Measurement of guided mode wave vectors by analysis of the transfer matrix obtained with multi-emitters and multi-receivers in contact.

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Abstract. Different quantitative ultrasound techniques are currently developed for clinical assessment of human bone status. This paper is dedicated to axial transmission: emitters and receivers are linearly arranged on the same side of the skeletal site, preferentially the forearm. In several clinical studies, the signal velocity of the earliest temporal event has been shown to discriminate osteoporotic patients from healthy subjects. However, a multi parameter approach might be relevant to improve bone diagnosis and this be could be achieved by accurate measurement of guided waves wave vectors. For clinical purposes and easy access to the measurement site, the length probe is limited to about 10 mm. The limited number of acquisition scan points on such a short distance reduces the efficiency of conventional signal processing techniques, such as spatio-temporal Fourier transform. The performance of time-frequency techniques was shown to be moderate in other studies. Thus, optimised signal processing is a critical point for a reliable estimate of guided mode wave vectors. Toward this end, a technique, taking benefit of using both multiple emitters and multiple receivers, is proposed. The guided mode wave vectors are obtained using a projection in the singular vectors basis. Those are determined by the singular values decomposition of the transmission matrix between the two arrays at different frequencies. This technique enables us to recover accurately guided waves wave vectors for moderately large array.

1. Introduction
Elastic waves are widely used in material characterization. For example, guided waves, such as Lamb waves, are used to characterize plates or tubes [1]. Structural and material properties can be characterized by fitting measured to theoretical guided mode wave vectors. The two dimensional spatio–temporal Fourier transform is one of the usual methods used to determine the guided mode wave vectors [2], [3]. Nevertheless this approach requires a large distance probed by the receivers. Practical constraints, as in clinical inspection of cortical bones, reduce the inspected spatial length and therefore the efficiency of this technique. The approach, presented in this paper, is based on the Singular Values Decomposition (or SVD) applied to the configuration of multi-emitter and multi-receiver arrays in contact with the object to be characterized. SVD, a widely used filtering tool [4], is adapted to the specific case of axial transmission. The aim of this paper is to introduce a method of measurement of the guided mode wave vectors in the context of ultrasonic cortical bone testing [5]-[8].

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2. Theory

2.1. Response matrix

A single probe consisting of two arrays is used in contact with a metallic plate, using coupling gel (figure 1). Superscripts E and R refer respectively to the emission and reception arrays. Thus the emission array contains $N_E^E$ emitters of position $x_i^E$, with $1 \leq i \leq N_E^E$. Likewise, the reception array contains $N_R^R$ receivers of position $x_j^R$, with $1 \leq j \leq N_R^R$. Define the $N_R^R \times 1$ vector $R_j(f)$ containing the Fourier transforms $r_j(t)$, with $f$ the temporal frequency. Likewise, the $N_E^E \times 1$ vector $E_i(f)$ contains the Fourier transforms $E_i(t)$. The relation between the two vectors $R$ and $E$ writes, at a given frequency

$$ R = H \cdot E, \quad (1) $$
with $H$ the $N_R^R \times N_E^E$ matrix containing the elements $H_{ji}(f)$ equal to the temporal Fourier transform of the impulse responses $h_{ji}(t)$. The $R$ matrix, called afterward “response matrix”, is characteristic of the “emission – scattering medium – reception” system.

![Figure 1. Geometry of the problem: the emission and reception arrays are in contact with the metallic plate.](image-url)

2.2. Singular vectors basis

The Singular Value Decomposition (SVD) of the transmission matrix $R$ writes

$$ R = \sum_{n=1}^{\min(N_R^R, N_E^E)} U_n \sigma_n V_n^*, \quad (2) $$
where the notations $'$ and * denote the transpose and conjugation operations. The number of experimental singular values $\sigma_n$ is equal to minimum size of the arrays, i.e. the minimum between $N_E^E$ and $N_R^R$, equal in the following to $N_E^E$. The notation $V_n$ refers to an emission singular vector, of dimension $N_E^E \times 1$, and $U_n$ to a reception singular vector, of dimension $N_R^R \times 1$. These two vectors are associated with the singular value $\sigma_n$.

The singular vectors $U_n$ (resp. $V_n$) are normalized and define an orthogonal basis of the received (resp. emitted) signals [12]. In particular any spatial plane wave can be expressed in the reception basis. Consider a spatial plane wave $e^{\text{pw}}(k)$ defined on the $j$th receiver as

$$ e^{\text{pw}}_j = \frac{1}{\sqrt{N_R^R}} \exp(ikx_j^R), \quad (3) $$
with $k$ the wave vector equal to $2\pi f / c$ and $c$ the phase velocity. The previous vector is defined normalized. The expression on the basis defined by the reception singular vectors $U_n$, is

$$ e^{\text{pw}} = \sum_{n=1}^{N_R^R} \langle e^{\text{pw}} | U_n \rangle U_n, \quad (4) $$
with the notation $\langle e^{\text{pw}} | U_n \rangle$ corresponding to the Hermitian scalar product, equal to $[e^{\text{pw}}]^* \cdot U_n$. 

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2.3. Noise and signal separation

One of the advantages of the SVD approach is its ability to separate noise and signal subspaces [10], [11]. Indeed, using an intermediate order \( m \) corresponding to the limit between the two subspaces, equation (4) can be rewritten with a truncated sum, with \( n \leq m \). The order \( m \) is defined at each frequency using a threshold \( t_1 \) applied to the singular values \( \sigma_n \). In the following, the signal singular vectors are retained (for \( \sigma_n \geq t_1 \)) whereas the noise singular vectors (for \( \sigma_n < t_1 \)) are eliminated. Thus the norm of the spatial plane wave on the basis of the signal subspace becomes

\[
\| e^{\text{pw}} \|_{U_{\text{pw}}} = \sqrt{\sum_{n=1}^{N_{\text{pw}}} |(e^{\text{pw}} | U_n)^2 |},
\]

with the notation | | used for the modulus of complex numbers. The value of the threshold \( t_1 \) is heuristically chosen, based on an estimation of the signal to noise ratio and on a trade off between the estimated number of modes and the noise. That threshold may also vary with frequency and attenuation. An example is shown in section 3 and figure 2.

2.4. Evaluation of the guided mode wave vectors

The norm of the spatial plane wave expressed on the signal (or retained) singular vectors in (5) is used to form an image in the \((k, f)\) plane. To enhance the contrast between the low and high values, the square of the norm is used in the following. As \( m \), the dimension of the signal subspace, is less than the number of receivers \( N_R \), i.e. the dimension of the vector \( e^{\text{pw}} \), the basis defined by the signal reception singular vectors \( U_{\text{pw}} \) is incomplete. It implies that the norm defined by (5) is less than 1. Consequently, each pixel \((k, f)\) of the image ranges from 0 to 1.

The value of the pixel reflects how the spatial plane wave is represented in the basis of the signal subspace. On the one hand, if the value is small compared to 1, the spatial plane wave is absent of the received signals. On the other hand, if the value is close to 1, the spatial plane wave is present in the received signal and probably due to a guided wave. A second threshold \( t_2 \) is then applied to the image in order to reduce the range from \( t_2 \) to 1. Thus a function, denoted \( \text{Norm} \), is defined in the \((k, f)\) plane as

\[
\text{Norm}(k, f) = \begin{cases} \| e^{\text{pw}} \|_{U_{\text{pw}}} & \text{if } \| e^{\text{pw}} \|_{U_{\text{pw}}} \geq t_2 \\ 0 & \text{if } \| e^{\text{pw}} \|_{U_{\text{pw}}} < t_2 \end{cases},
\]

The maxima of the function \( \text{Norm}(k, f) \) provide the wave vectors of the guided mode present in the signal subspace. In the examples shown in section 3, the value of that second threshold \( t_2 \) is heuristically chosen equal to 0.4 as illustrated in figure 4.

3. Experimental results and discussion

3.1. Experimental set up

The probe developed at the LIP (Laboratoire d’Imagerie Paramétrique) for the bone characterization, consists of \( N_E \) of emitters is equal to 5 and the number of receivers \( N_R \) is equal to 32. Thus the number of emitters, which is less than the number of receivers, corresponds to the number of experimental singular values. A coupling gel is used in order to improve the acoustical impedance matching. Emitted signals are pulses with bandwidth about 0.5–2 MHz. The reception array pitch is equal to 0.80 mm. The experimental scattering medium is a 2 mm thickness copper plate. The free plate guided modes are given by the Rayleigh-Lamb dispersion equation [1]. No attenuation is considered in this model. The modes are labelled considering their symmetry or anti-symmetry and cut off frequencies, i.e. \( S_n \) and \( A_n \).
3.2. Resolution and evaluation domain of the system

The resolution can be estimated considering the size of the main lobe of the sinc function, given by the scalar product between a reception singular vector $U_n$ corresponding to a guided mode $k_n$ and a plane wave $e^{i\omega(k)}$. With $L^R$ the size of the reception array, the resolution condition is

$$\Delta k \geq \frac{2\pi}{L^R}.$$  \hspace{1cm} (7)

Furthermore, the Shannon criterion is verified when $k$ is less than $\frac{\pi}{p}$, with $p$ the array pitch. Thus the range $2\pi L^R \leq k \leq \pi p$ corresponds to the evaluation domain, where the guided mode wave vectors $k_n$ can be evaluated.

3.3. Experimental results, comparison with the 2D Fourier transform

Figure 2 shows the experimental singular values $\sigma_n$ as function of the frequency $f$. The number of experimental singular values, i.e. 5, corresponds to the number of emitters. At low frequency, only the two first singular values are significant, whereas for $f > 1$ MHz, the number of significant singular values ranges from 3 to 5. It corresponds to the number $m$ of significant singular vectors used in (6). It depends on the signal to noise threshold $t_1$, shown with a curved line, heuristically fixed at a level as shown in Figure 2. The different peaks correspond to thickness resonance of guided modes. For example, the thin peak for $f = 1.1$ MHz corresponds to the ZGV (zero group velocity) resonance [2].

Figure 2. Singular values versus frequency (MHz) for a 2 mm thick copper plate. The signal to noise threshold $t_1$ is shown with a curved dashed line.

Figure 3(a) shows the image $\text{Norm}(k, f)$ defined by (6) using the signal singular vectors, i.e. the singular vectors associated with singular values bigger than the threshold $t_1$ shown in figure 2. The value of the pixels ranges from $10^{-8}$ to 0.96 is in agreement with the expected range, i.e. 0 to 1. The guided mode wave vectors are shown as trajectories in the $(k, f)$ plane. The width of each trajectory corresponds to the resolution given by (7). Figure 3(b) shows the two-dimensional spatio-temporal Fourier transform for comparison. Each line (or frequency) is normalized by its maximum.
Figure 3. $\text{Norm}(k,f)$ (a) and normalized spatio-temporal (2D) Fourier Transform (b) for a 2 mm thick copper plate. The maxima of the $\text{Norm}$ function are more clearly defined. The second threshold $t_2$ is indicated on the colorbar. The frequency indicated with an horizontal line is shown on figure 4(a).

The results given by the two methods are similar. However, the $\text{Norm}$ function allows a better ability to detect the guided mode wave vectors as illustrated in figure 4(a) for a fixed frequency. This frequency is shown as a dashed line in figure 3(a). The maxima of the $\text{Norm}$ function ranging from 0.4 to 0.9 are clearly defined, in contrast with the maxima of the normalized spatio-temporal Fourier transform depending on the relative amplitude of each guided modes. As discussed in paragraph 2.4, the values of the maxima of the $\text{Norm}$ function do not depend on the relative amplitude of the guide mode. The peak are more clearly defined compared to the sinc secondary maxima. Thus this original technique can be applied to noisy signals such as those acquired in cortical bone. The two methods are compared in the whole $(k,f)$ domain in figure 4(b). The results are in good agreement with the theoretical Lamb (computed with $e = 2$ mm, $c_L = 4700$ m.s$^{-1}$ and $c_T = 2300$ m.s$^{-1}$) modes for the two methods. The norm function give more experimental values than the spatio-temporal Fourier transform using the same second threshold equal to 0.4.
Figure 4. Comparison of both methods: \( \text{Norm}(k, f) \) (continuous line and circles) and normalized 2D Fourier transform (dashed line and dots), for \( f = 1.04 \) MHz \((a)\) and in the whole \((k, f)\) domain \((b)\). The frequency 1.04 MHz is shown as a dashed line in figure 3\((a)\). The second threshold \( t_2 \) is shown with a horizontal dashed line. The ability to detect phase velocity is improved using the \( \text{Norm} \) function as the maxima are better defined, compared to the sinc secondary maxima.
4. Conclusion

An original method for evaluating the guided mode wave vectors using arrays in contact (an emission array and a reception array) is exposed in this paper. The ability to detect the guided mode wave vectors is improved compared to the results obtained with the spatio-temporal Fourier transform as the values of the maxima of the Norm function do not depend on the relative amplitude of the guide mode. Further applications will concern in particular the evaluation of elastic properties of cortical bone with a reduced inspected spatial length due to practical constraints. These results open a perspective towards a good evaluation of the thickness and/or the transverse and longitudinal bulk wave velocities. Attenuation and non-isotropic medium will be taken into account.

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