Challenges for String Gas Cosmology

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Abstract

In spite of the phenomenological successes of the inflationary universe scenario, the current realizations of inflation making use of scalar fields lead to serious conceptual problems which are reviewed in this lecture. String theory may provide an avenue towards addressing these problems. One particular approach to combining string theory and cosmology is String Gas Cosmology. The basic principles of this approach are summarized.

1. Introduction

The current paradigm of early universe cosmology is the inflationary universe scenario (1, 2) (see also (3, 4) for earlier ideas). Most implementations of this scenario are based on the existence of a slowly rolling scalar field whose energy-momentum tensor is dominated by the contribution of the field potential energy which drives a period of accelerated (and in fact in most models almost exponential) expansion. In spite of the impressive phenomenological successes of this paradigm in predicting the spectrum of density perturbations and the angular power spectrum of cosmic microwave background (CMB) anisotropies, the scalar field-driven inflationary universe scenario suffers from some important conceptual problems (5, 6). Addressing these problems is one of the goals of superstring cosmology.

One of the conceptual problems of scalar field-driven inflation is the amplitude problem, namely the fact that in the simplest realizations of the model, a hierarchy of scales needs to be present in order to be able to obtain the observed small amplitude of the primordial anisotropies (see e.g. (7) for a fairly general analysis of this problem).

A more serious problem is the trans-Planckian problem (5). As can be seen from the space-time diagram of Figure 1, provided that the period of inflation lasted sufficiently long (for GUT scale inflation the number is about 70 e-foldings), then all scales inside of the Hubble radius today started out with a physical wavelength smaller than the Planck scale at the beginning of inflation. Now, the current theory of cosmological perturbations (the theory used to calculate the spectra of density fluctuations and microwave anisotropies) is based on Einstein’s theory of General Relativity coupled to a simple semi-classical description of matter. It is clear that these building blocks of the theory are inapplicable on scales comparable and smaller than the Planck scale. Thus, the key successful prediction of inflation (the theory of the origin of fluctuations) is based on an incomplete analysis since we know that new physics must enter into a correct computation of the spectrum of cosmological perturbations. The key question is as to whether the predictions obtained using the current theory are sensitive to the specifics of the unknown theory which takes over on small scales. Toy model calculations using modified dispersion relations (8, 9) have shown that the predictions are in fact sensitive to Planck-scale physics, thus opening up the exciting possibility to test Planck-scale and string-scale physics in current observations (see (10) for a review with references to other approaches...
towards exploring this “trans-Planckian window of opportunity”.

A third problem is the singularity problem. It was known for a long time that standard Big Bang cosmology cannot be the complete story of the early universe because of the initial singularity, a singularity which is unavoidable when basing cosmology on Einstein’s field equations in the presence of a matter source obeying the weak energy conditions (see e.g. [11] for a textbook discussion). Recently, the singularity theorems have been generalized to apply to Einstein gravity coupled to scalar field matter, i.e. to scalar field-driven inflationary cosmology [12]. It is shown that in this context, a past singularity at some point in space is unavoidable. Thus we know, from the outset, that scalar field-driven inflation cannot be the ultimate theory of the very early universe.

The Achilles heel of scalar field-driven inflationary cosmology is, however, the cosmological constant problem. We know from observations that the large quantum vacuum energy of field theories does not gravitate today. However, to obtain a period of inflation one is using the part of the energy-momentum tensor of the scalar field which looks like the vacuum energy. In the absence of a convincing solution of the cosmological constant problem it is unclear whether scalar field-driven inflation is robust, i.e. whether the mechanism which renders the quantum vacuum energy gravitationally inert today will not also prevent the vacuum energy from gravitating during the period of slow-rolling of the inflaton field. Note that the approach to addressing the cosmological constant problem making use of the gravitational back-reaction of long range fluctuations (see [13] for a summary of this approach) does not prevent a long period of inflation in the early universe.

Finally, a key challenge for inflationary cosmology is to find a well-motivated candidate for the scalar field which drives inflation, the inflaton. Ever since the failure of the model of old inflation [1, 2], it is clear that physics beyond the Standard Model of particle physics must be invoked.

It is likely that string theory will provide ideas which allow us to successfully address some of the abovementioned problems of the current versions of inflationary cosmology. Foremost, since one of its goals is to resolve space-time singularities, string theory has a good chance of providing a nonsingular cosmology (and string gas cosmology, the scenario explored below, indeed has the
potential of resolving the cosmological singularity). Since string theory should describe physics on all scales, it should be possible to compute the spectrum of cosmological perturbations in a consistent way within string theory, thus opening the trans-Planckian window of opportunity. Finally, string theory contains many light scalar fields, good candidates for inflation.

In this lecture, I present a review of String Gas Cosmology (SGC), an attempt to apply ideas of superstring theory to construct a new paradigm of early universe cosmology. The main idea of SGC is to apply new symmetries (T-duality) and new degrees of freedom (string winding modes) which are central to string theory and to apply them to early universe cosmology. In the following section, I will present the basics of SGC. Next, the equations which describe the dynamics of SGC will be discussed. I will end with one section describing recent progress in the context of SGC towards stabilizing all of the moduli which describe the volume and shape of the extra spatial dimensions which critical superstring theory predicts (this topic will be reviewed in more detail in a accompanying write-up [14]), and with a section listing some outstanding challenges.

2. Challenges for String Cosmology

An immediate problem which arises when trying to connect string theory with cosmology is the dimensionality problem. Superstring theory is perturbatively consistent only in ten space-time dimensions, but we only see three large spatial dimensions. The original approach to addressing this problem is to assume that the six extra dimensions are compactified on a very small space which cannot be probed with our available energies. However, from the point of view of cosmology, it is quite unsatisfactory not to be able to understand why it is precisely three dimensions which are not compactified and why the compact dimensions are stable. Brane world cosmology [15] provides another approach to this problem: it assumes that we live on a three-dimensional brane embedded in a large nine-dimensional space. Once again, a cosmologically satisfactory theory should explain why it is likely that we will end up exactly on a three-dimensional brane (for some interesting work addressing this issue see [16, 17, 18]).

Finding a natural solution to the dimensionality problem is thus one of the key challenges for superstring cosmology. This challenge has various aspects. First, there must be a mechanism which singles out three dimensions as the number of spatial dimensions we live in. Second, the moduli fields which describe the volume and the shape of the unobserved dimensions must be stabilized (any strong time-dependence of these fields would lead to serious phenomenological constraints). This is the moduli problem for superstring cosmology. As mentioned above, solving the singularity problem is another of the main challenges. These are the three problems which string gas cosmology [19, 20, 21] explicitly addresses at the present level of development.

In order to make successful connection with late time cosmology, any approach to string cosmology must also solve the flatness problem, namely make sure that the three large spatial dimensions obtain a sufficiently high entropy (size) to explain the current universe. Finally, it must provide a mechanism to produce a nearly scale-invariant spectrum of nearly adiabatic cosmological perturbations.

Since superstring theory leads to many light scalar fields, it is possible that superstring cosmology will provide a convincing realization of inflation (see e.g. [22] for reviews of recent work attempting to obtain inflation in the context of string theory). However, it is also possible that superstring cosmology will provide an alternative to cosmological inflation, maybe along the lines of the Pre-Big-Bang [23] or Ekpyrotic [24] scenarios. The greatest challenge for these alternatives is to solve
the flatness problem (see e.g. [25]).

3. Heuristics of String Gas Cosmology

In the absence of a non-perturbative formulation of string theory, the approach to string cosmology which we have suggested, string gas cosmology [19, 20, 21] (see also [26]), is to focus on symmetries and degrees of freedom which are new to string theory (compared to point particle theories) and which will be part of a non-perturbative string theory, and to use them to develop a new cosmology. The symmetry we make use of is T-duality, and the new degrees of freedom are string winding modes.

We take all spatial directions to be toroidal, with $R$ denoting the radius of the torus. Strings have three types of states: momentum modes which represent the center of mass motion of the string, oscillatory modes which represent the fluctuations of the strings, and winding modes counting the number of times a string wraps the torus. Both oscillatory and winding states are special to strings as opposed to point particles.

The energy of an oscillatory mode is independent of $R$, momentum mode energies are quantized in units of $1/R$, i.e.

$$E_n = n \frac{1}{R},$$

and winding mode energies are quantized in units of $R$:

$$E_m = m R,$$

where both $n$ and $m$ are integers.

The T-duality symmetry is a symmetry of the spectrum of string states under the change

$$R \rightarrow \frac{1}{R}$$

in the radius of the torus (in units of the string length $l_s$). Under such a change, the energy spectrum of string states is invariant: together with the transformation (3), winding and momentum quantum numbers need to be interchanged

$$(n, m) \rightarrow (m, n).$$

The string vertex operators are consistent with this symmetry, and thus T-duality is a symmetry of perturbative string theory. Postulating that T-duality extends to non-perturbative string theory leads [27] to the need of adding D-branes to the list of fundamental objects in string theory. With this addition, T-duality is expected to be a symmetry of non-perturbative string theory. Specifically, T-duality will take a spectrum of stable Type IIA branes and map it into a corresponding spectrum of stable Type IIB branes with identical masses [28].

We choose the background to be dilaton gravity. It is crucial to include the dilaton in the Lagrangian, firstly since the dilaton arises in string perturbation theory at the same level as the graviton (when calculating to leading order in the string coupling and in $\alpha'$), and secondly because it is only the action of dilaton gravity (rather than the action of Einstein gravity) which is consistent with the T-duality symmetry. Given this background, we consider an ideal gas of matter made up of all fundamental states of string theory, in particular including string winding modes.

Any physical theory requires initial conditions. We assume that the universe starts out small and hot. For simplicity, we take space to be toroidal, with radii in all spatial directions given by the
string scale. We assume that the initial energy density is very high, with an effective temperature which is close to the Hagedorn temperature, the maximal temperature of perturbative string theory.

Based on the T-duality symmetry, it was argued [19] that the cosmology resulting from SGC will be non-singular. For example, as the background radius $R$ varies, the physical temperature $T$ will obey the symmetry

$$T(R) = T(1/R)$$

and thus remain non-singular even if $R$ decreases to zero. Similarly, the length $L$ measured by a physical observer will be consistent with the symmetry [3], hence realizing the idea of a minimal physical length [19].

Next, it was argued [19] that in order for spatial sections to become large, the winding modes need to decay. This decay, at least on a background with stable one cycles such as a torus, is only possible if two winding modes meet and annihilate. Since string world sheets have measure zero probability for intersecting in more than four space-time dimensions, winding modes can annihilate only in three spatial dimensions (see, however, the recent caveats to this conclusion based on the work of [29]). Thus, only three spatial dimensions can become large, hence explaining the observed dimensionality of space-time. As was shown later [21], adding branes to the system does not change these conclusions since at later times the strings dominate the cosmological dynamics. Note that in the three dimensions which are becoming large there is a natural mechanism of isotropization as long as some winding modes persist [30].

4. Equations for String Gas Cosmology

The equations of SGC are based on coupling an ideal gas of all string and brane modes, described by an energy density $\rho$ and pressures $p_i$ in the $i$'th spatial direction, to the background space-time of dilaton gravity. They follow from varying the action

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} e^{-2\phi} \left[ \hat{R} + 4 \partial^\mu \phi \partial_\mu \phi \right] + S_m ,$$

where $g$ is the determinant of the metric, $\hat{R}$ is the Ricci scalar, $\phi$ is the dilaton, $\kappa$ is the reduced gravitational constant in ten dimensions, and $S_m$ denotes the matter action. The metric appearing in the above action is the metric in the string frame. In the case of a homogeneous and isotropic background given by

$$ds^2 = dt^2 - a(t)^2 dx^2 ,$$

the three resulting equations (the generalization of the two Friedmann equations plus the equation for the dilaton) in the string frame are [20] (see also [31])

$$- d\dot{\lambda}^2 + \dot{\phi}^2 = e^\phi E$$

$$\ddot{\lambda} - \dot{\phi} \dot{\lambda} = \frac{1}{2} e^\phi P$$

$$\ddot{\phi} - d\dot{\lambda}^2 = \frac{1}{2} e^\phi E ,$$

where $E$ and $P$ denote the total energy and pressure, respectively, $d$ is the number of spatial dimensions, and we have introduced the logarithm of the scale factor

$$\lambda(t) = \log(a(t))$$
and the rescaled dilaton

\[ \varphi = 2\phi - d\lambda. \]  

(12)

Note that the contribution to the pressure from winding modes is negative, whereas that from the momentum modes is positive. Thus, from the second of the above equations it follows immediately that a gas of strings containing both stable winding and momentum modes will lead to the stabilization of the radius of the torus: windings prevent expansion, momenta prevent the contraction. The right hand side of the equation can be interpreted as resulting from a confining potential for the scale factor. Note that this behavior is a consequence of having used dilaton gravity rather than Einstein gravity as the background. The dilaton is evolving at the time when the radius of the torus is at the minimum of its potential.

The above background equations thus demonstrate that, in order for any spatial dimensions to be able to grow large, the winding modes circling this dimension must be able to annihilate. In the case of three spatial dimensions, the interaction of string winding modes can be described in analogy with the interaction of cosmic strings. Two winding strings with opposite orientations can intersect, producing closed loops with vanishing winding as a final state. The equations which describe the energy transfer between winding and non-winding strings are given in analogy to the case of cosmic strings (see e.g. [32, 33, 34] for reviews). First, we split the energy density in strings into the density in winding strings

\[ \rho_w(t) = \nu(t)\mu t^{-2}, \]  

(13)

where \( \mu \) is the string mass per unit length, and \( \nu(t) \) is the number of strings per Hubble volume, and into the density in string loops

\[ \rho_l(t) = g(t)e^{-3(\lambda(t) - \lambda(t_0))}, \]  

(14)

where \( g(t) \) denotes the comoving number density of loops, normalized at a reference time \( t_0 \). In terms of these variables, the equations describing the loop production from the interaction of two winding strings are [35]

\[ \frac{d\nu}{dt} = 2\nu(t^{-1} - H) - c'^2\nu^{-2}t^{-1} \]  

(15)

\[ \frac{dg}{dt} = c'^1\mu t^{-3}\nu^2e^{3(\lambda(t) - \lambda(t_0))} \]  

(16)

where \( c' \) is a constant, which is of order unity for cosmic strings but which depends on the dilaton in the case of fundamental strings [39].

The system of equations [32, 33, 34, 35, 14] was studied in [35] (see also [36]). It was verified that the presence of a large initial density of string winding modes causes any initial expansion of \( a(t) \) to come to a halt. Thereafter, \( a(t) \) will decrease. The resulting increase in the density of winding strings will lead to rapid loop production, and the number of winding strings will decrease, then allowing \( a(t) \) to start expanding again. In [35], this initial evolution of \( a(t) \) was called “loitering”. In [35], the analysis was performed using a constant value of \( c' \). Taking into account the dilaton dependence of \( c' \), one finds [39] that the annihilation mechanism and resulting liberation of the three large dimensions only works if the initial value of the dilaton is sufficiently large.

5. Progress in String Gas Cosmology

A key issue in all approaches to string cosmology is the question of moduli stabilization. The challenge is to fix the shape and volume moduli of the compact dimensions and to fix the value of the dilaton. Moduli stabilization is essential to obtain a consistent late time cosmology.
There has recently been a lot of progress on the issue of moduli stabilization in SGC, progress which will be reviewed in detail in [14]. In a first study [37], the stabilization of the radii of the extra dimensions (the “radion” degrees of freedom) was studied in the string frame. It was shown that, as long as there are an equal number of string momentum and winding modes about the compact directions, the radii are dynamically stabilized at the self-dual radius. The dilaton, however, is in general evolving in time.

For late time cosmology, it is crucial to show that the radion degrees of freedom are stabilized in the Einstein frame. Obstacles towards achieving this goal were put forward in [38, 39, 40, 41]. However, if the spectrum of string states contains modes which are massless at the self-dual radius (which is the case for Heterotic but not for Type II string theory), then these modes generate an effective potential for the radion which has a minimum with vanishing energy at the self-dual radius and thus yields radion stabilization [42, 43] (see also [44]). As shown in these references, the radion stabilization mechanism is consistent with late time cosmology (e.g. fifth force constraints). The same massless modes also yield stabilization of the shape moduli [45, 46]. The outstanding challenge in this approach is to stabilize the dilaton (for some ideas see [47]).

There has been other important work on SGC. For example, in [48] it was shown that topologically stable one cycles (such as exist for a toroidal background) are not necessary for the success of SGC in predicting that only three spatial dimensions can become large. The results generalize to certain orbifold backgrounds for which the winding strings are long-lived but not absolutely stable. On the other hand, for other corners of the hypothetical M-theory moduli space, for example 11-d supergravity, the resulting cosmology may not single out three as the number of large spatial dimensions since there are no fundamental one-dimensional objects [49] (see, however, the ideas in [50]). The brane thermodynamics during the early stages of SGC was studied in [51]. The effects of wrapped branes were studied in [52], the effects of background fluxes were studied in [53], and generalizations to Calabi-Yau backgrounds and backgrounds with non-vanishing spatial curvature were considered in [54, 55]. For other work on moduli stabilization in SGC see [56].

An important concern is the danger that string windings lead to new instabilities towards fluctuations in the size of the radion as a function of the spatial coordinates of the large three-dimensional space. In [57, 58] it was shown that - at least before dilaton stabilization - no such instabilities arise.

Finally, recently there has been interest in the possibility that the massive modes of SGC could contribute to the dark matter [59, 38] and/or dark energy [60].

6. Problems for String Gas Cosmology

As mentioned in the previous section, an important outstanding problem in SGC is to find a mechanism to fix both the dilaton and the radion at the same time. This problem is presumably a reflection in SGC of the difficulties faced in other approaches to string cosmology, for example in the Type IIB flux compactification models [61] when attempting to fix all moduli (in those models, it is the volume modulus which is difficult to stabilize).

Since SGC assumes as initial conditions that all spatial sections are of string scale, it appears that inflation of the three large dimensions is required in order to solve the entropy problem [11]. Obtaining inflation from string gas ideas has proved to be difficult (see [62, 63, 64] for some recent ideas), and a major challenge is to obtain inflation consistent with moduli stabilization. In any cosmology which proposes an alternative to inflation (and it is possible that SGC will lead to such an
alternative) the most difficult problem is, once again, to solve the entropy problem.

It would also be nice to put SGC on a firmer footing. The current considerations are rather heuristic - e.g. they rely on an artificial splitting between a classical background space-time and stringy matter, a splitting which is bound to become a bad approximation on stringy distance scales, and it would be nice to base an improved scenario on a more consistent analysis of strings in non-trivial background space-times.

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