Numerical simulation of solitary wave attenuation by vegetation with non-hydrostatic model

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Abstract. In this paper solitary wave attenuation by vegetation is investigated by using numerical simulation. To take into account effects of dispersion, we use a dispersive and nonlinear wave mode; the non-hydrostatic with 1 vertical layer. The damping by vegetation is modelled by using mean drag force using Morison’s formula. The dissipative term is added in the momentum equation. The wave model is implemented numerically using finite volume with momentum conservative staggered grid scheme. To test the numerical scheme implementation, we simulate a physical experiment of solitary wave attenuation by vegetation of Huang et al. 2011. Comparison with experimental data shows that the results of simulation agree quite well.

1. Introduction
Tsunami waves are usually generated by earthquakes, underwater volcanic eruption and landslides. During its generation process, long waves with relatively small amplitudes, are generated. When reaching the shallower area, the wave becomes shorter but with an amplified wave height, i.e. shoaling effect, results in devastating damage in coastal area.

Coastal vegetation has been considered as a natural barrier for coastal protection. The 2004 Indian Ocean tsunami shows that mangrove and coastal vegetation may reduce tsunami wave runup [12]. Researches in this area are including approaches via physical experiment, field observatory, and numerical simulation. Irem et. al. 2009 [4], by using physical experiment, shows the coastal vegetation that are planted in a shore can reduce wave runup. Irish et al. 2014 [3] conducted laboratory experiments to model the coastal vegetation, where multiple small cylinder patches are used for representing coastal vegetation to study the tsunami wave runup. Huang et al. 2011 [2] investigates the interaction between solitary wave with rigid vegetation in a wave flume, both using physical experiment as well as numerical approach with Boussinesq model.

Approach by using numerical simulation is to model wave energy dissipation by the presence of the vegetation. Tang et al 2013 [13] models the damping effects of vegetation by using the non-dispersive Nonlinear Shallow Water Equations (NSWE), where solitary wave simulation and long wave runup are investigated. Kobayashi et al [5] study the wave attenuation by vegetation analytically, based on solution of continuity and linearized momentum equation, where small amplitude wave is considered. Yao et al 2018 [15] study the damping effects of vegetation on a sloping bottom. Here, the
vegetation damping is modeled by using drag force in the momentum equation in the fully nonlinear Boussinesq model.

The aim of this paper is to model the dissipation effect by the presence of the vegetation in a dispersive wave model. In this paper, instead of using non-dispersive NSWE and Boussinesq type of model, here we present an alternative model, which takes into account effects of dispersion, i.e. the non-hydrostatic model. The model is solved numerically by using finite volume with momentum conservative staggered grid as proposed by Stelling & Zijlema 2003 [11]. The dissipation effect by the presence of the vegetation is modeled by using Morison’s formula [7], by adding a source term in the momentum equation. To test the accuracy of the modelling and implementation, we reconstruct physical experiment of solitary wave propagation with emergent vegetation by Huang et al. 2011 [2]. The main difference of results in this paper with the numerical simulation that is performed by [2] is the wave model that is used. In [2], they use the Boussinesq type of model as a wave model, whereas in this paper is the non-hydrostatic model.

The content of this paper is as follows. In the next section, we discuss the non-hydrostatic wave model and its proposed numerical implementation by using predictor corrector method, as well as the implementation of a source term for representing drag force for the vegetation. In Section 3, we recall the experimental setup of Huang et al 2011 [2] and numerical simulation to reconstruct the experiment. Results and discussions regarding qualitative and quantitative comparison between results of simulation with experimental data are discussed in Section 4. Finally, we conclude the paper in Section 5.

2. Wave model and numerical implementation
2.1. Non-hydrostatic model
In this paper, to take into account effects of dispersion, we use the non-hydrostatic wave model. Let $x$ and $z$ denote the horizontal and vertical coordinates, respectively, and $t$ the time. We denote the surface elevation, the depth averaged horizontal velocity and the vertical velocity as $\eta(x,t)$, $u(x,t)$ and $w(x,t)$, respectively. The ideal fluid is bounded by an impermeable bottom at $z = -d(x,t)$ and the surface $z = \eta(x,t)$, with the total depth denoted as $h(x,t) = \eta(x,t) + d(x,t)$. In this paper, we only consider 1 vertical layer of non-hydrostatic model. Following [11,14], we recall the 1 dimensional (1D) non-hydrostatic wave model for 1 layer is given as the following system

$$\partial_t h + \partial_x (hu) = 0$$  \hspace{1cm} (2.1)

$$\partial_t u + g \partial_x \eta + u \partial_x u + R_{bot} + R_{veg} = \int_{-d}^{\eta} \partial_x P \, dz$$  \hspace{1cm} (2.2)

$$\partial_t w_1 = \frac{1}{h} \int_{-d}^{\eta} \partial_z P \, dz$$  \hspace{1cm} (2.3)

$$\partial_x u + \partial_z w = 0$$  \hspace{1cm} (2.4)

where $g = 9.81 \text{m/s}^2$ denotes the gravity acceleration and $P$ the hydrodynamic pressure. The first two equations are the continuity and horizontal momentum equations, respectively. The last two equations are the linearized vertical momentum and the Euler relation from incompressibility assumption, respectively. $R_{bot}$ and $R_{veg}$ denote dissipation terms by the bottom roughness and by vegetation. In this paper, the set of equations (2.1 – 2.4) is solved by using finite volume with staggered grid scheme as firstly introduced by [11], then later is used by [14] and [1]. The main idea for the proposed numerical implementation for solving the hydrodynamic pressure is to use the predictor-corrector method. The idea is as follows. Firstly, we solve the hydrostatic model; eq. (2.1) and (2.2), which means that the hydrodynamic pressure in the right hand side of (2.2) is set to be zero. From solving the hydrostatic model, we obtain prediction values for $u$ and $h$. Secondly, by using the predicted values, we use the equation (2.3) & (2.4) for calculating hydrodynamic pressure $P$ where the calculated pressure $P$ is then used for correcting the values of $u$ in (2.2). To implement the proposed procedure, we solve the system (2.1-2.4) by using staggered grid scheme.

Let us define $\eta^i = \eta(x_i, t^n)$ and $u^n_{i+1/2} \approx u(x_{i+1/2}, t^n)$ are approximation values for $\eta$ and $u$ at discretized horizontal domain $x \in [0, L]$, with discrete points $x_i$ with $i = 1, 2, ..., N$. Here, $x_i$ are
called as Full-grid and grid points that are in between the full-grid as half-grid, i.e. \( x_{i+1/2}, \ i = 1, 2, \cdots, N + 1 \). In the staggered grid scheme, variables \( \eta, h, P \) and \( w \) are placed in the full-grid, while \( u \) is in half-grid, see figure 1. \( P_{i,j} \) and \( w_{i,j} \) denote the hydrodynamic pressure and vertical velocity at the full grid \( i \) at layer-\( j \). For 1 layer non-hydrostatic model, pressure free condition is assumed at the surface and impermeability flow along the bottom, such that \( P_{1,1} = 0 \) and \( w_{1,2} = 0 \). We define the horizontal spatial discretization and temporal discretization as \( \Delta x \) and \( \Delta t \).

\[
\begin{align*}
\eta_{i+1/2} &= \eta_{i} + \frac{\Delta t}{\Delta x} \left( \frac{u_{i+1/2}^n + u_{i-1/2}^n - 2u_{i}^n}{2} \right) \\
u_{i+1/2}^n &= \frac{u_{i+1}^n + u_{i}^n}{2} + g \left( \frac{\eta_{i+1}^n - \eta_{i}^n}{\Delta x} \right) + \left( u \partial_x u \right)_{i+1/2}^n + R_{bot}(i+1/2) + R_{veg}(i+1/2) \\
\end{align*}
\]

The advection term \( u \partial_x u \) in (2.6) is calculated via a horizontal momentum \( q = h u \), i.e. \( u \partial_x u = \frac{1}{h} \left[ \partial_x (qu) - u \partial_x q \right] \). As proposed by [10], the advection term can be calculated by the following formula.

\[
(u \partial_x u)_{i+1}^n = \frac{1}{h_{i+1/2}^n} \left( \frac{u_{i+1}^n - u_{i}^n}{\Delta x} - u_{i+1/2}^n \frac{q_{i+1}^n - q_{i}^n}{\Delta x} \right)
\]

Where \( q \) and \( h \) are values at half grid that are defined by a simple averaging. The value of \( u_{i+1}^n \) is calculated by using upwind method, see [11, 14]. Now the hydrostatic model (2.5) & (2.6) can be solved directly for given initial conditions. For the second step, we need to include the hydrodynamic pressure \( P \) for representing non-hydrostatic model. To that end, by using a relation from the linearized vertical momentum (2.3), we need to solve eq. (2.4) in order to calculate \( P \). As derived in [14], by using Leibnitz’s integral rule and an approximation by using trapezoidal rule, the expression of hydrodynamic pressure in (2.2) can be expressed as \( \int_{-d}^{h} \partial_x P dz = -\frac{1}{2} \partial_x P_2 \), and is calculated as follows.

\[
\frac{1}{2} \partial_x P_2 \approx \frac{P_{i+1/2}^n - P_{i-1/2}^n}{\Delta x}.
\]

By using Keller-box scheme [14], the linearized vertical momentum eq. (2.3) leads to

\[
\frac{1}{2} (\partial_t w_1 - \partial_t w_2) = -\frac{P_{i-1}^n - P_{i}^n}{h}.
\]
Taken into account permeability condition at bottom and pressure free at surface, i.e. \( w_2 = 0 \) and \( P_1 = 0 \), then the discrete formula of (2.10) is given as follows
\[
\frac{w_{i-1}^{n+1} - w_{i}^{n+1}}{2\Delta t} = \frac{p_{i}^{n+1}}{h_{i}^{n+1}} \tag{2.11}
\]

Finally, the eq. (2.4) is calculated by using a Keller box scheme results in \( \partial_x u + w/t/h = 0 \), which can be calculated by using discrete formula as follows.
\[
\frac{u_{i+1/2}^{n+1} - u_{i-1/2}^{n+1}}{\Delta x} + \frac{w_{i}^{n+1}}{h_{i}^{n+1}} = 0 \tag{2.12}
\]

In summary, the hydrodynamic pressure \( P \) is obtained by solving (2.12) by using relation that is obtained from (2.11). The resulting pressure \( P \) is used for calculating hydrodynamic pressure in (2.9).

2.2. Damping by vegetation

In this subsection, we discuss an implementation of wave damping effect by vegetation. The Morison’s formula [7] is adopted for representing wave force impact on pile structure. Following [6], the resistance force by vegetation is modeled as force due to multiple piles structure. Average force by vegetation per unit volume is given by
\[
F = F_D + F_I = \frac{1}{2} \rho C_D b_v N u |u| + \rho C_M v_p t_v \partial u \tag{2.13}
\]

Where \( F_D \) and \( F_I \) are forces due to drag and inertia, respectively. Drag and inertia coefficients are denoted by \( C_D \) and \( C_M \), respectively. \( N \) denotes the number of stems per unit area or vegetation density, \( b_v \) and \( t_v \) are the width and thickness of the vegetation, respectively, \( \rho \) denotes the fluid density. The relative importance of inertial term in (2.9) can be approximated by the Keulegan & Carpenter (\( K_C \)) number as \( K_C = u_{max} T/d \), where \( u_{max} \) is maximum fluid velocity, \( T \) is the wave period and \( d \) is the diameter of pile, in this case it is the vegetation tree trunk. The larger \( K_C \) number, the drag force is more dominant than the inertia force. Since we only consider a solitary wave for representing tsunami wave, i.e. long wave period, then the inertia force in (2.9) can be neglected.

Following [2], for including drag force a body force in a dispersive wave model, we used grid-averaged velocity in \( u \), so that \( R_{\text{avg}} \) is defined as follows
\[
R_{\text{avg}} = \frac{1}{2} C_D d_t N_t u_p |u_p| \tag{2.14}
\]

Where \( d_t \) is the averaged diameter of tree trunk or the root, \( N_t \) is number of trees per unit bottom area, and \( u_p \) is the pore velocity, defined as \( u_p = u/(1 - \phi) \), with \( \phi \) is the solid volume portion of the trees. Formula (2.9) can be calculated in the half-grid as follows
\[
R_{\text{avg}} = \frac{1}{2} C_D d_t N_t u_p |u_p| \left| \left( u_p \right)_{i+1/2} \right| \approx \frac{1}{2} C_d d_t N_t \left| u_p \right|_{i+1/2} \left| u_p \right|_{i+1/2} \tag{2.15}
\]

To test the accuracy of the numerical implementation, we reconstruct a physical experiment of [2], i.e. a solitary wave attenuation by parameterized vegetation.

3. Experimental setup and numerical simulation

The experiments of Huang et al. 2011 [2] are performed in a glass-walled flume with 32m length and 0.55m wide. The vegetation is made from Perspex tubes with diameter of 0.01m with one block of the model is with length of 0.545m. Three configurations of model block are performed, i.e. with 1, 2 and 3 blocks of vegetation model, with vegetation widths are 0.545, 1.090, and 1.635m. Several wave probes for measuring wave signals are placed in the flume. An illustration of the locations of the probes and vegetation block configurations are shown in figure 2. A uniform water depth of 0.15m is set. In this paper, we only consider experiment B2, i.e. the experiment with solid volume portion \( \phi = 0.087 \), vegetation width 1.090 m, the incident wave height is 0.03 m, and length between Perspex tubes is 0.03 m. Here, \( N_t \) and \( d_t \) are 340.625 and 0.01, respectively.

An analytical solitary wave is placed with its center at \( x = -5 \) m. The solitary wave is obtained by using the following formula
\[ \eta(x, t) = H \sech^2 \left( \frac{3}{4} \frac{H}{d^2} (x - x_0 - ct) \right) \text{, with } C = \sqrt{g \frac{d}{H}} \left( 1 + \frac{H}{2d} \right) \]

Where \(H\), \(C\) and \(d\) are the incident wave height, wave speed and the water depth, respectively. For the reconstructing the experiment B2 in [2], we use the following values; \(H = 0.03 \text{ m}\), \(x_0 = -5 \text{ m}\), and \(d = 0.15 \text{ m}\). As suggested by [2], here we use a mean drag coefficient \(C_D = 1.45\) for the experiment B2.

![Figure 2](image)

**Figure 2.** Layout of experiment B2 by Huang et al. 2011 [2]. Blue pillars indicate the vegetation.

To perform a numerical simulation, we consider a domain of simulation with \(x \in [-12 \text{ m}, 10 \text{ m}]\). At both ends, sponge layers are placed as absorber, i.e. \(x_{\text{sponge}} \in [-12 \text{ m}, -7 \text{ m}]\) for the left boundary and \(x_{\text{sponge}} \in [5 \text{ m}, 10 \text{ m}]\) for the right boundary. For accurately simulate the propagation of the solitary wave, we use spatial discretization \(\Delta x = 0.02 \text{ m}\), with a temporal discretization that satisfies the CFL condition of the numerical implementation, i.e. \(\Delta t = 0.001 \text{ s}\). The simulation is performed for 15 s. In the simulation, we set \(R_{\text{bot}} = 0\), or no dissipation in the bottom.

4. Results and discussions

Results of simulation by using the proposed numerical scheme are shown in figure 3. Here snapshots of the solitary wave propagation at \(t = 5, 5.5, 6.5\) and 7 s are shown. Red dashed line shows the maximum wave height \((\eta_{\text{max}})\) during all time simulation, while green color indicates the location of the vegetation tubes. It is clear from the plot of \(\eta_{\text{max}}\) that solitary wave propagates with a relatively constant wave amplitude. The amplitude is slightly increasing when the wave hitting the vegetation, creating wave reflection that propagates leftward. After passing the vegetation, the solitary wave continues with relatively constant amplitude.

To test the accuracy of the simulation, we compare signals from the simulation with available measurement data at G1 \((x = 0 \text{ m})\) and at G5 \((x = 4.29 \text{ m})\). The comparisons are shown in figure 4. It can be seen that the amplitude of the solitary wave can be well represented by the simulation, while the tails of the solitary wave cannot be followed by the simulation.

| Position \(x\) (m) | Gauge | RMSE    | CorrCoef |
|-------------------|-------|---------|----------|
| 0.00              | G1    | 0.0017615 | 0.97704  |
| 4.29              | G5    | 0.0017032 | 0.95283  |
Figure 3. Snapshots of simulation at $t = 5, 5.5, 6.5, 7 \text{s}$. Dotted line denotes the maximum wave height and green rectangle indicates location of the vegetation.

Figure 4. Signal comparison between results of simulation (solid red line) and experiment B2 by Huang e.a. 2011 (black dots) at G1 and G5.

To get a quantitative comparison between results of the simulation and the experimental data, we compare the Relative Mean Square Error (RMSE) value and Correlation Coefficient (Corr) between two signals that are defined as follows

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_i - \hat{y}_i)^2}{N}} \quad \text{and} \quad Corr(y, \hat{y}) = \frac{\langle y, \hat{y} \rangle}{|y||\hat{y}|} \quad (4.1)$$

Here $y$ and $\hat{y}$ denote the experimental signal and signal from simulation, respectively. The notation $|.|$ and $\langle .. \rangle$ represent $L_2$ norm and inner-product, respectively. Results of the comparison are shown in table 1.
5. Conclusions
In this paper we investigated solitary wave attenuation due to vegetation by using a dispersive and nonlinear wave model, i.e. the non-hydrostatic wave model with 1 vertical layer. The model is implemented numerically by using finite volume with momentum conservative staggered grid scheme. Dissipation effect by vegetation is modelled by using mean drag force by Morison’s formula. The additional term is added into the momentum equation and calculated in the staggered grid scheme. We compare the performance of the numerical implementation by simulating physical experiment of solitary wave attenuation that is performed by Huang et al. 2011 [2]. Results of comparison show that the numerical implementation can represent effects of solitary wave attenuation by vegetation with relatively small error, i.e. RMSE of 0.0017 and correlation coefficient of 0.95.

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