Investigation of temperature hysteresis on tooth contact surface of hypoid gears using middle-infrared ray imagery based on thermal network model

Toshiki HIROGAKI*, Eiichi AOYAMA*, Raphael PIHET* and Koudai NIWA*
*Department of Mechanical Engineering, Doshisha University, 1-3, Tataramiyakodani, Kyotanabe, Kyoto, 610-0321, Japan
E-mail: thirogak@mail.doshisha.ac.jp

Received 17 December 2013

Abstract

Recently, infrared imaging technologies have been developing in various industrial fields rapidly. We therefore focus on the infrared ray imagery as a novel method to estimate the gear tooth meshing. In the present report, a high response infrared thermography, which could take a middle infrared ray image of around 4 μm wavelength, was used to investigate the temperature hysteresis on the tooth contact surface of hypoid gear under running conditions. We installed the high reflective mirror of infrared ray at the opposite side of the gear tooth meshing point to obtain the tooth surface images around the entire gear, and estimated the variation of tooth surface temperature at the duration from a tooth meshing to the next tooth meshing. Moreover, using a thermal network model, the modeling of the temperature variation of the gear tooth surface regarding the time is conducted as to predict the temperature after a certain number of rotations. The network model presented in this report is derived from the ones such as Forster network or Cauer network which are based on the concept of thermal capacitance and thermal resistance. This network is investigated to match the actual results on the temperature evolution. Finally, it is clear that the temperature on the tooth surface during a gear rotation can be modeled by first order time delay and the maximum temperature is affected by contact pressure, relative sliding speed between tooth surfaces and average peripheral speed of tooth surface. This proposed method is found to be effective to evaluate the temperature hysteresis on the tooth contact surface of hypoid gear.

Key words: Tooth surface temperature, Hypoid gear, Tooth contact, Image processing, Middle-infrared rays, Thermal network

1. Introduction

The gear noise reduction has been studied for several types of gears (spur gears, hypoid gears…). The gear noise is mainly caused by the accuracy of the tooth surface because the transmission error causes vibrations of the gears. Kubo, et al.(1991) managed to estimate the transmission error in cylindrical involute gears by a tooth contact pattern. They developed an index value (which estimates the magnitude of the meshing frequency component of the transmission error) to predict the transmission error. They showed that such an index was in good correlation with the meshing frequency component of the transmission error. A study on the effect of the contact ratio and profile corrections on the vibrations of rotational gear has been made by Sato, et al. (1983). They showed that a slight increasing in the total contact ratio could efficiently reduce the meshing transmission errors of production gears. In 2010, Kolivand, et al. tried to predict the mechanical gear mesh efficiency of hypoid gear pairs. This efficiency model could predict the sliding friction loss by seeking closed-form friction formula based on the actual lubrication analysis. They showed that the surface roughness amplitudes and pinion shaft offset impact significantly mechanical efficiency of the gear pair. In 2003, Podzharov, et al. focused on the design of high contact ratio spur gears to reduce static and dynamic transmission error. They managed to develop methods and programs to calculate the static and dynamic transmission errors under load in spur gear. That is, there are many reports dealing with the transmission error and the mechanical efficiency of
gear meshing. However, there have been few reports dealing with monitoring and investigating the flash temperature on the tooth contact surface under running conditions to analyze the tooth meshing.

In this study, a novel approach to analyze the temperature variation of hypoid gear using an infrared thermography is presented. The final aim of this research is to estimate the contact pressure distribution on the gear tooth surface during meshing using infrared imagery, which has never been investigated this way in this field. We selected a high speed, high response and high sensitivity infrared thermography with a function of frametrigger to take a clear tooth surface imagery at every rotation angle under running conditions. Moreover, we propose a thermal network model to estimate the temperature hysteresis on the tooth contact surface considering the obtained thermal imagery.

2. Experimental devices and procedure

In this experiment, Gleason-type hypoid gears were used with the specifications given in Table 1.

| Table 1 Specifications of Gleason-type hypoid gear pair |
|---------------------------------|-----------------|-----------------|
| Number of teeth                | Driving gear    | Driven gear     |
| Module [mm]                    | 13              | 39              |
| Pressure angle [deg.]          | 3.256           | 20              |
| Mean spiral angle [deg.]       | 50.37           | 31.02           |
| Face width [mm]                | 22.07           | 19              |
| Face angle [deg.]              | 27.6            | 67              |
| Pitch angle [deg.]             | 22.7            | 65.3            |
| Root angle [deg.]              | 21.1            | 60.5            |
| Addendum [mm]                  | 4.8             | 0.93            |
| Tooth depth [mm]               | 6.71            | 6.59            |
| Reference diameter [mm]        | 42              | 127             |
| Offset [mm]                    | 20              |                 |
| Contact ratio                  | 2.164           |                 |
| Material                       | SCM415          |                 |
| Heat treatment                 | Carburizing and quenching | |

In the driving experiment, ATM-460 (KOKUSAI Co., Ltd), which can also measure the meshing transmission error, was used. This measuring instrument can apply a load by controlling the rotational speed on the driving side. The maximum load is 100 Nm and the maximum rotational speed is 60 min⁻¹. The device is shown in Fig. 1. To photograph the tooth surface temperature, a middle-infrared ray thermography TVS-8500 (Japan Avionics Co., Ltd) was used. Its temperature resolution is 0.025°C and the flame time is 1/120 sec. The device is shown in Fig. 2.

![Fig. 1 Driving experiment](image1.png)

First, a differential oil (coefficient of friction μ = 0.07) is coated thinly on the teeth surface (both on gear and pinion). Then, the gears are rotated under the loaded condition. To photograph the same tooth surface at each rotation, a
A phototransistor-trigger is laid on the flange of the driving gear and a synchronized image using the trigger signal is read in. The temperature of the target tooth surface is detected using a subtracted image to reduce the effect of ambient light. Then, the temperature distribution can be evaluated. Moreover, to look at the over side of the gears at the same time, an optical flat mirror with coating the gold film is placed under the gears. The complete setup is shown in Fig. 3. We mainly look at the tooth surface of the pinion gear (the drive gear), as shown in Fig. 3 (b) and (c).

### 3. Theoretical expression of the relative sliding velocity on the tooth surface

The sliding velocity represents the relative velocity that is existing between the pinion and the gear. It is defined as the difference between the rolling velocities of the teeth in meshing. This kind of velocity is due to the involute profile of the teeth. The calculation of this velocity is important to determine the relationship between the temperature rise and the contact pressure distribution (which is explained later in this research). Two types of sliding velocity can be defined: one parallel to the pitch line and the other one perpendicular to this line. The definitions of the different parameters of the gears needed to obtain the expression of the sliding velocity are given in Fig. 4.

![Fig. 4 Definitions of the parameters of the gears](image)

First, the analysis of the pinion sliding velocity is performed. According to Coleman’s theory, the expression of the sliding velocity parallel to the pitch line is given in Eq. (1).

\[
V_{FP} = V_n \left( \tan \beta_{m1} + |z_0| \sin \alpha_n \left( \frac{\tan \beta_{m1}}{A_p \cos \delta_1} \right) - |z_0| \cos \alpha_n \left( \frac{1}{A_p \cos \beta_{m1}} \right) \right) \tag{1}
\]
Where $V_n$ is the rotation speed, $\beta_{m1}$ is the mean spiral angle, $z_0$ is the distance between the point P and the meshing position on the action line, $A_p$ is the distance between the summit point of the pitch cone and the point P, $\alpha_n$ is the pressure angle, and $\delta_1$ is the cone angle, as shown in Fig. 4.

Then, the expression of the sliding velocity perpendicular to the pitch line is given by Eq.(2).

$$V_{PP} = V_n \left\{ \sin \alpha_n + z_0 \left( \frac{1}{A_p \tan \delta_1} \right) \right\} \quad (2)$$

The global sliding velocity of the pinion is finally given by the simple Eq.(3).

$$V_1 = \sqrt{V_{FP}^2 + V_{PP}^2} \quad (3)$$

On the gear, the sliding velocity parallel to the pitch line is also given by Eq.(4).

$$V_{FG} = V_n \left\{ \tan \beta_{m2} - |z_0| \sin \alpha_n \left( \frac{\tan \beta_{m2}}{A \tan \delta_2} \right) - |z_0| \cos \alpha_n \left( \frac{1}{A \cos \beta_{m2}} \right) \right\} \quad (4)$$

Where $V_n$ is the rotation speed, $\beta_{m2}$ is the mean spiral angle, $z_0$ is the distance between the point P and the meshing position on the action line, $A$ is the distance between the summit point of the pitch cone and the point P, $\alpha_n$ is the pressure angle, and $\delta_2$ is the cone angle, as shown in Fig. 4.

Then, the expression of the sliding velocity perpendicular to the pitch line is given by Eq.(5).

$$V_{PG} = V_n \left\{ \sin \alpha_n - |z_0| \left( \frac{1}{A \tan \delta_2} \right) \right\} \quad (5)$$

Here, the global sliding velocity of the gear is finally simply given by Eq.(6) as well.

$$V_2 = \sqrt{V_{FG}^2 + V_{PG}^2} \quad (6)$$

Finally, two other sliding velocities are defined; the sliding velocity along the tooth depth direction $V_p = V_{PP} - V_{FG}$ and the sliding velocity along the tooth flank line direction $V_p = V_{PP} - V_{PG}$. The global sliding velocity between the gear and the pinion is then given by Eq.(7).

$$V = \sqrt{V_p^2 + V_p^2} \quad (7)$$

4. Experimental results and discussion

4.1. Tooth contact evaluation

For the driving experiment, the rotation speed and the torque are set to 60 min$^{-1}$ and 100 Nm respectively. The images recorded by the infrared thermography camera before the drive and after 20 revolutions are substracted to obtain the temperature distribution on the gear teeth surface, which means the rising temperature between before the drive and after 20 revolutions. The purpose is to evaluate the tooth contact area. In a previous experiment, this area has been determined using red lead painted on the tooth surface. The paint is then taken off by the meshing between the two gears. Here, we look at the tooth surface just after tooth meshing as shown in Fig. 5 (a). These results are shown in Figs. 5 (b) and (c), respectively.
It can be understood from these figures that the contact area corresponds to the main temperature rising area. Indeed, it can clearly be seen that, in the almost center of the tooth surface, the temperature has more increased than in other parts. Moreover, the area over 1.0ºC from the thermal image corresponds to the area where the red lead has been removed. The area with the higher temperature rise corresponds to the main meshing area. Furthermore, the temperature on the tooth surface is increasing within the time due to the meshing between the two gears until reaching some kind of maximum value (which is not completely true as the temperature continues to increase but very slowly). As this study focuses on the tooth surface temperature distribution due to the gear meshing, the thermal imagery is mainly used before the temperature reaches this maximum.

4.2. Sliding velocity distribution

The determination of the sliding velocity is conducted for the driving side (i.e., the pinion). Using Eqs. (1)–(7), the sliding velocity can be determined along the tooth surface. The results in rotation speed 60 min⁻¹ are given in Figs. 6(a) and (b). The different sliding velocities are represented in a cross-section of the surface of the pinion.
It can be considered from these figures that the sliding velocity along the tooth depth direction is not very affected by the position on the tooth surface whereas a clear shape can be seen for the sliding velocity along the tooth flank line direction. It goes from a maximum value on the tooth tip to become null on the pitch line and then increases again by going to the tooth root. The global sliding velocity has the same shape besides that the minimum value on the pitch line is not null. It can also be seen that the sliding velocity along the width of the tooth surface is mostly constant, especially along the pitch line where the sliding velocity is clearly straight.

4. Modeling of the temperature variation with an electronic analogy

4.1. First model

The objective of this modeling is to obtain a model that can represent and predict the temperature rise of the gears after a certain number of rotations. We therefore focus on the pixel (the point) at the maximum temperature on the tooth surface just after meshing teeth, because the temperature in certain pixel on the meshing surface is considered to be predicted by the same theory. First, it is necessary to investigate the behavior of the temperature variation by getting its values during the rotation of the gears using the infrared thermography at the same point on the tooth surface, as shown in Fig. 7. That is, the temperature plots in each rotation cycle are obtained from an image including a mirror image in such as Fig. 3 (c).

![Temperature variation from 20th to 29th rotations](image)

It can be seen that the temperature variation has an exponential-like behavior. That is, as the solid line shows in Fig. 7, the temperature increases due to the meshing and then decreases due to cooling until the next meshing in a rotation cycle, locally. Globally, as the broken line shows in Fig. 7, the maximum temperature rise after each meshing increases a little bit. This kind of behavior is close to the response of an RLC circuit when the voltage of the capacity is observed. Here, it is well known that it is possible to make an analogy between the mechanical field and the electronic field regarding the following correlation as shown in Table 2. Therefore, we attempt to investigate the temperature hysteresis from a view of this analogy.

| Mechanical field | Electronic field               |
|------------------|--------------------------------|
| \( \lambda \) : friction | \( R \) : resistance          |
| \( M \) : mass    | \( L \) : inductance           |
| \( k \) : stiffness| \( 1/C \) : inverse of capacity|

As a first basic approach, we focus on the capacity voltage behavior of an RLC circuit to represent the temperature variation of the gear during the meshing. In a first step, the calculation of the output temperature is calculated by a simple RLC circuit in a rotation cycle and, then, in a second step, the calculation regarding the loop of the temperature variation has to be taken in account globally, as we can see in Fig. 8.
In such thermal analogy as it is well known in the Foster network or the Cauer network, usually the R and C components are only taken into account. Here, in this model, the inductance $L$ is added to take an inertia component in the analogical model between mechanical, electrical and thermal model. Also, it gives a second order differential equation to represent the temperature variation due to the meshing, which adds more dynamic meaning to the phenomenon.

Moreover, Foster and Cauer networks are not considering a feedback loop in their model, but add the R and C components to improve the precision of their calculation. In this model, the feedback loop is chosen as to represent the actual dependence between the temperature rise at one rotation and the temperature rise at the previous ones.

Furthermore, it is necessary to take a look at the heat circle to consider the temperature variation both locally and globally. Indeed, the temperature rise of the rotation $n$ depends on the temperature rise of the rotation $n-1$, as shown in Fig. 9 based on Fig. 8 (b).

First, we look at a local thermal network model, which means to express a tooth mesh in one rotation. The first step consists in the calculation of the transfer function of the RLC circuit. The second order equation to solve is given as Eq. (8):

$$V_{in} = V_{out} + RC \frac{dV_{out}}{dt} + LC \frac{d^2V_{out}}{dt^2}$$  \hspace{1cm} (8)

The result is well known as a second order transfer function that can be written as:

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{1}{1+j\omega\tau - \frac{\omega^2}{\omega_0^2}} \text{ with } \tau = RC \text{ and } \omega_0 = \sqrt{LC}$$  \hspace{1cm} (9)

We can rewrite this expression using the notation of the mechanical field and the coefficient $\xi$ which represent the absorption of the system. Then the transfer function becomes Eq. (10).
\[ H(s) = \frac{1}{1 + \frac{2\xi s}{s_0} + \frac{s^2}{s_0^2}} \] \quad \text{with} \quad \xi = \frac{1}{2} \sqrt{\frac{L}{C}} \quad (10) \]

Then, the loop due to the gears rotation has to be taken into account. The global transfer function of the system is Eq. (11).

\[ H'(s) = \frac{S}{I} = \frac{H}{1-H} = \frac{\omega_0}{2\xi} \cdot \frac{1}{s + 1 + \tau s} \] \quad \text{with} \quad \tau = \frac{1}{2\xi \omega_0} \quad (11) \]

Taking the input function as an impulsion (immediate rising), we get the expression of \( V_{out}(t) \), the output of the global system, which is the temperature variation shown in Eq. (12).

\[ V_{out}(t) = LC \cdot \exp \left[-\frac{R}{L} t\right] \quad (12) \]

Second, we look at the global thermal network model, which means to take into account the temperature rise for each rotation. It means that the temperature variation of the rotation \( n \) is equal to the value of the temperature rise \( n-1 \) in addition with the temperature rise due to the contact between the gear and the pinion, as shown in Eq. (13).

\[ T_n(t) = \Delta T + T_{n-1}(t) \quad (13) \]

Let's note \( A = LC \) and \( B = R/L \). The expression of the first temperature variation is simply given by Eq. (14).

\[ T_0(t) = Ae^{-Bt} \quad \text{and} \quad D = T_0(t_f) \quad (14) \]

Where \( t_f \) represents the time when the next meshing will occur. Indeed, the rotations speed of the pinion \( N \) can be expressed as \( N = 1/t_f \). Therefore the value of \( n \) is given by \( n = \left[ \frac{t}{t_f} \right] \) where the brackets represent the Gauss sign and \( t \) the time.

The second expression of the temperature variation is given by Eq. (15).

\[ T_1(t) = T_0(t) - H(t - t_f)T_0(t) + H(t - t_f)[Ae^{-B(t-t_f)} + D] = H(t - t_f)[Ae^{-B(t-t_f)} + D] + [1 - H(t - t_f)]T_0(t) \quad (15) \]

Where the function \( H \) represents the Heaviside function. Here, it can be understood that each temperature rise will follow the same shape. Therefore, the expression of the temperature variation at the rotation \( n \) is shown in Eq. (16).

\[ T_n(t) = H(t - nt_f)[Ae^{-Bt} + nD] + \left[1 - H(t - nt_f)\right]T_{n-1}(t) \quad (16) \]

By iteration, the global expression of the temperature variation is shown as Eq. (17).

\[ T_n(t) = H(t - nt_f)[Ae^{-B(t-nt_f)} + nD] + \sum_{i=0}^{n-1} \prod_{j=i+1}^{n} \left[1 - H(t - jt_f)\right]H(t - it_f)[Ae^{-B(t-it_f)} + iD] \quad (17) \]

In fact, in this expression, \( n = p + 1 \) with \( p \) the number of rotations.

In general, the coefficients have to be identified from the experimental results at this kind of model. Therefore, experimental results of the temperature regarding the time have been used to determine the coefficient \( A, B \) and \( D \). The following values have been obtained (with \( t_f=1s \)):
Table 3 Data used to determine A, B and D

| Time [sec] | Temperature [deg.] |
|------------|---------------------|
| 0.15       | 1.81                |
| 0.23       | 1.61                |
| 0.31       | 1.42                |
| 0.54       | 1.13                |
| 0.62       | 1.08                |
| 0.69       | 0.96                |

We consider that the values of A, B and D are determined regarding the experimental values of the temperature rise after 21 revolutions because the torque becomes a constant only after 20 seconds (20 revolutions as the rotation speed is 60 min⁻¹). Thus, we count $p$ after 20 rotations at the beginning from now on in the experimental values.

We use the software Mathematica to visualize the results from Eq. (17). According to the previous definition of the variables and the constants, the temperature variation $T_n(t)$ is shown in Fig. 10.

![Fig. 10 Temperature rise by Eq. (17) for $n=10$](image)

As a partial conclusion, it can be considered here that the global shape of the actual temperature variation has been modeled. It can be clearly seen that the temperature rise of a rotation depends on the previous ones. Nevertheless, several remarks can be made on this first model. Indeed, here $\Delta T$ between each mesh is supposed as a constant, which means each teeth meshing gives the same temperature rise. This is actually insufficient because the temperature rise will reach a maximum value that depends on the gears properties, comparing Fig. 10 with Fig. 7. Moreover, the temperature rise due to the meshing has been supposed as an impulsion (immediate rising) whereas its shape is closer to a linear or “second order” increasing. Finally, the decreasing speed of the temperature rise at the end of the meshing also has to change because the higher the temperature of the gear is, the faster the temperature decreases.

Regarding these remarks, it can be considered that the electronic analogy can be used to represent the temperature variation due to the meshing but this model has to be improved to correctly agree with the experimental results.

4.2. Improvements of the modeling of the temperature rise

The first temperature rise keeps the same expression as before, as it can be seen in Eq. (14). From here, to improve and also simplify the expression of the temperature variation, it is decided to modify the coefficients $A$ and $B$. Indeed, these two coefficients actually depends on the number of revolutions $n$. Hence the expression of the temperature rise at the revolution $n$ is changed, as shown in Eq. (18).
The coefficient $A(n)$ corresponds to the temperature rising value at the revolution $n$. The coefficient $B(n)$ represents the temperature decreasing speed. They both depend on the gear properties. Finally, $t_f$ still corresponds to the time needed by the driving gear to make one revolution. By the experiment, it is observed that the coefficients $A(n)$ and $B(n)$ decrease regarding the number of revolutions $n$ and reach a minimum value. An important problem here is to determine the actual expressions for these two coefficients. A first possible approach is to choose an equation that can represent the behavior of $A(n)$ and $B(n)$, as shown in Eq. (19) and (20).

\begin{align}
A(n) &= \frac{1}{n+1}A_{max} + \frac{n}{n+1}A_{min} \\
B(n) &= \frac{1}{n+1}B_{max} + \frac{n}{n+1}B_{min}
\end{align}

The representation of the evolution of such equations can be seen in Fig. 11 for both $A(n)$ and $B(n)$, as their expressions are similar.

Hence the maximum and minimum values for these coefficients need to be determined by the experiment again. To achieve this identification, the experimental results have to be matched with the theoretical expression of the global temperature variation $T(t)$, which becomes the sum of all the temperature rise at each revolution $T_n(t)$. The expression of the global temperature variation can be seen in Eq. (21).

\begin{equation}
T(t) = \sum_{i=0}^{n} H(t - it_f)[A(i)e^{-B(i)(t-it_f)}]
\end{equation}

Using this proposed expression, the temperature increasing depends on the number of revolutions and is not a constant anymore, as shown previously. Moreover, the temperature rise will reach a maximum value and after driving, the temperature will decrease until its initial value. This proposed model is therefore considered closer to the real evolution of the temperature variation.

As to match this theoretical expression with the experimental results, the temperature has to be recorded by the infrared camera for many short times to obtain the temperature decreasing evolution just after the meshing of the gears.
This will allow to determine the coefficient $B(n)$. $A(n)$ will be able to be determined with a more global point of view by looking at the temperature rise at each revolution. Such investigations have not been made for the moment and are the next step to improve the previous temperature variation model.

Nevertheless, the evolution of theoretical expression of $T(t)$ given by Eq. (21) can be seen in Fig. 12 with random values for the different coefficients.

Fig. 12 Temperature rise by Eq. (21) for $n=10$

Comparing Fig. 12 based on Eq. (21) with Fig. 7, it can be seen that the improved evolution is very close to the experimental one. Therefore, it can be assumed that, by determining the coefficients of Eq. (21), the theoretical expression of $T(t)$ will agree with the experimental results. As a result, it is demonstrated that the proposed model considering a positive feedback loop is found to be effective to explain the global variation of the temperature on the tooth surface in driving condition, as the broken lines shown in Fig. 12.

5. Determination of the load distribution
5.1. Relationship between temperature, load, sliding speeds and time

The final aim of this study is to establish the link between the temperature distribution and the load distribution on the tooth surface. This can be easily understood by looking at the infrared imagery that the main temperature rising area corresponds to the contact area regarding the tooth surface pressure. However, an obtained imagery is composed of many different time delay pixels after tooth meshing, as the details are explained in the next section. The local temperature variation $\Delta T$ between each mesh (one rotation) have to be more related with various driving conditions to discuss this problem, which means to discuss Eq. (12) and Eq. (14) more theoretically.

Therefore, we here focus on the Deng and Nakanishi equation (2002) regarding the bulk temperature of spur gear tooth surface under driving condition, which managed to develop a relationship between the temperature and the load as it can be seen in Eq. (22).

$$\Delta T = UP^a V^b V_m^c$$

(22)

Where $\Delta T$ is the temperature rise (in °C) at the $n$th meshing, $P$ is the load per unit face width (in N/m) defined as equivalent contact pressure because this model is assumed as a line contact, $V$ is the sliding velocity (in m/s), $V_m$ is the average peripheral speed (in m/s) and $t$ is the time from meshing start during the duration between $n$ and $n+1$ rotation (in s). $U$, $a$, $b$ and $c$ represent constant numbers. The expression of the peripheral speed is given in Eq. (23).
\[ V_m = \frac{V_1' + V_2'}{2} \]  

(23)

Where \( V_1' \) is the peripheral speed on the driving side (in m/s) and \( V_2' \) is the peripheral speed on the driven side (in m/s). \( V \) is obtained by Eq. (7).

The factor \( U \) is a constant number that depends on the gear properties such as surface roughness, materials or thermal capacity and also depends on the lubrication method. Here, providing we look at one rotation duration, considering Eq. (14), Eq. (22) could be modified to calculate the temperature in detail during a rotation as follows.

\[ \Delta T = UP^a V^b V_m^c e^{dt} \]  

(24)

As to determine \( a, b, c \) and \( d \), the results obtained in this research for the hypoid gear are substituted in Eq. (23) and Eq. (24). Then, each constant number is determined using the least square method from the measured data, such as shown in Fig. 7, during the duration from 20th to 21th rotation under various driving conditions which are set the rotation speed 20 min\(^{-1}\) – 60 min\(^{-1}\) and the torque 1 Nm – 10 Nm. The results obtained for this determination are,

\[ U = 0.044, a = 0.92, b = 4.64, c = -4.20, d = -0.05 \]  

(25)

This is an empirical relationship as its physical meaning is not obvious as well as the Cauer thermal network model. Indeed, it can be considered that the contact pressure has clearly an influence on the temperature rise of the tooth surface and that the relative sliding velocity also can have an influence on it. Thus, Eq. (24) is considered to be useful as a practical formula, which seems to be the Cauer thermal network model to practically predict a complex temperature distribution.

Fig. 13 shows the relationship between the tooth surface temperature rise estimated by the previous equation (solid line) and the results obtained by the experiment (plot pointed).

![Fig. 13 Relationship between the tooth surface temperature rise estimated by the previous equation and the results obtained by the experiment](image)

It can be seen in this figure that the tooth surface temperature rise is in a good correlation with the values by Eq. (24) as proven by the linear proportion. Therefore, it can be concluded that Eq. (24) can be used to determine the load distribution on the tooth surface from the temperature rise with knowing the sliding velocity and the average peripheral speed.

5.2. Estimation and investigation of the pressure distribution on the tooth surface

Another important investigation is to determine the pressure distribution regarding the position on the tooth surface. Therefore, a system of coordinate (i, j) is used to determine the position on the tooth surface. The coordinate i represents the position on the contact line (which has an inclination angle of 46º to the pitch line) and the coordinate j represents
the position on the meshing length. The definition of these coordinates can be seen in Fig. 14. Each point defined in this way represents, in fact, a pixel of the image obtained by the infrared thermography. The tooth meshing starts at the toe and root side point, and then ends at the heel and tip side point. It can also be seen from Fig. 14 that there is a time difference between the meshing start and the end in an infrared imagery. Thus, this time delay at each contact line from the meshing start has to be considered in a tooth surface.

By these definitions and considering the time delay in a tooth surface $t_{ij}$, the expression of the load for each point can be expressed as in Eq. (26).

$$P_{ij} = \left( \frac{\Delta T_{ij}}{U \bar{V}_{ij} B \bar{V}_{c} c_{mij} e^{\Delta t_{ij}}} \right) \frac{1}{u}$$

(26)

The temperature distribution is already obtained directly from the infrared thermography image. Figures 15 (a) and (b) show the tooth contact area determined by peeling off red paint and the temperature distribution on the tooth contact area just after 20 revolutions respectively. Then, by calculating the sliding velocity and the average peripheral speed at each $P_{ij}$, the contact pressure distribution from Eq. (26) can be obtained as shown in Fig. 16. It can be seen from Fig. 15 that the outline of the tooth contact area is almost in agreement with one of the temperature rise, but a higher temperature distribution is located at the tip side from the pitch line in detail.

It can be seen from Fig. 16 that the contact pressure is higher along the pitch line (in red) and is decreasing gradually by going to the ends (the tip and root sides) on the tooth surface. To have a better representation of these phenomena from Fig. 15 (b), we estimate the evolutions of the temperature rise in the tooth profile direction at the center of the tooth width as shown in Fig. 17 and in the meshing length direction as shown in Fig. 19. From these results, the contact pressure can be calculated and represented using the previous procedure as shown in Fig. 18 and Fig. 20.

It can be seen in Fig. 18 that the maximum value of the contact pressure clearly appears on the pitch line. It can be seen in Fig. 20 that the pressure distribution increases from the toe until the middle of the tooth surface and then decreases until the heel. These two observations are in perfect correlation with the fact that the main meshing area (the main pressure area) is located in the middle of the tooth surface along the pitch line. As a result, it is necessary to consider a model of heat conduction to estimate the contact pressure in meshing teeth from an infrared imagery with a high response thermography.

Finally, we discuss the relationship between the loads derived from commanded torque and ones derived from experimental data. That is, we focus on the integrated load on each the contact line $m$ in Fig. 14 as a load experimental data, which means the integrated $x_{m}(i)$ load along the contact line $m$ derived from Eq. (26). Meanwhile, we derive the
Fig. 15 The image of target tooth contact area

(a) Tooth contact area  (b) Temperature distribution of tooth contact area

Fig. 16 Contact pressure distribution

Fig. 17 Temperature in tooth profile direction

Fig. 18 Contact pressure in tooth profile direction

Fig. 19 Temperature in meshing length direction

Fig. 20 Contact pressure in meshing length direction

Fig. 21 Relationship between loads derived from commanded torque and ones derived from experimental data
concentrated load at the center of the contact line $m$ from a commanded torque in the driving experiment as shown in Fig.1 and its distance from the center of gear shaft. These loads are estimated at the ten contact lines from meshing start point to end point on the meshing length in Fig. 14. These results are good agreement, shown in Fig. 20, and the derived load from Eq. (26) is confirmed to be reliable even though the estimation is from a macro-view point. Totally, a proposed model is considered to be effective to analyze the tooth contact problem.

6. Conclusion

We attempt to construct a novel approach to analyze the tooth surface temperature of hypoid gear with an infrared thermography, based on installing the high reflective mirror of infrared ray at the opposite side of the gear tooth meshing point to obtain the tooth surface images around the entire gear. Moreover, we investigate a thermal network model to estimate the temperature hysteresis on the tooth contact surface considering the obtained thermal imagery. The conclusion is as follows.

(1) Very small temperature rise due to the meshing on the tooth surface can be detected using a substraction process with a high response infrared thermography. The obtained area is in good agreement with the one by a red lead painting and peeling test method.

(2) A proposed thermal network model based on an electronic and mechanical analogy is found to be effective to estimate the temperature hysteresis on the tooth contact surface of hypoid gear. Especially, a model with a positive feedback loop is needed to predict the global and local variation of the temperature on the tooth surface in driving condition.

(3) We construct a practical formula to derive the contact pressure distribution from the meshing temperature considering thermal network model. It is demonstrated that the contact pressure on the meshing tooth surface could be estimated from an infrared imagery with a high response thermography.

References

Deng D., Nakanishi T. and Kato M., Surface Temperature Calculation and its Application to Surface Fatigue Strength Evaluation. Journal of Mechanical Design, Vol. 124 (2002), pp. 805–811

Kawamoto S., Hirogaki T., Ida T. and Ono A., A Study on Gear Noise Reduction Based on Helical-Gear Tooth Accuracy. SAE Transactions, Vol. 100 (1991), pp. 1775–1780

Kolivand M. and Kahraman A., Prediction of Mechanical Gear Mesh Efficiency of Hypoid Gear Pairs. Mechanism and Machine Theory, Vol. 45 (2010), pp. 1568–1582

Kubo A., Kuboki T. and Nonaka T., Estimation of Transmission Error of Cylindrical Involute Gears by Tooth Contact Pattern. JSME International Journal, Series III, Vol. 34, No. 2 (1991), pp. 252–259

Podzharov E., Mozuras A. and Alvarez Sanchez J. A., Design of High Contact Ratio Spur Gears to Reduce Static and Dynamic Transmission Error. Ingenieria Mecanica, Vol. 1 (2003), pp. 85–90

Sato T., Umezawa K. and Ishikawa J., Effects of Contact Ratio and Profile Correction on Gear Rotational Vibration. Bulletin of the JSME, Vol. 26, No. 221 (1983), pp. 221–224

Tsai M.-H. and Tsai Y.-C., A Method for Calculating Static Transmission Errors of Plastic Spur Gears Using FEM Evaluation. Finite Elements in Analysis and Design, Vol. 27 (1997), pp. 345–357

Winter, H. and Paul, M., Influence of Relative Displacements between Pinion and Gear on Tooth Root Stresses of Spiral Bevel Gears. Journal of Mechanisms, Transmissions, and Automation Design, Vol. 107 (1985), pp. 43–48

Zhang, Y., Litvin F. L. and Handschuh R. F., Computerized Design of Low-Noise Face-Milled Spiral Bevel Gears. Mech. Mach. Theory, Vol. 30 (1995), pp. 1171–1178