From quantum fusiliers to high-performance networks

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Our objective was to design a quantum repeater capable of achieving one million entangled pairs per second over a distance of 1000km. We failed, but not by much. In this letter we will describe the series of developments that permitted us to approach our goal. We will describe a mechanism that permits the creation of entanglement between two qubits, connected by fibre, with probability arbitrarily close to one and in constant time. This mechanism may be extended to ensure that the entanglement has high fidelity without compromising these properties. Finally, we describe how this may be used to construct a quantum repeater that is capable of creating a linear quantum network connecting two distant qubits with high fidelity. The creation rate is shown to be a function of the maximum distance between two adjacent quantum repeaters.

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INTRODUCTION

The twentieth century saw the discovery of quantum mechanics, a set of principles describing physical reality at the atomic level of matter. These principles have been used to develop much of today’s advanced technology including, for example, today’s microprocessors. Quantum physics also allows a new paradigm for the processing of information — a field known as quantum information processing [1, 2]. Over the last decade we have seen a huge worldwide effort to develop and explore quantum-information based devices and technologies [3, 4]. Quantum key distribution (QKD) enabled devices are already commercially available [4]. The next step after this is likely to be small scale processors, probably distributed in nature.

Quantum repeaters are a natural candidate to consider [4]. Their role is to enable the creation of entangled states between remote locations. Long-distance entanglement is achieved by placing a number of repeater nodes in-between two end points and creating entangled links between the adjacent nodes. Once a node has links both to the left and to the right, entanglement swapping within the nodes then allows longer-range entangled links to be formed. Once swapping operations have occurred at all the intermediate nodes an end-to-end entangled link will have been formed. These entangled pairs can then be used in QKD, quantum communication, or distributed quantum computation.

The current goal of many research groups is to produce a stream of entangled qubits over long distances, preferably with rates in the MHz range. There have been many proposals for how this could be achieved and a number of "in-principle" demonstrations have been performed. Such proposals have generally focused on the quantum components necessary to create entangled links between neighboring nodes, purification of these links, and swap operations to create longer-distance links [3, 8, 10, 11, 12, 13]. The entangled links are generally created by entangling an optical signal (appendix 1) with a qubit and then transmitting that signal over a channel to the neighboring node. Here the signal entangles with a qubit within that node and then a measurement is made on the quantum signal indicating successful generation or not [6, 7]. The probability of successfully generating the link scales at best as exp[−L/L0], where L is the distance between repeater nodes and L0 the attenuation length of the fiber.

The next step is to look at the overall design of the repeater network, in terms of both the quantum and classical components. The communication time for classical messages to be transmitted between nodes severely limits the performance of a repeater network. Messages generally need to be sent between nodes in all of the three key quantum stages of a repeater network: entanglement distribution, purification, and swapping. In this letter we will describe a pipe-lined architecture where one knows when the end-to-end entangled pairs are going to be available.

QUANTUM FUSILIERS AND FUSILANDS

The major issue affecting the performance for a quantum repeater is the probabilistic nature of the generation of entanglement between adjacent nodes and not knowing when such a link is going to be available. This issue means that a confirmation signal needs to be sent back from the receiver to the transmitter side and so the generation rate is ultimately limited by this round trip transmission time. With typical repeater nodes being separated by, say, 40km this would take on the order of 400 μs. Now with the probability of success for entanglement generation being below 25%, quite a number of attempts are going to be needed before we are "guaranteed" a link. A significant time delay results if the attempts are per-
formed sequentially. One could parallelize the operations but with significantly more resources. One must be able to do better!

A simpler design does indeed exist which we depict in Fig. (1). In this design each repeater node comprises two fundamental parts: a quantum fusillade containing multiple fusiliers (transmitters) and quantum fusilands (receivers). There are generally more fusiliers than fusilands and for the moment we will consider a single fusiland. The creation of a constant-time entanglement link begins by a classical pulse initiating all the fusiliers in that node to prepare individual quantum optical signals. These signals then interact and become entangled with the qubits in their respective fusilier cavities. The signals then propagate, temporally multiplexed together with the classical heralding pulse, along the fiber to the fusiland in the next repeater node. The classical pulse announces to the fusiland that a series of quantum signals are about to arrive and so the fusiland initializes the qubit into the appropriate state and then interacts with the first fusilier’s quantum signal. The signal is then measured to determine whether a successful entanglement-creation operation has occurred. If not, the fusiland qubit is re-prepared for the arrival of the second fusilier’s signal and the same interaction/measurement procedure is performed. This continues until a success is reported. A successful result triggers two operations: first, it stops any further signals interacting with the fusiland; and secondly, it dispatches a classical message back to the fusiland informing it of which fusilier was successful. The time taken from firing the fusillade to receiving the classical message is essentially the round trip time between two adjacent repeaters and is a constant. With enough fusiliers we can “guarantee” the entangled link exists. The failure probability is given by \( p_f = (1 - p)^n \), where \( n \) is the number of fusiliers and \( p \) is the success probability of a single fusilier/fusiland. With \( p = 0.25 \), 16 fusiliers are needed for \( p_f < 0.01 \).

With only one receiving fusiland we have to discard any further fusilier signals once a measurement has been successful. This is obviously a waste, but we could utilize a few extra fusilands. Once the first fusiland has been successfully entangled, we then route the remaining signals to the next fusiland. When that’s successful, we then go on to the next and so on. If we have \( n \) fusiliers and \( m \) fusilands then the probability that all \( m \) links have not been established is \( p_f(m) = \sum_{j=1}^{m} \binom{n}{j-1} p^{j-1} (1 - p)^{n-j+1} \). For \( p_f = 0.01 \) and \( p = 0.25 \), the numbers of fusiliers/fusilands needed are \((n=16, m=1)\), \((n=24, m=2)\), \((n=70, m=10)\), \((n=485, m=100)\) which in the asymptotic limit of large \( m \) goes to \((n=m/p, m)\). This clearly shows the advantage of having multiple fusilands in terms of resource efficiency.

With multiple entanglement links available between adjacent nodes, there are various possibilities for how these can be used. The simplest is just to use them in parallel to improve the overall network performance, however as our entangled links may not be perfect we need to be able to purify them. Normal purification protocols are problematic since they are probabilistic and require two-way communication to determine if we have succeeded or failed \([14, 15]\). Upon failure our entangled links are destroyed and we must start the link generation again. This is a major performance issue but it can be solved by using quantum error correction \([16]\).

The particular error-correction code to be used will depend on the errors induced in the entanglement generation process and on the failure rate of the quantum gates at each node. If we assume perfect local gates and that the predominant channel error (excluding loss) is a bit-flip error, then our entangled link can be represented by

\[
\rho[F] = F \frac{1}{2} |gg + ee\rangle\langle gg + ee| + \frac{1}{2} |ge + eg\rangle\langle eg + ge|,
\]

where \( F \) measures the fidelity (quality) of the entangled pairs one is trying to create. In this case, to create an entangled pair with fidelity \( F' > F \) we make use of a three-qubit repetition code, which corrects a single bit-flip error as follows: Given three copies of \( \rho(F) \) we perform non-destructive parity measurements on the first and second and then on the second and third fusiliers, recording the results \( p_{12} \) and \( p_{23} \). The second and third fusiliers are then measured out in the \( X \) basis. On the fusiland side identical parity measurements are performed with results, say, \( r_{12} \) and \( r_{23} \) and then the second/third fusilands are measured out in the \( X \) basis. The resulting entangled state is \( \rho[F'] = F^3 + 3F^2(1-F) \) up to a bit-flip correction determined by \( p_{12} \), \( p_{23} \), \( r_{12} \), and \( r_{23} \) and a phase-flip correction determined by the results of the four \( X \) measurements. These corrections simply update the Pauli frame and need only be communicated to one end of the network. This means we do not need to wait and so the fusiliers and fusilands can be further processed.

This simple protocol is quite effective at increasing the

![Fig. 1: Schematic representation of a quantum repeater node and its link to its next neighbor. The basic repeater is composed of two fundamental components: a quantum fusillade containing multiple fusiliers (transmission cavities each with a qubit within them) and a quantum fusiland (receiving cavity with a qubit within it and a signal detector).](image-url)
fidelity of the remaining pair relative to that of the initial pairs; for instance if we started with $F = 0.99$ we would have $F' \geq 0.99$. Importantly, the non-determinism inherent in purification-based schemes is not present in this scheme, allowing for pipe-lining of the overall repeater network. To extend entanglement beyond neighboring nodes we perform swap operations (achieved by parity gates) between local fusiliers and fusilands when the local fusiler is entangled to the right and the fusiland to the left. This removes those local fusiliers and fusilands and creates a longer range link. After the swap operation the quality of the new link is likely to have degraded and so more error correction may be required.

In the case of a general channel error and faulty local gates, to achieve fault tolerance whilst keeping with the spirit our design we can simply replace physical qubits with logical qubits encoded with a concatenated code such as the Bacon-Shor code [17]. Then error correction is performed at the same time as entanglement swapping without any need for additional protocols [18]. This contrasts with other recent schemes based on planar codes and cluster states [19, 20, 21]. Since logical Bell pairs are required to perform error correction, one promising approach is to produce many logical Bell pairs at each node, rejecting pairs when errors are detected, so that a high-quality pair is always available when required [22]. We expect that this will yield a scheme which has a high threshold (> 1%) for channel and gate errors whilst retaining the deterministic nature of the protocol. As with all error-correction schemes, the maximum error rate that is tolerable will depend ultimately on the number of entangled links we have available, the number of qubits at each node, and our target fidelity. However, we are confident that our method for establishing entanglement between repeater nodes gives us the flexibility to tailor error correction to communication tasks to ensure high fidelity entanglement with a practical amount of resources.

### A QUASI-ASYNCHRONOUS DESIGN

With all the quantum components available we now need to consider appropriate strategies for putting this network together and how it will operate. This will need to involve both the quantum resources and the classical communication resources. The two logical choices for how such a network could operate are basically either a synchronous or asynchronous scheme. A synchronous scheme requires all the individual repeater nodes to have a shared clock which in certain circumstances could be challenging. An asynchronous design does not require this and so it is the design we will focus on here. We depict such a scheme in Fig. 2 and note that an advantage of this design is that the distance between adjacent nodes need not be the same (some could be at 10km say, others at 40km).

![FIG. 2: Schematic representation of a quasi-asynchronous repeater network. Entanglement generation is initiated on the left-hand side (LHS) where the system clock is located. In this design the classical heralding pulse from the left-hand most node propagates to the furthermost right-hand node (RHS) initiating all the fusiliers as it propagates. Swap operations within local nodes occur when the local fusiliers and fusilands have links to their neighbors. The left-hand node can start its next entanglement generation cycle after the round trip time for an entanglement generation between neighboring nodes. The classical heralding pulse on this next round picks up the Pauli frame information as it propagates through the network and makes it available to the right end node as it arrives.](image)

The quasi-asynchronous design begins with the clock in the left hand network node initiating the classical heralding pulse that is going to propagate along the whole network from left to right. As it goes it will initiate the fusiliers to fire the signals to the fusiland in the adjacent node and thus we will see the fusiliers firing in temporal progression from the left hand side of the network to the right hand side. Each of the adjacent nodes reports by a classical message which fusilands were successful and when that node has a link both to the left and the right the entanglement swap operation is performed, creating a longer distance link and freeing the fusiliers and fusilands in that node for future operations. The results of the parity measurements and swap operations are then available at that local node. We propagate this information on the heralding pulse for the next round of long-range entanglement generation. It is important that the next heralding pulse arrives at the repeater nodes after the swapping operations have been performed as the herald will pick this information up. It also means we know exactly when the entanglement link is ready to use and so we have an efficient pipe-lined design.

### A BUTTERFLY DESIGN

As the entanglement generation is effectively flowing from left to right, the left-hand fusiler and the right-hand fusiland become entangled at quite different times. For QKD-like applications this is not an issue. For computational applications this could be an issue, but a simple
solution is to split the network into two halves. The actual location of the split depends on the topology of the network, but is chosen to maximize throughput and to balance the availability of the left and right qubits. Each side is going to see a generalized parity for its half of the network. The two halves can be simply connected by entanglement swapping and this information propagated to either the left- or right-hand end with the next heralding pulse. It also means these resources in the "central" node are freed relatively quickly and consequently we do not need exceptionally long lived qubits anywhere in the repeater network. This should significantly lessen the technological challenge inherent in distributed quantum information processing as the quantum resources now have to be good on time scales associated with the round trip time between adjacent nodes and not the propagation time over the whole network.

DISCUSSION

We have so far presented a highly optimized design for a quantum repeater and its associated use in a network where the key element is the construction of a constant-time, near-deterministic, high fidelity entanglement link generator between neighboring repeater nodes. This time is of the order of the round trip time between adjacent nodes, approximately 0.4ms for a 40km link (0.1ms for a 10km link) and so allows a maximum rate/fusiland of 2500 (10000) entangled pairs between adjacent nodes. With more fusilands per repeater node one can approach a MHz rate. By utilizing oneway error correction the near-deterministic nature can be maintained without any significant time cost. Finally by utilizing a butterfly network design the end nodes becomes entangled at roughly the same time with the classical generalized parity results arriving one cycle (round trip) later. This allows a highly efficient and pipe-lined architecture. While we have considered a linear design the network topology can be easily generalized.

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APPENDIX - ENTANGLEMENT LINKS

One of the core elements necessary in any repeater design is the creation of entanglement between nearest neighbor links. This entanglement will be created between two electronic spins placed in cavities at neighboring repeater stations with nuclear spins available for quantum storage. The electronic and nuclear-spin systems may be achieved, for example, by single electrons trapped in quantum dots, by neutral donor impurities in semiconductors or NV diamond centers. For a sufficient interaction between the electron and the light field, the system should be placed in a cavity resonant with the light. The mechanism for the entanglement between nodes generally fall into two categories.

- The heralded creation of very high fidelity entan-
gled links utilising single photon or weak coherent sources generally with a low probability of success. The qubit-light field can operate in a number of regimes including on-resonance and dispersive. Moderate to strong coupling regimes are generally required.

• The creation of moderate fidelity entangled links utilising strong coherent fields and homodyne detection generally with a moderate to high probability of success. The qubit-light field generally operate in the dispersive regime.

Which is better to use really depends on the physical system but the second approach can use the same qubit-photon interacts for the local gate operations necessary in purification and entanglement swapping.