Robust Power Allocation for NOMA Heterogeneous Networks with EH under Imperfect CSI

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Abstract:
A robust power allocation is proposed for downlink non-orthogonal multiple access (NOMA) heterogeneous networks with EH (Energy harvesting) under imperfect channel state information (CSI). In order to achieve green communication, an EH-aided scheme by leveraging energy from macro base station (MBS) signal and interference signal transmitted from other SBSs is proposed, which reduces the power burden and energy consumption of the SBS. In order to conform to the actual communication scenario, we construct an energy efficiency optimization function under imperfect CSI with considering the constraint of the outage probability interference power in macro cell user (MCU). However, the formulated optimization problem is non-convex due to the fractional form of the objective function and the probabilistic constraints of the outage probability limit. To cope with this problem, we propose a robust power allocation scheme. Firstly, the probabilistic problem is converted into a robust non-probabilistic problem by the minimax probability machine (MPM) and robust optimization theory. Then, the robust non-probabilistic problem can be transformed into the convex optimization problem via Dinkelbach method and sequential convex programming. Finally, the optimal transmission powers of the small cell users (SCUs) are obtained by Lagrange dual approach. The simulation results show that the robust power allocation scheme for NOMA heterogeneous networks with EH under imperfect CSI can significantly improve energy efficiency compared with traditional power allocation algorithms.

Key words: energy harvesting, heterogeneous networks, imperfect channel state information, power allocation

1. Introduction

Nowadays, with the explosive growth of traffic demand, such as the popularity of mobile terminal devices, there are more and more demands for high data rate and energy efficiency in the heterogeneous networks. [1]-[2]. And in order to further improve the spectrum efficiency, NOMA technology is integrated into heterogeneous networks.

The resource allocation algorithms can improve system performance in NOMA heterogeneous networks and have been widespread concerned. Then the energy-efficient resource allocation algorithm in the coordinated downlink multicell network system is studied with perfect CSI [3], the user scheduling, data rate adaptation, and power allocation are jointly designed to maximize the system energy efficiency under the maximum transmitted power constraint and the outage probability constraint. In [4], the energy-efficient power allocation and wireless backhaul bandwidth allocation in heterogeneous small cell networks is studied based on perfect CSI and the studied scheme maximizes energy efficiency by allocating both transmit power of each small cell base station to users and bandwidth for backhauling. The power allocation problem is studied in [5] for NOMA system with perfect CSI, where the difference of convex programming is applied as a suboptimal algorithm.

However, in the actual communication environment, due to the channel estimation error, quantization error and delay feedback, it is impossible to obtain perfect CSI [6]-[8]. The power allocation scheme deviating from the actual environment will lead to the wrong allocation of resources and reduce the system performance. Therefore, considering imperfect CSI, wireless communication networks have been extensively studied. An algorithm joint user scheduling and
power allocation is proposed in a downlink NOMA single-cell network with imperfect CSI [9], in which the non-convex problem is solved by the iterative algorithm. In [10], with the imperfect CSI, a power allocation algorithm about small cell and SCUs is proposed for downlink NOMA heterogeneous networks. To restrain the impact of parameter uncertainties for multiuser underlay cognitive radio networks, a robust adaptive power allocation algorithm is investigated in [11], and then an effective algorithm is developed that aims at maximizing the overall throughput of secondary users under the robust interference temperature and signal to interference and noise ratio constraints. In [12], a power adjustment algorithm is proposed for heterogeneous networks which considers varying QoS requirements of users. In order to maximize energy efficiency [13], a robust power allocation scheme for a downlink NOMA heterogeneous network is designed with considering imperfect CSI. A robust resource allocation scheme for maximizing the interference efficiency of users in heterogeneous networks is proposed [14], and the closed-form solution is obtained by using Dinkelbach’s method, the logarithmic transformation method, and the successive convex approximation method.

But the finiteness of energy resources and excessive energy consumption will cause a lot of pollution to the environment currently [15]–[17]. Traditional EH techniques allow mobile terminals or BS to harvest energy from wireless transmission environment, which is a very effective technique, and is extensive concerned [18]–[20]. In [21], the energy beamforming scheme for an EH multiple input single output (MISO) HetNets is proposed, which maximizes the sum harvested energy. In [22], an EH-rate maximization-based resource allocation (RA) scheme for heterogeneous macrocell-smallcell networks with simultaneous wireless information and power transfer (SWIPT) is addressed to obtain the optimal power splitting (PS) and time-switching variables where the minimum throughput constraint of the macrocell user is considered. A robust power allocation and PS scheme is proposed for downlink SWIPT-enabled HetNets with EH [23]. In [24], a robust secure transmission scheme is proposed for wireless information and power transfer in HeNets under the deterministic and stochastic CSI errors, respectively.

Based on above analysis, we can find that there are few schemes for jointly considering imperfect CSI and EH, so in this paper, we propose a robust power allocation for NOMA heterogeneous networks with EH under imperfect CSI. Different from [21]–[23], which installing an EH device at the mobile terminal to reduce the energy consumption of the system, but in this paper, the EH device is installed at the BS to collect energy. And in contrast to [11], [13], [15], we add the interference constraints about macro users to protect macro users. Then a robust power allocation algorithm is designed to maximize energy efficiency. First, we formulate the optimization problem with outage probabilistic and then convert probabilistic constraints to non-probabilistic constraints by MPM, at last, the original optimization problem is converted into convex problem by using the Dinkelbach method and the quadratic transformation approach and, the optimal solution is obtained by Lagrange algorithm.

The rest of the paper is organized as follows: Section 2 builds the system model of NOMA heterogeneous networks and formulates energy efficiency maximization optimization. Section 3 transforms the optimization problem into a robust optimization problem. Section 4 shows the power allocation algorithm. Section 5 demonstrates the numerical results. Finally, the paper is concluded in Section 6.

2. System model and problem formulation

As shown in Figure 1, we consider a two-layer heterogeneous networks. The first layer represents the macro cell, a macro base station (MBS) is deployed in the centre of a macro cell, and $M$ MCUs are randomly distributed in the macro cell, in here $m \in \{1,2,3.....M\}$ represents the $m$th user. The second layer represents the small cell, there are $K$ small cells is deployed in the coverage edge area of the macro cell, in here $k \in \{1,2,3.....K\}$ represents the $k$th small cell, and $N$ SCUs randomly distributed in each small cell, in here $n \in \{1,2,3.....N\}$ represents the $n$th user in the small cell. In the small cell,
multiple users can reuse the same frequency band via the NOMA technology. And there is an EH device in each small base station (SBS) that can harvest energy from the transmitted signal of MBS and other SBS, then the power is effectively supplied to the SBS. EH device can reduce the power consumption of base station and improve the life of SBS, which is conducive to the construction of green network.

![Figure1. System model](image)

Without loss of generality, we assume that the estimated channel gains in the small cell are sorted as $|\hat{h}_{1,k}| \leq |\hat{h}_{2,k}| \leq \ldots \leq |\hat{h}_{N_{k},k}|$. According to NOMA principle, the received signal at $n$th SCU from $k$th SBS can be given by

$$y_{n,k} = h_{n,k}p_{n,k}s_{n,k} + \sum_{r=1}^{N_{k}} p_{r,k}s_{r,k} + \sum_{m=1}^{M_{k}} h_{m,n}p_{m,n}s_{m} + \delta^{2}$$ (1)

where $s_{n,k}$ is the modulated symbols between SBS to the $n$th SCU and $s_{m}$ is the modulated symbols between MBS to the $n$th MCU, $p_{m}$ and $p_{n,k}$ are the transmission power of the $n$th MCU from MBS and the $n$th SCU from $k$th SBS.

$$h_{n,k} = g_{n,k}d_{n,k}^{-\alpha}$$ (2)

$$h_{m,n} = g_{M,n}d_{M,n}^{-\alpha}$$ (3)

Equation (2) and (3) denote channel gain from SBS to SCU and MBS to MCU, respectively. $g_{n,k} \sim \mathcal{CN}(0,1)$ and $g_{M,n} \sim \mathcal{CN}(0,1)$ are the Rayleigh fading coefficient from $k$th SBS and MBS to $n$th SCU. $d_{n,k}$ is the distance from $k$th SBS to $n$th user and $d_{M,n}$ is the distance from MBS to $n$th user. $\alpha$ is the path loss exponent, $\delta^{2}$ is the additive white Gaussian noise (AWGN).

From the equation (1), $h_{n,k}p_{n,k}s_{n,k}$ represents the desired signal from $k$th SBS to $n$th SCU, $h_{r,k}\sum_{r=1}^{N_{k}} p_{r,k}s_{r,k}$ represents the co-channel interference from SBS in the same small cell, $\sum_{m=1}^{M_{k}} h_{m,n}p_{m,n}s_{m}$ is the cross-tier interference from macro cell. The received signal-to-interference-plus-noise (SINR) of the $n$th SCU can be given by

$$r_{n,k} = \frac{h_{n,k}p_{n,k}}{h_{n,k}\sum_{r=1}^{N_{k}} p_{r,k} + \sum_{m=1}^{M_{k}} h_{m,n}p_{m,n} + \delta^{2}}$$ (4)

According to Shannon’s formula, the data rate of the $n$th SCU in the $k$th SBS can be written as

$$R = \frac{\sum_{k=1}^{K} \sum_{n=1}^{N_{k}} B \log(1 + r_{n,k})}{K}$$ (5)

The EH device on the $k$th small base station can collect the energy from other SBS and the MBS, and convert the collected energy into its own energy, which can compensate the power consumption of the base station. The collected energy of $k$th can be given by

$$Q_{k} = \gamma P_{m}g_{0,k} + \sum_{r=1}^{K} \sum_{n=1}^{N_{r}} p_{r,n}g_{r,k}$$ (6)

where $\gamma \in (0,1)$ represents the conversion efficiency of the received energy. $P_{m} = \sum_{k=1}^{K} p_{m}$ represents the total transmitting power of MBS, and $g_{0,k}$ and $g_{r,k}$ represent channel gain from the MBS and the $r$th SBS to the $k$th SBS, respectively. Therefore, the total power consumption can be described as

$$Q = \sum_{k=1}^{K} \sum_{n=1}^{N_{k}} p_{n,k} + P_{e} - \sum_{k=1}^{K} Q_{k}$$ (7)

According the equation (5) and (7), the optimization problem is formulated as

$$E = \frac{R}{Q} = \frac{\sum_{k=1}^{K} \sum_{n=1}^{N_{k}} B \log(1 + r_{n,k})}{\sum_{k=1}^{K} \sum_{n=1}^{N_{k}} p_{n,k} + P_{e} - \sum_{k=1}^{K} Q_{k}}$$

$$C1: \sum_{n=1}^{N_{k}} p_{n,k} \leq P_{k}$$

$$C2: p_{n,k} > 0$$

$$C3: R_{n,k} \geq R_{n,k}^{\text{min}}$$

$$C4: \sum_{k=1}^{K} \sum_{n=1}^{N_{k}} p_{n,k}h'_{n,k} \leq I_{th}$$

where $P_{k}$ is the maximum transmit power of the $k$th SBS, $h'_{n,k}$ denotes channel gain between $n$th SBS and $n$th MCU, and $I_{th}$ is the total interference threshold from all SBS to $n$th MCU. $C1$ is the transmission power constraint for the BS, $C2$ ensures that the power of SCU is non-negative, $C3$ ensures the QoS of SCU, and $C4$ limits the interference power received by the MCU.
3. Robust power allocation formulation

In order to provide more protection to the performance of MCUs in real communication environment, the channel uncertainties from the SBS to each MCU need to be considered. Considering imperfect CSI, we assume that each MCU can tolerate a certain outage probability, then we introduce the probability constraint $C_4$. Therefore, the problem of robust power allocation with the outage probability constraints is formulated as:

$$E = \frac{R}{Q} = \frac{\sum_{k=1}^{N} \sum_{n=1}^{K} B \log(1 + r_{n,k})}{\sum_{k=1}^{N} \sum_{n=1}^{K} p_{n,k} + P_e - \sum_{k=1}^{K} Q_{ck}}$$

$$C_1: \sum_{k=1}^{N} p_{n,k} \leq P_k$$

$$C_2: p_{n,k} > 0$$

$$C_3: R_{n,k} \geq R_{n,k}^{\text{min}}$$

$$C_4: \text{Pr}[\sum_{k=1}^{K} \sum_{n=1}^{K} p_{n,k} h_{m,n}^{*} \leq I_{\text{thr}}] \geq 1 - \varepsilon$$

where $\varepsilon \in [0,1]$ is the outage probability threshold. Due to the introduction of probability constraint $C_4$, the optimization problem (9) is an NP-hard problem, which is difficult to obtain the optimal value in a polynomial time. To solve this problem simply, we transform the probabilistic problem into a non-probabilistic problem.

Due to the channel uncertainty, $h_{m,n}^{*}$ in $C_4$ of (9) can be formulated as $h_{m,n}^{*} = \overline{h}_{m,n}^{*} + \Delta h_{m,n}^{*}$, where $\overline{h}_{m,n}^{*}$ is the estimated channel gain and $\Delta h_{m,n}^{*}$ represents estimated channel errors which is a random variable.

Besides, due to the influence of channel fading, the perfect CSI and accurate statistical models of these random parameters cannot be obtained directly. In order to solve the problem, the outage probability constraint of macro user interference is transformed into deterministic constraint by MPM method without knowledge of exact models of uncertainties [25].

We consider a probability-constraint problem as follows:

$$\inf_{y \sim \mathcal{F}(E)} \text{Pr}[a^T y \leq b] \geq 1 - \varepsilon$$

where inf means the lower bound operation, $a$ represents the optimal object, $y$ is the true value of the uncertain parameter, $\overline{y}$ is the estimated value of the uncertain parameter, $b$ is a constant value, $E$ denotes the variance of $y$, and $\varepsilon \in [0,1]$ represents the outage probability threshold. From inequation (10), we note that the maximum outage probability of $a^T y \geq b$ not exceed the threshold $\varepsilon$, namely $\text{Pr}[a^T y \geq b] \leq \varepsilon$, and thus problem (10) is converted into a sup form:

$$\sup_{y \sim \mathcal{F}(E)} \text{Pr}[a^T y \geq b] \leq \varepsilon$$

where sup means the upper bound operation. According to the principle of MPM, we transform the problem (10) into (12):

$$\sup_{y \sim \mathcal{F}(E)} \text{Pr}[a^T y \geq b] = \frac{1}{1 + d^2}$$

And

$$d^2 = \inf_{y \geq \overline{y}} (y - \overline{y})^2 = \frac{\text{max}(b - a^T \overline{y}, 0)^2}{a^T E a}$$

According to (11) and (12), we obtain

$$\frac{1}{1 + d^2} \leq \varepsilon$$

According to (13) and (14), we obtain the following inequation

$$k(\varepsilon)\sqrt{a^T E a} \leq \text{max}(0, b - a^T \overline{y})$$

where $k(\varepsilon) = \sqrt{(1 - \varepsilon) / \varepsilon}$ and $a^T \overline{y} \leq b$. Then, problem (10) is transformed into the convex form

$$a^T y + k(\varepsilon)\sqrt{a^T E a} \leq b$$

Based on the robust optimization theory, the channel uncertainty $\Delta h_{m,n}^{*}$ from SBS to MCUs is defined as the following bounded and constrained set, i.e.,

$$R = [\Delta h^T \Delta h \leq \varsigma^2, \forall i]$$

where $\Delta h = [\Delta h_{m,n}^{*}, \Delta h_{m,n}^{*} \ldots \Delta h_{N,m}^{*}]^T$, $\varsigma^2 \geq 0$ represents the maximum allowable threshold of uncertainties from all SCU in a small cell. Define $E = \text{diag}[\varsigma^2]$. $a = \{p_{n,k}, p_{1,k}, \ldots, p_{N,k}\}$, $y = [h_{m,n}^{*}, h_{m,n}^{*} \ldots h_{N,m}^{*}]$. According to the constraint $C_4$ in (9), we have

$$\inf_{\Delta h \in R} \text{Pr}[P^T \Delta h \leq I_{\text{thr}}] \geq 1 - \varepsilon$$

According to MPM and the inequation (16), we obtain

$$p^T \Delta h + k(\varepsilon)\sqrt{p^T E p} = p^T \overline{h} + k(\varepsilon)\sqrt{p^T p} \leq I_{\text{thr}}$$

where $\overline{p} = (\overline{y}) = [p_{1}, p_{1}, \ldots, p_{N}]$ and $\overline{h} = [h_{m,n}^{*}, h_{m,n}^{*} \ldots h_{N,m}^{*}]$ are the auxiliary variables, respectively. According to the Cauchy–Buniakowsky–Schwarz inequality, we obtain

$$\overline{p} \Delta h \leq \overline{p} \Delta h \leq \overline{p} \| \Delta h \| \leq \varepsilon \| \overline{p} \|$$

$$\overline{p} \Delta h \leq \overline{p} \Delta h \leq \overline{p} \| \Delta h \| \leq \varepsilon \| \overline{p} \|$$

$$\overline{p} \Delta h \leq \overline{p} \Delta h \leq \overline{p} \| \Delta h \| \leq \varepsilon \| \overline{p} \|$$

$$\overline{p} \Delta h \leq \overline{p} \Delta h \leq \overline{p} \| \Delta h \| \leq \varepsilon \| \overline{p} \|$$
According to (19) and (20), we obtain
\[
p^* \mathbf{H} + k(\xi) \mathbf{z}_m \sqrt{\sum_{n=1}^N p_{n,k}} \leq I_{th}
\] (21)

Due to \( \sum_{n} a_x^2 \leq \sum a_x^2 \) and \( p_{n,k} > 0 \), then \( C4 \) in (9) can be rewritten as
\[
\sum_{n=1}^N \sum_{k=1}^K p_{n,k} (\tilde{H}_{n,k} + \zeta_m \sqrt{1 - \frac{E}{E}}) \leq I_{th}
\] (22)

Thus, we can reformulate SINR as
\[
t'_{n,k} = \frac{p_{n,k}}{\sum_{i=1}^K p_{i,k} + \sum_{m=1}^M H_{m,n} p_m + \delta'}
\] (23)

where \( H_{m,n} = h_{m,n} / h_{n,k} \), \( \delta' = \delta / h_{n,k} \). Then the channel uncertainty of \( H_{m,n} \) is defined as the following bounded ellipsoid uncertainty set \( R_{H} \).

\[
R_{H} = \{ \Delta H_{m,n} | H_{n,k} = \bar{H}_{n,k} + \Delta H_{m,n} : (\Delta H_{m,n})^2 \leq (\sigma_{m,n})^2 \} \) (24)

According to (27), we can rewrite the data rate
\[
R' = \frac{\sum_{k=1}^K p_{n,k}}{\sum_{i=1}^K p_{i,k} + A_{n,k}}
\] (28)

where \( A_{n,k} = (\bar{H}_{n,k} + \sigma_{m,n}) \sum_{m=1}^M p_m + \delta' \)

From the above analysis, the robust optimization problem can be reformulated as
\[
E = \frac{\sum_{k=1}^K B \log(1 + r'_{n,k})}{Q} \leq \frac{\sum_{k=1}^K p_{n,k}}{\sum_{i=1}^K p_{i,k} + P_c - \sum_{k=1}^K Q_{th}} \leq I_{th}
\]

\( C1: \sum_{n=1}^N p_{n,k} \leq P_c \)
\( C2: p_{n,k} > 0 \) (29)
\( C3: R_{n,k}' \geq R_{n,k}^{\min} \)
\( C4: \sum_{n=1}^N p_{n,k} (\tilde{H}_{n,k} + \zeta_m \sqrt{1 - \frac{E}{E}}) \leq I_{th} \)

The robust optimization objective function (29) with respect to \( p_{n,k} \) is no-convex and it is difficult to obtain the optimal solution. In order to solve the optimization problem (29), we propose a reasonable sub-optimal power allocation algorithm to maximize the energy efficiency.

### 4 Power Allocation Algorithm

In this section, we can obtain an optimal allocated power of SCU by solving the optimization problem (29). Due to the optimization problem (29) is a nonlinear fractional form, it is NP-hard. In order to solve NP-hard problem, we transform optimization problem (29) into a linear and convex form via sequential convex programming and Dinkelbach method. Then, to solve the optimization problem (29), we can obtain optimal allocated power of each SCU by the Lagrangian dual method.

According to the sequential convex programming approach [26], we can transform the data rate of the \( n \)th SCU in the \( k \)th SBS via the lower bound of inequality, i.e
\[
R_{n,k}' = B \log(1 + r'_{n,k}) \geq R_{n,k} = B(\alpha_{n,k} \log_2 r_{n,k}^* + \beta_{n,k})
\] (30)

where \( \gamma_{n,k} = \gamma_{n,k} \), \( R_{n,k}' \approx R_{n,k} \), \( \alpha_{n,k} \) and \( \beta_{n,k} \) are respectively written as
\[
\alpha_{n,k} = \gamma_{n,k} \log_2 (1 + r_{n,k}^*) \] (31)
\[
\beta_{n,k} = \log_2 (1 + r_{n,k}^*) - \frac{\gamma_{n,k}}{1 + r_{n,k}^*} \log_2 \gamma_{n,k}^* \] (32)

Then, we can further transform the fractional objective function (29) into a tractable non-fractional form by the Dinkelbach method. The optimization objective function (29) is written as
\[
\max \theta = R' - \left( \sum_{k=1}^{K} \sum_{n=1}^{N} p_{n,k} + P_e - \sum_{k=1}^{K} Q_{ER} \right)
\]

\[C1: \sum_{n=1}^{N} p_{n,k} \leq P_k \]
\[C2: p_{n,k} > 0 \]
\[C3: R_{k}^* \geq R_{k}^{\text{min}} \]
\[\tilde{C}4: \sum_{k=1}^{K} \sum_{n=1}^{N} p_{n,k} (\tilde{r}_{n,m}^p + \xi_n \sqrt{\frac{1-e^u}{e}}) \leq I_{\text{thr}} \]

where the parameter \( t \) is introduced to measure the weight of the sum rate of SCUs and total power assumption.

Proposition: the optimization problem (33) is convex over \( p_{n,k} \).

Proof:
The objective function is transformed as
\[
R' - t \left( \sum_{k=1}^{K} \sum_{n=1}^{N} p_{n,k} + P_e - \sum_{k=1}^{K} Q_{ER} \right)
\]

Proof of Concavity: it is obviously observed (35) is linear function of \( p_{n,k} \)
\[
\nu \left( \sum_{k=1}^{K} \sum_{n=1}^{N} p_{n,k} + P_e - \sum_{k=1}^{K} Q_{ER} \right)
\]

Therefore, we focus on the concavity of the \( R' \), we can obtain
\[
\sum_{k=1}^{K} \sum_{n=1}^{N} B(\alpha_{n,k} \log_2 y_{n,k}^* + \beta_{n,k}) = \\
\sum_{k=1}^{K} \sum_{n=1}^{N} B(\alpha_{n,k} \log_2 \left( \frac{P_{n,k}}{\sum_{i=1}^{N} p_{i,k} + (H_{n,m} + \sigma_{n,m}) \sum_{m=1}^{M} p_{m} + \delta^*} \right) + \beta_{n,k}) \\
= \sum_{k=1}^{K} \sum_{n=1}^{N} \left( B \alpha_{n,k} \log_2(p_{n,k}) - \\
B \alpha_{n,k} \log_2 \left( \sum_{i=1}^{N} p_{i,k} + (H_{n,m} + \sigma_{n,m}) \sum_{m=1}^{M} p_{m} + \delta^* \right) + B \beta_{n,k} \right)
\]

From the equation (36), we can observe that \( B \beta_{n,k} \) is constant and \( B \alpha_{n,k} \log_2 (p_{n,k}) \) is a concave function of variable \( p_{n,k} \). Therefore, we need to prove (37) is a concave function of variable \( q_{n,k} \).
\[
B \alpha_{n,k} \log_2 \left( \sum_{i=1}^{N} p_{i,k} + (H_{n,m} + \sigma_{n,m}) \sum_{m=1}^{M} p_{m} + \delta^* \right) \]

Let us define
\[
f(Q) = B \alpha_{n,k} \log_2 \left( \sum_{i=1}^{N} p_{i,k} + (H_{n,m} + \sigma_{n,m}) \sum_{m=1}^{M} p_{m} + \delta^* \right)
\]

where \( Q = [q_1^T, q_2^T, ..., q_{N+1}^T, q_{N+2}^T, ..., q_{N+M}^T] \). The Hessian matrix \( H \) of \( f(Q) \) with respect to \( Q \) can be calculated as
\[
H = \frac{B \alpha_{n,k}}{\phi^2 \ln 2} \left[ \begin{array}{cccc}
\phi \theta_1 - \theta_1^2 & -\theta_1 \theta_2 & \cdots & -\theta_1 \theta_K \\
-\theta_1 \theta_1 & \phi \theta_2 - \theta_2^2 & \cdots & -\theta_2 \theta_K \\
\vdots & \vdots & \ddots & \vdots \\
-\theta_1 \theta_{K-1} & -\theta_2 \theta_{K-1} & \cdots & \phi \theta_K - \theta_K^2 \\
0 & 0 & \cdots & 0
\end{array} \right]
\]

where \( \theta = [\theta_1, ..., \theta_K] \), \( \phi = \sum_{i=1}^{N} \theta_i + (H_{n,m} + \sigma_{n,m}) \sum_{m=1}^{M} p_{m} + \delta^* \). Note that the 1st to the \((u - 1)\)th elements of \( \theta \) are zero. Assuming \( N \times 1 \) vector \( z = [z_1, ..., z_N]^T \)
\[
x'Hz = \frac{B \alpha_{n,k}}{\phi^2 \ln 2} \left[ \begin{array}{c}
\phi \sum_{i=1}^{N} z_i^2 \theta_i - (\sum_{i=1}^{N} z_i \theta_i)^2 \\
\vdots \\
\phi \sum_{i=1}^{N} z_i \theta_i - (\sum_{i=1}^{N} z_i \theta_i)^2
\end{array} \right]
\]

According to the Cauchy-Schwarz inequality \((a^T a)(b^T b) \geq (a^T b)^2 \), we set \( a = \sqrt{\theta} \), \( b = z' \sqrt{\theta} \). In particular, we can obtain \( x'Hz > 0 \), which means that the Hessian matrix \( H \) is positive semi-definite. Therefore, \( f(Q) \) is a convex function with respect to \( Q \). As a result, the objective function (38) is concave with respect to \( Q \). Furthermore, the objective function is concave and the constraints in this problem (33) are concave. Therefore, the optimization problem (33) is a concave optimization problem.

Proof end.

To solve the optimization problem (33), we can obtain the optimal allocated power of each SCU by the Lagrangian dual method. According to (33), we get the Lagrangian function as following
\[
L = \sum_{k=1}^{K} \sum_{n=1}^{N} B(\alpha_{n,k} \log_2 y_{n,k}^* + \beta_{n,k}) \\
-t(\sum_{k=1}^{K} \sum_{n=1}^{N} p_{n,k} + P_e - \sum_{k=1}^{K} Q_{ER}) \\
+ \sum_{k=1}^{K} \lambda_k (P_e - \sum_{n=1}^{N} p_{n,k}) \\
+ \sum_{k=1}^{K} \sum_{n=1}^{N} \phi_{n,k} (\alpha_{n,k} \log_2 y_{n,k}^* + \beta_{n,k} - R_{k}^{\text{min}}) \\
+ u(I_{\text{thr}} - \sum_{k=1}^{K} \sum_{n=1}^{N} p_{n,k} (\tilde{r}_{n,m}^p + \xi_n \sqrt{\frac{1-e^u}{e}}))
\]

where \( \lambda_k, \phi_{n,k} \) and \( u \) are corresponding Lagrange multipliers for the constraints \( C1 \), \( C3 \) and \( \tilde{C}4 \), respectively. Thus, the Lagrangian dual function is
given by
\[ g(\lambda_k, \varphi_k, u) = \max L(p_{1,k}, \lambda_k, \varphi_k, u) \]
\[ \text{s.t.} \ C1, C2, C3, C4 \]

And the dual problem can be expressed as
\[ \min g(\lambda_k, \varphi_k, u) \]
\[ \text{s.t.} \ \lambda_k, \varphi_k, u \geq 0 \]

Considering the complexity of the calculation, it is assumed that there are two SCUs in a small cell. Based on the KKT conditions, we can obtain the closed-form of power for each SCU.

\[ p_{1,k} = \frac{\alpha_{1,k}(1 + \varphi_{1,k})}{[t(1 - \gamma \sum_{j \neq k} g_{1,j}) + \lambda_k + u(\hat{R}_{n,m} + \zeta_n \sqrt{\frac{1 - \epsilon}{\epsilon}})]} \ln 2 \]  
(44)

\[ p_{2,k} = \frac{(-B_{1,k} + C_{1,k} - A_{1,k}D_{1,k})}{2D_{1,k}} \]
\[ + \sqrt{(D_{1,k}A_{1,k} + B_{1,k} - C_{1,k})^2 + 4D_{1,k}C_{1,k}A_{1,k}} \]
\[ B_{1,k} = \frac{\alpha_{1,k}(1 + \varphi_{1,k})}{\ln 2} \]  
(46)

\[ C_{1,k} = \frac{\alpha_{1,k}(1 + \varphi_{1,k})}{\ln 2} \]  
(47)

\[ D_{1,k} = t(1 - \gamma \sum_{j \neq k} g_{1,j}) + \lambda_k + u(\hat{R}_{n,m} + \zeta_n \sqrt{\frac{1 - \epsilon}{\epsilon}}) \]  
(48)

Based on the equation (44) and (45), we can get the optimal \( p_{1,k} \) and \( p_{2,k} \), then the Lagrangian dual variables are updated via the sub-gradient method.

\[ \lambda_{k+1}^\star = [\lambda_k^\star - \beta_k^\star \cdot (P_k^\star - \sum_{n=1}^N p_{n,k})]^+ \]  
(49)

\[ \mu_{k+1}^\star = [\mu_k^\star - \xi_k^\star \cdot (I_{P_k^\star} - \sum_{k=1}^K \sum_{m=1}^M p_{k,m} (\hat{R}_{n,m} + \zeta_n \sqrt{\frac{1 - \epsilon}{\epsilon}}))]^+ \]  
(50)

\[ \varphi_{k+1}^\star = [\varphi_k^\star - \xi_k^\star \cdot (\alpha_k \log_2 Y_{k,n}^\star + \beta_k - R_{\text{max}}^\star)]^+ \]  
(51)

where \([q]^+ = \max(0, q)\), \( \zeta_k^\star \), \( \xi_k^\star \) and \( \varphi_k^\star \) are the step sizes of each iteration, respectively.

The proposed algorithm is detailed in Algorithm 1.

**Algorithm 1:** The proposed algorithm

1: **Initialization:** Initialize \( p_{1,k} = p_{2,k} = P_l / 2 \) and input energy efficiency \( t \). Initialize iteration index \( l \), the maximum iteration number \( l_{\text{max}} \), and the convergence precision \( \Delta > 0 \).

2: **while**

\[ l < l_{\text{max}} \text{ or } \| R^l - t(\sum_{k=1}^K \sum_{n=1}^N p_{n,k} + P_l - \sum_{k=1}^K Q_{I_k}) \| > \Delta \]

3: **do**

4: Update \( \alpha \) and \( \beta \) according to (31) and (32)

5: Update Lagrange multipliers \( \lambda_k \), \( u \) and \( \varphi \) according to (49), (50) and (51);

6: Update \( p_{1,k} \) and \( p_{2,k} \) according to (44) and (45);

7: Set \( t = t + \frac{R^l}{\sum_{k=1}^K \sum_{n=1}^N p_{n,k} + P_l - \sum_{k=1}^K Q_{I_k}} \) and \( l = l + 1 \)

8: **end while**

9: Output \( p_{n,k} \)

### 5 Simulation results

In this section, the performance of the proposed power allocation scheme is evaluated by the simulation results. It is assumed that there are six low-power SBSs deployed in the marginal coverage area of an MBS, and SCUs and MCUs are randomly distributed in the coverage area of small cell and macro cell. Simulation parameters are shown in Table 1.

| parameter          | value   |
|---------------------|---------|
| Macro cell radius   | 250m    |
| Small cell radius   | 30m     |
| Transmission power of MBS | 40dBm  |
| Transmission power of SBS | 20dBm  |
| Circuit power consumption | 30dBm  |
| Path-loss exponent | 3       |
| User minimum data rate | 0.1bps/Hz |
| Outage probability  | 0.05    |

**Figure 2.** The total energy efficiency versus noise with different \( P_l \)

The comparison of the total energy efficiency with different circuit power consumption \( P_l \) is shown in Figure 2, where noise \( \delta^2 \) is varying from...
We set $\sigma = 0.01$, $\zeta = 0.01$ and $\gamma = 0.5$. From Figure 2, we can see that the proposed algorithm can decrease the energy efficiency performance with the increasing of the noise. Meanwhile, the energy efficiency performance of the proposed algorithm also degrades as the $P_c$ increases. For example, the energy efficiency with $P_c = 26dBm$ is 84.05% superior to that with $P_c = 28dBm$, and $P_c = 28dBm$ is 79.44% larger than that with $P_c = 30dBm$ as the $\delta^2 = -48dBm/Hz$.

Figure 3 displays the total energy efficiency versus the noise with different power schemes which include the proposed robust power allocation algorithm, equal power allocation (EPA) algorithm and fractional transmit power allocation (FTPA) algorithm. We assume $\sigma = 0.01$, $\zeta = 0.01$ and $\gamma = 0.5$. From Figure 3, we can see that the total energy efficiency of three scheme decreases with the increasing of $\delta^2$ and the output performance of the proposed algorithm is better than EPA algorithm and FTPA algorithm. This indicates that each SCU can obtain the optimal allocated power by the proposed scheme than EPA and FTPA by using fixed allocation.

Figure 4 displays the total energy efficiency performance of different transmission power of MBS $P_m$ versus the upper boundary of the uncertainty parameter $\sigma$. We set $\zeta = 0.01$ and $\gamma = 0.5$. From Figure 4, it is shown that the total energy efficiency of our scheme decreases with the increasing of $\sigma$. When $\sigma = 0.06$, the total energy efficiency of our scheme with $P_m = 36dBm$ is 4.748Mbit/J, $P_m = 38dBm$ is 3.507Mbit/J, $P_m = 40dBm$ is 2.734Mbit/J. And it demonstrates that the impact of $\sigma$ on the total energy efficiency is smaller when $P_m$ is larger.

The total energy efficiency versus $\gamma$ with different $P_c$ is shown in Figure 5, we set $\sigma = 0.01$ and $\zeta = 0.01$. From Figure 5, we note that the total of energy efficiency of the proposed algorithm improves with the increasing of energy conversion efficiency $\gamma$. That is because we use the energy conversion efficiency $\gamma$, the collected energy is increase when $\gamma$ is increasing, then the overall power consumption is decrease, and the system energy efficiency is increase.

Figure 6 shows the relationship between energy efficiency performance with different transmission power of SBS $P_k$. We assume the parameters $\sigma = 0.01$, $\zeta = 0.01$ and $\gamma = 0.5$. From Figure 6, we can observe that the energy efficiency of all small cells increases as the number of small cells increases. Meanwhile the proposed algorithm can improve the energy efficiency performance as the $P_k$ increases.
For example, the energy efficiency with $P_t = 22\,dBm$ is 5.03% superior to that with $P_t = 20\,dBm$, and $P_t = 20\,dBm$ is 3.45% larger than that with $P_t = 18\,dBm$ as the number of small cells is six.

**Figure 6.** The total energy efficiency versus number of small cells with different $P_t$

The total energy efficiency with two different techniques versus the number of small cells is shown in Figure 8. We set $\sigma = 0.01$, $\zeta = 0.01$ and $\gamma = 0.5$. We note that the output performance with NOMA is better as compared with OFDMA. For example, when the number of small cell is six, the energy efficiency with NOMA is 1.83Mbit/J larger than the energy efficiency with OFDMA. It is because NOMA technology can not only allow multiple users to reuse the same channel, but also allow the strong user to remove the interference from the weak users.

**Figure 7.** The total energy efficiency versus $P_m$ with different $\sigma$

**Figure 8.** The total energy efficiency versus different reuse techniques

### 6 Conclusion

In this paper, we propose a robust power allocation scheme for NOMA heterogeneous networks with EH under imperfect CSI. To compensate for the power consumption of the SBS in two-layer downlink NOMA heterogeneous networks, an EH-aided SBS scheme is proposed, in which the interference signals of other SBSs and MBS are harvested as energy by SBS. In order to maximize the total energy efficiency under the imperfect CSI, a robust power allocation scheme, which provide better robustness as compared with the conventional schemes. Based on the MPM method, the outage probability constraints are converted into deterministic ones without the well-known statistical distributions of uncertain parameters. Then we can transform the optimization problem into a convex problem by using the Dinkelbach method and the quadratic transformation approach. At last, the closed-form solution for power allocation to each small user is obtained via Lagrange dual approach, which has improved system energy efficiency. The superiority and efficiency of the proposed scheme with considering the EH-aided SBS is shown in simulation results compared with the traditional power allocation algorithms.
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Declarations

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