Evaluation and Decision-Making Method of Power Distribution Network Projects Based on Weighting with Game Theory and Implicit Enumeration

Lei Chen¹, Jing Jiang², Qiang Zhang³*, Lin Zhang⁴ and Hong Liu³
¹ State Grid Henan Jiyuan Power Supply Company, Jiyuan, Henan, 454650, China
² Economic and Technological Research Institute of State Grid Henan Electric Power Company, Zhengzhou, Henan, 450018, China
³ Key Laboratory of Smart Grid of Ministry of Education (Tianjin University), Tianjin, 300072, China
*E-mail: zhangqiang@tju.edu.cn

Abstract. Aiming at the problems of one-sided determination of index weight in the evaluation method of distribution network projects, and instability of project optimization results, a method of evaluation and decision-making of distribution network construction and renovation projects based on weighting with game theory and implicit enumeration is proposed. Firstly, the evaluation index system is constructed from two dimensions of technical efficiency and economic benefit. Secondly, the indicators are weighted by the weight set model based on game theory. After that, the decision optimization model is solved by the implicit enumeration method. The case study shows that the method is practical and effective, and can provide a scientific and reasonable reference for the construction and renovation projects investment of grid companies.

1. Introduction
Power distribution network is an important infrastructure to ensure the smooth progress of people's production and life. In the "Action Plan for Power Distribution Network Construction and Reform (2015-2020)" issued by the National Energy Administration, it is pointed out that more than 2 trillion yuan will be invested in power distribution network[1]. The distribution network construction and renovation projects involve multiple levels of high, medium and low voltage power, covering a wide range and a large amount. How to evaluate the projects effectively, and how to carry out the projects under the condition of limited funds, so as to maximize the reliability and quality of power supply and to realize the development of distribution network scientifically and orderly, all of which are important problems that the power grid enterprise needs to solve.

In recent years, some experts and scholars have put forward some feasible methods for scientific evaluation and decision-making of distribution network construction and renovation projects[2-6]. The main idea is to establish a distribution system project evaluation index system first, then to set the weight of the indicators, make comprehensive evaluation on the projects, and finally to establish an optimization model and obtain the solution by solving the preferred model. The methods to determine the weight of indicators mainly include subjective weighting method and objective weighting method, such as the Analytic Hierarchy Process adopted in literature[2], Delphi method in literature[3], all of which are
subjective weighting methods. The objective weighting method includes Principal Component Analysis, Grey Relational Analysis, etc. For example, literature[4] uses the Principal Component Analysis method, and literature[5] uses the Grey Relational Analysis method to calculate the index weight. Both subjective and objective weighting methods have their own advantages, but they also have disadvantages. Therefore, in order to avoid the one-sidedness of a single weighting method, some scholars have proposed the ‘combined’ weighting method[6, 7]. Compared with a single weighting method, the results of the combined weighting method are more stable. For example, literature[6] adopts the order relation method and Gini coefficient method to calculate the combined weight. Literature[7] proposed an AHP-entropy weight method to establish the combined weight.

In order to make up for the above work, this paper proposes a distribution network project evaluation and decision-making method based on weighting with game theory and implicit enumeration method. Firstly, the evaluation index system is constructed from two aspects of technical efficiency and economic benefits. Secondly, a weight determination model based on game theory is proposed. Then a decision optimization model is established and solved by the implicit enumeration method. Finally, an example is given to verify the rationality and effectiveness of the proposed method.

2. Construction of evaluation system for the distribution network projects

2.1. The basis for distribution network construction and renovation projects classification

By referring to the classification in the “13th Five-Year” distribution network rolling planning outlines of the State Grid Corporation, and from the aspects of engineering characteristics, engineering functions, and the relationship with the upper level power grid, the project units are divided into the following ten categories according to the attributes:

(1) Meet the new load requirements; (2) Resolve equipment overload; (3) Solve the low voltage station area problem; (4) Substation supporting delivery; (5) Solve the "bottleneck" problem; (6) Eliminate equipment safety hazards; (7) Strengthen the structure of the grid; (8) Distributed power access; (9) Renovation of high-loss distribution transformer; (10) Others.

The key point to evaluate the distribution network projects is whether the evaluation index system can comprehensively cover the influencing factors of the construction and renovation benefits, and can evaluate the projects by the available basic data.

Based on the basic principles of comprehensiveness, hierarchy, practicability, reliability, independence and openness, this paper constructs the index system of the projects from two aspects of technical efficiency and economic benefit, as shown in Table 1.

3. Index Weight determination based on game theory

3.1. The basic idea of using game theory to determine index weight

Whether the evaluation method is subjective or objective, it is difficult to avoid the one-sidedness and arbitrariness of a single weighting method in determining the index weight. Literature[8] proposed a combined weighting evaluation method based on game theory, which is introduced into the research field of comprehensive evaluation. The basic idea of this method is to seek agreement or compromise between different weights obtained by a single weighting method, and to minimize the deviation between the comprehensive weight and the basic weight as the goal, which can be classified as the multiplayer optimization problem in game theory.

3.2. The step for finding the coefficient weight

**Step 1** Construct the basic weight set and the possible weight set[8]. Set the number of indicators as $m$, use $N$ methods to weight the indicators, and get the index weight vector:

$$w_i = (w_{i1}, w_{i2}, \cdots, w_{im}), \quad i = 1, 2, 3, \cdots, N$$

(1)
Table 1. The evaluation index system of distribution network construction and renovation projects

| Level 1 indicators | Level 2 indicators | Level 3 indicators |
|--------------------|--------------------|--------------------|
| Grid structure     | B1                 | Improvement of overlimit rate of average power supply radius of 10kV line |
|                    |                    | Improvement of overlimit rate of average number of sections of 10kV overhead line |
| Technical efficiency | A1                | Improvement of standardized structure proportion of 10kV distribution network |
| Equipment level    | B2                 | Improvement of contact rate of 10kV line |
|                    |                    | Improvement of contact rate between stations at 10kV line |
|                    |                    | Improvement of N-1 pass rate of 10kV line |
| Power supply capability | B3               | Improvement of standardization rate of 10kV line section |
|                    |                    | Improvement of standardization rate of 10kV transformer capacity |
|                    |                    | Improvement of the over-limit rate of 10kV in the average operating life of the equipment |
| Equipment level    | B2                 | Improvement of 10kV line cable rate |
|                    |                    | Improvement of insulation rate of 10kV overhead line |
|                    |                    | Improvement of high loss distribution transformer proportion |
|                    |                    | Improvement of energy-saving distribution transformer proportion |
| Economic benefits  | A2                 | Improvement of 10kV line outlet interval utilization |
| Investment returns | B4                 | Improvement of 10kV line maximum load rate average |
| Financial Evaluation | B5             | Improvement of 10kV line load imbalance |
|                    |                    | Improvement of 10kV heavy load line proportion |
|                    |                    | Improvement of 10kV light load line proportion |
|                    |                    | Improvement of average improvement degree of maximum load rate of 10kV distribution transformer |
|                    |                    | Improvement of load unbalance degree of 10kV distribution transformer |
|                    |                    | Improvement of 10kV heavy load distribution transformer proportion |
|                    |                    | Improvement of 10kV light load distribution ratio improvement degree |

On this basis, the basic weight set \( \{w_1, w_2, \ldots, w_N\} \) is constructed from \( N \) index weight vectors, so that any linear combination of index weight vectors is:

\[
\mathbf{w} = \sum_{i=1}^{N} \alpha_i \mathbf{w}_i \quad (\alpha_i > 0, \sum_{i=1}^{N} \alpha_i = 1)
\]

\( \mathbf{w} \) is a possible weight vector, all of which forms a possible weight vector set \( \{\mathbf{w} | \mathbf{w} = \sum_{i=1}^{N} \alpha_i \mathbf{w}_i\} \).
Step 2 Determine the countermeasure model. Find the most satisfactory weight vector, that is, an optimization result of the $N$ linear combination coefficients $\alpha_i$ in equation (2). The goal is to minimize the dispersion, so the countermeasure model is:

$$\min \left\| \sum_{j=1}^{N} \alpha_j \mathbf{w}_j - \mathbf{w}_i^* \right\|_2, \quad i = 1, 2, \ldots, N$$ (3)

Step 3 Find the conditions to meet the optimal solution of the countermeasure model. According to the differential properties of the matrix, the first derivative condition when the countermeasure model obtains the optimal solution is analysed as follows:

$$\sum_{j=1}^{N} \alpha_j \mathbf{w}_j \mathbf{w}_j^T = \mathbf{w}_i \mathbf{w}_i^T, \quad i = 1, 2, \ldots, N$$ (4)

This is the following linear equation:

$$\begin{bmatrix} \mathbf{w}_1 \mathbf{w}_1^T & \mathbf{w}_1 \mathbf{w}_2^T & \cdots & \mathbf{w}_1 \mathbf{w}_N^T \\ \mathbf{w}_2 \mathbf{w}_1^T & \mathbf{w}_2 \mathbf{w}_2^T & \cdots & \mathbf{w}_2 \mathbf{w}_N^T \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{w}_N \mathbf{w}_1^T & \mathbf{w}_N \mathbf{w}_2^T & \cdots & \mathbf{w}_N \mathbf{w}_N^T \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{bmatrix} = \begin{bmatrix} \mathbf{w}_1 \mathbf{w}_1^T \\ \mathbf{w}_2 \mathbf{w}_2^T \\ \vdots \\ \mathbf{w}_N \mathbf{w}_N^T \end{bmatrix}$$ (5)

Step 4 Find the most satisfactory weight vector. In this paper, the improved comprehensive weight model of game theory proposed in literature[10] is adopted. After taking the absolute value, the normalization processing is performed:

$$\alpha_i^* = \frac{\alpha_i}{\sum_{k=1}^{N} |\alpha_k|}$$ (6)

Finally, the most satisfactory weight vector $\mathbf{w}^*$ is obtained:

$$\mathbf{w}^* = \sum_{i=1}^{N} \alpha_i^* \mathbf{w}_i$$ (7)

3.3. Evaluation model

$$P = \sum_{i=1}^{m} \sum_{j=1}^{N} u_{ij} \mathbf{w}^*$$ (8)

$P$ is the comprehensive score of the construction and renovation projects of a single distribution network, and $u_{ij}$ is the score of the specific indicator corresponding to the project, and $\mathbf{w}^*$ is the vector of the comprehensive weight.

4. Establishment and solution of decision-making optimization model

4.1. Decision optimization model

$$\max z = \sum_{k=1}^{n} \sum_{i=1}^{m_k} x_{ki} R \left( P_{ki} \right)$$ (9)
\[
\sum_{k=1}^{n} \sum_{i=1}^{m_k} x_{ki} C(P_{ki}) \leq C_{total} \\
\text{s.t.} \quad \sum_{i=1}^{m_k} x_{ki} C(P_{ki}) \leq C_k \quad (k = 1, 2, \ldots, n) \\
\delta_1 : \delta_2 : \cdots : \delta_n = C_1 : C_2 : \cdots : C_n \\
x_{ki} = 0 \text{ or } 1 \quad (i = 1, 2, \ldots, m_k)
\]

\( z \) is the objective function of the decision-making optimization model, where \( n \) is the total number of categories of the declared project, and \( m_k \) is the total number of declared items in the \( k \)-th category. \( R(P_{ki}) \) is the evaluation score of the \( i \)-th declared item in the \( k \)-th category, and \( x_{ki} \) is the selected variable of the \( i \)-th declaration in the \( k \)-th category. \( C(P_{ki}) \) is the investment quantity of the \( i \)-th declared project in the \( k \)-th category, \( C_{total} \) is the total investment constraint, \( C_k \) is the investment constraint of the \( k \)-th category, and \( \delta_k \) is the constraint of the allocation proportion of the project investment.

4.2. To solve the problem with implicit enumeration method

Implicit enumeration method is a simple and fast method to solve 0-1 integer programming problem[9]. The problem involved in this paper is not a simple 0-1 integer programming problem, so the implicit enumeration method is used twice to solve the problem. In order to meet the requirements of project investment allocation proportion, the first round of solution using implicit enumeration often leads to surplus in the investment budget. If the total surplus is more than the minimum project declaration amount, the optimization process should be carried on. The final project optimization set is obtained through the second round solution. Although the final result may not strictly meet the investment allocation ratio constraint, the economics of the overall investment could be guaranteed.

When the number of decision variables increases, the time complexity of implicit enumeration will increase correspondingly. Compared with genetic algorithm (GA), the optimal result of genetic algorithm is not stable enough to obtain the global optimal result, but by using the implicit enumeration method, the stable optimal result could be obtained.

5. Case study

5.1. Determine the weight of the index system

Taking the distribution network investment project of a certain region in J city, H province in 2018 as an example, the final result of the optimal project in this region is determined according to the total project investment, the investment allocation proportion of various projects, the evaluation score of each declared project and its investment amount. In this paper, the Delphi method[2] (Method1), principal component analysis method[7] (Method 2), and improved grey correlation method[8] (Method 3) are respectively used to empower the three-level indicators in the index system of the distribution network construction and renovation projects. The comprehensive weight of the three-level index system is calculated by using the weighting method based on game theory, as shown in Table 2, where \( L \) is for Level, \( M \) is for Method and \( CW \) is for comprehensive weight. The comprehensive weight is selected to evaluate the projects.

5.2. Project evaluation and optimization results

In 2018, 443 investment projects of distribution network in a certain region in J city, H province were declared. The total project budget is 100 million yuan, the investment ratio is 4.6:0.3:0.2:2.8:0.6:0.3:1.1, and the amount of the distributed power access projects and the high-loss distribution transformer are 0.
Table 2. The weight indicators of J city H province evaluation index system of distribution network construction and renovation projects

| L1 | M1 | M2 | M3 | CW | L2 | M1 | M2 | M3 | CW | L3 | M1 | M2 | M3 | CW |
|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
|    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
| B1 | 0.3377 | 0.3450 | 0.3341 | 0.3364 |
| C1 | 0.1646 | 0.1656 | 0.1666 | 0.1651 |
| C2 | 0.1620 | 0.1618 | 0.1665 | 0.1619 |
| C3 | 0.1604 | 0.1603 | 0.1662 | 0.1604 |
| C4 | 0.1660 | 0.1654 | 0.1666 | 0.1657 |
| C5 | 0.1673 | 0.1669 | 0.1688 | 0.1671 |
| C6 | 0.1797 | 0.1799 | 0.1673 | 0.1797 |
| C7 | 0.1416 | 0.1423 | 0.1429 | 0.1421 |
| C8 | 0.1400 | 0.1409 | 0.1428 | 0.1408 |
| C9 | 0.1518 | 0.1560 | 0.1431 | 0.1526 |
| B2 | 0.3243 | 0.2592 | 0.3317 | 0.3243 |
| C10 | 0.1408 | 0.1384 | 0.1428 | 0.1399 |
| C11 | 0.1377 | 0.1338 | 0.1426 | 0.1365 |
| C12 | 0.1474 | 0.1460 | 0.1430 | 0.1461 |
| C13 | 0.1408 | 0.1425 | 0.1428 | 0.1419 |
| C14 | 0.1030 | 0.1035 | 0.1109 | 0.1049 |
| C15 | 0.1096 | 0.1099 | 0.1111 | 0.1101 |
| C16 | 0.1114 | 0.1105 | 0.1111 | 0.1106 |
| C17 | 0.1157 | 0.1168 | 0.1112 | 0.1157 |
| B3 | 0.3380 | 0.3958 | 0.3342 | 0.3392 |
| C18 | 0.1145 | 0.1164 | 0.1112 | 0.1154 |
| C19 | 0.1096 | 0.1065 | 0.1111 | 0.1074 |
| C20 | 0.1102 | 0.1086 | 0.1111 | 0.1091 |
| C21 | 0.1133 | 0.1138 | 0.1112 | 0.1133 |
| C22 | 0.1127 | 0.1138 | 0.1112 | 0.1133 |
| B4 | 0.5155 | 0.5398 | 0.5071 | 0.5155 |
| C23 | 0.5007 | 0.5133 | 0.5000 | 0.5007 |
| C24 | 0.4993 | 0.4867 | 0.5000 | 0.4993 |
| A2 | 0.4455 | 0.4111 | 0.4373 | 0.4373 |
| C25 | 0.3233 | 0.2855 | 0.3313 | 0.3215 |
| C26 | 0.3440 | 0.3600 | 0.3353 | 0.3427 |
| C27 | 0.3326 | 0.3544 | 0.3334 | 0.3357 |

Table 3. The optimization results of J city H province distribution network projects

| Project categories | Number of declared projects | Number of projects selected for the 1st round | Investment selected for the 1st round | Number of projects selected for the 2nd round | Investment selected for the 2nd round | Total number of projects selected | Total investment |
|--------------------|----------------------------|---------------------------------|-------------------------------|---------------------------------|-------------------------------|-------------------------------|----------------|
| 1. New load        | 363                        | 302                             | 4579.99                       | 4                               | 186.00                        | 306                           | 4765.99         |
| 2. Substation      | 6                          | 4                               | 291.21                        | 0                               | 0                             | 4                             | 291.21           |
| supporting         |                            |                                 |                               |                                 |                               |                               |                 |
| 3. Low voltage     | 5                          | 2                               | 174.39                        | 0                               | 0                             | 2                             | 174.39           |
| station area       |                            |                                 |                               |                                 |                               |                               |                 |
| 4. "Bottleneck"    | 34                         | 16                              | 2830.26                       | 0                               | 0                             | 16                            | 2830.26          |
| problem            |                            |                                 |                               |                                 |                               |                               |                 |
| 5. Equipment       | 12                         | 9                               | 530.00                        | 0                               | 0                             | 9                             | 530.00           |
| overload            |                            |                                 |                               |                                 |                               |                               |                 |
| 6. Safety hazards  | 6                          | 4                               | 228.20                        | 0                               | 0                             | 4                             | 228.20           |
| 7. Strengthen the   | 17                         | 10                              | 1138.10                       | 1                               | 41.25                         | 11                            | 1179.35          |
| structure           |                            |                                 |                               |                                 |                               |                               |                 |
| Total              | 443                        | 347                             | 9772.15                       | 5                               | 227.25                        | 352                           | 9999.40          |
As shown in Table 3, the implicit enumeration method is used twice to find the optimal item set. After the second round optimization, a total of 352 optimal projects were selected, with a total investment of 99.994 million yuan, and the remaining 0.60 million yuan could not reach the minimum project investment.

6. Conclusion
This paper constructs an index system of distribution network construction and renovation projects from two aspects of technical efficiency and economic benefits, and puts forward an evaluation and optimization decision-making method of distribution network project based on weighting with game theory and implicit enumeration method. This method applies game theory to calculate the index weight, effectively avoiding the one-sidedness caused by a single subjective or objective weighting method. In addition, the optimization result to meet the classification properties of the projects is realized by using implicit enumeration method, which ensures the maximum investment efficiency of the projects. This method provides a scientific theoretical basis for the optimization decision of distribution network projects of power grid companies, and holds the accuracy and rationality of investment distribution from the overall perspective, and improves the technical and economic benefits of power grid companies.

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