A multidomain approach to asteroid families’ identification

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ABSTRACT
It has been shown that large families are not limited to what found by hierarchical clustering methods in the domain of proper elements \((a, e, \sin(i))\), which seems to be biased to find compact, relatively young clusters, but that there exists an extended population of objects with similar taxonomy and geometric albedo, which can extend to much larger regions in proper elements and frequencies domains: the family ‘halo’. Numerical simulations can be used to provide estimates of the age of the family halo, which can then be compared with ages of the family obtained with other methods. Determining a good estimate of the possible orbital extension of a family halo is therefore quite important, if one is interested in determining its age and, possibly, the original ejection velocity field. Previous works have identified families’ haloes by an analysis in proper elements domains, or by using Sloan Digital Sky Survey-Moving Object Catalog data, fourth release (SDSS-MOC4) multiband photometry to infer the asteroid taxonomy, or by a combination of the two methods. The limited number of asteroids for which geometric albedo was known until recently discouraged in the past the extensive use of this additional parameter, which is however of great importance in identifying an asteroid taxonomy. The new availability of geometric albedo data from the Wide-field Infrared Survey Explorer (WISE) mission for about 100 000 asteroids significantly increased the sample of objects for which such information, with some errors, is now known.

In this work, we proposed a new method to identify families’ haloes in a multidomain space composed by proper elements, SDSS-MOC4 \((a^\ast, i - z)\) colours, and WISE geometric albedo for the whole main belt (and the Hungaria and Cybele orbital regions). Assuming that most families were created by the breakup of an undifferentiated parent body, they are expected to be homogeneous in colours and albedo. The new method is quite effective in determining objects belonging to a family halo, with low percentages of likely interlopers, and results that are quite consistent in term of taxonomy and geometric albedo of the halo members.

Key words: Celestial mechanics – minor planets, asteroids: general.

1 INTRODUCTION
Asteroid families are groups of asteroids that are supposed to have a common origin in the collisional event that shattered the parent body. They are usually determined by identifying clusters of objects close in proper elements domain \((a, e, \sin(i))\). The hierarchical clustering method (HCM hereafter) as described by Bendjoua & Zappalà (2002) operates by identifying all objects that are closer than a given distance (cutoff) with respect to at least one other member of a family. If an object is closer than this distance, it is associated with the dynamical family, and the procedure is repeated until no new family members are found. The choice of this cutoff distance is then of paramount importance in determining the family. For small values of the cutoff, only the objects closest in proper element domain are identified as family members: the family ‘core’. At

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larger cutoff one is able to identify objects that, while still belonging to the collisional group, may have dynamically evolved since the family formation and drifted apart from the core: the family “halo”.\(^1\)

To illustrate this issue, Fig. 1 displays the family core (blue crosses) and halo (red circles), as obtained in Carruba (2013) for the Hygiea family, at cutoff of 66 and 76 m s\(^{-1}\), respectively. At cutoffs larger than 76 m s\(^{-1}\) the family merged with other dynamical groups in the region (such as the Veritas and Themis family) and with the local background, thus it was no longer identifiable as a separate entity. Numerical simulations can be used to provide estimates of the age of the family halo, as done, for instance, by Brož & Morbidelli (2013) for the Eos halo, that can then be compared with ages of the family obtained with other methods. Determining a good estimate of the possible orbital extension of a family halo is therefore quite important, if one is interested in determining its age and, possibly, the original ejection velocity field.

One problem in obtaining a good determination of a family halo is however the presence of objects in the orbital region of the halo that might not be connected with the local family. Assuming that most families were created by a breakup of an undifferentiated parent body, we would expect that most of its members should be homogeneous in colours and albedo. Objects that belong to the dynamical families but that differ in colours or albedo may possibly be asteroids of the local background that just happened to lie in the orbital region of a given family: the interlopers. An analysis of spectral properties of local asteroids may provide insights on the possible presence of such interlopers in groups found in proper elements domains, but such information is usually available only for 2 per cent or less of the main belt asteroids. Yet including too many interlopers into the family may change the perceived orbital structure of the group and cause to obtain distorted estimates of its properties. Recently, the Sloan Digital Sky Survey-Moving Object Catalog data, fourth release (SDSS-MOC4 hereafter; Ivezić et al. 2002), provided multiband photometry on a sample two order of magnitude larger than any available in any current spectroscopic catalogues (about 60 000 numbered objects). While for the purpose of deriving very reliable inferences about asteroid surface compositions, multiband photometry is not as precise as spectroscopy. Nesvorný et al. (2005) showed that the SDSS-MOC is a useful data set to study general statistical variations of colours of main belt asteroids. These authors used an automatic algorithm of principal component analysis (PCA) to analyse SDSS photometric data and to sort the objects into different taxonomical classes. In particular, PCA can be used to derive linear combinations of the five SDSS colours \((u, g, r, i, z)\), in order to maximize the separation between a number of different taxonomic classes in SDSS data. Two large separated complexes were found in the PCA first two components: the C/X complex and the S complex, with various subgroups identified inside the main complexes. A problem with this approach was however the large errors that affected colours in the ultraviolet band \(u\), and that propagated into the computation of the principal components. To avoid including the \(u\)-data, other authors (Ivezić et al. 2002; Parker et al. 2008) constructed a colour-code diagram in a \((a^*, i - z)\) plane, where

\[
a^* = C_1 \cdot (g - r) + C_2 \cdot (r - i) + C_3,
\]

and \(C_1\), \(C_2\) and \(C_3\) are numerical coefficients that depend on the colour values and on the number of observations in the given data base (Roig & Gil-Hutton 2006), and \(g, r, i\) and \(z\), the other SDSS colours, had an accuracy of about 0.03 mag, higher than the average errors in the \(u\)-band. As in the plane of \((PC_1, PC_2)\), asteroids divided in the \((a^*, i - z)\) plane into three fairly distinct groups, the C-complex \((a^* < 0)\), the S-complex \((a^* > 0, i - z > -0.13)\) and the V-type asteroids \((a^* > 0, i - z < -0.13)\;\text{Parker et al. (2008))}^2\)

Asteroid taxonomy is however also defined by the geometric albedo \(p_V\) (roughly, the ratio of reflected radiation from the surface to incident radiation upon it, at zero phase angle (i.e. as seen from the light source), and from an idealized flat, fully reflecting, diffusive scattering (Lambertian) disc with the same cross-section). C-type asteroids tend to have lower values of geometric albedo than S-type ones, and Tholen asteroid taxonomy (Tholen 1989) used values of \(p_V\) to distinguish classes of asteroids inside the X-complex, such as the M-, E- and P-types. Until recently, however, only about two thousand asteroids had reliable values of geometric albedo (see Tedesco et al. 2002). Initial results from the Wide-field Infrared Survey Explorer (WISE) (Wright et al. 2010), and the NEOWISE (Mainzer et al. 2011) enhancement to the WISE mission recently allowed us to obtain diameters and geometric albedo values for more than 100 000 main belt asteroids (Masiero et al. 2011), increasing the sample of objects for which albedo values were known by a factor of 50. Masiero et al. (2011) showed that, with some exceptions, such as the Nysa–Polana group, asteroid families typically show a characteristic albedo for all members, and that a strongly bimodal albedo distribution was observed in the inner, middle and outer portions of the main belt.

Previous works, such as Bus & Binzel (2002a,b), Nesvorný et al. (2005), found asteroid families in extended domains of proper elements and SDSS-MOC4 principal components data in order to minimize the number of possible interlopers, but such an analysis was not extended to asteroids’ geometrical albedo. Here, we take full advantage of the newly available WISE data and we introduce a new HCM [see Bendjoya & Zappalà (2002) for details of the method in proper elements space] in a multidomain space

\(^1\)The term halo was first introduced to describe this population of objects by Nesvorný et al. (2006), and then adopted by other authors such as Parker et al. (2008).

\(^2\)The \(a^*\) colour is nothing but the first principal component \(PC_1\) of the data distribution in the \((g - i)\) versus \((r - i)\) colour–colour diagram.
composed by asteroids proper elements \((a, e, \sin i)\), SDSS-MOC4 colours \((a'^{\ast}, i - z)\) and \(WISE\) geometric albedo \((p_V)\), to identify haloes associated with main belt asteroid families. The great advantage of this approach is that any group identified in these domains will most likely belong to the same taxonomical group, since its members have to share not only similar values of proper elements, but also of taxonomically related information, such as \((a'^{\ast}, i - z)\) and \(p_V\). A shortcoming of this approach is related to the more limited number of asteroids that have data in the three domains at the same time, when compared with the larger number of objects that have only proper elements and frequencies, only SDSS-MOC4 principal components data, only \(WISE\) albedo data or a dual combination of these three quantities. However, groups determined with this new approach may serve as a first step in determining the real orbital extension of the families’ cores and haloes, with a precision that other methods already in use in the literature may lack.

This work is so divided: in Section 2, we discuss the basics of our approach for finding haloes in the multidomain space. In Section 3, we compare the efficiency of this approach in finding low numbers of interlopers with the results of other methods used for identifying asteroid families. In Sections 4, 5 and 6, we apply our method to all the currently known major families in the inner, central and outer main belt. In Sections 7 and 8, we discuss the case of the asteroids in the Cybele group and in the Hungaria region. Finally, in Section 9, we present our conclusions.

2 METHODS

In this work, we are trying to make best use of all the new data on surface colours (SDSS-MOC4) and geometric albedo (\(WISE\) and NEOWISE) that is currently available to try to find the most possibly accurate determination of all major main belt family haloes. For this purpose, we determined the main belt asteroids with synthetic proper elements available at the AstDyS website http://hamilton.dm.unipi.it/cgi-bin/astdys/astibo, accessed on 2013 January 15 (Knežević & Milani 2003) that also have SDSS-MOC4 and \(WISE\) albedo data, and errors in proper elements \((a, e, \sin i)\) less than what described as ‘pathological’ in Knežević & Milani (2003), i.e. \(\Delta a > 0.01\) au, \(\Delta e > 0.1\) and \(\Delta \sin i > 0.03\). We computed the SDSS-MOC4 colours \((a'^{\ast}, i - z)\) and their errors, computed with standard propagation of uncertainty formulas under the assumption that the SDSS-MOC4-calibrated magnitude behave as uncorrelated variables. For our sample of 58 955 asteroids with SDSS colours, we found values of the coefficients \(C_1, C_2,\) and \(C_3\) in equation (1) of 0.939 0.342 08 and −0.6324, respectively. To avoid including data affected by too large uncertainties, we eliminated from our sample asteroids with errors in \(a'^{\ast}\) or \((i - z)\) larger than 0.1 mag. As a test of the validity of our approach we also computed \(PC_1, PC_2\) principal components according to the approach of Novaković, Cellino & Knežević (2011), with their errors, and also rejected objects with errors larger than 0.1. While 68.1 per cent of the asteroids in the SDSS-MOC4 sample passed the conversion into \((a'^{\ast}, i - z)\) colours and the rejection of noisy data, only 42.08 per cent of the same asteroids had errors in \(PC_1, PC_2\) less than 0.1.\(^3\)

\(^3\)The large rejection of noisy data in this later approach is due to the inclusion of the magnitudes in the \(r\) filter, which are affected by larger errors that the magnitudes in the other filters. An alternative approach based on principal components \(PC_1, PC_2\) only in \(g, r, i\) and \(z\) colours domain was also tried. 67.7 per cent of our data passed the conversion into this space with errors less than 0.1. Since the \((a'^{\ast}, i - z)\) approach was slightly more efficient and it provided results that are easier to analyse in terms of taxonomies than the principal component approach, in this work we have decided to opt for the \((a'^{\ast}, i - z)\) method.

on these results, we decided to work with \((a'^{\ast}, i - z)\) colours rather than principal components. We also eliminated from our sample asteroids with errors in \(p_V\) larger than 0.05 if \(p_V < 0.2\), and asteroids with errors in \(p_V\) larger than 0.1 if \(p_V > 0.2\). The stringent constraint on errors in geometric albedo \(p_V\) for low-albedo asteroids was required to better distinguish between CX-complex asteroids \((p_V < 0.1)\) and S-complex asteroids \((p_V > 0.1)\). Since some objects in the inner main belt and Hungaria region have values of albedo in the \(WISE\) survey that are too high (up to 0.8–0.9) and are possibly an artefact of the method used to calculate absolute magnitude (Masiero et al. 2011), we also eliminated all objects in these two regions (essentially those with semimajor axis smaller than that of the centre of the 3J:-1A mean-motion resonance, i.e. about 2.5 au) with \(p_V > 0.5\) from our data base.

We then defined a distance metrics between two asteroids in a multidomain space as

\[
d_{\text{ad}} = \sqrt{d_{\text{SPV}}^2 + C_{\text{SPV}}[\Delta(a'^{\ast})^2 + \Delta(i - z)^2 + \Delta p_v]^2].
\]

where \(\Delta a'^{\ast} = a'^{\ast}_1 - a'^{\ast}_2\) and similar relations hold for \(\Delta(i - z)\) and \(\Delta p_v\). Following the approach of Bus & Binzel (2002a,b) for a similar distance metric of proper elements and SDSS-MOC principal components (see also Nesvorný et al. 2005; Carruba & Michchenko 2007), \(C_{\text{SPV}}\) is a weighting factor equal to 10\(^8\) (other choices in a range between 10\(^4\) and 10\(^8\) have been tested without significantly changing the robustness of the results), and \(d\) is the standard distance metrics in proper element domain defined in Zappalà et al. (1995) as

\[
d = na \sqrt{k_1 \left(\frac{\Delta a}{a}\right)^2 + k_2 (\Delta e)^2 + k_3 (\Delta \sin i)^2},
\]

where \(n\) is the asteroid mean motion; \(\Delta x\) the difference in proper \(a, e\) and \(\sin i\); and \(k_1, k_2, k_3\) are weighting factors, defined as \(k_1 = 5/4, k_2 = 2, k_3 = 2\) in Zappalà et al. (1990, 1995). As first halo members, we selected asteroids that belong to the asteroids family, whose spectral type is compatible with that of the other members according to Mothé-Diniz et al. (2005), Nesvorný et al. (2006), Carruba (2009a,b, 2010b) and other authors, and that, of course, also have acceptable SDSS-MOC4 and \(WISE/NEOWISE\) data. For families not treated by these authors, we consulted the list of asteroid families available at the AstDyS website, and the Nesvorný (2012) HCM Asteroid Families V2.0, on the Planetary Data System, available at http://sbn.psi.edu/pds/resource/nesvornymfam.html, accessed on 2013 March 13. We then obtained dynamical groups using equation (2), for a value of cutoff \(d_{\text{ad}}\) a bit less than the value for which the family halo merges with the local background (and other families in the region). As an example of this procedure, we choose the case of the Themis family. Fig. 2 displays the total number of members of this group (blue line) and the number of new members of the group (green line, as new members we mean the number of objects that became part of the group at that given velocity cutoff), as a function of the velocity cutoff \(d_{\text{ad}}\). For \(d_{\text{ad}} = 315\) m s\(^{-1}\), the Themis halo merged with other Local Groups, such as the Hygiea family, so we choose in this case to work with a halo defined at \(d_{\text{ad}} = 310\) m s\(^{-1}\).
found in dynamical group encountered in proper elements (or frequencies) domains only. This can be verified by an analysis of the SDSS-MOC4 and WISE albedo data of the group so obtained. Again, for the case of the Themis family, Fig. 3 shows a projection in the \((a^*, i − z)\) plane (panel A) and a histogram of the relative distribution of \(p_V\) values (panel B) of members of the Themis halo obtained with this method. The Themis family is made mostly by asteroids with CX-complex taxonomy, which in the \((a^*, i − z)\) plane appear on the left of the vertical dotted line, but nine asteroids have colours incompatible with such classification and should be considered as interlopers. This is confirmed by an analysis of \(p_V\) values, where most of the albedos are below 0.1, the threshold for CX-complex asteroids, but there is a tail of objects with higher albedos. The percentage of possible interlopers found with this method, 1.30 per cent, is indeed quite inferior to the \(\simeq 10\) per cent statistically expected in dynamical families obtained only in proper elements domain \(d\) of Zappalà et al. (1995), with a metric in proper elements and SDSS-MOC4 colours domains, given by

\[
d_{\text{ind}} = \sqrt{d^2 + C_{\text{SPV}}[(\Delta a^*)^2 + (\Delta (i − z))^2]},
\]

where \(C_{\text{SPV}}\), as discussed in Section 2, is a weighting factor equal to \(10^6\), and with a newly defined distance metric in proper elements and WISE geometric albedo \(p_V\) domain, given by

\[
d_{\text{ind}} = \sqrt{d^2 + C_{\text{SPV}}(\Delta p_V)^2}.
\]

We determined families’ haloes with the standard metrics of Zappalà et al. (1995), and equations (4), (5) and (2) for several large families in the main belt. Table 1 summarizes our results for the Hygiea, Koronis, and Eos family haloes, where we report the value of the cutoff at which the family was found, the number of halo members, the percentage of SDSS-MOC4 and geometric albedo likely interlopers (see Section 2 for a definition of the concept of SDSS-MOC4 and geometric albedo likely interlopers), for the four methods that we used (we will refer to these methods as metrics \(D, DS, DPV\) and \(DSPV\)). The last column, which reports the sum of the percentage of SDSS-MOC4 and geometric albedo likely interlopers, gives a measure of the efficiency of the method in finding likely interlopers: the lower this index, the better the method is working in avoiding taxonomically uncorrelated asteroids to the family halo. Among the several large families’ haloes that we analysed, we choose to display the results for the Hygiea, Koronis and Eos groups because these are families for which the new multidomain method showed one of the best, medium and worse results in term of not finding interlopers when compared with the other methods, respectively.

The Hygiea family case was the one for which the new method had the best results among the families analysed, with an overall efficiency of only 9.86 per cent. In the case of the Koronis family,
the efficiency was lower (23.00 per cent), but the new method still provided the best results when compared with other approaches. The Eos family was a very peculiar case: most of the family members are K-type, an S-complex type whose $a'$ values are very close to zero, the limiting value separating CX-complex asteroids and S-complex ones. The family is surrounded by CX-complex asteroids, and an analysis only based on distance metric inevitably recognizes as family members many background objects not necessarily connected to the family. Only in the case of this family, we found an efficiency of the new method slightly inferior to the results of the $DS$ metric (69.25 per cent with respect to 67.07 per cent). Overall, the new approach was at its best a factor of 2 more efficient in eliminating interlopers than other methods, and at its worse provided comparable results to what obtained in the domain of proper elements and Sloan colours.

Having concluded that the method described by equation (2) is the most efficient in term of low numbers of interlopers, we are now ready to start our analysis of the main belt. We will do this by investigating asteroids family haloes in the inner main belt.

### 4 INNER MAIN BELT

The inner main belt is dynamically limited in semimajor axis by the 3J:-1A mean-motion resonance at high $a$ (Zappala et al. 1995). The 7J:-2A mean-motion resonance is sometimes used by some authors as the boundary between the inner main belt at high inclination and the region of the Hungaria asteroids. In this work, we will just use the upper limit in $a$ given by the 3J:-1A mean-motion resonance. The linear secular resonance $v_{6}$ separates the low-inclined asteroid region from the highly inclined area, dominated by the Phocaea family (see Carruba 2009b, 2010a for a discussion of the local families’ groups and dynamics). The Phocaea family is located in a stable island limited by the 7J:-2A and 3J:-1A in semimajor axis, and by the $v_{6}$ and $v_{5}$ secular resonances in inclination (Knežević & Milani 2003). We found 2366 objects that have proper elements and frequencies, SDSS-MOC4 ($a'$, $i$ – $z$) colours, WISE geometric albedo data in the inner main belt, and reasonable errors, according to the criteria defined in Section 2. We will start our analysis by studying the case of the Belgica family.

#### 4.1 The Belgica family

The Belgica group was a clump associated with the former Flora family and identified by Mothé-Diniz et al. (2005) as a small and sparse group of only 41 members at a cutoff in proper element domain of 57.5 m s$^{-1}$. Here, we found that the halo of the Belgica family merges with that of the Baptistina group already at a cutoff of 100 m s$^{-1}$. We will therefore treat the Belgica family together with the Baptistina cluster.

#### 4.2 The Baptistina family

The Baptistina family, as the Belgica group, was studied by Mothé-Diniz et al. (2005) and was part of the former Flora family. It is located in a very complex dynamical region (Michchenko et al. 2010), being crossed by powerful mean-motion resonances such as the 7J:-2A and interacting with secular resonances such as the $z_{2} = 2(g - g_{6}) + (s - s_{6})$. It has been obtained in the $(a, g, s)$ frequency domain by Carruba & Michchenko (2009) to study possible diffusion of its members in $s$–type resonances such as the $v_{17} + v_{4} + v_{5} = 2v_{6}$. Here, we identified a 56-member CX halo at a cutoff of 250 m s$^{-1}$. The taxonomical structure of the halo is indeed very complex and puzzling. The majority of the Baptistina halo members have SDSS-MOC4 data compatible with a CX-complex taxonomy, with only two members (3.6 per cent of the total) that are possible interlopers. The albedo data are however very puzzling, since 47 members (83.9 per cent of the total) have values of $pv > 0.1$, not usually associated with dark CX-complex asteroids. Baptistina family members seem to behave as the members of the Hungary group, a CX-complex family, characterized by large values of albedos (see Section 8.1). Understanding the properties of the Baptistina halo will require a much more in depth analysis than what we performed in this work.

#### 4.3 The Vesta family

The Vesta family is unique in the main belt, since it is made mostly by V-type asteroids that are associated with a basaltic composition, typical of differentiated objects with a crust. Of the many possible differentiated or partially differentiated asteroids that may have

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**Table 1. Efficiency of distance metrics in several domains in finding asteroid families’ halo members.**

| Family name | $d_{ld}$ cutoff value (m s$^{-1}$) | Number of members | Percentage of SDSS-MOC4 likely interlopers | Percentage of $pv$ likely interlopers | Metric efficiency |
|-------------|-----------------------------------|-------------------|-------------------------------------------|--------------------------------------|------------------|
| Hygiea halo |                                   |                   |                                           |                                      |                  |
| Metric $D$  | 80                                | 6152              | 7.58                                      | 15.59                                | 23.17            |
| Metric $DS$ | 225                               | 728               | 5.63                                      | 14.02                                | 19.65            |
| Metric $DPV$ | 140                             | 2977              | 8.63                                      | 6.45                                 | 15.08            |
| Metric $DSPV$ | 290                            | 426               | 1.41                                      | 8.45                                 | 9.86             |
| Koronis halo |                                  |                   |                                           |                                      |                  |
| Metric $D$  | 65                                | 6958              | 15.71                                     | 14.36                                | 30.07            |
| Metric $DS$ | 230                               | 1054              | 13.54                                     | 13.11                                | 26.65            |
| Metric $DPV$ | 80                              | 1566              | 14.32                                     | 21.74                                | 36.06            |
| Metric $DSPV$ | 215                            | 200               | 16.00                                     | 7.00                                 | 23.00            |
| Eos halo    |                                   |                   |                                           |                                      |                  |
| Metric $D$  | 40                                | 5322              | 52.70                                     | 17.35                                | 70.05            |
| Metric $DS$ | 135                               | 1886              | 55.99                                     | 11.08                                | 67.07            |
| Metric $DPV$ | 80                              | 846               | 57.01                                     | 18.94                                | 75.95            |
| Metric $DSPV$ | 165                            | 738               | 51.36                                     | 17.89                                | 69.25            |
existed in the primordial main belt, (4) Vesta is the largest remnant for which a basaltic crust is still present and was observed by a space mission (Russel et al. 2012). Many V-type objects are observed outside the limits of the traditional HCM family (Carruba et al. 2005, Nesvorný et al. 2008), making this family a test-bed for the application of methods on halo determinations.

We determined a 161-member halo at a cutoff of \(d_{\text{nd}} = 275\, \text{m s}^{-1}\). 46 halo members (28.6 per cent of the total, a considerable fraction of the halo) are possible SDSS-MOC4 interlopers, and 26 asteroids (16.2 per cent of the total) are possible albedo interlopers. Among the asteroids with \(a > 0.58\) per cent are in a region of the \((a', i = z)\) plane associated with V-type objects, according to the criteria defined in Section 1, and can be considered as possible V-type candidates.

How efficient is the new method in identifying V-type asteroids outside the Vesta family as members of the halo? Among the V-type asteroids not connected to the traditional HCM Vesta family listed in Carruba et al. (2005), only four objects are present in our multidomain catalogue: (3849), (3869), (4188) and (4434). Of these, two (50 per cent of the total), (3869) and (4188), were part of the Vesta halo as found by our method. Having such a limited sample of objects in our catalogue and in the halo, we are not able to achieve any conclusions on the validity of the method for the Vesta halo. The Sidwell, Somekawa, Henninghaack and Ausonia AstDyS family merge with the Vesta halo at a cutoff of less than 320 m s\(^{-1}\).

### 4.4 The Erigone family

The Erigone family was identified by Nesvorný et al. (2006) as a CX-complex group at a cutoff of 80 m s\(^{-1}\). Their analysis appears to be confirmed by our work: we found a 57 member CX-complex group at a cutoff \(d_{\text{nd}} = 400\, \text{m s}^{-1}\). No SDSS-MOC4 interlopers were found in the halo, and only one object (1.8 per cent of the total) was a possible albedo interloper. The Maartes AstDyS family merges with the Erigone halo at a cutoff of \(d_{\text{nd}} = 165\, \text{m s}^{-1}\).

### 4.5 The Massalia family

The Massalia family was identified by Nesvorný et al. (2006) as an S-complex family at a cutoff of 50 m s\(^{-1}\). In this work, we identified 19 S-complex members at a cutoff \(d_{\text{nd}} = 250\, \text{m s}^{-1}\). Seven SDSS-MOC4 interlopers (36.8 per cent of the total) were found in the halo, and 10 objects (52.6 per cent of the total) were possible albedo interlopers. Incidentally, (20) Massalia itself is a C-type asteroid, and quite likely an interloper in its own family.

### 4.6 The Nysa/Mildred/Polana family

The Nysa/Polana family was studied by Mothé-Diniz et al. (2005) that confirmed previous results about the dual structure of the family, made by an S-type member group around (878) Mildred, and an F-type group around (142) Polana. In this work, we find a CX-complex halo of 147 members at a cutoff \(d_{\text{nd}} = 280\, \text{m s}^{-1}\). The halo is dominated by the CX-complex Polana group, which is also made by the largest bodies in the area (Mothé-Diniz et al. 2005): we found no possible SDSS-MOC4 interlopers, and one (0.7 per cent of the total) albedo interloper. A smaller halo associated with the Mildred family merges with the larger Polana halo at about 150 m s\(^{-1}\), and the Clarissa Planetary Data System is englobed at a cutoff of 345 m s\(^{-1}\).

### 4.7 The Euterpe family

The Euterpe family is a low-inclination group listed by the Planetary Data System. In this work, we identified an S-complex halo at a cutoff \(d_{\text{nd}} = 335\, \text{m s}^{-1}\). Two objects (22.2 per cent of the total) were possible SDSS-MOC4 interlopers, and one asteroid (11.1 per cent of the total) is a possible albedo interloper.

### 4.8 The Lucienne family

The Lucienne family is a relatively high-inclination group listed by the Planetary Data System. Unfortunately, we could not find any member of this group in our multidomain sample of asteroids for the inner main belt, so no conclusions are possible to achieve on this cluster.

### 4.9 The Phocaea family

The Phocaea family has been studied by Knežević & Milani (2003) and by Carruba (2009b). It is located in a stable island bounded by the \(v_6\) and \(v_5\) secular resonances in inclination and the \(3J:2A\) and \(2J:1A\) mean-motion resonances in semimajor axis. Despite the peculiar dynamical configuration, Carruba (2009b) concluded that it was likely that the Phocaea family was a real S-complex collisional family, with an estimated age of about 2.2 billion years. In this work, we found an 80-member S-complex halo at a cutoff \(d_{\text{nd}} > 800\, \text{m s}^{-1}\), which is the value for which all asteroids in the stable island in our data base were found connected to the Phocaea family. 27 objects (33.8 per cent of the total) were possible SDSS-MOC4 interlopers, and 16 asteroids (20.0 per cent of the total) had values of \(p_v < 0.1\). Overall, we confirm the analysis of Carruba (2009b) on the possible reality of the Phocaea family as an S-complex collisional group.

### 4.10 The inner main belt: an overview

Our results for the inner main belt are summarized in Table 2, where we give information on the first halo member used to determine the family halo, the cutoff value used to identify the halo, the number of bodies in the halo, the spectral complex to which the majority of halo members belongs and the number of possible interlopers, according to SDSS-MOC4 and geometric albedo considerations.

Fig. 4, panel A, displays an \((a, \sin i)\) projection of asteroids in our multivariate sample in the central main belt. Vertical red lines identify the orbital position of the main mean-motion resonances in the area. Blue lines show the location of the main linear secular resonances, using the second order and fourth-degree secular perturbation theory of Milani & Knežević (1994) to compute the proper frequencies \(g\) and \(s\) for the grid of \((a, e)\) and \((a, \sin i)\) values shown in Fig. 4, panel A, and the values of angles \(\Omega, \omega, M\) and eccentricity of (25) Phocaea, the highly inclined asteroid associated with the largest family in the region (Carruba 2009b). The orbital position in the \((a, \sin i)\) plane of the first numbered asteroid in all the inner main belt groups is also identified in Fig. 4, panel A. In panel B of the same figure, we display a density map of the inner main belt, according to the approach described in Carruba & Michtchenko (2009). Density maps display regions characterized by strong mean-motion or secular resonances by a relatively low number of asteroids per unit bin. To quantitatively determine the local density of asteroids, we computed the log_{10} of the number of all asteroids with proper elements per unit square in a 22 by 67 grid in \(a\) (starting at \(a = 2.18\, \text{au}\), with a step of 0.015 au) and \(\sin i\).
Table 2. Asteroid families’ haloes in the inner main belt.

| First halo member | $d_{md}$ value (m s$^{-1}$) | Number of members | Spectral complex | Number of SDSS-MOC4 interlopers | Number of $p_V$ interlopers |
|-------------------|-----------------------------|-------------------|-----------------|-------------------------------|-----------------------------|
| (298) Baptistina: (4691) | 250 | 56 | CX | 2 | 47 |
| (4) Vesta: (2011) | 275 | 161 | S(V) | 46 | 26 |
| (163) Erigone: (9566) | 400 | 57 | CX | 0 | 1 |
| (20) Massalia: (10102) | 250 | 19 | S | 7 | 10 |
| (44) Nysa/Mildred/Polana: (1768) | 280 | 147 | CX | 0 | 1 |
| (27) Euterpe: (5444) | 335 | 9 | S | 2 | 1 |
| (25) Phocaea: (3322) | $>800$ | 80 | S | 27 | 16 |

Figure 4. Panel A: an ($a$, sin $i$) projection of inner main belt asteroids in our multivariate sample. Panel B: contour plot of the number density of asteroids in the proper element sample. Superimposed, we display the orbital location of asteroid families in the CX-complex (plus signs), and in the S-complex (circles). (starting at 0, with a step of 0.015). Superimposed to the density map, we also show the orbital projection of the haloes found in this work shown as plus signs for CX-complex families, and circles for S-complex families. The other symbols are the same as in Fig. 4, panel A.

Fig. 5 displays a projection in the ($a^{*}$, $i$) plane of all asteroids in our multidomain sample (panel A), and an ($a$, sin $i$) projection of the same asteroids, (panel B), where objects in the CX-complex are shown as blue circles, and asteroids in the S-complex are identified as red plus signs. The inner main belt is slightly dominated by S-complex asteroids, but with a significant minority of CX-complex bodies. V-type asteroids are mostly concentrated in the Vesta family, but with a significant population outside the dynamical group (see also Carruba et al. 2005).

This is confirmed by an analysis of WISE $p_V$ geometrical albedo data, a histogram of which is presented in Fig. 6, panel A. Fig. 6, panel B, displays an ($a$, sin $i$) projection of the same asteroids, where blue full dots are associated with asteroids with $p_V < 0.1$, red full dots display asteroids with $0.1 < p_V < 0.3$ and magenta full dots show asteroids with $p_V > 0.3$. The majority of asteroids in the inner main belt is made by high-albedo objects, associated with S-complex taxonomies, but with a significant minority of CX-complex bodies.

5 CENTRAL MAIN BELT

The central main belt is dynamically limited in semimajor axis by the 3J:-1A and 5J:-2A mean-motion resonances (Zappalà et al. 1995). The linear secular resonance $\nu_6$ separates the low-inclined asteroid region from the highly inclined area, dominated by the Hansa and Pallas families (Carruba 2010b). As discussed in Carruba (2010b), in the highly inclined region the local web of linear secular resonances and mean-motion resonances divided the region into six separated stable islands, each hosting one or more major families, and that can be considered as a stable archipelago. Of particular interest in this region is the Tina family, whose members are all in anti-aligned states of the $\nu_6$ linear secular resonance (Carruba & Morbidelli 2011). We found 3693 objects that have proper elements and frequencies, SDSS-MOC4 $a^{*}$ and $i - z$ colours, WISE geometric albedo data, and satisfy our error analysis criteria, in the central main belt, and we will start our analysis by studying the case of the Hestia family.

5.1 The Hestia family

The Hestia family was identified in Nesvorný et al. (2005) as a 154-member group with S-taxonomy at a cutoff in proper element domain of 80 m s$^{-1}$. Here, we obtained a CX-halo of 26 members at a cutoff $d_{md} = 360$ m s$^{-1}$. 11 objects (42.3 per cent of the total) were SDSS-MOC4 interlopers, and 12 (46.2 per cent of the total) had $p_V < 0.1$.

5.2 The Astraea family

The Astraea family is listed at the AstDyS. We identified a small CX-complex halo of four members at a cutoff of 320 m s$^{-1}$, with no interlopers.
Figure 5. Panel A: an \((a^*, i - z)\) projection of inner main belt asteroids in our multidomain sample. Panel B: an \((a, \sin(i))\) projection of the same asteroids, where objects in the CX-complex are shown as blue circles, and asteroids in the S-complex are identified as red plus signs.

Figure 6. Panel A: a histogram of number frequency values \(n_i/N_{Tot}\) as a function of geometric albedo \(p_V\) for inner main belt asteroids in our multidomain sample. Panel B: an \((a, \sin(i))\) projection of the same asteroids, where blue full dots are associated with asteroids with \(p_V < 0.1\), red full dots display asteroids with \(0.1 < p_V < 0.3\) and magenta full dots show asteroids with \(p_V > 0.3\).

5.3 The Aeolia family
The Aeolia family was identified in Nesvorný et al. (2005) as a group of 28 members at a cutoff of 20 m s\(^{-1}\) with no identifiable dominant taxonomy. Here, we obtained a halo of 14 members at a cutoff \(d_{md} = 320\) m s\(^{-1}\), all with CX-complex taxonomies. Two objects (14.3 per cent of the total) have values of \(p_V > 0.1\).

5.4 The Chloris family
The Chloris family was a group of 135 members identified in Nesvorný et al. (2005) at a cutoff in proper element domain of 120 m s\(^{-1}\). Most of the members of this group belonged to the C-class. In this work, we found a halo of 35 members at a cutoff of 340 m s\(^{-1}\). One object (2.9 per cent of the total) was a possible SDSS-MOC4 interloper, and eight objects (22.9 per cent of the total) had values of \(p_V > 0.1\).

5.5 The Misa family
The Misa family was a large C-class group of 119 asteroids identified at a cutoff of 80 m s\(^{-1}\) in proper element domain by Nesvorný et al. (2005). Here, we found a halo of 33 members at a cutoff \(d_{md} = 355\) m s\(^{-1}\) all belonging to the CX-complex. One object (3.0 per cent of the total) was a possible SDSS-MOC4 and albedo interloper. The Leonidas AstDyS group merges with this family at a cutoff of less than 150 m s\(^{-1}\).

5.6 The Brangane family
The Brangane group was a 30-member S-type cluster identified in proper element domain by Nesvorný et al. (2005) at a cutoff of 30 m s\(^{-1}\). In this work, we identified an S-complex halo of just three members at a cutoff \(d_{md} = 355\) m s\(^{-1}\). One member (33.3 per cent of the total) was a possible SDSS-MOC4 interloper, and, as observed for some other S-complex families, all objects had \(p_V < 0.1\).
5.7 The Bower family

The Bower family was a 82-member cluster identified by Nesvorný et al. (2005) at a cutoff of 100 m s$^{-1}$ with no dominant taxonomical information. In this work, we identified a 27-member halo at a cutoff $d_{\text{md}} = 260$ m s$^{-1}$. Most of the members belonged to the CX-complex, but six (22.2 per cent of the total) were possible SDSS-MOC4 interlopers, and seven objects (25.9 per cent of the total) had $p_V > 0.1$.

5.8 The Cameron family

The Cameron group was identified at a cutoff of 60 m s$^{-1}$ by Nesvorný et al. (2005). It was a 162 group made mostly by S-type asteroids. The halo that we identified in this work had three members at $d_{\text{md}} = 310$ m s$^{-1}$. Contrary to what was found by Nesvorný et al. (2005), all members have SDSS-MOC4 colours compatible with a CX-complex taxonomy, but two objects (66.7 per cent of the total) had $p_V > 0.1$. The Innes AstDys group merges with this family at cutoff lower than 150 m s$^{-1}$.

5.9 The Rafita family

The Rafita family was an S-complex group identified by Nesvorný et al. (2005) in the $(a, e, \sin i)$ proper elements domain at a cutoff of 100 m s$^{-1}$. Unfortunately, we could not identify any member of this family in our multidomain sample of proper elements, SDSS-MOC4 colours and geometric albedos. Therefore, we could not analyse this family halo.

5.10 The Eunomia family

The Eunomia family is the largest family in the central main belt. Moth-Diniz et al. (2005) analysed the spectra of 43 members of this family, most of which belonging to the S-complex, but with a large taxonomical diversity that suggested surface inhomogeneities or the action of space weathering. The presence of T- and X-class asteroids, classes these compatible with iron meteorites, suggested the possibility that the formation of the Eunomia family may have been the result of the catastrophic breakup of a differentiated (or partially differentiated) parent body. The identification of three V-type asteroids in the orbital proximity of the Eunomia family provided further hints for this possibility. Carruba, Michchenko & Lazzaro (2007) showed that it is possible to migrate from the Eunomia dynamical family to the current orbital location of (21238) 1995 WV7, the largest of the V-type asteroids in the Eunomia region, via the interplay of the Yarkovsky effect and the $v_5 - v_6 + v_{16}$ non-linear secular resonance, on time-scales of at least 2.6 Gyr.

In this work, we identified a halo with 52 members, at a cutoff $d_{\text{md}} = 90$ m s$^{-1}$. As found in Moth-Diniz et al. (2005), the Eunomia family halo is quite diverse, with a predominance of objects belonging to the S-complex, but with a fairly large majority of C- and X-complex asteroids. We found seven SDSS-MOC4 interlopers and seven asteroids with $p_V < 0.1$, which yields a percentage of 13.5 per cent likely interlopers. The Planetary Data System group of Schulhof merges with the Eunomia family at a cutoff of 195 m s$^{-1}$.

5.11 The Iannini family

The Iannini family was studied in Nesvorný et al. (2005), where it was identified in proper element domain at a cutoff of 30 m s$^{-1}$. The group was listed as an S-type, but here we found a 93-member halo dominated by CX-complex asteroids, at a cutoff $d_{\text{md}}$ of 305 m s$^{-1}$. The discrepancy with Nesvorný et al. (2005) spectral classification may possibly be caused by the low number (18) of objects found in this family at the time. There were no SDSS-MOC4 interloper, and six asteroids (6.5 per cent of the total) had $p_V > 0.1$.

5.12 The Gefion family

The Gefion family, previously identified as the Ceres family (Zappalà et al. 1995) and also as the Minerva/Gefion family (Mothé-Diniz et al. 2005), was identified in Mothé-Diniz et al. (2005) as a fairly homogeneous family, with members mostly belonging to the S-complex. Because of its orbital proximity to (1) Ceres, it was studied in Carruba et al. (2003) as a test case for chaotic diffusion caused by close encounters with massive asteroids. The Gefion family halo was identified at a cutoff $d_{\text{md}} = 210$ m s$^{-1}$, with 146 members. Mothé-Diniz et al. (2005) found that the local background of this family is mostly dominated by distinguished C-type asteroids. Indeed, our halo is contaminated by a minority of bodies belonging to the C-complex: we found 43 SDSS-MOC4 interlopers and 33 asteroids with $p_V < 0.1$, which yields a percentage of 29.5 and 22.6 per cent likely interlopers, respectively. The Minerva AstDys group merges with this family halo at cutoff lower than 150 m s$^{-1}$.

5.13 The Adeona family

The Adeona family was analysed by Mothé-Diniz et al. (2005) that found it to be a very homogeneous family, made mostly in its entirety by asteroids belonging to the CX-complex. Because of its orbital proximity to (1) Ceres, it was also studied in Carruba et al. (2003) to understand the long-term effects of diffusion caused by close encounters with massive asteroids. The Adeona family halo has been identified in this work at a cutoff $d_{\text{md}} = 295$ m s$^{-1}$, with 149 members. We found one SDSS-MOC4 interloper (0.7 per cent of the total), and four objects with geometric albedo (barely) larger than 0.1, which corresponds to a percentual of possible interlopers of 2.7 per cent. This high uniformity of the Adeona albedo confirms the results found in Mothé-Diniz et al. (2005).

5.14 The Maria and Renate families

The Maria family was analysed together with the Renate family in Mothé-Diniz et al. (2005), and both families had a majority of members with known taxonomies belonging to the S-complex, indistinguishable from the local background. Zappalà et al. (1997) analysed this family and found that the spectra of 10 family members were compatible with those of near-Earth asteroids (433) Eros and (1036) Ganymede, conclusion not supported by the work of Mothé-Diniz et al. (2005). The Maria family halo has been identified in this work at a cutoff $d_{\text{md}} = 240$ m s$^{-1}$ with 135 members. We found 21 objects that can be classified as SDSS-MOC4 interlopers, five of which barely in the area of the CX-complex, and 10 asteroids with $p_V < 0.1$, which yields a percentual of possible interlopers of 15.6 and 7.4 per cent, respectively. The Renate family, considered together with the Maria family in Mothé-Diniz et al. (2005), and also classified as an S-complex group in that work, merges with the Maria family at a cutoff of 225 m s$^{-1}$. For the purpose of halo analysis, the two families can be considered as an unique group.
5.15 The Padua family

This family, previously associated with the asteroid (110) Lydia, is made mostly by X-type asteroids indistinguishable from the local background, according to Moth-Đinizio et al. (2005). The family is very important from a dynamical point of view, since it is the second family, after the Agnia, to have most of its members in a non-linear secular resonance configuration. More than 75 per cent of its members, according to Carruba (2009a), are currently in a $z_1$ librating state. Conservation of the $K'_z = \sqrt{2 - e'^2} (2 \cos i)$ quantity associated with this secular allowed us to set limits on the original ejection velocity field, which was in agreement with result obtained with an alternative Monte Carlo model that included Yarkovsky and Yarkovsky-O’Keefe-Radzievsky-Paddack (YORP) semimajor axis mobility. The current spread of values in the $(\sigma, g - g_0 + s - s_0)$ plane, where $\sigma$ is the resonant argument of the $z_1$ resonance allowed to set a lower limit on the age of the family of 25 Myr, which was then used to set an upper limit on the effect of low-energy collisions. The Padua halo was identified at a cutoff of 130 m s$^{-1}$, with 31 members, and no interlopers. The Zdenekhorsky AstDyS group merges with the Padua halo at cutoff lower than 100 m s$^{-1}$.

5.16 The Juno family

The Juno family was identified in Nesvorný et al. (2005) as a 74-member S-type group. Here, we identified a halo of 61 members at a cutoff of 275 m s$^{-1}$, which, contrary to what published in Nesvorný et al. (2005), is made mostly by CX-complex bodies, with (3) Juno itself, an Sk object and a possible interloper. There were no SDSS-MOC4 interlopers, and four asteroids (6.6 per cent of the total) had values of $p_V > 0.1$.

5.17 The Dora family

The Dora family was classified by Moth-Đinizio et al. (2005) as a very homogeneous C-complex family, with the majority of members belonging to the Ch class, and five objects in the C and B classes. The family was very differentiated from the local background, made mostly by asteroids belonging to the S-complex. The Dora halo was identified at a cutoff of 265 m s$^{-1}$, with 108 members. Only two members were possible SDSS-MOC4 interlopers and had $p_V > 0.1$ (1.9 per cent), confirming the very homogeneous nature of this family, as found in Moth-Đinizio et al. (2005).

5.18 The Merxia and Nemesis family

The Merxia family was identified in Nesvorný et al. (2005) and was dominated by the two largest bodies, (808) Merxia, and (1327) Namaqua, the second of which was most likely an interloper because of its low albedo. The family is crossed by the 3J:1S:1A three-body mean-motion resonance, which divides it into two lobes and cause a depletion in the number of members at the centre of the family, and it was well differentiated from the local background, dominated by CX-complex objects, according to Moth-Đinizio et al. (2005). Nesvorný et al. (2005) also identified in the region the Nemesis family, but its halo merges with that of the Merxia family at a cutoff of $\simeq 200$ m s$^{-1}$, and we therefore decided to treat the two families as a single case. We found a CX-halo at a cutoff of 250 m s$^{-1}$, with 19 members, 5 of which could be SDSS-MOC4 interlopers and 9 of which have $p_V < 0.1$. The large percentual of possible interlopers (26.3 and 42.1 per cent) may be caused by the fact that, possibly, there is no Merxia halo, and the family is small and limited to the S-complex core found in Moth-Đinizio et al. (2005).

5.19 The Agnia family

The Agnia family, previously identified as the Liberatrix family, was the first group to be found having the majority of its members in $z_1$ librating states (Vokrouhlický et al. 2006b). Conserved quantities of the $z_1$ resonance and spread in the $(\sigma, g - g_0 + s - s_0)$ plane, as discussed for the case of the Padua family, were introduced in that work to obtain constraints on the family original ejection velocity field and age. The family, first analysed by Bus (1999), appears compatible with an S-complex taxonomy in Moth-Đinizio et al. (2005), while the local background is dominated by CX-complex bodies. Here, we determined a halo at a cutoff of 190 m s$^{-1}$, with 12 members. As for the Merxia family halo, we found a large number of possible interlopers: four SDSS-MOC4 CX-complex members and four $p_V < 0.1$ asteroids (33.3 per cent of the total), which may suggest that the actual Agnia family is small and with a limited halo.

5.20 The Astrid family

The Astrid family was identified in Bus (1999) and Moth-Đinizio et al. (2005) as a very tight clump, with most members belonging to the C-complex. No asteroid in the local background had taxonomical information at the time of Moth-Đinizio et al. (2005) analysis. Here, we found a very robust and isolated group, with a halo that was separated from the local background for cutoffs as large as 435 m s$^{-1}$, with six members, and no interlopers, confirming that this is a very homogeneous and robust group.

5.21 The Hoffmeister family

The Hoffmeister family was found to be a very compact and spectrally homogeneous CX-group in Moth-Đinizio et al. (2005). Here, we determined a halo of eight members at a cutoff of 210 m s$^{-1}$. No interlopers were detected, so confirming previous analysis of this group.

5.22 The Lavrov family

The Lavrov group, previously known as the Henan clump, is a small group formed mostly by L-type asteroids, that are also typical of the local background (Moth-Đinizio et al. 2005). We identified a halo of six members at a cutoff of 200 m s$^{-1}$. We identified only one possible SDSS-MOC interloper and two asteroids with $p_V < 0.1$ (12.5 and 25.0 per cent of the total, respectively), which confirms that this should be a fairly compact and robust L-class group.

5.23 The 1995 SU37 family

The 1995 SU37 group is listed at the Planetary Data System. We identified a small S-complex halo of four members, at a cutoff of 105 m s$^{-1}$, with no interlopers.

5.24 The Watsonia family

The Watsonia family is listed at the AstDyS. We identified a CX-complex halo of 10 members at a cutoff of 425 m s$^{-1}$. Three objects (30.0 per cent of the total) were possible SDSS-MOC4 interlopers, and five objects (50 per cent of the total) had $p_V > 0.1$. 

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5.25 The Ino, Atalante and Anacostia families
The Ino, Atalante and Anacostia families are listed at the AstDys. Unfortunately, we could not find any of their members in our multidomain data base. No information is therefore available for this family in this work.

5.26 The Gersuind family
With the Gersuind family we start the analysis of the highly inclined \( \sin i > 0.3 \) asteroid groups in the central main belt, that were the subject of the study of Carruba (2010b). The Gersuind family was studied in Gil-Hutton (2006). While having most of its members at \( \sin i > 0.3 \), it lies at lower inclinations than the centre of the \( \nu_3 \) resonance, and it is not therefore considered a proper high-inclination family by other authors, such as Machuca & Carruba (2011). The few objects with SDSS-MOC3 data in the family obtained by Carruba (2010b) were compatible with an S-complex taxonomy. Here, we found a halo at 310 m s\(^{-1}\) with seven members, the majority of which were compatible with an S-complex taxonomy. Three objects (42.9 per cent of the total) were SDSS-MOC4 interlopers, and two objects (28.6 per cent of the total) had albedos smaller than 0.1, confirming the analysis of Carruba (2010b). The Planetary Data System Emilkowalski group merges with this family at cutoff lower than 100 m s\(^{-1}\).

5.27 The Myriostos family
The Myriostos family is listed at the AstDys. We identified a 7-member CX-complex halo at a cutoff of 580 m s\(^{-1}\), with two SDSS-MOC4 interlopers (28.6 per cent of the total). Six objects (85.7 per cent of the total) have \( p_V > 0.1 \).

5.28 The Kunitaka family
The Kunitaka family is listed at the AstDys. We could not find any of its members in our multidomain data base, so no information is available on this group in this work.

5.29 The Hansa family
The Hansa family is the largest dynamical group among high-inclination families in the central main belt. The Hansa family was originally proposed by Hergenrother, Larson & Spahr (1996), studied in Gil-Hutton (2006) and re-analysed in Carruba (2010b) that found a large group compatible with an S-type taxonomy. The family is located in a stable island limited in inclination by the \( \nu_3 \) and \( \nu_5 \) secular resonances. This is confirmed by our analysis: we found a 20-member halo at a cutoff of \( >535 \) m s\(^{-1}\) (at cutoff as large as 1000 m s\(^{-1}\) the family does not yet connect with the local background), with all but one member (95 per cent of the total) with an S-complex taxonomy. We did not detect asteroids with \( p_V < 0.1 \). The 2001 YB113 AstDys group merges with this family halo for cutoff less than 150 m s\(^{-1}\).

5.30 The Brucato family
The Brucato family was first identified as a clump in proper elements domain and as a family in the \((n, g, g + s)\) proper frequencies domain in Carruba (2010b), Novaković et al. (2011) then re-obtained this group in proper element domains as a family, using a larger sample of asteroid proper elements. The family is located in a stable island limited in inclination by the \( \nu_3 \) and \( \nu_{16} \) linear secular resonances. The group was made mostly by CX-complex asteroids, and this is confirmed by the current analysis: we identify a family halo at a cutoff of \( >950 \) m s\(^{-1}\) with 32 members, all belonging to the CX-complex. Two albedo interlopers (6.3 per cent of the total) were identified in this family halo. The 1998 DN2, 1999 PM1, 1998 LF3 and 2004 EW7 AstDys groups merge with this family at cutoffs lower than 150 m s\(^{-1}\).

5.31 The Dennispalm family
The Dennispalm family is listed at the AstDys. We could not find any of its members in the multidomain data base, so no information is available on this group in this work.

5.32 The Barcelona family
The Barcelona family was first identified as a clump in Gil-Hutton (2006). Carruba (2010b) identified the group as a dynamical family, and this was confirmed by the later work of Novaković et al. (2011). The Barcelona family was made mostly by Sq asteroids. Very few objects were present in our multidomain sample of asteroids at this inclinations: we identified an S-complex halo of only one member at a cutoff of 730 m s\(^{-1}\).

5.33 The Tina family
The Tina family, first identified in Carruba (2010b), is unique in the Solar system because all of its members are in \( \nu_3 \) anti-aligned librating states, making it the only family currently known to lie in a stable island of a linear secular resonance. Carruba & Morbidelli (2011) studied its dynamics and obtained estimates of the family age and possible survival time before the family members escape from the stable island (both events happened and will happen on timescales of 150 Myr). (1222) Tina itself belongs to the X-complex. The one halo object that we identified at a cutoff of 890 m s\(^{-1}\) is compatible with such taxonomy. The very limited number of objects with known taxonomy does not, however, allow us to determine if the Tina’s group is a real family or a conglomerate of asteroids happening to be lying in the local stable island, yet.

5.34 The Gallia family
The Gallia family was first identified as a clump in Gil-Hutton (2006), and was re-obtained as a family in Carruba (2010b), and Novaković et al. (2011). It is located in a stable island limited in inclination by the \( \nu_3 \) and \( \nu_{16} \) linear secular resonances. Its taxonomy was compatible with an S-complex composition, according to the analysis of Carruba (2010b). Here, we identified a halo of just three members at a cutoff of \( >410 \) m s\(^{-1}\). All members were compatible with an S-complex taxonomy.

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4 See Carruba & Michtchenko (2007) for a more in depth discussion of frequency families.
5.35 The Pallas family
Williams (1992) first proposed the Pallas family, that was later reanalysed by Gil-Hutton (2006), Carruba (2010b) and Novakovic et al. (2011). Most of the Pallas family members have B-type taxonomies, but C-type objects are also observed in the orbital region. In this work, we identified a halo of eight members at a cutoff of >920 m s\(^{-1}\). No SDSS-MOC4 interloper was found, but, as observed for Hungary family members, all eight asteroids have large values of \(p_{\nu}\), in principle incompatible with a B- or C-type taxonomy.

5.36 The central main belt: an overview
Our results for the central main belt are summarized in Table 3, that has the same format as Table 2.

Fig. 7, panel A, displays an \((a, \sin(i))\) projection of asteroids in our multidomain sample in the central main belt. Blue lines show the location of the main linear secular resonances, using the second-order and fourth-degree secular perturbation theory of Milani & Knežević (1994) to compute the proper frequencies \(g\) and \(s\) for the grid of \((a, e)\) and \((a, \sin(i))\) values shown in Fig. 10, panel A, and the values of angles and eccentricity of (480) Hansa, the highly inclined asteroid associated with the largest family in the region (Carruba 2010b). Other symbols have the same meaning as in Fig. 4, panel A. In panel B of the same figure, we display a density map of the central main belt. We computed the \(\log_{10}\) of the number of all asteroids with proper elements per unit square in a 22 by 67 grid in \(a\) (starting at \(a = 2.500\) au, with a step of 0.015 au) and \(\sin(i)\) (starting at 0, with a step of 0.015). Superimposed on to the density map, we also show the orbital projection of the haloes found in this work shown as red plus signs for CX-complex families and as blue circles for S-complex families. The other symbols are the same as in Fig. 10, panel B. The reader may notice that regions with higher number density of asteroids are associated with the families’ haloes found in this work. The haloes that we determined do not have multiple membership, i.e. an asteroid in a given family halo is not present in another family halo.

Fig. 8 displays a projection in the \((a^*, i - z)\) plane of all asteroids in our multidomain sample (panel A), and an \((a, \sin(i))\) projection of the same asteroids, (panel B). We refer the reader to the caption of Fig. 6 for a more detailed description of this figure symbols. The central main belt is a transitional region, where CX-complex and S-complex asteroids are rather mixed. A greater proportion of S-type asteroids can be found at lower semimajor axis (and vice versa), but overall, no group dominates the local taxonomy.

This is confirmed by an analysis of \(WISE p_{\nu}\) geometrical albedo data, a histogram of which is presented in Fig. 9, panel A. Fig. 9, panel B, displays an \((a, \sin(i))\) projection of the same asteroids, where we used the same colour code as in Fig. 6, panel B.

One can notice the clearly bi-modal distribution of asteroid albedos, and the mixture of objects with low albedo and high albedo that dominates the central main belt. Having analysed the families of the central main belt, we are now ready to move on to the outer main belt.

6 OUTER MAIN BELT
The outer main belt is dynamically limited in semimajor axis by the 5J-2A and 2J-1A mean-motion resonances (Zappalà et al. 1995). The linear secular resonance \(\nu_6\) separates the low-inclined asteroid region from the highly inclined area, dominated by the Euphrosyne family (Machuca & Carruba 2011). We found 5385 objects that have proper elements and frequencies, SDSS-MOC4 colours, \(WISE\) geometric albedo data and satisfy our error analysis criteria in the outer main belt. We start our analysis of families’ haloes by investigating the Koronis family.

6.1 The Koronis family
The Koronis family is one of the most interesting families in the main belt. Bottke et al. (2002) explained its shape in eccentricity as originating by the interaction of asteroids evolving via Yarkovsky effect into secular resonances such as the \(2\nu_5 - 3\nu_6\). Carruba & Michtchenko (2007) showed that its upper boundary in eccentricity is limited by the \(\nu_5 + 2\nu_5 - 2\nu_7 + \nu_6\) non-linear secular resonance. The interesting subgroup of the Karin cluster, identified by Nesvorný et al. (2002a) opened new perspectives in the understanding of recent breakups among asteroids. Moth-Diniz et al. (2005) identified this family as predominantly belonging to the S-complex, with a few interlopers belonging to the C and D types, which predominate in the background. The presence of a few K-type asteroids in the family was somehow puzzling, and justified by Moth-Diniz et al. (2005) as a possible remnant of a pre-existing family.

In this work, we identified a halo with 200 members, at a cutoff \(d_{\text{min}}\) of 215 m s\(^{-1}\). 32 of the halo members have SDSS-MOC4 colours not compatible with an S-complex taxonomy, which yields to a 16.0 per cent fraction of possible interlopers in the halo, somewhat in agreement with the finding of Moth-Diniz et al. (2005). About 14 asteroids (7.0 per cent of the total) have values of \(p_{\nu}\) smaller than 0.1, which is the limit for S-complex asteroid albedos, and that may be related with the presence of the interloper population found in Moth-Diniz et al. (2005).

6.2 The Lau family
The Lau cluster is listed at the Planetary Data System. In this work, we identified a 10-object CX halo at a cutoff of 490 m s\(^{-1}\). No interloper was found in this group.

6.3 The (1993) FY12 family
The cluster around (18405) (1994 FY12) was identified for the first time in Nesvorný et al. (2005) at a cutoff of 50 m s\(^{-1}\) as an 11-asteroid X group. Here, we find a halo at a cutoff \(d_{\text{min}}\) of 355 m s\(^{-1}\) of just two members, one of which (50.0 per cent of the total) have \(p_{\nu}\) > 0.1. The possible X-type composition of this small group appears to be confirmed by our analysis.

6.4 The Fingella family
The Fingella family is listed at the Planetary Data System. In this work, we identified a 16-object CX halo at a cutoff of 480 m s\(^{-1}\). No interloper was found in this group.

6.5 The Naema family
The Naema group is another family discussed in Nesvorný et al. (2005), where it was visible at a cutoff of 40 m s\(^{-1}\) as a 64 C-type group. Here, we found a halo of 43 members at a cutoff of 390 m s\(^{-1}\), with all members belonging to the CX-complex. No interlopers were found in this group.
Table 3. Asteroid families’ haloes in the central main belt.

| First halo member | $d_{\text{max}}$ cutoff value (m s$^{-1}$) | Number of members | Spectral complex | Number of SDSS-MOC4 likely interlopers | Number of $p_V$ likely interlopers |
|-------------------|------------------------------------------|-------------------|-----------------|----------------------------------------|-----------------------------------|
| (46) Hestia: (7321) | 360                                      | 26                | CX              | 11                                     | 12                                |
| (5) Astrea: (4018)  | 320                                      | 4                 | CX              | 0                                       | 0                                 |
| (396) Aeolia: (76144)  | 320                                      | 14                | CX              | 0                                       | 2                                 |
| (410) Chloris: (9545) | 340                                      | 35                | CX              | 1                                       | 8                                 |
| (569) Misa: (2289)   | 355                                      | 33                | CX              | 1                                       | 1                                 |
| (606) Brangane: (56748) | 355                                      | 3                 | S               | 1                                       | 3                                 |
| (1639) Bower: (26703) | 260                                      | 27                | CX              | 6                                       | 7                                 |
| (2980) Cameron: (4067) | 310                                      | 3                 | CX              | 0                                       | 2                                 |
| (15) Eunomia: (630)  | 90                                       | 52                | S               | 7                                       | 7                                 |
| (4652) Iannini: (143366) | 305                                      | 93                | CX              | 0                                       | 6                                 |
| (1272) Gefion: (2373) | 210                                      | 146               | S               | 43                                      | 33                                |
| (145) Adeona: (1783) | 295                                      | 149               | CX              | 1                                       | 4                                 |
| (170) Maria/Renate: (4104) | 240                                      | 135               | S               | 21                                      | 10                                |
| (363) Padua: (2560)  | 130                                      | 31                | CX              | 0                                       | 0                                 |
| (3) Juno: (22216)    | 275                                      | 61                | CX              | 0                                       | 4                                 |
| (668) Dora: (1734)   | 265                                      | 108               | CX              | 2                                       | 2                                 |
| (808) Merxia/Nemesis: (3439) | 250                                      | 19                | CX              | 5                                       | 8                                 |
| (847) Agnia: (1020)  | 190                                      | 12                | S               | 4                                       | 4                                 |
| (1128) Astrid: (2169) | 435                                      | 6                 | CX              | 0                                       | 0                                 |
| (1726) Hoffmeister: (1726) | 210                                      | 62                | CX              | 0                                       | 0                                 |
| (2354) Lavrov: (2354) | 200                                      | 8                 | S               | 1                                       | 2                                 |
| (18466) 1995 SU37: (95534) | 105                                      | 4                 | S               | 0                                       | 0                                 |
| (729) Watsonia: (5492) | 425                                      | 10                | CX              | 3                                       | 5                                 |
| (680) Gersund: (14627) | 310                                      | 7                 | S               | 3                                       | 2                                 |
| (10000) Myriotos: (101897) | 580                                      | 7                 | CX              | 2                                       | 6                                 |
| (480) Hansa: (13617) | >535                                     | 20                | S               | 1                                       | 0                                 |
| (4203) Brucato: (4203) | 950                                      | 32                | CX              | 0                                       | 2                                 |
| (945) Barcelona: (11028) | 730                                      | 1                 | S               | 0                                       | 0                                 |
| (1222) Tina: (16257) | 890                                      | 1                 | CX              | 0                                       | 0                                 |
| (148) Gallia: (40853) | >410                                     | 3                 | S               | 0                                       | 0                                 |
| (2) Pallas: (24793)  | >920                                     | 8                 | CX              | 0                                       | 0                                 |

Figure 7. Panel A: an $(a, \sin i)$ projection of central main belt asteroids in our multivariate sample. Panel B: contour plot of the number density of asteroids in the proper element sample. Superimposed, we display the orbital location of asteroids of families in the CX-complex (plus signs) or in the S-complex (circles).

6.6 The Brasilia family

The Brasilia family was identified as a small clump of 96 members in Mothé-Diniz et al. (2005), of which only 4 had known spectral type, all belonging to the CX-complex. We identified a small halo of 24 objects, most belonging to the CX-complex, that merges with the local background at a cutoff of 440 m s$^{-1}$. While all but three halo members (12.5 per cent of the total) have SDSS-MOC4 colours compatible with a taxonomy in the CX-complex, the vast majority of these asteroids (17, 70.8 per cent of the total) has large albedos ($p_V > 0.1$), not usually associated with a CX-composition. The reason for the large-albedo values of these objects remains unexplained.
Figure 8. Panel A: an \((a^*, i - z)\) projection of central main belt asteroids in our multidomain sample. Panel B: an \((a, \sin(i))\) projection of the same asteroids, where objects in the CX-complex are shown as blue circles, and asteroids in the S-complex are identified as red plus signs.

Figure 9. Panel A: a histogram of number frequency values \(n_i/N_{\text{Tot}}\) as a function of geometric albedo \(p_V\) for central main belt asteroids in our multidomain sample. Panel B: an \((a, \sin(i))\) projection of the same asteroids, where blue full dots are associated with asteroids with \(p_V < 0.1\), red full dots display asteroids with \(0.1 < p_V < 0.3\), and magenta full dots show asteroids with \(p_V > 0.3\).

6.7 The Themis family

The Themis family is a rather large, homogeneous group, made mostly by asteroids of types belonging to the C/X-complex. Mothédiniz et al. (2005) discuss that the spectral type of the family members is not distinguishable from that of asteroids in the background, which is dominated by C and X-type objects. Tanga et al. (1999) suggest that this family originated from the breakup of a large \(\approx 370\) km in diameter) C-type parent body. In this work, we identified a halo with 700 members, at a cutoff \(d_{\text{md}}\) of 310 m s\(^{-1}\). Nine of the halo members have SDSS-MOC4 colours not compatible with a C/X-complex taxonomy, which yields a 1.3 per cent fraction of possible interlopers in the halo. 50 asteroids have values of \(p_V\) larger than 0.1, which is the limit for CX-complex asteroid albedos, which yields a percentage of possible albedo interlopers of 7.1 per cent. The AstDys Ashkova group merges with the Themis halo at a cutoff of 205 m s\(^{-1}\).

6.8 The Eos family

Vokrouhlický et al. (2006b) studied in detail the dynamical evolution of this asteroid family, also pioneering the use of Monte Carlo methods for asteroid families’ chronology. The Eos family is crossed by the powerful 9J:-4A mean-motion resonance, and interacts with the \(z_1, 2v_5 - 3v_6\) and \(2v_5 - 2v_6 + v_{16}\) secular resonances (see also Carruba & Michtchenko 2007). More recently, Brož & Morbidelli (2013) identified Eos halo members based on SDSS-MOC4 data, and obtained an estimate of the halo age, that is in agreement with what found by Vokrouhlický et al. (2006b). In this work, we identified a halo at a cutoff of 165 m s\(^{-1}\), with 738 members. In agreement with what was found by Mothédiniz et al. (2005), the taxonomy of the family is quite inhomogeneous, but well distinguished from the local background that is dominated by C and X asteroids. Most members of the Eos family have a characteristic K-type taxonomy, and we found that many members of the halo members have \((a^*, i - z)\) colours compatible with an S-complex taxonomy. But, as discussed in Section 3, the Eos family lies at the separation between CX-complex and S-complex asteroids in the \((a^*, i - z)\). Our simple criterion for identifying S-complex asteroids, \(a^* > 0\), does not apply well to the case of the Eos family. 379 asteroids (51.4 per cent of the total) have values of \(a^* < 0\) and may be considered interlopers. 132 Eos halo asteroids have values of \(p_V < 0.1\), which yields that 17.9 per cent of the halo asteroids...
may actually be albedo interlopers. The quite diverse mineralogy of bodies in the Eos family area provides challenges that should be confronted with more advanced tools than the one used in this paper, in our opinion. The Telramund cluster, which was identified as an S-type 70-member group at a cutoff of 60 m s\(^{-1}\) by Nesvorný et al. (2005), merges at very low cutoff with the Eos family, so we consider this group as a substructure of the larger family.

### 6.9 The Hygiea family

(10) Hygiea is the fourth most massive asteroid of the main belt, and the family associated with this body has been studied in Zappalà et al. (1995), Mothé-Diniz et al. (2005), and, more recently by Carruba (2013). It is made mostly by bodies belonging to the CX-complex, and, as the Themis family, is not easily distinguishable from the local background of objects. We identified a halo with 426 objects at a cutoff of 290 m s\(^{-1}\). The slightly higher number of objects that we found in the Hygiea halo with respect to Carruba (2013) might be due to the larger sample of bodies in our data set. Of the 426 possible halo members, 6 (1.4 per cent) may be interlopers based on SDSS-MOC4 colours analysis and 36 have \(p_v > 0.1\), which is incompatible with CX-complex asteroids. Overall, we found a maximum of 8.5 per cent of objects that are possibly not correlated with the Hygiea halo. The AstDys Filipenko group merges with this family halo at a cutoff of 125 m s\(^{-1}\). The AstDys Higson cluster merges with this halo at a cutoff of 255 m s\(^{-1}\), by Nesvorný et al. (2005), merges at very low cutoff with the Eos family, so we consider this group as a substructure of the larger family.

### 6.10 The Emma family

The Emma family was identified by Nesvorný et al. (2005) as a 76-member group at a cutoff of 40 m s\(^{-1}\) in proper elements domain. No information on its taxonomy was given in that paper. In this work, we found a family halo of 43 members at a cutoff \(d_{\text{md}}\) of 270 m s\(^{-1}\). No interlopers were identified in this family halo.

### 6.11 The Veritas family

The Veritas family is a relatively small group, made mostly by CX-complex asteroids. Milani & Farinella (1994) used for the first time chaotic chronology on this family to determine its relatively young age, later confirmed by other works. We identified a halo of 148 members at a cutoff of 240 m s\(^{-1}\). Two objects have colours not compatible with a CX-complex taxonomy, and nine have values of \(p_v > 0.1\). Overall, up to 6.1 per cent of the halo members encountered may be interlopers of the Veritas halo.

### 6.12 The Lixiaohua family

The Lixiaohua family was identified by Nesvorný et al. (2005) as a 97 CX-complex group at a cutoff of 50 m s\(^{-1}\). It was the subject of a dynamical study by Novaković, Tsiganis & Knežević (2010) that extensively studied the local dynamics and the diffusion in the \((e_p, i_p)\) plane using Monte Carlo modelling. In this work, we identified a 69-member CX halo at a cutoff of \(d_{\text{md}} = 255\) m s\(^{-1}\). Only one object (1.4 per cent of the total) was a possible SDSS-MOC4 interloper, and all asteroids in the halo had \(p_v < 0.1\). The AstDys Gaitsch group merges with this family at cutoffs lower than 50 m s\(^{-1}\).

### 6.13 The Aegle family

The Aegle family is an AstDys group. We identified a 21 CX-complex group at a cutoff of 290 m s\(^{-1}\). Two objects (9.5 per cent of the total) were possible SDSS-MOC4 interlopers, and all members had low albedos.

### 6.14 The Meliboea family

The Meliboea family was discussed by Zappalà et al. (1995) and Mothé-Diniz et al. (2005). It is a small group, mainly composed of asteroids belonging to the CX-complex. It is a rather inclined family \((i_p \approx 15^\circ)\), characterized by the presence of several weak mean-motion and secular resonances. We identified a halo with 73 members at a cutoff of 270 m s\(^{-1}\). As found by Mothé-Diniz et al. (2005), the Meliboea family is fairly homogeneous, with only three members of the halo with colours not compatible with a CX-complex taxonomy. Only one halo member has \(p_v > 0.1\), which yields a percentage of up to 4.1 per cent possible interlopers. The AstDys group of Inarradas merges with this family halo at a cutoff of 140 m s\(^{-1}\), while the Traversa cluster is englobed at a cutoff of 205 m s\(^{-1}\).

### 6.15 The Klumpkea/Tirela family

The Klumpkea family was identified in Machuca & Carruba (2011) and corresponds to the old Tirela family of Nesvorný et al. (2005). Nesvorný et al. (2005) listed the Tirela family as a D-group. Here, we found a halo at a cutoff of 290 m s\(^{-1}\) with 21 members, 2 of which (9.5 per cent of the total) have colours (barely) in the S-complex area. All members of the halo have \(p_v < 0.1\), which makes this family compatible with a C-complex taxonomy. The AstDys Zhanetskiij cluster is annexed by this halo at a cutoff of 165 m s\(^{-1}\), the Ursula family is englobed at 200 m s\(^{-1}\), and the Pannonia group merges at a cutoff of 245 m s\(^{-1}\).

### 6.16 The Theobalda family

The Theobalda family is also an AstDys group. We identified a 34-member CX-complex halo, with two (5.9 per cent of the total) possible albedo interlopers.

### 6.17 The Kartvelia family

The Kartvelia family is listed at the AstDys. We found a 26-member CX-complex halo at a cutoff of 280 m s\(^{-1}\), with just one (3.8 per cent of the total) possible SDSS-MOC4 interloper.

### 6.18 The Alauda family region

The orbital region of the Alauda family has been most recently analysed by Machuca & Carruba (2011) that found several small groups, among which the Alauda and Lutheria families, in the area. In this work, we identified a 215 CX-complex group at a cutoff of 420 m s\(^{-1}\). Two members, 1.3 per cent of the total, have colours in the S-complex region, and 19 objects, 12.0 per cent of the total, have \(p_v > 0.1\). The AstDys Higson cluster merges with this halo at a cutoff of 235 m s\(^{-1}\), the AstDys Moravia and Snelling groups merge at a cutoff of 240 m s\(^{-1}\), while the AstDys Vassar cluster is annexed at 255 m s\(^{-1}\).

### 6.19 The Euphrosyne family

Machuca & Carruba (2011) most recently analysed the orbital region of this highly inclined asteroid family. This family is characterized by its interaction with linear secular resonances. In particular,
13 of its members are in \( v_2 \) anti-aligned librating states, one in a \( v_5 \) anti-aligned librating state (242435), and one in a \( v_5 \) aligned librating state (2009 UL136), according to Machuca & Carruba (2011).

The long-term effect of close encounters of asteroids with absolute magnitude \( H < 13.5 \) with (31) Euphrosyne was recently studied in Carruba et al. (2013). As discussed in Machuca & Carruba (2011), the Euphrosyne family is separated by the near regions of (69032) and Alauda in inclination by areas with very low asteroid densities. It is a region with a relatively small population of objects, separated among them by large distances, which explains why the 75-member halo that we identified in this work is at the high cutoff value of 575 m s\(^{-1}\). All halo members have colours in the S-complex area, and three members have \( p_V > 0.1 \). As for the case of the Klumpkea family, this is a group highly compatible with a C-complex taxonomy, with a 4.0 per cent of possible interlopers.

### 6.20 The outer main belt: an overview

Having obtained estimates for the haloes of the main families in the outer main belt, we are now ready to outline our results. As done for previous asteroid regions, our results are summarized in Table 4.

Fig. 10, panel A, displays an \((a, \sin (i))\) projection of asteroids in our multivariate sample in the outer main belt, using the same symbols for analogous figure in the inner and central main belt. Here, however, blue lines show the location of the main linear secular resonances, using the second-order and fourth-degree secular perturbation theory of Milani & Knežević (1994) to compute the proper frequencies \( g \) and \( s \) for the grid of \((a, e)\) and \((a, \sin (i))\) values shown in Fig. 10, panel A, and the values of angles and eccentricity of (31) Euphrosyne, the highly inclined asteroid associated with the largest family in the region (Machuca & Carruba 2011). In panel B of the same figure, we display a density map of the outer main belt. To quantitatively determine the local density of asteroids, we computed the \( \log_{10} \) of the number of all asteroids with proper elements per unit square in a 67 by 67 grid in \( a \) (starting at \( a = 2.805 \text{ au} \), with a step of 0.015 au) and \( \sin (i) \) (starting at 0, with a step of 0.015). The other symbols are the same as in Fig. 4, panel A. The reader may notice that regions with higher number density of asteroids are associated with the families’ halo found in this work. Families’ haloes are indeed more extended in proper elements domain than the core families found with the standard HCM.

To study how the family haloes found in this work are related to the local taxonomy, we also plotted in Fig. 11 a projection in the \((a^*, i - z)\) plane of all asteroids in our multivariate sample (panel A), and an \((a, \sin (i))\) projection of the same asteroids, (panel B). The great majority of asteroids in the region belong to the CX-complex, but there is a sizeable minority of bodies belonging to the S-complex, that, as shown in Fig. 11, panel B, are mostly associated with the Eos and Koronis families.

This is confirmed by \( WISE p_V \) geometrical albedo data, a histogram of which is presented in Fig. 12, panel A. Fig. 12, panel B, displays an \((a, \sin (i))\) projection of the same asteroids, with the colour code used in Fig. 6, panel B. The great majority of objects have low albedos, characteristics of the dark C-type objects that predominate in the outer main belt, but there is a fraction of asteroids that are associated with the Eos and Koronis families, with medium and high values of geometric albedo.

Overall, we confirmed the results of the taxonomical analysis of Mothé-Diniz et al. (2005): the outer main belt is dominated by CX-complex dark asteroids, with the two notable exceptions of the Eos and Koronis families. S-complex asteroids in the local background may be escapers from these two large families. Dynamical studies on the orbital evolution of members of these families are however needed to confirm this conclusion.

### 7 CYBELE GROUP

The Cybele group, usually not considered part of the main belt, is located beyond the 2J:-1A mean motion resonance in semimajor axis and its orbital region is usually defined to lie between 3.27 and 3.70 au in proper \( a \) and to \( i < 30^\circ \). Currently, there are 1111 asteroid in the Cybele orbital region. The largest collisional family in the region is associated with (87) Sylvia, a triple asteroid (Vokrouhlický et al. 2010). The same authors investigated the orbital region of two other large binary asteroids in the region, (107) Camila and (121) Hermione, that are currently not part of any recognizable

| First halo member | \( d_{\text{max}} \) cutoff value (m s\(^{-1}\)) | Number of members | Spectral likely interlopers | Number of SDSS-MOC4 complex | Number of \( p_V \) likely interlopers |
|-------------------|--------------------------------------------|-------------------|-----------------------------|-------------------------------|-----------------------------------|
| (158) Koronis: (761) | 215 | 200 | S | 32 | 14 |
| (18405) 1993 FY12: (29959) | 355 | 2 | CX | 0 | 1 |
| (10811) Lau: (51707) | 490 | 10 | CX | 0 | 0 |
| (709) Fingella: (1357) | 480 | 16 | CX | 0 | 0 |
| (845) Naema: (21257) | 390 | 43 | CX | 0 | 0 |
| (293) Brasilia: (3985) | 440 | 24 | CX | 3 | 17 |
| (24) Themis: (981) | 310 | 700 | CX | 9 | 50 |
| (221) Eos: (320) | 165 | 738 | S | 379 | 132 |
| (10) Hygiea: (867) | 290 | 426 | CX | 6 | 36 |
| (283) Emma: (3369) | 270 | 43 | CX | 0 | 0 |
| (490) Veritas: (5592) | 240 | 148 | CX | 2 | 9 |
| (3556) Lixiaohua: (18477) | 255 | 69 | CX | 1 | 0 |
| (96) Aegle: (29579) | 290 | 21 | CX | 2 | 0 |
| (137) Meliboea: (1165) | 270 | 73 | CX | 3 | 1 |
| (1040) Klumpkea/Tirela: (18399) | 290 | 21 | CX | 2 | 0 |
| (778) Theobalda: (3432) | 310 | 34 | CX | 0 | 2 |
| (781) Kartivelia: (781) | 280 | 26 | CX | 1 | 0 |
| (702) Alauda/Luther: (11911) | 420 | 158 | CX | 2 | 19 |
| (31) Euphrosyne: (16712) | 575 | 75 | CX | 0 | 3 |
Figure 10. Panel A: An \((a, \sin(i))\) projection of outer main belt asteroids in our multivariate sample. Panel B: contour plot of the number density of asteroids in the proper element sample. Superimposed, we display the orbital location of asteroids of families in the CX-complex (plus signs) or in the S-complex (circles).

Figure 11. Panel A: an \((a^*, i - z)\) projection of outer main belt asteroids in our multidomain sample. Panel B: a \((a, \sin(i))\) projection of the same asteroids, where objects in the CX-complex are shown as blue circles, and asteroids in the S-complex are identified as red plus signs.

Figure 12. Panel A: a histogram of number frequency values \(n_i/N_{\text{tot}}\) as a function of geometric albedo \(p_V\) for outer main belt asteroids in our multidomain sample. Panel B: an \((a, \sin(i))\) projection of the same asteroids, where blue full dots are associated with asteroids with \(p_V < 0.1\), red full dots display asteroids with \(0.1 < p_V < 0.3\) and magenta full dots show asteroids with \(p_V > 0.3\).
family. They concluded that, while it is possible that Yarkovsky/YORP driven mobility in the orbital region of these asteroids may have depleted possible local collisional families in time-scales of 4 billion years, other mechanisms, such as resonance sweeping or other perturbing effects associated with the late Jupiter’s inward migration may have been at play in the region in order to justify the current lack of dynamical groups. The AstDyS report three more families in the region of the Cybele group: Huberta, Ulla and (2000) EK76. No halo group was found for the (2000) EK76 AstDyS cluster, which will not therefore be treated in this section. There were 128 asteroids in our multidomain sample in this region, and we will start our analysis by investigating the Sylvia family halo.

7.1 The Sylvia family

The Sylvia family was first identified in Nesvorný et al. (2006) and was the first dynamical group found in the Cybele region. Its dynamical evolution was studied in detail in Vokrouhlický et al. (2010). In this work, we identified a CX-complex halo of 12 members at a cutoff $d_{\text{cut}}$ of 395 m s$^{-1}$. There were no SDSS-MOC4 interlopers, and just one object (7.7 per cent of the total) was identified as a possible albedo interloper.

7.2 The Huberta family

The Huberta family is a group reported at the AstDyS website. We identified a CX-complex halo of four members at a cutoff of 495 m s$^{-1}$. One object (25.0 per cent of the total) was a possible SDSS-MOC4 and albedo interloper.

7.3 The Ulla family

The Ulla family is a very isolated group at relatively high inclination of $\sin(i) \approx 0.3$ and slightly lower than the centre of the $v_6$ secular resonance. It is listed as a dynamical family at the AstDyS website, and we identified a small CX-complex halo of four members for cutoffs larger than 220 m s$^{-1}$. No interlopers were identified in this halo.

7.4 Cybele group: an overview

We summarize our results for the Cybele group in Table 5 that has the same format as similar tables used for the inner, central and outer main belt.

Fig. 13, panel A, displays an ($a$, $\sin(i)$) projection of asteroids in our multivariate sample in the outer main belt, with the same symbols used for analogous figure for the inner, central and outer main belt. Here, however, blue lines show the location of the main linear secular resonances, using the second-order and fourth-degree secular perturbation theory of Milani & Knežević (1994) to compute the proper frequencies $g$ and $s$ for the grid of ($a$, $e$) and ($a$, $\sin(i)$) values shown in Fig. 13, panel A, and the values of angles and eccentricity of (87) Sylvia, the asteroid associated with the largest family in the region (Vokrouhlický et al. 2010). We also display the location of the $z_1$ secular resonance as a red line, since this resonance is important in the dynamical evolution of the Sylvia group. In panel B of the same figure, we display a density map of the outer main belt, according to the approach described in Carruba & Michtchenko (2009). To quantitatively determine the local density of asteroids, we computed the log$_{10}$ of the number of all asteroids with proper elements per unit square in a 67 by 67 grid in $a$ (starting at $a = 3.27$ au, with a step of 0.015 au) and $\sin(i)$ (starting at 0, with a step of 0.015). The other symbols are the same as in Fig. 4, panel B.

To study how the family haloes found in this work are related to the local taxonomy, we also plotted in Fig. 14 a projection in the $(a^*, i - z)$ plane of all asteroids in our multidomain sample (panel A), and an ($a$, $\sin(i)$) projection of the same asteroids, (panel B).

The majority of asteroids in the Cybele region belong to the CX-complex, but there is a sizeable minority of S-complex bodies. This is confirmed by WISE $p_v$ geometrical albedo data, a histogram of which is presented in Fig. 15, panel A. Fig. 15, panel B, displays an ($a$, $\sin(i)$) projection of the same asteroids, with the same colour code used in similar figures for the inner, central and outer main belt. The vast majority of asteroids in the Cybele region are dark objects, typical of CX-complex taxonomy. Overall, the predominance of dark, CX-complex asteroids in the Cybele group, confirms the taxonomical analysis performed by Vokrouhlický et al. (2010). The last region to be analysed in this work will be that of the Hungary asteroid family.

8 THE HUNGARIA REGION

The Hungary region is located at the inner edge of the asteroid main belt (at semimajor axis $a < 2$ au), and it is located at high inclinations and low to moderate eccentricities. The limitations in eccentricity allow for a perihelion large enough to avoid strong interactions with Mars, even considering secular changes in the Mars eccentricity. (Milani et al. 2010). The $v_3$ and $v_5$ secular resonances fix the dynamical limits of the Hungary region in inclination. Only one family has been so far positively identified in the Hungary orbital region, the namesake (434) Hungary group by Milani et al. (2010). Other authors (Cañada-Assandri et al. 2013, private communication) pointed out that the highly inclined Hungary population is dominated by S-type objects, and fairly distinguished from the C-complex population observed in the Hungary dynamical family. But no family in proper elements domain has yet been observed in this highly inclined region. We identified only 37 objects in our multidomain sample, with reasonable errors, and we will start our analysis of the Hungary region by studying the Hungary family halo.

8.1 The Hungary family

The most recent identification of the Hungary family was obtained by Milani et al. (2010), who also found no evidence for other

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Table 5. Asteroid families’ haloes in the Cybele group.

| First halo member | $d_{\text{cut}}$ (m s$^{-1}$) | Number of members | Spectral likely interlopers | Number of SDSS-MOC4 complex | Number of $p_v$ likely interlopers |
|------------------|-----------------------------|-------------------|---------------------------|-----------------------------|-------------------------------|
| (87) Sylvia       | 395                         | 13                | CX                        | 0                           | 1                             |
| (260) Huberta     | 495                         | 4                 | CX                        | 0                           | 1                             |
| (909) Ulla        | > 220                       | 4                 | CX                        | 0                           | 0                             |
Figure 13. Panel A: an $(a, \sin(i))$ projection of Cybele-group asteroids in our multivariate sample. Panel B: contour plot of the number density of asteroids in the proper element sample. Superimposed, we display the orbital location of asteroids of families in the CX-complex (plus signs).

Figure 14. Panel A: an $(a^*, i-z)$ projection of Cybele-group asteroids in our multidomain sample. Panel B: an $(a, \sin(i))$ projection of the same asteroids, where objects in the CX-complex are shown as blue circles, and asteroids in the S-complex are identified as red plus signs.

Figure 15. Panel A: a histogram of number frequency values $n_i/N_{tot}$ as a function of geometric albedo $p_V$ for Cybele-group asteroids in our multidomain sample. Panel B: an $(a, \sin(i))$ projection of the same asteroids, where blue full dots are associated with asteroids with $p_V < 0.1$, red full dots display asteroids with $0.1 < p_V < 0.3$ and magenta full dots show asteroids with $p_V > 0.3$. 
possible dynamical groups in the region (but identified possible sub-structures inside the Hungaria family). Here, we identified a 2-CX-complex halo at a cutoff of 590 m s\(^{-1}\). One object, 50 per cent of the total, was however more compatible with S-complex taxonomy and could therefore be a possible interloper. All asteroids in the halo had \(p_V > 0.1\), which is usually incompatible with a CX-complex taxonomy, but that seems typical of Hungaria objects, as discussed by Warner et al. (2009).

8.2 The highly inclined Hungaria population

No family has been currently identified in the highly inclined (\(i > 0.4\)) Hungaria region in the domain of proper elements. Cañada-Assandri et al. (2013) pointed out that the highly inclined Hungaria region is dominated by S-complex objects, and taxonomically fairly different than the members of the CX-complex Hungaria group. While no family is yet identifiable in the proper element domain in this region, we checked for the presence of a halo, possibly associated with hypothetical local frequency families. We identified a S-complex halo of seven members at a cutoff \(d_{\text{in}} = 655 \text{ m s}^{-1}\), around (2049) Grietje. One object, 14.3 per cent of the total, is a possible SDSS-MOC4 interloper, and there were no albedo interlopers.

8.3 The Hungaria region: an overview

We summarize our results for the Hungaria region in Table 6, which has the same format as similar tables used for the inner, central, outer main belt and Cybele group.

Fig. 16, panel A, displays an \((a, \sin i)\) projection of asteroids in our multivariate sample in the outer main belt, with the same symbols used for analogous figures in the inner, central and outer main belt, and the Cybele region. Here, however, blue lines show the location of the \(v_3\) linear secular resonances, using the second-order and fourth-degree secular perturbation theory of Milani & Knežević (1994) to compute the proper frequencies \(g\) and \(s\) for the grid of \((a, e, i, c)\) and \((a, \sin i)\) values shown in Fig. 13, panel A, and the values of angles and eccentricity of (434) Hungaria, the asteroid associated with the largest family in the region. We also display the location of the \(v_5\), \(v_3\) (blue lines), and \(v_{14}\) (red line) secular resonances, since this resonance is important in setting dynamical boundaries in the region. The orbital position in the \((a, \sin i)\) plane of the first numbered asteroid in all the Hungaria groups is also identified in Fig. 13, panel A. In panel B of the same figure, we display a density map of the outer main belt, according to the approach described in Carruba & Michtchenko (2009).

To quantitatively determine the local density of asteroids, we computed the \(\log_{10}\) of the number of all asteroids with proper elements per unit square in a 15 by 24 grid in \(a\) (starting at \(a = 1.8\) au, with a step of 0.015 au) and \(\sin i\) (starting at 0.25, with a step of 0.015). The other symbols are the same as in Fig. 4, panel B.

To study how the family haloes found in this work are related to the local taxonomy, we also plotted in Fig. 17 a projection in the \((a^*, i - z)\) plane of all asteroids in our multidomain sample (panel A), and an \((a, \sin i)\) projection of the same asteroids (panel B). The majority of asteroids in the Hungaria region in our sample belong to the S-complex, but there is a sizeable minority of CX-complex bodies.

The analysis of WISE \(p_V\) geometrical albedo data, a histogram of which is presented in Fig. 18, panel A, show some peculiarities. Fig. 18, panel B, displays an \((a, \sin i)\) projection of the same asteroids, with the same colour code used in similar figures for the inner, central and outer main belt. The vast majority of asteroids in the Hungaria region and the Hungaria family are very bright objects, typical of S-complex taxonomy. The fact that many asteroids in the Hungaria family show high albedo and CX-taxonomy remains yet to be explained.

Table 6. Asteroid families’ haloes in the Hungaria region.

| First halo member | \(d_{\text{in}}\) cutoff value (m s\(^{-1}\)) | Number of members | Spectral likely interlopers | Number of SDSS-MOC4 complex | Number of \(p_V\) likely interlopers |
|-------------------|----------------------------------------|------------------|-----------------------------|-----------------------------|-------------------------------|
| (434) Hungaria: (5968) | 590 | 2 | CX | 1 | 2 |
| (2049) Grietje: (3043) | 655 | 7 | S | 1 | 0 |

Figure 16. Panel A: an \((a, \sin i)\) projection of Hungaria-region asteroids in our multivariate sample. Panel B: contour plot of the number density of asteroids of families in the CX-complex (plus signs) and S-complex (circles).
9 CONCLUSIONS

In this work, we:

(i) Introduced a new method to obtain asteroid families and asteroid family haloes based on a distance metric in a multidomain composed of proper elements, SDSS-MOC4 \((a^*, i - z)\) colours and \textit{WISE} geometrical albedo \(p_V\).

(ii) Compared this new distance metric with other distance metrics in domain of proper elements, proper elements and SDSS-MOC4 colours, and proper elements and geometric albedo. The method is at best a factor of 2 more efficient in eliminating interlopers than other methods, and at worst it provides comparable results to groups found in domains of proper elements and SDSS-MOC4 colours only.

(iii) Applied this method to all the major known families in the asteroids’ main belt, and in the Cybele and Hungaria orbital regions. Overall, we identified 62 asteroid families’ haloes, of which 7 were in the inner main belt, 31 in the central main belt, 19 in the outer main belt, 3 in the Cybele group and 2 in the Hungaria region. We confirm the taxonomical analysis performed by Mothé-Diniz et al. (2005), Nesvorný et al. (2006), Carruba (2009a,b, 2010a,b) and other authors, with some small discrepancies for a few minor families in the central main belt.

Overall, apart from a few problematic cases such as the Eos family, our method appears to provide robust results in terms of asteroid family identification and in efficiency in eliminating interlopers from the clusters. While the sample of objects with data in all three domains is still limited, we believe that such an approach may be certainly more reliable than traditional HCM in identifying possible collisional groups. The possible future increase in the number of asteroids for which data in all three domains will be available, for instance because of the \textit{GAIA} mission, may provide in the future data bases for asteroid family identification much larger than the one used in this work.

Many other applications of this new approach are possible with current data bases. An analysis of asteroid families in domains of proper frequencies such as \((n, g, g + s)\) (Carruba & Michtchenko 2007, 2009), where \(g\) is the precession frequency of the longitude of pericentre, and \(s\) the precession frequency of the longitude of the

Figure 17. Panel A: an \((a^*, i - z)\) projection of Hungaria-region asteroids in our multidomain sample. Panel B: an \((a, \sin(i))\) projection of the same asteroids, where objects in the CX-complex are shown as blue circles and asteroids in the S-complex are identified as red plus signs.

Figure 18. Panel A: a histogram of number frequency values \(n_i/N_{Tot}\) as a function of geometric albedo \(p_V\) for Hungaria-region asteroids in our multidomain sample. Panel B: an \((a, \sin(i))\) projection of the same asteroids, where blue full dots are associated with asteroids with \(p_V < 0.1\), red full dots display asteroids with \(0.1 < p_V < 0.3\), and magenta full dots show asteroids with \(p_V > 0.3\).
node, SDSS-MOC4 colours and WISE albedo may provide useful insights on the secular evolution of asteroid families. Many exciting years of discoveries are still open, in our opinion, in the field of asteroid dynamics.

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