Student Obstacles in Solving Algebraic Thinking Problems

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Abstract. The aim of this research is to analyze the student obstacles on solving algebraic thinking problems in low grades elementary school. This research is a preliminary qualitative research, and involved 66 students of grade 3 elementary school. From the analysis student test results, most of student experience difficulty in solving algebraic thinking problems. The main obstacle is the student's difficulty in understanding the problem of generalizing the pattern because the students are not accustomed to see the rules that exist in generalize the pattern.

1. Introduction

The development of thinking skills in mathematics learning is strongly influenced by the role of teachers in presenting lessons that can cultivate mathematical skills. This relates to the role of the teacher as a learning designer, where the teacher has a task in determining what activities will be presented in the classroom that can facilitate the students in developing those skills. In addition, the success of developing mathematical skills in learning is also inevitably influenced by teacher's belief in mathematics. One such unproductive belief that students only need to learn and use the same standard computational algorithms and methods that are equally determined to solve algebraic problems \cite{1}. This belief must be replaced by productive belief that all students must have various strategies and approaches chosen in solving problems, but not limited to general methods, standard algorithms, and procedures. For that reason, algebraic thinking is introduced as an alternative thinking that students can use in solving problems.

Thinking algebra is considered as a very important thinking skill to be developed in elementary school. In some countries this algebraic study gets attention. The studies are conducted to see how far the curriculum of a country can facilitate elementary school students in developing algebraic thinking skills in their level. The researchers found that school curriculum The basics have several different ways of introducing algebraic thinking \cite{2, 3, 4, 5, 6}. Several studies have shown that students have been able to develop algebraic thinking in primary schools \cite{7, 8, 9, 10}. These results show that student elementary school solved problems with solutions obtained by students themselves in a variety of ways as a step in preparing to think algebra and have been had algebraic thinking non-symbolic

From some of these studies, researches have been done are more focusing in algebraic thinking development in elementary school level. However, the analysis of the barriers that arise when students develop algebraic thinking is something that is rarely studied. Though an analysis of these barriers is very important to do before developing the didactic design of algebraic thinking. Therefore, this study
aims to analyze the learning barriers of students in the development of algebraic thinking which results can later be used as a guide for teachers in designing learning that can assist students in developing algebraic thinking.

2. Literature Review
Algebraic reasoning is the manipulation of numerals and signs (e.g., $x + 5 = 12 - 4$) to solve for an unknown [11]. Thinking algebra or commonly called algebraic reasoning is the manipulation of numbers and signs to break something unknown. Thinking algebra introduced in elementary school is not to think about algebraically formally, but is related to teaching students about the basics to be able to think algebraically in solving problems. At elementary school level, unknown or unknown part is not symbolized in the form of letters, usually unknown symbolized by pictures or dots.

Therefore, algebraic thinking developed in elementary schools is non-symbolic, meaning it is not related to algebraic thinking by using algebraic symbols like in high school. Thinking algebra in elementary school begins with algebraic aspects such as equations and problem solving, generalizing patterns that include describing sequence terms based on the position of the pattern [12]. The algebraic thinking aspect developed in this research is to generalize the geometry pattern. This is because patterns can be found in numbers as well as in geometric situations [13]. So as to enable elementary school students to more easily develop algebraic thinking. To create a generalization scheme, the first student must solve the problem, usually by observing the problem part, creating a new problem, observing the pattern and making generalizations [14].

In this study, students are given situations related to the pattern, then asked to see the rules that exist in the pattern so that students can make a generalization of the given pattern. This rule is important in generalizing the pattern. Rules provide insight into how students first visualize different parts of geometric structures and in this process come to generalize the functional relationship explicitly by using correspondence between item numbers and measured components of the pattern structure [15]. To perform a learning barrier analysis in this study is based on three types of learning barriers [16]. First, the obstacles of ontogenic origins (knowledge of students who have a limited context). The second, obstacle of epistemological (knowledge of students who have limited application context). The third, the obstacles of didactical (due to the educational system).

3. Experimental Method
This research is preliminary qualitative study with design didactical research (DDR). This research was conducted at two elementary school in Sumedang, West Java Province, Indonesia. There were 66 students grade 3 (8-9 year old) as participants. Data was collected from student individual written test and interview about their difficulty. The problems given for student were three problems about algebraic thinking domain, i.e. generalization of patterns.

4. Result and Discussion
There were three problems given by researcher that used for looking student obstacles. The problems type were generalizations patterns. One was problem in Radford research. Others were problems created by researcher. For each problem, we gave three question that should be answered by student. Students were asked to provide answers by finding the rules in the pattern.
Figure 1. First problem given to student

Observe the figure below!

Figure 1 Figure 2 Figure 3 Figure 4

How many square in Figure 5? How do you know the square in Figure 5?
How many square in Figure 12? How do you know the square in Figure 12?
How many square in Figure 25? How do you know the square in Figure 25?"

Figure 2. Second problem given to student

Observe the figure below!

Figure 1 Figure 2 Figure 3 Figure 4

How many all triangles in Figure 7? How do you know all the triangles in Figure 7?
How many black triangles in Figure 10? How are you to know the many black triangles
in Figure 10?
How many white triangles in Figure 15? How do you know the many white triangles
in Figure 15?"

Figure 3. Third problem given to student

Based on data analysis from student test, we found some difficulty that student faced when they solved algebraic thinking problems. From 66 students, there was only one student who has shown the idea of algebra thinking. Other students showed difficulty finding algebraic ideas of seeing rules in generalizing patterns. From the results of the analysis, there are three kinds of identified learning obstacles, namely ontological obstacle, didactical obstacle, and epistemological obstacle.

Didactical obstacles found caused students to misinterpret problems and students had difficulty explaining the strategies used. Student difficulties in interpreting problems can occur due to inconsistencies in the use of the term or word used in the problem. For example, the words used in the context of the problem are Figure 1, Figure 2, Figure 3, and Figure 4, while the word question used is the 5th image, the 8th image, and the 10th image. This difference is likely to lead to multiple interpretations of students. In addition, students' difficulties in interpreting problems can be caused by the teacher's habits when asking students to solve problems. When students had difficulties with students when solving problems, students were not helped with questions or keywords that can help students understand the problem. However, the teacher directly noticed the intent of the given problem. This can make it difficult for students to interpret questions on questions because students were not accustomed to understanding them on their own. An example of an answer to one of the students experiencing this obstacle can be seen in Figure 4.

Figure 4. Examples of students answers (on problem 1) that experience didactical obstacle
In Figure 4 it is known that student answers and questions have nothing to do. Students did not seem to understand the question. From the interview results revealed that students mistakenly in interpreting the problems and questions given. The students understood the question in question number 1 as "Which picture of the many apples is 5?", "Which picture is the 8 apple?". So when students answered the picture that has many apples 8, the students added a lot of apples in pictures 1 and 2. From the answers can also be seen the student difficulty in expressing the way they use in solving problems. This is because communicating the reason they are having difficulty and expressing thinking is one that is difficult for students [17, 18].

The ontological obstacle found is due to the lack of knowledge and readiness of the students to study the pattern. Lack of developing of algebraic thinking in elementary schools made students to have no understanding of this material. The difficulties caused by these obstacles were the difficulty in understanding the problem and the students incorrectly using the settlement strategy. An example of the results of students who experience this obstacle can be seen in Figure 5.

![Figure 5](image5.png)

**Figure 5.** Examples of students answers (on problem 2) that experience ontological obstacle

In Figure 5, it is known that the students answered the first and second questions correctly, but the students still used the number of jump calculations. So when the student was faced with the question of determining the square number of the larger pattern, the student experienced a miscalculation. This shows that students’ algebraic thinking ability to generalize the pattern had not yet emerged. The problem should be solved by generalizing the pattern rule (i.e. the sequence number plus the sequence number plus one so as to produce 51). This can happen because students saw the relationships between patterns in the image rather than seeing the pattern relationship with the sequence of numbers [19]. Students found it difficult to find a rule that the existing pattern rules are multiplied, so students compared the number of images between patterns with other patterns to determine many images in the pattern in question.

The epistemological obstacle found is the student got difficulties in distinguishing questions that ask the whole and the part of the pattern. This difficulty occurs because students were not accustomed to solving problems that have different contexts, so that when students got different problems and questions, students found it difficult to understand. Examples of answers to students who experience this difficulty can be seen in Figure 6.

![Figure 6](image6.png)

**Figure 6.** Examples of students answers (on problem 3) that experience epistemological obstacle
In Figure 6 we can see students giving wrong answers when answering the number of black triangles in the 10th image and the number of white triangles in the 15th image. The exact answer is the number of black triangles in the 10th image is 10 and the number of white triangles in the 15th image is 16. From the student's answer it is seen that the students had difficulties in understanding the problem when faced with the question with different context. From the interview, the students answered by searching for all the triangles on each pattern asked. Difficulties of students in solving problems with different contexts can also be caused by students solving problems by looking at the pattern as a whole without looking at the questions asked. This was due to this tendency to occur because students tend to identify the whole object [20].

With the finding that the algebraic ability of students is still low and the difficulties experienced by students into consideration of this ability to be developed from elementary school low grade. Therefore, it is necessary to create learning activities that can facilitate it. Learning algebra thinking especially generalizing patterns needs to be preceded by continuing learning patterns with simple variables, such as continuing patterns by analyzing the sequence of colors. So students are expected to gradually be able to develop this ability.

5. Conclusion
Based on the results, the ability of grade 3 students to solve algebraic thinking problem is still low, but see the findings that occur, algebraic thinking skills can be developed in the low grade of elementary school. The main obstacle found is the student's difficulty in understanding the problem of generalizing the pattern because the students were not accustomed to see the rules that exist in generalize the pattern. For that, a learning design needs to be designed to facilitate the student in developing their algebraic thinking.

Acknowledgments
We thank student and teachers for their help and participation.

References
[1] NCTM 2014 Principles to Actions: Ensuring Mathematical Success for All. (Reston, V.A: NCTM)
[2] Moyer J, Hünker D A and Cai F 2004 Developing Algebraic Thinking in the Earlier Grades: A Case Study of the U.S. Investigations Curriculum. The Mathematics Educator vol 8 no 1 p 6-38.
[3] Fong N S 2004 Developing Algebraic Thinking in Early Grades: Case Study of The Singapore Primary Mathematics Curriculum. The Mathematics Educator vol 8 no 1 p 39-59.
[4] Schmittau J and Morris A 2004 The Developing of Algebra in Elementary Mathematics Curriculum of V.V. Davydov. The Mathematics Educator vol 8 no 1 p 60-87.
[5] Lew H C 2004 Developing Algebraic Thinking in Early Grades: Case Study of Korean Elementary School Mathematics. The Mathematics Educator vol 8 no 1 p 88-106.
[6] Cai J 2004 Developing Algebraic Thinking in Earlier Grades: Case Study of Chinese Elementary School Curriculum. The Mathematics Educator vol 8 no 1 p 107-130.
[7] Booker G and Windsor W 2010 Developing Algebraic Thinking: using problem-solving to build from number and geometry in the primary school to the ideas that underpin algebra in high school and beyond. Procedia Social and Behavioral Sciences 8 p 411–419.
[8] Radford L 2010 Elementary Forms of Algebraic Thinking in Young Students. In M. F. Pinto. & T. F. Kawasaki (Eds.), Proceedings of the 34th Conference of the International Group for the Psychology of Mathematics Education (Belo Horizonte, Brazil: PME) vol. 4 p 73-80.
[9] Radford L 2011 Embodiment, perception and symbols in the development of early algebraic thinking. In Ubuz, B. (Ed.), Proceedings of the 35th Conference of the International Group for the Psychology of Mathematics Education (Ankara, Turkey: PME) vol. 4 p 17-24
[10] Carraher D W, Schliemann A D and Brizuela B M 2006 Arithmetic and Algebra in Early Mathematics Education. *Journal for Research Mathematics Education*. Vol. 37 No. 2, 87-115.

[11] Powell S R and Fuchs L S 2014 Does Early Algebraic Reasoning Differ as a Function of Students’ Difficulty with Calculations versus Word Problems? *Learn Disabil Res Pract*, 29 3 p 106-116.

[12] Radford L 2012 *On the development of algebraic thinking*. PNA 64(1) p 117-133.

[13] Ling G W and Ghazali M 2014 A Study Of Malaysian Year 5 Pupils’ Pre-Algebraic Thinking. *Asia Pacific Journal of Educators and Education*. Vol. 29, 105–124.

[14] Čadež T H and Kolar V M 2015 Comparison of Types of Generalizations and Problem-Solving Schemas Used to Solve A Mathematical Problem. *Educ Stud Math*, 89:283–306.

[15] Wilkie K J and Clarke D M 2015 Developing Students’ Functional Thinking in Algebra Through Different Visualisations of A Growing Pattern’s Structure. *Math Ed Res J*. Now York: Springer.

[16] Brousseau G 2002 *Theory of Didactical Situations in Mathematics*. New York: Kluwer Academic Publishers.

[17] Barbosa A, Vale I and Palhares P 2012 Pattern Tasks: Thinking Processes Used By 6th Grade Students. *Relime*. Vol. 15 No.3 México November 2012.

[18] Cooper T J and Warren E 2011 Years 2 to 6 Students’ Ability to Generalise: Models, Representations, and Theory for Teaching and Learning. In J. Cai & E. Knuth (Eds.), *Early Algebraization: A Global Dialogue from Multiple Perspectives*. 187–214. Berlin, Germany: Springer-Verlag.

[19] Warren E and Cooper T 2008 Generalising the Pattern Rule for Visual Growth Patterns: Actions that Support 8 Year Olds’ Thinking. *Educational Studies in Mathematics*, 67, 171–185.

[20] Erdogan E O and Turan P 2014 The Primary School Students’ Pattern Seeking Process In the Spreadsheet Environment. *Education and Science*. Vol. 39, No. 173, hal. 182-197.