Dilaton-derived quintessence scenario leading naturally to the late-time acceleration of the Universe

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(March 19, 2022)

Quintessence scenarios provide a simple explanation for the observed acceleration of the Universe. Yet, explaining why acceleration did not start a long time ago remains a challenge. The idea that the transition from radiation to matter domination played a dynamical role in triggering acceleration has been put forward in various guises. We propose a simple dilaton-derived quintessence model in which temporary vacuum domination is naturally triggered by the radiation to matter transition. In this model Einstein’s gravity is preserved but quintessence couples non-minimally to the cold dark matter, but not to “visible” matter. Such couplings have been attributed to the dilaton in the low-energy limit of string theory beyond tree level. We also show how a cosmological constant in the string frame translates into a quintessence-type of potential in the atomic frame.

Recent astronomical observations of distant supernovae light-curves[3][4] suggest that the expansion of the universe has recently begun to accelerate. This observation has deep theoretical implications. Accelerated expansion is the hallmark of repulsive gravity, which according to Einstein’s theory of relativity can only be achieved with extreme forms of matter, such as a cosmological constant $\Lambda$ (the vacuum energy). The measurement of a non-zero cosmological constant vindicates Einstein’s greatest “blunder”, but leaves cosmology with severe fine-tuning problems. Normal forms of matter are diluted by expansion; $\Lambda$ is not. In order to achieve $\Lambda$ domination nowadays and not before, one has to tune the initial ratio between vacuum and other forms of energy to about a part in $10^{130}[4].$

Overall cosmologists would rather set $\Lambda = 0$, and hope that other, less extreme, forms of repulsive matter were behind the observed acceleration of the Universe. Quintessence[4][9], a scalar field $\phi$ endowed with a rolling potential, has become a popular alternative. Such potentials have appeared variously in the context of Kaluza-Klein, super-gravity, and string theories (see[8] Section IIB for an excellent review). Quintessence has the desirable property that its energy density “scales” (i.e. remains at constant fraction) or “tracks” the dominant form of matter in the Universe. Deviations from scaling eventually develop, following which quintessence starts behaving like a cosmological constant, leading to the observed acceleration of the Universe.

However, explaining why acceleration only starts nowadays, some 30 expansion times since the Universe became classical, still requires that quintessence be fine-tuned, either in the field’s initial conditions or in the parameters of its Lagrangian (see however[8][11]). In general any theory attempting to explain the cosmological acceleration has to explain what is special about the present epoch for acceleration to start now. We propose that the best explanation for the coincidence of observed acceleration nowadays is to associate it with our proximity to the cosmological transition from radiation to dust domination. This view was first proposed by Barrow and Magueijo[1] in a different context.

In[12] Armendariz-Picon, Mukhanov, and Steinhardt proposed $\kappa$-essence, a quintessence-type implementation of this idea. In such a model scaling is only possible in the radiation epoch, with $\Lambda$ type of behaviour triggered by the onset of the matter epoch. This type of behaviour is achieved with a Lagrangian containing a series of non-linear kinetic terms. As the authors themselves recognize, such a model serves to illustrate a point, rather than to provide the simplest and best motivated realization of such a dynamics. The purpose of this letter is to show that a similar dynamics may be realized in much simpler models, coincident with dilaton models appearing in the low energy limit of string theory beyond tree-level[13][4].

In non-minimal theories radiation and matter have differing effects on the dynamics of the quintessence field. These can be interpreted in two alternative ways. In one we may depart from Einstein’s gravity, and couple the field $\phi$ to the Ricci scalar $R$ (possibly in the form $g(\phi)R$) in the gravitational action. This amounts to identifying quintessence with the a Brans-Dicke field[14]. The field $\phi$ will then be driven by $R$ as well as its potential. Recalling that $R = 0$ for radiation contributions, but $R \propto \rho$, the energy density, for non relativistic matter, we see that the extra term could in principle push the field off scaling at an epoch close to nowadays, providing an “$R$-boost”[17]. Simple as this idea might be, it does not survive close scrutiny; the $R$-boost is in fact deep in the radiation epoch. One must remember that the gravitational equations for such a theory are also heavily modified, and indeed the work undertaken by[16][18] shows that more fine-tuning, if anything, is required in order to achieve acceleration nowadays.

Another possibility is to retain Einstein’s gravity, but to directly couple quintessence to the matter fields, via a coupling of the form $f(\phi)\mathcal{L}_m$. This corresponds to identifying quintessence with the Einstein’s frame formulation of the dilaton and generate field-dependent masses and polarisations. These couplings, and the general Brans-
In the Einstein conformal frame, given by:

\[ S = \int d^4x \sqrt{-g} \left( \frac{R}{2} + \mathcal{L}_\phi + \mathcal{L}_V + f(\phi)\mathcal{L}_I \right) \]

(1)

in which \( 8\pi G = 1 \), \( \mathcal{L}_V \) is the Lagrangian of “visible matter” (baryons, photons, and also baryonic and neutrino dark matter), and \( \mathcal{L}_I \) the Lagrangian of a dominant non-baryonic form of cold dark matter. As usual \( \mathcal{L}_\phi = -\partial_\mu \phi \partial^\mu \phi/2 - V(\phi) \) with \( V(\phi) = V_0 e^{-\lambda \phi} \) the standard quintessence potential. This theory clearly has the potential to behave in line with the dynamics sought - since it drives quintessence via invisible matter. In the radiation epoch invisible matter becomes subdominant, and we may expect the usual scaling solutions to be valid. In the vicinity of the transition to matter domination, the new driving term becomes significant and may induce deviations from scaling.

Actions with different couplings to each individual matter terms arise in full-loop expansion generalisations of an effective action for the massless modes of a dilaton, for example as considered by Damour and Polyakov \[ 14 \]. These give an action of the form

\[ S = \int d^4x \sqrt{-g} \left( \hat{B}_b(\Phi)(\hat{R}/2 - 2\Lambda) - \hat{B}_\Phi(\Phi)\partial_\mu \Phi \partial^\mu \Phi \right. \]

\[ + \sum_i \hat{B}_i(\Phi) \tilde{L}^{(i)} \]  

(2)

where \( i \) represent the different matter terms, and \( \Lambda \) is a string frame cosmological constant. In \[ 14 \] it was hoped that the couplings are not too different for different types of matter, so as not to conflict with the Eötvös experiment; however they could be very different for the dark matter of the Universe \[ 14, 21 \]. A further rationale for why this could be the case is that the dark matter may indeed be very exotic (e.g. super-symmetric dark matter), in which case we may expect the couplings to the dilaton to be very different than to ordinary matter.

Hence, we follow \[ 14 \] assuming a Universal coupling \( \hat{B}(\Phi) \) for gravity and all forms of visible matter, but follow \[ 14 \] taking the coupling to invisible matter to have a different strength. For example, the higher-order loop corrections to the string coupling could be non-negligible giving a coupling of the form \[ 14 \]

\[ B_I(\Phi) = e^{-2\Phi} + c_0 + c_1 e^{2\Phi} + c_2 e^{4\Phi} + \ldots \]

(3)

with \( c_i \neq 0 \) parameterizing the corrections beyond tree-level. Hence the action can be written

\[ S = \int d^4x \sqrt{-\hat{g}} \left( \sigma(\hat{R}/2 - 2\Lambda + \hat{L}_V) - (\omega/\sigma)\partial_\mu \sigma \partial^\mu \sigma \right. \]

\[ + B_I(\sigma) \hat{L}_I \]  

(4)

where \( \sigma = \hat{B}(\Phi) \), as defined in \[ 14 \].

Conformally transforming from the string frame to the Einstein frame we obtain the proposed action \[ 1 \], where the function \( f(\phi) \) can be expressed in terms of the coupling \( B_I(\Phi) \). The relevant transformation is \( g_{\mu\nu} = 2\sigma \hat{g}_{\mu\nu} \) and \( 2\sigma = e^{-\lambda \phi} \) with \( \lambda = (\omega + 3/2)^{1/2} \). We highlight the remarkable fact that a dilaton independent cosmological constant in the string frame is transformed into a quintessence potential \( V(\phi) = \hat{\Lambda} e^{-\lambda \phi} \) in the Einstein frame \[ 22 \]. Hence, the presence of a cosmological constant in the string frame allows one to identify the dilaton with the quintessence field. Note that the Einstein frame for our model is identical with the Jordan or atomic frame for visible matter, in which it follows geodesics; this is usually considered the physical frame \[ 13 \].

The coupling \( f(\phi) \) (and also all the \( B(\Phi) ) \) are expected to be approaching a minimum \[ 14, 23 \] characterised by \( \phi = \phi_0 \), say. Hence, for our purposes, the function \( f(\phi) \) may be approximated as a Taylor expansion about the minimum,

\[ f(\phi) = 1 + \sum \frac{1}{\beta!} \frac{\partial^\beta f}{\partial \phi^\beta} (\phi - \phi_0)^\beta \]

(5)

We therefore investigate a coupling of the form \( f(\phi) = 1 + \alpha(\phi - \phi_0)^\beta \) where \( \alpha \) and \( \beta \) reflect the concavity of the minimum.

Varying action \[ 1 \] with respect to the metric and \( \phi \) we obtain the field equations:

\[ G_{\mu\nu} = T^V_{\mu\nu} + T^\phi_{\mu\nu} + f(\phi)T_I^{\mu\nu} \]

\[ \square \phi = \frac{\partial V}{\partial \phi} - \frac{\partial f}{\partial \phi} \hat{L}_I \]  

(6)

(7)

where \( G_{\mu\nu} \) is the Einstein’s tensor and the various \( T_{\mu\nu} \) are stress-energy tensors. Heuristically, we may interpret the new term driving \( \phi \) as a contribution to an effective potential \( V_{eff} = V - f(\phi)\mathcal{L}_I \). Bianchi’s identities \( (\nabla_\mu G^\mu_\nu = 0) \) lead to integrability conditions:

\[ \nabla^\mu T^{\mu\nu}_V = 0 \]  

(8)

\[ \nabla^\mu T^{\mu\nu}_I = (g^{\nu\sigma} \mathcal{L}_I - T^{\nu\sigma}_I) f' \nabla_\sigma \phi \]  

(9)

to be contrasted with Amendola’s coupled quintessence \[ 20 \] (for which the interaction term is proportional to \( T \)).

Interestingly, the equations of motion depend on the Lagrangian, and so full divergences are no longer irrelevant leading to a wealth of possibilities. For a perfect fluid we may infer the Lagrangian from its constituent
FIG. 1. The evolution of $\Omega_\phi$ and $w_{tot}$ for a model with $\lambda = 8$, $\beta = 8$, $\alpha = 50$, and $\phi_0 = 32.5$. An early period of scaling is broken near the transition from radiation to matter, first with a period of kination, then inflation. At late times the universe returns to a matter dominated scaling solution.

particles (providing they do not interact). For a pressureless fluid each particle has Lagrangian

$$\mathcal{L}(x) = -\int d\lambda E_0 \frac{\delta(x - y(\lambda))}{\sqrt{-g}} \sqrt{-g_{\mu\nu}} \frac{dy^\mu}{d\lambda} \frac{dy^\nu}{d\lambda}$$

where $\lambda$ is the affine parameter (or proper time), $y(\lambda)$ is the particle’s trajectory, and $E_0$ its rest mass. Hence we have that for a homogeneous pressureless fluid $\mathcal{L} = -\rho$.

A similar argument applied to relativistic particles leads to $\mathcal{L} = 0$ for radiation fluids.

Specializing to a flat Friedmann model, with scale factor $a$, we find Friedmann equations:

$$3\left(\frac{\dot{a}}{a}\right)^2 = \rho_b + \rho_r + f(\phi)\rho_I + \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\dot{\rho}_I + 3\frac{\dot{a}}{a}\rho_I = -\frac{f'(\phi)}{f(\phi)}(\rho_I + \mathcal{L}_I) = 0$$

$$\rho_b + 3\frac{\dot{a}}{a}\rho_b = 0$$

$$\rho_r + 4\frac{\dot{a}}{a}\rho_r = 0$$

$$\ddot{\phi} + 3\frac{\dot{a}}{a}\dot{\phi} + V' = f'(\phi)\mathcal{L}_I = -f'(\phi)\rho_I$$

where dots represent derivatives with respect to proper time, and the prime (') indicates differentiation with respect to $\phi$.

In Figs. 1 and 2 we plot two typical examples of solutions for the cosmological evolution in this theory. We plot the fraction of energy in quintessence $\Omega_\phi = \rho_\phi/\rho_{tot}$, and the total equation of state $w_{tot} = p_{tot}/\rho_{tot}$ where $p_{tot}$ is the total pressure (induced by the radiation and $\phi$). We separate the radiation from the matter epoch (left and right panels), and indicate where nucleosynthesis and Nowadays lie. We see that the driving term $f(\phi)\mathcal{L}_I$, in the form proposed, can indeed kick $\phi$ off scaling in the vicinity of $a_{eq}$ with a transient regime lasting $4\exp$ times after and before $a_{eq}$. Typically the field is first pushed into kination to re-emerge into inflationary behaviour, the two events arranging themselves symmetrically around $a_{eq}$ along the log($a$) axis.

The acceleration produced in this model is always a transient phenomenon. The field $\phi$ produces inflation because the driving term $f(\phi)\rho_I$ induces a local minimum in the effective potential $V_{eff} = V + f\rho_I$ similar to the one in the potential proposed in [9]. However, as soon as inflation starts, $\rho_I$ is diluted, which in turn withdraws the extra driving force, leading the field back into scaling. As in [10], the observed spell of vacuum domination turns out to be a bluff, with a new matter epoch following the present $\Lambda$ dominance. This complex feedback process explains the fast oscillations preceding kination and inflation for some of the parameters of our model, such as the one in Fig. 2.

Our model illustrates the point that we do not need to have an inflationary attractor to explain the current acceleration of the Universe. Indeed, as shown in Fig. 3 the
structure of attractors in our model is the same as in standard quintessence. It is the motion of the system while moving between the two (matter and radiation) attractors which is new. Perhaps similar transient behaviour is present in some extended quintessence models; most of the work done so far has focused on attractors \[4\] \[5\].

We remark that the symmetry of kination and inflation around \(a_{eq}\) means that this model bypasses the nucleosynthesis constraints usually affecting standard quintessence \[6\]. This is because, coincidentally, nucleosynthesis, equality, and nowadays are roughly equally spaced along the \(\log(a)\) axis. Hence, typically kination occurs before nucleosynthesis if we want the field to inflate nowadays. This means that \(\Omega_\phi \approx 0\) during nucleosynthesis, invalidating the bound \(\lambda > 5\) derived in \[6\].

However, there are further constraints on this type of model, due to the fact that for many purposes it is \(f \rho_I\) what should be regarded as the matter density (since this is the gravitational mass of the invisible matter) and not \(\rho_I\) (which is the conserved mass). Deciding between the two is mostly a matter of language, dependent on whether to count \((f-1)\mathcal{L}_I\) as an interaction term or not. In any case, the transition between a radiation epoch (with \(a \propto t^{1/2}\)) and a matter epoch (with \(a \propto t^{2/3}\)) is determined by the redshift for which \(\rho_0 + \rho_I f = \rho_r\) and so is affected by the change in \(f\). In general, this pushes up the redshift of equality, since \(f\) is a decreasing function. A competing factor results from the reduction of the amount of \(\rho_I\) nowadays resulting from the current dominance of quintessence. This tends to reduce the equality redshift. The first effect is normally larger than the second, but can be made arbitrarily small by increasing \(\lambda\) - so that the change in \(\phi\) and \(f(\phi)\) is smaller.

More important still is the effect such a coupling may have on the growth of dark matter perturbations. It can be proved that, in the limit in which fluctuations in \(\phi\) are ignored, the equations for \(\delta_I = \delta \rho_I / \rho_I\) are unaffected. Hence, we expect the only effect on the matter power spectrum to result from the change in the redshift at equality. No obvious disastrous effect is present; and so the model we have proposed is not a priori inconsistent with observations of large scale structure. Nonetheless, more subtle effects are present due to fluctuations in \(\phi\), which induce new terms in the equations for \(\delta_I\) (cf. \[4\]), besides sourcing the perturbed potentials. A complete study of structure formation for our model is deferred to future work, but we suspect results not dissimilar to those found in standard quintessence scenarios \[2\].

In summary, we have found a bridge between dilaton and quintessence models, by noting that a string frame \(\hat{\Lambda}\) transforms into a rolling potential for the dilaton in the physical frame. The dilaton may couple with different strengths to visible and dark matter, a property we used to naturally trigger (transient) acceleration nowadays. The model is consistent with obvious constraints, but a careful study of its more subtle effects on structure formation is warranted.

Acknowledgements We would like to thank Luca Amendola, John Barrow, Michael Joyce, and Kelly Stelle for discussions. RB thanks the PPARC for support.

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A further transformation $\phi \to -\phi$ is required to bring the potential to its standard form.