Pion–Nucleon Scattering at Low Energies

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Abstract

We study pion–nucleon scattering at tree level with a chiral lagrangian of pions, nucleons, and Δ-isobars using a $K$-matrix unitarization procedure. Evaluating the scattering amplitude to order $Q^2$, where $Q$ is a generic small momentum scale, we obtain a good fit to the experimental phase shifts for pion center-of-mass kinetic energies up to 50 MeV. The fit can be extended to 150 MeV when we include the order-$Q^3$ contributions. Our results are independent of the off-shell Δ parameter.

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Pion–nucleon ($\pi N$) scattering is a fundamental hadronic process for which a large amount of data is available and it is important to understand this as completely as possible. Several relativistic models [1–5] exist which provide reasonably good fits to the experimental phase shifts. These models consider the $N$, $\pi$ and $\Delta$-resonance fields, the isoscalar-scalar $\phi$ (in some cases implicitly via a power series expansion), the $\rho$ meson and sometimes higher resonances, although these play a minor role. Our interest here is to examine whether a model which contains the minimal number of fields, namely the $N$, $\pi$ and $\Delta$, can yield equally good fits. Thus we effectively integrate out any other fields. For example, provided the center-of-mass (c.m.) energy is not too high, we can expand the $\rho$ propagator as $(m_\rho^2 - t)^{-1} = m_\rho^{-2}(1 +$
\( t/m_p^2 + \cdots \), where the Mandelstam variable \( t = (q - q')^2 \) and \( q \) and \( q' \) are the initial and final pion c.m. four-momenta. The series of terms can be absorbed into contact interactions in the lagrangian and it is clearly important to employ the most general set of such contact interactions which is consistent with the symmetries of quantum chromodynamics.

While the \( \Delta \) degree of freedom plays an important role in \( \pi N \) scattering, the \( Z \) parameter that specifies the form of the \( \pi N \Delta \) vertex has been controversial, see the discussion of Benmerrouche et al. [6]. Most of the papers cited above fit the \( Z \) parameter to the \( \pi N \) data. This is unsatisfactory since, as we showed recently [7], the scattering is independent of \( Z \) if the lagrangian contains the most general set of contact terms (we demonstrate this explicitly below). Thus results which depend on \( Z \) indicate that the contact terms have been implicitly constrained, whereas it is clearly preferable to employ a general lagrangian and allow the data itself to impose constraints.

We would like to employ a lagrangian which explicitly embodies chiral symmetry since this is known to be a fundamental symmetry at low energies. Such an approach was first taken by Peccei [8] to calculate the scattering lengths and this paper represents a modern extension of his work to study the phase shift data. In order to systematically enumerate the lagrangian we can be guided by Weinberg’s power counting arguments [9]. For this purpose we identify a generic small-momentum scale \( Q \). This is of the order of the pion three-momentum or the pion mass and therefore much smaller than the scale of the nucleon or the delta mass. Then according to the power counting, a Feynman tree diagram without loops contributes to \( \pi N \) scattering at order \( Q^\nu \) with

\[
\nu = 1 + \sum_i V_i \left( d_i + \frac{1}{2} n_i - 2 \right),
\]

where \( V_i \) is the number of vertices of type \( i \) characterized by \( n_i \) baryon fields and \( d_i \) pion derivatives or \( m_\pi \) factors. This suggests that we associate \( d_i + \frac{1}{2} n_i \) powers of \( Q \) to a term of type \( i \) in the lagrangian [10]. Also, Krause [11] argues that \( iP - M \) is of \( O(Q) \), as is a single factor of \( \gamma_5 \) (note \( \gamma_\mu \gamma_5 \) is of \( O(1) \)). Although we naively count \( \gamma_5 \) as \( O(Q) \) for organizing the lagrangian, we shall show later that this counting is not precise. Chiral symmetry
\((SU(2) \otimes SU(2))\), Lorentz invariance, and parity constrain the possible \(\pi N\) interactions and these can be found in Ref. [12]. For interactions involving the \(\Delta\) isobar we use the notation of our previous paper [7] and follow the discussion therein. We write the lagrangian up to quartic order as the sum of order \(Q^2\), \(Q^3\), and \(Q^4\) parts: \(\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4\).

The order \(Q^2\) part of the lagrangian is

\[
\mathcal{L}_2 = \overline{N}(i\mathcal{D} + g_A \gamma^\mu \gamma_5 a_\mu - M)N + \frac{1}{4} f_\pi^2 \text{tr}(\partial_\mu U^\dagger \partial^\mu U) + \frac{1}{4} m_\pi^2 f_\pi^2 \text{tr}(U + U^\dagger - 2)
\]

where the pion field arises in \(U(x) = \exp(2i\pi(x)/f_\pi)\) with \(\pi \equiv \frac{1}{2} \mathcal{P} \cdot \tau\) and the axial vector field \(a_\mu = \partial_\mu \pi/f_\pi + \cdots\), while the vector field \(v_\mu = -\frac{1}{2} i[\pi, \partial_\mu \pi]/f_\pi^2 + \cdots\). The trace is taken over the isospin matrices and the covariant derivative on the nucleon field is \(D_\mu N = \partial_\mu N + iv_\mu N\). As regards the \(\Delta\), the kernel tensor in the kinetic-energy term is

\[
\Lambda_{\mu\nu} = -i(\mathcal{D} - M_\Delta)g^{\mu\nu} + i(\gamma^\mu D^\nu + \gamma^\nu D^\mu) - \gamma^\mu (i\mathcal{D} + M_\Delta)\gamma^\nu .
\]

Here we have chosen the standard parameter \(A = -1\), because it can be modified by redefinition of the \(\Delta\) field with no physical consequences [13]. The covariant derivative is

\[
D_\mu \Delta_\nu = \partial_\mu \Delta_\nu + iv_\mu \Delta_\nu - v_\mu \times \Delta_\nu ,
\]

in which \(\Delta_\mu = T^a \Delta_\mu\), with \(T^a\) the standard 2 \times 4 isospin \(\frac{3}{2}\) to \(\frac{1}{2}\) transition matrices. The off-shell \(Z\) parameter appears in \(\Theta_{\mu\nu} = g_{\mu\nu} - \left(Z + \frac{1}{2}\right) \gamma_\mu \gamma_\nu\). We have simplified the \(\pi\Delta\Delta\) interaction in Eq. (2) by choosing the physically irrelevant parameters \(Z_2 = -\frac{1}{2}\) and \(Z_3 = 0\) (see Ref. [7]); this term does not contribute to the scattering amplitude at tree level.

The order \(Q^3\) part of \(\mathcal{L}\) is

\[
\mathcal{L}_3 = \frac{\beta_\pi}{M} \overline{N} N \text{tr}(\partial_\mu U^\dagger \partial^\mu U) - \frac{\kappa_\pi}{M} \overline{N} v_{\mu\nu} \sigma^{\mu\nu} N
\]

\[
+ \frac{\kappa_1}{2M^2} i \overline{N} \gamma_\mu \tilde{D}_\nu \text{Ntr}(a_\mu a_\nu) + \frac{\kappa_2}{M} m_\pi^2 \overline{N} N \text{tr}(U + U^\dagger - 2) + \cdots,
\]

where the dots represent terms that do not contribute to the \(\pi N\) scattering amplitude up to \(O(Q^3)\) and we have defined.
\[ \mathcal{D}_\mu = \partial_\mu - i v_\mu, \]
\[ v_{\mu\nu} = \partial_\mu v_\nu - \partial_\nu v_\mu + i [v_\mu, v_\nu]. \]

We have also applied naive dimensional analysis \cite{14} to factor out the dimensional factors so that the parameters are expected to be of order unity.

Finally, the order $Q^4$ part of $\mathcal{L}$ is

\[ \mathcal{L}_4 = \frac{\lambda_1}{M} m_\pi^2 \gamma_5 (U - U^\dagger) N + \frac{\lambda_2}{M^2} \gamma_\mu D^\mu v_{\mu\nu} N 
+ \frac{\lambda_3}{M^2} m_\pi^2 \gamma_\mu [a_\mu, U - U^\dagger] N + \frac{\lambda_4}{2M^3} i \gamma_\nu \mathcal{D}_\mu N \text{tr} (a^\mu a^\nu) 
+ \frac{\lambda_5}{16M^4} i \gamma_5 \{ \mathcal{D}_\mu, \mathcal{D}_\nu \} \tau^a N \text{tr} (\tau^a [D^\mu a^\mu, a^\nu]) + \cdots, \]

(8)

where the braces denote an anticommutator and

\[ D_\mu a_\nu = \partial_\mu a_\nu + i [v_\mu, a_\nu], \quad D^a v_{\mu\nu} = \partial^a v_{\mu\nu} + i [v^a, v_{\mu\nu}]. \]

Again the dots represent terms that do not contribute to the $\pi N$ scattering amplitude up to $O(Q^3)$, such terms include the usual fourth-order pion lagrangian.

Using the pion and nucleon equations of motion \cite{13,14,16}, we have simplified the contact terms listed in Ref. \cite{12}. For example, we reduce the $O(Q^3)$ term $\gamma_\pi \mathcal{D}_\mu \mathcal{D}_\nu N \text{tr} (a^\mu a^\nu)$ to the sum of the $O(Q^3)$ $\kappa_1$ term, the $O(Q^4)$ $\lambda_4$ term, and higher-order terms which we omit. As a result we have the minimum number of independent terms contributing to the $\pi N$ scattering amplitude up to $O(Q^3)$. As we have remarked, the isoscalar-scalar $\phi$ and isovector-vector $\rho$ fields as given in Ref. \cite{10} have been integrated out. Their effects show up in the contact terms $\beta_\pi$, $\kappa_2$ and $\lambda_2$. For example, in terms of the $\rho\pi\pi$ coupling ($g_{\rho\pi\pi}$) and the $\rho NN$ coupling ($g_\rho$), the rho gives a contribution to the $\lambda_2$ parameter of $-2g_{\rho\pi\pi}g_\rho M^2 f_\pi^2 / m_\rho^4$.

In Fig. 1 we show the tree Feynman diagrams for $\pi N$ scattering. The crossed diagrams for Figs. 1(b) and 1(c) are suppressed. The lagrangian $\mathcal{L}_2$ gives contributions to the $T$ matrix of $O(Q)$ from all three diagrams; note that the contact diagram is due to the Weinberg term $-\gamma_5 \gamma_\mu v_\mu N$. The interactions in $\mathcal{L}_3$ and $\mathcal{L}_4$ (except for the $\lambda_1$ term) give further contributions to Fig. 1(a) of order $Q^2$ and $Q^3$, respectively. In Fig. 1(b), each vertex can be
either a pseudovector $g_A$ vertex or a symmetry-breaking $\lambda_1$ vertex. As mentioned earlier, the appearance of $\gamma_5$ renders the $\lambda_1$ vertex of higher order than expected from the chiral counting of Eq. (1) so we have included it in $\mathcal{L}_4$. The reason for this extra power of $Q$ results from the following relation:

$$u(p')\gamma_5\frac{1}{\not{p} + \not{q} - M}\gamma_5 u(p) = -\frac{\pi(p')\gamma^5 u(p)}{(p + q)^2 - M^2}, \quad (10)$$

where $u(p)$ is the positive-energy free Dirac spinor. Thus, with one $\lambda_1$ and one $g_A$ vertex, Fig. (b) is of $O(Q^3)$; we include this contribution. With both vertices of $\lambda_1$ type the result is of $O(Q^4)$, whereas, associating an extra factor of $Q$ with each $\gamma_5$ as suggested by Ref. [11] and Eq. (8), we would expect $O(Q^5)$.

We follow the standard notation of Höhler [17] and Ericson and Weise [18] to write the $T$ matrix as

$$T_{ba} \equiv \langle \pi_b | T | \pi_a \rangle = T^+ \delta_{ab} + \frac{1}{2}[\tau_b, \tau_a]T^-, \quad (11)$$

where the isospin symmetric and antisymmetric amplitudes are

$$T^\pm = A^\pm + \frac{1}{2}(\not{q} + \not{q}')B^\pm. \quad (12)$$

Here $A^\pm$ and $B^\pm$ are functions of the Mandelstam invariant variables $s = (p + q)^2$, $t$, and $u = (p - q')^2$, where $p$ is the initial nucleon c.m. momentum. They are given by the sum.
The amplitudes arising from nucleon exchange are (see Ref. [17] for example). We list these contributions in the following for completeness.

\[ A_C^+ = \frac{2}{M f_\pi^2} \left[ \beta_\pi \left(2m_\pi^2 - t \right) - 2\kappa_2 m_\pi^2 + \lambda_4 \nu^2 \right] , \]

\[ B_C^+ = \frac{1}{M f_\pi^2} (\kappa_1 - 2\lambda_4) \nu \, , \]

\[ A_C^- = -\frac{2\kappa_\pi}{f_\pi^2} \nu \, , \]

\[ B_C^- = \frac{1}{2f_\pi^2} (1 + 4\kappa_\pi) - \frac{1}{M^2 f_\pi^2} \left( \frac{1}{2} \lambda_3 t + 4\lambda_3 m_\pi^2 - \lambda_5 \nu^2 \right) \, , \]

where \( \nu = (s - u)/4M \). The contributions from the nucleon- and \( \Delta \)-exchange are well-known (see Ref. [17] for example). We list these contributions in the following for completeness.

The amplitudes arising from nucleon exchange are

\[ A_N^+ = \frac{M}{f_\pi^2} g_A \left( g_A - 4\lambda_4 \frac{m_\pi^2}{M^2} \right) \, , \]

\[ B_N^+ = \frac{M}{f_\pi^2} g_A \left( g_A - 4\lambda_4 \frac{m_\pi^2}{M^2} \right) \frac{\nu}{\nu_B^2 - \nu^2} \, , \]

\[ A_N^- = 0 \, , \]

\[ B_N^- = -\frac{g_A^2}{2f_\pi^2} + \frac{M}{f_\pi^2} g_A \left( g_A - 4\lambda_4 \frac{m_\pi^2}{M^2} \right) \frac{\nu_B}{\nu_B^2 - \nu^2} \, , \]

where \( \nu_B = (t - 2m_\pi^2)/4M \). The amplitudes arising from \( \Delta \) exchange are

\[ A_\Delta^+ = \frac{2h_A^2}{9M f_\pi^2} \left[ \alpha_1 + \frac{3}{2}(M_\Delta + M)t \right] \frac{\nu_\Delta}{\nu_\Delta^2 - \nu^2} - \frac{4h_A^2}{9M_\Delta f_\pi^2} \left[ (E_\Delta + M)(2M_\Delta - M) + \left(2 + \frac{M}{2M_\Delta}\right) m_\pi^2 - (2m_\pi^2 - t)Y \right] \, , \]

\[ B_\Delta^+ = \frac{2h_A^2}{9M f_\pi^2} \left[ 2(E_\Delta + M)(E_\Delta - 2M) + \frac{3}{2} \right] \frac{\nu}{\nu_\Delta^2 - \nu^2} - \frac{16h_A^2}{9f_\pi^2} \frac{M}{M_\Delta} Z^2 \nu \, , \]

\[ A_\Delta^- = -\frac{h_A^2}{9M f_\pi^2} \left[ \alpha_1 + \frac{3}{2}(M_\Delta + M)t \right] \frac{\nu}{\nu_\Delta^2 - \nu^2} - \frac{8h_A^2}{9M_\Delta f_\pi^2} Y \nu \, , \]

\[ B_\Delta^- = -\frac{h_A^2}{9M f_\pi^2} \left[ 2(E_\Delta + M)(E_\Delta - 2M) + \frac{3}{2} t \right] \frac{\nu_\Delta}{\nu_\Delta^2 - \nu^2} + \frac{h_A^2}{9f_\pi^2} \left\{ \left(1 + \frac{M}{M_\Delta} \right)^2 + \frac{8M}{M_\Delta} Y + \frac{2}{M_\Delta} \left[(2m_\pi^2 - t)Z^2 - 2m_\pi^2 Z \right] \right\} \, , \]

where \( \nu_\Delta = (2M_\Delta^2 - s - u)/4M \), \( E_\Delta = (M_\Delta^2 + M^2 - m_\pi^2)/2M_\Delta \), and
\[ \alpha_1 = 2(E_\Delta + M)[M_\Delta(2E_\Delta - M) + M(E_\Delta - 2M)] , \quad (25) \]

\[ Y(Z) = \left(2 + \frac{M}{M_\Delta}\right)Z^2 + \left(1 + \frac{M}{M_\Delta}\right)Z . \quad (26) \]

Notice that in agreement with Ref. [7] only the nonpole terms in the \( \Delta \)-exchange diagram involve the off-shell parameter \( Z \). Therefore these contributions can be absorbed into the parameters of the contact terms according to

\[ \beta_\pi(Z) = \beta_\pi(-\frac{1}{2}) - \frac{h_\Delta^2}{18} \left[ 4Y(Z) + \frac{M}{M_\Delta} \right] \frac{M}{M_\Delta} , \quad (27) \]

\[ \kappa_\pi(Z) = \kappa_\pi(-\frac{1}{2}) - \frac{h_\Delta^2}{9} \left[ 4Y(Z) + \frac{M}{M_\Delta} \right] \frac{M}{M_\Delta} , \quad (28) \]

\[ \kappa_1(Z) = \kappa_1(-\frac{1}{2}) + \frac{4h_\Delta^2}{9} \left( 4Z^2 - 1 \right) \frac{M^2}{M_\Delta^2} , \quad (29) \]

\[ \lambda_2(Z) = \lambda_2(-\frac{1}{2}) - \frac{h_\Delta^2}{9} \left( 4Z^2 - 1 \right) \frac{M^2}{M_\Delta^2} , \quad (30) \]

\[ \lambda_3(Z) = \lambda_3(-\frac{1}{2}) + \frac{h_\Delta^2}{9} \left( Z^2 - Z - \frac{3}{4} \right) \frac{M^2}{M_\Delta^2} . \quad (31) \]

We shall quote parameters obtained with \( Z = -\frac{1}{2} \) and the parameters for other values of \( Z \) can be obtained from Eqs. (27) to (31). We have verified this numerically.

We use the standard labelling for isospin-spin partial wave channels, namely \( \alpha \equiv (l, 2I, 2J) \) where \( l \) is the orbital angular momentum, \( I \) is the total isospin, and \( J = l \pm \frac{1}{2} \) is the total angular momentum. The elastic scattering amplitude

\[ f_\alpha = \frac{1}{|q|} e^{i\delta_\alpha} \sin \delta_\alpha \quad (32) \]

is obtained from the amplitudes \( A^\pm \) and \( B^\pm \) by the usual partial wave expansion [19]. Here \( \delta_\alpha \) is the phase shift of the \( \alpha \) partial wave.

Unitarity requires \( f_\alpha \) to take the complex structure in Eq. (32). However, \( f_\alpha \) is real in a tree-level approximation to the scattering amplitude. We may recover unitarity by obtaining the phase shifts from two common methods. The first assumes that the calculated \( f_\alpha \) is simply the real part of Eq. (32). The second introduces a \( K \) matrix given by [18]

\[ f_\alpha = \frac{K_\alpha}{1 - i|q|K_\alpha} \quad \text{where} \quad K_\alpha = \frac{1}{|q|} \tan \delta_\alpha . \quad (33) \]
The calculated real tree-level amplitude \( f_{\alpha} \) is then assumed to actually be \( K_{\alpha} \), which is true for \( |q| \) small enough. For sufficiently small phase shifts, the two methods yield the same answer because \( \sin \delta_{\alpha} \approx \tan \delta_{\alpha} \approx \delta_{\alpha} \). However, near the resonance region where \( \delta_{\alpha} \sim \pi/2 \), the \( K \)-matrix method is preferred for the following simple reason. (We note that Goudsmit et al. \[5\] have proposed a justification for the \( K \)-matrix method.)

First, for energies near a resonance, the amplitude in the resonant channel takes the relativistic Breit-Wigner form. Taking the \( P33 \) channel as an example, we have \[18\]

\[
|q|f_{P33}^{BW} = \frac{M_{\Delta} \Gamma_{\Delta}}{M_{\Delta}^2 - s - iM_{\Delta} \Gamma_{\Delta}},
\]

where \( \Gamma_{\Delta} \) is the \( \Delta \) width. Eqs. (32) and (34) lead to

\[
\tan \delta_{P33} = \frac{M_{\Delta} \Gamma_{\Delta}}{M_{\Delta}^2 - s}.
\] (35)

Next, we expect that the tree-level amplitude can be obtained by setting the imaginary part of the denominator of Eq. (34) to zero:

\[
|q|f_{P33}^{tree} = \frac{M_{\Delta} \Gamma_{\Delta}}{M_{\Delta}^2 - s},
\] (36)

and this is indeed obtained by retaining only the pole contribution of Eqs. (21) to (24) and using the partial wave expansion. Finally, given the tree amplitude Eq. (36), the correct phase shift of Eq. (35) is obtained by the \( K \)-matrix method. Thus, while the two methods do not differ for small phase shifts in the nonresonant channels, the \( K \)-matrix method is also good on resonance. We therefore use the \( K \)-matrix method here.

In our calculations we choose the standard values \( M = 939 \text{ MeV}, M_{\Delta} = 1232 \text{ MeV}, \) and \( m_{\pi} = 139 \text{ MeV} \). We also take \[20\] \( f_\pi = 92.4 \text{ MeV} \) from charged pion decay, \( g_A = 1.26 \) from neutron \( \beta \) decay, and \( h_A = 1.46 \) from the \( \Delta \) width, \( \Gamma_{\Delta} = 120 \text{ MeV} \); allowing \( g_A \) and \( h_A \) to vary does not improve the fit. We first consider an \( O(Q^2) \) approximation to the \( T \) matrix which neglects \( \mathcal{L}_4 \). The four parameters listed in Table 1 were obtained by a \( \chi^2 \) fit to the data of Arndt \[21\] for pion c.m. kinetic energies between 10 and 150 MeV. Because negligible error bars are given in the data at low energies, we assign all the data points the
same relative weight. In Fig. 2, we plot the calculated $S$- and $P$-wave phase shifts (dashed curves), along with the data to which we fit, as a function of the pion c.m. kinetic energy; we also display older data from Bugg \[22\] and from Koch and Pietarinen \[23\]. The calculation is in good agreement with the data up to 50 MeV, but beyond this energy the fit deteriorates for three of the partial waves. The value of $\chi^2$ is unity for a relative weight of 15% which is a measure of the accuracy of the fit. The threshold (vanishing pion kinetic energy) $S$-wave scattering lengths ($a_{2s}$) and the $P$-wave scattering volumes ($a_{2l2j}$) are given in Table II. The difference between the data from Refs. \[21\] and \[23\] gives an indication of the error in the absence of a more reliable estimate. As regards theoretical predictions, apart from $a_{13}$ which is closer to the older value \[23\], the $O(Q^2)$ results agree nicely with Ref. \[21\] which is to be expected since they are the zero energy extrapolation of the data we have fitted.

We now include $L_4$, which involves five additional parameters ($\lambda_1$ to $\lambda_5$), to take the tree approximation to $O(Q^3)$. The results are indicated by the solid curve in Fig. 2 which gives a good fit (with a relative weight of 8% for $\chi^2 = 1$) out to 150 MeV. In fact only the $S_{11}$ and $P_{13}$ phase shifts deviate significantly from the data in the 150 –200 MeV range. Of course the rather precise agreement for $\delta_{P33}$ is strongly influenced by the phenomenological $K$-matrix unitarization. This forces the phase shift to be $\pi/2$ at $s = M_\Delta^2$ corresponding to a c.m. energy of 127 MeV. As regards the threshold results given in Table II, the predictions are a little closer to the data than at $O(Q^2)$ with the exception of $a_1$. In this connection it is instructive to examine the isoscalar and isovector $S$ wave scattering lengths, ($b_0$, $b_1$). A recent determination \[24\] gave $(-0.008 \pm 0.007, -0.096 \pm 0.007)$ in units of $m^{-1}$, in substantial agreement with Refs. \[21,23\]; note that Arndt favors a value of $b_0$ consistent with zero. At $O(Q^2)$ we obtain $(0.007, -0.081)$ and at $O(Q^3)$ $(-0.010, -0.077)$. Thus the isoscalar $b_0$, which is zero in the chiral limit, has improved by going to $O(Q^3)$, while the magnitude of $b_1$ remains too small.

Apart from $\lambda_3$ which has little influence on the fit, the $O(Q^3)$ parameters listed in Table II are of order unity although Eqs. (27) to (31) show that, while the fit is independent of $Z$, the actual parameter values will depend on $Z$. The pseudoscalar coupling with parameter
TABLE I. Parameters from fits to the $S$- and $P$-wave phase shifts.

| fit      | $\beta_\pi$ | $\kappa_\pi$ | $\kappa_1$ | $\kappa_2$ | $\lambda_1$ | $\lambda_2$ | $\lambda_3$ | $\lambda_4$ | $\lambda_5$ |
|----------|-------------|---------------|-------------|-------------|--------------|--------------|--------------|--------------|--------------|
| $O(Q^2)$ | -0.1960     | 0.5001        | 0.3061      | -0.9328     |              |              |              |              |              |
| $O(Q^3)$ | -0.1376     | 0.5301        | 0.7431      | -0.5799     | 0.3650       | -0.3239      | -0.0401      | 0.6334       | -0.4347      |

$\lambda_1$ allows the effective $\pi NN$ coupling constant to be adjusted in the $O(Q^3)$ fit. From the Goldberger-Treiman relation, our values for $g_A$ and $f_\pi$ correspond to a $\pi NN$ coupling, $g_{\pi NN} = 12.8$ which is a little lower than the value of 13.1 obtained by Arndt et al. $^{[25]}$. When the $\lambda_1$ term is included $g_{\pi NN}$ decreases slightly to 12.6. We will not comment on the sigma term since this requires extrapolation to the unphysical region which may not be reliable with this tree-level model.

With 9 parameters our $O(Q^3)$ calculation deviates from the data only beyond 150 MeV c.m. energy. At the higher energies we do a little better than Goudsmit et al. $^{[4]}$ who have 7 parameters and fit to 75 MeV. The calculation of Boffinger and Woolcock $^{[2]}$, which is an improved version of Ref. $^{[1]}$, contains 10 parameters and produces a fit which is similar to ours but a little better at energies $\sim 200$ MeV. The remaining models $^{[3][4]}$ have a larger number of parameters (14) and correspondingly fit to significantly higher energies.

In conclusion, we have discussed a chiral lagrangian involving just the basic $N$, $\pi$ and $\Delta$ fields, with a series of terms representing a momentum expansion. We find that a tree-level calculation with this model represents the data as well as other models with a similar number of parameters. Further we have confirmed by explicit calculation that the $Z$ parameter of the $\pi N\Delta$ vertex is irrelevant if a sufficiently general lagrangian is employed. Of course it would be more satisfactory if a unitary scattering amplitude emerged naturally, rather than being imposed phenomenologically. Such would be the case if loops were calculated in heavy baryon chiral perturbation theory and work in this direction is in progress.

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FIG. 2. The calculated $S$- and $P$-wave phase shifts as functions of the pion c.m. kinetic energy. The phase-shift data from Arndt \cite{21} (triangles), Bugg \cite{22} (squares), and Koch and Pietarinen \cite{23} (circles) are also shown.
TABLE II. The calculated $S$-wave scattering lengths and $P$-wave scattering volumes for the $O(Q^2)$ and $O(Q^3)$ fits compared with the data of Refs. [21] and [23]. The scattering lengths and volumes are in units of $m^{-1}$ and $m^{-3}$ respectively.

| length/volume | $O(Q^2)$ | $O(Q^3)$ | Ref. [21] | Ref. [23] |
|---------------|----------|----------|-----------|-----------|
| $a_1$         | 0.169    | 0.144    | 0.175     | 0.173     |
| $a_3$         | −0.074   | −0.087   | −0.087    | −0.101    |
| $a_{11}$      | −0.074   | −0.071   | −0.068    | −0.081    |
| $a_{13}$      | −0.032   | −0.031   | −0.022    | −0.030    |
| $a_{31}$      | −0.038   | −0.040   | −0.039    | −0.045    |
| $a_{33}$      | 0.212    | 0.209    | 0.209     | 0.214     |

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