Thermal tuning the optical cavity for 3 mode interaction studies using a $CO_2$ laser

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Abstract. Three mode interactions could induce parametric instability in advanced gravitational wave detectors with high optical power circulating in the cavities. One of the conditions for parametric instability to occur is when the cavity frequency difference between fundamental mode and the high order mode matches the test mass acoustic mode frequency. The optical mode spacing is a function of cavity g-factor (radius of curvature). At the Gingin High Optical Power Facility, we have an 80 meter optical cavity particularly designed for studying high optical power effects in advanced gravitational wave detectors such as parametric instabilities. Here we present the recent results of thermal tuning the cavity g-factor by heating the test mass surface with a $CO_2$ laser to investigate the 3-mode interactions. Observation of test mass thermal noise peaks above 160 kHz enhanced by 3 mode interaction is presented.

1. Introduction

Advanced gravitational wave detectors are aiming to detect frequent gravitational wave signals at least once a year, with the sensitivity able to reach a distance of 200MPc [1]. To achieve this sensitivity, the detectors will use extremely high optical power inside the arm cavities to reduce quantum shot noise limit. However, high optical power, together with low optical and mechanical loss test masses could induce parametric instability in a gravitational wave detector. At Gingin high optical power facility, an 80m optical cavity is used to investigate parametric instabilities and method for parametric instability control. Currently, spontaneous parametric instability has not been observed due to the low power level in the instrument. However, 3-mode interaction was observed [2] and possible control strategies was proposed [3,4,5]. Here we present further studies of 3-mode interaction using $CO_2$ laser thermal tuning method.

1.1. Three Mode Parametric Interaction and Parametric Instability

Gravitational wave detectors consist of a pair of Fabry-Perot (FP) cavities [6]. Thermal motion of the test mass will scatter the incoming fundamental optical mode of frequency ($\omega_0$) to higher order optical modes. The higher order optical modes provide a back action force on the test mass mirrors through radiation pressure creating a resonance condition which leads to further amplification of the higher order optical mode. If the beating frequency between a resonant higher order optical mode ($\omega_h$) and the fundamental optical mode matches the acoustic mode frequency ($\omega_m$) of the test mass mirror, as well as when the optical mode shape has a good spatial overlap with the acoustic mode then, 3-mode interaction will occur. This either amplifies...
or dampen the mechanical motion of the test mass. Amplification of the mechanical mode could lead to parametric instability, whereas damping leads to the cooling of the mechanical mode.

A quantitative discussion on parametric instability can be described using parametric gain \( R \) \cite{7}. Instability occurs when the value of \( R \) is greater than 1. The gain can be defined as

\[
R = \frac{4P_c Q_1 Q_m}{m_{\text{eff}} L c \omega_m^2} \cdot \frac{\Lambda}{1 + (\frac{\Delta \omega}{\delta})^2}
\]

where \( P_c \) is the power of the laser circulating within the cavity, \( Q_1 \) and \( Q_m \) are the quality factors of higher order optical mode and the acoustic mode, \( m_{\text{eff}} \) is the effective mass of the mirror acoustic mode, \( L \) is the length of the cavity, \( c \) is the velocity of light, \( \omega_m \) is the frequency of the acoustic mode, \( \Delta \omega = |\omega_0 - \omega_h| - \omega_m \), \( \delta \) is the half-line width of the higher order mode and \( \Lambda \) is the spacial overlap factor between the optical and the acoustic mode.

With a reasonably good spatial overlap, parametric gain \( R \) will be maximum at \( \Delta \omega = |\omega_0 - \omega_h| - \omega_m = 0 \), i.e the frequency difference between the higher order optical mode and the fundamental mode becomes equal to the acoustic mode of the test mass. The frequency difference (mode gap) of the fundamental mode and the high order mode is given by

\[
|\omega_0 - \omega_h| = \omega_m
\]

\[
\omega_0 - \omega_h = \frac{c}{L} \cos^{-1} \sqrt{(1 - \frac{L}{R_1}) \cdot (1 - \frac{L}{R_2})}
\]

where \( R_1 \) and \( R_2 \) are the radius of curvature of the input test mass (ITM) and the end test mass (ETM) respectively, and \( L \) is the length of the cavity. It can be seen that mode gap can be tuned by changing the radius of curvature of the cavity mirrors as shown in eqn.(3). By heating the test mass mirror, we change the radius of curvature of the test mass as shown in fig.1. We could either tune the \( |\omega_0 - \omega_h| \) away from \( \omega_m \) to reduce parametric instability in the case of \( R > 1 \), or tune \( |\omega_0 - \omega_h| \) close to \( \omega_m \) to cool the test mass mode.

\[\text{Figure 1.} \quad \text{Fig.a Frequency spacing between the modes when the cavity is cold. Fig.b Thermal tuning of the ETM changes the radius of curvature of the mirror. This change in radius of curvature tune the spacing between the optical frequencies to match the acoustic mode frequency.}\]

We use a \( CO_2 \) laser to perform a localized heating to create a deformation on the surface of the test mass which changes the radius of curvature as shown in the figure 1.
2. Experimental setup for thermal tuning

Experimental setup is shown in figure 2. A stabilized CO$_2$ laser heats one of the test masses and creates a localized deformation. We use two diagnostic tools to measure the radius of curvature change of the heated test mass. They are (i) Hartmann sensor and (ii) a beam profile CCD camera. Table I shows the optical cavity parameters. Calculation using equation (2) shows that the frequency difference ($\Delta \omega$) between the fundamental mode TEM$_{00}$ and the first higher order optical mode TEM$_{01}$ is 1.298 MHz ($f = \frac{\Delta \omega}{2 \pi} = 206\text{kHz}$).

Using FEA simulation, an acoustic mode at 181.6 kHz is identified. This acoustic mode has a good spatial overlap with the first order optical mode. By thermal tuning the radius of curvature of the mirror we can adjust the frequency spacing between the optical modes to match the acoustic mode frequency as shown in figure 1(a) and figure 1(b). To tune the frequency spacing from 206 kHz to 181.6 kHz, a radius of curvature change of 1.5 km is needed. The heating power and spot size of the heating beam needed for thermal tuning is estimated using the following equation:

$$R_c = \frac{4 \cdot \pi \cdot k \cdot a^2}{2 \cdot \alpha \cdot P}$$  \hspace{1cm} (4)

where $R_c$ is the radius of curvature change of the heated test mass, $k$ is the thermal conductivity of the test mass, $a$ is the heating spot size, $\alpha$ is coefficient of thermal expansion of the material and $P$ is the laser heating power.
2.1. Hartmann sensor

Our Hartmann sensor [8] consists of a metal plate 16x16mm with uniformly distributed holes of 151µm in diameter and hole to hole spacing 429µm. The plate is kept at a distance of 10mm from the CCD pixel photodiode array. A super luminescent laser emitting diode (SLED) source beam is collimated before illuminating the test mass, and the reflected beam is demagnified and collimated to fall on the Hartmann sensor using a telescopic arrangement as shown in figure 2. The CCD read the position of the illuminated holes and compares these positions with any new position that has formed during the deformation. Data obtained from the pixels are processed and the localized deformation of the test mass due to $CO_2$ heating is measured. A typical front surface deformation during $CO_2$ heating measured by the Hartmann sensor is shown in figure 3.

2.2. Beam profile camera

A beam profile camera measures the leaking transmitted light from the locked Fabry-Perot cavity. The image on the camera is proportional to the spot size of the beam on the ETM.

![Hartmann sensor: Deformation of ETM at the place heated using a $CO_2$ laser. The spot size falling on the Hartman sensor is a de-magnified image of the mirror surface illuminated using a Hartmann source (HS) beam.](image)

**Figure 3.** Hartmann sensor: Deformation of ETM at the place heated using a $CO_2$ laser. The spot size falling on the Hartman sensor is a de-magnified image of the mirror surface illuminated using a Hartmann source (HS) beam.

![Transient radius of curvature change measured using CCD camera (lower curve). The upper curve is the transmitted power. Region a to b shows the mirror before heating, b denotes the instant when $CO_2$ laser is switched ON, c and g denotes the equilibrium state attained by the mirror during heating, d and h denotes the region when the $CO_2$ laser is switched OFF and e denotes when the cavity is unlocked.](image)

**Figure 4.** Transient radius of curvature change measured using CCD camera (lower curve). The upper curve is the transmitted power. Region a to b shows the mirror before heating, b denotes the instant when $CO_2$ laser is switched ON, c and g denotes the equilibrium state attained by the mirror during heating, d and h denotes the region when the $CO_2$ laser is switched OFF and e denotes when the cavity is unlocked.
Using the Spiricon software we measure the horizontal and vertical crosssection of the spot on the camera. Figure 4 shows the variation in the spot size as ETM is heated with the $CO_2$ laser.

3. Results

Figure 5(a) and figure 5(b) shows the spectrum of the acoustic mode of 181.6 kHz without and with $CO_2$ thermal tuning. A frequency gap of 181.6 kHz of the optical cavity, corresponds to a radius of curvature of 923m of the ETM. $CO_2$ laser heating of the ETM showed that at 650mW heating power, the acoustic thermal noise peak value is maximum as shown in figure 5(c). The value of heating power is consistence with the radius of curvature obtained from equation 4.

![Graphs](a) Frequency spectrum observed without heating (b) Thermal noise peak of 181.6 kHz is observed due to the 3 mode interaction, (c) Variation of thermal noise peak of 181.6 kHz with $CO_2$ heating power.

4. Conclusion

We have observed 3-mode interactions with controlled thermal tuning using a $CO_2$ laser. The experimental result is consistent with the theoretical calculation. The $CO_2$ laser thermal tuning provides an effective method for 3-mode interaction study and possibly for parametric instability control. This research is supported by the Australian Research Council and is part of the Australian Consortium for Gravitational Astronomy.

5. References

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