The SELEX Measurements
in the Unified Picture for Hadron Spectra

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Abstract

We give an analysis of the experimental material presented by the SELEX Collaboration to find the true place for the SELEX state $D_{sJ}(2632)$ in the unified picture for hadron spectra developed early. It is found that the SELEX measurements are excellently incorporated in the unified picture for hadron spectra. Our analysis shows that the measured values for the masses of the SELEX state exactly coincide with the calculated masses of the states living in the corresponding KK towers. We also found quite a natural but rather model dependent explanation of the decay pattern for the SELEX state being dominated by the $D_{sJ}^+\eta$ decay mode.

1 Introduction

Exciting experimental measurements in hadron spectroscopy of last two years clearly open a new page in hadron physics which is certainly a starting page of a new era in particle physics. Indeed, there were discovered many new narrow hadronic states with unexpected properties. First of all, a series of new mesons have been discovered whose properties are in a strong disagreement with the predictions of conventional QCD-inspired quark potential models. New narrow meson $D_{sJ}^+(2317)$, decaying into $D_s^+\pi^0$ has been observed by BABAR Collaboration [1] in the first. This observation was soon confirmed by CLEO Collaboration [2], which have also established the existence of a new narrow state with a mass near 2.46 GeV in its decay to $D_s^+\pi^0$. Belle Collaboration [3] reported the first observation of the $D_{sJ}(2317)$ and $D_{sJ}(2457)$ in $B$ decays: $B \to \bar{D}D_{sJ}(2317)$ and $B \to \bar{D}D_{sJ}(2457)$ with a subsequent $D_{sJ}(2317)$ decay to $D_s\pi^0$ and $D_{sJ}(2457)$ decay to $D_s^+\pi^0$ and $D_s\gamma$ final states. Both CLEO and Belle observations of $D_{sJ}(2457)$ have been confirmed by BABAR [4]. Moreover, Belle Collaboration has reported [5, 6] the discovery of very narrow $X(3872)$-meson state ($\Gamma_{X(3872)}^{\text{tot}} < 2.3 MeV$) in the $J/\psi\pi^+\pi^-$ invariant mass distribution in the $B$ decay $B^\pm \to K^\pm J/\psi\pi^+\pi^-$. This observation of Belle Collaboration was soon confirmed by CDF at Fermilab [7]. The mass measurement presented by CDF $3871.4 \pm 0.7 \pm 0.4$ MeV is in agreement with the result of Belle. It should be noted, in particular, that the mass 2317 MeV is approximately 41 MeV below the $DK$ threshold but the mass 3872 MeV is very near the $D^0\bar{D}^{*0}$ threshold, while the $D^+D^{*-}$ channel with approximately 8 MeV higher threshold mass is forbidden for $X(3872)$ decay by phase

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space. What is remarkable here that all new narrow states have been observed at the masses which are surprisingly far from the predictions of conventional quark potential models. It is still more non-trivial that all new observed states are very narrow, their total widths being about a few MeV. The small widths were found to be in contradiction with quark model expectations.

Quite recently the SELEX Collaboration [8, 9] has reported the first observation of a charm-strange meson $D_{sJ}^+(2632)$ in the charm hadro-production experiment E781 at Fermilab. This state was seen in two decay modes, $D_{sJ}^+\eta$ and $D^0K^+$. In the $D_{sJ}^+\eta$ decay mode an excess of 49.3 events with a significance of 7.2 $\sigma$ at a mass of $2635.9 \pm 2.9$ MeV has been observed. There is a corresponding peak of 14 events with a significance of 5.3 $\sigma$ at a mass of $2631.5 \pm 1.9$ MeV in the decay mode $D^0K^+$. Upper limit for the decay width of this state, consistent with a width due only to resolution in the $D^0K^+$ decay mode, is $<17$ MeV at 90% confidence level. The relative branching ratio $\Gamma(D^0K^+)/\Gamma(D_{sJ}^+\eta)$ is 0.16 $\pm$ 0.06. The mechanism which keeps this state narrow is unclear. Its decay pattern being dominated by the $D_{sJ}^+\eta$ decay mode is also unusual to be explained in the conventional quark models. From the standard quark model point of view the $D_{sJ}^+(2632)$ meson which is 660 MeV heavier than the ground $D_s$ state and about 100 MeV below the quark models predictions of 2.73-2.81 GeV for the $2^3S_1$ $c\bar{s}$ state together with the $D_{sJ}(2317)$ and $D_{sJ}(2457)$ states mentioned above seem constitute quite a new group of the charm-strange family rather distinguished from the ordinary $D_s(1968)$ and $D^*_s(2112)$ mesons by unusual features in the masses, narrow widths and decay pattern. It is quite unclear how these newly discovered mesons could be understood as the states of a simple $c\bar{s}$ quark system. In fact, a description of new, recently discovered meson states with unexpectedly low masses, narrow total decay widths and unusual decay pattern is extremely problematic in the framework of phenomenology based on conventional quark models. Why these meson states are less massive and so narrow than earlier predicted in quark models is an open question so far. Certainly, all of that raise a challenge to the theory. In other words, this means either considerable modifications in the conventional quark models have to be introduced to reconsider the systematics of charmed and charmed-strange mesons spectroscopy and strong decays or that completely new approaches should be applied in hadron spectroscopy.

Recent discoveries of very narrow, manifestly exotic baryons lead us to the same conclusion. Here are the first observations of the $\Theta^+$ ($Q=1$, $S=1$) states with the simplest quark assignment ($uudd\bar{s}$) decaying into $nK^+$ and $pK_S^0$ [11] and the discovery of another exotic baryon $\Xi^{--}$ ($Q=-2$, $S=-2$), now denoted as $\Phi^{--}$ by PDG, with the quark assignment ($dssuu$) decaying into $\Xi^--\pi^-$ at the mass $M = 1862$ MeV with the width $\Gamma < 18$ MeV [12], but see also [13].

In Ref. [14], where some of our previous studies were partially summarized, it has been claimed that existence of the extra dimensions in the spirit of Kaluza and Klein together with some novel dynamical ideas may provide new conceptual issues for the global solution of the spectral problem in hadron physics to build up a unified picture for hadron spectra. We have shown [15], in particular, that all charmed and charmed-strange mesons, including the recently observed new states, have excellently been incorporated in the systematics provided by the created unified picture for hadron spectra. A thorough

\[2\] A comprehensive analysis of the different options for the SELEX state $D_{sJ}^+(2632)$ in quark models has been made in [10].
analysis [16] of many different experiments reported the observation of a new Θ baryon taken together allowed us to claim that many different Θ states have been discovered and all of them were excellently incorporated in the unified picture for hadron spectra developed early [14]. In this article we give the similar analysis of the experimental material presented by SELEX Collaboration to find the true place for the SELEX state $D_{sJ}^+(2632)$ in the unified picture for hadron spectra.

2 Understanding the SELEX state $D_{sJ}^+(2632)$ in the unified picture for hadron spectra

The SELEX experiment [8] have used the Fermilab charged hyperon beam at 600 GeV to produce charmed particles in a set of thin foil targets of Cu or diamond. The negative beam composition was approximately half $\Sigma^-$ and half $\pi^-$. The most important features of the experiment were the high-precision, highly redundant, vertex detector that provided an average proper time resolution of 20 fs for charm decays, a 10 m long Ring-Imaging Cerenkov (RICH) detector that separated $\pi$ from K up to 165 GeV, and a high-resolution tracking system that had momentum resolution of $\sigma_p/p < 1\%$ for a 150 GeV track. Photons have been detected in 3 lead glass photon detectors, one following each spectrometer magnet. The photon angular coverage in the center of mass of a production collision exceeded 2$\pi$. For the presented analysis the photon energy threshold was 2 GeV.

The experiment recorded data from $15.2 \times 10^9$ inelastic interactions and wrote $10^9$ events to tape using both positive and negative beams. The data set had 65% of the events induced by the $\Sigma^-$ beam with the balance split roughly equally between $\pi^-$ and protons. Previous SELEX $D_s$ studies have shown that most of the signal came from the $\Sigma^-$ beam, therefore it was restricted in the analysis within the $9.9 \times 10^9 \Sigma^-$-induced interactions. The details of the procedure how charm candidates have been selected and many other details can be found in original paper [8] and references therein.

The results of the search for the $D_{sJ}\eta$ decay mode were presented in the $M(KK\pi^\pm\eta) - M(KK\pi^\mp)$ mass difference distribution shown in Fig. 1 extracted from the original paper. In this plot the $\eta$ mass was fixed at the PDG value. A clear peak is seen for a mass difference of 667.4 ± 2.9 MeV. There is an excess of 43.9 events over an expected background of 51.7 events with a significance of 7.2 $\sigma$ at a mass of 2635.9 ± 2.9 MeV. The signal did not change with variations of ±2% in the photon energy scale.

The results of the search for the $D^0K^+$ decay mode were shown in the $M(K^-\pi^+K^+) - M(K^-\pi^-)$ mass difference distribution depicted in Fig. 2 extracted from the original paper too. The known $D_{sJ}(2573)$ state is clearly seen in this Figure, but there is another peak above the $D_{sJ}(2573)$. Both of the peaks were fitted with a Breit-Wigner convolved with a fixed width Gaussian using a constant background term. The Gaussian resolution was set to the simulation value of 4.9 MeV. The mass difference and width of the $D_{sJ}(2573)$ obtained by the fit agreed well with the PDG values. The fitted mass difference of the second Breit-Wigner was obtained of 767.0 ± 1.9 MeV, leading to a mass for the new peak of 2631.5 ± 1.9 MeV. For the Breit-Wigner width it was found that it was < 17 MeV at 90% confidence level. This signal has a significance of 5.3 $\sigma$. It was also pointed out that the mass difference between this signal and the one seen in the $D_s\eta$ mode is 3.2 ± 3.5 MeV, statistically consistent with being the same mass. For the relative branching ratio $\Gamma(D^0K^+)/\Gamma(D_s^+\eta)$ the value 0.16 ± 0.06 has been reported.
In conclusion the SELEX reported the measured peaks as the first observation of yet another narrow, high-mass $D_s$ state decaying strongly to a ground state charm plus a pseudoscalar meson. The mechanism which keeps this state narrow is unclear. The branching ratios for this state are also unusual. The $D_s\eta$ decay rate dominates the $D^0K^+$ rate by a factor of $\sim 6$ despite having half the phase space. To place this new state in the spectroscopy of the charm-strange meson system will require careful study from a number of experiments in the future.

Recently we suggested quite a new scheme of systematics for hadron states. The fundamental Kaluza-Klein hypothesis on existence of the extra dimensions with a compact internal extra space is a base of our approach to hadron spectroscopy. The observed hadron states occupy the storeys and live in the corresponding KK towers built in according to the established general physical law. Herewith the size of the internal compact extra space determines the global characteristics of the hadron spectra while the masses of the decay products are the fundamental parameters of the compound systems which appear as the elements of the global structure. Our approach to hadron spectroscopy has been verified with a large amount of experimental data on hadron states and received an excellent agreement; see [14, 16] and references therein for the details. What is remarkable that all new hadron states experimentally discovered last two years have been observed just at the masses predicted in our approach, and those states appeared to be narrow as predicted too. Here we apply our approach to analyse the experimental material presented by SELEX Collaboration.

First of all, it should be emphasized that the Kaluza-Klein tower of KK-excitations for the $DK$ system has already been built in Ref. [15]. The built Kaluza-Klein tower extracted from Ref. [15] is shown in Table 1. Clearly, the SELEX state shown by boldface number in Table 1 just occupies the 12th storey in the Kaluza-Klein tower. Some known experimental information has been presented in this Table as well.

We have also built the Kaluza-Klein tower of KK-excitations for the $D_s\eta$ system by the formula

$$M_n^{D_s\eta} = \sqrt{m_{D_s}^2 + \frac{n^2}{R^2}} + \sqrt{m_{\eta}^2 + \frac{n^2}{R^2}}, \quad (n = 1, 2, 3, ...),$$

(1)

where $R$ is the same fundamental scale established before; see [14] and references therein for the details. The such built Kaluza-Klein tower is shown in Table 2. As seen the SELEX measured state just lives in 8th storey within this Kaluza-Klein tower.

Here we would like to compare the theoretically calculated spectra with the experimental material presented by SELEX Collaboration.

The $D^0K^+$ invariant mass spectrum measured by SELEX is shown in Fig. 3 extracted from Ref. 9. We have plotted in Fig. 3 the spectral lines corresponding to KK excitations in the $D^0K^+$ system taken from Table 1. As it should be expected the spectral line corresponding to the $M_{12}^{D^0K^+}(2630.5)$-storey exactly coincided with a clear peak on the histogram. Here is also clear seen a strong correlation of the spectral lines with the other experimentally observed peaks, and we already have early found out that strong correlation more than once in our previous studies [16]. At least, it should be emphasized an evidence for the two other states corresponding to the spectral lines $M_{10}^{D^0K^+}(2554.5)$ and $M_{11}^{D^0K^+}(2591.4)$ with clear peaks on the histogram in Fig. 3. Obviously, a further,

\[^3\text{I thank A. Evdokimov for drawing my attention to Ref. 9 and sending me the pictures from his talk.}\]
much more careful experimental studies with a higher statistics and better mass resolution are very desired.

Figure 4 extracted from Ref. 9 too shows the $D^+_s\eta$ invariant mass spectrum measured by SELEX Collaboration as well. We have also plotted in Fig. 4 the spectral lines corresponding to KK excitations in the $D^+_s\eta$ system taken from Table 2. Again we found a remarkable correlation of the spectral lines with the peaks on the histogram. The reported state by SELEX just corresponds to the spectral line from $M_{D^+_s\eta}(2636.8)$-storey in KK tower for the $D^+_s\eta$ system given by Table 2. Here a further, careful experimental studies with a higher statistics and better mass resolution are also utterly important.

3 What could we say about the decay widths?

Our conservative estimate for the widths of KK excitations looks like [17]

$$\Gamma_n \sim \frac{\alpha}{2} \cdot \frac{n}{R} \sim 0.4 \cdot n \text{ MeV}, \quad (2)$$

where $n$ is the number of KK excitation, and $\alpha \sim 0.02$, $R^{-1} = 41.48 \text{ MeV}$ have been taken from our previous studies [17, 14]. This gives $\Gamma_{12}(D^0K^+) \sim 4.8 \text{ MeV}$ and $\Gamma_8(D^+_s\eta) \sim 3.2$ MeV. Thus, in that case, the ratio

$$\frac{\Gamma_{12}(D^0K^+)}{\Gamma_8(D^+_s\eta)} = 1.5 \quad (3)$$

is approximately 10 times larger than the experimental value. However, it’s clear, that account of some additional, model dependent dynamical assumptions may change that estimate. For this goal let us consider the simplest multidimensional model with an effective interaction

$$S_I = \int_{\mathcal{M}_{(4+d)}} d^{4+d}z \left[ G_{D^0\eta}(z) \Phi_{D^0}(z) \Phi_R(z) \Phi_K(z) + G_{D^0K^+}(z) \Phi_D(z) \Phi_{D^+_s}(z) \right], \quad (4)$$

where $\mathcal{M}_{(4+d)} = M_4 \times \mathcal{K}_d$, $M_4$ is the pseudo-Euclidean Minkowski space-time, $\mathcal{K}_d$ is a compact internal $d$-dimensional extra space with the characteristic size $R$, $z^M = \{x^\mu, y^m\}$, $(M = 0, 1, \ldots, 3 + d, \mu = 0, 1, 2, 3, m = 1, 2, \ldots, d)$ are local coordinates on $\mathcal{M}_{(4+d)}$ so that $x^\mu \in M_4$, $y^m \in \mathcal{K}_d$, $\Phi_P(z)$, $(P = D^0, K, D^+_s, \eta)$, are local multidimensional fields corresponding to the decay products. We choose a $d$-dimensional torus $\mathcal{T}^d$ with equal radii $R$ as an especially simple example of the compact internal space of extra dimensions $\mathcal{K}_d$. The eigenfunctions and eigenvalues of the Laplace operator on the internal space $\mathcal{K}_d$ in this special case have quite a simple analytical form

$$\Delta_{\mathcal{K}_d} Y_n(y) = -\frac{\lambda_n}{R^2} Y_n(y), \quad Y_n(y) = \frac{1}{\sqrt{V_d}} \exp \left( i \sum_{m=1}^d n_m y^m / R \right), \quad (5)$$

$$\lambda_n = |n|^2, \quad |n|^2 = n_1^2 + n_2^2 + \ldots + n_d^2, \quad n = (n_1, n_2, \ldots, n_d), \quad -\infty \leq n_m \leq \infty,$$

where $n_m$ are integer numbers, $V_d = (2\pi R)^d$ is the volume of the torus.

We write a harmonic expansion for the multidimensional fields $\Phi_P(z)$ to reduce the multidimensional theory to the effective four-dimensional one

$$\Phi_P(z) = \Phi_P(x, y) = \sum_n \phi_P^{(n)}(x) Y_n(y). \quad (6)$$
The coefficients $\phi_P^{(n)}(x)$ of harmonic expansion are called Kaluza-Klein (KK) excitations or KK modes, and they include the zero-mode $\phi_P^{(0)}(x)$, corresponding to $n = 0$ and the eigenvalue $\lambda_0 = 0$. Substitution of the KK mode expansion into action (4) and integration over the internal space $\mathcal{K}_d$ gives

$$S_I = \int_{M_4} d^4x \left[ g_{D^0 K} (\phi_{D^0}^{(0)}(x))^2 \phi_{K}^{(0)}(x) \phi_{K}^{(0)*}(x) + g_{D_s \eta} (\phi_{\eta}^{(0)}(x))^2 \phi_{D_s}^{(0)}(x) \phi_{D_s}^{(0)*}(x) + g_{D^0 K} \phi_{D^0}^{(0)}(x) \phi_{K}^{(0)}(x) \sum_{n \neq 0} \phi_{D^0}^{(n)}(x) \phi_{K}^{(n)*}(x) + g_{D_s \eta} \phi_{\eta}^{(0)}(x) \phi_{D_s}^{(0)}(x) \sum_{n \neq 0} \phi_{\eta}^{(n)}(x) \phi_{D_s}^{(n)*}(x) \right] + \ldots .$$

The coupling constants $g$ of the four-dimensional theory are related to the coupling constants $G_{(4+d)}$ of the initial multidimensional theory by the equation

$$g_{D^0 K} = \frac{G_{D^0 K}^{(4+d)}}{V_d}, \quad g_{D_s \eta} = \frac{G_{D_s \eta}^{(4+d)}}{V_d},$$

where $V_d$ is the volume of the compact internal extra space $\mathcal{K}_d$. The fundamental coupling constants $G_{(4+d)}$ have dimension $[\text{mass}]^{-d}$. So, the four-dimensional coupling constants $g$ are dimensionless, as it should be.

As we have established before \[17\]

$$\frac{1}{R} = 41.481 \text{MeV} \quad \text{or} \quad R = 24.1 \text{GeV}^{-1} = 4.75 \times 10^{-13} \text{cm} .$$

If we relate the strong interaction scale with the pion mass

$$G_{(4+d)} \sim \frac{10}{[m_\pi]^d},$$

then

$$g \sim \frac{10}{(2\pi m_\pi R)^d} ,$$

and

$$g(d = 1) \sim 0.5.$$

On the other hand

$$g_{\text{eff}} = g_{\pi NN} \exp(-m_\pi R) \sim 0.5, \quad (g_{\pi NN}^2/4\pi = 14.6).$$

So, $R$ has a clear physical meaning: size \[9\] just corresponds to the scale of distances where strong Yukawa forces in strength come close to electromagnetic ones \[14, 17\]. Physically this means that KK modes of strong interacting particles interact weakly. This fact allows us to apply the four-dimensional perturbation theory in the calculations and to use a quantum mechanical Schrödinger wave function to describe the configuration of a compound system with a non-zero KK modes as constituents. By this way in the lowest order over coupling constant in the above model we can easily calculate the decay widths.

As a result one obtains

$$\Gamma(D_{sJ} \rightarrow D^0 K^+) = C_0 \frac{\alpha_{D^0 K}}{M_{D_{sJ}}^2} \left| \psi_{Sch}^{D^0 K^+}(0) \right|^2, \quad \alpha_{D^0 K} = \frac{g_{D^0 K}^2}{4\pi} ,$$

\[13\]
and
\[ \Gamma(D_{sJ} \to D_s^+ \eta) = C_0 \frac{\alpha_{D_s \eta}}{M_{D_{sJ}}^2} \cdot |\psi_{Sch}^{D_s^+ \eta_0}(0)|^2, \quad \alpha_{D_s \eta} = \frac{g_{D_s \eta}^2}{4\pi}, \]
where \( C_0 \) is known, model dependent constant.

Now, let us consider a Coulomb-like wave function of the ground state as a Schrödinger wave function describing the configuration of the non-zero KK modes in a compound system
\[ \psi_C^{D_0 K^+}(\vec{x}) = \frac{1}{\sqrt{\pi}} \left( \frac{1}{2^{a_n}} \right)^{3/2} \exp(-|\vec{x}|/2a_n) = \frac{1}{\sqrt{\pi}} \left( \frac{\alpha_{D^0 K} \mu_n}{2} \right)^{3/2} \exp(-\alpha_{D^0 K} \mu_n |\vec{x}|/2), \]
\[ a_n = \frac{1}{\alpha_{D^0 K} \mu_n}, \quad \mu_n = \frac{m_n^{D^0} m_n^{K^+}}{m_n^{D^0} + m_n^{K^+}}, \quad m_n^{D^0} = \sqrt{m_{D^0}^2 + \frac{n^2}{R^2}}, \quad m_n^{K^+} = \sqrt{m_{K^+}^2 + \frac{n^2}{R^2}}. \]

and the similar wave function for the \( D_{sJ}^+ \eta_0 \) configuration
\[ \psi_C^{D_{sJ}^+ \eta_0}(\vec{x}) = \frac{1}{\sqrt{\pi}} \left( \frac{1}{2^{\tilde{a}_n}} \right)^{3/2} \exp(-|\vec{x}|/2\tilde{a}_n) = \frac{1}{\sqrt{\pi}} \left( \frac{\alpha_{D_{sJ} \eta} \tilde{\mu}_n}{2} \right)^{3/2} \exp(-\alpha_{D_{sJ} \eta} \tilde{\mu}_n |\vec{x}|/2), \]
\[ \tilde{a}_n = \frac{1}{\alpha_{D_{sJ} \eta} \tilde{\mu}_n}, \quad \tilde{\mu}_n = \frac{m_n^{D_{sJ}^+} m_n^{\eta}}{m_n^{D_{sJ}^+} + m_n^{\eta}}, \quad m_n^{D_{sJ}^+} = \sqrt{m_{D_{sJ}^+}^2 + \frac{n^2}{R^2}}, \quad m_n^{\eta} = \sqrt{m_{\eta}^2 + \frac{n^2}{R^2}}. \]

From Eqs. (13-16) it follows for the SELEX state
\[ \frac{\Gamma(D_{sJ} \to D^0 K^+)}{\Gamma(D_{sJ} \to D_s^+ \eta)} = \left( \frac{\alpha_{D^0 K}}{\alpha_{D_s \eta}} \right)^4 \cdot \left( \frac{\mu_{12}}{\tilde{\mu}_s} \right)^3 = 1.194 \cdot \left( \frac{\alpha_{D^0 K}}{\alpha_{D_s \eta}} \right)^4. \]

The SELEX measured value for the ratio \( \Gamma(D_{sJ} \to D^0 K^+)/\Gamma(D_{sJ} \to D_s^+ \eta) \) is obviously achieved if we take
\[ \frac{\alpha_{D^0 K}}{\alpha_{D_s \eta}} = 0.6 \quad \text{or} \quad \frac{g_{D^0 K}}{g_{D_s \eta}} = 0.77. \]

In conclusion of this section, we would like to emphasize that the unusual decay pattern observed by SELEX Collaboration, which is the most problematic in conventional quark models, can easily be explained in the unified picture for hadron spectra. No doubt, it would be very interesting to construct such, more general, model which the ratio (18) appeared to be as an intrinsic feature for.

4 Summary and Discussion

In this article we have presented an analysis of the experimental material recently reported by the SELEX Collaboration. It is shown that the SELEX measurements are excellently incorporated in the unified picture for hadron spectra developed early. The main advantage of our approach to hadron spectroscopy is that all calculated numbers for the masses of hadron states do not depend on a special dynamical model but follow from fundamental hypothesis on existence of the extra dimensions with a compact internal extra space. Our analysis shows that the measured values for the masses of the SELEX state exactly coincide with the calculated masses of the states living in the corresponding KK
towers. This is really a wonderful fact that the SELEX measurements even discriminate the masses of the discovered state for the different configurations which our systematics of hadron spectroscopy ascribe to, even though the existing statistics is not sufficiently well to allow a more reliable statement concerning that discrimination.

In our approach we also find quite a natural but rather model dependent explanation of decay pattern for the SELEX state being dominated by the $D_s^+\eta$ decay mode which is known as a great puzzle in the framework of conventional quark model phenomenology. Unfortunately, there is no other experiment with a confirmation of the SELEX measurements, but we hope that new experiments with better statistics and with higher mass resolution will appear in the near future to confirm these exciting measurements. In this respect we have paid attention to note made by SELEX Collaboration in original paper concerning a one-bin excess in the $D^0 K^+$ invariant mass spectrum at 2636 MeV observed by the CLEO Collaboration. That’s why we have also performed the spectral analysis of the CLEO measurements. The result of our analysis is presented in Fig. 5. As is seen from Fig. 5, there is really irregularity in the $D^0 K^+$ invariant mass spectrum around the $M_{12}^{D^0 K^+}(2630.5)$ spectral line. The peak at 2573 MeV in the $D^0 K^+$ invariant mass spectrum reported by the CLEO Collaboration is near the $M_{11}^{D^0 K^+}(2591)$ spectral line. The other irregularities in the $D^0 K^+$ invariant mass spectrum observed by the CLEO Collaboration are strongly correlated with the calculated spectral lines. The most striking manifestation of such correlation is exact coincidence of the $M_{4}^{D^0 K^+}(2392)$ spectral line with the observed very narrow peak at 2392 MeV. Here, it should be stressed that the similar coincidence has been observed by the SELEX Collaboration as well, see Fig. 3. Figure 3 also shows that the broad peak at 2573 MeV in the $D^0 K^+$ invariant mass spectrum reported by the CLEO Collaboration, in fact, contains two peaks corresponding to the $M_{10}^{D^0 K^+}(2554)$ and $M_{11}^{D^0 K^+}(2591)$ spectral lines. The further study of this signal region with improved statistics and better resolution is also necessary to confirm the double peak structure.

In summary, newly performed experimental studies by the SELEX Collaboration provide new, additional and excellent confirmation of our theoretical conception. Of course, this fact is quite fascinating and encouraging for us.

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Table 1: Kaluza-Klein tower of KK excitations for $DK$ system and experimental data.

| n  | $M^{D\pm K^-}_n$ MeV | $M^{D^0 K^-}_n$ MeV | $M^{DK}_{exp}$ MeV |
|----|-----------------------|----------------------|---------------------|
| 1  | 2369.26               | 2359.98              |                     |
| 2  | 2375.78               | 2366.54              |                     |
| 3  | 2386.53               | 2377.37              |                     |
| 4  | 2401.35               | 2392.28              |                     |
| 5  | 2420.03               | 2411.08              |                     |
| 6  | 2442.33               | 2433.51              |                     |
| 7  | 2468.00               | 2459.32              |                     |
| 8  | 2496.79               | 2488.25              |                     |
| 9  | 2528.45               | 2520.06              |                     |
| 10 | 2562.75               | 2554.51              |                     |
| 11 | 2599.47               | 2591.38              |                     |
| 12 | 2638.42               | 2630.48              |                     |
| 13 | 2679.43               | 2671.64              |                     |
| 14 | 2722.34               | 2714.68              |                     |
| 15 | 2767.00               | 2759.48              |                     |
| 16 | 2813.29               | 2805.90              |                     |
| 17 | 2861.09               | 2853.84              |                     |
| 18 | 2910.32               | 2903.19              |                     |
| 19 | 2960.87               | 2953.86              |                     |
| 20 | 3012.67               | 3005.78              |                     |
| 21 | 3065.64               | 3058.86              |                     |
| 22 | 3119.73               | 3113.06              |                     |
| 23 | 3174.85               | 3168.29              |                     |
| 24 | 3230.98               | 3224.52              |                     |
| 25 | 3288.04               | 3281.69              |                     |
| 26 | 3346.00               | 3339.75              |                     |
| 27 | 3404.82               | 3398.65              |                     |
| 28 | 3464.44               | 3458.37              |                     |
| 29 | 3524.84               | 3518.87              |                     |
| 30 | 3585.99               | 3580.10              |                     |
Table 1: Kaluza-Klein tower of KK excitations for $D_s\eta$ system and the SELEX state.

| n  | $M_{n}^{D_s\eta}$ MeV | $M_{exp}^{D_s\eta}$ MeV |
|----|------------------------|--------------------------|
| 1  | 2518.31                |                           |
| 2  | 2524.30                |                           |
| 3  | 2534.20                |                           |
| 4  | 2547.88                |                           |
| 5  | 2565.18                |                           |
| 6  | 2585.90                |                           |
| 7  | 2609.85                |                           |
| 8  | 2636.82                | 2635.9 ± 2.9             |
| 9  | 2666.58                |                           |
| 10 | 2698.96                |                           |
| 11 | 2733.74                |                           |
| 12 | 2770.75                |                           |
| 13 | 2809.84                |                           |
| 14 | 2850.85                |                           |
| 15 | 2893.64                |                           |
| 16 | 2938.10                |                           |
| 17 | 2984.11                |                           |
| 18 | 3031.58                |                           |
| 19 | 3080.41                |                           |
| 20 | 3130.53                |                           |
| 21 | 3181.86                |                           |
| 22 | 3234.32                |                           |
| 23 | 3287.87                |                           |
| 24 | 3342.44                |                           |
| 25 | 3397.99                |                           |
| 26 | 3454.46                |                           |
| 27 | 3511.81                |                           |
| 28 | 3570.00                |                           |
| 29 | 3629.00                |                           |
| 30 | 3688.76                |                           |
Figure 1: (a) $M(KK\pi^\pm \eta) - M(KK\pi^\pm)$ mass difference distribution. Charged conjugates are included. The shaded region is the event excess used in the estimation of signal significance. Results for the fit shown see in original paper [8]. (b) Mass difference distribution for mixed events as described in the original paper.
\[ \Delta M = M(K^-\pi^+K^+) - M(K^-\pi^+) \]

Figure 2: (a) $M(K^-\pi^+K^+) - M(K^-\pi^+)$ mass difference distribution. Charged conjugates are included. The shaded regions are the event excesses used in the estimation of signal significances. Results for the fit shown see in original paper [8]. (b) Wrong sign background $D^0 K^-$ events, as described in the original paper.
Figure 3: The $D^0 K^+$ invariant mass spectrum presented in Ref. [9]. The vertical (spectral) lines correspond to KK tower for $D^0 K^+$ system; see Table 1.
Figure 4: The $D_s^+\eta$ invariant mass spectrum presented in Ref. [9]. The vertical (spectral) lines correspond to KK tower for $D_s^+\eta$ system given by Table 2.
Figure 5: “Corrected” invariant mass of \((K^-\pi^+\pi^0)K^+)\) combinations presented in Ref. [18]. Data points are for \(K^-\pi^+\pi^0\) combinations in the \(D^0\) signal region; the histogram shows invariant mass of \((K^-\pi^+\pi^0)K^+)\) combinations where the \(K^-\pi^+\pi^0\) combinations were chosen in \(D^0\) sidebands. The vertical (spectral) lines correspond to KK tower for \(D^0K^+\) system given by Table 1.