Charged Particle Energization and Transport in Reservoirs throughout the Heliosphere: 1. Solar Energetic Particles

Edmond C. Roelof
Johns Hopkins University/Applied Physics Laboratory, Laurel MD 20723-6099, USA
E-mail: edmond.roelof@jhuapl.edu

Abstract. “Reservoirs” of energetic charged particles are regions where the particle population is quasi-trapped in large-scale (relative to the gyroradii) magnetic field structures. Reservoirs are found throughout the heliosphere: the huge heliosheath (90<r<110 AU) between the termination shock and the heliopause; the domain of PUIs and ACRs within the solar wind itself; the inner solar system (inside of r<5AU) when filled with SEPs or CIRs; closed structures dragged out from the solar corona by CMEs; and the ring currents of the larger planets (Earth, Jupiter, Saturn, etc.) when energized by magnetic storms. Energization (or cooling) of the charged particles within these reservoirs is produced by the interaction when the particle magnetic drifts have a component along the large-scale electric fields produced by plasma convection. The appropriate description of this transport is “weak scattering”, in which the particle’s first adiabatic invariant (magnetic moment) is approximately conserved while the particle itself moves rather freely along magnetic field lines. Considerable insight into the observed properties of energization processes can be gained from a remarkably simple equation that describes the particle’s fractional time-rate-of-change of momentum (d(lnp)/dt) which depends only upon its pitch angle, the divergence of the plasma velocity (V⊥) transverse to the magnetic field, and the inner product of (V⊥) with the curvature vector of the field lines. The possibilities encompassed in this simple (but general) equation are quite rich, so we restrict our application of it in this paper to the compressive acceleration of SEPs within CMEs.

1. Introduction
Here’s what this paper on energetic charged particle acceleration will and will not discuss. It will introduce a new expression for the fractional time-rate-of-change of momentum (d(lnp)/dt) for single particle motion in electromagnetic fields in which the first adiabatic invariant, or “magnetic moment” M=p²(1-μ²)/B, is conserved and parallel electric fields are neglected. This is the familiar “guiding center” approximation for individual particle motion. Consequently, it will not deal with the important microphysics of wave/particle interactions, acceleration within the immediate region of shocks, or magnetic reconnection. The results of this paper will therefore apply to regions away from such activity, although, if appropriate, those aforementioned processes could be also be considered as sources for energization regions where the guiding center acceleration approximation is valid. It also will not attempt to describe the formation of the “seed” populations, i.e., the energetic charged particle populations that exist in the corona prior to the lift-off of the CME and will then be accelerated by the mechanism to be described in this paper. This obviously is an important, but somewhat separate issue.

In this era of sophisticated computational plasma dynamics, it may seem rather naive to invoke a guiding center equation within a large region of space. However, observations over decades have pointed to large regions wherein the wave/particle interactions are either intrinsically weak, or wherein the effects of pitch-
angle scattering are dominated by the focusing effects of the large-scale magnetic fields. For instance, we have previously argued (based on observations) that the inner heliosphere (r<1AU) is just such a “weak-scattering” or nearly “scatter-free” region for energetic solar electrons and galactic cosmic rays [1]. The essential condition for applying guiding center theory to particle acceleration is that the magnetic moment is approximately conserved over the time scale (τ) of the acceleration itself, where 1/τ=dlnp/dt. An a priori validation of this assumption would require a simulation based on the actual plasma turbulence within the weak-scattering region. We don’t always have such measurements. On the other hand, even if we don’t have such plasma measurements, we can establish an a posteriori justification of the weak-scattering assumption in remote regions by demonstrating that the implications that follow from invoking the assumption are in agreement with observations. We will take the latter approach in this paper with regard to our use of the guiding center expression for dlnp/dt.

2. New form of the guiding center equation for individual particle acceleration

An equation was presented 15 years ago that provides considerable physical insight into the acceleration process for energetic charge particles [2,3]. It is very simple and yet quite general (requiring only the conservation of the magnetic moment with negligible parallel electric fields). It describes the evolution of the particle momentum (p) along a guiding center path. It is valid for both non-relativistic and relativistic energies and time-dependent plasma flows and magnetic fields.

\[ \frac{d\ln p}{dt} = \frac{(1-\mu^2)}{2} \frac{D\ln B}{dt} + \mu^2 \mathbf{V} \cdot \mathbf{\kappa} \]  

(1)

Here (\(\mathbf{\kappa}\)) is the curvature vector of the magnetic field (B) and the “transverse convective operator” is defined as

\[ \frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{V} \cdot \nabla \]  

(2)

Equations (1) and (2) were a re-formulation (in terms of momentum) by Roelof [2,3] of the well-known equation presented by Northrop [4] in terms of energy. That re-formulation is reproduced in the first part of Appendix A. The derivation neglects any effects due to the parallel component of the electric field along the magnetic field (\(\mathbf{E}=\mathbf{b} \cdot \mathbf{E}\)). The remaining transverse electric field is taken to be \(\mathbf{E}=\mathbf{\nabla} \times \mathbf{B}\). This is why only the transverse plasma velocity \(\mathbf{V}=\mathbf{V}-\mathbf{b}=(\mathbf{I}-\mathbf{bb})\mathbf{V}\) appears in the equation for guiding center motion; obviously, \(\mathbf{V} \cdot \mathbf{\kappa} = \mathbf{V} \cdot \mathbf{\kappa}\). As was pointed out by Northrop [4], the basic physical mechanisms for the acceleration are a combination of the well-known betatron acceleration (in a time-varying magnetic field) plus the work done on a charged particle when its guiding-center drift velocities (due to gradients and curvature of the magnetic field with spatial scales larger than the particle’s gyro-radius) have components that lie along the local transverse electric field. Consequently, the latter mechanism will operate unless pitch-angle scattering is so strong and rapid as to degrade the particle’s “memory” of the magnetic gradients and curvature. As stated in the Introduction, we therefore assume that this is not the situation in the regions that we are considering in this paper.

Two important points were made in the re-formulation. First, no properties of the particle itself (mass, charge, or energy) appear on the RHS of the momentum equation [2,3]. Therefore, every charged particle (protons, ions, or electrons) with the same pitch-cosine (\(\mu\)) gains (or loses) momentum at the same fractional rate, since \(d\ln p/dt=(1/p)(dp/dt)\). Second, through Faraday’s law, the time-dependence of the magnetic field can be eliminated from the transverse convective operator [3]. However, it was not realized at the time of writing how simple the remaining expression is when this is carried out. We derive the new equation in Appendix A. It contains a rigorous new expression for the transverse convective derivative

\[ \frac{D}{Dt} \ln B = -\mathbf{\nabla} \cdot \mathbf{V} - \mathbf{V} \cdot \mathbf{\kappa} \]  

(3)
Even though only spatial operators appear on the RHS, the new expression for the convective fractional derivative of the magnetic field strength (\(B\)) is nonetheless fully valid for time-dependent magnetic fields, just as long as the electric field is given by \(E = -\nabla \times B\). Simple substitution of Eq. (3) into Eq. (1) yields our new (general) equation for charged particle acceleration.

\[
d\ln p/dt = -(1/2) (\nabla \cdot V + \gamma \cdot \kappa) + \mu^2 \gamma \cdot \kappa
\]

\[
= -(1/3) \nabla \cdot V + (3\mu^2 - 1)/2 [(1/3) \nabla \cdot V + \gamma \cdot \kappa]
\]

The coefficient of the 2\(^{\text{nd}}\) term in Eq. (4') will be recognized as \(P_2(\mu)\), the 2\(^{\text{nd}}\) order Legendre polynomial. Notice the dependence on pitch cosine.

\[
d\ln p/dt = -(1/2) (\nabla \cdot V + \gamma \cdot \kappa) \quad \mu=0
\]

\[
d\ln p/dt = -(1/3) \nabla \cdot V \quad |\mu|=(1/3)^{1/2}
\]

\[
d\ln p/dt = \nabla \cdot \kappa \quad |\mu|=1
\]

Thus, the logarithmic acceleration rate behaves like 1\(^{\text{st}}\)-order Fermi (curvature) acceleration for field-aligned velocities \(|\mu|=1\) and like (transverse) compressive deceleration (“cooling”) at an intermediate pitch cosine \(|\mu|=(1/3)^{1/2}\), the root of \(P_2(\mu)\). There is an equal contribution from both terms for mirroring particles \(\mu=0\). Otherwise, there is a pitch-angle-weighted mixture of both at all other pitch cosines. Nonetheless, the simple average of \(d\ln p/dt\) over all pitch cosines is identical to the value at \(|\mu|=(1/3)^{1/2}\), because the average of \(P_2(\mu)\) over pitch cosines \((\mu)\) is zero.

The tabulation above reveals that Eq. (4) is very rich in possibilities for charged particle acceleration. For instance, the terms involving the field line curvature vector \((\kappa)\) immediately suggest an acceleration in “depolarization” events in planetary magnetospheres (Earth, Jupiter, Saturn) in which a “stretched” magnetic field relaxes to a more dipolar configuration by means of an inward plasma convection. In such a configuration we would have \(\kappa \cdot V=0\), and we would also expect an accompanying compression \((\nabla \cdot V<0)\), particularly near the plane of the dipole equator. In fact such acceleration may well favor the generation of strong bi-directional field-aligned beams of energetic electrons or ions, because for pitch angles near 0\(^{\circ}\) or 180\(^{\circ}\), the Fermi acceleration term dominates. This acceleration could operate on time scales less than a bounce time, because it requires only the conservation of the first adiabatic invariant (for the perpendicular energy), rather than the familiar (but possibly slower) conservation of the second adiabatic invariant \((dE/d\gamma_p)\) that is defined over a bounce time and (by definition) varies slowly.

However, in order to quantify such a suggestion for net acceleration, we need to know how long the particle experiences each possible pitch angle because \(d\ln p/dt\) depends strongly on pitch-cosine. This is equivalent to saying that we must have an equation that properly describes the global transport of the energetic particles in order to quantify the predictions of our individual-particle acceleration equation from Eq. (4). This is a non-trivial requirement. For instance, in magnetospheres, we must allow for pitch-angle anisotropies that vary strongly along an instantaneous magnetic field line (e.g., the bi-directional beam injection events). Consequently, we will leave the magnetospheric applications to future work. Right now we will concentrate on the simpler case of acceleration in which the curvature term does not come into play, because we have reason to believe that the pitch-angle distribution will be nearly isotropic. Then the contribution of \(P_2(\mu)\) averages out in Eq. (4'), leaving only the simple expression

\[
d\ln p/dt = -(1/3) \nabla \cdot V. \quad (4'')
\]

3. Description of general transport within energetic particle “reservoirs”
In a pair of papers, Roelof [5,6] derived a transport equation that describes the post-maximum phase of solar energetic particle events, when the combination of nearly scatter-free propagation along field lines and magnetic mirroring inside 1AU combined with back-scatter beyond 1AU reduces both the field-aligned intensity gradient and field-aligned pitch-angle anisotropy to small values. These conditions produce a particle “reservoir”. We will argue that similar conditions exist within the “nose” of an outgoing CME.

\[ \partial \ln f / \partial t + [\xi v + \mathbf{v} - \mathbf{r} \times (\mathbf{B}/\Omega)] \cdot \nabla \ln f - (1/3)(\nabla \cdot \mathbf{v}) \partial \ln f / \partial s = - \mathbf{v} \cdot \nabla (\xi / \mathbf{B}) \]

The omnidirectional (p.s.d.) phase-space density \( f_0(\mathbf{r}, p, t) \) and the parallel anisotropy function \( \xi_0(\mathbf{r}, p, t) \) are written here simply as \( f_0 \) and \( \xi_0 \), respectively, in Eq. (5). Since the p.s.d. is by definition non-negative, we have divided the original version (linear homogeneous in \( f \)) by \( f_0 \) itself, converting the dependent variable to \( \ln f \), thereby leaving a non-homogeneous term on the RHS that is independent of \( f \). The parallel spatial derivative along a field line \( \partial / \partial s = \mathbf{b} \cdot \nabla \) in the non-homogeneous term is defined by the unit vector \( (\mathbf{b} = \mathbf{B}/\mathbf{B}) \) direction of the magnetic field. It describes magnetic focusing of the pitch angle.

The physical content of the RHS of Eq. (5) is clear. The coefficient of the term describing particle acceleration \( \partial \ln f / \partial t \) is precisely the compressive acceleration term given by Eq. (4’), and the drift velocities depend on the parameter \( (\epsilon) \) itself, converting the dependent variable to \( \ln f \), thereby leaving a non-homogeneous term on the RHS that is independent of \( f \). The parallel spatial derivative along a field line \( \partial / \partial s = b \cdot \nabla \) in the non-homogeneous term is defined by the unit vector \( (b=B/B) \) direction of the magnetic field. It describes magnetic focusing of the pitch angle.

4. SEP acceleration: Observational characteristics

Certain simple patterns in the behavior of solar energetic particles (SEPs) emerge when only magnetically “well-connected” events are selected for study. These are the events in which the spacecraft happens to be on interplanetary field lines that are connected at the time of the first-arriving SEPs to the location marker on the Sun that is customarily taken as the site of the largest flare (observed in Ha, EUV, x-rays, etc.) having the closest onset time association with the SEP onset. The solar longitude of the foot-point of the magnetic field line \( (\phi_0) \) is usually estimated using the Parker relation \( \Phi_0 = \Phi + \Omega R / V \), where \( (\Phi) \) is the longitude of the spacecraft, \( (R) \) is its radial distance, and \( V \) is the solar wind velocity at the time of the observation. To good approximation (for observations at 1 AU), \( R \) is the distance from the center of the Sun, and it matters little whether ecliptic or heliospheric longitudes are used. As pointed out explicitly in the papers that first justified this connection relation [7,8], it does not require that the field line be a Parker spiral all the way back to the corona, but only that the plasma element (into which the field is “frozen”) did not undergo significant acceleration (or deceleration) during its transit to the spacecraft.

Customarily, “well-connected” is taken to mean that the longitude of the foot-point of the field line is within \( \sim 30^\circ \) of the flare longitude. On such field lines, the pitch-angle distribution of fast SEPs \((v>0.1c)\) during the rise-to-maximum phase is usually strongly anisotropic, with a beam-like intensity distribution that can be characterized by an exponential in pitch cosine \( j=j_0 \exp (a \mu) \), where the parameter \((a<1)\) is a measure of the anisotropy. Such a pitch-angle distribution was anticipated as a consequence of the “focusing” effect of the \( \sim 1/r^2 \) Parker field overcoming any pitch-angle scattering while the particle is en route to 1 AU [9]. The implication of such a p.a.d. is that most of the first-arriving particles are essentially
traveling along the field line ($v_//=\mu v=\nu$), and so they must have been emitted at a time earlier by $\Delta t=s/v$ than their arrival time. For 250 keV electrons (with $v=0.7c$), this is about 10 minutes at 1 AU (where $s=1.2$ AU). Thus, during the portion of the rise-to-maximum phase of a 250 keV electron event while these beam-like anisotropies are observed, we can construct the time histories of the injection process on a time scale considerably less than 10 minutes. Analysis of such events led to the discovery that these fast electrons commenced their interplanetary injection with delays of as much as 20 minutes after the type III radio emission from the associated flares [10]. An interpretation offered (and quantitatively supported by comparing SEP electron injection times with SOHO/LASCO observations of the velocity histories of the associated CMEs), was that the injection did not commence until the leading edge of the CME had attained an altitude of several solar radii into the corona [11].

![Figure 1](image.png)

**Figure 1.** A continuum of beam-like injection profiles observed for the ~200 magnetically well-connected 40-300 keV ACE/EPAM electron events during Solar Cycle 23. From [12].

Haggerty and Roelof [12] made a comprehensive study of the patterns that emerge from the compilation the properties of ~200 magnetically well-connected 40-300 keV electron events during Solar Cycle 23. Examples of electron beam-like pitch-angle anisotropies are presented in the upper panels of Figure 1, in which measured ACE/EPAM intensities are plotted for four different directions around a 30° cone in the Sunward hemisphere. Small intensity events often appeared as “spikes”, symmetric in their rise and fall and lasting <20 minutes in all. Moderate intensity events had a “pulse” shape, with a decay much slower than the rise. Large intensity events, although somewhat slower to rise than pulses, had a “ramp” shape, with a plateau or even a continued increase lasting at least an hour. When ~200 such well-connected beam-like electron events were analyzed over the complete Solar Cycle 23 [12], we found a continuum of such shapes, orderly enough for us to assign a “time profile index” ranging from 0 for a spike to 1 for a pulse and >2 for a ramp (as shown in the lower panel of Fig. 1). Figure 2 compares the distributions of peak intensity spectral index, peak intensity itself, injection
Figure 2. Essential properties of beam-like injection profiles (see Fig. 1). From [12].

Delays (at the Sun) from 14 MHz type III radio bursts, and plane-of-sky speed of the associated CME. Although spikes, pulses and ramps exhibit somewhat similar distribution of all these parameters, the ramps have harder spectra, higher peak intensities, longer injection delays, and are associated with the fastest CMEs.

Figure 3. Maximum intensities of near-relativistic electrons vs. those of 1.8-4.7 MeV protons in magnetically well-connected beam-like electron events. From [12].

How can we explain consistently such an over-all orderly pattern in this continuum of well-connected electron events? Interplanetary diffusion cannot have a significant effect on the event histories, because there are beam-like pitch-angle anisotropies throughout the rise-to-maximum phase of the well-connected events. Magnetic reconnection in the corona might be a viable explanation for the spikes, but because it is inherently a sporadic process, one would have to invoke a well-behaved “ensemble” of many such
reconnection events over several hours (lasting while the CME was in the high corona) in order to produce the relatively smooth profile of the ramps. On the other hand, CME-driven shocks might explain the ramps, but in order to produce the spikes, one would have to invoke a shock that “switched” on and off in only a few minutes. Finally, if one claimed that the sources of the spikes were magnetic connection while that of the ramps were high coronal CME-driven shocks, then why did we find a smooth continuum of time profiles from shocks through pulses to ramps throughout Solar Cycle 23?

Many aspects of the pattern found for the near-relativistic electrons were also found during Solar Cycle 23 in ~20 SEP well-connected beam-like proton events (20-100 MeV) within about the same velocity range (0.2c<v<0.5c) as the ACE/EPAM electrons. These strong proton pitch-angle anisotropies were measured by the two-dimension detector SOHO/Erne [12]. Detailed fitting of the rise-to-maximum of both the proton and electron intensities near 0° pitch angle (in the same ~20 events) revealed similar (although not identical) injection profiles, along with correlated injection delays. Even for lower energy ions (1.8-4.7 MeV, 0.006<v<0.010c), the peak intensities in well-connected events with beam-like near-relativistic electron anisotropies are correlated (r=0.842) to within an order of magnitude [12]. See Figure 3.

![Figure 3](image)

**Figure 3.** Snapshots of solar wind flow speed and pressure as functions of heliocentric distance at different times during the outward evolution of a high-speed stream driven by a slowly-increasing pressure pulse. Calculated from a simple 1-D gas dynamic code. Adapted from [13] and [14].

**5. Suggested mechanism:** Acceleration within coronal mass ejections
The implication of the previous Section is that the simple patterns observed in the injection histories for both near-relativistic electron and proton events demand a correspondingly simple and general acceleration/injection mechanism that must produce similar injection histories (and delays) for both protons and electrons. The rest of this paper will describe a mechanism that we believe can satisfy these nearly all-encompassing observational properties.

There is a fundamental property of the launch of CMEs through the corona out into the solar wind: they are driven by over-pressure that produces a radial plasma velocity profile that steadily steepens until (if it is fast enough) it drives a shock. Figure 4, adapted from [13] and [14], portrays a simple 1-D calculation of a relatively modest CME by Hundhausen [13] in which the temperature (and thereby the pressure) was increased and decreased by a factor of four at the inner boundary over 100 hours. The significance for us lies in the general shape of the radial velocity profile.

Consider a generic CME while it is only a solar radius or so above the solar surface. After it is launched, there will be a steady steepening of the velocity-radius profile. For fast CMEs, the steepening will eventually form a shock. What is important to us is that there is always a region inside the leading edge of the disturbance where \( \partial V_r/\partial r < 0 \). We will assume in this region that: (i) the SEP pitch-angle distribution is nearly isotropic; and (ii) the swept-up magnetic field is draped across the evolving front, i.e., it is strongly non-radial. Now consider the implications of our acceleration Eq. (4") and follow along with the aid of Figures 5 and 6.

**Figure 5.** The basic conditions for compressive acceleration of SEPs within the driver plasma of CMEs.

5.1 The acceleration time
Since \( \mathbf{V} = \mathbf{V}_\perp + \mathbf{V}_\parallel \mathbf{b} \), we can write

\[
\nabla \cdot \mathbf{V}_\perp = \nabla \cdot \mathbf{V} - B \partial (V_\parallel / B)
\]

so that
\[ \frac{d \ln p}{dt} \approx -\frac{1}{3} \nabla \cdot \mathbf{V} \approx -\frac{1}{3} \left( \frac{1}{r^2} \partial r (r^2 V_r) - B \partial (V / B) / \partial s \right) \]

In the nose region of the CME, the plasma velocity may be taken to be predominantly radial. Because of assumption (ii), we can assume that \( V \approx V_r \) and also that \( |V| \ll V_r \).

\[ \nabla \cdot V_r \approx \frac{1}{r^2} \partial r (r^2 V_r) - B \partial (V / B) / \partial s \]

\[ = \partial V / \partial r + 2 V_r / r - \partial V / \partial s + V \partial \ln B / \partial s \]  \( (6) \)

Since the nose region is broad, neither \( (V_r / B) \) nor \( \ln B \) should vary strongly along the field, and \( V_r \) is already a small quantity compared to \( V_r \). So we can neglect those terms in this important region.

The term \( \nabla \cdot V \) will therefore be **negative** (implying acceleration, not deceleration) if \( \partial V / \partial r < -2 V_r / r \), so if we define a gradient scale length \( (L_r) \) for the decreasing radial velocity by \( \partial \ln V_r / \partial r = -1/L_r \), then

\[ \nabla \cdot V_r \approx -V_r (1/L_r - 2/r) \]  \( (7) \)

Thus the SEPs will be **accelerated** (because \( \nabla \cdot V < 0 \)) in the nose region of the shock whenever \( L_r < r / 2 \) (half the radial distance from the center of the Sun).

We can get an estimate of the characteristic time for energization from the guiding center equation for \( d \ln p / dt \) (averaged over all pitch cosines for consistency with the assumed isotropy in the nose region). Defining this **growth** time as \( 1/\tau \), we see that

\[ (-1/3) \nabla \cdot V_r = 1/\tau = (-V_r / 3)(1/L_r + 2/r) = (V_r / 3 R_S) (R_S / L_r - 2 R_S / r) \]  \( (8) \)

To pick some numbers, let’s take \( V_r = 1000 \text{ km/s} \), so that \( V_r / 3 R_S = (1/2100) \). Then in a nose region where the radial velocity gradient scale was \( L_r = (1/10) R_S \) at \( r = 2 R_S \), we obtain the e-folding time for momentum increase:

**Figure 6.** The global topology of CMEs that favors compressive acceleration of SEP near the nose.
\[ \tau = \frac{(2100s)}{(10^{-1})} = 3.9 \text{ minutes} \]

The e-folding time for energy increase alone (non-relativistic) will be half of this (2 minutes), or the same (4 minutes) for fully relativistic particles. Of course, this guiding center estimate assumes that the particle has remained in the nose region for the entire e-folding time. The very magnetic drifts that energize (or de-energize) the particle will also transport the particle (transversely) within the energization region. We now turn to that important issue.

**Figure 7.** The basic properties of compressive SEP acceleration within CMEs that explain the simple patterns of magnetically well-connected SEP events (spikes, pulses and ramps) over the past Solar Cycle. See Figs 1, 2, and 3.
5.2 The “residence” times
The question of whether the net energization of the SEPs that escape upstream of the CME is positive or negative would be most properly answered by solving the time-dependent reservoir transport equation that includes processes such as escape from the region due to parallel anisotropy ($\tilde{\xi}$) or transverse motion across field lines due to magnetic curvature and gradient drifts. The $\mathbf{E} \times \mathbf{B}/B^2$ drift is, in this transverse (draped) magnetic field geometry, essentially the radial plasma velocity ($V_r$) that moves with the CME front. However, we can make some rough estimates of whether the SEPs are contained long enough to be significantly accelerated just from the guiding center equations. The usual expressions for the gradient and curvature drifts are

$$V_D = (pv/QeB) \mathbf{b} \times [(1-\mu^2)/2 \nabla \ln B + \mu^2 \mathbf{k}]$$  \hspace{1cm} (9)

We can express $pv/QeB=(A/Q)\gamma \beta^2 c^2/\Omega_0$, where $\Omega_0=eB/m_0$ is the non-relativistic gyro-frequency for protons (for electrons, multiply $\times 1836$). We’ll do the calculation for protons (or electrons), because the ion factor $(A/Q)$ can be folded in at the end. To good approximation we have the “rule-of-thumb” $\Omega_0(s^{-1}) \approx B(\text{nT})/10$, so $c^2/\Omega_0 \approx 9 \times 10^{11} \text{ km}^2 \text{s}^{-2}/\text{B(nT)}$. We make the rough estimates $|\nabla \ln B|/L_B$ and $|\mathbf{k}|=\kappa$, and we take an isotropic average over the pitch cosine. The application of the inequality $|\mathbf{b} \times (\mathbf{g}+\mathbf{h})|<|(\mathbf{g}+\mathbf{h})|<|\mathbf{g}|+|\mathbf{h}|$ to the vector cross-product yields

$$V_D \leq (\gamma \beta^2 c^2/\Omega_0)(1/L_B+\kappa)$$  \hspace{1cm} (10)

Then the distance a particle drifts due to magnetic field gradient and curvature during the (isotropic) acceleration time ($\tau$) estimated in the last section is bounded from below by

$$[V_D] \tau \leq (\gamma \beta^2 c^2/\Omega_0) (1/3)(1/L_B + \kappa)/[(V_r/3)(1/L_r - 2/r)]$$

$$= (\gamma \beta^2 c^2/\Omega_0 V_r) (L_r/L_B + \kappa L_r)/(1-2L_r/r)$$  \hspace{1cm} (11)

We have assumed again that the scale of the radial velocity gradient is much smaller than the radial distance of the acceleration region ($L_r << r$). Scaling the radial plasma velocity ($V_r$) by 1000 km/s (as in the previous section) and $V_D \tau$ by 1 $R_S=7 \times 10^9$ km, we obtain the lower limit

$$(V_D \tau)/R_S \leq (\gamma \beta^2)/(B(\text{nT})(9 \times 10^{10} \text{ km}^2 \text{s}^{-1})/[(1000 \text{ km}^3/s)(7 \times 10^9 \text{ km})](1000 \text{ km/s}/V_r) (L_r/L_B + \kappa L_r)$$

$$= (\gamma \beta^2) (130)/B(\text{nT})(1000 \text{ km}^3/V_r) (L_r/L_B + \kappa L_r)$$  \hspace{1cm} (12)

If the leading edge of the CME is inside of $r<0.1$ AU, then $B(\text{nT})>500$ nT and the factor $130/B(\text{nT})<0.3$. If we assume that all of the magnetic inhomogeneity scales ($L_B$ and $1/\kappa$) are comparable to the gradient scale of the radial velocity ($L_r$), then for a CME with $V_r=1000$ km/s well inside $r<0.1$ AU $\approx 20 R_S$ we conclude that the transverse magnetic drift distance ($V_D \tau$) during one momentum e-folding time ($\tau$) is

$$(V_D \tau)/R_S << \gamma \beta^2$$  \hspace{1cm} (13)

Consequently even a relativistic proton ($\gamma>1$ and $\beta=1$) should drift only a small fraction of a solar radius ($R_S$) in a momentum e-folding time. Non-relativistic particles will drift only a tiny fraction of a solar radius. For multiply-charged ions we just include the factor $(A/Q)$ on the RHS of Eq. (13). Therefore the only other confinement constraint involves the parallel (presumably weak-scattering) transport. However, the global topology of the leading portion of the CME is generally regarded as magnetically closed ($e.g.$,
“light-bulb”, “ice-cream cone”, lifting flux rope). Moreover, when the particle leaves the acceleration region in the leading portion of the CME (where $\partial \ln V_\perp / \partial r < -2/r$), it will encounter the much stronger magnetic fields at lower altitudes. Outside the nose region, this is a mirror confinement regime and is completely consistent with the reservoir requirement of weak field-aligned intensity gradients. Consequently, it would appear from the foregoing discussions that an efficient acceleration region for SEPs exists within the structure of outgoing CMEs.

6. Compression acceleration signatures in coronal images
We mention the intriguing possibility that the SEP acceleration time ($\tau$) could be extracted directly from coronal white light imagers. This possibility is justified in a companion paper from this Conference [15], in which we argue that an estimate of the plasma compression ($-\nabla \cdot V$) can be extracted from outward-propagating features within the CME, i.e., within the driver of the CME shock. The extraction follows simply from the integration of the equation for the electron density ($N$), i.e., for the conservation of electrons. Upon expansion and re-arrangement,

$$\frac{\partial N}{\partial t} + \nabla \cdot (nV) = 0 \quad \text{becomes} \quad D\ln N/ Dt = -\nabla \cdot V$$

(14)

where $D/ Dt = \partial / \partial t + V \cdot \nabla$ is the Lagrange convective derivative for a function co-moving (but evolving) with the (full) plasma velocity ($V$). Carrying out the implied integration, the average value of the compression along the trajectory of the centroid of the feature is

$$< -\nabla \cdot V > = (1/\Delta t) \ln \{ N[r(t+\Delta t), t+\Delta t] / N[r(t), t] \}$$

(15)

In the companion paper, the local electron densities are related to the brightness in the white light coronagraph pixels. We shall therefore proceed in this paper assuming that we have a measurement of the compression $< -\nabla \cdot V >$ for the outward-propagating density feature.

As we just showed in the previous Section, we have the identity

$$\nabla \cdot V = \nabla \cdot V + \nabla \cdot (BV / B) = \nabla \cdot V + B \partial (V \parallel / B) / \partial s$$

This equation becomes really useful if we can make an estimate of the divergence of the parallel plasma velocity, or even more simply (as in the geometry of the nose region of the CME), we can argue that $V \parallel$ is negligible compared to the radial velocity: $V \approx V_r$, so that $V \perp << V_r$. Then we can approximate the average value of the acceleration rate along the path of the density maximum in the brightest pixels of images taken over a time interval $\Delta t = t_2 - t_1$.

$$< 1/\tau > = (1/\Delta t) \int_0^2 dt / \tau [r(t), t] \approx (1/3\Delta t) \int_0^2 dt (-\nabla \cdot V) = -(1/3\Delta t) \ln (N_2 / N_1)$$

(16)

In the companion paper [15], we suggest how the electron densities $N_1 = N(r_1, t_1)$, $N_2 = N(r_2, t_2)$ can be extracted from the normalized pixel brightnesses in sequential coronagraph images. Note that any linear instrument calibration and response factors will cancel out of the density ratios. Also note that we do not have to estimate: either the plasma velocity ($V$); or the time derivative ($\partial N / \partial t$); or the spatial gradient ($V N$) of the electron density! All we have to do is track the feature’s normalized brightness in the images. Of course, we must still estimate of the maximum electron density in the feature. A method is suggested in [15]. If this approach proves feasible, it will give us observational “closure” for well-connected SEP events -- when we have concurrent coronagraph images that capture the radial brightness histories of density features in the associated CME.

7. An observation of compressive acceleration of SEPs at 1AU
We have been discussing compressive SEP acceleration within CMEs escaping through the corona, but we now believe that there was a fortuitous observation by the ISEE-3 spacecraft of an atypical SEP event (which happened to be among the largest of Solar Cycle 21) that was undergoing compressive acceleration within an ICME (interplanetary CME) at 1AU [16]. The report was titled “A Major Solar Shock-Associated Energetic Storm Particle Event Wherein the Shock Plays a Minor Role”, and the unusual configuration of the event is summarized in Figure 9. Because of the excellent angular coverage of the Low Energy Telescope, we could deduce from the beam-like pitch-angle distributions of the ~0.10-1.0 MeV ions that they were passing adiabatically through the shock, from the downstream region into the upstream region, with no signature of local shock acceleration. In addition, for the first quarter hour (~0.003 AU) behind the shock, the “slipped” layers of non-Parker-oriented magnetic field also contained ion beams that had to be moving along field lines that would eventually cross the shock. Further behind, the pitch-angle distributions were nearly isotropic, suggesting that this was the region that was the immediate source of the particle beams observed behind (and passing through) the shock.

**Figure 8.** Stylized visualization of the shock/multiple-magnetic-discontinuity ensemble that controls the SEPs. All structures are populated with ambient particle fluxes that are strongly anisotropic, while intense particle beams are observed both upstream and downstream of the shock. The measured pitch-angle distributions imply that the particles are passing adiabatically through the shock (with no acceleration). From [16].

We now (30 years later) know what to look for, and we find it in Figure 9 (taken from the original paper) [16]. The key is the solar wind velocity history (V_{SW}). It jumps from ~300 km/s (upstream) to only ~400 km/s immediately behind the shock. This small jump is consistent with the observed lack of measurable shock acceleration of energetic ions. However, the downstream velocity continues to rise during the next 2 hours (~0.030 AU behind the shock at an average velocity of 600 km/s) to as much as 750 km/s. This implies a strongly negative fractional velocity gradient \( \frac{\partial \ln V_r}{\partial r} = \frac{\ln(750/400)}{0.03} \) AU, yielding (in the
notation of the previous Section) a scale length for the negative gradient of $L_r \approx 0.05$, so that $L_r << r/2 = 0.50$ AU. We thus estimate that the SEPs were being accelerated in this interplanetary compression region with average characteristic time $\tau = L_r/V_r \approx (7.5 \times 10^6 \text{ km/s})/(600 \text{ km/s}) \approx 3.5 \text{ hr}$. Since this is very short compared to the transit time from the Sun for the ICME, we believe that it indicates efficient compressive acceleration in this ICME at 1AU, and we therefore take it as in situ evidence in support of the proposed mechanism of compressive acceleration in CMEs back at the Sun.

**Figure 9.** Energetic particle, magnetic field and plasma data averaged over 10-minute intervals from ISEE-3 during 6-7 June 1979. The estimate of the acceleration time ($\tau$) is explained in the text. From [16].

8. Discussion and Summary
We have introduced a general guiding center equation for fractional momentum acceleration \( (d\ln p/dt) \) that: is independent of particle identity (mass, charge, energy); is valid for non-relativistic and relativistic particles; and holds for time-dependent electric and magnetic fields. The acceleration is produced by a combination of betatron acceleration and work done on the particle when its drift velocity in the gradient and curvature of the magnetic field has a component along the transverse electric field. Under the conditions within a CME (e.g., pitch-angle isotropy), it takes the form \( d\ln p/dt = -(1/3)\nablav \cdot \mathbf{V} \), so that acceleration occurs as a result of the compression of the plasma velocity component transverse to the magnetic field. For reasonable CME parameters, we obtain acceleration rates \( 1/\tau = d\ln p/dt \) on the order of a few minutes. We have presented corroboration for this mechanism from the unusual circumstance in which it was observed \textit{in situ} by ISEE-3 during an SEP event at 1AU in June 1979.

Compressive acceleration, in its general concept, has appeared in many forms in the literature, \textit{e.g.}, [17] and [18], but it usually also invokes meandering of field lines or turbulent motions of the plasma and/or pitch-angle scattering (none of which are required for our proposed mechanism). Although they may indeed be operative in a CME, none of these manifestations makes the very simple prediction that the acceleration itself is \textit{independent} of all particle properties, depending only on the divergence of the (transverse) plasma velocity. In brief, we suggest that our new Eq. (4') has the very general properties required to explain the rather simple but broadly observed patterns in the rise-maximum phase of SEP events. The main variation in their time histories (spikes, pulses, ramps) is the result of how long the compression region lasts in the outgoing CME (see Fig.7), while the secondary variations in those profiles among different particle species may well be the consequence of their different “residence times” in the accelerating region which may be determined by their magnetic field drift velocities (which depend only on energy/charge). Finally, we should bear in mind the point made in the Introduction, that the final SEP spectrum may also depend on the properties of the “seed particle” population upon which the proposed compressive acceleration mechanism acts.

9. Acknowledgments.

The efforts of E. C. R. were supported by the NASA ACE and Voyager Interstellar missions under contracts NNX10AT75G and NNN06AA01C, respectively. This synthesis of observations was made possible by the many papers published in this century by Robert P. Lin and the University of California/Berkeley group, George M. Simnett and the University of Birmingham group, and our own group at the Johns Hopkins University/Applied Physics Laboratory.

10. Appendix. Derivation of the new equation for individual particle motion

Northrop’s formulation \[4\] of the guiding center equation for energy \((W)\) changes began with a non-relativistic equation

\[
\frac{dW}{dt} = qdR/dt \cdot \mathbf{E} + M_d\partial B/\partial t
\]

where the non-relativistic magnetic moment is \( M_0 = m_0 v_\parallel^2/2B = m_0 v_\parallel^2(1-\mu^2)/2B \) and the perpendicular drifts are

\[
\frac{dR}{dt} = \mathbf{E} \times \mathbf{B}/B^2 + (m_\parallel qB) \mathbf{b} \times (v_\perp^2/2 \nablav + v_\perp^2 \partial b/\partial s)
\]

Relativistically, the gradient and curvature drifts are increased by the relativistic factor \((\gamma)\) because of the corresponding increase in the particle’s gyroradius. In the limit of \( E \), “small”, \textit{i.e.}, \( V << c \), the dominant terms are still just the relativistic generalizations of the non-relativistic expressions.

\[
\frac{dR}{dt} = \mathbf{E} \times \mathbf{B}/B^2 + (\gamma m_\parallel qB) \mathbf{b} \times (v_\perp^2/2 \nablav + v_\perp^2 \partial b/\partial s)
\]
\[ \mathbf{E} \times \mathbf{B}/B^2 + (\gamma m_0 v^2/qB) \mathbf{b} \times [(1-\mu^2)/2 \mathbf{\nabla} \ln B + \mu^2 \partial \mathbf{b}/\partial s] \]

leading to

\[ \frac{dW}{dt} = qdR/\partial t + (M/\gamma\partial B/\partial t) \]

where the relativistic expression for the adiabatic invariant is stated to be

\[ M_2 = \gamma M_0 = \gamma^2 v^2/2m_0 B \]

In the re-formulation [2,3], we simply set \( dW = \mathbf{v}\mathbf{dp} \) (which holds both relativistically and non-relativistically), and we obtained (still assuming that \( E_\perp = 0 \))

\[ \mathbf{v}\mathbf{dp}/\partial t = qE \cdot (\gamma m_0 v^2/qB) \mathbf{b} \times [(1-\mu^2)/2 \mathbf{\nabla} \ln B + \mu^2 \partial \mathbf{b}/\partial s] + (\gamma m_0 v^2/2B)(1-\mu^2)\partial B/\partial t \]

This simple change of dependent variable (from \( W \) to \( p \)) introduces a common factor on the RHS of \( \gamma m_0 v^2 = \mathbf{p}v \), so our general result (relativistic or non-relativistic) is

\[ \frac{d\mathbf{p}}{dt} = (1/B) \mathbf{E} \cdot (1/B) \mathbf{b} \times (1-\mu^2)/2 \mathbf{\nabla} \ln B + \mu^2 \partial \mathbf{b}/\partial s] + (1/2B)(1-\mu^2)\partial B/\partial t \]

Please note carefully that since we have assumed that \( E_\parallel = 0 \), then we have \( \mathbf{E} = -\mathbf{V} \times \mathbf{B} \) for the total electric field, and therefore only \( \mathbf{V} \), should appear explicitly in our equations.

All that remains to be done is to complete the inner vector product \((1/B)\mathbf{E} \cdot \mathbf{b} \times \mathbf{A}\) for some vector \((\mathbf{A})\) with either \( \mathbf{A} = \mathbf{\nabla} \ln B \) or \( \mathbf{A} = \mathbf{b}/\partial s = \mathbf{k} \) (the curvature vector of the magnetic field lines). The “box product” identity is

\[ \frac{1}{B} \mathbf{E} \cdot \mathbf{b} \times \mathbf{A} = (\mathbf{b} \times \mathbf{V}) \cdot (\mathbf{b} \times \mathbf{A}) = \mathbf{V} \cdot \mathbf{A} -(\mathbf{b} \cdot \mathbf{V})(\mathbf{b} \cdot \mathbf{A}) = \mathbf{V} \cdot \mathbf{A} \]

After a trivial rearrangement of terms, we obtain the form of the guiding center equation for the fractional time-rate-of-change of the particle momentum.

\[ \frac{d\mathbf{p}}{dt} = (1/2)(1-\mu^2)(\partial \ln B/\partial t + \mathbf{V} \cdot \mathbf{\nabla} \ln B) + \mu^2 \mathbf{V} \cdot \mathbf{k} \]

Note that, although the perpendicular subscript \((\mathbf{V} \cdot \mathbf{k})\) is redundant in the term \((\mathbf{V} \cdot \mathbf{k})\) because the curvature is always transverse to \((\mathbf{B})\), it is necessary to indicate it explicitly in the term \((\mathbf{V} \cdot \mathbf{\nabla} \ln B)\), because the gradient of \((\mathbf{B})\) may have a parallel component.

The direct path to our new equation follows from realizing that with \( \mathbf{E} = \mathbf{E}_\perp = -\mathbf{V} \times \mathbf{B} \) we can immediately write \( \mathbf{E} = -\mathbf{V} \times \mathbf{B} \) when we substitute the electric field into Faraday’s law.

\[ \partial \mathbf{B}/\partial t = -\mathbf{\nabla} \times \mathbf{E} = \mathbf{\nabla} \times (\mathbf{V} \times \mathbf{B}) \]

\[ = \mathbf{V} \cdot (\mathbf{\nabla} \times \mathbf{B}) \times \mathbf{B} + (\mathbf{V} \times \mathbf{V}) \times \mathbf{V} \cdot \mathbf{\nabla} \mathbf{B} \]

\[ \partial \mathbf{B}/\partial t + \mathbf{V} \cdot \mathbf{\nabla} \mathbf{B} = \text{D} \mathbf{B}/dt = -\mathbf{B} (\mathbf{\nabla} \times \mathbf{V}) + \mathbf{B} \cdot \mathbf{\nabla} \mathbf{V} \]

Since \( \text{D}/dt \) is a linear differential operator, we can write

\[ \mathbf{B} \cdot \text{D} \mathbf{B}/dt = (1/2) \text{D} \mathbf{B}^2/dt = (1/2) \text{D} (\mathbf{B} \cdot \mathbf{B})/dt = \mathbf{B} \cdot \text{D} \mathbf{B}/dt \]
If (s) is the distance along the field line, then

$$\mathbf{B} \cdot (\mathbf{B} \cdot \nabla \mathbf{V}) = \mathbf{B} \cdot (\mathbf{B} \frac{\partial \mathbf{V}}{\partial s}) = \mathbf{B}^2 \frac{\partial \mathbf{V}}{\partial s}$$

$$\mathbf{b} \cdot (\nabla \mathbf{V}) = \partial (\mathbf{b} \cdot \mathbf{V}) / \partial s - \mathbf{V} \cdot \partial \mathbf{b} / \partial s = - \mathbf{V} \cdot \mathbf{\kappa}$$

This leaves only two terms

$$\partial \ln \mathbf{B} / \partial t = \nabla \cdot \mathbf{V} - \mathbf{V} \cdot \mathbf{\kappa}$$

Thus, using Faraday’s law to eliminate the explicit time dependence in the magnetic field, the guiding center equation finally takes the remarkably simple form, which is nonetheless valid for time-dependent variations in both $\mathbf{B}$ and $\mathbf{V}$.

$$\frac{dlnp}{dt} = (1 - \mu^2)/2 \frac{\partial \ln \mathbf{B}}{\partial t} + \mu^2 \mathbf{V} \cdot \mathbf{\kappa}$$

$$= -(1 - \mu^2)/2 (\nabla \cdot \mathbf{V} + \mathbf{V} \cdot \mathbf{\kappa}) + \mu^2 \mathbf{V} \cdot \mathbf{\kappa}$$

11. References

[1] Roelof, E. C. (2008) Scatter-free propagation of energetic solar electrons and galactic cosmic rays in the inner heliosphere, Proc. 7th Ann. Int’l. Astrophys. Conf., AIP Conf. Proc., 1039, 174-183.

[2] Kunow, H., et al. (including E. C. Roelof) (1999), Co-rotating Interaction Regions at High Latitudes, Space Sci. Rev., 91, 221-268.

[3] Roelof, E. C. (2000), High Latitude Observations of Corotating Interaction Regions: Remote Sensing Using Energetic Particles, 242-249 American Institute of Physics (New York), AIP Conf. Proc. No. 528, 242-249.

[4] Northrop, T. G. (1963) The Adiabatic Motion of Charged Particles, (Interscience, New York).

[5] Roelof, E. C. (2012a) Energetic particle reservoirs: 1. Derivation of the transport equation, Proc. 11th Ann. Int’l. Astrophys. Conf., AIP Conf. Proc., 1500, 174-179.

[6] Roelof, E. C. (2012b) Energetic particle reservoirs: 2. Solutions of the transport equation and comparison with data, Proc. 11th Ann. Int’l. Astrophys. Conf., AIP Conf. Proc., 1500, 180-185.

[7] Nolte, J. T. and E. C. Roelof (1973a) Large-scale structure of the interplanetary medium: 1, High coronal source longitude of the quiet-time solar wind, Solar Physics, 33, 241.

[8] Nolte, J. T. and E. C. Roelof (1973b) Large-scale structure of the interplanetary medium: 2, Evolving magnetic configurations deduced from multi-spacecraft observations, Solar Physics, 33, 483.

[9] Roelof, E. C. (1969) Propagation of solar cosmic rays in the interplanetary magnetic field, Ch. VII of Lectures in High Energy Astrophysics., H. Ogelman and J. R. Wayland, ed. NASA SP-199.

[10] Krucker, S., D. E. Larson, R. P. Lin, and B. J. Thompson (1999) On the origin of impulsive electron events observed at 1 AU Astrophys. J., 519, 864-875.

[11] Simnett, G. M., E. C. Roelof, and D. K. Haggerty (2002) Near-relativistic electrons accelerated by shocks associated with coronal mass ejections (CMEs), Astrophys. J., 579, 854-862.

[12] Haggerty, D. K. and E.C. Roelof (2009) Probing SEP acceleration processes with near-relativistic electrons, Proc. 8th Ann. Int’l’ Astrophys. Conf. AIP Conference Proceedings, 1183, 3-10.

[13] Hundhausen, A. J. (1973), Non-linear model of high speed solar wind streams, J. Geophys. Res., 78, 1528-1542.

[14] Gosling, J. T. and V. J. Pizzo (1999) Formation and evolution of corotating interaction regions and their three-dimension structure, Space Sci. Rev., 89, 21-52.
[15] Roelof, E. C. and Vourlidas, A. (2015), On extracting plasma compression signatures from white light coronal images, *This Conference.*

[16] van Nes, P., E. C. Roelof, R. Reinhard, T. R. Sanderson, and K.-P. Wenzel, (1985) A major shock-associated energetic storm particle event wherein the shock plays a minor role, *J. Geophys. Res.,* 90, 3981-3994.

[17] Giacalone, J. and J. R. Jokipii (2007), Adiabatic compression acceleration of fast charged particles, *Astrophys. J.,* 660, 336-340.

[18] Fisk, L. A., and G. Gloeckler (2014), The case for a common spectrum of particles accelerated in the heliosphere: Observations and Theory, *J. Geophys. Res.,* 119, 8733-8749.