Inelastic Final-State Interactions of $B \rightarrow VV \rightarrow \pi K$ Processes

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Abstract

We study the final-state interactions in $B \rightarrow \pi K$ decays through $B \rightarrow VV \rightarrow \pi K$ processes where the inelastic rescattering occurs via single pion exchange. The next-to-leading order low energy effective Hamiltonian and BSW model are used to evaluate the weak transition matrix elements and the final-state interactions. We found that the final-state interaction effects in $B \rightarrow \rho K^* \rightarrow \pi K$ processes are significant. The Fleischer-Mannel relation about the CKM angle $\gamma$ may be significantly modified.
1. Introduction

Final-state interactions (FSI) play a great role in many physical processes, especially in the weak decays which are the one of the focus of recent interests. Their effects lie in two aspects. First, strong phases in weak decay amplitudes are generated by the final-state interactions, they may contribute the strong phases for the direct CP asymmetries; second, FSI effects may significantly change the theoretical predications for certain quantities. Study on final-state interactions would definitely need information about the non-perturbative effects of low energy hadron interactions. Unless we can correctly evaluate the FSI effects, it is impossible to extract reliable information about the reaction mechanism or new physics from the data. Understanding the final-state interaction effects in weak decays is not only crucially important, but also is a challenging and difficult task in both theory and phenomenology. Up to now, definite quantitative analysis has not been accessible yet. The estimate of the final-state interactions is centered on some particular cases where the symmetry relations can be applied, or one can use some simplified models for example, the Regge pole or single-pion exchange etc. to do the job.

The origin of CP violation in the Standard Model comes from the complex phase of the Cabibbio-Kobayashi-Maskawa (CKM) matrix. In general, for three generation quark families the CKM matrix elements form a unitarity triangle. So, reasonable extraction of each angles ($\alpha, \beta, \gamma$) is extremly important for testing the Standard Model. Among the three angles, to extract $\gamma$ is most difficult. Some methods have been put forward for this purpose. But all the methods are either too complicated or impractical from experimental point of view. Last year, the CLEO collaboration reported the combined ratios for $B \to \pi K$ decays:

$BR(B^{\pm} \to \pi^{\mp}K) = (2.3^{+1.1+0.2}_{-1.0-0.2} \pm 0.2) \times 10^{-5}$

$BR(B_d \to \pi^{\mp}K^{\pm}) = (1.5^{+0.5+0.1}_{-0.4-0.1} \pm 0.1) \times 10^{-5}$

Fleischer and Mannel give a bound relation on the CKM angle $\gamma$ based on the above results:

$R \geq sin^2 \gamma$, where $R = \frac{BR(B_d \to \pi^{\mp}K^{\pm})}{BR(B^{\pm} \to \pi^{\mp}K)} = 0.65 \pm 0.40$, $\gamma \equiv Arg(V_{ub})$. In their work, final-state interactions were neglected. However, the final-state interactions in such channels may be
important and cannot be ignored. Some authors [4] have studied the final-state interaction effects on $B \to \pi K$ decays. Their studies are based on the Regge pole theory. In Ref. [5], the authors study the final-state interactions due to single pion exchange in $D \to VP$ processes. Their results show that the single pion exchange can be significant. It is believed that even though the two schemes describe the process based on different physical considerations, in practice, each of them works well, just because the uncertainties are partly compensated by proper selection of some phenomenological parameters in these schemes.

In this paper, we study the final-state interactions in $B \to VV \to \pi K$ due to single pion exchange. In Standard Model calculation, the decay of $B \to \rho K^*$ has the same order amplitude as that of $B \to \pi K$. In fact, both $K^* \to K\pi$ and $\rho \to \pi\pi$ are the dominant strong decay channels, so that the single pion exchange mechanism may dominate the final state interactions. In our processes, the exchanged pion is in the t-channel. There are other two-vector-meson states which can rescatter into the $\pi K$ final state by pion exchange. But these intermediate mesons have smaller couplings to $\pi K$ and $\pi\pi$ than $K^* \to \pi K$ and $\rho \to \pi\pi$; also they are heavier than $\rho$ and $K^*$ and will result in larger t-value which will further reduce their contribution. So, the final-state interactions in $B \to \rho K^* \to \pi K$ processes may be the largest in the processes of $B \to VV \to \pi K$. We use the method which was presented in Ref. [5]. The next-to-leading order low energy effective Hamiltonian and BSW model are used to evaluate the weak transition matrix elements and the final-state interactions. We find that the final-state interaction effects in $B \to \rho K^* \to \pi K$ are significant.

2. The formulation for FSI in effective Hamiltonian

To evaluate the decays of $B \to \pi K$, we use the next-to-leading order low energy effective Hamiltonian and BSW model. The next-to-leading order low energy effective Hamiltonian describing $|\Delta B| = 1$ transitions is given at the renormalization scale $\mu = O(m_b)$ as [3]:

$$
\mathcal{H}_{\text{eff}}(|\Delta B| = 1) = \frac{G_F}{\sqrt{2}} \left[ \sum_{q=u,c} v_q \left\{ Q_1^q C_1(\mu) + Q_2^q C_2(\mu) + \sum_{k=3}^{10} Q_k C_k(\mu) \right\} \right] + H.C. \quad (1)
$$
The CKM factors $v_q$ are defined as $v_q = V_{q\bar{q}}^* V_{q\bar{b}}$, where $q = u, c$.

The ten operators $Q^u_1$, $Q^u_2$, $Q_3, \ldots, Q_{10}$ are known as the following forms:

\begin{align*}
Q^u_1 &= (\bar{q}_u u_\beta) V_{A} (\bar{u}_\beta b_\alpha) V_{A} \\
Q^u_2 &= (\bar{q}_u) V_{A} (\bar{u} b) V_{A} \\
Q_{3(5)} &= (\bar{q}_b) V_{A} \sum_{q'} (\bar{q}' q') V_{A} (V_{A} + A) \\
Q_{4(6)} &= (\bar{q}_\alpha b_\beta) V_{A} \sum_{q'} (\bar{q}' q') V_{A} (V_{A} + A) \\
Q_{7(9)} &= \frac{3}{2} (\bar{q}_b) V_{A} \sum_{q'} e_q (\bar{q}' q') V_{A} (V_{A} + A) \\
Q_{8(10)} &= \frac{3}{2} (\bar{q}_\alpha b_\beta) V_{A} \sum_{q'} e_q (\bar{q}' q') V_{A} (V_{A} + A)
\end{align*}

where $Q^u_1$ and $Q^u_2$ are the current-current operators, and the current-current operators $Q^c_i$ and $Q^s_i$ can be obtained from $Q^u_i$ and $Q^s_i$ through the substitution of $u \rightarrow c$. $Q_3, \ldots, Q_6$ are the QCD penguin operators, whereas $Q_7, \ldots, Q_{10}$ are the electroweak penguin operators. The quark $q = s$ for $b \rightarrow s$ transition; ($V \pm A$) refer to $\gamma\mu(1 \pm \gamma_5)$.

The matrix elements are:

\begin{equation}
< Q^T(\mu) \cdot C(\mu) > = < Q^T >_0 \cdot C'(\mu)
\end{equation}

where $< Q >_0$ denote the tree level matrix elements of these operators, and $C'(\mu)$ are defined as

\begin{align*}
C'_1 &= \overline{C}_1, \\
C'_2 &= \overline{C}_2, \\
C'_3 &= \overline{C}_3 - P_s/3, \\
C'_4 &= \overline{C}_4 + P_s, \\
C'_5 &= \overline{C}_5 - P_s/3, \\
C'_6 &= \overline{C}_6 + P_s, \\
C'_7 &= \overline{C}_7 + P_e, \\
C'_8 &= \overline{C}_8, \\
C'_9 &= \overline{C}_9 + P_e, \\
C'_{10} &= \overline{C}_{10},
\end{align*}

where $P_s, e$ are given by

\begin{align*}
P_s &= \frac{\alpha s}{8\pi} \overline{C}_2(\mu) \left[ \frac{10}{9} - G(m_q, q, \mu) \right], \\
P_e &= \frac{\alpha_{em}}{8\pi} \left( 3\overline{C}_1 + \overline{C}_2(\mu) \right) \left[ \frac{10}{9} - G(m_q, q, \mu) \right], \\
G(m, q, \mu) &= -4 \int_0^1 dx x(1-x) \ln \left[ \frac{m^2 - x(1-x)q^2}{\mu^2} \right],
\end{align*}

here $q$ represents $u$, $c$, and $q^2 = m_b^2/2$. The numerical values of the renormalization scheme independent Wilson Coefficients $\overline{C}_i(\mu)$ at $\mu = O(m_b)$ are [5]

\begin{align*}
\bar{c}_1 &= -0.313, \\
\bar{c}_2 &= 1.150, \\
\bar{c}_3 &= 0.017, \\
\bar{c}_4 &= -0.037, \\
\bar{c}_5 &= 0.010, \\
\bar{c}_6 &= -0.046, \\
\bar{c}_7 &= -0.001 \cdot \alpha_{em}, \\
\bar{c}_8 &= 0.049 \cdot \alpha_{em}, \\
\bar{c}_9 &= -1.321 \cdot \alpha_{em}, \\
\bar{c}_{10} &= 0.267 \cdot \alpha_{em}.
\end{align*}
(1) Without the final-state interactions

In $B_u^- \to \pi^- K^0$ decay, only penguin diagrams contribute. As commonly agreed in the present literatures, the annihilation diagram contributions are neglected because of $V_{ub}$ and the form factor suppression. In $\bar{B}_d^0(b \bar{d}) \to \pi^+ K^-$ decay, both tree and penguin diagrams contribute. The amplitudes of these two decays are:

$$A_{\text{dir}}(B_u^- \to \pi^- K^0) = \frac{G_F}{\sqrt{2}} \sum q_{u,c} v_q [a_3 + \frac{2M^2_{K^0}}{(m_s+m_d)(m_b-m_d)} (a_5 - \frac{1}{2}a_7) - \frac{1}{2}a_9] M_{K^0\pi^-}$$

$$A_{\text{dir}}(\bar{B}_d^0 \to \pi^+ K^-) = \frac{G_F}{\sqrt{2}} \sum q_{u,c} v_q [a_1 \delta_{uq} + a_3 + \frac{2M^2_{K^-}}{(m_s+m_u)(m_b-m_u)} (a_5 + a_7) + a_9] M_{K^+\pi^-}$$

where “dir” means direct decay without final-state interactions, and

$$M_{K^0\pi^-} \equiv <\bar{K}^0|((\bar{s}d)_{V-A}|B_u^- > <\pi^-|((\bar{d}b)_{V-A}|B_u^- > = -if_K F_0^{B_u^- \pi}(M^2_{K^0})(M^2_{B_u^-} - M^2_{\pi^-})$$

$$M_{K^-\pi^+} \equiv <K^-|((\bar{s}u)_{V-A}|\bar{B}_d^0 > <\pi^+|((\bar{u}b)_{V-A}|\bar{B}_d^0 > = -if_K F_0^{B_d^0 \pi}(M^2_{K^-})(M^2_{\bar{B}_d^0} - M^2_{\pi^+})$$

and

$$a_{2i-1} = C_{2i-1}^c / N_c$$
$$a_{2i} = C_{2i}^c / N_c$$

Because $|v_c|/|v_u| >> 1$, the tree diagram contributions to the decay amplitude are small compared to the Penguin diagrams. The factorization approximation and BSW model are used to evaluate the matrix elements in Eq.(8). Table.1 gives the calculation results in the standard method for the above two direct decays. The non-factorization effects have been considered by the choice of $N_c = \infty, 3, 2$.

(2) With the final-state interactions

In $B \to \rho K^* \to \pi K$ processes, the exchanged pions can be neutral and charged as shown in Fig.1. For charged pion exchange the decay $B_u^\pm \to \pi^\pm K$ can get the tree diagram contribution in the intermediate state $B \to \rho K^* \to \pi K$.

We take the decay $B_u^- \to \rho^- K^{*0} \to \pi^- K^0$ as an example to show how to calculate the final-state interaction effects.
The amplitude for $B_u^- \to \rho^- K^{*0}$ decay is:

$$A(B_u^- \to \rho^- K^{*0}) = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} v_q (a_3 - \frac{1}{2} a_9) M K^{*0\rho^-}$$  \hspace{1cm} (10)$$

where

$$M K^{*0\rho^-} \equiv <K^{*0}|(\bar{s}d)_{V-A}|0> <\rho^-|(\bar{d}b)_{V-A}|B_u^- > = \frac{2M_{K^{*0}}}{M_{B_u} + M_{\rho^-}} f_K V B_u^\rho (M_{K^{*0}}^2) \epsilon_{\mu\nu\rho\sigma}^{K^{*0}} \epsilon_{\rho^-}^{\prime} p_{K^{*0}}^\mu p_{\rho^-}^\nu + i M_{K^{*0}} (M_{B_u} + M_{\rho^-}) f_K A_1 B_u^\rho (M_{K^{*0}}^2) (\epsilon_{\mu\nu\rho\sigma}^{K^{*0}} p_B)^\mu (\epsilon_{\rho^-}^{\prime} \cdot p_B)$$  \hspace{1cm} (11)

As in Ref.\[3\], the single pion exchange in the t-channel makes a significant contribution to the FSI and would dominate. To get the absorptive part of the loop as shown in Fig.1a, the way to make cuts is: let the $\rho^-$ and $K^{0\ast}$ be on-shell, and leave the exchanged pion to be off-shell.

In the center of mass frame of $B_u^-$ where $p_{Bu} = (M_{Bu}, 0)$, the matrix element in Eq.(11) is recast into the following form for the process $B_u^- \to \rho^- K^{*0} \to \pi^- \bar{K}^0$

$$M_{FSI}^{K^{*0}\pi^-} = \frac{1}{2} f \frac{d^3p_1}{(2\pi)^3} \frac{d^3p_2}{(2\pi)^3} (2\pi)^4 \delta^4(p_1 + p_2 - p_B) <\pi^- \bar{K}^0|S|\rho^- K^{*0} > M_{K^{*0}\rho^-}$$  \hspace{1cm} (12)

where $S$ is the S-matrix of strong interaction, $\theta$ is the angle between $\vec{p}_1$ and $\vec{p}_3$, and

$$H_1 = -4g_{\rho\pi\pi} g_{K^* K \pi} \left( \frac{p_3 \cdot p_4}{M_2^2} - \frac{(p_2 \cdot p_3)(p_2 \cdot p_4)}{M_2^2} \right) - \frac{(p_1 \cdot p_3)(p_1 \cdot p_4)}{M_1^2 M_2^2} + \frac{(p_1 \cdot p_2)(p_1 \cdot p_3)(p_2 \cdot p_4)}{M_1^2 M_2^2} (13)$$

$$H_2 = -4g_{\rho\pi\pi} g_{K^* K \pi} \left( \frac{p_3^0 p_4^0}{M_2^2} - \frac{(p_2^0 p_3^0)(p_2 \cdot p_4)}{M_2^2} \right) - \frac{(p_1^0 p_2^0)(p_1 \cdot p_3)(p_2 \cdot p_4)}{M_1^2 M_2^2} + \frac{(p_1^0 p_2^0)(p_2 \cdot p_3)(p_2 \cdot p_4)}{M_1^2 M_2^2}$$

We set $p_1 = p_{K^*}, p_2 = p_{\rho}, p_3 = p_{K}, p_4 = p_{\pi}, M_1 = M_{K^*}, M_2 = M_{\rho}, M_3 = M_{K}, M_4 = M_{\pi}$.

So, the amplitude of $B_u^- \to \rho^- K^{*0} \to \pi^- \bar{K}^0$ is:

$$A_{FSI}^{\rho\pi\pi}(B_u^- \to \pi^- \bar{K}^0) = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} v_q (a_3 - \frac{1}{2} a_9) M_{FSI}^{K^{*0}\pi^-}$$  \hspace{1cm} (14)$$

The factor $F(p_{\pi\pi}^2)$ in Eq.(12) is an off-shell form factor for the vertices $K^* K \pi$ and $\rho \pi \pi$. We take $F(p_{\pi\pi}^2) = \left( \frac{\Lambda^2 - m_{\pi\pi}^2}{\Lambda^2 - m_{\pi\pi}^2} \right)$ as in Ref.\[3\], where $\Lambda = 1.2 - 2.0 GeV$.  


3. The numerical results

The parameters such as meson decay constants, form factors and quark masses needed in our calculations are taken as:

meson decay constant [11] [9]:
\[ f_\pi = 0.13 GeV, \quad f_K = 0.16 GeV, \quad f_{\rho u}^{u\bar{u}} = 0.156 GeV, \quad f_{K^*} = 0.221 GeV; \]

form factor [3]:
\[ F_0^{B\pi}(0) = 0.333, \quad F_1^{B\pi}(0) = 0.333, \quad V^{BK^*}(0) = 0.369, \quad A_1^{BK^*}(0) = 0.328, \]
\[ A_2^{BK^*}(0) = 0.331, \quad V^{B\rho}(0) = 0.329, \quad A_1^{B\rho}(0) = 0.283, \quad A_2^{B\rho}(0) = 0.283; \]
effective strong coupling constants [5]:
\[ g_{K^*K\pi} = 5.8, \quad g_{\rho\pi\pi} = 6.1; \]
\[ \Lambda \text{ in the off-shell form factor } F(p^2_{\pi^0}): \quad \Lambda = 1.5 GeV; \]
quark mass [10]:
\[ m_u = 0.005 GeV, \quad m_d = 0.01 GeV, \quad m_s = 0.2 GeV, \quad m_c = 1.5 GeV, \quad m_b = 4.5 GeV; \]
the Wolfenstein CKM parameters [11]:
\[ \lambda = 0.22, \quad A = 0.8, \quad \eta = 0.34, \quad \rho = -0.12. \]

Due to the non-factorization effects, it is hard to choose the value of \( N_c \), so all three cases are taken into account: \( N_c = \infty, 3, 2. \)

The corresponding numerical results are presented in Table 1 and Table 2.

4. The Constraints on \( \gamma \) and \( A^{dir}_{\text{cp}} \)

(i) Without final-state interaction

For direct decays of \( B \to \pi K \), the amplitudes are:

\[
\begin{align*}
A^{dir}(B^+ \to \pi^+ K^0) &= A^+_{cs} - A^+_{us} e^{i\gamma} e^{i\delta}, \\
A^{dir}(B^- \to \pi^- \bar{K}^0) &= A^+_{cs} - A^+_{us} e^{-i\gamma} e^{i\delta}, \\
A^{dir}(B^0 \to \pi^- K^+) &= A^0_{cs} - A^0_{us} e^{i\gamma} e^{i\delta_0}, \\
A^{dir}(\bar{B}^0 \to \pi^+ K^-) &= A^0_{cs} - A^0_{us} e^{-i\gamma} e^{i\delta_0},
\end{align*}
\]
where $\delta_0$ and $\delta_+$ are CP-conserving strong phases.

The ratio $R$ is defined by:

$$R \equiv \frac{BR(B^0 \to \pi^- K^+) + BR(\bar{B}^0 \to \pi^+ K^-)}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- K^0)}$$

$$= \left(\frac{A_0^0}{A_{cs}}\right)^2 \frac{1 - 2r_0 \cos \gamma \cos \delta_0 + r_0^2}{1 - 2r_+ \cos \gamma \cos \delta_+ + r_+^2}$$

(16)

where $r_0 = A_{us}^0/A_{cs}^0$, $r_+ = A_{us}^+ / A_{cs}^+$, When neglecting the electro-weak penguin diagram contributions, $A_{us}^+ = 0$, i.e. $r_+ = 0$. According to SU(2) isospin symmetry of strong interaction. $A_{cs}^0 / A_{cs}^+ \approx 0$. So,

$$R = 1 - 2r_0 \cos \gamma \cos \delta + r_0^2.$$  

(17)

Following Fleischer and Mannel[3], we can obtain the inequality

$$R \geq \sin^2 \gamma$$  

(18)

This is the Fleischer-Mannel relation.

Direct CP violation in $B^\pm \to \pi^\pm K$ is defined through the CP asymmetry

$$A_{CP}^{dir} = \frac{BR(B^+ \to \pi^+ K^0) - BR(B^- \to \pi^- K^0)}{BR(B^+ \to \pi^+ K^0) + BR(B^- \to \pi^- K^0)} = \frac{2r_+ \sin \gamma \sin \delta_+}{1 - 2r_+ \cos \gamma \cos \delta_+ + r_+^2}.$$  

(19)

where this ”dir” means direct CP asymmetry. In direct decays, it is small from Eq.(19),

$$A_{CP}^{dir} \leq \mathcal{O}(\lambda^2).$$  

(20)

(ii) With the final-state interactions

When considering the final-state interactions, the amplitude of $B \to \pi K$ is changed to:

$$A(B^+ \to \pi^+ K^0)$$

$$= A^{dir}(B^+ \to \pi^+ K^0) + A^{FSI}(B^+ \to \rho^+ K^{*-} \to \pi^+ K^0) + A^{FSI}(B^+ \to \rho^0 K^{*-} \to \pi^+ K^0)$$

$$= (A_{cs}^+ - A_{us}^+ e^{i\gamma} e^{i\delta_+})(1 + A_1 e^{i\delta_1} + A_3 e^{i\gamma} e^{i\delta_3})$$

$$A(B^0 \to \pi^- K^+)$$

$$= A^{dir}(B^0 \to \pi^- K^+) + A^{FSI}(B^0 \to \rho^- K^{*+} \to \pi^- K^+) + A^{FSI}(B^0 \to \rho^0 K^{*0} \to \pi^- K^+)$$

$$= (A_{cs}^0 - A_{us}^0 e^{i\gamma} e^{i\delta_0})(1 + A_2 e^{i\delta_1})$$  

(21)
where $A_1$, $A_2$, $A_3$ are final-state interaction amplitudes, and $\delta_1$, $\delta_2$, $\delta_3$, are the strong phases caused by the final-state interactions, i.e. the phase shifts of the inelastic rescattering. The term $A_3 e^{i\gamma} e^{i\delta_3}$ is the contribution from the tree diagram of $B^+ \to \rho^0 K^{**} \to \pi^+ K^0$. Our numerical calculations give $A_1 = 0.5$, $A_2 = 0.15$, $A_3 = 0.05$, $\delta_1 \approx 90^0$, $\delta_2 \approx 90^0$. The strong phases are $90^0$, because we calculate only the absorptive part of the hadron loop caused by the final-state interactions. We will come back to this point later in the last section. $A_1 = 0.5 \geq A_2 = 0.15$ is encouraging for our mechanism, because it can explain that the experimental branching ratio of $B^\pm \to \pi^\pm K$ is larger than that of $B^0 \to \pi^\mp K^\pm$.

The ratio $R$ in Eq.(16) is changed into

$$ R = \frac{1 + A_2^2 + 2A_2 \cos \delta_2}{1 + A_1^2 + 2A_1 \cos \delta_1} (1 - 2r_0 \cos \gamma \cos \delta_0 + r_0^2) $$  \hspace{1cm} (22)

If we define $R'$

$$ R' = 1 - 2r_0 \cos \gamma \cos \delta_0 + r_0^2 $$  \hspace{1cm} (23)

then from Eq.(22), (23),

$$ R' = \frac{1 + A_1^2 + 2A_1 \cos \delta_1}{1 + A_2^2 + 2A_2 \cos \delta_2} R $$  \hspace{1cm} (24)

So, the Fleischer-Mannel relation is changed to

$$ R' \geq \sin^2 \gamma $$  \hspace{1cm} (25)

When $A_1 = 0.5$, $A_2 = 0.15$, $\delta_1 = 90^0$, $\delta_2 = 90^0$, $R' = 1.25R$. The bound relation in Eq.(18) is modified as much as 25%, when only the absorptive part of the hadron loop is under consideration. No doubt, the dispersive part of the loop will also contribute to the modification. Let us take an example, when the dispersive part of the amplitude is equal to the absorptive part, $A_1 = 0.5\sqrt{2}$, $A_2 = 0.15\sqrt{2}$, $\delta_1 = 45^0$, $\delta_2 = 45^0$, then $R' = 1.58R$. So, the bound relation is modified significantly as much as 58%.

For direct CP asymmetry,

$$ A_{CP}^{\text{dir}} \approx 2A_3 \sin \gamma \sin \delta_+ $$  \hspace{1cm} (26)
The final-state interaction can provide about 5% – 10% direct CP asymmetry. But since the relative sign cannot be fixed by the theory, we are unable to determine whether the correction is constructive or destructive.

5. Conclusion and discussion

From the numerical results shown in Table 1 and Table 2, one can notice that the final-state interactions due to the single pion exchange in \( B \rightarrow \rho K^* \rightarrow \pi K \) processes are 10% – 30% relative to the direct decay amplitude. This result is based on considering only the absorptive part of the hadron loop caused by the final-state interactions. The dispersive part of the loop is difficult to calculate because of the ultraviolet divergence. The elastic and inelastic rescattering caused by vector trajectory( \( \rho, \omega, \text{and } K^* \) ) exchange may give additional contributions, and there are many multiparticle intermediate states which cannot be neglected [4]. Because of existence of these uncertainties, the simple Fleischer-Mannel relation \( R \geq sin^2 \gamma \) is modified greatly by the final-state interactions. We think it is difficult to get reasonable information about weak angle \( \gamma \) in \( B \rightarrow \pi K \) processes. The 5% – 10% direct CP asymmetry can be generated by final-state interactions. Moreover, as discussed above, we only consider the absorptive part of the hadron loop in this work, there are still many uncertainties of the theoretical predictions on the constraint of \( \gamma \) and \( A_{\text{dir}}^{\text{CP}} \). At present stage, there is no reliable renormalization scheme for obtaining correct dispersive part of the hadron loop, and it is a well-known fact for evaluating the loops in the chiral Lagrangian theories. We will try some phenomenological ways to carry out the renormalization elsewhere. [12]

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References

[1] M. Gronau, J. Rosner, D. London, Phys.Rev.Lett. 73 (1994), 21.
   M. Gronau, D. Wyler, Phys.Lett. B 265 (1991), 172.

[2] J. Alexander, CLEO collaboration, talk given at the 2nd International Conference on B Physics and CP violation, Honolulu, Hawaii, 24-27 March 1997;
   R.F.Würthwein, CLEO collaboration, talk given at MPI Heidelberg and Private communication.

[3] R. Fleischer, T. Mannel, hep-ph/9704423.

[4] D. Delepine, J. Geraral, J. Pestieau and J. Weyers, hep-ph/9802361;
   A. Falk, A. Kagan, Y. Nir and A. Petrov, hep-ph/9712225;
   Neubert, hep-ph/97.

[5] X. Li, B. Zou, Phys.Lett. B 399 (1997), 297.

[6] A. Buras, M. Jamin, M. Lautenbacher and P. Weisz, Nucl.Phys. B 370 (1992) 69; Nucl.Phys. B 375 (1992) 501.

[7] N. Deshpande and X. He, Phys. Lett. B 336 (1994) 471; Phys. Rev. Lett. 26 (1995) 74;
   N. Deshpande, X. He and J. Trampetic, Phys. Lett. B 345 (1995) 547.

[8] M. Wirbel, B. Stech and M. Bauer, Z.Phys. C 29 (1985) 637; Z.Phys.C 34 (1987) 103.

[9] D. Du, L. Guo, Z.Phys. C 75 (1997) 9-15.

[10] Particle Data Group, Phys.Rev.D 54 (1996) 1.

[11] A. Ali, D. London, Preprint CERN-TH 7248/94.

[12] D. Du et al. in preparation.
Table 1. The amplitude and branching ratios of the direct \( B \to \pi K \) decays. Where "Tree" means only the tree diagram contribution; "Penguin" means only the Penguin diagram contribution. "Tree+Penguin" means tree plus Penguin diagram contributions.

| Decay Mode | \( N_c \) | \( A^{dir}(10^{-9}) \) | \( Br \) |
|------------|----------|-----------------|-------|
| \( B_u^- \to \pi^- K^0 \) | \( N_c = \infty \) | 0 | 6.53 + i36.1 | 6.53 + i36.1 | 0 | 1.18 \times 10^{-5} |
| \( B^0 \to \pi^+ K^- \) | \( N_c = \infty \) | -7.88+i2.78 | 6.51 + i34.8 | -1.36 + i37.6 | 5.93 \times 10^{-7} | 1.20 \times 10^{-5} |
| \( B_u^- \to \pi^- K^0 \) | \( N_c = 3 \) | 0 | 5.78 + i31.2 | 5.78 + i31.2 | 0 | 8.79 \times 10^{-6} |
| \( B^0 \to \pi^+ K^- \) | \( N_c = 3 \) | -7.16+i2.53 | 5.82 + i32.0 | -1.34 + i34.5 | 4.9 \times 10^{-7} | 1.01 \times 10^{-6} |
| \( B_u^- \to \pi^- K^0 \) | \( N_c = 2 \) | 0 | 4.87 + i27.8 | 4.87 + i27.8 | 0 | 6.94 \times 10^{-6} |
| \( B^0 \to \pi^+ K^- \) | \( N_c = 2 \) | -6.8+i2.4 | 5.01 + i29.7 | -1.79 + i32.1 | 4.43 \times 10^{-7} | 8.81 \times 10^{-6} |
Table 2. The amplitude for the final-state interactions $B \to \rho K^* \to \pi K$.

| Decay Mode | $N_c$ | $A^{FSI}(10^{-9})$ | $A^{FSI}/A^{dir}$ |
|------------|-------|-------------------|-------------------|
| $B_u^- \to \rho^- K^* \to \pi^- \bar{K}^0$ | $N_c = \infty$ | $0$ | $9.86 - i1.86$ | $9.86 - i1.86$ | $-0.27e^{i89.6^0}$ |
| $\bar{B}^0 \to \rho^+ K^*^- \to \pi^+ K^-$ | $N_c = \infty$ | $1.22 + i3.47$ | $9.35 - i1.86$ | $10.6 + i1.60$ | $-0.28e^{i96.5^0}$ |
| $B_u^- \to \rho^0 K^*^- \to \pi^- \bar{K}^0$ | $N_c = \infty$ | $0.57 + i1.6$ | $7.34 - i2.83$ | $7.9 - i1.22$ | $-0.22e^{i91.5^0}$ |
| $\bar{B}^0 \to \rho^0 K^*^0 \to \pi^+ K^-$ | $N_c = \infty$ | $-0.25 - i0.72$ | $-5.06 + i1.39$ | $-5.32 + i0.67$ | $0.14e^{i80.8^0}$ |
| $B_u^- \to \rho^- K^*^0 \to \pi^- \bar{K}^0$ | $N_c = 3$ | $0$ | $8.2 - i1.65$ | $8.2 - i1.65$ | $-0.26e^{i89.1^0}$ |
| $\bar{B}^0 \to \rho^+ K^*^- \to \pi^+ K^-$ | $N_c = 3$ | $1.11 + i3.15$ | $8.61 - i1.67$ | $9.72 + i1.49$ | $-0.28e^{i96.5^0}$ |
| $B_u^- \to \rho^0 K^*^- \to \pi^- \bar{K}^0$ | $N_c = 3$ | $0.81 + i2.31$ | $6.86 - i2.74$ | $7.68 - i0.44$ | $-0.24e^{i97.2^0}$ |
| $\bar{B}^0 \to \rho^0 K^*^0 \to \pi^+ K^-$ | $N_c = 3$ | $0.06 + i0.16$ | $-3.82 + i1.11$ | $-3.77 + i1.27$ | $0.12e^{i69.2^0}$ |
| $B_u^- \to \rho^- K^*^0 \to \pi^- \bar{K}^0$ | $N_c = 2$ | $0$ | $7.1 - i1.38$ | $7.1 - i1.38$ | $-0.26e^{i89.0^0}$ |
| $\bar{B}^0 \to \rho^+ K^*^- \to \pi^+ K^-$ | $N_c = 2$ | $1.06 + i2.99$ | $8.01 - i1.43$ | $9.06 + i1.56$ | $-0.29e^{i96.6^0}$ |
| $B_u^- \to \rho^0 K^*^- \to \pi^- \bar{K}^0$ | $N_c = 2$ | $0.94 + i2.66$ | $6.58 - i2.48$ | $7.52 + i0.18$ | $-0.27e^{i98.6^0}$ |
| $\bar{B}^0 \to \rho^0 K^*^0 \to \pi^+ K^-$ | $N_c = 2$ | $0.21 + 0.60$ | $-3.01 + i0.86$ | $-2.8 + i1.46$ | $0.10e^{i59.2^0}$ |
Fig. 1. Final-state interactions in $B \to \rho K^* \to \pi K$ due to single pion exchange. (a), (b) the neutral pion exchange case. (c), (d) the charged pion exchange case.