Students’ Ability Of Mathematical Problem-Solving Based On SOLO Taxonomy

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Abstract. One of the critical purposes of learning mathematics is to improve the problem-solving ability. To figure out the level of students’ ability in mathematical problem-solving, this study employed SOLO taxonomy. 32 students of VIII grade in SMP Muhammadiyah Bantul were involved. Their level of mathematical problem-solving ability was categorized based on SOLO taxonomy that consists of pre-structural, uni-structural, multi-structural, relational, and extended abstract. Using a qualitative descriptive method, data were collected by problem-solving test in a pattern of number material and interviews. The results revealed that most students are in the uni-structural level, both of upper and lower group students. This finding shows that the students need to be supported and well facilitated in order to reach higher levels, such as relational and extended abstract levels.

1. Introduction

Problem solving is the core of mathematics learning activities at all levels [1]. Moreover, problem solving is agreed as the heart of mathematics learning because it is not only for subject learning but emphasizes the development of thinking skills [2]. This is reinforced by the argument which states that one important objectives of mathematics education is the acquisition of learning skills how to maintain problems [3].

Problem solving is an attempt to achieve the desired goal and not automatically known the right way for that purpose [4]. Problem solving is a cognitive process to find solutions to certain problems or find solutions to achieve certain goals [5]. When problems are identified, problem solving is a process to find the relationship between goals and solutions in solving problems. In addition, problem solving is regarded as a process to invent a combination of a number of rules that can be applied to new situations. It means that problem solving is not just a form of ability to apply rules that have been mastered through activities that have been learned before, but more than that is a process to formulate new answers or to get a set of rules at a higher level [6].

Problem solving abilities are important because students need to know how to solve problems for the future, whether in mathematics or in everyday life. In addition, through problem solving students will learn to read mathematics, develop a sense of pride, become more critical and analytical with the problems presented, and solve problems [7]. Problem solving can also help students learn facts, skills, concepts, and mathematical principles [8]. Mathematical problem solving skills can help students analyze and apply problem solving strategies in diverse situations [8]. Problem solvers that are good in everyday life and the world of work will bring great benefits for both theirself and others [1].
Characteristics of problem solving involve a variety of cognitive activities in students by linking and using prior knowledge and experience [9]. The existence of various cognitive activities causes different levels of students' problem solving abilities. Given the importance of problem solving abilities, it is necessary to know about how far the problem solving abilities that have been achieved by students. One framework that can be used to investigate the level of students' problem solving abilities is SOLO (Structure of the Observed Learning Outcome) Taxonomy.

This Taxonomy was first introduced by Biggs & Coills in 1982 [10]. This taxonomy is suitable for measuring various types of students’ learning outcomes [11]. SOLO Taxonomy is one suitable framework to be used in developing systematic and objective assessment instruments. It is easily understandable by both teacher and student. For this reason, the taxonomy may be used as instructional as well as an evaluative tool [10]. In addition, the SOLO taxonomy can also illustrate how the structures of cognitive complexity or students' response from the existing level of thinking [12].

SOLO Taxonomy is a technique to determine the response in learning and is usually used in education, among others: 1) showing the individual's cognitive level objectively, 2) helping students analyze the results of their work and see how to improve it, 3) to determine levels, 4) determination, 5) ability predictor, 6) education in education. SOLO Taxonomy is a taxonomy that classifies the level of ability of students at five different and hierarchical levels, namely pre-structural, unistructural, multistructural, relational, expanded abstracts [13]. The five levels can be used to make student classification in solving problem [12].

In this study, the five levels are used to investigate students' problem solving abilities. The description of the data is divided based on the students’ initial ability level (upper and lower group). This is based on the findings that showed the relationships between SOLO and other learning factors such as prior academic abilities were positive [11]. This research will provide an overview of the extent to which the level of students' problem solving abilities is in accordance with SOLO Taxonomy. Thus, this study will provide empirical instructions to the teachers and education developers to improve students' problem solving abilities.

2. Research Method

2.1. Participants
This was a qualitative descriptive study which aimed to describe mathematical problem solving abilities based on SOLO taxonomy on number pattern material. The study was conducted at SMP Muhammadiyah Bantul with the 32 subjects from VIII grade students. Students were categorized into 2 groups: 16 upper group students and 16 lower group students. This group was divided by the students’ value of mathematics learning outcome in the report card of the previous semester.

2.2. Techniques and Instruments of Data Collection
Data collection techniques used were tests and interviews. Test was carried out to describe students' problem solving abilities. While the interview was conducted to check the data. Besides the researcher as the main instrument, this study developed problem-solving tests on number patterns material and interview guidelines. These instruments has been validated by experts in mathematics education.

2.3. Data Analysis
Student response data were analyzed using indicators compiled based on SOLO Taxonomy. This was including five indicators, namely pre-structural, uni-structural, multi-structural, relational, and expanded abstracts. This indicators used to analyze student answers in this study were adjusted to the framework developed by Chick [14], have been modified by Ekawati, Junaedi, and Nugroho and validated by experts [15]. The description and specifications for each indicator can be seen in table 1.
3. Result and Discussion

3.1. Result
As describe before, students were given two problem solving problems in the topic of pattern number. Although they have different levels of difficulty, the two problems arranged can be used to describe students' abilities into 5 levels of SOLO Taxonomy. The two problems were presented in Figure 1 and Figure 2 as follows.

The First Problem:
In a building there are 10 chairs in the front row. The number of seats in the next lines is always more than 4 seats compared to the previous row. Determine the number of seats in the building if there are 10 rows of seats.

Figure 1. The First Problem
The Second Problem:
Mr. Candra made several square-shaped pool designs. Each pool has a square shape in the water storage area and is given a blue tile. Around the pool is surrounded by a barrier that has white tiles, as shown below.

![Pool 1](image1)
![Pool 2](image2)
![Pool 3](image3)

How many white tiles, when the blue tile is 441?

Adapted from brainly.co.id
Translated from Bahasa

Figure 2. The Second Problem

Based on the results of the analysis, there are no students at the pre-structural level (0%). Beside that, this study revealed that for upper group students, the students respond in the first question showed that most students are at multi-structural level (53.125%). Whereas in the second problem, most students only reach in the uni-structural level (62.5%). While the response of lower class students in the first question and the second question shows that most students are at an uni-structural level (68.75% for the first problem) and (78.125% for the second problem). The summary of the analysis data can be seen in table 2.

| Response Level     | Upper Group | Lower Group |
|--------------------|-------------|-------------|
|                    | First Problem | Second Problem | First Problem | Second Problem |
| Pre-structural     | 0           | 0           | 0           | 0           |
| Uni-structural     | 37.5        | 62.5        | 68.75       | 78.125      |
| Multi-structural   | 53.125      | 31.25       | 28.125      | 21.875      |
| Relational         | 6.25        | 6.25        | 3.125       | 0           |
| Extended abstract  | 3.125       | 0           | 0           | 0           |

Table 2. Data analysis results

In the upper group students, the analysis results of the students answer indicate that the level of thinking of upper group students is generally at the multi-structural level. Besides that, it can also be seen that there are some upper group students who can reach the relational. Only some upper group students can meet in the extended abstract levels. Moreover, there is no student can reach the extended abstract level for the second problem. To provide an overview of the answers of upper group students, the following is an example of the results of student respond on the problem solving test provided. Those examples are presented in Figures 3 and Figure 4.
In Figure 3, it can be seen that the student uses some relevant data, including the number of seats in the first row and the difference between the rows of seats. Here, the student has passed the relational level, because besides using some relevant data/information, student also capable to connect the information so that relevant conclusions are obtained. More than that, the student answer presented in Figure 3 show that the student can generalize the number pattern of a given problem, so that this student can determine the number of seats in any row with only the pattern found. In this case, it can be said that this student has reached the extended abstract level.

In Figure 4, it can be seen that the student has been able to reach the multi-structural level because she can make a number of relationships from known or acquired data. The student can afford to think that the number of blue tiles is obtained from the square of the original number. However, she has not been able to draw relevant conclusions based on what she found. This was reinforced by the result of interview which she said that it was difficult to determine the relationship between many blue tiles and white tiles. That is, this student has not managed to attain the relational level.

Meanwhile, for lower group students, the results of the analysis of students' answers indicate that some students who are in the lower group are generally at the uni-structural level. Only some of the lower group students can achieve in multi-structural level. There are only very few students who can reach the relational level. In fact, there are no lower group students who can achieve the highest level, i.e. extended abstract level. Answer samples of lower group students can be observed in Figure 5 and Figure 6.
In Figure 5, it appears that this student can reach uni-structural levels to the relational level. This student is categorized to be at the uni-structural level because he has been able to use at least one information to solve problems. Furthermore, this student can determine relevant conclusions from some information obtained. This finding indicates that this student meet the relational level. However, unfortunately, this student has not demonstrated his ability to generalize the results in his answers sheet. Indeed, this method is correct. But with this situation, students must determine the number of seats on each row to determine the next row. So that when he encounters a row of chairs with large numbers, he will be troubled.

In Figure 6, it can be seen that the student is only able to obtain the structural level. This student cannot determine the relationship between several existing data. In her answer sheet, she only write down the existing information without explaining the relationships between the data. Accordingly, this student cannot found the solution and draw conclusions from the problems given.

3.2. Discussion
The results of this study indicate that students' problem solving abilities are not optimal. This does not only happen to lower student groups, but all students. From answers written by students, most students have not been able to choose relevant information and link known information. This will hamper the subsequent problem solving process. Moreover, they will be trouble to make relevant conclusions and generalize data from the results obtained. Therefore, teachers need to train students more about problem solving activities. Students need to be involved frequently in solving various problems, ranging from low taxonomy levels to extended abstract levels.

However, from this study, it seems that the achievement of the level of problem solving abilities based on solo taxonomy on both questions is different. This indicates that the level of difficulty of the problem also affects the level that can be achieved by students. Therefore, the problem of problem solving also need to be adjusted to the level of students [16]. Thus, this is to prevent the decline in student confidence and make them have a positive outlook on problem solving activities [16].

Moreover, this capability is certainly also influenced by other factors such as motivation, self-efficacy, attitude towards mathematics, self-esteem and also teachers' teaching behaviour [17]. Therefore, the teachers need to promote this ability and then engage their student to the activities in class that encourage students to have high motivation to learn mathematics and have good attitudes toward mathematical problem solving.

In addition, it should also be emphasized to students that in solving problems, the most important thing is not the final answer. This means that students may not find the right answer or have a different way. The teacher should suggest that the results of problem solving are mathematical concepts. For this to change, the teacher should compose a learning environment in which in the problem solving activity, the answer is not the key and sometimes there are multiple ways to attain an answer [17].

4. Conclusion
The results of this study indicate that in both the upper and lower group students, not many students can reach the high level. Especially at the extended abstract level, almost no students can reach this level. Therefore, this study suggests to develop the students' problem solving abilities, especially at the level of relational and extended abstract.

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