A Measurement of $R_b$ using a Vertex Mass Tag

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Abstract

We report a new measurement of $R_b = \frac{\Gamma_{Z^0 \rightarrow b\bar{b}}}{\Gamma_{Z^0 \rightarrow \text{hadrons}}}$ using a double tag technique, where the $b$ hemisphere selection is based on the reconstructed mass of the $B$ hadron decay vertex. The measurement was performed using a sample of 130k hadronic $Z^0$ events, collected with the SLD at the SLC. The method utilizes the 3-D vertexing abilities of the CCD pixel vertex detector and the small stable SLC beams to obtain a high $b$-tagging efficiency and purity. We obtain $R_b = 0.2142 \pm 0.0034\,(\text{stat.}) \pm 0.0015\,(\text{syst.}) \pm 0.0002(R_c)$. 

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We report a new measurement of $R_b$, the fraction of $Z^0 \rightarrow b\bar{b}$ events in hadronic $Z^0$ decays, collected with the SLD at SLC using a mass tag technique. The ratio $R_b$ is of special interest as a test of the Standard Model (SM), since it is sensitive to possible new physics effects which modify the radiative corrections to $Zb\bar{b}$ vertex. The vertex corrections are isolated because $R_b$ is a ratio between two hadronic rates, hence propagator (oblique), radiative and QCD corrections common to all quark flavors mostly cancel. Recent measurements yielded a world average $R_b$ value $3\sigma$ higher than that predicted by the SM [1]. Previous measurements [2] selected $b\bar{b}$ events based upon mainly the long $B$ hadron lifetime and were limited systematically by contamination in the sample from residual $c\bar{c}$ events. To avoid this limitation our $b$-tag exploits the large $B$ mass, since the mass distribution has a very small charm contamination beyond the charm mass cutoff. Taking advantage of SLD’s precise 3-D vertexing capability and the small and stable SLC beam spot, we achieve a very efficient and pure $b$ selection. We use a self-calibrating double tag technique [2], which allows one to measure both $R_b$ and the $b$-tag efficiency, $\epsilon_b$, simultaneously.

This measurement is performed using approximately 130K $e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}$ events collected during 1993-95. A detailed description of the detector can be found elsewhere [3]. We used the information from charged particle tracks measured with the CCD pixel Vertex Detector (VXD) along with the Central Drift Chamber. The event selection and the determination of the thrust axis use the the energy deposits measured with the Liquid Argon Calorimeter.

The luminous region of the SLC interaction point (IP) has a size of about $(1.5 \times 0.8) \mu$m in the x,y plane transverse to the beam direction and 700 $\mu$m along the beam direction. We use the average IP position of small groups of sequential hadronic events to determine the primary vertex (PV) in the x-y plane. The longitudinal position of the PV is determined for each event individually [3]. This results in a PV position measurement with uncertainties of 7 $\mu$m transverse to the beam axis and 35 $\mu$m (52 $\mu$m for $b\bar{b}$ events) along the axis. The measured track impact parameter resolution is $\sigma_{r\phi}[\mu m] = 11 \oplus 70/p \sin^{3/2}\theta$, $\sigma_{rz}[\mu m] = 37 \oplus 70/p \sin^{3/2}\theta$ where $p$ is the track momentum expressed in GeV/c.

The hadronic event selection is based on charged track multiplicity and track visible energy requirements as described in Ref. [3]. The event selection is studied with Monte Carlo (MC) events generated using JETSET 7.4 event generator [4], where the $B$ hadron decays are simulated using a model tuned to current $B$ and $D$ decay data [5]. A plane transverse to the thrust axis is used to divide the event into two hemispheres. In order to ensure that the events are well contained within the acceptance of the VXD, the polar angle of the thrust is required to be within $|\cos \theta_{\text{thrust}}| < 0.71$. In addition, to ensure the event hemisphere division is sensible and to reduce the contribution from events containing $g \rightarrow b\bar{b}$, we require that the event contain no more than three jets (defined using charged tracks and the JADE algorithm [6] with $y_{\text{cut}}=0.02$). A total of 72074 events were selected.

In each event well measured tracks [3] are used to search for a secondary vertex (SV). The SV are found by searching for areas of high track overlap density from the individual track resolution functions, in 3-D co-ordinate space [7]. The SV are required to be separated from
the $PV$ by at least 1 mm and to contain at least two tracks each with a 3-D impact parameter with respect to the $IP \geq 130 \mu m$, ensuring they originate from the decay of a particle with relatively long lifetime. Simulation studies show that secondary vertices are found in 50% of all $b$ hemispheres, in 15% of the charm and < 1% of the light quark hemispheres [7].

Due to the cascade structure of the $B$ decay, not all the tracks in the decay chain will come from a common decay point, thus the $SV$ is incomplete. We improve our estimate of the $B$ decay vertex mass by attaching to the $SV$ additional tracks that are consistent with the hypothesis of originating from the same $SV$. We illustrate this in Fig. 1(a). We define the vertex axis to be the straight line between the $PV$ and $SV$ centroids. For each track not in the $SV$ the 3-D distance of closest approach, $T$, and the distance from the $PV$ along the vertex axis to this point, $L$, are calculated. Tracks with $T < 1$ mm and $L/D > 0.25$, where $D$ is the distance from the $PV$ to the $SV$ are attached to the $SV$ to form a $B$ decay candidate. The invariant mass, $M_{ch}$, of the $B$ candidate is obtained assuming each track has the mass of a charged $\pi$; the distribution of $M_{ch}$ is shown in Fig. 2(a). We require $M_{ch}$ to be well above the charm mass, $M_{ch} > 2$ GeV/$c^2$, results in a $b$ hemisphere tagging efficiency of 28% with a purity of 98%.

We improve the $b$ tagging efficiency by applying a kinematic correction to the calculated $M_{ch}$. Due to the neglect of information about the neutral particles in the decay, the $SV$ flight path and the $SV$ momentum vector are typically acollinear. In order to compensate for the acollinearity we correct $M_{ch}$ using the minimum missing momentum ($P_t$) transverse to the $SV$ flight path. To reject non-$b\bar{b}$ events with an artificially large $P_t$ due to detector resolution effects, we define $P_t$ with respect to a vector tangent to the error boundaries of both the $PV$ and the $SV$, such that $P_t$ is minimized (see Fig. 2(b)). The ability to make this minimal correction is most effective at SLD due to the small and stable beams pot of the SLC and the high resolution vertexing. We then define the $P_t$-corrected mass, $\mathcal{M} = \sqrt{M_{ch}^2 + P_t^2 + |P_t|}$, and require $\mathcal{M} \leq 2 \times M_{ch}$ to reduce the contamination from fake vertices in light quark events. The distribution of $\mathcal{M}$ is shown in Fig. 3(b). By requiring $\mathcal{M} > 2 GeV/c^2$ we significantly raise our $b$-tag efficiency, yielding $\epsilon_b = 35.3\%$ for the same purity.

We measure $R_b$ and $\epsilon_b$ by counting the fraction of the event sample containing one tagged hemispheres, $F_s$, and the fraction containing both hemispheres tagged, $F_d$: 

$$R_b = \frac{(F_s - R_c(\epsilon_c - \epsilon_{uds}) - \epsilon_{uds})^2}{F_d - R_c(\epsilon_c - \epsilon_{uds})^2 + \epsilon_{uds}^2 - 2 F_s \epsilon_{uds} - \lambda_b R_b(\epsilon_b - \epsilon_b^2)};$$

$$\epsilon_b = \frac{F_d - R_c \epsilon_c(\epsilon_c - \epsilon_{uds}) - F_s \epsilon_{uds} - \lambda_b R_b(\epsilon_b - \epsilon_b^2)}{F_s - R_c(\epsilon_c - \epsilon_{uds}) - \epsilon_{uds}}.$$

The only term dependent upon $B$ production and decay modeling is the $b$ hemisphere tagging correlation, $\lambda_b = \frac{\epsilon_b^{double} - \epsilon_b^2}{\epsilon_b - \epsilon_b^2} = 0.59\%$, where we have used the simulation to estimate $\lambda_b$. Estimates of the hemisphere tagging rates of light quarks, $\epsilon_{uds} = 0.06\%$ and charm quarks, $\epsilon_c = 0.69\%$, are also derived from the simulation, and we assume $R_c = \frac{\Gamma_{Z \rightarrow c \bar{c}}}{\Gamma_{Z \rightarrow c \bar{c}} + \Gamma_{Z \rightarrow q \bar{q}}} = 0.171$. We measure $R_b = 0.2142 \pm 0.0034_{stat}$, which includes a correction of +0.0003 for the $e^+ e^- \rightarrow \gamma \rightarrow b\bar{b}$ contribution as calculated by ZFITTER [8]. The measured value of $\epsilon_b = 35.3\% \pm 0.6\%$ is in good agreement with the MC estimate of 35.5%. As a cross-check we repeated the
measurement using different $\mathcal{M}$ cuts; the results are summarized in Fig. 3(a). The $R_b$ results are consistent for values of $0 < \mathcal{M}_{\text{cut}} < 3 \text{ GeV}/c^2$.

The systematic uncertainty on $R_b$, given in detail in Table 1, results from a combination of detector related effects and physics uncertainties in the simulation which affect our estimates of $\epsilon_c$, $\epsilon_{uds}$ and $\lambda_b$. The physics systematic errors are assigned by comparing the nominal simulation distributions with an alternative set of distributions which reflect the uncertainties in the world average measurements of the MC physics parameters [9]. The two significant sources of systematic errors from light quark events come from the uncertainties in long lived strange particle production and gluon splitting into heavy quark pairs. The effects of strange particle production are studied by varying the $s\bar{s}$ production probability in jet fragmentation. The $g\rightarrow b\bar{b}$ and $g\rightarrow c\bar{c}$ production rates are varied based upon the OPAL $g\rightarrow c\bar{c}$ measurement [10] and the theoretical prediction for the ratio $g\rightarrow b\bar{b}/g\rightarrow c\bar{c}$ [9].

The various charmed hadron production rates and fragmentation parameters in $Z^0$ decays are varied within the present LEP measurement errors. Charmed hadron fragmentation is studied by varying the average scaled energy $\langle x_E \rangle$ in the Peterson fragmentation function [11], as well as by studying the difference between the Peterson and Bowler models [12] for the same values of $\langle x_E \rangle$. Charmed hadron decay lifetimes are varied according to the world average measurement errors [13]. The charmed hadron decay charged multiplicity and $K^0$ production rate systematic uncertainties are based on measurements by Mark-III [12]. Charmed hadron decays with fewer neutral particles have higher charged mass and are therefore more likely to be tagged. Thus, an additional systematic uncertainty is estimated by varying the rates of charmed hadron decays with no $\pi^0$’s by $\pm 10\%$.

The $B$ decay modeling uncertainty enters via the $\lambda_b$ estimation. It is studied by varying the $B$ lifetime, $B$ baryon production rate, $B$ fragmentation function and the $B$ decay charged multiplicity in a similar manner as for the charm systematic studies. Simulation uncertainties which affect the tagging efficiency are studied by comparing the angular distribution of the $b$-tagging rate between data and simulation and a systematic error is assigned to the difference. Hard gluon radiation effects are estimated from $\pm 30\%$ variation of the fraction of simulation events where both $B$ hadrons are contained within the same hemisphere and a hard gluon is in the other. Another systematic error is assigned to effects of $B$ hadron momentum correlation between the two hemispheres, mostly due to soft gluon radiation and fragmentation effects, which in turn translate to a $b$-tagging efficiency correlation. This is estimated by comparing the $B$ momentum correlation in the HERWIG [15] and JETSET [4] event generators.

As a cross check we decomposed the efficiency correlation into an independent set of components which represent all sources of correlation between the two $b$ hemispheres. The components we have studied and their contributions are: the PV measurement (-0.02%), the track resolution effect on the $IP$ determination (+0.04%), the detector non-uniformity via the tagging angular distribution dependence (+0.49%), the momentum distribution of the $B$ hadron in each hemisphere (+0.08%) and the effect of hard gluon emission forcing the two $B$ hadrons into one hemisphere (+0.07%). The estimated $\lambda_b$ (0.59 $\pm$ 0.11)% and that from the sum of the components (0.67%) are in good agreement.
A major source of detector systematic uncertainty is due to the discrepancy in modeling the track impact parameter resolution, mainly along the beam axis. In the simulation track $z$ impact parameters are smeared using a random Gaussian distribution of width $20 \, \mu m / \sin \theta$, as well as adjusted for $z$ impact parameter mean position shifts to match the data. The full difference in $R_b$ between the nominal and resolution-corrected samples is conservatively assigned to be the resolution systematic error. The difference between the measured and simulation charged track multiplicity as a function of $\cos \theta$ and momentum is attributed to an unsimulated tracking inefficiency correction. Both the tracking resolution and efficiency corrections require the use of a random number generator. After application of these corrections the result vary slightly with different random sequences. These fluctuations are included as an additional MC statistical uncertainty. The uncertainty on the primary vertex $xy$ location simulation is estimated from the effect of adding a Gaussian tail to the IP distribution of $100 \, \mu m$ width for 0.5% of the simulated events. The simulation shows that the $\leq 3$ jets requirement in the event selection favors $b\bar{b}$ over other $q\bar{q}$ events which biases our measurement by $+0.55\%$. We verified this bias in the data, by measuring $R_b$ with and without applying the $\leq 3$ jet criterion and found that our measured $R_b$ value only changed by 0.0001, which is consistent with a statistical fluctuation but nevertheless was included as a systematic error. Another bias of $+0.26 \pm 0.12\%$ is introduced by the other event selection criteria, thus the combined bias is $+0.82 \pm 0.13\%$ and was corrected. Fig. 3(b) shows the statistical and the detector, physics and $R_c$ systematic uncertainties versus the minimum $M$ cut.

In summary we have measured:

$$R_b = 0.2142 \pm 0.0034_{\text{stat.}} \pm 0.0015_{\text{syst.}} \pm 0.0002_{R_c}$$

which includes a correction of $+0.0003$ for the $e^+e^- \rightarrow \gamma \rightarrow b\bar{b}$ contribution. This value supersedes our previous $R_b$ measurements [3] and is in good agreement with the SM prediction of 0.2155. A new high precision measurement has recently been reported by ALEPH [16], which also incorporates mass information to improve a lifetime-based probability tag. With the new SLD and LEP measurements the gap between the SM prediction of $R_b$ and the world average has narrowed.

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Figure 1: (a) An illustration of the SV track attachment criteria. (b) Illustration of the $P_t$ derivation.
Figure 2: Distribution of (a) $M_{ch}$ and (b) $P_t$ corrected mass, $M$, for data (points) and MC which includes a breakdown of the $b$, $c$ and $uds$ contributions (open, hatched and crosshatched histograms respectively).
Figure 3: (a) Variation of $R_b$ with the $\mathcal{M}$ cut. The statistical error is represented by the boxes and the outer bars show the sum in quadrature of the statistical and systematic errors. (b) $R_b$ statistical and systematic uncertainties versus $\mathcal{M}$ cut.
| Light Quark Systematic ($\epsilon_{uds}$)                      | $\delta R_b$ |
|-------------------------------------------------------------|-------------|
| $g \rightarrow b\bar{b}$ 0.31±0.11%                        | -0.00033    |
| $g \rightarrow c\bar{c}$ 2.38±0.48%                        | -0.00004    |
| $K^0$ production ±10%                                       | -0.00003    |
| $\Lambda$ production ±10%                                   | -0.00002    |
| Total $uds$ physics systematic                               | 0.00034     |

| Charm Systematic ($\epsilon_c$)                             | $\delta R_b$ |
|-------------------------------------------------------------|-------------|
| $D^+$ production 0.259 ± 0.028                               | -0.00011    |
| $D_s$ production 0.115 ± 0.037                                | -0.00005    |
| c-baryon production 0.074 ± 0.029                             | 0.00011     |
| c-frag. $\langle x_E \rangle_D$ = 0.482 ± 0.008               | -0.00006    |
| c-frag. function shape                                        | -0.00001    |
| $D^0$ lifetime 0.415 ± 0.004 ps                               | -0.00003    |
| $D^+$ lifetime 1.057 ± 0.015 ps                               | -0.00001    |
| $D_s$ lifetime 0.467 ± 0.017 ps                               | -0.00002    |
| $\Lambda_c$ lifetime 0.200 ± 0.011 ps                        | -0.00001    |
| $D^0$ decay $\langle N_{ch} \rangle$ = 2.54 ± 0.05            | -0.00006    |
| $D^+$ decay $\langle N_{ch} \rangle$ = 2.50 ± 0.06            | -0.00006    |
| $D_s$ decay $\langle N_{ch} \rangle$ = 2.65 ± 0.33            | -0.00009    |
| $D^0 \rightarrow K^0$ production 0.401 ± 0.059                | +0.00015    |
| $D^+ \rightarrow K^0$ production 0.646 ± 0.078                | +0.00020    |
| $D_s \rightarrow K^0$ production 0.380 ± 0.06                 | +0.00002    |
| $D^0$ decay no-$\pi^0$ frac. 0.370 ± 0.037                    | +0.00005    |
| $D^+$ decay no-$\pi^0$ frac. 0.499 ± 0.050                    | -0.00008    |
| $D_s$ decay no-$\pi^0$ frac. 0.352 ± 0.035                    | <0.00001    |
| Total Charm Physics systematic                                | 0.00033     |

| B decay modeling ($\lambda_b$)                               | $\delta R_b$ |
|-------------------------------------------------------------|-------------|
| $B$ lifetime ±0.05 ps                                        | 0.00004     |
| $B$ decay $\langle N_{ch} \rangle$ = 5.73 ± 0.35             | 0.00003     |
| $b$ fragmentation                                            | 0.00019     |
| $\Lambda_b$ production fraction 0.074 ± 0.03                 | 0.00008     |
| Hard gluon radiation                                         | 0.00008     |
| $B$ momentum correlation                                     | 0.00029     |
| $b$-tag $\cos \theta$ dependency                            | 0.00001     |
| Total $bb$ Physics systematic                                | 0.00038     |

| Detector Systematic                                          | $\delta R_b$ |
|-------------------------------------------------------------|-------------|
| Tracking resolution                                          | 0.00096     |
| Tracking efficiency                                          | 0.00040     |
| $\langle IP \rangle_{xy}$ tail                              | 0.00010     |
| MC statistics                                               | 0.00091     |
| Event selection bias                                         | 0.00028     |
| Total detector and MC                                       | 0.00141     |
| $R_c = 0.171 \pm 0.006$                                     | 0.00021     |
| Total (excl. $R_c$)                                          | 0.00154     |

Table 1: Summary of systematic uncertainties for the $M > 2.0$ GeV/$c^2$ cut.