Multidisciplinary design optimization analysis of pin fins based on the Monte Carlo simulation

Z Y Zhao¹, W B Hu¹ and J B Wang²,³,⁴

¹School of civil and traffic engineering, Qinghai Nationalities University, Xi Ning, 810007, P.R. China
²School of Civil Engineering, Xi’an University of Architecture and Technology (Yan’ta Campus), Xi’an 710055, China
³School of Civil Engineering, Henan Polytechnic University, Jiaozuo 454001, China
E-mail: wjb@hpu.edu.cn

Abstract. For aero-engines, pin fins improve the heat exchange efficiency of blades but increase their mass. To reduce the mass of blades, three types of pin fin models (diamond, elliptical, and rectangular) were used in this paper to analyze the stress distribution under the combined action of temperature and centrifugal force. The maximum stress was used as the constraint condition in these models. With the aim of mass optimization, these models were subjected to the multidisciplinary design optimization based on the Monte Carlo simulation. After the optimization via diamond, elliptical, and rectangular models, the pin fin mass was reduced by 15.42, 19.42, and 16.03%, respectively.

1. Introduction

As vital cooling tools, pin fins are used to increase gas disturbance and heat transfer area, and improve heat exchange efficiency. Chiang and Chang [1] developed the response surface methodology to analyze and optimize the cooling efficiency of the circular pin fin model under the constraints of mass and space. Yang et al. [2] used the genetic algorithms to study and optimize the turbulent flow and heat transfer characteristics of the radiator with rectangular pin fins. It was found that the use of standard k-ε model helped obtain a better fluid flow and heat transfer performance. Obayopo et al. [3] carried out numerical analysis and optimization of proton exchange membrane (PEM) with horizontal pin fin array. In order to understand the effect of pin fins on thermal performance, Tullius et al.[4] carried out numerical analysis and optimization of six types of short pin fin arrays in the microchannel. Chen and Yan [5] used a 3D model to dynamically simulate and improve the heat dissipation process of the cylindrical pin fin radiator. Baby and Balaji [6] carried out an experimental study on the pin fin radiators made of different phase change materials and analyzed the thermal performance of the radiators by genetic algorithm.

At present, the research and optimization design of the pin fins are focused on the influence of the pin fin’s shape [4], height, arrangement [1, 5] and number [6] on its heat convection. For aero-engines, pin fins increase the mass of blades while increasing the heat exchange efficiency. In order to reduce the mass of blades, diamond, elliptical and rectangular pin fin models are established in this paper. Through strength analysis, the stress distribution of the three models under the combined action of temperature and centrifugal force is obtained. The Monte Carlo simulation (MCS), also known as a statistical simulation method, is a very important numerical calculation method proposed in the mid-1940s based on the probability statistics theory. It refers to the method that uses random numbers to solve numerous
computing problems. The MCS method is widely used in financial engineering, computational physics, nuclear engineering, and many other fields. With the MCS method, the multidisciplinary design optimization analysis of the three models is carried, which ultimately enables the models to be much lighter and more efficient.

2. Strength analysis

2.1. Analysis models

In this paper, three pin fin models – rectangular, diamond and elliptical models – are established. Figure 1 shows the pin fin models (including the solid domain model and the fluid domain model) built using Catia software, the specific dimensions of the solid domain model, and the arrangement of pin fins in it. Each model has five pin fin arrays. All three models have the same initial dimensions. They are made of K444 alloy.

2.2. Model parameterization

Figure 1 shows the parametric diagram of the models. $L1$ is the distance from the first pin fins of the first column to the long side, and $H1$ is the distance from this pin fins to the short side; $L2$ is the distance from the first pin fin of the second column to the long side, and $H2$ is the distance from this pin fins to the short side; $SX$ is the distance in the horizontal direction of the pin fins, and $SY$ is the distance in the vertical direction of the pin fins. In addition, the height of the pin fins is Height and the blade thickness is Thickness. The eight design variables are shared by all models, mainly to control the arrangement of the pin fins arrays. Table 1 gives the initial values for these design variables. Figures 1b) - d) provide the parametric design variables for the three types of pin fins models. IncludeAngle is an interior angle in the diamond pin fins, MajorSemiAxis is the long axis of the elliptical pin fins, MinorSemiAxis is the short axis of the elliptical pin fins, LongSide is the long side of the rectangular pin fins, and ShortSide is the rectangular pin fins. Table 1 and figure 2 depict the initial values for these design variables.

![Figure 1. Parametric diagram of the models.](image-url)
Table 1. Initial values for these design variables (mm).

| Parameters  | \( L1 \) | \( L2 \) | \( H1 \) | \( H2 \) | \( SX \) | \( SY \) | Thickness | Height |
|-------------|--------|--------|--------|--------|--------|--------|----------|--------|
| size        | 2.8    | 5.2    | 6      | 8.7    | 4.8    | 5.4    | 0.5      | 2.0    |

Figure 2. Initial sizes of the pin fins models.

2.3. Analysis process

During strength analysis, first the pin fin models were established using Catia, and then flow field analysis was carried out using the commercial software ANSYS-CFX to obtain the temperature distribution of the solid domain. At the same time, strength analysis was carried out using ABAQUS to generate the INP file, and the temperature data was then interpolated to get a new INP file. Finally, the stress of each model under the combined action of temperature and centrifugal force was calculated by ABAQUS, as shown in figure 3.

In the analysis, the inlet gas is the ideal gas which is incompressible, stable and laminar. Fluid flow is governed by the law of conservation. The basic conservation laws include the law of conservation of mass, the law of conservation of momentum and the law of conservation of energy. The governing equations are mathematical descriptions of these conservation laws. These laws are reflected in fluid mechanics by the corresponding continuity equations and the N-S equations.

According to the present study, the analytical results of the standard k-ε model show good agreement with the experimental data. Therefore, the k-ε model is used for analysis in this paper. Figure 4 shows the inlet and outlet of the model. The specific boundary conditions are as follows:

1) Inlet boundary conditions: inlet gas temperature is 784K, and gas flow rate is 12g/s.

2) Wall boundary conditions: no-slip boundary conditions. In addition, the external temperature is set to 1100K, and the heat transfer efficiency is 2100w/m2K.

3) Outlet boundary conditions: pressure at the outlet is set to 0.8Mpa.

2.4. Analysis of calculation results

Table 2 shows the effect of temperature on the strength of the pin fins models. As can be seen from table 2, temperature has a great impact on the strength of the model. In the three pin fins models, even the elliptical model with the smallest stress change is found with a stress increase by 15.596%.

Table 2. Maximum stress of the pin fins in different conditions.

| Pin fins models | Purely induced by centrifugal force (MPa) | Under combined action of temperature and centrifugal force (MPa) | Variation (%) |
|-----------------|------------------------------------------|---------------------------------------------------------------|--------------|
| Diamond         | 497.0                                    | 624.0                                                        | 25.553       |
| Elliptical      | 545.0                                    | 630.0                                                        | 15.596       |
| Rectangular     | 513.0                                    | 623.0                                                        | 21.442       |
Figure 3. Analysis process.
3. Optimization analysis

3.1. Optimization analysis process

Figure 5 shows an optimization flowchart for the entire optimization system. During optimization analysis, we need to record every step of the process.

![System optimization framework](image_url)

**Figure 5.** System optimization framework.

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**Figure 5.** System optimization framework.
3.2. Objective function and constraints

3.2.1. Objective function. In the simulation analysis, we hope to reduce the Mass of the model by changing the shape and arrangement of the pin fins.

\[ \min F_c = \text{Mass} \]  

(1)

3.2.2. Constraints. In general, reduction of the model mass may increase the model stress. To ensure that the model stress does not change greatly, the constraint shown in Eq. 8 is applied.

\[ \frac{\sigma'_{\text{max}} - \sigma_{\text{max}}}{\sigma_{\text{max}}} \leq 5\% \]  

(2)

where \( \sigma_{\text{max}} \) and \( \sigma'_{\text{max}} \) are the maximum stresses in the model before and after optimization, respectively. This equation provides the following specific stress constraints for the three models:

- **Diamond**: \( \sigma'_{\text{max}} \leq 655.2\text{Mpa} \)
- **Elliptical**: \( \sigma'_{\text{max}} \leq 661.5\text{Mpa} \)
- **Rectangular**: \( \sigma'_{\text{max}} \leq 654.15\text{Mpa} \)

The main function of pin fins is to increase the heat transfer area of the model. Therefore, in order to keep the heat transfer area of the model constant, the constraint imposed on the rectangular pin fins is:

\[ \text{LongSide} = 3.6 - \frac{\text{ShortSide}}{2} \]  

(4)

The constraint imposed on the elliptical pin fins is:

\[ \text{SemiMinorAxis} = (3.6 - \text{SemiMajorAxis} \times 2) / (\pi - 2) \]  

(5)

Heat transfer area constraint is imposed on the diamond pin fin, as long as the length of its four sides remains unchanged.

The spatial constraint applied in the analysis is:

\[ L1 = L2 - SX \times 0.5, H1 = H2 - SY \times 0.5 \]  

(6)

The constraint in Eq.6 is applied to ensure that the pin fins columns and rows are equally spaced.

3.3. Optimization results

Figures 6-8 show the Pareto charts for the three pin fins models. According to figure 6, for the diamond pin fins, blade Thickness has the greatest influence on the model mass, followed by the Height and Angle of the pin fins. In terms of model strength, Angle-SY has the greatest influence, followed by Angle^2. According to Figure 7, for the elliptical pin fins, blade Thickness has the greatest influence on model mass, followed by Height-L2 and SX-SY. In terms of model strength, SX-SY has the greatest effect, followed by Height-L2 and H2-L2. According to figure 8, for the rectangular pin fins, blade Thickness has the greatest influence on model mass, followed by SX-Thickness. In terms of model strength, Height-SX has the greatest effect, followed by SX^2 and L2-Thickness.

![Figure 6. Pareto chart of the diamond pin fins modal, a) mass, b) stress.](image)
By comparison, the only thing we know for sure is that blade thickness has the greatest effect on the mass of all three models. In terms of model stress, complicated factors are involved for different shapes of pin fin models.

Figure 9 shows the comparison of shapes before and after optimization. Table 3 shows the dimensions after optimization. Table 4 shows the changes in model mass.
Table 3. Dimensions before and after optimization (mm).

| Variables       | Before optimization | After optimization | Diamond | Elliptical | Rectangular |
|-----------------|---------------------|-------------------|---------|------------|-------------|
|                 |                     |                   |         |            |             |
| L1              | 2.8                 | 2.9446            | 3.9223  | 2.2919     |             |
| L2              | 5.2                 | 5.3657            | 5.8519  | 4.5185     |             |
| H1              | 6                   | 7.0405            | 4.7571  | 6.0907     |             |
| H2              | 8.7                 | 9.3866            | 7.225   | 8.7327     |             |
| SX              | 4.8                 | 4.8422            | 3.8592  | 4.4532     |             |
| SY              | 5.4                 | 4.6923            | 4.9358  | 5.2841     |             |
| Thickness       | 0.5                 | 0.45881           | 0.3712  | 0.3915     |             |
| Height          | 2                   | 1.7884            | 1.485   | 1.9363     |             |
| Angle           | 35°                 | 32.471°           | -       | -          |             |
| MajorSemiAxis   | 1.5                 | -                 | 1.53    | -          |             |
| MinorSemiAxis   | 0.5256              | -                 | 0.4722  | -          |             |
| LongSide        | 2.4                 | -                 | -       | 2.2676     |             |
| ShortSide       | 1.2                 | -                 | -       | 1.3324     |             |

Table 4. Transformation of the optimization objective.

| Shape         | Mass(t) Before optimization | Mass(t) After optimization | Variation (%) |
|---------------|------------------------------|----------------------------|---------------|
| Diamond       | 1.2415943E-05               | 1.0501E-05                 | 15.423        |
| Elliptical    | 1.1826868E-05               | 9.5305E-06                 | 19.417        |
| Rectangular   | 1.2245253E-05               | 1.0283E-05                 | 16.027        |

4. Summary
In this paper, Monte Carlo Simulation is used for multidisciplinary optimization analysis on the rectangular, diamond and elliptical pin fin models. The temperature distribution of the models is obtained through CFD analysis. The stress distribution of the models is obtained through strength
analysis. After that, the mass of the models is optimized. Over the course of the study, the key findings are as follows:

1. Through simulation analysis, stress distribution of the pin fin models under the combined action of centrifugal force and temperature is obtained. With the effect of both centrifugal force and temperature, the maximum stress in the model appears at the bottom, but the high stress area is much smaller. In addition, the low stress area not only appears at the top of the model, but also in the middle of the model. For some models, the low stress area even appears primarily in the middle. Compared to the centrifugal force alone, the combination of centrifugal force and temperature causes the stress of the diamond pin fin model to increase by 25.55%, that of the elliptical model by 15.59%, and that of the rectangular model by 21.44%.

2. The analysis results show that for the diamond pin fins, blade Thickness has the greatest influence on the model mass, followed by the Height and Angle of the pin fins. In terms of model strength, Angle-SY has the greatest influence, followed by Angle^2. For the elliptical pin fins, blade Thickness has the greatest influence on model mass, followed by Height-L2 and SX-SY. In terms of model strength, SX-SY has the greatest effect, followed by Height-L2 and H2-L2. For the rectangular pin fins, blade Thickness has the greatest influence on model mass, followed by SY-Thickness. In terms of model strength, Height-SX has the greatest effect, followed by SX^2 and L2-Thickness.

3. According to the research results, when using the Monte Carlo simulation for optimization design, all three models show significant change in mass before and after optimization. Among them the diamond pin fins shows the least variation of 15.42%.

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