Importance of closed quark loops for lattice QCD studies of tetraquarks

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Outline

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2 Approach

3 Efficient computation of $C_{jk}(t)$

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Light mesons

![Graph showing the spectrum of light mesons with their corresponding quantum numbers and mass values. The graph includes symbols for π, ρ, ω, and the a0 mesons.]
Tetraquark interpretation

Nonet of light scalar mesons \((J^P = 0^+)\) still poorly understood

- \(I = 1\) (two \(u/\bar{d}\) quarks) states \((a_0, f_0)\) are **heavier** than the \(I = 1/2\) \((u/\bar{d} + s\) quark) states \((\kappa)\)

- Tetraquark interpretation resolves the mass ordering of the \(0^+\) sector naturally
  
  \begin{itemize}
  \item \(a_0 \equiv us\bar{s}\bar{d}\) and \(\kappa \equiv u(u\bar{u} + d\bar{d})\bar{s}\)
  \item \(a_0(980) \rightarrow K\bar{K}[su][\bar{ds}] \text{ & } a_0(980) \rightarrow \eta_s\pi[ss][\bar{du}]\)
  \end{itemize}

Experimental results vs. conventional \(q\bar{q}\) pairs vs. tetraquark interpretation.
Approach

Study of **effective masses** from mesonic two-quark and four-quark operators.

- Information about possible stable states around threshold
- Composition of states from the solution of the generalized eigenvalue problem
- Relies on large operator basis, in particular 2 meson states

**Gauge configurations:**

- 2+1 dynamical clover fermions and Iwasaki gauge action
- generated by the PACS-CS Collaboration
  
  S. Aoki et al. [PACS-CS Collaboration], Phys. Rev. D 79 (2009) 034503 [arXiv:0807.1661 [hep-lat]].
- Lattice: $32^3 \times 64$, $a \approx 0.09$fm
- $\approx 500$ configurations at $M_\pi \approx 300$MeV
In our study: 6 operators with the quantum numbers of $a_0(980)$.

- $O^{q\bar{q}} = \sum_x (\bar{d}_x u_x)$
- $O^{K\bar{K}, \text{point}} = \sum_x (\bar{s}_x \gamma_5 u_x)(\bar{d}_x \gamma_5 s_x)$
- $O^{\eta_s \pi, \text{point}} = \sum_x (\bar{s}_x \gamma_5 s_x)(\bar{d}_x \gamma_5 u_x)$
- $O^{Q\bar{Q}} = \sum_x \epsilon_{abc} (\bar{s}_{x,b}(C\gamma_5)\bar{d}_{x,c})\epsilon_{ade} (u_{x,d}(C\gamma_5)s_{x,e})$
- $O^{K\bar{K}, \text{2-part}} = \sum_{x,y} (\bar{s}_x \gamma_5 u_x)(\bar{d}_y \gamma_5 s_y)$
- $O^{\eta_s \pi, \text{2-part}} = \sum_{x,y} (\bar{s}_x \gamma_5 s_x)(\bar{d}_y \gamma_5 u_y)$
$C_{jk}(t)$

$$C_{jk} = \langle O_j O_k^\dagger \rangle$$

|        | $O_q\bar{q}^\dagger$ | $O_{K\bar{K}}^\dagger$ | $O_{\eta_s\pi}^\dagger$ | $O_{Q\bar{Q}}^\dagger$ | $O_{K\bar{K}}$ | $O_{\eta_s\pi}$ |
|--------|-----------------------|--------------------------|--------------------------|------------------------|----------------|----------------|
| $O_q\bar{q}$ | -                     | -                        | +                        | -                      | -              | +              |
| $O_{K\bar{K}}$ | -                     | +                        | -                        | -                      | +              | -              |
| $O_{\eta_s\pi}$ | +                     | -                        | +                        | -                      | +              | -              |
| $O_{Q\bar{Q}}$ | -                     | -                        | -                        | +                      | -              | +              |
| $O_{K\bar{K}}$ | -                     | +                        | -                        | -                      | +              | -              |
| $O_{\eta_s\pi}$ | +                     | -                        | +                        | -                      | -              | +              |
| $O_{\eta_s\pi}$ | +                     | -                        | +                        | -                      | -              | +              |

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Efficiency of methods and combination of methods

\[ R(a), R(b) = \Delta C(a)(t) \cdot \sqrt{\tau(a)} \Delta C(b)(t) \cdot \sqrt{\tau(b)} \]
Efficiency of methods and combination of methods

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\[ R(a), R(b) = \Delta C(a)(t) \cdot \sqrt{\tau(a)} \]

\[ R(b) = \Delta C(b)(t) \cdot \sqrt{\tau(b)} \]
Efficiency of methods and combination of methods

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\[ R^{(a),(b)} = \frac{\Delta C^{(a)}(t) \cdot \sqrt{\tau^{(a)}}}{\Delta C^{(b)}(t) \cdot \sqrt{\tau^{(b)}}} \]
Evidence for the relevance of closed quark loops

- closed fermion loops not only required to include $O^{q\bar{q}}$ to the operator basis

- several matrix elements experience distinct changes in their characteristics (not only an addition of stochastic noise)

  - e.g. the correlation function for $O^{Q\bar{Q}}$
\[
\begin{align*}
0 & \equiv \mathcal{O}q\bar{q}, \quad 1 \equiv \mathcal{O}\frac{K}{\bar{K}} \text{point},
2 & \equiv \mathcal{O}\eta_s\pi \text{point}, \quad 3 \equiv \mathcal{O}Q\bar{Q}, \\
4 & \equiv \mathcal{O}\frac{K}{\bar{K}} \text{2part}, \quad 5 \equiv \mathcal{O}\eta_s\pi \text{2part}.
\end{align*}
\]
Analysis

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\[ 0 \overset{\sim}{=} \mathcal{O}(q\bar{q}), \quad 1 \overset{\sim}{=} \mathcal{O}(K\bar{K}\text{ point}), \quad 2 \overset{\sim}{=} \mathcal{O}(\eta_s\pi\text{ point}), \quad 3 \overset{\sim}{=} \mathcal{O}(Q\bar{Q}), \]

\[ 4 \overset{\sim}{=} \mathcal{O}(K\bar{K}\text{ 2part}), \quad 5 \overset{\sim}{=} \mathcal{O}(\eta_s\pi\text{ 2part}) \]
## Analysis

| no loops [1,2,4,5] | [1,2,4,5] | [1,2,3,4,5] | [0,1,2,4,5] | [0,1,2,3,4,5] |
|---------------------|-----------|-------------|-------------|---------------|
| ![Graph](chart1.png) | ![Graph](chart2.png) | ![Graph](chart3.png) | ![Graph](chart4.png) | ![Graph](chart5.png) |

\[
0 \overset{\text{ }}{=} \mathcal{O}q\bar{q}, \quad 1 \overset{\text{ }}{=} \mathcal{O}K\bar{K}_{\text{point}}, \quad 2 \overset{\text{ }}{=} \mathcal{O}\eta_s\pi_{\text{point}}, \quad 3 \overset{\text{ }}{=} \mathcal{O}Q\bar{Q}, \quad 4 \overset{\text{ }}{=} \mathcal{O}K\bar{K}_{\text{2part}}, \quad 5 \overset{\text{ }}{=} \mathcal{O}\eta_s\pi_{\text{2part}}.
\]

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\[ 0 \approx \mathcal{O}q\bar{q}, \quad 1 \approx \mathcal{O}K\bar{K}_{\text{point}}, \quad 2 \approx \mathcal{O}\eta_s\pi_{\text{point}}, \quad 3 \approx \mathcal{O}Q\bar{Q}, \]
\[ 4 \approx \mathcal{O}K\bar{K}_{\text{2part}}, \quad 5 \approx \mathcal{O}\eta_s\pi_{\text{2part}}. \]
Summary

- Study of **effective masses** from mesonic two-quark and four-quark operators.

- Investigation of methods and combination of methods to find the **optimal strategy** to compute each diagram of the correlation matrix.

- Computation of closed fermion loops expensive, but **essential**

- Analysis of states around the two particle threshold reveals **evidence for an additional state** ($a_0$, likely of $q\bar{q}$ structure)