Introduction to Inflationary Cosmology

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Abstract. The early universe according to the big bang and the grand unified theories is discussed. The shortcomings of big bang are summarized together with their resolution by inflationary cosmology. Inflation, the subsequent oscillation and decay of the inflaton, and the resulting ‘reheating’ of the universe are studied. The density perturbations produced by inflation and the temperature fluctuations of the cosmic background radiation are sketched. The hybrid inflationary model is described. Two ‘natural’ extensions of this model which avoid the disaster encountered in its standard realization from the overproduction of monopoles are presented.

1 Introduction

The observed Hubble expansion of the universe together with the discovery of the cosmic microwave background radiation (CMBR) had established hot big bang as a viable model of the universe. The success of nucleosynthesis in reproducing the observed abundance of light elements and the proof of the black body character of the CMBR then imposed hot big bang as the standard cosmological model. This model combined with grand unified theories (GUTs) provides the framework for discussing the early universe.

Despite its successes, the standard big bang (SBB) model had some long-standing shortcomings. One of them is the horizon problem. The CMBR received now has been emitted from regions which never communicated before sending light to us. The question then arises how come the temperature of the black body radiation from these regions is so finely tuned as the results of the cosmic background explorer (COBE) show. Another issue is the flatness problem. The present universe appears almost flat. This requires that, in its early stages, the universe was flat with a great accuracy. Also, combined with GUTs which predict the existence of superheavy monopoles, the SBB model leads to a catastrophe due to the overproduction of these monopoles. Finally, the model has no explanation for the small density perturbations required for the structure formation in the universe and the generation of the observed temperature fluctuations in the CMBR.

Inflation offers an elegant solution to all these problems of the SBB model. The idea is that, in the early universe, a real scalar field (the inflaton) was displaced from its vacuum value. If the potential energy density of this field happens to be quite flat, the roll-over of the field towards the vacuum can be very slow for a period of time. During this period, the energy density is
dominated by the almost constant potential energy density of the inflaton. As a consequence, the universe undergoes a period of quasi-exponential expansion, which can readily solve the horizon and flatness problems by stretching the distance over which causal contact is established and reducing any pre-existing curvature in the universe. It can also adequately dilute the GUT monopoles. Moreover, it provides us with the primordial density perturbations which are needed for explaining the large scale structure in the universe as well as the temperature fluctuations observed in the CMBR. Inflation can be easily incorporated in GUTs. It occurs during the GUT phase transition at which the GUT gauge symmetry breaks by the vacuum expectation value (vev) of a Higgs field, which also plays the role of the inflaton.

After the end of inflation, the inflaton oscillates about the vacuum. The oscillations are damped because of the dilution of the field energy density by the cosmological expansion and the inflaton decay into ‘light’ matter. The resulting radiation energy density eventually dominates over the field energy density and the universe returns to a normal big bang type evolution. The temperature at which this occurs is historically called ‘reheat’ temperature although there is neither supercooling nor reheating of the universe.

An important disadvantage of the early realizations of inflation is that they require tiny parameters in order to reproduce the COBE results on the CMBR. To solve this ‘naturalness’ problem, hybrid inflation has been introduced. The idea was to use two real scalar fields instead of one that was normally used. One field may be a gauge non-singlet and provides the ‘vacuum’ energy density which drives inflation, while the other is the slowly varying field during inflation. This splitting of roles between two fields allows us to reproduce the temperature fluctuations of the CMBR with ‘natural’ (not too small) values of the relevant parameters. Hybrid inflation, although initially introduced in the context of non-supersymmetric GUTs, can be ‘naturally’ incorporated in supersymmetric (SUSY) GUTs.

Unfortunately, the monopole problem reappears in hybrid inflation. The end of inflation, in this case, is abrupt and is followed by a ‘waterfall’ regime during which the system falls towards the vacuum manifold and performs damped oscillations about it. If the vacuum manifold is homotopically non-trivial, topological defects will be copiously formed by the Kibble mechanism since the system can end up at any point of this manifold with equal probability. So a disaster is encountered in the hybrid inflationary models which are based on a gauge symmetry breaking predicting monopoles.

One idea for solving the monopole problem of SUSY hybrid inflation is to include into the standard superpotential for hybrid inflation the leading non-renormalizable term. This term cannot be excluded by any symmetries and, if its dimensionless coefficient is of order unity, can be comparable with the trilinear coupling of the standard superpotential (whose coefficient is $\sim 10^{-3}$). Actually, we have two options. We can either keep both these terms or remove the trilinear term by a discrete symmetry
and keep only the leading non-renormalizable term. The pictures emerging in the two cases are different. However, they share a common feature. The GUT gauge group is broken during inflation and, thus, no topological defects can form at the end of inflation. So, the monopole problem is solved.

2 The Big Bang Model

We will start with an introduction to the salient features of the SBB model \[1\] and a summary of the history of the early universe in accordance to GUTs.

2.1 Hubble Expansion

At cosmic times \( t \geq t_P = M_P^{-1} \approx 10^{-44} \) sec \((M_P = 1.22 \times 10^{19} \text{ GeV is the Planck scale})\) after the big bang, the quantum fluctuations of gravity are suppressed and classical relativity is adequate. Strong, weak and electromagnetic interactions, however, require quantum field theoretic treatment.

We assume that the universe is homogeneous and isotropic. The strongest evidence for this cosmological principle is the observed \[3\] isotropy of the CMBR. The space-time metric then takes the Robertson-Walker form

\[
\text{ds}^2 = -dt^2 + a^2(t) \left( \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2 \theta \, d\varphi^2) \right),
\]

where \( r, \varphi \) and \( \theta \) are ‘comoving’ polar coordinates, which remain fixed for objects that just follow the general cosmological expansion. \( k \) is the ‘scalar curvature’ of the 3-space and \( k = 0, > 0 \) or \( < 0 \) corresponds to flat, closed or open universe. The dimensionless parameter \( a(t) \) is the ‘scale factor’ of the universe. We take \( a_0 \equiv a(t_0) = 1 \), where \( t_0 \) is the present cosmic time.

The ‘instantaneous’ radial physical distance is given by

\[
R = a(t) \int_0^r \frac{dr}{(1 - kr^2)^{1/2}},
\]

For flat universe \( (k = 0) \), \( \bar{R} = a(t) \bar{r} \) (\( \bar{r} \) is a ‘comoving’ and \( \bar{R} \) a physical radial vector in 3-space) and the velocity of an object is

\[
\bar{V} = \frac{d\bar{R}}{dt} = \frac{\dot{a}}{a} \bar{R} + a \frac{d\bar{r}}{dt},
\]

where overdots denote derivation with respect to \( t \). The second term in the right hand side (rhs) of this equation is the ‘peculiar velocity’, \( \bar{v} = a(t) \bar{v} \), of the object, i.e., its velocity with respect to the ‘comoving’ coordinate system. For \( \bar{v} = 0 \), \( 3 \) becomes

\[
\bar{V} = \frac{\dot{a}}{a} \bar{R} \equiv H(t) \bar{R},
\]

where \( H(t) \equiv \dot{a}(t)/a(t) \) is the Hubble parameter. This is the well-known Hubble law asserting that all objects run away from each other with velocities proportional to their distances and is the first success of SBB cosmology.
2.2 Friedmann Equation

In a homogeneous and isotropic universe, the energy momentum tensor takes the form \((T_{\mu \nu}) = \text{diag}(-\rho, p, p, p)\), where \(\rho\) is the energy density and \(p\) the pressure. Energy momentum conservation then yields the continuity equation

\[
\frac{d\rho}{dt} = -3H(t)(\rho + p),
\]  

(5)

where the first term in the rhs describes the dilution of the energy due to the Hubble expansion and the second term the work done by pressure.

For a universe described by the metric in (1), Einstein’s equations

\[
R_{\mu \nu} - \frac{1}{2} \delta_{\mu \nu} R = 8\pi G T_{\mu \nu},
\]  

(6)

where \(R_{\mu \nu}\) and \(R\) are the Ricci tensor and scalar curvature and \(G \equiv \frac{M_P^{-2}}{2}\) is the Newton’s constant, lead to the Friedmann equation

\[
H^2 \equiv \left(\frac{\dot{a}(t)}{a(t)}\right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2}.
\]  

(7)

Averaging \(p\), we write \(\rho + p = (1 + w)\rho \equiv \gamma \rho\) and (5) gives \(\rho \propto a^{-3\gamma}\). For a universe dominated by pressureless matter, \(\gamma = 1\) and, thus, \(\rho \propto a^{-3}\). This is interpreted as mere dilution of a fixed number of particles in a ‘comoving’ volume due to the Hubble expansion. For a radiation dominated universe, \(p = \frac{\rho}{3}\) and, thus, \(\gamma = \frac{4}{3}\), which gives \(\rho \propto a^{-4}\). The extra factor of \(a(t)\) is due to the red-shifting of all wave lengths by the expansion. Substituting \(\rho \propto a^{-3\gamma}\) in (7) with \(k = 0\), we get \(a(t) \propto t^{2/3\gamma}\) which, for \(a(t_0) = 1\), gives

\[
a(t) = \left(\frac{t}{t_0}\right)^{2/3\gamma}.
\]  

(8)

For ‘matter’ or ‘radiation’, we obtain \(a(t) = (t/t_0)^{2/3}\) or \(a(t) = (t/t_0)^{1/2}\).

The early universe is radiation dominated and its energy density is

\[
\rho = \frac{\pi^2}{30} \left(N_b + \frac{7}{8} N_f\right) T^4 \equiv c \ T^4,
\]  

(9)

where \(T\) is the cosmic temperature and \(N_b(f)\) the number of massless bosonic (fermionic) degrees of freedom. The quantity \(g_* = N_b + (7/8)N_f\) is called effective number of massless degrees of freedom. The entropy density is

\[
s = \frac{2\pi^2}{45} g_* \ T^3.
\]  

(10)

Assuming adiabatic universe evolution, i.e., constant entropy in a ‘comoving’ volume \((sa^3 = \text{constant})\), we obtain \(aT = \text{constant}\). The temperature-time relation during radiation dominance is then derived from (9) (with \(k = 0\):

\[
T^2 = \frac{M_P}{2(8\pi c/3)^{2/3}} t.
\]  

(11)
Classically, the expansion starts at $t = 0$ with $T = \infty$ and $a = 0$. This initial singularity is, however, not physical since general relativity fails for $t \leq t_P$ (the Planck time). The only meaningful statement is that the universe, after a yet unknown initial stage, emerges at $t \sim t_P$ with $T \sim M_P$.

### 2.3 Important Cosmological Parameters

The most important parameters describing the expanding universe are:

- **i.** The present value of the Hubble parameter (known as Hubble constant) $H_0 \equiv H(t_0) = 100\ h\ \text{km sec}^{-1}\ \text{Mpc}^{-1}$ ($h \approx 0.72 \pm 0.07$ [19]).

- **ii.** The fraction $\Omega = \sigma/\rho_c$, where $\rho_c$ is the critical density corresponding to a flat universe. From (7), $\rho_c = 3H^2/8\pi G$ and $\Omega = 1 + k/a^2 H^2$. $\Omega = 1$, $> 1$ or $< 1$ corresponds to flat, closed or open universe. Assuming inflation (see below), the present value of $\Omega$ must be $\Omega_0 = 1$ in accord with the recent DASI observations which yield $\Omega_0 = 1 \pm 0.04$. The low deuterium abundance measurements [21] give $\Omega_B h^2 \approx 0.020 \pm 0.001$, where $\Omega_B$ is the baryonic contribution to $\Omega_0$. This result implies that $\Omega_B \approx 0.039 \pm 0.077$. The total contribution $\Omega_M$ of matter to $\Omega_0$ can then be determined from the measurements [22] of the baryon-to-matter ratio in clusters. It is found that $\Omega_M \approx 1/3$, which shows that most of the matter in the universe is non-baryonic, i.e., dark matter. Moreover, we see that about 2/3 of the energy density of the universe is not even in the form of matter and we call it dark energy.

- **iii.** The deceleration parameter

\[
q = -\frac{(\ddot{a}/\dot{a})}{(\dot{a}/a)} = \frac{\rho + 3p}{2\rho_c},
\]

Equation (12) gives $q_0 = (\Omega_0 + 3w \Omega_X)/2$, where $\Omega_X = \rho_X/\rho_c$ and $w_X = p_X/\rho_X$ with $\rho_X$ and $p_X$ being the dark energy density and pressure. Observations prefer $w_X = -1$, with a 95% confidence limit $w_X < -0.6$ [24]. Thus, dark energy can be interpreted as something close to a non-zero cosmological constant (see below).

### 2.4 Particle Horizon

Light travels only a finite distance from the time of big bang ($t = 0$) until some cosmic time $t$. From (1), we find that the propagation of light along the radial direction is described by $a(t)dr = dt$. The particle horizon, which is the ‘instantaneous’ distance at $t$ travelled by light since $t = 0$, is then

\[
d_H(t) = a(t) \int_0^t \frac{dt'}{a(t')}.
\]
The particle horizon is an important notion since it coincides with the size of the universe already seen at time \(t\) or, equivalently, with the distance at which causal contact has been established at \(t\). Equations \(8\) and \(13\) give

\[
d_H(t) = \frac{3\gamma}{3\gamma - 2} t, \quad \gamma \neq \frac{2}{3},
\]

Also,

\[
H(t) = \frac{2}{3\gamma} t^{-1}, \quad d_H(t) = \frac{2}{3\gamma - 2} H^{-1}(t).
\]

For ‘matter’ (‘radiation’), these formulae become

\[
d_H(t) = 2H^{-1}(t) = 3t \quad (d_H(t) = H^{-1}(t) = 2t).
\]

Assuming matter dominance, the present particle horizon (cosmic time) is \(d_H(t_0) = 2H_0^{-1} \approx 6,000 \text{ h}^{-1} \text{ Mpc} (t_0 = 2H_0^{-1}/3 \approx 6.5 \times 10^9 \text{ h}^{-1} \text{ years})\). The present \(\rho_c = 3H_0^2/8\pi G \approx 1.9 \times 10^{-29} \text{ h}^2 \text{ gm/cm}^3\).

### 2.5 Brief History of the Early Universe

We will now summarize the early universe evolution according to GUTs \[3\]. We take a GUT gauge group \(G (= SU(5), SO(10), SU(3)^3, ...\) with or without SUSY. At a scale \(M_X \sim 10^{16} \text{ GeV}\) (the GUT mass scale), \(G\) breaks to the standard model gauge group \(G_S\) by the vev of an appropriate Higgs field \(\phi\). (For simplicity, we take this breaking occurring in one step.) \(G_S\) is, subsequently, broken to \(SU(3)_c \times U(1)_{em}\) at the electroweak scale \(M_W\).

GUTs together with SBB provide a suitable framework for discussing the early universe for \(t > 10^{-44}\) sec. They predict that the universe, as it expands and cools, undergoes \[25\] a series of phase transitions during which the gauge symmetry is gradually reduced and important phenomena occur.

After the big bang, \(G\) was unbroken and the universe was filled with a hot ‘soup’ of massless particles which included photons, quarks, leptons, gluons, \(W^\pm, Z^0\), the GUT gauge bosons \(X, Y, \ldots\) and several Higgs bosons. (In the SUSY case, the SUSY partners were also present.) At \(t \sim 10^{-37}\) sec \((T \sim 10^{16} \text{ GeV})\), \(G\) broke down to \(G_S\) and the \(X, Y, \ldots\) and some Higgs bosons acquired masses \(\sim M_X\). Their out-of-equilibrium decay could, in principle, produce \[26,27\] the observed baryon asymmetry of the universe. Important ingredients are the violation of baryon number, which is inherent in GUTs, and C and CP violation. This is the second (potential) success of SBB.

During the GUT phase transition, topologically stable extended objects \[15\] such as monopoles \[4\], cosmic strings \[28\] or walls \[29\] can also be produced. Monopoles, which exist in most GUTs, can lead into problems \[5\] which are, however, avoided by inflation \[13,3\] (see Sects. \[3\] and \[1.3\]). This is a period of an exponentially fast expansion of the universe which can occur during some GUT phase transition. Cosmic strings can contribute \[30\] to the density perturbations needed for structure formation \[4\] in the universe, whereas walls are \[29\] catastrophic and GUTs should be constructed so that they avoid them (see e.g., \[31\]) or inflation should extinguish them.
At $t \sim 10^{-10}$ sec or $T \sim 100$ GeV, the electroweak transition takes place and $G_S$ breaks to $SU(3)_c \times U(1)_{em}$. $W^\pm, Z^0$ and the electroweak Higgs fields acquire masses $\sim M_W$. Subsequently, at $t \sim 10^{-4}$ sec or $T \sim 1$ GeV, color confinement sets in and the quarks get bounded forming hadrons.

The direct involvement of particle physics essentially ends here since most of the subsequent phenomena fall into the realm of other branches. We will, however, sketch some of them since they are crucial for understanding the earlier stages of the universe evolution where their origin lies.

At $t \approx 180$ sec ($T \approx 1$ MeV), nucleosynthesis takes place, i.e., protons and neutrons form nuclei. The abundance of light elements ($D, ^3He, ^4He$ and $^7Li$) depends crucially on the number of light particles (with mass $\lesssim 1$ MeV), i.e., the number of light neutrinos, $N_\nu$, and $\Omega_B h^2$. Agreement with observations is achieved for $N_\nu = 3$ and $\Omega_B h^2 \approx 0.020$. This is the third success of SBB cosmology. Much later, at the so-called ‘equidensity’ point, $t_{eq} \approx 5 \times 10^4$ years, matter dominates over radiation.

At $t \approx 200,000$ $h^{-1}$ years ($T \approx 3,000$ K), the ‘decoupling’ of matter and radiation and the ‘recombination’ of atoms occur. After this, radiation evolves as an independent component of the universe and is detected today as CMBR with temperature $T_0 \approx 2.73$ K. The existence of the CMBR is the fourth success of SBB. Finally, structure formation starts at $t \approx 2 \times 10^8$ years.

### 3 Shortcomings of Big Bang

The SBB model has been successful in explaining, among other things, the Hubble expansion, the existence of the CMBR and the abundance of the light elements formed during nucleosynthesis. Despite its successes, this model had a number of long-standing shortcomings which we will now summarize:

#### 3.1 Horizon Problem

The CMBR, which we receive now, was emitted at the time of ‘decoupling’ of matter and radiation when the cosmic temperature was $T_d \approx 3,000$ K. The decoupling time, $t_d$, can be calculated from

$$\frac{T_0}{T_d} = \frac{2.73}{3,000} = \frac{a(t_d)}{a(t_0)} = \left(\frac{t_d}{t_0}\right)^\frac{2}{3}. \quad (16)$$

It turns out that $t_d \approx 200,000$ $h^{-1}$ years.

The distance over which the CMBR has travelled since its emission is

$$a(t_0) \int_{t_d}^{t_0} \frac{dt'}{a(t')} = 3t_0 \left[ 1 - \left(\frac{t_d}{t_0}\right)^\frac{2}{3} \right] \approx 3t_0 \approx 6,000 \, h^{-1} \, \text{Mpc}, \quad (17)$$

which coincides with $d_H(t_0)$. A sphere of radius $d_H(t_0)$ around us is called the ‘last scattering surface’ since the CMBR has been emitted from it. The
particle horizon at $t_d$, $3t_d \approx 0.168 \, h^{-1} \, \text{Mpc}$, expanded until now to become $0.168 \, h^{-1}(a(t_0)/a(t_d)) \, \text{Mpc} \approx 184 \, h^{-1} \, \text{Mpc}$. The angle subtended by this ‘decoupling’ horizon now is $\theta_d \approx 184/6,000 \approx 0.03$ rads. Thus, the sky splits into $4\pi/(0.03)^2 \approx 14,000$ patches which never communicated before emitting the CMBR. The puzzle then is how can the temperature of the black body radiation from these patches be so finely tuned as COBE requires.

### 3.2 Flatness Problem

The present energy density of the universe has been observed \[20\] to be very close to its critical value corresponding to a flat universe ($\Omega_0 = 1 \pm 0.04$). From (7), we obtain $(\rho - \rho_c)/\rho_c = (3(8\pi G \rho_c)^{-1}(k/a^2)) \propto a$ for ‘matter’. Thus, in the early universe, $(|\rho - \rho_c|/\rho_c) \ll 1$ and the question is why the initial energy density of the universe was so finely tuned to its critical value.

### 3.3 Magnetic Monopole Problem

This problem arises only if we combine SBB with GUTs \[2\] which predict the existence of monopoles. According to GUTs, the universe underwent \[25\] a (second order) phase transition during which an appropriate Higgs field, $\phi$, developed a non-zero vev and the GUT gauge group, $G$, broke to $G_S$.

The GUT phase transition produces monopoles \[3\]. They are localized deviations from the vacuum with radius $\sim M_X^{-1}$ and mass $m_M \sim M_X/\alpha_G$ ($\alpha_G = g_G^2/4\pi$, where $g_G$ is the GUT gauge coupling constant). The value of $\phi$ on a sphere, $S^2$, of radius $\gg M_X^{-1}$ around the monopole lies on the vacuum manifold $G/G_S$ and we, thus, obtain a mapping: $S^2 \rightarrow G/G_S$. If this mapping is homotopically non-trivial, the monopole is topologically stable.

The initial ‘relative’ monopole number density must satisfy the causality bound \[33\] $r_{M,\text{in}} = (n_M/T^3)_{\text{in}} \sim 10^{-10}$, which comes from the requirement that, at monopole production, $\phi$ cannot be correlated at distances bigger than the particle horizon. The subsequent evolution of monopoles is studied in \[5\]. The result is that, if $r_{M,\text{in}} \gtrsim 10^{-9}$ ($\lesssim 10^{-9}$), the final ‘relative’ monopole number density $r_{M,\text{fin}} \sim 10^{-9}$ ($\sim r_{M,\text{in}}$). This combined with the causality bound yields $r_{M,\text{fin}} \gtrsim 10^{-10}$. However, the requirement that monopoles do not dominate the energy density of the universe at nucleosynthesis gives

$$r_M(T \approx 1 \, \text{MeV}) \lesssim 10^{-19},$$

and we obtain a clear discrepancy of about nine orders of magnitude.

### 3.4 Density Perturbations

For structure formation \[6\] in the universe, we need a primordial density perturbation, $\delta \rho/\rho$, at all length scales with a nearly flat spectrum \[34\]. We also need an explanation of the temperature fluctuations of the CMBR observed by COBE \[3\] at angles $\theta \gtrsim \theta_d \approx 2^\circ$ which violate causality (see Sect. 3.1).
4 Inflation

The above four cosmological puzzles are solved by inflation. Take a real scalar field $\phi$ (the inflaton) with (symmetric) potential energy density $V(\phi)$ which is quite flat near $\phi = 0$ and has minima at $\phi = \pm \langle \phi \rangle$ with $V(\pm \langle \phi \rangle) = 0$. At high $T$'s, $\phi = 0$ due to the temperature corrections to $V(\phi)$. As $T$ drops, the effective potential tends to the $T = 0$ potential but a small barrier separating the local minimum at $\phi = 0$ and the vacua at $\phi = \pm \langle \phi \rangle$ remains. At some point, $\phi$ tunnels out to $\phi_1 \ll \langle \phi \rangle$ and a bubble with $\phi = \phi_1$ is created. The field then rolls over to the minimum of $V(\phi)$ very slowly (due to the flatness of $V(\phi)$) with the energy density $\rho \approx V(\phi = 0) \equiv V_0$ remaining practically constant for quite some time. The Lagrangian density

$$L = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi)$$

(19)

gives the energy momentum tensor

$$T_{\mu}^{\nu} = -\partial_\mu \phi \partial^\nu \phi + \delta_\mu^{\nu} \left( \frac{1}{2} \partial_\lambda \phi \partial^\lambda \phi - V(\phi) \right),$$

(20)

which during the roll-over becomes $T_{\mu}^{\nu} \approx -V_0 \delta_\mu^{\nu}$ yielding $\rho \approx -p \approx V_0$. So, the pressure is opposite to the energy density in accord with (5). $a(t)$ grows (see below) and the ‘curvature term’, $k/a^2$, in (8) diminishes. We thus get

$$H^2 \equiv \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} V_0,$$

(21)

which gives $a(t) \propto e^{Ht}$, $H^2 = (8\pi G/3)V_0 = \text{constant}$. So the bubble expands exponentially for some time and $a(t)$ grows by a factor

$$\frac{a(t_f)}{a(t_i)} = \exp H(t_f - t_i) \equiv \exp H\tau,$$

(22)

between an initial ($t_i$) and a final ($t_f$) time.

The scenario just described is known as ‘new’ inflation. Alternatively, we can imagine, at $t_P$, a region of size $\ell_P \sim M_P^{-1}$ (the Planck length) where the inflaton is large and almost uniform carrying negligible kinetic energy. This region can inflate (exponentially expand) as $\phi$ rolls down towards the vacuum. This scenario is called ‘chaotic’ inflation.

We will now show that, with an adequate number of e-foldings, $N = H\tau$, the first three cosmological puzzles are easily resolved (we leave the question of density perturbations for later).

4.1 Resolution of the Horizon Problem

The particle horizon during inflation

$$d_{H}(t) = e^{Ht} \int_{t_i}^{t} \frac{dt'}{e^{Ht'}} \approx H^{-1} \exp H(t - t_i),$$

(23)
for $t-t_i \gg H^{-1}$, grows as fast as $a(t)$. At $t_f$, $d_H(t_f) \approx H^{-1}\exp H\tau$ and $\phi$ starts oscillating about the vacuum. It then decays and ‘reheats’ the universe at a temperature $T_r \sim 10^9$ GeV after which normal big bang cosmology is recovered. $d_H(t_f)$ is stretched during the $\phi$-oscillations by a factor $\sim 10^9$ and between $T_r$ and now by a factor $T_r/T_0$. So, it finally becomes $H^{-1}\exp H\tau 10^9(T_r/T_0)$, which must exceed $2H_0^{-1}$ if the horizon problem is to be solved. This readily holds for $V_0 \approx M_X^4$, $M_X \sim 10^{16}$ GeV and $N > 55$.

### 4.2 Resolution of the Flatness Problem

The ‘curvature term’ of the Friedmann equation, at present, is given by

$$\frac{k}{a^2} \approx \left(\frac{k}{a^2}\right)_0 \exp \left(2H\tau\right) \left(\frac{10^{-13}\text{ GeV}}{10^9\text{ GeV}}\right)^2,$$

where the terms in the rhs are the ‘curvature term’ before inflation, and its growth factors during inflation, $\phi$-oscillations and after ‘reheating’. Assuming $(k/a^2)_0 \sim H^2$, we get $\Omega_0^{-1} = \frac{k}{a^2_0} H^2_0 \sim 10^{48} \exp -2H\tau \ll 1$ for $H\tau \gg 55$. Strong inflation implies that the present universe is flat with a great accuracy.

### 4.3 Resolution of the Monopole Problem

For $N \gg 55$, the monopoles are diluted by at least 70 orders of magnitude and become irrelevant. Also, since $T_r \ll m_M$, there is no monopole production after ‘reheating’. Extinction of monopoles may also be achieved by non-inflationary mechanisms such as magnetic confinement. For models leading to a possibly measurable monopole density see e.g., [38, 39, 40].

### 5 Detailed Analysis of Inflation

The Hubble parameter during inflation depends on the value of $\phi$:

$$H^2(\phi) = \frac{8\pi G}{3} V(\phi).$$

To find the evolution equation for $\phi$ during inflation, we vary the action

$$\int \sqrt{-\text{det}(g)} \ d^4x \left(\frac{1}{2} \partial_\mu \phi \partial^\mu \phi - V(\phi) + M(\phi)\right),$$

where $g$ is the metric tensor and $M(\phi)$ the (trilinear) coupling of $\phi$ to ‘light’ matter causing its decay. Assuming that this coupling is weak, one finds

$$\ddot{\phi} + 3H\dot{\phi} + \Gamma_\phi \dot{\phi} + V'(\phi) = 0,$$

where the prime denotes derivation with respect to $\phi$ and $\Gamma_\phi$ is the decay width of the inflaton. Assume, for the moment, that the decay time of $\phi$,
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$t_d = \Gamma^{-1}_\phi$, is much greater than $H^{-1}$, the expansion time for inflation. Then the term $\Gamma_\phi \dot{\phi}$ can be ignored and (27) becomes

$$\ddot{\phi} + 3H \dot{\phi} + V' = 0 \, .$$

(28)

Inflation is by definition the situation where $\ddot{\phi}$ is subdominant to the ‘friction’ term $3H \dot{\phi}$ (and the kinetic energy density is subdominant to the potential one). Equation (28) then reduces to the inflationary equation [43]

$$3H \dot{\phi} = -V' \, ,$$

(29)

which gives

$$\ddot{\phi} = -\frac{V''(\phi)}{3H(\phi)} + \frac{V'(\phi)}{3H^2(\phi)} H'(\phi) \, .$$

(30)

Comparing the two terms in the rhs of this equation with the ‘friction term’ in (28), we get the conditions for inflation (slow roll conditions):

$$\epsilon \, , \, |\eta| \leq 1 \, , \text{with} \quad \epsilon = \frac{M_P^2}{16\pi} \left( \frac{V'(\phi)}{V(\phi)} \right)^2 \, , \, \eta = \frac{M_P^2 V''(\phi)}{8\pi V(\phi)} \, .$$

(31)

The end of the slow roll-over occurs when either of these inequalities is saturated. If $\phi_f$ is the value of $\phi$ at the end of inflation, then $t_f \sim H^{-1}(\phi_f)$.

The number of e-foldings during inflation can be calculated as follows:

$$N(\phi_i \rightarrow \phi_f) \equiv \ln \left( \frac{a(t_f)}{a(t_i)} \right) = \int_{t_i}^{t_f} H dt = \int_{\phi_i}^{\phi_f} \frac{H(\phi)}{\phi} d\phi = -\int_{\phi_i}^{\phi_f} \frac{3H^2(\phi)d\phi}{V'(\phi)} \, ,$$

(32)

where (23), (29) were used. We shift $\phi$ so that the global minimum of $V(\phi)$ is displaced at $\phi=0$. Then, if $V(\phi) = \lambda \phi^\nu$ during inflation, we have

$$N(\phi_i \rightarrow \phi_f) = -\int_{\phi_i}^{\phi_f} \frac{3H^2(\phi)d\phi}{V'(\phi)} = -8\pi G \int_{\phi_i}^{\phi_f} \frac{V(\phi)d\phi}{V'(\phi)} = \frac{4\pi G}{\nu} (\phi_f^2 - \phi_i^2) \, .$$

(33)

Assuming that $\phi_i \gg \phi_f$, this reduces to $N(\phi) \approx (4\pi G/\nu)\phi^2$.

6 Coherent Oscillations of the Inflaton

After the end of inflation at $t_f$, the term $\ddot{\phi}$ takes over in (28) and $\phi$ starts performing coherent damped oscillations about the global minimum of the potential. The rate of energy density loss, due to ‘friction’, is given by

$$\dot{\rho} = \frac{d}{dt} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) = -3H \dot{\phi}^2 = -3H(\rho + p) \, ,$$

(34)

where $\rho = \dot{\phi}^2/2 + V(\phi)$ and $p = \dot{\phi}^2/2 - V(\phi)$. Averaging $p$ over one oscillation of $\phi$ and writing $\rho + p = \gamma \rho$, we get $\rho \propto a^{-3\gamma}$ and $a(t) \propto t^{1/3\gamma}$ (see Sect. 2.2).
The number \( \gamma \) can be written as (assuming a symmetric potential)

\[
\gamma = \int_0^T \dot{\phi}^2 dt = \frac{\int_0^{\phi_{\text{max}}} \dot{\phi} d\phi}{\int_0^{\phi_{\text{max}}} (\rho/\dot{\phi}) d\phi},
\]

(35)

where \( T \) and \( \phi_{\text{max}} \) are the period and the amplitude of the oscillation. From \( \rho = \dot{\phi}^2/2 + V(\phi) = V_{\text{max}} \), where \( V_{\text{max}} \) is the maximal potential energy density, we obtain \( \dot{\phi} = \sqrt{2(V_{\text{max}} - V(\phi))} \). Substituting this in (35) we get

\[
\gamma = \frac{2 \int_0^{\phi_{\text{max}}} (1 - V/V_{\text{max}})^{\gamma/2} d\phi}{\int_0^{\phi_{\text{max}}} (1 - V/V_{\text{max}})^{-\frac{1}{2}} d\phi},
\]

(36)

For \( V(\phi) = \lambda \phi^\nu \), we find \( \gamma = 2\nu/(\nu + 2) \) and, thus, \( \rho \propto a^{-6\nu/(\nu + 2)} \) and \( a(t) \propto t^{(\nu+2)/3\nu} \). For \( \nu = 2 \), in particular, \( \gamma = 1 \), \( a(t) \propto t^{2/3} \) and \( \dot{\phi} \) behaves like pressureless matter. This is not unexpected since a coherent oscillating massive free field corresponds to a distribution of static massive particles. For \( \nu = 4 \), we obtain \( \gamma = 4/3 \), \( \rho \propto a^{-4} \), \( a(t) \propto t^{1/2} \) and the system resembles radiation. For \( \nu = 6 \), one has \( \gamma = 3/2 \), \( \rho \propto a^{-9/2} \), \( a(t) \propto t^{4/9} \) and the expansion is slower (the pressure is higher) than in radiation.

7 Decay of the Inflaton

Reintroducing the ‘decay term’ \( \Gamma \dot{\phi} \), (27) can be written as

\[
\dot{\rho} = \frac{d}{dt} \left( \frac{1}{2} \dot{\phi}^2 + V(\phi) \right) = -(3H + \Gamma \phi) \dot{\phi}^2,
\]

(37)

which is solved \([10,44]\) by

\[
\rho(t) = \rho_f \left( \frac{a(t)}{a(t_f)} \right)^{-3\gamma} \exp[-\gamma \Gamma \phi(t - t_f)],
\]

(38)

where \( \rho_f \) is the energy density at \( t_f \). The second and third factors in the rhs of this equation represent the dilution of the field energy due to the expansion of the universe and the decay of \( \phi \) to ‘light’ particles respectively.

All pre-existing radiation (known as ‘old radiation’) was diluted by inflation, so the only radiation present is the one produced by the decay of \( \phi \) and is known as ‘new radiation’. Its energy density satisfies \([10,44]\) the equation

\[
\dot{\rho}_r = -4H \rho_r + \gamma \Gamma \phi \rho_r,
\]

(39)

where the first term in the rhs represents the dilution of radiation due to the cosmological expansion while the second one is the energy density transfer from \( \phi \) to radiation. Taking \( \rho_r(t_f) = 0 \), this equation gives \([10,44]\)

\[
\rho_r(t) = \rho_f \left( \frac{a(t)}{a(t_f)} \right)^{-4} \int_{t_f}^t \left( \frac{a(t')}{a(t_f)} \right)^{4-3\gamma} e^{-\gamma \Gamma \phi(t' - t_f)} \gamma \Gamma \phi dt'.
\]

(40)
For $t_f \ll t_d$ and $\nu = 2$, this expression is approximated by

$$\rho_r(t) = \rho_f \left( \frac{t}{t_f} \right)^{\frac{8}{3}} \int_0^t \left( \frac{t'}{t_f} \right)^{\frac{2}{3}} e^{-\Gamma_\phi t'} dt', \quad (41)$$

which can be expanded as

$$\rho_r = \frac{3}{5} \rho \Gamma_\phi t \left[ 1 + \frac{3}{8} \Gamma_\phi t + \frac{9}{88} (\Gamma_\phi t)^2 + \cdots \right], \quad (42)$$

with $\rho = \rho_f (t/t_f)^{-2} \exp(-\Gamma_\phi t)$ being the energy density of the field $\phi$.

The energy density of the ‘new radiation’ grows relative to the energy density of the oscillating field and becomes essentially equal to it at a cosmic time $t_d = \Gamma_\phi^{-1}$ as one can deduce from (42). After this time, the universe enters into the radiation dominated era and the normal big bang cosmology is recovered. The temperature at $t_d$, $T_r(t_d)$, is historically called the ‘reheat’ temperature although no supercooling and subsequent reheating of the universe actually takes place. Using (11), we find that

$$T_r = \left( \frac{45}{16 \pi^2 g_*} \right) \frac{1}{(\Gamma_\phi M_P)^{\frac{\nu+2}{\nu}}}, \quad (43)$$

where $g_*$ is the effective number of degrees of freedom. For $V(\phi) = \lambda \phi^\nu$, the total expansion of the universe during the damped field oscillations is

$$\frac{a(t_d)}{a(t_f)} = \left( \frac{t_d}{t_f} \right)^{\frac{\nu+2}{\nu}}, \quad (44)$$

## 8 Density Perturbations from Inflation

Inflation not only homogenizes the universe but also generates the density perturbations needed for structure formation. To see this, we introduce the notion of event horizon at $t$. This includes all points with which we will eventually communicate sending signals at $t$. Its ‘instantaneous’ radius is

$$d_e(t) = a(t) \int_t^\infty \frac{dt'}{a(t')} \quad (45)$$

This yields an infinite event horizon for ‘matter’ or ‘radiation’. For inflation, however, we obtain $d_e(t) = H^{-1} < \infty$, which varies slowly with $t$. Points, in our event horizon at $t$, with which we can communicate sending signals at $t$, are eventually pulled away by the exponential expansion and we cease to be able to communicate with them emitting signals at later times. We say that these points (and the corresponding scales) crossed outside the event horizon. Actually, the exponentially expanding (de Sitter) space is like a black hole
our present horizon turned inside out. Then, exactly as in a black hole, there are quantum fluctuations of the ‘thermal type’ governed by the Hawking temperature $T_H = H/2\pi$. It turns out [47,48] that the quantum fluctuations of all massless fields (the inflaton is nearly massless due to the flatness of the potential) are $\delta \phi = T_H$. These fluctuations of $\phi$ lead to energy density perturbations $\delta \rho = V'(\phi)\delta \phi$. As the scale of this perturbations crosses outside the event horizon, they become classical metric perturbations.

It has been shown [49] that the quantity $\zeta \approx \delta \rho/(\rho + p)$ remains constant outside the event horizon. Thus, the density perturbation at any present physical (‘comoving’) scale $\ell$, $(\delta \rho/\rho)_\ell$, when this scale crosses inside the post-inflationary particle horizon ($p=0$ at this instance) can be related to the value of $\zeta$ when the same scale crossed outside the inflationary event horizon (at $\ell \sim H^{-1}$). This latter value of $\zeta$ is found, using (48), to be

$$\zeta |_{\ell \sim H^{-1}} = \left( \frac{\delta \rho}{\rho} \right)_{\ell \sim H^{-1}} = \left( \frac{V'(\phi)H(\phi)}{2\pi \phi^2} \right)_{\ell \sim H^{-1}} = -\left( \frac{9H^3(\phi)}{2\pi V'(\phi)} \right)_{\ell \sim H^{-1}}. \tag{46}$$

Taking into account an extra $2/5$ factor from the fact that the universe is matter dominated when the scale $\ell$ re-enters the horizon, we obtain

$$\left( \frac{\delta \rho}{\rho} \right)_\ell = \frac{16\sqrt{6}\pi}{5} \frac{V^2(\phi_\ell)}{M_P^2 V'(\phi_\ell)}. \tag{47}$$

The calculation of $\phi_\ell$, the value of $\phi$ when the ‘comoving’ scale $\ell$ crossed outside the event horizon, goes as follows. A ‘comoving’ (present physical) scale $\ell$, at $T_r$, was equal to $\ell(a(t_d)/a(t_0)) = \ell(T_0/T_r)$. Its magnitude at $t_f$ was equal to $\ell(T_0/T_r)(a(t_f)/a(t_d)) = \ell(T_0/T_r)(t_f/t_d)^{(v+2)/3v} = \ell_{\text{phys}}(t_f)$, where the potential $V(\phi) = \lambda \phi^v$ was assumed. The scale $\ell$, when it crossed outside the inflationary event horizon, was equal to $H^{-1}(\phi_\ell)$. We, thus, obtain

$$H^{-1}(\phi_\ell)e^{N(\phi_\ell)} = \ell_{\text{phys}}(t_f), \tag{48}$$

which gives $\phi_\ell$ and, thus, $N(\phi_\ell) \equiv N_\ell$, the number of e-foldings the scale $\ell$ suffered during inflation. In particular, the number of e-foldings suffered by our present horizon $\ell = 2H_0^{-1} \sim 10^3$ Mpc turns out to be $N_Q \approx 50 - 60$.

Taking $V(\phi) = \lambda \phi^4$), [53], [47] and [18] give

$$\left( \frac{\delta \rho}{\rho} \right)_\ell = \frac{4\sqrt{6}\pi}{5} \lambda^\frac{1}{2} \left( \frac{\phi_\ell}{M_P} \right)^3 = \frac{4\sqrt{6\pi}}{5} \lambda^\frac{1}{2} \left( \frac{N_\ell}{\pi} \right)^\frac{3}{2}. \tag{49}$$

From the result of COBE $3$, $(\delta \rho/\rho)_Q \approx 6 \times 10^{-5}$, one can then deduce that $\lambda \approx 6 \times 10^{-14}$ for $N_Q \approx 55$. We thus see that the inflaton must be a very weakly coupled field. In non-SUSY GUTs, the inflaton is necessarily gauge singlet since otherwise radiative corrections will make it strongly coupled. This is not so satisfactory since it forces us to introduce an otherwise unmotivated very weakly coupled gauge singlet. In SUSY GUTs, however, the
inflaton could be identified \cite{51} with a conjugate pair of gauge non-singlet fields $\phi$, $\phi$ already present in the theory and causing the gauge symmetry breaking. Absence of strong radiative corrections from gauge interactions is guaranteed by the mutual cancellation of the D-terms of these fields.

The spectrum of density perturbations can be analyzed. For $V(\phi) = \lambda \phi^\nu$, we find $(\delta \rho/\rho)_{\ell} \propto \phi_{\ell}^{(\nu+2)/2}$ which, together with $N(\phi_{\ell}) \propto \phi_{\ell}^2$ (see \cite{53}), gives

$$\left(\frac{\delta \rho}{\rho}\right)_{\ell} = \left(\frac{\delta \rho}{\rho}\right)_{Q} \left(\frac{N_{\ell}}{N_{Q}}\right)^{\frac{\nu+2}{2}}.$$  \hfill (50)

The scale $\ell$ divided by the size of our present horizon ($2H_0^{-1} \sim 10^4$ Mpc) should equal $\exp(N_{\ell} - N_{Q})$. This gives $N_{\ell}/N_{Q} = 1 + \ln(\ell/2H_0^{-1})^{1/N_{Q}}$ which expanded around $\ell = 2H_0^{-1}$ and substituted in (50) yields

$$\left(\frac{\delta \rho}{\rho}\right)_{\ell} \approx \left(\frac{\delta \rho}{\rho}\right)_{Q} \left(\frac{\ell}{2H_0^{-1}}\right)^{\alpha_{s}},$$  \hfill (51)

with $\alpha_{s} = (\nu + 2)/4N_{Q}$. For $\nu = 4$, $\alpha_{s} \approx 0.03$ and, thus, the density perturbations are essentially scale independent. The customarily used spectral index $n = 1 - 2\alpha_{s}$ is about 0.94 in this case.

\section{Temperature Fluctuations}

The density inhomogeneities produce temperature fluctuations in the CMBR. For angles $\theta > 2^{\circ}$, the dominant effect is the scalar Sachs-Wolfe \cite{52} effect. Density perturbations on the ‘last scattering surface’ cause scalar gravitational potential fluctuations, which then produce temperature fluctuations in the CMBR. The reason is that regions with a deep gravitational potential will cause the photons to lose energy as they climb up the well and, thus, these regions will appear cooler.

Analyzing the temperature fluctuations from the scalar Sachs-Wolfe effect in spherical harmonics, we obtain the corresponding quadrupole anisotropy:

$$\left(\frac{\delta T}{T}\right)_{Q-S} = \left(\frac{32\pi}{45}\right) \frac{1}{\nu M_P^3} V'_{\phi}(\phi_{\ell}).$$  \hfill (52)

For $V(\phi) = \lambda \phi^\nu$, this becomes

$$\left(\frac{\delta T}{T}\right)_{Q-S} = \left(\frac{32\pi}{45}\right) \frac{1}{\nu M_P^3} \lambda^{\frac{\nu+2}{\nu}} \phi_{\ell}^{\frac{\nu+2}{\nu}} = \left(\frac{32\pi}{45}\right) \frac{1}{\nu M_P^3} \lambda^{\frac{\nu+2}{\nu}} \left(\frac{\nu M_P^2}{4\pi}\right)^{\frac{\nu+2}{\nu}} N_{Q}^{\frac{\nu+2}{\nu}}.$$  \hfill (53)

Comparing this with the COBE \cite{3} result, $(\delta T/T)_{Q} \approx 6.6 \times 10^{-6}$, we obtain $\lambda \approx 6 \times 10^{-14}$ for $\nu = 4$ and number of e-foldings suffered by our present horizon scale during the inflationary phase $N_{Q} \approx 55$. The ‘tensor’ fluctuations \cite{53}, which generally can also exist in the temperature of the CMBR, turn out to be negligible in all cases considered here.
10 Hybrid Inflation

10.1 The non-Supersymmetric Version

The main disadvantage of inflationary scenarios such as the ‘new’ [35] or ‘chaotic’ [36] ones is that they require tiny parameters in order to reproduce the results of COBE [3]. This has led Linde [11] to propose, in the context of non-SUSY GUTs, hybrid inflation which uses two real scalar fields $\chi$ and $\sigma$ instead of one. $\chi$ provides the ‘vacuum’ energy density driving inflation, while $\sigma$ is the slowly varying field during inflation. This allows us to reproduce the COBE results with ‘natural’ (not too small) values of the parameters.

The scalar potential utilized by Linde is

$$V(\chi, \sigma) = \kappa^2 \left( M^2 - \frac{\chi^2}{4} \right)^2 + \frac{\lambda^2 \chi^2 \sigma^2}{4} + \frac{m^2 \sigma^2}{2} , \quad (54)$$

where $\kappa, \lambda > 0$ are dimensionless constants and $M, m$ mass parameters. The vacua lie at $\langle \chi \rangle = \pm 2M, \langle \sigma \rangle = 0$. For $m=0$, $V$ has a flat direction at $\chi = 0$, where $V = \kappa^2 M^4$ and the mass$^2$ of $\chi$ is $m^2 = -\kappa^2 M^2 + \lambda^2 \sigma^2/2$. So, for $\chi = 0$ and $|\sigma| > \sigma_c = \sqrt{2} \kappa M/\lambda$, we obtain a flat valley of minima. For $m \neq 0$, the valley acquires a slope and the system can inflate rolling down this valley.

The $\epsilon$ and $\eta$ criteria (see (31)) imply that inflation continues until $\sigma$ reaches $\sigma_c$, where it terminates abruptly. It is followed by a ‘waterfall’, i.e., a sudden entrance into an oscillatory phase about a global minimum. Since the system can fall into either of the two minima with equal probability, topological defects (monopoles, cosmic strings or walls) are copiously produced [14] if they are predicted by the particular GUT employed. So, if the underlying GUT gauge symmetry breaking (by $\langle \chi \rangle$) leads to the existence of monopoles or walls, we encounter a catastrophe.

The onset of hybrid inflation requires [54] that, at $t \sim H^{-1}$, $H$ being the inflationary Hubble parameter, a region exists with size $\gtrsim H^{-1}$, where $\chi$ and $\sigma$ are almost uniform with negligible kinetic energies and values close to the bottom of the valley of minima. Such a region, at $t_P$, would have been much larger than the Planck length $\ell_P$ and it is, thus, difficult to imagine how it could be so homogeneous. Moreover, as it has been argued [55], the initial values (at $t_P$) of the fields in this region must be strongly restricted in order to obtain adequate inflation. Several possible solutions to this problem of initial conditions for hybrid inflation have been proposed (see e.g., [56,57,58]).

The quadrupole anisotropy of the CMBR produced during hybrid inflation can be estimated, using (52), to be

$$\left( \frac{\delta T}{T} \right)_Q \approx \frac{16\pi}{45} \left( \frac{\lambda \kappa^2 M^5}{M_P^2 m^2} \right)^{1/2} . \quad (55)$$

The COBE [3] result, $(\delta T/T)_Q \approx 6.6 \times 10^{-6}$, can then be reproduced with $M \approx 2.86 \times 10^{16}$ GeV, the SUSY GUT vev, and $m \approx 1.3 \kappa \sqrt{A} \times 10^{15}$ GeV. Note that $m \sim 10^{12}$ GeV for $\kappa, \lambda \sim 10^{-2}$. 


10.2 The Supersymmetric Version

Hybrid inflation is \[12\] ‘tailor made’ for globally SUSY GUTs except that an intermediate scale mass for \(\sigma\) cannot be obtained. Actually, all scalars acquire masses \(\sim m_{3/2} \sim 1\) TeV (the gravitino mass) from soft SUSY breaking.

Let us consider the renormalizable superpotential

\[
W = \kappa S(-M^2 + \phi \phi), \tag{56}
\]

where \(\tilde{\phi}, \phi\) is a pair of \(G_S\) singlet left handed superfields belonging to conjugate representations of \(G\) and reducing its rank by their vevs, and \(S\) is a gauge singlet left handed superfield. \(\kappa\) and \(M \sim 10^{16}\) GeV are made positive by field redefinitions. The vanishing of the F-term \(F_S\) gives \(\langle \phi \rangle \langle \phi \rangle = M^2\), and the D-terms vanish for \(\langle |\phi|\rangle = \langle |\phi|\rangle\). So, the SUSY vacua lie at \(\langle \phi \rangle = \pm M\) and \(\langle S\rangle = 0\) (from \(F_{\phi} = F_{\phi} = 0\)). Thus, \(W\) leads to the breaking of \(G\).

\(W\) also gives rise to hybrid inflation. The potential derived from \(W\) is

\[
V(\phi, \phi, S) = \kappa^2 |M^2 - \tilde{\phi}\phi|^2 + \kappa^2 |S|^2 (|\phi|^2 + |\phi|^2) + D - \text{terms}. \tag{57}
\]

D-flatness implies \(\tilde{\phi}^* = e^{i\theta} \phi\). We take \(\theta = 0\), so that the SUSY vacua are contained. \(W\) has a \(U(1)_R\) R-symmetry: \(\tilde{\phi}\phi \rightarrow \tilde{\phi}\phi, S \rightarrow e^{i\alpha} S, W \rightarrow e^{i\alpha} W\). Actually, \(W\) is the most general renormalizable superpotential allowed by \(G\) and \(U(1)_R\). Bringing \(\phi, \phi, S\) on the real axis by \(G\) and \(U(1)_R\) transformations, we write \(\phi = \phi \equiv \chi/2, S \equiv \sigma/\sqrt{2}\) where \(\chi, \sigma\) are normalized real scalar fields. \(V\) then takes the form in (54) with \(\kappa = \lambda\) and \(m = 0\). So, Linde’s potential is almost obtainable from SUSY GUTs but without the mass term of \(\sigma\).

SUSY breaking by the ‘vacuum’ energy density \(\kappa^2 M^4\) on the inflationary valley \(\langle \phi = 0, |S| > S_c \equiv M\) causes a mass splitting in the supermultiplets \(\tilde{\phi}, \phi\). We obtain a Dirac fermion with mass squared \(\kappa^2 |S|^2\) and two complex scalars with mass squared \(\kappa^2 |S|^2 \pm \kappa^2 M^2\). This leads \[13\] to one-loop corrections to \(V\) on the valley via the Coleman-Weinberg formula \[14\]:

\[
\Delta V = \frac{1}{64\pi^2} \sum_i (-)^{F_i} M_i^4 \ln \frac{M_i^2}{\Lambda^2}, \tag{58}
\]

where the sum extends over all helicity states \(i\), with fermion number \(F_i\) and mass squared \(M_i^2\), and \(\Lambda\) is a renormalization scale. We find that \(\Delta V(|S|)\) is

\[
\kappa^2 M^4 \frac{\kappa^2 N}{32\pi^2} \left(2 \ln \frac{\kappa^2 |S|^2}{\Lambda^4} + (z + 1)^2 \ln(1 + z^{-1}) + (z - 1)^2 \ln(1 - z^{-1})\right), \tag{59}
\]

where \(z = x^2 = |S|^2/M^2\) and \(N\) is the dimensionality of the representations to which \(\tilde{\phi}, \phi\) belong. These radiative corrections generate the necessary slope on the inflationary valley. Note that the slope is \(\Lambda\)-independent.

From (58), (59) and (59), we find the quadrupole anisotropy of the CMBR:

\[
\left(\frac{\delta T}{T}\right)_Q \approx \frac{8\pi}{\sqrt{N}} \left(\frac{N_Q}{45}\right)^{1/2} \left(\frac{M}{M_P}\right)^2 x_Q^{-1} y_Q^{-1} \Lambda(x_Q^2)^{-1}, \tag{60}
\]
with
\[ A(z) = (z + 1) \ln(1 + z^{-1}) + (z - 1) \ln(1 - z^{-1}) , \]  
(61)

\[ y_Q^2 = \int_1^{x_Q^0} \frac{dz}{z} A(z)^{-1}, \quad y_Q \geq 0 . \]  
(62)

Here, \( x_Q = |S_Q|/M \), with \( S_Q \) being the value of \( S \) when our present horizon crossed outside the inflationary horizon. Finally, from (59), one finds

\[ \kappa \approx \frac{8 \pi^2}{\sqrt{N N_Q}} y_Q \frac{M}{M_P} . \]  
(63)

The slow roll conditions for SUSY hybrid inflation are \( \epsilon, |\eta| \leq 1 \), where

\[ \epsilon = \left( \frac{\kappa^2 M_P}{16 \pi^2 M} \right)^2 \frac{N^2 x^2 - A(x^2)^2}{8 \pi}, \]  
(64)

\[ \eta = \left( \frac{\kappa M_P}{4 \pi M} \right)^2 \frac{N}{8 \pi} \left( (3z + 1) \ln(1 + z^{-1}) + (3z - 1) \ln(1 - z^{-1}) \right). \]  
(65)

These conditions are violated only \('\text{infinitesimally}' close\) to the critical point \( (x = 1) \). So, inflation continues until this point, where the ‘waterfall’ occurs.

Using COBE \( [3] \) and eliminating \( x_Q \) between (60) and (63), we obtain \( M \) as a function of \( \kappa \). The maximal \( M \) which can be achieved is \( \approx 10^{16} \text{ GeV} \) (for \( N = 8, N_Q \approx 55 \)) and, although somewhat smaller than the SUSY GUT vev, is quite close to it. As an example, take \( \kappa = 4 \times 10^{-3} \) which gives \( M \approx 9.57 \times 10^{15} \text{ GeV} \), \( x_Q \approx 2.633 \), \( y_Q \approx 2.42 \). The slow roll conditions are violated at \( x - 1 \approx 7.23 \times 10^{-5} \), where \( \eta = -1 \) (\( \epsilon \approx 8.17 \times 10^{-8} \) at \( x = 1 \)). The spectral index \( n = 1 - 6 \epsilon + 2 \eta [60] \) is about 0.985.

SUSY hybrid inflation is considered ‘natural’ for the following reasons:

i. There is no need of tiny coupling constants (\( \kappa \approx 10^{-3} \)).

ii. \( W \) in \( [50] \) has the most general renormalizable form allowed by \( G \) and \( U(1)_R \). The coexistence of the \( S \) and \( S\bar{\phi}\phi \) terms implies that \( \bar{\phi}\phi \) is ‘neutral’ under all symmetries and, thus, all the non-renormalizable terms of the form \( S(\bar{\phi}\phi)^n, n \geq 2 \), are also allowed \( [10] \). The leading term of this type \( S(\bar{\phi}\phi)^2 \), if its dimensionless coefficient is of order unity, can be comparable to \( S\bar{\phi}\phi \) (recall that \( \kappa \approx 10^{-3} \)) and, thus, play a role in inflation (see Sect. \( [1] \)). \( U(1)_R \) guarantees the linearity of \( W \) in \( S \) to all orders excluding terms such as \( S^2 \) which could generate an inflaton mass \( \gtrsim H \) and ruin inflation by violating the slow roll conditions.

iii. SUSY guarantees that the radiative corrections do not ruin \( [31] \) inflation, but rather provide \( [13] \) the necessary slope on the inflationary path.

iv. Supergravity corrections can be brought under control leaving inflation intact \( [53,58,61] \).

In summary, for all these reasons, we consider SUSY hybrid inflation (with its extensions) as an extremely ‘natural’ inflationary scenario.
11 Extensions of Supersymmetric Hybrid Inflation

Applying (SUSY) hybrid inflation to higher GUT gauge groups predicting monopoles, we encounter the following problem. Inflation is terminated abruptly as the system reaches the critical point and is followed by the ‘waterfall’ regime during which the scalar fields $\phi, \bar{\phi}$ develop their vevs starting from zero and the spontaneous breaking of the GUT gauge symmetry occurs. The fields $\phi, \bar{\phi}$ can end up at any point of the vacuum manifold with equal probability and, thus, monopoles are copiously produced through the Kibble mechanism leading to a disaster.

One of the simplest GUTs predicting monopoles is the Pati-Salam (PS) model with gauge group $G_{PS} = SU(4)_c \times SU(2)_L \times SU(2)_R$. These monopoles carry two units of ‘Dirac’ magnetic charge. We will present solutions of the monopole problem of hybrid inflation within the SUSY PS model, although our mechanisms can be extended to other gauge groups such as the ‘trinification’ group $SU(3)_c \times SU(3)_L \times SU(3)_R$, which predicts monopoles with triple ‘Dirac’ charge.

11.1 Shifted Hybrid Inflation

One idea for solving the monopole problem is to include into the standard superpotential for hybrid inflation (in (56)) the leading non-renormalizable term, which, as explained, cannot be excluded. If its dimensionless coefficient is of order unity, this term competes with the trilinear term of the standard superpotential (with coefficient $\sim 10^{-3}$). A totally new picture then emerges. There appears a non-trivial flat direction along which $G_{PS}$ is broken with the appropriate Higgs fields acquiring constant values. This ‘shifted’ flat direction acquires a slope again from radiative corrections and can be used for inflation. The end of inflation is again abrupt followed by a ‘waterfall’ but no monopoles are formed since $G_{PS}$ is already broken during inflation.

The spontaneous breaking of the gauge group $G_{PS}$ to $G_S$ is achieved via the vevs of a conjugate pair of Higgs superfields

$$H^c = (4, 1, 2) \equiv \begin{pmatrix} u^c_H & \bar{u}^c_H & \bar{d}^c_H & \bar{\nu}^c_H \\ d_H & \bar{d}_H & \bar{d}_H & \bar{e}_H \end{pmatrix},$$

$$\bar{H}^c = (\bar{4}, 1, 2) \equiv \begin{pmatrix} u_H & \bar{u}_H & \bar{d}_H & \bar{\nu}_H \\ d^c_H & \bar{d}^c_H & \bar{d}^c_H & \bar{e}^c_H \end{pmatrix},$$

in the $\bar{\nu}_H, \nu_H$ directions. The relevant part of the superpotential, which includes the leading non-renormalizable term, is

$$\delta W = \kappa S(-M^2 + \bar{H}^c H^c) - \beta S(\bar{H}^c H^c)^2 / M_S^2,$$

where $M_S \approx 5 \times 10^{17}$ GeV is the string scale and $\beta$ is taken positive for simplicity. D-flatness implies that $\bar{H}^c = e^{i\theta} H^c$. We restrict ourselves to the
direction with \( \theta = 0 \) (\( \hat{H}^c = H^c \)) containing the ‘shifted’ inflationary path (see below). The scalar potential derived from \( \delta W \) then takes the form

\[
V = \left[ \kappa (|H|^2 - M^2) - \frac{\beta |H|^4}{M_S^2} \right]^2 + 2\kappa^2 |S|^2 |H|^2 \left[ 1 - \frac{2\beta}{\kappa M_S^2} |H|^2 \right]^2. \tag{68}
\]

Defining the dimensionless variables \( w = |S|/M, y = |H^c|/M \), we obtain

\[
\tilde{V} = \frac{V}{\kappa^2 M^4} = (y^2 - 1 - \xi y^4)^2 + 2w^2 y^2 (1 - 2\xi y^2)^2, \tag{69}
\]

where \( \xi = \beta M^2/\kappa M_S^2 \). This potential is a simple extension of the standard potential for SUSY hybrid inflation (which corresponds to \( \xi = 0 \)).

For constant \( w \) (or \( |S| \)), \( \tilde{V} \) in (69) has extrema at

\[
y_1 = 0, \quad y_2 = \frac{1}{\sqrt{\xi}} \quad \text{and} \quad y_{3\pm} = \frac{1}{\sqrt{2\xi}} \sqrt{(1 - 6\xi w^2) \pm \sqrt{(1 - 6\xi w^2)^2 - 4(1 - w^2)}}. \tag{70}
\]

The first two extrema (at \( y_1, y_2 \)) are \( |S| \)-independent and, thus, correspond to flat directions, the trivial one at \( y_1 = 0 \) with \( \tilde{V}_1 = 1 \), and the ‘shifted’ one at \( y_2 = 1/\sqrt{\xi} \) = constant with \( \tilde{V}_2 = (1/4\xi - 1/2) \), which we will use as inflationary path. The trivial trajectory is a valley of minima for \( w > 1 \), while the ‘shifted’ one for \( w > w_0 = (1/8\xi - 1/2)^{1/2} \), which is its critical point. We take \( \xi < 1/4 \), so that \( w_0 > 0 \) and the ‘shifted’ path is destabilized before \( w \) reaches zero. The extrema at \( y_{3\pm} \), which are \( |S| \)-dependent and non-flat, do not exist for all values of \( w \) and \( \xi \), since the expressions under the square roots in (64) are not always non-negative. These two extrema, at \( w = 0 \), become SUSY vacua. The relevant SUSY vacuum (see below) corresponds to \( y_{3-} (w = 0) \) and, thus, the common vev \( v_0 \) of \( H^c, H^e \) is

\[
\left( \frac{v_0}{M} \right)^2 = \frac{1}{2\xi} (1 - \sqrt{1 - 4\xi}). \tag{71}
\]

We will now discuss the structure of \( \tilde{V} \) and the inflationary history for \( 1/6 < \xi < 1/4 \). For fixed \( w > 1 \), there exist two local minima at \( y_1 = 0 \) and \( y_2 = 1/\sqrt{\xi} \), which has lower potential energy density, and a local maximum at \( y_{3+} \) between the minima. As \( w \) becomes smaller than unity, the extremum at \( y_1 \) turns into a local maximum, while the extremum at \( y_{3+} \) disappears. The system then falls into the ‘shifted’ path in case it had started at \( y_1 = 0 \). As we further decrease \( w \) below \( (2 - \sqrt{30\xi - 5})^{1/2}/3\sqrt{\xi} \), a pair of new extrema, a local minimum at \( y_{3-} \) and a local maximum at \( y_{3+} \), are created between \( y_1 \) and \( y_2 \). As \( w \) crosses \( (1/8\xi - 1/2)^{1/2} \), the local maximum at \( y_{3+} \) crosses \( y_2 \) becoming a local minimum. At the same time, the local minimum at \( y_2 \) turns into a local maximum and inflation ends with the system falling into the local minimum at \( y_{3-} \) which, at \( w = 0 \), becomes the SUSY vacuum.

We see that, no matter where the system starts from, it passes from the ‘shifted’ path, where the relevant part of inflation takes place. So, \( G_{PS} \) is broken during inflation and no monopoles are produced at the ‘waterfall’.
After inflation, the system could fall into the minimum at $y_{3+}$ instead of the one at $y_{3-}$. This, however, does not happen since in the last e-folding or so the barrier between the minima at $y_{3-}$ and $y_2$ is considerably reduced and the decay of the ‘false vacuum’ at $y_2$ to the minimum at $y_{3-}$ is completed within a fraction of an e-folding before the $y_{3+}$ minimum even appears.

The only mass splitting within supermultiplets on the ‘shifted’ path appears between one Majorana fermion in the direction $(\bar{\nu}_H + \nu_H)/\sqrt{2}$ with $m^2 = 4\kappa^2 |S|^2$ and two real scalars $\text{Re}(\delta \bar{\nu}_H + \delta \nu_H)$ and $\text{Im}(\delta \bar{\nu}_H + \delta \nu_H)$ with $m^2 = 4\kappa^2 |S|^2 + 2\kappa^2 m^2$. Here, $m = M(1/4\kappa - 1)^{1/2}$ and $\delta \bar{\nu}_H = \bar{\nu}_H - v$, $\delta \nu_H = \nu_H - v$ where $v = (\kappa M_S^2/2\beta)^{1/2}$ is the value of $H^c$, $H^c$ on the path.

The radiative corrections on the ‘shifted’ path can be constructed and $(\tilde{T}/T)_Q$ and $\kappa$ can be evaluated. We find the same formulas as in (60) and (63) with $N = 2$ and $N = 4$ respectively and $M$ generally replaced by $m$.

COBE can be reproduced, for instance, with $\kappa \approx 4 \times 10^{-3}$, corresponding to $\xi = 1/5$, $v_0 \approx 1.7 \times 10^{16}$ GeV ($N_Q \approx 55$, $\beta = 1$). The scales $M \approx 1.45 \times 10^{16}$ GeV, $m \approx 7.23 \times 10^{15}$ GeV, the inflaton mass $m_{\text{infl}} \approx 4.1 \times 10^{13}$ GeV and the ‘inflationary scale’, which characterizes the inflationary ‘vacuum’ energy density, $v_{\text{infl}} = \kappa^{1/2} m \approx 4.57 \times 10^{14}$ GeV. The spectral index $n \approx 0.954$.

### 11.2 Smooth Hybrid Inflation

An alternative solution to the monopole problem of hybrid inflation has been proposed in [14]. We will present it here within the SUSY PS model of Sect. 11.1, although it can be applied to other semi-simple gauge groups too. The idea is to impose an extra $Z_2$ symmetry under which $H^c \rightarrow -H^c$. The whole structure of the model remains unchanged except that now only even powers of the combination $H^c H^c$ are allowed in the superpotential terms.

The inflationary superpotential in (67) becomes

$$\delta W = S \left( -\mu^2 + \frac{(H^c H^c)^2}{M_S^4} \right),$$

where we absorbed the dimensionless parameters $\kappa$, $\beta$ in $\mu$, $M_S$. The resulting scalar potential $V$ is then given by

$$\tilde{V} = \frac{V}{\mu^4} = (1 - \tilde{\chi}^4)^2 + 16\tilde{\sigma}^2 \tilde{\chi}^6,$$

where we used the dimensionless fields $\tilde{\chi} = \chi/2(\mu M_S)^{1/2}$, $\tilde{\sigma} = \sigma/2(\mu M_S)^{1/2}$ with $\chi$, $\sigma$ being normalized real scalar fields defined by $\bar{\nu}_H = \nu_H = \chi/2$, $S = \sigma/\sqrt{2}$ after rotating $\nu_H$, $\nu_H$, $S$ to the real axis.

The emerging picture is completely different. The flat direction at $\tilde{\chi} = 0$ is now a local maximum with respect to $\tilde{\chi}$ for all values of $\tilde{\sigma}$, and two new symmetric valleys of minima appear at

$$\tilde{\chi} = \pm \sqrt{\tilde{\sigma}} \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right)^{1/2} - 1 \right)^{1/2}. \quad (74)$$
They contain the SUSY vacua lying at $\tilde{\chi} = \pm 1, \tilde{\sigma} = 0$ and possess a slope already at the classical level. So, in this case, there is no need of radiative corrections for driving the inflaton. The potential on these paths is

$$\tilde{V} = 48\tilde{\sigma}^4 \left[ 72\tilde{\sigma}^4 \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right) \left( \left( 1 + \frac{1}{36\tilde{\sigma}^4} \right) \frac{1}{2} - 1 \right) - 1 \right].$$

The system follows a particular inflationary path and ends up at a particular point of the vacuum manifold leading to no production of monopoles.

The end of inflation is not abrupt since the inflationary path is stable with respect to $\tilde{\chi}$ for all $\tilde{\sigma}$'s. It is determined by using the $\epsilon$ and $\eta$ criteria.

This model allows us to take the vev $v_0 = (\mu M_S)^{1/2}$ of $\tilde{H}^c$, $H^c$ equal to the SUSY GUT vev. COBE then yields $M_S \approx 4.39 \times 10^{17}$ GeV and $\mu \approx 1.86 \times 10^{15}$ GeV for $N_Q \approx 57$. Inflation ends at $\sigma = \sigma_0 \approx 1.34 \times 10^{17}$ GeV, while our present horizon crosses outside the inflationary horizon at $\sigma = \sigma_Q \approx 2.71 \times 10^{17}$ GeV. Finally, $m_{\text{infl}} = 2\sqrt{2}\mu^2/v_0 \approx 3.42 \times 10^{14}$ GeV.

## 12 Conclusions

We summarized the shortcomings of SBB and their resolution by inflation, which suggests that the universe underwent a period of exponential expansion. This may have happened during the GUT phase transition at which the relevant Higgs field was displaced from the vacuum. This field (inflaton) could then, for some time, roll slowly towards the vacuum providing an almost constant ‘vacuum’ energy density. Inflation generates the density perturbations needed for the large scale structure of the universe and the temperature fluctuations of the CMBR. After the end of inflation, the inflaton performs damped oscillations about the vacuum, decays and ‘reheats’ the universe.

The early inflationary models required tiny parameters. This problem was solved by hybrid inflation which uses two real scalar fields. One of them provides the ‘vacuum’ energy density for inflation while the other one is the slowly rolling field. Hybrid inflation arises ‘naturally’ in many SUSY GUTs, but leads to a disastrous overproduction of monopoles. We constructed two extensions of SUSY hybrid inflation which do not suffer from this problem.

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