Long-term Dynamical Stability in the Outer Solar System. I. The Regular and Chaotic Evolution of the 34 Largest Trans-Neptunian Objects

Marco A. Muñoz-Gutiérrez1, Antonio Peimbert2, Matthew J. Lehner1,3,4, and Shiang-Yu Wang1

1 Institute of Astronomy and Astrophysics, Academia Sinica, 11F of AS/NTU Astronomy-Mathematics Building, No. 1, Section 4, Roosevelt Rd., Taipei 10617, Taiwan, Republic of China; mmunoz@asiaa.sinica.edu.tw
2 Instituto de Astronomía, Universidad Nacional Autónoma de México, Apdo. Postal 70-264, Ciudad Universitaria, Mexico
3 Department of Physics and Astronomy, University of Pennsylvania, 209 S. 33rd St., Philadelphia, PA 19104, USA
4 Center for Astrophysics | Harvard & Smithsonian, 60 Garden St., Cambridge, MA 02138, USA

Abstract

We carried out an extensive analysis of the stability of the outer solar system, making use of the frequency analysis technique over short-term integrations of nearly 100,000 test particles, as well as a statistical analysis of 200 I Gyr long numerical simulations, which consider the mutual perturbations of the giant planets and the 34 largest trans-Neptunian objects (we have called all 34 objects “dwarf planets,” DPs, even if probably only the largest of them are true DPs). From the frequency analysis, we produced statistical diffusion maps for a wide region of the a–e phase-space plane; we also present the average diffusion time for orbits as a function of perihelion. We later turned our attention to the 34 DPs, making an individualized analysis for each of them and producing a first approximation of their future stability. From the 200 distinct realizations of the orbital evolution of the 34 DPs, we classified the sample into three categories, including 17 stable, 11 unstable, and 6 resonant objects; we also found that, statistically, two objects from the sample will leave the trans-Neptunian region within the next gigayear, most likely being ejected from the solar system, but with a nonnegligible probability of going inside the orbit of Neptune, either to collide with a giant planet or even falling to the inner solar system, where our simulations are no longer able to resolve their continuous evolution.

Unified Astronomy Thesaurus concepts: Dwarf planets (419); Trans-Neptunian objects (1705); Solar system (1528); N-body simulations (1083); Kuiper belt (893)

1. Introduction

The outer regions of the solar system, those beyond the orbit of Neptune, are far from empty; instead, they contain vast amounts of large and small objects, the remnants of planet formation. Those remnants include millions of small icy planetesimals that constitute the reservoir of visible comets (Dones et al. 2015; Nesvorný et al. 2017), dozens of dwarf planet (DP)-sized objects (Brown 2008; Schwamb et al. 2014), and possibly even planetary-sized objects (Trujillo & Sheppard 2014; Batygin & Brown 2016).

In particular, the region immediately after Neptune and up to a few hundred au from the Sun, the trans-Neptunian region, constitutes the reservoir of short-period comets (Levison & Duncan 1997; Duncan et al. 2004; Volk & Malhotra 2008).

Strictly speaking, the trans-Neptunian region comprises everything beyond Neptune; however, it is commonly associated only with the Kuiper Belt and its extended region, which includes the detached scattered disk and extreme trans-Neptunian objects (TNOs) but excludes the Oort cloud. Commonly, TNOs are classified into several subpopulations forming the Kuiper Belt, namely, the cold and hot classical, resonant, and scattering populations (Gladman et al. 2008).

Despite the apparent stability of the Kuiper Belt, the mere existence of comets puts in evidence the belt’s continuous erosion, as cometary nuclei are driven toward the inner solar system due to perturbations produced by the giants and DPs acting on secular timescales (see, for instance, Levison & Duncan 1997; Volk & Malhotra 2008; Nesvorný et al. 2017; Muñoz-Gutiérrez et al. 2019). In this regard, the trans-Neptunian region is not absolutely stable, and this lack of stability will, in principle, apply to any object in its dominion, including the DPs themselves.

According to the IAU definition, a DP of the solar system is a celestial body that: “(a) is in orbit around the Sun, (b) has sufficient mass for its self-gravity to overcome rigid body forces so that it assumes a hydrostatic equilibrium (nearly round) shape, (c) has not cleared the neighborhood around its orbit, and (d) is not a satellite.” The existence of comets makes it clear that, although the orbital cleansing process in the trans-Neptunian region is a slow one, it is nonetheless an active and continuous one that, left to its own devices, could require hundreds of Gyr (or even Tyr) to be completed.

Following the IAU definition, there are currently five objects in the solar system recognized as DPs, namely, Ceres (in the asteroid belt), Eris, Pluto, Haumea, and Makemake (in the Kuiper Belt). By considering the IAU criteria, a number of other TNOs could potentially be classified as DPs. Tancredi & Favre (2008) gave a list of 18 potential trans-Neptunian DPs based on physical and dynamical criteria, including the four already classified as such (see also an updated list in Tancredi 2010). The apparent minimum threshold in radius for a TNO to effectively become a DP, i.e., to attain hydrostatic equilibrium shapes, is around 400 km (see Rambaux et al. 2017); this would imply that there are currently about a dozen DPs in the Kuiper Belt. On the other end, Valsecchi (2009) and later Margot (2015) gave criteria to differentiate between planets and DPs, basically the definition of a planet, establishing a clear limit above the size of Eris/Pluto.

We are used to thinking that members of the solar system (planets, satellites, asteroids, DPs, etc.) that are long familiar to us are stable and will remain as such for the age of the solar system.
Table 1
Main Orbital Parameters of All 34 DPs

| Object      | Mass ($\times 10^{-3} M_\oplus$) | $a$ (au) | $e$ | Inc. | $q$ (au) |
|-------------|----------------------------------|---------|-----|------|---------|
| Eris        | 2.7956                           | Measured| 67.82| 0.438 | 43.993  | 38.163  |
| Pluto       | 2.4467                           | Measured| 39.56| 0.250 | 17.141  | 29.673  |
| Haumea      | 0.6706                           | Measured| 67.10| 0.503 | 30.803  | 33.360  |
| 2007 OR10   | 0.6110                           | Measured| 43.14| 0.194 | 28.205  | 34.772  |
| Makemake    | 0.5531                           | Measured| 45.54| 0.159 | 29.002  | 38.286  |
| Quaoar      | 0.2343                           | Measured| 43.39| 0.037 | 7.991   | 41.773  |
| 2002 MS4    | 0.1369                           | Estimated| 41.78| 0.144 | 17.693  | 35.756  |
| Sedna       | 0.1254                           | Estimated| 515.06| 0.852 | 11.929  | 76.190  |
| Orcus       | 0.1073                           | Measured| 39.31| 0.223 | 20.568  | 30.553  |
| 2014 EZ51   | 0.1012                           | Estimated| 52.17| 0.227 | 10.272  | 40.311  |
| 2010 JQ179  | 0.0635                           | Estimated| 78.48| 0.499 | 32.043  | 39.356  |
| 2002 AW197  | 0.0606                           | Estimated| 47.26| 0.129 | 24.382  | 41.151  |
| 2014 KH162  | 0.0594                           | Estimated| 61.87| 0.332 | 28.860  | 41.339  |
| Varda       | 0.0446                           | Measured| 45.80| 0.142 | 21.511  | 39.281  |
| 2007 UK426  | 0.0415                           | Measured| 73.69| 0.490 | 23.357  | 37.559  |
| 2013 FY27   | 0.0391                           | Estimated| 58.80| 0.394 | 33.120  | 35.641  |
| 2003 AZ49   | 0.0349                           | Measured| 39.51| 0.178 | 13.565  | 32.472  |
| 2015 RR255  | 0.0331                           | Estimated| 81.99| 0.586 | 7.552   | 33.946  |
| 2003 OP22   | 0.0288                           | Estimated| 43.24| 0.106 | 27.161  | 38.654  |
| 2014 UZ224  | 0.0266                           | Estimated| 109.45| 0.650 | 26.785  | 38.312  |
| Ixion       | 0.0263                           | Estimated| 39.55| 0.244 | 19.631  | 29.898  |
| Varuna      | 0.0259                           | Measured| 43.00| 0.053 | 17.175  | 40.709  |
| 2005 RN43   | 0.0255                           | Estimated| 41.56| 0.024 | 19.272  | 40.578  |
| 2002 TC302  | 0.0239                           | Estimated| 55.40| 0.294 | 35.038  | 39.091  |
| 2010 RF33   | 0.0227                           | Estimated| 49.41| 0.246 | 30.643  | 37.253  |
| 2004 GV9    | 0.0218                           | Estimated| 42.00| 0.078 | 21.983  | 38.743  |
| 2002 UX25   | 0.0209                           | Measured| 42.73| 0.143 | 19.433  | 36.643  |
| 2010 KZ10   | 0.0167                           | Estimated| 45.27| 0.056 | 26.089  | 42.724  |
| 2005 UQ313  | 0.0121                           | Estimated| 43.37| 0.146 | 25.716  | 37.044  |
| 2012 VF13   | 0.0118                           | Estimated| 263.94| 0.695 | 24.052  | 80.528  |
| 2014 WK109  | 0.0105                           | Estimated| 51.09| 0.209 | 14.501  | 40.429  |
| 2005 QU1   | 0.0088                           | Estimated| 112.34| 0.671 | 14.030  | 36.921  |
| 2010 EK109  | 0.0054                           | Estimated| 68.62| 0.533 | 29.460  | 32.512  |
| 2002 TX200  | 0.0018                           | Measured| 43.31| 0.124 | 25.854  | 37.965  |

system and beyond. However, this is not necessarily the case. Actually, even from definition, a DP is an object that has been unable to clean its orbit; thus, the constant stirring of smaller objects around could result in destabilizing interactions for both sides. Essentially, similar to how the delivery of comets is an example of the slow but continuous process of orbital cleansing in the trans-Neptunian region, larger objects, such as DPs and potential DPs, are prone to experience the same fate and also be removed from the region within the age of the system (though likely at a significantly lower rate than comets; nonetheless, within a much more massive disk, this rate should be markedly larger; see, e.g., Silsbe & Tremaine 2018).

Previous works addressing the stability of the outer solar system have focused mainly on the delivery of comets, thus typically considering only massless particles in numerical simulations (e.g., Holman & Wisdom 1993; Duncan et al. 1995; Robutel & Laskar 2001). In this work, we intend to complement and shed light on the long-term evolution of larger objects, those that, in cases of being driven inward, would become a much more spectacular phenomenon than normal comets. We build upon our simulations from Muñoz-Gutiérrez et al. (2019) to explore the fate of the 34 largest TNOs (including the four known DPs). We find that out of the 10 more massive TNOs in our sample, four turn out to be unstable or highly unstable (namely, Haumea, 2007 OR10, 2002 MS4, and Orcus); in our view, this fact alone highlights the importance of considering the interaction among DPs to fairly account for the secular evolution of the outer solar system in general.

2. Our Sample: The 34 Largest TNOs

In a previous paper (Muñoz-Gutiérrez et al. 2019), we compiled a set of the 34 brightest TNOs then known (early 2019). Since brightness correlates with size and thus with mass, this represents a set of 34 of the most massive TNOs (with some bias for objects closer to the Sun). All four confirmed trans-Neptunian DPs are in this set, and at least one more is also clearly a DP; regardless, we call all members of this set DPs, indistinctly. In that work, we explored the dynamical influence that these objects, a greatly neglected component of the outer solar system, have on the orbital evolution of light TNOs. We specifically focused on the injection rate of cometary nuclei from the Kuiper Belt into the inner solar system; we followed the evolution of the light TNOs starting from their interaction with the DPs and continuing with their interaction with Neptune’s resonances, a strong interaction with Neptune itself, and up to their end stage as Jupiter family comets (JFCs).

In this work, we use the same simulations we used in the previous paper, but we focus our attention not on the evolution of the test particles perturbed by the DPs but rather on the long-term evolution of the DPs themselves. The data for all 34 members of our DP sample (masses and barycentric orbital elements) are listed in Table 1 (here we only recall the basic
the current locations of 22 of the DPs in our set are marked by black circles. The color indicates the value of the logarithm of the diffusion parameter of the orbit in that location, which translates into the stability of orbits; i.e., bluer regions (lowest values of $D$) represent the more stable orbits, and redder colors (highest values of $D$) represent the more unstable ones. The location and ratios of some MMRs with Neptune are labeled at the top of the figure. The solid and dashed green lines indicate the perihelion distances at the location of Neptune and $a_N + 2\sqrt{3}R_N$, respectively. Finally, the current locations of 22 of the DPs in our set are marked by black circles.

Aspects of the data compilation; for details, the reader is referred to the Appendix in Muñoz-Gutiérrez et al. 2019).

Due to their remoteness, the physical properties of TNOs are difficult to constrain. However, out of the 34 TNOs in our sample, 13 are known with enough detail to determine their size, mass, and density. Another 14 objects possess a confidently measured radius and albedo. In order to determine their mass, we need to know their densities; we use a formula derived in Muñoz-Gutiérrez et al. 2019 to assign a density (this formula includes a systematic and a random component) and thus derive the masses for these objects. Finally, for seven objects, only the absolute magnitudes were known; for them, we used formulae from Muñoz-Gutiérrez et al. 2019 to assign plausible densities and albedos in order to assign plausible radii and masses (both formulae include a systematic and a random component) in order to complete the required input data for our numerical simulations.

Since we ran our simulations, and the publication of our previous paper (Muñoz-Gutiérrez et al. 2019), there have been updates to the observed physical parameters of 2002 TC$_{302}$ (Ortiz et al. 2020). The absolute magnitude is found to be slightly dimmer (4.32 versus the old 3.90); also, the size was found to be slightly smaller (250 km versus our assumed 292 km). Therefore, the albedo was found to be slightly larger (0.147 versus the old 0.115). The new size also affects the expected density, which is approximately 15% smaller than our previous estimate. Overall, this would imply a mass of $0.0128 \times 10^{-3} M_\oplus$ instead of the $0.0239 \times 10^{-3} M_\oplus$ we used (Ortiz et al. 2020 preferred a slightly lower density and thus a slightly lower mass of $0.0088 \times 10^{-3} M_\oplus$). This difference is not statistically significant enough to demand new simulations.

3. Orbital Stability in the Outer Solar System

3.1. Introduction to Frequency Map Analysis

We begin our analysis by characterizing the overall orbital stability of the outer solar system. To this aim, we performed a frequency map analysis (FMA; Laskar 1990; Laskar et al. 1992) of wide regions of the $a$–$e$ phase-space plane (semimajor axis versus eccentricity). A similar analysis was performed earlier by Robutel & Laskar (2001). Here we reproduce their analysis, but we increase the resolution in the sampling of the phase-space values corresponding to the outer solar system, focusing on the regions occupied by the majority of the DPs used in this work.

To produce the diffusion maps of Figures 1–4, we performed short-term numerical integrations (for a duration of $\sim 1.31$ Myr, or approximately 8000 orbital periods of Neptune) of thousands of test particles distributed in different homogeneous grids that cover the heliocentric $a$–$e$ phase-space plane.

The test particles are subject to the gravitational potential generated by the Sun and the four giant planets, where the masses of all of the interior planets (including the Moon and Ceres) are added to that of the Sun. The initial conditions for all of the massive bodies, on Julian day 2,458,176.5 corresponding to 2018 February 27, were retrieved from JPL’s Horizons system.6

We create a diffusion map of the outer solar system by performing a frequency analysis of the evolution of each of our test particles. We focus on the quantity

$$z(t) = a(t)\exp(i\lambda(t)), \quad (1)$$

6 https://ssd.jpl.nasa.gov/horizons.cgi
where \(a\) and \(\lambda\) are the semimajor axis and mean longitude of the particle, respectively, while the complex value of \(z(t)\) represents a rough approximation of the position of the particle (the details regarding the relevance of \(z(t)\) can be found elsewhere; e.g., Robutel & Laskar 2001; Muñoz-Gutiérrez & Giuliatti Winter 2017).

The algorithm used for the FMA was that of Šidlichovský & Nesvorný (1996), which provides an approximate decomposition of \(z(t)\) as follows:

\[
    z(t) = \alpha_0 \exp(i\nu_0 t) + \sum_{k=1}^{N} \alpha_k \exp(i\nu_k t). \tag{2}
\]

For particles that lie in Keplerian orbits, \(\lambda(t) = nt\) (where \(n\) is the mean motion of the particle); also, \(N = 0\), \(a = |\alpha_0|\), and \(n = \nu_0\) will remain constant.

In general, \(N > 0\), while \(|\alpha_0|\) and \(\nu_0\) need not remain constant; but, for moderately stable orbits, \(|\alpha_0|\) and \(\nu_0\) will remain nearly constant, and \(a\) and \(n\) will remain close to them (i.e., \(\alpha_0 \gg \alpha_k\)). On the other hand, for unstable orbits, these values will evolve quickly over time.

In particular, \(\nu_0\) (the mean frequency of the particle) will change for unstable orbits; a measure of the stability of the orbits can then be defined as the diffusion parameter

\[
    D = \frac{|\nu_{01} - \nu_{02}|}{T}, \tag{3}
\]

where \(\nu_{01}\) and \(\nu_{02}\) are the main frequencies of a particle in each adjacent time interval of length \(T\). Small values of \(D\) indicate a stable trajectory, while larger values are indicative of unstable orbital evolution, resulting from strong variations of the main frequencies, which are characteristic of the orbital erratic nature. For a more detailed description of the FMA, the interested reader is referred to Laskar (1990); Laskar et al. (1992); Laskar (1993).

In order to compute the change of the main frequencies, we perform the frequency analysis for each particle in the intervals \(0 \text{ Myr} < T < 0.655 \text{ Myr}\) and \(0.655 \text{ Myr} < T < 1.31 \text{ Myr}\).

In the maps of Figures 1, 2, and 4, we color each \((a, e)\) pair according to the logarithm of the diffusion parameter of its test particle such that redder colors indicate more unstable orbits, while bluer colors stand for the more stable orbits. Particles that are lost from the simulation before it finishes, mainly due to ejections or collisions with a planet, are shown in white.

These diffusion maps let us identify regions where particles could survive on a long-term basis while permitting us to easily identify the location and width of mean-motion resonances (MMRs) with Neptune. In the maps, the solid green lines indicate orbits with a perihelion \(q = a_{\text{Neptune}}\) (i.e., it represents the crossing line of Neptune, with the implied risk of collision).
The dashed green lines indicate the distance from which the influence of Neptune starts to be dominant, as has been used in previous works (Gladman & Duncan 1990; Muñoz-Gutiérrez et al. 2019); numerically, the dashed green line can be expressed as \( q = a_N + 2\sqrt{3}R_{HN} \), where \( a_N \) and \( R_{HN} \) are the semimajor axis and Hill radius of Neptune, respectively.

### 3.2. Stability and Resonances in the Plane

Figures 1 and 2 show the diffusion maps produced from the integration of 34,162 particles distributed in two homogeneous grids with the following conditions: in Figure 1, the heliocentric \( a \) is sampled from 38 to 58 au with a step size \( \Delta a = 0.04 \) au, while \( e \) is sampled from zero to 0.3 with a step size \( \Delta e = 0.01 \). For Figure 2, \( a \) is sampled from 58 to 82 au, while \( e \) covers 0.3–0.6 in steps of the same size as in Figure 1.

In order to focus on the stability and shape of MMRs with Neptune, all of the remaining orbital elements—inclinations (from the plane of the ecliptic), \( i \); arguments of pericenter, \( \omega \); and mean anomalies, \( M \)—are set to zero. The longitudes of the ascending nodes, \( \Omega \), are also nominally set to zero; while meaningless when \( i = 0^\circ \), they are a necessary input for the Mercury integrator.

The initial locations of 30 of the 34 DPs in our sample are indicated by black dots in both maps; the other four DPs all have \( a > 100 \) au and thus were left off of these figures.

We immediately notice that, for the majority of objects in our sample, a long-standing stability is expected based solely on their location in the phase space. However, these maps were made for an \( i = 0^\circ \); when studying orbits with \( i \gtrsim 15^\circ \), the resonances move slightly outward. We can see, for example, how two of the Plutinos (Pluto and Ixion) appear to be outside of the 3:2 MMR in Figure 1; even more, they appear to be in a highly unstable region. However, using a map with a more appropriate inclination, it is easy to see that this is not the case.

In Figure 3, we present a small diffusion map using \( i = 17^\circ.12 \), the heliocentric inclination of Pluto; this map clearly shows that Pluto is firmly inside the 3:2 resonance. The shifting effect of the location of MMRs, due to the change in \( i \), will occur for all resonances. For example, we can see the opposite case for 2015 KH162. In Figure 2, it appears to be clearly located inside the 3:1 MMR; however, we were unable to identify a stable libration of any of the possible second-order resonant arguments for it. Therefore, despite its suggestive location on the map, due to its high inclination, 2015 KH162 is found to be clearly nonresonant.

Based on the diffusion maps, it is possible to understand that the low stability of some of the DPs in our simulations, for example, 2007 OR10 and 2015 RR345, are very clearly located in the broad unstable region (close to the dashed line; \( a_N + 2\sqrt{3}R_{HN} \)), while 2002 MS4 and 2002 UX25 are located in the unstable regions near both sides of the 5:3 MMR. On the other hand, 2005 RN43, Quaoar, Varuna, and 2010 KZ39 represent examples of objects with low \( e \) located in very stable regions.

### 3.3. Stability and Diffusion in the Kuiper Belt with Random Inclinations

In order to better understand the effect that a nonzero inclination would have on the general aspect and the stability picture drawn from the diffusion maps, we ran a set of simulations including 48,235 total test particles covering different patches of the \( a-e \) phase space. In this case, the inclination of each particle was assigned at random, with values between 0° and 50°, in order to represent the broad distribution of inclinations in our DP sample (where the minimum and maximum \( i \) values are 7°.552 and 43°993, respectively). Besides, the choosing of such values allows us to better
identify the resonances for objects that orbit far away from the plane.

Since each of the 34 DPs also has its own value for the other angular parameters, \( \omega, \Omega, \) and \( M \), we decided to assign random values between 0° and 360° for the corresponding initial conditions of each test particle in the map. The exact values used for these angles are probably not very important; but, where there is any difference, this approach has the advantage of showing us, locally, the range of stability available to objects with such \( a-e \) values. We will study a more representative determination for each of the objects of our sample in Section 3.4.

In Figure 4, we show the resulting diffusion map for random inclinations and angular elements, as described above, for a larger region of the phase space than those of Figures 1 and 2 (though both of these regions are covered in the simulations of this section). As in the previous maps, the solid green line delimits the collision region with Neptune, while the dashed green line delimits the region where Neptune’s gravity is dominant. The locations of 30 out of 34 DPs are again marked by black dots.

It is interesting to note how the general form of the MMRs changes with respect to the zero-inclination case. With random angles and inclinations, the definition of the resonances gets blurred; however, the stronger MMRs, whose ratios are labeled at the top of the figure, are still easily identifiable. It is evident that the locations of the resonances are perturbed by the value of the inclination; thus, we can see how both Pluto and Ixion are indeed located within the limits of the 3:2 MMR.

This diffusion map constitutes a better representation of the stability of the Kuiper Belt as a whole, not only for the DPs but for objects of all sizes in the various dynamical families present in the region covered by the map (namely, the classical belt, the scattered disk, and some resonant populations). From the map, it is possible to statistically estimate the diffusion time of Kuiper Belt objects as a function of their perihelion.

In Figure 5, we present the fraction of stable orbits as a function of their perihelion for diffusion times varying between 0.1 to 5 Gyr. The diffusion time, \( T_{\text{Diff}} \), is obtained from the diffusion parameter as \( T_{\text{Diff}} = 1/(\text{DP}) \), where \( P \) is the period of the orbit (see, for example, Gaslaci Gallardo et al. 2020 and Roberts & Muñoz-Gutiérrez 2021, for other recent applications of the diffusion time on different systems). To determine the stable fraction, we counted the number of particles within our sample for each perihelion band between 28 and 50 au whose orbits have diffusion times above six different time limits: 0.1, 0.2, 0.5, 1, 2, and 5 Gyr. These limits are significant in the sense that any object with a diffusion time below 0.1 Gyr is expected to be lost very quickly, and thus those orbits are not expected to have a meaningful contribution to the present-day population; on the other hand, a diffusion time above 5 Gyr represents longer than the lifetime of the solar system.

We note that our sample is incomplete; besides having a hard limit at a semimajor axis of 82 au, there are additionally a few gaps below 30 au and above 40.6 au. Nonetheless, the sample is numerous enough as to provide a good representation of the expected stable fraction of Kuiper Belt objects as a function of perihelion. For example, we expect that, from an initial population with perihelia of 38 au, almost all objects would be stable for more than 100 Myr, 10% of this initial population would be unstable in timescales of 1 Gyr, and nearly 30% of the population would be lost over the age of the solar system.

3.4. Specific Stability of the Objects of Our DP Sample

In order to obtain a better estimate regarding the specific stability of each object in our sample of large TNOs (i.e., beyond the global stability suggested by the general configuration of the diffusion map of Figure 4), we performed additional stability simulations of 100 massless clones of each DP. The clones were generated by randomly assigning orbital elements within the intervals \( a_{\text{DP}} \pm 0.04 \) au and \( e_{\text{DP}} \pm 0.01 \), while the angular elements \( \omega_{\text{DP}}, \alpha_{\text{DP}}, \Omega_{\text{DP}}, \) and \( M_{\text{DP}} \) were randomly generated within intervals of \( \pm 1^\circ \) around the initial values of each DP’s orbit.

We calculated the value of the diffusion parameter for each DP as the average of the parameters of its 100 clones. The results are shown in Table 2. In the last column of Table 2, we give a prediction of the stability of each object based on the value of its diffusion time.

Our definition of \( T_{\text{Diff}} \) represents an estimate of the timescale for a change of unity in the period of an orbit; from Kepler’s Third Law, it is clear that changes in period (or mean motion) originate from changes in the semimajor axis of the orbit. In this work, we assume that a drastic change in the orbital motion would be a quarter of this, and we will set our criteria by comparing our stability timescales with \( T_{\text{Diff}}/4 \).

Based on the previous argument, we call those orbits with \( T_{\text{Diff}} > 20 \) Gyr “very stable,” that is, those for which a significant change in the mean motion would not occur over the remaining life of the solar system. We call those orbits for which \( 4 \) Gyr \( < T_{\text{Diff}} < 20 \) Gyr “stable”; that is, an appreciable change in their orbits can occur over the age of the solar system but requires more than 1 Gyr. Finally, “unstable” are the orbits with \( T_{\text{Diff}} < 4 \) Gyr, i.e., orbits that will experience drastic changes in less than 1 Gyr of evolution.

The 1 Gyr limit may seem arbitrary; however, many dynamical works that consider a timescale for the study of continuous and stationary phenomena in the solar system, a so-called steady state, use this same scale of 1 Gyr (e.g., Levison & Duncan 1997; Tiscareno & Malhotra 2009; Nesvorný et al. 2017; Yu et al. 2018; Nesvorný et al. 2019). Indeed, the present study arises from 1 Gyr integrations, which were performed for the characterization of the contribution of DPs to the delivery of short-period comets, from the Kuiper Belt to the inner solar...
We know from previous works that, while cometary nuclei in the Kuiper Belt are generally stable, there is a constant supply of new objects toward the inner solar system sustained by instabilities in the trans-Neptunian reservoirs (see Dones et al. 2015, for a recent review on comets and their reservoirs).

From the previous arguments, it becomes clear that the proper study of the stability of the objects in the outer solar system requires two more things: longer integrations and a more complete dynamical model of the solar system.

To this aim, we will take advantage of 1 Gyr long simulations we ran to study the fate of cometary nuclei in the presence of the 34 largest trans-Neptunian DPs (Muñoz-Gutiérrez et al. 2019). A by-product of those simulations is a large amount of data on the fate of each individual DP, which will let us study their dynamical evolution on a statistical basis.

4. Long-term Dynamical Simulations

While illuminating and helpful in providing a general picture of the stability of the outer solar system, the short-term simulations of the previous sections only provide us with a prediction of the stability of the objects located in those regions. It is evident that this is not enough to observe the evolution of the orbits and their possible departure from their current regions of regular or quasi-regular evolution. On top of that, the previous simulations lacked the simultaneous presence of all 34 DPs, thus ignoring the effect that they can have on each other.

We ran a total of 200 1 Gyr long simulations using the hybrid symplectic integrator from the Mercury package (Chambers 1999). The initial conditions were the same as in Section 3.1.

Most of the simulations simulations are using (177 out of 200) were designed for the study of the evolution of cometary nuclei and each one included a different set of some hundreds of test particles; the results for the evolution of test particles on many of these simulations were reported in Muñoz-Gutiérrez et al. (2019).

The initial conditions of the massive objects in all 177 simulations were identical, but each simulation differed from the others by the specific set of test particles that were included; while the nominal time step for all integrations was 400 days, the presence of close interactions forced the integrator to modify the time steps when required. This implies that each specific set of test particles resulted in a different specific set of time steps for each simulation, which in turn resulted in different evolutions for the massive objects in each case. The differences in evolution are barely noticeable for the giant planets but sometimes resulted in wildly different positions for the DPs; since the error tolerances for the integrations were stringent (with a tolerance accuracy parameter of $10^{-10}$), these differences represent the evolution of very close orbits in a chaotic region of the phase space, and indeed, all of these results are consistent with the very same initial conditions within very small error bars.

To the previous sample, we added 23 extra simulations, to end up with an even 200. Each of the new simulations had the same initial conditions for the massive objects and included a token test particle at 100 au; instead of depending on interactions on test particles, which slow down the integrations, each new simulation has a slightly different initial time step (between 389 and 412 days), which again results in different evolutions for each of our DPs.

4.1. Dynamical Model of Our TNO Sample

Our dynamical model is composed, in all cases, of a central star with a mass equal to $(1+\epsilon) M_\odot$, where $\epsilon$ is a small quantity that represents the mass of all of the terrestrial planets plus the Moon and Ceres; also, we consider the gravitational influence of the four giant planets and 34 massive DPs.

We know from previous works that, while cometary nuclei in the Kuiper Belt are generally stable, there is a constant supply of new objects toward the inner solar system sustained by instabilities in the trans-Neptunian reservoirs (see Dones et al. 2015, for a recent review on comets and their reservoirs).

From the previous arguments, it becomes clear that the proper study of the stability of the objects in the outer solar system requires two more things: longer integrations and a more complete dynamical model of the solar system.

To this aim, we will take advantage of 1 Gyr long simulations we ran to study the fate of cometary nuclei in the presence of the 34 largest trans-Neptunian DPs (Muñoz-Gutiérrez et al. 2019). A by-product of those simulations is a large amount of data on the fate of each individual DP, which will let us study their dynamical evolution on a statistical basis.

4.1. Dynamical Model of Our TNO Sample

Our dynamical model is composed, in all cases, of a central star with a mass equal to $(1+\epsilon) M_\odot$, where $\epsilon$ is a small quantity that represents the mass of all of the terrestrial planets plus the Moon and Ceres; also, we consider the gravitational influence of the four giant planets and 34 massive DPs.

Most of the simulations we are using (177 out of 200) were designed for the study of the evolution of cometary nuclei and each one included a different set of some hundreds of test particles; the results for the evolution of test particles on many of these simulations were reported in Muñoz-Gutiérrez et al. (2019).

The initial conditions of the massive objects in all 177 simulations were identical, but each simulation differed from the others by the specific set of test particles that were included; while the nominal time step for all integrations was 400 days, the presence of close interactions forced the integrator to modify the time steps when required. This implies that each specific set of test particles resulted in a different specific set of time steps for each simulation, which in turn resulted in different evolutions for the massive objects in each case. The differences in evolution are barely noticeable for the giant planets but sometimes resulted in wildly different positions for the DPs; since the error tolerances for the integrations were stringent (with a tolerance accuracy parameter of $10^{-10}$), these differences represent the evolution of very close orbits in a chaotic region of the phase space, and indeed, all of these results are consistent with the very same initial conditions within very small error bars.

To the previous sample, we added 23 extra simulations, to end up with an even 200. Each of the new simulations had the same initial conditions for the massive objects and included a token test particle at 100 au; instead of depending on interactions on test particles, which slow down the integrations, each new simulation has a slightly different initial time step (between 389 and 412 days), which again results in different evolutions for each of our DPs.

4.1. Dynamical Model of Our TNO Sample

Our dynamical model is composed, in all cases, of a central star with a mass equal to $(1+\epsilon) M_\odot$, where $\epsilon$ is a small quantity that represents the mass of all of the terrestrial planets plus the Moon and Ceres; also, we consider the gravitational influence of the four giant planets and 34 massive DPs.

Most of the simulations we are using (177 out of 200) were designed for the study of the evolution of cometary nuclei and each one included a different set of some hundreds of test particles; the results for the evolution of test particles on many of these simulations were reported in Muñoz-Gutiérrez et al. (2019).

The initial conditions of the massive objects in all 177 simulations were identical, but each simulation differed from the others by the specific set of test particles that were included; while the nominal time step for all integrations was 400 days, the presence of close interactions forced the integrator to modify the time steps when required. This implies that each specific set of test particles resulted in a different specific set of time steps for each simulation, which in turn resulted in different evolutions for the massive objects in each case. The differences in evolution are barely noticeable for the giant planets but sometimes resulted in wildly different positions for the DPs; since the error tolerances for the integrations were stringent (with a tolerance accuracy parameter of $10^{-10}$), these differences represent the evolution of very close orbits in a chaotic region of the phase space, and indeed, all of these results are consistent with the very same initial conditions within very small error bars.

To the previous sample, we added 23 extra simulations, to end up with an even 200. Each of the new simulations had the same initial conditions for the massive objects and included a token test particle at 100 au; instead of depending on interactions on test particles, which slow down the integrations, each new simulation has a slightly different initial time step (between 389 and 412 days), which again results in different evolutions for each of our DPs.

4.2. Validity and Meaning of the Different Results

The statistical studies we present here are possible due to the different results obtained in different simulations, despite the
identical initial conditions used for the massive objects. Those differences could naïvely put into question the validity of our results or be attributed simply to the finite precision of the integrator; however, our usage of a highly reliable and widely accepted integrator (the Mercury package of Chambers 1999, with over 1000 citations) could easily quell at least the later argument.

On the other hand, it is understood that numerical errors are unavoidable in integrations; furthermore, while studying a very unstable system, any small imprecision may lead to important divergences over time, i.e., dynamical chaos (e.g., Murray & Dermott 1999). This does not invalidate the meaning of such integrations; in fact, the numerical errors in the short term are smaller that the uncertainties in the observational data for the initial conditions of the objects. In the longer term, however, one could argue that only the very stable orbits will exist in the real system, since any unstable orbit would be expected to have been expelled from the system long ago, leading to the false conclusion that the divergences found in our numerical experiments cannot have an observational equivalent nowadays. On the face of this argument, we must remember that the solar system has thousands of objects that we are not including in our simulations, which can chaotically interact at different levels, and a small interaction with another close or even not so close component may lead to a divergence more significant that those produced by numerical errors.

To these, we must add the presence of extrasolar perturbations that are not easily predictable or accounted for, such as moderately close stellar flybys or interstellar visitors ('Oumuamua, 2I/Borishov), any of which can also destroy the stable orbits expected in a perfectly steady-state system or the ones produced by a perfect hypothetical integrator.

In a sense, the instability of orbits in the trans-Neptunian region is hardly unexpected, since it has been long known that some apparently stable orbits, over time spans of gigayears, can suddenly and quickly become chaotic on time spans of megayears (Torbett 1989; Holman & Wisdom 1993; Duncan et al. 1995); this phenomenon supplies comets to the inner solar system. At first glance, the instability of larger objects, such as a DP, may seem surprising; however, there is nothing that prevents, in principle, such massive objects from suffering the same fate as cometary nuclei, at least in a statistical sense. To put it simply, the constant presence of new comets shows that the solar system, and in particular the trans-Neptunian region, is not in a perfectly smooth and stable steady state.

Of course, we do not expect all DPs to be unstable (see Table 2, as well as Section 5); some of the most stable orbits (such as Pluto’s) will not be destabilized by inaccuracies resulting from numerical errors. Still, for objects surrounded by a not-so-stable phase space, the presence of slight (but constant) perturbations can disrupt their stability.

5. Results and Discussion

5.1. Classification and Global Results

We now present the statistical results from our long-term simulations, that is, the results from 200 different realizations for each of the 34 DPs (for clarity, in the rest of this work, we will call each of the 200 realizations for the evolution of each DP an “orbit”; i.e., we will be studying 6800 different orbits). As we did in Section 3.4, we are interested in providing a general classification for each object (stable or unstable), as well as their possible evolutions, since, as we will show, some of the DPs are likely to leave the trans-Neptunian region during the remaining lifetime of the Sun.

An ideally stable object would present 200 regular orbits; that is, each orbit would have only small variations in its main orbital parameters during our entire simulation. This also implies that the orbits of all the 200 simulations would be equivalent.

In order to define what a regular (or irregular) orbit is, we need to determine the limits in the deviations of the orbital parameters that we will tolerate and still call an orbit “regular.” We first calculate the standard deviations of the orbital parameters $a$, $e$, and $i$ in the first 25 Myr of each integration; that is, for each object, we calculate the $\sigma_a$, $\sigma_e$, and $\sigma_i$ using the data of the first 25 Myr for all 200 of its orbits. In Figure 6, we plot the values of $\sigma_a/a_0$, $\sigma_e$, and $\sigma_i$ versus the initial orbital elements of each DP, that is, against $a_0$, $e_0$, and $i_0$. From Figure 6, we can see that most of the DPs have similar values of their corresponding dispersion. Using the values of Figure 6, we now define a characteristic deviation for each orbital element simply as the average of the above standard deviations; thus, we have $\langle \sigma_a/a_0 \rangle = 0.00372$, $\langle \sigma_e \rangle = 0.00836$, and $\langle \sigma_i \rangle = 1.148$. With these characteristic values, we are now able to determine, for each DP, how many of the orbits are regular in the corresponding parameter. We do this by defining an individual orbit as regular if its orbital parameters $(a, e, i)$ remain within five characteristic deviations from their original value for the entire simulation; i.e., they remain within the intervals $a_0(1 \pm 5(\sigma_a/a_0))$, $e_0 \pm 5(\sigma_e)$, and $i_0 \pm 5(\sigma_i)$ for 1 Gyr. Note that we do not use any restrictions on the evolution of $\omega$, $\Omega$, or $M$.

In Table 3, we show the number of regular orbits in each orbital parameter (out of 200) according to the previous procedure. We also include the number of orbits that were regular in all three parameters simultaneously. We will consider a DP to be stable if it fulfills the following two conditions: at least 80% of its orbits are stable, and none of its orbits end up being lost in the simulation. We consider three kinds of losses: objects whose $a$ increases beyond 10,000 au (escape from the solar system), objects whose distances to the Sun decrease under 1 au (big comet), or collisions with a planet.

Of the 15 objects classified as unstable in Table 3, 13 were classified as such for having between zero and 95 stable orbits (clearly less than 80%); the other two unstable objects, 2010 RF₄₃ and 2002 UX₁₅₂, were classified as such because, despite having 170 and 173 stable orbits, some of their orbits were lost (5 and 1, respectively).

An additional result of the analysis of unstable and semistable orbits is the presence of resonant orbits; we have added an additional category to allow for the specifics of the behavior of such orbits. There are six objects in Table 3 that have been classified as resonant; three of those (Pluto, 2010 JO₁₇₀, and 2003 AZ₁₄₄) could easily be considered stable, while the other three (Ixion, 2002 TC₁₃₀₂, and 2010 EK₁₃₉) would be considered unstable according to our previous selection criteria. It is worth noting that, although the last three objects could be classified as unstable, they are not really fully unstable, as they remain locked in their resonance 94%–100% of the time (as shown by the number of orbits regular in $a$). The reduced number of regular orbits for such objects (between zero and two) is actually due to their resonant nature,
which leads to large \(e\) variations (and sometimes of the inclination) above the 5\(\sigma\) level defined before. This new category highlights their resonant behavior, which dominates for most of their orbits and is the most defining characteristic of their evolution; it is for this reason that we have also included the three more stable resonant objects in this category.

In Figure 7, we present the initial and final perihelion versus eccentricity for all 200 orbits for 32 DPs (Sedna and 2012 VP\(_{113}\) perihelia lie far beyond the range of the figure). Colored open circles indicate the initial condition of each DP in this plane.

Figure 6. Standard deviations of the 200 realizations for each DP after 25 Myr of evolution. Green circles are the \(\sigma\) values for each of the main orbital parameters against the initial conditions for each DP. A dotted green line marks the average of all of the points in each panel, which correspond to one-fifth of our reference value, used to differentiate between regular and irregular orbits.

Figure 7. Scattering of final conditions in the eccentricity–perihelion plane. The colored dots (blue for stable, red for unstable, and green for resonant) mark the final values on each of the 200 realizations for 32 DPs (the Sedna and 2012 VP\(_{113}\) perihelia lie far beyond the range of the figure). Colored open circles indicate the initial condition of each DP in this plane.

which leads to large \(e\) variations (and sometimes of the inclination) above the 5\(\sigma\) level defined before. This new category highlights their resonant behavior, which dominates for most of their orbits and is the most defining characteristic of their evolution; it is for this reason that we have also included the three more stable resonant objects in this category.

In Figure 7, we present the initial and final perihelion versus eccentricity for all 200 orbits for 32 DPs (Sedna and 2012 VP\(_{113}\) have perihelia of 76.2 and 80.5 au, outside the range of this figure). We can see that for some DPs (the stable ones; blue), the final values for each orbit (dots) all lie very close to one another and also to the initial condition (circles); for the resonant DPs (green), we see that the initial and final values lie in long lines corresponding to constant \(a\), showing the evolution in \(e\) common in resonant orbits; and for both stable and resonant orbits, there are frequently a few stragglers far from the initial values. On the other hand, there are DPs (unstable DPs; red) where the final points change in both \(e\) and \(a\), sometimes spreading over large areas of this plane. This highlights the differences of the three classifications defined for Table 3.

The last column in Table 3 gives us an idea of the quality of the determinations presented in Table 2. Of the 28 nonresonant objects, 21 remain in the same category (considering “very stable” objects to be in the “stable” category). Of the other seven objects, 2010 RF\(_{43}\) changes but is near the limit of both classifications, while the other six (Orcus, 2013 FY\(_{27}\), 2015 RR\(_{245}\), 2014 UZ\(_{224}\), 2002 UX\(_{25}\), and 2005 QU\(_{182}\)) change category completely; in all seven cases, the new determination is unstable, while the old one was either “stable” (two cases) or “very stable” (five cases). This difference occurs mostly because of the limitations of the FMA, which extrapolates the amount that the orbit will change based on a short-term integration in which the orbit does not deviate substantially from its initial conditions; i.e., while the perturbations are small, their behavior can be modeled by the FMA, but when they grow large enough to reach the direct influence of Neptune (within 3 Hill radii of Neptune), the stability decreases dramatically, and the short-term estimates are no longer predictive.

Regarding the resonant DPs, we find that Pluto, 2010 JO\(_{179}\), and 2003 AZ\(_{64}\) were stable in the FMA and remain stable in the long-term integration. On the other hand, Ixion and 2002 TC\(_{302}\)
have regular evolution for $a$ and $i$, but their $e$ changes drastically (as is expected for many resonant objects). Finally, 2010 EK130 was unstable in the FMA and shows erratic behavior in both $e$ and $i$ in the long-term integrations while remaining in the resonance (with constant $a$).

The four Plutinos represent an interesting subset of DPs showing a wide variety of behaviors. While Pluto is completely (100%) stable, 2003 AZ$_{24}$ would be considered stable (94%), and Ixion would be considered unstable (0% stable); yet, out of the 600 orbits corresponding to these three DPs, only three orbits leave the resonance. Orcus, on the other hand, is truly unstable; not only do 39 orbits leave the resonance, but, out of those 39 orbits, 26 leave the simulation entirely (18 leaving the solar system and eight falling to the inner solar system). A final note should be made about the definition of stability: one could measure stability by the consistency in $e$ and $i$, the consistency of the orbital period (or $a$), or the ability to remain in the Kuiper Belt. The least stable object would be different in each case. Ixion is the DP with the smallest number of regular orbits (0%) while remaining in resonance, 2005 QU$_{182}$ is the DP that is less likely to remain close to its original $a$ (0%), and Haumea is the most unstable DP considering the number of orbits that remain in the solar system (37.5%).

Table 3 shows the list of objects ejected and the end state of their ejections. Overall, 378 (out of 6800; 5.56%) of the orbits were ejected from the solar system; this implies that the solar system is still evolving and 1 Gyr into the future will have lost about 20 km could, potentially, become a game changer. As far as life on Earth is concerned, any object larger than 20 km could, potentially, become a game changer.

5.2. Different Outcomes for Irregular Orbits

Table 4 shows the list of objects ejected and the end state of their ejections. Overall, 378 (out of 6800; 5.56%) of the orbits were ejected from the solar system; this implies that the solar system is still evolving and 1 Gyr into the future will have lost about 20 km could, potentially, become a game changer. As far as life on Earth is concerned, any object larger than 20 km could, potentially, become a game changer.

There are expected to be approximately 10,000 times more objects between 20 and 200 km than those in our sample.

Table 3

| Object | $M$ [10$^{-3} M_{\odot}$] | $a$ | $e$ | $i$ | All 3 | Classification |
|--------|------------------|-----|-----|-----|-------|----------------|
| Eris   | 2.7956           | 197 | 191 | 197 | 191   | Stable         |
| Pluto  | 2.4467           | 200 | 200 | 200 | 200   | Resonant (3:2) |
| Haumea | 0.6706           | 46  | 4   | 53  | 4     | Unstable       |
| 2007 OR10 | 0.6110           | 58  | 128 | 145 | 0     | Stable         |
| Varuna | 0.6706           | 191 | 187 | 200 | 187   | Resonant (3:2) |
| Quaoar | 0.2343           | 200 | 200 | 200 | 200   | Stable         |
| 2002 MS4 | 0.1369           | 129 | 95  | 133 | 95    | Unstable       |
| Sedna  | 0.1254           | 200 | 200 | 200 | 200   | Stable         |
| Orcus  | 0.1073           | 161 | 1   | 153 | 1     | Stable         |
| 2014 EZ31 | 0.1012           | 200 | 197 | 200 | 197   | Stable         |
| 2010 IO29 | 0.0635           | 200 | 188 | 200 | 188   | Resonant (21:5) |
| 2002 AW$_{97}$ | 0.0606 | 200 | 199 | 200 | 199   | Stable         |
| 2015 KH62 | 0.0594           | 200 | 200 | 200 | 200   | Stable         |
| Varda  | 0.0446           | 200 | 200 | 200 | 200   | Stable         |
| 2007 UK$_{26}$ | 0.0415 | 66  | 95  | 102 | 59    | Unstable       |
| 2013 FY$_{27}$ | 0.0391 | 122 | 82  | 130 | 82    | Unstable       |
| 2003 AZ$_{54}$ | 0.0349 | 197 | 188 | 196 | 188   | Unstable       |
| 2015 RR$_{33}$ | 0.0331 | 14  | 16  | 67  | 14    | Unstable       |
| 2003 OP$_{12}$ | 0.0288 | 200 | 200 | 200 | 200   | Stable         |
| 2014 UX$_{234}$ | 0.0266 | 97  | 180 | 163 | 88    | Unstable       |
| Ixion  | 0.0263           | 200 | 0   | 196 | 95    | Resonant (3:2) |
| Varuna | 0.0259           | 200 | 200 | 200 | 200   | Stable         |
| 2005 EN$_{31}$ | 0.0255 | 200 | 200 | 200 | 200   | Stable         |
| 2002 TC$_{322}$ | 0.0239 | 188 | 0   | 151 | 0     | Resonant (5:2) |
| 2010 RF$_{31}$ | 0.0227 | 186 | 170 | 190 | 170   | Stable         |
| 2004 GV$_{9}$ | 0.0218 | 200 | 168 | 200 | 168   | Stable         |
| 2002 UX$_{25}$ | 0.0209 | 199 | 173 | 199 | 173   | Unstable       |
| 2010 KZ$_{90}$ | 0.0167 | 200 | 200 | 200 | 200   | Stable         |
| 2005 UQ$_{131}$ | 0.0121 | 200 | 199 | 200 | 199   | Stable         |
| 2012 VP$_{13}$ | 0.0118 | 200 | 200 | 200 | 200   | Stable         |
| 2014 WK$_{109}$ | 0.0105 | 200 | 200 | 200 | 200   | Stable         |
| 2005 QU$_{182}$ | 0.0088 | 0   | 98  | 137 | 0     | Unstable       |
| 2010 EK$_{19}$ | 0.0054 | 198 | 2   | 2  | 2     | Resonant (7:2) |
| 2002 TX$_{100}$ | 0.0018 | 200 | 200 | 200 | 200   | Stable         |

Table 4

| Object | Total Ejections | Outer Space | Inner System | Collisions |
|--------|-----------------|-------------|--------------|------------|
| Haumea | 125             | 98          | 25           | 2(Jup)     |
| 2007 OR$_{23}$ | 80       | 70          | 9            | 1(Nep)     |
| 2002 MS$_{4}$ | 51         | 41          | 10           | 0          |
| 2015 RR$_{33}$ | 40        | 33          | 6            | 1(Nep)     |
| 2013 FY$_{27}$ | 30        | 26          | 4            | 0          |
| Orcus | 26              | 18          | 8            | 0          |
| 2007 UK$_{26}$ | 12        | 8           | 4            | 0          |
| 2010 RF$_{31}$ | 5         | 4           | 1            | 0          |
| 2002 TC$_{322}$ | 4        | 4           | 0            | 0          |
| 2003 AZ$_{54}$ | 3         | 1           | 2            | 0          |
| 2002 UX$_{25}$ | 1         | 1           | 0            | 0          |
| 2010 EK$_{19}$ | 1         | 1           | 0            | 0          |

Total for the sample: 378, 305, 69, 4.
(Fraser & Kavelaars 2009; Dones et al. 2015); this implies an infall rate of a few thousand objects larger than 20 km each gigayear (or, equivalently, a few every megayear).

Specifically, the most likely object to reach the inner solar system within the next gigayear is Haumea, while 2002 MS₄, 2007 OR₁₀, and Orcus have a modest probability and 2015 RR₄₅, 2013 FY₂₇, 2007 UK₁₀₂₆, 2003 AZ₄₄, and 2010 RF₄₃ have a minor probability of reaching the inner solar system (see Table 4). It must also be noted that if Haumea (and, to a slightly lesser extent, 2007 OR₁₀) does not reach the inner solar system within the next gigayear, it becomes less likely to do so, as it is also very likely to be ejected from the solar system. In general, a critical step for infalling orbits includes a close encounter with Neptune, but such an encounter is approximately four times more likely to result in an expulsion of the solar system than an infalling trajectory.

The most likely fate for objects ejected from the Kuiper Belt is expulsion from the solar system. Haumea and 2007 OR₁₀ are the objects more likely to do so, with a half-life of about 1 Gyr; 2002 MS₄, 2015 RR₄₅, 2013 FY₂₇, and Orcus are also quite likely to be ejected, with a half-life of 2–4 Gyr; and 2007 UK₁₀₂₆, 2010 RF₄₃, 2002 TC₃₀₂, 2003 AZ₄₄, 2002 UX₂₅, and 2010 EK₁₃₉ have a nonnegligible probability of ejection but with a half-life longer than that of the solar system. In summary, we expect about 1.5 DPs to be expelled from the solar system in the next gigayear.

5.3. Future Stability of the Outer Solar System

When trying to paint a picture of the Kuiper Belt 1 Gyr into the future, the most striking feature is the overall loss of approximately two objects (1.89 objects lost).

Another characteristic is that there is no object for which its 200 orbits converge at the same point after 1 Gyr (even for Sedna, the slowest and most stable of the DPs, the behavior of the different runs shows we cannot keep track of the mean anomaly beyond 10 Myr). If we ignore the mean anomaly, we can only have some confidence in the argument of pericenter and longitude of the ascending node for Sedna and 2012 VP₁₁₃; for all other objects, we lose coherence for both quantities before the 50 Myr mark. On the other hand, our simulations do not consider interactions with external potentials (such as stellar flybys or the effect of Planet 9); any such encounter would more severely affect precisely those objects that are particularly well behaved in our simulations.

This does not mean that we will be completely unable to recognize some of the orbital characteristics of the trans-Neptunian DPs in the next gigayear.

Focusing only on the six DPs we have classified as resonant, we see that they are extremely likely to remain in the solar system (5.96 out of six; 99.3%); in fact, they are likely to remain inside their current MMRs (5.915 out of six; 98.6%), but only slightly less than half will remain with similar values of their mean orbital parameters (2.89 out of six; 48.2%).

Although the 28 nonresonant objects are less likely to remain in the solar system (26.15 out of 28; 93.4%), they are more likely to maintain similar values of their main orbital parameters (20.135 out of 28; 71.9%).

Overall, by focusing on the main orbital parameters, we would be able to recognize about two-thirds of the DPs (23.025 out of 34; 67.7%), and about three-fourths (26.05 out of 34; 76.6%) if we allow for the resonant objects that remain inside their MMRs.

5.4. Effect of Additional Unseen Populations

When considering together this work (which deals with DPs and TNOs greater than about 400 km) and our previous work (Muñoz-Gutiérrez et al. 2019, which deals with small TNOs in the 2–10 km range), there is a gap for a population of intermediate-sized objects (20–400 km). These objects are able to also have a gravitational effect on other objects, but overall, we expect their disruptive effect to be much smaller than the one produced by our sample. We would expect that including 100 additional objects (the most massive within this population) would increase the disruption by less than 10% but with an increase of a factor of ~5 on the computational effort, while a massive simulation that includes the 1000 most massive objects of this population would include nearly all of the disruptive effect due to the rapid decay in the mass distribution and increase the disruption by less than 20% but with an egregious increase on the computer expense.

A different possibility would be the presence of additional large TNOs not included in our sample (larger than 400 or even 1000 km). For example, Schwamb et al. (2009) and Shankman et al. (2017) estimated between 40 and 80 Sedna-like objects in the outer solar system; these objects are expected to be similar in size, eccentricity, and perihelia to Sedna. Such a population would have very little effect on our 34 object sample. We can see that Sedna is very stable; this implies that no other body from our sample interacts with it. Equivalently, Sedna does not interact with any other object; also, with a perihelion of 76 au, it never gets close enough to perturb objects that could potentially get close enough to Neptune to be strongly disrupted. Also, most of the effect between DPs is secular in nature, and any object with such orbital characteristics would spend too little time close to other objects for the secular interactions to be significant. Interactions between pairs of Sedna-like objects would also be unimportant due to the large volume at such large distances from the Sun.

A potentially more disruptive population would be objects with semimajor axes similar to that of Sedna but with perihelia able to reach the classical Kuiper Belt region; any such object would be much less efficient than the DPs from our sample, since they spend only a very small fraction of their time within the Kuiper Belt range. Since only about 1% of their time would be spent in the classical Kuiper Belt region, approximately 100 such objects would be needed to contribute as much as any single object from our sample, and a few hundred would be needed to change our overall conclusions. While objects with these characteristics cannot be ruled out, such a large population is not expected to be hidden from us.

While the existence of additional large and distant DPs would decrease the stability of our sample, and our results thus represent a lower limit to the rate at which DPs will be lost, we do not expect that there are enough additional interactions to change our conclusions significantly.

5.5. Other Considerations

This study explores the future evolution of the trans-Neptunian region. It would be interesting to know how this region has evolved to reach its present configuration. It is obvious that the number of DPs is falling with time; this also means that the evolution speed is slowing down with time.

According to Muñoz-Gutiérrez et al. (2017), the evaporation rate is proportional to the square of the mass of the disk in
We performed a frequency analysis considering the simplest model of the solar system (i.e., Sun + giant planets), running thousands of orbits in a region broadly covering from 30 to 82 au in $a$ and 0 to 0.6 in $e$.

The FMA method allows us to estimate with some certainty the stability of the outer solar system using only very short numerical integrations. Most of the 34 DPs in our sample were found to be located in stable areas of the produced diffusion maps, both when considering 0° inclination or a random inclination between 0° and 50°.

The random inclination simulations also allowed us to determine, by means of the diffusion time, the stability of Kuiper Belt objects as a function of their perihelion. We found, as can be expected, that the greater the perihelion, the longer the stability time.

Overall, the large diffusion maps we used to study statistical stability are convenient for the analysis of comets (or any large set of objects), but, for our list of 34 specific DPs, a custom-made map for each object will give us a better perspective on the evolution of each of them. The analysis of the individual maps suggests that only five objects from our sample are unstable on timescales of 1 Gyr.

While an FMA will provide a certain degree of confidence for the evolution of any given particle, these are lacking due to the omission of the DPs. This omission impacts the results in two relevant areas: first, DPs are the second most important ingredient in the stability of the Kuiper Belt region; second, the effect of the DPs on the stability is slower to take hold than the one from the giant planets, and thus FMA are not entirely suited to predict the long-term behavior in every region of phase space.

A better strategy to study the stability of these DPs consists of running a more complete model of the solar system and integrating it for a longer period of time.

Since the Kuiper Belt region of the full model is chaotic, a statistical analysis is necessary; to do this, we decided to run an ensemble of models with identical initial conditions but random perturbations (arising from the time steps of the numerical integrator). We performed 200 long-term simulations considering the mutual perturbations of the giant planets and the 34 largest TNOs; our final analysis consists of 6800 1 Gyr long individual realizations (200 for each of the 34 DPs).

From our simulations, we divide the 34 DPs into three categories based on the long-term behavior of their 200 realizations. We find that six DPs are “resonant” (where most of their realizations remain within a known resonance); of the 28 nonresonant objects, 17 are “stable” (where most of their realizations present a very small evolution of $a$, $e$, and $i$), while 11 are “unstable” (where many of their realizations change drastically in at least one of their main orbital parameters).

We also find that 12 DPs present at least one realization where they are unable to finish the simulation, either by being ejected from the solar system, by colliding with a giant planet, or by falling to the inner solar system. Out of these 12 objects, we classified 10 as unstable and two as resonant.

When studying the global evolution of the Kuiper Belt over the next gigayear, we find that statistically, 23,025 DPs are expected to remain with recognizable orbital parameters, 9,085 are expected to have drastic transformations in at least one of their main orbital parameters, and 1,89 are expected to be lost altogether.

Of the 1,89 lost objects, 1,525 DPs are expected to leave the solar system altogether, 0.345 are expected to find their way to

6. Summary and Conclusions

In this work, we have explored the global stability of a vast region of phase space in the outer solar system, where the majority of the largest known TNOs are located. We have focused most of our work on the 34 largest known TNOs, which we indistinctly call DPs.
inner solar system (i.e., to reach distances of less than 5 au from the Sun), and 0.02 are expected to collide with a giant planet.

An unexpected result of this work is that objects like Haumea, 2007 OR10, and 2015 RR245 turn out to be in highly unstable orbits. Though it may be thought that these DPs are permanent fixtures of the solar system, in reality, it is highly unlikely that all three of them will be part of the solar system inventory 1 Gyr into the future. While interesting, we are leaving a detailed analysis of the chaotic nature of these and other DPs in our sample for a future paper.

We thank the anonymous referee for an insightful report, which helped to improve the quality of this paper. A.P. would like to express thanks for grant PAPIIT IG 100319.

ORCID iDs
Marco A. Muñoz-Gutiérrez @ https://orcid.org/0000-0002-0792-4332
Antonio Peimbert @ https://orcid.org/0000-0001-7042-2207
Matthew J. Lehner @ https://orcid.org/0000-0003-4077-0985
Shiang-Yu Wang (王祥宇) @ https://orcid.org/0000-0001-6491-1901

References
Agnor, C. B., & Hamilton, D. P. 2006, Natur, 441, 192
Batygin, K., & Brown, M. E. 2016, AJ, 151, 22
Brown, M. E. 2008, in The Solar System Beyond Neptune, ed. M. A. Barucci et al. (Tucson, AZ: Univ. Arizona Press), 335
Chambers, J. E. 1999, MNRAS, 304, 793
Dones, L., Brassier, R., Kaib, N., & Rickman, H. 2015, SSRv, 197, 191
Dubinski, J. 2019, Icar, 321, 291
Duncan, M., Levison, H., & Dones, L. 2004, in Comets II, ed. M. Festou, H. U. Keller, & H. A. Weaver (Tucson, AZ: Univ. Arizona Press), 193
Duncan, M. J., Levison, H. F., & Budd, S. M. 1995, AJ, 110, 3073
Fraser, W. C., & Kavelaars, J. J. 2009, AJ, 137, 72
Gaslac Gallardo, D. M., Giuliani Winter, S. M., Madeira, G., & Muñoz-Gutiérrez, M. A. 2020, Ap&SS, 365, 5
Glidden, B., & Duncan, M. 1990, AJ, 100, 1680
Glidden, B., Marsden, B. G., & Vanlaerhoven, C. 2008, in The Solar System Beyond Neptune, ed. M. A. Barucci et al. (Tucson, AZ: Univ. Arizona Press), 43
Holman, M. J., & Wisdom, J. 1993, AJ, 105, 1987
Laskar, J. 1990, Icar, 88, 266
Laskar, J. 1993, PhysD, 67, 257
Laskar, J., Froeschlé, C., & Celletti, A. 1992, PhysD, 56, 253
Levison, H. F., & Duncan, M. J. 1997, Icar, 127, 13
Liu, S.-F., Hori, Y., Müller, S., et al. 2019, Natur, 572, 355
Margot, J.-L. 2015, AJ, 150, 185
Muñoz-Gutiérrez, M. A., & Giuliani Winter, S. 2017, MNRAS, 470, 3750
Muñoz-Gutiérrez, M. A., Peimbert, A., Pichardo, B., Lehner, M. J., & Wang, S. Y. 2019, AJ, 158, 184
Muñoz-Gutiérrez, M. A., Pichardo, B., & Peimbert, A. 2017, AJ, 154, 17
Murray, C. D., & Dermott, S. F. 1999, Solar System Dynamics (Cambridge: Cambridge Univ. Press), 9
Nesvorný, D., Vokrouhlický, D., Dones, L., et al. 2017, ApJ, 845, 27
Nesvorný, D., Vokrouhlický, D., Stern, A. S., et al. 2019, AJ, 158, 132
Ortiz, J. L., Santos-Sanz, P., Sicardy, B., et al. 2020, A&A, 639, A134
Rambaux, N., Bugué, D., Chambat, F., & Castillo-Roger, J. C. 2017, ApJL, 850, L9
Roberts, A. C., & Muñoz-Gutiérrez, M. A. 2021, Icar, 358, 114201
Robutel, P., & Laskar, J. 2001, Icar, 152, 4
Schwamb, Megan E., Brown, Michael E., & Rabinowitz, David L. 2009, ApJ, 694, L45
Schwamb, M. E., Brown, M. E., & Fraser, W. C. 2014, AJ, 147, 2
Shankman, Cory, Kavelaars, J. J., Bannister, Michele T., et al. 2017, AJ, 154, 50
Šidlichovský, M., & Nesvorný, D. 1996, CeMDA, 65, 137
Silbbee, K., & Tremaine, S. 2018, AJ, 155, 75
Slattery, W. L., Benz, W., & Cameron, A. G. W. 1992, Icar, 99, 167
Tancredi, G. 2010, in IAU Symp., 263 (Icy Bodies of the Solar System), ed. J. A. Fernandez et al. (Cambridge: Cambridge Univ. Press), 173
Tancredi, G., & Favre, S. 2008, Icar, 195, 851
Tiscareno, M. S., & Malhotra, R. 2009, AJ, 138, 827
Torbett, M. V. 1989, AJ, 98, 1477
Trujillo, C. A., & Sheppard, S. S. 2014, Natur, 507, 451
Valsecchi, G. B. 2009, SuvAJ, 179, 1
Volk, K., & Malhotra, R. 2008, ApJ, 687, 714
Yu, T. Y. M., Murray-Clay, R., & Volk, K. 2018, AJ, 156, 33