Towards precise measurement of oscillatory domain wall by ferromagnetic Josephson junction

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We theoretically propose a principle for precise measurement of oscillatory domain wall (DW) by a ferromagnetic Josephson junction, which is composed of a ferromagnetic wire with DW and two superconducting electrodes. The current-voltage curve exhibits stepwise structures, only when DW oscillates in the ferromagnetic wire. The voltage step appears at \( V = n(h/2e)\omega_{DW} \) with the fundamental constant \( h/e \), integer number \( n \), and the DW frequency \( \omega_{DW} \). Since \( V \) can be determined in the order of \( 10^{-9} \) accuracy, the oscillatory DW will be measured more precisely than present status by conventional method.

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Nano-scale magnetic materials for spintronics devices are extensively studied due to many advantages such as enhanced operation speed, low power consumption, and high integration of memory cell.1,2 Non-volatile memory using a magnetic domain wall (DW) is one example of such devices,3,4 and many studies are devoted to control DW.5–13 Among those studies, the oscillatory DW is experimentally observed7–9 and is examined toward applications, e.g. high sensitive magnetic sensor, nano-scale telecommunication, rf-assisted writing of magnetic bit, and microwave generator.14–17 Once such devices are realized in a microscopic circuit, one needs to measure the DW frequency more precisely.

One example of precise measurement is the Josephson effect.18 Under irradiation of microwave to the Josephson junction, the current-voltage (I-V) curve shows step structures at \( V = n(h/2e)\nu \) with microwave frequency \( \nu \), integer \( n \), and the ratio of the Plank constant and the elementary charge \( h/e \). This structure called Shapiro step19 is adopted to the voltage standard around the world,20,21 since the voltage can be determined in the order of \( 10^{-9} \) accuracy by the precise values of the frequency and the fundamental constants.20,21

On the other hand, many authors studied the Josephson effect in a hybrid structure of superconductor and ferromagnet.22–27 In this ferromagnetic Josephson junction (FJJ), two superconducting electrodes are separated by a thin ferromagnetic layer, whose thickness is controlled in the order of nano meter.22–27 If DW is put into the ferromagnetic layer of FJJ, it might be possible to measure the oscillatory DW more precisely by using the Josephson effect.

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In this letter, by using FJJ, we theoretically propose a principle to measure the oscillatory DW more precisely than the present accuracy. As depicted in Fig.1(a), we use FJJ composed of a ferromagnetic Py wire with DW.
and two superconducting Nb electrodes. The interface is parallel to the \(xz\)-plane with width \(L_x\) and depth \(L_z\). A dc bias current is applied to the \(y\)-direction between the electrodes separated by height \(d\). We suppose the magnetic flux densities \(\mathbf{B}(r, t)\) due to DW as shown in Figs. 1 (b) and (c), where \(M_S\) is the saturation magnetization and \(r = (x, z)\). In such a magnetic flux density, a gauge invariant phase difference \(\theta(r, t)\) satisfies the differential equation,\(^28\) \(\nabla_x z \theta(r, t) = -\frac{\hbar}{2e} \mathbf{B}(r, t) \times \mathbf{n}\), where \(\nabla_x z = (\partial/\partial x, 0, \partial/\partial z)\) and \(\mathbf{n}\) is the unit vector perpendicular to the \(xz\)-plane. The solution is given by,

\[
\theta(r, t) = \varphi(t) - \frac{2e}{\hbar} M_S z d - \frac{e \lambda d}{\hbar} M_S, \quad -\frac{L_x}{2} < x < x_1, (1)
\]

\[
\theta(r, t) = \varphi(t) + \frac{2e}{\hbar} M_S (x - \frac{x_1 + x_2}{2}) d, \quad x_1 < x < x_2, (2)
\]

\[
\theta(r, t) = \varphi(t) + \frac{2e}{\hbar} M_S z d + \frac{e \lambda d}{\hbar} M_S, \quad x_2 < x < \frac{L_x}{2}, (3)
\]

where \(x_1\) and \(x_2\) are both ends of DW. Below, the DW thickness \(\lambda \equiv x_2 - x_1\) is assumed to be constant.

The DW oscillation is driven by the external ac voltage \(V_{AC}(t)\) parallel to the \(x\)-direction with a certain frequency. Here, we assume that the width of DW is much smaller than \(L_x\), and DW harmonically oscillates within the junction. Then, the position of \(x_1\) is expressed to be \(x_0 \sin(\omega_{DW} t)\), where \(x_0\) and \(\omega_{DW}\) are the oscillation amplitude and frequency of DW, respectively (see Figs. 1 (b) and (c)).

From Eqs. (1), (2), and (3), the Josephson current \(I_J(t)\) is given by

\[
I_J(t) = j_c \int_{-L_x/2}^{L_x/2} dx \int_{-L_z/2}^{L_z/2} dz \sin \theta(r, t),
\]

\[
= -2I_c \tilde{x}_1(t) \frac{\sin(\pi \varphi_x)}{\pi \varphi_x} \sin(\pi \varphi_z \lambda) \cos \varphi(t)
\]

\[
+ I_c \frac{\sin(\pi \varphi_x)}{\pi \varphi_x} \cos(\pi \varphi_z \lambda) + \frac{\lambda}{\pi \varphi_z \lambda} \sin(\varphi(t)
\]

\[
- I_c \lambda \frac{\sin(\pi \varphi_x)}{\pi \varphi_x} \sin(\varphi(t) + \pi \varphi_z \lambda), (4)
\]

where \(\varphi_i = \Phi_i/\Phi_0\) and \(\Phi_i = dL_i M_S\) for \(i = x, z\). The Josephson critical current and its density are denoted by \(I_c = j_c L_x L_z\) and \(j_c\), respectively. The position and the width of DW are normalized as, \(\tilde{x}_1 = x_1/L_x\) and \(\tilde{\lambda} = \lambda/L_x\).

We examine the \(I-V\) curve in FJJ by the resistively shunted junction (RSJ) model,

\[
I = I_J(t) + \frac{1}{R} \frac{\Phi_0}{2 \pi} \frac{d\varphi(t)}{dt}, (5)
\]

with the resistance of junction \(R\) and the flux quantum \(\Phi_0 = h/2e\). We numerically solve Eq. (5) and calculate the \(I-V\) curve by using the relation, \(\langle \dot{\varphi}(t) \rangle\) \(\partial\varphi(t)/\partial t\) \(= 2eV/h\), where \(\langle \cdots \rangle\) denotes a time average. Figure 2 shows the \(I-V\) curve for the following parameters; \(\omega_{DW}=160\,\text{MHz}\), \(\lambda = 100\,\text{nm}\), \(d=5\,\text{nm}\), \(I_c = 1\,\mu\text{A}\), \(M_S = 0.75\,\text{T}\), and \(R=1\,\Omega\).\(^{7,26,27,29,30}\) We find the stepwise structure similar to the Shapiro step.\(^{19}\) This structure, on the other hand, disappears without the DW oscillation (see the inset of Fig. 2). It is noted that ferromagnetism and roughness of interface suppress the proximity effect in FJJ and then leads to the suppression of \(I_c\).\(^{22-27}\) The obtained values in Fig. 2 are experimentally observable, i.e., in order of micro-ampere and micro-volt. In view of the measurement, the fabrication of flat interface and the choice of material is important.

To clarify the origin of the stepwise structure, we analytically examine the relation between the dc voltage and \(\omega_{DW}\) in the \(I-V\) curve. For this purpose, it is useful to note that Eq. (4) is written as,

\[
I_J(t) = -I_c \tilde{x}_0 \frac{\sin(\pi \varphi_x)}{\pi \varphi_x} \sin(\pi \varphi_z \lambda) \sum_{n=-\infty}^{\infty} J_n(\varphi_D) \times \text{Im} \left[ e^{i(\varphi_0 + ((n+1) \omega_{DW} + 2 \pi V)/2)} \right]
\]

\[
-(-1)^n e^{i(\varphi_0 + (-(n+1) \omega_{DW} + 2 \pi V)/2)}
\]

\[
+ I_c \frac{\sin(\pi \varphi_x)}{\pi \varphi_x} \cos(\pi \varphi_z \lambda) + \frac{\lambda}{\pi \varphi_z \lambda} \sin(\varphi(t)
\]

\[
- I_c \lambda \frac{\sin(\pi \varphi_x)}{\pi \varphi_x} \sin(\varphi(t) + \pi \varphi_z \lambda), (4)
\]

\[
\sum_{n=-\infty}^{\infty} J_n(\varphi_D) \text{Im} \left[ e^{i(\varphi_0 + (2 \pi V - n \omega_{DW})/2)} \right]
\]

\[
- I_c \lambda \frac{\sin(\pi \varphi_x)}{\pi \varphi_x} \sum_{n=-\infty}^{\infty} J_n(\varphi_D) \text{Im} \left[ e^{i(\varphi_0 + \pi \varphi_z \lambda + (2 \pi V - n \omega_{DW})/2 \lambda)} \right], (6)
\]

where \(J_n(\varphi_D)\) is the first kind Bessel function. To obtain Eq. (6), we suppose \(\varphi(t) = \varphi_0 + 2eVt/h + \delta \varphi(\sin(\omega_{DW} t))\) with a constant \(\varphi_0\). The time-dependent term \(2eVt/h\) is due to the applied bias voltage. The DW oscillation
appears at the following voltages, for 2\(e\) constants are known as, exhibits stepwise structures, only when DW oscillates in conducting electrodes. The current-voltage curve of FJJ of a ferromagnetic wire (FW) with DW and two supermagnetic Josephson junction (FJJ), which is composed of oscillatory domain wall (DW) by a ferromagnetic wire and so on. Therefore, we can precisely determine the frequency by a ferromagnetic wire and so on. Those possibilities and their more detailed treatments will be examined in forthcoming work.

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\[ V = \frac{n}{2e} \omega_{DW}, \] (7)

with integer \(n\). It is worth to note that Eq. (7) relates \(V\) to \(\omega_{DW}\) only by the fundamental constant \(h/e\) and \(n\). Therefore, we can precisely determine the frequency of DW oscillation by the \(I-V\) curve. Those fundamental constants are known as, \(ε=1.602 176 487 (40) \times 10^{-19}\) C and \(h=1.054 571 628 (53) \times 10^{-34}\) Js with accuracy in order of \(10^{-10}\). On the other hand, \(V\) is precisely determined by the conventional Josephson junction in the order of \(10^{-9}\) accuracy. Hence, the measurement on \(V\) can determine the DW frequency precisely. Accuracy of conventional measurement on the DW oscillation is limited in order of about \(10^{-2} \sim 10^{-3}\), since those methods depend on material parameters such as anisotropic field, saturation magnetization and so on. Therefore, our method will be more precise than the present accuracy by conventional one.

So far, we have discussed the \(I-V\) curve in the FJJ composed of the ferromagnetic Py wire and superconducting Nb electrodes. If \(j_c\) is larger than that of the Nb-based FJJ, one can expect a larger step height in the \(I-V\) curve, since those methods depend on material parameters such as anisotropic field, saturation magnetization and so on. Therefore, our method will be more precise than the present accuracy by conventional one.