Rigorous mathematical thinking based on gender in the real analysis course

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Abstract. Real analysis is a course that is abstract and high difficulty level. Male students are weak in understanding basic concepts, while women are always follow on examples of answers from lecturers. Consequently, their abstraction abilities do not develop. To provide appropriate treatment, the lecturer should know the rigour mathematical ability of the students. Therefore, this study aims to identify students' rigour mathematical abilities in Real Analysis courses. The research method is descriptive qualitative. The research instruments are test questions and interview sheets. The sample was six students. The researcher divides the students into three cognitive levels based on test results, i.e. two for low cognitive level, two for medium, and two for a high cognitive level. This study concludes that there is a difference of mathematical thinking of rigour of students based on gender-related to three levels of cognitive ability. Male students have a simple way of thinking impacting on a short answer pattern. The way of thinking of female students is systematic, and the pattern of the answer is long and sorted. The results of this study can be the basis of the treatment of students based on the level of cognitive ability and gender.

1. Introduction

The study of mathematics whose degree of difficulty is quite high is Real Analysis [1-3]. The difficulty in studying and solving mathematical problems was different for each gender [4-8]. The researcher refers more specifically to student learning difficulties for Real Analysis courses by interviewing ten students, each consisting of five women and men. The results of the interview revealed that male students prioritise the mastery of basic concepts rather than follow the lecturer's way of completing the proof of theorem or applicative matters. Also, they have difficulty in determining the initial idea when proving a theorem because the mastery of basic concepts is still lacking. While female students are less accustomed to using imagination or describe something that is abstract. They prefer to work on the problem or learn real analysis by following the lecturer's way of explaining or completing it during lectures. They are accustomed to solving the problem of proof or applicative with neat and systematic writing. Initial conclusion of this research is the mathematical ability of female students is higher than men. Devine et al. have initial conclusions in their research that reinforce the statement that female students have the potential in mathematics better than male [9].

The explanation of lecturers at the lecture has not been able to improve students' high-level mathematical ability effectively and efficiently. Therefore, one way to improve it is by doing the learning that can develop his rigour mathematical thinking ability. Hidayat et al. suggest that rigour mathematical thinking (RMT) emphasises the interaction and mediation between teachers and learners.
which results in a good understanding of matter to be transformed further conceptually through interrelated ideas [10]. Thus the ability of rigour mathematical thinking is needed to improve the accuracy and abstraction function of students' cognitive abilities [11]. There are three levels of student’s cognitive skills namely; high, moderate, and low [12,13]. These three levels distinguish the ability of students in studying and receiving lecture materials. Fitriyani [14] has studies connecting RMT and cognitive levels using different gender samples. The results of his research focused on male subjects using the three levels of cognitive abilities needed for RMT correctly.

Based on the above discussion, the purpose of this study is to determine the students' rigour mathematical thinking ability in the Real Analysis course on the topic of continuous function based on gender and students' cognitive level. The results of this study can be a reference for lecturers to apply a sufficient learning process. The process takes into account the level of cognitive ability and gender.

2. Method

This research uses descriptive qualitative method. The primary focus of this research is the subject. Researchers analysed the rigour mathematical abilities according to the three-level theory of cognitive skills associated with gender but did not elicit new methods [15]. The subjects of the study were six students from the 3rd level of academic year 2017/2018 from the Mathematics Education course that had contracted the Real Analysis course. Six people represent three levels of cognitive ability, i.e. each level consists of two people with high, moderate, and low-level criteria. Researchers conducted this research at the Faculty of Teacher Training and Education (FKIP) Unswagati Cirebon. The study period is October 2017 - March 2018. The research instruments are test sheets and interviews.

Researchers started this study by analysing the results of initial interviews of ten students from third-grade. Researchers select them randomly on condition that they have contracted Real Analysis courses. The discussions aim to know what difficulties they encounter during lectures and the reasons that support them. The researcher conducted a preliminary test with questions that had an RMT capability indicator. Researchers selected two students at random to represent each level of cognitive ability. The series of research procedures have been in line with Kinard's rigorous mathematical thinking theory in which RMT connects three levels of the cognitive function of the student, namely the level of qualitative, quantitative, and abstract relational thinking [11].

3. Results and discussion

Students take the test for Real Analysis problems of a continuous function. These questions want answers that relate some interrelated theories, the understanding of the material through the method of proof, and the student's ability to construct the solution in the form of a function graph. The researchers classified the results of the answers of all students into three levels of cognitive ability based on statements from Abdul Wahab et al, i.e. high with a score of 70-100, moderate with a score of 50-69, and low with a score of 0-49. Randomly selected two students with a different gender from each level of cognitive ability [16].

In the test, the student must show the function differential $f$ which is always present in the domain $D_f$ if the limit of the function's differential exists. The researcher analysed the answers of the six students. The following is the answer of two students with high cognitive abilities as seen in Figure 1.
Figure 1. Student's answer with high cognitive ability.
Based on Figure 1, it appears that both subjects were able to use all the level 3 cognitive functions, from relational abstract thinking abilities, i.e. they found the initial idea of completion of the answer, were able to activate prior knowledge by connecting and resolving the aspects they were thinking. That aspect is the value of $f'(a), \forall a \in D_f$ is not always exist. They can check by drawing a graph of the differential $f'(x)$ having two values. Both subjects are also able to provide initial clues up to logically prove the function of $f(x) = |x|$, where $D_f = \{x|x \in R\}$ has no derivative at point $x = 0$. They can also give examples that the signum $s(x)$ function is discontinuous at $x = 0$. Both are also proficient in a logic reinforcement of answers so that for those who read there is no different interpretation. The procedure answers are systematic and able to think hypotheses by involving definitions and the theorems of continuous functions. They are also capable of solving unique cases, which can infer the discontinuity of a function of two different of its continuity. Then they relate it to a defined function concept through consideration of the mathematical proportions of the two functions and then analyse them in detail.

In Figure 1 on the left side, the student is male, while the right side is a female. Although both are in the level 3 of cognitive function, there is still a difference in answering the problems. Female students look neater in painting function graph. They also detail in compile each stage of completion, patterns of answers lengthy, and all the answers are appropriate to the concepts of the lecturer. The male students are less neat in painting the function graph than a female. They typical of writing the short answer and not patterned. All their answers are slightly in line with the concepts of the lecturers, as students develop from their thought based on the references they have.

Here are the results of the answers of two students who have average cognitive skills. Both solutions are in Figure 2.

(a) Male student.
Based on Figure 2, the two students are at the RMT level 2 cognitive function, which is quantitative thinking with accuracy. It's evident from the answers of both of which demonstrate the ability to use contradictions to prove that $f'(a)$ always exist, which requires quite sophisticated analytical skills. Based on Figure 2, the two students are at the RMT level 2 cognitive function, which is quantitative thinking with accuracy. It’s evident from the answers of both of which demonstrate the ability to use contradictions to prove that (a) Both subjects know how to answer the problem by using an idea related to the continuous function, namely the concept of limit. Both are able to analyse the spatial relationship of the conclusion of the answer with the function graph. Students are skilled in selecting examples of functions for analysis, so they find the solution though simple. They are right in choosing correct assumptions to answer questions. Students can observe and graph the function graph of absolute value without searching for its coordinate point or other critical attributes. They are also skilled in summarising continuity through the graph of the absolute value function created. Finally, the two subjects can draw the right conclusions even though the description of the answer is simple.

In Figure 2 the left side is the answer of male students, while the right is the answer of women. Women tend to be more neat and complete in drawing graphs of continuous functions. They were able to arrange the problem-solving steps from the theory they learned from the lecturers, and then they applied to the answer, but not yet perfect. They choose the description of the sentence according to the examples of questions that they have from a lecturer. Male students are less neat in the preparation of answers than female. They prefer to explore their abilities from the references they have with their sentence descriptions using their language.

Furthermore, Figure 3 shows the student’s answers that have low cognitive abilities. The solution of the two students is as follow.
(a) Male students.

(b) Female students.

Figure 3. Student’s answer with low cognitive ability.
Based on Figure 3, it appears that both students enter RMT cognitive function at level 1, which is qualitative thinking. Both subjects are proficient in writing mathematical symbols of differential and function's domains. They understand the material connection between the properties of limits and the differential. Both of them can visualise the graph of continuous function even with the help of coordinate points. They can differentiate and compare two features involved in the answer, namely continuous and discontinuous functions. Both can search and collect information about the continuity. After that, it ends by completing the elements to answer the problem, although not yet capable of connecting between existing information. They were able to construct a simple hypothesis based on the relationship between the continuous function theorem, as well as the decoding of the code.

Although both students are at the last level of cognitive ability and enter the lower RMT cognitive function, both can read the intended purpose. They are then able to relate to which theory has a direct bearing on the answer. Differences remain visible from both student outcomes. In Figure 3 the left side for the student answers men, and right for women. Female students are more theoretical, following the existing stages than male. While men more exploring the answers to the references they have. Female students are systematic in answering the question of continuous function, while men are left behind some mathematical languages.

The interview reinforces analysis of the results of the sixth male student test above after the test results. All samples from male students representing all three levels of cognitive ability. The transcripts informed that they stated preferring to draw a function graph in proof of a theorem even when the appearance was incomplete. Students noted that the figure they were making was roughly without any completeness elements of a function graph. They do not commonly recite mathematical events logically with linguistic rules. Male students’ thinking is more straightforward than a female; they do not like answers with long sentences. They tend to have their way of proving a theorem without following the instructions or steps of the lecturer. In other words, male students have high confidence in math ability [17-19].

For a female student, the answer to proof is detailed and systematic. They can recite mathematical events logically with mathematics language. The thinking of female students tends to like a long explanation by recalling the various previous definitions and related theorems. They tend to follow the way lecturers are given in solving the proof problem, as it is challenging to self-explore from their references. In other words, female students have the confidence level of math ability [17-19].

4. Conclusion
This study concludes that there is a difference of mathematical thinking of rigour of students based on gender-related to three levels of cognitive ability (high, medium, low). Students with top cognitive skills are at the RMT level 3 cognitive function of abstract relational-thinking. Moderate cognitive skills are at level 2, which is quantitative-thinking with accuracy. Finally, the low cognitive skills are at level 1, namely qualitative-thinking. Male students of all three levels of cognitive ability have a simple way of thinking impacting on a short answer pattern. Their ability to solve the problem of proof is to use their approach based on the reference they have. While female students, the way they think is systematic. The pattern of the answer is long but sorted. They solve the problem of proof following the concepts and examples of completion of the lecturer. Male students' self-confidence in math ability is higher than female.

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