Emission spectrum of soft massless states from heavy superstring

Shoichi Kawamoto
(Tunghai Univ. → NCTS, Taiwan )

Phys. Rev. D87 (2013) 124001
with T. Matsuo (Anan NCT)
Why heavy (super)string?

Highly excited string is very small, heavy, and unstable

Decay modes, lifetime...?
(accessible by perturbation theory)

This decay may be seen as a stringy toy model of Hawking radiation.

e.g. Long-lived states and cosmological application
Blackhole/String correspondence

Highly excited string has enormous entropy

May explain Bekenstein-Hawking entropy?

\[ g_s \sim N^{-\frac{1}{4}} \]

\[ \sim GM \approx Mg_s^2 \alpha' \]

Corresponding point

\[ S_{\text{string}} \sim S_{\text{BH}} \sim \sqrt{N} \]

[Susskind ('93), Horowitz-Polchinski ('97)]
Decay process

Leading order in $g_s$: Splitting only once

Not so large momentum transfer: microscopic strings may fly away (soft emission)

Turns out that massless state emission is dominant
Setup

Rest frame

\[ P^{\mu}_{\text{ini}} = (\sqrt{2N}, 0, \cdots, 0) \]

\[ P^{\mu}_{\text{fin}} = (-\sqrt{\omega^2 + 2N'}, 0, \cdots, 0, -\omega) \]

\[ k^\mu = (-\omega, 0, \cdots, 0, \omega) \]

\( (\alpha' = 1/2) \)

- Observe a heavy string from far away \( \rightarrow \) Averaged initial state
- Not observe final states \( \rightarrow \) sum over the possible states
- Initial and final states are “heavy”

\[ N, N' \gg \omega \quad \rightarrow \quad N - N' = \sqrt{2N} \omega \]
Semi-inclusive decay

Decay rate: \[ \Gamma = \frac{\omega^{D-2} d\omega}{M^2} P(\Phi(N) \rightarrow V(k) + \Phi(N')) \]

\[ \frac{1}{g(N)} \sum_{\Phi(N)} \sum_{\Phi(N')} \sum_{\zeta} |\langle \Phi(N') | V(\zeta, k) | \Phi(N) \rangle|^2 \]

averaging \quad \text{sum over} \quad \text{A state at level } N

Exponentially many states at level \( N \rightarrow \) difficult to sum over

(Open superstring) density of states: \[ g(N) \sim N^{-\frac{11}{4}} e^{\pi \sqrt{8N}} \]
\[ \hat{P}_N = \int \frac{dw}{2\pi i w} w^{\hat{N} - N} \rightarrow \sum_{\Phi(N)} |\Phi(N)\rangle = \sum_{\phi=\text{All}} \hat{P}_N |\phi\rangle \]

projection operator onto level \(N\)

\[
\sum_{\Phi(N)} \sum_{\Phi(N')} |\langle \Phi(N')|V(k)|\Phi(N)\rangle|^2 = \sum_{\phi,\phi'} |\langle \phi'|\hat{P}_N V(k) \hat{P}_N |\phi\rangle|^2
\]

\[
= \int \frac{dv}{2\pi iv} v^{N-N'} \int \frac{dw}{2\pi i w} w^{-N} \text{tr} [V^+(k,1)V(k,v)w^{\hat{N}}]
\]

\[v^{\hat{N}} V(k,1)v^{-\hat{N}} = V(k,v)\]

Oscillator part is given by 1-loop calc.

23 August 2013
@ YITP QFT workshop 2013

S. Kawamoto (Tunghai U.)
Consider open/closed superstring massless state emission from heavy open/closed superstring.

Note: This $\text{tr}$ is not supertrace.

Fermion emission is discussed to be subleading effect.

[Iengo-Russo, Chen-Li-She]
Vertex Operators

Light-cone vertex operator: \( k^+ = 0 \)

\[
V_B(\zeta, k) = (\zeta^i(k)B^i - \zeta^-(k)p^+) e^{ik \cdot X}
\]

Massless boson emission

\[
B^i = \dot{X}^i - R^{ij} k^j
\]

\[
V_F(u, k) = \left( u^a(k)F^a + u^\dot{a}(k)F^{\dot{a}} \right) e^{ik \cdot X}
\]

Massless fermion emission

\[
F^a = \sqrt{p^+} S^a
\]

\[
F^{\dot{a}} = \frac{1}{\sqrt{p^+}} \left( (\gamma \cdot \dot{X}S)^{\dot{a}} + \frac{1}{3} : (\gamma^i S)^{\dot{a}} R^{ij} : k^j \right)
\]

\[
R^{ij} = \frac{1}{2} (S\gamma^{ij} S)(\tau) \quad : \text{generator of rotation}
\]
Basic traces

\[ \text{tr} \left( V_B(\zeta, 1) \dagger V_B(k, v) w^\hat{N} \right) = (\zeta^* i \zeta i \Omega(v, w) + \zeta^* - \zeta^-(p^+)^2) \theta_4(0|\tau)^{-8} \]

\[ \text{tr} \left( V_F(u, 1) \dagger V_F(u, v) w^\hat{N} \right) = 4 \left[ p^+ u^a u^a + u^a \gamma^i_{b \bar{a}} u^b p^i + u^a \gamma^i_{a b} u^i p^b + \frac{u^a(\bar{k}) u^a(k)}{p^+} ((p)^2 + \Omega(v, w)) \right] \Xi(v, w) \theta_4(0|\tau)^{-8} \]

1. \[ \int \frac{dv}{2\pi i v} v^{N-N'} \] integral is easy to carry out.

\[ v^{-N-N'} \] term survives \((N > N')\)

2. \[ p^+ \rightarrow \sqrt{N} \] : large-N factor

\[ \Omega(v, w) = \sum_{n=1}^{\infty} \frac{v^n + (w/v)^n}{1 - w^n} \]

\[ \Xi(v, w) = \frac{1}{2} + \sum_{n=1}^{\infty} \frac{v^n + (w/v)^n}{1 + w^n} \]

\[ \theta_4(0|\tau) = \prod_{n=1}^{\infty} \left( \frac{1 - w^n}{1 + w^n} \right) \]

\[ w = e^{i\pi \tau} \]
Open string state from Open string

$$P_{\text{boson, open}} = \sum_{\zeta} |\zeta|^2 \frac{N - N'}{\mathcal{G}(N)} \oint \frac{dw}{2\pi i w} \frac{w^{-N'} \theta_4^{-8}}{1 - w^{N - N'}}$$

$$P_{\text{fermion, open}} \approx 4 \sum_u |u|^2 \frac{\sqrt{N}}{\mathcal{G}(N)} \oint \frac{dw}{2\pi i w} \frac{w^{-N'} \theta_4^{-8}}{1 + w^{N - N'}}$$

Evaluate $w$-integral by saddle point method  \((N' : \text{large})\)

$$P_{\text{boson, open}} \approx \frac{N - N'}{\mathcal{G}(N)} e^{\pi \sqrt{8N'}} N'^{-11/4} \frac{1}{1 - e^{-\sqrt{2\pi} \frac{N - N'}{\sqrt{N'}}}}$$

$$P_{\text{fermion, open}} \approx \frac{\sqrt{N}}{\mathcal{G}(N)} e^{\pi \sqrt{8N'}} N'^{-11/4} \frac{1}{1 + e^{-\sqrt{2\pi} \frac{N - N'}{\sqrt{N'}}}}$$
Decay rate for open string emission

Using,

\[ G(N) \approx N^{-\frac{11}{4}} e^{\pi \sqrt{8N}} \]

\[ N - N' = \omega \sqrt{2N} \]

and

\[ \Gamma = \frac{\omega^7 d\omega}{M^2} P_{\text{boson or fermion}} \]

\[ \Gamma_{\text{boson, open}} \sim \frac{\omega^8 d\omega}{M^2} \frac{\sqrt{N}}{e^{2\pi\omega} - 1} \]

\[ \Gamma_{\text{fermion, open}} \sim \frac{\omega^7 d\omega}{M^2} \frac{\sqrt{N}}{e^{2\pi\omega} + 1} \]

Thermal distribution of Hagdorn temperature \( T_H = \frac{1}{2\pi} \)

23 August 2013
@ YITP QFT workshop 2013

S. Kawamoto (Tunghai U.)
Closed string emission

Closed string vertex operator: open × open

\[ V_{\zeta\bar{\zeta}}(k, \tau) = \int_{0}^{\pi} \frac{d\sigma}{\pi} V_{\zeta}(k/2; \tau + \sigma) V_{\bar{\zeta}}(k/2; \tau - \sigma) \]

\[ \tilde{\alpha}^i_n, \tilde{S}^a_n \quad \text{from closed} \]

\[ \alpha^i_n, S^a_n \quad \text{from open} \]
From Heavy closed string

Calculation is factorized

\[
\frac{1}{\mathcal{G}^{\text{cl}}(N)} \sum_{\Phi(N), \Phi'(N)} \left| \langle \Phi(N') | V_{\xi} | \Phi(N) \rangle \right|^2 = \frac{1}{\mathcal{G}(N)} \int \frac{dv}{2\pi iv} v^{N-N'} \int \frac{dw}{2\pi i\omega} w^{-N} \text{tr}(V_{\xi}(1)^\dagger V_{\xi}(v) w^{\hat{N}}) \\
\times \frac{1}{\mathcal{G}(N)} \int \frac{d\tilde{v}}{2\pi i\tilde{v}} \tilde{v}^{N-N'} \int \frac{d\tilde{w}}{2\pi i\tilde{w}} \tilde{w}^{-N} \text{tr}(V_{\tilde{\xi}}(1)^\dagger V_{\tilde{\xi}}(\tilde{v}) \tilde{w}^{\hat{N}})
\]

Product of the open result \( (\alpha' = 2) \)

For example,

\[
P_{\text{closed}}^{i,j} = \frac{\omega \sqrt{N}}{e^{2\pi \omega} - 1} \cdot \frac{\omega \sqrt{N}}{e^{2\pi \omega} - 1} = \frac{\omega^2 N (e^{4\pi \omega} - 1)}{(e^{2\pi \omega} - 1)^2} \cdot \frac{1}{e^{4\pi \omega} - 1}
\]

interpret

\[\text{Thermal distribution of Hagedorn temp. } T_H = \frac{1}{4\pi}\]

S. Kawamoto (Tunghai U.)
Closed states from open string

“Left” and “right” parts of closed vertex act on the same Fock space.

\[ P_{\text{closed from open}} = \frac{1}{\mathcal{G}(N)} \sum_{\Phi(N), \Phi'(N)} |\langle \Phi(N') | V_{\xi} \xi | \Phi(N) \rangle|^2 \]

\[ = \frac{1}{\mathcal{G}(N)} \int_0^\pi \frac{d\sigma}{\pi} \int_0^\pi \frac{d\tilde{\sigma}}{\pi} \int \frac{dv}{2\pi iv} v^{N-N'} \int \frac{dw}{2\pi iw} w^{-N} \text{tr}(\langle V_{\xi}(e^{i\sigma}) V_{\xi}(e^{-i\tilde{\sigma}}) \rangle^d (V_{\xi}(ve^{i\sigma}) V_{\xi}(ve^{-i\sigma}))w^{\hat{N}}) \]

4 vertex insertion

Leading to a bit complicated result...
Example: Boson – Boson case

After v-integral,

\[ P = \frac{1}{G(N)} \int \frac{dw}{2\pi iw} w^{-N} \left( \zeta^{ij}(\zeta^{ij*} + \zeta^{ji*}) P_1(w) \frac{1 + (-1)^L}{2} + \zeta^{ij}(\zeta^{ij*} - \zeta^{ji*}) \frac{2}{\pi^2} P_2(w) \frac{1 - (-1)^L}{2} \right) \theta_4^{-8} \]

\[ P_1(w) = \left( \frac{L}{2} \right)^2 \frac{w^L}{(1 - w^{\frac{L}{2}})^2} \]

\[ P_2(w) = 2 \sum_{n=1}^{\infty} \frac{n(n + L)}{(2n + L)^2} \frac{w^{L+n}}{(1 - w^n)(1 - w^{n+L})} + \sum_{n=1}^{L-1} \frac{n(L - n)}{(2n - L)^2} \frac{w^L}{(1 - w^n)(1 - w^{L-n})} \]

In this sum, \( n = (L + 1)/2 + O(1) \) part gives the dominant contribution.

Leading order part is the same as that from closed string!!

(\( \omega \) dependence)
Emission rates (summary)

Emission rates:

\[ M \approx \sqrt{N} \]

\[ \Gamma \sim \frac{\omega^8 d\omega}{M^2} \frac{\sigma(\omega)}{e^{\beta_H \omega} + 1} \]
\[ \beta_H = \pi \sqrt{8\alpha'} \]

Open from Open:

\[ \sigma_{\text{boson}} = g^2 \sqrt{N} \cdot 1 \]
\[ \sigma_{\text{fermion}} = g^2 \sqrt{N} \cdot \omega^{-1} \]

Closed from Open/Closed:

\[ \sigma_{BB} = g^4 N \cdot \frac{\omega(e^{\beta_H \omega} - 1)}{(e^{\beta_H \omega/2} - 1)^2} \]
\[ \sigma_{BF} = g^4 N \cdot \frac{e^{\beta_H \omega} + 1}{(e^{\beta_H \omega/2} - 1)(e^{\beta_H \omega/2} + 1)} \]
\[ \sigma_{FF} = g^4 N \cdot \frac{\omega^{-1}(e^{\beta_H \omega} - 1)}{(e^{\beta_H \omega/2} + 1)^2} \]
Observations I

Open string is like a blackbody.

Behaves like a cavity? (Once absorbed, hardly emitted)

Closed string emission from open/closed string takes the same form.

Locality of the interaction
Blackhole greybody factors

Greybody factors:  \[ \Gamma = \frac{\sigma_{js}(\omega)\omega^8 d\omega}{e^{\beta \omega} + 1} \]

s: spin, j: total ang. mon.
\[ j \geq s, \omega \ll 1 \]

Spherical BH in an asymptotically flat space

[Harmark-Natario-Schiappa ('07), Kanti-March-Russel(02)]

\[ \sigma_{j0} \propto \omega^{2j} : \text{scalar (s=0)} \Rightarrow \omega^0 \]
\[ \sigma_{j\frac{1}{2}} \propto \omega^{2j-1} : \text{Dirac fermion} \Rightarrow \omega^0 \]
\[ \sigma_{j1} \propto \omega^{2j} : \text{vector} \Rightarrow \omega^2 \]

Dominant \( j=s \) modes

Heavy string

\[ \omega^{-1} \]
\[ d^9k \rightarrow \omega^8 d\omega \]

\[ \omega^0 \] (j=0 ?)
Blackhole greybody factors

5D D5-D1-KK near extremal BH

\[ \sigma_{s=0} \propto \frac{\omega (e^{\beta_B \omega} - 1)}{(e^{\beta_B \omega/2} - 1)^2} \]

[Das-Mathur ('96), Maldacena-Strominger(97)]

\[ \sigma_{s=1/2} \propto \frac{\omega e^{\beta_B \omega} + 1}{(e^{\beta_B \omega/2} - 1)(e^{\beta_B \omega/2} + 1)} \]

[Hosomichi ('97)]

Closed string emission from Heavy superstring

Bosons

\[ \sigma \propto \frac{\omega (e^{\beta_H \omega} - 1)}{(e^{\beta_H \omega/2} - 1)^2} \]

\[ \sigma \propto \frac{\omega^{-1} (e^{\beta_H \omega} - 1)}{(e^{\beta_H \omega/2} + 1)^2} \]

Fermions

\[ \sigma \propto \frac{e^{\beta_H \omega} + 1}{(e^{\beta_H \omega/2} - 1)(e^{\beta_H \omega/2} + 1)} \]

Why these kinds of black holes?

23 August 2013
@ YITP QFT workshop 2013

S. Kawamoto (Tunghai U.)
Summary

- Calculate open/closed massless state emission from heavy open/closed superstring
- Open string state emission: blackbody like
- Closed string state emission: same for open/closed string
- Greybody factors are somehow blackhole like
  (Our setup is non-BPS)

- Numerical coefficient?
- Next order? Coupling constant vs. large-N