$x$- and $\xi$-scaling of the Nuclear Structure Function at Large $x$.

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Inclusive electron scattering data are presented for $^3$H and Fe targets at an incident electron energy of 4.045 GeV for a range of momentum transfers from $Q^2 = 1$ to 7 (GeV/c)$^2$. Data were taken at Jefferson Laboratory for low values of energy loss, corresponding to values of Bjorken $x \gtrsim 1$. The structure functions do not show scaling in $x$ in this range, where inelastic scattering is not expected to dominate the cross section. The data do show scaling, however, in the Nachtmann variable $\xi$. This scaling may be the result of Bloom Gilman duality in the nucleon structure function combined with the Fermi motion of the nucleons in the nucleus. The resulting extension of scaling to larger values of $\xi$ opens up the possibility of accessing nuclear structure functions in the high-$x$ region at lower values of $Q^2$ than previously believed.

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Deep inelastic electron scattering (DIS) from protons has provided a wealth of information on the parton structure of the nucleon. In general, the nucleon structure functions $W_1$ and $W_2$ depend on both the energy transfer ($\nu$) and the square of the four-momentum transfer ($-Q^2$). In the Bjorken limit of infinite momentum and energy transfer, the structure functions depend only on the ratio of $Q^2/\nu$ (modulo QCD scaling violations). Thus, when taken as a function of Bjorken $x = Q^2/2M\nu$, where $M$ is the mass of the proton), the structure functions are independent of $Q^2$. In the parton model, $x$ is interpreted as the longitudinal momentum fraction of the struck quark, and the structure function can be related to the quark momentum distributions. This scaling was observed in high energy electron-proton scattering at SLAC, confirming the parton picture of the nucleon. Violations of Bjorken scaling arise at low $Q^2$ due to effects coming from kinematic corrections and higher-twist effects. A better scaling variable for finite $Q^2$ comes from the operator product expansion treatment of DIS, as was shown in $\cite{10}$. Using the Nachtmann variable $\xi = 2x/(1 + \sqrt{1 + 4M^2x^2/Q^2})$ avoids additional scaling violations arising from finite $Q^2$ corrections to $x$-scaling (which is derived in the infinite momentum limit).

Scaling in $x$ should also be seen in electron-nucleus scattering as both $\nu$ and $Q^2$ approach $\infty$. Because $x$ represents a momentum fraction, it must be between 0 and 1 for scattering from a nucleon. When scattering from a nucleus, $x$ can vary between 0 and $A$, the number of nucleons, due to the nucleon momentum sharing. At finite $Q^2$ and large $x$ ($x \gtrsim 1$), additional scaling violations come from quasielastic (QE) scattering off of a nucleon in the nucleus, rather than scattering off of a single quasi-free quark. The quasielastic contribution to the cross section decreases with respect to the inelastic contributions as $Q^2$ increases due to the nucleon elastic form factor, but QE scattering dominates at very low energy loss (corresponding to $x > 1$) up to large values of $Q^2$.

Previous measurements of inclusive electron scattering from nuclei for $x \leq 3$ and $Q^2 \lesssim 3$ (GeV/c)$^2$ (SLAC experiment NE3 $\cite{11}$) showed scaling for $x \leq 0.4$, but a significant $Q^2$ dependence for larger $x$ values. For these $x$ values, the momentum transfer is low enough that quasielastic and resonance contributions to the scattering violate...
the expected scaling in $x$. When the structure function was examined as a function of $\xi$, the behavior was completely different. The data appeared to be approaching a universal curve as $Q^2$ increased, even in regions where the scattering was predominantly quasielastic. It was noted that this behavior is similar to the local duality observed by Bloom and Gilman \[3,4\] in the proton structure function. Local duality is essentially the observation that the structure function in the resonance region has the same behavior as the deep inelastic structure function, when averaged over a range in $\xi$. It was suggested \[2\] that in the nucleus, the nucleon momentum distribution could effectively perform this averaging of the structure function, causing the QE and DIS contributions to have the same $Q^2$ behavior. Later it was suggested \[3\] that the apparent scaling might instead come from an accidental cancellation of $Q^2$ dependent terms, and would occur only for a limited range of momentum transfers (up to $Q^2 \sim 7.0$ (GeV/c)$^2$). More recent measurements (SLAC experiment NE18) \[8\] showed continuing scaling behavior up to $Q^2 = 6.8$ (GeV/c)$^2$, but the data were limited to values of $x$ very close to 1.

The present data, from experiment E89-008 at Jefferson Lab, were taken with an electron beam energy of 4.045 GeV for scattering angles between 15 and 74 degrees, covering a $Q^2$ range from 1 to 7 (GeV/c)$^2$. The scattered electrons were measured in the High Momentum Spectrometer (HMS) and Short Orbit Spectrometer (SOS) in Hall C. Data were taken using cryogenic hydrogen and deuterium targets and solid targets of C, Fe, and Au. Details of the experiment and cross section extraction can be found in refs. \[7\] and \[8\].

For unpolarized scattering from a nucleus, the inclusive cross section (in the one-photon-exchange approximation) can be written as:

$$\frac{d\sigma}{d\Omega dE} = \sigma_{\text{Mott}} \left[ W_2 + 2W_1 \tan^2(\theta/2) \right], \quad (1)$$

where $\sigma_{\text{Mott}} = 4e^2 E^2 \cos^2(\theta/2)/Q^4$, $\theta$ is the scattering angle, and $W_1(\nu, Q^2)$, $W_2(\nu, Q^2)$ are the structure functions. An explicit separation of $W_1$ and $W_2$ requires performing a Rosenbluth separation, which involves measuring the cross section at a fixed $\nu$ and $Q^2$ while varying the incident energy and scattering angle. Because the data is taken at fixed beam energies, we make an assumption about the ratio of the longitudinal to transverse cross section, $R = \sigma_L/\sigma_T = (1 + \nu^2/Q^2) W_2/W_1 - 1$, to extract $W_2$. Given a value for $R$, we can determine the dimensionless structure function $\nu W_2$ directly from the cross section:

$$\nu W_2 = \frac{\nu}{1 + \beta} \frac{d\sigma}{d\Omega dE}, \quad (2)$$

where

$$\beta = 2 \tan^2(\theta/2) \left[ 1 + \frac{\nu^2}{Q^2} \right] / (1 + R). \quad (3)$$

For our analysis, we use the parameterization $R = 0.32/Q^2$ \[9\], and assign a 100% uncertainty to this value. This parameterization comes from the non-relativistic plane-wave impulse approximation (PWIA) for quasielastic scattering. It is also consistent with data taken in the DIS region (0.2 $< x <$ 0.5 for $Q^2$ up to 5 (GeV/c)$^2$) \[10,11\] and a measurement of $R$ near $x = 1$ in a $Q^2$ range similar to that of the present experiment \[8\].

For the HMS ($\theta \leq 55^\circ$), the systematic uncertainty in the cross section is typically 3.5-4.5%, dominated by acceptance, radiative corrections, and bin centering. For the high $x$ points, the systematic uncertainties become larger because of the strong kinematic dependence of the cross section, but are always smaller than the statistical uncertainties. The uncertainty in $R$ causes an additional uncertainty in the extracted structure function of 0.5-5.0%, which is largest for the largest scattering angles. For the SOS ($\theta = 74^\circ$), the total systematic uncertainty in the structure function is typically $\sim 12\%$ (due mostly to large background from pair production), somewhat larger at the highest values of $x$.

Figure 1 shows the extracted structure function for iron as a function of $x$. As in the previous data \[8\], scaling is seen only for values of $x$ significantly below one, where DIS dominates and resonance and QE contributions are negligible. However, when taken as a function of $\xi$ (Fig. 2), the structure function shows scaling for nearly all values of $\xi$. At low $\xi$, DIS dominates, and scaling behavior is expected from the parton model. For intermediate and high values of $\xi$, where the QE contributions can be significant or even dominate the cross section, the indications of scaling seen in previous data \[8\] are confirmed.

![FIG. 1. Structure Function per nucleon vs $x$ for Iron from the present measurement. The $Q^2$ values given are for $x = 1$. Errors shown are statistical only.](image-url)
The new data allow a more careful test of the suggestion that the scaling comes from an accidental cancellation of $Q^2$ dependent terms. In the PWIA, it was shown that the quasielastic portion of the scattering should show scaling in $y$ (where $y$ is the minimum allowed momentum of the struck nucleon along the direction of the virtual photon). Final-state interactions (FSI), neglected in the PWIA, can cause scaling violations and introduce a $Q^2$ dependence to the quasielastic scaling function, $F(y)$.

For very large values of $Q^2$ ($Q^2 \gg M_N$), $y$ can be written in terms of $\xi$, with corrections of order $1/Q^2$:

$$F(y) = F(y(\xi, Q^2)) = F(y_0(\xi) - \frac{M_N^2 \xi}{Q^2} + O(1/Q^4)),$$

where $y_0(\xi) \equiv M_N(1-\xi)$. The authors of ref. argued that these $1/Q^2$ corrections would introduce scaling violations that would be cancelled by final-state interactions. Their nuclear matter calculations indicated approximate cancellation of the $Q^2$ dependent terms from the FSI and the variable transformation, leading to an accidental scaling of the structure function in terms of $\xi$.

This cancellation occurs only for intermediate $Q^2$ values, and it was predicted that the scaling would break down at very large momentum transfers. We can test this model by directly examining the size of the scaling violations coming from FSI and from the variable transformation. The violations coming from the transformation from $y$ to $\xi$ can be very large. At $y = -0.3$ GeV/c, which corresponds to $\xi \approx 1.1$ for $Q^2 > 2$ (GeV/c)$^2$, the scaling violations from the exact transformation are $>200\%$ between $Q^2 = 2$ (GeV/c)$^2$ and $Q^2 = 4$ (GeV/c)$^2$, and $>50\%$ between $Q^2 = 4$ (GeV/c)$^2$ and $Q^2 = 6$ (GeV/c)$^2$. In order to see scaling in $\xi$, these large scaling violations would have to be cancelled by FSI. A $y$-scaling analysis of the new data indicates that final-state interactions produce $\sim 10\%$ deviations from scaling for these values of momentum transfer, far too small to cancel the transformation induced scaling violations.

While the data show that the cancellation suggested in ref. does not explain the observed scaling, the quality of the scaling indicates that there is some connection between the $y$-scaling picture of quasielastic scattering and the $\xi$-scaling picture of the DIS. The $\xi$-scaling analysis involves removing only the Mott cross section, while the $y$-scaling analysis also removes the strongly $Q^2$-dependent elastic form factor, yet both show scaling above $Q^2 \gtrsim 3$(GeV/c)$^2$ in the region of low energy loss. In this region, the cross section is dominated by quasielastic scattering and there is no expectation that $\xi$-scaling should be valid. While the connection between $\xi$-scaling and $y$-scaling in nuclei is not fully understood, it is essentially the same behavior as seen by Bloom and Gilman in resonance scattering from a free proton. They measured $\nu W_2$ as a function of an improved scaling variable, $x' = Q^2/(2M_N + M_f^2)$, and observed that while there was significant resonance scattering at high $x'$.
and low $Q^2$, the resonance structure, when averaged over a range in $x'$, agreed with the DIS limit of the structure function. The resonance peaks fall more rapidly with $Q^2$ than the DIS contributions, but at the same time move to larger values of $x'$. The DIS structure function falls with increasing $x'$, at a rate which almost exactly matches the falloff with $Q^2$ of the resonance (and elastic) form factors. This behavior also holds when examining the structure function in terms of $\xi$ instead of $x'$ [15,16] (note that in the Bjorken limit, $x' = \xi$).

In nuclei, this same behavior leads to scaling in $\xi$. When $\nu W_2^4$ is taken as a function of $\xi$, the QE peak falls faster with $Q^2$ than the deep inelastic scattering component, but also moves to larger values of $\xi$. In the case of the proton, the resonance behavior follows the scaling limit on average, but the individual peaks are still visible. In heavy nuclei, the smearing of the peaks due to the Fermi motion of the nucleon washes out the individual resonance and quasielastic peaks, leading to scaling at all values of $\xi$. Figure 4 shows the structure function versus $\xi$ for the deuteron. Because of the smaller Fermi motion in deuterium, the QE peak is still visible for all values of $Q^2$ measured and the scaling seen in iron is not seen in Deuterium near $x=1$ (indicated by the arrows in Fig. 4). Note that for $Q^2 \gtrsim 3\text{(GeV/c)}^2$, the data still show scaling in $\xi$ away from the QE peak.

The success of $\xi$-scaling beyond the deep inelastic region opens up an interesting possibility. In the Bjorken limit, the parton model predicts that the structure functions will scale, and that the scaling curves are directly related to the quark distributions. At finite (but large) $\nu$ and $Q^2$, scaling is observed and it is therefore assumed that the structure functions are sensitive to the quark distributions. It is not clear that this assumption must be correct, but the success of scaling is taken as a strong indication that it is true. In nuclei, we see a continuation of the DIS scaling even where the resonance strength is a significant contribution to the structure function. This opens up the possibility of measuring quark distributions in nuclei at lower $Q^2$ or higher $x$. If one requires that measurements be in the deep inelastic regime (typically defined as $W^2 > 4\text{(GeV/c)}^2$, where $W^2$ is the invariant mass squared of the final hadron state), data at large values of $x$ can only be taken at extremely high values of $Q^2$. Because the quark distributions become small at large $x$, and the cross section drops rapidly with $Q^2$, it can be very difficult to make these high-$x$ measurements in the DIS region. However, the observation of $\xi$-scaling indicates that one might be able to use measurements at moderate values of $Q^2$, where the contributions of the resonances are relatively small compared to the DIS contributions and where these contributions have the same behavior (on average) as the DIS.

A more complete understanding of $\xi$-scaling, through precision measurements of scaling in nuclei and local duality in the proton is required. High precision measurements of duality in the proton have been made recently at Jefferson Lab [15,16], and additional proposals have been approved that will extend these measurements to higher $Q^2$ [18]. There is also an approved experiment to continue $x > 1$ measurements at higher beam energies, which will extend the present study of $\xi$-scaling in nuclear structure functions to significantly higher $Q^2$ [19]. Finally, there is an approved experiment that will make a precision measurement of the structure function in nuclei as part of a measurement of the EMC effect [20], which will make a quantitative determination of how far one can extend scaling in nuclei when trying to extract high $x$ nuclear structure.

In conclusion, we have measured nuclear structure functions for $x \gtrsim 1$ up to $Q^2 \approx 7\text{(GeV/c)}^2$. The cross section for $x > 1$ is dominated by quasielastic scattering and, as expected, does not exhibit the $x$-scaling predicted for parton scattering at large $Q^2$. However the data do show scaling in $\xi$, hinted at in previous measurements. The $\xi$-scaling in nuclei at large $x$ can be interpreted in terms of local duality of the nucleon structure function, with nucleon motion averaging over the resonances. Measurements of $\xi$-scaling and local duality, combined with a more complete understanding of the theoretical underpinnings of duality and $\xi$-scaling may allow us to exploit this scaling to access high-$x$ nuclear structure functions, which can be difficult to obtain in the DIS limit.

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