Search for new physics effects in Bhabha scattering at $e^+e^-$ linear colliders

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Abstract

We study electron-electron contact-interaction searches in the process $e^+e^- \rightarrow e^+e^-$ at a future linear collider with longitudinally polarized beams, and evaluate the model-independent constraints on the contact interaction parameters, emphasizing the role of beam polarization.

1 Introduction

The possibility of constructing high energy polarized electron and positron beams is considered with great interest with regard to the physics programme at a linear collider (LC). Indeed, one of the most important advantages of initial beam polarization is that one can measure spin-dependent observables, which represent the most direct probes of the fermion helicity dependence of the electroweak interactions. Consequently, one would expect a substantial gain in the sensitivity to the features of possible non-standard interactions and, in particular, stringent constraints on the individual new coupling constants could be derived from the data analysis by looking for deviations of the observables from the Standard Model (SM) predictions.

Contact interaction Lagrangians (CI) provide an effective framework to account for the phenomenological effects of new dynamics characterized by extremely high intrinsic mass scales $\Lambda$, at the ‘low’ energies $\sqrt{s} \ll \Lambda$ attainable at current particle accelerators. The explicit parameterization of the four-fermion quark and lepton contact interactions is, a priori, somewhat arbitrary. In general, it must respect $SU(3) \times SU(2) \times U(1)$ symmetry, because the new dynamics are active well-beyond the electroweak scale. Furthermore, usually one limits to the lowest dimensional operators, $D = 6$ being the minimum, and neglects higher dimensional operators that are suppressed by higher powers of $1/\Lambda^2$ and therefore are expected to give negligible effects.

In this note, we consider the effects of the flavor-diagonal, helicity conserving $eeff$ contact-interaction effective Lagrangian [1]

$$L_{CI} = \frac{1}{1 + \delta_{ef}} \sum_{\alpha\beta} g_{eff}^2 \epsilon_{\alpha\beta} (\bar{e}_\alpha \gamma_\mu e_\alpha) \left( \bar{f}_\beta \gamma^\mu f_\beta \right),$$

in the Bhabha scattering process

$$e^+ + e^- \rightarrow e^+ + e^-,$$

at an $e^+e^-$ linear collider with c.m. energy $\sqrt{s} = 0.5\text{ TeV}$ and polarized electron and positron beams. In Eq. (1): $\alpha, \beta = L, R$ denote left- or right-handed fermion helicities, $f$ indicates the
fermion species, so that $\delta_{ef} = 1$ for the process (2) under consideration, and the CI coupling constants are parameterized in terms of corresponding mass scales as $\epsilon_{\alpha\beta} = \eta_{\alpha\beta}/\Lambda_{\alpha\beta}^2$. Actually, one assumes $g_{\text{eff}}^2 = 4\pi$ to account for the fact that the interaction would become strong at $\sqrt{s} \simeq \Lambda$, and by convention $|\eta_{\alpha\beta}| = \pm 1$ or $\eta_{\alpha\beta} = 0$, leaving the energy scales $\Lambda_{\alpha\beta}$ as free, a priori independent parameters.

For the case of the Bhabha process (2), the effective lagrangian interaction in Eq. (1) envisages the existence of three individual, and independent, CI models, contributing to individual helicity amplitudes or combinations of them, with a priori free, and non-vanishing, coefficients (basically, $\epsilon_{\text{LL}}, \epsilon_{\text{RR}}$ and $\epsilon_{\text{LR}}$ combined with the \pm signs). Correspondingly, in principle the most general, and model-independent, analysis of the data must account for the situation where all four-fermion effective couplings defined in Eq. (1) are simultaneously allowed in the expression for the cross section. Potentially, the different CI couplings may interfere and substantially weaken the bounds. Indeed, although the different helicity amplitudes by themselves do not interfere, the deviations from the SM could be positive for one helicity amplitude and negative for another, so that accidental cancellation might occur in the sought for deviations from the SM predictions for the relevant observables. It should be highly desirable to apply a general (and model-independent) approach to the analysis of experimental data, that allows to simultaneously include all terms of Eq. (1) as independent, non vanishing free parameters, and yet to derive separate constraints (or exclusion regions) on the values of the CI coupling constants, free from potential weakening due to accidental cancellations. Such an analysis is feasible with initial beam longitudinal polarization and allows to extract the individual helicity cross sections from suitable combinations of measurable polarized cross sections and, consequently, to disentangle the constraints on the corresponding CI constants.

2 Sensitivity to CI

With $P^-$ and $P^+$ the longitudinal polarization of the electron and positron beams, respectively, and $\theta$ the angle between the incoming and the outgoing electrons in the c.m. frame, the differential cross section of process (2) at lowest order, including $\gamma$ and $Z$ exchanges both in the $s$ and $t$ channels and the contact interaction (1), can be written in the following form [2, 3, 4]:

$$\frac{d\sigma(P^-, P^+)}{d\cos \theta} = (1 - P^- P^+) \frac{d\sigma_1}{d\cos \theta} + (1 + P^- P^+) \frac{d\sigma_2}{d\cos \theta} + (P^+ - P^-) \frac{d\sigma_P}{d\cos \theta}. \tag{3}$$

In Eq. (3):

$$\frac{d\sigma_1}{d\cos \theta} = \frac{\pi\alpha^2}{4s} \left[ A_+ (1 + \cos \theta)^2 + A_- (1 - \cos \theta)^2 \right],$$

$$\frac{d\sigma_2}{d\cos \theta} = \frac{\pi\alpha^2}{4s} 4A_0,$$

$$\frac{d\sigma_P}{d\cos \theta} = \frac{\pi\alpha^2}{4s} A_+^P (1 + \cos \theta)^2, \tag{4}$$

with

$$A_0(s, t) = \left( \frac{s}{t} \right)^2 \left| 1 + g_R g_L \chi_Z(t) + \frac{t}{s} \epsilon_{LR} \right|^2,$$
\[ A_+(s, t) = \frac{1}{2} \left| 1 + \frac{s}{t} + g_L^2 \left( \chi_Z(s) + \frac{s}{t} \chi_Z(t) \right) + 2 \frac{s}{\alpha} \epsilon_{LL} \right|^2 \]
\[ + \frac{1}{2} \left| 1 + \frac{s}{t} + g_R^2 \left( \chi_Z(s) + \frac{s}{t} \chi_Z(t) \right) + 2 \frac{s}{\alpha} \epsilon_{RR} \right|^2, \]
\[ A_-(s) = \left| 1 + g_R g_L \chi_Z(s) + \frac{s}{\alpha} \epsilon_{LR} \right|^2, \]
\[ A_{P}^+(s, t) = \frac{1}{2} \left| 1 + \frac{s}{t} + g_L^2 \left( \chi_Z(s) + \frac{s}{t} \chi_Z(t) \right) + 2 \frac{s}{\alpha} \epsilon_{LL} \right|^2 \]
\[ - \frac{1}{2} \left| 1 + \frac{s}{t} + g_R^2 \left( \chi_Z(s) + \frac{s}{t} \chi_Z(t) \right) + 2 \frac{s}{\alpha} \epsilon_{RR} \right|^2. \] (5)

Here: \( \alpha \) is the fine structure constant; \( t = -s(1 - \cos \theta)/2 \) and \( \chi_Z(s) = s/(s - M_Z^2 + i M_Z \Gamma_Z) \), \( \chi_Z(t) = t/(t - M_Z^2) \) represent the Z propagators in \( s \) and \( t \) channel, respectively, with \( M_Z \) and \( \Gamma_Z \) the mass and width of the Z; \( g_L = \tan \theta_W \), \( g_R = -\cot 2 \theta_W \) are the SM right- and left-handed electron couplings of the Z, with \( s_W^2 = 1 - c_W^2 \equiv \sin^2 \theta_W \).

With both beams polarized, the polarization of each beam can be changed on a pulse by pulse basis. This would allow the separate measurement of the polarized cross sections for each of the four polarization configurations RR, LL, RL and LR, corresponding to the four sets of beam polarizations \((P^-, P^+) = (P_1, P_2), (-P_1, -P_2), (P_1, -P_2)\) and \((-P_1, P_2)\), respectively, with \( P_{1,2} > 0 \). Specifically, with the simplifying notation \( d\sigma \equiv d\sigma/d\cos \theta \):

\[ d\sigma_{RR} \equiv d\sigma(P_1, P_2) = (1 - P_1 P_2) d\sigma_1 + (1 + P_1 P_2) d\sigma_2 + (P_2 - P_1) d\sigma_P, \]
\[ d\sigma_{LL} \equiv d\sigma(-P_1, -P_2) = (1 - P_1 P_2) d\sigma_1 + (1 + P_1 P_2) d\sigma_2 - (P_2 - P_1) d\sigma_P, \]
\[ d\sigma_{RL} \equiv d\sigma(P_1, -P_2) = (1 + P_1 P_2) d\sigma_1 + (1 - P_1 P_2) d\sigma_2 - (P_2 + P_1) d\sigma_P, \]
\[ d\sigma_{LR} \equiv d\sigma(-P_1, P_2) = (1 + P_1 P_2) d\sigma_1 + (1 - P_1 P_2) d\sigma_2 + (P_2 + P_1) d\sigma_P. \] (6)

To make contact to the experiment we take \( P_1 = 0.8 \) and \( P_2 = 0.6 \), and impose a cut in the forward and backward directions. Specifically, we consider the cut angular range \(|\cos \theta| < 0.9\) and divide it into nine equal-size bins of width \( \Delta z = 0.2 \) (\( z \equiv \cos \theta \)). We also introduce the experimental efficiency, \( \epsilon \), for detecting the final \( e^+e^- \) pair, and according to the LEP2 experience \( \epsilon = 0.9 \) is assumed.

The most natural choice of observables are the four, directly measurable, differential event rates integrated over each bin:

\[ N_{LL}, \ N_{RR}, \ N_{LR}, \ N_{RL}. \] (7)

where \((\alpha, \beta = L, R)\), and

\[ N_{a\beta}^{\text{bin}} = \frac{1}{4} \mathcal{L}_{\text{int}} \epsilon \int_{\text{bin}} (d\sigma_{a\beta}/dz)dz. \] (8)

In Eq. (8), \( \mathcal{L}_{\text{int}} \) is the time-integrated luminosity, which is assumed to be equally divided among over the four combinations of electron and positron beams polarization defined in Eqs. (5).

The current bounds on \( \Lambda_{a\beta} \), of the order of several TeV, are such that for the LC c.m. energy \( \sqrt{s} = 0.5 \) TeV the characteristic suppression factor \( s/\Lambda^2 \) in Eq. (5) is rather strong. Accordingly, we can safely assume a linear dependence of the cross sections on the parameters \( \epsilon_{a\beta} \). In this
regard, indirect manifestations of the CI interaction \([\text{I}]\) can be looked for, via deviations of the measured observables from the SM predictions, caused by the new interaction. The reach on the CI couplings, and the corresponding constraints on their allowed values in the case of no effect observed, can be estimated by comparing the expression of the mentioned deviations with the expected experimental (statistical and systematic) uncertainties.

To this purpose, assuming the data to be well described by the SM \((\epsilon_{\alpha\beta} = 0)\) predictions, i.e., that no deviation is observed within the foreseen experimental uncertainty, and in the linear approximation in \(\epsilon_{\alpha\beta}\) of the observables \((7)\), we apply the method based on the covariance matrix:

\[
V_{kl} = \langle (O_k - \bar{O}_k)(O_l - \bar{O}_l) \rangle
= \sum_{i=1}^{4} (\delta N_i)^2 \left( \frac{\partial O_k}{\partial N_i} \right) \left( \frac{\partial O_l}{\partial N_i} \right) + (\delta L_{\text{int}})^2 \left( \frac{\partial O_k}{\partial L_{\text{int}}} \right) \left( \frac{\partial O_l}{\partial L_{\text{int}}} \right)
+ (\delta P^-)^2 \left( \frac{\partial O_k}{\partial P^-} \right) \left( \frac{\partial O_l}{\partial P^-} \right) + (\delta P^+)^2 \left( \frac{\partial O_k}{\partial P^+} \right) \left( \frac{\partial O_l}{\partial P^+} \right). \tag{9}
\]

Here, the \(N_i\) are given by Eq. \((7)\), so that the statistical error appearing on the right-hand-side is given by

\[
\delta N_i = \sqrt{N_i}, \tag{10}
\]

and the \(O_i = (N_{LL}, N_{RR}, N_{LR}, N_{RL})\) are the observables. The second, third and fourth terms of the right-hand-side of Eq. \((9)\) represent the systematic errors on the integrated luminosity \(L_{\text{int}}\), polarizations \(P^-\) and \(P^+\), respectively.

Defining the inverse covariance matrix \(W^{-1}\) as

\[
(W^{-1})_{ij} = \sum_{k,l=1}^{4} \sum_{\text{bins}} (V^{-1})_{kl} \left( \frac{\partial O_k}{\partial \epsilon_i} \right) \left( \frac{\partial O_l}{\partial \epsilon_j} \right), \tag{11}
\]

with \(\epsilon_i = (\epsilon_{LL}, \epsilon_{LR}, \epsilon_{RR})\), model-independent allowed domains in the four-dimensional CI parameter space to 95% confidence level are obtained from the error contours determined by the quadratic form in \(\epsilon_{\alpha\beta}\) \([5, 6]\):

\[
(\epsilon_{LL} \epsilon_{LR} \epsilon_{RR}) W^{-1} \begin{pmatrix} \epsilon_{LL} \\ \epsilon_{LR} \\ \epsilon_{RR} \end{pmatrix} = 7.82. \tag{12}
\]

The value 7.82 on the right-hand-side of Eq. \((12)\) corresponds to a fit with three free parameters.

The quadratic form \((12)\) defines a three-dimensional ellipsoid in the \((\epsilon_{LL}, \epsilon_{LR}, \epsilon_{RR})\) parameter space. The matrix \(W\) has the property that the square roots of the individual diagonal matrix elements, \(\sqrt{W_{ii}}\), determine the projection of the ellipsoid onto the corresponding \(i\)-parameter axis in the three-dimensional space, and has the meaning of the bound at 95\% C.L. on that parameter regardless of the values assumed for the others. Conversely, \(1/\sqrt{(W^{-1})_{ii}}\) determines the value of the intersection of the ellipsoid with the corresponding \(i\)-parameter axis, and represents the 95\% C.L. bound on that parameter assuming all the others to be exactly known. Accordingly, the
ellipsoidal surface constrains, at the 95% C.L. and model-independently, the range of values of the CI couplings $\epsilon_{\alpha\beta}$ allowed by the foreseen experimental uncertainties.

The numerical results can be represented graphically. As an example, in Figs. 1–2 we show the planar ellipses that are obtained by projecting onto the two planes ($\epsilon_{LL}, \epsilon_{LR}$) and ($\epsilon_{LR}, \epsilon_{RR}$) the 95% C.L. allowed three-dimensional ellipsoid.

To appreciate the significant role of initial beam polarization we show in Figs. 1–2 the allowed bounds, obtained from unpolarized initial beams $P^- = P^+ = 0$ (dotted line), polarized electrons and unpolarized positrons $|P^-| = 0.8$, $P^+ = 0$ (dashed line), and both polarized beams with $|P^-| = 0.8$ and $|P^+| = 0.6$ (solid line). As can be seen from those figures, the electron polarization is obviously essential to dramatically reduce the allowed domains of CI parameters with respect to the case of unpolarized beams. The results presented in Figs. 1–2 show that the increase in the sensitivity to the parameters $\epsilon_{\alpha\beta}$ due to the additional availability of positron polarization is quite useful. One can obtain from Figs. 1–2 the corresponding bounds on $\Lambda_{\alpha\beta}$. For the case when both initial beams are polarized, we have: $\Lambda_{RR} > 38$ TeV, $\Lambda_{LL} > 38$ TeV and $\Lambda_{LR} > 49$ TeV.

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References

[1] E. Eichten, K. Lane and M. E. Peskin, Phys. Rev. Lett. 50 (1983) 811.
[2] B. Schrempp, F. Schrempp, N. Wermes and D. Zeppenfeld, Nucl. Phys. B 296 (1988) 1.
[3] M. Beccaria, F. M. Renard, S. Spagnolo and C. Verzegnassi, Phys. Rev. D 62 (2000) 053003 [hep-ph/000210].
[4] A. A. Pankov and N. Paver, preprint IC/2001/125, Trieste [hep-ph/0109203].
[5] F. Cuypers and P. Gambino, Phys. Lett. B 388 (1996) 211 [hep-ph/9606391].
[6] A. A. Babich, P. Osland, A. A. Pankov and N. Paver, Phys. Lett. B 518 (2001) 128 [hep-ph/0107159]. Phys.Lett.B518:128-136,2001
Figure 1: (a) Allowed areas at 95% C.L. on electron contact interaction parameters in the plane ($\epsilon_{LL}, \epsilon_{LR}$), obtained from differential polarized cross sections at $\sqrt{s} = 500$ GeV, $\mathcal{L}_{\text{int}} = 50$ fb$^{-1}$, $|P^-| = 0.8$ and $|P^+| = 0.6$ (solid line), $|P^-| = 0.8$ and $P^+ = 0$ (dashed line), and $P^- = P^+ = 0$ (dotted line). (b) Magnification of allowed domains presented in Fig. 1a.

Figure 2: Same as in Fig. 1, for ($\epsilon_{LR}, \epsilon_{RR}$) plane.