RELATIVISTIC QUANTUM MECHANICS

Part I

1. Kets, Bras and Operators
   Ket or state space, bra space and inner products. Linear operators. Product of operators, associative axiom of multiplication. Hermitian operators and observables. Representations in state space: orthonormal basis, completeness relation, representations of kets and bras, representations of operators, change of representation, unitary operators, transformation matrix. Eigenvalue equation and diagonalisation. Definition of observable, commuting observables, unitary equivalent observables, noncommuting observables.

2. Physical Postulates of Quantum Mechanics
   Description of the state of the system, physical quantities, measurement of physical quantities, spectral decomposition, probability interpretation. Expectation values. Heisenberg uncertainty relation.

3. Position, momentum and translation
   Continuous spectra, some useful properties of the Dirac delta function. Position eigenkets and position measurement. Translations: properties of the infinitesimal translation operator, momentum as generator of translation in quantum and classical mechanics. Finite translations. Canonical commutation relations. Wave functions in position and momentum space. Momentum operator in the position space, momentum space wave function.

4. Quantum Dynamics
   Time evolution. The Schrödinger equation for the time evolution of a state ket. Schrödinger versus Heisenberg picture: state kets and observables in the Schrödinger and Heisenberg pictures. The Heisenberg equation of motion: evolution of the mean value of an observable, Ehrenfest's theorem, time evolution of base kets and transition amplitudes. Schrödinger wave equation, conservation of probability revisited.

Part II

1. Lorentz tranformations

2. Relativistic Wave equations
   Klein-Gordon equation, The Dirac equation: continuity equation for the Dirac wave function, covariance of Dirac equation, covariance of probability current, transformations of spinors under space reflection, bilinear covariants. Nonrelativistic limit of the Dirac equation. Solutions of the free Dirac equation: plane wave solutions, polarised electrons in relativistic theory. Projection operators for energy and spin. Helicity, Wave packets of plane Dirac waves: Klein paradox. Negative energy solutions- the hole theory.
3. **Discrete symmetries in the Dirac theory**
   Charge conjugation: effect of charge conjugation on a negative energy electron at rest, charge conjugation of eigenstates with arbitrary spin and momentum. Space inversion or parity transformation. Time reversal.

4. **Right and left-handed electrons and neutrino equation**
   Right and left-handed electrons, two component neutrino theory.

**INTRODUCTION TO QUANTUM FIELD THEORY**

1. **Classical Theory**
2. **Canonical quantization**
3. **Quantized oscillators**
4. **Fermion and bosons**
5. **The forced oscillator**
6. **Perturbation theory**
7. **Free fields**
8. **Green's functions and Wick's theorem**
9. **Interacting fields**
10. **Symmetries in quantum field theory I (Poncaré Group)**
11. **Symmetries in quantum field theory II (Internal)**
12. **Cross sections**
13. **Perturbation theory (I)**
14. **Perturbation theory (II) - Feynman graphs**
15. **Fermions and Yukawa theory**
16. **Momentum space**
17. **Electrodynamics (I)**
18. **Electrodynamics (II)**
19. **Compton scattering**
20. **Ultraviolet divergences**
INTRODUCTION TO PARTICLE PHYSICS

Part I. Discoveries of particles

1. First discoveries.
   Electron, proton, neutron.
   Neutrino. Fermi theory of the weak interactions.

2. Detection of particles.
   Principles of detection of particles,
   Particles in magnetic fields.
   Radiation and ionization.
   Scintillation and Cherenkov radiation

3. Cosmic rays and discoveries of particles.
   Cosmic rays.
   Positron.
   Muon.
   Pion. Yukawa theory of strong interactions.
   Strange particles. Kaons.

4. Acceleration of particles.
   Linear accelerators.
   Cyclic accelerators.
   First accelerator experiments. Muon neutrino.
   Existing and future accelerators.

5. Quarks.
   Classification of hadrons.
   $SU(3)$-symmetry. Eightfold way.
   u, d and s quarks.
   Quark jets. Cross-section of annihilation $e^+e^->$hadrons.
   Cabibbo mixing.

6. Heavy leptons and quarks.
   Observations of resonances.
   Charmed particles. Discovery of $J/\psi$.
   $\tau$-lepton.
   Bottom quark.
   Top quark.
   Tau neutrino.

7. Discoveries of the gauge bosons.
   Intermediate bosons, $W, Z$.
   Introduction of Colour. Gluons.
Part II. Propagation and interaction of particles

1. Propagation of particles
   Equations of motion and their solutions.
   Static solutions and Waves.
   Equation of motion for massless fermion. Helicity.

2. Interactions
   Physical picture. Potentials.
   Virtual particles. Propagators.
   Hamiltonian of interaction.
   Evolution matrix and S-matrix
   Invariant amplitude. Rates of processes.
   Phase volume. Normalization of the wave functions.

3. Decay and scattering
   Decay probability.
   Scattering. Cross-section
   Optical theorem.
   Resonances.

Part III. Symmetries and interactions

1. Symmetries at classical and quantum level
   Symmetries and transformations.
   Conservation laws.
   Properties of matrix elements.

2. Parity
   Parity transformations.
   Parity conservation.
   Parity of pion and other particles.
   Parity violation.
   Pion decay.

3. Charge conjugation
   C-transformation.
   C-parity
   Violation of C-symmetry.

4. CP-violation
   CP-properties of $K^0$ and $K^0$ mesons.
   Discovery of CP-violation.
   Direct CP-violation.
   CP-violation in B-meson system.

5. Continuous symmetries.
   Conserved currents and conserved charges,
   Unitary symmetries.
6.  **$U(1)$-symmetry**
   - Baryon number.
   - Lepton number. Neutrinoless double beta decay.
   - Individual leptonic numbers. Neutrino oscillations.
   - B-L number.

7.  **$SU(2)$-symmetry**
   - $SU(2)$ transformations
   - Isospin.
   - Isospin symmetry in nucleon-nucleon scattering.
   - Isospin symmetry in pion-nucleon scattering.
   - Isospin and electric charge. Hypercharge.
   - G-parity.

8.  **$SU(2) \times U(1)$-symmetry. $SU(3)$-symmetry**
   - Strangeness and Hypercharge.
   - $SU(2) \times U(1)$.
   - $SU(3)$-symmetry.

**Part IV. Quarks and Hadrons.**

   - Quarks and Gluons.
   - Deep inelastic scattering. Partons.
   - Asymptotic freedom.

**Literature**

D. Perkins, "Introduction to high energy physics", Addison -Wesley, 2000.
B.R. Martin and G. Shaw, "Particle physics", Wiley, 1997.
G. Kane, "Modern elementary particle physics", Addison-Wesley, 1993.
F. Halzen and A.D. Martin, "Quarks and leptons".
I.S. Hughes, "Elementary particle physics", Cambridge Univpress.
M. Fukugita and T. Yanagida, "Physics of neutrinos", Springer-Verlag, Berlin Heidelberg, 2003.

**LIE GROUPS AND LIE ALGEBRAS**

**Part I**

1.  Introduction

2.  **Basics of Groups**
   - Definitions, the Groups $U(1)$ and $SO(2)$.

3.  **Infinitesimal Transformations, Symmetries and Conserved Charges**
4. The Groups $SU(2)$ and $SO(3)$
   $SU(2)$, it parameterization and its Lie algebra, infinitesimal generators of $SU(2)$, the Lie Algebra of $SU(2)$, counting equations, the group $SO(3)$, parameterization of $SO(3)$, the Lie Algebra of $SO(3)$, the ad action on the Lie Algebra.

5. Representations of the Lie Algebra of $SU(2)$
   Representations, reducible and irreducible representations, finite dimensional irreducible representations of Lie $SU(2)$, the quadratic Casimir, complex conjugate representation and an invariant inner product.

6. Tenor representations and young tableaux
   Tensors, tensors of a Lie algebra, symmetry and reducibility, young tableaux, representations of the Groups $SU(2)$ and $SO(3)$.

7. $SU(3)$ and its Lie Algebra
   The Lie Algebra of $SU(3)$, the Jacobi identity and the adjoint representation, the Cartan subalgebra and roots, fundamental representations of $SU(3)$, the eigenvectors of the adjoint action and the root diagram, a few words from physics, some representation theory, a few more words from physics, highest weights.

Part II

1. Introduction

2. Lie Algebras

3. Matrix groups and their Lie Algebras
   The general linear group $Gl(n)$, the special linear group $Sl(n)$, the orthogonal group $O(n)$, the special orthogonal group $SO(n)$, the unitary group $U(n)$, the special unitary group $SU(n)$, the symplectic group $SP(n)$, some accidental correspondences.

4. The Cartan-Killing Form
   The root space, the Cartan subalgebra and the roots of the $A_n$ series.

5. Simple and semi-simple Lie Algebras
   On the structure of simple Lie algebras, uniqueness of the Cartan-killing form.

6. Some Representation Theory
   Weights, the Weyl group

GENERAL RELATIVITY

Part I: Towards the Einstein equations

1. From the Einstein Equivalence Principle to Geodesics
Introduction, caveats and omissions, motivation: the Einstein equivalence principle, geodesics, metrics and coordinate transformation, Christoffel symbols, geodesics and coordinate transformations.

2. The Physics and Geometry of Geodesics
   The geodesic equation from a variational principle, Newtonian limit, gravitational redshift, locally inertial and Riemannian normal coordinates, more on geodesics and the variational principle, affine and non-affine parametrizations.

3. Tensor Algebra
   From the Einstein equivalence principle to the principle of general covariance, tensors, tensor algebra, tensor densities, a coordinate-independent interpretation of tensors.

4. Tensor Analysis
   Preliminary remarks, covariant derivative for vector fields, invariant interpretation of the covariant derivative, extension of the covariant derivative to other tensor fields, main properties of the covariant derivative, principle of minimal coupling, tensor analysis - some special cases, covariant differentiation along a curve, parallel transport and geodesics, generalisations.

5. Physics in a Gravitational Field
   Particle mechanics in a gravitational field revisited, electrodynamics in a gravitational field, conserved quantities from covariantly conserved currents. conserved quantities from covariantly conserved tensors?

6. The Lie derivative, Symmetries and Killing Vectors
   Symmetries (isometries) of a metric: preliminary remarks, Lie derivative for scalars, Lie derivative for vector fields, Lie derivative for other tensor fields, Lie derivative of the metric and Killing vectors, Killing vectors and conserved quantities.

7. Curvature I: The Riemann Curvature Tensor
   Curvature: preliminary remarks, the Riemann curvature tensor from the commutator of covariant derivatives, symmetries and algebraic properties of the Riemann tensor, the Ricci tensor and the Ricci scalar. An example: the curvature tensor of the two-sphere, Bianchi identities, another look at the principle of general covariance.

8. Curvature II: Geometry and Curvature
   Intrinsic geometry, curvature and parallel transport. Vanishing Riemann tensor and existence of flat coordinates. The Geodesic deviation equation.

9. Towards the Einstein Equations
   Heuristics. A more systematic approach. The weak-field limit. The Einstein equations. Significance of the Bianchi identities. Comments on the initial value problem and the canonical formalism. The cosmological constant. The Weyl tensor and the propagation of gravity.

10. The Einstein Equations from a Variational Principle
    The Einstein-Hilbert action. The matter Lagrangian. Consequences of the variational principle.
Part II: Selected Applications of General Relativity

11. The Schwarzschild Metric
   Introduction, Static isotropic metrics, solving the Einstein equations for a static isotropic metric. Basic properties of the Schwarzschild metric - the Schwarzschild radius. Measuring length and time in the Schwarzschild metric.

12. Particle and Photon Orbits in the Schwarzschild Geometry
   From conserved quantities to the effective potential. Timelike geodesics. The anomalous precession of the perihelia of the planetary orbits. Null geodesics, bending of light by a star.

13. Approaching and Crossing the Schwarzschild Radius
   Infinite gravitational redshift. Vertical free fall. Tortoise coordinates. Eddington-Finkelstein coordinates, black holes and event horizons. The Kruskal metric. Varia on black holes and gravitational collapse.

14. Cosmology I: Maximally Symmetric Spaces
   Preliminary remarks. The cosmological principle. Homogeneous, isotropic and maximally symmetric spaces, the curvature tensor of a maximally symmetric space. The metric of a maximally symmetric space I, II and III. The Robertson-Walker metric.

15. Cosmology II: Basics
   Olber's paradox. The Hubble expansion. Area measurements in a Robertson-Walker metric and number counts. The cosmological red-shift, the red-shift distance relation (Hubble's law).

16. Cosmology III: Basics of Friedman-Roberton-Walker Cosmology
   The Ricci tensor of the Robertson-Walker metric. The matter content: a perfect fluid. Conservation laws. The Einstein and Friedman equations.

17. Cosmology IV: Qualitative Analysis
   The critical density. The big bang, the age of the Universe. Long term behaviour. Density and pressure of the present universe.

18. Cosmology V: Exact Solutions
   Preliminary remarks. The Einstein universe. The matter dominated era, age and life-time of the Universe, the radiation dominated era, the vacuum dominated era.

19. Linearised Gravity and Gravitational Waves
   Preliminary remarks. The linearised Einstein equations. Gauge freedom and coordinate choices. The wave equation. The polarisation tensor. Physical effects of gravitational waves, detection of gravitational waves.

20. Kaluza-Klein Theory I
   Motivation. The basic idea: history and overview. The Kaluza-Klein miracle, the origin of gauge invariance. Geodesics. First problems: the equations of motion.
21. Kaluza-Klein Theory II
Masses from scalar fields in five dimensions, charges from scalar fields in five dimensions. Kinetics of dimensional reduction. The Kaluza-Klein ansatz revisited. Non-Abelian generalization and outlook.

QUANTUM FIELD THEORY II

1. Path Integrals and Quantum Mechanics
Schrödinger and Heisenberg pictures. The kernel, the kernel for small time. Convolution. The Path integral, convolution again. Dependence on $\varepsilon$. Matrix elements. Generating functional of correlation functions.

3. Many Degrees of Freedom and Field Theory Path Integrals
Generalisation to more than one degree of freedom. Quantum field theory. Euclidean space.

4. Perturbation Theory
Gaussian path integrals: finite dimensional integrals, passing to field theory. Turning on the interaction. The Feynman propagator. Leading order calculation of $Z[J]$, the normalisation. Relation to the standard perturbation theory and Wick's theorem, no vacuum bubbles.

5. Path Integrals for Fermions
Grassman variables, apples and pears, integration rules. Generating functionals. Quantum field theory of fermions

6. Gauge Theories
Local gauge symmetry. Vector potentials and covariant derivatives, gauge covariance. Gauge invariant action and field equations. Coupling of fermions.

7. Path Integrals for Gauge Theories
Need for a gauge choice for the photon. A simple model: determining $\Delta_{\varepsilon}(y,x)$, gauge choice independence. Gauge theory, the Fadeev-Popov trick. Fadeev-Popov ghosts.

8. The Standard Model
The Lagrangian of the Standard Model. The electroweak interaction of leptons. QCD Lagrangian and the electroweak interaction of quarks.

THE STANDARD MODEL

1. Introduction
Overview of the Standard Model, heuristic principles for construction of Standard Model. Symmetry. Electromagnetic interactions, the Lagrangian of electrodynamics, P and C symmetry QED Lagrangian, gauge symmetry of the EM interactions.

2. Unification of Weak and Electromagnetic Interactions
Similarity of the weak and EM interactions. Intermediate bosons. Unification and symmetry.

3. Symmetry of the Standard Model
SU(2). $B_{\mu}$ - Field, SU(2) x U(1)
4. \textbf{γ and Z\(^0\) boson.}
Weak mixing angle. Interaction of the right handed components. Other multiplets. Weak isospin and weak hypercharge.

5. \textbf{Gauge Invariance of the Standard Model}
W-bosons as gauge bosons. Gauging the symmetry, U(1) case. SU(2) gauge symmetry. Properties of Yang-Mills Lagrangian. Covariant derivative. Gauge invariant interaction of fermions. SU(2) x U(1) gauge theory. How to construct gauge invariant theory? Mass terms and symmetry breaking. Non-renormalizability due to explicit mass terms in the Lagrangian. High energy behaviour of theory.

6. \textbf{Spontaneous Symmetry Breaking}
Higgs mechanism. Higgs potential, two field example. U(1) invariance. Continuous set of degenerate ground states. Choice of ground state breaks symmetry. Wrong sign of mass term for scalar field. Flat direction and appearance of massless scalar field. Goldstone theorem. Spontaneous violation of discrete symmetry. More scalar fields.

7. \textbf{Goldstone Theorem}
Spontaneous violation of symmetry and Goldstone theorem at quantum level. Vacuum and symmetry. Vacuum expectation value (VEV). On the translation invariance of the vacuum. Spontaneous violation of symmetry and masses of particles. Proof of Goldstone theorem.

8. \textbf{Higgs mechanism}
Gauge interactions of the scalar boson. Adding gauge invariant Lagrangian for the scalar field. YM Lagrangian. Spontaneous violation of symmetry. Effects of gauge interactions: two important terms, mass term for gauge field, gauge condition. Gauge fixing, radiation gauge, gauge fixing Lagrangian. No massless boson in theory, would-be Goldstone boson. Unitary gauge. Summary.

9. \textbf{Patterns of Symmetry Breaking}
O(3) gauge symmetry breaking. Examples of symmetry breaking in SU(2). Two theorems. Field counting.

10. \textbf{Standard Model of the Electroweak Interactions}
Higgs mechanism in the Standard Model. Spontaneous violation of SU(2) x U(1). Masses of vector bosons. Masses, VEV, mixing, weak mixing angle. Masses of fermions. Concluding remarks about Higg's mechanism in SM. Summary of SM.

11. \textbf{Generations of Fermions. Fermion Mixing}
Generations of fermions. Charge currents and weak mixing. GIM cancellation. How to get mixing, mixing matrices. Quark mixing matrix. Mixing matrix and CP violation. Determination of the elements of the CKM-matrix. Semileptonic decay of hadrons. Properties of CKM-matrix, unitarity, hierarchy of mixing. Mixing in Leptonic Sector: non-zero neutrino masses and mixing in leptonic sector. Lepton universality. Other processes stipulated by charge currents.
12. Phenomenology of Neutral Currents
   Neutral currents in the SM. Another representation. Properties of NC couplings. 4-fermion interactions due to neutral currents. Processes stipulated by neutral currents, neutrino-electron scattering, neutrino nucleon scattering.

13. Physics of W and Z Bosons
   Masses of W and Z. Decays and production of W and Z. Production of W and Z in proton-antiproton colliders. Physics of Z: Invisible width, number of generations, asymmetries and parity violation, polarisation asymmetry, left-right asymmetry. Other parameters of Z.

14. Higgs Boson Physics
   Properties of the Higgs boson, mass, couplings and decays. Searches for Higgs boson, bounds on Higgs mass. Searches of Higgs at LEP. Higgs mass and precision tests of the SM. Future searches of Higgs. Theoretical bounds on Higgs mass. Corrections to Higgs potential. Triviality bound.

15. Anomalies and the Standard Model
   Anomalies in QED. Axial current anomaly. In Yang-Mills theories. Anomalies in the Standard Model. Anomalies and B and L.

16. Neutrino oscillations

SUSY FIELD THEORY

1. Introduction to Supersymmetry (SUSY)
   Spinor representation of the Lorentz group. Graded Lie Algebra of (N=1) supersymmetry. Component fields. SUSY transformation of fields. Multiplets.

2. Superfields
   Grassmann variables, superspace, superfields. Constraints. Scalar, chiral superfields, renormalizable supersymmetric action for scalar superfields. Field equations. R-invariance. Vector superfields.

3. Gauge transformations
   Gauge invariance. SUSY and U(1) gauge invariant action

4. Supersymmetry breaking
   Spontaneous SUSY breaking. O'Raifeartaigh mechanism. Fayet-Iliopoulos mechanism. SUSY QED.

5. Why SUSY?
   Softening of quantum divergences. Gauge hierarchy problem, chiral protection of fermion masses, gauge protection for the gauge boson masses, scalar masses protected in SUSY. supergravity as a good candidate for unification, SUSY as most general symmetry of S-matrix, Coleman-Mandula theorem.

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