EVOLUTION OF STAR CLUSTERS NEAR THE GALACTIC CENTER: FULLY SELF-CONSISTENT N-BODY SIMULATIONS

M. FUHL,1,2 M. IWASAWA,2,3 Y. FUNATO,3 AND J. MAKINO2

Received 2007 August 28; accepted 2008 June 29

ABSTRACT

We have performed fully self-consistent N-body simulations of star clusters near the Galactic center (GC). Such simulations have not been previously performed, because it is difficult to generate fast and accurate simulations of these systems using conventional methods. We used the Bridge code, which integrates the parent galaxy using a tree algorithm and the star cluster using a fourth-order Hermite scheme with individual time steps. The interaction between the parent galaxy and the star cluster is calculated with a tree algorithm. Therefore, the Bridge code can handle both the orbital and internal evolution of star clusters correctly at the same time. We here investigate the evolution of star clusters using the Bridge code, and compare the results with previous studies. We find that (1) the inspiral timescale of the star clusters is shorter than that obtained with “traditional” simulations, in which the orbital evolution of star clusters is calculated analytically using the dynamical friction formula, and (2) core collapse of the star cluster increases the core density and helps the cluster to survive. The initial conditions of star clusters are not so severe as previously suggested.

Subject headings: galaxies: star clusters — Galaxy: center — Galaxy: kinematics and dynamics — methods: numerical — stellar dynamics

1. INTRODUCTION

A few dozen very young and massive stars have been found in the central parsec of the Galaxy (Krabbe et al. 1995; Paumard et al. 2001, 2006). These stars are a few million years old (Paumard et al. 2001; Ghez et al. 2003) and lie on a disk (Lu et al. 2006) or two disks (Paumard et al. 2006). The disks rotate around the central black hole (BH) and are at large angles to each other. One disk rotates clockwise in projection, the other counterclockwise (Paumard et al. 2006). These disks are coeval to within 1 Myr. The orbits of the stars on the clockwise-rotating disk are circular (Paumard et al. 2006) or eccentric, with lower limit eccentricities of 0.0–0.8 (Lu et al. 2006), while those on the counterclockwise disk have high eccentricities, at around 0.8 (Paumard et al. 2006). In the central parsec, in situ formation of these stars seems problematic, because of the strong tidal field of the central BH. To overcome this difficulty, two possibilities have been suggested: (1) in situ star formation in a massive accretion disk, or (2) inspiraling young star clusters.

The accretion disk scenario was proposed by Levin & Beloborodov (2003). A dense gaseous disk is formed from a molecular cloud, which somehow fell into the neighborhood of the central BH. If the disk is sufficiently massive, it can become gravitationally unstable, leading to fragmentation and formation of stars. However, this scenario is problematic. Observations have shown that the two disks can be at large angles to each other, and that stars on the disks formed at almost the same time. In this scenario, the two disks must have existed simultaneously within 1 Myr. Moreover, it is difficult to make stars with eccentric orbits from accretion disks (Nayakshin et al. 2007).

The star cluster inspiral scenario was proposed by Gerhard (2001). In this scenario, a star cluster forms at a distance of tens of parsecs from the GC, and spirals into the GC through dynamical friction. This scenario is supported by the observational fact that two young dense star clusters, the Arches and Quintuplet clusters, are observed at distances of ~30 pc from the GC (Figer 2004). Although this scenario can explain two stellar disks without difficulty, numerical simulations have shown that it would take too long a time for the star cluster to inspiral to the central parsec unless it is very massive or its initial position is very near to the GC (Portegies Zwart et al. 2003, hereafter PZ03; Gürkan & Rasio 2005).

In PZ03 and Gürkan & Rasio (2005), the orbit of the cluster within its parent galaxy was calculated using the dynamical friction formula (Chandrasekhar 1943). However, this approach may overestimate the inspiral timescale. In Fujii et al. (2006), we performed fully self-consistent N-body simulations of a satellite galaxy within its parent galaxy and found that the orbital decay of the satellite is much faster than calculated analytically from the dynamical friction formula. This difference was caused by particles escaping from the satellite, via two mechanisms. First, the direct gravitational force from escaped particles works as effective drag force to the satellite. Second, escaped particles remain close to the body of the satellite and enhance the dynamical friction. These effects should also work in the case of star clusters. Therefore, a fully self-consistent N-body simulation is necessary to obtain correct results for the orbital evolution of star clusters.

Kim & Morris (2003, hereafter KM03) performed self-consistent N-body simulations of star clusters near the GC. Their results showed that if the initial central density of a star cluster is initially very high (~106 M⊙ pc−3), the cluster can deliver stars to the central parsec of the Galaxy. In these simulations, however, the internal evolution of the star clusters was neglected. The stars in their model of star clusters have an equal mass and a large softening length of 0.025 pc. Such star clusters experience neither mass segregation nor core collapse. However, if core collapse occurs, the core density of a star cluster will increase, and an initial high density for the

1 Department of Astronomy, Graduate School of Science, The University of Tokyo, 7-3-1 Hongo, Bunkyo, Tokyo 113-0033, Japan; fujii@cfca.jp.
2 Division of Theoretical Astronomy, National Astronomical Observatory of Japan, 2-21-1 Osawa, Mitaka, Tokyo, 181-8588, Japan; makino@cfca.jp.
3 Department of General System Studies, College of Arts and Sciences, The University of Tokyo, 3-8-1 Konaba, Meguro, Tokyo 153-8902, Japan; iwasawa@margaux.astron.s.u-tokyo.ac.jp, funato@artcompsci.org.
core will not be necessary. Thus, it is also important to solve the internal evolution correctly.

Such a fully self-consistent N-body simulation has been impossible with conventional numerical methods. While star clusters need a very accurate scheme, such as the combination of a fourth-order Hermite scheme and a direct force calculation, galaxies contain too many particles to use a direct force calculation. To solve this problem, we have developed a new tree-direct hybrid scheme, the “Bridge” scheme (Fujii et al. 2007). The Bridge scheme enables us to perform fully self-consistent N-body simulations of star clusters within their parent galaxies in a realistic time (less than 2 days with a single GRAPE-6 board).

We performed fully self-consistent N-body simulations of evolution of star clusters within their parent galaxies using the Bridge scheme. We also ran “traditional” N-body simulations, in which the orbital decay of the star cluster is calculated using the dynamical friction formula, for comparison. We found that the inspiral timescale of the star cluster is shorter than that obtained in previous studies. In addition, if the initial orbit of the star cluster is eccentric, the timescale of inspiral is much shorter than that for a cluster in the circular orbit of the same apocenter distance.

The eccentricities of the stars that escaped from the star cluster are distributed around the eccentricity of the star cluster. Thus, if a star cluster is initially in an eccentric orbit, it naturally explains the rather high eccentricities of the stars in the observed “disks.” In addition, because of the mass segregation effect, very massive stars (>10\(^7\) \(M_\odot\)) remain in the cluster and are brought very close to GC. Thus, we conclude that the timescale problem with the cluster inspiral scenario was partly the result of the treatment of the dynamical friction, and partly due to the limited assumption of a circular orbit, but that in fact it is not difficult to make central stellar disks from inspiraled star clusters.

We describe the simulation method and initial conditions in §2. In §3 we show the results of simulations. Section 4 presents a summary and discussions.

2. NUMERICAL SIMULATION

2.1. Models

We adopted a King model with \(W_0 = 3\) for the model of a star cluster. Its core radius, \(r_c\), half-mass radius, \(r_h\), and tidal radius, \(r_t\), are 0.087 pc, 0.13 pc, and 0.47 pc, respectively. It consists of 65536 stars, and we assigned each star a mass randomly drawn from a Salpeter (1955) initial mass function between 0.3 and 100 \(M_\odot\), irrespective of position. The total mass of the star cluster, \(M_{SC}\), is 7.9 \(\times 10^4\) \(M_\odot\). This model imitates the Arches cluster (Nagata et al. 1995), whose mass and velocity dispersion are 7 \(\times 10^4\) \(M_\odot\) within 0.23 pc and 22 km s\(^{-1}\), respectively (Figer et al. 2002). The Arches cluster is located at \(\sim 30\) pc from the GC. We, however, placed our cluster much closer to the GC, to compare our result with those of PZ03. In Table 1 we summarize the model parameters.

We used two galaxy models as a model of the central region of the Galaxy; one (galaxy 1) includes the central supermassive black hole (SMBH), and the other (galaxy 2) does not. For the galaxy model 1, we adopted a King model with nondimensional central potential of \(W_0 = 10\). We scaled the density and velocity dispersion of our model at 5 pc from the GC to the those of the Galaxy. The density at 5 pc is \(6.8 \times 10^4\) \(M_\odot\) pc\(^{-3}\), and the one-dimensional velocity dispersion is 64 km s\(^{-1}\) in our scales, while the observed density at 5 pc is \(\rho(5\text{ pc}) = 6.9 \times 10^3\) \(M_\odot\) pc\(^{-3}\) (Genzel et al. 2003) and the observed velocity dispersion is 54 km s\(^{-1}\) at \(\sim 4\) pc (Genzel et al. 2000). Figure 1 shows the enclosed masses of our model galaxy and the result of Genzel et al. (2003). In this model, the total mass of the galaxy, \(M_G\), is \(8 \times 10^7\) \(M_\odot\), and the core radius, \(r_{c,G}\), and the half-mass radius, \(r_{h,G}\), are 0.66 pc and 21 pc, respectively. We used \(2 \times 10^6\) particles to model the Galaxy. The mass of a particle of the galaxy is 40 \(M_\odot\). Since our galaxy model has the finite core size of 0.66 pc, the orbital evolution of the star cluster should be reasonably accurate as long as its distance from the GC is more than 1 pc.

The galaxy model 2 is a more realistic model than model 1. It is based on King model, and includes a central SMBH. We put a SMBH at the center of the galaxy with King model \(W_0 = 10\), and integrated it for approximately two crossing times. The central region of the galaxy evolves, and its density profile becomes cuspy. This model roughly represents the Galactic center between 0.1 and 5 pc. The enclosed mass is shown in Figure 1. The total mass and the particle mass of the galaxy are \(2.9 \times 10^7\) \(M_\odot\) and \(14\) \(M_\odot\), respectively.

We show the initial position and velocity of the star cluster are shown in Table 2. We calculated two orbits: one circular (model C) and one eccentric (model E) for galaxy model 1, and two eccentric

![Fig. 1. Enclosed mass of our model galaxy and the GC (Genzel et al. 2003).](image-url)
(models B1 and B2) for galaxy model 2. In model B1, the star cluster has almost the same eccentricity as the previous simulation, but without the SMBH. In model B2, the eccentricity of the star cluster is lower than in model B1. The orbital elements for the eccentric case were chosen so that the star cluster would survive for at least several orbits. At first, we tried the eccentric orbit with the same orbital energy as for the circular case. However, the disruption of the cluster was too fast unless the orbit is close to circular, so we made the eccentric orbit significantly wider.

2.2. Fully Self-Consistent N-Body Simulation

We performed fully self-consistent N-body simulations using the direct-tree hybrid Bridge code (Fujii et al. 2007). Only the internal motion of the star cluster is calculated by the direct scheme, with high accuracy; all other interactions are calculated by the tree algorithm. The splitting between the direct part and tree part is accomplished through splitting the Hamiltonian in a way similar to MVS (Wisdom & Holman 1991; Kinoshita et al. 1991) or RESPA (Tuckerman et al. 1990). Thus, the low-accuracy calculation of galaxy particles and their interaction with cluster particles is integrated with a time-symmetric leapfrog algorithm, resulting in small long-term error. Hence, we can treat a large-N system with embedded small-scale systems fully self-consistently and accurately.

The numerical parameters used for the time integration are summarized in Table 3. For the tree part, the Bridge code needs the same parameters as the Barnes-Hut tree code, modified for use with GRAPE hardware (Barnes & Hut 1986; Makino 2004). We used an opening angle $\theta = 0.75$ with a center-of-mass (dipole-accurate) approximation. The maximum group size for a GRAPE calculation (Makino 1991) is 8192. The step size of leapfrog integrator is $\Delta t = 1/512$ (in Heggie units; Heggie & Mattei 1986) $= 2.9 \times 10^{-4}$ Myr for galaxy 1 and $\Delta t = 1/1024$ (in Heggie units) $= 1.2 \times 10^{-4}$ Myr for galaxy 2. The potential is softened using Plummer softening. The softening length between galaxy particles and between cluster particles and galaxy particles are the same; both are $\epsilon_G = 3.9 \times 10^{-2}$ pc. For galaxy 2, we adopted a softening length between the SMBH and galaxy particles, $\epsilon_{G-BH}$, of 0.12 pc, and between the SMBH and star cluster particles, $\epsilon_{SC-BH}$, of 0.012 pc.

For the direct part, we used the fourth-order Hermite integrator with block time steps; the time step criterion is the standard Aarseth type (Makino & Aarseth 1992), with $\eta = 0.01$. We also used Plummer softening for the gravitational force between cluster particles, and we did not model physical collisions or binary formation in the calculation reported in this paper. The softening length between star cluster particles, $\epsilon_{SC}$, is $1.0 \times 10^{-2}$ pc.

We stopped the simulations at $T = 0.75-0.8$ Myr. Since we used softening and did not model the physical collisions and merging of stars, the structure of the star cluster after core collapse might not be expressed correctly. We also ignored stellar evolution, because we treat only very short time ($< 1$ Myr) in our simulations.

We used GRAPE-6 (Makino et al. 2003) for force calculation. The total energy was conserved better than $5 \times 10^{-5}$ for the circular orbit and $8 \times 10^{-5}$ for the eccentric orbit throughout the simulations.

2.3. N-Body Simulation with Artificial Dynamical Friction

To compare our result with those of previous works, for model C and E we also performed N-body simulations in which the Galaxy is modeled as a fixed potential and the dynamical friction to the Galaxy is calculated analytically. This treatment is the same as in PZ03, where the acceleration due to dynamical friction is calculated using Chandrasekhar’s dynamical friction formula (Chandrasekhar 1943; Binney & Tremaine 1987; McMillan & Portegies Zwart 2003),

$$ a_{df} = -4\pi \ln \chi G^2 \rho_G M_{SC} \frac{v_{SC}}{v_{SC}^2}, $$  \hspace{1cm} (1)

Here $M_{SC}$ and $v_{SC}$ are the mass and the center-of-mass velocity of the star cluster, $\rho_G$ is the local density of the Galaxy, $\ln \Lambda$ is the Coulomb logarithm, and

$$ \chi \equiv \frac{\text{erf}(X) - \frac{2X}{\sqrt{\pi}} \exp(-X^2)}, $$  \hspace{1cm} (2)

where $X = v_{SC}/\sqrt{2\sigma_G}$, and $\sigma_G$ is the local one-dimensional velocity dispersion of the Galaxy, assumed to be isotropic and locally Maxwellian. For $M_{SC}$, we adopted the bound mass, which is the total mass of the particles bound to the star cluster. We gave all bound stars the acceleration due to the dynamical friction from the above formula. Unbound stars were not affected by dynamical friction.

In equation (1), the Coulomb logarithm, $\ln \Lambda$, is given by

$$ \ln \Lambda = \ln (b_{\text{max}}/b_{\text{min}}), $$  \hspace{1cm} (3)

where $b_{\text{max}}$ and $b_{\text{min}}$ are the maximum and minimum impact parameters. This is often set as a constant, given by

$$ \ln \Lambda \sim \ln \left(\frac{R_{SC}}{\langle r_{SC} \rangle}\right), $$  \hspace{1cm} (4)

where $R_{SC}$ is the distance of the star cluster from the GC, and $\langle r_{SC} \rangle$ is the characteristic radius of the star cluster (roughly the half-mass radius). PZ03 adopted a constant value, $\chi \ln \Lambda = 1$. However, Hashimoto et al. (2003) found that a constant $\Lambda$ overestimates dynamical friction at pericenter, and proposed a variable $\Lambda$:

$$ \ln \Lambda = \ln \left(\frac{R_{SC}}{1.4r_{SC}}\right), $$  \hspace{1cm} (5)

where $r_{SC}$ is the size of the star cluster. We performed N-body simulations with artificial dynamical friction in two ways. The first is the same as used in PZ03 (constant $\Lambda$), and the second is as
proposed by Hashimoto et al. (2003) (variable Λ). We adopted the virial radius of the star cluster for $r_{SC}$.

3. SIMULATION RESULTS

3.1. Circular Orbits

The top panel of Figure 2 shows snapshots of the star clusters projected onto the $x$-$y$ plane. The top six panels are for the run with the circular initial orbit (model C), and the bottom six panels are for the run with the eccentric orbit (model E). Times are 0.122, 0.239, 0.356, 0.473, 0.591, and 0.708 Myr.

models C (circular orbit) and E (eccentric orbit), respectively. In model C, the star cluster is initially located at 2 pc from the GC. Due to the tidal field of the Galaxy, the star cluster becomes elongated. Particles stripped from the star cluster form tidal arms and make ringlike structures, finally forming a disklike structure.

We investigated the orbital and internal evolutions of the star cluster. Figure 3 shows the distance of the star cluster from the GC obtained by our N-body simulations. The solid curve shows
the result of the full $N$-body simulation. The dashed and dotted curves show the result of the traditional simulations in which the dynamical friction is calculated analytically from the formula with constant and variable $\Lambda$, respectively. The orbital decay in the full $N$-body simulation is faster by 30%–40% than that in other simulations. On the other hand, the evolution of the bound mass of the star cluster shown in Figure 4 is almost the same among the three. This result suggests that previous studies underestimated the inspiral timescale of star clusters. This effect is not very large, but it is not negligible.

In Figure 4, it seems that the mass loss in the very late stage (after more than 80% of the initial total mass is lost) seems to be significantly slower for the full $N$-body simulation than for the other two simulations, even though the cluster is much closer to the GC. This difference is probably due to the finite core size of our galaxy model ($r_c = 0.66$ pc) and not a real effect. In order to study the evolution after this stage, we need to use a more realistic model of the mass distribution of the central parsec of the galaxy, which includes the central massive black hole. In addition, stellar collisions and mergers within the star cluster must be modeled. We are currently working on such an extension of the Bridge code.

Figures 5 and 6 show the core radius and the core density of the star cluster. The core collapse occurred at around $T \approx 0.53$ Myr. The core collapse times are also the same among the three simulations.

### 3.2. Eccentric Orbits

The bottom panel of Figure 2 shows the snapshots of model E projected onto the x-y plane. The initial distance of the star cluster from the GC is 5 pc. The star cluster is elongated, and particles...
are stripped due to the tidal force of the Galaxy. The stripped particles form complex tidal tails.

Figure 7 shows the orbital evolution of the star cluster. The orbital decay of the full $N$-body simulation is faster than the traditional simulations, as was the case in the runs from the circular orbit. In this case also, the evolution of the bound mass is the same among the three simulations (see Fig. 8). Moreover, this result shows that variable $\Lambda$ works better than constant $\Lambda$.

Figures 9 and 10 show the core radius and the core density of the star cluster. These figures show that the core collapse occurred at around $T \simeq 0.55$ Myr. The core collapse time is almost the same as that for the case of the circular orbit, even though the bound mass at the collapse time is different by almost a factor of 2. The half-mass relaxation time at the time of collapse is 0.11 and 0.34 Myr in models C and E, respectively. Thus, if clusters are rapidly losing mass due to the tidal field, the apparent age measured by the present relaxation time could show large variations, even if the clusters started from the same initial conditions and collapsed at the same time. Furthermore, the core collapse times are the same as in the case without mass loss due to the tidal field. The core collapse time, $t_{cc}$, is estimated as $t_{cc} \simeq 0.20t_{rh}$, where $t_{rh}$ is half-mass relaxation time (Portegies Zwart & McMillan 2002). From this equation, we obtain the core collapse time of our model star cluster as 0.51 Myr. Our other results agree with this.

Figures 11–14 show the results of model B1 (dashed curves) and B2 (dotted curves), in which the Galaxy model has a central BH. Solid curves show the result of model E (with no BH; the same as the solid curve in Figures 7–10). The orbital evolution of the star cluster in models B1 and B2 is essentially similar to that in model E. However, the orbital decay in model B1 is somewhat slower than in model E. As is clear in Figure 12, in model B1, the
mass loss at the pericenter passage is much larger than in the case of model E. Therefore, core collapse did not occur, and the core density of the cluster did not increase. This is because the star cluster suffers a strong tidal force from the SMBH at the pericenter. The small mass of the star cluster slows the orbital evolution of the star cluster. Furthermore, the strong tidal field disrupts the star cluster much faster than in the case without a SMBH.

In model B2, the pericenter of the star cluster is farther out than in model B1. The evolution of the bound mass is the same as in the case without a SMBH; however, the orbital evolution is much slower than in model B1, because the pericenter of the star cluster is farther away.

The cluster model and its orbital evolution in model E and B1 are almost the same as simulation 8 of KM03. Their model of the Galaxy has a power-law density profile with a central SMBH, while our galaxy model 1 for model E has no SMBH. However, the enclosed mass of the galaxy in KM03 is very similar to ours between \(1\) and \(10\) pc. In their simulation, the star cluster was totally disrupted before \(1\) Myr, but in our simulation, the star cluster has \(30\%\) of its initial mass to the end (0.75 Myr). The difference is caused by the internal evolution of the star cluster. In our simulation, mass segregation and core collapse of the cluster made the central density much higher, which prevents the complete disruption of the cluster. In KM03, such an evolution was prevented by the numerical method they used.

On the other hand, the cluster in model B1 was disrupted by the strong tidal field of the central SMBH before core collapse could occur. At around \(1\) pc, the enclosed mass of galaxy model 2, which includes a SMBH, is twice as large as that of K04’s model. This difference in the enclosed masses is caused by the mass of the SMBH. We adopted \(3.6 \times 10^6 M_\odot\) (Eisenhauer et al. 2005), while KM03 used \(2.5 \times 10^6 M_\odot\).

Thus, the presence of a SMBH has a considerable effect on the evolution of star clusters. The main effect is that the tidal field near the GC becomes much stronger, resulting in faster mass loss from star clusters and preventing core collapse. In the models we tested, the core collapse time is about the same as the time for
complete disruption in model B1. If the core collapse time is somewhat faster, it is possible that the cluster survives and approaches the GC. In particular, if an intermediate-mass black hole (IMBH) forms within the star cluster, it would significantly help the cluster’s survival (Kim et al. 2004). We will study this aspect in more detail in forthcoming papers.

3.3. Eccentricities and Inclinations of the Escaped Stars

For some of the stars in the central parsec, projected positions and proper motions have been measured, and their orbital elements have been estimated (Paumard et al. 2006; Lu et al. 2006). Young and bright stars apparently belong to one of the two “disks” (clockwise and counterclockwise rotating disks), although the existence of the counterclockwise disk is controversial (Lu et al. 2006). The eccentricities of the stars on the counterclockwise-rotating disk are high, \( \sim 0.8 \) (Paumard et al. 2006). For the stars on the clockwise-rotating disk, Paumard et al. (2006) concluded that their orbits are circular, while Lu et al. (2006) concluded that the lower limits of their eccentricities distribute between 0.0 and 0.8. These high eccentricities are difficult to explain with the in situ formation scenario, and have been thought to be difficult to explain with the cluster inspiral scenario, since in both cases the stars would have close to circular orbits.

We investigated the eccentricities, \( e \), and inclinations, \( i \), of stars that have escaped from star cluster (i.e., unbound stars) for models C and E. Figures 15 and 16 show the eccentricities and inclinations of escaped stars and their evolution for models C and E, respectively. In these figures, the \( x \)-axis is the semimajor axis, \( a \), of the star. The eccentricity is defined as

\[
e = \frac{(r_a - r_p)}{(r_a + r_p)},
\]

where \( r_a \) and \( r_p \) are the apocenter and pericenter distances, respectively. We obtained the peri- and apocenter distances by integrating the orbits of the stars in the model potential. The inclination is defined as

\[
i = \cos^{-1}(h_z/h),
\]

where \( h \) and \( h_z \) are the angular momentum and its \( z \) component, respectively. The top panels show the eccentricities and inclinations of the stars that escaped before \( T = 0.15 \) Myr. The central gap corresponds to the semimajor axis of the star cluster, and the left and right wings are the stars on the leading and trailing arms, respectively. The distribution of the escaped stars on the \( a-e \) plains expands because of the time-varying gravitational force from the star cluster (see middle panels). The bottom panels show the distribution of all escaped stars at the last snapshot, \( T = 0.75 \) Myr. Even for a
circular cluster orbit (model C), the eccentricities of the escaped stars in innermost orbits can reach rather large values. However, for an eccentric cluster orbit (model E), the eccentricities of the escaped stars are even higher, ranging from 0.4 to 0.8, consistent with the observed values of stars in the two “disks.” Thus, if the cluster is initially in a highly eccentric orbit, the high eccentricities of observed stars are naturally explained.

3.4. The Evolution of the Mass Function

Figure 17 shows the evolution of the mass function (MF) of the stars bound to the star cluster for model C. We can see that the slope of the mass function for \( m > 10 \, M_\odot \) becomes flatter as the system evolves, while that for \( m < 10 \, M_\odot \) shows rather little change. To see this effect more clearly, in Figure 18 we plot the fraction of the mass retained in the cluster as the function of the stellar mass for model C. Stars with mass \( m > 30 \, M_\odot \) are almost perfectly retained in the cluster, and the retention rate quickly drops in the range of \( 10 \, M_\odot < m < 30 \, M_\odot \). For \( m < 10 \, M_\odot \), the retention rate becomes smaller for smaller mass, but the dependence becomes much weaker.

Figure 19 shows the evolution of the enclosed mass in \( r \), the distance from the center of the star cluster for model C. The enclosed masses are calculated for five mass ranges. Initially, all
mass ranges have the same profile. At $T = 0.15$ Myr, the most massive stars have sunk to the center, and the second most massive stars also show some central condensation. At $T = 0.45$ Myr, a large fraction of stars more massive than $10 \ M_\odot$ have sunk to the center (within radius 0.02 pc). However, the distribution of stars with mass less than $10 \ M_\odot$ has not changed significantly. Thus, stars heavier than $10 \ M_\odot$ remain in the star cluster.

We can estimate the critical mass of the star at which this change in behavior occurs, by calculating the mass of the star at which the dynamical friction just balances the two-body heating. First, we consider the energy change of the stars in the system consisting of two components with the masses $m_1$ and $m_2$. Assuming that the velocity dispersions of both components are Maxwellian, the mean energy change of a star of mass $m_1$ can be expressed as

$$\langle \frac{d}{dt} (m_1 E_1) \rangle = \frac{4}{3} \sqrt{\frac{\pi}{3}} G^2 m_1 m_2 n_2 \log \Lambda \frac{(E_1 + (E_2))^2}{(m_2 E_2 - m_1 E_1)},$$

where $\langle E_1 \rangle$ and $\langle E_2 \rangle$ are the mean specific kinetic energy of each component, $n_2$ is the number density of stars of mass $m_2$, and $\ln \Lambda$ is the Coulomb logarithm (Heggie & Hut 2003). At least in the initial model, the velocity dispersion of the stars is independent of mass, so we can set $\langle E_1 \rangle = \langle E_2 \rangle = 1/2 \sigma^2$. In this case, equation (6) can be rewritten as

$$\langle \frac{d}{dt} (m_1 E_1) \rangle = A n_2 m_1 m_2 (m_2 - m_1),$$

where $A$ is a constant. Now we consider the case of continuous mass distribution, in which the mass distribution of $m_2$ is given by

$$\frac{dn}{dm} \propto m^{-\alpha},$$

where $C$ is a constant. The number density of mass $m$, $n_2$, is expressed as

$$n_2 = C m_2^{-\alpha}.$$ 

By substituting equation (9) in equation (7) and integrating it over mass $m_2$, we can obtain the energy change of stars with mass $m_1$ as

$$\langle \frac{d}{dt} (m_1 E_1) \rangle = A' m_1 \int_{m_{\text{min}}}^{m_{\text{max}}} m_2^{-\alpha+1} (m_2 - m_1) \ dm_2$$

$$\propto m_1 \left[ m_2^{-\alpha+3} - \frac{m_2^{-\alpha+2} - m_1^{-\alpha+2}}{-\alpha+2} \right]_{m_{\text{min}}}^{m_{\text{max}}},$$

Fig. 18.—Fraction of the mass function of the bound stars to the initial mass function for model C. Times are the same as in Fig. 17.

Fig. 19.—Evolution of the enclosed mass of the bound stars for model C.
where $m_{\text{max}}$ and $m_{\text{min}}$ are the maximum and minimum mass of the MF, and $A'$ is a constant. If the right side of equation (11) is negative, the star with mass $m_1$ loses energy and sinks to the center of the star cluster. The minimum mass with a negative energy change, $m_{\text{sink}}$, is expressed as

$$m_{\text{sink}} = -\frac{\alpha + 2 m_{\text{max}} - m_{\text{min}}}{\alpha + 3 m_{\text{max}}^2 - m_{\text{min}}^2}$$

(12)

$$= f(\alpha) m_{\text{max}} \frac{1 - x^{-\alpha + 3}}{1 - x^{-\alpha + 2}}.$$  

(13)

where we defined $x \equiv m_{\text{min}}/m_{\text{max}}$ and $f(\alpha) \equiv (-\alpha + 2)(-\alpha + 3)$. With the values used for our model, $m_{\text{max}} = 100 M_{\odot}$, $m_{\text{min}} = 0.3 M_{\odot}$, and $\alpha = 2.35$, we obtain $m_{\text{sink}} = 7.9 M_{\odot}$. This value agrees well with the mass at which the power-law index of the MF breaks in Figure 18. If $x \ll 1$, we obtain

$$1 - x^{-\alpha + 3} = \begin{cases} \alpha > 3, \\ -x^{-\alpha - 2} & 2 < \alpha < 3, \\ x & \alpha < 2. \end{cases}$$

(14)

Therefore,

$$m_{\text{sink}} \approx \begin{cases} f(\alpha) m_{\text{min}} & \alpha > 3, \\ -f(\alpha)m_{\text{min}}^3 - \frac{\alpha - 3}{\alpha - 2} m_{\text{max}}^2 & 2 < \alpha < 3, \\ f(\alpha) m_{\text{max}} & \alpha < 2. \end{cases}$$

(15)

When $2 < \alpha < 3$, the value of $m_{\text{sink}}$ varies from $m_{\text{min}}$ to $m_{\text{max}}$.

The observed MF of the stars in the central parsec is much flatter than a Salpeter function (Paumard et al. 2006). The cluster inspiral model rather naturally explains this flat MF, since only the most massive stars remain bound to the cluster and are carried to the central region of the galaxy.

4. SUMMARY AND DISCUSSION

4.1. Summary

We have performed fully self-consistent $N$-body simulations of a star cluster within its parent galaxy and compare the orbital and internal evolutions of the star cluster with those obtained by “traditional” simulations, in which the orbital evolution of the star cluster is calculated from the dynamical friction formula. We confirm that the inspiral timescale of the star cluster is shorter than that obtained from the traditional simulations. Furthermore, our results show that core collapse increases the cluster’s core density and helps the cluster survive.

We performed simulations of circular and eccentric orbits of the star cluster. In previous studies, most of the simulations were from circular orbits (PZ03; Gürkan & Rasio 2005). We found, however, that eccentric orbits are more favorable to explaining the distribution of stars around the GC, for following reasons.

First, eccentric orbits are natural, if the formation of star clusters is triggered by collisions between gas clouds. Second, star clusters with eccentric orbits can approach the GC much faster than those with circular orbits (KM03). Third, Paumard et al. (2006) showed that many stars in the counterclockwise-rotating disk have high eccentricities ($e \simeq 0.8$), while the distribution of the eccentricities of the stars in the clockwise disk is very broad. Since the eccentricities of the escaped stars distribute around the eccentricity of the star cluster, the star cluster model with eccentric orbits can naturally explain the existence of high-eccentricity stars.

The power-law index of the MF of the bound stars to the star cluster breaks at around $7.9 M_{\odot}$. Stars heavier than this mass sink to the center of the star cluster due to mass segregation. Since tidal stripping removes the stars outside of the star cluster, massive stars selectively remain in the star cluster. As a result, the star cluster carries only massive stars to the GC. This star cluster scenario can reproduce the flat MF in the central parsec, without the need for a nonstandard initial mass function.

4.2. Realistic Model of the Galaxy

First, we showed that the orbits of star clusters decay faster in full $N$-body simulations than in traditional simulations, which treat the dynamical friction analytically using a King model $W_0 = 10$ as a model of the central region of the Galaxy. The model is sufficient for such comparisons, but not for a more realistic comparison between our model and the stars in the GC, because within $\sim 1$ pc the mass density of our model is much lower than that of the actual Galaxy. Next, we showed the case of a more realistic galaxy model with a central BH. The orbital evolution was similar to that in the model without a BH. For a comparison between our simulations and actual stars, however, we need more simulations in various initial conditions. We will report more detailed results of runs with central SMBHs in forthcoming papers.

4.3. Formation of an Intermediate-Mass Black Hole (IMBH)

IRS 13E consists of seven stars within a projected diameter of $\sim 0.02$ pc, and is located at $\sim 0.14$ pc in projection from the GC; these stars have very similar proper motions (Maillard et al. 2004; Paumard et al. 2006). Maillard et al. (2004) has suggested that IRS 13E is the remnant core of a star cluster that has fallen to the GC and dissolved there, and that the members of IRS 13E are bound by a central IMBH. From a proper-motion analysis, the minimum mass of the IMBH has been estimated as $1300 M_{\odot}$ (Maillard et al. 2004) to $10^6 M_{\odot}$ (Schödel et al. 2005).

In this paper we simulated the evolution of star clusters only before core collapse because of the limitation of our present code. Our code currently cannot treat the postcollapse evolution, since we use the softened potential. Collisions and mergers between stars would have occurred and an IMBH would have formed if our code were able to handle these events. The star cluster inspiral scenario might reveal the origin of IRS 13E. We are currently working to implement collisions and mergers in our code. The results will be reported in future papers.
REFERENCES

Barnes, J., & Hut, P. 1986, Nature, 324, 446
Binney, J., & Tremaine, S. 1987, Galactic Dynamics (Princeton: Princeton Univ. Press), 425
Chandrasekhar, S. 1943, ApJ, 97, 255
Eisenhauer, F., et al. 2005, ApJ, 628, 246
Figer, D. F. 2004, in ASP Conf. Ser. 322, The Formation and Evolution of Massive Young Star Clusters, ed. H. J. G. L. M. Larmers, L. J. Smith, & A. Nota (San Francisco: ASP), 49
Figer, D. F., et al. 2002, ApJ, 581, 258
Fujii, M., Funato, Y., & Makino, J. 2006, PASJ, 58, 743
Fujii, M., Iwasawa, M., Funato, Y., & Makino, J. 2007, PASJ, 59, 1095
Genzel, R., Pichon, C., Eckart, A., Gerhard, O. E., & Ott, T. 2000, MNRAS, 317, 348
Genzel, R., et al. 2003, ApJ, 594, 812
Gerhard, O. 2001, ApJ, 546, L39
Ghez, A. M., et al. 2003, ApJ, 586, L127
Gürkan, M. A., & Rasio, F. A. 2005, ApJ, 628, 236
Hashimoto, Y., Funato, Y., & Makino, J. 2003, ApJ, 582, 196
Heggie, D. C., & Hut, P. 2003, The Gravitational Million-Body Problem (Cambridge: Cambridge Univ. Press), 159
Heggie, D. C., & Mathieu, R. D. 1986, in The Use of Supercomputers in Stellar Dynamics, ed. P. Hut, and, S. McMillan (Berlin: Springer), 233
Kim, S. S., Figer, D. F., & Morris, M. 2004, ApJ, 607, L123
Kim, S. S., & Morris, M. 2003, ApJ, 597, 312 (KM03)
Kinoshita, H., Yoshida, H., & Nakai, H. 1991, Celest. Mech. Dyn. Astron., 50, 59
Krabbe, A., et al. 1995, ApJ, 447, L95
Levin, Y., & Beloborodov, M. 2003, ApJ, 590, L33
Lu, J. R., Ghez, A. M., Hornstein, S. D., Morris, M., Thompson, D. J., & Becklin, E. E. 2006, J. Phys. Conf. Ser., 54, 279
Maillard, J. P., Paumard, T., Stolovy, S. R., & Rigaut, F. 2004, A&A, 423, 155
Makino, J. 1991, PASJ, 43, 621
Makino, J., & Aarseth, S. J. 1992, PASJ, 44, 141
Makino, J., Fukushige, T., Koga, M., & Namura, K. 2003, PASJ, 55, 1163
McMillan, S. L. W., & Portegies Zwart, S. F. 2003, ApJ, 596, 314
Nagata, T., Woodward, C. E., Shure, M., & Kobayashi, N. 1995, AJ, 109, 1676
Nayakshin, S., Cuadra, J., & Springel, V. 2007, MNRAS, 379, 21
Paumard, T., Maillard, J. P., Morris, M., & Rigaut, F. 2001, A&A, 366, 466
Paumard, T., et al. 2006, ApJ, 643, 1011
Portegies Zwart, S. F., & McMillan, S. L. W. 2002, ApJ, 576, 899
Portegies Zwart, S. F., McMillan, S. L. W., & Gerhard, O. 2003, ApJ, 593, 352 (PZ03)
Salpeter, E. E. 1955, ApJ, 121, 161
Schödel, R., Eckart, A., Iserlohe, C., Genzel, R., & Ott, T. 2005, ApJ, 625, L111
Tuckerman, M. E., Martyna, G. J., & Berne, B. J. 1992, J. Chem. Phys., 97, 1990
Wisdom, J., & Holman, M. 1991, AJ, 102, 1528