Limiting energy density and gravity in
Riemann-Cartan space-time

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Abstract. The gravitational interaction is discussed within the framework of gauge gravita-
tional theory in the Riemann-Cartan space-time. In the case of spatially homogeneous isotopic
gravitating systems the gravitational repulsion at extreme conditions near limiting energy
density and the transition from gravitational repulsion to attraction in dependence on energy
density is studied. The conversion to Friedman regime and the transition to gravitational
repulsion at very small energy densities is analyzed.

Keywords: cosmic singularity, gravity, modified gravity, physics of the early universe

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1 Introduction

Within the framework of the general relativity theory (GR), there are no restrictions for permissible values of the energy density for gravitating matter. Since in the case of ordinary gravitating matter with positive energy density and non-negative pressure, the gravitational interaction has the character of attraction increasing together with the energy density the occurrence of singular states with divergent energy density which is unacceptable from a physical point of view is inevitable in GR. In cosmology, this leads to cosmological problem — the problem of the beginning of the Universe in time, inherent in the Big Bang model. Many attempts have been made to solve this problem both within the framework of the metric theory of gravitation and various alternative theories of gravity (see e.g. [1, 2], review [3]). However, most of the obtained partial regular solutions are unlikely to lead to the solution of the cosmological problem, since at the same time there are singular cosmological solutions that cannot be excluded from consideration for physical reasons. At the same time, there is an approach in the relativistic theory of gravitation that allows us to solve the cosmological problem due to the conclusion about the possible existence of a limiting (i.e. maximum allowable) energy density in the nature. We are talking about the gauge theory of gravity in the Riemann-Cartan space-time (GTRC), known in the literature as Poincaré gauge theory of gravity. The development of GTRC is connected with names T.W.B. Kibble, D.D. Ivanenko, D.W. Sciama, A. Trautman and others and at present this theory is one of the most important directions of the development of the theory of gravity (see for example [4]). GTRC is the theory based on generally accepted physical principles of classical field theory and theory of fundamental physical interactions, including the principle of local gauge invariance and it is a direct generalization of the metric theory of gravity when the group of tetrad Lorentz transformations is included in the gauge group corresponding to the gravitational interaction. Investigations of GTRC based on general expression of gravitational Lagrangian $L_g$ as function of gravitational field strengths — the curvature $F^{ik}_{\mu\nu}$ and torsion $S_{\mu\nu}$ tensors including both the scalar curvature and various invariants quadratic in the curvature and torsion tensors with indefinite parameters assuming spatial parity, show that this theory by certain restrictions on indefinite parameters satisfies the correspondence principle with GR in the case of gravitating systems with energy densities much less than limiting energy density and allows to solve certain principal problems of GR. The conclusion about possible existence of limiting energy density was obtained by investigation of isotropic cosmology built in the frame of GTRC (see e.g. [5–8]). Gravitational interaction at extreme conditions (extremely high energy densities $\varepsilon$
and pressures $p$) near limiting energy density is repulsive excluding the appearance of singular state with divergent energy density. The value of limiting energy density $\varepsilon_{\text{max}}$ depends on content of gravitating matter and indefinite parameters of gravitational Lagrangian $\mathcal{L}_g$: in the case of usual gravitating matter the dependence takes place on equation of state $p = p(\varepsilon)$; in the case of inflationary cosmological models the value $\varepsilon_{\text{max}}$ depends also on parameters characterising scalar fields near limiting energy density. Because the gravitational interaction near limiting energy density is repulsive, the value $\varepsilon_{\text{max}}$ should exceed the highest energy densities of existing astrophysical objects. The study of gravitational interaction near limiting energy density can be interesting not only for cosmology but also by investigation of superdense astrophysical objects.

The present paper is devoted to investigation of homogeneous isotropic gravitating systems (HIGS) filled by gravitating matter with equation of state $p = p(\varepsilon)$ in Riemann-Cartan space-time at extreme conditions. At first the gravitational equations for HIGS are given.

2 Equations for homogeneous isotropic gravitating systems in Riemann-Cartan space-time

Any HIGS in Riemann-Cartan space-time is described by three functions of time: the scale factor of Robertson-Walker metric $R(t)$ and two torsion functions — scalar function $S_1(t)$ and pseudoscalar function $S_2(t)$. Gravitational equations for HIGS obtained in the frame of GTRC take the following form [5, 10]

$$\frac{k}{R^2} + (H - 2S_1)^2 - S_2^2 = \frac{1}{6f_0Z} \left[ \varepsilon - 6bS_2^2 + \frac{\alpha}{4} \left( \varepsilon - 3p - 12bS_2^2 \right)^2 \right],$$  \hspace{1cm} (2.1)

$$\dot{H} - 2\dot{S}_1 + H(H - 2S_1) = -\frac{1}{12f_0Z} \left[ \varepsilon + 3p - \frac{\alpha}{2} \left( \varepsilon - 3p - 12bS_2^2 \right)^2 \right],$$  \hspace{1cm} (2.2)

where $H = \dot{R}/R$ (a dot denotes the differentiation with respect to $x^0 = ct$), $k = +1, 0, -1$ for closed, flat and open models respectively and $Z = 1 + \alpha (\varepsilon - 3p - 12bS_2^2)$. The torsion functions $S_1$ and $S_2$ are:

$$S_1 = -\frac{\alpha}{4Z} [\dot{\varepsilon} - 3\dot{p} + 12f_0\omega HS_2^2 - 12(2b - \omega f_0)S_2\dot{S}_2],$$  \hspace{1cm} (2.3)

$$S_2 = \frac{\varepsilon - 3p}{12b} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12b\alpha(1 - \omega/4)},$$  \hspace{1cm} (2.4)

where $X = 1 + \omega(f_0^2/b^2)[1 - (b/f_0) - 2(1 - \omega/4)\alpha(\varepsilon + 3p)] \geq 0$ and $f_0 = \frac{c^4}{16\pi G}$ ($G$ is Newton’s gravitational constant), $\alpha, \omega, b$ are indefinite parameters.

By using expressions for torsion functions and the equation of the energy conservation law, which has the same form as in GR

$$\dot{\varepsilon} + 3H (\varepsilon + p) = 0,$$  \hspace{1cm} (2.5)

we can express the Hubble parameter and its time derivative as functions of energy density and pressure. Really by using (2.3) and (2.4) we find that

$$S_1 = -\frac{3f_0\omega\alpha}{4bZ} HD,$$  \hspace{1cm} (2.6)
where
\[
D = \frac{1}{2} \left( \frac{3}{6} \frac{dp}{d\varepsilon} - 1 \right) (\varepsilon + p) + \frac{1}{3} (\varepsilon - 3p) - \frac{b}{6f_0\alpha (1 - \omega/4) \sqrt{X}} \\
+ \frac{1 - \omega (f_0/2\alpha)}{2\sqrt{X}} \left[ (3\frac{dp}{d\varepsilon} + 1) (\varepsilon + p) + \frac{1}{3\alpha(1 - \omega/4)} \right],
\]
(2.7)
and \( Z = \frac{-\omega/4 + (b/2f_0)(1 + \sqrt{X})}{1 - \omega/4} \). Then cosmological equation (2.1) leads to the Hubble parameter in the form:
\[
H = H_\pm = \pm \frac{\sqrt{A_1}}{1 + \frac{3f_0\omega\alpha}{2bZ} D},
\]
(2.8)
where
\[
A_1 = \frac{\varepsilon - 3p}{12b} + \frac{\varepsilon + 3p}{12f_0Z} + \frac{1 - (b/2f_0)(1 + \sqrt{X})}{12b\alpha(1 - \omega/4)} \left( 1 - \frac{b}{f_0Z} \right) \\
+ \frac{1 - (b/2f_0)(1 + \sqrt{X})^2}{24f_0\alpha Z(1 - \omega/4)^2} - \frac{k}{R^2}.
\]
(2.9)
By taking into account that \( H - 2S_1 = H(1 + \frac{3f_0\omega\alpha}{2bZ} D) \) and \( \dot{D} = -3H(\varepsilon + p)D_1 \), where \( D_1 \) is certain function of energy density
\[
D_1 = \frac{1}{6} \left( \frac{3}{6} \frac{dp}{d\varepsilon} - 1 \right) \left( \frac{3}{6} \frac{dp}{d\varepsilon} + 1 \right) + \frac{1}{2} (\varepsilon + p) \frac{\varepsilon^2}{6b\sqrt{X}} \left( 1 + \frac{3}{6} \frac{dp}{d\varepsilon} \right) \\
+ \frac{1 - \omega f_0}{2\sqrt{X}} \left[ \left( 1 + \frac{dp}{d\varepsilon} \right) \left( 1 + \frac{3}{6} \frac{dp}{d\varepsilon} \right) + 3(\varepsilon + p) \frac{dp}{d\varepsilon} \right] \\
+ \frac{1 - \omega f_0}{2X^{3/2}} \frac{f_0^2}{b^2} \omega \alpha (\varepsilon + p) \left( 1 + \frac{3}{6} \frac{dp}{d\varepsilon} \right)^2 (1 - \omega/4)
\]
(2.10)
we obtain the time derivative of the Hubble parameter as function of energy density
\[
\dot{H} = -H^2 + \left[ A_2 + \frac{9f_0\omega\alpha}{2bZ} (\varepsilon + p) H^2 \left( D + \frac{f_0\omega\alpha}{2bZ \sqrt{X}} \left( 1 + \frac{3}{6} \frac{dp}{d\varepsilon} \right) \right) \right] \left[ 1 + \frac{3f_0\omega\alpha}{2bZ} D \right]^{-1},
\]
(2.11)
where
\[
A_2 = -\frac{1}{12f_0Z} \left[ \varepsilon + 3p - \frac{(1 - (b/2f_0)(1 + \sqrt{X})^2)}{2\alpha(1 - \omega/4)^2} \right]
\]
(2.12)
and \( H, D, D_1 \) are given by (2.7)-(2.10). With the purpose to analyse HIGS at extreme conditions we transform obtained quantities and equations to dimensionless form by the following way:
\[
x^0 \rightarrow \tilde{x}^0 = x^0 / \sqrt{6f_0\omega\alpha}, \quad H \rightarrow \tilde{H} = H / \sqrt{6f_0\omega\alpha}, \\
\varepsilon \rightarrow \tilde{\varepsilon} = \omega \alpha \varepsilon, \quad p \rightarrow \tilde{p} = \omega \alpha p, \\
S_{1,2} \rightarrow \tilde{S}_{1,2} = S_{1,2} / \sqrt{6f_0\omega\alpha}, \quad b \rightarrow \tilde{b} = b / f_0, \\
R \rightarrow \tilde{R} = R / \sqrt{6f_0\omega\alpha}, \quad \varepsilon' + 3\tilde{H} (\varepsilon + \tilde{p}) = 0,
\]
(2.13)
where prim denotes the differentiation with respect to $x^0$. The quantities $X$ and $Z$ are dimensionless and can be written in the form:

$$X = 1 + \frac{\omega}{b} \left( \frac{1}{b} - 1 \right) - 2 \left( 1 - \omega/4 \right) \frac{1}{b^2} \left( \ddot{\varepsilon} + 3 \ddot{\rho} \right),$$

$$Z = \frac{-\omega/4 + (\ddot{b}/2) \left( 1 + \sqrt{X} \right)}{1 - \omega/4}. \quad (2.14)$$

By using dimensionless value $\tilde{D} = \omega \alpha D$ we can write dimensionless Hubble parameter in the form

$$\tilde{H} = \tilde{H}_\pm = \pm \sqrt{\frac{\tilde{A}_1}{1 + \frac{3}{2bZ} \tilde{D}}}, \quad (2.15)$$

where

$$\tilde{A}_1 = \frac{\ddot{\varepsilon} - 3\ddot{\rho} + \ddot{\varepsilon} + 3\ddot{\rho} + \omega \frac{1 - (\ddot{b}/2)(1 + \sqrt{X})}{b(1 - \omega/4)} \left( 1 - \frac{\ddot{b}}{Z} \right)}{2bZ} - \frac{k}{R^2}. \quad (2.16)$$

By taking into account that the quantity $D_1$ is dimensionless we write the time derivative of the Hubble parameter (2.11) in the following dimensionless form:

$$\dot{\tilde{H}} = -\tilde{H}^2 + \left[ \tilde{A}_2 + \frac{9}{2bZ} (\dddot{\varepsilon} + \dddot{\rho}) \tilde{H}^2 \left( D_1 + \frac{\tilde{D}}{2bZ \sqrt{X}} (1 + 3 \frac{d\tilde{p}}{d\tilde{\varepsilon}}) \right) \right] \left( 1 + \frac{3\tilde{D}}{2bZ} \right)^{-1}, \quad (2.17)$$

where

$$\tilde{A}_2 = -\frac{1}{2Z} \left[ \ddot{\varepsilon} + 3\ddot{\rho} - \omega (1 - \ddot{b}/2)(1 + \sqrt{X})^2 \right]. \quad (2.18)$$

By using obtained equations (2.15)-(2.18) we can investigate HIGS at extreme conditions near limiting energy density. To do this, it is necessary to have the state equation of matter and the values of indefinite parameters. As it was shown by investigation of isotropic cosmology in the frame of GTRC, the most important results can be obtained by the following restrictions on indefinite parameters:

$$0 < 1 - \frac{b}{f_0} \ll 1, \quad 0 < \omega \ll 1, \quad (2.19)$$

and the value of parameter $\alpha^{-1}$ corresponds to some high energy density, by which at cosmological asymptotics $\alpha \varepsilon \ll 1$ By these restrictions the value of limiting energy density $\varepsilon_{\text{max}}$ is of order $(\omega \alpha)^{-1}$.

3 Homogeneous isotropic gravitating systems at extreme conditions

Now we will investigate spatially flat HIGS by using equation of state for ultrarelativistic matter $\ddot{\rho} = \ddot{\varepsilon}/3$. By using restrictions (2.19) for parameters $b$ and $\omega$ we obtain the following

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2 It should be noted that the scale factor $R(t)$ of Robertson-Walker metric is dimensionless and the transformation of $R$ given above is connected with the fact that the coefficient $k$ in term $\frac{k}{\pi}$ does not change by considered transformation.
approximative expressions for $\tilde{H}$ and $\tilde{H}'$:  

$$
\tilde{H} = \tilde{H}_\pm = \pm \frac{\sqrt{2\epsilon X(1+\sqrt{X})}}{4\epsilon + \sqrt{X}(1+\sqrt{X})},
$$  

(3.1)

$$
\tilde{H}' + \tilde{H}^2 = -\frac{2\epsilon \sqrt{X}}{4\epsilon + \sqrt{X}(1+\sqrt{X})} + \frac{64\epsilon^2 X^{3/2}(1+\sqrt{X})}{[4\epsilon + \sqrt{X}(1+\sqrt{X})]^3} \left[ \frac{\epsilon}{X^{3/2}} + \frac{\epsilon}{X(1+\sqrt{X})} + \frac{1}{2\sqrt{X}} \right],
$$  

(3.2)

where $X = 1 - 4\tilde{\epsilon}$. Then the value of limiting energy density following from $X = 0$ is equal to $\tilde{\epsilon}_{\text{max}} = 0.25$ and we have from (3.1)-(3.2) at the first approximation with respect to $\sqrt{X}$ near the limiting energy density:

$$
\tilde{H} = \pm \frac{\sqrt{2X}}{2}, \quad \tilde{H}' = 1 - (3/2)\sqrt{X}.
$$  

(3.3)

that corresponds to the previously received estimate [11]. In the case under consideration ($\epsilon R^4 = \text{const}$) the implementation of the scale factor $R(t)$ can be represented near $\tilde{\epsilon}_{\text{max}}$ as: $R(t) = c_1 \exp(c_2 t^2)$ ($c_1$ and $c_2$ are constant). The state with limiting energy density is achieved in the process of compression of HIGS ($\tilde{H}_-$) being the initial state in the expansion process ($\tilde{H}_+$). The results of numerical analysis of expressions (3.1)-(3.2) near limiting energy density are presented in figures 1–3.

Figure 1. Parameter $\tilde{H} = \tilde{H}_\pm$ as function of $\tilde{\epsilon}$ near limiting energy density: $\tilde{H}_+$ (red line), $\tilde{H}_-$ (blue line).

It follows from figure 1 that the parameter $\tilde{H}_+$ ($\tilde{H}_-$) vanishing at the limiting energy density $\tilde{\epsilon}_{\text{max}}$ reaches its maximum (minimum) value $\tilde{H}_+ = 0.272$ ($\tilde{H}_- = -0.272$) at $\tilde{\epsilon}_1 = 0.57\tilde{\epsilon}_{\text{max}}$.

As follows from figure 2 the derivative $\tilde{H}'$ decreases from its maximum value 1 to zero at $\tilde{\epsilon}_1$. The acceleration parameter $\tilde{H}' + \tilde{H}^2$ is also reduced and vanishes at the energy density $\tilde{\epsilon}_2 = 0.397\tilde{\epsilon}_{\text{max}} < \tilde{\epsilon}_1$. In the interval for the energy density ($\tilde{\epsilon}_2$, $\tilde{\epsilon}_{\text{max}}$), the gravitational interaction has the character of repulsion, and at the density $\tilde{\epsilon}_2$ there is a transition from $\tilde{H}_-$-solution to $\tilde{H}_+$-solution.

3Figure 1 and figure 2 do not reflect the behavior of the corresponding quantities in the region of small values $\tilde{\epsilon} \to 0$, while the formulas are subject to correction. Numerical data are given with an accuracy of 0.001.

4The graphs in figure 2 are valid for both the $\tilde{H}_-$-solution and the $\tilde{H}_+$-solution.
gravitational repulsion to attraction. With a further decrease in the energy density, the negative acceleration parameter reaches its minimum value $\tilde{H}' + \tilde{H}^2 = -0.026$ at $\tilde{\epsilon}_3 = 0.204\tilde{\epsilon}_{\text{max}}$ corresponding to the maximum gravitational attraction force, which, as it decreases, approaches the gravitational attraction force of GR. The transition to the Friedman mode occurs when the value $\tilde{\epsilon}$ becomes much less than $\tilde{\epsilon}_{\text{max}}$ and the value $X$ is approaching 1; then according to (20) $\tilde{H} \sim \sqrt{\tilde{\epsilon}}$, approximately such a transition occurs when $\tilde{\epsilon} = \tilde{\epsilon}_4 \sim 0.001\tilde{\epsilon}_{\text{max}}$. By using the equation of energy conservation in dimensional form (see (13)) we obtain the dependence $\tilde{\epsilon} = \tilde{\epsilon}(\tilde{x}^0)$ at extreme conditions presented in figure 3. Assuming that limiting energy density corresponds to $\tilde{x}^0 = 0$, we find an estimate for the moments of time $\tilde{x}^0_1 = \pm 0.718$, $\tilde{x}^0_2 = \pm 1.056$, $\tilde{x}^0_3 = \pm 1.768$, $\tilde{x}^0_4 = \pm 31.344$ corresponding to $\tilde{\epsilon}_1$, $\tilde{\epsilon}_2$, $\tilde{\epsilon}_3$, $\tilde{\epsilon}_4$. By using obtained data we will estimate the time interval $\Delta t = \Delta x^0/c = (\Delta x^0/c)\sqrt{\tilde{H}_0\omega \alpha} (\Delta x^0 = 2\tilde{x}^0_4)$ of transition from the Friedman compression mode to the Friedman expansion mode. As noted
earlier, the magnitude $\varepsilon_{\text{max}}$ should exceed the energy density in the densest astrophysical objects and be less than the Planck energy density. If we assume that the magnitude of the limiting density is two orders of magnitude higher than the density of a neutron star ($\omega_\alpha \sim 10^{-37} \text{ (kg/ms)}^2$), then we find for the transition time from compression to expansion the following estimation $\Delta t \approx 0.8 \cdot 10^{-3} \text{s}$. With an increase in the limiting energy density up to the Planck energy density ($\omega_\alpha \sim 10^{-113} \text{ (kg/ms)}^2$), the transition time is reduced to a time interval exceeding the Planck time $t_P = 10^{-43} \text{s}$ by one or two orders of magnitude. It should be noted that the order of the estimates obtained does not depend practically on the equation of state of matter ($0 \leq p \leq \varepsilon/3$). After switching to the Friedman regime, the further evolution of HIGS when taking into account changes in the equation of state of matter will correspond to GR until the energy density decreases to values corresponding to the effective cosmological constant $\Lambda = \frac{(1 - b f_0)^2}{66\alpha}$ appearing in cosmological equations in asymptotics that take the form of Friedman equations with $\Lambda$ [9, 10]:

$$k \frac{R^2}{R} + H^2 = \frac{1}{6b} \left[ \varepsilon + \frac{1}{4\alpha} \left(1 - \frac{b}{f_0}\right)^2 \right], \quad (3.4)$$

$$\dot{H} + H^2 = -\frac{1}{12b} \left[ (\varepsilon + 3p) - \frac{1}{2\alpha} \left(1 - \frac{b}{f_0}\right)^2 \right]. \quad (3.5)$$

With the decrease $\varepsilon$ and reversal of the acceleration parameter to zero, the gravitational interaction becomes repulsive and the transition to accelerating mode occurs. The effective cosmological constant in (3.4)-(3.5) induced by the vacuum torsion $S_2^{(\text{vac})} = \frac{1 - b f_0}{12b\alpha}$ leads to the change of gravitational interaction, when energy density is small — the vacuum gravitational repulsion effect, which leads to accelerating cosmological expansion [11].

If the limiting energy density exists in the nature, this should lead to important physical consequences in astrophysics. First of all, we note that within the framework of GTRC the stable astrophysical objects can exist at energy densities significantly lower than the limiting density, at which the gravitational interaction has the character of attraction. The properties of dense astrophysical objects with energy densities comparable to the limiting energy density differ from what GR gives. The fundamental consequence from a physical point of view is to prevent collapse and exclude singular states with divergent energy density. Significant changes in the gravitational interaction in the case of astrophysical objects with energy densities small compared to the limiting energy density take place when their rotational moments interacting with torsion are taken into account. Thus, the interaction of vacuum torsion with the rotational moments of astrophysical objects (stars, galaxies) studied in the frame of minimum GTRC [12] leads to the appearance, in addition to the Newtonian gravitational attraction force, of an additional force caused by their interaction [11, 13, 14]. The magnitude of this force, as well as in general the physical consequences associated with the interaction of torsion with the rotational moments of astrophysical objects, depend on the restrictions imposed on the parameters $b$ and $\alpha$. Taking into account the value of the cosmological constant, following from observations, we obtain a restriction on the parameters $b$ and $\alpha$ while the condition $0 < 1 - \frac{b}{f_0} \ll 1$ takes place. As a result, only one of the parameters $\alpha$ and $b$, let’s say parameter $\alpha$, is undefined. As for the parameter $\omega$, which is important in the case of systems at extreme conditions and does not manifest itself in asymptotics, the restriction $0 < \omega \ll 1$ was used in this paper. It should be noted that, in principle, a theory with a limiting energy density can also be constructed in the case of $\omega \sim 1$. It becomes important to find the values of parameters $\alpha$ and $\omega$ at which GTRC agrees with the observational data.
4 Conclusion

Gauge gravitation theory in Riemann-Cartan space-time with a limiting energy density leads to fundamental physical consequences that make important changes in existing ideas about the world around us. Changing the gravitational interaction under extreme conditions near the limiting energy density makes it possible to solve the problem of cosmological singularity within the framework of classical theory without using quantum notions. The possible existence of a limiting energy density leads to significant changes in the description of dense astrophysical objects, usually identified with black holes. If the conclusion about the existence of a limiting energy density turns out to be true, another constant will be added to the number of fundamental physical constants, which determines the value of the limiting energy density. Within the framework of considered theory, it will be probably a constant $\alpha$.

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