Mathematical models of noise characteristics of high-speed digital-to-analog converters of radar signal generators

V V Romashov, L V Romashova, A N Doktorov, K A Yakimenko, N A Socheva

Vladimir State University, 87 Gorky Street, Vladimir, 600000, Russia

E-mail: romashovmurom@mail.ru

Abstract. A mathematical model of the power spectral density of phase noise of high-speed digital-to-analog converters in one side frequency band of tuning is obtained. Calculated using the developed mathematical model of the power spectral density of phase noise of high-speed digital-to-analog converters from Analog Devices in the efficiency mode in the NRZ baseband showed a small error in comparison with the experimental characteristics. It is shown that in the 2×NRZ oversampling mode, the phase noise level of the high-speed DAC is 7-8 dBc/Hz lower than in the main NRZ operation mode.

1. Introduction

Digital-to-analog converters (DACs) [1-3] have recently been widely used in radio engineering systems to generate high-frequency radio signals. They have the following advantages: the ability to form a radio signal at once at a high frequency with the required types of modulation, high accuracy of the synthesized frequency, digital control of the frequency and phase of the output signal, very high resolution in frequency and phase, high frequency tuning speed [4, 5]. Digital-to-analog converters are an integral part of direct digital synthesizers (DDS) [6-10], which are used to create reference signals in radio systems [11].

An important characteristic that determines the quality of the generated radio signals is the power spectral density (PSD) of phase noise in one sideband. This value is characterized by the ratio of the noise power at the frequency F of one side band in the frequency band 1 Hz to the power of the carrier signal [12].

PSD models of phase noise based on power functions are used for the theoretical analysis of the noise characteristics of radio devices [8]. The use of these models significantly simplifies the design of signal generators and other devices, allowing you to do without complex experimental studies. For digital computing synthesizers, similar models are proposed and considered in [13, 14].

Since the noise characteristics of the DAC are mainly determined by the noise of the DAC, we will take the model of the characteristics of the DAC as the basis for the PSD model of phase noise in a single sideband of the DAC. The aim of the work is to develop a mathematical model of the power spectral density of phase noise in one sideband of digital-to-analog converters in various operating modes.
2. The model of the power spectral density of phase noise in a single sideband of digital-to-analog converters

For a mathematical model of the power spectral density of phase noise in one sideband of the DAC, we use the model for the DAC [13], based on power functions of the form

\[ S_{DAC}(F) = K_{DAC} \left( \frac{10^{k_1}}{F^2} + \frac{10^{k_2}}{F} + 10^{k_3} \right) + 10^{k_4} + S_q, \]  

where coefficients \( k_1, k_2, k_3, k_4 \) determine the level PSD \( 1/F^2 \) phase noise, \( 1/F \) phase noise, the natural noise component of the input circuits and the natural noise component of the load resistance, respectively. \( F \) - offset from the carrier frequency, \( K_{DAC} = f_{out} / f_{CLK} \) - DAC transmission ratio, \( f_{out} \) and \( f_{CLK} \) - DAC output and clock frequencies, \( S_q = 2^{-2N-0.59} \left( \frac{f_{out}}{f_{CLK}} \right)^2 \) - DAC quantization noise, \( N \) - number of DAC bits.

The coefficient \( k_1 \) determines the level of flicker noise \( 1/F \) for the linear section of the PSD of phase noise in the offset frequency range 100-10000 Hz, when all other components are significantly smaller. Therefore, the expression for it for the average offset frequency \( F=1000 \) Hz is written as:

\[ k_1 = \lg \left( 10^{-S_{db}(F,f_{out min})/10} \right) \left( \frac{K_{DAC min}}{F^2} \right)^2 \]  

The expression \( S_{db}(F,f_{out min}) \) is determined by the PSD value of the phase noise in dBc/Hz, defined for the lowest output frequency \( f_{out min} \) DAC (smallest \( K_{DDSm in}=f_{out min}/f_{CLK} \)), for which there are experimental spectral characteristics.

Value \( k_2 \) determines the level of white frequency noise \( 1/F^2 \), which is defined for the minimum offset frequency \( F = 10 \) Hz.

The coefficients for the natural noise components are determined for the offset frequencies \( F \geq 1 \) MHz, when the flicker noise is zero, for the lowest output frequency of the DDS

\[ k_3 = \lg \left( 10^{-S_{db}(F,f_{out max})/10} \right) \left( \frac{K_{DAC max}}{F^2} \right)^2 \]  

and the \( k_4 \) coefficient is determined for the maximum output frequency of the synthesizer for \( K_{DAC max} = f_{out max} / f_T \).

\[ k_4 = \lg \left( 10^{-S_{db}(F,f_{out max})/10} \right) \left( \frac{K_{DAC max}}{F^2} \right)^2 \]  

The noise level of the DAC quantization is determined by the number of bits \( N \), for \( N \geq 14 \) quantization noise is significantly less than additive noise and can be ignored.

To create a model of the noise characteristics of digital-to-analog converters, we use the experimental noise characteristics of a digital-to-analog converter AD9164 [11] shown in figure 1. This DAC can operate in various modes: normal operation mode (non-return-to-zero (NRZ)), 2xNRZ - The DAC operates at twice the clock speed if new data samples are captured in the DAC core on both the leading and trailing edges, radiofrequency (RF) or mixmode - in this case, each clock pulse of the NRZ mode is represented by two different-polar pulses with a duration equal to half of the period.
Figure 1. Experimental spectral power densities of phase noise DAC AD9164 in a single sideband for \( f_{\text{CLK}} = 6 \text{ GHz} \)

In this case, the noise characteristics of the chip are given taking into account the noise of the clock generator.

\[
S_{\text{DAC, REF}}(F) = S_{\text{DAC}}(F) + S_{\text{REF}}(F) K_{\text{DAC}}^2,
\]

(6)

where \( S_{\text{DAC}}(F), S_{\text{REF}}(F) \) - models of the noise characteristics of the DAC itself and the reference frequency generator, \( K_{\text{DAC}} = f_{\text{out}} / f_{\text{CLK}} \) - the DAC transmission ratio is similar to the DAC transmission ratio in (1). Therefore, to calculate the approximation coefficients \( k_i \), we will determine the noise characteristics of the DAC itself \( S_{\text{DAC}}(F) \) from (6), shown for the two output frequencies in figure 2.

Figure 2. Noise characteristics DAC AD9164
Using formulas (2) – (4) and Fig. 3, we determine the coefficients of the model: $k_1 = -8.4$, $k_2 = -5.7$, $k_3 = -17.1$, $k_4 = -15.4$.

The results of simulating the PSD of the natural phase noise of the AD9164 DAC and the entire chip, taking into account the REF noise for the lowest output frequency of 70 MHz in the NRZ mode, are shown in figure 3.

![Figure 3](image-url)  
**Figure 3.** Experimental and simulated noise characteristics of the DAC AD9164 for $f_{out} = 70$ MHz and $f_{CLK} = 6$ GHz.

As can be seen, the error of the model, taking into account the error of digitizing the experimental data, is several dBc/Hz. Despite the large natural noise of the REF, the resulting values of the PSD of phase noise due to the small transmission coefficient $K_{DAC} = f_{out} / f_{CLK}$ for a small output frequency are mainly determined by the noise of the DAC itself.

For other frequencies, the simulation results shown in figure 4 were also satisfactory.

![Figure 4](image-url)  
**Figure 4.** Experimental and simulated noise characteristics of the DAC AD9164 in NRZ mode, taking into account REF noise at the output frequencies of 3900 MHz and 900 MHz at the clock frequency $f_{CLK} = 6$ GHz.
Similar calculations were made for the DAC AD9172. The approximation coefficients for this chip in accordance with the calculation are written as $k_1 = -7.25$, $k_2 = -4.87$, $k_3 = -15.8$, $k_4 = -13.6$. A comparison of the results of modeling noise characteristics with experimental characteristics in the NRZ mode is shown in figure 5. As you can see, the greatest error is observed at offset frequencies of more than 300 kHz. This is probably due to the error in the approximation of the initial experimental characteristics.

![Figure 5](image)

**Figure 5.** Experimental and simulated noise characteristics of the DAC AD9172 in NRZ mode, taking into account REF noise at the output frequencies of 3600 MHz, 1800 MHz and 900 MHz at the clock frequency $f_{CLK} = 12$ GHz

We will conduct a study of the noise characteristics of the DAC in the mode 2xNRZ. In this mode, the signal samples are made on the leading and trailing edges of the clock generator pulses, which is equivalent to doubling the clock frequency. The noise characteristics for the clock frequency $f_{CLK}=4000$ MHz shown in the datasheet of the AD9164 integrated DAC correspond to this mode and are shown in figure 6.

![Figure 6](image)

**Figure 6.** Experimental noise characteristics of the DAC AD9164 at the clock frequency 4000 MHz in the mode 2xNRZ
Let’s look at how the noise characteristics of the DAC change when the clock frequency changes. To do this, we add the DAC transmission coefficient to the expression for the mathematical model of noise characteristics (1), since it affects the characteristic at detuning frequencies of more than 0.4\( f_T \).

\[
S_{DAC}(F) = \left( \frac{f_{out}}{f_{CLK}} \right)^2 \left( \frac{10^{k_2}}{F^2} + \frac{10^{k_1}}{F} + 10^{k_4} \right) + \left( 10^{k_3} + S_q \right) \left( \frac{\pi f_{out} / f_{CLK}}{\sin(\pi f_{out} / f_{CLK})} \right)^2. \tag{7}
\]

The dependencies for the modes NRZ and 2xNRZ constructed using the model (7) (in the formula, double the clock frequency is substituted) are shown in figure 7.

![Figure 7. PSD of phase noise DAC AD9164 for the modes NRZ and 2xNRZ](image)

For a low output frequency of 70 MHz, the characteristics for the NRZ and 2xNRZ modes up to 10 kilohertz differ from each other by 6 dBc/Hz, with a larger offset, both dependencies are limited to thermal noise at -171 dBc/Hz.

For a frequency of 900 MHz, the dependencies for any detuning differ by 6 dBc/Hz, and with an increase in the output frequency to 3900 MHz, the DAC transmission coefficient and phase noise in the NRZ mode begin to affect significantly (by 20 dBc/Hz) increase.

This does not happen in 2xNRZ, since at twice the clock frequency, the effect of the DAC transmission coefficient on natural noise is insignificant.

The phase noise of the reference generator has a great influence on the formation of high-frequency signals. Taking into account the noise of the REF in accordance with (6), the obtained noise characteristics of the AD9164 are shown in figure 8.

As can be seen, the proposed PSD model of DAC phase noise approximates the experimental dependences with sufficient accuracy. A small outlier in the experimental characteristics is due to the PSD of the phase noise of the reference generator.
Figure 8. Experimental and simulated noise characteristics of the DAC AD9164 at the mode 2xNRZ taking into account the REF noise at the output frequencies of 3900 MHz, 900 MHz and 70 MHz at the clock frequency $f_{CLK} = 4$ GHz.

3. Conclusion
Thus, the proposed mathematical model of the power spectral density of phase noise in one sideband of digital-to-analog converters corresponds to the experimental characteristics with a sufficiently high accuracy and allows us to theoretically study the noise characteristics of high-speed DACs at any output frequencies.

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