (32)

$$\partial_x (f + h) = -\phi g \partial_x g + \frac{1}{2} \partial_x [g \partial_x g].$$  \hspace{1cm} (33)

The meaning of these coordinates becomes clear if we proceed to the additive limit $g(x) \to 1$: condition (32) indicates that the most probable value $\phi$ of the fluctuation amplitude of conjugate force for homogeneous systems has the sign opposite to that of $f + h$; according to Eq.(33), the “susceptibility” $(\partial_x^2 V)^{-1} = -1/\partial_x (f + h)$ in this case assumes an infinitely large value. Thus, node $S$ corresponds to the stationary state of a thermodynamic system which is unstable to a transition to the ordered phase. Then the phase portrait has the form shown in Fig.4a.
At temperatures $\tau > \tau_0$, the steady state with the coordinates

$$\phi = 0,$$  \hspace{1cm} (34)

$$2\sigma f_0 = g_0 \partial_x g_0.$$  \hspace{1cm} (35)

is formed at the point $C$. In the additive limit, this point corresponds to the state of thermodynamic equilibrium. Figure 4 shows that the corresponding phase portrait can be obtained not only upon an decrease in temperature (see Fig.4a). Here we stress that the system can pass from equilibrium (ordered) domain trough unstable (disordered) to stable if we increase the velocity of the noise growth (see Fig.1 and Fig.4b).

Naturally, a real thermodynamic system in the process of its evolution tends to the equilibrium state Eqs.(34, 35) rather than to the unstable state Eqs.(32, 33). The equilibrium state corresponds to small and large values of the exponent $a$ for any values of the parameter $\tau$. The domain of medium values of the exponent $a$ at small noise correlations corresponds to unstable states.

Comparing Fig.4a, Fig.4b we see that they are distinguished by the location of the attraction node on the axis $\phi = 0$. The general form of phase portraits shown in Fig.4 is characterized by the presence of two separatrices with branches $PQ$ and $MS_0N$. They divide the phase plane into three isolated regions corresponding to large, intermediate, and small values of the order parameter $x$. The first region is characterized by an indefinite increase in the values of the quantities $x$ and $\phi$ with time $t \to \infty$. It will be sown below that this is not realized in actual practice. The region of intermediate values of $x$ in which the system ultimately goes over to a stationary ordered state is most interesting. It is this region that determines the phase transition kinetics. The formation of the region corresponding to values $x \ll 1$ is associated with the multiplicative nature of noise. In this region, the order parameter $x(t)$ tends with time to the value $x = 0$.

Let us analyze the behavior of the system in each of these regions. For this purpose, we consider the probability of realization of a phase trajectory corresponding to different initial values $x_0 \equiv x(t = 0)$. In accordance with (18), the probability can be written in the form

$$P(x_0) \propto \exp \left\{ -\frac{1}{2} \int_{x_0}^2 g_2^2 dt \right\},$$  \hspace{1cm} (36)

where expressions (24) and (27) are taken into account and integration is carried out along the corresponding trajectory. The dependence $P(x_0)$ obtained for an exponent $a < 1/2$ is shown in Fig.5 (curves 1, 2). Apart from the trivial increase in probability (36) which approaches the origin for values of $x_0$ corresponding to separatrices, the jumps near which the value of $P(x_0)$ can increase insignificantly are observed. Outside the region bounded by the (outer) separatrix, we have $P = 0$ since $x(t), \phi(t) \to \pm \infty$ for $t \to \infty$ in this case.

Such a behavior of the probability $P(x_0)$ can be explained by the form of the time dependencies $x(t)$ and $\phi(t)$ during relaxation of the initial value. Far away from the region of $S_0$ (see Fig.4a), the quantities $\phi$ and $x$ rapidly change their values, the change slowing down as we approach this region. Such a behavior can be explained by the fact that the action $S\{x(t), \phi(t)\}$ changes much more slowly near this region than away from it. This can be visualized by associating the region $S_0CS$ with the bed of a large river [15].
For values of the exponent \( a > 1/2 \), the integrand in formula (36) diverges, and the probability \( P \) assumes zero value for \( x_0 \ll 1 \). Typically, this divergence is observed only in the region of phase portrait bounded by the separatrix branch \( S_0O \) in Fig.4b.

In order to explain the form of the dependence \( P(x_0) \), we analyze the behavior of the quantities \( x(t) \) and \( \phi(t) \) for various values of exponent \( a \). For this purpose, we put \( \dot{\phi} = 0 \) in Eq.(38). The obtained quadratic equation gives stationary values of conjugate momentum in the limit \( x \to 0 \): \[
\phi = \begin{cases} 
\frac{1}{2} \left( \frac{1}{2} - a \right)^{-1} x^{1-2a}, & a < \frac{1}{2}, \\
(a - \frac{1}{2})^{-1} x^{-1}, & a > \frac{1}{2}.
\end{cases}
\] (37)

Thus, for \( a < 1/2 \), the system tends with time to the origin \( x = \phi = 0 \), and the attraction node jumps to infinity (\( \phi \to \infty, x = 0 \)) as \( a \) exceeds the critical value \( a = 1/2 \). The corresponding integrand in distribution (38), i.e., \[
g^2 \phi^2 = \begin{cases} 
\frac{1}{2} \left( \frac{1}{2} - a \right)^{-2} x^{2(1-a)}, & a < \frac{1}{2}, \\
(a - \frac{1}{2})^2 x^{-2(1-a)}, & a > \frac{1}{2}.
\end{cases}
\] (38)
is characterized by the sign inversion in the exponent upon a transition through the critical value \( a = 1/2 \). Substituting Eq.(37) for \( a < 1/2 \) into the equation of motion (27) and retaining the leading term in it, we obtain the equation \( 2 \dot{x} = -ax^{2a-1} \) which gives the time dependence of the order parameter:

\[
x^{2(1-a)} = a(1-a)(t_0 - t), \quad t < t_0, \quad a < 1/2,
\] (39)

where \( t_0 \) is the integration constant defining the time during which the point gets to the axis \( x = 0 \). The substitution of dependence Eq.(33) into Eq.(38) at \( a < 1/2 \) and of the obtained expression into the integral in Eq.(36) shows that the probability \( P(x_0) \) of realization of a trajectory in the region \( x \ll 1 \) of the phase portrait differs from zero (see Fig.5, curves 1, 2).

A completely different situation is observed for an exponent \( a > 1/2 \). In this case, Eq.(27) can be reduced to \( 2 \dot{x} = -(1-a)x^{2a-1} \), leading to Eq.(39) in which the factor \( a(1-a) \) is replaced by \((1-a)^2\). However, for \( a > 1/2 \) the expression \( g^2 \phi^2 \) acquires an exponent with the opposite sign in accordance with Eq.(38) so that it assumes an infinitely large value for \( t \to t_0 \). As a result, the probability Eq.(38) becomes vanishingly small (see Fig.5, curves 3, 4). The physical reason behind such a behavior is that the system gets to the axis \( x = 0 \) over a finite time interval \( t_0 < \infty \), which ensures an infinitely large value of the conjugate momentum \( \phi \propto x^{-1} \propto (t_0 - t)^{-1/2(1-a)} \). This can be visualized as the precipitation of condensate (absorbing state) of configuration points from the phase portrait domain \( x \ll 1 \) onto the abscissa axis for \( \phi \to \infty \). Note that the condition \( t_0 < \infty \) is satisfied only below the separatrix branch \( S_0O \) in Fig.4b, while in the region \( NS_0O \) we have \( t_0 = \infty \), and the convergence of the integrand in (38) is not manifested. Consequently, the equality \( P = 0 \) holds only below the curve \( S_0O \) (see Fig.5 and Fig.4b).

In accordance with (33), the time dependence of the most probable magnitude \( x(t) \propto t^{H} \) is defined by the exponent \( H^{-1} = 2(1-a) \) whose magnitude determines the fractional dimension \( D \equiv H^{-1} \) characterizing the domain \( x \ll 1 \) of the phase portrait of a system with multiplicative noise (13). In the additive limit \( a = 0 \), we
have dimension $D = 2$ of the phase plane as expected. This means that as the time $t \to \infty$, the phase trajectories of the system fill the entire phase plane. The increase of the exponent $a > 0$ leads to a decrease in the fractional dimension $D$ which assumes the critical value $D = 1$ for $a = 1/2$. As the value of $a$ increases further, the fractional dimension of the set of points on the plane $x, t$, which is the law of motion $x(t)$, becomes smaller than unity. The physical reason behind such a behavior is the above mentioned absorption of configuration points by the axis $x = 0$ for $\phi \to \infty$.

Finally, it must be stressed that the noise correlation time can play role of master parameter as the temperature and produces transition to the ordered state. Moreover we shown that the increasing of the velocity of the noise intensity growth produces appearance of the domain of disordered states. This domain disappears if noise self-correlation time does not exceed value $\tau_c$. Path integral solution shown that absorbing state appearance is characterized by the vanishingly small probability of the system states realization. Asymptotic time dependence of the order parameter $x$ explains picture of absorbing states by the fractal dimension smaller than unity.

Acknowledgement

I am grateful to prof. A.I.Olemskoi for inspiring discussions and helpful comments.
References

[1] R. Dickman, in: Nonequilibrium Statistical Mechanics in One Dimension, ed. by V. Privman, Cambridge University Press, Cambridge, 1997.

[2] W. Horsthemke and R. Lefever, Noise Induced Transitions. Theory and Application in Physics, Chemistry, and Biology, Springer, Heidelberg, 1984.

[3] O. V. Gerashchenko, S. L. Ginzburg, and M. A. Pustovoit, Pis’ma Zh. Eksp. Teor. Fiz. 67 (1998) 945.

[4] M.A. Munoz, Phys.Rev.E 57 (1998) 1377.

[5] A.I. Olemskoi, D.O. Kharchenko, Fizika Tverdogo Tela 42, N3, (2000) 520.

[6] A.I. Olemskoi, D.O. Kharchenko, e-print cond-mat/9908092

[7] J. Zinn–Justin, Quantum Field Theory and Critical Phenomena, Clarendon Press, Oxford, 1994.

[8] A. I. Olemskoi, Physics-Uspechi 168 (1998) 287.

[9] S.E.Mangioni, R.R.Deza, R.Toral, H.Wio, e-print cond-mat/9908351

[10] V.E. Shapiro, Phys.Rev.E 48 (1993) 109.

[11] D.O. Kharchenko, Ukr.Fiz.Jurn. N5 (1999) 647.

[12] F.Castro, H.S.Wio, G.Abramson, Phys.Rev.E 51, (1995).

[13] N.G. van Kampen, Stochastic Processes in Physics and Chemistry, North–Holland Phys.Publ., 1984

[14] E. M. Lifshitz and L. P. Pitaevskii, Statistical Physics, Part 2 Nauka, Moscow, 1978

[15] A. I. Olemskoi and A. V. Khomenko, Zh.Eksp.Teor.Fiz. 110 (1996) 2144.

[16] A.I. Olemskoi,Fractals in Condensed Matter Physics, in: Physycs Reviews, 18, Part 1, ed. by I.M. Khalatnikov, Gordon & Breach, London, 1996.
FIGURE CAPTIONS

**Fig.1.** Phase diagram indicating appearance of the ordered phase \((x \neq 0)\): noise correlation time \(\tau\) vs exponent \(a\) (curves 1, 2 correspond to \(\varepsilon = 0.65, 0.7\)).

**Fig.2.** Bifurcation diagrams: stationary values of the order parameter vs noise self-correlation time at \(\varepsilon = 0.7\) (\(a = 0.2\) (a); \(a = 0.6\), (b)).

**Fig.3.** Phase diagram indicating the ranges of the parameters \(\tau_c, \varepsilon\).

**Fig.4.** Basic types of phase portraits in the ordered state: \(\varepsilon = 0.7, \tau = 0.4\) (\(a = 0.2\) (a), \(a = 0.8\) (b)). The notation for stationary points is the same as in Fig.2.

**Fig.5.** Dependence of the probability \(P\) of realization of various trajectories on the initial value of the order parameter \(x_0\) for \(\varepsilon = 0.7\) (curves 1, 2 correspond to \(a = 0.2\ \tau = 0.4, 0.6\), curves 3,4 correspond to \(a = 0.8\ \tau = 0.4, 0.6\). The initial value of the conjugate momentum \(\phi_0 = 0.5\).
