Decomposition of High-Frequency Electrical Conductivity into Extracellular and Intracellular Compartments based on Two-Compartment Model using Low-to-High Multi-$b$ Diffusion MRI

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Abstract

**Background:** As an object’s electrical passive property, the electrical conductivity is proportional to the mobility and concentration of charged carriers that reflect the brain micro-structures. The measured $M_b$-DWI data by controlling the degree of applied diffusion weights can quantify the apparent mobility of water molecules within biological tissues. Without any external electrical stimulation, magnetic resonance electrical properties tomography (MREPT) techniques have successfully recovered the conductivity distribution at a Larmor-frequency.

**Methods:** This work provides a non-invasive method to decompose the high-frequency conductivity into the extracellular medium conductivity based on a two-compartment model using multi-$b$ diffusion-weighted imaging ($M_b$-DWI). To separate the intra- and extracellular micro-structures from the recovered high-frequency conductivity, we include higher $b$-values DWI and apply the random decision forests to stably determine the micro-structural diffusion parameters.

**Results:** To demonstrate the proposed method, we conducted human experiments by comparing the results of reconstructed conductivity of extracellular medium and the conductivity in the intra-neurite and intra-cell body. Human experiments verify that the proposed method can recover the extracellular electrical properties from the high-frequency conductivity using a routine protocol sequence of MRI scan.

**Conclusion:** We have proposed a method to decompose the electrical properties in the extracellular, intra-neurite, and soma compartments from the high-frequency conductivity map, reconstructed by solving the electro-magnetic equation with measured B1 phase signals.

**Keywords:** Magnetic resonance electrical property tomography; High-frequency conductivity decomposition; Multi-$b$ diffusion weighted imaging; Low-frequency conductivity tensor; Random forest

**Background**

Using a conventional MRI scanner without any external electrical stimulation, magnetic resonance electrical properties tomography (MREPT) techniques have
been developed and successfully recover the conductivity distribution at Larmor-frequency (about 128 MHz at 3 T) [1, 2, 3, 4]. The electrical conductivity of biological tissues is proportional to the apparent concentration and mobility of ions in the intracellular and extracellular compartments. As the separated form of electrical conductivity of biological tissues, the low-frequency conductivity (< 1 kHz) is dominantly influenced by the apparent concentration and mobility of ions in the extracellular compartment.

To decompose the high-frequency conductivity into the extracellular and intracellular compartments using a conventional clinical MRI scanner, it requires microstructural parameters including the extracellular volume fraction (EVF), distributions of electrical charged molecules, and diffusion coefficient coefficients in the extracellular and intracellular compartments [5, 6]. For the model-based microstructure imaging based on the tissue micro architecture in the brain, the microstructural parameters for the intracellular and extracellular compartments have been studied, related with specific tissue micro-structure features from M\textsubscript{b-DWI} data [7, 8, 9]. Although a two compartment model is the simplest form, the determination of micro-structural parameters from measured decay MR DWI signals with respect to b-value is ill-posed because the estimation of parameters from the combination of smooth exponential curves is sensitive to noise in the measured DWI data. To stabilize the ill-posedness, two compartment models typically assume some restrictions: intrinsic diffusivity and/or water diffusing in elongated cellular fibres, based on the ball-and-stick model [10, 11]. For separating the low-frequency conductivity from the high-frequency conductivity, we need to quantify the microstructures of extracellular compartment. However, typical two compartment models based on the ball-and-stick model consider the diffusion signals from soma or other large cellular domains as those from the extracellular compartment. The reconstructed low-frequency conductivity using the ball-and-stick model overestimates...
EVF and, as a result, causes the biased conductivity values, especially in the gray matter region.

Various exponential diffusion models have been proposed to describe signal attenuation with DWIs using more than two $b$-values \cite{12, 13, 14, 15, 16, 17}. Using the exponential diffusion models, an electrodeless method providing the low-frequency electrical property imaging without any external hardware was proposed\cite{6, 18, 17}. The proposed methods mainly focus on separating the ion mobility and concentration in the extracellular space (ECS) from the recovered high-frequency conductivity. However, the developed methods still have difficulties in distinguishing the hindered diffusion of free water molecules and the diffusion-limited compartment. Recently, a biophysical model has been proposed for apparent cell body (soma) and neurite density imaging (SANDI), which tries to recover the soma size and density in addition to neurite density \cite{9}. In particular, in ECS, to decompose the electrical properties of each compartment, it is important to distinguish the micro-structural characteristics of the soma size and density belonging to the intracellular compartment. The SANDI model needs to include the direction-averaged DWI signal at high $b$-values ($\geq 3000$ s/mm$^2$) to detect the apparent soma size and density. Combining with the estimated DWI data for the higher $b$-values, we apply the SANDI model to estimate the micro-structures of extracellular compartment.

To determine the micro-structural parameters of biological tissues using the measured Mb-DWI data, we use the random forest regression, an ensemble machine learning algorithm by constructing a multitude of decision trees at training time \cite{19, 20}. The applied machine learning method builds a forest of uncorrelated trees, combined with randomized node optimization and bootstrap aggregating \cite{21}.

To demonstrate the electrical property decomposition from the recovered high-frequency conductivity, we generate the high-frequency conductivity using the convection-reaction partial differential equation with a small regularization parameter\cite{1}. 

The recovered high-frequency conductivity reflects the combined electrical properties by the intra- and extra-cellular compartments using a routine protocol sequence of MRI scan. We conducted human experiments by comparing the results of reconstructed conductivity of extracellular medium and the conductivity in the intra-neurite and intra-cell body. We extracted the apparent total ion concentration from the high-frequency conductivity map and the estimated micro-structural parameters. Human experiments indicate that the total ion concentration and the extracellular diffusion tensor can predict the extracellular electrical properties without externally injected currents. The accuracy and precision of the reconstructed low-frequency conductivity distribution in the extracellular compartment were evaluated.

**Results**

MRI measurements were performed with three healthy volunteers without a documented history of any disease were recruited. The participants were located inside the bore of a 3T MRI scanner with the head coil in transmit and a 32-channel RF head coil (Achieva TX, Philips Medical Systems, the Netherlands). All experimental protocols were approved by the institutional review board of Kyung Hee University (KHSIRB-16-033). All methods were carried out in accordance with the relevant guidelines and regulations and all participants provided written informed consent.

For MREPT imaging experiments, the multi-spin-echo pulse sequence with multiple refocusing pulses was adopted to minimize the measured noise. Before the data acquisition, we applied a volume shimming method with the volume defined to cover the brain region. Imaging parameters were as follows: TR/TE=1500/15 ms, number of echoes (NE)=6, number of excitation (NEX)=1, slice thickness=4 mm, field-of-view (FOV)=260×260 mm², imaging matrix size=128×128×5, and scan time=16 min. After MREPT scans, we performed DWI scans using the single-shot spin-echo echo planner imaging (SS-SE-EPI) pulse sequence. We applied the diffu-
sion weighting gradients in 15 directions with 4 $b$-values of 1000, 2200, 3000 and 3600 s/mm$^2$, respectively. Imaging parameters were as follows: TR/TE=2000/70 ms, $\delta/\Delta=21/33$ ms, NEX=2, slice thickness=4 mm, and acquisition matrix size =64×64×5. The scan time was about 6.2 min. The matrix size of 64×64×5 was extended to 128×128×5 to match the spatial resolution (2.03×2.03×4 mm$^3$) of MREPT experiment. We used only 3 orthogonal gradient directions in the experiments. An additional conventional T$\text{I}$ weighted scan of 2 min was included for anatomical reference.

To reduce the noise artifacts, we used odd echoes of six measured complex MREPT signals to avoid the background phase signal due to the consecutive 180° RF pulses. Since the accumulated noise artifacts in the phase signal is inversely proportional to MR magnitude intensity, $\tilde{S}_k$, $k = 1, 3, 5$, the measured phase signal was optimized as a weighted averaging using the weight of [22]

$$w_k = \frac{|\tilde{S}_k|^2}{|S_1|^2 + |S_2|^2 + |S_3|^2}, \quad k = 1, 3, 5$$

Figure 1 shows the estimated micro-structural parameter maps of SANDI model (8) for the first human subject: the estimated extracellular volume fraction $f_{ec}$, intra-neurite volume fraction $f_{ne}$, soma volume fraction $f_{so}$, extracellular diffusivity $D_{ec}$, and intra-neurite diffusivity $D_{in}$ from the first subject.

Figure 2 (a) and (b) show the MR magnitude and the B1 phase images at the third imaging slice of the first subject. For quantitative analyses, T$\text{I}$ image was segmented into cerebrospinal fluid (CSF), gray matter (GM), and white matter (WM) using the segmentation tool of Statistical Parametric Mapping (SPM 12) [23]. These regions are shown in Figure 2 (c).

Figure 3 shows the details of reconstructed conductivity images from the first subject. To estimate the high frequency conductivity, $\sigma_H$, with the acquired transceiver
phases of B1 maps, we solved the convection-reaction partial differential equation (PDE) in (5) with the diffusion term $c = 0.02$.

To estimated the extracellular ion concentration, $\bar{c}_{ec}$, in (15), we assume that the intra-neurite ion concentration and soma ion concentration are same, and further assume that $c_{in} = \beta \bar{c}_{ec}$ for some constant $\beta$. For the human brains, we set $\beta = 0.41$ as suggested in [6] by adopting reference values of intracellular and extracellular ion concentrations of four predominant ions ($\text{Na}^+$, $\text{Cl}^-$, $\text{K}^+$, and $\text{Ca}^{2+}$). Using the reference ratio value $\beta = 0.41$, the extracellular ion concentration $\bar{c}_{ec}$ can be estimated as

$$
\bar{c}_{ec} = \frac{\sigma_H}{\beta (f_{ne}D_{in} + f_{so}D_{is}) + f_{ec}D_{ec}}
$$

(2)
With the estimated high frequency conductivity and the parameters of the SANDI model for brain micro-structure, we recovered the extracellular ion concentration, $\bar{c}_{ec}$, extracellular conductivity, $\sigma_{ec}$, intra-neurite conductivity, $\sigma_{ne}$, and soma conductivity, $\sigma_{so}$.

Figure 4 shows the $3 \times 3$ extracellular conductivity tensor, $C_{ec}$, and intra-neurite conductivity tensor, $C_{ne}$, images. To estimate conductivity tensors, we used the water molecule diffusion tensors with the $b$ value of 1000 s/mm$^2$. We fixed the principal diffusion direction (eigenvector corresponding to the maximum eigenvalue of the diffusion tensor) and solved the equations (19) and (20) to obtain the extracellular diffusion tensor, $D_{ec}$, and intra-neurite diffusion tensor, $D_{ne}$, images. Using these diffusion tensors, we reconstructed the extracellular conductivity tensor, $C_{ec}$, and intra-neurite conductivity tensor, $C_{ne}$, in (21) and (22), respectively.

Table 1 summarizes the estimated values of high frequency conductivity, extracellular ion concentration, extracellular conductivity, and intra-neurite conductivity of each subject. In Table 1, longitudinal (L), transverse (T), and average (A) white matter conductivity values were found by computing the principal eigenvalue, the mean of the other two eigenvalues, and the mean of all eigenvalues of the conductivity tensor over all region, respectively.

The high-frequency conductivity values in CSF regions were between 1.43 and 1.58 S/m and the extracellular conductivity values were between 1.35 and 1.49 S/m. Note SANDI model does not take into account CSF compartment. For this
Figure 3  Estimated high frequency conductivity, $\sigma_H$, extracellular ion concentration, $\bar{c}_{ec}$, extracellular conductivity, $\sigma_{ec}$, intra-neurite conductivity, $\sigma_{in}$, and soma conductivity, $\sigma_{is}$, from the first subject.

Table 1  Estimated values of high-frequency conductivity, $\sigma_H$, extracellular ion concentration, $\bar{c}_{ec}$, extracellular conductivity, and intra-neurite conductivity within CSF, white matter, and gray matter regions.

| Subject | $\sigma_H$ | $\bar{c}_{ec}$ | Extracellular conductivity | Intra-neurite conductivity |
|---------|------------|----------------|----------------------------|----------------------------|
| CSF     | 1.43±0.70  | 0.96±0.35      | 1.35±0.72                  | 0.02±0.02                  |
| GM      | 0.51±0.30  | 0.45±0.24      | 0.29±0.20                  | 0.07±0.05                  |
| WM      | 0.40±0.19  | 0.38±0.16      |                            |                            |
| WM      | 0.39±0.15  | 0.40±0.14      | 0.29±0.17                  | 0.14±0.08                  |
| WM      | 0.39±0.17  | 0.41±0.16      | 0.29±0.20                  | 0.12±0.05                  |

reason, a slight difference between high-frequency conductivity values and extracellular conductivity values was found in CSF regions.

For GM regions, the high-frequency conductivity values were between 0.51 and 0.55 S/m and the extracellular conductivity values were in the range of 0.29 to 0.31 S/m. The extracellular conductivity values were higher than the neurite conductivity values (0.07 S/m in GM), as expected.
We found longitudinal WM extracellular conductivity values of 0.29~0.31 S/m. Transverse WM extracellular conductivity values were between 0.14 and 0.16 S/m. The average ratio between longitudinal conductivity values and transverse conductivity values was 1.93~2.07. For WM regions, longitudinal intra-neurite conductivity values were 0.15 ~0.18 S/m and transverse intra-neurite conductivity values were 0.08~0.09 S/m.

The extracellular conductivity values in GM regions were higher than the average extracellular conductivity values in WM regions, whereas the average intra-neurite conductivity values in WM regions were always higher than the intra-neurite conductivity values in GM regions.

Figure 5 shows the estimated extracellular conductivity tensor, $C_{ec}$, and intra-neurite conductivity tensor, $C_{ne}$, images represented by tri-axial ellipsoids, respectively, in the rectangular ROIs shown in extracellular conductivity, $\sigma_{ec}$, and intra-neurite conductivity, $\sigma_{ne}$, images. Since all of the conductivity tensors shared the same eigenvectors from the diffusion tensor, their orientations were same. The radii of each ellipsoid are proportional to the eigenvalues and their axes are oriented along
the directions of eigenvectors. As expected, the volume of intra-neurite conductivity ellipsoids appeared larger in WM regions.

**Discussion**

Literature conductivity values including those obtained from direct impedance measurements (IM), MREPT and diffusion tensor MR electrical impedance tomography (DT-MREIT), are summarized in Table 2. The conductivity values of brain tissues heavily depend on the biological tissue structures, participant’s age and pathology, frequency, and the measurement conditions (**in vivo**, **ex vivo**, and **in vitro**) [24, 25, 26]. Without external injection currents and using conventional MR pulse sequences minimizing magnetic field inhomogeneity, MREPT is a promising
research area for practical clinical medical devices. Comparing to MREPT, by injecting a dc current into the imaging subject, MREIT can reconstruct images of the internal low-frequency conductivity distribution. In this paper, we proposed a new way to decompose electrical properties in each compartment (extracellular, intra-neurite, and soma compartments) from the reconstructed high-frequency conductivity using MREPT technique and the micro-structural parameters using SANDI model. There are still many problems to overcome, but nevertheless, the method of extracting low-frequency electrical properties in a non-invasive way from the high-frequency conductivity is critical for clinical usefulness.

Table 2 Literature conductivity values

| Method | CSF | GM | WM | L | T | A | | |
|--------|-----|----|----|---|---|---| | |
| MREPT | - 0.69±0.62 | 0.30±0.63 | Human, 64–300 MHz [24] |
| IM | - 0.70 | 0.39 | Ovine, 131 MHz [27] |
| Meta-analysis | 1.71±0.30 | 0.47±0.24 | 0.22±0.14 | Human, <1 kHz [26] |
| DT-MREIT | 1.58, 1.53 | 0.29, 0.24 | 0.39, 0.49 | 0.13, 0.17 | 0.22, 0.28 | Human, 10 Hz [28] |
| Cat (spinal cord), 5–10 Hz [30] |

From the intrinsic noise in the MR measurements, MREPT reconstruction techniques have been proposed to improve the quality of the high-frequency conductivity map [31, 32, 33, 34]. The high conductivity values in Table 1 using the reconstruction algorithm in [1] also depend on the first term in (5), which is the diffusion term to stabilize the solution. The diffusion term acts as a low-pass filter, leading to some blurring of the final high-conductivity maps.

Because SANDI model does not take into account CSF compartment, a slight difference between the high-frequency conductivity values and the extracellular conductivity values was found in CSF region as was described in Result section. In future studies, it is worth exploring a new model that take into account CSF compartment.

At the low-frequency, the internal electrical current flow caused by external current stimulation occurs only in the extracellular space and CSF, excluding the intracellular space, due to the insulation properties of cell membrane [35, 36, 25]. Despite extensive researches for the electrical properties in ECS, the reported low
frequency conductivity values do not match and show considerable standard deviation. Recently, a meta-analysis of reported human head electrical conductivity values at low frequency (< 1 kHz) provides a recommended value estimated under suitable and realistic conditions, in which data acquisition techniques were categorized into five groups including directly applied current and MREIT. In Table 2, the reference values for low-frequency conductivity were 1.71±0.3 (CSF), 0.47±0.24 (GM), and 0.22±0.14 S/m (WM) with broadly similar results to ours [28].

At fairly low frequencies, the conductivity of brain tissue, particularly white matter, is known to be anisotropic [30]. Comparing to the anisotropy of intra-neurite compartment, the average ratio between longitudinal WM extracellular conductivity values and transverse WM conductivity values of 1.93~2.07 found here were lower than 3 and 5.9 reported in [29] and [30], respectively. The extracellular conductivity values in GM and WM regions exhibited much stronger frequency dependencies compared to CSF region, because of their complicated tissue structures. More rigorous analysis including geometric and viscous components of the tortuosity of ECS will be needed in the future work.

To compensate the difficulties of measuring DWI data for higher b-values, we adopt the hypothesis that the DWI data corresponding to the b-value range (1000-3600 s/mm²) reflects distinguishable diffusion signals between the soma and the extracellular space. To avoid the ill-posedness to determine the six unknowns, \( f_{in}, f_{ec}, D_{in}, D_{is}, D_{ec}, \) and \( r_{s} \), in the two compartment model (8) from the smoothly decayed exponential curves, the parameter of \( D_{is} \) was fixed as \( 2 \times 10^{-3} \) mm²/sec. However, the determination of parameters by matching the observed DWI data for Mb-DWI data and SANDI model was still very sensitive to measured noise artifacts.

Electrical brain stimulation (EBS) techniques, such as transcranial direct current stimulation (tDCS) and deep brain stimulation (DBS), are promising treatments for human disorders [37, 38, 39, 40, 41]. Since there is no clear explanation for
the mechanism, EBS studies have relied on computational modeling using reference conductivity values in the whole brain region. The proposed electrical property decomposition from the high-frequency conductivity can be a promising work for the EBS techniques.

**Conclusion**

We have proposed a method to decompose the electrical properties in the extracellular, intra-neurite, and soma compartments from the high-frequency conductivity map, reconstructed by solving the electro-magnetic equation with measured B1 phase signals. By decomposing the electrical conductivity into the product of mobility and charged carrier concentrations, voxel-wise micro-structures including the extracellular volume fraction and diffusivity were investigated using SANDI model by analyzing the Mb-DWI data based on a two compartment model. In the SANDI model, to distinguish the intra-soma compartment from the mixed diffusivity signals in ECS, the Gaussian phase distribution approximation of the tail was used. To determine the micro-structural parameters in each separated compartment using Mb-DWI data, a machine learning algorithm, random forests, was used by constructing a multitude of decision trees. Combining with the predicted DWI data and SANDI model, we separated the extracellular conductivity from the high-frequency conductivity using the decomposed micro-structural diffusion parameters. To verify the proposed method, we conducted human experiments to verify that the proposed method recovered the low-frequency electrical properties using a routine protocol sequence of MRI scan.

**Methods**

**High-frequency conductivity at Larmor frequency using B1 phase map**

The electrical conductivity of biological tissue as a function of frequency is complicated by the anisotropic nature of tissue, non-homogeneous natures in the extracellular and intracellular compartments, and randomly distributed cells sizes.
The high-frequency conductivity is dominantly isotropic because the electrical current flow tends to pass through the cell membrane. To visualize the high-frequency conductivity at Larmor frequency, by measuring the B1 phase map based on the Bloch-Siegert shift, the high-frequency electrical conductivity $\sigma_H$ and permittivity $\epsilon_H$ in biological tissues satisfy the following at Larmor frequency $\omega$:

$$\nabla^2 \mathbf{B}_1 = i \omega \mu_0 \gamma_H \mathbf{B}_1 - \frac{\nabla \gamma_H}{\gamma_H} \times (\nabla \times \mathbf{B}_1)$$  \hspace{1cm} (3)

where $\gamma_H = \sigma_H + i \omega \epsilon_H$, $\mathbf{B}_1$ denotes the B1 field, and $\mu_0 = 4\pi \times 10^{-7}$ N/A$^2$ is the magnetic permeability of free space [2].

For the positive (negative) rotating component of the transmit B1 field $B_1^+ = |B_1^+|e^{i\phi^+}$ ($B_1^- = |B_1^-|e^{i\phi^-}$), by assuming $\sigma_H \gg \omega \epsilon_H$, a phase-based MREPT formula has been proposed and conducted for numerous clinical studies[42, 43, 44, 45]:

$$\nabla \phi'^{tr} \cdot \nabla \tau_H + \tau_H \nabla^2 \phi'^{tr} - 2\omega \mu_0 = 0$$  \hspace{1cm} (4)

where $\tau_H$ denotes $\frac{1}{\sigma_H}$ and $\phi'^{tr} = \phi^+ + \phi^-$ is the measurable transceive phase using MRI[1].

Since the reaction-diffusion equation is sensitive to the measured noise, to stabilize the formula (4), after adding an artificial diffusion term, the equation (4) leads to

$$-c \nabla^2 \tau_H + \nabla \phi'^{tr} \cdot \nabla \tau_H + \tau_H \nabla^2 \phi'^{tr} = 2\omega \mu_0$$  \hspace{1cm} (5)

where $c$ is a constant diffusion coefficient.
Detection of the micro-structures of biological tissues based on intracellular and extracellular compartments

Diffusion of water molecule through biological tissue can be quantified using the apparent diffusion coefficient (ADC) using two magnetic field gradients with the same area between 180° RF pulse. The first gradient induces dephasing of water proton spins, while the second gradient refocuses the spins. The signal intensity $S_b$ by applying a diffusion encoding gradient is given by

$$S(b) = S(0) \exp(-bD)$$  \hspace{1cm} (6)

where $S_0$ is the signal obtained without diffusion gradient and $b$ denotes the diffusion-weighting factor depending on the gradient pulse used in the DWI sequence:

$$b = \gamma^2 \delta^2 G^2 \left( \Delta - \frac{\delta}{3} \right)$$  \hspace{1cm} (7)

where $\gamma = 26.75 \times 10^7 \text{rad/Ts}$ is the gyromagnetic ratio of hydrogen, $\Delta$ is the diffusion time interval, and $\delta$ and $G$ are the duration and amplitude, respectively, of the diffusion-sensitizing gradient pulse. By assuming that the mobility of electrical ions are proportional to the water molecular mobility, the investigation of ADC maps for multiple $b$-values provides the apparent ion mobility component of the electrical conductivity.

For the model-based micro-structure imaging based on the tissue micro-architecture in the brain, the model parameters for the intracellular and extracellular compartments are associated with specific tissue micro-structure characteristics from Mb-DWI data [7, 8, 9]. To distinguish the diffusion signals from ECS and the cell bodies of any brain cell type (collectively named soma) [9], SANDI model proposes
the following compartment model of brain tissue micro-structure:

\[ S_b = S_0 (f_{ic} A_{in}(b) + f_{is} A_{is}(b)) + f_{ec} A_{ec}(b) \]  

(8)

where \( f_{ic} \) and \( f_{ec} \) are the intracellular and extracellular volume fractions, \( f_{ic} + f_{ec} = 1 \); \( f_{in} \) and \( f_{is} \) are the neurite and soma relative volume fractions in the intracellular compartment, \( f_{in} + f_{is} = 1 \); \( A_{in} \) and \( A_{is} \) are the normalized signals for restricted diffusion within neurites and soma, respectively, and \( A_{ec} \) is the normalized signal of the extracellular compartment. To investigate the complicated micro-structural model (8), some assumptions are applied to each parameter. The diffusion of water molecules associated with the extracellular compartment is modeled as isotropic Gaussian diffusion:

\[ A_{ec}(b) = e^{-bD_{ec}} \]  

(9)

The diffusion signals \( A_{in} \) from neurites are assumed as a collection of sticks (long thin cylinders). The direction-averaged DWI signal \( A_{in} \) is computed as [11]

\[ A_{in}(b) = \sqrt{\frac{\pi}{4bD_{in}}} \text{erf} \left( \sqrt{bD_{in}} \right) \]  

(10)

where \( \text{erf} \) is the error function. The signal contribution, \( A_{is} \), for the inra-soma compartment is computed from the Gaussian phase distribution (GPD) approximation of the tail:

\[ A_{is}(b) = \exp \left\{ -\frac{2(\gamma G)^2}{D_{is}} \left( \sum_{m=1}^{\infty} \frac{\alpha_m^{-4}}{\alpha_m^2 \gamma_s^2 - 2} \right)^2 (2\delta - \Psi) \right\} \]  

(11)
where

$$\Psi = \frac{2 + e^{-\alpha_m D_{is}(\Delta-\delta)} - 2e^{-\alpha_m D_{is}\delta} - 2e^{-\alpha_m D_{is}\Delta} + e^{-\alpha_m D_{is}(\Delta+\delta)}}{\alpha_m^2 D_{is}},$$

$D_{is}$ is the bulk diffusivity of water in soma, $\alpha_m$ is the $m$-th root of the Bessel equation $(\alpha r_s)^{-1} J_2^2(\alpha r_s) = J_5^2(\alpha r_s)$.

The total unknown parameters to be determined are $f_{in}, f_{ec}, D_{in}, D_{is}, D_{ec},$ and $r_s$. To avoid the ill-posedness to determine the six unknowns from the smoothly decayed exponential curves, typically the parameter of $D_{is}$ is fixed as $2 \times 10^{-3}$ mm$^2$/sec [9].

**Model-parameter estimation using random forest regression**

Random forest is a popular machine learning algorithm using the bootstrap aggregating (bagging) with a tree model as the base model. Random forest regression is an ensemble supervised learning method that combines bagging decision trees with random subset sampling of the predictors for constructing each node split [21].

To perform the random forest regression, each individual decision tree produces a prediction individually and then predictions of all decision trees are combined to generate a prediction of the ensemble. The number of trees can be adapted to find the desired trade-off between accuracy and computational efficiency of the detection process.

The five parameters $f_{in}, f_{ec}, D_{in}, D_{ec},$ and $r_s$ in (8) are estimated by random forest regression using the scikit-learn python toolkit [46] as in [9]. We generated $13^5$ synthetic signals using the model (8) with $13^3$ combinations of the five parameters chosen uniformly distributed within the intervals: $f_{in}, f_{ec} \in [0.01, 0.99], D_{in}, D_{ec} \in [0.1, 3] \times 10^{-3}$ mm$^2$/sec, and $r_s \in [3, 20] \mu$m. We added rician-distributed noise to the synthetic signals. We split the synthetic signals into random train and test subsets with test size of 20%. Hyperparameter selection in random forest regression
was performed using a grid search for the number of decision trees (150, 180, 200, 230, 250, 280, 300 and 330 trees). The remaining hyperparameters were left to default in scikit-learn toolkit. By comparing the mean squared errors obtained from the different number of trees, the final random forest regressor was built with 300 trees. One decision tree in the random forest regressor is displayed in Figure 6. To produce an understandable image, the depth of the decision tree was limited to three.

High-frequency conductivity decomposition

The recovered high-frequency conductivity $\sigma_H$ at Larmor frequency, obtained by solving the equation (5), can be decomposed as the following compartment mode:

$$\sigma_H = (\sigma_{ne} + \sigma_{so}) + \sigma_{ec}$$  \hspace{1cm} (12)$$

where $\sigma_{ne}$ and $\sigma_{so}$ denote the conductivity in the intra-neurite and soma compartments, respectively, and $\sigma_{ec}$ is the conductivity in the extracellular compartment.

At each compartment, the apparent conductivities $\sigma_{ne}$, $\sigma_{so}$, and $\sigma_{ec}$ are expressed...
as the sum of products of concentration, charge carrier mobility, and the charger of carrier. For simplicity of notation, we write \( f_{ne} \) and \( f_{so} \) instead of \( f_{ic}f_{in} \) and \( f_{ic}f_{is} \), respectively:

\[
\sigma_{ne} = f_{ne} \sum_{j=1}^{N_e} z^j_n q c^e_j m^e_j = f_{ne} \sum_{j=1}^{N_e} z^j_n q c^e_j \left( \frac{r_w q}{r_j k_B T} \right) D_{in} \tag{13}
\]

\[
= f_{ne} \bar{c}_{in} D_{in}
\]

\[
\sigma_{so} = f_{so} \sum_{j=1}^{N_s} z^j_s q c^s_j m^s_j = f_{so} \sum_{j=1}^{N_s} z^j_s q c^s_j \left( \frac{r_w q}{r_j k_B T} \right) D_{is} \tag{14}
\]

\[
= f_{so} \bar{c}_{is} D_{is}
\]

\[
\sigma_{ec} = f_{ec} \sum_{j=1}^{N_{ec}} z^j_e q c^e_j m^e_j = f_{ec} \sum_{j=1}^{N_{ec}} z^j_e q c^e_j \left( \frac{r_w q}{r_j k_B T} \right) D_{ec} \tag{15}
\]

\[
= f_{ec} \bar{c}_{ec} D_{ec}
\]

where \( \bar{c}_{ec} := \sum_{j=1}^{N_{ec}} z^j_e q c^e_j \left( \frac{r_w q}{r_j k_B T} \right) \) denote the apparent ion concentrations with respect to the water molecule diffusivity in the extracellular compartments. The other symbols of the physical quantities are as follows: \( r_w \) and \( r_j \) are the Stoke’s radius of a water molecule and an ion, respectively, \( q = 1.6 \times 10^{-19} \text{C} \) is the electric charge carried by a single proton, \( k_B \) is the Boltzmann constant, and \( T \) is the absolute temperature. \( c^e_j, m^e_j z^j_e q, \) and \( N_{ec} \) are concentration, the charge carrier mobility, the charge of carrier, and the number of electrical charges in the extracellular space, respectively.

By the same argument, \( \sigma_{ne} = f_{ne} \bar{c}_{in} D_{in} \) and \( \sigma_{so} = f_{so} \bar{c}_{is} D_{is} \), where \( \bar{c}_{in} \) and \( \bar{c}_{is} \) denote the apparent ion concentrations with respect to the water molecule diffusivity in the intra-neurite, and soma compartments, respectively.

**Extracellular diffusion tensor**

Diffusion process is sensitive to intracellular, extracellular, and cell density. For a fixed \( b \) value, the measured non-singular diffusion tensor \( D_b \) can be diagonalized as
\[
\mathbf{D}_b = \mathbf{S}_D \hat{\mathbf{D}}_b \mathbf{S}_D^T \quad \text{with} \quad \hat{\mathbf{D}}_b = \begin{pmatrix} d_1^b & 0 & 0 \\ 0 & d_2^b & 0 \\ 0 & 0 & d_3^b \end{pmatrix}
\] (16)

where the column vectors of \( \mathbf{S}_D \) are the orthonormal eigenvectors of \( \mathbf{D}_b \), the superscript \( T \) denotes the transpose and \( d_1^b \geq d_2^b \geq d_3^b \) are the corresponding eigenvalues.

We separate the apparent diffusion tensor \( \mathbf{D}_b \) into the extracellular and intracellular compartments:

\[
\mathbf{D}_b = \mathbf{D}_{ec} + \mathbf{D}_{ne} + \mathbf{D}_{so}
\] (17)

where \( \mathbf{D}_{ec} \) and \( \mathbf{D}_{ne} \) denote the apparent diffusion tensors in ECS and intra-neurite compartment, respectively, and \( \mathbf{D}_{so} \) is the isotropic diffusion. By assuming that the diffusion tensors \( \mathbf{D}_{ec} \), \( \mathbf{D}_{ne} \), and \( \mathbf{D}_b \) share the eigenvectors, the intrinsic diffusion tensors \( \mathbf{D}_{es} \) and \( \mathbf{D}_{ne} \) can be expressed as

\[
\mathbf{D}_{ec} = \mathbf{S}_D \hat{\mathbf{D}}_{ec} \mathbf{S}_D^T \quad \text{and} \quad \mathbf{D}_{ne} = \mathbf{S}_D \hat{\mathbf{D}}_{ne} \mathbf{S}_D^T
\] (18)

where \( \hat{\mathbf{D}}_{ec} \) and \( \hat{\mathbf{D}}_{ne} \) are the diagonal matrices consist of the eigenvalues of \( \mathbf{D}_{ec} \) and \( \mathbf{D}_{ne} \), respectively. To translate the estimated intrinsic diffusivities \( D_{ec} \) and \( D_{in} \) to apparent diffusion tensors in each compartment, we define scale parameters as

\[
\eta_{ec} = f_{ec} \frac{3D_{ec}}{tr(\mathbf{D}_b)}, \quad \text{and} \quad \eta_{ne} = f_{ne} \frac{3D_{in}}{tr(\mathbf{D}_b)}
\] (19)

where \( tr(\mathbf{D}_b) = d_1^b + d_2^b + d_3^b \) denotes the trace of \( \mathbf{D}_b \). Under the hypothesis that the extracellular diffusion tensor \( \mathbf{D}_{ec} \), the diffusion tensor \( \mathbf{D}_{ne} \), and the diffusion tensor
\( \mathbf{D}_b \) share the eigenvectors, we can determine the decomposed diffusion tensors:

\[
\mathbf{D}_{ec} = \eta_{ec} \mathbf{D}_b \quad \text{and} \quad \mathbf{D}_{ne} = \eta_{ne} \mathbf{D}_b
\]  
(20)

From the relation (20), the conductivity tensors in ECS and the neurite compartment can be expressed as the following

\[
\mathbf{C}_{ec} = \bar{\mathbf{c}}_{ec} \mathbf{D}_{ec} = \bar{\mathbf{c}}_{ec} \eta_{ec} \mathbf{D}_b
\]  
(21)

and

\[
\mathbf{C}_{ne} = \bar{\mathbf{c}}_{in} \mathbf{D}_{ne} = \bar{\mathbf{c}}_{in} \eta_{ne} \mathbf{D}_b
\]  
(22)

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Abbreviations
Mb-DWI: multi-b diffusion-weighted imaging; MREPT: magnetic resonance electrical properties tomography; EVF: extracellular volume fraction; SANDI: soma and neurite density imaging; ECS: extracellular space; SS-SE-EPI: single-shot spin-echo echo planner imaging; CSF: cerebrospinal fluid; GM: gray matter; WM: white matter; IM: impedance measurements; DT-MREIT: diffusion tensor magnetic resonance electrical impedance tomography; EBS: electrical brain stimulation; tDCS: transcranial direct current stimulation; DBS: deep brain stimulation

Availability of data and materials
All data generated or analyzed during this study are included in this published article.

Ethics approval and consent to participate
All experimental protocols were approved by the institutional review board of Kyung Hee University (KHSIRB-16-033). All methods were carried out in accordance with the relevant guidelines and regulations and all participants provided written informed consent.

Competing interests
The authors declare that they have no competing interests.

Consent for publication
Not applicable.
Authors' contributions

OIK designed the research topic and experiments, analyzed and drafted the manuscript. MBL and HJK prepared and performed the experiments and wrote up the experimental and result sections. All authors read and approved the final manuscript.

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