ABSTRACT. One important frontier for astronomical adaptive optics (AO) involves methods such as multi-object AO and multi-conjugate AO that have the potential to give a significantly larger field of view than conventional AO techniques. A second key emphasis over the next decade will be to push astronomical AO to visible wavelengths. We have conducted the first laboratory simulations of wide-field, laser guide star AO at visible wavelengths on a 10 m class telescope. These experiments, utilizing the UCO/Lick Observatory’s multi-object/laser tomographic adaptive optics (MOAO/LTAO) test bed, demonstrate new techniques in wave front sensing and control that are crucial to future on-sky MOAO systems. We (1) test and confirm the feasibility of highly accurate atmospheric tomography with laser guide stars, (2) demonstrate key innovations allowing open-loop operation of Shack–Hartmann wave front sensors (with errors of ~30 nm) as will be needed for MOAO, and (3) build a complete error budget model describing system performance. The AO system maintains a performance of 32.4% Strehl ratio on-axis, with 24.5% and 22.6% at 10″ and 15″, respectively, at a science wavelength of 710 nm (R-band) over the equivalent of 0.8 s of simulation. The mean ensquared energy on-axis in a 50 mas spaxel is 46%. The off-axis Strehl ratios are obtained at radial separations 2–3 times the isoplanatic angle of the atmosphere at 710 nm. The MOAO-corrected field of view is ~25 times larger in area than that limited by anisoplanatism at R-band. The error budget we assemble is composed almost entirely of terms verified through independent, empirical experiments, with minimal parameterization of theoretical models. We find that error terms arising from calibration inaccuracies and optical drift are comparable in magnitude to traditional terms like fitting error and tomographic error. This makes a strong case for implementing additional calibration facilities in future AO systems, including accelerometers on powered optics, three-dimensional turbulators, telescopes, and laser guide star simulators, and external calibration ports for deformable mirrors. These laboratory demonstrations add strong credibility to the implementation of on-sky demonstrators of laser tomographic adaptive optics (LTAO) on 5–10 m telescopes in the coming years.

Online material: color figures

1. INTRODUCTION

1.1. Progression of Adaptive Optics Technologies

The advent of laser guide star (LGS) adaptive optics (AO) on 8–10 m class telescopes has enabled a new regime of diffraction-limited science at near-infrared wavelengths. AOs use deformable mirrors to rapidly (~1 kHz) correct atmospheric-turbulence-induced phase errors. This process requires a bright, pointlike phase reference, a luxury that is quite rare among the stars in the sky. In the 1980s and 1990s, the USAF (US Air Force) Starfire Optical Range, the MIT Lincoln Laboratory, the Lawrence Livermore National Laboratory (LLNL), and the University of Arizona pioneered use of the sodium-layer LGS to increase the sky visibility of AO systems (Fugate et al. 1991; Humphreys et al. 1991; Lloyd-Hart et al. 1995; Max et al. 1997). In LGS AO, a laser is tuned to the sodium D2 line (589 nm) and focused on the atmosphere’s sodium layer at 90 km, creating an “artificial star” in the sky (Foy & Labeyrie 1985).

The LGS has since contributed greatly to high spatial resolution astronomy at infrared (IR) wavelengths. Most 8–10 m class telescopes now have an LGS-AO program, and many exist on smaller telescopes. The LGS facility at Gemini North has been routinely used since 2007 February (Trujillo et al. 2007) to study the stellar populations of nuclear clusters in nearby galaxies (Seth et al. 2008) and resolve distant quasar hosts.
(Watson et al. 2008), for example. The Very Large Telescope’s (VLT) LGS system has been used with a near-infrared imager and integral field unit (IFU) spectrograph since 2007 (Bonaccini Calia et al. 2006) to investigate the kinematics of high-redshift galaxies (Genzel et al. 2008) and resolve gravitationally lensed quasars (Sluse et al. 2008). The LGS-AO system at the Keck Observatory (Wizinowich et al. 2006) is extremely productive scientifically, having been used to directly image extrasolar planets (Marois et al. 2008), track the motion of stars orbiting the Milky Way Galaxy’s central black hole (Ghez et al. 2005), localize the progenitors of Type II supernovae (Gal-Yam et al. 2007), and investigate the binarity of brown dwarfs (Liu et al. 2006). Including such systems on all telescopes, 60 refereed journal articles based on LGS-AO imaging and spectroscopy have been published (Liu 2008).

1.2. Rationale for Visible- and Wide-Field AO

The success of the current generation of LGS-AO systems in terms of peer-reviewed scientific articles is encouragement to push AO technologies further.

Space-based facilities such as the superb 2.4 m Hubble Space Telescope (HST) routinely deliver field sizes of 2–4′ for optical/near-IR imaging with a stable point spread function (PSF). HST has been refurbished in 2009 with the Wide Field Camera 3 (WFC3) as well as other new instruments, giving images with full width at half-maximum (FWHM) between 35 and 150 mas from the optical to H-band. The James Webb Space Telescope (JWST) will deliver diffraction-limited imaging as well as IFU spectroscopy at wavelengths of 2 μm or longer with a 6.5 m aperture. Despite the unprecedented optical/IR capabilities of these facilities, the availability of even larger ground-based primary mirrors is an attractive reason to pursue diffraction-limited imaging from the ground. The B and V passbands, which will not be imaged by JWST, are especially critical.

Wide-field, laser-driven visible-light AO on 5–10 m telescopes will be an exciting new capability. An 8–10 m telescope with 30% Strehl ratio in V would be several times more sensitive to point sources in the visible than any telescope existing today. This sensitivity would enable deeper studies of extragalactic globular clusters, supernovae, and quasars at high redshift, as well as bright OB stars, OII regions, and red giants in galaxies as distant as 30–100 Mpc. LGSs will be critical to these systems to enable good sky coverage. The great potential of large-aperture visible-light AO has been explored in various science case studies for future AO systems, including the Keck Next Generation AO System (Max et al. 2008) and PALM-3000 for the Palomar telescope (Bouchez et al. 2008).

1.3. Multi-Conjugate and Ground-Layer AO

Multi-conjugate adaptive optics (MCAO) is a technique of increasing the field of view of AO correction by nearly an order of magnitude. MCAO requires the use of multiple deformable mirrors conjugated to different altitudinal layers. This produces an angle-dependent correction that produces a contiguously corrected field of view. Closed-loop MCAO has been successfully demonstrated in the laboratory and on an 8 m telescope (Marchetti et al. 2008). On-sky, the Multi-Conjugate Adaptive Optics Demonstrator (MAD) has achieved between 20% and 35% Strehl ratio in K-band across a 2′ field with three bright natural guide stars (NGSs). Typical MCAO systems using 2–3 deformable mirrors of order comparable to deformable mirrors in today’s single-conjugate AO systems on 8–10 m class telescopes are predicted to achieve similar Strehl ratios (40%–60% in K) over an ∼2′ field of view.

In ground-layer adaptive optics (GLAO), a single deformable mirror at the ground is controlled by averaging the signals from multiple wave front sensors (WFSs) pointing in widely separated directions (2–5′). This effectively only corrects the ground layer of turbulence, achieving improved seeing over a wide field. GLAO has been demonstrated on-sky with Rayleigh LGSs at the Multiple Mirror Telescope, yielding 0.33″ resolution in K-band (Baranec et al. 2009) and 0.15″ more recently. In typical GLAO designs, the expected resolution is 0.15″ – 0.3″ in median seeing for the K- through J-bands. Although GLAO systems do not reach the diffraction-limit below 2 μm, expansion to wider fields of view (>4′) is less expensive than in MCAO systems, for which complex deformable mirror relays must be optimized.

1.4. Laser Tomographic Adaptive Optics

The performance of all AO systems is limited by errors in wave front estimation and phase correction. The “cone effect” is one such error inherent to the current generation of LGS-AO systems. This error term is present because the cone of atmosphere sampled by an LGS does not overlap sufficiently with the cylinder of turbulence sampled by astronomical objects. The cone effect error term increases with aperture size, reaching ∼150 nm rms on a 10 m aperture, which precludes laser-driven, diffraction-limited performance at visible wavelengths on 8–10 telescopes if single laser beacons are used.

To address this, multiple LGSs are envisioned (see Fig. 1) to provide wave front sensing of the entire cylinder of turbulence.
leading to the science object (Ragazzoni et al. 1999; Viard et al. 2000; Tokovinin et al. 2001). In such an arrangement, the integrated LGS wave fronts can be analyzed tomographically (Gavel 2004; Baranec et al. 2006; Gilles & Ellerbroek 2008) to compute turbulence phase maps at various heights in the atmosphere. The deformable mirror correction can then be optimized along a single line of sight toward an object of interest through the three-dimensional atmosphere. AO systems that use laser tomographic adaptive optics (LTAO) and solve the cone effect may reach higher Strehl ratios on-axis than traditional LGS-AO systems of equivalent subaperture size, correction rate, and total laser brightness.

To optimize LTAO system performance on-axis, the LGS constellation size is best set to geometrically fill the cylinder of turbulence sampled by an astronomical object (see the left-hand side of Fig. 2). This arrangement minimizes the error in tomographic wave front estimation.

1.5. Multi-Object Adaptive Optics

Multi-object AO (MOAO) is a wide-field technique in which multiple scientific sensors, either spectrographs or small imagers, probe the large “field of regard” sensed tomographically by the LGSs (Hammer et al. 2004; Assémat et al. 2004). Each of these sensors has a separate internal deformable mirror, applying a correction optimized for the direction in which it is pointed. MOAO systems require multiple sensors and deformable mirrors to take advantage of the large field of regard sensed by the LGSs (typically 2′–5′).

Each of the MOAO “arms” utilize three-dimensional wave front sensing information from the LGSs, removing the cone effect and realizing the theoretically higher Strehl ratio of LTAO than traditional single-conjugate LGS-AO. In MOAO, the LGS constellation may be widened beyond the optimal LTAO radius to increase the field size. However, the error in tomographic wave front sensing increases with constellation radius, so the LTAO Strehl ratio represents the upper limit to the MOAO system performance. Figure 2 sketches the major differences between the high-Strehl ratio LTAO and MOAO system designs.

LTAO and MOAO are currently being considered as pathways toward laser-driven, wide-field, visible-light AO on large-aperture telescopes. These techniques must be tested with integrated test beds to ensure their feasibility.

1.6. Goals of This Article

We have implemented an integrated MOAO/LTAO laboratory instrument in the Laboratory for Adaptive Optics at the University of California, Santa Cruz (UCSC), and now rigorously test its performance at R-band. The goals of this article are to:

1. Demonstrate the technical feasibility of the LTAO and MOAO modes;
2. Show that WFSs and deformable mirrors with $64 \times 64$ degrees of freedom are of sufficient order to extend MOAO correction into visible wavelengths, in the limit of bright laser and tip/tilt guide stars;

![Figure 2](https://example.com/figure2.png)

**Fig. 2.—Simplified schematic diagrams of high-Strehl LTAO and MOAO architectures.** The left-hand side displays an LTAO design and the right-hand side shows a MOAO design. LGS WFSs are shown as gray cylinders; science instruments and integrated deformable mirrors are shown as boxes. Folding optics and other optical relays are omitted. If just one science instrument is used in LTAO mode (as shown), the LGS constellation size can be set to fill the cylinder of turbulence illuminated by an object at infinity, optimizing on-axis Strehl ratios. In MOAO, this constellation size is widened to increase the field of regard. The field area in-between guide stars can be populated with the MOAO arms assigned to individual astronomical objects. See the electronic edition of the PASP for a color version of this figure.

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3. Demonstrate open-loop wave front sensing with calibrated Shack–Hartmann WFSs; and
4. Construct a time-dependent error budget that models both the spatial and temporal performance of the system.

Section 2 describes the optical layout of the test bed and includes an explanation of a calibration that enables open-loop WFS operation. The performance of the AO system for a 45.0” diameter LGS constellation and an optical science wavelength (710 nm) is presented in § 3. Section 4 assembles an error budget modeling total system performance, which is largely composed of empirically measured terms with some parameterization of theoretical dependencies. Section 5 concludes and discusses implications for MOAO/LTAO systems now being designed.

2. UCO/LICK MOAO/LTAO TEST BED

2.1. Summary

The UCO/Lick MOAO/LTAO test bed is an integrated laboratory simulator of a wide-field AO system on a 10–30 m telescope, with optical layout shown in Figure 3. The optical design of the test bed is summarized here and presented in more detail in §§ 2.2–2.9. Further details of the optical layout are also presented in earlier publications (Gavel 2006; Ammons et al. 2006, 2008).

The system has five star-oriented Shack–Hartmann WFSs pointing in the direction of five LGSs. These WFSs are set to 66 × 66 subapertures to enable high-order compensation. The design of the WFSs and a description of their open-loop performance are presented in §§ 2.2 and 2.3. The five LGSs are arranged in a box-five, or quincunx, constellation at an equivalent altitude of 90 km with a guide star diameter of 45.0”. Sections 2.4 and 2.5 describe the LGS constellation and its mechanical elongation. The atmospheric section above the telescope aperture contains 3–5 Galil motors for simple translation of atmospheric phase plates. The phase plates are glass, acrylic, or plastic slides with Kolmogorov turbulence etched into or sprayed onto them. Further details on the phase plates are in § 2.6. The test bed has three deformable mirrors, conjugated to different altitudinal layers. An ALPAO DM-52 woofer deformable mirror is conjugated to the ground layer, as is one Hamamatsu programmable phase modulator (PPM). A second PPM is conjugated to 9 km altitude. The deformable mirrors are described in § 2.8.

The optical layout and performance of the MOAO/LTAO test bed are intended to be analogous to a wide-field LGS AO system on a 10 m telescope. This is accomplished by matching WFS type, subaperture number, the geometry of atmospheric metapupils, and dimensionless atmospheric strength to the values expected for such an on-sky system. This matching of “similarity parameters” is discussed in § 2.10.

2.2. Multiplexed Wave Front Sensors

The Shack–Hartmann WFS pupils are multiplexed onto two Dalsa cameras, allowing for up to eight LGSs with up to 100 × 100 subapertures per pupil, suitable for simulating MOAO on a 30 m telescope. The WFS multiplexing, designed by B. Bauman, is accomplished with two pupil-steering quadrangles, with four off-axis sections of lenses glued together in each quad. This structure images each LGS pupil onto a

![Diagram of MOAO/LTAO test bed](image-url)

Fig. 3.—Optical layout for MOAO/LTAO test bed. The test bed is divided into four large sections by broad functionality: an atmospheric section, a deformable mirror section, a WFS array, and a quadrature-polarization truth interferometer. Five LGSs at 90 km equivalent altitude and seven science stars at infinity illuminate an atmospheric section with three translating atmospheric phase plates. The light then passes through three ports where deformable mirrors can be conjugated to different altitudinal heights. After exiting the deformable mirror section, the light is sent into one of several multiplexed WFSs, which can detect up to nine LGSs. The science light is sent to a far-field camera for Strehl ratio checking. The QPI is used to check the measurements of the Shack–Hartmann WFSs as well as to measure the total magnitude of super-Nyquist frequencies.
particular quadrant of the camera, regardless of the field position of the LGS. Because of features on the cut edges of the quad-lenses, the LGSs cannot be placed closer than 10.0° from each other. The multiplexing design saves camera space and has been shown to be feasible on other on-sky systems (Visible Light Laser Guide Star Experimental System [ViLLaGES], e.g., Gavel et al. 2008)

Each Shack–Hartmann subaperture has $4 \times 4$ pixels for centroiding. A Vitrum lenslet array with $f = 6.8$ mm is used to break up the pupil beam into subapertures, giving a Hartmann spot FWHM of 0.61 pixels. This small spot size is chosen to minimize spot interference, but Hartmann linearity problems become significant, as described in § 2.3.

2.3. Open-Loop Performance of Wave Front Sensors

For some architectures of laser tomographic AO systems, in which the wave front sensing unit is not placed downstream of the deformable mirror(s), the WFSs are required to operate in open loop. Open-loop WFS operation is particularly critical to MOAO systems. They must sense the phase in a particular direction to high accuracy with high dynamic range, which are not properties typically endemic to Shack–Hartmann WFSs. The low dynamic range of the Shack–Hartmann WFS is caused by nonlinearities inherent to centroiding unresolved structures with finite windows. This effect is exacerbated in systems with small Hartmann spot sizes, especially less than 1 pixel in FWHM. The various difficulties associated with open-loop wave front sensing are described in more detail in Ammons et al. (2007).

The Shack–Hartmann WFSs have been empirically calibrated to enable open-loop wave front sensing. Uncalibrated Shack–Hartmann sensors are subject to bias due to the effects of WFS nonlinearity, truncation error, nonuniform pupil illumination, and nonpupil beam vignetting. Our method of accounting for these effects and estimating true, unbiased atmospheric tilts is called a linearity calibration.

To perform this calibration, we raster scan the LGS constellation in the plane of the sky with all optics and focal plane blocks in place. The side length of the scan grid is one Hartmann spot FWHM, or 0.6 pixels. All center-of-mass centroids for each Hartmann spot in each WFS are recorded to form two-dimensional matrices of $x$-centroids and $y$-centroids for each grid position. The data are searched for glitches, or subapertures in which the measured spot range during the calibration is significantly less than the median spot range of the other subapertures in the pupil. The slope values in glitch subapertures are interpolated from nearest neighbors during normal WFS operation. Following the raster scan, the data are spline interpolated to increase the matrix sampling from 0.15 pixels (the raster scan interval) to 0.015 pixels. The matrices that relate $x$-centroid, $y$-centroid, and raster scan position are inverted to create lookup tables for each subaperture that return interpolated scan position when measured centroids are input. In this way, we have converted the nonlinear, biased centroid measurements for all subapertures into unbiased tip/tilt coordinates in the pupil plane.

The open-loop accuracy of the calibrated WFS is estimated by comparing the five wave front measurements of a common aberration placed at the ground layer. Each measured wave front is differenced with respect to the average of all wave fronts, and the rms of these residuals is averaged for all wave fronts to derive the open-loop error on the sensors. Without the calibration, this comparison gives $>100$ nm of wave front disagreement over $66 \times 66$ subapertures on a ground-layer atmosphere with 600 nm rms of piston/tip/tilt-removed aberration. With the linearity calibration, this comparison gives 25–30 nm rms disagreement. This open-loop error is found to monotonically increase with the total amplitude of atmosphere introduced. For the visible-light AO experiments presented in this article, this systematic error averages 39 nm. These errors reflect the open-loop disagreement of the sensors; this method does not estimate global wave front estimation error (common to all five WFSs) if it depends on the atmospheric phase in a quasi-static manner. Any common wave front estimation error that does not depend on the atmospheric phase is a static error, which is removed by image sharpening. Independent interferometric measurement of the wave front using a quadrature polarization interferometer (QPI) has confirmed that the open-loop error estimated from WFS disagreement is equivalent to the total open-loop error, setting the common, quasi-static component of the estimation error to less than 10 nm rms.

The linearity calibration procedure has no direct analog on a telescope-mounted LTAO system, as no internal calibrator could reproduce the size and shape of the real LGS spots in real time. However, it may be possible to dither the actual LGS constellation during real-time operation.

2.4. Guide and Science Stars

High-power light-emitting diodes (LEDs) at $\lambda = 650$ nm are used for LGSs, at an equivalent altitude of 90 km. These LEDs have a nonzero spectral width of $\sim 10–20$ nm, which decreases coherent interference between guide stars. We use five LEDs in a quincunx pattern of 22.5° radius. Seven science test stars are distributed within the LGS constellation, with off-axis angles up to 15.0°. The light from these natural test stars is collimated at the telescope pupil. The constellation is shown in Figure 4. The wavelength of the science stars is 658 nm. The error due to the difference in central wavelengths between the wave front sensing stars and the science test stars is less than 5 nm rms. In addition, the multispectral error due to the LED’s nonzero spectral width (arising from nonlinear dependencies between effective PPM phase and wavelength) is less than 2 nm rms.

This AO system has multiple science stars so that the Strehl ratio can be checked in different directions. A conventional MOAO system design has multiple sensors and multiple deformable mirrors to realize the highest Strehl ratios across the field of regard simultaneously. In this test bed, however,
there is only one ground-layer PPM deformable mirror. We simulate conventional MOAO by checking Strehl ratios on different science stars while the atmosphere is “frozen” during a single AO iteration. This involves optimizing the correction for seven different field points and applying seven different corrections with the ground-layer PPM during an AO iteration. Because the atmosphere is motionless during this process, this simulates simultaneous correction with multiple deformable mirrors in different directions. A two-phase paddle blocks either the LGSs during Strehl ratio measurement or the science stars during wave front sensing.

2.5. LGS Elongation

The LGS constellation is mechanically elongated by physically moving the LGS constellation by \( \sim 120 \) mm. Because geometric angles are magnified with respect to on-sky angles, this relatively large shift in height corresponds to an elongation of 0.6 Hartmann pixels (or one spot size of \( 2.0'' \)). A full-aperture image of the elongated spots is shown in Figure 5; notice that the spots are defocused at extrema, creating a characteristic peanut shape for side guide stars. The principal effect of elongation in on-sky systems is to reduce centroiding accuracy and increase measurement error. However, because this bench uses bright guide stars, the measurement error is largely not affected. The elongation increases the WFS systematic error by a small amount (9.7 nm rms, Ammons et al. 2007). As the effect of elongation is slight, we do not perform this step for the experiments presented in this article.

2.6. Atmospheric Phase Plates

We use four different types of phase plates: etched glass, acrylic-coated glass, CD cover, and hairspray-coated glass. All types have Kolmogorov power spectra extending to the spatial resolution limit of our test interferometer (5 pixels per Hartmann subaperture). The CD covers tend to have more non-Kolmogorov structure, including excesses of astigmatism and coma, but they have the advantage of reaching higher total rms variation in a 2 cm metapupil (0.5–1.0 \( \mu m \) rms) than the other types. They also display less amplitude variation than the other types. The etched glass plates were manufactured at LLNL and have pure Kolmogorov structure. However, these plates possess an excess of high spatial frequencies that introduce WFS measurement error. For the weaker plates, this error is small, as is the amplitude variation across the plate, so we use these weaker plates to simulate high altitude layers. We distribute the atmosphere in three layers (0, 4.5, and 9 km) with strengths that mimic Mauna Kea-type \( C_2^a \) atmospheric profiles (the exact distribution is presented in § 3).

The acrylic- and hairspray-coated plates were manufactured in the Laboratory for Adaptive Optics at UCSC. These plates largely possess Kolmogorov power spectra, but bubble-like features are frequent. These features are manifest as regions of low intensity in the WFSs, which introduces measurement error. These non-Kolmogorov bubbles are easily picked out and avoided.

The optimum plates for the ground layer are CD covers, as they are the only type with sufficient phase amplitude to model dominant layers. For layers at high equivalent altitude, it is important that total rms phase error be low (<150 nm) to reduce scintillation error terms. Total intensity variation must be low.

![Fig. 4.—LGS and science star constellation for MOAO experiments. Blue circles denote the positions of LGSs; diamonds denote the positions of science test stars. See the electronic edition of the PASP for a color version of this figure.](image1)

![Fig. 5.—Full-aperture images of LGSs during mechanical elongation. The left-hand panel is in log scale, and the inset is in linear scale. The principal effect of axial translation is defocusing rather than lateral motion for side guide stars (compared to on-sky LGS systems), as the geometric angles in this test bed are magnified compared to the angular dimensions of a 10 m telescope. This creates a characteristic peanut shape for side guide stars.](image2)
for all layers, as the Hartmann sensor is sensitive to intensity dropouts. Power spectra of phase for the various plates used are shown in previous publications (Ammons et al. 2007).

2.7. Insertable Field Stops

The high-order deformable mirrors (PPMs) produce a grid of diffraction spots due to their pixelated nature. The intensity in these diffraction peaks varies with the particular PPM used and its displayed phase shape. Unfortunately, the physical separation between the zeroth order and the first order is equivalent to 45.0°, approximately the same as the LGS separation. We rotate the LGS constellation 22° from the proper quincunx pattern to minimize the interference between the LGS diffraction spots and the PPM rotation axis. If these diffraction spots are allowed to propagate to the WFS, the interference creates biases in Shack–Hartmann spot centroiding. Powered PPMs increase diffraction intensity and cause higher residual error during closed-loop tests. In order to block the majority of these diffraction spots, we insert field stops with five holes of 1.6 mm diameter to pass the five LGSs during the wave front sensing. Although open-loop operation is less affected by this diffraction error, the masks are inserted in the interest of reducing all possible wave front errors. The focal plane block holes are several times larger than the spatial filters used to improve wave front sensing in other systems; for example, the Gemini Planet Imager (Macintosh et al. 2006) uses small spatial filters to eliminate high spatial frequencies that contribute to aliasing errors.

2.8. Deformable Mirrors and Phase Wrapping

Three deformable mirrors are present in this system: a ground-layer woofer (ALPAO DM-52), a Hamamatsu PPM at the ground-layer, and another PPM device at an equivalent altitude of 9 km. The PPMs have very high spatial resolution, with 768 × 768 independent controllable pixels across the screen. An approximately 5 × 5 region on the PPM maps to one Shack–Hartmann subaperture. The ground-conjugated woofer–tweeter pair and the second PPM at higher altitude can be used for closed-loop MCAO experiments, the results of which have been previously published (Laag et al. 2008).

We now use the system for MOAO experiments by flattening the ground-layer PPM and woofer during wave front sensing, so as to sense the atmosphere wholly in open loop. Both the PPM and woofer are highly repeatable on short timescales, so very little error is introduced by performing MOAO in this manner (<5 nm rms). For MOAO, the 9 km PPM is powered down at all times. The ground-layer PPM is the only deformable mirror that corrects introduced atmospheric turbulence; the woofer is only used to correct instrumental errors, as described subsequently.

The influence functions of the PPM pixels are much smaller in width than our Hartmann subapertures, so the transformation of desired phase into pixel commands is more direct than with traditional deformable mirrors. The desired phase is simply interpolated onto the PPM pixel map. The open-loop response of the PPMs to voltage signals has been calibrated across its full range with a QPI. These signals have been stored for each pixel, inverted, interpolated, and used to form a lookup table. The QPI measurement noise of ∼30 nm rms propogates into the PPM lookup tables; thus the PPMs can be used in open loop with a go-to error of ∼30 nm.

The PPMs also have a limited stroke of 600–900 nm. In order to create shapes with typical atmospheric amplitudes, we wrap the phase of the PPMs, typically 8–15 times in a pupil. This phase wrapping is essentially the same as performing a modulus operation on the desired phase map. This is optically permissible for two reasons: (1) the spectral width of the LGSs and science test stars is negligibly small, so the 2π phase jumps appear to be smooth transitions; and (2) the phase transitions are sharp because the PPM influence functions are narrow. As a result, this phase wrapping does not affect PSF formation.

The woofer mirror is used primarily in these experiments to introduce a field-dependent static correction, which does not change during the course of experimentation. It is not used to assist the ground-layer PPM in correcting the atmospheric turbulence. Each field point has a separate static correction wave front, which is only placed on the woofer when the Strehl ratio is checked on that star. The 9 km PPM is not used for static correction but is left unpowered. During wave front sensing, both the woofer and ground-layer PPM are flattened.

We store a field-dependent static correction, or low-order pattern, that cancels the astigmatism introduced by a 3 inch beam splitter and other misaligned optics within the system. The static correction is generated by a combination of two techniques: automated image sharpening of low-order modes, up to fourth Zernike order, and image sharpening via dithering of individual woofer actuators. The modal sharpener fits the science PSF to a perfect Airy model to estimate the merit of correction. The dithering sharpener attempts to maximize the Strehl ratio itself. We also use the QPI, coupled to the MOAO/LTAO system, to measure and cancel high-order aberration caused by the PPMs themselves. The combination of these methods gives Strehl ratios of 83% across the science field, when optimized for individual stars.

2.9. Software and Control

The control algorithms for this test bench are discussed in more detail in prior publications (Ammons et al. 2006, 2007; Laag et al. 2008) and summarized here. No part of the reconstruction or control is matrix-based. We use simple center-of-mass centroiding over 4 × 4 pixel subapertures, followed by a slope linearization lookup (see § 2.3). The slopes are reconstructed into phase using Fourier techniques (Poynier & Macintosh 2003). This reconstruction technique is performed iteratively to reduce edge effects. Mean piston and tip/tilt are then removed from the five measured pupils. The edge phase values are smeared several subapertures beyond the edge of the pupil to...
further reduce edge errors. The pupils are dewarped and registered before being analyzed tomographically. The tomographic analysis code is Tomography Spherical Wave (TSW), which applies a minimum variance preconditioned conjugate gradient (PCG) solver to the problem of back-projection tomography (Gavel 2004). We use alternating sets of 5 PCG and fixed iterations, for a total of 35 iterations, to provide the tightest convergence. The number and strength of “software” atmospheric layers in TSW correctly models the simulated atmosphere for all realizations.

Following the tomographic procedure, the optimized phase corrections are placed on the deformable mirror. For each science star, a different shape is placed on the deformable mirror to correct turbulence and field-dependent static errors. After checking Strehl ratios on all stars independently, the wind motors are driven and the next AO iteration begins.

2.10. Similarity Parameters

2.10.1. Geometric Scaling

The problem of constructing a test bed that simulates execution of wide-field AO on a 10 m telescope begins with several geometric constraints. High-order deformable mirrors with good go-to performance are commercially available with sizes of order 1–2 cm, restricting the simulated telescope aperture to nearly this size when large magnification factors are avoided. In addition, atmospheric turbulence plates are readily available with Fried parameters of 200–500 μm at optical wavelengths, permitting $D/r_0$ scales of 30–60 with a 1–2 cm pupil, suitable for investigating visible-light AO. However, the simulated atmospheric space must represent ~15 equivalent kilometers to fully simulate three-dimensional atmospheric profiles. Matching the transverse/longitudinal aspect ratio to that of a real 10 m telescope would require this space to be on the order of 15 m, occupying lab space unnecessarily. The MOAO/LTAO test bed compresses the height dimension in the atmosphere by a factor of 60 to address this. The height scale, LGS constellation mount, and atmospheric plate locations are set to mimic the beam footprints of a true LGS constellation at the relevant atmospheric heights.

Scintillation error effects, due to Fresnel propagation of phase errors at different atmospheric heights into amplitude variation, are worsened for scaled-down test beds. This is because the ratio of the size of the entrance pupil to the wavelength of light is much smaller than on real telescopes. As seen in the scintillation index equation in § 4.3.2, further compressing of height scales mitigates this effect.

For simulation of wide-field AO, it is most important that the following geometric characteristics of the system are analogous to those of a real telescope: (1) WFS geometry and subaperture number, (2) atmospheric strength and distribution across equivalent atmospheric heights, (3) deformable mirror actuator size and deformable mirror-lenslet mapping, (4) LGS and science star metapupils for all atmospheric layers, and (5) LGS spot size and range of motion as seen by Shack–Hartmann subaperatures. For the MOAO/LTAO system, these characteristics match a hypothetical wide-field AO system on a 10 m telescope with circular aperture, except that the LGS size is 0.61 pixels rather than ~1 pixel, which only affects Hartmann linearity.

The critical system parameters, in both physical laboratory units and on-sky equivalent units, are presented in Table 1.

2.10.2. Wavelength Scaling

We quantify AO performance at the laser wavelength used for the science stars, 658 nm. The performance (i.e., Strehl ratio) of the AO system is largely set by the atmospheric strength, specified by $D/r_0$, with $r_0$ given at the science wavelength. In turn, $r_0$ is a function of the simulated seeing strength ($r_0$ at 500 nm, or $r_{0.500}$) and the simulated science wavelength ($\lambda$). We simulate the AO performance at science wavelengths other than 658 nm by choosing a $D/r_0$ that corresponds to median seeing at a different wavelength. The following equation models the dependency of total atmospheric strength on $D/r_0$ (Hardy 1998):

$$\sigma = 0.366(D/r_0)^{5/6},$$

where $\sigma$ is the tip/tilt-removed rms deviation of phase in units of $\lambda$. $D/r_0$ is dependent on the seeing strength in the following way (Hardy 1998):

$$r_0 = r_{0.500}(\lambda/500 \text{ nm})^{6/5}.$$

Then

$$\sigma = 0.366(500 \text{ nm}/\lambda)(D/r_{0.500})^{5/6}.$$

With a 10 m telescope, a $\sigma$ of 9 radians (950 nm) rms corresponds to $r_0 = 21.4$ cm, simulating median seeing at optical red wavelengths ($r_{0.500} = 15.4$ cm for $\lambda = 658$ nm) or poor seeing at IR

| Parameter                      | Atmosphere | Lab          |
|--------------------------------|------------|--------------|
| Transverse magnification       | 1          | 0.00183      |
| Longitudinal magnification     | 1          | 1.67 x $10^{-5}$ |
| Atmospheric path length        | 9 km       | 150 mm       |
| Aperture, $D$                  | 10 m       | 18.3 mm      |
| Field diameter                 | 42.5"      | 42.5"        |
| Subaperture size, $d$          | 15.2 cm    | 278 μm       |
| Mean $r_0$ at 0.5 μm           | 16.2 cm    | 296 μm       |
| Mean $r_0$ at 0.5 μm           | 3.17"      | 3.17"        |
| Science wavelength             | 0.71 μm    | 0.658 μm     |
| $D/r_0$ at science $\lambda$   | 41.41      | 41.41        |
| Mean rms atmospheric strength  | 921 nm     | 854 nm       |
wavelengths \( r_{0,500} = 9.3 \text{ cm for } \lambda = 1.0 \mu \text{m} \). There are some inaccuracies inherent to this model. Most of the crucial error budget terms scale linearly with wavelength, but several do not, including the static uncorrectable errors (\( \sim 45 \text{ nm for the system} \)). As a result, simulations at wavelengths bluer than 658 nm overpredict the Strehl ratio and simulations at wavelengths redder than 658 nm underpredict. Because of these inaccuracies, we choose a science wavelength as close to 658 nm as possible while still preserving system performance. Setting the total amplitude of phase error to \( \sim 850 \text{ nm rms in turn sets the science wavelength to 710 nm for median seeing} (r_{0,500nm} = 16.2 \text{ cm}), permitting R-band tests.

2.10.3. Temporal Scaling

All physical processes that govern the AO performance, if they are controllable, can be slowed (or stopped and restarted) during operation without affecting performance. Real computation time can be freely extended without degrading performance as long as the atmosphere stays frozen during operation. In addition, we eliminate the delay errors present on-sky AO systems by keeping the atmosphere frozen during the wave front sensing, computation and deformable mirror actuation steps. Each AO iteration uses 8 s in real time to measure WFS signals, perform reconstruction and tomographic analysis, and apply deformable mirror shapes. Measuring Strehl ratios uses an additional 40 s, as different deformable mirror shapes must be applied and new science images taken for each of seven science stars. The length of equivalent time that each AO iteration simulates is set only by the physical speeds of the atmospheric plates (in microns per iteration) and the desired simulated wind speed (in meters per second). As discussed subsequently, we choose plate speeds that simulate AO operation at 1 kHz.

3. PERFORMANCE AT VISIBLE WAVELENGTHS

3.1. Experimental Makeup

Previous work with this test bed has demonstrated that open-loop operation decouples instantaneous AO performance from preceding AO iterations (Ammons et al. 2008). The only dependence of performance on simulation time point arises from a gradual degradation of Hartmann references and open-loop WFS calibration accuracy with time, caused by internal air movement and optical drift induced by slow temperature relaxation. These dependencies introduce small error terms, which we explicitly model in § 4.3.

In this section, we present experimental results from four individual manifestations of the atmosphere, with no continuous transitional evolution between the four states. This approach maximizes the atmospheric variation in the experiment. For each realization, we evolve the atmosphere with a variable Taylor frozen-flow model over 200 equivalent milliseconds. Although the wind speeds are set to simulate operation at 1 kHz, the instantaneous performance is independent of the Hartmann frame rate, unlike real AO systems on telescopes. This is because the bandwidth and delay error terms are not included in this simulation. We perform AO iterations every 5 ms, giving a record of atmospheric statistics and the performance of the tomographic algorithm at a 200 Hz rate; we check the Strehl ratio on the science stars every 10 ms to give AO performance data at a 100 Hz sampling rate. The trends in AO performance measured would be unchanged, only more densely sampled, if we performed AO iterations every 1 ms.

Overall, this amounts to nearly 1 full second of simulation data (800 ms) comprising 4 atmospheric realizations, 160 AO iterations, and 80 Strehl ratio measurements. This models the common practice of recording on-sky telemetry on random nights throughout a year of operation, as the phase in different atmospheric realizations is distinct.

3.2. System Performance

The atmospheric parameters and performance for the four realizations, including R-band Strehl ratios, ensquared energies, and wind speeds, are listed in Table 2. Over the course of these experiments, the system attains mean R-band Strehl ratios of 32.4% on-axis, 24.5% at a distance 10.0" off-axis, and 22.6% at a distance of 15.0" off-axis. The mean ensquared energy for 50 mas spaxels on-axis is 46%. The distributions of \( r_0 \), \( \tau_0 \), and Strehl ratios for all stars are shown in Figure 6. Notice that the Strehl ratios quickly rise to equilibrium values after the start of the simulation; this is a natural feature of open-loop AO operation.

Time-averaged PSFs are shown for three off-axis distances in Figure 7. The image FWHM’s range between 0.02" (the diffraction limit at this wavelength) and 0.03". Using the seven science
test stars, it is possible to map the nominal Strehl ratio as a function of position in the MOAO field and compare it to a normal single-conjugate AO system at this wavelength. Figure 8 displays MOAO and anisoplanatic Strehl maps. The MOAO Strehl map is obtained by interpolating through the mean Strehl ratios obtained in the experiments discussed previously. The map of anisoplanatic Strehl ratios is obtained in the following manner: a three-dimensional atmosphere is set up, as for MOAO, and a full tomographic wave front sensing analysis is performed. Line integrals are performed through the

![Figure 6](image_url)

**Fig. 6.—**$R$-band Strehl ratios and atmospheric parameters versus simulation time. Strehl ratios are shown in the four upper panels, and $r_0$ and $\theta_0$ are plotted in the lower panels. The $y$-axes units for $r_0$ are shown on the left-hand side of the plot, and the units for $\theta_0$ (in arcseconds) are shown on the right-hand side. Data for the four atmospheric realizations are presented left-to-right. The solid lines in the top plots denote on-axis Strehl ratios; dashed lines denote off-axis Strehl ratios (10.0″–15.0″). The solid lines in the lower panels plot the Fried parameter $r_0$ and the dashed lines plot the anisoplanatic angle $\theta_0$.

![Figure 7](image_url)

**Fig. 7.—**Inverted logarithmic images of mean PSFs at three off-axis distances. The left-hand panels show the PSF on-axis, the middle panels show the PSFs at 10.0″ off-axis, and the right-hand panels display the PSFs at 15.0″ off-axis. The top panels have image sizes of 0.45″ × 0.45″. The bottom panels have image sizes of 0.11″ × 0.11″. $R$-band Strehl ratios are displayed in the top panels.
reconstructed atmosphere for a large number of field points surrounding the central science star. When these integrated wave fronts are placed on the ground-layer deformable mirror, the Strehl ratio on the central star is checked and mapped as a function of the field point. This approximates the anisoplanatism at the chosen science wavelength (710 nm). The Strehl distribution (shown in the right-hand panel of Fig. 8) is not radially symmetric because it is not time averaged over multiple atmospheric realizations, but it is a useful indication of the magnitude of anisoplanatism. If an R-band Strehl cutoff of 15% is chosen to signify usable imagery, then the MOAO field of regard has been expanded to 0.5 square arcminutes, or ~25 times larger than that limited by anisoplanatism.

4. ERROR BUDGET

4.1. Summary

In this section we present a time-dependent error budget, comprising 13 separate static and parameterized error models, which we use to predict the spatial and temporal performance of the AO system. Each of the error terms has been verified with independent, empirical measurements, or justified with separate numerical simulations. The error terms and models are discussed in § 4.3. The full error budget is presented in Table 3, including on-axis cases in both laboratory and on-sky units and an off-axis case in on-sky units.

This test bed studies the idealized performance of an MOAO/LTAO system, with bright tip/tilt loops and no delay or bandwidth error. However, it is subject to a large number of static and calibration errors discovered in existing AO systems, including:

![MOAO Strehl and Strehl with Anisoplanatism](image)

**Fig. 8.—MOAO R-band Strehl map and anisoplanatic Strehl map.** The MOAO R-band Strehl map is an interpolation to the mean results from the four atmospheric realizations presented here. The stars indicate the locations of LGSs. The anisoplanatic Strehl map is obtained as described in the text for only one atmospheric realization. See the electronic edition of the PASP for a color version of this figure.

| Error Budget Term                  | On-Sky, Off-Axis | Lab, Off-Axis (12.5") | Lab, Off-Axis (658 nm) |
|-----------------------------------|------------------|------------------------|------------------------|
| Fitting error                     | 40.7             | 37.7                   | 37.7                   |
| WFS aliasing                      | 16.2             | 15.0                   | 15.0                   |
| Tomography error                  | 69.0             | 77.6                   | 63.9                   |
| WFS systematic error              | 41.5             | 38.5                   | 38.5                   |
| Field stop misalignment           | 10.0             | 10.0                   | 10.0                   |
| PPM lookup table error            | 30.0             | 30.0                   | 30.0                   |
| Static uncorrectable, S = 83%    | 48.6             | 45.0                   | 45.0                   |
| Scintillation                     | 7.1              | 11.7                   | 11.7                   |
| WFS scintillation                 | 10.0             | 24.8                   | 24.8                   |
| Photon error                      | 16.2             | 15.0                   | 15.0                   |
| WFS zero-point drift              | 10.8             | 10.0                   | 10.0                   |
| Linearity calibration drift       | 10.8             | 10.0                   | 10.0                   |
| Pupil registration drift          | 25.9             | 24.8                   | 24.8                   |
| Bright star tip/tilt residual     | 41.0             | –                      | –                      |
| Bandwidth/servo lag               | 47.0             | –                      | –                      |
| Multispectral error               | 5.0              | –                      | –                      |
| Asterism deformation              | 19.0             | –                      | –                      |
| Residual Na focus change          | 31.0             | –                      | –                      |
| Differential atmospheric refraction | 3.0             | –                      | –                      |
| Total rms                         | 131.6            | 127.3                  | 109.5                  |
| Predicted Strehl ratio (%)        | 25.7             | 28.1                   | 33.5                   |
| Measured Strehl ratio (%)         | –                | 23.6                   | 32.4                   |
| Relative error in model           | 16.0%            | –                      | 3.3%                   |

Each term represents the mean over all 160 AO iterations over 4 atmospheric realizations. All values are in units of nanometers, unless otherwise stated. The left-hand column gives error budget values in on-sky units on-axis. The middle column gives values in physical, laboratory units (unstretched, using $\lambda = 658$ nm) at an off-axis distance of 12.5°. The right-hand column displays error budget values in laboratory units for the on-axis case.
1. Static and dynamic uncorrectable modes (largely introduced by the PPM deformable mirrors),
2. Static and dynamic WFS zero-point calibration error,
3. Go-to control errors (the open-loop control error measured for the PPM is set by the measurement accuracy of the QPI, used to calibrate it),
4. A particular manifestation of deformable mirror finite stroke errors (errors due to insufficiency of PPM wrapping to completely describe a unwrapped wave front, due to the spatial smoothing of the phase on the PPM),
5. Deformable mirror-to-lenslet warping and misregistration error.

Using the known atmospheric strengths at every time point in the MOAO experiments, it is possible to predict the Strehl ratio as a function of time and space from this error budget. These predictions are shown in Figure 9 for the four realizations. These Strehl plots should be compared against the actual Strehl ratios measured. Figure 10 compares the predicted Strehl ratios against the measured Strehl ratios as a function of atmospheric realization and field angle, integrated over simulated time. Averaged over all realizations, the on-axis predictions match the measured Strehl ratios to 3%, but our error budget model overpredicts the off-axis Strehl ratio by \( \sim 16\% \). This is likely due to propagation of WFS systematic errors through the tomographic algorithm, which become manifest as rings of error at the edges of the upper-altitude metapupils. This propagation was not modeled in the error budget.

4.2. Errors Not Modeled

This test bench was built to investigate the idealized performance of LTAO and similar technologies. The photon error in the system (\( \sim 15\) nm rms per pupil) models 30 W sodium-layer LGSs sharpened to 0.8" and a low read-noise CCD. We are not subject to bandwidth or delay error because (1) the system does not pass or record science light between AO iterations and (2) there is no time delay between wave front measurement and correction.

Tip/tilt errors cannot be measured by LGSs because the emitted photons retrace their own upward path through the atmosphere. Multi-LGS systems require multiple NGSs to constrain global tip/tilt as well as quadratic modes unsensed by the LGSs. Because we are free to take individual science frames every AO iteration, we time average the science PSFs by shifting-and-adding individual images to remove tip/tilt errors explicitly. This models a case with a bright tip/tilt star, for which there is little residual tip/tilt jitter. All Strehl ratios quoted in this article are measured from instantaneous (not time-integrated) PSFs, although the time-averaged PSFs are displayed in Figure 6. The shifting-and-adding sufficiently models the global tip/tilt loop in the bright star case, but the test bed is still subject to coupling of low-order, high altitude quadratic modes into guide star tip/tilt. This introduces an error term in the overall error budget, which is included in the tomographic error term. It is envisioned that tip/tilt measurements from multiple NGSs will address this error term, as modeled in the split tomography algorithm of Gilles & Ellerbroek (2008), for example.

4.3. Individual Error Models

We now present the details of the time- and space-dependent error model used to predict the Strehl ratios in Figure 10. The mean values of each of these terms are shown in Table 3. We stress that the majority of these terms have been measured with
independent, empirical methods. We now group these error terms by their dependence on atmospheric strength (and therefore time) or field radius.

### 4.3.1. Static Terms

Nearly half of the error budget terms are static, meaning that they have no time- or space-dependence. The first of these is measurement error due to photon counting noise. The static value of $\sim 15$ nm is determined by measuring the rms disagreement between measured wave fronts from iteration to iteration. This value is quite low compared to current on-sky systems but similar to that expected in the modeled case of bright LGSs ($\sim 30$ W) sharpened by low-order uplink correction to the Hartmann pixel size ($0.8''$).

The second static term is the PPM lookup table error. This error reflects the measurement error of the QPI, used to characterize the PPM deformable mirrors. Because this error is manifest over one full wave of range, and we wrap phase by $2\pi$ to achieve greater stroke, it is not proportional to atmospheric strength. The third static term is due to uncorrectable modes in the AO system. The image sharpening techniques we utilize, described in § 2.7, cannot completely correct the modes introduced by the PPMs themselves, although the low-order components are canceled nearly completely by the woofer static correction. As the 83% static Strehl ratio (at 658 nm) is uniform across the field, this error term has no spatial dependence.

The fourth static term is due to minor misalignments in the field stops. These field stops have to be removed every AO iteration for checking Strehl ratios and put back into place for wave front sensing. The value quoted (10 nm) is derived from Fresnel simulations of wave fronts passing through spatial filters; if the holes are displaced by a minor amount, the overall wave front is filtered differently and changed. This error term is dependent on the repeatability of the filter wheels that mount the focal plane stops. They are estimated to be repeatable to the 100 $\mu$m level, while the size of the holes are 1.6 mm.

The fifth and sixth static terms are linearity calibration drift and WFS zero-point drift. The zero-point error is determined by comparing measured wave fronts with references saved at a fixed time in the past. Over the typical length of AO simulations (2 hr), the errors increase by $\sim 10$ nm. The linearity calibration drift error is determined by measuring the open-loop accuracy of the WFSs on long timescales. This is done by comparing the multiple LGS wave fronts on a common ground-layer. We find that this open-loop accuracy degrades by $\sim 10$ nm over 2 hr. It is assumed that both the WFS zero-point references and the linearity calibration data can be retaken before every experiment. Both of these drift terms are due to optical drift, isolated to several nonkinematic cube beam splitter mounts leading to the Shack–Hartmann WFS facility.

### 4.3.2. Time-Dependent Terms

The following error terms have a dependence on total atmospheric strength. The atmospheric fitting error is given by (Hardy 1998)

$$\sigma_{fit} = 0.53(d/r_0)^{5/6},$$

where $d$ is the subaperture size, and $r_0$ is the Fried parameter. The total atmospheric strength is given by

$$\sigma_a = 0.366(D/r_0)^{5/6},$$

where $D$ is the telescope diameter. Solving for $\sigma_{fit}$ for our system (66 subapertures across a 10 m pupil) gives

$$\sigma_{fit} = 0.0441 \sigma_a.$$  

Similarly, the WFS aliasing error can be expressed as a fraction of the fitting error (40% for Shack–Hartmann WFS) and thus the atmospheric error (Fusco et al. 2006):

$$\sigma_{alias} = 0.0176 \sigma_a.$$  

The error in estimating the wave front in open loop is labeled the WFS systematic error. This error is reduced from $\sim 80$ nm to $\sim 30$ nm through linearity calibration (Ammons et al. 2007). The open-loop error model is characterized by measuring the

![Figure 10](image-url)  

**Fig. 10.**—Strehl predictions plotted against measured Strehl ratios. Panels are arranged by atmospheric realization horizontally. Plotted on the x-axis is the field position in arcseconds and plotted on the y-axis is the Strehl ratio. The predicted values are shown as triangles, and the measured values are shown as diamonds. Note that the on-axis predictions match the measured values, but the off-axis predictions are $\sim 10\%$ higher than the measured values.
disagreement between adjacent LGSs on common ground-layer turbulent plates. The WFS systematic term is empirically measured to increase monotonically with the total atmospheric strength. We use the following fitted linear relationship to describe the WFS systematic error:

\[
\sigma_{\text{WFS}} = 0.045 \sigma_a.
\]

The scintillation term is calculated explicitly from theoretical models of scintillation (Stroud 1994). The scintillation index is given by

\[
\sigma^2_\chi = 0.288(\sqrt{\lambda h/r_{0,\text{layer}}})^{(5/3)},
\]

where \(\lambda\) is the test bed wavelength, \(h\) is the height of a particular atmospheric layer, and \(r_{0,\text{layer}}\) is the Fried parameter of that layer. The effect on the measured Strehl ratio is

\[
S = \exp(-\sigma^2_\chi) \exp(-\sigma^2_\lambda).
\]

Substituting our physical path lengths, we can express this error as a function of the atmospheric strengths at different altitudes:

\[
\sigma_\chi = 1.466(\lambda/a^2)^{(5/12)} \sqrt{h_1^{(5/6)} \sigma^2_{a,1} + h_2^{(5/6)} \sigma^2_{a,2}},
\]

where \(a\) is the physical telescope aperture size in meters, \(h_x\) is the height at layer \(x\) in meters, and \(\sigma_{a,x}\) is the atmospheric strength at that altitude. This error can be explicitly calculated at each simulation time point, as the relative strength at each altitude is a derived output of the tomographic algorithm (TSW).

The WFS scintillation error is the effect that scintillation has on Shack–Hartmann WFSs. Ideally, WFSs are not sensitive to amplitude variations, but in reality this is not true. We explicitly measure this error by comparing the wave fronts measured on a single plate when at different altitudes. We assume that this error has the same dependencies as pure scintillation: \(\sigma_\chi_{\text{WFS}} \propto h^{(5/12)}\) and \(\sigma_{\chi,\text{WFS}} \propto \sigma_{a,x}\). The final model includes both of these proportionalities and is calibrated via the previous empirical measurement. This model is

\[
\sigma_{\chi_{\text{WFS}}} = 0.235 \sqrt{h_1^{(5/6)} \sigma^2_{a,1} + h_2^{(5/6)} \sigma^2_{a,2}}.
\]

Note that both pure scintillation and WFS scintillation are severe for this test bed, as the physical aperture size is much smaller compared to the wavelength of light than a 10 m telescope. LGS elongation is expected to lower the magnitude of this error by a small amount, to approximately 23.5 nm rms for this test bed.

Finally, notice that the Strehl ratio slowly drops over the course of four realizations in Figure 6. This is due to a slow drift in pupil-to-lenslet registration, amounting to 0.5% of a sub-aperture per real hour. The simulations were taken over 2 days, so this amounts to a pupil shift of 25% of a subaperture (0.38% of the full aperture, or 68 \(\mu m\) physical shift at the telescope pupil). Using simulations of shifted Kolmogorov atmospheres, we find that this results in \(\sim 40\) nm of error after 2 days. Then

\[
\sigma_{\text{pupil}} = 0.89 \times t,
\]

where the units of \(\sigma_{\text{pupil}}\) are in nanometers, and \(t\) is the number of hours since pupil calibration. As the PPM pixels are 5 times smaller than the Hartmann subapertures, slow pupil drift is not a problem for software and control, as long as it can be calibrated frequently. Note that closed-loop AO systems would suffer a great deal from this magnitude of pupil shift (perhaps not converging) because wave fronts from previous AO iterations are erroneously shifted multiple times; a deformable mirror-to-lenslet registration with accuracy better than 5% of a subaperture is typically desired for closed-loop systems. Fortunately, open-loop AO systems have no feedback, so the only error introduced is that modeled by the previous equation.

In addition to a standard deformable mirror-to-lenslet calibration involving raising and centroiding 25 Gaussian functions on the PPM, we also perform a spatially variant, Strehl-optimal calibration to lock down the pupil drift. This involves placing a layer of turbulence at the ground, measuring the mean wave front, and varying the \(x\) - and \(y\)-shift of the applied wave front until the Strehl ratio on a particular science star is maximized. When performed on all seven science test stars individually, this surprisingly produces a set of offset vectors that extend radially outward from the central star, implying that the PPM and the ground layer are not perfectly conjugated. The magnitude of the pupil drift error (\(\sigma_{\text{pupil}}\)) is measured directly by repeating this calibration over long time baselines.

### 4.3.3. Tomographic Error

The only error term dependent on both space and time is the tomographic error. Tomographic estimation error is due to a number of contributors, including misestimation of layer heights, inadequate coverage of atmospheric space, and tomographic blind modes. In these experiments, we fix layer heights and ensure that no atmosphere is missed, so the dominant component is that due to tomographic blind modes. Tomographic blind modes are three-dimensional modes that are unsensed due to the geometry of the LGS constellation. It is expected that this error is directly proportional to atmospheric strength.

Tomographic error increases with off-axis distance, as less information is available to estimate the wave front. We calibrate this dependency explicitly with numerical simulations of atmospheric tomography, using the same solver used to analyze bench data (TSW). We match the LGS constellation parameters and Kolmogorov atmospheric strengths in these simulations to those in the laboratory experiments. This gives tomographic errors at different field points that correspond to individual
atmospheric realizations. The averages of these values are shown in Table 3.

5. CONCLUSIONS

5.1. Error Budget Meta-Analysis

The error budget terms describing the MOAO/LTAO bench can be grouped into three categories: traditional errors, calibration errors, and dynamic drift errors. The traditional error terms are those endemic to adaptive optics, limited by subaperture size and laser power, that cannot be improved with better hardware calibration. These include fitting error, WFS aliasing, tomographic error, scintillation, and photon error. The calibration errors include those terms that could nominally be reduced with increased hardware cost and calibration effort, including WFS systematic error, deformable mirror go-to error, static uncorrectable errors, and WFS scintillation. The dynamic drift errors include those calibration errors that have a time dependence, requiring a greater frequency of calibration. These include linearity calibration drift, pupil registration drift, WFS zero-point drift, and field stop misalignment. The total magnitudes of the errors in each of these categories are illustrated in Figure 11.

The total magnitude of calibration errors is comparable to the traditional AO errors, but the dynamic drift component has been greatly reduced by extensive, frequent calibration. In quadrature, the magnitude of all calibration errors is 49% of the total error budget for the on-axis case, with 42% arising from static errors. The remaining contributor of 51% is from traditional error terms that cannot be reduced without decreasing subaperture size and increasing laser power. This partitioning of errors, with static and dynamic calibration errors comparable to traditional error terms, is expected for high-order AO systems such as the MOAO/LTAO test bed.

The vibration amplitude on this test bed is much lower than that seen in typical telescope instruments, but the magnitude of pure optical drift is larger, which requires optical calibration every few hours. These calibrations include (1) dark frame measurement, (2) a Hartmann linearity calibration to enable open-loop WFS performance, (3) a deformable mirror-to-lenslet registration procedure to calibrate high-order optical warping (for both PPM and woofer), and (4) spatially variant image sharpening to increase static Strehl ratios throughout the field of view. We also use a single ground-layer turbulent plate to optimize (1) open-loop gain, (2) WFS registration between LGS pupils, and (3) global deformable mirror-to-lenslet registration as a function of science star position.

It will be important to include extensive calibration facilities when designing visible-light LTAO systems for routine use at astronomical observatories. Of particular utility will be accelerometers on fast optics susceptible to vibration, which can be used to diagnose the low- and high-order effects of telescope and instrument vibration. Powered optics vibrating parallel to the optical axis change the magnification ratio, which can affect AO performance by changing the imaging plate scale or changing the deformable mirror-to-lenslet registration magnification. Accelerometers can assist the diagnosis of these problems, which may otherwise be attributed to other sources of error, and either dampened or canceled in open loop. Full telescope simulators, including LGS elongation mechanisms and three-dimensional turbulators, will assist in daytime calibration. If the go-to characteristics of the deformable mirrors change with time, it may be necessary to periodically transfer them to accurate interferometers external to the instrument (external deformable mirror ports) for recalibration.

5.2. Extrapolation to On-Sky LTAO Systems

We now discuss the ramifications of these experiments for future AO systems on large telescopes. Some error budget terms that would be present in on-sky high-order LTAO systems are not simulated here. These include the total tip/tilt residual for a bright star, bandwidth/servo lag error, multispectral error, asteroid deformation error, residual sodium-layer focus change, and differential atmospheric refractive index. In addition, several effects of LGS elongation are not simulated. We present estimated values of these errors in the first column of Table 3, taken directly from the Keck Next Generation Adaptive Optics (KNGAO) error budget (Neyman & Dekany 2008). KNGAO is similar to this LTAO/MOAO system in implementation and order, except for simulated LGS brightness, which affects measurement error.

Some values of existing error budget terms are changed, including the PPM lookup table error, scintillation error, and WFS scintillation error. The PPM lookup table error is assumed not to be present in an analogous on-sky system, as some other type of deformable mirror with good go-to response would ideally be used (e.g., MEMS [micro electro-mechanical system]).
Scintillation and WFS scintillation errors are calculated using on-sky physical parameters, assuming a Mauna Kea atmosphere. Residual tip/tilt error is calculated using KNGAO values describing a bright NGS case, including tip/tilt measurement and bandwidth errors and excising long exposure and science instrument drift errors. Note that several terms accounting for calibration error and dynamic drift have not been changed, although it would be expected that an on-sky system would have less severe dynamic drift than the LTAO/MOAO test bed. This makes the on-sky Strehl estimate a lower limit.

5.3. Future Test Bed Plans

The coming decade brings exciting new possibilities for high-order AO on large telescopes. IR AO system designs for Extremely Large Telescopes (ELTs) and specialized high-contrast architectures on 8–10 m telescopes (GPI, Macintosh et al. 2006) have required the development of order $64 \times 64$ deformable mirrors. As these mirrors have been functionally demonstrated and even higher order deformable mirrors are being designed, there exists the potential to lower subaperture sizes on observatory AO systems to those suitable for visible-light AO correction. We have demonstrated in this article that order $64 \times 64$ for a 10 m telescope may be sufficient to realize visible-light correction in the bright tip/tilt star case, given a site with good median seeing ($0.65''$). However, increasing LGS power in step with decreasing subaperture size remains an expensive option. For wide-field correction in the visible, the tomographic error due to blind modes quickly becomes the dominant error term on 6–10 m telescopes. We intend to investigate one remedy to the tomographic error that does not sacrifice field size or require an order of magnitude more LGSs: wind prediction. We have performed preliminary experiments with wind prediction on the MOAO/LTAO test bed and found that it assures the buildup of tomographic error in sparsely sampled rings in upper-altitude metapupils, provided the atmosphere is 100% Taylor frozen flow (Ammons et al. 2008).

We believe the MOAO/LTAO experiments presented here demonstrate technologies necessary for wide-field, laser-driven, visible-light AO with high sky coverage. These include WFS calibrations that enable open-loop operation, open-loop deformable mirror characterization, and pupil registration calibration.

New technologies like uplink LGS correction may enable the power densities necessary for visible-light AO correction (Gavel et al. 2008). Hartmann centroiding accuracy for an LGS is proportional to the inverse square of its angular size, so measurement error can be greatly reduced by precorrecting the outgoing laser beam and sharpening its image on the sodium layer. Moderate sharpening of the LGS spot to Hartmann pixel sizes ($0.8''-1.0''$) preserves the use of Hartmann sensors, but delivers 4–5 times the effective power of the unsharpened LGS (typically 2.0'). Uplink correction is being tested on-sky with the ViLLaGES (Gavel et al. 2008).

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