Dusty Gas Accretion onto Massive Black Holes and Infrared Diagnosis of the Eddington Ratio

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Abstract

Evidence for dust around supermassive black holes (SMBHs) in the early universe is strongly suggested by recent observations. However, the accretion mechanism of SMBHs in dusty gas is not well understood yet. We investigate the growth of intermediate-mass black holes (IMBHs) of $\sim 10^4$–$10^6$ $M_\odot$ in dusty clouds by using one-dimensional radiative-hydrodynamics simulations. We find that the accretion of dusty gas onto IMBHs proceeds gently with small fluctuations of the accretion rate, whereas that of pristine gas causes more violent periodic bursts. At dust-to-gas mass ratios similar to the solar neighborhood, the time-averaged luminosity becomes smaller than that for primordial gas by one order of magnitude and the time-averaged Eddington ratio ranges from $\sim 10^{-4}$ to $\sim 10^{-2}$ in clouds with initial gas densities of $n_H = 10^{10}$ cm$^{-3}$. Our calculations show that the effect of dust opacity alone is secondary compared to the radiation pressure on dust in regulating the BH growth. We also derive spectral energy distributions at IR bands by calculating dust thermal emission and show that the opacity alone is secondary compared to the radiation pressure on dust in regulating the BH growth. We also derive spectral energy distributions at IR bands by calculating dust thermal emission and show that the opacity alone is secondary compared to the radiation pressure on dust in regulating the BH growth.

Key words: dust, extinction – galaxies: evolution – galaxies: high-redshift – quasars: supermassive black holes – radiative transfer

1. Introduction

Understanding the growth mechanism of massive black holes is one of the major challenges of modern astrophysics. It is known that the number density of supermassive black holes (SMBHs) decreases steeply in the early universe (e.g., Richards et al. 2006). Yet, recent observations have detected SMBHs with masses $\gtrsim 10^6$ $M_\odot$ even at cosmic times less than $\sim 1$ Gyr (Mortlock et al. 2011; Wu et al. 2015). However, the existence of SMBHs at high redshifts alone does not suggest clues about the initial seed mass and the subsequent growth history.

Various mechanisms have been suggested for the formation channel of SMBH seeds, which typically are assumed to have masses $M_{BH} \sim 10^5$ $M_\odot$: (1) growth from stellar-mass BHs of Population III star remnants by gas accretion (Alvarez et al. 2009; Jeon et al. 2012; Park & Ricotti 2013); (2) direct collapse of supermassive stars (Omukai 2001; Begelman et al. 2006; Volonteri & Begelman 2010; Agarwal et al. 2012; Latif et al. 2013; Inayoshi et al. 2014; Sugimura et al. 2014); (3) formation in dense star clusters via collisions among stars (Rees 1978; Portegies Zwart & McMillan 2002; Devecchi et al. 2012; Katz et al. 2015; Yajima & Khochfar 2016). After the formation of SMBH seeds, it is widely believed that further growth proceeds through gas accretion. However, the accretion mechanism has not been understood yet because the accreting gas is subject to feedback from radiation emitted near the accretion disks around BHs.

One of the mechanisms limiting the gas accretion rate is the balance between gravitational attraction and radiation pressure on free electrons, the so-called Eddington limit. Cosmological simulations showed that massive black holes of $\sim 10^5$ $M_\odot$ could grow up to supermassive ones of $\sim 10^9$ $M_\odot$ by $z \sim 6$ using the Eddington-limited Bondi–Hoyle accretion prescription and a simple thermal feedback model (Li et al. 2007; Di Matteo et al. 2008, 2012; Sijacki et al. 2009). Di Matteo et al. (2012) carried out cosmological simulations in large cosmological volumes of $(0.75 \, \text{Gpc})^3$ in comoving units and showed that the accretion rate of SMBHs in rare massive galaxies was near the Eddington limit most of the time, resulting in SMBHs with $\sim 10^9$ $M_\odot$ at $z \sim 7$. However, due to computational limitations, in such cosmological simulations gas dynamics at the Bondi radius (where the gravity of BHs is dominant) is not well resolved, thus requiring sub-grid feedback models. Therefore there is a large uncertainty in the estimation of the gas accretion rate.

Using high-resolution radiative-hydrodynamics simulations resolving the Bondi radius, Milosavljević et al. (2009) showed that the neighboring gas is ionized by the radiation from a central BH, and the thermal pressure of H II regions pushed gas away from the BH against gravity. As a result, the accretion rate was significantly suppressed even at lower luminosities than Eddington. Park & Ricotti (2011) showed that the gas accretion changed periodically and the time-averaged accretion rate was $\sim 1\%$ of the Bondi rate regardless of some parameters, e.g., radiative efficiency, black hole mass, background density (see also Park & Ricotti 2012).

Inayoshi et al. (2016) suggested that the gas accretion rate could exceed the Eddington limit when BHs accrete from extremely high-density gas clouds where the size of ionized bubbles is smaller than the Bondi radius (see also Park et al. 2014a; Sakurai et al. 2016). In addition, assuming anisotropic...
radiative feedback, Sugimura et al. (2017) showed that gas efficiently accretes onto a BH along the shadowed region and the accretion rate exceeds the Eddington limit. Thus, it is still unclear how much the growth of BHs is regulated by radiative feedback.

In local galaxies, it is well known that BH mass tightly correlates with bulge mass or velocity dispersion (Kormendy & Ho 2013). This implies coevolution of BHs with galaxies. Park et al. (2016) showed that the growth rate of massive BHs can be enhanced under the influence of the gravitational potential of the bulge. In addition, as star formation proceeds, gas surrounding a BH is enriched in metals through Type-I/II supernovae and stellar winds. Observations of high-redshift quasars at $z \gtrsim 6$ suggest that a large mass in dust exists around SMBHs. Dust is detected via its thermal emission or dust extinction (Bertoldi et al. 2003; Priddey et al. 2003; Maiolino et al. 2004; Wang et al. 2013). In addition, recent discoveries of high-redshift submillimeter galaxies indicate that some galaxies can become dust-rich at early times (Riechers et al. 2013; Watson et al. 2015). Theoretically, recent simulations show that the metallicity near the galactic centers could reach $\sim 0.01 Z_\odot$, even at $z \sim 10$ (e.g., Wise et al. 2012) or even higher depending on the halo mass (e.g., Ricotti & Gnedin 2005; Ricotti et al. 2016).

The metallicity and amount of dust of massive galaxies in an overdense region could reach the level of the solar neighborhood even at $z \gtrsim 6$ (e.g., Yajima et al. 2015). Therefore BHs were likely to grow in a dusty medium even in the early universe. If dust exists around a BH, radiation from the inner parts of an accretion disk around a BH can be obscured. This changes the observational properties of accreting BHs and the dynamics of accreting gas. In addition to dust opacity, the radiation force on dust can play a role in determining the growth rate of BHs (Ciotti & Ostriker 2007; Hensley et al. 2014; Numekata et al. 2014). However, the interplay between dust and the photoionization feedback, which is the main feedback mechanism suppressing the growth of stellar- or intermediate-mass BHs (IMBHs), has not been studied in sufficient detail. In this work, we investigate the impact of dust on the growth of BHs by using one-dimensional radiative-hydrodynamics simulations resolving both the Bondi radius and ionized bubbles simultaneously. We also estimate self-consistently the thermal emission from dust in our time-dependent models and show that the emission from a hot dust component at $\sim 20 \mu m$ is prominent only during luminosity bursts, while a warmer dust component produces a flux at $100 \mu m$ that is roughly proportional to the mean accretion rate. We thus conclude that the ratio of the flux at $20 \mu m$ to that at $100 \mu m$ is a good proxy for the Eddington ratio.

The paper is organized as follows. We describe our models in Section 2. In Section 3, we present simulation results that include time-averaged Eddington ratios, accretion histories with and without dust, dependences of the Eddington ratios on metallicity and BH mass, and spectral energy distributions (SEDs) in the infrared (IR) considering dust thermal emission. We discuss the dust destruction processes and the condition for hyper-accretion in Section 4, and summarize our results in Section 5.

2. Model

We solve the equations for the gas dynamics surrounding a BH including the effects of its radiative feedback using one-dimensional radiative-hydrodynamics simulations. In this work, we use the hydrodynamic code Zeus-MP (Stone & Norman 1992; Hayes et al. 2006). Park & Ricotti (2011) incorporated the radiative transfer of X-ray and UV photons and chemical reactions for primordial gas into Zeus-MP. Here we further incorporate radiative transfer of X-ray and UV photons and chemical reactions for dust in the code. In spherical symmetric coordinates the basic equations, i.e., the conservations of mass, momentum, and energy, are

$$\frac{\partial p}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho v) = 0,$$

$$\rho \left( \frac{\partial v}{\partial t} + v \frac{\partial v}{\partial r} \right) = - \frac{\partial p}{\partial r} - \frac{GM_B \rho}{r^2} + f_{\text{rad}},$$

$$\rho \left( \frac{\partial e}{\partial t} + v \frac{\partial e}{\partial r} \right) = - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 v) + \Gamma - \Lambda,$$

where $f_{\text{rad}}$ is radiation force, $\Gamma$ is the heating rate by photoionization of hydrogen and helium, and $\Lambda$ is the radiative cooling rate. In this work, we consider the radiation force on dust ($f_{\text{rad}}^{\text{dust}}$), free electrons ($f_e^{\text{rad}}$), and neutral hydrogen ($f_{\text{H}I}^{\text{rad}}$). For dust grains of uniform size the optical depth is

$$\tau_{d,\nu} = \frac{Q_{\nu} \pi a_d^2 D_m n_{\text{H}_2}}{m_d} = \frac{3Q_{\nu} D_m n_{\text{H}_2}}{4\rho_d a_d},$$

where $a_d$ is the radius of a dust grain, $m_d$ is its mass, $Q_{\nu}$ is the absorption coefficient to geometrical cross section, $D_m$ is hydrogen mass, $\rho_d$ is the mass density of a dust grain, and $D$ is the dust-to-gas mass ratio. We consider only the hydrogen mass in the definition of $D$, i.e., $D = M_{\text{dust}}/M_H$. If the wavelength of the radiation is shorter than $2\pi a_d$, then $Q_{\nu} \sim 1$ (Draine & Lee 1984). In this work we assume compact spherical dust grains, i.e., $m_d = 4\pi a_d^3 \rho_d / 3$, and we assume that the dust-to-gas mass ratio is proportional to the gas metallicity as $D = M_{\text{dust}}/M_H = 0.01 (Z/Z_\odot)$, motivated by observations of a nearly constant dust-to-metal mass ratio in local galaxies (Draine et al. 2007).

Observations of local galaxies indicate that dust has a continuum size distributions. For example, the dust in the Milky Way shows a power-law size distribution $dn_d/da_d \propto a_d^{-3.5}$, often referred to as the MRN distribution (Mathis et al. 1977). However, for the sake of simplicity we use a single-size dust model with a fiducial size $a_d = 0.1 \mu m$. The choice of the fiducial size is motivated below. At wavelengths $\lambda < 2\pi a_d$, where $Q_{\nu} = 1$, the opacity of a power-law distribution of dust radii $dn_d/da_d \propto a_d^{-\alpha}$ is the same as that of a single-size dust model for

$$a_{d,1} = \left( \frac{3 - \alpha}{4 - \alpha} \right)^\frac{1}{\frac{3 - \alpha}{a_{d,\text{max}}} - \frac{4 - \alpha}{a_{d,\text{min}}}} (a_{d,\text{max}} - a_{d,\text{min}}).$$

where $a_{d,1}$ is the dust size of the equivalent single-size model, $a_{d,\text{min}}$ and $a_{d,\text{max}}$ are minimum and maximum grain sizes in the grain distribution. Assuming realistic minimum and maximum dust sizes of $a_{d,\text{min}} = 8.1 \times 10^{-3} \mu m$ and $a_{d,\text{max}}^* = 1.0 \mu m$ in an MRN model (i.e., $\alpha = 3.5$), the equivalent single-size dust model has $a_{d,1} = 0.1 \mu m$.

Near active galactic nuclei (AGNs), the main components of dust are graphite and silicate because of their high sublimation temperature $T \gtrsim 1500$ K, whereas icy dust can be easily
Table 1

| Model   | $M_{BH}$ ($M_\odot$) | $n_\infty$ (cm$^{-3}$) | Dust Attenuation | $f_{\text{dust}}^{\text{rad}}$ |
|---------|----------------------|------------------------|------------------|------------------|
| M5-Z0   | $10^5$               | 100                    | ×                | ×                |
| M5-Z1   | $10^5$               | 100                    | ✓                | ✓                |
| M5-Z1rad| $10^5$               | 100                    | ✓                | ✓                |

Note. $n_\infty$ is the initial background density. $f_{\text{dust}}^{\text{rad}}$ is the radiation pressure on dust.

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Dust attenuation is the Lyman-limit frequency. However, the results can be easily scaled to different BH masses and different densities on dust. Red symbols refer to simulations with dust absorption of IR photons by dust is negligible. Therefore, we neglect the opacity of dust to IR light.

Within a fully ionized region, the radiation force on dust can be larger than that from Compton scattering on electrons by a factor

$$f_0 \equiv \frac{f_{\text{dust}}}{f_{\text{rad}}} = \frac{3Q_r D m_{\text{H}}}{4\rho_4 a_4 \sigma_T} = 7.1 \times 10^2 \left( \frac{a_4}{0.1 \mu m} \right)^{-1} \left( \frac{Z}{Z_\odot} \right).$$

UV photons absorbed by dust are re-emitted as IR photons via thermal emission from dust. However, the IR photons can escape from neighboring gas without further interaction with dust due to its lower absorption cross section at IR wavelengths. If the hydrogen column density exceeds $10^{22}$ cm$^{-2}$, the IR photons can be absorbed by dust and impart additional momentum to the gas. In this work, we focus on spatial scales up to an ionized bubble. On these scales the absorption of IR photons by dust is negligible. Therefore, we neglect the opacity of dust to IR light.

Aside from the addition of dust physics, the simulations have the same physics and initial conditions as the ones presented in Park & Ricotti (2012). For our fiducial simulations we investigate the gas dynamics around BHs of $10^5 M_\odot$ embedded in gas clouds with uniform initial gas density. The radiative luminosity is $L = \eta \dot{m} c^2$ with a constant radiative efficiency $\eta = 0.1$ and $\dot{m}$ estimated at the inner boundary in the logarithmically spaced radial grid. We assume a power-law spectrum $L_\nu \propto \nu^{-1.5}$ with the frequency range $\nu \gtrsim \nu_L$, where $\nu_L$ is the Lyman-limit frequency. However, the results can be easily scaled to different BH masses and different densities because the problem is basically scale-free. Park & Ricotti (2012) found that simulations with a fixed $M_{BH} n_{\text{H}}$ show identical behavior once length and timescales are renormalized appropriately. Park & Ricotti (2012) used the condition $M_{BH} n_{\text{H}} = 10^5 M_\odot$ cm$^{-3}$ as a fiducial run. For this combination of parameters, if gas accretes onto the BH at the Bondi rate, the luminosity is close to the Eddington luminosity. Here we use the same fiducial run with $M_{BH} n_{\text{H}} = 10^5 M_\odot$ cm$^{-3}$ to compare the impact of dust attenuation and radiation force on the gas accretion onto BHs in the dust-free case. Table 1 summarizes three different models we ran to understand the impact of dust: (1) dust-free case (M5-Z0), (2) dusty case with $Z = Z_\odot$ without radiation pressure on dust (M5-Z1), and (3) dusty case with $Z = Z_\odot$ including radiation pressure on dust (M5-Z1rad). These named runs have a BH of $10^5 M_\odot$ and initial background gas density of 100 cm$^{-3}$. In all simulations we include $f_{\text{dust}}^{\text{rad}}$ and $f_{\text{rad}}^{\text{H}_2}$. The inner and outer boundaries for simulations with $M_{BH} = 10^5 M_\odot$ are $2 \times 10^{-2}$ pc and $2 \times 10^3$ pc, respectively. For the cases with different BH masses, the boundaries are scaled by a factor of $M_{BH} / 10^5 M_\odot$. The value of the inner boundary is chosen to resolve the Bondi radius within the ionized gas and the radius of the outer boundary to capture the radius of the ionized bubble. We have confirmed that the simulation results did not change by reducing the radius of the inner boundary by a factor of a few. The number of cells in the radial direction is $n_{\text{cell}} = 256$. We have checked that our results are convergent and do not change when increasing the number of cells from 128 to 400. The cell size changes with radial distance with the equatorial ratio. To check that the results are indeed scalable to different BH masses we run simulations with BH masses of $10^3$, $10^4$, and $10^5 M_\odot$ keeping constant $M_{BH} n_{\text{H}} = 10^5 M_\odot$ cm$^{-3}$. In Section 3.6 we also present simulations for SMBHs of masses $M_{BH} = 10^8$–$10^9 M_\odot$, but for such massive BHs it is likely a poor assumption to neglect the effects of the galaxy bulge, star formation, and angular momentum.

3. Results

3.1. Time Evolution of Luminosity Under Radiative Feedback

Figure 1 shows the time-averaged BH luminosity $\langle \dot{L}_{\text{rad}} \rangle = \langle \dot{L} \rangle / \dot{L}_{\text{Edd}}$, in units of the Eddington luminosity $L_{\text{Edd}} \approx 1.3 \times 10^{33}$ erg s$^{-1}$ ($M_{BH} / 10^5 M_\odot$), for the simulations in Table 1. We calculate $\langle \dot{L}_{\text{rad}} \rangle$ at various metallicities ($Z = 0$–$1 Z_\odot$) and initial gas densities $n_\infty = 10$–$1000$ cm$^{-3}$. Regardless of the amount of dust, $\langle \dot{L}_{\text{rad}} \rangle$ increases almost linearly with gas density. This means that the accretion rate normalized by the Bondi rate is nearly constant as a function of the gas density.
The middle panel shows the luminosity of the M5-Z1 run (with dust). The burst cycle is shorter than that of the M5-Z0 run (without dust). This is because the cycle timescale is proportional to the sound crossing time over the ionized region, $\sim n_{\text{HII}}/c_s$ (Park & Ricotti 2012), and the size of the H II region ($R_{\text{HI}}$) is decreased due to the dust opacity. In order for the dust attenuation to work, the optical depth of dust in the H II region should be larger than unity. We can roughly estimate the critical metallicity (and therefore dust abundance) that can have an effect in reducing the size of the H II region and therefore the period between bursts. First we estimate the size of the Strömgren sphere:

$$r_{\text{HII}} = \left( \frac{3f_{\text{Edd}}L_{\text{Edd}}}{4\pi n_{\text{HI}}a_B\bar{E}_{\text{ion}}} \right)^{\frac{1}{3}}$$

$$= 33 \text{ pc} \left( \frac{f_{\text{Edd}}}{10^{-2}} \right)^{\frac{1}{3}} \left( \frac{n_{\text{HI}}}{10^2 \text{ cm}^{-3}} \right)^{-\frac{1}{3}} \left( \frac{M_{\text{BH}}}{10^7 M_\odot} \right)^{\frac{1}{3}}.$$  \hspace{1cm} (5)

where $a_B$ is the case-B recombination coefficient and $\bar{E}_{\text{ion}}$ is the mean energy of ionizing photons. We here set $T = 7 \times 10^4$ K as the temperature of the ionized region in the above estimation. For a power-law spectrum, $\bar{E}_{\text{ion}}$ is estimated as $\alpha^{-1} \times 13.6$ eV, where $\alpha$ is the slope of the power-law spectrum, and $\bar{E}_{\text{ion}} = 40.8$ eV for $\alpha = 1.5$. Using the estimated size of the ionized region above, we can derive the critical metallicity for the dust attenuation as

$$\tau = \frac{3 \times 10^{-2} Q_m m_{\text{H}}n_{\text{HI}}}{4\rho a_d} \left( Z_{\text{crit}}/Z_\odot \right) r_{\text{HII}} \approx \frac{1}{1}.$$  \hspace{1cm} (6)

Thus, the critical metallicity is given by

$$Z_{\text{crit}} \approx 0.2 Z_\odot \left( \frac{f_{\text{Edd}}}{10^{-2}} \right)^{\frac{1}{3}} \left( \frac{M_{\text{BH}}n_{\text{HI}}}{10^2 M_\odot \text{ cm}^{-3}} \right)^{\frac{1}{3}}$$

$$\approx 0.2 Z_\odot \left( \frac{M_{\text{BH}}n_{\text{HI}}}{10^2 M_\odot \text{ cm}^{-3}} \right)^{\frac{1}{3}}.$$  \hspace{1cm} (7)

If the metallicity is higher than $Z_{\text{crit}}$, the size of the H II region decreases and so does the period between bursts. Note that, even in the case with dust attenuation (but no radiation pressure on dust), the ionized bubble is larger than the Bondi radius. Therefore, the gas density inside the H II region is roughly the same as in the run with no dust (M5-Z0), and so is the mean accretion rate. However, the peak luminosities are somewhat smaller than in the M5-Z0 run due to the smaller H II bubbles. This might lead to a lower ($f_{\text{Edd}}$) than the cases with primordial gas by a factor $\sim 2$.

The bottom panel of Figure 2 shows the luminosity for the run that includes the effect of both dust opacity and radiation pressure on dust (M5-Z1rad). Unlike the M5-Z0 and M5-Z1 runs, the gas accretion of M5-Z1rad proceeds more gently and the luminosity varies between the burst and quiescent phases by one or two orders of magnitude (instead of five orders of magnitude as in the other runs). The maximum and minimum luminosities are $\sim 10^{-2}$ and $10^{-4}$ of $L_{\text{Edd}}$, respectively. Due to the high radiation force on dust, which as shown in Equation (4) at solar metallicity is nearly 700 times larger than Compton scattering on electrons, the gas inflow is significantly suppressed when $f_{\text{Edd}} > 10^{-3}$, resulting in a new luminosity burst.

Figure 2. Time evolution of the Eddington ratio $f_{\text{Edd}}$ for the models in Table 1. The top, middle, and bottom panels show $f_{\text{Edd}}$ for models M5-Z0, M5-Z1, and M5-Z1rad. The dashed lines are the time-averaged Eddington ratios shown in Figure 1.
than in the M5-Z0 run by one order of magnitude. In the M5-Z1rad run, the period between bursts is not determined by the sound crossing time of the H II region \((\rho_{\text{BH}}/c_s)\). As will be shown in Section 3.2, the H II bubble in the M5-Z1rad run maintains almost constant size, whereas the one in the M5-Z0 run recombines and collapses when the luminosity drops to the minimum (see also Park & Ricotti 2012). In the case of primordial gas, the expansion of the H II bubbles significantly reduces the gas density and accretion rate, and then the rapid gas inflow during the collapse of the ionized bubble allows the luminosity to reach nearly the Eddington limit. On the other hand, in the case of dusty gas, the dust attenuation regulates the expansion of the H II bubble, and the radiation force suppresses the gas inflow significantly. Thus, the dust plays a role in diminishing the large periodic variability of the luminosity and leads to more gentle gas accretion onto the BH.

Figure 3 presents the density profiles in the high- and low-luminosity phases for runs M5-Z0 and M5-Z1rad. In the high-luminosity phase, the density of the M5-Z0 run increases steeply at \(r \lesssim 1 \text{ pc}\), and it reaches \(n_{\text{H}} \sim 10^4 \text{ cm}^{-3}\) at the inner boundary. The inflow velocity exceeds the sound speed at \(\sim n_{\text{H}}/2\), and hence the density profiles are roughly \(\propto r^{-3/2}\) as in the Bondi profile (free-falling gas).

On the other hand, the density of M5-Z1rad increases slowly as the radial distance decreases. In the case with dust, the net inward force is significantly reduced due to the radiation force. By introducing an effective gravitational constant as \(G' = G(1 - f_{\text{rad}}/f_{\text{grav}})\), we estimate the transonic radius as \(r \sim G'M_{\text{BH}}/2c_s^2\). When \(f_{\text{rad}} \lesssim 10^{-3}\), the transonic radius is smaller than the inner boundary of the calculation box. As a result, the gas inflow of M5-Z1rad is always subsonic in our calculation box. In addition, the temperature at \(r \lesssim 0.1 \text{ pc}\) increases as the radial distance decreases, due to compression heating. Therefore, the force of the pressure gradient also works to suppress the gas inflow. Thus the gas density does not increase steeply.

As shown in the lower panel of the figure, the gas density is decreased due to the photoionization feedback during the quiescent low-luminosity phase. At the ionization front, the gas is almost in pressure equilibrium with the gas outside the H II region, i.e., \(2n_{\text{H}}/T_{\text{H II}} \sim n_{\text{H}}/T_{\text{H II}}\), where the factor \(\sim 2\) is due to the increased number of particles from ionization. Since the temperature of the H II region reaches \(\sim 7 \times 10^4 \text{ K}\) via photoionization of hydrogen and helium (Park & Ricotti 2011), the density of the region is lower than that of the ambient H I gas by roughly a factor \(\sim 14\). The gas density is \(\lesssim 100 \text{ cm}^{-3}\) even at the inner boundary in the simulation. Therefore the dust optical depth inside the H II region does not exceed unity, as will be discussed in Section 3.2.

Figure 4 shows the outward acceleration of the gas due to radiation pressure normalized by the gravitational acceleration. Both \(f_{\text{rad}}^{\text{dust}}\) and \(f_{\text{rad}}^{\text{e}}\) are roughly constant inside the H II region. The number ratio of dust grains to free electrons is almost constant inside the H II region. Therefore, \(f_{\text{rad}}^{\text{dust}}\) is higher than \(f_{\text{rad}}^{\text{e}}\) by a factor \(\sim 700\) as shown by Equation (4). During the burst of accretion the dust optical depth exceeds unity inside the H II region, because the gas density increases near the BH as shown in Figure 3. Therefore the radiation pressure decreases as the distance increases, due to the dust opacity. \(f_{\text{rad}}^{\text{dust}}\) drops sharply outside the ionizing front because the electron abundance decreases, while \(f_{\text{rad}}^{\text{e}}\) decreases gradually since dust exists both in the ionized and neutral gas. The radiation force on H I, \(f_{\text{rad}}^{\text{H I}}\), increases monotonically with the radial distance in the H II region. Where \(r < 1\), \(f_{\text{rad}}^{\text{H I}}/f_{\text{grav}}\) is simply proportional to the neutral fraction \(x_{\text{H I}}\). Inside the H II region, hydrogen is in ionization equilibrium:

\[
\int \frac{3x_{\text{H I}}L_{\nu}/\sigma_T e^{-\tau_{\nu}}}{4\pi r^2 h\nu} d\nu \sim c_B n_{\text{H}}(1 - x_{\text{H I}})^2.
\]

For \(r \ll 1\) and \(x_{\text{H I}} \ll 1\), \(x_{\text{H I}} \propto n_{\text{H}} r^2\), \(f_{\text{rad}}^{\text{H I}}\) increases somewhat more slowly than \(r^2\). This is due to the decrease in gas...
density as the radial distance increases at \( r \lesssim 10 \) pc. Near the ionization front, \( f_{\text{rad}}^{\text{HI}} \) increases steeply with increasing \( n_{\text{HI}} \), and it decreases exponentially in the neutral region.\(^5\) In this work, although the inner boundary is much larger than the size of accretion disks around massive BHs, we assumed that the gas entering the inner boundary emits radiation at a rate \( \eta \rho c^2 \) instantaneously. Since the inner boundary is smaller than the Bondi radius, the accreted gas exceeds the sound speed when the radiation pressure is not significant. In this case, the timescale of the gas transport from the inner boundary to the accretion disk of BHs is much shorter than the dynamical time of gas at the scale we focus on. On the other hand, when the metallicity is close to the solar abundance, the transonic point can be much nearer than the inner boundary. If the time delay is not negligible compared to the dynamical time, the time behavior of the accretion rate may change. Due to the radiation force on dust, the accretion rate of \( M_{\text{5}}-Z_{1\text{rad}} \) is suppressed to a maximum Eddington ratio of \( f_{\text{edd}} \sim 10^{-2} \) as shown in Figure 2. If the time delay becomes comparable to the timescale of periodicity of gas accretion, it may allow a higher accretion rate at the peaks, leading to larger time variations. Park & Ricotti (2012) have investigated the effects of a time delay. They showed that the time delay does not affect the accretion history significantly when the delay is smaller than the time interval between periodic bursts of luminosity. As the time delay becomes comparable to the interval between bursts, the period between bursts gets longer by a factor of a few. However, although the global properties of the accretion are not strongly affected by even a relatively long time delay, some detailed relationships such as those discussed in Section 3.5 may be modified. Since the parameter space to explore is very large, it is hard to make predictions without resorting to realistic simulations in the galactic environment. We leave the exploration of this effect to future work in which we will use 3D simulations with angular momentum and a more realistic environment for the galactic center.

### 3.2. Size of Ionized Bubble

The time evolution of the position of the ionization front (defined where \( x_e = 50\% \)) is presented in Figure 5. The size of the ionized bubble \( n_{\text{HI}} \) changes with the luminosity in the case of the \( M_{\text{5}}-Z_{0} \) run without dust. When the pressure equilibrium at the ionization front breaks due to the decreased density and luminosity, the neutral gas free-falls into the BH. In this case, the inflowing gas remains neutral until it is near the BH, resulting in the collapse of the ionized bubble just before the next bursts of luminosity.

On the other hand, in the dusty gas simulation (\( M_{\text{5}}-Z_{1\text{rad}} \)) the size of the ionized bubble remains nearly constant, with \( n_{\text{HI}} \lesssim 20 \) pc. Even during the high-luminosity phases, the size is several times smaller than that of the dust-free case (\( M_{\text{5}}-Z_{0} \)). This is because dust opacity regulates the evolution of the size of the ionized bubble. Figure 6 shows the optical depth of dust during the high- and low-luminosity phases. When the luminosity is high, the optical depth of dusty gas reaches unity at \( r \sim 10 \) pc (\( M_{\text{5}}-Z_{1\text{rad}} \)). Therefore the ionizing front cannot propagate far beyond 10 pc, whereas in the dust-free case (\( M_{\text{5}}-Z_{0} \)) the ionizing front (I-front) reaches \( \sim 40 \) pc. During the low-luminosity phase, the optical depth does not exceed unity in the ionized bubble due to the lower gas density, as shown in Figure 3. In this case, the size of the ionized bubble is determined by the ionizing luminosity. The gas density increases steeply over the transition from ionized region to neutral region. This jump in density significantly increases the optical depth due to the high-density dust in the neutral region. Therefore, regardless of different luminosities, the optical depth exceeds unity just behind the ionizing front.

### 3.3. Metallicity Dependence

As star formation proceeds, the interstellar medium in galaxies becomes enriched in metals/dust by Type-II supernovae (e.g., Wise et al. 2012). Recent observations indicated a

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\(^5\) We notice that there is a dip in \( f_{\text{rad}}^{\text{HI}} \) near the ionization front. At this radius, the flux decreases steeply as \( \exp(-N_{\text{HI}}/\rho_{\text{HI}}) \), while the neutral fraction increases at \( \sim 1-2 \) cells behind, producing the dip. This is due to the finite resolution. Increasing the number of cells makes this dip smaller. Since the radiation force on H I is negligible, this feature does not affect our results.
large dispersion in the amount of dust in high-redshift galaxies (Watson et al. 2015). Here we study the dependence of the BH accretion luminosity on the gas metallicity and therefore the dust abundance (since we assume a constant dust-to-metal mass ratio). Figure 7 shows \( f_{\text{Edd}} \) as a function of metallicity. As the metallicity increases, \( \langle f_{\text{Edd}} \rangle \) decreases monotonically and ranges from \( f_{\text{Edd}} \sim 10^{-2} \) at \( Z = Z_{\odot} \) to \( f_{\text{Edd}} \sim 10^{-3} \) at \( Z = Z_{\odot} \). We examine the metallicity dependence based on the assumption of steady spherical inflow of ionized gas (i.e., Bondi accretion inside the ionized bubble). In the case without dust, we estimate the accretion rate of Bondi-like inflow in the ionized region \( (M_{\text{BH,II}}) \) as follows:

\[
M_{\text{BH,II}} \sim 4\pi \lambda_{\text{II}} G^2 \alpha_{\text{HI}}^3 M_{\text{BH}}^2, \tag{9}
\]

where \( \lambda_{\text{II}} \) is the dimensionless mass accretion rate, \( \alpha_{\text{HI}} \) and \( \alpha_{\text{HII}} \) are the density and the sound speed of ionized gas. The value of \( \lambda_{\text{II}} \) depends on the polytropic index \( \gamma \) of the equation of state, i.e., \( P = K \rho^\gamma \), and ranges from \( \sim 1.12 \) for an isothermal gas to \( 1/4 \) for the adiabatic case. When we assume that the gas inflow is isothermal and consists of ionized hydrogen and helium at a temperature \( T_{\text{HII}} \sim 7 \times 10^4 \) K, the accretion rate from Equation (9) is \( \sim 770 \) times smaller than the Bondi rate calculated from the density and temperature of the ambient medium. This reduction in the accretion rate is roughly similar to our simulation results and previous works (Milosavljević et al. 2009; Park & Ricotti 2012; Sugimura et al. 2017). Note, however, that the inflow of primordial gas causes periodic bursts as shown in Section 3.1. These bursts can increase the time-averaged accretion rate with respect to that estimated above (see also Park & Ricotti 2012).

Next, we modify the above accretion rate by taking into account the radiation pressure on dust. The net inward force is reduced by radiation pressure. In the optically thin regime \( (\tau_d < 1) \), the fractional reduction of the inward gravitational force is independent of the radial distance, and depends only on the BH luminosity. In this case, the accretion rate is

\[
M \sim \frac{4\pi \lambda_{\text{II}} G^2}{4\pi GM_{\text{BH}}^2 \rho_{\text{II}} c_{\text{HII}}^3} \left(1 - \frac{f_{\text{Edd}} \sigma_T L}{4\pi GM_{\text{BH}}^2 \rho_{\text{II}} c_{\text{HII}}^3}\right)^2, \tag{10}
\]

By solving this second-order equation, we derive an analytical estimate of the accretion rate and luminosity. The analytically estimated Eddington ratios are shown as dashed \( (n_\infty = 100 \) cm\(^{-3} \) \) and dotted \( (n_\infty = 300 \) cm\(^{-3} \) \) lines in Figure 7. As shown in the figure, the modified Bondi rate considering the radiation force roughly explains the metallicity dependence of the accretion rate in the simulations with radiation pressure on dust. Note that \( f_{\text{Edd}} \) in the cases of high metallicity and high density differs from the value estimated above, whereas in the case with \( n_\infty = 100 \) cm\(^{-3} \) our analytical estimate is a good fit to the simulation results. When the density and metallicity are high, the size of the H II bubble is close to the Bondi radius (e.g., Park & Ricotti 2012). Therefore, the gas density in the ionized region can become somewhat higher than that estimated assuming pressure equilibrium across the ionization front due to the BH gravitational potential, resulting in a higher accretion rate.

3.4. Mass Dependence

Here we study dependence of the gas accretion rate on the BH mass for our fiducial runs with \( M_{\text{BH,HI}} = 10^3 M_{\odot} \) cm\(^{-3} \). Park & Ricotti (2012) showed that the accretion rate normalized by the Bondi rate did not change for different BH masses when \( M_{\text{BH,HI}} = 10^3 M_{\odot} \) cm\(^{-3} \). Figure 8 shows the time-averaged Eddington ratios for different BH masses. As shown in Park & Ricotti (2012), \( \langle f_{\text{Edd}} \rangle \sim 1\% \) independently of the BH mass in the case without dust. We find that the same result

\[ Z = 0.0 Z_{\odot} \]

\[ Z = 1.0 Z_{\odot} \]

\[ Z = 1.0 Z_{\odot} \]

\[ Z = 1.0 Z_{\odot} \]
Figure 9. Upper panel: dust temperature profile for the M5-Z1rad run. Solid and dashed lines represent the temperatures of graphite and silicate dust grains, respectively. The red and blue colors show the phases of high and low dust temperature, respectively. The red and blue colors show the phases of high and low dust temperature, respectively. The red and blue colors show the phases of high and low dust temperature, respectively. The red and blue colors show the phases of high and low dust temperature, respectively.

Due to the strong attenuation of UV flux outside the I-front, the dust temperature drops sharply at \( r \gtrsim 10 \) pc. In regions where \( T_d \gtrsim 100 \) K, the dust temperature of silicate is somewhat lower than that of graphite. This is because \( Q_c \) of silicates is higher than \( Q_c \) of graphite in that temperature range, thus the efficient thermal photon emissivity results in the lower temperature.

In order to better understand the radial profile of the dust temperature, we rewrite Equation (11) as

\[
F \sim 4 \sigma_{SB} T_d^4 \tilde{Q}(T_d),
\]

where \( F = \int F_\nu d\nu \) is radiation flux and \( \tilde{Q}(T_d) = \int B_{\nu}(T_d) Q_\nu d\nu / \int B_{\nu}(T_d) d\nu \) is the frequency-averaged dust absorption coefficient. Therefore, the dust temperature is

\[
T_d(r) = \left[ \frac{M_{BH}}{16 \pi \sigma_{SB} \tilde{Q}(T_d) r^2} \right]^{\frac{1}{4}}.
\]

For \( \tau \ll 1 \), this equation is scaled as follows:

\[
T_d(r) = 4.7 \times 10^{2} K \left( \frac{f_{\text{Edd}}}{10^{-2}} \right)^{\frac{1}{4}} \left( \frac{M_{BH}}{10^{5} M_\odot} \right)^{\frac{1}{2}} \times \left( \frac{Q_c}{10^{-2}} \right)^{-\frac{1}{2}} \left( \frac{r}{0.1 \text{ pc}} \right)^{-\frac{1}{2}}.
\]

As shown in the figure, the dust temperature roughly decreases as \( \propto r^{-1/2} \). However, note that \( \tilde{Q} \) decreases as the dust temperature decreases. Therefore, the radial profile of the dust temperature is somewhat shallower than \( r^{-1/2} \).

The lower panel of Figure 9 shows the radial dependence of the fraction of energy absorbed by the dust per unit radial bin, normalized by the total luminosity. During the quiescent accretion phase, most energy is absorbed by dust near the ionizing front. This is because the absorption energy is converted into recombination and cooling radiation, e.g., Ly\( \alpha \) photons. These recombination UV photons are eventually absorbed by dust. In addition, by considering only dust absorption, the IR properties do not change significantly even if we consider different choices for the SED from the accretion disk, e.g., a SED extending to the soft-UV range. Figure 9 shows the dust temperature as a function of distance from the BH. During the phase of high accretion, the dust temperature is \( \sim 800 \) K near the inner boundary of the simulation, and decreases as the distance increases due to the geometrical attenuation of the flux \( \propto r^{-2} \).

3.5. Thermal Emission from Dust

Dust releases the energy absorbed at UV wavelengths as thermal emission in the IR. Here we investigate the IR properties of the dusty gas accreting onto the BH. The dust temperature \( T_d \) can be estimated by assuming radiative equilibrium:

\[
\int \pi a_d^2 Q_\nu F_\nu d\nu = \int 4 \pi a_d^2 Q_\nu \pi B_\nu(T_d) d\nu.
\]

This equation assumes equilibrium between dust absorption of the radiation emitted by the BH (left side) and the thermal emission from dust (right side). By solving this equation, we estimate the dust temperature of each gas shell. We assume \( Q_\nu = 1 \) at UV wavelengths for the left side of the equation, and use \( Q_\nu \) estimated in Laor & Draine (1993) for the right side. In order to estimate the thermal emission by dust we post-process the simulation results, calculating the flux absorbed by dust. In the estimation of the dust thermal emission, we neglect the absorption of ionizing photons by hydrogen and helium. This is because the absorbed energy is converted into recombination and cooling radiation, e.g., Ly\( \alpha \) photons. These recombination UV photons are eventually absorbed by dust. In addition, by considering only dust absorption, the IR properties do not change significantly even if we consider different choices for the SED from the accretion disk, e.g., a SED extending to the soft-UV range. Figure 9 shows the dust temperature as a function of distance from the BH. During the phase of high accretion, the dust temperature is \( \sim 800 \) K near the inner boundary of the simulation, and decreases as the distance increases due to the geometrical attenuation of the flux \( \propto r^{-2} \).
to each other, these variations in the SEDs can be explained by the different phases of accretion and the Eddington ratios as explained above. An estimate of the masses and Eddington ratios of these hard X-ray-selected AGNs is currently in progress (T. Shimizu 2017, private communication). With the caveat of being able to isolate the AGN contribution from the stellar contribution in the IR SEDs, the results of this study can be used to test our model predictions.

We also show the contributions of graphite and silicate grains, separately. The stretching resonance of astronomical silicate grains produces the bump in the SED at $\lambda \sim 9.8 \mu m$. At shorter wavelengths ($\lambda \lesssim 8 \mu m$), the contribution from graphite grains is dominant. In this work, we have not considered reabsorption of the thermal emission by dust, because the absorption cross section of dust to IR photons is quite small and we assume that the ambient gas is optically thin at IR wavelengths. However, if the hydrogen column density of the gas reservoir fueling the BH is much higher than $10^{22}$ cm$^{-2}$, some IR radiation emitted by hot dust can be absorbed by the dust and reprocessed to thermal emission at lower dust temperature. This can somewhat suppress the IR flux at shorter wavelengths.

Figure 11 shows the relation between the Eddington ratio, $f_{\text{Edd}}$, and the flux density ratio between 14 and 140 $\mu m$ ($f_{14/140}$) multiplied by the square root of bolometric luminosity, $L_{\text{bol,40}}$ (in units of $10^{40}$ erg s$^{-1}$). As shown in Figure 10, the flux at $\lambda \lesssim 10 \mu m$ depends sensitively on the mass ratio of the different dust components. Therefore we choose the flux at $14 \mu m$ as diagnostic of the phase of high accretion. The wavelengths 14 and 140 $\mu m$ correspond to the peak emission of two blackbody spectra with temperatures of 2070 K and 20.7 K, respectively. We find that this flux ratio tightly correlates with $f_{\text{Edd}}$. The correlation between $f_{\text{Edd}}$ and $f_{14/140}$ varies with the BH mass and dust size, mainly because the temperatures of the warm and hot dust components depend on these physical parameters. However, $f_{\text{Edd}}$ tightly correlates with $f_{14/140}$ regardless of the assumed initial background density. Yet, we can use our knowledge of $L_{\text{bol,40}}$ to correct for these effects and recover $f_{\text{Edd}}$ regardless of the different physical parameters of the SMBH, which are generally unknown observationally. The different panels in the figure illustrate the tight correlation between $f_{\text{Edd}}$ and $L_{\text{bol,40}}$, $f_{14/140}$, which holds by varying different physical parameters by many orders of magnitude. We find that the correlation can be roughly fitted by the following relationship:

$$f_{\text{Edd}} \sim 3 \times 10^{-2} (L_{\text{bol,40}} f_{14/140})^{2/3}. \quad (15)$$

Therefore, bearing in mind that our model is still very simplistic, it may be possible to use IR observations of warm and hot dust obscuring SMBHs to estimate $L_{\text{bol,40}}$ and $f_{14/140}$ and therefore to derive $f_{\text{Edd}}$ and the SMBH mass as

$$M_{\text{BH}} \sim 8 \times 10^3 M_\odot L_{\text{bol,40}} f_{\text{Edd}}^{10^{-2}}. \quad (16)$$

Since $L_{\text{bol,40}} \equiv f_{\text{Edd}} L_{\text{Edd}} \propto f_{\text{Edd}} M_{\text{BH}}$, we can use Equation (15) to show that $f_{\text{Edd}} \propto M_{\text{BH}}^{1/2} f_{14/140}^{1/4}$. Therefore, for a fixed BH mass, we indeed find $f_{\text{Edd}} \propto f_{14/140}$. Instead, keeping $M_{\text{BH}}$ constant while varying the density and/or BH mass, we find $\eta_{\text{II}} \propto f_{14/140}^{1/3} M_{\text{BH}}$ (see Equation (5)). Using Equation (14), the dust temperature at $\eta_{\text{II}}$ is $T_d(\eta_{\text{II}}) \propto M_{\text{BH}}^{1/4}$ (here we ignore a weak dependence on $f_{\text{Edd}}$). Therefore, the weak mass dependence in the figure indicates that $f_{14/140}$ decreases as $T_d(\eta_{\text{II}})$ decreases with increasing BH mass (and $L_{\text{bol,40}}$).

We also study the SEDs for models with different dust sizes, $a_d = 0.05$ and 0.02 $\mu m$. The right panel of Figure 11 shows the correlations in the cases of different dust sizes. Unlike the tight correlations found for different BH masses and densities, models with different dust sizes produce a systematic change in the shape of the relationship. Since $Q_{\text{abs}}$ in the IR band decreases with dust size, smaller dust grains become hotter when irradiated by the same UV flux, as shown in Equation (14). Moreover, the size of the H II region becomes smaller as the dust size decreases (see Equation (2)). Thus, models with smaller dust tend to have higher $f_{14/140}$ for a fixed $f_{\text{Edd}}$. In the same panel we also show the correlation for the low-metallicity (low-dust) case with $Z = 0.1 Z_\odot$. As the metallicity decreases, the BH luminosity and the radius of the ionized bubble, $\eta_{\text{II}}$, increase. As a result, the dust temperature does not change significantly, producing the same correlation as in the fiducial case with $Z = 1 Z_\odot$. Therefore we suggest that the uncertainty of the metallicity does not need to be considered.

Can JWST or ALMA observe the thermal emission from the dust obscuring massive BHs in nearby galaxies? For low-redshift AGNs, the flux from hot dust at rest-frame $\lambda \sim 14 \mu m$ can be observed by JWST, while the warm dust emission at $\lambda \sim 140 \mu m$ is in the wavelength range covered by Spitzer. Even if we measure the flux density ratio between 14 and 450 $\mu m$ (instead of 140 $\mu m$), which is in the wave band covered by ALMA, a similar relationship between $f_{\text{Edd}}$ and the flux ratio is found. Therefore ALMA can also be used to trace the cold dust at $z \sim 0$. In this section we focused on the detectability of IMBHs of $10^5-10^6 M_\odot$, assuming the condition $M_{\text{BH}} H_\infty = 10^6 M_\odot$ cm$^{-3}$. The scenario we have in mind is that these IMBHs are at the centers of dwarf or normal star-forming galaxies. Some of the black holes in this mass range can be the
BH seeds of today’s SMBHs formed via gas accretion onto stellar-mass black holes, mergers of stars and BHs, or direct collapse of supermassive stars (Volonteri 2010).

Using the specific luminosities derived above, we estimate the flux densities from dust around massive BHs at a specific redshift $z$ as $F_\nu = (1 + z)L_{\nu,0}/4\pi D_L^2$, where $D_L$ is the luminosity distance, $\nu$ is the frequency in the observer’s rest frame, and $\nu_0 = \nu(1 + z)$ is the frequency in the galaxy’s rest frame. Here we artificially put BHs at $z = 0.05$–$10$ and discuss their observability by JWST or ALMA. Figure 12 shows the flux densities as a function of redshift. The dotted lines show the sensitivities of JWST and ALMA. We show the flux densities at $14 \mu m (F_{14})$, $140 \mu m (F_{140})$, and $450 \mu m (F_{450})$ in the observer frame. Due to the negative $K$-correction, the flux density at $450 \mu m$ decreases slowly with increasing redshift, whereas that at $14 \mu m$ decreases more steeply at $z \gtrsim 1$. At higher redshift, the flux at $14 \mu m$ in the observer’s rest frame corresponds to a wavelength in the galaxy’s rest frame of $14 \mu m/(1 + z)$, which can be shorter than the peak wavelength of a modified blackbody of even hot dust, resulting in the decrease in the flux. As a result, the flux density at $450 \mu m$ becomes higher than that at $14 \mu m$ at $z \gtrsim 1$. The relation between $F_{14}$ and $F_{450}$ changes depending on the BH mass and $f_{\text{Edd}}$. As shown above, $F_{14}$ is lower than $F_{140}$ or $F_{450}$ when $f_{\text{Edd}}$ is low. Even in the phase of high accretion ($f_{\text{Edd}} \sim 10^{-2}$), $F_{450}$ is higher than $F_{14}$ in the case of a massive BH with $10^9 M_\odot$. Under the constraint $M_{\text{BH}}/M_\odot = \text{const.}$, we have $n_{\text{H}} \propto M_{\text{BH}}$ (see Equation (5)). Therefore, the dust temperature at the ionization front is lower in massive BHs (see Equation (14)), leading to a higher $F_{450}$.

In summary, we find that only a massive BH of mass $\gtrsim 10^9 M_\odot$ at $z \lesssim 0.1$ can be observed by ALMA. On the other hand, JWST will allow us to observe the dust thermal emission from more distant massive BHs. It will be able to probe hot dust obscuring BHs of $10^6 M_\odot$ with $f_{\text{Edd}} \sim 10^{-2}$ up to $z \sim 0.5$. When
f_{Edd} \sim 10^{-2}$, even a $10^4 M_\odot$ BH at $z \lesssim 0.1$ can be observed. Thus we suggest that $JWST$ and ALMA will be powerful tools to probe IMBHs via the observation of dust thermal emission. In general, the IR flux increases as the product $M_{BH} n_\infty$ increases. For instance, for the case of a BH with $M_{BH} = 10^4 M_\odot$ accreting from gas with density $n_\infty = 100$ cm$^{-3}$ and metallicity $Z = 0.01 Z_\odot$, ALMA and $JWST$ can probe BHs up to $z \sim 0.5$. However, even upcoming new telescopes will have difficulty in observing IMBHs at $z \gg 1$ in the IR.

In our simulations, we have investigated the accretion dynamics of spherically symmetric clouds with an isotropic radiation field in which the accretion rate and luminosity can be suppressed significantly. Recently, Sugimura et al. (2017) suggested that the gas accreted efficiently onto a BH if the radiation is anisotropic, due to shadowing effects near the edge of the BH accretion disk or torus. In this case, $f_{Edd}$ can become $\gtrsim 1$, and even relatively massive BHs at high redshifts $z \gtrsim 1$ may be observable by $JWST$ or ALMA, although the dust temperature and SEDs can differ from our current work.

### 3.6. Extension to SMBH Masses

As discussed above, the observability of IMBHs via their dust thermal emission is limited to redshifts $z \lesssim 0.5$ even with $JWST$ or ALMA. On the other hand, SMBHs of $\gtrsim 10^8 M_\odot$ can be observed more easily at $z \sim 0$ and up to higher redshifts. Here we present the results of two sets of simulations for SMBHs with masses of $10^8$ and $10^9 M_\odot$ and gas metallicities $Z = 10^{-2}$ and $1.0 Z_\odot$. We consider dust absorption and thermal emission within a radius of 100 kpc. Figure 13 shows $f_{Edd}$ as a function of $L_{bol,40} / f_{14/140}$. We find that the same relationship found for IMBHs (dashed line) holds for SMBHs. For the cases with metallicity $Z = 10^{-2} Z_\odot$, the Eddington ratio $f_{Edd}$ can reach $\sim 0.2$–$0.5$ at the peaks of accretion. Therefore, the figure shows that the correlation is maintained over the wider range of $f_{Edd}$ from $10^{-4}$ to $\sim 1$.

---

**Figure 13.** Eddington ratio as a function of $L_{bol,40} / f_{14/140}$ as in Figure 11. Different symbols represent simulations with different BH masses and metallicities but $M_{BH} n_\infty = 10^4 M_\odot$ cm$^{-3}$.

**Figure 14.** Same as Figure 12 but for SMBHs with masses of $10^8 M_\odot$ (upper panel) and $10^9 M_\odot$ (lower panel). Different colors indicate different wavelengths in the observer’s rest frame: 14 $\mu$m (blue), 450 $\mu$m (red). Solid and dashed lines refer to the peak accretion rates for the cases with $Z = 1$ and $10^{-2} Z_\odot$, respectively. Horizontal dotted lines show 10 $\sigma$ detection limits with 10 hours integration at 14 $\mu$m by $JWST$ (F1500W filter) and at 450 $\mu$m by ALMA with 50 antennas.

Figure 14 shows the expected IR fluxes from SMBHs as a function of redshift. As shown before, when the BHs are more massive than $\sim 10^5 M_\odot$, the flux at 450 $\mu$m is higher than that at 14 $\mu$m. In the case of SMBHs, ALMA is able to probe to higher redshifts than $JWST$. We find that ALMA can detect SMBHs of $10^8 M_\odot$ at $z \sim 5$ for gas of solar metallicity, and at $z \sim 10$ for gas metallicity $Z = 10^{-2} Z_\odot$. Therefore ALMA can be a strong tool to probe dust-obscured SMBHs in the early universe.

Note that, in the cases of SMBHs, the sizes of ionized bubbles and the Bondi radii exceed 1 kpc. At these large spatial scales, the gravitational potential of a bulge can be dominant (e.g., Park et al. 2016) and gas angular momentum cannot be neglected, due to the rotation of the galaxy. In addition, star formation and stellar feedback drastically change the gas structure near the SMBH, and this may affect its accretion rate significantly (Ciotti & Ostriker 2007; Dubois et al. 2015). We will investigate the importance of these effects in the context of dust-obscured SMBHs in future work.

### 4. Discussion

#### 4.1. Dust Decoupling

In this work we have assumed that the motion of dust is completely coupled to the gas through collisions between gas and dust particles. Here we simply estimate the coupling
timescale. At first, we consider the momentum equation for the relative velocity $v$ between dust and gas:

$$\frac{dv}{dt} = -\frac{n_\text{H}}{m_\text{d}} \pi a_\text{d}^2 v^2.$$  \hspace{1cm} (17)

Therefore, the timescale for the dust coupling with gas is roughly

$$\tau \sim \frac{v}{|dv/dt|} = \frac{4}{3} \frac{\rho_\text{d} a_\text{d}}{n_\text{H} m_\text{H} v} = 6.0 \times 10^3 \text{yr} \left(\frac{a_\text{d}}{0.1 \mu\text{m}}\right) \left(\frac{n_\text{H}}{10^2 \text{cm}^{-3}}\right)^{-1} \left(\frac{v}{10 \text{ km s}^{-1}}\right)^{-1}. \hspace{1cm} (18)$$

This coupling timescale is much shorter than the dynamical timescale and the period between bursts in our current simulations. We should also note that the dust may be charged in the ionized gas due to photoelectric emission or collisions with free electrons (Spitzer 1962; Weingartner & Draine 2001). In this case the Coulomb drag further reduces the coupling timescale with respect to the calculation above for neutral dust (Draine & Salpeter 1979). Thus the assumption of complete coupling is reasonable.

### 4.2. Dust Destruction

The dust formation/destruction processes have not been considered in our simulations. Even at the innermost cell (=0.02 pc) the dust temperature does not exceed the dust sublimation temperature. For the sublimation temperatures of graphite grains, $T_{\text{sub}} \sim 1800 \text{ K}$, we estimate the destruction radius as follows:

$$r_{\text{sub}} = 6.8 \times 10^{-3} \text{ pc} \left(\frac{f_\text{Edd}}{10^{-2}}\right)^{1/2} \times \left(\frac{M_{\text{BH}}}{10^5 M_\odot}\right)^{1/2} \left[\frac{Q(T_{\text{sub}})}{0.2}\right]^{-1/2}. \hspace{1cm} (19)$$

Thus, the dust can survive sublimation by photoheating even inside the inner boundary of the calculation box.

On the other hand, the destruction by a thermal sputtering process, i.e., collisions between dust and gas, is likely to be effective near the BH because of the high density and temperature found at the innermost cells in our simulations. The timescale for destruction by thermal sputtering is estimated (Draine & Salpeter 1979; Draine 2011) as follows:

$$\tau_\text{sp} \sim 1 \times 10^5 \text{ yr} \left[1 + \left(\frac{T}{10^6 \text{ K}}\right)^{-3} \left(\frac{a_\text{d}/0.1 \mu\text{m}}{n_\text{H}/1 \text{ cm}^3}\right)\right]. \hspace{1cm} (20)$$

During the quiescent phase, the density, temperature, and inflow velocity at the innermost cell are $\sim 10^3 \text{ cm}^{-3}$, $3 \times 10^5 \text{ K}$, and $\lesssim 10 \text{ km s}^{-1}$, respectively. Therefore, during this phase in the duty cycle, the sputtering timescale is shorter than the inflow timescale, $\tau_\text{in} \sim r/v_{\text{in}}$, and the dust is likely to be destroyed inside the inner boundary of the calculation box. However, during the peaks of accretion, the temperature decreases somewhat and the velocity increases, resulting in a longer sputtering timescale. Therefore, during an accretion burst dust can survive even close to a BH, and can absorb UV radiation emitted during the burst. However, as shown in Figure 9, the contribution to the opacity from dust absorption at radii smaller than the inner boundary in our simulations is not significant. In addition, Compton heating increases the temperature of gas on such a scale (Park et al. 2014b), leading to enhancement of dust destruction. Yet, photoionization heating of metals is not included in this work. This heating can expand the destruction radius. These detailed heating processes will be considered in our future works.

In this work, we adopt a single-size dust model in the calculations. When considering a realistic dust size distribution, the destruction radius depends on the dust size. This is because small dust grains can have higher temperature, and hence they reach the sublimation temperature at a greater distance. In addition, the destruction timescale for thermal sputtering is proportional to the size. Thus, smaller dust grains are more easily destroyed near the BH than larger dust grains. This indicates that the dust size distribution changes with radial distance.

### 4.3. Revisited Condition for Hyper-Eddington Accretion

In this work we have investigated the density dependence up to $n_\text{H} = 1000 \text{ cm}^{-3}$. This condition allows the ionizing front to propagate far from the Bondi radius in the case without dust. Recently, Inayoshi et al. (2016) suggested that BHs could grow at a super-Eddington rate if they are embedded in very high-density gas clouds where the ionized bubble is smaller than the Bondi radius. This is because the ionized bubble is gradually shrunk as the gas density near the ionization front increases, due to the gravitational force of the BH. Finally the ionized bubbles disappear, resulting in no radiation force on electrons. As a result the growth rate of the BH becomes close to the Bondi rate, which can be much higher than the Eddington limit, i.e., so-called hyper-Eddington accretion. Inayoshi et al. estimated the critical gas density for hyper-Eddington accretion by comparing the Strömgren radius to the Bondi radius. By calculating the Strömgren radius for the density inside the ionized region (which is smaller than $n_\infty$ by a factor ~2($T_{\text{H}_2}/T_{\text{H}_1}$)), we derive the critical density:

$$n_\infty \gtrsim 10^5 \text{ cm}^{-3} \left(\frac{M_{\text{BH}}}{10^5 M_\odot}\right)^{-1} \left(\frac{T}{10^4 \text{ K}}\right)^{3/2},$$

where we have assumed $T_{\text{H}_2} = 7 \times 10^4 \text{ K}$. On the other hand, in the case of dusty gas, the size of the ionized region is regulated by the dust opacity if the metallicity is higher than a critical value (i.e., if $Z > 0.1Z_\odot$). In this regime, the size of the ionized region is close to the photon mean free path, i.e., $r_{\text{H}_2} \sim (d_\text{f}/\tau_\text{d} n_{\text{H}_2})^{-1}$. Thus, the critical density for super-Eddington accretion in a dusty gas is

$$n_\infty \gtrsim 2 \times 10^3 \text{ cm}^{-3} \left(\frac{M_{\text{BH}}}{10^5 M_\odot}\right)^{-1} \left(\frac{T}{10^4 \text{ K}}\right) \left(\frac{Z}{Z_\odot}\right)^{-1}. \hspace{1cm} (21)$$

This critical density is much lower than that for the case of primordial gas. However, the effect of dust in the regime of super-Eddington accretion, in addition to reducing the size of the ionized bubble, is to reduce the accretion rate due to the radiation pressure on dust. Radiation pressure is effective at radii larger than the size of the ionized region, as long as $\tau_\text{d} \lesssim 1$. Therefore, if the ram pressure force can overcome the
radiation force on dust at the radius of $r_d \sim 1$, the dusty gas may be able to efficiently accrete onto the BH. The accretion dynamics of high-density dusty clouds will be investigated in future work with further improvements to the code.

### 4.4. Effects of Multi-dimensionality

In this work we have considered gas accretion and the propagation of the ionizing front in spherical gas clouds using 1D radiative-hydrodynamics simulations. During the propagation of the ionization front, the cumulated thin shell forming in a D-type front can be subject to the Rayleigh–Taylor instability (RTI; Giuliani 1979; García-Segura & Franco 1996; Ricotti 2014), which cannot be followed by the 1D simulations.

Whalen & Norman (2008) have investigated the propagation of the ionizing front induced by Population III stars, and showed that the formation of hydrogen molecules caused a thin shell instability. Park et al. (2014a) investigated the growth of the RTI at the ionizing front using 2D radiative-hydrodynamics simulations. The code and setup of their simulations were nearly identical to what we used in this work, i.e., BHs accreting from a background of initially uniform density. Their simulations showed that the photoionization feedback suppressed the growth of the RTI, and the ionization front was stable when the radius of the ionized bubble was larger than the Bondi radius. Park & Ricotti (2011) have confirmed that the accretion history of 1D simulations was same as that of 2D simulations including the effect of the RTI. Finally, Park et al. (2017), using 3D simulations, also found similar results to the 1D simulations, but with enhanced turbulence at the I-front. Therefore, our simulation results are not likely to be affected by considering the RTI.

When accreting gas has angular momentum, the accretion is suppressed at a specific radius due to the centrifugal force. Then the accretion can be governed by the angular momentum transport due to the $\alpha$-viscosity in an accretion disk (Shakura & Sunyaev 1973). This process can cause a time delay of radiative feedback after gas enters the inner boundary of the calculation box. As discussed in Section 3.1, the time delay is not likely to affect the global results of our simulations significantly (see Park & Ricotti 2012). On the other hand, the gas accreted with angular momentum can cause anisotropic radiative feedback due to the shadowing effect as suggested by Sugimura et al. (2017). The anisotropy of radiative feedback can significantly change the accretion rate. However, resolving the accretion disk scales and the ionized bubbles simultaneously requires too large a dynamical range to be handled in 3D simulations, and even with current computational resources it is prohibitively expensive. The whole picture of the accreting gas with angular momentum will be investigated in future work by combining small-scale (accretion disk) and large-scale (Bondi radius and ionized bubble) simulations.

## 5. Summary

In this paper we have studied the accretion of dusty gas onto IMBHs by using one-dimensional radiative-hydrodynamics simulations. Park & Ricotti (2011) showed that, for the case of gas of primordial composition, the growth of a BH is strongly regulated by photoionization feedback. As star formation proceeds, high-redshift galaxies are enriched in metals/dust by Type-II supernovae. Recent observations have detected dust-rich galaxies even at $z \gtrsim 6$ (e.g., Riechers et al. 2013). In this paper, we investigate the effects of dust on the growth of BHs and observable diagnostics due to the thermal emission from heated dust. Dust affects the accretion rate and the period of the bursts mainly because of radiation pressure on dust but also because of the dust opacity that reduces the size of the ionized region.

By assuming as a fiducial model accretion onto a BH of $10^5 M_\odot$ embedded in a medium of uniform density, we investigate the dependence of the BH growth rate on the gas density (in the range $n_\infty = 10$ to $1000 \text{ cm}^{-3}$) and on the metallicity (in the range $Z = 0$ to $1 Z_\odot$). We find that the accretion of dusty gas onto IMBHs proceeds gently with small fluctuations of the accretion rate, whereas that of primordial gas causes periodic bursts. For dust-to-gas mass ratio similar to the solar neighborhood, the time-averaged luminosity becomes smaller than that for primordial gas by one order of magnitude. The time-averaged Eddington ratio is $\langle f_{\text{edd}} \rangle \sim 10^{-3}$ for the initial gas density $n_\infty = 100 \text{ cm}^{-3}$. Our calculations show that the effect of dust opacity is secondary with respect to radiation pressure on dust. Neglecting radiation pressure on dust but including the effect of dust opacity, the growth rate of IMBHs and $\langle f_{\text{edd}} \rangle$ are smaller than those of primordial gas by a factor $\sim 2$. For both primordial and dusty clouds, the Eddington ratio, $\langle f_{\text{edd}} \rangle$, increases linearly with the initial gas density. In addition, assuming the constraint $(M_{\text{BH}}/M_\odot)(n_\infty/\text{cm}^{-3}) = 10^7$, we study the dependence of the growth rate on the BH mass. We show that $\langle f_{\text{edd}} \rangle$ is constant for different BH masses in both the cases with and without dust.

Finally, we derive the SEDs at IR bands by calculating dust thermal emission. Our modeled SEDs show that the flux ratio at $\lambda \lesssim 20 \mu\text{m}$ and $\gtrsim 100 \mu\text{m}$ depends sensitively on the Eddington ratio, but is nearly independent of the other parameters in the problem. This is because at high Eddington ratios the thermal emission from hot dust near the BH produces a higher flux density at $\lesssim 20 \mu\text{m}$, while the emission at $\gtrsim 100 \mu\text{m}$, which is produced by warmer dust further out, near the ionization front, is nearly independent of $\langle f_{\text{edd}} \rangle$. Therefore, we suggest that the combination of mid-IR observations by JWST and far-IR observations by Spitzer or ALMA can provide a novel method to estimate the Eddington ratio of BHs throughout their duty cycle, including their short bursty phase and their longer quiescent phase.

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