Abstract: This paper proposes the design and analysis of \((2+\alpha)\) order low pass Bessel filter using different optimization techniques. The coefficients of the proposed filter are found out by minimizing the error between transfer functions of \((2+\alpha)\) order low pass filter and third-order Bessel approximation using simulated annealing (SA), interior search algorithm (ISA), and nonlinear least square (NLS) optimization techniques. The best optimization technique based on the error in gain, cut off frequency, roll-off, passband, stopband, and phase is chosen for designing the proposed filter. The stability analysis of the proposed filter has also been done in \(W\)-plane. The simulated responses of the best optimized proposed filter are obtained using the FOMCON toolbox of MATLAB and SPICE. The circuit realization of 2.5 order low pass Bessel filter is done using two DVCCs (differential voltage current conveyors), one generalized impedance converter (GIC) based inductor, and one fractional capacitor. The proposed filter is implemented for the cut off frequency of 10 kHz using a wideband fractional capacitor. Monte Carlo noise analyses are also performed for the proposed filter. The MATLAB and SPICE results are shown in good agreement.

Keywords: Bessel filter, optimization, DVCC, GIC, Monte Carlo.

1. Introduction

Recently, fractional order systems have shown great attraction to researchers in the field of science and engineering. These fields contain bioengineering, control systems, signal processing, nanotechnologies, biology, electrical engineering, medicine, finances, etc. The concepts of fractional calculus can be used to model various fractional order systems since it provides various novel features along with design flexibilities. The continuous progress of fractional order systems and circuits requires the study of their mathematical explanation as well as their physical implementation [1-2]. Various signal processing blocks such as a fractional order oscillator, filters, integrators, differentiators, multivibrators, etc. have been explored in the fractional order domain. Many definitions have been proposed for fractional order derivatives [3] such as

Caputo definition is as follows

\[
D^\alpha_0 f(t) = \frac{1}{\Gamma(m-\alpha)} \int_0^t (t-u)^{m-\alpha-1} f^{(m)}(u) du
\]

Riemann-Liouville is as follows

\[\]
\[
\begin{align*}
\frac{\alpha^m}{\alpha^m} D^\alpha_a f(t) &= \frac{d^m}{dt^m} \left[ \frac{1}{\Gamma(m-\alpha)} \int_0^t \left( (t-\tau) \right)^{m-\alpha-1} f(\tau) d\tau \right], \\
-1 < \alpha \leq m, m \in N.
\end{align*}
\]

Grunwald-Letnikov is as follows

\[
\begin{align*}
\frac{\alpha^m}{\alpha^m} D^\alpha_a f(t) &= \lim_{h \to 0} \sum_{k=0}^{[\frac{t}{h}]} \frac{(-1)^k}{k^\alpha} \left\{ \begin{array}{c}
\frac{t}{k}
\end{array} \right\} \left( t - k h \right), \alpha \in R, t - \alpha = nh
\end{align*}
\]

Where \( \Gamma(.) \) is gamma function, \( m \) is an integer and \( \alpha \) is fractional order.

Initially, fractional order filters have been designed for first and second order systems [4-5]. Further, the active and passive realization of fractional Butterworth filter has been done by Ali et al. [6]. Nowadays, the performance of fractional order filters is being improved by using optimization techniques [7-9]. Freeborn et al. realized fractional order Butterworth, Chebyshev, and Inverse Chebyshev filters using optimization techniques [10-14]. In addition to these, the comparison of different optimization techniques for designing fractional filters (Butterworth Chebyshev and Bessel) has also been done [15-18]. Thus, fractional order Butterworth, Chebyshev, Inverse Chebyshev, and Bessel filters have been designed using optimization techniques in the literature.

However, there is a need to design a higher order fractional filter. Here, higher order Bessel filter is designed using optimization techniques as it is not attempted previously. In the proposed work, \((2+\alpha)\) order low pass Bessel filter is approximated using SA, ISA, and NLS optimization techniques. The best technique out of these three is chosen and then the proposed filter is realized using DVCC based circuit. DVCC is an advanced and most effective block for realizing analog circuits. It has benefits of the differential difference amplifier and second generation current conveyor (CC-II).

This paper is organized as follows: Section 2 focuses on the optimization techniques used for the proposed filter. Section 3 presents the stability analysis in W-plane. Section 4 deals with the comparison of different optimization techniques based on various performance parameters. Section 5 emphasizes the analog realization of the proposed filter. Section 6 discusses results and finally, the main facts are summarized in section 7.

2. Optimization techniques for coefficient selection

In the proposed work, \((2+\alpha)\) order low pass Bessel filter coefficients are optimized using SA, ISA, and NLS. These optimization techniques have been shown using flow charts in Figs. 1-3. To approximate the passband behavior of the proposed filter, the frequency range from \( \omega \) equals \( 10^{-5} \) to \( 1.5 \) rad/sec is used to reduce the error function.

The transfer function of \((2+\alpha)\) order low pass filter is given as follows

\[
T_{LP}^{2+\alpha}(S) = \frac{a_0}{a_1 S^{2+\alpha} + a_2 S^{1+\alpha} + a_3 S + a_4 + 1}
\]

The 3rd order Bessel transfer function with cut off frequency \( 1 \) rad/sec is given by

\[
B_3(s) = \frac{0.2506}{s^3 + 3.4175s^2 + 4.8664s + 2.7718}
\]

Eq. 6 is used to minimize the error between Eq. 4 and Eq. 5 with SA, ISA, and NLS optimization techniques.
\[
\min_{\mathbf{x}} \left\| T(\mathbf{x}, \omega) - B_3(\omega) \right\|^2 = \min_{\mathbf{x}} \sum_{i=1}^{k} \left( |T(x, \omega_i) - B_3(\omega_i)| \right)^2
\]

subject to \( x > 0.1 \)

where \( \mathbf{x} \) is a vector of filter coefficients, \( T(\mathbf{x}, \omega) \) is magnitude response of Eq. 1 and \( B_3(\omega_i) \) is the third-order Bessel approximation at a frequency \( \omega_i \), and \( k \) is the total number of data points. SA, ISA, and NLS optimized filter coefficients \((a_0, a_1, a_2, a_3)\) are found out for \( \alpha \) value ranging from 0.1-0.9 and summarized in Table 1.

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Fig. 1. Flow chart of SA technique
Arbitrarily create the positions of elements between upper and lower bound and get their fitness values.

Fix the value of maximum number of iterations and initial population

Find the respective fitness values of all elements

Compare the value of element with the control parameter

Is element value less than control parameter?

Yes
Element enters into mirror group and find their fitness values.

No
Element enters into composition group and find their fitness values.

Compare new fitness value with old element value

Is new fitness value better than old value?

Yes
Accept the new fitness value

No
Reject the new fitness value and accept the old value.

Are number of iterations over?

Yes

No
End

Fig. 2. Flow chart of ISA technique
Form a matrix or vector of input \( x_{data} \).

Define Lower bound and upper bound in the form of vectors or matrices of same size as \( x \). Coefficients of \( x \) are to be found out using lscurvefit.

Define \( y_{data} \) observed output matrices or vectors.

Form \( F(x, x_{data}) \) in the vector form \([s, x_{data}(1), \ldots, s, x_{data}(k)]\). It is matrix-valued or vector-valued function of same size as \( x_{data} \).

Lsqcurvefit solve the problem using
\[
\min \| F(x, x_{data}) - y_{data} \|^2 = \min \sum (F(x, x_{data}) - y_{data})^2
\]

Analyze and compare the results of default trust-region-reflective algorithm and the levenberg-marquardt algorithm.

Are the results same?

- **Yes**: Any of the algorithm results can be chosen.
- **No**: Better results out of two can be taken into consideration.

End

**Fig. 3.** Flow chart of NLS technique
Table 1: Filter coefficients of (2+ α) order low pass Bessel filter using SA, ISA, and NLS

|   | SA      | ISA     | NLS     |
|---|---------|---------|---------|
| α | 0.2     | 0.5     | 0.8     |
| a | 0.2000  | 0.2000  | 0.2000  |
| a1| 0.9957  | 0.9997  | 1.0000  |
| a2| 0.9997  | 0.9988  | 0.9997  |
| a3| 0.9972  | 0.9981  | 0.9999  |
| a4| 1.0000  | 0.9997  | 0.9999  |

3. Stability Analysis

Stability analysis is an important aspect to confirm the possibility of analog realization of the proposed filter. To explore the stability of the proposed (2+α) order Bessel filter, conversion of s-plane transfer function into the W-plane transfer function is required [19-24]. This conversion is done in the following manner

i) Convert s=W^m and α=k/m.

ii) Choose k and m for the required value of α.

iii) Converted W-plane transfer function is solved for all poles.

iv) Evaluate the absolute pole angles |θ_m^w|, if all are greater than \( \frac{\pi}{2m} \) then the system is stable otherwise not.

Table 2 reported the minimum pole angle |θ_m^w| for (2+α) order low pass Bessel filter using SA, ISA, and NLS techniques. It can be seen that all pole angles (|θ_m^w|) are greater than 9 degrees (minimum value of \( \frac{\pi}{2m} \) for m=1). Hence, it confirms that all the techniques used for optimizing the proposed filter are physically realizable.

Table 2. Minimum root angles using GA, SA, ISA, and NLS techniques

| α | SA     | ISA     | NLS     |
|---|--------|---------|---------|
|   | θ_m^w(min)(degrees) |         |         |
| 0.2 | 12.7   | 13.5    | 12.2    |
| 0.5 | 11.5   | 12.2    | 11.8    |
| 0.8 | 10.5   | 11.1    | 11.7    |

4. Comparison of optimization techniques

SA, ISA, and NLS optimized filter coefficients are compared for errors such as gain, cut off frequency, roll-off, passband, stopband, and phase. The best optimization technique out of these three in terms of the above mentioned parameters is chosen for analog realization of the proposed filter.

The following factors have been used to compare the performance of the techniques used in the proposed work [25]:
i) Gain error: It is calculated by comparing the maximum gain of fractional order low pass Bessel filter with the maximum gain of the ideal Bessel filter.

ii) Cut off frequency error: It is the frequency at which the magnitude of the signal falls by 3 dB of its maximum value. Cut off frequency error is measured by comparing the cut off frequency of the proposed filter with the ideal Bessel filter (1 rad/sec).

iii) Roll-off error: It is the fall in dB/decade in the transition band. The roll-off error can be determined by comparing the roll-off rate of the proposed filter with the ideal Bessel filter.

iv) Passband error (PE): It is the error observed in the passband (till 1 rad/sec) when compared to the ideal Bessel response. PE can be calculated as follows:

\[ PE = 20 \times \log_{10} \left( \sum_{i=1}^{K} \left| \frac{f_{LP}^{2+\alpha}(\omega)}{K} - B_\alpha(\omega) \right|^2 \right) \text{dB} \]  

where \( K = 500 \) and \( 0.01 \leq \omega_i \leq 1 \)

v) Stopband error (SE): It is the error observed in the stopband (from 1 rad/sec to 10 rad/sec) when compared to the ideal Bessel response. It is calculated as follows:

\[ SE = 20 \times \log_{10} \left( \sum_{i=1}^{K} \left| \frac{f_{LP}^{2+\alpha}(\omega)}{K} - B_\alpha(\omega) \right|^2 \right) \text{dB} \]  

where \( K = 500 \) and \( 1 \leq \omega_i \leq 10 \)

vi) Phase error: It is the error observed in the phase response when compared to the ideal Bessel response. It is measured as follows:

\[ \text{Phase Error} = \frac{\sum_{i=1}^{K} \left| \tan^{-1} f_{LP}^{2+\alpha}(\omega) - \tan^{-1} B_\alpha(\omega) \right|}{K} \]  

where \( K = 500 \) and \( 0.01 \leq \omega_i \leq 10 \)

Table 3 shows the comparison of parameters for different optimization techniques (SA, ISA, and NLS). It has been observed that NLS gives the minimum error of all parameters (gain error, cut off frequency error, roll-off error, PE, SE, and phase error) as compared to SA, and ISA for \( \alpha \) equal to 0.2, 0.5, and 0.8. Gain and roll-off of ideal third-order Bessel filter are -20.9dB and -54.6dB/decade.

The frequency and phase response of SA, ISA, and NLS optimized (2+\( \alpha \)) order Bessel filters for the orders 2.2, 2.5, and 2.8 have been plotted in Fig. 4(a)-(c). Further, the frequency and phase responses have also been plotted for \( B_3(s) \) to show the deviation of fractional order filters. These responses show that the roll-off increases as the order are increasing from 2.2 to 2.8. The errors in gain, cut off frequency, roll-off, passband (PE), stopband (SE), and phase for (2+\( \alpha \)) order Bessel filters are calculated and tabulated in Table 3.
Fig. 4. Frequency and phase response of $(2+\alpha)$ order Bessel filter using (a) SA (b) ISA (c) NLS
Table 3. Simulated parameters of \((2+\alpha)\) fractional-order low pass Bessel filter

| \(\alpha\) | Parameters | SA       | ISA      | NLS      |
|------------|-----------|----------|----------|----------|
| 0.2        | Gain Error (dB) | 6.1      | 5.6      | 0.1      |
|            | Cut off frequency error (rad/sec) | 0.9364   | 0.9922   | 0.051    |
|            | Roll-off error (dB/dec) | 11.4     | 12.3     | 9.5      |
|            | Passband Error (dB) | 56.1936  | 66.1824  | 92.3605  |
|            | Stopband error(dB) | 87.6072  | 93.7984  | 92.5471  |
|            | Phase error(radians) | 2.5      | 2.77     | 1.12     |
| 0.5        | Gain error (dB) | 6.9      | 6.9      | 0        |
|            | Cut off frequency error (rad/sec) | 0.741    | 0.964    | 0.11     |
|            | Roll-off error (dB/dec) | 2.1      | 4.3      | 1.7      |
|            | Passband Error(dB) | 47.9189  | 60.2175  | 108.7552 |
|            | Stopband error(dB) | 80.8105  | 90.8799  | 112.7983 |
|            | Phase error(radians) | 0.515    | 0.62     | 0.381    |
| 0.8        | Gain error(dB) | 6.9      | 6.9      | 0        |
|            | Cut off frequency error (rad/sec) | 0.43     | 0.883    | 0.01     |
|            | Roll-off error(dB/dec) | 7.2      | 4.5      | 0.4      |
|            | Passband Error(dB) | 42.5550  | 57.2557  | 127.8340 |
|            | Stopband error(dB) | 67.3757  | 90.7401  | 137.4023 |
|            | Phase error(radians) | 0.445    | 0.587    | 0.122    |

5. Analog realization of the proposed filter

It has been discussed earlier that the NLS gives the least gain error, cut off frequency error, roll-off error, PE, SE, and phase error as compared to SA, and ISA techniques for the proposed filter. So, there is a requirement to verify the results obtained from the NLS optimization technique. Here, DVCC is chosen to design NLS optimized \((2+\alpha)\) order low pass Bessel filter. DVCC is defined using the following matrix:

\[
\begin{bmatrix}
I_{y1} \\
I_{y2} \\
V_x \\
I_z
\end{bmatrix} =
\begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 \\
0 & 0 & 1 & 0
\end{bmatrix}
\begin{bmatrix}
V_{y1} \\
V_{y2} \\
I_x \\
V_z
\end{bmatrix}
\]  

(10)

Fig. 5 shows the circuit diagram of the NLS optimized \((2+\alpha)\) order low pass Bessel filter using 2 DVCCs, 1 GIC based inductor, and 1 fractional capacitor. The internal structure of DVCC is given in Fig. 6 using 3 AD844 [26-28].
Fig. 5. DVCC based NLS optimized proposed filter

Fig. 6. The internal structure of DVCC

Fig. 7. The internal structure of GIC based inductor
GIC based inductor is used in the proposed circuit for L=1mH. In Fig.7, the desired value of the inductor (L=1mH) is achieved by choosing $R=1K$, $R_5=100\Omega$, and $C_4=0.01\mu F$. Equivalent input impedance to ground of above ckt. (Fig. 7) is given by

$$Z = \frac{Z_2Z_5}{Z_4}$$  \hspace{1cm} (11)$$

Using $Z_1=Z_2=Z_3=R$, $Z_4=1/sC_4$, $Z_5=R_5$, Eq. 11 becomes

$$Z = sC_4R_5$$  \hspace{1cm} (12)$$

$$L = C_4R_5$$  \hspace{1cm} (13)$$

DVCC based 2.5 order proposed filter is used $C_1$ as a fractional capacitor, this capacitor is used for a wide frequency range. It is made up of 10 resistances and nine capacitances. Fig. 8 shows the structure of a wideband fractional capacitor and Table 4 gives the values of resistances and capacitances used in $C_1$ [29].

![Fractional capacitor diagram](image)

**Fig. 8.** Wideband Fractional capacitor ($C_1$) for $\alpha=0.5$

**Table 4.** Values of resistances and capacitances used in the fractional capacitor for $\alpha=0.5$

| Resistors | Values (Ω) | Capacitors | Values (F) |
|-----------|------------|------------|------------|
| $R_2$     | 537.6      | -          | -          |
| $R_3$     | 394.1      | $C_3$      | 803.2 p    |
| $R_4$     | 974        | $C_4$      | 1506 p     |
| $R_5$     | 2153       | $C_5$      | 3164 p     |
| $R_6$     | 4665       | $C_6$      | 6779 p     |
| $R_7$     | 10.07 K    | $C_7$      | 14.58 n    |
| $R_8$     | 21.75 K    | $C_8$      | 31.33 n    |
| $R_9$     | 47.4 K     | $C_9$      | 66.7 n     |
| $R_{10}$  | 108.2 K    | $C_{10}$   | 135.6 n    |
| $R_{11}$  | 341.4 K    | $C_{11}$   | 84 n       |

To get the overall transfer function of DVCC based proposed filter (Fig. 5), the following steps are required:
\[
\frac{V_o}{V_{in}} = \frac{1}{LC_s s^{\alpha-1} + R_s s^{\alpha-1} + 1}
\]  
(14)

\[
\frac{V_{out}}{V_i} = \frac{R_s}{sC_s R_s + R_s + (R_2 + R_3)}
\]  
(15)

\[
\frac{V_{out}}{V_{in}} = \frac{R_s}{s^{\alpha-2}[LC_s R_2 R_1] + s^{\alpha-4}[R_2 R_3 R_1 C_1 + LC_s (R_2 + R_3)] + s^{\alpha-6}[R_4 R_5 R_3 + R_2 R_3] + s^{\alpha-8}[R_4 R_3 C_2] + [R_2 + R_3]}
\]  
(16)

After dividing the numerator and denominator of Eq. 16 by \(R_2+R_3\), then compare this equation with Eq. 4. The outcome of comparison gives the values of \(R_1=5163.9\Omega\), \(R_2=168200\Omega\), \(R_3=19070\Omega\) with \(C_1=2.5\text{F}\), \(C_2=0.02\text{µF}\), and \(L=1\text{mH}\). These values are used to get the magnitude response of DVCC based NLS optimized \((2+\alpha)\) order low pass Bessel filter, magnitude is scaled by 10000 and frequency shifted to 10 kHz.

### 6. Result and discussion

#### 6.1 SPICE simulated magnitude response

The SPICE simulated magnitude response of the proposed 2.5 order Bessel filter is shown in Fig. 9. The MATLAB and SPICE simulated results of 2.5 order Bessel filters have been compared. The absolute error in MATLAB and SPICE simulated results of gain and cut off frequency are 3.5 dB and 0.37 rad/sec, respectively. It specifies that the results of MATLAB and SPICE are close to each other as desired for realization at the circuit level using approximated fractional order capacitors.

In addition to it, the Monte Carlo analysis of 2.5 order DVCC based NLS optimized Bessel filter for all the resistances and capacitances used in the circuit (Fig. 5) within 5% tolerance has been done for \(n=100\) runs. The resultant plots are shown in Fig.10 (a)-(b). The maximum variation in gain, cut off frequency and roll-off rate for 2.5 order proposed filter are (-20.18 dB to -20.22 dB), (15.80 kHz to 16.33 kHz) and (-39.65 dB/decade to - 41.27dB/decade) respectively. Thus, it shows the reasonable variation in the above mentioned parameters.

![Fig. 9. SPICE simulated frequency response of 2.5 order DVCC based filter](image-url)
6.2. Noise Analysis

Noise analysis is an important aspect to see the impact of noise on the proposed circuit. There are different kinds of noise in any electronic circuit: shot noise, flicker noise, and thermal noise. The collective effect of all such noises on the proposed circuit (Fig. 5) is determined in the SPICE environment. The behavior of input and output noise voltage of 2.5 order proposed filter to frequency is shown in Fig. 11. As can be seen from this figure (Fig. 11) that both the input and output noise are low in the entire passband.
7. Conclusion
This work presents the designing of (2+α) order low pass Bessel filter using SA, ISA, and NLS techniques. These techniques are used to optimize the filter coefficients. Further, the best optimization technique based on gain error, cut off frequency error, roll-off error, PE, SE, and phase error has been chosen to design the proposed filter using DVCCs. The NLS optimized (2+α) order low pass Bessel filter gives good similarity with SPICE simulated DVCC based circuit. Therefore, MATLAB and SPICE results show a good similarity between results. This work can be further extended for other approximations of the filter.

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