The Nucleon Distribution Amplitudes and their application to nucleon form factors and the \( N \rightarrow \Delta \) transition at intermediate values of \( Q^2 \)

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We compare a recent lattice determination of the nucleon distribution amplitudes with other approaches and models. We study the nucleon distribution amplitudes up to twist 6 in next-to leading conformal spin and we also investigate conformal \( d \)-wave contributions to the leading twist distribution amplitude. With the help of light-cone sum rules one can relate the distribution amplitudes to the form factors of the nucleon or the \( N \rightarrow \Delta \) transition at intermediate values of the momentum transfer. We compare our results with experimental data in the range \( 1 \, \text{GeV}^2 \leq Q^2 \leq 10 \, \text{GeV}^2 \). Keeping in mind that we are working only in LO QCD and NLO-QCD corrections might be sizeable we already obtain a surprisingly good agreement for the nucleon form factors \( G_M^N, G_M^\Delta, G_A^N \) and \( G_p^R \) and for the \( N \rightarrow \Delta \) transition form factor ratios \( R_{EM} \) and \( R_{SM} \).

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I. INTRODUCTION

The nucleon distribution amplitudes represent the universal non-perturbative input to numerous exclusive reactions, see, e.g., \cite{1} for an early review. Taking corrections up to twist-6 \cite{2} into account we compare different non-perturbative methods to determine the nucleon distribution amplitudes, in particular lattice simulations \cite{3,4,5}. QCD sum rule estimates \cite{2,6} and a phenomenological model \cite{6}. For asymptotically large values of the momentum transfer \( Q^2 \) the form factors can be expressed as a convolution of two leading-twist distribution amplitudes with a hard-perturbatively calculable - scattering kernel \cite{7,8,9,10,11,12,13,14,15}. This approach (pQCD) is formally proven in the \( Q^2 \rightarrow \infty \) limit, and currently there is the consensus that pQCD is not valid at experimentally accessible values of the momentum transfer. In \cite{16} light-cone sum rules \cite{17,18} were worked out which relate the nucleon distribution amplitudes to the experimentally accessible form factors of the nucleon at intermediate momentum transfer. Form factors are interesting quantities per se, since they encode information about the structure of the investigated baryon. This interest rised a lot in recent years, in particular because new data from JLAB \cite{19,20,21,22} for the well-known electromagnetic form factors of the nucleon contradict common textbook-wisdom. See, e.g., \cite{23} for a review and further references. To our knowledge light-cone sum rules are the only theoretical approach to determine form factors at intermediate momentum transfer that incorporate consistently the purely perturbative approach (pQCD). This was explicitly shown in the case of the pion form factor \cite{24}. If one calculates the light-cone sum rules for the pion form factor to leading order and next-to-leading order in QCD one can show that the \( \alpha_s \)-corrections include the pQCD result in the \( Q^2 \rightarrow \infty \) limit. In the case of baryon form factors the pQCD result is expected to be included in the \( O(\alpha_s^2) \) corrections to the light-cone sum rule calculation. Currently only leading-order sum rules for the baryon form factors are known and a part of the NLO QCD corrections to the nucleon form factors. The paper is organized as follows. In section 2 we introduce the concept of distribution amplitudes, in section 3 we collect QCD sum rule predictions for the nucleon distribution amplitudes and in section 4 we shortly explain the lattice determination of the moments of the nucleon distribution amplitudes. All these approaches, including the numerical results, are discussed in section 5. The light-cone sum rule formalism is introduced in section 6 where we also give a short overview over the current literature on light-cone sum rules for baryonic form factors. In the next three sections we compare light-cone sum rule predictions with different models of the nucleon distribution amplitude for the form factors of the nucleon and for the \( N \rightarrow \Delta \) transition. In section 7 we use the nucleon distribution amplitudes including next-to-leading conformal spin contributions to determine the form factors, in section 8 we make use of some relations between twist-4 and twist-3 parameters and in section 9 we investigate the effect of the \( d \)-wave contributions to the leading twist distribution amplitude. We conclude and summarize our results in section 11. In the appendix we give for the first time the full expression for all nucleon distribution amplitudes up to twist-6 including also the \( d \)-wave contribution for the leading twist distribution amplitude.

II. THE NUCLEON DISTRIBUTION AMPLITUDES

The distribution amplitudes comprise the infrared behaviour in exclusive processes involving large momentum transfer. They remove the infrared divergences in the perturbative diagrams encoding the nonperturbative content,
that the number of independent distribution amplitudes is reduced compared to the general case. In particular, for the nucleon distribution amplitudes isospin symmetry and the presence of two quarks of the same type implies further details on distributions amplitudes (with complete expressions and definitions up to twist 6) are summarized in [2].

The leading-twist contribution to the nucleon distribution amplitudes has been determined long time ago including formulas we omit the gauge factors in order to simplify the notation.

Thus momentum conservation implies for the moments of the distribution amplitudes the relation

$$F_{i}^{n_{1}n_{2}n_{3}} = F_{i}^{n_{1}+1n_{2}n_{3}} + F_{i}^{n_{1}n_{2}+1n_{3}} + F_{i}^{n_{1}n_{2}(n_{3}+1)}.$$  

Further details on distributions amplitudes (with complete expressions and definitions up to twist 6) are summarized in the appendices.

For the nucleon distribution amplitudes isospin symmetry and the presence of two quarks of the same type implies that the number of independent distribution amplitudes is reduced compared to the general case. In particular,
the leading-twist nucleon distribution amplitudes can be expressed in terms of only one independent distribution amplitude which is usually taken as

\[
\varphi(x_1, x_2, x_3) = V_1(x_1, x_2, x_3) - A_1(x_1, x_2, x_3)
\]

and is equal to \(\Phi_3(x_1, x_2, x_3)\) in the notation of Ref. 2. The distribution amplitudes \(A_1\) and \(V_1\) are defined in the appendices. At leading twist the nucleon distribution amplitude \(\varphi(x_1)\) corresponds to the following form of the proton state \(29, 30\):

\[
| P, \uparrow \rangle = \int_0^1 \frac{dx}{\sqrt{2x(1-x)}} u^i(x_1) \left[ u^j(x_2) d^j(x_3) - d^j(x_2) u^i(x_3) \right].
\]

The first moments of \(\varphi(x_i)\) can be interpreted as the momentum fractions carried by the quarks. The leading-twist distribution amplitude depends at leading conformal spin on one non-perturbative parameter, the normalization constant \(f_N\), while for twist four we have two additional constants \(\lambda_1\) and \(\lambda_2\). In our approach no new parameters appear in leading conformal spin up to twist six. At next-to-leading conformal spin only two non-perturbative parameters \(V_i^d = \varphi_{001}^d\) and \(A_i^d = \varphi_{010}^d\) arise in the case of leading twist and at next-to-leading twist we have three non-perturbative parameters, \(f_i^d\), \(f_i^s\) and \(f_i^d\), for details see Ref. 2, 3. For the leading-twist distribution amplitude \(\varphi(x_i)\) we have also determined the next-to-next-to-leading conformal spin contributions which can be completely parametrized, e.g., by the moments \(\varphi_{101}^d, \varphi_{200}^d\) and \(\varphi_{002}^d\). The local matrix elements defining the non-perturbative parameters up to next-to-leading conformal spin are (see Ref. 3 for the corrected formulas from Ref. 2)

\[
\begin{align*}
(0)\varepsilon^{ijk} \left[ u^i C_\mu u^j \right] (0) [\gamma_5 \gamma^\mu d^k]_\delta(0) |P\rangle = f_N(P \cdot z) f_N(P), \\
(0)\varepsilon^{ijk} \left[ u^i C_\gamma u^j \right] (0) [\gamma_5 \gamma^\mu d^k]_\delta(0) |P\rangle = \lambda_1 m_N N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ u^i C_\sigma u^j \right] (0) [\gamma_5 \sigma^{\mu\nu} d^k]_\delta(0) |P\rangle = \lambda_2 m_N N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ u^i C_\gamma_{iz} D d^k \right] (0) [\gamma_5 k^z]_\delta(0) |P\rangle = f_N V_i^d (P \cdot z)^2 \gamma^z N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ u^i C_\gamma_{iz} D d^k \right] (0) [\gamma_5 k^z]_\delta(0) |P\rangle = -f_N A_i^d (P \cdot z)^2 \gamma^z N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ u^i C_{\gamma_{iz}} \gamma_5 D d^k \right] (0) [\gamma_5 k^z]_\delta(0) |P\rangle = \lambda_1 f_i^d (P \cdot z) M \gamma^z N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ u^i C_{\gamma_{iz}} \gamma_5 D d^k \right] (0) [\gamma_5 k^z]_\delta(0) |P\rangle = \lambda_2 f_i^d (P \cdot z) M \gamma^z N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ u^i C_{\gamma_{iz}} \gamma_5 D d^k \right] (0) [\gamma_5 k^z]_\delta(0) |P\rangle = \lambda_1 f_i^d (P \cdot z) M \gamma^z N_\delta(P),
\end{align*}
\]

with the nucleon spinor \(N_\delta(P)\), the nucleon mass \(m_N\), an arbitrary light-like vector \(z^\nu\) with \(z^2 = 0\) and \(\vec{D} = D - \vec{D}\). All derivatives act only on the quark fields and not on any explicit factor \(z\). The second moments of the nucleon distribution amplitudes are related to the following local operators:

\[
\begin{align*}
(0)\varepsilon^{ijk} \left[ u^i C_\mu u^j \right] (0) [\gamma_5 \gamma^\mu d^k]_\delta(0) |P\rangle = f_N \varphi_{002}^d(P \cdot z)^3 \gamma^z N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ (i\gamma_5 D^z u^j) C_\mu u^i \right] (0) [\gamma_5 \gamma^\mu d^k]_\delta(0) |P\rangle = f_N \varphi_{002}^d(P \cdot z)^3 \gamma^z N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ (i\gamma_5 D^z u^j) C_\mu u^i \right] (0) [\gamma_5 \gamma^\mu d^k]_\delta(0) |P\rangle = f_N \varphi_{002}^d(P \cdot z)^3 \gamma^z N_\delta(P), \\
(0)\varepsilon^{ijk} \left[ (i\gamma_5 D^z u^j) C_\mu u^i \right] (0) [\gamma_5 \gamma^\mu d^k]_\delta(0) |P\rangle = f_N \varphi_{002}^d(P \cdot z)^3 \gamma^z N_\delta(P),
\end{align*}
\]

The parameters used in this work with their twist and conformal spin are summarised in the table below

| Leading conformal spin | Next-to-leading conformal spin | Next-to-next-to-leading conformal spin |
|------------------------|-------------------------------|--------------------------------------|
| \(f_N\)                | \(V_i^d, A_i^d\)              | \(\varphi_{101}^d, \varphi_{200}^d, \varphi_{002}^d\) |

As in the meson case these parameters can be estimated with QCD sum rules (see, e.g., Refs. 32–34 for some recent work in the meson case) or with lattice simulations (see, e.g., Refs. 33, 36 for lattice works considering the same mesons).
The leading-twist distribution amplitude was investigated with QCD sum rules up to the second moments in [29, 37] and up to the third moments in [30] including perturbative contributions and terms proportional to the gluon condensate and to the four-quark condensate (several errors in [29] were corrected in [30]). The next-to-leading twist normalization constants \( \lambda_1 \) and \( \lambda_2 \) describe the coupling to the proton of two independent proton interpolating fields used in QCD sum rules, \( \lambda_1 \) is the coupling of the so-called Ioffe current [38], while \( \lambda_2 \) is the coupling of the interpolating nucleon field that was advocated in [39]. In [38] and [39] first QCD sum rule estimates for \( \lambda_{1,2} \) were presented. Higher dimensional condensates were included in [40]. Unfortunately these pioneering works contain several misprints, for a review with the correct expressions see, e.g., [41, 42]. \( \alpha_s \)-corrections were calculated by Jamin in [43]. They turned out to be very large (\( \approx +50\% \) for \( |\lambda_2| \)), corresponding to \( \approx +25\% \) for \( |\lambda_1| \)), but we will not take them into account, since we also do not have \( \alpha_s \)-corrections for the light-cone sum rules, connecting the distribution amplitudes with the nucleon form factors. In [4] also contributions of non-planar diagrams to the dimension 8 condensates were included. Putting all this together (for the first time) the QCD sum rule expression for \( \lambda_1 \) reads

\[
2(2\pi)^4 m_N^2 |\lambda_1| = e^{\frac{m_0^2}{2M_B^2}} \left\{ M_B^6 E_3 \left( \frac{s_0}{M_B^2} \right) L^{-\frac{3}{2}} \left( 1 + \frac{53}{12} + \frac{\gamma_E}{\alpha_s(M_B^2)} \right) \right\} + \frac{b}{4} M_B^6 E_3 \left( \frac{s_0}{M_B^2} \right) L^{-\frac{3}{2}} + \frac{a^2}{3} \left\{ 4 - \frac{4 m_0^2}{3 M_B^2} \right\},
\]

where \( M_B \) is the Borel parameter, \( s_0 \) is the continuum threshold and

\[
E_n(s_0/M^2) = 1 - e^{(-s_0/M^2)^n} \sum_{k=0}^{n-1} \frac{k!}{k!} \left( \frac{s_0}{M^2} \right)^k,
\]

where \( M_B \) is the Borel parameter, \( s_0 \) is the continuum threshold and

\[
L = \frac{\alpha_s(\mu^2)}{\alpha_s(M_B^2)},
\]

\[
a = -(2\pi)^2 \langle \bar{q}q \rangle \simeq 0.55 \text{ GeV}^3,
\]

\[
b = (2\pi)^2 \left( \frac{\alpha_s G^2}{\pi} \right) \simeq 0.47 \text{ GeV}^4,
\]

\[
m_0^2 = \langle \bar{q}Gq \rangle \langle \bar{q}q \rangle \simeq 0.65 \text{ GeV}^2.
\]

We have neglected in Eq. (22) the small \( \alpha_s \)-corrections to the four-quark contribution proportional to \( a^2 \). The corresponding formula for \( \lambda_2 \) can be found, e.g., in [4]. QCD sum rule estimates for the \( f_2^d \) defined in Eqs. (16) - (18) were first presented in [2] and updated in [6]. The parameter set which is obtained by QCD sum rules will be called sum-rule estimate in the following, we use the numerical values from [30] for the moments of the leading-twist distribution amplitude and the values from [6] for \( f_2^d \), \( f_2^d \), \( f_1^d \), \( f_1^d \) and \( f_2^d \). In our analysis we use two related parameter sets which are based on the QCD sum rule determination:

- Demanding that all higher conformal contributions vanish, fixes \( A_1^d, V_1^d, f_1^d, f_2^d \) and \( f_1^d \), while the values for \( f_N \), \( \lambda_1 \), \( \lambda_2 \) are taken from the QCD sum rule estimates or from the lattice calculation. This parameter set will be called asymptotic. In the case of the leading twist, one would be left with the asymptotic distribution amplitude \( \varphi(x_i, Q^2 \rightarrow \infty) = \varphi_{asy}(x_i) = 120 x_1 x_2 x_3 f_N \). The corresponding expressions for the higher twist distribution amplitudes can be found in [6].

- With the help of light-cone sum rules [6, 12, 14] one can express the nucleon form factors in terms of the eight non-perturbative parameters \( f_N, \lambda_1, \lambda_2, A_1^d, V_1^d, f_1^d, f_2^d \) and \( f_2^d \) (incuding twist-6 corrections and expanding the distribution amplitudes up to the next-to-leading conformal spin). Choosing values for these parameters in between the asymptotic and the sum-rule values, we got an astonishingly good agreement with the experimental numbers, see [6]. This procedure is obviously rather ad-hoc and has to be replaced by a real fit after \( \alpha_s \)-corrections to the light-cone sum rules have been calculated. The parameter set obtained in [6] will be called BLW.

### IV. LATTICE DETERMINATION OF THE NUCLEON DISTRIBUTION AMPLITUDES

Lattice QCD offers the possibility to perform non-perturbative computations in QCD without additional model assumptions. For example, one can evaluate hadron masses and matrix elements of local operators between hadron
states. In particular, the non-perturbative parameters $f_N, \ldots$ introduced above can be extracted from Monte Carlo simulations on the lattice as advocated in [45]. Recently, the QCDSF collaboration has performed such a calculation [4]. It is based on gauge field configurations generated with two dynamical flavours of quarks. For the gauge field the standard Wilson action was used, while the lattice action for the quarks was the so-called non-perturbatively $O(a)$ improved Wilson fermion action, also known as the clover fermion action. Although lattice artefacts seem to be small, a reliable continuum extrapolation could not be attempted, and we utilize here the data obtained on the finest lattice corresponding to a gauge coupling parameter $\beta = 5.40$. Setting the scale via a Sommer parameter of $r_0 = 0.467$ fm the lattice spacing turns out to be $a \approx 0.067$ fm.

On the lattice, matrix elements between the vacuum and a nucleon such as those needed here are computed from two-point correlation functions of the local operator $O_\alpha(x)$ under study and a suitable interpolating field $\bar{N}_\alpha(x)$ for the nucleon. Asymptotically this two-point function decays exponentially with the distance between the operators since the lattice calculations are performed in Euclidean space. Projecting onto definite momentum one finds for sufficiently large (Euclidean) times $t$:

$$
\sum_{\vec{x}} \sum_{\vec{y}} e^{-i\vec{P} \cdot \vec{x}} e^{i\vec{P} \cdot \vec{y}} \langle O_\alpha(\vec{x}, t)\bar{N}_\beta(\vec{y}, 0) \rangle = \frac{V_s \sqrt{Z}}{2E(P)} M_\alpha \left( E(P)\gamma_4 - i\vec{P} \cdot \vec{\gamma} + m_N \right) e^{-E(P)t}.
$$

(28)

Here $V_s$ denotes the spatial volume of the lattice and the matrix elements of $O_\alpha(x)$ and $\bar{N}_\alpha(x)$ have been represented as

$$
\langle 0 | O_\alpha(0) | P \rangle = M_\alpha N_\alpha(P),
$$

(29)

$$
\langle P | \bar{N}_\alpha(0) | 0 \rangle = \sqrt{Z} \bar{N}_\alpha(P).
$$

(30)

As the local operators $O_\alpha(x)$ used in the simulations are linear combinations of the operators appearing in (11)-(21), the constants $M_\alpha$ are directly related to moments of the distribution amplitudes. The operators $O_\alpha(x)$ need to be renormalized. In Ref. [4] a non-perturbative renormalization procedure has been chosen. As the space-time symmetry on the lattice is reduced to the finite (spinorial) hypercubic group, the mixing pattern of our three-quark operators is more complicated than in the continuum and the choice of the operators becomes an important issue. Guided by the group-theoretical classification of three-quark operators given in [16] the problematic mixing with lower-dimensional operators could however be completely avoided. Moreover, the freedom in the choice of the operators has been exploited in order to reduce the statistical uncertainties of the results.

Primarily, the combination of moments

$$
\phi_{n_1 n_2 n_3} = \frac{1}{3} \left( V_{n_1 n_2 n_3} - A_{n_1 n_2 n_3} + 2T_{n_1 n_2 n_3} \right) = \frac{1}{3} (2 \varphi_{n_1 n_2 n_3} + \varphi_{n_3 n_2 n_1})
$$

(31)

has been evaluated, from which the combination $\varphi_{n_1 n_2 n_3}$ usually used in sum rule calculations is readily obtained by

$$
\varphi_{n_1 n_2 n_3} = 2\phi_{n_1 n_2 n_3} - \phi_{n_3 n_2 n_1}.
$$

(32)

In the following sections we shall compare these lattice results with results obtained from other approaches and see what the lattice numbers imply for the nucleon form factors.

V. COMPARISON OF DIFFERENT METHODS TO DETERMINE THE DISTRIBUTION AMPLETTES

In Table II we compare different estimates for the moments of the leading-twist distribution amplitude at 1 GeV$^2$. It turns out that the BLW model, the BK model and the lattice evaluation give almost indential results, which are close to the asymptotic values, while the QCD sum rule estimates seem to overestimate the deviation from the asymptotic form, although the deviation goes in the right direction. The BLW model was inspired by this experience: One starts with the asymptotic form and goes then in the direction of the QCD sum rule estimate, but only for a fraction of the whole difference. Choosing this fraction to be $1/3$ one gets an astonishingly good agreement between light-cone sum rule predictions for the nucleon form factors and experiment, see [4]. In the same spirit one can make a BLW model for the second moments, also given in table II [117]. These values are again very close to the lattice values. The BK model [47] was also inspired by experiment, in particular the decay $J/\Psi \rightarrow NN$, the Feynman contribution to the nucleon form factor and the valence quark distribution function. In Table II we compare different estimates for $f_N$, ...
rule estimates the radiative corrections are expected to be sizeable, but these are only known for
expressed in terms of the parameters \( \alpha \) and \( \beta \) if radiative corrections to the sum rule estimates are inclu
ded, cf. Eq. (22), for
central lattice value of \( f_N \) via the relations

Moreover, the errors on \( f_N \) are about 30% larger than the QCD sum rule estimates. For
\( |\lambda_1| \) and \( |\lambda_2| \) are about 35% smaller than the QCD sum rule estimates, while the lattice results for \( |\lambda_1| \) and \( |\lambda_2| \) are about 30% larger than the QCD sum rule estimates. For \( \lambda_1 \) and \( \lambda_2 \) the discrepancy is reduced strongly, if radiative corrections to the sum rule estimates are included, cf. Eq. (22), for \( f_N \) - according to our knowledge - no
radiative corrections have been calculated yet.

The parameters \( \lambda_1 \) and \( \lambda_2 \) can also be extracted from the lattice calculation of the nucleon decay matrix elements (expressed in terms of the parameters \( \alpha \) and \( \beta \)) in [22, 23]. Using the relations

\[
\lambda_1 = \frac{4}{m_N} \alpha, \quad \lambda_2 = \frac{8}{m_N} \beta, \tag{35}
\]

| Asy     | QCD-SR | COZ | KS | BK | BLW | LAT         |
|---------|--------|-----|----|----|-----|-------------|
| \( \phi^{100} \) | \( \frac{1}{3} \approx 0.333 \) | 0.560(60) | 0.579 | 0.55 | \( \frac{4}{13} \approx 0.31 \) | 0.415 | 0.3999(37)(139) |
| \( \phi^{010} \) | \( \frac{1}{3} \approx 0.333 \) | 0.192(12) | 0.192 | 0.21 | \( \frac{4}{13} \approx 0.31 \) | 0.285 | 0.2986(11)(52)  |
| \( \phi^{001} \) | \( \frac{1}{3} \approx 0.333 \) | 0.229(29) | 0.229 | 0.24 | \( \frac{4}{13} \approx 0.31 \) | 0.300 | 0.3015(32)(106) |
| \( \phi^{200} \) | \( \frac{1}{7} \approx 0.143 \) | 0.350(70) | 0.369 | 0.35 | \( \frac{4}{13} \approx 0.18^* \) | 0.225 | 0.1816(64)(212) |
| \( \phi^{020} \) | \( \frac{1}{7} \approx 0.143 \) | 0.084(19) | 0.068 | 0.09 | \( \frac{4}{13} \approx 0.13^* \) | 0.121 | 0.1281(32)(106) |
| \( \phi^{002} \) | \( \frac{1}{7} \approx 0.143 \) | 0.109(19) | 0.089 | 0.12 | \( \frac{4}{13} \approx 0.13 \) | 0.132 | 0.1311(13)(382) |
| \( \phi^{111} \) | \( \frac{2}{13} \approx 0.095 \) | -0.030(30) | 0.027 | 0.02 | \( \frac{4}{13} \approx 0.08^* \) | 0.071 | 0.0613(89)(319) |
| \( \phi^{101} \) | \( \frac{2}{13} \approx 0.095 \) | 0.102(12) | 0.113 | 0.10 | \( \frac{4}{13} \approx 0.10^* \) | 0.097 | 0.1091(41)(152) |
| \( \phi^{110} \) | \( \frac{2}{13} \approx 0.095 \) | 0.090(10) | 0.097 | 0.10 | \( \frac{4}{13} \approx 0.10^* \) | 0.093 | 0.1092(67)(219) |

| Asy     | QCD-SR | BK | BLW | LAT         |
|---------|--------|----|-----|-------------|
| \( f_N \cdot 10^3 [\text{GeV}]^2 \) | 5.0(5) | 6.64 | 5.0(5) | 3.234(63)(86) |
| \( \lambda_1 \cdot 10^3 [\text{GeV}]^2 \) | -27(9) | -27(9) | -27(9) | -35.57(65)(136) |
| \( \lambda_2 \cdot 10^3 [\text{GeV}]^2 \) | 54(19) | 54(19) | 54(19) | 70.02(128)(268) |
| \( A_1^u \) | 0 | 0.38(15) | \( \frac{4}{13} \approx 0.07 \) | 0.13 | 0.1013(81)(298) |
| \( V_1^d \) | \( \frac{4}{13} \approx 0.333 \) | 0.23(3) | \( \frac{4}{13} \approx 0.31 \) | 0.30 | 0.3015(32)(106) |
| \( f_1^u \) | 0.30 | 0.40(5) | - | 0.33 | - |
| \( f_1^d \) | 0.10 | 0.07(5) | - | 0.09 | - |
| \( f_2^u \) | \( \frac{4}{13} \approx 0.267 \) | 0.22(5) | - | 0.25 | - |
| \( f_2^d \) | \( \frac{4}{13} \approx 0.267 \) | 0.22(5) | - | 0.25 | - |
we obtain from the results in [24, 26]
\[ \lambda_1 = -43.90 \pm 4.7 \pm 8.5 \cdot 10^{-3} \text{GeV}^2, \quad \lambda_2 = 93.96 \pm 10.2 \pm 22.7 \cdot 10^{-3} \text{GeV}^2, \]
at the renormalization scale 1 GeV$^2$. In that case the deviation from the QCD sum rule values is even more pronounced. In the non-relativistic limit one gets

\[ 2\lambda_1 + \lambda_2 = 0. \]

The estimates presented in table I fulfill this relation almost perfectly:

\[ \frac{2\lambda_1 + \lambda_2}{2\lambda_1 - \lambda_2} \bigg|_{\text{QCD-SR}} = 0 \pm 0.24, \]  \[ \frac{2\lambda_1 + \lambda_2}{2\lambda_1 - \lambda_2} \bigg|_{\text{LAT}} = 0.008 \pm 0.013. \]  

For the ratio $f_N/\lambda_1$ the differences between the central lattice and QCD sum rule estimates are even more enhanced:

\[ \left( \frac{f_N}{\lambda_1} \right)_{\text{QCD-SR}} = -0.185 \pm 0.064, \]  \[ \left( \frac{f_N}{\lambda_1} \right)_{\text{LAT}} = -0.0909 \pm 0.0054 \pm 0.0095. \]  

The QCD sum rule estimate is a factor of two larger than the lattice result. In the next section we will see that the electromagnetic form factors of the nucleon depend only on the ratio but not on the individual values of $f_N$ and $\lambda_1$, if the so-called Ioffe interpolating field is used, while the $N \to \Delta$ transition depends on the individual values.

**VI. LIGHT-CONE SUM RULES FOR FORM FACTORS**

Light-cone sum rules (LCSR) are an advancement of QCD sum rules [31] for intermediate values of the momentum transfer $Q^2$, i.e., $1 \text{ GeV}^2 < Q^2 < 10 \text{ GeV}^2$ in the case of nucleon form factors. They were introduced in [17, 18]. The starting point is a correlation function of the form

\[ T(P, q) = \int d^4x e^{-iqx} \langle 0|T\{\eta(0)j(x)\}|N(P)\rangle, \]  

which describes the transition of a baryon $B$ with momentum $P - q$ to the nucleon $N(P)$ via the current $j$. The baryon $B$ is created by the interpolating three-quark field $\eta$. If $B$ is a nucleon one can use, e.g., the Ioffe current [38] for the proton

\[ \eta_{\text{Ioffe}}(x) = \epsilon^{ijk} \left[ u^i(x) (C \gamma_\mu) u^j(x) \right] (\gamma_5 \gamma^\nu) \bar{d}_k(x). \]  

A typical example for $j$ is the electromagnetic current in the case of the electromagnetic form factors

\[ j^{em}_\mu(x) = e_u \bar{u}(x) \gamma_\mu u(x) + e_d \bar{d}(x) \gamma_\mu d(x). \]  

With the definitions in Eqs. [11], [12] the correlation function in Eq. [40] describes the electromagnetic form factors of the nucleon, which can be measured, e.g., in elastic electron-proton scattering. The basic idea of the light-cone sum rule approach is to calculate the correlation function in Eq. [40] both on the hadron level (expressed in terms of form factors) and on the quark level (expressed in terms of the nucleon distribution amplitudes). Equating both results and performing a Borel transformation to suppress higher mass states one can express the form factors in terms of the eight (taking only leading and next-to leading conformal spin into account) non-perturbative parameters of the nucleon distribution amplitudes, the Borel parameter $M_B$ and the continuum threshold $s_0$, for details see [3, 16].

We studied the electromagnetic nucleon form factors with the Chernyak-Zhitnitsky interpolating field ($\eta_{\text{CZ}}$) in [16]. In [44] we found that $\eta_{\text{CZ}}$ yields large unphysical isospin violating effects, therefore we introduced a new isospin respecting CZ-like current to determine the electromagnetic form factors. In [6] we also studied the Ioffe current for the nucleon and extended our studies from the electromagnetic form factors to axial form factors, pseudoscalar form...
In the class of nucleon to resonance transitions the following processes were considered: The $N \rightarrow \Delta$ transition was studied in this framework in [53] (for a similar approach for $Q^2 = 0$ see, e.g., [54]), the axial part of the $N \rightarrow \Delta$ transition was calculated in [53]. Very recently the form factors of the $N \rightarrow N^*(1535)$ transition were presented in [50]. In [51, 59] pion-electroproduction was investigated. Also decays of baryons can be described with that formalism: $\Lambda \rightarrow l\nu \gamma$ and $\Lambda \rightarrow \Lambda \gamma$ and $\Lambda \rightarrow \Lambda l^+ l^-$ were treated in [62] with the same formalism. Electromagnetic form factors of $\Sigma$ and $\Lambda$-baryons were estimated in [63, 64]. In [56] the transition $\Sigma \rightarrow N$ was investigated. Recently the rare decays $\Lambda_b \rightarrow \Lambda \gamma$ and $\Lambda_b \rightarrow \Lambda l^+ l^-$ were treated in [62] with the same formalism. So far all mentioned LCSR calculations for the baryon form factors were done in leading order QCD. One expects sizeable radiative corrections of up to 30%. In [65] a first step in calculating the full $O(a_s)$-corrections to the nucleon electromagnetic form factors was performed. The intrinsic final uncertainty of this approach is expected to be in the range of less than $\pm 20\%$, if QCD corrections are included. Comparing the theoretical predictions with experimental numbers one must be careful to distinguish between quantities directly calculated like $F_1$ and $F_2$ and quantities like $G_E = F_1 - Q^2/(4m_N^2)F_2$ for which cancellations might ruin the predictive power.

In the following we use the LO QCD light-cone sum rules of [6] for the electromagnetic form factors of the nucleon and the LO QCD results of [53] for the $N \rightarrow \Delta$ transition to compare the consequences for the form factors which the lattice results for the nucleon distribution amplitudes entail with those which result from different QCD sum rule estimates. Note, however, that the errors on the non-perturbative parameters of the nucleon distribution amplitudes will not be taken into account, because this would not make much sense due to the inherent uncertainty in the LO light-cone sum rules.

**VII. RESULTS FOR THE FORM FACTORS AT INTERMEDIATE MOMENTUM TRANSFER**

In this section we use light-cone sum rules to extract physical form factors from the nucleon distribution amplitudes, by taking into account conformal spin contributions up to the p-wave; $d$-wave effects will be discussed in section 9. We compare our theory results to the following experimental numbers. For the electromagnetic nucleon form factors we take data from:

- The magnetic form factor of the proton normalized to the dipole form factor $G_M^p/\mu_p G_D$ from [64, 67, 68, 69] with

$$G_D(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{0.71 \text{GeV}^2}\right)^2}, \quad \mu_p = 2.7928\ldots$$

(43)

The data of [68, 69, 70] are actually taken from the reanalysis in [71].

- The ratio of the electric and magnetic form factors of the proton $\mu_p G_E^p/G_M^p$ from Rosenbluth separation [66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 77, 78] and from polarization transfer [19, 20, 21, 22]. We would like to point out here that [73, 74] claimed already in the seventies a steeper $Q^2$ dependence of $G_E^p$ compared to $G_M^p$ for momentum transfers above 1 GeV$^2$. Currently the Rosenbluth separation data for $G_E$ are judged to be less reliable.

- The ratio of the proton form factors $F_1^p$ and $F_2^p$ given as $\sqrt{Q^2 F_1^p/((\mu_p - 1) F_2^p)}$ in [66, 67, 79].

- The magnetic form factor of the neutron normalized to the dipole form factor: $G_M^n/\mu_n G_D$ from [73, 80, 81, 82, 83, 84, 85, 86] with $\mu_n = -1.913\ldots$

- The electric form factor of the neutron normalized to the dipole form factor: $G_E^n/G_D$ from [73, 80, 81, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 97, 96, 97, 98, 99, 100]. The data are very well described by the so-called Galster fit [101].
we show in our plots the update of the Galster-fit from Kelly [102]:

\[
G_E^{\text{Galster}}(Q^2) = \frac{(1.70 \pm 0.04) \tau}{1 + (3.30 \pm 0.32) \tau} G_D(Q^2), \quad \text{with } \tau = \frac{Q^2}{4m_p^2}.
\]  

For the axial form factors we compare our result to the dipole formula [103]

\[
G_A(Q^2) = \frac{1.267}{\left(1 + \frac{Q^2}{(1.014\text{GeV})^2}\right)^2}.
\]

For more details see [6].

Finally we use the following data for the \(N \to \Delta\) transition:

- The magnetic form factor normalized to the dipole form factor \(G_M^*/(3G_D)\) from [110, 111].
- The ratio of the electric quadrupole to the magnetic form factor \(R_{EM}\) from [111, 112].
- The ratio of the Coulomb quadrupole to the magnetic form factor \(R_{SM}\) from [111, 112].

For more details see [53]. Since we compare the data with the LCSR predictions, which are expected to work best in the region \(1 \text{ GeV}^2 < Q^2 < 10 \text{ GeV}^2\), we are only interested in experiments where values of the form factors for momentum transfer above \(Q^2 = 1 \text{ GeV}^2\) are available.

For the theory prediction we will use six models (i.e. six determinations for the nonperturbative parameters \(f_N, \lambda_1, \lambda_2, A_1^V, V_1^d, f_1^u, f_1^d\) for the nucleon distribution amplitudes including s- and p-wave contributions:

1. QCD sum rule estimates (dotted red lines),
2. asymptotic form (dashed red lines),
3. BLW model (solid red lines),
4. lattice evaluation plus QCD sum rule estimate for \(f_1^p\) (dotted blue lines),
5. lattice evaluation plus asymptotic values for \(f_1^p\) (dashed blue lines),
6. lattice evaluation plus BLW estimate for \(f_1^p\) (solid blue lines).

Since \(f_1^d, f_1^d\) and \(f_2^d\) have not been determined on the lattice, we have to use in the lattice parameter set QCD sum rule estimates, asymptotic values or the BLW model for \(f_2^p\). For the nucleon form factors we use the LCSRs obtained in [6] and for the \(N \to \Delta\) transition we use the LCSRs obtained in [53].

To our accuracy, the sum rules for the nucleon form factors depend only on the five parameters \(f_N/\lambda_1, A_1^V, V_1^d, f_1^u\) and \(f_2^d\). Within the light-cone sum rule approach we determine the form factors \(F_1\) and \(F_2\) directly. The electric and the magnetic form factors \(G_E\) and \(G_M\) are linear combinations of \(F_1\) and \(F_2\):

\[
G_M(Q^2) = F_1(Q^2) + F_2(Q^2),
\]

\[
G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4m_N^2} F_2(Q^2).
\]  

As discussed above, Eq. (46) shows that in \(G_E\) cancellations occur. Therefore our predictions for \(G_E\) are less reliable than those for \(G_M\).

The light-cone sum rule predictions for the form factors are shown in Figs. 4 - 7. For \(G_M^B, G_M^A, G_M^N\) and \(G_E\) the differences between the lattice determinations and the other approaches (asymptotic, QCD sum rule and BLW) are smaller than the expected overall uncertainties, i.e., the pairs of parameter sets (1) - (4), (2) - (5) and (3) - (6) yield almost identical results. Since \(f_1^u\) and \(f_1^d\) were chosen to be identical within these pairs, differences can only occur due to \(V_1^d, A_1^V\) and \(f_N/\lambda_1\). The lattice values for \(V_1^d, A_1^V\) are very close to the BLW values, while \(f_N/\lambda_1\) is in the lattice determination about a factor of two smaller than the QCD sum rule estimate. In \(G_E, \sqrt{Q^2 F_2^p/((\mu_p - 1) F_1^p)}\) and \(G_E^B\) cancellations occur, so we expect much bigger theoretical uncertainties and also big differences between our data sets might be possible.

The data for \(G_M^p\) (Fig. 6) are very well described with the asymptotic and the BLW data sets, the differences between the parameter sets (2) and (5) and between (3) and (6) are negligible. The pure QCD sum rule estimates (set (1)
FIG. 1: LCSR results for the electromagnetic form factors (left: $G_M/(\mu_p G_D)$ vs. $Q^2$; right: $\mu_p G_E/G_M$ vs. $Q^2$) of the proton, obtained using the BLW model (red solid line), the asymptotic model (red dashed line) and the QCD sum rule model (red dotted line) of the nucleon distribution amplitudes. The corresponding results for the lattice values of the nucleon distribution amplitudes are given in blue. The red data points on the right picture are JLAB data, while the blue and the green ones are obtained via Rosenbluth separation. Currently the Rosenbluth separation data for $G_E$ are judged to be less reliable.

FIG. 2: LCSR results for the electromagnetic form factors of the neutron (left: $G_M/(\mu_n G_D)$ vs. $Q^2$; right: $G_E/(G_D)$ vs. $Q^2$), obtained using the BLW model (red solid line), the asymptotic model (red dashed line) and the QCD sum rule model (red dotted line) of the nucleon distribution amplitudes. The corresponding results for the lattice values of the nucleon distribution amplitudes are given in blue. The thin solid blue line represents the updated Galster fit.

FIG. 3: LCSR results (solid curves) for the axial form factor of the proton $G_A$ normalized to $G_D = g_A/(1 + Q^2/m_A^2)$ vs. $Q^2$ (left panel) and the tensor form factor $G_T$ normalized to $G_A$ vs. $Q^2$ (right panel), obtained using the BLW model (red solid line), the asymptotic model (red dashed line) and the QCD sum rule model (red dotted line) of the nucleon distribution amplitudes. The corresponding results for the lattice values of the nucleon distribution amplitudes are given in blue.
This holds for all parameters except the pure QCD sum rule estimates (set (1)). In our approach the interpolating nucleon field, the other by the nucleon distribution amplitude.

Finally we have the ratios

\[ \frac{G_{p}^{r}}{G_{D}} \]  

and (4) are about a factor of two too small. In the case of \( G_{n}^{r} \) (Fig. 2) one sees the same structure for the different models of the nucleon distribution amplitude as for \( G_{n}^{A} \), but now all theory predictions are shifted to lower values. For \( G_{p}^{r} \) (Fig. 3) we agree for \( Q^{2} \) values below 5 GeV\(^2\) very well with the dipole behavior, if we use the asymptotic or the BLW parameters; for higher \( Q^{2} \) we predict a slightly steeper fall-off. Again the pure QCD sum rule estimates are considerably worse.

An interesting test of our approach is whether the unphysical tensor form factor \( G_{T} \) (Fig. 3) is consistent with zero. This holds for all parameters except the pure QCD sum rule estimates (set (1)). In our approach \( G_{T}^{r} \) is not exactly zero, because we treat the initial proton state differently from the final proton state: one is described by an interpolating nucleon field, the other by the nucleon distribution amplitude.

Finally we have the ratios \( \frac{G_{p}^{r}}{G_{D}} \) (Fig. 2), \( \frac{G_{p}^{r}}{G_{M}^{r}} \) (Fig. 1), and \( \frac{Q^{2}F_{2}^{r}}{(\mu_{p} - 1) F_{1}^{p}} \) (Fig. 4), which are very sensitive to the explicit form of the nucleon distribution amplitudes due to cancellations in \( G_{E} \). If we just look at \( G_{p}^{r} \) our result would be consistent with zero and therefore describes the data well. If we investigate \( G_{n}^{r}/G_{D} \), we blow up the large \( Q^{2} \) contributions. Now we have a “perfect” agreement between the pure QCD sum rule parameters and the data. Our data set (4) is almost identical to the updated Galster fit. The BLW model is consistent with zero, while the asymptotic distribution amplitude yields negative values. The difference between the lattice values for the distribution amplitudes and the data sets (1) - (3) is visible, but not dramatic. In the case of \( \frac{G_{E}^{p}}{G_{M}^{p}} \), and \( \frac{Q^{2}F_{2}^{p}}{(F_{1}^{p} \ast 1.79)} \) we can make similar observations. The purely asymptotic values lie above (below) the data for \( \frac{G_{p}^{r}}{G_{M}^{r}} \) (\( \frac{Q^{2}F_{2}^{p}}{(F_{1}^{p} \ast 1.79)} \)), the BLW data set moves the results in the right direction, but not far enough. Our data set (1) is completely off, because it predicts a very small value for \( F_{1}^{p} \). Now we also have big differences between the lattice values of the distribution amplitudes and the pure QCD sum rule values.

Taking into account that the full O(\( \alpha_{s} \))-corrections to the LCSRs are not yet available, already a rough agreement of our approach with the data is a success. For \( G_{p}^{r}, G_{A}^{r}, G_{M}^{r} \) and \( G_{E}^{r} \) we get unexpectedly “good” results if we use the BLW form or the asymptotic form of the nucleon distribution amplitude. The corresponding lattice values (set (5) and (6)) give similar results. For \( G_{E}^{r}/G_{D}, G_{E}^{p}/G_{M}^{p}, \) and \( \sqrt{Q^{2}F_{2}^{p}}/(F_{1}^{p} \ast 1.79) \) the BLW model lies in the right ball park, but we would prefer a model for the distribution amplitudes which lies in the middle between the asymptotic and the pure QCD sum rule estimate (BLW is closer to the asymptotic value).

In the transition form factors of \( \gamma^{*}N \rightarrow \Delta \) all eight non-perturbative parameters appear, see ⁵³. The results are
shown in Fig. 5. As expected, now the differences between the parameter set pairs (1) - (4), (2) - (5) and (3) - (6) are more pronounced. In the case of $G_M^*$ the theory curves generally tend to be more flat than the experimental data. The form factors obtained with the lattice values for the nucleon distribution amplitude lie considerably above the data sets (1) - (3). Above $Q^2 \approx 3$ GeV$^2$ the asymptotic distribution amplitude and the BLW distribution amplitude are close to the data. The fact that $R_{EM}$ is close to zero is reproduced very well with the BLW parameters (sets (3) and (6)) and the lattice plus asymptotic values (set(5)), while positive values are obtained with the purely asymptotic form (set (2)). One gets a negative result with the QCD sum rule determination of the nucleon distribution amplitude (sets (1) and (4)). In the case of $R_{SM}$ the differences are not very pronounced, all values are close to zero. Altogether one has to conclude that while all approaches give the correct order of magnitude none gives a really convincing description of all data. However in view of the fact that the systematic uncertainties are even more pronounced than for the nucleon form factors more could not have been expected.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig5.png}
\caption{$\gamma^* N \rightarrow \Delta$ transition form factors (left:$G_M^*/(3G_D)$ vs. $Q^2$, middle:$R_{EM}$ vs. $Q^2$, right: $R_{SM}$ vs. $Q^2$) in the LCSR approach \cite{53} obtained using the BLW model (solid line), the asymptotic model (dashed line) and the QCD sum rule model (dotted line) of the nucleon distribution amplitudes. The corresponding form factors based on the lattice values for the nucleon distribution amplitudes are given in blue.}
\end{figure}

VIII. REDUCING THE NUMBER OF INDEPENDENT PARAMETERS OF THE DISTRIBUTION AMPLITUDES

In \cite{113} the following approximate relations between twist-4 and twist-3 parameters were derived:

\begin{align*}
\lambda_1 f_1^d &= \frac{3}{10} - \frac{1}{6} \frac{f_N}{\lambda_1}, \\
\lambda_1 f_1^u &= \frac{1}{10} - \frac{1}{6} \frac{f_N}{\lambda_1}
\end{align*}

Using these relations, we can express the nucleon form factors in terms of only three independent parameters, namely $f_N^d$, $f_N^u$ and $A_1^q$. For the comparison with the data we show now only two models for the remaining three parameters of the nucleon distribution amplitude:

(a) Lattice determination of the distribution amplitude - blue curve.

(b) BLW model - red curve.

We obtain the following values for $f_1^*$:

|      | (a) | (b) | asymptotic | BLW | QCD-SR |
|------|-----|-----|------------|-----|--------|
| $f_1^d$ | 0.11 | 0.13 | 1.00       | 0.09 | 0.07   |
| $f_1^u$ | 0.31 | 0.33 | 0.30       | 0.33 | 0.40   |

which are compared with our previous estimates for $f_1^d$ and $f_1^u$. In this approach $f_1^d$ lies between the asymptotic and the BLW value: $f_1^u$ is also close to the asymptotic or the BLW value, but its deviation from the asymptotic value is in the “wrong” direction. In Fig. 6 we show the electromagnetic form factors of the proton, in Fig. 7 the electromagnetic form factors of the neutron, in Fig. 8 the axial and the tensor form factor of the proton and finally in Fig. 9 the ratio of the form factors $F_2$ and $F_1$ of the proton. In Fig. 10 we show the three $N \rightarrow \Delta$ transition form factors. In all cases we obtain results which are very close the BLW results of the previous section, so it seems that the nucleon form factors are very sensitive to the values of $V_1^d$ and $A_1^q$, while the dependence on $f_N/\lambda_1$, $f_1^d$ and $f_1^u$ is less pronounced.
FIG. 6: LCSR results for the electromagnetic form factors (left: $G_M/(\mu_p G_D)$ vs. $Q^2$; right: $\mu_p G_E/G_M$ vs. $Q^2$) of the proton, obtained using the BLW model (red solid line) and the lattice prediction (blue solid line) for the nucleon distribution amplitudes. In both cases the parameters $f_1^D$ are determined from the twist-3 parameters, cf. Eq. (47). The red data points on the right picture are JLAB data, while the blue and the green ones are obtained via Rosenbluth separation.

FIG. 7: LCSR results for the electromagnetic form factors of the neutron (left: $G_M/(\mu_n G_D)$ vs. $Q^2$; right: $G_E/(G_D)$ vs. $Q^2$), obtained using the BLW model (red solid line) and the lattice prediction (blue solid line) for the nucleon distribution amplitudes. In both cases the parameters $f_1^N$ are determined from the twist-3 parameters, cf. Eq. (47). The thin solid blue line represents the updated Galster fit.

FIG. 8: LCSR results (solid curves) for the axial form factor of the proton $G_A$ normalized to $G_D = g_A/(1 + Q^2/m_A^2)$ vs. $Q^2$ (left panel) and the tensor form factor $G_T$ normalized to $G_A$ vs. $Q^2$ (right panel), obtained using the BLW model (red solid line) and the lattice prediction (blue solid line) for the nucleon distribution amplitudes. In both cases the parameters $f_1^T$ are determined from the twist-3 parameters, cf. Eq. (47).
FIG. 9: LCSR results (solid curves) for the ratio \( \sqrt{Q^2 F^2_2/(F^2_1 \times 1.79)} \) obtained using the BLW model (red solid line) and the lattice prediction (blue solid line) for the nucleon distribution amplitudes. In both cases the parameters \( f^{x}_1 \) are determined from the twist-3 parameters, cf. Eq. (47). Red symbols: experimental values obtained via Polarization transfer. Blue symbols: experimental values obtained via Rosenbluth separation.

FIG. 10: \( \gamma^* N \rightarrow \Delta \) transition form factors (left: \( G^*_{M}/(3G_D) \) vs. \( Q^2 \), middle: \( R_{EM} \) vs. \( Q^2 \), right: \( R_{SM} \) vs. \( Q^2 \)) in the LCSR approach [53] obtained using the BLW model (red solid line) and the lattice prediction (blue solid line) for the nucleon distribution amplitudes. In both cases the parameters \( f^x_1 \) are determined from the twist-3 parameters, cf. Eq. (47).

IX. EFFECTS OF HIGHER CONFORMAL SPIN CONTRIBUTIONS

In this section we include (in comparison to the previous sections) also contributions of the next-to-next-to leading conformal spin to the leading-twist distribution amplitude. These terms have been determined on the lattice [3, 4] and with QCD sum rules [29, 30, 37]. The explicit expressions for the leading-twist distribution amplitudes can be found in appendix C. The contributions of these higher moments to the electromagnetic form factors of the nucleon have already been estimated in [16], but only for the Chernya-Zhitnitsky interpolating field. Here we work out the contributions of the second moments to the light-cone sum rules for nucleon form factors using the Ioffe current. We will use the form in Eq. (C1) for the leading-twist distribution amplitude and the following parameter sets:

1. Asymptotic distribution amplitude (black lines).
2. BLW plus second moments from QCD sum rules (dotted red lines).
3. BLW plus second moments from the lattice (dashed red lines).
4. BLW plus second moments á la BLW (solid red lines).
5. Lattice evaluation plus QCD sum rule estimates for $f^p_y$ (dotted blue lines).
6. Lattice evaluation plus asymptotic values for $f^p_y$ (dashed blue lines).
7. Lattice evaluation plus BLW estimates for $f^p_y$ (solid blue lines).

In Fig. [11] we show the electromagnetic form factors of the proton, in Fig. [12] the electromagnetic form factors of the neutron, in Fig. [13] the axial and the tensor form factor of the proton and finally in Fig. [14] the ratio of the form factors $F_2$ and $F_1$ of the proton. In Fig. [14] we show the three $N \rightarrow \Delta$ transition form factors.

In almost all cases the second moments of the leading-twist distribution amplitude determined with QCD sum rules give huge corrections. We show these parameter sets in the plots, but we will not discuss them any further.

The magnetic form factor of the proton $G_M^n$ is very well described by the BLW model with second moments á la BLW (set(4)) or from the lattice (set(3)) and the lattice values for the distribution amplitude with $f^p_y$ from BLW (set (7)) or with the asymptotic values for $f^p_y$ (set (6)). This is not unexpected, since the second moments á la BLW and from the lattice are quite similar in size. This observation ensures, however, that there is not an unexpected strong sensitivity of the LCSR to the second moments. The theory predictions for the magnetic form factor of the neutron $G_M^n$ are again shifted to lower values. Apart from this fact, the predictions for the parameter sets (1), (3),

\[ \text{FIG. 12: LCSR results for the electromagnetic form factors of the neutron (left: } G_M/(\mu_u G_D) \text{ vs. } Q^2; \text{ right: } \mu_u G_E/G_M \text{ vs. } Q^2 \text{) of the proton, obtained using the asymptotic form (black solid line), the BLW model (red) with second moments from QCD sum rules (dotted), lattice (dashed) and BLW (solid) and the lattice determination (blue) with } f^p_y \text{ from QCD sum rules (dotted), the asymptotic model (dashed) and the BLW model (solid). The red data points on the right picture are JLAB data, while the blue and green ones are obtained via Rosenbluth separation.} \]
FIG. 13: LCSR results (solid curves) for the axial form factor of the proton $G_A$ normalized to $G_D = g_A/(1 + Q^2/m_A^2)^2$ vs. $Q^2$ (left panel) and the tensor form factor $G_T$ normalized to $G_A$ vs. $Q^2$ (right panel), obtained using the asymptotic form (black solid line), the BLW model (red) with second moments from QCD sum rules (dotted), lattice (dashed) and BLW (solid) and the lattice determination (blue) with $f^p$ from QCD sum rules (dotted), the asymptotic model (dashed) and the BLW model (solid).

FIG. 14: LCSR results (solid curves) for the ratio $\sqrt{Q^2} F^p_Y/(F^p + 1.79)$ obtained using the asymptotic form (black solid line), the BLW model (red) with second moments from QCD sum rules (dotted), lattice (dashed) and BLW (solid) and the lattice determination (blue) with $f^p$ from QCD sum rules (dotted), the asymptotic model (dashed) and the BLW model (solid). Red symbols: experimental values obtained via Polarization transfer. Blue symbols: experimental values obtained via Rosenbluth separation.

FIG. 15: $\gamma^*N \rightarrow \Delta$ transition form factors (left: $G_M^\gamma/(3G_D)$ vs. $Q^2$, middle: $R_{EM}$ vs. $Q^2$, right: $R_{SM}$ vs. $Q^2$) in the LCSR approach obtained using the asymptotic form (black solid line), the BLW model (red) with second moments from QCD sum rules (dotted), lattice (dashed) and BLW (solid) and the lattice determination (blue) with $f^p$ from QCD sum rules (dotted), the asymptotic model (dashed) and the BLW model (solid).
(4), (6), (7) lie relatively close together and they agree a little better with experiment, compared to the case where the \( d \)-wave contributions have been neglected. Also for \( G_A^p \) and \( G_T^p \) we get nice results, unless we use the QCD sum rule values of the second moments. As expected, in \( G_E^p / G_M^p \), \( G_E^p / G_D^p \) and \( F_2^p / F_1^p \) cancellations arise that lead to a strong dependence on the concrete form of the nucleon distribution amplitudes.

In the case of the \( N \to \Delta \) transition the inclusion of \( d \)-wave corrections leads to strong enhancements in the prediction of \( G_M^* \), while \( R_{EM} \) and \( R_{SM} \) agree now better with experiment.

**X. CONCLUSION**

We have compared a new determination of the nucleon distribution amplitudes based on lattice QCD with different values available in the literature. The non-perturbative parameters of non-leading conformal spin from the lattice evaluation turned out to be close to the asymptotic form and very close to the BLW model. For the leading conformal spin parameters \( f_N \), \( \lambda_1 \) and \( \lambda_2 \) the deviation between lattice and QCD sum rules is about 30\%, which is possibly due to neglected radiative corrections in the QCD sum rules estimates. Our models for the nucleon distribution amplitudes can be related to measurable form factors with light-cone sum rules. Despite the fact that the light-cone sum rules are only calculated to leading order in QCD and despite an intrinsic uncertainty of light-cone sum rules of about \( \pm 20\% \) we get a very good description of \( G_E^p \), \( G_M^p \), \( G_A^p \) and \( G_T^p \) at intermediate momentum transfer. In \( G_E^p \), \( G_M^p \) and \( F_2^p / F_1^p \) cancellations occur, which limit our predictive power. In general we found the following tendency: The asymptotic distribution amplitudes describe the data already amazingly well. Pure QCD sum rule estimates for the non-perturbative parameters overestimate the deviation from the asymptotic form, but the deviation goes in the right direction. The best results are obtained for the BLW values and the very similar lattice values. Including also \( d \)-wave contributions to the twist-3 distribution amplitude improves the description of \( G_M^* \), \( G_M^p \), \( G_A^p \) and \( G_T^p \) a little bit, but results also in bigger uncertainties in \( G_E^p \), \( G_M^p \) and \( F_2^p / F_1^p \). In the case of the \( N \to \Delta \) transition, we do not see the steep fall-off of \( G_M^* \), but the smallness of \( R_{EM} \) and \( R_{SM} \) is very well reproduced.

Further improvements on the theoretical side can be achieved by determining the NLO QCD corrections to light-cone sum rules, which connect the nucleon distribution amplitudes to the form factors. To match the NLO QCD accuracy also \( \alpha_s \)-corrections have then to be included in all QCD sum rule estimates of the moments of the nucleon distribution amplitudes.

**XI. ACKNOWLEDGMENT**

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APPENDIX A:NUCLEON DISTRIBUTION AMPLITUDES UP TO TWIST-6

For completeness we give in this appendix the full expressions for the nucleon distribution amplitudes up to twist-6, details can be found in [2, 6]. The general Lorentz decomposition of the matrix element defined in Eq. (5) reads [2]

$$\langle 0 + | P \rangle = S_1 m_N C_{\alpha \beta} (\gamma_5 N)_\gamma + S_2 m_N C_{\alpha \beta} (\not \! \! \! \! \! \! \gamma_5 N)_\gamma + P_1 m_N (\gamma_5 C)_{\alpha \beta} N_\gamma + P_2 m_N^2 (\gamma_5 C)_{\alpha \beta} (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma$$

$$+ \left( V_1 + \frac{x^2 m_N^2}{4} V_2^M \right) (P C)_{\alpha \beta} (\gamma_5 N)_\gamma + V_3 m_N (P C)_{\alpha \beta} (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma + V_4 m_N (\gamma_5 C)_{\alpha \beta} (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma$$

$$+ \left( A_1 + \frac{x^2 m_N^2}{4} A_1^M \right) (P \gamma_5 C)_{\alpha \beta} N_\gamma + A_2 m_N (P \gamma_5 C)_{\alpha \beta} (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma + A_3 m_N (\gamma_5 C)_{\alpha \beta} (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma$$

$$+ A_4 m_N^2 (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma + A_5 m_N^3 (\gamma_5 C)_{\alpha \beta} (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma + A_6 m_N^4 (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma$$

$$+ T_1 + \frac{x^2 m_N^2}{4} T_1^M \right) (P \gamma_5 C)_{\alpha \beta} (\not \! \! \! \! \! \! \not \! \! \! \! \! \! \gamma N)_\gamma + T_2 m_N (x^\mu P \gamma_5 C)_{\alpha \beta} (\gamma_5 N)_\gamma$$

$$+ T_3 m_N (\sigma \mu C)_{\alpha \beta} (\gamma_5 N)_\gamma + T_4 m_N (P \gamma_5 C)_{\alpha \beta} (\gamma_5 N)_\gamma$$

$$+ T_5 m_N (x^\mu \gamma_5 C)_{\alpha \beta} (\gamma_5 N)_\gamma + T_6 m_N (x^\mu P \gamma_5 C)_{\alpha \beta} (\gamma_5 N)_\gamma$$

$$+ T_7 m_N (x^\mu \gamma_5 C)_{\alpha \beta} (\gamma_5 N)_\gamma + T_8 m_N^3 (x^\mu \gamma_5 C)_{\alpha \beta} (\gamma_5 N)_\gamma,$$

(A1)

with

$$S_1 = S_1, \quad 2 (P \cdot x) S_2 = S_1 - S_2,$$

$$P_1 = P_1, \quad 2 (P \cdot x) P_2 = P_2 - P_1,$$

$$V_1 = V_1, \quad 2 (P \cdot x) V_2 = V_1 - V_2 - V_3,$$

$$2 V_3 = V_3, \quad 4 (P \cdot x) V_4 = -2 V_1 + V_3 + V_4 + 2 V_5,$$

$$4 (P \cdot x) V_5 = V_4 - V_3, \quad 4 (P \cdot x)^2 V_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6,$$

$$A_1 = A_1, \quad 2 (P \cdot x) A_2 = -A_1 + A_2 - A_3,$$

$$2 A_3 = A_3, \quad 4 (P \cdot x) A_4 = -2 A_1 - A_3 - A_4 + 2 A_5,$$

$$4 (P \cdot x) A_5 = A_3 - A_4, \quad 4 (P \cdot x)^2 A_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6,$$

$$T_1 = T_1, \quad 2 (P \cdot x) T_2 = T_1 + T_2 - 2 T_3,$$

$$2 T_3 = T_7, \quad 2 (P \cdot x) T_4 = T_1 - T_2 - 2 T_7,$$

$$2 (P \cdot x) T_5 = -T_1 + T_5 + 2 T_8, \quad 4 (P \cdot x)^2 T_6 = 2 T_2 - 2 T_3 - 2 T_4 + 4 T_5 + 2 T_7 + 2 T_8,$$

$$4 (P \cdot x) T_7 = T_7 - T_8, \quad 4 (P \cdot x)^2 T_8 = -T_1 + T_2 + T_3 - T_5 - T_6 + 2 T_7 + 2 T_8.$$

(A2)

The calligraphic notation is used for distribution amplitudes belonging to a simple Dirac structure, while the non-calligraphic functions denote distribution amplitudes of definite twist. Each distribution amplitude $F = V_i, A_i, T_i, S_i, P_i$ can be represented as

$$F(a_1, a_2, a_3, (P \cdot x)) = \int D x e^{-i (P \cdot x) \sum_i x_i a_i} F(x_i),$$

(A3)

where the functions $F(x_i)$ depend on the dimensionless variables $x_i, 0 < x_i < 1; \sum_i x_i = 1$ which correspond to the longitudinal momentum fractions carried by the quarks inside the nucleon.
APPENDIX B: EXPANSION OF THE NUCLEON DISTRIBUTION AMPLITUDES UP TO NEXT-TO LEADING CONFORMAL SPIN

In \[2\] the distribution amplitudes were expanded up to next-to leading order in the conformal spin. The twist-3 distribution amplitudes read

\[
\begin{align*}
V_1(x_i, \mu) &= 120 x_1 x_2 x_3 \left[ \phi^0_3(\mu) + \phi_3^+(\mu)(1 - 3x_3) \right], \\
A_1(x_i, \mu) &= 120 x_1 x_2 x_3 (x_2 - x_1) \phi_3^- (\mu), \\
T_1(x_i, \mu) &= 120 x_1 x_2 x_3 \left[ \phi^0_3(\mu) - \frac{1}{2} (\phi_3^+ - \phi_3^-)(\mu)(1 - 3x_3) \right].
\end{align*}
\]

The twist-4 distribution amplitudes read

\[
\begin{align*}
V_2(x_i, \mu) &= 24 x_1 x_2 \left[ \phi^0_4(\mu) + \phi_4^+(\mu)(1 - 5x_3) \right], \\
A_2(x_i, \mu) &= 24 x_1 x_2 (x_2 - x_1) \phi_4^- (\mu), \\
T_2(x_i, \mu) &= 24 x_1 x_2 \left[ \xi^0_4(\mu) + \xi_4^+(\mu)(1 - 5x_3) \right], \\
V_3(x_i, \mu) &= 12 x_3 \left[ \psi^0_4(\mu)(1 - x_3) + \psi_4^+(\mu)(1 - x_3 - 10x_1 x_2) \\
&\quad + \psi_4^- (\mu)(x_1^2 + x_2^2 - x_3(1 - x_3)) \right], \\
A_3(x_i, \mu) &= 12 x_3 (x_2 - x_1) \left[ (\psi^0_4 + \psi_4^+)(\mu) + \psi_4^- (\mu)(1 - 2x_3) \right], \\
T_3(x_i, \mu) &= 6 x_3 \left[ (\phi^0_4 + \phi_4^0 + \xi_4^0)(\mu)(1 - x_3) \\
&\quad + (\phi_4^+ + \psi_4^+ + \xi_4^+)(\mu)(1 - x_3 - 10x_1 x_2) \\
&\quad + (\phi_4^- - \psi_4^- + \xi_4^-)(\mu)(x_1^2 + x_2^2 - x_3(1 - x_3)) \right], \\
T_7(x_i, \mu) &= 6 x_3 \left[ (\phi^0_4 + \psi_4^0 - \xi_4^0)(\mu)(1 - x_3) \\
&\quad + (\phi_4^+ + \psi_4^+ - \xi_4^+)(\mu)(1 - x_3 - 10x_1 x_2) \\
&\quad + (\phi_4^- - \psi_4^- - \xi_4^-)(\mu)(x_1^2 + x_2^2 - x_3(1 - x_3)) \right], \\
S_1(x_i, \mu) &= 6 x_3 (x_2 - x_1) \left[ (\phi^0_4 + \phi_4^0 + \xi_4^0 + \phi_4^+ + \psi_4^+ + \xi_4^+)(\mu) \\
&\quad + (\phi_4^- - \psi_4^- + \xi_4^-)(\mu)(1 - 2x_3) \right], \\
P_1(x_i, \mu) &= 6 x_3 (x_1 - x_2) \left[ (\phi^0_4 + \psi_4^0 - \xi_4^0 + \phi_4^+ + \psi_4^+ - \xi_4^+)(\mu) \\
&\quad + (\phi_4^- - \psi_4^- - \xi_4^-)(\mu)(1 - 2x_3) \right].
\end{align*}
\]
The twist-5 amplitudes are given by

\[ V_4(x_i, \mu) = 3 \left[ \psi_5^0(\mu)(1 - x_3) + \psi_5^+(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) + \psi_5^-(\mu)(2x_1x_2 - x_3(1 - x_3)) \right], \]

\[ A_4(x_i, \mu) = 3(x_2 - x_1) \left[ -\psi_5^0(\mu) + \psi_5^+(\mu)(1 - 2x_3) + \psi_5^-(\mu)x_3 \right], \]

\[ T_4(x_i, \mu) = \frac{3}{2} \left[ (\phi_5^0 + \psi_5^0 + \xi_5^0)(\mu)(1 - x_3) + (\phi_5^+ + \psi_5^+ + \xi_5^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) + (\phi_5^- - \psi_5^- + \xi_5^-)(\mu)(2x_1x_2 - x_3(1 - x_3)) \right], \]

\[ T_8(x_i, \mu) = \frac{3}{2} \left[ (\phi_5^0 + \psi_5^0 - \xi_5^0)(\mu)(1 - x_3) + (\phi_5^+ + \psi_5^+ - \xi_5^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) + (\phi_5^- - \psi_5^- - \xi_5^-)(\mu)(2x_1x_2 - x_3(1 - x_3)) \right], \]

\[ V_5(x_i, \mu) = 6x_3 \left[ \phi_5^0(\mu) + \phi_5^+(\mu)(1 - 2x_3) \right], \]

\[ A_5(x_i, \mu) = 6x_3(x_2 - x_1)\phi_5^0(\mu), \]

\[ T_5(x_i, \mu) = 6x_3 \left[ \xi_5^0(\mu) + \xi_5^+(\mu)(1 - 2x_3) \right], \]

\[ S_2(x_i, \mu) = \frac{3}{2}(x_2 - x_1) \left[ - (\phi_5^0 + \psi_5^0 + \xi_5^0)(\mu) + (\phi_5^+ + \psi_5^+ + \xi_5^+)(\mu)(1 - 2x_3) + (\phi_5^- - \psi_5^- + \xi_5^-)(\mu)x_3 \right], \]

\[ P_2(x_i, \mu) = \frac{3}{2}(x_2 - x_1) \left[ - (\phi_5^0 - \psi_5^0 + \xi_5^0)(\mu) + (\phi_5^+ - \psi_5^+ + \xi_5^+)(\mu)(1 - 2x_3) + (\phi_5^- + \psi_5^- + \xi_5^-)(\mu)x_3 \right], \quad \text{(B3)} \]

and the twist-6 contributions are given by

\[ V_6(x_i, \mu) = 2 \left[ \phi_6^0(\mu) + \phi_6^+(\mu)(1 - 3x_3) \right], \]

\[ A_6(x_i, \mu) = 2(x_2 - x_1)\phi_6^0, \]

\[ T_6(x_i, \mu) = 2 \left[ \phi_6^0(\mu) - \frac{1}{2} (\phi_6^+ - \phi_6^-) (1 - 3x_3) \right]. \quad \text{(B4)} \]

The coefficients \( \phi_i^r, \psi_i^r \) and \( \xi_i^r \) (i stands for the twist) in the above expansions can be expressed in terms of the eight non-perturbative parameters \( f_N, \lambda_1, \lambda_2, f_1^a, f_1^d, f_2^d, A_t^\nu, V_1^d \), defined in section 2. The corresponding relations read, for the leading conformal spin:

\[ \phi_0^0 = \phi_6^0 = f_N, \quad \phi_0^+ = \phi_0^- = \frac{1}{2} (f_N + \lambda_1), \]

\[ \xi_0^0 = \xi_0^+ = \frac{1}{6} \lambda_2, \quad \psi_0^0 = \psi_0^- = \frac{1}{2} (f_N - \lambda_1). \quad \text{(B5)} \]

For the next-to-leading spin, for twist-3:

\[ \phi_3^- = \frac{21}{2} f_N A_t^\nu, \quad \phi_3^+ = \frac{7}{2} f_N (1 - 3V_1^d), \quad \text{(B6)} \]

for twist-4:

\[ \phi_4^+ = \frac{1}{4} \left[ f_N(3 - 10V_1^d) + \lambda_1(3 - 10f_1^d) \right], \]

\[ \phi_4^- = -\frac{5}{4} \left[ f_N(1 - 2A_t^\nu) - \lambda_1(1 - 2f_1^d + 4f_1^v) \right], \]

\[ \psi_4^+ = -\frac{1}{4} \left[ f_N(2 + 5A_t^\nu - 5V_1^d) - \lambda_1(2 - 5f_1^d - 5f_1^v) \right], \]

\[ \psi_4^- = \frac{5}{4} \left[ f_N(2 - A_t^\nu - 3V_1^d) - \lambda_1(2 - 7f_1^d + f_1^v) \right], \]

\[ \xi_4^+ = \frac{1}{16} \lambda_2(4 - 15f_2^d), \quad \xi_4^- = \frac{5}{16} \lambda_2(4 - 15f_2^d). \quad \text{(B7)} \]
for twist-5:

$$
\phi_5 = -\frac{5}{6} \left[ f_N(3 + 4V_1^d) - \lambda_1(1 - 4f_1^d) \right],
$$

$$
\phi_5 = -\frac{5}{3} \left[ f_N(1 - 2A_1^u) - \lambda_1(f_1^d - f_1^u) \right],
$$

$$
\psi_5^+ = -\frac{5}{6} \left[ f_N(5 + 2A_1^u - 2V_1^d) - \lambda_1(1 - 2f_1^d - 2f_1^u) \right],
$$

$$
\psi_5^- = \frac{5}{3} \left[ f_N(2 - A_1^u - 3V_1^d) + \lambda_1(f_1^d - f_1^u) \right],
$$

$$
\xi_5^+ = \frac{5}{36} \lambda_2(2 - 9f_2^d), \quad \xi_5^- = -\frac{5}{4} \lambda_2 f_2^d,
$$

(B8)

and for twist-6:

$$
\phi_6^+ = \frac{1}{2} \left[ f_N(1 - 4V_1^d) - \lambda_1(1 - 2f_1^d) \right],
$$

$$
\phi_6^- = \frac{1}{2} \left[ f_N(1 + 4A_1^u) + \lambda_1(1 - 4f_1^d - 2f_1^u) \right].
$$

(B9)

**$x^2$-corrections:**

Next we summarize the expressions for the $x^2$-corrections to the leading twist distribution amplitudes $V_1$, $A_1$ and $T_1$. These corrections have been determined in \[20, 21, 22, 23, 24\].

For $V_1$ we have

$$
V_1^{M(u)}(x_2) = \int_0^{1-x_2} dx_1 V_1^{M}(x_1, x_2, 1 - x_1 - x_2) = \frac{x_2^2}{24} \left( f_N C_f^u + \lambda_1 C_\lambda^u \right),
$$

$$
V_1^{M(d)}(x_3) = \int_0^{1-x_3} dx_1 V_1^{M}(x_1, 1 - x_1 - x_3, x_3) = \frac{x_3^2}{24} \left( f_N C_f^d + \lambda_1 C_\lambda^d \right)
$$

(B10)

with

$$
C_f^u = (1 - x_2)^3 \left[ 113 + 495x_2 - 552x_2^2 - 10A_1^u(1 - 3x_2) + 2V_1^d(113 - 951x_2 + 828x_2^2) \right],
$$

$$
C_\lambda^u = -(1 - x_2)^3 \left[ 13 - 20f_1^d + 3x_2 + 10f_1^u(1 - 3x_2) \right],
$$

$$
C_f^d = -(1 - x_3)^3 \left[ 1441 + 505x_3 - 3371x_3^2 + 3405x_3^3 - 1104x_3^4 
- 24V_1^d(207 - 3x_3 - 368x_3^2 + 412x_3^3 - 138x_3^4) \right] - 12(73 - 220V_1^d) \ln(x_3),
$$

$$
C_\lambda^d = -(1 - x_3)^3 \left[ 11 + 131x_3 - 169x_3^2 + 63x_3^3 - 30f_1^d(3 + 11x_3 - 17x_3^2 + 7x_3^3) 
- 12(3 - 10f_1^d) \ln(x_3) \right].
$$

(B11)

In the case of $A_1$ one finds

$$
A_1^{M(u)}(x_2) = \int_0^{1-x_2} dx_1 A_1^{M}(x_1, x_2, 1 - x_1 - x_2) = \frac{x_2^2}{24}(1 - x_2)^3 \left( f_N D_f^u + \lambda_1 D_\lambda^u \right),
$$

$$
A_1^{M(d)}(x_3) = \int_0^{1-x_3} dx_1 A_1^{M}(x_1, 1 - x_1 - x_3, x_3) = 0,
$$

(B12)

with

$$
D_f^u = 11 + 45x_2 - 2A_1^u(113 - 951x_2 + 828x_2^2) + 10V_1^d(1 - 30x_2),
$$

$$
D_\lambda^u = 29 - 45x_2 - 10f_1^u(7 - 9x_2) - 20f_1^d(5 - 6x_2).
$$

(B13)
Finally, for $T_1$ one has

$$T_1^{M(u)}(x_2) = \int_0^{1-x_2} dx_1 T_1^{M}(x_1, x_2, 1 - x_1 - x_2) = \frac{x_2^2}{48} \left(f_N E_f^u + \lambda_1 E_\lambda^u \right),$$

$$T_1^{M(d)}(x_3) = \int_0^{1-x_3} dx_1 T_1^{M}(x_1, 1 - x_1 - x_3, x_3) = \frac{x_3^2(1-x_3)^4}{4} \left(f_N E_f^d + \lambda_1 E_\lambda^d \right)$$

(B14)

with

$$E_f^u = - \left[ (1-x_2) \left( 3(439 + 71x_2 - 621x_2^2 + 587x_2^3 - 184x_2^4) 
+ 4A_1^u(1-x_2)^2(59 - 483x_2 + 414x_2^2) - 4V_1^d(1301 - 619x_2 - 769x_2^2 + 1161x_2^3 - 414x_2^4) \right) 
- 12(3 - 10f_1^d) \ln(x_2),
$$

$$E_\lambda^u = - \left[ (1-x_2)(5 - 211x_2 + 281x_2^2 - 111x_2^3 
+ 10(1+61x_2 - 83x_2^2 + 33x_2^3)f_1^d - 40(1-x_2)^2(2-3x_2)f_1^u) \right] - 12(3 - 10f_1^d) \ln(x_2),
$$

$$E_f^d = 17 + 92x_3 + 12(A_1^u + V_1^d)(3 - 23x_3),
$$

$$E_\lambda^d = -7 + 20f_1^d + 10f_1^u.$$

(B15)

**APPENDIX C: EXPANSION OF THE NUCLEON DISTRIBUTION AMPITUDEN OF TWIST-3 UP TO NEXT-TO-NEXT-TO LEADING CONFORMAL SPIN**

The expansion of the leading-twist distribution amplitude in a basis which is diagonal with respect to one-loop renormalization reads up to next-to-next-to-leading conformal spin

$$\phi(x_1, x_2, x_3, \mu) = 120 x_1 x_2 x_3 f_N(\mu_0) L^{\frac{1}{n_F}} \left\{ 1 + h_{10}(\mu_0)(x_1 - 2x_2 + x_3) L^{\frac{2}{n_F}} 
+ h_{11}(\mu_0)(x_1 - x_3) L^{\frac{3}{n_F}} 
+ h_{20}(\mu_0) \left[ 1 + 7(x_2 - 2x_1 x_3 - 2x_2^2) \right] L^{\frac{4}{n_F}} 
+ h_{21}(\mu_0)(1 - 4x_2)(x_1 - x_3) L^{\frac{5}{n_F}} 
+ h_{22}(\mu_0) \left[ 3 - 9x_2 + 8x_2^2 - 12x_1 x_3 \right] L^{\frac{6}{n_F}} \right\}$$

(C1)

with

$$L = \frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}, \quad \beta_0 = 11 - \frac{2}{3} n_F.$$

(C2)
The coefficients $h_{ij}$ can be expressed in terms of the moments by

$$h_{10}(\mu) = \frac{7}{2} (1 - 3\varphi^{010}(\mu))$$

$$= -\frac{7}{4} (1 - 3(A_1^a(\mu) + V_1^d(\mu)))$$

$$= -\frac{7}{4} \left( \delta^+_4(\mu) - \delta^-_3(\mu) \right),$$

$$h_{11}(\mu) = \frac{21}{2} (\varphi^{100}(\mu) - \varphi^{001}(\mu))$$

$$= \frac{21}{4} (1 + A_1^a(\mu) - 3V_1^d(\mu))$$

$$= \frac{1}{2} \left( 3\delta^+_3(\mu) + \delta^-_3(\mu) \right),$$

$$h_{20}(\mu) = \frac{18}{5} \left( h_{10}(\mu) + 4 - 7(3\varphi^{101}(\mu) + \varphi^{200}(\mu) + \varphi^{002}(\mu)) \right),$$

$$h_{21}(\mu) = 126 (\varphi^{200}(\mu) - \varphi^{002}(\mu)) - 9h_{11}(\mu),$$

$$h_{22}(\mu) = \frac{21}{5} (-h_{10}(\mu) - 4 + 6(\varphi^{101}(\mu) + 2\varphi^{200}(\mu) + 2\varphi^{002}(\mu))).$$

Of course, this form of $\varphi$ is not uniquely determined by the moments. The anomalous dimensions were obtained, e.g., in [114, 115, 116].

One can also write down the renormalization group equations for the moments $\varphi^{n_1n_2n_3} = V_1^{n_1n_2n_3} - A_1^{n_1n_2n_3}$ alone:

$$\varphi^{100}(\mu) = \frac{1}{21} \left( 7 + h_{10}(\mu_0) L^{\frac{n_0}{n_0}} + h_{11}(\mu_0) L^{\frac{n_0}{n_0}} \right),$$

$$\varphi^{010}(\mu) = \frac{1}{21} \left( 7 - 2h_{10}(\mu_0) L^{\frac{n_0}{n_0}} \right),$$

$$\varphi^{001}(\mu) = \frac{1}{21} \left( 7 + h_{10}(\mu_0) L^{\frac{n_0}{n_0}} - h_{11}(\mu_0) L^{\frac{n_0}{n_0}} \right),$$

$$\varphi^{101}(\mu) = \frac{1}{126} \left( 12 + 3h_{10}(\mu_0) L^{\frac{n_0}{n_0}} - 2h_{20}(\mu_0) L^{\frac{n_0}{n_0}} - h_{22}(\mu_0) L^{\frac{n_0}{n_0}} \right),$$

$$\varphi^{200}(\mu) = \frac{1}{252} \left( 36 + 9h_{10}(\mu_0) L^{\frac{n_0}{n_0}} + 9h_{11}(\mu_0) L^{\frac{n_0}{n_0}} + h_{20}(\mu_0) L^{\frac{n_0}{n_0}} + h_{21}(\mu_0) L^{\frac{n_0}{n_0}} + 3h_{22}(\mu_0) L^{\frac{n_0}{n_0}} \right),$$

$$\varphi^{002}(\mu) = \frac{1}{252} \left( 36 + 9h_{10}(\mu_0) L^{\frac{n_0}{n_0}} - 9h_{11}(\mu_0) L^{\frac{n_0}{n_0}} + h_{20}(\mu_0) L^{\frac{n_0}{n_0}} - h_{21}(\mu_0) L^{\frac{n_0}{n_0}} + 3h_{22}(\mu_0) L^{\frac{n_0}{n_0}} \right).$$

Next we determine $V_1$, $A_1$ and $T_1$ from $\varphi$ up to $d$-wave contributions. Including all anomalous dimensions we obtain (in the following we suppress the explicit renormalization scale dependence in the formulas)

$$V_1(x_1, x_2, x_3) = 120x_1x_2x_3 f_N L^{\frac{n_0}{n_0}} \left\{ 1 - \frac{h_{10}}{2} (1 - 3x_3)L^{\frac{n_0}{n_0}} + \frac{h_{11}}{2} (1 - 3x_3)L^{\frac{n_0}{n_0}} \right\}$$

$$- \frac{h_{20}}{2} (-2 + 7(x_1 + x_2 - 4x_1x_2)) L^{\frac{n_0}{n_0}}$$

$$- \frac{h_{21}}{2} [-1 + 8x_2 - 8x_2^2 - x_3 - 8x_2x_3 + 4x_3^2] L^{\frac{n_0}{n_0}}$$

$$+ \frac{h_{22}}{2} [6 - 21x_1 + 20x_1^2 - 21x_2 + 24x_1x_2 + 20x_2^2] L^{\frac{n_0}{n_0}} \right\},$$

$$A_1(x_1, x_2, x_3) = -60x_1x_2x_3 (x_1 - x_2) f_N L^{\frac{n_0}{n_0}} \left\{ 3h_{10} L^{\frac{n_0}{n_0}} + h_{11} L^{\frac{n_0}{n_0}} \right\}$$

$$+ (1 - 4x_3) \left( 7h_{20} L^{\frac{n_0}{n_0}} + h_{21} L^{\frac{n_0}{n_0}} + h_{22} L^{\frac{n_0}{n_0}} \right) \right\},$$
\[ T_1(x_1, x_2, x_3) = 120 x_1 x_2 x_3 f_N L^{\frac{3}{20}} \left\{ 1 + h_{10}(1 - 3 x_3) L^{\frac{8}{20}} \right. \]
\[ - h_{20} \left[ -1 + 14 x_1 x_2 - 7 x_3 + 14 x_3^2 \right] L^{\frac{14}{20}} \]
\[ - h_{22} \left[ -3 + 12 x_1 x_2 + 9 x_3 - 8 x_3^2 \right] L^{\frac{22}{20}} \right\} . \tag{C20} \]

In the light-cone sum rule determination of the nucleon form factors we need the distribution amplitudes at a certain renormalization scale \( \mu = \mu_0 \), therefore we give also the simplified expressions \( (L = 1) \) in the following.

Our expressions agree up to next-to-leading conformal spin with the corresponding ones of \cite{2}. In \cite{16} the vector function \( V_1 \) including the next-to-next-to-leading conformal spin was used with the following notation:

\[ V_1(x_1, x_2, x_3) = 120 x_1 x_2 x_3 f_N \left\{ 1 + \tilde{\phi}_3^d (\mu)(1 - 3 x_3) + \tilde{\phi}_3^{d4} [3 - 21 x_3 + 28 x_3^2] \right. \]
\[ + \tilde{\phi}_3^{d2} [5(x_1^2 + x_2^2) - 3(1 - x_3)^2] \right\} , \tag{C21} \]

\[ A_1(x_1, x_2, x_3) = 120 x_1 x_2 x_3(x_2 - x_1) f_N \left\{ \tilde{\phi}_3^- + (1 - 4 x_3) \tilde{\phi}_3^{d3} \right\} , \tag{C22} \]

\[ T_1(x_1, x_2, x_3, \mu) = 120 x_1 x_2 x_3 f_N \left\{ 1 + \frac{1}{2}(\tilde{\phi}_3^- - \tilde{\phi}_3^+) (1 - 3 x_3) \right. \]
\[ + \tilde{\phi}_3^{d4} [1 - 14 x_1 x_2 + 7 x_3 - 14 x_3^2] \]
\[ + \tilde{\phi}_3^{d5} [1 + x_1 x_2 - 8 x_3 + 11 x_3^2] \right\} , \tag{C23} \]

with

\[ \tilde{\phi}_3^{d1} = \frac{1}{10} (h_{20} - h_{21} + 3 h_{22}) \]
\[ = \frac{9}{10} (3 + 28 \varphi_0^{002} - 21 V_1^d) , \tag{C24} \]

\[ \tilde{\phi}_3^{d2} = \frac{1}{5} (-7 h_{20} + 2 h_{21} + 4 h_{22}) \]
\[ = -\frac{63}{5} (3 + 5 A_1^a - V_1^d - 2(\varphi_0^{002} + 5 \varphi_{101} + 5 \varphi_{200})) , \tag{C25} \]

\[ \tilde{\phi}_3^{d3} = \frac{1}{2} (7 h_{20} + h_{21} + h_{22}) \]
\[ = \frac{63}{2} (A_1^a + 4 V_1^d - 4(\varphi_0^{002} + 2 \varphi_{101})) , \tag{C26} \]

\[ \tilde{\phi}_3^{d4} = h_{20} + h_{22} \]
\[ = -\frac{9}{20} (3 + 7(A_1^a + V_1^d) - 56(\varphi_0^{002} - 2 \varphi_{101} + \varphi_{200})) , \tag{C27} \]

\[ \tilde{\phi}_3^{d5} = 2 h_{22} \]
\[ = -\frac{63}{10} (3 + 7(A_1^a + V_1^d) - 8(2 \varphi_0^{002} + \varphi_{101} + 2 \varphi_{200})) . \tag{C28} \]

Numerically one obtains for the COZ model \cite{30}

\[ \tilde{\phi}_3^{d1}(\mu = 1 \text{GeV}) = 0.61 , \tag{C34} \]

\[ \tilde{\phi}_3^{d2}(\mu = 1 \text{GeV}) = 3.7 . \tag{C35} \]

This agrees with the numbers quoted in \cite{16}. Using the lattice calculation we get

\[ \tilde{\phi}_3^{d1}(\mu = 1 \text{GeV}) = 0.51 , \tag{C36} \]

\[ \tilde{\phi}_3^{d2}(\mu = 1 \text{GeV}) = 0.71 . \tag{C37} \]

Here again the pure QCD sum rule calculation seems to overestimate the effects.
APPENDIX D: MODELS FOR THE LEADING-TWIST NUCLEON DISTRIBUTION AMPLITUDE

In this section we present concrete models for the leading-twist nucleon distribution amplitude including next-to-next-to-leading conformal spin at the renormalization scale 1 GeV. At twist-3 one independent distribution amplitude \( \varphi(x_1, x_2, x_3, \mu) \) arises, see, e.g., \(^2\):

\[
\varphi(x_1, x_2, x_3, \mu) = (V_1 - A_1)(x_1, x_2, x_3, \mu). \tag{D1}
\]

In \(^2\) this distribution amplitude was denoted by \( \Phi_3(x_1, x_2, x_3, \mu) \). From \( \varphi \) one easily gets \( V_1, A_1 \) and \( T_1 \), see, e.g., \(^2\):

\[
T_1(x_1, x_2, x_3) = \frac{1}{2}\left[ \varphi(x_1, x_3, x_2) + \varphi(x_2, x_3, x_1) \right], \tag{D2}
\]

\[
V_1(x_1, x_2, x_3) = \frac{1}{2}\left[ \varphi(x_1, x_2, x_3) + \varphi(x_2, x_1, x_3) \right], \tag{D3}
\]

\[
A_1(x_1, x_2, x_3) = \frac{1}{2}\left[ \varphi(x_2, x_1, x_3) - \varphi(x_1, x_2, x_3) \right]. \tag{D4}
\]

The asymptotic form - only the leading conformal spin contribution - of \( \varphi(x_1, x_2, x_3, \mu) \) reads

\[
\varphi_{Asy}(x_1, x_2, x_3, \mu) = 120x_1x_2x_3\phi_0^0(\mu), \quad (\phi_0^0 \equiv f_N). \tag{D5}
\]

Including next-to-leading conformal spin one gets \(^2\)

\[
\varphi(x_1, x_2, x_3, \mu) = \varphi_{Asy}(x_1, x_2, x_3, \mu) \left[ 1 + \tilde{\phi}_3^-(\mu)(x_1 - x_2) + \tilde{\phi}_3^+(\mu)(1 - 3x_3) \right], \tag{D6}
\]

with

\[
\tilde{\phi}_3^- = \frac{\phi_3^-}{\phi_3^0}, \quad \tilde{\phi}_3^+ = \frac{\phi_3^+}{\phi_3^0}. \tag{D7}
\]

In the literature also second moments of the leading-twist distribution amplitude were determined with QCD sum rules \(^2, 30, 37\). With this information one can build models for \( \varphi(x_1, x_2, x_3) \) at a certain renormalization scale \( \mu \), including next-to-next-to leading conformal spin. We will use the model from \(^30\) \( \varphi^{COZ}(x_1, x_2, x_3) \) and the model from \(^37\) \( \varphi^{KS}(x_1, x_2, x_3) \):

\[
\varphi^{COZ}(x_1, x_2, x_3) = \varphi_{Asy}(x_1, x_2, x_3)
\left[ 23.814x_1^2 + 12.978x_2^2 + 6.174x_3^2 + 5.88x_3 - 7.098 \right], \tag{D8}
\]

\[
\varphi^{KS}(x_1, x_2, x_3) = \varphi_{Asy}(x_1, x_2, x_3)
\left[ 20.16x_1^2 + 15.12x_2^2 + 22.68x_3^2 - 6.72x_3 + 1.68(x_1 - x_2) - 5.04 \right]. \tag{D9}
\]

Bolz and Kroll derived a very simple model using some experimental constraints \(^47\). Their model for the leading-twist distribution amplitude reads

\[
\varphi^{BK}(x_1, x_2, x_3) = \frac{1}{2}\varphi_{Asy}(x_1, x_2, x_3)(1 + 3x_3). \tag{D10}
\]

Based on the lattice calculations of \( \varphi^{100}, \varphi^{001}, \varphi^{101}, \varphi^{200} \) and \( \varphi^{002} \) in \(^2, 41\) and Eq. \((D1)\) we have obtained the model

\[
\varphi^{LAT}(x_1, x_2, x_3) = \varphi_{Asy}(x_1, x_2, x_3)(- 0.401 + 29.214x_1 - 44.542x_2 + 7.664x_3
\quad + 12.561x_2x_3 + 31.748x_1x_3 - 103.09x_2x_3
\quad - 41.880x_1^2 + 92.958x_2^2 + 17.836x_3^2). \tag{D11}
\]

[1] V. L. Chernyak and A. R. Zhitnitsky, “Asymptotic Behavior of Exclusive Processes in QCD,” Phys. Rept. 112 (1984) 173.

[2] V. Braun, R. J. Fries, N. Mahnke, and E. Stein, “Higher twist distribution amplitudes of the nucleon in QCD,” Nucl. Phys. B589 (2000) 381–409, arXiv:hep-ph/0007279.
QCDSF Collaboration, M. Göckeler et al., “Moments of nucleon distribution amplitudes from irreducible three-quark operators,” PoS LAT2007 (2007) 147, arXiv:0710.2489 [hep-lat].

M. Göckeler et al., “Nucleon distribution amplitudes from lattice QCD,” Phys. Rev. Lett. 101 (2008) 112002, arXiv:0804.1877 [hep-lat].

QCDSF Collaboration, V. M. Braun et al., “Nucleon distribution amplitudes and proton decay matrix elements on the lattice,” arXiv:0811.2712 [hep-lat].

V. M. Braun, A. Lenz, and M. Wittmann, “Nucleon form factors in QCD,” Phys. Rev. D73 (2006) 094019, arXiv:hep-ph/0604050.

V. L. Chernyak and A. R. Zhitnitsky, “Asymptotic Behavior of Hadron-Form-Factors in Quark Model,” JETP Lett. 25 (1977) 510.

V. L. Chernyak and A. R. Zhitnitsky, “Asymptotics of Hadronic Form-Factors in the Quantum Chromodynamics,” Sov. J. Nucl. Phys. 31 (1980) 544–552.

V. L. Chernyak, A. R. Zhitnitsky, and V. G. Serbo, “Asymptotic hadronic form-factors in quantum chromodynamics,” JETP Lett. 26 (1977) 594–597.

V. L. Chernyak, V. G. Serbo, and A. R. Zhitnitsky, “Calculation of asymptotics of the pion electromagnetic form-factor in the QCD perturbation theory,” Sov. J. Nucl. Phys. 31 (1980) 552–558.

A. V. Radyushkin, “Deep elastic processes of composite particles in field theory and asymptotic freedom,” arXiv:hep-ph/0410276.

A. V. Efremov and A. V. Radyushkin, “Asymptotical Behavior of Pion Electromagnetic Form-Factor in QCD,” Theor. Math. Phys. 42 (1980) 97–110.

A. V. Efremov and A. V. Radyushkin, “Factorization and Asymptotical Behavior of Pion Form-Factor in QCD,” Phys. Lett. 98B (1980) 245–250.

G. P. Lepage and S. J. Brodsky, “Exclusive Processes in Quantum Chromodynamics: Evolution Equations for Hadronic Wave Functions and the Form-Factors of Mesons,” Phys. Lett. 88B (1979) 359–365.

G. P. Lepage and S. J. Brodsky, “Exclusive Processes in Perturbative Quantum Chromodynamics,” Phys. Rev. D22 (1980) 2157.

V. M. Braun, A. Lenz, N. Mahnke, and E. Stein, “Light-cone sum rules for the nucleon form factors,” Phys. Rev. D65 (2002) 074011, arXiv:hep-ph/0112085.

I. I. Balitsky, V. M. Braun, and A. V. Kolesnichenko, “Radiative Decay $\Sigma^+ \rightarrow p\gamma$ in Quantum Chromodynamics,” Nucl. Phys. B312 (1989) 509–550.

V. L. Chernyak and I. R. Zhitnitsky, “B meson exclusive decays into baryons,” Nucl. Phys. B345 (1990) 137–172.

Jefferson Lab Hall A Collaboration, M. K. Jones et al., “$G_E^p/G_M^p$ ratio by polarization transfer in $e+p \rightarrow e+p$,” Phys. Rev. Lett. 84 (2000) 1398–1402, arXiv:nucl-ex/9910005.

O. Gayou et al., “Measurements of the elastic electromagnetic form factor ratio $\mu_p G_E^p/G_M^p$ via polarization transfer,” Phys. Rev. C64 (2001) 038202.

Jefferson Lab Hall A Collaboration, O. Gayou et al., “Measurement of $G_E^p/G_M^p$ in $e+p \rightarrow e+p$ to $Q^2 = 5.6\text{GeV}^2$,” Phys. Rev. Lett. 88 (2002) 092301, arXiv:nucl-ex/011010.

V. Punjabi et al., “Proton elastic form factor ratios to $Q^2 = 3.5\text{GeV}^2$ by polarization transfer,” Phys. Rev. C71 (2005) 055202, arXiv:nucl-ex/0501018.

C. F. Perdrisat, V. Punjabi, and M. Vanderhaeghen, “Nucleon electromagnetic form factors,” Prog. Part. Nucl. Phys. 59 (2007) 694–764, arXiv:hep-ph/0612014.

V. M. Braun, A. Khodjamirian, and M. Mau, “Pion form factor in QCD at intermediate momentum transfers,” Phys. Rev. D61 (2000) 073004, arXiv:hep-ph/9907495.

Y. Aoki, C. Dawson, J. Noaki, and A. Soni, “Proton decay matrix elements with domain-wall fermions,” Phys. Rev. D75 (2007) 014507, arXiv:hep-lat/0607002.

Y. Aoki et al., “Proton lifetime bounds from chirally symmetric lattice QCD,” arXiv:0806.1031 [hep-lat].

M.-q. Huang and D.-W. Wang, “Light-cone QCD sum rules for the semileptonic decay $\Lambda_b \rightarrow pl\bar{\nu}$,” Phys. Rev. D69 (2004) 094003, arXiv:hep-ph/0401094.

A. J. Lenz, “Form factors of baryons within the framework of light-cone sum rules,” AIP Conf. Proc. 964 (2007) 77–83, arXiv:0708.0633 [hep-ph].

V. L. Chernyak and I. R. Zhitnitsky, “Nucleon Wave Function and Nucleon Form-Factors in QCD,” Nucl. Phys. B246 (1984) 52–74.

V. L. Chernyak, A. A. Ogloblin, and I. R. Zhitnitsky, “The wave functions of the octet baryons,” Z. Phys. C42 (1989) 569.

M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, “QCD and Resonance Physics. Sum Rules,” Nucl. Phys. B147 (1979) 385–447.

V. M. Braun and A. Lenz, “On the SU(3) symmetry-breaking corrections to meson distribution amplitudes,” Phys. Rev. D70 (2004) 074020, arXiv:hep-ph/0407282.

P. Ball, V. M. Braun, and A. Lenz, “Higher-twist distribution amplitudes of the K meson in QCD,” JHEP 05 (2006) 004, arXiv:hep-ph/0603063.

P. Ball, V. M. Braun, and A. Lenz, “Twist-4 Distribution Amplitudes of the K and $\phi$ Mesons in QCD,” JHEP 08 (2007) 099, arXiv:0707.1201 [hep-ph].

V. M. Braun et al., “Moments of pseudoscalar meson distribution amplitudes from the lattice,” Phys. Rev. D74 (2006) 074501, arXiv:hep-lat/0606012.
J. J. Kelly, “Simple parametrization of nucleon form factors,” Phys. Rev. C 70 (2004) 015206, arXiv:nucl-ex/0401030.

I. A. Qattan et al., “Precision Rosenbluth measurement of the proton elastic form factors,” Phys. Rev. Lett. 94 (2005) 142301, arXiv:nucl-ex/0410010.

W. Bartel et al., “Measurement of proton and neutron electromagnetic form-factors at squared four momentum transfers up to 3 (GeV/c)^2,” Nucl. Phys. B588 (1973) 429–475.

L. E. Price et al., “Backward-angle electron-proton elastic scattering and proton electromagnetic form-factors,” Phys. Rev. D4 (1971) 45–53.

J. Arrington, “How well do we know the electromagnetic form factors of the proton?,” Phys. Rev. C68 (2003) 034325, arXiv:nucl-ex/0305009.

C. B. Crawford et al., “Measurement of the proton electric to magnetic form factor ratio from H-1(pol.)(e(pol.),e' p),” Phys. Rev. Lett. 98 (2007) 052301, arXiv:nucl-ex/0609007.

Bates FPP Collaboration, B. D. Milbraith et al., “A comparison of polarization observables in electron scattering from the proton and deuteron,” Phys. Rev. Lett. 80 (1998) 452–455, arXiv:nucl-ex/9712006.

A1 Collaboration, T. Pospischil et al., “Measurement of G(E(p))/G(M(p)) via polarization transfer at Q^2 = 0.4 (GeV/c)^2,” Eur. Phys. J. A12 (2001) 125–127.

M. Jones private communication.

A. Lung et al., “Measurements of the electric and magnetic form-factors of the neutron from Q^2 = 1.75 (GeV/c)^2 to 4 (GeV/c)^2,” Phys. Rev. Lett. 70 (1993) 718–721.

G. Kubon et al., “Precise neutron magnetic form factors,” Phys. Lett. B524 (2002) 26–32, arXiv:nucl-ex/0107016.

H. Anklin et al., “Precise measurements of the neutron magnetic form factor,” Phys. Lett. B428 (1998) 248–253.

E93-038 Collaboration, R. Madey et al., “Measurements of G(E)(n)/G(M)(n) from the H-2(e(pol.),e' n(pol.))(H-1) reaction to Q^2 = 1.45 (GeV/c)^2,” Phys. Rev. Lett. 91 (2003) 122002, arXiv:nucl-ex/0308007.

W. Xu et al., “The transverse asymmetry A(T') from quasielastic polarized He-3(pol.)(e(pol.),e') process and the neutron magnetic form factor,” Phys. Rev. Lett. 85 (2000) 2900–2904, arXiv:nucl-ex/0008003.

Jefferson Lab E95-001 Collaboration, W. Xu et al., “PWIA extraction of the neutron magnetic form factor from quasi-elastic He-3(pol.)(e(pol.),e') at Q^2 = 0.3 (GeV/c)^2 to 0.6 (GeV/c)^2,” Phys. Rev. C67 (2003) 012201, arXiv:nucl-ex/0208007.

H. Anklin et al., “Precision measurement of the neutron magnetic form factor,” Phys. Lett. B336 (1994) 313–318.

E93026 Collaboration, H. Zhu et al., “A measurement of the electric form-factor of the neutron through d(pol.)(e(pol.),e' n(pol.)) at Q^2 = 0.5 (GeV/c)^2,” Phys. Rev. Lett. 87 (2001) 081801, arXiv:nucl-ex/0105001.

D. Rohe et al., “Measurement of the neutron electric form factor G(en) at 0.67 (GeV/c)^2 via He-3(pol.)(e(pol.),e' n),” Phys. Rev. Lett. 83 (1999) 4257–4260.

C. Herberg et al., “Determination of the neutron electric form factor in the D(e,e' n)p reaction and the influence of nuclear binding,” Eur. Phys. J. A5 (1999) 131–135.

M. Ostrick et al., “Measurement of the neutron electric form factor G(E,n) in the quasi-free H-2(e(pol.),e' n(pol.))p reaction,” Phys. Rev. Lett. 83 (1999) 276–279.

J. Becker et al., “Determination of the neutron electric form factor from the reaction He-3(e,e' n) at medium momentum transfer,” Eur. Phys. J. A6 (1999) 329–344.

T. Eden et al., “Electric form-factor of the neutron from the H-2 (e, polarized), e-prime n (polarized) ) H-1 reaction at Q^2 = 0.255 (GeV/c)^2,” Phys. Rev. C50 (1994) 1749–1753.

Jefferson Laboratory E93-038 Collaboration, B. Plaster et al., “Measurements of the neutron magnetic form factor ratio G(E(n))/G(M(n)) via the H-2(e(pol.),e' n(pol.))(H-1) reaction to Q^2 = 1.45 (GeV/c)^2,” Phys. Rev. C73 (2006) 025205, arXiv:nucl-ex/0511025.

BLAST Collaboration, E. Geis et al., “The Charge Form Factor of the Neutron at Low Momentum Transfer from the ^2H(e,e' n)p Reaction,” arXiv:0803.3827 [nucl-ex].

D. I. Glazier et al., “Measurement of the Electric Form Factor of the Neutron at Q^2 = 0.3 – 0.8(GeV/c)^2,” Eur. Phys. J. A24 (2005) 101–109, arXiv:nucl-ex/0410026.

Jefferson Lab E93-026 Collaboration, G. Warren et al., “Measurement of the electric form factor of the neutron at Q^2 = 0.5 (GeV/c)^2 and 1.0 (GeV/c)^2,” Phys. Rev. Lett. 92 (2004) 042301, arXiv:nucl-ex/0308021.

J. Bermuth et al., “The neutron charge form factor and target analyzing powers from He-3(pol.)(e(pol.),e' n) scattering,” Phys. Lett. B564 (2003) 199–204, arXiv:nucl-ex/0303015.

I. Passchier et al., “The charge form factor of the neutron from the reaction H-2(pol.)(e(pol.),e' n)p,” Phys. Rev. Lett. 82 (1999) 4988–4991, arXiv:nucl-ex/9907012.

J. Golak, G. Ziemer, H. Kamada, H. Witala, and W. Gloeckle, “Extraction of electromagnetic neutron form factors through inclusive and exclusive polarized electron scattering on polarized He-3 target,” Phys. Rev. C63 (2001) 034006, arXiv:nucl-th/0008008.

R. Schiavilla and I. Sick, “Neutron charge form factor at large q^2,” Phys. Rev. C64 (2001) 041002, arXiv:nucl-ex/0107004.

S. Galster et al., “Elastic electron - deuteron scattering and the electric neutron form-factor at four momentum transfers 5 fm^-2 < q^2 < 14 fm^-2,” Nucl. Phys. B32 (1971) 221–237.

J. J. Kelly, “Simple parametrization of nucleon form factors,” Phys. Rev. C70 (2004) 068202.

A. Bodek, S. Avvakumov, R. Bradford, and H. Budd, “Vector and Axial Nucleon Form Factors: A Duality Constrained...
Parameterization,” Eur. Phys. J. C53 (2008) 349–354, arXiv:0708.1946 [hep-ex].
[104] P. Stoler, “Baryon form-factors at high Q^2 and the transition to perturbative QCD,” Phys. Rept. 226 (1993) 103–171.
[105] L. M. Stuart et al., “Measurements of the Delta(1232) transition form factor and the ratio sigma(n)/sigma(p) from inelastic electron proton and electron deuteron scattering,” Phys. Rev. D58 (1998) 032003, arXiv:hep-ph/9612416.
[106] W. Bartel et al., “Electroproduction of pions near the Delta(1236) isobar and the form-factor of G*(M)(q^2) of the (gamma N Delta) vertex,” Phys. Lett. B28 (1968) 148–151.
[107] S. Stein et al., “Electron Scattering at 4-Degrees with Energies of 4.5-GeV - 20-GeV,” Phys. Rev. D12 (1975) 1884.
[108] F. Foster and G. Hughes, “Electroproduction of nucleon resonances,” Rept. Prog. Phys. 46 (1983) 1445–1489.
[109] S. S. Kamalov, S. N. Yang, D. Drehsel, O. Haustein, and L. Tiator, “γ*N−−>Δ transition form factors: A new analysis of the JLab data on p(e,e' p)π0 at Q^2 = 2.8 (GeV/c)^2 and 4.0 (GeV/c)^2,” Phys. Rev. C64 (2001) 032201, arXiv:nucl-th/0006068.
[110] J. C. Alder et al., “π0 electroproduction at the first resonance at momentum transfers Q^2=0.6, 1.0 and 1.56 GeV^2,” Nucl. Phys. B46 (1972) 573–592.
[111] V. V. Frolov et al., “Electroproduction of the Delta(1232) resonance at high momentum transfer,” Phys. Rev. Lett. 82 (1999) 45–48, arXiv:hep-ex/9808024.
[112] CLAS Collaboration, K. Joo et al., “Q^2 dependence of quadrupole strength in the gamma*p → Delta(1232)+ → p pi0 transition,” Phys. Rev. Lett. 88 (2002) 122001, arXiv:hep-ex/0110007.
[113] V. M. Braun, A. N. Manashov, and J. Rohrwild, “Baryon Operators of Higher Twist in QCD and Nucleon Distribution Amplitudes,” arXiv:0806.2531 [hep-ph].
[114] M. Bergmann and N. G. Stefanis, “Evolution effects on the nucleon distribution amplitude,” arXiv:hep-ph/9403210.
[115] M. Bergmann, W. Schroers, and N. G. Stefanis, “Large-order trend of the anomalous-dimensions spectrum of trilinear twist-3 quark operators,” Phys. Lett. B458 (1999) 109–116, arXiv:hep-ph/9903339.
[116] V. M. Braun, S. E. Derkachov, G. P. Korchemsky, and A. N. Manashov, “Baryon distribution amplitudes in QCD,” Nucl. Phys. B553 (1999) 355–426, arXiv:hep-ph/9902375.
[117] To be precise, we have determined \( \phi_{000}^2, \phi_{011}^2 \) and \( \phi_{011}'^2 \) from the BLW description and \( \phi_{200}^2, \phi_{020}^2 \) and \( \phi_{011}^2 \) from momentum conservation.