Nonlinear magnetoresistance of an irradiated two-dimensional electron system

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Nonlinear magnetotransport of a microwave-irradiated high mobility two-dimensional electron system under a finite direct current excitation is analyzed using a dc-controlled scheme with photon-assisted transition mechanism. The predicted amplitudes, extrema and nodes of the oscillatory behavior of resistance and differential resistance versus the magnetic field and the current density, are in excellent agreement with the recent experimental observation [Hatke et al. Phys. Rev. B 77, 201304(R) (2008)].

PACS numbers: 73.50.Jt, 73.40.-c, 73.43.Qt, 71.70.Di

The prediction\(^1\) and detection\(^2\) of radiation induced magnetoresistance oscillation (RIMO) in two-dimensional (2D) electron systems (ES), especially the discovery of the zero-resistance state\(^3\), have stimulated intensive experimental\(^6,7,8,9,10,11,12,13,14,15,16,17,18\) and theoretical\(^20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35\) studies on this extraordinary transport phenomenon of electrons in very high Landau levels.

Despite the fact that basic features of RIMO have been established and the understanding that it stems from impurity scattering has been reached, so far there has been no common agreement as to the accurate microscopic origin of these giant resistance oscillations. Presented in different forms, many theoretical models\(^6,30,31\) have considered RIMO to arise from electron transitions between different Landau states due to impurity scattering accompanied by absorbing and emitting photons. This origin is called the "photon-assisted transition" or "displacement" mechanism. A different origin, called the "inelastic" or "distribution function" mechanism\(^4,30,31\) considers RIMO to arise from a microwave-induced nonequilibrium oscillation of the time-averaged isotropic electron distribution function in the density-of-states (DOS) modulated system. Both mechanisms exist in a real 2D semiconductor and have been shown to produce magneto-resistance oscillations qualitatively having the observed period, phase and magnetic field damping. The "displacement" mechanism predicts a well-defined photoresistivity with given impurity scattering and Landau-level broadening, while the "inelastic" mechanism yields an additional factor proportional to the ratio of the inelastic scattering time \(\tau_{\text{in}}\) to the impurity-induced quantum scattering time \(\tau_q\).\(^{23}\) Since the inelastic scattering time \(\tau_{\text{in}}\) or the thermalization time \(\tau_{\text{th}}\)\(^2\) being the property of a nonequilibrium state and contributed by the direct Coulomb interactions between electrons and by all other possible impurity- and phonon-scattering mediated effective electron-electron scatterings\(^2\) is very hard to determine theoretically or to measure experimentally, the sharp controversy whether \(\tau_{\text{in}}/\tau_q \gg 1\) or \(\tau_{\text{in}}/\tau_q \ll 1\), i.e. which mechanism plays the dominant role in the experimental systems\(^23,30\) has been an unsolved issue.

The detailed comparison between theoretical predictions and experiments may provide a useful way to distinguish them.

Introducing additional parameters into microwave-illuminated 2DESs, such as dc excitations, can be of help to distinguish different models and mechanisms. It has been shown that a finite current alone, can also induce substantial magnetoresistance oscillation and zero-resistance without microwave radiation.\(^38,39,40,41,42,43\) Simultaneous application of a direct current and a microwave radiation leads to very interesting and complicated oscillatory behavior of resistance and differential resistance.\(^36,37,44\) Recent careful measurements\(^45,46\) disclosed further details of such nonlinear magnetotransport in a high-mobility 2D semiconductor under both ac and dc excitations, allowing a careful comparison with theoretical predictions.

Our examination is based on a current-controlled scheme of photon-assisted transport\(^23\) which deals with a 2DES of short thermalization time having \(N_e\) electrons in a unit area of the \(x\)-\(y\) plane and subject to a uniform magnetic field \(B = (0, 0, B)\) in the \(z\) direction. When an electromagnetic wave with incident electric field \(E_0 e^{\text{sin} \omega t}\) irradiates perpendicularly on the plane together with a dc electric field \(E_0\) inside, the steady transport state of this 2DES is described by the electron drift velocity \(v_0\) and an electron temperature \(T_e\), satisfying the force and energy balance equations\(^24\)

\[
N_s e E_0 + N_e (v_0 \times B) + F_0 = 0, \quad (1)
N_s e E_0 \cdot v_0 = S_p - W = 0. \quad (2)
\]

Here, the frictional force resisting electron drift motion,

\[
F_0 = \sum_{q_i} |U(q_i)|^2 \sum_{n=-\infty}^{\infty} q_i J_0^2(\xi) \Pi_2(q_i, \omega_0 + n\omega), \quad (3)
\]

is given in terms of the electron density correlation function \(\Pi_2(q_i, \Omega)\), the effective impurity potential \(U(q_i)\), a radiation-related coupling parameter \(\xi\) in the Bessel function \(J_n(\xi)\), and \(\omega_0 \equiv q_i \cdot v_0\). The electron energy absorption from the radiation field, \(S_p\), and the electron energy dissipation to the lattice, \(W\), are given in Ref.\(^23\)\(^{23}\) The nonlinear longitudinal resistivity and differential resistivity in the presence of a radiation field are obtained from Eq.\(^1\) by taking \(v_0\) and the current density \(J = N_s e v_0\) in the \(x\) direction, \(v_0 = (0, 0, 0)\) and \(J = (J, 0, 0)\),

\[
R_{xx} = -F_0/(N_s^2 e^2 v_0), \quad r_{xx} = -(\partial F_0/\partial v_0)/(N_s^2 e^2). \quad (4)
\]
We have calculated the differential resistivity $r_{xx}$ from above equations (taking up to three-photon processes) under different magnetic fields $B$ and bias drift velocities $v_0$ for a GaAs-based heterosystem with carrier density $N_c = 3.7 \times 10^{15}/m^2$ and low-temperature linear mobility $\mu_0 = 1200 m^2/Vs$ at lattice temperature $T = 1.5 K$, irradiated by a linearly $x$-polarized microwave of frequency $\omega/2\pi = 69 GHz$ with incident amplitude $E_0 = 3.6 V/cm$. The elastic scatterings are assumed due to a mixture of short-range and background impurities, and the Landau-level broadening $\Gamma$ is taken to be a Gaussian form with a broadening parameter $\alpha = 7.23$.

Figure 1 presents the calculated $r_{xx}$ versus $\epsilon_\omega = \omega/\omega_c$ at fixed bias current densities from $J = 0$ to $J = 0.16 A/m$, in 0.01 A/m increments. (a) Differential magnetoresistivity $r_{xx}$ vs $\epsilon_\omega$ = $\omega/\omega_c$ under fixed bias current densities from $J = 0$ to $J = 0.16 A/m$, with incident amplitude $E_0 = 3.6 V/cm$. The maxima of differential resistivity $r_{xx}$ at low temperatures linear mobility $\mu_0$, and low-temperature linear mobility $\mu_0 = 1200 m^2/Vs$ at lattice temperature $T = 1.5 K$, irradiated by a linearly $x$-polarized microwave of frequency $\omega/2\pi = 69 GHz$ with incident amplitude $E_0 = 3.6 V/cm$. The elastic scatterings are assumed due to a mixture of short-range and background impurities, and the Landau-level broadening $\Gamma$ is taken to be a Gaussian form with a broadening parameter $\alpha = 7.23$.

Figure 1 presents the calculated $r_{xx}$ versus $\epsilon_\omega = \omega/\omega_c$, $\omega_c = eB/m$ is the cyclotron frequency) at fixed bias drift velocities from $2v_0/\nu_p = 0$ ($v_p$ is the Fermi velocity) to $2 \times 10^{-3}$ in steps of $1.25 \times 10^{-4}$, corresponding to current densities $J = 0$ to 0.16 A/m in steps of 0.01 A/m. The $J = 0$ case exhibits typical RIMO with a sequence of resistance maxima ($1^+, 2^+, 3^+, 4^+$ and $5^+$) and negative values around the resistance minima $1^-$ and $2^-$. With increasing $J$ to 0.16 A/m, the maxima $2^+, 3^+$ and $4^+$ (minima $2^-, 3^-$ and $4^-$) evolve into minima (maxima) having seemingly little change in the $B$ positions. Further, all the curves cross approximately at $\epsilon_\omega \approx 1.5, 2, 2.5, 3, 3.5$ and 4.5, indicating that $J$ at this range has little effect on photoresistance there. These and other features of Fig. 1 reproduce what was exactly observed in Ref.40.

Figure 2(a) shows the calculated $r_{xx}$ versus $\epsilon_j$ at $\omega = 2k_Fv_0$, $k_F$ is the Fermi wave vector of the 2D electron system) at fixed $\epsilon_j$ from 2 to 3.5 in steps of 0.0625. The electron density correlation function $\Pi(q, \Omega)$ in Eq. (3). The electron density correlation function $\Pi(q, \Omega)$ is essentially a multiplication of two energy-$\Omega$ shifted periodically modulated DOS functions of electrons in the magnetic field. Its periodicity with changing frequency $\Omega \to \Omega + \omega$ at low temperatures and high Landau-level occupations, determines the main periodic behavior of magnetoresistance. The previous examination of the node positions of the oscillatory peak-valley pairs of $R_{xx}$, which appear periodically roughly along the lines $\epsilon_\omega + \eta \epsilon_j = m = 1, 2, 3, 4, ...$ in the $\epsilon_\omega-\epsilon_j$ plane, where $\epsilon_\omega \equiv \omega/\omega_c$ is the control parameter of RIMOs, and $\epsilon_j \equiv \omega_j/\omega_c$ is the control parameter of current-induced magnetoresistance oscillations, and $\eta \lesssim 1$, dependent on the scattering potential.

The maxima of differential resistivity $r_{xx}$ show up at lower values in the $\epsilon_j$ axis in comparison with the node positions of related valley-peak pairs of $R_{xx}$, and its appearance exhibits a periodicity $\Delta \epsilon_j = 1.2$. In the $\epsilon_\omega-\epsilon_j$ plane, the differential resistance maxima are expected to show up roughly in the vicinity along the lines $\epsilon_\omega + \lambda \epsilon_j = m = 1, 2, 3, 4, ...$ (5) with $\lambda \gtrsim 1$, dependent on the scattering potential ($\lambda = 1.04$ for the system on discussion). Eq. (6) qualitatively accounts for the periodic change of $r_{xx}$ in a large scale in steps of $\Delta (\epsilon_\omega + \lambda \epsilon_j) = 1$.

Under strong microwave irradiation, as in the present case, the role of virtual photon process [the $n = 0$ term in the sum of Eq. (9)] is negligible due to samll $J_0^0(\xi)$ and main contributions to resistivity come from $n = \pm 1, \pm 2, ...$ terms (single- and multiple-photon processes). Noticing that the frequency differentiate $\Pi^1_2(q, \Omega)$ is an...
even function of $\Omega$ and considering contributions from scatterings parallel and antiparallel to the drift velocity $v_0$ and from $\pm|n|$ terms, we see that, in the case of finite bias current, the $r_{xx}$ behavior is determined by the sum of two terms: (a) $\Pi(q_1)|n|\omega + q_1 v_0 \cos \theta$ and (b) $\Pi(q_1)|n|\omega - q_1 v_0 \cos \theta$. Depending on the $\Pi(q_{1,2})$ function behavior in the vicinity of $\Omega = |n|\omega$, effects of these two terms can be cancelled or added, completely or partly, at different locations of $\epsilon_c$. $\Pi(q_{1,2})$ function reaches maxima (positive) at around $\Omega/\omega_c = N - 1/2$, reaches minima (negative) at around $\Omega/\omega_c = N + 1/2$, and passes through zero (changing sign) at around $\Omega/\omega_c = N$ and $N + 1/2$ for all integers $N \geq 2$. Thus, at $\epsilon_c \approx l$ or $l + 1/2$ ($l = 2, 3, 4, ...$) with which all involved $|n|\omega$ frequencies are located around $N\omega$, or $(N + 1/2)\omega_c$, contributions from (a) and (b) are almost cancelled out for modest $v_0$. In the case of $\epsilon_c \approx l - 1/4$, $\epsilon_c \approx l + 1/4$, there always exists a term of frequency $(N - 1/2)\omega_c$, $(N + 1/2)\omega_c$ in $|n|\omega$, and contributions from (a) and (b) are positively [negatively] additive. These clearly account for the suppression of the current effect at $\epsilon_c \approx l$ and $l + 1/2$, and the enhancement of it around $\epsilon_c \approx l - 1/4$ and $\epsilon_c \approx l + 1/4$.

Above discussions are general. The accurate behavior of resistivity $r_{xx}$ inside a period scale is relevant to the detailed shape of the DOS function. Figs. 1 and 2 represent the result of a Gaussian-type DOS. The good quantitative agreement with experiment without adjusting parameters indicates that the present current-controlled scheme of photon-assisted transport captures the main physics of RMOs in the discussed quasi-2D system.

This work was supported by the projects of the National Science Foundation of China, and the Shanghai Municipal Commission of Science and Technology.

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The amplitude maxima will be at $\omega \approx 2.25, 2.75$ and 3.25 if one considers only up to single-photon process ($|n| \leq 1$).