An Efimov trimer near the atom-dimer threshold can increase the atom loss rate in ultracold trapped atoms through the avalanche mechanism proposed by Zaccanti et al. A 3-body recombination event creates an energetic atom and dimer, whose subsequent elastic collisions produce additional atoms with sufficient energy to escape from the trapping potential. We use Monte Carlo methods to calculate the average number of atoms lost and the average heat generated by recombination events in both a Bose-Einstein condensate and a thermal gas. We take into account the energy-dependence of the cross sections and the spatial structure of the atom cloud. We confirm that the number of atoms lost can be much larger than the naive value 3 if there is an Efimov trimer near the atom-dimer threshold. This does not produce a narrow loss feature, but it can significantly affect the determination of Efimov parameters.

In a mixture of atoms and shallow dimers, a narrow loss feature can also be caused by an Efimov trimer near the atom-dimer threshold. We will refer to a scattering length \(a_s\) for which an Efimov trimer is exactly at the threshold as an atom-dimer resonance. For \(a_s\), there is resonant enhancement of both the elastic scattering of an atom and the shallow dimer and their inelastic scattering into an atom and a deep dimer (strongly-bound molecule). The large binding energy of the deep dimer gives the outgoing atom and dimer large enough kinetic energies to escape from the trapping potential. The resulting peak in the atom loss rate near \(a_s\) was first observed in a mixture of \(^{133}\text{Cs}\) atoms and dimers [8].

There have also been observations of enhanced loss rates near \(a_s\) in systems consisting of atoms only. Zaccanti et al. observed a narrow loss peak near the predicted position of an atom-dimer resonance in a Bose-Einstein condensate (BEC) of \(^{39}\text{K}\) atoms [9]. They also observed a loss peak in a thermal gas near the next atom-dimer resonance, at a scattering length larger by a factor of about 22.7. Pollack et al. observed a loss peak near the predicted position of an atom-dimer resonance in a BEC of \(^{7}\text{Li}\) atoms [10]. Machtey et al. observed such a loss peak in a thermal gas of \(^{7}\text{Li}\) atoms [11]. These loss features near the atom-dimer resonance are puzzling, because the equilibrium population of shallow dimers is expected to be negligible in these systems.

Zaccanti et al. proposed an avalanche mechanism for the enhancement of the atom loss rate near \(a_s\) in systems consisting of atoms only [12]. The loss of atoms is initiated by a 3-body recombination event that produces an atom and a shallow dimer with kinetic energies large enough to escape from the trap. If they both escape, there would be 3 atoms lost. If the dimer instead scatters inelastically, the scattered atom is also lost. However the dimer can undergo multiple elastic collisions before ultimately....
escaping or suffering an inelastic collision, and it may deliver enough energy to the scattered atoms to allow them to escape from the trap. These atoms may also undergo multiple elastic collisions, resulting in an avalanche of additional lost atoms. Near $a_*$, the resonant enhancement of the atom-dimer cross sections increases both the probability for the dimer to initiate an avalanche and the probability for an inelastic collision. The resulting increase in the number of atoms lost per recombination event could produce an observable loss feature.

In this paper, we present a quantitative analysis of the avalanche mechanism for atom loss. We use Monte Carlo methods to generate avalanches of atoms initiated by recombination events with the appropriate probability distribution. We calculate the average number of atoms lost and the average energy converted into heat from an avalanche. We use the results to calculate the atom loss rate constant for both a Bose-Einstein condensate and a thermal gas of trapped atoms.

We consider identical bosons of mass $m$ with a large positive scattering length $a$. The 2-body physics in our Monte Carlo model for the avalanche mechanism consists of the binding energy $E_d = \hbar^2/(m a^2)$ for the shallow dimer and the cross section $\sigma_{AA} = 8\pi a^2/(1 + a^2 k_{cm}^2)$ for elastic atom-atom scattering with center-of-mass wavenumber $k_{cm}$. The 3-body physics in our model consists of the rate constants $\alpha_{\text{shallow}}$ and $\alpha_{\text{deep}}$ for 3-body recombination at threshold into the shallow dimer and into deep dimers and the cross sections $\sigma_{\text{AD}}^{(\text{el})}$ and $\sigma_{\text{AD}}^{(\text{in})}$ for elastic and inelastic atom-dimer scattering. In the zero-range limit, these reaction rates are determined by $a$ and two Efimov parameters: the atom-dimer resonance $a_*$ and a dimensionless parameter $\eta_*$ that controls the decay width of an Efimov trimer [7]. Analytic expressions for $\alpha_{\text{shallow}}$ and $\alpha_{\text{deep}}$ are given in Ref. [1]. They are more conveniently expressed in terms of a parameter $a_{20}$ that differs from $a_*$ by a universal ratio: $a_{20}/a_* = 4.4724$. The 5 digits of accuracy in the ratio are obtained by combining universal results from Refs. [12,13]. Parameterizations of $\sigma_{\text{AD}}^{(\text{el})}$ and $\sigma_{\text{AD}}^{(\text{in})}$ from the atom-dimer threshold up to the dimer-breakup threshold can be obtained from Ref. [1].

We make several simplifying approximations in our model. We ignore the effects of potential energies on the cross sections for the dimer and the atoms. In the case of a BEC, we also ignore mean-field energies. We also approximate the trajectories of the atoms and the dimer between collisions as straight lines. In the first collision of the atom or dimer from the recombination event with a stationary atom, the center-of-mass wavenumber $k_{cm}$ is $1/(\sqrt{3}a)$ or $2/(3\sqrt{3}a)$, respectively. In the second collision with a stationary atom, the typical $k_{cm}$ is smaller by a factor of $1/\sqrt{2}$ for the atom and $\sqrt{3}/\sqrt{2}$ for the dimer. As the number of elastic collisions increases, $k_{cm}$ decreases towards 0. The universal cross sections for the first collision, a typical second collision, and after many elastic collisions ($k_{cm} \to 0$) are shown in Fig. 1 for $\eta_* = 0.2$. At $k_{cm} = 0$, $\sigma_{\text{AD}}^{(\text{el})}$ and $k_{cm} \sigma_{\text{AD}}^{(\text{in})}$ have dramatic peaks with maxima near $a_*$ and 22.7 $a_*$. The elastic cross section also has deep minima near 0.38 $a_*$ and 8.6 $a_*$. For the first few collisions, $\sigma_{\text{AD}}^{(\text{in})}$ still has deep minima but there are no dramatic peaks.

The experimental inputs in our Monte Carlo model are the number $N_0$ of trapped atoms, the frequencies $\nu_x$, $\nu_y$, and $\nu_z$ of the harmonic trapping potential, the temperature $T$ of the atoms, the scattering length $a$ (which can be controlled by varying the magnetic field near a Feshbach resonance), and the trap depth $E_{\text{trap}}$, which should be much larger than the energies of the trapped atoms. Atoms and dimers that reach the edge of the atom cloud are assumed to be lost if their energies exceed $E_{\text{trap}}$ and 2$E_{\text{trap}}$, respectively. The role of the remaining experimental inputs is to determine the number density $n(x,y,z)$ of the trapped atoms. We consider two simple cases: a BEC of atoms at zero temperature in the Thomas-Fermi limit and a thermal gas of atoms above the critical temperature for BEC. The rate at which the number $N$ of atoms in a thermal gas decreases due to 3-body recombination is

$$
\frac{dN}{dt} = - (n_{\text{loss}} \alpha_{\text{shallow}} + 3 \alpha_{\text{deep}})(n^2)N,
$$

(1)
where $\langle n^2 \rangle$ and $\langle N_{\text{lost}} \rangle$ are spatial averages weighted by $n(r)$ and $n^3(r)$, respectively. The right side must be multiplied by $1/3!$ if the system is a BEC. Atoms produced by the avalanche that have energy less than $E_{\text{trap}}$ can never escape from the trapping potential and their kinetic energy will ultimately be transformed into heat. The recombination heating rate in a thermal gas is $\langle E_{\text{heat}} \rangle \phi_{\text{shallow}} (n^2) N$, where $\langle E_{\text{heat}} \rangle$ is the average heat from a single avalanche.

The development of an avalanche can be decomposed into discrete steps corresponding to the recombination event and the subsequent scattering events. For each event, the subsequent state of the avalanche has a simple probability distribution. (a) The position $(x, y, z)$ of the recombination point has a distribution proportional to $n^3(x, y, z)$. (b) The momenta of the outgoing particles from an event have a distribution that is isotropic in the center-of-mass frame. (c) An atom or dimer flies beyond the edge of the atom cloud with probability $\exp(-\sigma \int n \, \text{d}t)$, where $\sigma$ is $\sigma_{AA}$ for an atom and $\sigma_{AD}^{(e)} + \sigma_{AD}^{(i)}$ for a dimer and where $\int n \, \text{d}t$ is the column density integrated from the position of the previous collision out to infinity along a straight path in the direction of the momentum. If the random number determines that an atom or dimer fails to reach the edge of the atom cloud, it is also used to determine the position where it scatters. (d) Given that a dimer scatters, it scatters inelastically with probability $\sigma_{AD}^{(i)} / (\sigma_{AD}^{(e)} + \sigma_{AD}^{(i)})$. All these simple probability distributions together determine the probability distribution of avalanches.

We generate avalanches with the appropriate probability distribution using a Monte Carlo method that produces a binary tree whose nodes represent events. The branches represent the two outgoing particles from each event. There are also terminal nodes that correspond to atoms and dimers whose ultimate fate is determined. The conditions for a terminal node depend on the kinetic energy $E$ of the particle. (a) If an atom has $E < E_{\text{trap}}$, it remains trapped. (b) If an atom that reaches the edge of the atom cloud has $E > E_{\text{trap}}$, it is lost. (c) If a dimer has an inelastic collision, both it and the scattered atom are lost. (d) If a dimer that reaches the edge of the cloud has $E > 2E_{\text{trap}}$, it is lost. (e) If a dimer that reaches the edge of the cloud has $E < 2E_{\text{trap}}$, it will return to the cloud and eventually suffer an inelastic collision. The terminal nodes give contributions to the number of atoms lost and to the heat of the remaining atoms. Adding these contributions from all the terminal nodes, we get $N_{\text{lost}}$ and $E_{\text{heat}}$ for the avalanche. We calculate $\langle N_{\text{lost}} \rangle$ and $\langle E_{\text{heat}} \rangle$ by averaging over many avalanches. More than 100,000 avalanches are sometimes required to get smooth results for $\langle N_{\text{lost}} \rangle$ and $\langle E_{\text{heat}} \rangle$ as functions of $a$.

Zaccanti et al. developed a simple probabilistic model for the avalanche process [9]. In the Zaccanti model, the avalanche is reduced to a discrete sequence of dimer scattering events. A variable number of elastic collisions is followed either by the escape of the dimer from the trap or by a final inelastic collision. There is one lost atom for each elastic collision up to a maximum number that is determined by the trap depth $E_{\text{trap}}$. The relative probability for each sequence of scattering events is determined by the mean column density and by $\sigma_{AD}^{(i)}$ and $k_{\text{cm}} \sigma_{AD}^{(in)}$. The Zaccanti model is greatly simplified in several ways compared to our Monte Carlo model: (a) The spatial structure of the avalanche is ignored. (b) Elastic scattering of the atoms is not considered. (c) The energy dependence of $\sigma_{AD}^{(i)}$ and $k_{\text{cm}} \sigma_{AD}^{(in)}$ is not taken into account. Zaccanti et al. used their model to calculate $\langle N_{\text{lost}} \rangle$ for their experiment with $^{39}$K atoms [9]. It predicts that $\langle N_{\text{lost}} \rangle$ increases from its background value of 3 to about 13 near the atom-dimer resonance. The resulting prediction for the atom loss rate agrees qualitatively with the loss feature they observed near 30.4 $a_0$.

Macht et al. developed an alternative probabilistic model for the avalanche process [14]. They made the same simplifications as in the Zaccanti model, but they used different probabilities for the sequences of scattering events. Macht et al. did not introduce the trap depth $E_{\text{trap}}$, so they could not calculate $\langle N_{\text{lost}} \rangle$. Instead they calculated the average number $\bar{N}$ of dimer collisions. They suggested that the loss feature near $a_0$ might be associated with the maximum of $\bar{N}$.

In the experiment with $^{39}$K atoms in Ref. [9], the loss feature near 30.4 $a_0$ is at a scattering length that may be too small for universal predictions to be reliable. We therefore focus on the experiments with $^7$Li atoms. In Ref. [10], the data near the atom-dimer resonance were obtained using a BEC of $^7$Li atoms in the $|1, +1\rangle$ hyperfine state with $E_{\text{trap}} \approx 0.5 \, \text{µK}$. They measured the rate constant $L_3 = 3(\alpha_{\text{shallow}} + \alpha_{\text{deep}})$ as a function of
the recombination event are the most important for generating an avalanche. As shown in Fig. 1 the maxima in the elastic cross sections for the first few collisions occur well above $a_\star$. Decreasing $\eta_\star$ by a factor of 5 does not give dramatic changes in $\langle N_{\text{lost}} \rangle$ and $\langle E_{\text{heat}} \rangle$.

If the avalanche mechanism is taken into account, as in Eq. (1), the rate constant is $L_3 = (N_{\text{lost}})\alpha_{\text{shallow}} + 3\alpha_{\text{deep}}$. In Fig. 2 we show the predictions for both experiments for $L_3$ as a function of $a$ using $a_\star = 282$ $a_0$ and $\eta_\star = 0.2$. If $\eta_\star$ were decreased to 0.04, the changes in the curves would be obvious only near $a_{\alpha_0} \approx 1260$ $a_0$, where the minimum would be about a factor of 5 below the data. There is no narrow loss feature near $a_\star$, but instead there is a broad enhancement in $L_3$ in the region between $a_\star$ and $a_{\alpha_0}$. The enhancement is large enough that it could affect the fitted values of the Efimov parameters. We are unable to get a narrow loss feature like that in the data in Fig. 2 for any values of the parameters.

Our results for the avalanche mechanism also suggest that the effects of heating should be taken into account differently in the analysis of atom loss data at positive scattering lengths. In addition to the recombination heating associated with $E_{\text{heat}}$, one must take into account the disappearance of the three atoms that undergo recombination [17]. In a thermal gas, the resulting rate of change in the temperature is

$$dT/dt = (\langle E_{\text{heat}} \rangle/3kT + \langle N_{\text{lost}} \rangle - 2) \alpha_{\text{shallow}} + \alpha_{\text{deep}} \langle n^2 \rangle T.$$  

This has the same dependence on $T$ as that assumed in Ref. [17]. In the coefficient of $\alpha_{\text{shallow}}$, the $1/kT$ term is determined by $\langle E_{\text{heat}} \rangle$ and the additive constant $\langle N_{\text{lost}} \rangle - 2$ can differ from the naive value 1. The effects of heating are usually taken into account by using the coupled equations for $dN/dt$ and $dT/dt$ to extrapolate to the initial value of $dN/dt$ [17]. The $\langle N_{\text{lost}} \rangle$ and $\langle E_{\text{heat}} \rangle$ terms in Eqs. (1) and (2) may have a significant effect on this extrapolation.

We have found that the avalanche mechanism does not produce a narrow loss feature near the atom-dimer resonance, but instead a broad enhancement in $L_3$ between $a_\star$ and $a_{\alpha_0}$. This can have a significant effect on the determination of Efimov parameters from data at positive scattering lengths. The heating from the avalanche mechanism can also be important in the extrapolation to the initial atom loss rate. All experiments on Efimov loss features at positive $a$ should probably be reanalyzed to take into account the effects of the avalanche mechanism. If the observed loss features near $a_\star$ in $39K$ and $^7\text{Li}$ atoms survive such a reanalysis, the mechanism for these loss features will remain a puzzle.

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