Multi-objective Optimization for Cruising Phase in Civil Aircraft's Trajectory Planning

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Abstract. Cruising phase is the important stage for aircraft performance optimization. This paper introduces a method of simultaneous optimization of multi-objective such as flight fuel cost, time cost, and shortest re-planning path under the constraints of hazardous environment, into trajectory planning for cruising phase. A dynamic optimization model of the route under hazardous weather and a performance optimization model for cruising phase of the vertical profile are established, and intelligent algorithms are used to solve them. Taking a certain route as an example, a dynamic real-time optimization route is generated by using the dynamic route optimization model. Combining with the performance data of a certain type of aircraft, the performance optimization model of the cruising phase is used to optimize its performance of cruising phase. In the end, this paper verifies the optimization results of the multi-objective optimization algorithm for route and performance.

1. Introduction

Since the 1970s, fuel prices have been rising due to energy shortages. Air flight will consume a lot of fuel resources, and fuel consumption cost has become a major part of flight cost. At the same time, a waste of time is a waste of resources. "The least time" is the main reason why people choose airplanes as a means of transportation. When hazardous weather such as thunderstorms and typhoons affect the planned flight route, a large number of flight delays will undoubtedly consume a lot of time costs for the crew and passengers, and ultimately cause huge economic losses. In addition, hazardous weather is also one of the important causes of aviation accidents, and flights must be diverted and circled outside the safe boundary.

Therefore, how to ensure the normal operation of flights in the hazardous weather environment, how to save fuel, reduce flight time, shorten flight distance, improve economy, and achieve green flight are the focus of airlines and the entire industry.

In the research on restricted no-flight areas such as aircraft avoiding temporary military areas and hazardous weather areas like thunderstorms and typhoons, Richards, A. et al. use the MILP method to solve the optimal obstacle avoidance trajectory [1]. Krozel J. et al. propose a new dynamic diversion algorithm to avoid hazardous weather [2]. McCrea M.V. et al. establish and verify the re-routing planning model [3]. Taylor C. et al. use simulated annealing algorithm to dynamically generate diversion path [4]. Agust A. et al. deeply analyse the random probability model and get the optimal diversion path [5]. Wang F. et al. study a re-routing planning method bases on Maklink graph and GA algorithm [6].
In the research on the vertical flight profile, the papers [7-9] complete four-dimensional trajectory optimization with the goal of fuel minimization. Jensen L. L. et al. study height optimization method to reduce fuel consumption for cruising phase [10]. Miller L. E. obtains the optimal cruising altitude and speed through trajectory analysis [11]. Antoine S. et al. take the cost factor as the target and determine the optimal speed profile for flight trajectory [12]. Liu J. et al. simulate the continuous descent track with the arrival time as the optimization goal [13].

In summary, scholars at home and abroad have conducted in-depth studies on saving fuel consumption, reducing flight time cost, proposing algorithm of rerouting planning under the constraint of hazardous environment, but there are relatively few studies on trajectory planning based on all the above factors. This paper comprehensively considers the multi-objective optimization of flight fuel cost, time cost and the shortest path of re-routing and re-planning under the constraint of hazardous environment. The cruising phase is the main phase of the flight. In order to achieve the overall optimization of the objective function, the paper focuses on the research of cruising speed, cruising altitude and flight path to minimize the weighted sum of various costs.

2. Multi-objective optimization model

2.1. Route optimization model under the restricted no-flight area

This paper adopts DP (Dynamic Programming) algorithm to optimize the real-time dynamic route under the restricted no-flight area. DP is a common method to solve the optimization problem of the multistage decision-making process. Its advantage is that it can obtain global optimization results and solve some online optimal control problems [14], and can be corrected when the flight deviates from the route.

To establish a diversion model with the goal of the shortest path, the objective function is:

\[
J_N(x_N) = \min \{d[x_k, u_N(x_k)] + J_{N-1}(u_N(x_k))\}
\]

Along with: \( J_1 = d(x_{i_k}, F) \) (2)

Where, \( d(x_{i_k}, F) \) represents the distance from \( x_{i_k} \) to the terminal point [15].

Here, the input waypoints are latitude and longitude of geographical coordinates, which need to be converted into plane coordinates in solving process. Gauss Kruger projection method is used to convert.

Suppose the latitude and longitude of the target geographic coordinates are \((\lambda_o, \varphi_o)\), and the geographic coordinates of the origin of a certain ground rectangular coordinate system are \((\lambda_0, \varphi_0)\). Because the projection formula is relatively complicated and the calculation amount is large, when programming the solution, the projection range \((\Delta \lambda, \Delta \varphi)\) is set to 0°, and only the first item of the projection formula is selected [16], namely:

\[
\begin{align*}
    x &= \Delta S = S(\varphi) - S(\varphi_0) \\
    y &= \frac{a \cos \varphi}{\sqrt{1 - e^2 \sin^2 \varphi}} \cdot \frac{\Delta \lambda}{\rho}
\end{align*}
\]

Where, semi-major axis of the earth \( a = 6378245.0 \text{m} \), oblateness of the earth \( e^2 = 0.006693421623 \), conversion factor \( \rho = 180^\circ / \pi \), \( \Delta \lambda = \lambda - \lambda_o \), \( S(\varphi) = a(1 - e^2) \cdot \left( \frac{A \varphi \cdot B}{2} \sin 2\varphi + C \sin 4\varphi - \frac{D}{6} \sin 6\varphi + \cdots \right) \),

\[
A = 1.0050517739, B = 0.00506237764, C = 0.00001062451, D = 0.00000002081
\]

2.2. Performance optimization model for cruising phase with vertical profiles

The trajectory optimization with vertical profile is based on the state equation of the aircraft as the basic constraint, and certain control variables like thrust and angle of attack are selected to meet the performance indicators.

The energy state model of aircraft particle motion is:
\[
\begin{aligned}
\dot{E} &= V(T - D)/mg \\
x &= V \cos \gamma + V_w
\end{aligned}
\] (4)

Among them, the state variables are energy \(E\) and distance \(x\), the control variables are speed \(V\) and thrust \(T\), \(V_w\) is wind speed, \(g\) is gravitational acceleration the earth, \(D\) is drag, and \(\gamma\) is flight path angle.

If the performance index functional is expressed as DOC (Direct Operation Cost), the study mainly discusses how to control the direct use cost related to flying hours during flight, so as to minimize the cost of this part. Which is:

\[
J = \int_{t_i}^{t_f} \left( C_f \dot{f} + C_t \right) dt
\] (5)

Where, \(C_f, C_t\) are the cost per unit mass of fuel and the cost per unit time respectively, and \(\dot{f}\) is the fuel consumption rate.

Applying the principle of minimum value, the Hamiltonian function of 4D flight profile optimization can be written as:

\[
H_{4D} = C_f \dot{f} + \lambda_E (T - D)V/mg + \lambda_x (V \cos \gamma + V_w) - \lambda_m f
\] (6)

In the above formula, \(\lambda_E, \lambda_x, \lambda_m\) are the co-state variables of energy, distance, and mass, respectively.

It can be deduced that the value \(H_{4D}\) of the optimal trajectory should be the smallest, and when \(t\) is not obvious in \(H\) and \(t\), \(H_{4D}\) remains constant for the optimal trajectory:

\[
\min H_{4D} = H_0 = \text{const}
\] (7)

If \(C_t = -H_b\) is selected, the Hamiltonian function is:

\[
H_{4D} = C_f \dot{f} + C_t + \lambda_E (T - D)V/mg + \lambda_x (V \cos \gamma + V_w) - \lambda_m f = H_{4D} - H_0 = 0
\]

Then the calculation model of the optimal cost for cruising flight is:

\[
\psi(E_x) = -\lambda(E_x) = \min_{V} C_f \cdot f + C_t / V + V_w
\] (8)

According to the optimization model obtained by formula (8), the optimization index for cruising phase (cost/price) can be expressed as:

\[
\min C_T = \min_{V} C_{\text{ approves}} \left| \frac{m}{V + V_w} + C_t \right|
\] (9)

Where, \(C_{\text{ approves}}\) is the unit fuel price, \(m\) is the aircraft weight change and fuel consumption.

Definition of cost index \(CI (=\text{time cost/fuel cost})\):

\[
CI = C_f / (100C_{\text{ approves}})
\] (10)

The relationship between fuel consumption and engine thrust is:

\[
\dot{m} = -C_{\text{ sfec}} \cdot T / g
\] (11)

Suppose the expression of \(C_{\text{ sfec}}\) (unit: \(\text{lb}/(\text{lb} \cdot \text{hr})\)) is:

\[
C_{\text{ sfec}} = C_{\text{ sfec0}} + a \cdot m / m_0 + b \left( H_0 - H \right) / H_0
\] (12)

Where, \(m_0\) is the initial weight of the aircraft, \(m\) is the weight of the aircraft, \(a, b\) are the aircraft mass and altitude correction parameters, respectively.

Substituting formulas (10), (11) and (12) into formula (9), the optimization goal becomes:

\[
\min C_T = \min \left[ C_{\text{ approves}} \left( C_{\text{ sfec0}} + a \frac{m}{m_0} + b \frac{H_0 - H}{H_0} \right) T / g + 100CI \right] \left( V + V_w \right)
\] (13)
During the cruising phase, the flight is in a steady state, and the relationship is balanced by the cruising force, then the thrust is:

$$T = D = \frac{1}{2} \rho V_r^2 S C_D = \frac{1}{2} \rho V_r^2 S (C_{D0} + KC_L^2)$$ (14)

Where, $C_D, C_L$ are drag and lift coefficients respectively, $C_{D0}$ is parasitic drag coefficients, and $K$ is induced drag factor.

Therefrom, the optimization objective for cruising phase is obtained. We can optimize the speed $V_r$, aircraft quality $m$ and altitude $H$ to minimize the cruising cost $C_r$.

3. Program and simulation

3.1. Simulation of dynamic route optimization under restricted no-flight area

Take the domestic route as an example for simulation. Since the longitude and latitude of the waypoints on the main domestic routes are mostly between east longitude 116 and 119 degrees, north latitude 31 and 35 degrees, we take (E117, N33) as the origin of the coordinates.

The input waypoints are shown in Figure 1. Assume that we add two constraint regions, with which both radiuses are 100Km and the circle centres are (E118.9, N32) and (E117.9, N34.7) respectively [15], and we program and simulate on the VC++ platform. The display results are shown in Figure 1.

In Figure 1, the green line is original optimization route with the goal of the shortest path. After adding two orange circle of restricted no-flight areas, the shortest path of re-routing and re-planning is the red line.

| Waypoints | Longitude/ Latitude |
|-----------|---------------------|
| ZSPD      | E121.40/N31.20      |
| ZSYA      | E119.26/N32.23      |
| YTY       | E119.54/N32.30      |
| ZSWX      | E120.18/N31.34      |
| ZSLY      | E118.20/N35.03      |
| ZSSH      | E119.09/N33.30      |
| ZSLG      | E119.10/N34.36      |
| ZSBZ      | E118.02/N37.22      |
| ZSZB      | E118.03/N36.48      |
| ZSCZ      | E116.52/N38.18      |
| ZBTJ      | E117.20/N39.13      |
| ZSHU      | E117.21/N38.21      |
| ZBAA      | E116.60/N40.10      |

Figure 1. The simulation result of dynamic real-time optimization.

From the simulation results, we can see that the route from Shanghai to Beijing is selected for optimization simulation in this section, and the shortest flight distance that we get is 1120.45 Km, which is approximately the straight-line distance between the airports of the two cities (about 1113.62 Km). After adding two restricted no-flight areas, the route has been diverted and re-routed. The dynamic real-time optimization path is realized.

3.2. Simulation of the performance optimization for cruising phase with vertical profile
Here, we use a one-dimensional search method when optimizing the cruising cost of the formula (13). Specifically, Fibonacci method is adopted.

In the following content, \( C_{\text{unit}} = 5.19 \) CNY/kg, Mach number \( M = V_i/V_a \) and \( V_a \) is sound velocity.

In the cruising trajectory mode with constant altitude and changing Mach number, the parameter optimization method is adopted. We simulate with height \( H = 12000 \) m, cost index \( C_I = 30 \), flight distance \( x = 2240.9 \) Km \((1120.45 \times 2\) , which is from the simulation results in section 3.1). The simulation diagram is shown in Figure 2, and the simulation data is shown in Table 1.

![Figure 2. Optimal (a) DOC and (b) flight mass at constant altitude.](image)

| Weight (Kg) | Mach number | Time (h) | Distance (Km) | Fuel cost (CNY) | Time cost (CNY) | Total cost (CNY) |
|-------------|-------------|----------|---------------|----------------|----------------|-----------------|
| Changing Mach number | 56000 | 0.738808 | 0.3823 | 300 | 3875.4 | 5952.4 | 9827.8 |
| 55253 | 0.742197 | 0.6342 | 500 | 6269.5 | 9874.5 | 16144 |
| 54045 | 0.745586 | 0.8207 | 650 | 7787.7 | 12778 | 20566 |
| 52545 | 0.748975 | 0.6285 | 500 | 5666.8 | 9785.7 | 15453 |
| 51453 | 0.752364 | 0.3640 | 290.9 | 3163.2 | 5667.5 | 8830.7 |
| Total | 2.8297 | 2240.9 | 26763 | 44058 | 70822 |
| Keeping Mach number | 56000 | 0.738808 | 2.8554 | 2240.9 | 28945 | 44459 | 73404 |

From the simulation results, changing Mach number at constant altitude can reduce fuel consumption and time costs, and thereby reduce direct operating costs. The DOC is reduced by \( 100\% \times (73404-70822)/73404 = 3.52\% \). We can see that the impact of changing Mach number on the cost is obvious, which is also the main reason for the current cruising optimization to consider changing speed.

Similarly, in the trajectory mode with constant Mach number and changing altitude, the minimum direct operating cost is also used as the Performance function. We simulate with \( M = 0.73689 \), cost index \( C_I = 30 \), and flight distance \( x = 2240.9 \) Km \((1120.45 \times 2\) . The simulation diagram is shown in Figure 3, and the simulation data is shown in Table 2.
Figure 3. Optimal (a) DOC and (b) flight mass at constant Mach number.

Table 2. Simulation results with changing altitude and keeping altitude unchanged.

| Height (m) | Weight (Kg) | Velocity (m/s) | Time (h) | Distance (Km) | Fuel cost (CNY) | Time cost (CNY) | Total cost (CNY) |
|------------|-------------|----------------|----------|---------------|-----------------|----------------|-----------------|
| 9600       | 56000       | 222.4930       | 0.1436   | 115           | 1658            | 2235.9         | 3893.9          |
| 10200      | 55681       | 220.5619       | 0.1511   | 120           | 1657.1          | 2352.6         | 4009.7          |
| 10800      | 55362       | 218.6138       | 0.1588   | 125           | 1662.5          | 2472.5         | 4135            |
| 11400      | 55042       | 216.6481       | 0.1652   | 130           | 1659.3          | 2572.2         | 4231.5          |
| 12000      | 54722       | 214.6644       | 2.2657   | 1750.9        | 21941           | 35277          | 57218           |
| Total      |             | 2.8844         | 2240.9   | 28578         | 44910           | 73488          |                 |

From the simulation results, changing altitude at constant Mach number can reduce fuel consumption and direct operating costs. Comparing with the cruising altitude of 9600 m, the DOC is reduced by $100\% \times (75863 - 73488)/75863=3.13\%$. We can see that changing the altitude reduces the flight cost. It conforms to the current concept for cruising optimization: the aircraft is loaded with full fuel at the beginning of the flight and usually flies at a relatively low cruising altitude first. With the passage of time and the consumption of fuel, the weight of the aircraft becomes lighter, and then the aircraft climbs to a higher altitude.

4. Conclusions

Taking a certain route as an example, the route has been diverted and the shortest flight path has been re-planned under the restricted no-flight area, the dynamic real-time optimization route has been realized, and the optimization effect has been demonstrated on the VC++ simulation platform. Combining with the performance data of a certain type of aircraft, the goal of performance optimization to reduce fuel consumption and time cost for cruising phase can be achieved by selecting the best flight speed and altitude, and the optimization results are obvious.

In the future work, we can further study the optimization characteristics for climbing and descent. At the same time, due to the complexity and uncertainty of atmospheric environment changes, it is necessary to conduct more systematic and accurate modelling and analysis of aircraft atmospheric environment models, such as wind field models, environmental temperature, and environmental pressure. Engine and automatic throttle system, as an important part of aircraft thrust model and speed control, need to be further improved.
References

[1] Richards, A., How J. P. (2002) Aircraft Trajectory Planning with Collision Avoidance Using Mixed Integer Linear Programming In: Proceedings of the American Control Conference. Alaska. pp. 1936-1941.

[2] Krozel J., Lee C., Mitchell J.S.B. (2006) Turn-constrained route planning for avoiding hazardous weather. Air Traffic Control Quarterly, 14(2): 159-182.

[3] Mccrea M.V., Sherali H D, Trani A A. (2008) A probabilistic framework for weather-based rerouting and delay estimations within an Airspace Planning model. Transportation Research Part C: Emerging Technologies, 16(4): 410-431.

[4] Taylor C., Wanke C. (2012) Dynamically Generating Operationally Acceptable Route Alternatives Using Simulated Annealing. Air Traffic Control Quarterly, 20(1): 416-425.

[5] Agust A., Alonso-Ayuso A., Escudero L.F., et al. (2012) On air traffic flow management with rerouting. Part II: Stochastic case. European Journal of Operational Research, 219(1): 167-177.

[6] Wang F., Wang H. (2014) A Re-routing Path Planning Method Based on Maklink Graph and GA Algorithm. Journal of Transportation Systems Engineering and Information Technology, 14(5): 154-160.

[7] Soler M., Olivares A., Staffetti E., et al. (2012) Framework for Aircraft Trajectory Planning Toward an Efficient Air Traffic Management. Journal of Aircraft, 49 (1): 341-348.

[8] Bonami P., Olivares A., Soler M., et al. (2013) Multiphase Mixed-Integer Optimal Control Approach to Aircraft Trajectory Optimization. Journal of Guidance Control and Dynamics, 36 (5):1267-1277.

[9] Soler M., Olivares A., Staffetti E. (2015) Multiphase Optimal Control Framework for Commercial Aircraft Four-Dimensional Flight-Planning Problems. Journal of Aircraft, 52 (1): 274-286.

[10] Jensen L. L., John Hansman R. (2014) Commercial Airline Altitude Optimization Strategies for Reduced Cruise Fuel Consumption. In: 14th AIAA Aviation Technology, Integration, and Operations Conference. Los Angeles.

[11] Miller L.E. (2014) Optimal cruise performance. Journal of Aircraft, 30(3):403-405.

[12] Antoine S., Wissem M., Francois S. (2017) Optimal Speed-Profile Determination for Aircraft Trajectories[J]. Aerospace Science and Technology, 67: 327-342.

[13] Liu J., Zhang J., Dai X., Zu H., (2018) Research on Trajectory Generation and Optimization in Continuous Descent Operations. In: 2018 Aviation Technology, Integration, and Operations Conference. Atlanta, pp.1-13.

[14] Yong E., Chen L., Tang G. (2008) A Survey of Numerical Methods for Trajectory Optimization of Spacecraft. Journal of Astronautics, 29(2):397-406.

[15] Wang S., Yang Y., Jing Z. (2012) Optimal Flight Path Planning of Cruising Phase with No-Fly Zone Constraints Based on Dynamic Programming Algorithm. Materials Science and Information Technology, 433-440:5911-5917.

[16] Liu J., Liu G.F. (2005) Algorithm of Coordinates Conversion in Gauss- Kruger Projection. Computer Simulation, 22(2):119-124.