FORMATION AND HARDENING OF SUPERMASSIVE BLACK HOLE BINARIES 
IN MINOR MERGERS OF DISK GALAXIES

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ABSTRACT

We model for the first time the complete orbital evolution of a pair of supermassive black holes (SMBHs) in a 1:10 galaxy merger of two disk-dominated gas-rich galaxies, from the stage prior to the formation of the binary up to the onset of gravitational wave (GW) emission when the binary separation has shrunk to 1 mpc. The high-resolution smoothed particle hydrodynamics (SPH) simulations used for the first phase of the evolution include star formation, accretion onto the SMBHs as well as feedback from supernovae explosions, and radiative heating from the SMBHs themselves. Using the direct N-body code φ-GPU, we evolve the system further without including the effect of gas, which in the mean time has been mostly consumed by star formation. We start at the time when the separation between two SMBHs is ~700 pc and the two black holes are still embedded in their galaxy cusps. We use three million particles to study the formation and evolution of the SMBH binary until it becomes hard. After a hard binary is formed, we reduce (reselect) the particles to 1.15 million and follow the subsequent shrinking of the SMBH binary due to three-body encounters with the stars. We find approximately constant hardening rates and that the SMBH binary rapidly develops a high eccentricity. Similar hardening rates and eccentricity values were reported in earlier studies of SMBH binary evolution in the merging of dissipationless spherical galaxy models. The estimated coalescence time is ~5.5 Gyr, significantly smaller than a Hubble time. We discuss why this timescale should be regarded as an upper limit. Since 1:10 mergers are among the most common interaction events for galaxies at all cosmic epochs, we argue that several SMBH binaries should be detected with currently planned space-borne GW interferometers, whose sensitivity will be especially high for SMBHs in the mass range considered here.

Key words: black hole physics – galaxies: evolution – galaxies: interactions – galaxies: nuclei – gravitational waves

Online-only material: color figures

1. INTRODUCTION

Central supermassive black holes (SMBHs) are ubiquitous and are found in a variety of galaxies, ranging from low-mass galaxies to the most massive early-type galaxies (Ferrarese & Ford 2005; Gültekin et al. 2009). Within our current cosmological picture of hierarchical structure formation, galaxies form through continuous mergers. If both candidate galaxies harbor a central SMBH before the merger, the evolution of the latter is thought to be as follows (Begelman et al. 1980): the SMBHs of the merging galaxies sink toward the center of the merger remnant due to dynamic friction and form a gravitationally bound binary system. The further evolution of the SMBH binary is governed by interactions with stars and gas. If the binary semi-major axis value shrinks to one where emission of gravitational waves (GWs) efficiently takes away energy and angular momentum from the binary, the coalescence of SMBHs becomes inevitable. In fact, there is growing observational evidence for this process: there are reports about two widely separated SMBHs in a single galaxy (e.g., Komossa et al. 2003; Rodriguez et al. 2006; Fabbiano et al. 2011) as well as indirect evidence for binary SMBHs (e.g., Merritt & Ekers 2002; Liu et al. 2003; Valtonen et al. 2008; Boroson & Lauer 2009; Iguchi et al. 2010).

Binary SMBHs are of particular interest in astrophysics and general relativity. If SMBH binary evolution leads to coalescence following a galaxy merger, such an event would give rise to one of the loudest possible bursts of GWs detectable for future space-borne low-frequency laser interferometers (Hughes 2003; Barack & Cutler 2004; Amaro-Seoane et al. 2012). However, the exact pathway to SMBH coalescence and especially the associated timescales, and hence the chances for possible detection during any satellite mission run time, are still matters of debate.

Numerical N-body simulations considering the decay of a pair of SMBHs in major mergers of massive elliptical galaxies due to interaction with the stellar background only show that the critical separation for GW emission should be reached in about 1 Gyr as long as the galaxies deviate sufficiently from sphericity, a configuration which allows for centrophilic orbits (Sridhar & Touma 1999; Poon & Merritt 2001; Merritt & Poon 2004; Merritt & Vasiliev 2011) that efficiently refill the loss cone (Berczik et al. 2006; Berentzen et al. 2009; Khan et al. 2011, 2012; Preto et al. 2011). Khan et al. (2012) showed that SMBHs with masses ranging from 10^6 M⊙–10^7 M⊙, e.g., at the center of faint elliptical galaxies or in the bulge of spiral galaxies, coalesce well within a Gyr after the merger of the two...
The gravitational softening adopted in the study of Callegari et al. (2011) was $\epsilon = 45$ pc for dark matter and baryonic...
Figure 1. Density ($3.3 \times 10^{10} M_\odot \text{kpc}^{-3}$) distribution of the dark matter (top panels) and the stellar component (bottom panels) in the $x$–$y$ (left column) and $y$–$z$ (right column) planes. The two high-density regions are clearly visible around the two SMBHs (black dots) in the center. The size of each box is 4 kpc. (A color version of this figure is available in the online journal.)

particles in the larger galaxy, whereas for the smaller galaxy it was 20 pc. The SMBH was introduced at the center of each galaxy represented by a point-mass particle. The adopted masses for the SMBHs were $6 \times 10^5 M_\odot$ and $6 \times 10^4 M_\odot$ for the primary and the satellite galaxy, respectively, consistent with the $M_\bullet$–$M_{\text{bulge}}$ relation (Merritt & Ferrarese 2001; H{"a}ring & Rix 2004). The simulation also included the effects of star formation and accretion onto the SMBHs, as well as feedback from both processes. The final separation of the two SMBHs at the end of the simulation was $\sim 30$ pc, which is comparable to the softening that was used in the simulations. For more details of the construction of the galaxy models and the setup for the initial orbit of the galaxy merger, we refer the reader to Callegari et al. (2011). Here it suffices to say that the merger was assumed to start at $z \sim 3$–5, and was completed at a time corresponding to $z \sim 1$–2, a choice motivated by the optimal detection window of planned laser interferometers such as eLISA. The choice of galaxy types and sizes, as well as of the masses of the SMBHs just reported, was thus made based on the characteristic densities expected in the concordance ΛCDM model for galaxies having the same circular velocity as the Milky Way, $V_c \sim 200$ km s$^{-1}$.

In this study, we choose a snapshot of the system at the time 2.56 Gyr of Callegari et al. (2011), when the separation between the two black holes is 700 pc and the black holes are still embedded in the individual cusps (Figure 1). The masses of the SMBHs are $M_{\bullet 1} = 1.5 \times 10^6 M_\odot$ and $M_{\bullet 2} = 4.9 \times 10^5 M_\odot$, corresponding to a mass ratio $q = M_{\bullet 2}/M_{\bullet 1} = 0.3$. We select all particles in the central 5 kpc region of the merger remnant and add all those particles which have a pericenter passage smaller than 3 kpc.

Most of the gas in the central region is already converted into stars. The remaining gas particles, which contribute only a few percent in mass in the central region (Figure 2), are also treated as stars in our simulations. The total mass of the selected sample is $3.3 \times 10^{10} M_\odot$.

Although they are state-of-the-art hydrodynamic simulations, the Callegari et al. (2011) mergers use a few million particles to represent the stellar component of a galaxy; this is still several
orders of magnitude smaller than the actual number of stars in a real galaxy. Because of the small number of dark matter particles, the mass of one particle in the primary galaxy is comparable to the mass of the SMBH in the satellite galaxy in Callegari et al. (2011). This is a potential problem for continuing the calculation to higher resolution in a regime in which the interaction with the background of stars and dark matter might dominate, since it can lead to unrealistic mass segregation of the dark matter and non-physically large kicks of the two SMBHs. To avoid such high-mass particle encounters with the SMBH binary, we split each dark matter particle in the primary galaxy into 10 particles. Each child particle has a mass 1/10 of that of the parent particle. The child particles are randomly distributed over a 10 pc sphere corresponding to the softening of the parent dark matter particle used in our simulations (see Section 3.2). The child particles have the same velocities as their parent particles. A similar technique of particle splitting, which causes an enormous slow-down because of the small time steps required to resolve the orbit of the binary, phi-GPU does not include the regularization (Mikkola & Aarseth 1998) which causes an enormous slow-down because of the small time steps required to resolve the orbits of the binaries. phi-GPU does not include the regularization (Mikkola & Aarseth 1998) of close encounters or binaries, so we use softening to avoid the formation of tight binaries. For the interactions between two particles, we adopt the following criterion for the gravitational softening:

$$
\epsilon_{ij} = (\epsilon_s^2 + \epsilon_i^2)/2.
$$

where \(\epsilon_{bh} = 0\) for SMBHs, \(\epsilon_i = 0.01\) pc for stars, and \(\epsilon_{dm} = 10\) pc for dark matter particles.

During the late phase of the SMBH binary evolution (Run 3), we reduce the star–BH interaction softening by an additional factor of ten to resolve the stellar encounters during the pericenter passage of the SMBH binary.

### 3. NUMERIC METHODS

#### 3.1. Simulation Software

We use the direct \(N\)-body code \(\phi\)-GPU (GPU: graphics processing unit) with a fourth-order Hermite integration scheme and hierarchical block time steps for our \(N\)-body simulations.

The code is written from scratch in C++ and is based on an earlier C version of \(\phi\)-GRAPE\(^8\) designed for GRAPE6a clusters (Harfst et al. 2007). In the present version of the \(\phi\)-GPU code, we use native GPU support and direct code access to the GPUs using only the NVIDIA CUDA library.\(^9\) The multi-GPU support is achieved through message passing interface (MPI) parallelization. Each MPI process uses only a single GPU, but we usually start two or more MPI processes per node (to effectively use the multi-core CPUs and the multi-GPUs on our clusters).

The \(\phi\)-GPU code is fully parallelized using the MPI library. The MPI parallelization was done in the same “\(j\)” particle parallelization scheme as in the earlier \(\phi\)-GRAPE code. All the particles are divided equally between the working nodes (using the \texttt{MPI_Bcast()} command), and in each node we calculate only the fractional forces for the particles in the current time step, i.e., the so-called active or \(i\) particles. We get the full forces from all the particles acting on the active particles after the global \texttt{MPI_Allreduce()} communication routine is applied.

Besides the fourth-order Hermite integration scheme used, \(\phi\)-GPU additionally supports a sixth-order and even an eighth-order Hermite integration. The numeric integration of the particle orbits as well as the time step criterion (see Section 3.3 for details) is based on the (serial CPU) \(N\)-body code YEBISU (Nitadori & Makino 2008).

More details about the \(\phi\)-GPU code can be found in Berczik et al. (2011).

The present version of the code used here is extensively modified to handle the computational challenges required for our current project.

#### 3.2. Gravitational Softening

We use a gravitational (Plummer) softening between all particles. \(\phi\)-GPU supports the use of different softening lengths for different components and even individual softening for the particles. The softening between the SMBH particles is set equal to zero. The use of zero softening for the stars and dark matter leads to the formation of tight binaries in the system, which causes an enormous slow-down because of the small time steps required to resolve the orbits of the binaries. \(\phi\)-GPU does not include the regularization (Mikkola & Aarseth 1998) of close encounters or binaries, so we use softening to avoid the formation of tight binaries. For the interactions between two particles, we adopt the following criterion for the gravitational softening:

\(\epsilon_{ij}^2 = (\epsilon_s^2 + \epsilon_i^2)/2\).
3.3. Time Step Criterion

The time step criterion (Nitadori & Makino 2008) applied for individual particles is

$$\Delta t = \eta \left( \frac{A^{(1)}}{A^{(p-3)}} \right)^{1/(p-3)},$$

(2)

where

$$A^{(k)} = \left( |a^{(k-1)}| + |a^{(k+1)}| + |a^{(k)}|^2 \right)^{1/2}.$$  

(3)

Here, $p$ is the order of the integrator, $a^{(k)}$ is the $k$th-order derivative of the acceleration, and $\eta$ is the time-step parameter.

$\phi$-GPU can employ different time-step parameters $\eta$ for different components (BH, stars, and dark matter). We adopt $\eta = 0.1$ for all components. In the late phase of the SMBH binary evolution (Run 4), we reduce this parameter for SMBHs to $\eta = 0.03$ to integrate the binary orbit with smaller time steps, hence achieving higher accuracy.

3.4. GPU Clusters

The $N$-body integrations are carried out on three GPU high-performance computing clusters. lano has 172 GPUs at the Center of Information and Computing at National Astronomical Observatories, Chinese Academy of Sciences. titan has 32 GPUs at the Astronomisches Rechen-Institut in Heidelberg, and accre employs 192 GPUs at Vanderbilt University, Nashville, TN.

We use up to 64 GPUs for our runs, with a total CPU wall-clock time of approximately one year.

4. SMBH Binary Evolution

We start our high-resolution run at time $T = 0$, when the two SMBHs are still embedded in their respective galaxy cusps. Figure 1 shows the dark matter (top) and stellar volume mass densities (bottom) with views on the $x$–$y$ (left) and $y$–$z$ (right) planes. For visualization, we use the open-source glnemo2 software package.10 Two high-density regions around the two SMBHs are clearly visible in the figure, which suggests that individual galaxy cusps are still in the process of merging at the beginning of our run. The evolution of the SMBH binary can be described by the three distinct phases discussed in the following subsections.

4.1. Dynamical Friction

In the first phase, the two black holes centered in their respective galaxy cusps are unbound to each other and move independently in the potential of the merger remnant. Dynamical friction against the background dark matter and stars is very efficient in bringing the two SMBHs closer. At about $T = 40$ Myr, the individual cusps are already merged into one, and the two SMBHs are located in a single cusp (see Figure 3).

Earlier studies show that SMBHs form a binary when their relative separation $\Delta R_{BH}$ reaches $\sim r_h$ ($r_h$ is the gravitational influence radius defined as the radius of a sphere around the two black holes enclosing stellar mass equal to twice the SMBH masses). Figure 4 shows the evolution of the binary separation. At the same time, when the two cusps merge ($T = 40$ Myr), the separation between the two SMBHs is roughly about $r_h = 15$ pc and an SMBH binary system is formed.

4.2. Stellar-Dynamical Hardening

The subsequent evolution of the binary is governed by stellar encounters, predominantly three-body encounters. For spherical galaxy models, the subsequent hardening is reported to depend on the particle number (Makino & Funato 2004; Berczik et al. 2005). For a realistic particle number $N$, which is several orders of magnitude larger than current state-of-the-art simulations can accommodate, the binary should stall when its semi-major axis $a \sim a_h$ for these models. Here, $a_h$ is the semi-major axis of a “hard binary,” as defined by

$$a_h = \frac{q}{(1+q)^2} \frac{r_h}{4}$$

(e.g., Merritt et al. 2007). In our model $r_h$ is about 15 pc, and $a_h \approx 0.66$ pc with $a_h^{-1} \approx 1.5$ pc$^{-1}$.

Figure 5 shows the evolution of the SMBH binary’s inverse semi-major axis and eccentricity. The eccentricity grows to a very high value of $e \approx 0.9$ soon after the formation of the SMBH binary. The inverse semi-major axis of the binary evolution is roughly constant in time, and the phase of “hard binary” evolution is reached quickly. This behavior is consistent with the findings of Khan et al. (2012), who noticed that for shallow cusps ($\gamma = 0.5, 1.0$) $1/a$ of the binary evolves at a constant rate immediately after its formation (see Figure 2 of Khan et al. 2012). In fact, we find $\gamma = 1 = \gamma$ for the stellar density profile in our simulation. For steep cusps ($\gamma = 1.5, 1.75$), the SMBH binary undergoes a rapid phase of evolution before entering the hard binary regime, presumably corresponding to the clearing of the loss cone (Yu 2002). The long-term evolution of the SMBH binary is discussed later. Here, we describe in detail the different runs that we carried out in order to reach an SMBH separation where GW emission starts becoming important.

At $T = 200$ Myr, we reduce the particle number from $\sim 3$ million to $\sim 1.15$ million by selecting the particles which have their estimated pericenters in the inner $1$ kpc to increase the computational speed (Run 2). In order to see whether or not our selection has introduced some changes in the mass distribution, we plot the cumulative mass distribution at various time steps after the new selection of particles (Figure 6). The cumulative mass profile looks very stable in the inner parts. In the outer parts, however, there is a small expansion of the profile as expected due to the new cutoff. We also start the new run (Run 2) 20 Myr earlier to see if the evolution of the binary as it happened in the earlier run (Run 1) can be recovered. Figure 5 shows that both the inverse semi-major axis and the eccentricity evolution are well reproduced for the period where the two runs overlap.

We stop Run 2 when the value of the inverse semi-major axis is roughly $40$ pc$^{-1}$ or $a \approx 25$ mpc. The value of the eccentricity at the end of Run 2 is $e \approx 0.96$. For these parameters, the value of the pericenter $r_p$ for the massive binary is $r_p = (1 - e)a = 1$ mpc, which is smaller than the softening for the star–BH interaction (7 mpc). In order to resolve accurately the star–BH encounters at the pericenter passage of the binary, we reduce the softening for star–BH encounters by an additional factor of 10 and start a new run (Run 3) at an earlier time of 520 Myr. For Run 3, we again split the dark matter particles, this time selecting only those having a pericenter smaller than 50 pc from the center of the massive binary. As for the earlier splitting, each dark matter particle is split into ten particles and spread over a 10 pc sphere retaining the velocities of the parent particles. We employ the new particle splitting to avoid the unphysical

10 http://projets.oamp.fr/projects/glnemo2
Figure 3. Same as in Figure 1 but at $T = 40$ Myr. Both SMBHs are embedded in a single cusp. (A color version of this figure is available in the online journal.)

Figure 4. Relative separation between the two SMBHs as a function of time. The red arrow shows the estimated value of the influence radius $r_h$. (A color version of this figure is available in the online journal.)

jumps in the binary semi-major axis caused by massive dark matter particles that occur from time to time. Again, our new particle splitting does not introduce noticeable changes in the central mass profile. For Run 4, we reduce the $\eta$ parameter for the SMBHs from $\eta = 0.1$ to $\eta = 0.03$ to achieve higher accuracy in the integration of the SMBH binary orbit (there are roughly $10^4$ orbits in one model time unit). We evolve the SMBH binary until about 1.2 Gyr with Run 4. The inverse semi-major axis value is $50 \text{ pc}^{-1}$ or $a = 20 \text{ mpc}$. The eccentricity value is 0.955, which leads to the pericenter distance of 0.9 mpc. The binary evolution for Run 2 is very similar to Run 3 and Run 4. The binary’s inverse semi-major axis evolves at a constant rate (top panel of Figure 5), which is consistent with our earlier studies in which we followed the evolution of the SMBH binary by merging two spherical galaxies (Khan et al. 2011, 2012). We fit a straight line to calculate the binary’s hardening rate $s = (d/dt)(1/a)$ in the late phase of Run 3 and Run 4 (Figure 5, top panel). The value of the hardening rate is 115.7 in model units and 44.5 $\text{kpc}^{-1} \text{Myr}^{-1}$ in physical units. This value of the hardening rate is similar to those obtained by merging two spherical galaxies (see top
Figure 5. Evolution of the binary’s inverse semi-major axis (top) and eccentricity (bottom). The red arrow in the upper panel of the figure points to value of $1/a$, which corresponds to the estimated semi-major axis $a_h$ of the hard binary. The thin blue line shows the linear fit to estimate the constant SMBH binary hardening rate.

(A color version of this figure is available in the online journal.)

Figure 6. Mass profile after the selection of the new particle sample at $T = 182$ Myr. The black dotted line represents the theoretical Hernquist model $\gamma = 1.0$. Clearly, the cumulative mass profile is very stable in the inner kpc.

(A color version of this figure is available in the online journal.)

Figure 7. Top: intermediate-to-major and minor-to-major axis ratios as a function of distance from the center of the SMBH binary at $T = 200$ Myr. Bottom: the time evolution of axis ratios calculated at a distance of 0.2 kpc from the center of the SMBH binary.

(A color version of this figure is available in the online journal.)

panel of Figure 8 of Khan et al. (2011) having a similar profile ($\gamma = 1$) as adopted for the galaxy bulges in the merger study of Callegari et al. (2011). In Khan et al. (2011), the high value of the hardening rate was attributed to the non-spherical shape of the merger remnant supporting a large fraction of stars on centrophilic orbits (see also Berczik et al. 2006). The value of the hardening rate is approximately six times higher for the same $N$ when compared to the value for SMBHs of similar mass in spherical galaxy models (top panel of Figure 3 of Khan et al. 2011). For the merger remnant of two late-type galaxies under consideration in our current study, we analyze the shape by calculating axis ratios defined for a homogeneous ellipsoid with the same tensor of inertia. Figure 7 shows the intermediate-to-major and minor-to-major axis ratios at various distances from the center (top panel) and also for different times (bottom panel). We can see from the top panel of Figure 7 that deviations from spherical symmetry extend all the way to the center (few tens of parsec). The merger remnant is considerably flattened, which can also be seen from Figure 3, when compared to those which result after the merger of two spherical galaxy progenitors. The flattening increases as we move to larger distances and becomes more or less constant at a distance of about 1 kpc. We also
calculate the axis ratio at different times of evolution for the merger remnant at a distance of 200 pc from the center, as most of the centrophilic orbits are expected to come at about this distance. As seen in the bottom panel of Figure 7, the axis ratios remain constant during the whole time of the evolution of the SMBH binary. Due to the non-spherical shape of the merger remnant, we expect that the SMBH binary should evolve at a constant rate supported by the centrophilic orbit family of stars rather than the relaxation effects alone. Therefore, it is reasonable to extrapolate our results for the merger of late-type galaxies to a realistic number of star particles. Hence, we can predict the coalescence time for the SMBHs using the estimated hardening rates in the stellar-dynamical phase plus those in the GW-dominated regime.

4.3. Relativistic Regime

At small enough separation, the value of which depends on the mass and eccentricity of the SMBH binary, GWs extract energy and angular momentum efficiently from the binary, thus making its coalescence inevitable.

As shown in earlier studies (Berentzen et al. 2009; Khan et al. 2012), the estimated coalescence time obtained using a constant hardening rate $s$ in the stellar-dynamical regime and the formula of Peters (1964) for hardening in the GW-dominated regime agree remarkably well with $T_{\text{coal}}$ obtained from simulations that follow the binary evolution until coalescence using post-Newtonian ($\mathcal{P}\mathcal{N}$) terms in the equation of motion of the SMBH binary. However, these simulations are computationally very expensive due to additional $\mathcal{P}\mathcal{N}$ terms in the equation of motion of the binary and the small softening needed to resolve the star–BH interactions at pericenter until the SMBH binary enters into the GW-dominated regime. In order to further evolve the binary to the full coalescence of the SMBHs, we again need to reduce the softening and also add $\mathcal{P}\mathcal{N}$ terms in the equation of motion of the binary, which would drastically increase the computational time (by several months).

Further evolution of the SMBH binary can be estimated from

$$\frac{da}{dt} = \left( \frac{da}{dt} \right)_{\text{NB}} + \left( \frac{da}{dt} \right)_{\text{GW}} = -sa^2(t) + \langle \frac{da}{dt} \rangle_{\text{GW}}. \quad (5)$$

The orbit-averaged expressions—including the lowest-order $2.5 \mathcal{P}\mathcal{N}$ dissipative terms—for the rates of change of a binary’s semi-major axis and eccentricity due to GW emission are given by Peters (1964):

$$\langle \frac{da}{dt} \rangle_{\text{GW}} = \frac{64}{5} G^3 M_1 M_2 (M_1 + M_2) \frac{a^3 e^5 (1 - e^2)^{5/2}}{d^3} \times \left( 1 + \frac{73}{24} e^2 + \frac{37}{96} e^4 + \frac{1}{2} e^6 \right), \quad (6)$$

$$\langle \frac{de}{dt} \rangle_{\text{GW}} = -\frac{304}{15} \frac{G^3 M_1 M_2 (M_1 + M_2)}{a^5 e^3 (1 - e^2)^{5/2}} \times \left( 1 + \frac{121}{304} e^2 \right). \quad (7)$$

Our estimates show that already for the parameters of the SMBH binary at $T \sim 1.2$ Gyr, the contribution to the hardening of the SMBH binary can be as large as 10%. This is the reason that we stop our Run 4 at this point.

We now solve the coupled Equations (5)–(7) numerically to follow the SMBH binary evolution. For a numerical solution of the coupled equations, the semi-major axis of the binary was chosen at time $T \sim 500$ Myr to have a significant overlap with the $N$-body evolution of the massive binary. The eccentricity value was chosen to be 0.95, and we assume that the eccentricity remains constant during the stellar-dynamical hardening phase. This assumption is supported by the eccentricity evolution shown in Figure 5 (bottom), which shows that the value of $e$ remains more or less constant from time $T \sim 600$ Myr onward.

The estimated evolution is shown in Figure 8. We can see that the coalescence time of the SMBH binary is $T_{\text{coal}} \sim 2.9$ Gyr. The coalescence time of 2.9 Gyr, though longer when compared to Khan et al. (2011), Preto et al. (2011), and Khan et al. (2012), is still short enough to have a handful of 1:10 merger cases for detection with eLISA. From Khan et al. (2012), we know that binary hardening rates depend strongly on the adopted density profile. For steep density cusps with an inner power-law density index $\gamma = 1.75$, their study shows four times higher values of $s$ when compared to $\gamma = 1.0$. In the current study, the adopted density profile at the start of the merger simulation was a Hernquist profile, which has $\gamma = 1$. This slope is observed in bright elliptical galaxies which host SMBHs having masses $\sim 10^5$–$10^9 M_\odot$. The faint bulges/ellipticals which host smaller SMBHs with masses $\sim 10^6$–$10^7 M_\odot$ have typically steep...
cusps ($\gamma \sim 1.5–1.75$). It is conceivable that the prolonged effect of gas dissipation at higher mass and spatial resolution in the last stages of the merger, beyond the starting point of the direct $N$-body calculation, would have led to a steeper baryonic cusp at small scales (see Mayer et al. 2007 for the dependence of the inner density profile of merger remnants on the numerical resolution of the gas component). In addition, the slope of the initial bulge profile in the galaxy models could be steeper and yet still consistent with the observed distribution of bulge slopes. Hence, we can easily expect that typical coalescence times for SMBH binaries in gas-rich mergers can be shorter and comparable to Khan et al. (2012).

5. SUMMARY AND CONCLUSIONS

Starting from the results of Callegari et al. (2011), we study the orbital evolution of a pair of SMBHs in a minor merger of disk galaxies (with 30% gas fraction in the disk), from an initial separation of 60 kpc to a final separation of less than a milli-parsec (binary’s pericenter distance). Initially, the mass ratio between the galaxies and SMBHs is 0.1. During the merger, the two SMBHs accrete gas, increasing their masses in the process. The mass of the SMBH in the satellite galaxy increases almost eightfold as the gas in the secondary galaxy is funneled toward the center due to the tidal force of the primary galaxy at each pericenter passage. The perturbations produced by the passages of the secondary galaxy are not significant for the primary galaxy, so the SMBH in the primary galaxy accretes gas steadily, and the mass of the SMBH here grows by a factor of two. At the end of SPH simulations, the mass ratio between the two SMBHs is approximately $q = 0.3$ (see Figure 1 of Callegari et al. 2011).

At the start of our direct $N$-body simulations, the separation between the two SMBHs is roughly 700 pc, and the binary has yet to form. Gas particles, which contribute only a few percent to the mass of the selected central region, are treated as star particles. We use particle splitting to reduce the mass of dark matter particles to avoid both mass segregation and unphysical encounters of high-mass dark matter particles with the SMBHs. Dynamical friction is very efficient in bringing the two SMBHs to a point where they form a binary at a separation of roughly 15 pc. The subsequent hardening, which happens at a constant rate, is governed by individual stars interacting with the massive binary. We artificially suppress the contribution of dark matter to the hardening of the SMBH binary by introducing a large softening ($\epsilon_{\text{dm}} = 10$ pc). The shape analysis of the merger product shows that the system is predominantly triaxial from the periphery to the center. The SMBH binary evolves at a constant rate and the hardening rate is high, which suggests that the stalling of the SMBH binary should not be an issue in realistic galaxy mergers such as those considered here. The eccentricity is very high ($e \sim 0.95$), as was observed for the shallow density profile ($\gamma \lesssim 1.0$) galaxy merger simulations performed in our earlier studies (Khan et al. 2011, 2012). The dependence on eccentricity of the coalescence time under GW emission is $T_{\text{coal}} \sim (1 - e^2)^{7/2}$. For very eccentric SMBH binaries, this could easily account for a decrease of an order of magnitude. Accordingly, it will be very important to further investigate the dependence of the eccentricity evolution under different values of $\gamma$ and $q$. Currently, we are carrying out direct $N$-body simulations of galaxy mergers with galaxies having both steep and shallow density profiles at the centers together with an initial mass function to address this question.

Another issue that should be mentioned here is the impact of gas dynamics on the evolution of the SMBH binary’s eccentricity. In gas-dynamical simulations, of both individual nuclear disks and galaxy mergers, published work has largely reported fast circularization due to dynamical friction, whether the two black holes are already moving in the same plane and co-rotating in a nuclear disk (e.g., Dotti et al. 2006, 2007) or the relative inclination of the spin of the two galaxies is less than 90° (as in the works of Callegari et al.), so that torques can bring the two galaxies and their embedded black holes into a nearly coplanar, pro-grade configuration. In nearly retrograde configurations, circularization is not expected to be effective (Nixon et al. 2011), although such configurations might hinder orbital decay already at 1 kpc scales in the first place (Callegari et al. 2011). Other effects, such as gap opening at small binary separations (Cuadra et al. 2009) and non-axisymmetric modes in the nuclear disk transferring angular momentum to and from the orbit of the two holes, can also lead to eccentric orbits. One example of the latter are the major merger SPH simulations of Mayer et al. (2007), later confirmed by the adaptive mesh refinement (AMR) calculations of Chapron et al. (2011), in which the eccentricity remains high to parsec scale separations in a massive, self-gravitating nuclear disk supporting strong spiral modes. It is difficult to reach firm conclusions regarding the effect of gas versus stars on the eccentricity because, as the aforementioned studies show, more than one outcome is possible. The strong dependence of $T_{\text{coal}} \cdot \text{GW}$ on eccentricity for high $e$ means that if gas dynamics circularize the massive binary, then it may enter the GW regime and merge on a timescale longer than 2.9 Gyr. However, the presence of gas would give rise to competing effects concerning the timescale. For example, if a significant cold gas mass is present at separations less than 700 pc, accretion onto the black holes would continue down to smaller separations. Accretion would further increase the masses of the SMBHs, possibly leading to an even higher final mass ratio, and consequently speed up the merger. If significant accretion happens, our results on the coalescence timescales are conservative and should be regarded as upper limits. Also, if gas is effective in circularizing the orbit of the SMBH binary, then the hardening rate would also be higher, resulting in speed-up of the merger.

In our current study, we evolve the SMBH binary to a separation (0.9 mpc at pericenter of the SMBH binary) where the contribution to the hardening rate of the SMBH binary due to the emission of GWs becomes important (roughly 10%). Using the constant value of the hardening rate in the Newtonian regime and the formula of Peters (1964) for GW emission from two point masses orbiting each other, in the relativistic regime, we estimate the coalescence time of two SMBHs to be 2.9 Gyr after the merger of the two galaxies. The total coalescence time of 2.6 Gyr + 2.9 Gyr = 5.5 Gyr. Although it is longer when compared to the times obtained for similar mass binaries in Khan et al. (2012), is still short enough to have a few 1:10 mergers of SMBHs in late-type galaxy mergers in the range at which eLISA is most sensitive. From Khan et al. (2012), we know that binary hardening rates depend strongly on the density profile adopted. For steep density cusps observed at the center of faint bulges/ellipticals, we can expect the coalescence times to be much shorter, comparable to the ones that were obtained in Khan et al. (2012) for the merger of steep power-law density profile galaxies.

The current work should be regarded mainly as a proof-of-concept, since we have considered only one particular initial condition and have not computed directly the binary SMBH...
shrinking in the post-Newtonian phase. Nevertheless, it shows for the first time that the coalescence of the two SMBHs on a timescale sufficiently short to be astrophysically relevant does indeed take place as a result of a quite realistic galaxy merger with previous effects of dissipation taken into account. In the future, we will explore a wider range of initial conditions motivated by cosmological simulations of galaxy formation, and we will carry out the direct computation of the binary shrinking to much smaller separations.

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