A New Model to Calculate Oil-Water Relative Permeability of Shaly Sandstone

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Oil-water relative permeability curves are the basis of oil field development. In recent years, the calculation of oil-water relative permeability in sandstone reservoirs by resistivity logging data has received much attention from researchers. This article first analyzed the existing mathematical models of the relationship between relative permeability and resistivity and found that most of them are based on Archie formula, which assumes the reservoir is clean sandstone. However, in view of the fact that sandstone reservoir is commonly mixed with shale contents, this research, based on the dual water conductivity model, Poiseuille’s equation, Darcy’s law, and capillary bundle model, derived a mathematical model (DW relative permeability model) for shaly sandstone reservoir, which calculates the oil-water relative permeability with resistivity. To test and verify the DW relative permeability model, we designed and assembled a multifunctional core displacement apparatus. The experiment of core oil-water relative permeability and resistivity was designed to prove the effectiveness of the DW relative permeability model in shaly sandstone reservoirs. The results show that the modified Li model can well express the transformational relation between resistivity and relative permeability in sandstone reservoir with low clay content. Compared with the modified Li model and the Pairoys model, the DW relative permeability model is more helpful to collect better results of relative permeability in shaly sand. These findings will play a significant role in the calculation of oil-water relative permeability in reservoirs based on resistivity logging data and will provide important data and theory support to the shaly sandstone reservoir characterized oil field development.

1. Introduction

The evaluation of tight reservoir has always been an important part of petroleum geology research, while oil-water relative permeability, which is vital to the evaluation of fluid flow in porous media, is used in all aspects of the reservoir engineering [1, 2]. Traditionally, relative permeability is obtained in laboratory. However, in many cases, especially in low permeability reservoirs, or when phase transformation or mass transfer happens with the change of pressure, oil-water relative permeability experiments are difficult, expensive, and time consuming simultaneously [3, 4]. Alternately, it is difficult to maintain the samples the same as in the reservoirs; moreover, relative permeability is almost impossible to obtain in real time. Despite all these difficulties, experiment serves as the main method to calculate relative permeability curves for oil fields.

Conventional resistivity logging data, which is the basic information of oil and gas well standard logging, is in large amount and available. In recent years, more and more scholars indicated that there is a relationship between relative permeability and resistivity [5–9]. Cai et al. [10] presented a review of the electrical conductivity models using fractal, percolation, and effective medium theories. In another article,
Cai et al. [11] proposed a combined model including pore-throat ratio, tortuosity, and connectivity, exactly estimating the influence of complex pore structure on the transport behavior associated with electrical parameters. Li [12] based on Archie formula put forward a mathematical model that uses resistivity to calculate the relative permeability of gas-water, and Li verified the model with experimental data. Li et al. [13, 14] conducted a lot of research on this field. Li together with Horne and Williams worked out methods to calculate two-phase relative permeability with resistivity logging data in uniform medium. Mohammed and Birol [15] modified Li model by taking the fluid viscosity and the average water saturation at the time of water breakthrough into consideration. Alex et al. [16] proposed a method using resistivity to calculate relative permeability in dual porosity model, but the model has not been verified experimentally. Pairoys et al. [17] verified the Li model and Brooks-Corey model [6] with gas-water relative permeability experiments and found that the Li model works better than Brooks-Corey model in that situation. Then, the Li model is modified by replacing the pore size distribution index $\lambda$ to index saturation exponent $n$ Li model [18]. Pairoys [19] analyzed the change of resistivity under different frequency in the process of unsteady two-phase flow displacement, based on which the Li model was verified again with gas-water relative permeability experimental data and oil flooding data by Bian and Li [20]. The above-modified models based on the Li model were established under the condition of homogeneous clean sandstone reservoir. However, most real sandstone reservoirs contain shale contents, which influence rock resistivity and relative permeability significantly.

This study, based on the dual water conductivity model, Poiseuille’s equation, Darcy’s law, and capillary bundle model, proposed and verified a mathematical model (DW relative permeability model) to calculate relative permeability using resistivity. To improve the Li model, a new model named “dual water relative permeability model (DW model)” was proposed in the consideration of better expressing the transformational relations between resistivity and relative permeability in the shaly sand reservoir. According to experiments, the DW model achieved the goal of reflecting the relation between resistivity and relative permeability in a better way than the modified Li model and Pairoys model, which is helpful in both the calculation of oil-water relatively permeability in shaly sand reservoir based on resistivity and the oil field development.

2. Mathematical Background

2.1. Relationship between Water Saturation and Relative Permeability. There exist many relationship models between water saturation and relative permeability, among which the most common one is as shown below [20]:

$$k_{\text{rw}} = k_{\text{rwmax}} \left( S_{w}^{*} \right)^{n},$$

(1)

$$k_{\text{rwn}} = k_{\text{rwnmax}} \left( 1 - S_{w}^{*} \right)^{n_{\text{rwn}}},$$

(2)

where $k_{\text{rw}}$ and $k_{\text{rwn}}$ are relative permeabilities of the wetting and nonwetting phase, $S_{w}$ and $S_{\text{rwn}}$ are the saturation and the irreducible saturation of wetting phase, $k_{\text{rwmax}}$ is the maximum $k_{\text{rw}}$ when $S_{w} = 1 - S_{\text{rwn}}$, $k_{\text{rwnmax}}$ is the maximum $k_{\text{rwn}}$ when $S_{w} = S_{\text{rwn}}$, $S_{\text{rw}}$ is the residual saturation of nonwetting phase, and $S_{w}^{*}$ is the normalized saturation of wetting phase.

2.2. Li Model for the Relationship between Relative Permeability and Resistivity. Fluid flow in porous media is similar to current flow in conductive media [13]. According to the Li model, gas/water relative permeability is calculated using resistivity:

$$S_{w} = \frac{S_{w} - S_{\text{wr}}}{1 - S_{w} - S_{\text{rwr}}},$$

(3)

where $k_{\text{rw}}$ and $k_{\text{rwn}}$ are relative permeabilities of the wetting and nonwetting phase, $S_{w}$ and $S_{\text{wr}}$ are the saturation and the irreducible saturation of wetting phase, $k_{\text{rwmax}}$ is the maximum $k_{\text{rw}}$ when $S_{w} = 1 - S_{\text{rwn}}$, $k_{\text{rwnmax}}$ is the maximum $k_{\text{rwn}}$ when $S_{w} = S_{\text{rwn}}$, $S_{\text{rw}}$ is the residual saturation of nonwetting phase, and $S_{w}^{*}$ is the normalized saturation of wetting phase.

2.3. Pairoys Model. Many modified models grew out of the Li model that is suitable for gas-water two-phase flow and oil flooding, but not for water flooding. Pairoys worked out the following model after analyzing water flooding situation [19]:

$$S_{w} = \frac{S_{w} - S_{\text{wc}}}{1 - S_{w} - S_{\text{or}}},$$

(4)

$$k_{\text{rw}}^{*} = \frac{S_{w}^{*}}{I},$$

(5)

$$k_{\text{rwn}}^{*} = \left( S_{w}^{*} \right)^{2+\lambda/\lambda},$$

(6)

$$k_{\text{rwn}}^{*} = \left( 1 - S_{w}^{*} \right)^{2+\lambda/\lambda},$$

(7)

where $I$ is the resistivity index, and $k_{\text{rwn}}^{*}$ is the wetting-phase normalized relative permeability.

2.4. Modified Li Model. Based on the Li model, Bian and Li proposed a model for the relationship between resistivity and oil-water relative permeability of water wet sandstone reservoirs with low shaly contents [20]:

$$S_{w}^{*} = \frac{S_{w} - S_{\text{wc}}}{1 - S_{\text{wc}} - S_{\text{or}}},$$

(8)

$$k_{\text{rw}}^{*} = \frac{S_{w}^{*}}{I_{\text{max}}},$$

(9)

where $S_{w}$ is the saturation of the wetting phase, $S_{\text{wc}}$ is the irreducible saturation of the wetting phase, $S_{\text{or}}$ is the residual saturation of the nonwetting phase, $S_{w}^{*}$ is the normalized saturation of the wetting phase, $I$ is the resistivity index, when $S_{w} = 1 - S_{\text{rwn}}$, and $k_{\text{rw}}^{*}$ and $k_{\text{rwn}}^{*}$ are normalized relative permeabilities of the wetting and nonwetting phases $I_{\text{max}}$ is the resistivity index.
where $R_{or}$ is the formation resistivity when $S_w = 1 - S_{or}$ and $k_{rw}^*$ and $k_{ro}^*$ are normalized relative permeabilities of the water and oil phases.

3. Relationship between Resistivity and Relative Permeability of Shaly Sand Reservoir

The relationship models of resistivity and relative permeability mentioned above all assume that the reservoirs are homogeneous and pure sandstone. However, in reality, most sandstone reservoirs contain shale contents. Therefore, in order to calculate the relative permeability in shaly sand reservoirs accurately, a new model suitable for shaly sandstone should be established.

The cross-sectional area, length, and volume of the water wet shaly sandstone are $A_i$, $L_i$, and $V_i$, respectively (as shown in Figure 1(a)). The effective pore space of rock is considered to be composed of $n$ large bore capillary columns with equal cross-sectional area and $m$ small bore capillary columns with equal cross-sectional area. The large columns are filled with movable water and oil, while the small columns are filled with immovable water (irreducible water) and residual oil. The relationship models of resistivity and relative permeability of the model are analyzed below. The resistivity of free water is $R_w$, while the resistivity of clay water is $R_{cl}$. In the $j$th ($j = 1, 2, \cdots, m$) small capillary columns, the cross-sectional area, length, and volume of the bound free water are $A_{w,j}$, $L_{w,j}$, and $V_{w,j}$, respectively (as shown in Figure 1(b)).

When the water saturation of the rock is $S_w$, in the $i$th ($i = 1, 2, \cdots, n$) large capillary columns, the oil cross-sectional area and oil cross-section radius are $A_{of}$ and $r_{of}$, while the cross-sectional area, length, and volume of the movable water are $A_{wf,i}$, $L_{wf,i}$, and $V_{wf,i}$, respectively. In the $j$th ($j = 1, 2, \cdots, m$) small capillary columns, the oil cross-sectional area and oil cross-section radius are $A_{or}$ and $r_{or}$, while the cross-sectional area, length, and volume of the immovable water are $A_{wc,i}$, $L_{wc,i}$, and $V_{wc,i}$, respectively. Due to the existence of shale contents, it is assumed that the immobile water in the small capillary columns contains clay water. The cross-sectional area, length, and volume of the clay water are $A_{wb,i}$, $L_{wb,i}$, and $V_{wb,i}$, respectively (as shown in Figure 1(c)).

When the core sample is saturated with water, according to Poiseuille flow formula, the liquid flow in the $i$th ($i = 1, 2, \cdots, n$) large capillary column $q_{a,i}$ is calculated as follows.

$$q_{a,i} = \frac{\pi r_{a,i}^4 \Delta p}{8 \mu L_a},$$  

where $\Delta p$ is the pressure difference, and $\mu$ is the fluid viscosity.

The total flow of water in the rock $q$ is

$$q = \sum_{i=1}^{n} \frac{\pi r_{a,i}^4 \Delta p}{8 \mu L_a} + \sum_{f=1}^{m} \frac{\pi r_{w,f}^4 \Delta p}{8 \mu L_{w,f}}.$$  

According to Darcy’s formula,

$$Q = k \frac{\Delta p}{\mu L}.$$  

Suppose that the length of the large capillary column is equal to that of the small capillary column. The permeability $k$ is obtained as follows.

$$k = \frac{1}{8} \left( \phi_a \frac{r_a^2}{L_a} + \phi_b \frac{r_b^2}{L_b} \right).$$  

Similarly, when the water saturation is $S_w$ ($S_w \leq S_w \leq 1 - S_{or}$), the small capillary columns are filled with bound water and residual oil, while the large capillary columns are filled with movable water and movable oil. According to the Poiseuille flow formula, the total flow of movable water in the rock $q_{wf}$ is

$$q_{wf} = \sum_{i=1}^{n} \frac{A_{wf,i}^2 \Delta p}{8 \pi \mu L_{wf,i}}.$$  

According to Darcy’s formula,

$$Q_{wf} = k_w \frac{\Delta p}{\mu L}.$$  

So, the permeability $k_w$ is

$$k_w = \frac{1}{8} \phi \frac{S_{wf} (r_w^2 - r_{wf}^2)}{r_{wf}^2}.$$  

And the water relative permeability $k_{rw}$ is obtained as follows.

$$k_{rw} = \frac{\phi S_{wf} (r_w^2 - r_{wf}^2)}{r_{wf}^2 (\phi_a (r_a^2 - r_{a,i}^2) + \phi_b (r_b^2 - r_{b,j}^2))}.$$  

The electrical conductivity of the model is analyzed below. The resistivity of free water is $R_w$, while the resistivity of clay water is $R_{cl}$. In the $j$th ($j = 1, 2, \cdots, m$) small capillary columns, the cross-sectional area, length, and volume of the bound free water are $A_{wc,j}$, $L_{wc,j}$, and $V_{wc,j}$, respectively. When the core sample is saturated with
water, the resistivity is $R_0$. When the saturation is $S_w$ and the resistivity is $R_t$, there is the following equation.

$$\frac{1}{R_t(L/A)} = \sum_{i=1}^{n} \frac{1}{R_w(L_{wt}/A_{wt})} + \sum_{j=1}^{m} \frac{1}{R_w(L_{wt}/(A_b - A_{wb}))}$$

Assuming that all the capillary columns have the same length,

$$L_a = L_b = L_A,$$  \hspace{1cm} (23)

$$L_{wf} = L_{wc} = L_{wz} = L_{wb} = L_w.$$  \hspace{1cm} (24)

The resistivity index $I$ is

$$I = \frac{R_t}{R_0}.$$  \hspace{1cm} (25)

So, the water relative permeability $k_{rw}$ can be obtained.

$$k_{rw} = \frac{(S_w - S_{wc})^2}{(1 - S_{wc} - S_{or})^2 + (S_{wc} + S_{or})^2} \frac{L_A^2}{r_A^2}.$$  \hspace{1cm} (26)

Define the resistivity $R_b$.

$$\frac{S_{wc}}{R_b} = \frac{S_{wc} - S_{wb}}{R_w} + \frac{S_{wb}}{R_{wb}}.$$  \hspace{1cm} (27)

Define $SR$ can be calculated as follows.

$$SR = \frac{R_b}{R_w} \frac{1 - S_{or} - S_{wc}}{S_{wc}}.$$  \hspace{1cm} (28)

Therefore, the normalized water relative permeability in the dual water relative permeability model can be expressed as follows:

$$k^{*}_{rw} = \frac{k_{rw}}{k_{rw}(S_w = 1 - S_{or})},$$  \hspace{1cm} (29)

$$k^{*}_{rw} = \left( \frac{S_w'}{S_w} \right)^2 \left( \frac{R_{wb}/R_b}{(SR + 1) - 1} \right) \frac{R_t}{I(S_{S_w} + 1) - 1}.$$  \hspace{1cm} (30)

When $S_{wc} = S_{wc}$, $k^{*}_{rw} = 0$. When $S_{w} = 1 - S_{or}$, $S_{w}' = 1$ and $k^{*}_{rw} = 1$, which satisfies the boundary condition.

When the clay water content is 0 ($S_{wb} = 0$), the model is simplified to a clean sandstone model, and the normalized water relative permeability is as follows.

$$k^{*}_{rw} = \left( \frac{S_w'}{S_w} \right)^2 \frac{R_{wb}/R_b}{(1 - S_{or}) - R_0 S_{wc}}.$$  \hspace{1cm} (31)

3.1. Determination of the Parameters in the DW Relative Permeability Model

3.1.1. Calculation of the Resistivity of Clay Water $R_{wb}$

Diffusion factor of Na$^+$ ion diffusion layer $\alpha$ is calculated as follows [21]:

\[ \text{FIGURE 1: Capillary bundle model.} \]
\alpha = \begin{cases} 
1, & \text{when } P_w > P_{w_0}, \\
\frac{P_{w_{0}}}{P_w}, & \text{when } P_w \leq P_{w_0}, 
\end{cases} \quad (32)

where $P_w$ is the salinity of formation water, $P_{w_0}$ is the salinity of formation water when $x_d = x_{d1}$, and $x_d$ is the thickness of Na$^+$ ion diffusion layer (10$^{-8}$ cm).

The pore volume occupied by clay water $V_Q$ when $Q_v = 1$ mmol/cm$^3$ is calculated as follows:

$$V_Q = \frac{1}{2.853 + 0.019T(°C)}. \quad (33)$$

The equivalent conductivity $\beta$ of compensation Na$^+$ ion in clay water (S/m) (mmol/L) is calculated as follows:

$$\beta = 0.0857T(°C) - 0.143. \quad (34)$$

The clay water resistivity $R_{wb}$ is calculated as follows:

$$R_{wb} = \frac{\alpha V_Q}{\beta}, \quad (35)$$

$$R_{wb} = \alpha \frac{0.0857T(°C) - 0.143}{2.853 + 0.019T(°C)}. \quad (36)$$

It can be seen from the above formula that the resistivity of formation water is affected by both $\alpha$ and temperature $T$. When the water salinity is high, $\alpha = 1$. Therefore, the resistivity of clay water $R_{wb}$ is independent from the equilibrium concentration and clay types.

### 3.1.2. Calculation of the Clay Water Saturation $S_{wb}$

$$S_{wb} = \alpha V_Q Q_v. \quad (37)$$

Substitute equation (33) into equation (37),

$$S_{wb} = \frac{\alpha Q_v}{2.853 + 0.019T(°C)}. \quad (38)$$

From the above formula, it can be seen that the clay water saturation $S_{wb}$ increases with the increase of $\alpha$. As the temperature $T$ increases, $S_{or}$ decreases. With the increase of $Q_v$, $S_{wb}$ increases.

### 3.1.3. Calculation of the Resistivity $R_{wb}$

$$\frac{S_{wb}}{R_{wb}} = \frac{S_{wb} - S_{or}}{R_w} + \frac{S_{wb}}{R_{wb}}. \quad (39)$$

### 3.1.4. Calculation of the Parameter SR

$$SR = \frac{R_w}{R_{wb}} \frac{1 - S_{or} - S_{wc}}{S_{wb}}. \quad (40)$$

### 3.1.5. Calculation of the Normalized Water Saturation $S_{w}^{′}$

$$S_{w}^{′} = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}. \quad (41)$$

### 3.1.6. Calculation of the Normalized Water Relative Permeability $k_{rw}^{′}$

$$k_{rw}^{′} = \left(S_{w}^{′}\right)^2 \frac{(R_{or}/R_w)(SR + 1) - 1}{I \left(SRS_{or} + 1\right) - 1}. \quad (42)$$

In conclusion, the DW relative permeability model can be expressed as follows.

$$\begin{align}
(1) & \alpha = \begin{cases} 
1, & \text{when } P_w > P_{w_0}, \\
\frac{P_{w_{0}}}{P_w}, & \text{when } P_w \leq P_{w_0}, 
\end{cases} \\
(2) & R_{wb} = \alpha \frac{0.0857T(°C) - 0.143}{2.853 + 0.019T(°C)}, \\
(3) & S_{wb} = \frac{\alpha Q_v}{2.853 + 0.019T(°C)}, \\
(4) & S_{wc} = S_{wc} - S_{wb} + \frac{S_{wb}}{R_{wb}}, \\
(5) & SR = R_w \frac{1 - S_{or} - S_{wc}}{R_w}, \\
(6) & S_{w}^{′} = \frac{S_w - S_{wc}}{1 - S_{wc} - S_{or}}, \\
(7) & k_{rw}^{′} = \left(S_{w}^{′}\right)^2 \frac{(R_{or}/R_w)(SR + 1) - 1}{I \left(SRS_{or} + 1\right) - 1}, \\
(8) & k_{or}^{′} = \left(1 - S_{w}^{′}\right)^2 \left(1 - k_{rw}\right).
\end{align}$$

### 3.2 Sensitivity Analysis of Parameters of DW Model of Shaly Sandstone

#### 3.2.1. The Effect of Irrducible Water Saturation $S_{wc}$ on $k_{rw}^{′}$

Suppose that $S_{or} = 0.2$, $S_{wb} = 0.05$, $R_w = 30 \Omega \cdot m$, $R_{or} = 35 \Omega \cdot m$, $R_w = 0.5 \Omega \cdot m$, and $R_{wb} = 0.1 \Omega \cdot m$, oil-water relative permeability curves under different irradiated water saturation $S_{wc}$ are shown in Figure 2. It indicates that the normalized water relative permeability $k_{rw}^{′}$ decreases while $S_{wc}$ increases. The reason is that the movable water saturation $S_w$ decreases under the same normalized water saturation $S_{wc}$ as $S_{wc}$ increases, thus the normalized relative permeability of water phase $k_{rw}$ decreases.

#### 3.2.2. The Effect of Residual Oil Saturation $S_{or}$ on $k_{rw}^{′}$

Suppose that $S_{wc} = 0.2$, $S_{wb} = 0.05$, $R_w = 30 \Omega \cdot m$, $R_{or} = 35 \Omega \cdot m$, $R_w = 0.5 \Omega \cdot m$, and $R_{wb} = 0.1 \Omega \cdot m$, the oil-water relative permeability curves under different residual oil saturation $S_{or}$ are shown in Figure 3. It illustrates that the normalized water relative permeability $k_{rw}^{′}$ decreases with the increase of $S_{or}$.
because the movable water saturation $S_{wM}$ decreases under the same normalized water saturation $S_w$ when $S_w$ increases, which leads to the decrease of normalized relative permeability of water phase $k_{rw}$.

3.2.3. The Effect of Cation Exchange Capacity $Q_C$ on $k_{rw}$. Suppose that there are a set of cores with the same parameters as follows. $T = 20^\circ C$, $P_w = 8000$ ppm, $S_{wc} = 0.5$, $S_o = 0.2$, $R_w = 5 \, \Omega \cdot m$, $R_o = 33 \, \Omega \cdot m$, and $R_{oc} = 35 \, \Omega \cdot m$. Normalized oil-water relative permeability curves under different $Q_C$ are shown in Figure 4. It illustrates that the normalized water relative permeability $k_{rw}$ decreases with the increase of $Q_C$.

Studies [22–24] show that clay mineral content is one of the main factors affecting the shape of oil-water relative permeability curve of rock. When water is injected into the core sample, it first enters into larger pores, where the relative permeability of the water phase increases rapidly. Soon after the injection, water gradually enters into small pores, where the flow resistance increases. At the same time, the oil in large pore paths is separated into small oil droplets by the water. If the oil droplets migrate to the vicinity of the pore throat, the so-called “liquid resistance effect” will emerge when the diameter of the oil droplets is similar to that of the pore throat. In this case, the capillary force of the oil-water interface throat must be overcome if the oil droplets want to move [25, 26]. The hydrophilic particles in the pores will move to the pore throat and cause blockage. With the increase of water saturation $S_w$, the amount of plugging particles will increase, and the relative permeability of water phase will decrease accordingly.

3.2.4. The Effect of Total Salinity $P_w$ on $k_{rw}$. Suppose that $T = 20^\circ C$, $Q_s = 0.25$ mmol/L, $S_{wc} = 0.5$, and $S_{oc} = 0.3$. Figure 5
shows the normalized oil-water relative permeability curves under different $P_w$. It illustrates that the normalized water relative permeability $k_{rw}^*$ increases with increased $P_w$.

In shaly sand reservoir, with the decrease of salt content in free water, the salinity in free water decreases, clay mineral crystal layer expands, and the formation permeability continues to decline, which is called the salt-sensitive phenomenon (Meng, 2012). There are a large number of clay minerals in shaly sand reservoir. Therefore, with the decrease of free water salinity, the salt content in free water decreases, the salt-sensitive phenomenon gets worse, and the relative permeability of water phase goes down.

4. Experimental Verification of DW Relative Permeability Model

4.1. Experiments. In order to verify the relationship between resistivity and relative permeability in DW Model, a multifunctional core displacement experiment device was designed.
(Figure 6). The resistivity and relative permeability of core samples with different water saturations were measured. Core samples from Wells A and B are tested to explore the relationship between resistivity and relative permeability. Core samples are divided into two groups. Group I contains less clay and smaller Qv than group II. The basic physical parameters of cores are shown in Table 1. The salinity of formation water in wells A and B are

| Well | Core | Water Saturation | Relative Permeability |
|------|------|------------------|-----------------------|
| A    | A-7  | 0.2              | k_rw:experiment       |
|      | A-8  | 0.4              | k_ro:experiment       |
|      | A-9  | 0.6              | k_rw:Li model         |
|      | A-10 | 0.8              | k_ro:Li model         |
|      |      | 1.0              | k_rw:modified Li      |
|      |      |                   | k_ro:modified Li      |

Figure 7: Comparison of relative permeability curves in sandstone reservoir.
7500 ppm and 8000 ppm, respectively. The density of brine used is 1.02 g/cm³. The oil viscosity in both wells is 8.8 mPa s at 20°C, and its density is 0.845 g/cm³.

4.2. Verification of Relationship Models between Resistivity and Relative Permeability of Sandstone. The Li model, modified Li model, and Pairoys model are verified with the
experimental data of core samples in group I. Figure 7 shows the oil-water relative permeability curves of group I cores. In Figure 7(a), the solid blue triangular and pink dots are the normalized water and oil relative permeability obtained from the water displacing oil experiment, respectively; the brown and sky blue chain dotted lines are the normalized water and oil relative permeability calculated with resistivity based on the Li model; the green and purple broken line are the normalized water and oil relative permeability calculated with resistivity based on Pairoys model; and the blue and red solid line are the ones calculated with resistivity based on the modified Li model. Figure 7(a) indicates that the Li model does not work well in the data process of water flooding. The problems of Pairoys model is that the normalization oil relative permeability it calculates is nonnegligibly larger than the experimental value. However, the normalization water and oil relative permeabilities calculated by the modified Li model are in good agreement with experimental data.

Figure 7(b) shows the oil-water relative permeability curve of cores in group I. The water relative permeabilities calculated by the Li model and Pairoys model are smaller than the experimental data, while the oil relative permeabilities are larger. The oil and water relative permeabilities calculated by the modified Li model fit well with the experimental data in sandstone reservoir with less shale contents.

4.3. Verification of Relationship Models between Resistivity and Relative Permeability of Shaly Sandstone. Experiment is designed to measure the resistivity and relative permeability of shaly sandstone samples in group II. Figure 8 shows the comparison of the experimental results and the model calculated results.

Figure 8(a) shows the normalized relative permeability curves, and Figure 8(b) shows the relative permeability curves. The filled dots are the unsteady oil-water relative permeability experiment data, the broken lines are the relative permeability curves calculated with resistivity by the modified Li model, and the solid lines are calculated with resistivity by the DW model. As shown in Figure 8, the Li model, the water relative permeability calculated by the modified Li model fits well with the experimental data, but the calculated oil relative permeability curve is smaller than the experimental data. Meanwhile, the relative permeability curves of oil and water calculated by the DW relative permeability model better fit the experimental data.

5. Conclusions

This study established the relationship model between the resistivity and oil-water relative permeability of the shaly sandstone reservoir based on the rock physics experiment and the logging response of the shaly sandstone reservoir. According to existing research results, the following main conclusions can be drawn.

(1) In view of the influence of shale, the DW relative permeability model, suitable for shaly sandstone reservoir, was derived to calculate oil-water relative permeability using resistivity based on the dual water conductivity model, Poiseuille’s equation, and Darcy’s law.

(2) According to the sensitivity analysis, with other conditions being the same, the relative permeability of water phase will decrease as the irreducible water saturation increases, residual oil saturation increases, cation exchange capacity of rock increases, or free water salinity decreases.

(3) With the core water flooding experimental device, the resistivity and oil-water relative permeability of two groups of sandstone samples with different shale contents were tested. The experimental results show that the modified Li model is suitable for clean sandstone reservoirs, and the DW relative permeability model is suitable for shaly sandstone reservoirs.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declared that they have no conflicts of interest to this work.

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