Thermoelastic stress analysis of a 2D stress field using a single detector infrared scanner and lock-in filtering

G Pitarresi, A Normanno and L D’Acquisto

Dipartimento di Meccanica, Università degli Studi di Palermo, Viale delle Scienze, 90128 Palermo, Italy

E-mail: pitarresi@dima.unipa.it

Abstract. A low resolution, low cost, fast infrared scanner is used to acquire the temperature change along a line on the surface of cyclically loaded samples. The temperature signal is sampled versus time by exploiting the raster scanning movement of the thermocamera single detector. The temperature data is then post-processed by a Fast Fourier Transform based lock-in algorithm implemented in MATLAB®, in order to filter out the thermoelastic effect induced temperature change. A procedure is also implemented in order to extend the data sampling time by opportunely synchronising successively grabbed data frames. The effectiveness of such synchronisation procedure is first demonstrated by performing the signal processing analysis on unidirectional tensile samples made of polycarbonate. The ability of the implemented one-dimensional FFT lock-in algorithm to analyse also two-dimensional stress fields is then assessed by processing the temperature data acquired on a polycarbonate disc under cyclic compression loads.

1. Introduction

The Thermoelastic Effect describes the temperature changes in loaded matter as induced by the volume changes under linear elastic behavior [1]. When such transformation is adiabatic and the material is homogenous and isotropic, a simple linear relationship can be derived where the temperature change, measured on the free surface of a body, is proportional to the change of the first stress invariant as follows [2,3]:

$$\Delta T = -T_0 \cdot K \cdot \Delta (\sigma_x + \sigma_y) \quad \text{with} \quad K = \alpha/\rho C_p$$

where $T_0$ is the absolute temperature and $K$ the thermoelastic constant function of the coefficient of thermal linear expansion $\alpha$, the density $\rho$ and the specific heat $C_p$ of the bulk material. Thermoelastic Stress Analysis techniques (TSA) are aimed at acquiring stress information by measuring temperature changes on the surface of bodies when simple thermoelastic laws of the like of eq. 1 apply [4]. Three most common features in many TSA set-ups include: the application of cyclic loads for the material to achieve adiabatic conditions, the use of high thermal resolution IR sensors for non-contact full field temperature measurements, the use of lock-in signal analysis techniques in order to filter out the thermoelastic effect induced temperature change from other noisy components [5-8]. The special needs associated with the last two features have advised TSA practitioners to using only on purpose...

1 To whom any correspondence should be addressed.
developed commercial hardware, so far available at elevated costs and with a limited choice of suppliers and configurations.

Aim of this work is to present an experimental methodology using fast linear IR scanners to measure the thermoelastic signal on the free surface of a body with a generic stress field. These systems, non conventional in TSA applications, use a single detector to measure the temperature versus time along a chosen line. A fast raster scanning mode is realized by means of an oscillating mirror which changes the optical path between the sensor and the measured spot. The use of a single detector and of a simpler optical hardware, as compared for instance to early single detector TSA systems such as SPATE, proposes such IR systems as a potential tool for TSA, with costs that can be estimated as three to four times lower than actual high thermal resolution focal plane array systems.

The ability of fast single detector IR cameras to sample temperature versus time with a resolution adequate for deriving the thermoelastic signal was first demonstrated in [9]. The methodology proposed was though limited to one-dimensional stress field analyses. Moreover the high scanning speed of the IR camera used in [9] requires the sample to be stressed at very high frequency loads (over 100 Hz). The possibility to overcome these limitations was recently discussed in [10], by means of the use of slower linear IR scanners whose detector movement is kept sweeping along the same line in space. One further feature of the approach presented in [10] was the adoption of a lock-in correlation algorithm more effective in filtering out the thermoelastic signal component.

This work implements two substantial improvements on the methodology presented in [10]: the synchronization of different acquired frames to extend the time window of sampled data, and the adoption of a lock-in filter implementing a one-dimensional Fast Fourier Transform (FFT) analysis to measure the thermoelastic signal on a surface with a generic two-dimensional stress field. Both these features will be further discussed in the following sections and supported with experimental analyses performed on two polycarbonate samples: a tensile specimen loaded in traction and a circular compression loaded disc developing a 2D stress field. The experimental results obtained clearly demonstrate the validity of the proposed methodology for measuring the thermoelastic signal.

2. Description of the IR equipment
The IR thermocamera employed in this study is a Varioscan 3022 by JENOPTIK GmbH. This system employs a single IR detector, a thermoelectrically cooled MCT sensor with a thermal resolution of 0.12 K [11]. Two rotating mirrors allow the signal to be collected from different points, horizontal-wise and vertical-wise, collecting the signal on a full field frame array of 360×240 pixels. The system acquires and save each frame in about 0.9 seconds, and a small temporal gap is spent between the acquisition of two succeeding frames during which the signal is not acquired. One feature of the employed system is the possibility to fix the mirror which enables the vertical scanning so that each row in the data frame is acquired on the same horizontal line in the field of view. The rotational movement of the mirror sweeping the horizontal line consists of an oscillation of the mirror around its axis. In the first half period of this alternating rotation the line is swept from left to right, during which the signal from 360 points is acquired in 1/270 sec, and the same number of points is acquired in the second half period moving now from right to left (figure 1a). With this raster scanning mechanism 240 rows from the same line are collected and piled up to build one data frame array. All acquired frames can then be automatically captured, stored and displayed on a PC by means of a proprietary software (Irbis® V.2.0) allowing a complete remote control of the system through an Ethernet connection.

It can be noted that the time employed to acquire one single measure is very fast. It is also known from IR thermography theory that a good signal to noise ratio, end hence thermal resolution, depends on the time the sensor remains focused on the measuring spot. Despite the low thermal resolution of the system available in this work, poor thermal resolution should not be an issue in this approach since higher resolution fast IR scanners are available. For instance thermal resolution values of 0.03 K are achieved by the Varioscan 3021 models [11] or by the AGEMA system employed in [9]. This resolutions, achieved with the fast scanning mechanism described above, are close to the values reached by commercial focal plane array systems routinely employed in TSA [7,8].
2.1. Time-temperature sampling.

In this work the fixed-line scan mode is used. The temperature from a pixel on the scanned line is acquired periodically whenever the optical path of the sensor focuses on it. The information from one pixel location is then stored in a column of the data frame array. The column-wise information is then a measure of temperature versus time. This enables a procedure by which the temperature-time variation is sampled over 360 aligned points for a total time of 0.9 sec (i.e. the frame rate). If the body is loaded at sufficiently slow frequencies, the thermoelastic effect induced temperature change can be sampled over a finite number of periods during a single frame acquisition. Figure 1b shows two thermograms from a tensile sample loaded at 3.38 Hz and 6 Hz. It is possible to distinguish alternating darker and lighter horizontal fringes caused by the thermoelastic effect induced temperature change. The horizontal orientation of such thermal fringes is caused by the high speed of the sensor line swap, such that the time shift on the thermoelastic signal from different columns is non distinguishable. In the case of a cyclic loading frequency $f_L$, a simple relationship can be established between the number of horizontal thermal fringes shown in one frame, $n_S$, and the camera frame rate $f_R$:

$$n_S = f_L / f_R = f_L \cdot 0.888 \text{ sec}$$

where 0.888 s is the exact time spent by the camera employed in this work to acquire a frame [11].

3. Lock-In algorithm

The thermoelastic signal from each of the 360 points on the scanned line is derived by performing a lock-in filtering operation of the temperature versus time data. A scheme of this lock-in procedure is shown in figure 2. In order to make the filtering mechanism clear, some equations are reported on the scheme, which describe the trigonometric operations of signal processing performed if the signal is represented by continuous harmonics. In reality the signal is digitized by the sampling mechanism described above and the filtering operations are performed by applying FFT analyses. As shown in figure 2 the core of the lock-in technique consists in mixing the measured signal, named $S$, with a reference signal which is a carrier of the loading frequency, i.e. the useful frequency. This frequency named $\omega_R$ can be assigned as an input data or can be derived from $S$ if the corresponding harmonic is
easily identified on the frequency domain with the FFT analysis. Once \( \omega_R \) is obtained, two pure cyclic reference signals named \( F \) and \( G \), with \( F \) in quadrature with \( G \), are built and multiplied to \( S \). This operation results in two signals which contain the information of interest as DC components, and hence a low-pass filtering operation performed with a new FFT analysis is able to filter out such components named \( X \) and \( Y \). These represent the thermoelastic signal components in phase and in quadrature with the reference signal and by combining them the thermoelastic signal amplitude \( A \) is readily obtained.

The above procedure is implemented in this work with an algorithm using the Fast Fourier Transform functions available in Matlab®. The approach here proposed implements a 1D FFT analysis of each column of the data frame array, differing from the global 2D FFT approach proposed in [10]. This 1D lock-in treatment allows the thermoelastic signal to be derived on each scanned point independently, enabling the immediate application of this analysis to the case of a 2D stress field. In fact although the 2D FFT based treatment from [10] is believed to be more effective in noise rejection, it is much more elaborate for analyzing 2D stress fields.

![Scheme of the implemented lock-in analysis.](image)

### 3.1. Synchronisation of data frames.

The FFT analysis requires that the sampled data are equally spaced in time. This is not the case for the data along the columns of the acquired frame arrays. In fact from figure 1a it is possible to follow the movement of the mirror acquiring the signal. Let’s consider the first three measures of column 2 in this scheme: the time spent by the mirror to go from point 1 (row1,col2) to point 2 (row2,col2) is longer than from point 2 to point 3 (row3,col2). This is due to the longer distance swept by the mirror from point 1 to 2. By further proceeding along column 2, it is observed that the time interval is the same every two measures, i.e. considering alternate points. In light of this the sampled data useful for the FFT analysis need to be taken by considering only odd rows or, alternatively, even rows. The total number of samples is then reduced from 240 to 120. The corresponding time interval \( \Delta t \) becomes equal to one oscillation period of the mirror, i.e. \( \Delta t = 2 \times (1/270) \) sec, and the sampling frequency is then \( f_S = 1/\Delta t = 135 \) Hz. The maximum loading frequency applicable for the FFT analysis to work, i.e. the Nyquest frequency, will be half the sampling frequency, \( f_{Ny} = f_S/2 = 67.5 \) Hz. The total number of points sampled in one frame, \( N=120 \), and the time interval between two succeeding acquisitions on the same point, \( \Delta t \), affects the value of the fundamental frequency \( f_F \). This is the first order harmonic frequency determining the discrete resolution of the frequency domain from the FFT analysis, and is given by:

\[
 f_F = \frac{1}{N \cdot \Delta t} = \frac{135}{120} = 1.125 \text{ Hz} 
\]  

The thermoelastic signal in TSA is usually heavily corrupted by noise. The small number of points from a single frame, useful for the FFT analysis to extract the thermoelastic signal, is likely not
enough to perform the filtering operation very effectively. It is observed that if the total sampling period were extended, a higher \( N \) value and a smaller \( f_R \) would be obtained (see eq. 3), with further benefits being a smaller influence of border effects, and a more effective noise to signal reduction.

With the IR scanner in use in this work, in order to sample the signal over a longer time window, the information from more than one acquired frame has to be opportually collected. As noticed before, this is not straight forward because there is a small time gap between two succeeding frames during which the camera is not acquiring the signal. So if these are joined by simply attaching one frame after the other, a new array is formed with its columns doubled in length, but a discontinuity is then inserted in the temperature versus time data at the point of attachment of the frames. This discontinuity introduces further noise which can affect the analysis. An algorithm is proposed in this work to solve the above problem and to be able to attach more frames in a sequence, in order to build a longer array. The steps of this so called synchronization algorithm is here described and graphically supported by considering a data frame consisting of a pure sinusoidal signal with a frequency of 3 Hz (see figure 3):

\begin{enumerate}
    \item[i)] Frame 2 is attached with frame 1 starting from row 1 of frame 2;
    \item[ii)] The FFT analysis of frame 1+2 is performed and the amplitude of the harmonics of the resulting signal plotted in the frequency domain. The amplitude of the harmonic at the reference frequency \( f_L \) is determined in particular, and named \( A(1) \);
    \item[iii)] Steps i and ii are repeated attaching frame 2 to frame 1 starting from row2 of frame 2;
    \item[iv)] Steps from i to iii are repeated for \( n \)-times, and at each iteration frame 2 is attached to frame 1 starting from the \( n \)-th line of frame 2;
    \item[v)] For each combination of attached frames an FFT analysis is performed as done in step ii, and the power distribution in the frequency domain plotted. The amplitude of the harmonic at the \( f_L \) frequency from this plot is \( A(n) \);
    \item[vi)] The optimum number of lines to remove from frame 2 to get the best synchronisation with frame 1 is \( n^* \) such that \( A(n^*) = \max(A(n)) \).
    \item[vii)] Steps from i to vi are repeated to synchronise frame 2 with frame 3, frame 3 with frame 4 and so on. For each couple of frames the optimum number \( n^* \) of lines to remove is obtained.
    \item[viii)] When the total number of couples of frames have been synchronised, the final array is built up by simply joining together all the frames removing from each frame its first \( n^* \) rows as determined in step vii.
\end{enumerate}

![Figure 3](image.png)

**Figure 3.** Main steps of the synchronization algorithm applied to join two pure sinusoidal signal frames. (a,b) \( n<n^* \); (c) \( n=n^* \); (d) \( n>n^* \).
The logic behind this procedure is that the best synchronisation is achieved when the harmonic of the thermoelastic signal has a maximum amplitude, because this means that a smooth attachment is obtained and the power is not dispersed over other harmonics as would be the case if a steep jump is present at the attached location. The effectiveness of this procedure is demonstrated in figure 3 for a pure sinusoidal signal, and in figure 4 for synchronised experimental data frames from tensile samples.

Figure 4. Example of synchronised frames from experimental data.

4. Experimental results and discussion
The temperature-time sampling mechanism based on the fast linear IR scanner, the 1D FFT based lock-in filtering algorithm and the synchronization procedure of acquired frames are the main features of the TSA technique presented in this work. These have been implemented for the experimental evaluation of the thermoelastic signal on cyclically loaded polycarbonate samples. This material has a high thermoelastic constant (e.g. $K = 46 \times 10^{-6}$ MPa$^{-1}$) and a relatively wide elastic range enabling the possibility to generate high values of the thermoelastic signal. This is evaluated and plotted in figure 5a in uncalibrated units corresponding to grey scale levels. In fact the acquired frames are 8 bit gray scale images. Since the IR system in use gave no other possibility to access the measured raw data, the bitmap images were imported in Matlab after an optimization of the brightness level and contrast range parameters performed with the Irbis© software [11].

Two types of samples have been tested by applying cyclic sinusoidal loads, generating one and two dimensional stress fields as described below.

4.1. One-dimensional stress field analysis.
This stress case was investigated by determining the thermoelastic signal on tensile samples at different traction-traction load amplitudes. A plot of the stress amplitude versus the thermoelastic signal amplitude is shown in figure 5a. In this plot the thermoelastic signal is reported for each stress amplitude and data are then interpolated with a linear regression curve. The correlation coefficient of the linear regression line is used as a parameter to quantify the linear behavior as foreseen by the theory (eq. 1), and used as an index of the quality of the analysis results. Data plotted in figure 5a are in particular derived applying the lock-in analysis to a single column on an extended array of data obtained by joining 24 acquired frames with the synchronization procedure of section 3.1. Figure 5b,c shows the correlation coefficient of the linear regression performed on the thermoelastic signal versus stress amplitude results from all the points scanned. In particular the results in figure 5b are referred to an extend sampling time of 24 synchronized frames, while figure 5c consider the analysis on a single grabbed frame. It is seen that performing the analysis on a higher number of synchronized frames dramatically improves the linear trend between the thermoelastic signal and stress amplitude. This is confirmed in figure 5d which plots the average correlation coefficient and its standard deviation variation range at increasing number of synchronized frames. It must be noted that for the present case of a uniform stress field each scanned point should deliver the same thermoelastic signal. The analysis can then be improved by averaging the thermoelastic signal obtained from each point. Indeed it has been found that this further averaging operation improves results, but these are not shown or further considered since the final scope of the work is to analyse two-dimensional stress fields for which each scanned point deliver different thermoelastic signals and so the quality of the analysis should rely only
on the filtering of the data measured on that point. Tests at frequencies of 3 Hz and 9 Hz, have also
been performed showing very similar results to those reported in figure 5.

Figure 5. (a) Thermoelastic signal measured versus stress amplitude; (b,c) Correlation
coefficient of the linear regression as shown in figure 5a, for all sampled points; (d) Correlation
coefficient at varying number of synchronised frames (average value and standard
deviation calculated from all sampled points on the scanned line).

4.2. Two-dimensional stress field analysis.
For this stress case a polycarbonate disc (100 mm diameter per 7.5 mm thickness) was used.
Concentrated compression loads were applied varying sinusoidally (average load of -2 kN and load
amplitude of 1.8 kN). The elastic theoretical solution of the in-plane stress field for this shape and load
case is readily available in the literature. Figure 6a shows a contour map of the isopachs normalized
by dividing for the absolute value of the stress invariant at the center of the disc (about 6 MPa in this
analysis). The experimental analysis consisted in deriving the thermoelastic signal from the points
along the diametral line aligned with the load (marked in figure 6a). The disc was loaded at a
frequency of 3 Hz and the IR camera was tilted on its side by a 90° angle to allow the scanned line to
coincide with the diametral loading direction of the sample. The analysis considered only half
diameter of the disc, the one resting against the fixed plate, to minimize the influence of the
deformation movement of the disc, bigger on the side of the disc resting against the moving actuator.
One further reason was an evident disturbance of the heat coming from the moving actuator affecting
the signal on the area of high stress concentration near the loading point.

The thermoelastic signal was determined on a built up array of 40 synchronized grabbed frames.
The infrared image of the disc was used to position the line scan and to identify the pixels
Correlation Coefficient
implemented technique to analyse this 2D stress field. The thermoelastic signal evaluation is plotted up to about 2.5 mm away from the border of the disc. In fact nearer points did not follow correctly the theoretical trend. This was likely due to some unknown influences such as local plasticity and flattening deformation on the contact point changing the local theoretical stress distribution.

![Diagram](image)

Figure 6. (a) Contour map of isopachichs in a compression disc (scale normalised by the absolute value of the isopachich at the center of the disc); (b) comparison of theoretical and experimental normalised isopachics along the half diametral load line marked in figure 6a.

5. Conclusions
A new thermoelastic stress analysis set-up has been implemented based on three main features: a temperature-time sampling technique exploiting the raster scanning mechanism of fast linear IR scanners, a 1D-FFT based lock-in filtering algorithm, and a procedure for synchronizing several successively acquired frames to extend the sampling time. Based on these features a thermoelastic stress experimental analysis of 1D and 2D stress fields has been performed. In both cases the measured thermoelastic signal agreed well with the theoretical predictions when the data sampling and analysis was performed on more grabbed frames. The intrinsic quality of the results obtained is to a certain extent affected by the low thermal resolution of the IR scanner available for this work. This should though not be a concern for the methodology presented since IR scanners with higher thermal resolution are available in the market, using the same scanning principle. It is also believed that a customization of such IR scanners to better suit the scopes and the features of the present thermoelastic stress analysis could easily determine further improvements in both performances and hardware costs.

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