Summary: The conformal loop ensemble CLE_κ is the canonical conformally invariant probability measure on noncrossing loops in a properly simply connected domain in the complex plane. The parameter κ varies between 8/3 and 8; CLE_8/3 is empty while CLE_8 is a single space-filling loop. In this work, we study the geometry of the CLE gasket, the set of points not surrounded by any loop of the CLE. We show that the almost sure Hausdorff dimension of the gasket is bounded from below by $2 - (8 - \kappa)(3\kappa - 8)/(32\kappa)$ when $4 < \kappa < 8$. Together with the work of O. Schramm et al. [Commun. Math. Phys. 288, No. 1, 43–53 (2009; Zbl 1187.82044)] giving the upper bound for all κ and the work of S. Nacu and W. Werner [J. Lond. Math. Soc., II. Ser. 83, No. 3, 789–809 (2011; Zbl 1223.28012)] giving the matching lower bound for κ ≤ 4, this completes the determination of the CLE_κ gasket dimension for all values of κ for which it is defined. The dimension agrees with the prediction of B. Duplantier and H. Saleur ["Exact fractal dimension of 2D Ising clusters", Phys. Rev. Lett. 63, 2536–2537 (1989; doi:10.1103/PhysRevLett.63.2536)] for the FK gasket.

MSC:
60J67 Stochastic (Schramm-)Loewner evolution (SLE)
60D05 Geometric probability and stochastic geometry
28A78 Hausdorff and packing measures

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