Sum rules in the superpartner spectrum of the minimal supersymmetric standard model

YOSHIHARU KAWAMURA †,∗
Physik Department, Technische Universität München
D-85747 Garching, Germany
TATSUO KOBAYASHI
Institute of Particle and Nuclear Studies
High Energy Accelerator Research Organization, Tanashi, Tokyo 188, Japan
JISUKE KUBO **
Physics Department, Faculty of Science,
Kanazawa University, Kanazawa, 920-11 Japan

Abstract
Assuming that the string inspired, universal sum rules for soft supersymmetry-breaking terms, which have been recently found both in a wide class of four-dimensional superstrings and in supersymmetric gauge-Yukawa unified gauge models, are satisfied above and at the grand unification scale, we investigate their low energy consequences and derive sum rules in the superpartner spectrum of the minimal supersymmetric standard model.

† Humboldt Fellow.
* On leave from: Department of Physics, Shinshu University, Matsumoto, 390 Japan.
**Partially supported by the Grants-in-Aid for Scientific Research from the Ministry of Education, Science and Culture (No. 40211213).
One of the most important issues in realistic supersymmetric theories is to understand how supersymmetry is broken and then to relate it with low energy physics. Though the recent exiting theoretical developments in supersymmetric gauge theories as well as superstrings [1], this problem has not been solved in a satisfactory fashion yet. It is however widely accepted that the supersymmetry breaking, whatever its origin is, appears as soft supersymmetry-breaking (SSB) terms in low energy effective theories, because the softness is a desirable property for not spoiling the supersymmetric solution of the naturalness problem of the standard model [2].

One might hope that the SSB terms have a minimal structure as it is suggested by $N = 1$ minimal supergravity [2], on one hand. It may be worthwhile, on the other hand, to find out relations among the SSB terms that have least dependence of the mechanism of supersymmetry breaking and are satisfied in a wide class of models. Phenomenological investigations and consequences based on these relations certainly would have a more general validity than those based on the assumption of the so-called universal SSB terms. At first sight one might think that once we deviate from the universalality of the SSB terms, we would fall into the chaos of varieties [3]. Recent investigations on the SSB terms in 4D superstrings [4]–[8], however, have shown that it is possible to do systematic investigations of the SSB terms, and it has turned out to be also possible to parametrize the SSB terms in a simple way [6]–[8] so that one can easily find relations among them that are independent of the detailed nature of supersymmetry breaking. It has been then shown [9] that there exist symmetries in the effective $N = 1$ supergravity, which, along with a simple assumption on Yukawa couplings, lead to these relations. These symmetries happen to coincide with the S- and T- dualities, which are usually present in 4D superstrings [10]. Moreover it has turned out [4] that these relations are renormalization group (RG) invariant at the lowest nontrivial order in perturbation theory in all gauge-Yukawa unified (GYU) models [12], so that once they are satisfied at the string scale, they are satisfied at the grand unification scale, too. We call these relations sum rules for the SSB terms, which may be summarized as [8, 9, 11]

$$h^{ijk} = -M Y^{ijk}, \quad (1)$$

$$M^2 = m_i^2 + m_j^2 + m_k^2. \quad (2)$$

Here $M$ and $m_i^2$ stand for the unified gaugino mass and the soft scalar mass squared of the chiral superfield $\Phi_i$, respectively. $Y^{ijk}$ is the dimensionless Yukawa coupling for the $\Phi_i \Phi_j \Phi_k$ term in the superpotential, while the $h^{ijk}$ is the dimensional coupling for the trilinear term.
of the corresponding scalar components. Higher order corrections to the above sum rules are model-dependent in general. We, furthermore, would like to recall that the assumption on the gauge-Yukawa unification in supersymmetric grand unified theories (GUTs), especially in the third generation sector, leads to a successful prediction of the top quark mass \[13\].

In this letter we are motivated by the desire to find out low energy consequences of the sum rules (1) and (2) which are assumed to be satisfied for the third generation sector of SU(5) type GUTs at the GUT scale \(M_{\text{GUT}}\). We will assume that between \(M_{\text{GUT}}(\sim 10^{16} \text{ GeV})\) and the supersymmetry braking scale \(M_S(< 1 \text{ TeV})\) the minimal supersymmetric standard model (MSSM) describes particle physics, and we will derive sum rules in the superpartner spectrum (Eq. (13)) from the string inspired, universal relations (1) and (2). These sum rules are independent on the details of the SSB parameters as long as the sum rules (1) and (2) are satisfied at \(M_{\text{GUT}}\). Needless to say that these sum rules could be tested by future experiments, e.g., at LHC.

Before we present the details of our investigations, we would like to briefly outline the basic nature that leads to the sum rules (1) and (2), both in 4\(D\)-superstring-inspired supergravity models and GYU models.

To analyze how the sum rules (1) and (2) within the framework of effective \(N = 1\) supergravity can be realized, one considers a non-canonical Kähler potential of the general form

\[
K = \tilde{K}(\Phi_a, \Phi^{*a}) + \sum_i K_i^i(\Phi_a, \Phi^{*a})|\Phi^i|^2,
\]

where \(\Phi_a\)'s and \(\Phi_i\)'s are chiral superfields in the hidden and visible sectors, respectively. The basic assumptions are: (1) Supersymmetry is broken by the \(F\)-term condensations \((\langle F_a \rangle \neq 0)\) of the hidden sector fields \(\Phi_a\). (2) The gaugino mass \(M\) stems from the gauge kinetic function \(f\) which depends only on the hidden sector fields, i.e. \(f = f(\Phi_a)\). (3) We consider only those Yukawa couplings that have no field dependence. (4) The vacuum energy \(V_0\) vanishes. For the sum rules (1) and (2) to be satisfied under these assumptions, a certain relation among the Kähler potential \(\tilde{K}\) in the hidden sector, the gauge kinetic function \(f\) and the Kähler metric has to exist, i.e.,

\[
K_{(T)}(\Phi_a, \Phi^{*a}) \equiv \ln(K_i^i K_j^j K_k^k) = \tilde{K} + \ln \text{Re} f + \text{const.}
\]

for all \(\{i, j, k\}\) appearing in the sum rules (1) and (2), implying that the theory has two types of symmetries: The first one corresponds to the Kähler transformation together with the chiral
rotation of the matter multiplets,

\[
\begin{align*}
\Phi_i & \to e^{M_i} \Phi_i, \quad \Phi^{*i} \to e^{\overline{M_i}} \Phi^{*i}, \\
K_i & \to K_i e^{-(M_i + \overline{M_i})}
\end{align*}
\]

(5)

\[
K{(T)} \to K{(T)} - M - \overline{M}, \quad f(\Phi_a) \to f(\Phi_a), \quad W \to e^M W,
\]

(6)

where \(M_i\) is a function of \(\Phi_a\) and has to satisfy the constraint \(M_i + M_j + M_k = M\) for all possible set of \(\{i, j, k\}\) appearing in the sum rules (1) and (2). The second one is the invariance of the Kähler metric \(K_a{(S)}b\) under the \(SL(2, R)\) transformation of the gauge kinetic function \(f(\Phi_a)\), where \(K{(S)} = -\ln(\overline{f(\Phi_a) + f(\Phi^{*a})})\). For 4D string models, these symmetries appear as the target-space duality invariance and S-duality [10], respectively. In fact, Brignole et. al. [8] have already found these sum rules in their explicit computations in various orbifold models.

In case that gauge symmetries break, we generally have \(D\)-term contributions to the soft scalar masses. Such \(D\)-term contributions, however, do not appear in the sum rules, because each \(D\)-term contribution is proportional to the charge of the matter field \(\Phi_i\) [14].

The basic assumption in GYU models is that the Yukawa couplings \(Y^{ijk}\) are expressed in terms of the gauge coupling \(g\):

\[
Y^{ijk} = \rho^{ijk} g + \ldots,
\]

(7)

where \(\rho^{ijk}\) are constant independent of \(g\) and \(\ldots\) stands for higher order terms. Eq. (7) is the power series solution to the reduction equation [13] \(\beta^Y_{ij} = \beta_g dY^{ijk}/dg\), where \(\beta^Y_{ij}\) and \(\beta_g\) stand for the \(\beta\) functions of \(Y^{ijk}\) and \(g\), respectively. The next assumption is that the coefficients \(\rho^{ijk}\) satisfy the diagonality relation \(\rho_{ipq}\rho^{pql} \propto \delta^j_i\). This implies that the one-loop anomalous dimensions \(\gamma_i^{(1)j}/16\pi^2\) for \(\Phi_i\)’s become diagonal if the reduction solution (7) is inserted, i.e., \(\gamma_i^{(1)j} = \gamma_i \delta^j_i g^2\), where \(\gamma_i\) are constant independent of \(g\). It can be then shown that the sum rule (1) as well as the relation \((m^2)^j_i = m_i^2 \delta^j_i = \kappa_i M^2 \delta^j_i\) with \(\kappa_i = \gamma_i/(T(R) - 3C(G))\) are RG invariant in one-loop order, where \(T(R)\) and \(C(G)\) are the Dynkin index of the matter representation \(R\) and the quadratic Casimir of the adjoint representation of \(G\), respectively.

The sum rule (2) then follows from the consequence of the reduction of \(Y^{ijk}\), i.e., \(\gamma_i + \gamma_j + \gamma_k = T(R) - 3C(G)\) for \(\{i, j, k\}\) appearing in the sum rule.

To come to our main result of this letter, let us first describe the parameter space. Since we assume an \(SU(5)\) type GYU in the third generation, Eq. (7) takes the form

\[
g_t = \rho_t g, \quad g_b = g_r = \rho_b g,
\]

(8)

at $M_{\text{GUT}}$, where $g_i \ (i = t, b, \tau)$ are the Yukawa couplings for the top, bottom quarks and the tau, and we ignore the Cabibbo-Kabayashi-Maskawa mixing of the quarks. For a given model, the $\rho$’s are fixed, but here we consider them as free parameters. It is more convenient to go from the parameter space $(\rho_t, \rho_b)$ to another one ($k_t \equiv \rho_t^2$, $\tan \beta$), because we use the (physical) top quark mass $M_t$ as input, i.e., $M_t = (175.6 \pm 5.5)$ GeV. Therefore, the unification condition of the gauge couplings of the MSSM, along with $\alpha_{\text{EM}}^{-1}(M_Z) = 127.9 + (8/9 \pi) \ln(M_t/M_Z)$ and also the tau mass $M_{\tau} = 1.777$ GeV as low energy input parameters, fixes the allowed region in the $k_t - \tan \beta$ space, which is shown in Fig. 1, where we have used $M_S = 300$ GeV. In the following analyses when varying $\tan \beta$, we use $k_t$ for $M_t = 175$ GeV, while ignoring the bottom quark mass.

![FIG. 1: $M_t$ in the $k_t - \tan \beta$ space](image)

The parameter space in the SSB sector is constrained at $M_{\text{GUT}}$ due to unification:

$$
\begin{align*}
  h_t &= -M g_t = -M \rho_t g, \quad h_b = h_{\tau} = -M g_b = -M \rho_b g, \\
  m^2_{\tilde{t}_R} &= m^2_{\tilde{t}_L} = m^2_{\tilde{b}_L} = m^2_{\tilde{t}_R}, \quad m^2_{\tilde{b}_R} = m^2_{\tilde{\tau}_L}, \\
  m^2_{\Sigma(t)} &\equiv m^2_{\tilde{t}_R} + m^2_{\tilde{t}_L} + m^2_{\tilde{H}_2}, \quad m^2_{\Sigma(b,\tau)} \equiv m^2_{\tilde{b}_R,\tilde{\tau}_R} + m^2_{\tilde{b}_L,\tilde{\tau}_L} + m^2_{\tilde{H}_1}.
\end{align*}
$$

The unification condition for the gaugino mass is: $M_1 = M_2 = M_3 = M$ at $M_{\text{GUT}}$. We note that in the one-loop RG evolution of $m^2_{\Sigma}$’s in the MSSM only the same combinations of the
sum of $m_i^2$’s enter. Therefore, as far as we are concerned with the evolution of $m^2_{\Sigma}$’s, we have only one additional parameter $M$, which we further identify with $M_S$. In Fig. 2, we show the evolution of $m^2_{\Sigma(t,b,\tau)}/M^2_3$ as function of $Q/M_S$ for $\tan \beta = 50.$

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.pdf}
\caption{The evolution of $m^2_{\Sigma}/M^2_3$ for $\tan \beta = 50$.}
\end{figure}

The figure above shows that $m^2_{\Sigma(t,b)}/M^2_3$ is stable near $\ln Q/M_S \simeq 0$. As for $m^2_{\Sigma(\tau)}/M^2_3$, we find $\Delta(m^2_{\Sigma(\tau)}/M^2_3) \simeq \pm 0.1$ for $\Delta(\ln Q/M_S) \simeq \pm 5$ at $\tan \beta = 50$. These features in the stability of the evolution of $m^2_{\Sigma(t,b,\tau)}/M^2_3$ do not change very much for the entire range of $\tan \beta$ between 2 and 50.

To derive the announced sum rules for the superpartner spectrum, we further define

$$s_i \equiv m^2_{\Sigma(i)}/M^2_3 \hspace{1em} (i = t, b, \tau) \hspace{1em} \text{at} \hspace{1em} Q = M_S.$$  \hspace{1em} (12)

These parameters do not depend on the value of $M$ (which is defined at $M_{GUT}$) in one-loop order, but they do on $\tan \beta$. This dependence is shown in Fig. 3.
We then require that the electroweak gauge symmetry is correctly broken at $M_S$ to obtain the sum rules,

$$
-\cos 2\beta \ m_A^2 = (s_b - s_t) M_3^2 + 2(\hat{m}_t^2 - m_t^2) - 2(\hat{m}_b^2 - m_b^2)
= (s_\tau - s_t) M_3^2 + 2(\hat{m}_t^2 - m_t^2) - 2(\hat{m}_\tau^2 - m_\tau^2),
$$

(13)

where $m_A^2$ is the neutral pseudoscalar Higgs mass squared, and $\hat{m}_i^2$ stands for the arithmetic mean of the two corresponding scalar superparticle mass squared. From the sum rules we also obtain $(s_b - s_\tau) M_3^2 \simeq 2(\hat{m}_b^2 - \hat{m}_\tau^2)$, which yields $\hat{m}_b^2 > \hat{m}_\tau^2$ because $s_b - s_\tau > 0$. Note that $s_\tau$ becomes negative for $\tan \beta > 33$, which gives a bound on $m_{H_1}^2$ for a given $M_3$, $2\hat{m}_\tau^2 = s_\tau M_3^2 - (M_3^2 \cos 2\beta)/2 - m_{H_1}^2 > 0$. If this is not satisfied, the $U(1)_{EM}$ is broken.

The $s_i$'s are relatively stable against the deviation from the GUT scale sum rules (1) and (2), which is shown in Fig. 4 for $m_{\Sigma(t)}/M_3^2$, where we have varied the initial values at $M_{GUT}$. Fig. 4 shows a weak infrared attractiveness of $m_{\Sigma(t)}/M_3^2$'s [16], which is the reason of the stability. So, the weak infrared attractiveness works for suppressing this uncertainty at $M_{GUT}$, but it is not strong enough to wash out the information at $M_{GUT}$ [17].
We have also analyzed the infrared attractiveness [16] of $m_{\Sigma(b,\tau)}^2/M_3^2$ and $A_{b,b,\tau}$ for different values of $\tan \beta$ and found that the infrared attractiveness is indeed a general tendency [16], but its degree differs among the quantities and depends on $\tan \beta$.

As we have emphasized, the sum rules (13) are satisfied under very general assumptions. Before we close we summarize the most important ones. The first one is that the Yukawa couplings $Y_{ijk}$ in question are field-independent in 4D string models. The next one is that below the string scale the gauge-Yukawa unification is realized so that the string inspired sum rules are RG invariant below the string scale and are satisfied down to $M_{\text{GUT}}$. We do not need this assumption if these sum rules have a strong infrared attractiveness [16] (which is model-dependent, of course). It is then assumed that we have an $SU(5)$ type GUT and below $M_{\text{GUT}}$ the effective theory is the MSSM.

The sum rules could be tested by future experiments, e.g., at LHC, and experimental verifications of them would give important hints on the nature of unification and supersymmetry breaking.
[1] See, for instance, W. Lerche, CERN-TH/96-332, hep-th/9611190, 1996.

[2] For a review, see H.P. Nilles, Phys. Rep. 110, 1 (1984); H.E Haber and G.L. Kane, Phys. Rep. 117, 75 (1985).

[3] Phenomenology based on the non-universal SSB terms is briefly summarized in J. Amundson et al., Report of the snowmass Supersymmetry Theory Working Group, hep-ph/9609374 (1996), and references therein.

[4] L.E. Ibáñez and D. Lüst, Nucl. Phys. B382, 305 (1992).

[5] V.S. Kaplunovsky and J. Louis, Phys. Lett. B306, 269 (1993).

[6] A. Brignole, L.E. Ibáñez and C. Muñoz, Nucl. Phys. B422, 125 (1994); [Erratum: B436, 747 (1995)].

[7] T. Kobayashi, D. Suematsu, K. Yamada and Y. Yamagishi, Phys. Lett. B348, 402 (1995).

[8] A. Brignole, L.E. Ibáñez, C. Muñoz and C. Scheich, Z. f. Phys. C74, 157 (1997); A. Brignole, L.E. Ibáñez and C. Muñoz, CERN-TH/97-143, hep-ph/9707209 (1997).

[9] Y. Kawamura, T. Kobayashi and J. Kubo, Phys. Lett. B405, 64 (1997).

[10] For a review, see, for instance, A. Giveon, M. Porrati and E. Rabinovici, Phys. Rep. 244, 77 (1994); S. Förste and J. Louis, hep-th/9612192 (1996).

[11] T. Kobayashi, J. Kubo, M. Mondragon and G. Zoupanos, KEK Preprint 97-103, hep-ph/9707425 (1997).

[12] D. Kapetanakis, M. Mondragón and G. Zoupanos, Zeit. f. Phys. C60, 181 (1993); J. Kubo, M. Mondragón and G. Zoupanos, Nucl. Phys. B424, 291 (1994).

[13] J. Kubo, M. Mondragón, M. Olechowski and G. Zoupanos, Nucl. Phys. B479, 25 (1996).

[14] Y. Kawamura, Phys. Rev. D53, 3779 (1996); Y. Kawamura and T. Kobayashi, Phys. Lett. B375, 141 (1996).
[15] W. Zimmermann, Com. Math. Phys. 97, 211 (1985); R. Oehme and W. Zimmermann Com. Math. Phys. 97, 569 (1985).

[16] M. Lanzagorta and G.G. Ross, Phys. Lett. B349, 319 (1995); ibid. B364, 163 (1995); P.M. Ferreira, I. Jack and D.R.T. Jones, Phys. Lett. B357, 359 (1995); S.A. Abel and B.C. Allanach, hep-ph/9707436 (1997).

[17] See also the results of N. Polonsky and A. Pomarol, Phys. Rev. Lett. 73, 2292 (1994); Phys. Rev. D51, 6532 (1995).