TEM turbulence optimisation in stellarators

J H E Proll, H E Mynick, P Xanthopoulos, S A Lazerson and B J Faber

Abstract
With the advent of neoclassically optimised stellarators, optimising stellarators for turbulent transport is an important next step. The reduction of ion-temperature-gradient-driven turbulence has been achieved via shaping of the magnetic field, and the reduction of trapped-electron mode (TEM) turbulence is addressed in the present paper. Recent analytical and numerical findings suggest TEMs are stabilised when a large fraction of trapped particles experiences favourable bounce-averaged curvature. This is the case for example in Wendelstein 7-X (Beidler et al 1990 Fusion Technol. 17 148) and other Helias-type stellarators. Using this knowledge, a proxy function was designed to estimate the TEM dynamics, allowing optimal configurations for TEM stability to be determined with the STELLOPT code without extensive turbulence simulations. A first proof-of-principle optimised equilibrium stemming from the TEM-dominated stellarator experiment HSX (Anderson et al 1995 Fusion Technol. 27 273) is presented for which a reduction of the linear growth rates is achieved over a broad range of the operational parameter space. As an important consequence of this property, the turbulent heat flux levels are reduced compared with the initial configuration.

Keywords: stellarator, turbulence, transport, optimisation

I. Introduction
The development of stellarators has taken great strides since their inception by Lyman Spitzer Jr in 1951 [1]. Stellarators are inherently 3-dimensional, so that the configuration space of possible equilibria is very large. Different techniques of optimisation have been employed to find equilibria with certain desired features within this configuration space. The reduction of neoclassical transport down to levels of tokamaks for example has been achieved through the introduction of quasi-symmetries [2, 3] or variants of omnigeneity [4, 5], which made stellarators competitive with tokamaks regarding the expected levels of transport. In addition, other stellarator design features are being addressed via optimisation, too, such as the confinement of fast particles [6]. In neoclassically optimised stellarators such as Wendelstein 7-X (W7-X) [7, 8] or the quasi-symmetric stellarators HSX (Helically Symmetric Experiment) [9] and NCSX (National Compact Stellarator Experiment) [10], the turbulent transport is expected to be the dominant transport channel in a large part of the plasma, just as in tokamaks. While NCSX has not been built and W7-X is only about to start operation, HSX is already running and has shown that the neoclassical transport is indeed reduced thanks to the quasi-symmetry [11] and the turbulent transport gains importance. This turbulent transport is thought to be driven by microinstabilities like the ion temperature gradient mode (ITG) or the trapped-electron mode (TEM). An optimisation to reduce these kinds of microinstabilities in stellarators is not trivial, especially since analytical predictions regarding the actual nonlinear behaviour of microturbulence in general geometry are rather sparse. For instance, only very recently Plunk et al [12] published a theory on the saturation...
of ITG turbulence whereas an analogous theory for TEMs has yet to be achieved. Comprehensive numerical simulations of microturbulence in general geometry on the other hand have become available, although they are computationally very demanding—a typical well-resolved turbulence simulation in flux-tube geometry and including kinetic electrons needs roughly half a million CPUh to reach a saturated state. For this matter, an optimisation based on calculating the nonlinear heat flux as a figure of merit for every configuration along the path of optimisation would be desirable, but it is evident that this procedure is currently not viable. We must therefore find simplified expressions to represent the nonlinear heat flux as a figure of merit. Ideally, these ‘proxies’ would be based on analytical theory of the linear or even nonlinear instabilities. The reduction of ITG turbulence has been theoretically demonstrated via this method [13–16] and in this paper we will tackle the reduction of TEM turbulence. Ultimately, one would want to combine all methods of optimisation into one grand scheme to find the point in configuration space where ‘the ideal stellarator’ lives. A code that would be capable of carrying out such an ambitious task is STELLOPT [17], which we will also use for our TEM optimisation. In the next section we briefly explain how STELLOPT works and how a new optimisation is implemented. In section III we review what we already know about TEMs in general geometry using analytical and linear numerical findings. Section IV then shows how we can use this knowledge to come up with simple expressions for the proxy. In section V we present a proof-of-principle equilibrium where we optimised starting from HSX towards reduced TEM turbulence and we comment on the applicability to experimentally feasible TEM-optimised equilibria. Section VI contains the main conclusions, together with future plans related to this work.

II. Optimising with STELLOPT

The STELLOPT code is designed to optimise 3D MHD equilibria created by VMEC [18] by minimising the difference between certain features of the equilibrium and their targeted value. Each design feature $i$—this can be the neoclassical transport, the turbulent transport, ballooning stability, the major radius, to name only a few—is associated with a target value $f_i^{\text{target}}$, and the difference between this target value and the actual value of the final ‘optimised’ equilibrium $f_i^{\text{equilibrium}}$ should be as small as possible. Since usually more than only one design feature shall be targeted at once, each design feature gets assigned a tolerance $\sigma_i$. This acts as a weight $(1/\sigma_i^2)$ when all design features are eventually combined into one function $\chi^2$ that has to be minimised:

$$\chi^2 = \sum_i \frac{(f_i^{\text{equilibrium}} - f_i^{\text{target}})^2}{\sigma_i^2}. \quad (1)$$

How $f_i^{\text{equilibrium}}$ for a given design feature in a given equilibrium is determined depends very much on the design feature itself. For the neoclassical transport for example, STELLOPT is coupled to the NEO code [20] to calculate the neoclassical effective ripple $\epsilon_{\text{eff}}$ [19]. For the turbulent transport it would be ideal to use the turbulent heat fluxes stemming from gyrokinetic simulations. However, since these are CPU-intensive, simpler proxy functions that can substitute for the turbulent heat flux and that are ideally based on analytical theory are sought. The STELLOPT code can be used to optimise any combination of VMEC input parameters to any set of target figures of merit (FOM), subject to the constraints of a given optimisation method. When utilised for stellarator design the boundary harmonics are treated as the free parameters, although enclosed toroidal flux, net toroidal current and a pressure scaling factor may be included as well. As the VMEC boundary representation (R and Z harmonics) is non-unique, the initial configuration is converted to either Hirshman–Breslau [21] or Garabedian [22] representation. These harmonics are the quantities varied by STELLOPT and converted back to the VMEC representation for evaluation of the configuration by VMEC. Once an optimum shape has been computed, codes like NESCOIL [23] and COILOPT [24, 25] are utilised to generate a coil set consistent with that equilibrium. To explore the space of accessible configuration in a given device, VMEC may be run in free boundary mode and STELLOPT set to treat the vacuum coil currents as free parameters. Such a capability allows exploration of a given device’s capabilities, as was done for NCSX [26]. Additionally, the inclusion of plasma profiles, synthetic MSE diagnostics, and magnetic diagnostics [27] has allowed the code to provide a 3D equilibrium reconstruction capability [28, 29]. In this paper, a modified Levenberg–Marquardt [30] algorithm was used to find the minimum of $\chi^2$ in configuration space. This method guarantees that the found optimised equilibrium is at least at a local minimum in configuration space. STELLOPT is also equipped with stochastic algorithms (e.g. differential evolution [31], particle swarm [32]) that have not been applied to this work.

III. The density-gradient-driven TEM in general geometry

To tackle the optimisation of stellarators towards reduced TEM turbulence we should first assess what we already know about the TEM in general geometry. The TEM [33, 34] can be regarded as a drift wave that is driven unstable by a resonance with the precessional drift of trapped particles. Generally, the higher the trapped-particle fraction in a given configuration, the more unstable the TEM becomes. It is destabilised by increasing the density gradient and/or the electron temperature gradient. In a collisional plasma, trapped particles can become detrapped due to collisions, which usually leads to a stabilisation of the TEM [35]. Since this work addresses the suppression of the worst-case instability, collisions will be neglected from here on. First, we revisit some analytical theory regarding the stability properties of TEMs. Because of the temporal and spatial scales involved, the gyrokinetic framework is employed. We then look at different stellarator equilibria with very different geometric properties and present linear simulation results that confirm the analytical findings.
More details on the calculations can be found in previously published papers [36–39].

### III.A. Analytical theory

The stability analysis of TEMs in general geometry via a dispersion relation is not as accessible as in tokamaks. However, it is possible to define a rate of gyrokinetic energy transfer \( P_e \) from the fluctuating electric field to the electrons [36–38], which, at the point of marginal stability where the growth rate \( \gamma \) approaches zero, can be written as

\[
P_e = \frac{\pi e^2}{T_e} \int \frac{d\ell}{B} \int d^3 \phi (\omega - \omega_{de}) \sigma_{de} (\omega_{de} - \omega_{e0})^2 |e\phi| |f_{de}| .
\]

(2)

Here, \( T_e \) is the electron temperature, \( \omega \) is the real frequency of the mode and \( \sigma_{de} = \frac{\kappa \eta}{1 - \kappa^2 \eta} \) denotes the bounce-averaged precessional drift frequency of the electrons, whose drift velocity is given by \( \mathbf{v}_d \). The velocity-dependent diamagnetic frequency is given by \( \omega_{de} = \omega_{de0} \left[ 1 + \eta_e \left( \frac{\mathcal{E}}{T_e} - \frac{2}{3} \right) \right] \), where \( \mathcal{E} \) denotes the energy of the particle, \( \eta_e = \frac{d\ln T_e}{d\ln n} \) gives the ratio between the scale lengths of electron temperature gradient and density gradient and the diamagnetic frequency is defined as \( \omega_{de} = \frac{\kappa \eta}{\eta - 1} \mathbf{B} \times \mathbf{k}_0 \). In addition, \( J_0 \) denotes the Bessel function, \( \phi \) is the electrostatic potential and \( f_{de} \) is the Maxwellian distribution function of the electrons. For the electrons to have a destabilising influence, the energy transfer rate must be negative, \( P_e < 0 \). This means that \( \sigma_{de} \omega_{de} > 0 \) (if the temperature gradient is small, \( \eta_e < 2/3 \)) at least for some particles in velocity space, because all the other terms are positive definite. Thus, for some of the trapped electrons, the precessional drift must be resonant with the propagation of drift waves. In which direction the trapped electrons precess depends on the curvature they sample along their path in the magnetic field:

\[
\sigma_{de} \propto \int_{z_1}^{z_2} \kappa \left( 1 - \frac{\mathcal{L} B(z) / 2}{\sqrt{1 - \mathcal{L} B(z)}} \right) dz, \tag{3}
\]

where the integration is taken along the particle path along a field line, with the pitch angle like coordinate \( \mathcal{L} = v^2 / \beta^2 B \), bounce points \( z_1 \) and \( z_2 \) and, most importantly, the local radial curvature \( \kappa \), which can have positive and negative values, depending on the direction of the drift. Here, the diamagnetic frequency is chosen to be negative, \( \omega_{de} < 0 \), which means that for a resonance to exist the precessional drift must also be negative, \( \sigma_{de} (\kappa \eta / (\eta - 1)) < 0 \). For a particle to assume such a negative precessional drift it must sample mainly negative local curvature along its path, see figure 1—so-called ‘bad curvature’. This local ‘bad curvature’ has long been recognised as the drive for interchange instabilities [40, 41] and ITGs, and it does play an important role for TEMs, but there its average over the bounce motion determines the stability properties. Many of the trapped particles will have averaged bad curvature \( \sigma_{de} < 0 \) if the particles are mainly trapped in regions of local bad curvature, i.e. if the magnetic field and the local curvature are in phase. Magnetic configurations where this is the case should therefore be characterised by destabilising electrons, \( P_e < 0 \), and should thus be prone to TEM instabilities. Configurations where the magnetic field and the local curvature are even partially out of phase, on the other hand, should have reduced TEM activity. (Another possibility to achieve mainly good average curvature would be to improve the local curvature all-together, for example by having a high plasma pressure \( \beta \) [42], but our goal is to also optimise the vacuum configurations, so increasing \( \beta \) is not an option.) In the limit where all particles experience good average curvature, \( \sigma_{de} > 0 \), as is the case in quasi-isodynamic stellarators [43, 44] with the maximum-J-property (\( J \) is the action integral of the bounce motion of trapped particles and constant on flux surfaces, the maximum of \( J \) being at the plasma centre), it can be shown that TEMs and trapped-particle modes are stable in large regions of parameter space, i.e. if the electron temperature gradient is small, \( \eta_e < 2/3 \).

### III.B. Linear simulation results

Configurations with most of the particles experiencing good average curvature can also benefit from enhanced TEM stability, as can be shown with linear simulations. The simulations are performed with the GENE code [45] in the collisionless and electrostatic limit. The geometry of the different configurations is incorporated into GENE via the GIST geometry interface [46], and we chose to study three very different stellarator equilibria: the quasi-axisymmetric stellarator design NCSX (National Compact Stellarator Experiment, nowadays designated QUASAR [47], see figure 2 on the left), the quasi-helically symmetric stellarator experiment in Madison, Wisconsin, HSX (Helically Symmetric Experiment, see figure 2 in the middle), and the stellarator Wendelstein 7-X.
(W7-X, see figure 2 on the right), which approaches quasi-isodynamicity. For each of the configurations two stellarator-symmetric flux tubes were chosen from the flux surface at half toroidal flux, $s = 0.5$,—one where the binormal coordinate $\alpha = 0$ in the midplane, and the second one at $\alpha = \pi N$ where $N$ denotes the number of periods. In all configurations, the flux tube with $\alpha = 0$ is centered around the bean-shaped poloidal cross section and is therefore referred to as ‘bean flux tube’. The poloidal cross section at the center of the $\alpha = \pi N$ flux tube is either triangle-shaped (in W7-X and HSX) or bullet-shaped (in NCSX) and therefore called ‘triangle flux tube’ or ‘bullet flux tube’, respectively. The two quasi-symmetric devices NCSX and HSX display a strong overlap of the magnetic trapping well and the region of bad local curvature. This is not the case though for W7-X, especially at the centre of the flux tube. The analytical theory therefore suggests that W7-X should have lower TEM growth rates than both NCSX and HSX. On the other hand, it should be noted that in NCSX, the region of bad local curvature, though it overlaps with the magnetic trapping well, is very small compared with the large region of good local curvature, especially in the bullet flux tube. NCSX might therefore benefit from enhanced TEM stability, too. We simulated purely density-gradient-driven TEMs, thus choosing both ion and electron temperature profiles to be flat. For each value of the normalised density gradient $a/L_\rho$, where $a$ denotes the minor radius and $L_\rho^{-1} = -\frac{\text{d}\ln n_\rho}{\text{d}r}$ the density gradient scale length, several wave numbers $k_y \rho_s$ ($\rho_s$ is the ion sound radius) around the expected most unstable mode were simulated, and the highest growth rate was then recorded. The predicted behaviour of the different configurations is indeed born out in the simulations: W7-X and NCSX have the lowest linear growth rates, whereas HSX has the highest, see figure 3. The bullet flux tube of NCSX is more stable than the bean flux tube, which can be explained by the region of bad local curvature being even smaller in the bullet flux tube than in the bean flux tube. The fact that the bean flux tube in HSX has higher growth rates than the triangle flux tube can be attributed to the fact that there is a magnetic trapping region with bad local curvature at zero ballooning angle, which should enhance the mode. This is very much in line with the analytical predictions and previous linear simulation results [39]. To summarise this section: it was expected from analytical calculations and also shown via gyrokinetic simulations that configurations where fewer particles have average bad curvature benefit from enhanced stability of density-gradient-driven TEMs. Very recent nonlinear results confirm these findings [48]. More extensive nonlinear data will be published in a
Figure 3. Linear growth rates of density-gradient-driven TEMs in each of the simulated flux tubes in NCSX, HSX and W7-X. At each simulated density gradient $a/L_n$, where $a$ is the minor radius of the device and $L_n$ the density gradient scale length, the growth rate of the most unstable mode is displayed.

We thus average the bounce averaged curvature of particles with good average curvature have a minimum sign is introduced to make a configuration with a majority of the magnetic field along a given field line. The minus curvature. In order to obtain the improved proxy function, the central finding from the analytical theory discussed above was that it is beneficial for a configuration to have as few trapped particles as possible with bad average curvature. The calculation can be performed for many different equilibria during the process of the optimisation. We remember the analytical expression for the gyrokinetic energy transfer rate (equation (2)) and use this as inspiration for our proxy function. The central finding from the analytical theory discussed above was that it is beneficial for a configuration to have as few trapped particles as possible with bad average curvature. The minus curvature. In order to obtain the improved proxy function $Q_{\text{bounce}}$ we thus average the bounce averaged curvature $Q_{\text{bounce}}(\lambda)$ of a particle with pitch angle $\lambda$ over all trapped particles, i.e. over all pitch angles $\lambda$, equivalent to how the average is done in equation (2):

$$Q_{\text{bounce}} = - \int_{1/R_{\text{min}}}^{1/R_{\text{max}}} \varpi_{\text{bounce}}(\lambda) d\lambda,$$

(4)

with

$$\varpi_{\text{bounce}}(\lambda) = \int_{0}^{\epsilon_{\text{f}}} H\left(\frac{1}{\lambda} - B(\epsilon)\right) \omega_0(\lambda, \epsilon) d\epsilon$$

and where $B_{\text{min}}$ and $B_{\text{max}}$ denote the minimum and maximum of the magnetic field along a given field line. The minus sign is introduced to make a configuration with a majority of particles with good average curvature have a minimum $Q_{\text{bounce}}$, which seems more intuitive from an optimisation point of view. If we plot the maximum growth rate obtained from TEM simulations with a pure density gradient for various configurations and flux tubes versus the corresponding proxy value we see that the proxy correlates well with the linear growth rates, see figure 4. Especially if two configurations are very different, the proxy correctly predicts which one is the more stable. For configurations that are very similar, however, for example different W7-X configurations that mainly differ by their mirror ratio (HM being high mirror, LM being low mirror, and SC being the standard configuration), a lower proxy value does not necessarily mean a lower TEM growth rate. This means an optimisation will probably need to make large steps in proxy value $Q$ to ensure that the found optimised equilibrium indeed has lower levels of TEM activity.

V. The proof-of-principle configuration

A first attempt at an optimisation was made with HSX as the starting equilibrium. For this first proof-of-principle optimisation STELLOPT’s fixed boundary mode was chosen, which means the accessible configuration space was very large. This was indeed necessary. The constraints of fixed aspect ratio and low neoclassical transport prevented STELLOPT from finding an equilibrium with lower proxy, i.e. better average curvature.

### Table 1. Targets for HSX optimisations.

| Optimised quantity | Target count | Target | Weight $1/\sigma$ |
|--------------------|--------------|--------|------------------|
| Neoclassical transport $\epsilon_{3/2}$ | 127 | 0 | 0.001 |
| Turbulent transport $Q_{\text{bounce}}$ | 25 | 0 | 1000 |
| Major radius $R_0$ | 1 | 1.22 | 10 |

Note: The turbulent and neoclassical values are evaluated at multiple radial locations.
In order to see any change in the proxy the requirement of low neoclassical transport had to be relaxed significantly, which means that the weight for the neoclassical transport was chosen to be very small compared with the weight for our proxy (see table 1). The resulting TEM-optimised equilibrium shown in figure 5 on the right has lost the helical symmetry. This leads to a significant increase in the neoclassical transport—the neoclassical effective ripple went up by an order of magnitude. Moreover, the magnetic field along the field line of this preliminary equilibrium is very jagged. This should of course be avoided when trying to find a truly optimised configuration. In this case, however, our primary focus is to show that the proxy works. To test this, we first performed linear GENE simulations. A scan over the binormal wave vector $k_y\rho_i$ for a purely density-gradient-driven TEM with a density gradient $a/L_n = 3$ and no temperature gradient shows that the linear growth rates are indeed reduced for the optimised equilibrium, at least for the scales where turbulence is generated (figure 6). This stabilisation also holds for a large range of density gradients, as can be seen in figure 7. For these simulations, a scan over various wave numbers was performed and the highest growth rate for each gradient is displayed. These linear results lead to the expectation that a nonlinear simulation of this proxy-optimised configuration would also show reduced transport. A test with pure density-gradient-driven TEM turbulence at a density gradient of $a/L_n = 3$ shows that the nonlinear electron heat flux went from $Q/Q_{GB} = 1.05$ to $Q/Q_{GB} = 0.62$, where the heat fluxes are measured in Gyro-Bohm units $Q_{GB} = nTcc_s^2/\rho_i^2$, with the density $n$, the ion sound speed $c_s$, and the sound Larmor radius $\rho_i$. This means a reduction of about 40% was achieved. However, the high neoclassical transport remains a handicap, and, since the fixed boundary mode of STELLOPT was chosen to create this optimised equilibrium, the magnetic field is not realisable by simply adjusting the currents in the existing coils of the HSX experiment. Additional STELLOPT runs should therefore be used to try to find actually optimised but experimentally realisable configurations using the free-boundary mode.

VI. Conclusions and outlook

In this paper we have presented a method to optimise stellarators for density-gradient-driven TEM turbulence using the optimisation code STELLOPT. The optimisation for TEM turbulence complements ongoing efforts to optimise stellarators not only for neoclassical transport but also for turbulent transport. We used analytical theory and linear flux-tube simulations performed with the GENE code to guide us in devising a proxy function that can stand in for the expected turbulent

Figure 5. Comparison of the initial helically-symmetric HSX and the derived optimised equilibrium produced with STELLOPT. Shown are the magnetic field strength on the outermost flux surface at the top and the magnetic field strength and local bad curvature (on the left and right axis, respectively) along the bean flux tube at the surface with half flux $s = 0.5$. The optimised equilibrium is not helically symmetric anymore.
heat flux. Both analytical theory and the linear simulations suggested that configurations with a lower fraction of particles with bounce-averaged bad curvature should be less unstable to TEMs. The bounce-averaged curvature averaged over all trapped particles was thus chosen for the proxy. The comparison between the proxy and linear growth rates for TEMs in various configurations revealed that the proxy is well suited to predict the relative stability of a configuration. Assuming that the linear growth rates are correlated with the turbulent transport levels, the proxy should thus be able to guide the optimiser towards configurations with lower TEM turbulence levels. A first proof-of-principle configuration where this was indeed achieved was presented. There, the linear growth rates were reduced compared with the starting equilibrium of HSX, as was the turbulent heat flux. This configuration was,
however, not realisable with HSX’s given coil set. The presentation of an experimentally feasible TEM-optimised configuration is deferred to a future publication.

One possible improvement of the current proxy would be to include further weighting of the deeply trapped particles by taking into account the mode structure of the linear modes via the electrostatic potential $|\phi|^2$, as it is included also in the equation for the energy transfer rate, equation (2). Another possibility that has proven fruitful in the reduction of ITG turbulence is to include the distance of the flux surfaces in the optimisation, trying to find configurations where this distance is particularly large, which would result in a smaller effective gradient and thus possibly large regions of stability in parameter space. Combining the TEM optimisation with the ITG optimisation where the local curvature is minimised might also lead to better results for TEM turbulence. The simultaneous reduction of both ITGs (and interchange instabilities in general) and TEMs might already be happening with our proxy, when the reduction of the bounce averaged bad curvature is achieved by making the local curvature better. However, there might be configurations where this is not feasible, but shifting the local bad curvature away from the magnetic wells is.

In addition to improving the proxy function and thus hopefully reducing the turbulence at high gradients other problems in turbulence optimisation could and should be addressed. One of these problems is to increase the critical gradient for the onset of turbulence, which is particularly important if the turbulence is very ‘stiff’, i.e. if the heat flux increases dramatically once the critical gradient is exceeded. Ideally, in addition to reducing the turbulent heat flux, one would achieve an improvement in the particle flux to flush out impurities from the plasma, but a deeper understanding of turbulence is required before this challenge can be tackled. In general it remains to be seen to what extent the simultaneous optimisation of several different aspects (neoclassical and turbulent transport, fast-particle confinement, divertor etc) can be successful. Until then the TEM optimisation presented here will serve as a useful tool to learn more about the influence of geometry on TEM stability and will certainly guide future efforts.

Acknowledgments

The authors thank T Görler, G W Hammett, P Helander, D R Mikkelsen, J N Talmadge and M C Zarnstorff for many fruitful discussions as well as Y Turkin for providing the MCviewer for displaying the magnetic geometry and S P Hirshman for access to the VMEC code.

Some of these simulations were performed on the HELIOS supercomputer, Japan. One of the authors (J H E Proll) gratefully acknowledges funding from the Max Planck/Princeton Center for Plasma Physics. This work has been carried out within the framework of the EURofusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under the grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[1] Spitzer L Jr 1951 US Atomic Energy Commission Report No. NYO-993 (PM-S-1)
[2] Nührenberg J and Zille R 1988 Phys. Lett. A 129 113
[3] Boozer A H 1995 Plasma Phys. Control. Fusion 37 A103
[4] Hall L S and McNamara B 1975 Phys. Fluids 18 552
[5] Mynick H E, Chu T K and Boozer A H 1982 Phys. Rev. Lett. 48 322
[6] Drevlak M, Beidler C D, Geiger J, Helander P and Turkin Y 2014 Quasi-isodynamic configuration with improved confinement 41st EPS Conf. on Plasma Physics (Berlin, Germany)
[7] Beidler C D et al 1990 Fusion Technol. 17 148
[8] Klinger T et al 2013 Fusion Eng. Des. 88 461
[9] Anderson F S B, Almagni A F, Anderson D T, Mathews P G, Talmadge J N and Shoheit J L 1995 Fusion Technol. 27 273
[10] Zarnstorff M C et al 2001 Plasma Phys. Control. Fusion 43 A237
[11] Canik J M, Anderson D T, Anderson F S B, Likin K M, Talmadge J N and Zhai K 2007 Phys. Rev. Lett. 98 085002
[12] Plunk G G, Bañón A and Jenko F 2015 Plasma Phys. Control. Fusion 57 044005
[13] Mynick H E, Pomphrey N and Xanthopoulos P 2010 Phys. Rev. Lett. 105 095004
[14] Mynick H E, Pomphrey N and Xanthopoulos P 2011 Phys. Plasmas 18 056101
[15] Mynick H E, Xanthopoulos P, Faber B J, Lucia M, Rorvig M and Talmadge J N 2014 Plasma Phys. Control. Fusion 56 094001
[16] Xanthopoulos P, Mynick H E, Helander P, Turkin Y, Plunk G G, Jenko F, Görler T, Told D, Bird T and Proll J H E 2014 Phys. Rev. Lett. 113 155001
[17] Spong D A et al 2001 Nucl. Fusion 41 711
[18] Hirshman S P, van Rij W I and M erkel P 1986 Comput. Phys. Commun. 43 143
[19] Beidler C D and Maa H 2001 Plasma Phys. Control. Fusion 43 1131
[20] Nemov V V, Kasilov S V, Kernbichler W and Heyn M F 1999 Phys. Plasmas 6 4625
[21] Hirshman S P and Breslau J 1998 Phys. Plasmas 5 2664
[22] Bauer F, Betancourt O and Garabedian P R 1981 Phys. Fluids 24 48
[23] Merkel P 1987 Nucl. Fusion 27 867
[24] Strickler D J, Berry L A and Hirshman S P 2002 Fusion Sci. Technol. 41 107
[25] Zhen J, Song Y, Breslau J and Neilson G H 2014 Fusion Eng. Des. 89 487
[26] Pomphrey N, Boozer A H, Brooks A B and Hatcher R 2007 Fusion Sci. Technol. 51 181
[27] Lazerson S A, Sakakibara S and Suzuki Y 2013 Plasma Phys. Control. Fusion 55 025014
[28] Lazerson S A and the DIII-D Team 2015 Nucl. Fusion 55 1
[29] Schmitt J C, Bialek J, Lazerson S A and Majeski R 2014 Rev. Sci. Instrum. 85 11E817
[30] Marquardt D W 1963 J. Soc. Ind. Appl. Math. 11 431
[31] Goldberg D E 1989 Genetic Algorithms in Search, Optimization and Machine Learning (Reading, MA: Addison-Wesley)
[32] Kennedy J and Eberhard R 1995 Particle swarm optimization Proc. of IEEE Int. Conf. on Neural Networks IV pp 1942–8
[33] Kadomtsev B B and Pogutse O P 1967 J. Exp. Theor. Phys. 24 1172
[34] Dannert T and Jenko F 2005 Phys. Plasmas 12 072309
[35] Romanelli M, Regnolli G and Bourdelle C 2007 Phys. Plasmas 14 082305
[36] Proll J H E, Helander P, Plunk G G and Connor J W 2012 Phys. Rev. Lett. 108 245002
[37] Helander P et al 2012 Plasma Phys. Control. Fusion 54 124009
[38] Proll J H E, Helander P and Plunk G G 2013 Phys. Plasmas 20 122505
[39] Proll J H E, Xanthopoulos P and Helander P 2013 Phys. Plasmas 20 122506
[40] Jenko F and Dorland W 2001 Plasma Phys. Control. Fusion 43 A141
[41] Kadomtsev B B 1966 Reviews of Plasma Physics vol 2 ed M A Leontovich (New York: Consultants Bureau) p 153
[42] Bourdelle C, Dorland W, Garbet X, Hammett G W, Kotschenreuther M, Rewoldt G and Synakowski E J 2003 Phys. Plasmas 10 2881
[43] Gori S, Lotz W and Nührenberg J 1996 Theory of Fusion Plasmas (Bologna: Editrice Compositori)
[44] Subbotin A A et al 2006 Nucl. Fusion 46 921
[45] Jenko F, Dorland W, Kotschenreuther M and Rogers B N 2000 Phys. Plasmas 7 1904
[46] Xanthopoulos P, Cooper W A, Jenko F, Turkin Y, Runov A and Geiger J 2009 Phys. Plasmas 16 082303
[47] Neilson G H, Gates D A, Heitzenroeder P J, Breslau J, Prager S C, Stevenson T, Titus P, Williams M D and Zarnstorff M C 2014 IEEE Trans. Plasma Sci 42 489
[48] Helander P, Bird T, Jenko F, Kleiber R, Plunk G G, Proll J H E, Riemann J and Xanthopoulos P 2015 Nucl. Fusion 55 053030