Effect of stress paths on the particle crushing

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ABSTRACT

In previous studies, almost all they had carried out the crushing tests under usual tri-axial compression, Ko and simple shear conditions. However, it is necessary to perform the crushing test under general stress conditions in ground with controlled each principal stresses individually to find the mechanical characteristics of the particle crushing. Therefore, the purpose of this study is to confirm the effect for particle crushing under various stress paths in the combination of principal stresses. Authors plan the particle crushing tests with combinations of principal stresses operating the mean stress p, deviatoric stress q and Lode angle θ up to the Shear Failure Line (SFL) on the p-q plane. These particle crushing tests were carried out using the high pressure true tri-axial compression apparatus under the planned stress paths. As a result, the degree of Lode angles has little traction to the progress of the particle crushing and authors found that deviatoric stress q has essential effect on the particle crushing. In addition, the degree of particle crushing are little effected by the various monotonic stress paths on p−q plane when the maximum values of p and q are same.

Keywords: particle crushing, stress path, mean stress, deviatoric stress, Lode angle

1 INTRODUCTION

Particle crushing is interesting phenomenon and important topic in the geotechnical engineering point of view. Therefore, many experimental and theoretical studies were conducted before now (Vesic and Clough 1968, Miura and O. Hara 1979, Hardin 1985, Fukumoto 1992, Nakata et al. 1999, Hyodo et al. 2002, Yu. Fangwei 2017). In some cases, the particle crushing should be considered for the design and simulation of the pile foundation (Yasufuku et al. 2001, Kuwajima et al. 2009, Wu et al. 2013). The unstable bearing capacity of crushable sandy soil is particularly severe problem. In addition, stress conditions are not only high pressure but also complicate stress distributions in the ground near the pile tip. In fact, the stress distribution just beneath the center of pile is close to Ko condition and the shear stress is distinguished in the ground near corner of pile tip.

By the way, particle crushing tests were performed on the stress conditions of usual tri-axial compression, Ko and simple shear in the experimental studies. However, it requires attention that there are limited stress paths in these experiments. This study aims to investigate the effect of stress paths to the particle crushing in the three-dimensional stress space. Therefore, the particle crushing tests are performed under various stress paths using specialized tri-axial high pressure compression apparatus. The conditions of particle crushing progresses under each differing stress paths are compared in this paper.

2 OUTLINE OF TESTS

The specialized tri-axial high pressure compression apparatus (Yokura et al. 2015) was used to control the various stress paths under high pressure conditions. Figure 1 shows the plane view of the apparatus used in this study, and Fig.2 shows a cubic test specimen with the shape of edges cutting 50 x 50 x 50 mm. This specimen is installed in the center of this apparatus and
loaded by oil jack with maximum stress up to 200 MPa through the rigid quenching steeled loading square (40 x 40 mm) columns. Toyoura silica sand is used in this study and compacted to be initial relative density of 90% or more.

Mean stress $p$, deviatoric stress $q$ and Lode angle $\theta$ are controlled independently in this study, and it is necessary to make differences only in the stress path with the same attainment mean stress $p$ and deviatoric stress $q$. For that purpose, it is easy to understand considering the three-dimensional principal stress space represented by $\sigma_1$, $\sigma_2$, $\sigma_3$. Therefore, experiments are conducted by setting a stress path on the $p$-$q$ plane and $\pi$ plane. Figure 3 shows the relationships between $p$-$q$, $\pi$ plane and the three-dimensional principal stresses, and Fig.4 shows the stress path on the $\pi$-plane. The definitions of mean stress $p$ and deviatoric stress $q$ are shown below.

$$p = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3) \quad (1)$$

$$q = \frac{1}{\sqrt{2}} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \quad (2)$$

In previous study (Keigo Fukuda et al. 2018), authors found little influence of the Lode angle $\theta$ on the crushing property from the experimental results of particle crushing tests under various Lode angles $\theta$ as shown Fig.4. Therefore, the effects of stress paths on $p$-$q$ plane are described specifically in this paper (in case of Lode angle $\theta = 0$). The particle crushing experiments are conducted by setting the stress paths on the $p$-$q$ plane as shown Fig.5. First, three $\pi$-planes with mean stress $p$=14.5, 30 and 50 MPa are fixed under considering the loading capacity of the apparatus. Experiments were conducted by stress paths with Lode angles $\theta = 0^\circ$, $20^\circ$, $40^\circ$, $60^\circ$ for each deviatoric stress levels and the SFL condition (stress path pattern: $P_1$ in Fig.5), and the isotropic compression cases without deviatoric stress (stress path pattern: IP in Fig.5). In total, 51 cases of experiments (3 IP + 48 $P_1$) were conducted as shown Fig.6. Then, the test of $p = 14.5$ MPa is named A test, the test of $p = 30.0$ MPa is named B test and the test of $p = 50.0$ MPa is named C test.
For example, experiment with the case of p = 14.5 MPa, the path of θ = 60° and unloading point of R1 is expressed as “A-60°-R1”. Some other crushing tests were conducted too on the stress path of p-q plane with Lode angles θ = 0° (stress path pattern: P2, P3, and P4 in Fig.5).

3 TEST RESULTS AND DISCUSSIONS

First, the sieving tests were conducted to determine the particle size distribution after particle crushing and the relative breakages Br (Hardin 1985) were fixed. Volumetric strain εv - p and deviatoric strain εd - q relationships of one test case are shown in Fig.7. The plastic work Wp are calculated by adding up the area of each painted place. In addition, plastic volumetric strain and deviatoric strain, elastic volumetric strain and deviatoric strain are shown in the figures. Since this plastic volumetric strain εvp is closely related to the residual volume reduction of the sample, it is considered to be related to the relative breakage Br.

Figure 8 shows the relationship between the Lode angles θ and Br, and the relationship between the Lode angles θ and Wp on the test case of stress path pattern P1. According to Br and Wp obtained by the test, almost no difference by the stress path of Lode angle θ was found. It is also understood that Br and Wp fluctuates depending on the magnitude of deviatoric stress q. It is considered to the consume energy when particles break. Therefore, it can be inferred that there is a unique relationship between relative breakage Br and Plastic

![Fig. 7. Definition of volumetric and deviatoric strain components.](image1)

![Fig. 8. Relative breakage Br and the plastic work Wp – Lode angles θ on the stress path pattern P1.](image2)

![Fig. 9. Relationship of Br – Wp (stress path pattern IP+P1).](image3)

![Fig. 10. Relationship of Br – εvp (stress path pattern IP+P1).](image4)
work \( W_p \). Figure 9 shows the relationship between plastic work \( W_p \) and relative breakage \( B_r \). The proximate curve is convex downward as shown in the figure. Also, since the increment of \( B_r \) gradually decreases, it seems that \( B_r \) has an upper limit. The relationship between plastic volumetric strain \( \varepsilon_v^p \) and relative breakage \( B_r \) is nearly proportional as shown in Fig.10, and the order of approximation is good \((R^2=0.939)\).

\[ P_1, P_2, P_3 \text{ and } P_4 \text{ represents the stress paths as shown in Fig.5.} \]

\[ \text{and } R_1, R_2, R_3 \text{ and SFL represents the magnitude of deviatoric stress as shown Fig.6.} \]

\[ \text{Figure 11 shows the grain size accumulation curves before and after crushing on the C tests with the case of Lode angles } \theta = 0^\circ. \]

\[ \text{Although stress path patterns are different in each figure, the grain size accumulation curves are almost same when the maximum deviatoric stress } q \text{ is an equal value.} \]

\[ \text{As the fine particles increase gradually with increase in maximum deviatoric stress } q, \text{ deviatoric stress } q \text{ has a certain effect to the particle crushing.} \]

\[ \text{These tendencies are same even where A and B tests.} \]

\[ \text{The relative breakages } B_r \text{ are estimated based on these figures.} \]

\[ \text{The relationships between mean stress } p \text{ and relative breakages } B_r \text{ are shown in Fig.12.} \]

\[ \text{The increasing tendency of relative breakage } B_r \text{ gradually increases as the mean stress } p \text{ increases in the case where the deviatoric stress } q \text{ is small.} \]

\[ \text{On the other hand, it can be seen that the increasing tendency of relative breakage } B_r \text{ gradually decreases with increase in the mean stress } p \text{ in the case where the deviatoric stress } q \text{ is large.} \]

\[ \text{In addition, it can find that the influence on the increase of particle crushing is larger in the deviatoric stress } q \text{ than the mean stress } p. \]
Photographs of soil particle before and after crushing tests are shown in Fig.13 and Fig.14, respectively. These photos were taken by digital microscope to the particle shape analyses. The difference in the appearance of sand before and after is remarkable. Most of particles become finer after crushing tests, while some of them still remain its original size. It may be concluded that there are not only particles fractured entirely but also particles which are scraped the edges off. In addition, if the hardness of some particles is larger than that of around particles before crushing, then the somewhat hard particles still remain easily its original size because the contact force decrease with the contact points increase by the prior fracture of around particles during crushing tests.

4 EXPERIMENTAL FORMULA

Based on the above experimental results (Fig.12), an approximate expression of $B_r$ - (p, q) relationship shown in the Eq. (3) and (4) are established with the following assumptions in case of the monotonic stress paths loading.

$$B_r = A_0 \times \left( 1 - \cos \left( \frac{\pi \times r}{r_c} \right) \right)$$  \hspace{1cm} (3)

$$r^2 = p^2 + (c \times q)^2$$  \hspace{1cm} (4)

Assumptions:

a) $B_r$=0, when $p=q=0$.
b) The limit of $B_r$ exists as “$A_0$” at $r=r_c$.
c) The effects of p and q to the particle crushing can be estimated as root-mean-square: “r”. However, the effectiveness level is considered as “c”.
d) The relationship between $B_r$ and r is expressed as the cosine function.
e) The effects of stress path patterns can be neglected.

A three-dimensional image of this experimental

Fig. 13. Appearance of the sand before crushing tests (x40).

Fig. 14. Appearance of the sand after crushing tests (x40).
formula is shown in Fig. 15.

As shown Table 1, the combinations of stress path patterns were configured to estimation of unknown parameters ($A_0$, $r_c$, and $c$) by using the least-squares method. However, in case of only stress path pattern $P_4$ was omitted because the pattern $P_4$ has only there tests due to the limitation of SFL within the range of $p$-$q$ combinations in this study. In Fig. 16, the estimated $B_r$ are compared with experimental data of each stress path combinations. Although only $P_2$ case is slightly larger than other cases, the order of approximation are totally good by using this relational expression and the relative breakage $B_r$ can be estimated by $p$ and $q$ without of consideration of stress path. In addition, plastic work $W_p$ and volumetric strain $\varepsilon_p$ can be estimated by inputting Eq. (3) to the results of Fig. 9 and Fig. 10, respectively.

5 CONCLUSIONS

In this experimental study on the particle crushing of Toyoura silica sand, it was found that there is little influence of the Lode angle $\theta$ for the particle crushing progression. The relative breakage $B_r$ and the plastic work $W_p$ increase in proportion to the combination of mean stress $p$ and deviatoric stress $q$, and have almost no relation to the stress path patterns in the monotonic loading. Based on the above experimental results, an approximate expression of $B_r \approx (p, q)$ relationship was derived. This expression has good approximate performance of the relative breakage $B_r$, and the plastic work $W_p$ and the volumetric strain $\varepsilon_p$ can be estimated additionally too in consideration of particle crushing.

ACKNOWLEDGEMENTS

This study was supported by JSPS KAKENHI Grant Number JP16K06575.

REFERENCES

1) Fukuda, K. and Yamamoto, H. (2018): Crushing Characteristic of Soil Particle on The Effect of Stress Path, Streamlining Information Transfer between Construction and Structural Engineering, ASEA SEC-4, GFE-02.
2) Fukumoto, T. (1992): Particle Breakage Characteristics of Granular Soil, Soils and Foundations, 32(1), 26-40.
3) Hardin, B. O. (1985): Crushing of Soil Particles, Journal of Geotechnical Engineering, 111(10), 1177-1192.
4) Hyodo, M., Hyde, A.F.L., Aramaki, N. and Nakata, Y. (2002): Undrained Monotonic and Cyclic Shear Behaviour of Sand under Low and High Confining Stresses, Soils and Foundations, 42(3), 63-76.
5) Kuwajima, K., Hyodo, M. and Hyde, A.F.L. (2009): Pile Bearing Capacity Factors and Soil Crushability, Journal of Geotechnical and Geoenvironmental Engineering, 135(7), 901-913.
6) Miura, N. and O-Hara, S. (1979): Particle-Crushing of a Decomposed Granite Soil under Shear Stresses, Soils and Foundations, 19(3), 1-14.
7) Nakata, Y., Hyde, A.F.L., Hyodo, M. and Murata, H. (1999): A Probabilistic Approach to Sand Particle Crushing in the Triaxial Test, Geotechnique, 49(5), 567-583.
8) Vesic, A. and Clough, G.W. (1968): Behavior of Granular Materials under High Stresses, Journal of the Soil Mechanics and Foundation Division, 94(3), 661-688.
9) Wu, Y., Yamamoto, H. and Yao, Y.P. (2013): Numerical Study on Bending Behavior of Pile Considering Sand Particle Crushing, Geomechanics and Engineering, 5(3), 241-261.
10) Yasufuku, N., Ochiai, H. and Ohno, S. (2001): Pile End-Bearing Capacity of Sand Related to Soil Compressibility, Soils and Foundations, 41(4), 59-71.
11) Yokura, K., Yamamoto, H. and Wu, Y. (2015): Crushing tests of soil particles by high pressure true tri-axial compression apparatus, The 6th Japan-China Geotechnical Symposium, Sapporo, Japan, 51-56.
12) Yu, F. (2017): Particle Breakage and the Drained Shear Behavior of Sands, International Journal of Geomechanics, 17(5), 1-11.