Novel Fuzzy Assignment Problem Using Hexagonal Fuzzy Numbers

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Abstract: In this paper, a hexagonal fuzzy assignment problem employing a fuzzy new ranking technique is applied. This method requires less number of iterations to succeed in an optimality solution compared to the prevailing methods. Numerical examples show the effectiveness of the proposed fuzzy hexagonal assignment problem.

Key words: Hexagonal fuzzy number, Centroids of Centroid ranking methods, Assignment

AMS Subject Classification: 90C05.

1. INTRODUCTION

In real life, the problem of allocating different persons/workers to different jobs may play a major role in different fields. Not everybody has a similar capacity to execute a similar assignment and these distinctive abilities are communicated as far as benefit/cost/time required in executing designated work. Therefore, it is necessary to decide how to assign different jobs to different workers to reduce the cost of performing the job. The allocation problem is one of the main research combinatorial optimization problems.

Fuzzy logic was proposed by Lotfi Zadeh [21] in 1965. The fuzzy hexagon number is a generalized fuzzy number in which the information is incomplete and more complex. Distribution issues play an important role in industry and other real-world applications. In the distribution problem, "m" jobs must be completed by "m" people who can try to work on their own. In this question, $M_{ij}$ refers to the matrix of the cost of assigning the $i^{th}$ job to the $j^{th}$ person. The problem is to find the best recruitment solution to reduce the total cost of performing all the work or to increase profits completely. Most authors use the Hungarian method to solve the allocation problem.

A.Srinivasan and G. Geetharamani [3] first transformed the Fuzzy number into crisp one and using Robust ranking method, then formed a linear programming problem (LPP) and solved using Ones Assignment Method. A. Srinivasan and G. Geetharamani [3] first used a robust method to convert fuzzy numbers into crisp numbers, and then formed a linear programming problem (LPP) and used the mapping method to solve. A. Nagoor Gani and V.N Mohamed [1] transformed a problem with fuzzy parameter to crisp version in the LPP form and solved by simplex methods. A. Nagoor Gani and
R. Abdul Saleem [2] using centroid of centroid of two triangles and rectangle introduced hexagonal fuzzy ranking method and solved Sequential LPP. C.Muralidaran and Dr.B.Venkateswarlu [9], using left and right spread ranking functions found improvised solutions than existing ones. A.Thamaraiselvi and R.Santhi [4] newly introduced Hexagonal Intuitionistic Fuzzy to deal IFTP. Intuitionistic fuzzy problems with six parameters that can be solved by HIFNs. Y. L. P. Thorani, N. Ravi Shankar [23], developed a new algorithm for fuzzy assignment problem based on ranking method.

On the basis of this idea, a distortion method is adopted for the above points. This paper uses the centroid of the centroid method to transform the Hexagonal fuzzy matrix into a crisp matrix, and studies a Hexagonal fuzzy cost matrix with allocation problems, the more realistic problem. The ambiguity allocation problem is reformulated as an obvious problem, so that traditional solutions to the allocation problem can also be applied.

2. PRELIMINARIES

2.1 FUZZY NUMBER [8][10][30]

Generalized regular real number with membership function $\mu_{\tilde{A}} : X \rightarrow [0,1]$ is a fuzzy number $\tilde{A}$ and satisfying the following conditions.

$\tilde{A}$ is a normal, convex set.

$\mu_{\tilde{A}}(x) = 1 & \text{at least } x \in R$ exists.

$\mu_{\tilde{A}}(x)$ is a continuous or piecewise continuous in the range closed interval $[0,1]$

3. HEXAGONAL FUZZY NUMBER [1][2][4][6][24]

Let $\tilde{A}_{HF} = (A_{a_1}, A_{a_2}, A_{a_3}, A_{a_4}, A_{a_5}, A_{a_6}; \nu)$ or $\tilde{A}_{HF} = (A_{a_1}, A_{a_2}, A_{a_3}, A_{a_4}, A_{a_5}, A_{a_6})$ where $A_{a_1} \leq A_{a_2} \leq A_{a_3} \leq A_{a_4} \leq A_{a_5} \leq A_{a_6}$. Real numbers satisfy $A_{a_5} - A_{a_4} \leq A_{a_4} - A_{a_3}$ and $A_{a_6} - A_{a_5} \leq A_{a_5} - A_{a_4}$ if its membership function of hexagonal fuzzy number is $\mu_{\tilde{A}_{HF}}(x)$.

$$
\mu_{\tilde{A}_{HF}}(x) = \begin{cases} 
0, & \text{for } x < A_{a_1} \\
\frac{1}{2} \left( \frac{x - A_{a_1}}{A_{a_2} - A_{a_1}} \right), & \text{for } A_{a_1} \leq x \leq A_{a_2} \\
\frac{1}{2} \left( \frac{x - A_{a_2}}{A_{a_3} - A_{a_2}} \right), & \text{for } A_{a_2} \leq x \leq A_{a_3} \\
1, & \text{for } A_{a_3} \leq x \leq A_{a_4} \\
\frac{1}{2} \left( \frac{x - A_{a_4}}{A_{a_5} - A_{a_4}} \right), & \text{for } A_{a_4} \leq x \leq A_{a_5} \\
\frac{1}{2} \left( \frac{x - A_{a_5}}{A_{a_6} - A_{a_5}} \right), & \text{for } A_{a_5} \leq x \leq A_{a_6} \\
0, & \text{for } x > A_{a_6}
\end{cases}
$$

3.1 GENERALIZED HEXAGONAL FUZZY NUMBER: [1][2][4][6][18][20][21][22].

A generalized hexagonal fuzzy number is defined as 

$\tilde{A}_{HF} = (A_{a_1}, A_{a_2}, A_{a_3}, A_{a_4}, A_{a_5}, A_{a_6}; \nu)$ or $\tilde{A}_{HF} = (A_{a_1}, A_{a_2}, A_{a_3}, A_{a_4}, A_{a_5}, A_{a_6}; \nu)$ where $A_{a_1}, A_{a_2}, A_{a_3}, A_{a_4}, A_{a_5}, A_{a_6}$ are real and $\nu$ is its maximum degree of membership function. The membership function is given below:
3.2 DEFINITION

The set $\mathcal{A}_{3\varphi}$ in the normal real number set is described as a generalized hexagonal fuzzy number, and its membership function has the following characteristics. [6] [7] [8] [18] [20] [21]

Left incremental function, range

Right decrement function, range

Remark 3.2.1. If $\omega = 1$, it is called a normal hexagonal fuzzy number.

Remark 3.2.2. The membership functions $\mu_{\mathcal{A}_{3\varphi}}(x)$ are continuous functions.

Remark 3.2.3. The Hexagonal fuzzy number $\mathcal{A}_{3\varphi}$ is the ordered quadruple

Length of $\mathcal{G}_1$ and length of $\mathcal{G}_2$

Figure 1: Normal Hexagonal fuzzy numbers

4. RANKING OF HEXAGONAL FUZZY NUMBERS [6]

Considering the center of the midpoint of the triangle is $\mathcal{G}_1, \mathcal{G}_2$ and the center of rectangle $\mathcal{G}_3$, the center of the triangle with vertices $\mathcal{G}_1, \mathcal{G}_2$ and $\mathcal{G}_3$ is $(\bar{x}, \bar{y}) = \left( \frac{2\bar{a}_1 + 4\bar{a}_2 + 3\bar{a}_3 + \sqrt{6} \bar{a}_4 + 4\bar{a}_5 + 2\bar{a}_6}{18} \right)$. Ranking function of, $\mathcal{A}_{3\varphi} = (\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3)$.
\[ A_{a_x} / A_{a_y} / A_{a_z} \text{ maps the set of all fuzzy number to a set of real number as } \rho(A) = \sqrt{x_{A}^2 + y_{A}^2} \]

\[ \text{Figure 2: Generalized Hexagonal fuzzy numbers} \]

Find \( \rho(A) \) and \( \rho(B) \) then use the following to arrange the obscured numbers. \( A \) is equivalent to \( B \), when \( \rho(A) \) is equivalent to \( \rho(B) \). \( A \) is less than or equal to \( B \) when \( \rho(A) \) is less than or equal to \( \rho(B) \). \( A \) is greater than or equal to \( B \) when \( \rho(A) \) is greater than or equal to \( \rho(B) \).

\[ \text{MATHEMATICAL FORMULATION OF ASSIGNMENT PROBLEM} \]

Let
\[ X_{ij} = \begin{cases} 1 & \text{if } i^{th} \text{ job is assigned to } j^{th} \text{ facility} \\ 0 & \text{if } i^{th} \text{ job is not assigned to } j^{th} \text{ facility} \end{cases} \]

Subject to
\[ \sum_{j=1}^{m} x_{ij} = 1 \quad j = 1, 2, \ldots, m \]  
\[ \sum_{i=1}^{n} x_{ij} = 1 \quad i = 1, 2, \ldots, m \]  
\[ x_{ij} = 0 \text{ or } 1 \]

If \( C_{ij} \) is the cost or efficiency of assigning \( i^{th} \) task to the \( j^{th} \) machine, \( x_{ij} \) must be a positive integer or zero, and there can only be one integer. Therefore, the position \( x_{ij} = 0 \) or \( 1 \) is automatically satisfied.

Each allocation problem has a matrix called cost matrix \( C_{ij} \), where \( C_{ij} \) is the cost of allocating the first function to capacity. In this article, we call it the distribution matrix and express it as follows.

\[ \begin{bmatrix} C_{11} & C_{12} & \cdots & C_{1n} \\ C_{21} & C_{22} & \cdots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1} & C_{m2} & \cdots & C_{mn} \end{bmatrix} \]

Therefore, the allocation problem can be solved in a simplex way. When each \( ai = bi = 1 \), it also happens to be a \( n \times n \) transport problem. However, since the mapping problem is largely solved, it can be frustrating to try to solve it in a simple way. In fact, there is a very convenient iterative process to solve the allocation problem.
6. PROPOSED METHOD

In order to use the T K method to solve the fuzzy assignment problem, each assignment problem contains a matrix, the rows of the matrix represent personnel, and the work or tasks are performed in columns. The cost of each specific task is the unit element in the matrix. Set the action of the suggested method.

Step-1: Given Hexagonal Fuzzy matrix, using ranking method, convert into crisp matrix, then converting it to a balanced one if it is not.

Step 2: Next, in the cost matrix, the smallest element of each row must be subtracted from all items in the corresponding row, and the smallest element of each column should also be subtracted from all elements in the corresponding column.

Step 3: Then, each row and each column will have at least one zero in the matrix. Determine the first zero or column that appears in the cost matrix. Assume that (i, j) chooses zero. Count the total number of zeros in i\textsuperscript{th} row and j\textsuperscript{th} column (except for the selected number). Select the next zero and count the full number of zeros in the corresponding row and column in the same way. Repeat this process for all zeros in the cost matrix. The allocation will be allocated to the minimum number of zeros obtained using this method. Delete the rows and columns corresponding to the custom cell.

Step 4: Repeat the above process for the remaining expenses in the original plan. Repeat the process until all appointments are completed.

Note: Calculate the number of zeros in the final table. If there is only one zero in the corresponding row and column, map it at this position. If the lowest zero value is repeated in multiple positions, the minimum value is determined from the original cost matrix at the zero position and set at the zero position.

ILLUSTRATION

Four row involves A, B, C and D, and the four column represents Job-1, Job-2, Job-3 and Job-4. A cost matrix \([C_{ij}]\) is given, the elements of which are Hexagonal numbers. The problem is to find a suitable position, so the total cost of recruitment becomes minimal.

| Persons | Jobs       |
|---------|------------|
|         | Job-1      | Job-2       | Job-3       | Job-4       |
| A       | (28,29,30,31,32,1) | (3,5,7,9,10,12,1) | (5,7,8,11,14,27,1) | (3,7,11,15,19,24,1) |
| B       | (2,4,6,9,11,13,1)   | (2,3,4,5,7,9,1)   | (10,12,14,16,20,24,1) | (1,3,9,12,15,17,1) |
| C       | (7,9,11,14,18,22,1) | (6,7,9,11,13,16,1) | (9,11,13,12,18,20,1) | (2,4,6,9,12,15,1) |
| D       | (6,9,12,15,20,25,1) | (23,24,25,25,26,27,1) | (18,19,20,20,21,22,1) | (11,14,17,21,25,30,1) |

\[
\begin{align*}
A & = \begin{bmatrix} 30 & 7.7 & 11.4 & 13.1 \\
B & = & 7.3 & 5 & 15.9 & 9.5 \\
C & = & 13.4 & 10.2 & 13.8 & 8 \\
D & = & 14.4 & 25 & 19.9 & 19.6 \\
\end{bmatrix}
\]

Subtract the smallest number of each row in the corresponding row
Subtract the smallest number of each column in the corresponding column

\[
\begin{bmatrix}
  22.3 & 0 & 3.7 & 5.4 \\
  2.3 & 0 & 10.9 & 4.5 \\
  5.4 & 2.2 & 5.8 & 0 \\
  0 & 10.6 & 5.5 & 5.2
\end{bmatrix}
\]

Subtract the smallest number of each column in the corresponding column

\[
\begin{bmatrix}
  I_1 & I_2 & I_3 & I_4 \\
  22.3 & 0 & 0 & 5.4 \\
  2.3 & 0 & 7.2 & 4.5 \\
  5.4 & 2.2 & 2.1 & 0 \\
  0 & 10.6 & 1.8 & 5.2
\end{bmatrix}
\]

| Persons | Job | No of zero R + C | Value of Zero\(^{th}\) Position |
|---------|-----|------------------|-------------------------------|
| A       | I₂  | 2                | -                             |
| A       | I₃  | 1                | -                             |
| B       | I₂  | 1                | -                             |
| C       | I₄  | 0                | 8 (least)                     |
| D       | I₁  | 0                | 14.4                          |

Now C - I₄ and D - I₁ is having no other zero. Corresponding C - I₄ and D - I₁ Position value are 8 and 14.4 respectively.

Given the minimum value between the actual values of the two zero patterns C - I₄ and D - I₁, it is clear that C - I₄ has the lowest value. Therefore, to put the task in this position, delete the C\(^{th}\) row and I₄ column, and then simplify the cost matrix to

\[
\begin{bmatrix}
  I_1 & I_2 & I₃ \\
  22.3 & 0 & 0 \\
  2.3 & 0 & 7.2 \\
  0 & 10.6 & 1.8
\end{bmatrix}
\]

Repeat the same process by subtracting the smallest element in each row and column of the above matrix, it will reduce to

\[
\begin{bmatrix}
  I_1 & I_2 & I₃ \\
  22.3 & 0 & 0 \\
  2.3 & 0 & 7.2 \\
  0 & 10.6 & 1.8
\end{bmatrix}
\]

| Persons | Job | No of zero R + C | Value of Zero\(^{th}\) Position |
|---------|-----|------------------|-------------------------------|
| A       | I₂  | 2                | -                             |
| A       | I₃  | 1                | -                             |
| B       | I₂  | 1                | -                             |
| D       | I₁  | 0                | -                             |

Here D - I₁ is not another zero. Therefore, to perform the task at this position D - I₁, delete the columns D\(^{th}\) row and I₁\(^{th}\) and then simplify the cost matrix to

\[
\begin{bmatrix}
  I_2 & I₃ \\
  0 & 0 \\
  0 & 7.2
\end{bmatrix}
\]

Repeat the same process by subtracting the smallest element in each row and column of the above matrix, it will be reduced to

\[
\begin{bmatrix}
  I_2 & I₃ \\
  0 & 0 \\
  0 & 2.037
\end{bmatrix}
\]

| Persons | Job | No of zero R + C | Value of Zero\(^{th}\) Position |
|---------|-----|------------------|-------------------------------|
| A       | I₂  | 2                | -                             |
| A       | I₃  | 1                | 11.4                          |
| B       | I₂  | 1                | 5 least                       |
Here, $A - J_2$ and $B - J_3$ have only one zero, and the corresponding values of positions $A - J_2$ and $B - J_3$ are 11.4 and 5 respectively. The lowest value is located at $B - J_2$, then after deleting the $B^2$th and $J_2^2$th columns, the remaining parent matrix will only be reduced to one position $A - J_3$, and then will be allocated at that position.

We can assign assignment solutions is $A - J_3, B - J_2, C - J_4$ and $D - J_1$.

The fuzzy optimal total cost $= \bar{a}_{13} + \bar{a}_{22} + \bar{a}_{34} + \bar{a}_{41} = (5,7,8,11,14,27) + (2,3,4,5,7,9) + (2,4,6,9,12,15) + (6,9,15,20,25) = (15,23,33,40,53,76)$

7. CONCLUSION

This study uses the centroid method to transform the Hexagonal fuzzy assignment problem into crisp assignment problem, and directly uses the traditional knowledge method to obtain the optimal solution. This new method takes less time to solve task problems that are easy to understand and apply. No optimization test is needed because it always obtains the best solution by assigning a function to each row and each column. Numerical examples illustrate the proposed new method and prove that the obtained solution is optimal.

REFERENCES

[1] A. Nagoor Gani and R. Abdul Saleem “A New Ranking Approach on Fuzzy Sequential Linear Programming Problem” International Journal of Pure and Applied Mathematics Volume 117. No. 11 2017, 345-355.

[2] A. NagoorGani and V.N Mohamed, “Solution of a fuzzy Assignment problem by Using a New Ranking Method”, International Journal of Fuzzy Mathematical Archive, Vol.2, 2013,8-16.

[3] A. Srinivasan, G Geetharamani, “Method for solving fuzzy assignment problem using ones assignment method and Robust's ranking technique”, Applied Mathematical Sciences,2013.

[4] A. Thamaraiselvi and R. Santhi “On Intuitionistic Fuzzy Transportation Problem Using Hexagonal Intuitionistic Fuzzy Numbers” International Journal of Fuzzy Logic Systems (IJFLS) Vol.5, No.1, January 2015

[5] A. SahayaSudha and K.R. Vijayalakshmi “A Modern Approach in Ranking of Symmetric Hexagonal Fuzzy Number” Aryabhatta Journal of Mathematics and Informatics [AJMI] Vol.8, Issue2, July-Dec, 2016.

[6] A. Thiruppathi, and Dr. C. K. Kirubhashankar “New Ranking Of Generalized Hexagonal Fuzzy Number Using Centroids Of Centroided Method” Advances in Mathematics: Scientific Journal 9 (2020), no.8, 6229–6240 ISSN: 1857-8365 (printed); 1857-8438 (electronic) https://doi.org/10.37418/amsj.9.8.90.

[7] A. Thiruppathi, and Dr. C. K. Kirubhashankar “Optimal Solution of Novel Fuzzy Assignment Problem Using Hexagonal Fuzzy Numbers” proceedings of the National Conference on Emerging Trends In Mathematical Sciences (NCETMS - 2016)-on 22nd December 2016.

[8] A. Thiruppathi and D. Iranian “An Innovative Method for Finding Optimal Solution to Assignment Problems “International Journal of Innovative Research in Science, Engineering and Technology, Vol. 4, Issue 8, August 2015

[9] C. Muralidaran and Dr. B. Venkateswarlu “Accuracy Ranking Function for Solving Hexagonal Fuzzy Linear Programming Problem” International Journal of Pure and Applied Mathematics Volume 115 No. 9 2017, 215-222 [https://acadpubl.eu/jsi/2017-115-9/articles/9/17.pdf].

[10] Cheng C.H, “A New Approach for Ranking Fuzzy Numbers by Distance Method”, Fuzzy Sets and Systems Vol. 95 (1998), 307-317.

[11] Chu T.C. and Tsao. C T. “Ranking Fuzzy Numbers with an Area between the Centroid Points and the Original Point”. Computers and Mathematics with Applications 43 (2002)111-117.

[12] D. S. Dinagar, K. Thrirupurasundari, “A New Method for Finding the Cost of Fuzzy Assignment Problem Using Genetic Algorithm of Artificial Intelligence”, International Electronic Journal of Pure and Applied Mathematics, 2014.

[13] Dr. M. S. Annie Christi, Mrs. Malini. D, “Solving Transportation Problems with Hexagonal Fuzzy Numbers Using Best Candidates Method and Different Ranking Techniques” Int. Journal of
Engineering Research and Applications ISSN: 2248-9622, Vol. 6, Issue 2, (Part - 4) February 2016, pp.76-81, www.ijera.com.

[14] Dr. Mrs. A.Sahaya Sudha and Mrs.M. Revathy “A New Ranking on Hexagonal Fuzzy Numbers” International Journal of Fuzzy Logic Systems (IJFLS) Vol.6, No.4, October 2016.

[15] G. Nirmala and R. Anju, “cost Minimization Assignment Problem Using Fuzzy Quantifier”, Internal Journal of computer Science and Information Technologies, Vol. 5(6), 2014, pp. 7948-7950.

[16] L.A.Zadesh Fuzzy sets Information and control. Vol. 8, 1965, PP. 338-353.

[17] P.Malini, M.Ananthanarayanan. "Solving Fuzzy Assignment Problem using Ranking of Generalized Trapezoidal Fuzzy Numbers", Indian Journal of Science and Technology, 2016.

[18] P.Rajarajeswari, A.SahayaSudha (2014) Ranking of Hexagonal Fuzzy Numbers using Centroid, AARJMD Vol1 No 17 Pg 265-277.

[19] P.Rao, PhaniBushan, and N.Ravi Shankar,"Ranking fuzzy numbers with an area method using Circumcenter of centroids", Fuzzy Information and Engineering, 2013.

[20] P.Rajarajeswari and A.SahayaSudha “A New Approach for Ranking of Fuzzy Numbers using the Incentre of Centroids” Intern. J. Fuzzy Mathematical Archive Vol.4, No.1,2014,52-60 ISSN:2320–3242(P),2320–3250 Published on 21 April 2014.

[21] P.Rajarajeswari, A.SahayaSudha and R.Karthika, “A New Operation on Hexagonal Fuzzy Number”, International Journal of Fuzzy Logic Systems,3(3),2013,15-26.[27]

[22] S. Narayanaamoorthy, V. Annapoorani and M. Santhiya “A Method for Solving Fuzzy and Intuitionistic Fuzzy Assignment Problem using Ones Assignment Method with Fuzzy Numbers” International Journal of Pure and Applied Mathematics Volume 117 No. 14 2017, 91-99.

[23] Y.L. P.Thorani, N. Ravi Shankar. "Fuzzy assignment problem with generalized fuzzy numbers", Applied Mathematical Sciences, 2013.