Intrinsic mechanism of phase locking in two-dimensional Josephson junction networks in presence of an external magnetic field

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Abstract

We present numerical simulations of the dynamics of two-dimensional Josephson junction arrays to study the mechanism of mutual phase locking. We show that in the presence of an external magnetic field two mechanisms are playing a role in phase locking: feedback through the external load and internal coupling between rows due to microwave currents induced by the field. We have found the parameter values (junction capacitance, cell loop inductance, impedance of the external load) for which the interplay of both these mechanisms leads to the in-phase solution. The case of unshunted arrays is discussed as well.

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Josephson junctions are natural voltage-controlled oscillators [1]. By building up one-dimensional (1D) and two-dimensional (2D) arrays of the junctions one can hope to obtain the necessary output power which is required for many practical applications. This goal will be achieved when all junctions of the array are phase-locked. While a detailed theoretical analysis of phase locking in 1D systems has been performed several years ago [2],[3] and has been supported by promising recent experiments [4] the full examination of the problems connected with the formation of stable phase locking in 2D arrays has not been achieved up to now. Several authors have shown that 2D arrays are more stable against non-uniformities in the critical currents [5]-[8]. Nevertheless, it is not understood so far whether 2D arrays possess an internal mechanism of coupling which could be a potential advantage. Even a qualitative observation regarding the mechanism could provide useful criteria for the problem of optimum array design.

In this letter we address these issues by means of simulations of the dynamical properties of 2D Josephson networks with and without external load in presence of an external magnetic field.

We investigate arrays consisting of active junctions connected by superconducting inductances in the direction perpendicular to the bias current. Such a structure has been accepted as a standard design now [8]-[10]. We have \( N \) rows of \( M \) junctions. The behavior of every Josephson junction is simulated using the RSJ model. Within this model the current through the \( k \)th junction is given by

\[
\beta_c \ddot{\varphi}_k + \dot{\varphi}_k + \sin \varphi_k = i_k, \tag{1}
\]

\[
i_k = I_k / I_C.
\]

Here \( \beta_c = 2\pi C I_C R_N^2 / \Phi_0 \) is the McCumber parameter, \( \varphi_k \) is the Josephson phase difference over the junction, \( \Phi_0 \) stands for the magnetic flux quantum. \( C \), \( R_N \) and \( I_C \) are the junction capacitance, normal resistance and critical current, respectively. \( I_k \) is the total current flowing through the \( k \)th junction. Introducing dimensionless parameters, we can formulate flux quantization conditions for every cell containing junctions \( m \) and \( n \):

\[
\varphi_m - \varphi_n = \varphi_{ext}^{(m,n)} + l_0 \sum_{p=1}^{4} \mu_p i_p^{(m,n)}, \tag{2}
\]

\[
\sum_{p=1}^{4} \mu_p = 1, \tag{3}
\]

where we introduced the parameters \( l_0 = 2\pi I_c L_0 / \Phi_0 \) and \( \varphi_{ext}^{(m,n)} = 2\pi \Phi_{ext}^{(m,n)} / \Phi_0 \). Furthermore, \( i_p^{(m,n)} = I_p^{(m,n)} / I_c \) is the normalized current flowing through the respective inductance of the \( p \)th branch. The quantity \( \mu^p \) is the contribution to the loop inductance.
from the \( p \)th branch of the cell. The frequency of the Josephson oscillations is

\[
\omega_j = \overline{\nu}_j; \quad (4)
\]

here \( \overline{\nu}_j \) indicates the normalized DC component of the voltage across junction \( j \). We restrict our simulations to the case of a uniform external magnetic flux \( \varphi_{\text{ext}}^{(m,n)} = \varphi_{\text{ext}} \). The resistor in the external load \( r_s \) has been chosen to match the array normal resistance \( r_{\text{array}} = N r_N / M \). The first and second harmonics of the current flowing through the external load have been calculated as

\[
i_{n\omega} = 1/T \int_0^T i(t) \exp(-in\omega t) \, dt, \quad n = 1, 2 \quad (5)
\]

with the averaging time \( T \geq 1000/\omega \). In the simulations we also introduced a small spread of the critical currents \( \sigma \) of about 5\% with the product \( V_c = I_c R_N \) being the same for all junctions. The arrays have been investigated by means of the PSCAN program [11],[12].

The most effective phase locking mechanism for both 1D and 2D arrays is high-frequency coupling between an array and external load, as has been discussed by several authors before [1]. In dependence on inductive or capacitive character of the load one can expect different types of coherent solutions. The in-phase state of a 1D array is characterized by conditions

\[
\varphi_i - \varphi_j = 0 \quad (6)
\]

for the phase difference between any two junctions. This and other types of coherent solutions for the 1D have been described in the papers [3],[13]. For the 2D array with \( \varphi_{\text{ext}} = 0 \) and all junctions being identical we can write down the same relations for any phases in both "transverse" (first index) and "longitudinal" (second index) directions

\[
\varphi_{i,k} - \varphi_{i,l} = 0. \quad (7)
\]

The situation changes when we consider 2D structures with nonzero magnetic field within the cells. In this case the array symmetry is broken and we can no longer expect uniform oscillations of all junctions within a row. However, we still can define an in-phase solution

\[
\varphi_{i,k} - \varphi_{i,l} = 0. \quad (8)
\]

with respect to the "longitudinal" direction. In the following we have to find conditions under which an external magnetic flux does not disturb this in-phase solution. We show that the 2D array standard design itself can not assure this solution and must be improved. For the sake of clarity we start with the four junction array (Fig. 1) and the traditional design of the unit cell. In this case the additional branch \( A \) is absent. Only the external load \( B \) must control the phase locking of the whole system.

In Fig. 2 we show the results of simulations which have been performed for the value \( l_0 = 0.5 \) of the loop inductance and two different types of external load. For the
capacitive load (solid line in Fig. 2a) we have obtained the anti-phase solution within the whole range of the external magnetic field. We find no first harmonic in the net oscillation. Consequently, the microwave current through the load is characterized by the second harmonic component essentially (Fig. 2b). The phase relations between the junctions in each column remain the same for all values of the external field. On the other hand, in the inductive load regime (filled triangles in Fig. 2a) we have got in-phase oscillations within the regions $\varphi_{\text{ext}} < 2.0$ and $\varphi_{\text{ext}} > 4.2$ and the anti-phase solution for $2.0 < \varphi_{\text{ext}} < 4.2$. It is worth noting that an increase of the magnetic flux ($0 \leq \varphi_{\text{ext}} \leq \pi$) leads to an increasing oscillation frequency. As a result, the transition observed can not be caused by the action of the external load which remains in the inductive regime.

In order to explain the result we turn to the properties of the unshunted array first. Here, the microwave currents induced by external magnetic fields can contribute not only to the phase locking within one row but also to the phase locking between neighbouring rows. A stability analysis for zero Josephson junction capacitance $\beta_c = 0$ and $l_0 \ll 1$ has been performed analytically [14]. As a result one observes that even small magnetic fields favour the stability of the anti-phase state.

For non-zero capacitance of the Josephson junctions and for the unit cell inductance $l_0 \approx 1$ the frequency dependence of both in- and anti-phase states is more complex. The boundaries of the stability regions can be determined numerically only. In Fig. 3 we show these regions for the network without external load and $\varphi_{\text{ext}} = 1.5$. One recovers that the point $l_0 = 0.5$ and $\beta_c = 0.0$ (which are our parameters for Fig. 2) lies within the region where the unshunted array exhibits a stable anti-phase solution.

The state chosen by the array with shunt depends on the interplay of the two high-frequency currents, flowing through a given junction of the array, i.e. the shunt current and the circulating current. For a small value of the external field the shunt current is strong enough to support the in-phase state. The increasing magnetic field within the cells produces a shift of the oscillation phases along the vertical direction and leads to a decreasing of the microwave current through the external load (Fig. 2b). The mechanism providing the in-phase solution weakens and for some value $\varphi_{\text{ext}}^\text{lim}$ fails.

Our simulations for the shunted array with a larger number of junctions (up to 6 in every row) have shown the following: The value of the critical flux $\varphi_{\text{ext}}^\text{lim}$ for which switching into anti-phase state takes place becomes smaller with the number $M$ of Josephson junctions per row growing. From a practical point of view this means that the increasing of the array size makes the in-phase state less tolerant toward the external field. For a $4 \times 4$ shunted array with the loop inductance $l_0 = 1.0$ this limiting value is $\varphi_{\text{ext}}^\text{lim} = 0.7$. After switching the array reveals a different behavior: The phase differences of the junction voltage oscillations are equal to $\pi$ for nearest neighbours in each column. Large microwave currents are flowing along the transverse inductances.

Another conclusion we have made from the simulations of the arrays with $M, N \geq 2$ is
that the boundaries of stability of the in-phase and anti-phase solutions for the unshunted array at the \( l_0, \beta_c \)-plane do not depend on \( M \) and \( N \). One possible explanation is that the circulating microwave currents choose the nearest ways to flow and consequently only the short-range coupling between neighbouring cells determines the stability conditions.

The mechanism assuring the in-phase state depends mainly on the impedance in the vertical direction (in contrast to the other one created by circulating currents). This way one can hope to influence the relative strength of both mechanisms by changing the impedance of the cell into the direction perpendicular to the bias current. We have done so by including the additional branch \( A \). Fig. 4 presents the boundaries of the in-phase and anti-phase solutions for the unshunted (stars) and shunted (filled boxes) arrays in comparison. The additional branch \( A \) improves the stability of the in-phase state within the region \( l_0 \leq 1.0, \beta_c \leq 1.0 \) for the unshunted array (this set of parameters is supposed to be the most promising for obtaining maximum output power). As a result, the shunted array remains phase locked for all values of the external flux (Fig. 5).

In conclusion, we have shown that the magnetic field in two-dimensional arrays provides an internal mechanism of phase locking. This mechanism yields the stability of the anti-phase state for the array with inductances only being used for the transverse connections. A capacitive shunt parallel to these inductances improves the stability of the in-phase state.

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References

[1] K.K. Likharev, Dynamics of Josephson Junctions and Circuits (Gordon and Breach, Philadelphia, 1991).

[2] A.K. Jain, K.K. Likharev, J.E. Lukens, and J.E. Sauvageau, Phys. Rep. 109 (1984) 310.

[3] P. Hadley, M.R. Beasley, and K. Wiesenfeld, Phys. Rev. B 38 (1988) 8712.

[4] S. Han, A.H. Worsham, and J.E. Lukens, IEEE Trans. Supercond. 3 (1993) 2489.

[5] M. Octavio, C.B. Whan, and C.J. Lobb, Appl. Phys. Lett. 60 (1992) 766.

[6] K. Wiesenfeld, S.P. Benz, and P.A.A. Booi, J. Appl. Phys. 76 (1994) 3835.

[7] M. Darula, P. Seidel, J. v. Zameck Glyscinski, A. Darulova, F. Busse, and S. Benacka, in: Applied Superconductivity, H.C. Freyhardt(Ed.), DGM Informationsgesellschaft, Oberursel, (1993) 1245.

[8] R.L. Kautz, IEEE Trans. Appl. Supercond. 5 2704 (1995).

[9] S.P. Benz and C.J. Burroughs, Supercond. Sci. Technol. 4 (1991) 561.

[10] S.P. Benz and C.J. Burroughs, Appl. Phys. Lett. 58 (1991) 2162.

[11] A.A. Odintsov, V.K. Semenov, and A.B. Zorin, IEEE Trans. Magn. MAG–23 (1987) 763.

[12] S.V. Polonsky, V.K. Semenov, and P.N. Shevchenko, Proc. ISEC’91 (1991) 160.

[13] P. Hadley and M. Beasley, Appl. Phys. Lett. 50 (1987) 621.

[14] M. Basler, W. Krench, and K.Yu. Platov, to be published (cond-mat/9603041)
FIGURE CAPTIONS

Fig. 1. An array of $2 \times 2$ Josephson junctions with an additional branch $r_x, c_x$ (denoted by $A$) and external shunt $r_s, c_s, l_s$ ($B$); $l_x = (1/4)l_0$, $l_y = (1/4)l_0$.

Fig. 2. (a) The frequency $\omega$ of a shunted array with inductive (filled triangles) and capacitive (solid line) shunt as a function of the external flux $\varphi_{ext}$ (Parameters: $i_{dc} = 1.5$, $\beta_c = 0.0$, $l_0 = 0.5$, $l_s = 1.0$, $r_s = 1.0$). (b) The first and second harmonics of the shunt current $i_\omega$ (solid line), $i_{2\omega}$ (dot line) for the inductive shunt ($c_s = 2.0$) and the second harmonic (open triangles) of the shunt current for the capacitive shunt ($c_s = 0.2$) as a function of the external flux $\varphi_{ext}$ (Parameters: $i_{dc} = 1.5$, $\beta_c = 0.0$, $l_0 = 0.5$, $l_s = 1.0$, $r_s = 1.0$).

Fig. 3. The boundary of the in-phase and anti-phase solutions at the $\beta_c, l_0$-plane for the unshunted array from Fig. 1 (Parameters: $i_{dc} = 1.5$, $\varphi_{ext} = 1.5$).

Fig. 4. The boundary of the in-phase and anti-phase solutions at the $\beta_c, l_0$-plane for unshunted array (stars) and shunted array (filled boxes) with the additional branch $A$ (Parameters: $i_{dc} = 1.5$, $\varphi_{ext} = 1.5$, $l_s = 1.0$, $r_s = 1.0$, $c_s = 2.0$, $c_x = 6.0$, $r_x = 0.1$).

Fig. 5. The frequency $\omega$ of the shunted array without (stars) and with (solid line) additional branch $A$ as a function of the external flux $\varphi_{ext}$ (Parameters: $i_{dc} = 1.5$, $\beta_c = 0.0$, $l_0 = 0.5$, $l_s = 1.0$, $r_s = 1.0$).
The diagram illustrates the relationship between $\phi_{ext}$ and $\varepsilon$. The graph shows two distinct lines, one represented with a dashed line at $c_s = 2.0$ and the other with a solid line at $c_s = 0.2$. The graph indicates a comparison of two different cases, highlighting the change in $\varepsilon$ as $\phi_{ext}$ varies from 0 to 7.
\[ i_{\omega}, c_s = 2.0 \]
\[ i_{2\omega}, c_s = 2.0 \]
\[ i_{2\omega}, c_s = 0.2 \]
unshunted array
shunted array

in-phase
anti-phase
