Effects of junctional correlations in the totally asymmetric simple exclusion process on random regular networks

Yongjoo Baek,1,* Meesoon Ha,2,† and Hawoong Jeong3,4,‡

1Natural Science Research Institute, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea
2Department of Physics Education, Chosun University, Gwangju 501-759, Korea
3Department of Physics and Institute for the BioCentury, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea
4APCTP, Pohang, Gyungbuk 790-784, Korea

(Dated: December 8, 2014)

We investigate the totally asymmetric simple exclusion process on closed and directed random regular networks, which is a simple model of active transport in the one-dimensional segments coupled by junctions. By a pair mean-field theory and detailed numerical analyses, it is found that the correlations at junctions induce two notable deviations from the simple mean-field theory which neglects these correlations: (1) the narrower range of particle density for phase coexistence and (2) the algebraic decay of density profile with exponent 1/2 even outside the maximal-current phase. We show that these anomalies are attributable to the effective slow bonds formed by the network junctions.

PACS numbers: 02.50.-r, 89.75.Hc, 05.60.Cd, 64.60.-i

I. INTRODUCTION

Various transport phenomena involve self-driven, hard-core particles moving unidirectionally along one-dimensional (1D) segments. The totally asymmetric simple exclusion process (TASEP), in which 1D lattice gas particles randomly hop one step forward if and only if the next site is vacant, is one of the simplest models of such phenomena. The model has an important advantage of being exactly solvable in homogeneous 1D systems [1], with very well-understood dynamical phases [2]. Besides its original purpose as the model of mRNA translation by ribosomes [3], the TASEP has been applied in modified forms to numerous examples of vehicular, pedestrian, and biological transport [4].

In many of these examples, the 1D segments are not isolated from each other but coupled by junctions, where the particles can randomly switch from one segment to another. The motor proteins on cytoskeletal networks [5, 6] provide one interesting example of such behavior. In order to clarify the effects of junctions, the TASEP has been studied in various systems consisting of coupled 1D segments, including open (i.e. connected to particle reservoirs) segments with a single junction [7], closed (i.e. conserving the particles) 1D loops with a single junction [8, 9], closed 1D loops with a shortcut [10], periodic hexagonal lattices [11], and closed and directed random regular (CDRR) networks [12]. For lack of exact solutions, all these studies rely on the simplifying assumption that the correlations between each junction and its neighboring sites are negligible. This approach is also called the defect mean-field (DMF) theory [13], because it was originally proposed to address the effects of a single local defect in the 1D TASEP [14, 15]. The theory allows one to approximate every segment as an open 1D system whose boundary conditions are given by occupancies of the junctions at both ends. Thus, in the limit of infinitely long segments, the dynamical phases of different parts of the system can be analytically predicted from the well-established knowledge of the 1D systems.

While the predictions of the DMF theory agree qualitatively well with the numerical results, they are also known to be quantitatively inexact for the single-junction cases [8, 15] due to the neglected correlations at junctions. When the system has a large number of junctions, to our knowledge there has been no systematic test of the quantitative agreement in the proper asymptotic limit. Using the CDRR networks studied by the authors of Ref. [12], in this study we analytically and numerically show that the neglected junctional correlations induce nonzero corrections to the DMF predictions. On the analytical side, we develop a defect pair mean-field (DPMF) theory which takes into account the pair correlations between junctions and their neighboring sites. On the numerical side, we detect non-DMF behaviors in the steady-state properties of the current and the density profile of particles through extensive Monte Carlo simulations at different segment lengths. The observed non-DMF behaviors reveal interesting connections to the unresolved issues of how a single slow bond affects the 1D transport [14–18].

The rest of the paper is organized as follows. We first introduce the model in Sec. II and explain its DMF description in Sec. III. Then we show by a DPMF argument that there are nonzero corrections to this DMF description in Sec. IV. Numerical evidence for these corrections is presented in Sec. V, whose similarities with the 1D TASEP with a single slow bond are discussed in

* Present address: Department of Physics, Technion–Israel Institute of Technology, Haifa 32000, Israel
† Corresponding author: msha@chosun.ac.kr
‡ hjjeong@kaist.edu
theory, each link can be regarded as an open 1D system whose entry and exit rates are given by
\[ \tilde{\alpha} = \rho_u/c, \quad \tilde{\beta} = 1 - \rho_v, \] (1)
respectively [see Fig. 1(b)]. Then the steady-state current \( J_{uv}(\tilde{\alpha}, \tilde{\beta}) \) and the bulk density \( \rho_{uv} \) of link \( uv \) can be obtained from the exact solutions of 1D TASEP with open boundaries [1].

In the steady state, each junction satisfies the continuity equation
\[ \sum_{v} J_{uv} = \sum_{v'} J_{v'u}, \] (2)
which is also called Kirchhoff’s current law in the context of electrical circuits. This equation is automatically satisfied if all junctions have the same mean occupancy \( \rho_u \), so that the link indices (e.g. \( uv \)) can be dropped from all quantities mentioned so far. Then the global current \( J \) is equal to the current through each link, which gives
\[ J = \min \left[ (\rho_u/c)(1 - \rho_u/c) \right] \]
\[ = \begin{cases} (\rho_u/c)(1 - \rho_u/c) & \text{if } \rho_u < c/(c+1), \\ c/(c+1)^2 & \text{if } \rho_u = c/(c+1), \\ \rho_u(1 - \rho_u) & \text{otherwise}. \end{cases} \] (3)

Similarly, the global particle density \( \rho \) is equal to the bulk density of particles in each link, which satisfies
\[ \rho = \begin{cases} \rho_u/c & \text{if } \rho_u < c/(c+1), \\ \rho_u & \text{if } \rho_u > c/(c+1). \end{cases} \] (4)

When \( \rho_u = c/(c+1) \), \( \rho \) can assume any value between \( \rho_u/c \) and \( \rho_u \), since each link has two coexistent blocks with different bulk densities whose interface can fluctuate back and forth [19]. Hence the density–current relation is obtained as
\[ J = \begin{cases} c/(c+1)^2 & \text{if } \rho_{\text{DMF}}^* < \rho < 1 - \rho_{\text{DMF}}^*, \\ \rho(1 - \rho) & \text{otherwise}, \end{cases} \] (5)
where \( \rho_{\text{DMF}}^* \equiv 1/(c+1) \) and \( 1 - \rho_{\text{DMF}}^* \) become the phase boundaries. Here \( \rho < \rho_{\text{DMF}}^* \) corresponds to the low-density phase, \( \rho > 1 - \rho_{\text{DMF}}^* \) to the high-density phase, and the current plateau in the middle to the phase coexistence regime. The predictions of Eq. (5) are shown by the dashed lines in the main plot of Fig. 2.

IV. DEFECT PAIR MEAN-FIELD THEORY

In this section we propose the defect pair mean-field (DPMF) theory, which takes into account the pair correlations between junctions and their adjacent sites. As in the DMF case, we expect all links to have the same statistics, which implies that the link indices can be left
The entry (exit) rate of a link is given by the occupancy \( \rho \) of each junction. (c) The DPMF theory assumes that only the pairs consisting of a junction and its adjacent site are correlated.

FIG. 2. (Color online) Fundamental diagram of the density–current relation for the CDRR networks with \( c = 2 \) (red/gray) and \( c = 3 \) (black). The DMF predictions (dashed lines), the DPMF predictions (solid lines), and the simulation results obtained at \( L = 100 \) (×), 200 (○), 400 (△), 800 (□) are shown. (Inset) Covariance \( \langle \sigma_{ab} \equiv \langle \tau_a \tau_b \rangle - \langle \tau_a \rangle \langle \tau_b \rangle \rangle \) between a junction and its adjacent site in an incoming link (upper two curves) and in an outgoing link (lower two curves) at \( L = 800 \), where the lines are guide to the eye. The CDRR networks with \( N = 10^4 \) are used.

out. Thus we let \( \tau_i \) denote the state of the \( i \)-th inner site from the entrance junction, whose mean occupancy is denoted by \( \rho_i \equiv \langle \tau_i \rangle \).

The DPMF theory assumes that the only nonzero correlations among \( \tau_L, \tau_u \), and \( \tau_1 \) are the pair correlations between \( \tau_L \) and \( \tau_u \) and those between \( \tau_u \) and \( \tau_1 \). These correlations are related to the conditional probabilities

\[
\begin{align*}
\mu^{\tau_L \tau_u}_u &\equiv P(\tau_L | \tau_u), & \mu^{\tau_u \tau_1}_u &\equiv P(\tau_1 | \tau_u), \\
\nu^{\tau_u \tau_L}_L &\equiv P(\tau_L | \tau_u), & \nu^{\tau_1 \tau_u}_u &\equiv P(\tau_u | \tau_1).
\end{align*}
\]

For example, the three-point correlation \( \langle \tau_L(1 - \tau_u)\tau_1 \rangle \) can be written as

\[
\langle \tau_L(1 - \tau_u)\tau_1 \rangle = P(\tau_L = 1, \tau_u = 0, \tau_1 = 1) = \rho_L \mu^{10}_u = \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u),
\]

\[
\langle \tau_L(1 - \tau_u)\tau_1 \rangle = P(\tau_L = 1, \tau_u = 0, \tau_1 = 1) = \rho_L \mu^{10}_u = \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u),
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\]

\[
\langle \tau_L(1 - \tau_u)\tau_1 \rangle = P(\tau_L = 1, \tau_u = 0, \tau_1 = 1) = \rho_L \mu^{10}_u = \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u),
\]

where the third equality follows from the assumption that there is no correlation between \( \tau_L \) and \( \tau_1 \). The theory also assumes that these \( \tau_L, \tau_u \), and \( \tau_1 \) are uncorrelated with the rest of the system, so that the other inner sites of each link can be regarded as forming an open 1D system whose entry and exit rates are given by

\[
\tilde{\alpha} = \rho_1 / c, \quad \tilde{\beta} = 1 - \rho_L,
\]

respectively [see Fig. 1(c)].

The correlations between a junction and its adjacent sites evolve according to

\[
\frac{d}{dt} \langle \tau_L \tau_1 \rangle = c \langle \tau_L(1 - \tau_u)\tau_1 \rangle - \langle \tau_u \tau_L(1 - \tau_1) \rangle,
\]

\[
\frac{d}{dt} \langle \tau_L \tau_u \rangle = c \langle \tau_L(1 - \tau_u)\tau_u \rangle - \langle \tau_u \tau_L(1 - \tau_1) \rangle,
\]

\[
\frac{d}{dt} \langle \tau_L \tau_1 \rangle = c \langle \tau_L(1 - \tau_u)\tau_1 \rangle - \langle \tau_u \tau_L(1 - \tau_1) \rangle,
\]

where \( \tau_1 \) and \( \tau_1' \) (\( \tau_L \) and \( \tau_L' \)) denote the states of the first (last) inner sites belonging to different outgoing (incoming) links. In the steady state, through manipulations similar to Eq. (7), these equations can be rewritten as

\[
c \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u) = \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u),
\]

\[
c \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u) = \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u),
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\[
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\]

\[
c \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u) = \rho_L \mu^{10}_u \mu^{11}_1 (1 - \rho_u),
\]

From the steady-state conditions of the single-site occupancies and the definitions of conditional probabilities
given by Eq. (6), we obtain the following useful identities:

\[
J = (1 - \rho_u)\mu_1^0 = \rho_L\mu_1^0 = \rho_u\mu_1^0/c \\
= \rho_L(1 - \rho_L) = \rho_1(1 - \rho_2), \\
\mu_1^1 = 1 - \mu_1^0, \quad \mu_1^2 = 1 - \mu_1^1, \\
\mu_1^1 = \frac{\rho_1 - \rho_u + cJ}{1 - \rho_u}, \quad \mu_1^0 = \frac{\rho_u - \rho_L + J}{1 - \rho_L}. \tag{11}
\]

Using these identities, Eq. (10) can be rewritten as

\[
\frac{c(\rho_1 - \rho_u + cJ)}{1 - \rho_u} = \frac{\rho_u - cJ}{\rho_1} + \frac{c(\rho_1 - \rho_u)}{\rho_u} \\
\frac{\rho_u - \rho_L + J}{1 - \rho_L} = \frac{c(\rho_1 - J)}{\rho_u} = -(\rho_2 - J). \tag{12}
\]

At the phase boundary \(\rho_{DPMF}^*\) between the low-density phase and the coexistence regime, we expect that the sites adjacent to the entrance (exit) junction have the occupancy equal to the bulk density of the low-density (high-density) phase. In order to satisfy the steady-state condition, both bulk densities must correspond to the same value of current, which implies

\[
\rho_1 = 1 - \rho_L = \rho_{DPMF}^*, \quad J = \rho_{DPMF}^*(1 - \rho_{DPMF}^*). \tag{13}
\]

Plugging this condition into Eq. (12), we obtain a system of equations for two unknown variables \(\rho_{DPMF}^* \) and \(\rho_u\). They can be solved for \(\rho_{DPMF}^*\) as

\[
\rho_{DPMF}^* = \frac{1 + 4c - \sqrt{1 + 4c + 4c^2 - 4c^3 + 4c^4}}{2c(1 + 2c - c^2)} \\
\approx \rho_{DMF}^* \left[ 1 + \frac{1}{2} \left( \frac{1}{c + 1} \right) - \frac{1}{8} \left( \frac{1}{c + 1} \right)^2 \right]. \tag{14}
\]

Thus, the DPMF theory suggests that junctional correlations induce nonzero corrections to the DMF predictions in the asymptotic limit: the phase coexistence regime spans a narrower range of \(\rho\) and has a larger value of \(J\) than the predicted by the DMF theory, which qualitatively agrees with the corrections predicted by our DPMF argument.

\[
\frac{\rho_{DPMF}^*}{\rho_{DMF}^*} = \frac{1 + 4c - \sqrt{1 + 4c + 4c^2 - 4c^3 + 4c^4}}{2c(1 + 2c - c^2)} \approx \rho_{DMF}^* \left[ 1 + \frac{1}{2} \left( \frac{1}{c + 1} \right) - \frac{1}{8} \left( \frac{1}{c + 1} \right)^2 \right].
\]

Thus, the DPMF theory suggests that junctional correlations induce nonzero corrections to the DMF predictions in the asymptotic limit: the phase coexistence regime spans a narrower range of \(\rho\) and has a larger value of \(J\) (see the solid lines in the main plot of Fig. 2). This is reminiscent of the observation that the current plateau of the TASEP on closed loops with a single junction seems to be underestimated by the DMF prediction [8]. We numerically verify these corrections for the CDRR networks in the next section.

V. NUMERICAL RESULTS

A. Density–current relation

The density–current relation obtained by the DMF and the DPMF theories are compared with the simulation results obtained at different values of the segment length \(L\) in Fig. 2. As \(L\) increases, the current plateau becomes narrower in the \(\rho\)-axis and higher in the \(J\)-axis than predicted by the DMF theory, which qualitatively agrees with the corrections predicted by our DPMF argument.

\[
\sigma_{\nu L} \equiv \langle \nu_L \nu_u \rangle - \langle \nu_L \rangle \langle \nu_u \rangle, \\
\sigma_{\nu 1} \equiv \langle \nu_u \nu_1 \rangle - \langle \nu_u \rangle \langle \nu_1 \rangle. \tag{15}
\]

The inset of Fig. 2, which shows the covariance between each junction and its adjacent sites defined by

\[
\sigma_{\nu L} \equiv \langle \nu_L \nu_u \rangle - \langle \nu_L \rangle \langle \nu_u \rangle, \\
\sigma_{\nu 1} \equiv \langle \nu_u \nu_1 \rangle - \langle \nu_u \rangle \langle \nu_1 \rangle,
\]

also shows that the junctional correlations are generally non-negligible.

We also check the accuracies of the theories by observing how the deviations of \(J\) from the predictions of each theory scale with \(L\). For the CDRR networks with \(c = 2\), the inset of Fig. 3 shows that deviations from the DMF predictions constantly decrease with \(L\) only for \(\rho = 0.3\) and 0.7. The DMF theory obviously fails to predict the asymptotic values of \(J\) for the other values of \(\rho\), all of which are supposed to be in the phase coexistence regime according to the theory. Meanwhile, the main plot of Fig. 3 shows that the DPMF predictions...
The steady-state density profile averaged over all links of the system, where \( \Delta_i = \rho_i - \bar{\rho} \) de-
notes the difference between the mean occupancy of the $i$-th inner site and the effective bulk density $\hat{\rho}$ estimated by the relation $J = \hat{\rho}(1 - \hat{\rho})$.

The upper panel of Fig. 5 shows the density profile of particles in the low-density phase with $\rho = 0.34$. Near the entrance, the particle density increases algebraically to the bulk density with the exponent close to $1/2$, which can be written as

$$\Delta_i \sim i^{-1/2}.$$  \hspace{1cm} (18)

Meanwhile, near the exit, the particle density decreases exponentially to the bulk density as

$$\Delta_i \sim \exp\left[-(L-i)/\xi\right].$$  \hspace{1cm} (19)

While Eq. (19) is qualitatively consistent with the 1D result, Eq. (18) is not. The latter is against the property of the open 1D system that the algebraic decay with the exponent $1/2$ appears only in the maximal-current phase [2].

Similar difference from the homogeneous 1D TASEP is also observed in the high-density phase, as shown in the lower panel of Fig. 5 for $\rho = 0.66$. In this case, the density profile has an inverted shape so that

$$\Delta_i \sim (L-i)^{-1/2}$$  \hspace{1cm} (20)

near the exit and

$$\Delta_i \sim \exp(-i/\xi)$$  \hspace{1cm} (21)

near the entrance. Again, Eq. (20) shows a behavior that cannot occur outside the maximal-current phase. These properties of the density profiles indicate that the TASEP on the CDRR networks has anomalies that cannot be explained by any analytical approaches relying on the effective rate assumptions.

VI. SLOW-BOND EFFECTS

The algebraic behavior of density profiles outside the maximal-current phase was previously reported in the context of the 1D TASEP with open boundaries and a single slow bond of hopping rate $r$ in the middle [17]. The study found that the particle density decays to the bulk values algebraically with exponent $1/2$ before and after the slow bond if $r < r_c \approx 0.80(2)$, which agrees with our observation presented in the previous section.

What is the origin of this agreement? In the model defined in Sec. II, every site of the system rather than every bond is updated at an equal rate. Both update schemes are equivalent for 1D systems, where every site has the same number of bonds attached to it. However, if the bonds are unevenly distributed among sites and if the system is updated sitewise, then the bonds attached to the sites with high connectivity become effectively slow. This is indeed what happens to the bond between the entrance junction and the first inner site of our model. Because of these slow bonds, many properties of the 1D slow-bond problem are still retained in the network case.

The slow-bond effects explain the deviations from the DMF theory other than the algebraic boundary behavior. First, the particle–hole asymmetry observed at finite size (asymmetry with respect to the $\rho \rightarrow 1 - \rho$ transformation observed in Figs. 2 and 3) originates from the asymmetric arrangement of slow bonds. Since the particles have a slow bond near the entrance and the holes have their own near the exit, the particle–hole symmetry is violated on a microscopic level. Second, the correlations at the entrance and exit junctions (see the inset of Fig. 2) show the influence of the slow bonds. The covariance between the entrance junction and the first inner site measures the correlation between the endpoints of a slow bond, which is naturally bound to be negative. On the other hand, the covariance between the exit junction and the last inner site changes sign depending on the direction of information flow near the exit, which is determined by the group velocity $v_g = 1 - 2\tilde{\rho}_{\text{exit}}$ of density waves [19]. Here $\tilde{\rho}_{\text{exit}}$ denotes the effective bulk density near the exit. When the information is flowing from the bulk (low-density phase near the exit), the covariance becomes positive due to the accumulation of particles before the slow bond. On the other hand, when the information is flowing from the nearest slow bonds (high-density phase near the exit), the covariance becomes negative due to their influence.

These connections to the slow-bond problem also have interesting implications for the effects of junction regulations. A recent numerical study about the TASEP on closed loops sharing a single junction [9] showed that the range of the coexistence regime can be reduced by pumping the particles out of junctions. It was observed that if the junctions are updated $c$ times as fast as the ordinary sites, then the coexistence regime completely disappears, so that the maximal current $J = 1/4$ can be achieved. Such behavior makes sense because the pumping rate of $c$ eliminates the effective slow bonds next to junctions.

However, is $c$ the minimal pumping rate for the elimination of the current plateau? This question is related to the unresolved problem of the critical hopping rate $r_c$ of the slow bond, above which the slow bond cannot create a macroscopic traffic congestion. Some studies [17, 18] report $r_c < 1$, while others support $r_c = 1$ [14–16, 22]. If the former is true, then the minimal pumping rate required for the maximal current might be lower than $c$, which would be another important consequence of the junctional correlations.

VII. CONCLUSIONS

We have discussed the effects of correlations between junctions and their adjacent sites in the TASEP on the closed and directed random regular networks. Our defect pair mean-field theory showed that the range of the phase coexistence regime must be narrower and that the height of the current plateau must be higher than the
simple mean-field predictions neglecting the junctional correlations. The numerically obtained fundamental diagram (i.e., density–current relation) and the scaling behaviors of the steady-state current confirmed the existence of those corrections. Moreover, we also observed the algebraic convergence of the density profiles to bulk density values with exponent close to 1/2, which is attributable to the slow-bond effects. Implications of those slow-bond effects for the TASEP in more heterogeneous network structures and the optimal junction regulations remain interesting subjects for future studies.

ACKNOWLEDGMENTS

This work was supported by the National Research Foundation of Korea (NRF) funded by the Korea government [Grant No. 2014R1A1A4A01003864 (Y.B., M.H.) and No. 2011-0028908 (Y.B., H.J.)].

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