Finite Size Scaling Behavior of Dissipative Tunnel Junctions

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We present the results of a series of Quantum Monte Carlo calculations of the temperature dependent conductance of dissipative non-superconducting tunnel junctions. Finite size scaling methods are used to demonstrate the absence of coherent transport in Ohmic tunnel junctions. However, a quantum phase transition between coherent and incoherent behavior is found in sub-Ohmic tunnel junctions. The critical conductance is found to be temperature independent and the critical exponents, $\nu$ and $\eta$, are found to be in agreement with renormalization group results by Kosterlitz.

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In spite of the significant effort to understand the effect of ohmic dissipation on normal (non-superconducting) tunnel junctions, there remains a conflict between analytical studies and quantum Monte Carlo simulations of these systems. In particular, analytical treatments such as renormalization group, large N approximation, and variational methods generally conclude that no quantum phase transition occurs in tunnel junctions subjected to a purely Ohmic environment. In contrast, it has been argued that quantum Monte Carlo studies of Ohmic tunnel junctions exhibit a quantum phase transition between an ordered phase which exhibits coherent tunneling and a disordered phase which does not.

If such a phase transition existed, it would be a remarkable example of a quantum phase transition driven by dissipation. Moreover, the existence of such a transition would be of considerable importance to the electronic and transport properties of a variety of systems including double quantum dots and metallic granular composites. For these reasons, we believe that it is essential to resolve this discrepancy. Towards this end, we will reexamine the simulation data using using finite size scaling methods. This approach will allow us to identify quantum phase transitions which may occur in Ohmic and sub-Ohmic tunnel junctions.

Our discussion is based on the generalized Ben Jacob Mottola Schon (BMS) model which describes the effects of dissipation and Coulomb blockade phenomenon on non-superconducting tunnel junctions. The BMS model is defined by the effective action

$$S[\phi] = \int d\tau d\tau' \alpha(\tau - \tau')[1 - \cos(\phi(\tau) - \phi(\tau'))]$$  \(1\)

where the phase $\phi(\tau)$ is defined in terms of the time-dependent voltage across the tunnel junction, $V(\tau)$, such that $V(\tau) = \frac{\hbar}{\pi e} \phi(\tau)$. The generalized BMS model is a long range XY model with spin-spin interactions of the form $\alpha(\tau) = \alpha_0 \tau_Q^{-\epsilon} \left( \frac{\pi k_B T}{2} \right)^{2-\epsilon}$. $\tau_Q$ is defined in terms of the charging energy $E_Q = e^2/2C$ by $\tau_Q = \hbar/E_Q$. Charging effects are introduced to the model by working on a lattice such that $\phi(\tau)$ is defined for $\tau = j\tau_Q$ where $j = 1 \ldots N$ and where $N = \hbar/\beta/k_B T$ is the total number of timeslices. $\epsilon$ is a parameter, discussed below, which characterises dissipative bath coupled to the tunnel junction. When $\epsilon = 0$, the coupling parameter, $\alpha_0$, is related to $R_T$, the $T \to \infty$ limit of the resistance, by the expression $\alpha_0 = \hbar/(2\pi e^2 R_T)$. More generally $\alpha_0$ is proportional to $|t|^2 N_L N_R$ i.e. the squared tunneling matrix element multiplied by the density of states of the left and right electrodes.

When one derives this action from a more microscopic model, the long range interaction, $\alpha(\tau)$, is generated by integrating out the modes associated with a dissipative bath. These modes might, for instance, represent, particle-hole excitations occurring in the tunnel junction electrodes. In the absence of manybody effects, the particle-hole excitations give rise to Ohmic dissipation ($\epsilon = 0$). However, in some situations, Fermi edge singularity effects such as the orthogonality catastrophe and the exciton effect may become important. In such cases, the dissipative bath is not Ohmic i.e. $\epsilon \neq 0$ and the tunnel junction may exhibit a variety of interesting effects including non-linear $I(V)$ characteristics.

As discussed by Drewes et al., the renormalization group theory predicts a quantum phase transition for sub-Ohmic tunnel junctions (i.e. $\epsilon > 0$). This transition occurs at $\alpha_0 = \alpha_c(\epsilon)$ where $\alpha_c(\epsilon) \to \infty$ as $\epsilon \to 0$. This result appears to be in conflict with the Monte Carlo simulations of Ohmic ($\epsilon = 0$) tunnel junctions by Scalici et al. The latter study interpreted the observed $\alpha_0$ dependence of the correlation function $g(\tau) = \exp i\phi(\tau) \exp -i\phi(0)$ as evidence for a quantum phase transition at $\alpha_0^* \approx 0.7$.

In this paper, we will reconsider the simulation results using finite size scaling methods. Using these methods we have obtained the following results: (1.) No quantum phase transition occurs in Ohmic tunnel junctions with $\alpha_0 < 10.0$; (2.) The sub-Ohmic tunnel junctions exhibit a quantum phase transition characterized by a temperature independent critical conductance of order $e^2/h$; (3.) Values for the $\eta$ and $\nu$ critical exponents are found to be consistent with the analytical results of Kosterlitz.
and Nickel et al. \cite{2}.

A Quantum Phase Transition: We begin our discussion, by considering the tunnel junction conductance which can be written as \cite{1,2}

\[ G = \frac{2\pi a_0}{\hbar^2 R Q} \int_0^{k' B} \gamma_c(\tau)(\cos(\phi(\tau) - \phi(0))) d\tau \quad (2) \]

where \( \gamma_c(\tau) = [\pi(k_B T/Q)/\sin(\pi k_B T \tau)]^{-\epsilon} \). We evaluate \( G \) using \( g(\tau) = (\cos(\phi(\tau) - \phi(\tau'))) \) obtained from a series of simulations. The simulations involved the standard Metropolis Rosenbluth Teller Teller algorithm of the long range XY model on a 1-D lattice of \( N = h/\tau Q \) spins and periodic boundary conditions. Our simulations included \( 10^5 \) cycle runs for the 10, 20, and 32 timeslice systems and \( 2 \times 10^5 \) cycle runs for the 64 and 128 timeslice systems.

The results for \( \epsilon = 0.2 \) are presented in fig. 1 and the results for \( \epsilon = 0 \) are presented in fig. 2. Interestingly, in fig. 1 the \( N \) dependence of the conductance curves reverses at the point of intersection at \( \alpha_0 \approx 0.9 \). For \( \alpha_0 > \alpha_c \) one observes metallic behavior i.e. a regime where \( G \) increases as \( k_B T = E_Q/N \to 0 \). Conversely, for \( \alpha_0 < \alpha_c \), \( G \) decreases as \( T \to 0 \). Hence, we may identify the crossing point as a quantum phase transition between two phases. Hereafter, these two phases will be referred to as sub-Ohmic (\( \alpha_0 > \alpha_c \)) or insulating (\( \alpha_0 < \alpha_c \)). The sub-Ohmic phase exhibits a conductance which increases with increasing \( N \propto 1/k_B T \) whereas the insulating exhibits a conductance which decreases with increasing \( N \).

Remarkably the conductance curves in fig. 1 don’t simply exhibit a set of crossings. Instead all six curves cross at a single point. This observation suggests that the critical conductance is temperature independent. The existence of a temperature independent critical conductance can be understood using the finite size scaling ansatz

\[ < e^{i\phi(\tau)} e^{-i\phi(0)} >= \left( \frac{T_Q}{\tau} \right)^{d-2+\eta} K_{\pm} \left( \frac{L_T}{\xi_T} \frac{\tau}{\xi_T} \right) \quad (3) \]

where \( d = 1 \) is the dimensionality of the model, \( T_Q = \hbar/E_Q \) is the size of the time slices, and where \( \xi_T \) is the correlation time. Eq. 3 together with eq. 2 determine the scaling behavior of the conductance:

\[ G = \frac{e^2}{\hbar} \left( \frac{k_B T}{\Delta} \right)^{\eta-1-\epsilon} F_{\pm} \left( \frac{k_B T}{\Delta} \right) \quad (4) \]

where \( \Delta = h/\xi_T \) and where \( F_{\pm}(x) \) and \( F_{-}(x) \) are the \( \alpha_0 < \alpha_c \) and \( \alpha_0 > \alpha_c \) branches of some universal scaling function. Now Fisher, Ma and Nickel \cite{2} have shown that the long range spin-spin interactions cause \( \eta = 1 + \epsilon \). Because of this, eq. 4 implies a temperature independent critical conductance, \( G_c(\epsilon) \), of order \( e^2/\hbar \). This result has been confirmed using a large \( N \) approximation of the model \cite{2}. According to that treatment

\[ G_c(\epsilon) = 2\pi(1-\epsilon)\cotn \left( \frac{\pi}{2} \right) \left( \frac{e^2}{\hbar} \right) \quad (5) \]

This result shows that \( G_c \to 0 \) as \( \epsilon \to 1 \) which indicates that the disordered phase is absent at \( \epsilon = 1 \). Similarly eq. 4 shows that \( G_c \to \infty \) as \( \epsilon \to 0 \). This suggests that

![Figure 1](image1.png)

**FIG. 1.** Conductance for \( \epsilon = 0.2 \). Notice that the conductance curves cross at \( G_c \approx 2e^2/h \). The crossing separates a phase where the conductance exhibits sub-Ohmic from a phase where it exhibits insulating behavior.

![Figure 2](image2.png)

**FIG. 2.** Conductance for \( \epsilon = 0.0 \).
the ordered phase is absent at $\epsilon = 0$. These interpretations are supported by the observed $\epsilon$ dependence of $\alpha_c(\epsilon)$ given in fig. 3.

As was mentioned in the introduction, the absence of an ordered phase in the $\epsilon \to 0$ limit is not only a prediction of the large $N$ approximation but also is predicted by Kosterlitz’ renormalization group treatment of the long-range XY model. Additional evidence may be found in figs. 2 and 4. For $10 > \alpha_0 > 0.1$, $G$ decreases with decreasing temperature, $k_B T = E_Q/N$. This strongly suggests that no metallic or coherent phase is occurs in this range of $\alpha_0$. In addition, these figures do not exhibit any crossings which might indicate a phase transition.

**Finite Size Scaling Behavior:** We have seen that the existence of the temperature independent conductance provides indirect evidence of the finite size scaling behavior proposed in eq. 6. To obtain more direct evidence for finite size scaling, one must be able to collapse the conductance data using

$$G = \frac{e^2}{\hbar} F_\pm \left( N^{1/\nu} |\alpha_0 - \alpha_c| \right)$$

This is done in fig. 5. In that figure, we used the value of $\alpha_c$ from eq. 6 and adjusted $\nu$ to minimize the scatter. In this manner we obtained data collapse with $\nu = 4.2 \pm 0.6$ for $\epsilon = 0.2$. This may be compared with the one loop renormalization group result that $\nu = 1/\epsilon[1 + O(\epsilon)]$.

Next consider the $\epsilon = 0$ limit. In this limit, the scaling behavior indicated in eqn. 6 reduces to

$$G = \frac{e^2}{\hbar} L(\alpha_0 - \mu \ln N)$$

where $\mu = \lim_{\epsilon \to 0} \alpha_c/\nu$ is assumed to be finite. The resulting data collapse occurs is displayed in fig. 6. A comparison between figs. 5 and 6 indicates that only the monodecreasing branch of the scaling function is present in the $\epsilon = 0$ data. This means that for all $\alpha_0$, the conductance of Ohmic tunnel junctions decreases with increasing $N \propto 1/k_B T$. Again we conclude that, as predicted by Kosterlitz’s renormalization group treatment, a coherent (ordered) phase is does not occur in Ohmic tunnel junctions.

**Summary:** We have performed a finite size scaling analysis of the conductance of dissipative Ohmic and sub-Ohmic tunnel junctions. Consistent with the predictions of renormalization group and large $N$ theories, the sub-Ohmic tunnel junction exhibits a quantum phase transition between sub-Ohmic and insulating phases. The results for Ohmic tunnel junctions are also consistent with the RNG predictions: In particular, little evidence for a metallic phase is found in Ohmic tunnel junctions for $\alpha_0 < 10$. However, the conductance does exhibit finite size scaling consistent with a quantum phase transition which, according to renormalization group theory and large $N$ approximation, occurs at $\alpha_c^{-1} = 0$ in the Ohmic ($\epsilon = 0$) limit.

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FIG. 5. The conductance at $\epsilon = 0.2$. Two branches occur. The upper branch is metallic (increasing with $N$) and the lower branch is insulating (decreasing with $N$). Data collapse validates scaling form in eq. 1.

FIG. 6. Conductance for $\epsilon = 0.0$. Note: The conductance is monotonically increasing function of temperature, $k_B T = E_Q/N$. This implies insulating behavior at low temperatures.

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