An IND-CCA2 Secure Public Key Cryptographic Protocol using Suzuki 2-Group

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Abstract

Objectives: The public key cryptographic protocol is one of the most important fields in computer security. These new public key cryptographic protocols provide high security as compared to past results in the same field. Methods/Statistical Analysis: Public key cryptographic is a protocol of transferring private info and data through open network communication, so only the receiver who has the secret key can read the encrypted messages which might be documents, phone conversations, images or other form of data. To implement privacy simply by encrypting the information intended to remain secret can be achieved by using methods of public key cryptography. Findings: In this study, we propose the new IND-CCA2 secure public key cryptographic protocol using the concept of integral coefficient ring polynomial based on Suzuki 2-group. We demonstrated the security of proposed public key cryptographic protocol in the adaptively chosen cipher text secure (IND-CCA2) in the random oracle model. Application/Improvements: We discussed the new strategy with change over an IND-CPA public key cryptographic protocol into an IND-CCA2 cryptographic protocol.

Keywords: IND-CCA2, Public Key Cryptography, Ring Polynomial, Random Oracle, Suzuki 2-Group

1. Introduction

The conception of Public Key Cryptography (PKC) first brought in public domain and introduced6. Since then various public key cryptographic protocols have been developed but could not take desired results. It is a one-way functions show the significant roles in the conception of public key cryptographic protocols. On the apparent difficulty of specific predicaments specifically huge finite commutative rings, these days most prosperous public key cryptographic protocols are established.

To outline public key cryptographic protocol using the undesirable word issue for groups and semi-groups is proposed. The thought is really not in view of word issue, but rather on another, comparatively easier, introduce issue. For a new public key cryptographic protocol which depends on finitely gave assemblies hard word problem. One of successful key establishment protocol came up with a compact algebraic structure. The establishment of their strategy included in the difficulty of explaining conditions over arithmetical structure. Subsequently the first proposed new public key cryptographic protocol is used by braid groups. The security foundation is that when the framework parameters, for example, braid index and the canonical length of the working braids, are chosen legitimately, the Conjugator Search Problem (CSP) is unmanageable. A new public key cryptographic protocol built on finite non-abelian groups was published. Their strategy depends on the discrete logarithm problem in which the inner automorphism group is defined by means of the conjugate action. Later, their system was developed and improves to the so-called MOR systems. In the interim, utilizing one-way functions and trapdoors in finite groups developed new approaches to design public key cryptographic protocol.

Homomorphic public key cryptographic protocol was developed for the first time for non-abelian groups. Afterwards, the extended and expanded their process.

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to discretionary nonidentity finite groups in view of the difficulty of the participation issue for groups of integer matrices. Edified thought the number-crunching key exchange, proposed a new public key cryptographic protocol using polycyclic groups. 

Generic algebraic systems are especially a non-commutative one which is creating its significances, making its marks and attracting many among the above public key cryptographic protocols. There are some difficulties of resolving CSP over certain non-abelian groups using non-commutative algebraic systems. In spite of the fact that there are algorithms for understanding a few variations of CSP in specific groups, such as braid groups with respect to the system parameters none of them can resolve CSP itself defined over general non-abelian group in polynomial time. However, non-commutative acts favorably and unfavorably from one perspective, it makes CSP significant; then again, it brings some bother for planning public key cryptographic protocols. Rectifying the problem, making it favorable is the key concern for developing public key cryptographic protocol over non-commutative algebraic systems.

1.1 Organization

In this article, we establish new ideas for scheming adaptively chosen cipher-text secure (IND-CCA2) secure public key cryptographic protocol using the concept of dihedral group. The main idea of our purpose is to define the technique in polynomials and take them as the fundamental work structure for a given dihedral group. By doing so, it is much easy to implement the effective IND-CCA2 secure public key cryptographic protocol in the random oracle model.

1.2 The Structure of the Article

This paper is sorted out as takes after. In Section 2, preliminaries are presented; In Section 3, we demonstrated some extension over dihedral group; In Section 4, we proposed new IND-CCA2 secure public key cryptographic protocol using dihedral group. In Section 5, we demonstrated supporting example for proposed new public key cryptographic protocol. The security of proposed public key cryptographic protocol is discussed in Section 6. Finally, concluding remarks are made in Section 7.

2. Preliminaries

In this segment, we demonstrated required basic definition of integer coefficient ring polynomials and its properties.

2.1 Integral Coefficient Ring Polynomials (ICRP)

Assume \((\text{\scriptsize \{}\text{\scriptsize \mathbb{R}}, \text{\scriptsize \cdot}, \text{\scriptsize 1}\text{\scriptsize \}})\) is algebraic structure for ring \(\text{\scriptsize \mathbb{R}}\) with multiplicative operation \(\cdot\) of non-commutative semi group and \((\text{\scriptsize \mathbb{R}}, \text{\scriptsize +}, \text{\scriptsize 0}\text{\scriptsize \})\) is algebraic structure with additive operation \(\text{\scriptsize +}\) of commutative group. Now we consider Integral Coefficient Polynomials (ICP) with ring assignment as follows:

For \(\mathbb{z} \in \mathbb{Z}_{\geq 0}\) and \(\theta \in \mathbb{R}\),

\[
\mathfrak{z}\theta \triangleq \theta + \cdots + \theta \quad (l \text{ times})
\]

When \(\mathbb{z} \in \mathbb{Z}_{>0}\), we can define

\[
\mathfrak{z}\theta \triangleq (-\mathbb{z})(-\theta) = (-\theta) + \cdots + (-\theta) \quad (-\mathbb{z} \text{ times})
\]

For \(\mathfrak{z} = 0\), it is normal to define \(\mathfrak{z}\theta = 0\).

Property 1.

\[
(\alpha)\theta^i \cdot (\beta)\theta^j = (\alpha\beta)\theta^{i+j}\quad \forall \alpha, \beta, i, j \in \mathbb{Z}_{>0}
\]

2.1.1 Proof

As indicated by the definition of the distributive of multiplication, scale multiplication concerning commutative of addition and addition, this statement is finished up instantly.

Remark. In non-commutative ring \(\mathbb{R}\), we get

Presently, let us continue to define positive ICRP \((\alpha)\theta. (\beta) \neq (\beta)k.(\alpha)\theta\) for \(\theta \neq k\).

\[
\mathcal{A}(u) = a_0 + a_1u + \cdots + a_ju \in \mathbb{Z}_{>0}\{u\}
\]

We can allocate this polynomial by utilizing a component in \(\mathbb{R}\) and finally get

\[
\mathcal{A}(\theta) = \sum_{a_0}^{\theta} (a_0)\theta^0 = (a_0)1 + (a_1)\theta + \cdots + (a_j)\theta^j.
\]

This is a component in \(\mathbb{R}\), obviously. Advance, in the event that we view as a component in \(\mathbb{R}\), then
An abelian group is a group in which the operation is commutative. A normal subgroup of a group is a subgroup that is invariant under the group's conjugation operation. A component of order 4 in a group is a component that when multiplied by itself three times gives the identity element. The subgroup generated by a component of order 4 is a normal subgroup of the group.

The smallest common multiple of the order of the component is known as the exponent of the component. It is realized that the component is rudimentary abelian. The set of all components that do not generate a normal subgroup as a group is called a rudimentary subgroup.

The Fridani subgroup of a group is a subgroup that is generated by every one of the components of the arrangement. The Fridani subgroup of a group is a normal subgroup of the group. The Fridani subgroup of a group is generated by the components of the arrangement.

The Fridani subgroup of a group is known as the exponent of the component. If the exponent of the component is equal to 4, then the component is known as a component of order 4.
we can transfer these outcomes to general Suzuki 2-group.

Now, given a Suzuki 2-group \((G_2, \cdot, 1)\). Assume that there is a ring \((\otimes, +, 1)\) and a monomorphism \(\varphi: (G_2, \cdot, 1) \rightarrow (\otimes, +, 1)\). Then, the inverse map \(\varphi^{-1}: \varphi(G_2) \rightarrow G_2\) is also a well-defined monomorphism and for \(\alpha, \beta \in G_2\), if \(\varphi(\alpha) + \varphi(\beta) \in \varphi(G_2)\), we can allot another component \(u \in G_2\) as

\[ u = \varphi^{-1}(\varphi(\alpha) + \varphi(\beta)), \quad (5) \]

and call \(u = \alpha \otimes \beta\) as the quasi_sum of \(\alpha\) and \(\beta\). Correspondingly, for \(\kappa \in \otimes\) and \(\alpha \in G_2\), \(\alpha \in G_2\), if \(\kappa \cdot \varphi(\alpha) \in \varphi(G_2)\), then we can allot another component \(u \in G_2\) as

\[ u = \varphi^{-1}(\kappa \cdot \varphi(\alpha)), \quad (6) \]

and call \(\kappa = \alpha \otimes \alpha\) as the \(\kappa\) quasi_multiple of \(\alpha\).

At that point, we can see that the monomorphism \(\varphi\) is linear in sense of that the accompanying equalities hold

\[ \varphi(\kappa \otimes \alpha \otimes \beta) = \varphi(\kappa) \cdot \varphi(\alpha \otimes \beta) \]

for \(\alpha, \beta \in G_2\) and \(\kappa \cdot \varphi(\alpha) + \varphi(\beta) \in \varphi(G_2)\).

Further, for \(\varphi(\alpha) = x_0 + x_1 \cdot t + \cdots + x_n \cdot t^n \in \varphi(G_2)\) and \(\alpha \in G_2\), if

\[ \varphi(\varphi(\alpha)) = z_0 + z_1 \cdot t + \cdots + z_n \cdot t^n \in \varphi(G_2), \]

then for new member \(u \in G_2\) as

\[ w = \varphi^{-1}(\varphi(\varphi(\alpha))) = y^{-1}(z_0 + z_1 \cdot t + \cdots + z_n \cdot t^n), \quad (7) \]

and call \(w = \varphi(\alpha)\) as the quasi_polynomial of \(\alpha\) on \(\varphi\).

Clearly, for arbitrary \(\alpha, \beta \in G_2\), \(\kappa \in \otimes\) and \(\varphi(\alpha) \in \varphi(G_2)\), \(\alpha \otimes \beta, \kappa \otimes \alpha\) and \(\varphi(\alpha)\) are not always well-defined. But, we can prove that the following theorem holds.

3.1 Theorem

For some \(\alpha \in G_2\) and some \(\varphi(\alpha) \in \varphi(G_2)\), if \(\varphi(\alpha)\) and \(\varphi(\beta)\) are well-defined, then

\[ i. \varphi(\varphi(\alpha)) = \varphi(\varphi(\alpha)), \]

\[ ii. \varphi(\alpha \cdot \varphi(\beta)) = \varphi(\alpha) \cdot \varphi(\beta). \]

3.1.1 Proof

(i) \(\varphi(\varphi(\alpha)) = \varphi(\varphi(\alpha))\) is straightforward from the definition of quasi_polynomial.

(ii) \[\varphi(\alpha \cdot \varphi(\beta)) = \varphi(\alpha) \cdot \varphi(\beta)\]

4. An IND-CCA2 Secure Public Key Cryptographic Protocol

In18 introduced a method to translate an IND-CPA encryption protocol into an IND-CCA2 scheme18. By using concept of Fujisaki and Okamoto18, we convert IND-CPA public key cryptographic technique based on Suzuki 2-group in IND-CCA2 public key cryptographic technique based on Suzuki 2-group.

The technique described as follows:

4.1 Setup

- We assume that SDP on \(G_2\) for a given Suzuki 2-group \((G_2, \cdot, 1)\).
- Select two random integers \(i, j \in \mathbb{Z}\).
- Select two component c and d from \(G_2\).
- Let \(\mathcal{H}_1, \mathcal{H}_2\) are two hash functions define (cryptographic) as \(\mathcal{H}_1: \{0,1\}^* \rightarrow \mathbb{Z}\) and \(\mathcal{H}_2: \{0,1\}^* \rightarrow \mathbb{Z}\).
- Let \(\mathcal{H}_1\) and \(\mathcal{H}_2\) are two hash functions define (cryptographic) as \(\mathcal{H}_1: \{0,1\}^* \rightarrow \mathbb{Z}\) and \(\mathcal{H}_2: \{0,1\}^* \rightarrow \mathbb{Z}\).

The public parameters of the technique is given by the tuple \(\{G_2, c, d, i, j, \mathcal{H}_1, \mathcal{H}_2\}\).

4.2 Key Generation

- Each entity selects an arbitrary polynomial \(\varphi(\varphi(\alpha)) \in \varphi(G_2)\) and then takes \(\varphi(\alpha)\) as his/her private key.
- Calculates \(c \cdot \varphi(\alpha) \cdot \mathcal{H}_1 \cdot \mathcal{H}_2 = \varphi(\alpha)\) as his/her public key.

4.2.1 Proof

(i) \(\varphi(\alpha)\) and \(\varphi(\beta)\) are well-defined.

(ii) \(\varphi(\alpha \cdot \varphi(\beta)) = \varphi(\alpha) \cdot \varphi(\beta)\).
4.3 Encryption

For given a $M \in \mathcal{M}$ and Receiver’s key $(c, d, c_\mathcal{N})$, the sender

- Selects a random component $w \in \{0, 1\}^{d \times e}$.
- Selects extracts polynomial $f(c) = f_1(M \parallel w) \in \mathbb{Z}[u]$ such that $f(c) \neq 0$.
- Calculates $s = f(c)^{i} \cdot d \cdot f(c)^{j}$, $t = (M \parallel w) \Theta f_1(f(c)^{i} \cdot c_\mathcal{N} \cdot f(c)^{j})$.

Finally outputs the cipher-text $C = (s, t) \in \mathcal{G}_2 \times \{0,1\}^{n \times w_0}$.

4.4 Decryption

Upon getting a $C$, the receiver utilizing his/her private key $h(c)$, calculates

$M' = \Theta f_2 (f(c)^{i} \cdot d \cdot f(c)^{j})$

Finally, extracts $f(c) = f_1(M') \in \mathbb{Z}[u]$ and checks whether $s = f(c)^{i} \cdot d \cdot f(c)^{j}$ holds. Assuming this is the case, yields the starting $c_\mathcal{N}$ bits of $M'$; generally, yields empty string, which implies that the given cipher-text is invalid.

5. Concrete Examples

In this segment, we illustrate example for supporting our proposed new public key cryptographic technique based on Suzuki 2-group.

Let us the class of Suzuki 2-group with order $p^2$. Utilizing Higman’s documentation, a Suzuki 2-group of order $p^2$ will be indicated by $\mathcal{S}(\eta, \theta)$. Assume $p = 2^n$ where $\eta \geq 3$ belongs natural number such as an extent that the field $F_p$ has nontrivial automorphism $\theta$ of non-even order. This infers $\eta$ is not a force of 2. At that point the gatherings $\mathcal{S}(\eta, \theta)$ do exist.

Honest, in case we describe $\mathcal{G}_2 = \{f(1 ... 1) \in F_p\}$, where $f(1 ... 1)$ is a $3 \times 3$ - matrix over $F_p$.

Give us a chance to delineate our technique by utilizing a Suzuki 2-group: $M_2 \left\{ F_p \right\}$, where $\mathcal{N} = q \cdot p$ while $q$ and $p$ are two extensive secure primes. We have strong motivation to trust that symmetrical decomposition problem over $M_2 \left\{ GL \left( 3, p \right) \right\}$ is immovable, since it is infeasible to extract.

$$y = \begin{pmatrix} i & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in M_2 \left\{ GL \left( 3, p \right) \right\} \subset M_2 \left\{ F_p \right\} \subset M_2 \left\{ \mathcal{N} \right\}$$

Form

$$y^2 = \begin{pmatrix} i^2 \mod \mathcal{N} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \in M_2 \left\{ GL \left( 3, p \right) \right\} \subset M_2 \left\{ F_p \right\} \subset M_2 \left\{ \mathcal{N} \right\}.$$
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(1) Take note of that if \( f(u) \) does not satisfy the condition of \( f(c) \neq 0 \), we ought to at first amend \( f(u) \) to \( f(u) + \Delta \). Where

\[
\Delta = \min \{ \zeta \in \mathbb{F}_2 : f(c) + \zeta \cdot \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \neq 0 \}
\]

Luckily, in this illustration. The above extracted \( f(u) \) meets the necessity of \( f(c) \neq 0 \), i.e., \( \Delta = 0 \). At that point, then cipher-text combine is

\[
s = f(c) \cdot d \cdot f(c)^3 = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} = \begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix} = \begin{pmatrix} 2 & 4 & 8 \\ 8 & 2 & 15 \\ 4 & 8 & 8 \end{pmatrix}.
\]

and

\[
t = (M \perp \omega) \oplus f_2(f(c)^2 \cdot d \cdot f(c)^3) = \begin{pmatrix} 2 & 4 & 9 \\ 5 & 3 & 1 \\ 8 & 2 & 15 \end{pmatrix} \oplus f_2(\begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix}) = \begin{pmatrix} 2 & 4 & 9 \\ 5 & 3 & 1 \\ 8 & 2 & 15 \end{pmatrix} \oplus f_2(\begin{pmatrix} 8 & 0 & 0 \\ 0 & 8 & 0 \\ 0 & 0 & 8 \end{pmatrix}) = \begin{pmatrix} 2 & 4 & 9 \\ 5 & 3 & 1 \\ 8 & 2 & 15 \end{pmatrix} \oplus f_2(\begin{pmatrix} 6 & 1 & 1 \\ 4 & 6 & 1 \\ 6 & 6 & 6 \end{pmatrix}) = \begin{pmatrix} 4 & 5 & 8 \\ 1 & 5 & 0 \\ 14 & 4 & 9 \end{pmatrix}.
\]

6. Security Analysis and Discussion

In 1999, \( \Lambda \) acquainted a strategy with change over an IND-CPA cryptographic technique into an IND-CCA2 cryptographic technique. For self-containing, we practice their principle thought as takes after:

Assume that \( \Lambda \rightarrow \{ \Omega, \bar{d}_H, \bar{d}_c \} \) is an IND-CPA secure public-key cryptographic technique with key generation procedure \( \Omega(1^\chi) \), encryption procedure \( \bar{d}_H(\mathbf{M}, \mathbf{S}) \) and decryption procedure \( \bar{d}_c[H,C](y) \), where \( \Omega(1^\chi) \) and \( \bar{d}_H \) are a private key and the confirming public key, \( \mathbf{M} \) a message with \( \theta + \theta_0 \) bits, \( \mathbf{S} \) a random string with \( \theta \) bits and \( \mathbf{C} \) a cipher-text. The transformed public-key cryptographic technique \( \Lambda \rightarrow \{ \Omega, \bar{d}_H, \bar{d}_c \} \) is defined by \( \Omega(1^\chi) \rightarrow \Omega(1^{\theta + \theta_0}) \), \( \bar{d}_H(\mathbf{M}, \mathbf{S}) \rightarrow \bar{d}_H(\mathbf{M}, \mathbf{S}) \), \( \bar{d}_c[H,C](y) \rightarrow \bar{d}_c[H,C](y) \), \( \bar{d}_c[H,C](y) \rightarrow \bar{d}_c[H,C](y) \)

where \( f_i : \{0,1\}^{\theta + \theta_0} \rightarrow \{0,1\}^\chi \) is a random function of, \( m \) is a message with \( \theta \) bits, \( \omega \) an arbitrary string with \( \theta_0 \) bits and \( \| \) denotes concatenation.

6.1 Theorem 6.1

Assume that \( \Lambda \rightarrow \Omega(1^{\theta + \theta_0}) \) is the first IND-CPA secure cryptographic technique and \( \Lambda \) is changed over technique. In the event that \( \exists \) a \( \{ \omega, \bar{d}_H, \bar{d}_c, \bar{d}_{\bar{d}_a} \} \)-adversary \( \mathcal{A} \) for \( \Lambda(1^\chi) \) in the sense of IND-CCA2 in the ROM, \( \exists \) a \( \{ \omega', \bar{d}_H, \bar{d}_c, \bar{d}_{\bar{d}_a} \} \)-adversary \( \mathcal{A}' \) for \( \Lambda(1^{\theta + \theta_0}) \) and constant \( c \), where

\[
\bar{d}_{\bar{d}_a}(y) := \begin{pmatrix} d_{\bar{d}_a}(y) \end{pmatrix} + \begin{pmatrix} d_{\bar{d}_a}(y) \end{pmatrix} \quad \text{for} \quad \mathcal{A}' \equiv \mathcal{A} \quad \text{for} \quad \Lambda(1^{\theta + \theta_0}) \quad \text{and} \quad \text{constant} \quad c,
\]

\[
t' = t + \left( c \ast k + T_k(\omega) \right) \cdot \bar{d}_H
\]

Here, \( \{ \omega', \bar{d}_H, \bar{d}_c, \bar{d}_{\bar{d}_a} \} \)-adversary \( \mathcal{A} \), casually, implies that \( \mathcal{A} \) stops inside \( t \) stages, prevails with probability in any event, makes at most \( q_H \) inquiries to \( \mathcal{A} \), and most \( q_{\bar{d}_a} \) inquiries to decryption oracle \( d_{\bar{d}_a}(y) \)
The computational time of the encryption procedure \( \hat{E}_{\mathcal{A}}(\cdot) \) is \( T_{\mathcal{E}}(k) \) and

\[
L_0 \rightarrow \log_2 \left( \min_{m \in \{0,1\}^{\nu+\nu_0}} \left[ \#\{\hat{E}_{n,\mathcal{A}}(M, \omega) | \omega \in \{0,1\}^\nu \} \right] \right).
\]

Proof. See Theorem 3 of [3].

As indicated in [3], we can changeover our fundamental public key cryptographic technique into more secure new public key cryptographic technique, which comes to IND-CCA2 security, with sacrificing of \( \nu_0 \) bits plaintext.

6.2 Theorem 6.2

Let \( f_j \) be a random oracle and \( \mathcal{A} \) be an IND-CPA foe that has advantage against the purpose fundamental technique inside \( t \) iterations. Assume that \( \mathcal{A} \) makes a \( q_{f_j} \) total of inquiries to \( f_j \). Then there is a procedure \( A \) that resolves polynomial Diffie-Hellman problem over \( d_{\nu'} \) with advantage at least \( \epsilon' \) within \( t' \) iterations, where

\[
\epsilon' = \frac{2\epsilon_{\nu_0}}{d_{f_j}} \quad \nu' = O(\Omega).
\]

Proof. See the Theorem 6 of [3].

6.3 Theorem 6.3

Assume that \( f_{j_1} \) and \( f_{j_2} \) are two random oracles. Then the presented public key cryptographic technique is an IND-CCA2 accepting polynomial Diffie-Hellman problem over the Suzuki 2-group \( G_{S} \) is hard. All that has been assumed is an IND-CCA2 foe \( \mathcal{A} \) that has advantage against the presented public key cryptographic technique inside \( t \) steps. Assume that adversary \( \mathcal{A} \) makes at most \( q_D \) decryption inquiries, and at most \( q_{f_{j_1}}, q_{f_{j_2}} \) inquiries to the hash functions \( f_{j_1} \) and \( f_{j_2} \) respectively. Then there is a technique \( B \) which can solve polynomial Diffie-Hellman problem with the probability at least \( 0 \) inside \( t_0 \) steps, where

\[
\epsilon'' = \frac{2\epsilon_{\nu_0}}{d_{f_{j_1}}} \left[ \epsilon_{\nu_0} \cdot 2^{-\nu_0} - 1 \right] q_D + 2^{\nu_0 - 1} q_{f_{j_2}}
\]

\[
t' = \mathcal{O}(t + (ck + T_{\mathcal{E}}(\nu))q_{f_{j_2}})
\]

where \( c \) is a constant and \( T_{\mathcal{E}}(\nu) \) represents the computational time of the encryption process \( \hat{E}_{n,\mathcal{A}}(\cdot) \) in our purposed public key cryptographic technique, and \( \nu_0 \rightarrow \log_2 (\min_{m \in \{0,1\}^{\nu+\nu_0}} \left[ \#\{\hat{E}_{n,\mathcal{A}}(M, \omega) | \omega \in \{0,1\}^\nu \} \right] \)
6.8 Collision-Resisting

The above correcting procedure is not to abuse the property of collision resistance. In fact, the collision resistance of $f_1$ is established in the one-wayness of $f'$.

7. Conclusions

In this study, we demonstrated new approach for designing the public key cryptographic technique using the concept of general non-commutative algebraic system such as Suzuki 2-group. Also we discussed the new strategy with change over an IND-CPA cryptographic technique into an IND-CCA2 cryptographic technique. By using this new strategy we change our past IND-CPA public key cryptographic technique in to more secure IND-CCA2 public key cryptographic technique. The principle thought in our suggestion lies that we consider polynomials on given non-commutative arithmetical framework as the major work structure for creating cryptographic arrangements. Consequently, we can get some commutative sub-structures for the given non-commutative scientific systems.

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