Characterization of electrostatic actuators for suspended mirror control with modulated bias

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Abstract. Electrostatic actuators are one of the most promising devices for mirror control in advanced gravitational waves detectors. An accurate characterization of such actuators is required for a correct design, able to satisfy the requirement of the control system, both in term of low noise content as well as to fit the required dynamic range. To this aim a simple and effective experimental set-up was developed, consisting in a suspended mirror which displacement, induced by an electrostatic actuator, is measured by using an optical lever. The effect of stray patch charge on the mirror was minimized by using an alternate voltage as bias reference for the actuator. Different working conditions were investigated, in particular by varying the mirror-actuator distance and the bias amplitude. The experimental results were compared to the prediction of a numerical model taking into account the actuator geometry and the working conditions.

1. Introduction

The first generation of interferometric detectors for gravitational waves detection have currently reached their target performances. In particular the VIRGO \cite{1}, LIGO \cite{2} and GEO 600 \cite{3} detectors have been involved, in the last years, in cooperative and long term scientific data taking, while the TAMA \cite{4} detector is upgrading to enhance its sensitivity in low frequency range.

In about one year from now, most of the detectors are expected to start their upgrade to the \textit{Advanced} configuration, with the aim to enhance their sensitivity by about one order of magnitude. This second generation of detectors will take advantage from some technical solution that allow to reduce the effect of some fundamental noises on the interferometer sensitivity. Nevertheless, at the same time, it is necessary to reduce the effect of other contributions, like control and environmental noises.

To this aim, a non-negligible contribution could come from the use of electrostatic actuators (EA) for the control of the last suspended mass, a solution already applied for the GEO 600 detector \cite{5} \cite{6} that is under study also for other detectors, currently adopting coil-magnets pair actuators. In particular, the use of EA allows to keep the mirrors under control without the need of gluing the magnets on the mirror bulk, saving, in this way, the mechanical quality of the test masses and, as a consequence, reducing the final thermal noise \cite{7}. Another advantage is the strong reduction of the coupling with external magnetic fields, that is an important issue for magnetic actuators, since no direct coupling is anymore possible in the case of EA.
In this paper a simple electrostatic actuator, working on one degree of freedom, is described. The characterization is performed by using a suspended mass with a simple optical lever as position sensor. The experimental set-up is placed under vacuum to reduce the charging effects due to the air. The characterization was performed for different actuator-mass distances and in two different bias conditions; this in order to fulfill a double target: to verify the effectiveness of a modulated bias in reducing the effect of the residual charges present on the test mass, and to check the validity of a theoretical model available for the computation of the actuator capacitance.

2. Operation theory

The working principle of an electrostatic actuator is simply described by standard electrostatic, giving, for a device of capacitance $C$, polarized at fixed voltage $V$, a resulting force along the $x$ axis equal to:

$$F_x = -\frac{1}{2} \left| \frac{dC}{dx} \right| V^2$$ (1)

where the capacitance is supposed to vary by changing the system characteristics along the main axis while the minus sign is due to the characteristic of such forces, that are always attractive. In the case of actuator for suspended dielectric mirrors, such devices mainly consist in a set of close conductive strips, arranged in a suitable geometry, alternately polarized at two different voltages. The strips, together with the dielectric suspended test mass, placed at distance $x$ with respect to the actuator plane, constitute a capacitor, with a capacitance variable by changing the distance of the test mass with respect to the actuator.

The theoretical expression of the capacitance is described in [8], here we want just to recall the most important characteristics. For the simplest geometry, i.e. a set of $N$ parallel conductive strips with period $b$, rectangular in shape, of length $L$ and width $a$, laying on a substrate with relative dielectric constant $\epsilon_s$, placed at distance $x$ from the test mass having a relative dielectric constant $\epsilon_m$, the capacitance can be written as:

$$C(x) = C_\infty \alpha_m (\tilde{a}, \tilde{x}, \epsilon_m)$$ (2)

where $\tilde{a} = a/b$ is the normalized strip width, $\tilde{x} = x/b$ is the normalized distance and $\alpha_m$ is a function of the listed parameters describing the effect of the mirror at distance $x$, while $C_\infty$ is:

$$C_\infty = N\pi^2 L \epsilon_s + \frac{1}{4} \epsilon_0 \alpha_s (\tilde{a})$$ (3)

is the capacitance of the isolated actuator, being $\alpha_s$ a function of the normalized strip width that is proportional to the capacitance linear density of the electrodes in free space, while the first term, depending on $\epsilon_s$, takes into account the effect of the actuator substrate. It is important to underline that the expression (2) is calculated in the approximation of infinitely long strips and taking into account only the contribution of the first image charges, both for the substrate and for the mirror. As a consequence the capacitance of real devices become different from the this value for small values of $x$ with respect to $b$ due to the increasing weight of border effects and image charges as the distance decreases [8].

By substituting the capacitance (2) in the expression of the force (1), assuming that the displacement of the test mass is small with respect to the static distance $d$ from the actuator, one obtains:

$$F_x = -N\pi^2 L \epsilon_s + \frac{1}{8} \epsilon_0 \alpha_s \left| \frac{d\alpha_m}{dx} \right|_{x=d} V^2 = -\alpha V^2$$ (4)

being $\alpha$ the coupling constant of the electrostatic actuator in $(N/V^2)$ where all characteristics, but the polarization voltage, are included.
The last expression has to be modified to consider also the presence of a stray electric charge \( q \) on the dielectric mass. In this case, by making the simple approximation that the electric field is proportional to the polarization voltage applied to the actuator, it is possible to write:

\[
F_x = -\alpha V^2 + \beta V
\]

(5)

where the factor \( \beta \) is, in general, a function of the charge \( q \), the distance \( x \) and the geometry of the actuator. The effects of this term were already observed on a similar set-up [9], and some techniques for its mitigation were already developed [10].

The main purpose of the EA, like any actuation system for the test masses, is the mirror driving to keep the interferometer locked on the right working point, but the expression (5) clearly shows a non linear behavior of the force with respect to the applied voltage. Since in the framework of standard control systems it is more effective to work with linear actuation systems, a simple approach consists in applying the square root of the driving signal, resulting in the desired linear driving force because of the square voltage in equation (5). Moreover, to reduce the effect of the stray charges, it is possible to modulate the driving signal, to obtain a zero averaged contribution of the linear term of the actuation force even in presence of charges on the test masses. This driving technique was already successfully experimented in the control of a bench top Michelson interferometer with a suspended mirror controlled by a such EA [11].

To clarify this approach, let \( A(t) \) be the driving signal we want to apply on the test mass, \( A_{DC} \) the voltage bias and \( 2\pi \omega_M \) the modulation frequency of the full driving signal. The square root is computed and sent, with the modulation, to the actuator driver. In this way the voltage applied to the actuator is:

\[
V = G\sqrt{A_{DC} + A(t) \cos \omega_M t}
\]

(6)

where \( G \) is the gain of the EA driver. With this voltage, the force exerted on the test mass becomes:

\[
F = -\frac{1}{2} \alpha G^2 (A_{DC} + A(t))(1 + \cos 2\omega_M t) + \beta G \sqrt{A_{DC} + A(t) \cos \omega_M t}
\]

(7)

If the modulation frequency is chosen at enough high frequency to have negligible effects on the test mass motion and the frequency content of the driving signal is much smaller with respect to \( \omega_M \), the force (7) only consists of a DC bias term and of a term proportional to the original driving signal \( A(t) \).

3. Experiment

The experimental set-up used to characterize the actuator consists of a suspended dielectric mass with a mirror attached on the front face. The mass of the suspended element is \( m = 1.312 \) Kg, while the length of the suspension wires is \( l = 0.18 \) m. On the rear face a simple electrostatic actuator, composed by \( N = 20 \) strips, with length \( L = 8 \) cm, width \( a = 3.2 \) mm, and period \( b = 4 \) mm, was placed. The EA substrate has a relative dielectric constant \( \epsilon_s = 4.47 \), while the suspended dielectric mass has \( \epsilon_m = 3.7 \). The actuator was mounted on a translation stage in order to easily change the relative distance from the suspended mass. The mass position was sensed with an optical lever, using as source a 1 mW superluminescent LED emitting at 850 nm. The reflected beam impinges on a lens and then on a 2D position sensing photodiode (PSD) placed in the image plane of the beam spot on the mirror. In this way the displacement signal detected by the PSD is dominated by the mass translation along the optical axis, while the contribution of the rotation around the vertical axis is negligible.

Both the driving and position signals are generated or processed by a digital DAQ system, running at a sampling frequency \( F_s = 1 \) kHz. The ADC and DAC resolutions are 16 bit, while their range is \( \pm 10 \) V. The driving signal (6), is digitally generated by the CPU managing the
Figure 1. Schematic of the experimental set-up.

Figure 2. Comparison between the model and the force measured, in different bias conditions, for a excitation with $f = 0.1$ Hz.

DAQ system, and is sent, through the DAC, to a commercial HV amplifier, with a gain $G = 200$, a maximum output voltage of 2 kV and a bandwidth of 1 kHz. Because of the limited sampling rate, the modulation frequency for the driving signals was fixed at $f_M = 100$ Hz, that is not a very high value with respect to the suspended mass resonance frequency $f_0$ (around 1 Hz), but it is high enough for our purposes. The signals coming from the PSD were processed by using an amplifier that also allowed to minimize the effect of the quantization noise due to the analog to digital converter [12].

All the system, including the PSD, was placed in a vacuum chamber. The probe beam of the optical lever was sent in the chamber by using an optical fiber and guided to the mirror attached on the test mass by means of a suitable collimator. A schematic view of the experimental set-up is shown in figure 1. The characterization of the actuator was performed by applying, to the HV amplifier, a voltage of the form indicated in equation (6), with a simple line at frequency $f = 2\pi \omega$ as driving signal:

$$V = \sqrt{A_{DC} + A_{AC} \cos \omega t \cos \omega_M t}$$

(8)

In this way the component at angular frequency $\omega$ of the force applied on the suspended mass is:

$$F_\omega = \frac{1}{2} \alpha G^2 A_{AC} = \frac{1}{2} \alpha V_{AC}^2$$

(9)

where

$$V_{AC} = G \sqrt{A_{AC}}$$

(10)

The effect of this force was measured by calculating the displacement spectrum of the mass, using the data coming from the PSD. Finally the displacement was converted in force by taking into account the theoretical transfer function of the suspended mass.

The measurements were performed at fixed distance between the actuator and test mass of $d = 3.5$ mm, using a fixed bias $A_{DC} = 50$ V$^2$, that corresponds to a force equal to the half of the maximum force for the given output range of the HV amplifier. Two excitation frequencies were used, namely 0.1 Hz and 20 Hz, that are both far enough from $f_0$ to allow us to neglect the transfer function details around the resonance of the suspended mass. For each frequency a set of excitation amplitudes $A_{AC}$, in the range $5 \div 45$ V$^2$ were used. All the measurements were repeated in DC bias ($f_M = 0$) and alternate bias ($f_M = 100$ Hz) conditions. In particular, for each set of measurements in DC bias, also the sign of the voltage applied to the HV amplifier was changed to enhance the effects due to the stray charges on the mass, following the suggestions given in [10]. This change can be formally taken into account by changing the
sign of $G$ in equation (7). A change in the electrostatic force should arise only in the case $\beta \neq 0$ and $\omega_M = 0$.

4. Results and Discussion

The most interesting results are related to some observed deviation from the theoretical model developed in section 2. The measurements performed at $f = 0.1$ Hz are shown in figure 2. This set shows the best signal to noise ratio, since that frequency is well below the resonance frequency $f_0$. The filled dots represent the force measured in AC bias, both with positive or negative $G$, while the open circles are the force measured in DC bias with different $G$ sign. The deviation from the foreseen behavior, clearly visible for all the points in DC bias, in particular in the case of negative $G$, is due to the presence of spurious charges on the dielectric suspended mass. In fact in this case, equation (7) becomes:

$$F = -\alpha G^2 (A_{DC} + A_{AC} \cos \omega t) + \beta G \sqrt{A_{DC} + A_{AC}} \cos \omega t$$

and a not negligible contribution could arise from the last term. Moreover this contribution depends on $G$ as confirmed by the results. The two opposite polarizations, for the AC bias, give instead the same results, as the experimental measurements are practically overlapped. This confirms the effectiveness of the alternate bias technique that is insensitive to any static stray charge present on the test mass. It is important to underline that the points reported in figure 2, representing the measurements in AC bias, were multiplied by a factor 2 to take into account the presence of the modulation at $\omega_M$ that introduce a factor $1/2$ in the force expression (eq. 9). In this way it is possible to compare the two different bias condition on a single graph. Of course all the measurements were performed in the same run, without opening the vacuum chamber to not intentionally change the charge distribution on the test mass.

Following the (11) a larger disagreement would be expected also for the case of DC bias with positive $G$, but one should consider that the description of the electrical field between the EA and the test mass is very roughly approximated in the model; moreover a slight dependence of $\beta$ from the sign of $G$ is expected. More investigations are need in this direction which also require some upgrade in the experimental set-up, as the possibility to change the distance between the EA and the test mass without opening the chamber and changing, in this way, the amount of charges on the mass. Similar result were obtained also for the $f = 20$ Hz excitation, but in this case the measurements were affected by a larger error, since the suspension filter out most of the force and the resulting displacement, at 20 Hz, is not very large respect to the noise floor and the finale error on the force estimation is of the order of 20% making very difficult to decide if the points follow the model or not. The displacement spectra, in case of measurements in DC bias conditions, also show additional lines placed at multiple frequency respect to the one injected by the signal. These lines disappear if the measurement is performed in AC bias as shown in figure 3. This behavior is easily explained by the form of the driving signal (8) that corresponds to the force in equation (11). By calculating the Fourier coefficient of such force, it is possible to prove that the last term of this equation can be written as:

$$\beta G \sqrt{A_{DC} + A_{AC}} \cos \omega t = G \beta \gamma \sqrt{1 + a \cos \omega t} = G \beta \gamma \left( c_0 + 2 \sum_{n=1}^{+\infty} c_n \cos n \omega t \right)$$

where $\gamma = \sqrt{A_{DC}}$, $a = A_{AC}/A_{DC}$ and the generic Fourier coefficient is:

$$c_n = -\frac{\Gamma(n - \frac{1}{2})}{2\sqrt{\pi} \Gamma(n + 1)} \left( -\frac{a}{2} \right)^n 2F1 \left( \frac{n}{2} - \frac{1}{4}, \frac{n}{2} + \frac{1}{4}, n + 1, a^2 \right)$$

where $\Gamma$ is the gamma function and $2F1$ is the Gauss Hypergeometric function. Since for real cases $a < 1$, this relation foresees decreasing contribution for all the harmonics of the injected...
signal. In fact those lines are clearly visible in the spectrum at least until n = 3. Equation (12) also foresees a contribution on the fundamental frequency, that corresponds to the disagreement between the DC and AC measurements reported in figure 2.

Thanks to this behavior, the chosen form of the driving, allows to easily detect the presence of charges on the test mass, simply by looking for the harmonics of the injected line.

In figure 4 the measured displacements ratio between the 2nd and 1st harmonics (upper graph) and the 3rd and 1st harmonics (lower graph) are reported as a function of the voltage \( V_{AC} \). The measurements were done both for positive and negative \( G \). The comparison with the model, based on the expression (13) is also given: \( R_i = c_i/c_1 \). The large error do not allow to state any definite conclusion, but the measurements largely follow the model.

5. Conclusion
A driving technique for electrostatic actuators, making use of alternate bias to minimize the effect of stray charges present on the dielectric test mass, was developed and characterized. The result confirms the effectiveness of this technique compared with the standard DC bias. A device driven in this way show a very good agreement with the theoretical model of electrostatic actuation. Moreover this technique also allows to easily detect the presence of spurious electric charges on the test mass and gives quantitative indications on the level of charging.

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