The Monostatic Radar and Jammer Games Based on Signal-to-Jamming-plus-Noise Ratio

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ABSTRACT In modern electronic warfare, both the radar and the jammer are capable of changing their waveforms to achieve better performance. The conflict relationship between the radar and jammer can be modeled as a non-cooperative game. In this paper, the optimal strategies of a monostatic radar and a jammer are investigated in terms of both Stackelberg game and symmetric game. Instead of using the widely used mutual information as the utility function, signal-to-jamming-plus-noise ratio (SJNR) is used in formulating the utility functions which is directly related to the target detection performance. In Stackelberg game, the Stackelberg equilibria solutions are derived when the leader is the radar and the jammer, respectively. In symmetric game, the existence of Nash equilibrium (NE) is investigated and the corresponding strategies are obtained when Nash equilibrium exists. If Nash equilibrium does not exist, the Stackelberg equilibria strategies are still useful as they can be regarded as the robust strategies which could optimize the worst case performance. The relationship between the mutual information utility function and the proposed SJNR utility function is investigated as well. Simulation results show the effectiveness of the radar and jammer strategies. The investigation in this paper can provide optimal waveform solutions for the monostatic radar and the jammer in the confrontation scenario.

INDEX TERMS equilibrium, Stackelberg game, symmetric game, signal-to-jamming-plus-noise ratio (SJNR).

I. INTRODUCTION

GAME theory is a branch of mathematics that models and analyzes the interaction of decision-makers, called players, under the assumptions of the rationality of players and the strategic interdependence. Each player aims to maximize its utility as a best response to the actions of the other players [1]. With the development of high speed signal processing hardware and various machine learning algorithms, the radar and the jammer are both becoming more adaptive and intelligent. Cognitive radar [2] could sense the environment and make optimal decisions such as changing its waveform or allocating its resources to achieve better performance, so does the cognitive jammer [3]. In the scenario where both a cognitive radar and a cognitive jammer exist, game theoretic methods are appropriate to model the interaction of them and finding the state, called equilibrium, where the performance is maximized for the radar and jammer from their own perspective.

Various game theoretic approaches have been proposed to analyze the strategies of the smart radar and the jammer. For example, the interactive of a radar and a missile has been examined through differential games [4]. The radar aims to minimize the uncertainty of the missile’s position by changing the filter gain, while the missile tries to maximize this uncertainty. In [5], a zero-sum game is modeled where the radar tries to change its polarization strategies to detect the target, while the target uses different types of aerial vehicles to avoid being detected. In [6], a two-person-zero-sum (TPZS) game is modelled. The effects of jamming on target detection performance of the radar with a constant false alarm rate processor are investigated. In [7], the interaction between a smart target and a smart MIMO radar is modeled using unilateral game, hierarchical game and symmetric game. Mutual information is used as the utility function and
the equilibria solutions are derived. In [8], a Bayesian game is considered for a statistical MIMO radar and a jammer using mutual information as the utility function. The target information is only known by the jammer while the receiver noise information is only known by the MIMO radar. In [9], the MIMO radar and jammer Stackelberg game based on mutual information is considered in the presence of clutter. In [10], the two players are the monostatic radar and the jammer. Mutual information is still used as the utility function.

For the purpose of low probability of interception, the radar transmission power is usually minimized. In [11], the power allocation game between a MIMO radar and multiple jammers are investigated. The objective of the radar is to minimize the total power while ensuring that the signal-to-interference-plus-noise ratio (SINR) is greater than the detection threshold. The jammers aim at minimizing the SINR to decrease the radar performance. SINR is also used as the utility for the resource allocation in radar networks to avoid the interference from other radars [12]–[14]. In communications, the game between cognitive radio and jammer has been investigated. Capacity is used in [15] which has the similar expression as mutual information. SINR is used in [16] as the utility for wireless communication networks.

From the state-of-the-art literatures mentioned above, it is easy to conclude that utility function is one of the most import issues in game model as it defines what the rational player is interested in or concerned about. Among those criteria, mutual information has acquired extensive attention which is an information theoretic criterion that was firstly proposed by Bell [17] in optimal waveform design. Mutual information between the received echo and the target impulse response is regarded as a way of evaluating whether the radar has a better capacity in characterizing the target in a contaminated environment. Larger mutual information indicates better target estimation performance. Also note that SINR is another import criteria which is related to the target detection performance.

In this paper, considering the target detection performance, the signal-to-jamming-plus-noise ratio (SJNR) is used in formulating the utility functions. The two players are the monostatic radar and the jammer. The monostatic radar aims at maximizing the SJNR to achieve better detection performance of the target, while the jammer intends to minimizing the SJNR to protect the target from being detected.

The main contributions are as follows. Firstly, the TPZS game is modeled using SJNR as the utility function between the monostatic radar and the jammer. In most of the recent game theoretic methods for radar and jammer strategies design, mutual information is widely used. We focus on the target detection task in this paper. So SJNR is considered. Secondly, the Stackelberg equilibria strategies are obtained when the game leader is the radar and the jammer, respectively. The existence of Nash equilibrium (NE) is analyzed. Thirdly, the comparisons between the widely used mutual information criterion and the proposed SJNR criterion are made.

The paper is organized as follows. In section I, the related works are summarized. In Section II, the signal model of a monostatic radar detecting a target is given as well the the game theoretic formulation. In Section III, Stackelberg game is used to find out the optimal radar and jammer strategies at the equilibrium. In Section IV, the problem of symmetric game is investigated. It is shown that Nash equilibrium does not always exist. Simulations are given in Section V. In Section VI, the comparisons between the SJNR utility and the mutual information utility are shown. Finally, conclusions and future works are given in Section VII.

II. PROBLEM FORMULATION

A. SIGNAL MODEL

The signal model is depicted in Fig. 1. $x(t)$ is the complex-valued baseband signal of the transmit waveform of the radar. The target is a deterministic extended target. $h(t)$ is the complex-valued baseband target impulse response. Let $X(f)$ and $H(f)$ denote the Fourier transforms of $x(t)$ and $h(t)$, respectively. Let $r(t)$ denote the complex-valued receiver filter impulse response and $n(t)$ be a complex-valued, zero-mean channel noise process with the power spectral density (PSD) $S_{nn}(f)$. $j(t)$ denotes the barrage jamming transmitted by the jammer, which is a complex-valued, zero-mean Guassian random process. The PSD of $j(t)$ is $J(f)$. The variables in boldface letters denote random processes whereas others are deterministic. The received signal at the radar receiver is

$$y(t) = r(t) * (x(t) * h(t) + j(t) + n(t)),$$  (1)

where

$$y_s(t) = r(t) * x(t) * h(t)$$  (2)

is the signal component,

$$y_n(t) = r(t) * (j(t) + n(t))$$  (3)

is the jamming and noise component. So the SJNR at $t_0$ is

$$SJNR_{t_0} = \frac{\|y_s(t_0)\|^2}{E[|y_n(t_0)|^2]},$$  (4)

The corresponding expression of SJNR in frequency domain is

$$SJNR_{t_0} = \frac{\int_{-\infty}^{\infty} R(f)H(f)X(f)e^{j2\pi ft_0}df^2}{\int_{-\infty}^{\infty} |R(f)|^2 [J(f) + S_{nn}(f)]df}.$$  (5)
According to Schwartz inequality, if and only if the receiver filter is
\[ R(f) = \frac{[kH(f)X(f)e^{j2\pi f t_0}]^*}{J(f) + S_{nn}(f)} , \] (6)
SJNR achieves the maximum value, i.e.,
\[ (SJNR)_{t_0} = \int_{-\infty}^{\infty} \frac{|H(f)X(f)|^2}{J(f) + S_{nn}(f)} df. \] (7)
Suppose that the transmitted signal and jamming PSD are essentially limited to the frequency band of BW. Then, the approximated SJNR at the output of the radar receiver can be denoted by
\[ SJNR\left(\left|X(f)\right|^2, J(f)\right) \approx \int_{BW} \frac{|X(f)H(f)|^2}{J(f) + S_{nn}(f)} df. \] (8)
Strictly speaking, \( x(t) \) is not limited in the frequency domain but the approximate bandwidth BW can be acquired by confining the majority of the signal energy to that frequency interval. From (8), the SJNR is related with the radar waveform \( X(f) \), the jamming and noise PSD \( J(f) \) and \( S_{nn}(f) \), as well as the target frequency response \( H(f) \).

The target detection performance is closely related with the SJNR at the radar receiver. Greater SJNR implies better target detection probability. In this paper, we will use SJNR as the utility function.

**B. GAME THEORY BACKGROUND**

In game theory, zero-sum game is a kind of competitive game and the sum of the outcomes of all the players is equal to zero, which means that the players are in a strict competitive situation. In the scenario where a radar and a jammer exist, there are only two persons and the two are thoroughly competitive. Therefore, TPZS game is modeled.

The radar and jammer TPZS game can be completely characterized as
\[ G = \langle P, A, U \rangle . \] (9)
\( P \) is the player set, \( P = \{\text{radar, jammer}\}. \) \( A = A_r \times A_j \) is the action set, where the radar and jammer actions are their transmitted waveforms, \( U = \{u_r, u_j\} \) is the utility function set, and \( u_r = -u_j \). In this paper, SJNR is used as the utility function for the radar, i.e., \( u_r = SJNR \). So the utility function for the jammer is \( u_j = -SJNR \).

Let \( SJNR(a, b) \) be the utility function of the game, where \( a \) is the strategy of the minimizer (the jammer) and \( b \) be the strategy of the maximizer (the radar). If the two players act in a hierarchical manner, the game turns to a Stackelberg game. In the Stackelberg game, the two players are asymmetric. One player will dominate the decision process, which is called the leader. The leader knows that his strategy will be intercepted by his opponent. The other player is the follower, which will take actions rationally to the leader’s strategy. With conservativeness and rationality assumptions, the leader will adopt a safe strategy which can avoid the worse case and the game will lead to a Stackelberg Equilibrium (SE). When the minimizer is the leader, the SE strategies of \( a \) and \( b \) can be calculated by solving (10). When the maximizer is the leader, the SE strategies can be solved by (11).
\[ \min_{a \in A_r} \max_{b \in A_j} SJNR(a, b), \] (10)
\[ \max_{b \in A_r} \min_{a \in A_j} SJNR(a, b). \] (11)
where the only difference is that the "minmax" is replaced by "maxmin". In the former hierarchical games, the available information for the two players is asymmetric. In the case of symmetric information, neither of the player has knowledge of the other’s strategy. In such circumstances, the NE is a good tool to analyze the outcome of the strategic interaction. If a game is competitive and has unique pure-strategy NE, all the players prefer to stay at NE under the assumptions of conservativeness and rationality. As for the TPZS game with utility function \( SJNR(a, b) \), the pure-strategy NE \( (a^*, b^*) \) is defined as
\[ SJNR(a, b^*) \geq SJNR(a^*, b^*) \geq SJNR(a^*, b) . \] (12)
Informally speaking, the NE of a TPZS game on a continuous space is the saddlepoint of its utility function. At the saddlepoint, no player would like to unilaterally deviating from the NE strategy. That is to say, \( (a^*, b^*) \) should satisfy the following equation
\[ (a^*, b^*) = \arg \min_{a \in A_r} \max_{b \in A_j} SJNR(a, b) \]
\[ = \arg \max_{b \in A_r} \min_{a \in A_j} SJNR(a, b) . \] (13)

**III. STACKELBERG GAME AND THE SE STRATEGIES**

In most of the cases, the radar or the jammer knows that its strategies will be intercepted by its opponent. So we start from the asymmetric case and Stackelberg game is formulated.

**A. RADAR AS THE LEADER**

Suppose that the radar is the leader, who wants to maximize the SJNR. The jammer is the follower, who can intercept the radar strategies and minimize the SJNR. The "maxmin" optimization problem can be formulated as
\[ \max_{|X(f)|^2} \min_{J(f)} \int_{BW} \frac{|X(f)H(f)|^2}{J(f) + S_{nn}(f)} df, \]
s.t.
\[ \int_{BW} |X(f)|^2 df = E_x \]
\[ \int_{BW} J(f) df = P_j \]
\[ J(f) \geq 0, \] (14)
where \( E_x \) is the transmission energy of the radar waveform, and \( P_j \) is the jamming power limit.

After solving the optimization problem in (14), the result is the SE of the game, which is
\[ \left|X^*(f)\right|^2_{SE} = \frac{\lambda_1 |H(f)|^2}{4\lambda_2^2}, \] (15)
\( J^r(f)_{SE} = \max \left[ 0, \frac{1}{2\lambda_2} |H(f)|^2 - S_{nn}(f) \right] \). \tag{16}

where \( \lambda_1 \) and \( \lambda_2 \) are two constants and can be obtained by

\[
\int_{BW} |X(f)|^2 \, df = E_x,
\]

\[
\int_{BW} J(f) \, df = P_J.
\]

(17)

The proof of the above result is as follows.

**Proof.** The optimization problem in (14) can be divided into two stage optimization problems as follows.

- The first stage is the inner optimization problem given that the radar strategy \( |X(f)|^2 \) is known by the jammer, i.e.,

\[
\min_{J(f)} \int_{BW} |X(f)H(f)|^2 \, df,
\]

\[\text{s.t.} \int_{BW} J(f) \, df = P_J, J(f) \geq 0, \]

(18)

The second order derivative of the integrand in (18) with respect to \( J(f) \) is greater than 0. Therefore, the integrand is strictly convex with respect to \( J(f) \). As a result, the optimal jamming PSD given that the radar waveform is known can be solved by the Lagrangian multiplier method.

We invoke the Lagrangian multiplier technique yielding an objective function

\[
K(J(f), \lambda_1) = \int_{BW} \frac{|X(f)H(f)|^2}{J(f) + S_{nn}(f)} \, df
\]

\[+ \lambda_1 \left[ P_J - \int_{BW} J(f) \, df \right], \]

(19)

where \( \lambda_1 \) is the Lagrangian multiplier. This is equivalent to minimizing \( k(J(f)) \) with respect to \( J(f) \), where \( k(J(f)) \) is given by

\[
k(J(f)) = \frac{|X(f)H(f)|^2}{J(f) + S_{nn}(f)} - \lambda_1 J(f). \tag{20}
\]

Taking the derivation of \( k(J(f)) \) with respect to \( J(f) \) and setting it to 0 yields the optimal \( J(f) \), i.e.,

\[
J(f) = \max \left[ 0, \sqrt{\frac{|X(f)|^2 |H(f)|^2}{\lambda_1^2} - S_{nn}(f)} \right], \tag{21}
\]

where \( \lambda_1' = -\lambda_1 \) is a constant, and it is determined by \( \int_{BW} J(f) \, df = P_J \).

- The next stage is the maximization problem for the radar under the assumption that the radar knows the jammer will respond as in (21), i.e.,

\[
\max_{|X(f)|^2} \int_{BW} \frac{|H(f)X(f)|^2}{J(f) + S_{nn}(f)} \, df
\]

\[\text{s.t.} \left\{ \begin{array}{l}
\int_{BW} |X(f)|^2 \, df = E_x \\
J(f) = \max \left[ 0, \sqrt{\frac{|X(f)|^2 |H(f)|^2}{\lambda_1^2} - S_{nn}(f)} \right]
\end{array} \right\} \tag{22}
\]

Substituting the expression of \( J(f) \) into the objective function and we can obtain a strictly concave function with respect to \( |X(f)|^2 \). The Lagrangian multiplier method can also be applied to find the optimal radar strategy \( |X(f)|^2 \).

\[
K \left( |X(f)|^2, \lambda_2 \right) = \int_{BW} \frac{|H(f)X(f)|^2}{J(f) + S_{nn}(f)} \, df
\]

\[+ \lambda_2 \left( E_x - \int_{BW} |X(f)|^2 \, df \right), \tag{23}
\]

where \( \lambda_2 \) is the Lagrangian multiplier. This is equivalent to maximizing \( k \left( |X(f)|^2 \right) \) with respect to \( |X(f)|^2 \), where \( k \left( |X(f)|^2 \right) \) is given by

\[
k \left( |X(f)|^2, \lambda_2 \right) = \frac{|H(f)X(f)|^2}{J(f) + S_{nn}(f)} - \lambda_2 |X(f)|^2. \tag{24}
\]

Substituting (21) into the former equation and get

\[
k \left( |X(f)|^2, \lambda_2 \right) = \sqrt{\lambda_1 |X(f)H(f)|^2 - \lambda_2 |X(f)|^2}. \tag{25}
\]

Taking the derivation of \( k \left( |X(f)|^2 \right) \) with respect to \( |X(f)|^2 \) and setting it to 0 yields the optimal radar strategy, i.e.,

\[
|X(f)|^2 = \frac{\lambda_1' |H(f)|^2}{4\lambda_2^2}. \tag{26}
\]

Substituting (26) into (21) yields

\[
J(f) = \max \left[ 0, \frac{1}{2\lambda_2} |H(f)|^2 - S_{nn}(f) \right]. \tag{27}
\]

Therefore, the radar and jammer strategies as SE when the radar is the leader are

\[
|X^r(f)|^2_{SE} = \frac{\lambda_1' |H(f)|^2}{4\lambda_2^2}, \tag{28}
\]

\[
J^r(f)_{SE} = \max \left[ 0, \frac{1}{2\lambda_2} |H(f)|^2 - S_{nn}(f) \right]. \tag{29}
\]

Firstly, the jamming power constraint can be used to obtain the constant \( \lambda_2 \). Then the constant \( \lambda_1' \) can be obtained using the radar energy constraint.

Therefore, the SE strategies are obtained when the radar is the leader.

From the SE strategies, we can conclude that both the radar waveform \( |X(f)|^2 \) and the jammer PSD \( J(f) \) increase with the increase of the target frequency response \( H(f) \), i.e., both the radar and the jammer prefer to concentrate more energy/power into the frequency bands with larger target power. Note that the SE result pair \( \left\{ |X^r(f)|^2_{SE}, J^r_{SE}(f) \right\} \) is unique.
B. JAMMER AS THE LEADER

Suppose that the jammer is the leader which intends to minimize the SJNR. The radar is the follower. Then the optimization problem can be formulated as the following “minmax” problem, i.e.,

$$\min_{J(f)} \max_{X(f)} \int_{BW} \frac{|X(f)H(f)|^2}{J(f) + S_{nn}(f)} df,$$

s.t.

$$\int_{BW} |X(f)|^2 df = E_x$$

$$J(f)df = P_J$$

$$J(f) \geq 0,$$

After solving the optimization problem in (30), the radar and the jammer SE results are

$$|X^j(f)|^2_{SE} = E_x \delta(f - f_i),$$

where

$$f_i = \arg \max_{f} \frac{|H(f)|^2}{S_{nn}(f)}.$$

Note that

$$\lambda_4 = \sqrt{\frac{|H(f)|^2 E_x}{X_3}},$$

is a constant. Let

$$J(f) = \max \{0, \lambda_4 - S_{nn}(f)\}.$$  (37)

Therefore, the SE for the radar and the jammer is

$$|X^j(f)|^2_{SE} = E_x \delta(f - f_i),$$

$$J^j(f)_{SE} = \max \{0, \lambda_4 - S_{nn}(f)\},$$

respectively, where

$$f_i = \arg \max_{f} \frac{|H(f)|^2}{S_{nn}(f)}.$$  (38)

The jammer’s SE strategy is a water-filling solution. If the noise PSD $S_{nn}(f)$ is constant in the frequency band $BW$, i.e., $S_{nn}(f) = \sigma_n^2$, the Lagrangian multiplier

$$\lambda_4 = \frac{P_J}{\sigma_n^2 BW}.$$  (39)

The jammer has a flat spectral in the frequency band $BW$. And $f_i = \arg \max_{f} H(f)$. The radar prefers to put the energy into the largest target scattering mode and the smallest jamming and noise mode.

Proof. Firstly, given the leader’s strategy, namely $J(f)$, the optimization problem for the follower is considered, which is

$$\max_{|X(f)|^2} \int_{BW} \frac{|X(f)H(f)|^2}{J(f) + S_{nn}(f)} df,$$

s.t.

$$\int_{BW} |X(f)|^2 df = E_x.$$

It is obvious that the integrand in the objective function is an increasing function with respect to $|X(f)|^2$. Therefore, the optimal result for problem (33) is

$$|X(f)|^2 = E_x \delta(f - f_i),$$

where $\delta(\cdot)$ is the Dirac Delta function which equals to 1 if and only if the argument equals to 0. $f_i$ is the frequency which maximizes the function $F(f) = \frac{|H(f)|^2}{S_{nn}(f)}$, i.e., $f_i = \arg \max_{f} F(f)$. It indicates that the radar prefers to put its energy into the largest target scattering mode and the smallest jamming and noise mode.

Secondly, the outer minimization problem for the leader is as follows,

$$\min_{J(f)} \int_{BW} \frac{|H(f)X(f)|^2}{J(f) + S_{nn}(f)} df,$$

s.t.

$$|X(f)|^2 = E_x \delta(f - f_i),$$

$$f_i = \arg \max_{f} \frac{|H(f)|^2}{S_{nn}(f)}$$

$$\int_{BW} J(f)df = P_J, J(f) \geq 0.$$

The integrand is a strictly concave function with respect to $J(f)$. According to the Lagrangian multiplier method, the result is

$$J(f) = \max \left[0, \sqrt{\frac{|H(f)|^2 E_x \delta(f - f_i)}{X_3}} - S_{nn}(f)\right].$$  (36)

IV. GAMES WITH SYMMETRIC INFORMATION

In the Stackelberg game model, the two players, namely, the radar and the jammer, are asymmetric. In this section, the case with symmetric information is studied where neither of the radar and the jammer has knowledge of the other’s strategy. NE is the result for the symmetric game if equilibrium exists.

Denote $(|X^r(f)|^2, J^r(f))$ as the solution for the SE game when radar is the leader, and $(|X^j(f)|^2, J^j(f))$ as the solution for the SE game when jammer is the leader. If NE exists, then

$$SJNR\left(|X^r(f)|^2, J^r(f)\right) = SJNR\left(|X^j(f)|^2, J^j(f)\right).$$  (40)

According to whether the target frequency response $H(f)$ and the noise PSD $S_{nn}(f)$ are constant numbers in the frequency band $BW$ or not, there are two cases as follows.

A. THE TARGET FREQUENCY RESPONSE AND THE NOISE PSD ARE NOT CONSTANTS

In this case, inequality

$$SJNR\left(|X^r(f)|^2, J^r(f)\right) < SJNR\left(|X^j(f)|^2, J^j(f)\right)$$

holds. NE does not exist. The proof is as follows.

Proof. Firstly, the following inequality is proved to be true.

$$SJNR\left(|X^r(f)|^2, J^r(f)\right) < SJNR\left(|X^r(f)|^2, J^j(f)\right).$$  (42)
According to the "maxmin" case, the radar is the leader. When the radar strategy is given as $|X^r(f)|^2$, the jamming PSD $J^r(f)$ is the optimal jamming strategy that minimizes SJNR. Therefore, $SJNR( |X^r(f)|^2, J^r(f) ) \leq SJNR( |X^j(f)|^2, J^j(f) )$ holds true. In the previous proof, the integrand in (18) is strictly convex with respect to $J(f)$, i.e., if $J^r(f) \neq J^j(f)$, then $SJNR( |X^r(f)|^2, J^r(f) ) < SJNR( |X^j(f)|^2, J^j(f) )$. Obviously, when the target frequency response $H(f)$ and the noise PSD $S_{nn}(f)$ are not constant in BW, $|J^r(f)|^2 \neq |J^j(f)|^2$ is true. Therefore, $SJNR( |X^r(f)|^2, J^r(f) ) < SJNR( |X^j(f)|^2, J^j(f) )$ is proved to be true.

Secondly, the following inequality is proved to be true.

$$SJNR \left( |X^r(f)|^2, J^j(f) \right) < SJNR \left( |X^j(f)|^2, J^j(f) \right).$$

(43)

According to the "minmax" case, the jammer is the leader. Given the jamming PSD $J^j(f)$, the optimal radar strategy is $|X^j(f)|^2$ which maximizes the SJNR. Therefore, $SJNR( |X^r(f)|^2, J^j(f) ) \leq SJNR( |X^j(f)|^2, J^j(f) )$. The integrand in (33) is a monotone increasing function with respect to $|X(f)|^2$. If $|X^r(f)|^2 \neq |X^j(f)|^2$, the inequality holds true. Obviously, when the target frequency response $H(f)$ and the noise PSD $S_{nn}(f)$ are not constant in BW, $|X^r(f)|^2 \neq |X^j(f)|^2$ is true. Therefore, $SJNR( |X^r(f)|^2, J^j(f) ) < SJNR( |X^j(f)|^2, J^j(f) )$ is proved to be true.

According to (42) and (43),

$$SJNR \left( |X^r(f)|^2, J^r(f) \right) < SJNR \left( |X^r(f)|^2, J^j(f) \right) \leq SJNR \left( |X^j(f)|^2, J^j(f) \right).$$

(44)

Therefore, (41) is proved. NE does not exist.

\[ \square \]

B. THE TARGET FREQUENCY RESPONSE AND THE NOISE PSD ARE BOTH CONSTANTS

In this case, (40) can be proved. NE exists.

\[ \square \]

Proof. Let $|H(f)|^2 = \sigma_H^2$, $S_{nn}(f) = \sigma_n^2$ are constants in BW. The SE strategies when the radar is the leader are

$$|X^r(f)|^2 = \frac{X^r}{\lambda^2} \sigma_H^2 = \frac{E_v}{\pi \lambda^2},$$
$$J^r(f) = \max \left[ 0, \frac{1}{\lambda^2} \sigma_H^2 - \sigma_n^2 \right] = \frac{P_v}{\pi \lambda^2}.$$  

(45)

The SE strategies when the jammer is the leader are

$$|X^j(f)|^2 = \frac{E_v}{\pi \lambda^2},$$
$$J^j(f) = \frac{P_v}{\pi \lambda^2}.$$  

(46)

Therefore, $J^j(f) = J^r(f)$, $SJNR \left( |X^r(f)|^2, J^r(f) \right) = SJNR \left( |X^r(f)|^2, J^j(f) \right)$ holds true.

$$|X^r(f)|^2 = |X^j(f)|^2, SJNR \left( |X^r(f)|^2, J^j(f) \right) = SJNR \left( |X^j(f)|^2, J^j(f) \right)$$ holds true as well. Therefore,

$$SJNR \left( |X^r(f)|^2, J^j(f) \right) = SJNR \left( |X^j(f)|^2, J^j(f) \right) = SJNR \left( |X^j(f)|^2, J^j(f) \right).$$  

(47)

NE exists.

\[ \square \]

V. NUMERICAL RESULTS

In this section, simulation results are provided and the basic simulation parameters and assumptions are given below. Suppose that there is a monostatic radar and a target equipped with a self-screening jammer. The frequency band of the transmitted waveform of the radar is $|f_0 - BW/2, f_0 + BW/2|$, where $f_0$ is the carrier frequency which is assumed to be 1GHz, BW is the bandwidth which is set to be 10MHz. As a result, the frequency interval is $[0.995GHz, 1.005GHz]$.

Suppose that there are two kinds of target frequency responses. The square of the target frequency spectrum $|H(f)|^2$ of the first one is given by

$$|H(f)|^2 = \beta \exp \left\{ -\alpha (f - f_0)^2 \right\},$$

(48)

where $\alpha$ and $\beta$ are parameters that describe the characteristics of the target frequency spectrum. According to [10], $\alpha$ is related with the radar bandwidth. If the bandwidth is 10MHz, $\alpha$ is set to be $10^{-13}s^2$. Note that $\alpha$ describes how fast $|H(f)|^2$ decreases with the increase of $|f - f_0|$. $\beta$ describes the amplitude of the target frequency response. For a target with unit RCS, $\beta$ is the ratio of the received radar power and the transmitted radar power, which is defined as $\beta = \frac{\Delta \mu}{\beta_0} = \frac{\lambda^2 \Delta \sigma}{4 \pi \lambda^2 R^2}$, where $\lambda$ is the radar wavelength, $R$ is the target distance from the radar, $\Delta \sigma = 1m^2$ is the unit target RCS, $A_e$ is the effective antenna aperture which can be calculated by $A_e = \frac{\lambda^2}{4\pi R^2}$, $G$ is the radar antenna gain, suppose that $G = 30dB$, $\beta$ can be obtained as $4.5354 \times 10^{-15}$.

The second target frequency response is supposed to be a constant number within the frequency band of BW, i.e., $\alpha = 0, |H(f)|^2 = \beta$. Without loss of generality, we set $\beta$ to be the same as the first case. So the square of the target frequency response of the second case is $|H(f)|^2 = 4.5354 \times 10^{-15}$. In this case, the target is supposed to be a point target with an impulse shape function as its target impulse response, which has a flat frequency response within BW as its counterpart.

Using the above parameters, the target frequency responses under the two assumptions are shown in Fig.2.

The receiver noise is assumed to be additive white Gaussian noise and the PSD of which equals to $kT_s$, where $k$ is the Boltzmann’s constant and $T_s = 300K$ is the effective noise temperature. Therefore, $S_{nn}(f) = kT_s$ can be calculated.
A. RADAR AND JAMMER GAME WITH TARGET IN CASE 1

In this case, the target frequency response is not a constant which is shown in Fig.2(a). The Stackelberg game and symmetric game between the radar and the jammer are considered. SJNR is analyzed and we show that NE does not exist.

1) Stackelberg game: radar as the leader

Suppose that the radar is the leader. The "maxmin" problem is solved and the SE strategies of the radar and the jammer are obtained. In Fig.3, the constraints of the radar energy and the jamming power are changed and the SE results are shown. In Fig.3(a), the jamming power is fixed to be $P_J = 10W$, while the radar energy is set to be 1kJ, 10kJ and 100kJ, respectively. The SE strategies of the radar are shown. In Fig.3(b), the radar transmission energy is fixed to be $E_x = 10kJ$, while the jamming power constraint is set to be 1W, 10W and 100W, respectively. The SE strategies of the jammer are shown.

From the SE strategies when the radar is the leader, both the radar and jammer strategies have the similar shape to the target frequency response in Fig.2(a). They both prefer to allocating more energy/power into the frequency bands with larger target energy. With the increase of the energy/power constraints, the radar and the jammer strategies increase as well.

2) Stackelberg game: jammer as the leader

Suppose that the jammer is the leader. The "minmax" problem is solved and the SE strategies of the radar and the jammer are obtained. In Fig.4, the constraints of the radar energy and the jamming power are changed and the SE results are shown. In Fig.4(a), the jamming power is fixed to be $P_J = 10W$, while the radar energy is set to be 1kJ, 10kJ and 100kJ, respectively. The SE strategies of the radar are shown. In Fig.4(b), the radar transmission energy is fixed to be $E_x = 10kJ$, while the jamming power constraint is set to be 1W, 10W and 100W, respectively. The SE strategies of the jammer are shown.

From the SE strategies when the jammer is the leader, the radar strategies have a Dirac delta function shape at the frequency of the largest target frequency response. The SE strategies of the jammer are flat spectral because the noise PSD is a constant. With the increase of the energy/power constraints, the strategies of the radar and the jammer will both increase.

3) Symmetric game

It is obvious that the radar and jammer strategies are different when the leader is the radar and the jammer, respectively. Therefore, NE does not exist.
4) SJNR analyzes

The SJNR is analyzed as follows. When the radar is the leader, the SJNR at the SE is:

\[
SJNR\left(\vert X^r(f)\vert^2, J^r(f)\right) = \int_{BW} \frac{|H(f)X^r(f)|^2}{J^r(f) + S_{nn}(f)} df = \int_{BW} \frac{|H(f)|^2 \lambda_1 |H(f)|^2}{4\lambda_2} df = \frac{\lambda_1}{2\lambda_2} \int_{BW} |H(f)|^2 df = \frac{\lambda_1}{2\lambda_2} E_t,
\]

where \( \int_{BW} |H(f)|^2 df = E_t \) is the target energy. The constants \( \lambda_1 \) is related with the radar transmitted energy, and \( \lambda_2 \) is related with the jammer power. Therefore, if the radar and the jammer strategies are the SE strategies in (15) and (16), the SJNR is related only with the radar energy, jamming power, as well as the target energy.

According to the radar energy constraint,

\[
E_x = \int_{BW} |X(f)|^2 df = \int_{BW} \frac{\lambda_1}{4\lambda_2} |H(f)|^2 df = \frac{\lambda_1}{4\lambda_2} E_t,
\]

we can get \( \frac{\lambda_1}{4\lambda_2} = \frac{E_x}{E_t} \).

According to the jammer power constraint,

\[
P_J = \int_{BW} \left( \frac{1}{2\lambda_2} |H(f)|^2 - S_{nn}(f) \right) df = \frac{1}{2\lambda_2} E_t - P_n,
\]

so, \( 2\lambda_2 = \frac{E_x}{P_J + P_n} \), where \( P_n = \int_{BW} S_{nn}(f) df \) is the noise power. Therefore,

\[
\frac{\lambda_1}{2\lambda_2} = \frac{\lambda_1}{4\lambda_2} 2\lambda_2 = \frac{E_x}{E_t} \frac{E_t}{P_J + P_n} = \frac{E_x}{P_J + P_n}. \quad (52)
\]

Substituting (52) into (49), we can get

\[
SJNR\left(\vert X^r(f)\vert^2, J^r(f)\right) = \frac{E_x E_t}{P_J + P_n}. \quad (53)
\]

When the jammer is the leader, the SJNR at the SE is:

\[
SJNR\left(\vert X^j(f)\vert^2, J^j(f)\right) = \int_{BW} \frac{|H(f)X^j(f)|^2}{J^j(f) + S_{nn}(f)} df = \int_{BW} \frac{E_x |H(f)|^2}{P_J + \sigma_n^2} df = \frac{E_x |H(f)|^2}{P_J + \sigma_n^2}, \quad (54)
\]

where \( S_{nn}(f) = \sigma_n^2 \) is a constant number in BW.

Suppose there are 100 different radar transmission energies sampled uniformly from 1kJ to 15kJ. And there are also 100 different jamming powers sampled uniformly from 1J to 10J. Stackelberg game model is used to model the interaction of the radar and the jammer. The SJNR at the SE when the radar is the leader is shown in Fig. 5(a), while the SJNR at the SE when the jammer is the leader is shown in Fig. 5(b). The SJNR when the jammer is the leader minus the SJNR when the radar is the leader is the SJNR difference, which is shown in Fig. 5(c). The SJNRs are different and NE does not exist.

Although there is no NE, the SE strategies provide the gain-floor for the radar when the radar is the leader and the loss-ceiling for the jammer when the jammer is the leader.

When the radar is the leader, in Fig. 6(a), the SJNR of using the SE strategy, the uniform strategy and the random strategy for the jammer is shown. Suppose that the radar adopts the SE strategy under different radar transmission energy constraint, which ranges from 1kJ to 100kJ. The jammer adopts three kind of strategies with \( P_J = 1W \), namely the SE strategy for the jammer, the uniform power allocation strategy and the
random power allocation strategy. The SJNR of all the three cases are compared. In Fig. 6(a), with the increasing of $E_x$, the SJNRs of all the three cases increase linearly. The SJNR of the SE strategy is indeed a gain-floor for the radar, which can be interpreted as the worst-case for the radar. If the radar adopts the SE strategy, no matter what strategy the jammer adopts, the SJNR will be no worse than the case when the jammer adopts the SE strategy. As the radar maximizes the worst-case performance, the gain-floor is optimized.

When the jammer is the leader, in Fig. 6(b), the SJNR of using the SE strategy, the uniform strategy and the random strategy for the radar is shown. Suppose that the jammer adopts the SE strategy under different jamming power constraint, which ranges from 1W to 100W. The radar adopts three kind of strategies with $E_x = 1kW$, namely the SE strategy for the radar, the uniform energy allocation strategy and the random energy allocation strategy. The SJNRs of all the three cases are compared. In Fig. 6(b), with the increasing of $P_J$, the SJNRs of all the three cases decrease linearly. The SJNR of the SE strategy is the loss-ceiling for the jammer. If the radar adopts the SE strategies, using the SE strategy of the jammer will lead to the highest SJNR, which is the worst-case for the jammer. If the radar adopts other strategies, SJNR will reduce and the jamming performance will be improved.

Note that the uniform strategy and the random strategy are used as examples. No matter what other strategies are used, the conclusion remains unchanged.

(a) Gain-floor for radar.
(b) Loss-ceiling for jammer.

FIGURE 6. Gain-floor and loss-ceiling for the radar and the jammer respectively.

### B. RADAR AND JAMMER GAME WITH TARGET IN CASE 2

In this case, the target frequency response is a constant as shown in Fig. 2(b). The radar and jammer strategies are investigated using Stackelberg game and symmetric game.

1) Stackelberg game

If the target frequency response is a constant, the radar strategy at SE is the same no matter the leader is the radar or the jammer, so does the jammer strategy. The radar and jammer strategies are shown in Fig. 7. Both the radar and the jammer uniformly allocate their energy/power into the frequency band.
2) Symmetric game and SJNR

The SJNR is

\[ SJNR = \int_{BW} \frac{|H(f)X(f)|^2}{J(f) + S_{nn}(f)} df \]

\[ = \int_{BW} \frac{E_x}{P_J BW} \frac{|H(f)|^2 \sigma_n^2}{P_J + \sigma_n^2} df \]

\[ = \frac{|E_x|}{P_J BW} \frac{\sigma_n^2}{P_J + \sigma_n^2} \nu, \tag{55} \]

which is the same when the radar or the jammer is the leader. Therefore, NE exists.

Suppose there are 100 different radar transmission energies sampled uniformly from 1kJ to 15kJ. And there are also 100 different jamming powers sampled uniformly from 1J to 10J. The SJNRs under different energy/power constraints are shown in Fig. 8. It can be concluded that the SJNR difference is zero.

Therefore, if the target frequency response is in case 2 in Fig. 2(b), NE exists.

We compare the SJNR of using the NE strategies with other existing strategies. Firstly, if the jammer uses the NE strategy in (45) and (46), which is \( J_{NE}(f) = \frac{P_J}{BW} \). The

\[ SJNR(|X(f)|^2, J_{NE}(f)) = \int_{BW} \frac{|X(f)|^2 |H(f)|^2}{P_J BW + S_{nn}(f)} df. \tag{56} \]

Because both the target frequency response and the noise PSD are constants, the above SJNR can be written as

\[ SJNR(|X(f)|^2, J_{NE}(f)) = \int_{BW} \frac{|X(f)|^2 \sigma_n^2}{P_J BW + \sigma_n^2} df \]

\[ = \frac{\sigma_n^2}{P_J BW + \sigma_n^2} \int_{BW} |X(f)|^2 df \]

\[ = \frac{\sigma_n^2}{P_J BW + \sigma_n^2} E_x, \tag{57} \]
which is not related with the radar waveform. Secondly, if the radar uses the NE strategy in (45) and (46), which is 
\[ |X_{NE}(f)|^2 = \frac{E}{T} \]. The SJNR will be
\[ SJNR(|X_{NE}(f)|^2, J(f)) = \int_{BW} \frac{E}{J(f) + S_{nn}(f) df}. \]

(58)

We can compare the SJNR of using the NE strategy of the jammer with other exiting strategies. Because the NE strategy of the jammer is the uniform strategy. We compare the SJNR of using the NE strategy of the jammer with the random jamming strategy and the Dirac Delta function shape jamming strategy which only allocates the jamming power into the frequency of the largest target mode. Suppose that the jamming power constraint ranges from 1W to 100W, the SJNR of using the three jamming strategies is shown in Fig. 9.

Apparently, the SJNR of using the NE strategy of the jammer is the smallest, which means that the jamming performance is optimized. With the increase of the jamming power, the SJNR of using the NE strategy as well as the random jamming strategy decreases. Note that the random strategy and the Dirac Delta strategy are used as examples. No matter what other jamming strategies are used, the conclusion remains unchanged, which indicates that the optimal jamming strategy is the NE strategy.

VI. COMPARISON BETWEEN THE SJNR UTILITY AND THE MUTUAL INFORMATION UTILITY

Mutual information is a widely used criterion for radar waveform optimization. When using mutual information as a criterion, the extended target is usually modelled as a stochastic target whose random process \( h(t) \) is a finite-energy finite-duration random process. \( h(t) \) can be defined by multiplying a stationary Gaussian random process \( g(t) \) with a rectangular window function \( a(t) \) with duration \( T_h \), i.e., \( h(t) = a(t)g(t) \). As is shown in Fig. 10, \( g(t) \) is a wide-sense stationary process. \( h(t) \) is local stationary in the support region \([0, T_h]\). Therefore, \( h(t) \) is not a true stationary Gaussian random process and has limited energy. In this case, PSD is not available to describe the scattering characteristic of the extended target.

We denote \( H(f) \) as the Fourier transform counterpart of \( h(t) \), and then an energy spectral density (ESD) can be defined as follows
\[ \xi_H(f) = E\left[|H(f)|^2\right], \]

(59)

where \( E[\cdot] \) is the expectation. Suppose that \( \mu_H(f) = E[H(f)] \) is the mean of \( H(f) \). The energy spectral variance (ESV) is
\[ \sigma_H^2(f) = E\left[|H(f) - \mu_H(f)|^2\right]. \]

(60)

Just as PSD can characterize the power spectral characteristic of an infinite-time wide-sense stationary random process, ESV can be used to characterize the average energy of a finite-time zero-mean random process. Without loss of generality, \( \mu_H(f) = 0 \), the ESV and ESD functions are equivalent.

The SJNR of detecting the random target can be approximated by
\[ SJNR \approx \frac{1}{T_h} \int_{BW} \frac{|X(f)|^2 \sigma_H^2(f)}{T_h[S_{nn}(f) + J(f)]} df. \]

(61)

The mutual information between the received signal \( y(t) \) and the target random impulse response \( h(t) \) given a deterministic transmit waveform \( x(t) \) is [10]
\[ I(h(t); y(t)|x(t)) \approx \frac{1}{T_h} \int_{BW} \ln \left(1 + \frac{|X(f)|^2 \sigma_H^2(f)}{T_h[S_{nn}(f) + J(f)]}\right) df, \]

(62)

where \( T_h \) is the length of the echo \( y(t) \).

Define
\[ R_{SJNR}(f) = \frac{|X(f)|^2 \sigma_H^2(f)}{T_h[S_{nn}(f) + J(f)]} \]

(63)

as the SJNR spectral density function. Define
\[ R_{MI}(f) = \ln \left(1 + \frac{|X(f)|^2 \sigma_H^2(f)}{T_h[S_{nn}(f) + J(f)]}\right) \]

(64)

as the mutual information spectral density function. According to the Shannon capacity equation
\[ I = \int_{BW} \ln(1 + R_{SJNR}(f)) df, \]

(65)

where \( I \) is the information rate. According to (63) and (64),
\[ R_{MI}(f) = \ln(1 + R_{SJNR}(f)). \]

(66)
The mutual information spectral density is a function of the SJNR spectral density. When $R_{SJNR}(f) \to \infty$, $R_{MI}(f) \approx \ln(R_{SJNR}(f))$. When $R_{SJNR}(f) \to 0$, $R_{MI}(f) \approx R_{SJNR}(f)$ using Taylor approximation. Therefore, the two criteria have implicit relationship.

From the derivation in [10], the SE strategies when the radar is the leader are

$$|X^\nu(f)|_{MI}^2 = \left(\frac{2\hat{T}^2}{\lambda_1 \sigma_H^2(f)} + \left(\frac{\hat{T}}{\lambda_2}\right)^2 - \frac{2\hat{T}^2}{\lambda_1 \sigma_H^2(f)}\right),$$

$$J^\nu(f)_{MI} \approx \max \left[0, \frac{1}{2\lambda_2} \sigma_H^2(f) - S_{nn}(f)\right].$$

(67)

The SE strategies when the jammer is the leader are

$$|X^j(f)|_{MI}^2 = \max \left[0, \frac{T}{\lambda_3} - \frac{\max \left[S_{nn}(f), \frac{1}{\lambda_4}\right]}{\sigma_H^2(f)}\right],$$

$$J^j(f)_{MI} = \max \left[0, \frac{1}{\lambda_4} - S_{nn}(f)\right].$$

(69)

where $\lambda_1, \lambda_2, \lambda_3$ and $\lambda_4$ are constants that satisfy the radar waveform energy and jamming power constraints.

If random extended targets are considered, the SE strategies of the SJNR utility in (15), (16) when the radar is the leader can be rewritten as

$$|X^\nu(f)|_{SJNR}^2 = \frac{\lambda_1 \sigma_H^2(f)}{4\lambda_2^2},$$

$$J^\nu(f)_{SJNR} = \max \left[0, \frac{1}{2\lambda_2} \sigma_H^2(f) - S_{nn}(f)\right].$$

(71)

The SE strategies of the SJNR utility in (31), (32) when the jammer is the leader can be rewritten as

$$|X^j(f)|_{SJNR}^2 = E_x \delta(f - f_i),$$

$$J^j(f)_{SJNR} = \max \left[0, \lambda_4 - S_{nn}(f)\right].$$

(73)

It is obvious that $J^\nu(f)_{MI}$ and $J^\nu(f)_{SJNR}$ are the same. To compare the SJNR and MI criterion, we suppose that the random target ESV $\sigma_H^2(f)$ has the same shape as the target frequency response in case 1 (Fig.2(a)). If the noise PSD $S_{nn}(f)$ is a constant number in $BW$ as the assumption in this paper, the jamming strategies have the similar shape with the target ESV when the radar is the leader using both the SJNR criterion and the MI criterion, as is shown in Fig.3(b). However, $|X^\nu(f)|_{MI} \neq |X^\nu(f)|_{SJNR}$. The radar strategies under different transmit energy constraints using MI criterion are shown in Fig.11(a). Compare it with the result in our paper in Fig.3(a), it is apparent that they have the similar shape. Note that the SE pair $(|X^\nu(f)|_{MI}^2, J^\nu(f)_{MI})$ may not be unique. But the SE pair $(|X^\nu(f)|_{SJNR}^2, J^\nu(f)_{SJNR})$ is unique.

It is also apparent that $J^j(f)_{MI}$ and $J^j(f)_{SJNR}$ are the same. To compare the SJNR and MI criterion, we also suppose that the random target ESV $\sigma_H^2(f)$ has the same shape as the target frequency response in case 1 (Fig.2(a)). If the noise PSD $S_{nn}(f)$ is a constant number in $BW$ as the assumption in this paper, the jamming strategies are constants when the jammer is the leader using both the SJNR criterion and the MI criterion, as is shown in Fig.4(b). However, $|X^j(f)|_{MI} \neq |X^j(f)|_{SJNR}$. The radar strategies under different transmit energy constraints using MI criterion are shown in Fig.11(b). Compare it with the result in our paper in Fig.4(a), it is apparent that $|X^j(f)|_{MI}^2$ allocates the energy into the frequency bands with larger target frequency.
response. With the increasing of the radar energy constraint, the frequency band becomes wider and the amplitude becomes larger. \( |X^2(f)|^2_{\text{SJNR}} \) allocates the energy at a single frequency with the largest target mode. With the increasing of the radar energy constraint, only the waveform amplitude increases. Note that in practice, the impulse shape radar waveform spectral are impossible. There is always a frequency band. So the SJNR performance would deteriorate to some extent. Note that the SE pair \( |X^2(f)|^2_{M1}, |J^2(f)|_{M1} \) and \( |X^2(f)|^2_{\text{SJNR}}, |J^2(f)|_{\text{SJNR}} \) are unique.

In Fig. 11(c), the radar strategy at SE when the jammer is the leader is shown keeping the radar transmission energy to be 1kJ while changing the jamming power constraints. It has the same results as in [10]. With the increase of the jamming power, the radar tends to allocate its energy in a narrower frequency interval.

If the target ESV is a constant in \( BW \) as in case 2, the NE strategies of the radar and the jammer based on MI exist, which are just distributing the energy and the power evenly in the given bandwidth [10], so are the strategies based on SJNR. Therefore, the SJNR of using the NE strategies based on MI is the same as that of using the NE strategies based on SJNR. So is the MI utility.

VII. CONCLUSION

In this paper, the interaction of the monostatic radar and the jammer is modeled as a TPZS game. SJNR is used as the utility function which is related to the target detection performance. Stackelberg game as well as symmetric game are considered. The SE and NE strategies of the radar waveform and the jammer PSD are analyzed which are useful guidelines for radar waveform design and jamming strategy optimization in the cognitive electronic warfare. We found out that NE only exists when both the target frequency response and the noise PSD are constants. In general cases, SE strategies are also useful despite no NE, because they can be regarded as the optimization of the worst case performance of the leader. We also compare our results with the widely used MI utility function and illustrate the relationship and the difference of them.

In future, more practical methods will be considered in the game such as the observation error of the players [18] and other radar jamming suppression methods will be included [19], [20].

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