RSE production amplitude and possible evidence of a (pseudo)scalar boson at about 57 GeV

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March 6, 2023

Abstract
Threshold enhancements predicted by the Resonance-Spectrum-expansion (RSe) production amplitude and observed by the BaBaR Collaboration in open-bottom production above the \( B\bar{B} \) threshold, as well as by several collaborations in open-charm production above the \( D\bar{D} \) threshold, can also be seen in diphoton amplitudes at energies above 100 GeV. One such threshold effect is visible in \( \tau\tau \) and \( \mu\mu \) data of the L3 Collaboration at LeP, and in the diphoton and four-lepton data of the aTLaS and CMS Collaborations at LHC, as it is enhanced by the nearby presence of the Higgs resonance. This supports the assumption of pair production at about 115 GeV. An accumulation of single-photon and dimuon data around 28 GeV observed by the L3 and the CMS Collaborations, respectively, give further credit to the hypothesis of the existence of a (pseudo)scalar boson with a mass of about 57 GeV.

Keywords: beyond Standard Model, compositeness, spectroscopy, particle and resonance production, threshold effects, (pseudo)scalar weak-boson partner
1 Introduction

The suggestion that the intermediate vector bosons, often called weak bosons, may be composite is not new [1]. Possible spin-zero partners have been studied in numerous works (see e.g. Refs. [2–10]). To date, no experimental evidence of their existence has been reported. However, the interest in weak substructure has since been renewed [11–15], also due to the observation of the Higgs boson.

In several previous papers [16–20] we argued that experiment hints at the possible existence of a (pseudo)scalar boson with a mass of about 57 GeV. In Ref. [16] we focused on diphoton data published by the CMS [21] and ATLAS [22] Collaborations at LHC. In Ref. [17] we showed that our view on the CMS and ATLAS diphoton data in the mass region 110–135 GeV is also corroborated by four-lepton signals published by CMS [23] and ATLAS [24], as well as by data for $\tau\tau$ in $e^+e^- \rightarrow \tau\tau(\gamma)$ and $\mu\mu$ in $e^+e^- \rightarrow \mu\mu(\gamma)$ published by the L3 Collaboration [25]. Furthermore, experimental data on $Z \rightarrow 3\gamma$ events measured by the L3 Collaboration [26] also provide a very modest confirmation of the existence of a (pseudo)scalar boson of about 57 GeV. In Ref. [19] we studied a rather convincing hint supporting the existence of such a boson, owing to the observation of event excesses above background near dimuon masses of 28 and 57 GeV measured by the CMS Collaboration [27].

The organisation of this paper is as follows. First we discuss the RSE production amplitude in Sect. 2. Then, in Sect. 3, we display our initial material on the hypothesis of the existence of a boson with a mass of about 57 GeV. Subsequently experimental data on the three single-photon CM energies of the 87 candidate $Z \rightarrow 3\gamma$ events measured by the L3 Collaboration [26], assuming $\sqrt{s} = M_Z$, are discussed in Sect. 4. In Sect. 5 we elaborate on how the 28 GeV dimuon data accumulation [27] supports the existence of a new boson at 57 GeV. A final discussion is presented in Sect. 6.
2 RSE production amplitude

Two-body subamplitudes in processes of strong decay are often analysed under the spectator assumption \cite{28-30}. The two-body production subamplitude $P$ may then be expressed as a linear combination of elements of the two-body scattering amplitude $T$ \cite{31-34}, where $T$ contains the full two-body dynamics supposed to be known, either from experiment \cite{35-45}, or from theoretical considerations \cite{46-52}.

The RSE amplitude $T$ for two-meson scattering \cite{53} describes, assuming quark-pair creation, an in principle infinite but in practice finite set of possibly overlapping meson-meson resonances, being all either intrinsic or dynamically generated. In Ref. \cite{54} we deduced a relation between a subamplitude $P$, describing a meson pair emerging from the products of a strong three-meson decay process, and the corresponding two-meson scattering amplitude $T$, reading

$$P = \Im \langle m | C \rangle + T C.$$ (1)

The separation of the two-body dynamics from the kinematics is extremely useful for data analysis.

The complex coefficients in $C$ are smooth and completely known functions of the two-body CM energy (see Ref. \cite{54}). As they are of a purely kinematic origin and therefore do not carry any information on the two-body interactions, all fitting freedom is restricted to complex couplings. The power of this approach has already been demonstrated \cite{55} in the simple one-channel case, applied to production processes involving the light scalar mesons $f_0(500)$ and $K^*_0(700)$.

In Ref. \cite{31} it was found, by also employing the OZI rule \cite{56} and the spectator picture, that the production amplitude can be written as a linear combination of the elastic and inelastic two-body scattering amplitudes, with coefficients that do not carry any singularities, but are rather supposed to depend smoothly on the total CM energy of the system. Hence, the conclusions of Ref. \cite{31} are in perfect agreement with our result (1).

A seeming conflict between Eq. (1), with a complex matrix $C$, and the postulate that the production amplitude should be given by a real linear combination of the elements of $T$ was satisfactorily sorted out in Ref. \cite{57}.

Now, the contribution of $\Im \langle m | C \rangle$ in Eq. (1) has the property of giving rise to an enhancement at threshold in the production amplitude, which is often quite pronounced because of the nearby presence of a true resonance. In Ref. \cite{58} we discussed several of such threshold enhancements, among others for the reactions of electron-positron annihilation into open-bottom mesons, for data obtained by the BABAR Collaboration \cite{59}, and into open-charm mesons, for data obtained by the BES Collaboration \cite{60}. In particular, in Ref. \cite{61} we determined the mass and width of the $\Upsilon(4S)$, finding very different values from those given in the tables of the Particle Data Group \cite{62}. However, recently a bottomonium resonance with comparable mass and width has been observed \cite{63}.  

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In the following, we will concentrate on the enhancements observed by BABAR for the reaction $e^+e^- \rightarrow b\bar{b}$. In Fig. [1] we depict the data measured and analysed by BABAR [59]. As also remarked in their paper, the large statistics and the small energy steps of the scan make it possible to clearly observe the two dips at the opening of the thresholds corresponding to the $B\bar{B}^* + \bar{B}B^*$ and $B^*\bar{B}^*$ channels. Near the $BB^*$ threshold, we observe that the data suggest a vanishing of the $B\bar{B}$ signal, whereas the $BB^*$ signal becomes stronger, just above threshold. At the $B^*\bar{B}^*$ threshold, this phenomenon repeats itself, now with respect to the $BB^*$ signal.

Unfortunately it is not easy to measure dips in amplitudes. It certainly depends on the bin sizes applied to the analysis of the experimental results, which in their turn depend on the amount of available data. As a result, several threshold enhancements in meson-meson data are usually interpreted as meson resonances [58].

Figure 1: Experimental data for the reaction $e^+e^- \rightarrow b\bar{b}$ measured by the BaBaR Collaboration [59]. The vertical lines indicate the $BB^*$ and $B^*\bar{B}^*$ thresholds, as indicated in the figure. The eye-guiding lines reflect our interpretation of the data and do not represent fits.
3 The dip at 115 GeV in $\gamma\gamma$, four-lepton and $\tau\tau$
LHC data and in $\mu\mu$ LEP data

In this section we search for similar effects in the gauge-boson and Higgs sectors of the Standard Model (SM). For that purpose we shall study LEP and LHC data for possible consequences of weak compositeness.

But first, some words are due on applying our above production formalism, employed with excellent results in meson spectroscopy, to a hypothetical and unknown substructure in the weak-boson sector. In the case of strong meson decay, the empirically successful $^3P_0$ model was used as a basis for describing the necessary quark-pair creation, which implies a transition interaction between the original $q\bar{q}$ pair and the two-meson final state. Due to the relative $P$-wave of the created $q\bar{q}$ pair, that potential peaks at a certain distance away from the origin. In the RSE approach, this potential is modelled with a spherical delta-shell function, which in momentum space becomes a spherical Bessel function and is the standard non-resonant lead term in the RSE production amplitude. More generally, in QCD and owing to asymptotic freedom, one expects a similar peaked structure from string breaking that leads to meson decay, as confirmed on the lattice [64]. Of course, we do not know if a possible weak substructure is governed by a non-Abelian gauge-type interaction similar to QCD, but it is certainly not an unrealistic assumption. Therefore, expecting comparable threshold enhancements from such a hypothetical substructure appears to be reasonable as well.

More than two decades ago, the ALEPH and L3 Collaborations reported an excess of data in the reaction $e^+e^- \rightarrow Z^* \rightarrow HZ$, consistent with the production of Higgs bosons with a mass of about 114 GeV [65,66]. Here, we elaborate on the idea that the signal at 114 GeV could be the onset of a threshold enhancement, corresponding to the creation of a pair of spin-zero bosons with a mass of about 57 GeV.
Fortunately, we have several sets of experimental results at our disposal. In Ref. 25 the L3 Collaboration published data, obtained at LEP, on $\tau^+\tau^-$ production in electron-positron annihilation. The CMS Collaboration collected diphoton events corresponding to an integrated luminosity of $4.8 \text{ fb}^{-1}$ 21. A similar analysis was performed by the ATLAS Collaboration 22, with slightly better statistics, though lower resolution. The ATLAS 22 and CMS 67 Collaborations have also published experimental data on four-lepton production in $pp$ collisions, by selecting events with two pairs of isolated and oppositely charged same-flavour (either $e^+e^-$ or $\mu^+\mu^-$) leptons. The combined data of the above experiments, scaled in order to exhibit comparable amplitudes, are shown in Fig. 2.

![Invariant Mass Distributions](image)

Figure 2: Diphoton signals published by CMS 68 (•) and ATLAS 24 (●), four-lepton signals by CMS 23 (⋆) and ATLAS 24 (●), invariant-mass distributions for $\tau\tau$ in $e^+e^- \rightarrow \tau\tau(\gamma)$ (◆) and for $\mu\mu$ in $e^+e^- \rightarrow \mu\mu(\gamma)$ (●) by L3 25.

As is clear from Fig. 2 each one of the data sets has insufficient statistics to confirm a dip in the data at about 115 GeV. But combined these data show a consistent picture and besides a 5–7 $\sigma$ enhancement at about 125 GeV they also exhibit a clear dip at about 115 GeV. Nevertheless, one needs much improved statistics for data distributions in order to conclude whether a weak substructure has been discovered or just a Higgs-like boson. If we assume that the structure at 115 GeV manifests the onset of a threshold enhancement from particle-antiparticle pair production, then each partner of the pair must have a mass of about 57.5 GeV.
4 A very modest signal from $Z \rightarrow 3\gamma$ events

In this section we will concentrate on the reaction

$$Z \rightarrow \gamma Z_0(57) \rightarrow \gamma \gamma \gamma \, .$$

(2)

Assuming that the $Z_0(57)$ mass is 57.5 GeV, the single-photon CM energy for the $\gamma$ in the process $Z \rightarrow \gamma Z_0(57)$ equals about 28 GeV.

The L3 Collaboration produced experimental data during the 1991–1993 runs at LEP for the reaction (2) [26], using 65.8 pb$^{-1}$ of data on top of and around the $Z$ peak, for CM energies between 88.5 and 93.7 GeV. In Fig. 3a/b we depict the L3 data on the three single-photon CM energies for each of the candidate events. The L3 Collaboration also produced the QED prediction for the data distribution in their experiment, using a Monte-Carlo simulation for the expected number of events, resulting in the blue histogram in Fig. 3a.

Figure 3: (a): experimental data (red dots and error bars) for the three single-photon CM energies of the 87 candidate $Z \rightarrow 3\gamma$ events measured by the L3 Collaboration [26], assuming $\sqrt{s} = M_Z$. The histogram (blue) was obtained by L3 from a Monte-Carlo simulation for the expected number of events predicted by QeD. With the green band we indicate where we expect photons from the radiative process $Z \rightarrow \gamma Z_0(57)$ for the case that $Z_0(57)$ has a mass of 57.5 GeV. (b): The same data as shown in (a), but now measured events divided by QED-expected events. The blue line represents the QED prediction published by L3.

The L3 Collaboration expressed the single-photon CM energies as a function of $M_\gamma / \sqrt{s}$. Here, we have converted that information into $M_\gamma$, while assuming $\sqrt{s} = M_Z$. 

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10 20 30 40
1.0
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M_\gamma (GeV)
data/QED
(b)
Moreover, we indicate, by a green band, where we expect an accumulation of photons from the radiative process $Z \to \gamma Z_0(57)$. The width of the green band expresses the width of the $Z$ boson resonance. One observes that most of the L3 data agree well with the expectation from QED. Nevertheless, be it a coincidence or not, in the mass region where we expect a signal from $Z \to \gamma Z_0(57)$ events, we observe a small enhancement. In Fig. 3b we depict the ratio of the measured signal over the QED prediction. Now one clearly observes a modest enhancement in the single-photon distribution at about 28 GeV, as expected for a $Z_0(57)$ mass of 57.5 GeV. Notice furthermore the data deficits data below and above the signal at 28 GeV, which is a common feature of resonances.

Again, the 87 candidate $Z \to 3\gamma$ events are by far not enough to obtain a high-statistics signal. Nevertheless, for what it is worth, the largest excess of data comes exactly with the single-photon CM energy as expected when assuming the reaction of Eq. (2). The CMS and ATLAS Collaborations probably have sufficient data at their disposal for $3\gamma$ events with total invariant masses in the $Z$-boson region, in order to be able to improve on the L3 statistics. We would warmly welcome an analysis of such data.
5 A possible interpretation of the CMS dimuon enhancements

More recently the CMS Collaboration reported \cite{27} on an excess of events above the background near a dimuon invariant mass of 28 GeV, with a significance of 4.2 standard deviations. That result appears to support the hypothesis of a $Z \rightarrow \gamma Z_0(57)$ decay process, with a $Z_0(57)$ mass of 57.5 GeV and a photon energy of about 28 GeV.

The data had been collected in 2012 with the CMS detector in proton-proton collisions at the LHC, for centre-of-mass (CM) energies of 8 TeV and with an integrated luminosity of 19.7 fb$^{-1}$. The event selection required a $b$-quark jet, with at least one jet in the central and the forward pseudorapidity region, respectively. The result is depicted in Fig. 4.

It would be unfair to the reader not to mention here further information on the above result. Namely, in a different dimuon data selection with higher statistics, CMS again obtained a signal near 28 GeV but now with a significance of only 2.9 standard deviations. Moreover, in related dimuon events selected from data collected at the LHC in 2016, for proton-proton collisions at CM energies of 13 TeV and corresponding to an integrated luminosity of 35.9 fb$^{-1}$, CMS found near 28 GeV signals of 2.0 standard deviations and a 1.4 standard-deviation deficit for the two mutually exclusive dimuon-event categories. Accordingly, CMS concluded \cite{27} that more data and additional theoretical input are required to understand the results. Consequently, the data in Fig. 4 are to be considered with some caution.

A closer look at the data depicted in Fig. 4 reveals a second accumulation of data near 57 GeV. Now, if we assume that the $Z_0(57)$ does exist, then in the reaction $Z \rightarrow \gamma Z_0(57)$ the $Z_0(57)$ and the intermediate photon have masses of 57.5 and 28 GeV, respectively. Moreover, both particles couple to dimuons. This might then explain why the data show two enhancements, i.e., one near 28 GeV and one near 57 GeV. We say...
“might”, because one would expect the \( Z_0(57) \) to be much less likely to decay into muon pairs than the photon. Actually, one expects the \( Z_0(57) \) to dominantly couple to \( \gamma \gamma \).

Note that recently several studies on the issue of the 28 GeV dimuon enhancement have been published \[69 \text{–} 72\]. However, none of these works relate that phenomenon to the L3 \( Z \rightarrow \gamma \gamma \gamma \) result, or to the 115 GeV dip which can be observed in diphoton signals published by CMS \[68\] and ATLAS \[24\], four-lepton signals by CMS \[23\] and ATLAS \[24\], invariant-mass distributions for \( \tau \tau \) in \( e^+e^- \rightarrow \tau \tau (\gamma) \) and \( \mu\mu \) in \( e^+e^- \rightarrow \mu\mu (\gamma) \) by L3 \[25\] as shown in Fig. 2.
6 Conclusions

Summarising, the striking coincidences in very different data sets we presented in this paper strongly suggest that a high-statistics analysis of a $Z$ boson decaying into three photons or into a lepton pair plus a photon would be very opportune, in view of a possible confirmation of weak-boson compositeness. Very recently, Higgs boson decays to a lepton pair and a photon have been studied by ATLAS [73], for $m_{\ell\ell} < 30$ GeV ($\ell = \text{either } e \text{ or } \mu$), and by CMS [74], for $m_{\ell\ell} > 50$ GeV. The latter study concludes that the main contribution to the $\ell\ell\gamma$ final state is from Higgs boson decays to a $Z$ boson and a photon.

7 Acknowledgements

We are grateful for the precise measurements and data analyses of the L3, BABAR, CMS, and ATLAS Collaborations, which made the present analysis possible. We acknowledge support from CFisUC and FCT through the project UID/FIS/04564/2020.
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