Theoretical investigation of hybrid foil-magnetic bearings on operation mode and load sharing strategy

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Abstract
This paper presents a theoretical investigation on operation mode and load sharing strategy of hybrid foil-magnetic bearings (HFMBs). According to the inherent characteristics of gas foil bearings (GFBs) and active magnetic bearings (AMBs), six possible work scopes are discussed to understand the operation mode of HFMBs. A numerical model coupling the calculations of the film pressure in GFBs and the magnetic forces in AMBs is conducted to predict the operation performance of HFMBs. The accuracy of the model is proved by comparing the predicted and tested friction torques. Analysis on the static and dynamic performance of HFMBs within three representative cases is conducted by varying the load sharing ratio. The dynamic supporting stiffness and operation position of HFMBs can be obviously enhanced by adjusting the operating mode and load sharing strategy. Results show that the direct stiffness and dynamic damping of HFMBs adopting hybrid mode are significantly higher than those adopting AMB or GFB independent mode.

Keywords
Hybrid foil-magnetic bearing, operation mode, load sharing strategy, prediction model, experimental investigation, static and dynamic performance

Date received: 7 September 2021; accepted: 29 December 2021

Introduction
Advanced energy and power equipment with high operating speeds, high energy density, and high working efficiency require oil-free bearings to have more comprehensive performance and adaptability to severe environment. Gas foil bearings (GFBs) and active magnetic bearings (AMBs), as two innovative types of oil-free bearings, are able to meet the requirements of most high-speed machines and have been developed and investigated for several generations.1

GFBs, as a type of compliant, self-acting hydrodynamic fluid film bearings using ambient gas as lubricant, have shown many advantages including inherent simple structure, no complex oil lubrication and sealing system, excellent high-speed operation characteristics and tolerance for high temperatures.2 To date, GFBs have been successfully used in many oil-free machinery applications, such as air cycle machine in aircrafts,3 turbochargers,4,5 turbojet engine,6,7 micro gas turbine engine.8–10 However, there are still some issues that hinder the development of GFBs. A special friction-resistant coating is needed on the surface of top foil to extend the applicable bearing life.11 Due to its poor damping property, extra coulomb damping is usually required to increase the high-speed stability of GFBs during the design of the compliant structure.12

Generating contact-free magnetic field forces by actively controlling the dynamics of a magnetic is the principle of AMBs which is actually used most often among the magnetic suspensions.13 The dynamics of the contact-free hovering depends mainly on the implemented control law.14–19 Current industrial applications of AMBs include vacuum and cleanroom systems,20,21 machine tools,22 medical devices,23,24 turbo-machinery,25,26 and Danfoss turbocor compressors.27,28 However, in high-speed applications, AMBs are easily affected by structural resonance, which is the primary reason leading to the
control system instability. Moreover, the lack of reliable and durable auxiliary bearing is also an issue that should be solved when AMBs are operated under high speeds.

Hybrid foil magnetic bearings (HFMBs), as shown in Figure 1, combine the above two oil-free bearing technologies. The unique coupling method enables HFMBs to make full use of the advantages of both bearings while effectively overcoming the disadvantages. Swanson et al. designed a test rig supported by a HFMB system to simulate the rotor dynamics of a small gas turbine engine. The hybrid system tripled the load capacity of the magnetic bearing alone and can offer a significant reduction in total bearing weight compared to a comparable magnetic bearing. Jeong et al. built an experiment set-up to analyze the performance of rigid rotor supported by combined smart bearings on critical speeds with PD control. The shaft vibration in a critical speed is significantly suppressed by the CSB (Combined smart bearing) rather than AFB (air foil bearing). Tian et al. presented a searching algorithm to determine the steady-state working position of a hybrid foil-magnetic bearing. The system simulation was performed and the results show that the steady-state controller with the area searching algorithm can locate the steady-state working position accurately and efficiently. Pham and Ahn conducted an experimental investigation on the parameter optimization of hybrid foil–magnetic bearings to support a flexible rotor. The optimized controller would not only improve vibration performance by 26% at bending critical speed, but also save the energy consumption of driving motor by 50% during run up test. Yang et al. presented a theoretical method to calculate the load capacity, dynamic stiffness and damping coefficients of HFMBs under a specified load sharing factor $\lambda$ with the predetermined operational state. The results demonstrate that the HFMBs show significant nonlinear property with considering of the shaft position. Jeong and Lee tested a rigid rotor supported by HFMBs to elucidate the effect of initial eccentric position on the vibration response of rotor. The magnetic control force was remarkably effective in reducing the subsynchronous vibration of the rotor supported by the HFMB. Jeong et al. developed a 225 kW class turbo blower by including hybrid foil–magnetic bearings. The HFMB technology results in superior vibration stability for unbalance vibration and aerodynamic instability in the range 12,000–15,000rpm (200–250Hz). Tian and Sun presented an adaptive control method to simplify the controller design and improve the performance of the rotor-HFMB system. The stiffness and equivalent viscous damping change with the excitation frequency. Jeong and Lee used a control algorithm to reduce the sudden imbalance vibration amplitudes of a rigid rotor which was operated at up to 12,000rpm. It was experimentally verified that using the HFMB made sudden imbalance vibration control possible during rotor operation with an air foil bearing. Basumatary et al. presented a dynamic model that coupled dynamics of GFBs and electromagnetic actuators to discuss the effect of electromagnetic actuators on the stability of a rotor supported on GFBs. The sub-synchronous vibration present in case of GFBs is reduced and the stability band of the rotor is increased due to the implementation of electromagnetic actuator. However, the most contributory research work on HFMBs is proposed in 2000. Heshmat et al. presented an experimental investigation on HFMBs and proposed the earliest control algorithms for the load assignment of on the two bearings, GFBs and AMBs. Although the paper provides an ideal method of load distribution, it does not tell how to determine the load ratio and how to apply hybrid foil magnetic bearings.

The paper aims to provide guidance on the application of hybrid foil magnetic bearings at any possible working conditions. According to the load capacity and speed threshold of the GFB and the AMB, the working area of the HFMBs is divided into 6 parts. A numerical model coupling the calculations of the film pressure in GFBs and the magnetic forces in AMBs is conducted to calculate the static and dynamic performance of HFMBs. The friction torque experiment is used to prove the accuracy of the mathematical model. The influence of operation modes and load sharing ratio on bearing performance in different working conditions is theoretical analyzed by the case study, which purpose is to prove that by changing load sharing ratios and operation modes the HFMB can achieve optimized static and dynamic performance. This work is to be used as a preliminary design study to develop turbomachinery supported by HFMBs.

**Characteristics of hybrid foil-magnetic bearings**

A HFMB is configured as a nested design by inserting a GFB component in the radial gap between a AMB poles, as shown in Figure 2. The AMB directly acts on the rotor by magnetic force, while the GFB supports the rotor through hydrodynamic pressure in the gas film. The performance of the HFMB is the result of the joint action of the two bearings.

Figure 3 shows the suitable work scope of GFBs, AMBs as well as HFMBs. The work scope of AMBs

![Figure 1. The structure of hybrid foil magnetic bearings.](image-url)
consists of scope 1, scope 2 and scope 3 in consideration of the specific load capacity of the bearing depending on the type of ferromagnetic material and the design of the bearing magnet. It will be about 20 N/cm² and can be as high as 40 N/cm². The designed load capacity of AMBs is usually less than that of GFBs in a hybrid system in view of larger air gap and more compact structure. However, the main factors limiting the work scope of GFBs are load capacity and bearing stability. The work scope of GFBs consists of scope 2 and scope 4. The bearing load capacity of GFBs increases with the rise of the rotational speed. Meanwhile, GFBs will become unstable as the rotational speed increases.

The work scope of HFMBs is divided into six parts on the basis of the relationship between load capacity and speed of GFBs and AMBs.

Scope 1: As the rotational speed is low, the gas pressure generated between the top foil and the rotor is not enough to support the rotor. Therefore, it is suggested to use AMB independent mode in this scope (no need of using hybrid mode).

Scope 2: As the increase of rotational speed, the rotor may be separately supported by the GFB or the AMB. That means in this scope HFMBs can adopt three different operation modes (AMB independent mode, GFB independent mode and hybrid mode).

Scope 3: With the further increase of rotational speed, GFBs may have instability issue. However, due to the addition of the active control system (AMBs), the dynamic performance of HFMBs is enhanced, and the stable scope expands towards the high-speed direction. HFMBs can adopt AMB independent mode or hybrid mode.

Scope 4: The load exceeds the load capacity of AMBs, but is still within the capacity of GFBs. GFB independent mode and hybrid mode are the two possible methods in this scope.

Scope 5: As the rotational speed increases, GFBs steps into its unstable range and have the possibility of instability. By applying electromagnetic force to increase the bearing eccentricity, GFBs can be stabilized again. Hybrid mode is the only feasible operation mode in this scope.

Scope 6: The bearing load exceeds both the load capacity of GFBs or AMBs. Thus, hybrid model is the only option for HFMBs.
Theoretical model of hybrid foil-magnetic bearings

Gas foil bearing model

Figure 4 schematically presents the fundamental configuration and nomenclature for self-acting hydrodynamic GFBS. In GFBS, the top foil and bump foil act as supporting elements for the air film. The air and the supporting foil structure operate in series. i and j respectively represent the number of bumps and number of the calculated nodes in the axis direction.

Based on the continuity equation, Navier-Stokes equation and gas state equation, the prediction of the air pressure distribution between rotor and top foil can be solved by the dimensionless Reynolds equation using the finite difference method:\(^4^4\):
\[
\frac{\partial}{\partial \theta} (p \frac{\partial p}{\partial \theta}) + \frac{\partial}{\partial z} (p \frac{\partial p}{\partial z}) = \frac{\partial (\rho h)}{\partial \theta}
\]  
(1)

In the static calculation process of GFBS, the specific bearing load and speed correspond to a certain eccentricity and attitude angle of the rotor center under the action of gas film force. In the dimensionless process of Reynolds equation of compressible gas, the following dimensionless parameters are considered:
\[
\bar{p} = \frac{p}{p_a}, \quad \bar{h} = \frac{h}{C}, \quad \bar{z} = \frac{(z / R) \Lambda}{R} = \frac{6 \rho \Omega}{p_a} R \frac{(R / C)^2}{(1 - \nu^2)}
\]  
(2)

Referring to Eq. (2), it should be noted that the gas film thickness depends not only on the initial eccentricity and attitude angle, but also on the elastic deformation of the supporting structure. Therefore, the gas film thickness equation can be obtained:
\[
h = C + e \cdot \cos(\theta - \theta_0) + \delta
\]  
(3)

\(\delta\) is the compliant surface deformation. It can be seen from Figure 4 that zero slopes at both ends are an appropriate approximation because the top foil is continuous and the slope at each bump support is close to zero. The deflection of the top foil at the center between two adjacent bumps can be found as:\(^4^5\):
\[
v_{2i} = \frac{s^2 \Delta z}{1920 EI} (3p_{2i+1,j} + 3p_{2i-1,j} - p_{2i,j})
\]  
(4)

Where, \(EI\) is the bending stiffness of the top foil segment. Note that the local deflection function above was found assuming each bump is rigid, hence the total top foil deflection along the radial direction can be calculated by adding the local deflection of top foil to the bump deflection. The bumps are considered as springs with a constant structural stiffness, so the normalized foil deformation is given by:\(^4^6\):
\[
\left\{\begin{array}{l}
\delta_{2i-1,j} = 2s \left(\frac{l_0}{E}\right)^3 \left(1 - \nu^2\right) (p - p_a) \\
\delta_{2i,j} = 2s \left(\frac{l_0}{E}\right)^3 \left(1 - \nu^2\right) (p - p_a) \\
+ \frac{s^2 \Delta z}{1920 EI} (3p_{2i+1,j} + 3p_{2i-1,j} - p_{2i,j})
\end{array}\right.
\]  
(5)

The solution of Reynolds equation must be based on the deflection of foil structure and the distribution of gas film pressure. At both ends of GFBS along the axial direction, the gas film boundary is connected with the atmosphere, which can be regarded as the same as the atmospheric pressure. The gas film at the starting point and the ending point is also connected with the atmosphere. The boundary condition of Eq. (1) is:
\[
\left\{\begin{array}{l}
p = p_a, \quad z = \pm L / 2 \\
p = p_a, \quad \theta = 0, \quad 2\pi
\end{array}\right.
\]  
(6)

3.2 Active magnetic bearing model

Figure 5 depicts a simple example of a magnetic bearing control loop though comprising all the necessary components of a “standard” Active magnetic bearing (AMB) system. A magnetic bearing system can usually be well controlled by a linear control scheme despite strong nonlinear electromagnetic force. The force/displacement and the force/current dependencies of the magnet force \(F_x\) have to be linearized at operating point JSIP170164 which denotes the desired equilibrium position:\(^1^3\):
\[
F_x(i, x) = F_{x1}(i_{x1}, x_{x1}) - F_{x2}(i_{x2}, x_{x2}) \\
= F_{x1}(i_{x1}, x_{x1}) - F_{x2}(i_{x2}, x_{x2}) \\
+ [k_{x1}(i_{x1} - i_{x1}) - k_{x2}(i_{x2} - i_{x2})] \\
- [k_{x1}(x_{x1} - x_{x1}) - k_{x2}(x_{x2} - x_{x2})]
\]  
(7)

The constant \(k_{x1}(N / A)\) and \(k_{x2}(N / m)\) in Eq. (7) are commonly identified as force/current factor and force/displacement factor. The equilibrium position may alter in HFMBs when the rotor speed or load are varied and corresponding equilibrium displacement \(x_{x1}, x_{x2}\) and current \(i_{x1}, i_{x2}\) can be described as:
The magnetic force along X direction of HFMBs can be written simplistically:

\[
F_x(i, z) = F_{x1}(i_1, x_1) - F_{x2}(i_2, x_2) = C_{cur} - (k_{ix1} + k_{ix2})\Delta i + (k_{xx1} + k_{xx2})\Delta x
\]  

(8)

The first part of Eq. (9) represents a constant value determined by bias current after confirming the static equilibrium position of the rotor. The equilibrium position of the electromagnetic system causes the changes of force/current factor and force/displacement factor of four groups of magnets as well as the electromagnetic force at the static equilibrium position. Different Taylor series at different equilibrium positions lead to different linearized mathematical models of electromagnetic force.

Hybrid foil magnetic bearing model

The schematic diagram of HFMBs system is shown in Figure 6. The load of an off-center operation HFMBs is divided into two parts. One is borne by the hydrodynamic pressure between the top foil and the rotor, while the other is borne by the magnetic force generated by a pair of poles along the X-axis direction.

The calculation process of static and dynamic performance of HFMBs is shown in Figure 7. In the prediction of static performance of HFMBs, it is usually necessary to determine the center position of rotor according to the hydrodynamic pressure effect. Then, the dynamic properties of the electromagnetic force can be obtained. It is assumed that the PID controller gains will not change when the rotational speed is below 60 krpm. According to the load assigned to GFBs, the finite difference method is used to solve the static Reynolds equation with the consideration of the elastic deflection of the foil structure. The small perturbation method is used to predict the dynamic coefficients of GFBs. On the basis of the static results of GFBs and the bias current of AMBs according to the rotor center, the dynamic stiffness and damping coefficients of AMBs can be calculated. The dynamic parameters of the HFMBs are obtained by summing the stiffness and damping coefficients of GFBs and AMBs.

The gas film force is expanded by Taylor series at the equilibrium position, and the following formula is obtained:

\[
\begin{bmatrix}
F_{x1} \\
F_{y1}
\end{bmatrix} = \begin{bmatrix}
F_{x10} \\
F_{y10}
\end{bmatrix} + \frac{\partial}{\partial x} \begin{bmatrix}
F_{x10} \\
F_{y10}
\end{bmatrix} \Delta x + \frac{\partial}{\partial y} \begin{bmatrix}
F_{x10} \\
F_{y10}
\end{bmatrix} \Delta y + o(\Delta x^2, \Delta y^2, \Delta x^2, \Delta y^2)
\]  

(10)

As the perturbation term \(\Delta x, \Delta y, \Delta \dot{x}, \Delta \dot{y}\) approaches zero, \(o(\Delta x^2, \Delta y^2, \Delta x^2, \Delta y^2)\) is an infinitely small quantity of higher order and Eq. (10) can be reduced to a linear equation.

\[
\begin{bmatrix}
K_{xx} & K_{xy} \\
K_{yx} & K_{yy}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial F_{x10}}{\partial x} & \frac{\partial F_{y10}}{\partial y} \\
\frac{\partial F_{x10}}{\partial x} & \frac{\partial F_{y10}}{\partial y}
\end{bmatrix}
\]  

(11)

\[
\begin{bmatrix}
D_{xx} & D_{xy} \\
D_{yx} & D_{yy}
\end{bmatrix} = \begin{bmatrix}
\frac{\partial F_{x10}}{\partial x} & \frac{\partial F_{y10}}{\partial y} \\
\frac{\partial F_{x10}}{\partial x} & \frac{\partial F_{y10}}{\partial y}
\end{bmatrix}
\]  

(12)

The dynamic performance of AMBs depends on the control system. By taking the Laplace transform of the electromagnetic force and substituting the transfer function of the controller, the generalized stiffness of the electromagnetic system can be obtained.
\[ \begin{align*}
K(j\omega) &= \frac{C_{cur}}{j\omega} - F_s(j\omega) \times X(j\omega) \\
&= (k_{ix1} + k_{ix2})c_p - (k_{xx1} + k_{xx2}) + j(k_{ix1} + k_{ix2})(\omega^2 + \frac{c_i}{\omega}) \\
&= k_{mxt} + k_{mxt}X(j\omega) \\
&= k_{mxt} + k_{mxt}(\omega^2 + \frac{c_i}{\omega}) \\
&= k_{mxt} + k_{mxt}(\omega^2 + \frac{c_i}{\omega})
\end{align*} \]

Eq. (13) establishes the functional relationship between the time domain stiffness and damping of electromagnetic system according to PID controller gains. The real part and imaginary part of the frequency domain stiffness is generally considered as the time domain stiffness and time domain damping separately.

In order to simplify the calculation, the non-symmetric coupling terms \( D_{xy}, D_{yx} \) and the non-symmetric stiffness coefficients \( k_{xy}, k_{yx} \) in AMBs are neglected. Therefore, the dynamic stiffness and damping coefficients of HFMBs are the superposition of stiffness and damping of AMBs and GFBs, which are respectively calculated according to different working states.

\[
K = \begin{bmatrix}
K_{axx} + K_{mxt} & K_{axy} \\
K_{axy} & K_{ayy} + K_{mty}
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
D_{axx} + D_{mxt} & D_{axy} \\
D_{axy} & D_{ayy} + D_{mty}
\end{bmatrix}
\]

**Comparison of test results and numerical predictions**

Figure 8 presents schematic diagram of the test rig which is used to measure the friction torque of HFMBs. The test rig is driven by a high-speed motor supported by ball bearings. An oil/gas lubricating system is used to reduce the friction of the ball bearings. The driving motor can rotate up to 24krpm. An eddy current sensor is used to monitor the vertical position of the bearing. The displacement signal becomes the control PWM signal after the operation of the controller. Three signals (force signal,
speed signal and displacement signal) are collected by the PXI chassis.

Figure 9 display the front photograph and schematic diagram of the test rig for friction torque. A shaft with a diameter of 30 mm was installed on the high-speed motor. A HFMB with a weight of 1.65kg is supported by the shaft. A rod which is connected with bearing housing is used to measure the friction torque. The friction force generated in the bearings is recorded by the load sensor. Note that a balance weight on the left provides a pre-tightening force to the load sensor. Thus, the friction torque can be obtained by the product of the rod length and the change in measured value of the load sensor. A typical example of friction torque test is given in Appendix A. The test friction torque (18.17 N·mm) with AMB load ratio of 0 is obtained by subtracting the initial torque (0 N·mm) from the average torque (18.17 N·mm) at a steady speed of 24krpm. The average friction torque of the other set of test data is 21.86N, and the load ratio of the AMB is 0.2.

In the process of experiments, the load ratio of HFMBs is changed by adjusting the bias current of magnetic poles. Table 1 shows the relationship between the bias current and the sharing load ratios. Figure 10 compares the predictions and tested result of the friction torque when the load ratio of the AMB is increased from −0.5 to 0.7 at two rotational speeds. The predicted friction torque gradually decreases from 16.7344 N·mm with the AMB load ratio of −0.5 to 13.1805 N·mm with the AMB load ratio of 0.7 and the rotational speed of 24 krpm. Similarly, when the rotational speed is 18krpm, the predicted friction torque changes from 13.6017 N·mm to 10.2233 N·mm. With the increase of the load ratio of the AMB, the supporting force generated by fluid dynamic pressure of the GFB gradually decreases, as well as the friction torque.

The experimental results and predicted values showed a similar downward trend. When the load ratio of AMBs is less than or equal to 0, the experimental result is greater than the predicted value. The lower the load ratio, the greater the difference. However, when the load ratio is greater than 0, this trend is just the opposite. It is an unavoidable installation error that the parallelism of the bearing and the journal is insufficient. Therefore, regardless of whether the AMB provides a positive load or a negative load, as the load rate of the AMB increases, the deviation between the measured torque value and the predicted value will always become larger. The smaller the load ratio of the AMB, the more accurate the experimental measurement results. In addition, the tested results at the two rotational speeds are also given for comparison.

**Case study—parametric discussion**

A set of stand conditions of HFMBs structure and PID controller gains are given in Table 2, and the corresponding structure is shown in Figure 11. The film thickness varies in both $\theta$ and $Z$ directions due to the nature of GFBs, and the gap between rotor and magnetic poles only depends on the rotor position. The film thickness is the performance of the load capacity relating to the compliant surface forced by dynamic pressure, which has no direct relationship with AMBs gap.

According to the above description, the operation modes of HFMBs are divided into three categories: GFB independent mode, AMB independent mode and hybrid mode. It is worth noting that in HFMBs, GFBs operate independently from the influence of AMBs, while the independent operation of AMBs will be affected by GFBs. Considering the large radial clearance of HFMBs and the working load of the bearing does not exceed 10N, the design load of AMBs is set at 10N, the AMB design method is shown in Appendix B.

Three typical operation conditions, shown in Table 3, are used to study the influence of the operation mode and load sharing ratio of HFMBs on their static and dynamic performance under different working conditions. For two reasons, working scope 2 and working scope 4 are the most representative of HFMB’s six working scopes. On the one hand, considering the reduction of the power consumption generated by the AMB bias current, the GFB independent mode of scope 2 and scope 4 should be adopted as much as possible when designing the HFMB according to the operation conditions. On the other hand, the scope 2 and scope 4 can adopt any of the three operation modes and the load sharing ratio can be adjusted arbitrarily, so the other scopes with fewer operation modes and limited range of load sharing ratio can be considered as special cases of scope 2 and scope 4. It is worth noting that another way to improve the stability is to apply a reverse electromagnetic force on the journal, which will be discussed in condition 3.

**Operation condition in scope 2**

**Static characteristics**

Figure 12 shows the variation trend of the rotor center according to the change of AMBs load ratio. The...
rotational speed of the bearing is 30krpm and the total load is 8N, which is in scope 2. With the increase of the load ratio of AMBs, the rotor center is gradually approaching the bearing center from eccentric position. When AMBs load ratio is 0, the rotor is located at the position with the maximum eccentricity and the load is completely loaded by GFBs. In comparison, when AMBs load ratio is 1, the rotor is at bearing center and the load is completely loaded by AMBs, which can be defined as the independent operation of AMB in HFMBs. Adjusting the sharing load ratio is bound to change the load supported by GFBs and result in the deviation of the rotor center. Therefore, the determination of the rotor center of HFMBs depends on the load of GFBs.

The distribution curve of gas film pressure of HFMBs (on the middle surface) and the variation curve of differential current are shown in Figures 13(a) and 13(b), separately, with different AMBs load ratios. It is obvious that the gas film pressure gradually decreases with the increase of AMBs load ratio. As a cooperative component, the differential current along the \(X\) direction increases linearly, while the differential current along the \(Y\) direction shows a parabola descent. The differential current in both directions is slightly greater than zero when AMBs load ratio is zero, which is caused by the eccentricity of rotor. When the rotor reaches the bearing center, the air film pressure is close to the atmospheric pressure in the circumferential direction. Owing to the \(X\) direction differential current generates a separate load on the rotor, the differential current along the \(X\) direction reaches its maximum, and the differential current along the \(Y\) direction drops to zero. The significance of static

| AMB sharing ratio | AMB sharing ratio | AMB sharing ratio | Bias current | Bias current | Bias current |
|-------------------|-------------------|-------------------|--------------|--------------|--------------|
| 0                 | 0 A               | 0.4               | 1.45 A       | 0.8          | 2.05 A       |
| 0.2               | 1.03 A            | 0.6               | 1.78 A       | 1            | 2.3 A        |
| 0.3               | 1.26 A            | 0.7               | 1.92 A       |              |              |

The distribution curve of friction torque with rotational speed of 18krpm and 24krpm.

![Figure 9. Photograph and schematic diagram of friction torque test rig.](image)

![Figure 10. Bearing friction torque versus AMB load ratio with rotational speed of 18krpm and 24krpm.](image)

![Table 1. Bias current varies with load sharing ratio.](image)

| AMB sharing ratio | AMB sharing ratio | AMB sharing ratio | Bias current | Bias current | Bias current |
|-------------------|-------------------|-------------------|--------------|--------------|--------------|
| 0                 | 0 A               | 0.4               | 1.45 A       | 0.8          | 2.05 A       |
| 0.1               | 0.73 A            | 0.5               | 1.63 A       | 0.9          | 2.18 A       |
| 0.2               | 1.03 A            | 0.6               | 1.78 A       | 1            | 2.3 A        |
| 0.3               | 1.26 A            | 0.7               | 1.92 A       |              |              |

| Gas foil bearing (GFB) | Born length | Bearing housing radius | Top foil thickness | Bump foil thickness | Bump pitch | Bump half length | Bump height |
|------------------------|-------------|------------------------|--------------------|---------------------|------------|------------------|-------------|
| 40 mm                  | 31.6 mm     | 0.1 mm                 | 1.55 mm            | 0.4 mm              |

| Active magnetic bearing (AMB) | Bias current | Area of pole | Coil turns | Air gap thickness | \(K_p\) | \(K_i\) | \(K_d\) |
|-----------------------------|--------------|--------------|------------|------------------|--------|--------|--------|
| 1.5 A                       | 202 mm\(^2\) | 198          | 0.8 mm     | 1.70             | 7.60   | 0.035  |

The distribution curve of gas film pressure of HFMBs (on the middle surface) and the variation curve of differential current are shown in Figures 13(a) and 13(b), separately, with different AMBs load ratios. It is obvious that the gas film pressure gradually decreases with the increase of AMBs load ratio. As a cooperative component, the differential current along the \(X\) direction increases linearly, while the differential current along the \(Y\) direction shows a parabola descent. The differential current in both directions is slightly greater than zero when AMBs load ratio is zero, which is caused by the eccentricity of rotor. When the rotor reaches the bearing center, the air film pressure is close to the atmospheric pressure in the circumferential direction. Owing to the \(X\) direction differential current generates a separate load on the rotor, the differential current along the \(X\) direction reaches its maximum, and the differential current along the \(Y\) direction drops to zero. The significance of static...
parameters lies in that the rotor center can indirectly reflect HFMBs dynamic property in real time so as to modify and adjust the load sharing ratio.

**Dynamic characteristics**

Direct stiffness coefficient and cross-coupled stiffness coefficient of the HFMBs (rotational speed: 30krpm, bearing load: 8N) are shown in Figure 14 with respect to load sharing ratio. The direct stiffness coefficient $K_{xx}$ of HFMBs decreases sharply with the increase of AMBs load ratio, and the direct stiffness coefficient $K_{yy}$ also decreases significantly, as shown in Figure 14a. On the one hand, the gas film pressure of GFBs will decrease with the increase of AMBs load ratio, which will lead to a decrease in the dynamic stiffness coefficient of GFBs and finally cause the decrease of the total dynamic stiffness coefficient of HFMBs. On the other hand, the direct stiffness of AMBs increases with the increase of force/current factor and decreases with the increase of force/displacement factor. The increase of force/displacement factor with the increase of AMBs load ratio is slightly greater than force/current factor. Therefore, the direct stiffness coefficient of AMBs will also decrease significantly. It is easy to understand the reason that direct dynamic stiffness coefficient of HFMBs along the $X$ direction has a significantly stronger downward trend than that along the $Y$ direction.

It can be clearly seen from Figure 14 that the direct stiffness of AMB independent mode is greater than that of AMB because of the hydrodynamic pressure influence.
on AMBs. In terms of operation modes, the direct stiffness $K_{xx}$ is the largest with hybrid mode adopting AMBs sharing ratio of 0, followed by GFB independent mode and finally AMB independent mode. In contrast, the order of direct stiffness $K_{yy}$ from large to small is hybrid mode, GFB independent mode and AMB independent mode. The result indicates the direct stiffness of the HFMB support with the hybrid mode is always the largest in both the X direction and Y direction.

As can be seen from Figure 14(b), the cross-coupled stiffness coefficient of HFMBs decreases gradually with the increase of AMBs load ratio. Since the cross-coupled stiffness coefficient of AMBs is neglected, the instability of HFMBs can be attributed to GFBs. Both direct stiffness coefficient and cross-coupled stiffness coefficient of HFMBs decrease with the increase of AMBs load ratio.

Direct dynamic damping coefficient of HFMBs at the operation condition (rotational speed 30kprm, bearing load 8N) when different load sharing ratios are adopted, as shown in Figure 15. The diagram illustrates the change of direct dynamic damping coefficient of HFMBs with AMBs load ratio, and compares with that of GFBs and AMBs. It should be reminded that at this time, the dynamic damping of AMBs in the X direction and that in the Y direction are equal. Direct damping coefficient of HFMBs decreases slightly with the increase of AMBs load ratio because of direct damping coefficient of GFBs decrease. When AMBs load ratio is 0, the direct stiffness coefficient is the maximum as well as the cross-coupled stiffness coefficient. At 30kprm, the
dynamic damping of hybrid mode is the greatest, followed by AMB independent mode, and finally GFB independent mode.

**Operation condition in scope 4**

**Static characteristics**

Figure 16 describes the static characteristics of HFMBs in scope 4. The corresponding rotational speed is 30krpm and the load is 15N. The static performance of the bearing in scope 4 is extremely similar to that in scope 2. The static characteristics reflect the load sharing strategy of HFMBs, and it will not substantially change with the change of working condition.

**Dynamic characteristics**

Figure 17 presents the direct stiffness coefficient, cross-coupled stiffness coefficient and dynamic damping coefficient vs. AMBs load ratio in scope 4 (rotational speed 30krpm, bearing load 15N). The design bearing capacity of AMBs is 10N, which means the load ratio of AMBs should not exceed 0.67 as shown in Figure 17. The
direct stiffness coefficient of HFMBs along the X direction turns negative when AMBs load ratio exceeds 0.7. When AMBs show negative stiffness coefficient, HFMBs are out of control. The bearing stiffness of AMBs is generated by linear control, and the larger load ratio will make HFMBs show the essential negative stiffness. On the other hand, within the allowable operating range, the direct stiffness and cross-coupled stiffness of hybrid mode are the maximum when AMBs load ratio is 0. The dynamic damping hardly changes with the load sharing ratio. In terms of operation mode, AMBs cannot operate independently.

**Operation condition with higher rotational speeds**

**Static characteristics**

Figure 18 describes the static characteristics of HFMBs with higher rotational speed. The rotational speed is 42k rpm and the load is 15N. When the magnetic force provides a positive load, the rotor center gradually approaches the bearing center with the increase of AMBs load ratio, which is the same as other operation conditions. However, when the magnetic force provides a reverse load to enhance the rotor mass, the rotor center gradually moves away from the bearing center with the decreasing of AMBs load ratio. In both cases, the center trajectory of the rotor together forms a complete curve. In Figure 18b and Figure 18(c), corresponding to Figure 18(a), the gas film pressure of HFMBs independently supporting the rotor is further increased due to the reverse loading of magnetic force. When the magnetic force acts in the opposite way, the differential current along the X direction also changes from positive to negative, while the differential current along the Y direction still shows a parabolic decline. It can be concluded that using AMBs to increase the rotor mass is actually an extended application of the original operation mode.

**Dynamic characteristics**

Reverse loading (AMBs load ratio is negative) is another way to change the dynamic performance of HFMBs. Figure 19 describes the dynamic characteristics of HFMBs with reverse load sharing strategies. The stiffness and damping coefficient of bearings supported by GFBs or AMBs separately are also added for comparison. As can be seen from Figure 19a, when AMBs provide a positive effect, its direct stiffness coefficient curve changes with the load ratio similar to that of the first two operation conditions. The direct stiffness coefficient along X and Y direction decreases with the increase of AMBs load ratio. However, when AMBs act the opposite way, the direct stiffness coefficient along the X direction shows a short rise followed by a rapid decline, while the direct stiffness along the Y direction keeps rising. The maximum value of the direct stiffness coefficient along X direction occurs when AMBs load ratio is around −0.1. The reason is that the rise of differential current of AMBs will reduce the stiffness coefficient, even if it is an opposite effect of the magnetic force. Therefore, there is always a maximum value for the direct stiffness coefficient of HFMBs along the X direction. When the load ratio of AMBs is −0.1, the direct stiffness of HFMBs is the largest. When the load ratio of AMBs is −0.5, the cross-coupled stiffness of HFMBs is the maximum. Dynamic damping is almost unaffected by the load ratio. The direct stiffness, cross-coupled stiffness and dynamic damping of hybrid mode are greater than that of GFB independent mode.

**Conclusion**

In order to guide the conceptual application of HFMBs, this paper presented a theoretical and experimental investigation on operation mode and load sharing strategy. Because GFBs and AMBs have totally different physical properties and suitable work scope, the hybrid bearing, HFMBs, can be designed into six work scopes. By cross correlating Reynolds equation, foil structure deflection and magnetic force, a numerical model was presented. The accuracy of the prediction model is verified by comparing the friction torque of the test data and the numerical prediction with different AMBs load ratio. The bearing performance is not just determined by the rotational speed and load but also related to the operation mode and load sharing strategy. Thus, three representative
cases were selected to study the influence of load sharing ratio and operation mode on HFMBs.

According to the six work scopes, HFMBs can adopt different operation modes. When the load is less than the design load of AMBs, HFMBs can usually adopt the hybrid mode, GFB independent mode and AMB independent mode. When the actual load is greater than the design load of AMBs, HFMBs adopt the hybrid mode and GFB independent mode, while the AMB independent mode may be out of control. In all the three cases, the direct stiffness and the dynamic damping of hybrid mode are the largest, which reflects the strong dynamic performance advantage of hybrid mode. Although the cross-coupled stiffness will decrease as the load ratio of the AMB increases, the change in the cross-coupled stiffness is negligible due to the substantial increase in direct stiffness and damping in the hybrid mode.

With the increase of AMBs load ratio, the center position of the rotor gradually moves from the eccentric position to the bearing center. The direct stiffness and cross-coupled stiffness of HFMBs are greatly affected by load sharing ratio and operation mode, but the dynamic damping is only affected by operation mode. There is usually a maximum value for the direct stiffness as a function of AMBs load ratio, which is usually equal to zero or slightly less than zero. The cross-coupled stiffness will decrease with the increase of AMBs load ratio, and dynamic damping is almost unaffected by load sharing ratio. In the following work, a test rig supported by HFMBs will be built and the influence of PID controller gains on the performance of HFMBs will be discussed.

Appendix

Appendix A. Experimental measurement of friction torque

Figure 20 shows the shaft rotational speed and the measured bearing friction torque in a lift-off test cycle. The applied bearing load is 16.5N and the AMB load ratio is 0. As the rotational speed increases, a sharp peak appears in the bearing friction torque, which indicates the dry friction between the shaft and the top foil. The friction torque rapidly decreases with the increase of the speed. The bearing lifts off by generating a thin gas film which separates the rotor from the top foil, and electromagnetic force generated by the upper and lower poles share a portion of the load. Therefore, at a steady speed of 24krpm, the friction torque becomes much smaller. The friction torque becomes 18.17N mm with an AMBs load ratio of 0. The purpose of the test is to study the effect of AMB load ratio (-0.5 to 1) on friction torque of HFMBs. The final friction torque is obtained by subtracting the initial torque from the average torque at steady speed (18krpm and 24krpm). Another experimental measurement of friction torque with AMB load ratio of -0.2 is provided in Figure 21.

Appendix B. Structure and load design of electromagnetic bearings

Electromagnetic force formula:

\[ F_{\text{mag}} = \frac{\mu_0 N^2 A i^2}{4 \pi^2} \]  

(16)

When the structure of the AMB is determined, the electromagnetic force is proportional to the square of the current and inversely proportional to the square of the air gap length. Where,

\[ Ni = \lambda A_{cu} \]  

(17)

\( J \) is mainly determined by the cooling condition and insulation level of the coil.

\[ J = 3 \text{ A / mm}^2 \]  

(18)

The electromagnetic force becomes

\[ F_{\text{mag}} = \frac{\lambda^2 \mu_0 J^2}{4 \pi^2} \frac{AA_{cu}^2}{4x^2} \]  

(19)
In the eight-pole electromagnetic bearing, the electromagnetic force is determined according to the coordinate direction of the magnetic pole.

\[
F_{mag} = \frac{\lambda^2 \mu_0 J^2}{4\pi^2} A A_{cu}^2 \cos 22.5^\circ
\]

(20)

According to the geometric relationship of the bearing structure

\[
A = D_0 \arcsin \frac{b}{D_0}
\]

(21)

\[
A_{cu} = \frac{4(8 + \pi)b^2 - 4(\pi D + 4D - 4D_0) + \pi(D^2 - D_0^2)}{32}
\]

(22)

The calculation parameters of the electromagnetic bearing structure are as follows

\[
\begin{align*}
F_{mag} &= \frac{\lambda^2 \mu_0 J^2}{4\pi^2} A A_{cu}^2 \cos 22.5^\circ \\
A_{cu} &\leq \frac{28b_0}{\lambda J \mu_0} \\
A &= D_0 \arcsin \frac{b}{D_0} \\
A_{cu} &= \frac{4(8 + \pi)b^2 - 4(\pi D + 4D - 4D_0) + \pi(D^2 - D_0^2)}{32}
\end{align*}
\]

(23)

According to the above formula, the design method is as follows:

(a) Determine the magnetic flux density \(B\) when the stator is magnetically saturated.
(b) Determine the air gap \(x_0\) according to the structure of the GFB.
(c) Determine occupancy \(\lambda\) according to coil winding method.
(d) Determine \(J\) according to cooling condition.
(e) Calculate the coil cavity area \(A_{cu}\) required for the stator material to reach the magnetic saturation.
(f) Calculate the magnetic pole area \(A\) of the AMB according to the calculated \(A_{cu}\).
(g) Calculate the bearing capacity of the AMB based on the \(A_{cu}\) and \(A\). When the bearing capacity meets the design requirements, improve the parameters and repeat the above steps. Through constant adjustment of parameters, the result that the designer wants is finally achieved.

Authors’ contributions
HZ wrote the whole manuscript, completed the simulation analyses and drawings; QY, HG, YC assisted with simulation analyses; KF was in charge of the whole analyses and revised the final manuscript. All authors read and approved the final manuscript.

Funding
The authors acknowledge the financial support from the National Key R&D Program of China (2021YFF0600208), the National Natural Science Foundation of China (51875185), and the Foundation of Hunan Province (2020RC4018, 2020GK2069).

Declaration of Conflicting Interests
The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding
The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work was supported by the National Key R&D Program of China (2021YFF0600208), the National Natural Science Foundation of China (51875185), and Foundation of Hunan Province (grant number 2020GK2069, 2020RC4018, 2021YFF0600208, 51875185).

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Nomenclature

\( A \) cross-sectional area of magnetic circuit
\( A_{cu} \) coil cavity area
\( C \) radial clearance
\( C_{cur} \) equilibrium force
\( c_p \) proportional constant
\( c_d \) differential constant
\( c_i \) integral constant
\( D \) damping of HFMBs
\( D_{mx} \) damping of AMBs
\( D_{axx} D_{a xy} D_{b xy} \) damping of GFBs
\( E I \) bending stiffness of the top foil segment
\( e \) eccentricity
\( h \) film thickness
\( h (h/C) \)
\( i \) coil current
\( i_{x1} i_{x2} \) current in the X-axis direction
\( i_{0x1} i_{0x2} \) equilibrium current in the X-axis direction

\( J \) cooling factor
\( K \) stiffness of HFMBs
\( k_{x1} k_{x2} \) force/current factor
\( k_{xx1} k_{xx2} \) force/displacement factor
\( k_{mx} \) stiffness of AMBs
\( K_{axx} K_{a xy} K_{dxx} K_{dxy} \) stiffness of GFBs
\( l_0 \) half-length of bump in \( \theta \) direction
\( N \) Coil turns
\( p \) pressure
\( p_a \) ambient pressure
\( q_0 \) attitude angle
\( R \) distance between bearing or journal
\( s \) distance between computational grid points in circular direction
\( x \) air gap
\( x_{x1} x_{x2} \) displacement in the X-axis direction
\( x_{0x1} x_{0x2} \) equilibrium displacement in the X-axis direction
\( z \) axial coordinate
\( \bar{z} \) \( (z/R) \)
\( \Lambda \) \( \frac{s_{cur}}{2} \left( \frac{\bar{z}}{R} \right)^2 \)
\( \delta \) local top foil deflection
\( \Delta z \) distance between computational grid points in axial direction
\( \lambda \) occupancy rate
\( \mu \) absolute viscosity
\( \mu_0 \) air permeability
\( \theta \) angular coordinate
\( \theta_0 \) angular position of \( h_{min} \)
\( \nu \) position ratio
\( \Omega \) angular velocity
\( \omega \) Input frequency