Regularised dynamic optimal transportation of electric vehicles over networks considering strategic charging pricing

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Abstract
A dynamic optimal transport problem of electric vehicles (EVs) over a network is investigated. The EVs are considered to be transported from their initial locations to the destination nodes for charging purposes. In our framework, the operators of charging stations are strategic, and each of them designs their charging pricing optimally to maximise the revenue. Since EVs play an essential role as power loads at the charging stations, the designed transport strategy by the EV operator has an impact on the market energy price which in turn influences the charging prices. Therefore, to design an efficient transport plan, the EV operator needs to take into account its influence on the charging pricing and the market energy price due to their complex interplay. To achieve this goal, a unified framework is proposed for optimal EV transportation by considering factors including the delay, charging cost, and real-time social demand of EVs over a finite-time horizon. The balanced dynamic optimal transport strategy is enabled through a combined quadratic and entropic regularisation. To compute the equilibrium pricing for all charging stations, an iterative particle-swarm optimisation scheme is designed which addresses a high-dimensional nonlinear optimisation problem. Finally, case studies are used to illustrate and corroborate the obtained results.

1 | INTRODUCTION

Electric Vehicles (EVs) play an important role in the pollution reduction for sustainable development especially in the urban areas, since their operations can solely depend on the clean energies from solar and wind instead of fossil fuels. Moreover, the technologies including EV battery lifetime and charging efficiency become increasingly mature. Therefore, there is a foreseeable growth of the number of EVs replacing traditional gasoline vehicles to fulfil the smart city initiative.

Similar to the rental bike services such as Citi bikes, it can be a profitable business model for commercial companies to provide EV rental services. Based on the report [1], the global EV rental market was $10.22 billion in 2019, and it is projected to reach $17.36 billion by 2024 with an over 11% compound annual growth rate. In this rental market, the customers can rent an EV at some station and return it to some other station based on their own needs. Thus, EV rental service can provide a convenient way for people when the travel distance is farther than the one accomplished by rental bikes. One of the critical problems for the EV rental service provider (SP), which is absent in the bike rental services, is to recharge those EVs whose battery charge is almost depleted. Since, not every EV rental station is equipped with charging facilities due to the building and maintenance costs, the EV rental SP needs to recharge the EVs at some specific locations that provide charging service. Moreover, due to the high social demand of rental EVs, the EV rental SP needs to schedule the EV charging in a fast and economical way.

An EV transport problem in a city is considered by taking heterogeneous practical factors into account. The city transportation system can be modelled as a mesh network consisting of nodes and links. Among all the nodes, some of them are close to the EV charging stations. The EV rental SP’s goal is to re-dispatch those EVs that are near to completing power to the charging stations. Those EVs to be recharged are geographically distributed over the network at some rental stations. Due to the complex network structure and the massive amount of EVs to be reallocated, it is imperative for the EV rental SP to make an effective and economical transport strategy. The first factor to be considered is the transportation cost. The EV rental SP aims to transport the EVs to
a charging station which is close to the original location. Second, the SP is aware of the time spent on transportation, and thus he takes into account the congestion on the transport route that results in delay. To capture the charging time at the station, the SP also needs to reallocate the EVs to different stations instead of to a single one, since the charging capacity of each station can be limited. Furthermore, the number of EVs to be transported is massive. Therefore, the EV rental SP can transport the EVs dynamically in a finite-time horizon instead of in a single instance. Another critical factor to be considered is the social demand of rental service. It is natural that EV demands at some locations, for example, parks and financial districts, in the city are higher than others. To accomplish this goal, the SP prefers to recharge more EVs at the stations that have a higher EV rental requirements. Thus, after the completion of charging, the EVs can be redistributed to the rental service market quickly.

To this end, a hybrid-regulserised approach is proposed that achieves the dynamic optimal transport of EVs over networks. Specifically, a quadratic cost of the traffic flow is adopted on each transportation link to capture the delay. This traffic cost function indicates that the EV rental SP is sensitive to the incurred delay. Furthermore, the quadratic form has a reasonable choice since the EVs should be charged and re-dispatched in a fast manner to meet the social demand of EVs. In addition, an entropic penalisation term is imposed between the designed final EV distribution and the desired one which is given as a prior information from social demand of EVs at each station. Therefore, the SP has an incentive to allocate the EVs optimally which is consistent with the desired distribution. However, note that due to transport costs (e.g., distance and delay on links), the desired EV final distribution may not be achieved.

To capture the aforementioned transport factors, an objective function is considered including the cost in each time horizon during the transport window. It also captures the fact that the EV rental SP needs to design a transport strategy for each time step by splitting the total amount of EVs to be transported into different horizons strategically. Another important factor to be considered is the charging price at the stations. Note that at different time instances, the energy price is dynamically changing when considering the real-time electricity pricing scheme. This factor further introduces an optimisation consideration for the SP, where it takes into account the EV charging cost when designing the dynamic transport strategy. The final consideration of the established problem is the influence of EVs on the charging pricing. Note that each charging station aims to maximise its own revenue by providing the EV charging services which creates a noncooperative game between different charging stations. Therefore, the charging stations need to determine their own real-time EV charging prices strategically. Furthermore, during peak hours (power usage is high in the considered area), the charging price at each station becomes higher due to the increase in public electricity price. This pricing pattern will influence the charging behaviour of EVs and hence the transport strategy of the SP. Note that the massive EV fleets in the future rental market will play an important role as a load in the energy system. Different from those EVs owned by individuals and charged at home, these rental EVs may be charged simultaneously at some specific charging stations in the urban area managed by the rental companies. Thus, the total EV charging load, together with some other major loads (e.g., the aggregated residential and commercial loads), will in turn affect the power pricing. To design an optimal transport scheme, the charging pricing game between stations as well as the interplay between EVs and power pricing should be analysed holistically.

The contributions of this study are summarised as follows.

1. Establishment of a holistic framework for economical electric vehicle (EV) transportation over a mesh network by considering delay, charging cost, and real-time social demand over a time horizon.
2. Development of a balanced dynamic optimal transport strategy of EVs through hybrid quadratic and entropic (KL-divergence) regularisation.
3. Integration of the transportation network with a demand-aware power market, and achievement of the equilibrium charging pricing for profit maximisation of each charging station through an intelligent particle-swarm optimisation scheme.

1.1 Related works

Transport over networks has been investigated in many fields including mathematics [2], operations research [3], and engineering [4]. In the original formulation without regularisation, the transport problem becomes a linear assignment problem which can be solved efficiently by linear programing techniques [5,6]. The problem considered here is related to the regularised optimal transport literature. A typical regulariser that has been investigated recently is the entropic regularisation which greatly enhances the computational efficiency to obtain the optimal transport strategy [7]. Our work leverages this convex entropic regulariser to quantify the difference between targeted and obtained final distributions of EVs. In addition, we use another quadratic regulariser to penalise the traffic delays in the network. In transportation engineering, a great number of previous works have been focussed on obtaining the user equilibrium in traffic assignment by assuming that each agent is selfish in maximising their own utility [8,9]. Extension of the static traffic assignment problem to the dynamic setting has also been investigated such as [10,11]. Furthermore, the authors in [12] have investigated the strategic path planning and charging for EV owners over a network under time-varying traffic conditions and dynamic location-based electricity pricing. [13] has addressed a joint routing and charging scheduling optimisation problem for an Internet of electric vehicle network and developed an approximate algorithm to compute the solution in a distributed manner. A vehicle routing problem with mixed conventional and electric vehicles has been considered in [14], and the authors have developed heuristic search methods to address the formulated
mixed integer programming that minimises the total travel and charging costs. Different to the discussed selfish routing problems, in our work, we establish a hybrid-regularised dynamic optimal transport framework and design the EV transport strategy in a centralised way. In addition, different to the EV routing problems that regard the charging pricing as an exogenous variable, our transport optimisation formulation considers an active interaction with the charging stations where the charging pricing and the transport strategies are intertwined.

There have been a great number of previous works on investigating the pricing of EV charging. For example, the authors in [15] have developed stochastic dynamic pricing and energy management policy for EV charging service providers by considering the profit, customer satisfaction, and the impact on power grid under the uncertain renewable energy penetration holistically. A game-theoretic model has been proposed in [16] to design the optimal EV charging pricing with considerations of EV users’ self-interested charging behaviour, traffic pattern and congestion and charging costs. Furthermore, a competitive charging pricing scenario between stations has been investigated in [17,18] using game theory. To deal with the increasingly complex charging system’s objectives and uncertain EV user’s behaviours, artificial intelligence techniques have also been widely leveraged for optimal EV charging pricing. For example, in [19,20], a reinforcement learning approach has been proposed to determine the optimal charging scheduling and pricing strategies for an EV charging station in an online fashion [21] has developed a multi-modal adaptive dynamic programing based approach to decide the dynamic charging/discharging policies of EVs by considering dynamic pricing and stochastic EV arrival/departure schedule. Herein, we also model the pricing at EV charging stations using a noncooperative game. In contrast, the major focus here is on the development of optimal dynamic EV transport plan by considering the transport and charging costs which is inherently coupled with the pricing game played among charging stations.

With the increasing usage of EVs, the transportation system and the power system become more coupled due to their natural dependencies [22–24]. In [25], the authors showed that the EV charging patterns have a significant impact on the energy price. In addition, the timing of EV charging and its impacts on the power system stability received a great amount of attention and various charging methods were proposed including the off-peak and coordinated charging [26–28]. On the other hand, the electricity pricing determined by the power system operator has an influence on the EV charging behaviour [29,30]. The authors in [31,32] have investigated the interplay between transportation and power systems from a pricing perspective and proposed a scheme in which two systems can achieve a socially optimum operating point. Furthermore, [33–35] have developed holistic optimisation frameworks for the planning and operation of power and transportation systems from a collaborative viewpoint. Our considered model is different from this paradigm. We investigate the decentralised operations where each system operator optimises their own objective. Furthermore, we study the system’s interplay from a economics perspective where the EV load has an influence on the charging pricing at charging stations and in turn the price affects the EV transport strategy design.

1.2 | Organisation of the study

Herein, Section 2 formulates a general static optimal transportation problem of EVs over networks. Section 3 extends the framework to a dynamic one with a consideration of flexible final EV distribution. Section 4 models the interplay between the EV transport operator and the charging stations and designs an algorithm to compute the solution. Section 5 corroborates the established framework and the designed algorithm through extensive case studies. Section 6 provides the conclusion.

2 | PROBLEM FORMULATION

An optimal transport problem of EVs is considered for charging over a network. EVs are initially distributed at nodes in the network. The nodes can refer to the EV rental stations in a city, and the links of the network represent the physical transportation routes. Since not every station is installed with charging equipment, the EVs need to be reallocated to charge at some specific stations. To meet the social demand for EV renting, the EV system operator has a predefined number of EVs to be transported and charged at each charging station.

2.1 | Transportation of EVs over networks

Denote the transportation network by $G = (V, E)$, where $V$ is the set of nodes and $E$ is the set of directed links. We also denote $V_c \subseteq V$ as the set of nodes associated with EV charging stations. Further, denote $C \in \mathbb{R}^{V_c \times V_c}$ as the cost matrix where the element $C_{ij}$ is the shortest path from node $i$ to node $j$, for $i, j \in V$. Furthermore, denote the initial distribution of the EVs over the transportation network as $\rho_0 \in [0,1]^{V_c}$. Our target is to reallocate the EVs to a final distribution $\rho_1 \in [0,1]^{V_c}$. Here, $\rho_0$ and $\rho_1$ are known information, that is, $\rho_0$ and $\rho_1$ are available data to the EV system operator. Naturally, $\rho_0$ can be obtained by calculating the amount of EVs to be transported at each node/station, and $\rho_1$ represents the EVs’ locations after transportation determined by the EV system operator. The EV reallocation problem is to find an optimal joint distribution $\mu \in \mathbb{R}^{V_c \times V_c}$ that transports $\rho_0$ to $\rho_1$. Specifically, the problem can be formulated as

Herein, EV system operator refers to EV rental SP, and we use them interchangeably.
where one is a $|V|$-dimensional vector with all one entries. The constraints in (OP) capture the balanced transport of EVs that the total number of EVs is maintained after the reallocation.

Note that (OP) has $|V|^2$ number of decision variables which increases quadratically in terms of the number of nodes. In our problem, the transportation network exhibits a sparse structure, that is, $|E| \ll |V|^2$. Hence we consider an alternative equivalent formulation of (OP) in the following.

**Remark:** (OP) is suitable in some cases in the sense that we can eliminate some paths that are out of EV’s capacity by imposing conditions on the cost matrix $C$. Further, a large amount of traffic over a particular link leads to a congestion on that route. To take into account this practical delay factor, we need to devise an optimal transport strategy that utilises as many links as possible while minimising the overall transportation cost simultaneously.

### 2.2 Congestion consideration

To capture the congestion effect, we aim to obtain a solution $f$ which is not sparse. Hence, we add a delay cost term into the objective function. Specifically, the cost function becomes

$$J_c(f|\rho_0, \rho_1) = \sum_{e \in E} c_e f_e + \frac{\epsilon}{2} \sum_{e \in E} f_e^2,$$

where $f_e : \mathbb{R}_+^{|E|} \rightarrow \mathbb{R}_+$, and $c_e \$/unit is the cost of transport on link $e \in E$. The optimal EV transportation problem on minimising the overall cost can be formulated as follows:

$$\text{(OP - Q)} : W_{1,e}(\rho_0, \rho_1) = \begin{cases} \min_{f_e, \ e \in E} \sum_{e \in E} c_e f_e \\ \text{s.t.} \\ Df = \rho_0 - \rho_1, \\ f_e \geq 0, \ \forall e \in E. \end{cases}$$

The formulated problem (OP') captures several major features of EV transportation tasks managed by the rental company. First, the EV rental company aims to minimise the overall transportation cost (reflected through the minimisation of $\sum_{e \in E} c_e f_e$). Second, the EV charging locations are considered through the final EV distribution $\rho_1$. Note that $\rho_1$ is an $|N|$-dimensional vector. Therefore, the elements in $\rho_1$ associated with those nodes without charging stations are assigned with a value 0. As shown in the case study later (Section 5), the charging stations are distributed in the city, and the EV rental company needs to transport EVs to these charging stations over the network (captured by matrix $D$) strategically.

Note that (OP') is a linear programming (LP) problem with $|E|$ decision variables which generally admits a sparse solution. Equivalently speaking, the EVs are transported over a set of links while the rest of links are of no transportation flow. Furthermore, a large amount of traffic over a particular link leads to a congestion on that route. To take into account this practical delay factor, we need to devise an optimal transport strategy that utilises as many links as possible while minimising the overall transportation cost simultaneously.

### 3 Regularised Dynamic Optimal Transportation

In this section, we first extend the problem (OP - Q) by considering flexible final distributions, and then present the regularised dynamic optimal transportation problem of EVs.
3.1 | Flexible final distribution of EVs

The formulated framework in Section 2 can be extended to capture more practical considerations. For example, the final distribution $\rho_1$ can be considered non-prespecified. Specifically, the support of $\rho_1$ is known, that is, the charging stations are fixed. Since more EVs at one charging station take more time to complete the charging task, the number of EVs at each charging station after transport needs to be determined strategically. Moreover, the EV rental SP can have an anticipation of desirable final distribution $\rho_1$ that matches the demand pattern of EVs. Denote this prior anticipated final distribution by $\tilde{\rho}_1$, which can be estimated based on the historical data of EV demands. Note that $\tilde{\rho}_1$ is a socially optimal re-distribution of EVs in terms of the demand of EV renting services, but it does not capture the transportation cost of moving $\rho_0$ to $\tilde{\rho}_1$. Therefore, the optimal transportation strategy can yield a final distribution of EVs which is not the same as $\tilde{\rho}_1$. Note that $\tilde{\rho}_1$ can be learnt from historical EV renting data.

To this end, Kullback-Leibler (KL) divergence is adopted to quantify the difference between the real obtained final distribution $\rho_1$ and the anticipated/desired one $\tilde{\rho}_1$ which is

$$D_{KL}(\rho_1 \| \tilde{\rho}_1) = \sum_{i=1}^{V} \rho_{1,i} \log \frac{\rho_{1,i}}{\tilde{\rho}_{1,i}}$$

where $\rho_{1,i}$ and $\tilde{\rho}_{1,i}$ are the $i$-th element in $\rho_1$ and $\tilde{\rho}_1$, respectively. Furthermore, denote the feasible set of $\rho_1$ by $\Xi^1$. Then, the problem with entropic regularisation is formulated as:

$$(\text{OP - E}) : \min_{\rho_1 \in \Xi^1, f_e \geq 0, \forall e \in E} \sum_{e \in E} c_e f_e + \frac{\delta}{2} \sum_{e \in E} f_e^2 + \delta D_{KL}(\rho_1 \| \tilde{\rho}_1)$$

s.t. $Df = \rho_0 - \rho_1$, $f_e \geq 0, \forall e \in E$,

where $\delta$ $$/\text{unit}$ captures the criticality of desired final distribution and it is non-negative.

Remark: When $\delta$ goes to zero, $\rho_1$ can be freely chosen in the set $\Xi^1$, and each charging station is assumed to have a sufficient capacity to charge all the EVs.

3.2 | An Illustrative example

A numerical example is presented below to illustrate the optimal transportation under regularisation. Consider a transportation network containing five nodes and six links which is shown in Figure 1. The network structure gives an incidence matrix

$$D = \begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & -1 & 1 & 0 \\
0 & -1 & 0 & 0 & -1 & 1 \\
0 & 0 & -1 & 0 & 0 & -1
\end{bmatrix}$$

For demonstration purpose, consider the normalized cost of each link as $c = [1, 2, 3, 1.5, 0.5, 0.8] /\text{unit}$. Further, the numbers of EVs at node one and node two are assumed to be 50 and 100, respectively. Thus, $\rho_0 = [0, 50, 0, 0, 0]$ unit. The goal is to reallocate the EVs to nodes three and five for battery charging. The efficiency of each charging station can be different. In the case study, the prior desired final distribution is assumed to be $\tilde{\rho}_1 = [0, 0.5, 0, 0, 0]$ unit, that is, the charging efficiency of node 5 is higher than that of node 3. In other words, the minimum charging time is achieved when 60 and 90 EVs go to node 3 and node 5, respectively. However, due to the congestion and transport cost, the desired $\tilde{\rho}_1$ may not be achieved.

3.2.1 | Optimal strategy

In the example, we assume the coefficients $\epsilon = 1.5 /\text{unit}^2$ and $\delta = 2 /\text{unit}$ which quantify the congestion cost and the deviation from desired final distribution, respectively. The optimal solution to (OP-E) is shown in Figure 2. The optimal link flow on links is $f^* = [0, 0.33, 0.15, 0.52, 0, 0.33]$ unit. The number of EVs on each link can be obtained by multiplying $f^*$ by the total number of EVs to be reallocated. In addition, the optimal final distribution of EVs is $\rho_1^* = [0, 0, 0.52, 0, 0.48]$ units. It can be seen that the optimal final distribution does not coincide with the desired one $\tilde{\rho}_1$.

3.2.2 | Effects of congestion

Next, we study the impact of congestion on the optimal transportation strategy. Specifically, we assume $\delta = 1 /\text{unit}$ and we design the strategy with varying $\epsilon$. The result is shown in Figure 3. With a larger $\epsilon$, the congestion has a higher impact on the optimal transportation plan. Specifically, as $\epsilon$ increases, the flows on links become more evenly distributed.
3.2.3 | Effects of charging time

We next investigate the impact of desired final distribution on the optimal transportation decision. Specifically, we assume $\epsilon = 5 \$/unit$^2$ and design the strategy with varying $\delta$. The result is shown in Figure 4. With a larger $\delta$, the system operator has a higher valuation of social demand of EVs on the decision-making. Therefore, the optimal final distribution converges to the desired one as $\delta$ increases.

3.3 | Dynamic optimal transportation with regularisation

Since EV demands in different time slots are not fixed, the number of EVs to be allocated for charging is dynamically changing. To design a more practical operation strategy, the framework needs to be extended by considering the transport problem over a time horizon $T$. We denote the initial distribution of EVs at time $t$ by $\rho_{0,t}$. Here, $\rho_{0,t}$ is a decision variable since the system operator needs to decide the number of EVs to be transported from the initial stations at time $t$. We denote the feasible set of $\rho_{0,t}$ by $\Xi^t$. In a finite-time period $T$, the total number of EVs to be transported at each station is known, that is $\sum_{t=1}^{T} \rho_{0,t} = \bar{\chi}_0$, where $\bar{\chi}_0$ is a constant vector with nonnegative elements satisfying $\Gamma \bar{\chi}_0 = \Gamma_0$, and it captures the fact that we extend the single-period transport problem to a $T$-horizon one. The EV flow on link $e$ at time $t$ is denoted by $f_{e,t}$ and the flows on all links is captured by $f_t$. The desired final distribution at time $t$ is $\bar{\rho}_{1,t}$ which captures the real-time demand of EVs from the society. Furthermore, except for the EVs, the traditional gasoline vehicles (GVs) are also considered which play an essential role on the road traffic and congestion. The GV flow on link $e \in E$ at time $t$ is denoted by $g_{e,t}^v$. Note that $f_{e,t}$ can be obtained from historical data and hence it can be seen as a known constant. To this end, the dynamic optimal transport problem is formulated as:

**FIGURE 2** (a) The optimal traffic flow on each link. (b) The optimal final distribution of EVs

**FIGURE 3** (a) The optimal traffic flow on each link with varying $\epsilon$. (b) The optimal final distribution of EVs with varying $\epsilon$
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FIGURE 4 (a) The optimal traffic flow on each link with varying \( \delta \). (b) The optimal final distribution of EVs with varying \( \delta \)

\[
\begin{align*}
\min_{\rho_{0,t} \in \mathbb{R}^+, \rho_{1,t} \in \mathbb{R}^+} & \quad \sum_{t=1}^{T} \left( \sum_{e \in E} c_e f_{e,t} + \frac{e}{2} \sum_{e \in E} (f_{e,t} + f_{e,t}^*)^2 \right) \\
& + \delta D_{KL}(\rho_{1,t} || \tilde{\rho}_{1,t}) \\
\text{s.t.} & \quad D f_t = \rho_{0,t} - \rho_{1,t}, \quad \forall t, \\
& \quad f_{e,t} \geq 0, \quad \forall e \in E, \quad \forall t, \\
& \quad \sum_{t=1}^{T} \rho_{0,t} = \bar{x}_0
\end{align*}
\]

The above formulation captures the dynamic feature of EV transportation over a network. Note that this transport optimisation problem is solved in a centralised manner by the EV operator. This centralised decision-making mechanism is facilitated by the assumption that the EV operator has a complete information of the network topology (which is the transportation network) and EV distributions. Our next goal is to integrate the critical charging cost with the established framework.

\section{INTEGRATION OF STRATEGIC CHARGING PRICING}

In this section, the strategic charging pricing is modelled through a game-theoretic framework. The optimal transportation problem of EVs is reformulated by including the charging costs. Note that the charging price can be horizontally different at various stations and vertically different across time within a station itself. Furthermore, the equilibrium charging pricing at all charging stations is computed by designing an artificial intelligence based algorithm.

4.1 Influence of EVs on the market energy pricing

To meet the demand of massive EVs, the charging stations need to purchase sufficient amount of power from the utility. Each charging station acts as a node in the power distribution network. The real-time power demand at each station is reflected by \( \rho_{1,t} \). In addition to the EV load, the network also includes the normal load from residents. Another critical factor to be considered is the energy price which is denoted by \( \lambda_t \) \$/kWh. For convenience, we denote \( \lambda = [\lambda_1, \ldots, \lambda_T] \) as the energy price over the entire time periods. Note that the energy price is determined by the utility company which is dependent on the overall demand of the network. Moreover, the energy price in the system increases with the aggregated demand. Therefore, the energy price is chosen to be governed by the equation:

\[
\lambda_t = \psi L_t^2, \tag{5}
\]

where \( \psi \) \$/kWh\(^2\) is a positive parameter quantifying the electricity demand to price, and \( L_t^2 \) is the total load demand of the system at time \( t \). Note that if the EV load is substantial, for example massive EVs are in the market can influence the energy price through (5). If the EV load is not comparable with the normal loads from the residential and other commercial loads, then the market power price \( \lambda_t \) is not significantly influenced by the EVs. To be general, we include the market power pricing factor in our framework. Also note that (5) is not unique, and other forms of electricity pricing can be possible.

During peak hours, the energy price tends to increase. Hence, the charging price at each station becomes higher. This pricing mechanism provides an approach to shift the charging pattern from peak hours to off-peak slots. In addition, as mentioned above, since the massive EVs play an important role in the power market, its charging load may in term affects the market energy price and the EV charging prices, and thus makes
the optimal transportation of EVs a nontrivial decision making problem.

4.2 Non-cooperative game for charging stations

Each charging station aims to maximise its own revenue by providing the EV charging services. Denote the charging price of the station \( i \) at time \( t \) by \( p_{i,t} > 0 \). For convenience, the pricing profile that station \( i \) needs to determine is \( p_i = [p_{i,1}, ..., p_{i,T}] \). Therefore, the utility of the charging station at node \( i \) is

\[
U_i(p_i, p_i^j) = \sum_{t=1}^{T} \left( \gamma p_{i,t} - L_{i,t} \rho_i \right),
\]

where \( U_i : \mathbb{R}_+^T \times [0, 1]^T \rightarrow \mathbb{R}_+ \), \( p_i^j \in [0, 1]^T \) is a profile including all stages' final distribution at node \( i \) (with \( p_{i,t}^j \) denoting the \( t \)-th element), \( \Gamma > 0 \) is a constant that maps the distribution of EVs to the amount of charged power at stations, and \( L_{i,t} \) is the total EV charging load of station \( i \) at time \( t \). \( L_{i,t} \) here is determined by the EVs' transportation strategy, which is naturally affected by the charging pricing. Thus, the charging pricing and transport decision are inherently coupled. Besides, we assume that the amount of charging power to be approximately the same for each EV, since these EVs are near to running out of power, and the EV operator needs to reallocate them to the charging stations to refill the battery for the next cycle of renting service.

To this end, a non-cooperative game between the EV charging stations is formulated. In this game, the players include all the nodes \( i \in V_c \). Each node \( i \) determines its pricing profile \( p_i \in \mathcal{P}_i \) to maximise its revenue (6), where \( \mathcal{P}_i \) is the feasible set of \( p_i \). This pricing game between charging stations is denoted by \( \mathcal{G} \). Note that \( p_i^j \) is influenced by all players' pricing strategies. Hence, player \( i \)'s utility \( U_i \) is determined by \( \{p_i^j\}_{i \in V_c} \). With a slightly modification, we rewrite player's utility as \( U_i(\{p_i^j\}_{i \in V_c}) \). A natural solution concept of game \( \mathcal{G} \) is Nash equilibrium (NE) which is defined as follows.

**Definition 1** A strategy profile \( \{p_i^j\}_{i \in V_c} \), where \( p_i^j \in \mathcal{P}_i \), constitutes an NE of game \( \mathcal{G} \) if it satisfies the inequalities

\[
U_i(p_i, p_i^j) \geq U_i(p_i, p_i^{*j}), \quad \forall i \in V_c, \quad \forall p_i \in \mathcal{P}_i,
\]

where \( p_{-i} \) denotes the strategies of all players in set \( V_c \) excluding player \( i \).

Note that at the NE, no player can increase its utility by unilateral deviation from the current strategy. Another remark is that the decision-making scheme at charging stations is decentralised, which is naturally enabled by the game-theoretic modelling. However, determining strategic charging pricing is also not fully distributed, as each charging node needs to respond to all others' decisions as shown in the Definition 1.

4.3 Optimal transportation problem reformulation

Denote \( \Lambda = [p_{1,t}; ..., p_{N,t}] \) as the charging price profile including all nodes at time \( t \). For the nodes without charging station, the corresponding price is set to zero. By capturing the EV charging costs, the EV rental SP needs to solve the following problem:

\[
\min_{\rho_{0,t} \in \mathbb{R}^+, \rho_{1,t} \in \mathbb{R}^+} \sum_{t=1}^{T} \left( \sum_{e \in E} c_e f_{e,t} + \frac{\epsilon}{2} \left( \sum_{e \in E} f_{e,t} + f_{e,t}^p \right)^2 \right) + \delta D_X L(p_{1,t}|p_{1,t}) + \eta \rho_{1,t} \Lambda_t
\]

s.t. \( D_f = \rho_{0,t} - \rho_{1,t}, \quad \forall t, \)
\( f_{e,t} \geq 0, \quad \forall e \in E, \quad \forall t, \)
\( \sum_{t=1}^{T} \rho_{0,t} = \bar{X}_0, \)

where \( \eta \) kWh/unit is a positive constant. Note that \( \langle OP − D' \rangle \) is convex and hence can be solved efficiently.

The problem formulation \( \langle OP − D' \rangle \) captures multiple objectives of a practical EV rental SP in a holistic fashion: minimising the EV transportation cost and the associated delay cost, and the discrepancy between the desired social optimal final distribution (which could be obtained from the historical EV rental data) and the achieved final distributions of EVs.

4.4 Computing the equilibrium charging pricing

When the charging price of each station is determined, the EV system operator responds to it and addresses \( \langle OP − D' \rangle \) which further results in a new EV load distribution. Therefore, the charging stations need to update their pricing strategies. Our goal is to find an NE at which the pricing profile does not change and the optimal transportation plan is obtained.

For each charging station, its best response to the other charging stations' strategies can be obtained through

\[
p_{i,t}^{br} = \arg \max_{p_i \in \mathcal{P}_i} U_i(p_i, p_{-i}^{*}).
\]

Due to the high dimension of \( p_i \) and complicated utility function of charging station \( i \in V_c \), especially the nonlinear dependence of \( U_i \) on the other stations' strategies, solving (8) is
not a straightforward task. To enable a fast computation of the best response strategy, the particle swarm optimisation (PSO) is adopted which is an artificial intelligence driven method [36]. PSO is an algorithm that can iteratively update the obtained solution towards an optimal one. In PSO, each candidate solution is referred as a particle representing a point with an appropriate dimension. In our problem, the search space is of \( T \)-dimension, and hence each particle has \( T \) variables to determine where each variable is the charging price in a particular time slot. Consider that \( N \) particles constitute the swarm. For convenience, denote the position of particle by \( x_i \) and its velocity by \( v_i \), \( j = 1, \ldots, N \). Each particle is evaluated through the value of the objective function which is called the fitness in PSO. For each particle \( j \), its best solution obtained until the current time is referred as the personal best solution \( x_j^{pb} \). Similarly, the best solution obtained so far for the overall particles is referred as the global best solution \( x^{gb} \). Specifically, there are two objects to be updated in the PSO for each particle \( j \), referred to as the velocity \( v_j \) and position \( x_j \), as follows:

\[
\begin{align*}
v_j(t+1) &= w(t)v_j(t) + c_1R_1(x_j^{gb} - x_j(t)) \\
&+ c_2R_2(x^{gb} - x_j(t)), \\
x_j(t+1) &= x_j(t) + v_j(t+1), \\
w(t+1) &= 0.5 + \gamma, \quad \gamma \sim U(0,1),
\end{align*}
\]

where \( c_1 \) and \( c_2 \) are real-valued acceleration constants which are usually chosen in the range \( 0 \leq c_1, c_2 \leq 4 \); \( R_1 \) and \( R_2 \) are two \( T \)-dimensional diagonal random matrices and the entries are generated according to the uniform distribution \( U(0,1) \); \( w \) is an inertial parameter to avoid the velocity explosion.

For each charging station \( i \in V_c \), it solves the problem (8) to obtain the best response strategy \( p_i^{br} = [p_{i,1}^{br}, \ldots, p_{i,T}^{br}] \). The pricing strategy \( p_i \) can be regarded as a \( T \)-dimensional position where each element in \( p_i \) is a coordinate in the space. The detailed PSO algorithm is summarised in Algorithm 1.

Due to the large search space of \( p_i \in \mathcal{P}_i \), finding the exact NE satisfying (7) can be computationally intractable especially over large-scale network and long horizon \( T \). A practical method is to use the approximate NE as the solution. Specifically, the approximate NE satisfies the conditions

\[
U_i(p_i^s, p_{-i}^s) \geq U_i(p_i, p_{-i}^s) - \theta, \quad \forall i \in V_c, \quad \forall p_i \in \mathcal{P}_i,
\]

where \( \theta > 0 \) is a small constant. Obtaining approximate NE becomes computationally easier comparing with the exact NE counterpart, while the approximate NE maintains a satisfying performance. In the case studies in Section 5, this practical consideration of approximation reduces the number of iterations by several orders as players cease to update their strategies when the resulting utility improvement is marginal.

Together with the PSO scheme, the best response dynamics of all charging stations in computing the approximate NE is summarised in Algorithm 2. Therefore, Algorithm 2 provides a tractable way to obtain the strategic charging pricing at all stations as well as the dynamic transportation policies.

\section{CASE STUDIES}

In this section, the established framework for EV optimal transportation is illustrated using various case studies and the effectiveness of designed algorithm is verified.

\subsection{System models and simulation setup}

The case studies are based on the major transportation network of Midtown Manhattan in New York City as depicted in Figure 5. The considered model includes 42 nodes and 68 links, and thus the transportation network is sparse which is

\begin{algorithm}[h]
\caption{PSO Scheme for Utility Maximisation}
1: Initialise all \( N \) particles’ positions \( x_i(0) \) and velocities \( v_i(0) \) of \( T \)-dimension in the feasible space, for \( j = 1, \ldots, N \), and set particles’ best position \( x_j^{pb} \) as its initial position.
2: Calculate the fitness of each particle according to (6), and set \( x^{gb} \) as the one that achieves the largest fitness.
3: Repeat the following:
4: Each particle updates its velocity \( v_j(t+1) \) and position \( x_j(t+1) \) according to (9)–(11).
5: Project \( v_j(t+1) \) and \( x_j(t+1) \) into the feasible regime if the coordinates are out of bounds.
6: Evaluate the fitness of the particles with updated position.
7: If the current best solution of all particles is better than the stored personal best solution, update \( x_j^{pb} \) with \( x_j(t+1) \)
8: Until meeting a stopping criterion.
9: return \( x^{gb} \)
\end{algorithm}

\begin{algorithm}[h]
\caption{Best Response Dynamics for Computing Approximate NE}
1: Initialise the pricing strategies for all charging stations \( i \in V_c \)
2: for \( i \in V_c \) do
3: Each charging station \( i \) updates its best response pricing scheme \( p_i^{br} \) through PSO Algorithm 1 by considering its effects on dynamic transportation and charging strategies of EVs and the electricity price
4: end for
5: If the pricing strategies constitutes an approximated NE, go to step 6; otherwise, go to step 2
6: return the pricing strategies
\end{algorithm}
suitable for the established framework in (OP'). In addition, three major EV charging stations are allocated at nodes 2, 8, and 11. The investigated three EV charging stations are integrated with the distribution system at nodes 11, 21, and 31, respectively. Besides the EV load at the charging stations, each node in the distribution system also supplies power to the residents in the corresponding area. Since we focus on the dynamic EV transportation and charging price design, a daily power consumption pattern of a typical resident is necessary which can be found in [37]. The unit time scale is 1 h, and hence the horizon of the problem is $T = 24$. We further assume that there are 50,000 residents in the considered area and they could be approximately treated as uniformly distributed. Those residents constitute the residential load of each node in the distribution system. The purpose of considering distribution system is to quantify the impact of EV load on the market power pricing when the network contains massive EVs.

The EV system operator needs to design an optimal transportation strategy to reallocate the EVs from the original nodes to the nodes with charging stations. We assume that the total number of EVs to be transported is 480, in which 1/5, 3/10, and 1/2 proportions are initially distributed at nodes 40, 15, and 34 in Figure 5, respectively. Note that this distribution can be chosen with other values. Since all EVs to be transported are near to full battery discharge, we assume that each EV consumes approximately 40 kWh at the charging station before re-entering the EV rental market. Depending on the social demand of EVs at each station, the desired final distribution at each time $t, t = 1, \ldots, T$, is assumed to be 1/4, 1/2, and 1/4 units at nodes 2, 8, and 11 in Figure 5, respectively. In addition to the EV traffic flow, we also consider the gasoline vehicle (GV) flow in the transportation network that contributes to the delay on each link. The hourly GV traffic flow $f_{e,t}^g, \forall e \in E, t = 1, \ldots, T$, in the following case studies are obtained from the dataset typical for New York City [38]. Comparing with the normal power loads in the considered area, the EV load does not significantly affect the market power pricing. Then, for fast computation of equilibrium pricing in this scenario, each charging station can determine their dynamic EV charging pricing by regarding the market power price as static external information. However, note that our established framework is general enough to investigate the active role of market power pricing in the presence of massive EVs. Some other weighting parameters are chosen as $\epsilon = 28$/unit$^2$, $\delta = 6$ $$/unit$, and $\eta = 5$ kWh/unit.

5.2 Results of dynamic optimal transportation

The optimal transport strategy is first illustrated when the charging pricing is flat during the horizon. Therefore, we solve (OP − $D'$) by regarding $\lambda_t$ as a constant vector. Specifically, for charging station located at node 2, 8, and 11, the charging pricing is assumed to be 0.5 $$/kWh, 0.8 $$/kWh, and 0.6 $$/kWh, respectively. Note that the strategic design of the charging prices are presented in the Section 5.3. In the current case, the charging pricing is higher than the energy price, since each charging station provides the EV charging services to make profit and thus the price needs to take into account the facility and operation costs. Figure 6 shows the corresponding results after solving (OP − $D'$). Note that solving (OP − $D'$) takes less than 0.02 s in MATLAB on a Lenovo ThinkPad X1 Carbon Gen 7 laptop with 1.8 GHz processor and 16.0 GB RAM. It can be seen from 6(a) that the transport strategy is dynamically changing over the time horizon. From Figure 6(b), EVs at node 15 are transported to the charging stations at an earlier stage over the considered time horizon than those EVs initially located at node 34. Furthermore, Figure 6(c) indicates that the optimal final distribution of transported EVs is not too far away from the desired one, where the moderate amount of deviations are caused by the consideration of congestion costs on links and different charging costs at stations.

5.3 Results of optimal transportation and charging pricing

By considering the active role of charging station operators in the framework, the EV charging prices are dynamically changed accordingly. To this end, each charging station becomes strategic in determining the charging price to maximise their own revenue. First, it is critical to show the effectiveness of Algorithm 2 in computing the equilibrium charging price. Note that during the best response updates, each charging station is aware of the influence of its price changes on the EV transportation. For example, by fixing the charging prices at nodes 8 and 11, the station at node 2 optimises its own objective and determines the best pricing decision by anticipating its impact on the transportation systems. The obtained results of coupled system are depicted in Figure 7. The charging pricing is not flat anymore at each station as shown in Figure 7(c). This dynamic changing pattern of charging prices yields a better revenue for charging stations in the network. As shown in Figure 7(a) and 7(b), the optimal initial and final distributions of EVs encounter more small range of adjustments comparing to those in Figure 6. Due to the entropic regularisation, the final distribution of EVs is still close to the desired one. The revenue of each charging station is depicted in Figure 7(d). It can be seen that all three utility values converge to steady states using the designed Algorithm 2. Note that at every update step, each charging station determines its best response pricing strategy using the PSO scheme. Regarding the computational cost of the algorithm, it takes around 38.2 s for the algorithm to yield both the dynamic optimal transport plan and the charging pricing in the considered case study, which is reasonably fast considering the complex competitions between charging stations and interactions between transportation system operator and charging service providers. In summary, the designed algorithm can successfully compute the consistent optimal transport strategies of EVs as well as the strategic charging pricing for each charging nodes in the network.
5.4 Comparison results with other strategies

For comparison, a shortest path transport strategy is considered which does not take into account the congestion and charging costs, and hence the EVs to be transported at each node have a predefined route. Another strategy for comparison is called EV demand-aware strategy. Specifically, the system operator values the most this obtained final distribution of EVs which coincides with the desired one at every time step. Hence, the number of transported EVs from nodes 40, 15, and 34 is fixed at each time t according to their initial distribution, and the final distribution of EVs at nodes 2, 8, 11 is equal to 0.2, 0.3, and 0.5 units respectively. Note that each charging station is still strategic to maximise its own revenue. The obtained results are shown in Figure 8. As expected, only three routes are used by the EVs in the shortest-path scenario which is depicted in Figure 8(b). Specifically, all EVs initially located at node 40, 15, and 34 will take their shortest paths to the charging stations at node 2, 8, and 11, respectively. The final distribution of EVs in three cases is shown in Figure 8(a) which demonstrates significant difference. To verify the advantage of the proposed dynamic transport framework, we compare the total costs of EV operator under three operating scenarios, and the result is shown in Figure 8(c). It can be concluded that using the dynamic transport approach, the cost of EV operator has a considerable decrease comparing with shortest path and demand-aware methods.

6 CONCLUSIONS

The regularised dynamic optimal transport of electric vehicles over networks with a consideration of strategic charging pricing...
is investigated. Specifically, a holistic framework for EV transportation has been established by including its delay, charging cost, and real-time social demand of EVs. By combining quadratic and entropic regularisation, a balanced dynamic optimal transport strategy of EVs is proposed. Furthermore, the charging stations are modelled as active players in determining their charging pricing to maximise the profit. To compute the equilibrium charging pricing for all charging stations, an iterative algorithm based on particle-swarm optimisation scheme has been proposed that solves a challenging high-dimensional optimisation programme. The obtained strategy has successfully captured the interactions between the EV system operator and the demand-aware charging stations across the entire network. The developed dynamic transport strategy has been shown to be effective in reducing the operational cost of EV operators over two other benchmark strategies. As for the future work, the framework might be extended to capture the uncertainty of desired social EV demands at stations instead of a static one, in which the optimal transport strategy should consider the stochastic nature of EV final distribution. Another direction of future work is to extend the current centralised optimal transportation framework to a distributed one and develop efficient distributed algorithms, where each node determines its own optimal transport strategy based on the possibly incomplete information.

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**FIGURE 7** (a) and (b) The optimal initial and final distributions of EVs, respectively. (c) The strategic charging pricing which is dynamic at all three nodes. (d) The utility update trajectories of charging stations, and all converge to their equilibrium states

**FIGURE 8** (a) The final distribution of EVs at stations using three different strategies. (b) The traffic flow on each link under the shortest path strategy. (c) The total cost of EV operators under different operating strategies where the proposed approach shows its advantages over the others
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How to cite this article: Feng R, Czarkowski D. Regularised dynamic optimal transportation of electric vehicles over networks considering strategic charging pricing. IET Energy Syst. Integr. 2021;3:73–85. https://doi.org/10.1049/esi2.12005