We discuss the possible applications of string theory for the construction of generalizations of the $SU(3) \times SU(2) \times U(1)$ standard model of strong and electroweak interactions. This includes an investigation of effective $d = 4$ dimensional supergravity theories that could be derived from higher dimensional string theories ($d = 10$) and M-theory ($d = 11$). It is shown how the question of unification of gauge and gravitational coupling constants could find a solution within this framework. The mechanism of hidden sector supersymmetry breakdown and its consequences for the pattern for soft supersymmetry breaking terms are discussed in detail.

1 Introduction

The supersymmetric extension of the standard model of weak, electromagnetic and strong interactions still contains many parameters whose origin has to be understood. It is the general hope that an explanation of these parameters can be found in a more complete and fundamental theory. In such a theory one would hope to understand how the various coupling constants are unified. One also expects hints to understand the nature of supersymmetry breakdown and its consequences for the soft breaking parameters of the theory. Such a theory could be string theory. Especially in the framework of the heterotic string one is confident to have identified a candidate theory that could manifest itself as the supersymmetric standard model in its low energy limit. In these lectures, we shall try to discuss such questions.

TASI97 has been devoted to the various aspects of supersymmetric models. Not much has been said about string theory so far. So I have to introduce the basics of string theory in these lectures. I shall not repeat this discussion here in the written up version, since it has already been published elsewhere. A pretty complete picture of the status of string theory can be found in last years TASI lectures, which I encourage you to consult. To follow the lectures here, it might be useful to consider the notations and conventions. Especially, it might be useful to learn the basics of constructing $d = 4$ low energy effective supergravity theories from higher dimensional string theories. In our

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ON THE LOW-ENERGY LIMIT OF STRING AND M-THEORY∗

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We discuss the possible applications of string theory for the construction of generalizations of the $SU(3) \times SU(2) \times U(1)$ standard model of strong and electroweak interactions. This includes an investigation of effective $d = 4$ dimensional supergravity theories that could be derived from higher dimensional string theories ($d = 10$) and M-theory ($d = 11$). It is shown how the question of unification of gauge and gravitational coupling constants could find a solution within this framework. The mechanism of hidden sector supersymmetry breakdown and its consequences for the pattern for soft supersymmetry breaking terms are discussed in detail.

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discussion we shall try to concentrate on the main points avoiding technical complications. We shall see that some of the aspects of the field are not so difficult to understand as one might have previously thought.

The part that is presented in this written version of the lectures concerns mainly two important aspects of the field that are at the center of current research: the question of the unification of gauge and gravitational coupling constants and the question of supersymmetry breakdown. In the first part we shall discuss gauge coupling constants in string theory. In general, they are not constants but functions of so-called moduli fields, whose vacuum expectation values (vevs) are undetermined at the classical level. An understanding of the actual value of the couplings will thus require a determination of these vevs and require the consideration of supersymmetry breakdown. Still we shall first discuss these questions at the classical level. We start with the (weakly coupled) heterotic $E_8 \times E_8$ string. The $d = 4$ dimensional effective low energy theory is discussed in the approximation obtained through the method of reduction and truncation. The method is very simple, but allows us to understand the qualitative properties of these models. At the classical level the gauge coupling constants are determined through the so-called dilaton field and one obtains a definite relation between the gauge coupling and the gravitational coupling constant. We comment on this relation and discuss explicitly how quantum effects can modify this classical relation.

The absolute value of the coupling constants requires knowledge about the vev of the dilaton field and thus we have to consider the breakdown of supersymmetry. Here the mechanism of gaugino condensation is discussed in detail. While some aspects of this mechanism look very encouraging, there remain some problems, most notably the run-away behaviour of the dilaton. We discuss some attempts to shed light on that problem. After that we explicitly discuss various aspects of gauge coupling unification in the heterotic string.

With the recent progress in string dualities, a new picture of unification might emerge. It is closely connected to a conjectured $d = 11$ dimensional theory known as M-theory. The $E_8 \times E_8$ version of that theory on an 11-dimensional interval might be especially interesting in that respect. The second part of these lectures is devoted to a discussion of the possible low energy manifestations of that theory. Again, we first review its implications for the question of unification, that look even more promising than the one found in the previously considered version of the heterotic string. The question of the breakdown of supersymmetry, though, looks very similar in both cases, with one exception concerning the soft breaking parameters. It solves a long standing problem of the smallness of the observable sector gaugino masses. We comment on the various phenomenological properties of the models obtained
in this framework. This will include a discussion of a possible nonuniveralit
ty of the soft scalar masses and its relevance for flavour changing neutral currents,
the size of the neutralino masses and the question of the lightest supersymmetric
particle in connection with the critical density of the universe.

2 Dynamical coupling constants

Most physics models contain coupling constants as free parameters that can be
adjusted to fit the experimental values. In more complete theories one could
imagine that the values of these parameters are determined dynamically by
the theory itself. The question arises, how and why the coupling parameters
take the values that are observed in nature. String theory provides an ex-
ample for such a class of models. At first sight, one would then expect that
it is rather easy to see whether a given string model has a chance to be re-
alistic or not. Just compute the physical coupling constants and then check
whether they coincide with the measured values. Looking more closely we
find, however, that the situation is more complicated. First of all, it would be
very difficult to do the computations needed. Secondly, the coupling constants
might not be determined yet at the classical level. In string theory, the cou-
pling constants are functions of so-called moduli fields and the actual values
of the coupling parameters are determined through the vacuum expectation
values (vev) of these fields. This is not only true for the gauge couplings, but
also for the Yukawa couplings as well as the radii and other properties of the
compactification parameters.

We shall concentrate here on the gauge coupling constants first. The het-
erotic string gives a definite prediction for the gauge coupling function as well
as the functional relation between gauge couplings and gravitational couplings.
At tree level the universal gauge coupling constant $g_{\text{string}}$ is determined by the
vev of the dilaton field $S$:

$$S = \frac{4\pi}{g_{\text{string}}^2} + i \frac{\theta}{2\pi}.$$  \hspace{1cm} (1)

Nonuniversalities can appear at the one-loop level and depend on further mod-
uli fields $T$, $U$ or $B$ that parametrize the properties of the compactification
parameters as well as the Wilson lines.

$$\frac{1}{g_a^2(\mu)} = \frac{k_a}{g_{\text{string}}^2} + \frac{b_a}{16\pi^2} \ln \frac{M_{\text{string}}^2}{\mu^2} - \frac{1}{16\pi^2} \Delta(T, U, B \ldots).$$  \hspace{1cm} (2)

Given this situation we have then to see how a theory with a realistic set of
gauge coupling constants can emerge.

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We would therefore like to connect such a theory with a low energy effective field theory describing the known particle physics phenomena. A prime candidate is a low energy supergravitational generalization of the standard model of strong and electroweak interactions, with supersymmetry broken in a hidden sector. Unification of observable sector gauge couplings might appear as required in supersymmetric grand unified theories (SUSY-GUTs). The vevs of the moduli fields $S, T, \ldots$ should then determine gauge couplings in hidden and observable sector, including the correct values of the QCD coupling $\alpha_s$ and the weak mixing angle $\sin^2 \theta_W$.

In string theories with unbroken supersymmetry the vevs of the moduli fields are undetermined. A first step in the determination of gauge coupling constants requires therefore the discussion of supersymmetry breakdown. We then have to see how the vevs of the moduli are fixed. Of course, not any value of the moduli will lead to a satisfactory model. In fact we shall face some generic problems concerning the actual values of the coupling constants. We have to understand why the value of $\alpha_{\text{string}} = \frac{g_{\text{string}}^2}{4\pi} \approx \frac{1}{20},$ (3) whereas a natural expectation for the vev of $S$ would be a number of order 1 or maybe 0 or even infinity, as happens in many simple models. A related question concerns the possibility of so-called string unification leading to the correct prediction of the weak mixing angle $\sin^2 \theta_W$. Naively one would expect string unification to appear at $M_{\text{string}} \approx 4 \times 10^{17}\text{GeV}$, while the correct prediction of $\sin^2 \theta_W$ seems to lead to a scale that is a factor of 20 smaller. We then have to face the question how such a situation can be achieved with natural values of the vevs of the moduli fields.

These are the questions we want to address in the next sections. We shall need to start with the discussion of supersymmetry breakdown. Here we shall concentrate in the framework of gaugino condensation. This will then lead us and include a discussion of the problem of a "runaway" dilaton that any attempt of a dynamical determination of coupling constants has to face. The next question then concerns the value of $<S> \approx 1$ and its compatibility with weak coupling. Finally we shall discuss new results concerning string threshold corrections and the question of string unification in the framework of the weakly coupled heterotic string. After that we shall discuss alternative possibilities.
3 Gaugino Condensation

One of the prime motivations to consider the supersymmetric extension of the standard model is the stability of the weak scale ($M_W$) of order of a TeV in the presence of larger mass scales like a GUT-scale of $M_X \approx 10^{16}$ GeV or the Planck scale $M_{Pl} \approx 10^{18}$ GeV. The size of the weak scale is directly related to the breakdown scale of supersymmetry, and a satisfactory mechanism of supersymmetry breakdown should explain the smallness of $M_W/M_{Pl}$ in a natural way. One such mechanism is based on the dynamical formation of gaugino condensates that has attracted much attention since its original proposal for a spontaneous breakdown of supergravity\cite{78,79}. In the following we shall address some open questions concerning this mechanism in the framework of low energy effective superstring theories\cite{60,61}.

Before discussing these detailed questions let us remind you of the basic facts of this mechanism. For simplicity we shall consider here a pure supersymmetric ($N=1$) Yang-Mills theory, with the vector multiplet $(A_\mu, \lambda)$ containing gauge bosons and gauge fermions in the adjoint representation of the nonabelian gauge group. Such a theory is asymptotically free and we would therefore (in analogy to QCD) expect confinement and gaugino condensation at low energies\cite{95}. We are then faced with the question whether such a non-trivial gaugino condensate $<\lambda\lambda> \neq 0$ leads to a breakdown of supersymmetry. A first look at the SUSY-transformation on the composite fermion $\lambda\sigma^\mu A_\mu$ might suggest a positive answer, but a careful inspection of the multiplet structure and gauge invariance leads to the opposite conclusion. The bilinear $\lambda\lambda$ has to be interpreted as the lowest component of the chiral superfield $W^\alpha W_\alpha = (\lambda\lambda, \ldots)$ and therefore a non-vanishing vev of $\lambda\lambda$ does not break SUSY\cite{77}. This suggestion is supported by index-arguments\cite{99} and an effective Lagrangian approach\cite{96}. We are thus lead to the conclusion that in such theories gaugino condensates form, but do not break global (rigid) supersymmetry.

Not all is lost, however, since we are primarily interested in models with local supersymmetry including gravitational interactions. The weak gravitational force should not interfere with the formation of the condensate; we therefore still assume $<\lambda\lambda> = \Lambda^3 \neq 0$. This expectation is confirmed by the explicit consideration of the effective Lagrangian of ref.\cite{78} in the now locally supersymmetric framework. We here consider a composite chiral superfield $U = (u, \psi, F_u)$ with $u = <\lambda\lambda>$. In this toy model\cite{78,79} we obtain the surprising result that not only $<u> = \Lambda^3 \neq 0$ but also $<F_u> \neq 0$, a signal for supersymmetry breakdown. In fact
\[ <F_\alpha> = M_S^2 = \frac{\Lambda^3}{M_{Pl}}, \quad (5) \]

consistent with our previous result that in the global limit \( M_{Pl} \to \infty \) (rigid) supersymmetry is restored. For a hidden sector supergravity model we would choose \( M_S \approx 10^{11} \text{ GeV} \).

Still more information can be obtained by consulting the general supergravity Lagrangian of elementary fields determined by the Kähler potential \( K(\Phi_i, \Phi^*_j) \), the superpotential \( W(\Phi_i) \) and the gauge kinetic function \( f(\Phi_i) \) for a set of chiral superfields \( \Phi_i = (\phi_i, \psi_i, F_i) \). Non-vanishing vevs of the auxiliary fields \( F_i \) would signal a breakdown of supersymmetry. In standard supergravity notation these fields are given by

\[ F_i = \exp(G/2)(G^{-1})^j_i G_j + \frac{1}{4} \frac{\partial f}{\partial \Phi_k} (G^{-1})^k_i \lambda \lambda + \ldots, \quad (6) \]

where the gaugino bilinear appears in the second term. This confirms our previous argument that \( \langle \lambda \lambda \rangle \neq 0 \) leads to a breakdown of supersymmetry, however, we obtain a new condition: \( \partial f / \partial \Phi_i \) has to be nonzero, i.e. the gauge kinetic function \( f(\Phi_i) \) has to be nontrivial. In the fundamental action \( f(\Phi_i) \) multiplies \( W_\alpha W^\alpha \) which in components leads to a form \( \text{Re} f(\phi_i) F_{\mu \nu} F^{\mu \nu} \) and tells us that the gauge coupling is field dependent. For simplicity we consider here one modulus field \( M \) with

\[ <\text{Re} f(M)> \approx 1/g^2. \quad (7) \]

This dependence of \( f \) on the modulus \( M \) is very crucial for SUSY breakdown via gaugino condensation. \( \partial f / \partial M \neq 0 \) leads to \( F_M \approx \Lambda^3 / M_{Pl} \) consistent with previous considerations. The goldstino is the fermion in the \( f(M) \) supermultiplet. In the full description of the theory it might mix with a composite field, but the inclusion of the composite fields should not alter the qualitative behaviour discussed here. As we shall see in a moment, an understanding of the mechanism of SUSY breakdown via gaugino condensation is intimately related to the question of a dynamical determination of the gauge coupling constant as the vev of a modulus field. We would hope that in a more complete theory such questions could be clarified in detail.

The candidate at our disposal for such a theory is the \( E_8 \times E_8 \) heterotic string. The second \( E_8 \) (or a subgroup thereof) could serve as the hidden sector gauge group and it was soon found that there we have nontrivial \( f = S \) where \( S \) represents the dilaton superfield. The heterotic string thus contains all the necessary ingredients for a successful implementation of the mechanism.
of SUSY breakdown via gaugino condensation\textsuperscript{[3,4].} Also the question of the dynamical determination of the gauge coupling constant can be addressed. A simple reduction and truncation\textsuperscript{8} from the $d=10$ theory leads to the following scalar potential\textsuperscript{24}

\[ V = \frac{1}{16S_R T_R} \left[ |W(\Phi) + 2(S_R T_R)^{3/2}(\lambda\lambda)|^2 + \frac{T_R}{3} \left| \frac{\partial W}{\partial \Phi} \right|^2 \right], \quad (8) \]

where $S_R = \text{Re}S, \ T_R = \text{Re}T$ is the modulus corresponding to the overall radius of compactification and $W(\Phi)$ is the superpotential depending on the matter fields $\Phi$. The gaugino bilinear appears via the second term in the auxiliary fields (6). To make contact with the dilaton field, observe that $<\lambda\lambda> = \Lambda^3$ where $\Lambda$ is the renormalization group invariant scale of the nonabelian gauge theory under consideration. In the one-loop approximation

\[ \Lambda = \mu \exp \left( -\frac{1}{b g^2(\mu)} \right), \quad (9) \]

with an arbitrary scale $\mu$ and the $\beta$-function coefficient $b$. This then suggests

\[ \lambda\lambda \approx e^{-f} = e^{-S} \quad (10) \]

as the leading contribution (for weak coupling) for the functional $f$-dependence of the gaugino bilinear\textsuperscript{\[.\]}

In the potential (8) we can then insert (10) and determine the minimum. In our simple model (with $\partial W/\partial T = 0$) we have a positive definite potential with vacuum energy $E_{\text{vac}} = 0$. Suppose now for the moment that $<W(\Phi)> \neq 0$\textsuperscript{\[‡\]}. $S$ will now adjust its vev in such a way that $|W(\Phi) + 2(S_R T_R)^{3/2}(\lambda\lambda)| = 0$, thus

\[ |W(\Phi) + 2(S_R T_R)^{3/2} \exp(-S)| = 0. \quad (11) \]

This then leads to broken SUSY with $E_{\text{vac}} = 0$ and a fixed value of the gauge coupling constant $g^2 \approx <\text{Re}S>^{-1}$. For the vevs of the auxiliary fields we obtain $F_S = 0$ and $F_T \neq 0$ with important consequences for the pattern of the soft SUSY breaking terms in phenomenologically oriented models\textsuperscript{1}, which we shall discuss later.

\[ \dagger \text{Relation } (10) \text{ is of course not exact. For different implementations see } [27, 94, 10]. \text{ The qualitative behaviour of the potential remains unchanged.} \]

\[ \dagger \text{In many places in the literature it is quoted incorrectly that } <W(\Phi)> \text{ is quantized in units of the Planck length since } W \text{ comes from } H, \text{ the field strength of the antisymmetric tensor field } B \text{ and } H = dB - \omega_{3Y} + \omega_{3L} (\omega \text{ being the Chern-Simons form). Quantization is expected for } <dB> \text{ but not necessarily for } H. \]
Thus a satisfactory picture seems to emerge. However, we have just discussed a simplified example. In general we would expect also that the superpotential depends on the moduli, $\partial W/\partial T \neq 0$ and, including this dependence, the modified potential would no longer be positive definite and one would have $E_{\text{vac}} < 0$.

But even in the simple case we have a further vacuum degeneracy. For any value of $W(\Phi)$ we obtain a minimum with $E_{\text{vac}} = 0$, including $W(\Phi) = 0$. In the latter case this would correspond to $<\lambda \lambda> = 0$ and $S \to \infty$. This is the potential problem of the runaway dilaton. The simple model above does not exclude such a possibility. In fact this problem of the runaway dilaton does not seem just to be a problem of the toy model, but more general. It always appears in models that are continuously connected to a regime that is asymptotically free in the ultraviolet. The problem could then be avoided only when there are other minima for finite $S$. Alternatively one could consider a situation which is not connected to infinitely weak coupling (e.g. a theory that not asymptotically free). But such a situation we have excluded in this lectures from the beginning.

One attempt to avoid this problem with additional minima at finite $S$ was the consideration of several gaugino condensates, but it still seems very difficult to produce satisfactory potentials that lead to a dynamical determination of the dilaton for reasonable values of $<S>$. In absence of a completely satisfactory model it is then also difficult to investigate the detailed phenomenological properties of the approach. Here it would be of interest to know the actual size of the vevs of the auxiliary fields $<F_S>$, $<F_T>$ and $<F_U>$. In the models discussed so far one usually finds $<F_T>$ to be the dominant term, but it still remains a question whether this is true in general.

4 Fixing the dilaton

This is a very general problem that is unsolved at the moment. In the following we give a speculation of how the problem could be solved within the assumptions made above. It is not the only attempt to solve the problem, but it might point out some aspects of the problem that open a new way to look at it in the framework of the recent developments in string theory.

It seems that we need some new ingredient before we can understand the mechanism completely. The resolution of all these problems might come with a better understanding of the form of the gauge kinetic function $f[\Phi]$. In all the previous considerations one assumed $f = S$. How general is this relation? Certainly we know that in one loop perturbation theory $S$ mixes with $T$, but
this is not relevant for our discussion and, for simplicity, we shall ignore that
for the moment. The formal relation between \( f \) and the condensate is given
through \( \Lambda^3 \approx e^{-f} \) and we have \( f = S \) in the weak coupling limit of string
theory. In fact this argument only implies that

\[
\lim_{S \to \infty} f(S) = S. \tag{12}
\]

Nonperturbative effects could lead to the situation that \( f \) is a very complicated
function of \( S \). In fact a satisfactory incorporation of gaugino condensates in
the framework of string theory might very well lead to such a complication.
In \( \S \) we suggested that a nontrivial \( f \)-function is the key ingredient to better
understand the mechanism of gaugino condensation. We still assume \( \S \) to make contact with perturbation theory. How do we then control \( e^{-f} \) as a
function of \( S \)? In absence of a determination of \( f(S) \) by a direct calculation one
might use symmetry arguments to make some progress. Let us here consider
the presence of a symmetry called \( S \)-duality which in its simplest form is given
by a \( SL(2,Z) \) generated by the transformations

\[
S \to S + i, \quad S \to -1/S. \tag{13}
\]

Such a symmetry might be realized in two basically distinct ways: the
gauge sector could close under the transformation (type I) or being mapped
to an additional ‘magnetic sector’ with inverted coupling constant (type II).
In the second case one would speak of strong-weak coupling duality, just as in
the case of electric-magnetic duality \( \S \). Within the class of theories of type I,
however, we could have the situation that the \( f \)-function is itself invariant \( \S \)
under \( S \)-duality; i.e. \( S \to -1/S \) does not invert the coupling constant since the
gauge coupling constant is not given by Re\( S \) but \( 1/g^2 \approx \text{Re} f \). In view of \( \S \) we would call such a symmetry weak-weak coupling duality. The behaviour
of the gauge coupling constant as a function of \( S \) is shown in Fig. 1. Our
assumption \( \S \) implies that \( g^2 \to 0 \) as \( \text{Re} S \to \infty \) and by \( S \)-duality \( g^2 \) also
vanishes for \( S \to 0 \), with a maximum somewhere in the vicinity of the self-dual
point \( S = 1 \). Observe that \( S \approx 1 \) in this situation does not necessarily imply
strong coupling, because \( g^2 \approx 1/\text{Re} f \) and even for \( S \approx 1 \), \( \text{Re} f \) could be large
and \( g^2 \ll \ll 1 \), with perturbation theory valid in the whole range of \( S \). Of
course, nonperturbative effects are responsible for the actual form of \( f(S) \).

\[\S\]
More complicated choices of transformation properties for \( f \) are possible and lead to
similar results as obtained in our simple toy model.
Fig. 1 - Coupling constant $g^2$ as the function of $S$ in type-I models (dashed) vs $g^2$ given by $f = S$

To examine the behaviour of the scalar potential in this approach, let us consider a simple toy model, with chiral superfields $U = Y^3 = (\lambda \lambda, \ldots)$ as well as $S$ and $T$. We have to choose a specific example of a gauge kinetic function which is invariant under the $S$-duality transformations. Different choices are possible, the simplest is given by

$$f = \frac{1}{2\pi} \ln(j(S) - 744),$$

(14)

$j(S)$ being the usual generator of modular invariant functions. This function behaves like $S$ in the large $S$-limit. If we assume a type I-model where the gauge sector is closed under $S$-duality, then we also have to assume that the gaugino condensate does not transform under $S$-duality (because of the $f W^a \bar{W}_a$-term in the Lagrangian)\footnote{1}. Under these conditions an obvious candidate for the superpotential is just the standard Veneziano-Yankielowicz superpotential (extended to take into account the usual $T$-duality, which we assume to be completely independent from $S$-duality)\footnote{2}.

\footnote{1}For type I-models it was shown in \footnote{2} that one can always redefine the gauge kinetic function and condensate in such a way that this holds.
\[ W = Y^3 (f + 3b \ln \frac{Y \eta^2(T)}{\mu} + c). \] (15)

This is clearly invariant under $S$-duality. Therefore we then cannot take the conventional form for the Kähler potential which would be given by

\[ K = -\ln(S + \bar{S}) - 3 \ln(T + \bar{T} - Y \bar{Y}), \] (16)

since it is not $S$-dual. To make it $S$-dual one could introduce an additional $\ln |\eta(S)|^4$ term, giving e.g.

\[ K = -\ln(S + \bar{S}) - 3 \ln(T + \bar{T} - Y \bar{Y}) - \ln |\eta(S)|^4. \] (17)

Because the only relevant quantity is

\[ G = K + \ln |W|^2, \] (18)

we can as well put this new term (which is forced upon us because of our demand for symmetry) into the superpotential and take the canonical Kähler function instead, which gives

\[ K = -\ln(S + \bar{S}) - 3 \ln(T + \bar{T} - Y \bar{Y}), \] (19)

\[ W = \frac{Y^3}{\eta^2(S)} (f + 3b \ln \frac{Y \eta^2(T)}{\mu} + c), \] (20)

where the remarkable similarity to the effective potential for $T$-dual gaugino condensation $W = W_{inv}/\eta^6(T)$ can be seen more clearly.

This model exhibits a well defined minimum at $<S> = 1$, $<T> = 1.23$ and $<Y> \approx \mu$. Supersymmetry is broken with the dominant contribution being $<F_T> \approx \mu^3$. The cosmological constant is negative.

In contrast to earlier attempts this model fixes the problem of the runaway dilaton and breaks supersymmetry with only a single gaugino condensate. Previous models needed multiple gaugino condensates and (to get realistic vevs for the dilaton) matter fields in complicated representations. We feel that the concept of a nontrivial gauge kinetic function derived (or constrained) by a symmetry is a much more natural way to fix the dilaton and break supersymmetry, especially so because corrections to $f = S$ are expected in any case.

Earlier models which included $S$-duality in different ways (both with and without gaugino condensates) were able to fix the vev of the dilaton but did not succeed in breaking supersymmetry. An alternative mechanism to fix the vev of the dilaton has been discussed in.
Of course there are still some open questions not solved by this approach. The first is the problem of having a vanishing cosmological constant. Whereas early models of gaugino condensation often introduced \textit{ad hoc} terms to guarantee a vanishing vacuum energy, it has been seen to be notoriously difficult to get this out of models based on string inspired supergravity. The only way out of this problem so far has been to introduce a constant term into the superpotential, parameterizing unknown effects. This approach does not even work in any arbitrary model, but at least in our model the cosmological constant can be made to vanish by adjusting such a constant.

Another question not addressed in this toy model is the mixing of $S$ and $T$ fields which happens at the one-loop level. It is still unknown whether one can keep two independent dualities in this case. In a consistent interpretation our toy model should describe an all-loop effective action. If it is considered to be a theory at the tree-level then the theory is not anomaly free. Introducing terms to cancel the anomaly which arises because of demanding $S$-duality will then destroy $S$-duality. At tree-level the theory therefore cannot be made anomaly free.

An additional interesting question concerns the vevs of the auxiliary fields, i.e. which field is responsible for supersymmetry breakdown. In all models considered so far (multiple gaugino condensates, additional matter, $S$-duality) it has always been $F_T$ which dominates all the other auxiliary fields. It has not been shown yet that this is indeed a generic feature. The question is an important one, since the hierarchy of the vevs of the auxiliary fields is mirrored in the structure of the soft SUSY breaking terms of the MSSM$^{11}$. We want to argue that there is at least no evidence for $F_T$ being generically large in comparison to $F_S$, because all of the models constructed so far (including our toy model) are designed in such a way that $\langle F_S \rangle = 0$ by construction at the minimum (at least at tree-level for the other models). In fact, if one extends our model with a constant in the superpotential (see above), then $\langle F_S \rangle$ increases with the constant (but does not become as large as $\langle F_T \rangle$).

Of course there are still some assumptions we made by considering this toy model. We assumed that there is weak coupling in the large $S$ limit. This is an assumption because the nonperturbative effects are unknown (at tree-level it can be calculated that $f = S$). In addition it is clear that the standard form we take for the Kähler potential does not include nonperturbative effects and thus could be valid only in the weak coupling approximation (this is of course related to our choice of the superpotential). Of course, an equally valid assumption would be that nonperturbative effects destroy the calculable tree-level behaviour even in the weak coupling region. The model of ref. $^{21}$ could be re-interpreted in that sense (they do not consider gaugino condensates and
the gauge kinetic function, but their $S$-dual scalar potential goes to infinity for $S \to \infty$). We choose not to make this assumption, because it is equivalent to the statement that the whole perturbative framework developed so far in string theory is wrong.

Again it should be emphasized here that the $S$-duality considered is not a strong-weak coupling duality but a weak-weak coupling duality. Even if the present model does not seem fully satisfactory, we are convinced that the idea of weak-weak coupling duality might be of more general relevance. In type II-models one has a duality between strong and weak coupling\(^6\). At the moment it is not completely clear how to incorporate that in a realistic model.

5 $S = 1$ and weak coupling

A problem could be the actual size of the gauge coupling constant. If $f = S$ and $<S> = 1$ then the large value of the gauge coupling constant does not fit the low scale of gaugino condensation necessary for phenomenologically realistic supersymmetry breaking ($10^{13}\,\text{GeV}$). However if $f = S$ only in the weak coupling limit then one can have $<f> >> 1$ and thus $g^2 << 1$ even in the region $S = O(1)$. Therefore in our model $<S> = 1$ is consistent with the demand for a small gauge coupling constant, whereas in models with $f = S$ a much larger (and therefore more unnatural) $<S>$ is needed. Of course, it still has to be understood how such a large value of $f(S = 1)$ can appear.

To summarize we find that the choice of a nontrivial $f$-function (motivated by a symmetry requirement) gives rise to a theory where supersymmetry breaking is achieved by employing only a single gaugino condensate. The cosmological constant turns out to be negative, but can be adjusted by a simple additional constant in the superpotential. The vevs of all fields are at natural orders of magnitude and due to the nontrivial gauge kinetic function the gauge coupling constant can be made small enough to give a realistic picture.

Turning our attention to the observable sector we see that a small (grand unified) coupling constant is a necessity and the above mechanism is required for a satisfactory description of the size of the observed coupling constants like e.g. $\alpha_{\text{QCD}}$. But this alone might not be sufficient for a realistic model. String theory should predict all low energy coupling constants correctly and should also give the correct ratio of electroweak and strong coupling constants.

LEP and SLC high precision electroweak data give for the minimal supersymmetric Standard Model (MSSM) with the lightest Higgs mass in the range $60\,\text{GeV} < M_H < 150\,\text{GeV}$
\[
\sin^2 \hat{\theta}_W(M_Z) = 0.2316 \pm 0.0003
\]
\[
\alpha_{em}(M_Z)^{-1} = 127.9 \pm 0.1
\]
\[
\alpha_S(M_Z) = 0.12 \pm 0.01
\]
\[
m_t = 160^{+11+6}_{-12-5}\text{GeV},
\]
for the central value \(M_H = M_Z\) in the \(\overline{MS}\) scheme. This is in perfect agreement with the recent CDF/D0 measurements of \(m_t\). Taking the first three values as input parameters leads to gauge coupling unification at \(M_{\text{GUT}} \sim 2 \cdot 10^{16}\text{GeV with } \alpha_{\text{GUT}} \sim \frac{1}{26}\) and \(M_{\text{SUSY}} \sim 1\text{TeV}\). Slight modifications arise from light SUSY thresholds, i.e. the splitting of the sparticle mass spectrum, the variation of the mass of the second Higgs doublet and two–loop effects. Whereas these effects are rather mild, huge corrections may arise from heavy thresholds due to mass splittings at the high scale \(M_{\text{heavy}} \neq M_{\text{GUT}}\) arising from the infinite many massive string states. In the following sections we shall discuss this question of string unification in detail.

6 Gauge coupling unification

In heterotic superstring theories all couplings are related to the universal string coupling constant \(g_{\text{string}}\) at the string scale \(M_{\text{string}} \sim 1/\sqrt{\alpha'}\), with \(\alpha'\) being the inverse string tension. It is a free parameter which is fixed by the dilaton vacuum expectation value \(g_{\text{string}}^2 = \frac{S + S}{2}\). In general this amounts to string unification, i.e. at the string scale \(M_{\text{string}}\) all gauge and Yukawa couplings are proportional to the string coupling and are therefore related to each other. For the gauge couplings (denoted by \(g_a\)) we have

\[
g_a^2 k_a = g_{\text{string}}^2 = \frac{\kappa^2}{2\alpha'}.
\]

Here, \(k_a\) is the Kac–Moody level of the group factor labeled by \(a\). The string coupling \(g_{\text{string}}\) is related to the gravitational coupling constant \(\kappa^2\). In particular this means that string theory itself provides gauge coupling and Yukawa coupling unification even in absence of a grand unified gauge group.

To make contact with the observable world one should construct the field theoretical low–energy limit of a string vacuum. This is achieved by integrating out all the massive string modes corresponding to excited string states as well as states with momentum or winding quantum numbers in the internal dimensions. The resulting theory then describes the physics of the massless string excitations at low energies \(\mu < M_{\text{string}}\) in field–theoretical terms. If one wants to state anything about higher energy scales one has to take into account
threshold corrections $\Delta_a(M_{\text{string}})$ to the bare couplings $g_a(M_{\text{string}})$ due to the infinite tower of massive string modes. They change the relations (22) to:

$$g_a^{-2} = k_a g_{\text{string}}^{-2} + \frac{1}{16\pi^2} \Delta_a ,$$  \hspace{1cm} (23)

The corrections in (23) may spoil the string tree–level result (22) and split the one–loop gauge couplings at $M_{\text{string}}$. This splitting could allow for an effective unification at a scale $M_{\text{GUT}} < M_{\text{string}}$ or destroy the unification.

The general expression of $\Delta_a$ for heterotic tachyon–free string vacua is given in 52. Various contributions to $\Delta_a$ have been determined for several classes of models: First in 29 for two $\mathbb{Z}_3$ orbifold models with a (2,2) world–sheet supersymmetry. This has been extended to fermionic constructions in 3. Threshold corrections for (0,2) orbifold models with quantized Wilson lines have been calculated in 49. Threshold corrections for the quintic threefold and other Calabi–Yau manifolds with gauge group $E_6 \times E_8$ can be found in 9, 54. In toroidal orbifold compactifications moduli dependent threshold corrections arise only from N=2 supersymmetric sectors. They have been determined for some orbifold compactifications in 19 and for more general orbifolds in 72. The full moduli dependence of threshold corrections for (0,2) orbifold compactifications with continuous Wilson lines has been derived in 73, 74. These models contain continuous background gauge fields in addition to the usual moduli fields. In most of the cases these models are (0,2) compactifications. In all the above orbifold examples the threshold corrections $\Delta_a$ can be decomposed into three parts:

$$\Delta_a = \tilde{\Delta}_a - b_a^{N=2} \Delta + k_a Y .$$  \hspace{1cm} (24)

Here the gauge group dependent part is divided into two pieces: The moduli independent part $\tilde{\Delta}_a$ containing the contribution of the N=1 supersymmetric sectors as well as scheme dependent parts which are proportional to $\tilde{b}_a$. This prefactor $\tilde{b}_a$ is related to the one–loop $\beta$–function: $\beta_a = b_a g_a^2 / 16\pi^2$. Furthermore the moduli dependent part $b_a^{N=2}$ with $b_a^{N=2}$ being related to the anomaly coefficient $\delta_a$ by $b_a^{N=2} = \delta_a - k_a \delta_{\text{GS}}$. The gauge group independent part $Y$ contains the gravitational back–reaction to the background gauge fields as well as other universal parts. They are absorbed into the definition of $g_{\text{string}}$: $g_{\text{string}}^{-2} = \frac{s+5}{2} + \frac{1}{16\pi^2} Y$. The scheme dependent parts are the IR–regulators for both field– and string theory as well as the UV–regulator for field theory. The latter is put into the definition of $M_{\text{string}}$ in the $\overline{\text{DR}}$ scheme.

\footnote{A lowest expansion result in the Wilson line modulus has been obtained in 9.}
\[
M_{\text{string}} = 2 \frac{e^{(1-\gamma_E)/2}3^{-3/4}}{\sqrt{2\pi \alpha'}} = 0.527 \ g_{\text{string}} \times 10^{18} \text{ GeV}.
\] (25)

The constant of the string IR–regulator as well as the universal part due to gravity were recently determined in [34].

The identities (23) are the key to extract any string–implication for low–energy physics. They serve as boundary conditions for our running field–theoretical couplings valid below \(M_{\text{string}}\). Therefore they are the foundation of any discussion about both low–energy predictions and gauge coupling unification. The evolution equations \(*\) valid below \(M_{\text{string}}\)

\[
\frac{1}{g^2_a(\mu)} = \frac{k_a}{g^2_{\text{string}}} + \frac{b_a}{16\pi^2} \ln \frac{M^2_{\text{string}}}{\mu^2} - \frac{1}{16\pi^2} b^N_{\text{string}} \triangle,
\] (26)

allow us to determine \(\sin^2 \theta_W\) and \(\alpha_S\) at \(M_Z\). After eliminating \(g_{\text{string}}\) in the second and third equations one obtains

\[
\sin^2 \theta_W(M_Z) = \frac{k_2}{k_1 + k_2} - \frac{k_1}{k_1 + k_2} \frac{\alpha_{\text{em}}(M_Z)}{4\pi} \left[ A \ln \left( \frac{M^2_{\text{string}}}{M^2_Z} \right) - A' \triangle \right],
\]

\[
\alpha_S^{-1}(M_Z) = \frac{k_3}{k_1 + k_2} \left[ \frac{\alpha_{\text{em}}^{-1}(M_Z)}{4\pi} + \frac{1}{4\pi} B \ln \left( \frac{M^2_{\text{string}}}{M^2_Z} \right) + \frac{1}{4\pi} B' \triangle \right],
\] (27)

with \(A = \frac{k_2}{k_1} b_1 - b_2, B = b_1 + b_2 - \frac{k_1 + k_2}{k_3} b_3\) and \(A', B'\) are obtained by exchanging \(b_1 \rightarrow b'_1\). For the MSSM one has \(A = \frac{28}{5}, B = 20\). However to arrive at the predictions of the MSSM (21) one needs huge string threshold corrections \(\triangle\) due to the large value of \(M_{\text{string}}\) \((3/5k_1 = k_2 = k_3 = 1)\):

\[
\triangle = \frac{A}{A'} \left[ \ln \left( \frac{M^2_{\text{string}}}{M^2_{GUT}} \right) + \frac{32\pi \delta_{\sin^2 \theta_W}}{5\alpha_{\text{em}}(M_Z)} \right].
\] (28)

At the same time, the N=2 spectrum of the underlying theory encoded in \(A', B'\) which enters the threshold corrections has to fulfill the condition

\[
\frac{B'}{A'} = \frac{B}{A} \frac{\ln \left( \frac{M^2_{\text{string}}}{M^2_{GUT}} \right) + \frac{32\pi \delta_{\sin^2 \theta_W}}{36\alpha_{\text{em}}(M_Z)} \alpha_S^{-1}}{\ln \left( \frac{M^2_{\text{string}}}{M^2_{GUT}} \right) + \frac{32\pi \delta_{\sin^2 \theta_W}}{36\alpha_{\text{em}}(M_Z)} \alpha_S^{-1}},
\] (29)

\* We neglect the N=1 part of \(\triangle_a\) which is small compared to \(b^N_{\text{string}} \triangle\).
where $\delta$ represents the experimental uncertainties appearing in (21). In addition $\delta$ may also contain SUSY thresholds.

For concreteness and as an illustration let us take the $\mathbb{Z}_8$ orbifold example of (21) with $A' = -2, B' = -6$ and $b_1' + b_2' = -10$. It is one of the few orbifolds left over after imposing the conditions on target–space duality anomaly cancellation (46). To estimate the size of $\Delta$ one may take in eq. (25) $g_{\text{string}} \sim 0.7$ corresponding to $\alpha_{\text{string}} \sim \frac{1}{26}$, i.e. $M_{\text{string}}/M_{\text{GUT}} \sim 20$. Of course this is a rough estimate since $M_{\text{string}}$ is determined by the first eq. of (21) together with (22). Nevertheless, the qualitative picture does not change. Therefore to predict the correct low–energy parameter (27) eq. (28) tells us that one needs threshold correction of considerable size:

$$-17.1 \leq \Delta \leq -16.3.$$  

(30)

7 String thresholds

The construction of a realistic unified string model boils down to the question of how to achieve thresholds of that size. To settle the question we need explicit calculations within the given candidate string model. There we can encounter various types of threshold effects. Some depend continuously, others discretely on the values of the moduli fields. For historic reasons we also have to distinguish between thresholds that do or do not depend on Wilson lines. The reason is the fact that the calculations in the latter models are considerably simpler and for some time were the only available results. They were then used to estimate the thresholds in models with gauge group $SU(3) \times SU(2) \times U(1)$ and three families, although as a string model no such orbifold can be constructed without Wilson lines. Therefore, the really relevant thresholds are, of course, the ones found in the (0,2) orbifold models with Wilson lines (73) which may both break the gauge group and reduce its rank. We will discuss the various contributions within the framework of our illustrative model. However the discussion can easily applied for all other orbifolds. The threshold corrections depend on the $T$ and $U$ modulus describing the size and shape of the internal torus lattice. In addition they may depend on non–trivial gauge background fields encoded in the Wilson line modulus $B$.

Moduli dependent threshold corrections $\Delta$ can be of significant size for an appropriate choice of the vevs of the background fields $T, U, B, \ldots$ which enter these functions. Of course in the decompactification limit $T \to i\infty$ these corrections become always arbitrarily huge. This is in contrast to fermionic string compactifications or $N=1$ sectors of heterotic superstring compactifications. There one can argue that moduli–independent threshold corrections

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cannot become huge at all [4]. This is in precise agreement with the results found earlier in [2, 3]. In field theory threshold corrections can be estimated with the formula [5, 6, 7, 8]:

\[ \Delta = \sum_{n,m,k} \ln \left( \frac{M_{n,m,k}^2}{M_{\text{string}}^2} \right) , \]  

with \( n, m \) being the winding and momentum, respectively and \( k \) the gauge quantum number of all particles running in the loop. The string mass in the \( N = 2 \) sector of the \( \mathbb{Z}_8 \) model we consider later with a non-trivial gauge background in the internal directions is determined by [9, 10, 11, 12]:

\[ \alpha' M_{n,m,k}^2 = 4|p_R|^2 \]

\[ p_R = \frac{1}{\sqrt{Y}} \left[ \frac{T}{2\alpha'} U - B^2 \right] n_2 + \frac{T}{2\alpha'} n_1 - U m_1 + m_2 + B k_2 \]

\[ Y = -\frac{1}{2\alpha'} (T - T^*) (U - U^*) + (B - B^*)^2 . \]  

(32)

In addition a physical state \( |n,m,k,l\rangle \) has to obey the modular invariance condition

\[ m_1 n_1 + m_2 n_2 + k_1^2 - k_1 k_2 + k_2^2 - k_2 k_3 - k_3 k_4 + k_4^2 + k_5^2 = 1 - N_L - \frac{1}{4} k_5^2 . \]

Therefore the sum in (31) should be restricted to these states. This also guarantees its convergence after a proper regularization. In (31) cancellations between the contributions of various string states may arise. E.g. at the critical point \( T = i = U \) where all masses appear in integers of \( M_{\text{string}} \) such cancellations occur. They are the reason for the smallness of the corrections calculated in [2, 3] and in all the fermionic models [4]. Let us investigate this in more detail. The simplest case \( (B = 0) \) for moduli dependent threshold corrections to the gauge couplings was derived in [5]:

\[ \Delta(T, U) = \ln \left[ \frac{-iT + iT^*}{2\alpha'} |\eta \left( \frac{T}{2\alpha'} \right)|^4 \right] + \ln \left[ (-iU + iU^*) |\eta(U)|^4 \right] . \]  

(33)

Formula (33) can be used for any toroidal orbifold compactifications, where the two-dimensional subplane of the internal lattice which is responsible for the \( N=2 \) structure factorize from the remaining part of the lattice. If the latter condition does not hold, (33) is generalized [4].
Table 1: Lowest mass $M^2$ of particles charged under $G_A$ and threshold corrections $\Delta(T,U)$.

| $T/2\alpha'$ | $U$ | $M^2\alpha'$ | $\ln(M^2\alpha')$ | $\Delta^{II}$ |
|--------------|-----|--------------|------------------|--------------|
| $I_a$        | $i$ | $i$          | $1$              | $-0.72$      |
| $I_b$        | $1.25i$ | $i$ | $\frac{4}{3}$ | $-0.22$ | $-0.76$ |
| $I_c$        | $4.5i$ | $4.5i$ | $\frac{4}{37}$ | $-3.01$ | $-5.03$ |
| $I_d$        | $18.7i$ | $i$ | $\frac{10}{187}$ | $-2.93$ | $-16.3$ |

In Table 1 we determine the mass of the lowest massive string state being charged under the considered unbroken gauge group $G_A$ and the threshold corrections $\Delta(T,U)$ for some values of $T$ and $U$.

The influence of moduli dependent threshold corrections to low–energy physics [entailed in eqs. (27)] has until now only been discussed for orbifold compactifications without Wilson lines by using (33). In these cases the corrections only depend on the two moduli $T,U$. However to obtain corrections of the size $\Delta \sim -16.3$ one would need the vevs $\frac{T}{2\alpha'} = 18.7, U = i$ which are far away from the self–dual points. It remains an open question whether and how such big vevs of $T$ can be obtained in a natural way in string theory.

A generalization of eq. (33) appears when turning on non–vanishing gauge background fields $B \neq 0$. According to (32) the mass of the heavy string states now becomes $B$–dependent and therefore also the threshold corrections change. This kind of corrections were recently determined in (34). The general expression there is

$$\Delta^{II}(T,U,B) = \frac{1}{12} \ln \left[ \frac{Y^{12}}{1728^8} |C_{12}(\Omega)|^2 \right] ,$$

where $B$ is the Wilson line modulus, $\Omega = \left( \frac{T}{2\alpha'} \frac{B}{-B U} \right)$ and $C_{12}$ is a combination of $g = 2$ elliptic theta functions explained in detail in (34). It applies to gauge groups $G_A$ which are not affected by the Wilson line mechanism. The case where the gauge group is broken by the Wilson line will be discussed later (those threshold corrections will be singular in the limit of vanishing $B$). Whereas the effect of quantized Wilson lines $B$ on threshold corrections has already been discussed in (32) the function $\Delta^{II}(T,U,B)$ now allows us to study the effect of a continuous variation in $B$. 

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We see in Fig. 2 that the threshold corrections change very little with the Wilson line modulus $B$. They are comparable with $\Delta = -0.72$ corresponding to the case of $B = 0$. In this case eq. (34) becomes eq. (33) for $T_{2\alpha} = i = U$.

So far all these calculations have been done within models where the considered gauge group $G_A$ is not broken by the Wilson line and its matter representations are not projected out. To arrive at SM like gauge groups with the matter content of the MSSM one has to break the considered gauge group with a Wilson line.

From the phenomenological point of view, the most promising class of string vacua is provided by (0,2) compactifications equipped with a non-trivial gauge background in the internal space which breaks the $E_6$ gauge group down to a SM–like gauge group. Since the internal space is not simply connected, these gauge fields cannot be gauged away and may break the gauge group. Some of the problems present in (2,2) compactifications with $E_6$ as a grand unified group like e.g. the doublet–triplet splitting problem, the fine–tuning problem and Yukawa coupling unification may be absent in (0,2) compactifications. It is important that these properties can be studied in the full string theory, not just in the field theoretic limit. The background gauge fields give rise to a new class of massless moduli fields again denoted by $B$ which have quite different low–energy implications than the usual moduli.
arising from the geometry of the internal manifold itself. In this framework the question of string unification can now be discussed for realistic string models. The threshold corrections for our illustrative model take the form

\[ \Delta^I(T, U, B) = \frac{1}{10} \ln \left[ \left( \frac{1}{128} \prod_{k=1}^{10} \theta_k(\Omega) \right)^4 \right], \]  \tag{35} \]

where \( \theta_k \) are the ten even \( g = 2 \) theta–functions. Equipped with this result we can now investigate the influence of the B–modulus on the thresholds and see how the conclusions of ref. 47 might be modified. The results for a representative set of background vevs is displayed in Fig.3.

From this picture we see that threshold corrections of \( \Delta \sim -16.3 \) can be obtained for the choice of \( \frac{T}{\alpha'} \sim 4.5i \sim U \) and \( B = \frac{1}{2} \). This has to be compared to the model in ref. 48 where such a value was achieved with \( T = 18.7i \) and \( B = 0 \). This turns out to be a general property of the models under consideration. With more moduli, sizeable threshold effects are achieved even with moderate values of the vevs of the background fields.
8 Heterotic string unification

Equipped with these explicit calculations of string threshold corrections we can now ask the question how string theory might lead to the correct prediction of gauge coupling constants. We also hope to deduce information on the spectrum of theories that lead to successful gauge coupling unification.

The modulus plays the role of an adjoint Higgs field which breaks e.g. the $G_A = E_6$ down to a SM like gauge group $G_a$. According to eq. (82) the vev of this field gives some particles masses between zero and $M_{\text{string}}$. This is known as the stringy Higgs effect. Such additional intermediate fields may be very important to generate high scale thresholds. Sizeable threshold corrections $\Delta$ can only appear if some particles have masses different from the string scale $M_{\text{string}}$ and where cancellations between different states as mentioned above do not take place. In particular some gauge bosons of $G_A$ become massive receiving the mass:

$$\alpha' M_I^2 = \frac{4}{\alpha'} |B|^2 .$$  \hspace{1cm} \text{(36)}

As before let us investigate the masses of the lightest massive particles charged under the gauge group $G_a$. For our concrete model we have $M_{\text{string}} = 3.6 \cdot 10^{17}$GeV.

| $T/2\alpha'$ | $U$ | $B$ | $M_I$ [GeV] | $\ln(M_I^2\alpha')$ | $\Delta^I$ |
|-----------------|--------|--------|----------------|----------------------|----------------|
| $IIa$ | $i$ | $i$ | $\frac{1}{10}$ | $8.4 \cdot 10^{12}$ | $-23.0$ | $-10.03$ |
| $IIb$ | $i$ | $i$ | $\frac{1}{2}$ | $4.2 \cdot 10^{17}$ | $-1.39$ | $-1.72$ |
| $IIc$ | $1.25i$ | $i$ | $\frac{1}{2}$ | $3.7 \cdot 10^{17}$ | $-1.61$ | $-2.12$ |
| $IId$ | $4i$ | $i$ | $\frac{1}{2}$ | $2.1 \cdot 10^{17}$ | $-2.78$ | $-7.86$ |
| $IIe$ | $4.5i$ | $4.5i$ | $\frac{1}{2}$ | $9.3 \cdot 10^{16}$ | $-4.39$ | $-16.3$ |
| $IIf$ | $18.7i$ | $i$ | $\frac{1}{2}$ | $1.1 \cdot 10^{16}$ | $-4.31$ | $-43.3$ |

Table 2: Lowest mass $M_I$ of particles charged under $G_a$ and threshold corrections $\Delta^I$ for $B \neq 0$.

Whereas $\Delta^{II}$ describes threshold corrections w.r.t. to a gauge group which is not broken when turning on a vev of $B$, now the gauge group is broken for
$B \neq 0$ and in particular this means that the threshold $\Delta^I$ shows a logarithmic singularity for $B \to 0$ when the full gauge symmetry is restored. This behaviour is known from field theory and the effects of the heavy string states can be decoupled from the former: Then the part of $\Delta^I_a$ in (23) which is only due to the massive particles becomes

$$b_A - b_a \ln \frac{M_{\text{string}}^2}{|B|^2} - \frac{b'_A}{16\pi^2} \ln \left| \eta \left( \frac{T}{2\alpha'} \right) \eta(U) \right|^4,$$

where the first part accounts for the new particles appearing at the intermediate scale of $M_I$ and the other part takes into account the contributions of the heavy string states. One of the questions of string unification concerns the size of this intermediate scale $M_I$. In a standard grand unified model one would be tempted to identify $M_I$ with $M_{\text{GUT}}$. While this would also be a possibility for string unification, we have in string theory in addition the possibility to consider $M_I > M_{\text{GUT}}$. The question remains whether the thresholds in that case can be big enough, as we shall discuss in a moment. Let us first discuss the general consequences of our results for the idea of string unification without a grand unified gauge group. Due to the specific form of the threshold corrections in eq. (23) unification always takes place if the condition $AB' = A'B$ is met within the errors arising from the uncertainties in (21). It guarantees that all three gauge couplings meet at a single point $M_X$.

$$M_X = M_{\text{string}} e^{\frac{1}{2} A'} \Delta.$$

For our concrete model this leads to $M_X \sim 2 \cdot 10^{16}\text{GeV}$. Given these results we can now study the relation between $M_I$ and $M_X$, which plays the rôle of the GUT–scale in string unified models. As a concrete example, consider the model $IIe$ in Table 2. It leads to an intermediate scale $M_I$ which is a factor 3.9 smaller than the string scale, thus $\sim 10^{17}\text{GeV}$, although the apparent unification scale is as low as $2 \times 10^{16}\text{GeV}$. We thus have an explicit example of a string model where all the non–MSSM particles are above $9.3 \cdot 10^{16}\text{GeV}$, but still a correct prediction of the low energy parameters emerges. **Thus string unification can be achieved without the introduction of a small intermediate scale.**

Of course, there are also other possibilities which lead to the correct low–energy predictions. Instead of large threshold corrections one could consider a non–standard hypercharge normalization, i.e. a $k_1 \neq 5/3$. This would maintain gauge coupling unification at the string scale with the correct values of $\sin^2 \theta_W(M_Z)$ and $\alpha_S(M_Z)$. However, it is very hard to construct such models. A further possibility would be to give up the idea of gauge coupling unification
within the MSSM by introducing extra massless particles such as \((3, 2)\) w.r.t. \(SU(3) \times SU(2)\) in addition to those of the SM\(^{24}\). A careful choice of these matter fields may lead to sizable additional intermediate threshold corrections in \((27)\) thus allowing for the correct low–energy data \((21)\). Unfortunately the price for that is exactly an introduction of a new intermediate scale of \(M_I \sim 10^{12–14}\)GeV. It seems to be hard to explain such a small scale naturally in the framework of string theory. In some sense such a model can be compared to the model \(IIa\) in table 2. Other possible corrections to \((27)\) may arise from an extended gauge structure between \(M_X\) and \(M_{\text{string}}\). However this might even enhance the disagreement with the experiment \(^{25}\). Finally a modification of \((27)\) appears from the scheme conversion from the string– or SUSY–based \(\overline{DR}\) scheme to the \(\overline{MS}\) scheme relevant for the low–energy physics data \((21)\). However these effects are shown to be small \(^{24}\).

Let us stress here the important message that string unification can, in principle, be achieved with moduli dependent threshold corrections within (0,2) superstring compactification. The Wilson line dependence of these functions is comparable to that on the \(T\) and \(U\) fields thus offering the interesting possibility of large thresholds with background configurations of moderate size. All non–MSSM like states can e.g. be heavier than \(1/4\) of the string scale, still leading to an apparent unification scale of \(M_X = \frac{1}{27} M_{\text{string}}\). We do not need vevs of the moduli fields that are of the order 20 away from the natural scale, neither do we need to introduce particles at a new intermediate scale that is small compared to \(M_{\text{string}}\).

The situation could be even more improved with a higher number of moduli fields entering the threshold corrections: They may come from other orbifold planes giving rise to \(N=2\) sectors or from additional Wilson lines. We think that the actual moderate vevs of the underlying moduli fields can be fixed by non–perturbative effects as e.g. gaugino condensation. Of course, unification can be achieved in different ways, as the introduction of an intermediate scale. This does not seem to be very natural, because it postulates a new scale that is a factor \(10^4\) smaller than the GUT scale in order to explain a factor 20 discrepancy in the difference of the unification scale and the string scale. One might also argue that in the framework of string theory one should consider models with a grand unified group unbroken at the string scale but broken at the GUT scale. This might lead to interesting models and consequences, but it does not contribute to an explanation of the difference of the string scale and the GUT scale.

An alternative view of unification might arise according to recent developments in nonperturbative string theory. We shall discuss that in the following sections.
9 Recent developments: M-Theory

From all the new and interesting results in string dualities, it is the heterotic M–theory of Hoˇ rava and Witten that seems to have immediate impact on the discussion of the phenomenological aspects of these theories. One of the results concerns the question of the unification of all fundamental coupling constants and the second one the properties of the soft terms (especially the gaugino masses) once supersymmetry is broken. In both cases results that appeared problematic in the weakly coupled case get modified in a satisfactory way, while the overall qualitative picture remains essentially unchanged. In these lectures we shall therefore concentrate on these aspects of the new picture.

The heterotic M–theory is an 11–dimensional theory with the gauge fields living on two 10–dimensional boundaries (walls), respectively, while the gravitational fields can propagate in the bulk as well. A $d = 4$ dimensional theory with $N = 1$ supersymmetry emerges at low energies when 6 dimensions are compactified on a Calabi–Yau manifold. The scales of that theory are $M_{11}$, the $d = 11$ Planck scale, $R_{11}$ the size of the $x^{11}$ interval, and $V \sim R^6$ the volume of the Calabi–Yau manifold. The quantities of interest in $d = 4$, the Planck mass, the GUT–scale and the unified gauge coupling constant $\alpha_{\text{GUT}}$ should be determined through these higher dimensional quantities. The fit of ref. identifies $M_{\text{GUT}} \sim 3 \cdot 10^{16}$ GeV with the inverse Calabi–Yau radius $R^{-1}$. Adjusting $\alpha_{\text{GUT}} = 1/25$ gives $M_{11}$ to be a few times larger than $M_{\text{GUT}}$. On the other hand, the fit of the actual value of the Planck scale can be achieved by the choice of $R_{11}$ and, interestingly enough, $R_{11}$ turns out to be an order of magnitude larger than the fundamental length scale $M_{11}^{-1}$. A satisfactory fit of the $d = 4$ scales is thus possible, in contrast to the case of the weakly coupled heterotic string, where naively the string scale seemed to be a factor 20 larger than $M_{\text{GUT}}$.

As we have seen before, otherwise the heterotic $E_8 \times E_8$ string looks rather attractive from the point of view of phenomenological applications. One seems to be able to accommodate the correct gauge group and particle spectrum. The mechanism of hidden sector gaugino condensation leads to a breakdown of supersymmetry with vanishing cosmological constant to leading order. With a condensate scale $\Lambda \sim 10^{13}$ GeV, one obtains a gravitino mass in the TeV range and soft scalar masses in that range as well. In the simplest models this type of supersymmetry breakdown is characterized through the vacuum expectation value of moduli fields other than the dilaton, giving a small problem with the soft gaugino masses in the observable sector: they turn out to be too small, generically some two orders of magnitude smaller than the soft scalar masses. It is again in the framework of heterotic M–theory
that this problem is solved\textsuperscript{42}. gaugino masses are of the same size as (or even larger than) the soft scalar masses.

The mechanism of hidden sector gaugino condensation itself can be realized in a way very similar to the weakly coupled case. This includes the mechanism of cancellation of the vacuum energy, which in the weakly coupled case arises because of a cancellation of the gaugino condensate with a vacuum expectation value of the three index tensor field $H$ of $d = 10$ supergravity. This cancellation is at the origin of the fact that supersymmetry breakdown is dominated by a $T$ modulus field rather than the dilaton ($S$). Hořava\textsuperscript{43} observed that this compensation of the vacuum expectation values of the condensate and $H$ carries over to the M–theory case. In\textsuperscript{86} this has been explicitly worked out for the mechanism of gaugino condensation in the heterotic M–theory and the similarity to the weakly coupled case was shown. Now the gaugino condensate forms at the hidden 4–dimensional wall and is cancelled locally at that wall by the vacuum expectation value (vev) of a Chern–Simons term. This also clarifies some questions concerning the nature of the vev of $H$ that arose in the weakly coupled case.

In the remainder of these lectures we want to discuss the phenomenological properties of the heterotic M–theory. This includes a presentation of the full effective four–dimensional $N = 1$ supergravity action in leading and next–to–leading order, the mechanism of hidden sector gaugino condensation and its explicit consequences for supersymmetry breaking and the scalar potential and finally the resulting soft breaking terms in the 4–dimensional theory. Although some of the issues have already been discussed earlier, we shall at each step first explain the situation again for the weakly coupled theory and then compare it to the results obtained in the M–theory case.

These results are obtained using the method of reduction and truncation that has been successfully applied to the weakly coupled case\textsuperscript{100, 24, 82}. It is a simplified prescription that shows the main qualitative features of the effective $d = 4$ effective theory. In orbifold compactification it would represent the fields and interactions in the untwisted sector. We compute Kähler potential ($K$), superpotential ($W$) and gauge kinetic function ($f$) both in the weakly and strongly coupled regime and explain similarities and differences.

The results in leading order had been obtained previously\textsuperscript{6, 66, 32, 31}. These papers mainly focused on the breakdown of supersymmetry via a Scherk–Schwarz mechanism which we shall not discuss here in detail. It remains to be seen, if and how such a mechanism can be related to the mechanism of gaugino condensation.

The remainder of the lectures will proceed as follows. First we discuss the scales and the question of unification as suggested in\textsuperscript{101} and compare
the two cases. Then we derive the effective $d = 4$ action of M–theory using the method of reduction and truncation. In this case we have to deal with a nontrivial obstruction first encountered in [10]. It leads to an explicit $x^{11}$ dependence of certain fields, which is induced by vevs of antisymmetric tensor fields at the walls. To obtain the effective action in $d = 4$ we have to integrate out this dependence. This then leads to corrections to $K$ and $f$ in next to leading order, which are very similar compared to those in the weakly coupled case. We also discuss the appearance and the size of a critical radius for $R_{11}$. The phenomenological fit presented in our discussion of unification implies that we are not too far from that critical radius. We then turn again to the question of supersymmetry breakdown. We start with the weakly coupled case and investigate the nature of the vev of the $H$–field (concerning some quantization conditions) and the cancellation of the vacuum energy. In the strongly coupled case we shall see that such a cancellation appears locally at one wall. This supports the interpretation that the gaugino condensate is matched by a nontrivial vev of a Chern–Simons term. We then explicitly identify the mechanism of supersymmetry breakdown and the nature of the gravitino. The goldstino turns out to be the fermionic component of the $T$ superfield that represents essentially the radius of the 11th dimension. It is a bulk field, with a vev of its auxiliary component on one wall. Integrating out the 11th dimension we then obtain explicitly the mass of the gravitino.

The remainder deals with the induced soft breaking terms in the observable sector: scalar and gaugino masses. We shall see a strong model dependence of the scalar masses and argue that they are not too different from the gravitino mass. This all is very similar to the situation in the weakly coupled case. We then compute the soft gaugino masses and see that in the strongly coupled case they are of the order of the gravitino mass. This comes from the fact that we are quite close to the critical radius and represents a decisive difference to the weakly coupled regime.

10 Scales and unification

As we have seen, models of particle physics that are derived as the low energy limit of the $E_8 \times E_8$ heterotic string are particularly attractive. They seem to be able to accommodate the correct gauge group and particle spectrum to lead to the supersymmetric extension to the $SU(3) \times SU(2) \times U(1)$ standard model. It is exactly in this framework that a unification of the gauge coupling constants is expected to appear at a scale $M_{GUT} = 3 \cdot 10^{16}$ GeV. As we know, this heterotic string theory (weakly coupled at the string scale) gives a prediction for the relation between gauge and gravitational coupling constants.
To see this explicitly let us have a look at the low energy effective action of the $d = 10$–dimensional field theory:

$$L = -\frac{4}{(\alpha')^3} \int d^{10}x \sqrt{g} \exp(-2\phi) \exp\left(\frac{1}{(\alpha')^3} R + \frac{1}{4} \text{tr} F^2 + \ldots\right),$$

(39)

where $\alpha'$ is the string tension and $\phi$ the dilaton field in $d = 10$. A definite relation between gauge and gravitational coupling appears because of the universal behaviour of the dilaton term in eq. (39). The effective $d = 4$–dimensional theory is obtained after compactification on a Calabi–Yau manifold with volume $V$:

$$L = -\frac{4}{(\alpha')^3} \int d^{4}x \sqrt{g} \exp(-2\phi) V \left(\frac{1}{(\alpha')^3} R + \frac{1}{4} \text{tr} F^2 + \ldots\right).$$

(40)

Thus a universal factor $V \exp(-2\phi)$ multiplies both the $R$ and $F^2$ terms. Newton’s and Einstein’s gravitational coupling constants are related as

$$G_N = \frac{1}{8\pi \kappa_4^2} = \frac{1}{M_{\text{Planck}}^2},$$

(41)

with $M_{\text{Planck}} \approx 1.2 \cdot 10^{19}$ GeV. From eq. (40) we then deduce:

$$G_N = \frac{\exp(2\phi)(\alpha')^4}{64\pi V},$$

(42)

as well as

$$\alpha_{\text{GUT}} = \frac{\exp(2\phi)(\alpha')^3}{16\pi V},$$

(43)

leading to the relation

$$G_N = \frac{\alpha_{\text{GUT}} \alpha'}{4}.$$  

(44)

Putting in the value for $M_{\text{Planck}}$ and $\alpha_{\text{GUT}} \approx 1/25$ one obtains a value for the string scale $M_{\text{string}} = (\alpha')^{-1/2}$ that is in the region of $10^{18}$ GeV. This is apparently much larger than the GUT–scale of $3 \cdot 10^{16}$ GeV, while naively one would like to identify $M_{\text{string}}$ with $M_{\text{GUT}}$. The discrepancy of the scales is sometimes called the unification problem in the framework of the weakly coupled heterotic string. We have discussed it in the previous sections. There we have seen that the above argumentation is rather simple minded and that more sophisticated (threshold) calculations are needed to settle this issue. In any case, the natural appearance of $M_{\text{string}} \sim M_{\text{GUT}}$ would have been desirable. Let us now see how the situation looks in the case of heterotic string theory at stronger coupling.
We now consider the $E_8 \times E_8$ M–theory. The effective action of the strongly coupled $E_8 \times E_8$ – M–theory in the “downstairs” approach is given by (we take into account the numerical corrections found in [3])

\[
L = \frac{1}{\kappa^2} \int_{M_{11}} d^{11}x \sqrt{g} \left[ -\frac{1}{2} R - \frac{1}{2} \bar{\psi}_I \Gamma^{IJK} D_J \left( \frac{\Omega + \hat{\Omega}}{2} \right) \psi_K - \frac{1}{48} G_{IJKL} G^{IJKL} \\
- \frac{\sqrt{2}}{384} (\bar{\psi}_I \Gamma^{IJKLMN} \psi_N + 12 \bar{\psi}_J \Gamma^{KL} \psi^M) \left( G_{JKLM} + \hat{G}_{JKLM} \right) \\
- \frac{\sqrt{2}}{3456} \epsilon_{I_1 I_2 \ldots I_{11}} C_{I_1 I_2 I_3} G_{I_4 \ldots I_{11}} \right]
\]

(45)

where $M_{11}$ is the “downstairs” manifold while $M_i^{10}$ are its 10–dimensional boundaries. In the lowest approximation $M_{11}$ is just a product $M_4 \times X^6 \times S^1/Z_2$. Compactifying to $d = 4$ in such an approximation we obtain [4]

\[
G_N = \frac{\kappa^2}{8\pi} = \frac{\kappa^2}{8\pi R_{11} V},
\]

(46)

with $V$ the volume of the Calabi–Yau manifold $X^6$ and $R_{11} = \pi \rho$ the $S^1/Z_2$ length.

The fundamental mass scale of the 11–dimensional theory is given by $M_{11} = \kappa^{-2/9}$. Let us see which value of $M_{11}$ is favoured in a phenomenological application. For that purpose we identify the Calabi–Yau volume $V$ with the GUT–scale: $V \sim (M_{GUT})^{-6}$. From (47) and the value of $\alpha_{GUT} = 1/25$ at the grand unified scale, we can then deduce the value of $M_{11}$

\[
V^{1/6} M_{11} = (4\pi)^{1/9} \alpha_{GUT}^{-1/6} \approx 2.3,
\]

(48)

to be a few times larger than the GUT–scale. In a next step we can now adjust the gravitational coupling constant by choosing the appropriate value of $R_{11}$ using (10). This leads to

\[
R_{11} M_{11} = \left( \frac{M_{\text{Planck}}}{M_{11}} \right)^2 \frac{\alpha_{GUT}}{8\pi (4\pi)^{2/3}} \approx 2.9 \cdot 10^{-4} \left( \frac{M_{\text{Planck}}}{M_{11}} \right)^2.
\]

(49)
This simple analysis tells us the following:

- In contrast to the weakly coupled case (where we had a prediction \(M_{\text{Planck}}\)), the correct value of \(M_{\text{Planck}}\) can be fitted by adjusting the value of \(R_{11}\).

- The numerical value of \(R_{11}^{-1}\) turns out to be approximately an order of magnitude smaller than \(M_{11}\).

- Thus the 11th dimension appears to be larger than the dimensions compactified on the Calabi–Yau manifold, and at an intermediate stage the world appears 5-dimensional with two 4-dimensional boundaries (walls).

We thus have the following picture of the evolution and unification of coupling constants. At low energies the world is 4-dimensional and the couplings evolve accordingly with energy: a logarithmic variation of gauge coupling constants and the usual power law behaviour for the gravitational coupling. Around \(R_{11}^{-1}\) we have an additional 5th dimension and the power law evolution of the gravitational interactions changes. Gauge couplings are not affected at that scale since the gauge fields live on the walls and do not feel the existence of the 5th dimension. Finally at \(M_{\text{GUT}}\) the theory becomes 11-dimensional and both gravitational and gauge couplings show a power law behaviour and meet at the scale \(M_{11}\), the fundamental scale of the theory. It is obvious that the correct choice of \(R_{11}\) is needed to achieve unification. We also see that, although the theory is weakly coupled at \(M_{\text{GUT}}\), this is no longer true at \(M_{11}\). The naive estimate for the evolution of the gauge coupling constants between \(M_{\text{GUT}}\) and \(M_{11}\) goes with the sixth power of the scale. At \(M_{11}\) we thus expect unification of the couplings at \(\alpha \sim O(1)\). In that sense, the \(M\)-theoretic description of the heterotic description gives an interpolation between weak coupling and moderate coupling. In \(d = 4\) this is not strong–weak coupling duality in the usual sense. We shall later come back to these questions when we discuss the appearance of a critical limit on the size of \(R_{11}\).

These are, of course, rather qualitative results. In order to get a more quantitative feeling for the range of \(M_{11}\) and \(R_{11}\), let us be a bit more specific and write the relation of the unification scale \(M_{\text{GUT}}\) to the characteristic size of the Calabi–Yau space as:

\[ V^{1/6} = aM_{\text{GUT}}^{-1} . \]  

The above formula corresponds to the situation in which we identify the unification scale with the radius, \(R\), of \(X^6\) which volume is given by \(V = (aR)^6\). We expect the parameter \(a\) to be somewhere in the range from 1 to \(2\pi\). Using
the above identification and the value of $M_{\text{GUT}} = 3 \cdot 10^{16} \text{ GeV}$ we obtain:

$$M_{11} \approx \frac{2.3}{a} M_{\text{GUT}}. \quad (51)$$

As said before, the scale $M_{11}$ occurs to be of the order of the unification scale $M_{\text{GUT}}$. However, we do not expect $M_{11}$ to be smaller than $M_{\text{GUT}}$ because we need the ordinary logarithmic evolution of the gauge coupling constants up to $M_{\text{GUT}}$. In fact, $M_{11}$ should be somewhat bigger in order to allow for the evolution of $a$ from its unification value $1/25$ to the strong regime. Thus, we expect the parameter $a$ to be quite close to 1. Putting the above value of $M_{11}$ into eq. (49) we get the length of $S^1/Z_2$:

$$R_{11} \approx 9.2 a^2 M_{11}^{-1} \approx 4 a^3 M_{\text{GUT}}^{-1}. \quad (52)$$

It is about one order of magnitude bigger than the scale characteristic for the 11–dimensional theory. This is the reason for the relatively large value of the $d = 4$ Planck Mass. Of course $R_{11}$ can not be too large. For $a$ between 1 and 2.3 (values corresponding to $M_{11} > M_{\text{GUT}}$) we obtain $R_{11}^{-1}$ in the range $(6.2 \cdot 10^{14} - 7.4 \cdot 10^{15}) \text{ GeV}$ (as we discussed, the parameter $a$ should not be too different from 1 so the upper part of the above range is favoured). Smaller values of $R_{11}^{-1}$ seem to be very unnatural. Trying to push $R_{11}^{-1}$ to smaller values would need a redefinition of $M_{11}$. For that purpose in [49] a definition $m_{11} = 2 \pi (4 \pi k^2)^{-1/9}$ was used. This allows then to push $a$ to the extreme limit of $2 \pi$. With these extreme choices of both $a$ and $m_{11}$ one would then be able to obtain $R_{11}^{-1}$ as small as $3 \cdot 10^{13} \text{ GeV}$. Values smaller than that (like values of $10^{12} \text{ GeV}$ as sometimes quoted in the literature) cannot be obtained. In any case, even values in the lower $10^{13} \text{ GeV}$ range seem to be in conflict with the critical value of $R_{11}$, as we shall see later.

11 The effective action in $d = 4$

We now want to work out more explicitly the effective action in $d = 4$ as obtained using the method of reduction and truncation.

We shall first consider again the $d = 10$ effective field theory for the heterotic string (in more detail as given previously):

$$L = -\frac{4}{(\alpha')^3} \int d^{10}x \sqrt{g} \exp(-2 \phi) \left( \frac{1}{(\alpha')} R + \frac{1}{4} \text{tr} F^2 + \frac{1}{12} \alpha' H^2 + \ldots \right), \quad (53)$$

where we have included the three index tensor field strength

$$H = dB + \omega^YM - \omega^L. \quad (54)$$
$B$ is the two–index antisymmetric tensor while

$$
\omega^Y = \text{Tr}(AF - \frac{2}{3}A^3)
$$

(55)

and

$$
\omega^L = \text{Tr}(\omega R - \frac{2}{3}\omega^3)
$$

(56)

are the Yang–Mills and Lorentz–Chern–Simons terms, respectively. The addition of these terms in the definition of $H$ is needed for supersymmetry and anomaly freedom of the theory.

To obtain the effective theory in $d = 4$ dimensions we use as an approximation the method of reduction and truncation explained in ref.\cite{100}. It essentially corresponds to a torus compactification, while truncating states to arrive at a $d = 4$ theory with $N = 1$ supersymmetry. In string theory compactified on an orbifold this would describe the dynamics of the untwisted sector. We retain the usual moduli fields $S$ and $T$ as well as matter fields $C_i$ that transform nontrivially under the observable sector gauge group. In this approximation, the Kähler potential is given by

$$
G = -\log(S + S^*) - 3\log(T + T^* - 2C_i^*C_i) + \log|W|^2
$$

(57)

with superpotential originating from the Chern–Simons terms $\omega^Y$

$$
W(C) = d_{ijk}C_iC_jC_k
$$

(58)

and the gauge kinetic function is given by the dilaton field

$$
f = S.
$$

(59)

For a detailed discussion of this method and the explicit definition of the fields see the review\cite{82}. These expressions for the $d = 4$ effective action look quite simple and it remains to be seen whether this simplicity is true in general or whether it is an artifact of the approximation. Our experience with supergravity models tells us that the holomorphic functions $W$ and $f$ might be protected by nonrenormalization theorems, while the Kähler potential is strongly modified in perturbation theory. In addition we have to be aware of the fact that the expressions given above are at best representing a subsector of the theory. In orbifold compactification this would be the untwisted sector, and we know that the Kähler potential for twisted sectors fields will look quite different. Nonetheless the used approximation turned out to be useful for the discussion of those aspects of the theory that determine the dynamics of the $T$– and $S$– moduli. When trying to extract, however, detailed masses and other properties
of the fields one should be aware of the fact, that some results might not be true in general and only appear as a result of the simplicity of the approximation.

So far the classical action. What about loop corrections? Not much can be said about the details of the corrections to the Kähler potential. This has to be discussed on a model by model basis. The situation with the superpotential is quite easy. There we expect a nonrenormalization theorem to be at work. The inclusion of other sectors of the theory will lead to new terms in the superpotential that in general have $T$–dependent coefficients. Such terms can be computed in simple cases by using e.g. methods of conformal field theory.

The situation for $f$, the gauge–kinetic function is more interesting. Symmetries and holomorphicity lead us to believe, that although there are non-trivial corrections at one–loop, no more perturbative corrections are allowed at higher orders. The existence of such corrections at one loop seems to be intimately connected to the mechanism of anomaly cancellation in the $d = 10$ theory. To see this consider one of the anomaly cancellation counter–terms introduced by Green and Schwarz:

$$\epsilon^{NEGAWSKLOV} B_{V} \text{Tr} F^2_{\text{LKS}W} F^2_{\text{AGEN}}.$$ (60)

We are interested in a $d = 4$ theory with $N = 1$ supersymmetry, and thus expect nontrivial vacuum expectation values for the curvature terms $\text{Tr} R^2$ and field strengths $\text{Tr} F^2$ in the extra six dimensions. Consistency of the theory requires a condition for the 3–index tensor field strength. For $H$ to be well defined, the quantity

$$dH = \text{Tr} F^2 - \text{Tr} R^2$$ (61)

has to vanish cohomologically. In the simplest case (the so–called standard embedding leading to gauge group $E_6 \times E_8$) one chooses equality pointwise $\text{Tr} R^2 = \text{Tr} F^2$. Let us now assume that $\text{Tr} F^2_{\text{agen}}$ is nonzero. The Green–Schwarz term given above by eq. (60) then leads to

$$\epsilon^{mn} B_{mn} \epsilon^{\mu\nu\sigma\tau} \text{Tr} F_{\mu\nu} F_{\rho\sigma}$$ (62)

in the four–dimensional theory. An explicit inspection of the fields tells us that $\epsilon^{mn} B_{mn}$ is the pseudoscalar axion that belongs to the $T$–superfield. Upon supersymmetrization the term in eq. (62) will then correspond to a one–loop correction to the holomorphic $f$–function that is proportional to $T$ with the coefficient fixed entirely by the anomaly considerations. This is, of course, nothing else than a threshold correction. In the simple case of the standard embedding with gauge group $E_6 \times E_8$ one obtains e.g.

$$f_6 = S + \epsilon T ; \quad f_8 = S - \epsilon T .$$ (63)
respectively, where $\epsilon$ is the constant fixed by the anomaly. These results can be backed up by explicit calculations in string theory. In cases where such an explicit calculation is feasible, many more details about these corrections can be deduced. The above result (63) obtained in $d = 10$ field theory represents an approximation of the exact result in the large $T$–limit. For a detailed discussion of these calculations and the limiting procedure see $^1$. We have here mainly concentrated on that limit, because it represents a rather model independent statement.

Thus we have seen that there are corrections to the gauge–kinetic function at one loop. Their existence is found to be intimately related to the mechanism of anomaly cancellation. The corrections found are exactly those that are expected by general symmetry considerations $^8$. In (63) we have given the result for the standard embedding. Coefficients might vary for more general cases, but the fact that they have opposite sign for the two separate groups is true in all known cases.

Superpotential and $f$–function should not receive further perturbative corrections beyond one loop. This implies that the knowledge of $f$ at one loop represents the full perturbative result. Combined with the fact that the coefficients are fixed by anomaly considerations one would then expect that this result for the $f$–function might be valid even beyond the weakly coupled limit. Not much can be said about the Kähler potential beyond one loop.

We now turn to the calculation in the M-theory case $^8$. In the strongly coupled case we have to perform a compactification from $d = 11$ to $d = 4$. Again we use the method of reduction and truncation. For the metric we write

$$g_{MN}^{(11)} = \begin{pmatrix} c_4 e^{-\gamma} e^{-2\sigma} g_{\mu\nu} & e^\sigma g_{mn} e^{2\gamma} e^{-2\sigma} \\ e^{-\gamma} e^{-2\sigma} g_{\mu\nu} & e^{-2\gamma} e^{-2\sigma} \end{pmatrix}$$

with $M, N = 1 \ldots 11; \mu, \nu = 1 \ldots 4; m, n = 5 \ldots 10$ and $\det(g_{mn})=1$. This is the frame in which the 11–dimensional Einstein action gives the ordinary Einstein action after the reduction $d = 4$:

$$-\frac{1}{2\kappa^2} \int d^{11}x \sqrt{g^{(11)}} R^{(11)} = -\frac{c_4 V_7}{2\kappa^2} \int d^4x \sqrt{g} R + \ldots$$

where $V_7 = \int d^7x$ is the coordinate volume of the compact 7–manifold and the scaling factor $c_4$ describes our freedom to choose the units in $d = 4$. The most popular choice in the literature is $c_4 = 1$. This however corresponds to the unphysical situation in which the 4–dimensional Planck mass is determined by the choice of $V_7$ which is just a convention. With $c_4 = 1$ one needs further
rescaling of the 4–dimensional metric. We instead prefer the choice

\[ c_4 = \frac{V_7}{\hat{V}_7} \]  

(66)

where \( V_7 = \int d^7 x \sqrt{g^{(7)}} \) is the physical volume of the compact 7–manifold. This way we recover eq. (46) in which the 4–dimensional Planck mass depends on the physical, and not coordinate, volume of the manifold on which we compactify. As a result, if we start from the product of the 4–dimensional Minkowski space and some 7–dimensional compact space (in the leading order of the expansion in \( \kappa^{2/3} \)) as a ground state in \( d = 11 \) we obtain the Minkowski space with the standard normalization as the vacuum in \( d = 4 \).

To find a more explicit formula for \( c_4 \) we have to discuss the fields \( \sigma \) and \( \gamma \) in some detail. In the leading approximation \( \sigma \) is the overall modulus of the Calabi–Yau 6–manifold. We can divide it into a sum of the vacuum expectation value, \( \langle \sigma \rangle \), and the fluctuation \( \tilde{\sigma} \). In general both parts could depend on all 11 coordinates but in practice we have to impose some restrictions. The vacuum expectation value can not depend on \( x^\mu \) if the 4–dimensional theory is to be Lorentz–invariant. In the fluctuations we drop the dependence on the compact coordinates corresponding to the higher Kaluza–Klein modes. Furthermore, we know that in the leading approximation \( \langle \sigma \rangle \) is just a constant, \( \sigma_0 \), while corrections depending on the internal coordinates, \( \sigma_1 \), are of the next order in \( \kappa^{2/3} \). Thus, we obtain

\[ \sigma(x^\mu, x^m, x^{11}) = \langle \sigma \rangle(x^m, x^{11}) + \tilde{\sigma}(x^\mu) = \sigma_0 + \sigma_1(x^m, x^{11}) + \tilde{\sigma}(x^\mu). \]  

(67)

To make the above decomposition unique we define \( \sigma_0 \) by requiring that the integral of \( \sigma_1 \) over the internal space vanishes. The analogous decomposition can be also done for \( \gamma \). With the above definitions the physical volume of the compact space is

\[ V_7 = \int d^7 x \langle e^{2\sigma} e^{\gamma} \rangle = e^{2\sigma_0} e^{\gamma_0} \hat{V}_7 \]  

(68)

up to corrections of order \( \kappa^{4/3} \). Thus, the parameter \( c_4 \) can be written as

\[ c_4 = e^{2\sigma_0} e^{\gamma_0}. \]  

(69)

The choice of the coordinate volumes is just a convention. For example in the case of the Calabi–Yau 6–manifold only the product \( e^{3\sigma} \hat{V}_6 \) has physical meaning. For definiteness we will use the convention that the coordinate volumes are equal 1 in \( M_{11} \) units. Thus, \( \langle e^{3\sigma} \rangle \) describes the Calabi–Yau volume in these units. Using eqs. (48, 49) we obtain \( e^{3\sigma_0} = VM_{11}^6 \approx (2.3)^6 \), \( e^{\gamma_0} e^{-\sigma_0} = R_{11} M_{11} \approx 9.2a^2 \). The parameter \( c_4 \) is equal to the square of the 4–dimensional Planck mass in these units and numerically \( c_4 \approx (35a)^2 \).
At the classical level we compactify on \( M^4 \times X^6 \times S^1/\mathbb{Z}_2 \). This means that the vacuum expectation values \( \langle \sigma \rangle \) and \( \langle \gamma \rangle \) are just constants and eq. (67) reduces to

\[
\sigma = \sigma_0 + \tilde{\sigma}(x^\mu), \quad \gamma = \gamma_0 + \tilde{\gamma}(x^\mu).
\]

(70)

In such a situation \( \sigma \) and \( \gamma \) are 4–dimensional fields. We introduce two other 4–dimensional fields by the relations

\[
\frac{1}{4!} e^{6\sigma} G_{11\lambda\mu\nu} = \epsilon_{\lambda\mu\nu\rho} (\partial^\rho D),
\]

(71)

\[
C_{11\bar{a}b} = C_{11} \delta_{\bar{a}b}
\]

(72)

where \( x^a \ (x^{\bar{b}}) \) is the holomorphic (antiholomorphic) coordinate of the Calabi–Yau manifold. Now we can define the dilaton and the modulus fields by

\[
S = \frac{1}{(4\pi)^{2/3}} \left( e^{3\sigma} + i24\sqrt{2}D \right),
\]

(73)

\[
T = \frac{1}{(4\pi)^{2/3}} \left( e^\gamma + i6\sqrt{2}C_{11} + C^*_i C_i \right)
\]

(74)

where the observable sector matter fields \( C_i \) originate from the gauge fields \( A_M \) on the 10–dimensional observable wall (and \( M \) is an index in the compactified six dimensions). The Kähler potential takes its standard form as in eq. (57)

\[
K = -\log(S + S^*) - 3\log(T + T^* - 2C^*_i C_i).
\]

(75)

The imaginary part of \( S \) (Im\( S \)) corresponds to the model independent axion, and with the above normalization the gauge kinetic function is \( f = S \). We have also

\[
W(C) = d_{ijk} C_i C_j C_k
\]

(76)

Thus the action to leading order is very similar to the weakly coupled case.

Before drawing any conclusion from the formulae obtained above we have to discuss a possible obstruction at the next to leading order. For the 3–index tensor field \( H \) in \( d = 10 \) supergravity to be well defined one has to satisfy \( dH = \text{tr}F_2^2 + \text{tr}F_2^2 - \text{tr}R^2 = 0 \) cohomologically. In the simplest case of the standard embedding one assumes \( \text{tr}F_2^2 = \text{tr}R^2 \) locally and the gauge group is broken to \( E_6 \times E_8 \). Since in the M–theory case the two different gauge groups live on the two different boundaries (walls) of space–time such a cancellation point by point is no longer possible. We expect nontrivial vacuum expectation values (vevs) of

\[
(dG) \propto \sum_i \delta(x^{11} - x_i^{11}) \left( \text{tr}F_i^2 - \frac{1}{2} \text{tr}R^2 \right)
\]

(77)
at least on one boundary ($x^{11}$ is the position of $i$–th boundary). In the case of the standard embedding we would have $\text{tr}F_2^2 = 1/2 \text{tr}R^2 = 1/4 \text{tr}R^2$ on one and $\text{tr}F_2^2 = 1/2 \text{tr}R^2 = -1/2 \text{tr}R^2$ on the other boundary. This might pose a severe problem since a nontrivial vev of $G$ might be in conflict with supersymmetry ($G_{11ABC} = H_{ABC}$). The supersymmetry transformation law in $d = 11$ reads

$$
\delta \psi_M = D_M \eta + \frac{1}{288} G_{IJKL} (\Gamma_M^{IJKL} - 8 \delta^I_M \Gamma^{JKL} ) \eta + \ldots
$$

(78)

Supersymmetry will be broken unless e.g. the derivative term $D_M \eta$ compensates the nontrivial vev of $G$. Witten has shown\[^{101}\] that such a cancellation can occur and constructed the solution in the linearized approximation (linear in the expansion parameter $\kappa^{2/3}$). This solution requires some modification of the metric on $M^{11}$:

$$
g^{(11)}_{MN} = \begin{pmatrix}
(1 + b) \eta_{\mu \nu} & (g_{ij} + h_{ij}) \\
(g_{ij} + h_{ij}) & (1 + \gamma')
\end{pmatrix}
$$

(79)

$M^{11}$ is no longer a direct product $M^4 \times X^6 \times S^1/Z_2$ because $b$, $h_{ij}$ and $\gamma'$ depend now on the compactified coordinates. The volume of $X^6$ depends on $x^{11}$:

$$
\frac{\partial}{\partial x^{11}} V = - \frac{\sqrt{2}}{8} \sqrt{g} \omega^{AB} \omega^{CD} G_{ABCD}
$$

(80)

where the integral is over the Calabi–Yau manifold $X^6$ and $\omega$ is the corresponding Kähler form. The parameter $(1 + b)$ is the scale factor of the Minkowski 4–manifold and depends on $x^{11}$ in the following way

$$
\frac{\partial}{\partial x^{11}} b = \frac{1}{2} \frac{\partial}{\partial x^{11}} \log v_4 = \frac{\sqrt{2}}{24} \omega^{AB} \omega^{CD} G_{ABCD}
$$

(81)

where $v_4$ is the physical volume for some fixed coordinate volume in $M^4$. In our simple reduction and truncation method with the metric $g^{(11)}_{MN}$ given by eq. (64) we can reproduce the $x^{11}$ dependence of $V$ and $v_4$. The volume of $X^6$ is determined by $\sigma$:

$$
\frac{\partial}{\partial x^{11}} \log V = \frac{\partial}{\partial x^{11}} (3 \langle \sigma \rangle) = 3 \frac{\partial}{\partial x^{11}} \sigma
$$

(82)

while the scale factor of $M^4$ can be similarly expressed in terms of $\sigma$ and $\gamma$ fields:

$$
\frac{\partial}{\partial x^{11}} \log v_4 = - \frac{\partial}{\partial x^{11}} (2 \langle \gamma \rangle + 4 \langle \sigma \rangle) = - \frac{\partial}{\partial x^{11}} (2 \gamma + 4 \sigma)
$$

(83)
Substituting ⟨σ⟩ with σ in the above two equations is allowed because, due to our decomposition (67), only the vev of σ depends on the internal coordinates (the same is true for γ). The scale factor b calculated in ref. 101 depends also on the Calabi–Yau coordinates. Such a dependence can not be reproduced in our simple reduction and truncation compactification so we have to average eq. (81) over X6. Using equations (80–83) after such an averaging we obtain (to leading order in the expansion parameter κ2/3)

\[ \frac{\partial \gamma}{\partial x^{11}} = \frac{\partial \sigma}{\partial x^{11}} = \frac{\sqrt{2}}{24} \int d^6x \sqrt{|g|} \omega^{AB} \omega^{CD} G_{ABCD} \int d^6x \sqrt{|g|}. \tag{84} \]

Substituting the vacuum expectation value of G found in 102 we can rewrite it in the form

\[ \frac{\partial \gamma}{\partial x^{11}} = \frac{\partial \sigma}{\partial x^{11}} = \frac{2}{3} \alpha \kappa^{2/3} V^{-2/3} \tag{85} \]

where

\[ \alpha = \frac{\pi c}{2(4\pi)^{2/3}} \tag{86} \]

and c is a constant of order unity given for the standard embedding of the spin connection by

\[ c = V^{-1/3} \left| \int \omega \wedge \text{tr}(R \wedge R) \right| \frac{8\pi^2}{8}. \tag{87} \]

Our calculations, as those of Witten, are valid only in the leading nontrivial order in the κ2/3 expansion. The expression (83) for the derivatives of σ and γ have explicit factor κ2/3. This means that we should take the lowest order value for the Calabi–Yau volume in that expression. An analogous procedure has been used in obtaining all formulae presented in this paper. We always expand in κ2/3 and drop all terms which are of higher order than our approximation. Taking the above into account and using our units in which M11 = 1 we can rewrite eq. (85) in the simple form:

\[ \frac{\partial \gamma}{\partial x^{11}} = \frac{\partial \sigma}{\partial x^{11}} = \frac{2}{3} \alpha e^{-2\sigma_0}. \tag{88} \]

Eqs. (84–88) have been derived in ref. 86. As we will see in the following, these results contain all the information to deduce the effective action, i.e. Kähler potential, superpotential and gauge kinetic function of the 4–dimensional effective supergravity theory.

It is the above dependence of σ and γ on x11 that leads to these consequences. One has to be careful in defining the fields in d = 4. It is obvious, that the 4–dimensional fields S and T can not be any longer defined by eqs.
because now \( \sigma \) and \( \gamma \) are 5-dimensional fields. We have to integrate out the dependence on the 11th coordinate. In the present approximation, this procedure is quite simple: we have to replace \( \sigma \) and \( \gamma \) in the definitions of \( S \) and \( T \) with their averages over the \( S^1/Z_2 \) interval. With the linear dependence of \( \sigma \) and \( \gamma \) on \( x^{11} \) their average values coincide with the values taken at the middle of the \( S^1/Z_2 \) interval

\[
\bar{\sigma} = \sigma \left( \frac{\pi \rho}{2} \right) = \sigma_0 + \tilde{\sigma}(x^\mu),
\]

(89)

\[
\bar{\gamma} = \gamma \left( \frac{\pi \rho}{2} \right) = \gamma_0 + \tilde{\gamma}(x^\mu).
\]

(90)

When we reduce the boundary part of the Lagrangian of M–theory to 4 dimensions we find exponents of \( \sigma \) and \( \gamma \) fields evaluated at the boundaries. Using eqs. (67) and (88) we get

\[
e^{-\gamma} \big|_{M^{10}} = e^{-\gamma_0} \pm \frac{1}{3} \alpha e^{-3\sigma_0},
\]

(91)

\[
e^{3\sigma} \big|_{M^{10}} = e^{3\sigma_0} \pm \alpha e^{\gamma_0}.
\]

(92)

The above formulae have very important consequences for the definitions of the Kähler potential and the gauge kinetic functions. For example, the coefficient in front of the \( D_\mu C_i^* D^\mu C_i \) kinetic term is proportional to \( e^{-\gamma} \) evaluated at the \( E_6 \) wall where the matter fields propagate. At the lowest order this was just \( e^{-\gamma_0} \) or \( \langle T \rangle^{-1} \) up to some numerical factor. From eq. (91) we see that at the next to leading order also \( \langle S \rangle^{-1} \) is involved with relative coefficient \( \alpha/3 \). Taking such corrections into account we find that at this order the Kähler potential is given by

\[
K = -\log(S + S^*) + \frac{2\alpha C_i^* C_i}{S + S^*} - 3\log(T + T^* - 2C_i^* C_i)
\]

(93)

with \( S \) and \( T \) now defined by

\[
S = \frac{1}{(4\pi)^{2/3}} \left( e^{3\sigma} + i24\sqrt{2}D + \alpha C_i^* C_i \right),
\]

(94)

\[
T = \frac{1}{(4\pi)^{2/3}} \left( e^{\tilde{\gamma}} + i6\sqrt{2}C_{11} + C_i^* C_i \right)
\]

(95)

where bars denote averaging over the 11th dimension. It might be of some interest to note that the combination \( \langle S \rangle \langle T \rangle^3 \) is independent of \( x^{11} \) even before this averaging procedure took place. The solution above is valid only for terms
at most linear in $\alpha$. Keeping this in mind we could write the Kähler potential also in the form

$$K = -\log(S + S^* - 2\alpha C_i^* C_i) - 3\log(T + T^* - 2C_i^* C_i). \quad (96)$$

Equipped with this definition the calculation of the gauge kinetic function(s) from eqs. (88, 92) becomes a trivial exercise. In the five–dimensional theory $f$ depends on the 11–dimensional coordinate as well, thus the gauge kinetic function takes different values at the two walls. The averaging procedure allows us to deduce these functions directly. For the simple case at hand (the so–called standard embedding) eq. (92) gives

$$f_6 = S + \alpha T; \quad f_8 = S - \alpha T. \quad (97)$$

It is a special property of the standard embedding that the coefficients are equal and opposite. The coefficients vary for more general cases. This completes the discussion of the $d = 4$ effective action in next to leading order, noting that the superpotential does not receive corrections at this level.

The nontrivial dependence of $\sigma$ and $\gamma$ on $x^{11}$ can also enter definitions and/or interactions of other 4–dimensional fields. Let us next consider the gravitino. After all we have to show that this field is massless to give the final proof that the given solution respects supersymmetry. Its 11–dimensional kinetic term

$$-\frac{1}{2}\sqrt{g}\bar{\psi}_I \Gamma^{IJK} D_J \psi_K \quad (98)$$

remains diagonal after compactification to $d = 4$ if we define the 4–dimensional gravitino, $\psi_\mu^{(4)}$, and dilatino, $\psi_{11}^{(4)}$, fields by the relations

$$\psi_\mu = e^{-(\sigma - \sigma_0)/2}e^{-(\gamma - \gamma_0)/4} \left( \psi_\mu^{(4)} + \frac{1}{\sqrt{6}} \Gamma_\mu \psi_{11}^{(4)} \right), \quad (99)$$

$$\psi_{11} = -\frac{2}{\sqrt{6}} e^{-(\sigma - \sigma_0)/2}e^{-(\gamma - \gamma_0)/4} \Gamma_{11} \psi_{11}^{(4)}. \quad (100)$$

The $d = 11$ kinetic term $(98)$ gives after the compactification also a mass term for the $d = 4$ gravitino of the form

$$\frac{3}{8} e^{\sigma_0} e^{-\gamma_0} \frac{\partial \gamma}{\partial x^{11}} = \frac{\sqrt{2}}{64} e^{\sigma_0} e^{-\gamma_0} \int d^6 x \sqrt{|g|} e^{AB} \omega^{CD} G_{ABCD} = \frac{1}{4} e^{\sigma_0} e^{-\gamma_0}. \quad (101)$$

The sources of such a term are nonzero values of the spin connection components $\omega_\mu^{11}$ and $\omega_m^{11}$ resulting from the $x^{11}$ dependence of the metric. It is a
constant mass term from the 4–dimensional point of view. This, however, does not mean that the gravitino mass is nonzero. There is another contribution from the 11–dimensional term

$$\left. - \frac{\sqrt{2}}{384} \sqrt{g} \bar{\psi}_J \Gamma^{IJKLMN} \psi_N \left( G_{JKLM} + \hat{G}_{JKLM} \right) \right. . \tag{102}$$

After redefining fields according to (99,100) and averaging the nontrivial vacuum expectation value of $G$ over $X^6$ we get from eq. (102) a mass term which exactly cancels the previous contribution (101). The gravitino is massless – the result which we expect in a model with unbroken supersymmetry and vanishing cosmological constant. Thus, we find that our simple reduction and truncation method (including the correct $x^{11}$ dependence in next to leading order) reproduces the main features of the model.

The factor $\langle \exp(3\sigma) \rangle$ represents the volume of the six–dimensional compact space in units of $M_{11}^{-6}$. The $x^{11}$ dependence of $\sigma$ then leads to the geometrical picture that the volume of this space varies with $x^{11}$ and differs at the two boundaries:

$$V_{E_8} = V_{E_6} - 2\pi^2 \rho \left( \frac{\kappa}{4\pi} \right)^{2/3} \left| \int \omega \wedge \frac{\text{tr}(F \wedge F) - \frac{1}{2} \text{tr}(R \wedge R)}{8\pi^2} \right| . \tag{103}$$

where the integral is over $X^6$ at the $E_6$ boundary. In the given approximation, this variation is linear, and for growing $\rho$ the volume on the $E_8$ side becomes smaller and smaller. At a critical value of $\rho$ the volume will thus vanish and this will provide us with an upper limit on $\rho$:

$$\rho < \rho_{\text{crit}} = \left( \frac{4\pi}{c^2} \right)^{2/3} \frac{M_{11}^3 V_{E_6}^{2/3}}{2^{1/6}} \tag{104}$$

where $c$ was defined in eq. (87). To estimate the numerical value of $\rho_{\text{crit}}$ we first recall that from eq. (100) we obtained 

$$M_{11} V_{E_6}^{1/6} = \left( \alpha_{\text{GUT}} (4\pi)^{-2/3} \right)^{-1/6} \approx 2.3 . \tag{105}$$

Thus, we get

$$\rho^{-1} > \rho_{\text{crit}}^{-1} \approx 0.16 e V_{E_6}^{-1/6} . \tag{106}$$

\[ \text{††} \text{With } V \text{ depending on } x^{11} \text{ we have to specify which values should be used in eqs. (106,107,108). The appropriate choice in the expression for } G_N \text{ is the average value of } V \text{ while in the expressions for } \alpha_{\text{GUT}} \text{ and for the } V–M_{\text{GUT}} \text{ relation we have to use } V \text{ evaluated at the } E_6 \text{ wall.} \]
The numerical value of $V$ at the $E_6$ boundary depends on what we identify with the unification scale $M_{GUT}$ via eq. (50):

$$V_{E_6}^{-1/6} = a M_{GUT}^{-1}$$  \hspace{1cm} (107)

with $a$ somewhere between 1 and about 2. Thus, the bound (106) can be written in the form

$$R^{-1}_{11} > 0.05 \frac{c}{a} M_{GUT}.$$  \hspace{1cm} (108)

For the phenomenological applications we have to check whether our preferred choice of $6.2 \cdot 10^{14}$ GeV $< R^{-1}_{11} < 7.4 \cdot 10^{15}$ GeV that fits the correct value of the $d = 4$ Planck mass satisfies the bound (108). In a rather extreme case of $c = 1$ and $a = 2.3$ we find that the upper bound on $R^{-1}_{11}$ is of the order of $6.5 \cdot 10^{14}$ GeV. Even for $c = 1$ this bound goes up to about $1.5 \cdot 10^{15}$ GeV if we identify $V^{-1/6}$ with $M_{GUT}$. Although some coefficients are model dependent we find in general that the bound can be satisfied, but that $R^{-1}_{11}$ is quite close to its critical value. Values of $R^{-1}_{11}$ about $10^{12}$ GeV as necessary in $E_8$ seem to be beyond the critical value, even with the modifications discussed before. In any case, models where supersymmetry is broken by a Scherk–Schwarz mechanism seem to require the absence of the next to leading order corrections in (97), i.e. $\alpha = 0$. It remains to be seen whether such a possibility can be realized.

Inspection of (63) and (97) reveals a close connection between the strongly and weakly coupled case. The variation of the Calabi–Yau manifold volume as discussed above is the analogue of the one loop correction of the gauge kinetic function (63) in the weakly coupled case and has the same origin, namely a Green–Schwarz anomaly cancellation counterterm. In fact, also in the strongly coupled case this leads to a correction for the gauge coupling constants at the $E_6$ and $E_8$ side. As seen, gauge couplings are no longer given by the (averaged) $S$–field, but by that combination of the (averaged) $S$ and $T$ fields which corresponds to the $S$–field before averaging at the given boundary leading to

$$f_{6,8} = S \pm \alpha T$$  \hspace{1cm} (109)

at the $E_6$ ($E_8$) side respectively. The critical value of $R_{11}$ will correspond to infinitely strong coupling at the $E_8$ side $S - \alpha T = 0$. Since we are here close to criticality a correct phenomenological fit of $\alpha_{GUT} = 1/25$ should include this correction $\alpha_{GUT}^{-1} = S + \alpha T$ where $S$ and $\alpha T$ give comparable contributions. This is a difference to the weakly coupled case, where in $f = S + \epsilon T$ the latter contribution was small compared to $S$. This stable result for the corrections to $f$ when going from weak coupling to strong coupling is only possible because of the rather special properties of $f$. $f$ does not receive further perturbative
corrections beyond one loop, and the one loop corrections are determined by the anomaly considerations. The formal expressions for the corrections are identical, the difference being only that in the strongly coupled case these corrections are as important as the classical value.

12 Supersymmetry breaking at the hidden wall

We shall now discuss the question of supersymmetry breakdown within this framework. We consider the breakdown of supersymmetry in a hidden sector, transmitted to the observable sector via gravitational interactions. Such a scenario was suggested in [1] after having observed that gaugino condensation can break supersymmetry in $d = 4$ supergravity models. As we have seen, a nontrivial gauge kinetic function $f$ seems to be necessary for such a mechanism to work. In the heterotic string both ingredients, a hidden sector $E_8$ and a nontrivial $f$, were present in a natural way and a coherent picture of supersymmetry breakdown via gaugino condensation emerged. In the strongly coupled case, such a mechanism can be realized as well. In fact the notion of the hidden sector acquires a geometrical interpretation: the gaugino condensate forms at one boundary (the hidden wall) of spacetime. We shall now discuss this mechanism in detail. First we remind you of some relevant formulae in the weakly coupled case. Our aim then is to compare the strong coupling regime with the weak coupling regime and clarify similarities as well as differences. For the weakly coupled case we start with the action of $d = 10$ supergravity. Supersymmetry transformation laws for the $d = 10$ gravitino fields $\psi_M$ and the dilatino field $\lambda$ are written

$$
\delta \lambda = \frac{1}{8} \varphi^{-3/4} \Gamma^{MNP} H_{MNP} + \frac{\sqrt{2}}{384} \Gamma^{MNP} \bar{\chi}^a \Gamma_{MNP} \chi^a + \ldots,
$$

$$
\delta \psi_M = \frac{\sqrt{2}}{32} \varphi^{3/4} (\Gamma^Q_M - 9 \delta^Q_M \Gamma^P Q) H_{NPQ}
+ \frac{1}{256} (\Gamma_{MNPQ} - 5 \delta_{MNPQ} \Gamma_{PQ}) \bar{\chi}^a \Gamma^{NPQ} \chi^a + \ldots,
$$

(110)

implying that a condensate of gauginos $\bar{\chi} \chi$ and/or non–vanishing vevs of the $H$ fields may break supersymmetry. Here we assume the appearance of the gaugino condensate in the hidden sector

$$
\langle \bar{\chi}^a \Gamma_{mnp} \chi^a \rangle = \Lambda^3 \epsilon_{mnp},
$$

(111)

[1] Here we use the conventions of [12], where the Lagrangian is given in the Einstein frame. To recover the effective action (53) in the string frame, one has to make a proper Weyl transformation and identify $\varphi = \exp (\phi/3)$. 
with $\Lambda$ being the gaugino condensation scale and $\epsilon_{mnp}$ the covariantly constant holomorphic three–form. The perfect square structure seen in the Lagrangian

$$-\frac{3}{4} \phi^{-3/2}(H_{MNP} - \sqrt{2}\phi^{3/4}\bar{\chi}^\alpha \Gamma_{MNP} \chi^\alpha)^2$$

will be a very important ingredient to discuss the quantitative properties of the mechanism. When reducing to the $d = 4$ effective action we will find a cancellation of the vevs of the $H$ field and the gaugino condensate at the minimum of the potential such that the term in eq. (112) vanishes. Before we look at this in detail, let us first comment on such a possible vev of $H$ and a possible quantization condition of the antisymmetric tensor. In it was shown, that an antisymmetric tensor field $H = dB$ has a quantized vacuum expectation value. In many subsequent papers this has been incorrectly taken as an argument for the quantization of the vev of $H = dB + \omega^{YM} - \omega^L$ as given in eq. (54). The correct way to interpret this situation is to have a cancellation of the gaugino condensate with the vev of a Chern–Simons term, for which such a quantization condition does not hold. After all the Chern–Simons term $\omega^{YM}$ contains the superpotential of the $d = 4$ effective theory. This cancellation leads to a certain combination of $\psi_M$ and $\lambda$ as the candidate goldstino that will provide the longitudinal component of the gravitino. While in $d = 10$ this looks rather complicated, it simplifies tremendously once one reduces to $d = 4$. Qualitatively the scalar potential takes the following form at the classical level (for the detailed factors see):

$$V = \frac{1}{ST} \left[ | W - 2(ST)^{3/2}(\bar{\chi}\chi)|^2 + \frac{T}{3} \left| \frac{\partial W}{\partial C} \right|^2 \right].$$

(113)

We observe the important fact that the potential is positive and vanishes at the minimum. Thus we have broken supersymmetry with a vanishing cosmological constant at the classical level. The first term in the brackets of eq. (113) corresponds to the contribution from eq. (112) once reduced to $d = 4$ and vanishes at the minimum. In the $d = 4$ theory it represents the auxiliary component $F_S$ of the dilaton superfield $S$. Thus we have $F_S = 0$ and supersymmetry is broken by a nonvanishing vev of $F_T$. The goldstino is then the fermion in the $T$–multiplet and we are dealing with a situation that has later been named moduli–dominated supersymmetry breakdown. This fact has its origin in the special properties of the $d = 10$ action (the term in eq. (112)) and seems to be of rather general validity. The statement $F_S = 0$ is, of course, strictly valid only in the classical theory. The corrections discussed in section 3, eq. (63) will slightly change these results as we shall discuss later.

Having minimized the potential and identified the goldstino we can now
compute the gravitino mass according to the standard procedure. The result has a direct physical meaning because we are dealing with a theory with vanishing vacuum energy. We obtain
\[ m_{3/2} \sim \frac{F_T}{M_{Planck}} \sim \frac{\Lambda^3}{M_{Planck}^2}. \]  

(114)

A value of \( \Lambda \sim 10^{13} \text{ GeV} \) will thus lead to a gravitino mass in the TeV region.

Next we turn to supersymmetry breaking in the strongly coupled case \((d = 11, \text{ M–theory picture})\) and start with the \( d = 11 \) action. Supersymmetry transformation laws for the gravitino fields in this case are given by
\[
\delta \psi_A = D_A \eta + \frac{\sqrt{2}}{288} G_{IJKL} (\Gamma_A^{IJKL} - 8 \delta_A^I \Gamma^{JKL}) \eta
\]
\[
- \frac{1}{1152 \pi} \left( \frac{\kappa}{4 \pi} \right)^{2/3} \delta(x^{11}) (\chi^a \Gamma_{BCD} \chi^a) \left( \Gamma_A^{BCD} - 6 \delta_A^B \Gamma_C^D \right) \eta + \ldots \quad \text{(115)}
\]
\[
\delta \psi_{11} = D_{11} \eta + \frac{\sqrt{2}}{288} G_{IJKL} (\Gamma_{11}^{IJKL} - 8 \delta_{11}^I \Gamma^{JKL}) \eta
\]
\[
+ \frac{1}{1152 \pi} \left( \frac{\kappa}{4 \pi} \right)^{2/3} \delta(x^{11}) (\chi^a \Gamma_{ABC} \chi^a) \Gamma^{ABC} \eta + \ldots \quad \text{(116)}
\]

where gaugino bilinears appear in the right hand side of both expressions. Again we consider gaugino condensation at the hidden \( E_8 \) boundary
\[
\langle \chi^a \Gamma_{ijk} \chi^a \rangle = g_8^2 \Lambda^3 \epsilon_{ijk}. \]  

(117)

The \( E_8 \) gauge coupling constant appears in this equation because the straightforward reduction and truncation leaves a non–canonical normalization for the gaugino kinetic term. An important property of the weakly coupled case \((d = 10, \text{ Lagrangian})\) was the fact that the gaugino condensate and the three–index tensor field \( H \) contributed to the scalar potential in a full square. Ho\-ra\-va made the important observation that a similar structure appears in the \( \text{M–theory} \) Lagrangian as well:\n\[
- \frac{1}{12 \kappa^2} \int_{M^{11}} d^{11}x \sqrt{g} \left( G_{ABC11} - \frac{\sqrt{2}}{32 \pi} \left( \frac{\kappa}{4 \pi} \right)^{2/3} \delta(x^{11}) \chi^a \Gamma_{ABC} \chi^a \right)^2 \quad \text{(118)}
\]

with the obvious relation between \( H \) and \( G \). Let us now have a closer look at the form of \( G \). At the next to leading order we have
\[
G_{11ABC} = (\partial_{11} C_{ABC} + \text{permutations})
\]
\[
+ \frac{1}{4 \pi \sqrt{2}} \left( \frac{\kappa}{4 \pi} \right)^{2/3} \sum_i \delta(x^{11} - x_i^{11}) (\omega_{ABC}^Y - \frac{1}{2} \omega_{ABC}^L). \quad \text{(119)}
\]
Observe, that in the bulk we have $G = dC$ with the Chern–Simons contributions confined to the boundaries. Formula (118) suggests a cancellation between the gaugino condensate and the $G$–field in a way very similar to the weakly coupled case, but the nature of the cancellation of the terms becomes much more transparent now. In the former case we had to argue via the quantization condition for $dB$ that the gaugino condensate is cancelled by one of the Chern–Simons terms. Here this becomes obvious. The condensate is located at the wall as are the Chern–Simons terms, so this cancellation has to happen locally at the wall and $dC$ should vanish for $G$ not to have a vev in the bulk. In any case there is a quantization condition for $dC$ as well.

So this cancellation is very similar to the one in the weakly coupled case. At the minimum of the potential we obtain $G_{ABCD} = 0$ everywhere and

$$G_{ABC11} = \frac{\sqrt{2}}{32\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta(x^{11}) \chi^a \Gamma_{ABC} \chi^a$$

at the hidden wall. Eqs. (115) and (116) then become

$$\delta \psi_A = D_A \eta + \ldots$$

$$\delta \psi_{11} = D_{11} \eta + \frac{1}{384\pi} \left( \frac{\kappa}{4\pi} \right)^{2/3} \delta(x^{11}) (\chi^a \Gamma_{ABC} \chi^a) \Gamma^{ABC} \eta + \ldots$$

An inspection of the potential shows that $\delta \psi_{11}$ is nonvanishing and supersymmetry is spontaneously broken. Because of the cancellation in eq. (118), the cosmological constant vanishes to leading order. Recalling supersymmetry transformation law for the elfbein

$$\delta e^m_I = \frac{1}{2} \bar{\eta} \Gamma^m \psi_I,$$

one finds that the superpartner of the $T$ field plays the role of the goldstino. Again we have a situation where $F_S = 0$ (due to the cancellation in (118)) with nonvanishing $F_T$. But here we find the novel and interesting situation that $F_T$ differs from zero only at the hidden wall, although the field itself is a bulk field. In general it would be interesting to consider also situations where the goldstino is not a bulk but a wall field.

At that wall our discussion is completely 4–dimensional although we are still dealing effectively with a $d = 5$ theory. To reach the effective theory in $d = 4$ we have to integrate out the dependence of the $x^{11}$ coordinate. As in the previous section this can be performed by the averaging procedure explained there. With the gaugino condensation scale $\Lambda$ sufficiently small compared to the compactification scale $M_{GUT}$, the low–energy effective theory is well described by four dimensional $N = 1$ supergravity in which supersymmetry is
spontaneously broken. In this case, the modes which remain at low energies will be well approximated by constant modes along the $x^{11}$ direction. This observation justifies our averaging procedure to obtain four dimensional quantities. Averaging $\delta \psi_{11}$ over $x^{11}$, we thus obtain the vev of the auxiliary field $F_T$

$$F_T = \frac{1}{2} \int \frac{dx^{11} \sqrt{g_{11}} \delta \psi_{11}}{\int dx^{11} \sqrt{g_{11}}}.$$

(124)

Note that this procedure allows for a nonlocal cancellation of the vev of the auxiliary field in $d = 4$. A condensate with equal size and opposite sign at the observable wall could cancel the effect and restore supersymmetry. Using $\int dx^{11} \sqrt{g_{11}} \delta (x^{11}) = 1$, the auxiliary field is found to be

$$F_T = \frac{1}{32\pi(4\pi)^{2/3}} \frac{g_s^2 \Lambda^3}{R_{11} M_{11}^4}.$$

(125)

Similarly one can easily show that $F_S$ as well as the vacuum energy vanish. This allows us then to unambiguously determine the gravitino mass, which is related to the auxiliary field in the following way:

$$m_{3/2} = \frac{F_T}{F_T + F_S} = \frac{1}{64\pi(4\pi)^{2/3}} \frac{g_s^2 \Lambda^3}{R_{11} M_{11}^4} = \frac{\pi}{2} \frac{\Lambda^3}{M_{Planck}^2}.$$

(126)

As a nontrivial check one may calculate the gravitino mass in a different way. A term in the Lagrangian

$$-\frac{\sqrt{2}}{192\kappa^2} \int dx^{11} \sqrt{g_{11}} \Gamma^{IJKLMN} \psi_I \psi_J G_{JKLM},$$

(127)

becomes the gravitino mass term when compactified to four dimensions. Using the vevs of the $G_{IJK11}$ given by eq. (120), one can obtain the same result as eq. (126). This is a consistency check of our approach and the fact that the vacuum energy vanishes in the given approximation.

It follows from eq. (126), that the gravitino mass tends to zero when the radius of the eleventh dimension goes to infinity. When the four–dimensional Planck scale is fixed to be the measured value, however, the gravitino mass in the strongly coupled case is expressed in a standard manner, similar to the weakly coupled case as can be seen by inspecting (126) and (114). To obtain the gravitino mass of the order of 1 TeV, one has to adjust $\Lambda$ to be of the order of $10^{13}$ GeV when one constructs a realistic model by appropriately breaking the $E_8$ gauge group at the hidden wall.

In the minimization of the potential we have implicitly used the leading order approximation. As was explained in a previous section, the next to
leading order correction gives the non-trivial dependence of the background metric on $x^{11}$. Then the Einstein–Hilbert action in eleven dimensions gives additional contribution to the scalar potential in the four-dimensional effective theory, which shifts the vevs of the $G_{IJKL}$. As a consequence, $F_S$ will no longer vanish. Though this may be significant when we discuss soft masses, it does not drastically change our estimate of the gravitino mass (126) and our main conclusion drawn here is still valid after the higher order corrections are taken into account.

13 Soft supersymmetry breaking terms

In the previous section, we have shown that the gaugino condensation breaks supersymmetry both in the weakly coupled heterotic string and in the heterotic $M$–theory. We chose $\Lambda$ in such a way that the gravitino mass appeared in the TeV–range. In this section we shall discuss the soft supersymmetry breaking terms that appear in the low–energy effective theory as a consequence of this nonzero gravitino mass.

We first give the relevant formulae for gaugino and scalar masses in the observable sector. Given the gauge kinetic function $f_6$ in the observable sector, the gaugino mass is calculated to be

$$m_{1/2} = \frac{\partial f_6}{\partial \phi^i} \frac{F_i}{2 \text{Re} f_6},$$

where $\phi^i$ symbolically denote hidden sector fields responsible for supersymmetry breakdown. Writing the Kähler potential

$$K = \hat{K}(\phi^i, \phi^*_i) + Z(\phi^i, \phi^*_i)C \ast C + \text{(higher orders in } C, C^*)$$

one can also calculate the mass of a matter field $C$

$$m_0^2 = m_{3/2}^2 - F_i F_j \frac{Z^*_i Z_j - Z_i Z^{-1} Z_j}{Z}.$$ (130)

Here a vanishing cosmological constant is assumed.

Using the classical approximation naively, these formulae lead to a surprising result. All soft masses vanish. At the basis of this fact it had been suggested that the gravitino mass could be arbitrarily high, still leading to softly broken supersymmetry in the TeV range. It has been observed meanwhile that this surprising result is an artifact of the approximation and it is now commonly accepted that generically the soft masses tend to be of the order of the gravitino mass or at least not arbitrarily small compared to it.
In general the result for the soft scalar masses is strongly model dependent. We shall see in the following that the situation concerning the gaugino mass is less model dependent but varies when we go from the weakly to the strongly coupled case.

We start again with the weakly coupled case. At the leading order (tree level), the gauge kinetic function for the observable sector is simply

\[ f_6 = S, \]

whereas the gaugino condensation gives

\[ F_S = 0, \quad F_T = m_{3/2}(T + T^*). \]

Thus, at this level, the gaugino mass vanishes. As was discussed earlier in these lectures, the gauge kinetic function receives corrections at one–loop order. Using eq. (63), the gaugino mass is explicitly written as

\[ m_{1/2} = \frac{F_S + \epsilon F_T}{2\text{Re}(S + \epsilon T)}. \]

(131)

Note that \( F_T/(T + T^*) \sim m_{3/2} \). Also we expect \( F_S \) to be of the order of \( \epsilon T m_{3/2} \) due to the one–loop corrections. Plugging them into the above expression, we obtain

\[ m_{1/2} \sim \frac{\epsilon T}{S} m_{3/2}. \]

(132)

Since in the weakly coupled case the ratio \( \epsilon T/S \) is small, the gaugino becomes much lighter than the gravitino.

Let us now consider the scalar masses. At the tree level, the Kähler potential is

\[ K = -\ln(S + S^*) - 3\ln(T + T^*) + (T + T^*)^n C^* C + \text{(higher orders in } C^* C), \]

(133)

where \( n \) denotes the modular weight of a field \( C \). For a field with \( n = -1 \) (un–twisted sector in an orbifold construction), which naturally appears in the simple truncation procedure, we recover the previous formula (57). From eq. (130), it follows that

\[ m_0^2 = m_{3/2}^2 + \frac{|F_T|^2}{(T + T^*)^2} = (1 + n)m_{3/2}^2. \]

(134)

A scalar field with the modular weight \( -1 \) has a vanishing supersymmetry breaking mass at the leading order. It is an artifact of the approximation of reduction and truncation (i.e. torus compactification) that the fields have modular weight \( -1 \). A field whose modular weight is different from \( -1 \) has a mass comparable to the gravitino mass. Though, as discussed in section 3, corrections at the one–loop level are model dependent, one expects they are of the order of \( \epsilon T/S m_{3/2}^2 \). Summarizing these contributions, one obtains

\[ m_0^2 = (1 + n)m_{3/2}^2 + O\left(\frac{\epsilon T}{S} m_{3/2}^2\right), \]

(135)
where the actual value of the second term depends on the model one considers. A conclusion we can draw from eqs. (132) and (135) is that the gaugino masses tend to be much smaller than the scalar masses:

\[ m_{1/2} \ll m_0 \leq O(m_{3/2}). \]  

(136)

Phenomenologically this relation might be problematic. Requiring that the gaugino masses are at the electro–weak scale, eq. (136) would then imply that the masses of the squarks and sleptons should be well above the 1 TeV region, which raises the fine–tuning problem to reproduce the Fermi scale. Another potential problem is the relic abundance of the lightest superparticles (LSPs) which are likely the lightest neutralinos in the present case. With the parameters characterized by (136), the standard computation of the relic abundances shows that too many LSPs would (if stable) still be around today, resulting in the overclosure of the Universe.

Thus in the weak coupling regime, one can conclude that, though the gaugino condensation realizes supersymmetry breaking, it tends to lead to a picture where gaugino masses are generically smaller than gravitino and scalar masses. A satisfactory situation might only be achieved, if one fine–tunes the scalar masses in a way that they become comparable to the gaugino masses.

Next we want to discuss how the situation changes when one considers the strongly coupled case (heterotic M–theory).

As in the weakly coupled heterotic string theory, the gaugino mass vanishes at the leading order of the \( \kappa^{2/3} \) expansions, because \( f_6 = S \) and \( F_S = 0 \). Again the next to the leading order is important. The analogue of eq. (131) in the strongly coupled case is

\[ m_{1/2} = \frac{F_S + \alpha F_T}{2 \text{Re}(S + \alpha T)}. \]

(137)

Thus we obtain, as before

\[ m_{1/2} \sim \frac{\alpha T}{S} m_{3/2}. \]

(138)

A crucial difference in this case, however, is the fact that the ratio \( \alpha T / S \) is not a small number, but can be as large as unity. This is because the values of \( S \) and \( T \) inferred from our input variables (see section 2.2) suggests that we are rather close to criticality (in which case the ratio becomes unity). Thus we can conclude that, unlike the weakly coupled case, the gaugino mass in the strongly coupled regime is comparable to the gravitino mass. This observation confirms the expectation that the gravitino mass should be in the TeV–region.
and the gaugino condensation scale $\Lambda \sim 10^{13}$ GeV. Because of the simplicity of the mass formula (128) and the fact that the gauge-kinetic function $f$ is stable in higher order perturbation theory, the statement concerning the soft gaugino masses is rather model independent.

The situation is more complicated in the case of the scalar masses which we consider now in the framework of heterotic $M$–theory. At the leading order we arrive at the same conclusions as in the weak coupling case, since the Kähler potential is identical in both cases. In section 4, we calculated the corrections to the Kähler potential at the next to leading order, which reads

$$\hat{K} = -\ln(S + S^*) - 3\ln(T + T^*)$$

$$Z = \frac{6}{T + T^*} + \frac{2\alpha}{S + S^*}$$

where the latter is valid for a field with the modular weight $-1$. Now using the formula (130) one may be able to calculate the scalar masses, with the result

$$m_0^2 = m_{3/2}^2 - \frac{2}{1 + \delta} \frac{|F_T|^2}{(T + T^*)^2} - \frac{\delta(2 - \delta)}{1 + \delta} \frac{|F_S|^2}{(S + S^*)^2}$$

$$- \frac{\delta}{(1 + \delta)^2}(F_S^* F_T^* + F_S F_T)$$

where

$$\delta = \frac{\alpha}{3} \frac{T + T^*}{S + S^*}.$$  

We can clearly see from this expression that the structure obtained in the leading order is badly violated. Given the fact that the expansion parameter $\alpha(T + T^*)/(S + S^*)$ is of order unity it is no longer possible to fine tune the scalar masses (by choosing modular weight $-1$ for all of them) to a small value and then hope that the corrections respect this fine tuning. In addition the scalar masses depend strongly on the form of the Kähler potential which, in contrast to the gauge kinetic function, receives further corrections in higher order. Thus detailed statements about the scalar masses are very model dependent. It remains to be seen whether any sensible quantitative statement can be made about the scalar masses with the formulae given above. The results for the gaugino masses are more reliable since $f$ does not receive corrections in higher order.

In summary we can, however, conclude with the qualitative statement that in the strong coupling regime,

$$m_{1/2} \sim m_0 \sim m_{3/2}.$$  

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This contrasts with the relation (136) for the weak coupling regime and represents an important improvement concerning phenomenological applications. In the strongly coupled case, the difference between dilaton– and moduli– dominated supersymmetry breakdown seems less pronounced than it is in the weakly coupled case.

14 Some phenomenological consequences

We have presented a consistent framework of supersymmetry breaking and soft breaking terms triggered by the gaugino condensate at the hidden wall. In the strongly coupled case, in complete analogy to the weakly coupled case, the gravitino mass \( \frac{m_3}{\sqrt{2}} \) is related to the gaugino condensation scale \( \Lambda \) as

\[
\frac{m_3}{\sqrt{2}} \approx \frac{\Lambda}{M_{\text{Planck}}^2}.
\]

Furthermore, as explained in detail, the soft masses are of the order of the gravitino mass. This implies that these masses should be in the TeV range in order to solve the naturalness problem of the Higgs boson mass in the supersymmetric framework. This requires that \( \Lambda \) should be around \( 10^{13} \) GeV, three orders of magnitude smaller than the GUT scale (the compactification scale) and thus the 11D Planck scale as well. The gauge coupling constant at the \( E_8 \) wall, where the gaugino condensate is supposed to occur, is larger than the one at the \( E_6 \) wall. If the eleventh dimensional radius \( \rho \) approaches the critical radius \( \rho_{\text{crit}} \), the \( E_8 \) gauge coupling constant becomes strong at a scale as large as the GUT scale, and the running coupling constant will blow up at that scale already. Then the gaugino condensation scale \( \Lambda \), which is approximately identified with the blow–up energy scale, would become too large. For a value of \( \Lambda \sim 10^{13} \) GeV, \( \rho \) should (although close) not be too close to the critical value so that the gauge coupling constant does not blow up immediately. This gives a constraint on the constant \( \alpha \) (defined in (86)), which depends on the detailed properties of the Calabi–Yau manifold under consideration. In any case it is probably necessary to break the hidden \( E_8 \) to a smaller group to obtain a smaller coefficient of the \( \beta \)-function. These considerations should be kept in mind when one attempts to construct a realistic model.

The fact that the gravitino mass cannot be arbitrarily large, but should lie in the TeV range in the heterotic \( M \)-theory regime suggests that the theory might share a problem already encountered in the weakly coupled case. Late time decay of the gravitinos would upset the success of the standard big–bang nucleosynthesis scenario. This problem is rather universal in most of the supergravity models where breakdown of supersymmetry is mediated through
gravity. Indeed this is not really a serious difficulty, but just implies that the universe underwent inflationary expansion followed by reheating at a relatively low temperature \( T < 10^9 \text{ GeV} \) for \( m_{3/2} = 1 \text{ TeV} \), in which the gravitino number density is diluted by the inflation and the low reheat temperature suppresses gravitino production after that.

A main difference between the weakly and the strongly coupled case manifests itself when we consider phenomenological issues associated with the soft masses. In the weakly coupled string case, the gaugino condensation scenario gives a very small gaugino mass compared to the scalar masses. For a typical size of the compactification radius of the 6D manifold, the gaugino mass is shown to be more than one order of magnitude smaller than the scalar mass (see for example eqs. (7.20) and (7.24) (with \( \sin \theta \rightarrow 0 \) limit) of ref. 11 for more detail). This hierarchy among the soft masses obviously raises a naturalness problem. With gaugino masses of the order of 100 GeV, the scalar masses would be far above 1 TeV, requiring fine tuning to obtain the electroweak symmetry breaking scale. This causes problems for explicit model building. Another phenomenological difficulty caused by the small gaugino mass arises in the context of relic abundances of the lightest superparticles (LSPs). Under the assumption of \( R \)-parity conservation, the LSP is stable and remains today as a dark matter candidate. Given the superparticle spectrum in the weak coupling regime, the bino, the superpartner of the \( U(1)_Y \) gauge boson, is most likely to be the LSP. To evaluate the relic abundances of the bino, one has to know its annihilation cross section (see ref. and references therein). In our case, the bino pair annihilates into fermion (quarks and leptons) pairs via \( t \)-channel scalar (squarks and sleptons) exchange. The cross section is roughly proportional to

\[
\sigma \propto \frac{m_B^2}{m_f^4}
\]

where \( m_B \) is the bino mass and \( m_f \) represents a scalar mass. As the scalar becomes heavier, the cross section is suppressed, yielding a larger relic abundance. Indeed when the scalar mass is more than an order of magnitude larger than the gaugino mass, a standard calculation shows that the relic abundance exceeds the critical value of the universe. This overclosure is a serious problem in the weakly coupled case.

In the strong coupling regime, the gaugino acquires a mass comparable to the gravitino mass and the scalar masses. Thus the above two problems do not appear. All the soft masses are in the same range. If this is not far from the electroweak scale, one can naturally realize the electroweak symmetry breaking at the correct scale without fine tuning. Moreover in this scenario,
the annihilation cross section of the bino becomes larger, and thus we can obtain a relic abundance compatible with the observations. In some regions of parameter space we may even realize a situation where the LSP is the dominant component of the dark matter of the universe.

A characteristic of the mechanism of gaugino condensation is the fact that it is the $T$ field that plays the dominant role in the breakdown of supersymmetry. In this scenario scalar fields with different modular weight will have different masses, which may cause problems with flavor changing neutral currents (FCNC). In the strong coupling case, the situation may be improved through the presence of a large gaugino mass which contributes to the scalar masses at low energies through radiative corrections that can be computed via renormalization group methods. In a situation where scalar masses at the GUT scale are small enough, this universal radiative contribution might wash out nonuniversalities and avoid problems with FCNC. Details of the superparticle phenomenology in the strongly coupled case, including the issues outlined above, will be discussed elsewhere.

Eqs. (63) (in the weak coupling case) and (97) (in the strong coupling case) show that the imaginary part of the complex scalar fields, $S$ and $T$, has an axion–like coupling to the gluon fields. In the weakly coupled case, world–sheet instanton effects and possibly other non–perturbative effects give non–negligible contributions to the potential. Then the axion candidates receive masses comparable to the gravitino mass, and they do not solve the strong $CP$ problem. However, in the strongly coupled case, it has been argued that these non–perturbative contributions originated at high energy physics might be suppressed to a negligible level. If this is the case, a linear combination of the Im$S$ and Im$T$ will play a role of the axion, whose potential is dominated by the QCD contribution. Then this axion, referred to as the $M$–theory axion, will be able to solve the strong $CP$ problem. A word of caution should be added here, since a reliable calculation of these world sheet nonperturbative effects has only been performed in the weakly coupled case. The above argumentation in the $M$–theory framework uses the implicit assumption that those Yukawa couplings remain as weak as in the case of the weakly coupled string, an assumption that might not be necessarily correct. Apart from that, the axion decay constant in this case becomes as large as $10^{16}$ GeV, which leads to the potential problem that the energy density of the coherent oscillation of the axion field exceeds the critical energy density of the universe. This problem could be solved if the entropy production occurs after the QCD phase transition when the axion gets massive, or if this world is almost $CP$ conserving and the initial displacement of the axion field is very small. The direct detection of the relic axions with such a large decay
constant would be extremely difficult. However the $M$–theory axion may give
a significant contribution to the isocurvature density fluctuations during the
inflationary epoch, which may be detectable in future satellite observations\[57\].
It remains to be seen whether this mechanism leads to a satisfactory solution
of the strong CP–problem.

15 Summary and outlook

In any case we have seen that the M–theoretic version of the heterotic string
shows some highly satisfactory phenomenological properties concerning the
unification of fundamental coupling constants as well as the nature of the soft
supersymmetry breaking parameters.

Still there remain some problems that still resist attempts for a satisfactory
solution. Certainly one of them is the question of fixing the vev of the dilaton.
One would like to see whether the M-theoretic approach to the problem might
give us some new hints in that direction.

In the last years there has been revolutionary progress in the understanding
of nonperturbative aspects of string theory. Here we have discussed the first
consequences of phenomenological interest that could be derived from this new
insights. Let us hope that other aspects of that field might also be of relevance
for this questions and increase our understanding of the low-energy effective
actions that could be derived from string theory.

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