Lorentz Violation and Spacetime Supersymmetry

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Abstract. Supersymmetry and Lorentz invariance are closely related as both are spacetime symmetries. Terms can be added to Lagrangians that explicitly break either supersymmetry or Lorentz invariance. It is possible to include terms which violate Lorentz invariance but maintain invariance under supersymmetric transformations. I illustrate this with some simple extensions of the original Wess-Zumino model.

INTRODUCTION

The understanding of spacetime symmetries has grown remarkably in the last century. The mass-momentum behavior is associated with the requirement that theories respect translation invariance, while the intrinsic spin and the existence of antiparticles are consequences of the Lorentz group. The theoretical discovery of supersymmetry is a major achievement of twentieth century physics, and it marked another expansion of our concept of spacetime symmetries. The advent of supersymmetry was followed by its application to gauge theories and the Standard Model and a local version (supergravity) that incorporates general relativity. This is a clear indication that supersymmetry and Lorentz invariance are closely linked. The introduction of fermionic generators that relate fermions and bosons resulted largely from motivations that were theoretical, but it is clear that supersymmetry has come to dominate particle physics phenomenology.

A large part of the history of physics and particle physics in particular involves the introduction of symmetries and the understanding of the spontaneous breaking of these symmetries. The longstanding point of view has been that spacetime symmetry is one of the most fundamental. The familiar symmetries regarding translations, angular momentum, and Lorentz boosts that constitute the Poincaré group are usually assumed to be unbroken symmetries. While there is great theoretical appeal for these symmetries to be unbroken, this issue is of course a question that can only be decided by experiment. In any case the amount of breaking of the Lorentz symmetry must be very small, if it is indeed nonzero, since it has so far escaped experimental detection. Supersymmetry,
if it exists, comprises another part of the overall spacetime symmetry, and it is clearly broken. In fact, it must be broken badly compared to the scale at which we perform experiments, so much so that we have currently not detected any of the supersymmetric partners to the Standard Model particles. However, from a more theoretical point of view, the breaking of supersymmetry is also very small since the breaking scale is very much suppressed in comparison to the Planck scale (where the true nature of spacetime presumably emerges).

The Poincaré algebra involves the generator of translations $(P_{\mu})$ and the generator of rotations and Lorentz boosts $M_{\mu\nu}$ in the following way

\begin{align}
\left[ P_{\mu}, P_{\nu} \right] &= 0 \\
\left[ P_{\mu}, M_{\rho\sigma} \right] &= i(\eta_{\mu\rho} P_{\sigma} - \eta_{\mu\sigma} P_{\rho}) \\
\left[ M_{\mu\nu}, M_{\rho\sigma} \right] &= i(\eta_{\nu\rho} M_{\mu\sigma} - \eta_{\nu\sigma} M_{\mu\rho} - \eta_{\mu\rho} M_{\nu\sigma} + \eta_{\mu\sigma} M_{\nu\rho}) ,
\end{align}

(1)

Explicit terms can be added to a Lagrangian that violate the Poincaré algebra. If one wants to preserve energy-momentum conservation, then any Lorentz violation continues to respect the first of these equations. The set of terms which can be added to the Standard Model have been categorized [1, 2] in an extended Lagrangian. These terms are thought to arise in a more fundamental theory like string theory [3, 4] which is nonlocal, but in the context of the Standard Model extension they are viewed as phenomenological parameters.

Supersymmetry involves extending this algebra with a fermionic generator $Q$,

\begin{align}
\left[ Q, P_{\mu} \right] &= 0 \\
\{ Q, \overline{Q} \} &= 2\gamma^{\mu} P_{\mu} .
\end{align}

(2)

This part of the algebra is not respected by supersymmetry breaking terms in a Lagrangian. The breaking is usually required to be soft meaning that quadratic divergences continue to be avoided even though supersymmetry is explicitly broken. It is well-known that, for the required terms to be soft, they must be superrenormalizable, and any non-renormalizable terms are expected to be suppressed by powers of the Planck mass $M_{P}^{-1}$ or some other cutoff scale associated with new physics. While motivated by more fundamental theories and arising from some mechanism of spontaneous breaking of supersymmetry presumably in some nonperturbative sector that is “hidden” by making it completely chargeless under the Standard Model gauge group. One can abandon any attempt to derive these supersymmetry breaking terms from a more fundamental theory, add them to a supersymmetric model in the most general way, and then view them in a purely phenomenological way.

For the purposes of this talk, when I say a theory is supersymmetric, I mean that the relevant Lagrangian respects some modified version of the transformations in Eq. (2) but that some portion of Poincaré transformations in Eq. (1) is not respected. Specifically, the models presented here involve violations associated with the Lorentz generator $M_{\mu\nu}$.

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2 The four-component notation for spinors makes the relationship with the Standard Model Lorentz-violating extension [1, 2] transparent.
so that one can say the models respect both supersymmetry and translation invariance. Previous discussion of the superPoincaré algebra has been with regard to its exact realization, or to cases where the subalgebra involving the fermionic generator \( Q \) is broken.

This talk is based on work done with V. Alan Kostelecký [5].

**LORENTZ-VIOLATING WESS-ZUMINO MODEL**

The first four-dimensional supersymmetric field theory was written down by Wess and Zumino [6]. A modest goal is to determine whether it is possible to have an unbroken supersymmetry even in the presence of broken Lorentz symmetry. Can terms that explicitly violate the Lorentz symmetry be added to the Wess-Zumino model that still preserve some version of the supersymmetry? This approach of adding explicit terms is in the same spirit as the Lorentz-violating Standard Model extension in which Lorentz violation is added to the Standard Model[3]. Consider the following Lagrangian,

\[
\mathcal{L} = \frac{1}{2} \partial_{\mu} A \partial^{\mu} A + \frac{1}{2} \partial_{\mu} B \partial^{\mu} B + \frac{1}{2} i \bar{\psi} \partial A \psi + \frac{1}{2} F^2 + \frac{1}{2} G^2 + m \left( -\frac{1}{2} \bar{\psi} \psi + A F + B G \right)
+ \frac{g}{\sqrt{2}} \left( F(A^2 - B^2) + 2 G A B - \bar{\psi}(A - i \gamma_5 B) \psi \right)
+ k_{\mu \nu} \partial^\mu A \partial^\nu A + k_{\mu \nu} \partial^\mu B \partial^\nu B + \frac{1}{2} i k_{\mu \nu} \bar{\psi} \gamma^\mu \partial^\nu \psi
+ \frac{1}{2} k_{\mu \nu} k_{\rho \sigma}(\partial^\nu A \partial^\rho A + \partial^\nu B \partial^\rho B),
\]

(3)

where one recognizes the first three lines as the original Wess-Zumino model. The last two lines involve the Lorentz-violating coefficients \( k_{\mu \nu} \) either linearly or quadratically. These couplings are simply numbers (with Lorentz index labels) that do not change under a particle Lorentz transformation, which boosts or rotates local field configurations within a fixed inertial frame. Since there are an even number of indices these terms do not violate the CPT invariance. Without loss of generality, \( k_{\mu \nu} \) can be taken to be a real symmetric, traceless, coefficient.

Since the supersymmetric transformation relates fermions to bosons, there is a non-trivial relationship between the coupling coefficients involving the scalars and the fermion. The supersymmetric transformation forces a relationship (namely the common \( k_{\mu \nu} \)) on the Lorentz-violating terms of Eq. (3) that is similar to the common mass and couplings that are a well-known consequence of supersymmetric theories.

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3 Local field theories that are derived from underlying nonlocal theories have well-known problems with causality and positivity. These problems can emerge at a high-energy scale determined by the Planck mass [3]. The correct point of view is to regard these field theories as effective theories, and the inconsistencies will be reconciled when the full underlying fundamental theory is taken into account. In fact such arguments can be used to establish upper bounds on the size of the Lorentz violation.
If one modifies the supersymmetric transformations of the Wess-Zumino model by adding new terms involving the coefficients $k_{\mu\nu}$ coefficients,

$$
\begin{align*}
\delta A &= i\gamma_5 \psi, \\
\delta B &= i\gamma_5 \psi, \\
\delta \psi &= -(i\partial + i k_{\mu\nu} \gamma^\mu \partial^\nu) (A + i\gamma_5 B) \varepsilon + (F + i\gamma_5 G) \varepsilon, \\
\delta F &= -\bar{\varepsilon} (i\partial + i k_{\mu\nu} \gamma^\mu \partial^\nu) \psi, \\
\delta G &= \bar{\varepsilon} (\gamma_5 \partial + k_{\mu\nu} \gamma^\mu \partial^\nu) \psi,
\end{align*}
$$

one finds that the Lagrangian is invariant up to a total derivative.

The commutator of two supersymmetry transformations in Eq. (4) yields

$$
[\delta_1, \delta_2] = 2i\gamma^\mu \varepsilon_2 \partial_\mu + 2ik_{\mu\nu} \gamma^\mu \varepsilon_2 \partial^\nu,
$$

which involves the generator of translations. A modified supersymmetry algebra therefore exists, and the lagrangian in Eq. (3) provides an explicit example of an interacting model with both exact supersymmetry and Lorentz violation.

One can also show that a modification of the supersymmetry transformation in Eq. (4) cannot be modified, say by changing the transformation of the scalar fields $A$ and $B$, because any such modification would not result in a closure of the supersymmetry algebra. In fact, one can understand the modification of the Lagrangian and the supersymmetric transformations as a global substitution of the form $i\partial_\mu \rightarrow i\partial_\mu + ik_{\mu\nu} \partial^\nu$. The translation generator $P_\mu$ commutes with itself (first equation in Eq. (1)) and satisfies the first equation in Eq. (2), so it then follows that

$$
[Q, p^2] = 0.
$$

Since the superpotential containing the mass and coupling terms is unaffected by the Lorentz violation, analogues should exist for various conventional results on supersymmetry breaking [8, 9, 10].

One consequence of the supersymmetric Lagrangian is the relationship between the fermionic and scalar propagators. The fermionic propagator is

$$
\frac{i}{p_\mu (\gamma^\mu + k_{\mu\nu} \gamma^\nu) - m}.
$$

Rationalizing this propagator by multiplying by $p_\mu (\gamma^\mu + k_{\mu\nu} \gamma^\nu) + m$, one gets the denominator

$$
p^2 + p_\mu p_\rho (k_{\mu\nu} \gamma^\nu \gamma_\rho + k_\rho_\sigma \gamma_\mu \gamma_\sigma) + k_{\mu\nu} k_\rho_\sigma p^\mu p^\rho \gamma^\nu \gamma^\sigma - m^2.
$$

For $k_{\mu\nu}$ symmetric, the second term is $2k_{\mu\nu} p^\mu p^\nu$, and the third term is $k_{\mu\nu} k^\mu_\rho p^\nu p^\rho$. Consequently, the scalar and fermion, which are related by supersymmetry, have propagators with the same structure.

One might try to eliminate the Lorentz violation by a choice of coordinates such that $x^\mu \rightarrow x^\mu + k_{\mu\nu} x_\nu$. However, the metric would then no longer have the usual Minkowski form, and the Lorentz violation is simply moved into the new metric. The Lorentz violation of the theory is physical.
CPT-ODD LORENTZ BREAKING

It is well-known that a local Lorentz-invariant quantum field theory preserves the combination $CPT$ where $C$ is charge conjugation, $P$ is parity, and $T$ is time reversal. If Lorentz-violating terms are added to the Lagrangian, however, then this result no longer needs to hold and nonlocality as well as CPT violation is permitted.

A $CPT$-violating term can be added to the Wess-Zumino model in the following way,

$$L = \frac{1}{2} \partial_\mu A \partial^\mu A + \frac{1}{2} \partial_\mu B \partial^\mu B + \frac{1}{2} i \bar{\psi} \gamma^5 \partial_\mu \psi + \frac{1}{2} F^2 + \frac{1}{2} G^2 + k_\mu (A \partial_\mu B - B \partial_\mu A) - \frac{1}{2} k_\mu \bar{\psi} \gamma_5 \gamma^\mu \psi + \frac{1}{2} k^2 (A^2 + B^2).$$ (9)

The coefficient $k_\mu$ has mass dimension one, and a unique supersymmetry transformation can be established on dimensional grounds to be

$$\delta A = \bar{\epsilon} \psi,$$

$$\delta B = i \bar{\epsilon} \gamma_5 \psi,$$

$$\delta \psi = - (i \partial - \gamma_5 k) (A + i \gamma_5 B) \epsilon + (F + i \gamma_5 G) \epsilon,$$

$$\delta F = - \bar{\epsilon} (i \partial - \gamma_5 k) \psi,$$

$$\delta G = \bar{\epsilon} (\gamma_5 \partial + i k) \psi,$$  \hspace{1cm} (10)

and the Lagrangian again transforms into a total derivative.

The Lagrangian can be derived from the original Wess-Zumino model by applying a field redefinition,

$$\psi \rightarrow e^{-i \gamma_5 k \cdot x} \psi$$

$$A \pm iB \rightarrow e^{\pm i k \cdot x} (A \pm iB).$$ (11)

The terms involving the coupling $k_\mu$ respect $C$ and $T$ but violate $P$, giving an overall $CPT$ violation. Since parity is violated by the terms involving $k_\mu$ (both in the Lagrangian and in the transformations) one obtains a supersymmetric transformation that acts differently on the left- and right-handed states,

$$\delta \psi_L = (-i \partial + k) (A + iB) \epsilon_R + (F - iG) \epsilon_L,$$

$$\delta \psi_R = (-i \partial - k) (A - iB) \epsilon_L + (F + iG) \epsilon_R,$$  \hspace{1cm} (12)

where the chiral components are defined as usual: $\psi_L = P_L \psi, \epsilon_L = P_L \epsilon, P_{L/R} = (1 \mp \gamma_5)/2$.

The mass and coupling terms in Eq. (9) are not supersymmetric. The parity-odd mass and coupling terms are also not consistent with supersymmetry. For example, the parity-violating mass term

$$m (i \bar{\psi} \gamma_5 \psi + 2AG - 2BF),$$ (13)
cannot be added to the Wess-Zumino theory because it is not invariant under the modified supersymmetry transformations in Eq. (10). It is not surprising that they cannot be reconciled with supersymmetry since the mass and coupling terms do not respect the field redefinition.

In addition to the $k_{\mu\nu}$ and $k_\mu$-dependent terms described above, the simple field content of the Wess-Zumino model admits additional renormalizable Lorentz-violating term to its Lagrangian: $(A^2 \partial B \pm B^2 \partial A)$, $\phi \overline{\gamma}_5 \gamma^\mu \psi$, and $\overline{\psi} \sigma^\mu \gamma^\nu \partial^\lambda \psi$. However, there do not appear to be supersymmetric interpretations for these terms.

**SUPERFIELD FORMULATION**

The elegant method of superspace [11] allows one to combine all the component fields of a supersymmetric multiplet into one superfield. By extending the four dimensions of spacetime to include fermionic dimensions as well, this technique highlights the role of supersymmetry as a spacetime symmetry. The fermionic coordinates, $\theta$, that are added to the usual spacetime coordinates highlight the role of supersymmetry as a spacetime symmetry. The CPT transformation maps the components of a chiral superfield $\Phi$ into themselves, while the parity transformation alone maps left-chiral superfields into right-chiral superfields and vice versa. Define

$$\phi = \frac{1}{\sqrt{2}} (A + iB), \quad \mathcal{F} = \frac{1}{\sqrt{2}} (F - iG).$$

(14)

In terms of these complex scalars, the left-chiral superfield appropriate for the model in Eq. (3) is

$$\Phi(x, \theta) = \phi(x) + \frac{i}{\sqrt{2}} \overline{\theta} \gamma_\mu \theta \phi(x) + \frac{i}{\sqrt{2}} \overline{\theta} \gamma_5 \gamma^\mu \theta \phi(x) + \frac{i}{\sqrt{2}} \overline{\theta} \gamma_5 \gamma^\mu \theta \frac{1}{2} (1 - \gamma_5) \theta \mathcal{F}(x)$$

$$+ \frac{1}{2} i \overline{\theta} \gamma^\mu \theta \phi(x) + \frac{i}{\sqrt{2}} \overline{\theta} \gamma_5 \gamma^\mu \theta \phi(x) + \frac{i}{\sqrt{2}} \overline{\theta} \gamma_5 \gamma^\mu \theta \frac{1}{2} (1 - \gamma_5) \theta \mathcal{F}(x)$$

$$- \frac{i}{\sqrt{2}} \overline{\theta} \theta \phi(x) + \frac{i}{\sqrt{2}} \overline{\theta} \gamma^\mu \theta \phi(x)$$

$$- \frac{i}{\sqrt{2}} \overline{\theta} \gamma_5 \gamma^\mu \theta \frac{1}{2} (1 - \gamma_5) \theta \mathcal{F}(x) - \frac{i}{\sqrt{2}} \overline{\theta} \gamma_5 \gamma^\mu \theta \frac{1}{2} (1 - \gamma_5) \theta \mathcal{F}(x).$$

(15)

Here, the subscript $L$ denotes projection with $\frac{1}{2} (1 - \gamma_5)$. The lagrangian in Eq. (3) can then be expressed as

$$\mathcal{L} = \phi^* \phi |_D + \left( \frac{1}{2} m \phi^2 |_F + \frac{1}{2} g \phi^3 |_F + \text{h.c.} \right),$$

(16)

where the symbols $|_D$ and $|_F$ refer to projections onto the $D$- and $F$-type components of the (holomorphic) functions of $\Phi(x, \theta)$. The theory can therefore be represented as an action in superspace.

A supersymmetry transformation on $\Phi(x, \theta)$ generated as $\delta_Q \Phi(x, \theta) = -i \overline{\theta} Q \Phi(x, \theta)$ where

$$Q = i \partial_{\overline{\theta}} - \gamma^\mu \theta \partial_\mu - k_{\mu\nu} \gamma^\mu \theta \partial_\nu.$$ 

(17)

It is easy to check that the application of the supersymmetry transformation to the resulting chiral superfield yields the transformation given in Eq. (4), and that the supersymmetry algebra closes. The superpotential is not modified and gives rise to the mass and
coupling terms in Eq. (3) in the usual way by projecting out the usual $F$-term component of the holomorphic functions of the chiral superfield $\Phi$. The generators $Q$ and $P_{\mu} = i \partial_{\mu}$ satisfy

$$[Q, P_{\mu}] = 0, \quad \{Q, \overline{Q}\} = 2 \gamma^\mu P_{\mu} + 2k_{\mu\nu} \gamma^\mu p^\nu,$$

which represents a new superalgebra explicitly containing the Lorentz violation parameter $k_{\mu\nu}$.

**CONCLUSIONS**

Lorentz-violating extensions of the Wess-Zumino model have been found that remain exactly supersymmetric. Nontrivial relationships exist between terms in the Lagrangian involving scalars and terms involving fermions reminiscent of the common masses and couplings in other supersymmetric theories. The supersymmetry algebra closes in a modified way and the momentum generator continues to generate translations. Therefore a representation of supersymmetry has been found that does not respect the Lorentz invariance of the superPoincaré algebra. Many of the appealing features of supersymmetry are preserved even if Lorentz violation is allowed. One might extend the considerations discussed here to other representations of supersymmetry. For example the vector superfield should have a straightforward generalization to include Lorentz violation. One could then conceive of phenomenological models (like supersymmetric QED or the Minimal Supersymmetric Standard Model) that include explicit terms that break supersymmetry, Lorentz invariance, or both.

The Lorentz symmetry has long been viewed by many as one which is unlikely to be broken. However, one should be wary of arguments that say since a dimensionless number is constrained to be very small, it must be exactly zero. The prominent recent example is the observed size of a “cosmological constant” compared to known particle physics scales. Furthermore large hierarchies exist between observed physical scales in particle physics: there is a large gap of at least six orders of magnitude between the electron mass and the heaviest neutrino, so we can confidently say that a “desert” unpopulated by massive states has already been confirmed experimentally. Whether Lorentz violation exists and is characterized by very small dimensionless numbers or ratios is a question that can ultimately only be decided by experiment.

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