Global Fits of the CKM Matrix

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Abstract. We report upon the present status of global fits to Cabibbo-Kobayashi-Maskawa matrix.

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1 Introduction

The three-family Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix is a key element of the Standard Model (SM). The nine complex CKM elements are completely specified by three mixing angles and one phase that is responsible for CP violation in the SM. Measuring the CKM matrix elements in various ways provides consistency tests of the matrix elements itself and with unitarity. Any significant inconsistency with the SM would indicate the CKM dependence expressed as functions of Wolfenstein parameters. As an example, consider the charmless semileptonic branching fraction for $B \to \rho \ell \nu$, which is predicted to be $\mathcal{B}(B \to \rho \ell \nu) = |V_{ub}|^2 \cdot \Gamma_{\rho \ell \nu} \cdot \tau_B$, where $\tau_B$ is the $B^0$ lifetime and $\Gamma_{\rho \ell \nu}$ is the reduced rate affected by non-gaussian uncertainties. This analysis treats eleven theoretical parameters with non-gaussian errors, the reduced inclusive semileptonic rates $\tilde{\Gamma}_{X_s \ell \nu}$ and $\tilde{\Gamma}_{X_d \ell \nu}$, the form factor for $B \to D^* \ell \nu$ at zero recoil, $\mathcal{F}_D(1)$, the bag factors of the $K^0$ and $B^0$ systems, $B_K$ and $B_B$, the $B^0$ decay constant $f_B$, $\xi^2 = f_B^2 / f_{B_s}^2 B_{B_s} / B_{B_d}$ and the QCD parameters $\eta_1, \eta_2, \eta_3$ and $\eta_B$.

We perform a $\chi^2$ minimization based on a frequentist approach by selecting a specific value for each $T_i$ within the allowed range (called a model). We perform individual fits for many models scanning over the allowed non-gaussian ranges of the $T_i$ parameter space. The QCD parameters are not scanned; their small errors are treated in the $\chi^2$ as gaussian. For theoretical quantities calculated on the lattice, which have gaussian errors (e.g., the masses of the $t$-quark, $c$-quark, and $W$-boson), we add specific systematic errors to the predicted values.

2 The Scan Method

The scan method is an unbiased procedure for extracting $A, \rho, \eta$ from measurements. We select observables that allow us to factorize their predictions in terms of theoretical quantities $T_i$ that have an \textit{a priori} unknown (and likely non-gaussian) error distribution ($\Delta_i$), other observables, and the CKM dependence expressed as functions of Wolfenstein parameters. As an example, consider the charmless semileptonic branching fraction for $B \to \rho \ell \nu$, which is predicted to be $\mathcal{B}(B \to \rho \ell \nu) = |V_{ub}|^2 \cdot \Gamma_{\rho \ell \nu} \cdot \tau_B$, where $\tau_B$ is the $B^0$ lifetime and $\Gamma_{\rho \ell \nu}$ is the reduced rate affected by non-gaussian uncertainties. This analysis treats eleven theoretical parameters with non-gaussian errors, the reduced inclusive semileptonic rates $\tilde{\Gamma}_{X_s \ell \nu}$ and $\tilde{\Gamma}_{X_d \ell \nu}$, the form factor for $B \to D^* \ell \nu$ at zero recoil, $\mathcal{F}_D(1)$, the bag factors of the $K^0$ and $B^0$ systems, $B_K$ and $B_B$, the $B^0$ decay constant $f_B$, $\xi^2 = f_B^2 / f_{B_s}^2 B_{B_s} / B_{B_d}$ and the QCD parameters $\eta_1, \eta_2, \eta_3$ and $\eta_B$.

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We consider a model to be consistent with the data if the fit probability yields $P(\chi^2) > 5\%$. We determine the best estimate for each of the 17 fit parameters and plot a 95% confidence level (C.L.) contour in the $\rho - \eta$ plane. We overlay the $\rho - \eta$ contours of all accepted fits. In order to study correlations among the $T_i$ and constraints the data impose we perform global fits with non-gaussian theory errors scanned over a $\pm 5 \Delta$ wide range (see section 5).

\[ V = \begin{pmatrix} 1 - \frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4), \]
function, we use a new approach that is based upon the
\[ \Delta m = S \chi \Delta m \]

2.1 Treatment of \( \Delta m_{B_s} \)

Since \( B_s^0 \bar{B}_s^0 \) oscillations have not been observed yet, a lower limit on \( \Delta m_{B_s} \) at 95% C.L. has been determined by combining analyses of different experiments using the amplitude method \( [5] \). To incorporate \( \Delta m_{B_s} \) into the \( \chi^2 \) function, we use a new approach that is based upon the significance of a \( \Delta m_{B_s} \) measurement \( [9] \):

\[
S = \sqrt{N} \frac{f_{B_s}(1-2w)e^{-\frac{1}{2}(\Delta m, \sigma_i)^2}}{2},
\]

where \( N \) is the sample size, \( f_{B_s} \) is the \( B_s \) purity, \( w \) is the mistag fraction, and \( \sigma_i \) is the resolution. Substituting

\[
C \sqrt{\frac{1}{2} f_{B_s}(1-2w)}
\]

and interpreting \( S \) as the number of standard deviations by which \( \Delta m_{B_s} \) differs from zero, \( S = \Delta m_{B_s}/\sigma_{\Delta m_{B_s}} \), we may define a contribution to the \( \chi^2 \) from the \( \Delta m_{B_s} \) measurements as:

\[
\chi^2_{\Delta m_{B_s}} = C^2 \left( 1 - \frac{\Delta}{\Delta m_{B_s}} \right) e^{-(\Delta m_{B_s}, \sigma_i)^2},
\]

where \( \Delta \) is the best estimate according to experiment. The values of \( (\Delta, C^2, \sigma_i) \) are chosen to give a minimum at 17 ps\(^{-1} \), and a \( P(\chi^2) = 5\% \) at \( \Delta m_{B_s} = 14.4 \) ps\(^{-1} \). In the region of small \( \chi^2 \), this function exhibits similar general features as that used in our previous global fits \( [10] \), while it does not suffer from numerical instabilities arising from multiple minima. The two functions deviate at large values of \( \chi^2 \), where in any case poor fits result.

### Table 1. Measurement inputs used in \( \chi^2 \) minimization

| Observable | Value | Comment |
|------------|-------|---------|
| \( B(B \to X_s \ell \nu) \) | \( (2.03 \pm 0.22_{\text{exp}} \pm 0.31_{\text{bkg}}) \times 10^{-3} \) | \( \Upsilon(4S) \) |
| \( B(B \to X_s \ell \nu) \) | \( (1.71 \pm 0.48_{\text{exp}} \pm 0.21_{\text{bkg}}) \times 10^{-3} \) | LEP |
| \( B(B \to X_s \ell \nu) \) | 0.1070 \pm 0.0028 | \( \Upsilon(4S) \) |
| \( B(B \to X_s \ell \nu) \) | 0.1042 \pm 0.0026 | LEP |
| \( B(B \to \rho \ell \nu) \) | \( (2.68 \pm 0.43_{\text{exp}} \pm 0.5_{\text{bkg}}) \times 10^{-3} \) | CLEO/BABAR |
| \( |V_{cb}|F(1)\) | 0.0388 \pm 0.005 \pm 0.009 | LEP/CLEO/Belle |
| \( \Delta m_{B_s} \) | (0.503 \pm 0.006)ps\(^{-1} \) world average |
| \( \Delta m_{B_s} \) | > 14.4 ps\(^{-1} \) at 95\% C.L. |
| \( |\epsilon_K| \) | (2.271 \pm 0.017) \times 10^{-3} | PDG 2000 [7] |
| \( \sin 2\beta \) | 0.731 \pm 0.055 | BABAR/Belle |
| \( \lambda \) | 0.2241 \pm 0.0033 | world average |

### Table 2. Theoretical parameter with non-gaussian errors

| Parameter | Value | Comment |
|-----------|-------|---------|
| 0.87 \( \leq F_B(1) \leq 0.95 \) | 38.0 \( \leq \Gamma(\ell \nu) \leq 41.5 \) ps\(^{-1} \) |
| 12.0 \( \leq F_B(\rho \ell \nu) \leq 22.2 \) ps\(^{-1} \) | 54.8 \( \leq \Gamma(\ell \nu) \leq 79.6 \) ps\(^{-1} \) |
| 0.72 \( \leq B_K \leq 1.0 \) | 0.06 (gaussian) |
| 211 \( \leq f_{B_s} \sqrt{B_{B_s}} \leq 235 \) MeV | 33 MeV (gaussian) |
| 1.18 \( \leq \xi \leq 1.30 \) | 0.04 (gaussian) |
| 0.54 \( \leq \eta_B \leq 0.56 \) | 1.0 \( \leq \eta_L \leq 1.64 \) |
| 0.564 \( \leq \eta_B \leq 0.584 \) | 0.43 \( \leq \eta_B \leq 0.51 \) |

### Table 3. Results of 95\% C.L. range for \( \bar{\rho}, \bar{\epsilon}, \alpha \) and \( \gamma \) from the global fits shown in figure [1]. For comparison results from RFIT and the Bayesian method are also given.

| Parameter | Scan method | RFIT \([9]\) | Bayesian \([9]\) |
|-----------|-------------|-------------|-------------|
| \( \bar{\rho} \) | -0.13 to 0.40 | 0.091 to 0.317 | 0.137-0.295 |
| \( \bar{\epsilon} \) | 0.22 to 0.48 | 0.273 to 0.408 | 0.295-0.409 |
| \( \alpha \) | 50.4° to 126.6° | |
| \( \gamma \) | 34.4° to 91.7° | 42.1° to 75.7° | 47.0° to 70.0° |

### 3 Results of the global Fit

Figure [1] shows the result of scanning all \( T_i \) simultaneously within \( \pm 1\Delta \) of their allowed range except for the QCD parameters. We have used the input measurements summarized in table [1] and ranges for the \( T_i \) listed in table [2]. The black points represent the best estimates of \( (\bar{\rho}, \bar{\epsilon}) \) for each model that is consistent with the data. The grey region shows the overlay of all corresponding 95\% C.L. \( \bar{\rho} - \bar{\epsilon} \) contours. For reference, the light ellipse depicts a typical contour. To guide the eye the 95\% C.L. bounds on \( V_{ub}/V_{cb} \), \( |\epsilon_K| \), \( \Delta m_{B_s} \) and \( 2\beta \) as well as the lower bound on \( \Delta m_{B_s} \) are also plotted. From these fits we derive 95\% C.L. ranges for \( \bar{\rho}, \bar{\epsilon}, \alpha \) and \( \gamma \) that are listed in table [3]. For comparison, recent results from two other global fits (RFIT [1], Bayesian fit [9]) are also shown.

Using the same source of inputs, several differences exist between the scan method and the other two approaches. First, we scan separately over the inputs of exclusive and inclusive \( b \to u \ell \nu \) and \( b \to c \ell \nu \) measurements. Second, we use a different approach to incorporate \( \Delta m_{B_s} \). While in the Bayesian method theoretical quantities are parameterized in terms of gaussian and uniform distribu-
4 Search for New Physics

The decay $B \rightarrow \phi K_S^0$ that proceeds via a $b \rightarrow s(s')$ penguin loop is expected to produce $\sin 2\beta$ in the SM to within $\sim 4\%$. New physics contributions, however, may introduce a new phase $\theta_\rho$ that may change the CP asymmetry $a_{\phi K_S^0}$ significantly from $a_{J/\psi K_S^0}$. The $BABAR$/Belle average of $S_{\phi K_S^0} = -0.39 \pm 0.41$ has been updated this summer yielding $S_{\phi K_S^0} = -0.14 \pm 0.33$ [13]. The deviation from $\sin 2\beta$ has remained at $\sim 2.6\sigma$. In our global fit we introduce a new phase $\theta_\rho$. Figure 2 shows the overlay of all resulting contours in the $\theta_\rho - \rho$ plane that have acceptable fit probabilities. Presently, the phase is consistent with zero as expected in the SM.

Physics beyond the SM may affect $B_d^0 B_d^0$ mixing and CP violation in $B \rightarrow J/\psi K_S^0$ and $B \rightarrow \pi\pi$. Using a model-independent analysis [12] we can introduce a scale parameter, $r_d$, for $B_d^0 B_d^0$ mixing and an additional phase, $\theta_d$, for parameterizing $a_{\phi K_S^0}$. In the SM we expect $r_d = 1$ and $\theta_d = 0$. With present uncertainties $r_d$ and $\theta_d$ are consistent with the SM expectations (see [10]).

5 Visualizing the role of theoretical errors

In addition to the global fits in the $\tilde{\rho} - \tilde{\eta}$ plane, we explore the impact of measurements on the theoretical parameters and their correlations. We typically scan theory parameters within $\pm 5\Delta$ and denote them with $\sim$. Presently, we use either exclusive or inclusive $\bar{V}_{ub}$, $\bar{V}_{cb}$ and $\bar{B}_{K}$ information and plot contours for three of the five scanned theoretical parameters for different conditions. An example is shown in Figure 2 where we have scanned inclusive $\bar{V}_{ub}$, inclusive $\bar{V}_{cb}$, $\bar{B}_{K}$, $f_{B_d} \sqrt{B_{B_d}}$, and $\xi$. For $\bar{V}_{ub}$, $\bar{V}_{cb}$ and $\bar{B}_{K}$ we plot two-dimensional contours on the surface of a cube. In each plane five contours are visible. The outermost contour (solid black) results from requiring a fit probability of $> 32\%$. The next contour (also solid black) is obtained by restricting all other undisplayed theory parameters to their allowed range of $\pm 1\Delta$. The third solid line results by fixing the parameter orthogonal to plane to the allowed range, while the outer dashed line is found if the latter parameter is fixed to its central value. The internal dashed black line is obtained by fixing all undisplayed parameters to their central values. Further details, other combination plots and results for exclusive $\bar{V}_{ub}$ and $\bar{V}_{cb}$ scans are discussed in [10].

6 Conclusion

The scan method provides a conservative, robust method that treats non-gaussian theoretical uncertainties in an unbiased way. This reduces conflicts with the SM resulting from unwarranted assumptions concerning the theoretical uncertainties, which is important in searches for new physics. The scan methods yields significantly larger ranges for the $\tilde{\rho} - \tilde{\eta}$ plane than the Bayesian method. Presently, all measurements are consistent with the SM expectation due to the large theoretical uncertainties. The deviation of $a_{\phi K_S^0}$ from $\sin 2\beta$ measured in charmonium $K_S^0 (K_L^0)$ modes is interesting but not yet significant. Model-independent parameterizations will become important in the future when theory errors are further reduced.

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