Pre-Service Mathematics Teachers Divisibility Construction

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Abstract. Divisibility concept is fundamental to the development of another concept in elementary number theory. A study of the construction of the divisibility advanced mathematics teacher was done to provide empirical evidence of early genetic decomposition design. Through the question, we designed in the form of test sheets and interview, we explore the construction of 3 students. The result of this qualitative study illustrated that construction of the divisibility concept on the problem solving of number theory represented in algebra.

1. Introduction
The divisibility concept is a basic concept in the theory of elementary number. The divisibility concept is a particular case of the idea of division where the remnant of the division process is zero. Brown et al. [1] revealed that the basis for the development of the theory of number is the concept of divisibility. The concept is likely to be found in discussions of other concepts on the theory of number. It is the principal reason why researchers prefer the idea of the divisibility of the various concepts discussed in the theory of elementary number.

At the bachelor level, the concept of divisibility can be represented in both number and algebra while these two representations are different but have the same basic concept of involving the relationship between the divisible number, the divided number, and the resulting. If the relationship is represented in algebra, then the algebra form can be considered a function. So, to construct the concept of divisibility, students will review the basic concept of algebra in a new context.

In the construction of the concept of divisibility, students begin by constructing the concepts in the representation of the number expression and then to an algebraic expression. The division of the relationship between arithmetic and algebra is a way of thinking about the relationship between concrete and abstract [2]. Study of Demana & Leitzel shows that learning numerical calculations and problem solving first allow students to understand the basic concept of algebra [3].

The design of construction can be done in a APOS theoretical framework. Realistically, it is a mental construct of a person striving to understand or have an understanding of a mathematical idea [4]. The APOS theory provides tools for analyzing the construction of a concept. The APOS theory explains how the mental structure of actions, processes, objects and schemes and mental mechanisms such as interiorization, encapsulation, coordination, de-encapsulation and construction form a mathematical concept [5]. Actions are an external transformation of an object or objects understood, a process is an internal transformation without having to perform explicit steps, an object is a static mental structure in which action can be implemented, while a collection of actions, processes, and coherent objects forms a mathematical concept scheme. Mental mechanisms build mental structures. Schemes one already can be simulated into a cognitive object in which action can be implemented. The process is
made from interiorization mechanisms and coordination, the object is built from an encapsulation mechanism, while a thematization mechanism creates schemes. Coordination combines two processes for de-encapsulation results. The APOS theory has been much used as a tool in analysis of how the construction of the student concept in various mathematical concepts includes the association of limits function, expressions of algebra, fractions, chain rules, and the vector room [3,6–9].

In the application of the APOS theory, constructive analysis of the concept of mathematics follows a genetic decomposition previously proposed. Genetic decomposition is a description of mental structures and mechanisms [10]. The question that comes up with designing genetic decomposition is how genetic decomposition is designed and what is needed in designing genetic decomposition. Some genetic decomposition is designed with consideration of experience, researchers as teachers, text material, research data, observations of a student studying concept, observations of a student’s work, development of a concept and data from a student interview [5]. In designing genetic decomposition, some consideration such as research experiences, material text studies, predetermined research data is used to give researchers an initial consideration in designing genetic decomposition. Theoretical construction of the concept of divisibility that students may have are described in the following 4 steps.

a. Actions of integer division according to the mathematical requirements needed to get the quotient.

b. The interiorization of action to the process is being aware of and able to make decisions about the divisibility of a certain mathematical situation without carrying out explicit division.

c. Encapsulation the process to the object is to recognize the relationship between the division, the divided, and the quotient, and the properties that are applicable in that relationship.

d. Thematization Object to the scheme is awareness and can use a link between division algorithms and divisibility in solving math problems.

In completing this design of genetic decomposition, researchers need field data with student interviews that provide empirical evidence for the construction of the concept of divisibility. It would need to generate early genetic decomposition. The purpose of this study, therefore, is to explore the construction of the college divisibility concept based on the genetic decomposition designed for such consideration.

2. Method
This study was a qualitative study with an exploratory type of research [11]. The problem that was designed was presented to 30 advanced math teacher who had completed the elementary theory of number. Researchers selected 3 people an interview subject based on group criteria in response to the following divisibility issues:

For \( n \in \mathbb{Z} \), prove that \( \frac{n(n+1)(2n+1)}{6} \) is an integer.

Interview questions are designed to respond to the responses students give in tests. It means each student gets a question interview which may be different from each other. The goal of an interview to explore more deeply how the construction of the divisibility concept and confirm some of the responses given.

3. Result and Discussion
The construction of the divisibility concept is obtained through the analysis of the student responses to problems presented on tests and interviews based on the proposed genetic decomposition. Of the 30 students who took the test, none of the students could get the problem right. The five types of responses given to the divisibility issues are: (1) not answer or answer but not related to problems; (2) take some integer \( n \) to be substitution on \( n(n+1)(2n+1) \) then showed that \( \frac{n(n+1)(2n+1)}{6} \) an integer; (3) use mathematical induction to show that the statement is true but the induction step was not completed; (4) use mathematical induction to show that the statement is true and the induction step is complete; Also (5) misinterpreting a given mathematical situation, giving a false example.

The response group 1 could not be explored because the group did not provide a response or provide a response unrelated to problems so that it could not be analyzed using the APOS theory. The three students to who the subject was subjected were subject 1 (M1), subject 2 (M2), and subject 3 (M3),
giving different responses demonstrating the difference in concept construction. M1 is a subject in response group 2, M2 is a subject in response group 3 and, M3 is a subject in response group 4.

Through the issues of divisibility, students are expected to make a correlation between division algorithms and divisibility and use the attribute of divisibility to indicate that \( \frac{n(n+1)(2n+1)}{6} \) is an integer if \( n \in \mathbb{Z} \). In interview sessions, it was uncovered that students didn’t study division algorithms. Thus, researchers focus interviews on student skills using the inhibitions in solving the divisibility issues. M1 stitched several integer \( n \) on \( n(n+1)(2n+1) \) to show that an expression is a round number. It supplies \( n \) value \( n = 2 \) into form \( \frac{n(n+1)(2n+1)}{6} \) so it is got \( \frac{2(2+1)(2(2)+1)}{6} = \frac{30}{6} = 5 \) and 5 is an integer. The answers indicated that M1 was just carrying out a procedural activity. Here’s quote from his interview:

\[ R : \text{You answered problem 2 by substitution n value } n = 2 \text{ and got 5. According to you, what is the meaning of } \frac{n(n+1)(2n+1)}{6} \text{ an integer?} \]

\[ M1 : \text{[Stop] So if I substitute } n = 2 \text{ then I got 5, it means that it is an integer.} \]

\[ R : \text{Has it answered the problem?} \]

\[ M1 : \text{No, mom.} \]

The interview revealed that M1 denotes \( \frac{n(n+1)(2n+1)}{6} \) as an integer” as the final result of \( \frac{n(n+1)(2n+1)}{6} \) if \( n \) replaced by a number is an integer. Bachelor realized that he had not answered the question, but he could not think of any other way to explain it. From the M1 response, both in tests and interviews, it appears that the construction of the concept of division M1 is action oriented.

M2 uses mathematical induction to prove a statement. He started basic stage by using math induction to prove the statement. He started basic stage by showing that to \( n = 1 \) so that \( P(1) \) correct is \( \frac{n(n+1)(2n+1)}{6} = \frac{1(1+1)(2(1)+1)}{6} = 1 \). On induction steps, he wrote a statement for \( n = k \) then \( \frac{k(k+1)(2k+1)}{6} \).

But in the proving step for \( n = k + 1 \), it could not complete the proofing. He just wrote that \( \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6} = \frac{(k+1)(k+2)(2k+3)}{6} = \frac{(k^2+2k+k+2)(2k+3)}{6} = \frac{2k^3+3k^2+4k^2+3k+4k+6}{6} = \frac{2k^3+7k^2+7k+6}{6} \) is true. Here is a citation for an interview with M2:

\[ R : \text{You choose mathematical induction to answer this problem. Why?} \]

\[ M2 : \text{This is like the similar problem on my notebook.} \]

\[ R : \text{What is your reason to say that } \frac{2k^3+7k^2+7k+6}{6} \text{ is true?} \]

\[ M2 : \text{[Stop] I confuse, so I just write like that.} \]

\[ R : \text{In this case, how can you say that } \frac{2k^3+7k^2+7k+6}{6} \text{ is true?} \]

\[ M2 : \text{[Silent]} \]

\[ R : \text{Well. Why for } n = 1, \text{ you conclude that } \frac{1(1+1)(2(1)+1)}{6} = 1 \text{ is true?} \]

\[ M2 : \text{Oh yes, because } \frac{6}{6} = 1 \text{ and 1 is an integer.} \]

\[ R : \text{On induction steps, is } \frac{k(k+1)(2k+1)}{6} \text{ true?} \]

\[ M2 : \text{[Stop] yes} \]

\[ R : \text{Is } \frac{(k+1)(k+2)(2k+3)}{6} = 2k^3 + 7k^2 + 7k + 6? \text{ Can you recheck it?} \]

\[ M2 : \text{[Time the factors in the form } (k+1)(k+2)(2k+3) \text{]. There is an error mom. It should be } 2k^3 + 9k^2 + 12k + 6. \]

\[ R : \text{Can you try to use that algebra to prove } \frac{2k^3+9k^2+13k+6}{6} \text{ is an integer. Change the form } 2k^3 + 9k^2 + 12k + 6 \text{ so that } k(k+1)(2k+1) \text{ seen.} \]

\[ M2 : \text{May I change the form } k(k+1)(2k+1) \text{ first?} \]

\[ R : \text{Yes, please.} \]

\[ M2 : \text{[Change the forms } k(k+1)(2k+1) \text{ become } 2k^3 + 3k^2 + k, \text{ then manipulate again the form } 2k^3 + 9k^2 + 12k + 6]. \text{ So } 2k^3 + 3(3k^2 + 4k) + 6. \text{ I confuse... Hmm... if I} \]
substitute a number [stop] for example 1, so that $2(1)^3 + 3(1)^2 + (1) + 6(1)^2 + 12(1) + 6 = 30$. It is divided 6 getting 5. But, [Silent]

R : Well... Enough. Is induction step used to giving the prove on the problem?

M2 : [Silent]

M2 thinks that these problems can be answered with mathematical induction. But he didn't understand mathematical induction. Skemp notes that the use of mathematical rules in resolving issues without knowing whether these rules are valid or not is an instrumental understanding [12].

Construction of the concept of m2 is in the process stage. It can be seen from the tendency of M2 indicating $\frac{n(n+1)(2n+1)}{6}$ an integer by thinking about a particular integer that might meet, as she believes, she substituted the number. But she was not able to prove $\frac{2k^3+9k^2+13k+6}{6}$ as an integer. The ability of M2 to manipulate algebra is good enough to factor in multiplication, but it lacks meticulous care. The ability of M2 understanding the structure in this algebraic expression is still low. He chose an improper manipulation when he changed the form $2k^3 + 9k^2 + 13k + 6$ into another equivalent form. It inhibits M2 in describing answers in mathematical induction measures.

M3 uses mathematical induction to prove a statement. Just like M2, he starts a basic step by demonstrating that for $n = 1$ then $P(1)$ is true $\frac{n(n+1)(2n+1)}{6} = \frac{1(1+1)(2+1)}{6} = 1$. On induction steps, he wrote a statement for $n = k$ then $P(n)$ is true $\frac{k(k+1)(2k+1)}{6} = \frac{2k^3+3k^2+k}{6}$. On the proving steps he was able to show that for $n = k + 1$, it will be proved $P(k + 1)$ is true $\frac{(k+1)(k+1+1)(2(k+1)+1)}{6}$. She outline the prove:

$\frac{(k+1)((k+1)+1)(2(k+1)+1)}{6} = \frac{(k+1)(k+2)(2k+3)}{6} = \frac{2k^3+9k^2+13k+6}{6} = \frac{2k^3+3k^2+6k^2+k+12k+6}{6} = \frac{2k^3+3k^2+6k^2+k+12k+6}{6} = \frac{2k^3+3k^2+k+6k^2+12k+6}{6} = \frac{2k^3+3k^2+k+6k^2+12k+6}{6} = \frac{2k^3+3k^2+k}{6}$ is an integer. Here is a quote from a research interview with M3:

R : Why the expression $\frac{2k^3+3k^2+k}{6}$ is an integer?

M3 : From taking value $n = k$ so the expression $\frac{2k^3+3k^2+k}{6}$ is an integer.

R : Could the mathematical induction steps be used for giving the prove on the problem?

M3 : Hmmmm.... [Speaking in whispering voice: it is started from $n = 1$, but for $n < 1$…] I think for $n < 1$ not to belong in this proofing.

R : Is there any way so you can prove the statement for $\forall n \in Z$?

M3 : [Stop] No idea, mom.

The above interviews indicate that M3 can already use the traits of divisibility to solve divisibility issues. He realized that mathematical induction he was using had not answered the problem. But the limited context he had left him with no other solution to solve problems.

M1, M2, and M3 have the construction of the concept of divisibility at the stage of action, process and object. M1 is at the action stage. He can perform only divisions against a specific number. Nevertheless, M1 understanding of the issues presented allows it to set an appropriate example. M2 that tried to answer with mathematical induction turned out to have an instrumental understanding for mathematical induction concepts. Maybe she has enough skills to manipulate algebraic expression, but she cannot see the structure in that expression. The ability to look for structures in algebraic expression is called structure sense [13].

The results of the Dreyfus &Hock study [14] found that many students failed to implement the structure sense when they solved their advanced algebra problem. Also, the divisibility concept of an object he has yet to have made it unable to prove the point of divisibility so that the induction step was incomplete. M3 also answers with mathematical induction and can finish well. Besides, he has skills in manipulating both algebra and structure sense and can use the attributes of divisibility. This means the manipulation and structure sense skills have influenced their performance in addressing the problem. Even so, M3 contexts in division algorithms prevented her from properly answering the divisibility issues.
4. Conclusion

Construction of the divisibility concept of the subject interviews vary in stages of action, process and object. No subject has constructed the concept of divisibility up to the stage of the scheme. The conception of action demonstrated by external procedural activity in the decision of divisibility. M1 shows the construction of the concept of divisibility at this stage. M2 shows a conception of process, in which the disconnect decision is taken without actually doing any explicit division. It is the way it chooses number \( n = 1 \). The pause in the interview shows that he thinks for a moment before deciding on the number. While construction of the concept of divisibility at the stage of the object is indicated by the performance of M3 in answering questions in either a test or an interview. It can use the properties of divisibility in solving problems.

The subject knowledge of the basic concept of algebra also affects the construction of the divisibility they have. Subjects having a more advanced rudimentary algebra concept, could be more easily applying the concept of divisibility he has. Campbell stated that the theory of number is part of algebra [2]. Hence, the rudimentary form of algebra is needed in constructing the concept of divisibility.

The concept of divisibility is a special case in the division algorithm. The study found that students have a limited context in developing their divisibility concept schematics because they do not construct the division algorithm.

Based on the results of this study, the suggestions for learning the divisibility concept in the theory of number are: (1) the division algorithm must be constructed before constructing the concept of divisibility; (2) in constructing the concept of divisibility, the instructor needs to give emphasis to the concept of division and multiplication in weighing the problem of divisibility and; (3) in constructing the concept of divisibility, requiring cultivating the way of thinking in algebra so that students can practice skills in algebraic manipulation and structure sense. Moreover, the problems presented in algebra gave them the experience of re-discussing the concept in a new context and thus developing a deeper understanding.

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