ABSTRACT: At the present time, there are hints of new physics (NP) in several observables involving $b \to c\ell \bar{\nu}_\ell$ decays. In this talk, I describe measurable angular distributions for $\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ and $\bar{B} \to D^* \tau^- (\to \pi^- \nu_\tau) \bar{\nu}_\tau$ decays, including the most general NP contributions. These angular distributions contain enough information to pin down the Lorentz structure of the NP, which will help to identify it. They also have the ability to reveal the presence of non-SMEFT (non-decoupling) NP.
$\bar{B} \to D^* \ell^- \bar{\nu}_\ell$ Decays: Angular Distributions and New Physics

David London

\[ b \to c \tau^- \bar{\nu}_\tau \text{ Anomalies} \] — Several observables have been measured that involve $b \to c \ell^- \bar{\nu}_\ell$ decays:

\[ R_D^{(\ell)} = \frac{B(\bar{B} \to D^{(*)} \tau^- \bar{\nu}_\tau)}{B(\bar{B} \to D^{(*)} \ell^- \bar{\nu}_\ell)}, \quad \ell = e, \mu \] \quad \text{and} \quad \frac{B(B_c \to J/\psi \tau \nu)}{B(B_c \to J/\psi \mu \nu)}.

(1)

The simplest explanation is that there is new physics (NP) in these decays, which differ from the SM predictions by a $3 \sigma$ deviation. It was found that, for the SM, the $p$-value of the fit is $\sim 0.1\%$, corresponding to a discrepancy of 3.3σ.

New-Physics Explanations — There are two different approaches to examining possible NP explanations of these anomalies. The first is to construct specific NP models, and here three classes of models have been proposed [1], those involving (i) a new $W'$ boson, (ii) a leptoquark, or (iii) a charged Higgs boson.

The second is an effective field theory (EFT) approach. Assuming a left-handed (LH) neutrino, the most general effective Hamiltonian for $b \to c \tau^- \bar{\nu}_\tau$ is

\[ H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{cb} \left\{ \left[ (1 + C^L_V)(\bar{c} \gamma^\mu P_L \ell b) + C^R_V(\bar{c} \gamma^\mu P_R b) \right] (\bar{\tau} \gamma_\mu P_L \nu) + C_T(\bar{c} \sigma^{\mu\nu} P_L b)(\bar{\tau} \sigma_{\mu\nu} P_L \nu) \right\} + h.c. \]

(3)

Here, the coefficients $C^L,R_{V,S,T}$ are generated only through NP.

The question then is: how can we distinguish among these NP contributions? In this talk, I suggest using angular distributions, as these are sensitive to different Lorentz structures.

Angular Distributions — Consider $B$ decays of the form $B \to V_1 (\to P_1 P'_1) V_2 (\to P_2 P'_2)$. Examples of these include $B_d \to \phi K^*$ and $B_s^0 \to \phi \phi$. In these decays, there are three helicity amplitudes, $A_0$, $A_1$, and $A_\perp$. Their presence leads to a number of different angular functions in $|A(B \to V_1 (\to P_1 P'_1) V_2 (\to P_2 P'_2))|^2$. Specifically, the differential decay rate is a function of three angles, $\theta_1$, $\theta_2$, and $\Phi$, shown in Fig. 1.

The angular distributions in these decays are used mainly to search for CP-violating effects. Since such effects are predicted to be small in the SM, the measurement of CP violation in any of these decays would be a smoking-gun signal of NP.

CP violation arises from the interference of two amplitudes. Consider two helicity amplitudes $A_\perp$ and $A_i$ ($i = 0$ or $\perp$). These can be written $A_\perp = |A_\perp| e^{i\phi_\perp} e^{i\delta_\perp}$ and $A_i = |A_i| e^{i\phi_i} e^{i\delta_i}$, where the $\phi_{\perp,i}$ are weak (CP-odd) phases and the $\delta_{\perp,i}$ are strong (CP-even) phases. Strong phases are...
typically produced due to QCD effects, and since $B \rightarrow V_1(\rightarrow P_1 P'_1)V_2(\rightarrow P_2 P'_2)$ decays are purely hadronic, the helicity amplitudes may well have different strong phases.

When one computes $|A|^2$, the term $\epsilon_{\mu\nu\rho\sigma} p_1^\mu p_2^\nu p_3^\rho p_4^\sigma$ arises. In the rest frame of, for example, particle #1, this equals $m_1 \vec{p}_2 \cdot (\vec{p}_3 \times \vec{p}_4)$, which is a triple product (TP). Its coefficients (for $i = 0, ||$) are

$$\text{Im}(A \times A^*) = |A||A_i|(\sin(\phi_\perp - \phi_l) \cos(\delta_\perp - \delta_l) + \cos(\phi_\perp - \phi_l) \sin(\delta_\perp - \delta_l)). \tag{4}$$

These are also the coefficients of the P-odd terms in the angular distribution, which are proportional to $\sin \Phi$ ($\Phi$ is the angle between the $V_1 \rightarrow P_1 P'_1$ and $V_2 \rightarrow P_2 P'_2$ decay planes, see Fig. 1). This implies that the corresponding terms in the angular distribution of the CP-conjugate decay have the opposite sign.

Note that a nonzero TP is not necessarily a signal of CP violation. If there is no CP violation in the decay ($\phi_{\perp,0||} = 0$), there can still be a nonzero TP if $\delta_\perp - \delta_l \neq 0$ (see the CP-even term in Eq. (4) above), in which case we expect that TP(decay) = $-\text{TP}$ (CP-conjugate decay). Thus, a signal of CP violation would be TP(decay) + TP (CP-conjugate decay) $\neq 0$. To be specific, this isolates the CP-odd term above, and indicates that there are NP contributions to the decay.

$\bar{B} \rightarrow D^* \mu^- \bar{\nu}_\mu$ — In the SM, this decay takes place via $\bar{B} \rightarrow D^* (\rightarrow D \pi) W^* (\rightarrow \mu^- \bar{\nu}_\mu)$. This is similar to $B \rightarrow V_1(\rightarrow P_1 P'_1)V_2(\rightarrow P_2 P'_2)$, but there are two important differences: (i) $\bar{B} \rightarrow D^* \mu^- \bar{\nu}_\mu$ is a semileptonic decay, implying that the hadronic and leptonic currents factorize (unlike $B \rightarrow V_1(\rightarrow P_1 P'_1)V_2(\rightarrow P_2 P'_2)$), and (ii) the $W^*$ is virtual, leading to four helicity amplitudes: $A_0, A_\parallel, A_\perp, A_l$ (timelike).

Ref. [8] examines the effect of adding NP to this decay. The analysis assumes a LH neutrino, but is otherwise completely general. It uses an EFT approach: the decay is generically described by $\bar{B} \rightarrow D^* (\rightarrow D \pi) N^* (\rightarrow \mu^- \bar{\nu}_\mu)$, and the NP effects are included in an effective Hamiltonian similar to Eq. (3):

| coupling | quarks | leptons | spin | type |
|----------|--------|---------|------|------|
| $(1 + C_{L}^T)$ | $\bar{c}\gamma_\mu P_L b$ | $\bar{\mu}\gamma^\mu P_L \gamma_\mu$ | vector | SM + NP |
| $C_{V}^R$ | $\bar{c}\gamma_\mu P_R b$ | $\bar{\mu}\gamma^\mu P_L \gamma_\mu$ | vector | NP |
| $C_{S}^R$ | $\bar{c}\gamma_\rho b$ | $\bar{\mu}\gamma_\mu P_L \gamma_\mu$ | scalar | NP |
| $C_{L}^S$ | $\bar{c}\gamma_\rho b$ | $\bar{\mu}\gamma_\mu P_L \gamma_\mu$ | tensor | NP |
The addition of general NP leads to four new helicity amplitudes: $A_{SP}, A_{0,T}, A_{||,T}, A_{\perp,T}$. The helicity amplitudes depend on the NP parameters as follows:}

| Helicity Amplitude | Coupling |
|--------------------|----------|
| $A_0, A_{||}, A_{\perp}, A_{SP}$ | $1 + C_V^L - C_V^R$ |
| $A_{\perp}$ | $1 + C_V^L + C_V^R$ |
| $A_{SP}$ | $C_P \equiv \frac{1}{2}(C_S^R - C_S^L)$ |
| $A_{0,T}, A_{||,T}, A_{\perp,T}$ | $C_T$ |

Now, $C_V^L, C_V^R, C_P$ and $C_T$ each have a magnitude and a weak (CP-odd) phase. However – and this is the key point – because $\Bar{B} \to D^*$ is the only hadronic transition in the decay, all helicity amplitudes are expected to have the same strong (CP-even) phase. Thus, this process involves only seven NP parameters: the four magnitudes of $1 + C_V^L, C_V^R, C_P$ and $C_T$, and their three relative weak phases.

The differential decay rate can be computed as a function of the eight helicity amplitudes. It is found that (i) the differential decay rate is a function of $q^2$ and three angles, $\theta^*, \theta_\ell, \chi$ (see Fig. 2), and (ii) the angular distribution contains twelve independent angular functions. Nine terms are CP-conserving and are present in the SM. There are also three CP-violating terms. These are TPs, proportional to $\sin \chi$, and arise only in the presence of NP. Furthermore, since the strong phases of all helicity amplitudes are equal, the CP-even piece of Eq. (4) vanishes, so the measurement of a TP in $\Bar{B} \to D^* \mu^- \bar{\nu}_\mu$ is itself a signal of CP violation, i.e., of NP.

The bottom line is that, for each $q^2$ bin, there are twelve observables (the coefficients of the angular functions). All of these are functions of seven NP parameters. With more observables than unknowns, in principle, it is possible to extract all the unknown parameters (this assumes that the value of $V_{cb}$ is known).

I want to emphasize the following: because $B \to V_1 (\to P_1 P'_1) V_2 (\to P_2 P'_2)$ decays are purely hadronic, their angular distribution can only be used to detect the presence of CP-violating NP. On the other hand, since the $\Bar{B} \to D^* \mu^- \bar{\nu}_\mu$ decay is semileptonic, it is much cleaner. Here, the angular distribution can also be used to detect the presence of CP-conserving NP.

In Ref. [9], Belle did an angular analysis of $\Bar{B} \to D^* \ell^- \bar{\nu}_\ell$ with the purpose of extracting $V_{cb}$. The fit did not allow for NP contributions. To be specific, TP terms were not included. Our suggestion is to redo the analysis, including all NP terms, both CP-conserving and CP-violating.
With a full fit, one might be able to measure or at least constrain the seven $b \to c\mu^-\bar{\nu}_\mu$ NP parameters.

Note: one might think that this is a lot of work for a decay process that shows no hints of NP. However, in Ref. [10], the Belle $\bar{B} \to D^*\ell\bar{\nu}_\ell$ data was reanalyzed, and it was found that there is some tension, possibly suggesting a violation of $e-\mu$ universality. So perhaps it is worthwhile to search for NP in this decay.

The measurement of the $\bar{B} \to D^*\ell\bar{\nu}_\ell$ angular distribution requires the knowledge of $p_\ell$. For this reason, this method cannot be applied to $\bar{B} \to D^*\tau^-\bar{\nu}_\tau$: because the $\tau^-$ decays, $p_\tau$ is not measurable due to the missing $\bar{\nu}_\tau$. In Ref. [11], an alternative angular distribution is proposed that uses the decay $\tau^- \to \pi^-\bar{\nu}_\tau$. The process is then a series of two-body decays: $\bar{B} \to D^*\pi^*, D^* \to D\pi'. N^* \to \tau^-\bar{\nu}_\tau, \tau^- \to \pi^-\bar{\nu}_\tau$.

The $\tau$ and $\pi^-$ are on-shell, so that the final state is described by six kinematic parameters. Due to the lack of knowledge of $p_\tau$, not all of these are measurable. Even so, an angular distribution can be constructed using the $\pi^-$ from the $\tau^-$ decay. $E_\pi$ and $q^2$ (the square of the momentum of the $\tau^-\bar{\nu}_\tau$ pair) are measurable. Also measurable are $\theta_\pi$, $\chi_\pi$ and $\theta^*$ (see Fig. 3). The remaining unmeasurable angle can be integrated over.

The differential decay rate is then a function of $q^2$, $E_\pi$ and three angles, $\theta^*$, $\theta_\pi$, $\chi_\pi$. The angular distribution can be written as follows:

$$\sum_{i=1}^{9} f_i^R(q^2, E_\pi)\Omega_i^R(\theta^*, \theta_\pi, \chi_\pi) + \sum_{i=1}^{3} f_i^I(q^2, E_\pi)\Omega_i^I(\theta^*, \theta_\pi, \chi_\pi). \quad (5)$$

The nine $f_i^R\Omega_i^R$ terms are CP-conserving and are present in the SM. The $f_i^R$ contain $|A_i|^2$ and $\text{Re}[A_i^*A_i]$ pieces. The three $f_i^I\Omega_i^I$ terms are CP-violating; the $f_i^I$ contain $\text{Im}[A_i^*A_i^T]$ pieces. These terms are TPs, proportional to $\sin\chi_\pi$, and are smoking-gun signals of NP.

The point is that the seven $b \to c\tau^-\bar{\nu}_\tau$ NP parameters can be extracted. Experimentalists will of course determine which type of analysis is best – $q^2/E_\pi$ bins, a fit to all the data, etc. If the discrepancy with the SM persists, this will tell us which NP parameters are nonzero. In turn, the knowledge of the Lorentz structure will be very helpful in identifying the NP.
Non-SMEFT NP [12] — But there’s more.

Whatever it is, the NP is heavy, with a mass of $O(\text{TeV})$ or higher. When it is integrated out, the EFT that is produced must necessarily respect the $SU(2)_L \times U(1)_Y$ electroweak gauge symmetry. This symmetry can be realized linearly, i.e., it is broken via the Higgs mechanism, in which case we have SMEFT. Alternatively, it can be realized nonlinearly, i.e., it is broken in another way, and this corresponds to HEFT, for example. Since the discovery of the Higgs boson, SMEFT is the default assumption, but HEFT is still possible.

It can only be determined experimentally whether a linear or nonlinear realization provides a better description of Nature. These two options can be distinguished through their predictions for the size of certain low-energy dimension-six four-fermion operators.

In particular, consider the operator $C_{\ell}^R \sim O(1)/\Lambda_{\text{NP}}^2$, like the other NP coefficients. On the other hand, SMEFT predicts $|C_{\nu}^R| \sim O(1)/\Lambda_{\text{NP}}^2 v^2/\Lambda_{\text{NP}}^2$, where $v$ is the Higgs vev. This is much smaller than the other NP coefficients.

As discussed above, it is possible to measure all NP parameters in the angular distribution of $\bar{B} \to D^* \tau^-(\to \pi^- \nu_\tau)\bar{\nu}_\tau$. If it is found that $|C_{\nu}^R|$ is much larger than the SMEFT prediction, this will point to non-SMEFT (non-decoupling) NP.

Conclusions — In summary, I have described measurable angular distributions for $\bar{B} \to D^* \mu^- \bar{\nu}_\mu$ and $\bar{B} \to D^* \tau^- (\to \pi^- \nu_\tau)\bar{\nu}_\tau$, including the most general NP contributions. These semileptonic decays are described by seven NP parameters.

Although unrelated by lepton flavour universality – the $\tau$ decays, while the $\mu$ does not – the two distributions have some similarities: both contain twelve independent angular functions. Nine of these are CP-conserving and are present in the SM. Three terms are CP-violating (TPs) and are smoking-gun signals of NP. In both cases, if the angular distributions can be measured, the seven NP parameters can be extracted. In the case of $\bar{B} \to D^* \tau^- (\to \pi^- \nu_\tau)\bar{\nu}_\tau$, this is particularly interesting, as it would give us information about the NP hinted at in the $b \to c\tau^- \bar{\nu}_\tau$ anomalies.

Finally, these angular distributions have the ability to reveal the presence of non-SMEFT (non-decoupling) NP.

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