study on settlement prediction of soft soil subgrade based on grey theory

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abstract. the settlement of subgrade is one of the dominant diseases of road engineering in soft soil areas. the settlement prediction of soft soil subgrade is beneficial to improve the level of prevention and treatment of road engineering diseases in soft soil areas. this paper takes the settlement data of highway subgrade in a soft soil area, which is located at yunnan province, as the research object. the grey gm(1,1) model of roadbed settlement is established, and the sinusoidal residual error corrects the calculation model. it is shown that the model has high accuracy by comparing the predicted data with the monitored data. the model can be applied to the settlement prediction of highway subgrade in soft soil area. it can provide the necessary theoretical basis for road maintenance and management, and ultimately realize the treatment of settlement disease in the highway roadbed.

1. introduction

with the continuous development of the "one-belt and one-way" construction, yunnan province has been a gateway and link to china's southeast asian countries, which has been greatly supported and helped by the chinese government. road engineering is one of the key development objects in the construction of "one belt and one road," and its operational safety has also become the focus. the settlement of subgrade will seriously endanger driving safety and greatly reduce the service life of roads [1]. therefore, the settlement controlling of the subgrade is the necessary guarantee for road operation safety. to timely and effectively adopt roadbed settlement control measures, achieve greater economic benefits and achieve better control effects, it is necessary to make reasonable predictions for severe soft soil subgrade.

the settlement and deformation mechanism of the subgrade is complex, and the settlement may be affected by many factors, such as construction [2], load [3], geological conditions [4]. it is a great challenge to analyze and predict the settlement of subgrade by mechanical method. the grey theory provides a new solution for the prediction of complex problems. jin [5] applied the grey prediction model to the settlement prediction of high fill highway subgrade. comparing the predicted data with the measured data, it was showed that the grey theory has high accuracy in the settlement prediction of subgrade. tang [6] used an unequal time interval grey model to predict roadbed settlement, which verified that the model was suitable for settlement prediction of the subgrade. liu [7] put forward a grey forecasting model to optimize the background value and proved the feasibility of this method in the settlement prediction of highway subgrade.

based on the grey system theory, the lagrange difference method is adopted to correct the original data in this paper. the grey gm(1,1) model is established, and the sinusoidal (cosine) curve fitting and superposition of the periodic variation of residual error are carried out. the above methods predict the
settlement of soft soil subgrade. Taking a highway of a soft soil subgrade in Yunnan Province as an example, the settlement prediction of the subgrade is compared with monitoring data. It is proved that the model has high prediction accuracy and can be used to predict the settlement of soft soil subgrade.

2. Establishment of Grey Prediction Model

2.1. The establishment of grey predicted mode

The general grey model is GM(n, h) model, which n represents order differential equation and h represents variables [8]. The basis of grey model sequence prediction is based on cumulative generated sequence [9], whose purpose is to weaken the randomness by accumulating of sequence, and enhance the stability of sequence required by model. The grey GM(n, h) model of settlement [10] is consistent with Asaoka's governing differential equation of settlement derived from Mikasa's one-dimensional consolidation equation [11], which reflects the nature of settlement [12]. A higher order differential equation can be established to solve the problem in theory. Nevertheless, the higher order differential equation brings difficulties in solving the differential equation, which makes it difficult and inconvenient to use the model. Therefore, this paper uses GM(1, 1) model to predict the settlement of soft soil subgrade.

There are a series of original settlement with the number of n, and they are recorded as follows

\[ X^{(0)} = (x^{(0)}(1), \ldots, x^{(0)}(i), \ldots, x^{(0)}(n)) \]  

where \( x^{(0)}(i) \) is the i-th data in the original sequence. A new sequence \( X^{(1)} \) is generated by the first-order accumulation of the sequence, which is called as 1-AGO.

\[ X^{(1)} = (x^{(1)}(1), \ldots, x^{(1)}(i), \ldots, x^{(1)}(n)) \]

where \( x^{(1)}(i) \) is the i-th data in the newly generated sequence after first-order accumulation. On this basis, the first order linear differential equation of the conventional GM(1,1) model can be established.

\[ \frac{dx^{(1)}}{dt} + ax^{(1)} = u, \]

where a is the development coefficient of the model, and u is the coordination coefficient of the model, which can be obtained by the least square method. The differential equation is solved, and the prediction model is obtained.

\[ X_a^{(1)}(k + 1) = \left( x^{(0)}(1) - \frac{u}{a} \right) e^{-at} + \frac{u}{a} \]  

2.2. The sinusoidal residual correction

Because of the fluctuation of the sequence curve of subgrade settlement, when the grey GM (1,1) model is used to fit the calculation, it can be seen that the settlement increases slowly with volatility. The traditional grey prediction is suitable for the trend of monotonous increasing or monotonous decreasing. To predict the settlement of subgrade accurately, the sinusoidal residual correction is introduced. The residual sinusoidal function is as follows.

\[ \varepsilon(k) = A \sin \frac{2\pi (k+\frac{nT}{2})}{T} \]

where \( \varepsilon(k) \) is prediction residual for the k-th month, A is the average of the absolute residual value, and T is the number of months which spans by the first cycle. If the trend of the original sequence is increasing, the n is odd. Otherwise, the n is even.

Finally, the corrected residual values are superimposed on the corresponding prediction value, and the corrected predicted values can be obtained.

\[ X_b^{(0)}(k) = X_a^{(0)}(k) - \varepsilon(k) \]
where $X_b^{(0)}(k)$ is the predicted value after residual correction, $X_a^{(0)}(k)$ is the initial predictive value of the model, $E_a(t_k)$ is a residual value after periodic correction, which can be considered as the error of the GM(1,1) model in mathematics.

It not only reduces the residual value generally, but also makes the residual value fluctuate. It makes the fitting curve more approximate to the original data curve. Therefore, the accuracy of the model is improved. The period of future sequence can be determined by the trend of the residual period of known sequence during forecasting. The magnitude of variation can be reasonably selected and controlled according to the allowable or possible variation of the system. After determining the period and amplitude of the future sequence, the residual correction values in the future can be calculated. Then, it is superimposed on the corresponding prediction values.

3. The model application and error analysis

3.1. The establishment of model

This paper takes the observation data of subgrade in a soft soil area of Yunnan Province as an example. The observational time of deformation is about 21 days per month. Although the date varies individually, the errors are within ±4 days. It satisfies the requirement of the equal time interval for data in the grey system. Therefore, the data is considered to be isochronal. The first monitoring point was on October 6, and assumes that the initial settlement is 0. The Lagrange difference method was adopted to correct the data to October 21. The other data are measured data, and the prediction results of the settlement are shown in Table 1.

According to the calculated data, the average residual absolute value is 2.78, and the period of $T$ is 12. With regard to the cumulative settlement adopted in this paper, the original sequence is an increasing trend. Therefore, the $n$ is odd, $n = 1$. The sinusoidal correction function of residual error is obtained: $\epsilon(k) = 2.78 \sin(2\pi(k + 6)/12)$. The model residual correction is shown in Figure 1, and the model prediction results before and after the residual correction are shown in Figure 2.

| Date       | Observative settlement | Initial prediction value | Initial residual value | Correction of residual value | Final prediction value |
|------------|------------------------|--------------------------|------------------------|------------------------------|------------------------|
| 2017-10-21 | 6                      | 6.0                      | 0.0                    | 0.0                          | 6.0                    |
| 2017-11-19 | 23                     | 22.7                     | -0.3                   | -1.4                         | 24.1                   |
| 2017-12-20 | 29                     | 25.2                     | -3.8                   | -2.4                         | 27.6                   |
| 2018-1-21  | 33                     | 27.9                     | -5.1                   | -2.8                         | 30.7                   |
| 2018-2-22  | 33                     | 30.9                     | -2.1                   | -2.4                         | 33.3                   |
| 2018-3-21  | 33                     | 34.2                     | 1.2                    | -1.4                         | 35.6                   |
| 2018-4-17  | 34                     | 37.9                     | 3.9                    | 0.0                          | 37.9                   |
| 2018-5-22  | 35                     | 41.9                     | 6.9                    | 1.4                          | 40.5                   |
| 2018-6-24  | 44                     | 46.4                     | 2.4                    | 2.4                          | 44.0                   |
| 2018-7-19  | 52                     | 51.4                     | -0.6                   | 2.8                          | 48.6                   |
| 2018-8-19  | 62                     | 57.0                     | -5.0                   | 2.4                          | 54.6                   |
| 2018-9-18  | 65                     | 63.1                     | -1.9                   | 1.4                          | 61.7                   |
3.2. The error analysis

By analyzing the forecasting results above, it can be discovered that the initial forecasting error of the traditional grey GM(1,1) model is 7.39%. The final prediction error of the model corrected by sinusoidal residual error is only 6.34%. Compared with the traditional grey model, the error is reduced by 14.2%. The results show that the grey model modified by the sinusoidal residual error has higher prediction accuracy and its prediction results are credible. The prediction error is shown in Table 2.
Table 2. The error analysis of model.

| Data       | Initial prediction | Final prediction |
|------------|--------------------|------------------|
|            | Settlement         | Error            | Settlement | Error  |
| 2017-10-21 | 6                  | 0                | 6          | 0      |
| 2017-11-19 | 23                 | -1.30            | 24.1       | 4.78   |
| 2017-12-20 | 29                 | -13.10           | 27.6       | -4.83  |
| 2018-1-21  | 33                 | -15.45           | 30.7       | -6.97  |
| 2018-2-22  | 33                 | -6.36            | 33.3       | 0.91   |
| 2018-3-21  | 33                 | 3.64             | 35.6       | 7.88   |
| 2018-4-17  | 34                 | 11.47            | 37.9       | 11.47  |
| 2018-5-22  | 35                 | 19.71            | 40.5       | 15.71  |
| 2018-6-24  | 44                 | 5.45             | 44.0       | 0.00   |
| 2018-7-19  | 52                 | -1.15            | 48.6       | -6.54  |
| 2018-8-19  | 62                 | -8.06            | 54.6       | -11.94 |
| 2018-9-18  | 65                 | -2.92            | 61.7       | -5.08  |

The average error 7.39 6.34

4. Conclusion
Based on Grey theory, the settlement prediction model of soft soil subgrade is constructed. The model needs fewer data and has high operability and high prediction accuracy, which can be used to predict the settlement of soft soil subgrade. On this basis, the sinusoidal residual error of the model is corrected according to the characteristics of settlement variation. According to the existing analysis rules, the characteristics of residual distribution in the future are predicted. Thus, the purpose of improving the prediction accuracy of the traditional GM (1,1) model is achieved. When the residual is sinusoidally corrected, the appropriate period should be selected according to the variation characteristics of residual sequence and the engineering characteristics of soft soil subgrade. Thus, the final prediction results are more reliable and accurate. From the test results, it is feasible to adopt the sinusoidal residual correction method.

References
[1] Chen R, Cheng W, Jia R, et al. (2016) Numerical study of high speed railway subgrade settlement induced by underwater lowering. Procedia Engineering, 143:1350-1357.
[2] Biwen Z. (2013) Analysis and control of high-speed railway subgrade settlement caused by shallow-shield tunnel construction. Modern Tunnelling Technology, 50(2): 109-113,126.
[3] Yu H, Wang Y, Chao Z, et al. (2017) Study on subgrade settlement characteristics after widening project of highway built on weak foundation. Arabian Journal for Science & Engineering, 42(1): 1-10.
[4] Gao B L, Wang J C, Yong L I. (2015) Research on the salt soluble disasters of saline lake subgrade along the qinghai-tibet railway in Chaerhan salt lake region. Journal of Railway Engineering Society, 32(5): 6-11, 17.
[5] Jin S J, Zhang D S, Shu Z, et al. (2011) Grey model theory used in prediction of subgrade settlement. Applied Mechanics and Materials, 105-107: 1576-1579.
[6] Tang J, Liu J, Jiang J. (2014) Fill subgrade settlement analysis based on prediction model of non-isometric. Journal of Central South University, 45(6):2054-2061.
[7] Liu J W, Zhong Z X. (2015) Prediction of expressway subgrade settlement based on an improved gray prediction model. Journal of Railway Science and Engineering, 2015.

[8] Liu S F, Xie N M. (2014) Gray system theory and its application (7th edition). Science Press, Nanjing.

[9] Bu P, Li C K, Liao M G. (2016) The application of the improved grey prediction model on buildings deformation monitoring. Bulletin of Surveying and Mapping, 12: 60-63.

[10] Lv Y J, Zhou S Q. (2018) Applied research on the grey system GM(1,1) model in settlement forecast of soft soil foundation. Transportation Science & Technology, 2: 5-7.

[11] Urzua A, Ladd C C, Christian J T. (2016) New approach to analysis of consolidation data at early times. Journal of Geotechnical and Geoenvironmental Engineering, 142(10).

[12] Zhang Y P, Yu Y N, Zhang T Q, et al. (2002) Grey theory and Asaoka method in settlement prediction [J]. System Engineering Theory & Practice, 22(9): 141-144.