Precise focusing in the focus point scenario
toward the natural Higgs boson in the MSSM

Bumseok Kyae(a)*, and Chang Sub Shin(b)†

(a) Department of Physics, Pusan National University, Busan 609-735, Korea
(b) New High Energy Theory Center,
Department of Physics and Astronomy,
Rutgers University, Piscataway NJ 08854, USA

Abstract

A small Higgs mass parameter $m_{h_u}^2$ can be insensitive to various trial heavy stop masses, if a universal soft squared mass is assumed for the chiral superpartners and the Higgs boson at the grand unification (GUT) scale, and a focus point (FP) of $m_{h_u}^2$ appears around the stop mass scale. The challenges in the FP scenario are (1) a too heavy stop mass ($\approx 5$ TeV) needed for the 126 GeV Higgs mass and (2) the too high gluino mass bound ($\gtrsim 1.4$ TeV). For a successful FP scenario, we consider (1) a superheavy right handed (RH) neutrino and (2) the first and second generations of hierarchically heavier chiral superpartners. The RH neutrino can move a FP in the higher energy direction in the space of $(Q, m_{h_u}^2(Q))$, where $Q$ denotes the renormalization scale. On the other hand, the hierarchically heavier chiral superpartners can lift up a FP in that space through two-loop gauge interactions. Precise focusing of $m_{h_u}^2(Q)$ is achieved with the RH neutrino mass of $\sim 10^{14}$ GeV together with an order one ($0.8 - 1.4$) Yukawa coupling to the Higgs, and the hierarchically heavy masses of $15 - 25$ TeV for the heavier generations of superpartners, when the $U(1)_R$ breaking soft parameters, $m_{1/2}$ and $A_0$ are set to be 1 TeV at the GUT scale. Those values can naturally explain the small neutrino mass through the seesaw mechanism, and suppress the flavor violating processes in supersymmetric models.

PACS numbers: 12.60.Jv, 14.80.Ly, 11.25.Wx, 11.25.Mj
Keywords: Focus point scenario, Right handed neutrino, Effective SUSY

* email: bkyae@pusan.ac.kr
† email: changsub@physics.rutgers.edu
I. INTRODUCTION

The naturalness problem of the electroweak scale (EW) and the Higgs boson mass has been the most important issue for last four decades in theoretical particle physics community. It has provided a strong motivation to study various theories beyond the standard model (SM). Particularly, the minimal supersymmetric SM (MSSM) has been regarded as the most promising candidate among new physics models beyond the SM. However, any evidence of new physics beyond the SM including supersymmetry (SUSY) has not been observed yet at the large hadron collider (LHC), and experimental bounds on SUSY particles are increasing gradually. Nonetheless, a better new idea that can replace the present status of SUSY doesn’t seem to appear yet. Accordingly, it would be worth while to explore a breakthrough within the SUSY framework.

Concerning the radiative Higgs mass and EW symmetry breaking, the top quark Yukawa coupling of order unity ($y_t$) plays the key role in the MSSM: through the sizable top quark Yukawa coupling, the top quark and stop make a dominant contribution to the renormalization of the soft mass parameter of the Higgs ($m_{h_u}^2$) as well as the radiative physical Higgs mass squared ($\Delta m_{H}^2$) [1]:

$$\Delta m_{H}^2 \approx \frac{3y_t^4}{4\pi^2}\sin^4\beta v_h^2 \log\left(\frac{\tilde{m}_t^2}{m_t^2}\right) + \cdots,$$

(1)

$$\Delta m_{h_u}^2 \approx \frac{3y_t^2}{8\pi^2}\tilde{m}_t^2 \log\left(\frac{\tilde{m}_t^2}{\Lambda^2}\right) + \cdots,$$

(2)

where $m_t$ ($\tilde{m}_t$) denotes the top quark (stop) mass, and $v_h$ is the vacuum expectation value (VEV) of the Higgs, $v_h \equiv \sqrt{\langle h_u \rangle^2 + \langle h_d \rangle^2} \approx 174$ GeV with $\tan\beta \equiv \langle h_u \rangle / \langle h_d \rangle$. $\Lambda$ means a cut-off scale. A messenger scale of SUSY breaking is usually adopted for it. Here we set the left-handed and right-handed stops’ squared masses, $m_{q_3}^2$ and $m_{u_3}^2$, equal to $\tilde{m}_t^2$ for simplicity. Note that $\Delta m_{h_u}^2$ can be a large negative value for a large stop mass and a high messenger scale.

As seen in Eq. (1), a large stop mass can raise the radiative Higgs mass. According to the recent analysis based one three-loop calculations [2], a 3–4 TeV or 5 TeV stop mass is necessary for explaining the recently observed 126 GeV Higgs mass [3] without a stop mixing effect. From Eq. (2), however, such a heavy stop mass is expected to significantly enhance the renormalization effect on $m_{h_u}^2$, and eventually it gives rise to a fine-tuning problem associated with naturalness of the EW scale. It is because a negative $m_{h_u}^2$ triggers the EW symmetry breaking, and eventually determines the $Z$ boson mass in the MSSM, as seen in the extremum condition of the MSSM Higgs potential [1]:

$$\frac{1}{2}m_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2\beta}{\tan^2\beta - 1} - |\mu|^2,$$

(3)

where $m_Z^2$ denotes the $Z$ boson mass and $\mu$ is the “$\mu$-term” coefficient in the MSSM superpotential. If $-m_{h_u}^2$ is excessively large, thus, it should be compensated with $|\mu|^2$. Thus, a
fine-tuning of $10^{-3}$–$10^{-4}$ doesn’t seem to be avoidable in the MSSM, unless the messenger scale $\Lambda$ is low enough. Due to the reason, a relatively smaller stop mass ($\ll 1$ TeV) has been assumed for naturalness of the EW scale, and various extensions of the Higgs sector have been proposed for explaining the observed 126 GeV Higgs mass \cite{4}. Unfortunately, however, the stop mass bound has already reached 700 GeV \cite{5}, which starts threatening the traditional status of SUSY as a solution to the naturalness problem of the EW phase transition.

In fact, the renormalization of $m_{\tilde{h}}^2$, Eq. (2) is necessarily affected by ultraviolet (UV) physics. For a more complete expression of it, thus, the full renormalization group (RG) equations should be studied for a given UV model, even though Eq. (2) would not be much sensitive to a UV physics in SUSY models. Unlike the expectation based on low energy physics, however, it was claimed that the $Z$ boson and Higgs masses at low energy is quite insensitive to the stop mass in the “focus point (FP) scenario” \cite{4,7}: under the simple initial condition for the stops and Higgs squared masses, $m_{\tilde{q}_3}^2 = m_{\tilde{u}_3}^2 = m_{\tilde{h}}^2 = \cdots \equiv m_0^2$ at the grand unification (GUT) scale, the RG solution of $m_{\tilde{h}}^2$ turns out to be almost independent of $m_0^2$ at the EW scale unlike those of $m_{\tilde{q}_3}^2$ and $m_{\tilde{u}_3}^2$. It is because the coefficient of $m_0^2$ in the RG solution of $m_{\tilde{h}}^2$ at the EW scale turns out to be quite small. Accordingly, $m_{\tilde{h}}^2$ can remain small enough even for relatively large trial $m_0^2$s ($\sim$ multi-TeV) unlike other superparticles in the chiral sector. Interestingly enough, moreover, the FP scenario favors the simplest version of SUSY model with the minimal field contents and the universal initial condition for the soft squared masses at the GUT scale: many careless extensions of the MSSM at low energy would destroy the FP mechanism.

The insensitivity of $m_{\tilde{h}}^2$ to $m_0^2$ or stop masses implies that Eq. (2) is effectively canceled by other ingredients. One might expect that a fine-tuning for smallness of $m_{\tilde{h}}^2$ would be hidden somewhere in this scenario. This guess is actually true. As will be seen later, the smallness of the coefficient of $m_0^2$ in $m_{\tilde{h}}^2$ originates from the fact that

$$e^{-\frac{3}{4\pi^2}} \int_{t_0}^{t_{W}} dt' y_t^2 \approx \frac{1}{3}. \quad (4)$$

Here $t$ parametrizes the renormalization scale $Q$, $t - t_0 = \log \frac{Q}{M_G}$. $t_W$ and $t_0$ correspond to the EW and GUT scale $M_G$ ($\approx 2 \times 10^{16}$ GeV), respectively. Actually, Eq. (4) is an accidental relation in some sense. Just the quarks and leptons’ masses, the low energy values of the SM gauge couplings, and the MSSM field contents completely determine $y_t(t)$, and the $Z$ boson mass scale and the gauge coupling unification scales provide exactly the needed energy interval. In the sense that Eq. (4) is not artificially designed, but Nature might permit it, we will call it “Natural tuning.” Of course, there might exist a deep reason for it. In this paper, however, we will not attempt to explain the origin, but take a rather pragmatic attitude: we will just accept, utilize, and improve it.

However, the recently observed 126 GeV Higgs mass is challenging also in the FP scenario. Since the FP scenario works with the minimal field contents, the Higgs mass can be raised
only through the radiative correction by the quite heavy stop, $\tilde{m}_t \sim 5$ TeV [2]. To get a heavier stop mass, we need a larger $m_{h_0}^2$. In order that $m_{h_u}^2$ still remains insensitive even to much larger $m_{\tilde{s}}^2 \geq (5 \text{ TeV})^2$, a more precise focusing is quite essential. That is to say, the coefficient of $m_{\tilde{s}}^2$ in the $m_{h_u}^2$’s RG solution should be much closer to zero. Moreover, $m_{h_u}^2$ doesn’t follow the original FP scenario below the stop mass scale, because the stops are decoupled there. For a predictive EW scale, thus, the FP should appear around the stop mass scale rather than the conventional EW or $Z$ boson mass scale. The present heavy gluino mass bound at the LHC, $M_3 \gtrsim 1.4$ TeV [8] also spoils the success of the FP scenario [9–11]. The heavy gluino leads to a too large negative $m_{h_u}^2$ at the EW scale through RG evolution. Such an RG effect by a heavy gluino mass should be compensated properly for a small enough $Z$ boson mass.

In this paper, we will attempt just to trim the FP scenario such that the FP is made located around the stop mass scale and the heavy gluino effect becomes mild. In order to accomplish that goal, we will consider a superheavy right-handed (RH) neutrino [12, 13], and the two-loop gauge interactions by the hierarchically heavier first and second generations of chiral superpartners (sfermions) [14, 15]. Hierarchically heavy masses for the first two generations of sfermions ($\gtrsim 15$ TeV) could also sufficiently suppress unwanted SUSY flavor and SUSY CP violating processes as in the “effective SUSY model” [16]. Once the location of the FP is successfully modified to a desirable position, even quite heavy stop mass could still be naturally compatible with the $Z$ boson mass scale, and the 126 GeV Higgs mass can be supported dominantly by the radiative correction from such a heavy stop.

This paper is organized as follows: we will review the FP scenario and discuss the problems associated with the recent experimental results in section II. In section III we will explore the ways to move the location of the FP into a desirable position in the space of $(Q, m_{h_u}^2(Q))$. Section IV will be a conclusion. For convenience of our discussion in the main text, we will leave the details of the full RG equations and derivation of some semi-analytic solutions to them in Appendix.

II. FOCUS POINT SCENARIO

Based on our semi-analytic solutions to the RG equations, let us discuss first the RG behaviors of soft parameters associated with the Higgs and the third generation of sfermions. When $\tan \beta$ is small enough, the top quark Yukawa coupling, $y_t$ dominantly drive the RG running of $\{m_{h_u}^2, m_{h_d}^2, m_{\tilde{g}}^2, A_t\}$, while the bottom quark and tau lepton’s Yukawa couplings, $y_b$ and $y_\tau$ are safely ignored. For small $\tan \beta$, thus, the one-loop RG equations for
\[m^2_{h_u}, m^2_{u_3}, m^2_{q_3}, A_t\] are written as

\[
16\pi^2 \frac{d}{dt} m^2_{h_u} = \frac{g_1^2}{4} (X_t + A_t^2) - \frac{32}{15} g^2_1 M_1^2 - \frac{6}{5} g_1^2 M_1^2, \tag{5}
\]

\[
16\pi^2 \frac{d}{dt} m^2_{u_3} = 4g_1^2 (X_t + A_t^2) - \frac{32}{3} g^2_3 M_3^2 - \frac{32}{15} g_1^2 M_1^2, \tag{6}
\]

\[
16\pi^2 \frac{d}{dt} m^2_{q_3} = 2g_1^2 (X_t + A_t^2) - \frac{64}{3} g^2_3 M_3^2 - \frac{6}{5} g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2, \tag{7}
\]

\[
8\pi^2 \frac{d}{dt} A_t = 6g_1^2 A_t - \frac{16}{3} g^2_3 M_3 - \frac{32}{3} g_2^2 M_2 - \frac{13}{15} g_1^2 M_1, \tag{8}
\]

where \(X_t \equiv m^2_{h_u} + m^2_{u_3} + m^2_{q_3}\), and \(A_t\) denotes the “A-term” coefficient corresponding to the top quark Yukawa coupling. \(t\) parametrizes the renormalizations scale \(Q\), \(t - t_0 = \log \frac{Q}{M_G}\). \(g_{3,2,1}\) and \(M_{3,2,1}\) in the above equations stand for the three MSSM gauge couplings and gaugino masses. Our semi-analytic solutions to them are approximately given by

\[
m^2_{h_u}(t) \approx m^2_{h_u,0} + \frac{X_0}{2} \left[ \frac{2}{3} \int_{t_0}^t dt' \frac{1}{2} \left( \frac{m_{1/2}}{g_0^2} \right)^2 \right] - \frac{3}{4} \left( \frac{m_{1/2}}{g_0^2} \right)^2 \left( g^4_2(t) - g^4_0 \right), \tag{9}
\]

\[
m^2_{u_3}(t) \approx m^2_{u_3,0} + \frac{X_0}{3} \left[ \frac{2}{3} \int_{t_0}^t dt' \frac{1}{2} \left( \frac{m_{1/2}}{g_0^2} \right)^2 \right] + \frac{8}{9} \left( \frac{m_{1/2}}{g_0^2} \right)^2 \left( g^4_3(t) - g^4_0 \right), \tag{10}
\]

\[
m^2_{q_3}(t) \approx m^2_{q_3,0} + \frac{X_0}{6} \left[ \frac{2}{3} \int_{t_0}^t dt' \frac{1}{2} \left( \frac{m_{1/2}}{g_0^2} \right)^2 \right] + \frac{8}{9} \left( \frac{m_{1/2}}{g_0^2} \right)^2 \left( g^4_3(t) - \frac{3}{2} g^4_2(t) + \frac{11}{18} g^4_0 \right), \tag{11}
\]

\[A_t(t) = \frac{3}{4\pi^2} \int_{t_0}^t dt' G_A e^{-\frac{3}{4\pi^2} \int_{t_0}^{t'} dt'' A^2_t A^4_t A^6_t - \frac{1}{4\pi^2} \left[ \int_{t_0}^{t'} dt'' G^2_\chi e^{-\frac{3}{4\pi^2} \int_{t_0}^{t''} dt'' A^2_t A^4_t A^6_t} \right]} \tag{12}
\]

where we ignored the bino mass \(M_1\) and the relevant U(1)_Y gauge contributions due to their smallness. For the complete expressions and derivation of the above solutions, refer to Appendix (setting \(\tilde{m}^2 = 0\)). Here, \(\{m^2_{h_u,0}, m^2_{u_3,0}, m^2_{q_3,0}, A_0\}\) denote the values of \(\{m^2_{h_u}(t), m^2_{u_3}(t), m^2_{q_3}(t), A_t(t)\}\) at the GUT scale, and \(X_0 \equiv m^2_{h_u,0} + m^2_{u_3,0} + m^2_{q_3,0}\). \(g_0\) and \(m_{1/2}\) are the unified gauge coupling and gaugino mass at the GUT scale, respectively. \(F(t)\) in the above solutions is defined as

\[
F(t) \equiv \frac{3}{4\pi^2} e^{\frac{3}{4\pi^2} \int_{t_0}^t dt' y^2_t A^4_t e^{-\frac{3}{4\pi^2} \int_{t_0}^{t'} dt'' A^2_t A^4_t A^6_t}} \frac{1}{4\pi^2} \left[ e^{\frac{3}{4\pi^2} \int_{t_0}^{t'} dt'' y^2_t A^2_t A^4_t A^6_t} e^{-\frac{3}{4\pi^2} \int_{t_0}^{t''} dt''' y^2_t A^2_t A^4_t A^6_t} - \frac{1}{4\pi^2} \left[ e^{\frac{3}{4\pi^2} \int_{t_0}^{t''} dt''' y^2_t A^2_t A^4_t A^6_t} e^{-\frac{3}{4\pi^2} \int_{t_0}^{t''''} dt'''' y^2_t A^2_t A^4_t A^6_t} - \frac{1}{4\pi^2} \right] \right]. \tag{13}
\]

\(G_A\) in Eq. (12) and \(G^2_\chi\) in Eq. (13) are given, respectively, by

\[
G_A(t) \equiv \frac{16}{3} g^2_3 M_3 + 3g^2_2 M_2 + \frac{13}{15} g^2_1 M_1 = \left( \frac{m_{1/2}}{g_0^2} \right) \left[ \frac{16}{3} g^4_3 + 3g^4_2 + \frac{13}{15} g^4_1 \right], \tag{14}
\]

\[
G^2_\chi(t) \equiv \frac{16}{3} g^2_3 M_3^2 + 3g^2_2 M_2^2 + \frac{13}{15} g^2_1 M_1^2 = \left( \frac{m_{1/2}}{g_0^2} \right)^2 \left[ \frac{16}{3} g^6_3 + 3g^6_2 + \frac{13}{15} g^6_1 \right]. \tag{15}
\]

Note that \(F(t)\) is independent of \(\{m^2_{h_u,0}, m^2_{u_3,0}, m^2_{q_3,0}\}\). So \(\{m^2_{h_u,0}, m^2_{u_3,0}, m^2_{q_3,0}\}\) appear only in the first three terms in the above RG solutions, Eqs. (9), (10), and (11).
\[ F(t) \text{ depends on } \tan \beta \text{ in principle. But it turns out to be almost insensitive to } \tan \beta. \]
For instance, \( F(t) \) at \( Q = 5 \text{ TeV} \) \((= F(t_W))\) is estimated as
\[ F(t_W) \approx \{-1.03, -1.02\} \times \left( \frac{m_{1/2}}{g_0^2} \right)^2 \quad (16) \]
for \( \{\tan \beta = 5, \tan \beta = 50\} \). Here the numerical estimation for \( \tan \beta = 50 \) was performed by including \( y_b \) and \( y_t \) effects. For the complete RG equation we used, see Appendix. Thus, the last three terms in Eq. (19) at \( Q = 5 \text{ TeV} \) yield \( \{-1.43, -1.41\} \times m_{1/2}^2 \) for \( \{\tan \beta = 5, \tan \beta = 50\} \). Note that the \( F(t) \) term dominates over the last two terms in Eq. (19) at \( Q = 5 \text{ TeV} \).
Although the last two terms provide a positive coefficient of \( m_{1/2}^2 \), thus, the large gluino mass effect contained in \( F(t) \) flips the sign.

If the gauge sector’s contributions proportional to \( m_{1/2}^2 \) were relatively suppressed, \( A_t(t) \) and \( F(t) \) are simplified as follows:
\[ A_t(t) \approx A_0 e^{\frac{1}{18}} \int_0^t dt' y_t^2, \quad \text{and} \quad F(t) \approx A_0^2,e^{\frac{1}{18}} \int_0^t dt' y_t^2 \left[ e^{\frac{1}{18}} \int_0^t dt' y_t^2 - 1 \right]. \quad (17) \]
In this case, thus, \( \{m_{h_u}^2(t), m_{u_3}^2(t), m_{q_3}^2(t)\} \) reduce to
\[ m_{h_u}^2(t) \approx m_{h_u}^2 + \frac{X_0}{2} \left[ e^{\frac{1}{18}} \int_0^t dt' y_t^2 - 1 \right] + \frac{A_0^2}{2} e^{\frac{1}{18}} \int_0^t dt' y_t^2 \left[ e^{\frac{1}{18}} \int_0^t dt' y_t^2 - 1 \right] + \cdots, \quad (18) \]
\[ m_{u_3}^2(t) \approx m_{u_3}^2 + \frac{X_0}{3} \left[ e^{\frac{1}{18}} \int_0^t dt' y_t^2 - 1 \right] + \frac{A_0^2}{3} e^{\frac{1}{18}} \int_0^t dt' y_t^2 \left[ e^{\frac{1}{18}} \int_0^t dt' y_t^2 - 1 \right] + \cdots, \quad (19) \]
\[ m_{q_3}^2(t) \approx m_{q_3}^2 + \frac{X_0}{6} \left[ e^{\frac{1}{18}} \int_0^t dt' y_t^2 - 1 \right] + \frac{A_0^2}{6} e^{\frac{1}{18}} \int_0^t dt' y_t^2 \left[ e^{\frac{1}{18}} \int_0^t dt' y_t^2 - 1 \right] + \cdots, \quad (20) \]
where “...” doesn’t contain \( m_0^2 \) and \( A_0 \). As emphasized in Eq. (4), the most important notice should taken here that \( e^{\frac{1}{18}} \int_0^t dt' y_t^2 \approx \frac{1}{3} \) for \( t = t_0 + \log \frac{10^2 \text{GeV}}{M_G} \) \((\equiv t_W)\) when \( \tan \beta \) is small enough \([6]\). Thus, if a universal soft squared masses are assumed, \( m_{h_u}^2 = m_{u_3}^2 = m_{q_3}^2 \equiv m_0^2 \), and \( A_0 = 0 \) is set at the GUT scale, Eqs. (18)–(20) are recast into \([6]\)
\[ m_{h_u}^2(t_W) \approx \frac{3m_0^2}{2} \left[ e^{\frac{1}{18}} \int_{t_W}^t dt' y_t^2 - \frac{1}{3} \right] + \cdots \approx 0.006 m_0^2 + \cdots, \quad (21) \]
\[ m_{u_3}^2(t_W) \approx \frac{3m_0^2}{2} \left[ \frac{2}{3} e^{\frac{1}{18}} \int_{t_W}^t dt' y_t^2 + 0 \right] + \cdots \approx \frac{1}{3} m_0^2 + \cdots, \quad (22) \]
\[ m_{q_3}^2(t_W) \approx \frac{3m_0^2}{2} \left[ \frac{1}{3} e^{\frac{1}{18}} \int_{t_W}^t dt' y_t^2 + \frac{1}{3} \right] + \cdots \approx \frac{2}{3} m_0^2 + \cdots, \quad (23) \]
where “...” doesn’t contain \( m_0^2 \). Hence, \( m_{h_u}^2(t) \) almost vanishes at the EW sale \((t \approx t_W)\).
It means that \( m_{h_u}^2 \) can be light enough at the EW scale, almost independent of \( m_0^2 \), only if the “...” in Eq. (21) was also suppressed. Since \( m_{h_u}^2 \) is very insensitive to \( m_0^2 \), even a large enough \( m_0^2 \) guarantees the smallness of \( m_{h_u}^2 \) at the EW scale, whereas it makes stop masses quite heavy: \( m_{u_3}^2(t_W) \approx m_0^2/3 \) and \( m_{q_3}^2(t_W) \approx 2m_0^2/3 \). In the FP scenario, therefore, the naturalness of the EW scale and the Higgs mass is based on Natural tuning.
Although $A_0$ is comparable to other soft parameters, $m_{h_u}^2$ can still remain light at the EW scale, if $(m_{h_u0}^2, m_{u30}^2, m_{q30}^2, A_0^2)$ are very specially related, satisfying e.g. $m_0^2 (1, 1 + x - 3y, 1 - x, 9y)$ at the GUT scale, where $x$, $y$ are arbitrary numbers \[17\]. However, such a relation looks hard to realize in a supergravity (SUGRA) model. For simplicity, we will assume in this paper that $|x|, |y| \ll 1$, namely, $A_0$ is quite suppressed compared to $m_0^2 (= m_{h_u0}^2 = m_{u3}^2 = m_{q30}^2)$. Actually, this is possible e.g. in the gauge mediated SUSY breaking scenario with a GUT scale messenger. To get a universal soft squared mass in the gauge mediation, the SM gauge group should be embedded in a simple group at the GUT scale. However, the effect by non-vanishing $A_0$ on $m_{h_u}^2$ can be compensated by another ingredient introduced later. Hence, the gravity mediated SUSY breaking scenario with the universal soft squared mass and $A_0 \neq 0$ can also be consistent with the FP scenario.

Unlike the naive expectation, the low energy value of $m_{h_u}^2$ is not sensitive to the stop masses in the FP scenario. Apparently, hence, the naturalness of the Higgs boson seems to be guaranteed in this framework. It is a result of

1. the employed initial conditions, $m_{h_u0}^2 = m_{u30}^2 = m_{q30}^2 = m_0^2$ and $A_0 = 0$, and
2. the accidental result, $e^{\frac{\alpha}{4\pi} \int_0^{T_W} dt' y_t^2} \approx \frac{1}{3}$ (“Natural tuning”).

The first condition is associated with a model-building problem. Actually, it can easily be realized in a large class of simple SUGRA models. However, the second condition would be a kind of fine-tuning condition, because the top quark Yukawa coupling $y_t(t)$ and the interval of the energy scales between the EW and the GUT scales should specially be related. But it is not artificially designed. As mentioned in Introduction, we will simply accept such a natural tuning phenomenon.

However, the recent experimental results at the LHC seem to spoil the nice picture of the original FP scenario. Most of all, the gauge contributions in Eqs. \[9\]–\[12\] cannot be ignored any longer, since the mass bound for the gluino has been increased, $M_3 \gtrsim 1.4$ TeV \[8\]. As a result, the unified gaugino mass $m_{1/2}$ should be heavier than at least 550 GeV. Since a large $m_{1/2}$ leads to a large negative $m_{h_u}^2$ and large positive $m_{u3}^2$ and $m_{q3}^2$ at low energy, as seen in Eqs. \[9\]–\[11\] and \[12\], $-m_{h_u}^2$ cannot be small enough at the EW scale. A too large negative $m_{h_u}^2$ should be finely tuned with $|\mu|^2$ to be matched to $M_Z^2$ in Eq. \[3\]. Moreover, the observed Higgs mass, 126 GeV is somewhat heavy as a SUSY Higgs mass. Once we suppose $A_0 \approx 0$, quite heavy stop masses ($\sim 5$ TeV) are needed for explaining the observed Higgs mass \[2\]. A very large $m_{1/2}$ for 5 TeV would require a serious fine-tuning between $m_{h_u}^2$ and $|\mu|^2$ or $m_{1/2}$ and $m_0^2$. Alternatively, one can try to extend the MSSM for raising the Higgs mass. However, many extensions of the MSSM Higgs sector end up with ruin of the FP scenario, as will be commented later.

Since the stops are decoupled around 5 TeV ($t \equiv t_T$), $m_{h_u}^2$ follows the RG running of the SM below $t \approx t_T$. Hence, the FP mechanism based on the SUSY RG equations would not work well any more. The heavy fields’ correction to the RG solution can be estimated using the formula on the Coleman-Weinberg’s effective potential \[18\]. In fact, RG solution is a
Thus, the low energy value of \( m_{h_u}^2 \) is dominated by the heavy fields. The signs of the both loop effects are opposite. The result of one-loop effects by massless fields, while the Coleman-Weinberg’s one-loop effective potential is dominated by the heavy fields. The signs of the both loop effects are opposite. Thus, the low energy value of \( m_{h_u}^2 \) below the stop decoupling scale is roughly estimated as [19]:

\[
m_{h_u}^2(t_W) \approx m_{h_u}^2|_{\Lambda T} + \frac{3|y_t|^2}{8\pi^2} \left[ (\tilde{m}_t^2 + m_t^2) \left\{ \log \frac{\tilde{m}_t^2 + m_t^2}{\Lambda_T^2} - 1 \right\} - m_t^2 \left\{ \log \frac{m_t^2}{\Lambda_T^2} - 1 \right\} \right]
\]

where \( m_t (\tilde{m}_t) \) denotes the top quark (stop) mass, and the cut-off \( \Lambda_T \) is the scale where the stops are decoupled, and so \( m_{h_u}^2|_{\Lambda T} = m_{h_u}^2(t_T) \). Here we set \( m_{u_3}^2 \approx m_{q_3}^2 \approx \tilde{m}_t^2 \) for simple estimation. Note that \( \frac{3|y_t|^2}{8\pi^2} \tilde{m}_t^2 \approx (975 \text{ GeV})^2 \). Accordingly, \( m_{h_u}^2 \) at \( t = t_T \) (or \( m_{h_u}^2|_{\Lambda T} \)) should be smaller than \((1 \text{ TeV})^2 \) in order for \( m_{h_u}^2 \) at the EW scale to be smaller than \((1 \text{ TeV})^2 \). Since \( t = t_T \) is more or less far from \( t_W \), however, the coefficient of \( m_0^2 \) in Eq. \( \text{(24)} \) is not suppressed enough, \( m_{h_u}^2(t_T) \approx 0.1 m_0^2 - \cdots \), where \( m_0^2 > (5 \text{ TeV})^2 \) for obtaining 5 TeV stop masses. Hence, \( m_{h_u}^2(t_T) \) is quite sensitive to \( m_0^2 \), and it should be tuned with \( m_{1/2}^2 \) in Eq. \( \text{(24)} \) and/or \( |\mu|^2 \). For a predictively small \( m_{h_u}^2 \), thus, the FP should somehow appear around the stop decoupling scale \([10, 11]\). That is to say, the coefficient of \( m_0^2 \) should be much closer to zero around the stop mass scale, as mentioned in Introduction.

Fig. 1-(a) and (b) display the RG behaviors of \( m_{h_u}^2 \) for \( m_0^2 = (7 \text{ TeV})^2 \), \( (5 \text{ TeV})^2 \), \( (3 \text{ TeV})^2 \), when \( m_{1/2} = 1 \text{ TeV} \), \( A_0 = 0 \), \( \tan \beta = 5 \) [Fig. 1-(a)] and \( \tan \beta = 50 \) [Fig. 1-(b)]. Note
that $m_{1/2} = 1$ TeV yields the gluino mass of 2.5 TeV at TeV scale, which is well-above the present experimental lower bound 1.4 TeV [8]. Although we presented the simple RG equations valid for small $\tan \beta$ in Eqs. (5)–(7), the figures in Fig. 1 are based on the full one-loop RG equations including $y_b$ and $y_t$. Fig. 1(a) and (b) show that the FP is located at a slightly higher (lower) energy scale for a small (large) $\tan \beta$. Table I lists the values of \{m_{q_3}^2, m_{u_s}^2, m_{h_u}^2\} at $t = t_T$ (i.e. at $Q = 5$ TeV) in these cases. It shows that $m_{h_u}^2(t_T)$ is quite sensitive to $m_0^2$, as mentioned above. For $\tan \beta = 50$, particularly, the fine-tuning measure defined in Refs. [20] is estimated as

$$\Delta m_0^2 = \left| \frac{\partial \log m_Z^2}{\partial \log m_0^2} \right| = \left| \frac{m_Z^2}{m_0^2} \frac{\partial m_Z^2}{\partial m_0^2} \right| \approx 1729 \tag{25}$$

around the $m_0^2 = (7 \text{ TeV})^2$. It is quite large. It is because the location of the FP is too far from the point $(t = t_T, m_{h_u}^2 = 0)$.

In order to get $m_{h_u}^2$ that is small enough and insensitive to $m_0^2$, the location of the FP needs to be moved somehow to a position around the stop mass scale. See Fig. 2 $\epsilon$ in Fig. 2(a) and (b) should be as small as possible for a predictable $m_{h_u}^2$ at the EW scale. In addition, at a location of the FP near $t = t_T$, $m_{h_u}^2$ should be in the range of $0 \lesssim m_{h_u}^2 \lesssim (1 \text{ TeV})^2$. Since the heavy gluino makes a large negative contribution to $m_{h_u}^2(t_T)$, we need some other ingredients to overcome the heavy gluino effect. Below $t = t_T$, $m_{h_u}^2$ further decreases by $\sim (975 \text{ GeV})^2$ down to $t = t_W$, as discussed in Eq. (24).

We will discuss how to move the FP to such desirable positions in the next section. We intend to argue that the Higgs mass happens to be 126 GeV by 5 TeV stop mass, after $m_{h_u}^2$ at $t = t_T$ is made insensitive to $m_0^2$. It would be a way to trim the original idea of the Natural tuning.

### III. PRECISE FOCUSING

As $\tan \beta$ increases, the size of the top quark Yukawa coupling decreases. As a consequence, the factor $\left[ e^{-\frac{3}{4} \int_0^{t_T} dt' y_t^2} - \frac{1}{3} \right]$ in Eq. (21) vanishes at a lower energy scale $t (< t_W)$ for a smaller
FIG. 2: Desirable locations of the focus point in the \((t, m_{h_u}^2)\) space. The straight lines sketch different RG evolutions of \(m_{h_u}^2\) for various \(m_{0u}^2\). \(t_T\) corresponds to the assumed stop decoupling scale \((Q = 5\,\text{TeV})\). \(\epsilon\) needs to be as small as possible.

\[ y_t. \] It implies that the FP moves to a lower energy scale for a larger \(\tan\beta\) \cite{6}. The numerical analysis including \(y_t\) and \(y_t\), Fig. 1(a) and (b) confirm such a behavior of the FP. Since we intend to move the FP in the higher energy direction, a large \(\tan\beta\) is disfavored.

A much larger top quark Yukawa coupling \(y_t(t)\) at higher energy scales can move the FP to a new location at a higher energy scale. Actually, \(y_t(t)\) can be easily raised at higher energy scales e.g. by introducing a new Yukawa coupling of the Higgs. For instance, let us consider a coupling between \(h_u\) and a new singlet \(S\) in the next-to-MSSM (NMSSM):

\[ W_S = \lambda S h_u h_d + \cdots. \] (26)

In this case, the RG equations of \(y_t\) and \(\lambda\) are given by

\[ 8\pi^2 \frac{d}{dt} y_t^2 = y_t^2 \left[ \lambda^2 + 6y_t^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right], \] (27)

\[ 8\pi^2 \frac{d}{dt} \lambda^2 = \lambda^2 \left[ 4\lambda^2 + 3y_t^2 - 3g_2^2 - \frac{3}{5} g_1^2 \right] \] (28)

for small \(\tan\beta\). Due to the additional positive contribution by \(\lambda^2\) to the RG equation of \(y_t\), \(y_t^2\) becomes larger than that in the absence of \(\lambda\). Moreover, the \(\lambda\) coupling introduces a positive contribution also to the RG equation for \(m_{h_u}^2\):

\[ 16\pi^2 \frac{d}{dt} m_{h_u}^2 = 2\lambda^2 (X_\lambda + A_\lambda) + 6y_t^2 (X_t + A_t^2) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2, \] (29)

where \(X_\lambda \equiv (m_{h_u}^2 + m_{\tau_3}^2 + m_{u_3}^2)\). It turns out, however, that the FP’s location is too sensitive to \(\lambda\). According to our analysis, \(\lambda\) should be smaller than at least 0.1. Otherwise, the FP moves too far away in the high energy direction. For example, \(\lambda = 0.6\) and \(\tan\beta = 3\) moves the location of FP to \(10^{13}\) GeV energy scale. Hence, the parameter window satisfying the 126 GeV Higgs mass and the Landau pole constraint in the NMSSM, \(0.6 \lesssim \lambda \lesssim 0.7\) and \(1 < \tan\beta \lesssim 3\) \cite{21} cannot be compatible with the FP scenario. As seen in this example,
extensions of the MSSM Higgs sector with a new sizable Yukawa coupling e.g. for raising the Higgs mass could result in ruin of the FP scenario. Such a $\lambda$ coupling effect on $y_t$ can be reduced just by assuming that $S$ is superheavy and so decoupled at a very high energy scale.

One well-motivated superheavy particle is the RH neutrino ($N^c$), which is introduced to explain the smallness of the active neutrino mass through the seesaw mechanism [22] by the superpotential,

$$W_N = y_N l_3 h_u N^c + \frac{1}{2} M_N N^c N^c,$$

where $l_3$ is a lepton doublet in the MSSM. We assume that the Majorana mass of $N^c$ is $M_N \approx 2 \times 10^{14}$ GeV. If the RH neutrino is embedded in a multiplet of a GUT, Eq. (30) can be naturally obtained from the non-renormalizable term in GUTs, $W \supset \langle H_G \rangle \langle H_G \rangle N^c N^c/M_P$, where $\langle H_G \rangle$ and $M_P$ are a VEV of a GUT breaking Higgs ($\sim 10^{16}$ GeV) and the reduced Planck mass ($\approx 2.4 \times 10^{18}$ GeV), respectively. For $M_N \sim 10^{14}$ GeV, the Yukawa coupling $y_N$ should be of order unity to get a neutrino mass of order 0.1 eV. Here, we suppose that only one Yukawa coupling with $h_u$, $y_N$ is of order unity: for simplicity, we assume that other Yukawa couplings of $h_u$ to other RH neutrinos are small enough. Accordingly, other RH neutrinos should be relatively lighter than $M_N$. Since $N^c$ would be decoupled at a very high energy scale ($Q = M_N \approx 2 \times 10^{14}$ GeV), its RG effect on $y_t$ could be mild, and the FP would relatively slowly move as $y_N$ varied. Consequently, $m_{h_u}^2$ at $t = t_1$ could become less sensitive to $m_{h_u}^2$ [13]. If the heaviest RH neutrino was lighter than $\sim 10^{15}$ GeV, its RG effect on $y_t$ would be negligible, because the required Yukawa coupling becomes too small.

Similar to Eq. (29), the RG evolution of $m_{h_u}^2$ between $Q = M_G$ and $Q = M_N$ is described by

$$16\pi^2 \frac{d}{dt} m_{h_u}^2 = 2 y_N^2 (X_N + A_N^2) + 6 y_L^2 (X_t + A_t^2) - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2,$$

where the $y_N^2 X_N = y_N^2 (m_{h_u}^2 + m_{N^c}^2 + m_{l_3}^2)$ and $y_N^2 A_N^2$ terms are additional positive contributions coming from the RH neutrino. On the other hand, the RG equations for $m_{u_1}^2$ and $m_{q_1}^2$ maintain the same forms with those in the absence of the RH neutrino, Eqs. (6) and (7). They are just affected only through the modified value of $y_t^2 (X_t + A_t^2)$, which appear also in Eq. (31). For the complete form of the RG equations, refer to Appendix. Due to the $y_N^2 (X_N + A_N^2)$ terms in Eq. (31), $m_{h_u}^2/m_{u_1}^2$ and $m_{h_u}^2/m_{q_1}^2$ more rapidly decreases from $Q = M_G$ to $Q = M_N$ than the case without the RH neutrino. Below $Q = M_N$, however, the RH neutrino becomes decoupled, and so $m_{h_u}^2$, $m_{u_1}^2$ and $m_{q_1}^2$ respect the same RG equations with Eqs. (5) - (7).

Considering Eq. (9), one can see that the RG solution of $m_{h_u}^2$ valid only below $Q = M_N$
\[ (t < t_I) \text{ should be written as} \]
\[ m_h^2(t) = m_{h_u}^2(t) + X_I \left[ e^{\frac{t}{T}} \int_{t}^{t_I} dt' y_I^2 - 1 \right] + \ldots \]
\[ = \frac{X_I}{2} \left[ e^{\frac{t}{T}} \int_{t}^{t_I} dt' y_I^2 - \left( 1 - \frac{2m_{h_u}^2}{m_{h_u}^2 + m_{u_t}^2 + m_{q_t}^2} \right) \right] + \ldots, \]  
(32)

where \( \{ m_{h_u}^2, m_{u_t}^2, m_{q_t}^2 \} \) denote the values of \( \{ m_{h_u}^2, m_{u_t}^2, m_{q_t}^2 \} \) at \( Q = M_N \), respectively, and \( X_I \equiv m_{h_u}^2 + m_{u_t}^2 + m_{q_t}^2 \). Note that “…” in Eq. (32) does not contain the dependence of \( \{ m_{h_u}^2, m_{u_t}^2, m_{q_t}^2 \} \). Comparing with Eq. (13), \( \{ m_{h_u}^0, m_{u_t}^0, m_{q_t}^0 \} \) and \( X_0 \) are replaced by \( \{ m_{h_u}^2, m_{u_t}^2, m_{q_t}^2 \} \) and \( X_I \) in Eq. (32). On the contrary, \( y_I^2 \) in Eq. (32) is the same as \( y^2 \) of Eq. (9) for \( t < t_I \), because \( y_I^2 \) should be set to explain the top quark mass at low energy and undergoes the same RG evolution as the case of Eq. (9). The RH neutrino makes \( y_I^2 \) larger only above \( Q = M_N \). Since \( m_{h_u}^2/m_{u_t}^2 \) and \( m_{h_u}^2/m_{q_t}^2 \) are more suppressed at \( Q = M_N \) by the RH neutrino effect above \( Q = M_N \), \( 1 - 2m_{h_u}^2/(m_{h_u}^2 + m_{u_t}^2 + m_{q_t}^2) \) or \( 1 - 2m_{h_u}^2/X_I \) in Eq. (32) is larger than that evaluated at \( Q = M_N \) in the absence of the RH neutrino. As a result, \( \exp[\frac{t}{T} \int_{t}^{t_I} dt' y_I^2] - (1 - 2m_{h_u}^2/X_I) \) vanishes at a \( t \) larger than \( t_W \). It implies that a FP must still exist and appear at a scale higher than \( t_W \). Therefore, we can move the FP to around \( t = t_F \) using a sizable \( y_N \). We will discuss it again later.

Toward the desirable FP location, we need to somehow lift up the FP in the \((t, m_{h_u}^2(t))\) space as mentioned before. As a trial, let us turn on a small \( A_0 \) in Eq. (12), keeping \( m_{h_u}^2 = m_{u_t}^2 = m_{q_t}^2 \). Then Eq. (13) yields \( m_{h_u}^2(t_W) \approx -A_0^2/9 \). So the FP moves in the opposite direction to our desire. From Eqs. (9) and (16), increase of \( m_{1/2}^2 \) also moves the FP in the negative direction. Because of the experimental gluino mass constraint \( (M_3 \gtrsim 1.4 \text{ TeV}) \), however, one cannot decrease \( m_{1/2}^2 \) sufficiently.

Indeed, the largest negative contribution to \( m_{h_u}^2 \) comes from the gluino mass \( M_3 \), as seen from Eqs. (13)–(16): Eq. (13) is dominated by the \( g_3^2 M_3 \) and \( q_3^2 M_{3}^2 \) terms in Eqs. (14) and (15), which eventually give a negative \( F(t_F) \) as seen in Eq. (10). A too large negative \( m_{h_u}^2 \) at the EW scale should be fine-tuned with \( |\mu|^2 \) to yield the desired size of \( m_{2}^2 \). One way to compensate the negative gluino mass effect on \( m_{h_u}^2 \) is to cancel it with the positive contribution from the wino mass effect, sacrificing the gaugino mass unification, \( M_3^2 \approx M_2^2 \) at the GUT scale \([9, 10]\). A fine-tuning between \( m_{2}^2 \) and \( m_{1/2}^2 \) could also leave a light enough \( m_{h_u}^2 \), as seen in Eqs. (9) and (16): a FP achieved through such a fine-tuning can remain insensitive e.g. to the scaling of \( (m_0^2, m_{1/2}^2) \rightarrow \lambda^2(m_0^2, m_{1/2}^2) \), keeping the ratio between \( m_0^2 \) and \( m_{1/2}^2 \) \([11]\). However, the idea of Natural tuning is lost in this mechanism.

In this paper, we propose to consider the two-loop gauge effects by the first and second generations of hierarchically heavier sfermions, maintaining the gaugino mass unification. For simplicity, we suppose a universal heavy mass for them \( (\equiv \tilde{m}) \). If \( \tilde{m}^2 \gg m_{1/2}^2 \), RG running of \( \tilde{m}^2 \) is negligible. Then, the gauge contributions to the RG equations for the soft
masses of the Higgs and sfermions are modified as
\[ 16\pi^2 \frac{d}{dt} m_f^2 = -8 \sum_{i=3,2,1} C_i^f \left( g_i^2 M_i^2 - \tilde{m}^2 - \frac{\tilde{\mu}^2}{4\pi^2} g_i^4 \right) + \cdots \]
\[ = -8 \sum_{i=3,2,1} C_i^f \left( \left( \frac{m_{1/2}}{g_0} \right)^2 g_i^6 - \tilde{m}^2 - \frac{\tilde{\mu}^2}{4\pi^2} g_i^4 \right) + \cdots, \tag{33} \]

where \( f = h_u, u_3, q_3, \) etc., and \( C_i^f \) denotes the Casimir for \( f \). With the universal soft mass condition, the contributions by the “D-term” potential to Eq. (33) vanish. Since \( g_i^2 M_i^2 \)s are always accompanied with \( -\tilde{\mu}^2 g_i^4 \) in Eqs. (5)–(7), they all should be modified into \( g_i^2 M_i^2 - \tilde{m}^2 g_i^4 \). As a result, the heavy gluino effect can be compensated to be milder by the \( \tilde{m}^2 \) terms [15]. If \( \tilde{m} \) is much heavier than the gluino, moreover, it can be comparable to or even dominate over it. Thus, a heavy enough \( \tilde{m}^2 \) could raise \( m_{h_u}^2 \) up even to a positive value at \( t = t_T \). Note that \( \tilde{m}^2 \) doesn’t appear in \( X_0 \) in Eq. (9): the heavier sfermions’ effects on Eqs. (5)–(8) via the Yukawa interactions are extremely tiny. So \( \tilde{m}^2 \) doesn’t touch the FP mechanism. Indeed, any Yukawa couplings and tan \( \beta \) are not involved in \( g_i^2 M_i^2 - \tilde{m}^2 g_i^4 \). Since the both contributions originate from the gauge interactions, their relation could be more easily realized in a UV model [24] than the relation between \( m_{1/2}^2 \) and \( m_0^2 \). Note that they leave intact the A-term RG equation Eq. (8). For the full expressions of the semi-analytic solutions, refer to Appendix.

The hierarchical mass pattern between the first/second and the third generations can be realized by employing the two different SUSY breaking mediations, e.g. the gravity or gauge mediation and U(1)’ mediation. For instance, the first two generations of matter could carry non-zero [but opposite] U(1)’ charges and they could receive additional U(1)’ SUSY breaking mediation effects proportional to their charge squareds [25] for their hierarchically heavier masses [13, 26]. Their desired relation could be achieved from the hierarchy between \( g_0 \) and the U(1)’ gauge coupling, and also the messengers’ masses with a common SUSY breaking source. In such a setup, a relation between \( \tilde{m}^2 \) and \( m_{1/2}^2 \) could also be obtained [24]. A tuning introduced in this process might be under control in a UV theory.

To summarize our discussion so far, in Table II we present the FP’s movements for the various variations of parameters. We can move the FP into the desirable positions of Fig. 2 by using e.g. \( y_N \) and \( \tilde{m}^2 \).

| Variations | \( \tan \beta \uparrow \) | \( y_1^2 \uparrow \), \( \lambda^2 \uparrow \), \( y_N^2 \uparrow \) | \( A_0^2 \uparrow \) | \( m_{1/2}^2 \uparrow \) | \( \tilde{m}^2 \uparrow \) |
|------------|----------------|----------------|----------------|----------------|----------------|
| Focus Point | \( \leftarrow \) | \( \Rightarrow \) | \( \downarrow \) | \( \downarrow \) | \( \uparrow \) |

**TABLE II**: Movement of the focus point for increases of the various parameters in the \( (t, m_{h_u}(t)) \) space
FIG. 3: RG evolutions of $m_{h_u}^2$ for $m_0^2 = (9\text{ TeV})^2$ [Red], $(7\text{ TeV})^2$ [Green], and $(5\text{ TeV})^2$ [Blue], and for (a) $y_{NI} = 0.8$, $\tilde{m}^2 = (15\text{ TeV})^2$ and (b) $y_{NI} = 1.0$, $\tilde{m}^2 = (20\text{ TeV})^2$, when $\tan\beta = 5$ and $m_{1/2} = A_0 = 1\text{ TeV}$. The unit of the vertical axis is $(\text{GeV})^2$. Below the seesaw scale, $t = t_1 \approx 25.3\{Q \approx 2 \times 10^{14}\text{ GeV}\}$, the RH neutrino is decoupled. The dotted lines at $t \approx 0.92$ denote the assumed stop decoupling scale, $Q = 5\text{ TeV}$. Below the stop decoupling scale, the above RG runnings must be modified. The above figures show that the (extrapolated) FP appears at desirable locations.

| $m_0^2$ $(\text{TeV})^2$ | $m_{q_3}^2(t_T)$ $(\text{TeV})^2$ | $m_{u_3}^2(t_T)$ $(\text{TeV})^2$ | $m_{h_u}^2(t_T) (\text{TeV})^2$ |
|--------------------------|--------------------------|--------------------------|--------------------------|
| $y_{NI} = 0.8$            | $y_{NI} = 1.0$            |
| $\tilde{m} = 15\text{ TeV}$ | $\tilde{m} = 20\text{ TeV}$ |
| tan $\beta = 5$          | tan $\beta = 5$          |

TABLE III: Soft squared masses of the stops and Higgs at $t = t_T \approx 0.92\{Q = 5\text{ TeV}\}$ for $m_0^2 = (9\text{ TeV})^2$, $(7\text{ TeV})^2$, and $(5\text{ TeV})^2$, when tan $\beta = 5$ and $m_{1/2} = A_0 = 1\text{ TeV}$. The left [right] four columns correspond to the results of $\{y_{NI} = 0.8, \tilde{m}^2 = (15\text{ TeV})^2\}$ [$(y_{NI} = 1.0, \tilde{m}^2 = (20\text{ TeV})^2)\] .

Fig. 3(a) and (b) show the RG evolutions of $m_{h_u}^2$ for $m_0^2 = (9\text{ TeV})^2$, $(7\text{ TeV})^2$, and $(5\text{ TeV})^2$, when $\{y_{NI} = 0.8, \tilde{m}^2 = (15\text{ TeV})^2\}$ and $\{y_{NI} = 1.0, \tilde{m}^2 = (20\text{ TeV})^2\}$, respectively. Here, $y_{NI}$ means $y_N$ evaluated at the RH neutrino decoupling scale ($Q = M_N \approx 2 \times 10^{14}\text{ GeV}$). $y_N$ of $y_{NI} = 0.8\{1.0\}$ reaches 0.95\{1.2\} at the GUT scale, while its RG evolution becomes frozen below $Q = M_N$. In both cases, we set tan $\beta = 5$ and $m_{1/2} = A_0 = 1\text{ TeV}$. Note that $m_{1/2}$ and $A_0$ are U(1)$_R$ breaking parameters. Thus, e.g. if U(1)$_R$ breaking scale is relatively lower than the SUSY breaking scale, they can be smaller than other soft SUSY breaking parameters, $m_0^2$ and $\tilde{m}^2$ as desired. In Ref. [27], conformal sequestering was considered to suppress them. In “pure gravity mediation,” $m_{1/2}$ and $A_0$ are suppressed at the tree-level [28]. Below the seesaw scale, $t = t_1 \approx 25.3\{Q \approx 2 \times 10^{14}\text{ GeV}\}$, the RH neutrino is decoupled. Thus, $m_{h_u}^2$ in Fig. 3(a) and (b) follow the RG equations without the RH
neutrino below $t = t_I$, while they are governed by the full RG equations including the RH neutrino between $t = t_0$ and $t = t_I$. For the analyses in Fig. 3-(a) and (b), we used the full RG equations in Appendix.

In Fig. 3-(a) [(b)], the FP appears at a slightly lower [higher] scale than the stop decoupling scale ($t = t_T \approx 0.92$). Since $m^2_{h_u}$ is well-focused in the both cases, $m^2_{h_u}(t_T)$ is quite insensitive to the various trial $m^2_0$s as seen in Table III for $5 \text{ TeV} < m^2_0 < 9 \text{ TeV}$ at the GUT scale, $m^2_{h_u}$ just changes from $-0.3 \text{ (TeV)}^2$ to $0.9 \text{ (TeV)}^2$ at the stop decoupling scale. For precise focusing, hence, it is required that

$$0.8 \lesssim y_{NI} \lesssim 1.0 \quad \text{and} \quad (15 \text{ TeV})^2 \lesssim \tilde{m}^2 \lesssim (20 \text{ TeV})^2,$$

(34)

when $\tan \beta = 5$ and $m_{1/2} = A_0 = 1 \text{ TeV}$. Under the situation that $m^2_{h_u}$ at $t = t_T$ is insensitive to $m^2_0$ and stop masses, $m^2_0$ can happen to be around $(8 \text{ TeV})^2$ at the GUT scale, which leads to $5 \text{ TeV}$ stop masses and the $126 \text{ GeV}$ Higgs mass at the EW scale. However, if a larger $y_{NI}$ is taken, e.g. $y_{NI} = 1.4$, the FP emerges around $t \approx 3$ ($Q \approx 40 \text{ TeV}$). For $\tilde{m}^2 \gtrsim (24 \text{ TeV})^2$ and $y_{NI} = 1.0$, the EW symmetry breaking does not arise, because $m^2_{h_u}(t_T) > (1 \text{ TeV})^2$. Hence, the above range of $y_N$ and $\tilde{m}^2$ for a desirable FP need to be supported by a UV model. Once $M_N$ is fixed by a GUT as explained above, however, the above range of $y_{NI}$ could be regarded as another Natural tuning, since $y_N$ can be determined by the active neutrino mass. The tuning issue introduced for the desired $\tilde{m}^2$ could be converted to a model-building problem [24].

Similarly, Fig. 4-(a), (b), and Table IV present the results of $m^2_{h_u}$ for $m^2_0 = (9 \text{ TeV})^2$, ...
Moreover, the initial value of stop's squared masses at the GUT scale, and IV, however, such a thing doesn't occur. It is because the gluino mass is quite heavy in squared too small or even negative at the EW scale via RG evolutions. As seen in Table III (15–25 TeV) can solve the SUSY flavor and SUSY CP problems. In Ref. [14], it was pointed the effective SUSY, the hierarchically heavy masses for the first two generations of sfermions

| m_0^2 | tan β = 50 | y_{NI} = 1.0 | ˜m = 15 TeV | m_0^2 | tan β = 50 | y_{NI} = 1.2 | ˜m = 20 TeV |
|-------|-------------|-------------|-------------|-------|-------------|-------------|-------------|
| m_0^2 (t_T) | (9 TeV)^2 | (7 TeV)^2 | (5 TeV)^2 | m_0^2 (t_T) | (9 TeV)^2 | (7 TeV)^2 | (5 TeV)^2 |
| m_0^2 (t_T) | (6.3 TeV)^2 | (4.8 TeV)^2 | (3.1 TeV)^2 | m_0^2 (t_T) | (5.9 TeV)^2 | (4.2 TeV)^2 | (2.1 TeV)^2 |
| m_0^2 (t_T) | (5.9 TeV)^2 | (4.4 TeV)^2 | (2.9 TeV)^2 | m_0^2 (t_T) | (5.5 TeV)^2 | (3.9 TeV)^2 | (2.1 TeV)^2 |
| m_0^2 (t_T) | (1.2 TeV)^2 | (0.8 TeV)^2 | (0.4 TeV)^2 | m_0^2 (t_T) | (0.7 TeV)^2 | (0.7 TeV)^2 | (0.7 TeV)^2 |

TABLE IV: Soft squared masses of the stops and Higgs at t = t_T ≈ 0.92 (Q = 5 TeV) for m_0^2 = (9 TeV)^2, (7 TeV)^2, and (5 TeV)^2, when tan β = 50 and m_{1/2} = A_0 = 1 TeV. The left [right] four columns correspond to the results of \{y_{NI} = 1.0, ˜m^2 = (15 TeV)^2\} [\{y_{NI} = 1.2, ˜m^2 = (20 TeV)^2\}]

(7 TeV)^2, and (5 TeV)^2, when tan β = 50 and m_{1/2} = A_0 = 1 TeV. Here, we take \{y_{NI} = 1.0, ˜m^2 = (15 TeV)^2\} and \{y_{NI} = 1.2, ˜m^2 = (20 TeV)^2\} in Fig. IV(a) and (b), respectively.

y_N of y_{NI} = 1.0 (1.2) reaches 1.25 (1.6) at the GUT scale. Thus, the parameter ranges required for precise focusing are

\[ 1.0 \lesssim y_{NI} \lesssim 1.2 \] [1.4] \quad \text{and} \quad (15 \text{ TeV})^2 \lesssim ˜m^2 \lesssim (20 \text{ TeV})^2 \quad \text{[[(25 \text{ TeV})]^2}, \quad (35)\]

when tan β = 50 and m_{1/2} = A_0 = 1 TeV. Here, the \{y_{NI} = 1.4, ˜m^2 = (25 \text{ TeV})^2\} case was not displayed in Fig. IV and Table IV. Particularly, \{y_{NI} = 1.2, ˜m^2 = (20 \text{ TeV})^2\} leads to a quite exact focusing, and so m_0^2 (t_T) is almost invariant under variation of m_0^2. Again, m_0^2 \approx (8 \text{ TeV})^2 at the GUT scale happens to yield 5 TeV stop masses and eventually the 126 GeV Higgs boson mass. Around m_0^2 = (8 \text{ TeV})^2 [[(9 \text{ TeV})]^2], the fine-tuning measure is estimated as

\[ \Delta m_0^2 = \left| \frac{\partial \log m_0^2}{\partial \log m_0^2} \right| \approx 368 [490], \quad \text{and} \quad 9 \] (36)

for \{y_{NI} = 1.0, ˜m^2 = (15 \text{ TeV})^2\} [\{y_{NI} = 1.4, ˜m^2 = (25 \text{ TeV})^2\}], and \{y_{NI} = 1.2, ˜m^2 = (20 \text{ TeV})^2\}, respectively. They are remarkably small compared to Eq. (25). For the case of a small enough \Delta m_0^2, \Delta m^2 would become dominant over it [29].

According to the “effective SUSY” (or “more minimal SUSY”), the masses of the first two generations of sfermions are required to be about 5–20 TeV in order to avoid the SUSY flavor and SUSY CP problems, while the third ones and gauginos are lighter than 1 TeV for naturalness of the Higgs. In our case, the third generations of sfermions are heavier than 1 TeV, but the naturalness problem can be addressed depending on the FP scenario. As in the effective SUSY, the hierarchically heavy masses for the first two generations of sfermions (15–25 TeV) can solve the SUSY flavor and SUSY CP problems. In Ref. [14], it was pointed out that such heavy masses for the first two generations of sfermions drive the stop mass squared too small or even negative at the EW scale via RG evolutions. As seen in Table III and IV, however, such a thing doesn’t occur. It is because the gluino mass is quite heavy in our case. Moreover, the initial value of stop’s squared masses at the GUT scale, m_0^2 can be
quite large without a serious fine-tuning, only if $m^2_{h_u}(t)$ is well-focused near the stop mass scale.

Since all the sfermions are very heavy in this model, the pair annihilation cross section of the lightest neutralino is quite suppressed, and so it would overcloses the universe. However this problem could be resolved, e.g. if a sufficient amount of entropy is somehow produced after thermal freeze-out of the neutralino \cite{13}. In this paper, we don’t discuss this issue in details.

IV. CONCLUSION

According to the recent analysis based on three-loop calculations, the radiative correction by 5 TeV stop masses can support the 126 GeV Higgs mass without a large stop mixing effect. The 5 TeV stop mass decoupling scale is much higher than the FP scale determined in the original FP scenario. As a result, $m^2_{h_u}$ evaluated at low energy becomes sensitive to $m_0^2$ chosen at the GUT scale, and so to the low energy value of stop mass, unlike the original FP scenario. Moreover, the present high gluino mass bound ($\gtrsim 1.4$ TeV) results in a too large negative $m^2_{h_u}$ at low energy, which gives rise to a serious fine-tuning problem in the MSSM Higgs sector.

In this paper, we have studied how the location of the FP changes under various variations of parameters. In particular, we noted that the FP can move to the desirable location under increases of both the Yukawa coupling of a superheavy RH neutrino to the Higgs, and the masses of the first and second generations of sfermions. On the other hand, the “$\lambda$” coupling in the NMSSM should be more suppressed than 0.1 to be consistent with the FP scenario, if it is introduced.

We have shown that an order one Yukawa coupling ($0.8 - 1.4$) of the superheavy RH neutrino ($\sim 10^{14}$ GeV) at the seesaw scale can move the FP to the desired stop decoupling scale, and two-loop gauge interactions by the hierarchically heavy masses ($15-25$ TeV) of the first two generations of sfermions can effectively compensate the heavy gluino effects in the RG evolution of $m^2_{h_u}$. Here we set the $U(1)_R$ breaking soft parameters, $m_{1/2} = A_0 = 1$ TeV at the GUT scale. The gaugino mass unification is maintained in this setup. Such heavy masses of the RH neutrino and the first two generations of sfermions can provide also a natural explanation of the small active neutrino mass via the seesaw mechanism, and suppress the flavor violating processes in SUSY models. At the new location of the FP, $m^2_{h_u}$ can be insensitive to $m_0^2$ or trial heavy stop squared masses, restoring the naturalness of the small EW scale. Under this setup, the 126 GeV Higgs mass can be naturally explained by an accidentally selected $m_0^2$ of about $(8$ TeV$)^2$, which gives 5 TeV stop mass at low energy.
Acknowledgments

B.K. thanks Department of Physics and Astronomy in Rutgers University for the hospitality during his visit to Rutgers University. B.K. is supported by the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Grant No. 2013R1A1A2006904, and also in part by Korea Institute for Advanced Study (KIAS) grant funded by the Korean government. C.S.S. is supported in part by DOE grants [d0e-sc0010008, DOE-ARRA-SC0003883, and DOE-DE-SC0007897.

V. APPENDIX

In Appendix, we present the full RG equations utilized in our analyses and some semi-analytic solutions on which the discussions in the main text are based. The notations here follow those of the main text of this paper.

A. The full RG equations

The RG equations for the gauge couplings, $g_{3,2,1}$ and gaugino masses, $M_{3,2,1}$ are integrable. The RG solutions for them are given by

$$g_i^2(t) = \frac{g_0^2}{1 - \frac{g_0^2}{8\pi^2} b_i (t-t_0)}, \quad \text{and} \quad \frac{M_i(t)}{g_i^2(t)} = \frac{m_{i/2}}{g_0^2},$$

(37)

where $b_i$ ($i = 3, 2, 1$) denotes the beta function coefficients for the case of the MSSM field contents, $(b_3, b_2, b_1) = (-3, 1, \frac{33}{5})$. $t$ parametrizes the normalizations scale $Q$, $t-t_0 = \log_2 \frac{Q}{M_G}$.

The relevant superpotential in this paper is

$$W \ni y_t q_3 h_u + y_b q_3 h_d + y_t l_3 h_e + y_N l_3 h_N + 1 \frac{1}{2} M_N N c N c,$$

(38)

where $q_3$ ($l_3$) and $\{ u_3^c, d_3^c \}$ ($e_3^c$) stand for the third generations of quark (lepton) doublet and singlets. The Majoran mass of the RH neutrino $N c$ is assumed to be $M_N \approx 2 \times 10^{14}$ GeV. Below the energy scale of $M_N$, thus, the RH neutrino $N c$ is decoupled from dynamics. The one-loop RG equations for the above dimensionless Yukawa couplings are given by

$$8\pi^2 \frac{dy_t}{dt} = y_t^2 \left[ 6y_t^2 + y_b^2 + y_N^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{13}{15} g_1^2 \right],$$

(39)

$$8\pi^2 \frac{dy_b}{dt} = y_b^2 \left[ y_t^2 + 6y_b^2 + y_\tau^2 - \frac{16}{3} g_3^2 - 3g_2^2 - \frac{7}{15} g_1^2 \right],$$

(40)

$$8\pi^2 \frac{dy_\tau}{dt} = y_\tau^2 \left[ 3y_b^2 + 4y_\tau^2 + y_N^2 - 3g_2^2 - \frac{9}{5} g_1^2 \right],$$

(41)

$$8\pi^2 \frac{dy_N}{dt} = y_N^2 \left[ 3y_t^2 + y_\tau^2 + 4y_N^2 - 3g_2^2 - \frac{3}{5} g_1^2 \right],$$

(42)
and the RG equations of the “A-term” coefficients corresponding to the Yukawa couplings of Eq. (35) are

\[
8\pi^2 \frac{dA_t}{dt} = 6y_t^2 A_t + y_b^2 A_b + y_N^2 A_N - \frac{16}{3} g_3^2 M_3 - 3g_2^2 M_2 - \frac{13}{15} g_1^2 M_1, \quad (43)
\]

\[
8\pi^2 \frac{dA_b}{dt} = y_t^2 A_t + 6y_b^2 A_b + y_r^2 A_r - \frac{16}{3} g_3^2 M_3 - 3g_2^2 M_2 - \frac{7}{15} g_1^2 M_1, \quad (44)
\]

\[
8\pi^2 \frac{dA_r}{dt} = 3y_b^2 A_b + 4y_r^2 A_r + y_N^2 A_N - 3g_2^2 M_2 - \frac{9}{5} g_1^2 M_1, \quad (45)
\]

\[
8\pi^2 \frac{dA_N}{dt} = 3y_t^2 A_t + y_r^2 A_r + 4y_N^2 A_N - 3g_2^2 M_2 - \frac{3}{5} g_1^2 M_1. \quad (46)
\]

Below the scale of \( M_N \), the RG evolutions of \( y_N \) and \( A_N \) become frozen, and they should be decoupled from the above equations.

The RG evolutions for the soft squared masses are governed by the following equations:

\[
16\pi^2 \frac{dm_{h_u}^2}{dt} = 6y_t^2 (X_t + A_t^2) + 2y_b^2 (X_N + A_N^2) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + \frac{\bar{m}_t^2}{4\pi^2} \left[ 6g_2^4 + \frac{6}{5} g_1^4 \right], \quad (47)
\]

\[
16\pi^2 \frac{dm_{b}^2}{dt} = 4y_t^2 (X_t + A_t^2) - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1 + \frac{\bar{m}_t^2}{4\pi^2} \left[ \frac{32}{3} g_3^4 + \frac{32}{15} g_1^4 \right], \quad (48)
\]

\[
16\pi^2 \frac{dm_{A}^2}{dt} = 2y_t^2 (X_t + A_t^2) + 2y_b^2 (X_b + A_b^2) - \frac{32}{3} g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 \quad (49)
\]

\[
+ \frac{\bar{m}_t^2}{4\pi^2} \left[ \frac{32}{3} g_3^4 + 6g_2^4 + \frac{2}{15} g_1^4 \right],
\]

\[
16\pi^2 \frac{dm_{h_d}^2}{dt} = 6y_b^2 (X_b + A_b^2) + 2y_r^2 (X_r + A_r^2) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + \frac{\bar{m}_t^2}{4\pi^2} \left[ 6g_2^4 + \frac{6}{5} g_1^4 \right], \quad (50)
\]

\[
16\pi^2 \frac{dm_{d}^2}{dt} = 4y_b^2 (X_b + A_b^2) - \frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 + \frac{\bar{m}_t^2}{4\pi^2} \left[ \frac{32}{3} g_3^4 + \frac{8}{15} g_1^4 \right], \quad (51)
\]

\[
16\pi^2 \frac{dm_{A}^2}{dt} = 4y_r^2 (X_r + A_r^2) - \frac{24}{5} g_1^2 M_1^2 + \frac{\bar{m}_t^2}{4\pi^2} \left[ \frac{24}{5} g_1^4 \right], \quad (52)
\]

\[
16\pi^2 \frac{dm_{h_u}^2}{dt} = 2y_r^2 (X_r + A_r^2) + 2y_N^2 (X_N + A_N^2) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + \frac{\bar{m}_t^2}{4\pi^2} \left[ 6g_2^4 + \frac{6}{5} g_1^4 \right], \quad (53)
\]

\[
16\pi^2 \frac{dm_{N}^2}{dt} = 4y_N^2 (X_N + A_N^2), \quad (54)
\]

where \( X_t, X_b, X_r, \) and \( X_N \) are defined as \( X_t \equiv m_{h_u}^2 + m_{u_3}^2 + m_{q_3}^2, \) \( X_b \equiv m_{h_d}^2 + m_{d_3}^2 + m_{q_1}^2, \) \( X_r \equiv m_{h_u}^2 + m_{u_3}^2 + m_{l_1}^2, \) and \( X_N \equiv m_{h_u}^2 + m_{N_2}^2 + m_{l_1}^2, \) respectively. The \( \bar{m}_t^2 \) terms denote the contributions coming from the two-loop gauge interactions by the first and second generations of sfermions, which are assumed to be hierarchically heavier than the third ones. Here we suppose a universal soft mass for the first two generations of sfermions, which eliminates the contributions by the “D-term” potential from the above equations. Since these effects are comparable to the one-loop gaugino mass terms, we take them into account. \( m_{N}^2 \) and \( X_N \) as well as \( y_N \) and \( A_N \) are dropped out from the above equations below \( Q = M_N. \)
B. Semi-analytic RG solutions

Let us present our semi-analytic solutions to the RG equations. When $\tan(\beta)$ is small enough and the RH neutrino is decoupled, the RG evolutions of the soft mass parameters, $m_{h_u}^2$, $m_{u_3}^2$, $m_{q_1}^2$, and $A_t$ are approximately simplified as

\begin{align}
16\pi^2 \frac{dm_{h_u}^2}{dt} &= 6y_t^2 (X_t + A_t^2) - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + \frac{\tilde{m}^2}{4\pi^2} \left[ 6g_2^4 + \frac{6}{5} g_1^4 \right], \\
16\pi^2 \frac{dm_{u_3}^2}{dt} &= 4y_t^2 (X_t + A_t^2) - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 + \frac{\tilde{m}^2}{4\pi^2} \left[ \frac{32}{3} g_3^4 + \frac{32}{15} g_1^4 \right], \\
16\pi^2 \frac{dm_{q_1}^2}{dt} &= 2y_t^2 (X_t + A_t^2) - \frac{32}{3} g_3^2 M_3^2 - 6g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 + \frac{\tilde{m}^2}{4\pi^2} \left[ \frac{32}{3} g_3^4 + 6g_2^4 + \frac{2}{15} g_1^4 \right], \\
8\pi^2 \frac{dA_t}{dt} &= 6y_t^2 A_t - \frac{16}{3} g_3^2 M_3 - 3g_2^2 M_2 - \frac{13}{15} g_1^2 M_1 \equiv 6y_t^2 A_t - G_A.
\end{align}

Summation of Eqs. (55), (56), and (57) yields the RG equation for $X_t$:

\begin{equation}
\frac{dX_t}{dt} = \frac{3y_t^2}{4\pi^2} (X_t + A_t^2) - \frac{1}{4\pi^2} G_X.
\end{equation}

In Eqs. (58) and (59), $G_A$ and $G_X$ are defined as

\begin{align}
G_A(t) &\equiv \left( \frac{m_{1/2}}{g_6^2} \right) \left[ \frac{16}{3} g_3^4 + 3g_2^4 + \frac{13}{15} g_1^4 \right], \\
G_X(t) &\equiv \left( \frac{m_{1/2}}{g_6^2} \right)^2 \left[ \frac{16}{3} g_3^6 + 3g_2^6 + \frac{13}{15} g_1^6 \right] - \frac{\tilde{m}^2}{4\pi^2} \left[ \frac{16}{3} g_3^4 + 3g_2^4 + \frac{13}{15} g_1^4 \right],
\end{align}

respectively, assuming $\frac{M_{1/2}(t)}{g_6(t)} = \frac{m_{1/2}}{g_6^2}$ $(i = 3, 2, 1)$.

The solutions of $A_t$ and $X_t$ are given by

\begin{align}
A_t(t) = e^{\frac{3}{4\pi^2} \int_{t_0}^{t} dt' y_t^2} \left[ A_0 - \frac{1}{8\pi^2} \int_{t_0}^{t} dt' G_A e^{-\frac{3}{4\pi^2} \int_{t_0}^{t} dt' y_t^2} \right], \\
X_t(t) = e^{\frac{3}{4\pi^2} \int_{t_0}^{t} dt' y_t^2} \left[ X_0 + \int_{t_0}^{t} dt' \left( \frac{3}{4\pi^2} y_t^2 A_t^2 - \frac{1}{4\pi^2} G_X \right) e^{-\frac{3}{4\pi^2} \int_{t_0}^{t} dt' y_t^2} \right],
\end{align}

where $A_0$ and $X_0$ denote the GUT scale values of $A_t$ and $X_t$, $A_0 \equiv A_t(t = t_0)$, and $X_0 \equiv X_t(t = t_0) = m_{h_{1,0}}^2 + m_{u_{3,0}}^2 + m_{q_{1,0}}^2$. 

20
With Eqs. (59), (62), and (63), one can solve Eqs. (55), (56), and (57):

\[
m_{h_u}(t) = m_{h_u,0}^2 + \frac{X_0}{2} \left[ e^{\frac{3}{4\pi^2} \int_{t_0}^t dt' y_i^2} - 1 \right] + \frac{1}{2} F(t) \\
- \left( \frac{m_{1/2}}{g_0^2} \right)^2 \left[ \frac{3}{2} \{ g_2^4(t) - g_0^4 \} + \frac{1}{22} \{ g_4^4(t) - g_0^4 \} \right] \\
+ \left( \frac{\tilde{m}^2}{4\pi^2} \right) \left[ \frac{3}{2} \{ g_2^2(t) - g_0^2 \} + \frac{1}{11} \{ g_4^2(t) - g_0^4 \} \right],
\]

(64)

\[
m_{u_3}^2(t) = m_{u_3,0}^2 + \frac{X_0}{3} \left[ e^{\frac{3}{4\pi^2} \int_{t_0}^t dt' y_i^2} - 1 \right] + \frac{1}{3} F(t) \\
+ \left( \frac{m_{1/2}}{g_0^2} \right)^2 \left[ \frac{8}{9} \{ g_3^4(t) - g_0^4 \} - \frac{8}{9g_0^4} \{ g_4^4(t) - g_0^4 \} \right] \\
- \left( \frac{\tilde{m}^2}{4\pi^2} \right) \left[ \frac{16}{9} \{ g_3^2(t) - g_0^2 \} - \frac{16}{9g_0^2} \{ g_4^2(t) - g_0^2 \} \right],
\]

(65)

\[
m_{q_3}^2(t) = m_{q_3,0}^2 + \frac{X_0}{6} \left[ e^{\frac{3}{4\pi^2} \int_{t_0}^t dt' y_i^2} - 1 \right] + \frac{1}{6} F(t) \\
+ \left( \frac{m_{1/2}}{g_0^2} \right)^2 \left[ \frac{8}{9} \{ g_3^4(t) - g_0^4 \} - \frac{3}{2} \{ g_2^4(t) - g_0^4 \} - \frac{1}{198} \{ g_4^4(t) - g_0^4 \} \right] \\
- \left( \frac{\tilde{m}^2}{4\pi^2} \right) \left[ \frac{16}{9} \{ g_3^2(t) - g_0^2 \} - \frac{3}{2} \{ g_2^2(t) - g_0^2 \} - \frac{1}{99} \{ g_4^2(t) - g_0^2 \} \right],
\]

(66)

where \( F(t) \) is defined as

\[
F(t) \equiv e^{\frac{3}{4\pi^2} \int_{t_0}^t dt' y_i^2} \int_{t_0}^t dt' \frac{3}{4\pi^2} y_i^2 A_i^2 e^{-\frac{3}{4\pi^2} \int_{t_0}^{t'} dt'' y_i^2} \\
- \frac{1}{4\pi^2} \left[ e^{\frac{3}{4\pi^2} \int_{t_0}^t dt' y_i^2} \int_{t_0}^t dt' G_X^2 e^{-\frac{3}{4\pi^2} \int_{t_0}^{t'} dt'' y_i^2} - \int_{t_0}^t dt' G_X^2 \right].
\]

(67)

Note that \( F(t) \) in Eq. (67) is independent of the initial values for the squared masses, \( m_{h_u,0}^2, m_{u_3,0}^2, \) and \( m_{q_3,0}^2 \). Using Eqs. (37), one can obtain the following useful results:

\[
\int_{t_0}^t dt' g_i^2 M_i^2 = \frac{4\pi^2}{b_i} \left( \frac{m_{1/2}}{g_0^2} \right)^2 \{ g_i^4(t) - g_0^4 \},
\]

(68)

\[
\int_{t_0}^t dt' g_i^2 M_i = \frac{8\pi^2}{b_i} \left( \frac{m_{1/2}}{g_0^2} \right) \{ g_i^2(t) - g_0^2 \},
\]

(69)

\[
\int_{t_0}^t dt' g_i^4 = \frac{8\pi^2}{b_i} \{ g_i^2(t) - g_0^2 \}.
\]

(70)

[1] For a review, for instance, see M. Drees, R. Godbole and P. Roy, “Theory and phenomenology of sparticles: An account of four-dimensional \( N=1 \) supersymmetry in high energy physics,” Hackensack, USA: World Scientific (2004) 555 p. References are therein.
[2] J. L. Feng, P. Kant, S. Profumo and D. Sanford, Phys. Rev. Lett. 111 (2013) 131802 [arXiv:1306.2318 [hep-ph]].

[3] G. Aad et al. [ATLAS Collaboration], Phys. Lett. B 716 (2012) 1 [arXiv:1207.7214 [hep-ex]]; S. Chatrchyan et al. [CMS Collaboration], Phys. Lett. B 716 (2012) 30 [arXiv:1207.7235 [hep-ex]].

[4] See, for instance, B. Kyae and J. -C. Park, Phys. Rev. D 86 (2012) 031701 [arXiv:1203.1656 [hep-ph]]; B. Kyae and J. -C. Park, Phys. Rev. D 87 (2013) 075021 [arXiv:1207.3126 [hep-ph]]; B. Kyae and C. S. Shin, JHEP 1306 (2013) 102 [arXiv:1303.6703 [hep-ph]]; B. Kyae, arXiv:1401.1878 [hep-ph].

[5] ATLAS collaboration, ATLAS-CONF-2013-024; S. Chatrchyan et al. [CMS Collaboration], Eur. Phys. J. C 73 (2013) 2677 [arXiv:1308.1586 [hep-ex]].

[6] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. Lett. 84 (2000) 2322 [hep-ph/9908309].

[7] J. L. Feng, K. T. Matchev and T. Moroi, Phys. Rev. D 61 (2000) 075005 [hep-ph/9909334].

[8] ATLAS collaboration, ATLAS-CONF-2013-061.

[9] H. Abe, T. Kobayashi and Y. Omura, Phys. Rev. D 76 (2007) 015002 [hep-ph/0703044]; D. Horton and G. G. Ross, Nucl. Phys. B 830 (2010) 221 [arXiv:0908.0857 [hep-ph]]; J. E. Younkin and S. P. Martin, Phys. Rev. D 85 (2012) 055028 [arXiv:1201.2989 [hep-ph]]; H. Abe, J. Kawamura and H. Otsuka, PTEP 2013 (2013) 013B02 [arXiv:1208.5328 [hep-ph]]; I. Gogoladze, F. Nasir and Q. Shafi, Int. J. Mod. Phys. A 28 (2013) 1350046 [arXiv:1212.2593 [hep-ph]].

[10] T. T. Yanagida and N. Yokozaki, Phys. Lett. B 722 (2013) 355 [arXiv:1301.1137 [hep-ph]]; T. T. Yanagida and N. Yokozaki, JHEP 1311 (2013) 020 [arXiv:1308.0536 [hep-ph]].

[11] A. Delgado, M. Quiros and C. Wagner, arXiv:1402.1735 [hep-ph].

[12] K. Kadota and K. A. Olive, Phys. Rev. D 80 (2009) 095015 [arXiv:0909.3075 [hep-ph]].

[13] M. Asano, T. Moroi, R. Sato and T. T. Yanagida, Phys. Lett. B 708 (2012) 107 [arXiv:1111.3506 [hep-ph]].

[14] N. Arkani-Hamed and H. Murayama, Phys. Rev. D 56 (1997) 6733 [hep-ph/9703259].

[15] J. -H. Huh and B. Kyae, Phys. Lett. B 726 (2013) 729 [arXiv:1306.1321 [hep-ph]].

[16] A. G. Cohen, D. B. Kaplan and A. E. Nelson, Phys. Lett. B 388 (1996) 588 [hep-ph/9607394].

[17] J. L. Feng and D. Sanford, Phys. Rev. D 86 (2012) 055015 [arXiv:1205.2372 [hep-ph]].

[18] S. R. Coleman and E. J. Weinberg, Phys. Rev. D 7 (1973) 1888.

[19] M. S. Carena, M. Quiros and C. E. M. Wagner, Nucl. Phys. B 461 (1996) 407 [hep-ph/9508343].

[20] J. R. Ellis, K. Enqvist, D. V. Nanopoulos and F. Zwirner, Mod. Phys. Lett. A 1 (1986) 57; R. Barbieri and G. F. Giudice, Nucl. Phys. B 306 (1988) 63.

[21] M. Masip, R. Munoz-Tapia and A. Pomarol, Phys. Rev. D 57 (1998) R5340 [hep-ph/9801437]; R. Barbieri, L. J. Hall, A. Y. Papaioannou, D. Pappadopulo and V. S. Rychkov, JHEP 0803 (2008) 005 [arXiv:0712.2903 [hep-ph]]; L. J. Hall, D. Pinner and J. T. Ruderman, JHEP 1204
(2012) 131 [arXiv:1112.2703 [hep-ph]]; E. Hardy, J. March-Russell and J. Unwin, JHEP 1210 (2012) 072 [arXiv:1207.1435 [hep-ph]].

[22] P. Minkowski, Phys. Lett. B 67 (1977) 421; M. Gell-Mann, P. Ramond and R. Slansky, Conf. Proc. C 790927 (1979) 315 [arXiv:1306.4669 [hep-th]]; T. Yanagida, Conf. Proc. C 7902131 (1979) 95; R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912.

[23] I. Jack, D. R. T. Jones, S. P. Martin, M. T. Vaughn and Y. Yamada, Phys. Rev. D 50 (1994) 5481 [hep-ph/9407291].

[24] B. Kyae and C. S. Shin, work in progress.

[25] P. Langacker, G. Paz, L. -T. Wang and I. Yavin, Phys. Rev. Lett. 100 (2008) 041802 [arXiv:0710.1632 [hep-ph]]; P. Langacker, G. Paz, L. -T. Wang and I. Yavin, Phys. Rev. D 77 (2008) 085033 [arXiv:0801.3693 [hep-ph]].

[26] K. S. Jeong, J. E. Kim and M. -S. Seo, Phys. Rev. D 84 (2011) 075008 [arXiv:1107.5613 [hep-ph]].

[27] R. Ding, T. Li, F. Staub and B. Zhu, arXiv:1312.5407 [hep-ph].

[28] M. Ibe and T. T. Yanagida, Phys. Lett. B 709 (2012) 374 [arXiv:1112.2462 [hep-ph]]; J. L. Evans, M. Ibe, K. A. Olive and T. T. Yanagida, Eur. Phys. J. C 73 (2013) 2468 [arXiv:1302.5346 [hep-ph]].

[29] S. Antusch, L. Calibbi, V. Maurer, M. Monaco and M. Spinrath, JHEP 01 (2013) 187 [JHEP 1301 (2013) 187] [arXiv:1207.7236].