Phase noise coherence of two continuous wave radio frequency signals of different frequency

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Abstract: A method is proposed for determining the correlated and uncorrelated parts of phase noise spectra (PNS) of two continuous wave radio signals of different frequencies, $\omega_1$ and $\omega_2$. The PNS of the two signals and of mixed signals are measured. The PNS are modelled as having a correlated part that is the same for both signals, except for a multiplicative factor, and uncorrelated parts, that are different for the two signals. A property of the model that the PNS of some mixing products are linear combinations of the PNS of the signals at $\omega_1$, $\omega_2$, and $\omega_1 - \omega_2$ is experimentally verified. The difference of the PNS at $\omega_1 + \omega_2$ and $\omega_1 - \omega_2$ is proportional to the correlated part of the PNS and is a part of auxiliary functions that are used for finding the multiplicative factor and the correlated, partly correlated, and uncorrelated phase noise at different offset frequencies. A conventional spectrum analyser was used to characterise two signal generators, a phase-coherent and a non-phase-coherent one. For the phase-coherent generator the phase noise of two signals was found to be correlated for offset frequencies below 10 Hz, partly correlated for 10 Hz–1 kHz and uncorrelated above 1 kHz.

1 Introduction

Phase noise is a fundamental property of oscillators and is important for the performance of wireless communication systems [1], in frequency modulated continuous wave radar systems [2], and in radio astronomy applications [3]. Phase noise can be measured by most modern spectrum analysers. The power spectral density (PSD), normalised by the signal’s power is then measured. The spectrum analyser’s phase noise should naturally be smaller than that of the tested signal [4]. There are several techniques and many modifications for measuring the phase noise of radio frequency (RF) signals of continuous wave (CW) oscillators; some recent reviews on measuring phase noise are [4–7] and on modelling is [8]. These techniques are not used for measuring the coherent and non-coherent parts of signals of different frequencies. Phase and amplitude noise can be separated using a phase detector [6] and advanced spectrum analysers have built in techniques for amplitude noise rejection.

In a number of recent applications in wireless communications multiple signal of different frequencies are fed through a system: In concurrent multiple band amplifiers, modulated signals are up-converted to different centre frequencies and concurrently amplified by the amplifier [9, 10]. In emerging technologies in satellite communication multiple modulated signals at different centre frequency are fed through the same satellite transponder [11]. In receivers for cognitive or software defined radio, a number of channels are down-converted by carriers of different frequency [12]. The effect of phase noise coherence on the system performance has – to our knowledge – not been investigated in these applications.

In digital predistortion of multiple-input multiple-output transmitters for signals of the same frequency it has been shown that coherent phase noise results in significantly smaller residual distortion than partially non-coherent phase noise [13]. In order to extend that investigation to signals of different frequency a description of partially coherent phase noise for such signals is required. It was observed in a dual two-tone test for the characterisation of memory effects in concurrent dual band amplifier that the standard deviation of the intermodulation products was sensitive to the phase noise [14]. Two different carrier frequencies were used and a phase coherent RF generator [15] resulted in smaller standard deviation than a non-phase-coherent RF generator. A detailed error analysis would require a model for the coherence of two RF carriers of different frequency.

Recently, RF synthesisers that can generate multiple channel-to-channel coherent RF signals of different frequency have emerged [15]. In a recent paper on phase noise [16] a method is presented that – similarly to the method presented here – mixes two signals of different frequencies. The signals are, however, totally uncorrelated and the residual phase noise of the device under test is determined, and not the signals’ phase noise.

In this paper, we present a technique for measuring the coherent and non-coherent parts of two CW RF signals of different frequency. The two test signals are mixed and the phase noise spectra (PNS) near the mixing frequencies are measured using a conventional spectrum analyser. The PNS around several mixing frequencies are then combined to determine the coherent and non-coherent parts of the signals. Two commercially available signal generators are analysed, a non-phase-coherent one and a phase-coherent one.

2 Theory

A general background and notation are first given in this section. We then give the expressions for PNS of mixed signals; we introduce the description of correlated and uncorrelated phase noise and show how different PNS are linear combination of others as a result of the description of correlated and uncorrelated phase noise. At the end, we introduce auxiliary functions that are used for obtaining bounds of a function in the model for correlated and uncorrelated phase noise.

2.1 General background

We analyse two CW RF signals, $y_1(t)$ and $y_2(t)$ and their mixing products. Each signal is described by a nominal angular frequency,
where \( \phi_i(t) \) is the phase noise of phase noise or phase deviation of signal \( i \), where \( i = 1 \) or \( 2 \), and \( V_i = \sqrt{2Z_0P_i} \) the amplitude, where \( P_i \) is the power and \( Z_0 \) is the characteristics impedance \[18\]. We have omitted the amplitude noise in (1); it is typically small compared to the phase noise \[4, 19\] and can in measurements be neglected by modern spectrum analysers. We make the conventional assumptions that \( \phi_i(t) \ll 1 \) and with power that is much smaller than the power of the nominal signal \[20\]. The one sided PSD of \( \psi_i(t) \) is

\[
S_{\psi_i}(\omega) = \frac{V_i^2}{2} \delta(\omega - \omega_i) \]  \tag{2}

where \( S_{\psi_i}(\omega - \omega_i) \) is the PSD of the phase noise. The PSD of the phase noise is given by \[7\]

\[
S_{\psi_i}(\omega) = E \left \{ \frac{1}{T} \lim_{T \to \infty} \left| \phi_i(t) \right|^2 \right \} \]  \tag{3}

where \( T \) is the measurement time, \( \phi_i(\omega) \) is the finite-time Fourier transform of the phase noise, and \( E \) denotes the expectation operator. Phase noise is often presented as the single sideband PSD \( L_{\text{ps}}(\omega) = S_{\psi_i}(\omega)/2 \) \[17\] in the vicinity of \( \omega_i \) with \( \omega_i = \omega - \omega_o \) being the offset frequency from \( \omega_0 \). For simplicity we use the notation \( S_{\psi_i}(\omega_o) = \left| \phi_i(\omega_o) \right|^2 \) for the PSD of the phase noise and obtain the common expression \[17\]

\[
L_{\psi_i}(\omega_o) = \frac{1}{2} \left| \phi_i(\omega_o) \right|^2. \]  \tag{4}

The PNS \( L_{\text{ps}}(\omega_o) \) is conventionally given in dBc/Hz.

### 2.2 Correlated and uncorrelated phase noises

A mixer’s output signal, \( \psi(t) \), is proportional to the product of the two input signals (see ch. 13 in \[21\]). Mixing two signals \( \psi_1(t) \) and \( \psi_2(t) \) given by (1) we obtain after using the trigonometric identity for the product of two cosines of different arguments

\[
\psi(t) = C \times \psi_1(t) \times \psi_2(t) = \frac{V_1 V_2}{2} \cos((\omega_1 + \omega_2)t + (\phi_1(t) + \phi_2(t))) + \frac{V_1 V_2}{2} \cos((\omega_1 - \omega_2)t + (\phi_1(t) - \phi_2(t))), \]  \tag{5}

where \( C \) is a constant that takes into account losses in the mixer. The signal \( \psi(t) \) in (5) has frequency components at \( \omega_1 + \omega_2 \) and \( \omega_1 - \omega_2 \). From (5), (1), and (4) we get that the PNS of the mixed signal, \( \psi(t) \), in the vicinity of \( \omega_1 + \omega_2 \) and \( \omega_1 - \omega_2 \) can be written as

\[
L_{\psi_1 \pm \psi_2}(\omega_o) = \frac{1}{2} \left| \phi_1(\omega_o) \pm \phi_2(\omega_o) \right|^2, \]  \tag{6}

where \( \omega_o = \omega - (\omega_1 \pm \omega_2) \). The contribution to the phase noise at \( \omega_1 \pm \omega_2 \) from the signal at \( \omega_1 + \omega_2 \) is neglected in (6), since the PNS decreases fast with \( \omega_o \) for RF oscillators and the frequency separation of the signals at \( \omega_1 + \omega_2 \) and \( \omega_1 - \omega_2 \) is much greater than \( \omega_o \), i.e., \( 2\omega_0 \gg \omega_o \), for the case \( \omega_1 < \omega_2 \). In (6), \( \phi_1(\omega_o) \) and \( \phi_2(\omega_o) \) can be correlated, partially correlated, or uncorrelated random variables. In the special case that \( \phi_1(\omega_o) \) and \( \phi_2(\omega_o) \) are uncorrelated random variables, (6) simplifies to

\[
\left| \phi_1(\omega_o) \right|^2 + \left| \phi_2(\omega_o) \right|^2 / 2, \]  \tag{7}

where the cross-PSD of uncorrelated random variables approaches zero \[18\].

Creating the mixing products \( \psi_1(t) \times \psi_1(t) \times \psi_2(t) \) and \( \psi_1(t) \times \psi_2(t) \times \psi_2(t) \) we obtain the PNS in the vicinity of \( 2\omega_1 \pm \omega_2 \) and \( 2\omega_2 \pm \omega_1 \), respectively,

\[
L_{\psi_1 \pm \psi_2}(\omega_o) = \frac{1}{2} \left| \phi_1(\omega_o) \pm \phi_2(\omega_o) \right|^2, \]  \tag{8a}

\[
L_{\psi_1 \pm \psi_2}(\omega_o) = \frac{1}{2} \left| \phi_1(\omega_o) \pm \phi_2(\omega_o) \right|^2, \]  \tag{8b}

where \( \omega_o = \omega - (2\omega_1 \pm \omega_2) \) and \( \omega_o = \omega - (2\omega_2 \pm \omega_1) \) in (7a) and (7b), respectively. We now define the phase noise \( \phi_1(t) \) and \( \phi_2(t) \) as composed of one uncorrelated part, \( \phi_{11}(t) \) and \( \phi_{22}(t) \), respectively, and one correlated part \( \phi_{12}(t) \). In order to be general we also assume that \( \phi_{12}(t) \) is convolved by the functions \( k_1(t) \) and \( k_2(t) \), respectively,

\[
\phi_1(t) = k_1(t) * \phi_{11}(t) + \phi_{12}(t), \]  \tag{9a}

\[
\phi_2(t) = k_2(t) * \phi_{22}(t) + \phi_{22}(t). \]  \tag{9b}

where * denotes convolution. The formulation in (8) corresponds to the standard model for partially correlated additive noise signals \[7\] except for \( k_1(t) \) and \( k_2(t) \). An ideal frequency doubler with \( \psi_1(t) \) as input and \( \psi_2(t) \) as output would, e.g., have \( k_1(t) = 1 \) and \( k_2(t) = 2 \). The convolution in (8) enables that different frequency components of \( \phi(t) \) are amplified or damped differently. Functions corresponding to \( k_1(t) \) and \( k_2(t) \) have been used in relations between random variables like the surface topography of thin film interfaces, in which case significant frequency dependence was found \[22\]. The Fourier transform of (8a) and (8b) gives

\[
\phi_1(\omega) = k_1(\omega) * \phi_{11}(\omega) + \phi_{12}(\omega), \]  \tag{10a}

\[
\phi_2(\omega) = k_2(\omega) * \phi_{22}(\omega) + \phi_{22}(\omega). \]  \tag{10b}

We set \( k_1(\omega) = 1 \) but allow \( k_2(\omega) \neq 1 \), i.e. we use \( \phi_1(\omega) \) as the reference for the correlated phase noise. The choice of reference signal is of course arbitrary. Setting both \( k_1(\omega) \neq 1 \) and \( k_2(\omega) \neq 1 \) is not meaningful. For the case \( \phi_1 = 0 \) and \( k_1 = 0 \) \( \phi_1(\omega) \) and \( \phi_2(\omega) \) are not defined. We denote \( k_2 = k \) and omit the (offset) frequency dependence below. Notice that \( k = k_1 + jk_1 \) is a complex valued function in the general case. We here refrain from making any further assumption on \( k(t) \) based on a priori knowledge of the signal generators in order to keep our method general. In Section 4, it is found that a real valued frequency independent \( k \) well describes the experimental data.

Putting (9a) and (9b) into (4) gives

\[
L_{\psi_1}(\omega_o) = \frac{1}{2} \left| \phi_{11}(\omega_o) \right|^2, \]  \tag{11a}

\[
L_{\psi_1}(\omega_o) = \frac{1}{2} \left| \phi_{22}(\omega_o) \right|^2. \]  \tag{11b}

In the same way, putting (9a) and (9b) into (6) gives

\[
L_{\psi_1 \pm \psi_2}(\omega_o) = \frac{1}{2} \left( \left| \phi_{11}(\omega_o) \right|^2 + \left| \phi_{22}(\omega_o) \right|^2 \right). \]  \tag{11c}

Equations (9a) and (9b) into (7a) and (7b), respectively, give

\[
L_{\psi_1 \pm \psi_2}(\omega_o) = \frac{1}{2} \left( \left| \phi_{11}(\omega_o) \right|^2 + 4 \left| \phi_{22}(\omega_o) \right|^2 \right). \]  \tag{12a}

\[
L_{\psi_1 \pm \psi_2}(\omega_o) = \frac{1}{2} \left( \left| \phi_{22}(\omega_o) \right|^2 + 4 \left| \phi_{22}(\omega_o) \right|^2 \right). \]  \tag{12b}

From (10a), (10b) and (11) we obtain

\[
L_{\psi_1 \pm \psi_2} = 2L_{\psi_1} + 2L_{\psi_2} - L_{\psi_1 \pm \psi_2}. \]  \tag{13}

Using (10a) and (10b) with (11) and (12a) and (12b), respectively,
we obtain
\[ \mathcal{L}_{2\omega_1 + \omega_2} = 6 \mathcal{L}_{\omega_1} + 3 \mathcal{L}_{\omega_2} - 2 \mathcal{L}_{\omega_1 - \omega_2}, \] (14a)
\[ \mathcal{L}_{2\omega_2 + \omega_1} = 3 \mathcal{L}_{\omega_1} + 6 \mathcal{L}_{\omega_2} - 2 \mathcal{L}_{\omega_1 - \omega_2}. \] (14b)

Using (11), (13), (10a) and (10b) into (12a) and (12b), respectively, give
\[ \mathcal{L}_{\omega_2 - \omega_1} = 2 \mathcal{L}_{\omega_2} - \mathcal{L}_{\omega_1} + 2 \mathcal{L}_{\omega_1 - \omega_2}, \] (15a)
\[ \mathcal{L}_{\omega_1 - \omega_2} = - \mathcal{L}_{\omega_1} + 2 \mathcal{L}_{\omega_2} + 2 \mathcal{L}_{\omega_1 - \omega_2}. \] (15b)

Thus, \( \mathcal{L}_{\omega_1 + \omega_2}, \mathcal{L}_{\omega_1 - \omega_2}, \) and \( \mathcal{L}_{2\omega \pm \omega} \) are linear combinations of \( \mathcal{L}_{\omega_1}, \mathcal{L}_{\omega_2}, \) and \( \mathcal{L}_{\omega_1 - \omega_2} \). Notice that (13)–(15b) are valid for any \( k_1 \) and \( k_2 \), i.e. we have five model parameters, but only three independent equations. To determine \( |\phi_{1,1}|^2 \), \( |\phi_{1,2}|^2 \), and \( |\phi_{2,1}|^2 \) directly from experimental data is possible only if \( k_1 \) and \( k_2 \) are known. Some information about \( k \) can, however, be obtained if we make some reasonable assumptions as we will see below.

### 2.3 Auxiliary functions

Taking the difference of the equations in (11) yields
\[ \rho_n(\omega_n \pm \omega_2) = \rho_n(\omega_n \pm \omega_1) = \frac{\mathcal{L}_{\omega_n + \omega_2} - \mathcal{L}_{\omega_n - \omega_2}}{4 \mathcal{L}_{\omega_1}} = \frac{\mathcal{k}_1 |\phi_{1,1}|^2}{|\phi_{1,1}|^2 + |\phi_{1,2}|^2}, \] (17)
for \( n = 1 \) or 2, with \( k_1 = 1 \) and \( k_2 = k \). We also define
\[ \rho_n(2\omega_n, \omega_2) = \frac{\mathcal{L}_{2\omega_n + \omega_2} - \mathcal{L}_{2\omega_n - \omega_2}}{8 \mathcal{L}_{\omega_1}}, \] (18a)
\[ \rho_n(\omega_n, 2\omega_2) = \frac{\mathcal{L}_{2\omega_2 + \omega_n} - \mathcal{L}_{2\omega_2 - \omega_n}}{8 \mathcal{L}_{\omega_1}}. \] (18b)

Combining (12a) and (12b) with (10a) and (10b) gives
\[ \rho_n(\omega_n, \omega_2) = \rho_n(2\omega_n, \omega_2) = \rho_n(\omega_n, 2\omega_2). \] (19)

Notice that \( \rho_1 = \rho_2 \) for the case of two signals with identical correlated phase noise, i.e. \( k = 1 \), and uncorrelated phase noise with the same statistical properties, i.e. \( |\phi_{1,1}|^2 = |\phi_{1,2}|^2 \). We use that \( |\phi_{1,1}|^2 \geq 0 \) in (17) for \( \rho_1 \) and obtain a bound for \( \mathcal{k} \)
\[ |k_1| \leq |\mathcal{k}_1| \leq |\rho_1|, \]
\[ \text{sign}(k_1) = \text{sign}(\rho_1). \] (20)

In the same way using that \( |\phi_{2,1}|^2 \geq 0 \) in (17) for \( \rho_2 \) gives a second bound
\[ |k_2| \leq |\mathcal{k}_2| \leq \frac{1}{|\rho_2|}, \]
\[ \text{sign}(k_2) = \text{sign}(\rho_2). \] (21)

For the case of completely correlated signals, \( |\phi_{1,1}|^2 = |\phi_{1,2}|^2 = 0 \), and \( k_1 = 0 \), one gets \( k_2 = 1/\rho_1 \). We then summarise the experiments performed to measure PNS.

### 3 Experimental

We first describe the experimental set-up and then how it was characterised. We measure the phase noise in the mixing products mentioned above. The mixers are wide band to make the set-up useful for different oscillator frequencies. Lower conversion loss could be obtained if narrow band mixers were used. The two carrier frequencies used in the experiments were \( f_1 = 1.1 \) GHz and \( f_2 = 1.6 \) GHz. These frequencies were selected to have a relatively large frequency spacing and for the available mixers to be within their operational frequency range. An outline of the experimental set-up used for creating mixing products at \( \omega_1 - \omega_2 \) and \( \omega_1 + \omega_2 \) using one mixer is shown in Fig. 1a. In Fig. 1b, it is outlined how mixing products at \( 2\omega_1 - \omega_2 \) and \( 2\omega_1 + \omega_2 \) are created using two mixers. Mixing products at \( 2\omega_1 - \omega_2 \) and \( 2\omega_1 + \omega_2 \) were created as shown in Fig. 1b, but switching generators 1 and 2.

Two different RF signal sources were investigated. The first was a Rohde & Schwarz SMW 200A vector signal generator with two separate RF channels, denoted the non-phase-coherent generator below. The second was a three channel Holzworth instrumentation H90003A RF synthesiser with its standard internal reference, designed to generate phase coherent RF signals operating at the same or different RF frequencies. It is denoted the phase-coherent generator below. For the phase-coherent generator only two of the three channels were used; for the non-phase-coherent generator both RF channels were used. Both the tested signal generators were designed for laboratory use and have low phase noise.

### 3.2 Set-up characterisation

In the presented method, signals are mixed and the PNS of the mixing products are measured. Mixing results in power losses and the measured signal are therefore of different power. Signals of low power may be affected by additive noise of the spectrum analyser. Notice in (2)–(4) that the PNS are independent of the signal power; additive noise of the spectrum analyser will affect the measured PNS at low signal powers. The PNS were measured for different signal power levels in order to find the levels at which the phase noise is the dominating noise source. In Fig. 2a, \( L_{\omega_n} \) is shown for power levels of +10 to −30 dBm for the non-phase-coherent generator at \( f_1 = 1.1 \) GHz. Below 10 kHz \( L_{\omega_n} \) does not change with the power level. The measured spectra in Fig. 2a are well above the specifications of the instrument. In Fig. 3, the phase noise versus signal power is shown at given offset frequencies. For offset frequencies up to 1 kHz the phase noise is independent of the power, i.e. a zero slope. At these offset frequencies the additive noise is negligible at the used power levels; the experimental data are due to the generator’s phase noise. For higher offset frequencies, the spectra have a slope of −1 versus power for low powers and become power independent at higher powers. For an offset frequency of 10 MHz the experimental data are dominated by phase noise down to −40 dBm, but corrupted by additive noise for lower power levels. For offset frequencies of 1 MHz, 100 kHz, and 10 kHz, the corresponding powers are −5, −20 and −20 dBm, respectively.

Mixers use active components and can add phase noise [23]. In order to test if the used mixers add phase noise we mixed a signal with itself and created a mixing product (second harmonic) at \( \omega = 2\omega_1 \). Mixing the signal with itself we get \( \phi_1 = \phi_2 \) in (6) and, hence
\[ \mathcal{L}_{2\omega_1} = 4 \mathcal{L}_{\omega_1}. \] (22)
which predicts a $4 \times (6 \, \text{dB})$ increase in the PNS at the second harmonic frequency, if there are no additional phase noise sources. In Fig. 2a, the noise spectra at $f_1$ and $2f_1$ for $f_1 = 1.1$ GHz are shown. The power levels of the respective signals are given in the figure. It was not possible to measure the signal at $2f_1$ at a higher power level because of the mixer's specified power levels. In Fig. 2b, the ratio $L_{2f_1}/L_{2f_2}$ is shown. The ratio is $-6 \, \text{dB}$ in agreement with (22), up to an offset frequency of 10 kHz, at which the intrinsic noise of the spectrum analyser becomes significant, as shown in Figs. 2a and 3. The mixer does not add any significant phase noise up to an offset frequency of 10 kHz. Above 10 kHz, we cannot with safety say that the mixers' noise contribution is insignificant. The increase and decrease of the ratio vs. offset frequency in Fig. 2b are caused by different additive noise power at $f_1 = 1.1$ GHz and $2f_1 = 2.2$ GHz.

In order not to introduce different delays between the signals $Y_1$ and $Y_2$ cables of the same length were used in the connections to the mixers. If $Y_2$ is delayed by a time $T_D$ relative $Y_1$, (11) becomes

$$L_{2f_1} = \frac{1}{2} \left( 1 \pm \exp \left( -j\omega_T T_D \right) \right) \left( \phi_{1_1}^2 + \phi_{2_1}^2 \right).$$

(23)

A difference in cable length, $D$, gives $T_D = D/v_p$, with $v_p$ being the phase velocity, which is $v_p = c_0/\sqrt{\varepsilon \mu}$ (see ch. 2 in [21]). We use $c_0 = 3 \times 10^8 \, \text{m/s}$, $\mu = 1$, and $\varepsilon = 2.3$ (RG223 coaxial cables with polyethylene were used), and estimate that $D = 1 \, \text{mm}$. We get that $\omega_T T_D \ll 1^\circ$ even for an offset frequency of 1 MHz. We therefore neglect this effect in the analysis below.
It was shown above that \( L_{v_1 + v_2} \) and \( L_{2v_1 + v_2} \), \( v_1 + v_2 \), and \( L_{2v_1 + 2v_2} \) are linear combinations of \( L_{v_1}, L_{v_1}, \) and \( L_{v_1} \). In Figs. 6a and b the experimentally determined PNS \( L_{v_1 + v_2}, L_{2v_1 + v_2}, \) and \( L_{2v_1 + 2v_2} \) are shown together with the same PNS as determined from the linear combinations of \( L_{v_1}, L_{v_1}, \) and \( L_{v_1} \) in (13)–(15b). The ratios of the directly measured PNS and those from linear combinations are shown in Figs. 6a and d. The directly measured PNS and those obtained from linear combinations of others are in excellent agreement up to offset frequencies of 10–100 kHz. The deviations occur at the same offset frequencies where the PNS were affected by additive noise, as shown in Figs. 2 and 3, above. We conclude that the phase noise model for signals of different frequencies in (9a) and (9b) well describes the signals of the two generators.

3.3 Measurements

The PNS of the signals at \( f_1 = 1.1 \) GHz and \( f_2 = 1.6 \) GHz were measured, as well as the PNS of the mixing products for both the RF generators. In Table 1, the nominal frequency and signal power levels, as measured by the spectrum analyser, of the measured signals are given. The frequencies and power levels were the same for both tested signal generators. Notice that the power levels of the signals vary by almost 30 dB. The main losses are due to the mixers. The measured spectra for the different mixing products have different levels of additive noise, as shown in Fig. 2.

4 Results, analysis, and discussion

We present and analyse the results in three stages. In the first stage, we present and analyse the experimentally determined PNS and verify the relations that some PNS are linear combinations of others. In the second stage, we analyse the auxiliary functions, \( \rho_1 \) and \( \rho_2 \), for the experimental data. In the third stage, we determine the correlated and uncorrelated phase noise of the two RF generators’ signals. We then give a brief discussion of the method.

4.1 Phase noise spectra

In Fig. 4, the PNS in Table 1 are shown for the non-phase-coherent generator. The PNS in Table 1 for the phase-coherent generator are shown in Fig. 5. In the \( L_{v_1} \) and \( L_{v_2} \) spectra in Figs. 4 and 5, the additive noise is negligible and the phase noise is dominating up to \( \sim 1 \) MHz, as seen if we compare the spectra with Fig. 2a for the power levels given in Table 1. The PNS \( L_{v_1 + v_2} \) are phase noise dominated up to \( \sim 0.1 \) MHz; at higher offset frequencies the additive noise dominates the PNS, as seen in Fig. 2 and the power levels in Table 1. The spectra for \( L_{v_1} \) and \( L_{v_2} \) in Fig. 5 are in good agreement with the manufacturers datasheet. For \( L_{v_1 + v_2} \) the PNS in Figs. 4 and 5 are due to phase noise up to \( \sim 50 \) kHz (\( \sim 10 \) kHz) and at higher offset frequencies the additive noise is dominating.

For the PNS \( L_{v_1} \) and \( L_{v_2} \) in Fig. 4 the slope is \( -10 \) dB/decade for 1 to 100 Hz. It increases to \( -20 \) dB/decade for 100 Hz to 100 kHz. Such a frequency dependence is indicative of a phase locked loop source (here the non-phase-coherent generator), where the phase noise is attenuated below a certain offset frequency (the loop bandwidth) [1, 4]. The \( L_{v_1} \) and \( L_{v_2} \) PNS for the phase-coherent generator in Fig. 5 have the typical offset frequency dependence of a free running oscillator. It decreases with a slope of \( -30 \) dB/decade for small offset frequencies below 100 Hz. The slope decreases with increasing offset and reaches a noise floor at an offset of 10 kHz [1, 4].

In Fig. 4 one sees that for the non-phase-coherent signal generator \( L_{v_1 + v_2} \sim L_{v_1 + v_2} \). It indicates that its two signals are uncorrelated, since it requires that \( \phi_1^2 \approx 0 \) in (11). In Fig. 5 the phase-coherent generator has PNS \( L_{v_1 + v_2} \sim L_{v_1 + v_2} \) up to \( \sim 1 \) kHz, which is in agreement with (11) if \( \phi_1^2 \) and \( \phi_2^2 \) are negligible and \( |1 + k|^2 \approx |1 - k|^2 \). The auxiliary functions are used to analyse the properties of \( k \) further below. Furthermore, Fig. 4 indicates that \( \phi \) is negligible, since putting \( \phi_1 = 0 \) in (12a) and (12b) gives \( L_{v_1 + v_2} \sim L_{v_1 + v_2} \), which is seen in the spectra in Fig. 4. In the same way the spectra in Fig. 5 indicate that \( \phi \) is negligible above \( \sim 1 \) kHz.

Table 1 Frequency and power of measured PNS signals

| PNS | Frequency, GHz | Power, dBm |
|-----|----------------|------------|
| \( L_{v_1} \) | 1.1/1.6 | \( -1.0/1.5 \) |
| \( L_{v_1 - v_2} \) | 0.5/2.7 | \( -8.1/9.5 \) |
| \( L_{2v_1 - v_2} \) | 0.8/3.8 | \( -20.5/23.5 \) |
| \( L_{3v_1 - 2v_2} \) | 2/1.4 | \( -24.9/27.0 \) |

4.2 Auxiliary functions

The function \( \rho_2 \) as obtained from (17), (18a), and (18b), respectively, is shown in Fig. 7 for the two analysed RF generators. Also shown is \( 1/\rho_2 \). Equation (17) shows that for signals with completely uncorrelated phase noise \( \rho = 0 \) and \( 1/\rho \rightarrow \infty \). Hence, Fig. 7a shows that the two signals of the non-phase-coherent generator are predominantly uncorrelated at all frequencies.

For the phase-coherent generator’s signals in Fig. 7b \( \rho_2 \simeq 1/\rho_2 \simeq 1.4 \) for offset frequencies below 10 Hz. Putting \( \phi_1 > \phi_1^2 \) and \( \phi_1 > \phi_1^2 \) in (17) gives \( \rho_1 \simeq k_1 \) and \( 1/\rho_2 \simeq k_2 \), respectively. If \( k_1 \gg k_2 \), we set that \( \rho_1 \simeq 1/\rho_2 \). Thus, \( k_1 \simeq k_2 \simeq 0 \) describe the results in Fig. 7b for offset frequencies below 10 Hz. In the range 10–100 Hz \( \rho_1 \) and \( 1/\rho_2 \) are diverging which indicates that the influence from the uncorrelated phase noise increases. In this region we cannot use \( \rho_1 \) and \( 1/\rho_2 \) for quantitative determination of \( k \), but \( k \) must be within the bounds given in (20) and (21). Above 1 kHz, \( \rho_1 \) and \( 1/\rho_2 \) are at the same levels as for the signals of the non-phase-coherent generator. Here the correlated phase noise is negligible.

4.3 Correlated and uncorrelated phase noise

In this section we determine the correlated and uncorrelated parts of the phase noise. We use the experimentally determined PNS and the value for \( k \) obtained from the analysis of \( \rho_1 \) and \( \rho_2 \). For the non-phase-coherent generator we set \( k = 1 \), i.e., we include the correlated phase noise term, \( \phi_1^2 \). Using the a priori knowledge of the signal generator one could have set \( \phi_1^2 = 0 \). For the phase-coherent generator we set \( k = 1.4 \) for all offset frequencies, i.e., we extrapolate the value determined in the range 1–10 Hz to higher offset frequencies. We estimate the correlated, \( \phi_1^2 \), and uncorrelated, \( \phi_2^2 \), components by minimising the least square error

\[
\| \begin{bmatrix} 2L_{v_1} \\ 2L_{v_2} \\ \vdots \end{bmatrix} - \begin{bmatrix} 1 \\ 1 - k^2 \end{bmatrix} \begin{bmatrix} \phi_1^2 \\ \phi_2^2 \end{bmatrix} \|_2^2 = 0,
\]

where \( \| \cdot \|_2 \) denotes the norm 2, subject to all \( \phi_1^2 \geq 0 \). This is a convex problem, the solution to which is well established [24] and it can be obtained using reliable numerical solvers [25].

In Fig. 8 all \( \phi_1^2, \phi_2^2, \) and \( \phi_1^2 \) are shown as determined from (24) for the two generators, with \( k = 1 \) and \( k = 1.4 \), respectively. For the non-phase-coherent generator’s signals, Fig. 8a, \( \phi_1^2, \phi_2^2 \), and \( \phi_1^2 \) are negligible for all offset frequencies, as expected. Putting these inequalities into (10a) and (10b) we get that \( L_{v_1} \simeq 0.5L_{v_1} \) and \( L_{v_2} \simeq 0.5L_{v_2} \). Fig. 8b shows that for the phase-generator’s signals at small offset frequencies (\( <10 \) Hz), the correlated phase noise \( \phi_1^2 \) is larger than the uncorrelated, \( \phi_2^2 \), and \( \phi_1^2 \) and \( \phi_2^2 \) are in agreement with (10a) and (10b) for the case that \( \phi_1^2 > |\phi_2^2| \) and \( |\phi_1^2| > |\phi_2^2| \). The
correlated phase noise, $|\phi_1|^2$, decreases faster with increasing offset frequency, than $L_{u1}$ and $L_{v1}$ in Fig. 5, at offset frequencies higher than 100 Hz, $|\phi_1|^2 < |\phi_2|^2$ and $|\phi_3|^2 < |\phi_4|^2$, and $L_{u2}$ and $L_{v2}$ are dominated by uncorrelated phase noise. In Fig. 8b, most power of the PNS is at small offset frequencies, where $|\phi_1|^2$ is dominating; hence, the signals can be said to be coherent.

For the signals of the phase-coherent generator, we see in Fig. 8b that $|\phi_1|^2$, $|\phi_3|^2$, and $|\phi_2|^2$ can be modelled by the empirical equations

$$|\phi_1|^2 = K_c f_{m1}^{-3},$$
$$|\phi_3|^2 = K_{u2} f_{m1}^{-2},$$

where $n = 1$ or 2 and $K_c$ and $K_{u2}$ are empirical constants, which is typical for a free running oscillator [1]. In Fig. 8b, the straight lines correspond to $K_c = 1.26 \times 10^{-7}$, $K_{u2} = 1.26 \times 10^{-7}$, and $K_{v2} = 6.2 \times 10^{-7}$. We can now combine (25a), (25b), and (17) and get model expressions for $p_1$ and $1/p_2$. In Fig. 7b, model spectra for $p_1$ and $1/p_2$ are plotted. These spectra follow well the experimental data, which corroborates that $k_r = 1.4$ could be extrapolated to the region 10–1000 Hz.

The relative contributions to $L_{u1}$ and $L_{v1}$ from $|\phi_1|^2$, $|\phi_3|^2$, and $|\phi_2|^2$, respectively, are given by [cf. (9a), (9b), (10a), and (10b)]

$$R_{ua}^{(1)} = \left(\frac{k_r}{4}\right) \frac{|\phi_1|^2}{(2 L_{u1})},$$
$$R_{vu}^{(1)} = \frac{|\phi_3|^2}{(2 L_{v1})},$$

where $n = 1$ or 2, and are shown in Fig. 9 for the two generators. For the non-phase-coherent generator’s signals the ratios $R_{ua}^{(1)}$ and $R_{vu}^{(1)}$ are close to unity for all frequencies and $R_{ua}^{(3)}$ and $R_{vu}^{(3)}$ are close to zero. For the phase-coherent generator’s signals, the ratios $R_{ua}^{(1)}$ and $R_{vu}^{(1)}$ are close to unity for offset frequencies below 50 Hz. There is then a gradual decrease to zero at ~1 kHz. We use the data of the modelled slopes in Fig. 8b in (25a) and (25b) to obtain model data for the ratios in Fig. 9. The model data are in good agreement with the experimental data which lends credence to the model in (25a) and (25b).

### 4.4 Discussion

The assumption that $2\omega_1 \gg \omega_m$ in the derivation of (6) is valid for RF oscillators. The proposed method would not be suitable for measuring the phase noise coherence of one RF oscillator and one low frequency oscillator. The function $k(\omega_m)$ could not be determined except for upper and lower bounds in offset frequency ranges with partially correlated phase noise. However, using the functions $\rho_n$ and making the reasonable assumption that $k$ is limited and does not have strong frequency dependence, a model for the phase noise coherence could be obtained also in these ranges. In our case the tested generators were ‘black boxes’. If one knows the design of a tested generator $k$ may be known a priori. We notice that for the phase-coherent generator we found that $k \approx k_r \approx 1.4$, which is close to the ratio of the signals’ frequencies, $f_2/f_1 \approx 1.45$.

An alternative to the proposed method would be to measure the signals directly with a sample rate of at least twice the highest carrier frequency.
Fig. 6  Experimental data for the PNS $L_{v_1 + v_2}, L_{v_1 + v_2}$, and $L_{v_1 + v_2}$, and the corresponding PNS from linear combinations of $L_{v_1}, L_{v_2},$ and $L_{v_1 - v_2}$ (upper figures). For clarity, the curves are shifted upwards by 0, 20, 40, 60, and 80 dB, respectively. Ratios of the measured PNS and PNS from linear combinations (lower figures); the markers correspond to the same PNS in the upper and lower figures

a Measured PNS and PNS from linear combinations for the non-phase-coherent generator
b Measured PNS and PNS from linear combinations for the phase-coherent generator
c Ratios of measured PNS and PNS from linear combinations from the non-phase-coherent generator
d Ratios of measured PNS and PNS from linear combinations from the phase-coherent generator

Fig. 7  Auxiliary functions, $p_1$ and $1/p_2$ against offset frequency

a Non-phase-coherent generator
b Phase-coherent generator; the solid lines are from the model in (25)
frequency. In the case above that would have required a sample frequency of 3.2 GHz. One could then calculate the FFTs and determine the cross-PSD relative to the offset frequency. The two channels would have to be measured with synchronous sampling and with quantisation noise below the phase noise level. That would be challenging even with state-of-the-art digital oscilloscopes. Another approach would be to down-convert the signals to baseband or low frequencies by mixing with auxiliary RF signals at frequencies close to those of the analysed signals. The down-converted signals should be digitised with synchronous sampling. FFT cross-correlation

Fig. 8 Estimated correlated, $|\phi_v(s_m)|^2$, and uncorrelated, $|\phi_{1,v}(s_m)|^2$ and $|\phi_{2,v}(s_m)|^2$, phase noise components against offset frequency

a Non phase-coherent generator where $k=1$ was used in (24)
b Phase-coherent generator, where $k=1.4$ was used in (24). Slopes for the empirical model in (25) are shown

Fig. 9 Relative contributions of the correlated and uncorrelated parts to the PNS $\mathcal{L}_{\nu_2}$ and $\mathcal{L}_{\nu_1}$

a Non phase-coherent generator where $k=1$ was used in (24)
b Phase-coherent generator, where $k=1.4$ was used in (24). Curves where the empirical model in (25) has been used are also shown
technique could be used to determine the correlated phase noise. The auxiliary RF signals used for down-conversion would have to have lower phase noise than the investigated signals.

The presented method could be extended to more than two signals. In that case one would have to consider that the correlated phase noise of signals one and two could have a correlated and an uncorrelated part in signal three and so on. The number of parameters to determine would increase fast with the number of signals and one would have to measure the PNS at frequencies of mixing products of three or more signals.

The concept of partially coherent phase noise of RF signals at different frequencies could be included in models for phase noise. Such models could then be included in system simulators or computer-aided-design tools used for analysing the effect from phase noise on the performance of communication systems where multiple frequencies are used.

5 Conclusions

A method is proposed for determining the correlated and uncorrelated parts of PNS of two signals of different frequencies. The method is based on a model for the phase where there is one correlated part and one uncorrelated part. The correlated part is the same for the two signals, except for a multiplicative constant. Measured PNS of the mixing products of the two signals are used to determine the correlated and uncorrelated parts. According to the model the PNS of some mixing products are linear combinations of others, which is also seen in experimental data, which corroborates the proposed model. The proposed method is to our knowledge the first method for determining the coherence between the phase noise of two RF signals of different frequencies. The method can be used with standard spectrum analysers and mixers. Naturally, the phase noise of the analysed signals should be higher than the intrinsic noise of the used spectrum analyser and the used mixers’ additive noise; the reference oscillator of the instrument should have lower phase noise than the analysed signals.

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