Research Article
A Robust Adaptive Control Algorithm for Multimissile Cooperative Formation

Zheng Gong,1 Mingang Wang,1 Jingang He,2 and Jin Zhang3

1Northwestern Polytechnical University, Xi’an 710072, China
2Luoyang Optoelectro Technology Development Center, Luoyang 471000, China
3Shanghai Electro-Mechanical Engineering Institute, Shanghai 201109, China

Correspondence should be addressed to Zheng Gong; gongz@mail.nwpu.edu.cn

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To improve the probability of missile coordinated detection, this paper investigates the control method of multimissile cooperative formation. Based on the missile’s nonlinear dynamics modeling, the formation control algorithm based on the position of the leader missile (or virtual leader missile) is studied. Meanwhile, considering the influence of the communication time delay, a robust adaptive cooperative controller satisfying the time delay network is designed. The simulation verification result indicates that cooperative formation can be achieved quickly with small velocity and position tracking errors when the missiles are fully connected. Therefore, the multimissile cooperative formation control algorithm designed in this paper can meet the requirements of multimissile coordinated detection, thus improving the missile’s cooperative attack capability.

1. Introduction

With the widespread application of information and network technology in modern warfare, the cooperation of multiaircraft weapon systems has attracted increasing attention. The research and application of multimissile cooperative attacks in modern warfare become crucial. Various countries are actively studying multiaircraft coordinated detection and target tracking, multisource information fusion, cooperative guidance, and formation control technology.

As for multimissile coordinated detection, to better complete combat tasks, it is necessary to consider the mutual coordination and cooperation between multiple missiles. Therefore, the optimal design of multimissile coordinated detection is essential to multimissile cooperative attacks. In the midcourse guidance phase, the satellite system can only obtain the area where the target may appear instead of the motion information of the target, and due to the limitation of maneuverability, the detectable area of a single missile is limited, which makes it difficult to intercept the target successfully. If multimissile coordinated detection is applied, based on the cooperative design of missiles in space position or detection time, the detectable areas of multiple missiles can cover the area where the target may appear as much as possible, thereby ensuring that the target can be intercepted during the handover phase of the mid-terminal guidance. Overall, the formation flight of air-to-air missiles can overcome the shortcomings of a single missile and complete the tasks that a single missile cannot accomplish, such as expanding the detection range, improving the detection accuracy, and enhancing the coordinated detection capability. This has an important influence on the successful interception of the target during the handover phase of the mid-terminal guidance.

Multimissile formation tasks include several processes, such as initial construction, maintenance, shrinkage, expansion, and reconstruction. Moreover, there exist severe nonlinear couplings in the translational dynamics and rotational dynamics of the missiles with external disturbances. Furthermore, the communication delay is also involved in the multi-missile formation tasks. These difficulties make it challenging to design an adaptive control algorithm to address the problem of multi-missile formation flying. In order to achieve high precision formation flying mission, it is necessary
to carry out effective cooperative control for each formation member. In the research field of unmanned aerial vehicle (UAVs) formation control, scholars from various countries have proposed many methods in recent years, such as the leader-follower method [1, 2], behavior-based control method [3, 4], and virtual structure method [5, 6]. At present, these methods have been widely used in robot formation [7], aircraft formation [8–10], spacecraft formation [11, 12], and missile formation [13].

For the problem of missile formation coordination, most literatures mainly centered on the cooperative guidance based on the consistent arrival time [14–16]. The main idea is to use the consistency algorithm to design the guidance law so that all missiles can reach the target at the same time, that is, to realize the coordinated attack on the target. In recent years, the research on cooperative guidance of multiple missiles has attracted wide attention of scholars. In [17], the cooperative attack with multiple missiles on a stationary target was considered, and a two-step procedure was proposed. In [18], Jeon et al. combined the classical proportional guidance law with the additional attack time error correction term and proposed the guidance law of cooperative attack target. In [19], Lee et al. designed a cooperative guidance law with attack time and angle constraints, but the attack time and angle should be given in advance. However, Jeon et al. adjusted the difference of the time-to-go of multiple missiles in the process of attacking the target and took attack time as the coordination variable to ensure that multiple missiles intercept the target at the same time in [15]. The coordination variable is the minimum amount of information required to achieve coordination between missiles. At present, in the cooperative guidance design of multiple missiles, the selection of coordination variable is mainly time [20, 21]. In [22], Wei et al. used linear quadratic optimal control theory to design the formation optimal holding controller for multimissile formations. The designed controller can ensure the rapid and stable adjustment and maintenance of missile formation during missile cooperative operation. Zhang et al. [23] designed a nonlinear optimal controller for missile cooperative attack formation by solving Riccati matrix differential equation. The formation maintenance of multiple unmanned aerial vehicles based on proximity behavior was explored in [24]. Individual decision-making was conducted according to the expected UAV formation structure and the position, velocity, and attitude information of other UAVs in the azimuth area. The problem of formation tracking control for the multiple flight vehicle (MFV) system considering actuator saturation constraints was investigated in [25]. Cooperative guidance problems for multiple missiles with directed interaction topologies are dealt with in [16]. In [26], an adaptive output-feedback neural tracking controller was designed for a class of multiple-input multiple-output system. In [27], the problem of asymptotic tracking control for a type of nonlinear stochastic systems was investigated. Although the research achievements of cooperative guidance are increasing year by year, many problems remain to be solved due to the late start.

In the design of missile formation controller mentioned above, the interference of complex external environment is not considered. However, in the course of missile flight, the external disturbances such as airflow and strong wind are often not negligible; so, the requirement of suppressing the disturbances must be considered in the design of missile formation controller [28, 29]. Considering the influence of such external disturbance is one of the problems to be studied in this paper. Meanwhile, in order to implement formation, missiles often carry out cooperative control through communication, and communication delay may occur in this process. Therefore, the influence of communication delay must be considered when designing the controller [30].

For the missile formation flight system, a robust adaptive missile formation cooperative control strategy is proposed, and the Lyapunov stability of the closed-loop system is analyzed strictly. Firstly, an adaptive law is proposed to estimate the upper bound of the disturbance without communication delay, and a missile formation control algorithm is further designed. On this basis, considering the external disturbance and communication delay, the conditions for asymptotic stability of the system are given by Lyapunov stability analysis. Finally, the effectiveness of the proposed robust adaptive controller is verified by numerical simulation.

2. Cooperative Formation Control Scheme for Multiple Missiles

To ensure that the multimissile cooperative search strategy has the highest interception probability at the beginning of the mid-terminal guidance handover, it is necessary to maintain a certain formation flight before the mid-terminal handover and during the handover process. Thus, based on the specific missile formation with the highest probability of target interception (Figure 1), the missile group formation flight control law can be derived.

Generally, formation control refers to the application of multiple missiles to generate a certain formation configuration during the formation flight and maintain or shrink it in real-time. Formation members communicate with each other and conduct cooperative work, controlling the relative distance and direction between the formation members and maintaining the required geometric configuration. From the aforementioned analysis, the distance and azimuth information of the follower missile based on the leader missile has been obtained from the aforementioned optimization problem before the mid-terminal handover. This also means that the expected formation of the missile group is known. Furthermore, through the leader-follower missile control strategy, the missile group formation control is converted to the design of a controller for each missile so that it can track the expected position and velocity information to achieve the purpose of formation maintenance.

The leader-follower missile control strategy needs to adjust the multimissile cooperative search strategy in time according to the number of missiles in the missile group, and then it changes the selection of leader missile. The specific control process is as follows.

Based on the herringbone formation of five missiles with the highest probability of target interception, the position
that always points to the center of the target maneuvering domain is selected as the center point for the formation control. The velocity is the expected velocity of the entire formation, and it needs to meet the velocity requirement of the seeker for interception. Other missiles track the relative position and velocity of the formation center following the cooperative search strategy to achieve the purpose of multimissile formation flight. For example, when five missiles are used for cooperative search, $M_1$ is selected as the leader missile, and $M_2-M_5$ are selected as the follower missiles. $M_1$ always points to the center of the target maneuvering domain, and the position of $M_2-M_5$ is calculated based on the optimization problem of the maximum interception probability. In the multimissile formation flight control, let $M_1$ be the leader missile, and its expected information tracks the center point of the formation. $M_2-M_5$ separately tracks the velocity and the expected relative position of $M_1$. For instance, when the four missiles are used for cooperative search, the maximum probability of intercepting the target may be achieved through the overlap and coverage of the seeker’s field of view. At this time, the virtual leader missile is selected to always point to the center of the target maneuvering domain, and other missiles track the velocity and the expected relative position of the virtual leader missile.

### 3. Design of the Control Algorithm for Multimissile Cooperative Formation

#### 3.1. Relative Nonlinear Dynamic Model of Formation Movement

In three-dimensional space, a formation flight system is composed of $n$ missiles with close velocity. The movement of the $i$-th missile can be described as

$$
\begin{align*}
\dot{x}_i &= V_i \cos \theta_i \cos \psi_i, \\
\dot{y}_i &= V_i \sin \theta_i, \\
\dot{z}_i &= -V_i \cos \theta_i \sin \psi_i, \\
\dot{V}_i &= (F_{x_i} - X_i)/m_i - g \sin \theta_i + d_{i1}, \\
\dot{\theta}_i &= (F_{y_i} - m_i g \cos \theta_i)/(m_i V_i) + d_{i2}, \\
\dot{\psi}_i &= -F_{z_i}/(m_i V_i \cos \theta_i) + d_{i3},
\end{align*}
$$

where $(x_i, y_i, z_i)$ represents the position of the missile; $V_i, \theta_i,$ and $\psi_i$ represent the missile velocity, ballistic inclination, and deflection angles of the missile, respectively; $m_i$ is the mass of the missile; $X_i$ is the resistance suffered by the missile, and it is related to the resistance coefficient $C_r,$ the current air density $\rho,$ the velocity $V_i,$ of the missile, and the characteristic area $S_i$ of the missile. The calculation formula of $X_i$ is

$$X_i = \frac{1}{2} C_r \rho V_i^2 S_i. \quad (2)$$

Let $F_i = [F_{x_i}, F_{y_i}, F_{z_i}]^\top$ represent the missile control force vector, and $d_i = [d_{i1}, d_{i2}, d_{i3}]^\top$ represent the disturbance suffered by the missile. Assume that $d_i$ is bounded and satisfies $\|d_i\|_\infty \leq \delta_i,$ also, let $p_i = [x_i, y_i, z_i]^\top$ represent the position of the missile, and $v_i = [\dot{x}_i, \dot{y}_i, \dot{z}_i]^\top$ represent the velocity of the missile. Then, formula (1) can be expressed as

$$
\begin{align*}
\dot{p}_i &= v_i, \\
\dot{v}_i &= a_i(p_i, v_i) + b_i(p_i, v_i) F_i + d_i,
\end{align*}
$$

where

$$
\begin{align*}
a_i(p_i, v_i) &= \begin{bmatrix}
-c_r \rho V_i^2 S_i/m_i & \cos \theta_i & \cos \psi_i \\
-c_r \rho V_i^2 S_i/m_i & \sin \theta_i & -g \\
c_r \rho V_i^2 S_i/m_i & \cos \theta_i & \sin \psi_i
\end{bmatrix}, \\
b_i(p_i, v_i) &= \begin{bmatrix}
1/m_i & \cos \theta_i & \cos \psi_i & -1/m_i & \sin \theta_i & \cos \psi_i & 1/m_i & \sin \psi_i \\
1/m_i & \sin \theta_i & 0 & 1/m_i & \cos \theta_i & 0 & 1/m_i & \cos \psi_i
\end{bmatrix}.
\end{align*}
$$

For the convenience of analysis, $d_i$ in the above formula represents the interference component on each channel.

#### 3.2. Description of Formation Tracking Control Problems

In missile formation flight control, the formation is usually maintained according to actual conditions. Assume that the expected position of the $i$-th formation missile is $p_{EF}^i = p_{EF}^a + p_{EF}^b$, $p_{EF}^a$ is the expected position of the formation center, i.e., the position of the leader missile or virtual leader missile, and it corresponds to the center of the target maneuvering domain. $p_{EF}$ is the expected position of the $i$-th missile relative to the center of the formation. It should be noted that $p_i \rightarrow p_{EF}^i$ and $p_i \rightarrow p_{EF}$ mean that the tracking requirements are fulfilled. Meanwhile, we have $p_i - p_{EF} \rightarrow p_{i} - p_{i, EF}$ and...
\[
\dot{p}_i - \dot{p}_{iF} \rightarrow p_i - p_{iF},
\]
indicating that the formation maintaining requirements are fulfilled. Therefore, based on this principle, the controller \( F_i \) is designed for the missile formation motion model, such that for \( \forall i, j = 1, 2, \ldots, n \), when \( t \rightarrow \infty \), we have \( p_i \rightarrow p_i^d \) and \( v_i \rightarrow v_i^d \). Meanwhile, in the transient convergence process of the tracking errors of \( p_i - p_i^d \) and \( v_i - v_i^d \), it is guaranteed to a certain extent that \( p_i - p_{iF} \rightarrow p_i - p_{iF} \) and \( v_i - v_{iF} \rightarrow v_i - v_{iF} \).

In the missile formation system, the missile obtains the status of other missiles through information interaction to complete the formation task. This information interaction can be described by graph theory. The graph \( G = (V, \xi, C) \) is composed of a node set \( V = \{1, 2, \ldots, n\} \), an edge set \( \xi \), and a weighted adjacency matrix \( C = [c_{ij}] \in \mathbb{R}^{n \times n} \). In the graph, the edge \((j, i) \in \xi \) refers to the information connection from node \( j \) to node \( i \). As for the element \( c_{ij} \) in \( C \), if \((j, i) \in \xi \), then \( c_{ij} > 0 \); otherwise, \( c_{ij} = 0 \). As for the path \( v_1, \ldots, v_n \) in the graph, for any \( 1 \leq i \leq r \), \((v_i, v_{i+1}) \in \xi \). If the edges in the graph are not directed, and any two nodes can be connected in turn by paths, this graph is called a connected undirected graph. If the edges in the graph are directed, and any two nodes can be connected in turn by paths, this graph is called a strongly connected directed graph. If the direction of the edges is not considered, and any two nodes can be connected at the same time by paths, this graph is called a weakly connected directed graph.

3.3. Formation Controller Design. To obtain the expected configuration of the missile formation to complete special tasks, each missile needs to track its expected position \( p_i^d \) and expected velocity \( v_i^d \). Firstly, a robust adaptive cooperative controller with no communication time delay is given. Then, considering the influence of the communication delay, a robust adaptive cooperative controller with time delay is designed.

Denote the tracking error as \( e_i = p_i - p_i^d \), then we have \( \dot{e}_i = v_i - v_i^d \). The linear sliding surface is selected as

\[
s_i = \dot{e}_i + l_i e_i,
\]

where \( l_i > 0 \) is a constant. For the convenience of stability analysis, the following lemma is provided.

**Lemma 2.** For any real number \( x \) and nonzero real number \( y \), the following inequality holds.

\[
0 \leq |x| (1 - \tanh(|x/y|)) \leq \alpha |y|,
\]

where \( \alpha > 0 \). The minimum value \( \alpha^* \) satisfies \( \alpha^* = x^* (1 - \tanh(x^*)) \), and \( x^* \) satisfies the equation \( e^{-2x^*} + 1 - 2x^* = 0 \).

Assume that the mass of the missile is known and there is no communication delay in the information interaction between the missiles, that is, the missile can obtain the information of adjacent missiles in real-time. According to formulas (3) and (6), we have

\[
\dot{s}_i = a_i(p_i, v_i) + b_i(p_i, v_i)F_i + d_i - p_i^{d, i} + l_i (v_i - v_i^d).
\]

To deal with unknown disturbances in the multimeasure system, an adaptive parameter \( a_i^d \) is introduced, and a robust adaptive controller is designed as

\[
F_i = b_i^{-1}(p_i, v_i) \left( -a_i(p_i, v_i) - l_i (v_i - v_i^d) + p_i^{d, i} - k_i s_i \right.
\]

\[
- \sum_{j=1}^{n} c_{ij}(s_i - s_j) - \tilde{d}_i \tanh (s_i/q_i^2) \bigg),
\]

\[
\dot{\tilde{d}}_i = y_d ||s_i||, \tag{10}
\]

\[
\frac{d}{dt} q_i^2 = -3aq_i q_i^2, \tag{11}
\]

where \( k_i > 0 \) is a constant, \( c_{ij} \) represents the element in row \( i \) and column \( j \) of the weighted adjacency matrix \( C, \tilde{d}_i \) is the estimated value of the disturbance upper bound \( d_i \), and \( \tilde{d}_i(0) > 0, y_d > 0 \) is a constant, \( q_i \) is a time-varying parameter, and \( q_i(0) > 0 \), and the constant \( y_d \) is as defined in Lemma 2. Substituting formula (9) into formula (8), the closed-loop equation of the system can be obtained as

\[
\dot{s}_i = d_i - k_i s_i - \sum_{j=1}^{n} c_{ij}(s_i - s_j) - \tilde{d}_i \tanh (s_i/q_i^2). \tag{12}
\]

**Theorem 3.** For the missile formation system (3), robust controllers (9)--(11) are designed. If the communication topology between missiles is a strongly connected directed graph, then the system states \( p_i \) and \( v_i \) are globally consistent and bounded. Besides, when \( t \rightarrow \infty \), \( e_i \rightarrow 0 \) and \( e_i \rightarrow 0 \) hold. Also, \( p_i \rightarrow p_{iF} \) and \( v_i \rightarrow v_{iF} \) hold. In this case, the missile can track the expected trajectory while forming and maintaining the expected formation.

**Proof.** The inequality (7) in Lemma 2 can be written as

\[
-x/y \cdot \tanh (x/y) \leq \alpha - |x/y|. \tag{13}
\]
According to formula (10) and $\tilde{d}_i(0) > 0$, for any $t \geq 0$, $\tilde{d}_i(t) > 0$ holds. Considering that $\tilde{d}_i > 0$, $|d_{ij}| \leq ||d_i||_{\infty} \leq \tilde{d}_i$, and formula (13), we have
\[
s_i^T \tilde{d}_i - \tilde{d}_i s_i^T \tanh \left( s_i/q_i^2 \right) \leq ||d_i||_{\infty} ||s_i||_1 - \tilde{d}_i q_i^2 \cdot s_i^T / q_i^2 \cdot \tanh \left( s_i/q_i^2 \right) \\
\cdot \tanh \left( s_i/q_i^2 \right) \leq \tilde{d}_i ||s_i||_1 + \tilde{d}_i q_i^2 \sum_{j=1}^{n} a_i - |s_i| / q_i^2 \\
\cdot \tanh \left( s_i/q_i^2 \right) \leq \tilde{d}_i ||s_i||_1 + \tilde{d}_i q_i^2 \sum_{j=1}^{n} (|a_i| - |s_i| / q_i^2) \\
= \tilde{d}_i ||s_i||_1 + \tilde{d}_i q_i^2 (3|a_i| - |s_i| / q_i^2) = 3\tilde{a} \tilde{d}_i q_i^2 + \tilde{d}_i ||s_i||_1,
\]
(14)

where $\tilde{d}_i = d_i - \bar{d}_i$, $d_i, \bar{d}_i$, and $s_{ij}$ represent the l-th components of vectors $d_i$ and $s_i$ ($l = 1, 2, 3$), respectively.

The selected Lyapunov function is
\[
V = \sum_{i=1}^{n} \frac{1}{q_i^2} \eta_i \bar{s}_i^T \bar{s}_i + \sum_{i=1}^{n} \frac{1}{2}\sum_{j=1}^{n} \eta_i \bar{d}_i \bar{d}_j + \sum_{i=1}^{n} \eta_i \frac{d_i}{dt} q_i^2
\]
where $\eta_i > 0$ is a constant, and it is defined in Lemma 1. Based on the derivation of formula (15) and the use of formulas (10)–(12), we have
\[
\dot{V} = \sum_{i=1}^{n} \eta_i \bar{s}_i \bar{s}_i + \sum_{i=1}^{n} \eta_i \bar{d}_i \bar{d}_j + \sum_{i=1}^{n} \frac{d_i}{dt} q_i^2
\]
\[
= \sum_{i=1}^{n} \eta_i \bar{s}_i \left( -k_i s_i - \sum_{j=1}^{n} c_{ij} (s_i - s_j) - \tilde{d}_i \tanh \left( s_i/q_i^2 \right) + \bar{d}_i \right)
\]
\[
- \sum_{i=1}^{n} \eta_i \bar{s}_i ||s_i||_1 - \sum_{i=1}^{n} \frac{3\eta_i \bar{d}_i q_i^2}{2} = -\sum_{i=1}^{n} \eta_i k_i s_i^T s_i
\]
\[
- \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_i c_{ij} \left( s_i - s_j \right)^T (s_i - s_j) - \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_i c_{ij} \left( s_i - s_j \right) \bar{s}_i \bar{s}_j
\]
\[
+ \sum_{i=1}^{n} \eta_i \bar{s}_i \left( -\tilde{d}_i \tanh \left( s_i/q_i^2 \right) + \bar{d}_i \right) - \sum_{i=1}^{n} \eta_i \tilde{d}_i ||s_i||_1 - \sum_{i=1}^{n} \frac{3\eta_i \bar{d}_i q_i^2}{2}.
\]
(16)

Substituting formula (14) into formula (16), equation (17) can be obtained.
\[
\dot{V} \leq -\sum_{i=1}^{n} \eta_i k_i s_i^T s_i - \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_i c_{ij} \left( s_i - s_j \right)^T (s_i - s_j)
\]
\[
- \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_i c_{ij} \left( s_i - s_j \right)^T (s_i - s_j) - \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_i c_{ij} \left( s_i - s_j \right) \bar{s}_i \bar{s}_j
\]
\[
- \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_i c_{ij} \left( s_i - s_j \right) \bar{s}_i \bar{s}_j
\]
\[
= -\sum_{i=1}^{n} \eta_i k_i s_i^T s_i - \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_i c_{ij} \left( s_i - s_j \right)^T (s_i - s_j) - \frac{1}{2} \eta_i LY,
\]
(17)

where $Y = [\bar{s}_i^T \bar{s}_i \bar{s}_j \bar{s}_j \bar{s}_k \bar{s}_k]^T$. Since the communication topological graph between missiles is a strongly connected directed graph, the following formula can be obtained from Lemma 1.
\[
\dot{V} \leq -\sum_{i=1}^{n} \eta_i k_i s_i^T s_i - \sum_{i=1}^{n} \sum_{j=1}^{n} \eta_i c_{ij} \left( s_i - s_j \right)^T (s_i - s_j) \leq 0.
\]
(18)

It can be seen from formula (18) that $V$ is bounded. Thus, the system states $p_i$ and $v_i$ are globally consistent and bounded.

Because $V$ is bounded, $s_i, \tilde{d}_i \in L_\infty$. Since the disturbance $d_i$ is bounded, $\tilde{d}_i \in L_\infty$ can be obtained from formula (12). Besides, since $V$ is bounded and $V \leq 0$, we have $\lim_{t \to \infty} V(t) \to 0$ and $s_i \to L_2$. According to the Barbalat Lemma, when $t \to \infty$ and $s_i \to 0$, it can be obtained from the definition of $s_i$ that $e_i \to 0$ and $\dot{e}_i \to 0$. Since $e_i = p_i - \bar{p}_i = p_i - \bar{p}_i - \bar{p}_i$ and $e_i = p_j - \bar{p}_j - \bar{p}_j$ holds, and so does $v_i - \bar{v}_i \to 0$. Thus, the robust formation controllers (9)–(11) can track the expected trajectory of the missile, while forming and maintaining the expected formation. It can be known from formula (11) that the parameter term $p_i^2$ decreases monotonically. Too small $p_i^2$ makes the hyperbolic tangent function of the controller close to the sign function. However, the real-time switching of the sign function can easily cause high-frequency vibrations in the unmodeled dynamics of the system, thereby resulting in system instability. In this case, a smaller $\gamma_i$ can be selected to make $\tilde{d}_i$ smaller so that the parameter $p_i^2$ decreases more slowly. Based on this, the designed controller is a continuous function with good robustness, and it can simultaneously suppress disturbances and model parameter uncertainty.

Remark 1. In this section, the robust adaptive control scheme is designed to address the multimissile formation problem subject to nonlinear dynamics, uncertain parameters, and communication delay. An adaptive parameter is introduced to identify unknown configurations of the control system. Moreover, the closed-loop stability of the derived control system is proven based on Lyapunov theory. Unlike the common sliding mode method or other traditional robust control methods, the proposed approach is continuous which is simple to be implemented.

4. Simulation Verification of Formation Controller

To verify the validity of the controller, the simulation verification is conducted under $\theta = 4^\circ$, $d_{max} = 20km$, $\sigma_x = 500$, $\sigma_y = 850$, $\sigma_z = 1200$, and the formation parameter $(r, \xi, \varsigma) = (2714.31, 54.82, 49.37)$. The formation parameter is converted to the coordinates of the ballistic coordinate system of the leader missile $(x, y, z) = (2218.5, 1018.3, 1186.8)$. Here, the formation flight of five missiles is taken as an
Table 1: Initial conditions of formation missiles.

| Missile   | Position (m)     | Velocity (m/s) | Relative expected position (m) |
|-----------|------------------|----------------|---------------------------------|
| Missile 1 | (10, 10, -10)    | 800 m/s        | (0, 0, 0)                       |
| Missile 2 | (-2100, 1200, -1000) | 780 m/s     | (-2218.5, 1018.3, -1186.8)     |
| Missile 3 | (-2000, 1000, -1000) | 790 m/s    | (-2218.5, 1018.3, 1186.8)      |
| Missile 4 | (-2400, -1200, -1000) | 810 m/s   | (-2218.5, -1018.3, -1186.8)    |
| Missile 5 | (-2200, -1000, 1200)  | 815 m/s   | (-2218.5, -1018.3, 1186.8)     |

Figure 2: Relative trajectory of formation missiles.

Figure 3: Missile position tracking error.
example. Let $\mathbf{w}_i = [V_i, \beta_i, \psi_i]^T$, where the subscript $i$ represents the $i$-th missile. For the convenience of the simulation, the resistance suffered by the missile is ignored. Besides, considering the influence of gravity, the maximum overload of each missile in the missile group is 10 g.

The weights of the missiles are $m_1 = 150$ kg, $m_2 = 200$ kg, $m_3 = 180$ kg, $m_4 = 160$ kg, and $m_5 = 170$ kg. The missile’s initial ballistic inclination angle and ballistic deflection angle are both $0^\circ$. The other initial conditions are listed in Table 1.

In the process of missile formation flight, to quantitatively describe the accuracy of formation tracking and formation maintenance, the formation tracking error $\mu_1$ and the formation keeping error $\mu_2$ are defined as
the formation center is set to the following:

\[ \mu_2 = \frac{1}{5} \sum \| c_i \|_2, \]  

(19)

\[ \mu_2 = \left\| p_1 - p_2 \right\|_2 + \left\| p_1 - p_3 \right\|_2 + \left\| p_2 - p_4 \right\|_2. \]  

(20)

It can be seen that a smaller \( \mu_1 \) and \( \mu_2 \) make the formation configuration performance of the missile meet the requirements better.

The initial state and velocity of the formation missile are set to the following:

\[
\begin{align*}
p_1(0) &= [10 \ 10 \ -10]^T m, \\
p_2(0) &= [-2100, 1200, -1000]^T m, \\
p_3(0) &= [-2000, 1000, -1000]^T m, \\
p_4(0) &= [-2400, -1200, 1200]^T m, \\
p_5(0) &= [-2200, -1000, 1200]^T m, \\
w_1(0) &= [800 \ 0 \ 0]^T m/s, \\
w_2(0) &= [780 \ 0 \ 0]^T m/s, \\
w_3(0) &= [790 \ 0 \ 0]^T m/s, \\
w_4(0) &= [810 \ 0 \ 0]^T m/s, \\
w_5(0) &= [815 \ 0 \ 0]^T m/s.
\end{align*}
\]  

(21)

The expected position of the formation missile relative to the formation center is set to the following:

\[
\begin{align*}
p_{SF} &= [0 \ 0 \ 0]^T m, \\
p_{2F} &= [-2218.5, 1018.3, -1186.8]^T m, \\
p_{3F} &= [-2218.5, -1018.3, -1186.8]^T m, \\
p_{4F} &= [-2218.5, -1018.3, 1186.8]^T m, \\
p_{5F} &= [-2218.5, 1018.3, 1186.8]^T m, \\
v_{IF} &= v_{IF}, i = 1, \cdots, 5.
\end{align*}
\]  

(22)

The expected position and velocity of the formation center is \( p_0^d = [800 \ 60 \ \sin(t/60) \ 0]^T m, \) \( v_0^d = \dot{p}_0^d. \)

Then, the expected position and velocity to be tracked by the formation missile is \( p_i^d = p_{IF} + p_i^d, v_i^d = v_{IF} + v_i^d, \) and \( i = 1, \cdots, 5. \)

The parameters of the controller and the corresponding initial values are \( \lambda_i = 2, \ k_i = 0.2, \ q_i(0) = 0.001, \) \( \gamma_i = 0.00001, \) \( q_i^T(0) = 1, \) and \( i = 1, \cdots, 4. \)

The missiles in the missile group are fully connected, and the weighted adjacency matrix is as follows:

\[
C = [c_{ij}]_{4 \times 4} = \begin{bmatrix}
0 & 1 & 1 & 1 \\
1 & 0 & 1 & 1 \\
1 & 1 & 0 & 1 \\
1 & 1 & 1 & 0
\end{bmatrix}.
\]  

(23)

The trajectories of five missiles are shown in Figure 2. The position and velocity tracking error are depicted in Figures 3 and 4, respectively. It can be observed from Figures 3 and 4 that the position and velocity tracking error converge to the origin in 10 s. The adaptive parameters \( \hat{d}_i \) and \( q_i^T \) curves are shown in Figure 5. It can be obtained from Figure 5 that \( \hat{d}_i \) converges to a certain constant ultimately.
and monotonically decreased. Since $q_i$ declines much slower than the position and velocity tracking errors converge to zero, the robustness is guaranteed, and the oscillation of the control system is avoided. Moreover, the formation tracking error defined in (20) is depicted in Figure 6. It is shown that the formation tracking error converges sharply and is less than 2 m in Figure 6, which suggests that the proposed control scheme is of high accuracy. Finally, the missile control force is depicted in Figure 7.

5. Conclusions

To meet the requirements of multimissile cooperative guidance technology, missiles perform cooperative search under a specific space configuration. Meanwhile, it is also necessary to design a multimissile formation control method that meets the constraints of the specific space configuration. In this paper, a robust adaptive cooperative controller is designed according to the requirement of coordinated detection under a specific space formation configuration. The controller can meet the formation control requirements in the fully connected time-delay network and achieve the cooperative formation fast with small velocity and position tracking errors. The future work will further investigate the design of the formation controller for the local and intermittently connected network in a complex environment.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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