Landauer’s principle at zero temperature

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The second law

- The 1st law puts heat and work on similar footing and says that, in principle, one can be interconverted into the other.

- For a system coupled to two baths, for instance, we have:

  \[
  \frac{dU}{dt} = \dot{Q}_h + \dot{Q}_c + \dot{W}
  \]

- Not all such processes are possible, however:
  - This is the purpose of the 2nd law.
• The 2nd law deals with entropy.
  
  • *Entropy, however, does not satisfy a continuity equation.*

• There can be a flow of entropy from the system to the environment, which is given by the famous Clausius expression $\dot{Q}/T$.

• But, in addition, there can also be some entropy which is spontaneously *produced* in the process. The entropy balance equation thus reads

$$ \frac{dS}{dt} = \dot{\Sigma} + \frac{\dot{Q}_h}{T_h} + \frac{\dot{Q}_c}{T_c} $$

• The quantity $\dot{\Sigma}$ is called the *entropy production rate*.

• The second law can now be formulated mathematically as:

$$ \dot{\Sigma} \geq 0 $$
Why entropy production matters

- 1st and 2nd laws for a system coupled to two baths:

\[
\frac{dU}{dt} = \dot{Q}_h + \dot{Q}_c + \dot{W} = 0
\]

\[
\frac{dS}{dt} = \dot{\Sigma} + \frac{\dot{Q}_h}{T} + \frac{\dot{Q}_c}{T} = 0
\]

Carnot’s statement of the 2nd law

“The efficiency of a quasi-static or reversible Carnot cycle depends only on the temperatures of the two heat reservoirs, and is the same, whatever the working substance. A Carnot engine operated in this way is the most efficient possible heat engine using those two temperatures.”

- Entropy production is therefore the reason the efficiency is smaller than Carnot:

\[
\eta = \eta_C - \frac{T_c}{\dot{Q}_h} \dot{\Sigma}
\]
Entropy and information

- In information theory, *entropy* acquires a different interpretation.
  - $S$ = amount of ignorance (lack of information) one has about the system.

- If the system is described by a set of states $n = 1, 2, 3, \ldots$, each with probability $p_n$, the Shannon entropy is defined as

$$S = - \sum_{n} p_n \ln p_n$$

- The quantity $-\ln p_n$ is called the “surprise” of the state:
  - It measures how surprised we are to observe the system in state $n$.
    - If $p_n \sim 1 \rightarrow -\ln p_n \sim 0$ (no surprise at all)
    - If $p_n \sim 0 \rightarrow -\ln p_n \gg 1$ (huge surprise)

- Entropy is thus the “average surprise”. 😞
• To have a concrete example, consider a particle which can be found in 1 of 2 sides of a box:

• Suppose we have some ignorance about the system:
  • We do not know which side the particle is.
• Let \( (p_R, p_L) \) be the probabilities of finding it on the left or on the right.
• Consequently, the "state" of the particle (i.e., left or right) has some entropy associated to it:

\[
S = -p_R \ln p_R - p_L \ln p_L
\]

• e.g. maximum ignorance: \( p_R = p_L = 1/2 \) and thus

\[
S = \ln 2
\]
Consider now the following procedure:

Irrespective of the initial state, the final state is always “left”:

\[(p_R, p_L) = (0, 1)\]

Hence the final entropy is zero:

\[S = 0\]

We call this information erasure because any initial information about where the particle was is now forever lost.
Landauer’s principle

- Landauer’s principle states that there is a fundamental heat cost associated with information erasure.

  - To erase information one must pay an energy bill:

    \[ \Delta Q_E \geq -T\Delta S_S \]

  - \( \Delta S_S \) is the change in entropy of the system

  - \( \Delta Q_E \) is the amount of heat flowing to the environment
    \((\Delta Q_E > 0 \text{ when energy leaves the system})\).

- Landauer’s bound: minimum heat cost for erasing information.

  - For instance, if \( \Delta S_S = 0 - \ln 2 \), we find

    \[ \Delta Q_E \geq T \ln 2 \]

- Landauer worked for IBM. In computing terms, this is the energy cost required to erase one bit of information.
Information is physical

• Landauer’s principle is often used to argue that “information is physical”.
  
  • Information is physical because information is *stored* in physical systems and *communicated* using physical systems.

• Information theory is thus not purely mathematical.

• Landauer’s principle also highlights a fundamental *irreversibility* of physical processes:
  
  • If $\Delta S_S \geq 0$ then $\Delta Q_E \geq -T\Delta S_S$ does not impose any restrictions.
    
    • There is no energy cost to acquire information.

  • But there is an energy cost to erase it.
Landauer’s principle is a consequence of the 2nd law
Landauer from the 2nd law

- Recall the 2nd law for a system coupled to a single bath:
  \[
  \frac{dS_S}{dt} = \dot{\Sigma} - \frac{\dot{Q}_E}{T}, \quad \dot{\Sigma} \geq 0
  \]
- I changed the sign of \( \dot{Q}/T \) here because \( \dot{Q}_S = -\dot{Q}_E \)
- We integrate over some interval of time, leading to
  \[
  \Sigma = \Delta S_S + \frac{\Delta Q_E}{T} \geq 0
  \]
- This looks exactly like Landauer’s principle:
  - It is a direct consequence of the 2nd law \( \Sigma \geq 0 \).
Microscopic formulation of Landauer’s principle

• But this is tricky because:

  • Landauer’s bound is defined for the Shannon entropy and for systems of arbitrary size, such as a single bit.

  • The second law uses the thermodynamic entropy and holds only for macroscopic bodies.

• To connect the two universes we must construct a microscopic theory of thermodynamics that is capable of extending the 2nd law to the microscopic domain.

• In this talk I will focus on the quantum version, as it encompasses the classical theory as a particular case.

  • This is the theory we call Quantum Thermodynamics.
Entropy production in quantum systems

- All information about a quantum system is contained in its density matrix $\rho$.

- Entropy is now quantified by the von Neumann entropy:
  $\quad S(\rho) = - \text{tr}(\rho \ln \rho)$

- We consider two quantum systems, $S$ and $E$ (the "environment") in arbitrary states $\rho_S$ and $\rho_E$.
  - $S$ and $E$ can have any size: generalization of the bath concept.

- The two then interact with a unitary $U$, leading to
  $\quad \rho'_{SE} = U(\rho_S \otimes \rho_E) U^\dagger$

- This is the quantum version of a system interacting with a bath.

- What is the entropy production?
• The dynamics is unitary and so in principle one could say it is reversible.
  
  • Indeed, if all of $\rho_{SE}'$ is accessible, everything would be reversible.

• “Irreversibility” depends on which degrees of freedom become inaccessible after the interaction.

• There are many possibilities:
  
  • System-bath correlations become, in practice, inaccessible.
  
  • Any changes we make in the bath may also not be recoverable.
  
  • If measurements are done in the system, quantum coherence may also be lost.
  
  • etc.

• Each of these features can be gauged using a certain information-theoretic quantifier.

M. Esposito, K. Lindenberg, C. Van den Broeck, “Entropy production as correlation between system and reservoir”. New Journal of Physics, 12, 013013 (2010).

G. Manzano, J. M. Horowitz, J. M. R. Parrondo, “Quantum fluctuation theorems for arbitrary environments: adiabatic and non-adiabatic entropy production”, Physical Review X, 8, 031037 (2018).
• The choice of entropy production which is closest to the classical formulation is:

\[ \Sigma = I'(S : E) + D(\rho_E' || \rho_E) \]

where

\[ I'(S : E) = S(\rho_S') + S(\rho_E') - S(\rho_{SE}') = \text{SE correlations} \]

\[ D(\rho_E' || \rho_E) = \text{tr}(\rho_E' \ln \rho_E' - \rho_E' \ln \rho_E) = \text{change in the bath} \]

• This formula can be taken as a general definition of entropy production for an arbitrary system+bath interaction process.

  • The system and bath can have any size.
  
  • Their states are arbitrary, except that they start in any (product) state.
  
  • They interact with an arbitrary unitary \( U \).

• Assuming that the bath is in a thermal state, one finds:

\[ \Sigma = I''(S : E) + D(\rho_E' || \rho_E) = \Delta S_S + \beta \Delta Q_E \geq 0 \]
In the last 5 years there have been several papers which generalized/improved Landauer’s original result:

- J. Goold, M. Paternostro and K. Modi, *Phys. Rev. Lett.* **114**, 060602 (2015).

- G. Guarnieri, S. Campbell, J. Goold, S. Pigeon, B Vacchini and M. Paternostro, *NJP*, **19**, 103038 (2017).

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- P. Strasberg, G. Schaller, T. Brandes and M. Esposito, *Phys. Rev. X.*, **7**, 021003 (2017).

- S. Campbell, G. Guarnieri, M. Paternostro and B. Vacchini, *Phys. Rev. A.*, **96**, 042109 (2017).
Experimental verification of Landauer’s principle linking information and thermodynamics
Antoine Bérut, Artak Arakelyan, Artyom Petrosyan, Sergio Ciliberto, Raoul Dillenschneider & Eric Lutz

Quantum Landauer erasure with a molecular nanomagnet
R. Gaudenzi, E. Burzuri, S. Maegawa, H. S. J. van der Zant & F. Luis

Experimental test of Landauer’s principle in single-bit operations on nanomagnetic memory bits
Jeongmin Hong, Brian Lambson, Scott Dhuey, Jeffrey Bokor

High-Precision Test of Landauer’s Principle in a Feedback Trap
Yonggun Jun, Momčilo Gavrilo, and John Bechhoefer
Department of Physics, Simon Fraser University, Burnaby, British Columbia V5A 1S6, Canada
(Received 15 August 2014; published 4 November 2014)
Quantum Landauer erasure with a molecular nanomagnet

R. Gaudenzi, E. Burzuri, S. Maegawa, H. S. J. van der Zant and F. Luis

Experimental demonstration of information to energy conversion in a quantum system at the Landauer limit

J. P. S. Peterson, R. S. Sarthour, A. M. Souza, I. S. Oliveira, J. Goold, K. Modi, D. O. Soares-Pinto and L. C. Céleri

Single-Atom Demonstration of the Quantum Landauer Principle

L. L. Yan, T. P. Xiong, K. Rehan, F. Zhou, D. F. Liang, L. Chen, J. Q. Zhang, W. L. Yang, Z. H. Ma and M. Feng
Trouble at $T \to 0$
Trouble at $T \rightarrow 0$

- Landauer’s bound becomes trivial in the limit $T \rightarrow 0$:

  $$\Delta Q_E \geq -T\Delta S_S \quad \rightarrow \quad \Delta Q_E \geq 0$$

- All it says is that the heat cost for erasure is non-negative. But otherwise, it is \textit{independent} of the amount of erasure.

- Take, as an example, spontaneous emission:
  
  - To erase information about an atom, we must emit a photon.
  
  - Energy therefore \textit{was} emitted. But the bound does not capture it.
A tighter Landauer bound

- Now I would like to show how it is possible to derive a modified bound, which:
  - Is always tighter than the original.
  - Tends to it at high temperatures.
  - But yields non-trivial information when $T \to 0$.
- Landauer’s bound stems from the positivity of:
  \[ \Sigma = I'(S : E) + D(\rho'_E || \rho_E) \geq 0 \]
- Instead, we focus only on the positivity of:
  \[ I'(S : E) \geq 0 \]
Derivation

• The initial bath state is thermal.
  • But its final state $\rho'_E = \text{tr}_S(\rho'_{SE})$ is not.

• Define a reference thermal state $\rho_E(T')$ which is thermal, but at a temperature $T'$ such that

$$\text{tr}\{H_E\rho_E(T')\} = \text{tr}\{H_E\rho'_E\} := E_E(T')$$

• From the MaxEnt principle $S(\rho_E(T')) \geq S(\rho'_E)$ so that

$$\Delta S_S + \Delta S_E^{\text{th}} \geq \Delta S_S + \Delta S_E$$

$$= I'(S : E)$$

$$\geq 0$$

• Thus

$$\Delta S_S + \Delta S_E^{\text{th}} \geq 0$$

• This is the 1st result we will need.
• Next define the functions

\[ Q(T') = \Delta Q_E = \int_T^{T'} C_E(\tau) d\tau \quad \text{and} \quad S(T') = \Delta S_E^{\text{th}} = \int_T^{T'} \frac{C_E(\tau)}{\tau} d\tau \]

• Here \( C_E(T) \) is the equilibrium heat capacity of the bath.

• We may then write

\[ \Delta S_E^{\text{th}} = S(T') = S(Q^{-1}(\Delta Q_E)) \]

• Plugging this in \( \Delta S_S + \Delta S_E^{\text{th}} \geq 0 \) we then finally get

\[ S(Q^{-1}(\Delta Q_E)) \geq - \Delta S_S \]

• Finally, inverting \( S \) and \( Q \), we get:

\[ \Delta Q_E \geq Q(S^{-1}(-\Delta S_S)) \]
About this modified bound

• Our new bound is:

\[ \Delta Q_E \geq \mathcal{Q}(\mathcal{S}^{-1}(-\Delta S_S)) \]

• It is identical in spirit to Landauer’s original bound (same assumptions).
  
  • Landauer’s is universal because it assumes only 1 thing: that the bath is thermal and has a temperature \( T \).

• Compared to Landauer, we require only one additional piece of information: the bath equilibrium heat capacity \( C_E(T) \).
  
   • However, our bound is always always tighter.

     • Requires a bit more info, but is also always better.
Applications
Cavity QED

- Consider a 2-level atom interacting with an optical cavity field via the Rabi model:

\[
H = \hbar \omega a^\dagger a + \frac{\hbar \omega}{2} \sigma_z + \hbar g (a + a^\dagger) \sigma_x
\]

- The atom is the system and the cavity is the bath.

\[
\frac{T}{\omega} = 0.01 \quad \frac{T}{\omega} = 0.1 \quad \frac{T}{\omega} = 0.4
\]
Next consider the emission of the atom into a 1D waveguide of length $L$.

The waveguide is characterized by a continuum of modes with dispersion relation $\omega_k = ck$. Hence

$$E_E(T) = \sum_k \frac{\hbar \omega_k}{e^{\beta \omega_k} - 1} = \frac{\pi L}{12 \hbar c} T^2$$

Similarly, the entropy can be found to be

$$S_E(T) = \frac{\pi L}{6 \hbar c} T$$

Thus

$$\Delta Q_E \geq -T \Delta S_S + \frac{3 \hbar c}{\pi L} \Delta S_S^2$$

The first term is the original bound; it vanishes when $T \to 0$

But the 2nd remains.
Heat capacity examples

- We can also provide explicit forms for the bound by assuming different scalings for the bath’s heat capacity.

- We focus on $T = 0$.

- Phonons: $C_E = aT^3$

\[ \Delta Q_E \geq \frac{3^{4/3}}{4} \frac{(-\Delta S_S)^{4/3}}{a^{1/3}} \]

- Gapped system (e.g. BCS superconductor): $C_E = be^{-\delta/T}$

\[ \Delta Q_E \geq \delta \frac{(-\Delta S_S)}{\ln(-b/\Delta S_S)} \]
Summary

- Landauer’s bound $\Delta Q_E \geq - T \Delta S_S$ provides a fundamental link between thermodynamics and information.

- The bound is powerful because it is universal.
  - Assumes minimal information about the system and process.

- But it trivializes when $T \to 0$.

- In this talk I showed how one can derive a modified bound which
  - Is always tighter than the original.
  - Tends to it at high temperatures.
  - Yields non-trivial information when $T \to 0$.

- The bound has exactly the same spirit as the original and assumes only 1 additional ingredient: knowledge of the bath’s heat capacity.
Landauer’s Principle at Zero Temperature

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Thank you ❄️🔄🔥🙃
Extra slides
Flow of heat

• The 2nd law reads

\[ \dot{\Sigma} = -\frac{\dot{Q}_h}{T_h} - \frac{\dot{Q}_c}{T_c} \geq 0 \]

• But if there is no work involved,

\[ \dot{Q}_c = -\dot{Q}_h \]

\[ \therefore \dot{\Sigma} = \left(\frac{1}{T_c} - \frac{1}{T_h}\right) \dot{Q}_h \geq 0 \]

• Heat flows from hot to cold.

Clausius’ statement of the 2nd law

“Heat can never pass from a colder to a warmer body without some other change, connected therewith, occurring at the same time.”
Work from a single bath

- Finally, suppose there is only one bath present:

\[ \dot{W} = -\dot{Q}_h \]

\[ \dot{\Sigma} = -\frac{\dot{Q}_h}{T_h} = \frac{\dot{W}}{T_h} \geq 0 \]

- Positive work (in my definition) means an external agent is *doing* work on the system.

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Kelvin-Planck statement of the 2nd law

“It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work.”
Information content

• Suppose you have an object in your hand, such as a coin or a deck of cards.
  • What is the information content of this object?

• Call a friend:
  • The two of you share some background information to make information possible (like establishing what a “coin” is).
  • But your friend does not know the state of the object (e.g. if the coin is Head or Tails).
  • Information content = size of the set of instructions that your friend requires to be able to reconstruct the state of the object.

M. B. Plenio, V. Vitelli, “The physics of forgetting: Landauer’s erasure principle and information theory”, Contemporary Physics 42, 25 - 60 (2001).
• We can understand Landauer’s principle using basic thermodynamics.

• The particle is an ideal gas so $pV = T$

  ![Diagram of particle compression]

• The work in an isothermal compression is

  $$W = - \int_{V_1}^{V_2} pdV = T \ln \frac{V_1}{V_2}$$

• Minimum work is when one compresses up to the middle of the partition, so $V_1 = 2V_2$

  $$W_{\text{min}} = T \ln 2$$

• The energy of the particle $U(T)$ remains constant, so this work must be converted into heat:

  $$\Delta Q_E = W_{\text{min}} = T \ln 2$$
Maxwell demon

Seem to violate the 2nd law.

- Bennet used Landauer’s principle to solve this paradox:
- There is no energy cost for the demon to acquire information.
- But there is an energy cost to erase it!

Kelvin-Planck statement of the 2nd law

“It is impossible to devise a cyclically operating device, the sole effect of which is to absorb energy in the form of heat from a single thermal reservoir and to deliver an equivalent amount of work.”
Thermal case

• Let us check that we indeed recover the standard expressions when the bath is thermal.

• Since the dynamics is unitary

\[ S(\rho'_{SE}) = S(\rho_S \otimes \rho_E) = S(\rho_S) + S(\rho_E) \]

• Whence

\[ I'(S : E) = S(\rho'_S) + S(\rho'_E) - S(\rho'_{SE}) \]

\[ = S(\rho'_S) + S(\rho'_E) - S(\rho_S) - S(\rho_E) \]

\[ = \Delta S_S + \Delta S_E \]

• The entropy production will then be

\[ \Sigma = \Delta S_S + S(\rho'_E) - S(\rho_E) + D(\rho'_E || \rho_E) \]
• Opening up the last terms:

\[
S(\rho'_E) - S(\rho_E) + D(\rho'_E | | \rho_E) = - \text{tr}(\rho'_E \ln \rho'_E) + \text{tr}(\rho_E \ln \rho_E) \\
+ \text{tr}(\rho'_E \ln \rho'_E - \rho'_E \ln \rho_E) \\
= \text{tr}\{(\rho_E - \rho'_E)\ln \rho_E\}
\]

• This result is still general. Now we assume that the bath is in a thermal state

\[
\rho_E = \frac{e^{-\beta H_E}}{Z_E}
\]

• This is then seen to be the heat entering the bath:

\[
\text{tr}\{(\rho_E - \rho'_E)\ln \rho_E\} = \beta \Delta Q_E = \beta(\langle H_E' \rangle - \langle H_E \rangle)
\]

• Whence:

\[
\Sigma = \Delta S_S + \beta \Delta Q_E
\]