Detecting and Locating Disturbances in High Voltage Electrical Networks

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Abstract—The safe operation of any engineered system relies on, in particular, an efficient identification of malfunctions. The case of the high voltage electrical networks is particularly challenging due to their size and their complex structure. We propose a simple method to identify and locate disturbances in the power grid, relying only on voltage phases measurements and on the knowledge of the Kron-reduced network structure. The strength of the approach lies in its simplicity paired with the ability to precisely locate disturbances and even to differentiate between line and node disturbances. If we have access to measurement at only a subset of nodes, our method is still able to identify the location of the disturbance if the disturbed nodes are measured. If not, we manage to identify the region of the network where the disturbance occurs.

Index Terms—Kron reduction, control oscillation, line disturbance, node disturbance, detection and localization.

I. INTRODUCTION

Despite their long history, electrical networks are still an active topic of applied and fundamental research. Our understanding of such large networked system is still incomplete and advances in network science help to clarify some of its complex behaviors [1], [2]. Furthermore, the current changes in the operation of electrical grids (following the change in energy sources from conventional to new renewables) put it in unprecedented operating states [3], [4]. A safe and efficient operation of the grid then requires an accurate and reliable assessment of its state in real time.

Recently, applications of results from network science and dynamical systems theory allowed developing new techniques aiming at a fast assessment of the current state of power grids. Such improvements are based on the measurement of some quantities such as voltage amplitudes, phases, and frequencies, which are nowadays widely accessible on a high time resolution thank to Phasor Measurement Units (PMUs). Part of the work on state estimation, such as [5], [6], [7], is mostly focused on the estimation of the current state of the whole system. This is performed using various techniques, such as estimation of the eigenmodes of the network through probing signals [5] or resonance methods [7], or based on the measurement of the response of the system to ambient noise [6]. Closer to our interest in this manuscript, Refs. [8], [9], [10], [11], [12] proposed some approaches aiming at locating the source of a disturbance. For networks composed of areas with weak inter-area connections and strong intra-area connections, Ref. [8] uses the residues of an estimated transfer function in order to locate a nodal disturbance. Other approaches rely on Discrete Wavelet Transform (DWT) [9], [10] or logistic regression [11] of PMU measurements in order to identify the source of a disturbance. Most of these methods are designed to locate a nodal disturbance and do not cover line disturbances.

The case of disturbances located on electrical lines led to fewer results compared to power disturbances [13], [14]. This is partly due to the difference in the way such disturbances can be incorporated in the model. Nodal perturbations, occurring mostly as variation of power injections/consumptions, act as an additive perturbation, whereas disturbances on lines, which are modifications of the line capacity, are represented as multiplicative disturbances in the model, which are much harder to tackle analytically. Nevertheless, line perturbations are at least as important as nodal perturbation and, in our opinion, should be investigated in details. Methods based on the propagation of the disturbance [12] would be able to determine the area where a disturbed line is located, but to the best of our knowledge, there are no methods specifically designed to locate line disturbances.

In this manuscript, we cover the case of disturbed lines (e.g., modeling a malfunctioning transformer) as well as distributed nodes (e.g., modeling a faulty generator). Such malfunctions typically behave as time varying admittance for lines and time varying power injections for generators [15], [16], [17]. Assuming that the amplitude and frequency of these variations are not too large, we propose a way to locate the faulty element in the network, based on measurement of the voltage angles. If we have access to measurement at the faulty element, we are able to precisely locate it, even though such slow disturbance will spread out throughout all the network. If we do not have access to measurements at all nodes, we are nevertheless able to determine the area of the network where the faulty element is located.

The manuscript is organized as follows. We first recall some preliminary tools in Sec. [II].
results about it are covered in Sec. [III] In Sec. [IV] we detail the method locating the faulty element, and validate it numerically in Sec. [V] Finally, Sec. [VI] concludes the manuscript.

II. PRELIMINARIES

Throughout this manuscript, we denote by $e_i$, the $i$th vector of the canonical basis, with 1 at index $i$ and zero everywhere else, and $e_{ij} := e_i - e_j$. We write $\text{diag}(\{m_i\}) \in \mathbb{R}^{n \times n}$ the diagonal matrix $M$ with elements $m_1, m_2, \ldots, m_n$ on its diagonal and 1 the vector of ones of length $n$.

A. The Sherman-Morisson formula

Our results rely heavily on the Sherman-Morisson formula [18, Sec. 2.1.4], giving an explicit formulation for the inverse of the rank-1 perturbation of an invertible matrix $A$,

$$
(A + uv^\top)^{-1} = A^{-1} - \frac{A^{-1}uv^\top A^{-1}}{1 + v^\top A^{-1}u},
$$

(1)

where $u$ and $v$ are vectors characterizing the rank-1 perturbation of $A$. We emphasize that the Sherman-Morisson formula applies to the pseudo inverse of Laplacian matrices (even though such matrices are singular), provided that $u$ and $v$ are orthogonal to the constant vector $1$.

B. The Kron reduction

It is possible to rewrite the power flow equations with respect to a fewer number of voltage variables through Kron reduction [19]. This is done by taking the Shur complement [18, Sec. 3.2.11] of the Laplacian matrix with respect to a subset of nodes and adapting the power injections accordingly. Partitioning the nodes in two sets $I_g = \{1, \ldots, n_g\}$ (the nodes that are not reduced) and $I_c = \{n_g + 1, \ldots, n + n_c\}$ (the ones that are reduced), we write the angle and power vectors as well as the Laplacian matrix in block form (reordering indices if necessary)

$$
\theta = \begin{pmatrix} \theta^g \\ \theta^c \end{pmatrix}, \quad P = \begin{pmatrix} P^g \\ P^c \end{pmatrix}, \quad L = \begin{pmatrix} L_{gg} & L_{gc} \\ L_{cg} & L_{cc} \end{pmatrix}.
$$

(2)

The Kron-reduced Laplacian matrix is then the Schur complement of $L_{cc}$ in $L$,

$$
L_{cc}^r = L_{gg} - L_{gc}(L_{cc})^{-1}L_{cg},
$$

(3)

and the power vector is modified as

$$
P_{cc}^r = P^g - L_{gc}(L_{cc})^{-1}P^c.
$$

(4)

The Kron-reduced power flow equations are then given by

$$
P_{cc}^r = L_{cc}^r \theta^g.
$$

(5)

III. MODEL

We model the voltage dynamics in the lossless line approximation, which standard for high voltage transmission grids [20]. On time scales ranging from few AC cycles to approximately 10–20 seconds, the transient dynamics is governed by the swing equations [20],

$$
m_i \dot{\omega}_i + d_i \omega_i = P_i - \sum_j b_{ij} \sin(\theta_i - \theta_j).
$$

(6)

This set of differential equations describes the dynamics of the voltage angles $\theta_i$ and frequencies $\omega_i = \dot{\theta}_i$ at each network bus, labeled $i = 1, \ldots, N$, in a frame rotating at the nominal grid frequency of the grid (50 or 60 Hz). Parameters $m_i$ and $d_i$ denote the inertia and damping/primary control at node $i$ respectively, and $P_i$ is the active power produced ($P_i > 0$) or consumed ($P_i < 0$) at bus $i$. Buses are connected to one another via lines with susceptances $b_{ij}$. The operating state is given by a stable fixed point $\{t^*\}$ of (6), i.e., the solution of the power flow equations, $P_i = \sum_j b_{ij} \sin(\theta_i - \theta_j)$. When angle differences are small enough, such solutions are well approximated by the solutions of the linearized power flow equations (also called DC power flow equations [21, Sec. 9.7]),

$$
P = L \theta.
$$

(7)

A. Malfunctioning elements

We consider malfunctions either at nodes, yielding the time varying power

$$
P_i(t) = P_i + \xi(t),
$$

(8)

or on lines, described as varying effective susceptance of transformers

$$
b_{ij}(t) = b_{ij} + \xi(t),
$$

(9)

where $\xi(t)$ can be any function of time, whose characteristic time scale is typically smaller than any intrinsic time scale of the system Eq. (6). Formally, we require that the characteristic time of the disturbance is larger than the time scale on which its impact is damped at the oscillators and larger than the time it requires to spread throughout the network, i.e., for all $i, j$,

$$
\left[ \max_t |\xi(t)| \right]^{-1} \gg \max \left\{ \frac{m_i}{d_i}, \sqrt{\frac{m_i}{\lambda_j}}, \frac{d_i}{\lambda_j} \right\}.
$$

(10)

In order to guarantee that the system remains in the vicinity of the initial fixed point, we also require that the amplitude of $\xi$ is not too large,

$$
\max_t |\xi(t)| \ll \begin{cases} P_i, & \text{for a disturbance at node } i, \\ b_{ij}, & \text{for a disturbance at line } (i, j). \end{cases}
$$

(11)

These last assumptions allow us to consider that the system remains in the linear approximation regime. We will assume Eqs. (10) and (11) to be satisfied throughout the manuscript.
B. Simulation setting

In the numerical confirmation of Sec. [V] we will consider a oscillating disturbance,

\[ \xi(t) = \xi_0 \sin(\omega_m t) . \]  

(12)

From a practical point of view, this allows us to tune the time scale of the disturbance, which is the oscillation frequency \( \omega_m \) in order to guarantee that it is sufficiently slow with respect to all time scales of the networks.

Furthermore, such disturbance are typically observed when there is a malfunction at an element of the grid. In Eq. [6], transformer are modeled as lines and a malfunctioning transformer would translate as a line with oscillating susceptance. Similarly, a malfunctioning generator is modeled as a time varying power source, i.e., exactly our setting with \( \xi_0 \) given in Eq. (12).

IV. Locating the faulty element

Our method kicks in once the grid operator has realized that something is going wrong in the network. This can be done in various ways. Oscillations are typically revealed by Fourier Transform of the time series measured, performed at regular time intervals. Our method would be one of the tools that grid operators would use once they observe such oscillations in the network, in order to identify its origin and fix it. We use the same method to identify malfunctions at nodes or on lines. In order to perform this we need to know the Laplacian matrix of the network, possibly Kron-reduced in some nodes are not measured.

Assume that we know the Kron-reduced matrix of the network \( L' = L_{cc} - L_{cc} (L_{cc})^{-1} L_{eq} \), with measurements at each non-reduced node (compare notations with [13]). Note that everything works the same if the set of Kron-reduced nodes is empty, i.e., we consider directly the full Laplacian matrix. Assuming that the voltage angles are small enough in order to linearize the power flow equations, a reasonable approximation of a stable fixed point of Eq. (6) is

\[ \theta^*_y = (L')^\dagger P_r , \]  

(13)

where \( A^\dagger \) denotes the Moore-Penrose pseudo-inverse of the matrix \( A \) [18, Sec. 5.5.2].

If we have access to the time series of \( \theta_y(t) \) and the knowledge of \( L' \), we can define

\[ \psi(t) := L' \theta_y(t) . \]  

(14)

We will show now that the time series of \( \psi(t) \) are able to locate the malfunctioning element with as much accuracy as can be expected.

We distinguish three cases: a malfunctioning line between two nodes that are not Kron-reduced; a malfunctioning line with at least one end-node that is Kron-reduced; and a malfunctioning node (which can be reduced or not).

A. Malfunctioning line with non-reduced end-nodes

This is the most interesting case because we are able to locate the faulty line exactly. It also covers the case where no Kron-reduction is applied. Here the faulty line has no influence on the reduced power vector and is a rank-1 perturbation of the reduced Laplacian matrix,

\[ P_r(t) = P_r^0 \]

\[ L'(t) = L_0^c + \xi(t)e_{ij}e^\dagger_{ij} , \]  

(16)

where we adapted the size of \( e_{ij} \) and the indices \( i \) and \( j \) in order to comply with the Kron reduction. Writing the time-evolving version of Eq. (13), one gets

\[ \theta^*_y(t) = [L'(t)]^\dagger P_r , \]  

(17)

which, after left multiplication by \( L_0^c \), yields

\[ \psi(t) := [L_0^c] \xi(t)e_{ij}e^\dagger_{ij} \]

\[ = L_0^c \left( L_0^c \xi(t)e_{ij}e^\dagger_{ij} \right) \]

\[ = P_r + \alpha(t) \left( e^\dagger_{ij} L_0^c e_{ij} \right) , \]

where we used Sherman-Morrisson Eq. (1) at the second line and

\[ \alpha(t) = \frac{\xi(t)}{1 + \xi(t)e^\dagger_{ij} L_0^c e_{ij}} . \]  

(19)

Remember that for \( L \) a Laplacian matrix, \( LL^\dagger = \mathbb{I} + \eta^{-1} \mathbb{1} \mathbb{1}^\top \), implying that \( LL^\dagger e_{ij} = e_{ij} \), and that the Kron reduction \( L_0^c \) of \( L \) is a (weighted) Laplacian matrix. One sees that the only time varying components of \( \psi(t) \) are at the two end points of the faulty transformer, which allows to locate it exactly.

This case is illustrated in Fig. [XI (d)], which shows the time series of \( \psi_i(t) \) for nodes 1 to 9 of the network shown in Fig. [XI (a)], when the capacity of the green edge is varying with time. One sees that the extremities of the faulty transformer are unambiguously identified.

B. Malfunctioning line with at least one reduced end-node

In this situation, the exact location of the faulty transformer cannot be determined based on the measurements because at least one of the end points is hidden to the observer. We detail the case where the two end points of the faulty transformer are in the Kron-reduced set. The mixed case where one of the end points is reduced and the other is not is very similar. All of our computations rely on [13, Secs. V.1 and V.2]. Assuming that \( i \) and \( j \) are in the Kron-reduced set, one gets time-varying reduced power vector and time-varying reduced Laplacian matrix

\[ P_r(t) = P_{r,0} - \beta(t) \left( e^\dagger_{ij} L'^c \right)^{-1} P_c \]

\[ = P_r^0 - \beta(t) \mathbb{1} w w^\top , \]  

(20)

(21)

where we defined

\[ \beta(t) = \frac{\xi(t)}{1 + \xi(t)e^\dagger_{ij} (L'^c)^{-1} e_{ij}} . \]  

(22)
and

\[ w = L^{gc}(L^{cc})^{-1} e_{ij}, \quad (23) \]

and again, \( e_{ij} \) is adapted according to the Kron reduction. Now in the same spirit as before, we compute

\[
\psi(t) = L_0^t [L^t(t)]^I P_r(t) = L_0^t \left[ (L_0^t)^I - \beta(t)(L_0^t)^I w w^\top (L_0^t)^I \right] \frac{1 + \beta(t)w^\top (L_0^t)^I w}{1 + \beta(t)w^\top (L_0^t)^I w} \cdot \left[ P_{r,0} - \beta(t) (e_{ij}^\top (L^{cc})^{-1} P_e) w \right] = P_{r,0} + \beta'(t) w, \quad (24)
\]

where again, we used the Sherman-Morisson formula, Eq. (1), and where we gathered all time-varying quantities in \( \beta'(t) \).

For the case where node \( i \) is not reduced and node \( j \) is, a similar calculation gives

\[
\psi(t) = P_{r,0} + \gamma(t) \tilde{w}, \quad (25)
\]

where

\[
\tilde{w} = e_i + L^{gc}(L^{cc})^{-1} e_j, \quad (26)
\]

with \( e_i \) and \( e_j \) being adapted to the Kron-reduced system, and where, again, we assembled all time-varying quantities in \( \gamma(t) \).

In these two cases, the effect of the faulty line will be measured on the non-reduced nodes connected to the reduced component containing the faulty line. This is seen in Figs. (e) and (f), showing the time series of \( \psi_i(t) \), when the faulty transformer is at the blue and orange lines respectively. One cannot identify precisely its location, but it is possible to identify the reduced component to which it belongs, which is the most we could expect from the available measurements. In Fig. (e), where the faulty line connects a reduced node to a non-reduced one, we observe that the amplitude of \( \psi(t) \) is much larger at the non-reduced end of the faulty transformer than at any other non-reduced node. This makes sense if line capacities are all similar, the \( i \)th component of \( \tilde{w} \) [Eq. (26)] is likely to be significantly larger than its other components, which will translate as a larger amplitude of variation in \( \psi_i(t) \). However, in full generality, we cannot guarantee that the amplitude of \( \psi_i(t) \) will be larger than all other components of \( \psi(t) \).

C. Malfunctioning node

This case is a bit easier to treat. The Laplacian matrix is constant in time and only one component of the vector of powers is time-varying. The time series of \( \psi(t) \) are then given by

\[
\psi(t) = L^t(L^r)^I P_r(t) = P_r(t) - \frac{1}{n} \sum_{i=1}^{N_g} P_{r,0} \cdot (27)
\]

There are now two possible cases. If the malfunctioning node is not reduced, we can locate it exactly. Indeed, in this case, the reduced power vector has the form

\[
P_r(t) = P_g(t) - L^{gc}(L^{cc})^{-1} P_e(0) = P_g(0) + \xi(t)e_i, \quad (28)
\]

which, once plugged into Eq. (27), yields

\[
\psi(t) = P_r(0) + \xi(t)(e_i - n^{-1}1). \quad (29)
\]

Provided that \( n \) is not too small, the amplitude of \( \psi(t) \) will be significantly larger at the malfunctioning node, allowing then to identify it.

If the malfunctioning node is Kron-reduced, the reduced power vector is expressed as

\[
P_r(t) = P_g(0) - L^{gc}(L^{cc})^{-1} P_e(t). \quad (30)
\]

Again, plugging this into Eq. (27), we see that all non-reduced nodes that are neighbors of the reduced component
are significantly impacted by the disturbance. We can then identify the reduced component in which the faulty node is located, but we cannot identify the node exactly. This is not surprising as we have no measurements at the disturbed node.

Whereas this approach works in theory, it appears that in the current state of transmission grids, the right-hand side of Eq. (10) is dominated by $\sqrt{m_i/\lambda}$. One can verify that, for the limit of Eq. (10) to be satisfied, it would require a unreasonably large amplitude of power oscillation at the disturbed node. It was then not possible to apply this method to node disturbances in the PanTaGruEl model. Nevertheless, one can expect that with decreasing number of rotating machines following the energy transition, and consequently the decrease of inertia in the system, the limit of Eq. (10) will be easier to satisfy in the future.

V. Numerical validation

In order to validate our approach, we simulated the dynamics Eq. (6) with an oscillating capacity at a transformer, on the PanTaGruEl model of the interconnected European grid [23], [24]. This model consists of 3809 buses and 7343 lines or transformers. It is built on publicly available data of geolocalization of power system elements, and parameters (admittances, productions, loads,...) are reconstructed based on standard assumptions.

In the following, we compare the fault detection using our method relying on $\psi_i(t)$ or by simply measuring the voltage frequencies $\omega_i(t)$. In the left panels of Fig. 2 show the maximal amplitude of $\psi_i(t)$ for the oscillating disturbance of five different lines of the PanTaGruEl model, shown in Fig. 3. The right panels show the maximal amplitude of the frequencies $\omega_i(t)$ for the same time series. The red circle indicate the indices of the two extremities of the disturbed line. Our method identifies unequivocally the endpoints of the disturbed lines whereas the time series of frequencies are not able to do so.
Fig. 4. (a) Evolution of $\psi_i(t)$. Two components stand out from the start. (b) Evolution of frequencies $\omega_i(t)$. For the first seconds, ordering disturbance is impossible. Even when the system has stabilized, multiple frequencies $\omega_i$ oscillate.

VI. CONCLUSION

We proposed an elegant method to identify and locate disturbances in the electrical network. Our method relies on time series of voltage phases and is able to differentiate between line disturbances and nodal disturbances. In the case of partial measurements, we are able to locate precisely the fault if the nodes where the fault occurs are measured. Otherwise, we can determine the area of the network where the faulty, unmeasured nodes/lines are located, using the Kron-reduced network.

The main condition required for our method to work out well is that the disturbance changes much slower than the intrinsic time scales of the system.

We hope that the idea raised in this manuscript will help the development of more efficient tools to detect and locate disturbances more accurately. Future work will aim at proposing a scheme for online detection and localization of disturbances, which requires the approach to be fast and light.

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