On the occurrence of Berezinskii-Kosterlitz-Thouless behavior in highly anisotropic cuprate superconductors

T. Schneider

Physikinstitut, University of Zurich - Winterthurerstrasse 190, 8057 Zurich, Switzerland

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Abstract - The conflicting observations in the highly anisotropic \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta\), evidence for BKT behavior emerging from magnetization data and smeared 3D-\(xy\) behavior, stemming form the temperature dependence of the magnetic in-plane penetration depth are traced back to the rather small ratio, \(\xi^c/\xi^0 = \xi^c/\xi^0 \simeq 0.45\), between the \(c\)-axis correlation length probed above \(T^c\) and below \(T_c\), and the comparatively large anisotropy. The latter leads to critical amplitudes \(\xi^c/\xi^0\) which are substantially smaller than the distance between two \(\text{CuO}_2\) double layers. In combination with \(\xi^c/\xi^0 \simeq 0.45\) and in contrast to the situation below \(T_c\), the \(c\)-axis correlation length \(\xi^c\) exceeds the distance between two \(\text{CuO}_2\) double layers only. Below this narrow temperature regime where 3D-\(xy\) fluctuations dominate, there is then an extended temperature regime where the units with two \(\text{CuO}_2\) double layers are nearly uncoupled so that 2D thermal fluctuations dominate and BKT features are observable.

Since the pioneering work of Berezinskii [1], Kosterlitz and Thouless [2] (BKT) on the BKT transition in the two-dimensional (2D) \(XY\) model, much efforts have been devoted to observe the universal behavior characteristic of the KT transition, as the universal jump of the superfluid density [3], measured in \(^4\text{He}\) superfluid films, or the non-linear \(I\)-\(V\) characteristic, observed in thin films of conventional superconductors [4,5]. Signatures of BKT physics can be expected also in layered superconductors with weak inter-layer coupling. Potential candidates are underdoped cuprate superconductors where the anisotropy increases with reduced transition temperature \(T_c\) [6]. Recent studies of the \(I\)-\(V\) characteristic [7], the frequency-dependent conductivity [8], the Nernst signal [9], the magnetization [10,11], and of the resistance [12] have been interpreted as signatures of BKT behavior. On the other hand, several experiments [13–21] failed to observe any trace of the universal jump in the superfluid density around \(T_c\). Indeed, a systematic finite-size scaling analysis of in-plane penetration depth data taken on films and single crystals of the highly anisotropic \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta\) reveal a smeared transition, consistent with an inhomogeneity-induced finite-size effect [22]. Furthermore, there is evidence that in small thin-film samples on insulating substrates, edge effects modify the vortex-vortex interaction making it short range, unlike the logarithmic long-range interaction needed for the BKT transition. This appears to make the BKT transition impossible in thin films of any size if they are supported by a non-superconducting substrate [23].

In this work we attempt to unravel the conflicting observations on the highly anisotropic \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_8+\delta\): evidence for BKT behavior emerging from the magnetization data of \(\text{Li et al.} [10]\) and evidence for smeared 3D-\(xy\) behavior, stemming form the temperature dependence of the magnetic in-plane penetration depth [13,16,22]. Although \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\) is highly anisotropic [24], resistivity measurements [25] uncover clearly the three-dimensional nature of the transition in this extreme type-II superconductor. Accordingly, sufficiently close to \(T_c\) homogeneous samples are expected to exhibit 3D-\(xy\) critical behavior. To explore how close this should be, we consider the magnetic in-plane penetration data shown in fig. 1, derived from the complex conductivity measurements of \(\text{Osborn et al.} [16]\) on epitaxially grown \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\) films, using a two-coil inductive technique. For comparison we included the leading 3D-\(xy\) behavior given by the universal relation [5,22,26,27]

\[
\frac{1}{\xi^2(T)} = \frac{16\pi^3k_B T}{\Phi_0^2\xi^c(T)}, \quad \xi^c = \xi^c(t^{-\nu}) ,
\]

(1)
with
\[ t = T/T_c - 1, \quad \nu \simeq 2/3. \tag{2} \]

Apparently, there is a rounded transition point to a finite-size effect, preventing the c-axis correlation length \( \xi^c \) to grow beyond the limiting length \( L_c \) set by inhomogeneities. Indeed the extreme in \( \text{d} \text{Re}(\rho)/\text{d}T \) exhibits an inflection point at \( T = T_{p_0} \simeq 83.77 \text{K} \), where \( \xi^c \) attains the limiting length \( L_c \). As shown previously [22], eq. (1) yields together with \( T_{c} = 84 \text{K} \), \( T_{p_0} \simeq 83.77 \text{K} \), \( \text{Re}(\rho(T_{p_0}))/\text{Re}(\rho(T = 0)) = 0.011 \), \( \text{Re}(\rho(T = 0)) = 0.6 \text{K} \), \( \text{Re}(\rho(T_{p_0})) = 13.66 \text{K} \), \( \lambda_{ab}(0) = 265 \text{nm} \), and \( L_c = \xi^c(T_{p_0}) \) the estimates
\[ \xi^{c,0} \simeq 2 \text{Å}, \quad L_c \simeq 93 \text{Å}, \tag{3} \]

for the critical amplitude of the c-axis correlation length below \( T_c \) and the limiting length \( L_c \) set by inhomogeneities. The rather small value of \( \xi^{c,0} \), reflecting the high anisotropy, \( \gamma = \xi^{c,0}/\xi^{s,0} \), was confirmed in other \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) films and single crystals in terms of a detailed finite-size scaling analysis [22].

Although the c-axis correlation length \( \xi^c \) increases by approaching \( T_c \), the occurrence of 3D thermal fluctuations requires that in \( \text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta} \) it exceeds \( s = 15 \text{Å} \), the distance between two \( \text{CuO}_2 \) double layers. In fig. 2, we depicted the temperature dependence of \( \xi^c \). Even though it exceeds \( 15 \text{Å} \) around \( 80 \text{K} \), a glance to fig. 1 shows that consistency with 3D-xy behavior is achieved much closer to \( T_c \) only, namely above \( T \approx 83.25 \text{K} \). Accordingly, the occurrence of 3D-xy behavior requires \( \xi^c \) to exceed the distance between two \( \text{CuO}_2 \) double layers by at least a factor of three. Above \( T_c \), the situation is even worse because the ratio between the correlation lengths above \( (\xi^s) \) and below \( (\xi^c) \) \( T_c \) is a universal quantity and in the 3D-xy universality class given by
\[ \xi^{s,0}/\xi^{c,0} \simeq 0.45. \tag{4} \]

This ratio implies a substantial shrinkage of the 3D-xy fluctuation-dominated regime above \( T_c \). To illustrate this point we included in fig. 2 the temperature dependence of \( \xi^c \). Taking again \( \xi^c \gtrsim 3s = 45 \text{Å} \) to locate the regime where 3D-xy fluctuations dominate we obtain roughly \( T \gtrsim 83.8 \text{K} \) which is rather close to the \( T_c \approx 84 \text{K} \) of the fictitious homogeneous system.

Given the reduced 3D-xy critical regime above \( T_c \), the system corresponds to a stack of nearly independent units with two \( \text{CuO}_2 \) double layers as long as \( \xi^c \lesssim 3s \). In this intermediate regime, 2D and in particular BKT features are then expected. To check this expectation quantitatively we consider the 3D-xy scaling expression for the susceptibility in the limit \( T \gtrsim T_c \), and \( H_c \rightarrow 0 [5,28–30] \).

\[ \frac{m}{H_c} = -\frac{Q^+C_3^+g_B T (\xi_{ab}^c)^2}{\Phi_{0}^{2}C_4^+}, \tag{5} \]

where \( m = M/V \) is the magnetization per unit volume and \( Q^+C_3^+ \approx 0.9 \) a universal number. In type-II superconductors, exposed to a magnetic field \( H_i \) in direction \( i \), there is also the magnetic-field–induced limiting length \( L_{H_i} = \sqrt{\Phi_0/(a H_i)} \) with \( a \approx 3.12 \) [31], related to the average distance between vortex lines [26,31]. As the magnetic field increases, the density of vortex lines becomes greater, but this can continue indefinitely, the limit is roughly set on the proximity of vortex lines by the overlapping of their cores. Due to these limiting length the phase transition is rounded and occurs smoothly. Indeed, approaching \( T_c \) from above the correlation lengths combination \( \xi^c \) increases but is bounded by \( L_{H_i}^2 = \Phi_0/(a H_k) \) where \( i \neq j \neq k \). In this context it is important to recognize that the confinement effect of the magnetic field in direction \( i \) on fluctuations within a region \( L_{H_i} \) acts only in the plane perpendicular to the field. Therewith eq. (5) reduces for
from the magnetization data of Li et al. [10] with H applied along the c-axis. The solid line is \( M(T = 84 \text{K})/H = 6.6/H \) corresponding to eq. (7).

\[
T \simeq T_c \text{ and } H_c \to 0 \quad \text{to} \quad \frac{m}{H_c} = -\frac{Q^+ C^+_3 k_B T}{\Phi_0^{1/2} a^{1/2} H_c^{1/2}},
\]

because \( \xi^+_{ab}/\xi^+_c = \gamma \) and the growth of \( \xi^+_{ab} \) is limited by \( \xi^+_c = L_H \simeq \sqrt{\Phi_0/(a H_c)} \). On the other hand, in the strict 2D case \( \xi^+_{ab} \) cannot grow beyond the thickness \( d_a \) of the system. Consequently, as \(-m/H_c \) and \( \xi^+_{ab} \) initially increase with reduced temperature in a fixed field they then saturate due to the magnetic-field–induced finite-size effect. In this case eq. (5) reduces to

\[
\frac{m}{H_c} = -\frac{Q^+ C^+_3 k_B T}{\Phi_0 d_a H_c}.
\]

In fig. 3 we show \(-M/H \) vs. \( H \) for Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ close to \( T_c \) at \( T = 84 \text{K} \) derived from the magnetization data of Li et al. [10], with \( H \) applied along the c-axis. The comparison with the characteristic 2D-behavior (7) reveals remarkable agreement, extending over nearly two decades of the applied magnetic field. Noting that down to the lowest applied magnetic field, \( H_c = 5 \text{Oe} \), there is no sign of leveling off, arising from an inhomogeneity-induced finite-size effect, it follows that the length of the homogenous regions in the \( ab \)-plane, \( L_{ab} \), exceeds the attained magnetic-field–induced limiting length \( L_{H_c} = \sqrt{\Phi_0/(a H_c)} = 1.14 \times 10^{-4} \text{cm} \). This behavior does not confirm the occurrence of intermediate 2D critical behavior in sufficiently anisotropic systems above \( T_c \) only. In addition, it uncover that in the \( ab \)-plane excellent homogeneity can be achieved and an applied magnetic field leads to the outlined finite-size effect.

To substantiate the occurrence of the magnetic-field–induced finite-size effect further, we consider the temperature dependence of the magnetization at fixed magnetic field. When \( \xi^+ < 3s \) and \( L_{H_c} \gg \xi^+_{ab} \) eq. (5) reduces to

\[
\frac{m}{T H_c} = -\frac{Q^+ C^+_3 k_B (\xi^+_{ab})^2}{\Phi_0^2 d_a}.
\]

In this regime the in-plane correlation length \( \xi^+_{ab} \) is expected to exhibit the characteristic BKT-behavior in the reduced temperature \( t = T/T_c - 1 \), namely \( \xi^+_{ab} \propto \exp(bt^{-1/2}) \) [2], with \( b \approx 1 \) a non-universal constant and \( a \) related to the vortex core diameter. However, as \( t \) decreases and with that \( \xi^+_{ab} \), approaches the limiting length \( L_{H_c} = \sqrt{\Phi_0/(a H_c)} \) a finite-size effect sets in and \( \xi^+_{ab} \) saturates in the limit \( L_{H_c} \ll \xi^+_{ab} \). To describe the resulting crossover we introduce the finite-size scaling function \( S \) [32] in terms of \( (\xi^+_{ab})^2 = (\xi^+_{ab})^2 S \times \left( H_c (\xi^+_{ab})^2/\Phi_0 \right) \) where \( S \left( (\xi^+_{ab})^2 / (a L_{H_c}^2) \right) \rightarrow 1 \) for \( L_{H_c} \gg \xi^+_{ab} \) and \( S \left( (\xi^+_{ab})^2 / (a L_{H_c}^2) \right) \rightarrow sa L_{H_c}^2/(\xi^+_{ab})^2 \) for \( L_{H_c} \ll \xi^+_{ab} \).

In fig. 4 we depicted \(-M/(TH_c) \) vs. \( t = T/T_c - 1 \) at \( H_c = 10 \text{Oe} \) and the inset shows the finite-size scaling function in terms of \(-M/(TH_c) \) vs. \( \exp(2bt^{-1/2}) \). It is seen that by approaching the transition temperature \(-M/(TH_c) \) increases but saturates to \(-M/(TH_c) = 8.7 \times 10^{-3} \) in agreement with the behavior shown in fig. 3. Sufficiently away from \( T_c \) a crossover to the BKT behavior, requiring \( L_{H_c} \gg \xi^+_{ab} \) and indicated by the solid line, can be anticipated. As the crossover
extends over a rather extended temperature regime, agreement with the leading BKT behavior is limited and the 3D-$xy$ critical regime is not accessible. Nevertheless, the finite-size scaling function reveals the flow to BKT correlation length probed above ($\xi$) and pronounced anisotropy it is not an artefact of $T_\xi$ as this behavior relies on the universal ratio fluctuations dominate and BKT features are observable.

We have seen that the conflicting observations in the highly anisotropic Bi$_2$Sr$_2$CaCu$_2$O$_{6.5}$, evidence for BKT behavior emerging from the magnetization data of Li et al. [10] and smeared 3D-$xy$ behavior, stemming form the temperature dependence of the in-plane superfluid density underdoped La$_{2-x}$Sr$_x$CaCu$_2$O$_{6.5}$, which are substantially smaller than the distance between two CuO$_2$ double layers. In combination with $\xi_c/\xi_0 \approx 0.45$ and in contrast to the situation below $T_c$ the c-axis correlation length $\xi_c$ exceeds the distance between two CuO$_2$ double layers very close to $T_c$ only. Below this narrow temperature regime where 3D-$xy$ fluctuations dominate, there is then an extended temperature regime left where the units with two CuO$_2$ double layers are nearly uncoupled so that 2D thermal fluctuations dominate and BKT features are observable. As this behavior relies on the universal ratio $\xi_c/\xi_0 \approx 0.45$ and pronounced anisotropy it is not an artefact of Bi$_2$Sr$_2$CaCu$_2$O$_{6.5}$, but a generic feature of sufficiently anisotropic cuprate superconductors. Examples include underdoped La$_{2-x}$Sr$_x$CuO$_4$ and YBa$_2$Cu$_3$O$_7$-$\delta$ where the temperature dependence of the in-plane superfluid density does not reveal any trace of the universal jump [13–21].

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