Exclusive $D_2^*$ weak decays and the experimental potential

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The decay width of the $D_2^*$ meson is dominated by the electromagnetic mode, and it is thus the longest-lived charged vector meson. In light of this point, we perform the first QCD LCSR calculations of $D_2^* \to \phi$ helicity form factors and discuss the experiment potential of discovering exclusive $D_2^*$ weak decays. The main result is the partial decay widths, which read as $\Gamma_{D_2^*\to\ell\nu} = 2.44 \times 10^{-12}$ GeV, $\Gamma_{D_2^*\to\phi\ell\nu} = (1.37^{+0.29}_{-0.17}) \times 10^{-13}$ GeV, $\Gamma_{D_2^*\to\phi\ell\nu} = (0.39 \pm 0.03) \times 10^{-13}$ GeV and $\Gamma_{D_2^*\to\phi\ell\nu} = (1.41^{+0.54}_{-0.32}) \times 10^{-13}$ GeV. We show that these channels are promising in the near future, serving as the first experimental observation of weak decays of a vector meson, and would open up a new playground for precision test of the standard model.

Introduction.—The unitarity of the Cabibbo-Kobayashi-Maskawa (CKM) matrix is a crucial criterion of the validity of the Standard Model. Besides the well known unitarity triangles, which indicate the orthogonality between different rows and columns, the CKM unitarity can also be tested by the normalization conditions of individual rows and columns. Least square fitings according to explicit polarizations of the weak currents, the strange quark, and also between different QCD power diations of the photon from the charm quark and from the strange quark, and also between different QCD power corrections. From another perspective, the $g_{D_2^*D_s\gamma}$ coupling is very sensitive to different contributions, so the indirect determination from weak $D_2^\pm$ decays will subsequently act as an important benchmark to probe the involved dynamics.

Evaluating the weak $D_2^*$ decays requires the input of the corresponding heavy-to-light form factors, which are basic physical quantities charactering the momentum redistribution of partons after the weak interaction. In this paper we study the $D_2^* \to \phi$ form factors from QCD light-cone sum rules (LCSRs) approach, which has been widely applied to calculate form factors in charmed meson decays [8-11], and this work is its first implementation in a vector-to-vector transition. Different helicity form factors according to explicit polarizations of the weak current and the $\phi$ meson are calculated. From the small momentum transfer region $0 \leq Q^2 \leq 0.4$ GeV$^2$ where the LCSRs predictions are reliable, proper parametrization of the form factors is inevitable to extend them to the large region $0.4 \leq Q^2 \lesssim 1.2$ GeV$^2$. We employ both the simplified $z$-series expansion formalism [12] and the two-pole parametrization [13], and it turns out that the parametrization scheme does not bring additional considerable uncertainties. With the helicity form factors, we obtain the partial decay widths of $D_2^*$ weak decays considered here, they are $\Gamma_{D_2^*\to\phi\ell\nu} = (1.37^{+0.29}_{-0.17}) \times 10^{-13}$ GeV, $\Gamma_{D_2^*\to\phi\ell\nu} = (0.39 \pm 0.03) \times 10^{-13}$ GeV and $\Gamma_{D_2^*\to\phi\ell\nu} = (1.41^{+0.54}_{-0.32}) \times 10^{-13}$ GeV. These predictions, together with the partial decay width of the leptonic mode $\Gamma_{D_2^*\to\ell\nu} = 2.44 \times 10^{-12}$ GeV, promote the experiments to measure the weak decay of a vector meson with the great potential in the near future.

$D_2^* \to \phi$ helicity form factors.—To perform the LCSRs calculation of the $D_2^* \to \phi$ form factors, we start with the

\begin{equation}
|V_{us}|^2 + |V_{cs}|^2 + |V_{cb}|^2 = 1.026 \pm 0.022,
|V_{cd}|^2 + |V_{ca}|^2 + |V_{cb}|^2 = 1.025 \pm 0.022, \tag{1}
\end{equation}

with the uncertainties both dominated by that of $|V_{cs}| = 0.987 \pm 0.011$ [1]. The $|V_{ca}|$ values are typically extracted from semileptonic $D$ decays and leptonic $D_s$ decays, and other independent channels, such as weak $D_2^*$ decays, are highly anticipated to reduce the uncertainty.

Weak $D_2^*$ decays can also provide a platform for examining the heavy quark symmetry, which is the foundation of the heavy quark effective theory [2]. The heavy quark spin symmetry relates the ground-state pseudoscalar and vector mesons, e.g. $D(s)$ and $D_s(s)$ mesons. It has been checked through the relation between the semileptonic decays $B \to D\ell\nu$ and $B \to D^*\ell\nu$ [3,4], where different spin states appear in the final states. The weak $D_2^* \to \phi\ell\nu$ decay, together with $D_2^* \to \phi\ell\nu$, will create the first chance to test the heavy quark spin symmetry with heavy mesons in the initial states.

Practically, $D_2^\pm$ might be the first vector meson whose weak decays will be discovered, because it is the longest-lived charged vector meson indicated by the lattice evaluation of the partial width of its dominant decay channel $D_2^* \to D_s\gamma$ [6]. Once the branching ratio of a weak decay channel is measured, it can be used to indirectly determine the total decay width of $D_2^\pm$ with the theoretical calculation of the weak decay width as an input, for which only an experimental upper limit is currently given as $\Gamma_{D_2^*} < 1900$ keV [1]. Meanwhile, the electromagnetic decay width can also be indirectly determined, from which the electromagnetic coupling $g_{D_2^*D_s\gamma}$ can be extracted.
In the rest frame of the heavy meson $D_s^*$, the vector current $j_{\mu}^V = \bar{c} \gamma_{\mu} s$ and the weak current $\bar{J}_{\mu}^W = \bar{s} \gamma_{\mu} (1 - \gamma_5) c$ carry four momenta $p_{1a} = (m_{D_s^*}, 0)$ and $q_{\mu} = (q_0, -\mathbf{p})$, and the $\phi$ meson momentum $p_2 = p_1 - q$. Multiplying both sides of Eq. (2) by the polarisation vector of the weak current, we decompose the modified correlation function in terms of invariant helicity amplitudes,

$$\tilde{e}^\mu F_{\mu a}(q, p_1) = \sum_{i, j=0, \pm} \epsilon_{i a, i'}^{\mu} F_{ij}(q^2, p_1^2),$$

where the subscripts $i, j$ and $i' = i + j$ denote the polarisation directions of the weak current, $\phi$ meson and vector current, respectively.

In the view of LCSRs, correlation functions can be formulated in twofold ways, namely, at the quark level and the hadronic level. Firstly, they can be evaluated directly at the quark-gluon level in the Euclidean momenta space, with the result as a convolution of hard amplitudes with various LCDA at different twists [17]. The QCD calculation of Eq. (2) is carried out with negative $q^2$, and the operator product expansion (OPE) is applicable in the kinematical region $|q^2| \in [0, m_s^2 - 2m_c \lambda] \sim [0, 0.4]\text{GeV}^2$, where $\lambda \sim 0.5\text{GeV}$ is a typical hadronic scale. In this region, the operator product of the $c$-quark fields in the correlation function can be expanded near the light cone $x^2 \sim 0$ due to the large virtuality, which at leading order deduces to the free quark propagator. The OPE amplitudes can be further rewritten in a dispersion integral over the invariant mass of the interpolating heavy meson,

$$F_{ij}^{\text{OPE}}(q^2, p_1^2) = \frac{1}{\pi} \int_{m_2^2}^\infty ds \frac{u^2}{|u^2 m_\phi - q^2 + m_{D_s^*}^2|^n} \sum_n \text{Im} F_{ij}^{\text{OPE}}(q^2, s) u^{n} |s - p_1|^n, \quad (4)$$

in which $s \equiv (u^2, u) = um_\phi^2 + (m_s^2 - uq^2)/u$. As an example, we present the leading-power contribution ($n = 1$) to the OPE amplitude with the $00$ helicity configuration as,

$$\frac{1}{\pi} \text{Im} F_{ij}^{\text{OPE}}(q^2 < 0, u) = \frac{\sqrt{m_{D_s^*}^2 + m_{D_s^*}^2} m_\phi \phi_{\parallel} (u)}{2m_{D_s^*} \sqrt{|q^2|}} + \frac{\sqrt{\lambda (um_{D_s^*}^2 + uq^2)} \phi_{\parallel} \phi_{\parallel} (u)}{2m_{D_s^*} \sqrt{|q^2|}}$$

$$- \frac{\sqrt{\lambda (m_{D_s^*}^2 - q^2)} \phi_{\parallel} (u) - \phi_{\parallel} (u)}{2m_{D_s^*} \sqrt{|q^2|}} + \frac{\sqrt{\lambda f_{\parallel} m_{D_s^*}^2 \phi_{\parallel} (u)}}{4m_{D_s^*} \sqrt{|q^2|}},$$

where $\phi_{\parallel}, \phi_{\perp}, \phi_{\parallel}^\parallel, \phi_{\parallel}^\perp$ are LCDA of the $\phi$ meson at different twists [18, 20], the functions $\phi(u) = \int du \phi(u)$ satisfy the boundary conditions $\phi(0) = \phi(1) = 0$, and $\lambda$ refers to $(m_{D_s^*}^2, m_{D_s^*}^2, q^2)$ with the källén function defined by $\lambda(a, b, c) \equiv a^2 + b^2 + c^2 - 2ab - 2ac - 2bc$.

When $q^2$ shifts from deeply negative to positive, the typical distance grows between the two currents in Eq. (2), hence the long-distance quark-gluon interaction begins to form hadrons. In this respect, the correlation function can be understood by the sum of contributions from all possible intermediate states with appropriate subtractions. The dispersion relation of invariant amplitudes in variable $p_1^2 > 0$ reads

$$F_{ij}(q^2, p_1^2) = \frac{1}{\pi} \int_{m_2^2}^\infty ds \frac{\text{Im} F_{ij}(q^2, s)}{s - p_1^2}, \quad (6)$$

By inserting a complete set of hadronic states with the quantum number of the $c\bar{c}$ current, the spectral function of the ground state is obtained from the optical theorem and written by means of two detached matrix elements

$$\epsilon_{i a, i'}^{\mu} \rho_{ij}^a(q^2) = \epsilon_{i a, i'}^{\mu} \langle \phi|J_{\mu, j}^W(x)|D_s^*(0)\rangle \langle D_s^*(0)|J_{\mu, i}^W(0)\rangle, \quad (7)$$

in which the latter one is parametrized by the $D_s^*$ decay constant, and the former one is written in terms of the $D_s^* \rightarrow \phi$ transition form factors associated with orthogonal Lorentz structures [21, 22]. In order to clarify the contribution from each helicity configuration, we introduce the helicity form factors $H_{ij} \equiv \epsilon_{i a, i'}^{\mu} \langle \phi|J_{\mu, j}^W(x)|D_s^*(0)\rangle$ and write down the helicity invariant amplitudes as

$$F_{ij}(q^2, p_1^2) = \frac{m_{D_s^*}}{m_{D_s^*} - p_1^2} H_{ij} + \int_{s_0}^\infty ds \rho_{ij}^a(q^2, s) \frac{s - p_1^2}{s}. \quad (8)$$

Based on the quark-hadron duality, Eqs. (4) and (8) describe the same correlation function from two parallel views, so in principle we can solve the helicity form factors by matching the two equations if we know the spectral functions $\rho_{ij}^a(1)$. We take the semi-local duality to offset the contributions from large $s > s_0$ regions in the two dispersion relation integrals, because the magnitude of timelike form factor is close to the spacelike one when the momentum transfer is far away from the resonant state regions, and they converge to equal in the QCD limit [23, 24]. The threshold value is selected by the rule of thumb that it is larger than the mass square of ground state $s_0 \gtrsim m_{D_s^*}^2$, and close to the outset of the first excited state. We Borel-transform both sides of the residual contributions below $s_0$ to suppress the pollutions from excited resonant states and continuum spectral, and arrive at the sum rules of the helicity form factors (at the leading power approximation with $n = 1$),

$$m_{D_s^*} f_{D_s^*} H_{ij}(q^2) = \frac{1}{\pi} \int_{m_2^2}^{s_0} ds u(s) \text{Im} F_{ij}^{\text{OPE}}(q^2, s) e^{-(s + m_{D_s^*}^2)/M^2}. \quad (9)$$

Note that the timelike polarisation of leptonic current $\tilde{e}_\mu(t) = (\eta_0, 0, 0, -|\mathbf{p}|)/\sqrt{|\mathbf{p}|} \propto q_0$ does not contribute in the semi leptonic decaying processes with massless leptons, and the other three polarisations, picking up the spin-one part of the off-shell $W$ boson, satisfy $q^2 \epsilon_\mu = 0$. 

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To extrapolate to the whole kinematic region $[0, q_0^2 = (m_{D^*_s} - m_\rho)^2]$, we adopt the simplified $z$-series expansion \cite{14} which is required to (a) reproduce the result obtained from LCSRs calculation in the lower interval $[0, 0.4]$ GeV$^2$ with good accuracy, (b) provide an extrapolation to the large interval $[0.4$ GeV$^2, q_0^2]$ with the expected analytical properties of the helicity form factors. With taking \( m_{D^*_s} = 2.112 \text{ GeV} \) \cite{11} and \( f_{D^*_s} = 0.274 \text{ GeV} \) \cite{6}, we depict the helicity form factors in figure 4 where the result obtained directly from LCSRs calculation is shown by gray bands, and the extrapolation by $z$-series parameterisation is shown by red bands. The results reveal the gentle behaviour of the longitudinal helicity form factors, and meanwhile the linear-like rise of the helicity form factors associated with the transversally polarised weak current. The quick drop of $H_{00}$ in small recoiled regions is understudied by the phase space suppression from $\lambda$. We also test the Becirevic and Kaidalov (BK) parameterisation \cite{10} and find that it gives almost the same result as the $z$-series parameterisation in the small recoiled regions.

**Exclusive $D_s^*$ weak decays.**—The leptonic decays $D_s^* \rightarrow \ell \nu$ ($\ell = e, \mu$) have the decay width

\[
\Gamma_{D_s^* \rightarrow \ell \nu} = \frac{G_F^2}{12\pi} |V_{cs}|^2 f_{D_s^*}^2 m_{D_s^*}^3 = 2.44 \times 10^{-12} \text{ GeV} \tag{10}
\]

if we accept the lattice result of the decay constant $f_{D_s^*} = 0.274$ GeV \cite{6} and neglect the lepton masses. The differential decay width of semileptonic decays of a particular polarization mode is written as

\[
\frac{d\Gamma_{ij}}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{1/2} q^2}{192\pi^3 m_{D_s^*}^3} |H_{ij}(q^2)|^2. \tag{11}
\]

With the helicity form factors obtained above, we obtain the spin averaged total decay width

\[
\Gamma_{D_s^* \rightarrow \ell \nu q} = \frac{1}{3} \int_0^{q_0^2} dq^2 \sum_{i,j=0,\pm} d\Gamma_{ij} = (1.37^{+0.29}_{-0.17}) \times 10^{-13} \text{ GeV}. \tag{12}
\]

The leptonic and semileptonic $D_s^*$ weak decays, meanwhile, extent the investigation of lepton flavour universe (LFU) study \cite{23} \cite{28}.

Under the naive factorisation hypothesis with considering only the color singlet operator at tree level, the

![Graph](image-url)

**FIG. 1.** The helicity form factors $H_{ij}(q^2) \equiv \sqrt{q^2} H_{ij}(q^2)$, with the uncertainties come from LCSRs parameters $M^2 = 1.50 \pm 0.05$ GeV$^2$ and $s_0 = 7.50 \pm 0.50$ GeV$^2$. 
decay amplitudes of $D_s^* \rightarrow \phi \pi, \phi \rho$ channels are detached into two matrix elements,

$$A_{D_s^{*+} \rightarrow \phi \pi^+} = (-i) G_F \frac{V_{cs} a_1 m_\pi f_\pi}{\sqrt{2}} \sum_{j=0, \pm} \mathcal{H}_{0j}(m_{s}^2), \quad (13)$$

$$A_{D_s^{*+} \rightarrow \phi \rho^+} = G_F \frac{V_{cs} a_1 m_\rho f_\rho}{\sqrt{2}} \sum_{i,j=0, \pm} \mathcal{H}_{ij}(m_{s}^2). \quad (14)$$

Considering the wilson coefficient $a_1 = 0.999$ at the factorisation scale $\mu = (m_{D_s}^2 - m_{\pi}^2)^{1/2}$ and the decay constants of light mesons, say, $f_{\pi} = 0.130$ GeV [1] and $f_{\rho} = 0.210$ GeV [20], we obtain the partial widths of nonleptonic decays as

$$\Gamma_{D_s^{*+} \rightarrow \phi \pi^+} = (0.39 \pm 0.03) \times 10^{-13} \text{ GeV}, \quad (15)$$

$$\Gamma_{D_s^{*+} \rightarrow \phi \rho^+} = (1.41^{+0.54}_{-0.32}) \times 10^{-13} \text{ GeV}. \quad (16)$$

The dominant source of uncertainties comes from the LCSRs predictions of the helicity form factors, which vary about 10% as showing in the $\phi \rho$ channel. The extrapolation by the simplified z-series expansion brings another uncertainty and the $\phi \rho$ channel has very little uncertainty with varying about 30%. If we take the total width $\Gamma_{D_s} = (7.0 \pm 2.8) \times 10^{-8}$ GeV evaluated from lattice QCD [6], the branching fractions of $D_s^*$ weak decays are

$$B(D_s^* \rightarrow \ell \nu) = (3.49 \pm 0.14) \times 10^{-5},$$

$$B(D_s^* \rightarrow \phi \pi) = (1.96^{+0.41}_{-0.24} \times 0.78) \times 10^{-6},$$

$$B(D_s^* \rightarrow \phi \rho) = (5.57 \pm 0.42 \pm 2.29) \times 10^{-7},$$

$$B(D_s^* \rightarrow \phi \pi) = (2.01^{+0.77}_{-0.46} \pm 0.80) \times 10^{-6}. \quad (17)$$

Let us give a brief discussion on the experimental potential of $D_s^*$ weak decays. The integrated luminosity at Belle II would achieve 10 ab$^{-1}$ and hence the number of $D_s$ mesons would be reconstructed from the $\phi \pi$ channel [31], and hence the number of $D_s (D_s^*)$ production can be expected at order $O(10^8)$ by considering the branching fraction $B(D_s \rightarrow \phi \pi) = (4.5 \pm 0.4)\%$. In this respect, Belle II has excellent potential to study the exclusive $D_s^*$ weak decays. Meanwhile, about $3.07 \times 10^6 D_s^*$ mesons have been collected by BESIII with the integrated luminosity 3.2 fb$^{-1}$ at 4.178 GeV [32]. They are directly produced from the $e^+e^-$ collision at the $D_sD_s^*$ threshold with lower background, and it provides a good chance to measure the leptonic decays $D_s^* \rightarrow \ell \nu$ and to further determine $\Gamma_{D_s}$. Note that the photon-radiation effect is tiny in the leptonic $D_s^*$ decays since these channels are not helicity suppressed in contrast to the $D_s \rightarrow \ell \nu$ decays. For the hadronic decay channels, we hope LHCb, with the excellent particle identification to distinguish $K, \pi$ and $\mu$, would study the $D_s^* \rightarrow \phi (K\bar{K})\pi$ channel with $D_s^*$ producing from semileptonic decay $B_s \rightarrow D_s^* \ell \nu$.\footnote{Corresponding author: scheng@hnu.edu.cn}

Summary. - In this letter we calculate the $D_s^* \rightarrow \phi$ helicity form factors from LCSRs with the accuracy up to two-particle twist-5 DAs of the $\phi$ meson at the leading order of $\alpha_s$, with which we study the experimental potential of discovering $D_s^*$ weak decays. The result shows that the leptonic decays $D_s^* \rightarrow \ell \nu$ are the most hopeful channels to be measured at BESIII, the semileptonic decays $D_s^* \rightarrow \phi \ell \nu$ could be accessible at Belle II after the phase 3 running, and the hadronic $D_s^* \rightarrow \phi \pi \pi$ decays are promising at LHCb. The measurement of purely leptonic decays would determine the total width of the $D_s^*$ meson and hence clarify some fundamental properties of the $D_s^*$ meson, such as the electromagnetic and strong couplings $g_{D_s^*D_s^*}^{\phi}$. It is highly hopeful that these channels will promote the first observation of weak decays of a vector meson, opening up a new playground to test the standard model and pushing us to higher precision studies.

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