Dc Electrical Current Generated by Upstream Neutral Modes

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Quantum Hall phases are gapped in the bulk but support chiral edge modes, both charged and neutral. Here we consider a circuit where the path from the source of electric current to the drain necessarily passes through a segment consisting solely of neutral modes. We find that upon biasing the source, a dc electric current is detected at the drain, provided there is backscattering between counter-propagating modes under the contacts placed in certain locations. Thus, neutral modes carry information that can be used to nonlocally reconstruct a dc charge current. Our protocol can be used to detect any neutral mode that counterpropagates with respect to all charge modes. Our protocol applies not only to the edge modes of a quantum Hall system, but also to systems that have neutral modes of non-quantum Hall origin. We conclude with a possible experimental realization of this phenomenon.

I. INTRODUCTION

The quantum Hall effects (QHE)1 are the earliest known example of topological insulators2. They have a charge gap in the bulk, and all currents are carried by edge/surface modes, which can be either charged (with fractional charge in the fractional QHE) or neutral chiral modes. While the charge modes produce quantized electrical conductance, neutral modes are a manifestation of topology, electron-electron interactions, and possibly disorder, and contribute to heat transport. Neutral edge modes in quantum Hall systems have been detected by shot noise experiments3 and also by their quantized heat transport coefficients4. Apart from quantum Hall systems, neutral (e.g. Goldstone) modes arise in systems in which a continuous symmetry is broken spontaneously.

In this work, we design a geometry where the unique current path from the source to the drain is forced to pass through a segment consisting of neutral modes only. We assume that the $U(1)$ symmetry of each channel is broken by the contacts; thus backscattering between channels is present under them. The breaking of these $U(1)$ symmetries results in a non-zero dc current at the drain D. This protocol can be used either as a transformer, which converts charge current to neutral current, and then back to charge current, or as an efficient detector of neutral modes as long as the neutral counterpropagates with respect to all charge modes.

The proposed geometry is shown in Fig. 1. The relevant physics can be extracted by focusing on regions II, III, and IV. The solid black line at the top is a right-moving chiral charged mode, arising from a $\nu = 1$ quantum Hall system extending above the figure, which is the “probe” system. Charges are injected at the source S and detected at the drain D. The “test” system extends below the figure, and has two counterpropagating neutral modes (dashed black lines), and possibly other charged chiral modes (dash-dotted orange lines), which all have to be right-moving for our scheme to be relevant. The edges of the two systems overlap only in regions II and IV, separated from the active region III by boundaries $B_2$, $B_3$. Density-density interactions between the chiral modes of the top and bottom systems exist only in regions II and IV, which also host the contacts $C_1$ and $C_2$. Regions I and V are present to specify boundary conditions. Tunneling/scattering between the chiral modes occurs solely under the contacts. Reflection and transmission of a right-moving charge injected at S is shown schematically at $B_3$, while a similar process for a left-moving neutral excitation is shown at $B_2$.

FIG. 1. A single right-moving chiral charged mode (solid black line) represents the edge of a $\nu = 1$ quantum Hall system extending above the figure, which is the “probe” system. Charges are injected at the source S and detected at the drain D. The “test” system extends below the figure, and has two counterpropagating neutral modes (dashed black lines), and possibly other charged chiral modes (dash-dotted orange lines), which all have to be right-moving for our scheme to be relevant. The edges of the two systems overlap only in regions II and IV, separated from the active region III by boundaries $B_2$, $B_3$. Density-density interactions between the chiral modes of the top and bottom systems exist only in regions II and IV, which also host the contacts $C_1$ and $C_2$. Regions I and V are present to specify boundary conditions. Tunneling/scattering between the chiral modes occurs solely under the contacts. Reflection and transmission of a right-moving charge injected at S is shown schematically at $B_3$, while a similar process for a left-moving neutral excitation is shown at $B_2$. with neutral modes only, such as an XXZ chain. Electrons are injected from the source S via tunnelling into the probe chiral edge mode and detected at the drain D.
The source and drain are separated by a grounded contact G. Clearly, current cannot flow from S to D along the right-moving, chiral top edge. The edge modes of the probe and test systems overlap, and thus interact, only in regions II and IV. The interaction is of the density-density form, with separate number conservation in the “bare” charged and neutral modes. These interactions renormalize the bare charged and neutral modes such that, generically, all three renormalized eigenmodes have nonuniversal charge. Regions II and IV also host the contacts C₁ and C₂ respectively, which we assume can be decoupled from the interacting modes at will. Finally, regions I and V are semi-infinite “free” regions, where the edges of the probe and test systems are fully decoupled and are present to fix the asymptotic boundary conditions.

Before proceeding we discuss the notion of ideal contacts. The latter refers to terminals connected to the edge modes, which absorb the entire impinging current with no detectable signal away from the contacts. Ideal contacts have been discussed in Refs. 7, 9, 11 in the absence of interactions, and, in the presence of interactions, in 7, where it was shown that a microscopic realization of an ideal contact for counterpropagating edge modes requires backscattering between them.

Our results can be encapsulated in two ways: Firstly, neutral modes can carry information about the charge current, information that can be used to reconstruct the charge current at a different location. Secondly, one can use the charge chiral mode (top mode of Fig. 1) as a “probe”, and apply it to a “test” system (bottom of Fig. 1). In this functionality, our device can be used to detect coherently propagating bosonic neutral modes in the test system. A dc charge current at the drain is direct evidence for neutral modes.

More concretely, let us assume there is at least one left-moving neutral mode in the test system. When electrons are injected at the source S if both C₁ and C₂ are coupled to the modes in region II and IV respectively, a dc current will be detected at D, regardless of whether the test system has (right-moving) charge chirals or not. The presence of right-moving chiral charge modes in the test system will not change this conclusion qualitatively.

Let us understand the physics in two extreme limits, when (i) both the contacts are coupled, or (ii) both of the contacts are decoupled.

Case (i) Both contacts coupled: Assuming no charge chiral modes in the test system, consider a charge (positive by fiat) injected into the probe chiral at S, which travels to the boundary B₁. There, a lump of positive neutral density (a neutralon) is reflected into the left-moving neutral mode in region III and lumps of nonuniversal charge are transmitted into the two right-moving modes in region IV to be fully absorbed at C₂. The left-moving neutralon in III travels to B₂, at which point a positive (electrically) charged lump is reflected into the probe chiral, and an equal and opposite charge is transmitted into the left-moving mode in the region II, to be fully absorbed at C₁. There will also be a neutralon reflected into the right-moving neutral chiral in region III, which travels to B₃. As usual, this will undergo transmission and reflection, with the transmitted part being completely absorbed at C₂. The reflected neutralon part has the same sign as the original neutralon, and repeats the process described earlier with a smaller amplitude. With both contacts coupled, an infinite sequence of charge lumps of the same sign is detected at D. Thus, a dc current at S implies a dc current of the same sign (but with a nonuniversal magnitude) at D.

This is already an instance of the effect we are looking for. Now we add (right-moving) charge chirals to the test system. All proceeds as before until the left-moving neutralon impinges on B₂. Now, in addition to the reflected neutral lump, charge lumps will be transmitted into the nonuniversal charge modes in region II (to be absorbed at C₁), and reflected into the probe and test charge chirals. The magnitude and sign of the charges are determined by the interaction parameters in region II. Recall that the reflection/transmission is deterministic because no tunneling between the different modes is involved. Thus, there is a dc current at D.

To summarize, when both contacts are coupled, if a left-moving neutral is present in the test system, there is always a dc current at D, as long as the charge chirals (if any) of the test system are all right-moving.

Case (ii) Both contacts decoupled: Initially, let us assume that no charge chiral modes are present in the test system. The first step (the injected lump of electric charge traveling from S to B₁) results in the reflection of a neutralon and transmission of lumps of nonuniversal charge in the two right-moving chirals in region IV) is the same as before. However, now the right-moving lumps in region IV travel to B₁ and undergo repeated partial reflection and transmission. Similarly, the left-moving neutralon, upon arriving at B₂, results in a charge lump in the probe charge chiral in III, and a left-moving charge lump in region II. This latter lump will undergo partial transmission/ reflection at B₁. This leads to multiple scattering at all the boundaries. However, we can assert, based on charge conservation, that no dc current is observed at D. Since no left-moving charge modes enter region III, the entire charge injected at S has to proceed to region V (after multiple scattering in region IV). Any charge detected at D is initiated by a neutralon arriving at B₂ via the left-moving neutral in III and its descendants via multiple scattering. Since no total (time-integrated) charge enters region III from either of regions II or IV, the time-integrated charge entering the drain D must vanish. Evidently, charge noise will be detected at D. Similar logic ensures that the dc charge current exiting region IV into region V is the entire charge current injected at S.

These conclusions do not change when we allow (right-moving) charge modes in the test system. Since the interactions in regions II and region IV are density-density interactions, the total U(1) “charge” (which is completely
independent of electric charge) of each mode has to be conserved in the dc limit. Thus, we conclude, that in the presence of (right-moving) charge chiral modes in the test system, we still need both the contacts to be coupled in order to have a non-zero current at the drain D.

In what follows, we will present an outline of the calculations leading to our results, relegating straightforward mathematical details to the supplemental material (SM[13]). For simplicity, we will focus on the case where the test system has neutral modes only.

We model the neutrals by an XXZ spin chain and the interaction between the spin chain and the spin-polarized charged mode as a spin-spin interaction. The model is described by the action in Eq. 1 where the probe charged mode is represented by the bosonic field $\phi_1$, the right-moving test neutral by $\phi_2$ and the left-moving test neutral by $\phi_3$. The interaction between the neutrals $\phi_2$ and $\phi_3$ is denoted by $\lambda_{23} (x)$. The interaction between the charged mode and the spin chain, (the same for both the left- and right-moving neutrals), is denoted by $\lambda_{12}(x) (= \lambda_{13}(x))$

$$S = \frac{1}{4\pi} \int dx dt \left[ -\partial_x \phi_1 (\partial_t \phi_1 + v_1 \partial_x \phi_1) - \partial_x \phi_2 (\partial_t \phi_2 + v_2 \partial_x \phi_2) - \partial_x \phi_3 (\partial_t \phi_3 - v_2 \partial_x \phi_3) - 2\lambda_{12}(x) \partial_x \phi_1 (\partial_t \phi_2 + \partial_x \phi_2) - 2\lambda_{23}(x) \partial_x \phi_2 \partial_x \phi_3 \right].$$ (1)

Assuming the interactions are turned on abruptly in regions II and IV, we calculate the reflection ($r_{ij}^{B_n}$) and transmission coefficients ($t_{ij}^{B_n}$) at B$_2$ and B$_3$, which allows us to compute the current at D as a function of time via multiple reflections[13].

![Image](image_url)

**FIG. 2.** The dc current at D as a function of the $\lambda_{12}$ and $\lambda_{23}$ for two different values of $v_1,v_2$, when C$_1$ C$_2$ is coupled.

**III. EXPERIMENTAL REALIZATION**

We now discuss an experimental realization of our setup. For monolayer graphene, Hartree-Fock calculations suggest[17] that at charge neutrality ($\nu = 0$), there is a quantum phase transition between a canted antiferromagnetic (CAF) phase, stabilized for purely perpendicular magnetic field, and a spin-polarized phase which can be stabilized by increasing the Zeeman energy $E_Z$ with an in-plane $B$ field. The spin-polarized phase has a fully gapped bulk and a pair of gapless helical edge modes[18–20], whereas the CAF phase breaks $U(1)$ spin-rotation symmetry and has a neutral Goldstone mode in the bulk, but no gapless charged edge mode[21].

Experimentally, the phase transition has been seen[22,23], but evidence that the phase at purely perpendicular $B$ is the CAF phase is indirect, via the detection of magnon transmission above the Zeeman energy[23]. Indeed, recent scanning tunneling spectroscopy measurements indicate that the ground state has bond-order[24,25]. To confirm that the system has CAF order one would need to detect gapless collective excitations, as has been done recently in bilayer graphene[27].

A potential experimental realization of the central idea of this paper is shown in Fig. 3. A sheet of graphene in a perpendicular $B$ field is gated such that the left half is at filling $\nu = 1$, while the right half is at $\nu = 0$. In the central part of the $\nu = 0$ region, we overlay graphene with a ferromagnetic insulator, whose exchange field makes the graphene under it fully polarized and gapped. However, the annular periphery of $\nu = 0$ region is in the putative...
CAF phase, with a gapless Goldstone mode. No topological edge modes exist between the two phases at $\nu = 0$. Confinement in the “radial” direction in the $\nu = 0$ region will reconstruct the continuum of bulk Goldstone modes into bands of clockwise-moving and anticlockwise-moving neutral modes. The lowest two bands will be gapless, and represent the counterpropagating neutral modes in Fig. 1. These counter-propagating neutral modes interact with the charge edge mode of the $\nu = 1$ quantum Hall phase on the left in the regions where they are proximate (Fig. 3). Adding the source S, drain D, and grounded contact G at appropriate locations realizes the setup of Fig. 1 and provides a way to unambiguously detect the gapless neutral Goldstone mode of the CAF.

![Fig. 3](image)

FIG. 3. A sheet of graphene in a perpendicular $B$-field is gated to have $\nu = 1$ on the left and $\nu = 0$ on the right. The central region of $\nu = 0$ is overlain by an insulating ferromagnet, inducing the fully polarized phase of $\nu = 0$ graphene in this region. The periphery of $\nu = 0$ is presumed to be in the CAF state, with gapless Goldstone modes. The lowest sub-band of the radially confined Goldstone modes interacts with the $\nu = 1$ edge mode and is detected by the scheme described in the text.

IV. SUMMARY AND OUTLOOK

In this work we have proposed a setup that has two functionalities: (i) Given a system known to have a neutral mode (the bottom system in Fig. 1), we encode information about the charge current into the neutral current, and subsequently read it out as a dc charge current at a different spatial location. (ii) Given a test system suspected of having coherently propagating, bosonic, neutral edge modes when all the charge modes in the test system are gapped, and gapless modes represent spin/valley fluctuations. Trivial insulators with spontaneous symmetry breaking of a continuous symmetry, such as the putative CAF phase of graphene at charge neutrality, are prime examples of such systems. Moreover, our setup will detect coherently propagating, bosonic, neutral edge modes in QH systems as well, as long as two conditions are met: (i) all chiral charge modes of the test QH system are co-propagating, and (ii) there is at least one neutral mode which counter-propagates with respect to the charge modes. For example, the neutral mode of $\nu = 2/3$ at the Kane-Fisher-Polchinski fixed point could be detected by our setup. Using monolayer graphene for the probe system allows one to reverse the propagation direction of the probe charge chiral in situ by gating to obtain $\nu = \pm 1$ in order to realize the geometry of Fig. 1. It must be noted that pairs of neutral edge modes can be generated by edge reconstructions in quantum Hall systems. We emphasize that our setup can detect coherently propagating bosonic neutral modes regardless of their physical origin.

Let us compare our setup with previous approaches to neutral mode detection. In one approach, the passage of upstream neutral modes through a quantum point contact was detected through the generation of charge noise. More recently, measurements of heat transport “upstream” as compared to charge transport have been employed. Not only are these hard measurements, (they require a precise determination of the temperature at a given contact), but they cannot determine whether the heat propagating upstream reflects coherent neutral modes rather than incoherent transport (e.g., due to diffusive modes). The latter is the result of charge and heat equilibration, and also leads to upstream charge noise.

A second theoretical approach for detecting neutral modes in certain quantum Hall systems via dc currents depends on tunneling between QH edges at quantum point contacts, and only specific neutral modes in specific configurations lead to dc currents. In our proposal, tunneling between different chiral modes occurs only under the contacts.

There are a few unresolved issues of broad import: (i) How does one understand the formulation of linear and non-linear response in the charge-neutral-charge circuit? (ii) Certain exotic spin systems are believed to have neutral Majorana modes as the $\nu = 5/2$ state. Our proposed device can detect bosonic neutral modes, but can some extension thereof be used to detect Majorana modes as well?
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Supplement to Dc Electrical Current Generated by Upstream Neutral Modes

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Here we describe the details of the calculation in supplement to the main text.

In this set of supplemental materials, we provide details of the action (Section SI), how the reflection and transmission coefficients are computed (Section SII), and how the current and the current noise at the drain are computed (Section SIII). While most of our analysis is in the case when the test system contains neutral modes only, we also analyze (Section SIV) an interesting special case when the “test” system (bottom of Fig. 1 in the main text) is the fractional quantum Hall state at ν = 2/3, and thus has charge as well as neutral edge modes.

SI. ACTION AND EIGENMODES

In this section, we present details of the case considered in the main text, which assumes that there are no charge modes in the test system.

We define eigenmodes in each region, depending on the interaction strengths, as a linear combination of the bare modes. In regions where the test charged chiral mode is coupled (regions of II and IV) we write the bare fields φα in terms of the eigenmodes of Eq. S1

\[ \tilde{\phi}_\alpha = M_{\alpha\beta} \phi_\beta. \]  

(S2)

Similarly for region j = I, III we can write the bare fields in terms of the eigenmodes (φβj) as,

\[ \phi_\alpha = N_{\alpha\beta} \phi_\beta. \]  

(S3)

We will use these definitions to calculate the reflection and transmission coefficients at every boundary. The matrix N depends on a single parameter because only the two bare neutral modes of the test system are mixed. However, the matrix M is more complex, and can be written as a real member of the group SO(2,1). More explicitly,

\[
M = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cosh(\xi_1) \sinh(\xi_1) & 0 \\
0 & \sinh(\xi_1) \cosh(\xi_1) & 0
\end{bmatrix} \cdot \begin{bmatrix}
\cosh(\xi_2) & 0 & \sinh(\xi_2) \\
0 & 1 & 0 \\
\sinh(\xi_2) & 0 & \cosh(\xi_2)
\end{bmatrix}.
\]

(S4a)

\[
N = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cosh(\xi) \sinh(\xi) & 0 \\
0 & \sinh(\xi) \cosh(\xi) & 0
\end{bmatrix}.
\]

(S4b)

The θ,ξi appearing in these matrices can be determined in a straightforward manner from the action.

FIG. S1. The geometry of our detector, for the case, when the test system has no charge chiral edge modes. In regions II and IV there are density-density interactions between the test charge chiral and the test neutral chiral. C1 and C2 are perfect ohmic contacts while S, D are the source and drain separated by the grounded contact G.

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SII. FINDING REFLECTION AND TRANSMISSION

The equations of motion resulting from the action of Eq. S1 are

\[ \frac{\partial^2 \phi_1}{\partial x \partial t} + v_1 \frac{\partial^2 \phi_1}{\partial x^2} + \frac{d}{dx} \left[ \lambda_{12}(x) \left( \frac{\partial \phi_2}{\partial x} + \frac{\partial \phi_3}{\partial x} \right) \right] = 0 \]  
(S5a)

\[ \frac{\partial^2 \phi_2}{\partial x \partial t} + v_2 \frac{\partial^2 \phi_2}{\partial x^2} + \frac{d}{dx} \left[ \lambda_{12}(x) \frac{\partial \phi_1}{\partial x} + \lambda_{23} \frac{\partial \phi_3}{\partial x} \right] = 0 \]  
(S5b)

\[ \frac{\partial^2 \phi_3}{\partial x \partial t} - v_3 \frac{\partial^2 \phi_3}{\partial x^2} - \frac{d}{dx} \left[ \lambda_{12}(x) \frac{\partial \phi_1}{\partial x} + \lambda_{23} \frac{\partial \phi_2}{\partial x} \right] = 0. \]  
(S5c)

For future convenience, we have kept the notations \( v_2 \) and \( v_3 \), though for the purposes of this section \( v_2 = v_3 \).

Fourier transforming them to the frequency domain results in the following equations.

\[ -i \omega \phi_1' + v_1 \phi_1' + [\lambda_{12}(x) (\phi_2' + \phi_3')] = 0 \]  
(S6a)

\[ -i \omega \phi_2' + v_2 \phi_2' + [\lambda_{12}(x) \phi_1' + \lambda_{23} \phi_3'] = 0 \]  
(S6b)

\[ -i \omega \phi_3' - v_3 \phi_3' - [\lambda_{12}(x) \phi_1' + \lambda_{23} \phi_2'] = 0. \]  
(S6c)

Here prime represents derivative with respect to \( x \). Now we integrate across the boundary, assuming \( \phi_i \) to be continuous across it. This leads to the following boundary conditions across an arbitrary boundary where the interaction strengths change abruptly:

\[ v_1 [\phi_1']^0_+ + [\lambda_{12}(x) (\phi_2' + \phi_3')] = 0 \]  
(S7a)

\[ v_2 [\phi_2']^0_+ + [\lambda_{12}(x) \phi_1' + \lambda_{23} \phi_3'] = 0 \]  
(S7b)

\[ v_3 [\phi_3']^0_+ + [\lambda_{12}(x) \phi_1' + \lambda_{23} \phi_2'] = 0. \]  
(S7c)

Here \( 0\pm \) represents across the boundary, and depending on the boundary, the values of \( \lambda_{ij} \) change.

A. Boundary type I

This type of boundary corresponds to the probe chiral not interacting with anything to the left, while it interacts with the test chirals on the right, as in the cases of boundaries B₁ and B₃. If the boundary is at \( x = 0 \) we have

\[ \lambda_{12}(x) = \lambda_{12} \theta(-x). \]  
(S8)

We now write the boundary conditions for the three incoming channels as

Case I: Source coming from the left through channel 1, i.e.

\[ (\tilde{\phi}_1')_{0-} = (\tilde{\phi}_1')_{0+} = 1, \]  
(S9a)

\[ (\tilde{\phi}_2')_{0-} = N_{2\beta}^{-1} (\tilde{\phi}_{\beta}')_{0-} = 0 \]  
(S9b)

Using these boundary conditions with Eq. S7 we solve for the reflection and transmission coefficients. We will denote the transmission coefficient \( (t_{αβ}^{B_j}) \) and reflection coefficients \( (r_{αβ}^{B_j}) \) where the respective transmission and reflection happens from \( α \) mode to \( β \) mode at boundary \( B_j \). We will use this notation from now on.

Case II: Source coming from the left through channel 2, i.e.

\[ (\tilde{\phi}_2')_{0-} = N_{2\beta}^{-1} (\tilde{\phi}_2')_{0-} = 1 \]  
(S10a)

\[ (\tilde{\phi}_1')_{0+} = M_{1\beta}^{-1} (\tilde{\phi}_2')_{0+} = 0 \]  
(S10b)

Case III: Source coming from the right through channel 3, i.e.

\[ (\tilde{\phi}_3')_{0+} = M_{3\beta}^{-1} (\tilde{\phi}_3')_{0+} = 1 \]  
(S11a)

\[ (\tilde{\phi}_2')_{0-} = N_{2\beta}^{-1} (\tilde{\phi}_2')_{0-} = 0 \]  
(S11b)

B. Boundary type II

In this type of boundary, the probe chiral interacts with the test chiral for \( x < 0 \), but does not interact for \( x > 0 \). Examples are B₂ and B₄, where we have

\[ \lambda_{12}(x) = \lambda_{12} \theta(x). \]  
(S12)

For this let us again write down different boundary conditions,

Case I: Source coming from the left through channel 1, i.e.

\[ (\tilde{\phi}_1')_{0-} = M_{1\beta}^{-1} (\tilde{\phi}_1')_{0-} = 1 \]  
(S13a)

\[ (\tilde{\phi}_2')_{0+} = M_{2\beta}^{-1} (\tilde{\phi}_2')_{0+} = 0 \]  
(S13b)

Case II: Source coming from the left through channel 2, i.e.

\[ (\tilde{\phi}_2')_{0-} = M_{2\beta}^{-1} (\tilde{\phi}_3')_{0-} = 1 \]  
(S14a)

\[ (\tilde{\phi}_3')_{0+} = N_{3\beta}^{-1} (\tilde{\phi}_3')_{0+} = 0 \]  
(S14b)
Case III: Source coming from the right through channel 3, i.e., \( (\tilde{\phi}_j)′ \bigg|_{0^+} = N_{3\beta}^{-1} (\phi_{\beta})′ \bigg|_{0^+} = 1 \) and
\[
\begin{align*}
(\tilde{\phi}_1)′ \bigg|_{0^+} &= M_{1\beta}^{-1} (\phi_{\beta})′ \bigg|_{0^+} = 0 \\
(\tilde{\phi}_2)′ \bigg|_{0^+} &= M_{2\beta}^{-1} (\phi_{\beta})′ \bigg|_{0^+} = 0
\end{align*}
\]
(S15a) (S15b)

III. CURRENT AND NOISE AT D

![Graph showing current and noise as a function of \( \lambda_{12} \) for \( \lambda_{33} = 0.5 \) for velocities \( v_1 = 1.0 \), \( v_2 = 1.1 \) when both contacts are coupled.]

FIG. S2. Dc current and noise as a function of \( \lambda_{12} \) for \( \lambda_{33} = 0.5 \) for velocities \( v_1 = 1.0 \), \( v_2 = 1.1 \) when both contacts are coupled.

First let us consider the case when the C1 and C2 are both coupled. Because they are perfect contacts [1] they absorb all the currents (charge or neutral) that reach region II and IV respectively. We will calculate the total fraction of the current injected at the source S that reaches the drain D. Different paths between S and D can be classified by the number of reflections at the boundaries B2 and B3 (these two numbers have to be equal). The greater the number of reflections, the longer the time to reach the drain. Therefore, the total current fraction at D, as a function of time is,

\[
r_D(t) = \sum_{n=0}^{\infty} \Delta_n \delta(t - t_n).
\]

(S16)

Here \( \Delta_n = \left(\frac{r_{32} r_{23} B_3}{r_{13} r_{31} B_2}\right)^n \) and

\[
t_n = t_0 + n\Delta T,
\]

(S17)

with \( t_0 \) being the shortest time to reach the drain and \( \Delta T \) being the total time for one set of reflections at B2 and B3. Taking the Fourier transform, the current fraction at D as a function of \( \omega \) is

\[
r_D(\omega) = \frac{e^{\frac{i B_3 B_2}{r_{13} r_{31}}}}{1 - \left(\frac{r_{32} r_{23} B_3}{r_{13} r_{31} B_2}\right)e^{i\omega \Delta T}}
\]

(S18)

We can now easily calculate the fraction of average tunneling current reaching the drain by summing over the all the charge packets reaching the drain as the zero frequency limit of this expression.

\[
f = R_{B_3 B_2} \left[ 1 + r_{32} r_{23} B_3 + \left( r_{32} r_{23} B_3 \right)^2 + \left( r_{32} r_{23} B_3 \right)^3 + \ldots \right]
\]

\[
= \frac{r_{B_3 B_2}}{1 - r_{32} r_{23} B_3}
\]

(S19)

Next we compute the zero temperature noise following Ref. 2 for this problem.

\[
L(t, t') = \langle I_D(t)I_D(t') \rangle + \langle I_D(t')I_D(t) \rangle - 2\langle I_D(t)\rangle \langle I_D(t') \rangle
\]

(S20)

Using \( I_D(t) = I_D(t)I_1(t) \) we can write,

\[
L(t, t + \tau) = r_D(t)\Gamma_D(t + \tau)S(\tau),
\]

(S21)

with \( S(\tau) \) being the conventional partitioning noise [2, 3]. Averaging over \( t \) we get the noise

\[
N_D(\tau) = h(\tau)S(\tau)
\]

(S22)

where \( h(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T r_D(t)\Gamma_D(t + \tau)dt \)

(S23)

and \( S(\tau) = \frac{e^2 \Gamma^2}{2\pi^2 a^2} \cos(V_0\tau) \frac{2}{(1 + \frac{e^2 V_0^2}{\pi a^2})^2} \)

(S24)

Here the average tunneling current is \( \langle I_{un} \rangle = \frac{e^2 V_0}{2\pi^2 a^2} \Gamma \), where \( \Gamma \) is the tunneling amplitude from the source into the probe charge chiral, \( V_0 \) is the potential difference between the source and the probe charge chiral, and \( a \) is the UV cutoff (\( a \rightarrow 0 \) being the UV limit) [2]. Note that the dimension of \( N_D(\tau) \) is the same as that of \( S(\tau) \). Therefore, \( N_D(\tau) \) has the dimensions of current squared and \( N_D(\omega) = \langle I_{un} \rangle \) has the dimensions of charge. Now, using Eq. S16 we can simplify \( h(\tau) \) as,

\[
h(\tau) = \left(\frac{B_2 B_3}{r_{13} r_{31}}\right)^2 \sum_{n, n' = 0}^{\infty} \Delta_n \Delta_{n'} \delta(\tau + (t_n - t_{n'})).
\]

(S25)

Thus the noise at zero frequency will be

\[
N_D(\omega = 0) = \left(\frac{B_2 B_3}{r_{13} r_{31}}\right)^2 \sum_{n, n' = 0}^{\infty} \Delta_n \Delta_{n'} S(t_{n'} - t_n)
\]

(S26)

Now, using the expression for \( S(\tau) \) from Eq. S24 in the limit \( V_0 \rightarrow 0 \) (small voltage difference) and \( a \rightarrow 0 \) we can easily find,

\[
N_D(\omega = 0) = \frac{e^2 \Gamma^2 V_0}{2\pi^2 a^2} \left(\frac{B_2 B_3}{r_{13} r_{31}}\right)^2 \langle I_{un} \rangle.
\]

(S27)
Thus we can plot the noise to $e\langle I_{\text{tun}} \rangle$ ratio as a function of the interaction strength. We show one illustrative case in Fig. S2. When both contacts are coupled, noise is present at D but weak.

Next let us consider the case where the $C_1$ is coupled but $C_2$ is decoupled. Previously, when both contacts were coupled, all signals transmitted at $B_3$ were absorbed at $C_2$. Now, with $C_2$ decoupled, there will be multiple reflections in region IV. However, since there is no tunneling in any region, the $U(1)$ “charge” of each mode will be conserved. Thus the total reflected “charge” of the neutral (including all possible multiple reflections) from region IV to region III will vanish. Thus the dc current at the drain D will be zero.

SIV. AN EXAMPLE WITH A CHARGED MODE IN THE TEST SYSTEM

As we understand that in the absence of the contacts $C_1$ & $C_2$ there will be no current at the drain D due to the charge conservation. This also guarantees that the total current at D will be zero if we remove either of the contacts. However, we can again calculate the total current at D when both the contacts are coupled. The first packet that boundary $B_3$ via mode 1 will reflect back to mode 2 of region III with a fraction $r_{B_3}^{B_1}$. The part that transmits to region IV will get absorbed by $C_2$. The part that reflected into the mode 2 travels to boundary $B_2$ and one part reflects to mode 1 going towards the drain D with weight $r_{B_2}^{B_1}$, one part reflects to mode 3 travelling to boundary $B_3$ with weight $r_{B_2}^{B_3}$, and a part transmits to region II and gets absorbed by $C_1$. The part that travelled to boundary $B_3$ will reflect back to mode 2 with weight $r_{B_3}^{B_2}$ and the same process as the previous step starts. Thus the fraction of total current reaching the drain will be a series with multiple reflection in region III

$$f = r_{B_2}^{B_3} r_{B_1}^{B_2} \sum_{n=0}^{\infty} \left( r_{B_2}^{B_2} r_{B_3}^{B_2} \right)^n = \frac{r_{B_2}^{B_3} r_{B_1}^{B_2}}{1 - r_{B_2}^{B_2} r_{B_3}^{B_2}}.$$  \hspace{1cm} (S28)

As we understand that in the absence of the contacts $C_1$ & $C_2$ there will be no current at the drain D due to the charge conservation. This also guarantees that the total current at D will be zero if we remove either of the contacts. However, we can again calculate the total current at D when both the contacts are coupled. The first packet that boundary $B_3$ via mode 1 will reflect back to mode 2 of region III with a fraction $r_{B_3}^{B_1}$. The part that transmits to region IV will get absorbed by $C_2$. The part that reflected into the mode 2 travels to boundary $B_2$ and one part reflects to mode 1 going towards the drain D with weight $r_{B_2}^{B_1}$, one part reflects to mode 3 travelling to boundary $B_3$ with weight $r_{B_2}^{B_3}$, and a part transmits to region II and gets absorbed by $C_1$. The part that travelled to boundary $B_3$ will reflect back to mode 2 with weight $r_{B_3}^{B_2}$ and the same process as the previous step starts. Thus the fraction of total current reaching the drain will be a series with multiple reflection in region III

$$f = r_{B_2}^{B_3} r_{B_1}^{B_2} \sum_{n=0}^{\infty} \left( r_{B_2}^{B_2} r_{B_3}^{B_2} \right)^n = \frac{r_{B_2}^{B_3} r_{B_1}^{B_2}}{1 - r_{B_2}^{B_2} r_{B_3}^{B_2}}.$$  \hspace{1cm} (S28)

FIG. S3. A modified picture of the problem where the bottom region also has a right moving charged mode and a left moving neutral mode [4].

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