I. INTRODUCTION

Two-dimensional magnetic systems have been a subject of intensive investigation for almost half a century now. Both ferro- and anti-ferromagnetic systems have been studied extensively, experimentally as well as theoretically, revealing a myriad of interesting properties including the discovery of high-temperature superconductivity in doped two-dimensional cuprates\cite{1}. One of the important aspects of these systems which has continued to attract interest is the magnetic excitations which are of fundamental relevance to understand the spin dynamics. Knowledge of the collective spin wave excitations can provide valuable insight into their dynamical response as well as their thermodynamic behavior. The advancement of experimental techniques, such as ferromagnetic resonance spectroscopy and inelastic neutron scattering, have been of immense help in exploring this field meticulously\cite{2, 3}. Inelastic neutron scattering is one of the most powerful and versatile techniques, available at that time, was also reported. However, spatially dependent extended couplings were assumed be- effective medium approximation\cite{9}. In this case, however, the futility of using the low moments associated with the spectral densities to evaluate the magnon dispersions and lifetimes.

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Spin dynamics in low dimensional magnetic systems has been of fundamental importance for a long time and has currently received an impetus owing to the emerging field of nanoelectronics. Knowledge of the spin wave lifetimes, in particular, can be favorable for future potential applications. We investigate the low-temperature spin wave excitations in two-dimensional disordered ferromagnetic systems, with a particular focus on the long wavelength magnon lifetimes. A semi-analytical Green’s functions based approach is used to determine the dynamical spectral functions, for different magnetic impurity concentrations, from which the intrinsic linewidth is extracted. We obtain an unambiguous $q^5$ scaling of the magnon linewidth which is ascribed to the disorder induced damping of the spin waves, thereby settling a longstanding unresolved issue on the wave-vector dependence. Our findings are also in good agreement with previous theoretical studies on Heisenberg ferromagnets. Additionally, we demonstrate the futility of using the low moments associated with

in Heisenberg ferromagnets, in the low-energy limit, were studied theoretically as early as the sixties by Murray\cite{8}. The author calculated the spin wave energies and the scattering cross section, within the Born approximation, and reported a $q^5$ scaling of the magnon lifetime. The exchange interactions, in this case, were restricted to nearest neighbors only. Similar $q^5$ dependence was also found in amorphous Heisenberg ferromagnets, in the low-temperature and long wavelength limit, by using an effective medium approximation\cite{9}. In this case, however, spatially dependent extended couplings were assumed between the magnetic sites. Based on Green’s functions calculations, Mano\cite{10} also predicted an identical behavior of the lifetime in the long wavelength limit. The finite linewidth of the excitations, which increased rapidly with decreasing wavelength, was attributed to the randomness in the magnitude of the spins. Also the discrepancy between the observed magnetization behavior and that predicted by elementary spin wave theory was believed to originate from this finite linewidth of the spin waves. On the contrary, similar spin wave studies in amorphous ferromagnets by Kaneyoshi\cite{11} led to a $q^7$ dependence of the linewidth. This was an outcome of using a quasi-crystalline approximation, which is essentially a virtual-crystal-like approach. Within this approximation, the magnon dispersion reduces to that of an ideal crystal, wherein the disorder effects are completely neglected. In yet another study, based on the two-magnon interaction theory of Heisenberg ferromagnets, a leading order $q^2$ scaling of the magnon lifetime was suggested by Ishikawa et al.\cite{12}. However, in most of the aforementioned studies, good agreement with numerical calculations, available at that time, was also reported. However, despite the existence of innumerable studies, one important feature which has eluded understanding, over the decades, is the wave-vector dependence of the magnon lifetime (inversely proportional to the linewidth), especially in the long wavelength limit ($q \to 0$). Spin waves
obtained a $d+2$ ($d > 1$, is the dimensionality) scaling of the magnon linewidth. Although the discussion was extended to the more relevant site-diluted systems, the exchange interactions were again restricted to nearest neighbors only. Thus, the lack of a general accord on the issue of linewidth scaling becomes apparent from the widely varying predictions available in the literature. Moreover, considerable attention and interest have also been devoted to the case of anti-ferromagnets, including even recently[14]. This served as a further motivation behind the current study of the magnetic excitations in disordered (diluted) ferromagnetic systems, with a particular focus on the long wavelength magnon lifetimes. A proper knowledge of the lifetimes is not only of fundamental interest but can also be of practical importance, as we shall discuss later. In this article, we provide a comprehensive and detailed analysis of the low-temperature spin wave excitations in two-dimensional site-diluted ferromagnets, in the presence of extended exchange interactions. The calculations have been performed on sufficiently large system sizes and a proper statistical sampling over disorder is also taken into account. We lay special emphasis on the correct evaluation of the magnon linewidths in the long wavelength limit. In the process, we demonstrate that determining the correct wave-vector dependence of the lifetimes constitutes a non-trivial task. In addition, we also discuss the nature of the magnon density of states, the spectral functions, as well as the magnon dispersion over a relatively broad concentration range.

II. HEISENBERG MODEL AND EXCHANGE COUPLINGS

We start with the Hamiltonian describing $N_{\text{imp}}$ interacting spins ($S_i$) randomly distributed on a square lattice of $N$ sites, given by the dilute Heisenberg model

$$H = -\sum_{i,j} J_{ij} p_i p_j S_i \cdot S_j$$

where the sum $i,j$ runs over all sites and the random variable $p_i$=1 if the site is occupied by an impurity or otherwise zero. We consider classical spins ($|S_i|=S$) on a square lattice, with lattice spacing $a$, and with periodic boundary conditions. All calculations, in the present work, are performed at $T=0$ K. The concentration of magnetic impurities in the system is denoted by $x$ ($=N_{\text{imp}}/N$). The Hamiltonian, Eq. (1), is treated within the self-consistent local random phase approximation (SC-LRPA), which is essentially a semi-analytical approach based on (finite temperature) Green’s functions. Within this approach, the retarded Green’s functions are defined as

$$G_{ij}^{\text{r}}(\omega) = \int_{-\infty}^{\infty} G_{ij}^{\text{r}}(t)e^{i\omega t} dt$$

where $G_{ij}^{\text{r}}(t)= -i\theta(t)\langle [S_i^+(t),S_j^-](0)\rangle$, describe the transverse spin fluctuations, and $\langle \cdots \rangle$ denotes the expectation value, and ‘c’ the disorder configuration index. After performing the Tyablikov decoupling[15–17] (assuming magnetization along the z-axis) of the higher-order Green’s functions which appear in the equation of motion of $G_{ij}^{\text{r}}(\omega)$, we obtain

$$\omega I - H_{\text{eff}}^{\text{c}} G^{\text{r}} = D$$

where $H_{\text{eff}}^{\text{c}}$, $G^{\text{c}}$, and $D$ are $N_{\text{imp}} \times N_{\text{imp}}$ matrices. The effective Hamiltonian matrix elements are

$$H_{ij}^{\text{eff}} = -\langle S_i^+ \rangle J_{ij} + \delta_{ij} \sum_l \langle S_l^+ \rangle J_{lj}$$

and the diagonal matrix

$$D_{ij} = 2\langle S_i^z \rangle \delta_{ij}.$$
FIG. 1. Average magnon DOS $\rho_{\text{avg}}$ as a function of energy $\omega$ plotted for different concentrations $x$.

III. MAGNON DENSITY OF STATES AND SPECTRAL FUNCTION

From the retarded Green’s functions defined above one can calculate the average magnon density of states (DOS), which is given by $\rho_{\text{avg}}(\omega) = (1/N_{\text{imp}}) \sum_i \rho_i(\omega)$, where $\rho_i(\omega) = -1/(2\pi S) \Im[G_{ii}(\omega)]$ is the local magnon DOS. Fig. 1 shows the average magnon DOS as a function of the energy for different impurity concentrations. The DOS have been averaged over a hundred disorder configurations, although it was found that typically 25 configurations were sufficient for each impurity concentration. We observe irregular features in the DOS which become more pronounced with increase in dilution. On decreasing the concentration from $x = 0.1$ to $x = 0.02$, a significant increase in weight around the low energy end of the spectrum is observed. This increase in weight is attributed to the increase in the fraction of impurities which are weakly connected to the rest. These isolated impurity regions have their own zero-energy modes which in turn contribute to the DOS at the low energies. In order to gain a better insight into this behavior we look at the distribution of the local DOS shown in Fig. 2(a) and 2(b), at two different energies $2.2 J_0 S^2$ and $3.2 J_0 S^2$, respectively for $x = 0.1$. Here we can clearly identify certain impurity regions, of typically two or three impurities, which are weakly coupled to the surrounding impurities. These can be seen to make a higher contribution to the DOS. (For more details see Fig. 8, App.). Note that the distribution shown corresponds only to a part of the lattice from a $200a \times 200a$ system. With increasing dilution the average separation between the spins increases and hence the effective coupling decreases. This accounts for the increase in the irregular features observed in the DOS for $x = 0.02$. The dynamical spectral function, also known as the structure factor, provides valuable insight into the underlying spin dynamics of a system. Experimentally this can be probed by inelastic neutron scattering and ferromagnetic resonance to a good accuracy. The averaged spectral function is defined by

$$A(\mathbf{q}, \omega) := -\left\langle \frac{1}{2\pi S} \Im[G^r(\mathbf{q}, \omega)] \right\rangle_c,$$  \hspace{1cm} (6)

where $G^r(\mathbf{q}, \omega)$ is the Fourier transform of the retarded Green’s function $G^r_{ij}(t)$, and $\langle \ldots \rangle_c$ denotes the configuration average. Fig. 3 shows the averaged spectral functions as a function of energy for four different concentrations. The $A(\mathbf{q}, \omega)$’s are averaged over a few hundred disorder configurations, and the results are plotted only in the $[1 0]$ direction of the Brillouin zone, for progressively increasing momentum $\mathbf{q}$, since the focus is on the long wavelength regime here. It should be noted that the $[0 1]$ direction is equivalent to the $[1 0]$ direction in this case, due to the lattice symmetry. Also note that for $q \gg 2\pi/(La)$ the deviation from rotation invariance is not negligible. Well-defined excitations are found to exist only for small values of $\mathbf{q}$, in each case. For increasing $\mathbf{q}$, the excitation peaks become broader and develop a tail extending toward the higher energies. On decreasing the concentration from 0.08 to 0.02 the zone of stability of the well-defined magnon modes is found to decrease by almost one order of magnitude. Also the excitations become increasingly asymmetric with increase in the momentum. This increase in asymmetry is associated with the crossover from propagating low-energy spin waves to localized or quasi-localized excitations (fractons)[24] at higher energies. Here, the term localized implies that the excitations are quite broad in energy at fixed wavevectors, or rather quantitatively the excitation energy is much larger than the linewidth (i.e. the full-width at half-maximum). The nature of the spectral functions is similar to what was observed by neutron scattering...
experiments in \(\text{Mn}_x\text{Zn}_{1-x}\text{F}_2\)[25, 26], which is a three-dimensional randomly diluted anti-ferromagnet. The authors measured sharp spin waves near the zone center which broadened progressively with the wave vector approaching the zone-boundary. These findings were attributed to a crossover from low-energy extended spin waves (magnons) to localized high-energy excitations (fractons), which was further consistent with the theoretical conjecture of fractons in disordered percolating networks[24, 27]. A recent numerical study[28] on site-diluted two-dimensional anti-ferromagnets also reveal the existence of localized excitations at high energies. The authors evaluated the inverse participation ratio (IPR), for different dilutions and different system sizes in order to establish the nature (extended or localized) of the states, although the largest system size studied was only \(32a \times 32a\). These studies provide relevance and also additional motivation to study the two-dimensional ferromagnets from this aspect. The proper and accurate evaluation of the spectral functions, as we shall see in what follows, constitutes a vital task since the magnon dispersion as well as the lifetime can be directly extracted from them.

IV. MOMENTS ANALYSIS

Before embarking into further details of the long wavelength magnon properties, we define the moments associated with the spectral density. The \(n\)-th moment is defined by

\[
m_n(q) = \int_{0}^{\infty} \omega^n A(q, \omega) d\omega
\] (7)

In the limit \(q \to 0\), it can be shown that \(m_1(q) \approx D_0 q^2[32]\), where we call \(D_0\) as the effective spin wave stiffness. It is also well known that in the long wavelength limit the dispersion in ferromagnetic systems is quadratic in \(q\), \(\omega(q) \approx Dq^2\), where \(D\) denotes the spin stiffness coefficient. The moments, as sometimes found in the literature[30, 31], are used in the spectral function analyses as a good approximation to estimate the excitation energy and linewidth, especially in the presence of disorder. Nonetheless, the accuracy and the viability of this assumption is subject to further examination. In order to address this, as a first step, we numerically calculated the dispersion from the first moment and then compared it to the real excitation energy \(\omega(q)\) extracted from the \(A(q, \omega)\) peaks shown in Fig. 3. The results for the particular case of \(x = 0.03\) are plotted in the inset of Fig. 4. As can be seen, in the small \(q\) limit, both \(m_1(q)\) and \(\omega(q)\) are linear in \(q^2\) but the first moment
fairly overestimates the real magnon energies. This is better reflected when we extract the respective spin stiffness coefficients, $D_0$ from $m_1(q)$ and $D$ from $\omega(q)$, and plot them against the concentration as shown in Fig. 4. For all considered $x$, the effective spin stiffness is larger than the actual spin stiffness, overestimating by 15-20% in each case. This clearly demonstrates that the first moment is not a reliable quantity to evaluate the spin stiffness in these diluted systems as it fails to reproduce the magnon energies precisely. The other relevant quantities of interest is the intrinsic linewidth of the magnetic excitations. The linewidth gives a measure of the excitations’ broadening due to disorder, which maybe magnetic or structural disorder, or due to the magnon-magnon interactions. One can obtain the linewidth from the second moment from the relation

$$\gamma_0(q) = \sqrt{m_2(q) - m_1^2(q)}$$

where $\gamma_0(q)$ is the effective linewidth. In Fig. 5 we have plotted this effective linewidth as a function of $q$ for four different impurity concentrations. We find that in the small-$q$ limit the linewidth is linear in $q$ for all considered $x$. The same holds true for all other intermediate concentrations, which are not shown here. Consequently, we end up with $\omega \propto q^2$ and $\gamma_0 \propto q$, in the limit $q \to 0$. This indicates that the magnetic excitations are incoherent or localized around the $\Gamma$-point, since $\gamma_0 > \omega$. However, this is somehow contrary to what we have observed in the spectral functions shown in Fig. 3, where the excitations are well defined for small $q$ values. Hence, we can safely assume that the effective linewidth obtained from the moments does not correspond to the real linewidth of the excitations. The same discrepancy was also demonstrated for the case of Ga$_{1-x}$Mn$_x$As[32], a well-known III-V diluted ferromagnetic semiconductor, where the lattice has an fcc structure. Note that similar linear $q$-dependence, obtained from the moments analysis, was reported by the authors in disordered double-exchange systems[31]. Determining the correct $q$-dependence of the intrinsic linewidth, in the long wavelength limit, requires further detailed analysis which is elucidated in the following.

V. SCALING OF MAGNON LIFETIME

We extract the linewidth, which is the full-width at half-maximum, directly from the magnon spectral functions (Fig. 3) corresponding to the first non-zero $q$ values from different system sizes. The extracted linewidths are plotted as a function of the wave-vector in Figs. 6(a) and 6(b), for $x = 0.03$ and 0.05, respectively. In order to have sufficiently small $q$ values, and also check for the probable finite-size effects we have performed the calculations on system sizes ranging from $200a \times 200a$ up to $500a \times 500a$. The linewidth data are averaged over one hundred disorder configurations and the error bars corresponding to the standard deviation are contained within the symbols. Now, since we are interested in the $q \to 0$ regime, we focus on a restricted region of the $q$ values, (highlighted by the shaded regions in the plots), in order to give more weight to the smallest available $q$’s. We remark that the limit considered for the shaded regions only serve as an approximate value and not as a clear demarcation of the $q$ regime, defining the long wavelength limit. Note that the value of $\ln'(qa) \approx -4$ corresponds to a value $qa \approx 0.02$. To determine the $q$-dependence we use a linear fit of the form $n \ln'(qa) + C$, (with $n = 3, 4,$ and 5) for the data within these shaded regions. As can be clearly seen for both cases, $x = 0.03$ and 0.05, it is the
provide a linear moments overestimate the real spin stiffness as well as determine the linewidth or the excitation energies. The associated with the spectral function are inappropriate to previous studies, we have shown that the moments as-sulted systems is a non-trivial task. Contrary to some wave-vector dependence of the magnon linewidth in di-point. It is demonstrated that determining the correct a restricted region of the Brillouin zone, around the Γ-point. Thermal fluctuations also play an important role in these systems, but since we focus only on the low-temperature excitations we can neglect the thermal effects here. Further experimental studies to quantitatively examine the linewidth in these compounds could prove to be very useful.

VI. CONCLUSION

We have addressed the low temperature spin excitations in two-dimensional diluted Heisenberg systems, with a particular focus on the long wavelength limit. A self-consistent Green’s functions based approach is used to evaluate the magnon DOS and the dynamical spectral functions. Well-defined excitations are observed only in a restricted region of the Brillouin zone, around the Γ-point. It is demonstrated that determining the correct wave-vector dependence of the magnon linewidth in diluted systems is a non-trivial task. Contrary to some previous studies, we have shown that the moments associated with the spectral function are inappropriate to determine the linewidth or the excitation energies. The moments overestimate the real spin stiffness as well as provide a linear q-dependence of the linewidth, implying incoherent excitations in the limit q → 0. However, this is found to be inconsistent with the stiffness and the linewidth extracted from the calculated spectral functions. In the long wavelength limit, the linewidth in fact scales as q^4 in two-dimensional systems, for a wide range of impurity concentrations. The discrepancy arises due to the inability of the moments to reproduce the asymmetry in the excitation peaks. The origin of this asymmetry is ascribed to the disorder induced broadening of the spin waves. Hence, this underlines the importance of the disorder effects in these systems and we emphasize that the failure to properly account for them will possibly result in an incorrect wave vector dependence of the linewidth.

Most data storage devices, in nowadays spintronics, try to utilize the dynamical motion of spins. From this perspective, a precise knowledge of the excitations’ life-
time (inversely proportional to the linewidth) could be of practical relevance. For instance, a short lifetime is important for memory devices to leave a bit in a steady state after a read-in or read-out operation. On the other hand, a longer lifetime is advantageous for the unhindered transmission of signals in inter-chip communications. It would be equally interesting to look into the temperature effects on the spin dynamics, in particular the linewidth, where in addition to disorder the thermal effects also play a vital role. However, this is beyond the scope of the current work. The present findings provide qualitative insights into the low temperature excitations and the magnon lifetimes in two-dimensional ferromagnets, and could serve as a firm basis for future research on complex disordered magnets. More experimental studies oriented in this direction are also highly desirable to resolve the controversy arising from the numerous theoretical proposals.

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Appendix: Impurity Configuration Energies

The peaks in the averaged magnon DOS in the diluted case, Fig. 1, can be related to different configurations of some few impurities in coordinate space on the square lattice. This can be motivated by considering the distribution of the local magnon DOS in Fig. 2(a),2(b) which reveals clusters of less than four impurities up to an energy of \(\omega/(J_0 S^2) \approx 3\). Thus, the relevant small clusters which give rise to the magnon DOS peaks on a square lattice are identified to be those plotted in Fig. 7. The energies corresponding to the configurations (a)-(g) are given by

\[
E_{(a);1} = 2 \quad (A.1)
\]

\[
E_{(b);1} = 2^{-\alpha/2}E_{(a);1} \quad (A.2)
\]

\[
E_{(c);1} = 3 \quad (A.3)
\]

\[
E_{(c);2} = 1 + 2^{1-(\alpha/2)} \quad (A.4)
\]

\[
E_{(d);1} = 3 \quad (A.5)
\]

\[
E_{(d);2} = 1 + 2^{1-\alpha} \quad (A.6)
\]

\[
E_{(e);1/2} = 1 + 2^{-\alpha/2} + 5^{-\alpha/2} \pm 10^{-\alpha/2}[2^\alpha - 10^{\alpha/2}(1 + 2^{\alpha/2}) + 5^{\alpha}(1 - 2^{\alpha/2} + 2^\alpha)]^{\frac{1}{2}} \quad (A.7)
\]

where the energy \(E_{(\cdot);p}\) is given in values of \(J_0 S^2/\alpha^p\), with \(\alpha\) being the lattice constant and the index \(p\) the eigenvalue number (the ground-state energy zero has been excluded). The energies \(E_{(\cdot);p}\) are indicated in Fig. 8 by vertical lines. Fig. 8 shows the magnon DOS of a 10a x 10a system with three impurities averaged over all possible configurations. A comparison of this results with the \(x = 0.02\) case in Fig. 1 (system size of \(L = 1340a\)) reveals indeed that the relevant energies \(E \gtrsim 0.5J_0 S^2\) are given by the Eqs. (A.1)-(A.11), with \(\alpha = 3\). Many of the other configurations can be generated by a simple isotropic rescaling of the configurations shown in Fig. 7. The corresponding energies are indicated by dashed and solid lines in Fig. 8(b). To answer the question how specific the choice of the exponent \(\alpha = 3\) is, we plotted the energies Eq. A.1- A.11 of the magnon DOS peaks for different values of \(\alpha\) in Fig. 9. We recall that a two-dimensional system with monotonically decaying interactions cannot have ferro- or anti-ferromagnetic long-range order at \(T > 0\), if \(\alpha \geq 4\). The plot shows that in the range of \(2 < \alpha < 4\) the energies of the individual impurity configurations do not intersect each other. Thus, one can assume to have no significant and quantitative change in the physics if the exponent is varied within this range.

FIG. 7. Basic impurity configurations on a square lattice which give a high contribution to the magnon DOS.

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FIG. 8. Averaged magnon DOS $\rho_{\text{avg}}$ of a small system $L = 10\xi$ with only three impurities.
(a) For specific maxima we show the corresponding impurity configuration in coordinate space on the square lattice. The energy corresponding to a configuration inset is on its LHS indicated by a solid line, according to Eq. A.1- A.11 with $\alpha = 3$.
(b) Zoom of plot (a) at small $\omega$ values. The solid lines indicate the energies after rescaling the lattice constant by two, the dashed lines a rescaling by $\sqrt{5}$.

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