Spontaneous breaking of Lorentz invariance, black holes and perpetuum mobile of the 2nd kind

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Abstract

We study the effect of spontaneous breaking of Lorentz invariance on black hole thermodynamics. We consider a scenario where Lorentz symmetry breaking manifests itself by the difference of maximal velocities attainable by particles of different species in a preferred reference frame. The Lorentz breaking sector is represented by the ghost condensate. We find that the notions of black hole entropy and temperature loose their universal meaning. In particular, the standard derivation of the Hawking radiation yields that a black hole does emit thermal radiation in any given particle species, but with temperature depending on the maximal attainable velocity of this species. We demonstrate that this property implies violation of the second law of thermodynamics, and hence, allows construction of a perpetuum mobile of the 2nd kind. We discuss possible interpretation of these results.

1 Introduction

It is highly non-trivial that the laws of thermodynamics hold in the presence of gravitational interactions. Indeed, gravitating systems are generically unstable against collapse resulting in the formation of black holes, \textit{i.e.}, curvature singularities shielded by event horizons. Classical no-hair theorems state that black holes are characterized by just a few parameters (mass, electric charge and angular momentum). So one may worry that entropy can be lost behind the black hole horizons invalidating the second law of thermodynamics. However, as first suggested by Bekenstein \cite{Bekenstein}, it is natural to assign to black holes the entropy proportional to the horizon area. With this assignment the net entropy of a black hole and the outer region never decreases: in this way the second law of thermodynamics holds for systems including black holes in general relativity (GR).
The Bekenstein proposal acquires a remarkable physical justification due to the Hawking effect [2]. A black hole with mass $M$ emits thermal radiation with temperature $T_H = (8\pi M)^{-1}$ (we set the Newton constant equal to one, $G = 1$). It is important that the temperature of the radiation is universal for all species of particles. This allows to consider black hole as a body with well-defined temperature $T_H$ and the Bekenstein entropy $S_B$ is related to the black hole energy (mass) in the usual way,

$$dM = T_H dS_B.$$ 

These properties of black holes in GR are believed to reflect the fundamental principles of quantum theory. In particular, the second law of thermodynamics follows from unitarity of quantum physics (see e.g. [3]). The validity of thermodynamical description of black holes in GR is consistent with a possibility of constructing a UV completion of GR in terms of the microscopic quantum theory with conventional properties, where presumably the Bekenstein entropy would be reproduced by counting of the microscopic states of the black hole. Indeed, this was achieved [4, 5] in string theory for certain classes of extremal black holes.

One may wonder whether these properties are specific to GR or persist in its extensions as well. Stated otherwise, it is worth exploring what principles of GR are crucial for the validity of thermodynamical description of black holes. Better understanding of these issues may shed light on the very basic principles of quantum gravity and help to explain why we observe gravity the way it is.

In this paper we explore the role of microscopic Lorentz invariance. Recent observation of the cosmic acceleration motivated attempts to modify gravity in the infrared and a number of consistently looking effective field theories doing this job were constructed [6]–[14]. Except for the Dvali–Gabadadze–Porrati model [6] all these theories introduce spontaneous breaking of Lorentz invariance. In other words, in all these models a non-trivial tensor (vector) condensate different from the metric is present in the ground state. It is natural to ask how spontaneous Lorentz symmetry breaking affects black hole physics.

It should be stressed that all the models with IR modification of gravity mentioned above are lacking UV completion so far. In some cases, there are arguments indicating that UV completion is not possible in terms of conventional field theory or weakly coupled string theory [15]. However, arguments presented in [15] directly apply only to theories possessing stable Lorentz-invariant vacuum, whereas in the proposed theories with spontaneous Lorentz symmetry breaking the would be Lorentz-invariant vacuum suffers from the ghost instability and it is unclear whether it makes sense to include it in the space of the physical states at all.
Black hole thermodynamics provides an alternative way to probe the UV physics of Lorentz violating models. Indeed, it is known for a long time that black holes are unique IR probes of the microscopic theory. For instance, in agreement with string theory, black hole physics implies that global symmetries cannot be exact in quantum gravity. An interesting extension of this statement to the case of gauge symmetries was suggested recently in [16]. It is definitely of interest to find more examples where black hole physics can provide information about UV completion of the effective theories. If it turns out that Lorentz violating models are not capable of reproducing the success of black hole thermodynamics in GR, this will indicate that there are fundamental difficulties with their embedding into microscopic theory and may suggest hints on necessary properties of the would-be UV completion (if it exists).

We consider modification of the black hole thermodynamics due to spontaneous Lorentz symmetry breaking. More specifically, we study what happens if Lorentz symmetry breaking manifests itself by the difference of maximal velocities attainable by particles of different species in a preferred reference frame. Note that if one assumes that Lorentz symmetry is spontaneously broken in a hidden sector, then, generically, leading operators mediating Lorentz symmetry breaking into the visible sector result precisely in this effect. This scenario has been extensively studied in flat space-time [17, 18]. To study dynamics of such models in nonlinear gravitational backgrounds one needs to specify the structure of the Lorentz breaking sector. Here we consider a setup where hidden sector is described by the simplest of the Lorentz symmetry breaking models, namely by the ghost condensate model of Ref. [7].

## 2 Setup

The hidden sector contains a single scalar field $\phi$ with the action

$$S_\phi = \Lambda^4 \int \sqrt{-g} P(X) d^4 x,$$

(1)

where $X = \partial_\mu \phi \partial^\mu \phi$, and $P(X)$ is a function with the minimum at $X = 1$. In the action (1) we dropped terms with more than one derivative acting on $\phi$, which are generically present in the ghost condensate action. These terms are not important for our purposes and we will comment on their effects later. We assume that the effective cosmological constant is fine-tuned to zero, i.e., $P(1) = 0$. Then the system (1) has a family of solutions with $X = 1$ and vanishing energy-momentum tensor. We are interested in two solutions from this family: the

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$^1$We use the $(+,-,-,\cdots)$ signature of the metric.
vacuum solution, preserving spatial isotropy and homogeneity, and the black hole solution. The vacuum solution is

\[ g_{\mu\nu} = \eta_{\mu\nu} \quad (2a) \]
\[ \phi = t \quad (2b) \]

Note, that the shift symmetry of the action (1)

\[ \phi \rightarrow \phi + c \quad (3) \]

implies time translation invariance of the vacuum (2).

Let the visible sector be represented by a massless minimally coupled scalar field \( \psi \). The shift symmetry (3) and the reflection symmetry \( \phi \rightarrow -\phi \) fix the form of the leading operator mediating Lorentz symmetry breaking to the visible sector. With this operator included the action for the field \( \psi \) takes the form,

\[ S_\psi = \int \sqrt{-g} \left( \frac{(\partial_\mu \psi)^2}{2} + \frac{\varepsilon (\partial_\mu \phi \partial^\mu \psi)^2}{2} \right) d^4x \quad (4) \]

where \( \varepsilon \) is a dimensionless parameter. We do not need to assume that the parameter \( \varepsilon \) is small; rather, we consider \( |\varepsilon| \lesssim 1 \) which is enough for the validity of the effective action (4) up to the cutoff scale \( \Lambda \). It follows from the form of the action (4) that the field \( \psi \) propagates in the effective metric\(^2\)

\[ \tilde{g}^{\mu\nu} = g^{\mu\nu} + \varepsilon \partial^\mu \phi \partial^\nu \phi \quad (5) \]

Consequently, the propagation velocity of the field \( \psi \) in the vacuum (2) is equal to

\[ v = \frac{1}{\sqrt{1 + \varepsilon}} \quad (6) \]

Positive (negative) values of \( \varepsilon \) correspond to subluminal (superluminal) propagation of the field \( \psi \).

Solution describing a black hole of mass \( M \) in the coordinate system regular at the horizon has the form [19]

\[ ds^2 = d\tau^2 - \frac{2MdR^2}{r(\tau, R; M)} - r^2(\tau, R; M)d\Omega^2 \quad (7a) \]
\[ \phi = \tau \quad (7b) \]

\(^2\)We take into account that for configurations with \( X = 1 \) there is a proportionality between \( \text{det} g_{\mu\nu} \) and \( \text{det} \tilde{g}_{\mu\nu} \) with constant coefficient, \( \text{det} g_{\mu\nu} = (1 + \varepsilon) \text{det} \tilde{g}_{\mu\nu} \).
where
\[ r(\tau, R; M) = \left( \frac{3}{2} \sqrt{2M (R - \tau)} \right)^{2/3} \]
(8)

The metric (7a) is nothing but the ordinary Schwartzschild metric with the mass \( M \) in the reference frame of free falling observers (Lemaitre reference frame). Again, the shift symmetry (3) implies that this solution is stationary. Note that the \( \phi \)-field configuration is smooth for this solution so that the effective theory (1) breaks down only in the vicinity of the black hole singularity.

The effective metric felt by the field \( \psi \) in the black hole background (7) has the form
\[ ds^2 = \frac{d\tau^2}{(1 + \varepsilon)} - \frac{2MdR^2}{r(\tau, R; M)} - r^2(\tau, R; M)d\Omega^2 . \]
(9)

The coefficient in front of \( d\tau^2 \) can be absorbed by the rescaling
\[ \tau \mapsto \tilde{\tau} = \frac{\tau}{\sqrt{1 + \varepsilon}} . \]
(10)

The subsequent rescaling
\[ M \mapsto \tilde{M} = (1 + \varepsilon)M , \quad R \mapsto \tilde{R} = \frac{R}{\sqrt{1 + \varepsilon}} \]
(11)
casts the effective metric (9) into the form (7a) with \( \tau, R \) and \( M \) replaced by \( \tilde{\tau}, \tilde{R} \) and \( \tilde{M} \).

One observes that the metric felt by the field \( \psi \) in the coordinates \( \tilde{\tau}, \tilde{R} \) is again the Lemaitre metric, but now corresponding to the black hole with the mass \( \tilde{M} \). In particular, the effective metric has a horizon at \( r(\tilde{\tau}, \tilde{R}; \tilde{M}) \equiv r(\tau, R; M) = 2\tilde{M} \). Consequently, as one could have expected, the black hole horizon appears larger for subluminal particles and smaller for superluminal. This is a clear signal that black hole thermodynamics in this setup is crucially different from the conventional case. Indeed, the horizon area is not a universal notion any longer — different particle species see different black hole horizons and it is unclear what entropy should be assigned to the black hole.

Moreover, the coincidence of the effective metric in the coordinates \( \tilde{\tau}, \tilde{R} \) with the metric of a black hole with the mass \( \tilde{M} \) enables one, at least naively, to carry out the common derivation (see e.g. [2, 20, 21, 22, 23]) of the Hawking radiation. One obtains that the black hole emits thermal radiation of \( \psi \)-particles characterized in the tilded coordinate frame by effective temperature \( \tilde{T}_\psi = \frac{8\pi M}{(1 + \varepsilon)^{-1}T_H} \) where \( T_H = \frac{8\pi M}{(1 + \varepsilon)^{-1}} \) is the usual Hawking temperature of the black hole with the mass \( M \). The effective temperature \( \tilde{T}_\psi \) is
defined with respect to the rescaled time $\tilde{\tau}$. Rescaling back to the physical time $\tau$ we obtain the physical temperature of the emitted $\psi$-radiation

$$T_{\psi} = \frac{T_H}{(1 + \varepsilon)^{3/2}} = \psi^3 T_H.$$  \hfill (12)

This result implies that the temperature of the Hawking radiation emitted by the black hole in particles of a given type depends on the (maximal attainable) velocity of the propagation of these particles. Therefore, with Lorentz invariance being spontaneously broken the notion of a black hole temperature becomes ill-defined, as the temperature depends on the type of particles used to measure it. This is an alarming property, and, indeed, we are going to show that it leads to the violation of the second law of thermodynamics and thus opens up a possibility to construct a perpetuum mobile of the second kind. Later, we will discuss some possible loopholes in the naive derivation of the Hawking radiation suggested above.

3 Perpetuum mobile of the 2nd kind: construction manual

We are going to present a counterexample to the following formulation of the second law of thermodynamics: a process, whose only result is the transfer of energy from a cold body to a hot body, is impossible. In the setup of Sec.2, let us consider two types of particles, $\psi_1$ and $\psi_2$ with different speeds, $v_2 > v_1$. Let us take a black hole and surround it by two shells, $A, B$. We assume that the shell $A$ interacts only with the field $\psi_1$, while the shell $B$ — only with the field $\psi_2$. We choose the temperatures of the shells to satisfy

$$T_2 > T_B > T_A > T_1,$$ \hfill (13)

where $T_1, T_2$ are the temperatures of the Hawking radiation of the black hole in particles $\psi_1, \psi_2$, respectively. Now, since $T_A > T_1$ there will be a net flux of energy $F_1(T_A, T_1) > 0$ from the shell $A$ into the black hole carried by the particles $\psi_1$. On the other hand, as $T_B < T_2$, the net flux $F_2(T_B, T_2)$ of energy from the shell $B$ into the black hole, carried by the particles $\psi_2$, is negative, $F_2(T_B, T_2) < 0$. In the conventional case without Lorentz violation the functions $F_i, i = 1, 2$, are given by the Steffan-Boltzman formula,

$$F_i(T, T') = \frac{\pi^3}{15} (2M)^2 \left( \Gamma_i(T)T^4 - \Gamma_i(T')T'^4 \right),$$ \hfill (14)

where $\Gamma_i(T)$ are the grey body factors which depend on the spin of the particles and slowly vary with temperature. We do not need the explicit form of the functions $F_i$ in the case of
Lorentz violation. It is sufficient for our argument that $F_i$ fulfill the following requirements:

\[ F_i(T, T') = 0 \text{ at } T = T', \]
\[ F_i(T, T') \text{ grow with } T \text{ at fixed } T'. \]

Then one can choose the temperatures of the shells in such a way that the two energy fluxes compensate each other, $F_1(T_A, T_1) + F_2(T_B, T_2) = 0$, and the black hole mass stays constant. This can be satisfied simultaneously with the inequalities (13). So, for an outer observer the state of the black hole does not change, and the only result of the process under consideration is the transfer of energy from the shell $A$ to the shell $B$. As $T_A < T_B$, the second law of thermodynamics is violated by this process. In other words, an outer observer is forced to conclude that the entropy of the system decreases.

4 Discussion

Definitely, the above conclusion is puzzling, so at this point one may wonder whether our derivation is too superficial. At any rate, it is worth asking what conclusion is to be drawn from our observation. It is instructive to divide the potential explanations of the strange behavior found here into the following three classes (not necessarily excluding each other).

(i) The presented description of the Hawking radiation in the ghost condensate is correct, but there is some subtle way in which a low energy effective theory forces our perpetuum mobile to change its state so that the entropy actually increases.

(ii) The derivation of the Hawking radiation using only low energy theory is incorrect.

(iii) The presented description of the Hawking radiation in the ghost condensate is correct, and the violation of the second law of thermodynamics within a low energy effective theory is a physical effect. According to the discussion in Introduction this means that the UV completion of the ghost condensate, if it exists at all, has very unusual properties.

Let us start with discussing the possibility (i). First, a possible objection to the scheme of the perpetuum mobile presented above could be that it does not take into account the Hawking radiation of gravitons. One way to get around this objection is to introduce a mildly large number $N \sim |\varepsilon|^{-1}$ of fields $\psi_1$ and $\psi_2$ so that the entropy transferred between the shells is much larger than the entropy radiated in gravitons. In this way the effect of gravitons is made negligible. Note that the needed number of species $N$ is independent of the mass of the black hole and is determined solely by the parameter $\varepsilon$. Another way to get around the above objection is to include gravitons into consideration and make them play the role of one of the fields, say $\psi_2$ (then, the field $\psi_1$ must be subluminal). The shell $A$
can be made sufficiently thin to interact weakly with gravitons, while the shell $B$, on the contrary, can be made sufficiently massive to absorb all the gravitons emitted by the black hole.\footnote{It should be noted, though, that it is not obvious that a shell $B$ with the needed properties can indeed be constructed (c.f. "gravity is the weakest force" conjecture \cite{16}).} So, we do not find it plausible that taking into account gravitons allows to avoid the conclusion that the second law of thermodynamics is violated.

For simplicity, let us assume in the rest of the discussion that only the field $\psi_1$ has a non-zero value of $\varepsilon$. Note that during the work of our perpetuum mobile there is a non-zero flux of $\psi_1$-particles into the black hole. One may argue that because of the direct coupling between $\psi_1$ and the ghost condensate, the ghost condensate profile outside the black hole “remembers” about the amount of $\psi_1$ particles swallowed by the black hole. In other words, there can be extra entropy available for the outside observer which is contained in the perturbations of the ghost condensate. To see that this is not the case, recall that there is a convenient fluid analogy \cite{24} for the ghost condensate. Namely, one introduces a unit four-vector

$$u_\mu = \frac{\partial_\mu \phi}{\sqrt{X}}.$$  \hspace{1cm} (15)

Then the field equations of the ghost condensate coincide with the hydrodynamical equations describing dynamics of the irrotational relativistic fluid consisting of two components which cannot mix with each other. The fluid four-velocity is given by $u_\mu$, $P(X)$ plays the role of the pressure, while $(2XP' - P)$ is the energy density. One component has $X \geq 1$ and positive energy density, while the other has $X < 1$ and negative energy density. For instance, the field equation of the ghost condensate

$$\nabla_\mu (P' \nabla^\mu \phi) = 0$$ \hspace{1cm} (16)

is a conservation law for the fluid current. This equation changes in the presence of the field $\psi_1$ coupled to the effective metric (5). One obtains

$$\nabla_\mu (P' \nabla^\mu \phi) = -\frac{\varepsilon}{2\Lambda^4} \nabla_\mu (\nabla^\mu \psi_1 \nabla_\nu \psi_1 \nabla^\nu \phi).$$ \hspace{1cm} (17)

Consequently, $\psi_1$-field plays the role of the source for the ghost current. At moderate $\varepsilon$, $(1 + \varepsilon) \sim 1$, the expectation value $\langle \nabla^\mu \psi_1 \nabla_\nu \psi_1 \rangle$ can be estimated in the coordinate system corresponding to the metric (7a) as

$$|\langle \nabla^\mu \psi_1 \nabla_\nu \psi_1 \rangle| \sim T^4_{\psi_1}.$$ \hspace{1cm} (18)
Thus, the source in (17) can be considered as a small perturbation as long as

$$|\varepsilon| T_{\psi_1}^4 \Lambda^4 \ll 1,$$

(19)

which is true for a large enough black hole. Note that unperturbed solution (7b) corresponds to a spherically symmetric flow of zero energy fluid into the black hole. Any small perturbation of the ghost condensate fluid induced by the $\psi_1$ particles will be carried into the black hole by the background flow so that at the end there is no memory left outside about the number of $\psi_1$ particles emitted by the black hole.

Another possible objection to our construction may be that the ghost condensate action (1) is just a low energy effective action and in general it also contains terms with more derivatives acting on $\phi$. One effect of these terms is that the ghost condensate exhibits Jeans instability [24]. So, the device described above is not, strictly speaking, a perpetuum mobile. However, the characteristic time of the growth of the instability is

$$t_J \sim \frac{1}{\Lambda^3},$$

(20)

and can be very large\(^4\). The requirement that the device can operate only for the period of time shorter than $t_J$ together with the inequality (19) places a rather mild upper bound on the amount of entropy and energy which can be transferred between the shells. For instance, an estimate for the latter is given by (c.f. Eq. (14))

$$E \sim |\varepsilon| (2\tilde{M})^2 T_{\psi_1}^4 t_J,$$

(21)

Taking into account the constraint (19) one obtains

$$E < E_{\text{max}} = \sqrt{|\varepsilon|}.$$

(22)

This bound is not restrictive, for $\varepsilon = 0.01$, $\Lambda = 10$ MeV we obtain $E_{\text{max}} = 10^{36}$ erg which is a huge amount of energy.

Another effect of the higher derivative terms is that they modify the black hole solution (7) and lead to the accretion of ghost condensate energy into black hole. However, as shown in [19], the accretion rate is very slow. The change in the black hole mass over the time $t_J$ is negligible for black holes heavier than the Planck mass. So it appears that higher derivative terms generically do not prevent violation of the second law of thermodynamics as well.

\(^4\)It is longer than the lifetime of the Universe if $\Lambda \lesssim 10$ MeV.
The above arguments do not completely exclude the possibility \((i)\) and further study is needed. However, to our opinion, this possibility is unlikely, as the very fact that a notion of horizon does not have a universal meaning strongly suggests the breakdown of the standard black hole thermodynamics. Actually, this is already suggested by the observation that perturbations in the ghost condensate can carry negative energy and thus violate the null energy condition. The latter is known to be related to the entropy bounds \([26]\), and consequently, to the black hole thermodynamics. So let us turn to the other two possible options mentioned above.

We proceed to the option \((ii)\). There are several derivations of the Hawking effect in GR. Let us discuss subtleties which potentially may affect these approaches in our setup. First, recall that the original derivation due to Hawking \([2]\) applies to the collapsing body, while we considered an eternal black hole. Fluid picture makes it likely that in our case the metric of the collapsing black hole is not different from the usual case. However, a ghost field singularity (caustic) is likely to be present at the origin even before the horizon forms (see \([7, 25]\) for a discussion of these caustics). At the caustic the gradients of the ghost field are discontinuous, and the effective metric (5) is ill-defined. The discontinuity of the gradients is supposed to be smoothed out by the higher derivative terms in the ghost condensate action. So, strictly speaking, the presence of these caustics makes it impossible to apply the Hawking derivation for a field \(\psi\) having a non-vanishing coupling to the ghost condensate field without knowing UV completion of the ghost condensate theory. On the other hand, it has been argued that the caustics are resolved in an extension of the ghost condensate model dubbed gauged ghost condensate \([24, 28]\). It would be interesting to see how our analysis is modified in the case of the gauged ghost condensate.

Another well-known subtlety of the Hawking derivation is the so called trans-Planckian problem (see, e.g., \([27]\) for a recent discussion). Indeed, the presence of the Hawking radiation results from the Bogolyubov transformation between the modes of the in- and out-vacua. But the outgoing wave of the Hawking radiation gets infinitely blue-shifted in the region near the black hole horizon, so this calculation implicitly assumes that one knows the form of the vacuum modes up to the trans-Planckian frequencies. This assumption is also present in the approach of Ref. \([20]\), which makes use of the eternal black hole metric and thus avoids the subtlety with the ghost condensate caustics. The above assumption is justified in the standard case because the vacuum modes with high frequencies are related to the soft modes by Lorentz boosts. However, in the presence of the ghost condensate one has a preferred reference frame (rest frame of the ghost condensate), and the statement that a wave
has large frequency in this frame has an invariant physical meaning. Strictly speaking the
effective action (1) applies only to modes with frequencies smaller than \( \Lambda \) in this reference
frame, so again we conclude that the Hawking radiation may be sensitive to the details of the
UV completion in our setup. If true, this conclusion is rather interesting because Hawking
radiation was shown to be independent of the details of UV completion under rather general
assumptions, even allowing for Lorentz invariance breaking in the UV (while preserving it
in the IR) [27].

A subtlety of the approach [21] based on the Euclidean continuation of the black hole
metric is that in our setup the analytic continuation is to be performed differently for different
species, because they feel different effective metrics. Moreover, an explicit time-dependence
of the ghost condensate background makes it somewhat unclear how to perform Euclidean
continuation of the full theory. However, at the quadratic order where the interaction of
the field \( \psi \) with the ghost condensate is described by the effective metric (5) there are no
apparent obstructions to the analytic continuation of the effective metric, and the thermal
circle is present in the Euclidean time. This reasoning would be incorrect if higher order
terms describing interaction with the ghost condensate in the action for the field \( \psi \) could
not be neglected; this would again mean that the Hawking radiation in our setup is sensitive
to the UV physics. Another consequence of this option would be that the existence of
the thermal Euclidean circle in the low energy action does not imply thermal behavior
in Minkowski signature. Recently, an example of a completely different system where this
happens was discussed in [29] with a possible conclusion that such a behavior is an indication
of non-locality in time.

We see that the latter possibility is closely related to the option (iii), namely that a UV
completed theory should be very unusual. As a concrete scenario of a microscopic theory
where the conflict with the second law of thermodynamics is resolved one can imagine a the-
ory containing an infinite tower of fields \( \psi_i \) with indefinitely growing (maximal) propagation
velocities. Such a theory would not possess any black hole horizons at all — one would be
capable of probing the entire black hole interior using fields with higher and higher velocities.
Then one would be able to see that the particle content of the "black hole" changes in our
process, and calculate the entropy directly without relying on the Bekenstein formula, thus
avoiding a conflict with the second law of thermodynamics.

It would be interesting to understand which of the options (i)-(iii) is actually realized.

To conclude, it is worth stressing that we concentrated on the scenario where the Lorentz
violating sector is represented by the ghost condensate just for the sake of simplicity. We
expect our conclusion that the standard black hole thermodynamics breaks down in the
presence of spontaneous Lorentz symmetry breaking to be rather generic and apply to other
models as well.

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