Correlated imaging through atmospheric turbulence

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Correlated imaging through atmospheric turbulence is studied, and the analytical expressions describing turbulence effects on image resolution are derived. Compared with direct imaging, correlated imaging can reduce the influence of turbulence to a certain extent and reconstruct high-resolution images. The result is backed up by numerical simulations, in which turbulence-induced phase perturbations are simulated by random phase screens inserting propagation paths. © 2019 Optical Society of America

OCIS codes: 270.0270, 010.1330, 110.0115

As correlated imaging develops well in recent years [1–5], more attention has been focused on how to apply this technique to practical applications to overcome the limits in conventional optical systems. For an imaging system which must look through the atmosphere, turbulence-induced wavefront distortions distort the point spread function (PSF) of the system from its ideal diffraction-limited shape, which leads to the degradation of image resolution [6]. To mitigate turbulence effects, a number of methods, such as speckle imaging and adaptive optics techniques [6], have been proposed and applied in optical astronomy. Nonetheless, each of these techniques has its own set of performance limits, hardware and software requirements. New approaches to the problem of reducing these effects are still of much interest. Here we investigate the performance of correlated imaging through atmospheric turbulence and find that the influence of turbulence can be weakened by the second-order intensity correlation.

A schematic of correlated imaging through the atmosphere is depicted in Fig. 1. The beam splitter (BS) divides thermal light into two beams propagating through two distinct optical paths. One is test arm which includes an unknown object and a telescope setup consisting of a lens with focal length f and a detector $D_t$. The object is located at a distance $d_1$ from the source as well as $d_2$ to the telescope setup. The other is the reference arm where another telescope setup consisting of a lens and a detector $D_r$ is placed at $d_0 = d_1 + d_2$ from the source.

For remote sensing (i.e., $d_1, d_2 \gg f$), the detector $D_t$ (or $D_r$) generally lies close to the back focal plane of the lens (i.e., $d_3 \approx f$). The test arm is imbedded in the atmosphere, and turbulence-induced wavefront fluctuations in propagation paths $d_1$ and $d_2$ are represented by $\Psi_1$ and $\Psi_2$, respectively. While the reference arm is said to be a free-space propagation through the distance $d_0$ by assuming that there exists no turbulence. The assumption is based on the fact that the optical field in the reference arm is totally predictable if the field distribution of the source is well known [1, 2].

In the test arm, the field $E_t(x_1)$ in the detector $D_t$ can be given by

$$E_t(x_1) = \int dx_1 E_s(x_1) h_1(\xi, x_1) t(\xi) h_2(x_1, \xi),$$  

where $E_s(x)$ corresponds to the source field, and $t(\xi)$ denotes the transmission function of the object. $h_1(\xi, x)$, $h_2(x_1, \xi)$ are the impulse response functions from the source to the object and from the object to the detector $D_t$, respectively.

Furthermore, according to the extended Huygens-Fresnel integral [7], $h_1(\xi, x)$ and $h_2(x_1, \xi)$ have the forms

$$h_1(\xi, x) = \frac{1}{\sqrt{\pi d_1}} e^{\frac{ik}{\lambda d_1}(x-\xi)^2 + \Psi_1(x, \xi)},$$

$$h_2(x_1, \xi) = \frac{1}{\lambda \sqrt{d_2 d_3}} \int d\eta e^{-\frac{ik}{2\lambda}[(\xi-x_1)/M] \eta + \Psi_2(\xi, \eta)},$$

where $k = 2\pi/\lambda$ is the wave number with $\lambda$ being the wavelength, and $M = -d_1/d_2$ is the magnification of the telescope setup. $\Psi_1(x, \xi)$ and $\Psi_2(\xi, \eta)$ account for the random parts (due to atmospheric turbulence) of the complex phases of the fields in the propagation paths $d_1$ and $d_2$, respectively.

![Fig. 1. Schematic of correlated imaging through atmospheric turbulence.](image-url)
The field $E_r(x_r)$ in the detector $D_r$ is connected to the source field $E_s(x)$ by the Fresnel diffraction integral

$$E_r(x_r) = \frac{1}{\sqrt{3\lambda d_1|M|}} \int dx E_s(x)e^{ik(x-r/M)^2}.$$  \hspace{1cm} (3)

It's worth pointing out that the apertures of the lenses are regarded as large enough, and the diffraction limit of the lenses has been neglected here.

Performing the intensity correlation measurement between the test arm and the reference arm, we get

$$G(x_t, x_r) = \langle I_t(x_t)I_r(x_r) \rangle - \langle I_t(x_t) \rangle \langle I_r(x_r) \rangle$$

$$= c_0 \int dx dx'dx''dx'''d\xi d\xi' \langle E_s(x)E_s^*(x'') \rangle$$

$$\times \langle E_s^*(x')E_s^*(x'''') \rangle \langle h_1(\xi, x)h_1^*(\xi', x') \rangle$$

$$\times \langle h_2(x_t, \xi)h_2^*(x_t, \xi') \rangle t(\xi)t^*(\xi')$$

$$\times e^{ikL[(x''-x_r/M)^2-(x''''-x_r/M)^2]}$$

$$\times e^{2\pi i\frac{d}{M}((x''-x_r/M)^2-(x''''-x_r/M)^2)}.$$  \hspace{1cm} (4)

where $c_0$ is a constant ($\lambda^3d_1d_2d_3|M|$)$^{-1}$, and $I_t(x_t)$, $I_r(x_r)$ represent the intensity distributions in $D_t$ and $D_r$, respectively. Here, we have supposed that the thermal field, and the two turbulent regions are statistically independent of each other.

If the source is fully spatially incoherent and its intensity distribution is of the Gaussian type, the first-order correlation function of the source has the form

$$\langle E_s(x)E^*_s(x') \rangle = I_0 e^{-\frac{2d^2}{\alpha^2}} \delta(x-x')$$

$$\text{where } I_0 \text{ denotes the mean intensity at the center of the source, and } r_c \text{ is the } 1/e^2 \text{ intensity radius. With the help of Eqs. (2a), (2b), and (5), Eq. (4) can be rewritten as}$$

$$G(x_t, x_r) = I_0^2 \int dx dx'dy dy' d\xi d\xi' t(\xi)t^*(\xi')$$

$$\times e^{-\frac{2(\xi^2+\xi'^2)}{\alpha^2}} e^{ikL[(x''-x_r/M)^2-(x''''-x_r/M)^2]}$$

$$\times e^{2\pi i\frac{d}{M}[(x''-x_r/M)^2-(x''''-x_r/M)^2]} e^{\psi(\xi, x)+\psi^*(\xi', x')}$$

$$\times e^{\psi(\xi, y)+\psi^*(\xi', y')}.$$  \hspace{1cm} (6)

The ensemble average of phase variations arising from turbulence can be approximated by [7]

$$\langle e^{-\frac{i\pi}{2}((x''-x_r)^2+(x''''-x_r)^2)(\xi-\xi')+(\xi-\xi')^2} \rangle$$

$$\approx e^{-\rho_1(x''-x_r)^2+(x''''-x_r)^2)(\xi-\xi')+(\xi-\xi')^2}.$$  \hspace{1cm} (7)

where $\rho_1 = (0.545C_n^2)^{2(i)}k^2d_1^{-3/5}$ $(i = 1, 2)$ is the coherence length of a spherical wave propagating in the turbulent medium and $C_n^2(i)$ corresponds to the refractive-index structure constants describing the strength of atmospheric turbulence in the propagation path $d_i$. It's worth emphasizing that we have adopted a quadratic approximation of the Rytov's phase structure function in Eq. (7) to obtain the analytical formula, and this approximation has been used widely in literatures [4, 7].

Substituting Eq. (7) to Eq. (6) and integrating over $\eta, \eta', x, x'$, we have

$$G(x_t, x_r) = \frac{\sqrt{\pi}I_0^2c_0}{\alpha^2d_2(\alpha+2\beta_1)} \int d\xi t(\xi)^2$$

$$\times e^{-\frac{\alpha^2}{2\alpha^2}(x''-x_r/M)^2} e^{-\frac{\beta_1^2}{2}(x''-x_r/M)^2},$$  \hspace{1cm} (8)

where $A = k/2d_1$, $B = k/2d_2$, $\alpha = r_e^{-2}/2$, $\beta_1 = \rho_1^{-2}$.

By making $x_r = x_t$ in Eq. (8), we carry out a special point-to-point intensity correlation [8] and obtain the PSF of the correlated imaging system

$$h_s(x_t, \xi) = e^{-\frac{\alpha^2}{2\alpha^2}(x''-x_r/M)^2} e^{-\frac{\beta_1^2}{2}(x''-x_r/M)^2}.$$  \hspace{1cm} (9)

For the sake of comparison, we also present the intensity distribution in $D_t$,

$$I_t(x_1) = \frac{\sqrt{\pi}I_0c_0}{\alpha^2d_2} \int d\xi t(\xi)^2 e^{-\frac{\alpha^2}{2\alpha^2}(x''-x_r/M)^2},$$  \hspace{1cm} (10)

and the PSF of the test arm

$$h_t(x_1, \xi) = e^{-\frac{\alpha^2}{2\alpha^2}(x''-x_r/M)^2}.$$  \hspace{1cm} (11)

From Eqs. (9) and (11), we can see that the full widths at half maximum (FWHM) of $h_s$ and $h_t$ both broaden with the increase of $\beta_1$ (apart from the influence of the size of the source), which indicates that the resolution, whether for correlated imaging or direct imaging, is degraded by atmospheric turbulence. Additionally, and most importantly, $h_s$ has a narrower FWHM compared to $h_t$, which means that correlated imaging is helpful to reduce turbulent effects and achieve high-resolution images.

In simulations, we consider correlated imaging through horizontal paths in the atmosphere, and thus $C_n^2$ can be regarded as constant in the whole turbulent regions. The numerical model of light propagation in turbulence has been developed well [9, 10]. The spatial power spectral density of the index of refraction fluctuations can be described by the Von Karman spectrum [9],

$$\Phi_n(K, z) = 0.033C_n^2(z)(K^2 + L_0^{-2})^{-11/6}e^{-(K^2/2n)^2},$$  \hspace{1cm} (12)

where $K^2 = K_x^2 + K_y^2 + K_z^2$, $z$ is the propagation distance from the source, $L_0$ and $l_0$ represent the outer scale and inner scale of the turbulence, respectively. By using the spectrum in Eq. (12) to filter a complex Gaussian pseudorandom field and inverse transforming the result, one obtains a two-dimensional phase screen which has the same statistics as the turbulence-induced phase variations [9]. For long atmospheric paths, the multiple phase-screen model [10] has been used in simulations. The turbulent region with the propagation length $d_i$ is broken into a number of layers with a thickness $\Delta z$. Phase fluctuations in each layer are represented by a phase screen inserting in the middle of the layer. The effect of field propagation through these continuous layers can be calculated separately and then combined to characterize
Fig. 2. Simulated (open circles) and theoretical (solid line) on-axis irradiance variance versus the propagation distance. The outer scale and inner scale of turbulence are $L_0 = 3$ m and $l_0 = 1$ cm, respectively.

Fig. 3. The reconstructed images (from left to right) via the correlation in the atmosphere with turbulent level $C_n^2 = 10^{-12}$ m$^{-2/3}$. (a) was obtained by the test arm, and (b) was extracted from the correlation. The normalized horizontal sections of the images are plotted in (c), where open circles correspond to the simulated data and solid lines show the theoretical predictions from Eqs. (8) and (10), respectively.

Fig. 4. The acquired images of the double slit in the atmosphere with turbulent level $C_n^2 = 10^{-13}$ m$^{-2/3}$. (a) was obtained by the test arm, and (b) was extracted from the correlation. The normalized horizontal sections of the images are plotted in (c), where open circles correspond to the simulated data and solid lines show the theoretical predictions from Eqs. (8) and (10), respectively.

To compare direct imaging and correlated imaging, a simple double slit (slit width 10 cm and center-to-center separation 20 cm) was used. After statistics over $10^4$ samples, we obtained a blurred image detected by the test arm directly [see Fig. 4(a)] and a clear image reconstructed through the correlation [see Fig. 4(b)]. This confirms the analytical result that ghost imaging could reduce turbulent effects and improve resolution.

In summary, by taking advantage of the extended Huygens-Fresnel integral, we have presented the theoretical expressions that describes how atmospheric turbulence corrupts the image resolution. Meanwhile, the analytical calculations and the numerical simulations have demonstrated that correlated imaging can provide imaging performance superior to direct imaging through the atmosphere. As an unique image-formed method, correlated imaging can be effectively combined with conventional phase compensating techniques (e.g., adaptive optics) to further eliminate turbulent effects.

This work is supported by the Hi-Tech Research and Development Program of China (Grant No. 2006AA12Z115), Shanghai Fundamental Research Project (Grant No. 09JC1415000), and the National Natural Science Foundation of China (Grant No. 6087709).

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