What to Expect When You’re Expecting: Femtoscopy at the LHC

Mike Lisa

Department of Physics, Ohio State University, 1040 Physics Research Building, 191 West Woodruff Ave., Columbus, OH 43210, USA

A huge systematics of femtoscopic measurements have been used over the past 20 years to characterize the system created in heavy ion collisions. These measurements cover two orders of magnitude in energy, and with LHC beams imminent, this range will be extended by more than another order of magnitude. Here, I discuss theoretical expectations of femtoscopy of $A + A$ and $p + p$ collisions at the LHC, based on Boltzmann and hydrodynamic calculations, as well as on naive extrapolation of existing systematics.

Keywords: LHC, HBT, femtoscopy, predictions, hydrodynamics, Boltzmann cascade, heavy ions, RHIC

I. INTRODUCTION

What distinguishes ultrarelativistic heavy ion physics from particle physics is its focus on geometrically large systems. The desire is not to understand fundamental processes, as in the latter field, but to create and probe new states of matter and access the only phase transition associated with a fundamental interaction (QCD). The geometrically-sensitive, bulk properties are the crucial ones, and these are reflected in the soft ($p_T \sim \Lambda_{QCD}$) sector.

In soft sector observables, long-term baselines have been established over a large energy range. Prior to first data at RHIC, it was commonly speculated (and hoped) that large deviations from these systematics (e.g. $\pi^0/\pi^+$ ratios, sidewards flow, strangeness enhancement, total multiplicity) would signal clearly the qualitatively different nature of the system created there \cite{2}. In femtoscopic systems, rather generic arguments led to expectations \cite{2, 3} of a rapid increase, with $\sqrt{s_{NN}}$, in the pion “HBT radii” $R_{out}$ and $R_{long}$, reflecting relatively long timescales of the transition from deconfined QGP to confined hadronic matter.

Such dramatic speculations are largely absent today, in anticipation of LHC collisions. Soft-sector, global observables at RHIC are only quantitatively different than they are at lower energies. Even in the high-$p_T$ sector, where jet suppression and partonic energy loss measurements have generated huge excitement, energy scans at RHIC reveal that the data indicate more of an evolution than a revolution.

This is all to the good. Discoveries via sharp jumps à la superfand visible in Fig. 1. The number of refereed-journal publications reporting new femtoscopy results in relativistic heavy ion collisions. Beginning of availability of new beams are indicated in yellow boxes.

FIG. 1: The number of refereed-journal publications reporting new femtoscopy results in relativistic heavy ion collisions. Beginning of availability of new beams are indicated in yellow boxes.

In the next Section, we consider the case of simple extrapolation of measured femtoscopic trends, with no reference to physics per se. In Section III, we consider predictions of Boltzmann/cascade transport calculations, and in Section IV those of hydrodynamical models. Finally, we discuss speculations on the physics behind femtoscopic measurements in $p + p$ collisions, which will, in fact, be the first results available at the LHC. At the end we summarize.

II. NOTHING NEW UNDER THE SUN (NNUS) SCENARIO

Femtoscopic measurements display rich, multidimensional and nontrivial systematic dependences upon kinematic ($p_T$, $y$, etc) variables and particle species \cite{5, 7}. The dependence upon global variables such as $\sqrt{s_{NN}}$ and impact parameter, however, appears significantly more trivial. Schematically characterizing the measured femtoscopic length scales as a multidimensional function, evidence thus far indicates an overall factorization

\[
R \left( \sqrt{s_{NN}}, A, B, |\vec{b}|, \phi, y, m_T, m_1, m_2 \right)
\]

\[
= R_s \left( \sqrt{s_{NN}}, A, B, |\vec{b}| \right) \cdot F_k (\phi, y, m_T, m_1, m_2)
\]
where $F_k$ is a dimensionless function containing, e.g. decreasing “HBT radii” with particle $m_T$. The dimensional scale $R_g$ is determined by global observables. However, as indicated by the second equality of Equation 1, to good approximation the only relevant global observable is the total multiplicity $M$ of the collision. In fact, this multiplicity dominance well apply to all soft-sector observables [5].

There are at least three caveats to the above statement. Firstly, the CERES [9] collaboration has shown that the scale depends also on the freeze-out chemistry (baryon-to-meson ratio) in addition to the multiplicity. This is important at low (AGS) energies. For collisions above top SPS energy, $\sqrt{s_{NN}} \sim 17$ GeV, the chemical evolution is sufficiently weak that one may consider multiplicity only. Secondly, as seen in Figure 2, there is some residual dependence of the outward radius on $\sqrt{s_{NN}}$ in addition to multiplicity; in this Section, we ignore this potentially important detail. Finally, the azimuthal ($\phi_{rms}$ as determined relative to the reaction plane) dependence [10, 11, 12, 13] likely at some point violates the factorization. While it has not been experimentally tested, two collisions producing the same multiplicity, one very peripheral (i.e. spatially anisotropic in the entrance channel) at high energy and the other very central at low energy, presumably generate freezeout distributions with different spatial anisotropy, which is then reflected in the azimuthally-sensitive femtoscopic [14]. See Section IV for further discussion.

These caveats stated, however, the factorization of Equation 1 is probably our best, zero-new-physics guide to simple extrapolation of femtoscopic trends measured over two orders of magnitude in $\sqrt{s_{NN}}$ and from from the lightest ($p+p$) to the heaviest ($Pb+Pb$) systems. Figure 2 suggests a simple form $R_g(M) \propto M^{1/3}$; this ignores the finite offset $\sim 1$ fm when extrapolating $M \to 0$, but this is negligible for high multiplicity. This relation may reflect that a constant freezeout density drives the femtoscopic scales [9], though this neglects any dynamic effects. Assuming that this simple proportionality continues, then, we know $R_g(M)$ and determining femtoscopic expectations boils down to anticipating the multiplicity at the LHC.

A naive extrapolation [7, 8] of systematics suggests that $dN/dy$ at the LHC will be 60% larger than that observed at RHIC. Thus, the zeroth-order expectation is that length scales at the LHC will be 17% $(1.61^{1.5} = 1.17)$ larger than those measured at RHIC, for all kinematic selections and particle species, according to Equation 1.

Going beyond simple extrapolation to include a physical picture, saturation-based calculations [15] give much higher multiplicity—roughly triple that at RHIC. This leads to expectations of length scales 45% higher than those at RHIC. Thus, $R_{long}$ for pions at midrapidity and low $p_T$ in central collisions would be $1.45 \times 10$ fm.

Multiplicity predictions based on Boltzmann/cascade calculations can be significantly higher yet. Selecting two for which femtoscopic predictions also exist (Section III), A Multi-Phase Transport (AMPT) calculation [16] and the Hadronic Rescattering Model (HRM) [17] predict $5 \times$ and $7 \times$ RHIC multiplicity, respectively. Thus, femtoscopic scales at LHC may be as much as 90% higher than at RHIC. Depending on the final-state interaction which produces the two-particle correlation function, measuring length scales of $\sim 15$ fm may challenge experimental two-track resolutions. For two-pion correlations, such scales are within the capabilities of the ALICE detector [18].

### III. BOLTZMANN TRANSPORT CALCULATIONS

More interesting than simple scaling relations are models with real physics and dynamics, such as transport calculations. Boltzmann/cascade transport models generally reproduce “HBT radii” at RHIC better than do hydrodynamic calculations [5]. The reasons behind this include different physics in the models, a more detailed description of the kinetic freezeout, and the use of more appropriate methods of calculating the radii [19]. Predictions of pion HBT radii with each of the transport calculations discussed in Section II reveal predictions more subtle than the simple multiplicity-scaling discussed above.

For an infinite and boost-invariant system (only an approximation of reality, of course), the longitudinal HBT radius $R_{long}$ is proportional to the system evolution time (i.e. between interpenetration of the ions and kinematic freezeout of the products) [5] [20]. Naturally, this is not a unique, system-wide time, but a distribution. An example is seen in Figure 3 in which the pion freeze-out time distribution for collisions at RHIC and LHC are compared in the HRM calculation. The LHC timescales are roughly double those at RHIC. Although HRM is not explicitly a boost-invariant model, we see in Figure 4 that $R_{long}$ reflects this timescale increase, roughly doubling when the energy is increased from RHIC to LHC energies. The $\sim 70\%$ increase in $R_{long}$ is roughly consistent,
then, with expectations from both a timescale and from the multiplicity-scaling point of view. This is certainly not a coincidence, as the increased timescale is due in large part to the increased multiplicity.

On the other hand, there is more going on. The predicted $p_T$-dependence of both $R_\text{long}$ and $R_\text{side}$ are steeper at the LHC than at RHIC. Also, the increase in $R_\text{side}$ is significantly less than 90%. Both of these effects are consistent with a freezeout scenario with significantly increased transverse flow [14]. Indeed, transverse momentum distributions predicted by HRM are significantly harder (less steep) at the LHC than those at RHIC. Since the $p_T$ dependence of HBT radii [5] and spectra in the soft sector are observed to change very little between $\sqrt{s_{NN}} = 20 \div 200$ GeV, it will be interesting to see whether this trend is broken at the LHC, as predicted by HRM.

The HRM model is a deliberate effort to use the simplest (often criticized as too simplistic) physics picture, free of novel phases like QGP. It is a pure hadron-based transport calculation, though the initial conditions may be taken from Pythia or Saturation-based scenarios [17]. On the other side of the “simplicity spectrum” is AMPT, an attempt to describe the various stages of the system’s evolution in terms of the most appropriate model for that stage [21].

Similar to HRM, AMPT predicts stronger transverse flow at the LHC, as compared to RHIC, leading to steeper $p_T$-dependence of HBT radii. In terms of scale, the transverse (longitudinal) radii are predicted to increase by 10% (30%). This is more modest than the predictions of HRM (30% and 70%, respectively), and much more modest than pure-multiplicity scalings of Section II.

Thus the dynamical physics, in these models, lead to expected details significantly beyond simple extrapolation of lower-energy results.

**IV. HYDRODYNAMICAL CALCULATIONS**

As mentioned, hydrodynamical models tend to reproduce femtoscopic measurements more poorly than do Boltzmann/cascade calculations. On the other hand, they have enjoyed huge success in reproducing momentum-space observables such as elliptic flow. Furthermore, the conditions at LHC are likely to provide an even better approximation than at RHIC to the zero-mean-free-path assumptions of pure hydrodynamics. Finally, the direct connection between hydrodynamics and the Equation of State of strongly-interacting matter (color-confined or not) remains a compelling reason to explore soft-sector, bulk consequences of the model.

**A. Source Length Scales**

Recently, Eskola and collaborators [22] coupled a pQCD+saturation-based prediction for initial conditions at LHC to their $1+1$-dimensional hydro calculation. The Equation-of-State featured a first-order phase transition between an ideal QGP at high temperature and a hadron resonance gas at low temperature.

As shown in Figure 5, the initial energy density at which hydrodynamics is assumed to take over expected to roughly an order of magnitude larger at the LHC than at RHIC, due both to increased gluon production and to shorter system formation (thermalization) time $\tau_0$ at the higher energy. Since the initial transverse scale changes only little, the pressure gradients will likewise be much higher at LHC, leading to increased transverse flow. These effects place competing pressures on the space-time evolution of the system, and on the femtoscopic scales at freezeout, as discussed below.

The increased energy density (directly associated with entropy density and thus multiplicity) tends to produce longer timescales at the LHC. Longitudinal expansion tends to cool the system towards freezeout conditions. However, especially at the LHC, the large transverse flow generated by the intense pressure gradients cannot be ignored. Eskola and collaborators [22], estimate that the time required to cool from the maximum energy density (at $r = 0$ in Figure 5) to the critical energy density ($e_c = 1.93\text{GeV/fm}^3$) would be 6 fm/c (20 fm/c) at RHIC (LHC), due to longitudinal expansion alone. How-
ever, when transverse dynamics are included, the cooling times become 5 fm/c (7.5 fm/c) at RHIC (LHC). The evolution time to kinematic freezeout—say until $T \approx 140$ MeV—is $\tau_0 \sim 12 - 14$ fm/c in both cases; this is the timescale most directly probed by femtoscopy. This is dramatic—the effect of transverse flow on cooling timescales can almost be neglected at RHIC, while it is dominant at the LHC. This is reminiscent of the cascade calculations discussed in Section III; the much stronger flow may well lead to deviations from the invariants of the cascade calculations discussed in Section III; this is rem

FIG. 5: Initial energy density distribution in the transverse plane, calculated by Eskola et al [22].

FIG. 6: Left: freeze-out space-time hypersurface in collisions with finite impact parameter calculated by Heinz and Kolb [23]. The “IPES” calculation is an estimate of the system created at the LHC. The impact parameter is defined to lie in the $\hat{z}$-direction. Right: time-$z$- and momentum-integrated freeze-out shapes in the transverse plane.

Due to the preferential in-plane expansion, as the system evolves the spatial configuration should become increasingly in-plane extended (equivalently, decreasingly out-of-plane extended). Thus, knowledge of the entrance-channel shape (e.g. though Glauber model calculations) and measurement of the exit-channel shape (through femtoscopy) provide “boundary conditions” on the dynamical spacetime evolution of the anisotropic system, and probe the evolution timescale. The extracted timescale is model-dependent, requiring in principle a detailed time evolution of the flow. However, a simple estimate $25$ of the timescale extracted through shape measurements and that extracted from blast-wave fits $14, 26$ to azimuthally-integrated HBT radii are roughly consistent.

Measurements of the freezeout shape at the AGS $11$ and RHIC $12, 13$ indicate an out-of-plane-extended configuration. Consistent with the fact that preferential in-plane expansion (i.e. elliptic flow) is stronger at RHIC, the configuration at the higher bombarding energy is rounder. This is shown in Figure $7$ in which the transverse anisotropy is characterized by $\epsilon \equiv (R_2^x - R_2^y)/(R_2^x + R_2^y)$ ($x$ is in the reaction plane).

The anisotropic shapes and corresponding azimuthally-selected HBT radii have been calculated in two dynamical models. At the AGS, the transport code RMHD $27$ reproduces the overall scale $28$ and the anisotropy $10, 11$ of the source reasonably well, as shown in Figure $7$. At RHIC, the $2 + 1$ hydro code $23$ reproduces the shape quite well, while missing the scale. At the LHC, the latter calculation predicts—again—a qualitative change in the freezeout distribution. As shown in the right panel of Figure $6$ the source is expected to evolve to an in-plane configuration ($\epsilon < 0$).

However, again, this huge flow has other dramatic and qual-

B. Source Shape

Azimuthally-sensitive pion interferometry—the measurement of spatial scales as a function of emission angle relative to the reaction plane—probes the shape of the freezeout configuration, in addition to its scale $10, 14, 24$. At finite impact

\[ R_{\text{long}} \equiv \frac{R_x^2 - R_y^2}{R_x^2 + R_y^2} \]

where $R_x$ and $R_y$ are the source radii in the $x$ and $y$ directions, respectively.
STAR ALICE

Remarkably, Gaussian fits to correlation function, identical coordinate systems and iden-
d+Au, and p+p collisions, using the same detector, same en-

tfirst direct comparison of pion HBT radii in Au+Au, Cu+Cu,

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direct “apples-to-apples” comparison between results from

properties, one might well hope for qualitative differences when

FIG. 7: Transverse spatial freezeout anisotropy $\varepsilon$ as a function of

collision energy, as estimated with azimuthally-sensitive pion fem-
toscopy, for collisions with impact parameter $\sim 7$ fm. Round sources

It is well-recognized that $p + p$ collisions serve as a valuable reference
to heavy ion analyses in the “hard” (high-

V. PROTON COLLISIONS

Before heavy ions are accelerated at the LHC, proton col-
cisions at $\sqrt{s} = 1.4$ TeV will be measured. While the thrust of

Soft-sector analyses, too, should be performed for systems
from the smallest to the largest, and the results compared.

Since such analyses are assumed to measure the bulk prop-
ties, one might well hope for qualitative differences when

While pion HBT measurements have been common in both
the high-energy and heavy-ion communities for many years,
a direct “apples-to-apples” comparison between results from

FIG. 8: Full markers: predicted $p_T$-dependence of $R_{inv}$ and

The increase of HBT radii with multiplicity has also been
observed previously in $p + \bar{p}$ collisions by the E735 Collabora-
tion [32]. While in $A + A$ collisions, this is naturally related
to increasing length scales in the entrance channel geometry,
Pać and Skowroński [33] postulate that jet dynamics, rather
than bulk properties, drive this dependence in the $p + \bar{p}$ sys-
tem. Within a simple model of hadronization, they can repro-
cude the E735 multiplicity dependence, and make predictions
of similar multiplicity dependence for $p + p$ collisions at the
top LHC energy. However, since the expectation of increas-
ing length scales with multiplicity seems to be rather generic
to all scenarios, it will be interesting to see these predictions
expanded to more differential measures—say, the multiplicity
and $p_T$ dependence, probing both aspects of Equation 1

This should allow a more discriminating comparison between
models, allowing some to be ruled out.

Such differential predictions have very recently
been performed by Humanic [31], in the context of a
Pythia+hadronic rescattering (through HRM) scenario. It is
found that, contrary to some expectations, hadronic rescat-
tering is crucial to understand the $M \otimes p_T$ dependence of the
E735 data. Reproducing the data requires the assumption
of a surprisingly short hadronization timescale ($\sim 0.1$ fm); longer
timescales do not allow sufficient hadronic rescattering
needed to describe the $p_T$-dependence. The prediction of
the model for the highest multiplicity $p + p$ collisions at
$\sqrt{s} = 1.4$ TeV are shown in Figure 8.
VI. SUMMARY

Two decades’ worth of femtoscopic systematics [5] in heavy ion collisions reveals a strikingly consistent and simple structure. The kinematic and particle-species dependences, which reflect dynamic substructure, decouple from the global scale, which depends (almost) solely on multiplicity. The assumption that the factorization of Equation 1 persists—i.e. that \( F_k \) remains unchanged at the LHC—combined with the assumption \( R_k(M) \sim \sqrt{M} \), is the essence of the NNUS scenario.

In the NNUS picture, all femtoscopic length scales measured at RHIC will be reproduced at LHC, only scaled up by 20-90\%, depending on the multiplicity prediction. Thus, the “pion HBT radius” \( R_{\text{long}} \) at low \( p_T \) might be expected in the range \((1.2 \div 1.9) \times (7 \text{ fm}) = 8.4 \div 13 \text{ fm} \), while the average shift between pions and kaons (about 6 fm at RHIC [34]) would be in the range \( 7.2 \div 11.5 \text{ fm} \). However, dynamical models generally predict interesting violations of NNUS at the LHC. The significant dispersion between predictions holds out the possibility that the data will eliminate some models.

In the HRM and AMPT Boltzmann/cascade calculations, increased rescattering due to the higher density at the LHC generates much stronger global space-momentum correlations. This leads to flatter \( p_T \) spectra for high mass particles, and to a steeper \( p_T \)-dependence of the femtoscopic length scales; that is, \( F_k \) would pick up a \( \sqrt{\tau} \) dependence, violating NNUS factorization. Transverse (longitudinal) scales are expected to increase 10-30\% (30-70\%), relative to RHIC values.

In hydrodynamical calculations, much higher energy densities and pressure gradients at the LHC may generate qualitatively new femtoscopic signals. Contrary to the situation at RHIC, the transversely explosive nature of the source at the LHC severely shortens the time until freezeout. The freezeout hypersurface is of a qualitatively different shape in transverse position and time; while transverse radii may increase by 40\% relative to RHIC values, the longitudinal ones should expand little, and may even decrease.

The evolution timescale may also be probed by measuring the anisotropic shape of the source in coordinate space, for non-central collisions. Here again, a qualitative difference is predicted by hydrodynamics between RHIC and LHC collisions. In particular, the greatly increased flow and somewhat increased evolution time lead to predictions of an in-plane extended source, producing HBT radius oscillations \( 180^\circ \) out of phase with those seen at lower energy.

Probably as important as soft-physics analyses in heavy ion systems are parallel ones for \( p + p \) collisions. First preliminary “apples-to-apples” comparisons of Gaussian HBT radius measurements at RHIC suggest that NNUS factorization continues to hold even for these smallest systems; it remains to be seen whether this conclusion survives more sophisticated treatment of non-femtoscopic correlations in the data, presently underway. In the context of two simple models, pion HBT radii at the LHC depend strongly on the hadronization scenario. Both predict an increase in femtoscopic freezeout scales with increasing multiplicity, which in itself will not distinguish these models from any other. However, in one, the \( p_T \) dependence is found to depend strongly on the hadronization time and degree of subsequent hadronic scattering. Such scattering is usually ignored in treatments of \( p + p \) collisions; indeed, the lack of significant rescattering is believed to be their primary virtue as a reference measurement. As hinted at by the first RHIC measurements, maybe they are not so different from \( A + A \) collisions after all. More detailed measurements at the LHC may, in fact, spur a re-evaluation of ideas of the spacetime evolution of both heavy ion and hadronic collisions.

In any case, we may confidently expect considerable activity and excitement as the next mountain forms on Figure 1.