Transverse Momentum Distributions in p-Pb collisions and Tsallis Thermodynamics.

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Abstract. The transverse momentum distributions of charged particles in p-Pb collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV measured by the ALICE collaboration are fitted using Tsallis statistics. The use of a thermodynamically consistent form of this distribution leads to an excellent description of the transverse momentum distributions for all rapidity intervals. The values of the Tsallis parameter \( q \), the temperature \( T \) and the radius \( R \) of the system do not change within the measured pseudorapidity interval.

1. Introduction
It is by now standard to parameterize transverse momentum distributions with functions having a power law behavior at high momenta. This has been done by the STAR \[1\] and PHENIX \[2,3\] collaborations at RHIC and by the ALICE \[4,5\], ATLAS \[6\] and CMS \[7,8\] collaborations at the LHC using a form of the Tsallis \[9\] distribution. This distribution was proposed as a generalization of the Boltzmann-Gibbs distribution and introduces a new parameter \( q \). In the limit \( q = 1 \) it coincides with the latter. It has been shown repeatedly that the Tsallis distribution gives an excellent description of \( p-p \) collisions \[10,11,12\] resulting in a value for the \( q \) parameter which is around 1.1 - 1.2. In this paper we extend this analysis to transverse momentum spectra obtained in \( p-Pb \) collisions at \( \sqrt{s_{NN}} = 5.02 \) TeV by the ALICE collaboration \[13\]. We conclude that the Tsallis distribution gives an excellent fit to \( p-Pb \) collisions with values for \( q \) and \( R \) which are consistent with those obtained in \( p-p \) collisions but with a temperature which is clearly higher, around 112 MeV.

2. Tsallis Distribution
In the framework of Tsallis statistics \[10,9,14,15,16\] the entropy \( S \), the particle number, \( N \), the energy density \( \epsilon \) and the pressure \( P \) are given by integrals over the Tsallis distribution:

\[ f = \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{-\frac{1}{q-1}}. \]  

(1)
It can be shown (see e.g. [16]) that the relevant thermodynamic quantities are given by:

\[
S = -gV \int \frac{d^3p}{(2\pi)^3} [\text{\[1\]} \ln_q f - f], \tag{2}
\]

\[
N = gV \int \frac{d^3p}{(2\pi)^3} f^q, \tag{3}
\]

\[
\epsilon = g \int \frac{d^3p}{(2\pi)^3} Ef^q, \tag{4}
\]

\[
P = g \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{3E} f^q. \tag{5}
\]

where \(T\) and \(\mu\) are the temperature and the chemical potential, \(V\) is the volume and \(g\) is the degeneracy factor. We have used the short-hand notation

\[
\ln_q(x) \equiv \frac{x^{1-q} - 1}{1 - q}, \tag{6}
\]

often referred to as q-logarithm. It is straightforward to show that the relation

\[
\epsilon + P = Ts + \mu n \tag{7}
\]

(where \(n, s\) refer to the densities of the corresponding quantities) is satisfied. Thermodynamics gives rise to the following differential relations:

\[
de \epsilon = T ds + \mu dn, \tag{8}
\]

\[
dP = s dT + n d\mu. \tag{9}
\]

Since these are total differentials, thermodynamic consistency requires the following Maxwell relations to be satisfied:

\[
T = \left. \frac{\partial \epsilon}{\partial s} \right|_n, \tag{10}
\]

\[
\mu = \left. \frac{\partial \epsilon}{\partial n} \right|_s, \tag{11}
\]

\[
N = V \left. \frac{\partial P}{\partial \mu} \right|_T, \tag{12}
\]

\[
S = V \left. \frac{\partial P}{\partial T} \right|_\mu. \tag{13}
\]

It can be easily verified that this is indeed the case.

Following from Eq. (3), the momentum distribution is given by:

\[
\frac{d^3N}{d^3p} = gV \left[ 1 + (q - 1) \frac{E - \mu}{T} \right]^{-q/(q-1)}, \tag{14}
\]

or, expressed in terms of transverse momentum, \(p_T\), transverse mass, \(m_T \equiv \sqrt{p_T^2 + m^2}\), and rapidity \(y\)

\[
\frac{d^2N}{dp_T dy} = gV p_T m_T \cosh y \left[ 1 + (q - 1) \frac{m_T \cosh y - \mu}{T} \right]^{-q/(q-1)}. \tag{15}
\]
Table 1. Values of the Tsallis parameters for \( p - Pb \) collisions for various intervals of the pseudorapidity.

| Pseudorapidity Interval | q          | T (MeV)      | Radius (fm) |
|-------------------------|------------|--------------|-------------|
| \(-0.3 < \eta < 0.3\)   | 1.140 ± 0.001 | 112.81 ± 2.35 | 4.03 ± 0.12 |
| \(0.3 < \eta < 0.8\)    | 1.139 ± 0.001 | 113.35 ± 2.47 | 4.07 ± 0.13 |
| \(0.8 < \eta < 1.3\)    | 1.139 ± 0.001 | 111.92 ± 2.63 | 4.19 ± 0.14 |

At mid-rapidity, \( y = 0 \), and for zero chemical potential, as is relevant at the LHC, this reduces to:

\[
d^2N \left. \frac{d^2N_{\text{charged}}}{dp_T\,dy} \right|_{y=0} = gV p_T mt \left[ 1 + (q - 1) \frac{mt}{T} \right]^{-q/(q-1)}. \tag{16}
\]

In the limit where the parameter \( q \) goes to 1 it is well-known that this reduces to the standard Boltzmann distribution [9].

The parameterization given in Eq. (15) is close to the one used by various collaborations [1, 2, 4, 5, 6, 7, 8]. The differences have been spelled out explicitly in [17] and will not be repeated here.

3. Details of Transverse Momentum Distributions

The transverse momentum distributions of charged particles in \( p - Pb \) collisions were fitted using a sum of three Tsallis distributions, the first one for \( \pi^+ \), the second one for \( K^+ \) and the third one for protons, \( p \). The relative weights between these were determined by the corresponding degeneracy factors, i.e. 1 for \( \pi^+ \) and \( K^+ \) and 2 for protons. The fit was taken at mid-rapidity and for \( \mu = 0 \) using the following expression:

\[
\frac{1}{2\pi p_T} \frac{d^2N_{\text{charged}}}{dp_T\,dy} \Bigg|_{y=0} = \frac{2V}{(2\pi)^3} \sum_{i=1}^{3} g_i m_{T,i} \left[ 1 + (q - 1) \frac{m_{T,i}}{T} \right]^{-q/(q-1)}, \tag{17}
\]

where \( i = (\pi^+, K^+, p) \) and \( g_\pi = 1, g_K = 1 \) and \( g_p = 2 \). The factor 2 in front of the right hand side of this equation takes into account the contributions of the antiparticles \( (\pi^-, K^-, \bar{p}) \).

The Tsallis distribution describes the transverse momentum distributions of charged particles in \( p - Pb \) collisions as obtained by the ALICE collaboration [13] at \( \sqrt{s_{NN}} = 5.02 \) TeV in all pseudorapidity intervals remarkably well as shown in Fig. 1.

The Tsallis parameter \( q \) needed to describe the transverse momentum distributions of charged particles is shown in Fig. 2. It is identical to the value obtained in \( p - p \) collisions [17].

The Tsallis parameter \( T \) needed to describe the transverse momentum distributions of charged particles is shown in Fig. 3. It is larger than the value obtained in \( p - p \) collisions [17]. In order to clarify this it would be helpful to have data at lower transverse momentum as the distribution is very sensitive to this region.

The radius \( R \) is given by:

\[
R \equiv \left( \frac{3V}{4\pi} \right)^{1/3}, \tag{18}
\]

The values of \( R \) are shown in Fig. 4 and are consistent (albeit slightly larger) than those deduced from \( p - p \) collisions [17]. No noticeable dependence on pseudorapidity can be seen which could be due to the fact that the pseudorapidity region covered is always fairly central.
4. Conclusions
In conclusion, the Tsallis distribution, Eq. (16), leads to an excellent description of the transverse momentum distributions in high energy $p − Pb$ collisions as measured by the ALICE collaboration [13] at $\sqrt{s_{NN}} = 5.02$ TeV. Comparing the results to those obtained in $p − p$ collisions it can be noted that the values of $q$ are compatible, the values of $R$ are marginally larger while $T$ is substantially higher. The values of $T$ and $R$ are very sensitive to the low $p_T$ part of the transverse momentum distribution and extending the measurements to lower $p_T$ could bring much clarification here.

The values obtained for the Tsallis parameter $q$, the temperature $T$ and the radius $R$ are consistent over all pseudorapidity intervals, a feature which does not become apparent when using other forms of parameterization.

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Figure 1. Fits to transverse momentum distributions of charged particles in \( p - Pb \) collisions at \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \) measured by the ALICE collaboration \[13\] using the Tsallis distribution.

Figure 2. Values of the Tsallis parameter \( q \), as a function of pseudorapidity interval, obtained from fits to the transverse momentum distributions of charged particles obtained by the ALICE collaboration \[13\] in \( p - Pb \) collisions at \( \sqrt{s_{NN}} = 5.02 \text{ TeV} \).
Figure 3. Values of the Tsallis parameter, $T$, as a function of pseudorapidity interval, obtained from fits to the transverse momentum distributions of charged particles obtained by the ALICE collaboration [13] in $p-Pb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV.

Figure 4. Values of the Tsallis parameters $R$, as a function of pseudorapidity interval, obtained from fits to the transverse momentum distributions of charged particles obtained by the ALICE collaboration [13] in $p-Pb$ collisions at $\sqrt{s_{NN}} = 5.02$ TeV.