Nothing but Gold: Complexities in terms of Non-difference and Identity. Part 2. Contrasting Equivalence, Equality, Identity, and Non-difference

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Abstract The present paper is a continuation of a previous one by the same title, the content of which faced the issue concerning the relations of coreference and qualification in compliance with the Navya-Nyāya theoretical framework, although prompted by the Advaita-Vedānta enquiry regarding non-difference. In a complementary manner, by means of a formal analysis of equivalence, equality, and identity, this section closes the loop by assessing the extent to which non-difference, the main issue here, cannot be reduced to any of the former. The following sections of this study will focus on the assessment of the eventual possibility of causation and transformation in non-difference.

Keywords Non-difference · Equivalence · Equality · Identity · Nyāya

Abbreviations

- Primitive term (lowercase italics)
- Abstraction functor, expressing the Sanskrit suffix -tva or -tā (e.g., \(a_t = a\)-hood)
- Set A (capital)
- Extension of an abstract; \(|a| = A\)
- Relation R (capital italics)
- Relational abstract (bold capital italics)
- Relation \(R'\) interpreted as \(R\), salva veritate
- The relation \(R\) set of destination; for \(R\): \(A \hookrightarrow B\), \(\text{dom}R \subseteq A\), \(\text{ran}R \subseteq B\), and \(R[A] = B\)
- Avacchedaka operator; identifying the limitor of a relational abstract
- Nirūpaka operator; identifying the conditioner of a relational abstract

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Niṣṭha operator; connecting an abstract to a primitive term

Tadviparyayena operator (‘vice versa’); expressing a symmetrical relation

Yaṭhā-tathā operator (‘just like-so’); capable of expressing the coordination of a relation with its inverse (\(R \land R^{-1}\)). It always preserves the distinction between abstract properties and primitives terms of the anuyogin and pratiyogin positions

TvN Tadvatta-Nyāya (‘Axiom of Possession’)

SVN Samānādhikaraṇa-Viśiṣṭatva-Nyāya (‘Principle of Coreferential Qualification’)

\*φ ‘It is false that \(φ\)’

(t) ‘…’ Tātparya (purport of an expression)

As stated in the first part of this investigation (P1), non-difference (\(2\))—closely linked to the notion of coreference (sāmānādhikaranya, \(N\))—cannot be reduced to identity or equality. In the following sections I will try to definitely demonstrate why this is the case, but not before having discussed how non-difference cannot be subsumed to the relation of equivalence, either.¹

**Equivalence**

In an axiomatic theory of sets, equivalence (\(E\)) is a binary relation capable of formally expressing the naive concept ‘possessing the same property’.² In Nyāya Kośa (NK), equivalence is described sub voce tulyatva\(_{1kha}\).³ In this manner, \(x\) is

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¹ For the sake of clarity, formulas numbering follows directly from Anrò 2021, P1, allowing easier intertextual references. The notational system adopted here is in compliance with the ‘Navya-Nyāya Formal Language’ or NL (cf. Anrò, forthcoming); a descriptive table is provided at the end of the article.

² Grishin (2014), referring to: N. Bourbaki, Théorie des Ensembles. Eléments de mathématiques 1. In this article I chose to use the symbol ‘\(E\)’ to express equivalence, in keeping with NL notation (where relations are expressed with italic capital letters). I reserve tilde (‘\(\sim\)’) for negation (cf. P1 fn. 23).

³ Following the indexing proposed by NK, I make explicit the index clue ‘\(1[kha]\)’ to distinguish this particular sense of the term tulyatva from the following ones, instead referable to the concept of ‘equality’. The meanings \(1[ka]\) and \(1[kha]\) are explicitly reported as analogous to sādṛśya (similitude): NK: 333: tulyatva—\(1[ka]\) sādṛśyavat asyārtho ‘nusamādhityah’ l. Cf. NK: 991: sādṛśyam—\(1[kha]\) tadbhinnatve satī tadgataḥbhūyordharmavatvam l. Although distinct, two objects are said to be ‘similar’ because they share multiple common features. Moreover, in light of the truth conditions laid out (cf. infra), I see myself as obliged to introduce some differences in relation to Ingalls’ translation: samanīyatatva is ‘equality’ and not ‘equivalence’ here, while ‘equivalence’ is ‘tulyatva\(_{1kha}\)’. This is because, according to Ingalls: “Equality is a relation between classes. Equivalence is a truth function connecting statements or formulæ. Identity is a relation between individuals” (Ingalls 1951, p. 67). Here, on the contrary, equivalence is a relation connecting distinct instances of a given property; equality is a relation connecting statements or formulæ (and only in this sense is it, possibly, a relation between classes); and identity is a relation between individuals. According to the theory of sets: “\(R\) is an equivalence relation on \(A\) iff \(R\) is a binary relation on \(A\) that is reflexive on \(A\), symmetric, and transitive” (Enderton 1977, p. 56). Equality and identity, on the other hand, are equivalence relations under more restrictive conditions. This holds true to a great extent in this context as well.
equivalent (tulya) to y (⟨x, y⟩ ∈ E) if it shares with y a common property (dharma-vattva) even while keeping itself distinct from it (bhinnatva).4

Be it considered, for instance, the indefinite generic statement: gaur gāṁ janayatī (‘A cow gives birth to a cow’), or the following indefinite non-generic one: gāṁ ānaya (‘Fetch a cow’). In all of these cases, by reason of their indefinite character, if a cow (g) possesses the property cow-ness (gotva, g), then a second cow (g') might be said to be equivalent to g with respect to the property gotva. ‘That cow is equivalent to this one’—so gaur etasya gos tulyaḥ—will appear in NL as:

\[ ⟨g, g⟩ \in E \]

[8] (g', g) \in E \ \land \ (g, g) \in E

yad tulyatvam idam-go-nisṭha-gotva(vattva)-āvachinnam tad adah-go-nisṭha-gotva(vattva)-nirūpi tam; ‘Equivalence, conditioned by cow-ness in that cow, is limited by cow-ness in this cow’; iff (g, g') ∈ (lg, l = G) (dharma-vattva = gotvavattva); cf. fn. 3: NK, p. 991) \land \ (g, g') \in E; ‘Cow-ness in cow g' is a sub-set of What is equivalent to Cow-ness in cow g’; that is, (g, g') \in E_{g}.

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4 NK, p. 334: tulyatvam—[kha] bhinnatve sati dharma-vattva | caitreṇa caitrasya vā tulya ity ādau | atra tulyaśaḍabdāḥrāthaniṣṭe ca bhede trīḍyādharthasya pratiyogītvasya dharma cādheyatvasya anyayāt caitravācchinnatve sati caitravītitdharmavāṇa ity arthaḥ l. Indeed, in possession of a property (dharma-vattva) appears as an excessively vague condition to define equivalence: both pyramids and apples possess at least one property each; it does not follow they can be said to be equivalent. Nevertheless, NK declares that tulyatvam[kha] is analogous to sādṛśya: ‘bhūyor-dharma-vattva’, ‘possessing multiple [common] properties’ (NK, p. 991; cf. previous fn.). Here, significantly, the term ‘multiple’ (bhūyas) is omitted. In fact, ‘possession of multiple properties’ (bhūyordharma-vattva) appears as either too vague or singularly inappropriate for a technical use of the term ‘equivalent’. If two sisters are said to be alike, then their likeness (sādṛśya) must be further articulated: a single feature is picked out and then claimed to be common; say, their nose, their voice, etc. In the sense of ‘possessing [at least a common] property [singularly considered]’, the apparently-lacking definition of tulyatvam[kha] could thus be considered—by virtue of its being connected to sādṛśya (bhūyordharma-vattva)—a case of lāghava (lightness in definition). Note also that equivalence can be expressed with either a genitive or instrumental case; yet, NK specifies, the genitive is advisable according to the way grammarians use it: NK, p. 334: [kha] evam caitreṇa caitrasya vā sādṛśyam ity ādāv api draṣṭavyam | atra viśeṣo jñeyāḥ pānīṁyāḥ tulyopamayor yogyo trīḍyām nechchanti iti l.

5 Cf. the Axiom of Possession (Tadvattva-Nyāya, TVN) formulation, P1.§3: tadvattvatm (or taddharmavattvam) tad eva, ‘What possesses the property of being that, is that’. Thus, in [8] gotvavattva = gotva, i.e. the property possessing cow-hood = cow-hood; while, gotvavatta = gotva. For this reason, [8] reads the simplified version and gotva appears instead of gotvavatta. It is well-known that for any equivalence relation R on the set A, it is possible to obtain a partition of A. In this sense, we can obtain a partition of the class (jāti) cow-ness (gotva, g) with respect to a particular quality (guna)—for instance, colour. In set theory, if xEG (i.e. x is a cow) and ‘possessing a colour’ is ‘r’; (rāgavattva; say, sūklatva, whiteness), then the class of equivalence of the element x on G, with respect to the equivalence relation (E) ‘possessing the same colour’ (samarāgavattva), is [x]E = {y | y ∈ G \ {y}, xE_{r},}; i.e. the partitions of the cow set G, according their colour. ‘That cow is equivalent to this one, because of their colour’—so gaur etasya gos tulyaḥ, rāgavattvāt—in NL: (⟨g', g⟩) \in E \ \land \ (g, g) \in E \ \land \ (g', g) \in E \ \land \ (g, g) \in E, that is, (g, g') \in E_{r}.

Cf. Enderton (1977, p. 57): ‘The set [x]_{E} is defined by [x]_{E} = \{t | xRt\}. If R is an equivalence relation and x E_{FLD} (R) [’field’], then [x]_{E} is called the equivalence class of x (modulo R). […] The status of [x]_{E} as a set is guaranteed by a sub set axiom, since [x]_{E} \subseteq \text{ran}(R) [’range’]. Furthermore, we can construct a set of equivalence classes such as \{x \in A | x \in [x]_{E}\}; since this set is included in (ran R);’ where, “for any set a, the power set a is the set whose members are exactly the subset of a”, Enderton (1977, p. 19).
As shown by the previous example, in general the relation of equivalence appears as necessarily bound to domain multiplicity—setting aside, for the moment, the trivial case of the equivalence of an element with itself (reflexive equivalence). In this respect, let us consider the definition of jāti or sāmānya: “[...] sāmānya iti | tallakṣaṇaṁ tu nityatve saty anekasamavetvam”; “[...] The ‘universal’. While its definition is: the property which, being constant, is inherent in many [particulars]” (NSM 1988, pp. 97–98).

It follows that all individuals (vyakti) belonging to a given jāti are by definition equivalent to each other with respect to the jāti to which they belong. On the contrary, let us now consider the first jāti-bādhaṅka (‘blocker’ or ‘opposing agent of the universal’): vyakten aḥbhedā [bhedaḥbhāva], “‘the oneness of the individual’ or ‘indivisibility of the individual’ [or ‘radical absence of any possible distinction’], that is when exists only one member of any category, an individual alone” (Pellegrini 2016, p. 79). For instance, according to the Nyāya analysis, kāla (time) or dik (space) are radically one and therefore, qua singular substances, they cannot have equivalents in the sense that a cow, with respect to another cow, can.

In the utterance vaidyam ānaya (‘Fetch a doctor!’), the implied meaning appears, reasonably enough, to be: mad-rogamukti-विशया eka-kuśala-vaidyasya anya-kuśala-vaidyas tulyah; aṭha kuśala-vaidyam ānaya (‘In order to heal my disease, a skilled physician as well; so, fetch one!’). So—for v: vaidya, a physician; vt: vaidyatra, the property being a physician; V: the set Physicians; and for (v, vt)∈V and |V|=v—we could obtain the meaningful assertion: [8b] (v′, v) ∈ E⊥ (v, v′), for v′ ≠ v. However, [8c] (v′, v) ∈ E⊥ (v′, v), for v′ = v, that is, etat-kuśala-vaidyasya etat-kuśala-vaidyas tulyah (‘This very physician is equivalent to this very physician’) is true either in the secondary and here pointless sense of an individual being equivalent to himself; or even, taken as a negation, with a completely opposite meaning. This last sentence, in fact, could be interpreted as etat-vaidyasya na kaścit tulyah, tasya uttamattvāti: ‘This physician has no equivalent, because he is the best’ or *[8c-1] (v′, v) ∈ E⊥ (v, v), which is nevertheless a patent contradiction for (v, v′)∈E(V=|V|) (‘false’; cf. P1). The truth values thus suggest that the properties involved cannot be the same. Instead, for instance v′ = ‘Being an average physician’, and v′t = ‘Being the finest physician’, with a significative change in the truth values: a) v′ ≠ v′t (v′ ∈ |V|, ) ⊆ (v ∈ |V|); b) the domain of |V| = V will be now greater than one (card(V) ≥ 2) if we assume that there exists at least one average doctor apart from the outstanding one; and c) the domain of |V| = V will necessarily be equal to one, because there is by definition only one utmost exemplar (card(V′) = 1; for the concept of cardinality of a set, cf. infra). In sum, two distinct elements a and b, both belonging to the generic set Z = |Z|, could be said to be equivalent to the generic property zt. This is impossible with respect to the two distinct properties zt and z′t, however, because x and y now no longer belong to the same reference domain; i.e. an average physician is not equivalent to the best physician. Moreover, a reflexive equivalence could be either true but trivial or paradoxically negative and highly

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6 Along the same lines: TrS (2007): nityam ekam anekānugatam sāmānya l; “The universal is constant, one and recurrent in many particulars”. Śivāditya (1934, p. 50): sāmānya nityam ekam anekāsamavetam ll 62; Udāyaṇācārya (1989, p. 120): nityam ekam anekāvṛtti sāmānya. Quotations and translations from Pellegrini (2016, p. 76). Cf. also NS 2.2.67-68 (2009, pp. 522–523).
context-sensitive, although still formally true—e.g. ‘This physician is only equivalent to himself, because he has no equivalent’.

Equivalence appears, in light of the above, to be closely linked to domain multiplicity. Yet what could multiplicity mean in this context? In modern times, Navya-Nyāya—and Raghunātha Śiromāni (c. 1510), in particular—moved beyond the theory of number as an inherent quality (guna) in “adjectival function” (Ganeri, 1996, p. 111), through the logically primitive relational concept of paryāpti-sambandha in the sense of ‘completion’.7 This new conception “bears a close resemblance to the recent concept in Western logic of number as a class of classes” (Ingalls 1951, p. 76).8 Framed in this way, number becomes an imposed property (upādhi) related by paryāpti to the set of objects being numbered: indeed, “paryāpti is the relation by which numbers reside in wholes rather than the particulars of wholes”, so that “the loci of two-ness and of three-ness are mutually exclusive” (Ingalls 1951, p. 77). In this manner, “a trio of men, for example, is an instance of number 3, and the number 3 is an instance of number; but the trio is not an instance of number […; because] a number is something that characterises certain collections, namely, those that have that number” (Russell

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7 ‘Completion’, ‘thoroughness’, or ‘wholeness’, as translated in Ingalls (1951, pp. 76–77) and Guha (1979, pp. 50–56). Also, paryavasāna or sākalya. In other words, paryāpti is “a one-to-many relation […]. It relates numbers to pluralities of objects, but not to objects taken individually” (Ganeri 1996, p. 113). Number is thus a vyāsa-ya-ṛti-dharma, that is, a “property that occur in loci (e.g., a U b) whose parts (a, b) adhere to each other (i.e., are inseparable)” (Ingalls 1951, p. 78); or a “property which occurs jointly [and thus not distributively]” or a “collective property” (Ganeri 1996, p. 115). Regarding the flaws of the number-as-guna account (in particular, self-inherence and cross-categoricity) cf. Ganeri 2001, pp. 414–418. Phillips (1997, p. 361): “Numbers larger than one are cognition-dependent in a strong sense, in that they are created and last only by the act of counting”. See also Shaw (1982) and Jha (1992, pp. 49–60). About Frege’s criticism on the adjectival account, cf. Dummett (1991, pp. 72–81) and Frege (1953: § 22, 28; § 29, pp. 39–40). Acceptance of paryāpti-sambandha was far from unanimous; for a synthetic description of Raghunātha’s innovations and the associated debate, see Ingalls (1951, pp. 76–77); Ganeri (2011, pp. 181–199); and Guha (1979, pp. 169–201) about “the technique of the insertion of paryāpti”. For Raghunātha, “[paryāpti] is a special kind of self-linking relation” (svartipa-sambandha-viśesa), thus not reducible to inheritance; translated by Ganeri (1996, pp. 112–113), quoting Jagadīśa (1977, pp. 38–39).

8 Cf. also: “I would like to observe a point of similarity between the Nyāya theory and Russell’s definition of the number n as the class of all classes of n objects”. Russell (1919, p. 14): “It is clear that number is a way of bringing together certain collections, namely, those that have a given numbers of terms. We can suppose all couples is one bundle, all trios in another, and so on. In this way we obtain various bundles of collections, each bundle consisting of all the collections that have a certain number of terms. Each bundle is a class whose members are collections, i.e. classes; thus each is a class of classes”. Ganeri (1996, p. 120) notes that, in Nyāya approach, “numbers are relations taken in extension, not in intension. This means that the Nyāya has no need for Russell’s ‘axiom of infinity’, the postulate that there are infinite objects in the universe”; for a first survey on the Axiom of Infinity (Axiom des Unendlichen) in Zermelo-Fraenkel, cf. Jech (2006, pp. 12–13); for its formulation: Zermelo (1907, pp. 266–7). Nevertheless, the issue appears even more nuanced. For Russell’s own admission: “Of these two kinds of definitions [definition of a number by extension or by intension], the one by intension is logically more fundamental. This is shown by this issue appears even more nuanced. For Russell’s own admission: “Of these two kinds of definitions [definition of a number by extension or by intension], the one by intension is logically more fundamental. This is shown by this
Thereby, numbers could be conceived as “n-fold relations of mutual distinction: ‘The planets are (at least) three’ is ‘logically equivalent’ to: (¥)(¥)(¥) (Planet (x) & Planet (y) & Planet (z) & x≠y & y≠z & z≠x)”. In a nutshell, the condition laid down by the NK definition of equivalence—bhinnatve sati—requires that the cardinality of the reference domain be greater than one (condition-a) and, therefore, not trivially reflexive (condition-b). Equivalence in a full sense thus only exists between two distinct elements of a given set, which in turn is the reference domain of that property to which these elements are declared equivalent.

For the limited purposes of this article, we are dealing exclusively with natural numbers (viz., not negative integers); I thus propose to express the paryāpti relation in NL through the natural numbers symbol (‘N’), leaving the possibility of expanding the system open to further investigation. Consequently, being ‘two’ linked to the property two-ness (dvitva, 2; cf. NK, p. 381), the statement dvau gāva (‘Two cows’) could be expressed in NL as:

\[ [9] \ (g, g') \in \mathbb{N} \cup 2, \]

yat paryāptitvam dvi-go-niṣṭha-gotvāvacchinnam tad dvitva-nirūpitaṃ; ‘The relational abstract completion-ness, conditioned by two-ness, is limited by cow-hood in two cows’. 12

Now, NK explicitly states that tulyatva means sharing a given property (dharma-vatvā) in the context of a mutual distinction (bhinnatva). However, this very distinction cannot but imply multiplicity—as we have seen, an “n-fold relations of mutual distinction”. Therefore, equivalence can only be conceived as a relation the cardinality of which is strictly greater than one: \( \text{card}(E)>1 \). 13 Thus—being the
condition ‘greater than one’ expressible as $dvitvādi$ (‘two, etc.’, or $\geq 2$)—our first example concerning equivalence to gotva now begs a further truth condition which was previously solely implicit. This means that the reference domain must possess more than one element (i.e., there is more than one cow; condition-a); and that the relation involves two distinct elements of this multiple reference domain ($g'$≠$g$; i.e., we are talking about two different cows; condition-b). In NL:

$$[8_a] \left( (g', g) \cap E \subseteq (g, g) \right) \cap \mathbb{N} \subseteq (\geq 2)$$

yat paryāptitvam go-niṣṭha-tulyatvāvacchinnam tad dvitvādi-nirūpītam, etad eva tulyatvam ca idam-go-niṣṭha-gotvāvacchinnam adah-go-niṣṭha-gotvānirūpītma ca; ‘Equivalence, conditioned by cow-ness in that cow, and limited by cow-ness in this cow, for card$\left(E\right) \geq 2$.\(^{14}\)

In general—being ‘Possessing a particular property’ expressible as $taddharmavattva$ ($td$, cf. fn. 3)—the statement ‘Equivalence between the generic element $a$ and $b$ is a relation whose cardinality is strictly greater that one’ will now appear in NL as:

$$[10] \left( b \cdot td \cap E \subseteq (a \cdot td) \right) \cap \mathbb{N} \subseteq (\geq 2)$$

yat paryāptitvam tulyatvāvacchinnam tad dvitvādi-nirūpītam, etad eva tulyatvam ca idam-niṣṭha-taddharmavattva-avacchinnam adah-niṣṭha-taddharmavattva-nirūpītma ca; The relational abstract completion-ness, conditioned by two-ness, etc., is limited by equivalence, which is in turn conditioned by a particular property in a generic element, and limited by the same property occurring in another element; if $\left( a \neq b \right)$ and $\left( (a, b) \in \left[ td \right] \wedge \text{card}(E) \geq 2 \right)$.

The truth conditions and cardinality of a $tulyatva$ relation undoubtedly show that this cannot, except in a secondary sense, concern the relation of non-difference ($abheda$). A gold crown is undeniably one and, in this sense, the crown ($m$) is non-different from the gold ($h$). This state of affairs has been provisionally expressed in P1 through the relation of $sāmānādhikaranya$ ($N$): $\left[ 2_a \right] \left( h, h \right) \cap \mathbb{N} \subseteq (m, m)$. In all evidence, the cardinality of $\left[ 2_a \right]$ is equal to one (there is but one crown) and thus incompatible with the requested cardinality of $tulyatva$ expressed in [10]. Moreover—in violation of the definition of both $tulyatva$ (cf. dharmavattva) and condition-a (cf. supra)—a well-formed equivalence formula cannot be provided, by substitution, starting from $\left[ 2_a \right]$. The same properties do not appear on both sides of the

Footnote 13 continued
of elements is always $k$. Enderton (1977, p. 132): “Equinumerosity has the property of being reflexive (on the class of all sets), symmetric, and transitive. But it cannot be represented by an equivalence relation, because it concerns all sets”. Enderton (1977: 133): “A set is finite iff it is equinumerous to some natural number. Otherwise it is infinite. Here we rely on the fact that in our construction of [i.e., infinite], each natural number is the set of all smaller natural numbers. For example, any natural number is itself a finite set”. Cf. also Moschovakis (2006, pp. 7–18). Thereby, $\text{card}(R) > 1 \iff \left\langle x, y \right\rangle R \wedge (x \neq y)$; i.e, the cardinality of a generic relation $R$ is strictly greater than one if and only if the two relata $x$ and $y$ stand in relation $R$ and $x$ is different from $y$. Conversely, if $\left\langle x, y \right\rangle R \wedge x \neq y \iff \text{card}(R) > 1$; i.e., if $x$ stands in relation $R$ with $y$ and $x$ is distinct from $y$, therefore ($\circ$) the cardinality of relation $R$ is greater than one.

\(^{14}\) Note that $[8_a]$ is a composed relation; that is, there appears a chief relation ($\left( N \right)$) whose limitor (avacchedaka) is another relation ($E$), in turn composed of its own limitor and conditioner. Parenthesis highlight in NL chief relations.
equivalence relation, that is, the two relata do not belong to the same reference domain: *[2] \((h,h) \not\in E\cdot (m,m)\), false because \(h \in \|h\|\) \((an\ instance\ of\ gold\ belongs\ to\ the\ set\ Gold)\), but \(m \in \|m\|\) \((a\ crown\ belongs\ to\ the\ set\ Crowns)\). So, hāṭakasya na mukuṭam tulyam \(\text{('A\ crown\ is\ not\ equivalent\ to\ gold', } \langle h, m \rangle \not\in E)\).

In a further countercheck, we could state that crown and gold are nonetheless equivalent: mukuṭasya hāṭaka tulyam. What could be the meaning implied here? Firstly, that they are two. This immediately gives rise to a second question: with respect to what property? Uttered by a merchant, it could mean that they are equivalent to their value (mālya) or their ‘purchasing power’ (krayana): krayanāya hāṭaka-mukuṭa-bhūṣaṇasya piṇḍa-rūpa-hāṭakam tulyam \(\text{('For\ the\ purpose\ of purchasing,\ a\ golden\ accessory,\ such\ as\ a\ crown\ (m),\ is\ equivalent\ to\ raw\ forms\ of\ gold,\ such\ as\ a\ nugget\ (p)')}.\)

It follows that there is at least a crown and a nugget, and that both of them are equivalent to the set to which they belong, defined by the property ‘gold-ness’: \([11_a] (m,h) E (p,h)\)

\(yad\ tulyatvam\ mukuṭa-niśtha-hāṭakatva-avacchedakāvacchinnam\ tad\ piṇḍa-niśtha-hāṭakatva-nirūpitam\; \text{‘Equivalence, conditioned by gold-ness in a nugget, is limited by gold-ness in a crown’; iff (} (m, p) \in \|h\|) \land (m \neq p) \land (m,h) \subseteq \|E\cdot (p,h)\|,\) that is, \(\langle g, p \rangle \in E_{ht}\).

Equality

Equality gives rise to challenging questions which are not altogether easy to answer. Is it a relation? A relation between objects, or between names or signs of objects? In [his] Begriffsschrift [Frege] assumed the latter”, and here I do as well.\(^{15}\) According to NK, the relation of equality (samaniyatatva)\(^{16}\) consists in a mutual pervasion or invariable

\(^{15}\) Frege (1966, p. 56): On Sense and Reference, first published in Zeitschrift für Philosophie und philosophische Kritik, vol. 100, 1892, pp. 25–50. The reference is to: Begriffsschrift, eine der aritmetichen nachebildete Formelsprache des reinen Denkens, Halle, 1879.

\(^{16}\)
concomitance (vyāpti) in which the pervaded (vyāpyatva) is also the pervader (vyāpakatva): vyāpyatve sati vyāpakatvam.\(^{17}\) NK advances the classical example concerning cow-hood (gotva) and possessing dewlap, etc. (sānādīmatvā): these properties must be said to be equal since every instance of the former is an instance of the latter, and vice versa, and because—according to the Axiom of Extensionality (AE)—if two sets have exactly the same members then they are equal.\(^{18}\) The very concept is expressed in NK sub voce ‘tulyatva\(_{2ka-kha}^{}\)’: anyūnānatirikta-vyākitvatvam, “x is equal to y when x has all the manifestations (vyākītī) of and no other manifestation than y” (Ingalls 1951, p. 67); as in the case of ghaṭatva and kalaśatva, both translatable as potness and whose manifestations are nothing but pots; or as in the further case of buddhītva (intellection) and jñānatva (cognition).\(^{19}\) Tulyatva\(_{2ka-kha}^{}\) is explicitly mentioned by NK as a ‘blocker’ (bādhaka) impeding the establishment of distinct general properties; it follows that the same individual manifestations (vyākītī) cannot but point to the very same jāti, even if they are expressed with different terms.

\(^{16}\) NK, p. 957. Correspondingly: “niyata-tva, the state of being pervaded”, Jha (2001, p. 224). The term ‘niyata’—closely related to ‘niyama’, ‘restriction’—is a kta-pratayaya (past passive participle; cf. kṛt-prataya or primary derivative) from the root ni-tyām (‘to restrict’). In case of an invariable concomitance (vyāpti), a pervaded (vyāpya, e.g. smoke) is related to a pervader (vyāpaka, e.g. fire), while the reverse relation is usually not allowed (vyabhicāra; lit. ‘deviating’). The vyāpaka (tire) is said to be adhikā-desatyātī: viz., occurring in a greater number of instances; the vyāpya (smoke), on the contrary, nyūna-desatvītī occurs in a smaller number of instances. This is the case of a viśama-niyama, an unequal distribution of occurrences between vyāpaka and vyāpya. However, in case of samavāptī, samanīyama, samanīyata (lit., ‘equal restriction’), sāhacarya-niyama, or sāhacarya-niyata, both vyāpaka and vyāpya occur in the same number of instances or loci; i.e., a co-extension of pervader and pervaded is given. Cf. NK, p. 964: samavāpyāttvam—samanīyatatvam.\(^{2}\) NK, p. 1017: sāhacarya—[1] sāhītīyam [2] sāmānādhikāranyam [3] sāmabhīvāhārahīhl cf. Govardhana, Nyāyabodhīni (TrS, p. 92): sāhacarām nāma sāmānādhikāranyam. Potter (1968, p. 717) translates samanīyatva as “co-extensiveness” and significantly connects it to samavāpyātī or “equal pervasion”, a proposal perfectly suited to the the interpretation outlined here. Cf. also Matilal (1964, p. 87): “The word samanīyata contains the notion of niyama which is usually explained as a vyāk庞大的-reflation (cf. niyamaś cātra vyāpyakātā). Thus, samanīyatatvam has been analysed by the Nāyāvikas as follows: x is samanīyata with y if and only if x is pervaded by y and also the pervader of y (tatsamanīyatatvam tad-vyāpyatvam sati tad-vyāpakatvam”).

\(^{17}\) NK, p. 957. Ingalls (1951, p. 67): “a relation of x to y such that x pervades y and is pervaded by y; x and y may belong to any category”. Ingalls (1951, p. 86): “Gaṅgeśa defines ‘pervasion of x with y’ in the Pañcalakṣaṇī of TC as ‘non-deviation of x with respect to y’, which is further explained as ‘non-occurrence of x in the locus of absence of y’”. Cf. Matilal (1968, pp. 79–80): “pervasion of x with y is ‘x’s concurrence with such a y as is not the counterpositive of an absence which occurs in the locus of x’ (see: hetuman-nīṭha-virāhāprātyyoginī śādhīyena hetor aikādhiharanyam vyāptī ucayate), Viśvanātha, Bhāṣāpariccheda, v. 69)”. See also Matilal (1964, p. 87).

\(^{18}\) NK, p. 957: yathā lakṣyatāvāccedakasamaniyato dharman asādharānaṇadharman ityādau gar lakṣānasaya sānādīmattvāsva lakṣyatāvāccedākābhūtāgatvasamanīyatatvam l. Regarding AE, see Jech (2006, p. 3): “[1.1. Axiom of Extensionality [Axioms of Zermelo-Fraenkel]. If X and Y have the same elements, then X=Y].” Cf. also Enderton (1977, p. 2): “If A and B are sets such that for every object t, t ∈ A iff t ∈ B, then A = B”. In standard notation, with respect to the generic properties P and Q, (∀x) [(P(x) ↔ Q(x))] = (P(x) Q(x)). Samanīyatatva is therefore a binary, reflexive, symmetric, and transitive relation ruled by the logical biconditional (↔), in the sense of ‘both or neither’—as in the case of the properties ‘being an equilateral triangle’ and ‘being an equiangular trilateral’.

\(^{19}\) NK, p. 335: tulyatva—[2]ka anyūnānatirikta-vyākitvatvam | yathā nyāynamate buddhitva-janātavayor ghaṭavakalasātvayor v yāujitvam i dām tu ghaṭavakalasātvādānām bhede bhinnajātītā vā bādhakām iti bodhyam l. 2[ka]: tulyavaktvatvam l svabhinnajātisamanīyatavam iti phalito rthah l yathā ghaṭavakalasātvayor tulyatvam l. Cfr. Mahādeva Bhāṭṭa, Dinakaru, NSM, p. 103-104: tulyatvam tulyavaktvatvam ghaṭavakalasātvādānām jātīnām bhede. See also Jha (2001, p. 182).
“[Vyakter] tulyatvam, the sameness [of the individual]” operates as a blocker; therefore, “the substrate (adhidharaṇa) of the first property is nothing but the substrate of the second one, and vice versa.” More precisely: tulyatvam ca na jātibhādhakahām | kintu jātibhedabhādhakahām; “sameness [of the individuals] is not an universal blocker, but a blocker of the difference between universals”, which are thus, stricto sensu, equal. What follows (phalita) is the very same extension (samaniyatatva) of properties which differ only linguistically.

If samaniyatatva and tulyatva ṣaka-khaṣa define the relation of equality between distinct expressions both of which can refer to the same property, then: iyam gau iti iyam sāsnāmati iti vā, samaniyatatvāt (“This cow or this [animal] possessing dewlap, by virtue of equality”) or ghaṭatvakaḷaśatvayos tulyatvam (“Pot-ness is equal to pitcher-ness”). Equality expresses an identity of reference (vācyā or artha) between distinct signs and expressions (vācaka or pada). Samaniyatatvam vāgāḷambanāṃ nāmādheyaṃ vā: equality is a matter of words; it is a mere verbal difference regarding names or denominations. Thus, there is equality between signs and expressions, but identity regarding the object. In Frege’s words: “Equality. I use this word in the sense of identity and understand ‘a = b’ to have the sense of ‘a is the same as b’ or ‘a and b coincide’ ” (Frege 1966, p. 56, fn. *). Along the same lines, Quine opportune notes that “confusion and controversy have resulted from the failure to distinguish clearly between object and its name. […] The trouble comes […] in forgetting that a statement about an object must contain a name of the object rather than the object itself” (Quine 1981, p. 24). It is thus necessary to plainly distinguish between “Use versus Mention” (Quine 1981: §4, pp. 23–26; cf. also 1987, pp. 231–235). “The name of a name or other expression is commonly formed by putting the named expression in single quotation marks […] We mention x by using a name of x; and a statement about x [inescapably] contains a name of x” (Quine 1981, p. 23). In this sense, in defining the relation of identity as \( x = y \) iff \( (z=x) \leftrightarrow (z=y) \), Quine himself makes use of three different names for the object under examination—while ‘the object under examination’ constitutes a fourth expression. Only the names of x (i.e., its mentions) are distinct, however, because: ‘\( x \neq y \)’ ≠ ‘\( z \neq x \)’ ≠ ‘\( y \neq z \)’ ≠ ‘the object under examination’ (all in single quotation marks); while the use of the names, stricto sensu, allows the affirmation that \( x = y = z \) = the object under examination (all without single quotation marks).22

20 Pellegrini (2016, pp. 79–80), on differentiating jāti and upadhi by means of the concept of ‘blocker’ (bādhaka). [Vyakter] tulyatvam is traditionally counted as the second bādhaka. Cf. also, Phillips (1997, pp. 60–63).

21 Setu commentary on Kiranāvali, quoted by Śastṛi (cf. Udayana 1980, pp. 323–324, fn. 2); cf. Pellegrini (2016, p. 79, fn. 30). Apart from adding the translation slightly, substituting ‘sameness’ for ‘equivalence’.

22 It reads: ‘x is equal to y if and only if, for every z, \( z \) is equal to x and to y’. Quine’s definition might not be immediately intelligible for the reader not conversant with his notation; that is why I have chosen to roughly simplify his account. I am aware that the proposed provisional definition is boldly circular, defining ‘=’ via ‘=’ (and not via ‘E’ as Quine does). I hope the reader will understand the point of this simplification. Nevertheless, here is Quine’s original text: “We turn now to the problem of defining ‘\( x = y \)’ in terms of ‘E’ and our other primitives, that it will carry the intended sense x and y are the same object […]: (x)(z E x . \( \equiv \). z E y), when \( x \) and \( y \) are classes, since classes are the same when their members are the same”; Quine (1981, p. 134). “Let us use ‘\( \zeta \)’, ‘\( \eta \)’ […] to refer in general to any terms. […] The general definition of identity [is thus expressed] as follows […]: \( \zeta = \eta \) for \( \forall (\alpha) \alpha \in \zeta . \equiv \cdot \alpha \in \eta \)”; Quine (1981, pp. 135–6).
Words—variously: pada, śabda, vācaka, or nāman—are said to possess a peculiar primary referential power (śakti; together with its related abstract, śakyatā) by virtue of which they stand solely for certain defined entities (sattva) or meaning-relata (artha, vādyva or vācya) and not others. The issue is particularly complex and surely beyond the scope of this paper; yet, roughly speaking, the pada ‘go’ refers to its artha—the animal called ‘cow’—and not to a chair precisely because of that śakti: the power to point at the specific quality which distinguishes cows from chairs, that is, the pravṛtti-nimitta, the basis or grounds for using that term and not another.23 In this sense, two different expressions in possession of the very same grounds for use (pravṛtti-nimitta) could be said to be equal: vaṭavyṛṣa = nyagrodhapādapa because their primary referential power (śakti) points at the very same referent or artha (i.e., in a third expression, ficus benghalensis). In other terms, I assume that equality, in its proper sense, concerns first and foremost the padapadārtha-sambandha. Samaniyatatva must be conceived as a matter of śakyatā because it provides information about the use of the names of x (viz. about ‘x’, or about its mentions)—while establishing relations of co-extensionality, co-reference or synonymity (samabhivyāhāra; cf. NK, p. 957) between expressions.24 Consequently, I suggest that identity, stricto sensu, must concern the referent in question and not its names—being a statement about x and not about ‘x’ (cf. infra, § 7.).

Let us now analyse the NK example in NL involving the non-symmetric relation of invariable concomitance or pervasion (vyāpti) (cf. Anrö, forthcoming §4.4-5). Let samaniyatatva the relational abstract of equality (Q); gotva (g) the property cow-hood relative to the set Cows (lg,l=G); and sāsnāmatva (s1) the property possessing-dewlap referred to the set Living beings possessing-dewlap (ls,l=S).25

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23 NK, p. 855 śaktih—[ḥa] padapadārthayor vādyavācakabhāvaniyāmakaṃ sambandhāntaram śaktih; “The primary referential power is another [kind of] relation, which defines the relation between term and referent, that is, between expression and what is to be expressed”. NK, p. 580 pravṛttinimittam—[ka] padaśakyatāvuccedakam | yathā ghātavatam ghātapatasya pravṛttinimittam l; “the ground for use is the limitor of the primary meaningfulness of a term. Thus, the grounds for use of the term ‘pot’ is pot-ness”. NK, p. 860 sākyatvam—1. viṣayatāsambandhena śaktiyārayatvam | yathā gavāder arthasya gopadāsakyatvam l; “primary meaningfulness is the property ‘being the locus’ of the primary referential power, in virtue of the relation of content-ness. Thus, the primary meaningfulness of the term ‘cow’ is the referent cow, etc.”; cf. Govardhana, Nyāyabodhini (TrS, p. 129): viṣayatāsambandhena śaktiyārayatvam śakyatvam. For a general survey, cf. Ganeri (2006, pp. 9–48).

24 Nonetheless, as starting points for a discussion of the delicate issue related to interchangeability and cognitive synonymy, cf. among the others: Quine (1951), Carnap (1955). For a purely nominalist and extensional account, where no two different expressions in a language are synonymous, cf. also: Goodman (1949).

25 A cow is usually defined as ‘sāsnādīmat’, ‘possessing dewlap, etc.’; in order to avoid possible confusion, I prefer here to omit ‘ādi’ (lit. ‘beginning from’; e.g. ‘possessing cloven hoofs’), focusing on the property ‘possessing dewlap’ only. Thus: ‘sāsnāmat’ (sāsnāmati gauḥ).
The equality of expressions and the identity of their reference could thus be conveyed as:

\[ (g_{i} \land Q \land s_{i}) \land (s_{i} \land Q \land g_{i}) \leftrightarrow (g_{i} \land N \land s_{i}) \land (s_{i} \land N \land g_{i}) \]

yadi sāsnāmattvam gotvam vyāpnoti evam gotvam sāsnāmattvam vyāpnoti, tarhi sāsnāmattvagotve samaniyate; ‘If cow-ness pervades possessing-dewlap-ness and possessing-dewlap-ness pervades cow-ness, then cow-ness and possessing-dewlap-ness are equal’. Or, in full expression: yadi yad vyāpītvam gotv-avacchinam tat sāsnāmattva-nirūpītam evam yat vyāpītvam sāsnāmattva-avacchinam tad gotva-nirūpītam, tarhi yad yat samaniyatatvam gotv-avacchinam tat tat sāsnāmattva-nirūpītam, athāvā yad yat samaniyatatvam sāsnāmattva-avacchinam tat tad gotva-nirūpītam.

NL calls for a further operator here to express a symmetrical—that is, reversible—relation. For this purpose, be introduced the symbol ‘⇌’ in the straightforward meaning of: tadviparyayena (‘vice versa’, hereafter ‘&vv’). Consequently, [13] will now turn into:

\[ (g_{i} \Rightarrow Q \land s_{i}) \leftrightarrow (g_{i} \Rightarrow N \land s_{i}) \]

yadi sāsnāmattvam gotvam vyāpnoti tadviparyayena ca, tarhi ete samaniyate; ‘If the property cow-ness pervades the property possessing dewlap, &vv, then these properties are equal’. Iff \((G \subseteq S) \land (S \subseteq G) \Rightarrow (G = S)\).

With [14] we have definitely clarified that \(g_{i}\) and \(s_{i}\) have the same extension. Consequently—lest they not mean what they mean—they are in possession of the same ground of use (pravṛttinimitta), which is the limitor of their property of primary meaningfulness (sākyatā, Ś).

\[ (g_{i} \Rightarrow S \land g_{i}) \]

yā sākyatā go-pada-avacchinā sā gotva-nirūpītā, ‘The primary meaningfulness is limited by the term cow, while conditioned by cow-ness’. Iff \(l'g' = g \in \{l|l\}=G\), where single brackets in formulas such as [15] do mean the word (pada) \(x\); thereby, ‘The extension of the word ‘cow’ is a cow, qua instance of cow-ness and belonging to the set Cows’. Analogously, for ‘sāsnāmat’: ‘\(s' \Rightarrow S \land s'\), for \(l's' = s \in \{s\}=S\).

However, in [14]: \((G = S)\), and in [15]: \((l'g' = (g\in G)) \land (l's' = (s\in S)) \Rightarrow (l'g' = l's')\).

Therefore, we can conclude that equality relation—as padapadārtha (or vācyavācaka) sambandha and with respect to the terms ‘go’ and ‘sāsnāmat’ (sāsnādimat)—might be fully interpreted as:

\[ \text{For an example of the use of the locution ‘tadviparyayena’, cf. the incipit of Śāṅkara’s Brahma-sūtra-bhāṣya: […] tadviparyayena viṣayāṇas taddharmānām ca viṣaye ‘dhyāsah […]’; VM-B, pp. 7–9. For alternative formulations in the same meaning, consider also: vilomata, mithas, viparyak, anyonya-tas; cf. Bō. (VI: 117; V: 78; VI: 102; I: 67). It goes without saying that equivalence (E) is a symmetric relation as well. Extensionally, operator ‘⇌’ thus has the meaning of: sāmnādikaranya or anyonyādikratvā, respectively ‘coreferentiality’ or ‘being reciprocally sustained one on another’ (gotva-sāsnāmattve samānādikarane bhavataḥ). Formulas [8]–[12], §1, could therefore be improved and rephrased by means of the ‘⇌’ operator. For this usage, cf. § 4–5.}
In light of the above, the cardinality of the relation of equality will be greater than or equal to one \((\text{card}(Q) \geq 1)\). Firstly, because of the intrinsic plurality of manifestations of a general term (cf. supra). Secondarily, because a term could clearly refer to a singular, as in cases such as ‘dik’ (‘space’, cf. § 1) or in sentences such as ‘ayodhyā-kumāra rāmaḥ’.

It follows that, being the condition \(\geq 1\) expressed as ekatvāḍi \((\geq 1t\), lit., ‘oneness, etc.’), the equality between ‘gotva’ and ‘sāsnātta’ needs its cardinality truth condition to be made explicit, that is: \([16a]\) \((‘g’ ∼ Sₗ·g) \Leftrightarrow Qₗ (‘s’ ∼ Sₗ·s)) \Leftrightarrow ℕ \(≥ 1t\)), for \((\text{card}Q) \geq 1\). In more general terms, the equality between this (etat; ‘a’) and that (tat; ‘b’) generic expression—in relation to their common grounds of use (pravṛt tinimitta), expressed by the same generic property (taddharmavattva, \(td_t\), cf. fn. 3)—as a symmetric relation whose cardinality is greater than or equal to one, will now appear in NL as:

\[ [17a] (‘a’ ∼ Sₗ·tdₗ) \Leftrightarrow Qₗ (‘b’ ∼ Sₗ·tdₗ)) \Leftrightarrow ℕ \(≥ 1t\) \]

Identity

“Identity, we will say, is the relation that each thing has to itself and nothing else. […] The concept of identity is so basic to our conceptual scheme that it is hopeless to attempt to analyse it in terms of more basic concepts” (Hawthorne 2003, p. 99). The problem is that, “roughly speaking, to say of two things that they are identical is nonsense, and to say of one thing that it is identical with itself is to say nothing at

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27 Thereby: ‘dik’ ≠ ‘ākāśa’ ≠ ‘vyoman’, but dik = ākāśa = vyoman. In the same manner: ‘Rāma’ ≠ ‘ayodhyā-kumāra’, but Rāma = ayodhyā-kumāra (‘Rāma is the prince of [the city of] Ayodhyā’, or ‘Rāma, the prince of Ayodhyā’).
all”. A first move in the attempt to figure out this puzzle could be recognising that “a thing is identical with itself and with nothing else”, however obvious it may sound; consequently, to admit that “the identity relation comprises all and only the repetitious pairs, \( \langle x, x \rangle \)” ; nevertheless, and this is the key point, “\( \langle x, x \rangle \) is still not to be confused with \( x \)” (Quine 1987, pp. 89–90). Along exactly the same lines, NK defines the relation of identity—sub voce ‘\( \text{tādātmya}_2 \)’—as referring to a singularity (\( \text{aikya} \)) that cannot but be declared identical to itself precisely because it is that very singularity. Vācaspati Miśra (VM) likewise seems to accept this definition of identity: in negative terms, where there is not difference there is unit or singularity (ekatva): na cet, ekatvam evāsti, na ca bhedāḥ (cf. fn. 49 and P1). Similarly, \( \text{tādātmya}_1 \)kha suggests that identity could also be conceived as an idiosyncratic feature (\( \text{dharma} \)) by virtue of being ‘not-common’ (\( \text{asādhārana} \)) and ‘self-referring’ (\( \text{svavṛtti} \)); thus, radically singular (ekamāttra). This idiosyncratic feature has individuality (vyaktivṛti) as its form (\( \text{rūpa} \)). Thereby, in case of a blue pot, identity—grammatically expressed through the notion of \( \text{sāmānādihikaranya} \)—is precisely that particular individuality in (\( \text{nīśtha} \)) that very pot. In this sense, identity could thus be defined as a relation the cardinality of which is strictly equal to one. Obviously, I am not arguing here that the concept of unit completely parallels that of identity. Rather, I propose that identity is usefully describable through the cardinality one of the ordered couple it consists of; consequently, cardinality one must compose the definition of identity as a decisive factor.

Bhāsarvajña (c. 950) maintains that numbers stand for relations of identity (\( \text{abheda} \)) and difference (\( \text{bheda} \)). “Identity and difference depend on sameness \( \text{svātmāpeksā} \) and distinctness \( \text{parātmāpeksā} \) in colour and so on, and so are not considered to be qualities \( \text{guṇa} \). Further, it is a tautology \( \text{paryāya} \) to say ‘the one is identical’ \( \text{ekam abhinnam} \) or ‘the many are different’ \( \text{anekāṇa bhinnam} \)”.  

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28 Wittgenstein (2001, 5.5303); or, as Quine puts it: “evidently to say of anything that it is identical with itself is trivial, and to say that it is identical with anything else is absurd” (Quine 1987, p. 90). Cf. also Potter (1977, p. 54): “Strictly speaking, identity cannot be a relation within the \( \text{[Nyāya-Vaiśeṣika]} \) system, since the system may contain no two identical things […]. A relation must relate two distinct things, and it must be distinct from them” (cf. P1 fn. 32).

29 NK, p. 328 2 \( \text{tādātmya} \) \( \text{aikyam} \). Straightforwardly: ‘identity is singularity’.

30 Enterton (1977, p. 40): “Let be the set \( \{0, 1, 2, \ldots \} \), […] The identity relation on is \( I_0 = \{ \langle n, n \rangle \mid n \in \omega \} \)”.

31 NK, p. 328 1[kha] \( \text{tādātmya} \) \( \text{svavṛtyasādhāraṇo dharmāḥ} \) \( \text{tādṛṣyadharmas tadvyaktivṛdrūpāḥ} \) \( \text{yathā nilo ghaṭa ity ādau prathamāvibhakter abhedārdhakavatarāme nilāṇiṣṭhaḥadvakyāvṛtiḥ eva nilapaddottaraprathamāvibhaktarthas tādātmyaṃ} \) \( \text{atrāsādhāraṇyam caikamātravṛttivān} \i; \text{cf. Gadaḍhara (2005, p. 37): satyam—abhedas tādātmyaṃ \| tuc ca svavṛtyasādhāraṇo dharmah \| \text{asādhārans ca} \text{ekamātravṛttivān} \i}.

32 Given the generic set \( A \), the identity relation consists in the Cartesian product \( A \times A = \{ (x, x) \mid x \in A \} \) whose cardinality is equal to one. Cf. supra, Quine’s quotation (1987, pp. 89–90). Ingalls interprets identity—also expressed as ‘\( x \ y-svarūpa \)', ‘\( x \ y-tādātmya \)', ‘\( x \ y \text{ eva} \)—as a form of equality referring to individuals. The expressed concept is clear, all things considered; however, it seems to me that the chosen lexicon is highly misleading. Cf. Ingalls (1951, p. 68, 67 fn. 40).

33 “Bhāsarvajña is a radical Naiyāyika who rejects the classical Vaiśeṣika theory that numbers are qualities \( \text{guṇa} \)”; Ganeri (2001, p. 418).

34 \( \text{abhedabheda} \) ca \( \text{svātmāpārātmāpeksā} \) \( \text{rūpādīṣv api bhavata iti na tayor guṇatvādikalanetī} \) \( \text{yathā caikam abhinām iti paryāyas tathānekam bhinnam iti ca paryāyas tataḥ ca dviśvādī apya anekākaparāyaḥ [...]} \i; \text{Bhāsarvajña (1968, p. 159), as translated by Ganeri (2001, p. 418); square brackets are mine.}
“The statement ‘a and b are one’ is synonymous with ‘a = b’. […] On the other hand, the statement ‘a and b are two’ asserts that a ≠ b. […] Indeed, it is now standard to formalise sentences of the form ‘there are n Fs’ by means of non-identity’ […]” (Ganeri 2001, p. 418). In short, “number is but another name for diversity. Exact identity is unity, and with difference arises plurality.”

If x and y are meant as identical, “the intended sense [is that] ‘x and y are the same object’” (Quine 1981, p. 134). Therefore, being that very object, x and y are one. However, we have already seen that the definition of identity, according to Quine, likely sounds like: ‘x is y iff x is z and z is y’. Apparently, defining or even simply talking about identity—which is oneness—necessarily implies a panoply of multiple symbols and expressions; that is, any discourse about the identity of x makes use of the relation of equality between the different names for x—for instance, x is y; then x is y via z, etc. And yet, what about the relation between x and z, used as a medium between x and y? Multiplicity and the proliferation of names and relations are therefore paradoxically introduced where there was nothing but oneness.

To express the difficulties language encounters in dealing with identity—a structurally binary relation, by virtue of the very fact of being a relation, although radically converging on one—what could come to our aid is Frege’s premise about the problem of unit, as expressed in the context of his scrutiny of unit as the building-block of numbers and as the alleged result of abstraction (my glosses about identity appear in square brackets): “If we try to produce the number by putting together different distinct objects [or, in our case, to express identity from the combination of distinct expressions; e.g. ‘Scott = author of Waverley’], the result is an agglomeration in which the objects contained remain still in possession of precisely those properties which serve to distinguish them from one another [and, similarly, we obtain but an agglomeration once again comprising exactly those properties that differentiate the distinct expressions we used: ‘Scott’ and ‘author of Waverley’]; and this is not a number [or identity]. But if we try to do it in the other way, by putting together identicals [or, in our case, if we reaffirm the identity by a combination of identical expressions; e.g. Scott = Scott], the result runs perpetually together into one and we never reach a plurality [or, this constantly coalesces into trivial tautology, and we never achieve any informative expression]. […] The word ‘unit’ is admirably adapted to conceal this difficulty [and so is the term ‘identity’].”

How, then, to solve this conundrum? A negative, counterfactual, formulation could be attempted. Be ‘∄’ the relation of ‘constant or absolute absence’

35 A quotation by W.S. Jevons in Frege (1953, p. 46; § 35).

36 Frege (1953, p. 50, § 39). Dummett (1991, p. 86): “[A] number will be independent of the particular objects counted, being determined, as it ought to be, solely by how many of those objects there are […] It seems to be possible to guarantee this only if no trace of individuality is retained by the units […]”. The problem is that “if every unit is identical with any (other) unit, there can only be one unit”. Cf also, Frege (1953: § 35, 46) quoting W.S. Jevons: “It has often been said that units are units in respect of being perfectly similar to each other; but though they may be perfectly similar in some respect, they must be different in at least one point [for Jevons: “the empty form of difference”, cf. § 44, 56], otherwise they would be incapable of plurality”. Regarding the ambiguity of one, cf. also: Frege (1953: § 29).

Bhāsarvajña’s recursive definition of number (1968, p. 159) faces the same difficulties in distinguishing the unit: “So it is said that one is the initial integer [abhinnā], two is that [one] together with another identical, four is those [three] together with another identical, and so on”; trans. Ganeri (2001, p. 419).
(atytābhāva), ‘constant absence-hood’ (atytābhāvatva; ⌞) its relational abstract, and ‘constant absentee-hood’ (atytābhāvyā-pratīyogitā; ⌦⁻¹) the inverse of this latter—where, in general: (⌜v⌞) = (v ⌦⁻¹). Consequently, the traditional example bhūtale ghaṭa na (‘There is no pot on the ground’) can be expressed in NL as:

\[18\] \( g \cap ⌦⁻¹ (L⁻¹ \cap b) \)

yā atytābhāvyā-pratīyogitā ghaṭatvavacchinnā sā ādheyatā-nirūpītā, saiva ādheyatā bhūtala-nirūpītā; ‘The constant absentee-hood, with respect to the property being over (L⁻¹) a ground (b), is limited by pot-ness (g)’; iff \( |g| \cap L⁻¹ \cap b = \emptyset \) (‘The intersection of the set Pots and the set Superstrata of a certain ground is empty’); in s.n. (∃x, ∀y | Bx, Gy) ( łx, y łL).

Let us now use the same approach to analyse a second classical assertion: ghaṭah pato na (‘A pot is not a cloth’). To avoid any confusion with the relation of equality, concerning expressions, I will introduce here a specific notation for identity (I) and its negation (∁), absolutely abandoning the equality-identity overlap and radically embracing the account according to which “Nyāya conceives of identity as obtaining between objects, not between symbols” (Matilal 1968, p. 46). So, let anyonyābhāva (∁) be the symmetrical relation of mutual absence; anyonyābhāvatva (I) its relational abstract, i.e. the mutual absent-hood; and anyonyābhāvyā-pratīyogitā (I⁻¹) the converse of the latter, i.e. mutual absentee-hood. Accordingly, ghaṭah pato na will turn into:

\[19\] \( p \equiv I⁻¹ \cap g \)

yā anyonyābhāvyā-pratīyogitā paṭa-niṣṭhā sā ghaṭa-nirūpītā, tadviparyayena ca; ‘Mutual absentee-hood, conditioned by a pot, is limited by a cloth, &v&v’; iff (g ∈ G) ∧ (p ∈ P) ∧ (G ∩ P = ∅).

The relation of difference or mutual absence can easily be transformed into a ‘negation of identity between the relata’: anyuyogī-pratīyogi-tādāmya-pratisedha. Thus, for the same truth conditions, [19] can be rephrased in: [20] ⌦(p \equiv I⁻¹ \cap g), where the identity relation (I) between g and p is said to be absent. In accordance with [18], it could be stated that:

\[21\] \( p \equiv ⌦⁻¹ (I⁻¹ \cap g) \)

yā paṭa-niṣṭha-atyantābhāvyā-pratīyogitā sā tādāmyatā-nirūpītā, saiva tādāmyatā ghaṭa-nirūpītā, tadviparyayena ca; ‘Constant absentee-hood, limited by a cloth, is conditioned by mutual absentee-hood, in turn conditioned by a pot, &v&v’; iff (G ∩ P = ∅).

37 Cf. Matilal (1968, pp. 52–61). Regarding the expression of [18] in fourteen different NL permutations, see Anrō (forthcoming). Moreover, relations [19]–[21]—for paṭaḥ pratīyogī and ghaṭo ‘nuyogī in ⌦⁻¹, and the opposite in ⌦⁻¹ —could be symmetrically construed, with the same results, i.e. for ghaṭah pratīyogī and paṭo ‘nuyogī in ⌦⁻¹, and the opposite in ⌦⁻¹⁻¹. However, paying homage to the syntax of the sentence (vākyamaryādī)—which reads ‘ghaṭah pato na’ and not ‘paṭo ghaṭa na’—the former reading could be considered ‘verbally intelligible’ (śabdalahbya), while this latter is only implicit (tātparyal-abhya); see Pellegrini (2015, pp. 152–153).
Keeping in mind the elements laid out in these introductory examples, let us now move to the analysis of the counterfactual definition of identity. As mentioned above, identity is defined in terms of oneness.\(^{38}\) Now, Gadādhara (c. 1650) maintains that “the meaning of ‘one *F*’ [*eka-śabda*] is: an *F* qualified by being-alone [*kaivalya*; i.e. ‘being a unit’], where ‘being-alone’ [or ‘being a unit’] means ‘not being the counterpositive of a difference resident in something of the same kind’ [*svasajāṭīya*]”.\(^{39}\) This ‘uniqueness’ (*kaivalya*), Gadādhara overtly states, radically excludes multiplicity: *kaivalya* in the meaning of *svasajāṭīya-dvitiyā-rāhiya*, ‘being devoid of a second one of the same kind’. If a second one of the same kind were presumed here, the postulated relation would collapse into equivalence—as in the case of two manifestations of the same property. Therefore, the expression “‘one *F*” is to be analysed as saying of something which is *F* that no *F* is different to it. If this is paraphrased in a first order language as *Fx & ¬(*) (Fy & y≠x)*, then it is formally equivalent to a Russellian uniqueness clause: *Fx & (Vy) (Fy → y = x)*” (Ganeri 2001, p. 419). In other words, “to deny that an object a is numerically different from an object b is tantamount to saying that a is identical with b” (Matilal 1968, p. 46; cf. also, NK, p. 186, *ekatva*).

Let us proceed step by step. The definition opens by claiming that this ‘unit’ is the *pratiyogin* of a relation of difference (*bheda*). Accordingly, a single pot *g*, e.g., must appear in the *pratiyogin* position with respect to difference or mutual absence relation (*bheda = anyonyābhāva, I ; and therefore as anuyogin in *F*-1), just as in the assertion *paṭo ghaṭo na: g · *F*-1 ⊨ p* (*‘A cloth is not a pot’*).\(^{40}\)

Now, this difference could be said to momentarily occur (*niṣṭha*) in ‘Something which is the same’ (*svasajāṭīya*): let us call it *g*. Consequently: *(g, *F*-1 ⊨ g’)*, *‘Something which is the same is different from this (e.g., a pot)’*. As the third step, this relation is subsequently negated; because the object under examination must not be the *pratiyogin* of such a relation: “*a-pratiyogin*”, the text states. Thus: *(g, *F*-1 ⊨ g’)*. In light of the above examples, this last assertion can easily be transformed into:

\[
[22] g \equiv \text{ } F^{-1} \sqsubseteq (F^{-1} \sqsubseteq g')
\]

\(\text{yā ghaṭa-niṣṭhā-atyantābhāvīya-pratīyogitā sā anyonābhāvīya-pratīyogitā-}
\text{nirūpītā, saiva anyonābhāvīya-pratīyogitā svasajāṭīya-ghaṭa-nirūpītā,}
\]

\(^{38}\) Cf. also, Russell (1919, p. 181): “Number 1 is to be defined as the class of all unit classes, i.e. of all that have just one member, as we would say but for the vicious circle”.

\(^{39}\) *ekaśabdasaya kaivalyādviśisṭe saktih, kaivalyaṇ ca svasajāṭīya-dvitiyārāhiyaṃ, tac ca svasajāṭīyanubhāvahapratīyogitvam, svasajāṭīyaṇ ca uddeśavaiśesvayacakalakṣābdāt kaivalyaghaṭatvakena prakṛtavidheṣwartyavartvāṇaṃ pratīyat*; Gadādhara (1929, p. 167), as translated by Ganeri (2001, p. 419); square brackets are mine. As a general rule: “*yasyābhāvah sa pratiyogī* (counterpositive is that whose absence [is spoken of]); Matilal (1968, p. 52, fn. 2).

\(^{40}\) In our example, we are talking about a pot (*g*). Now, Gadādhara states that ‘oneness’ (*ekatva*) = ‘uniqueness’ (*kaivalya*) = ‘Being devoid of a second of the same kind’ (*svasajāṭīya-dvitiyā-rāhiyaṃ*) = ‘Not being the counterpositive of a difference with respect to something which is the same’ (*svasajāṭīya-niṣṭha-bhedāpratīyogitvā*) (cf. previous fn.). Now, where do all these more and more defined properties occur? It is easy to understand that singularity cannot but occur (*niṣṭha*) in the pot (*g*) we are talking about: because ‘this pot is this pot’ (*g*, *p*)\(\notin\)I, cf. supra Quine (1987, p. 98–90). Consequently, the remaining three properties cannot but concern and be referred to this very object. Thus, this pot (*g*) is not the *pratiyogin* of the claimed relation of difference, because the property ‘not being the counterpositive’ (*a-pratiyogitvā*) occurs in this.
tadviparyayena ca; (1)'This pot is the limitor of the constant absence of the mutual absence with respect to something which is the same, &vv'.

This assertion is true iff $g \notin \mathcal{I} \setminus g'$, because the constant absence (atvantābhāva, तद्विपर्यय) of $g$ occurs in $\mathcal{I} \setminus g'$. And yet $\mathcal{I} \setminus g' = G'$, i.e. the set Everything which is not $g'$, in which $G'$ is a singleton containing $g'$ solely: i.e. $G' = \{g'\}$. Therefore, if $g \notin G'$, then $g \in G'$; but $G' = \{g'\}$, so: $g = g'$ or, better, $\langle g, g' \rangle \in I$ (i.e. the two linguistically different expressions 'g' and 'g'' refer to the very same singular extension). In other words, if $g$ does not belong to the set Everything which is not $g'$ (i.e. $G'$), then $g$ cannot but belong to $G'$; however, $G'$ is a singleton whose unique element is $g'$. Thereby, $g$ and $g'$ are one and the same. In brief: $g = g'$, even if '$g' $\neq 'g''(name vs. mention); $\langle g, g' \rangle \in I$; and $(g, g') \in G'$ (as imposed by the definition: svasajātiya) for $\text{card}(G')=1$. So: ghāto aneko na bhāsate, ghātaikatvāt, 'No pot-multiplicity appears, because there is but one pot'. That is, ghāta-svasajātiya-ghaṭayos tādāntyam, 'Identity between this pot and what is the same thing as this pot', because these two expressions point to the very same pot. Therefore, if [22], then:

\[\text{[23]} \ (g \Rightarrow \mathcal{I} \setminus g') \land N_{\mathcal{I}}(I) \]

\[
yat \ tādāntyatā-avacchedaka-avacchiniya-parāyāptītavām \ tad ekatva-nirūpitam, ghāta-kaivalyād; saiva ghāta-niṣṭha-tādāntyatā svasajātiya-ghaṭa-nirūpitā, tadviparyayena ca; (1)'Pot' is identical to pot, &vv, for $\text{card}(I) = 1'$; iff $\langle g, g' \rangle \in I$.\]

The question potentially remains, why must the cardinality necessarily be equal to one? Firstly, for textual reasons: because Gādāhari himself imposes this condition when discussing the meaning of 'the term one' (ekāśabda). Secondly, for logical reasons. Indeed, what if the above analysis (cf. [21]–[23]) were repeated in terms of general properties, e.g. pot-ness ($g_t$)? The result would then be:

\[\text{[24]} g_t \Rightarrow \mathcal{I} \setminus g'_t \]

\[
yā \ ghātatvāvacchiniya-atvantābhāviya-pratijogita śā anyanyakūdyā-pratijogita-nirūpitā, saiva anyanyakūdyā-pratijogita svasajātiya-ghaṭatva-nirūpitā, tadviparyayena ca; whose purport is: (1)'Pot-ness ($g_t$) identical to pot-ness' ($g'_t$), &vv'.
\]

Formula [24] is true iff ($g_t \notin \mathcal{I}$) \wedge ($g'_t \notin \mathcal{I}$) \wedge ($g_t \in \mathcal{I} \setminus g'_t \in \emptyset$); but, $\mathcal{I} \setminus g'_t = G'$; therefore, $G \cap G' = \emptyset$. It follows that $\langle g_t, g'_t \rangle \in Q$ and $\langle G, G' \rangle \in I$, i.e. the expression 'pot-ness' is equal to the expression 'pot-ness' and the set Pot-ness is identical to the set Pot-ness' because they are the very same set (AE; cf. fn. 18).

Now, if we chose to distinguish $g_t$ and $g'_t$ call them ghātatva and kālaśatva, from a linguistic perspective the application of Gādāhari’s definition to a general property

\[\text{etadghāta 'nayapadāheheho bhinnah ('This pot is distinct from whatever else anything is'). Thus: (} \ (g \Rightarrow \mathcal{I} \setminus p') \land N_{\mathcal{I}}(\geq 2), \text{ ghāta-paṭayor bhinnate sati, yad anyanyakūdyā-pratijogyāvacchiniya-parāyāptītavām tad dvitvādi-nirūpitam, saiva ghāta-niṣṭha-pratijogita paṭa-nirūpitā, tadviparyayena ca; iff } (g \neq p) \land (\text{card}(\mathcal{I}) \geq 2).\]

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such as $g$, collapses significantly into equality ($Q$; cf. § 2). If, on the contrary, the same name, say ghātāvya, were retained, this would be just a reflexive case of equality. If equality primarily concerns different names with the same reference, identity should first and foremost concern reference and not its names, otherwise the one would collapse into the other. What could identity mean with regards to a set, if it is not a matter of names? Once again, the key is to think in terms of relations on the Cartesian plane. What is at stake here is the set of ordered couples belonging to the relation $\langle G, G \rangle \in I$ (i.e. $G \times G$ according to relation $I$), in which every single element of $G (g^1, g^2, \ldots, g^n)$ stands in relation $I$ to itself: $\langle g^n, g^n \rangle \in I$. In other words, the extensional interpretation of identity, with respect to a general property such as ghātava ($g$), turns out to be the set of the ordered couples stating the identity of all the elements of the dominion with themselves. It goes without saying that each of these couples clearly has a cardinality equal to one, as expressed in [23].

The cases of dik and Rāma have already been discussed in § 2: ‘dik’ ≠ ‘ākāśa’ ≠ ‘vyoman’, and ‘Rāma’ ≠ ‘ayodhya-kumāra’, are meant as different names (nāman, śabda, or pada) for the same reference, the space and the hero called Rāma. In light of § 3, it is now clear that from a linguistic point of view (śabdataḥ), referring to the same artha (object or reference), the name ‘dik’ is said to be equal to ‘ākāśa’ and ‘Rāma’ to ‘ayodhya-kumāra’; however, from an extensional point of view (arthataḥ) there is nothing but space, or nothing but Rāma. Thereby, arthataḥ, and starting from the assertion rāmo ‘yodhya-kumāra naiti na (‘It is false that Rāma is not the prince of Ayodhya’):

[25] $((ayodhya-kumāra) \Leftrightarrow \exists^{-1} \cap (I^{-1} \cap (Rāma))) \forall \mathbb{N} \setminus I,$
yad antābhāvīya-pratīyogitā-avacchedakāvavchinnā-paryāpti(vam tad ekatva-nirūputam, saiva ayodhya-kumāra-niṣṭha-antābhāvīya-pratīyogitā anyonyābhāvīya-pratīyogitā-nirūputā, saiva anyonyābhāvīya-pratīyogitā rāma-nirūputā, tadviparyayena ca; (1)‘The prince of Ayodhya is not the counterpositive of a difference occurring in Rāma, &vv; for card=1’.

That is, rāmo ‘yodhya-kumāraḥ in the meaning of rāmāyodhya-kumārayos tadātmyam (‘Rāma is identical to the prince of Ayodhya’), because: $\langle Rāma, ayodhya-kumāra \rangle \in I$, or $\langle Rāma, Rāma \rangle \in I$, or $\langle ayodhya-kumāra, ayodhya-kumāra \rangle \in I$, and card($I$) radically equal to one. In parallel, śabdataḥ (cf. supra [17]):

[26] ‘Rāma’- $\mathfrak{S}$-etapuruṣa $\Leftrightarrow Q$- (‘ayodhya-kumāra’- $\mathfrak{S}$-etapuruṣa)) $\forall \mathbb{N} \setminus (≥1)$, yad samanīyatavata-avacchedakāvavchinnā-paryāptivam tad ekatvā-di-nirūpīta, tatra yad eva rāma-pada-avacchinnā-etapuruṣa-nirūpīta-śakyatā-avacchedakāvavchinnā-samanīyatavatvaṃ tat ayodhya-kumāra-pada-avacchinnā-śakyatā-nirūpītaṃ, esāva śakyatā etatvā-avacchinnā-etapuruṣa-nirūpitā, tadviparyayena ca. Whose purport is: (1)‘Two distinct verbal expressions—‘Rāma’ and ‘prince of Ayodhya’—referring to the very same individual (card($Q$sub[26])=1).”
In conclusion, the constant counterpositive-ness (atyantābhāvya-pratīyogitā) of identity (tādātmya) has proved to be mutual absence (anyonyābhāva) or diversity (bheda). Conversely, the constant counterpositive-ness of mutual absence is nothing but identity.\(^{42}\) What has been obtained in this section is thus a counterfactual redefinition of identity in terms of oneness, mutual exclusion and constant absence; or, from a purely extensional perspective, its redefinition in terms of membership relation, complement and the cardinality of a set.\(^{43}\)

Interpreting Non-difference

VM has openly stated that the relation of non-difference (abheda, abhinna; \(\exists\)) is linguistically expressible in terms of sāmānādhi-karanya (N), syntactical homogeneity or coreferentiality (cf. P1). Yet, how to interpret in detail this relation? Could non-different relata be also said at once equivalent, equal, or identical? In the light of previous paragraphs, it will be argued that none of these interpretations is viable.

Interpreting a given relation means here to explore when this is true with respect to other relations. In this sense, if two relations are at once true, in every possible cases, they are one and the same, as e.g. syntactical homogeneity and coreferentiality are: \((\forall x) (\langle x, y\rangle \in R \land \langle x, y\rangle \in R') \rightarrow (R=R').\) In parallel, two relations are completely distinct if they never are simultaneously true: \((\forall x) (\langle x, y\rangle \in R \land \langle x, y\rangle \in R') \rightarrow (R\neq R').\) If a relation is always true when a second one is true, but not vice versa, the former is included in the latter: if \((\forall x) (\langle x, y\rangle \in R' \rightarrow \langle x, y\rangle \in R) \land (\exists x) (\langle x, y\rangle \in R \land \langle x, y\rangle \in R'),\) then \(R' \subseteq R.\) In a fourth case, two relations could share some common pairs and, in this sense, they could be said just ‘resembling’ (\(\equiv\)): \((\exists x) (\langle x, y\rangle \in R \land \langle x, y\rangle \in R') \rightarrow (R \equiv R');\) or \(R \cap R' \neq \emptyset.\(^{44}\)"

\(^{42}\) Cf. the analogous counterfactual definitions: Viṣvanātha’s NSM 12: anyonyābhāvatvat tādātmya-sambandhāvakacchāna-pratīyogitābhāvatvat (quoted by Ingalls as an instance of ‘essential identity’; Ingalls 1951, p. 68 fn. 134); NK, p. 328: I[ka]: tādātmya-sambandhāvakacchāna-pratīyogitāko yah abhāvah so nyonyābhāvah; cf. TrS (2007, p. 172): tādātmya-sambandhāvakacchāna-pratīyogitāko ‘nyonyābhāvah. See also Bālayutpattih (2012, p. 13; cf. also Pellegrini 2015, pp. 152–153): tādātmya-sambandhāvakacchāna-pratīyogitābhāvatvat anyonyābhāvasya lakṣaṇam. The same definition is also discussed in Matilal (1968, pp. 46–47).

\(^{43}\) Ingalls (1951, p. 71) defines identity in the same manner: “If \(x\) occurs whenever \(y\) occurs and vice versa, then \(x\) and \(y\) are essentially identical”; in Ingalls’ notation: ‘\(-\sim x \neq x’\), i.e. constant absence of mutual absence (bhedabhava) of \(x\), identical (\(\equiv\)) to \(x\). About bhedabhava as a matter of controversy, cf. Ingalls (1951, pp. 71–72).

\(^{44}\) E.g., the relation \(R\) (‘Having the same number of sides’) and \(R'\) (‘Having the same number of vertices’) identify the very same ordered pairs of two-dimensional polygons; thus, \(R=R'.\) On the contrary, the relation \(Q\) (‘Having the same squared root’) and \(Q'\) (‘Having the same shoe size’), on domains Numbers and Humans, do not apparently share any pair; thus, \(Q\neq Q'.\) Yet, on the domain Kids, the relations \(Z\) (‘Sibling’), \(Z'\) (‘Having the same surname’) reasonably could be said to resemble each other, albeit modulated by conventions and contingencies. Thus, \(Z \neq Z'\), also assuming here, for the sake of the discussion, that \(Z'\) and \(Z''\) resemble each other more than \(Z\) and \(Z'\). The concept of relational resemblance (\(\equiv\)) aims to highlight resemblances (common instances) between relations; these resemblances may then make it possible to interpret one relation via another. Relational interpretation aims to act as a tool in defining non-difference by means of overlapping and distinction with other relations. On resemblance as a tool to detect overlapping similarities and crisscrosses (e.g. games resemblances), cf. family resemblances (Familienähnlichkeit), Wittgenstein (2009: § 66).
Let us consider again the case of a golden crown. In [3] \( h, h \in V^N \land m \in m \) (ruled by SVN, Samānādhikaraṇa-Viśiṣṭatva-Nyāya or ‘Principle of Coreferential Qualification’; cf. P1), it has been shown that every case of coreference \( (N) \) can properly be interpreted in terms of qualification (viśeṣya-viśeṣaṇa-saṃsarga, \( V \)). Moreover, assertions [2]–[3] can stand as proper interpretations of mukūṭa-hāṭakayor abhiṣedah (‘Non-difference between crown and gold’), because ostensibly: \( (2) \subseteq (N) \subseteq (V) \).\(^{45}\)

\[ 27 \quad h, 2 \in m \]

\( \text{yā abhinnatā hāṭaka-niṣṭhā sā mukūṭa-nirūpitā & yā abhinnatā hāṭaka-niṣṭha-hāṭakatvāvacchinā sā mukūṭa-niṣṭha-mukūṭatva-nirūpitā; ‘Non-difference-ness in gold, conditioned by a crown’. Iff } \langle m, h \rangle \in 2 \land \langle m, h \rangle \in N \land \langle m, h \rangle \in V. \]

In assertions \(*2_b^2 \) and [11]–[12] (cf. §2), it has already been shown that [2]–[3]—and consequently [27]—cannot be interpreted as instances of equivalence on the model of [10]. Neither could they be interpreted as instances of equality on the model of [17]: \(*17_a^2 \) \( ((m \cdot ̂ S \cdot m) \subseteq Q \cdot ̂ (h \cdot ̂ S \cdot h)) \subseteq N \subseteq (1) \), which is false because: \( (h \in \text{G} \text{M}) \land (m \in \text{G} \text{G}) \), therefore \( G \neq M \). It follows that only its negation can be true, i.e.:

\[ 28 \quad ((m \cdot ̂ S \cdot m) \nsubseteq ̂ Q \cdot ̂ (h \cdot ̂ S \cdot h) ) \subseteq N \subseteq (1) \]

yad atyantābhāvīya-pratīyogitā-avacchedaka-vacchinā-paryāptitvam tad ekatvād-nirūpitam, tatra yā mukūṭa-pada-avacchinā-mukūṭatva-nirūpita-sākyatā-avacchedaka-vacchinā-atyantābhāvīya-pratīyogitā sā samaniyatvatva-nirūpitā, tad eva samaniyatvatvam sākyatā-nirūpitam, saiva sākyatā hāṭaka-pada-avacchinā hāṭakatva-nirūpitā, tadvipārayeṇa ca.

It is clear that (1) ‘The term gold is not equal to crown, simply because that which is a crown is not indifferently called gold, &vvd’. Let us consider the assertions: ‘Gold is mined’ or ‘In the periodic table, the chemical element known as gold has the atomic number 79’. Here, any substitution would clearly be nonsense because crowns are not mined, nor are they chemical elements in the periodic table, nor do they have an atomic number.\(^{46}\) The grounds for the use (pravṛtti-nimittā) of the terms ‘mukūṭa’ and ‘hāṭaka’ is plainly distinct, thus the two terms cannot be coextensive.

Moreover, equality is unquestionably a symmetrical relation since it identifies coreferentiality between terms, as in formulas such as [16] and as expressly stated

\footnotesize

\(^{45}\) For instance, the assertion dāndi puruṣaḥ (‘A staff holder’) qualifies \( V \) a man by means of a staff, though that does not imply that there is a relation of coreference \( (N) \) between the two relata—despite the fact that it is linguistically expressed as a case of syntactic homogeneity. The same goes for ghatavad-bhūtalum, ‘A ground qualified by a pot’ (lit. ‘A pot-possessing ground’) or kākavad-ghram, ‘A house qualified by a crow [on its roof]’. Since there are cases in which \( V \) is true but \( N \) is false, qualification appears to be more general and co-reference a more specific interpretation of the former (e.g., excluding all instances of qualification by contact, sanyoga-sambandha).

\(^{46}\) The substitution—in every assertion and also in [17]—would instead be perfectly sound with truly coextensive terms such as ‘svuarna’, ‘kanaka’, ‘kāṅkana’, etc. or, say, with the chemical symbol ‘Au’. It is well known that the analysis could be pushed forward as advanced, among others, by Putnam in his ‘Twin Earth thought experiment’ about the analogous case of ‘water’ and ‘\( H_2O \)’; cf. Putnam (1973). For the present purposes, these further issues are voluntary set aside. Regarding Substitutivity test, cf. fn. 50.
by the operator ‘\(\equiv\)’ (\(g = \text{sāsnādimat as well as sāsnādimat} = go\)). Since \(\neg g, \neg \ell_s, \ell_s \vdash S \land (G \equiv S)\), equality is a relation having set \(G\) — that is \(S\) — as its reference domain and range (i.e., \(Q^{\sub[16]}\)). \(G \mapsto G\) or \(S \mapsto S\). The same is not true for \(V\) and \(\supset\), which are consequently not symmetric. Consider the case of ‘A smiling man’ (\(\text{smayan puruṣāḥ}\)): while this man is qualified by his smile, it is harder to accept that a smile is qualified by this man who smiles — just as in the case of blueness qualifying a pot, which simply cannot be qualified by pot-ness. Thus, relation \(V\) openly appears to be not-symmetric and requires its proper inverse \(\left(\supset^{-1}\right)\) to be reversed.\(^47\) Syntactic homogeneity (\(\text{sāmānādhikaranya,} N\)) is, on the contrary, too vague a notion to be considered symmetrical or not. In fact, its possible symmetry depends on its interpretation: if \(N\) means equality — as in the sentence \(\text{sāsnādimatī gauh}\) — then it will be transitive and symmetrical. As shown, however, if it was interpreted as a general instance of qualification, it could no longer be said to be either symmetrical or transitive — just as in \(\text{nīlo ghaṭaḥ}\) (cf. also fn. 47). The issue might not be quite so predictable with regard to non-difference. In the golden crown case, if \(\supset\) is interpreted as a \(\text{viśiṣṭa-jñāna}\) — in which the crown is non-different (\(\supset\)) from the gold by which it is qualified \((V)\) — then \text{abheda}\ will clearly be non-symmetrical. Moreover, if non-difference were then further interpreted as ‘consisting of’ or ‘being made of’, it would be newly non-symmetrical. Indeed, it can safely be stated that a pot is ultimately clay (cf. Chandogya Up. 6.1.4–6), but it is harder to accept that clay is a pot or consists of a pot. Along the same lines, VM’s interpretation explicitly puts \text{abheda} in contact with causation (\(\text{kāryakāraṇabhāva,} K\)) in general and with material cause \(\text{upādānakāraṇa,} ^{43}K\) in particular (VM-B, p. 72–73). Thus, if \(k \rightarrow K \rightarrow r\), \(y\) \(\text{upādānakāraṇata kāraṇa-avacchinna sā kārya-nirūpitā}\) (‘Material causeness, conditioned by the effect \(r\)’, occurring in the cause \(k\)); its symmetric form is clearly false: \(*r \rightarrow K \rightarrow k, *y\) \(\text{upādānakāraṇata kāryaavacchinna sā kāraṇa-nirūpitā}\) (‘Material causeness, conditioned by the cause, occurring in the effect’). Then, the effect \(k\) could be said, once proved, to be non-different from the cause \(k\) from which it derives: \(k \supset \supset \supset r\). However, merely switching the relata is nothing but nonsense here as well: \(*r \rightarrow k \rightarrow k\) (‘The cause is non-different from the effect’). A negation of symmetry could be also achieved by interpreting non-difference as a case of ‘part and whole relation’, since what possesses parts (\(\text{avayavān}\) might be conceived as non-different from the parts \(\text{avayava}\) it possesses, but not vice versa. Thus, while it is reasonable to say that ‘A horse is not different from a limb of itself’, ‘A limb is not-different from a horse’ sounds slightly stranger in some way. In the form of joke, one of the Buddha’s teeth is not the Buddha.\(^48\)

\(^47\) Thereby: \(\text{pot, blueness} \in V\), i.e., ‘A pot qualified by blueness’, true for \(V\): Pots \(\mapsto\) Properties of Pots; while \(\text{blueness, pot} \in V^{-1}\), i.e., ‘Blueness qualifying a pot’, true for \(V^{-1}\): Properties of Pots \(\mapsto\) Pots. In other words, be it considered that the relation \(B\) (‘\(x\) is brother of \(y\)’); \(B\) is clearly symmetrical, because: \((x, y) \in B\) is true as well as \((y, x) \in B\), having the set Brothers as its domain and range \((B: \text{Brothers} \mapsto \text{Brothers})\). Consider now that the relation \(F\) ‘\(x\) is the father of \(y\)’; \(F\) is patently not symmetric, because \((x, y) \in F\) is true but \(* (y, x) \in F\) is false \((y\) is not the father of \(x\), simply because \(x\) is the father of \(y\)\). The only way to make \(* (y, x) \in F\) true is to construe its inverse relation \(F^{-1}\): ‘\(y\) is son of \(x\)’. Thereby: \((x, y) \in F\), true for \(F: \text{Fathers} \mapsto \text{Sons}\); while, \((y, x) \in F^{-1}\), true for \(F: \text{Sons} \mapsto \text{Fathers}\). The relation \(\supset\) has to be treated in the same way (cf. infra).

\(^48\) While without hooves or pectoral muscles there is no horse (and therefore ‘A horse is not-different from its hooves or pectoral muscles’), pectoral muscles are not a horse. Similarly, a pot is non-different
Moreover, while equality is a transitive relation, non-duality is not—and neither is $V$. If $hātaka = suvārna$ and $suvārna = kanaka$, then $hātaka = kanaka$ (cf. fn. 46), since these padas have one and the same grounds of use. And yet, being $b$, the property $kaṭakatvā$ (‘bracelet-hood’, for $|b| = B$), given $h.N \vdash b$ (A golden bracelet) and $[2] h.N \vdash m$ (A golden crown)—or $h. 2 \vdash b$ (A bracelet not-different from gold) and $[27] h. 2 \vdash m$ (A crown not-different from gold)—it patently does not follow that $*b.N \vdash m$ (A crown which is a bracelet) or $*b. 2 \vdash m$ (A crown non-different from a bracelet). In other words, if the crown is golden and so is the bracelet, it does not follow that the crown is a bracelet. One last remark about equality: it is surely licit to use it reflexively, but such a use appears somehow secondary in that it is lacking any informative value. Indeed, whereas it could be of some use to state that ‘gold = suvārna = Au (in the periodic table)’, it is much less interesting to repeat that ‘gold = gold’. The same holds for non-difference: it is safe to assert that ‘$m. 2 \vdash m$’ (A crown non-different from a crown), but such an assertion is utterly uninteresting.

To summarize, it turns out that, even though the crown is in fact gold, it cannot be said to be equal to gold, nor crown-ness to gold-ness. Nonetheless, this crown is still gold, a fact which renders the assertion ‘The crown is not gold’ ($*m \neq h$) also concurrently false. VM openly declares that non-difference is never reducible to a relation of reciprocal absence ($parasparābhdāvā$; i.e. $\not{\vdash}$). If that were the case, there would exist only simple difference and not any kind of non-difference. This eventuality is simply impossible ($asambhava$), however, because it would be directly contradictory ($virodha$) to non-difference: by hypothesis, the two properties do co-exist ($saha-avasthānā$) in the very same locus. If simple difference ($*m \neq h$) were the case, then the relation between gold-ness and crown-ness in a golden crown would be assimilable to a relation to whatever other property, say, horsehood: if $\{m, h\} \not{\in} 2$ was read as $m \neq h$, then a crown would also be not-different from a horse. In other terms, if non-duality was conceived as equality or diversity, we would be pushed back to the starting contradiction (cf. P1): $*m = h$ is false, as is $*m \neq h$. Thus, the crown is (i.e., $N$, $V$, and $2$) surely gold, yet not in the sense implied by equality or difference.

Footnote 48 continued
from its incurved sides, because if the sides were taken away there would be no pot left. In parallel, a side of a pot cannot store water, thus revealing that it is not a pot: i.e. a pot is non-different from a side of itself, but not vice versa. Potter (1977, p. 74–75): “In Nyāya-Vaiśeṣika a whole is produced from its parts, but is not constituted by them. Favourite examples in the literature are the pot which is produced by its halves, and the cloth which is produced from the threads which compose it. The pot and the cloth are not aggregates of sherds or threads; the pot is an unified substance, of medium dimension, with its own qualities and relations, a different entity from the sum or collection of its components” (italics added; because what I am trying to argue, in this paper, is that a pot is neither different from nor identical to its parts, simply because it is non-different (abhīma) from them). Phillips (1997, p. 147 and fn 84): “Logicians from the earliest period defend […] the position that the whole is more than the sum of its part (excluding heaps, collections, and the like)” See also, NS 4.2.4-17 (2009, pp. 698–706).

$49$ VM-B, p. 73: atrocyate kah punar ayam bheda nāma, yah sahābhedenaikatra bhavet? parasparābhāvā iti cet, kim ayam kāryakāraṇavah kaṭakahātakayor asti na vā? na cet, ekatvam evāsti, na ca bhedaḥ āstī cat bheda eva, nābhedaḥ na ca bhāvābhāvayor avirodhaḥ, sahāvasthānāsambhavāt 1 sambhāve vā kaṭakavardhamānayor api tattvenābhedaprasāngāḥ, bhedasyābhedāvirodhāt 1.
Now, could non-difference be interpreted as a relation of identity? Let us try to interpret the assertion hāṭakaṃ mukuṭam in terms of identity following the model of [22]–[24]—the crown is (N) gold, in the sense that the crown should be said to be identical (I) to gold:

\[ h \leftrightarrow \text{I.m} \lor [30] h \cdot \text{m}^{-1} \subseteq (\text{I}^{-1} \subseteq m) \]

That is, according to the counterfactual definition of identity, the crown should not be the counterpositive of an absolute absence of a mutual absence with respect to something which is that very entity, i.e. the gold. Here, a first important point: [30] is true for h∈I^1∩m, i.e. ‘An instance of gold (h) is meant to belong to the singleton |m| = \{m\’, which is indeed the case (‘A crown is not the counterpositive of an absolute absence of a mutual absence with respect to an instance of gold’).

What we are talking about is this crown, which is (i.e., V, N, 2, and I) this gold: what is at stake here is this very singleton. Non-difference fits the counterpositive definition of identity because these two relations ontologically focus on the very same artha. So far, non-difference seems to coalesce dangerously into identity.

However, let us now consider two additional points: on the one hand, the so-called Principle of the Indiscernibility of Identicals (sometimes called Leibniz’s Law, LL): for all x and y, if x = y (i.e. \langle x, y \rangle \in I), then x and y have the same properties—which is commonly considered quite uncontroversial. On the other hand, what is known as the Principle of Identity of Indiscernibles (PII): for all x and y, if x and y have the same properties, then x = y (i.e. \langle x, y \rangle \in I)—which, on the contrary, is highly controversial. Whether or not PII functions, this principle does not apply here anyway. In the assertion under examination stating that hāṭakaṃ mukuṭam, there is no trace of the commonality of properties, much less of indiscernibility. And yet, the situation regarding LL is even worse: if LL applied here, then crown and gold would display the same properties, which they do not—simply because we are still dealing with two fully distinct properties (cf. Leibniz 1989, p. 42 and 1981, p. 230).

Let us take a step forward. If non-difference were identity tout court and the indiscernibility of property followed for LL, then non-difference would pass the Substitutivity Test (ST). Still, consider the following case: if *[29] *m⇒I.m (The crown is identical to the gold), then obviously, by substitution: m⇒I.m and h⇒I.h (The crown is identical to the crown, the gold to the gold). The same holds true for a golden bracelet (kaṭaka, b): if *b⇒I.h, then b⇒I.b and h⇒I.h. In this case, however, it would follow—again by substitution between identical indiscernibles—that: *b⇒I.m (This bracelet is identical to this crown), which is pure nonsense—simply because a bracelet, perfectly discernible from a crown, is not a crown. Thereby, non-difference clearly fails the ST and, since fallacies are generated, it appears to be non-reducible to identity tout court. Moreover, this last example is a clear case of non-transitivity: non-difference has thus proven to be a non-transitive relation, while identity of course is—if x is identical to y, y is identical to z, z is identical to x (cf. supra, Quine 1981, pp. 134–136).

50 For an initial survey of identity, substitutivity, and Leibniz’s law, cf. Hawthorne (2003: § 2.3, pp. 108–131). In this regard, it is worth noting that “the totality of properties in an individual is always different
Interpreted as a case of qualified cognition \((V)\), non-difference does not even appear as a symmetric relation, and this is because \(V\) is certainly not one. It has been shown that, for SVN, the property Gold-ness in crowns is a subset of the set Properties of crowns: \(\{h\} \subseteq V^{(N)}(\{M\})\) (cf. P1), for \(V^{(N)}: M \rightarrow V^{(N)}(\{M\})\) and \(V^{(N)}(\{M\}) \subseteq M\). Non-difference can analogously be construed as a relation whose domain is \(M\) (Crowns) and whose range is \(2^M\) (What is non-different from crowns, e.g. gold-ness, heaviness, etc.): i.e. relation \(2: M \rightarrow 2^M\), for \(2^M \subseteq M\). What is at stake here is the gold-ness occurring in a crown. Inasmuch as the reference domains are distinct, by virtue of \(V\), the relata cannot be simply inverted as in case of symmetry; what is needed instead is a fully fledged inverse relation. The same is clearly true for different kinds of non-difference interpretations as well, such as causation, ‘part and whole’, ‘consisting of’, etc.\(^{51}\)

Cardinality also could help in distinguishing between non-difference and identity. Indeed, it has been shown that the cardinality of identity is strictly equal to one (\(\text{card}(I) = 1\); cf. [23]). I will argue here that non-difference can bear a cardinality equal to one and greater than one (\(\text{card}(2) \geq 1\)). The assertion \(\text{mukuta-hātakayor abhedāḥ}\) clearly begs for a cardinality equal to one, since there is but one crown here, a golden one:

Footnote 50 continued
from the totality of properties in any other individual. In this sense, the totality of properties also becomes a differentiating feature of an individual (fn. 99). [...] Is an individual identically with a bundle of properties without a separate substratum for those properties, or is it different from those properties and serves as their substratum, locus, or receptacle? Ultimately, like Nāyāyikas, Mīmāṃsakas also maintain that an individual (= substance) is different from its properties”, Deshpande (1992, pp. 30–31), quoting in fn. 99: Tantra-vārttika by Kumārila (comm. on Bhāṣya by Saṅkara, in his turn comm. on Jaimini’s Mīmāṃsasūtra), Banaras 1903, pp. 250–251; italics added, cf. fn. 48. For the reason alluded to by Deshpande *[29] is false but [30] is true. Indeed, \(I\)-relation implies the totality of properties and generates inconsistencies, while its counterfactual redefinition regarding a single property leaves open the possibility to claim [30].

\(^{51}\) In the example of crown and gold, for SVN, ‘A crown is not-different from the gold [it is qualified by]’ \((m, h) \in 2\); for \(2: M \rightarrow 2^M [M]\) and \(2^M [M] \subseteq M\), but not vice versa: \((h, m) \in 2\), for \(2: M \rightarrow 2^M [M]\) and \(2^M [M] \subseteq M\), which is clearly illicit. A well-formed inverse relation would instead be: \((h, m) \in 2^{-1}\), for \(2^{-1}: M \rightarrow [M]\), for \(2^M [\{M\}] \subseteq M\); let us say in active form, ‘A specimen of gold does not differ from the crown [it qualifies]’; QED. For the same reason, in the case of a blue pot, ‘A pot \((g)\) is non-different from blue-ness \((n)\) [by which it is qualified \((V^{(N)})\)]’, but not vice versa; for \((g, n) \in V^{(N)}\), \(V^{(N)}: G \rightarrow V^{(N)}[G]\), \(V^{(N)}[G] \subseteq G\); \((g, n) \in 2\), for \(2: G \rightarrow 2 [G]\), \(2 [G] \subseteq G\); and \((g, n) \in 2^{-1}\), for \(2^{-1}, 2^-1: G \rightarrow 2 [G]\), \(2 [G] \subseteq G\). This last case concerning \(guna\) is revelatory. In general, it has been shown that in \(V^{(N)}\) the viṣeṣya is the avacchedaka of the attributed viśeṣa (cf. P1, [4]–[7]). This feature occurs in what is qualified, indeed: viṣeṣya-vacchinchā-viṣeṣaḥ. A pot (dravya) is non-different from blueness (guna) because blueness occurs in the pot, and not pot in blue-ness. Therefore, in naming blueness we are talking about a qualification of the pot; in other words, there is no blueness but in the pot and, for this reason, the pot is non-different from one of its qualifications. SVN displays its heuristic power here. Hanging onto domain-range truth conditions, one must not yield to the temptation to be pulled back to the start and reinterpret non-difference as a vague notion of ‘being’. It is true that ‘The pot is blue’ because \(V^{(N)}: G \rightarrow V^{(N)}[G]\); hence, only pots exist (that is why: \(2: G \rightarrow 2 [G]\)). But, ‘Bleuiness is non-different from the pot’ is false because it relies on: \(2: N \rightarrow 2 [N]\), an interpretation which, in turn depends on: \(V^{(N)}: N \rightarrow V^{(N)}[N]\), a relation having Blue as its domain and connecting this quality with that by which it is qualified, here a pot (i.e. ‘A blueness qualified by a pot’; which is quite a piece of nonsense). So, the second temptation to resist, here made evident, is that of relying on gunas. Indeed, there is nothing but a pot, here. (cf. fn. 47). On Nyāya-Vaiśeṣika ontology, cf. Potter (1977, pp. 38–146) and Phillips (1997, pp. 44–51).
However, let us try to interpret $\mathcal{Z}$ as an *avayavāyavin* relation (‘Part and whole’) in which it turns out that multiplicity is structurally embedded: *āsva svāṅgābhinnah* (‘Non-difference between a horse ($a$) and its own limbs ($i$)’); for $\langle a, i \rangle \in \mathcal{Z}$ or *ghaṭah kapāladvayābhinnah*, (‘Non-difference between a pot ($g$) and its own halves ($k$)’; for $\langle g, k \rangle \in \mathcal{Z}$). That, just because *avayavī-avayavābheda* (‘Non-difference between the whole ($i$) and its constituents ($v$)’; for $\langle i, v \rangle \in \mathcal{Z}$). Thus:

\[
(a \cdot 2^{-1} \land i) \land N \geq I \lor (g \cdot 2^{-1} \land k) \land N \geq I \lor (i \cdot 2^{-1} \land v) \land N \geq I
\]

Looking closer, even interpretations based on *upādānakāraṇa* (‘K) or *viśeṣaṇa-viśeṣya-bhāva* (V) might display the same feature. Moreover, all of the above cases are ‘one-to-many’ relations. Multiplicity might be introduced into the domain as well, however, thereby obtaining ‘many to one’ and ‘many to many’ relations of non-difference. For instance, *vahnyabhinne prakāśanadāhakārye*, ‘The effects of making light and heat are non-distinct from fire’, or *bāspābhinnā meghāḥ*, ‘Clouds are non-different from water vapour’. Let us take now a step forward by considering, e.g., the 88 notes corresponding to the standard 88 piano keys ($K=\{k_1, \ldots, k_{88}\}$, for card($K$)=88). Now, a non-difference relation can be construed having as its domain every possible piano piece, written or not-yet-written, potentially counting infinite notes ($P=\{p_1, \ldots, p_n\}$ for card($P$) = $\aleph_0$; i.e. aleph-zero, the cardinality of the set of all natural numbers); thus, having dom($\mathcal{Z}$) = $P$ and ran($\mathcal{Z}$) = $K$, i.e. $\mathcal{Z} : P \rightarrow K$. Although it is pointless to say that every possible piano piece is equivalent, equal or identical to the 88 notes corresponding to the 88 piano keys, it

\[52\] The relation between the whole and the totality of its components appears a particularly complex case; i.e. ($\forall y \cdot 2 \cdot y \rightarrow \mathcal{Z} \rightarrow N \geq I$), in s.n. ($\exists x, \forall y \mid \exists x, \forall y \in \mathcal{Z}$). For instance, could a horse non-different from all its limbs be also said identical to them? Would this case pass ST? In order to avoid these difficulties I have chosen a more nuanced solution: ‘A horse non-different from one or some of its limbs’; the aforesaid quantification begs for the introduction of the bizarre non-difference inverse: *vapārītābheda* ($\mathcal{Z}^{-1}$). Cf. also fn. 51. Further investigations are required.
could be perfectly sound to state that the former are non-different from the latter. In standard notation: \((\forall x, \forall y \mid P(x, y)) \left(\langle x, y \rangle \in 2\right)\). Once more, a symmetric inversion of the *relata* is not possible. It is simply false that the 88 notes corresponding to the 88 piano keys are non-different from every possible piano piece: i.e., \(*\langle y, x \rangle \in 2\) is false, since only \(\langle y, x \rangle \in 2 - 1\) is true. Obviously, the same could be said about writing systems and literature or about the five DNA-RNA nitrogenous bases and living beings.53 A novel is non-different from, say, the Latin alphabet, but not vice-versa (i.e., \(\langle\text{novel}, \text{alphabet}\rangle \in 2 \) and \(\langle\text{alphabet}, \text{novel}\rangle \in 2 - 1\) are true, but \(*\langle\text{alphabet}, \text{novel}\rangle \in 2\) is false). In the same way, organisms are non-different from their nitrogenous basis, whereas these latter cannot be said to be simply non-different from the former (i.e., \(\langle\text{organisms}, \text{n-basis}\rangle \in 2 \) and \(\langle\text{n-basis}, \text{organisms}\rangle \in 2 - 1\) are true, but \(*\langle\text{n-basis}, \text{organisms}\rangle \in 2\) is false).

This last remark might cast new light—from an *advaita* perspective—on a classical issue concerning identity. The case of Rāma and his description as ‘prince of Ayodhya’ has already been discussed above. The case is analogous to the famous ‘Scott = author of Waverley’. It is well known that this case and its potentially paradoxical consequences have been analysed in detail, firstly through the distinction between names, descriptions, and denotations.54 Yet, it might still be usefully rephrased in terms of non-difference: ‘Rāma is non-different from the prince of Ayodhya’—just as ‘Scott is non-different from the author of Waverley’. It will come as little surprise that these assertions cannot pass the ST, since they involve properties which are distinct and highly informative (‘being called Rāma’ and ‘being the prince of a city called Ayodhya’) even if referring to the very same referent (the man called Rāma). Nonetheless, pushing the argument even further and assuming that Daśarathi Rāma was a real living human being—just as sir W. Scott was—it could be said that Rāma is non-different from his DNA-RNA nitrogenous bases or his biochemical bases in general—and the same for Scott.

\[
[r \cdot 2^{-1} \sqcup b_i] \sqsubseteq 2 \sqcup (\geq 1_t)
\]

\(\text{yad viparitābhinnatā-avacchedakāvacchinna-paryāptitvam tad ekatvādi-nirūpitam, saiva viparitābhinnatā rāma-niṣṭhā mahābhūtatva-nirūpitā;}
\]

(\(r\)-Rāma \(r\) is non-different from his biochemical bases \(b\)).55

53 Regarding the three pyrimidines (cytosine, thymine, uracil) and the two purines (adenine, guanine) and their role in composing nucleic acids (DNA and RNA), cf. Carey (2008, pp. 1164–1166).

54 Cf. end of § 2 and fn. 27. Russell (1905, p. 483): ‘If we say ‘Scott is the author of Waverley’, we assert an identity of denotation with a difference of meaning’; Russell (1919, pp. 173–175): ‘[…] a consideration of the difference between a name and a definite description. Take the proposition, ‘Scott is the author of Waverley’. […] A name is a simple symbol […]. On the other hand, ‘the author of Waverley’ is not a simple symbol […] but a description, which consists of several words, whose meaning are already fixed, and from which results whatever meaning is to be taken as the ‘meaning’ of the description’.

55 The inverse \((2^{-1})\) is required in reason of quantification, cf. fn. 51–52. Gross elements \((\text{mahābhūta})\) —ākāśa (ether or space), vāyu (air), tejas (fire), āpās (water), prthivī (earth)—are almost universally accepted in Indian cosmologies, starting with the Sāmkhya system; in this passage, I make free use of it. On Sāmkhya cosmology, cf. Larson (1987, pp. 65–72); regarding “material substances” in Nyāya-Vaiśeṣika system, see Potter (1977, p. 73).
The same holds for Scott as well, because every human could be said to be non-different from his/her biology.

\[ [33_\text{a}] (p, r \in \mathbb{2} \setminus b, t) \subseteq \mathbb{N} \cup \{1\} \]
yad viparīṭābhinnatā-avacchedāvavacchinā-paryāpītivān tad ekatvād-nirūpītam, saiva viparīṭābhinnatā puruṣatva-avacchedāvavacchinā mahā-bhūtatva-nirūpītā; ‘Humanhood (puruṣatva, \( p_t \)) is non-different from its biochemical-base-hood (\( b_t \));

\(|p_t| \subseteq 2^{-1} \setminus b_t| (\text{card}(2) \geq 1); \text{in s.n.} (\forall x, \forall y | P_{x, y} \setminus P_{x, y}) \langle \{x, y\} \in \mathbb{2} \rangle.\]

Clearly \([33_\text{a}]\) has nothing to do with the identity we evoked when talking about Rāma or Scott, since it involves general properties and no longer deals with a singularity, much less defined descriptions. Being distinctly relational in nature, \([33_\text{a}]\) could not be straightforwardly reduced to a predicative schema either, nor does it claim that ‘Humankind is its biology’—only that the former is non-distinct from the latter.

**Conclusions**

The assertion ‘A golden crown’ displays an evident case of sāmānādikaranya (\( N \)), syntactical homogeneity and coreferentiality. The notion of \( N \)-relation is nevertheless extremely vague and requires further interpretation. It has been shown that: \( N \neq E; N \subseteq V; Q \subseteq N; I \subseteq N; \not\exists \subseteq N. \) Thus, \( N \) might or might not be said to be reflexive, symmetric, or transitive, depending on the chosen interpretation. For instance, if \( N \) is supposed to be a particular case of \( V \)—as the assertion ‘A golden crown’ suggests—it will be non-symmetrical, non-transitive and reflexive only in a secondary, uninformative, sense.

It has also been shown that equivalence (tulyatva; \( E \)) first and foremost entails one shared property (taddharmavattva, \( td \)) among many. It has also proven to be a symmetric (\( \leftrightarrow \)) and transitive relation whose cardinality is strictly greater than one. According to [10], the equivalence between generic elements \( a \) and \( b \) can be expressed in NL as: \(((b, td) \equiv E\setminus(a, td)) \cap N \cup \{2\} \); if \((a \neq b) \land ((a, b) \in td) \land (\text{card}(E) \geq 2). \) In keeping with these truth conditions, interpretations of equivalence show that: \( E \neq N; E \neq Q; E \neq I; E \neq 2. \) That is, an equivalence relation, stricto sensu, is to be considered distinct from coreferentiality, equality, identity, and non-difference.

\[ 56 \text{ Equivalence can be said to be reflexive only in a secondary, uninformative, and highly context-sensitive sense. Only in this secondary reflexive case could it be said that } Q \subseteq E \text{ and } E \subseteq E \text{ (cf. reservations expressed in §5 and examples about dik and vaidya). In this case, a radical change in truth conditions (i.e. card}(E) \geq 1) \text{ occurs; this could be considered equivalence lato sensu. Nonetheless, } E \subseteq V. \text{ Let us try to interpret equivalence [8] (i.e.}, \text{ so gaur etasya gos tulyah) under qualification: } (g', g) \equiv V\setminus(g, g), \text{ ‘The qualifier-ness, conditioned by cow-ness in this cow, is limited by cow-ness in that cow’, if } (g, g') \in E \text{ and } g' \neq g \text{; which is a trivial, but true, case of reflexive qualification (‘Cow-ness qualified by that very cow-ness’) in two distinct instances.} \]

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Equality (samaniyatatva; Q) has proven to mean invariable concomitance, or mutual pervasion, with regards to names or expressions. It appears to be a reflexive (albeit in a secondary sense, because in this case it lacks any informative value), symmetric, and transitive relation, whose cardinality is greater than or equal to one. According to [17] the equality between two generic expressions ‘a’ and ‘b’—in relation to their primary meaningfulness (śakyatā, Ṣ) and individual manifestations with respect to a given generic property (taddharmavattva, td)—can be expressed in NL as: (‘a’ ∋ Ṣ ∪ td) → Q ∋ (‘b’ ∋ Ṣ ∪ td)) ∨ N ≥ (I); iff (l ‘a’ = b ∈ (ltd) ∩ T) ∧ (l ‘b’ = a ∈ (ltd) ∨ T) ∧ (card(TD) ≥ 1). The above relation of equality between terms can be promptly interpreted as a case of identity (I) and non-difference (⊇) of their extension. If {‘a’,‘b’}∈Q, then for AE also {‘a’,‘b’}∈I and {‘a’,‘b’}∈⊇. If linguistically {‘gold’, ‘Au’}∈Q, then ‘gold’ is extensionally non-different from ‘Au’ (‘gold’, ‘Au’∈⊇): every case of equality is, arthatah, also non-duality, but not the other way around (‘gold’, crown)∈⊇, yet ‘gold’≠ crown ∧ l ‘gold’ ≠ l ‘crown’⁻¹). In summary: Q ≠ E (cf. §1 and fn. 56); Q ⊆ ⊇, and consequently Q ⊆ (N ⊆ V); lastly, I is Q arthatah, while Q is I śabdatah (cf. § 2–3 and formulas [22]–[26]).

Identity (tādātmya; I) appears as a reflexive (although somehow paradoxically, cf. supra fn. 28), symmetric, and transitive relation whose cardinality is strictly equal to one. Identity, as ‘not the counterpositive of a difference resident in something of the same kind’, can be expressed, for the generic primitive a, as (cf. [22]-[25]): (a=∈I⁻¹(1⁻¹(a’) ∨ N ≥ (I); iff (a∈(1⁻¹ a’ = A⁻¹)) ∧ (A‘={a’}). In brief: I ≠ E (cf. §1 and fn. 56); I ⊆ ⊇, and thus I ⊆ (N ⊆ V); again, I and Q are the artha and śabda sides of the same coin (cf. previous point).

Non-difference (abheda; ⊇) has been shown to be a reflexive (although in a secondary sense), non-symmetric, and non-transitive relation whose cardinality is greater than or equal to one. Every instance of non-difference appears to be a case of co-reference and qualified cognition (viśiṣṭa-jñāna), but not the other way around. Indeed, the assertion daṇḍi puruṣaḥ qualifies a man by means of a staff, though it does not follow this man is non-different from his staff (cf. fn. 45). Thereby: ⊇ ⊆ (N ⊆ V). It follows that SVN rules ⊇, for ⊇[A] ⊆ A. That is, non-difference is an instance of closure as well, because the set ‘Non-different from what belongs to the generic set A’ is A-closed under the relation ⊇ (i.e. ⊇: A ⊆ ⊇[A]). On one hand, in cognitions connecting a pot and its colour (niło ghatah, a case of V(N)) or a crown and its material (hātakam mukutam, a case of “K; cf. end of §4), the relata in both cases are to be understood as non-different, yet in a different sense. The same clearly holds true for ‘part and whole’ relations (A, avayavāvayavi-bhāva). On the other hand, non-difference cannot even be reduced to identity; although they could appear as highly resembling each other, they do not collapse into other. In fact, their cardinality prevents such coalescence (card(I)=1 vs. card(⊇)≥1), together with the fact that I is always symmetrical and transitive while ⊇ never is (cf. interpretations under V, “K, or A), but also with the structural involvement of distinct properties (e.
g., being gold and being a crown) in \( \mathcal{Z} \). To sum up, it could be stated that: \((A \cong "K\) \subseteq \mathcal{Z}) \subseteq (\mathcal{N} \subseteq \mathcal{V}); \mathcal{Z} \neq E \) (cf. supra and fn. 56); \( \mathcal{Q} \subseteq \mathcal{Z}, I \subseteq \mathcal{Z} \)\(^{57}\)

In light of the above, let us take now a step forward. Non-difference between two generic properties \( a_i \) and \( b_i \) was expressed in §4 (cf. [31]–[33]) as: \((b, \mathcal{Z} \leftarrow a) \leftarrow \mathcal{N} \leftarrow (\geq I_i)\). Nevertheless, this definition can be further developed through [22]–[25] (i.e. the Gadañḍhara’s counterfactual definition of identity), the application of SVN, and the plain reading of the literal meaning of \( a\)-bheda (i.e., ‘non-difference’).

Difference, as shown, is expressed as \( \text{pa} \text{t} \text{o} \text{ g} \text{ha} \text{t} \text{o} \text{n} \text{a} \text{ na} \text{ g} \text{.} \text{ } \mathcal{I} \leftarrow \mathcal{P} \) (‘A cloth is not a pot’; cf. [19]–[21]). However, non-difference clearly negates difference. Since ‘The crown is gold’, the assertion ‘The crown is not gold’ will be false: \( \text{muku} \text{t} \text{a} \text{m} \text{ } \text{h} \text{a} \text{t} \text{a} \text{k} \text{a} \text{m} \text{ } \text{n} \text{e} \text{t} \text{ } \text{n} \text{a} \text{ } \text{g} \text{y} \text{.} \text{ } \mathcal{N} \leftarrow \mathcal{M} \) (recall here steps [19]–[23]). Abheda thus proves to be a peculiar relation which negates difference. Yet, it involves more than one property (e.g. the generic \( a_i \) and \( b_i \)) referring to the same potentially more than one locus (card \( \geq 1 \)). As has been said, abheda cannot collapse into mere identity, which involves, as we have seen, ‘the same kind’ (svasadajīva) and a cardinality equal to one. A counterpositive definition of non-difference might be more of the same: \( \text{sam} \text{ānādhikaraṇa-dharmāntara-avacchihna-bheda-apratīyogitvam abheda}, \) ‘Not being the counterpositive of a difference occurring in another co-occurring property’\(^{58}\).

\[ [34] ((b \mid N \leftarrow a_i) \iff \mathcal{B}^{-1} \leftarrow (\mathcal{I} \leftarrow a_i) \leftarrow \mathcal{N} \leftarrow (\geq I_i) \]

\( \text{yad} \text{ atyantābhāvyapratiyogitā-avacchedakāvavacchihna-paryāpti transparent td ekatvādī-nirūpata; tatra yāva atyantābhāvyapratiyogitā sāmānādhikaranatā-avacchedakāvavachhnā sā anyonyābhāvyapratiyogitā-nirūpata, etc.}\]

\( \text{āsāva anyonyābhāvyapratiyogitā etaddharmavattva-nirūpata, tadviparyayena ca; yathā yāva taddharma-niṣṭha-sāmānādhikaranatā sā etaddharmavattva-nirūpata, tathā yaitaddharmam-niṣṭha-sāmānādhikaranatā sā taddharmavattva-nirūpata; ‘The relational abstract absolute absentee-hood (\( \mathcal{B}^{-1} \)), conditioned by the}\]

\(^{57}\) Relations \( A \) and “\( K \) share some occurrences (\( A \cong "K\)), although not all; because, e.g., if mahābhūtas are simultaneously part (avayava) and material cause (upādāna-kāraṇa) of a living being, yet for Nayāvikas halves are not the material cause of a pot—which instead, in satkāryavāda sense, is the clay—but the samavāyikāraṇa (‘causal substrate’ or ‘substantial cause’; Matilal 1975, p. 44). Moreover, although they are expressible in oblique cases also (so not every instance of \( A \) and “\( K \) can be said, syntactically homogeneous from a linguistic point of view), they, nevertheless, appear as radical extensional instances of coreference; e.g. avāsa aṅgāni (the limbs of a horse), or tilāt tailam (oil from sesame [seeds]). Cf. fn. 45 for the opposite case of dāndī puruṣah.

\(^{58}\) This formulation might sound somehow paradoxical at first; nonetheless, regarding this peculiar notion of a kind of non-reflexive identity (abheda), which is at once not reducible to reflexive identity stricto sensu, as well as ‘compatible with difference’ (bhedasahīṣṣu) or rather with the coreference of different properties—see among the others Mohanty (2000, pp. 55–56): “[for an advaitin], the ‘blue’ and the ‘lotus’—in ‘the blue lotus’—are fundamentally identical. The quality is of the nature of the substance. […] It would seem, then, that for Śaṅkara there is only one category, namely substance, and one relation, namely tādāmya (being its essence), which is a form of identity that ‘tolerates’ differences (bhedasahīṣṣu)”; and Chakrabarti (2001, p. 219): “In the Advaita (and the Bhāṭṭa Mimāṃsā) view […] the relation between a substance [guṇī] and its qualia [guṇa] is that of identity in and through difference (bheda-sahīṣṣu-abheda). That is, a substance and its qualia are neither utterly identical nor utterly different”.

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mutual absentee-hood (\(I^1\)), in turn conditioned by this [generic] property \((\alpha_i)\), is limited by coreferenceness \((N)\), and vice versa; moreover, just like the coreferenceness, conditioned by this [generic] property \((\alpha_i)\), is limited by at least one specimen of that [generic] property \((b \in \{b_l\})\), so the inverse coreferenceness \((N^{-1})\), conditioned by that [generic] property \((b_l)\), is limited by at least one specimen of this [generic] property \((a \in \{a_l\})\); for a cardinality greater than or equal to one” \(\iff (|a| \neq |b|) \land (|a| \cap |b| \neq \emptyset)\) (i.e., \(a_i\) and \(b_l\) are not the same property but the intersection of their domains is not empty); in s. n. \((\forall x, \forall y \mid A_x, B_y) (\langle x, y \rangle \in \mathcal{N}) \leftrightarrow ((A \neq B) \land (\langle x, y \rangle \in \mathcal{N}))\).

In conclusion, non-difference seems to peculiarly reverse the claims of both Leibniz’s law (LL) and the Principle of Identity of Indiscernibles (PII). Apparently, abheda does not claim (as LL does) that the same referent must have the same properties it already has, which would coalesce into mere identity—which, although true, might even sound like a linguistic short circuit, as Wittgenstein has pointed out. Nor does it claim (as PII does) that what possesses the same properties is the very same referent, since different properties are at stake here. What abheda appears to claim—at first glance generating another linguistic short circuit just as identity might—is that distinct properties referring to the same locus cannot be said to be fully different. This is a crown, surely; but this crown is nothing but gold. What cognition has—etymologically—abstracted from the referent must indeed be located there again. The application of this analysis—prompted in the first instance by VM—to the issues of language, knowledge and knowledgeability, causation, and first and foremost to the relation between manifestation (jagat) and brahman, requires further investigation. Such investigation will be attempted in the following part of this article.

**Compliance with ethical standards**

**Conflict of interest** The author states that there is no conflict of interest.

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59 Regarding the yathā-tathā operator (‘just like-so’; \(\|\)), in order to express in NL the non-empty intersection between two sets, cf. Anrò (forthcoming: § 4.3).
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