Description of the newly observed $\Xi^{0}_c$ states as molecular states

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Abstract. Very recently, three new structures $\Xi_c(2923)^0$, $\Xi_c(2938)^0$, and $\Xi_c(2964)^0$ at the invariant mass spectrum of $A^+_c K^-$ observed by the LHCb Collaboration trigger a hot discussion about their inner structure. In this work, we study the strong decay mode of the newly observed $\Xi_c$, assuming that the $\Xi_c$ is a $D\Lambda - D\Sigma$ molecular state. With the possible quantum numbers $J^P = 1/2^+$ and $3/2^+$, the partial decay widths of the $D\Lambda - D\Sigma$ molecular state into the $A^+_c K^-$, $\Sigma_c^0 K^-\pi^-$, and $\Xi^0_c\pi^-$ final states through hadronic loop are calculated with the help of the effective Lagrangian. By comparison with the LHCb observation, the current results of total decay width support the $\Xi_c(2923)^0$ as $D\Lambda - D\Sigma$ molecule while the decay width of the $\Xi_c(2938)^0$ and $\Xi_c(2964)^0$ cannot be well reproduced in the molecular state picture. In addition, the calculated partial decay widths with $S$ wave $D\Lambda - D\Sigma$ molecular state picture indicate that allowed decay modes, $\Xi^0_c\pi^-$, may have the biggest branching ratios for the $\Xi_c(2923)$. The experimental measurements for this strong decay process could be a crucial test for the molecule interpretation of the $\Xi_c(2923)$.

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1 Introduction

During the past several decades, many narrow baryons with a heavy charm quark, a light up or down quark, and a strange quark have been reported by the LHCb, CDF Collaboration and so on \cite{1}. Very recently, three other neutral resonances $\Xi^{0}_c$ named $\Xi_c(2923)^0$, $\Xi_c(2939)^0$, and $\Xi_c(2965)^0$ have been observed in the $K^- A^+_c$ mass spectra by the LHCb Collaboration \cite{2}. The observed resonance masses and widths are

\[
\begin{align*}
M &= 2923.04 \pm 0.25(stat) \pm 0.20(syst) \pm 0.14(A^+_c) \text{ MeV} \\
\Gamma &= 7.1 \pm 0.8(stat) \pm 1.8(syst) \text{ MeV}, \\
M &= 2938.55 \pm 0.21(stat) \pm 0.17(syst) \pm 0.14(A^+_c) \text{ MeV} \\
\Gamma &= 10.2 \pm 0.8(stat) \pm 1.1(syst) \text{ MeV}, \\
M &= 2964.88 \pm 0.26(stat) \pm 0.14 \pm (syst)0.14(A^+_c) \text{ MeV} \\
\Gamma &= 14.1 \pm 0.9(stat) \pm 1.3(syst) \text{ MeV},
\end{align*}
\]

respectively. From the observed decay mode, the isospin of these three states are $1/2$. Although the quantum numbers of these states remain undetermined, it is very helpful to understand the spectroscopy of the heavy baryons containing $c$ and $s$ quark.

Due to their observed decay mode, the new structures $\Xi_c(2923)^0$, $\Xi_c(2938)^0$, and $\Xi_c(2964)^0$ contain at least three different valence quark components. In other word, these states may be candidates of conventional three-quark state. Indeed, the QCD sum rule suggests that the newly observed states $\Xi_c(2923)^0$, $\Xi_c(2938)^0$, and $\Xi_c(2964)^0$ are most likely to be considered as the $P$-wave $\Xi_c^-$ baryons with the spin-parity $J^P = 1/2^-$ or $3/2^-$.\cite{3} In Ref.\cite{4} the $\Xi_c(2923)^0$, $\Xi_c(2938)^0$, and $\Xi_c(2964)^0$ were suggested to be $1P$ $\Xi_c^-$ state with spin-parity $J^P = 3/2^-$ or $5/2^-$ in the chiral quark model. In Ref.\cite{5} the two-body strong decays of the $\Xi_c(2923)^0$, $\Xi_c(2938)^0$, and $\Xi_c(2964)^0$ were calculated by employing the $^3P_0$ approach with the conclusion that the $\Xi_c(2923)^0$ and $\Xi_c(2938)^0$ can be $1P$ $\Xi_c^-$ states, and the $\Xi_c(2964)^0$ can be regarded as the $2S$ $\Xi_c^-$ state. The lattice QCD calculation was also performed and try to determine their quantum numbers \cite{6}.

Although the authors in Refs.\cite{3,4,5} try to assign these states into the conventional three-quark frames, it is obvious that the inner structure of $\Xi_c(2923)^0$, $\Xi_c(2938)^0$, and $\Xi_c(2964)^0$ are not finally determined. And another interpretation is treating them as $D\Lambda - D\Sigma$ molecular states, because the smallest mass gaps between the newly observed $\Xi_c$ baryons and the ground $\Xi_c$, about 450 MeV, is large enough to excite a light quark-antiquark pair to form a hadronic molecular. Indeed, it is shown in Refs.\cite{7,8,9} that the interaction between $D$ meson and $A$ or $\Sigma$...
baryon is strong enough to form a bound state with a mass point as 2930 MeV.

The key point in this work is to explain whether the \( \Xi_c(2923)^0 \), \( \Xi_c(2938)^0 \), and \( \Xi_c(2964)^0 \) can be considered as a molecular state. Here, we will consider the strong \( \Xi^0 \rightarrow \Lambda^+ K^- + \Sigma^0 K^- \), and \( \Xi^0 \rightarrow \pi^- \) decays of the \( \Xi_c(2923)^0 \), \( \Xi_c(2938)^0 \), and \( \Xi_c(2964)^0 \) with the possible quantum numbers \( J^P = 1/2^+ \) and \( 3/2^+ \) using an effective Lagrangian approach. The approach is based on the hypothesis that the \( \Xi_c^0 \) is a hadronic molecular state of \( DA-D\Sigma \). The coupling of the \( \Xi_c^0 \) to the constituents is described by the effective Lagrangian. The corresponding coupling constant \( g_{\Xi_c^0 AD} \) and \( g_{\Xi_c^0 \Sigma D} \) are determined by the compositeness condition and the renormalization constant of the hadron wave function is set equal to zero. By constructing a phenomenological Lagrangian including the couplings of the bound state to its constituents and the constituents with other particles we calculated one-loop diagrams describing different decays of the molecular states.

This work is organized as follows. The theoretical formalism is explained in Sec. 2. The predicted partial decay widths are presented in Sec. 3 followed by a short summary in the last section.

## 2 FORMALISM AND INGREDIENTS

In the molecular scenario, the details of the calculations for \( \Xi^0 \rightarrow \Lambda^+ K^- \), \( \Xi^0 \rightarrow \Sigma^0 K^- \), and \( \Xi^0 \rightarrow \Xi^0 \pi^- \) are presented for \( \Xi_c \) state with two different total angular momenta \( J^P \). The molecular structure of the \( \Xi_c^0 \) baryon with quantum numbers \( J^P = 1/2^\pm \) is described by the Lagrangian

\[
\mathcal{L}_{\Xi_c^0}(x) = g_{\Xi_c^0 AD} \int d^4y \Phi(y^2) D^0(x + \omega_{AB}) \mathcal{A}(x - \omega_{D\Sigma}) \\
\times \bar{\Xi}_c^0(x) + g_{\Xi_c^0 \Sigma D} \int d^4y \Phi(y^2) \sqrt{\frac{2}{3}} D^0(x + \omega_{\Sigma \Sigma}) \\
\times \Gamma \Sigma^0(x - \omega_{D\Sigma}) - \sqrt{\frac{1}{3}} D^+(x + \omega_{\Sigma \Sigma}) \\
\times \Gamma \Sigma^-(x - \omega_{D\Sigma}) \right] \Xi_c^0(x),
\]

while for the choice \( J^P = 3/2^\pm \) the Lagrangian contains a derivative

\[
\mathcal{L}_{\Xi_c^0}(x) = g_{\Xi_c^0 AD} \int d^4y \Phi(y^2) D^0(x + \omega_{AB}) \mathcal{A}(x - \omega_{D\Sigma}) \\
\times \bar{\Xi}_c^0(x) + g_{\Xi_c^0 \Sigma D} \int d^4y \Phi(y^2) \sqrt{\frac{2}{3}} D^0(x + \omega_{\Sigma \Sigma}) \\
\times \Gamma \partial_\mu \Sigma^0(x - \omega_{D\Sigma}) - \sqrt{\frac{1}{3}} D^+(x + \omega_{\Sigma \Sigma}) \\
\times \Gamma \partial_\mu \Sigma^-(x - \omega_{D\Sigma}) \right] \Xi_c^0(x),
\]

where \( \Gamma \) is the corresponding Dirac matrix related to the spin-parity of the \( \Xi_c^0 \). In particular, for \( J^P = 1/2^+ \), \( 3/2^- \) we have \( \Gamma = \gamma^5 \) while for \( J^P = 1/2^-, 3/2^+ \) the Dirac structure \( \Gamma = 1 \). In the Lagrangian, an effective correlation function \( \Phi(y^2) \) is introduced to reflect the distribution of two constituents in the hadronic molecular \( \Xi_c^0 \) state. The introduced correlation function also makes the Feynman diagrams finite in the ultraviolet region of Euclidean space, which indicates that the Fourier transformation of the correlation function should drop fast enough in the ultraviolet region. Here we choose the Fourier transformation of the correlation in the Gaussian form

\[
\Phi(y^2) = \exp(-y^2/\alpha^2)
\]

with \( \alpha \) being the size parameter which characterize the distribution of components inside the molecule. The value of \( \alpha \) could not be determined from first principles, it is usually chosen to be about 1 GeV in the literature.[10][11][12][13][14]. In this work, we set \( \alpha = 1.0 \) GeV.

With the help of the effective Lagrangian in Eq. (1) and Eq. (2), we can obtain the self energy of the \( \Xi_c^0 \)

\[
\Sigma^{1/2}_{\Xi_c^0}(k_0) = \int \frac{d^4k}{(2\pi)^4} \left\{ g_{\Xi_c^0 AD} \Phi^2[(k_1 - k_0\omega_3)\mathcal{A} k_1 + m_A]\frac{1}{k_1^2 - m_A^2} \\
\times \left[ \frac{1}{(k_1 - k_0)^2 - m_D^2} + g_{\Xi_c^0 \Sigma D}^2 \frac{2}{3} \Phi^2[(k_1 - k_0\omega_3)\Sigma^0\mathcal{A} k_1 - m_D^2] \right] \right\}
\]

\[
\Sigma^{3/2}_{\Xi_c^0}(k_0) = \int \frac{d^4k}{(2\pi)^4} \left\{ g_{\Xi_c^0 AD} \Phi^2[(k_1 - k_0\omega_3)\mathcal{A} k_1 + m_A]\frac{1}{k_1^2 - m_A^2} \\
\times \left[ \frac{1}{(k_1 - k_0)^2 - m_D^2} + g_{\Xi_c^0 \Sigma D}^2 \frac{2}{3} \Phi^2[(k_1 - k_0\omega_3)\Sigma^0\mathcal{A} k_1 - m_D^2] \right] \right\}
\]

where \( k_0^2 = m_{\Xi_c^0}^2 \) with \( k_0, m_{\Xi_c^0} \) denoting the four momenta and the mass of the \( \Xi_c^0 \), respectively, \( k_1, m_D \), and \( m_A, m_{\Sigma} \) are the four-momenta, the mass of the \( D \) meson, and the mass of the \( A \) baryon, respectively. The coupling constant \( g_{\Xi_c^0 AD} \) and \( g_{\Xi_c^0 \Sigma D} \) is determined by the compositeness condition.[15][16]. It implies that the renormalization constant of the hadron wave function is set equal to zero with

\[
Z_{\Xi_c^0} = x_{DA} + x_{DS} - \frac{d\Sigma^{1/2}_{\Xi_c^0}(2\pi)^4-T}{dk_0}|_{k_0=m_{\Xi_c^0}} = 0.
\]

where \( x_{AB} \) is the probability to find the \( \Xi_c^0 \) in the hadronic state \( AB \) with the normalization \( x_{DA} + x_{DS} = 1.0 \). And the \( Z^{3/2}_{\Xi_c^0} \) is the transverse part of the self-energy operator \( Z^{3/2}_{\Xi_c^0} \) related to \( \Sigma^{3/2}_{\Xi_c^0} \) via

\[
Z^{3/2}_{\Xi_c^0}(k_0) = (g_{\mu\nu} - k_0^2/k_0^2)\Sigma^{3/2}_{\Xi_c^0} + \cdots.
\]
Fig. 2 shows the hadronic decay of the $\Lambda D - \Sigma D$ molecular state into the $\Lambda^+_3 K^- \Xi^+_c \pi^-$, and $\Sigma^+_3 K^-$ final states occurring by exchanging nucleon, $D^*_s$ meson, and $D^*$ meson. To compute the amplitudes of the diagrams shown in Fig. (2), we need the effective Lagrangian densities for the relevant interaction vertices. In Refs. [17][18], coupling of the vector meson to charm baryons are described from effective Lagrangians, which are obtained using the hidden gauge formalism and assuming SU(4) symmetry:

\[ B^{121} = p, \quad B^{122} = n, \quad B^{132} = \frac{1}{\sqrt{2}} \Sigma^0 - \frac{1}{\sqrt{6}} A, \]

\[ B^{213} = \sqrt{\frac{2}{3}} A, \quad B^{231} = \frac{1}{\sqrt{2}} \Sigma^0 + \frac{\sqrt{3}}{\sqrt{2}} A, \quad B^{232} = \Sigma^-, \]

\[ B^{233} = \Xi^-, B^{311} = \Sigma^+, \quad B^{313} = \Xi^0, \quad B^{141} = -\Sigma^{++}, \]

\[ B^{142} = \frac{1}{\sqrt{2}} \Xi^+_c + \frac{1}{\sqrt{6}} A_c, \quad B^{143} = \frac{1}{\sqrt{2}} \Sigma^+_c, \quad B^{242} = \Sigma^0_c, \]

\[ B^{243} = \frac{1}{\sqrt{2}} \Xi^+_c - \frac{1}{\sqrt{6}} A_c, \quad B^{244} = \Omega^0_c, \]

\[ B^{144} = \Xi^{++}, \quad B^{244} = -\Xi^{++}, \quad B^{344} = \Omega^{0}_c, \]

where the indices $i, j, k$ of $B^{ijk}$ denote the quark content of the baryon fields with the identification $1 \leftrightarrow u, 2 \leftrightarrow d, 3 \leftrightarrow s, 4 \leftrightarrow c$.

To evaluate the diagrams in Fig. (2), in addition to the Lagrangian in Eq. (11), Eq. (12), and Eq. (5), the following effective Lagrangians, responsible for vector mesons and pseudoscalar mesons interactions are needed as well [17] :

\[ \mathcal{L}_{PV} = \frac{i}{4} g_{h} (\partial_{\mu} P, P) |V_{\mu}|, \ (11) \]

where $P$ is the SU(4) pseudoscalar meson matrices, and $|\ldots|$ in the trace over the SU(4) matrices. The meson matrices are [17]

\[ P = \sqrt{2} \left( \begin{array}{cccc} \rho^+ & \pi^+ & K^+ & D^0 \\ \rho^- & -\pi^- & K^0 & -D^- \\ K^* & K^0 & 0 & D^0 \\ D^0 & -D^0 & 0 & 0 \end{array} \right) \]

(12)

The coupling $g_{h}$ is fixed from the strong decay width of $K^* \rightarrow K \pi$. With the help of Eq. (12), the two-body decay width $\Gamma(K^{*+} \rightarrow K^0 \pi^+)$ is related to $g_{h}$ as

\[ \Gamma(K^{*+} \rightarrow K^0 \pi^+) = \frac{g_{h}^2}{6 \pi m_{K^{*+}}} P_{P}^3 K^{*+} = \frac{2}{3} \Gamma_{K^{*+}}, \ (13) \]
where $P_{\pi K}$ is the three-momentum of the $\pi$ in the rest frame of the $K^*$. Using the experimental strong decay width ($\Gamma_{K^{*+}} = 50.3 \pm 0.8$ MeV) and the masses of the particles needed in the present work are listed in Table 1. We obtain $g = 4.64$ [1].

### Table 1. Masses of the particles needed in the present work (in units of MeV).

| $\Lambda$ | $\Lambda^*$ | $\Sigma^+$ | $\Sigma^0$ | $\Sigma^+$ | $D^+$ |
|-----------|-------------|-------------|-------------|-------------|-------|
| 1115.683  | 2286.46     | 2452.90     | 2453.75     | 2467.93     | 1864.83 |
| 938.272   | 939.565     | 497.611     | 493.68      | 139.57      | 1869.65 |
| $\Xi^0$   | $D_s^+$     | $D_s^*$     | $D_s^+$     | $\Xi^+$     | $\Lambda^*$ |
| 1115.683  | 2286.46     | 2452.90     | 2453.75     | 2467.93     | 1864.83 |
| 938.272   | 939.565     | 497.611     | 493.68      | 139.57      | 1869.65 |
| 2470.91   | 2112.1      | 2010.26     | 2006.85     | 1189.37     | 1192.642 |

The vertexes for the meson-baryon interaction are needed and the form in the $SU(3)$ sector is given by the chiral Lagrangian [19]

$$\mathcal{L}_{\phi \Sigma} = \frac{F}{2} \left( \bar{B} \gamma_{\mu} \gamma_5 \{u^\mu, B\} + \frac{D}{2} \bar{B} \gamma_{\mu} \gamma_5 \{u^\mu, B\} \right),$$

(14)

where $F = 0.51, D = 0.75$ [19] and at lowest order in the pseudoscalar field $u^\mu = \sqrt{2} \phi f / f$, with $f = 93$ MeV. And $B$ and $\phi$ is now the $SU(3)$ matrix of the baryon octet and meson, respectively,

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{\sqrt{6}}{\sqrt{2}} \Lambda & \frac{1}{\sqrt{2}} \Sigma^+ - \frac{\sqrt{6}}{\sqrt{2}} \Lambda & \frac{1}{\sqrt{2}} \Sigma^- - \frac{\sqrt{6}}{\sqrt{2}} \Lambda \\ \frac{1}{\sqrt{2}} \Sigma^0 - \frac{\sqrt{6}}{\sqrt{2}} \Lambda & \frac{1}{\sqrt{2}} \Sigma^+ + \frac{\sqrt{6}}{\sqrt{2}} \Lambda & \frac{1}{\sqrt{2}} \Sigma^- + \frac{\sqrt{6}}{\sqrt{2}} \Lambda \\ \frac{1}{\sqrt{2}} \Sigma^0 - \frac{\sqrt{6}}{\sqrt{2}} \Lambda & \frac{1}{\sqrt{2}} \Sigma^+ + \frac{\sqrt{6}}{\sqrt{2}} \Lambda & \frac{1}{\sqrt{2}} \Sigma^- - \frac{\sqrt{6}}{\sqrt{2}} \Lambda \end{pmatrix},$$

(15)

and

$$\phi = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda & \frac{1}{\sqrt{2}} \pi^+ - \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda & \frac{1}{\sqrt{2}} \pi^- - \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda \\ \frac{1}{\sqrt{2}} \pi^0 - \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda & \frac{1}{\sqrt{2}} \pi^+ + \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda & \frac{1}{\sqrt{2}} \pi^- + \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda \\ \frac{1}{\sqrt{2}} \pi^0 - \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda & \frac{1}{\sqrt{2}} \pi^+ + \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda & \frac{1}{\sqrt{2}} \pi^- - \frac{\sqrt{6}}{\sqrt{2}} \phi_k \Lambda \end{pmatrix}. $$

(16)

Moreover, the effective Lagrangians for the $\Lambda_\mathcal{A}$ and $D\Sigma_\mathcal{C}$ couplings are [20][21]

$$\mathcal{L}_{\Lambda_\mathcal{A}D^\mathcal{B}} = ig_{\Lambda_\mathcal{A}D^\mathcal{B}} \Lambda_\mathcal{A}D^\mathcal{B},$$

(17)

$$\mathcal{L}_{D\Sigma_\mathcal{C}} = -ig_{D\Sigma_\mathcal{C}}D\Sigma_\mathcal{C},$$

(18)

with $\mathcal{T}$ being the usual Pauli matrices. The coupling constant $g_{\Lambda_\mathcal{A}D^\mathcal{B}} = 10.7_{-1.3}^{+3}$ and $g_{D\Sigma_\mathcal{C}} = -2.69$ are borrowed from Refs. [20][21][22], where we take the central values $g_{\Lambda_\mathcal{A}D^\mathcal{B}} = 10.7$ in our calculation.

With the above vertexes, the amplitudes of the triangle diagrams of Fig. 3, evaluated in the center of mass frame of final states, are

$$\mathcal{M}_\alpha = (i)^3 \frac{g_{\Xi^0 \Xi^2 D^0}}{4} \int \frac{d^4 q}{(2\pi)^4} \phi [(k_1 \omega_A - k_2 \omega_D)^2]$$

$$\times \bar{\Lambda}_p(p_2) \gamma_{\mu} \left( \frac{k_2 + m_{\Sigma^0}}{k_2 - m_A} \right)^2 \Gamma \left( u(k_0), i k_2 \nu^0(k_0) \right)$$

$$\times \frac{1}{k_1^2 - m_D^2} \left( p_2^\mu + k_4^\mu \right)^2 q^2 - m_{D^+}^2, $$

(19)

$$\mathcal{M}_\beta = -i \frac{1}{4} \left( D_3 F_{\Xi^0 \Xi^2 D^0} \right) \int \frac{d^4 q}{(2\pi)^4} \phi [(k_1 \omega_A - k_2 \omega_D)^2]$$

$$\times \phi [(k_1 \omega_A - k_2 \omega_D)^2] \bar{u}(p_2) \gamma_{\mu} \left( \frac{k_2 + m_{\Sigma^0}}{k_2 - m_A} \right)^2 \Gamma \left( u(k_0), i k_2 \nu^0(k_0) \right)$$

$$\times \frac{1}{k_1^2 - m_D^2} \left( p_2^\mu + k_4^\mu \right)^2 q^2 - m_{D^+}^2, $$

(20)

$$\mathcal{M}_\epsilon = -i \frac{1}{4} \frac{g_{\Xi^0 \Xi^2 D^0}}{4} \int \frac{d^4 q}{(2\pi)^4} \phi [(k_1 \omega_A - k_2 \omega_D)^2]$$

$$\times \bar{\Lambda}_p(p_2) \gamma_{\mu} \left( \frac{k_2 + m_{\Sigma^0}}{k_2 - m_A} \right)^2 \Gamma \left( u(k_0), i k_2 \nu^0(k_0) \right)$$

$$\times \frac{1}{k_1^2 - m_D^2} \left( p_2^\mu + k_4^\mu \right)^2 q^2 - m_{D^+}^2, $$

(21)

$$\mathcal{M}_\delta = -i \frac{1}{4} \left( D_3 F_{\Xi^0 \Xi^2 D^0} \right) \int \frac{d^4 q}{(2\pi)^4} \phi [(k_1 \omega_A - k_2 \omega_D)^2]$$

$$\times \phi [(k_1 \omega_A - k_2 \omega_D)^2] \bar{u}(p_2) \gamma_{\mu} \left( \frac{k_2 + m_{\Sigma^0}}{k_2 - m_A} \right)^2 \Gamma \left( u(k_0), i k_2 \nu^0(k_0) \right)$$

$$\times \frac{1}{k_1^2 - m_D^2} \left( p_2^\mu + k_4^\mu \right)^2 q^2 - m_{D^+}^2, $$

(22)
$\mathcal{M}_{g_2} = (i)^4 \sqrt{\frac{1}{3}} \left( \frac{F-D}{2\pi^2} \right) g_{\Xi_0^+D} g_{\Xi_0^+D^+} \int \frac{d^4q}{(2\pi)^4} \times \Phi[(k_1 \omega_{-} - k_2 \omega_{+})^2] \frac{q-m_n}{q^2-m_n^2} \times \hat{p}_1 \frac{k_2 + m_\Sigma}{k_2^2 - m_\Sigma^2} \Gamma \{a(k_0), ik_2 \omega^p(k_0)\} \times \frac{1}{k_1^2 - m_{D^+}^2}.$

Once the amplitudes are determined, the corresponding partial decay widths can be obtained, which read,

$$\Gamma(\Xi_0^+ \rightarrow MB) = \frac{1}{2J+1} \left| \frac{p_1}{8\pi m_\Xi^0} |\mathcal{M}|^2 \right|,$$

where $J$ is the total angular momentum of the $\Xi_0^+$ state, the $|p_1|$ is the three-momenta of the decay products in the center of mass frame, the overline indicates the sum over the polarization vectors of the final hadrons, and $MB$ denotes the decay channel of $MB$, i.e., $\Lambda\bar{K}$, $\Xi_0^+\pi$, $\Xi_0^+K$.

### 3 RESULTS AND DISCUSSIONS

![Fig. 3. The coupling constants of the $\Xi_0^+$ state with different $J^P$ assignments as a function of the parameter $x_{DA}$. The $x_{DA}$ is the probability to find the $\Xi_0^+$ in the hadronic state $DA$.](image)

![Fig. 4. The total decay width with different spin-parity assignments for the various $\Xi_0^+$ as a function of the parameter $x_{DA}$. The cyan bands denote the experimental total width.](image)

We show the dependence of the total decay width on the $x_{DA}$ in Fig. 4. The total decay widths increase with $x_{DA}$ for the $J^P = 1/2^-$ and $J^P = 3/2^-$ assignments. For the $J^P = 1/2^-$ assignments, we found that the line shape of the total decay widths are huge different and the total decay widths first increases, then decreases but very slowly. A possible explanation for this may be that for an $S$-wave loosely bound state the effective coupling strength of the bound state to its components is more sensitive to the inner structure than the effective coupling strength of another possible molecular state, such as $P$-wave molecular state, relative to their inner component. This is why people often focus on the bound state from $S$-wave interaction and assume the $P$- and $D$-wave bound state should be difficult to form from hadron-hadron interaction.

From the Fig. 3 one find that the predicted total decay widths for the $\Xi_0^+(2964)$ state and $\Xi_0^+(2938)$ state in the four spin-parity assignments are all smaller than the experimental total width. Such results disfavor the assignment of these two states as $DA - D\Sigma$ molecular state. For the $\Xi_0^+(2923)$ state, the predicted total decay width...
is much smaller than the experimental total width in the case of $J^P = 1/2^+$, which disfavors such a spin-parity assignment for the $\Xi_c(2923)$ in the $DA - D\Sigma$ molecular picture. For the case of $J^P = 3/2^+$, since the estimated total decay widths is much smaller than the experimental total width, this case can be completely excluded as well. The $J^P = 3/2^+$ case is also disfavored due to the smallest width predicted. Hence, only the $\Xi_c(2923)$ can be considered as the molecular states composed of $D\Lambda$ and $D\Sigma$ components by comparison with the total decay width experimentally measured. The results in Fig. 4 also show that the total decay width of the $\Xi_c(2923)$ can not be reproduced when only consider $\Xi_c(2923)$ as pure $D\Lambda$ or pure $D\Sigma$ molecular state.

![Fig. 5. Partial decay widths of $\Xi_c(2923)^0 \to \Lambda^+_c K^-$ (red solid line) and $\Xi_c(2923)^0 \to \Xi_c \pi$ (black dash line) with $J^P = 1/2^-$ as a function of the parameter $x_{DA}$. The cyan bands denote the experimental total width. [4].](image)

The partial decay widths of $\Xi_c(2923)^0 \to \Lambda^+_c K^-$ and $\Xi_c(2923)^0 \to \Xi_c \pi$ with $J^P = 1/2^-$ assignment as a function of the parameter $x_{DA}$ are presented in Fig. 5. It is found that the transition $\Xi_c(2923)^0 \to \Xi_c \pi$ is main decay channel, which almost saturates the total width of $\Xi_c(2923)$. However, the transition $\Xi_c(2923)^0 \to \Lambda^+_c K^-$ give minor contributions. The decay width $\Xi_c(2923)^0 \to \Xi_c \pi$ is very different from that in the constituent quark model [4,15] if we assign the S-wave $DA - D\Sigma$ bound state as $\Xi_c(2923)$. Future experimental measurements of such a process can be quite useful to test the different interpretations of the $\Xi_c(2923)$.

4 Summary

In this work, the S-wave $DA - D\Sigma$ molecular states were studied by calculating their allowed two body strong decay to investigate whether the three new narrow $\Xi_c^*$ baryons, $\Xi^*_c(2938)$, $\Xi^*_c(2938)$, and $\Xi^*_c(2964)$ can be understood as $DA - D\Sigma$ molecules. With the coupling constants obtained by the composition condition, the decays through hadronic loops are calculated in a phenomenological effective Lagrangian approach. The total decay widths can be well reproduced with the assumption that the $\Xi^*_c(2923)$ as S-wave $DA - D\Sigma$ bound state with $J^P = 1/2^-$, which decay channels are $\Lambda^+_c K^-$ and $\Xi_c \pi$. The other newly reported $\Xi^*_c$ states cannot be accommodated in the current molecular picture. If the $\Xi^*_c(2923)$ is pure $DA - D\Sigma$ molecule, the transition strength of $\Xi_c(2923)^0 \to \Xi_c \pi$ is quite different from that in the constituent quark model [4,15] and the decay width almost saturates the total width of $\Xi_c(2923)$. Future experimental measurements of such a process can be quite useful to test the different interpretations of the $\Xi_c(2923)$.

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