Interference Effect in Multi-level Transport through a Quantum Dot

Hisashi Aikawa*, Kensuke Kobayashi, Akira Sano, Shingo Katsumoto, and Yasuhiro Iye

Institute for Solid State Physics, University of Tokyo, 5-1-5 Kashiwanoha, Chiba 277-8581

We present experimental results and a model to solve the problem of “in-phase Coulomb peaks” observed in transport through a quantum dot. In a marginal region between Coulomb-blockade and open-dot, we have observed Fano-type interference through two energy levels inside the dot, which manifest themselves in two overlapped Coulomb-diamond-like structures in the excitation spectrum. One of the two levels is strongly coupled to the leads and the phase of traversing electrons is locked to it. We have detected the phase change at the vertices and the centers of the larger diamonds through the sign of the Fano’s asymmetric parameters supporting the above deduction.

KEYWORDS: quantum dot, coherent transport, phase shift, Fano effect

A particle scattering experiment gives us information of the scatterer through the cross section and the phase shift. Electric conduction through a quantum dot (QD) can be viewed as scattering process and the phase shift and the transmission amplitude furnish with useful pieces of information on the electronic states in the QD. One of the ways to extract the phase shift is a double-slit experiment. Such an experiment was first carried out by Yacoby et al. using an Aharonov-Bohm (AB) ring with a QD. The AB phase shift was found to jump by $\pi$ just at every Coulomb peak due to the two-terminal configuration of the sample. The experiment adopting four-terminal configuration demonstrated Breit-Wigner-type gradual phase change by $\pi$ at each peak rather than a jump. A surprising finding, however, was that the phase additionally jumps by $\pi$ between adjacent peaks and the phase returns to the starting value after a single cycle of Coulomb oscillation. This “phase lapse” problem can be divided into two: (A) Why do the additional phase changes of $\pi$ appear between Coulomb peaks? (B) Why do they appear as “jumps” even in the four-terminal geometry? Here we call (A) the problem of “in-phase Coulomb peaks”, which we treat in this Letter. The same phenomenon has been observed as peaks with the same sign of Fano’s asymmetric parameter.

A number of theoretical models have been proposed to explain this problem. All of them seem to reproduce the main experimental features, and it is a task of further experiments to find out which of them or another mechanism is appropriate. We have experimentally proven that the in-phase Coulomb peaks appear in a quantum wire with a side-coupled QD, which rules out a certain class of models that base themselves on the specific geometry of the AB resonator.

Here we introduce a possible model based on a simple single-electron model motivated by a numerical simulation by Nakanishi et al. A quantum state inside a rectangular QD is labeled by a wave number $k_{l,n} = (\frac{2\pi}{L_x}, \frac{2\pi}{L_y})$ ($L_x > L_y$), where the indices $l$ and $n$ are positive integers. Consider the situation where two leads are attached to the QD via tunnel barriers along the $x$-direction. In the transport, the single-electron levels indexed by $(l,n)$ appear as Coulomb peaks. For a given $l$, the $(l,1)$ state among the series $(l,n)$ most strongly couples with the leads because the kinetic energy along the $x$-direction is highest, which makes the effective barrier height lowest. Hence the QD states are classified into a small number of strong coupling states (SCSs) and the others (weak coupling states, WCSs). Note that the SCSs correspond to classical trajectories that directly cross the dot while the WCSs correspond to those with many bounces along $y$.

The existence of such SCSs in more realistic dot potentials with leads is reported in numerical simulations. The strong couplings occur mainly through a connection of the inlet and the outlet by a large amplitude part of the wave function, which has a corresponding classical trajectory (a kind of “scare”). Hence the situation in the general models is essentially the same as that in the simple symmetric model besides the phase shifts as we see below.

Let us go back to the rectangular model and introduce some distortion (or disorder) expressed by a perturbation potential $V$, which causes an intermixing of the $x$- and $y$-motions. The wave function of a WCS after the mixing is expressed in the first order as

$$ \psi_j \approx \psi_j^0 + \psi_N \langle \psi_j^0 | V | \psi_N \rangle \frac{E_j^0 - E_N}{E_j^0}, $$

where $N$ is the index of SCS closest to the energy level $E_j^0$ of the unperturbed state $\psi_j^0$. The contributions from other WCSs can be ignored from the viewpoint of transport. Because $\psi_N$ has much stronger coupling with the lead states, the transport through $\psi_j$ is dominated by the second term in the right hand side of Eq. (1). This leads to a series of in-phase peaks and the phase changes only when the closest SCS takes over due to the energy denominator in Eq. (1). Note that this discussion is applicable to any dot shape if $\langle \psi_j^0 | V | \psi_N \rangle$ is finite. It should also be noted that this model only explains why there are series of in phase Coulomb peaks. The problem how the phase goes back to the initial value (i.e., problem (B)) between the peaks is out of our scope here.
Because an SCS has a large level broadening due to the strong coupling, the one closest to \( E_F \) gives “background” conduction with constant phase shift like a continuum to that through the WCS in question. In such systems (a discrete energy level plus the continuum), Fano showed that there appears a characteristic distortion in the transition probability (the Fano effect). In the case of transport through a quantum dot, the lineshape of the conductance \( G \) to the gate voltage \( V_g \) is given as,

\[
G(V_g) = A\left(\epsilon + q\right)^2/\left(\epsilon^2 + 1\right),
\]

where \( \epsilon \equiv \alpha(V_g - V_{res})/(\Gamma/2) \), \( A \) is the amplitude, \( \alpha \), gate voltage-energy conversion factor, \( V_{res} \), resonance position, \( \Gamma \), width of the resonance, and \( q \), Fano’s asymmetric parameter. The parallel conduction in the QD should thus lead to the Fano-type interference. This Fano-type distortions in transmission spectra through single quantum dots without external interference circuit have been reported, though no explanation has been given to the origin of parallel conduction. In Ref. 17, we have demonstrated that Fano effect can be utilized for the detection of phase shift variation without AB geometry. This can be easily understood by considering that the origin of the Fano distortion is the rapid variation of the phase shift by \( \pi \) around the resonance, i.e., rapid change of the interference sign from constructive to destructive (or vice versa). Hence the direction and the degree of the distortion, which is represented by Fano’s parameter \( q \), is sensitive to the variation of the phase shift. Therefore investigation of the single-dot Fano effect should be the touchstone of the above model to account for “in phase Coulomb peaks”.

In this Letter, we report systematic experiments on the single-dot Fano interference in a multi-level transport regime. We have observed clear trace of SCSs and changes of sign of the Fano parameter \( q \) in accordance with the interference. The present result provides a simple explanation for the long-standing “in phase Coulomb peaks” puzzle.

We prepared a QD from 2DEG formed at GaAs/AlGaAs hetero-structure (sheet carrier density \( 3.8 \times 10^{15} \text{ m}^{-2} \)), mobility \( 80 \text{ m}^2/\text{Vs} \) by using electron beam lithography followed by deposition of metallic gates and wet etching. The inset of Fig. 1(a) shows the gate configuration. The sample was cooled in the dilution refrigerator with a base temperature of 30 mK and was measured by standard lock-in techniques in a two-terminal setup. In order to enhance the interference effect, we chose the side gate (SG1 and SG2) voltages to keep the total conductance around \( e^2/h \) where the QD is at the border between the Coulomb blockade regime and the open-dot regime.

Figure 1(a) shows the conductance \( G \) of the QD as a function of \( V_g \). A fine oscillation is superposed on a slow background oscillation (BO). It appears as a sequence of sharp dips at the lowest temperature, which is the sign of reversed Coulomb oscillation. These dips rapidly changes to ordinary Coulomb peaks with decreasing the gate voltage.

![Fig. 1. (a) QD conductance \( G \) vs gate voltage \( V_g \) in the multi-level transport regime. Rapid oscillations that originate from the WCSs are superimposed on the slow background oscillation. The inset shows a scanning electron micrograph of the sample. The white regions are metallic gates that define a QD as illustrated by the black circle (200 nm diameter). The side gates SG1 and SG2 determine the coupling to the leads and the center gate \( G \) modifies the electrostatic potential of the QD. The lower region is etched off (not well resolved in this picture). (b) Gray-scale plot of \( G \) as a function of bias voltage \( V_{sd} \) and \( V_g \). Fine stripe-like features are Coulomb diamonds of WCS. On top of these fine structures, high conductance (white) regions show systematic change, leading to the large Coulomb-diamond-like structure (dashed lines).](image-url)
cause the effective QD size is reduced by increasing the negative gate voltage.

Figure 2 shows a magnification of a part of Fig. 1. Clear Fano distortion is observed and each dip can be fitted by Fano’s asymmetric line shape Eq. (2). This confirms that the two-level transport model is a good approximation for the present case. Figure 2(b) shows an expanded view of Fig. 1(b). Outside the borders of clear inverted Coulomb diamonds, parallel lines due to excitation to a higher level are observed around $V_g = -0.95$ V. The Fano distortion is observed only around $V_{sd} = 0$ again in accordance with the previous result.

Figure 3(a) displays the temperature dependence of the zero-bias conductance oscillation. The Fano interference are smeared out at around 500 mK. We have attributed a smearing of the line shape of a side-coupled QD to the thermal broadening and quantum decoherence. Essentially the same discussion is applicable here. In Fig. 3(b), we present the $V_{sd}$ dependence of $G$ taken at fixed $V_g$ as indicated by the arrows $\alpha$ and $\beta$ in Fig. 3(a), where the interference is constructive and destructive, respectively. In the destructive case (the lower in Fig. 3(b)), a simple resonance dip appears at zero-biases while in the constructive case (the upper in Fig. 3(b)), additional side peaks appear at low temperatures. The interference with an SCS with higher energy may be responsible for the side peaks though at present we have no concrete idea. Figure 3(d) shows the temperature dependence of the BO, the amplitude of which simply diminishes with increasing temperature as expected.

So far we have investigated how multi-level transport appears in the conductance. Now we examine the phase shifts at the Coulomb peaks. The previous works have established that information on the phase shift at a QD can be obtained from the sign of $q$ and so does the simulation. Note that the Breit-Wigner type phase shift is assumed, as was experimentally proven. Hence the problem we can address in the present two-terminal configuration is limited to (A) (“in-phase Coulomb peaks”): We are not concerned here with the phase-jump (B) due to the phase-locking. What we expect here are the following. i) An SCS dominates the perturbations over a range of WCSs, of which the Coulomb peaks are in-phase; ii) The sign of $q$ of a Coulomb peak reflects the phase, provided that the reference phase is unchanged. The turnover of the dominant SCS occurs at the valleys of the BO. In order to analyze the data in Fig. 3(d) along the above line, the sign of $q$ should be obtained for all the Coulomb dips. Unfortunately they are significantly distorted not only by the Fano effect but also by the BO itself hence if we adopt fitting of Eq. (2) to obtain $q$, the error bar crosses zero at a considerable number of dips. Here, instead, we first determined the centers of reso-
nances from the Coulomb diamonds as presented by the dotted vertical lines in Fig. 3(c) and obtained the signs of $q$’s from those of the gradients in conductance at the resonance position. This method gives correct sign of $q$ at a peak (dip) though the absolute value still has a large ambiguity.

We can see the expected behavior in Fig. 3(d), where the zero-crossing points of $q$ are indicated by the arrows. They are placed at the peaks and the valleys of the BO. Especially the changes of the sign at the peaks (labeled as A, B, and C) are clear.

The crossing points can be detected more clearly when the BO is smaller. We adjusted the coupling strength by the side-gate voltages so as to make $|q| > 1$. This adjustment makes the sign change of $q$ clearer, while it sacrifices the clarity of the diamond-like structure in the BO because the SCSs are shrunk and the modulation of $C_{\text{eff}}$ becomes weaker.

Figure 4(a) shows the Coulomb oscillation at the weaker coupling conditions (the average conductance $\sim 0.5 \, e^2/h$). The peaks show clear Fano distortion and have large enough $|q|$ for their sign to be distinguished. In Fig. 4(b), $q$ is plotted as a function of $V_g$ obtained by the fitting. The $V_g$ positions where the sign of $q$ changes are indicated by the vertical dotted and dashed lines, and they are again placed at the peaks and the valleys of the BO, respectively. In Fig. 4(c), we show a gray-scale plot of $G$ as a function of both $V_g$ and $V_{sd}$. Though larger diamonds are not so clear as in Fig. 1(c), the modulation of widths of the small Coulomb diamonds is still noticeable. In order to see the modulation clearer, we connect the edges of black regions that are distorted from the diamond shape due to the Fano effect with the white broken line. The vertical broken lines where the sign changes go through the narrowest points and broadest points of the region enclosed by the white line. This adds a further support to the above discussion.

In summary, we have observed Fano interference arising from the transmission through two energy levels inside a QD. One of the levels has stronger coupling to the leads and dominates the phase at the Coulomb peak, which was confirmed from the $V_{sd}$ dependence and the analysis of the Fano resonance. Our results support the theoretical models that consider the existence of SCSs as the origin of in-phase Coulomb peaks.

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