Service Guarantees Countering Renewable Generation Uncertainty in Multi-microgrids

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Abstract—With increased penetration of Renewable Energy Systems (RES), the conventional distribution grid is advancing towards Interconnected multi-microgrid systems (IMMG) supervised by a Distribution Network Operator (DNO). However, the inherent uncertainty of RES poses a challenge in meeting the power demand of critical infrastructures in the microgrids unless sufficient battery storage is maintained. Yet, maintaining expensive battery storage increases the operating cost. In this article, we propose a dynamic energy resource allocation strategy to optimize the battery reserve requirement while ensuring that critical demand is met with a provable guarantee. Our solution is built upon stochastic control techniques. Under our proposed scheme, the DNO responds to the evolving uncertainty by dynamically balancing the RES and battery resources and eliminates the risk of over or underproduction. We derive battery reserve allocation strategy for multi-microgrid systems in two settings: when microgrids can (interconnected) and, cannot (individualized) share power amongst each other. We present numerical simulations under different scenarios with detailed comparison of the performance of the proposed algorithm for individualized and shared settings. The simulation results demonstrate the efficacy of our algorithm. In particular, it quantifies value of connecting microgrids through savings in battery requirements under IMMG over individualized microgrids.

Index terms: Multi-microgrid, optimization, partial differential equations, renewable energy sources, resource allocation, stochastic process, uncertainty minimization, wind energy.

I. INTRODUCTION

In recent times, the proliferation of distributed energy resources (DER), propelled by a mounting interest towards the generation and utilization of clean renewables, has fostered a paradigm change in the operation and control of power distribution networks. Renewable energy sources (RES), like solar and wind energy systems, reduce the adverse environmental effects of traditional energy sources; integration of DERs also offer improved grid assistance, efficiency and reliability. As renewables are expected to contribute 60% of total generation by 2050 [1], a rapid growth of ‘prosumers’, equipped with distributed RES, is envisaged to transform the conventional energy systems to an ecosystem of multiple individual microgrids (MGs). Here, due to the intermittent nature of RES, the MGs are exposed to the risk of underproduction if extra measures such as mechanisms for ancillary services are not present [2]. Similarly, uncertainty of RES, exacerbated by excess provisioning of ancillary services, such as energy storage, can result into overproduction which increases the operating cost. Since both overproduction and underproduction are undesirable, an important question that needs to be answered is: how can the MGs guarantee this demand supply balance? In this context, energy storage allocation for demand-supply balance, considering fluctuating renewable generation, is of significant interest [3]–[7]. However, works which are primarily focused on guaranteeing load demand of multiple critical facilities under generation uncertainty, in a multi-microgrid framework, are still in their nascent stages. In the context of system reliability, various optimization techniques [5]–[9] are used. Analytical approaches to improve power balance in systems with multiple MGs are devised in [10]–[11]. Prior works have also considered game theoretic approaches [12]–[13], stochastic programming [14], and, model-predictive control [15] to improve system resiliency with high penetration of RES, however without explicitly modelling the RES output. A battery charging and discharging policy is proposed in [16], to reduce the power imbalance due to RES uncertainty with explicit stochastic model for RES. However, no provable guarantee of meeting the demand is provided.

The problem of assuring the demand-supply balance is coupled with proper provisioning of battery energy storage. Maintaining excess battery storage increases the investment and operating cost. Therefore, determining the optimal amount of energy storage to be maintained, with uncertain RES, is an important issue [17]–[21]. In an Individualized Multi-microgrid (IDMMG) system, energy sharing among MGs is not allowed which leads to large energy reserve requirement. In this context, collaborative operation across multiple facilities allows for risk pooling between RES and reduce the storage needed to maintain the guarantee of demand-supply balance. This leads to an integrated and flexible framework of Interconnected Multi-microgrids (IMMGs). However, the additional benefits of IMMG over IDMMG must be quantified to justify the utility of IMMG. Various optimization algorithms focusing on cost benefit of multi-microgrid architectures are proposed in [22]–[25]. Note that guarantee of supplying critical loads under unforeseen generation outage is difficult to obtain here as the methodologies depend on RES forecast or historical values. A careful observation reveals that most of the existing works either focus on cost optimization for energy trading in IMMG without addressing the supply guarantee required by the critical infrastructures (e.g. hospitals), or, solely consider energy storage optimization without exploring...
the benefit of cooperation under RES uncertainty. In summary, assurance of meeting the power demand of critical infrastructures in multi-microgrid setting along with quantitative analysis of cooperation benefit is limited in literature.

To this end, in this article we focus on an IMMG framework consisting of centralized battery storage and multiple small-scale MGs with critical infrastructures that require a guarantee of supplying the demand requested at a pre-specified future time. Each MG is equipped with RES which are uncertain in nature, thus posing a risk of not being able to meet the local critical demand. To mitigate this uncertainty, the Distribution network operator (DNO), which supervises the IMMG, allocates battery storage to support the MGs while managing how they share renewable energy with each other. The focus of the article is on the optimal allocation of battery storage and dynamic re-balancing by the DNO to meet the power demand of each MG at the requested time to create a reliable grid-independent IMMG system. The major contributions are summarized below:

(i) We study an IMMG managed by a DNO, which controls the energy sharing among the MGs. We provide a solution to the problem of initial battery storage provisioning with the objective of meeting power demand of each prosumer at a future pre-specified time with provable guarantees, in presence of stochasticity of the RES. Such a guarantee is essential for critical applications under catastrophic events. Here, we establish that the optimal initial provisioning of energy storage decided by our algorithm is such that (i) any smaller provisioning will lead to scenarios where demand cannot be met, (ii) any larger provisioning will lead to scenarios where generation exceeds the demand. We emphasize that with our optimal initial provisioning both cases (i) and (ii) above are avoided almost surely, that is, with probability one. Thus initial battery provisioning per our solution eliminates risks of underproduction and overproduction at the future pre-specified time, when critical demand is requested, with provable guarantee that the demand is met even in the presence of uncertainty in RES.

(ii) We provide exact solutions for continuous time resource allocations for the DNO to manage renewable and battery resources. Our solution builds upon stochastic control techniques. We provide a practical algorithm for discrete time battery resource management. We remark that such a solution is pertinent for supporting critical infrastructures.

(iii) We present a comparative study with a IDMMG based approach where energy sharing among the prosumers is not possible and the DNO is solely responsible to meet the critical load demands of the prosumers by dynamic allocation of renewable and battery storage. The solution is compared to the IMMG framework to show the advantage of cooperation in IMMG with high penetration of DER and critical loads.

The rest of the paper is organized as follows: The description of the IMMG is provided in Section II. A problem formulation is introduced in Section III. Section IV describes the proposed resource allocation algorithms for both IDMMG and IMMG and the simulation results are shown in Section V. Section VI presents the concluding remarks.

II. SYSTEM DESCRIPTION

Renewable based large distribution networks, with centralized battery storage [26], consist of several small-scale MGs geographically apart from each other, under the supervision of DNO. We consider such an islanded distributed framework constituting an IMMG, where the DNO can operate the network more efficiently than IDMMG by utilising the ability of cooperation between the MGs. Fig. 1 shows the hierarchical architecture of the IMMG under consideration, which contains \( N_m \) microgrids and a DNO which controls the energy transaction among the microgrids and dispatches the centralized battery units, based on the requirements, to maintain system reliability. Each MG has RES to supply its local demands. However, the uncertain nature of RES poses the risk of either power deficit, when generation is less than demand, or excess power that forces curtailment, when generation exceeds demand. In IDMMG, this risk is minimized by using independent energy storage systems such as batteries for individual MG, thus requiring large ancillary battery energy storage systems (BESS) [27]. In contrast, an IMMG, as shown in Fig. 1 enables MGs to share energy among each other in a controlled fashion to maintain demand-supply balance, where excess generation from one MG is utilized to minimize the deficit of another, instead of power curtailment, thus reducing the amount of BESS. DNO controls the energy transactions and the dispatch of batteries. A communication layer is used to transmit real-time power measurements of individual MG to aid the DNO for efficient resource allocation. To reduce the energy cost, the DNO needs to estimate optimal battery storage while ensuring that the load demands of the MGs are met, irrespective of the irregularities of RES. In the subsequent sections we formally introduce the problem and provide a solution along with a comparison with IDMMG.

![IMMG architecture with centralized BESS and microgrids with RES.](image)

Fig. 1. IMMG architecture with centralized BESS and microgrids with RES.

III. PROBLEM FORMULATION

Let \( \mathcal{I} = \{1, 2, \ldots, N_m\} \) denote the set of microgrids present in the distribution network. Each microgrid has its own renewable generation unit (ReGU) producing \( P_{gi}(t), i \in \mathcal{I} \), units of power at time \( t \), which is stochastic in nature and is modeled by Geometric Brownian Motion (GBM) given by:

\[
dP_{gi}(t) = \mu_{gi} P_{gi}(t) dt + \sigma_{gi} P_{gi}(t) dW_{ti},
\]

with an initial condition, \( P_{gi}(0) > 0 \), for all \( i \in \mathcal{I} \). The constants \( \mu_{gi} \) and \( \sigma_{gi} \) are the drift and volatility terms respectively and \( W_{ti} \) denotes a Weiner process. We empirically verify this
model assumption with RES generation data in Section V. The Weiner processes \( W_{i1}, W_{ij} \) are correlated according to
\[
dW_{i1}dW_{ij} = \rho_{ij}dt, \quad i,j \in I, \tag{2}
\]
where \( \rho_{ij} \) is the correlation coefficient indicative of correlation between RES output of closely sited microgrids. Therefore,
\[
dP_{g1}dP_{gj} = \sigma_{s_j} \rho_{ij} \hat{P}_{g1} \hat{P}_{gj}dt, \quad i,j \in I. \tag{3}
\]
Each microgrid also postulates a constant critical power demand of \( D_{ci} \) units, \( i \in I \), at a future time denoted as \( T_f \). In IMMG, the objective of the system is to satisfy power demand and supply balance at \( T_f \), almost surely with minimum amount of resources collectively. In case of \( P_{gi}(T_f) \geq D_{ci} \) for microgrid \( i \), it can ensure sustained operation of its own load demand. In addition, the excess generation of \( D_{ci} - P_{gi}(T_f) \) units can be used to supply the load demands of other microgrids, thus requiring less overall energy storage capacity to maintain grid reliability. Note, that this excess generation will need to be curtailed in IDMMG. On the other hand when \( P_{gi}(T_f) < D_{ci} \), to avoid the risk of microgrid \( i \) not being able to meet the power demand of its critical loads, it can be supported by either extra renewable generation from other microgrids or the energy storage maintained by the DNO. To avail these benefits of IMMG, microgrids participate in a contract with the DNO, which controls the energy transactions and battery allocation for the whole system by independently maintaining a portfolio consisting of fractions of ReGUs from each participant microgrid and centralized battery units. In particular, as shown in Fig. 1, the ReGUs in the portfolio of the DNO are physically located at the premises of the individual microgrid, thus following same dynamics (4), while the battery storage, being shared by all the microgrids, is fully controlled and allocated by the DNO. Considering the battery storage has the ability to dispatch in smaller units, each producing constant \( P_b \) units of power, the total available power in the portfolio maintained by the DNO at any time \( t \) is given by,
\[
V(P_{g1}(t), \ldots, P_{gN_m}(t), t) = \sum_{i=1}^{N_m} [a_i(t)P_{gi}(t)] + b(t)P_b, \tag{4}
\]
where, \( a_i(t), i \in I \), is the number of ReGUs allocated from microgrid \( i \) and \( b(t) \) is the number of battery units in the portfolio. At time \( t = T_f \), we assume that a total of \( m \), with \( 0 \leq m \leq N_m \), microgrids will have enough individual generation to supply their local demand. We denote the set of indices of the microgrids with sufficient generation at time \( T_f \) as \( I_s \), with \( |I_s| = m \), and let \( I_d \) be the set of indices of the microgrids not capable of supplying their own local demand by renewable generation individually at time \( T_f \); thus \( |I_d| = N_m - m \). We have \( P_{gi}(T_f) \geq D_{ci} \) if \( i \in I_s \) and \( P_{gi}(T_f) < D_{ci} \) if \( i \in I_d \). The DNO is responsible for supporting the \( j \)th microgrid, \( j \in I_d \), by utilizing either the surplus power (if available) from other microgrids, or the battery units in its portfolio. We remark that, the knowledge of the sets \( I_s \) and \( I_d \) are unknown \textit{a priori} at initial time. Therefore, a focus on the time evolution of the ReGUs and the battery units from \( t = 0 \) till the time \( t = T_f \) is required to guarantee that the demand is met at the requested time \( T_f \). At time \( t = T_f \), there are two possibilities:
\[
(1) \sum_{i=1}^{N_s} P_{gi}(T_f) \geq \sum_{i=1}^{N_s} D_{ci} \tag{5}
\]
In this case, collectively the microgrids have sufficient renewable generation to supply critical load demands of each individual microgrid. Hence, no additional power from battery units is required.
\[
(2) \sum_{i=1}^{N_m} P_{gi}(T_f) < \sum_{i=1}^{N_m} D_{ci} \tag{6}
\]
Here, the total deficit of \( \sum_{i=1}^{N_s} D_{ci} - P_{gi}(T_f) \) units of power is required to be maintained in the portfolio of the DNO to support the critical load demand of microgrid \( j \in I_d \), from the combination of battery units and surplus power, if any, from microgrid \( i, i \in I_s \). Note that, if microgrid \( j \in I_d \) also has a local battery storage, capable of supplying \( b_j \) kW of power, it can adjust its local demand from \( D_{cj} \) kW to \((D_{cj} - b_j)\) kW, which would be requested from the DNO, if required.

To address both the possibilities with minimum number of energy resources, the DNO needs to balance its own power portfolio (4). In this context, we introduce a constraint of \textit{rated power conservation} of the portfolio, which requires:
\[
\sum_{i=1}^{N_m} [a_i(t)P_{gi}(t)] + b(t)P_b = 0, \tag{7}
\]
for \( t \in (0, T_f] \) to ensure that the change in total power output of the generation portfolio due to the changes in the number of battery units and the number of ReGUs is zero; thus the change in total power generated by the portfolio maintained by DNO is only due to the change of renewable generation of individual microgrid. Thus, after initial allocation \( a_i(0), i \in I_s \), and \( b(0) \), the DNO, at a future time \( t \), can change the number of battery units but has to ensure that the power change is compensated by exchanging with the ReGUs in the portfolio.

Therefore, the problem from the perspective of the DNO is:
\[
\text{Determine the number, } b(t) \text{ of battery blocks and number, } a_i(t) \text{ of renewable generation units based on } P_{gi}(t), i \in I_s, \text{ such that } \sum_{i=1}^{N_m} [a_i(t)P_{gi}(t)] + b(t)P_b = 0, \tag{5}
\]
\[
\sum_{i=1}^{N_m} [a_i(T_f)P_{gi}(T_f)] + b(T_f)P_b = \sum_{i=1}^{N_m} r_i[D_{ci} - P_{gi}(T_f)], \tag{6}
\]
for \( t \in (0, T_f) \), where \( r_i = 1 \) if \( i \in I_d \) and \( r_i = 0 \) if \( i \in I_s \).

Note that, under the \textit{rated power conservation} constraint, finding the amount of initial battery units and initial ReGUs is essential for the DNO as having a lesser number of batteries and ReGUs has a risk of not being able to provide \( D_{ci} \) units of power at time \( T_f \) for all \( i \in I_d \), whereas provisioning more may result in excess energy produced at \( t = T_f \) that will lead to a loss of revenue to the DNO. Section V of this article presents the proposed strategy to find the resource allocation policy in real-time under both IDMMG and IMMG framework. Our proposed strategy can be utilized to support critical infrastructures by providing guaranteed supply of critical demand even in islanded mode of operation.

IV. Solution Methodology

A. Individualized Multi-microgrid System

First, we consider the case of IDMMG where power sharing among the microgrids is not possible and each microgrid is treated independently by the DNO \textit{i.e.} available power in the portfolio maintained by the DNO for each microgrid is:
\[
\hat{V}_i(P_{gi}(t), t) = \hat{a}_i(t)P_{gi}(t) + \hat{b}_i(t)P_b, \quad i \in I, \tag{8}
\]
Theorem 1.\footnote{See Appendix B} Under the rated power conservation constraint \((\ref{eq:power_conservation})\), suppose \(\hat{a}_i(t)\) and \(\hat{b}_i(t)\) are given by:

\[
\hat{a}_i(t) = -\Phi \left[ \frac{\ln \left( \frac{P_{gi}(T_f)}{2\sigma_i^2} \right) - \frac{\sigma_i^2}{2}(T_f - t)}{\sigma_i \sqrt{T_f - t}} \right],
\]

\[
\hat{b}_i(t) = \frac{D_{ci}}{P_{gi}} \Phi \left[ \frac{\ln \left( \frac{P_{gi}(T_f)}{2\sigma_i^2} \right) + \frac{\sigma_i^2}{2}(T_f - t)}{\sigma_i \sqrt{T_f - t}} \right], \quad t \in [0, T_f)
\]

where, \(\Phi(\cdot)\) is the standard normal cumulative distribution function, then the terminal condition specified by \((\ref{eq:terminal_condition})\) is satisfied almost surely at time \(T_f\).

Proof. Please see Appendix B. \(\Box\)

It follows that, in case of IDMMG, the total amount of battery units required by the DNO for all \(t \in [0, T_f)\) is:

\[
\hat{b}(t) = \sum_{i=1}^{N_m} \frac{D_{ci}}{P_{gi}} \Phi \left[ \frac{\ln \left( \frac{P_{gi}(T_f)}{2\sigma_i^2} \right) + \frac{\sigma_i^2}{2}(T_f - t)}{\sigma_i \sqrt{T_f - t}} \right], \quad t \in [0, T_f)
\]

while the total portfolio power value at \(t = T_f\) is:

\[
\hat{V}(P_{g1}(T_f), \ldots, P_{gN_m}(T_f), T_f) = \sum_{i=1}^{N_m} \max(D_{ci} - P_{gi}(T_f), 0).
\]

B. Interconnected Multi-microgrid System

Let us now consider the case of IMMG where the power available in the portfolio maintained by the DNO is given by \((\ref{eq:total_power_portfolio})\). In contrast to the IDMMG, here a single power portfolio is maintained instead of \(N_m\) independent portfolios. The IMMG needs to ensure that the DNO is able to make up for the deficit \(D_{ci} - P_{gi}(T_f)\), \(i \in I_d\) at \(t = T_f\) from the surplus generation \(P_{gj}(T_f) - D_{cj}, j \in I_s\), thus requiring less amount of centralized battery units \(b(t)\). In the following sections, we provide a policy of maintaining the number of shared battery units, \(b(t)\) and ReGUs, \(a_i(t), i \in I\) for all \(t \leq T_f\), that ensures that the power deficit is optimally met with guarantees given the knowledge of \(I_d\) and \(I_s\) is unknown a priori. Let the value of power portfolio, \(V\) at any time \(t \in [0, T_f)\) be denoted by \(V_t\). In case of IMMG, the terminal condition is given by,

\[
V_{T_f} = \max \left( \sum_{i=1}^{N_m} (D_{ci} - P_{gi}(T_f)), 0 \right).
\]

Finding an analytical solution to \(a_i(t), b(t)\) for all \(t \in [0, T_f)\) under the terminal condition \((\ref{eq:terminal_condition})\) is intractable due to higher dimensionality of \((\ref{eq:imbalance})\) \(\Box\)

Let, \(\eta_{gi} \equiv \frac{\mu_{gi}}{\sigma_{gi}^2}\) and consider a function \(\gamma_{ti}(x) \equiv e^{-\eta_{gi}x - \frac{x^2}{2}\eta_{gi}^2}}\. In the following sections, we provide a numerical solution in the following sections.

Let, \(\hat{W}_{ti}(x) \equiv \gamma_{ti}(x)d\hat{W}_{ti}(u) = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{x} e^{-\frac{(u - \eta_{ti}(u))^2}{2t}} du\), for all \(i \in I\). Then define,

\[
\hat{W}_{ti}(x) := \frac{1}{2\pi t^2} \int_{-\infty}^{x} e^{-\frac{(u - \eta_{ti}(u))^2}{2t}} du.
\]

\[
F_{\hat{W}_{ti}}(x) \equiv \int_{-\infty}^{x} \gamma_{ti}(u)d\hat{W}_{ti}(u) = \frac{1}{\sqrt{2\pi t}} \int_{-\infty}^{x} e^{-\frac{(u - \eta_{ti}(u))^2}{2t}} du.
\]

\[
F_{\hat{W}_{ti}}(x) \text{ is the distribution function of the stochastic process } \hat{W}_{ti} := \hat{W}_{ti} + \eta_{ti}, \text{ where } \hat{W}_{ti} \sim N(0,t) \text{ for all } i \in I. \gamma_{ti}(x) \text{ can be used to define a transformed probability measure given by } (\ref{eq:transformed_measure}). \text{ Let us denote the expected value of a random variable under this transformed probability measure by } \hat{E}[]\.

The following theorem provides the solution of \(V_t\) for all \(t \in [0, T_f)\) under this transformed probability measure.

Theorem 2. For any \(t \leq T_f\) the expected value of the portfolio at \(t_f\), which can be calculated numerically \((\ref{eq:expected_value})\), as described below, to find the provisioning policy of the ReGUs and battery units, \(a_i(t), i \in I\) and \(b(t)\) respectively, for \(t \in [0, T_f]\).

Let the time horizon \([0, T_f]\) be partitioned into \(N\) equal intervals \(0 = t_0 < t_1 < \ldots < t_N = T_f\) such that \(t_n = n\Delta t, n \in \{0, 1, \ldots, N\}\) where \(\Delta t = T_f/N\) denotes the length of each interval. The vector of renewable generation values at \(t = t_n\) is denoted by \(P_{G}(t_n) = [P_{g1}(t_n) \ldots P_{gN_m}(t_n)]^T\). We assume that at the end of each time interval, each ReGU output can either increase or decrease by a constant factor of \(u_i(\geq 1), d_i(< 1)\) respectively, with respect to the current values, with certain probabilities. Here, \(P_{G}(t_{n+1})\) can take one of the \(2^{N_m}\) possible values with corresponding probabilities \(P_k \in \mathbb{P} \in \mathbb{R}^{2^{N_m} \times 1}\). The time evolution of \(P_{G}(t_n)\) is shown in Fig. 3 for two consecutive time intervals and \(N_m = 2\) for brevity of exposition. Without loss of generality, imposing the constraint, \(u_1d_1 = 1, i \in I\), we can estimate the values of \(P_k, u_i, d_i\), given \(P_{G}(t_n), \mu_{gi}, \sigma_{gi}, \rho_{gi}, i \in I\), by comparing the expectation, variance and covariance of the stochastic processes, under the transformed probability measure, with that from the discrete \(N_m\)-ary tree. The detailed procedure is given in Appendix A. Let \(U \in \mathbb{R}^{2^{N_m} \times N_m}\) denote a matrix
where $k^{th}$ row of $U$ contains the movement factors of $P_G(t_n)$ associated with probability $P_k$, $k \in \{1, 2, \ldots, 2^{N_m}\}$. Once the values of $P_k, u_i, d_i$ are determined, we divide the algorithm in three sequentially executed parts as explained below.

1) **Forward propagation**: Let us consider that our objective is to find the values of $a_i(t_n), i \in I$ and $b(t_n)$ where $t_n < T_N = T_f$. Given the current value of the renewable generation vector, $P_G(t_n)$, first we estimate all possible values of $P_G(t_N)$ with corresponding probabilities as shown in Fig. 2. The general algorithm is given in Algorithm 1.

2) **Backward propagation**: Using the results obtained in the forward propagation step and values of $D_{ei}, i \in I$, the value of $V_{t_n}$ can be computed using (14) for all possible points at $t = T_n$. To estimate $V_{t_n}$, the portfolio value is back propagated from $t = T_N$ till $t_n$, using (14) at each $t_k, k \in \{N-1, N-2, \ldots, n\}$. For example, in case of $N_m = 2$, as shown in Fig. 2, $V_{t_n} = \sum_{i=1}^{N} P_i V_{t_{n+1}}$, where $V_{t_{n+1}i}$ are estimated from preceding back propagation steps. The backpropagation algorithm is given in Algorithm 3.

3) **Resources calculation**: Once $V_{t_n}$ is obtained, we can compute the values of $a_i(t_n)$ and $b(t_n)$ calculating the change in portfolio and renewable generation values obtained from forward and backward propagation as shown in Algorithm 3.

The entire algorithm is summarized in Algorithm 1. Each node in the implementation of tree data structure, shown in Fig. 2, has elements $P_G$, to contain ReGU output associated with the node, $V$, to store portfolio value of the node, $p$, which contains probability of encountering the node starting from initial time $t_n$, $ohp$, containing one hop probability of running into the node from its parent and an unique ID, $id$, assigned to each node. In the next section, we provide the simulation results of the algorithm where a IMM with two microgrids and a DNO, operating in hour-ahead market, is considered.

**Algorithm 1: Dynamic Resource Allocation**

**Input:** $P_G(t_n) = \{P_G(t_n)_{i=1}^{N_m}\}, D_e = \{D_{ei}\}_{i=1}^{N_m}, a = \{a_i(t_n)_{i=1}^{N}, P_b, T_f, N, P, U\};$

**Output:** $r = \{r_{i=1}^{N_m+1}\}$ // Resource values

/* Initialization */$
\Delta t \leftarrow T_f/N; n \leftarrow t_n/\Delta t; L, L_n \leftarrow [ ];$
$v \leftarrow 0 \in \mathbb{R}^{2^{N_m}}; r \leftarrow 0 \in \mathbb{R}^{(N+1)\times1};$
$A \leftarrow 0 \in \mathbb{R}^{2^{N_m} \times (N+1)}; A[i, N_m+1] \leftarrow P_b;$
/* Forward Propagation */$
L \leftarrow \mathbb{F}(N - n, U, P, P_G(t_n));$
/* Backpropagation */$
V_{t_n}, L_n \leftarrow \mathbb{B}(L, D_e);$ 
/* Resources Calculation */

if $|L_n| \neq 1$ then
  for $j = 1$ to $2^{N_m}$ do
    $c \leftarrow L_n[j]; v[j] \leftarrow V$ value of $c;$
    $p_g \leftarrow P_G$ value of $c; A[j, 1 : N_m] \leftarrow p_g T;$ 
  end
  $r \leftarrow A^{T} b;$
else
  $r[1 : N_m] \leftarrow a;$
  $r[N_m+1] \leftarrow V_{t_n} - r[1 : N_m] P_G(t_n);$ 
end
return $r;$

**Algorithm 2: Forward Propagation**

**Function** $\mathbb{F}(nStep, U, P, P_G):$

create root node with id = 0, $P = P_G; R_f \leftarrow \text{root};$

for $i = 1$ to $nStep$ do
  for $j = 1$ to $(2^{N_m})^i$ do
    parent $\leftarrow$ get node with id $= pid;$
    $pr \leftarrow p$ value of parent node;
    $C_G \leftarrow P_G$ value of parent node;
    for $k = 1$ to $N_m$ do
      child $\leftarrow$ create new node;
      child. $P_G = C_G \otimes U[k, :]$;
      child. $p \leftarrow pr \otimes P[k];$
      child. id $\leftarrow (2^{N_m} \times 0) + k;$
      child. $ohp \leftarrow P[k];$
      append child to parent;
    if $i = nStep$ then
      | append child to $R_f; $
    end
    $pid \leftarrow pid + 1;$ 
  end
end

return $R_f;$

**Algorithm 3: Backpropagation**

**Function** $\mathbb{B}(L, D_e):$

foreach $c \in L$ do
  $p_g$ $\leftarrow$ member $P_G$ of node $c;$
  $tv \leftarrow$ max $\sum_{i=1}^{2^{N_m}} D_e[i] - p_g[i], 0;$
  update value of member $V$ of $c$ with $tv;$
end
$L_c, L_n \leftarrow L; end \leftarrow |L_c|; ev \leftarrow tv;$
while end $\neq 1$ do
  $L_p \leftarrow \text{root}; numNodes \leftarrow end;$
  while numNodes $> 0$ do
    $s \leftarrow 2^{N_m}$ siblings from $L_c;$
    $v \leftarrow V$ values of each node in $s;$
    $p_s \leftarrow ohp$ values of each node in $s;$
    $parent \leftarrow$ parent node of siblings in $s;$
    $parent. V \leftarrow \sum_{i=1}^{2^{N_m}} v[i] p_s[i];$
    append parent to $L_p; numNodes \leftarrow numNode - 2^{N_m};$
  end
  $L_c \leftarrow L_p; end \leftarrow |L_c|;$
  if end $= 2^{N_m}$ then
    | $L_n \leftarrow L_c;$
  end
end

return ev, $L_n;$

V. Simulation results

A. Model assumption verification

First we analyse the empirical wind power time series data to validate the model assumption (1) and estimate the parameters of the stochastic process. Wind generation data from the northwestern part of USA is taken from [30] for the
month of June, 2020 at an interval of 5 minutes. As typical wind generation dynamics vary over different hours of a day \cite{31}, it is necessary to modify the parameters of \cite{1} according to period of interest during a day. In our case, wind data from 10 a.m.-5 p.m. is chosen and maximum likelihood estimator is used to find drift and volatility parameters. The estimated drift and volatility parameters are 0.007 and 0.027 respectively with chi-square goodness of fit score of 18.86 including 13 degrees of freedom. The p-value, calculated from the goodness of fit test with significance level $\alpha=0.05$, is 0.128 which justifies the validity of GBM model of wind power dynamics \cite{32}.

\section*{B. Parameters and sample size determination}

For the simulation, two MGs with renewable generations, $P_{g1}(t)$ and $P_{g2}(t)$ with parameters $\mu_{g1} = 0.006$, $\sigma_{g1} = 0.03$ and $\mu_{g2} = 0.005$, $\sigma_{g2} = 0.04$ respectively, are considered and realized using discrete approximation of \cite{1}. We also assume that the generations are correlated with a correlation coefficient of $\rho_{12} = 0.6$ \cite{31}. We assume that the battery unit power is $P_b = 1$ kW. The critical load demands, $D_{c1} = 20$ kW and $D_{c2} = 25$ kW, of the two MGs respectively, are to be met at $T_f = 5$ hrs. We use Algorithm \cite{1} to compute optimal provisioning of initial resources and resource updates at each time instant, under rated power conservation constraint, that satisfies power balance at $T_f$. We assume that the ReGUs and the battery units are adjusted at an interval of one hour.

We first focus on the number of samples needed to arrive at meaningful statistical inferences. Fig. 3 shows the density histogram plot of $P_{g1}(t)$ for different time instants for two sample sets each containing 10000 samples of $P_{g1}(t)$ drawn from numerical simulation of the dynamic equation \cite{1} with $\mu_{g1} = 0.006$, $\sigma_{g1} = 0.03$ and initial condition, $P_{g1}(0) = 20$ kW. Both sample sets consist of samples of $P_{g1}(t)$ with the terminal case of $P_{g1}(T_f) > D_{c1}$. We compare the distribution of the samples, at $t = 1, 2, \ldots, 5$, in the two sample sets, using Two-sample Kolmogorov–Smirnov (KS) test \cite{33} which produces KS statistics values $0.008$, $0.01$, $0.0062$, $0.0079$ and $0.0094$ respectively. For sample size of 10000 in each set, the critical value of KS statistics is $0.019$. The lower values of KS statistics than the critical value indicates that simulation of our algorithm for 10000 independent runs can produce highly stable results. Similar conclusions can be drawn for other terminal cases of $P_{g1}(T_f)$ and $P_{g2}(t)$, such as, $P_{g1}(T_f) \leq D_{c1}$, $P_{g2}(T_f) > D_{c2}$ and $P_{g2}(T_f) \leq D_{c2}$.

Here, in figures, Fig. 4 simulation results are shown for three sets of 10000 runs, each set satisfying different terminal conditions of $P_{g1}(T_f)$ and $P_{g2}(T_f)$ w.r.t $D_{c1}$ and $D_{c2}$ respectively, with mean values plotted for renewable generations along with standard deviations in shaded regions. For all other resources, mean values along with 95\% bootstrap confidence interval (CI) around the mean, with 10000 bootstrap samples, are plotted. The narrow 95\% CI around the means for battery requirement, power portfolio and BESS reduction, as seen from Fig. 4 indicate high confidence of the outcomes of our proposed algorithm. Implementation is done in Python 3.8.

\section*{C. Case study 1: $P_{g1}(T_f) \geq D_{c1}$, $P_{g2}(T_f) \geq D_{c2}$}

Fig. 4\textsuperscript{a} illustrates the scenario where the renewable generations of both the MGs are sufficient to meet respective load demands at $t = T_f$. We remark that the algorithm has no knowledge of the terminal state \textit{a priori} at any time $t < T_f$. We observe that the battery units requirement and the power portfolio value become 0 at $t = T_f$ as expected for both IDMMG and IMMGG. The trajectories of $b(t)$, $b(t)$ and average reduction in battery units in IMMGG (shown in Table \ref{tab:table1} and Fig. 5) reveal that IMMGG enables DNO to reduce battery

\begin{table}
\centering
\caption{Sample set 1 and 2 comparison}
\begin{tabular}{|c|c|c|c|}
\hline
Sample & Set 1 & Set 2 & Difference \\
\hline
\textit{p} & 0.05 & 0.07 & 0.02 \\
\hline
\end{tabular}
\end{table}
Fig. 4. (a) Both MGs are capable of supplying their own demand at $T_f$, requiring no battery and power to be maintained in portfolio for both IDMMG ($V(t)$, $b(t)$) and IMMG ($\hat{V}(t)$, $\hat{b}(t)$). (b) Excess generation from MG 1 is utilized to supply demand of MG 2 in IMMG at $T_f$, whereas in IDMMG, average battery units’ requirement is 25 to guarantee that the demand is met. (c) Both MGs are not capable of supplying own demand requiring same battery amount for both IDMMG and IMMG. For all cases, power required in the portfolio is lower in IMMG than IDMMG. Standard deviation of 10,000 realizations of $P_{g1}(t)$ and $P_{g2}(t)$ are shown in shaded regions with mean values plotted with solid lines. Zoomed inset, showing an example of 95% CI around the mean of battery requirement for IMMG at $t = 2$ hrs, indicates high confidence in average battery requirement of our proposed scheme.

requirement by 36.94% and the average reduction in initial battery requirement is 13.5% over IDMMG.

D. Case study 2: $P_{g1}(T_f) \geq D_{c1}, P_{g2}(T_f) < D_{c2}$

Fig. 5. Reduction in battery storage in IMMG as a percentage of the same in IDMMG for three case studies. For all $t < T_f$, in IMMG, DNO requires less amount of battery units compared to IDMMG while ensuring the demand is met at $T_f$ for all cases. The narrow 95% CI around the mean battery reduction percentage, shown in zoomed inset at $t = 2$ hrs for case 2 as example, indicates high confidence in average battery reduction in our proposed algorithm.

Table I

| Time (hrs.) | $P_{g1}(T_f) \geq D_{c1}$ | $P_{g1}(T_f) \geq D_{c1}$ | $P_{g1}(T_f) < D_{c1}$ |
|-------------|---------------------------|---------------------------|---------------------------|
|             | $P_{g2}(T_f) \geq D_{c2}$ | $P_{g2}(T_f) < D_{c2}$   | $P_{g2}(T_f) < D_{c2}$   |
| 0           | 13.50                     | 13.50                     | 13.50                     |
| 1           | 24.80                     | 20.43                     | 14.66                     |
| 2           | 43.96                     | 32.19                     | 15.20                     |
| 3           | 70.41                     | 51.57                     | 15.49                     |
| 4           | 68.98                     | 77.85                     | 15.31                     |
| 5           | 0                         | 100                       | 0                         |
| Overall     | 36.94                     | 49.26                     | 12.36                     |

Fig. 6. Incorrect initial battery allocation leads to the risk of overproduction or underproduction. $V_d(t)$ denotes the power portfolio trajectory when initial battery allocation is 20% more than optimal, $b(0)$, derived in Case 3. Here, at the end of the horizon, DNO has overproduction. $V_d(t)$ shows the scenario when initial battery allocation is 20% less than optimal. Here, DNO cannot produce required power to support the critical loads at the requested time $T_f$. 

Fig. 5 illustrates the scenario where microgrid 1 is capable of supplying its load demand of $D_{c1} = 20$ kW but renewable generation of microgrid 2 is not sufficient to meet $D_{c2} = 25$ kW at $t = T_f$ (here too the terminal state is not available a priori at any time $t < T_f$). Here the power deficit of
microgrid 2 is more than the surplus generated from microgrid 1. The plot shows that at the end of the time horizon, IMMG enables DNO to utilize the surplus thus requiring no battery unit at $t = T_f$ whereas in case of IDMMG, average battery units’ requirement is 25 to guarantee that the demand is met. Moreover, average reduction in battery units, for the entire time horizon, is 49.26%, as shown in Table I and Fig. 5. The value of the power portfolio, as shown in Fig. 4b, becomes 0 at $T_f$ in case of IMMG while IDMMG requires DNO to have the power deficit of microgrid 2 in its power portfolio.

**E. Case study 3:** $P_{g1}(T_f) < D_{c1}, P_{g2}(T_f) < D_{c2}$

Fig. 4c captures the scenario where both the microgrids are not capable of meeting their load demands at $t = T_f$. In this case the power portfolio value required by the DNO is equal to the deficit of the microgrids for both IDMMG and IMMG settings. The battery units requirement is nearly equal to the net load demand of the whole system while in case of IMMG, the average reduction over IDMMG, on initial battery units requirement, thus enhancing grid reliability and efficiency.

Next we demonstrate the optimality of the initial amount of battery reserve allocation derived by our proposed algorithm. Fig. 6 illustrates the scenario, under case 3 in IMMG, when initial battery allocation is not optimal as derived in Section IV-B. In Fig. 6, $V_u(t)$ denotes the power portfolio trajectory when initial battery allocation is 20% more than optimal and $V(t)$ denotes the optimal power portfolio. We can see that, at the end of the horizon, DNO has overproduction. $V_d(t)$ denotes the scenario when initial battery allocation is 20% less than optimal. In this case, DNO cannot provide the critical loads at the end of the horizon, as the power in the portfolio is not sufficient to support the combined production deficit of the MGs. Similar plots can be shown for case 1 and 2 also, which are omitted to conserve space. Thus our algorithm enables DNO to provide assurance of meeting the demand of the MGs, without overproduction or underproduction, which enhances the system reliability.

The results corroborate the efficacy and utility of our resource allocation strategy in IMMG with respect to BESS requirements while providing provable guarantees of meeting the power needs at the end of the prescribed horizons, which is an indispensable requirement for critical facilities. Therefore, it can be concluded that our proposed algorithm enables DNO to mitigate the uncertainty of supplying the demand of the microgrids at the requested time while reducing the battery requirement, thus enhancing grid reliability and efficiency.

**VI. CONCLUSION**

This article develops a novel energy management strategy for an IMMG containing multiple microgrids, equipped with DER and critical infrastructure loads, and a DNO with centralized battery storage, with the focus of efficient energy storage allocation in real-time, to meet the load demands of the microgrids at the requested time with *almost sure guarantee*. Explicit stochastic model of wind generation is considered to represent the uncertainties of renewable generations. The proposed algorithms mitigate the uncertainties of meeting the load demands of the microgrids, due to intermittent renewable generations, at the requested time, with *almost sure* guarantee which is an absolute requirement for critical infrastructures under catastrophic events. We also derive the energy storage allocation policy for IMMG with no energy sharing facilities and compare the results with IMMG. The simulation study shows that IMMG enables DNO to have lower amount of battery storage than IDMMG, while still maintaining the guarantee of meeting the load demands of each microgrid.
Note that, if \( \tilde{a}_i = \frac{\partial \tilde{V}}{\partial \tilde{P}_{gi}} \), then \( \tilde{a}_i \sigma_{gi} \tilde{P}_{gi} \frac{\partial \tilde{V}}{\partial \tilde{P}_{gi}} - \tilde{a}_i \sigma_{gi} \tilde{P}_{gi} = 0 \), which eliminates the uncertainty. Thus,
\[
\frac{\partial \tilde{v}}{\partial t} + \frac{1}{2} \sigma_{gi}^2 \tilde{P}_{gi} \frac{\partial ^2 \tilde{v}}{\partial \tilde{P}_{gi}^2} = 0.
\]

(20)

Now, to ensure that the critical demand is met almost surely at the requested time \( T_f \), we need to solve (20) with the terminal condition (10) and find \( \tilde{a}_i(t) \), \( \tilde{b}_i(t) \).

Let \( \tau = T_f - t \), \( x_i = \ln(P_{gi}) \) for all \( i \in I \). Therefore, \( \frac{\partial \tilde{v}}{\partial \tau} = -\frac{\partial \tilde{v}}{\partial t} \), \( \frac{\partial ^2 \tilde{v}}{\partial \tau^2} = \frac{1}{2} \sigma_{gi}^2 \frac{\partial ^2 \tilde{v}}{\partial \tilde{P}_{gi}^2} = \frac{1}{2} \sigma_{gi}^2 \left[ \frac{\partial ^2 \tilde{v}}{\partial \tilde{P}_{gi}^2} - \frac{\partial \tilde{v}}{\partial \tilde{P}_{gi}} \right]. \)

From (20),
\[
\frac{\partial \tilde{v}}{\partial \tau} - \frac{1}{2} \sigma_{gi}^2 \frac{\partial ^2 \tilde{v}}{\partial \tilde{P}_{gi}^2} = A_i \frac{\partial ^2 \tilde{v}}{\partial \tilde{P}_{gi}^2} + B_i \frac{\partial \tilde{v}}{\partial \tilde{P}_{gi}},
\]

(21)

where, \( A_i = \frac{1}{2} \sigma_{gi}^2 \) (\( A_i > 0 \)), \( B_i = -\frac{1}{2} \sigma_{gi}^2 \). From (10), the final condition on power portfolio \( \bar{V}_i(P_i(t), t = T_f) \), or equivalently, initial condition on \( V_i(x_i, \tau = 0) \) is given as:
\[
\bar{V}_i(x_i, 0) = \begin{cases} 
0 & \text{if } x_i \geq \ln(D_{ci}) \\
D_{ci} - x_i & \text{if } x_i < \ln(D_{ci})
\end{cases}
\]

(22)

Let \( V_i(x_i, \tau) = e^{-(\alpha_i + \beta_i) \tau} \tilde{u}_i(x_i, \tau) \), where \( \alpha_i, \beta_i \in \mathbb{R} \). Then, \( \frac{\partial \bar{u}_i}{\partial \tau} = e^{-\alpha_i x_i - \beta_i \tau} \frac{\partial \bar{u}_i}{\partial t} - \beta_i \bar{u}_i \), \( \frac{\partial ^2 \bar{u}_i}{\partial \tau^2} = e^{-\alpha_i x_i - \beta_i \tau} \left( \frac{\partial ^2 \bar{u}_i}{\partial x_i^2} - 2 \alpha_i \frac{\partial \bar{u}_i}{\partial x_i} + \alpha_i^2 \bar{u}_i \right) \).

Therefore, using (21),
\[
\frac{\partial \bar{u}_i}{\partial \tau} - \beta_i \bar{u}_i = A_i \frac{\partial ^2 \bar{u}_i}{\partial x_i^2} + B_i \frac{\partial \bar{u}_i}{\partial x_i} - \alpha_i \bar{u}_i,
\]

and thus,
\[
\frac{\partial \bar{u}_i}{\partial \tau} = A_i \frac{\partial ^2 \bar{u}_i}{\partial x_i^2} + [B_i - 2 \alpha_i A_i] \frac{\partial \bar{u}_i}{\partial x_i} + [\beta_i + \alpha_i^2 A_i - \alpha_i B_i] \bar{u}_i.
\]

Choosing \( \alpha_i = \frac{B_i}{2 A_i} \), \( \beta_i = \frac{B_i^2}{4 A_i} \), we get, \( \frac{\partial \bar{u}_i}{\partial \tau} = A_i \frac{\partial ^2 \bar{u}_i}{\partial x_i^2} \).

Solution of this partial differential equation is given by \( \bar{u}_i(x_i, \tau) = \frac{1}{\sqrt{4\pi \sigma_{gi}^2}} \int_{-\infty}^{\infty} \bar{u}_i(y, 0) e^{-\frac{(x_i-y)^2}{4 \sigma_{gi}^2}} \) dy. Since, \( \tilde{V}_i(0, \tau) = e^{-\alpha_i \tau} \tilde{u}_i(x_i, \tau) \), and \( \alpha_i = \frac{B_i}{2 A_i} \), \( \beta_i = \frac{B_i^2}{4 A_i} \),
\[
\tilde{v}(x_i, \tau) = \frac{1}{\sqrt{4\pi \sigma_{gi}^2}} \int_{-\infty}^{\infty} e^{-\frac{(y-B_i^2/4 A_i)^2}{4 \sigma_{gi}^2}} \tilde{u}_i(y, 0) dy
\]

(23)

Therefore, \( \tilde{V}_i(x_i, \tau) \) is given by \( e^{-\alpha_i x_i + \beta_i \tau} \bar{u}_i(x_i, \tau) \), which can be further expanded as:
\[
\frac{1}{\sqrt{4\pi \sigma_{gi}^2}} \int_{-\infty}^{\infty} e^{-\frac{(y-B_i^2/4 A_i)^2}{4 \sigma_{gi}^2}} \tilde{V}_i(0, \tau) dy
\]

(24)

Applying the initial condition (22) to \( \tilde{V}_i(y, 0) \) we get:
\[
\tilde{V}_i(x_i, \tau) = \frac{1}{\sigma_{gi} \sqrt{2\pi \tau}} \int_{-\infty}^{\ln(D_{ci})} e^{\frac{1}{2} \frac{(y-y_0)^2}{\sigma_{gi}^2}} (D_{ci} - e^y) \ dy
\]

(25)

Let, \( z_i = \frac{1}{\sigma_{gi} \sqrt{2\pi \tau}} (y + \frac{\sigma_{gi}}{2} \tau - x_i) \) \( \implies \) \( dz_i = \frac{1}{\sigma_{gi} \sqrt{2\pi \tau}} dy \). Then,
\[
I_1 = \frac{\ln(D_{ci})}{\sigma_{gi} \sqrt{2\pi \tau}} \int_{-\infty}^{\ln(D_{ci})} e^{\frac{1}{2} \frac{(y-y_0)^2}{\sigma_{gi}^2}} e^{-\frac{1}{2} \frac{(y-y_0)^2}{\sigma_{gi}^2}} \ dy
\]

(26)

where, \( I_1 = \Phi\left( \frac{\ln(D_{ci})}{\sigma_{gi} \sqrt{2\pi \tau}} \right) \) is the Cumulative Distribution Function (CDF) of the standard Gaussian random variable \( \sim N(0,1) \). Similarly,
\[
I_2 = \frac{\ln(D_{ci})}{\sigma_{gi} \sqrt{2\pi \tau}} \int_{-\infty}^{\ln(D_{ci})} e^{\frac{1}{2} \frac{(y-y_0)^2}{\sigma_{gi}^2}} \ dy
\]

(27)

Comparing equation (26) with equation (8),
\[
\frac{d}{dt} = -\Phi \left( \frac{\ln(D_{ci})}{\sigma_{gi} \sqrt{2\pi \tau}} \right)
\]

(28)

This completes the proof.

**Appendix C**

**Derivation of stochastic differential equation of \( V_i \) in IMM for RES uncertainty mitigation**

From (1) and for constant \( P_b \) (i.e. \( dP_b = 0 \)), and under the rated power conservation (5), applying Ito’s lemma (34) on differential of \( V_i \) and considering infinitesimal dt, we obtain:
\[
\frac{d}{dt} + \sum_{i=1}^{N_{eq}} \frac{\partial V_i}{\partial P_{gi}} dP_{gi} + \frac{N_m}{2} \sum_{i=1}^{N_{eq}} \frac{\partial^2 V_i}{\partial P_{gi}^2} dP_{gi} dP_{jg} = \sum_{i=1}^{N_m} a_i dP_{gi}
\]

(1)

Using (1) and (3),
\[
\left[ \frac{\partial V_i}{\partial P_{gi}} + \sum_{i=1}^{N_{eq}} \sum_{j=1}^{N_{eq}} \sigma_{gi} \sigma_{gj} \rho_{gij} \frac{\partial V_i}{\partial P_{gj}} \right] dP_{gi} + \frac{N_m}{2} \sum_{i=1}^{N_{eq}} \sigma_{gi} \sigma_{gj} \rho_{gij} \frac{\partial^2 V_i}{\partial P_{gi} \partial P_{jg}} dP_{gi} dP_{jg}
\]

(29)

Note that, if \( a_i = \frac{\partial V_i}{\partial P_{gi}} \), for all \( i \in I \), uncertainty associated with all the renewable generation is eliminated, and we have:
\[
\frac{\partial V_i}{\partial P_{gi}} + \frac{N_m}{2} \sum_{i=1}^{N_{eq}} \sum_{j=1}^{N_{eq}} \sigma_{gi} \sigma_{gj} \rho_{gij} \frac{\partial V_i}{\partial P_{gj}} dP_{gi} dP_{jg} = 0.
\]

(30)

Note that, if there are numerous MGs present in the IMM (\( N_{eq} \gg 1 \)), theoretical solution of (30) to find \( a_i(t), b(t) \), under the terminal condition (12), becomes intractable.
APPENDIX D

PROOF OF THEOREM 1

Given the dynamics of RES $i$, we can find the value of $V_t$, $t \in [0,T]$, given the terminal condition (12), by changing the probability measure using (13). From (13),
\[
dP_{gi}(t) = \mu_{gi} P_{gi}(t) dt + \sigma_{gi} P_{gi}(t) (dW_{t_i} - \eta_{gi} dt)
\]
\[
= \sigma_{gi} P_{gi}(t) dW_{t_i}.
\] (29)

Applying differentiation operator to and since $P_{b}$ is constant, $dV_t = \sum_{i=1}^{N} a_i dP_{gi} = \sum_{i=1}^{N} a_i \sigma_{gi} P_{gi} dW_{t_i}$. Therefore, for any $t \in [0,T]$, $V_{T_f} = V_t + \int_{t}^{T_f} \sum_{i=1}^{N} a_i \sigma_{gi} P_{gi} dW_{t_i}$. Hence, under the transformed probability measure, the expected value of $V_{T_f}$, calculated at $t$, where $t \in [0,T]$, is,
\[
\mathbb{E}[V_{T_f}] = V_t + \mathbb{E}\left[\int_{t}^{T_f} \sum_{i=1}^{N} a_i \sigma_{gi} P_{gi} dW_{t_i}\right]
\]
\[
= V_t + \int_{t}^{T_f} \sum_{i=1}^{N} a_i \sigma_{gi} P_{gi} \mathbb{E}[dW_{t_i}] = V_t,
\]
where the last equality follows from the fact that $\mathbb{E}[dW_{t_i}] = 0$, under the transformed probability measure (13). From (12),
\[
V_t = \mathbb{E}\left[\max_{i=1}^{N} \left(D_{ci} - P_{gi}(T_f)\right), 0\right].
\]

This completes the proof.

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