TRAINING INDIVIDUALLY FAIR ML MODELS WITH SENSITIVE SUBSPACE ROBUSTNESS

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ABSTRACT

We consider training machine learning models that are fair in the sense that their performance is invariant under certain sensitive perturbations to the inputs. For example, the performance of a resume screening system should be invariant under changes to the gender and/or ethnicity of the applicant. We formalize this notion of algorithmic fairness as a variant of individual fairness and develop a distributionally robust optimization approach to enforce it during training. We also demonstrate the effectiveness of the approach on two ML tasks that are susceptible to gender and racial biases.

1 INTRODUCTION

Machine learning (ML) models are gradually replacing humans in high-stakes decision making roles. For example, in Philadelphia, an ML model classifies probationers as high or low-risk (Metz & Satariano, 2020). In North Carolina, “analytics” is used to report suspicious activity and fraud by Medicaid patients and providers (Metz & Satariano, 2020). Although ML models appear to eliminate the biases of a human decision maker, they may perpetuate or even exacerbate biases in the training data (Barocas & Selbst, 2016). Such biases are especially objectionable when it adversely affects underprivileged groups of users (Barocas & Selbst, 2016).

In response, the scientific community has proposed many mathematical definitions of algorithmic fairness and approaches to ensure ML models satisfy the definitions. Unfortunately, this abundance of definitions, many of which are incompatible (Kleinberg et al., 2016; Chouldechova, 2017), has hindered the adoption of this work by practitioners. There are two types of formal definitions of algorithmic fairness: group fairness and individual fairness. Most recent work on algorithmic fairness considers group fairness because it is more amenable to statistical analysis (Ritov et al., 2017). Despite their prevalence, group notions of algorithmic fairness suffer from certain shortcomings. One of the most troubling is there are many scenarios in which an algorithm satisfies group fairness, but its output is blatantly unfair from the point of view of individual users (Dwork et al., 2011).

In this paper, we consider individual fairness instead of group fairness. Intuitively, an individually fair ML model treats similar users similarly. Formally, an ML model is a map \( h : \mathcal{X} \to \mathcal{Y} \), where \( \mathcal{X} \) and \( \mathcal{Y} \) are the input and output spaces. The leading notion of individual fairness is metric fairness (Dwork et al., 2011); it requires

\[
d_y(h(x_1), h(x_2)) \leq L d_x(x_1, x_2) \quad \text{for all } x_1, x_2 \in \mathcal{X},
\]

where \( d_x \) and \( d_y \) are metrics on the input and output spaces and \( L \geq 0 \) is a Lipschitz constant. The fair metric \( d_x \) encodes our intuition of which samples should be treated similarly by the ML model. We emphasize that \( d_x(x_1, x_2) \) being small does not imply \( x_1 \) and \( x_2 \) are similar in all respects. Even if \( d_x(x_1, x_2) \) is small, \( x_1 \) and \( x_2 \) may differ in certain problematic ways, e.g. in their protected/sensitive attributes. This is why we refer to pairs of samples \( x_1 \) and \( x_2 \) such that \( d_x(x_1, x_2) \) is small as comparable instead of similar.

Despite its benefits, individual fairness was dismissed as impractical because there is no widely accepted fair metric for many ML tasks. Fortunately, there is a line of recent work on learning the
fair metric from data [Ilvento 2019] [Wang et al. 2019]. In this paper, we consider two data-driven
choices of the fair metric: one for problems in which the sensitive attribute is reliably observed, and
another for problems in which the sensitive attribute is unobserved (see Appendix B).

The rest of this paper is organized as follows. In Section 2, we cast individual fairness as a form of
robustness: robustness to certain sensitive perturbations to the inputs of an ML model. This allows us
to leverage recent advances in adversarial ML to train individually fair ML models. More concretely,
we develop an approach to audit ML models for violations of individual fairness that is similar to
adversarial attacks (Goodfellow et al., 2014) and an approach to train ML models that passes such
audits (akin to adversarial training (Madry et al., 2017)). We justify the approach theoretically (see
Section 3) and empirically (see Section 4).

2 FAIRNESS THROUGH (DISTRIBUTIONAL) ROBUSTNESS

To motivate our approach, imagine an auditor investigating an ML model for unfairness. The auditor
collects a set of audit data and compares the output of the ML model on comparable samples in
the audit data. For example, to investigate whether a resume screening system is fair, the auditor
may collect a stack of resumes and change the names on the resumes of Caucasian applicants to
names more common among the African-American population. If the system performs worse on
the edited resumes, then the auditor may conclude the model treats African-American applicants
unfairly. Such investigations are known as correspondence studies, and a prominent example is
Bertrand & Mullainathan’s celebrated investigation of racial discrimination in the labor market. In a
correspondence study, the investigator looks for inputs that are comparable to the training examples
(the edited resumes in the resume screening example) on which the ML model performs poorly. In
the rest of this section, we formulate an optimization problem to find such inputs.

2.1 FAIR WASSERSTEIN DISTANCES

Recall $\mathcal{X}$ and $\mathcal{Y}$ are the spaces of inputs and outputs. To keep things simple, we assume that the ML
task at hand is a classification task, so $\mathcal{Y}$ is discrete. We also assume that we have a fair metric $d_x$
of the form

$$d_x(x_1, x_2)^2 \triangleq (x_1 - x_2, \Sigma(x_1 - x_2))^\frac{1}{2},$$

where $\Sigma \in S^{d \times d}$. For example, suppose we are given a set of $K$ “sensitive” directions that we wish
the metric to ignore; i.e., $d(x_1, x_2) \ll 1$ for any $x_1$ and $x_2$ such that $x_1 - x_2$ falls in the span of
the sensitive directions. These directions may be provided by a domain expert or learned from data (see
Section 4 and Appendix B). In this case, we may choose $\Sigma$ as the orthogonal complement projector
of the span of the sensitive directions. We equip $\mathcal{X}$ with the fair metric and $\mathcal{Z} \triangleq \mathcal{X} \times \mathcal{Y}$ with

$$d_z((x_1, y_1), (x_2, y_2)) \triangleq d_x(x_1, x_2) + \infty \cdot 1\{y_1 \neq y_2\}.$$ 

We consider $d_z^2$ as a transport cost function on $\mathcal{Z}$. This cost function encodes our intuition of which
samples are comparable for the ML task at hand. We equip the space of probability distributions on $\mathcal{Z}$
with the fair Wasserstein distance

$$W(P, Q) = \inf_{\Pi \in \mathcal{C}(P, Q)} \int_{\mathcal{Z} \times \mathcal{Z}} c(z_1, z_2) d\Pi(z_1, z_2),$$

where $\mathcal{C}(P, Q)$ is the set of couplings between $P$ and $Q$. The fair Wasserstein distance inherits
our intuition of which samples are comparable through the cost function; i.e. the fair Wasserstein
distance between two probability distributions is small if they are supported on comparable areas of
the sample space.

2.2 AUDITING ML MODELS FOR ALGORITHMIC BIAS

To investigate whether an ML model performs disparately on comparable samples, the auditor
collects a set of audit data $\{(x_i, y_i)\}_{i=1}^n$ and solves the optimization problem

$$\max_{P:W(P, P_\epsilon) \leq \epsilon} \int_{\mathcal{Z}} \ell(z, h) dP(z),$$

where $\ell: \mathcal{Z} \times \mathcal{H} \to \mathbb{R}_+$ is a loss function, $h$ is the ML model, $P_\epsilon$ is the empirical distribution of
the audit data, and $\epsilon > 0$ is a small tolerance parameter. We interpret $\epsilon$ as a moving budget that the
auditor may expend to discover discrepancies in the performance of the ML model. This budget forces
the auditor to avoid moving samples to incomparable areas of the sample space. We emphasize that equation 2.1 detects aggregate violations of individual fairness. In other words, although the violations that the auditor’s problem detects are individual in nature, the auditor’s problem is only able to detect aggregate violations. We summarize the implicit notion of fairness in equation 2.1 in a definition.

**Definition 2.1 (distributionally robust fairness (DRF)).** An ML model \( h : \mathcal{X} \rightarrow \mathcal{Y} \) is \((\epsilon, \delta)\)-distributionally robustly fair (DRF) WRT the fair metric \( \delta \) if

\[
\max_{p, w(p, P_n) \leq \epsilon} \int_{\mathcal{X}} \ell(z, h) dP(z) \leq \delta.
\]  

(2.2)

Although equation 2.1 is an infinite-dimensional optimization problem, it is possible to solve it exactly by appealing to duality. Blanchet & Murthy showed that the dual of equation 2.1 is

\[
\sup_{p, w(p, P_n) \leq \epsilon} \mathbb{E}_P [\ell(Z, h)] = \inf_{\lambda \geq 0} \{ \lambda \epsilon + \mathbb{E}_{P_n} [\ell_h^\delta(Z, h)] \},
\]

\[
\ell_h^\delta((x_i, y_i), h) \triangleq \sup_{x \in \mathcal{X}} \ell((x, y_i), h) - \lambda d_2^\epsilon(x, x_i).
\]  

(2.3)

This is a univariate optimization problem, and it is amenable to stochastic optimization. We describe a stochastic approximation algorithm for equation 2.3 in Algorithm 1. Inspecting the algorithm, we see that it is similar to the PGD algorithm for adversarial attack.

**Algorithm 1 stochastic gradient method for equation 2.3**

**Require:** starting point \( \lambda_1 \), step sizes \( \alpha_t > 0 \)

1: repeat
2: draw mini-batch \( (x_t^1, y_t^1), \ldots, (x_t^B, y_t^B) \sim P_n \)
3: \( x_{t+1}^* \leftarrow \arg \max_{x \in \mathcal{X}} \ell((x, y_t), h) - \lambda d_2(x_{t+1}, x) \), \( b \in [B] \)
4: \( \lambda_{t+1} \leftarrow \max \{ 0, \lambda_t - \alpha_t (\epsilon - \frac{1}{B} \sum_{b=1}^B d_2^2(x_{t+1}, x_{t+1}^*)) \} \)
5: until converged

It is known that the optimal point of equation 2.1 is the discrete measure \( \frac{1}{n} \sum_{i=1}^n \delta_{(T_\lambda(x_i), y_i)} \), where \( T_\lambda: \mathcal{X} \rightarrow \mathcal{X} \) is the unfair map

\[
T_\lambda(x_i) \leftarrow \arg \max_{x \in \mathcal{X}} \ell((x, y_i), h) - \lambda d_2^\epsilon(x, x_i).
\]  

(2.4)

We call \( T_\lambda \) an unfair map because it reveals unfairness in the ML model by mapping samples in the audit data to comparable areas of the sample space that the system performs poorly on. We note that \( T_\lambda \) may map samples in the audit data to areas of the sample space that are not represented in the audit data, thereby revealing disparate treatment in the ML model not visible in the audit data alone. We emphasize that \( T_\lambda \) more than reveals disparate treatment in the ML model; it localizes the unfairness to certain areas of the sample space.

We present a simple example to illustrating fairness through robustness (a similar example appeared in Hashimoto et al. (2018)). Consider the binary classification dataset shown in Figure 1. There are two subgroups of observations in this dataset, and (sub)group membership is the protected attribute (e.g. the smaller group contains observations from a minority subgroup). In Figure 1a we see the decision heatmap of a vanilla logistic regression, which performs poorly on the blue minority subgroup. The two subgroups are separated in the horizontal direction, so the horizontal direction is the sensitive direction. Figure 1b shows that such classifier is unfair with respect to the corresponding fair metric, i.e. the unfair map equation 2.4 leads to significant loss increase by transporting mass along the horizontal direction with very minor change of the vertical coordinate.

**Comparison with metric fairness** Before moving on to training individually fair ML models, we compare DRF with metric fairness equation 1.1. Although we concentrate on the differences between the two definitions here, they are more similar than different: both formalize the intuition that the outputs of a fair ML model should perform similarly on comparable inputs. That said, there are two main differences between the two definitions. First, instead of requiring the output of the ML model to be similar on all inputs comparable to a training example, we require the output to be similar to the training label. Thus DRF not only enforces similarity of the output on comparable inputs, but also accuracy of the ML model on the training data. Second, DRF considers differences
Figure 1: Figure (a) depicts a binary classification dataset in which the minority group shown on the right of the plot is underrepresented. This tilts the logistic regression decision boundary in favor of the majority group on the left. Figure (b) shows the unfair map of the logistic regression decision boundary. It maps samples in the minority group towards the majority group. Figure (c) shows an algorithmically fair classifier that treats the majority and minority groups identically.

between datasets instead of samples by replacing the fair metric on inputs with the fair Wasserstein distance induced by the fair metric. The main benefits of this modifications are (i) it is possible to optimize equation 2.1 efficiently, (ii) we can show this modified notion of individual fairness generalizes.

2.3 FAIR TRAINING WITH SENSITIVE SUBSPACE ROBUSTNESS

We cast the fair training problem as training supervised learning systems that are robust to sensitive perturbations. We propose solving the minimax problem

\[
\inf_{h \in H} \sup_{P: W(\mathcal{P}, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, h)] = \inf_{h \in H} \inf_{\lambda \geq 0} \lambda \epsilon + \mathbb{E}_{P_n}[\ell_\lambda(Z, h)],
\]

where \(\ell_\lambda\) is defined in equation 2.3. This is an instance of a distributionally robust optimization (DRO) problem, and it inherits some of the statistical properties of DRO. To see why equation 2.5 encourages individual fairness, recall the loss function is a measure of the performance of the ML model. By assessing the performance of an ML model by its worse-case performance on hypothetical populations of users with perturbed sensitive attributes, minimizing equation 2.5 ensures the system performs well on all such populations. In our toy example, minimizing equation 2.5 implies learning a classifier that is insensitive to perturbations along the horizontal (i.e. sensitive) direction. In Figure 1b, this is achieved by the algorithm we describe next.

To keep things simple, we assume the hypothesis class is parametrized by \(\theta \in \Theta \subset \mathbb{R}^d\) and replace the minimization with respect to \(H\) by minimization with respect to \(\theta\). In light of the similarities between the DRO objective function and adversarial training, we borrow algorithms for adversarial training (Madry et al., 2017) to solve equation 2.5 (see Algorithm 2).

Algorithm 2 Sensitive Subspace Robustness (SenSR)

Require: starting point \(\hat{\theta}_1\), step sizes \(\alpha_t, \beta_t > 0\)
1: repeat
2: sample mini-batch \((x_1, y_1), \ldots, (x_B, y_B) \sim P_n\)
3: \(x_{tb}^* \leftarrow \arg \max_{x \in X} \ell((x, y_{tb}), \theta) - \lambda_t d_b^2(x_{tb}, x), b \in [B]\)
4: \(\lambda_{t+1} \leftarrow \max\{0, \lambda_t - \alpha_t (\epsilon - \frac{1}{B} \sum_{b=1}^B d_b^2(x_{tb}, x_{tb}^*))\}\)
5: \(\hat{\theta}_{t+1} \leftarrow \hat{\theta}_t - \frac{\beta_t}{B} \sum_{b=1}^B \partial_\theta \ell((x_{tb}^*, y_{tb}), \hat{\theta}_t)\)
6: until converged

Related work Our approach to fair training is an instance of distributionally robust optimization (DRO). In DRO, the usual sample-average approximation of the expected cost function is replaced by \(\mathbb{E}_{P \in \mathcal{U}} \mathbb{E}_P[\ell(Z, h)]\), where \(\mathcal{U}\) is a (data dependent) uncertainty set of probability distributions. The uncertainty set may be defined by moment or support constraints (Chen et al., 2007; Delage & Ye, 2010; Goh & Sim, 2010), \(f\)-divergences (Ben-Tal et al., 2012; Lam & Zhou, 2007).
We start by stating our assumptions on the ML task: in this section, we regard the (transportation) cost function to study the effect of a data-driven choice of $W$ where we study how quickly the uniform convergence error $\omega_n(n)$, supposed to be $\sup_{\theta \in \Theta} \| \mathbb{E}_{P}(\ell(Z, \theta) - \mathbb{E}_{\sup_{\theta \in \Theta} \sup_{L_{\mathcal{Z}}} \mathbb{E}_{P}(\ell(Z, \theta))} \| \leq \delta_n$, (3.1)

where $W_{\ast}$ is the Wasserstein distance on $\Delta(\mathcal{Z})$ with a transportation cost function $c_{\ast}$ that is possibly different from $c$, vanishes, and $P_{\ast}$ is the true distribution on $\mathcal{Z}$. We permit some discrepancy in the (transportation) cost function to study the effect of a data-driven choice of $c$. In the rest of this section, we regard $c_{\ast}$ as the exact cost function and $c$ as a cost function learned from human supervision. We start by stating our assumptions on the ML task:

(A1) the feature space $\mathcal{X}$ is bounded: $D \equiv \max\{\text{diam}(\mathcal{X}), \text{diam}_{\ast}(\mathcal{X})\} < \infty$;

(A2) the functions in the loss class $L = \{\ell(\cdot, \theta) : \theta \in \Theta\}$ are non-negative and bounded: $0 \leq \ell(z, \theta) \leq M$ for all $z \in \mathcal{Z}$ and $\theta \in \Theta$, and $L$-Lipschitz with respect to $d_{\mathcal{Z}}$:

$$\sup_{\theta \in \Theta} \left( \sup_{(x, y), (x', y') \in \mathcal{Z}} |\ell((x_1, y), \theta) - \ell((x_2, y), \theta)| \right) \leq Ld_{\mathcal{Z}}(x_1, x_2);$$

(A3) the discrepancy in the (transportation) cost function is uniformly bounded:

$$\sup_{(x_1, y), (x_2, y) \in \mathcal{Z}} |c((x_1, y), (x_2, y)) - c_{\ast}((x_1, y), (x_2, y))| \leq \delta_{c} D^2.$$

Assumptions A1 and A2 are standard (see [Lee & Raginsky 2015] Assumption 1, 2, 3) in the DRO literature. We emphasize that the constant $\delta_{c}$ in Assumption A2 is not the constant $L$ in the definition of metric fairness; it may be much larger. Thus most models that satisfy the conditions of the loss class are not individually fair in a meaningful sense.

Assumption A3 deserves further comment. Under A1, A3 is mild. For example, if the exact fair metric is $d_{\mathcal{Z}}(x_1, x_2) = (x_1 - x_2)^T \Sigma_{\ast}(x_1 - x_2)$, then the error in the transportation cost function is at most

$$|c((x_1, y), (x_2, y)) - c_{\ast}((x_1, y), (x_2, y))| = |(x_1 - x_2)^T \Sigma_{\ast}(x_1 - x_2) - (x_1 - x_2)^T \Sigma_{\ast}(x_1 - x_2)| \leq D^2 \|\Sigma - \Sigma_{\ast}\|_{2},$$

3 **SENSR TRAINS INDIVIDUALLY FAIR ML MODELS**

One of the main benefits of our approach is it provably trains individually fair ML models. Further, it is possible for the learner to certify that an ML model is individually fair a posteriori. As we shall see, both are consequences of uniform convergence results for the DR loss class. More concretely, we study how quickly the uniform convergence error

$$\omega_n \equiv \sup_{\theta \in \Theta} \left( \sup_{P:W_{\ast}} \mathbb{E}_{P}(\ell(Z, \theta)) - \sup_{P:W_{\ast}} \mathbb{E}_{P}(\ell(Z, \theta)) \right) \leq \delta_n,$$

where $W_{\ast}$ is the Wasserstein distance on $\Delta(\mathcal{Z})$ with a transportation cost function $c_{\ast}$ that is possibly different from $c$, vanishes, and $P_{\ast}$ is the true distribution on $\mathcal{Z}$. We permit some discrepancy in the (transportation) cost function to study the effect of a data-driven choice of $c$. In the rest of this section, we regard $c_{\ast}$ as the exact cost function and $c$ as a cost function learned from human supervision. We start by stating our assumptions on the ML task:

(A1) the feature space $\mathcal{X}$ is bounded: $D \equiv \max\{\text{diam}(\mathcal{X}), \text{diam}_{\ast}(\mathcal{X})\} < \infty$;

(A2) the functions in the loss class $L = \{\ell(\cdot, \theta) : \theta \in \Theta\}$ are non-negative and bounded: $0 \leq \ell(z, \theta) \leq M$ for all $z \in \mathcal{Z}$ and $\theta \in \Theta$, and $L$-Lipschitz with respect to $d_{\mathcal{Z}}$:

$$\sup_{\theta \in \Theta} \left( \sup_{\ell((x_1, y), \theta) - \ell((x_2, y), \theta)} \leq Ld_{\mathcal{Z}}(x_1, x_2);$$

(A3) the discrepancy in the (transportation) cost function is uniformly bounded:

$$\sup_{(x_1, y), (x_2, y) \in \mathcal{Z}} |c((x_1, y), (x_2, y)) - c_{\ast}((x_1, y), (x_2, y))| \leq \delta_{c} D^2.$$
We see that the error in the transportation cost function vanishes in the large-sample limit as long as \( \Sigma \) is a consistent estimator of \( \Sigma_* \).

We state the uniform convergence result in terms of the entropy integral of the loss class: 
\[
\mathcal{E}(\mathcal{L}) = \int_0^\infty \sqrt{\log N_{\infty}(\mathcal{L}, r)} dr,
\]
where \( N_{\infty}(\mathcal{L}, r) \) as the \( r \)-covering number of the loss class in the uniform metric. The entropy integral is a measure of the complexity of the loss class.

**Proposition 3.1 (uniform convergence).** Under Assumptions A1–A3, equation 3.1 satisfies
\[
\delta_n \leq \frac{48\mathcal{E}(\mathcal{L})}{\sqrt{n}} + \frac{48LD^2}{\sqrt{n}\varepsilon} + \frac{L\delta_\varepsilon D^2}{\sqrt{\varepsilon}} + M\left(\frac{\log \frac{2}{N}}{2n}\right)^{\frac{1}{2}}
\]
with probability at least \( 1 - t \).

We note that Proposition 3.1 is similar to the generalization error bounds by Lee & Raginsky (2017). The main novelty in Proposition 3.1 is allowing error in the transportation cost function. We see that the discrepancy in the transportation cost function may affect the rate at which the uniform convergence error vanishes: it affects the rate if \( \delta_n \) is \( \omega_P\left(\frac{1}{\sqrt{n}}\right) \).

A consequence of uniform convergence is SenSR trains individually fair classifiers (if there are such classifiers in the hypothesis class). By individually fair ML model, we mean an ML model that has a small gap
\[
\sup_{P,W,(P,P_*) \leq \varepsilon} E_P[\ell(Z, \theta)] - E_{P_*}[\ell(Z, \theta)]
\]
The gap is the difference between the optimal value of the auditor’s optimization problem equation 2.1 and the (non-robust) risk. A small gap implies the auditor cannot significantly increase the loss by moving samples from \( P_* \) to comparable samples.

**Proposition 3.2.** Under the assumptions A1–A3, as long as there is \( \tilde{\theta} \in \Theta \) such that
\[
\sup_{P,W,(P,P_*) \leq \varepsilon} E_P[\ell(Z, \tilde{\theta})] \leq \delta
\]
for some \( \delta > 0 \), \( \tilde{\theta} \in \arg\min_{\theta \in \Theta} \sup_{P,W,(P,P_*) \leq \varepsilon} E_P[\ell(Z, \theta)] \) satisfies
\[
\sup_{P,W,(P,P_*) \leq \varepsilon} E_P[\ell(Z, \tilde{\theta})] - E_{P_*}[\ell(Z, \tilde{\theta})] \leq \delta + 2\delta_n
\]
where \( \delta_n \) is the uniform convergence error equation 3.1.

Proposition 3.2 guarantees Algorithm 2 trains an individually fair ML model. More precisely, if there are models in \( \mathcal{H} \) that are (i) individually fair and (ii) achieve small test error, then Algorithm 2 trains such a model. It is possible to replace equation 3.4 with other conditions, but a condition to its effect cannot be dispensed with entirely. If there are no individually fair models in \( \mathcal{H} \), then it is not possible for equation 2.5 to learn an individually fair model. If there are individually fair models in \( \mathcal{H} \), but they all perform poorly, then the goal of learning an individually fair model is futile.

Another consequence of uniform convergence is equation 3.3 is close to its empirical counterpart
\[
\sup_{P,W,(P,P_*) \leq \varepsilon} E_P[\ell(Z, \theta)] - E_{P_*}[\ell(Z, \theta)]
\]
In other words, the gap generalizes. This implies equation 3.5 is a certificate of individual fairness; i.e. it is possible for practitioners to check whether an ML model is individually fair by evaluating equation 3.5.

**Proposition 3.3.** Under the assumptions A1–A3, for any \( \varepsilon > 0 \),
\[
\sup_{\theta \in \Theta} \left\{ \sup_{P,W,(P,P_*) \leq \varepsilon} E_P[\ell(Z, \theta)] - E_{P_*}[\ell(Z, \theta)] - (\sup_{P,W,(P,P_*) \leq \varepsilon} E_P[\ell(Z, \theta)] - E_{P_*}[\ell(Z, \theta)]) \right\}
\leq 2\delta_n \text{ w.p. at least } 1 - t.
\]

## 4 Computational results

In this section, we present results from using SenSR to train individually fair ML models for two tasks: sentiment analysis and income prediction. We pick these two tasks to demonstrate the efficacy of SenSR on problems with structured (income prediction) and unstructured (sentiment analysis) inputs and in which the sensitive attribute (income prediction) is observed and unobserved (sentiment analysis). We refer to Appendix C and D for the implementation details.
4.1 FAIR SENTIMENT PREDICTION WITH WORD EMBEDDINGS

Problem formulation We study the problem of classifying the sentiment of words using positive (e.g., 'smart') and negative (e.g., 'anxiety') words compiled by [Hu & Liu (2004)]. We embed words using 300-dimensional GloVe [Pennington et al. (2014)] and train a one-layer neural network with 1000 hidden units. Such classifier achieves 95% test accuracy, however it entails major individual fairness violation. Consider an application of this sentiment classifier to summarizing customer reviews, tweets or news articles. Human names are typical in such texts and should not affect the sentiment score, hence we consider fair metric between any pair of names to be 0. Then sentiment score for all names should be the same to satisfy the individual fairness. To make a connection to group fairness, following the study of [Caliskan et al. (2017)] that reveals the biases in word embeddings, we evaluate the fairness of our sentiment classifier using male and female names typical for Caucasian and African-American ethnic groups. We emphasize that to satisfy individual fairness, the sentiment of any name should be the same.

Comparison metrics To evaluate the gap between two groups of names, \( N_0 \) for Caucasian (or female) and \( N_1 \) for African-American (or male), we report \( \frac{1}{|N_0|} \sum_{n \in N_0} (h(n)_1 - h(n)_0) - \frac{1}{|N_1|} \sum_{n \in N_1} (h(n)_1 - h(n)_0) \), where \( h(n)_k \) is logits for class \( k \) of name \( n \) (\( k = 1 \) is the positive class). We use list of names provided in [Caliskan et al. (2017)], which consists of 49 Caucasian and 45 African-American names, among those 48 are female and 46 are male. The gap between African-American and Caucasian names is reported as Race gap, while the gap between male and female names is reported as Gend. gap in Table 1. As in [Speer (2017)], we also compare sentiment difference of two sentences: "Let’s go get Italian food" and "Let’s go get Mexican food", i.e., cuisine gap (abbreviated Cuis. gap in Table 1), as a test of generalization beyond names. To embed these sentences we average their word embeddings.

Sensitive subspace We consider embeddings of 94 names that we use for evaluation as sensitive directions, which may be regarded as utilizing the expert knowledge, i.e. these names form a list of words that an expert believes should be treated equally. Fair metric is then defined using an orthogonal complement projector of the span of sensitive directions as we discussed in Section 2.1. When expert knowledge is not available, or we wish to achieve general fairness for names, we utilize an orthogonal complement projector of the span of sensitive directions, which may be regarded as utilizing the expert knowledge, i.e. these names form a list of names that we use for evaluation. Embeddings of these names define a group of comparable samples that we use to learn sensitive directions with SVD (see Appendix B.2 and Algorithm 3 for details). We take top 50 singular vectors to form the sensitive subspace. It is worth noting that, unlike many existing approaches in the fairness literature, we do not use any protected attribute information. Our algorithm only utilizes training words, their sentiments and a vanilla list of names.

Results From the box-plots in Figure 2, we see that both race and gender gaps are significant when using the baseline neural network classifier. It tends to predict Caucasian names as “positive”, while the median for African-American names is negative; the median sentiment for female names is higher than that for male names. We considered three other approaches to this problem: the algorithm of [Bolukbasi et al. (2016)] for pre-processing word embeddings; pre-processing via projecting

Table 1: Sentiment prediction experiments over 10 restarts

| Method    | Acc. % | Race gap | Gend. gap | Cuis. gap |
|-----------|--------|----------|-----------|-----------|
| SenSR     | 94±1   | 0.30±0.05| 0.19±0.03 | 0.23±0.05 |
| SenSR-E   | 93±1   | 0.11±0.04| 0.04±0.03 | 1.11±1.5  |
| Baseline  | 95±1   | 7.01±0.44| 5.59±0.37 | 4.10±0.44 |
| Project   | 94±1   | 1.00±0.56| 1.99±0.58 | 1.70±0.41 |
| Sinha+    | 94±1   | 3.88±0.26| 1.42±0.29 | 1.33±1.18 |
| Bolukb.+  | 94±1   | 6.85±0.53| 4.33±0.46 | 3.44±0.29 |

Figure 2: Box-plots of sentiment scores

1 titled ‘Popular Baby Names’ and available from [https://catalog.data.gov/dataset/]
out the sensitive subspace that we used for training SenSR (this is analogous to Prost et al. (2019)); training a distributionally robust classifier with Euclidean distance cost (Sinha et al., 2017). All approaches improved upon the baseline, however only SenSR can be considered individually fair. Our algorithm practically eliminates gender and racial gaps and achieves the notion of individual fairness as can be seen from almost equal predicted sentiment score for all names. We remark that using expert knowledge (i.e. evaluation names) allowed SenSR-E (E for expert) to further improve both group and individual fairness. However we warn practitioners that if the expert knowledge is too specific, generalization outside of the expert knowledge may not be very good. In Table [1] we report results averaged across 10 repetitions with 90%/10% train/test splits, where we also verify that accuracy trade-off with the baseline is minor. In the right column we present the generalization check, i.e. comparing a pair of sentences unrelated to names. Utilizing expert knowledge led to a fairness over-fitting effect, however we still see improvement over other methods. When utilizing SVD of a larger dataset of names we observe better generalization. Our generalization check suggests that fairness over-fitting is possible, therefore datasets and procedure for verifying fairness generalization are needed.

4.2 Adult

Problem formulation  Demonstrating the broad applicability of SenSR outside of natural language processing tasks, we apply SenSR to a classification task on the Adult (Dua & Graff, 2017) data set to predict whether an individual makes at least $50k based on features like gender and occupation for approximately 45,000 individuals. Models that predict income without fairness considerations can contribute to the problem of differences in pay between genders or races for the same work. Throughout this section, gender (male or female) and race (Caucasian or non-Caucasian) are binary.

Comparison metrics  Arguably a classifier is individually unfair if the classifications for two data points that are the same on all features except demographic features are different. Therefore, to assess individual fairness, we report spouse consistency (S-Con.) and gender and race consistency (GR-Con.), which are measures of how often classifications change only because of differences in demographic features. For S-Con (resp. GR-con), we make 2 (resp. 4) copies of every data point where the only difference is that one is a husband and the other is a wife (resp. difference is in gender and race). S-Con (resp. GR-Con) is the fraction of corresponding pairs (resp. quadruples) that have the same classification. We also report various group fairness measures proposed by De-Arteaga et al. (2019) with respect to race or gender based on true positive rates, i.e. the ability of a classifier to correctly identify a given class. See Appendix D.5 for the definitions. We report Gap_{RMS}, Gap_{G}, Gap_{R}^{max}, and Gap_{G}^{max} where R refers to race, and G refers to gender. We use balanced accuracy (B-acc) instead of accuracy^2 to measure predictive ability since only 25% of individuals make at least $50k.

Sensitive subspace  Let \( \{ (x_i, x_{g_i}) \}_{i=1}^{m} \) be the set of features \( x_i \in \mathbb{R}^D \) of the data except the coordinate for gender is zeroed and where \( x_{g_i} \) indicates the gender of individual \( i \). For \( \gamma > 0 \), let \( w_g = \arg \min_{w \in \mathbb{R}^D} \frac{1}{m} \sum_{i=1}^{m} -x_{g_i} (w^T x_i) + \log(1 + e^{w^T x_i}) + \gamma \| w \|_2 \), i.e. \( w_g \) is the learned hyperplane that classifies gender given by regularized logistic regression. Let \( e_g \in \mathbb{R}^D \) (resp. \( e_r \)) be the vector that is 1 in the gender (resp. race) coordinate and 0 elsewhere. Then the sensitive subspace is the span of \([w_g, e_g, e_r]\). See Appendix B.1 for details.
See Table 2 for the average of each metric on the test sets over ten 80%/20% train/test splits for Baseline, Project (projecting features onto the orthogonal complement of the sensitive subspace before training), CoCL (De-Arteaga et al., 2019), Adversarial Debiasing (Zhang et al., 2018), and SenSR. With the exception of CoCL (De-Arteaga et al., 2019), each classifier is a 100 unit single hidden layer neural network. The Baseline clearly exhibits individual and group fairness violations. While SenSR has the lowest B-acc, SenSR is the best by a large margin for S-Con. and has the best group fairness measures. We expect SenSR to do well on GR-consistency since the sensitive subspace includes the race and gender directions. However, SenSR’s individually fair performance generalizes: the sensitive directions do not directly use the husband and wife directions, yet SenSR performs well on S-Con. Furthermore, SenSR outperforms Project on S-Con and group fairness measures illustrating that SenSR does much more than just ignoring the sensitive subspace. CoCL only barely improves group fairness compared to the baseline with a significant drop in B-acc and while Adversarial Debiasing also improves group fairness, it is worse than the baseline on individual fairness measures illustrating that group fairness does not imply individual fairness.

5 Summary

We consider the task of training ML systems that are fair in the sense that their performance is invariant under certain perturbations in a sensitive subspace. This notion of fairness is a variant of individual fairness (Dwork et al., 2011). One of the main barriers to the adoption of individual fairness is the lack of consensus on a fair metric for many ML tasks. To circumvent this issue, we consider two approaches to learning a fair metric from data: one for problems in which the sensitive attribute is observed, and another for problems in which the sensitive attribute is unobserved. Given a data-driven choice of fair metric, we provide an algorithm that provably trains individually fair ML models.

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References

Solon Barocas and Andrew D. Selbst. Big Data’s Disparate Impact. *SSRN Electronic Journal*, 2016. ISSN 1556-5068. doi: 10.2139/ssrn.2477899.

Rachel K. E. Bellamy, Kuntal Dey, Michael Hind, Samuel C. Hoffman, Stephanie Houde, Kalapriya Kannan, Pranay Lohia, Jacquelyn Martino, Sameep Mehta, Aleksandra Mojsilovic, Seema Nagar, Karthikeyan Natesan Ramamurthy, John Richards, Diptikalyan Saha, Prasanna Sattigeri, Moninder Singh, Kush R. Varshney, and Yunfeng Zhang. AI Fairness 360: An extensible toolkit for detecting, understanding, and mitigating unwanted algorithmic bias, October 2018. URL https://arxiv.org/abs/1810.01943.

Aharon Ben-Tal, Dick den Hertog, Anja De Waegenaere, Bertrand Melenberg, and Gijs Rennen. Robust Solutions of Optimization Problems Affected by Uncertain Probabilities. *Management Science*, 59(2):341–357, November 2012. ISSN 0025-1909. doi: 10.1287/mnsc.1120.1641.

Marianne Bertrand and Sendhil Mullainathan. Are Emily and Greg More Employable Than Lakisha and Jamal? A Field Experiment on Labor Market Discrimination. *American Economic Review*, 94(4):991–1013, September 2004. ISSN 0002-8282. doi: 10.1257/0002828042002561.

Jose Blanchet and Karthyek R. A. Murthy. Quantifying Distributional Model Risk via Optimal Transport. *arXiv:1604.01446 [math, stat]*, April 2016.

Jose Blanchet, Yang Kang, and Karthyek Murthy. Robust Wasserstein Profile Inference and Applications to Machine Learning. *arXiv:1610.05627 [math, stat]*, October 2016.

The standard error is reported in the supplement. Each standard error is within $10^{-2}$.
Tolga Bolukbasi, Kai-Wei Chang, James Zou, Venkatesh Saligrama, and Adam Kalai. Man is to Computer Programmer as Woman is to Homemaker? Debiasing Word Embeddings. arXiv:1607.06520 [cs, stat], July 2016.

Aylin Caliskan, Joanna J. Bryson, and Arvind Narayanan. Semantics derived automatically from language corpora contain human-like biases. Science, 356(6334):183–186, April 2017. ISSN 0036-8075, 1095-9203. doi: 10.1126/science.aal4230.

Nicholas Carlini and David Wagner. Towards Evaluating the Robustness of Neural Networks. arXiv:1608.04644 [cs], August 2016.

Xin Chen, Melvyn Sim, and Peng Sun. A Robust Optimization Perspective on Stochastic Programming. Operations Research, 55:1058–1071, 2007. doi: 10.1287/opre.1070.0441.

Alexandra Chouldechova. Fair prediction with disparate impact: A study of bias in recidivism prediction instruments. arXiv:1703.00056 [cs, stat], February 2017.

Maria De-Arteaga, Alexey Romanov, Hanna Wallach, Jennifer Chayes, Christian Borgs, Alexandra Chouldechova, Sahin Geyik, Krishnaram Kenthapadi, and Adam Tauman Kalai. Bias in bios: A case study of semantic representation bias in a high-stakes setting. arXiv preprint arXiv:1901.09451, 2019.

Erick Delage and Yinyu Ye. Distributionally Robust Optimization Under Moment Uncertainty with Application to Data-Driven Problems. Operations Research, 58:595–612, 2010. doi: 10.1287/opre.1090.0741.

Dheeru Dua and Casey Graff. UCI machine learning repository, 2017. URL [http://archive.ics.uci.edu/ml]

Cynthia Dwork, Moritz Hardt, Toniann Pitassi, Omer Reingold, and Rich Zemel. Fairness Through Awareness. arXiv:1104.3913 [cs], April 2011.

Harrison Edwards and Amos Storkey. Censoring Representations with an Adversary. arXiv:1511.05897 [cs, stat], November 2015.

Peyman Mohajerin Esfahani and Daniel Kuhn. Data-driven Distributionally Robust Optimization Using the Wasserstein Metric: Performance Guarantees and Tractable Reformulations. May 2015.

Sahaj Garg, Vincent Perot, Nicole Limtiaco, Ankur Taly, Ed H Chi, and Alex Beutel. Counterfactual fairness in text classification through robustness. In Proceedings of the 2019 AAAI/ACM Conference on AI, Ethics, and Society, pp. 219–226. ACM, 2019.

Joel Goh and Melvyn Sim. Distributionally Robust Optimization and Its Tractable Approximations. Operations Research, 58(4-part-1):902–917, August 2010. ISSN 0030-364X, 1526-5463. doi: 10.1287/opre.1090.0795.

Ian J. Goodfellow, Jonathon Shlens, and Christian Szegedy. Explaining and Harnessing Adversarial Examples. December 2014.

Tatsunori B. Hashimoto, Megha Srivastava, Hongseok Namkoong, and Percy Liang. Fairness Without Demographics in Repeated Loss Minimization. arXiv:1806.08010 [cs, stat], June 2018.

Minqing Hu and Bing Liu. Mining and Summarizing Customer Reviews. In Proceedings of the Tenth ACM SIGKDD International Conference on Knowledge Discovery and Data Mining, pp. 10, Seattle, WA, August 2004.

Christina Ilvento. Metric Learning for Individual Fairness. arXiv:1906.00250 [cs, stat], June 2019.

Harini Kannan, Alexey Kurakin, and Ian Goodfellow. Adversarial Logit Pairing. arXiv:1803.06373 [cs, stat], March 2018.

Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. arXiv preprint arXiv:1412.6980, 2014.
Jon Kleinberg, Sendhil Mullainathan, and Manish Raghavan. Inherent Trade-Offs in the Fair Determination of Risk Scores. *arXiv:1609.05807 [cs, stat]*, September 2016.

Alexey Kurakin, Ian Goodfellow, and Samy Bengio. Adversarial Machine Learning at Scale. November 2016.

H. Lam and Enlu Zhou. Quantifying uncertainty in sample average approximation. In *2015 Winter Simulation Conference (WSC)*, pp. 3846–3857, December 2015. doi: 10.1109/WSC.2015.7408541.

Jaeho Lee and Maxim Raginsky. Minimax Statistical Learning with Wasserstein Distances. *arXiv:1705.07815 [cs]*, May 2017.

David Madras, Elliot Creager, Toniann Pitassi, and Richard Zemel. Learning Adversarially Fair and Transferable Representations. *arXiv:1802.06309 [cs, stat]*, February 2018.

Aleksander Madry, Aleksandar Makelov, Ludwig Schmidt, Dimitris Tsipras, and Adrian Vladu. Towards Deep Learning Models Resistant to Adversarial Attacks. *arXiv:1706.06083 [cs, stat]*, June 2017.

Cade Metz and Adam Satariano. An Algorithm That Grants Freedom, or Takes It Away. *The New York Times*, February 2020. ISSN 0362-4331.

Takeru Miyato, Shin-ichi Maeda, Masanori Koyama, Ken Nakae, and Shin Ishii. Distributional Smoothing with Virtual Adversarial Training. *arXiv:1507.00677 [cs, stat]*, July 2015.

Hongseok Namkoong and John C. Duchi. Stochastic Gradient Methods for Distributionally Robust Optimization with $\mathbb{F}$-divergences. In *Proceedings of the 30th International Conference on Neural Information Processing Systems*, NIPS’16, pp. 2216–2224, Barcelona, Spain, 2016. Curran Associates Inc. ISBN 978-1-5108-3881-9.

Nicolas Papernot, Patrick McDaniel, Somesh Jha, Matt Fredrikson, Z. Berkay Celik, and Ananthram Swami. The Limitations of Deep Learning in Adversarial Settings. November 2015.

Jeffrey Pennington, Richard Socher, and Christopher Manning. Glove: Global vectors for word representation. In *Proceedings of the 2014 conference on empirical methods in natural language processing (EMNLP)*, pp. 1532–1543, 2014.

Flavien Prost, Nithum Thain, and Tolga Bolukbasi. Debiasing Embeddings for Reduced Gender Bias in Text Classification. *arXiv:1908.02810 [cs, stat]*, August 2019.

Ya’acov Ritov, Yuekai Sun, and Ruofei Zhao. On conditional parity as a notion of non-discrimination in machine learning. *arXiv:1706.08519 [cs, stat]*, June 2017.

Soroosh Shafieezadeh-Abadeh, Peyman Mohajerin Esfahani, and Daniel Kuhn. Distributionally Robust Logistic Regression. September 2015.

Aman Sinha, Hongseok Namkoong, and John Duchi. Certifying Some Distributional Robustness with Principled Adversarial Training. *arXiv:1710.10571 [cs, stat]*, October 2017.

Robyn Speer. How to make a racist ai without really trying. 2017.

Christian Szegedy, Wojciech Zaremba, Ilya Sutskever, Joan Bruna, Dumitru Erhan, Ian Goodfellow, and Rob Fergus. Intriguing properties of neural networks. December 2013.

Hanchen Wang, Nina Grgic-Hlaca, Preethi Lahoti, Krishna P. Gummadi, and Adrian Weller. An Empirical Study on Learning Fairness Metrics for COMPAS Data with Human Supervision. *arXiv:1910.10255 [cs]*, October 2019.

Brian Hu Zhang, Blake Lemoine, and Margaret Mitchell. Mitigating Unwanted Biases with Adversarial Learning. *arXiv:1801.07593 [cs]*, January 2018.
### A PROOFS

#### A.1 PROOF OF PROPOSITION 3.1

By the duality result of [Blanchet & Murthy (2016)](https://www.sciencedirect.com/science/article/pii/S0022041317300831), for any $\epsilon > 0$,

$$
\sup_{P:W_1(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] - \sup_{P:W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)]
= \inf_{\lambda \geq 0} \{\lambda \epsilon + \mathbb{E}_P[\ell^*_{\lambda}(Z, \theta)]\} - (\lambda_n \epsilon + \mathbb{E}_P[\ell^*_{\lambda_n}(Z, \theta)])
\leq \mathbb{E}_P[\ell^*_{\lambda_n}(Z, \theta)] - \mathbb{E}_P[\ell^*_{\lambda}(Z, \theta)];
$$

where $\lambda_n \in \arg \min_{\lambda \geq 0} \{\lambda \epsilon + \mathbb{E}_P[\ell^*_{\lambda}(Z, \theta)]\}$. By assumption A3,

$$
|\ell^*_{\lambda_n}(z, \theta) - \ell^*_{\lambda}(z, \theta)|
= \left| \sup_{x_2 \in X} \ell((x_2, y), \theta) - \lambda_n c_s((x, y), (x_2, y)) - \sup_{x_2 \in X} \ell((x_2, y), \theta) - \lambda_n c((x, y), (x_2, y)) \right|
\leq \sup_{x_2 \in X} \lambda_n |c_s((x, y), (x_2, y)) - c((x, y), (x_2, y))|
\leq \lambda_n \delta_c \cdot D^2.
$$

This implies

$$
\sup_{P:W_1(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] - \sup_{P:W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)]
\leq \mathbb{E}_P[\ell^*_{\lambda_n}(Z, \theta)] - \mathbb{E}_P[\ell^*_{\lambda}(Z, \theta)] + \lambda_n \epsilon \cdot D^2.
$$

This bound is crude; it is possible to obtain sharper bounds under additional assumptions on the loss and transportation cost functions. We avoid this here to keep the result as general as possible.

Similarly,

$$
\sup_{P:W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] - \sup_{P:W_1(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)]
\leq \mathbb{E}_P[\ell^*_{\lambda_n}(Z, \theta)] - \mathbb{E}_P[\ell^*_{\lambda_n}(Z, \theta)] + \lambda_n \epsilon \cdot D^2;
$$

where $\lambda_n \in \arg \min_{\lambda \geq 0} \{\lambda \epsilon + \mathbb{E}_P[\ell^*_{\lambda}(Z, \theta)]\}$.

**Lemma A.1 (Lee & Raginsky (2017)).** Let $\tilde{\lambda} \in \arg \min_{\lambda \geq 0} \lambda \epsilon + \mathbb{E}_P[\ell^*_{\lambda}(Z, \theta)]$. As long as the function in the loss class are $L$-Lipschitz with respect to $d_2$ (see Assumption A2), $\tilde{\lambda} \leq \frac{L}{\sqrt{\epsilon}}$.

**Proof.** By the optimality of $\tilde{\lambda}$,

$$
\tilde{\lambda} \epsilon \leq \tilde{\lambda} \epsilon + \mathbb{E}_P\left[\sup_{x_2 \in X} \ell((x_2, Y), \theta) - \tilde{\lambda} d_2(X, x_2)^2 - \ell((X, Y), \theta)\right]
= \tilde{\lambda} \epsilon + \mathbb{E}_P[\ell^*_{\tilde{\lambda}}(Z, \theta) - \ell(Z, \theta)]
\leq \lambda \epsilon + \mathbb{E}_P[\ell^*_{\lambda}(Z, \theta) - \ell(Z, \theta)]
= \lambda \epsilon + \mathbb{E}_P\left[\sup_{x_2 \in X} \ell((x_2, Y), \theta) - \ell((X, Y), \theta) - \lambda d_2(X, x_2)^2\right]
$$

for any $\lambda \geq 0$. By Assumption A2, the right side is at most

$$
\tilde{\lambda} \epsilon \leq \lambda \epsilon + \mathbb{E}_P\left[\sup_{x_2 \in X} L d_2(X, x_2) - \lambda d_2(X, x_2)^2\right]
\leq \lambda \epsilon + \sup_{t \geq 0} Lt - \lambda t^2
$$

We minimize the right side WRT $t$ (set $t = \frac{L}{\sqrt{\epsilon}}$) and $\lambda$ (set $\lambda = \frac{L}{\sqrt{\epsilon}}$) to obtain $\tilde{\lambda} \epsilon \leq L \sqrt{\epsilon}$. \qed
By Lemma A.1, we have
\[
\sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] - \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] \leq \mathbb{E}_P[\ell_{\lambda_1}^*(Z, \theta)] - \mathbb{E}_P[\ell_{\lambda_2}^*(Z, \theta)] + \frac{L_\lambda D^2}{\sqrt{\epsilon}}.
\]
We combine the preceding bounds to obtain
\[
\left| \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] - \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] \right| \leq \sup_{f \in \mathcal{L}^c} \left| \int_Z f(z) d(P_n - P_\lambda)(z) \right| + \frac{L_\lambda D^2}{\sqrt{\epsilon}},
\]
where \( \mathcal{L}^c = \{ \ell_{\lambda}^c(\cdot, \theta) : \lambda \in [0, \frac{1}{4\epsilon}], \theta \in \Theta \} \) is the DR loss class. In the rest of the proof, we bound \( \sup_{f \in \mathcal{L}^c} \left| \int_Z f(z) d(P_n - P_\lambda)(z) \right| \) with standard techniques from statistical learning theory. Assumption A2 implies the functions in \( \mathcal{F} \) are bounded:
\[
0 \leq \ell((x_1, y_1), \theta) - \lambda d_1(x_1, x_1) \leq \ell_{\lambda_1}^c(z_1, \theta) \leq \sup_{x_2 \in \mathcal{X}} \ell((x_2, y_1), \theta) \leq M.
\]
This implies has bounded differences, so \( \delta_n \) concentrates sharply around its expectation. By the bounded-differences inequality and a symmetrization argument,
\[
\sup_{f \in \mathcal{L}^c} \left| \int_Z f(z) d(P_n - P_\lambda)(z) \right| \leq 2\mathcal{R}_n(\mathcal{L}^c) + M \left( \frac{\log \frac{2}{\epsilon}}{2n} \right)^{\frac{1}{2}}.
\]
WP at least \( 1 - t \), where \( \mathcal{R}_n(\mathcal{F}) \) is the Rademacher complexity of \( \mathcal{F} \):
\[
\mathcal{R}_n(\mathcal{F}) = \mathbb{E} \left[ \sup_{f \in \mathcal{F}} \frac{1}{n} \sum_{i=1}^n \sigma_i f(Z_i) \right].
\]
**Lemma A.2.** The Rademacher complexity of the DR loss class is at most
\[
\mathcal{R}_n(\mathcal{L}^c) \leq \frac{24 \mathbb{E}(\mathcal{L})}{\sqrt{n}} + \frac{24 LD^2}{\sqrt{n} \epsilon}.
\]
**Proof.** To study the Rademacher complexity of \( \mathcal{L}^c \), we first show that the \( \mathcal{L}^c \)-indexed Rademacher process \( X_f \triangleq \frac{1}{n} \sum_{i=1}^n \sigma_i f(Z_i) \) is sub-Gaussian WRT to a pseudometric. Let \( f_1 = \ell_{\lambda_1}^c(\cdot, \theta_1) \) and \( f_2 = \ell_{\lambda_2}^c(\cdot, \theta_2) \). Define
\[
d_{\mathcal{L}^c}(f_1, f_2) \triangleq ||\ell(\cdot, \theta_1) - \ell(\cdot, \theta_2)||_{\infty} + D^2 |\lambda_1 - \lambda_2|.
\]
We check that \( X_f \) is sub-Gaussian WRT \( d_{\mathcal{L}^c}: \)
\[
\mathbb{E}[\exp(t(X_{f_1} - X_{f_2}))]
\]
\[
= \mathbb{E}[\exp\left( \frac{t}{n} \sum_{i=1}^n \sigma_i (\ell_{\lambda_1}^c(Z_i, \theta_1) - \ell_{\lambda_2}^c(Z_i, \theta_2)) \right)]
\]
\[
= \mathbb{E}[\exp\left( \frac{t}{n} \sigma(\ell_{\lambda_1}^c(Z, \theta_1) - \ell_{\lambda_2}^c(Z, \theta_2)) \right)]^n
\]
\[
= \mathbb{E}[\exp\left( \frac{t}{n} \sigma \left( \sup_{x_1 \in \mathcal{X}} \inf_{x_2 \in \mathcal{X}} \left( \ell((x_1, y_1), \theta_1) - \lambda_1 d_1(x_1, X)^2 - \ell((x_2, y_1), \theta_2) + \lambda_2 d_2(X, x_2)^2) \right) \right) \] ^n
\]
\[
\leq \exp\left( \frac{t}{2} d_{\mathcal{L}^c}(f_1, f_2) \right).
\]
Let \( N(\mathcal{L}^c, d_{\mathcal{L}^c}, \epsilon) \) be the \( c \)-covering number of \( (\mathcal{L}^c, d_{\mathcal{L}^c}) \). We observe
\[
N(\mathcal{L}^c, d_{\mathcal{L}^c}, \epsilon) \leq N(\mathcal{L}, \| \cdot \|_{\infty}, \frac{\epsilon}{2}) \cdot N([0, \frac{1}{\sqrt{\epsilon}}], \| \cdot \|, \frac{D^2}{2\delta^2}) \quad (A.1)
\]
By Dudley’s entropy integral,

\[
\mathcal{R}_n(\mathcal{C}) \leq \frac{12}{\sqrt{n}} \int_0^\infty \log N(\mathcal{L}, d_{\mathcal{C}}, \epsilon) \frac{1}{\epsilon} d\epsilon
\]

\[
\leq \frac{12}{\sqrt{n}} \int_0^\infty \left( \log N(\mathcal{L}, \| \cdot \|_\infty, \frac{2}{\sqrt{n}}) + N([0, \frac{2}{\sqrt{n}}], \| \cdot \|_\infty) \right) \frac{1}{\epsilon} d\epsilon
\]

\[
\leq \frac{12}{\sqrt{n}} \left( \int_0^\infty \log N(\mathcal{L}, \| \cdot \|_\infty, \frac{2}{\sqrt{n}}) \frac{1}{\epsilon} d\epsilon + \int_0^\infty N([0, \frac{2}{\sqrt{n}}], \| \cdot \|_\infty) \frac{1}{\epsilon} d\epsilon \right)
\]

\[
\leq 24\mathcal{E}(\mathcal{L}) + \frac{24LD^2}{\sqrt{n}2^\epsilon} \int_0^\infty \log(\frac{1}{2}) d\epsilon
\]

where we recalled equation (A.1) in the second step. We evaluate the integral on the right side to arrive at the stated bound: \( \int_0^\infty \log(\frac{1}{2}) d\epsilon < 1 \). 

By Lemma A.2

\[
\sup_{f \in \mathcal{C}_*} \left| \int_Z f(z) d(P_n - \hat{P})(z) \right| \leq \frac{48\mathcal{E}(\mathcal{L})}{\sqrt{n}} + \frac{48LD^2}{\sqrt{n}2^\epsilon} + M\left( \frac{\log \frac{2}{2^\epsilon}}{2n} \right)^{\frac{1}{2}},
\]

which implies

\[
\left| \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \hat{\theta})] - \sup_{P : W(P, \hat{P}_*) \leq \epsilon} \mathbb{E}_P[\ell(Z, \hat{\theta})] \right|
\]

\[
\leq \frac{48\mathcal{E}(\mathcal{L})}{\sqrt{n}} + \frac{48LD^2}{\sqrt{n}2^\epsilon} + \frac{L\delta_n D^2}{\sqrt{\epsilon}} + M\left( \frac{\log \frac{2}{2^\epsilon}}{2n} \right)^{\frac{1}{2}}.
\]

WP at least 1 - \( t \).

A.2 PROOFS OF PROPOSITIONS 3.2 AND 3.3

Proof of Proposition 3.2 It is enough to show

\[
\sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \hat{\theta})] \leq \delta^* + 2\delta_n
\]

because the loss function is non-negative. We have

\[
\sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \hat{\theta})] \leq \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \hat{\theta})] + \delta_n
\]

\[
\leq \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \hat{\theta})] + \delta_n
\]

\[
\leq \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \hat{\theta})] + 2\delta_n
\]

\[
\leq \delta^* + 2\delta_n.
\]

Proof of Proposition 3.3

\[
\sup_{P : W(P, P_n) \leq \epsilon} \left( \mathbb{E}_P[\ell(Z, \theta)] - \mathbb{E}_P[\ell(Z, \theta)] \right) - \sup_{P : W(P, P_n) \leq \epsilon} \left( \mathbb{E}_P[\ell(Z, \theta)] - \mathbb{E}_P[\ell(Z, \theta)] \right)
\]

\[
= \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] - \sup_{P : W(P, P_n) \leq \epsilon} \mathbb{E}_P[\ell(Z, \theta)] + \mathbb{E}_P[\ell(Z, \theta)] - \mathbb{E}_P[\ell(Z, \theta)]
\]

\[
\leq \delta_n + \mathbb{E}_P[\ell(Z, \theta)] - \mathbb{E}_P[\ell(Z, \theta)]
\]

The loss function is bounded, so it is possible to bound \( \mathbb{E}_P[\ell(Z, \theta)] - \mathbb{E}_P[\ell(Z, \theta)] \) by standard uniform convergence results on bounded loss classes.
B DATA-DRIVEN FAIR METRICS

B.1 LEARNING THE FAIR METRIC FROM OBSERVATIONS OF THE SENSITIVE ATTRIBUTE

Here we assume the sensitive attribute is discrete and is observed for a small subset of the training data. Formally, we assume this subset of the training data has the form \( \{(X_i, K_i, Y_i)\}\), where \( K_i \) is the sensitive attribute of the \( i \)-th subject. To learn the sensitive subspace, we fit a softmax regression model to the data

\[
P(K_i = l \mid X_i) = \frac{\exp(a_i^T X_i + b_l)}{\sum_l \exp(a_i^T X_i + b_l)}, \quad l = 1, \ldots, k,
\]

and take the span of \( A = [a_1 \ldots a_k] \) as the sensitive subspace to define the fair metric as

\[
dx(x_1, x_2)^2 = (x_1 - x_2)^T (I - P_{\text{ran}(A)}) (x_1 - x_2).
\]

This approach readily generalizes to sensitive attributes that are not discrete-valued: replace the softmax model by an appropriate generalized linear model.

In many applications, the sensitive attribute is part of a user’s demographic information, so it may not be available due to privacy restrictions. This does not preclude the proposed approach because the sensitive attribute is only needed to learn the fair metric and is neither needed to train the classifier nor at test time.

B.2 LEARNING THE FAIR METRIC FROM COMPARABLE SAMPLES

In this section, we consider the task of learning a fair metric from supervision in a form of comparable samples. This type of supervision has been considered in the literature on debiasing learned representations. For example, method of [Bolukbasi et al. (2016)] for removing gender bias in word embeddings relies on sets of words whose embeddings mainly vary in a gender subspace (e.g. (king, queen)).

To keep things simple, we focus on learning a generalized Mahalanobis distance

\[
dx(x_1, x_2) = (\varphi(x_1) - \varphi(x_2))^T \tilde{\Sigma} (\varphi(x_1) - \varphi(x_2)),
\]

where \( \varphi(x) : \mathcal{X} \rightarrow \mathbb{R}^d \) is a known feature map and \( \tilde{\Sigma} \in S_{+}^{d \times d} \) is a covariance matrix. Our approach is based on a factor model

\[
\varphi_i = A_s u_i + B_s v_i + \epsilon_i,
\]

where \( \varphi_i \in \mathbb{R}^d \) is the learned representation of \( x_i \), \( u_i \in \mathbb{R}^K \) (resp. \( v_i \in \mathbb{R}^L \)) is the sensitive/irrelevant (resp. relevant) attributes of \( x_i \) to the task at hand, and \( \epsilon_i \) is an error term. For example, in [Bolukbasi et al. (2016)], the learned representations are the embeddings of words in the vocabulary, and the sensitive attribute is the gender bias of the words. The sensitive and relevant attributes are generally unobserved.

Recall our goal is to obtain \( \tilde{\Sigma} \) so that equation [B.2] is small whenever \( v_1 \approx v_2 \). One possible choice of \( \tilde{\Sigma} \) is the projection matrix onto the orthogonal complement of \( \text{ran}(A) \), which we denote by \( P_{\text{ran}(A)} \).

Indeed,

\[
dx(x_1, x_2)^2 = (\varphi_1 - \varphi_2)^T (I - P_{\text{ran}(A)}) (\varphi_1 - \varphi_2)
\approx (v_1 - v_2)^T B_s^T (I - P_{\text{ran}(A)}) B_s (v_1 - v_2),
\]

which is small whenever \( v_1 \approx v_2 \). Although \( \text{ran}(A) \) is unknown, it is possible to estimate it from the learned representations and groups of comparable samples by factor analysis.

The factor model attributes variation in the learned representations to variation in the sensitive and relevant attributes. We consider two samples comparable if their relevant attributes are similar. In other words, if \( \mathcal{I} \subset [n] \) is (the indices of) a group of comparable samples, then

\[
H\Phi_\mathcal{I} = HU_\mathcal{I}A_*^T + HV_\mathcal{I}B_s^T + HE_\mathcal{I} \approx 0
\Rightarrow HU_\mathcal{I}A_*^T + HE_\mathcal{I} \approx 0
\Rightarrow HU_\mathcal{I}A_*^T + HE_\mathcal{I} \approx 0.
\]


where $H = I_{|\mathcal{I}|} - \frac{1}{|\mathcal{I}|^2} 1_{|\mathcal{I}|} 1_{|\mathcal{I}|}^T$ is the centering or de-meaning matrix and the rows of $\Phi_{\mathcal{I}}$ (resp. $U_{\mathcal{I}}, V_{\mathcal{I}}$) are $\phi_i$ (resp. $u_i, v_i$). If this group of samples have identical relevant attributes, i.e. $V_{\mathcal{I}} = 1_{|\mathcal{I}|} v^T$ for some $v$, then $HV_{\mathcal{I}}$ vanishes exactly. As long as $u_i$ and $\epsilon_i$ are uncorrelated (e.g. $\mathbb{E}[u_i \epsilon_i^T] = 0$), equation B.5 implies

$$\mathbb{E}[\Phi_{\mathcal{I}}^T H \Phi_{\mathcal{I}}] \approx A \mathbb{E}[U_{\mathcal{I}}^T H U_{\mathcal{I}}] A^T + \mathbb{E}[E_{\mathcal{I}}^T H E_{\mathcal{I}}],$$

This suggests estimating $\text{ran}(A)$ from the learned representations and groups of comparable samples by factor analysis. We summarize our approach in Algorithm 3.

### Algorithm 3 estimating $\hat{\Sigma}$ for the fair metric

1: **Input:** $\{\phi_i\}_{i=1}^n$, comparable groups $\mathcal{I}_1, \ldots, \mathcal{I}_G$
2: $\hat{A}^T \in \arg\min_{W, A} \left\{ \frac{1}{2} \sum_{g=1}^{G} \|H_g \Phi_{\mathcal{I}_g} - W_g A^T\|^2_F \right\}$ \textit{\textcopyright factor analysis}
3: $Q \leftarrow \text{qr}(\hat{A})$ \textit{\textcopyright get orthonormal basis of ran($\hat{A}$)}
4: $\hat{\Sigma} \leftarrow I_d - QQ^T$

### C SenSR Implementation Details

This section is to accompany the implementation of the SenSR algorithm and is best understood by reading it along with the code implemented using TensorFlow\footnote{https://github.com/IBM/sensitive-subspace-robustness}. We discuss choices of learning rates and few specifics of the code. Words in italics correspond to variables in the code and following notation in parentheses defines corresponding name in Table 3, where we summarize all hyperparameter choices.

#### Handling class imbalance

Datasets we study have imbalanced classes. To handle it, on every epoch ($E$) (i.e. number of epochs) we subsample a batch size ($B$) training samples enforcing equal number of observations per class. This procedure can be understood as data augmentation.

#### Perturbations specifics

Our implementation of SenSR algorithm has two inner optimization problems — subspace perturbation and full perturbation (when $\epsilon > 0$). Subspace perturbation can be viewed as an initialization procedure for the attack. We implement both using Adam optimizer (Kingma & Ba [2014]) inside the computation graph for better efficiency, i.e. defining corresponding perturbation parameters as Variables and re-setting them to zeros after every epoch. This is in contrast with a more common strategy in the adversarial robustness implementations, where perturbations (i.e. attacks) are implemented using tf.gradients with respect to the input data defined as a Placeholder.

#### Learning rates

As mentioned above, in addition to regular Adam optimizer for learning the parameters we invoke two more for the inner optimization problems of SenSR. We use same learning rate of 0.001 for the parameters optimizer, however different learning rates across datasets for subspace step ($s$) and full step ($f$). Two other related parameters are number of steps of the inner optimizations: subspace epoch ($se$) and full epoch ($fe$). We observed that setting subspace perturbation learning rate too small may prevent our algorithm from reducing unfairness, however setting it big does not seem to hurt. On the other hand, learning rate for full perturbation should not be set too big as it may prevent algorithm from solving the original task. Note that full perturbation learning rate should be smaller than perturbation budget $\epsilon$ (i.e. $\epsilon/10$). In general, malfunctioning behaviors are immediately noticeable during training and can be easily corrected, therefore we did not need to use any hyperparameter optimization tools.
Table 3: SenSR hyperparameter choices in the experiments

|       | E   | B  | s  | se | ϵ  | f  | fe |
|-------|-----|----|----|----|----|----|----|
| Sentiment | 4K  | 1K | 0.1| 10 | 0.1| 0.01| 10 |
| Adult   | 12K | 1K | 10 | 50 | 10⁻³| 10⁻⁴| 40 |

Table 4: Summary of Adult classification experiments over 10 restarts

|          | Accuracy | B-TPR | Gap\(_{RMS}\)         | Gap\(_{RMS}\)         | Gap\(_{max}\)         | Gap\(_{max}\)         |
|----------|----------|-------|-----------------------|-----------------------|-----------------------|-----------------------|
| SenSR    | .787±.003| .789±.003| .068±.004 | .055±.003 | .087±.005 | .067±.004 |
| Baseline | .813±.001| .829±.001| .179±.004 | .089±.003 | .216±.003 | .105±.003 |
| Project  | .813±.001| .827±.001| .145±.004 | .064±.003 | .192±.004 | .086±.004 |
| Adv. Debias. | .812±.001| .815±.002| .082±.005 | .070±.006 | .110±.006 | .078±.005 |
| CoCL     | -        | .790  | .163      | .080      | .201      | .109      |

D. ADDITIONAL ADULT EXPERIMENT DETAILS

D.1 PREPROCESSING

The continuous features in Adult are the following: age, fnlwgt, capital-gain, capital-loss, hours-per-week, and education-num. The categorical features are the following: workclass, education, marital-status, occupation, relationship, race, sex, and native-country. See Dua & Graff (2017) for a description of each feature. We remove fnlwgt and education but keep education-num, which is an integer representation of education. We do not use native-country, but use race and sex as predictive features. We treat race as binary: individuals are either White or non-White. For every categorical feature, we use one hot encoding. For every continuous feature, we standardize, i.e., subtract the mean and divide by the standard deviation. We remove anyone with missing data leaving 45,222 individuals.

This data is imbalanced: 25% make at least $50k per year. Furthermore, there is demographic imbalance with respect to race and gender as well as class imbalance on the outcome when conditioning on race or gender: 86% of individuals are white of which 26% make at least $50k a year; 67% of individuals are male of which 31% make at least $50k a year; 11% of females make at least $50k a year; and 15% of non-whites make at least $50k a year.

D.2 FULL EXPERIMENTAL RESULTS

See Tables 4 and 5 for the full experiment results. The tables report the average and the standard error for each metric on the test set for 10 train and test splits.

D.3 SENSITIVE SUBSPACE

To learn the hyperplane that classifies females and males, we use our implementation of regularized logistic regression with a batch size of 5k, 5k epochs, and .1 \(\ell_2\) regularization.

Table 5: Summary of individual fairness metrics in Adult classification experiments over 10 restarts

|          | Spouse Consistency | Gender and Race Consistency |
|----------|-------------------|-----------------------------|
| SenSR    | .934±.012         | .984±.000                   |
| Baseline | .848±.008         | .865±.004                   |
| Project  | .868±.005         | 1±0                         |
| Adv. Debias. | .807±.002        | .841±.012                   |
D.4 Hyperparameters and Training

For each model, we use the same 10 train/test splits where use 80% of the data for training. Because of the class imbalance, each minibatch is sampled so that there are an equal number of training points from both the “income at least $50k class” and the “income below $50k class.”

D.4.1 Baseline, Project, and SenSR

See Table 3 for the hyperparameters we used when training Baseline, Project, and SenSR (Baseline and Project use a subset). Hyperparameters are defined in Appendix C.

D.4.2 Adversarial Debiasing

We used Zhang et al. (2018)’s adversarial debiasing implementation in IBM’s AIF360 package (Belzamery et al., 2018) where the source code was modified so that each mini-batch is balanced with respect to the binary labels just as we did with our experiments and dropout was not used. Hyperparameters are the following: adversary loss weight = .001, num epochs = 500, batch size = 1000, and privileged groups are defined by binary gender and binary race.

D.5 Group Fair Metrics

Let $\mathcal{C}$ be a set of classes, $A$ be a binary protected attribute and $Y, \hat{Y} \in \mathcal{C}$ be the true class label and the predicted class label. Then for $a \in \{0, 1\}$ and $c \in \mathcal{C}$ define $TPR_{a,c} = P(\hat{Y} = c | A = a, Y = c);\quad\text{Gap}_{A,c} = TPR_{0,c} - TPR_{1,c};\quad\text{Gap}_{A}^{\text{RMS}} = \sqrt{\frac{1}{|\mathcal{C}|} \sum_{c \in \mathcal{C}} \text{Gap}_{A,c}^2};\quad\text{Gap}_{A}^{\text{max}} = \arg \max_{c \in \mathcal{C}} \text{Gap}_{A,c}.$

Balanced Acc $= \frac{1}{|\mathcal{C}|} \sum_{c \in \mathcal{C}} P(\hat{Y} = c | Y = c).$

For Adult, we report $\text{Gap}_{R}^{\text{RMS}}, \text{Gap}_{G}^{\text{RMS}}, \text{Gap}_{R}^{\text{max}},$ and $\text{Gap}_{G}^{\text{max}}$ where $\mathcal{C}$ is composed of the two classes that correspond to whether someone made at least $50k, R$ refers to race, and $G$ refers to gender.