Optical system of 3D AOTF-based microscopic imager

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Abstract. In this paper, we propose a novel approach of calculating optical stereoscopic systems containing acousto-optic tunable filters. Each channel is designed separately as a simple imaging system. It is formed by an entrance lens, a radiation receiver lens, and an acousto-optic filter. The optimization of the depth of field and the magnification of the optical system is carried out during the calculations. The numerical values of the overall parameters obtained satisfy the given initial requirements. The Zemax simulation of the built optical system is in strong agreement with the calculation.

1. Introduction
Nowadays there are many ways to obtain spectral information from a studied object. Spectral imaging is a technique that produces a $I(x, y, \lambda)$ data cube. It is often used for spectral contrasting, pattern recognition, and spectral measurements [1, 2]. This technique can be improved by supplementing it with information about the geometric shape of the object $z(x, y)$. In this case, it becomes possible to obtain spectral information from a point with coordinates $x, y, z$ on the surface of the object, so one has an ability to obtain the $I(x, y, z, \lambda)$ distribution.

One of the options of obtaining 4D information $I(x, y, z, \lambda)$ is the combining of spectral imaging and stereoscopy. We need to underline it is necessary to save information about the spatial distribution of an object's spectral properties. Acousto-optic tunable filter (AOTF or simply AOF) is a well-know solution for this kind of problem. It has high spatial ($1000 \times 1000$ elements) and spectral (up to 0.1 nm) resolution, fast tuning time and arbitrary spectral addressing. Also, their small size and full software controllability make them easily combined with most visualization devices.

In practice, there is a small number of instrumental solutions that combine the capabilities of obtaining geometric (3D) and spectral information about an object at the same time [3, 4]. Such devices are important in tasks, which require an achievement of the maximum information per one measurement. An example of such research can be remote sensing [5, 6], monitoring of dynamic biological systems [7, 8], etc. Therefore, the development of the design algorithms for devices that combine the principles of three-dimensional and spectral visualization is an actual challenge.

In this article, we consider a device that combines the principles of spectral imaging and stereoscopy – an acousto-optic stereoscopic spectrometer. It is based on acousto-optic filtration of two light beams that form the stereoscopic image of an object. The concept of optical scheme is presented in figure 1. It consists of an entrance lens (1), after which the light from the object is divided into two beams, one per
stereo channel. AO filters (2), (3) are integrated into each channel. Prism (4) and the lenses (5, 6) create an image on the matrix sensors (7), (8).

Figure 1. The concept of the stereoscopic spectrometer. 1 – entrance lens; 2, 3 – AOTF; 4 – prism-based split system; 5, 6 – matrix radiance sensor lenses; 7, 8 – matrix radiance sensors.

During the design of such an optical system, it is necessary to take into account the size of the object, the distance to the object, the parallax of the stereoscopic system, the resulting spectral stereo image quality, and the depth of field of the image space [9]. The last factor significantly affects the accuracy of the reconstruction of the 3D coordinates on the object surface. In the case of shallow depth of field, there is a large number of reconstruction errors. This leads to the partial or complete destruction of the reconstructed 3D image of the object.

2. Depth of the field. Dependencies analysis

2.1. Initial data and assumptions
The primary optical system calculation makes it possible to find the initial approximations for the main parameters of the optical system [10]. This data is important in the further optimization of an optical system and, consequently, in the search for an optimal solution. Unfortunately, taking into account all the features of the optical system at this stage is complicated. Therefore, to simplify the primary analysis, we propose several simplifying assumptions (A1–A5).

A1. The left and right channels of the stereoscopic system are identical. This allows carrying out the calculations for only one channel and using the result for both of them.

A2. The optical system of the channel is presented in the form of a projection (imaging) system of three components:
• entrance lens;
• AOF is presented in the form of a plane-parallel plate;
• lens of the matrix radiation sensor.

A3. The entrance pupil of the AO filter is the aperture for this system. Actually, the entrance pupil of AOF is a material diaphragm on the entrance surface of AO cell.

A4. The optical system of the channel is considered to be linear, i.e. reflecting components are not taken into account.

A5. The object under study is placed in the focus of the input lens, and the matrix sensor is placed in the focus of the matrix sensor lens.
A simplified scheme of one channel of a stereoscopic spectrometer is shown in figure 1. The object placed into position P2 within the depth of field in image space.

![Figure 1](image1.png)

Figure 2. Optical scheme of the stereoscopic spectrometer channel. L1 – entrance lens, Ap – aperture diaphragm, EP – entrance pupil, AOC – acousto-optic cell, L2 – matrix sensor lens, S – matrix sensor, P1-P3 – several possible positions of an object.

There is a range of parameters for the presented scheme.
- \( L \) – distance from the entrance lens plane to the aperture diaphragm
- \( X \) – distance from the entrance lens plane to the entrance pupil
- \( f_1 = f'_1 \) – focal length of the entrance lens
- \( f_2 \) – focal length of the sensor’s lens
- DAO – AO entrance pupil diameter and diameter of aperture diaphragm
- DEP – optical system entrance pupil diameter
- \( -p \) – distance from the object plane to the entrance pupil
- \( \Delta \) – depth of field in image space (DOF)

We propose the following characteristics as initial data for further calculations.
- AOF angular field of view (FOV) \( \omega_x \times \omega_y = 3^\circ \times 3^\circ \)
- The maximum diameter of the input beam for AOF is 5 mm
- Size of the matrix radiation sensor \( W_s \times H_s = 4.8 \times 3.6 \text{ mm} \)
- Pixel size of the matrix radiation sensor \( a_{px} = 6 \mu \text{m} \)
- Allowable size of the scattering spot \( \delta' \leq 3a_{px} \) or \( \delta \leq 18 \mu \text{m} \)
- The range of sizes for the studied objects \( 2y_{\text{min}} \ldots 2y_{\text{max}} = 0.5 \ldots 5 \text{ mm} \)

2.2. Obtaining DOF equation
During the design of the stereoscopic system, it is necessary to take into account the distance from the main plane of the entrance lens to the object \(-p\), the depth of field in the image space \( \Delta \), and the minimum parallax of the stereoscopic system. These parameters are of critical value in the reconstruction of 3D coordinates on an object’s surface. The last factor is taken into account only at the stage of combining channels into a stereoscopic system.
Let us reduce the parameters \( p \) and \( A \) of the optical system to a form of \( f(f_1', f_2', D_{AO}) \), where \( f_1' \) is the focal length of the entrance lens, \( f_2' \) is the focal length of the sensor lens, \( D_{AO} \) is the diameter of the entrance pupil of the AO filter and, according to assumption A3, the diameter of the aperture.

Based on the figure 1, taking into account the assumption A5, the simple formula (2.1) is valid for the distance from the entrance pupil plane.

\[
-p = f_1' + x,
\]

where \( x = x(f_1', L) \) can be found via Gauss formula in the form below.

\[
x(f_1', L) = \frac{f_1' \cdot L}{L - f_1'}.
\]

Combining equations (2.1) and (2.2) and performing some simplifications, we get the equation (2.3) for \( p(f_1', L) \).

\[
p(f_1', L) = -\left( f_1' + x(f_1', L) \right) = \frac{f_1'^2}{L - f_1'}.
\]

In this form, the distance \( p \) does not depend on \( f_2' \) and \( D_{AO} \), and increases in absolute value with increasing of \( f_1' \).

After these short manipulations, it becomes possible to get an expression for the DOF \( A \) depending on \( f_1' \) and \( f_2' \). According to [11], for imaging systems, the depth of field can be written as (2.4):

\[
\Delta = \Delta(f_1', f_2', D_{EP}) = \frac{d_{EP} p(\delta)}{(\frac{\delta}{p})^2 - d_{EP}^2(\delta)^2},
\]

where \( \beta = f_2'/f_1' \) — linear magnification of an overall optical system.

The entrance pupil of the overall system is the image of the aperture diaphragm through the first component. Its size can be obtained via equation (2.5)

\[
D_{EP} = D_{AO} - |\beta p| = \frac{D_{AO} f_1'}{L - f_1'}
\]

Using equations (2.4) and (2.5) and performing a series of mathematical transformations, we obtain the following expression (2.6) for the depth of field in image space.

\[
\Delta(f_1', f_2', D_{AO}) = \frac{2D_{AO} f_1'^2 f_2'^2 \delta'}{D_{AO}^2 f_1'^2 - 2 \delta'^2 + 2 \delta f_1'^2 - f_2'^2 \delta'^2 - \delta'^2}.
\]

The dependence of DOF on the parameter \( \delta' \) in the region \( 0 \sim 20 \delta'_{px} \) is close to linear: the larger the allowable size of the scattering spot \( \delta' \), the larger the depth of field. The dependence on the parameter \( L \) is practically not observed due to \( \delta' \ll L \). So we have to analyze the influence of the parameters \( f_1', f_2', \) and \( D_{AO} \) on the DOF.

2.3. DOF vs. \( f_1' \) or \( f_2' \)

We visualize two-parameter dependences when fixing the third parameter and analyze their influence on the DOF. To determine the influence of \( f_1' \) and \( f_2' \) on the DOF, let’s fix the \( D_{AO} = 10 \text{ mm} \). In addition, we use \( L = 10 \text{ mm} \) and \( \delta' = 3 \delta' = 18 \mu\text{m} \). As a result, we’ve got the two-parametric function \( \Delta(f_1', f_2') \), shown in the figure 3.

Simple conclusion from figure 3: to achieve the maximum of DOF one has to minimize \( f_2' \) and maximize \( f_1' \).

2.4. DOF vs. \( D_{AO} \)

To determine the affection of \( D_{AO} \) on the DOF, let’s visualize the dependence of the DOF from the parameters \( f_1', f_2', \) and \( D_{AO} \) when fixing the parameters \( L = 10 \text{ mm} \) and \( \delta' = 3 \delta' = 18 \mu\text{m} \). The dependence
is plotted in the coordinates $(\Delta, f'_1, f'_2)$ for different values of $D_{AO}$. The result of the visualization is shown in figure 4.

![Figure 3. Visualization of the depth of field function with fixed $D_{AO}$.](image)

Visualization confirms the conclusion obtained by analyzing DOF vs. $f'_1$ and $f'_2$: the greater the focal length of the entrance lens, the greater the DOF itself. Also, the plot shows that minimizing the size of the aperture diaphragm $D_{AO}$ causes an increase in DOF. Finally, visualization makes obvious the upper bound of DOF, obtained with $D_{AO} = 1$ mm, $f'_1 = 200$ mm, and $f'_2 \approx 5$ mm. It is about 40 mm.

![Figure 4. Visualization of the depth of field function with different values of $D_{AO}$ ($D_{AO} = 1$ mm, 2 mm, 5 mm).](image)

When one has calculated the optimal value of the focal length $f'_2$, taking into account the limitation for the parameters $p$ and $D_{AO}$, it becomes possible to obtain the values of other parameters for the primary synthesis of the optical system. The next step is the optimization of an optical scheme to minimize...
aberrations arising in the optical and acousto-optical components of the device [12]. Estimation of the arising aberrations will enable us to conclude the ability of 4D reconstruction \( I(x, y, z, \lambda) \) of the object’s surface.

3. Applying the algorithm to the real data
First of all, it is necessary to determine the focal length of the \( L_2 \) component. The field of view of this lens must match with the AOF field of view, therefore we will assume that the \( L_1 \) field of view of is numerically equal to the AOF field of view.

\[
\omega_x = \omega_y = \omega_{L2} = 3^\circ.
\]

Also, let’s consider that the image is inscribed in the sensor’s area, so image height \( 2y' \) is numerically equal to the height of the sensor.

\[
2y' = H_d = 3.6 \text{ mm}. \tag{3.2}
\]

In this case, the required \( L_1 \) focal length can be calculated via equation below.

\[
f_2' = y' \cdot (\tan(\omega_{L2}/2))^{-1} = 1.8 / \tan(1.5^\circ) \approx 68.739 \text{ mm}. \tag{3.3}
\]

According to the analytical conclusions of the previous chapter, the \( f_2' \) should be minimized to achieve the maximum DOF. Decreasing \( f_2' \) while constant AOF field of view causes the decrease of image size. A slight change is acceptable because the edge areas of the image are not informative for the obtaining of the 3D image of the object [13]. In this case, one can accept \( f_2' = 60 \text{ mm} \).

As the maximum size of an object under study is known, one can find the initial assumption for linear magnification of the system.

\[
\beta_0 = 2y'/2y_{\text{max}} = 3.6 / 5 = 0.72^\circ. \tag{3.4}
\]

This results in the ability to find \( L_1 \) focal length.

\[
\beta_0 = f_2'/f_1' \Rightarrow f_1' = f_2' / \beta_0 = 60 / 0.72 \approx 83.3 \text{ mm}. \tag{3.5}
\]

As the \( L_1 \) focal length increases, the maximum size of the object under study and the DOF increases too. Slight change is acceptable, so let’s consider \( f_1' = 90 \text{ mm} \).

The next step is choosing the diameter of the aperture diaphragm \( D_{AO} \). The analysis presented in the previous chapter showed that DOF maximizing can be performed by minimizing \( D_{AO} \).

Let’s assume aperture diaphragm diameter \( D_{AO} = 1 \text{ mm} \). In this case, the value of DOF obtained via the equation (2.6) is:

\[
\Delta(f_1', f_2', D_{AO}) = \Delta(90,60,1) = 4.863 \text{ mm}. \tag{3.6}
\]

After \( f_1', f_2' \) and \( D_{AO} \) values is calculated, one has an ability to find other necessary parameters of an optical system. For example, distance from \( L_1 \) to object plane is equal to focal length \( f_1' = 90 \text{ mm} \). Linear magnification of the overall system (via equation (3.5)) \( \beta = f_2' / f_1' \approx 0.67^\circ \) (close to initial \( \beta_0 = 0.72^\circ \)).

4. Numerical simulation
In order to be sure the considered model is correct, we’ve performed a numerical simulation using Zemax software. To exclude other factors during the verification of the calculation results, we use models of ideal aberration-free lenses. AOF, according to A2, is represented in the form of a plane-parallel plate with a refractive index of 2.3.

The system is built in three configurations, differenced by the object’s position (positions P1–P3 on figure 1). Positions P1 and P3 match the far and near limits of DOF respectively, and position P2 matches the entrance lens focus. Then the distance from P1 to P3 is the DOF by definition. The visualization of the spot scatter for the three object locations P1, P2, P3 is shown in figures 5(a), 5(b), 5(c) respectively.

The diameters of the scattering spot for the positions P1–P3 are given in table 1. Here the estimated spot diameter is the maximum size of the scattering spot for positions P1–P3 according to the proposed
algorithm, and Zemax spot diameter is the maximum size of scattering spot calculated during Zemax simulation for the positions P1–P3.

Figure 5. Visualization of spot scatter diagram. (a) – object in position P1, (b) – object in position P2, (c) – object in position P3.

The inconsiderable difference between estimated and Zemax spot diameters in position P1 is supposed to be a Zemax calculation inaccuracy and is not of great importance. So the simulation data is almost identical to the proposed algorithmic calculations. This confirms the efficiency of the applied calculation method for designing optical systems of stereoscopic spectrometers.

| Position | Estimated spot diameter, μm | Zemax spot diameter, μm |
|----------|-----------------------------|-------------------------|
| P1       | 18.000                      | 17.998                  |
| P2       | 0.000                       | 0.000                   |
| P3       | 18.000                      | 18.000                  |

5. Discussion

The shown design algorithm allows one to obtain the main parameters of an optical system for one channel of a stereo system. Here the depth of field is the key factor. It is influenced by three main parameters: \( f'_1 \), \( f'_2 \), and \( D_{AO} \). According to the analytical conclusions of the previous chapters, in order to achieve the maximum of DOF, one should minimize \( f'_2 \) and \( D_{AO} \). At the same time, \( f'_1 \) should be maximized. However, there are some limitations for each of these parameters.

The \( f'_1 \) parameter has an upper limitation. According to the equation (2.3), an increase in \( f'_1 \) entails an increase in the distance from the entrance pupil plane to the object plane. As a result, with a constant stereoscopic base, the parallax of the stereoscopic system decreases. This can lead to the impossibility of the 3D reconstruction of coordinates on the object’s surface.

The \( f'_2 \) parameter is limited from below since the angular field of the \( L_1 \) lens must be in agreement with the angular field of the AOF. A decrease in \( f'_2 \) while the angular field is a constant leads to a decrease in the image size, and, as a consequence, to an increase in the number of errors in the 3D reconstruction. Therefore, the image should cover the maximum possible area on the matrix. Choosing a smaller sensor with a smaller pixel size is not a solution because the DOF decreases proportionally to the pixel size.

There is one more limitation on the \( f'_2 / f'_1 \) relation. Increasing \( f'_1 \) and decreasing \( f'_2 \) simultaneously results in a decrease of linear magnification of the overall optical system. This parameter can be varied in a narrow range since it is calculated from matrix sensor size and the maximum size of the inspected object. The last parameter only depends on the task being solved.
The aperture diameter value $D_{AO}$ is also limited from below. Reducing the aperture negatively affects the light transmission features of the system. There are 2 ways of possible compensation: increasing the exposure time at the radiation receiver, or increasing the gain at the radiation receiver. Using the first compensation method can deprive us of the ability to acquire images of dynamic objects. The second method is associated with a decrease in signal-to-noise ratio and, consequently, with an increase in the number of errors in the 3D reconstruction.

From the presented discussion, it’s obvious that the choice of the optimal parameters of the OS is rather complicated. Depending on the specific task, the permissible range of variation of the overall OS parameters is determined. However, as soon as the values of the parameters $f_1', f_2'$ and $D_{AO}$ are obtained, one can easily determine the remaining parameters of the OS. Then the resulting optical scheme can be optimized to become most suitable for a specific task.

6. Conclusion

We presented a universal method of design for an optical system of stereoscopic spectrometer channels. It is shown that the characteristics of the optical system significantly depend on three parameters: the focal length of the entrance lens ($f_1'$), the focal length of the matrix sensor lens ($f_2'$), and the entrance pupil diameter of an AO filter ($D_{AO}$). The variability of these parameters makes it possible to adapt the optical system for different tasks. In particular, to increase the depth of field, it is necessary to decrease $f_2'$ and $D_{AO}$ and increase $f_1'$.

However, to take into account all possible factors, further development of the method is required. First of all, it is necessary to take into account aberrations of real components (including AOTF) during the optical design. Also, one has to consider the features associated with the stereoscopic structure of the system (divergence between channels, the volume of intersection of the fields of view, etc.). Finally, such an algorithm should be formalized by the merit function. The injection of such a function would make it possible to take into account a huge amount of the requirements for the optical system. Therefore, further improvement of the algorithm seems to us to be a promising direction.

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