On the irreversible thermodynamic of a gas influenced by a thermal radiation force generated from a heated rigid flat plate

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Abstract
In the present paper, the effect of the nonlinear thermal radiation (TR) on the neutral gas mixture in the unsteady state is investigated for the first time. The unsteady BGK technique of the Boltzmann kinetic equation (BKE) for a non-homogeneous neutral gas (NHNG) is solved. The solution of the unsteady case makes the problem more general than the stationary case. For this purpose, the moments’ method, together with the traveling wave method, is applied. The temperature and concentration are calculated for each gas component and mixture for the first time. Furthermore, the study is held for aboard range of temperatures ratio parameter and a wide range of the molar fraction. The non-equilibrium distribution function (NEDF) is calculated for each gas component and the gas mixture. The significant non-equilibrium irreversible thermodynamic characteristics the entire system is acquired analytically. That technic allows us to investigate our model consistency with Boltzmann’s H-theorem, Le Chatelier’s principle, and thermodynamics laws. Moreover, the ratios among the further participation of the internal energy change (IEC) are evaluated via the Gibbs formula of total energy. The final results are utilized to the argon-helium NHNG at different magnitudes of radiation force (RF) strength and molar fraction parameters. 3D-graphics are presented to predict the behavior of the calculated variables, and the obtained results are theoretically discussed. The significance of this study is due to its vast applications in numerous fields, such as satellites, commercial, and various industrial applications.

Keywords
Fluid mechanics, kinetic theory, thermodynamics, statistical analysis, travelling wave method, moment method, boltzmann kinetic equation, radiation, internal energy

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Introduction
Thermal radiation is a deep-rooted part of our environment. Also, at high temperatures, the radiative heat transfer is a remarkable incomplete system analysis. Some application where TR transfer is of primary significance includes cryogenic fuel storage systems, spacecraft cooling systems, boilers and furnaces, and solar collectors. Although, the statistical-mechanical study of an NHNG in a non-equilibrium situation is a fascinating subject from theoretical as well as experimental viewpoint. Few papers deal with the gas mixture, compared with the significant number of studies in a single neutral gas.
It is well known that the atoms of an NHNG radiate and absorb thermal energy. Therefore, in NHNG, the interaction with TR is remarkable, unlike solids and liquids, capable of undergoing conspicuous volume alteration. Considering this interaction makes the NHNG way more realistic. Further, radiative heat transfer in NHNG has remarkable applications from combustion processes to atmospheric modeling operations. The BKE is valid for all ranges of Knudsen number, while the Navier-Stokes (NS) system is suitable to give us acceptable results for the continuum flow only, (where \( Kn = \frac{L}{\lambda} \) is the Knudsen number that measures the rarefaction of any gas molecules and represents the ratio between the mean free path \( \lambda \) to a characteristic length \( L \)).

For this purpose, Abourabia and Abdel Wahid\(^9\) presented a new approach for investigating the effect of the TR force on the NHNG. For steady problem, this idea was applied on a half-space filled off by an NHNG bounded by a fixed heated solid plane. The present paper is extended to investigate the non-stationary problem for an NHNG affected by a TR force. Therefore, we should solve two systems of non-linear partial differential equations (PDEs) instead of one system of ordinary differential equations (ODEs), as presented in.\(^2\) Abdel Wahid\(^2\) study the behavior of a single monatomic neutral gas under the effect of non-linear TR field in the unsteady state. He investigated the thermodynamic properties of the system and approved that the applied model is compatible with the laws of thermodynamics, Boltzmann’s H-theorem, and Le Chatelier’s principle. In,\(^4\) Abdel Wahid presented an approximate solution of the influence of a linear RF on a neutral gas mixture using the traveling wave method and the moment method. He used the approximation form of linear radiation force. He tested the computability of his mathematical model with Boltzmann’s H-theorem, Le Chatelier’s principle, and the laws of thermodynamics and approved it. Abourabia and Abdel Wahid\(^5\) presented an exact mathematical solution for \( \frac{\partial y}{\partial T} \) the non-stationary and compressible. In a frame co-moving with the fluid,\(^2,5\) the manner of the NHNG is studied under the following conditions:

\( \frac{\partial g_A}{\partial T} + \frac{\partial g_A}{\partial y} + \frac{f_y}{m_A} \frac{\partial g_A}{\partial \xi_y} = \omega_{AA}(g_A - g_A) \)  
\( + \omega_{AB}(g_B - g_A) \)  

**The physical and mathematical formulation of the problem**

Consider an NHNG consisting of two species, for example, an A-Kind and B-Kind. The gas fills in the upper half-space in the system (\( y = 0 \)) and bounded by an illimitable immobile plane solid plate (\( y = 0 \)), in a constant pressure \( P \).\(^2,5\) The solid plate is suddenly heated. That is generating a TR force. The flow is non-stationary and compressible. In a frame co-moving with the fluid,\(^2,5\) the manner of the NHNG is studied under the following conditions:

(I) The two velocities of the atoms (incident and reflected) are equals at the fixed plane solid plate but opposite signs. That result happens as a result of Maxwell’s formula of momentum defuses reflection. Furthermore, the exchange will be generated by the temperature difference among the atoms and the heated plane solid plate. That is taking the form of full energy accommodation.\(^8\)

(II) A TR force is acting from the heated plane solid plate on the NHNG, written in vector form\(^2,5,8\) as

\[
\mathbf{f} = -4\sigma_s \nabla T^4(y, t) \text{ here } f_y = \frac{-16\sigma_s T^3 \partial T(y, t)}{3n_c} \frac{\partial y}{\partial y}
\]  

Here \( \sigma_s \), \( n_c \), \( T \), and \( f_y \) are the Stefan-Boltzmann constant, the gas concentration at the plane solid plate surface, the velocity of light, the temperature, and the TR force component along the y-axis direction, respectively. For non-stationary motion, the operation in the entire system subject to \( f_y \) (the TR force) can be formulated in terms of the BGK-type in the following form.\(^4,7\)

\[
\frac{\partial g_A}{\partial T} + \xi_y \frac{\partial g_A}{\partial y} + \frac{f_y}{m_A} \frac{\partial g_A}{\partial \xi_y} = \omega_{AA}(g_A - g_A)
\]

\[
+ \omega_{AB}(g_B - g_A),
\]
\[ \frac{\partial g_0}{\partial t} + \xi_y \frac{\partial g_0}{\partial y} + \frac{f_v}{m_B} \frac{\partial g_0}{\partial \xi_y} = \omega_{BB}(g_{0B} - g_0) + \omega_{BA}(g_{0A} - g_0). \]  
\[ (3) \]

Here \( \xi_y \), \( g_0 \), and \( m_B \) are the molecular velocity of the NHNG particle component along the y-axis, the two-sided Maxwell NEDF, and the mass of atoms of the \( \nu \) species. The four quantities \( \omega_{AA}, \omega_{AB}, \omega_{BB}, \) and \( \omega_{BA} \) are the collision frequencies that are mentioned in 4,7 as:

\[ \omega_{\gamma \gamma} = n_{xy} \frac{\pi d_M^2}{4} \left[ \frac{SKT}{\pi m_y} \right], \quad \omega_{\gamma v} = n_{xy} \pi \left( \frac{d_y + d_v}{2} \right)^2 \left[ \frac{SKT}{\pi m_y} \right], \]

\( \mu_{\gamma v} = \left( \frac{m_v}{m_y + m_v} \right) \), here \( \gamma \) and \( v=A \) or \( B \) where \( d \), \( \mu_{\gamma v} \), and \( n_{\gamma v} \) are the effective collisions sphere diameter, the reduced mass, and an NHNG concentration at the solid plate surface, respectively. The local Maxwell NEDFs \( g_{0\nu} \) is denoted by:

\[ g_{0\nu} = \frac{n_{\nu}}{(2\pi RT_0^3)^{\frac{3}{2}}} \exp \left[ -\frac{\xi_y^2}{(2RT_0)} \right], \]

where \( \xi_y^2 = \xi_x^2 + \xi_v^2 + \xi_z^2 \).

Lee’s moment method 15-18 is employed here to gain the solution of the BGK kinetic equation. By adding heat to a NHNG, allowing it to expand, it is made rarer than the non-homogeneous gas neighbor sections. It continues to create an upward stream of the heated NHNG, which is usually followed by a flow in the reverse direction by the more distant parts of the NHNG. The fresh portions of NHNG are carried into the reverse direction by the more distant parts of the heated NHNG, which is usually followed by a flow in the reverse direction by the more distant parts of the NHNG. The four quantities \( n_{xy}, \omega_{AA}, \omega_{AB}, \) and \( \omega_{BA} \) are the collision frequencies that are mentioned in 4,7 as:

\[ \omega_{\gamma \gamma} = n_{xy} \frac{\pi d_M^2}{4} \left[ \frac{SKT}{\pi m_y} \right], \quad \omega_{\gamma v} = n_{xy} \pi \left( \frac{d_y + d_v}{2} \right)^2 \left[ \frac{SKT}{\pi m_y} \right], \]

\( \mu_{\gamma v} = \left( \frac{m_v}{m_y + m_v} \right) \), here \( \gamma \) and \( v=A \) or \( B \) where \( d \), \( \mu_{\gamma v} \), and \( n_{\gamma v} \) are the effective collisions sphere diameter, the reduced mass, and an NHNG concentration at the solid plate surface, respectively. The local Maxwell NEDFs \( g_{0\nu} \) is denoted by:

\[ g_{0\nu} = \frac{n_{\nu}}{(2\pi RT_0^3)^{\frac{3}{2}}} \exp \left[ -\frac{\xi_y^2}{(2RT_0)} \right], \]

where \( \xi_y^2 = \xi_x^2 + \xi_v^2 + \xi_z^2 \).

The obtained equation (5) are known as the general equations of transfer. 16 Applying the non-dimensional forms by assuming:

\[ y = y^* \left( \tau_{AB} \sqrt{2RT_1} \right), \quad t = t^* \tau_{AB}, \]

\[ \xi_y^* = \xi_y \sqrt{2RT_1}, \quad g_{0\nu} = \frac{g_{0\nu}^* (2\pi RT_1)^{\frac{3}{2}}}{n_{\nu}}, \quad i = 0, 1, 2. \]

\[ (6) \]

\[ N_v = \frac{16\pi r_{T_1}^3}{3n_{\nu}c_m}, \quad T_1 = T_{1v}, T_s, \]

\[ n_{1v} = n_{1v}^* n_{s}, \quad n_{2v} = n_{2v}^* n_{s}, \quad dU_v = dU_v^* K_v T_s. \]

here \( \tau_{AB}, N_v, \) and \( dU_v \) are the relaxation time between collisions of the A-B species, a non-dimensional constant, and the IEC of the NHNG species \( v \), respectively. When the values of \( g_{01}, g_{11}, \) and \( g_{21} \) are calculated, then the macroscopic quantities can be evaluated as:

The bulk velocity of the gas:

\[ u_{\nu}(y, t) = \frac{1}{n_B} \int \xi_y g_{\nu}(y, t, \xi_y) d\xi_y = \frac{(n_{1v} \sqrt{T_{1v}} - n_{2v} \sqrt{T_{2v}})}{(n_{1v} + n_{2v})}. \]

The number density:

\[ n_{\nu} = \int g_{\nu} d\xi_y = \frac{(n_{1v} + n_{2v})}{2}. \]

The temperature:
\[ T_v = \frac{1}{3n_v} \int \xi^2 g_v d\xi = \left( \frac{n_{1v}T_{1v} + n_{2v}T_{2v}}{n_{1v} + n_{2v}} \right). \]  
(9)

The static pressure:
\[ P_{3yv} = \int \xi^2 g_v d\xi = \frac{1}{2} (n_{1v}T_{1v} + n_{2v}T_{2v}). \]  
(10)

The y-component heat flux:
\[ Q_{yv} = \int \xi^2 g_v d\xi = \left( \frac{n_{1v}T_{1v}^2 - n_{2v}T_{2v}^2}{n_{1v} + n_{2v}} \right). \]  
(11)

The functions \( T_{1v}(y, t), T_{2v}(y, t), n_{1v}(y, t) \) and \( n_{2v}(y, t) \) in equation (4), which are unknowns to be calculated later in this paper for each component of the NHNG. Therefore, we should examine the problems in a phase space coordinate system where the velocity \( u \) is at the origin.\(^2,5,6\)

Now, let \( \psi_1 = \xi^2 \) and \( \psi_2 = \frac{1}{2} \xi^2 \). Substituting form equation (4) into equation (5), taking into account (6), we acquire the moments of the BGK-type in non-dimensional form. After dropping the stars, we have eight PDEs for atoms of the \( v = A, B \) species.

The energy conservation equation is:
\[
\frac{\partial}{\partial t} \left( n_{1v}T_{1v} + n_{2v}T_{2v} \right) + \frac{\partial}{\partial y} \left( n_{1v}T_{1v}^2 - n_{2v}T_{2v}^2 \right) \\
+ \frac{3n_v}{2} \left( n_{1v}T_{1v} + n_{2v}T_{2v} \right) \left( n_{1v}T_{1v} + n_{2v}T_{2v} \right) = 0,
\]  
(12)

The y-direction component of the heat flux has the form:
\[
\frac{\partial}{\partial t} \left( n_{1v}T_{1v}^2 - n_{2v}T_{2v}^2 \right) + \frac{5}{4} \frac{\partial}{\partial y} \left[ \left( n_{1v}T_{1v}^2 + n_{2v}T_{2v}^2 \right) \right] \\
+ \frac{3n_v}{2} \left( n_{1v}T_{1v} + n_{2v}T_{2v} \right) \left( n_{1v}T_{1v} + n_{2v}T_{2v} \right) = \\
\left[ \frac{\omega_{vA}}{\omega_{AB}} \left( n_{1v}T_{1v}^2 - n_{2v}T_{2v}^2 \right) \right] + \frac{\omega_{vA}}{\omega_{AB}} \left( n_{1v}T_{1v}^2 - n_{2v}T_{2v}^2 \right).
\]  
(13)

The equations of state complement the above equations:
\[ P_v = n_v T_v = \text{constant}, \]  
(14)

Thus, we acquire the fourth equation in the form:
\[ \left( n_{1v}T_{1v}^2 - n_{2v}T_{2v}^2 \right) = 0 \]  
(15)

Traveling wave solution method\(^17,18\) is utilized to solve the problem, by assuming
\[ \vartheta = ky - \Omega t. \]  
(16)

That will replace dependent variables from functions in \((y, t)\) to functions in \(\vartheta\) with the transformation constants \(k\) and \(\Omega\). Applying equation (16), we acquire the derivatives:
\[
\frac{\partial}{\partial \vartheta} = -\Omega \frac{\partial}{\partial \vartheta}, \quad \frac{\partial}{\partial y} = k \frac{\partial}{\partial \vartheta} \quad \text{and} \quad \frac{\partial^2}{\partial \vartheta^2} = k^2 \frac{\partial^2}{\partial \vartheta^2},
\]  
(17)

where \(n\) is a positive integer. Substituting from equations (16) and (17) into equations (12) and (13) we acquire:
\[
-\Omega \frac{\partial}{\partial \vartheta} \left( n_{1v}T_{1v}^2 + n_{2v}T_{2v}^2 \right) + k \frac{\partial}{\partial \vartheta} \left( n_{1v}T_{1v}^2 - n_{2v}T_{2v}^2 \right) \\
+ kN_v \left( n_{1v}T_{1v} + n_{2v}T_{2v} \right) \left( n_{1v}T_{1v} + n_{2v}T_{2v} \right) = 0
\]  
(18)

Know, we intend to solve equations (14, 15, 18, and 19) with the boundary value problem to evaluate \( T_{1v}, T_{2v}, n_{1v} \) and \( n_{2v} \), as follows; equation (15) gives:
\[ n_{2v}\sqrt{T_{2v}} = n_{1v}\sqrt{T_{1v}}. \]  
(20)

Substituting from equations (14) and (20) into equation (18), taking into consideration equation (10), we get:
\[
\frac{k}{\partial \vartheta} \left( n_{1v}T_{1v}^2 - n_{2v}T_{2v}^2 \right) \\
= k \frac{\partial}{\partial \vartheta} \left( n_{2v}\sqrt{T_{2v}}(T_{1v} - T_{2v}) \right) = 0
\]  
(21)

Integrating equation (21), for \(\vartheta\), we get after factorization
After solving this equation, we obtain eight roots for $H_v(\vartheta)$, by using any symbolic software. We take into account the root, which preserves the positive signs of both $n$ and $T$.

The magnitudes of the integrated constants can be evaluated with the initial and boundary condition $(y, t) = (0, 0) \Rightarrow \vartheta = 0$:

$$
\left( n_1 H_1 \right)^2 - n_2 H_2 \right)^2
\left( n_1 \left( \vartheta = 0 \right) + n_2 \left( \vartheta = 0 \right) \right) = C_v, \quad \text{where}
$$

$$
C_v = \frac{n_v}{n_s} \text{ is the molar fraction.}
$$

Moreover, $C_{2v}$ is the integration constants. We can easily show that $\theta_{1v}, \theta_{2v}$ are constants too.

To make our calculation simpler, and make better use of the equation (20), let us introduce the function $H_v(\vartheta)$ in the following form:

$$
H_v(\vartheta) = n_2 \sqrt{T_{2v}} = n_1 \sqrt{T_{1v}}.
$$

From equations (23) and (24) we can obtain:

$$
T_{1v}(\vartheta) = \frac{\left( \theta_{1v} + \theta_{2v} H_v \right)^2}{4 H_v^2},
$$

$$
T_{2v}(\vartheta) = \frac{\left( \theta_{1v} - \theta_{2v} H_v \right)^2}{4 H_v^2},
$$

$$
n_{1v}(\vartheta) = \frac{2 H_v^2}{(\theta_{1v} + \theta_{2v} H_v)} \quad \text{and}
$$

$$
n_{2v}(\vartheta) = \frac{2 H_v^2}{(\theta_{1v} - \theta_{2v} H_v)}.
$$

Now, we can integrate equation (19) w.r.t. $\vartheta$ after performing some necessary algebraic manipulations, taking into consideration equations (14) and (22), to get:

$$
\frac{5}{4} k \left( n_1 H_1 \right)^2 + n_2 H_2 \right)^2 + \frac{3}{2} k N_e \left( n_1 H_1 \right)^2 \right)^5
\left( \frac{\omega_{1v}}{\omega_{AB}} + \frac{\omega_{2v}}{\omega_{AB}} \right) \vartheta + \Omega \right) \theta_{1v} \theta_{2v} + \theta_{2v},
$$

here $\theta_{2v}$ is the constant of integration. Substituting from equation (25) into equation (26), we have:

$$
\left( H_v^{-8} \right) \left( 3 k N_e \theta_{1v}^9 - 12 k N_e \theta_{1v}^7 \theta_{2v}^2 H^2 + 18 k N_e \theta_{1v}^5 \theta_{2v}^4 H^4 + 4 k \theta_{1v}^3 H^6 \left( 80 - 3 N_e \theta_{2v}^6 \right) \right)
$$

After solving this equation, we obtain eight roots for $H_v(\vartheta)$, by using any symbolic software. We take into account the root, which preserves the positive signs of both $n$ and $T$.

The obtained formulae represent the initial and boundary conditions of the problem.

Substituting the calculated quantities $T_{1v}, T_{2v}, n_1$ and $n_2$ into the two-side Maxwell NEDF in the form (4).

We can acquire the sought NEDFs, which enable us to investigate the manner of the NHNG atoms. That was not possible if we utilized the NS equations system instead of the BGK-type.
the following relation:20–24

\[ dS(y, t) = \frac{\partial S(y, t)}{\partial t} + \nabla \cdot \mathbf{J}(y, t). \]

According to the theory of thermodynamics,4,7,13 the TFS can be evaluated:

1. The first is \( X_q \) that related to the modification in the concentration \( X_q(y, t) = \frac{\Delta Y}{\partial y} \).

2. The second TF is \( X_T \) that related to the alteration in the temperature,

\[ X_T(y, t) = \frac{\Delta T}{T(y, t)} \frac{\partial T(y, t)}{\partial y}. \]

3. The third TF is \( X_R \), which is related to the modification in the RF energy, \( X_R(y, t) = \frac{\Delta U}{\partial y} \frac{\partial U(y, t)}{\partial y} \), where \( U_R(y, t) = \frac{16\sigma T^4}{\gamma K_B T} \). The non-dimensional RF energy that affected the NHNG atoms and \( \Delta Y \) is the thickness of the layer near to the solid plane plate that has units as the mean free path.

After calculating the entropy generation and the TFS, the kinetic coefficients \( L_{ij} \) can be obtained from the following relation:20–24

\[ \sigma_S(y, t) = \sum_i \sum_j L_{ij}X_iX_j = \begin{pmatrix} X_1 & X_2 & X_3 \end{pmatrix} \begin{pmatrix} L_{11} & L_{12} & L_{13} \\ L_{21} & L_{22} & L_{23} \\ L_{31} & L_{32} & L_{33} \end{pmatrix} \begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \geq 0. \]

The Gibbs relation for the IEC \( dU(y, t) \) utilized to the system is: \( dU(y, t) = dU_S(y, t) + dU_Y(y, t) + dU_R(y, t) \). Moreover, the IEC related to the extensive variables modification (entropy \( dU_S \), volume \( dU_V \), and besides the temperature gradient generated by the TR force \( dU_R \)), are respectively donated for an NHNG as follows:

\[ dU_S(y, t) = \rho^{-1} \sum_{\nu=a}^B \rho_{\nu} dU_{S\nu}(y, t), \text{ here } dU_{S\nu}(y, t) = T_s dS_{\nu}(y, t), \]

\[ dU_V(y, t) = \rho^{-1} \sum_{\nu=a}^B \rho_{\nu} dU_{V\nu}(y, t), \text{ here } dU_{V\nu}(y, t) = -P_{\nu} dV_{\nu}(y, t), \]

\[ dU_R(y, t) = \rho^{-1} \sum_{\nu=a}^B \rho_{\nu} dU_{R\nu}(y, t), \text{ here } dU_{R\nu}(y, t) = \frac{16\sigma T^4}{3cn_K_B T_s}. \]

The pressure and modification in the volume are \( P_{\nu} = n_{\nu} T_v \) and \( dV_{\nu} = -\frac{\partial n_{\nu}}{\partial y} \), respectively, and

\[ dS_{\nu}(y, t) = \frac{\partial S_{\nu}(y, t)}{\partial t} + \frac{\partial S_{\nu}(y, t)}{\partial y} \text{ and } \delta y = 2.5, \delta t = 1. \]

\section*{Results and discussion}

The manner of the NHNG was examined under the influence of a TR force in the non-stationary state of a plane heat transfer problem in the entire system (NHNG + heated solid plane plate). Moreover, the TR is presented in the force term of the BKE for an NHNG subject to the inequality \( \lambda >> \delta >> d \), where \( \delta = n^{-1/3} \) is the average spacing among molecules, both \( \lambda \) and \( d \) are the mean free path and the molecular diameter, respectively. In all computation and graphs, we take the magnitudes of the following parameters from24,25 for the argon-helium NHNG, where the atoms of the A and B components are the argon and helium NHNGs, respectively:

\[ T_s = 1200K; \sigma_r = 5.670367 \times 10^{-8}kgs^{-3}K^{-4}; \]

\[ R = 8.3145J.K^{-1}.mol^{-1}; K_B = 1.3807 \times 10^{-23}J.K^{-1}; \]

\[ c = 2.9979 \times 10^8m.sec^{-1}; \]

\[ n_s = 3 \times 10^{18}m^{-3}; m_A = 39.948m_u; m_B = 4.0026m_u; \]

\[ m_u = 1.993 \times 10^{-26}Kg \text{ is atomic mass unit; } \]

\[ d_A = 3.84 \times 10^{-10}m; \lambda_A = 1.017m; \]

\[ V_{TA} = 7.067 \times 10^2m.sec^{-1}; \]

\[ \tau_A = 1.439 \times 10^{-3}sec; \]

\[ d_B = 6.2 \times 10^{-11}m; \lambda_B = 39.356m; \]

\[ V_{TB} = 2.232 \times 10^3m.sec^{-1}; \tau_B = 1.748 \times 10^{-2}sec; \]

for a fixed \( C_4 = 0.5 \), where \( K_B, \ V_T, \) and \( \tau_V \) are Boltzmann constant, thermal velocity, and the relaxation time of the NHNG species of the \( \nu \) species, respectively. Using the idea behind the shooting method,2 we
evaluate the transformation constants to obtain $k = 1.5$ and $\Omega = 3$. Dimensionless $N$ has the values $N_A(1200K) = 1.5355, N_B(1200K) = 2.628$. All figures clarify that all variables satisfy the equilibrium condition:

1- Wherein the equilibrium as $(y, t) = (0, 0)$:

- $n(y = 0, t = 0) = 1, T(0, 0) = 1,$
- $S(0, 0) = H's$ maximum value, $\sigma(0, 0) = 0,$
- $dU_5(0, 0) = 0, X_7(0, 0) = 0, X_8(0, 0) = 0,$
- $X_9(0, 0) = 0; \text{ see Figures 1, 2(c), and 3–11}$

2- Wherein the equilibrium as $x = 1$:

- $n(x = 1) = 1, T(x = 1) = 1,$
- $S(x = 1) = \text{Its maximum value,}$
- $\sigma(x = 1) = 0, dU_7(x = 1) = 0,$
- $dU_8(x = 1) = 0, dU_9(x = 1) = 0,$
- $X_7(x = 1) = 0, X_8(x = 1) = 0, X_9(x = 1) = 0; \text{ see Figures 12–22.}$

Now, we will discuss the manner of the NHNG atoms in the non-equilibrium state:

Figure 1(a)–(c) clarify the meaning that the gas concentration is an extensive parameter, while Figure 2(a)–(c) elucidate the implication that the gas concentration is an intensive parameter. A similar manner was introduced by Figures 23 and 24(a)–(c) and Figures 12 and 13(a)–(c), while the number density $n$ decreases, with time, the temperature $T$ growing, these happen for all magnitudes of $t$. That is, according to the fact of the uniform pressure, check Figures 1 and 2(c). Similarly, while the number density $n$ decreases, with time, the temperature $T$ growing, these happen for all magnitudes of $CA$. That is according to the fact of the uniform pressure; see Figures 12 and 13(c).

Figure 25(a)–(c) shed light upon an increment in $g_{1y}$ and a decrement in $g_{2x}$ try to compensate each other, compared with $g_{xy}$ for each gas and gas mixture. That denoted an excellent qualitative agreement in the
manner with \(^{21}\) of the NEDFs. That is explained by a trip for the non-equilibrium state, where the gas atoms, having temperature \(T_1\) and density \(n_1\), after being heated, are replaced by a reverse current of NHNG atoms having temperature \(T_2\) and density \(n_2\). This behavior agrees with Le-Chateliers principle.

Figure 2. (a) Temperature \(T_A\) versus \(y\) and \(t\) for fixed \(\chi = 0.5\), and \(C_A = 0.5\), (b) temperature \(T_B\) versus \(y\) and \(t\) for fixed \(\chi = 0.5\), and \(C_A = 0.5\), and (c) temperature \(T\) versus \(y\) and \(t\) for fixed \(\chi = 0.5\), and \(C_A = 0.5\).

Figure 3. Entropy \(S\) versus \(y\) and \(t\) for fixed \(\chi = 0.5\), and \(C_A = 0.5\).

Figure 4. Entropy production \(\sigma_s\) versus \(y\) and \(t\) for fixed \(\chi = 0.5\), and \(C_A = 0.5\).
The entropy $S$ always growing with time, and the entropy production $\sigma$ has non-negative magnitudes for all values of time $t$, temperature ratio $x$, and the molar fraction $C_A$. That gives a complete consistent with the second law of thermodynamics and the Boltzmann H-theorem. This behavior compatible with Le Chatelier’s principle; see Figures 4, 5, 14, 15, 26, and 27.

The manner of the various participants of the IEC can be clarified in Figures 7–9, 17–19, and 28–30.)
Gibbs formula derives the numerical ratios among the diverse involvement of the IEC based upon extensive parameters total derivatives. Bear in mind their different tendencies, the maximum numerical magnitudes of the all contribution at different RF strengths, are in quantity ordered as follows:

(i) For fixed values \( (x = 0.5, \text{ and } C_A = 0.5) \) and variable magnitudes of \( y \) and \( t \) in the considered range \( (0 < t < 1 \) and \( 0 < y < 2.5) \), we have \( dU_s(1200K) \): \( dU_f(1200K) \): \( dU_R(1200K) \) \( \approx 1.0 : 10^{-2} : 0.4 \), see Figures 7–9. The participation of \( dU_R \) reaches the maximum absolute value at \( y = 2.5 \) that represents the far distance from the heated solid plate. That is due to the \( X_T \) manner related to each component in the NHNG; see Figure 31. The \( X_T \) will have the reverse direction to the \( X_T \) see Figures 10 and 11. That gives a qualitative agreement in the manner with the TFs determined in \(^{24}\) comparing with the same NHNG (Ar-He) at the same molar fraction \( C_A = 0.5 \).

(ii) For a fixed value \( (x = 0.5) \) and variable magnitudes of \( C_A \) in the considered range \( (0.2 < C_A < 0.95) \), we have \( dU_s(1200K) \):

![Figure 11](image1.png)

**Figure 11.** Thermodynamic force \( X_A \) versus \( y \) and \( t \) for fixed \( x = 0.5, \text{ and } C_A = 0.5 \).

![Figure 12](image2.png)

**Figure 12.** (a) Concentrations \( n_A \) versus \( t \) and \( x \) for a fixed \( y = 0.4, \text{ and } C_A = 0.5 \), (b) concentrations \( n_B \) versus \( t \) and \( x \) for a fixed \( y = 0.4, \text{ and } C_A = 0.5 \), and (c) concentrations \( n \) versus \( t \) and \( x \) for a fixed \( y = 0.4, \text{ and } C_A = 0.5 \).
Figure 13. (a) Temperature $T_A$ versus $t$ and $x$ for a fixed $y = 0.4$, and $C_A = 0.5$, (b) temperature $T_B$ versus $t$ and $x$ for a fixed $y = 0.4$, and $C_A = 0.5$, and (c) temperature $T$ versus $t$ and $x$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 14. Entropy $S$ versus $t$ and $x$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 15. Entropy production $\sigma_s$ versus $t$ and $x$ for a fixed $y = 0.4$, and $C_A = 0.5$. 
Figure 16. Kinetic Inequality versus $t$ and $X$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 17. $dU_s$ versus $t$ and $X$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 18. $dU_v$ versus $t$ and $X$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 19. $dU_R$ versus $t$ and $X$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 20. Thermodynamic force $X_T$ versus $t$ and $X$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 21. Thermodynamic force $X_n$ versus $t$ and $X$ for a fixed $y = 0.4$, and $C_A = 0.5$. 

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represents the concentrations of the heavier NHNG (argon) indirectly and the lighter NHNG (helium), respectively. That means that the more, the lighter mass of the NHNG species, the more the efficiency of the TR energy participation in the entire system total energy modification; see Figure 30. That is according to the $X_R$ related to each component in the NHNG see Figure 32. The $X_T$ will have the reverse direction to the $X_R$; see Figures 33 and 34. That gives a qualitative agreement in the manner with the TF determined in,\textsuperscript{17} comparing with the same NHNG (Ar-He) at the same molar fraction $C_d = 0.5$.

(iii) For a steady value $C_d = 0.5$ and variable values of $\chi$ in the considered range ($0.1 < \chi < 1$), we have

$$dU_R(1200K) : dU_T(1200K) : dU_d(1200K) \approx 1 : 10^{-1} : 1.2;$$ see Figures 17–19.

Figure 21 indicated that, $dU_R$ reaches a maximum absolute value at $\chi = 0.1$ which means the more the temperature difference (among the fixed plate surface temperature and the NHNG temperature), the more
Figure 24. (a) Temperature $T_A$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $x = 0.5$, (b) temperature $T_B$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $x = 0.5$, and (c) temperature $T$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $x = 0.5$.

Figure 25. (a) The comparison between the combined perturbed velocity distribution functions $f_A[f_1,f_2]$ and equilibrium velocity distribution function $f_{oA}$ at ($x = 0.5$) for a fixed $y$ value ($y = 0.4$), (b) the comparison between the combined perturbed velocity distribution functions $f_B[f_1,f_2]$ and equilibrium velocity distribution function $f_{oB}$ at ($x = 0.5$) for a fixed $y$ value ($y = 0.4$), and (c) the comparison between the combined perturbed velocity distribution functions $f[f_1,f_2]$ and equilibrium velocity distribution function $f_o$ at ($x = 0.5$) for a fixed $y$ value ($y = 0.4$).
will be the active participation of the TR energy in the entire system total energy modification. That is attributed to the manner of the correspondent magnitudes of $X_R$ itself; see Figure 20.
Figure 32. Thermodynamic force $X_R$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $\chi = 0.5$.

Figure 33. Thermodynamic force $X_T$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $\chi = 0.5$.

Figure 34. Thermodynamic force $X_n$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $\chi = 0.5$.

Figure 35. Kinetic coefficient $L_{11} = 0$ versus $y$ and $t$ for fixed $\chi = 0.5$, and $C_A = 0.5$.

Figure 36. Kinetic coefficient $L_{33} = 0$ versus $y$ and $t$ for fixed $\chi = 0.5$, and $C_A = 0.5$.

Figure 37. Kinetic coefficient $L_{12} = L_{21}$ versus $y$ and $t$ for fixed $\chi = 0.5$, and $C_A = 0.5$. 
Figure 38. Kinetic coefficient $L_{33} = L_{32}$ versus $y$ and $t$ for fixed $x' = 0.5$, and $C_A = 0.5$.

Figure 39. Kinetic coefficient $L_{31} = L_{13}$ versus $y$ and $t$ for fixed $x' = 0.5$, and $C_A = 0.5$.

Figure 40. Kinetic coefficient $L_{11} \approx 0$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $x' = 0.5$.

Figure 41. Kinetic coefficient $L_{22} \approx 0$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $x' = 0.5$.

Figure 42. Kinetic coefficient $L_{33} \approx 0$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $x' = 0.5$.

Figure 43. Kinetic coefficient $L_{12} = L_{21}$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $x' = 0.5$. 
Figure 44. Kinetic coefficient $L_{23} = L_{32}$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $\chi = 0.5$.

Figure 45. Kinetic coefficient $L_{31} = L_{13}$ versus $t$ and $C_A$ for a fixed $y = 0.4$, and $\chi = 0.5$.

Figure 46. Kinetic coefficient $L_{11} \approx 0$ versus $t$ and $\chi$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 47. Kinetic coefficient $L_{22} \approx 0$ versus $t$ and $\chi$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 48. Kinetic coefficient $L_{33} \approx 0$ versus $t$ and $\chi$ for a fixed $y = 0.4$, and $C_A = 0.5$.

Figure 49. Kinetic coefficient $L_{12} = L_{21}$ versus $t$ and $\chi$ for a fixed $y = 0.4$, and $C_A = 0.5$. 
From our investigation, the constraints imposed on the kinetic coefficients $L_{ij}$ are consistent with our new model. $L_{11} \geq 0$, $L_{22} \geq 0$, and $L_{33} \geq 0$, for all magnitudes in the considered ranges of both $\chi$ and $C_A$. Onsager-Casimir’s reciprocal relations are fulfilled, as we have $(L_{12}=L_{21}, L_{13}=L_{31},$ and $L_{32}=L_{23})$, for all magnitudes in considered ranges of $t, \chi,$ and $C_A$; see Figures 31 and 35–51. Using our model, we find that Onsager-Casimir’s kinetic inequality fluctuates in the order of $\pm 10^{-14}$, $\pm 10^{-13}$, and $\pm 10^{-11}$; see Figures 6, 16 and 52, respectively, which is a very acceptable error in our calculations.

**Conclusion**

The calculated results were demonstrated to conclude the following:

(a) The predominant vital factors that are directly proportional to the efficiency of the radiation energy in the total energy modifications of the entire system are:

(i) The ratio between the temperature of the plane solid plate surface and the NHNG atoms.

(ii) The ratio between the mass of each component of the NHNG.

(iii) The molar fraction of each component of the NHNG.

(b) At a temperature ($T \geq 1200$K), the radiation energy participation in the total IEC becomes the predominant one.

(c) The lighter NHNG component, helium, is influenced by a TR force more than the more massive gas component, argon.

(d) The first laws of thermodynamics (notably, the second law), the Le-Chatelier’s rule, the Boltzmann H-theorem, and the Onsager-Casimir reciprocal relations are all consistent with our new model.

(e) The negative sign at specific kinetic coefficients, related to cross effects, implies in such cases that according to the imposed TF (gradient), there is a heat flux in the reverse direction to the main flux. For instance, the negative sign in front of $L_{12} = L_{12}$ and $L_{32} = L_{32}$ implies that there is a flow caused by the temperature gradient, from a lower to a higher temperature, known as thermal diffusion (or effect of Soret), which gives a qualitative agreement with the investigation results.17,24

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Data Availability
No data, models, or code were generated or used during the study.

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Nomenclature

\( C \)
\( d \)
\( f_y \)
\( J_y \)
\( K_B \)
\( K_n \)
The velocity of the gas particles.
The effective collisions sphere diameter.
The thermal radiation force component along y-axis direction.
The entropy flux component along y-axis direction.
Boltzmann constant.
Hydrodynamic Knudsen number.
The kinetic coefficients. $L_{ij}$

The pressure. $P$

The pressure at the plate surface. $P_s$

The gas constant. $R$

Entropy per unit mass. $S$

The mean temperature. $T$

The temperature at the plate surface. $T_s$

The temperature related to $g_1$. $T_1$

The temperature related to $g_2$. $T_2$

The flow (bulk) velocity. $u$

The internal energy of the gas. $U$

The IEC. $dU$

The radiation energy change. $dU_R$

The IEC due to variation of entropy. $dU_S$

The IEC due to variation of concentration. $dU_V$

The gas volume. $V$

Thermal gas velocity. $V_T$

The TF corresponding to variation of concentration. $X_1$

The TF corresponding to variation of temperature. $X_2$

The TF corresponding to variation of radiation field energy. $X_3$

The velocity of the light. $c$

The DF. $g$

The local Maxwellian equilibrium DF. $g_0$

DF for upward going particles $\xi_y > 0$. $g_1$

DF for downward going particles $\xi_y < 0$. $g_2$

The particle mass. $m$

Mass of atoms of the $v$ species. $m_v$

The mean density. $n$

The concentration at the plate surface. $n_s$

The concentration related to $g_1$. $n_1$

The concentration related to $g_2$. $n_2$

Displacement variable. $y$

The velocity of the light. $c$