Shubnikov de Haas Effect and Electron Mobilities in the Isomorphic InGaAs Quantum Well With the InAs Insert on the InP Substrate

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Abstract. The Shubnikov – de Haas (SdH) effect at T=4.2 and 8.4 K and the Hall effect have been investigated in isomorphic In₀.₅₂Al₀.₄₈As/In₀.₅₃Ga₀.₄₇As/In₀.₅₂Al₀.₄₈As/InP quantum well with InAs inserts to the center of the quantum well. The structures were both-sides delta-doped by silicon. The effective mass m* was measured by SdH effect by a new method which allowed experimentally to determine m* in the every dimensionally quantized subband using the temperature dependence of the SdH effect amplitude. Central InAs inserts in a quantum well lead to a decreasing of m* by about 20% as compared with the uniform In₀.₅₂Ga₀.₄₈As lattice-matched quantum well. We also calculated the transport and quantum mobility of electrons in dimensionally quantized subbands using the SdH effect. The calculated and experimentally determined values are in a good agreement.

Introduction

One of the most important problems of ultra-high-frequency (UHF) electronics is the optimization of base material of the transistor. In the InAlAs/InGaAs/InAlAs quantum well on the InP substrate, two-dimensional electron gas with a high electron concentration up to ns=3*10¹² cm⁻²,
has high mobility $\mu=10,000 \text{ cm}^2/(\text{Vs})^{-1}$ at room temperature [1,2] and highly saturated electron drift velocity. These structures are extensively used for making UHF devices for millimeter and submillimeter wave applications. The lattice-matched $\text{In}_y\text{Al}_{1-y}\text{As}/\text{In}_x\text{Ga}_{1-x}\text{As}/\text{In}_y\text{Al}_{1-y}\text{As}$ ($y=0.52$) structure on the InP substrate has a high InAs content in a quantum well $\text{In}_y\text{Ga}_{1-y}\text{As}$ ($x=0.53$). It leads to a decrease of the effective electron mass $m^*$ as compared with GaAs, the stronger capture of electrons in a deep $\text{In}_x\text{Ga}_{1-x}\text{As}$ quantum well and a significant increase of the electron mobility $\mu$. However, with increasing $x$ the thickness of strained layer $\text{In}_y\text{Ga}_{1-y}\text{As}$ has to decrease to avoid dislocations and hence the electron mobility drops. According to results [3-6] the thin InAs layer in the quantum well $\text{InGaAs}$ leads to the electron mobility enhancement as compared with the uniform quantum well. Thus on the one hand InAs insert results in the growth of $\mu$ due to an increase of the energy gap between dimensional quantization subbands and decreasing of $m^*$ in a quantum well. On the other hand, the InAs layer thickness $d$ is limited by a critical value [7]. Exceeding the critical thickness results in a degradation of the insert layer and the formation of lattice misfit dislocations.

Here we report the results of the experimental determination of the electron effective mass separately in different dimensionally quantized subbands by exploring the Shubnikov–de Haas (SdH) effect at different $T$. We developed a new method of a digital separation of SdH frequencies at different temperatures in the case of several filled subbands. It gave the possibility experimentally to measure the electron effective mass separately in the every subband using the temperature dependence of the SdH oscillations amplitude. The SdH effect also was used to evaluate the subband quantum and transport mobilities. In this study for reducing the electron effective mass and mobility enhancement we introduce the InAs insert to the quantum well center.

**Experimental**

A schematic diagram of the $\text{In}_{0.52}\text{Al}_{0.48}\text{As}/\text{In}_x\text{Ga}_{1-x}\text{As}/\text{In}_{0.52}\text{Al}_{0.48}\text{As}$ structure with a central InAs insert on InP substrate is shown in Fig. 1. The samples were grown by the molecular beam epitaxy on InP (100) substrates. All samples have $\text{In}_{0.52}\text{Al}_{0.48}\text{As}$ buffer matched (isomorphic) to InP. Samples have a double-side delta-doping by Si. The spacer thickness was 4.7 nm (for a sample #4 – 7.3 nm). A cap layer was undoped $\text{In}_{0.53}\text{Ga}_{0.47}\text{As}$. Temperature dependence of the resistance and Hall effect were measured in the temperature interval 4.2 K<$T<$300 K.

| In$_{0.53}$Ga$_{0.47}$As | 54 Å |
|-------------------------|------|
| In$_{0.52}$Al$_{0.48}$As | 173 Å |
| $\delta$2-Si             | 2.9·$10^{12}$ cm$^{-2}$ |
| In$_{0.52}$Al$_{0.48}$As | 47 Å |
| In$_{0.53}$Ga$_{0.47}$As (168 Å - d)/2 | InAs |
| δ1-Si                   | 0.88·$10^{12}$ cm$^{-2}$ |
| In$_{0.52}$Al$_{0.48}$As | 2440 Å |
| InP (100)                |      |

At $T=4.2$ K the SdH effect was investigated in magnetic field provided by a superconducting solenoid up to 6 T. We found that the Hall mobility depends on the width of the InAs insert to a quantum well. Some experimental parameters of samples at $T=4.2$ K are listed in table 1. The sample 4 was taken as the reference sample with $d=L$, that is this sample has the InAs strained quantum well. The maximal value of mobility was observed in the sample #3 with wide InAs insert. The mobility in the sample 1 without the insert and the reference sample #4 with InAs quantum well is less and close each to other.

Fig. 1 Sample structure. A quantum well is shown by a grey colour.
Table 1. The InAs insert width \(d\), the quantum well width \(L\), the Hall electron concentration \(N_{\text{Hall}}\) and the mobility \(\mu_{\text{Hall}}\) at \(T=4.2\) K, SdH electron concentration in 2 filled subbands \(n_{\text{SdH}}\)

| Sample # | \(L\), Å | \(d\), Å | \(N_{\text{Hall}}\), \(10^{12}\) cm\(^{-2}\) | \(\mu_{\text{Hall}}\), cm\(^2\)/(Vs) | \(n_{\text{SdH}}\), \(10^{12}\) cm\(^{-2}\) |
|----------|----------|---------|-----------------|-----------------|-----------------|
| 1        | 168      | 0       | 4.03            | 27600           | 2.76; 1.29      |
| 2        | 168      | 20      | 2.99            | 26000           | 2.3; 0.75       |
| 3        | 168      | 34      | 3.03            | 29500           | 2.44; 0.6       |
| 4        | 97       | 97      | 1.30            | 28400           | 1.29            |

As we see in table 1 the summary of electron concentration in both filled subbands are in a very good agreement with the Hall concentration of electrons.

Results and discussion

Amplitude of magnetoresistance oscillations (SdH effect) increases with temperature decrease. This gives the opportunity to determine the electron effective mass \(m^*\) at extremal sections of Fermi surface. The relation between amplitudes of oscillations \(A(T_1)\) and \(A(T_2)\) determined at temperatures \(T_1\) and \(T_2\) \((T_1 > T_2)\) for the same value of magnetic field \(B = B_n\) in case of Dingle temperature \(T_D\) independence on temperature is equal to [8]

\[
\frac{A(T_2)}{A(T_1)} = \frac{T_2}{T_1} \frac{\text{sh}(2\pi^2 k_B T_1 / \hbar \omega_{c})}{\text{sh}(2\pi^2 k_B T_2 / \hbar \omega_{c})}.
\]

(1)

The formula (1) is a transcendental equation for the determination \(m^*_c\). However, at \(T_1 = 2T_2\) this equation simplifies and it’s possible to get an expression for \(m^*_c\):

\[
m^*_c = \frac{eB_n}{2\pi^2 k_B T_2} \text{Arch}\left[\frac{A(T_2, B_n)}{A(T_1, B_n)}\right]
\]

(2)

Fig.2. Shubnikov – de Haas oscillations at two temperatures for sample #1 (a) ant its Fourier spectra (b) at \(T=4.2\) K (points – experiment, solid line – fitting)
This formula may be used only in the case of a single oscillation frequency. We observed two frequencies in all samples except sample #4 in which only one subband is filled by electrons. As an example the SdH oscillations at two temperatures for sample #1 are given in Fig. 2a. Two frequencies in the SdH effect correspond to two filled subbands. We use conventional Fourier transform (FT) of the SdH oscillations to obtain the electron subband concentrations $n_i$ from the frequencies of oscillations. Fast Fourier transform spectra are shown in Fig. 2b.

In order to measure accurately the amplitude of the SdH oscillation corresponding to every frequency we digitally separated observed frequencies by an original program using an idea developed in Ref. [9]. In fig. 3 we plotted SdH oscillation at two temperatures separately for high (Fig.3a) and low (Fig.3b) frequencies. We used separated high and low frequencies of the SdH oscillations at two temperatures using formula 2 for the experimental determination of the electron effective masses listed in table 2.

![Fig. 3 The SdH oscillations in sample 1 at two temperatures separately for high (a) and low (b) frequencies](image)

Table 2. Electron effective masses in the first ($m_1^*$) and the second ($m_2^*$) subbands

| sample # | $m_1^*$ | $m_2^*$ |
|----------|---------|---------|
| 1        | 0.059   | 0.060   |
| 2        | 0.050   | 0.057   |
| 3        | 0.048   | 0.049   |
| 4        | 0.042   | –       |

As we see in table 2 the effective mass in the second upper subband higher than in the first subband. This fact pointed to the some nonparabolicity of the electron energy spectrum.

Reliable electron densities and mobilities can be derived from Shubnikov-de Haas oscillations, for which each subband has an oscillation with its own period. The part of the density of states oscillating in magnetic field, $\Delta g$, can be expressed as [10,11]

$$
\frac{\Delta g(E_F)}{g_0} = 2\sum_{\sigma=1}^{\infty} \exp\left(-\frac{\pi s}{\mu_\sigma B}\right) \cos\left[\frac{2\pi s(E_F - E_i)}{\hbar\omega_c} - \frac{\pi r}{2}\right] \frac{(2\pi^2 s^2 k_B T/h\omega_c)}{\sinh(2\pi^2 s^2 k_B T/h\omega_c)},
$$

(3)
which leads to the following expressions for the conductivity tensor components in the 2-D case (the Landau level width is assumed to be independent of energy and magnetic field):

\[
\sigma_{xx} = \frac{e n_0 \mu_t}{1 + \mu_t^2 B^2} \left[ 1 + \frac{2 \mu_t^2 B^2}{1 + \mu_t^2 B^2} \frac{\Delta g(E_F)}{g_0} \right],
\]

(4)

\[
\sigma_{xy} = -\frac{e n_0 \mu_t^2 B}{1 + \mu_t^2 B^2} \left[ 1 - \frac{3 \mu_t^2 B^2 + 1}{\mu_t^2 B^2 (1 + \mu_t^2 B^2)} \frac{\Delta g(E_F)}{g_0} \right],
\]

(5)

where \(\mu_t\) is the transport mobility at \(B=0\), \(g_0\) is the density of states at zero magnetic field \(B\), \(\mu_q\) is the quantum mobility, \(n_s\) is the electron density, and \(e\) is the absolute value of the electron charge. The oscillation frequency \(F\) in the reciprocal magnetic field determines the two-dimensional electron density: \(n_s = e F/\pi \hbar\), and Fermi-energy is \(E_F = (\pi \hbar^2 / m^*) n_s\). Scattering through small angles makes a negligible contribution to transport relaxation time. If scattering is large at small angles (in the case of Coulomb scattering), the transport relaxation time may be an order of magnitude larger than quantum one. Hence the transport mobility is larger than quantum one. Analysis of the oscillation amplitude as a function of magnetic field and temperature yields the transport and quantum mobility’s of 2D electrons in each dimensional subband. By varying \(\mu_t\) and \(\mu_q\) in each subband, one can fit the theoretical dependence (3-5) of oscillation on magnetic field and FT to experimental data (see Fig. 2b). As an example the fit to the experimental data for the sample 1 is shown in Fig. 2b by a solid line. The resulting transport \(\mu_t\) and quantum \(\mu_q\) mobility’s are listed in table 3. Scattering through small angles makes a negligible contribution to transport relaxation time.

Table 3 Quantum \(\mu_q\) transport \(\mu_t\) electron mobility in the first and the second subbands and Hall mobility \(\mu_{Hall}\), at 4.2 K for all samples

| sample # | \(\mu_q\) cm\(^2\)/(V·s) | \(\mu_t\) cm\(^2\)/(V·s) | \(\mu_{Hall}\) cm\(^2\)/(V·s) |
|----------|-----------------|-----------------|-----------------|
| 1        | 2               | 2100            | 24900           | 27600           |
|          | 1               | 2200            | 27600           |                 |
| 2        | 2               | 2200            | 26800           | 26000           |
|          | 1               | 3000            | 28700           |                 |
| 3        | 2               | 1900            | 30700           | 29500           |
|          | 1               | 3100            | 36200           |                 |
| 4        | 1               | 2300            | 27600           | 28400           |

For all structures, the electron wavefunctions \(\psi_i(z)\) and the energies \(E_i\) were determined in the effective mass approximation by the self-consistent solution of the one-dimensional Schrödinger and Poisson equations [10,12]. We used the following values of the effective masses at the bottom of the conduction band in \(\Gamma\)-valley: in InAs – \(m^* = 0.03\ m_0\), in In\(_{0.53}\)Ga\(_{0.47}\)As – \(m^* = 0.043\ m_0\), in GaAs – \(m^* = 0.067\ m_0\) and in InAlAs – \(m^* = 0.075\ m_0\). The conduction band discontinuities were equal to \(\delta E_c = -0.7\) eV between InGaAs and InAlAs and \(\delta E_c = -0.45\)–0.3 eV between InGaAs and InAs and InGaAs and GaAs [3,13-15]. The value of a surface potential for all heterostructures was equal to \(\Phi_s = 0.5\) eV. Fig. 4 shows the band diagrams of the sample #1 and #3 with the wide InAs insert. By inserting a wide InAs layer the lowest electron energy level \(E_1\) falls down to the bottom of the quantum well and forms level bounded mainly with insert. Effective width of the quantum well determined as a half-width of electron wave function decreases. The distance between \(E_1\) and the next level \(E_2\) increases. On the one hand, it leads to an increase of the electron mobility \(\mu\) due to
the weakening of the inter-subband scattering and decreasing of electron effective mass in the quantum well. On the other hand, lateral roughness scattering increases in a narrow quantum well. Inserted InAs layers are strained as compared to the isomorphic In$_{0.53}$Ga$_{0.47}$As layer and electron mobility can increase only up to some critical width of the InAs insert. Thus it is necessary to choose the optimal width of an insert. As it is seen in table 1 the maximal electron mobility was observed in sample 3 with the wide InAs insert.

### Conclusion

We have measured the electron effective masses $m^*$ separately in the every subbands and found that $m^*$ decreases due to InAs insert to the center of the quantum well. We observed the electron mobility enhancement due to the InAs insert of the center of the quantum well. Our numerical calculations indicate that the electron mobility decreases with the subband number. The mobility in the 2nd subband is less because the wave function spread to the delta-layer where charged impurity scattering is higher. The calculated transport mobility $\mu_t$ is much higher than the quantum mobility $\mu_q$. This fact indicates the essential role of the small-angle scattering which is a characteristic of the Column scattering in the investigated structures.

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