A Modified Elastic Plate Model for the Thickness of Reinforced Soil of Shield Tunnel End

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Abstract: The soil body of the shield end could be unstable and should be reinforced, and its thickness affects the stability and economy of engineering. The existing elastic thin plate model was too conservative for design. A modified elastic plate model was proposed for the calculation. Based on elastic finite element analysis data, optimal fitting was conducted for the proposed elastic model, and the coefficients of the model were obtained. A case study showed that the elastic thin plate model is too conservative when used to calculate the thickness of the reinforced soil body. The actual reinforced thickness in practice is between the values of the elastic thin plate model and the modified elastic plate model.

Key words: shield tunnel; soils of tunnel end; reinforced thickness of soil; modified elastic plate model

1. Introduction

With the development and upgrade of infrastructures, the shield method is widely used in the construction of city and underwater tunnels. During the construction of the shield tunnel, the diaphragm wall of the tunnel end shaft must be broken during shield launching and arrival, leading to the soil collapse of the end. This is a high-risk process of the construction of the shield tunnel. Figure 1 shows a case of ground surface collapse and fractures at the shield tunnel end because of the diaphragm wall breaking and instability of the end soils. An analysis and evaluation of the end soils and subsequent soil reinforcement are significant to the shield launching and arrival. It is critical in the shield tunnel construction.

Issues regarding the shield tunnel end soil can be divided into soil failure mode, soil stability, reinforced range, reinforcement methods, and strength \textsuperscript{[1-3]}. Based on the practical conditions and experiences of previous engineering, several viewpoints regarding the reinforced thickness of the tunnel end soil exist (Figure 1). (1) The reinforced thickness of the tunnel end soil is 6.0 m, no matter the length of the principal machine of the shield. (2) The reinforced thickness of the tunnel end soils is 3.0–3.5 m, depending on the existing waterproof board at the hole door and combined action of the shield and waterproof board. (3) When the shield tunnel is below the groundwater table, the reinforced thickness of the end soils should be the sum of the length of the shield’s principal machine and cover thickness (1.5–2.0 m). The properties of the ground also influence the reinforced thickness. When the
shield tunnel is in cohesive soils, the reinforced thickness is generally taken as 3.5 m. When the shield tunnel is in sandy soils, one method is the sum of the length of the shield’s principal machine and cover thickness (1.0–1.5 m). Another method is also taken as 3.5 m, but double liquid grouting at the shield tail must be developed synchronously.

Reviewing domestic and international development, related studies on the stability and reinforced thickness of the tunnel end soil are mainly from (1) theory of the whole plate model in the code of the Japan Association of JET GROUT (JJGA)[4], in which reinforced soil is assumed as a whole elastic plate; (2) theory of soil slip instability in Japan[5], in which a mass-sliding mode of the soil is assumed. A combined slip face of line and arc is assumed to calculate the slipping moment and is used to analyze end soil stability. Based on the theory of soil slip instability in Japan, Lai Jianming and Bai Yun[5] developed a numerical simulation of soft soil stability in Shanghai using the three-dimensional (3D) nonlinear finite element method and developed a combined slip face of line and arc of the end soil. Working on the soft soil layer in Shanghai, Wu Dao[6] analyzed the strength and stability of the reinforced body based on the model of document[4-5] and developed a comprehensive method to calculate the reinforced range of the end soil. Subsequently, the stress distribution and deformation law of the reinforced body are also simulated by FEM when the shield is launching. YinLiming[7] developed a combined slip mode of the vertical and declining line when he studied the reinforced soils of sandy pebble, and the mode is used to calculate the reinforced thickness of the end soil.

Based on studies, JiangYusheng and LuoFurong[1,8] conducted a comprehensive and detailed study on the failure and reinforcement of the end soil. A mechanical model of equivalent load is proposed to conduct a theoretical and engineering study on the strength and reinforced thickness of the tunnel end soil. To determine the relationship between reinforced range and effect, XinZhensheng[9,10] built a numerical simulation model to analyze the pre-reinforcement range of the shield tunnel end. The reinforced range is then adjusted to achieve the desired result, providing a scientific reference for the problem of the reinforced range of the tunnel end.

HuXinpeng and SunMou[11] developed conducted studies on the reinforced range, strength, and waterproof requirements for the Nanjing metro tunnel construction. They proposed a reinforcement mode to the tunnel end for the special sandy ground in Nanjing. Regarding domestic shield tunnel construction, SunZhenchuan[12] introduced empirical values of the reinforced thickness and width of end soils. The strength and stability of the end soils are checked using existing theoretical methods. For the problems of reinforcing the end soils, ZhuShiyou and LinZhibin[13] built a database of reinforcement plans for end soils. Based on the plan database, the stability of the end soils can be
inferred, and the reinforcement plan and measures can also be advised according to the specific engineering conditions. Based on the above studies, Song Kezhi and Wang Mengshu\textsuperscript{[1,2]} developed a limit-equilibrium model of the combination of the decline line and logarithmic spiral and applied it for calculating the reinforced thickness of the end soils; better results are gained in practice.

With the development of shield tunnel technologies in China, the above studies are critical in exploring the failure mode of the end soils, calculating reinforced thickness, and guaranteeing the stability of the soils. However, engineering accidents occur frequently in shield tunnel construction, which could be related to the ground properties and corresponding reinforced thickness of the end soils. To ensure the soil’s stability, the end soils must be reinforced in a determined range. However, the reinforced thickness cannot be too large, or else it is unfavorable for shield driving and engineering economy. To further discuss a more scientific and rational calculation model for the reinforced thickness of the soils, this study developed a modified elastic model based on the elastic thin plate model.

2. Modification of elastic thin plate model

2.1 Elastic thin plate model

The reinforced thickness of the end soils is calculated using the elastic thin plate model in Japan\textsuperscript{[4]}. It assumes that the reinforced body is an elastic thin plate supported peripherally and resisting soil and water pressure (Figure 2). This model is currently used to calculate the reinforced body thickness theoretically in the world\textsuperscript{[1,6,7,8,10-13]}.

According to the elastic thin plate model, the maximum bend stress at the plate center is expressed as

$$\sigma_{\text{max}} = \frac{3(3 + \mu) p_0}{32} \left( \frac{D}{t} \right)^2 \leq \sigma_t,$$

(1)

where $\mu$ is the Poisson ration of the body, $p_0$ is the unit uniform load vertical to the body face, $D$ is the diameter of the circular plate, $t$ is the plate thickness, and $\sigma_t$ is the ultimate tensile strength of the reinforced soils. Considering the engineering safety factor is $K_0$, the thickness of the elastic plate can be derived using equation (2).

$$t = K_0 \left[ \frac{3(3 + \mu) p_0 D^2}{32\sigma_t} \right]^{\frac{1}{2}},$$

(2)
2.2 Modified elastic plate model

Considering the thickness of the plate, if the ratio of the thickness to the minimum feature size along the plane is greater than 1/5, it is a thick plate. If the ratio of the thickness to the minimum feature size along the plane is between 1/80 and 1/5, it is a thin plate [14,15].

In the elastic thin plate model, shear deformation and normal stress are neglected. This is a plane stress condition, and there is no interactive compression among every thin layer parallel to the neutral layer in the plate. If the size of the plate is within the thin plate definition, the elastic thin plate model could produce a rational result. However, it becomes more conservative to the thick plate. Regarding thick bend, mathematical solutions are more complicated; therefore, a series solution or elastic FEM solution is used.

The reinforced thickness of the tunnel end soil varies with different engineering methods. In general, the reinforced thickness is relatively thinner because of its higher strength built by the freezing method. However, jet grouting and mixing piles increase the reinforced thickness. Whatever the reinforced methods, the reinforced thickness is in the range of the thick plate.

To modify the elastic thin plate model and explain the feasibility of the modified model, elastic FEM analysis is launched on plates of different thicknesses. The ratio of plate thickness and diameter is distributed at 0.1 and 1.0 in the FEM model. The boundary condition of the elastic plate is peripheral simple support. The FEM parameters are calculated using engineering practice, where $E = 100 \text{ MPa}$, $\mu = 0.25$, and $p_0 = 1.0$. Figure 3 shows the FEM model of the plate. The maximum bending stress at the plate center of all cases is solved. Table 1 shows the results of the comparison of both models.

![Figure 3 FEM elastic plate model](image)

| $t/D$ | Solution of elastic thin plate model | Solution of FEM | Safety degree |
|---|---|---|---|
| 0.10 | 30.47 | 12.18 | 2.50 |

Table 1 Comparison between the elastic thin plate model and FEM (maximum bending stress at the plate center)
| $t/D$ | $\alpha$ | $\sigma_{max}$ | $\alpha$ |
|-------|--------|--------------|--------|
| 0.15  | 13.54  | 5.67         | 2.39   |
| 0.20  | 7.62   | 3.39         | 2.25   |
| 0.25  | 4.88   | 2.01         | 2.44   |
| 0.30  | 3.39   | 1.32         | 2.56   |
| 0.35  | 2.49   | 0.91         | 2.74   |
| 0.40  | 1.90   | 0.63         | 3.03   |
| 0.45  | 1.50   | 0.54         | 2.80   |
| 0.50  | 1.22   | 0.46         | 2.64   |
| 0.55  | 1.01   | 0.38         | 2.66   |
| 0.60  | 0.85   | 0.32         | 2.64   |
| 0.65  | 0.72   | 0.29         | 2.47   |
| 0.70  | 0.62   | 0.28         | 2.20   |
| 0.75  | 0.54   | 0.28         | 1.97   |
| 0.80  | 0.48   | 0.26         | 1.81   |
| 0.85  | 0.42   | 0.25         | 1.72   |
| 0.90  | 0.38   | 0.24         | 1.59   |
| 0.95  | 0.34   | 0.22         | 1.52   |
| 1.00  | 0.30   | 0.21         | 1.45   |

Based on engineering experience\cite{1,11,13}, the values of $t/D$ of the model in Table 1 extend most reinforced sizes in common construction methods. The calculated results show that the bending stress of the elastic thin plate model is larger than that of FEM. Therefore, the theory of elastic thin plate is too conservative to calculate the reinforced thickness, and the conservative degree varies with the value of $t/D$.

The elastic FEM analysis data shows that the existing elastic thin plate model will be modified.

Using equation (1) of the elastic thin plate model, the calculation formula for the maximum bend stress at the plate center is set as

$$\sigma_{max} = k \cdot p_0 \left(\frac{t}{D}\right)^\alpha$$

where we set $p_0 = 1.0$.

Take the natural logarithm on both sides of equation (3), and we set $x_i = \frac{t}{D}$, equation (3) becomes

$$\ln \sigma_{max} = \ln k + \alpha \ln x_i$$

(4)
where we set $\beta = \ln k$. The maximum bend stress $\sigma_i$ at the plate center can be obtained from FEM considering the different values of $t/D$. By using the least-square method, the sum of the squares of errors between FEM data and the fitting equation result is

$$Q_e = \sum_i^n (\ln \sigma_i - \ln \sigma_{max})^2 = \sum_i^n (\ln \sigma_i - \beta - \alpha \ln x_i)^2 .$$

(5)

To solve the minimum value of $Q_e$, we take a partial derivative regarding $\alpha$ and $\beta$, and $\alpha$ and $\beta$ can be solved as

$$\alpha = \frac{\sum_{i=1}^n \ln x_i \ln \sigma_i - \frac{1}{n} \sum_{i=1}^n \ln x_i \sum_{i=1}^n \ln \sigma_i}{\sum_{i=1}^n (\ln x_i)^2 - \frac{1}{n} (\sum_{i=1}^n \ln x_i)^2} .$$

(6)

$$\beta = \ln \sigma_{max} - \alpha \cdot \ln x_i .$$

By analyzing the data in Table 1, we can calculate that $\alpha = -1.804$, $\beta = -1.840$, then

$$\ln \sigma_{max} = -1.840 - 1.804 \ln x_i .$$

(7)

A significance test for equation (7) must be conducted to evaluate its significance. The regression sum of squares and residual sum of squares can be obtained using the above data, $U = 26.1$, $Q_e = 0.608122$, and $n = 19$.

The formula of the F-test is

$$F = \frac{U}{Q_e/(n-2)} \sim F(1, n-2) .$$

(8)

Then, $F = 729.6$ for test level of $\alpha = 0.01$, $F_{1-0.01}(1,n-2) = F_{0.99}(1,17) = 8.4$. Therefore, $F > F_{0.99}(1,17)$, so $H_0$ is rejected, and the regression equation (9) is significant.

The coefficient of $k = e^{-1.840} = 0.1588$, then, the modified model is as equation (9)

$$\sigma_{max} = 0.1588 \cdot \left(\frac{t}{D}\right)^{-1.804} .$$

(9)

Considering the common parameter of reinforced soil in practice and the Poisson ratio $\mu = 0.25$, equation (9) is translated into the mode of equation (1) as

$$\sigma_{max} = 0.04886 \left[3 + \mu \right] p_0 \left(\frac{D}{t}\right)^{-1.804} .$$

(10)

Finally, the results of the elastic thin plate, elastic FEM, and modified elastic plate models are
compared (Figure 4). Figure 4 shows that compared with the elastic thin plate model, the maximum bending stress of the modified elastic plate model is relatively low. The modified elastic plate model is not as conservative as the elastic thin plate model.

Equation (10) shows that the reinforced thickness of the end soils under the pressure of water and soil can be calculated using equation (11).

\[
t = K_0 \left[ k_0 (3 + \mu) p D^{1.804} \right]^\frac{1}{1.804},
\]

where, \( K_0 \) is the safety factor of reinforced thickness with a coefficient of \( K_0 = 0.04886 \).

![Figure 4 Comparison results of the three models](image)

3. An example analysis

Practical engineering takes the basic parameters of a shield tunnel as the hole door diameter \( D = 6.0 \) m, cover depth \( h_s = 20.0 \) m, ground overload \( q = 20 \) kPa, and average buoyant unit weight of the soils \( \gamma = 18.5 \) kN/m\(^3\). Reinforced by high-pressure jet grouting, the strength of reinforced soils is evaluated using core sampling. We take the cohesive force as \( C_u = 300 \) kPa, inner friction angel \( \varphi_u = 35^\circ \), unconfined compressive strength \( q_u = 1500 \) kPa, and tensile strength as 10% of the unconfined compressive strength \( \sigma_t = 150 \) kPa.

Therefore, the water pressure at the tunnel portal center can be obtained as

\[
p_w = \gamma_w h_w = 230 \text{kPa},
\]

and the soil pressure at the tunnel portal center can be obtained as

\[
p_a = (\gamma h_s + q) \cdot \tan(45^\circ - \frac{\varphi_u}{2}) = 115.3 \text{kPa}.
\]

Then, the resultant pressure at the tunnel portal center can be obtained as

\[
p = p_a + p_w = 345.3 \text{kPa}.
\]

Taking the parameters into equation (2) and equation (13), respectively, the reinforced thickness can be obtained as follows:

According to the elastic thin plate model
\[ t = \left[ \frac{3(3 + \mu) p D^2}{32 \sigma_t} \right]^{\frac{1}{2}} = 5.02\, \text{m}. \]

According to the modified elastic plate model
\[ t = \left[ \frac{0.04886(3 + \mu) p D^{1.804}}{\sigma_t^{1.804}} \right] = 3.43\, \text{m}. \]

Following engineering empirical, we assume that the safety factor \( K_0 = 1.5 \), the reinforced thickness is 7.54 m for the elastic thin plate model and 5.15 m for the modified elastic plate model. During construction, the reinforced thickness of the end soils is taken as 6.0 m. It is between the values of the elastic thin plate and modified elastic plate models. It is convinced that the elastic thin plate model is conservative and uneconomical to engineering.

Based on the example’s parameters, sensitivity analysis is conducted. The relationships between the reinforced thickness and water and soil pressure, tunnel diameter, and limit tensile strength of the reinforced soils can be obtained (Figure 5). From the figure, we can conclude that the larger the tunnel diameter, the thicker the reinforced soils needed, the higher the pressure, the thicker the reinforced soils needed, and the higher the limit tensile strength, the thinner the reinforced soils needed.

**Figure 5** Influence on the reinforced thickness by varying parameters

The case has manifested that the factors of water and soil pressure, tunnel diameter reinforced methods, and reinforced strength should be considered to calculate the reinforced thickness of the end soils. The above two models should be simultaneously used theoretically. Finally, the reinforced thickness is determined by comparing the two models’ results and considering similar engineering practices.

### 4. Conclusions

1. Based on the existing elastic thin plate model, a modified elastic plate model for calculating the reinforced thickness of the end soils is developed. The modified model is fitted using elastic FEM data, and the model’s coefficients are obtained. An example is given to evaluate the effectiveness of the model.
2. The maximum bending stress of the elastic thin plate model is higher than that of the modified elastic plate model, as shown in the calculations. The elastic thin plate model is more conservative and its degree is calculated from the ratio of thickness to the diameter of the plate \((t/D)\). The final value of the thickness in practice is between the values of the elastic thin plate model and the modified elastic plate model. The suggested model is useful for engineering practices and extends theoretical methods for calculating the reinforced thickness.
3. In engineering practices, the elastic thin plate and modified elastic plate models should be used to
calculate the reinforced thickness synthetically by comparing the two models’ results, considering similar practices, and finally determined the thicknesses.

References:

[1] JIANG Yusheng, WANG Chunhe, JIANG Hua, et al. Shield launching and arriving—Theoretical study and engineering practice on end reinforcement[M]. Beijing: People’s Communications Publishing House, 2011: 56-185. (in Chinese)

[2] SONG Kezhi, WANG Mengshu, SUN Mou. Limit equilibrium analysis of soils stability at shallow tunnel end upon shield excavation[Chinese][J]. Journal of Rock Mechanics and Engineering, 2015, 34(2): 407-413. (in Chinese)

[3] SONG Kezhi, HOU Hongchao, LIU Guibin, et al. Meso-mechanic behavior of soil damage of geotechnical engineering using DEM[J]. Journal of basic science and engineering, 2016, 24(3): 618-631. (in Chinese)

[4] Japanese Jet Grout Association. Jet Grout Construction Method [M]. Tokyo: Publishing conference of Kajima Institute, 1991: 17-24. (in Japanese)

[5] LAI Jianming, BAI Yun. 3D Nonlinear Analysis of the failure of the soil after the open for shield or pipe jacking machine to be pushed in or out[J]. Geotechnical Engineer, 1994, 6(1): 11–15. (in Chinese)

[6] WU Dao. Research on construction technique and mechanism of reinforced soil for department and reception of large scale shield[M. S. thesis][D]. Shanghai: Tongji University, 2006. (in Chinese)

[7] YIN Liming. Study on the reinforcement of soil for shield launching in sandy pebble stratum[M. S. thesis][D]. Changsha: Central South University, 2013. (in Chinese)

[8] LUO FuRong, JIANG Yusheng, JIANG Hua. Theoretical modeling and sensitivity analysis of improved sandy stratum at TBM portal areas with strength and stability theories [J]. Journal of Engineering Geology, 2011, 19(3): 364–369. (in Chinese)

[9] XIN Zhenxing, WANG Jinan. Study on the stability of typical construction sections in underwater shield subway tunnel projects[M. S. thesis][D]. Beijing: University of Science and Technology Beijing, 2007. (in Chinese)

[10] XIN Zhenxing, WANG Jin’an, MA Haitao, et al. Study on rational scope of pre-reinforcement at starting position for shield excavation[J]. Chinese Journal of Underground Space and Engineering, 2007, 3(3): 513–518. (in Chinese)

[11] HU Xinpeng, SUN Mou, WANG Junlan. Discussions on criteria for soil reinforcement during EPB shields’ launching and arrival construction in soft ground[J]. Tunnel Construction, 2006, 26(5): 11–13. (in Chinese)

[12] SUN Zhenchuan. Study on end soil reinforcement technology for city metro shield tunnel in soft soil stratum[J]. West China Exploration Engineering, 2003, (10): 81–83. (in Chinese)

[13] ZHU Shiyou, LIN Zhibin, GUI Changlin. Reinforcement technology for stability analysis on end soil in shield launching and arrival[J]. Tunnel Construction, 2012, 32(6): 788–795. (in Chinese)

[14] QU Qingzhang. Theory of Elastic Plates [M]. Beijing: People’s Communications Publishing House, 2000: 15-18. (in Chinese)
[15] WANG Minzhong. Advanced Theory of Elasticity [M]. Beijing: Peking University press, 2002: 26-27. (in Chinese)