Spectral Flattening at Low Frequencies in Crab Giant Pulses

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Abstract

We report on simultaneous wideband observations of Crab giant pulses with the Parkes radio telescope and the Murchison Widefield Array (MWA). The observations were conducted simultaneously at 732 and 3100 MHz with Parkes and at 120.96, 165.76, and 210.56 MHz with the MWA. Flux density calibration of the MWA data was accomplished using a novel technique based on tied-array beam simulations. We detected between 90 and 648 giant pulses in the 120.96–210.56 MHz MWA subbands above a 5.5σ threshold, while in the Parkes subbands we detected 6344 and 231 giant pulses above a threshold of 6σ at 732 and 3100 MHz, respectively. We show, for the first time over a wide frequency range, that the average spectrum of Crab giant pulses exhibits a significant flattening at low frequencies. The spectral index, α, for giant pulses evolves from a steep, narrow distribution with a mean α = −2.6 and width σα = 0.5 between 732 and 3100 MHz to a wide, flat distribution of spectral indices with a mean α = −0.7 and width σα = 1.4 between 120.96 and 165.76 MHz. We also comment on the plausibility of giant pulse models for fast radio bursts based on this spectral information.

Key words: pulsars: general – pulsars: individual (PSR J0534+2200) – instrumentation: interferometers

1. Introduction

The Crab pulsar (PSR J0534+2200) was discovered through its giant pulse emission (Staelin & Reifenstein 1968). Giant pulses are short-duration bursts of emission, lasting for ≲ 1 ns to ~10 μs, that appear only within a small fraction of the normal pulse phase window (Hankins et al. 2003; Popov & Stappers 2007; Bhat et al. 2008). Individual giant pulses are observed to have brightness temperatures in the range T_b ~ 10^{30–32} K, implying a coherent emission mechanism. At extremely high time resolution, Crab giant pulses have been observed to reach brightness temperatures of 10^{41} K, corresponding to a peak flux density of S_{peak} = 2.2 MJy at 9 GHz (Hankins & Eilek 2007). Giant pulses are therefore invaluable tools for understanding pulsar emission and, more generally, astrophysical coherent emission mechanisms from a variety of objects.

It has been established that the occurrence of giant pulse energies follows a power-law distribution (e.g., Argyle & Gower 1972; Cordes et al. 2004; Bhat et al. 2008; Oronsaye et al. 2015), while normal pulse energies tend to exhibit an exponential or log-normal distribution (e.g., Burke-Spolaor et al. 2012). There are six pulsars known to exhibit giant pulses. These include two young pulsars (PSRs J0534+2200 and J0540–6919) and four millisecond pulsars (PSRs J0218+4232, J1823–3021A, J1824–2452A, and J1939+2134; Knight et al. 2006), all of which have high magnetic field strengths at the light-cylinder radius (B_{LC} ~ 10^{6–8} G). The giant pulses from these six objects occur within a confined phase location, are of intrinsically short duration (microseconds or less), and exhibit a power-law pulse energy distribution. In the literature, there are several other pulsars that emit large-amplitude pulses, often referred to as “giant pulses” (e.g., B0950+08: Singal & Vats 2012; Tsai et al. 2015, 2016; J1752+2359: Ershov & Kuzmin 2005). It is not necessarily clear if the emission from these pulsars shares the distinctive characteristics exhibited by the above six confirmed cases.

The physics responsible for producing these coherent bursts of radio emission is unknown but is thought to be a broadband, nonlinear plasma process (e.g., Eilek & Hankins 2016; Melrose & Yuen 2016) that is able to produce detectable emission from radio to γ-ray frequencies (e.g., Abdo et al. 2010). While Crab giant pulses appear to be a broadband phenomenon, detectable across the full observing bandwidth in most observations, they are not expected to always be detected simultaneously over multiple widely separated frequency bands (e.g., Sallmen et al. 1999; Oronsaye et al. 2015).

The flux density spectrum of normal pulsar emission is typically described by a simple power-law model S ∝ ν^α, where ⟨α⟩ = −1.8 ± 0.2, for observing frequencies > 100 MHz (e.g., Sieber 1973; Lorimer et al. 1995; Maron et al. 2000). The underlying distribution of pulsar spectral indices, based on Monte Carlo simulations of pulsar surveys, has also been modeled as a Gaussian distribution with a mean of ⟨α⟩ = −1.4 ± 1 (Bates et al. 2013). Only a handful of cases (≈10%) are known where a different spectral shape is observed, such as a broken power law or flat spectrum. There are also the peculiar “gigahertz peaked spectra” (GPS) pulsars (Kijak et al. 2007, 2011), where the spectrum peaks and turns over at ~1 GHz. The spectral shape of these GPS pulsars is believed to be a consequence of the pulsar local environment (e.g., Dembska et al. 2012; Rajwade et al. 2016). For giant pulses, spectral flattening or a turnover has not yet been directly observed. Oronsaye et al. (2015) suggested, via Monte Carlo analysis, that there was an ~5% flattening of the spectral index distribution mean between 193 and 1382 MHz. More simultaneous wideband observations are therefore necessary to constrain the spectral behavior of giant pulses, both individually and statistically, for the population.
Multifrequency simultaneous observations of the Crab have previously been undertaken, though typically only between two frequencies (e.g., Bhat et al. 2008; Orosay et al. 2015) or over a narrow frequency range (e.g., Karuppusamy et al. 2012; Eftekhari et al. 2016). In order to further constrain the giant pulse emission mechanism, wideband simultaneous observations with intermediate-frequency coverage such as that conducted by Mikami et al. (2016) are required to uncover the broadband spectral behavior.

With the advent of the fast radio burst (FRB) phenomenon, especially the repeating FRB 121102 (Spitler et al. 2014, 2016; Scholz et al. 2016), several theories have been put forth suggesting that at least some FRBs may originate from extragalactic giant pulses (e.g., Connor et al. 2016; Cordes & Wasserman 2016; Lyutikov et al. 2016). Determining the spectral behavior of simultaneously detected Crab giant pulses over a wide frequency range will also provide clues regarding a giant pulse origin of FRBs, especially given the paucity of low-frequency detections.

In this article, we report on simultaneous observations of giant pulses from the Crab pulsar conducted with the Parkes radio telescope and the Murchison Widefield Array (MWA; Tingay et al. 2013). The Parkes 64 m radio telescope is well known for pulsar science and facilitated our high-frequency observations (732 MHz and 3.1 GHz). The MWA is a low-frequency (70–300 MHz) Square Kilometre Array precursor located in Western Australia at the Murchison Radio-astronomy Observatory. With the high time resolution Voltage Capture System (VCS; Tremblay et al. 2015), the MWA provided our low-frequency observations. We present the detection and analysis of simultaneous giant pulses from the MWA and the Parkes radio telescope covering 120–3100 MHz with one to three intermediate observing bands.

This paper is organized as follows. In Section 2, we describe the setup for the MWA and Parkes observations, and, in Section 3, we describe the postprocessing and data calibration. Section 4 describes the methods used to detect simultaneous giant pulses from both instruments and details the results of the analysis, focusing on giant pulse spectra. In Section 5, we discuss the implications of our results for the giant pulse emission mechanism and briefly comment on the applicability of a giant pulse model to explain the emission observed from FRBs. We summarize and conclude in Section 6. Throughout, we adopt $S_\nu$ to represent flux densities and $F_\nu$ to represent fluxes at frequency $\nu$.

2. Observations

The Crab pulsar was observed simultaneously with Parkes and the MWA on 2014 November 7. Parkes observed the pulsar at 732 and 3100 MHz for 1.4 hr. The MWA/VCS data collection was split into two distinct observations totaling 1.3 hr. The first 20 minute observation was conducted at a central frequency of 184.96 MHz. Immediately following this, the second observation lasted for 1 hr and was designed such that the MWA bandwidth was split into four subbands distributed between 120.96 and 278.40 MHz. Observation details are summarized in Table 1.

2.1. Parkes

We observed the Crab pulsar using the coaxial 1050 cm receiver on the 64 m Parkes radio telescope, which is capable of simultaneously recording signals at 732 MHz (64 MHz bandwidth) and 3100 MHz (1024 MHz bandwidth). Both systems are sensitive to linear polarization. Data were recorded with the mark-3 and mark-4 versions of the Parkes digital filterbank spectrometers (PDFB3 and PDFB4) for a duration of $\approx 5000$ s. The spectrometers employ polyphase digital filters, with PDFB3 recording data with 512 channels across the 64 MHz low-frequency band and PDFB4 recording data with 512 channels across the 1024 MHz high-frequency band. Data were recorded in polarimetric search mode. For each channel, four coherency products (the power from each probe and the complex-valued correlated power between the two) were detected and averaged over 256 $\mu$s before being written to disk with 8-bit precision.

The decorrelation bandwidths ($\Delta \nu_{\text{DISS}}$) due to diffusive scintillation at 732 and 3100 MHz are 35 kHz and 6 MHz, respectively, assuming $\Delta \nu_{\text{DISS}} = 2.3$ MHz at 2.33 GHz (Cordes et al. 2004) and a scaling of $\Delta \nu_{\text{DISS}} \propto \nu^{3.6}$ (e.g., Ellingson et al. 2013; Eftekhari et al. 2016, F. Kirsten et al. 2017, in preparation). Over the respective bandwidths of the observing frequency bands, these contributions are negligible. The refractive timescales are 2 days and $\approx 7$ hr, respectively. On the timescales we are probing, we do not expect any significant contribution from scintillation to the giant pulse flux densities in the 732 MHz band. In the 3100 MHz band, we expect that the small contribution from scintillation will be dominated by the measurement scatter in giant pulsar flux densities.

2.2. MWA

The MWA is a low-frequency array composed of 128 tiles, with each tile consisting of 16 dipoles evenly spaced in a regular $4 \times 4$ m grid. The MWA has 30.72 MHz instantaneous bandwidth that can be separated into 24 independent 1.28 MHz subbands, which can be distributed across the 70–300 MHz observing range.

The VCS is the high time and frequency resolution observing system for the MWA, capable of capturing the tile voltages after the channelization stage within the MWA signal processing pipeline. This allows critically sampled complex tile voltages (100 $\mu$s time resolution, 10 kHz frequency resolution) to be recorded to on-site disks at a data rate of $\approx 28$ TB hr$^{-1}$. Using the VCS, we recorded $\approx 4826$ s of data from the array pointed toward the Crab pulsar (see Table 2). As previously mentioned, this observing run was split into two observations. The first 20 minutes were with the full 30.72 MHz of bandwidth centered at 184.96 MHz. The remaining 60 minutes were observed with the bandwidth split into four 7.68 MHz subbands, distributed to center frequencies of 120.96, 165.76, 210.56, and 278.40 MHz.

At MWA frequencies, the decorrelation bandwidths due to diffusive scintillation are in the range $\Delta \nu_{\text{DISS}} \approx 50–1000$ Hz at the observed MWA bands. The refractive timescales are between 8 and 25 days. Therefore, we do not expect any contribution from scintillation to be significant in our intensity estimates for the Crab giant pulses at MWA frequencies.

3. Data Processing and Calibration

3.1. Parkes

Absolute flux density calibration was performed by observing the radio galaxy Hydra A (3C 218) as part of the Parkes
Pulsar Timing Array (PPTA) project (Manchester et al. 2013). Polarization calibration was conducted by injecting a linearly polarized signal into the feeds. This allowed us to measure the frequency-dependent differential gain and phase of the two feeds. We did not correct for feed ellipticity or cross-coupling.

The 732 MHz data were incoherently dedispersed and folded using DSPSR (van Straten & Bailes 2011) with an ephemeris from the Jodrell Bank monthly monitoring. A more accurate pulsar ephemeris was produced from these data, fitting for the optimum period, period derivative, and dispersion measure (DM) with TEMPO2 (Hobbs et al. 2006). The dispersion measure calculated by this process was 56.7762 pc cm\(^{-3}\) and is henceforth taken as the nominal dispersion measure for the Crab pulsar.

Data from both Parkes bands were then reprocessed using DSPSR and the updated ephemeris, subdivideing the data streams into individual pulses. The pulses were flux density- and polarization-calibrated using PSRCHIVE (Hotan et al. 2004) routines. Radio frequency interference (RFI) was removed using the PAZ routine, flagging the edge 5% of each band and running the built-in median-smoothed difference excision algorithm.

### 3.2. MWA

Calibrating the MWA data is nontrivial, especially in the case of VCS recorded data for which there is currently no dedicated automatic calibration pipeline. The Crab Nebula was selected as the calibrator source for both the 184.96 MHz full-bandwidth observation and the split-bandwidth observation. Visibilities for each observation were created using an online version of the MWA correlator (which performs the same function as the online version; Ord et al. 2015). For each band, a calibration solution (amplitude and phase) for each tile was calculated from the visibilities using the Real Time System (RTS; Mitchell et al. 2008). The output from the RTS is a calibration solution for each coarse channel containing the calibration information for each MWA tile; thus, there is a set of 24 solutions per observation. Due to poor-quality calibration solutions, data from eight of the 128 tiles for the full-bandwidth observation were discarded, while 21 of the 128 tiles were discarded for the split-bandwidth observation.

The MWA tiles and beam models are less well characterized at higher frequencies (\(\nu \sim 300\) MHz); moreover, there are increased levels of satellite-based RFI, making calibration significantly more difficult. Owing to the poor calibration solution quality at the 278.40 MHz band, the data were discarded, leaving us with three usable subbands (120.96–210.56 MHz) and one band at 184.96 MHz (see Figure 1).

The MWA uses analog beam formers to set the pointing direction of each tile; thus, there are a discrete set of delays available. For our observations, this means that the tile beam is never pointed directly at the Crab, and so we are never at full sensitivity. The MWA tile beam is very complex; thus, in some

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**Table 1**

| Parameter                      | MWA    | Parkes |
|--------------------------------|--------|--------|
| Center frequency (MHz)         | 120.96 | 184.96 |
| Bandwidth (MHz)                | 7.68   | 30.72  |
| FWHM (arcmin)                  | 2.63   | 2.36   |
| Time resolution (\(\mu\)s)     | 100    | 100    |
| Frequency resolution (MHz)     | 0.01   | 0.01   |
| Dispersion delay across bandwidth\(^a\) (ms) | 2048.48 | 2319.09 |
| Dispersion delay in lowest channel\(^b\) (ms) | 2.93 | 0.96 |
| Start time (UTC)               | 17:14:00 | 17:14:00 |
| Observation duration (s)       | 3663   | 3663   |

**Notes.**

\(^a\) The 278.4 MHz subband was excluded due to poor-quality calibration solutions (see the text).

\(^b\) Assuming a nominal dispersion measure of 56.7762 pc cm\(^{-3}\).

**Table 2**

| Pointing Center (Az., El.) (deg, deg) | Center Frequency (MHz) | Observation Efficiency | Receiver Temperature \(T_{rec}\) (K) |
|--------------------------------------|------------------------|------------------------|-------------------------------------|
| (18.43, 41.42)                       | 120.96                 | 0.980                  | 39                                  |
| (18.43, 41.42)                       | 165.76                 | 0.976                  | 32                                  |
| (26.56, 37.31)                       | 184.96                 | 0.980                  | 23                                  |
| (18.43, 41.42)                       | 210.56                 | 0.981                  | 34                                  |

**Figure 1.** Schematic of the MWA and Parkes frequency coverage vs. the mean system equivalent flux density (SEFD). The orange bars correspond to the split-bandwidth observations with 7.69 MHz bandwidth. The gray bar is the full-bandwidth observation with 30.72 MHz bandwidth. The green bar represents the 732 MHz Parkes band with 64 MHz bandwidth, and the blue bar represents the 3100 MHz Parkes band with 1024 MHz bandwidth.

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\(^a\) http://www.jb.man.ac.uk/pulsar/crab.html
cases, the Crab is not within a well-understood region of the beam. Throughout the 120.96, 165.76, and 184.96 MHz observations, the Crab is always within the half-power point of the beam, for which we have the most confidence in the beam modeling. At 210 MHz, the beam is such that the Crab is only barely within the half-power point for ~one-third of the full observation. We therefore have less confidence in the ability to accurately flux-calibrate the data at that particular frequency band using the method outlined here.

3.2.1. Tied-array Beam Forming

The tied-array beam is formed by coherently summing individual tile voltages (i.e., sum tiles in phase and then detect power). Theoretically, this process yields a factor of $\sqrt{N_{\text{co}}}$ improvement in sensitivity over an incoherent sum (i.e., detect tile power and sum; see Oronsaye et al. 2015), where $N_{\text{co}}$ is the number of tiles used to create the tied-array beam. This corresponds to a potential order of magnitude increase in sensitivity for the MWA. In reality, this is not the case, and we see an improvement by a factor of between 4.2 and 5.4, depending on the frequency. The discrepancy is primarily due to the pointing of the telescope (i.e., the MWA beam pattern is less well characterized as we diverge from a zenith pointing) and the calibration solution quality.

For MWA/VCS data, a tied-array (coherent) beam is created by a postprocess beam-forming pipeline (S. M. Ord et al. 2017, in preparation) implemented on the Galaxy cluster at the Pawsey Supercomputing Centre. The coherent beam-forming pipeline involves incorporating the individual tile polarimetric response, both cable and geometric delay models, and complex gain information (amplitude and phase) for each tile, per frequency channel, based on the calibration solutions. The tile weights used to create the tied-array beam are effectively determined by solving for the minimum $\chi^2$ error between the target data and the calibration model from the solutions.

3.2.2. Tied-array System Temperature and Gain

For a tied-array beam, the field of view is significantly smaller than that of the tile beam, approximating the naturally weighted synthesized beam of the array—nominally, FWHM $\sim 1.27\lambda/D$, where $\lambda$ is the observing wavelength and $D$ is the maximum baseline of the array. The scaling factor of 1.27 derives from the MWA being dominated by shorter baselines. This means that neither the integrated sky temperature nor the system gain will be the same as for the tile beam.

The overall system temperature ($T_{\text{sys}}$) for each frequency band is a combination of the receiver temperatures ($T_{\text{rec}}$), antenna temperatures ($T_{\text{ant}}$), and ambient temperature ($T_0$) and is calculated as

$$T_{\text{sys}} = \eta T_{\text{ant}} + (1 - \eta)T_0 + T_{\text{rec}},$$

where $\eta$ is the frequency- and direction-dependent radiation efficiency of the array. Efficiencies and receiver temperatures for each subband are given in Table 2. The receiver temperatures are well characterized across the nominal observing frequency range of the MWA. The ambient temperature weighting is $1 - \eta$, where $\eta \approx 1$ means that the contribution is negligible compared to the sky, and we therefore assume the ambient temperature is $T_0 \approx 290$ K. This contributes $\approx 5-7$ K to the total system temperature.

In order to calculate the antenna temperature in Equation (1), we require an adequate understanding of the tied-array synthesized beam pattern. In this case, the tied-array beam power pattern is the product of an individual MWA tile power pattern and the array factor. The tile pattern is simulated using the formalism set out by Sutinjo et al. (2015), while the array factor encapsulates the phase information required to point the tied array at the target source. For a full description of the formulation of the array factor, see the Appendix. This procedure was used to create the tied-array beam pattern at multiple times throughout the observation.

We use the global sky model (GSM) of de Oliveira-Costa et al. (2008) as our sky temperature map and scale it to our observing frequencies. The GSM was modified in the region of the Crab Nebula with the scaling $S_{\text{CN}} = 955\nu^{-0.27}$ Jy (Apparae 1973; Bietenholz et al. 1997) to more accurately represent the contribution from the nebula. Convolving the tied-array beam pattern with the GSM and integrating over the sky (see, e.g., Sokolowski et al. 2015), we produce an estimate of the antenna temperature (see Appendix A.1). Using these antenna temperatures and Equation (1), we calculate a system temperature estimate multiple times during the observation for each band. Fitting a second-order polynomial to the results from the separate evaluations of $T_{\text{sys}}$, we estimate a system temperature curve as a function of time.

We also calculate the gain, $G$ (see Appendix A.2), at the same intervals as the system temperature. The gains are relatively stable over the duration of the observation; thus, we fit a linear slope to create a gain curve as a function of time for the entire observation. The system temperature and tied-array gain curves are shown in Figure 2. Note that these estimates include the assumption of ideal sensitivity increase (i.e., by a factor of $\sqrt{N_{\text{co}}}$). This is corrected, given that we do not see the theoretical increase in sensitivity, in the following section.

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5 https://www.pawsey.org.au/
3.2.3. Flux Density Estimation

The output of the coherent beam-forming pipeline (see Section 3.2.1) is a set of PSRFITS files (Hotan et al. 2004), one file per 200 s per 1.28 MHz coarse channel. The individual channels can be combined into one 200 s file, reducing the number of data files by a factor of 24. These PSRFITS data were then incoherently dedispersed and subdivided into single-pulse archives using DSPSR and the ephemeris derived from the Parkes 732 MHz data. Each coarse channel’s edges were flagged (fine channels 0–19 and 108–127) to mitigate the effects of aliasing introduced during the channelization process, and the PSRCHIVE routine PAZ was used to apply the built-in median-smoothed difference excision algorithm. Finally, the archives were collapsed in polarization and frequency and written to a time series using PDV, without automatic baseline removal.

To compensate for the fact that the beam simulations assume the ideal $\sqrt{N_{\text{co}}}$ improvement in sensitivity, we estimate a coherency factor, $f_c$, by evaluating

$$f_c = \sqrt{N_{\text{co}}} \left( \frac{(S/N)_{\text{co}}}{(S/N)_{\text{inc}}} \right)^{-1},$$

where $(S/N)_{\text{co}}$ is the signal-to-noise ratio of a bright pulse in the coherently beam-formed data and $(S/N)_{\text{inc}}$ is the signal-to-noise ratio of the same pulse in the incoherently summed data. This quantity defines how well the coherent beam-forming pipeline performed compared to the theoretical expectation. The system temperature and gain calculations are used to convert the time series data from arbitrary power units to flux density units using

$$S = (S/N) \times \frac{f_c T_{\text{sys}}}{G \sqrt{n \Delta v \Delta t}},$$

where $S/N$ is the sample signal-to-noise ratio, $f_c$ is the coherency factor as in Equation (2), $n$ is the number of polarizations summed (in this case $n = 2$), $\Delta v$ is the observing bandwidth, and $\Delta t$ is the sample integration time. For an individual MWA tile, the SEFD is typically $\sim 2 \times 10^4$ Jy; however, for the coherently beam-formed, we find (for this set of subbands and pointings) the SEFD to be $\sim 2-3 \times 10^3$ Jy.

4. Analysis and Results

After postprocessing, we produced five time series with $\Delta t = 261.241 \mu s$ time resolution. This was achieved by rebinning the data into 129 phase bins per pulse period, ensuring that both the MWA and Parkes data had a sample time greater than the Parkes intrinsic sampling time of 256 $\mu s$. We use fluence (integrated flux density over the pulse width) as a direct measure of the pulse energy, given that peak or mean flux densities are less informative at MWA frequencies where giant pulses are typically scattered over several pulse periods.

4.1. Detecting Giant Pulses

Due to frequency-dependent propagation effects, the Parkes and MWA data were processed differently. As the data were incoherently dedispersed, the dispersive smearing across individual channels was not removed. While this delay is large at MWA frequencies ($\sim 1$%–10% of a pulse period), the dominating factor is still the multipath scattering that broadens an individual giant pulse across several pulse periods (see Section 4.2). Not only does this scattering make pulse detection and cross-matching more difficult, it also requires a more complicated method of measuring the pulse fluences. An example of a giant pulse detected simultaneously across all five frequencies, shown in Figure 3, illustrates the pulse-shape evolution with frequency due to multipath scattering.

A summary of the detected main pulse (MP) and interpulse (IP) giant pulses from each frequency band is presented in Table 3. Every giant pulse detected is recorded in a table format, including the pulse number, phase position, and fluence estimate.

4.1.1. Parkes

The calibrated single-pulse archives for the 732 and 3100 MHz data were summed in polarization and frequency to produce total intensity profiles. To find giant pulses in the

![Figure 3. Simultaneous giant pulse detected in all five observing bands: (a) 3100 MHz, (b) 732 MHz, (c) 210.56 MHz, (d) 165.76 MHz, and (e) 120.96 MHz. The effects of multipath scattering are most obvious at MWA frequencies, introducing a significant exponential tail to each giant pulse, while the Parkes pulses are 6 functions with the recorded time resolution. The 120.96 MHz pulse also has a visible rise time compared to the other frequencies.](image-url)
single-pulse archives, we used PSRCHIVE’s single-pulse analysis routine PSRSPA to search for candidate events with S/N \( \geq 6 \). The candidate lists were filtered to remove events with large pulse widths.\(^6\) The time of arrival was calculated for each giant pulse candidate with the ephemeris used during the folding process. The giant pulse positions in the rotation phase were then examined using TEMPO2. At 732 and 3100 MHz, there were 179 and 39 outliers (main pulse and interpulse combined) discarded, respectively.

This produced a list of 231 pulses at 3100 MHz and 6344 pulses at 732 MHz. From the final list of candidates, on-pulse peak flux densities were recorded for each single-pulse archive. The giant pulse fluences were then calculated as the product of the peak flux density and the time series bin width. The fluence errors were calculated from the off-pulse rms value.

Assuming Gaussian noise, the probability, \( P_n \), of a false detection above some S/N \( n \sigma \) is

\[
P_n(x > n\sigma) = \int_{n\sigma}^{\infty} P(x)dx = \frac{1}{\sqrt{2\pi}} \text{erfc}\left(\frac{n}{\sqrt{2}}\right),
\]

where \( \mu \) is the mean noise level, \( \sigma \) is the rms noise, and \( \text{erfc}(x) \) is the complementary error function. The S/N threshold when searching for single pulses in both the 732 and 3100 MHz bands was 6\( \sigma \), which corresponds to a false-detection likelihood of \( P_n \approx 1 \times 10^{-9} \). The number of false positives (\( N_th \)) is then the product of \( P_n \) and the number of observed pulsar rotations (\( \approx 1.5 \times 10^5 \)). We calculate this number to be significantly less than unity (\( N_th \approx 1.5 \times 10^{-4} \)) and therefore do not expect any giant pulse candidates with S/N > 6 to be spurious. After removing the RFI, ensuring that pulses were recorded only if they occurred in the main pulse and interpulse phase windows and by excluding candidates with pulse widths greater than one sample, we assert that all Parkes giant pulse candidates used in the following analysis are real.

4.1.2. MWA

As MWA giant pulses are severely scattered, some custom software was developed specifically for searching for scattered pulses in the time series. The input to this code is the time series created in Section 2.2. For each time series, the data were smoothed using a Savitzky–Golay low-pass filter with a window length of nine samples and a third-order fitting polynomial. Typical pulse widths are >40 samples; thus, the smoothing window length will not adversely affect the local pulse shape. This mitigated the high-frequency noise without reducing the fidelity of the individual pulses. The baselines for each time series were then removed by subtracting a linear fit over adjacent 10\(^7\) sample windows.

Local peaks were detected above a threshold of 5.5\( \sigma \)\(^7\) in partially overlapping sections of the time series. Any new candidate peaks recorded with the same sample number as a previously detected peak were discarded. Around each of the peaks, between 500 and 1000 samples (from the highest to the lowest frequency, respectively) were retrieved before and after the peak to ensure that the entire scattered pulse was captured in the time series window.

In order to further constrain the pulse position and extent, we fitted a pulse-broadening function (PBF) to each time series window. The thick, finite extent scattering-screen PBF proposed by Williamson (1972) was chosen, as it models both the significant rise time and exponential scattering tail present at low frequencies. The sample selections were fitted with the corresponding PBF form,

\[
g(t) = A \left(\frac{\pi \tau_d}{4(t - t_0)^3}\right)^{1/2} \exp\left[-\frac{\pi^2 \tau_d}{16(t - t_0)}\right],
\]

where \( A \) is a constant amplitude scaling, \( t_0 \) is the start time of the leading edge of the pulse, and \( \tau_d \) is the characteristic scattering time. Pulse numbers and phase were calculated based on the best-fitting pulse starting time, \( t_0 \). The pulse candidates were then selected based on whether their fitted \( \tau_d \) values fell within a predetermined range based on the approximate scattering time measured at each frequency (see Section 4.2). This distinguishes bona fide candidates with sensible scattering-time estimates from spurious detections. We note that the fitting was used only as a filtering process, and, because we know that, for the Crab, none of the standard PBFs fit correctly (see F. Kirsten et al. 2017, in preparation), the resulting scattering times may be somewhat less reliable. Each fitted pulse was also inspected by eye so that any questionable candidates were removed. For the full-bandwidth observation (184.96 MHz), this produced a list of 407 pulses. For the split-bandwidth observation, we recorded 90 pulses at 120.96 MHz, 386 pulses at 165.76 MHz, and 648 pulses at 210.56 MHz.

For each real candidate pulse, we define the start of the pulse as the best-fit \( t_0 \) and the end of the pulse as six \( \epsilon \)-folds past the PBF peak (i.e., \( t_0 + \pi^2 \tau_d/4 + 6\tau_d \)). In this case, \( \tau_d \) is the median scattering timescale as in Table 4, while \( \tau_d \) is the best-fitting scattering time for the individual pulse. We define this window as the actual pulse from which to calculate the fluence. For each candidate, we then integrate over the pulse window and record that as the pulse fluence, along with the fluence from the fitted PBF. The fluence uncertainty for each pulse is calculated by integrating under the fitted PBF model, scaled such that the peak amplitude is equal to the local rms value.

Detections near the threshold limit may have underestimated fluences, given that the giant pulses (specifically the scattered tail) would be dominated by noise and fall within the baseline rms well before a brighter counterpart pulse at a different frequency. Fluence estimates and, consequently, the calculated spectral indices (see Section 4.5) in those cases may be less reliable, especially for weaker pulses. Additionally, the software searches only for simple PBF forms; thus, giant pulses with significantly different structure (e.g., a second pulse within the scattering tail) may be discarded, especially if the structure is such that the estimated scattering timescales are outside the nominally expected range. At 210.56, 184.96, 165.76, and 120.96 MHz, this results in \( \sim 7\% \), \( \sim 0.4\% \), \( \sim 10\% \), and \( \sim 1\% \) of candidates being flagged, respectively. The 184.96 MHz fraction is significantly smaller due to both the sensitivity (i.e., the noise characteristics are typically better behaved) and the observation duration (i.e., we are less likely to observe, for instance, a giant pulse within the scattering tail of another).

\(^6\) The Parkes data are limited by the time resolution; thus, giant pulses appear as events with a width of one sample only.

\(^7\) A lower threshold than that used for the Parkes observations is enforced for the MWA data because of the significantly quieter RFI environment at the Murchison Radio-astronomy Observatory.
The determined scaling index is

\[ \alpha_d = -3.73 \pm 0.45 \]

significantly shallower than what is expected from a Kolmogorov model, which is \( \alpha_d = -4.4 \). This result is consistent with what is reported in the literature at low frequencies (e.g., Bhat et al. 2007; Ellingson et al. 2013; Eftekhari et al. 2016). Extrapolating using the above scaling index, we also estimate the scattering expected in the Parkes subbands in Table 4. Given the time variability of the characteristic scattering times observed for the Crab and the dependence of the estimated scattering time on the chosen PBF, discrepancies of as much as a factor of \( \sim 2 \) are not uncommon between similar frequencies. Our values from the MWA subbands are roughly consistent with those quoted in the literature (e.g., Staelin & Sutton 1970; Popov et al. 2006; Oronsaye et al. 2015). A more detailed examination of the scattering behavior of the Crab and other pulsars within the MWA observing frequency range will be reported in a forthcoming publication (F. Kirsten et al. 2017, in preparation). In particular, there is discussion of the difficulties in correctly characterizing the pulse broadening seen in Crab giant pulses at low frequencies and reconciling this with a variety of theoretical scattering-screen models.

4.2. Pulse Broadening

At both Parkes subbands, we cannot directly determine the scattering timescale \( (\tau_d) \), since we are limited by the time resolution \( (261.241 \mu s) \) of our recorded data. At MWA frequencies, from the rudimentary fitting performed when detecting the pulses, we can estimate the pulse broadening. We report the median scattering timescales in Table 4. Furthermore, we calculated the scattering spectral index \( (\alpha_d) \) using the MWA data. Using a least-squares minimization approach, we fitted a power law \( (\tau_d \propto \nu^{\alpha_d}) \) to the MWA scattering timescales (see Figure 5).

The determined scaling index is \( \alpha_d = -3.73 \pm 0.45 \), significantly shallower than what is predicted from a Kolmogorov model, which is \( \alpha_d = -4.4 \). This result is consistent with what is reported in the literature at low frequencies (e.g., Eftekhari et al. 2016).

### Table 4: Pulse-broadening Timescales in the Literature, Including This Work

| Center Frequency (MHz) | \( \tau_d \) (ms) | Reference | Key |
|------------------------|------------------|----------|-----|
| 28                     | 417 ± 284        | Eftekhari et al. (2016) | E+16 |
| 40                     | 132 ± 73         | Eftekhari et al. (2016) | E+16 |
| 44                     | 978 ± 287        | Ellingson et al. (2013) | E+13 |
| 60                     | 768 ± 275        | Ellingson et al. (2013) | E+13 |
| 60                     | 73 ± 45          | Eftekhari et al. (2016) | E+16 |
| 76                     | 48 ± 29          | Eftekhari et al. (2016) | E+16 |
| 76                     | 439 ± 122        | Ellingson et al. (2013) | E+13 |
| 115                    | 13 ± 5           | Staelin & Sutton (1970) | SS70 |
| 120.96                 | 26.1 ± 4.4       | This work                    |     |
| 157                    | 3.8 ± 1.3        | Staelin & Sutton (1970) | SS70 |
| 165.76                 | 7.8 ± 1.5        | This work                    |     |
| 173.25\(^a\)           | 1.5 ± 0.4        | Karuppasamy et al. (2012) | K+12 |
| 184.96                 | 5.1 ± 1.1        | This work                    |     |
| 192.64                 | 6.1 ± 1.5        | Oronsaye et al. (2015) | O+15 |
| 210.56                 | 3.4 ± 0.7        | This work                    |     |
| 300                    | 1.3 ± 0.2        | Salmen et al. (1999)        | S+99 |
| 600                    | 0.095 ± 0.005    | Salmen et al. (1999)        | S+99 |
| 732                    | ∼0.03\(^b\)      | This work                    |     |
| 3100                   | ∼0.0002\(^b\)    | This work                    |     |

Notes.

\(^a\) Extrapolated from 184.96 MHz, assuming \( \tau_d \propto \nu^{-3.7} \).

\(^b\) Scattering time estimated from Figure 6 in Karuppasamy et al. (2012).

We tested the noise statistics for coherently beam-formed, dedispersed, baseline-removed MWA data for normality. This was achieved by selecting five evenly spaced samples, each containing 1000 data points, from each subband time series and fitting a normal distribution. From these samples, the noise statistics are consistent with Gaussian noise (see Figure 4 for an example); therefore, we can use Equation (4) to calculate the false-detection likelihood for MWA data. The S/N threshold when searching through MWA data was 5.5\( \sigma \); thus, the false-detection probability is \( P_{\nu} \approx 2 \times 10^{-8} \). The number of pulsar rotations during the MWA split-bandwidth observations is \( \approx 1.1 \times 10^{5} \); thus, the number of false detections expected is again significantly less than unity \( (N_{\text{false}} \approx 2.2 \times 10^{-3}) \). For the full-bandwidth observation, the number of pulsar rotations is \( \approx 3.5 \times 10^{4} \), and the number of expected false detections is \( N_{\text{false}} = 7 \times 10^{-4} \)—again much less than unity. We claim that no MWA giant pulses are spurious detections, given the statistics and the filtering performed during the candidate selection process.

Figure 4. A 1000 sample example of the noise characteristics for coherently beam-formed, dedispersed, baseline-removed MWA data. The red solid lines are a fitted normal probability density function.

Figure 5. Pulse-broadening times from the four MWA bands. The median scattering timescales (circles), their respective errors, and the fitted power law (red dashed line) are plotted on a log-log scale. The scaling index, \( \alpha_d = -3.73 \pm 0.45 \), is significantly shallower than what is expected from a Kolmogorov model but consistent with other estimates at similar frequencies in the literature. Given the variability of Crab pulse broadening, it is not surprising that many of the scattering times from the literature do not fall on the fitted power law. The keys in the legend are the same as those in Table 4.
4.3. Simultaneous Giant Pulses

For every giant pulse found in Sections 4.1.1 and 4.1.2, a pulse number was recorded. We used these pulse numbers and the phase (to discriminate between MP and IP giant pulses) of each giant pulse to cross-match across the five frequency bands. The cross-matching was achieved by using routines from the Starlink Tables Infrastructure Library Tool Set (STILTS; Taylor 2006), which is designed for robust and efficient processing of tabular data. The tools are implemented for generic manipulation of tabulated data sets but are typically used for astronomical object catalog analysis, in particular cross-matching of large data sets based on user-specified selection criteria. The results of cross-matching the giant pulse samples from each subband are summarized in Table 5.

Between the two Parkes frequencies, we find that there are 157 simultaneous main pulses and nine simultaneous interpulses. These numbers correspond to approximately 72% and 64% coincidence for main pulses and interpulses, respectively, based on the number of pulses detected in the 3100 MHz band.

Between the MWA full-bandwidth observation and the 732 MHz band, there are 140 simultaneous main pulses and 33 simultaneous interpulses, corresponding to 41% and 50% based on the total numbers from the 184.96 MHz band. Across all three bands, we detected 10 simultaneous main pulse giant pulses and two simultaneous interpulse giant pulses.

Within the MWA bands (Table 1), the full-bandwidth and split-bandwidth observations have no overlap in time; thus, we focus only on the three subbands at 120.96, 165.7, and 210.56 MHz. Between the highest and middle bands, there are 269 simultaneous main pulses and 42 interpulses, corresponding to 80% and 84% based on the number of pulses detected in the 165.76 MHz band. Between the lowest and middle bands, there are 68 simultaneous main pulses and eight interpulses, corresponding to 87% and 72% correlation based on the number of pulses detected in the 120.96 MHz band. There are seven giant pulses detected simultaneously across all five bands: six main pulses and one interpulse.

For the brightest ~10% of pulses (combining main pulses and interpulses) in each band, we checked for pulses that had no counterpart in adjacent frequency bands. At 210.56 MHz, there are 68 pulses with fluences greater than 1.5 Jansky seconds (Jy s), of which there are only 42 counterparts at 732 MHz and 67 counterparts at 165.76 MHz. Inspecting the MWA time series, we found that the missing giant pulse in the 165.76 MHz band is below the detection threshold. At 165.76 MHz, there are 38 pulses with fluences greater than 5 Jy s, with 37 counterparts at 210.56 MHz and 26 counterparts at 120.96 MHz. The missing counterpart at 210.56 MHz is relatively clear in the time series; however, it is actually two giant pulses combined (and therefore discarded during the candidate processing): a main pulse and an interpulse in adjacent rotations. At 732 MHz, the main pulse is detected, but the interpulse in the subsequent rotation is not. Of the 12 missing counterparts at 120.96 MHz, eight have a visible counterpart below the 5.5σ detection threshold. For another three, there are no visible counterparts. For one pulse, there are no 120.96 MHz data at the corresponding time because of the dispersion delay.

In light of the “double giant pulse” (i.e., a main pulse and an interpulse occurring within one rotation), we searched for other examples across all frequency bands. At 3100 MHz, there is one marginal case (~0.4% of detected pulses), while at 732 MHz, there are 85 clear examples (~1% of detected pulses). Within the MWA bands, the pulse broadening makes robustly identifying double giant pulses difficult; however, we find ~one to two marginal examples per MWA band. The double giant pulses at one band do not necessarily coincide with double giant pulses at any other.

4.4. Giant Pulse Fluence Distributions

In Figure 6, we plot the complementary cumulative distribution function, also known as the survival function, of pulses as a function of fluence for each subband. The clustering at low frequencies suggests that there is some degree of flattening of the spectral indices occurring at the lowest frequencies. This also provides estimates for subpopulations of giant pulses and rates of occurrence as a function of frequency and fluence. Listed in Table 6 are some basic quantities describing the fluences of all detected giant pulses in each observed band.

Typically, the fluence distributions are assumed to follow a power law, \( N(>F) \propto F^{-\beta} \). In the literature, the standard approach is to estimate a power-law cutoff \((\lambda_{\text{min}})\); see Figure 7\) by eye and use a least-squares approach to only fit data beyond that limit. This approach may introduce significant biases in the power-law index estimation and assumes that the data are independent and identically sampled.

To avoid subjectivity, we chose to use the “powerlaw” Python module (Alstott et al. 2014), which appropriately treats several heavy-tailed distributions, particularly focusing on power laws. The best-fitting power-law distribution index \((\beta)\) and power-law cutoff \((\lambda_{\text{min}})\) are determined by finding the minimum Kolmogorov–Smirnov distance between the data and model (see, e.g., Figure 7). The data are used to evaluate
The spectral index for giant pulse emission is typically assumed to be a power law, where $S_\nu \propto \nu^{\beta}$. We find that a simple power law is unable to accurately model the observed giant pulse spectrum between 120.96 and 3100 MHz. Figure 8, we plot the spectral index distributions between each consecutive frequency pair, separated into main pulses and interpulses. From 732–3100 MHz, 75% of the simultaneous main pulses have a spectral index between $-3.3$ and $-2.1$. Between 732 and 167.76 MHz, the same fraction of giant pulses exhibits a spectral index in the range $-1.8$ to $-0.4$. The distribution between the two lowest MWA bands is wider and flatter, with 75% of the pulses within $-2.5$ to $0.7$. Using the 184.96 MHz data, we also calculated the spectral index distribution for a similar sample of giant pulses (with an $S/N \geq 11$, which accounts for the factor of 2 sensitivity improvement provided by 4 times the bandwidth). This produces a distribution with a mean $\alpha = -0.8$ and a width of 0.6, with 75% of the pulses between $-1.5$ and $-0.1$. Given the sparse interpulse distributions, we did not calculate the above intervals, though we can say that they appear to follow a similar trend of flattening.

As discussed in Section 3.2, the trustworthiness of the 210.56 MHz beam, and hence the fluence estimates, is questionable. We calculate a spectral index from the data in Table 6 between 210.56 and 732 MHz to be $\alpha \approx -0.6$ with a width of 0.5, while between 210.56 and 167.76 MHz, $\alpha \approx -4.7$ with a distribution width of $\sim 3$. The 210.56 MHz data are therefore not used in the following analysis.

Karuppusamy et al. (2010) reported spectral index distributions between 1300 and 1450 MHz centered around $-1.44 \pm 3.3$ and $-0.6 \pm 3.5$ for the main pulse and interpulse giant pulses, respectively, though the distribution width ranges from approximately $-15$ to $+10$. Mikami et al. (2016) also estimated spectral indices in the range $-15$ to $+10$ based on their fluence calculations between 1586 and 1696 MHz. We therefore do not find it surprising that our spectral index distributions are relatively wide, especially between MWA subbands.

Our observations indicate that the spectral index for simultaneous giant pulses is flattening over the sampled frequency range. If we use the median fluences from Table 6, the computed main pulse spectral index is $\sim -1.4$ across most bands, except between 120.96 and 167.76 MHz, where it steepens to $\sim -3.3$, and between the Parkes bands. In part, this is due to the smaller lever arm available between MWA bands; however, it also indicates that the detected simultaneous pulses (which have a slightly shallower spectral index) are more consistent tracers of the spectral flattening.

In Figure 9, we plot three different samples of giant pulses based on the frequency bands in which they were detected. In general, these spectra also show a tendency of flattening at the lower frequencies. An archetypal synthetic giant pulse spectrum based on the spectral index distributions is shown in Figure 10, which demonstrates the expected pulse spectral
shape given a 3100 MHz fluence of 0.013 Jy s. The shaded error region is calculated using the median absolute deviation of the spectral index distribution instead of the standard deviation, as it is less sensitive to the existence of extreme values (see Figure 8). The power laws are fits to the two Parkes bands and the 165.76 and 120.96 MHz MWA subbands.

The mean spectral index between 3100 and 732 MHz from the synthetic spectrum is $\alpha_{3100} = -2.7$ with a width of 0.4. Between 732 and 165.76 MHz, the synthetic spectral index becomes shallower with $\alpha_{732} = -1.1$ and a width of 0.4. Between 165.76 and 120.96 MHz, it is estimated to be $\alpha_{120} = -0.8$ with a distribution width of 2.5. The large error in $\alpha_{120}$ is due to a combination of relatively large errors in fluence estimates and the fact that the frequencies are relatively close together; hence, there is a wide distribution of spectral indices and therefore a less well-constrained mean. Spectral index information between each of the bands and from the synthetic spectrum are shown in Table 8.

The synthetic spectrum, in addition to the spectral index values, becomes shallower with a distribution width of 2.5. The large error region is calculated using the median absolute deviation on the values recorded in Table 6. Additionally, if we assume that the distributions follow the same power-law behavior and our noise statistics remain unchanged, then we can calculate the number of detectable giant pulses ($N_f$) above the extrapolated median fluences ($F_f$) using

$$N_f = N_0 \left( \frac{F_0}{F_f} \right)^{-\hat{\beta}},$$

where $N_0$ is the measured number of pulses above the measured median fluence $F_0$ and $\hat{\beta}$ is the measured power-law exponent. For main pulses only, Equation (6) yields an expected $\sim 5 \times 10^6$ detectable pulses at 210.56 MHz, $\sim 4 \times 10^5$ at 165.76 MHz, and $\sim 9 \times 10^3$ at 129.6 MHz. These predictions are a factor of $\sim 10^{3-4}$ times larger than the recorded number of main pulses. It is therefore implausible that the spectrum continues with the steep index to low frequencies.

### 4.6. Non-giant Pulse Emission

For the Parkes data, we also attempted to recover the non-giant pulse emission from the Crab. For this, we essentially treated all pulses with a detection below a 3.5$\sigma$ threshold as being non-giant pulse emission. All such pulses were synchronously averaged to construct an “integrated profile.” At 732 MHz, the MP and IP components of such a profile are approximately equal in amplitude ($S_{\text{peak}} \sim 19$ Jy), whereas in the constructed giant pulse profile (detections $\geq 6\sigma$), the MP is $\sim 6$ times brighter than the IP. At 3100 MHz, the giant pulse

### Table 6

| Center Frequency (MHz) | Main Pulse $F_{\nu}$ | Interpulse $F_{\nu}$ |
|-----------------------|----------------------|----------------------|
|                       | Median (Jy s) | Std. dev. (Jy s) | Min. (Jy s) | Max. (Jy s) | Median (Jy s) | Std. dev. (Jy s) | Min. (Jy s) | Max. (Jy s) |
| 120.96                | 5.62       | 3.71              | 2.57        | 20.42       | 4.91       | 2.34              | 2.66        | 10.91       |
| 165.76                | 1.99       | 2.46              | 0.78        | 19.96       | 2.01       | 1.52              | 1.12        | 6.77        |
| 184.96                | 0.52       | 0.78              | 0.16        | 9.54        | 0.61       | 0.58              | 0.31        | 3.57        |
| 210.56                | 0.64       | 0.80              | 0.29        | 7.89        | 0.61       | 0.54              | 0.31        | 3.53        |
| 732                   | 0.22       | 0.28              | 0.07        | 5.77        | 0.17       | 0.22              | 0.08        | 3.58        |
| 3100                  | 0.009      | 0.009             | 0.004       | 0.077       | 0.008      | 0.008             | 0.004       | 0.028       |

Note.

- Adjusting to account for the bandwidth difference produces a median of 1.7 Jy s for main pulses and 1.5 Jy s for interpulses.
5. Discussion

5.1. Spectral Flattening

Our analysis identifies a spectral flattening at low frequencies in Crab giant pulses. A flattening spectrum was also hinted at by Oronsaye et al. (2015), whose analysis showed that the spectrum becomes shallower by ~5% at lower frequencies based on Monte Carlo simulations of observations at 193 and 1382 MHz. We note, however, that the flucences presented by Oronsaye et al. (2015) are significantly different (by orders of magnitude) from those we calculate here. Reexamining the Parkes data used, we estimate that the flux densities are a factor of ~10–100 larger than quoted and attribute this to an error in the flux density calibration in the original processing. This discrepancy is also noted by Mikami et al. (2016), whose observing bands are at a frequency similar to those used by Oronsaye et al. (2015). The MWA flucences we calculate herein are roughly consistent with the estimates made by Oronsaye et al. (2015), which, together with the reevaluated Parkes flucences, implies that the flattening observed is more significant than the authors stated.

The two power-law slopes we identify behave similarly to those broken-type spectra (Maron et al. 2000; Bates et al. 2013), where $|\alpha_{low}| < |\alpha_{high}|$. The average spectral indices we see from our giant pulse samples ($\alpha_{165}^\text{MP} = -0.7 \pm 1.4$, $\alpha_{3100}^\text{MP} = -2.6 \pm 0.5$) are consistent with the estimates of Maron et al. (2000) for normal pulse emission, $\langle \alpha_{low} \rangle = -0.9 \pm 0.5$ and $\langle \alpha_{high} \rangle = -2.2 \pm 0.9$. Mikami et al. (2016) reported a main pulse spectral index between 325 and 2250 MHz of $\alpha_{2250}^\text{MP} = -2.44 \pm 0.47$, which is consistent with our estimated main pulse high-frequency spectral index.

We acknowledge that we have only four spectral points; thus, there is not enough information to robustly determine the actual spectral index values and uncertainties in the synthetic spectrum. The uncertainty in the MWA flucences is generally the most significant source of error, especially at the lowest frequency where the pulses tend to be scattered, and appropriately characterizing the pulses is difficult.

There is an increasing amount of evidence for a slightly flatter or even inverted spectrum at low frequencies (e.g., Bhat et al. 2007; Karuppusamy et al. 2010; Oronsaye et al. 2015; Eftekhar et al. 2016). In contrast, Popov et al. (2006) calculated giant pulse spectral indices between $-3.1$ and $-1.6$ for 111–600 MHz and $-3.1$ and $-2.5$ for 23–111 MHz, both with a mean of $-2.7 \pm 0.1$; however, note that these values are subject to selection effects. In addition to this, their errors in flucence and spectral index are likely optimistic given that at 23 MHz, the giant pulse rise time alone would be several tens or hundreds of pulse periods.

While there is indeed a wide spread in the spectral indices quoted in the literature, the general trend is a shallower spectral index at low frequencies (see Figure 11). Since our data are from simultaneous observations, we are able to confidently assert that the spectral index tends to be shallower at low frequencies. If we only use the values from the literature, a direct comparison is difficult, as they are from different instruments and measured at widely separated epochs (sometimes spanning decades).

The implication for the giant pulse emission mechanism is that we would need some process or propagation effect (possibly within the magnetosphere) that allows for a flattening and eventual turnover (which likely occurs at $\nu \ll 100$ MHz) in the spectrum. As with the GPS pulsars, this effect is perhaps caused by the surrounding environment of the pulsar (i.e., the Crab Nebula in this case). However, Oronsaye et al. (2015) showed that at MWA frequencies, free–free absorption from within the nebula (e.g., Bietenholz et al. 1997) is not able to explain the flattening they observe, with free–free absorption coefficients on the order of $10^{-25}$ cm$^{-2}$. Given that our flattening is more apparent than represented previously, free–free absorption alone causing the flattening is unlikely. Structures in the nebula and the intervening interstellar medium (e.g., Smith & Terry 2011) may be capable of attenuating the flucence estimates by a few percent but would require 10–100 such filaments to be intercepted. Not only is the chance alignment of filaments unlikely, but the dispersion measure of the pulsars would be increased by ~few pc cm$^{-3}$, which is unphysical.

### Table 7

| Frequency (MHz) | $\beta^a$ | $\gamma^a$ | $\delta^a$ | $\epsilon^a$ | $\zeta^a$ | $\eta^a$ | $\theta^a$ | $\psi^a$ | $\chi^a$ | $\phi^a$ | $\alpha^a$ | $\beta^a$ | $\gamma^a$ | $\delta^a$ | $\epsilon^a$ | $\zeta^a$ | $\eta^a$ | $\theta^a$ | $\psi^a$ | $\chi^a$ | $\phi^a$ | $\alpha^a$ |
|-----------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 120.96          | 3.73 ± 0.54 | 6.81 | 24       | 3.70 ± 0.86  | 3.94  | 10       |
| 165.76          | 2.69 ± 0.11  | 1.60 | 242      | 2.84 ± 0.29  | 1.47  | 40       |
| 184.96          | 2.88 ± 0.12  | 0.44 | 234      | 3.10 ± 0.29  | 0.49  | 52       |
| 210.56          | 2.90 ± 0.09  | 0.51 | 434      | 3.14 ± 0.25  | 0.49  | 71       |
| 732             | 3.30 ± 0.09  | 0.46 | 719      | 3.16 ± 0.09  | 0.15  | 658      |
| 3100            | 3.19 ± 0.17  | 0.01 | 82       | 2.15 ± 0.31  | 0.004 | 13       |

Notes:
- The uncertainties quoted are the standard error in the power-law index estimation.
- In this case, the evaluated $\gamma^a$ for the main pulses is relatively high, excluding ~85% of the detected pulses. See text for details.
5.2. Emission Mechanism

The giant pulse fluence dependence on frequency, particularly the flattening at low frequencies, is not predicted in detail in any of the current models. The spectral behavior provides important information about what physical processes are producing the emission.

The coherent radio emission mechanism for pulsars is still unknown (see, e.g., Melrose 1995 for a review), especially given the complexity of modeling pulsar magnetospheres (e.g., Spitkovsky 2006; Li et al. 2012; Tchekhovskoy et al. 2013) and the myriad emission models in the literature. There are several models that are able to address individual aspects of giant pulse emission (Eilek & Hankins 2016), though none are able to explain all of the characteristics alone, possibly because they

![Figure 8](image_url)

Figure 8. Distributions of spectral indices for each combination of giant pulses with the mean ($\mu$), median ($m$), and standard deviation ($\sigma$) included. The top row contains only the main pulse simultaneous matches, and the bottom row contains only the interpulse matches. The spectrum of the simultaneous giant pulses appears to be flattening at low frequencies. There are only a handful of interpulses; thus, the mean and median estimates are not as meaningful but tend to be steeper between the Parkes bands and similar to the main pulses at the lower frequencies.

![Figure 9](image_url)

Figure 9. Selected samples of giant pulses based on their simultaneous detections. Giant pulses with simultaneous detections in all four bands are plotted in panel (a). Giant pulses detected simultaneously without a 3100 MHz detection are shown in panel (b), while pulses with only a 3100, 732, and 165.76 MHz simultaneous detection are shown in panel (c). Panel (d) contains giant pulses detected only between 732 and 3100 MHz.

![Figure 10](image_url)

Figure 10. Archetypal average spectrum of detected giant pulses. Each spectral point is calculated based on the mean spectral index between the two frequencies. This is the expected shape of a giant pulse spectrum for a reference value of 0.013 Jy s at 3100 MHz. The gray shaded error region represents the median absolute deviation for each spectral index distribution. The power-law fits are based on the two Parkes bands (red dotted line) and two lowest MWA bands (blue dashed line).
A free-electron maser model involves the interaction of relativistic particle beams with plasma waves to induce charge bunching, leading to strong coherent bursts of radiation. The emission frequency, assuming the plasma is at rest (e.g., Benford 1992), is $\nu_{\text{FEM}} \sim 2 \gamma_b \nu_p$, where $\gamma_b$ describes the speed of the driving particle beam. For radio frequency emission, this requires a density enhancement similar to that of the strong plasma turbulence, $10^2 \lesssim \lambda \gamma_b^2 \lesssim 10^5$.

The flattening spectrum then raises the question of what is driving the nanoshot emission in the regions where conditions translate to emission at low frequencies. Crab giant pulse radio emission is suspected to originate higher in the magnetosphere, perhaps near the light cylinder. This is based on the relative enhancements required for radio emission in comparison to pair-production plasma models (e.g., Arendt & Eilek 2002; Eilek & Hankins 2016). High-altitude emission is also supported by multimeter observations of the Crab identifying that the high-energy and radio profiles are very close in pulse longitude, implying that they originate from similar regions within the magnetosphere (Abdo et al. 2010). While the strong plasma turbulence model has a shallow predicted scaling for nanoshot flux density that supports a flatter spectrum, it is unclear how that scaling translates into the regime where we are observing the superposition of many nanoshots. If $S_p \propto \nu^{-1}$ is representative for unresolved emission, then the model is unable to explain the steep spectral index typically observed above $\sim 300$ MHz, even though the model is able to describe the nanoshot timescales and frequency structure.

If we assume that in fact both phenomena are present within the magnetosphere, then the relative dominance of the processes would depend on, for example, the driving beam densities and ambient plasma characteristics. Typically, one can assume that the charged particles streaming from the pulsar are accelerated along the electric fields as they move away from the neutron star surface. In most models, $\gamma_\delta > 100$ and $\gamma_b^2 \sim 10$–100 are required in order to match the observed nanoshot frequency-time product (Eilek & Hankins 2016). In this way, one could imagine that strong plasma turbulence begins to dominate in the region where the low radio frequency emission is produced in the upper magnetosphere, where particles are further away from the star and therefore traveling faster.

Without further exploration of these models (and others), in terms of observational emission characteristics, it is difficult to say more. How the nanoscale attributes translate to millisecond timescales and predictions for the flux density frequency scaling are critical for meaningful comparison to observations. At low frequencies, there is the additional complication of pulse broadening, which distorts the intrinsic emission.

### 5.3. FRBs as Extragalactic Super-giant Pulses

Wideband observations are able to provide the limits of FRB spectral index distributions (e.g., Burke-Spolaor et al. 2016). Typically, the measured spectral indices of FRBs are poorly constrained. For example, the measured spectral index $(1.214–1.537$ GHz) of FRB 121102 ranges between $-10$ and $+14$ (Spitler et al. 2016), and for other FRBs the range is approximately $-8$ to $+6$ (e.g., Lorimer et al. 2007; Keane et al. 2012; Ravi et al. 2015; Burke-Spolaor et al. 2016). These values are consistent with the large spread in spectral indices measured for Crab giant pulses, including those calculated...
herein. If some FRBs are “super-giant” pulses from extragalactic pulsars, and assuming our low-frequency spectral index \((\alpha = -0.7 \pm 1.4)\) is representative, it is possible to estimate the number of expected FRB detections at MWA frequencies. Based on the calculations of Trott et al. (2013), we would expect to see somewhere between \(\sim 0.1\) and 100 FRBs per 10 hr of observing with the MWA above an S/N of 7, depending on scattering effects and data processing.

Given that no low-frequency instrument has claimed an FRB detection to date (e.g., Coenen et al. 2014; Karastergiou et al. 2015; Tingay et al. 2015; Rowlinson et al. 2016), there are two obvious constraints we can make. If FRBs are close enough to be detectable (\(\leq\)few hundred Mpc), then the nondetections thus far would suggest that the spectrum has turned over or been detectable sufficiently for the giant pulses to become undetectable. From our results, this seems at least plausible, assuming that the emission originates from a Crab-like pulsar. However, if the spectrum has not inverted, then the nondetections perhaps suggest that these objects are much more far away than assumed in the giant pulse FRB models. The latter is supported by the localization of FRB 121102 (Chatterjee et al. 2017) at \(\sim 1\) Gpc and the stable DM that FRB 121102 exhibits (see, e.g., Lyutikov 2017). With these results in mind, a super-giant pulse origin for FRBs seems less likely.

6. Conclusions

We have reported on simultaneous observations of the Crab pulsar conducted with the MWA (120.96, 165.76, 184.96, and 210.56 MHz) and Parkes radio telescope (732 and 3100 MHz). Our observations sampled from 120 to 3100 MHz (a factor of \(\sim 30\) in frequency) and thus simultaneously spanned low, middle, and high frequencies, which provided a unique view of the giant pulse spectrum. Giant pulses were detected in all bands, ranging from 90 at 120 MHz to 6344 at 732 MHz. Seven giant pulses (six main pulses and one interpulse) were detected simultaneously in five of the observing bands (excluding 184.96 MHz due to no time overlap with the 120.96, 165.76, and 210.56 MHz bands). The correlation of detected pulses between bands varies, ranging from \(\sim 40\%\) (184.96–732 MHz, relative to 184.96 MHz detections) to \(\sim 87\%\) (120.96–165.76 MHz, relative to 120.96 MHz detections).

The mean spectral index for the sample of simultaneous giant pulses tends to flatten at low frequencies, from \(\alpha = -2.6 \pm 0.5\) (732–3100 MHz) to \(-0.7 \pm 1.4\) (120.96–165.76 MHz). By creating a synthetic spectrum based on the distributions of spectral indices, we also saw that the evolution in spectral shape is not well characterized by a single power law. Furthermore, we compared our simultaneous wideband results with spectral index measurements from the literature, which further reinforced the observed spectral flattening. This flattening is unlikely to be caused by propagation effects within the nebula.

The emission mechanism required to explain this phenomenon is currently not well understood. Further work is required to extend current giant pulse emission models in order to determine how the flux density spectrum changes and how the intrinsic nanoshot characteristics translate to observing their superposition.

We also measured the characteristic pulse-broadening times for giant pulses in the MWA subbands. Specifically, we calculated a frequency scaling index of \(\alpha_{D} = -3.7 \pm 0.4\) which is consistent with the literature relating to the scattering characteristics of Crab giant pulses.

We also commented on the plausibility of a giant pulse origin of some FRBs. Considering the localization of FRB 121102 and the flattening spectrum that we observed, it appears that a giant pulse emission origin for FRBs (assuming that the Crab is typical) is less likely. This is supported by the nondetections of FRBs from any low-frequency telescope to date.

Investigations of giant pulse spectra over wide frequency ranges, especially extending below \(\sim 100\) MHz, have not been attempted for other giant pulse–emitting pulsars. Such studies are particularly important to check whether the Crab is a special case or typical in terms of giant pulse emission. We also emphasize the important role of simultaneous observations in this endeavor.

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Appendix

Array Factor Calculation

An antenna element in isolation has a complex voltage pattern given by some frequency-dependent function \(D(\theta, \phi)\), where \(\theta\) is the zenith angle and \(\phi\) is the azimuth. The function \(D(\theta, \phi)\) is called the element pattern and gives the signal strength received by the element for any given direction, assuming it is positioned at the origin, \(r = (0, 0, 0)\). The coordinate system used here is such that the azimuth (\(\phi\)) is defined with \(0^\circ\) directly east and increases in an anticlockwise direction. The zenith angle (\(\theta\)) is defined in the normal convention.

For an array of \(N\) elements, we define each element voltage pattern as \(D_n(\theta, \phi)\). The tied-array beam pattern will be the sum of each element pattern in response to a wave, \(v_s\), impinging on the array. Given that the source is in the far field, this wave will be planar. It is practical to express the planar wave in terms of the coordinate system we have adopted; thus,
\( \psi_n \) can be written as
\[
\psi_n = \exp(ik \cdot r_n) \equiv \exp\left[ \frac{2\pi i}{\lambda} (x_n \sin \theta \cos \phi + y_n \sin \theta \sin \phi + z_n \cos \theta) \right],
\]
(7)
where \( k \) is the three-dimensional wave vector, \( r_n = (x_n, y_n, z_n) \) is the position of the \( n \)th element relative to the center of the array, and \( \lambda \) is the observing wavelength.

We also apply weights, \( w_n \), on a per-element basis. For the MWA, when calculating the beam pattern for an individual tile (which consists of 16 dipole elements), these weights incorporate information about the cable losses and port currents required to accurately model the mutual coupling between dipoles and polarization characteristics (Sutinjo et al. 2015). On the tied-array scale, each element is now one MWA tile, and the weights encode the phase-delay information required to correctly point the array at a given sky position.

The tied-array voltage pattern is
\[
D_{\text{array}}(\theta, \phi) = \frac{1}{N} \sum_{n=1}^{N} w_n D_n(\theta, \phi) \psi_n.
\]
(8)
If we assume that the array elements are identical, then we can move the element factor out of the summation, and Equation (8) becomes
\[
D_{\text{array}}(\theta, \phi) = D(\theta, \phi) \frac{1}{N} \sum_{n=1}^{N} w_n \psi_n.
\]
(9)
Given that we have two separable factors in Equation (9), one of which is the element pattern, we define the other as the array factor,
\[
f(\theta, \phi) = \frac{1}{N} \sum_{n=1}^{N} w_n \psi_n(\theta, \phi),
\]
(10)
The array factor represents the response of an array of identical elements and encompasses the interference effects from the individual element patterns in response to the received radiation from the visible sky.

To point the tied-array radiation pattern, we adjust the complex weights \( w_n \). In this case, we require the array factor to be unity at the desired pointing center; thus, the weights are expressed as the complex conjugate of \( \psi_n \) evaluated only at the target position. Thus, the array factor pointed at some target zenith angle \( (za) \) and azimuth \( (az) \) is given by
\[
f(\theta, \phi; za, az) = \frac{1}{N} \sum_{n=1}^{N} \psi_n(za, az) \psi_n^*(\theta, \phi),
\]
(11)
where \( \psi_n^* \) denotes the complex conjugate of \( \psi_n \). This ensures that the array factor power pattern, \( |f(\theta, \phi)|^2 \), will be unity only at the pointing center and in the range \([0, 1]\) elsewhere.

The phased array power pattern is then
\[
B_{\text{array}}(\theta, \phi) = |D_{\text{array}}(\theta, \phi)|^2 = |D(\theta, \phi)|^2 |f(\theta, \phi)|^2,
\]
(12)
which is evaluated over \( \theta = [0, \pi/2] \) and \( \phi = [0, 2\pi] \) to recover the array response to the sky visible to the elements. Both the element factor and array factor are also functions of frequency, \( \nu \); therefore, the tied-array beam pattern is a function of frequency and direction.

This process effectively recreates the naturally weighted synthesized beam for the array. The element pattern, \( D(\theta, \phi) \), for the MWA has a grid-like morphology due to the MWA tiles being a regularly spaced grid of dipoles; thus, we find that for some frequency and pointing combinations, the tile pattern side lobes can be more sensitive than the main lobe. For a pseudo-random array, the tied-array beam pattern grating lobes will be randomly distributed across the sky for each pointing and frequency; thus, the element pattern dominates the sensitivity pattern on the sky. Contrary to our assumption, each tile is not necessarily identical, with some instances of individual dipoles failing, which reduces the tile sensitivity by \( \sim 1/16 \). This effect is not accounted for in the beam simulations.

As an example, Figure 12 shows a simulated MWA tile beam pattern and tied-array beam pattern at 210.56, 165.76, and 120.96 MHz. An important note here is that both the tile beam and tied-array beam models are theoretical, and, in reality, the true beam patterns will have features not described here.

### A.1. Antenna Temperature

The antenna temperature \( T_{\text{ant}}(\nu, \theta, \phi) \) is calculated as the product of the antenna pattern \( B_{\text{array}}(\nu, \theta, \phi) \) and the sky temperature \( T_{\text{sky}}(\nu, \theta, \phi) \) via the convolution
\[
T_{\text{ant}}(\nu, \theta, \phi) = \int \frac{B_{\text{array}}(\nu, \theta, \phi) T_{\text{sky}}(\nu, \theta, \phi)}{2\pi} d\Omega.
\]
(13)
The tied-array beam pattern was output in the necessary format for software used by Sokolowski et al. (2015) to compute the above integral with the GSM, which is natively produced in HEALPix\(^8\) format.

### A.2. Tied-array Gain

To calculate the tied-array gain, we first determine the beam solid angle from the array factor power pattern in the standard way,
\[
\Omega_A = \int |f(\theta, \phi)|^2 \sin \theta \, d\theta \, d\phi.
\]
(14)
The tied-array effective area is then
\[
A_e = \eta \left( \frac{4\pi\lambda^2}{\Omega_A} \right).
\]
(15)
where \( \eta \) is the same frequency- and pointing-dependent efficiency as in Equation (1) and \( \lambda \) is the observing wavelength.

Here we note a divergence in the terminology used. The gain of an aperture array is defined as \( G = 4\pi A_e / \lambda^2 = 4\pi\eta / \Omega_A \) in standard antenna theory. We use a different definition (albeit one common in radio astronomy), such that the gain is
\[
G = \frac{A_e}{2k_B},
\]
(16)
which relates directly to the system equivalent flux density of the array, \( \text{SEFD} = T_{\text{sys}} / G \). In convenient radio astronomy
\[^8\text{http://healpix.sourceforge.net/}\]
Figure 12. MWA tile pattern (left) and tied-array beam pattern (right) for each frequency: (a) 210.56 MHz, (b) 165.56 MHz, and (c) 120.96 MHz. The grayscale background gradient and the colored contours denote the zenith-normalized power for the beam. The magenta cross marks the tile beam-pointing center (azimuth = 18°43', zenith angle = 48°57'). In the case of the 210.56 MHz beam, the highest tile beam-sensitivity region actually exists in the side lobe. The red circles highlight the target position on each of the tied-array beam patterns.
units (K Jy$^{-1}$, where 1 Jy = $10^{-26}$ W m$^{-2}$ Hz$^{-1}$), this becomes simply

$$G = \frac{A_e}{2k_B} \times 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1},$$

where $k_B$ is Boltzmann’s constant and $A_e$ is in units of m$^2$.

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