THE AUTHENTIC ACTIVITIES AND MATHEMATICAL MODELLING PROCESSES FOR PREPARATORY SCHOOL PUPILS

INTRODUCTION
Mathematics has made a significant contribution to the rise and development of nations due to its prominent role in the scientific breakthrough, where the use of modern technology has been greatly extended over the last years and now it is applied to technological, social and human sciences. As many scientific discoveries over the ages were mainly based on mathematics, the sphere of technology also effectively used mathematical knowledge for placing satellites and spacecraft on the Earth’s orbit by using orbital mechanics (Astrodynamics), which is a core discipline within space mission design and control (BERTACHINI A. PRADO, MADEMONT, ZANARDI, WINTER, YOKOYAMA, GOMES, 2013). In addition, mathematics was actively exploited as a major tool to achieve particular goals:

- In weather forecasting: In using atmospheric dispersion modeling,
- In vehicle design: Operations Research is used in manufacturing,
- In medicine: In using five-year survival rates,
- In demographics: In using the statistical study of populations, especially human beings, etc. (QUARTERONI, 2018).

Also, the necessity of possessing significant skills that employees strongly need in the twenty first century, such as building a sense of complex systems or working in teams on various projects, can be supported and strengthened through the study of mathematics and the predominant role of it in real life.

Educational activities are considered one of the privileged fields in the process of teaching of mathematics due to their possible contribution to achieving the objectives of the subject and changing the negative view of mathematics in the way that mathematics is often regarded as an abstract subject full of different symbols and disconnected from the real life (SAYED, 2012). The more realistic these activities are and the more they are related to pupils’ lives, the more they help achieve these objectives.

Smith and Morgan (2016) identify three main rationales in orientations of curricula to use of real-world contexts or educational authentic activities in mathematics, namely: “mathematics as a tool for everyday life, the real world as a vehicle for learning mathematics, and engagement with the real-world as a motivation to learn mathematics”. In teaching mathematics, educational activities are the appropriate option to understand a real-life and work to generate and innovate real and mathematical models from students’ life and society, and this vision is to be reinforced through implementing educational activities, which based on mathematical modeling and models (HALLSTRÖM, SCHÖN Born, 2019).

MATHEMATICAL MODELLING
Mathematical modelling is considered to be an essential connector between mathematics and the real world. The use of mathematics to solve problem situations in the real world or what is called mathematical modelling can be regarded as a complex process that includes several stages: understanding the situation, building a mathematical model that describes the elements and relations between them, working on the mathematical model to determine what results come from it and explaining the outcomes of the work to find a solution for the realistic situations, evaluating the interpreted outcomes related to the original situation and dealing with the results “preparing a report on the solution”. These stages can be considered to be a
single chain of many processes (YILMAZ, DEDE, 2016). We believe that the practically applied aspect of mathematical modelling starts from reality and also ends within this reality.

Modelling helps achieve important goals of teaching mathematics in the twenty first century as it enhances pupils’ ability to form the existing knowledge, interact with real models in a mathematical way in order to make the process of learning meaningful and effective. As a result, it helps pupils develop their mathematical thinking (BLUM, 2015). Due to its importance, mathematical modelling has been one of the most discussed issues all around the world in education. In fact, it was included in the standards of mathematics curriculum in many countries during the period of mathematical curricula reform in the early 2000s.

Analyzing the results of consideration of educational normative documents (Mathematical educational standards) of Singapore, Germany and the USA, we note that in each of them special attention is paid to teaching mathematical modeling, where in mathematical educational standards of Singapore processes of mathematical modeling are highlighted and included in the curriculum, and in mathematical educational standards of Germany mathematical modeling is attributed to significant learning outcomes, and in mathematical educational standards of USA modeling issues are conceptually separated into a separate standard, and are also included in all other standards. The development of skills of pupils in processes mathematical modeling are highlighted in accordance, examples of real situations are given, which can be introduced to pupils.

There are also several international conferences focused on the issues of mathematical modelling which are held on a regular basis. For instance, International Conference on the Teaching of Mathematical Modelling and Applications (ICTMA) presents many research papers on mathematical modelling, its application in the workplace and the use of technology in teaching of mathematics, as well as International Congress on Mathematical Education (ICME), which provides research papers and booklets concerning the ways of teaching mathematical modelling and its application in real life by using various methods.

The development of science has led to the existence of several generally accepted approaches, including those ones connected with mathematics curriculum. Besides its impact on teaching and learning of mathematics it is clear that practical application of knowledge has become a vital aspect of mathematics curriculum at schools, and then modelling becomes an essential part of the curriculum on all educational levels and every classes. It's so necessary to show a definition to a mathematical model regarding it as an interpretation of a situation, a phenomenon, or a real problem in the language of mathematics or a mathematical representation of it. The process of converting a problem situation into a mathematical model is called mathematical modelling, or in other words, mathematical modelling is a process of describing phenomena in the form of mathematical equations (TREFFERT-THOMAS, VIIRMAN, HERNANDEZ-MARTINEZ, ROGOVCHENKO, 2017), and it’s important when practicing mathematical modeling to use real-world problems or authentic problems (VORHÖLTER, 2018).

Blum distinguishes mathematical modelling from its application (BLUM, 2015). He reckons that modelling mainly focuses on the «life-to-mathematics» track and emphasizes the processes involved in it, while application of mathematical modelling focuses on the opposite “mathematics-to-life” track and highlights the processed things, especially those parts of the real world which can be expressed in the following figure:

**Figure 1.** Mathematical applications and mathematical modelling (author’s figure)

[Diagram showing the relationship between mathematical modeling and mathematical application, connecting mathematics to life through the process of converting a problem situation into a mathematical model.]

Source: Search data.
The stages of mathematical modelling process

The Programme for International Student Assessment defined a series of mathematical modelling processes consisting of five stages including the following: simplifying data from the real world, moving to a mathematical problem, solving the mathematical problem, explaining and validating results in the real world (YULIANI, KUSUMAH, 2018). And Blum also developed a pattern of mathematical modelling processes according to the following figure (BLUM, 2015; STILLMAN, 2019).

Figure 2. Mathematical modelling process (BLUM, 2015)

According to this pattern, the process begins with real-life problems. They must firstly be simplified and reformulated to put forward the life model, then this particular model must be modified into mathematical one through its translation into the language of mathematics. We need to take into consideration the fact that the difference between real-life and mathematical models is not always clear because the use of numbers sometimes directly leads to the mathematical model without going through the real-life model stage. Afterwards we start working on the mathematical model using special methods to find a mathematical solution that must be explained in the context of real-life situation. Finally, the modelling process is completed and the solution is verified. The modelling process can be repeated if necessary by making some other assumptions or using various mathematical methods.

Saeki and Matsuzaki (SAEKI, MATSUZAKI, 2013) used this idea to design two similar tasks that could be used in teaching, and Blum and Leiß (BLUM, LEIß, 2007) approaches elaborate this essential cycle with enhanced pedagogy in mind but not all cycles have been constructed with the logic of the modelling process in mind. By presenting both of these models, this research suggests the following model for mathematical modelling process, which can be adopted for pupils, where this model is quite simplified in the way that helps students complete the mathematical modelling process without complications:
Figure 3. Suggested mathematical modelling process

According to the model, it consists of five main operations (The author suggested) on the way to mathematical modelling as following:

- **Simplifiable**: at this stage real-life situation is studied in order to determine the most important information and exclude additional information. Afterwards the situation must be clarified by using this basic information to reach the real-life model;

- **Mathematizable**: the main information about a real-life problem is translated into a mathematical form, then we choose appropriate laws and theories to work with and finally come at an appropriate mathematical model;

- **Constructing a model**: appropriate mathematical operations are used to construct a model in order to reach appropriate mathematical results;

- **Explanation**: the results need to be verbally explained based on the outcomes of the model;

- **Validation**: at this stage we validate results in the context of information provided by a real-life situation and in the context of its consistency with particular mathematics theories.

**Authentic activities**

Implementing *authentic activities* based on *modelling* are contributing to integrated science, technology, engineering, and mathematics literacy (HALLSTRÖM, SCHÖNBORN, 2019). Authentic activities mean this type of activities that describes real-life situations similar to those ones that pupils have to face in their everyday life and practices.

Authentic Activities play a crucial role in our perception of mathematics as it represents a natural way to show mathematics in more realistic and more accessible way to everyone (ORMROD, 2008). And students become more active and interesting in an educational environment that connects students to participate in authentic activities and realistic contexts (YONG, KARJANTO, GATES, CHAN, 2020).

Fesakis, Karta and Kozas (2018) and Siswono, Kohar, Rosyidi, Hartono and Masriyah (2018) recommend to focus on authentic activities, in which children do math and they are gradually led to mathematization. A number of studies confirmed that treating pupils with unauthentic Activities in math classes leads to misconceptions that impede their success in effective dealing with the modelling process. Among the most important misconceptions are the following statements (SAYED, 2012):
There is only one right (accurate and precise) solution to each text-problem;
We can find a solution by performing one or more mathematical operations on all the
objects in the problem;
The text-problem includes all necessary information that we need to find a correct
solution;
Every problem is presented by a teacher or it can be found in the textbook; the
problem must be possible to solve and have realistic meaning

It is common knowledge that characters, things, places and drawings in text-problems are
often different from those ones that can be seen in real-life situations. So do not worry if
information obtained in your own experience contradicts the information included in the
designed activities. The presented literary works attributed the growth of these beliefs to two
main reasons:

The stereotyped and shortened presentation of common text-problems in
mathematics lessons, textbooks, and tests that are often done by using one or more
arithmetic operations on the given numbers.
Current teaching practices and culture related to text-problems. Some studies have
shown that teachers’ knowledge and conceptions about the role of knowledge in the
real world in word-problem solving has a strong impact on their teaching behavior and
as a result on the beliefs and educational outcomes of their students.

That is why we prefer using Authentic Activities that help address the negative effects of this
type of problems and change pupils’ negative perception of mathematics at school instead of
the stereotyped approach to text-problems. The Main Features of Authentic Activities in
Mathematics (SAYED, 2012):

Taking into consideration pupils’ age when choosing and creating Authentic Activities;
Making sure that chosen activities are appropriate for students and contribute to the
development of mathematical modelling processes;
Making sure that chosen activities have as many characteristics of real-life situations as
possible;
Diversification of teaching methods used in teaching process according to the
requirements of the educational situation so that it enhances effectiveness of students
as well as develops a spirit of cooperation among them in the classroom;
Making sure that activities vary from easy to difficult ones in order to motivate students
and encourage them to participate in the activity, work together and enhance their
self-confidence;
Be sure that selected activities and teaching methods push students towards studying
mathematics and changing their negative perception of the subject.

The purpose of the study
Despite the fact that mathematical modelling is an extremely important and significant skill in
the modern world, preparatory school students in Egypt still have some difficulties while
dealing with the mathematical modelling processes. The researcher has assumed that the main
reason for it is connected with an extremely low points (Egypt didn’t show on table ranking) of
math’s learning mainly, where This is evident through 15-year-old school pupils’ scholastic
performance on mathematics of the PISA 2018. And one of the aspects of this test is designed
to check up pupils on mathematical literacy and the application of maths knowledge in real
life. Secondly, there was a pilot study conducted on a group of girls in the second year of
preparatory school on their abilities in mathematical modelling processes. The results revealed
that most of the girls dealt with test situations using their assumptions without building logical processes to model these situations.

Finally, the previous studies in teaching approach in Egypt (HANAA, 2010) urged the necessity of introducing mathematical modeling into mathematics curricula, and Diab (2015) urged the use of educational realistic activities in the development of mathematical modeling. English and Watson (DIAB, 2015) have enhanced the mathematical literacy of Year 6 students through modeling with authentic data, and Egupova and Elsaidi (2020) urged the use of mathematical exercises that have a pupils environment, as they help in developing mathematical modeling processes as well as they consider it to be one of the important goals of math’s education in the twenty-first century. So the purpose of this study was to examine the effectiveness of using Authentic Activities in developing mathematical modeling processes for preparatory school pupils, and we assume the following hypothesis is correct: there is a statistically significant difference between the averages of the study group in the pre- and post-measurements to check mathematical modelling operations in favor of the post-measurement.

RESEARCH METHODOLOGY
Design authentic activities
In order to select and create Authentic Activities we did the following:

- Reading «Encyclopedia of Logic, Mathematics, and Reasoning Gas» by Badr Al-Bassam to choose some suitable for children activities;
- Reading the book titled «California Mathematics: Concepts, Skills, and Problem Solving» which is considered to be one of the best and modern books on designing life activities for mathematics;
- Dividing activities into worksheets where paper includes one particular activity.

While choosing the program activities, we mainly drew on the features of Authentic Activities that we had previously found within the theoretical framework. The researcher often takes the idea of the activity from the previous studies and adjusts it so that it fits with the Egyptian environment on the one hand and matches the interests of pupils on the other hand. We also designed new activities for the Egyptian realities, as it adds an atmosphere of vitality and excitement to students. In the light of this, the program consisted of 19 worksheets that concerned different areas of the pupils’ lives, such as football matches, training at gyms, participation in public libraries, the cost of traveling by bus, the cost of holding birthday parties, selling and buying activities and some others.

One of activities, which is from Egyptian environment: A teacher maths went with his students on an educational touring to Luxor (city in Egypt), and they saw very high archaeological monument (Obelisk), but no one knows its real height. the teacher wants to know the height of this monument, but there is no suitable instrument for measuring its height directly. How can a teacher find out the height of a monument by using a two-meter stick? Note that at noon the sun’s rays are vertical to the planet earth.

Teaching methods of authentic activities
We used several methods to teach the authentic activities, such as the method of discussion (for describing activities), cooperative learning (for working in small groups and students learning cooperatively can capitalize on one another’s skills), and discovery learning (to solve the activities by themselves). On table (BLUM, 2015) we present the subjects and numbers of activities and classes.

| Subject                              | Number of authentic activities | Number of classes |
|--------------------------------------|-------------------------------|-------------------|
| Solving of algebraic equations (algebra) | 12                            | 3                 |
| Similarity (geometry)                | 7                             | 2                 |

Source: Search data.
Methods of educational assessment students in teaching authentic activities

The methods for evaluating in teaching authentic activities are as follows:

- **Formative assessment**: it was through the teaching process, where a teacher asked questions to pupils, to attract the attention towards the goals of the activities and tried to connect between steps of solving activities and mathematical modeling processes.

- **Summative assessment**: it was generally after the completion of teaching authentic activities, and that was through mathematical modeling processes test.

| Type of evaluating | Example |
|--------------------|---------|
| Questions on formative assessment | Identifying Similar Right Triangles on the following figure. |
| Mathematical modeling processes test | Carpet cleaner A charges 28.25 $ plus 18 $ a room. Carpet cleaner B charges 19.85 $ plus 32 $ a room. Explain how you could find the number of rooms for which the total cost of both carpet cleaners is the same? |

Source: Search data.

**Mathematical modeling processes test**

We prepared a mathematical modeling processes test according to the following steps:

- **Aim of the test**: The test aimed to assess the ability of preparatory school pupils to use mathematical modeling processes.

- **Preparation of the test questions**: we selected the test questions, which were realistic situations similar in difficulty to those included in the activities' worksheets, and the test content of 6 questions and 5 points for each question (one point for every mathematical modeling process, and by it pupils solved questions), so all points of the test were 30.

- **The validity and reliability**: we presented the worksheets and test to a group of experts (professors in curricula and mathematics teaching methods) to make notes on it.

- **About worksheets**: they recommended dividing the worksheets in the solving step into the five stages that represent the modeling processes, to guide pupils towards using these processes in the solving.

- **And about the test questions**: they agreed on the test questions, except one of them who indicated the ease of some of the test questions. This required questions with various difficulty levels.

- Regarding the calculation of the **reliability coefficient** of the test, we used the prior measurement on a group of second-year lower middle school pupils, and we calculated the reliability coefficient according to the test-retest reliability, where we applied the test, then we repeated applied it after 2 weeks on the pre-group (20 pupils from 2/2 class at Samira Mousa Experimental Language School). Then we calculated the correlation coefficient (by using Pearson correlation coefficient) between the results of the test and the results of the retest, the correlation coefficient was 0.72, it means that the test is based on an acceptable reliability coefficient.
Participants
The study group was chosen from preparatory school pupils from 2/1 class at Samira Mousa Experimental Language School in the 2020/2021 academic year. The study group consisted of 45 girls because we followed the design of one group with pre-measurement (before application of Study) and post-measurement (after application of Study).

During the study the girls worked in small groups consisting of four or three members in each group in all activities, where this number of members suitable for area of class. They were divided into groups according to their cultural background, where they live in the same society and abilities so that all groups were in equal position. However, the most important factor of division was to make sure that the members of each group can work together successfully, where they chose each other’s by themselves. In order to achieve this, the views of the teachers were also taken into consideration, where applied of activities and study test was by online learning on application “Microsoft Teams”, where meetings provided 2 times every week for 1.5 hours on link (https://teams.microsoft.com/l/team), and this is due to the epidemiological situation “COVID-19”.

Pre-measurement of test study
The study instrument (testing of mathematical modelling processes) was applied to the girls of the study group on 05/11/2020, before applying Authentic Activities to them to obtain the necessary pre-data to test the effect of activities on the development of mathematical modelling processes.

Application of authentic activities
Authentic Activities were applied to the study group. The researchers gave copies of the working papers to the girls so that every four or three of them could participate together and think about the activity for a while. Then they were asked to apply five mathematical modelling processes to this activity. In fig. 5 we show an example for activity.

Figure 4. Problem of Obelisk
A teacher maths went with his students on an educational touring to Luxor (city in Egypt), and they saw very high archaeological monument (Obelisk), but no one knows its real height. The teacher wants to know the height of this monument, but there is no suitable instrument for measuring its height directly. How can a teacher find out the height of a monument by using a two-meter stick? Note that at noon the sun’s rays are vertical to the planet earth

1. Identify the most important information in question (Simplifiable)
2. Draw a geometric shape that represents the question (Mathematizable)
3. Identify the theories that can be used to solve the question (Constructing a model)
4. Solve the question (Explanation)
5. Validate results in the context of information provided (Validation)

Source: Search data.

After that there was a discussion between the teacher and the girls and he chose one of them to simplify the problematic situation, the second one to develop the model, the third to solve the mathematical model, the fourth one to interpret the solution, and one more girl to validate the model and the solution.

Post application of the study instrument
After the end of the program, the study instrument was re-applied (modelling operations testing). On 30/11/2020, the students were to obtain the necessary data to check the impact of activities on the development of mathematical modelling processes.

Researcher’s notes during the experiment:
The researcher made several notes during the application of study instrument which explains the extent of pupils’ willingness to participate in the activities and their feeling about these activities. The most important notes are the following:
(1) The teacher noticed that many girls wanted to participate in Authentic Activities, that is why sometimes it was difficult to manage the class because they were eager to participate and write the solution on the blackboard;

(2) Some girls insisted on keeping thinking on the activity and they asked the teacher not to disturb them by explaining particular steps of the process. This fact demonstrates that children enjoyed the activity and they want to accomplish it themselves;

(3) The teacher noticed that at the beginning some girls were reluctant to participate in the activity, and when its importance was clarified to them, their wish to participate and interact with the activities became stronger;

(4) Some pupils insisted on keeping a copy of the activities that were studied during the class. When the teacher asked them "Why?", the answer was: "To think on the activity more carefully at home";

(5) After the experiment ended some girls asked the teacher to continue holding similar activities at least once a week for their enjoyment and in order to gain benefit from them.

RESULTS AND DISCUSSION

Here we deal with statistical treatment of data to come to results necessary to validate the hypothesis of study:

(1) We used $T$-test to verify statistically significant difference between two related averages by using SPSS software for counting $T$, where we entered group results on SPSS in the two cases (pre- and post- measurements) and compared between two means by using independent-simple $t$-test, and compared between $T$-counted and $T$-tabular, and if $T$-counted $> T$-tabular, this means that the results are statistically significant.

(2) Then we counted the impact size ($\eta^2$) in the pre- and post-applications to test mathematical modelling operations on the study group by using SPSS software, The significance of the value of impact size ($\eta^2$) is as follows:

(3) If the value of $\eta^2 = 0.01$ to less than 0.06, the size of the effect is small. Therefore, in this case, the experiment or survey should be repeated, to ascertain whether there is an effect or that the result is of practical significance.

(4) If the value of $\eta^2 = 0.06$ to less than 0.14, the effect size is moderate, and this indicates that the experiment or survey will lead to results of practical significance.

(5) If the value of $\eta^2 = 0.14$ and more, the size of the effect is large, as the results are of significance and practical importance.

The following table shows the results of $t$-test, the impact size ($\eta^2$) in the pre- and post-applications to test mathematical modelling operations on the study group.

| Application | N  | $\bar{x}$ | S. D. | $T$-counted | $T$-tabular | Significance | $\eta^2$ | The Impact Size |
|-------------|----|----------|-------|-------------|------------|-------------|--------|----------------|
| Pre-Application | 4 | 0.667 | 0.854 | 10.9 | 2.73 | Significant at the level of 0.01 | 0.79 | Large |
| Post-Application | 5 | 13.09 | 6.687 |

Note : the maximum score is 20, N = the number of pupils in Application, $\bar{x}$ = the average scores, S. D. = standard deviation.

Source: Search data.

It is clear from the table (3), that $T$-counted $> T$-tabular (at the level of 0.01), it means that results are statistically significant value at the level of 0.01 and then there is a statistically significant difference between the average scores of the students in the pre- and post-measures of the
The authentic activities and mathematical modelling processes for preparatory school pupils

The authentic activities and mathematical modelling processes for preparatory school pupils

The authentic activities and mathematical modelling processes for preparatory school pupils

test in favor of the post-application. The researcher also calculated the impact size through the value of $\eta^2$ which was 0.79. It is considered a high value and indicates that the Authentic Activities made a great impact on the development of mathematical modelling processes for the students in the study group.

Through the main effect on pupils' mathematical modeling processes, we do observe that this has been harmonious with what we expected from using the Authentic Activities on the experimental design. Because of connecting mathematics with pupils' lives and including activities that connect daily life with what is being studied, which gives pupils the importance of studying mathematics because it has an effect on their lives. We also observe pupils on the experimental design liked spending more time on the activities, which indicates the activities that increase the motivation of the pupils towards learning. It is possible that increasing time of studying authentic activities allows pupils to better integrate with each other and with mathematics, resulting in the increased ability to acquire mathematical concepts correctly that were demonstrated in the classroom.

**Limitations**
The data for the classroom discussed in this study were collected at school and with two different teachers. Thus, there was not a consistent teacher for all pupils in academic year 2020/2021, and we suggest that the findings may be more broadly applicable, though both groups of pupils share common characteristics as they both primarily draw pupils from the same region and them educational attainment in last year. Since the learning activities were completed, the classroom environment likely only has minimal impact; however, the course involved in all activities of this study focuses on pupils developing a deep conceptual understanding of mathematics and its applications in real life.

It is also worth recognizing that these materials are designed to provide authentic activities. In this manner, they are meant to provide a foundational experience that teachers can then build upon. As such, they are not intended to provide full coverage of all content that pupils would likely encounter in most general mathematics courses.

**CONCLUSION**
The use of Authentic Activities is a new and unfamiliar method that makes a great contribution to attracting pupils' attention while studying mathematics as they realize that math can present something new and useful to them. This means that the application of Authentic Activities provides a new and innovative space for learning by merging and using mathematical knowledge with Authentic Activities in the real world. Authentic Activities are based on the real environment (the pupils' environment) and attaching it to mathematical knowledge. Actually, it leads to the development of mathematical modelling processes among the pupils because Authentic Activities provides pupils with opportunity to study real-life problems, analyze them from various aspects and provide a correct picture of the problem that contributes to the increase of students' awareness of what they are learning, enhances its importance for them and develops their mathematical thinking.

**FUNDING**
The research is funded by a scholarship [ID: EGY-6328/17] under the Joint Executive Program between the Arab Republic of Egypt and Russia Federation.

**REFERENCES**
BERTACHINI A. PRADO, A. F.; MASDEMONT, J. J.; ZANARDI, M. C.; WINTER, S. M. G.; YOKOYAMA, T.; GOMES, V. M. Mathematical Methods Applied to the Celestial Mechanics of Artificial Satellites 2013, *Mathematical Problems in Engineering*, 2013, p. 1-5. Available at: https://doi.org/10.1155/2013/854317 Access: August 14, 2021.

BLUM, W. Quality Teaching of Mathematical Modelling: What Do We Know, What Can We Do? In: Cho S. (Ed.), *The Proceedings of the 12th International Congress on Mathematical Education* p. 71-96. Springer, Cham, 2015. Available at: https://doi.org/10.1007/978-3-319-12688-3_9 Access: August 14, 2021.
BLUM, W.; LEIß, D. How do students and teachers deal with modelling problems? In C. Haines, P. Galbraith, W. Blum, S. Khan (Eds.), Mathematical modelling (iCTMA12): Education, engineering and economics (pp. 222-231). Chichester: Horwood, 2007.

COMMON CORE STATE STANDARDS INITIATIVE: USA Standards for Mathematical Practice “Geometry”. Available at: http://www.corestandards.org/Math/ Access: August 14, 2021.

DIAB, R. A Suggested Proposal for Mixing between Brain-Based Learning and TRIZ Theory for Developing Geometric Sense and Creative Thinking of Second Stage Preparatory Students (Doctoral dissertation). Faculty of Education, Beni-Suef University, Egypt, 2015. Available from Egyptian Universities Libraries Union. (Document ID 12430143).

EGUPOVA, M.; ELSAIDI, M. Teaching mathematics in grades 7-9 in Egypt and in Russia: advantages and disadvantages, Mathematics at school, 2020, 3, p. 52-65. Available at: Google Scholar Access: August 14, 2021.

ENGLISH, L. D.; WATSON, J. Modelling with authentic data in Grade 6. ZDM Mathematics Education, 2018, 50(1-2), p. 103-115. Available at: DOI: 10.1007/s11858-017-0896-y Access: August 14, 2021.

FESAKIS, G.; KARTA, P.; KOZAS, K. Designing Math Trails for Enhanced by Mobile Learning Realistic Mathematics Education in Primary Education, International Journal of Engineering Pedagogy, 2018, 8(2), p. 49-63. Available at: https://online-journals.org/index.php/i-jep/article/view/8131/4950

GERMAN EDUCATIONAL STANDARDS. Available at: https://www.kmk.org/themen/qualitaetssicherung-in-schulen/bildungsstandards.html#c5035 Access: August 14, 2021.

GRONMO, L. S. The Role of Algebra in School Mathematics. In: Kaiser G., Forgasz H., Graven M., Kuzniak A., Simmt E., Xu B. (Ed.) Invited Lectures from the 13th International Congress on Mathematical Education. ICME-13 Monographs. Springer, Cham, 2018.

HALLSTRÖM, J.; SCHÖNBORN, K. J. Models and modelling for authentic STEM education: reinforcing the argument, International Journal of STEM Education, 2019, 6, p. 22. Available at: https://doi.org/10.1186/s40594-019-0178-z Access: August 14, 2021.

HANAA, H. The effectiveness of teaching a geometric unit includes realistic applications using mathematical modeling in the development of achievement and solving applied problems for preparatory school students. Faculty of Education, Ain Shams University, Egypt, 2010). Available from Egyptian Universities Libraries Union. (Document ID 11207792).

ORMROD, J. E. Educational Psychology: Developing Learners. New Jersey, Pearson/Prentice Hall, 2008. ISBN-13: 9780132974424.

QUARTERONI, A. The role of statistics in the era of big data: A computational scientist’ perspective, Statistics & Probability Letters, 2018, 136, p. 63-67. Available at: DOI: 10.1016/j.spl.2018.02.047

SAEKI, A.; MATSUZAKI, A. Dual modelling cycle framework for responding to the diversities of modelers. In G. A. Stillman, G. Kaiser, W. Blum, & J. P. Brown (Eds.), Teaching mathematical modelling: Connecting to research and practice (p. 89-99). Dordrecht: Springer, 2013. Available at: https://link.springer.com/chapter/10.1007/978-94-007-6540-5_7 Access: August 14, 2021.

SAYED, M. An Authentic Activities Based Program to Develop Mathematical Modelling Processes and Interests towards Mathematics for Prep. School Students, Journal of Mathematics Education (Egyptian Council of Mathematics), 2012, 15 (2). Available at: DOI:10.13140/RG.2.2.22147.14889 Access: August 14, 2021.
SINGAPORE EDUCATIONAL STANDARDS. Available at: https://www.moe.gov.sg/education/syllabuses/sciences Access: August 14, 2021.

SISWONO, T. Y. E.; KOHAR, A. W.; ROSYIDI, A. H.; HARTONO, S.; MASRIYAH. Searching for authentic context in designing PISA-like mathematics problem: From indoor to outdoor field experience, *Journal of Physics: Conference Series*, 2018, 953 (1), p. 012197. IOP Publishing. Available at: DOI: 10.1088/1742-6596/953/1/012197 Access: August 14, 2021.

SMITH, C.; MORGAN, C. Curricular orientations to real-world contexts in mathematics, *The Curriculum Journal*, 2016, 27 (1), p. 24-45. Available at: https://doi.org/10.1080/09585176.2016.1139498 Access: August 14, 2021.

STILLMAN, G.A. State of the Art on Modelling in Mathematics Education—Lines of Inquiry. In: Stillman G., Brown J. (eds) *Lines of Inquiry in Mathematical Modelling Research in Education*. ICME-13 Monographs. Springer, Cham. 2019. Available at: https://doi.org/10.1007/978-3-030-14931-4_1 Access: August 14, 2021.

TREFFERT-THOMAS, S.; VIIRMAN, O.; HERNANDEZ-MARTINEZ, P.; ROGOVCHENKO, YU. Mathematics lecturers’ views on the teaching of mathematical modelling, *Nordic Studies in Mathematics Education*, 2017, 22 (4), p. 121-145. Available at: http://ncm.gu.se/wp-content/uploads/2020/06/22_4_121146_treffert_thomas.pdf

VORHÖLTER, K. Conceptualization and measuring of metacognitive modelling competencies: Empirical verification of theoretical assumptions, *ZDM: the international journal on mathematics education*, 2018, 50 (3), p. 343-354. Available at: DOI:10.1007/s11858-017-0909-x Access: August 14, 2021.

YILMAZ, S.; DEDE, A. T. Mathematization competencies of pre-service elementary mathematics teachers in the mathematical modelling process, *International Journal of Education in Mathematics, Science and Technology*, 2016, 4 (4), p. 284-298. Available at: DOI:10.18404/ijemst.39145 Access: August 14, 2021.

YONG, S.-T.; KARJANTO, N.; GATES, P.; CHAN, A. Let us rethink how to teach mathematics using gaming principles, *International Journal of Mathematical Education in Science and Technology*, 2020, p. 1-20. Available at: DOI:10.1080/0020739X.2020.1744754 Access: August 14, 2021.

YULIANI, A.; KUSUMAH, Y. S. Analysis of mathematical modelling ability of line equations of junior high school students, *Journal of Physics: Conference Series*, 2018, 1132, p. 1-8. Available at: https://iopscience.iop.org/article/10.1088/1742-6596/1132/1/012045 Access: August 14, 2021.
The authentic activities and mathematical modelling processes for preparatory school pupils

As atividades autênticas e processos de modelagem matemática para alunos da escola preparatória

Las actividades auténticas y los procesos de modelización matemática para los alumnos de las escuelas preparatorias

**Resumo**

O objetivo deste estudo do efeito de atividades autênticas no ensino de matemática em processos de modelagem matemática: As atividades autênticas foram utilizadas pela realização do experimento, onde o trabalho foi com 45 alunos de 13 anos de escola preparatória (é a segunda etapa das etapas educacionais no Egito, e é a próxima etapa após o ensino fundamental) em Al Qalyubia (Uma das cidades do Egito) no ano letivo de 2020/2021 e estudando esse efeito. Os resultados mostraram que o valor do tamanho do impacto através do valor de $\eta^2$ foi calculado 0,79, e isso significou o forte efeito das atividades autênticas no desenvolvimento de processos de modelagem matemática, onde atividades autênticas levam ao desenvolvimento de processos de modelagem matemática entre os alunos, pois atividades autênticas proporcionam aos alunos a oportunidade de estudar problemas da vida real, analisá-los de vários aspectos e proporcionar uma visão correta do problema que contribui para o aumento da consciência dos alunos sobre o que estão aprendendo.

**Palavras-chave:** Atividades autênticas. Modelagem matemática. Ensinar matemática. Modelos reais. Aplicações matemáticas.

**Abstract**

The aim of this study of the effect of authentic activities in teaching mathematics on mathematical modeling processes: The authentic activities were used by conducting the experiment, where work was with 45 pupils 13th years old of preparatory school (It’s the second stage of the educational stages in Egypt, and it’s the next stage after Primary school) in Al Qalyubia (One of the cities in Egypt) in the 2020/2021 academic year and studying this effect. The results showed, that value of the impact size through the value of $\eta^2$ was calculated 0.79, and this meant the strong effect of the authentic activities on the development of mathematical modeling processes, where authentic activities lead to the development of mathematical modeling processes among the pupils because Authentic Activities provides pupils with the opportunity to study real-life problems, analyze them from various aspects and provide a correct picture of the problem that contributes to the increase of students’ awareness of what they are learning.

**Keywords:** Authentic activities. Mathematical modelling. Teaching mathematics. Real models. Mathematical applications.

**Resumen**

El objetivo de este estudio del efecto de las actividades auténticas en la enseñanza de las matemáticas en los procesos de modelado matemático: Las actividades auténticas se utilizaron al realizar el experimento, donde el trabajo fue con 45 alumnos de 13 años de la escuela preparatoria (es la segunda etapa de las etapas educativas en Egipto, y es la siguiente etapa después de la escuela primaria) en Al Qalyubia (una de las ciudades de Egipto) en el año académico 2020/2021 y estudiando este efecto. Los resultados mostraron, ese valor del tamaño del impacto a través del valor de $\eta^2$ se calculó 0.79, y esto significó el fuerte efecto de las actividades auténticas en el desarrollo de procesos de modelado matemático, donde las actividades auténticas conducen al desarrollo de procesos de modelado matemático entre los alumnos porque Actividades Auténticas brinda a los alumnos la oportunidad de estudiar problemas de la vida real, analizarlos desde diversos aspectos y proporcionar una imagen correcta del problema que contribuye al aumento de la conciencia de los estudiantes de lo que están aprendiendo.

**Palabras-clave:** Actividades auténticas. Modelización matemática. Enseñanza de las matemáticas. Modelos reales. Aplicaciones matemáticas.