Is the $J^P = 2^-$ assignment for the $X(3872)$ compatible with the radiative transition data?

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Abstract

The very recent analysis by BABAR Collaboration indicates that the $X(3872)$ may favor the quantum number $J^{PC} = 2^{--}$ rather than the previously assumed $1^{++}$. By pretending the $\eta_c(1D)$ charmonium to be the $X(3872)$, we study the parity-even radiative transition processes $\eta_c(1D) \rightarrow J/\psi(\psi') + \gamma$ within several phenomenological potential models. We take the $^3D_1$ admixture in $\psi'$ into account, and consider the contributions from the magnetic dipole ($M1$), electric quadrupole ($E2$), and magnetic octupole ($M3$) amplitudes. It is found that the ratio of the branching fractions of these two channels, as well as the absolute branching fraction of $\eta_c \rightarrow \psi' \gamma$, are in stark contradiction with the existing BABAR measurements. This may indicate that the $2^{--}$ assignment for the $X(3872)$ is highly problematic.

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The rising of a dozen of new charmonium resonances in recent years, most of which are above the $D\bar{D}$ mass threshold, has greatly reinvigorated the field of hadron spectroscopy. Among them, the $X(3872)$ particle, perhaps being the most intensively studied one from both theory and experiment, has occupied the central stage\.[1] The $X(3872)$ was first discovered in 2003 by BELLE Collaboration in the decay $B^+ \rightarrow J/\psi\pi^+\pi^-K^+$\.[2], subsequently confirmed by BABAR\,[3], as well as CDFII\,[3], D0\,[4] in inclusive $p\bar{p}$ collision experiments. Several unusual properties of this particle, \textit{i.e.} its mass in extreme proximity to the $D^0\bar{D}^{*0}$ threshold, the rather narrow width, and the large isospin violation seen in its decay pattern, have persuaded many authors to believe that, rather than being a conventional charmonium\,[5], the $X(3872)$ may be of exotic nature, \textit{e.g.}, a loosely-bound $D^0\bar{D}^{*0}$ molecule\,[7], or a diquark-antidiquark cluster\,[8], or a hybrid\,[9].

Aside from its mass, the most important property about the $X(3872)$ is its $J^{PC}$ quantum number. The even $C$-parity of the $X(3872)$ has been firmly established since the recent observations of its decay to $J/\psi\rho^0$\,[10] and to $J/\psi\gamma$\,[11]. By analyzing the decay angular distribution in the process $X(3872) \rightarrow J/\psi\pi^+\pi^-$, CDF Collaboration narrowed the possibilities of the $J^P$ down to $1^+$ or $2^-$\.[12]. The former assignment seems more appealing from the theoretical perspective, which is particularly congenial to a $S$-wave $D^0\bar{D}^{*0}$ molecule interpretation. Although the smoking gun from the experimental side is still not available yet, the $1^{++}$ assignment for the $X(3872)$ has already been tacitly accepted in most of the recent phenomenological works.

However, very recently there comes a quite unexpected, and, perhaps disquieting, news from BABAR Collaboration. The latest analysis of the decay $B \rightarrow J/\psi\omega K$ by BABAR indicates that the $P$-wave orbital angular momentum for the $J/\psi\omega$ system is more favored than the $S$-wave, which implies that the $X(3872)$ may favor $J^{PC} = 2^{++}$ instead of the universally-believed $1^{++}$\.[13]. Therefore, we are compelled to re-scrutinize the properties of the $X(3872)$. If future experiments will confirm the result of\.[13], our perception on the nature of this particle would have to be profoundly changed.

In light of the latest BABAR analysis\,[13], the most natural candidate for $X(3872)$ would be the $\eta_c (1D)$ meson. This $D$-wave spin-singlet (denoted by the spectroscopic symbol $^1D_2$) charmonium has been extensively studied in quark potential models for decades. Its predicted mass is scattered in the range 3760-3840 MeV\,[6]. The width of $\eta_c$ is believed to be narrow, since the decay into $D\bar{D}$ is forbidden by parity, and the energy conservation does not allow it to disintegrate into $D\bar{D}$. It can only decay through the strong and electromagnetic transitions, exemplified by $\eta_c \rightarrow \eta_c\pi\pi$ and $\eta_c \rightarrow h_c\gamma$, as well as through the OZI-forbidden annihilation $\eta_c \rightarrow gg$. Each of these processes is expected to have a partial width of a few hundred keV.

One of the strongest objections to identifying the $X(3872)$ with the $\eta_c$ charmonium probably comes from the electromagnetic transitions $\eta_c \rightarrow J/\psi(\psi') + \gamma$\,[13]. Such parity-conserving transitions flip the quark spin and change the orbital angular momentum by two units, so there must be strong multipole suppression, and one would expect a rather small branching fraction for such decay processes. Thus, one is puzzled by the fact why the BABAR Collaborations were able to observe these radiative decay channels several years ago\,[11], with only a limited statistics of the $X(3872)$ samples?

The motif of this paper is to quantify this objection, by presenting a detailed study for the radiative transition processes $\eta_c \rightarrow J/\psi(\psi') + \gamma$. We will employ potential nonrelativistic QCD (pNRQCD) as a convenient calculational device, and work with several phenomenological potential models, as well as take the $^3S_1-^3D_1$ admixture effect for $\psi'$ into account.
We will also identify the contributions from several different multipoles, e.g., the magnetic dipole (M1), electric quadrupole (E2), and magnetic octupole (M3) amplitudes.

Our key finding is that, no matter the mixing effect for $\psi'$ is taken into account or not, the predicted branching ratio for $\eta_{c2} \rightarrow \psi' + \gamma$ is orders of magnitude smaller than the lower bound that can be inferred from the BABAR measurement \[15\]. Moreover, the ratio of the branching fraction of $\eta_{c2} \rightarrow \psi' + \gamma$ to that of $\eta_{c2} \rightarrow J/\psi + \gamma$ is orders of magnitude smaller than the corresponding BABAR measurement for the $X$ particle \[15\]. These qualitative conclusions do not vary with specific potential model. We thus tend to conclude that, if the BABAR experiment is correct, the $\eta_{c2}$ assignment for the $(3872)$ particle would become highly unlikely. We hope that future higher-statistics experiments will help to clarify the situation.

Before launching into the calculation, we first recall some related experimental facts. BABAR has recently measured the following products of two branching fractions \[15\]:

\[
\begin{align*}
\mathcal{B}[B^\pm \rightarrow X(3872)K^\pm] & \mathcal{B}[X(3872) \rightarrow J/\psi + \gamma] = (2.8 \pm 0.8 \pm 0.1) \times 10^{-6}, \\
\mathcal{B}[B^\pm \rightarrow X(3872)K^\pm] & \mathcal{B}[X(3872) \rightarrow \psi' + \gamma] = (9.5 \pm 2.7 \pm 0.6) \times 10^{-6},
\end{align*}
\]

with 3.6$\sigma$ and 3.5$\sigma$ significance, respectively. Therefore one can deduce the ratio of the branching fraction of $X \rightarrow \psi'\gamma$ to that of $X \rightarrow J/\psi\gamma$:

\[
\frac{\mathcal{B}[X(3872) \rightarrow \psi' + \gamma]}{\mathcal{B}[X(3872) \rightarrow J/\psi + \gamma]} = 3.4 \pm 1.4.
\]

This measurement is in serious conflict with the predictions made from some specific $D^0\overline{D}^0$-molecule models \[7\], but the calculational framework underlying those models may be questionable. Interestingly, this ratio seems roughly compatible with the canonical $\chi_{c1}(2P)$ interpretation of the $(3872)$, which decays to $J/\psi(\psi') + \gamma$ through the dominant electric dipole ($E1$) transition \[6,16\].

Very recently BELLE Collaborations have also analyzed these two radiative transition channels \[19\]. Their preliminary results are

\[
\begin{align*}
\mathcal{B}[B^\pm \rightarrow X(3872)K^\pm] & \mathcal{B}[X(3872) \rightarrow J/\psi + \gamma] = (1.78^{+0.48}_{-0.44} \pm 0.12) \times 10^{-6}, \\
\mathcal{B}[B^\pm \rightarrow X(3872)K^\pm] & \mathcal{B}[X(3872) \rightarrow \psi' + \gamma] < 3.4 \times 10^{-6}.
\end{align*}
\]

Their first measurement is consistent with BABAR’s result, \(1a\). But BELLE has not observed any $B^\pm \rightarrow \psi'\gamma K^\pm$ signals. Consequently, BELLE is only able to place an upper bound on the ratio of these two branching fractions:

\[
\frac{\mathcal{B}[X(3872) \rightarrow \psi' + \gamma]}{\mathcal{B}[X(3872) \rightarrow J/\psi + \gamma]} < 2.1,
\]

at 90% confidence level.

Thus far, our knowledge is only limited to the product of two branching fractions, and it will be certainly useful to know the absolute branching fraction of $X \rightarrow J/\psi(\psi')\gamma$. Using a

\[1\] It seems that the $^3S_1 - ^3D_1$ mixing effect has not been incorporated in most phenomenological analysis of $\chi_{c1}(2P) \rightarrow \psi'\gamma$. 

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missing mass technique, BABAR was able to set an upper bound for the absolute branching fraction of $B^\pm \to X(3872)K^\pm$ some time ago [20]:

$$B[B^\pm \to X(3872)K^\pm] < 3.2 \times 10^{-4},$$

at 90% confidence level.

Combining the results of (1), (3) together with (5), we are able to place some lower bounds on the absolute branching fractions of the $X(3872)$ decays to $J/\psi(\psi') + \gamma$:

$$B[X(3872) \to J/\psi + \gamma] > 5.9 \times 10^{-3}, \quad B[X(3872) \to \psi' + \gamma] > 1.9 \times 10^{-2}, \quad \text{BABAR}$$

(6a)

$$B[X(3872) \to J/\psi + \gamma] > 3.8 \times 10^{-3}, \quad \text{BELLE}$$

(6b)

We are unable to establish a meaningful inequality for $X \to \psi' + \gamma$ from the latest BELLE data.

Our central task then is to explicitly examine whether the decays $\eta_{c2} \to J/\psi(\psi') + \gamma$ are consistent with the experimental constraints listed in (2), (4), and (6).

In passing, we note that there have already existed some theoretical studies on the decay $\eta_{c2} \to J/\psi(\psi') + \gamma$ by Sebastian and his coworkers [21–23]. These authors worked in the traditional first-quantized quantum-mechanical framework. Notably, some of serious inconsistencies seem to exist amongst a sequence of their papers 2. Therefore we feel it timely and obligatory to conduct an independent investigation on these transition processes.

A modern effective-field-theory framework of dealing with the single-photon transition in quarkonium has recently been put forward in Ref. [24], by coupling the potential nonrelativistic QCD (pNRQCD) [25] with the electromagnetism. In [24], the $M1$ transition process with $\Delta l = 0$, exemplified by $J/\psi \to \eta_c \gamma$, has been systematically analyzed. In this work, we will utilize the same formalism to tackle the $\Delta l = 2$ magnetic transition process $\eta_{c2} \to J/\psi(\psi') \gamma$. Our final results turn out to significantly differ from those in [21–23].

To account for the radiative transitions, it is convenient to promote the gauge group of NRQCD to $SU(3)_c \times U(1)_{em}$. One then matches this NRQCD action onto an even lower energy effective field theory, pNRQCD, by further integrating out the quantum fluctuations of virtuality of order $m^2 v^2$. pNRQCD is an ideal formalism to tackle radiative transitions, because the active degrees of freedom, i.e., the dynamical gluons and the emitted photon, are both ultrasoft ($k^\mu \sim m v^2$) and can be treated on an equal footing. Since they possess a wavelength much longer than the typical quarkonium radius, the corresponding fields need be multipole-expanded. By this way one can elegantly implement the multipole expansion, the standard treatment of the electromagnetic transitions in quantum mechanics textbooks.

Out starting point is the following pNRQCD Lagrangian density [24]:

$$\mathcal{L}_{pNRQCD} = \int d^3 r \, \text{Tr} \left\{ S^\dagger \left( i \partial_0 + \frac{\nabla^2 R}{4 m} + \frac{\nabla^2 r}{m} - V_S^{(0)}(r) \right) S \right\} + \mathcal{L}_{\text{light}} + \mathcal{L}_{\gamma pNRQCD},$$

(7)

where $S$ represents the field for a composite system made of the heavy quark and the heavy antiquark, depending on the center-of-mass coordinate, $\mathbf{R}$, and the relative coordinate, $\mathbf{r}$. $S$

\footnote{For example, the partial width of $\eta_{c2} \to J/\psi + \gamma$ was originally predicted to be 2.13 keV [21]. This prediction later shifted to an abnormally large value, 62.6 keV [22]. In their last publication [23], this prediction has shrunk to 0.699 keV.}
is a $2 \times 2$ matrix in spinor space and a singlet under color and $U(1)_{\text{em}}$ transformations. The $V^{(0)}_R$ denotes the static singlet $Q\bar{Q}$ potential. The first term in (1), describing the dynamics of the $Q\bar{Q}$ pair dictated by the strong interaction, resembles very much the traditional potential model. $\mathcal{L}_{\text{light}}$ represents the lagrangian for light degrees of freedom, e.g., light quarks, gluons as well as photons. The particularly relevant to this work is the third term, $\mathcal{L}_{\gamma_{pNRQCD}}$, which depicts the spin-dependent interaction between the $Q\bar{Q}$ pair and a photon:

$$\mathcal{L}_{\gamma_{pNRQCD}} = \int d^3 r \, \text{Tr} \left\{ e e Q S^i r \cdot E^{\text{em}} S + \frac{c_F^{\text{em}} e e Q}{2 m} \{ S^i, \sigma \cdot B^{\text{em}} \} S \right. \\
+ \frac{c_F^{\text{em}} e e Q}{16 m} \left\{ S^i, \sigma \cdot (r \cdot \nabla R)^2 B^{\text{em}} \right\} S + \frac{e e Q}{8 m^2} r V^{(0)}_S \left\{ S^i, \sigma \cdot \hat{r} \times (\hat{r} \times B^{\text{em}}) \right\} S \\
- \frac{c_S^{\text{em}} e e Q}{16 m^2} \left\{ S^i, \sigma \cdot [-i \nabla_r \times, E^{\text{em}}] \right\} S - \frac{c_S^{\text{em}} e e Q}{16 m^2} \left\{ S^i, \sigma \cdot [-i \nabla_r \times, (r \cdot \nabla R) E^{\text{em}}] \right\} S \\
+ \frac{c_W^{\text{em}} e e Q}{4 m^3} \left\{ S^i, \sigma \cdot B^{\text{em}} \right\} \nabla^2 S + \frac{c^{\text{em}}_{\gamma^p} e e Q}{4 m^3} \left\{ S^i, \sigma \cdot B^{\text{em}} \right\} \nabla_r^i \nabla_r^j S \right\}, \tag{8}$$

where the trace is over the spin indices. The $E^{\text{em}}$ and $B^{\text{em}}$ signify the electric and magnetic field strengths, which depend only on $R$. The ubiquitous occurrence of the Pauli matrix $\sigma$ (the first operator, responsible for the $E1$ transition, is an exception), signals that the concerned radiative transition is of the spin-dependent type, i.e., with quark spin flipped. The appearance of $r$ is a consequence of multipole-expanding the electromagnetic field, while the $\hat{r} \equiv r/r$ represents the unit radial vector. Notice that (8) is manifestly gauge invariant.

The coefficients $c_F^{\text{em}}$, $c_S^{\text{em}}$, $c_W^{\text{em}}$, and $c_{\gamma^p}^{\text{em}}$ in (8) are the matching coefficients that were directly inherited from the NRQCD with enlarged gauge group. They satisfy some exact relations dictated by reparametrization (or Poincaré) invariance [26]:

$$c_S^{\text{em}} = 2 c_F^{\text{em}} - 1, \quad c_W^{\text{em}} = c_W^{(1)} - c_W^{(2)} = 1, \quad c_{\gamma^p}^{\text{em}} = c_F^{\text{em}} - 1. \tag{9}$$

All these matching coefficients are known at the one loop level [26]. In particular, we have

$$c_F^{\text{em}} \equiv 1 + \kappa_Q^{\text{em}} = 1 + \frac{4 \alpha_s}{3 2 \pi} + \mathcal{O}(\alpha_s^2), \tag{10}$$

where $\kappa_Q^{\text{em}}$ is usually dubbed the *anomalous magnetic moment* of the heavy quark. The one-loop perturbative contribution to $\kappa_Q^{\text{em}}$ in (10) seems insignificant, less than 10% for charm. However, it is often conjectured that the magnetic moment of the bound quark may receive a large contribution due to some nonperturbative mechanism. For simplicity we will suppress such a possibility (see [24] for a discussion on whether such a contribution can naturally emerge from the first principle of QCD). Rather we will content ourselves with taking $\kappa_Q$ as its short-distance value.

The $\sigma \cdot \hat{r} \times (\hat{r} \times B^{\text{em}})$ term in (8) is also worth some remarks. As discussed in [24], unlike the remaining operators listed in (8), this spin-dependent transition operator receives some nontrivial corrections when descending from NRQCD to pNRQCD. This operator is intimately related to the quark spin-orbit potential, whose coefficient is also protected by the reparametrization (or Poincaré) invariance (Gromes relation) [27].

Prior to giving concrete expression for $\eta_c \to J/\psi(\psi')\gamma$, we note that there have been some phenomenological inclinations that $\psi'$ may not be a pure $S$-wave vector charmonium.
Rather it may have some admixture of $^3D_1$ component [28]:

$$|\psi'| = \cos \phi |2^3S_1\rangle - \sin \phi |1^3D_1\rangle,$$

(11)

usually with $\phi$ taken to be around 12°.

To compute the radiative transition process in pNRQCD, it is necessary to know the corresponding vacuum-to-quarkonium pNRQCD matrix elements. To the purpose of this work, we need know the radial Schrödinger wave function of a quarkonium. The symbol $e^*(\lambda)$ represents the polarization vector of any $J$ quarkonium state, satisfying the orthogonality condition $e^*(\lambda) \cdot e(\lambda') = \delta_{\lambda\lambda'}$, whereas the symbol $h^{ij}(\lambda)$ denotes the polarization tensor of the $n^1D_2$ state, which is symmetric and traceless, obeying the orthogonality condition $\text{tr} [h^*(\lambda)h(\lambda')] = \delta_{\lambda\lambda'}$. The overall normalization factors for each entity are chosen such that all the quarkonium states are nonrelativistically normalized.

With the knowledge of (8) and (12), it is then a straightforward exercise to calculate the transition amplitude for $\gamma$ quarkonium states, which is symmetric and traceless, obeying the orthogonality condition $\text{tr} [h^*(\lambda)h(\lambda')] = \delta_{\lambda\lambda'}$. The overall normalization factors for each entity are chosen such that all the quarkonium states are nonrelativistically normalized.

The first two entries have been constructed in [24], and the last two are new. Here $R_{nl}(r)$ denotes the radial Schrödinger wave function of a quarkonium. The symbol $e(\lambda)$ represents the radial Schrödinger wave function of a quarkonium. The symbol $e^*(\lambda)$ represents the polarization vector of any $J$ quarkonium state, satisfying the orthogonality condition $e^*(\lambda) \cdot e(\lambda') = \delta_{\lambda\lambda'}$, whereas the symbol $h^{ij}(\lambda)$ denotes the polarization tensor of the $n^1D_2$ state, which is symmetric and traceless, obeying the orthogonality condition $\text{tr} [h^*(\lambda)h(\lambda')] = \delta_{\lambda\lambda'}$. The overall normalization factors for each entity are chosen such that all the quarkonium states are nonrelativistically normalized.

With the knowledge of (8) and (12), it is then a straightforward exercise to calculate the transition amplitude for $^1D_2 \rightarrow ^3S_1 + \gamma$ and $^1D_2 \rightarrow ^3D_1 + \gamma$. The latter is a regular $M1$ transition between the $D$-wave spin singlet and triplet, which is included here because it can contribute to $\eta_{c2} \rightarrow \psi' + \gamma$ through the mixing mechanism. After completing the angular integration, the desired results are

$$\mathcal{M}[^1D_2 \rightarrow ^3S_1 + \gamma] = \frac{ee_q}{2m_Q} \sqrt{\frac{2}{15}} \left\{ \frac{e^* \cdot k \times e^*_j k^i h^{ij} k^j}{|k|^2} \epsilon^e F \epsilon^m J_1 + k^i h^{ij} \left( e^* \times e^*_j \right)^j (\epsilon^e F \epsilon^m - 1) J_2 + e^{*i} h^{ij} \left( k \times e^*_j \right)^j (J_2 + J_4 - \epsilon^q p^j \epsilon^m J_3) \right\},$$

(13a)

$$\mathcal{M}[^1D_2 \rightarrow ^3D_1 + \gamma] = \frac{ee_q}{2m_Q} 6 \sqrt{15} e^{*i} h^{ij} \left( k \times e^*_j \right)^j \epsilon^e F \epsilon^m J_0,$$

(13b)

where $k$ and $e^*_j$ represent the three-momentum and polarization vector of the emitted photon, respectively, and $e^*$ stands for that of the outgoing $J = 1$ charmonium. The
involved dimensionless overlap integrals are given by

\begin{align}
J_0 &= \int_0^\infty dr R_{3D_1}(r)R_{1D_2}(r) r^2, \\
J_1 &= -\frac{|k|^2}{4} \int_0^\infty dr R_{n^3S_1}(r)R_{1D_2}(r) r^4, \\
J_2 &= \frac{|k|}{2m_Q} \int_0^\infty dr R'_{n^3S_1}(r)R_{1D_2}(r) r^3, \\
J_3 &= -\frac{1}{m_Q^2} \int_0^\infty dr \left(\frac{R'_{n^3S_1}(r) - R_{n^3S_1}(r)}{r}\right) R_{1D_2}(r) r^2, \\
J_4 &= \frac{1}{2m_Q} \int_0^\infty dr R_{n^3S_1}(r)V_s^{(0)\prime}(r)R_{1D_2}(r) r^3.
\end{align}

To facilitate the comparison with the expressions in Ref. [21, 22], we have introduced the same \( J_i \ (i = 0, \ldots, 4) \) as theirs, except a different normalization factor for \( J_1 \) is adopted.

Some remarks on deriving the amplitude (13a) are in order. Unlike in the case of an ordinary \( M1 \) transition process with \( \Delta l = 0 \), not all of the operators in (8) can make a nonvanishing contribution for such a \( \Delta l = 2 \) transition process. For example, the leading magnetic-dipole operator, the \( \sigma \cdot B^{em} \) term, and the \( \sigma \cdot [-i\nabla_R \times B^{em}] \) term, as well as the \( \sigma \cdot B^{em} \nabla^2 \) term, simply cannot connect a \( D \)-wave state to the \( S \)-wave state owing to their spherically-symmetrical nature. It turns out that only four terms in (8) can directly make a nonzero contribution.

Equation (13a) has also embedded an interesting piece of contribution, the so-called final-state recoil correction [20]. As a manifestation of the relativistic effect to a moving \( ^3S_1 \) state, its wave function can develop a nonvanishing overlap with a spin-singlet \( P \)-wave component, therefore the transition \( ^1D_2 \to ^3S_1 \) can be effective realized through a \( E1 \) transition from the parent to this small \( ^1P_1 \) component. Some subtle gauge-invariance issue related to this Lorentz boost effect has been clarified [24]. Although the recoil correction and the spin-orbit-potential-related term are not separately invariant under the \( U(1)_{em} \) transformation, their sum is. We have explicitly verified that, for the reaction \( ^1D_2 \to ^3S_1 + \gamma \), the sum of the recoil correction and the correction induced by the spin-orbit-potential-related operator, is indeed independent of the redefinition of the pNRQCD field \( S \) by implementing a \( U(1)_{em} \) gauge link.

For the decay \( ^1D_2 \to ^3D_1 + \gamma \), which is an ordinary allowed \( M1 \) transition, we only consider the leading contribution and not include the relativistic correction in (13d).

Squaring the amplitudes in (13), and summing over all possible polarizations, it is easy to obtain the spin-averaged partial width \( \Gamma[\eta_{c2} \to J/\psi(\psi') + \gamma] \). Nevertheless, (13) encodes much richer polarization information. In this work, we would like to proceed to extract the helicity amplitude [30, 31] and the multipole amplitude [32, 33] from this equation. These two types of amplitudes can in principle be extracted experimentally. It is worth noting that, CLEO-c experiment has recently extracted the higher-order multipole amplitudes associated the cascade decay process \( \psi' \to \chi_{c1,2}\gamma \to J/\psi + \gamma\gamma \), by performing a maximum likelihood fit of the joint angular distributions of the two photons [34].

For the decay \( \eta_{c2}(\nu) \to \psi(\mu) + \gamma(\lambda) \), we signify the projection of the angular momentum of \( \eta_{c2} \) along the moving direction of the \( \gamma \) by \( \nu \), and denote the helicities of \( \psi \) and \( \gamma \) by \( \mu \) and \( \lambda \). With this specific choice of the quantization axis, we have \( \nu = \lambda - \mu \). There are in total three independent helicity amplitudes \( A_{\mu,\lambda} \), and the other three can be related by parity
invariance. We introduce the shorthand \( A_\nu \) for \( A_{\mu,\lambda} \) with \( \lambda \) fixed to be +1. Substituting the explicit representation of polarization tensors in (13), it is easy to obtain:

\[
A_0 \equiv A_{1,1} = -A_{-1,-1} = \frac{2c_F^{em}J_1 - (2c_S^{em} - 1)J_2 + c_{p'^p}^{em}J_3 - J_4 + 3\sqrt{2}c_F^{em}J_0 \sin \phi}{\sqrt{6}}, \quad (15a)
\]

\[
A_1 \equiv A_{0,1} = -A_{0,-1} = \frac{-c_S^{em}J_2 + c_{p'^p}^{em}J_3 - J_4 + 3\sqrt{2}c_F^{em}J_0 \sin \phi}{\sqrt{2}}, \quad (15b)
\]

\[
A_2 \equiv A_{-1,1} = -A_{-1,-1} = -J_2 + c_{p'^p}^{em}J_3 - J_1 + 3\sqrt{2}c_F^{em}J_0 \sin \phi, \quad (15c)
\]

where \( \phi \) is the \( ^3S_1 - ^3D_1 \) mixing angle, which equals 0 for \( J/\psi \), and may be put 12° for \( \psi' \) on phenomenological ground. In deriving the helicity amplitudes for \( \psi' + \gamma \), we have approximated \( \cos \phi \approx 0.978 \approx 1 \) for simplicity.

From (15), it is straightforward to obtain the spin-averaged partial width:

\[
\Gamma[\eta_{c2} \rightarrow J/\psi(\psi')+\gamma] = \frac{2\alpha c_0^2|k|^2}{75m_c^2} \left(|A_0|^2 + |A_1|^2 + |A_2|^2\right), \quad (16)
\]

According to [32, 33], the helicity amplitudes \( A_\nu (\nu = 0, 1, 2) \) are connected to the multipole amplitudes \( a_{J' \gamma} \) through the following orthogonal transformation:

\[
A_\nu = \sum_{J_\gamma} \sqrt{\frac{2J_\gamma + 1}{2J_{\eta_{c2}} + 1}} a_{J_\gamma} \langle J_\gamma, 1; 1, \nu - 1 | J_{\eta_{c2}}, \nu \rangle, \quad (17)
\]

where the Condon-Shortley notation for the Clebsch-Gordan coefficients, \( \langle j_1, m_1; j_2, m_2 | J, M \rangle \), has been adopted [35]. \( J_{\eta_{c2}} = 2 \) is the spin of the \( \eta_{c2} \) meson. \( J_\gamma \) represents the angular momentum carried by the photon, obeying \( 1 \leq J_\gamma \leq J_{\eta_{c2}} + 1 \), and \( a_1, a_2 \) and \( a_3 \) correspond to the \( M1, E2, \) and \( M3 \) multipole amplitudes, respectively.

The inverse transformation of (17) can be readily obtained:

\[
\begin{pmatrix}
  a_1 \\
  a_2 \\
  a_3
\end{pmatrix}
= \begin{pmatrix}
  1
  \sqrt{10} & \sqrt{1/10} & \sqrt{2/15} \\
  \sqrt{1/2} & \sqrt{1/6} & -\sqrt{2/15} \\
  \sqrt{2/5} & 2\sqrt{2/15} & \sqrt{2/15}
\end{pmatrix}
\begin{pmatrix}
  A_0 \\
  A_1 \\
  A_2
\end{pmatrix}. \quad (18)
\]

From (15), we can readily deduce these three multipole amplitudes:

\[
\begin{align*}
a_1 &= \frac{2c_F^{em}J_1 - 5(1 + c_S^{em})J_2 + 10c_{p'^p}^{em}J_3 - 10J_4 + 30\sqrt{2}c_F^{em}J_0 \sin \phi}{2\sqrt{15}}, \quad (19a) \\
a_2 &= \frac{2c_F^{em}J_1 - 3(c_S^{em} - 1)J_2}{2\sqrt{3}}, \quad (19b) \\
a_3 &= \frac{2c_F^{em}J_1}{\sqrt{15}}. \quad (19c)
\end{align*}
\]

\(^3\) Note that this transformation matrix is identical to the corresponding one in the \( \chi_{c2} \rightarrow J/\psi + \gamma \) process [34]. But for such a \( E1 \)-dominant process, \( a_1, a_2, a_3 \) should be identified with the electric dipole (\( E1 \)), magnetic quadrupole (\( M2 \)), and electric octupole (\( E3 \)) amplitudes, respectively.
As expected, the $S-D$-wave mixing effect can only enter into the $M1$ contribution. Note the $M3$ amplitude solely receives the contribution from the $\sigma \cdot (r \cdot \nabla R)^2 B^{am}$ operator in [8], which comes from multipole-expanding the leading magnetic transition operator to the second order in $r$.

We are now in a position to compare our expressions for the three multipole amplitudes [19] with those reported in [22, 23]. Somewhat surprisingly, it seems that our expressions disagree with theirs for each multipole, except for the $J_0$ and $J_1$ pieces. The authors of [22, 23] have not included the anomalous magnetic moment of the charm quark, and in their formulas, all the overlap integrals, $J_1$ through $J_4$, contribute to the $M1$ and $E2$ amplitudes. Taking $\kappa_Q = 0$ in [19], we find that $J_0$ would disappear from the $M1$ amplitude, and the $E2$ amplitude will only depend on $J_1$, which are in diametric contradiction to the equations in [22, 23].

We can express the spin-averaged transition width for $\eta_c \rightarrow J/\psi(\psi') + \gamma$ in terms of three multipole amplitudes:

$$\Gamma[\eta_c \rightarrow J/\psi(\psi') + \gamma] = \frac{2\alpha e^2 Q |k|^3}{75m_c^2} \left|a_1|^2 + |a_2|^2 + |a_3|^2\right|,$$

which is equivalent to [16], since the transformation [17] preserves the norm.

To make concrete predictions for the transitions $\eta_c \rightarrow J/\psi(\psi') + \gamma$, we need know the radial wave functions of each involved charmonium from some phenomenological potential models. To ensure that our conclusion not to rest heavily upon one specific model, we choose to study with five different potential models. All of them only differ in the way of parameterizing the static potential $V_S^{(0)}$. Specifically, the potentials we choose are Cornell type [36], the Buchmüller-Tye (BT) type [38], NR potential by Barnes et al. [37], the screened confinement potential by Li and Chao [17], and the potential proposed by Fulcher [39]. We solve the Schrödinger equation with these potentials numerically, with the input parameters taken from the aforementioned papers.

In Table I we tabulate the predictions from the various models for the overlap integrals $J_i$ ($i = 1, \cdots, 4$). When evaluating these integrals defined in [14], we have used $m_c = 1.5$ GeV, and determined the photon momentum by physical kinematics, i.e., we assume $\eta_c$ with a mass of 3872 MeV and use the physical masses of $J/\psi$ and $\psi'$ as input. Thus we obtain $|k| = 698$ MeV for $\eta_c \rightarrow J/\psi\gamma$ and 181 MeV for $\eta_c \rightarrow \psi'\gamma$. From Table I it is reassuring that these overlap integrals are not very sensitive to the different models. On the other hand, the overlap integral $J_0$ can be safely put to be unity, since to the intended accuracy, both the $\eta_c(1D)$ and the $\psi(1D)$ have degenerate radial wave function.

In Table II we give the explicit predictions for the decay $\eta_c \rightarrow J/\psi(\psi') + \gamma$, such as the partial width, the helicity amplitudes and the (normalized) multipole amplitudes for each decay channel within various potential models. We have taken $\alpha = 1/137$ and the electric charge of the charm quark, $e_c = 2/3$. We have used the one-loop perturbative value for the anomalous magnetic momentum of the $c$, as indicated in [10]. If $\alpha_s(m_c) = 0.35$ is used, we then get $\kappa_c = 0.074$. Detailed numerical checks reveal that all the predictions in Table II only change modestly if we set $\kappa_c = 0$.

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4 One may argue that, the 700 MeV photon in $\eta_c \rightarrow J/\psi\gamma$ is a little bit too energetic, and it is perhaps more appropriate to count $|k| \sim mv$ in this case. This may cast some doubt on the validity of multipole expansion, the underlying basis of this work. It might be illuminating to apply the hard-scattering mechanism, recently developed in [40], to reinvestigate this channel.
From Table II, one finds that the predicted partial width for $\eta_c^2 \rightarrow J/\psi\gamma$ from various potential models ranges between 3.11 and 4.78 keV, that for $\eta_c^2 \rightarrow \psi'\gamma$ without considering $^3S_1 - ^3D_1$ mixing ranges from 0.017 to 0.029 keV, and that for $\eta_c^2 \rightarrow \psi'\gamma$ incorporating the mixing effect ($\phi = 12^\circ$) ranges from 0.49 to 0.56 keV. This seems to indicate that different potential models give rise to reasonably consistent predictions for each channel.

**TABLE I:** The overlap integrals $J_i$ for the electromagnetic transition $\eta_c^2 \rightarrow J/\psi(\psi')\gamma$ in various potential models. Since $J_0 = 1$, we have not listed its value.

| Potential Models | Cornell [36] | Screened [17] | NR [37] | BT [38] | Fulcher [39] |
|------------------|--------------|--------------|---------|---------|-------------|
|                  | $J/\psi$ $\psi'$ | $J/\psi$ $\psi'$ | $J/\psi$ $\psi'$ | $J/\psi$ $\psi'$ | $J/\psi$ $\psi'$ |
| $J_1$            | -0.600 0.123  | -0.710 0.180  | -0.728 0.161  | -0.757 0.153  | -0.763 0.147  |
| $J_2$            | -0.376 0.051  | -0.347 0.063  | -0.365 0.056  | -0.383 0.048  | -0.390 0.044  |
| $J_3$            | -0.304 -0.256 | -0.227 -0.160 | -0.242 -0.191 | -0.245 -0.212 | -0.249 -0.225 |
| $J_4$            | 0.136 -0.243  | 0.121 -0.218  | 0.128 -0.231  | 0.144 -0.244  | 0.173 -0.291  |

**TABLE II:** The predictions of $\eta_c^2 \rightarrow J/\psi(\psi') + \gamma$ from various potential models. The mixing angle $\phi$ has been taken for both $0^\circ$ and $12^\circ$ for $\psi'$. We have taken $\alpha = 1/137$, and $\kappa_c = 0.074$ by using $\alpha_s(m_c) = 0.35$. In addition to the spin-averaged partial width, the helicity amplitudes and the (normalized) multipole amplitudes for each decay channel have also been given.

| Potential Models | $\phi$ $A_0$ $A_1$ $A_2$ $a_1$ $a_2$ $a_3$ $|a_2/a_1|$ $|a_3/a_1|$ Width (keV) |
|------------------|-------------|-------------|-------------|------|------|------|-------------|-------------|----------------|
| Cornell          | $J/\psi$ $\psi'$ | $-0.39$ $0.19$ $0.22$ $0.31$ $-0.66$ $-0.68$ | 2.15 | 2.21 | 3.11 |
|                  | $\psi'$     | $0^\circ$ $0.17$ $0.12$ $0.17$ $0.93$ $0.26$ $0.25$ | 0.28 | 0.27 | 0.017 |
|                  |             | $12^\circ$ $0.56$ $0.79$ $1.12$ $0.99$ $0.05$ $0.05$ | 0.05 | 0.05 | 0.50  |
| Screened         | $J/\psi$ $\psi'$ | $-0.50$ $0.18$ $0.21$ $0.19$ $-0.70$ $-0.70$ | 3.72 | 3.70 | 4.22  |
|                  | $\psi'$     | $0^\circ$ $0.21$ $0.09$ $0.14$ $0.85$ $0.38$ $0.37$ | 0.45 | 0.44 | 0.017 |
|                  |             | $12^\circ$ $0.60$ $0.76$ $1.09$ $0.99$ $0.07$ $0.07$ | 0.07 | 0.07 | 0.49  |
| NR               | $J/\psi$ $\psi'$ | $-0.50$ $0.19$ $0.22$ $0.20$ $-0.69$ $-0.69$ | 3.49 | 3.49 | 4.45  |
|                  | $\psi'$     | $0^\circ$ $0.20$ $0.11$ $0.16$ $0.89$ $0.33$ $0.32$ | 0.38 | 0.36 | 0.018 |
|                  |             | $12^\circ$ $0.59$ $0.78$ $1.11$ $0.99$ $0.06$ $0.06$ | 0.06 | 0.06 | 0.50  |
| BT               | $J/\psi$ $\psi'$ | $-0.53$ $0.20$ $0.22$ $0.18$ $-0.70$ $-0.69$ | 3.76 | 3.76 | 4.78  |
|                  | $\psi'$     | $0^\circ$ $0.20$ $0.12$ $0.18$ $0.91$ $0.30$ $0.29$ | 0.33 | 0.31 | 0.020 |
|                  |             | $12^\circ$ $0.59$ $0.79$ $1.13$ $0.99$ $0.06$ $0.06$ | 0.06 | 0.06 | 0.51  |
| Fulcher          | $J/\psi$ $\psi'$ | $-0.54$ $0.18$ $0.20$ $0.14$ $-0.70$ $-0.70$ | 5.16 | 5.16 | 4.77  |
|                  | $\psi'$     | $0^\circ$ $0.22$ $0.16$ $0.23$ $0.94$ $0.24$ $0.23$ | 0.26 | 0.24 | 0.029 |
|                  |             | $12^\circ$ $0.60$ $0.83$ $1.18$ $0.99$ $0.05$ $0.05$ | 0.05 | 0.05 | 0.56  |

In contrast to the minor role played by retaining $\kappa_c$, the effect of $^3S_1 - ^3D_1$ mixing is enormous for the decay $\eta_c^2 \rightarrow \psi'\gamma$, as can be clearly seen from Table II. The predicted partial widths with $\phi = 12^\circ$ are about 25 times larger than those without including the mixing for $\psi'$. This can be attributed to the fact that the transition $^1D_2 \rightarrow ^3D_1\gamma$ is an allowed $M1$ transition, with the overlap integral $J_0 = 1$. However for the transition $^1D_2 \rightarrow ^2S_1 + \gamma$,
due to the existence of a node in the 2S radial wave function, the corresponding overlap integrals are generally small. Therefore, even with a relatively small $\phi$ angle, the $S$-$D$-wave mixing can already play a very important role.

In Table III, we also list the magnitudes of the $M1$, $E2$, $M3$ amplitudes for each transition process. For clarity, we have renormalized $a_i$ ($i = 1, 2, 3$) in (19) such that the new multipole amplitudes satisfy $|a_1|^2 + |a_2|^2 + |a_3|^2 = 1$. It is a very interesting observation that $E2$ and $M3$ amplitudes are almost identical in all the decay channels. For $B_{c2} \rightarrow J/\psi \gamma$, these two higher-order multipoles are even more important in magnitude than the $M1$ amplitude! This pattern no longer holds true for the $\eta_{c2} \rightarrow \psi' \gamma$ channel. Nevertheless, it is also interesting to note that, for $\eta_{c2} \rightarrow \psi' \gamma$, the ratios $|a_2/a_1|$ and $|a_3/a_1|$ decrease significantly after the mixing effect is included. This can also be understood by the fact that, since the allowed $M1$ transition $1D_2 \rightarrow 3D_1 \gamma$ makes more pronounced contribution than $1D_2 \rightarrow 3S_1 \gamma$, including the mixing effect thus can greatly amplify the importance of the $M1$ amplitude relative to other two multipoles.

Surveying Table III, one may be able to place the following saturation bound for the ratio of the two branching fractions:

$$\frac{\mathcal{B}[\eta_{c2} \rightarrow \psi' + \gamma]}{\mathcal{B}[\eta_{c2} \rightarrow J/\psi + \gamma]} \leq 0.16, \quad \phi = 12^\circ \quad (21a)$$

$$\frac{\mathcal{B}[\eta_{c2} \rightarrow \psi' + \gamma]}{\mathcal{B}[\eta_{c2} \rightarrow J/\psi + \gamma]} \leq 6.1 \times 10^{-3}, \quad \phi = 0^\circ \quad (21b)$$

where the maximum ratio in first equation comes from the prediction in Cornell potential, and that in second equation is obtained from Fulcher’s potential model. These results seem to seriously conflict with the corresponding BABAR measurements, (2), no matter the mixing effect for $\psi'$ is taken into account or not. This seems to be a strong evidence to disfavor the $\eta_{c2}$ assignment for the $X(3872)$. At this stage, these upper limits seem still compatible, but orders of magnitude less, with the preliminary BELLE upper bound on this ratio, (1).

It is certainly interesting to examine whether the absolute branching fractions of $\eta_{c2} \rightarrow J/\psi(\psi') \gamma$ are compatible with the $B$ factory measurements or not. To realize this goal, one first needs know the full width of the $X(3872)$. BELLE has set an upper limit for the total width of $X(3872)$: $\Gamma_X < 2.3$ MeV at 90% confidence level [2]. The 2008 PDG compilation has estimated the total width of the $X(3872)$ to be $\Gamma_X = 3.0^{+2.5}_{-1.4}$ MeV [35].

We hope to establish an upper bound for the predicted branching ratios of $\eta_{c2} \rightarrow J/\psi(\psi') \gamma$. To this purpose, we have taken a conservative attitude, by assuming $\Gamma_X = 1.3$ MeV, the lower end of the PDG estimate. Such a value by itself is consistent with the full width of the $\eta_{c2}$. From Table III, one can see that, among all the potential models analyzed in this work, the BT potential model gives the largest prediction to the partial width of $\eta_{c2} \rightarrow J/\psi \gamma$, 4.78 keV. We thus estimate

$$\mathcal{B}[\eta_{c2} \rightarrow J/\psi + \gamma] \leq 3.7 \times 10^{-3} \quad (22)$$

It is interesting to contrast this theoretical upper bound with those inequalities inferred from the two $B$ factory experiments, (4). Equation (22) seems to mildly conflict with the experimental lower bound from either BABAR or BELLE. Due to large theoretical uncertainties in $\Gamma[\eta_{c2} \rightarrow J/\psi + \gamma]$ and $\Gamma_X$, we are inclined not to make strong claim just based on the analysis for this channel.

The decay $\eta_{c2} \rightarrow \psi' \gamma$ poses a much more stringent challenge to the data. Among all the potential models, Fulcher’s potential model yields the largest prediction to the partial width
of $\eta_{c2} \to \psi'\gamma$, with or without including the mixing effect. Taking the corresponding partial widths from Table III we may place the following inequalities:

$$B[\eta_{c2} \to \psi + \gamma] \leq 4.3 \times 10^{-4}, \quad \phi = 12^\circ$$  \hspace{1cm} (23a)

$$B[\eta_{c2} \to \psi' + \gamma] \leq 2.2 \times 10^{-5}. \quad \phi = 0^\circ$$  \hspace{1cm} (23b)

Comparing these theoretical upper bounds with the lower bound imposed by the BABAR measurement, $1.9 \times 10^{-2}$, which is given in (6), one observes the glaring discrepancies, no matter the mixing effect for $\psi'$ is included or not. If BABAR result can be trusted, this may also be viewed as a strong evidence against assigning the $X(3872)$ as the $\eta_{c2}$.

**TABLE III:** The predictions of the overlap integral $\langle r \rangle$ and the partial width for the electric transition $\eta_{c2} \to h_c + \gamma$ in various potential models. The mass of $h_c$ is taken to be 3525 MeV [35].

| Potential Models | Cornell | Screened | NR | BT | Fulcher |
|------------------|---------|----------|----|----|---------|
| $k$ (MeV)        | $\langle r \rangle_{21}$ (GeV$^{-1}$) (keV) | $\langle r \rangle_{21}$ (GeV$^{-1}$) (keV) | $\langle r \rangle_{21}$ (GeV$^{-1}$) (keV) | $\langle r \rangle_{21}$ (GeV$^{-1}$) (keV) | $\langle r \rangle_{21}$ (GeV$^{-1}$) (keV) |
| 331              | 3.06    | 587      | 3.54 | 786 | 3.39    |

For the sake of completeness, finally we would like to contrast the $\eta_{c2} \to J/\psi(\psi') + \gamma$ processes with the dominant electromagnetic transition of $\eta_{c2}$, the parity-changing electric transition $\eta_{c2} \to h_c + \gamma$. With the knowledge of the leading electric-dipole operator in (8) and the vaccum-to-$h_c$ matrix element in [121], one can readily reproduce the well-known $E1$ transition formula:

$$\Gamma[\eta_{c2} \to h_c + \gamma] = \frac{8\alpha e^2}{15} |\langle r \rangle_{21}|^2,$$  \hspace{1cm} (24a)

$$\langle r \rangle_{21} = \int_0^\infty dr R_1 P_1(r) R_1 D_2(r) r^3.$$  \hspace{1cm} (24b)

The contributions from the higher-order multipoles, $M2$ and $E3$, as well as the relativistic corrections, neither of which are expected to be significant, have been neglected for simplicity. From Table IIII one can see that the predicted partial width for $\eta_{c2} \to h_c + \gamma$ in five potential models ranges from 600 to 800 keV. As expected, this transition rate is several orders of magnitude greater than that of $\eta_{c2} \to J/\psi(\psi') + \gamma$.

In summary, we have presented a detailed analysis to the radiative transitions $\eta_{c2} \to J/\psi(\psi') + \gamma$ within various phenomenological potential models, employing the pNRQCD formalism as an elegant and efficient calculational framework. The major motivation of this study is to examine whether identifying the $X(3872)$ with the $\eta_{c2}(1D)$ charmonium is compatible with the $B$ factory measurements of the radiative decay $X(3872) \to J/\psi(\psi') + \gamma$. Our study reveals such an assignment would cause severe contradictions with the available BABAR measurements, either for the absolute branching fraction of $X \to \psi' + \gamma$, or the ratio of this branching fraction to that of $X \to J/\psi + \gamma$. As a result, if the BABAR measurements

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5 The $E1$ transition rates tabulated in Table IIII seem to be somewhat larger than the often quoted 464 keV [4].
measurements \cite{15} are trustable, the possibility for the \(X(3872)\) to be identified with the \(\eta_{c2}\) then becomes rather low. In our opinion, the \(X(3872)\) is most likely to carry \(J^{PC} = 1^{++}\).

A nuisance is that the preliminary BELLE results \cite{10} on the decay \(X(3872) \rightarrow \psi' \gamma\) seem to conflict with the BABAR measurement \cite{15}. It is important and urgent to resolve this experimental discrepancy, in order to unambiguously unravel the nature of the \(X(3872)\).

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