Primordial non-Gaussianity from the large-scale structure

V Desjacques$^{1}$ and U Seljak$^{1,2}$

$^1$ Institute for Theoretical Physics, University of Zürich, Winterthurerstrasse 190, CH-8057 Zürich, Switzerland
$^2$ Physics and Astronomy Department, University of California, and Lawrence Berkeley National Laboratory, Berkeley, CA 94720, USA

E-mail: dvince@physik.uzh.ch and seljak@physik.uzh.ch

Received 1 February 2010, in final form 25 March 2010
Published 27 May 2010
Online at stacks.iop.org/CQG/27/124011

Abstract

Primordial non-Gaussianity is a potentially powerful discriminant of the physical mechanisms that generate the cosmological fluctuations observed today. Any detection of non-Gaussianity would have profound implications for our understanding of cosmic structure formation. In this paper, we review past and current efforts in the search for primordial non-Gaussianity in the large-scale structure of the Universe.

PACS numbers: 98.80.−k, 98.80.Cq

(Some figures in this article are in colour only in the electronic version)

1. Introduction

In generic inflationary models based on the slow roll of a scalar field, primordial curvature perturbations are produced by the inflaton field as it slowly rolls down its potential [1–4]. Most of these scenarios predict a nearly scale-invariant spectrum of adiabatic curvature fluctuations, a relatively small amount of gravity waves and tiny deviations from Gaussianity in the primeval distribution of curvature perturbations [5–7]. While the latest measurements of the cosmic microwave background (CMB) anisotropies favor a slightly red power spectrum [8], no significant detection of a $B$-mode or of primordial non-Gaussianity (NG) has thus far been reported from CMB observations.

While the presence of a $B$-mode can only be tested with CMB measurements [9, 10], primordial deviations from Gaussianity can leave a detectable signature in the distribution of CMB anisotropies and in the large-scale structure (LSS) of the Universe. Until recently, it was widely accepted that measurement of the CMB furnished the best probe of primordial non-Gaussianity [11]. However, these conclusions did not take into account the scale dependence of the galaxy power spectrum and bispectrum arising for primordial NG of the local $f_{NL}^{loc}$ type.
These theoretical results, together with rapid developments in observational techniques that will provide large amount of LSS data, will enable us to critically confront predictions of non-Gaussian models. In particular, galaxy clustering should provide independent constraints on the magnitude of primordial non-Gaussianity as competitive as those from the CMB and in the long run may even give the best constraints.

The purpose of this work is to provide an overview of the search for a primordial non-Gaussian signal in the large-scale structure. We will begin by briefly summarizing how non-Gaussianity arises in inflationary models (section 2). Next we will discuss the impact of primordial non-Gaussianity on the mass distribution in the low-redshift Universe (section 3). The main body of this review is section 4, where we describe in detail a number of methods exploiting the abundance and clustering properties of observed tracers of the LSS to constrain the amount of initial non-Gaussianity. We conclude with a discussion of present and forecasted constraints achievable with LSS surveys (section 5).

2. Models and observables

Single-field slow-roll models led to a very small level of primordial non-Gaussianity [6, 7, 14]. This is because they assume (i) a single dynamical field (the inflaton), (ii) canonical kinetic energy terms (i.e. perturbations propagate at the speed of light), (iii) slow roll (i.e. the timescale over which the inflaton field changes is much larger than the Hubble rate) and (iv) an initial adiabatic Bunch–Davies vacuum. The lowest order statistics sensitive to non-Gaussian features in the initial distributions of scalar perturbations \( \Phi(x) \) (we shall adopt the standard CMB convention in which \( \Phi_1(x) \) is the Bardeen’s curvature perturbation in the matter era) is the three-point function or bispectrum \( B_\Phi(k_1, k_2, k_3) \), which is a function of any triangle \( k_1 + k_2 + k_3 = 0 \) (as follows from statistical homogeneity which we assume throughout this paper). It has been shown that, in the squeezed limit \( k_3 \ll k_1 \approx k_2 \), the bispectrum of any single-field slow-roll inflationary model asymptotes to the local shape [15–17]. The corresponding nonlinear parameter predicted by these models is \( f_{\text{NL}}^{\text{loc}} = \frac{1}{16} (1 - n_s) \approx 0.017 \), where \( n_s \) is the tilt or spectral index of the power spectrum \( P_\Phi(k) \), which is accurately measured to be \( n_s \approx 0.960 \pm 0.013 \) [8]. Therefore, any robust measurement of \( f_{\text{NL}}^{\text{loc}} \) well above this level would thus rule out single-field slow-roll inflation as defined above.

2.1. The shape of the primordial bispectrum

Large, potentially detectable amounts of Gaussianity can be produced when at least one of the assumptions listed above is violated, i.e. by multiple scalar fields [18, 19], nonlinearities in the relation between the primordial scalar perturbations and the inflaton field [7, 14], interactions of scalar fields [20], a modified dispersion relation or a departure from the adiabatic Bunch–Davies ground state [21]. Generation of a large non-Gaussian signal is also expected at reheating [22] and in the ekpyrotic scenario [23]. Each of these physical mechanisms leaves a distinct signature in the primordial three-point function \( B_\Phi(k_1, k_2, k_3) \), a measurement of which would thus provide a wealth of information about the physics generating the primordial fluctuations. Although the configuration shape of the primordial bispectrum can be extremely complex in some models, there are broadly three classes of shape characterizing the local, equilateral and folded type of primordial non-Gaussianity [24, 25]. The magnitude of each template ‘X’ is controlled by a dimensionless nonlinear parameter \( f_X^{\text{NL}} \) which we seek to constrain using CMB or LSS observations.

Any non-Gaussianity generated outside the horizon induces a three-point function that is peaked on squeezed or collapsed triangles for realistic values of the scalar spectral index. The
resulting non-Gaussianity depends only on the local value of the Bardeen’s curvature potential and can thus be conveniently parameterized up to third order by [7, 11, 14, 26]

$$\Phi(x) = \phi(x) + \int_{\text{loc}}^{k_{\text{loc}}} \phi^2(x) + g_{\text{loc}}^{\text{max}} \delta(x),$$

(1)

where $\phi(x)$ is an isotropic Gaussian random field and $f_{\text{loc}}^{\text{loc}}, g_{\text{loc}}^{\text{loc}}$ are dimensionless, phenomenological parameters. Since curvature perturbations are of magnitude $O(10^{-5})$, the cubic order correction should always be negligibly small compared to the quadratic one when $O(f_{\text{loc}}^{\text{loc}}) \sim O(g_{\text{loc}}^{\text{loc}})$. However, this condition is not satisfied by some multifield inflationary models such as the curvaton scenario, in which a large $g_{\text{loc}}^{\text{loc}}$ and a small $f_{\text{loc}}^{\text{loc}}$ can be simultaneously produced [19]. The quadratic term generates the three-point function at leading order:

$$B_{\phi}^{\text{loc}}(k_1, k_2, k_3) = 2 f_{\text{loc}}^{\text{loc}}[P_\phi(k_1)P_\phi(k_2) + (\text{cyc})],$$

(2)

where (cyc.) denotes all cyclic permutations of the indices and $P_\phi(k)$ is the power spectrum of the Gaussian part $\phi(x)$ of the Bardeen potential. The cubic-order terms generate a trispectrum $T_\phi(k_1, k_2, k_3, k_4)$ at leading order.

Equilateral type of non-Gaussianity, which arises in inflationary models with higher derivative operators such as the DBI model, is well described by the factorizable form [27]

$$B_{\phi}^{\text{eq}}(k_1, k_2, k_3) = 6 f_{\text{loc}}^{\text{loc}}[-(P_\phi(k_1)P_\phi(k_2) + (\text{cyc})) - 2(P_\phi(k_1)P_\phi(k_2)P_\phi(k_3))^{2/3} + (P_\phi^{1/3}(k_1)P_\phi^{2/3}(k_2)P_\phi(k_3) + (\text{perm.}))].$$

(3)

It can be easily checked that the signal is largest in the equilateral configurations $k_1 \approx k_2 \approx k_3$, and suppressed in the squeezed limit $k_3 \ll k_1 \approx k_2$. Note that, in single-field slow-roll inflation, the three-point function is a linear combination of the local and equilateral shape [15].

As a third template, we consider the folded or flattened shape which is maximized for $k_2 \approx k_3 \approx k_1/2$ [28]:

$$B_{\phi}^{\text{fold}}(k_1, k_2, k_3) = 6 f_{\text{loc}}^{\text{loc}}[(P_\phi(k_1)P_\phi(k_2) + (\text{cyc.})) + 2(P_\phi(k_1)P_\phi(k_2)P_\phi(k_3))^{2/3} - (P_\phi^{2/3}(k_1)P_\phi^{1/3}(k_2)P_\phi(k_3) + (\text{perm.}))].$$

(4)

and approximate the non-Gaussianity due to the modification of the initial Bunch–Davies vacuum in the canonical single-field action (although the latter peaks on squashed or collinear triangles). As in the previous example, $B_{\phi}^{\text{eq}}$ is suppressed in the squeezed configurations. Unlike $B_{\phi}^{\text{loc}}$, however, the folded shape induces a scale-dependent bias at large scales (see section 4.3).

2.2. Statistics of the linear mass density field

The Bardeen’s curvature potential $\Phi(x)$ is related to the linear density perturbation $\delta_0(k, z)$ at the redshift $z$ through the relation

$$\delta_0(k, z) = M(k, z)\Phi(k),$$

(5)

where

$$M(k, z) = \frac{2k^2T(k)D(z)}{3\Omega_m H_0^2}. $$

(6)

Here $T(k)$ is the matter transfer function normalized to unity as $k \to 0$, $\Omega_m$ is the present-day matter density, $D(z)$ is the linear growth rate normalized to $1 + z$. Equation (5) is important as it provides the connection between the primeval curvature perturbations and the low-redshift
mass density field. The \(n\)-point correlator of the linear matter density field can thus be related to those of \(\Phi(x)\):

\[
\langle \delta_0(k_0) \cdots \delta_0(k_n) \rangle = \left( \prod_{i=1}^{n} \mathcal{M}(k_i) \right) \langle \Phi(k_1) \cdots \Phi(k_n) \rangle.
\]

Smoothing unavoidably arises when comparing observations of the large-scale structure with theoretical predictions. Perturbation theory (PT), which is valid only in the weakly nonlinear regime [29] or the spherical collapse model, which ignores the strongly nonlinear internal dynamics of the collapsing regions [30, 31], requires that the small-scale nonlinear fluctuations be smoothed out. For this reason, we introduce the smoothed linear density field \(\delta_R\):

\[
\delta_R(k, z) = M(k, z) W_R(k) / \Phi_1(k) \equiv M_R(k, z) / \Phi_1(k),
\]

where \(W_R(k)\) is a (spherically symmetric) window function of the characteristic radius \(R\) or mass scale \(M\). We will assume a top-hat filter in configuration space throughout. Furthermore, since \(M\) and \(R\) are equivalent variables, we shall indistinctly use the notation \(\delta_R\) and \(\delta_M\) in what follows.

### 2.3. Topological defects models

In addition to inflationary scenarios, there is a whole class of models, known as topological defect models, in which cosmological fluctuations are sourced by an inhomogeneously distributed component which contributes a small fraction of the total energy–momentum tensor [32, 33]. The density field is obtained as the convolution of a discrete set of points with a specific density profile. Defects are deeply rooted in particle physics as they are expected to form at a phase transition. Since the early Universe may have plausibly undergone several phase transitions, it is rather unlikely that no defects at all were formed. Furthermore, high-redshift tracers of the LSS may be superior to CMB at finding non-Gaussianity sourced by topological defects [34]. However, CMB observations already provide stringent limits on the energy density of a defect component [8], so we shall only minimally discuss the imprint of these scenarios in the large-scale structure.

### 3. Evolution of the matter density field with primordial NG

In this section, we summarize a number of results relative to the effect of primordial NG on the mass density field. These will be useful to understand the complexification that arises when considering biased tracers of the density field (see section 4).

#### 3.1. Setting up non-Gaussian initial conditions

Investigating the impact of non-Gaussian initial conditions (ICs) on the large-scale structure traced by galaxies, etc, requires simulations large enough so that many long wavelength modes are sampled. At the same time, the simulations should resolve the dark matter halos hosting the observed galaxies or quasars (QSOs), so that one can construct halo samples whose statistical properties mimic as closely as possible those of the real data. This favors the utilization of pure \(N\)-body simulations, for which a larger dynamical range can be achieved, rather than computationally more expensive hydrodynamical simulations.

The evolution of the matter density field with primordial non-Gaussianity has been studied in series of large cosmological \(N\)-body simulations seeded with Gaussian and non-Gaussian initial conditions, see e.g. [13, 35–44]. For generic non-Gaussian (scalar) random fields, we
face the problem of setting up numerical simulations with a prescribed correlation structure [45]. For the equilateral and folded type of non-Gaussianity, this task is not easily accomplished (because it requires the calculation of a number of convolutions which are computationally demanding). However, for primordial NG described by a local mapping such as the $f_{\text{NL}}^{\text{loc}}$ model, this is a rather straightforward operation. This is the reason why most numerical studies of structure formation with inflationary NG have so far implemented the local shape solely.

### 3.2. Mass density probability distribution

In the absence of primordial NG, the probability distribution function (PDF) of the initial smoothed density field, i.e. the probability that a randomly placed cell of volume $V$ has some specific density, is Gaussian. Namely, all normalized or reduced smoothed cumulants $S_j$ of order $J \geq 3$ are zero. An obvious signature of primordial NG would thus be an initially non-vanishing skewness $S_3 = \langle \delta^3_R \rangle / \langle \delta^2_R \rangle^3$ or kurtosis $S_4 = \langle \delta^4_R \rangle / \langle \delta^2_R \rangle^2 - 3 / \langle \delta^2_R \rangle$ [37, 46, 47]. Here, the subscript $c$ denotes the connected piece of the $n$-point moment that cannot be simplified into a sum over products of lower order moments. At the third order, for instance, the cumulant of the smoothed density field is an integral of the three-point function

$$\langle \delta^3_R \rangle_c = \int \frac{d^3 k_1}{(2\pi)^3} \int \frac{d^3 k_2}{(2\pi)^3} \int \frac{d^3 k_3}{(2\pi)^3} B_R(k_1, k_2, k_3, z), \quad (9)$$

where

$$B_R(k_1, k_2, k_3, z) = M_R(k_1, z)M_R(k_2, z)M_R(k_3, z)B_0(k_1, k_2, k_3) \quad (10)$$

is the bispectrum of the smoothed linear density fluctuations at the redshift $z$. Note that, while $S_3(R, z) \propto D(z)^{-1}$, the product $\sigma S_3(R)$ does not depend on the redshift. Over the range of scale $0.1 \lesssim R \lesssim 100 \, h^{-1} \text{Mpc}$ accessible to LSS observations, $\sigma S_3^{\text{NL}}(R) \equiv \sigma S_3(R, f_{\text{NL}}^2 = 1)$ is a weakly monotonically decreasing function of $R$ that is of magnitude $\sim 10^{-4}$ for the local, equilateral and folded templates discussed above. Strictly speaking, all reduced moments should be specified to fully characterize the density PDF, but a reasonable description of the density distribution can be achieved with moments up to the fourth order.

Numerical and analytic studies generally find that a density PDF initially skewed toward positive values produces more overdense regions, whereas a negatively skewed distribution produces larger voids. Gravitational instabilities also generate a positive skewness in the density field, reflecting the fact that the evolved density distribution exhibits an extended tail toward large overdensities [48–53]. This gravitationally induced signal eventually dominates the primordial contribution such that, at fixed normalization amplitude, non-Gaussian scenarios deviate more strongly from the fiducial Gaussian model at high redshift. More precisely, the time evolution of the normalized cumulants $S_j$ can be worked out for generic Gaussian and non-Gaussian ICs using PT or the simpler spherical collapse approximation. For Gaussian ICs, PT predicts that the normalized cumulants be time independent to the lowest non-vanishing order, with a skewness $S_3 \approx 34/7$, whereas for non-Gaussian ICs, the linear contribution to the cumulants decays as $S_j(R, z) = S_j(R, \infty) / D^{j-2}(z)$ [54–57].

The persistence of the primordial hierarchical amplitude $S_j(R, \infty)$ sensitively depends upon the magnitude of $S_N$, $N \geq J$, relative to unity. For example, an initially large non-vanishing kurtosis could source skewness with a time dependence and amplitude similar to that induced by nonlinear gravitational evolution [54]. Although there is an infinite class of non-Gaussian models, we can broadly divide them into weakly and strongly non-Gaussian. In weak NG models, the primeval signal in the normalized cumulants is rapidly obliterated by gravity-induced non-Gaussianity. This is the case of hierarchical scaling models where $n$-point
correlation functions satisfy $\xi_n \propto \xi_2^{n-1}$ with $\xi_2 \ll 1$ at large scales. By contrast, strongly NG initial conditions dominate the evolution of the normalized cumulants. This occurs when the hierarchy of correlation functions obeys the dimensional scaling $\xi_n \propto \xi_2^{n/2}$, which arises in the particular case of $\chi^2$ initial conditions [58] or in defect models such as texture [38, 59, 60]. These scaling laws have been successfully confronted with numerical investigations of the evolution of cumulants [38, 39].

Although gravitational clustering tends to erase the memory of initial conditions, numerical simulations of non-Gaussian initial conditions show that the occurrence of highly underdense and overdense regions is very sensitive to the presence of primordial NG. In fact, numerical simulations of non-Gaussian initial conditions show that the occurrence of highly underdense and overdense regions is very sensitive to the presence of primordial NG. In fact, the imprint of primordial NG is best preserved in the low-density tail of the PDF $P(\rho)$ smoothed at the scale $R$ [41, 61]. A satisfactory description of this measurement can be obtained from an Edgeworth expansion of the initial mass density field. At high densities $\rho \gg 1$, the non-Gaussian modification approximately scales as $\rho^{5/3}$ whereas at low densities, $\rho \simeq 0$, the deviation is steeper and behaves as $\rho^{5/2}$ [62].

3.3. Power spectrum and bispectrum

Primordial non-Gaussianity also imprints a signature in Fourier space statistics of the matter density field as positive values of $f_{NL}^2$ tend to increase the small-scale matter power spectrum $P_0(k)$ [12, 41, 63] and the large-scale matter bispectrum $B_0(k_1, k_2, k_3)$ [12, 64].

In the weakly nonlinear regime where one-loop PT applies, the Fourier mode of the density field for growing mode initial conditions reads [49, 65]

$$\delta(k, z) = \delta_0(k, z) + \frac{1}{(2\pi)^3} \int d^3q_1 d^3q_2 \delta_D(k - q_1 - q_2) F_2(q_1, q_2) f_0(q_1, z) f_0(q_2, z).$$  (11)

The kernel $F_2(k_1, k_2) = 5/7 + \mu(k_1/k_2 + k_2/k_1)/2 + 2\mu^2/7$, where $\mu$ is the cosine of the angle between $k_1$ and $k_2$, describes the nonlinear second-order evolution of the density field. It is nearly independent of $\Omega_m$ and $\Omega_s$. At one-loop PT, the second term on the right-hand side of equation (11) generates a correction to the mass power spectrum,

$$P_0^{NG}(k, z) = P_0(k, z) + \{P_{(22)}(k, z) + P_{(13)}(k, z)\} + \Delta P_0^{NG}(k, z).$$  (12)

Here, $P_0(k)$ is the linear matter power spectrum at the redshift $z$, $P_{(22)}$ and $P_{(13)}$ are the standard one-loop contributions in the case of Gaussian ICs [65, 66] and

$$\Delta P_0^{NG}(k, z) = 2 \int \frac{d^3q}{(2\pi)^3} F_2(q, k - q) B_0(-k, q, k - q, z)$$  (13)

is the correction due to primordial NG [63]. This last term scales as $D^3(z)$ such that the effect of non-Gaussianity is largest at the low redshift. Most importantly, $F_2(k_1, k_2)$ vanishes in the squeezed limit $k_1 = -k_2$ as a consequence of the causality of gravitational instability. This strongly suppresses equation (13) at small wavenumbers, even in the local $f_{NL}^{loc}$ model for which $B_0(-k, q, k - q)$ is maximized in the squeezed limit $|k| \to 0$. For $f_{NL}^{loc} \sim O(10^2)$, the magnitude of the correction is at a percent level in the weakly nonlinear regime $k \lesssim 0.1 \ h \ Mpc^{-1}$, in good agreement with simulations [42, 44, 67]. Extensions of the renormalization group description of dark matter clustering [68] to non-Gaussian initial density and velocity perturbations can improve the agreement up to wavenumbers $k \lesssim 0.25 \ h \ Mpc^{-1}$ [69, 70].

To the second order in PT, the matter bispectrum $B_0(k_1, k_2, k_3)$ is the sum of a primordial contribution and of two terms induced by gravitational instability [49, 71] (here and henceforth
we omit the explicit $z$-dependence for brevity):

$$
B_3(k_1, k_2, k_3) = B_0(k_1, k_2, k_3) + [2 F_2(k_1, k_2) P_0(k_1) P_0(k_2) + \text{(cyc.})]
+ \int \frac{d^3q}{(2\pi)^3} [F_2(q, k_3 - q) T_0(q, k_3 - q, k_1, k_2) + \text{(cyc.})],
$$

(14)

where $T_0(k_1, k_2, k_3, k_4)$ is the primordial trispectrum of the density field. Note that a similar expression can be derived for the matter trispectrum, which turns out to be less sensitive to gravitationally induced nonlinearities [72]. The reduced bispectrum $Q_3$ is conveniently defined as

$$
Q_3(k_1, k_2, k_3) = \frac{B_3(k_1, k_2, k_3)}{[P_3(k_1) P_3(k_2) + \text{cyclic}]}. \quad (15)
$$

For Gaussian initial conditions, $Q_3$ is independent of time and, at tree-level PT, is constant and equal to $Q_3(k, k, k, k) = 4/7$ for equilateral configurations [49]. For general triangles, moreover, it approximately retains this simple behavior, with a dependence on triangle shape through $F_2(k_1, k_2)$ [12]. The inclusion of one-loop corrections greatly improves the agreement with the numerical data [73]. An important property of the matter bispectrum is the fact that the primordial part scales as $Q_3 \propto 1/M^2_R(k)$ for approximately equilateral triangles and under the assumption that $f^\text{loc}_{NL}$ is scale independent [12]. This ‘anomalous’ scaling considerably raises the ability of the matter bispectrum to constrain primordial NG of the local $f^\text{loc}_{NL}$ type. Unfortunately, neither the matter bispectrum nor the power spectrum are directly observable with the large-scale structure of the Universe. Temperature anisotropies in the redshifted 21 cm background from the pre-reionization epoch could in principle furnish a direct measurement of these quantities [74–76], but foreground contamination may severely hamper any detection. Weak lensing is another direct probe of the dark matter, although we can only observe it in projection along the line of sight [77].

As we will see shortly, however, a similar scaling is also present in the power spectrum and bispectrum of observable tracers of the large-scale structure such as galaxies. It is this unique signature that will make future all-sky LSS surveys competitive with forthcoming CMB experiments.

4. LSS probe of primordial non-Gaussianity

Discrete and continuous tracers of the large-scale structure such as galaxies, the Ly$\alpha$ forest, the 21 cm hydrogen line, etc, provide a distorted image of the matter density field. In CDM cosmologies, galaxies form inside overdense regions [78] and this introduces a bias between the matter and galaxy distributions [79]. As a result, distinct samples of galaxies trace the matter distribution differently, the most luminous galaxies preferentially residing in the most massive DM halos. This biasing effect, which concerns most tracers of the large-scale structure, is still poorly understood. Models of galaxy clustering assume for instance that the galaxy biasing relation only depends on the local mass density, but the actual biasing could be more complex [80, 81]. Because of biasing, tracers of the large-scale structure will be affected by primordial non-Gaussianity in a different way than the mass density field. In this section, we describe a number of methods exploiting the abundance and clustering properties of biased tracers to constrain the level of primordial NG. We focus on galaxy clustering as it provides the tightest limits on primordial NG (see section 5).
4.1. Halo finding algorithm

Locating groups of bound particles, or DM halos, in simulations is central to the methods described below. In practice, one aims at extracting halo catalogs with statistical properties similar to those of observed galaxies or QSOs. This, however, proves to be quite difficult because the relation between observed galaxies and halos is somewhat uncertain. Furthermore, there is freedom at defining a halo mass.

A important ingredient is the choice of the halo identification algorithm. There are two categories of halo finder: (i) spherical overdensity (SO) finder \[82\] with the overdensity threshold \( \Delta_{\text{so}}(z) \sim 200 \) and (ii) friends-of-friends (FOF) finder with a linking length \( b \sim 0.15-0.2 \) \[83\]. The mass of a SO halo is defined by the radius at which the inner overdensity exceeds \( \Delta_{\text{so}}(z) \) times the background density \( \bar{\rho}(z) \), whereas the mass of a FOF halo is the number of linked particles. Here, we will present results mostly for SO halos, as their mass estimate is more closely connected to the predictions of the spherical collapse model, on which most of the analytic formulas presented in this section are based. The question of how the spherical overdensity masses can be mapped onto friends-of-friends masses remains a matter of debate \[84\]. Clearly, however, since the peak height depends on the halo mass \( M \) through the variance \( \sigma(M) \), any systematic difference will be reflected in the value of \( \nu \) associated with a specific halo sample.

4.2. Abundances of voids and bound objects

A departure from Gaussianity can significantly affect the abundance of highly biased tracers of the mass density field, as their frequency sensitively depends upon the tails of the initial density PDF \[85–87\]. The (extended) Press–Schechter approach has been extensively applied to ascertain the magnitude of this effect. Because it depends only on the skewness, it is weakly sensitive to the shape of the primordial bispectrum.

4.2.1. Press–Schechter approach. The Press–Schechter theory \[88\] and its extensions based on excursion sets \[89–91\] predict that the number density \( n(M, z) \) of halos of mass \( M \) at the redshift \( z \) is entirely specified by a multiplicity function \( f(\nu) \):

\[
\begin{align*}
n(M, z) &= \frac{\bar{\rho}}{M^2} f(\nu) \frac{d\ln v}{d\ln M} \\
\text{(16)}
\end{align*}
\]

where the peak height \( v(M, z) = \delta_c(z)/\sigma(M) \) is the typical amplitude of fluctuations that produce those halos. Here and henceforth, \( \sigma(M) \) denotes the variance of the initial density field \( \delta_M \) smoothed on the mass scale \( M \propto R^3 \) and linearly extrapolated to present epoch, whereas \( \delta_c(z) \approx 1.68D(0)/D(z) \) is the critical linear overdensity for (spherical) collapse at the redshift \( z \). In the standard Press–Schechter approach, \( n(M, z) \) is related to the level excursion probability \( P(>\delta_c, M) \) that the linear density contrast of a region of mass \( M \) exceeds \( \delta_c(z) \): \( f(\nu) = -\frac{2}{M} \frac{dP}{dM} = \sqrt{\frac{2}{\pi}} \nu e^{-\nu^2/2}, \) \( \text{(17)} \)

where the last equality assumes Gaussian initial conditions. The factor of 2 is introduced to account for the contribution of low-density regions embedded in overdensities at scale \( >M \). In the extended Press–Schechter theory, \( \delta_M \) evolves with \( M \) and \( f(\nu) \) is the probability that a trajectory is absorbed by the constant barrier \( \delta = \delta_c \) (as is appropriate in the spherical collapse approximation) on the mass scale \( M \). In general, the exact form of \( f(\nu) \) depends on the barrier shape \[92\] and the filter shape \[93\]. Note also that \( \int d\ln v f(\nu) = 1 \), which ensures that all the mass is contained in halos.
Despite the fact that the Press–Schechter mass function overpredicts (underpredicts) the abundance of low(high)-mass objects, it can be used to estimate the fractional deviation from Gaussianity. In this formalism, the non-Gaussian fractional correction to the multiplicity function is \( R(v, f_{\text{NL}}^X) \equiv f(v, f_{\text{NL}}^X)/f(v, 0) = (dP/dM)(\delta_c, M, f_{\text{NL}}^X)/(dP/dM)(\delta_c, M, 0) \), which is readily computed once the non-Gaussian density PDF \( P(\delta_M) \) is known. In the simple extensions proposed by [94, 95], \( P(\delta_M) \) is expressed as the inverse transform of a cumulant generating function. In [95], the saddle-point technique is applied directly to \( P(\delta_M) \). The resulting Edgeworth expansion is then used to obtain \( P(\delta_c, M) \). Neglecting cumulants beyond the skewness, one obtains

\[
R_{\text{LV}}(v, f_{\text{NL}}^X) \approx 1 + \frac{1}{6} \sigma S_3(v^3 - 3v) - \frac{1}{6} \frac{d(\sigma S_3)}{d\ln v} \left( v - \frac{1}{2} \right) \tag{18}
\]

after integration over regions above \( \delta_c(z) \). In [94], it is the level excursion probability \( P(\delta_c, M) \) that is calculated within the saddle-point approximation. This approximation asymptotes to the exact large mass tail, which exponentially deviates from the Gaussian tail.

To enforce the normalization of the resulting mass function, one may define \( \nu_* = \delta_*/\sigma \) and use \( f_{\nu, \delta}(v, \nu_* f(\nu_*)) \approx 1 + \frac{1}{6} \sigma S_3(v^3 - 3v) - \frac{1}{6} \frac{d(\sigma S_3)}{d\ln v} \left( v - \frac{1}{2} \right) \).

The fractional deviation from the Gaussian mass function then becomes

\[
R_{\text{MVJ}}(v, f_{\text{NL}}^X) \approx \exp\left( \frac{v^3}{6} \sigma S_3 \right) \left[ 1 - \frac{1}{2} \sigma S_3 - \frac{v}{6} \frac{d(\sigma S_3)}{d\ln v} \right]. \tag{20}
\]

Both formulas have been shown to give reasonable agreement with numerical simulations of non-Gaussian cosmologies [42, 97, 98] (but note that [13, 99] have reached somewhat different conclusions). Expanding \( \delta_* = \delta_c\sqrt{1 - S_3\delta_c/3} \) at the first order shows that these two theoretical expectations differ in the coefficient of the \( \nu S_3 \) term. Therefore, it is also instructive to consider the approximation [100]

\[
R(v, f_{\text{NL}}^X) \approx \exp\left( \frac{v^3}{6} \sigma S_3 \right) \left[ 1 - \frac{\nu}{2} \sigma S_3 - \frac{v}{6} \frac{d(\sigma S_3)}{d\ln v} \right]. \tag{21}
\]

which is designed to match better the Edgeworth expansion of [95] when the peak height is \( \nu \sim 1 \). Note that when the primordial trispectrum is large (which, in the local model, would happen if \( g_{\text{NL}}^\text{loc} \sim 10^6 \)), terms involving the kurtosis should also be included [94, 95, 100, 101]. In this case, it is also important to take into account a possible renormalization of the fluctuation amplitude, \( \sigma_k \rightarrow \sigma_k + \delta\sigma_k \), to which the high-mass tail of the mass function is exponentially sensitive [100]. Finally, note also that [13, 43] parametrize the fractional correction using N-body simulations.

Figure 1 shows the effect of primordial NG of the local \( f_{\text{NL}}^\text{loc} \) type on the halo mass function. The dotted-dashed curve represents the approximation (equation (21)). While the agreement is reasonable for the SO halos (top panel), the theory significantly overestimates the deviation measured in the FOF mass function with linking length \( b = 0.2 \) (middle panel). A similar effect is noted in [98], who makes the replacement \( \delta_c \rightarrow \delta_c q \) with \( q \approx 0.75 \) to fit their measurement of \( R(v, f_{\text{NL}}^X) \) based on FOF halos. References [102, 103] provide a physical motivation of this replacement by demonstrating that the diffusive nature of the collapse barrier introduces a similar factor. However, an overlooked but important fact is that the FOF and SO mass estimates systematically deviate from each other. In figure 1, in particular, the FOF mass is on average 20% larger than the SO mass. As shown in the bottom panel of figure 1,
Figure 1. Fractional deviation from the Gaussian mass function as a function of the peak height \( \nu = \delta_c/\sigma \). Different symbols refer to different redshifts as indicated. The solid curve is the theoretical prediction (equation (21)) at \( z = 0 \) based on an Edgeworth expansion of the dark matter probability distribution function. At the top panel, halos are identified using a spherical overdensity (SO) finder with a redshift-dependent overdensity threshold \( \Delta_{\text{vir}}(z) \). In the middle panel, a friends-of-friends (FOF) finding algorithm with linking length \( b = 0.2 \) is used. The bottom panel shows the effect of decreasing the FOF mass by 20% (see the text). In all panels, error bars denote Poisson errors. For illustration, \( M = 10^{15} \, M_\odot/h \) corresponds to \( \nu = 3.2, 5.2, 7.7 \) at the redshift \( z = 0, 1 \) and \( 2 \), respectively. Similarly, \( M = 10^{14} \, M_\odot/h \) and \( 10^{13} \, M_\odot/h \) correspond to \( \nu = 1.9, 3, 4.5 \) and \( 1.2, 1.9, 2.9 \), respectively.

Rescaling the FOF mass to account for this difference removes most of the discrepancy with the FOF data. This illustrates an important point: the impact of primordial NG on the statistics of DM halos is sensitive to systematics caused by the choice of the halo finder. As we will see below, this may also be true for the non-Gaussian halo bias.

More sophisticated formulations based on extended Press–Schechter (EPS) theory and/or modifications of the collapse criterion look promising since they can reasonably reproduce both the Gaussian halo counts and the dependence on \( f_X^{\text{NL}} \) [102, 104, 105]. The probability of first upcrossing can, in principle, be derived for any non-Gaussian density field and any choice of the smoothing filter [106, 107]. For a general filter, the non-Markovian dynamics generates additional terms in the non-Gaussian correction to the mass function that arise from three-point correlators of the smoothed density \( \delta_M \) at different mass scales [102]. However, large error bars still make it difficult to test for the presence of such sub-leading terms. For generic moving barriers \( B(\sigma) \) such as those appearing in models of triaxial collapse [108, 109], the leading contribution to the non-Gaussian correction approximately is [104]

\[
R(\nu, f_X^{\text{NL}}) \approx 1 + \frac{1}{6} \sigma S_3 H_3 \left( \frac{B(\sigma)}{\sigma} \right),
\]

(22)

where \( H_3(\nu) \equiv \nu^3 - 3\nu \) and agrees well with the observed deviation [105].
4.2.2. Clusters abundance. Rich clusters of galaxies trace the rare, high-density peaks in the initial conditions and thus offer the best probe of the high-mass tail of the multiplicity function. To infer the cluster mass function, the x-ray and millimeter windows are better suited than the optical-wave range because selection effects can be understood better.

Following early theoretical [85, 86, 110–112] and numerical [36, 113–115] work on the effect of non-Gaussian initial conditions on the multiplicity function of cosmic structures, the abundance of clusters and x-ray counts in non-Gaussian cosmologies has received much attention in the literature. At a fixed normalization of the observed abundance of local clusters, the proto-clusters associated with high-redshift (2 < z < 4) Lyα emitters are much more likely to develop in strongly non-Gaussian models than in the Gaussian paradigm [40, 99, 116]. Considering the redshift evolution of cluster abundances can thus break the degeneracy between the initial density PDF and the background cosmology. In this regard, simple extensions of the Press–Schechter formalism (similar to those considered above) have been shown to capture reasonably well the cluster mass function over a wide range of redshift for various non-Gaussian scenarios [117]. Scaling relations between the cluster mass, x-ray temperature and Compton y-parameter calibrated using theory, observations and detailed simulations of cluster formation [118, 119] can then be exploited to predict the observed distribution functions of x-ray and SZ signals and assess the capability of cluster surveys to test the nature of the initial conditions [120–128].

An important limitation of this method is that, for a realistic amount of primordial NG, the non-Gaussian signal imprinted in cluster abundances is small compared to the systematics plaguing current and upcoming surveys [129–131]. Given the current uncertainties in the redshift evolution of clusters (one commonly assumes that clusters are observed at the epoch they collapse [130]), the selection effects in the calibration of x-ray and SZ fluxes with halo mass, the freedom in the definition of the halo mass, the degeneracy with the normalization amplitude \( \sigma_8 \) (for positive \( f_{X_{NL}} \), the mass function is more enhanced at the high-mass end, and this is similar to an increase in the amplitude of fluctuations \( \sigma_8 \) [132]) and the low-number statistics, the prospects of using the cluster mass function only to place competitive limits on \( f_{X_{NL}} \) with the current data are small. A two-fold improvement in cluster mass calibration is required to provide constraints comparable to CMB measurements [131].

4.2.3. Voids abundance. The frequency of cosmic voids, which is strongly sensitive to the low-density tail of the initial mass PDF, offers another probe of non-Gaussian initial conditions [133]. The Press–Schechter formalism can also be applied to ascertain the magnitude of this effect. Voids are defined as regions of mass \( M \) whose density is less than some critical value \( \delta_v \leq 0 \) or, alternatively, as regions for which the three eigenvalues of the tidal tensor [134] lie below some critical value \( \lambda_v \leq 0 \) [62, 105, 133, 135]. An important aspect in the calculation of the mass function of voids is the over-counting of voids located inside collapsing regions. This voids-in-clouds problem, as identified by [136], can be solved within the excursion set theory by studying a two-barrier problem: \( \delta_v \) for halos and \( \delta_v \) for voids. Including this effect reduces the frequency of the smallest voids [105]. Neglecting this complication notwithstanding, the differential number density of voids of radius \( R \) is [133, 135]

\[
\frac{dn}{dR} \approx \frac{9}{2\pi^2} \left( \frac{\pi}{2} \right) \left( \frac{|v_v|}{R^2} \right)^{1/2} e^{-|v_v|^2/2} \frac{d\ln |v_v|}{d\ln M} \left[ 1 - \frac{1}{6} \sigma S_N H_3(|v_v|) \right].
\]

(23)

where \( v_v = \delta_v/\sigma_M \). While a positive \( f_{X_{NL}} \) produces more massive halos, it generates fewer large voids [105, 133]. Hence, the effect is qualitatively different from a simple rescaling of the normalization amplitude \( \sigma_8 \). A joint analysis of both abundances of clusters and cosmic voids might thus provide interesting constraints on the shape of the primordial three-point
function. There are, however, several caveats to this method, including the fact that there is no unique way to define voids [133, 137]. Clearly, voids identification algorithms will have to be tested on numerical simulations [138] before a robust method can be applied to real data.

4.3. Galaxy two-point correlation

In Gaussian cosmologies, correlations of galaxies and clusters can be amplified relative to the mass distribution [79]. Before this was realized, it was argued that primordial fluctuations need to be non-Gaussian [139, 140] to explain the observed strong correlation of Abell clusters [141, 142]. Along these lines, [143] pointed out that primordial non-Gaussianity could significantly increase the amplitude of the two-point correlation of galaxies and clusters on large scales. However, except from [144] who showed that correlations of high-density peaks in non-Gaussian models are significantly stronger than those in the Gaussian model with the identical mass power spectrum, subsequent work focused mostly on abundances (section 4.2) or higher order statistics such as the bispectrum (section 4.4). It is only recently that [13] have demonstrated the strong scale-dependent bias arising in non-Gaussian models of the local \( f_{NL} \) type.

4.3.1. The non-Gaussian bias. In the original derivation of [13], the Laplacian is applied to the left- and right-hand side of \( \Phi = \phi + f_{NL} \phi^2 \) to show that, upon substitution of the Poisson equation, the overdensity in the neighborhood of density peaks is spatially modulated by a factor proportional to the local value of \( \phi \). Taking into account the coherent motions induced by gravitational instabilities, the scale-dependent bias correction reads

\[
\Delta b_k(k, f_{NL}) = 3 f_{NL} [b_1(M) - 1] \Omega_m H_0^2 \frac{1}{k^2 T(k) D(z)},
\]

where \( b_1(M) \) is the linear, Gaussian halo bias. The original result of [13] missed out a multiplicative factor of \( T(k) \) which was introduced subsequently by [145] upon a derivation of equation (24) in the limit of high-density peaks. The peak-background split approach [92, 146, 147] promoted by [148] shows that the scale-dependent bias applies to any tracer of the matter density field whose (Gaussian) multiplicity function depends on the local mass density only. In this approach, the Gaussian piece of the potential is decomposed into short- and long-wavelength modes, \( \phi = \phi_l + \phi_s \). This provides an intuitive explanation of the effect in terms of a local rescaling of the small-scale amplitude of matter fluctuations by a factor \( 1 + 2 f_{NL} \phi_l \) (see also [13, 67, 149]). As emphasized in [13], the scale dependence arises from the fact that the non-Gaussian curvature perturbations \( \Phi(x) \) are related to density fluctuations through the Poisson equation (5). There is no such effect in the \( \chi^2 \) model [11, 150] nor in texture-seeded cosmologies [151] for instance.

The derivation of [145], based on the clustering of regions of the smoothed density field \( \delta_M \) above threshold \( \delta_c(z) \), is formally valid for high-density peaks only. However, it is general enough to apply to any shape of primordial bispectrum [152]. In the high-threshold limit \( \nu \gg 1 \), the two-point correlation function of the level excursion set can be expressed as [110]

\[
\xi_{>\nu}(r) = -1 + \exp \left( \sum_{n=2}^{\infty} \sum_{j=1}^{n-1} \frac{1}{j!(n-j)!} \int \sigma^{-n} \xi_{(n)}(x_1, \ldots, x_1, x_2, \ldots, x_2) \right),
\]

where \( r = x_1 - x_2 \). For most non-Gaussian models in which the primordial three-point function is the dominant correction, this expansion can be truncated at the third order and Fourier transformed to yield the non-Gaussian correction \( \Delta P_{>\nu}(k) \) to the power spectrum.
Assuming a small level of primordial NG, we can also write $\Delta P_{s\nu}(k) \approx 2b_L\Delta b_L P_H(k)$, where $b_L = b_L(M) - 1 \approx \nu^2/\delta_c$ is the Lagrangian bias, and eventually obtain

$$\Delta b_X(k, f_{NL}^X) = b_\nu(k, f_{NL}^X) = \left( \frac{2b_L \delta_c(z)}{M_R(k, 0)} \right) \mathcal{F}(k, f_{NL}^X).$$  \hspace{1cm} (26)$$

The dependence on the shape of the three-point function is encoded in the function $\mathcal{F}(k, f_{NL}^X)$ [145, 152]:

$$\mathcal{F}(k, f_{NL}^X) = \frac{1}{16\pi^2\sigma^2} \int_0^\infty \! dk_1 k_1^3 M_R(k_1, 0) \int_{-1}^{+1} \! d\mu \, M_R(\sqrt{\alpha}, 0) \frac{B_\nu(k_1, \sqrt{\alpha}, k)}{P_\nu(k)},$$

where $\alpha = k^2 + k_1^2 + 2\mu kk_1$. Note that, for $f_{NL}^X < 0$, this first-order approximation always breaks down at sufficiently small $k$ because $\Delta P_{s\nu}(k) < 0$.

The scale-dependent bias induced by the equilateral and folded bispectrum shape is computed in [152]. To get insights into the large-scale behavior of $\Delta b_X(k, f_{NL}^X)$, let us identify the dominant contribution to $\mathcal{F}(k, f_{NL}^X)$ in the limit $k \ll 1$. Setting $M_R(\sqrt{\alpha}, 0) \approx M_R(k_1, 0)$ and expanding $P_\nu(\sqrt{\alpha})$ at the second order in the ratio $k/k_1$, we arrive at

$$\mathcal{F}(k, f_{NL}^X) \approx f_{NL}^{loc}(k, f_{NL}^X).$$  \hspace{1cm} (28a)$$

$$\mathcal{F}(k, f_{NL}^X) \approx 3 \int_0^\infty \! dk k^{2n_s - 4/3} M_R(k, 0)^2 P_\nu(k),$$  \hspace{1cm} (29)$$

assuming no running scalar index, i.e. $dn_s/d\ln k = 0$. The auxiliary function $\Sigma_R(n)$ is defined as

$$\Sigma_R(n) = \frac{1}{2\pi^2} \int_0^\infty \! dk \, k^{2n_s} M_R(k, 0)^2 P_\nu(k).$$

Hence, we have $\Sigma_R(0) \approx \sigma^2$. For a nearly scale-invariant spectrum $n_s \approx 1$, we obtain $\mathcal{F}(k, f_{NL}^X) \propto k$ and $\mathcal{F}(k, f_{NL}^X) \propto k^2$, such that the non-Gaussian bias is $\Delta b_X \propto k^{-1}$ and $\Delta b_X = \text{const}$ for the folded and equilateral bispectrum, respectively. Therefore, at large scales, the scale dependence of the non-Gaussian bias is much smaller for the folded template, and nearly absent for the equilateral shape. This makes them much more difficult to detect with galaxy surveys [152]. However, the equilateral and folded non-Gaussian bias depend sensitively upon the mass scale $M$ through the multiplicative factor $\sigma^{-2}$. For example, choosing $R = 5 \, h^{-1}$ Mpc instead of $R = 1 \, h^{-1}$ Mpc would increase the effect by a factor of $\sim 3$. In the high-peak limit, $\sigma^{-2} \approx b_L/\delta_c(z)$, which cancels out the dependence on redshift but enhances the sensitivity to the halo bias, i.e. $\Delta b_c \propto b_L^2$. By contrast, $\Delta b_c \propto b_L$ for the local $f_{NL}^{loc}$ model.

At this point, it is appropriate to mention a few caveats to these calculations. First, equation (26) assumes that the tracers form after a spherical collapse, which may be a good approximation for the massive halos only. If one instead considers the ellipsoidal collapse dynamics, in which the evolution of a perturbation depends upon the three eigenvalues of the initial tidal shear, $\delta_c(0)$ should be replaced by its ellipsoidal counterparts $\delta_{\text{loc}}(0)$ which is always larger than the spherical value [108]. In this model, the scale-dependent bias $\Delta b_c$ is thus enhanced by a factor $\delta_{\text{loc}}(0)/\delta_c(0)$ [149]. Second, equation (26) assumes that the biasing of the surveyed objects is described by the peak height $\nu$ only or, equivalently, the hosting halo mass $M$. However, this may not be true for quasars whose activity may be triggered by merger of halos [153, 154]. Reference [148] used the EPS formalism to estimate the bias correction $\Delta b_{\text{merger}}$ induced by recent mergers:

$$\Delta b_{\text{merger}} = \delta_c^{-1},$$  \hspace{1cm} (30)$$
so the factor \( b_1(M) - 1 \) should be replaced by \( b_1(M) - 1 - \delta_c^{-1} \approx b_1(M) - 1.6 \). The validity of this result should be evaluated with cosmological simulations of quasars formation. In this respect, semi-analytic models of galaxy formation suggest that merger-triggered objects such as quasars do not cluster much differently than other tracers of the same mass [155]. However, this does not mean that the same should hold for the non-Gaussian scale-dependent bias. Still since the recent merger model is an extreme case it seems likely that the actual bias correction is \( 0 < \Delta b_{\text{merger}} < \delta_c^{-1} \). Third, the scale-dependent bias has been derived using the Newtonian approximation to the Poisson equation, so one may wonder whether general relativistic (GR) corrections to \( M_R(k)^{-1} \) suppress the effect on scales comparable to the Hubble radius. Reference [156] showed how large-scale primordial NG induced by GR corrections propagates onto small scales once cosmological perturbations reenter the Hubble radius in the matter-dominated era. This effect generates a scale-dependent bias comparable, albeit of the opposite sign, to that induced by local NG [152]. However, this issue deserves further clarification as [157] have recently argued that there are no GR corrections to the non-Gaussian bias and that the scaling \( \Delta b_c \propto k^{-2} \) applies down to the smallest wavenumbers.

Finally, we can also ask ourselves whether higher order terms in the series expansion (25) furnish corrections to the non-Gaussian bias of magnitude comparable to equation (24). In the \( f_{\text{NL}}^{\text{loc}} \) model, the power spectrum of biased tracers of the density field can also be obtained from a local Taylor series in the evolved (Eulerian) density contrast \( \delta \) and the Gaussian part \( \phi \) of the initial (Lagrangian) curvature perturbation [67, 158]. Using this approach, it can be shown that the halo power spectrum arising from the first-order terms of the local bias expansion can be cast into the form [158]

\[
P_h(k) = \left[ b_1(M) + f_{\text{NL}}^{\text{loc}} \delta_c(k) \right]^2 P_R(k).
\]

Hence, we obtain a second-order term proportional to \( \left( f_{\text{NL}}^{\text{loc}} \right)^2 M_R^{-2} P_R(k) = \left( f_{\text{NL}}^{\text{loc}} \right)^2 P_{\phi}(k) \) which, however, contributes only at very small wavenumber \( k \lesssim 0.001 \, h^{-1} \, \text{Mpc} \). There is another second-order correction to the halo power spectrum that reads [100]

\[
\Delta P_h(k) = \frac{4}{3} \left( f_{\text{NL}}^{\text{loc}} \right)^2 \left[ b_1(M) - 1 \right]^2 \delta_c^2(z) S_3^{(1)}(M) M_R(k, 0) P_R(k).
\]

Its magnitude relative to the term linear in \( f_{\text{NL}}^{\text{loc}} \), equation (24), is approximately 0.03 at the redshift \( z = 1.8 \) and for a halo mass \( M = 10^{13} \, M_\odot / h \). Although this contribution becomes increasingly important at higher redshift, it is fairly small for realistic values of \( f_{\text{NL}}^{\text{loc}} \). All this suggests that equation (24) is the dominant contribution to the non-Gaussian bias in the wavenumber range \( 0.001 \lesssim k \lesssim 0.1 \, h \, \text{Mpc}^{-1} \).

### 4.3.2. Comparison with simulations

In order to fully exploit the potential of forthcoming large-scale surveys, a number of studies have tested the theoretical prediction (equation (24)) against the outcome of large numerical simulations [13, 42–44, 67, 98].

At the lowest order, there are two additional albeit relatively smaller corrections to the Gaussian bias which arise from the dependence of both the halo number density \( n(M, z) \) and the matter power spectrum \( P_h(k, z) \) on primordial NG [42]. Firstly, assuming the peak-background split holds, the change in the mean number density of halos induces a scale-independent offset which we denote as \( \Delta b_1(f_{\text{NL}}^{\text{loc}}) \). In terms of the non-Gaussian fractional correction \( R(v, f_{\text{NL}}^{\text{loc}}) \) to the mass function, this contribution is

\[
\Delta b_1(f_{\text{NL}}^{\text{loc}}) = -\frac{1}{\sigma} \frac{\partial}{\partial v} \ln[R(v, f_{\text{NL}}^{\text{loc}})].
\]

It is worth noticing that \( \Delta b_1(f_{\text{NL}}^{\text{loc}}) \) has a sign opposite to that of \( f_{\text{NL}}^{\text{loc}} \), because the bias decreases when the mass function goes up. Secondly, we also need to account for the change in the matter power spectrum (see section 3). As a result, non-Gaussianity of the \( f_{\text{NL}}^{\text{loc}} \) type adds a
Figure 2. Halo–halo and halo–matter power spectra $P_h(k)$ and $P_{h\delta}(k)$ measured in simulations of the Gaussian model and of the local $f_{loc}^{NL}$ type with $f_{loc}^{NL} = \pm 100$. Halos of mass $M > 2 \times 10^{13} \, M_\odot / h$ were identified at the redshift $z = 2$ with a SO finder. The linear Gaussian bias of this sample is $b_1(M) = 2.53$. The error bars represent the scatter among eight realizations. The solid and dashed curves show the theoretical $P_h(k)$ and $P_{h\delta}(k)$ obtained with the non-Gaussian bias correction (equation (34)). For $f_{NL}^{loc} = -100$, the cross-power spectrum is negative on scales $k \lesssim 0.005 \, h \, \text{Mpc}^{-1}$, in good agreement with the theoretical prediction.

**correction** $\Delta b_1(k, f_{loc}^{NL})$ to the bias $b(k)$ of dark matter halos that reads [42]

$$
\Delta b_1(k, f_{loc}^{NL}) = \Delta b_1(k, f_{loc}^{NL}) + \Delta b_1(f_{loc}^{NL} + b_1(M) \left( \frac{P_h(k, f_{loc}^{NL})}{P_h(k, 0)} - 1 \right)
$$

at the first order in $f_{loc}^{NL}$. The linear bias $b_1$ needs to be measured accurately as it controls the strength of the scale-dependent bias correction $\Delta b_1$. In this respect, the ratio $P_{h\delta}(k) / P_h(k)$, where $P_{h\delta}(k)$ is the halo–matter cross-power spectrum, is a better proxy for the halo bias as it is less sensitive to shot noise.

Reference [42] finds that the inclusion of these extra terms substantially improves the comparison between the theory and the simulations. Considering only the scale-dependent shift $\Delta b_1$ leads to an apparent suppression of the effect in simulations relative to the theory. Including the scale-independent offset $\Delta b_1$ considerably improves the agreement at all scales. Finally, adding the scale-dependent term $b_1(M) P_h(k, f_{loc}^{NL}) / P_h(k, 0) - 1$ further adjusts the match at small scale $k \gtrsim 0.05 \, h \, \text{Mpc}^{-1}$ by making the non-Gaussian bias shift less negative. Taking into account second- and higher order corrections could extend the validity of the theory up to scales $k \sim 0.1 - 0.3 \, h \, \text{Mpc}^{-1}$ [67].

Auto- and cross-power analyses may not agree with each other if the halos and dark matter do not trace each other on scale $k \lesssim 0.01 \, h \, \text{Mpc}^{-1}$ where the non-Gaussian bias is large, i.e. if there is stochasticity. Figure 2 shows $P_{h\delta}(k)$ and $P_h(k)$ averaged over eight realizations of the models with $f_{NL}^{loc} = 0, \pm 100$. The same Gaussian random seed field $\phi$ was used in each set of runs so as to minimize the sampling variance. Measurements of the non-Gaussian bias correction obtained with the halo–halo or the halo–matter power spectrum are in a good agreement with each other, indicating that non-Gaussianity does not induce stochasticity and
the predicted scaling equation (24) applies equally well for the auto- and cross-power spectrum. However, while a number of numerical studies of the $f_{NL}^{loc}$ model have confirmed the scaling $\Delta b_k (k, f_{NL}^{loc}) \propto M_R(k)^{-1}$ and the redshift dependence $\propto D(z)^{-1}$ [13, 42, 43, 98], the exact amplitude of the non-Gaussian bias correction remains somewhat debatable. Reference [42], who use SO halos and include the scale-independent offset $\Delta b_I$, finds satisfactory agreement with the theory. By contrast, [43, 98], who use FOF halos, find that equation (24) is a good fit to the simulations only upon replacing $\delta_c$ by $q\delta_c$ with $q \simeq 0.75$. Part of the discrepancy may be due to the fact that [43, 98] do not include $\Delta b_I$, which leads to an apparent suppression of the effect. Another possible source of discrepancy may be the choice of the halo finder. In this regard, figure 3 shows the non-Gaussian bias correction obtained with FOF halos. The best-fit values of $f_{NL}^{loc}$ are somewhat below the input values of $\pm 100$, in agreement with the findings of [43, 98]. This indicates that the choice of halo finder also affects the magnitude of the non-Gaussian halo bias. Discrepancies have also been observed between the theoretical and measured non-Gaussian bias corrections in non-Gaussian models of the local cubic-order coupling $g_{NL}^{loc} \phi^3$ [100]. Understanding these results will clearly require a better theoretical modeling of halo clustering.

**4.3.3. Redshift distortions.** Peculiar velocities generate systematic differences between the spatial distribution of data in real and redshift space. These redshift distortions must be properly taken into account in order to extract $f_{NL}^{loc}$ from redshift surveys. On the linear scales of interest, the redshift space power spectrum of biased tracers reads [159, 160]

$$P_b^s(k, \mu) = \left[ b_I^2 P_3(k) + 2b_1 f \mu^2 P_{60}(k) + f^2 \mu^4 P_6(k) \right],$$

(35)

where $P_3$ and $P_6$ are the density–velocity and velocity divergence power spectra, $\mu$ is the cosine of the angle between the wavemode $k$ and the line of sight and $f$ is the logarithmic derivative of the growth factor. For $P_6$, the one-loop correction due to primordial NG is identical to equation (13) provided $F_2(k_1, k_2)$ is replaced by the kernel $G_2(k_1, k_2) = 3/7 + \mu(k_1/k_2 + k_2/k_1)/2 + 4\mu^2/7$ describing the second-order evolution of the velocity divergence [58]. For $P_{60}$, this correction is
\[ \Delta P_{\text{NG}}^{\text{NL}}(k) = \int \frac{d^3q}{(2\pi)^3} \{ F_2(q, k - q) + G_2(q, k - q) \} B_0(k, q, -k - q). \] (36)

Again, causality implies that \( G_2(k_1, k_2) \) vanishes in the limit \( k_1 = -k_2 \). For unbiased tracers with \( b_1 = 1 \), the linear Kaiser relation is thus recovered at large scales: \( k \lesssim 0.01 \text{ h Mpc}^{-1} \) (see also [61]). For biased tracers, we still expect the Kaiser formula to be valid, but the distortion parameter \( \beta \) should now be equal to \( \beta = f/(b_1 + \Delta b_\kappa) \), where \( \Delta b_\kappa(k, f_X^{\text{NL}}) \) is the scale-dependent bias induced by the primordial non-Gaussianity.

4.3.4. Mitigating cosmic variance and shot noise. Because of the finite number of large-scale wavemodes accessible to a survey, any large-scale measurement of the power spectrum is limited by the cosmic (or sampling) variance caused by the random nature of the wavemodes. For discrete tracers such as galaxies, the shot noise is another source of error. For weak primordial NG, the relative error on the power spectrum \( P \) is \( \sigma_P/P \approx 1/\sqrt{N(1 + \sigma_n^2/P)} \), where \( N \) is the number of independent modes measured and \( \sigma_n^2 \) is the shot noise [161]. Under the standard assumption of Poisson sampling, \( \sigma_n^2 \) equals the inverse of the number density \( 1/\bar{n} \) and causes a scale-independent enhancement of the power spectrum. The extent to which one can improve the observational limits on the nonlinear will strongly depends on our ability to minimize the impact of these two sources of errors. By comparing differently biased tracers of the same surveyed volume [162, 163] and suitably weighting galaxies (by the mass of their host halo for instance) [164, 165], it should be possible to circumvent these problems and considerably improve the detection level.

Figure 3 illustrates how the impact of sampling variance on the measurement of \( f_{\text{NL}}^{\text{loc}} \) can be mitigated. Namely, the data points show the result of taking the ratio \( P_b(k, f_{\text{NL}}^{\text{loc}})/P_\delta(k, f_{\text{NL}}^{\text{loc}}) \) for each set of runs with the same Gaussian random seed field \( \phi \) before averaging over the realizations. This procedure is equivalent to the multi-tracer’s method advocated by [162]. Here, \( P_b \) can be thought as mimicking the power spectrum of a nearly unbiased tracer of the mass density field with high-number density. Although, in practical applications, using the dark matter field works better [166], in real data \( P_b \) should be replaced by a tracer of the same surveyed volume different than the one used to compute \( P_\delta \). Figure 3 also shows that, upon taking out most of the cosmic variance, there is some residual noise caused by the discrete nature of the dark matter halos. As shown recently [165], however, weighting the halos according to their mass can dramatically reduce the shot noise relative to the Poisson expectation, at least when compared against the dark matter. Applying such a weighting may thus significantly improve the error on the nonlinear parameter \( f_{\text{NL}}^{\text{loc}} \), but this should be explored in realistic simulations of galaxies, especially because the halo mass \( M \) may not be easily measurable from real data [166]. This approach undoubtedly deserves further attention as it has the potential to substantially improve the extraction of the primordial non-Gaussian signal from galaxy surveys.

To conclude this section, we note that while the PDF of power values \( P(k) \) has little discriminatory power (for large surveyed volume, it converges toward the Rayleigh distribution as a consequence of the central limit theorem) [167], the covariance of power spectrum measurements (which is sensitive not only to the selection function but also to correlations among the phase of the Fourier modes) may provide quantitative limits on certain type of non-Gaussian models [161, 168].

4.4. Galaxy bispectrum and higher order statistics

Higher statistics of biased tracers, such as the galaxy bispectrum, are of great interest as they are much more sensitive to the shape of the primordial three-point function than the power spectrum [12, 44, 64, 169, 170]. Therefore, they could break some of the degeneracies affecting
the non-Gaussian halo bias (for example, the leading order scale-dependent correction to the Gaussian bias induced by the local quadratic and cubic coupling are fully degenerate \[100\]).

4.4.1. Normalized cumulants of the galaxy distribution. The skewness of the galaxy count probability distribution function could provide constraints on the amount of non-Gaussianity in the initial conditions. As discussed in section 3, however, it is difficult to disentangle the primordial and gravitational causes of skewness in low-redshift data unless the initial density field is strongly non-Gaussian. The first analyses of galaxy catalogs in terms of count-in-cells densities all reached the conclusion that the skewness (and higher order moments) of the observed galaxy count PDF is consistent with the value induced by gravitational instabilities of initially Gaussian fluctuations \[50, 54, 171–174\]. Back then, however, most of the galaxy samples available were not large enough to accurately determine the cumulants \[S_J\] at large scales \[175\]. Despite the two orders of magnitude increase in surveyed volume, these measurements are still sensitive to cosmic variance, i.e. to the presence of massive super-clusters or large voids. Nevertheless, the best estimates of the first normalized cumulants \[S_J\] of the galaxy PDF strongly suggest that high-order galaxy correlation functions follow the hierarchical scaling predicted by the gravitational clustering of Gaussian ICs \[176\]. There is no evidence for strong non-Gaussianity in the initial density field as might by seeded by cosmic strings or textures \[177\].

The genus statistics of constant density surfaces through the galaxy distribution measures the relative abundance of low- and high-density regions as a function of the smoothing scale \(R\) and, therefore, could also be used as a diagnostic tool for primordial non-Gaussianity. While for a Gaussian random field the genus curve (i.e. the genus number as a function of the density contrast) is symmetric about \(\delta_R = 0\) regardless the value of \(R\), primordial NG and nonlinear gravitational evolution can disrupt this symmetry \[178\]. The effect of non-Gaussian ICs on the topology of the galaxy distribution has been explored in a number of papers \[36, 179–182\]. For large values of \(R\) and a realistic amount of primordial NG, the genus statistics can also be expanded in a series whose coefficients are the normalized cumulants \(S_J\) of the smoothed galaxy density field. Therefore, the genus statistics essentially provides another measurement of the (large-scale) cumulants of the galaxy distribution \[183, 184\].

4.4.2. Galaxy bispectrum. Most of the scale dependence of the primordial \(n\)-point functions is integrated out in the normalized cumulants, which makes them weakly sensitive to primordial NG. However, while the effect of non-Gaussian initial conditions, galaxy bias, gravitational instabilities, etc are strongly degenerated in the \(S_J\), they imprint distinct signatures in the galaxy bispectrum \(B_h(k_1, k_2, k_3)\), an accurate measurement of which could thus constrain the shape of the primordial three-point function.

In the original derivation of \[169\], the large-scale (unfiltered) galaxy bispectrum in the \(f^\text{loc}_\text{NL}\) model is given by

\[
B_h(k_1, k_2, k_3) = b_1^3 B_0(k_1, k_2, k_3) + b_2^3 b_2 [P_0(k_1) P_0(k_2) + \text{(cyc.)}] \\
+ 2b_1^2 [F_2(k_1, k_2) P_0(k_1) P_0(k_2) + \text{(cyc.)}].
\]

Equation (37)

Here, \(b_1\) and \(b_2\) are the first- and second-order bias parameters that describe the galaxy biasing relation assumed to be local and deterministic \[185\]. The first term on the right-hand side is the primordial contribution which, for equilateral configurations and in the \(f^\text{loc}_\text{NL}\) model, scales as \(M(k, z)^{-1}\) like in the matter bispectrum, equation (14). The two last terms are the contribution from nonlinear bias and the tree-level correction from gravitational instabilities, respectively. They have the smallest signal in squeezed configurations.
As recognized by [64, 170], equation (37) misses an important term that may significantly enhance the sensitivity of the galaxy bispectrum to non-Gaussian initial conditions. This contribution is sourced by the trispectrum \( T_R(k_1, k_2, k_3, k_4) \) of the smoothed mass density field:

\[
\frac{1}{2} b_1^2 b_2^2 \int \frac{d^3 q}{(2\pi)^3} T_R(k_1, k_2, q, k_3 - q) + (2 \text{ perms.}) ,
\]

(38)

and reduces at large scales to the sum of the linearly evolved primordial trispectrum \( T_0(k_1, k_2, k_3, k_4) \) and a coupling between the primordial bispectrum \( B_0(k_1, k_2, k_3) \) (linear in \( f_{\text{NL}} \)) and the second-order PT corrections (through the kernel \( F_2(k_1, k_2) \)). In the case of local non-Gaussianity and for equilateral configurations, the first piece proportional to \( T_0 \) scales as \( (f_{\text{NL}}) k^{-4} \) times the Gaussian tree-level prediction, with the same redshift dependence.

Hence, it is similar to the second-order correction \( (f_{\text{NL}}) k^2 M_{-2}^R P_R(k) \) that appears in the halo power spectrum (see equation (31)). The second piece linear in \( f_{\text{NL}} \) generates a signal at large scales for essentially all triangle shapes in the local model as well as in the case of equilateral NG. This second contribution is maximized in the squeezed limit (where it is one order of magnitude larger than the result obtained by [169]) which helps disentangling it from the Gaussian terms. Note that a strong dependence on triangle shape is also present in other NG scenarios such as the \( \chi^2 \) model [58].

These newly derived contributions are claimed to lead to more than one order of magnitude improvement in certain limits [170], but it is not yet clear whether these gains can be realized in any realistic survey. To accurately predict the constraints that could be achieved with future measurements of the galaxy bispectrum, a comparison of these predictions with the halo bispectrum extracted from numerical simulations is highly desirable. To date, the only numerical study [44] has measured the halo bispectrum for some isosceles triangles \((k_1 = k_2)\). While the shape dependence is in reasonable agreement with the theory, the observed \( k \)-dependence appears to depart from the predicted scaling.

4.5. Intergalactic medium and the Ly\textsubscript{\alpha} forest

Primordial non-Gaussianity also affects the intergalactic medium (IGM) as a positive \( f_{\text{NL}}^N \) enhances the formation of high-mass halos at early times and, therefore, accelerates reionization [186–188]. At lower redshift, small box hydrodynamical simulations of the Ly\textsubscript{\alpha} forest indicate that non-Gaussian initial conditions could leave a detectable signature in the Ly\textsubscript{\alpha} flux PDF, power spectrum and bispectrum [189]. However, while differences appear quite pronounced in the high-transmissivity tail of the flux PDF (i.e. in underdense regions), the Ly\textsubscript{\alpha} 1D flux power spectrum seems little affected. Given the small box size of these hydrodynamical simulations, it is worth exploring the effect in large \( N \)-body cosmological simulations using a semi-analytic modeling of the Ly\textsubscript{\alpha} forest [190]. Figure 4 shows the imprint of local-type NG on the Ly\textsubscript{\alpha} 3D flux power spectrum (which is not affected by projection effects) extracted from a series of large simulations at \( z = 2 \). The Ly\textsubscript{\alpha} transmitted flux is calculated in the Gunn–Peterson approximation [191]. A clear signature similar to the non-Gaussian halo bias can be seen and, as expected, it is of opposite sign since the Ly\textsubscript{\alpha} forest is anti-biased relative to the mass density field (overdensities are mapped onto relatively low-flux transmission). A detection of this effect, although challenging in particular because of continuum uncertainties, could be feasible with future data sets. The Ly\textsubscript{\alpha} could thus provide interesting information on the non-Gaussian signal over a range of scale and redshift not easily accessible to galaxy and CMB observations [189, 190].
5. Current limits and prospects

As the importance of primordial non-Gaussianity relative to the non-Gaussianity induced by gravitational clustering and galaxy bias increases toward high redshift, the optimal strategy to constrain the nonlinear coupling parameter(s) with LSS is to use large-scale, high-redshift observations [34].

5.1. Existing constraints on primordial NG

The non-Gaussian halo bias presently is the only LSS method that provides a robust limit on the magnitude of a primordial three-point function of the local shape. It is a broadband effect that can be easily measured with photometric redshifts. The authors of [148] have applied equation (24) to constrain the value of $f_{\text{loc}}^{\text{NL}}$ using a compilation of large-scale clustering data. Their constraints arise mostly from the QSO sample at the median redshift $z = 1.8$, which covers a large comoving volume and is highly biased, $b_1 = 2.7$. They obtain

$$-29 < f_{\text{loc}}^{\text{NL}} < +69$$

at 95% confidence level. These limits are competitive with those from CMB measurements, $-10 < f_{\text{loc}}^{\text{NL}} < +74$ [192]. It is straightforward to translate this 2-$\sigma$ limit into a constraint on the cubic order coupling $g_{\text{loc}}^{\text{NL}}$ since the non-Gaussian scale-dependent bias $\Delta b_b(k, g_{\text{loc}}^{\text{NL}})$ has the same functional form as $\Delta b_b(k, f_{\text{loc}}^{\text{NL}})$ [100]. Assuming $f_{\text{loc}}^{\text{NL}} = 0$, one obtains

$$-3.5 \times 10^5 < g_{\text{loc}}^{\text{NL}} < +8.2 \times 10^5.$$  

These limits are comparable with those inferred from an analysis of CMB data using $n$-point distribution functions, $-5.6 \times 10^5 < g_{\text{loc}}^{\text{NL}} < 6.4 \times 10^5$ [193] (see also [194]).

Measurements of the galaxy bispectrum in several redshift catalogs have shown evidence for a configuration shape dependence in agreement with that predicted from gravitational instability, ruling out $\chi^2$ initial conditions at the 95% C.L. [195, 196]. Recent analyses of the

Figure 4. Ratio between the Lyman-$\alpha$ 3D flux power spectrum extracted from simulations of Gaussian and non-Gaussian initial conditions at the redshift $z = 2$. The mean transmission is set to $\bar{F} = 0.8$ (figure taken from [190]).
SDSS LRGs catalog indicate that the shape dependence of the reduced three-point correlation $Q_3 \sim \xi_3/\langle \xi_3 \rangle^2$ is also consistent with Gaussian ICs [197], although a primordial (hierarchical) non-Gaussian contribution in the range $Q_3 \sim 0.5–3$ cannot be ruled out [198]. Other LSS probes of primordial non-Gaussianity, such as the abundance of massive clusters, are still too affected by systematics to furnish tight constraints on the shape and magnitude of a primordial three-point function, although the observation of a handful of unexpectedly massive high-redshift clusters has been interpreted as evidence of a substantial degree of primordial NG [199–201].

5.2. Future prospects

Improving the current limits will further constrain the physical mechanisms for the generation of cosmological perturbations.

The non-Gaussian halo bias also leaves a signature in cross-correlation statistics of weak cosmic shear (galaxy–galaxy and galaxy–CMB) [202, 203] and in the integrated Sachs–Wolfe (ISW) effect [148, 149]. Measurements of the lensing bispectrum could also constrain a number of non-Gaussian models [204]. However, galaxy clustering will undoubtedly offer the most promising LSS diagnostic of primordial non-Gaussianity. The detectability of a local primordial bispectrum has been assessed in a series of papers. It is expected that future all-sky galaxy surveys will achieve constraints of the order of $\Delta f_{\text{NL}}^{\text{loc}} \sim 1$ assuming all systematics are reasonably under control [95, 137, 148, 149, 158, 205–207]. Realistic models of cubic-type non-Gaussianity [100], modifications of the initial vacuum state or horizon-scale GR corrections [152] should also be tested with future measurement of the galaxy power spectrum.

Upcoming observations of high-redshift clusters will provide increased leverage on measurement of primordial non-Gaussianity with abundances and possibly put limits on any nonlinear parameter $f_{\text{NL}}^{\text{NL}}$ at the level of a few tens [127]. Combining the information provided by the evolution of the mass function and power spectrum of galaxy clusters can yield constraints with a precision $\Delta f_{\text{NL}}^{\text{loc}} \sim 10$ for a wide field survey covering half of the sky [201]. Alternatively, using the full covariance of cluster counts (which is sensitive to the non-Gaussian halo bias) can furnish constraints of $f_{\text{NL}}^{\text{loc}} \sim 1–5$ for a dark energy survey-type experiment [208, 209].

As emphasized in section 4, however, the exact magnitude of the non-Gaussian halo bias is still uncertain at the $\sim 20\%$ level, partly due to the freedom at the definition of the halo mass. Understanding this type of systematics will be crucial to set reliable constraints on a primordial non-Gaussian component. To fully exploit the potential of future galaxy surveys, it will also be essential to extend the theoretical and numerical analyses to other bispectrum shapes than the local template used so far. Ultimately, the gain that can be achieved will critically depend on our ability to minimize the impact of sampling variance and shot noise. In this regard, multi-tracer methods combined with optimal weighting schemes should deserve further attention as they hold the promise to become the most accurate method to extract the primordial non-Gaussian signal from galaxy surveys [162–165].

Acknowledgments

We give our special thanks to Nico Hamaus and Shirley Ho for sharing with us material prior to publication. We would also like to thank Martin Crocce, Nico Hamaus, Christopher Hirata, Shirley Ho, Ilian Iliev, Tsz Yan Lam, Patrick McDonald, Nikhil Padmanabhan, Emiliano Sefusatti, Ravi Sheth and Anze Slosar for their collaboration on these issues, and Tobias Baldauf for comments on an early version of this manuscript. This work was supported by
the Swiss National Foundation (contract no 200021-116696/1) and made extensive use of the NASA Astrophysics Data System and the arXiv.org preprint server.

References

[1] Mukhanov V F and Chibisov G V 1981 Quantum fluctuations and a nonsingular universe Sov. J. Exp. Theor. Phys. Lett. 33 532
[2] Starobinsky A A 1982 Dynamics of phase transition in the new inflationary universe scenario and generation of perturbations Phys. Lett. B 117 175–8
[3] Hawking S W 1982 The development of irregularities in a single bubble inflationary universe Phys. Lett. B 115 295–7
[4] Guth A H and Pi S-Y 1982 Fluctuations in the new inflationary universe Phys. Rev. Lett. 49 1110–3
[5] Allen T J, Grinstein B and Wise M B 1987 Non-Gaussian density perturbations in inflationary cosmologies Phys. Lett. B 197 66–70
[6] Falk T, Rangarajan R and Srednicki M 1992 Dependence of density perturbations on the coupling constant in a simple model of inflation Phys. Rev. D 46 4232–4
[7] Gangui A, Lucchin F, Matarrese S and Mollerach S 1994 The three-point correlation function of the cosmic microwave background in inflationary models Astrophys. J. 430 447–57
[8] Komatsu E et al 2009 Five-year Wilkinson microwave anisotropy probe observations: cosmological interpretation Astrophys. J. Suppl. 180 330–76
[9] Seljak U and Zaldarriaga M 1997 Signature of gravity waves in the polarization of the microwave background Phys. Rev. Lett. 78 2054–7
[10] Kamionkowski M, Kosowsky A and Stebbins A 1997 A probe of primordial gravity waves and vorticity Phys. Rev. Lett. 78 2058–61
[11] Verde L, Wang L, Heavens A F and Kamionkowski M 2000 Large-scale structure, the cosmic microwave background and primordial non-Gaussianity Mon. Not. R. Astron. Soc. 313 141–7
[12] Scoccimarro R, Sefusatti E and Zaldarriaga M 2004 Probing primordial non-Gaussianity with large-scale structure Phys. Rev. D 69 103513
[13] Dalal N, Doré O, Huterer D and Shiromok A 2008 Imprints of primordial non-Gaussianities on large-scale structure: scale-dependent bias and abundance of virialized objects Phys. Rev. D 77 123514
[14] Salopek D S and Bond J R 1990 Nonlinear evolution of long-wavelength metric fluctuations in inflationary models Phys. Rev. D 42 3936–62
[15] Maldacena J 2003 Non-gaussian features of primordial fluctuations in single field inflationary models J. High Energy Phys. JHEP05(2003)13
[16] Acquaviva V, Bartolo N, Matarrese S and Riotto A 2003 Gauge-invariant second-order perturbations and non-Gaussianity from inflation Nucl. Phys. B 667 119–48
[17] Creminelli P and Zaldarriaga M 2004 A single-field consistency relation for the three-point function J. Cosmol. Astropart. Phys. JCAP10(2004)16
[18] Linde A and Mukhanov V 1997 Non-Gaussian isocurvature perturbations from inflation Phys. Rev. D 56 535
[19] Lyth D H, Ungarelli C and Wands D 2003 Primordial density perturbation in the curvaton scenario Phys. Rev. D 67 023503
[20] Falk T, Rangarajan R and Srednicki M 1993 The angular dependence of the three-point correlation function of the cosmic microwave background radiation as predicted by inflationary cosmologies Astrophys. J. Lett. 403 L1–3
[21] Lesgourgues J, Polarski D and Starobinsky A A 1997 Quantum-to-classical transition of cosmological perturbations for non-vacuum initial states Nucl. Phys. B 497 479–508
[22] Dvali G, Gruzinov A and Zaldarriaga M 2004 Cosmological perturbations from inhomogeneous reheating, freeze-out, and mass domination Phys. Rev. D 69 083505
[23] Lehners J-L and Steinhardt P J 2008 Non-Gaussian density fluctuations from entropically generated curvature perturbations in ekpyrotic models Phys. Rev. D 77 063533
[24] Babich D, Creminelli P and Zaldarriaga M 2004 The shape of non-Gaussianities J. Cosmol. Astropart. Phys. JCAP8(2004)19
[25] Fergusson J R and Shellard E P S 2009 Shape of primordial non-Gaussianity and the CMB bispectrum Phys. Rev. D 80 043510
[26] Komatsu E and Spergel D N 2001 Acoustic signatures in the primary microwave background bispectrum Phys. Rev. D 63 063002
[27] Creminelli P, Nicolas A, Senatore L, Tegmark M and Zaldarriaga M 2006 Limits on non-Gaussianities from WMAP data J. Cosmol. Astropart. Phys. JCAP5(2006)4
[28] Meerburg P D, van der Schaar J P and Stefano Corasaniti P 2009 Signatures of initial state modifications on bispectrum statistics J. Cosmol. Astropart. Phys. JCAP(2009)18
[29] Bernardeau F, Colombi S, Gaztañaga E and Scoccimarro R 2002 Large-scale structure of the Universe and cosmological perturbation theory Phys. Rep. 367 1–248
[30] Gunn J E and Gott J R, III 1972 On the infall of matter into clusters of galaxies and some effects on their evolution Astrophys. J. 176 1
[31] Bond J R and Myers S T 1996 The peak-patch picture of cosmic catalogs: I. Algorithms Astrophys. J. Suppl. 103 1
[32] Durrer R 1999 Topological defects in cosmology New Astron. Rev. 43 111–56
[33] Vilenkin A and Shellard E P S 2000 Cosmic Strings and Other Topological Defects (Cambridge: Cambridge University Press)
[34] Verde L, Jimenez R, Kamionkowski M and Matarrese S 2001 Tests for primordial non-Gaussianity Mon. Not. R. Astron. Soc. 325 412–8
[35] Moscardini L, Matarrese S, Lucchin F and Messina A 1991 Non-Gaussian initial conditions in cosmological N-body simulations: II. Cold dark matter models Mon. Not. R. Astron. Soc. 248 424–38
[36] Weinberg D H and Cole S 1992 Non-Gaussian fluctuations and the statistics of galaxy clustering Mon. Not. R. Astron. Soc. 259 652–94
[37] Coles P, Moscardini L, Lucchin F and Messina A 1993 Skewness as a test of non-Gaussian primordial density fluctuations Mon. Not. R. Astron. Soc. 264 749
[38] Gaztanaga E and Maeloheon P 1996 Large-scale clustering from non-Gaussian texture models Astrophys. J. Lett. 462 L1
[39] White M 1999 Higher order moments of the density field in a parametrized sequence of non-Gaussian theories Mon. Not. R. Astron. Soc. 310 511–6
[40] Mathis H, Silk J, Griffiths L M and Kunz M 2004 Constraining cosmic microwave background consistent primordial voids with cluster evolution Mon. Not. R. Astron. Soc. 350 287–97
[41] Gros0 M, Branchini E, Dolag K, Matarrese S and Moscardini L 2008 The mass density field in simulated non-Gaussian scenarios Mon. Not. R. Astron. Soc. 390 438–46
[42] Desjacques V, Seljak U and Iliev I T 2009 Scale-dependent bias induced by local non-Gaussianity: a comparison to N-body simulations Mon. Not. R. Astron. Soc. 396 85–96
[43] Pillepich A, Porciani C and Hahn O 2010 Halo mass function and scale-dependent bias from N-body simulations with non-Gaussian initial conditions Mon. Not. R. Astron. Soc. 402 191–206
[44] Nishimichi T, Taruya A, Koyama K and Sabiu C 2009 Scale dependence of Halo bispectrum from non-Gaussian initial conditions in cosmological N-body simulations arXiv:0911.4768
[45] Vio R, Andreani P and Wamsteker W 2001 Numerical simulation of non-Gaussian random fields with prescribed correlation structure Pub. Astron. Soc. Pac. 113 1009–20
[46] Luo X and Schramm D N 1993 Kurtosis, skewness, and non-Gaussian cosmological density perturbations Astrophys. J. 408 33–42
[47] Lokas E L, Juszkiewicz R, Weinberg D H and Bouchet F R 1995 Kurtosis of large-scale cosmic fields Mon. Not. R. Astron. Soc. 274 730–44
[48] Peebles P J E 1980 The Large-Scale Structure of the Universe (Princeton, NJ: Princeton University Press)
[49] Fry J N 1984 The Galaxy correlation hierarchy in perturbation theory Astrophys. J. 279 499–510
[50] Coles P and Frenk C S 1991 Skewness and large-scale structure Mon. Not. R. Astron. Soc. 253 727–37
[51] Bouchet F R, Juszkiewicz R, Colombi S and Pellat R 1992 Weakly nonlinear gravitational instability for arbitrary Omega Astrophys. J. Lett. 394 L5–8
[52] Juszkiewicz R, Bouchet F R and Colombi S 1993 Skewness induced by gravity Astrophys. J. Lett. 412 L9–L12
[53] Lahav O, Itoh M, Inagaki S and Suto Y 1993 Non-Gaussian signatures from Gaussian initial fluctuations—evolution of skewness and kurtosis from cosmological simulations in the highly nonlinear regime Astrophys. J. 402 387–97
[54] Fry J N and Scherrer R J 1994 Skewness in large-scale structure and non-Gaussian initial conditions Astrophys. J. 429 36–42
[55] Juszkiewicz R, Weinberg D H, Amsterdamski P, Chodorowski M and Bouchet F 1995 Weakly nonlinear Gaussian fluctuations and the Edgeworth expansion Astrophys. J. 442 39–56
[56] Chodorowski M J and Bouchet F R 1996 Kurtosis in large-scale structure as a constraint on non-Gaussian initial conditions Mon. Not. R. Astron. Soc. 279 557–63
[57] Gaztañaga E and Fosalba P 1998 Cosmological perturbation theory and the spherical collapse model: II. Non-Gaussian initial conditions Mon. Not. R. Astron. Soc. 301 524–34
[58] Scoccimarro R 2000 Gravitational clustering from $\chi^2$ initial conditions Astrophys. J. 542 1–8
[59] Turok N and Spergel D N 1991 Scaling solution for cosmological sigma models at large N Phys. Rev. Lett. 66 3093–6
[60] Durrer R, Juszkiewicz R, Kunz M and Uzan J-P 2000 Skewness as a probe of non-Gaussian initial conditions Phys. Rev. D 62 021301
[61] Lam T Y, Desjacques V and Sheth R K 2010 The non-linear redshift space probability distribution function in models with local primordial non-Gaussianity Mon. Not. R. Astron. Soc. 402 2397–402
[62] Lam T Y and Sheth R K 2009 The non-linear probability distribution function in models with local primordial non-Gaussianity Mon. Not. R. Astron. Soc. 395 1743–8
[63] Taruya A, Koyama K and Matsubara T 2008 Signature of primordial non-Gaussianity on the matter power spectrum Phys. Rev. D 78 123534
[64] Sefusatti E 2009 One-loop perturbative corrections to the matter and galaxy bispectrum with non-Gaussian initial conditions Phys. Rev. D 80 123002
[65] Goroff M H, Grinstein B, Rey S-J and Wise M B 1986 Coupling of modes of cosmological mass density fluctuations Astrophys. J. 311 6–14
[66] Makino N, Sasaki M and Suto Y 1992 Analytic approach to the perturbative expansion of nonlinear gravitational fluctuations in cosmological density and velocity fields Phys. Rev. D 46 585–602
[67] Giannantonio T and Porciani C 2010 Structure formation from non-Gaussian initial conditions: multivariate biasing, statistics, and comparison with N-body simulations Phys. Rev. D 81 063530
[68] Matarrese S and Pietroni M 2007 Resumming cosmic perturbations J. Cosmol. Astropart. Phys. JCAP6(2007)026
[69] Izumi K and Soda J 2007 Renormalized Newtonian cosmic evolution with primordial non-Gaussianity Phys. Rev. D 76 083517
[70] Bartolo N, Beltran Almeida J P, Matarrese S, Pietroni M and Riotto A 2010 Signatures of primordial non-Gaussians in the matter power-spectrum and bispectrum: the time–RG approach J. Cosmol. Astropart. Phys. JCAP03(2010)011
[71] Catelan P and Moscardini L 1994 Kurtosis and large-scale structure Astrophys. J. 426 14–8
[72] Verde L and Heavens A F 2001 On the trispectrum as a Gaussian test for cosmology Phys. Rev. Lett. 92 211301
[73] Cooray A 2005 Large-scale non-Gaussians in the 21-cm background anisotropies from the era of reionization Mon. Not. R. Astron. Soc. 363 1049–56
[74] Pillepich A, Porciani C and Matarrese S 2007 The bispectrum of redshifted 21 centimeter fluctuations from the dark ages Astrophys. J. 662 1–14
[75] Bartelmann M and Schneider P 2001 Weak gravitational lensing Phys. Rep. 340 291–472
[76] White S D M and Rees M J 1978 Core condensation in heavy halos—a two-stage theory for galaxy formation and clustering Mon. Not. R. Astron. Soc. 183 341–58
[77] Kaiser N 1984 On the spatial correlations of Abell clusters Astrophys. J. Lett. 284 L9–12
[78] Desjacques V 2008 Baryon acoustic signature in the clustering of density maxima Phys. Rev. D 78 103503
[79] McDonald P and Roy A 2009 Clustering of dark matter tracers: generalizing bias for the coming era of precision LSS J. Cosmol. Astropart. Phys. JCAP8(2009)20
[80] Lacey C and Cole S 1994 Merger rates in hierarchical models of galaxy formation: part two. Comparison with N-body simulations Mon. Not. R. Astron. Soc. 271 676
[81] Davis M, Efstathiou G, Frenk C S and White S D M 1985 The evolution of large-scale structure in a universe dominated by cold dark matter Astrophys. J. 292 371–94
[82] Luki´c Z, Reed D, Habib S and Heitmann K 2009 The structure of Halos: implications for group and cluster cosmology Astrophys. J. 692 217–28
[83] Lucchin F and Matarrese S 1988 The effect of non-Gaussian statistics on the mass multiplicity of cosmic structures Astrophys. J. 330 535–44
[84] Colafrancesco S, Lucchin F and Matarrese S 1989 The mass function from local density maxima—groups and clusters of galaxies Astrophys. J. 345 3–11
[85] Coles P and Barrow J D 1987 Non-Gaussian statistics and the microwave background radiation Mon. Not. R. Astron. Soc. 228 407–26
[86] Press W H and Schechter P 1974 Formation of galaxies and clusters of galaxies by self-similar gravitational condensation Astrophys. J. 187 425–38
[87] Peacock J A and Heavens A F 1990 Alternatives to the Press–Schechter cosmological mass function Mon. Not. R. Astron. Soc. 243 133–43
[88] Cole S 1991 Modeling galaxy formation in evolving dark matter halos Astrophys. J. 367 45–53
[89] Bond J R, Cole S, Efstathiou G and Kaiser N 1991 Excursion set mass functions for hierarchical Gaussian fluctuations Astrophys. J. 379 440–60
[92] Sheth R K and Tormen G 1999 Large-scale bias and the peak background split Mon. Not. R. Astron. Soc. 308 119–26
[93] Maggiore M and Riotto A 2010 The Halo mass function from excursion set theory: I. Gaussian fluctuations with non-Markovian dependence on the smoothing scale Astrophys. J. 711 907
[94] Matarrese S, Verde L and Jimenez R 2000 The abundance of high-redshift objects as a probe of non-Gaussian initial conditions Mon. Not. R. Astron. Soc. 308 119–26
[95] Lo Verde M, Miller A, Shandera S and Verde L 2008 Effects of scale-dependent non-Gaussianity on cosmological structures J. Cosmol. Astropart. Phys. JCAP(2008)14
[96] Matarrese S, Verde L and Jimenez R 2000 The abundance of high-redshift objects as a probe of non-Gaussian initial conditions Mon. Not. R. Astron. Soc. 308 119–26
[97] Grossi M, Dolag K, Branchini E, Matarrese S and Moscardini L 2007 Evolution of massive haloes in non-Gaussian scenarios Mon. Not. R. Astron. Soc. 382 1261–7
[98] Grossi M, Verde L, Carbone C, Dolag K, Branchini E, Iannuzzi F, Matarrese S and Moscardini L 2009 Large-scale non-Gaussian mass function and halo bias: tests on N-body simulations Mon. Not. R. Astron. Soc. 398 321–32
[99] Kang X, Norberg P and Silk J 2007 Can a large-scale structure probe cosmic microwave background-constrained non-Gaussianity? Mon. Not. R. Astron. Soc. 376 343–7
[100] Desjacques V and Seljak U 2010 Signature of primordial non-Gaussianity of $\phi^3$ type in the mass function and bias of dark matter haloes Phys. Rev. D 81 023006
[101] Maggiore M and Riotto A 2009 The Halo mass function from excursion set theory with a non-Gaussian trispectrum arXiv:0910.5125
[102] Maggiore M and Riotto A 2009 The Halo mass function from excursion set theory: II. The diffusing barrier arXiv:0903.1250
[103] Maggiore M and Riotto A 2009 The Halo mass function from excursion set theory: III. Non-Gaussian fluctuations arXiv:0903.1251
[104] Lam T Y and Sheth R K 2009 Halo abundances in the $f_{\sigma_8}$ model Mon. Not. R. Astron. Soc. 398 2143–51
[105] Lam T Y, Sheth R K and Desjacques V 2009 The initial shear field in models with primordial local non-Gaussianity and implications for halo and void abundances Mon. Not. R. Astron. Soc. 399 1482–94
[106] Avelino P P and Viana P T P 2000 The cloud-in-cloud problem for non-Gaussian density fields Mon. Not. R. Astron. Soc. 314 354–8
[107] Inoue K T and Nagashima M 2002 Analytic approach to the cloud-in-cloud problem for non-Gaussian density fluctuations Astrophys. J. 574 9–18
[108] Sheth R K, Mo H J and Tormen G 2001 Ellipsoidal collapse and an improved model for the number and spatial distribution of dark matter haloes Mon. Not. R. Astron. Soc. 323 1–12
[109] Desjacques V 2008 Environmental dependence in the ellipsoidal collapse model Mon. Not. R. Astron. Soc. 388 638–58
[110] Matarrese S, Lucchin F and Bonometto S A 1991 Non-Gaussian initial conditions in cosmological N-body simulations: III—Groups in cold dark matter models Mon. Not. R. Astron. Soc. 253 35–46
[111] Park C, Spergel D N and Turok N 1991 Large-scale structure in a texture-seeded cold dark matter cosmogony Astrophys. J. Lett. 372 L53–7
[112] Borgani S and Bonometto S A 1989 Galaxy density in biased theories of galaxy origin Astron. Astrophys. 215 17–20
[113] Matarrese S, Lucchin F, Messina A and Moscardini L 1991 Non-Gaussian initial conditions in cosmological N-body simulations: III—Groups in cold dark matter models Mon. Not. R. Astron. Soc. 253 35–46
[114] Park C, Spergel D N and Turok N 1991 Large-scale structure in a texture-seeded cold dark matter cosmogony Astrophys. J. Lett. 372 L53–7
[115] Borgani S, Coles P, Moscardini L and Plionis M 1994 The angular distribution of clusters in skewed CDM models/cold dark matter Mon. Not. R. Astron. Soc. 266 524–44
[116] Mathis H, Diego J M and Silk J 2004 The case for non-Gaussianity on cluster scales Mon. Not. R. Astron. Soc. 353 681–8
[117] Robinson J and Baker J E 2000 Evolution of the cluster abundance in non-Gaussian models Mon. Not. R. Astron. Soc. 311 781–92
[118] Stanek R, Evrard A E, Böhringer H, Schuecker P and Nord B 2006 The x-ray luminosity–mass relation for local clusters of galaxies Astrophys. J. 648 956–68
[119] Nagai D 2006 The impact of galaxy formation on the Sunyaev–Zel’dovich effect of galaxy clusters Astrophys. J. 650 538–49
[120] Chiu W A, Ostriker J P and Strauss M A 1998 Using cluster abundances and peculiar velocities to test the Gaussianity of the cosmological density field Astrophys. J. 494 479
[121] Koyama K, Soda J and Taruya A 1999 Constraints on a non-Gaussian (χ²) cold dark matter model Mon. Not. R. Astron. Soc. 310 1111–8
[122] Robinson J, Gawiser E and Silk J 2000 Constraining primordial non-Gaussianity with the abundance of high-redshift clusters Astrophys. J. 532 1–16
[123] Benson A J, Reichardt C and Kamionkowski M 2002 Statistics of Sunyaev–Zel’dovich cluster surveys Mon. Not. R. Astron. Soc. 331 71–84
[124] Sadeh S, Rephaeli Y and Silk J 2006 Impact of a non-Gaussian density field on Sunyaev–Zeldovich observables Mon. Not. R. Astron. Soc. 368 1585–98
[125] Sadeh S, Rephaeli Y and Silk J 2007 Cluster abundances and Sunyaev–Zel’dovich power spectra: effects of non-Gaussianity and early dark energy Mon. Not. R. Astron. Soc. 380 637–45
[126] Sefusatti E, Vale C, Kadota K and Frieman J 2007 Primordial non-Gaussianity and dark energy constraints from cluster surveys Astrophys. J. 658 669–79
[127] Fedeli C, Moscardini L and Matarrese S 2009 The clustering of galaxy clusters in cosmological models with non-Gaussian initial conditions: predictions for future surveys Mon. Not. R. Astron. Soc. 397 1125–37
[128] Roncarelli M, Moscardini L, Branchini E, Dolag K, Grossi M, Iannuzzi F and Matarrese S 2009 Imprints of primordial non-Gaussianities in x-ray and SZ signals from galaxy clusters arXiv:0909.4714
[129] Oukbir J, Bartlett J G and Blanchard A 1997 X-ray galaxy clusters: constraints on models of galaxy formation. Astron. Astrophys. 320 365–77
[130] Henry J P 2000 Measuring cosmological parameters from the evolution of cluster x-ray temperatures Astrophys. J. 534 565–80
[131] Amara A and Refregier A 2004 Power spectrum normalization and the non-Gaussian halo model Mon. Not. R. Astron. Soc. 351 375–83
[132] Melott R and Seljak U 2007 A robust lower limit on the amplitude of matter fluctuations in the universe from cluster abundance and weak lensing J. Cosmol. Astropart. Phys. JCAP6(2007)24
[133] Kamionkowski M, Verde L and Jimenez R 2009 The void abundance with non-Gaussian primordial perturbations J. Cosmol. Astropart. Phys. JCAP1(2009)10
[134] Oukbir J, Bartlett J G and Blanchard A 1997 X-ray galaxy clusters: constraints on models of galaxy formation. Astron. Astrophys. 320 365–77
[135] Doroshkevich A G 1970 The space structure of perturbations and the origin of rotation of galaxies in the theory of fluctuation Astrofizika 6 581–600
[136] Song H and Lee J 2009 The mass function of void groups as a probe of primordial non-Gaussianity Astrophys. J. Lett. 701 L25–8
[137] Sheth R K and van de Weygaert R 2004 A hierarchy of voids: much ado about nothing Mon. Not. R. Astron. Soc. 350 517–38
[138] Colberg J M et al 2008 The Aspen–Amsterdam void finder comparison project Mon. Not. R. Astron. Soc. 387 933–44
[139] Dressler A 1980 Galaxy morphology in rich clusters—implications for the formation and evolution of galaxies Astrophys. J. 236 351–65
[140] Peebles P J E 1983 The sequence of cosmogony and the nature of primeval departures from homogeneity Astrophys. J. 274 1–6
[141] Bahcall N A and Soneira R M 1983 The spatial correlation function of rich clusters of galaxies Astrophys. J. 270 20–38
[142] Klypin A A and Kopylov A I 1983 The spatial covariance function for rich clusters of galaxies Sov. Astron. Lett. 9 41
[143] Grinstein B and Wise M B 1986 Non-Gaussian fluctuations and the correlations of galaxies or rich clusters of galaxies Astrophys. J. 310 19–22
[144] Andrada A P A, Ribeiro A L B and Wuensche C A 2006 High order correction terms for the peak-peak correlation function in nearly-Gaussian models Astron. Astrophys. 457 385–91
[145] Matarrese S and Verde L 2008 The effect of primordial non-Gaussianity on halo bias Astrophys. J. Lett. 677 L77–80
[146] Bardeen J M, Bond J R, Kaiser N and Szalay A S 1986 The statistics of peaks of Gaussian random fields Astrophys. J. 304 15–61
[147] Cole S and Kaiser N 1989 Biased clustering in the cold dark matter cosmogony Mon. Not. R. Astron. Soc. 237 1127–46
[148] Slosar A, Hirata C, Seljak U, Ho S and Padmanabhan N 2008 Constraints on local primordial non-Gaussianity from the large scale structure J. Cosmol. Astropart. Phys. JCAP8(2008)31
[149] Afshordi N and Tolley A J 2008 Primordial non-Gaussianity, statistics of collapsed objects, and the integrated Sachs–Wolfe effect Phys. Rev. D 78 123507
[150] Peebles P J E 1999 An isocurvature cold dark matter cosmogony: II. Observational tests Astrophys. J. 510 531–40
[151] Cen R Y, Ostriker J P, Spergel D N and Turok N 1991 A hydrodynamic approach to cosmology—texture-seeded cold dark matter and hot dark matter cosmogonies Astrophys. J. 383 1–18
[152] Verde L and Matarrese S 2009 Detectability of the effect of inflationary non-Gaussianity on halo bias Astrophys. J. Lett. 706 L91–5
[153] Hattimimoaiglou E, Mathez G, Solanes J-M, Manrique A and Salvador Solé E 2003 Major mergers of haloes, the growth of massive black holes and the evolving luminosity function of quasars Mon. Not. R. Astron. Soc. 343 692–704
[154] Hopkins P F, Hernquist L, Cox T J, Di Matteo T, Martini P, Robertson B and Springel V 2005 Black holes in galaxy mergers: evolution of quasars Astrophys. J. 630 705–15
[155] Bonoli S, Shankar F, White S, Springel V and Wyithe S 2009 On merger bias and the clustering of quasars arXiv:0909.0003
[156] Bartolo N, Matarrese S and Riotto A 2005 Signatures of primordial non-Gaussianity in the large-scale structure of the universe J. Cosmol. Astropart. Phys. JCAP10(2005)10
[157] Wands D and Slosar A 2009 Scale-dependent bias from primordial non-Gaussianity in general relativity Phys. Rev. D 79 123507
[158] McDonald P 2008 Primordial non-Gaussianity: large-scale structure signature in the perturbative bias model Phys. Rev. D 78 123519
[159] Kaiser N 1987 Clustering in real space and in redshift space Mon. Not. R. Astron. Soc. 227 1–21
[160] Hamilton A J S 1998 Linear redshift distortions: a review The Evolving Universe (Astrophysics and Space Science Library vol 231) ed D Hamilton (Dordrecht: Kluwer)
[161] Feldman H A, Kaiser N and Peacock J A 1994 Power-spectrum analysis of three-dimensional redshift surveys Astrophys. J. 426 23–37
[162] Seljak U 2009 How to evade the sample variance limit on measurements of redshift-space distortions J. Cosmol. Astro-Particle Phys. JCAP3(2009)4
[163] Hamaus N et al 2010 in preparation
[164] Fan Z and Bardeen J M 1995 Distributions of Fourier modes of cosmological density fields Phys. Rev. D 51 6714–21
[165] Stirling A J and Peacock J A 1996 Power correlations in cosmology: limits on primordial non-Gaussian density fields Mon. Not. R. Astron. Soc. 283 L99+
[166] Sefusatti E and Komatsu E 2007 Bispectrum of galaxies from high-redshift galaxy surveys: primordial non-Gaussian and nonlinear galaxy bias Phys. Rev. D 76 083004
[167] Jeong D and Komatsu E 2009 Primordial non-Gaussianity, scale-dependent bias, and the bispectrum of galaxies Astrophys. J. 703 1230–48
[168] Gaztanaga E 1992 N-point correlation functions in the CFA and SSRS redshift distribution of galaxies Astrophys. J. Lett. 398 L17–20
[169] Gaztanaga E and Yokoyama J 1993 Probing the statistics of primordial fluctuations and their evolution Astrophys. J. 403 450–65
[170] Bouchet F R, Strauss M A, Davis M, Fisher K B, Yahil A and Huchra J P 1993 Moments of the counts distribution in the 1.2 Jansky IRAS Galaxy Redshift Survey Astrophys. J. 417 36
[171] Matsubara T and Suto Y 1996 Nonlinear evolution of genus in a primordial random Gaussian density field Astrophys. J. 443 469–78
[172] Croton D J et al 2004 The 2dF Galaxy Redshift Survey: higher-order galaxy correlation functions Mon. Not. R. Astron. Soc. 352 1232–44
[173] Friith W J, Outram P J and Shank J 2006 High-order 2MASS galaxy correlation functions: probing the Gaussianity of the primordial density field Mon. Not. R. Astron. Soc. 373 759–68
[174] Matsubara T and Suto Y 1996 Nonlinear evolution of genus in a primordial random Gaussian density field Astrophys. J. 460 51
[175] Coles P, Moscardini L, Platini M, Lucchin F, Matarrese S and Messina A 1993 Topology in two dimensions: IV. CDM models with non-Gaussian initial conditions Mon. Not. R. Astron. Soc. 260 572–88
[180] Matsubara T and Yokoyama J 1996 Genus statistics of the large-scale structure with non-Gaussian density fields Astrophys. J. 463 409

[181] Hikage C, Tariya A and Sato Y 2001 Genus statistics for galaxy clusters and nonlinear biasing of dark matter halos Astrophys. J. 556 641–52

[182] Hikage C, Coles P, Grossi M, Moscardini L, Dolag K, Branchini E and Matarrese S 2008 The effect of primordial non-Gaussianity on the topology of large-scale structure Mon. Not. R. Astron. Soc. 385 1613–20

[183] Gott J R, III, Hambrick D C, Vogelezang M S, Kim J, Park C, Choi Y-Y, Cen R, Ostriker J P and Nagamine K 2008 Genus topology of structure in the Sloan Digital Sky Survey: model testing Astrophys. J. 675 16–28

[184] James J B, Colless M, Lewis G F and Peacock J A 2009 Topology of non-linear structure in the 2dF Galaxy Redshift Survey Mon. Not. R. Astron. Soc. 394 454–66

[185] Friy N and Gaztanaga E 1993 Biasing and hierarchical statistics in large-scale structure Astrophys. J. 413 447–52

[186] Chen X, Cooray A, Yoshida N and Sugiyama N 2003 Can non-Gaussian cosmological models explain the WMAP high optical depth for reionization? Mon. Not. R. Astron. Soc. 346 L31–5

[187] Avelino P P and Liddle A R 2006 Cosmic reionization constraints on the nature of cosmological perturbations Mon. Not. R. Astron. Soc. 371 1755–9

[188] Hikage C, Taruya A and Suto Y 2001 Genus statistics for galaxy clusters and nonlinear biasing of dark matter halos Astrophys. J. 463 409

[189] Hikage C, Coles P, Grossi M, Moscardini L, Dolag K, Branchini E and Matarrese S 2008 The effect of primordial non-Gaussianity on the topology of large-scale structure Mon. Not. R. Astron. Soc. 385 1613–20

[190] Gott J R, III, Hambrick D C, Vogelezang M S, Kim J, Park C, Choi Y-Y, Cen R, Ostriker J P and Nagamine K 2008 Genus topology of structure in the Sloan Digital Sky Survey: model testing Astrophys. J. 675 16–28

[191] James J B, Colless M, Lewis G F and Peacock J A 2009 Topology of non-linear structure in the 2dF Galaxy Redshift Survey Mon. Not. R. Astron. Soc. 394 454–66

[192] Fry J N and Gaztanaga E 1993 Biasing and hierarchical statistics in large-scale structure Astrophys. J. 413 447–52

[193] Chen X, Cooray A, Yoshida N and Sugiyama N 2003 Can non-Gaussian cosmological models explain the WMAP high optical depth for reionization? Mon. Not. R. Astron. Soc. 346 L31–5

[194] Frieman J A and Gaztañaga E 1999 The projected three-point correlation function: theory and observations Astrophys. J. Lett. 521 L83–6

[195] Scoccimarro R, Feldman H A, Fry J N and Frieman J A 2001 The bispectrum of IRAS redshift catalogs Astrophys. J. 546 652–64

[196] Kulkarni G V, Nichol R C, Sheth R K, Seo H-J, Eisenstein D J and Gray A 2007 The three-point correlation function of luminous red galaxies in the Sloan Digital Sky Survey Mon. Not. R. Astron. Soc. 378 1196–206

[197] Gaztañaga E, Cabrér A, Castander F, Crocce M and Fosalba P 2009 Clustering of luminous red galaxies: III. Baryon acoustic peak in the three-point correlation Mon. Not. R. Astron. Soc. 399 774–82

[198] Frieman J A and Gaztañaga E 1999 The projected three-point correlation function: theory and observations Astrophys. J. Lett. 521 L83–6

[199] Sartoris B, Borgani S, Fedeli C, Matarrese S, Moscardini L, Rosati P and Weller J 2010 The potential of x-ray cluster surveys to constrain primordial non-Gaussianity with WMAP 5-Year data arXiv:1003.0841

[200] Jimenez R and Verde L 2009 Implications for primordial non-Gaussianity (fNL) from weak lensing masses of high-z galaxy clusters Phys. Rev. D 80 123527

[201] Carbone C, Mena O and Verde L 2010 Cosmic shear statistics in cosmologies with non-Gaussian initial conditions arXiv:0912.4112

[202] Oguri M 2009 Self-calibrated cluster counts as a probe of primordial non-Gaussianity Phys. Rev. Lett. 102 211301

[203] Cunha C, Huterer D and Dore O 2010 Primordial non-Gaussianity from the covariance of galaxy cluster counts arXiv:1003.2416