The LHC di-photon Higgs signal predicted by little Higgs models

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Abstract

Little Higgs theory naturally predicts a light Higgs boson whose most important discovery channel at the LHC is the di-photon signal $pp \rightarrow h \rightarrow \gamma\gamma$. In this work we perform a comparative study for this signal in some typical little Higgs models, namely the littlest Higgs model (LH), two littlest Higgs models with T-parity (named LHT-I and LHT-II) and the simplest little Higgs modes (SLH). We find that compared with the Standard Model prediction, the di-photon signal rate is always suppressed and the suppression extent can be quite different for different models. The suppression is mild ($\lesssim 10\%$) in the LH model but can be quite severe ($\approx 90\%$) in other three models. This means that discovering the light Higgs boson predicted by the little Higgs theory through the di-photon channel at the LHC will be more difficult than discovering the SM Higgs boson.

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I. INTRODUCTION

The little Higgs \[1\] is proposed as an elegant mechanism of electroweak symmetry breaking with a naturally light Higgs sector. So far various realizations of the little Higgs symmetry structure have been proposed \[2–4\], which can be categorized generally into two classes \[5\]. One class use the product group, represented by the littlest Higgs model (LH) \[3\], in which the SM SU(2)\textsubscript{L} gauge group is from the diagonal breaking of two (or more) gauge groups. The other class use the simple group, represented by the simplest little Higgs model (SLH) \[4\], in which a single larger gauge group is broken down to the SM SU(2)\textsubscript{L}. Further, to relax the constraints from the electroweak precision tests \[4, 6\], a discrete symmetry called T-parity is proposed \[7\], which can also provide a candidate for the cosmic dark matter. For the LH there are two different implementations of T-parity in the fermion sector, called respectively LHT-I and LHT-II \[8, 9\]. A characteristic difference between LHT-I and LHT-II is that the top quark partner responsible for canceling the one-loop quadratic divergence of Higgs mass contributed by the top quark is T-even for the former and T-odd for the latter. The implementation of T-parity in the SLH has also been tried \[10\].

To test the little Higgs theory at the LHC, the Higgs phenomenology will play an important role \[11\]. At the LHC different search strategies will be applied for different mass ranges. For a light Higgs boson below about 140 GeV the di-photon signal \(pp \rightarrow h \rightarrow \gamma\gamma\) is the most important discovery channel because the narrow \(\gamma\gamma\) peak can be reconstructed to distinguish the signal from the backgrounds. In contrast, the dominant channel \(pp \rightarrow h \rightarrow \bar{b}b\) cannot be utilized for discovery because of the overwhelming QCD backgrounds. Recently the ATLAS collaboration reported their di-photon search results with 209 \(pb^{-1}\) of data collected early 2011 and excluded a signal rate of 4.2-15.8 times the SM prediction for 110 GeV \(\leq m_h \leq 140 \text{ GeV} \) \[12\]. With a luminosity of 2 \(fb^{-1}\) the ongoing LHC will be able to use the di-photon signal to exclude a light SM Higgs boson. So the di-photon Higgs channel will be a sensitive probe for new physics models like the little Higgs theory.

So far the di-photon signal has been studied in some new physics models \[13–16\]. Although some little Higgs models have also been discussed \[14–16\], these previous studies are performed separately in different frameworks. To show the difference of model predictions, it is necessary to perform a comparative study for various models. Further, the study for the SLH has not been reported in detail in the literature. In this work we consider all these
models (LH, LHT-I, LHT-II and SLH) to perform a comparative study.

Our work is organized as follows. In Sec. II we recapitulate the models. In Sec. III we calculate the rate of \( pp \rightarrow h \rightarrow \gamma\gamma \) at the LHC in these models. Finally, we give our conclusion in Sec. IV.

II. LITTLE HIGGS MODELS

A. Littlest Higgs model (LH)

The LH model \([3, 17]\) is based on a non-linear \(\sigma\) model in the coset space of \(SU(5)/SO(5)\) with additional local gauge symmetry \([SU(2) \otimes U(1)]^2\). The vacuum expectation value (VEV) of an \(SU(5)\) symmetric tensor field breaks the \(SU(5)\) to \(SO(5)\) at the scale \(f\). The top quark partner T-quark, heavy gauge bosons \((W_H, Z_H, A_H)\) and triplet scalar \((\Phi^{++}, \Phi^+, \Phi^0, \Phi^P)\) are respectively introduced to cancel the Higgs mass one-loop quadratic divergence contributed by the top quark, gauge bosons and Higgs boson in SM.

The top quark and T-quark can give the dominant contributions to the effective coupling \(hgg\). Their Higgs couplings are given by

\[
\mathcal{L}_t \simeq -\lambda_1 f \left[ \frac{s_\Sigma}{\sqrt{2}} \bar{u}_L u_R + \frac{1 + c_\Sigma}{2} \bar{U}_L U_R \right] - \lambda_2 f \bar{U}_L U_R + \text{h.c.,}
\]

where \(c_\Sigma \equiv \cos \frac{\sqrt{2}(v+h)}{f}\) and \(s_\Sigma \equiv \sin \frac{\sqrt{2}(v+h)}{f}\), with \(h\) and \(v\) being the neutral Higgs boson field and its VEV, respectively. After diagonalization of the mass matrix in Eq. (1), we can get the mass eigenstates \(t\) and \(T\) as well as their couplings with the Higgs boson \([14]\),

\[
\mathcal{L} = -\frac{m_t}{v} y_t \bar{t}t h - \frac{m_T}{v} y_r \bar{T}T h,
\]

where

\[
m_T = \frac{m_t f}{s_t c_t v}, \quad y_t = 1 + \frac{v^2}{f^2} \left[ -\frac{2}{3} + \frac{x}{2} - \frac{x^2}{4} + c_t^2 s_t^2 \right], \quad y_r = -c_t^2 s_t^2 v^2.
\]

The parameter \(x\) is a free parameter of the Higgs sector proportional to the triplet VEV \(v'\) and defined as \(x = \frac{4f v'}{m_t}\). The \(c_t\) and \(s_t\) are the mixing parameters between \(t\) and \(T\),

\[
r = \frac{\lambda_1}{\lambda_2}, \quad c_t = \frac{1}{\sqrt{r^2 + 1}}, \quad s_t = \frac{r}{\sqrt{1 + r^2}}.
\]
In addition to the Higgs couplings with charged fermions, the Higgs couplings with the charged bosons also contribute to the effective coupling $h\gamma\gamma$, which are given as 

$$
\mathcal{L} = 2\frac{m_W^2}{v} y'_W W^+ W^- h + 2\frac{m_H^2}{v} y_{H^\pm} W^+_H W^-_H h - 2\frac{m^2}{v} y_{\phi^+} \Phi^+ h - 2\frac{m^2}{v} y_{\phi^{++}} \Phi^{++} h,
$$

(5)

where

$$
m_{W_H} = \frac{gf}{2\pi c}, \quad m_\Phi = \frac{\sqrt{2} m_h}{\sqrt{1-x^2}},
$$

$$
y_{W_L} = 1 + \frac{v^2}{f^2} \left[ -\frac{1}{6} - \frac{1}{4}(c^2 - s^2)^2 \right], \quad y_{W_H} = -s^2 c^2 \frac{v_f^2}{f^2},
$$

$$
y_{\Phi^+} = \frac{v^2}{f^2} \left[ \frac{1}{3} + \frac{1}{4} x^2 \right], \quad y_{\Phi^{++}} = \frac{v^2}{f^2} \mathcal{O}(x^2 \frac{v_f^2}{f^2}, \frac{1}{16\pi^2}).
$$

(6)

The $c$ and $s$ are the mixing parameters in the gauge boson sector. Since the $h\Phi^{++}\Phi^{--}$ coupling is very small, the contributions of the doubly-charged scalar can be ignored. In the littlest Higgs model, the relation between $G_F$ and $v$ is modified from its SM form, which can induce [14]

$$
v \simeq v_{SM}[1 - \frac{v^2_{SM}}{f^2}(\frac{5}{24} + \frac{1}{8} x^2)],
$$

(7)

where $v_{SM} = 246$ GeV is the SM Higgs VEV.

**B. Littlest Higgs models with T-parity (LHT)**

The LHT-I and LHT-II have the same kinetic term of $\Sigma$ field where the T-parity can be naturally implemented, requiring that the coupling constant of $SU(2)_1 (U(1)_1)$ equals to that of $SU(2)_2 (U(1)_2)$. This will make the four mixing parameters in gauge sector $c$, $s$ ($\equiv \sqrt{1-c}$), $c'$ and $s'$ ($\equiv \sqrt{1-c'}$) equal to $1/\sqrt{2}$, respectively. Under T-parity, the SM bosons are T-even and the new bosons are T-odd. Therefore, the coupling $H^\dagger \phi H$ is forbidden, leading the triplet VEV $v' = 0$ and $x = 0$. Since the correction of $W_H$ to the relation between $G_F$ and $v$ is forbidden by T-parity, the Higgs VEV $v$ is modified as [15, 16]

$$
v \simeq v_{SM}(1 + \frac{1}{12} \frac{v^2_{SM}}{f^2}),
$$

(8)

The Higgs couplings with charged bosons of LHT-I and LHT-II can be obtained from the Eq. (5) and Eq. (6) by taking $c = s = 1/\sqrt{2}$ and $x = 0$.

For each SM quark (lepton), a heavy copy of mirror quark (lepton) with T-odd quantum number is added in order to preserve the T-parity. In the LHT-I [8, 15, 18], the T-parity
is simply implemented by adding the T-parity images for the original top quark interaction to make the Lagrangian T-invariant, so that the top quark partner canceling the one-loop quadratic divergence of Higgs mass is still T-even. Inspired by the way that the quadratic divergence given by top quark is canceled in the SLH, ref. [9] takes an alternative implementation of T-parity in LHT-II, where all new particles including the heavy top partner responsible for canceling the SM one-loop quadratic divergence are odd under T-parity.

In the LHT-I, the Higgs couplings with the heavy quarks are given by

$$L \kappa \simeq -\sqrt{2} \kappa f \left[ \frac{1 + c_\xi}{2} \bar{u}_L u'_R - \frac{1 - c_\xi}{2} \bar{u}_L q_R - \frac{s_\xi}{\sqrt{2}} \bar{u}_L \chi_R \right] - m_q \bar{q} q - m_\chi \bar{\chi} \chi + h.c.,$$

(9)

$$L_t \simeq -\lambda f \left[ \frac{s_\Sigma}{\sqrt{2}} \bar{u}_L u_R + \frac{1 + c_\xi}{2} \bar{U}_L u_R \right] - \lambda_2 f \bar{U}_L U_R + h.c.,$$

(10)

where $c_\xi \equiv \cos \frac{\theta + \phi}{\sqrt{2}}$ and $s_\xi \equiv \sin \frac{\theta + \phi}{\sqrt{2}}$. After diagonalization of the mass matrix in Eq. (9), we can get the T-odd mass eigenstates $u_-$, $q$ and $\chi$. In fact, there are three generations of T-odd particles, and we assume they are degenerate. The mass eigenstates $t$ and $T$ can be obtained by mixing the interaction eigenstates in Eq. (10), and their Higgs couplings are the same to those of LH with $x = 0$.

In the LHT-II, the Higgs couplings with the first two generations of heavy quarks are given by

$$L_{q}^{1,2} \simeq -\sqrt{2} \kappa f \left[ \frac{1 + c_\xi}{2} \bar{u}_L u'_R - \frac{1 - c_\xi}{2} \bar{u}_L q_R + \frac{s_\xi}{\sqrt{2}} \bar{u}_L \chi_R \right] - m_q \bar{q} q - m_\chi \bar{\chi} \chi + h.c.,$$

(11)

The mass eigenstates of $u_-$, $q$ and $\chi$ and their Higgs couplings can be obtained by the diagonalization of the mass matrix in Eq. (11).

The Higgs couplings with the third generation of heavy quarks are given by

$$L_{q}^3 \simeq -\sqrt{2} \kappa f \left[ \frac{1 + c_\xi}{2} \bar{u}_L u'_R - \frac{1 - c_\xi}{2} \bar{u}_L q_R - \frac{s_\xi}{\sqrt{2}} \bar{U}_L q_R - \frac{s_\xi}{\sqrt{2}} \bar{U}_L u'_R + \frac{c_\xi}{\sqrt{2}} \bar{U}_L \chi_R \right]
+ c_\xi \bar{\chi} \chi + m_q \bar{q} q - \lambda f \left[ s_\Sigma \bar{u}_L u_R + \frac{1 + c_\xi}{\sqrt{2}} \bar{U}_L U_R \right] + h.c.,$$

(12)

where $c_t$ is taken as $1/\sqrt{2}$. After diagonalization of the mass matrix in Eq. (12), we can get the mass eigenstates $t$, $T_-$, $u_-$, $q$ and $\chi$ as well as their Higgs couplings.

For the SM down-type quarks (leptons), the Higgs couplings of LHT-I and LHT-II have
two different cases [15]

\[
\frac{g_{hdd}^\text{SM}}{g_{hdd}^\text{SM}} \simeq 1 - \frac{1}{4} \frac{v_{\text{SM}}^2}{f^2} + \frac{7}{32} \frac{v_{\text{SM}}^4}{f^4} \quad \text{for Case A},
\]

\[
\simeq 1 - \frac{5}{4} \frac{v_{\text{SM}}^2}{f^2} - \frac{17}{32} \frac{v_{\text{SM}}^4}{f^4} \quad \text{for Case B}.
\]

The relation of down-type quark couplings also applies to the lepton couplings.

C. Simplest little Higgs model (SLH)

The SLH [4] model is based on \([SU(3) \times U(1)_X]^2\) global symmetry. The gauge symmetry \(SU(3) \times U(1)_X\) is broken down to the SM electroweak gauge group by two copies of scalar fields \(\Phi_1\) and \(\Phi_2\), which are triplets under the \(SU(3)\) with aligned VEVs \(f_1\) and \(f_2\).

The gauged \(SU(3)\) symmetry promotes the SM fermion doublets into \(SU(3)\) triplets. The Higgs couplings with the quarks are given by

\[
\mathcal{L}_t \simeq - f \lambda^t_2 \left[ x^t c \beta t^c_1 (-s_1 t^c_L + c_1 T^c_L) + s_\beta t^c_2 (s_2 t^c_L + c_2 T^c_L) \right] + \text{h.c.},
\]

\[
\mathcal{L}_d \simeq - f \lambda^d_2 \left[ x^d c \beta d^c_1 (s_1 d^c_L + c_1 D^c_L) + s_\beta d^c_2 (-s_2 d^c_L + c_2 D^c_L) \right] + \text{h.c.},
\]

\[
\mathcal{L}_s \simeq - f \lambda^s_2 \left[ x^s c \beta s^c_1 (s_1 s^c_L + c_1 S^c_L) + s_\beta s^c_2 (-s_2 s^c_L + c_2 S^c_L) \right] + \text{h.c.},
\]

where \(f = \sqrt{f_1^2 + f_2^2}\), \(t_\beta \equiv \tan \beta = \frac{f_2}{f_1}\), \(c_\beta = \frac{f_1}{f}\), \(s_\beta = \frac{f_2}{f}\), and

\[
s_1 \equiv \sin \frac{t_\beta (h + v)}{\sqrt{2} f}, \quad s_2 \equiv \sin \frac{(h + v)}{\sqrt{2} t_\beta f}, \quad s_3 \equiv \sin \frac{(h + v)(t_\beta^2 + 1)}{\sqrt{2} t_\beta f}.
\]

(16)

After diagonalization of the mass matrix in Eqs. (13), (14) and (15), we can get the mass eigenstates \((t, T), (d, D)\) and \((s, S)\), which was performed numerically in our analysis, and the relevant couplings with Higgs boson can be obtained.

The Higgs coupling with the charged bosons is given by [19],

\[
\mathcal{L} = 2 \frac{m_{W}^2}{v} y_w W^+ W^- h + 2 \frac{m_{W'}^2}{v} y_{W'} W'^+ W'^- h,
\]

(17)

where

\[
m_{W^+}^2 = \frac{g^2}{2} f^2, \quad y_w \simeq \frac{v}{v_{\text{SM}}} \left[ 1 - \frac{v_{\text{SM}}^2}{4 f^2} \frac{t_\beta^4 - t_\beta^2 + 1}{t_\beta^4} + \frac{v_{\text{SM}}^4}{36 f^4} \frac{(t_\beta^2 - 1)^2}{t_\beta^4} \right], \quad y_{W'} \simeq \frac{-v^2}{2 f^2}.
\]

(18)

The Yukawa and gauge interactions break the global symmetry and then provide a potential for the Higgs boson. However, the Coleman-Weinberg potential alone is not sufficient
since the generated Higgs mass is too heavy. Therefore, one can introduce a tree-level $\mu$ term which can partially cancel the Higgs mass,

$$-\mu^2(\Phi_1^+\Phi_2 + h.c.) = -2\mu^2 f^2 s_\beta c_\beta \cos \left( \frac{\eta}{\sqrt{2} s_\beta c_\beta f} \right) \cos \left( \frac{\sqrt{H^\dagger H}}{f c_\beta s_\beta} \right).$$ \hspace{1cm} (19)

Where $\eta$ is a pseudo-scalar boson, whose mass is determined by the parameter $\mu$.

The Coleman-Weinberg potential involves the following parameters

$$f, x_\lambda, t_\beta, \mu, m_h, v.$$ \hspace{1cm} (20)

Due to the modification of the observed $W$-boson mass, $v$ is defined as \[19\]

$$v \approx v_{SM} \left[ 1 + \frac{v_{SM}^2 t_\beta^4 - t_\beta^2 + 1}{12 f^2} \frac{t_\beta^4 - t_\beta^2 + 1}{180 f^4} \right].$$ \hspace{1cm} (21)

Assuming that there are no large direct contributions to the potential from physics at the cutoff, we can determine other parameters in Eq. \[20\] from $f, t_\beta$ and $m_h$ with the definition of $v$ in Eq. \[21\].

### III. THE DI-PHOTON $pp \to h \to \gamma\gamma$ SIGNAL AT LHC

#### A. Calculations

At the LHC the cross section of the single Higgs production via gluon -gluon fusion can be given

$$\sigma(pp \to (gg \to h)X) \equiv \sigma(gg \to h) = \tau_0 \int_{\tau_0}^1 \frac{dx}{x} f_g(x, \mu_F^2) f_g(\frac{\tau_0}{x}, \mu_F^2) \hat{\sigma}(gg \to h),$$

$$\hat{\sigma}(gg \to h) = \Gamma(h \to gg) \frac{\pi^2}{8m_h^2},$$ \hspace{1cm} (22)

where $\tau_0 = \frac{m_h^2}{s}$ with $\sqrt{s}$ being the center-of-mass energy of the LHC and $f_g(x, \mu_F^2)$ is the parton distributions of gluon. The Eq. \[22\] shows that the $\sigma(gg \to h)$ has the strong correlation with decay width $\Gamma(h \to gg)$.

Now we discuss the Higgs decays in little Higgs models. For the tree-level decays $h \to XX$ where $XX$ denotes $WW$, $ZZ$ or the SM fermion pairs, the little Higgs models give the correction via the corresponding modified couplings

$$\Gamma(h \to XX) = \Gamma(h \to XX)_{SM} \left( g_{hXX}/g_{hXX}^{SM} \right)^2.$$ \hspace{1cm} (23)
\( \Gamma(h \rightarrow XX)_{SM} \) is the SM decay width, and \( g_{hXX} \) and \( g_{hXX}^{SM} \) are the couplings of \( hXX \) in the little Higgs models and SM, respectively.

For a low Higgs mass, the loop-induced decay \( h \rightarrow gg \) will be important. The general expression for the effective coupling \( hgg \) are shown in Appendix A. In the SM, the main contributions are from the top quark loop, and the little Higgs models give the corrections via the modified couplings \( h t \bar{t} \). In addition, the decay width of \( h \rightarrow gg \) can be also corrected by the loops of heavy partner quark \( T \) quark in LH (\( T, D \) and \( S \) in SLH) (new T-even and T-odd quarks in LHT-I and LHT-II).

The general expression for the effective coupling \( h \gamma \gamma \) are shown in Appendix A. In the SM, the top quark loop and \( W \)-boson loop give the main contributions to the decay \( h \rightarrow \gamma \gamma \). The little Higgs models give the corrections via the modified couplings \( h t \bar{t} \) and \( h WW \). In addition to the loops of the heavy quark mentioned in the decay \( h \rightarrow gg \), the decay width of \( h \rightarrow \gamma \gamma \) can be also corrected by the loops of \( W_H, \Phi^+, \Phi^{++} \) in the LH, LHT-I and LHT-II (\( W' \) in the SLH). Note that in the lepton sector, LHT-I, LHT-II and SLH also predict some neutral heavy neutrinos, which do not contribute to the couplings of \( h \gamma \gamma \) at the one-loop level. Although the charged heavy leptons are predicted in LHT-I and LHT-II, they do not have direct couplings with the Higgs boson.

In addition to the SM decay modes, the Higgs boson in the LHT-I, LHT-II and SLH has some new important decay modes which are kinematically allowed in the parameter space. In the LHT-I the breaking scale \( f \) may be as low as 500 GeV [20], and the constraint in LHT-II is expected to be even weaker [9]. For a lower value of \( f \), the lightest T-odd particle \( A_H \) may have a light mass, so that the decay \( h \rightarrow A_H A_H \) can be open, whose partial width is

\[
\Gamma(h \rightarrow A_H A_H) = \frac{g_{hA_H A_H}^2 m_h^3}{128 \pi m_{A_H}^4 \sqrt{1-x_{A_H}}} \left(1 - x_{A_H} + \frac{3}{4} x_{A_H}^2\right),
\]

where \( x_{A_H} = 4m_{A_H}^2/m_h^2 \), and \( g_{hA_H A_H} \) is the coupling constants of \( h A_H A_H \). However, in the LH the electroweak precision data requires \( f \) larger than a few TeV [6] and thus the decay \( h \rightarrow A_H A_H \) is kinematically forbidden.

In the SLH, the new decay modes are \( h \rightarrow \eta \eta \) and \( h \rightarrow Z \eta \), whose partial widths are
given by

\[
\Gamma(h \rightarrow \eta \eta) = \frac{\lambda^2 v^2}{8\pi m_h} \sqrt{1 - x_\eta},
\]
\[
\Gamma(h \rightarrow Z \eta) = \frac{m_h^3}{32\pi f^2} \left( t_\beta - \frac{1}{t_\beta} \right)^2 \lambda^{3/2} \left( 1, \frac{m_Z^2}{m_h^2}, \frac{m_\eta^2}{m_h^2} \right),
\]  

(25)

where \( x_\eta = 4m_\eta^2/m_h^2 \) and \( \lambda(1, x, y) = (1 - x - y)^2 - 4xy \).

B. Numerical results and discussions

In our calculations the SM input parameters involved are taken from [21]. For the SM decay channels, the relevant higher order QCD and electroweak corrections are considered using the code Hdecay [22]. We focus on a light SM-like Higgs boson, whose mass is taken in the range of 110-140 GeV.

In the LH model the new free parameters are \( f, c, c', c_t \) and \( x \), where

\[
0 < c < 1, \quad 0 < c' < 1, \quad 0 < c_t < 1, \quad 0 < x < 1.
\]  

(26)

Taking \( f = 1 \) TeV, \( f = 2 \) TeV and \( f = 4 \) TeV, we scan over these parameters in the above ranges and show the scatter plots. The parameter \( c_t \) can control the Higgs couplings with \( t \), \( T \) and \( m_T \), which is involved in the calculation of \( \Gamma(h \rightarrow t \bar{t}) \), \( \Gamma(h \rightarrow gg) \) and \( \Gamma(h \rightarrow \gamma \gamma) \). For a light Higgs boson, the decay mode \( h \rightarrow t \bar{t} \) is kinematically forbidden. For the \( \Gamma(h \rightarrow gg) \) and \( \Gamma(h \rightarrow \gamma \gamma) \), the \( c_t \) dependence of top-quark loop and T-quark loop can cancel each other to a large extent (see Eq. (3)). Therefore, the rate \( \sigma(gg \rightarrow h) \times BR(h \rightarrow \gamma \gamma) \) is not sensitive to \( c_t \) for a light Higgs boson.

The rate \( \sigma(pp \rightarrow h) \times BR(h \rightarrow \gamma \gamma) \) for the LH model is shown in Fig. 1 normalized to the SM prediction. We can see that the LH model always suppresses the rate \( \sigma(pp \rightarrow h) \times BR(h \rightarrow \gamma \gamma) \), but the suppression can only reach about 10% for the small value \( f \). As the increasing of \( f \), the magnitude becomes small, and the rate is not sensitive to the parameters \( c, c', c_t \) and \( x \). For example, for \( f = 4 \) TeV, the scatter plots are shown in line with the rate being around 99.6 percent of SM prediction.

In LHT-I and LHT-II, the parameters \( c, c' \) and \( x \) are fixed as \( c = c' = \frac{1}{\sqrt{2}} \) and \( x = 0 \). Similar to the LH model, the result is not sensitive to \( c_t \) in LHT-I and LHT-II. Taking \( c_t = 1/\sqrt{2} \) can simplify the top quark Yukawa sector in the LHT-II [9, 16], and this choice is also favored by the electroweak precision data [20]. The new heavy quarks can contribute
FIG. 1: Scatter plots for the rate $\sigma(pp \rightarrow h) \times BR(h \rightarrow \gamma\gamma)$ at the LHC normalized to the SM prediction in the LH model.

to the decay widths of $h \rightarrow gg$ and $h \rightarrow \gamma\gamma$ via the loop, which are not sensitive to the actual values of their masses as long as they are much larger than half of the Higgs boson mass $m_H$.

The rate $\sigma(pp \rightarrow h) \times BR(h \rightarrow \gamma\gamma)$ for LHT-I and LHT-II is shown in Fig. 2 normalized to the SM prediction. We can see that LHT-I and LHT-II always suppress the rate, and the suppression is much more sizable than that of LH. For each model the rate in Case A is smaller than the rate in Case B because the coupling $hb\bar{b}$ in Case A is less suppressed than in Case B. Besides, we see that for $f = 500$ GeV and $m_h$ in the range of 130 GeV - 140 GeV, the rate in both models drops drastically. The reason for such a severe suppression is that the new decay mode $h \rightarrow A_H^+A_H^-$ is open and dominant in these parameter space and thus the total decay width of Higgs boson becomes much larger than the SM value.

In the SLH the new free parameters are $f$, $t_\beta$, $\lambda^d_x(m_D)$ and $\lambda^s_x(m_S)$. As shown above, the parameters $\lambda^d_x$, $\mu(m_\eta)$ can be determined by $f$, $t_\beta$, $m_h$ and $v$ with the assuming that the physics at the cutoff does not give the large direct contributions to the potential. Ref. [4] shows that the LEP-II data requires $f > 2$ TeV, and ref. [23] gives a lower bound of $f > 4.5$ TeV from the oblique parameter $S$. A recent studies about $Z$ leptonic decay and $e^+e^- \rightarrow \tau^+\tau^-\gamma$ process at the $Z$-pole show that the scale $f$ should be respectively larger than 5.6 TeV and 5.4 TeV [24]. Here, we assume the new flavor mixing matrices in lepton and quark sectors are diagonal [5, 25], so that $f$ and $t_\beta$ are free from the experimental constraints of the lepton and quark flavor violating processes. In addition, the contributions to the
FIG. 2: The rate $\sigma(pp \to h) \times BR(h \to \gamma\gamma)$ at the LHC normalized to the SM prediction in the LHT. The curves from bottom to top correspond to $f = 500$ GeV, 600 GeV, 700 GeV, 800 GeV, 1 TeV and 2 TeV, respectively.

Electroweak precision data can be suppressed by the large $t_\beta$. For the perturbation to be valid, $t_\beta$ cannot be too large for a fixed $f$. If we require $\mathcal{O}(v_0^4/f^4)/\mathcal{O}(v_0^2/f^2) < 0.1$ in the expansion of $v$, $t_\beta$ should be below 10, 20, and 28 for $f = 2$ TeV, 4 TeV, and 5.6 TeV, respectively. The small masses of the $d$ quark and $s$ quark require that $x_\Lambda^d$ and $x_\Lambda^s$ are very small, respectively, so there is almost no mixing between the SM down-type quarks and their heavy partners, and the results are not sensitive to them. We take $x_\Lambda^d = 1.1 \times 10^{-4}$ ($x_\Lambda^s = 2.1 \times 10^{-3}$), which can make the masses of $D$ and $S$ in the range of 0.5-2 TeV with other parameters fixed as in our calculation.

The rate $\sigma(pp \to h) \times BR(h \to \gamma\gamma)$ for the SLH at the LHC is shown in Fig. 3. We can see that the SLH always suppresses the rate, and the suppression is more sizable for a large $t_\beta$. When $t_\beta$ is large enough, such as $t_\beta = 10$ for $f = 2$ TeV ($t_\beta = 20$ for $f = 4$ TeV or
FIG. 3: The rate $\sigma(pp \rightarrow h) \times BR(h \rightarrow \gamma\gamma)$ at the LHC normalized to the SM prediction in SLH. The incomplete lines for the small values of $\tan \beta$ show the lower bounds of Higgs mass.

If $t_\beta = 25$ for $f = 5.6$ TeV, the new mode $h \rightarrow \eta\eta$ is open and dominant, which can further suppress the rate (the suppression can be up to 90%).

FIG. 4: Scatter plots for the rate $\sigma(pp \rightarrow h) \times BR(h \rightarrow \gamma\gamma)$ normalized to the SM prediction in SLH.

If the ultraviolet completion of the theory can give the sizable contributions to the Coleman-Weinberg potential, the correlation of the parameters $x_\lambda$, $\mu$ ($m_\eta$), $f$, $t_\beta$, $m_h$ and $v$ can be loosened greatly. In Fig. 4 we scan the following parameter space,

$$1 \text{ TeV} < f < 6 \text{ TeV}, \quad 0.5 \text{ TeV} < m_T < 2 \text{ TeV},$$

$$0.5 \text{ TeV} < m_D (m_S) < 2 \text{ TeV}, \quad 1 < t_\beta < 30,$$

where the parameter $x_\lambda$ is replaced with $m_T$ and the bound $\mathcal{O}(v_0^4/f^4)/\mathcal{O}(v_0^2/f^2) < 0.1$ is still
TABLE I: The value of $y_{f_i}$ corresponding to the quark $f_i$ and $Br(h \to \gamma\gamma)$ normalized to SM prediction in these little Higgs models, respectively. The parameters are fixed as $c = c' = c_t = \frac{1}{\sqrt{2}}$ and $x = 0$ in the LH, and $t_\beta=10$, $m_T=450$ (798) GeV, $m_D=548$ (1039) GeV, $m_S=597$ (1132) GeV and $m_\eta=42.7$ (179) GeV for $f=2$ (4) TeV in the SLH.

| $m_h=120$ GeV | t-quark | t-quark partner | other heavy quarks | $\sum y_{f_i}$ | $\frac{Br(h \to \gamma\gamma)}{Br(h \to \gamma\gamma)_{SM}}$ |
|---------------|---------|----------------|-------------------|----------------|----------------------------------|
| LH $f=1$ TeV  | 0.974   | -0.016         | -                 | 0.958          | 1.040                            |
| $f=2$ TeV     | 0.994   | -0.004         | -                 | 0.990          | 1.009                            |
| LHT-I $f=0.5$ TeV | 0.895   | -0.080         | -0.195            | 0.620          | 1.094                            |
| Case A $f=1$ TeV | 0.975   | -0.016         | -0.046            | 0.913          | 1.024                            |
| LHT-I $f=0.5$ TeV | 0.895   | -0.080         | -0.195            | 0.620          | 1.836                            |
| Case B $f=1$ TeV | 0.975   | -0.016         | -0.046            | 0.913          | 1.130                            |
| LHT-II $f=0.5$ TeV | 0.745   | -0.338         | -0.164            | 0.243          | 1.381                            |
| Case A $f=1$ TeV | 0.940   | -0.084         | -0.034            | 0.822          | 1.089                            |
| LHT-II $f=0.5$ TeV | 0.745   | -0.338         | -0.164            | 0.243          | 2.36                             |
| Case B $f=1$ TeV | 0.940   | -0.084         | -0.034            | 0.822          | 1.204                            |
| SLH $f=2$ TeV | 0.776   | -0.117         | 0.000             | 0.659          | 0.155                            |
| $f=4$ TeV     | 0.978   | -0.047         | 0.000             | 0.931          | 0.998                            |

To avoid that the rate is suppressed by the new decay modes $h \to \eta\eta$ and $h \to \eta Z$, we take $m_\eta > 2m_h$, so that the result is independent of the parameter $m_\eta (\mu)$. Fig. 14 shows that, compared to the SM prediction, the SLH still suppresses the rate $\sigma(pp \to h) \times BR(h \to \gamma\gamma)$ in more general parameter space.

From our above results we see that compared to the SM prediction the rate $\sigma(pp \to h) \times BR(h \to \gamma\gamma)$ is always suppressed in these typical little Higgs models. Now we analyze such a suppression in detail. Eq. (A11) and Eq. (A8) show that the effective coupling $hgg$ is not sensitive to the heavy quark masses as long as they are much larger than half of the Higgs boson mass. Therefore, according to the Eq. (A9) and Eq. (A10), the effective coupling $hgg$ is approximately proportional to $(\sum y_{f_i})^2$, where $y_{f_i}$ is defined in Eq. (A2). Table 1 shows the value of $y_{f_i}$ corresponding to the quark $f_i$ and $Br(h \to \gamma\gamma)$ normalized to SM prediction in these little Higgs models, respectively.
Because of the sizable suppression of the coupling $hbb$, $Br(h \rightarrow \gamma\gamma)$ is generally not suppressed, unless the new decay mode is open and dominant, as shown for the SLH with $f = 2 \text{TeV}$ in Table 1 (the new mode is $h \rightarrow \eta\eta$). In these little Higgs models, $\sum y_{fi}$ can be respectively less than 1, which shows that the $\sigma(pp \rightarrow h)$ is suppressed compared the SM prediction. There are some common reasons for these models: (i) All the models are based on the non-linear sigma models, the Yukawa coupling $ht\bar{t}$ is suppressed with the expansion of the non-linear sigma fields. (ii) The top quark partner cancels the quadratic divergence of Higgs mass contributed by top quark, which will induce that the Yukawa couplings of top quark and its partner have the opposite sign.

The forthcoming measurement of the di-photon signal at the LHC will allow for a probe of these little Higgs models. For example, if the signal rate is found to be above the SM prediction, these little Higgs models will be immediately disfavored. If the signal rate is found to be much lower than the SM prediction, then the SLH and LHT will be favored. However, due to the free parameters involved in the signal rate for each model, it will be hard for the LHC to clearly discriminate these different little Higgs models. For the precision test of different models, the ILC collider is necessary [27].

IV. CONCLUSION

We performed a comparative study for the LHC di-photon signal by considering four different little Higgs models, namely the LH, LHT-I, LHT-II and SLH. We obtained the following observations: (i) Compared with the SM prediction, the di-photon signal rate is always suppressed in these models; (ii) The suppression extent is different in different models, which is below 10% in the LH but can reach 90% in the LHT-I, LHT-II and SLH, especially in the parameter space with new decay modes ($h \rightarrow \eta\eta$ for the SLH and $h \rightarrow A_H A_H$ for the LHT-I and LHT-II) are open and dominant. Therefore, discovering the light Higgs predicted by these little Higgs models through the di-photon channel at the LHC will be more difficult than discovering the SM Higgs boson.
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Appendix A: The effective couplings of Higgs-photon-photon and Higgs-gluon-gluon

The effective Higgs-photon-photon coupling can be written as \[14, 28\]

\[
\mathcal{L}_{h\gamma\gamma}^{\text{eff}} = -\frac{\alpha}{8\pi v} F_{\mu\nu} F^{\mu\nu} h, \tag{A1}
\]

where \( F_{\mu\nu} \) is the electromagnetic field strength tensor. With the Higgs boson couplings to the charged fermion \( f_i \), vector boson \( V_i \) and scalar \( S_i \) given by

\[
\mathcal{L} = \sum_{f_i} -\frac{m_{f_i}}{v} y_{f_i} \bar{f}_i f_i h + \sum_{V_i} 2 \frac{m_{V_i}^2}{v} y_{V_i} V_i V_i h + \sum_{S_i} -2 \frac{m_{S_i}^2}{v} y_{S_i} S_i S_i h, \tag{A2}
\]

the factor \( I \) in Eq. (A1) can be written as

\[
I = \sum_{f_i} Q_{f_i}^2 N_{cf_i} y_{f_i} I_{\frac{1}{2}}(\tau_{f_i}) + \sum_{V_i} Q_{V_i}^2 y_{V_i} I_1(\tau_{V_i}) + \sum_{S_i} Q_{S_i}^2 y_{S_i} I_0(\tau_{S_i}), \tag{A3}
\]

where \( Q_X \) (\( X \) denotes \( f_i \), \( V_i \) and \( S_i \)) is the electric charge for a particle \( X \) running in the loop, and \( N_{cf_i} \) is the color factor for \( f_i \). The dimensionless loop factors are

\[
I_{\frac{1}{2}}(\tau_{f_i}) = -2\tau_{f_i}[1 + (1 - \tau_{f_i}) f(\tau_{f_i})], \tag{A4}
\]

\[
I_1(\tau_{V_i}) = 2 + 3\tau_{V_i} + 3\tau_{V_i}(2 - \tau_{V_i}) f(\tau_{V_i}), \tag{A5}
\]

\[
I_0(\tau_{S_i}) = \tau_{S_i}[1 - \tau_{S_i} f(\tau_{S_i})], \tag{A6}
\]

where \( \tau_x = 4m_{X}^2/m_h^2 \) and

\[
f(\tau_X) = \begin{cases} 
\sin^{-1}(1/\sqrt{\tau_X})^2, & \tau_X \geq 1 \\
-\frac{1}{4}[\ln(\eta_+/\eta_-) - i\pi]^2, & \tau_X < 1 
\end{cases} \tag{A7}
\]

with \( \eta_\pm = 1 \pm \sqrt{1 - \tau_X} \). When the masses of particles in the loops are much larger than half of the Higgs boson mass, we can get

\[
I_{\frac{1}{2}}(\tau_{f_i}) \simeq -4/3, \quad I_1(\tau_{V_i}) \simeq 7, \quad I_0(\tau_{S_i}) \simeq -1/3. \tag{A8}
\]
The effective Higgs-gluon-gluon coupling can be written as \[ L_{hgg}^{\text{eff}} = -\frac{a_s}{12\pi v} I_{hgg} G^\alpha_{\mu\nu} G^\alpha_{\mu\nu} h, \] (A9)

where \( G^\alpha_{\mu\nu} = \partial_\mu g_\alpha^\mu - \partial_\nu g_\alpha^\mu \) and the factor \( I_{hgg} \) from the contributions of quarks running in the loops is given by

\[ I_{hgg} = \sum q_i \frac{3}{4} y_{q_i} I_1(\tau_{q_i}), \] (A10)

with \( \tau_{q_i} = \frac{4m_{q_i}^2}{m_h^2} \).

Once the interactions in Eq. (A2) are given, we can obtain the effective \( h\gamma\gamma \) and \( hgg \) couplings from the above formulas. The relevant Higgs interactions in the LH, LHT-I and LHT-II and SLH are listed in the Sec. II. Here the Higgs interactions with the light fermions are not given since their contributions can be ignored.

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