Probing String Theory with Modulated Cosmological Fluctuations

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Abstract

Superstring theory, models with extra dimensions and other SUSY models generically predict that the coupling constants are in fact vacuum expectation values of fields like the dilaton, moduli etc. Assuming some of these fields are light during inflation, we get generation of small classical inhomogeneities in these fields from inflation. Consequently, coupling constants inherit small inhomogeneities at scales much larger than the causal horizon in the early universe. After the moduli get pinned down to their minima, the spatial variations of coupling constants in the late time universe will be erased. However, inhomogeneities in coupling constants in the very early universe would generate modulated large scale fluctuations in all relic species that are produced due to interactions and freezing out. Moreover (p)reheating of the inflaton field results in modulated curvature fluctuations. Even if the standard inflaton fluctuations are suppressed, in this picture we may have pure curvature cosmological fluctuations entirely generated by the modulated spatial variations of the coupling constants during preheating.

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I. INTRODUCTION

Early universe cosmology extends up to energy scales where we expect superstring/M theory to operate. It is of great interest to understand, even in principle, the potential signatures of superstring theory on cosmological observables.

The primordial inflationary stage of the now standard cosmological paradigm acts like a universal amplifier and stretcher of fluctuations. Indeed, vacuum fluctuations of all light (minimally coupled) degrees of freedom are unstable during inflation and appear in subsequent cosmological evolution as classical fluctuations with wavelengths on cosmological scales.

In inflationary models in $3 + 1$ dimensional theory, a scalar, or curvature mode of the metric fluctuations $\Phi$ is generated from quantum fluctuations of the inflaton field $\delta \phi$ \cite{1} while quantum production of gravitational waves (gravitons) results in a tensor mode of metric fluctuations $h_{\mu \nu}$ \cite{2}. In the simplest inflationary models both modes have almost scale free power spectra of fluctuations. Gravitational waves generated from inflation with a Hubble parameter $H$ have amplitude $k^{3/2} h_k \simeq H/M_P$, while the amplitude of the scalar mode depends on the inflaton potential $V(\phi)$, $k^{3/2} \Phi_k \simeq \sqrt{V/\dot{\phi}}$. In simple inflationary models, including chaotic inflation or hybrid inflation, the amplitude of the scalar mode is larger than that of the tensor mode by a factor of ten or more.

The dominant scalar mode and subdominant tensor mode both contribute to the observed CMB temperature anisotropy with magnitude $\Delta T \sim 10^{-5}$, while only the scalar mode contributes to large scale matter clustering. Independently, the tensor mode generates a very small transversal-traceless (B) component of CMB polarization anisotropy. The ever-growing precision of CMB anisotropy measurements in combination with large scale structure observations will open up the possibility of testing effects in cosmological fluctuations at the next, more precise level. I will try to motivate consideration of isocurvature cosmological fluctuations at this level. As an illustration we may use a parallel with basic atomic spectra and their fine structure.

In the context of inflationary cosmological fluctuations, one area of recent interest in string cosmology has been trans-planckian effects which could cause very small changes in primordial cosmological fluctuations at very small level if any at all. Cosmologists working with the braneworlds were looking for the features of extra dimensions in the $T/S$ ratio and details of the power spectra. Notice, however, that similar effects with respect to individual tests (say, the ratio $T/S$) may be attributed to other choices of inflationary models or other (tuned) effects during inflation.

In this paper I suggest a rather different idea \cite{3} of how string/brane cosmology physics may enter in the theory of cosmological fluctuations. Some of the effects we describe here may be tested observationally at a magnitude of the order of the gravitational wave amplitude $\sim \frac{H}{M_P}$ or even higher, depending on a model-dependent numerical pre-factor. The main suggestion, however, will be a new mechanism of generation of almost scale free curvature fluctuations which we will call modulated curvature fluctuations for reasons explained below. This opens a new possibilities to construct the amplitude and the spectra of primordial scalar fluctuations, rather detached from the inflation model building.

Superstring/M theory, supergravity and many phenomenological models of high energy
physics are formulated in various numbers of dimensions \( D > 4 \). The extra dimensions of fundamental theories have their most obvious relevance in application to cosmology. In effective four dimensional theory, compactified extra dimensions emerge as scalar fields, moduli, which we denote collectively as \( \chi_a \). Scalar fields also arise from bulk scalar fields like the dilaton and from projections of other fields. These moduli fields must be stabilized by some mechanism, although moduli with runaway potentials are not excluded. The fields \( \chi_a \) are thought to acquire masses \( m_a \) due to SUSY breaking sometime in the early universe.

Moduli fields in cosmology pose a problem. If the high energy and low energy moduli potentials are different, then after inflation the moduli will acquire large amplitudes. Residual moduli field oscillations will have a matter equation of state with energy density \( \varepsilon \propto 1/a^3 \), and they will thus quickly become dominant, before Big Bang Nucleosynthesis. Even if this problem is resolved, one encounters another problem. Moduli couple to matter gravitationally, so their time decay \( \tau \sim \Gamma^{-1} \approx M_P^2 / m_{\chi} \sim 10^5 (\text{Tev})^3 \) seconds. The products of Tev mass scale moduli decay may destroy successful BBN.

We will dare to ignore the dangerous effects of moduli on cosmology, assuming they are somehow resolved\(^1\), and focus instead on potentially useful consequences of these fields. The mass scale \( m_a \) and the SUSY breaking energy scale are smaller than the energy scale of inflation \( M \sim 10^{16} \text{ Gev} \), \( V \sim M^4 \), \( H^2 \sim V/M_P^2 \), \( H \sim 10^{13} \text{ Gev} \). As long as the moduli fields or dilaton are lighter than \( 10^{13} \text{ Gev} \), the outer space inflation can generate long-wavelength classical spatial inhomogeneities of the moduli fields \( \delta \chi_i \sim H \). We do not expect any impact of moduli inhomogeneities after they are pinned down to their minima at some time \( t_p \).

However, spatial inhomogeneities in the moduli fields present between the end of inflation and BBN may have significant impact if they are imprinted in the coupling constants \( \alpha_i \), making them spatially varying at scales larger than the casual horizon, \( \frac{\delta \alpha}{\alpha} \sim \frac{\delta \chi}{\chi} \sim \frac{H}{M_P} \). Such inhomogeneities would generate large scale isocurvature cosmological fluctuations in all components that are generated due to interactions and freezing out, including radiation, cold dark matter, baryons, etc. before \( t_p \). We will call these modulated fluctuations.

A more radical extension of this idea is to have not only modulated isocurvature but modulated curvature fluctuations. Indeed, if scalar fluctuations of the inflaton field are suppressed, moduli fluctuations may generate significant fluctuations in the matter created during (p)reheating after inflation by spatial modulation of the coupling constants between inflaton and matter.

II. MODULI FLUCTUATIONS FROM INFLATION

To establish this story, consider the pure gravity sector of low energy string theory \( S = \frac{1}{16\pi \kappa_5^2} \int d^D x \ e^{-2\phi} \sqrt{-G} \mathcal{R} \). The usual (toy model) dimensional reduction from \( D \) dimensional space \((x^\mu, y^i)\) to \( 3 + 1 \) space-time \( x^\mu \) gives us (e.g., [6])

\(^1\)This could be done, for instance, by choosing large \( m_a \), or by choosing a larger gravitational coupling as in theories with large extra dimensions, or by allowing runaway moduli without stabilization, see [4].
Here we collectively denoted moduli fields as $\chi_a$. For instance, the graviton in the $D$-dimensional bulk has $\frac{1}{2}D(D - 3)$ degrees of freedom. The massless $3 + 1$ dimensional KK projection contain two degrees of freedom in the usual tensor mode, $2(D - 4)$ gravi-vector degrees of freedom and $\frac{1}{2}(D - 3)(D - 4)$ gravi-scalars. The matter lagrangian $\mathcal{L}_m$ also contains moduli fields (including the dilaton). Most important for us will be the fact that the coupling constant in $\Lambda_m$ depends on $\chi_a$ [6]. We will have similar construction in the braneworld scenario, where one of the $\chi_a$ will be a radion, coupling with matter at the brane [5].

Suppose $3 + 1$ dimensional inflation, which is described by the outer spacetime de Sitter geometry $ds^2 = -dt^2 + e^{2Ht}d\vec{x}^2$, is driven by an inflaton field $\phi$. Consider vacuum fluctuations of the moduli field $\chi_a$ in this geometry. The eigenmode functions $\chi_k(t)e^{i\vec{k}\vec{x}}$ satisfy

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left(\frac{k^2}{a^2} + m^2\right)\chi_k = 0.$$ (2)

For positive frequency vacuum initial conditions in the far past its solutions are given in terms of the Hankel functions $\chi_k(\tau) = \frac{\sqrt{\pi}}{2}H|\tau|^{3/2}H^{(2)}_\lambda(k\tau)$, with the index $\lambda = \sqrt{\frac{9}{4} - \frac{m^2}{H^2}}$, and conformal time $\tau = \int \frac{dt}{a}$. For very light modes $m_a \ll H$, the amplitude of fluctuations is initially oscillating and then is frozen out at the level

$$\chi_k \approx \frac{H}{\sqrt{2k^{3/2}}}$$ (3)

when the physical wavelength of the fluctuations exceeds the Hubble radius $a(t_k)/k > H^{-1}$. This effect means that at the end of inflation there will be an inhomogeneous classical scalar field $\chi(\vec{x})$ that is a realization of a random gaussian field, which is a superposition of standing waves with random phases $\theta_k$ and normally distributed amplitudes with the variance given by the square of (3). In the simplest case the spectrum of fluctuations is scale free, $<\chi^2> = \int d^3k|\chi_k|^2 = \int \frac{dk}{k^3}$. The upper limit of integration corresponds to the latest mode at the end of inflation $k_{\text{max}} \approx H$. The lower limit defines the overall variance of fluctuations. If it is too large, the moduli field may become dominant and play the role of an inflaton field [7]. We assume that there is no significant value of $<\chi^2>$, that is to say that there is no significant deformation of the geometry of the compact dimensions (for instance, bulk gravitons or the radion in braneworlds). For the sake of generality we may have also an initial homogeneous component $\chi(t) = \langle \chi(t, \vec{x}) \rangle$ whose amplitude is frozen during inflation. Notice, however, that varying $m_a$ during inflation we may design the spectrum of fluctuations $\chi_k$, independently on the inflaton potential.

For massive modes with $m \leq H$ the effect is absent. Inflaton field fluctuations $\delta\phi_k(t)$ are generated similarly provided the effective inflaton mass is smaller than $H$. It is not mandatory for moduli fields to be light ($m_a \ll H$) during inflation. Indeed, if one assumes $N = 1$ supergravity with the minimal Kahler potential $K = \frac{\chi^a\chi^a}{M_p^2}$, then inflation induces moduli masses $m_a \sim H$ [8]. However, the same mechanism would make the inflaton field massive, which we would like to exclude in order to save standard inflaton fluctuations. This
can be done by tuning the Kahler potential (or using D-term inflation). To continue, we assume that the same cure is extended to fluctuations of some moduli fields.

To follow the time evolution of the field \( \chi(t, \vec{x}) \) after inflation, we assume \( \chi(t, \vec{x}) = \chi_0(t) + \delta \chi(t, \vec{x}) \). There are different possibilities, depending on the stabilization options. For quadratic moduli potentials \( V(\chi) = \frac{1}{2} m^2 \chi^2 \) the homogeneous and inhomogeneous parts are frozen until the decreasing Hubble parameter \( H \sim 1/t \) drops below \( m_a \). After that \( \chi(t) \) oscillates around its minimum with decreasing amplitude \( \chi(t) \simeq \chi_0 \cos \frac{mt}{m_a^2/2} \), where \( \chi_0 \) is the amplitude of the initial (homogeneous) displacement. For runaway moduli potentials the field \( \chi \) stays frozen.

Now we can proceed to the next step regarding coupling constant fluctuations \( \delta \alpha_i / \alpha_i \), which we will discuss very soon below.

### III. SPATIAL VARIATIONS OF COUPLING CONSTANTS

Next we will consider how spatial variations of moduli fields (which are present in the time interval between the end of inflation and the time \( t_p \) when moduli are pinned down) induce spatial variations of coupling constants. Effective low energy string theory, braneworld models and in general 3 + 1 effective descriptions of theories with extra dimensions have coupling constants \( \alpha \) that depend on various dilaton/moduli/radion fields, see (1) for illustration. Assuming spatial/temporal variations of these fields, the coupling constants \( \alpha_i \) also vary

\[
\frac{\delta \alpha_i}{\alpha_i} \simeq \sum_a \frac{\partial \log \alpha_i}{\partial \chi_a} \delta \chi_a(t, \vec{x}) .
\] (4)

In the standard model all three couplings are related through the RG flow equation

\[
\frac{1}{\alpha_i} = \frac{1}{\alpha} + \frac{b_i}{2\pi} \log \frac{E}{\Lambda} ,
\] (5)

where \( b_i = (33, 1, 3) \). Suppose that \( \alpha(\chi_a) \) depends on some of the moduli fields. The variations of all coupling constants are related \( \delta \alpha_1 / \alpha_1 = \delta \alpha_2 / \alpha_2 = \delta \alpha_3 / \alpha_3 \). Notice, however, that it is possible to have variations of the \( \alpha \)-s decoupled from each other [9]. Temporal variations of \( \alpha \) as well as joint time variations of all three \( \alpha_i \)-s in late-time cosmology were considered in papers [?]. Here we introduce a combination of two new elements. First, we concentrate not on time evolution but on the cosmologically large scale spatial variations of \( \alpha \)-s. Second, we consider observational consequences of spatial \( \alpha \) variations not at present, but in the very early universe, before the epoch when the \( \alpha \)-s may have been pinned down to their constant low-energy values.

Spatial variations of the coupling constants in the very early universe lead to the generation of large scale isocurvature cosmological perturbations in the matter and radiation components constituting the universe. Consider species of particles that are initially in thermal equilibrium with the hot plasma. Suppose these particles drop out of equilibrium in an expanding universe and their abundance is frozen out when the rate of their interaction with the hot plasma \( \Gamma \) begins to exceed the rate of expansion \( \Gamma > H \). One example would be weakly interacting neutralinos as CDM particles, which are frozen out at \( t_{CDM} \sim 10^{-6} \) sec.
Another example would be the generation of the baryon number due to interactions with baryon number violation. Spatial variation of the couplings of these interactions \( \alpha_i \) at scales larger than the Hubble radius results in a slightly different freeze out time of these components in different causally connected regions (the Hubble patches). This leads to slightly different energy densities \( \rho \) of these components in different Hubble patches

\[
\frac{\delta \rho}{\rho} \approx \frac{\delta \alpha}{\alpha} \sim \frac{\partial \log \alpha}{\partial \chi} \delta \chi.
\]

Thus we may have isocurvature fluctuations in the CDM component, isobaryonic fluctuations etc, modulated by the moduli field(s) fluctuations. The amplitude of modulated isocurvature fluctuations depends on a few factors, and our estimate of it bifurcates depending on the way the moduli are pinned down. Immediately after inflation we estimate \( \frac{\partial \log \alpha}{\partial \chi} \delta \chi \sim \frac{H}{M_P} \). If the moduli have a runaway potential, the spatial variation of \( \alpha \) stays at its initial value. In this case all species of relic particles which are frozen out of thermal equilibrium will have isocurvature fluctuations \( \frac{\delta \rho}{\rho} \) defined by the initial value of the \( \alpha \) variation.

For moduli potentials with a minimum, both the homogeneous component \( \chi \) and its fluctuations \( \chi_k \) at some point begin to oscillate. In the limiting case of a vanishing homogeneous moduli component \( \chi(t) = 0 \), the amplitude of the inhomogeneous fluctuations \( \delta \chi \) decreases with time \( \delta \chi \sim 1/a^{3/2} \), as does the \( \delta \alpha \) variation. In this case, again, the \( \frac{\delta \rho}{\rho} \) amplitude depends on how late the relic component is freezeed out and effect can be negligible. Interesting modulated isocurvature perturbations may arise in baryonic component if baryon assymetry is generated very soon after inflation. Interesting modulated isocurvature perturbations in CDM component are possible if CDM particle freeze out very soon after inflation. If moduli responsible for both curvature and isocurvature fluctuations, they can be correlated. Correlated curvature and isocurvature fluctuations can produce interesting combine effects [10], although there are observational constraint on isocurvature fluctuations [11]

IV. MODULATED CURVATURE PERTURBATIONS

This mechanism of generation of modulated perturbations works wherever different species are out of equilibrium and couplings are important, and coupling constants are inhomogeneous at the scales larger than the horizon. Let us apply it to the origin of matter from (p)reheating after inflation. Indeed, in the theory of inflation the subsequent radiation domination stage takes place after the decay of the inflaton field \( \phi \) due to its coupling with other particles (for instance, in the process of preheating after inflation [12]). Suppose the coupling is modulated by the spatial inhomogeneities of the moduli field. Then the transition from the inflaton dominated to the radiation dominated stage occurs at slightly different times in different Hubble patches, and we will have modulated isocurvature fluctuations in radiation after inflation. Since radiation becomes gravitationally dominant, its isocurvature fluctuations will turn into curvature fluctuations soon after inflation.

There is an interesting limiting case of this scenario. Let us assume that the scalar fluctuations from inflation are suppressed (say, by our choice of the parameters of the inflaton potential). What will be left will be the moduli fluctuations, which will trigger modulated
isocurvature fluctuations in radiation. These will be transferred into curvature fluctuations of the amplitude $\sim \delta \chi_a$. This is an alternative mechanism to obtain scale free curvature fluctuations from inflation, which does not invoke the details of the inflaton potential. We will call these fluctuations modulated curvature fluctuations. This idea is easily illustrated in perturbative reheating after inflation. Indeed, let $\Gamma_{tot}$ be the rate of inflaton decay into radiation. The reheating temperature is $T_r \simeq \sqrt{\Gamma_{tot} M_p}$. Variations in $\Gamma_{tot}$ due to the variation of the coupling constant results in entropy fluctuations $\delta T_r$, which convert into curvature fluctuations after inflaton decay.

Typically, however, reheating goes through a non-perturbative, non-linear preheating stage [13] and the dependence of the reheat temperature on the couplings is rather complicated [14]. However, on general grounds we still expect that at large scales spatial variations in the final reheat temperature $\frac{\delta T_r}{T_r}$ will be linear with respect to the couplings, $\frac{\delta T_r}{T_r} \simeq \frac{\delta \alpha}{\alpha}$.

V. DISCUSSION

We suggest a new mechanism for generating cosmological fluctuations from inflation. The main idea is to modulate coupling constants by large scale spatial variations of the moduli fields. We can obtain modulated isocurvature fluctuations and modulated curvature fluctuations and even a correlated mixture of them. This may require significant tuning. However, there is a particularly interesting case of generation of modulated curvature fluctuations, which does not require tuning.

This mechanism has some common features with the curvaton scenario [15]. However, in our scheme there are no additional assumptions about the decay of the curvaton, since the inflaton field decays anyway. In this respect, the model of modulated curvature fluctuations is more economic. If the inflaton decay goes gravitationally, then our mechanism is related to spatial variation of the effective Planckian mass (dilaton), considered in [16]. The mechanism of curvature fluctuations due to coupling constant variations similar to ours was independently suggested in [17].

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