Neutron halos in heavy nuclei-relativistic mean field approach

Andrzej Baran, Krzysztof Pomorski, Michał Warda

Institute of Physics, Maria Curie-Skłodowska University, PL-20-031 Lublin, Poland

Received: 18 June 1996/Revised version: 12 August 1996
Communicated by W. Weise

Abstract. Assuming a simple spherical relativistic mean field model of the nucleus, we estimate the width of the antiproton-neutron annihilation ($\langle I_n \rangle$) and the width of antiproton-proton ($\langle I_p \rangle$) annihilation, in an antiprotonic atom system. This allows us to determine the halo factor $f$, which is then discussed in the context of experimental data obtained in measurements recently done on LEAR utility at CERN. Another quantity which characterizes the deviation of the average nuclear densities ratio from the corresponding ratio of the homogeneous densities is introduced too. It was shown that it is also a good indicator of the neutron halo. The results are compared to experimental data as well as to the data of the simple liquid droplet model of the nuclear densities. The single particle structure of the nuclear density tail is discussed also.

PACS: 21.60.-n; 21.10.Gv; 36.10-k; 25.43.+t

I Introduction

The LEAR measurements at CERN [1,2], show large neutron halo factors for selected nuclei. The observations of this factor were performed by counting those nuclei which were born in annihilation reactions in antiprotonic atoms. The following processes were observed: 

$$\bar{p} + \frac{A}{2}X_N \rightarrow \frac{A-1}{2}X_N + \{\pi\},$$

$$\bar{p} + \frac{A}{2}X_N \rightarrow \frac{A-1}{2}X_{N-1} + \{\pi\}.$$

Here, $\frac{A}{2}X_N$ denotes the nucleus with the atomic number $Z$, the mass number $A$ and the number of neutrons $N$. The annihilation products consist of a few (\langle N_\pi \rangle = 5) pions \{\pi\}.

In reactions shown the resulting nuclei have the number of neutrons or the number of protons diminished by one as compared to the mother nucleus. Counting the annihilation products resulting from nuclear neutrons $\langle I_n \rangle$ and from nuclear protons $\langle I_p \rangle$, one can calculate the following halo factor $f = (Z/N_\pi)\langle I_n \rangle / \langle I_p \rangle$. For the case of "cold" annihilation reactions which take place at the peripheral region of the nucleus and do not excite the final nuclei, the factor $f$ is supposed to depend strongly on the ratio of neutron and proton densities.

In order to describe the phenomenon of annihilation of the antiprotonic atomic state on the mother nucleus we have done the calculation of the halo factor $f$ and the density ratios deviation $h$, which we shall define later. As a basis of our calculation serves the relativistic mean field model in which the field equations are solved numerically. These solutions are exact even far outside the nuclear surface. This precision is critical for the calculation of the quantities of interest. Both quantities $f$ and $h$ have been considered for nuclei: $^{58}$Ni, $^{96}$Zr, $^{98}$Ru, $^{130}$Te, $^{144}$Sm, $^{154}$Sm, $^{176}$Yb, $^{232}$Th, $^{238}$U. All these nuclei were studied in [1,2]. The halo factor $f$ shows some dependence on the separation energies of the last nucleons (proton or neutron).

This paper is organized as follows. In the next section we define and discuss the model independent functions $f$ and $h$ which are supposed to characterize halo phenomena. An example, the droplet model calculation of $h$ is shown. The third section contains a short presentation of the relativistic mean field model which we have used to determine $f$ and $h$ factors.

In the Summary of the paper we show and comment on the results of our calculations.

II Basic definitions

In this section we define basic quantities which are suitable in discussing halo phenomena. The first subsection collects the approximations made in halo factor definitions. The second subsection introduces new quantities. These are average densities calculated from the density distribution and the deviation of average densities ratio from the homogeneous neutron and proton densities ratio. An example of this is shown and reviewed.
A Halo factor

The simple expression for the antiproton absorption width for the antiproton which occupies a state \( s \) of an atom is

\[
\Gamma_n^{\pm(p)} \sim \int \rho_{n(p)} |\Psi_n^{\pm}(r)|^2 P(r) r^2 \, dr. \tag{1}
\]

The subscripts \( n \) and \( p \) are for neutrons and protons respectively. The wave function \( \Psi^\pm(r) \) of antiproton is determined in the Schrödinger model of hydrogen-like antiprotonic atom within a point nucleus. The approximation of the point like nucleus is motivated by the fact of a large mean distance \( B \) of the antiproton from the nucleus. Taking the mass ratio of the antiproton to the electron \( m_p/m_e \sim 2000 \), the main (supposed) quantum number \( n \) of antiproton in the atom e.g., \( n = 8 \), and the charge of the nucleus in question, is \( B \approx n^2 B_0/2000Z \), where \( B_0 \) is the Bohr radius of the electron in the hydrogen atom. For the cases which we are considering here this gives an average antiproton-nucleus distance of the value \( B \approx 30–40 \, \text{fm} \). As it is seen, this distance exceeds a few times past the nuclear radius \( R \approx A^{1/3} \, \text{fm} \). This motivates our point nucleus approximation. It is possible to solve the Dirac or the Schrödinger equation for the more complicated case of finite or even deformed nucleus. However, this will not change significantly the relative measures which we shall introduce and apply in this present paper.

The factor \( P(r) \), in the definition of \( f \) describes pion escape probability plus other effects and it will be discussed later. Here, we may say only, that in the present model it does not influence the calculation and with very high accuracy it may be assumed as \( P(r) \approx \text{const} \).

We define the halo factor as \([3]\]

\[
f \approx \frac{Z \sum_n \Gamma_n^s}{N \sum_p \Gamma_p^s}. \tag{2}
\]

The summation run through all the antiprotonic atomic states for which one expects the large annihilation widths. Usually it is assumed that only one state with \((n,l)\) ranging from \((6,5)\) for nickel nucleus to \((9,8)\) to uranium nucleus contributes to these widths \([4]\). In the present calculation we sum a few (three to five) states only. The summation stops if the addition of the successive \( \Gamma^s \) term does not change the value of the ratio.

It is worthwhile to mention the weak dependence of our \( f \) definition on the structure of the antiproton-nucleon \((\bar{p}N)\) interaction. It is assumed that both proton and neutron \( \Gamma^s \) widths depend on the imaginary part of the complex optical potential \( W(r) \) which is responsible for the absorption of the antiproton. We use the approximation that the optical potential is proportional to the density of nucleons: \( W(r) = a \rho(r) \). Here the parameter \( a \) is the \( \bar{p}N \) interaction length and is taken as \( a = a_r + i a_i \), where both real \( a_r \) and imaginary \( a_i \) parts of \( a \) are constants \([5]\).

In this way we obtain

\[
\Gamma_{n(p)} \sim \sqrt{m \, a_{n(p)}}. \tag{3}
\]

Because both \( \sqrt{\sum_n \Gamma^s_n} \) and \( \sqrt{\sum_p \Gamma^s_p} \) in the definition of halo factor \( f \) (Eq. 2) appear in a ratio and thus depend weakly on the specific antiprotonic atomic state one concludes that the optical potential dependence of \( f \) is washed out. In this sense \( f \) is the \( \bar{p}N \) interaction independent measure of the annihilation ratio and describes rather the geometrical properties of the nucleon density distributions.

In fact, the definition of \( f \) consists of \( \sqrt{m \, a_n}/\sqrt{m \, a_p} \), which measures the cross sections annihilation ratio \( \sigma(\bar{p}n)/\sigma(\bar{p}p) \) for the antiproton annihilation on nucleons in a nuclear medium. The value of this ratio for a given nucleus is very hard to estimate in an experiment or on theoretical grounds. In an early paper on nucleon-antinucleon optical potential by Bryan and Philips \([6]\), the relative antiproton \( \bar{p} \)-capture rates on neutrons and protons in singlet and triplet states was estimated from measurements of \( \bar{p} \) annihilation at rest in hydrogen and deuterium. From these one can calculate the ratio of capture on neutrons to the capture on protons. In singlet states it is close to 0.6 whereas in triplet states it exceeds 1. The measurements by Bugg et al. \([7]\) show that this ratio is close to 0.65 in case of the nucleus \(^{12}\text{C} \). Other authors \([8]\) give us an imaginary potential ratio the value \( \sim 80\% \). In the paper by Kalogeropoulos and Tzanakos \([9]\), one can find a similar value \( \sigma(\bar{p}n)/\sigma(\bar{p}p) = 3/4 \) as measured in deuterium. These different numbers suggest that the annihilation ratio may change from one nucleus to the other and in our opinion, it is not possible to scale the results using these numbers – the ratio may show a dependence on proton and neutron numbers \( N \) and \( Z \) as well as the state in which the annihilation takes place.

Therefore, in the calculations which follow we do not scale our results. As we shall see the results show some similarity to the experimental data. This supports our choice of the RMF model calculation and shows that the ratio \( \sqrt{m \, a_n}/\sqrt{m \, a_p} \) is close to unity for our model.

To select the peripheral annihilation acts (which one observes in \( \bar{p}X \) experiment) one should correct the definition of the halo factor on the pion escaping probability \( P_{\text{esc}}(r) \) and the deep hole creation probability \( P_{\text{dh}}(r) \). One has to take into account only this part of the annihilation width \( \Gamma \) which is responsible for the cold annihilation – the annihilation in which pions escape to infinity without exciting the rest of the system. On pure geometrical grounds this is shown to be a power function of \( r \) which agrees roughly with very accurate calculations presented in \([4]\). Assuming the dependence \( P_{\text{esc}} \sim r^k \), where \( k > 0 \), one can see from \( 2 \) that the summation of widths \( \Gamma \) will run over nearly the same spectrum of powers of \( r \) as without this factor. It is therefore allowed to stay with the old formulas in which \( P(r) \) is constant.

Another factor which probably enters the expression for \( \Gamma \) is a deep hole creation factor \( P_{\text{dh}}(r) \) \([4]\). Its role is again the preservation of cold acts of annihilation. The physics behind this is the following. A number of annihilation acts may occur on a deeply situated nucleon levels. This leads to a rearrangement of the nucleonic orbits and it may happen that the final system will show very small or positive (!) binding energies of the last nucleons. After its emission, the nucleus in question goes out of the observation range and cannot be counted as the peripheral halo product. The dependence of the function \( P_{\text{dh}} \) on \( r \) seems to satisfy the power law analogous to pion escaping probability \( P_{\text{esc}} \sim r^k \), and as it was said before, we do not
introduce this into our calculation. These are all the approximations which we have assumed in our paper while evaluating the halo factor.

\[ B \text{ Average densities} \]

Consider an average density \( \tilde{\rho} \) defined by the integral
\[
\tilde{\rho} = \frac{1}{N} \int \mathrm{d}r \phi^2(r),
\]
where \( \rho(r) \) is the spherically symmetric nucleon density distribution and \( \mathrm{d}r \) is the integration volume element. \( N \) is the number of particles in the system and is given by
\[
N = \int \mathrm{d}r \rho(r),
\]
In the case of pure Fermi distribution
\[
\rho(r) = \rho_0 \left( 1 + \exp \left( -\frac{r - R}{a} \right) \right)^{-1},
\]
the quantity \( \tilde{\rho} \) can be evaluated approximately with assumed accuracy. In the above formula, \( a \) is the width of the nuclear surface, \( R \) is the radius and \( \rho_0 \) is the central density of the nucleus. The expression valid to second order in expansion parameter \( a/R \) can be obtained from the Elton’s formula (pages 106–107 of [10]). To use this formula one has to modify the expression for the average density to the following form
\[
\tilde{\rho} \sim F_2(k) - 2F_1(k),
\]
where \( k = R/a \) and \( F_1(k) \) the Elton’s integral is defined by
\[
F_1(k) = \int_0^k \frac{x^a \mathrm{d}x}{\left( 1 + \exp(x - k) \right)}.
\]
From the formula mentioned before, one has to second order in \( a/R \)
\[
\tilde{\rho} \approx \rho_0 \left( 1 - \frac{3}{2} \frac{a}{R} + \frac{\pi^2}{2} \left( \frac{a}{R} \right)^2 \right).
\]
Fig. 1. The deviation \( h \) of the neutron to proton density ratio for Fermi density distributions in the case of liquid droplet model. The impulses show exact (full line) and approximate (dashed line) results. The data corresponds to the following nuclei: \( \text{\textrm{^{58}}Ni, \text{^{92}}Zr, \text{^{98}}Ru, \text{^{136}}Te, \text{^{144}}Sm, \text{^{176}}Yb, \text{^{232}}Th, \text{^{238}}U} \)

\[ \text{III Relativistic mean field model} \]

In this section we describe very shortly the relativistic mean field (RMF) model which we have used in the calculations of the neutron and proton densities entering both halo factor \( f \) and the average density deviation.

The RMF theory starts from a lagrangian consisting of nucleonic and mesonic degrees of freedom [15]. In some sense it seems to be more fundamental than other mean field models like Skyrme-Hartree-Fock and Gogny-Hartree-Fock models. It gives the relativistic treatment of nucleonic and mesonic variables and a proper description of the spin-orbit interactions. Nevertheless it is still an effective phenomenological method based on the local densities and fields.

The RMF theory was successfully used to reproduce parameters of the nuclear matter and some properties of finite nuclei like binding energies, mean square charge radii and quadrupole moments [16, 17].

This theory is based on the following field lagrangian density [15, 16, 18–20]
\[
\mathcal{L} = \bar{\psi}_i \left[ i \gamma^\mu \partial_\mu - M \right] \psi_i + \frac{1}{2} \sigma^{\mu\nu} \partial_\mu \sigma - \frac{1}{2} m_\sigma \sigma^2 - g_\sigma \bar{\psi}_i \psi_i \sigma
\]
The fields belong to nucleons (Dirac spinor field \( \psi \), the low mass isovector-vector meson (\( \rho^a; R_{\mu \nu} \)), isoscalar-vector (\( \omega^{\mu \nu}; \Omega_{\mu \nu} \)), scalar \( \sigma \) and to the massless photon vector field (\( A_\mu; F_{\mu \nu} \)). The field tensors are given by

\[
\begin{align*}
\Omega^{\mu \nu} &= \partial^\mu \omega^\nu - \partial^\nu \omega^\mu, \\
\tilde{R}^{\mu \nu} &= \partial^\mu \tilde{\rho}^\nu - \partial^\nu \tilde{\rho}^\mu - g_\rho (\tilde{\rho}^\mu \times \tilde{\rho}^\nu), \\
F^{\mu \nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu.
\end{align*}
\]

The Dirac spinors \( \psi \), of the nucleon and the fields of \( \sigma \), \( \rho \) and \( \omega \) mesons are solutions of the coupled Dirac and Klein–Gordon equations which are obtained from Eq. (12) by means of the classical variational principle and they are then solved by iteration for the case of the spherically-symmetric systems of nucleons. This iteration goes through the following steps: we start from an estimate of meson and electromagnetic fields and we solve the Dirac equation getting the spinors \( \psi \). The spinors are then used to obtain the densities. The latter serve as source to solve the Klein Gordon equations and achieve the new estimations of the meson and electromagnetic fields. The parameters used in our calculation are the same as in [18,19,21].

The simple version of the RMF model as given by the lagrangian density in Eq. (12) does not contain the Boguta’s nonlinear scalar meson potential \( U(r) = \alpha \sigma^3 + \beta \sigma^4 \) [22,23]. These nonlinear scalar \( \sigma \)-field modifies the surface properties of the nucleus. However, the omission of the nonlinear terms does not influence the neutron to proton densities ratio as well as the halo factors which we calculate in the present paper.

IV Results

On the basis of the relativistic mean field model which was presented in the previous section and is described in detail in [15,18,19,21,20,16] we have calculated the nucleon densities entering the Eq. (1) for the absorption width and in \([15, 18,19, 21, 20, 16]\) we have calculated the nucleon densities. This ratio which we define as

\[
l(r) = \log(\rho_n(r)/\rho_p(r)).
\]

is a sensitive quantity and \( n \) suits to relate densities especially in a peripheral nuclear region where both neutron and proton densities are small. Figure 2 shows this ratio for all nuclei under consideration.

In Fig. 2 one can see nuclei for which the logarithm \( l(r) \), for \( r > 2R \), has the value of few units. This means that the neutron density is much larger than the proton density. This indicates a possible large halo factor \( f \).

One has to remember that the microscopic density \( n \) is the sum of the single particles densities \( n \), over all occupied states

\[
\rho(r) = \sum \rho_v(r).
\]

It is interesting to study the dependence on \( r \) of the contribution of the single particle orbitals \( v = (n\ell j) \) to the total density. Or in other words what is the collectivity of the nuclear density tail? The results of the RMF calculation for some nuclei in which the halo factor is discussed are presented in Fig. 3. It is seen that for large distances of 10–14fm only a few valence orbitals contribute to the density tail. Even more, for some nuclei in which \( f \) is large, one neutron orbital exhausts 90% of the nuclear density. Only in the case of \( ^{144}\text{Sm} \) in addition to the neutron orbital 1h\( _{11/2} \) there appears 2d\( _{5/2} \) proton orbital with the comparable amplitude. In this nucleus one really observes very small neutron halo factor \( f \) (proton halo).

We have to add from our numerical experience that the asymptotic behaviour of the single particle density depends dramatically on the single particle energy. For the orbitals less bounded the large \( r \) tail of the density is longer. So the halo effect will not be the only probe of the surface width of the neutron and proton distributions but also a crucial test of the single particle structure foreseen by theoretical models. This also means the inclusion of a nuclear shape deformation into our model could be important.

Figure 4 shows the dependence of the logarithm of the total density of baryons against the distance \( r \) measured from the center of the nucleus. The logarithm of the density in the peripheral area of the nucleus (\( r > 8 \)) takes very small values and is nearly linear function of \( r \) with a steep negative slope. All the slopes for different nuclei are similar.
We now consider another two quantities which characterize the real density distribution. The first is the deviation of the density ratio from the homogeneous density ratio (10). In contrary to the case shown already in Fig. 1, this is calculated for real density distributions determined from the RMF model. In analogy to Eq. (10), it is defined through the average densities calculated in RMF model. The second quantity we considered is the celebrated halo factor $f$ (Eq. 2). Both quantities give the measure of the density distribution in the nucleus. These are shown in Fig. 5 and Fig. 6 respectively.

One can easily identify a halo nuclei. In Fig. 5 you can see an interesting case of Ru nucleus for which the relative density deviation $h$ is negative. This suggests the lack of the neutron halo for this nucleus. At the same time the halo factor $f$ is $\approx 2$. We concluded that the deviation $h$ is rather a rough indication of the neutron halo in nuclei.

The calculated halo factors $f$ are shown in Fig. 6 and in Table 1 where in addition we have also displayed the experimental values $f_{\text{exp}}$. These values were extracted from [2]. Except for extreme cases of $^{96}$Ru, $^{144}$Sm and $^{176}$Yb one can observe a good agreement between calculated factor $f_{\text{RMF}}$ and the experimental data. The case of $^{176}$Yb it is a special one. It shows a too low theoretical halo factor 3.1 and a very high measured value of 8.1. This is the case discussed also in [4]. The case of samarium nucleus, $^{144}$Sm, shows $f$ which probably indicates the proton halo.
Table 1. Experimental and calculated halo factors

| Nucleus | $f_{\text{exp}}$ | $f_{\text{RMF}}$ |
|---------|----------------|----------------|
| $^{58}\text{Ni}$ | 0.9 | 1.2 |
| $^{96}\text{Zr}$ | 2.6 | 2.3 |
| $^{96}\text{Ru}$ | 0.8 | 2.3 |
| $^{130}\text{Tc}$ | 4.1 | 3.5 |
| $^{144}\text{Sm}$ | <0.4 | 1.5 |
| $^{154}\text{Sm}$ | 2.0 | 3.0 |
| $^{176}\text{Yb}$ | 8.1 | 3.6 |
| $^{232}\text{Th}$ | 5.7 | 5.5 |
| $^{238}\text{U}$ | <6.0 | 5.0 |

instead of neutron one. In our calculations we obtained in this case $f = 1.5$. The similar situation of large theoretical $f$ is observed in the nucleus $^{96}\text{Ru}$.

V Summary

In the relativistic mean field model we have calculated the halo factor $f$ and the factor $h$ – the average density ratio deviation from the ratio of homogeneous density distributions, for nuclei in which we observed the neutron halo.

Both quantities $f$ and $h$ indicate the possible halo effects in most of the studied nuclei but in some cases (see e.g., Yb nucleus) they differ from experimental data significantly. This fact shows that one has to extend the RMF nuclear density model. The extension may include e.g., the deformation of the nuclear systems and/or the pairing interaction. The finite deformation in all considered cases may change the picture of the neutron halo.

The new parameter characterizing the peripheral properties of the nuclear density distributions, which we have called the average density ratio deviation $h$, is proportional to the difference of the proton and the neutron surface diffuseness parameters. Like $f$, it points out these nuclei which show the neutron halo properties.

We have shown that the nuclear density tail manifest a single particle nature. Therefore, the neutron halo is the single particle effect and not a collective one.

Some of our predictions, as compared to the experimental data, fail but most of them show correlations which are very promising ones. It is hard to explain the source of existing divergences. A possible explanation may be the lack of deformation in our model calculations. It is also possible that the inclusion of pairing interaction to this theory may improve the results. Such calculations are in progress.

We cordially acknowledge the discussion with Professor J. Jastrzębski from the Warsaw University and Professor S. Wycech from SINS, Warsaw.

References

1. J. Jastrzębski, H. Daniel, T. von Egidy, A. Grabowska, Y.S. Kim, W. Kuruczewicz, P. Lubinski, G. Riepe, W. Schmid, A. Stolarz, S. Wycech: Nucl. Phys. A 558, 405c–414c (1993)
2. P. Lubinski, J. Jastrzębski, A. Grochulksa, A. Stolarz, A. Trzezińska, W. Kuruczewicz, F.J. Hartmann, W. Schmid, T. von Egidy, J. Skalski, R. Smolanczuk, S. Wycech, D. Hilscher, D. Polster, H. Rossner: Phys. Rev. Lett. 73, 3199 (1994)
3. A. Baran and K. Pomorski, Int. Conf. on ENAM’95 Conference, Arles, June 18–23, 1995, p. 281
4. S. Wycech, et al.: Phys. Rev. C54, 1832 (1996). S. Wycech, R. Smolanczuk, Nuclear Haloes as Seen by Antiprotons, preprint hep-ph/9507281
5. C.J. Batty: Phys. Lett. B189, 393 (1987)
6. R.A. Bryan, R.J.N. Phillips: Nucl. Phys. B5, 201 (1968)
7. W.M. Bugg, G.T. Condo, E.L. Hart, H.O. Cohn, R.D. McCulloch: Phys. Rev. Lett. 31, 475 (1973)
8. R. Bizzarri, P. Guidoni, F. Marcelja, F. Marzano, E. Castelli, M. Sessa: Il Nuovo Cimento 22A, 225 (1974)
9. T.E. Kalogeropoulos, G.S. Tzanakos: in: Antinucleon-Nucleon Interaction, Proc. Third European Symp., Stockholm, July 9–13, 1976, G. Ekspong, S. Nilsson (eds.) p. 29, Oxford: Per- gamon Press 1977
10. L.R.B. Elton: Nuclear Sizes, Oxford: Oxford University Press 1961
11. W.D. Myers, W.J. Swiatecki: Ann. Phys. (N.Y.) 84, 186 (1974)
12. W.D. Myers: Nucl. Phys. A204, 465 (1973)
13. W.D. Myers, K.-H. Schmidt: Nucl. Phys. A410, 61 (1983)
14. R.W. Hasse, W.D. Myers: Geometrical Relationships of Macroscopic Nuclear Physics, Berlin: Springer 1988
15. B.D. Serot, J.D. Walecka: The Relativistic Nuclear Many-Body Problem, in: Advances in Nucl. Phys., J.W. Negele, Erich Vogt (eds.) Vol. 16, p. 1, New York: Plenum Press 1986
16. Y.K. Gambhir, P. Ring, T. Thimet: Ann. Phys. 198, 132 (1990)
17. A. Baran, L.L. Egidio, B. Nerlo-Pomorska, K. Pomorski, P. Ring, L.M. Robledo: J. Phys. G, Nucl. Part. Phys. 21, 657 (1995)
18. C.J. Horowitz, D.B. Serot: Nucl. Phys. A 368, 503 (1981)
19. C.J. Horowitz, D.B. Serot: Nucl. Phys. A 399, 529 (1981)
20. A. Bouyssy: Nucl. Phys. A 381, 445 (1982)
21. C.J. Horowitz, D.P. Murdock, B. Serot: in: Computational Nuclear Physics I: Nuclear Structure, p. 129, K. Langanke, J. Mahrus, S.E. Koonin (eds.) Berlin: Springer 1991
22. J. Boguta, A.R. Bodmer: Nucl. Phys. A 292, 413 (1977)
23. J. Boguta, S.A. Moszkowski: Nucl. Phys. A 403, 445 (1983)