Studying the Effects of Disturbance Torques on a 2U CubeSat in Low Earth Orbits

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Abstract. Disturbance torques in Low Earth Orbits may present a harsh environment for the successful operation of small satellite missions. The effects of gravity-gradient and aerodynamic torques are studied individually at various altitudes for low Earth orbit. Results show that for a basic 2U CubeSat, the gravity-gradient torque is highly perturbing, whereas the effect of aerodynamic torque is established to be so dominant that it offers attitude stabilization in lower altitudes; and may be used as a means of passive attitude stabilization.

1. Introduction
CubeSats, a class of small low earth orbiting satellites, are based on standardized units of mass and volume. The basic CubeSat unit measures 10x10x10 cm$^3$ (often called “1U”), and weighs around 1 kg. A combination of two units forms a larger 2U CubeSat (20x10x10 cm$^3$). CubeSats typically piggyback on the launch of larger satellite missions, and are mostly deployed into Low Earth Orbits (LEO) [1]. For such small satellites, LEO presents a harsh space environment with various forces that could act as disturbing forces on the CubeSat. The most prominent disturbance torques in LEO are the magnetic disturbance torques due to interaction with Earth’s magnetic field; gravity-gradient torques, which are a result of the differences in the Earth’s gravitational pull on the CubeSat body; and the aerodynamic torques due to aerodynamic drag, which is predominant in lower orbits. These disturbance torques act on the CubeSat and ultimately alter its orientation. Maintaining the CubeSat’s orientation in space to some desired attitude is important for it to fulfil its mission requirements. Therefore, the attitude of a CubeSat needs to be controlled or maintained in a certain manner. Passive attitude stabilization techniques such as gravity-gradient stabilization, passive magnetic control, and aerodynamic stabilization make use of these disturbance torques to control the CubeSat’s attitude.

This paper studies the effects of the gravity-gradient and aerodynamic disturbance torque individually. The magnetic disturbance torques are not studied independently due to an inadequate model of the Earth’s magnetic field, as only a simple dipole model was used. In reality, the Earth’s magnetic field model is one that is highly variable and unpredictable. Also, magnetic torque disturbance, if modelled accurately, will vary with orbit inclination, which is not considered in this study. Therefore, effects of only gravity-gradient and aerodynamic disturbance torques are considered. The main aim is to study how each disturbance torque plays a role in influencing the CubeSat attitude stability. This may further aid in changing design parameters to implement passive attitude stabilization.
2. Modelling

2.1. 2U CubeSat Dynamic Model

A basic 2U CubeSat is modelled as a rigid body (three degrees of freedom), with its center of mass coinciding with its center of gravity. Figure 1 shows the modelled dimensions and attitude rotation of 2U CubeSat. For attitude analysis, the CubeSat’s overall dynamics equation is given in Equation 1 [2].

\[ \mathbf{I} \ddot{\omega} + \dot{\omega} \times (\mathbf{I} \omega) = \tau_{total} \]  

(1)

where \( \tau_{total} \) is the sum of all the disturbance torques in Nm, \( \omega = [\omega_x \ \omega_y \ \omega_z]^T \) represents the angular velocities (degrees/sec) in roll, pitch and yaw; and \( \mathbf{I} \) is the inertia matrix about the center of gravity.

![Figure 1. 2U CubeSat model showing dimensions and Euler angle representations.](image)

2.2. Disturbance Torques

The major disturbance torques considered are: the magnetic torque, gravity-gradient torque, and aerodynamic torque. \( \tau_{total} \) in Equation 1 is the sum of all these torques, which will affect the CubeSat attitude stabilization.

2.2.1. Magnetic Torque. For simplification purposes, the Earth’s magnetic field is modelled as a dipole. The magnitude of the magnetic field is given in Equation 2

\[ B = \frac{\mu_o}{R^3} \]  

(2)

where \( B \) is magnetic field strength (Tesla), which varies with altitude and latitude, \( \mu_o \) is permeability of free space (\( \mu_o = 4\pi \times 10^{-7} H/m \)), and \( R \) is the distance from the center of Earth to the CubeSat in meters. If \( D \) is the sum of all magnetic moments on the CubeSat, the magnetic torque acting on the CubeSat, \( \tau_m \) (Nm), will be given by

\[ \tau_m = D \times B \]  

(3)

where \( \tau_m \) is the magnetic torque (Nm) on the CubeSat.

2.2.2. Gravity-Gradient Torque. The gravity-gradient torque is caused by the variation in Earth’s gravitational forces exerted on the CubeSat body. It is therefore in the z-direction, and the gravity-gradient torque is given by [3]:

\[ \tau_g = 3\left(\frac{\mu}{R^3}\right) z_o \times \mathbf{I} \cdot z_o \]  

(4)

where \( z_o \) is the z-axis in the orbit frame, as shown in figure 1.

2.2.3. Aerodynamic Torque. The expression for aerodynamic torque [2] is given as:

\[ \tau_a = \rho V_{orbit} [V_{orbit} A_{drag} c_p V_{orbit} - (\mathbf{I} + \dot{V}_{orbit} J) \omega] \]  

(5)
where $\rho$ is atmospheric density ($kg/m^3$), $V_{orbit}$ is orbital velocity of CubeSat ($m/s$), $A_{drag}$ is the area perpendicular to the velocity vector ($10cm \times 10cm$), $c_p$ is centre of pressure, and $J$ is the new moment of inertia matrix for drag. The atmospheric density and orbital velocity both vary with altitude. Within the same altitude, the atmospheric density also tends to vary slightly. For simulation and analysis of worst-case scenario, the maximum atmospheric density values are considered in the calculation of aerodynamic torque.

3. Results and Discussion

According to Equation 1, all the available disturbance torques act on the CubeSat body at any given time. However, for this study, the disturbance torques are simulated separately to study their effects on the CubeSat attitude. The Earth magnetic field model is incorporated in both the gravity-gradient and aerodynamic analyses. Simulations were carried out using Matlab/Simulink as it offers a convenient graphical environment to design the dynamic models and disturbance torques. A simple detumbling controller for CubeSats was incorporated into the model, so detumbling may be achieved and the attitude stabilization analysis is improved further [4]. The effects of disturbances on the CubeSat are studied without the intervention of any attitude control. The simulation is carried out for 6 orbits. Results of the simulations presented in figures 2-7, show the Euler angles for the CubeSat attitude versus number of orbits. In order to study of the effects of individual disturbance torque, the following altitudes were considered: 400 km, 700 km, and 1000 km.

3.1. Effects of Gravity-Gradient Torque

Figures 2-4 are plots of simulations for attitude (roll, pitch, and yaw) in degrees of the 2U CubeSat in the presence of gravity-gradient disturbance torque, at varying altitudes.

![Roll, Pitch, and Yaw for 2U CubeSat at an altitude of 400 km](image)

**Figure 2.** Plot of 2U CubeSat attitude (roll, pitch, and yaw) evaluated at altitude of 400 km, including gravity-gradient disturbance torque.
As can be seen in figures 2-4, there is high angular displacement in the roll and yaw axes. However, these high perturbations are not observed in the pitch axis (normal to orbit plane). This is evident due to the effect of gravity-gradient which is influenced by the configuration of the CubeSat. There were no significant changes in attitude performance for the different altitudes. This suggests that the CubeSat cannot be passively stabilized via gravity-gradient torque for the configuration considered.

3.2 Effects of Aerodynamic Torque

Figures 5-7 are plots of simulations for attitude (roll, pitch, and yaw) in degrees of the 2U CubeSat in the presence of gravity-gradient disturbance torque, at varying altitudes.
Figure 5. Plot of 2U CubeSat attitude (roll, pitch, and yaw) evaluated at altitude of 400 km, including aerodynamic disturbance torque.

Figure 6. Plot of 2U CubeSat attitude (roll, pitch, and yaw) evaluated at altitude of 700 km, including aerodynamic disturbance torque.

Figure 7. Plot of 2U CubeSat attitude (roll, pitch, and yaw) evaluated at altitude of 1000 km, including aerodynamic disturbance torque.

In figure 5, it can be seen that the effect of aerodynamic torque is highly stabilizing. This is due to high atmospheric density at low altitude, contributing to larger aerodynamic torques. This observation is consistent in figures 6 and 7 where aerodynamic torque becomes less dominant with an increase in altitude. Therefore, this may be used as a means of passive control alone for coarse pointing requirements for a 2U CubeSat.
4. Conclusion
Having considered the presence of magnetic torque for both analyses, the results establish a general idea of the effects of gravity-gradient and aerodynamic disturbances on the attitude of a 2U CubeSat at different altitudes in LEO. The effect of aerodynamic torque was seen to be more dominant. Consequently, passive aerodynamic stabilization may be achieved at lower altitudes, whereas gravity-gradient alone could not be used to stabilize attitude for this CubeSat configuration.

References
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