A Virgo Environmental Survey Tracing Ionised Gas Emission (VESTIGE). XI. Two dimensional Hα kinematics of the edge-on ram pressure stripped galaxy NGC 4330*

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ABSTRACT

Using the VESTIGE survey, a deep narrow-band Hα imaging survey of the Virgo cluster carried on at the CFHT with MegaCam, we have discovered a long and diffuse tail of ionised gas in the edge-on late-type galaxy NGC 4330. This peculiar feature witnesses an ongoing ram pressure stripping event able to remove the gas in the outer region of the disc. Tuned hydrodynamic simulations suggest that the ram pressure stripping event is occurring almost face-on, making NGC 4330 the ideal candidate to study the evo-

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1. Introduction

The environment plays a major role in shaping galaxy evolution (e.g. Peng et al. 2010). Rich clusters are dominated by quiescent systems (morphology-segregation effect, Dressler 1980), and the few spirals which inhabit these environments are generally deficient in all the components of their interstellar medium (HI - Cayatte et al. 1990, Solanes et al. 2001, Gavazzi et al. 2005, Chung et al. 2009; H2 - Fumagalli et al. 2009, Boselli et al. 2014a; dust - Cortese et al. 2010, Cortese et al. 2012b, Longobardi et al. 2020). Their star formation is also reduced with
Different physical mechanisms have been proposed in the literature to explain these results, as extensively reviewed in Boselli & Gavazzi (2006, 2014). These include gravitational interactions with other cluster members (Merritt 1983) or with the gravitational potential well of the cluster (Byrd & Valtonen 1990), including their combined effect, generally called galaxy harassment (Moore et al. 1998), or the hydrodynamic interaction of the interstellar medium (ISM) of galaxies with the hot and dense intracluster gas (thermal evaporation, Cowie & Songaila 1977; starvation, Larson et al. 1980; ram pressure stripping, Gunn & Gott 1972).

Several observations seem to indicate that while gravitational perturbations are dominant in relatively small systems such as groups thanks to the low velocity dispersion of galaxies within the dark matter halo, hydrodynamic interactions are dominant in massive clusters, where the high velocity dispersion and the density of the gas make them particularly efficient (e.g. Vollmer et al. 2001; Boselli et al. 2008a,b, 2014a; Gavazzi et al. 2013). It is nevertheless clear that all these different perturbing mechanisms might be acting jointly on galaxies in rich environments, with different effects on their evolution.

Despite this evolutionary picture is becoming clearer, there are still several aspects of the galaxy transformation in rich environments which are still poorly known. Among these is the fate of the ISM removed during a ram pressure stripping event. The stripped ISM is often observed in spectacular tails of atomic (e.g. Chung et al. 2007), ionised (e.g. Gavazzi et al. 2001; Yagi et al. 2010; Boselli et al. 2016a), or hot gas (e.g. Sun et al. 2007). Indeed, it has been observed that while in some objects the stripped gas collapses within the tail to form giant molecular clouds (Jáchym et al. 2014, 2017, 2019; Moretti et al. 2020) and stars (Fossati et al. 2016; Poggianti et al. 2019), in other systems it remains diffuse (e.g. Boissier et al. 2012). In these objects the cold gas stripped from the disc does not form stars because it changes of phase becoming first ionised than hot (e.g. Boselli et al. 2016a). The detailed analysis of a few nearby systems with high quality multifrequency spectroscopic and imaging data seem to suggest that the gas collapses into giant molecular clouds only whenever its velocity dispersion is sufficiently low (e.g. Fossati et al. 2016; Boselli et al. 2018b). Otherwise, the cold gas changes of phase for heat conduction, turbulence, magneto-hydrodynamic waves or shocks. It thus becomes first ionised and then hot gas visible in the X-rays spectral domain (Gavazzi et al. 2001; Sun et al. 2007; Fossati et al. 2016; Boselli et al. 2016a). The kinematics of the stripped gas is thus a crucial parameter for understanding the process of star formation within the tails. Observations and simulations of ram pressure stripped galaxies consistently indicate that a ram pressure stripping process is able to drastically perturb the velocity field of a spiral galaxy, displacing its kinematic centre with respect to the photometric one, and affecting its rotation curve (Vollmer et al. 2001, 2004, 2006, 2008; Kronberger et al. 2008; Merluzzi et al. 2013; Fumagalli et al. 2014; Consolandi et al. 2017; Sheen et al. 2017; Bellhouse et al. 2017, 2019; Boselli et al. 2021b). Despite the advent of extraordinary new instrumentation in the optical (MUSE at the VLT) and radio (ALMA) domain, multifrequency data at high angular and spectral resolution of nearby ram pressure stripped galaxies, where the proximity allows us to study the star formation process down to the scale of individual HII regions, are only sporadically available. To understand the effects of the stripping process on the kinematics of the gas, observations and simulations of galaxies spanning a wide range in stellar mass and with different impact parameters are necessary. A particularly interesting case are edge-on galaxies which allow us to investigate as the gas is stripped out from the disc plane in the z-direction. Among these, there are only a few systems undergoing a ram pressure stripping event which have been studied in detail: UGC 6697 in the cluster A1367 (Consolandi et al. 2017), and JO204 in the further cluster A957 (z ∼ 0.052, Gullieuszik et al. 2017). The kinematic properties of the stripped gas are significantly different in the two galaxies given their impact parameters, UGC 6697 suffering an edge-on stripping event while JO204 a face-on one.

The Virgo Environmental Survey Tracing Ionised Gas Emission (VESTIGE, Boselli et al. 2018c), a blind Hz narrow-band imaging survey of Virgo, allows us to identify a large number of galaxies undergoing a ram pressure stripping event in the closest cluster of galaxies. The presence of ionised gas tails without any stellar counterpart is the clearest demonstration that a system is undergoing a hydrodynamic interaction with the surrounding environment. Indeed, gravitational perturbations act differently on the stellar and gaseous components, also producing tidal structures visible in deep broad-band imaging data. Several galaxies undergoing a ram pressure stripping event have been identified so far (Boselli et al. 2016a, 2018a, 2021b; Fossati et al. 2018; Longobardi et al. 2020; Junais et al. 2021). Among them, NGC 4330 is an edge-on galaxy with evident tails of atomic (Chung et al. 2007, 2009) and ionised (Abramson et al. 2011; Fossati et al. 2018) gas indicating that the galaxy is entering the cluster from the south and impacting the intra cluster medium almost face on. It is thus an ideal candidate to study in great detail the effects of the perturbation on the kinematics of the perturbed gas on the disc of the galaxy and within the tail.

In this paper we use two different sets of high spectral resolution (up to R = 10000) Fabry-Perot (FP) data to study the kinematics of the ionised gas in this representative object. We combine these new sets of observations with tuned hydrodynamic simulations developed to reproduce the structural and spectro-photometric properties of this galaxy (Vollmer et al. 2021). Unfortunately most of the gas in the tail has a very low surface brightness, and thus it is out of reach of our dedicated observations which detect only a few, high surface brightness star forming regions. The proximity of the galaxy (16.5 Mpc, Gavazzi et al. 1999; Mei et al. 2007), however, and the adopted spectral resolution allow us to study in detail the kinematics of the gas of these compact regions down to scales of ∼100 pc in angular resolution and velocity dispersion σ ∼ 13 km s⁻¹. Such resolution unreached in the more distant UGC 6697 and JO204 despite their extraordinary MUSE data.

The paper is structured as follows: in Sect. 2 we describe the MegaCam CFHT narrow-band imaging and the FP high-resolution spectroscopic observations along with the multifrequency data used in the analysis. In Sect. 3 and 4 we analyse the imaging data and construct the rotation curve of the galaxy, while in Sect. 5 we study the 2-D kinematics of the gas and the specific kinematics of the gas out of the plane in Sect. 6. The results of the analysis are compared to the hydrodynamic simulations of Vollmer et al. (2021) in Sect. 7. The discussion and conclusions are given in Sect. 8.
Table 1. General properties of NGC 4330

| Variable | Value | Ref. |
|----------|-------|------|
| Alternative names (1) | HRS T24 | a |
| α (J2000) (2) | 12h23m17.10s | b |
| δ (J2000) (3) | +11°22'57".7 | b |
| Type | Scd | a |
| Distance | 16.5 Mpc | c |
| Dist. from M87 (4) | 2.1 ′ / 604.8 kpc | TW |
| D25 | 5.86 arcmin / 28.1 kpc | d |
| lpsys | 11.925 mag | d |
| Vhelio | 1551 km s^{-1} | TW |
| Vmax | 140 km s^{-1} | e |
| PA | 59±1° | TW |
| Morphological Inclination | 89.5° | TW |
| Mstar | 5 × 10^9 M_☉ | f |
| M(HI) | 4.73 ± 1.04 × 10^8 M_☉ | e |
| M(H2) | 1.27 ± 0.05 × 10^8 M_☉ | g |

Notes. (1) UGC 7456, VCC 630, IRAS 12207+1138. (2) Right ascension and (3) declination of the galaxy. (4) Projected distance from M87.

References. TW: This work; a: NED; b: NGVS, Ferrarese et al. (2012); c: Gavazzi et al. (1999); Mei et al. (2007); d: Cortese et al. (2012a); e: Chung et al. (2009); f: Boselli et al. (2015); g: Lee et al. (2017).

2. Observations and data reduction

2.1. VESTIGE narrow-band imaging

Deep Hα narrow-band imaging of NGC 4330 has been gathered with MegaCam at the CFHT by the VESTIGE survey (Boselli et al. 2018c), and the first imaging results have been presented in Fossati et al. (2018). Briefly, the galaxy has been observed using the narrow-band filter MP9063 (λc = 6591 Å; Δλ = 106 Å), which includes the Hα line and the two [NII] lines at λ 6548 Å and λ 6584 Å. The galaxy has been observed during the main VESTIGE survey but also during a pilot run. The data used in this work are the combination of these two datasets, and are thus deeper than those analysed in Fossati et al. (2018), which were limited to the pilot observations. To secure the subtraction of the stellar continuum, the galaxy was also observed in the broad r-band filter with shorter exposures to avoid saturation within the nucleus. The different exposures were gathered using a large dither pattern (15 arcmin in RA and 20 arcmin in Dec) to minimise the effects of scattered light and reflections in the flat-fielding, thus optimising the detection of low surface brightness features in the ionised gas component. A slightly different pattern was used for the pilot observations. The total integration time is 21060 sec in the narrow-band filter and 1506 sec in the broad-band r filter. These deep observations have a typical sensitivity of f(Hα) ∼ 2 × 10^{-17} erg s^{-1} cm^{-2} (5σ) for point sources and 5σ(Hα) ∼ 10^{-18} erg s^{-1} cm^{-2} arcsec^{-2} (1σ after smoothing the data to ∼ 3 arcsec resolution) for extended sources, thus approximately a factor of 1.7 deeper than the mean sensitivity of VESTIGE. NGC 4330 has been observed during excellent seeing conditions (FWHM = 0.73 arcsec).

The data have been reduced using Elixir-LSB (Ferrarese et al. 2012), a data reduction pipeline especially designed to detect the diffuse emission of extended low surface brightness structures formed during the interaction of galaxies with their surrounding environment (e.g. Boselli et al. 2016b). The astrometric and photometric calibrations have been done using the SDSS and the PanSTARRS surveys by means of the MegaPipe pipeline (Gwyn 2008). The typical photometric uncertainty in the data is of 0.02-0.03 mag in both bands. The emission of the ionised gas has been derived after subtracting the stellar continuum as described in Boselli et al. (2019). This is done by combining the r-band image with the g-band frame obtained during the Next Generation Virgo Cluster Survey (NGVS, Ferrarese et al. 2012) to take into account the colour of the stellar continuum (Spector et al. 2012).

2.2. Fabry-Perot spectroscopy

High resolution IFU spectroscopy was obtained using two different FP interferometers, PUMA attached at the 2.1 m telescope at San Pedro Mártir, Baja California, México, and GHASP, mounted on the 1.93 m telescope at the Observatoire de Haute Provence, France. A detailed description of the instruments, of the observations, and of the data reduction is given in Appendix A. Briefly, PUMA is a FP spectrograph with a field of view of ∼10×10 arcmin^2 coupled with a 512×512 CCD camera, with pixel size of 1.27×1.27 arcsec^2 and has a spectral resolution R ∼ 6500. GHASP has a field of view of 5.8×5.8 arcmin^2, is coupled with a 512×512 Image Photon Counting System with pixel size of 0.68×0.68 arcsec^2, and has a spectral resolution R ∼ 10000.

2.3. Multifrequency data

The FP data were combined to those gathered during other surveys of the Virgo cluster for the following analysis, as extensively described in Fossati et al. (2018). Namely, the distribution of the old stellar population has been derived using the very deep i-band image obtained during the NGVS survey of the cluster (Ferrarese et al. 2012). HI data are also available from the Imaging of Virgo Spirals in Atomic Gas (VIVA), gathered at the VLA in C-short array configuration (Chung et al. 2009). These data have been obtained with a velocity sampling of ∼ 10 km s^{-1} and a spatial resolution of ∼ 15 arcsec. The same set of data has been combined with other VLA data obtained in D configuration to increase the sensitivity to low column density, extended features by Abramson et al. (2011) with a final velocity resolution of 10.4 km s^{-1} and an angular resolution of FWHM of 26.4×24.0 arcsec^{-2}. 12CO (2−1) observations obtained with the Submillimeter Array (SMA), with a synthesized beam of 6.35×4.47 arcsec^2 and a spectral resolution of 5 km s^{-1}, have been presented in Lee et al. (2017).

3. Imaging analysis

3.1. Stellar distribution

The NGVS i-band image (see Fig. 1) traces the bulk of the stellar population and can thus be used to derive the main morphological and structural parameters of the galaxy. It can also be used as reference to identify the different ionised gas structures removed from the disc plane during the interaction. This image shows an...
edge-on disc with a symmetric and unperturbed stellar distribution down to the surface brightness limit of the NGVS survey ($\mu_i = 27.4$ mag arcsec$^{-2}$), proving that the galaxy is not undergoing any major gravitational perturbation. The galaxy has a small bulge without any evident bar (e.g. Abramson et al. 2011).

We derived the photometric properties of the stellar disc through the elliptical isophote fitting method using the IRAF task ellipse (Jedrzejewski 1987). In this task, ellipses are drawn to match the isophotes. The surface brightness along the ellipse is expanded in Fourier series:

$$\mu(\theta) = \mu_0 + \sum_{n=1,0} (A_n \sin n\theta + B_n \cos n\theta),$$

where $\mu_0$ is the surface brightness averaged over the ellipse, $A_n$ and $B_n$ the higher order Fourier coefficients. If an isophote is a perfect ellipse, all the $A_n$ and $B_n$ coefficient would be exactly zero. The term $A_1$, $B_1$, $A_2$, and $B_2$ indicate errors in the fitting procedure. The terms $A_3$ and $B_3$ give the “egg-shaped” isophote but the most interesting is $B_4$. If $B_4 > 0$, the galaxy has discy-shape isophotes, while if $B_4 < 0$ boxy-shaped isophotes. The ellipse task provides a number of parameters that describe the surface brightness of the galaxy as a function of the semi-major axis, including the ellipticity ($\varepsilon$), the position angle (PA), morphological inclination ($i_{\text{ph}}$), and the fourth degree cosine coefficient of the Fourier series ($B_4$) parameters. The red and blue vertical lines correspond to the ellipses drawn on the upper left panel and indicate the radius $R_{23.5}(i)$ ~145.0 arcsec (~11.6 kpc) and the effective radius $R_\text{e}(i)$ ~66.5 arcsec (~5.3 kpc), respectively.

The isophotal radius derived using the NGVS $i$-band image is $R_{23.5}(i)$ ~145.0 arcsec (~11.6 kpc). This value is slightly larger than the one derived by Cortese et al. (2012a) using the SDSS-$i$-band image ($R_{23.5}(i)$ ~140.3 arcsec, ~11.2 kpc). The difference with this previous measurement might be due to the presence of dust and to the adopted sigma clipping selection in the ellipse task. The effective radius is $R_\text{e}(i)$ ~66.5 arcsec (~5 kpc) and the effective surface brightness $\mu_\text{e}(i)$ ~21.5 mag arcsec$^{-2}$, consistently with those given in Cortese et al. (2012a). The photometric centre is located at RA(J2000) = 12h23m17.26s, Dec = +11°22'07".9, with an uncertainty of ~0.9 arcsec (~76 pc). The sudden jump of the parameters observed in Fig. 1 at a...
Fig. 2. Images of NGC 4330. Panel (a): full resolution VESTIGE continuum-subtracted Hα narrow-band image of NGC 4330 in linear scale. Panel (b): NGVS i-band image. Panels (c) and (d): PUMA and GHASP FP Hα monochromatic images, respectively. Panel (e): NGVS i-band image, the dashed black ellipse and the inner ellipse indicate the mean surface brightness, $\mu(i)=23.5$, and the effective surface brightness, $\mu_e(i)=21.5$ mag arcsec$^{-2}$, respectively, the green bullet indicates the photometric centre (see Fig. 1). Panel (f): NGVS i-band image with marks on the zones of interest labelled by letters (A to M) and two parallel dashed lines at $r_0 \sim 30$ arcsec ($\sim 2.4$ kpc) which divide the galactic disc in three main regions: northeastern, central and southwestern (see Sect. 4). The cyan, red, and blue contours in panels (a) to (d) show the VESTIGE surface brightness $\Sigma(H\alpha)=0.3, 1.0$ and $12.5 \times 10^{-17}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$, respectively. The red contours in panel (e) and (f) show the VESTIGE surface brightness $\Sigma(H\alpha)=10^{-17}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$. The blue contours in the lowest left (e) panel indicate the surface brightness limit of PUMA $\Sigma(H\alpha) \sim 3.5 \times 10^{-17}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$, while those in panel (f) show the surface brightness limit of GHASP $\Sigma(H\alpha) \sim 10^{-17}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$.
radius of $\sim 5$ arcsec ($\sim 0.4$ kpc) is due to the dusty region around the centre.

Consistently with e.g. Barbosa et al. (2015) we estimate the morphological inclination of the galaxy using the relation:

$$\cos i_{ph} = \frac{1 - \epsilon^2}{1 - q_0^2},$$

where $q_0 = 0.1$ is the intrinsic flattening of a Scd galaxy (Haynes & Giovanelli 1984). Figure 1 indicates that the morphological inclination of the galaxy converges to $i_{ph} = 89.5^\circ$ and the photometric position angle to $PA_{ph} = 59 \pm 1^\circ$ for $R(i) \geq 75$ arcsec ($\sim 6$ kpc) and the rising and the flat part of the RC (see Fig. 3). The output of the fit of Eq. B.4 (ETM) provides similar terminal velocities reached around $\sim 30$ arcsec ($\sim 2.4$ kpc) as well as similar transition radii between the rising and the flat part of the RCs compared to the IPM method. After this transition, the RCs computed with the ETM method are flat up to the photometric radius $R_{23.5} \sim 140$ arcsec ($\sim 11.6$ kpc), which is represented by the red line in panels (a) and (c) of Fig. 3. The approaching and receding sides of the RCs measured with the IPM method start to differ at a radius of $\sim 30$ arcsec ($\sim 2.5$ kpc) and do not have a single point of matching where the two sides overlap until the photometric radius. The rotation velocity obtained for each dataset with the ETM differs by $8$ km s$^{-1}$. This number is within the uncertainties due to the differences in resolution between the two instruments and could be partly due to a different threshold for the PVD extraction.

Both datasets show that the RCs derived using the ETM are steeper than those derived using the IPM in the inner regions. This difference is due to the fact that the inner regions and the HI regions are situated (see Fig. 2). The H$\alpha$ disc is truncated with respect to the stellar disc, indicating that the activity of star formation has been quenched at its edges (Fossati et al. 2018). The northeastern edge of the star forming disc is bent to the south in a "hook" shaped structure (e.g. Abramson et al. 2011), while a few compact and bright HI regions are present just south to the stellar disc at the southwestern edge. There is also a very extended, diffuse and low surface brightness emission ($\Sigma(H\alpha) \sim 3 \times 10^{-18}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$) towards the south of the disc, produced by the gas stripped during the ram pressure episode, indicating that the galaxy is moving northward on the plane of the sky within the cluster. Associated to the diffuse emission, the VESTIGE H$\alpha$ image shows a few weak compact regions with a surface brightness over $\Sigma(H\alpha) \sim 10^{-17}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$, which corresponds to the GHASP detection limit where star formation is probably taking place. Not all the features located out of the plane of the galaxy and above this threshold limit are clearly detected in the monochromatic FP image of NGC 4330, neither in the PUMA data nor in the deeper GHASP data. The bent region at the northeast edge of the disc (labelled A in Fig. 2f) is clearly detected as well as a few regions on the southwest extension of the disc (labelled K, L and M). A few ionised regions have also been detected at the limit of the stellar disc (labelled E, G and F).

4. Rotation curves

Rotation curves (RC) of galaxies are generally derived using the intensity-peak method (IPM). However, this method hardly works for galaxies with high inclination because the velocity profiles in a given position integrate the whole flux along the line of sight (LoS), thus at different radii on the galactic plane (Sancisi & Allen 1979; Swaters et al. 1997; García-Ruiz et al. 2002; Rosado et al. 2013). To overcome this problem, two different methods are generally used: the envelope-tracing method (ETM) (Sofue et al. 1997, 1999a; García-Ruiz et al. 2002) and the iteration method (ITM) (Swaters et al. 1997; Takamiya & Sofue 2000; Heald et al. 2006, 2007). We apply these three methods to NGC 4330, following the methodology described in detail in Appendix B.

4.1. Comparison of the three methods

Figure 3 shows the RCs derived using three different methods: (1) the intensity-peak method (IPM), (2) the envelope-tracing method (ETM), and (3) the iteration-method (ITM). The RCs obtained with the IPM using both GHASP and PUMA datasets show that NGC 4330 is rotating as a solid-body inside a radius of $\sim 30$ arcsec. This particular shape becomes more evident in the saturated boxy shape outside of the effective radius, $R_e$ shown in Fig. 1. This particular shape becomes more evident in the saturated boxy shape outside of the effective radius, $R_e$ shown in Fig. 1. This phenomenon might explain the peculiar morphology of NGC 4330, with a boxy shape particularly evident in the southwest direction, where the stripping is more efficient. Unfortunately, we lack of stellar kinematical data necessary to further probe this scenario.

3.2. Warm gas distribution

The ionised gas distribution of NGC 4330 has been extensively described in Fossati et al. (2018). Here we summarise the main characteristics, referring the reader to that work for a more detailed description. The ionised gas emission mainly comes from the inner regions of the stellar disc, where most of the HII regions are situated (see Fig. 2). The H$\alpha$ disc is truncated with respect to the stellar disc, indicating that the activity of star formation has been quenched at its edges (Fossati et al. 2018). The northeastern edge of the star forming disc is bent to the south in a "hook" shaped structure (e.g. Abramson et al. 2011), while a few compact and bright HI regions are present just south to the stellar disc at the southwestern edge. There is also a very extended, diffuse and low surface brightness emission ($\Sigma(H\alpha) \sim 3 \times 10^{-18}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$) towards the south of the disc, produced by the gas stripped during the ram pressure episode, indicating that the galaxy is moving northward on the plane of the sky within the cluster. Associated to the diffuse emission, the VESTIGE H$\alpha$ image shows a few weak compact regions with a surface brightness over $\Sigma(H\alpha) \sim 10^{-17}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$, which corresponds to the GHASP detection limit where star formation is probably taking place. Not all the features located out of the plane of the galaxy and above this threshold limit are clearly detected in the monochromatic FP image of NGC 4330, neither in the PUMA data nor in the deeper GHASP data. The bent region at the northeast edge of the disc (labelled A in Fig. 2f) is clearly detected as well as a few regions on the southwest extension of the disc (labelled K, L and M). A few ionised regions have also been detected at the limit of the stellar disc (labelled E, G and F).
Fig. 3. PUMA (panels (a) and (b)) and GHASP (panels (c) and (d)) rotation curves (RCs, panels (a) and (c)) and position-velocity diagrams (PVDs, panels (b) and (d)) of NGC 4330. Panels (a) and (c): RCs derived using three different methods: the intensity-peak method (IPM), the envelope-tracing method (ETM) and the iteration-method (ITM) (see Sect. B.3 for a further explanation on the methodology). In the case of the IPM (black squares), an azimuthal sector of $\pm 5^\circ$ in the sky plane has been taken around the major axis to compute the rotation velocities; filled and empty black squares indicate the receding and the approaching sides, respectively. The upper-left insert shows the azimuthal sector and the location of the regions pointed on the rotation curve. For the ETM, the rotation velocities have been derived from the PVD along the major axis ($z=0$), shown in panels (b) and (d). The terminal velocities are represented by empty blue circles for the receding side and the filled red circles for the approaching side; the pink line corresponds to the best fit to these points using $\eta = 0.3$ and the shaded area shows the rotation velocity amplitude for $0.2 < \eta < 0.5$ (see relation B.3). The ITM based on a cylindrical model (described in section B.4) is represented by the black line. Panels (b) and (d): position-velocity diagram (PVD) showing the LoS velocities computed with the ETM from which the rotation velocities are computed in the upper panels. On top of the PVD, the filled red and open blue circles indicate the extremal LoS velocities and the pink curve the arctan model fitting those velocities. The red and blue vertical dotted lines in panels (a) and (c), show the photometric and effective radius, the gray dotted lines in the lowest panels the position of $r_0$, see Sect. 4.

The IPM allows us to analyse the rotation on both the receding and approaching sides of the galaxy using the pixels within an angular sector of $\pm 5^\circ$ from the kinematic major axis. The RCs derived using this method show that at the photometric radius the rotation starts to be dominated by streaming motions. This occurs mainly at the southwest of the disc, where the gas has non-circular motions and where the ionised disc is distorted, hence the projection parameters may be no longer valid in these outer regions. These peculiar motions are not modelled with the other methods. Nevertheless, the ETM and ITM withdraw the effects of LoS to the rotational velocity on the inner disc, and allow us to obtain a reliable measure of the maximal rotational velocity of the galaxy, we fix it to the average value of the four velocities given in table B.1 for the two datasets: $V_{\text{rot}}^{\text{max}} = 158 \pm 12$ km s$^{-1}$, the uncertainty being estimated from their standard deviation.

4.2. The dynamical mass

The RC of NGC 4330 is not flat because the gas which is used as a mass-tracer is subject to ram pressure stripping outside the transition radius. Indeed, taking into account its size, morphological type, and maximum rotation velocity, NGC 4330 is an intermediate-mass galaxy which is supposed to have a flat RC (e.g. Sofue et al. 1999b), as previously described in this section by different models. Despite the fact that NGC 4330 is located at $\sim 0.4$ virial radius (e.g. Chung et al. 2007), its mass distribution is probably not affected by the cluster environment (Amram et al. 1992) and can be considered as unperturbed. Following Lequeux (1983), the actual dynamical mass of a spiral galaxy that contains a stellar disc, a spherical stellar bulge, and a spherical dark matter halo ranges in the mass interval given by $M(R) = \alpha RV_{\text{rot}}^2(R)/G$, where $G$ is the universal gravity constant, $V_{\text{rot}}$ the flat rotation velocity, and $\alpha$ a parameter spanning between the lowest value of $\alpha=0.6$ for a galaxy dominated by a flat disc component and the higher value of $\alpha=1.0$ if the spherical halo components dominate. Inside the radius
4.3. Comparison with HI and CO kinematical data

The HI velocity field is the most extended with a diameter of 2.4 kpc (20 arcsec). The RCs from CO and HI data were computed using the envelope-tracing method (ETM). The solid lines correspond to the best fit to the points of the terminal velocity using \( \eta = 0.3 \) and the shaded areas show the rotation velocity amplitude for \( \eta = [0.2, 0.5] \) (see relation B.3); the dashed lines show the RC computed from the PVD maximum intensity. For comparison, the Hz RCs shown in Fig. 3 are also reported on this plot. The green, purple and red vertical lines indicate the CO extension (~47 arcsec, ~3.8 kpc), the HI radius (~141 arcsec, ~11.3 kpc) and the outermost photometric radius (~2.5 ~145 arcsec, ~11.6 kpc), respectively. The top-right insert indicates the angular resolution of each dataset.

In Fig. C.4 we show the HI velocity fields from Chung et al. (2009) and the RCs we derive from the PVDs. As expected, due to beam smearing effects, the RCs obtained using the minimum and maximum HI surface brightnesses strongly differ. These data are used to compute the ETM RC, which matches fairly well with the Hz RC, except maybe within the first two kpc where the HI velocities have probably been overestimated (see Fig. 4). The comparison between the Hz, HI and CO velocity fields are shown in Figs. C.5 and C.6 in the appendix, respectively. The HI velocity field is the most extended with a diameter of ~282 arcsec (~22.6 kpc) from the northeastern top to the southwestern lower corner. The Hz velocity field has been measured on a diameter of ~205 arcsec (~16.4 kpc), and exceeds by more than a factor of two the one derived using CO data (~94 arcsec, ~7.5 kpc). Despite these differences, they might be partly due to the different sensitivities in the data combined with their different angular resolution (15 arcsec for HI and 6 arcsec for CO), the three velocity fields are comparable indicating a similar kinematics of the three components. We applied the ETM to the \(^{12}\)CO \((2−1)\) data of Lee et al. (2017) and to the HI data of Chung et al. (2009, see Sect. 2.3). From the PVD extracted with a pseudo-slit along the kinematic major-axis we computed the terminal velocity using \( \eta = 0.3 \), where \( \eta \) is a fraction of maximum intensity to set the envelope intensity (see Eq. B.3), then we fitted the data to the arc-tangent model (Eq. B.4) using a least-squares minimization. The HI, CO, and Hz ETM RCs are plotted in Fig. 4, which also includes the Hz RCs computed with the ITP and the IPM.

For the HI RC, the ETM fit gave a coefficient of determination \( R^2 = 0.97 \), with \( r_0 = 18 \pm 2 \) arcsec and \( r_t = 6 \pm 2 \) arcsec, providing a RC with the steepest inner slope, represented by the purple line in Fig. 4. This cuspy inner slope is due to the low HI surface brightness, which is reached thanks to the low-resolution VLA HI data (~25 arcsec, see Sect. 2.3). This low-resolution induces strong beam smearing effects and an overcorrection at the lowest HI levels. The ETM provides also a way to estimate of the lowest limit of the RC using only the maximum intensity \( I_{\text{max}} \) (in setting \( \eta = I_{\text{min}} = 0 \), see Eq. B.3). This curve is represented by the purple dashed line in Fig. 4. The interval between the two purple lines delimits all the possible HI RCs that could be computed and gives inner slopes compatible with those obtained at other wavelengths. The maximum HI rotation velocity is ~10 km s\(^{-1}\) lower than those obtained with Hz data.

Since the CO is detected only in the inner regions, we have extrapolated its ETM rotation curve, using the parameters of the arc-tan function fitted to the observational data (see Sect. B.3), to compare it with those computed at other wavelengths using the same method. The CO RC is represented by the green dashed line in Fig. 4. The fit of the CO RC, which was obtained with a coefficient of determination \( R^2 = 0.76 \), gave \( r_0 = 19 \pm 10 \) arcsec, and the transition between the rising and flat part of the RC at \( r_t = 33 \pm 26 \) arcsec. These values are in agreement with those determined for the Hz RCs using the same method. At large radii, the CO and the HI RCs match and have values ~10 km s\(^{-1}\) lower than those derived using the Hz data. We recall, however, that due to its limited extension, we do not have any observational constrain on the CO RC outside ~50 arcsec (~0.4 kpc).

5. Kinematical analysis from 2D-maps

5.1. Velocity fields

Figure 5 shows the velocity field of the ionised gas of NGC 4330 derived from the PUMA and GHASP FP datasets. The velocity fields of both datasets are consistent, they display a rigid body shape inside of a diameter of ~60 arcsec (~4.8 kpc) where the isovelocitiy contours are parallel to the minor axis of the galaxy. We have traced on these maps two parallel dashed lines at at \( r_0 \pm 30 \) arcsec (±2.4 kpc) enclosing this region. Outside this diameter the isovelocitiy contours become almost parallel to the major axis in the NW hook (region A) and on its counterpart, between regions I and J. However, these maps differ in the southern tail, i.e. in regions K and L (see point 5 hereafter). These last features are the signature of non-circular motions that indicate strong perturbations in the kinematics of the regions affected by the stripping process. The two-dimensional velocity field obtained with high-resolution FP interferometry allows us also to measure the kinematic of the gas in very low surface brightness regions. These regions, labelled from A to M, were previously identified in the monochromatic image and they are now identified on the velocity fields of both datasets in Fig. 5. Our data allowed us to measure the LoS velocities of the stripped gas in the southern re-
regions, in region F located on the compression front on the north-easter side of the disc and in region I on the disc plane. The LoS velocities of all these regions are given in Table 5.4 and will be further discussed in Sect. 5.4.

We notice that:

1. The detection and estimation of the LoS velocity of the gas over the galaxy disc, and more specifically inside of regions A, E, F, G, H, I, J (westernmost part) are robust since derived from independent sets of data with consistent results.

2. The gas inside region A has a radial velocity of $\sim 1650 \text{ km s}^{-1}$ that is $\sim 25 \text{ km s}^{-1}$ lower than that of the disc at the same galactocentric distance.

3. If we compare the radial velocities in the southeastern regions of the galaxy with respect to the radial velocities in the plane of the disc, either with respect to their position perpendicular to the plane of the disc, or with respect to the direction of the wind as deduced from the direction of the tails and predicted by simulations (e.g. Vollmer et al. 2021), we notice that the gas inside regions B, C, D, and H has radial velocities significantly different ($\sim \pm 50 \text{ km s}^{-1}$) from those measured in the disc from where the gas has been stripped. The radial velocities of the gas regions B, D, and C is lower than that in the disc, while the radial velocity measured in region H is higher by the same amount.

4. Despite the fact that they are above the FP detection limit observed in other regions, the detection of ionised gas in regions B, C, D, M and in the western tail of region J could have been affected by night skyline contamination. Indeed, the night-line subtraction was very difficult because the OH-radical has a strong emission line at $\sim 6597 \text{ Å}$ (e.g. Osterbrock et al. 1996), roughly at the same wavelength that the redshifted H$\alpha$ emission. On the other hand, the LoS velocity of the gas in region H, which has been probably originated in the approaching side of the disc, is lower and different from the LoS velocity of the gas measured in regions B, C, and D, in the receding side. This difference suggests that these estimates are not largely affected by the residual night skyline emission.

5. Ionised gas is detected in regions K and L with similar LoS velocities. A velocity difference of $\sim 50 \text{ km s}^{-1}$ is observed between both datasets, which could also be due to parasitic night skylines. Thus, despite this difference in the two datasets, these velocities are consistent with streaming motions caused by the stripping process.

6. The gas detected in regions J, K, and H has a velocity similar to that measured at the edge of the rotating disc ($1450 \text{ km s}^{-1}$), suggesting a flat rotation curve.

7. Figure 5 also shows two dashed lines at $\sim 30 \text{ arcsec}$ ($\sim 2.4 \text{ kpc}$) from the galactic centre. These dashed lines enclose the disc section where the ionised gas shows a rigid-body rotation as indicated by the velocity contours parallel to the major axis. To the southwest, region I marks the limit where the velocity contours start to become parallel to the major axis. Its counterpart to the northeast is the beginning of the hook shape where the isovelocities become not parallel to the minor axis. Region I might represent the outer layer of the vertical disc stratification, at the interface with the surrounding medium, where ram pressure begins its action. Since the stripping pressure is not perfectly vertical on the disc, the I region suffers a radial component of the stripping force.

In order to check the highly uncertain velocity of region D gathered with the FP data, we gathered a long-slit spectrum on February the 17, 2021 using the low-resolution ($R=700$) MISTRAL spectrograph installed on the 193 cm OHP (Adami et al. 2018). We took four independent exposures of 15 minutes each, for a total of 60 minutes. The exposures were gathered using a slit of width 1.9 arcsec oriented North-South to include at the same time region D and the disc of the galaxy. The low-resolution spectrum of region D is shown in Fig. C.2. Because of the night skyline contamination, the H$\alpha$ line is marginally detected with a signal-to-noise ratio $\sim 2$ and a mean LoS velocity of $1458\pm 92 \text{ km s}^{-1}$, only marginally consistent with the FP measurements of $1580\pm 30 \text{ km s}^{-1}$. Figure C.2 also shows the corresponding MISTRAL spectrum obtained on the galaxy disc north of region D, where the average velocity is $1629\pm 46 \text{ km s}^{-1}$, in
very good agreement with the FP data. As for the extraplanar regions detected in the FP data, the velocity in the stripped gas is lower than that measured in their corresponding regions to the north on the galaxy disc.

The gas stripping process removes the gas from the disc and transfers and dilutes it into the intracluster medium. The only velocity component that we can measure is the one along the LoS. For edge-on galaxies it is straightforward to interpret the LoS velocity as rotation velocity in the galaxy disc plane. The intergalactic wind might change the game, in adding a velocity component related to the direction of motion of the galaxy within the cluster. Nevertheless, in the specific case of NGC 4330, the wind is almost perpendicular to the galaxy disc and has a large component in the sky plane, as suggested by the numerical simulations (see Sect. 7). For these reasons the observed velocities of the gas expelled from the disc are not expected to dramatically change. Thus, the gas continues to rotate outside of the disc and shows the same LoS velocities than those it had before being stripped. Since regions B, C, D (H) have LoS velocities lower (higher) than their corresponding region in the disc, this suggests that here the gas partly lost its angular momentum during the ram pressure stripping event.

5.2. Velocity dispersion

Figure 6 shows the velocity dispersion of the galaxy as measured from the PUMA and GHASP datasets. Following Boselli et al. (2021b) we checked that the velocity profiles have a single peak, i.e. that the barycentric method applied to measure the velocity dispersions does not introduce any strong systematic bias in the results.

At first order, the velocity dispersion of the gas is fairly constant over the disc of the galaxy, with \( \sigma \sim 35+10 \text{ km s}^{-1} \) and \( \sigma \sim 21+7 \text{ km s}^{-1} \) for the PUMA and the GHASP data, respectively. The PUMA velocity dispersion is higher than the one measured in the GHASP data because of its reduced spectral resolution. It is indeed difficult to measure velocities far below the instrumental resolution (\( \sigma_{\text{LSF}} \approx 19.5 \) and \( 13.1 \text{ km s}^{-1} \), respectively) as it has been recently discussed in Boselli et al. (2021b) in comparing VLT/MUSE data with FP data obtained with GHASP.

At higher spatial frequencies, fluctuations in the velocity dispersion are observed on scales comparable to or larger than the one set by the seeing. These fluctuations are smoother in the GHASP data than in the PUMA ones. This is due to the difference in the method used to compute the map (Gaussian smoothing versus Voronoi tessellation, respectively).

Panels (c) and (e) of Fig. C.3 show intensity-velocity dispersion diagnostic diagrams and panels (a) and (d) the histograms for the intensities and of the velocity dispersions. These diagnostic diagrams are generally used to characterise the dynamics of the ISM in extragalactic giant HII regions (Munoz-Tunon et al. 1996) and also to investigate the dynamic of HII galaxies (Moiseev & Lozinskaya 2012; Carvalho & Plana 2018). Moiseev & Lozinskaya (2012) show a sketch where different features and regions in the plot can be interpreted in terms of (case 1) HII regions (constant velocity dispersion region with high H\( \alpha \) intensity), (case 2) shell (inclined bands in the plot), (case 3) low density turbulent ISM (triangular shape with a base spanning from low to high velocity dispersion and a triangle height ranging from low to moderate H\( \alpha \) intensity).

Shells (case 2) are not observed in the diagnostic diagrams shown in panels (c) and (e), probably because of the lack of spatial resolution for this relatively distant galaxy. The triangular shape distribution of the diagnostic diagram (case 3) is asymmetric, with a deviation toward the lowest velocity dispersions at low intensity and toward larger velocity dispersions at intermediate intensities, indicating the supersonic behaviour of the ISM in NGC 4330. In panel (e) we selected four areas, represented by ellipses, of different pixel density (from high to low mean pixel density: red, green, dark, and light blue), and identified their locations in the galaxy in panel (b). The red pixels with subsonic velocities are mainly located on the outskirts of the galaxy while the dark blue ones, with supersonic velocities, are a little bit on the inner side (case 3). The supersonic green pixels are located along the middle plane of the galaxy while the light blue with the highest intensities (case 1), match with the nucleus of the galaxy and the brightest HII regions. Their velocity dispersion is between 20 km s\(^{-1}\) and 26 km s\(^{-1}\).

5.3. The residual velocity fields

Two-dimensional (2D) residual velocity fields are generally used to optimise the free parameter determination of RCs that are computed from a 2D velocity fields. In general, because the RCs are based on axisymmetric models, a pattern observed on the residual velocity field would be due to incorrect parameter determination. As a consequence, when the free parameters of the rotation curve are optimised, the residual velocity field is pattern-free and the dispersion in the 2D residual velocity is minimum (see Warner et al. 1973; Fuentes-Carrera et al. 2004; Erroz-Ferrer et al. 2012).

The PUMA and GHASP residual velocity maps (\( V_{\text{res}} = V_{\text{LoS}} - V_{\text{mod}} \)) shown in Fig. 7 were obtained after subtracting the iteration model velocity field (\( V_{\text{mod}} \)) from the observed one (\( V_{\text{LoS}} \)). Once we identified the regions presenting a large scatter with the mean velocity dispersion of the residual velocity field, which are not supposed to be in rotation in the plane of the disc, they were masked from the observed velocity field and the axisymmetric model was recomputed. This is because the axisymmetric model is not supposed to be observationally constrained by non-axisymmetric motions.

The residual velocity maps given in Fig. 7 do not show any systematic pattern in the galactic disc and have a velocity dispersion of \( \sigma_{\text{res}} \sim 19 \text{ km s}^{-1} \) for the GHASP dataset and \( \sigma_{\text{res}} \sim 24 \text{ km s}^{-1} \) for the PUMA dataset. The residual velocity is \( \sim 0 \text{ km s}^{-1} \), suggesting that the adopted cylindrical model (described in section B.4) reproduces well the axisymmetric rotation of the disc modelled by the rotation curve and its associated parameters.

An accurate examination of Fig. 7 indicates how the kinematical properties of the various extraplanar ionised gas features created after a ram pressure stripping event differ from those properties predicted by a purely axisymmetric cylindrical model. The most discrepant regions in terms of relative velocity of the gas are regions G, H, and L for the PUMA data and G, H, and from J to M on the southwest side of the galaxy, for the GHASP data. Here the mean gas velocity is \( \sim 50 \text{ km s}^{-1} \) higher than that observed in the model. A less pronounced difference in the same direction is observed in region E (\( \sim 20 \text{ km s}^{-1} \)), which is located in projection close to the kinematic centre of the disc. On the contrary, the gas inside the regions D (PUMA) and B, C, D (GHASP) has a mean velocity \( \sim 40 \text{ km s}^{-1} \) higher than that of the cylindrical model. Additional analysis about these individual regions is given in Sect. 5.4.
5.4. Peculiar motions

Table 5.4 gives the main kinematical properties of the gas in regions A to M. The GHASP data indicate that the gas inside most of these particular regions has a velocity dispersion near to the mean value measured over the galactic disc, and this is probably due to the Gaussian smoothing used to create the map of the velocity dispersion. The PUMA data rather indicate that the gas inside all the same regions has a velocity dispersion higher than the one measured within the galaxy disc, with the higher velocity dispersions measured in regions E, G, and K. On the other hand, the residual velocity field derived from both sets of data shows that the gas in regions A, E, F, and I follows the rigid-body rotation with a low residual velocity. This result is expected for the gas in regions E, F, and I because they are still associated to the galactic disc, but not for the region A which has been perturbed by the stripping process.

In region A, the velocity contours from both datasets indicate that the gas is rotating with a slight bending with respect to the minor axis. The PVD obtained with the PUMA dataset shows that the gas at the northeast, where the Hα monochromatic map has a hook feature, has higher radial velocities than expected from the model. For this reason, we could expect that here the gas has a higher velocity dispersion due to non-circular...
At the southern edge of the galactic disc, the velocity contours are bent with respect to the minor axis. This is particularly evident for the gas associated to region G, where the velocity dispersion and residual velocity increase smoothly with galactocentric distance, reaching their largest value in the detached cloud at the southwester edge of the region. The same cloud at the edge of region G has also a higher LoS velocity than its counterpart on the disc, which means a lower rotation velocity because this cloud is located on the receding side of the disc. Regions B, C, D and H show a similar behaviour, they display lower LoS velocities, thus lower rotation velocities because they are approaching to the observer with respect to the galaxy centre. Then, we can infer that the gas inside regions J to M, with a low velocity dispersion but a high residual velocity, has evolved in a similar way than the gas in regions B, C, D and H, i.e. having lost angular momentum once stripped from the disc. Finally, the gas in region L has a higher velocity dispersion that the one observed around regions K and M. This does not mean that it gets kinetics energy from the wind because the line broadening is probably due to the presence of a star forming region (see Fig. 2).

6. Kinematics of the gas out the plane

In this section we focus on the PVDs at different locations in the galaxy disc, parallel and perpendicular to the major axis of the galaxy, in order to study the kinematics of the gas out of the plane of the stellar disc. Figures 8 and 10, on which we have also located regions A to M, show the PVDs extracted from the PUMA data cube measured along the major and the minor axis respectively, at different distances from disc plane (e.g. Epinat et al. 2008a; Rosado et al. 2013).

6.1. PVDs along the major axis

Figure 8 shows the PVDs along the major axis at different distances from the disc plane. The pseudo-slits are placed along the z-axis from +3.7 arcsec (∼0.3 kpc) to the northern direction (up wind direction) of the kinematic major axis, up to -11 arcsec (∼0.9 kpc) to the southern direction (tail direction), and they are separated each other by ∼1.8 arcsec(-0.1 kpc, see Sect. B.1.). Figure 8 shows that the rotating gas is less extended along the disc upwind (+1.8 arcsec), where it reaches 50 arcsec (∼4 kpc) in both the northeastern and southwest directions, than in the tail (70 arcsec to the northeast and 60 arcsec to the southwest). This result is expected in a ram pressure stripping scenario, where the gas is removed outside-in (e.g. Quilis et al. 2000; Vollmer et al. 2008a; Rosado et al. 2013; Boselli et al. 2006, 2011b; Roediger & Hensler 2005; Tonnissen & Bryan 2009).

To identify the main kinematic features resulting from the ram pressure stripping event, we compare the predicted values by theoretical fittings with the observed data. We have applied the ETM on each PVD measured along the major axis at different distances from the disc plane. In the left panels of Fig. 8 the best fit to the terminal velocity of each PVD is overlaid on its correspondent plot. In addition, in the context of the iteration method (ITM), we built a first order approximation data cube model to compute the rotation curve (RC) of NGC 4330 (see in Sect. 4). Here we compare the PVDs from the observational data with the PVDs at the same position obtained from that data cube model in the central panels of Fig. 8. We notice that the cylindrical model follows the emission intensity of the galaxy in velocity and matches with the kinematical behaviour of the major axis PV diagram.

Table 2. Velocities of peculiar regions derived from the PUMA and GHASP datasets.

| Region | \( V_{\text{rad}} \) (km s\(^{-1}\)) | \( \langle V_{\text{rad}} \rangle \) (km s\(^{-1}\)) | \( V_{\text{res}} \) (km s\(^{-1}\)) | \( \langle V_{\text{res}} \rangle \) (km s\(^{-1}\)) |
|--------|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| A      | 1659 35 -8                      | 1673 19 8                       |                                  |                                  |
| B      | ... ... ...                      | 1564 9 -41                      |                                  |                                  |
| C      | ... ... ...                      | 1601 0 -21                      |                                  |                                  |
| D      | 1580 37 -54                     | 1585 11 34                      |                                  |                                  |
| E      | 1562 38 20                      | 1579 19 2                       |                                  |                                  |
| F      | 1543 38 21                      | 1533 20 -2                      |                                  |                                  |
| G      | 1477 53 48                      | 1493 19 22                      |                                  |                                  |
| H      | 1548 37 109                     | 1496 21 51                      |                                  |                                  |
| I      | 1460 37 13                      | 1482 25 3                       |                                  |                                  |
| J      | 1435 33 19                      | 1542 14 116                     |                                  |                                  |
| K      | 1412 45 9                       | 1480 15 72                      |                                  |                                  |
| L      | 1450 38 49                      | 1501 21 99                      |                                  |                                  |
| M      | ... ... ...                      | 1541 18 139                     |                                  |                                  |

Notes. (1) Mean line-of-sight velocity in km s\(^{-1}\). (2) Mean velocity dispersion in km s\(^{-1}\). (3) Mean residual velocity in km s\(^{-1}\).
Fig. 8. Position-velocity diagrams (PVDs) parallel to the kinematic major axis extracted from the PUMA dataset. The distance of the pseudo-slit form the disc plane along the z-axis (positive to the north) is given in the upper-left corner of each panel. The positions of all the pseudo-slits are shown on the H\(\alpha\) monochromatic map in the right panel. From the top at +3.7 to -5.5 arcsec (from \(\sim\)0.3 to \(\sim\)0.4 kpc) the intensity of the contour level is at 1.5 \(\sigma\), from -7.4 to -11 arcsec (from \(\sim\)0.6 to \(\sim\)0.9 kpc) the intensity level is at 1.2 \(\sigma\). In the left panels, coloured lines overlaid on the PVD represent the best fit of the terminal velocity (relation B.3 with \(\eta = 0.3\)) using the arc-tangent model (Eq. B.4). There is a correspondence in colour between the terminal velocities and the pseudo-slits plotted on the monochromatic map in the right panel. In the middle panels, the PVDs derived from the data cube model, that was built in the context of the iteration method to compute the rotation curve (see Sect. B.4), are overlaid on the observed PVD in gray. All the PVDs extracted from the data cube model are at 8% of the maximum intensity of the model. The two parallel dashed lines are at \(r_0 = \pm 30\) arcsec (\(\pm 2.4\) kpc) and indicate the three main disc regions: northeast, central and southwest. The different features of interest are indicated with letters A to H.
Some low surface brightness plumes of Hα emission were detected on top of the dust lane inside region F. These plumes, which can be seen in the upper two panels of Fig. 8, show an increment in velocity by \( \sim 30 \text{ km s}^{-1} \) breaking the symmetry of the PVDs. If the Hα emission, after being compressed on the front side of the galaxy during the stripping event, is pulled as a whole to the southwest of the stellar disc, these plumes represent gas dragged that is still resisting to change, giving us a clue of the initial state of the disc.

The PVD along the major axis presents a kinematic symmetry with respect to the kinematic centre without any plume or knot with important variations in velocity with respect to the model. The kinematic major axis matches the bent dust lane in the central region, within a radius of \( \sim 30 \text{ arcsec} (\sim 2.4 \text{ kpc}) \), enclosed by the vertical dashed lines. As it is suggested by the bending of the velocity contours (see Fig. 5), the radial velocities tend to be constant \( \sim 150 \text{ km s}^{-1} \) to the southwest, while to the northeast, the PVDs show velocities that change by \( \sim \pm 25 \text{ km s}^{-1} \) perpendicular to the disc. To the northeast, the gas in region A has a radial velocity of \( \sim 200 \text{ km s}^{-1} \). As indicated by the residual velocity map (see Sect. 4), although the gas inside region E has a peculiar shape, it rather follows the rotation of the galaxy. On the contrary, the gas in region G has a lower velocity than in the disc.

![Fig. 9. Envelopes corrected from the morphological inclination (89.5°) of the different PVDs shown in Fig. 8. The vertical blue dotted line indicates the effective radius \( R_e(i) \).](image)

### 6.2. The Lag Gradient

Kinematical studies of edge-on galaxies revealed the presence of a vertical velocity gradient in the rotation curve moving away from the disc plane. This gradient has been observed at optical (e.g. Heald et al. 2006; Rosado et al. 2013; Bizyaev et al. 2017; Levy et al. 2019) and radio (e.g. Miller & Veilleux 2003; Fraternali & Binney 2006; Zschaechner & Rand 2015) wavelengths, including in the Milky Way (e.g. Kalberla et al. 2014). This phenomenon has been explained by a combination of vertical movements due to energy input from galactic fountains, radial movements of the gas due to the pressure gradient of the halo and a declining rotation velocity due to the conservation of the angular momentum (Fraternali & Binney 2006; Heald et al. 2007). This vertical velocity gradient can be modified whenever
a galaxy such as NGC 4330 is suffering an external perturbation. In NGC 4330 the amplitude of the rotation curve does not decrease sufficiently with the vertical distance from the disc plane to explain the observed lags. In this galaxy, which is suffering an almost face-on ram pressure stripping event, galactic fountains were probably suppressed, in particular in the upwind side of the disc where the external pressure is significantly increased.

The lag gradient of NGC 4330 can be estimated by measuring the change in velocity along the $z$-axis using the PVDs parallel to the major axis. Once corrected from the morphological inclination of the galaxy (89.5°), the envelopes of the different PVDs shown in Fig. 8 become the rotation curves given in Fig. 9. The rotation curve with the highest velocity amplitude is obtained in the plane of the disc ($z=0$ arcsec). To the southwest, at the effective radius, the velocity decreases by $\Delta V \approx 9$ km s$^{-1}$ each $\Delta z = 1.8$ arcsec ($-0.14$ kpc) from the kinematic major axis up to $z = -5.5$ arcsec ($-0.4$ kpc), where, because the ram pressure stripping compresses the gas, there is no change in velocity along the $z$-axis up to $-11$ arcsec ($-0.9$ kpc), where the most diffuse gas is. To derive the velocity gradient along the $z$-axis of NGC 4330 we used the fact that the asymptotic velocity $V_0$ of the relation (B.4) is one of the free parameters of the fitting to the terminal velocity of each PVD, so that, from $z = 0$ to $-5.5$ arcsec ($-0.4$ kpc) there is a lag gradient of $\Delta V/\Delta z \sim 64 \pm 9$ km s$^{-1}$ kpc$^{-1}$. The rotation in the north along the $z$-axis has been modified by the wind, it is slower by $\Delta V = 49$ km s$^{-1}$ with respect to the one measured at $z = 0$ arcsec. The difference in velocity between the northern RCs is $\Delta V = 16$ km s$^{-1}$. Although the rotation velocity decreases with the distance from the stellar disc, as indeed expected, it is not possible to determine a constant lag gradient in this direction.

6.3. PVDs along the minor axis

The ram pressure stripping event, which is acting almost face-on in NGC 4330, has reduced the vertical height of the ionised gas disc in the northern direction (upwind). We thus study the PVDs along the kinematic minor axis to determine whether galactic fountains or inflows have been suppressed by the external pressure (e.g. Heald et al. 2006, 2007; Rosado et al. 2013). Figure 10 shows the PVDs perpendicular to the kinematic major axis obtained from the PUMA dataset. The pseudo-slits are located in specific places to highlight the main features of the disc and its asymmetries, and they cross the regions labelled from A to M identified in Sect. 2. We also compare the PVDs from the observational data with the PVDs at the same position obtained from that data cube model computed with the iteration method (ITM, see Sect. 4).

In general, these PVDs show that the brightest Hα emitting gas is compressed to the galactic mid plane and that it follows mostly the rotation predicted by the cylindrical model. Upwind (northeast) there is no Hα emitting gas beyond $z = 5$ arcsec (0.4 kpc). Only near the midplane of the disc, downwind (southwest) the gas decreases its brightness along the $z$-axis following the exponential structure used to construct the model. Here the velocity of the most diffuse gas generally increases with $z$ all over the galactic disc. This increase in velocity with $z$ is not reproduced by the ITM model.

In region A, at 65.5 arcsec (5.2 kpc) from the kinematic centre, the ionised gas has exactly the same velocity of the cylindrical model. At 31.7 arcsec (2.5 kpc), the low radial velocity observed for the gas in region D in the velocity field is now evident as long as its corresponding position on the disc follows the model. We only detected some diffuse gas to the northeast along the $z$-axis inside region F at a radius of 4.1 arcsec (0.3 kpc) from the kinematic centre, where the diffuse gas rotates a little faster than the model. The pseudo-slit at the kinematic centre shows that the gas in region E is completely compressed to the galactic mid plane. To the southwestern edge of the disc, the most diffuse gas inside regions G, H, J, and K rotates slower than what expected from the model, in agreement with the residual velocity map. Finally, we have observed that the gas inside region L (-121.5 and -130.9 arcsec, 9.7 and 10.5 kpc respectively) has a high velocity dispersion, radial and residual velocity likely because the gas is moving with non-circular motions. Figure 10 also shows that the diffuse gas in region L has a velocity higher by $\sim 200$ km s$^{-1}$ than the one expected from the model.

Instead of a heart-shaped profile, which would indicate the presence of an inflow in the galaxy (e.g. Fraternali & Binney 2006; Rosado et al. 2013; Zschaechner et al. 2015), the PVDs suggest features similar to the intermediate- and high-velocity clouds of the Milky Way (e.g. Wakker & van Woerden 1997; Fraternali & Binney 2006) whose origin is attributed to the interaction with the Magellanic Clouds. This suggests that the kinematics of the extraplanar diffuse ionised gas of NGC 4330 is likely dominated by the ram-pressure stripping process.

7. The dynamical model

We present here the kinematical results of the simulation of NGC 4330 presented in Vollmer et al. (2021). They have been obtained using a N-body code, described in Vollmer et al. (2001), which consists of a non-collisional component plus a collisional one. The non-collisional component of 81 920 particles simulates the stellar disc, the stellar bulge and the dark halo. The collisional component simulates the ISM as an ensemble of 20 000 gas clouds that have inelastic collisions (sticky particles) that might coalesce or fragment, which evolve in the gravitational potential of the galaxy. A scheme for star formation was implemented, in which stars are formed during cloud collisions to further evolve as non-collisional particles. The star formation rate is proportional to the cloud collision rate. Ram pressure has been included as an additional acceleration on the sticky particles that are not protected by other particles. Once stripped out from the galactic plane, the warm gas clouds become diffuse if their density falls below a critical density (Vollmer et al. 2021). The model consists of two components: the HII regions ionised by young massive stars with ages less than 20 Myr and the diffuse gas ionised by the stellar UV radiation or by strong shocks induced by the ram pressure stripping process. As discussed in Vollmer et al. (2021), collisional ionization through the thermal electrons of the ambient ICM that confines the filaments is the most probable ionization mechanism in the extraplanar ionised gas filaments detected in the VESTIGE narrow-band image.

In Figs. C.8 and 11, we compare the observed Hα data to the simulated ones. For each figure we provide three panels, the top one corresponds to the data whereas the middle and bottom ones to the models. Figure 11 (panels (b) and (e)) and C.8 (panel (b)) give the lowest surface brightness achieved by the model while Figs. 11 (panels (c) and (f)) and C.8 (panel (c)) show the model at a depth comparable to the sensitivity of our observations. The plotted Hα emission map is the one extracted from the model for the snapshots of highest goodness for the diffuse gas. This corresponds to a peak of ram pressure occurred 140 Myr ago. Similarly to the deep Hα observations presented in Fossati et al. (2018), the model shows extra-planar linear filaments in the downwind region of the galactic disc, which correspond to the low surface brightness filaments located in the southwest tail and close to the...
Fig. 10. Position-velocity diagrams parallel to the kinematic minor axis obtained from the PUMA dataset. The locations of the pseudo-slits are in the upper-left corner of each panel and they are traced on the Hα monochromatic map in the left panel with colour correspondence. The distance increases from the south to the north of the disc, with zero at the kinematic centre. The black dotted ellipse on the Hα monochromatic map indicates the i-band μe(i) = 23 mag arcsec² isophote. The intensity of the contour levels is 1.3, 1.4 and 1.6 σ from −45.5 to +63 arcsec (~ −3.6 to 5 kpc) and 1, 1.1 and 1.25 σ from −130.9 to −52 arcsec (~ −10.5 to −4.2 kpc). The PVDs extracted from the data cube model (black contours) are at 1, 10 and 50% of the maximum intensity of the model.

downturn in the northeast (Vollmer et al. 2021). These filaments indicate the direction of the motion on the plane of the sky of the galaxy within the cluster.

The agreement between the gas distribution in the observations and in the simulations is overall very satisfactory. The brightest Hα knots detected in the FP observation (panel (a)) correspond to the brightest Hα regions in the simulation (panels (b) and (c)). The diffuse Hα emission in the model, given (in log scale) in panel (b), is far too faint to be detected in the FP observations, but it matches fairly well the diffuse emission observed using the Hα narrow-band filter by Fossati et al. (2018). While the tuning is very good at the northeast edge of the disc, the simulated southwest tail has a larger curvature than that observed in the diffuse ionised gas detected by VESTIGE. This difference might be due to the fact that given the orientation of NGC 4330 and its deduced orbit within the cluster, the northeast
edge of the disc is the region of the galaxy which first suffered the stripping process.

As suggested by the simulations, all the diffuse gas component (mainly HI gas) has been already stripped, while that in the southwest direction, which was perturbed later, is still partly present, as suggested by the diffuse HI tail observed by Chung et al. (2007). This effect is probably combined with an asymmetric distribution of the star forming regions over the disc of the galaxy, possibly formed after the external perturbation, with more compact HII regions along the southwest edge of the disc. Given their compactness and high density, these regions are less subject to the external perturbation than the diffuse gas. The difference between observations and simulations might thus result from a combined effect of sensitivity, gas density, and time elapsed since the beginning of the perturbation. Indeed, as shown in Fig. 19 of Abramson et al. (2011), the southwest curvature of the gas is more pronounced in the cold and diffuse atomic phase (the first one stripped) than in the compact HII, more resistant to the external pressure.

The Hα line-of-sight velocity fields are shown in panel (a) of Fig. 11. In order to compare the model to the data, a systemic velocity of 1551 km s$^{-1}$ has been added to the model. The velocity amplitudes are the same for the three panels. Hα isocontours from Fossati et al. (2018), corresponding to a surface brightness of $\Sigma$(Hα) $\approx 2 \times 10^{-18}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$, have been displayed in order to facilitate the comparison between the panels. We observe a very good agreement between the simulation and the observations. Where the simulations can be compared to the data, the amplitudes of the velocity fields are consistent. We note that if we include the low surface brightness tails, unfortunately not reached by the shallow FP observations, the velocity range in the simulations is even larger.

On the other hand, the HII velocity field (Fig. C.5, bottom panel) seems to show that the LoS velocities increase at the edge of the tail, although this raising is not very sharp, probably because of the lack of resolution in the HI data. The same excellent agreement is present also between the observed and simulated unfolded RCs shown in Fig. C.7 on which the simulated RC measured along the major axis of the galaxy has been compared to the GHASP observed and modelled RCs. The RC extracted from the simulations well predict the observed asymmetries between the receding (northeast) and approaching (southwest) sides of the disc up to $\sim 70$ arcsec ($\sim 5.6$ kpc). Since the RC extracted from the simulations is taken on the major axis, we do not have any data outside this radius because the ionised gas emission is sent to the southwest.

The Hα line-of-sight velocity dispersion fields are compared in the right panels of Fig. 11 using the same velocity range in the three panels. The mean velocity dispersion in the data is $\sim 20.5$ km s$^{-1}$, with a standard deviation of $\sim 6.7$ km s$^{-1}$, while in the model those quantities are respectively equal to $\sim 10.6$ and $\sim 6.7$ km s$^{-1}$. The smaller mean velocity dispersion in the model with respect to the observations (which have an instrumental limiting resolution of $\sim 13$ km s$^{-1}$), is due to the fact that the model estimate is strongly weighted by the diffuse gas component. This component is not reached by the observations. If we consider only the area of the model corresponding to the one detected in the data, the mean model velocity dispersion becomes $\sim 17.5$ km s$^{-1}$, with a standard deviation of $\sim 6.4$ km s$^{-1}$, and matches fairly well the observations. Striking is the similarity between models and observations around RA $= 12^h 23^m 15.5^s$, Dec $= 11^\circ 21' 42''$, where the velocity dispersion increases by one sigma with respect to other regions along the galaxy disc up to 28 - 45 km s$^{-1}$ in the data and 30 - 40 km s$^{-1}$ in the model (in red in both cases).

8. Discussion and conclusion

Located in the nearby Virgo cluster, where 1 arcsec corresponds to $\sim 80$ pc, the edge-on late-type NGC 4330 is the perfect candidate to study the effects of ram pressure stripping in the direction perpendicular to the disc plane of a perturbed system with an angular and spectral resolution unreachable elsewhere. The presence of extended tails in the atomic (Chung et al. 2007) and ionised (Fossati et al. 2018) gas and in the dust component (Lobgobardi et al. 2020) observed in this object witness an ongoing stripping process able to remove the different phases of the ISM in the outer regions and producing a truncated disc.

Using two different and independent sets of FP observations with a spectral resolution up to $R \sim 10000$, combined with our own tuned hydrodynamic simulations (Vollmer et al., 2021), we have studied the kinematics of the ionised gas. Although the diffuse emission of the ionised gas in the stripped tails, which has a surface brightness of $\Sigma$(Hα) $\approx 3 \times 10^{-18}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$, has not been reached by the present observations, limited to $\Sigma$(Hα) $\sim 10^{-17}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$, our data allows us to study the kinematics of the gas along the disc and in a few HII regions located outside the disc plane and probably formed within the stripped gas after the interaction.

The Fourier transform analysis of the deep NGVS i-band image reveals that the B1 parameter is negative, indicating that the galaxy disc has a boxy shape outside a radius of $\sim 60$ arcsec ($\approx 4.8$ kpc). This boxy structure is present mostly in the southwest direction, where the tail of stripped material is more pronounced. Simulations suggests that a thicker disc could result from the perturbation of the gravitational potential well due to the displacement of the gaseous component from the disc plane after a ram pressure stripping event (Farouki & Shapiro 1980; Clarke et al., 2017; Safarzadeh & Scannapieco 2017; Steyerleithner et al. 2020). The boxy shape would thus result from the induced perturbation on the stellar orbits. The datasets used in this work, which are sensitive only to the kinematics of the ionised gas, do not allow us to test this hypothesis. However, in the central regions, the kinematic major axis matches with the dust lane position angles, indicating that the perturbation probably had only a moderate effect on the dynamics of the inner gas.

It is thus conceivable that the dynamics of the stars are also moderately perturbed. Indeed, within the inner stellar disc ($r \lesssim 2$ kpc), the velocity field of the galaxy dominated by the emission of HII regions is characterised by isovelocities parallel to the minor axis. The analysis of the rotation curves and of the PVDs consistently indicate a solid-body rotation. Outside this radius the isovelocities are bent, becoming parallel to the major axis and show a low velocity gradient in the southwestern edges of the disc indicating non circular motions.

The FP data allowed us to detect several ionised gas features formed after the dynamical interaction of the galaxy with the surrounding ICM (ram pressure) and located at the edges or outside the stellar disc. These are i) a hook-like structure at the north-east of the galaxy in the region which first got in contact with the ICM, ii) several bright and extended regions formed along the downstream direction of the wind, principally located in the southwest of the galaxy, including a few HII regions well outside the disc and located along the filaments of stripped gas.

The analysis of the velocity field, of the velocity dispersion, of the rotation curves (derived using the ETM necessary to correct for beam smearing effects due to the velocity superposition along the line-of-sight), and of the PVDs consistently indicate that these regions do not follow the solid-body rotation but have peculiar velocities indicative of streaming motions out of the
Fig. 11. GHASP (panel (a) and (d)) and simulated (panels (b), (c), (e) and (f)) Hα velocity (left column) and velocity dispersion (right column) fields of NGC 4330. The simulated Hα velocity and velocity dispersion fields are given down to the faintest levels used in the middle panel of Fig. C.8 to show the kinematical properties of the low column density ionised gas reached by the simulations (panels (b) and (e)) and at a higher surface brightness level corresponding to the sensitivity of the FP observations ($\Sigma(H\alpha) \approx 2 \times 10^{-17}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$, panel (c)). The contours on the velocity fields indicate the Hα surface brightness $\Sigma(H\alpha) \sim 2 \times 10^{-18}$ erg s$^{-1}$ cm$^{-2}$ arcsec$^{-2}$ taken from the VESTIGE data (Fossati et al. 2018).

plane of the galaxy. We also observe a small decrease of the rotation of the stripped gas with the increasing distance from the galaxy disc.

This observational evidence can be explained considering that the stripping process is mainly perpendicular to the disc plane, as indeed suggested by our hydrodynamic simulations. Under this geometrical configuration, the gas is expected to keep its rotation with only a partial and gradual loss of angular momentum. Furthermore, the similarity of the Hα, CO, and HI velocity fields and rotation curves, particularly in the inner disc, indicate that here ram pressure stripping significantly affects the gas kinematics. Since in this scenario the acceleration caused by ram pressure is inversely proportional to the stripped gas surface density, the observed similarity in the kinematical perturbations of the different gas components (ionised, molecular, atomic) is expected given their comparable surface density.

Overall, the kinematical properties of the galaxy and of all these extraplanar features formed after the ram pressure stripping event are well reproduced by our simulations, at least up to the limited sensitivity of our FP observations. Nevertheless, it is hard to drive general conclusions on the effects of ram pres-
sure stripping on the kinematical properties of galaxies and of the stripped material from the analysis of a single object.

The present results, however, combined with those obtained from the analysis of the kinematical properties of a few other well resolved galaxies with available IFU data (e.g., Chemin et al. 2006; Merluzzi et al. 2013; Fumagalli et al. 2014; Consolandi et al. 2017; Boselli et al. 2021b), consistently suggest that the gas removed during a ram pressure stripping event tends to keep its rotation although with a possible loss of angular momentum. The perturbation can affect the kinematics of the gas even inside the disc of the perturbed system (Boselli et al. 2021a). Although we are still lacking of any direct evidence, indirect observations and simulations consistently indicate that a ram pressure stripping event can also perturb, although to a lower extent, the kinematics of the stellar component. Deep IFU spectroscopic observations with a sufficient spectral resolution of perturbed systems will be required to probe this scenario.

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Sardaneta et al.: VESTIGE
Appendix A: Fabry-Perot observations

Appendix A.1: PUMA observations

Fabry-Perot 3D spectroscopic observations of NGC 4330 were gathered using PUMA at the 2.1 m telescope at the Observatorio Astronómico Nacional in San Pedro Mártir, Baja California, México (OAN-SPM) (Rosado et al. 1995). The PUMA focal reducer hosts a FP interferometer with a field of view of $\sim 10 \times 10$ arcmin$^2$. The camera is a 512 x 512 CCD detector with a pixel scale of $\sim 1.27 \times 1.27$ arcsec$^2$.

NGC 4330 was observed during dark time, in February 2017 (see Table 1), as part of the FP survey of the Herschel Reference Survey (Rosell et al. 2010; Gómez-López et al. 2019). To increase the signal-to-noise ratio, the pixels have been electronically rebinned by $4 \times 4$, leading to a final spatial sampling of $\sim 1.3$ arcsec. The galaxy was observed during poor seeing conditions (FWHM $\sim 2.9 \pm 0.3$ arcsec) but excellent transparency with a total exposure time of 96 min, with 120 seconds per channel. The spectral domain has been selected using a FWHM = 0.6533 Å (i.e., a total exposure time of 96 min, with 120 seconds per channel).

The spectral domain has been selected using a FWHM = 0.6533 Å (i.e., a total exposure time of 96 min, with 120 seconds per channel). The instrumental line spread function is $\sigma_{ILS} \sim 0.43$ Å ($\sim 19.5$ km s$^{-1}$), the thermal broadening $\sigma_T \sim 0.18$ Å ($\sim 9.1$ km s$^{-1}$), and the intrinsic mean velocity dispersion $\sigma_v \sim 0.44$ Å ($\sim 20.0$ km s$^{-1}$), the model indicates that the observed $H_\alpha$ barycentre and the velocity dispersion should be corrected by $-0.20$ Å ($-8.9$ km s$^{-1}$) and $-0.12$ Å ($-5.3$ km s$^{-1}$), respectively.

We computed a parabolic phase map from the calibration cube in order to obtain the reference wavelength for the line profile observed inside each pixel. This phase map provides the shift that has to be applied to the spectral dimension to every pixel of the interferogram cube to bring all channels to the same wavelength. The wavelength-sorted data cube is then created by applying the phase map correction to the interferogram data cube. A detailed explanation about the data reduction process for data obtained at OAN-SPM with PUMA can be found in Fuentes-Carrera et al. (2004).

In order to work with profiles and maps with a similar signal-to-noise ratio (SNR) and at the same time optimise the angular resolution, we applied a Voronoi tessellation with a constant SNR = 9 to the data cube (Cappellari & Copin 2003; Daigle et al. 2006a,b). The brightest HII regions are not affected by this smoothing procedure because their SNR per pixel is larger than 9 and are thus displayed at full resolution.

The resulting data cubes are used to extract the monochromatic, continuum, line-of-sight (LoS) radial velocity and velocity dispersion maps of the galaxy. The continuum map is computed considering the average of the 3 lowest intensities of the 48 channels of the cube. For the monochromatic image, the intensity of the $H_\alpha$ line is obtained by integrating the flux of the line profile for each pixel. Following Daigle et al. (2006a), we calculate the radial velocity in each pixel by measuring the barycentre of the profile of the $H_\alpha$ line. This barycentre is computed as the intensity weighted centroid of the spectral bins falling into the emission line boundaries, with the continuum subtracted. Radial velocities and velocity dispersions are extracted using a single emission-line detection algorithm. In the case where more than one velocity component is present in the spectrum, only the strongest emission line is taken into account. When two emission lines are spectrally close and have comparable amplitudes, they might be taken as a single one with a larger velocity dispersion.

Assuming that all the $H_\alpha$ emission-profiles are described by Gaussian functions, from the emission profiles width map the velocity dispersion $\sigma$ was estimated as

$$\sigma = (\sigma_{\text{LS}}^2 - \sigma_{\text{ILS}}^2 - \sigma_v^2)^{1/2},$$

where $\sigma_{\text{LS}}$ is the observed LoS velocity dispersion directly measured from the data cube corrected (i) from the instrumental line-spread function (LSF; e.g., Valdez-Gutiérrez et al. 2001; Rosado et al. 2013; Cárdenas-Martínez & Fuentes-Carrera 2018), which

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3 The PUMA CCD has in fact $2048 \times 2048$ px$^2$, which are electronically binned $4 \times 4$ to increase the SNR per pixel by a factor 16 and to take into account the mean seeing of the site.

4 Additional deeper PUMA observations of NGC 4330 could not be made due to the lockdown in spring 2020.

5 https://www.atnf.csiro.au/computing/software/karma/

6 ‘Image Reduction and Analysis Facility’, http://iraf.noao.edu/; IRAF is distributed by the National Optical Astronomy Observatories, which are operated by the Association of Universities for Research in Astronomy, Inc., under cooperative agreement with the National Science Foundation.

7 ADHOCw, http://cesam.lam.fr/fabryperot/index/software, developed by J. Boulesteix.

8 https://projects.lam.fr/projects/computeeverything

9 https://projects.lam.fr/projects/freducwiz
is modelled using the instrumental finesse F and the free spectral range FSR by \( \sigma_{\text{LSRF}} = \sqrt{2 \ln 2} \times 19.5 \ \text{km s}^{-1} \) (see Table A.1) and (ii) from the thermal broadening \( \sigma_T = (kT/\mu m_p)^{1/2} \approx 9.1 \ \text{km s}^{-1} \), assuming an electronic temperature of HII regions of \( T_e = 10^4 \ \text{K} \) (Osterbrock & Ferland 2006).

Appendix A.2: GHASP observations

An independent set of FP spectroscopic observations was taken using the GHASP instrument on the 1.93 m telescope at the Observatoire de Haute Provence (OHP) in spring 2021. These observations were gathered using the same configuration extensively described in Gómez-López et al. (2019). The GHASP focal reducer is equipped with a FP with a field of view of 5.8 × 5.8 arcmin\(^2\) and a 512 × 512 Imaging Photon Counting System\(^{10}\) with a pixel scale of 0.68 × 0.68 arcsec\(^2\) (Gach et al. 2002). The observations were taken during poor seeing conditions (2.5 < FWHM < 4.0 arcsec), with medium transparent sky, and with a total integration time of 645 minutes. The FSR of the FP at the redshifted wavelength of the galaxy (379 km s\(^{-1}\)) was scanned through 32 channels, with a typical spectral resolution of \( R \approx 9704 \) at Hα redshifted. NGC 4330 was observed using two different narrow-band interference filters. Indeed, the mean redshifted Hα wavelength of the galaxy is \( \sim 6597 \ \text{Å} \), which unfortunately falls just in the middle of two available interference filters centred on 6591 Å and 6601 Å. Both filters have the same FWHM \( \sim 15 \ \text{Å} \) useful to overcome [NII] line contamination. To cover the whole velocity field of the galaxy, the first filter was used for the blueshifted part of the galaxy, the second one for the redshifted side. This does not exactly double the observing time because approximatively the two third of the velocity amplitude was covered by the two filters that overlap by \( \sim 5 \ \text{Å} \). Similarly to the PUMA observation, the wavelength calibrations were done using the NeI emission line at \( \lambda = 6598.95 \ \text{Å} \). The data reduction procedure adopted to reduce the GHASP data have been extensively described in Epinat et al. (2008b) and Gómez-López et al. (2019). The LoS velocity dispersion has been corrected from the line spread function (LSF) and thermal broadening using the method and the relations described in the previous section (Sect. A.1). The main observational parameters of the FP observations are summarised in Table A.1.

We decided not to combine the FP observations obtained with GHASP and PUMA because of their different spatial and spectral resolutions. On the contrary, we decided to keep them separated to i) check the consistency of the results in particular in the low surface brightness regions where the signal-to-noise is limited, ii) take benefit of the slightly better angular resolution of the PUMA data for the study of the extraplanar gas, and iii) of the higher spectral resolution of the GHASP data for the study of the velocity dispersion of the gas, and finally iv) use the deeper GHASP data for the comparison with the output of the simulations. We notice that for these purposes the observations were analysed differently, i.e. using a Voronoi adaptive spatial smoothing for the PUMA data in order not to thicken the almost edge-on disc of the galaxy, while using a Gaussian smoothing for the GHASP observations to detect the weakest structures.

The different data cubes and maps presented in the next sections were thresholded using the VESTIGE image; all pixels below a surface brightness limit of 1 and 3.5 × 10\(^{-17}\) erg s\(^{-1}\) cm\(^{-2}\) arcsec\(^{-2}\), for GHASP and PUMA respectively, have been masked. Nevertheless, because of the night sky lines contamination, for the GHASP data, even above this threshold limit, it was not possible to calculate a consistent line flux nor velocity, and velocity dispersion for each pixel in the regions labelled B, C, D, H, J (western tail only) and M. For each region, the profiles were summed to give average values, which in turn lead to average flux, LoS and velocity dispersions.

Appendix B: Derivation of the rotation curve

Appendix B.1: Position velocity diagrams

In order to study the velocity distribution as a function of the Hz surface brightness, we built position-velocity diagrams (PVDs) from the FP data cube after having subtracted the stellar continuum (e.g. Epinat et al. 2008b; Rosado et al. 2013). Using a pseudo-slit of 2 pixels width (i.e. \( \sim 2.6/1.4 \) arcsec for PUMA/GHASP, close to or below the seeing) centred at the photometric centre and aligned to the major axis defined in Section 3, we extracted the emission intensity projected along the major axis of the galaxy after rotating the data cube in order to align the position angle of the galaxy with the North-South direction (see Figure 8). We modified the PA and the centre of the pseudo-slit in order to obtain the most symmetric PVD in intensity, the final values correspond to the kinematic major axis (\( PA_{\text{kin}} \)) and kinematic centre (see Table B.1).

Appendix B.2: The intensity-peak method

The intensity-peak method is based either on a barycentric measurement of the line profiles for each pixel or by fitting it with a theoretical function such as a Gaussian (e.g. Sofue & Rubin 2001; Rosado et al. 2013). With this method, the tangential velocity in the galactic plane, \( V_{\text{rot}} \), is computed under the assumption that the galaxy has an infinitesimally flat disc, inclined by an angle \( i \) with respect to the sky plane and dominated by rotational motions around an axis perpendicular to the galactic plane. Assuming an axial symmetry about the galactic centre and considering that the velocity field is dominated by circular motions, we can measure the velocity of the pixels that are at the same distance from the kinematic centre along the major axis by dividing the galaxy on the sky-plane in rings. The rotation velocity \( V_{\text{rot}}(r) \) for a ring of radius \( r \) is given as the azimuthal average over the ring of the deprojected velocities:

\[
V_{\text{rot}}(r, \theta) = \frac{V_{\text{LoS}}(r, \theta) - V_{\text{sys}}}{\cos \theta \sin i}
\]

where \( r \) and \( \theta \) are the radial and angular coordinates in the plane of the galaxy (Mihalas & Binney 1981) and \( V_{\text{LoS}}(r, \theta) \) is the line-of-sight velocity extracted from each pixel of the FP velocity field (e.g. Fuentes-Carrera et al. 2004; Epinat et al. 2008a; Cárdenas-Martínez & Fuentes-Carrera 2018; Sardana et al. 2020).

This method has been used for the PUMA and GHASP datasets; the results are shown by the black filled and open squares in Fig. 3. In both cases, the RCs were computed considering all the pixels located within an angular sector of \( \pm 5^\circ \) from the kinematic major axis, \( V_{\text{rot}} \) have been computed within circular rings of 6.5 arcsecond a galaxy kinematical inclination \( i = 88^\circ \). Both sides of each RC show a slowly rising solid-body shape despite the fact that both sides do not match outside the inner radius \( r_0 \sim 30 \) arcsec (\( \sim 2.4 \) kpc), but the increasing trend...
is now called Ruiz et al. (2002), the rotation velocity is defined as:

\[ V_{\text{rot}} \sin i = (V_{\text{env}} - V_{\text{sys}}) - (\sigma_T^2 + \sigma_{LSF}^2)^{1/2}, \]  

where \( V_{\text{sys}} \) is the systemic velocity, \( i \) is the galaxy inclination with respect to the sky plane, \( \sigma_{LSF} \) is the instrumental velocity dispersion, \( \sigma_T \) is the thermal broadening of the gas, and \( V_{\text{env}} \) is the terminal velocity characterised by the intensity of the envelope of an observed PVD along the line profile\(^\text{11}\). The intensity of the envelope is defined as:

\[ I_{\text{env}} = (\eta I_{\text{max}}^2 + (I_{\text{min}}^2)^{1/2}, \]  

where \( I_{\text{max}} \) is the maximum intensity in the line profile, \( I_{\text{min}} \) is a minimum value of the contour, typically taken to be \( 3\times \text{rms} \) noise in the PVD, and \( \eta \) is a constant that determines the fraction of maximum intensity to set the envelope intensity, that is usually taken in the range \( 0.2\sim0.5 \) (see references above). An arc-tangent model reproduces fairly well the shape of the envelope of the PVD with the smallest number of arguments and provides an adequate match to most rotation curves (Courteau 1997). This function is given by:

\[ V_{\text{env}}(r) = V_{\text{sys}} + V_0 \tan^{-1}\left(\frac{r - r_0}{r_t}\right) + C \]  

\(^\text{B.4}\)

where \( V_{\text{sys}}, V_0, r_0, r_t \) and \( C \) are five free parameters: \( V_{\text{sys}} \) is the systemic velocity, \( V_0 \) is the velocity amplitude with respect to the centre, \( r_0 \) is the position of maximum intensity of the envelope of an observed PVD along the line profile, \( r_t \) is a parameter that determines the range of radial velocities where the function is valid, and \( C \) is an adjustable constant.

**Table A.1. Set-up of the Fabry-Perot observations**

| Parameter                      | OAN-SPM       | OHP          |
|--------------------------------|---------------|--------------|
| Telescope                      | 2.1 m         | 1.93 m       |
| Instrument                     | PUMA\(^{\text{(1)}}\) | GHASP        |
| Seeing                         | 2.9 arcsec    | 2.5 - 4.5 arcsec |
| Galaxy Hα wavelength range    | 6593 - 6600 Å | 1380 - 1672 km s\(^{-1}\) |
| velocity range                 |               |              |
| Observation date               | 2017-02-23    | 2021-01-14 to 18 |
|                                |               | 2021-01-16 to 18 |
| Number of cycles               | 1             | 80           |
| Total exposure time            | 96 min        | 427 min      |
| Scanning FP interferometer     |                |              |
| Calibration (Neon line)        | 6598.95 Å     |              |
| Finesse (F)                    | 19.9          | 12.3±0.3\(^{\text{(3)}}\) |
| Resolution power (pF) at Hα\(^{\text{(5)}}\) | 330 (328)     | 793 (789)    |
| Free spectral range at Hα\(^{\text{(5)}}\) | 20.05 Å /911 km s\(^{-1}\) | 8.35 Å / 379 km s\(^{-1}\) |
| Spectral resolution at Hα\(^{\text{(5)}}\) | 1.01 Å /45.8 km s\(^{-1}\) | 0.68 Å / 30.9 km s\(^{-1}\) |
| Spectral sampling at Hα\(^{\text{(5)}}\) | 0.42 Å /19.0 km s\(^{-1}\) | 0.26 Å / 11.9 km s\(^{-1}\) |
| Line Spread Function (σ\(_{LSF}\)) | 0.43 Å /19.5 km s\(^{-1}\) | 0.29 Å / 13.1 km s\(^{-1}\) |
| Scanning steps                 | 48            | 32           |
| Exposure time per channel      | 120 s         | 10 s         |
| Detector Name                  | Spectral 2 \(^{\text{(6)}}\) | IPCS (GaAs)  |
| Detector size                  | 512 × 512 pixels\(^2\) | 512 × 512 pixels\(^2\) |
| Field-of-view                  | 10 × 10 arcmin\(^2\) | 5.8 × 5.8 arcmin\(^2\) |
| Image scale                    | 1.27\(^{\text{(7)}}\) arcsec pix\(^{-1}\) | 0.68\(^{\text{(8)}}\) arcsec pix\(^{-1}\) |

**Notes.** \(^{\text{(1)}}\) https://www.astrossp.unam.mx/instrumentos/interferometria/puma/docs/manualespuma/puma.html. \(^{\text{(2)}}\) Queensgate Instruments, which is now called IC Optical Systems. \(^{\text{(3)}}\) Mean and standard deviation of the finesse (F) measured from eight different calibration cubes. \(^{\text{(4)}}\) p is the interference order. \(^{\text{(5)}}\) Hα\(_0\) is the wavelength of the Hα-line at rest, and Hα the wavelength at the galaxy heliocentric line-of-sight velocity (see Table 1). \(^{\text{(6)}}\) https://www.astrossp.unam.mx/en/users/ccd-s. \(^{\text{(7)}}\) After 4×4 electronic binning on the CCD. \(^{\text{(8)}}\) After 2×2 electronic binning on the IPCS.

of the curve continue up to ~95 arcsec (~8 kpc). We have indicated \( r_0 \) by dashed lines in all the maps and PV diagrams all over the paper. Beyond this radius, differences are observed between the receding and approaching sides. Discreant points are associated to several regions previously identified in Fig. 2: region A on the hook-like structure and region J located at about the same galacto-centric distance show velocity discrepancy of ~110 km s\(^{-1}\) in the GHASP RC and ~50 km s\(^{-1}\) in the PUMA RC; regions K and L in the southwest tail do not have a counterpart on the other side of the galaxy. The deepest GHASP data show velocities fairly constant (\( V_{\text{rot}} \sim 122 \pm 10 \) km s\(^{-1}\)) in the southwest tail, consistent with a streaming motion.

**Appendix B.3: The envelope-tracking method**

The envelope-tracking method (ETM) uses the position-velocity diagram (PVD) traced along the major axis of the galaxy in simulating a pseudo-slit having a width equal to two pixels (see Sect. B.1) and adopts all the kinematic parameters (centre, position angle, and systemic velocity, see Table B.1) derived from it, with the exception of the inclination which cannot be determined using this method. Following Sofue et al. (1997, 1999a); García-Ruiz et al. (2002), the rotation velocity is defined as:

\[ V_{\text{rot}} \sin i = (V_{\text{env}} - V_{\text{sys}}) - (\sigma_T^2 + \sigma_{LSF}^2)^{1/2}, \]  

where \( V_{\text{sys}} \) is the systemic velocity, \( i \) is the galaxy inclination with respect to the sky plane, \( \sigma_{LSF} \) is the instrumental velocity dispersion, \( \sigma_T \) is the thermal broadening of the gas, and \( V_{\text{env}} \) is the terminal velocity characterised by the intensity of the envelope of an observed PVD along the line profile\(^{\text{11}}\). The intensity of the envelope is defined as:

\[ I_{\text{env}} = (\eta I_{\text{max}}^2 + (I_{\text{min}}^2)^{1/2}, \]  

where \( I_{\text{max}} \) is the maximum intensity in the line profile, \( I_{\text{min}} \) is a minimum value of the contour, typically taken to be \( 3\times \text{rms} \) noise in the PVD, and \( \eta \) is a constant that determines the fraction of maximum intensity to set the envelope intensity, that is usually taken in the range \( 0.2\sim0.5 \) (see references above). An arc-tangent model reproduces fairly well the shape of the envelope of the PVD with the smallest number of arguments and provides an adequate match to most rotation curves (Courteau 1997). This function is given by:

\[ V_{\text{env}}(r) = V_{\text{sys}} + V_0 \tan^{-1}\left(\frac{r - r_0}{r_t}\right) + C \]  

\(^{\text{B.4}}\)

where \( V_{\text{sys}}, V_0, r_0, r_t \) and \( C \) are five free parameters: \( V_{\text{sys}} \) is the systemic velocity, \( V_0 \) is the velocity amplitude with respect to the centre, \( r_0 \) is the position of maximum intensity of the envelope of an observed PVD along the line profile, \( r_t \) is a parameter that determines the range of radial velocities where the function is valid, and \( C \) is an adjustable constant.

\(^{\text{11}}\) Sofue & Rubin (2001) provide a slightly different expression that corrects for instrumental and thermal broadening after deprojecting from inclination. Despite their work is often cited as reference since then, we believe this was a typo since the term that needs to be corrected for broadening is the observable, \( V_{\text{env}} - V_{\text{sys}} \), as done in the previous work of Sofue et al. (1997).
the velocity $C$ at $r = r_0$, $r_0$ and $r_t$ are two characteristic radii that shape the curve marking the radius between the rising and flat part of the RC and the sharpness of this transition respectively. $r_0$ is indeed the inflexion point of the function visible on the pink curve on Fig. 3, and represented by two dashed lines on the Hα velocity fields (Fig. 5). This arc-tangent model has also been used by other authors (Weiner et al. 2006; Drew et al. 2018; Zhao et al. 2021). Because the PVD is traced along the major axis, relation (B.4) can be transformed into a rotation curve using relation (B.2). Therefore, to apply the ETM, we determined the terminal velocity $V_{\text{env}}$ by fitting the arc-tangent model to both the maximum $I_{\text{max}}$ and minimum $I_{\text{min}}$ intensity line profiles. We corrected the velocities for $\sigma_T = 9.1 \text{ km s}^{-1}$ to account for the dispersion due to the gas temperature along the LoS (see Sect. 2). The results of the ETM are shown in Fig. 3. The rotation velocities and its best fit are given on the upper panels (a) and (c) while the PVDs from which the rotation velocities have been computed are shown on the lowest panel (b) and (d). Using Eq. (B.3), we computed the intensity of the envelope ($I_{\text{env}}$) for $\eta=0.2-0.5$ and then we applied relation (B.2). The shaded area corresponds to $\eta=0.2-0.5$ and the pink curve to $\eta=0.3$. For $\eta=0.3$, the maximum coefficient of determination is $R^2 = 0.8$ and was obtained with the parameters given in Table B.1.

Appendix B.4: The iteration method

The intensity-peak and the envelope-tracing methods are limited by the observational parameters such as the seeing, the spectral resolution, the slit width (or pseudo-slit width in case of 2D velocity fields). They also depend on the physical properties of the galaxy such as its inclination, the velocity dispersion of the gas and its distribution within the disc. To overcome these sources of uncertainties, Takamiya & Sofue (2000) proposed a different model, the iteration method (ITM), to generate a model data cube using specific radial and vertical density profiles of the ionised gas (e.g., exponentially declining), rotation curves, velocity dispersions, and viewing angles. First, to provide the initial conditions from which a first model data cube is computed, we used a rotation curve derived from a position-velocity diagram. Second, the ETM is used to derive a new rotation curve from the first model data cube, which is compared to the data. Third, the differences between these two RCs are added to the input rotation curve to generate a new model. This procedure is repeated until the difference between the rotation curves derived from the data and the final model satisfies a convergence criterion or until a maximum number of iterations has been performed (e.g. Takamiya & Sofue 2000; Sofue & Rubin 2001; Heald et al. 2006, 2007). As suggested by the intensity-weighted and the envelope-tracing rotation curves of NGC 4330, the inner regions of the disc within $R_{23}$ show a solid-body shape (see Fig. 3 and 5).

Therefore, following Swaters et al. (1997); Heald et al. (2006); Rosado et al. (2013) and Moiseev (2014), we applied to NGC 4330 the classically called cylindrical model. This model assumes, as a first grade approximation, that the galaxy has the shape of a cylinder that rotates as a rigid body around the $z$-axis perpendicular to the $x$, $y$-plane, which defines the disc. In order to compute the model data cube, we proposed an initial rotation curve that describes the rotation of the cylindrical shaped rigid body based on the arc-tangent model of a rotation curve (Eq. B.4). Using the non-linear least-squares minimization of the library LMFIT from PYTHON, we computed a model data cube for each FP data set whose emission profiles were represented by Gaussian functions. The velocity dispersion of the emission peaks was fixed for all profiles to the medium value of the velocity dispersion of the galaxy ($\sigma_{\text{LoS}}$). For the amplitude of the Gaussian functions, we assumed that the gas has an exponentially declining distribution (Mo et al. 2010; Takamiya & Sofue 2002) within the disc of the galaxy along the $z$-direction, $B(z) = B_0 \exp(-|z|/z_d)$, and we measured the vertical disc length $z_d = 6$ arcsec using the Hα surface brightness distribution. We used the value of the kinematic centre of each data set as the systemic velocity of the galaxy ($V_{\text{sys}}$) and the kinematic $PA$ obtained from the PVDs. This implies that the free parameters of the model are: the transition radius between the rising and flat part of the RC ($r_t$), the asymptotic velocity ($V_0$), and the inclination ($i$) of the galaxy.

Appendix C: Additional figures

The intensity-peak and the envelope-tracing methods are limited by the observational parameters such as the seeing, the spectral resolution, the slit width (or pseudo-slit width in case of 2D velocity fields). They also depend on the physical properties of the galaxy such as its inclination, the velocity dispersion of the gas and its distribution within the disc. To overcome these sources of uncertainties, Takamiya & Sofue (2000) proposed a different model, the iteration method (ITM), to generate a model data cube using specific radial and vertical density profiles of the ionised gas (e.g., exponentially declining), rotation curves, velocity dispersions, and viewing angles. First, to provide the initial conditions from which a first model data cube is computed, we used a rotation curve derived from a position-velocity diagram. Second, the ETM is used to derive a new rotation curve from the first model data cube, which is compared to the data. Third, the differences between these two RCs are added to the input rotation curve to generate a new model. This procedure is repeated until the difference between the rotation curves derived from the data and the final model satisfies a convergence criterion or until a maximum number of iterations has been performed (e.g. Takamiya & Sofue 2000; Sofue & Rubin 2001; Heald et al. 2006, 2007). As suggested by the intensity-weighted and the envelope-tracing rotation curves of NGC 4330, the inner regions of the disc within $R_{23}$ show a solid-body shape (see Fig. 3 and 5).

Therefore, following Swaters et al. (1997); Heald et al. (2006); Rosado et al. (2013) and Moiseev (2014), we applied to NGC 4330 the classically called cylindrical model. This model assumes, as a first grade approximation, that the galaxy has the shape of a cylinder that rotates as a rigid body around the $z$-axis perpendicular to the $x$, $y$-plane, which defines the disc. In order to compute the model data cube, we proposed an initial rotation curve that describes the rotation of the cylindrical shaped rigid body based on the arc-tangent model of a rotation curve (Eq. B.4). Using the non-linear least-squares minimization of the library LMFIT from PYTHON, we computed a model data cube for each FP data set whose emission profiles were represented by Gaussian functions. The velocity dispersion of the emission peaks was fixed for all profiles to the medium value of the velocity dispersion of the galaxy ($\sigma_{\text{LoS}}$). For the amplitude of the Gaussian functions, we assumed that the gas has an exponentially declining distribution (Mo et al. 2010; Takamiya & Sofue 2002) within the disc of the galaxy along the $z$-direction, $B(z) = B_0 \exp(-|z|/z_d)$, and we measured the vertical disc length $z_d = 6$ arcsec using the Hα surface brightness distribution. We used the value of the kinematic centre of each data set as the systemic velocity of the galaxy ($V_{\text{sys}}$) and the kinematic $PA$ obtained from the PVDs. This implies that the free parameters of the model are: the transition radius between the rising and flat part of the RC ($r_t$), the asymptotic velocity ($V_0$), and the inclination ($i$) of the galaxy.
| Parameter | Photometric | Kinematic |
|-----------|-------------|-----------|
| \( C_\alpha \) (J2000) \(^1\) | \( 12^h23^m17.1^s \) | \( 12^h23^m17.0^s \) | \( 12^h23^m17.0^s \) |
| \( C_\delta \) (J2000) \(^1\) | \(+11^\circ22'5.7''\) | \(+11^\circ22'06.7''\) | \(+11^\circ22'06.7''\) |
| PA (deg) \(^2\) | 59.0 ± 0.1 | 56 ± 1 | 59 ± 1 |
| \( i \) (deg) \(^3\) | 89.5 ± 0.1 | 88 ± 2 | 88 ± 2 |
| \( V_{sys} \) (km s\(^{-1}\)) \(^4\) | ... | 1569 ± 2 | 1563 ± 2 |
| \( V_{rot} \) (km s\(^{-1}\)) \(^5\) | ... | 164 ± 11 | 158 ± 8 |
| \( \langle \sigma_{obs} \rangle \) (km s\(^{-1}\)) \(^7\) | ... | 162 ± 8 | 150 ± 21 |
| \( \sigma_{LSF} \) (km s\(^{-1}\)) \(^8\) | ... | 35 ± 10 | 21 ± 7 |
| \( V_0 \) (km s\(^{-1}\)) \(^9\) | ... | 19.5 | 13.1 |
| \( r_0 \) (arcsec) \(^10\) | ... | 46 ± 8 | 34 ± 5 |
| \( r_0 \) (arcsec) \(^11\) | ... | 27 ± 2 | 28 ± 2 |
| \( C \) (km s\(^{-1}\)) \(^12\) | ... | 17 ± 4 | 15 ± 4 |
| \( V_0 \) (km s\(^{-1}\)) \(^13\) | ... | 98 ± 3 | 100 ± 2 |
| \( r_0 \) (arcsec) \(^14\) | ... | 107 ± 5 | 110 ± 15 |
| \( r_0 \) (arcsec) \(^15\) | ... | 8 ± 3 | 20 ± 1 |

**Notes.** (1): Right ascension and declination of the galaxy centre. The photometric and the kinematic centre differ by less than half the seeing disc. (2): Position angle of the major axis. (3): Column 2: photometric inclination computed with the ellipse task from IRAF. Columns 3 and 4: kinematic inclination computed with the iteration method (ITM). (4): Systemic velocity. (5): Rotation velocity measured with the RC computed with the envelope-tracing method (ETM) at \( R_{23.5} \). (6): Rotation velocity measured with the RC computed with the iteration method (ITM) at \( R_{23.5} \). (7): Mean velocity dispersion. (8): Instrumental velocity dispersion (also called the line-spread function, LSF). (9) to (14): Best fit parameters of equation B.4 to compute the RC with the ETM (see text).
Fig. C.1. Model of PUMA spectrum. The Hα plus the two [NII]6548, 6583 lines, transmitted by the filter, are represented by Gaussian functions of width $\sigma_{FSR}$ (see Table 1). Normalised units to the Hα line are used for the y-axis. The relative intensity between the Hα and the [NII] lines are given in Sect. (A.1). The distance between the two green dashed vertical lines represents the Free Spectral Range (FSR $\sim 20$ Å). Top panel: the redshifted Hα and [NII]6548, 6583 lines are located at their right position. The distance between the two gray dashed vertical lines represents the filter width (FWHM $\sim 90$ Å). Bottom panel: the redshifted lines are located at their apparent position due to the FSR recovering: the [NII]6548 line is shifted by plus one FSR and the [NII]6583 is shifted by minus one FSR. The green curve represents the sum of the Hα plus [NII]6583 lines whereas the red one represents the Gaussian which best fit the sum of the two lines (Best Fit Method).

Fig. C.2. Long-slit spectra obtained with MISTRAL at the 1.93 m OHP telescope. The Hα lines of region D (red spectrum) and the position of the orthogonal projection of region D on the major axis of the galaxy (blue spectrum) are fitted by Gaussian functions, with intensities in normalised units. The two vertical dashed lines on Hα indicate the measured velocity of region D (1458±46 km s$^{-1}$) and on the disc (1629±46 km s$^{-1}$).
Fig. C.3. GHASP velocity dispersion properties of NGC 4330: (a) intensity histogram; (b) loci, in the galaxy, of the different areas shown in panel (e) with different colors (red, green, blue and light blue) on top of which Hα isocontours have been superimposed; (c) velocity dispersion vs. emission intensity diagnostic diagram, each black dot represents a pixel and the blue dots represent the mean of the intensity in each velocity dispersion bin; (d) velocity dispersion histogram; (e) pixel density map of the velocity dispersion vs. intensity diagram, the color bar represents pixel density. The dashed line in plots (c) and (d) represents the Line Spread Function $\sigma_{LSF}$. 

Fig. C.4. HI emission data of NGC 4330 from Chung et al. (2009). Panel (a): Cold gas velocity field on which are superimposed the positions of pseudo-slits (28 arcsec wide) parallel to the kinematic major axis and the ellipse fitted to the isophote of surface brightness $\mu(i) = 23.5$ mag arcsec$^{-2}$ of the NGVS $i$-band image, which traces the stellar disc. Panels (b) and (d): outermost contour of the position-velocity diagrams (PVDs). Panel (b) corresponds to the three pseudo-slits positions drawn in panel (a) and panel (d) corresponds to the central pseudo-slit position, the red and blue circles indicate the position of the minimum intensity of the receding and approaching side, respectively, while the dashed line indicates the maximum intensity. Both entities are used to estimate the terminal velocity and to compute the RC using the ETM method. Panel (c): Rotation curve (RC) computed using the ETM, where the terminal velocity is represented by empty blue circles for the receding side and the filled red circles for the approaching side. The black solid line indicates the best fit to these points using $\eta = 0.3$, the shaded area shows the rotation velocity amplitude for $\eta = [0.2, 0.5]$, the purple dashed line the RC computed from the PVD maximum intensity, and the red and blue vertical lines indicate the photometric and effective radius, respectively.
Fig. C.5. Comparison between the GHASP Hα (upper panel) and the HI (lower panel, Chung et al. 2009) velocity fields of NGC 4330. The inner and outer black contours are at \( \Sigma(H\alpha) = 3 \times 10^{-18} \) and \( 1.25 \times 10^{-16} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ arcsec}^{-2} \), respectively, while the red contour \( \Sigma(H\alpha) = 10^{-17} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ arcsec}^{-2} \). The black diamond, the cross, and the star indicate the position of the CO, Hα, and HI emission kinematic centres, respectively. To align the maps horizontally, both are rotated \( i = 59^\circ \), the photometric PA of the galaxy. The grey ellipses encapsulated in a box located in the upper-right corner of each panel represent the sizes of the Hα mean seeing disc (upper panel) and of the HI beam (upper panel).

Fig. C.6. Comparison between the \(^{12}\text{CO}\) (upper panel, Lee et al. 2017) and the PUMA Hα (lower panel) velocity fields of NGC 4330. The inner black contours and the red outer contours are at \( \Sigma(H\alpha) = 10^{-17} \) and \( 1.25 \times 10^{-16} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ arcsec}^{-2} \), respectively. The black diamond and the cross indicate the position of the CO and Hα emission kinematic centres, respectively. To align the maps horizontally, both are rotated \( i = 59^\circ \), the photometric PA of the galaxy. The grey ellipses located in the upper-right corner of each panel represent the sizes of the CO beam (upper panel) and of the Hα mean seeing disc (lower panel).

Fig. C.7. Comparison between observed and simulated unfolded rotation curves. The simulated velocity field shown in the panel (c) of Fig. 11 has been used to measure the rotation curve along the major axis of the galaxy, the receding and approaching sides are respectively represented by filled orange and opened cyan stars (⋆) and superimposed to the GHASP observed and modelled rotation curves already displayed in the panel (c) of Fig. 3, using the same symbols, colours and lines. The blue vertical lines indicate the position of the effective radius, respectively.
**Fig. C.8.** Panel (a) GHASP and (panels (b) and (c)) simulated Hα surface brightness of NGC 4330, with scales identical to those used in Fig. 2 (d), i.e. with a threshold limit $\Sigma(H\alpha) \approx 10^{-17} \text{erg s}^{-1} \text{cm}^{-2} \text{arcsec}^{-2}$. The low level cuts used to display the simulated Hα images have been chosen to show the low column density ionised gas reached by the simulations (panel (b)) and at higher surface brightness levels ($\Sigma(H\alpha) \approx 2\times10^{-17} \text{erg s}^{-1} \text{cm}^{-2} \text{arcsec}^{-2}$, panel (c)), matching the typical surface brightness limit of VESTIGE and Fabry-Perot observations, respectively.