Temporal-Logic-Based Reward Shaping for Continuing Learning Tasks

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Abstract

In continuing tasks, average-reward reinforcement learning may be a more appropriate problem formulation than the more common discounted reward formulation. As usual, learning an optimal policy in this setting typically requires a large amount of training experiences. Reward shaping is a common approach for incorporating domain knowledge into reinforcement learning in order to speed up convergence to an optimal policy. However, to the best of our knowledge, the theoretical properties of reward shaping have thus far only been established in the discounted setting. This paper presents the first reward shaping framework for average-reward learning and proves that, under standard assumptions, the optimal policy under the original reward function can be recovered. In order to avoid the need for manual construction of the shaping function, we introduce a method for utilizing domain knowledge expressed as a temporal logic formula. The formula is automatically translated to a shaping function that provides additional reward throughout the learning process. We evaluate the proposed method on three continuing tasks. In all cases, shaping speeds up the average-reward learning rate without any reduction in the performance of the learned policy compared to relevant baselines.

1 Introduction

Reinforcement learning (RL) is a popular method for autonomous agents to learn optimal behavior through repeated interactions with the environment. Most RL algorithms aim to optimize the total discounted reward received by the learner. However, in cases involving infinite-horizon or continuing tasks, a discount factor can often lead to undesirable behaviors since the agent sacrifices long-term benefits for short-term gains (Mahadevan, 1996). Hence, the natural quantity to optimize for many continuing tasks is the average reward.

Many robotics applications for RL have delayed or sparse rewards, slowing down learning significantly due to long stretches of possibly uninformative exploration (Mahadevan, 1996). Reward shaping (Gullapalli & Barto, 1992; Mataric, 1994) is a common way to inject domain knowledge to guide exploration, but requires care so as not to change the underlying RL problem. Randlov & Alstrom (1998) added an intuitive signal to the reward of an agent learning to ride a bicycle, but had the unintended effect of causing the agent to ride in circles. Ng et al. (1999) considered additional rewards that are expressed as the difference of a potential function between the agent’s current state and its next state and showed that this so-called potential-based reward shaping (PBRS) does not affect the optimal policy. Wiewiora et al. (2003) further extended PBRS to include shaping functions of actions as well as states and Devlin & Kudenko (2012) allow time-varying shaping functions. To the best of our knowledge, all of the work in reward shaping concentrates on the discounted-reward setting, and often for Q-learning in particular. In this paper, we develop a reward shaping framework for average-reward RL.

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In general, manually constructing a potential function is difficult since it requires detailed knowledge of the task and the reward structure (Grzes & Kudeenko, 2010). To address this challenge, previous works provide methods of learning shaping functions online (Grzes & Kudeenko, 2010) or offline (Marthi, 2007). In contrast, our focus is on incorporating side-information or domain knowledge into the shaping function. For example, a user may wish to advise a robot tasked with cleaning to spend more time in one room compared to another. While this statement is intuitive for humans, it can be non-trivial to translate such a statement to an appropriate shaping function. We introduce a method to synthesize the shaping function directly from a temporal logic formula representing the domain knowledge. Compared to manually constructing a shaping function, providing a temporal logic formula is a more systematic and less error-prone. Furthermore, manually constructing a shaping function requires taking into account the relative magnitudes of rewards. In contrast, our proposed method allows for a qualitative input of domain knowledge and does not depend on knowledge of the original reward values and the underlying RL algorithm.

We test the proposed framework in three continuing learning tasks: continual area sweeping (Ahmadi & Stone, 2005; Shah et al., 2020), control of a cart pole in OpenAI gym (Brockman et al., 2016), and motion-planning in a grid world (Mahadevan, 1996). We compare with two benchmarks, the baseline R-learning without reward shaping, and shielding, which is a learning method where the learning agent never violates the given temporal logic formula (Alshiekh et al., 2018). The proposed framework improves learning performance from the baseline R-learning in terms of sample efficiency. We also show that when the provided domain knowledge is inaccurate, the proposed method still learns the optimal policy and is faster than the baseline R-learning. At the same time, shielding fails to learn the optimal policy as the learner is forced to strictly follow the inaccurate domain knowledge.

Our contributions are as follows: 1) we develop a reward shaping method for average-reward RL, 2) we remove the need for potential function engineering by allowing domain knowledge to be specified as a temporal logic formula, 3) we compare the approach to standard R-learning without reward shaping as well as shielding to illustrate the advantage of our method.

2 Related Work

There are several existing areas of research that develop techniques to learn from human knowledge. Learning from demonstration is the problem of learning a policy from examples of a teacher performing a task (Argall et al., 2009; Hussein et al., 2017). Knowledge from a human trainer can be transferred through real-time feedback to improve learning performance (Thomaz & Breazeal, 2006; Knox & Stone, 2009; Warnell et al., 2018). These techniques usually optimize some combination of inferred human rewards and task rewards. This work focuses on optimizing natural environmental rewards regardless of the quality of knowledge. For continuing tasks, demonstrations or real-time feedback data can be difficult to collect. Our approach takes a temporal-logic specification as input and does not require the human to perform the task or observe the learner during training.

In recent years, there has been growing interest in using temporal logic in RL. For example, Icarte et al. propose LTL formulas (Toro Icarte et al., 2018b) and later reward machine (Toro Icarte et al., 2018a), a type of Mealy machine to specify tasks by encoding the reward functions. Similar to reward shaping, reward machines improve sampling efficiency with optimality guarantees. The main difference is that reward shaping guides learning by providing additional rewards, while reward machines rely on exposing the reward structure to the learning agent. Some other approaches use temporal logic formulas as constraints that must be satisfied to guarantee safety during learning (Alshiekh et al., 2018; Jansen et al., 2018). To that end, the actions violating the safety formula are overwritten and never taken by the learning agent. In our setting, these actions are not inherently unsafe but simply “not recommended” based on the provided domain knowledge. Not allowing the agent to take these actions can be overly restrictive, and in cases where the provided domain knowledge is not exactly correct, can lead to sub-optimal policies.

3 Preliminaries

In this section, we introduce the definitions and concepts of average-reward reinforcement learning, reward shaping, and temporal logic used throughout the paper.
3.1 Average-Reward Reinforcement Learning

A Markov decision process (MDP) $\mathcal{M} = (S, s_I, A, R, P)$ is a tuple consisting of a finite set $S$ of states, an initial state $s_I \in S$, a finite set $A$ of actions, reward function $R : S \times A \times S \to \mathbb{R}$, and a probabilistic transition function $P : S \times A \times S \to [0, 1]$ that assigns a probability distribution over successor states given a state and an action. For a stationary policy $\pi : S \to A$, the expected average reward is defined as

$$\rho^\pi_M := \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{k=0}^{n-1} R(s_k, \pi(s_k), s_{k+1}) \right].$$

(1)

We are interested in reinforcement learning (RL) problems where the objective is to maximize an average reward over time. More specifically, given an MDP $\mathcal{M}$, where the transition function $P$ and/or reward function $R$ are unknown a priori, the objective is to learn an optimal policy $\pi^*$ such that $\rho^\pi_M \geq \rho^\pi_M$ for any stationary policy $\pi$.

Given a policy $\pi$, the $Q$-function $Q^\pi_M(s, a)$ is the value of taking action $a$ in state $s$ and thereafter following $\pi$, and defined as

$$Q^\pi_M(s, a) := \mathbb{E} \left[ \sum_{t=0}^{\infty} R(s_t, a_t, s_{t+1}) - \rho^\pi_M | s_0 = s, a_0 = a \right].$$

(2)

The optimal $Q$-function $Q^*_M$ satisfies the Bellman equation [Sutton et al., 1998]:

$$Q^*_M(s, a) = \mathbb{E}[R(s, a, s') + \max_{a' \in A}(Q^*_M(s', a')) | P(s, a, s') > 0] - \rho^*_M, \forall s \in S, a \in A,$$

(3)

where $\rho^*_M = \rho^*_M$, the expected average reward of executing the optimal policy $\pi^*$. From $Q^*_M$, the optimal policy $\pi^*$ can be determined by $\pi^*(s) = \arg \max_{a \in A}(Q^*_M(s, a))$.

R-learning is a classical approach for learning $Q^*_M$ and $\rho^*_M$, where the standard action-value update step in Q-learning is replaced with one derived from $\rho^*:

$$Q_{t+1}(s_t, a_t) \leftarrow Q_t(s_t, a_t) + \alpha (r_t + \max_{a_{t+1}} Q_t(s_{t+1}, a_{t+1}) - \rho_t(s_t) - Q_t(s_t, a_t)),
\rho_{t+1}(s_t) \leftarrow r_t = \beta (r_t + \max_{a_{t+1}} Q_t(s_{t+1}, a_{t+1}) - \rho_t(s_t) - Q_t(s_t, a_t)),$$

where $\alpha$ and $\beta$ are the learning rates [Schwartz, 1993; Mahadevan, 1996].

3.2 Potential-Based Reward Shaping and Potential-Based Advice

Potential-based reward shaping (PBRS) augments the reward function $R$ by a shaping reward function $F$, where $F(s, s') = \gamma \Phi(s') - \Phi(s)$, $\Phi : S \to \mathbb{R}$ is a potential function, and $0 < \gamma < 1$ is the discount factor. The resulting MDP is $\mathcal{M}' = (S, s_I, A, R', P)$ with $R' = R + F$. It has been proven, for the discounted and episodic settings, that $\mathcal{M}$ and $\mathcal{M}'$ have the same optimal policy [Ng et al., 1999]. The goal is to learn the optimal policy in $\mathcal{M}'$ with improved sample efficiency compared to learning in $\mathcal{M}$. Potential-based look-ahead advice [Wiewiora, 2003] further extends PBRS such that the potential function also depends on actions: $F(s, a, s', a') = \gamma \Phi(s', a') - \Phi(s, a)$, where $a'$ is chosen by the current policy.

3.3 Temporal Logic

We first define some basic notations. The set of finite sequences $w$ over a set $\Sigma$ is denoted $\Sigma^*$, and the set of infinite sequences is denoted $\Sigma^\omega$.

Given an MDP $\mathcal{M} = (S, s_I, A, R, P)$ and a policy $\pi$, a path $\sigma = s_0s_1s_2\cdots \in \Sigma^* = \Sigma^\omega$ is a sequence of states with $s_0 = s_I$ and $P(s_i, \pi(s_i), s_{i+1}) \geq 0$ for $i \geq 0$. Given a set $\text{AP}$ of atomic propositions (Boolean variables), we introduce the labelling function $L : S \to 2^{\text{AP}}$. For a path $\sigma = s_0s_1s_2\cdots \in \Sigma^\omega$, its corresponding label sequence is $w = w_0w_1\cdots \in (2^{\text{AP}})^\omega$, where $w_t = L(s_t)$.

A linear temporal logic (LTL) formula $\varphi$ constrains all finite and infinite label sequences $w \in (2^{\text{AP}})^\omega \cup (2^{\text{AP}})^*$ corresponding to paths in $\mathcal{M}$. For the full semantics of LTL, we refer the reader
to (Baier & Katoen, 2008). We translate the LTL formula $\varphi$ to an equivalent deterministic finite automaton (DFA) (Kupferman & Vardi, 2001). A DFA is a tuple $\mathcal{T} = (Q, q_0, \delta, \mathcal{AP}, \mathcal{T})$ where $Q$ is a finite set of states, $q_0 \in Q$ is the initial state, $\delta : \mathcal{AP} \times Q \to Q$ is a deterministic transition function and $H \subseteq Q$ is a set of accepting states. A run $\omega$ of $\mathcal{T}$ over label sequence $w$ is an infinite sequence $q_0q_1 \cdots \in Q^\omega$ where $q_0 = q_1$ and $q_i+1 = \delta(q_i, w_i)$. The run is accepted by $\mathcal{T}$ if $q_t \in H$ for all $t \geq 0$. A label sequence $w$ satisfies $\varphi$ iff the induced run is accepted by $\mathcal{T}$. Only a subclass of LTL formulas can be translated to an equivalent DFA with acceptance condition as presented above. We informally refer to such formulas as safety formulas and the translated DFA as a safety automaton (Kupferman & Vardi, 2001).

**Example 1.** Consider a cleaning robot operating in the environment with a human as shown in Figure 1(a) which we model as an MDP on a discrete gridworld shown in Figure 1(b). Trash can appear in any state in the grid with a given probability, and the robot is rewarded when it finds trash. Suppose we wish to provide the domain knowledge “the presence of humans tends to increase the probability of trash appearance”. We provide this knowledge in the form of advice as an LTL formula “Always human visible” written as $\Box \text{human\_visible}$. We use this formula to guide the agent during learning to more quickly associate human presence with the appearance of trash. The robot has line-of-sight with a range of 5. We have $\mathcal{AP} = \{\text{human\_visible}\}$, and labelling function $L$ that labels all states in visible range as $\{\text{human\_visible}\}$ and all other states as $\{\neg \text{human\_visible}\}$. The corresponding DFA is shown in Figure 1(c).

![Image](image-url)

(a) Gazebo environment for cleaning robot example. (b) Gridworld representation of (a). (c) DFA for $\varphi = \Box \text{human\_visible}$. Figure 1: A service robot example where the robot is rewarded for detecting trash that can appear at any state in the gridworld. Both robot and human can move in any of the cardinal directions.

In the above example, trash appears in states with a higher probability if the state is occupied by a human. However, the given formula $\varphi$ to always keep the human visible, if never violated by the robot, will likely result in the robot learning suboptimal behavior for the house cleaning task since trash could accumulate in other areas as well. In the next sections, instead of relying on LTL formulas to constrain the RL agent’s behavior, we discuss how to use a given LTL formula to guide the learning of an optimal policy that may violate the formula.

### 4 Reward Shaping for the Average-Reward Setting

The main contribution of this paper is a novel approach to provide shaping rewards based on an LTL formula to guide a reinforcement learning agent in the average-reward setting while allowing the optimal policy to be learned even if it violates the given formula. We first define a potential-based shaping function, and prove that the optimal policy can be recovered given any potential function (see supplementary material for the proof).

**Theorem 1.** Let $F : S \times A \times S \to \mathbb{R}$ be a shaping function of the form

$$F(s, a, s') = \Phi\left(s', \arg\max_{a'} Q^{*}_M(s', a')\right) - \Phi(s, a),$$

where $\Phi : S \times A \to \mathbb{R}$ is a real-valued function and $Q^{*}_M$ is the optimal average-reward $Q$-function satisfying (9). Define $\hat{Q}_M : S \times A \to \mathbb{R}$ as

$$\hat{Q}_M(s, a) := Q^{*}_M(s, a) - \Phi(s, a).$$

(4)
Then $\hat{Q}_{\mathcal{M}'}$ is the solution to the following modified Bellman equation in $\mathcal{M}' = (S, s_I, A, R', P)$ with $R' = R + F$:

$$\hat{Q}_{\mathcal{M}'}(s, a) = \mathbb{E}[R'(s, a, s') + \hat{Q}_{\mathcal{M}'}(s', a^*)] - \rho_{\mathcal{M}'}^\star,$$

where $a^* = \arg\max_a (Q_{\mathcal{M}'}(s', a') + \Phi(s', a'))$, and $\rho_{\mathcal{M}'}^\star$ is the expected average reward of the optimal policy in $\mathcal{M}$, $\pi_{\mathcal{M}'}$ executed in $\mathcal{M}'$.

In MDPs with finite state and action spaces, $\hat{Q}_{\mathcal{M}'}$ is the unique solution to (12) because it forms a fully-determined linear system. After learning $Q_{\mathcal{M}'}$ with the shaping rewards in $\mathcal{M}'$, it follows from (11) that the optimal policy $\pi^\star$ can be recovered as:

$$\pi^\star(s) = \arg\max_{a \in A} (\hat{Q}_{\mathcal{M}'}(s, a) + \Phi(s, a)).$$

It is important to note that our goal is not to learn the optimal policy of $\mathcal{M}'$. Rather, we learn a value function $\hat{Q}_{\mathcal{M}'}$ from which it is possible to recover the optimal policy of $\mathcal{M}$. Note that $\hat{Q}_{\mathcal{M}'}$ is not the same as $Q_{\mathcal{M}'}^\star$ where $\hat{Q}_{\mathcal{M}'}$ satisfies (12) but $Q_{\mathcal{M}'}^\star$ satisfies the following Bellman optimality equation:

$$Q_{\mathcal{M}'}^\star(s, a) = \mathbb{E}[R(s, a, s') + \max_{a' \in A} Q_{\mathcal{M}'}(s', a')]] - \rho_{\mathcal{M}'}^\star.$$ 

In practice, the shaping rewards in (10) require knowledge of $Q_{\mathcal{M}'}^\star(s', a')$, which can be bootstrapped by $\hat{Q}_{\mathcal{M}'}(s', a') + \Phi(s', a')$ as its estimate.

### 5 Potential Function Synthesis

We now present an algorithm for constructing a potential function $\Phi$ from a given temporal logic formula $\varphi$. Informally, the reward shaping function captures the information in the formula $\varphi$ by penalizing the learning agent for visiting the states from which a violation of $\varphi$ can occur with a non-zero probability. We illustrate this concept with an example.

**Example 2.** Consider again the example shown in Figure 2. In order to not violate the formula $\varphi = \square\text{human\_visible}$, the robot must “plan ahead” to ensure that no matter what the human does in the future, the robot will still not lose visibility of them. Figure 2 shows a case where the current state does not cause a violation of $\varphi$ as the robot can still see the human. However, there is a non-zero probability that the human will move south into a room, and the robot will not be able to maintain visibility and hence will violate $\varphi$.

Informally, we compute the set of state-action pairs from which the probability of violating $\varphi$ is 0. We refer to this set $W$ as the almost-sure winning region. To this end, we define a product MDP $\mathcal{M}_\varphi = \mathcal{M} \times T^\varphi$. Given an MDP $\mathcal{M} = (S, s_I, A, R, P)$, a DFA $T^\varphi = (Q, q_I, 2^{NP}, \delta, H)$, and a labelling function $L$ a product MDP is defined as $\mathcal{M}_\varphi = (S \times T^\varphi = (V, v_I, A, \Delta, \overline{H})$ where $V = S \times Q$ is the joint set of states, $v_I = (s_I, Q_I) = (s_I, \delta(q_I, L(s_I)))$ is the initial state, $\Delta : V \times S \times \rightarrow [0, 1]$ is the probability transition function such that $\Delta((s, q), a, (s', q')) = P(s, a, s')$, if $\delta(q, L(s')) = q'$, and 0, otherwise, and $\overline{H} = (S \times H) \subseteq V$ is the set of accepting states.

In order to compute the almost-sure winning region $W \subseteq S \times A$ in the MDP $\mathcal{M}$, we first compute the set $W_0^{\min}(\overline{H}) \subseteq V \times A$ of states and corresponding actions that have a minimum probability of 0 of reaching $\overline{H}$. This set can be computed using graph-based methods in $O(V \times A)$ (Baier & Katoen 2008). An algorithm for computing the set is provided in the supplementary material. We can then extract $W$ from $W_0^{\min}(\overline{H})$ by defining $W := \{(s, a) \in S \times A \mid (s, q, a) \in W_0^{\min}(\overline{H})\}$.

Given $W$, we construct $\Phi$ as:

$$\Phi(s, a) = \begin{cases} 0 & (s, a) \in W \\ C(s, a) & (s, a) \notin W \\ \end{cases}$$

5
where $C \in \mathbb{R}$ is an arbitrary constant, and $d : S \times A \to \mathbb{R}$ is any function such that $d(s, a) < C$ for all $s, a$.

**Example 3.** Continuing the cleaning robot example, we set $C = 1$ and $d(s, a)$ as the negative of the distance between $s$ and the closest state in $W$. The results are detailed in Section 6.1.

**Remark 1.** We note that while the method presented in this section can be applied for any safety formula, the synthesized potential function will not, in general, be Markovian. Since the goal of this paper is to help the agent learn an optimal policy with respect to a Markovian reward function $R(s, a)$, it suffices to consider Markovian potential functions. Restricting $\varphi$ to a subclass of safety formulas called invariant formulas will guarantee $\Phi$ is Markovian (Baier & Katoen, 2008).

### 6 Experiments

We evaluate our temporal-logic-based reward shaping framework in three continuing RL benchmarks. Due to space constraints, this section focuses on the two more complex domains: continual area sweeping and cart pole. The results of a set of additional continuing gridworld experiments can be found in the supplementary material. In all scenarios, we compare against a standard R-learning approach that we refer to as the baseline and the same R-learning approach where actions violating the given temporal logic formula are disallowed using the method in (Alshiekh et al., 2018). We refer to the latter as shielding, and we refer the framework presented in this paper as shaping. Furthermore, we study cases when the optimal learned policy must violate the given formula - in such situations, we refer to the provided formula as imperfect advice.

We show that 1) our reward shaping framework speeds up baseline average-reward RL, 2) shaping cannot outperform shielding with perfect advice, but 3) when the advice is imperfect, shaping still learns the optimal policy faster than the baseline, whereas shielding fails to learn the optimal policy.

#### 6.1 Continual Area Sweeping

The problem of a robot sweeping an area repeatedly and non-uniformly for some task has been formalized as continual area sweeping, where the goal is to maximize the average reward per unit time without assuming the distribution of rewards (Shah et al., 2020). We run the set of experiments on the gridworld in Figure 1B where the robot has a maximum speed of 3, i.e. it can move to any cell within a Manhattan distance of 3. We study two cases. First, the robot operates without a human and is given the formula $\Box \text{kitchen}$ where the kitchen is the bottom right room of the gridworld in Figure 1B. Second, a human is also present, and the robot is given the formula $\Box \text{human-visible}$ as described in Examples 1-3. In both scenarios, the baseline we compare our framework against is a deep R-learning approach on double DQN introduced by Shah et al. (2020).

**Always kitchen** In this scenario, the kitchen has the most cleaning needs, and the given formula is to always stay in the kitchen. State-action pair $(s, a) \in W$ if $L(s) = \{\text{kitchen}\}$ and $L(s') = \{\text{kitchen}\}$ for all $s'$ such that $P(s, a, s') > 0$. The potential function $\Phi$ is constructed as Equation 1 where $C = 1$ and $d(s, a)$ is the negative of the minimal distance between $s$ and the kitchen and plus 1 if $a$ gets closer to the kitchen. For the shielding approach, $(s, a)$ is allowed only if $(s, a) \in W$ or $a$ will decrease the distance to the kitchen from $s$.

Experiments are conducted in the following two scenarios: a) cleaning is required only in the kitchen and nowhere else, and b) cleaning can be required in the kitchen and some states that are randomly selected from the right half of the corridor. Each cell that might require cleaning is assigned a frequency between $1/20$ and $1/10$ so that the robot has to learn an efficient sweeping strategy among those cells. At each time step, there is also a 0.2 probability that a dirty cell no longer needs cleaning (such as trash gets picked up by people, the wet floor dries as time passes).

Figure 3a reports the learning curves when the formula is perfect, i.e., trash only appears in the kitchen. While shielding performs well from the beginning, shaping quickly catches up, and both learn significantly faster than the baseline. Figure 3b reports the results in the second case where there are unexpected rewards outside of the specified region. Shielding gets a head-start by blocking actions away from the region, but it fails to discover the unexpected reward outside the region. Shaping performs better than the baseline at the beginning and converges to a better policy that sweeps in the corridor too.
Always keep human visible  Consider the scenario described in Examples 1-3. At every time step, there is a 0.2 probability that the current position of the human needs cleaning. There is also a 0.2 probability that a dirty cell becomes clean by itself with every step. The human moves randomly between the corridor and the top left room and has a speed of 1 cell per step. $\Phi$ is constructed as described in Example 3. When the human is invisible to the robot, the winning region is unknown, and $d(s,a)$ is set to $-6$.

Figure 3 shows the results when the human is the only source of trash appearance. Shielding outperforms shaping and the baseline by forcing the agent to always follow the human. The learning curves of shaping and baseline methods do not converge to the optimal average reward because softmax action selection is used throughout learning. The exploratory actions can lose sight of the human, which is very costly in this task. Figure 3b shows the results when each cell in the corridor is assigned a frequency between $1/20$ and $1/10$ to require cleaning besides the cell that the human occupies at each step. In this case, the optimal policy is to not follow the human into the top left room and to always sweep in the corridor. As Figure 3d shows, the shielding method fails to learn the optimal policy because it is forced to strictly follow the human and misses the reward of staying the corridor; the shaping method is resilient to the imperfect advice while still using it to outperform the baseline initially.
6.2 Cart Pole

Cart pole is a widely studied classic control problem in the reinforcement learning literature. The agent is a cart with a pole attached to a revolute joint. Situated on a horizontal track, the agent must keep its pole balanced upright by applying a force to the right or the left. Each state is composed of the position \( x \), the linear velocity \( \dot{x} \), the pole angle \( \theta \), and the angular velocity \( \dot{\theta} \). At each step, the agent receives reward of 1 if \( x \in [-2.4, 2.4] \) and \( \theta \in [-\pi/15, \pi/15] \). Otherwise, the episode terminates. We modify the cart pole environment in OpenAI Gym (Brockman et al., 2016) to a continuing task by removing the termination conditions. We allow the cart to move anywhere on the \( x \)-axis and the pole to be at any angle, and restrict the max linear velocity by \( |\dot{x}| \leq 1 \). Other aspects of the motion model are kept the same, and reward is given under the same conditions. Therefore, the agent has to learn a policy that keeps the cart position in the scoring range (i.e., \( x \in [-2.4, 2.4] \)) and swings the pole up when it falls down. The source code of the continuing cart pole environment is available as supplementary material.

We compare our shaping framework with the same baseline deep R-learning algorithm and shielding. The specification is to keep the cart position in the scoring range. We assume the agent does not have access to the full dynamics model, and approximate the winning region by predicting the next \( x \) position as \( x_{t+1} = x_t + \dot{x}_t \Delta t \) where \( \Delta t \) is the time between steps. The shield blocks the action in the direction of \( x_{t+1} \) if \( x_{t+1} \) is not in the scoring range. \( \Phi(s, a) \) is constructed with \( C = 1 \) and \( d(s, a) \) as the minimal distance between the scoring range and \( x_{t+1} \) (and multiplied by a constant).

Figure 4a plots the learning performance when the accurate scoring range \([-2.4, 2.4]\) is given to shaping and shielding. Figure 4b shows the results when an inaccurate scoring range \([-2, 2]\) is used. The baseline fails to learn to keep the cart in the scoring range in most trials. Shaping and shielding are both able to significantly improve the learning performance, but when the knowledge is inaccurate, shielding over-restricts action choices and achieves worse average reward than shaping.

7 Conclusion

We provide a reward shaping framework to construct shaping functions from a temporal logic specification to improve reinforcement learning performance in the average-reward setting. We prove and empirically show that our shaping framework speeds up the learning rate, and allows the optimal policy to be learned when the provided advice is imperfect.

For future work, we are interested in studying adversarial advice - i.e., formulas provided to actively hinder the learning process. We are also interested in expanding the learning problem to include both non-Markovian reward structures and shaping functions.
Broader Impact

The guiding philosophy behind the work in this paper is that the barrier to entry for reinforcement learning is too high in terms of domain expertise and data availability. We believe this is particularly exacerbated in robotics applications as robotics has become integrated in various research fields, many of which involve no computer scientists and data collection is costly. RL as a tool has huge potential to unlock the use of robotics for research in many fields, but often requires a significant amount of tuning, training, and expertise to take advantage of properly. Our goal is to make this paper be the first of a series of papers aimed at facilitating high-level interaction with RL algorithms without requiring significant RL expertise and improving the sampling efficiency.

Since the work at its current stage is still theoretical, there is little direct negative societal consequences. However, this work, as is the case for most robotics-related research, contributes to the increased adoption of robots and autonomous systems in society. In particular, RL allows autonomous systems to achieve complex tasks that they were previously not capable of. In this paper, we use cleaning robots as the motivating case study throughout the paper. Autonomous systems being able to reliably complete complex tasks has the potential to displace the service industries, surveillance, and search and rescue, where the tasks are repetitive or even dangerous.

While we acknowledge the potential job loss that can occur from maturation of such technology, we believe the unlocking the potential of RL to non-experts can lead to the acceleration of research in many fields.

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Use unnumbered first level headings for the acknowledgments. All acknowledgments go at the end of the paper before the list of references. Moreover, you are required to declare funding (financial activities supporting the submitted work) and competing interests (related financial activities outside the submitted work). More information about this disclosure can be found at: https://neurips.cc/Conferences/2020/PaperInformation/FundingDisclosure

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Proof of Theorem 1

We first restate the following equations to be used in the proof.

For a stationary policy \( \pi : S \to A \), the expected average reward is defined as

\[
\rho_\pi^\pi := \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{k=0}^{n-1} R(s_k, a_k, s_{k+1}) + F(s_{k+1}, a_{k+1}) \right].
\] (8)

The optimal Q-function \( Q_\pi^* \) for average reward satisfies the Bellman equation (Sutton et al., 1998):

\[
Q_\pi^*(s, a) = \mathbb{E}[R(s, a, s') + \max_{a' \in A} (Q_\pi^*(s', a')) | P(s, a, s') > 0] - \rho_\pi^*, \forall s \in S, a \in A,
\] (9)

**Theorem 2.** Let \( F : S \times A \times S \to \mathbb{R} \) be a shaping function of the form

\[
F(s, a, s') = \Phi \left( s', \arg \max_{a' \in A} (Q_\pi^*(s', a')) \right) - \Phi(s, a),
\] (10)

where \( \Phi : S \times A \to \mathbb{R} \) is a real-valued function and \( Q_\pi^* \) is the optimal average-reward Q-function satisfying (9). Define \( \hat{Q}_{\pi'} : S \times A \to \mathbb{R} \) as

\[
\hat{Q}_{\pi'}(s, a) := Q_\pi^*(s, a) - \Phi(s, a).
\] (11)

Then \( \hat{Q}_{\pi'} \) is the solution to the following modified Bellman equation in \( \mathcal{M}' = (S, s_1, A, R', P) \) with \( R' = R + F \):

\[
\hat{Q}_{\pi'}(s, a) = \mathbb{E}[R'(s, a, s') + \hat{Q}_{\pi'}(s', a')] - \rho_{\hat{\pi}_{M'}}^*,
\] (12)

where \( a^* = \arg \max_a (\hat{Q}_{\pi'}(s', a') + \Phi(s', a')) \), and \( \rho_{\hat{\pi}_{M'}}^* \) is the expected average reward of the optimal policy in \( \mathcal{M} \), \( \hat{\pi}_{M'} \), executed in \( \mathcal{M}' \).

**Proof.** We first show that \( \rho_{\hat{\pi}_{M'}}^* = \rho_{\pi'}^\pi \), which means the expected average reward of \( \pi^* \) does not change with the additional rewards from \( F \).

From (8), we know that

\[
\rho_{\hat{\pi}_{M'}}^* = \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{k=0}^{n-1} R(s_k, a_k, s_{k+1}) + F(s_{k+1}, a_{k+1}) \right]
\]

\[
= \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{k=0}^{n-1} R(s_k, a_k, s_{k+1}) + \Phi(s_{k+1}, \arg \max_{a' \in A} (Q_\pi^*(s_{k+1}, a'))) - \Phi(s_k, a_k) \right].
\]

Since \( a_{k+1} = \pi^*(s_{k+1}) = \arg \max_{a'} (Q_\pi^*(s_{k+1}, a')) \), we have

\[
\rho_{\hat{\pi}_{M'}}^* = \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{k=0}^{n-1} R(s_k, a_k, s_{k+1}) + \Phi(s_{k+1}, a_{k+1}) - \Phi(s_k, a_k) \right]
\]

\[
= \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{k=0}^{n-1} R(s_k, a_k, s_{k+1}) - \Phi(s_0, a_0) \right]
\]

\[
= \lim_{n \to \infty} \frac{1}{n} \mathbb{E} \left[ \sum_{k=0}^{n-1} R(s_k, a_k, s_{k+1}) \right]
\]

\[
= \rho_{\pi'}^*.
\] (13)

Subtracting \( \Phi(s, a) \) from both sides of the Bellman optimality equation (9) gives us

\[
Q_\pi^*(s, a) - \Phi(s, a) = \mathbb{E}[R(s, a, s') - \Phi(s, a) + \max_{a' \in A} (Q_\pi^*(s', a'))] - \rho_{\hat{\pi}_{M'}}^*
\] (14)
Substituting \( \hat{Q}_M'(s, a) := Q^*_M(s, a) - \Phi(s, a) \) and \( a^* = \arg \max_{a'} (\hat{Q}_M'(s', a') + \Phi(s', a')) \) in (14), we get

\[
\hat{Q}_M'(s, a) = E[R(s, a, s') - \Phi(s, a) + \max_{a' \in A} (\hat{Q}_M'(s', a') + \Phi(s', a'))] - \rho^*_M
\]

\[
= E[R(s, a, s') + \Phi(s', a^*) - \Phi(s, a) + \hat{Q}_M'(s', a^*)] - \rho^*_M
\]

\[
= E[R(s, a, s') + F(s, a, s') + \hat{Q}_M'(s', a^*)] - \rho^*_M
\]

\[
= E[R'(s, a, s') + \hat{Q}_M'(s', a^*)] - \rho^*_M.
\]
Algorithm to compute almost-sure winning region

Correction to typo: In the body of the main paper, in Section 5, we state the following: "set $W_{0}^{\min}(\overline{H}) \subseteq V \times A$ of states and corresponding actions that have a minimum probability of 0 of reaching $\overline{H}$". This should in fact read: "states and corresponding actions that have a minimum probability of 0 of leaving $\overline{H}$". We apologize for the confusion.

In the following, we present the algorithm to compute the almost-sure winning region. Informally, the goal of the algorithm is to compute a set of state-action pairs from which there is a minimum probability 0 of violating the safety formula $\varphi$. We repeat some definitions here for self-completion.

Given an MDP $M = (S, s_I, A, R, P)$, a DFA $T = (Q, q_I, 2^A, \delta, H)$, and a labelling function $L$, a product MDP is defined as $M_{\varphi} = (M \times T)$ where $V = S \times Q$ is the joint set of states, $v_I = (s_I, q_I) = (s_I, \delta(q_I, L(s_I)))$ is the initial state, $\Delta : V \times \Sigma \times V \to [0, 1]$ is the probability transition function such that $\Delta((s, q), a, (s', q')) = P(s, a, s')$, if $\delta(q, L(s')) = q'$, and 0, otherwise, and $H = (S \times H) \subseteq V$ is the set of accepting states.

In order to compute the almost-sure winning region $W \subseteq S \times A$ in the MDP $M$, we first compute the set $W_{0}^{\min}(\overline{H}) \subseteq V \times A$ of states and corresponding actions that have a minimum probability of 0 of reaching $V \setminus H$. This set can be computed using graph-based methods in $O(V \times A)$ (Baier & Katoen, 2008) and the algorithm is given below. We then extract $W$ from $W_{0}^{\min}(\overline{H})$ by defining $W := \{(s, a) \in S \times A \mid (s, q, a) \in W_{0}^{\min}(\overline{H})\}$.

We introduce some additional notation. Let $Act(v) \subseteq A$ be the set of all available actions $a \in A$ available at $v$.

Result: Set $W_{0}^{\min}(\overline{H})$ of state-action pairs with minimum probability 0 of reaching $V \setminus H$
initialize $\overline{W} = (V \setminus H) \times A$, $\overline{H} = V \setminus H$
while True do
    forall $v \in \overline{H}$ do
        forall $a \in Act(v)$ do
            if $\exists v' \in \overline{H}$ such that $\Delta(v, a, v') > 0$ then
                $\overline{W} = \overline{W} \cup \{(v, a)\}$
            end
        end
        if $(v, a) \in \overline{W}$ \forall $a \in Act(v)$ then
            $\overline{H} = \overline{H} \cup \{v\}$
        end
    end
end
$W_{0}^{\min}(\overline{H}) = (V \times A) \setminus \overline{W}$
return $W_{0}^{\min}(\overline{H})$

Algorithm 1: Almost-sure winning set construction

We note that the algorithm above is a variation of the algorithm to compute the set of states with $Pr_{0}^{min}(s \models \diamond B)$ found in (Baier & Katoen, 2008). Informally, the algorithm computes the set of states from which there is a minimum 0 probability of eventually reaching set $B$. 

12
Gridworld Experiments

Figure 5: Variations on the grid world case study from Mahadevan (1996). Agent receives a reward when it moves to the green cell and is then transported to a random cell.

We evaluate our method against standard R-Learning in a continuing grid world environment as studied by Mahadevan (1996). Figure 5 shows the 6x6 grid with a +100 reward in the green cell. The agent can move to a neighboring cell in one of the four directions deterministically at every step, and gets “transported” to a random cell when it reaches the green cell and when it fails to reach the green cell after 100 steps.

The following four methods are compared: our temporal-logic-based reward shaping, hand-crafted reward shaping (Ng et al., 1999), shielding, and baseline R-learning. Shielding blocks actions up and left and only allows actions down and right. For the temporal-logic-based method, the winning region consists of all states, but only with the actions down and right. Then the potential function $\Phi$ is constructed with $C = 1$ and $d(s,a) = -1$. The hand-crafted potential function is taken from the grid world experiments by Ng et al. (1999), where $\Phi(s)$ is defined as the negative of the distance from $s$ to the green cell.

The correctness of the advice is varied by adding the wall from (2, 2) to (2, 5) as shown in Figure 5. When the agent is directly above the wall, it cannot reach the green cell by only moving down or right. This experiment evaluates our temporal-logic-based reward shaping framework when the environment is different from what the advice is given for.

Figure 6 compares the learning performance of each method in the two conditions. The observations are consistent with the results reported in other benchmarks in the paper. Shielding outperforms other methods when the advice is perfect, i.e., there is no wall, and fails to reach the maximum average reward when the wall is added. Our temporal-logic-based reward shaping method performs similarly to the hand-crafted shaping function while allowing the reward shaping to be automatically constructed. Both shaping methods still outperform the R-learning baseline when the advice is imperfect.

Figure 6: Learning performance comparison of temporal-logic-based reward shaping (ours), hand-crafted reward shaping (Ng et al.), shielding, and baseline. The average reward of every 100 steps is plotted, averaged by 100 runs.
References

Ahmadi, M. and Stone, P. Continuous area sweeping: A task definition and initial approach. In ICAR’05. Proceedings., 12th International Conference on Advanced Robotics, 2005., pp. 316–323. IEEE, 2005.

Alshiekh, M., Bloem, R., Ehlers, R., Könighofer, B., Niekum, S., and Topcu, U. Safe reinforcement learning via shielding. In Thirty-Second AAAI Conference on Artificial Intelligence, 2018.

Argall, B. D., Chernova, S., Veloso, M., and Browning, B. A survey of robot learning from demonstration. Robotics and autonomous systems, 57(5):469–483, 2009.

Baier, C. and Katoen, J.-P. Principles of model checking. MIT press, 2008.

Brockman, G., Cheung, V., Pettersson, L., Schneider, J., Schulman, J., Tang, J., and Zaremba, W. OpenAI Gym. arXiv e-prints, art. arXiv:1606.01540, June 2016.

Devlin, S. M. and Kudenko, D. Dynamic potential-based reward shaping. In Proceedings of the 11th International Conference on Autonomous Agents and Multiagent Systems, pp. 433–440. IFAAMAS, 2012.

Grzes, M. and Kudenko, D. Online learning of shaping rewards in reinforcement learning. Neural Networks, 23(4):541–550, 2010.

Gullapalli, V. and Barto, A. G. Shaping as a method for accelerating reinforcement learning. In Proceedings of the 1992 IEEE International Symposium on Intelligent Control, pp. 554–559, 1992.

Hussein, A., Gaber, M. M., Elyan, E., and Jayne, C. Imitation learning: A survey of learning methods. ACM Computing Surveys (CSUR), 50(2):1–35, 2017.

Jansen, N., Könighofer, B., Junge, S., and Bloem, R. Shielded decision-making in mdps. CoRR, abs/1807.06096, 2018. URL http://arxiv.org/abs/1807.06096

Knox, W. B. and Stone, P. Interactively shaping agents via human reinforcement: The TAMER framework. In The Fifth International Conference on Knowledge Capture, September 2009.

Kupferman, O. and Vardi, M. Y. Model checking of safety properties. Formal Methods in System Design, 19(3):291–314, 2001.

Mahadevan, S. Average reward reinforcement learning: Foundations, algorithms, and empirical results. Machine learning, 22(1-3):159–195, 1996.

Marthi, B. Automatic shaping and decomposition of reward functions. In Proceedings of the 24th International Conference on Machine Learning, ICML ’07, pp. 601–608, New York, NY, USA, 2007. Association for Computing Machinery. ISBN 9781595937933. doi: 10.1145/1273496.1273572. URL https://doi.org/10.1145/1273496.1273572

Mataric, M. J. Reward functions for accelerated learning. In Machine Learning Proceedings 1994, pp. 181–189. Elsevier, 1994.

Ng, A. Y., Harada, D., and Russell, S. J. Policy invariance under reward transformations: Theory and application to reward shaping. In Proceedings of the Sixteenth International Conference on Machine Learning, pp. 278–287. Morgan Kaufmann Publishers Inc., 1999.

Randlo\'v, J. and Alstr\'om, P. Learning to drive a bicycle using reinforcement learning and shaping. In Proceedings of the Fifteenth International Conference on Machine Learning, ICML ’98, pp. 463–471, San Francisco, CA, USA, 1998. Morgan Kaufmann Publishers Inc. ISBN 1558605568.

Schwartz, A. A reinforcement learning method for maximizing undiscounted rewards. In Proceedings of the Tenth International Conference on International Conference on Machine Learning, pp. 298–305. Morgan Kaufmann Publishers Inc., 1993.

Shah, R., Jiang, Y., Hart, J., and Stone, P. Deep R-Learning for Continual Area Sweeping. arXiv e-prints, art. arXiv:2006.00589, May 2020.
Sutton, R. S., Barto, A. G., et al. *Introduction to reinforcement learning*, volume 135. MIT press Cambridge, 1998.

Thomaz, A. L. and Breazeal, C. Reinforcement learning with human teachers: Evidence of feedback and guidance with implications for learning performance. In *Proceedings of the 21st National Conference on Artificial Intelligence - Volume 1*, AAAI’06, pp. 1000–1005. AAAI Press, 2006. ISBN 9781577352815.

Toro Icarte, R., Klassen, T., Valenzano, R., and McIlraith, S. Using reward machines for high-level task specification and decomposition in reinforcement learning. In *International Conference on Machine Learning*, pp. 2107–2116, 2018a.

Toro Icarte, R., Klassen, T. Q., Valenzano, R., and McIlraith, S. A. Teaching multiple tasks to an rl agent using ltl. In *Proceedings of the 17th International Conference on Autonomous Agents and MultiAgent Systems*, pp. 452–461. International Foundation for Autonomous Agents and Multiagent Systems, 2018b.

Warnell, G., Waytowich, N., Lawhern, V., and Stone, P. Deep TAMER: Interactive agent shaping in high-dimensional state spaces. In *Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence*, February 2018.

Wiewiora, E. Potential-based shaping and q-value initialization are equivalent. *Journal of Artificial Intelligence Research*, 19:205–208, 2003.

Wiewiora, E., Cottrell, G. W., and Elkan, C. Principled methods for advising reinforcement learning agents. In *Proceedings of the 20th International Conference on Machine Learning (ICML-03)*, pp. 792–799, 2003.