Suppression of microbunching instability with magnetic bunch length compression in a linac-based free electron laser

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(Received 7 October 2008; published 20 January 2010)

The aim of this paper is to investigate the effect of the magnetic bunch length compression on the development of the microbunching instability. It will be shown that, after removing the linear energy chirp required for the compression, an additional and properly tuned $R_{56}$ transport matrix element is able to dilute the initial energy modulation and to suppress the current spikes created by the microbunching instability without affecting the bunch length. A by-product of the study is the observation that a single compressor is more effective than the two-compressors scheme in reducing the unwanted modulations caused by the microbunching instability. The study is based on analytical calculations and on the simulation code ELEGANT.

DOI: 10.1103/PhysRevSTAB.13.010702 PACS numbers: 29.27.Bd, 29.27.Eg, 29.27.Fh, 41.60.Cr

I. INTRODUCTION

Linacs for the free electron laser (FEL) typically involve at least two magnetic chicanes, here called BC1 and BC2, for the bunch length compression; this scheme provides the flexibility needed to manipulate the current profile in order to satisfy the requirement of a high FEL power [1–3]. Unfortunately, this scheme also works like a powerful amplifier of any small initial energy or density modulation, thus driving the so-called microbunching instability [4–7].

In fact, an energy modulation induced by longitudinal space charge (LSC) [8–11] upstream of the compressor is converted by the chicane dispersive path into density modulation. This mechanism is further amplified by the LSC acting between BC1 and BC2 and by the dispersion function in BC2. Coherent synchrotron radiation (CSR) [12–15], emitted in the BC1 and BC2 bending magnets, also vehicles the conversion of energy into density modulation, thus strengthening the instability. The growth of final energy and density modulations may have a damaging impact on the FEL performance.

This paper explores the impact of magnetic compression on the development of the microbunching instability. It is organized as follows. Section II provides the background and motivations that led to the subsequent study. Section III is an analytical description of the physics of microbunching. Suppression of the instability by particle longitudinal phase mixing is investigated for several bunching amplitudes and wavelengths. Section IV draws on the results of Sec. III to show that a single compressor is superior to a double compressor as far as the microbunching instability is concerned. In Sec. V we show that the performance of a single compressor can be improved with a second chicane. This chicane does not add any further compression, but it is effective in damping the microbunching instability.

II. STUDY BACKGROUND

Let us start with a conventional double compression scheme; BC1 and BC2 contribute with their nominal compression factors to the total bunch length reduction. If the uncorrelated energy spread is included in the physics, the larger the negative $R_{56}$ in BC1 is, the bigger the uncorrelated energy spread out of the chicane will be by virtue of the preservation of the longitudinal emittance, the smaller the compression factor in BC2 should be to maintain the nominal total compression action. If the particle longitudinal crossover in BC1 is efficient enough, it is expected that no density clusters be enhanced by the dispersive motion in the chicane. Unfortunately, the above statement becomes less and less valid the longer the wavelengths in question.

Basing on these considerations, the succeeding sections will show that two schemes can alternatively be adopted to smear an initial density modulation. The first is the single compression scheme. In the second, BC2 is reintroduced in the layout but the energy chirp required by the compression in BC1 has to be removed at the entrance of BC2. This approach requires that the sign of $R_{56}$ be equal in BC1 and in BC2. In practice, the scheme evolves towards a single compression in which the final bunch length is determined by BC1 only, while the modulation washing out is made even more effective by BC2.

III. LONGITUDINAL PHASE MIXING

A. Analytical model

We want to calculate the bunching factor at the exit of the first stage of compression. We will show that, for the same compression factor, the bunching reduction depends on the degree of rotation of the longitudinal phase space or, in other words, on the strength of the particle longitudinal phase mixing in the first chicane.
The analytical model described in [5] is here considered for the introduction of the LSC instability in the discussion. When the beam exits the photoinjector, the space-charge oscillation period is typically of the order of meters [7] and any beam density modulation is practically frozen. Thus, without loss of generality, the microbunching instability is assumed to start at the photoinjector exit from a pure density modulation (caused by shot noise or unwanted modulation in the photoinjector laser temporal profile). As the beam travels along the linac to reach BC1, the density modulation leads to an energy modulation via the LSC wake. This is equal to the free space-charge wake for the wavelength of interest:

\[ \lambda_m \ll 2\pi \frac{d}{\gamma}, \]  

(1)

d being the transverse size of the vacuum chamber and \( \gamma \) the Lorentz factor. The expression for the LSC impedance is [7]

\[ Z(k) = \frac{iZ_0}{\pi k r^2_b} \left[ 1 - \frac{k r_b}{\gamma} K_1 \left( \frac{k r_b}{\gamma} \right) \right], \]  

(2)

where \( Z_0 = 377 \ \Omega \) is the free space impedance, \( r_b \) is the radius of the transverse cross section for a uniform distribution, and \( K_1 \) is the modified Bessel function. The current spectrum is characterized by a bunching factor [7]:

\[ b(k) = \frac{1}{N e c} \int I(z) e^{-i k z} d z, \]  

(3)

where \( N \) is the total number of electrons. \( b(k) \) couples with the LSC impedance along a path \( L \) to produce the energy modulation [7]:

\[ \Delta \gamma(k) = -\frac{I_0 b(k)}{I_\lambda} \int_0^L 4\pi Z(k, s) Z_0 d s, \]  

(4)

\( I_\lambda = 17 \ \text{kA} \) is the Alfvén current.

When the bunch length is compressed in an achromatic magnetic chicane characterized by an \( R_{56} \), the density modulation can be expressed through the bunching factor at the compressed wavelength [5]:

\[ b_1(k_1) = \left[ b_0(k_0) - i k_1 R_{56,1} \frac{\Delta \gamma(k_0)}{\gamma} \right] \times \exp \left[ -\frac{1}{2} \left( k_1 R_{56,1} \frac{\sigma_\gamma}{\gamma} \right)^2 \right], \]  

(5)

where \( k_1 = 2\pi/\lambda_1 = k_0/(1 + h R_{56,1}) \) is the wave number of the modulation after compression; it is equal to the initial wave number \( k_0 \) times the linear compression factor \( C = 1/(1 + h R_{56,1}) \), \( h \) being the linear energy chirp.

The result in (5) is valid for an initial Gaussian energy distribution. The exponential term in (5) shows that the particle longitudinal phase mixing can contribute to the suppression of the instability if the initial uncorrelated energy spread \( \sigma_\gamma/\gamma \) is larger than the energy modulation amplitude \( \Delta \gamma/\gamma \).

### B. Case study

To calculate the bunching factor at the exit of the first stage of compression, we first consider BC1 only with the nominal compression factor of 3.5. Then, we consider an additional positive \( R_{56} \) transport matrix element (called DC) immediately downstream of BC1. In the latter case, \( R_{56} \) in BC1 is stronger than in the former case but the total compression factor is reestablished by the DC element. The calculation is based on the parameters listed in Tables I and II.

Several studies [10,16,17] indicate that density modulations generated by shot noise or nonuniformities of the photocathode laser temporal profile are of the order of 0.01% at shorter wavelengths and up to 1% at longer wavelengths. Accordingly, two sinusoidal density modulations of amplitude 0.03% at 10 \( \mu \text{m} \) and 1% at 100 \( \mu \text{m} \) are considered. The density modulations are superimposed to the initial beam current profile shown in Fig. 1.

In order to make the analysis as realistic as possible, the expression of the bunching factor for a generic initial energy distribution is used [5]:

\[ b_1(k_1) = \left[ b_0(k_0) - i k_1 R_{56,1} \frac{\Delta \gamma(k_0)}{\gamma} \right] \int d \frac{\delta \gamma}{\gamma} V_0 \left( \frac{\delta \gamma}{\gamma} \right) \times \exp \left( -i k_1 R_{56,1} \frac{\delta \gamma}{\gamma} \right), \]  

(6)

\( V_0(\delta \gamma/\gamma) \) is the particle energy distribution function at the entrance of BC1; \( R_{56,1} \) is the BC1 transport matrix element. In the case study considered here, \( V_0(\delta \gamma/\gamma) \) is very close to the hyperbolic function described in Eq. (10) of [7].

| Parameter | Value | Units |
|-----------|-------|-------|
| Initial energy | 95 | MeV |
| Energy at BC1 and DC | 230 | MeV |
| Charge | 0.8 | nC |
| Initial bunch length, rms | 1.04 | mm |
| Initial peak current | 80 | A |
| Uncorrelated energy spread, rms | 10 | keV |

| Compression scheme | \( R_{56} \) [mm] | Compression factor |
|--------------------|----------------|-------------------|
| BC1 only | -31 | 3.5 |
| BC1 + DC | -33.8, +2.9 | 4.5, 1/1.3 |
The bunching factor after DC is

\[ b_2(k_z) = \left[ b_0(k_0) - i k_1 R_{56,1} \frac{\Delta \gamma(k_0)}{\gamma} \right] \int d \frac{\delta \gamma}{\gamma} V_0(\Delta \gamma) \]

\[ \times \exp \left( -i k_1 R_{56,1} \frac{\delta \gamma}{\gamma} + -i k_2 R_{56,2} \frac{\Delta \gamma(k_z)}{\gamma} \right) \]

\[ \times \int d \frac{\delta \gamma}{\gamma} V_1(\Delta \gamma) \exp \left( -i k_2 R_{56,2} \frac{\delta \gamma}{\gamma} \right) \]

where the suffix 2 refers to the DC element. According to (4), the energy modulation amplitude in front of BC1 is

\[ \Delta \gamma(k_0) = -\frac{I_0 b_0(k_0)}{I_A} \int_0^{BC1} \frac{4 \pi Z(k_0, s)}{Z_0} ds \]

while that in front of DC is

\[ \Delta \gamma(k_1) = -\frac{I_0 b_1(k_1)}{I_A} \int_{BC1}^{DC} \frac{4 \pi Z(k_f, s)}{Z_0} ds \].

It should be noticed that, unlike in the high gain approximation assumed in [4], in this case (7) must carry all terms of the equation; more explicitly, the bunching term at the entrance of DC (term in square brackets times the integral of \( V_0 \)) is comparable to the DC chicane contribution (term with \( R_{56,2} = 2.9 \text{ mm} \)).

C. Quantitative results

Table III shows the bunching factor calculated with (6) and (7), respectively, at the end of compression in the BC1 only and in the BC1 + DC scheme. The total compression factor is 3.5 in both cases. As expected, the DC option reduces the bunching by a factor 4.5 at 10 \( \mu \text{m} \), while it is almost inefficient at 100 \( \mu \text{m} \) because the longitudinal phase mixing is no longer effective at such longer wavelength.

| Initial density modulation | BC1 only | BC1 + DC |
|---------------------------|---------|----------|
| 0.03% at 10 \( \mu \text{m} \) | \( 9 \times 10^{-3} \) | \( 2 \times 10^{-3} \) |
| 1% at 100 \( \mu \text{m} \) | 0.146 | 0.188 |
| 0.03% at 100 \( \mu \text{m} \) | \( 8 \times 10^{-3} \) | \( 11 \times 10^{-3} \) |

As an additional check, an initial density modulation of 0.03% at 100 \( \mu \text{m} \) was studied. The final bunching is still not affected by the DC option, so demonstrating that the effect of phase mixing is more sensitive to the wavelength than to the modulation amplitude.

The implications of this section lead one to consider the advantages of a single compressor over the one with two compressors. This is discussed in the next section.

IV. ADVANTAGES OF A SINGLE COMPRESSION SCHEME

On the basis of the results in Table III, one might think of further increasing the positive \( R_{56} \) in DC while reducing the negative \( R_{56} \) in BC1; in this way the total compression factor is kept constant and the longitudinal phase mixing becomes effective even at longer wavelengths. As a matter of fact, this scheme is equivalent to a two-stage compressor with unequal weight between BC1 and BC2: the stronger the first chicane is, the more effective the instability suppression becomes. As a limiting case, the single compression scheme optimizes the suppression of the instability with respect to the double compression for two reasons: first, the phase mixing is more effective in BC1 only due to the larger \( R_{56} \) than in the BC1 + BC2 scheme. Second, the absence of the high energy compressor (BC2) does not provide the opportunity to transform the energy modulation accumulated by LSC downstream of BC1 into a current modulation.

These considerations are of general validity in the field of high energy linac-based FELs, but for a quantitative discussion we will focus on the FERMI@elettra project [1]. Some positive aspects of the single compression scheme have already been recognized for FERMI@elettra and reported in [18,19].

A single compression minimizes the amount of uncorrelated energy spread required to suppress the microbunching instability. This energy spread is obtained by a laser heater system [4]. The single compression scheme allows FERMI to operate with a beam heating of 10 keV rms, instead of 15 keV rms required by the case of double compression. The final slice energy spread is 120 and 180 keV rms in the two cases, respectively [19]. Owing to the sensitivity of harmonic cascade FELs to this parameter [20]—and of FERMI in particular [21,22]—it is important to reduce the final energy spread to the minimum allowed by the microbunching instability.
It was demonstrated that the instability gain in FERMI@elettra is of the order of unity if driven by CSR only, while it grows by 2 orders of magnitude when LSC is included [23]. This means that, in practice, the contribution of CSR to the microbunching instability is marginal for an initial bunch like the one described in Table IV and for a total compression factor equal or smaller than 10. In spite of the great advantage of suppressing the microbunching instability with minimum energy spread, the shortcoming of a single compression is that a short bunch is affected by strong longitudinal wakefield along a longer path than in the two-stage option, where the path to a short final bunch proceeds in two stages. Wakefields corrupt the longitudinal phase space by increasing the energy spread, by reducing the average beam energy, and by inducing nonlinearities in the energy distribution. A manipulated current profile, already shown in Fig. 1, was successfully studied to overcome this problem [24].

To demonstrate the attraction of the single compression scheme for a realistic model of linac-based FEL, the 3D tracking of a $5 \times 10^6$ particle file is here shown according to the parameters in Table IV. Figures 2 and 3 depict the properties of the FERMI@elettra bunch, compressed by a factor 10 in BC1 ($R_{56} = -46 \text{ mm}$). The ELEGANT code [25] was used that adopts a 1D CSR and LSC impedance model; more details about 3D simulations can be found in [26]. An initial density modulation of 0.03% at 30 $\mu$m and 10 keV rms uncorrelated energy spread was added to the initial beam. The modulation is washed out at the linac end: the bunching factor, calculated as in (3) for initial (uncompressed) and final (compressed) wavelength of 30 $\mu$m and 3 $\mu$m, respectively, reduces from $4 \times 10^{-2}$ before compression to $1 \times 10^{-3}$ after compression.

### V. SINGLE COMPRESSION WITH ENHANCED LONGITUDINAL PHASE MIXING

#### A. 1D linear mixing

This section purports to show an alternative scheme that is even more effective than a pure single compressor in suppressing the beam microbunching. It requires that the
correlated energy spread, needed for bunch length compression in BC1, be removed before passing through a second chicane. When this happens, the rotation of the longitudinal phase space can be made large enough so that phase mixing is maximized in the 2nd chicane. At the same time, the 2nd chicane does not change the overall bunch length that therefore depends on the $R_{56}$ in BC1 only. Its only task is to smear the microbunching modulations.

In this case the energy and density modulation washing out is more efficiently provided by two magnetic chicanes having $R_{56}$ of the same sign. In fact, the energy modulation smearing is induced by a complete rotation of the longitudinal phase space; the two chicanes must therefore stretch the particles in the same direction.

To illustrate this process, we consider a line charge that has a Gaussian energy distribution with $\langle E \rangle = 230$ MeV and $\sigma_E = 10$ keV. A linear energy chirp $\frac{dE}{Edt} = 0.036$ ps$^{-1}$

![Image](image.png)

**FIG. 4.** (Color) 1D linear tracking of the longitudinal phase space of a bunch portion before (left) and after (right) the linear energy chirp removal.

**FIG. 5.** (Color) 1D linear tracking of the longitudinal phase space. (a) An energy modulation was superimposed to the linear energy chirp entering BC1; 10 keV rms uncorrelated energy spread is present. (b) As a result of the bunch length compression (by a factor of 10) in BC1, the longitudinal phase space apparently rotates. (c) The longitudinal phase space is flattened by removing the linear energy chirp. (d) A further “rotation” allows the particles to longitudinally crossover.
and an energy modulation with amplitude $A_E = 1\%$ and wavelength $\lambda_E = 30 \, \mu m$ are superimposed to it. Linear transport matrix formalism is used to propagate the line charge through drift sections and $R_{56}$ elements.

Figure 4 shows that, after the linear energy chirp is removed, the residual energy chirp changes sign over one modulation period, so that particles lying on opposite fronts of the modulation can be (de)compressed by the same factor: the longitudinal phase space becomes folded and the initial energy modulation is removed, turning into an almost totally uncorrelated energy spread, as shown in Fig. 5. At the same time, the particle crossover in the $z$ coordinate damps the initial current spikes, therefore suppressing the microbunching instability.

Figure 6 shows the instability gain function (the ratio between the final and the initial bunching factor) for the single compression scheme (where $R_{56} = 0$ in BC2) and for the enhanced phase mixing (where $R_{56} = -30 \, \text{mm}$ in BC2). The FERMI@elettra layout is considered since it usefully includes two magnetic chicanes, BC1 at 230 MeV and BC2 at 570 MeV, separated by a 30 m long S-band linac. By virtue of the off-crest phasing of this linac, the energy chirp required for compression in BC1 is removed before the beam enters BC2. In the latter case, the gain is clearly reduced and it practically goes to zero for initial wavelengths shorter than $100 \, \mu m$.

**B. 3D tracking with collective effects**

Particle tracking was carried out with ELEGANT in order to obtain a complete and realistic picture of the dynamics discussed so far, including LSC, CSR, and longitudinal structural wakefields, and to support the analytical result in

![Image of Figure 6](image_url)

**FIG. 6.** (Color) Instability gain vs compressed modulation wavelength. The dotted line is for the single compression scheme. The solid line is for the linear energy chirp removed at BC2; this is made active only for additional rotation of the longitudinal phase space.

![Image of Figure 7](image_url)

**FIG. 7.** Longitudinal phase space (top) and current profile (bottom) of the bunch core at the entrance of BC2.

Fig. 6. The FERMI linac rf phasing was readjusted to cancel the linear chirp at BC2. Figures 7 and 8 show a portion of the bunch core at the entrance and at the exit of BC2 with $R_{56} = -30 \, \text{mm}$, respectively. At the end, the longitudinal phase space becomes folded, the energy spread is uncorrelated, for any practical purposes, on the slice scale of microns, and the charge clusters are largely suppressed.

To make the dynamics more evident, initial modulation amplitude of 1% was introduced at 30 $\mu m$ wavelength, corresponding to an initial bunching factor of $7 \times 10^{-2}$. After BC2, the bunching factor calculated for 3 $\mu m$ wavelength shrinks to approximately $3 \times 10^{-5}$.

![Image of Figure 8](image_url)

**FIG. 8.** Longitudinal phase space (top) and current profile (bottom) of the bunch core downstream of BC2 with $R_{56} = -30 \, \text{mm}$.
In addition to this, a different promising configuration shows up. If the linear energy chirp at the exit of BC1 could be removed by dedicated accelerating structures, then particle longitudinal crossover is enhanced in BC2. The effect is optimized by two chicanes, BC1 and BC2, with the same sign of $R_{56}$. As a result, the energy modulation is damped, together with the associated current spikes.

The FERMI@elettra case study has been analyzed in some details. The 1D beam transport was verified against the 3D particle tracking performed with ELEGANT, including collective effects, for a compression factor of 10 in BC1. The study shows that this alternative scheme of compression is successful and even more efficient than the single compression for initial modulations whose wavelength is of the order of tens of $\mu$m. It may even be applicable to longer wavelengths and/or higher compression factors, although these more extreme situations need to be confirmed by further investigations.

VI. CONCLUSIONS

The effect of magnetic bunch length compression on the microbunching instability development in a linac-based FEL has been investigated. In particular, the reduction of the instability gain by particle longitudinal phase mixing has been demonstrated analytically. It has been shown that the efficiency in removing the beam microbunching is much more sensitive to the initial modulation wavelength than to the amplitude.

The natural consequence of this dynamics is the adoption of a single compression scheme. The feasibility of this scheme has been demonstrated by means of a 3D particle tracking for the FERMI@elettra FEL, with promising results for the preservation of the beam quality.

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