A Multi-Objective Model with Probability Constraints of Trans-Provincial Hydropower Consumption Considering Dynamic Stability Limit

Ke Yang¹, Rui Mao¹, Xu Wen¹²³, Chuncheng Gao⁴⁵, Miao Huang³ and Wei Yan²

¹Southwest Subsection of State Grid, Chengdu, China
²State Key Laboratory of Power Transmission Equipment & System Security and New Technology(Chongqing University), Chongqing, China
³College of Automation, Chongqing University of Posts and Telecommunications, Chongqing, China
⁴Beijing Ke Dong Electric Power Control System Co., Ltd. Beijing, China
⁵Nari Group Corporation/State Grid Electric Power Research Institute, Nanjing, China

E-mail: wenxu@cqu.edu.cn

Abstract. With the rapid development of clean energy and serious delay of grid construction, water abandonment is becoming more and more severe. Under this condition, this paper proposed a multi-objective model with probability constraints of trans-provincial hydropower consumption considering dynamic stability limit. Firstly, a mathematical expression about the dynamic stability limit of trans-provincial channel is constructed. Secondly, a risk assessment index of abandoned water is established on the basis of semi-absolute deviation. Then, based on the framework of stochastic programming, a multi-objective model with probability constraints is proposed for hydropower transmission in trans-provincial trade. By means of genetic algorithm with embedded Monte-Carlo simulation technology, the model can be solved efficiently. Finally, data of Tibet power grid is calculated to verify the validity and universality of the research. The results show that the established model can achieve optimal economic benefit between grid companies and generation enterprises, and also effective management of water abandonment risk.

1. Introduction
With the gradual shortage of primary energy, clean energy such as hydropower has been rapidly developed [1]. However, the reverse distribution of electricity supply and demand in China determines that the large-scale hydropower resources in southwestern China must be optimized in the whole country. However, due to many factors, such as the incoordination between network and source planning, large-scale water abandonment in southwest region is becoming more and more serious [2]. Therefore, it is of great practical significance to fully explore the capacity of trans-provincial channels, maximize economic benefits and reduce abandoned water risks by means of market-oriented methods. Foreign researches emphasize more on the coordination of the income and economic risk from the perspective of purchasing or selling power [3, 4, 5]. However, resource endowment, power system, network structure and market reform process in foreign countries are different from ours. Moreover, their market subjects don’t take the responsibility of electric quantity balance and safe grid operation,
which make it is generally not necessary to consider the influence of network constraints on the decision-making of purchasing and selling. Therefore, foreign research results cannot be applicable to our "province as entity" transaction decision-making under China's electric power system.

Considering the contradiction between high hydropower supply and low demand, the requirement of abandoned water management, existing literatures mainly adopt definite indexes, such as installed capacity, peak shaving, to evaluate and abandoned water. However, most hydropower outputs are stochastic, the above indexes are only suitable for "ex-post" deterministic assessment. They can’t meet the requirements of "ex-ante" risk assessment and management of abandoned waters in random environment.

In view of this, from the perspective of promoting the optimal allocation of clean energy across the province, under the unified national electricity market, this paper carries out pioneering research in the following three aspects: 1) establishes the mathematical expression of dynamic stability limit of trans-provincial transmission channel, which realizes the transformation from "static" to "dynamic" management in trans-provincial transaction; 2) constructs a risk assessment index based on semi-absolute deviation for abandoned water, which extends the assessment from certainty to uncertainty and puts forward the concept of risk in advance. 3) proposes a risk of management and control model of transferring hydropower considering dynamic stability limit, which realizes the effective management of abandoned water risk. Based on the dynamic stability limits processing technique of trans-provincial transmission channel, the multi-objective genetic algorithm of embedded monte-carlo simulation technique is used to solve the model. Take Tibet power grid as an example, this paper verifies the effectiveness of the above work. Finally, universality of the study is discussed.

2. Establishment of "dynamic" stability limit expression of trans-provincial channel and risk assessment index of abandoned water

2.1. Equation of "dynamic" stability limit of trans-provincial channel

In the existing trans-provincial trading model, it is assumed that the trans-provincial transmission channel is static. In fact, the influencing factors of the stability limit of the trans-provincial transmission channel are extremely complex, including the number of starting units (or) boot capacity, the cutting capacity of key units, the output of key units, and the transmission power of associated coupling section, etc. At the same time, the influence factors of the same trans-provincial transmission channel are different in opposite power directions. In order to realize the "dynamic" management of trans-provincial transmission channel limit, the dynamic equation is constructed.

\[
P_{ex,k,n} = \begin{cases} 
  P_{ex,k,n,\text{min}} & \text{if } N_G = 1; \\
  P_{ex,k,n,\text{max}} & \text{if } N_G \neq 1; \\
  R_{ij,\text{min}} > P_i > P_{ij,\text{min}}; \\
  R_{ij,\text{max}} > P_i > P_{ij,\text{max}}; \\
  P_{g,\text{max}} > P_g > P_{g,\text{min}}; \\
  P_{g,\text{min}} > P_g > P_{g,\text{max}}; \\
  \ldots \end{cases}
\]

In the formula, \( P_{ex,k,n,\text{max}} \) represents the dynamic quota value of the k-th inter-provincial channel under the influence of the nth factor; \( N_G \) represents the number of key units that affect the quota of inter-provincial channel; \( P_g \), \( P_{g,\text{max}} \) and \( P_{g,\text{min}} \) respectively represent the starting output and corresponding upper and lower limits of key units that affect the inter-provincial channel limit; \( P_{ij,\text{max}} \) and \( P_{ij,\text{min}} \) respectively represent the transmission power and corresponding upper and lower limits of key units that affect the inter-provincial channel limit.
lower limits of the associated channels that affect the inter-provincial channel quota; 

\( p_{c,Q} \cdot p_{c,Q,\text{max}} \) and \( p_{c,Q,\text{min}} \) respectively represent the cutting capacity and corresponding upper and lower limits that affect the trans-provincial channel limit.

2.2. Construction of a risk assessment index based on semi-absolute deviation for abandoned water

The factors affecting abandoned water are extremely complex. From the consumption of electricity market perspective, the existing assessment indicators are deterministic[2]. However, most hydropower output is characterized by randomness. Those indicators are only suitable for post-assessment, which can’t meet the requirements of prior risk assessment and management in the random environment. Therefore, a risk assessment index considering the randomness of hydropower output and referring to the basic concept of semi-absolute deviation risk[6] is constructed as follows:

\[
E \left[ \sum_{k=1}^{N_k} (W_{u,k} - W_{u,k,\text{m}}) \right] 
\]

In the formula, \( E[\cdot] \) represents the expectation operator; \( N_w \) represents the number of hydropower units; \( W_{u,k,\text{m}} \) represents the monthly random electric quantity of the \( k \)-th hydropower unit; \( W_{u,k} \) represents the expected monthly electric quantity of the \( k \)-th hydropower unit.

It can be known from equation (2), when the actual electric quantity of the hydropower unit is lower than the expected value, that is, there is the risk in abandoned water. It can be seen that the constructed risk assessment index inherits the definition method of risk severity degree of semi-absolute deviation and at the same time, this index extends the existing assessment of abandoned water from certainty to uncertainty. Based on the above, this paper puts forward the concept of risk management of abandoned water, which can realize the pre-assessment and pre-control.

3. Establishment of a risk management and control model of transferring hydropower in electricity market

3.1. Modeling ideas and assumptions

Considering more random factors and also fully exploring the market optimization space and trans-provincial transmission channel capacity, the model is built within the framework of stochastic programming.

Referring to the literature[7], the influence of monthly electric power, electric quantity and unit output on power network security and market trading decisions are described in typical periods, which is divided into monthly peak, flat and valley, of monthly market. Meanwhile, the following assumptions are also made: 1) Random factors, such as load and electricity price, are all subjected to follow normal distribution; 2) Network losses are ignored in the trade; 3) The annual priority of power generation plan has been deducted from the total load of the monthly plan and is no longer shown separately; 4) Each hydropower station and thermal power plant is equivalent to one unit.

3.2. Model construction

3.2.1 Objective function. Aiming at minimizing the expectation of the total purchase cost and the abandoned water risk in monthly trans-provincial and inter-provincial market, the risk management and control model of abandoned water is established as follows:

\[
\min \sum_{k=1}^{N_k} u_{u,k} W_{u,k} + \sum_{k=1}^{N_k} u_{z,k} W_{z,k} + \sum_{k=1}^{N_k} u_{t,k} W_{t,k} 
\]

\[
\min E \left[ \sum_{k=1}^{N_k} (W_{u,k} - W_{u,k,\text{m}}) \right] 
\]
In the formula: \( u_{WT, k}, u_{TT, k} \) and \( u_{WT, k} \) respectively represent the unit price expectations of the k-th hydropower, thermal power and transferring power; \( N_T, N_{ex} \) respectively represent the number of thermal power units and transferring power units.

### 3.2.2 Constraints

1) Opportunity constraint of "dynamic" stable limit in trans-provincial channel

\[
\alpha_{s,ex} = Pr\left\{ P_{s,ex,k} \geq P_{s,ex,k,max} \right\} \leq \alpha_{s,ex,max} \quad s = 1, 3
\]

In the formula: \( Pr \) represents the probability operator; When \( s \) is set as 1, 2 and 3 respectively, it corresponds to peak, plat and valley load; \( P_{s,ex,k} \) and \( P_{s,ex,k,max} \) respectively represent the k-th unit power of transmission under \( s \) load and the corresponding stability limit of transmission channel, as shown in equation (1).

2) Opportunity constraints of monthly electricity balance considering hydropower transmission

\[
\gamma_1 = Pr\left\{ W_{d,max} \leq \sum_{k=1}^{N_W} W_{w,k,n} + \sum_{k=1}^{N_T} W_{t,k} - \sum_{k=1}^{N_{ex}} W_{ex,k} \right\} < \gamma_{1,max} \quad (6)
\]

\[
\gamma_3 = Pr\left\{ W_{d,min} \geq \sum_{k=1}^{N_W} W_{w,k,n} + \sum_{k=1}^{N_T} W_{t,k} - \sum_{k=1}^{N_{ex}} W_{ex,k} \right\} < \gamma_{3,max} \quad (7)
\]

In the formula: \( \gamma_s \) is the risk probability value that beyond the limit in the monthly electricity balance (\( s=1 \) means it beyond the upper limit, \( s=3 \) means it beyond the lower limit); \( W_{d,max} \) and \( W_{d,min} \) represent the maximum and minimum monthly electric quantity respectively; \( \gamma_{1,max} \) and \( \gamma_{3,max} \) respectively represent the limit value of the probability of monthly electricity balance which beyond the limit.

In addition, the remaining constraints of the model include: peak shaving in the state of monthly peak and valley load, monthly typical branch power flow, power balance equation of dc power flow under monthly peak and valley load, power equation of branch power flow under peak and valley load, upper and lower limits of power of thermal power and hydropower units, upper and lower limits of electric quantity of thermal power and hydropower units, etc.

### 4. Algorithm for solving the model

In essence, the established model is a multi-objective model with probability constraints[8,9]. By referring to the literature[10], the model is solved within the framework of genetic algorithm, and the fitness of multi-objective function is calculated by the method of relative superiority of targets. The steps of the algorithm include input of basic data, generation of initial population, calculation of stability limit of trans-provincial channel, opportunity constraints test, calculation of fitness function, operation of selection, cross and mutation, acquisition of hydropower transmission scheme and water abandonment risk, etc. Due to the limitation of space, this paper focuses on the algorithm processing skills, such as calculating dynamic stability limit of trans-provincial channel, which cannot be processed in literature[6,7].

1) Generation of each iteration population. On the basis of generating population with upper and lower constraints of hydropower and thermal power units, each external power transmitted can be
obtained by the power balance equation of typical load in each month. This method guarantees non-random generation to avoid seeds that do not meet the equation constraints, and also improves the solving efficiency indirectly.

(2) Calculation of dynamic stability limit of trans-provincial channel. In the process of each iteration of the population, dc power flow equation is used to calculate the branch power flow. According to the output state of each unit, the key information, such as the number of key units starting up, the output of key units, and the transmission power of associated channels that affect the trans-provincial channel limit, and formula (1), the dynamic stability limit of trans-provincial channel corresponding to each iteration of the population is obtained.

(3) Inspection of opportunity constraints. In order to improve the inspection efficiency, it is suggested to use Latin hypercube sampling technology [11] to generate random samples, such as hydropower output in typical load, and to check whether the opportunity constraints meet the conditions.

5. Verification of calculation examples

5.1. Basic date
The basic data of July 2017 of Tibet power grid was used to verify the validity of the research. The basic data are set as follows: Monthly electric quantity of load is 400 million kilowatt-hours; Monthly maximum and minimum load is 1 million, 0.5 million kilowatts respectively; Standard deviation of electric quantity of each month is determined as 20% of expected value; The probability of each opportunity constraint is 10.0%. The maximum stability limit of Chaila DC is 430,000 kilowatts, and the maximum stability limit of Chuanzang AC 130,000 kilowatts. Latin hypercube sampling scale is 500 times; The population size of genetic algorithm is 80, the crossover probability is 0.60, the mutation probability is 0.10, and the maximum iteration number is 300. The criterion of iteration termination is that the optimal individual remains unchanged for 30 consecutive generations or reaches the maximum iteration number.

5.2. Analysis of the impact of dynamic stability limit of trans-provincial channel on clean energy consumption
Based on the simulation conditions in section 4.1, the proposed model is compared with the treatment method in stability limit of trans-province channel in literature [6, 7], and the results are shown in Table 1.

| Solution | Power purchase cost /One hundred million RMB | Electricity of abandoned water/one million kilowatt-hours | Consumption of transmission/one million kilowatt-hours |
|----------|---------------------------------------------|--------------------------------------------------------|--------------------------------------------------------|
| Literature[6] | 1.40                                       | 0.53                                                   | 1.81                                                   |
| Literature[7] | 1.46                                       | 0.67                                                   | 1.89                                                   |
| In this paper | 1.58                                       | 0.83                                                   | 1.61                                                   |

As can be seen from table 1, because the model established in this paper takes the dynamic stability limit of the trans-provincial channel into account, the electricity purchase cost and electricity of abandoned water increase greatly, while the electricity consumption of transmission decreases. The essential reason is that this paper strictly considers the influence of the dynamic stability limit of trans-provincial channel on clean energy external consumption. The stability limit of the trans-provincial channel is no longer set as the fixed number, but is changed dynamically with the boot combination, cutting capacity and other factors. Traditional literatures do not consider the dynamic change of trans-provincial channel limit, which is not consistent with the actual operation of power grid.
5.3. Validation of the effectiveness of risk management and control of water abandonment and analysis of its impact on the external consumption of clean energy

Keep the simulation conditions unchanged, make comparisons with the two schemes: scheme 1 considers the index of water abandonment risk, while scheme 2 doesn’t consider that index. The results are shown in Tab.2 which validates the management of model on abandoned water risk and its influence on the external consumption of clean energy.

**Table 2. Comparison of consumption of hydropower transmission considering water abandonment risk**

| Scheme | Power purchase cost /One hundred million yuan | Electricity of abandoned water/one million kilowatt-hours | Risk of abandoned water/one million kilowatt-hours | Consumption of transmission/one million kilowatt-hours |
|--------|---------------------------------------------|--------------------------------------------------------|--------------------------------------------------|-----------------------------------------------------|
| 1      | 1.58                                        | 0.83                                                   | 0.32                                             | 1.61                                                |
| 2      | 1.46                                        | 0.77                                                   | 0.18                                             | 1.82                                                |

As can be seen from table 2, after considering the risk assessment indexes of abandoned water, both the electricity purchase cost and the abandoned water risk are reduced, but the electricity consumption is increased. This result reflects that the introduction of abandoned water risk management promotes wider consumption of clean energy. The essential reason can be explained as follows: When the abandoned water risk assessment is introduced and the load demand is unchanged, the trading decision optimization space of the provincial grid will extend to external transmission in order to reduce the risk of abandoned water. These not only guarantees more benefits of power selling for the grid company, but also increases the total economic benefit and reduces the abandoned water risk. Therefore, both the grid companies and the generation enterprises can make profits.

5.4. Validation of the effectiveness of risk management and control of water abandonment and analysis of its impact on the external consumption of clean energy

Keeping the simulation conditions unchanged, three schemes of 5.0%, 10.0% and 15.0% of the probability of opportunity constraint were adopted to verify their influences on the risk of abandoned water and the consumption of clean energy. The results are shown in Tab.3.

**Table 3. Comparison of hydropower transmission considering different probability of opportunity constraint**

| The probability of opportunity constraint | Power purchase cost /One hundred million RMB | Electricity of abandoned water/one million kilowatt-hours | Risk of abandoned water/one million kilowatt-hours | Consumption of transmission/one million kilowatt-hours |
|------------------------------------------|---------------------------------------------|--------------------------------------------------------|--------------------------------------------------|-----------------------------------------------------|
| 5.0%                                     | 1.65                                        | 0.89                                                   | 0.42                                             | 1.53                                                |
| 10.0%                                    | 1.58                                        | 0.83                                                   | 0.32                                             | 1.61                                                |
| 15.0%                                    | 1.49                                        | 0.77                                                   | 0.28                                             | 1.72                                                |

As can be seen from Tab.3, with the increasing probability value of opportunity constraint, both electricity purchase cost and abandoned water risk are reduced, but the external electricity consumption is increased. This result reflects that different probability of opportunity constraint has an impact on promoting large-scale consumption of clean energy. The essential reason can be explained as follows: In the mathematical way, the increment of probability of opportunity constraint means the expansion of space for optimization model. In the physical way, it means that the trading decision optimization space of the provincial grid will extend to external transmission in order to get more profits when the load demand is unchanged. This extension leads to increment in the total economic benefit and reduction in the abandoned water risk.

6. Conclusion
1) Since the stability limit of trans-provincial channel involves many complicated factors, such as the boot capacity of the key unit, boot combination, generation output, associated channel limit etc., it means that the stability limit of trans-provincial channel will change dynamically with different grid operation. Therefore, it is more reasonable to extend stability limit of trans-provincial channel from static to dynamic.

2) Considering the uncertainty of hydropower output, the built risk assessment index of abandoned water extends the existing assessment of water abandonment from certainty to uncertainty, and effectively describes the severity of water abandonment risk. The proposed risk management concept of abandoned water is conducive to wider consumption of clean energy, and is more suitable for the prior control of water abandonment risk than the existing indexes.

3) The established risk control model of water abandonment consumption for hydropower transmission, which considers the dynamic stability limit of trans-provincial channel, achieves the optimal economic benefit and effective management of water abandonment risk. In addition, it makes profits for both grid companies and generation enterprises. Therefore, it is of practicability and promotion value.

Acknowledgments
The Project supported by National Natural Science Foundation of China (51677012) and scientific and technological research project of Chongqing municipal education commission (Grant No. KJ1704101).

References
[1] KAHRL F and WILLIAMS J H 2013 The political economy of electricity dispatch reform in China. Energy Policy, 9(8) 361-369.
[2] CHEN Yuehui, XIANG Meng and ZHANG Bin 2018 Optimal dispatch modes of cross-provincial tie-line and early-warning strategy. Power System Protection and Control, 46(18) 100-107.
[3] Liuyuan and Guan X. H 2003 Purchase allocation and demand bidding in electric power marks. IEEE Transactions on Power systems, 18(1) 106-112.
[4] Dehua Zheng, Abinet Tesfaye Eseye, Jianhua Zhang, et al 2017 Short-term wind power forecasting using a double-stage hierarchical ANFIS approach for energy management in microgrids. Protection and Control of Modern Power Systems, 2(2) 33-39.
[5] Vilim M 2014 Wind power bidding in electricity markets with high wind penetration. Applied Energy, 118 141-155.
[6] YAN Wei, WEN Xu, WANG Junmei, et al 2012 An optimal monthly power purchasing model considering differences between Internal and external grid. Automation of Electric Power Systems, 36(17): 56-60.
[7] WEN Xu, WANG Junmei, GUO Lin 2013 A monthly Power Purchasing Risk Management Model for Provincial Power Grid Considering Wind Power Uncertainty. Electric Power Automation Equipment, 33(7) 27-33
[8] LIU Gushuai, XIAO Yiyao, HE Yuqiang, et al 2017 Multi-objective optimal method considering types of grid connected new energy of electric power system. Power System Protection and Control, 45(10) 31-37.
[9] YAN Jun, YAN Feng 2018 Location method of charging and swapping service facilities based on a two-step planning. Power System Protection and Control, 46(14) 48-56.
[10] LIN Ye 2011 Studies on optimization of global generation and transmission maintenance scheduling and its security correction. Chongqing: Chongqing university.
[11] YU H, CHUNG C Y, WONG K P, et al 2009 Probabilistic load flow evaluation with hybrid Latin hypercube sampling and Cholesky decomposition. IEEE Transactions on Power Systems, 24(2) 661-667.