THE MODULATION EFFECT FOR SUPERSYMMETRIC DARK MATTER DETECTION WITH ASYMMETRIC VELOCITY DISPERSION.

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The detection of the theoretically expected dark matter is central to particle physics cosmology. Current fashionable supersymmetric models provide a natural dark matter candidate which is the lightest supersymmetric particle (LSP). Such models combined with fairly well understood physics like the quark substructure of the nucleon and the nuclear form factor and/or the spin response function of the nucleus, permit the evaluation of the event rate for LSP-nucleus elastic scattering. The thus obtained event rates are, however, very low or even undetectable. So it is imperative to exploit the modulation effect, i.e. the dependence of the event rate on the earth’s annual motion. In this paper we study such a modulation effect both in non-directional and directional experiments. We calculate both the differential and the total rates using symmetric as well as asymmetric velocity distributions. We find that in the symmetric case the modulation amplitude is small, less than 0.07. There exist, however, regions of the phase space and experimental conditions such that the effect can become larger. The inclusion of asymmetry, with a realistic enhanced velocity dispersion in the galactocentric direction, yields the bonus of an enhanced modulation effect, with an amplitude which for certain parameters can become as large as 0.46.

1 Introduction

It is known that that dark matter is needed to close the Universe. It is also known that one needs two kinds of dark matter. One composed of particles which were relativistic at the time of structure formation. These constitute the hot dark matter component (HDM). The other is made up of particles which were non-relativistic at the time of freeze out. This is the cold dark matter component (CDM). The COBE data suggest that CDM is at least 60%. On the other hand recent data from the Supernova Cosmology Project suggest that there is no need for HDM and the situation can be adequately described by $\Omega < 1$, e.g. $\Omega_{CDM} = 0.3$ and $\Omega_{\Lambda} = 0.6$. In a more recent analysis Turner gives $\Omega_m = 0.4$.

Since the non exotic component cannot exceed 40% of the CDM, there is room for the exotic WIMP’s (Interacting Massive Particles). Recently the DAMA experiment has claimed the observation of one signal in direct detection of a WIMP, which with better statistics has subsequently been interpreted as a modulation signal.
In the currently favored supersymmetric extensions of the standard model the most natural WIMP candidate is the LSP, i.e. the lightest supersymmetric particle, whose nature can be described in most supersymmetric (SUSY) models to be a Majorana fermion, a linear combination of the neutral components of the gauginos and Higgsinos.

Since this particle is expected to be very massive, \( m_\chi \geq 30 \text{GeV} \), and extremely non relativistic with average kinetic energy \( T \leq 100 \text{KeV} \), it can be directly detected only via the recoiling of a nucleus \( (A,Z) \) in the elastic scattering process:

\[
\chi + (A,Z) \rightarrow \chi + (A,Z)^* \tag{1}
\]

(\( \chi \) denotes the LSP). In order to compute the event rate one proceeds with the following steps:

1) Write down the effective Lagrangian at the elementary particle (quark) level obtained in the framework of supersymmetry as described in Refs. 1, Bottino et al. 23 and 26.

2) Go from the quark to the nucleon level using an appropriate quark model for the nucleon. Special attention in this step is paid to the scalar couplings, which dominate the coherent part of the cross section and the isoscalar axial current, which, as we will see, strongly depend on the assumed quark model 13, 27, 28.

3) Compute the relevant nuclear matrix elements using as reliable as possible many body nuclear wave functions hoping that, by putting as accurate nuclear physics input as possible, one will be able to constrain the SUSY parameters as much as possible.

4) Calculate the modulation of the cross sections due to the earth’s revolution around the sun by a folding procedure assuming some distribution of velocities for LSP.

The purpose of our present review is to focus on the last point of our above list along the lines suggested by our recent letter 22, expanding our previous results and giving some of the missing calculational details. For the reader’s convenience, however, we will give a brief description on how to calculate LSP-nucleus scattering cross section, without elaborating on how one gets the needed parameters from supersymmetry. For the calculation of these parameters from representative input in the restricted SUSY parameter space, we refer the reader to the literature, e.g. Bottino et al. 23, Kane et al. 19, Castano et al. and Arnowitt et al. 24. Then we will specialize our study in the case of the nucleus \(^{127}\text{I}\) which is one of the most popular targets 19-9. To this end we will include the effect of the nuclear form factors. We will consider both a symmetric Maxwell-Boltzmann distribution as well as asymmetric distributions like the one suggested by Drukier 18. We will examine
the effect modulation in the directional as well as the non directional detection, both in the differential as well as the total event rates. We will present our results as a function of the LSP mass, $m_\chi$, for various detector energy thresholds, in a way which can be easily understood by the experimentalists.

2 The Basic Ingredients for LSP Nucleus Scattering

Because of lack of space we are not going to elaborate here further on the construction of the effective Lagrangian derived from supersymmetry, but refer the reader to the literature \cite{11,12,21,23,29}. The effective Lagrangian can be obtained in first order via Higgs exchange, s-quark exchange and Z-exchange. We will use a formalism which is familiar from the theory of weak interactions, i.e.

$$L_{eff} = -\frac{g_F}{\sqrt{2}}\{((\bar{\chi}_1)^{\lambda}\gamma^5\chi_1)J_\lambda + (\bar{\chi}_1\chi_1)J\}$$

(2)

where

$$J_\lambda = \bar{N}\gamma_\lambda(f_0^V + f_1^V\tau_3 + f_0^A\gamma_5 + f_1^A\gamma_5\tau_3)N$$

(3)

and

$$J = \bar{N}(f_s^0 + f_s^1\tau_3)N$$

(4)

We have neglected the uninteresting pseudoscalar and tensor currents. Note that, due to the Majorana nature of the LSP, $\bar{\chi}_1\gamma^\lambda\chi_1 = 0$ (identically). The parameters $f_0^V, f_1^V, f_0^A, f_1^A, f_0^S, f_1^S$ depend on the SUSY model employed. In SUSY models derived from minimal SUGRA the allowed parameter space is characterized at the GUT scale by five parameters, two universal mass parameters, one for the scalars, $m_0$, and one for the fermions, $m_{1/2}$, as well as the parameters $tan\beta$, one of $A_0$ and $m_0^{pole}$ and the sign of $\mu$. Deviations from universality at the GUT scale have also been considered and found useful \cite{25}. We will not elaborate further on this point since the above parameters involving universal masses have already been computed in some models \cite{11,29} and effects resulting from deviations from universality will be published elsewhere \cite{4} (see also Arnowitt \textit{et al} in Ref. \cite{4} and Bottino \textit{et al} in Ref. \cite{23}). For some choices in the allowed parameter space the obtained couplings can be found in a previous paper \cite{31}.

The invariant amplitude in the case of non-relativistic LSP can be cast \cite{11} in the form

$$|\mathcal{M}|^2 = \frac{E_fE_i - m^2}{m^2_x} + \frac{P_i \cdot P_f}{m^2_x} |J_0|^2 + |J|^2 \simeq \beta^2 |J_0|^2 + |J|^2 + |J|^2$$

(5)
where $m_x$ is the LSP mass, $|J_0|$ and $|J|$ indicate the matrix elements of the time and space components of the current $J$, respectively, and $J$ represents the matrix element of the scalar current $J$ of Eq. (4). Notice that $|J_0|^2$ is multiplied by $\beta^2$ (the suppression due to the Majorana nature of LSP mentioned above). It is straightforward to show that

$$|J_0|^2 = A^2 |F(q^2)|^2 \left( f_0 - f_{1/2} \frac{A - 2Z}{A} \right)^2$$

(6)

$$J^2 = A^2 |F(q^2)|^2 \left( f_0^2 - f_{1/2}^2 \frac{A - 2Z}{A} \right)^2$$

(7)

$$|J|^2 = \frac{1}{2J_f + 1} |\langle J \rangle [f_0^2 \Omega_0(q) + f_{1/2} \Omega_1(q)] |J_i|^2$$

(8)

with $F(q^2)$ the nuclear form factor and

$$\Omega_0(q) = \sum_{j=1}^{A} \sigma(j) e^{-iq \cdot x_j}, \quad \Omega_1(q) = \sum_{j=1}^{A} \sigma(j) \tau_3(j) e^{-iq \cdot x_j}$$

(9)

where $\sigma(j)$, $\tau_3(j)$, $x_j$ are the spin, third component of isospin ($\tau_3 |p\rangle = |p\rangle$) and coordinate of the j-th nucleon and $q$ is the momentum transferred to the nucleus.

The differential cross section in the laboratory frame takes the form

$$\frac{d\sigma}{d\Omega} = \frac{\sigma_0 (\mu_r / m_N)^2 \xi \beta^2 |J_0|^2 [1 - 2\eta + 1/(1 + \eta)^2 \xi^2] + |J|^2 + |J|^2}{1 + \eta}$$

(10)

where $m_N$ is the proton mass, $\eta = m_x/m_N A$, $\xi = \hat{p}_1 \cdot \hat{q} \geq 0$ (forward scattering) and

$$\sigma_0 = \frac{1}{2\pi} (G_F m_N)^2 \simeq 0.77 \times 10^{-38} \text{cm}^2$$

(11)

The reduced mass $\mu_r$ is given by

$$\mu_r = \frac{m_x}{1 + \eta}$$

(12)

For the evaluation of the differential rate, which is the main subject of the present work, it will be more convenient to use the variables $(v, u)$ instead of the variables $(v, \xi)$. Thus integrating the differential cross section, Eq. (10), with respect to the azimuthal angle we obtain

$$d\sigma(u, v) = \frac{du}{2(\mu_r b u)^2} [\Sigma_S + \Sigma_V \frac{v^2}{c^2} F^2(u)] + \Sigma_{\text{spin}} F_{11}$$

(13)
with
\[ \bar{\Sigma}_S = \sigma_0 \left( \frac{\mu_r}{m_N} \right)^2 \left\{ A^2 \left[ (f_0^0 f_1^1 A - 2Z_A)^2 \right] \right\} \] (14)
\[ \bar{\Sigma}_{\text{spin}} = \sigma_0 \left( \frac{\mu_r}{m_N} \right)^2 \left[ f_0^0 \Omega_0(0) \right]^2 + \left[ f_1^0 \Omega_0(0) \right]^2 \left[ \frac{F_{00}(u)}{F_{11}(u)} + \frac{2f_1^0 f_1^1 \Omega_0(0) \Omega_1(0)}{F_{11}(u)} \left( f_1^1 \Omega_1(0) \right)^2 \right] \] (15)
\[ \bar{\Sigma}_V = \sigma_0 \left( \frac{\mu_r}{m_N} \right)^2 A^2 \left( f_0^0 - f_1^1 \frac{A - 2Z_A}{A} \right)^2 \left[ 1 - \frac{1}{(2\mu_r b)^2} \frac{2\eta + 1}{(1 + \eta)^2} \frac{2u}{v^2} \right] \] (16)

We should remark that even though the quantity \( \bar{\Sigma}_{\text{spin}} \) can be a function of \( u \), in actual practice it is independent of \( u \). The same is true of the less important term \( \bar{\Sigma}_V \). In the above expressions \( F(u) \) is the nuclear form factor and
\[ F_{\rho \rho'}(u) = \sum_{\lambda, \kappa} \frac{\Omega_{\rho \kappa}^{(\lambda, \kappa)}(u)}{\Omega_{\rho}^{(\lambda, \kappa)}(0)} \frac{\Omega_{\rho'}^{(\lambda, \kappa)}(u)}{\Omega_{\rho'}^{(\lambda, \kappa)}(0)} \] (17)
are the spin form factors with
\[ u = q^2 b^2/2 \] (18)
b being the harmonic oscillator size parameter and \( q \) the momentum transfer to the nucleus. The quantity \( u \) is also related to the experimentally measurable energy transfer \( Q \) via the relations
\[ Q = Q_0 u, \quad Q_0 = \frac{1}{A m_N b^2} \] (19)
The detection rate for a particle with velocity \( v \) and a target with mass \( m \) detecting in the direction \( e \) will be denoted by \( R(\rightarrow e) \). Then one defines the undirectional rate \( R_{\text{undir}} \) via the equations via the equations
\[ R_{\text{undir}} = \frac{dN}{dt} = \frac{\rho(0) m}{m_N \chi m_N} \sigma(u, v) \left[ |v \cdot \hat{e}_x| + |v \cdot \hat{e}_y| + |v \cdot \hat{e}_z| \right] \] (20)
\( \rho(0) = 0.3 \text{GeV/cm}^3 \) is the LSP density in our vicinity. This density has to be consistent with the LSP velocity distribution (see next section).
The differential undirectional rate can be written as
\[ dR_{\text{undir}} = \frac{\rho(0) m}{m_N \chi m_N} d\sigma(u, v) \left[ |v \cdot \hat{e}_x| + |v \cdot \hat{e}_y| + |v \cdot \hat{e}_z| \right] \] (21)
where \( d\sigma(u, v) \) is given by Eq. (13).
The directional rate in the direction \( \hat{e} \) takes the form:

\[
R_{\text{dir}} = R(\rightarrow e) - R(\rightarrow -e) = \frac{\rho(0)}{m_X} \frac{m}{A m_N} v.e \sigma(u, v) \tag{22}
\]

and the corresponding differential rate is given by

\[
dR_{\text{dir}} = \frac{\rho(0)}{m_X} \frac{m}{A m_N} v.e \ d\sigma(u, v) \tag{23}
\]

### 3 Convolution of the Event Rate

We have seen that the event rate for LSP-nucleus scattering depends on the relative LSP-target velocity. In this section we will examine the consequences of the earth’s revolution around the sun (the effect of its rotation around its axis is expected to be negligible) i.e. the modulation effect. This can be accomplished by convoluting the rate with the LSP velocity distribution. Hitherto such a consistent choice can be a Maxwell distribution

\[
f(v') = \left(\sqrt{\frac{\pi}{3}} v_0\right)^{-3} e^{-\left(v'/v_0\right)^2} \tag{24}
\]

where \( v_0 \) is the velocity of the sun around the center of the galaxy. In the present paper following the work of Drukier, see Ref. 18, we will assume that the velocity distribution is only axially symmetric, i.e. of the form

\[
f(v', \lambda) = N(y_{\text{esc}}, \lambda)\left(\sqrt{\pi} v_0\right)^{-3} \left[f_1(v', \lambda) - f_2(v', v_{\text{esc}}, \lambda)\right] \tag{26}
\]

with

\[
f_1(v', \lambda) = \exp\left[-\frac{(v'_{x})^2 + (1 + \lambda)(v'_{y})^2 + (v'_{z})^2}{v_0^2}\right] \tag{27}
\]

\[
f_2(v', v_{\text{esc}}, \lambda) = \exp\left[-\frac{v_{\text{esc}}^2 + \lambda(v'_{y})^2 + (v'_{z})^2}{v_0^2}\right] \tag{28}
\]

where \( v_{\text{esc}} \) is the escape velocity in the gravitational field of the galaxy, \( v_{\text{esc}} = 625 \text{Km/s} \). In the above expressions \( \lambda \) is a parameter, which describes the asymmetry and takes values between 0 and 1 and N is a proper normalization constant given by

\[
\frac{1}{N(\lambda, y_{\text{esc}})} = \frac{1}{\lambda + 1} \left[\text{erf}(y_{\text{esc}}) - e^{-(\lambda+1)y_{\text{esc}}^2} \frac{\text{erf}(i \sqrt{\lambda} y_{\text{esc}})}{i \sqrt{\lambda}}\right]
\]

\[
- \frac{e^{-y_{\text{esc}}^2}}{\lambda} \frac{2}{\sqrt{\pi}} y_{\text{esc}} - e^{-\lambda y_{\text{esc}}^2} \frac{\text{erf}(i \sqrt{\lambda} y_{\text{esc}})}{i \sqrt{\lambda}} \tag{29}
\]
with $y_{esc} = \frac{v_{esc}}{v_0}$ and erf(x) the error function given by

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x dt e^{-t^2}$$

(30)

For $y_{esc} \to \infty$ we get the simple expression $N^{-1} = \lambda + 1$

The z-axis is chosen in the direction of the disc’s rotation, i.e. in the direction of the motion of the sun, the y-axis is perpendicular to the plane of the galaxy and the x-axis is in the radial direction. In the above frame we find that the position of the axis of the ecliptic is determined by the angle $\gamma \approx 29.80$ (galactic latitude) and the azimuthal angle $\omega = 186.3^0$ measured on the galactic plane from the $\hat{z}$ axis. Thus, the axis of the ecliptic lies very close to the y, z plane and the velocity of the earth around the sun is

$$v_E = v_0 + v_1 = v_0 + v_1(\sin \alpha \hat{x} - \cos \alpha \cos \gamma \hat{y} + \cos \alpha \sin \gamma \hat{z})$$

(31)

where $\alpha$ is the phase of the earth’s orbital motion, $\alpha = 2\pi(t - t_1)/T_E$, where $t_1$ is around second of June and $T_E = 1$ year.

We are now in a position to express the above distribution in the laboratory frame, i.e.

$$f(v, \lambda, v_E) = f_3(v, v_E, \lambda) - f_4(v, v_{esc}, \lambda)$$

(32)

with

$$f_3(v, v_E, \lambda) = \exp[-\left(\frac{(v_x + v_1 \sin \alpha)^2}{v_0^2}\right]$$

$$\times \exp[-\left(1 + \lambda\right)(\frac{v_y + v_1 \cos \gamma \sin \alpha)^2 + (v_z + v_0 + v_1 \sin \gamma \cos \alpha)^2}{v_0^2}]$$

(33)

$$f_4(v, v_{esc}, \lambda) = \exp[-\left(\frac{v_{esc}^2 + \lambda(v_y + v_1 \cos \gamma \sin \alpha)^2 + (v_z + v_0 + v_1 \sin \gamma \cos \alpha)^2}{v_0^2}\right]$$

(34)

4 Expressions for the Differential Event Rate in the Presence of Velocity Dispersion

We will begin with the undirectional rate.
4.1 Expressions for the Undirectional Differential Event Rate

The mean value of the undirectional event rate of Eq. (22), is given by

$$\langle \frac{dR_{\text{undir}}}{du} \rangle = \rho(0) \frac{m}{m_N} \int f(v, v_E) \| v \hat{e}_x \| + | v \hat{e}_y | + | v \hat{e}_z | \frac{d\sigma(u, v)}{du} d^3v$$

(35)

From now on we will omit the subscript undir in the case of the undirectional rate. The above expression can be more conveniently written as

$$\langle \frac{dR}{du} \rangle = \rho(0) \frac{m}{m_N} \sqrt{\langle v^2 \rangle} \langle d\Sigma \rangle \frac{d\Sigma}{du}$$

(36)

where

$$\langle d\Sigma \rangle = \int \frac{\| v \hat{e}_x \| + \| v \hat{e}_y \| + \| v \hat{e}_z \|}{\sqrt{\langle v^2 \rangle}} f(v, v_E) \frac{d\sigma(u, v)}{du} d^3v$$

(37)

It is convenient to work in spherical coordinates. But even then the angular sun is small. Thus introducing the parameter

$$\delta = \frac{2\nu_1}{\nu_0} = 0.27,$$

(38)

expanding in powers of $\delta$ and keeping terms up to linear in it we can manage to perform the $\phi$ integration using standard contour integral techniques and express the result in terms of the two modified Bessel functions $I_m(\frac{\lambda v^2}{2(1-t^2)})$ with $t = \cos \theta$ and $m=0,1$. Thus the angular integration of Eq. (22) yields

$$\tilde{M}_i(\lambda, y) = 2\pi \times \exp[-(y^2 + 1)(1 + \lambda)] \tilde{\Lambda}_i(\lambda, y) - \exp[-(y_{esc}^2 + \lambda y^2)] \tilde{\Lambda}'_i(\lambda, y), i = 1, 2$$

(39)

where $\tilde{\Lambda}_i, \tilde{\Lambda}'_i$ come from $f_3, f_4$ respectively. We find

$$\tilde{\Lambda}_i(\lambda, y) = \tilde{\Lambda}_{1,1}(\lambda, y) + \tilde{\Lambda}_{1,2}(\lambda, y) + \tilde{\Lambda}_{1,3}(\lambda, y)$$

(40)

with

$$\tilde{\Lambda}_{1,3}(\lambda, y) = \int_{-1}^{1} dt \exp[(\zeta^2/2 - 2(\lambda + 1)yt)] \frac{1}{\sqrt{\pi}} \frac{I_0(\zeta^2/2)}{i \zeta}$$

(41)

$$\tilde{\Lambda}_{1,2}(\lambda, y) = \frac{1}{\sqrt{\pi}} \int_{-1}^{1} dt \exp[-2(\lambda + 1)yt](1 - t^2)^{1/2} \frac{ef(i \zeta)}{i \zeta}$$

(42)
\[ \mathcal{A}_{1,1}(\lambda, y) = \frac{1}{\sqrt{\pi}} \int_{-1}^{1} dt \exp\left[-2(\lambda + 1)yt\right](1 - t^2)^{1/2} \frac{\exp\left(\zeta^2\right)}{\zeta} \operatorname{erf}(\zeta) \] (43)

where \( \zeta = y(\lambda (1 - t^2)^{1/2}) \) and the function \( \operatorname{erf}(i\zeta) \) given by Eq. 30.

Furthermore
\[ \mathcal{A}_{2}(\lambda, y) = -(\lambda + 1) \mathcal{A}_{1}''(\lambda, y) \] (44)

where \( \mathcal{A}_{1}''(\lambda, y) \) is obtained from \( \mathcal{A}_{1}(\lambda, y) \) by adding in the integrands the extra factor \( ty + 1 \). We should mention that in the last integral we have omitted the numerical factor \( \delta \cos \alpha \sin \gamma \).

The formulas for the second term in Eq. 39 for \( \mathcal{A}_{i}(\lambda, y) \) are obtained by a mere replacement of the expression \( \lambda + 1 \) by \( \lambda \). In all the above expressions \( y = \left(\frac{\nu}{\nu_0}\right) \) (not to be confused with the y-coordinate).

The functions \( \mathcal{G}_{i} \) have been obtained by considering the leading non vanishing term in the zeta expansion of the integrands of the expressions (41)-(43). Thus
\[ \mathcal{G}_{0}(0, y) = 0, \quad \mathcal{G}_{1}(0, y) = 0 \] (47)
\[ \bar{F}_{0}(\lambda, x) = (\lambda + 1)^{-2} \left[ x \sinh(x) - \cos(x) + 1 + x I_1(x) \right] \] (48)
\[ \bar{F}_{1}(\lambda, x) = (1 + \lambda)^{-2} \left[ (2 + \lambda)((x^2/(2(2 + \lambda)) + 1) \cosh(x) - x \sinh(x) - 1) + x^2 I_2[x] - (\lambda + 1) x I_1(x) \right] \] (49)

note that here \( x = (\lambda + 1)2y \). \( I_m(x) \) is the modified bessel function of order m. The functions \( \mathcal{G} \) cannot be obtained analytically, but they can easily be expressed as a rapidly convergent series in \( y = \frac{\nu}{\nu_0} \), which will not be given here.

Similarly
\[ \mathcal{G}_{i}'(\lambda, y) = 2y^2 \mathcal{A}'_{i}(\lambda, y), \quad i = 1, 2 \] (50)

Thus the folded non-directional event rate takes the form
\[ \frac{dS}{du} = \bar{\Sigma}_S \bar{F}_{0}(u) + \frac{\langle \nu^2 \rangle}{c^2} \bar{\Sigma}_V \bar{F}_{1}(u) + \bar{\Sigma}_{\text{spin}} \bar{F}_{\text{spin}}(u) \] (51)
where the $\bar{\Sigma}_i, i = S, V, \text{spin}$ are given by Eqs. (14) - (16).

The quantities $F_0, F_1, F_{\text{spin}}$ are obtained from the corresponding form factors via the equations

$$
\bar{F}_k(u) = F^2(u)\Phi_k(u)\frac{(1 + k)a^2}{2k + 1}, \quad k = 0, 1
$$

$$
\bar{F}_{\text{spin}}(u) = F_{11}(u)\Phi_0(u)a^2
$$

$$
\bar{\Psi}_k(u) = [\bar{\psi}(0), k(a\sqrt{u}) + 0.135 \cos \alpha \bar{\psi}(1), k(a\sqrt{u})]
$$

with

$$
a = \frac{1}{\sqrt{2}\mu_b v_0}
$$

and

$$
\tilde{\psi}^{(l),k}(x) = N(y_{\text{esc}}, \lambda)e^{-\lambda(e^{-1}\tilde{\Phi}^{(l),k}(x) - e^{y_{\text{esc}}}[e^{-1}\tilde{\Phi}'^{(l),k}(x)])}
$$

$$
\tilde{\Phi}^{(l),k}(x) = \frac{2}{\sqrt{6}\pi} \int_x^{y_{\text{esc}}} dy y^{2k-1}\exp(-(1 + \lambda)y^2)(\tilde{F}_1(\lambda, (\lambda + 1)y) + \tilde{G}_1(\lambda, y)))
$$

$$
\tilde{\Phi}'^{(l),k}(x) = \frac{2}{\sqrt{6}\pi} \int_x^{y_{\text{esc}}} dy y^{2k-1}\exp(-\lambda y^2)(\tilde{G}'_1(\lambda, y))
$$

The undirectional differential rate takes the form

$$
\langle \frac{dR}{du} \rangle = \bar{R}T(u)[(1 + \cos \alpha H(u)]
$$

In the above expressions $\bar{R}$ is the rate obtained in the conventional approach by neglecting the folding with the LSP velocity and the momentum transfer dependence of the differential cross section, i.e. by

$$
\bar{R} = \frac{\rho(0)}{m_X} m \frac{m}{Am_N} \sqrt{\langle v^2 \rangle} [\bar{\Sigma}_S + \bar{\Sigma}_{\text{spin}} + \langle \frac{\langle v^2 \rangle}{\langle v^2 \rangle} \bar{\Sigma}_V]
$$

where $\bar{\Sigma}_i, i = S, V, \text{spin}$ have been defined above, see Eqs (14) - (16).

The factor $T(u)$ takes care of the $u$-dependence of the unmodulated differential rate. It is defined so that

$$
\int_{u_{\text{min}}}^{u_{\text{max}}} du T(u) = 1.
$$
i.e. it is the relative differential rate. $u_{\text{min}}$ is determined by the energy cutoff due to the performance of the detector. $u_{\text{max}}$ is determined by the escape velocity $v_{\text{esc}}$ via the relations:

$$u_{\text{max}} = \frac{y_{\text{esc}}^2}{a^2}$$

(63)

On the other hand $H(u)$ gives the energy transfer dependent modulation amplitude. The quantity $t$ takes care of the modification of the total rate due to the nuclear form factor and the folding with the LSP velocity distribution. Since the functions $\tilde{F}_0(u), \tilde{F}_1$ and $\tilde{F}_{\text{spin}}$ in principle have a different dependence on $u$, the functions $T(u), H(u)$ and $t$ in principle depend on the SUSY parameters. If, however, we ignore the small vector contribution and assume (i) the scalar and axial (spin) dependence on $u$ is the same or (ii) only one mechanism (S, V, spin) dominates the, parameter $\bar{R}$ contains the dependence on all SUSY parameters. The other factors depend only on the LSP mass and the nuclear parameters. More specifically considering only the scalar interaction we get $\bar{R} \rightarrow \bar{R}_{S}$ and

$$t \, T(u) = a^2 \tilde{F}^2(u) \tilde{\psi}_{(0),0}(a \sqrt{u})$$

(64)

For the spin interaction we get a similar expression except that $\bar{R} \rightarrow \bar{R}_{\text{spin}}$ and $\tilde{F}^2 \rightarrow \tilde{F}_{\rho,\rho'}$. Finally for completeness we will consider the less important vector contribution. We get $\bar{R} \rightarrow \bar{R}_{V}$ and

$$t \, T(u) = F^2(u)[\tilde{\psi}_{(0),1}(a \sqrt{u}) - \frac{1}{(2\mu b)^2} \frac{2 \eta + 1}{(1 + \eta)^2} u \, \tilde{\psi}_{(0),0}(a \sqrt{u})] \frac{2 a^2}{3}$$

(65)

The quantity $T(u)$ depends on nuclear physics through the form factors or the spin response functions and the parameter $a$. The modulation amplitude takes the form

$$H(u) = 0.133 \frac{\tilde{\psi}_{(1),k}(a \sqrt{u})}{\tilde{\psi}_{(0),k}(a \sqrt{u})}, \, l = 1, 3$$

(66)

Thus in this case the $H(u)$ depends only on $a \sqrt{u}$, which coincides with the parameter $x$ of Ref. 19, i.e. only on the momentum transfer, the reduced mass and the size of the nucleus.

Returning to the differential rate it is sometimes convenient to use the quantity $T(u)H(u)$ rather than $H$, since $H(u)$ may appear artificially increasing function of $u$ due to the decrease of $T(u)$ (in obtaining $H(u)$ we have divided $T(u)$)

Before concluding this subsection we should mention that the above angular integrations can also be done even if the velocity distribution is triaxial. We will not explore this further since one has too many parameters.


4.2 Expressions for the Directional Differential Event Rate

The mean value of the directional differential event rate of Eq. (23), is defined by

\[
\langle \frac{dR}{du} \rangle_{\text{dir}} = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} \int f(v, v_E) v_e \frac{d\sigma(u, v)}{du} d^3v
\]

(67)

where \(\hat{e}\) is the unit vector in the direction of observation. It can be more conveniently expressed as

\[
\langle \frac{dR}{du} \rangle_{\text{dir}} = \frac{\rho(0)}{m_\chi} \frac{m}{A m_N} \sqrt{\langle v^2 \rangle} \langle \frac{d\Sigma}{du} \rangle_{\text{dir}}
\]

(68)

where

\[
\langle \frac{d\Sigma}{du} \rangle_{\text{dir}} = \int \frac{v_e}{\sqrt{\langle v^2 \rangle}} f(v, v_E) \frac{d\sigma(u, v)}{du} d^3v
\]

(69)

Working as in the previous subsection, i.e by expanding in powers of \(\delta\) and keeping terms up to linear in it we can manage to perform the \(\phi\) integration using standard contour integral techniques and express the result in terms of the two modified Bessel functions \(I_m(\frac{\lambda y^2}{2}(1 - t^2))\) with \(t = \cos \theta\) and \(m=0,1\). Thus the angular integration of Eq. (32) yields

\[
M_i(\lambda, y) = 2\pi \times \exp[-(y^2 + 1)(1 + \lambda)] \Lambda_i(\lambda, y) - \exp[-(y_{\text{esc}}^2 + \lambda y^2)] \tilde{\Lambda}_i(\lambda, y), i = 1, 4
\]

(70)

where \(\Lambda_i, \tilde{\Lambda} - i\) come from \(f_3, f_4\) respectively and are given by

\[
\Lambda_1(\lambda, y) = \int_{-1}^{1} dt \exp[-((\lambda/2)y^2(1-t^2)+2(\lambda+1)yt)] t I_0((\lambda/2)y^2(1-t^2))
\]

(71)

\[
\Lambda_2(\lambda, y) = \int_{-1}^{1} dt \exp[-((\lambda/2)y^2(1-t^2)+2(\lambda+1)yt)] t^2 I_0((\lambda/2)y^2(1-t^2))
\]

(72)

\[
\Lambda_3(\lambda, y) = \int_{-1}^{1} dt \exp[-((\lambda/2)y^2(1-t^2)+2(\lambda+1)yt)] (1-t^2) I_0((\lambda/2)y^2(1-t^2))
\]

(73)

\[
\Lambda_4(\lambda, y) = \int_{-1}^{1} dt \exp[-((\lambda/2)y^2(1-t^2)+2(\lambda+1)yt)] (1-t^2) I_1((\lambda/2)y^2(1-t^2))
\]

(74)

and analogous expressions for \(\Lambda'\) with the mere replacement of the expression \(\lambda + 1\) by \(\lambda\). Again in the above expressions \(y=(v/v_0)\) (not to be confused
with the y-coordinate). The above integrals can be expressed in terms of hypergeometric functions as follows

$$\Lambda_1(\lambda, y) = -\sum_{k=0}^{\infty} \frac{2(2(\lambda + 1)y)^{(2k+1)}}{(2k + 3)((2k + 1)!)} F_1^1\left(\frac{1}{2}, \frac{2k + 5}{2}; \lambda y^2\right)$$  \hspace{1cm} (75)

$$\Lambda_2(\lambda, y) = (\lambda + 1) \sum_{k=0}^{\infty} \frac{2(2(\lambda + 1)y)^{(2k)}}{(2k + 3)((2k)!)} F_1^1\left(\frac{1}{2}, \frac{2k + 5}{2}; \lambda y^2\right)$$  \hspace{1cm} (76)

$$\Lambda_3(\lambda, y) = (\lambda + 1) \sum_{k=0}^{\infty} \frac{2(2(\lambda + 1)y)^{(2k)}}{(2k + 3)((2k)!)} F_2^2\left(\frac{1}{2}, 2, 1, \frac{2k + 5}{2}; \lambda y^2\right)$$  \hspace{1cm} (77)

$$\Lambda_4(\lambda, y) = (\lambda + 1) \lambda y^2 \sum_{k=0}^{\infty} \frac{2(2(\lambda + 1)y)^{(2k)}}{(2k + 3)(2k + 5)((2k)!)} F_1^1\left(\frac{3}{2}, \frac{2k + 7}{2}; \lambda y^2\right)$$  \hspace{1cm} (78)

$$\Lambda'_1(\lambda, y) = -\sum_{k=0}^{\infty} \frac{2(2\lambda y)^{(2k+1)}}{(2k + 3)((2k + 1)!)} F_1^1\left(\frac{1}{2}, \frac{2k + 5}{2}; \lambda y^2\right)$$  \hspace{1cm} (79)

$$\Lambda'_2(\lambda, y) = \lambda \sum_{k=0}^{\infty} \frac{2(2\lambda y)^{(2k)}}{(2k + 3)((2k)!)} F_1^1\left(\frac{1}{2}, \frac{2k + 5}{2}; \lambda y^2\right)$$  \hspace{1cm} (80)

$$\Lambda'_3(\lambda, y) = \lambda \sum_{k=0}^{\infty} \frac{2(2\lambda y)^{(2k)}}{(2k + 3)(2k + 5)((2k)!)} F_2^2\left(\frac{1}{2}, 2, 1, \frac{2k + 5}{2}; \lambda y^2\right)$$  \hspace{1cm} (81)

$$\Lambda'_4(\lambda, y) = (\lambda y)^2 \sum_{k=0}^{\infty} \frac{2(2\lambda y)^{(2k)}}{(2k + 1)(2k + 3)(2k + 5)((2k)!)} F_1^1\left(\frac{3}{2}, \frac{2k + 7}{2}; \lambda y^2\right)$$  \hspace{1cm} (82)

It is more convenient to define the functions $F_i$ and $G_i$, $i = 0, 4$ as follows

$$-2y^2 \Lambda_1(\lambda, y) = F_0(2(\lambda + 1)y) + G_0(\lambda, y)$$  \hspace{1cm} (83)

$$2y^2(\Lambda_1(\lambda, y) + y \Lambda_2(\lambda, y)) = F_1(\lambda, 2(\lambda + 1)) + G_1(\lambda, y)$$  \hspace{1cm} (84)

$$4y^3(\Lambda_3(\lambda, y) - \Lambda_4(\lambda, y)) = F_2(2(\lambda + 1)) + G_2(\lambda, y)$$  \hspace{1cm} (85)

$$4y^3(\Lambda_3(\lambda, y) + \Lambda_4(\lambda, y)) = F_3(2(\lambda + 1)) + G_3(\lambda, y)$$  \hspace{1cm} (86)

The functions $F_i$ are obtained by keeping the leading terms in the expansion of the confluent hypergeometric functions of Eqs. (73)-(78). Thus we find

$$F_i(\chi) = \chi \cosh \chi - \sinh \chi , \quad i = 0, 2, 3$$  \hspace{1cm} (87)
\[ F_1(\lambda, \chi) = 2(1 - \lambda) \left[ \frac{(\lambda + 1)\chi^2}{4(1 - \lambda)} + 1 \right] \sinh \chi - \chi \cosh \chi \]  

(88)

The purely asymmetric quantities \( G_i \) satisfy

\[ G_i(0, y) = 0, \ i = 0, 4 \]  

(89)

They are expressed as rapidly convergent series in \( y \), but they are not going to be given here. Similarly we define the functions \( G' \) via the equations

\[ G'_0(\lambda, y) = -2y^2\Lambda'_1(\lambda, y) \]  

(90)

\[ G'_1(\lambda, y) = 2y^2(\Lambda'_1(\lambda, y) + y\Lambda'_2(\lambda, y)) \]  

(91)

\[ G'_2(\lambda, y) = 4y^3(\Lambda'_3(\lambda, y) - \Lambda'_4(\lambda, y)) \]  

(92)

\[ G'_3(\lambda, y) = 0 \]  

(93)

Thus the folded directional event rate takes the form

\[ \langle \frac{d\Sigma}{du} \rangle_{dir} = \frac{1}{2} a^2 [\bar{\Sigma} S F_0(u) + \langle \nu^2 \rangle c^2 \bar{\Sigma} V F_1(u) + \bar{\Sigma} spin F_{spin}(u)] \]  

(94)

where the \( \bar{\Sigma}_i, i = S, V, spin \) are given by Eqs. (14)-(16). The quantities \( F_0, F_1, F_{spin} \) are now obtained from the corresponding form factors via the equations

\[ F_k(u) = F^2(u) \Psi_k(u) \frac{(1 + k)a^2}{2k + 1}, \ k = 0, 1 \]  

(95)

\[ F_{spin}(u) = F_{11}(u) \Psi_0(u)a^2 \]  

(96)

\[ \Psi_k(u) = \frac{1}{2} \left[ (\psi(0), k(a \sqrt{u}) + 0.135 \cos \alpha \psi(1), k(a \sqrt{u}) e_z \cdot e \right. \]  

\[ - 0.117 \cos \alpha \psi(2), k(a \sqrt{u}) e_y \cdot e + 0.135 \sin \alpha \psi(3), k(a \sqrt{u}) e_x \cdot e \]  

(97)

with

\[ \psi(0), k(x) = N(y_{esc}, \lambda)e^{-\lambda}(e^{-1}\Phi(0), k(x) - e^{\frac{y^2}{y_{esc}}} \Phi'(0), k(x)) \]  

(98)

\[ \Phi(0), k(x) = \frac{2}{\sqrt{6\pi}} \int_x^{y_{esc}} dy y^{k-1} e^{-y^2} (F_1(2y) + G_1(\lambda, y)) \]  

(99)

\[ \Phi'(0), k(x) = \frac{2}{\sqrt{6\pi}} \int_x^{y_{esc}} dy y^{k-1} e^{-y^2} G'_1(\lambda, y) \]  

(100)
If we consider each mode (scalar, spin vector) separately the directional rate takes the form

\[
\langle \frac{dR}{du} \rangle_{\text{dir}} = \frac{\bar{R}}{2} t^0 R^0 [(1 + \cos \alpha H_1(u))e_x \cdot e - \cos \alpha H_2(u)e_y \cdot e + \sin \alpha H_3(u)e_z \cdot e]
\]  

(101)

In other words the non directional differential modulated amplitude is described in terms of the three parameters, \( H_l(u), l=1,2 \) and 3. The unmodulated one is \( R^0(u) \), which is again normalized to unity. It is the relative differential rate, i.e. the differential rate divided by the total rate, in the absence of modulation, i.e. The parameter \( t^0 \) entering Eq. (101) takes care of whatever modifications are needed due to the convolution of the non modulated total rate with the LSP velocity distribution in the presence of the nuclear form factors.

From Eqs. (93) - (101) we see that if we consider each mode separately the differential modulation amplitudes \( H(l) \) take the form

\[
H_l(u) = 0.135 \frac{\psi_k^{(l)}(a\sqrt{u})}{\psi_k^{(0)}(a\sqrt{u})}, \ l = 1,3 \ ; \ H_2(u) = 0.117 \frac{\psi_k^{(2)}(a\sqrt{u})}{\psi_k^{(0)}(a\sqrt{u})}
\]  

(102)

Thus in this case the \( H_l \) depend only on \( a\sqrt{u} \), which coincides with the parameter \( x \) of Ref. 19. We note that in the case \( \lambda = 0 \) we have \( H_2 = 0.117 \) and \( H_3 = 0.135 \), so that there remains This means that, if we neglect the coherent vector contribution, which, as we have mentioned, is small, \( H_l \) essentially depends only on the momentum transfer, the reduced mass and the size of the nucleus.

Returning to the differential rate it is sometimes convenient to use the quantity \( R_l \) rather than \( H_l \) defined by

\[
R_l = R^0 H_l, \ l = 1,2,3.
\]  

(103)

The reason is that \( H_l \), being the ratio of two quantities, may appear superficially large due to the denominator becoming small. once again if one mechanism dominates the parameters \( R_0 \) and \( R_l \) are independent of the particular SUSY model considered, except the LSP mass. In fact we find for the scalar interaction we get \( \bar{R} \to \bar{R}_S \) and

\[
t^0 R^0(u) = a^2 F^2(u) \psi_0^{(0)}(a\sqrt{u})
\]  

(104)

For the spin interaction we get a similar expression except that \( \bar{R} \to \bar{R}_{spin} \) and \( F^2 \to F_{p,p'} \). Finally for completeness we will consider the less important
vector contribution. We get \( \bar{R} \rightarrow \bar{R}_V \) and

\[
\int^0 R^0(u) = F^2(u)[\psi_1^{(0)}(a\sqrt{u}) - \frac{1}{(2\mu_r b)^2} \frac{2\eta + 1}{(1 + \eta)^2} u \psi_0^{(0)}(a\sqrt{u})] \frac{2a^2}{3} \tag{105}
\]

The quantity \( R_0 \) depends on nuclear physics through the form factors or the spin response functions.

Equation (101) deviates from the simple trigonometric expression. The dependence on the phase of the earth is complicated. If we imagine, however, that one can sum up all three directional rates, with the inclusion of \( H_2 \) and \( H_3 \), the maximum does not occur at \( \alpha = 0 \), but at \( \alpha = \alpha_H \) with

\[
\alpha_H = tan^{-1}\left[ \frac{H_3(u)}{H_1(u) + H_2(u)} \right] \tag{106}
\]

and the modulation at this value of the phase of the earth takes the value

\[
H_{max} = [\left( H_1 + H_2 \right)^2 + H_3^2]^{\frac{1}{2}} \tag{107}
\]

There exists one minimum at \( \alpha = \pi \), i.e. around Dec.2 and takes the value

\[
H_{min} = H_2 - H_1 \tag{108}
\]

Whenever \( H_1 > H_2 \) there exist two more minima \( \alpha = \pi/2 \) and \( 3\pi/2 \), equal to \( H_3 \) and two secondary maxima. In all cases considered in this work \( H_3 > H_2 - H_1 \) so that the interesting quantities are given by Eqs (107 - 108). In any case it is useful to know the difference between the maximum and the minimum, which takes the form

\[
H_m = [\left( H_1 + H_2 \right)^2 + H_3^2]^{\frac{1}{2}} - Min(H_1 - H_2, H_3) \tag{109}
\]

5 The Total Modulated Event Rates

Once again we will distinguish two possibilities, namely the directional and the non directional case. Integrating Eq. (101) we obtain for the total undirectional rate

\[
R = \bar{R} t \left[ (1 + h(a, Q_{min}) \cos \alpha) \right] \tag{110}
\]

where \( Q_{min} \) is the energy transfer cutoff imposed by the detector. The modulation of the non-directional total event rate can be described in terms of the parameter \( h \).

The effect of folding with LSP velocity on the total rate is taken into account via the quantity \( t \). The SUSY parameters have been absorbed in
From our discussion in the case of differential rate it is clear that strictly speaking the quantities $t$ and $h$ also depend on the SUSY parameters. They do not depend on them, however, if one considers the scalar, spin etc. modes separately.

Let us now examine the directional rate. Integrating Eq. (95) we obtain

$$R_{\text{dir}} = \bar{R}(t^0/2) \left[ (1 + h_1(a, Q_{\text{min}}) \cos \alpha) e_z \cdot e - h_2(a, Q_{\text{min}}) \cos \alpha e_y \cdot e + h_3(a, Q_{\text{min}}) \sin \alpha e_x \cdot e \right]$$

(111)

Furthermore if we somehow manage to measure the directional rate in all directions we obtain:

$$R_{\text{dir, all}} = \bar{R}(t^0/2) \left[ 1 + h_1(a, Q_{\text{min}}) \cos \alpha \right] + h_2(a, Q_{\text{min}}) |\cos \alpha| + h_3(a, Q_{\text{min}}) |\sin \alpha|$$

(112)

We see that the modulation of the directional total event rate can be described in terms of three parameters $h_l = 1, 2, 3$. In the special case of $\lambda = 0$ we essentially have one parameter, namely $h_1$, since then we have $h_2 = 0.117$ and $h_3 = 0.135$.

The effect of folding with LSP velocity on the total rate is taken into account via the quantity $t^0$. All other SUSY parameters have been absorbed in $\bar{R}$, under the same assumptions discussed above in the case of undirectional rates.

Given the functions $h_l(a, Q_{\text{min}})$ one can plot the expression in Eq. (112) as a function of the phase of the earth $\alpha$. For a gross description one can follow the procedure outlined above making the substitution $H \rightarrow h$. Thus the maximum occurs at $\pm \alpha_h$ with

$$\alpha_h = \tan^{-1}\left[ \frac{h_3(a, Q_{\text{min}})}{h_1(a, Q_{\text{min}}) + h_2(a, Q_{\text{min}})} \right]$$

(113)

The difference between the maximum and the minimum is now given by

$$h_m = \left[ (h_1 + h_2)^2 + h_3^2 \right]^{1/2} - \text{Min}(h_1 - h_2, h_3)$$

(114)

In all cases considered here $h_3 > |h_1 - h_2|$

6 Discussion of the Results

We have calculated the differential as well as the total event rates (directional and non directional) for elastic LSP-nucleus scattering for the target $^{127}I$, including realistic form factors. Only the coherent mode due to the scalar interaction was considered. The spin contribution will appear elsewhere. Special attention was paid to the modulation effect due to the annual motion of
the earth. To this end we included not only the component of the earth’s velocity in the direction of the sun’s motion, as it has been done so far, but all of its components. In addition both spherically symmetric as well only axially symmetric LSP velocity distributions were examined. Furthermore we considered the effects of the detector energy cutoffs, by studying two typical cases $Q_{\text{min}} = 10$ and $20$ KeV both on the modulated and the unmodulated amplitudes. We focused our attention on those aspects which do not depend on the parameters of supersymmetry other than the LSP mass.

The parameter $\bar{R}$, normally calculated in SUSY theories, was not considered in this work. The interested reader is referred to the literature and, in our notation, to our previous work.

6.1 The Undirectional Rates

Let us begin with the total rates, i.e. the quantities $t$ and $h$. In Table 1 we show the dependence of $h$ on the components of the earth’s velocity, in the symmetric case ($\lambda = 0$). We see that the modulation amplitude increases for about 50% when all components of the earth’s velocity are included.

In Table 2 we show how the quantities $t$ and $h$ depend on the detector energy cutoff and the LSP mass for the symmetric case. In tables 3 and 4 we show the same quantities for $\lambda = 0.5$ and $\lambda = 1.0$ respectively. From these tables we see a dramatic increase of the modulation when the realistic axially symmetric velocity distribution is turned on. This means that the modulation amplitude can be exploited by the experimentalists. We further notice that the modulation amplitude increases somewhat with cutoff energy. This is due to the fact that the modulation amplitude decreases less rapidly with the cutoff energy $Q_{\text{min}}$ than the unmodulated amplitude. This effect may be of use to the experimentalists, even though it occurs at the expense of the total rate.

Let us now examine the differential rates, which are described by the functions $T(u)$, $H(u)$ and $T(u) \times H(u)$. These are shown for various LSP masses and $Q_{\text{min}}$ in Fig. 1 ($\lambda = 0.0$), Fig. 2 ($\lambda = 0.5$) and Fig. 3 ($\lambda = 1.0$).
Table 1: The dependence of the modulation amplitude $h$ on the velocity of the earth in the symmetric case $\lambda = 0$ and $Q_{\text{min}} = 0$.

| Velocity | 10     | 30     | 50     | 80     | 100    | 125    | 250    |
|----------|--------|--------|--------|--------|--------|--------|--------|
| $z$-com  | 0.0453 | 0.0320 | 0.0179 | 0.0075 | 0.0041 | 0.0015 | -0.0033|
| all      | 0.0723 | 0.0558 | 0.0383 | 0.0252 | 0.0208 | 0.0173 | 0.0112 |

Table 2: The quantities $t$ and $h$ for $\lambda = 0$ in the case of the target $^{53}I$ for various LSP masses and $Q_{\text{min}}$ in KeV (for definitions see text). Only the scalar contribution is considered.

| Quantity | $Q_{\text{min}}$ | 10     | 30     | 50     | 80     | 100    | 125    | 250    |
|----------|------------------|--------|--------|--------|--------|--------|--------|--------|
| $t$      | 0.0              | 1.599  | 1.134  | 0.765  | 0.491  | 0.399  | 0.328  | 0.198  |
| $h$      | 0.0              | 0.072  | 0.056  | 0.038  | 0.025  | 0.021  | 0.017  | 0.011  |
| $t$      | 10.0             | 0.000  | 0.276  | 0.307  | 0.236  | 0.200  | 0.170  | 0.108  |
| $h$      | 10.0             | 0.000  | 0.276  | 0.307  | 0.236  | 0.200  | 0.170  | 0.108  |
| $t$      | 20.0             | 0.000  | 0.117  | 0.110  | 0.098  | 0.086  | 0.058  | 0.058  |
| $h$      | 20.0             | 0.000  | 0.084  | 0.044  | 0.024  | 0.017  | 0.013  | 0.005  |

remind the reader that the dimensionless quantity $u$ is related to the energy transfer $Q$ via Eq. (20) with $Q_0 = 60KeV$ for $^{127}I$. The curves shown correspond to LSP masses as follows: i) Solid line $\Leftrightarrow m_\chi = 30$ GeV. ii) Dotted line $\Leftrightarrow m_\chi = 50$ GeV. iii) Dashed line $\Leftrightarrow m_\chi = 80$ GeV. v) Intermediate dashed line $\Leftrightarrow m_\chi = 100$ GeV. vi) Fine solid line $\Leftrightarrow m_\chi = 125$ GeV. vi) Long dashed line $\Leftrightarrow m_\chi = 250$ GeV. If some curves of the above list seem to have been omitted, it is understood that they fall on top of vi). Note that, due to our normalization of $T$, the area under the corresponding curve is unity. This normalization was adopted to bring the various graphs on scale since the absolute values may change much faster as a function of the LSP mass.

In order to understand the dependence of the total and differential rates on $\lambda$, we will examine the functions $f(y)$, which are equal to the quantity $N(\lambda, y) \sqrt{6\pi}$ multiplied by the integrand of Eq. (58). The latter crucially depends on the functions $F_i(\lambda, 2(\lambda + 1)y)$ and $G_i(\lambda, y)$, $i = 1, 2$. These functions $f(y)$ are
Figure 1: The quantities $T, H$ and $TH$ entering the undirectional differential rate for $\lambda = 0.0$ and various values of energy cut off in $GeV$. For definitions see text. The energy transfer $Q$ is given by $Q = uQ_0$, $Q_0 = 60K eV$.

($\lambda = 0.0$, $Q_{min} = 00.$)

($\lambda = 0.0$, $Q_{min} = 00.$)

($\lambda = 0.0$, $Q_{min} = 10.$)

($\lambda = 0.0$, $Q_{min} = 10.$)

($\lambda = 0.0$, $Q_{min} = 20.0$)

($\lambda = 0.0$, $Q_{min} = 20.$)

($\lambda = 0.0$, independent of $Q_{min}$)
Figure 2: The same as in the previous figure for $\lambda = 0.5$

$(\lambda = 0.5, Q_{\text{min}} = 0.0)$

$(\lambda = 0.5, Q_{\text{min}} = 10.)$

$(\lambda = 0.5, Q_{\text{min}} = 20.0)$

$(\lambda = 0.5, \text{independent of } Q_{\text{min}})$
Figure 3: The same as in the previous figure for $\lambda = 1.0$, independent of $Q_{min}$
Table 3: The same quantities with table I for $\lambda = 0.5$

| Quantity | $Q_{\text{min}}$ | LSP mass in GeV |
|----------|------------------|-----------------|
|          | 10   | 30   | 50   | 80   | 100  | 125  | 250  |
| $t$      | 1.690| 1.241| 0.861| 0.558| 0.453| 0.372| 0.224|
| $h$      | 0.198| 0.151| 0.107| 0.083| 0.076| 0.071| 0.063|

Table 4: The same quantities with table I for $\lambda = 1.0$

| Quantity | $Q_{\text{min}}$ | LSP mass in GeV |
|----------|------------------|-----------------|
|          | 10   | 30   | 50   | 80   | 100  | 125  | 250  |
| $t$      | 1.729| 1.299| 0.919| 0.600| 0.487| 0.399| 0.240|
| $h$      | 0.314| 0.247| 0.181| 0.141| 0.131| 0.123| 0.112|

shown in Fig. 4 for $\lambda = 0, 1$. We see that in the case of $\lambda = 1.0$ the positive section of the function is enhanced.

6.2 The Directional Rates

Once again we distinguish two cases, the total and the differential rates.

6.2.1 The Directional Total Event Rates

The directional total event rates, which arise by summing the directional rates in all three directions is beyond the goals of the present experiments. We will, however, include it in the present discussion. The unmodulated rates can be can be parameterized in terms of the parameter $t^0$. This describes the modification of the total directional non modulated event rate due to the convolution with the velocity distribution. The modulation is now described by the three parameters $h_1, h_2, h_3$. These are shown in tables 5-7. It is clear
( \( f(y) \leftrightarrow \tilde{F}_1, \tilde{G}_1, \tilde{F}_1 + \tilde{G}_1 \) )

Figure 4: The quantities \( f(y) \) described in the text associated with \( \tilde{F}_i(\lambda, 2(\lambda + 1)y) \) and \( \tilde{F}_i(\lambda, 2(\lambda + 1)y) + \tilde{G}_i(\lambda, y) \), \( i = 1, 2 \). Thick solid line corresponds to \( \tilde{F}_i \), \( i = 1, 2 \), the finest line to \( \tilde{G}_i \), \( i = 1, 2 \) and the dashed line to the sum of the two. The intermediate thickness line corresponds to \( \lambda = 0 \), in which case \( \tilde{G}_i = 0 \), \( i = 1, 2 \).

from Eq. (112) that the modulation of the total rate is no longer given by a simple sinusoidal function. For some interesting cases the situation is shown in Fig. 5.

An idea about what is happening can be given by \( h_m \) and \( \alpha_m \). The first gives the difference between the maximum and the minimum values of modulated amplitude. The second involves the phase shift from the second of June, which is no longer the date of the maximum.

The second of June gives the location of the maximum, when only the component of the earth’s velocity along the sun’s direction of motion is considered or when \( h_3 \) is neglected. In almost all cases considered in this work, however, \( h_3 \) is important and in fact the obtained shift is on the average about \( \pm 35 \) days from the second of June.

From tables 5-7 we see that without detector energy cutoff, \( Q_{min} = 0 \), \( t^0 \) is a decreasing value of the LSP mass. It takes values of about 2 for low LSP mass and is decreased by an order of magnitude as we go to higher LSP masses. This is due to the nuclear form factor effects, which are present but not so severe for this intermediate mass nucleus. It is not a sensitive function of the asymmetry parameter \( \lambda \).

As expected, in the presence of energy cut off, \( t^0 \) is greatly reduced and becomes unobservable for light LSP. As the LSP mass increases \( t^0 \) increases. (see tables 5-7. It reaches a maximum at about 80 GeV and it starts decreasing. But even for the heaviest LSP this reduction is not much larger than 1/3, even for \( Q_{min} = 20KeV \). In other words a heavy LSP can cause sufficient energy transfer to partially compensate for the loss of phase space.

We also see that in all cases \( h_m \) is much larger than \( 2h_1 \), suggesting that when it comes to directional detection all components of the earth’s velocity are important. We also notice that for those LSP masses, which give a detectable
total rate, the modulation amplitude does not appreciably change with the LSP mass. It is not greatly affected by the energy cutoff $Q_{\text{min}}$. It is enhanced, however, by about a factor of two in going from $\lambda = 0$, no asymmetry, to $\lambda = 1$, maximum asymmetry.

From the above discussion it is clear that one needs all three modulation parameters, $h_1, h_2, h_3$. We remind the reader that in Eq (110) the z-axis has been chosen in the direction of the sun's velocity, the y-axis is perpendicular to the plane of the galaxy and the x-axis in the radial (galactocentric) direction. We mention again that $h_2$ and $h_3$ are constant, 0.117 and 0.135 respectively, in the symmetric case. On the other hand $h_1$ and $h_3$ substantially increase in the presence of asymmetry.

The precise value of the directional rate depends on the direction of observation. One can find optimal orientations, but we are not going to elaborate further.

### 6.2.2 The Directional Differential Event Rates

The directional differential rate is also very hard to detect, but perhaps a bit more practical than the total rates described in the previous subsection. It can be in terms of four functions of $u$, namely $R_0(u)$ and $H_i(u), i = 1, 2, 3$.

The situation is rather complicated and following our discussion of the
Table 5: The quantities \( t^0 \), \( h_1 \) and \( h_m \) for \( \lambda = 0 \) in the case of the target \( ^{53}I^{127} \) for various LSP masses and \( Q_{\text{min}} \) in KeV (for definitions see text). Only the scalar contribution is considered. Note that in this case \( h_2 \) and \( h_3 \) are constants equal to 0.117 and 0.135 respectively.

| Quantity | \( Q_{\text{min}} \) | 10 | 30 | 50 | 80 | 100 | 125 | 250 |
|----------|----------------------|----|----|----|----|-----|-----|-----|
| \( t^0 \) | 0.0                  | 1.960 | 1.355 | 0.886 | 0.552 | 0.442 | 0.360 | 0.212 |
| \( h_1 \) | 0.0                  | 0.059 | 0.048 | 0.037 | 0.029 | 0.027 | 0.025 | 0.023 |
| \( h_m \) | 0.0                  | 0.164 | 0.144 | 0.124 | 0.111 | 0.107 | 0.104 | 0.100 |
| \( t^0 \) | 10.0                | 0.000 | 0.365 | 0.383 | 0.280 | 0.233 | 0.194 | 0.119 |
| \( h_1 \) | 10.0                | 0.000 | 0.086 | 0.054 | 0.038 | 0.033 | 0.030 | 0.025 |
| \( h_m \) | 10.0                | 0.000 | 0.214 | 0.155 | 0.127 | 0.119 | 0.113 | 0.104 |
| \( t^0 \) | 20.0                | 0.000 | 0.080 | 0.153 | 0.136 | 0.116 | 0.102 | 0.065 |
| \( h_1 \) | 20.0                | 0.000 | 0.123 | 0.073 | 0.048 | 0.041 | 0.036 | 0.028 |
| \( h_m \) | 20.0                | 0.000 | 0.282 | 0.190 | 0.145 | 0.132 | 0.123 | 0.109 |

previous section we will give gross description of the modulation using the functions \( a_H(u) \) and \( H_m(u) \). The phase shift \( \alpha_H \) has been found to be a constant and about 0.7, which corresponds to a shift about \( \pm 35 \) days from June the second. Since \( H_m \) is defined as the ratio of two quantities, it can appear large because the denominator (non modulated rate) becomes small. Thus, following the strategy of the previous subsection we also present the quantity \( R_m = R_0 H_m \). These functions are shown in Fig. 6 for LSP masses in the range 30-250 GeV, \( \lambda = 0 \) and \( Q_{\text{min}} = 0, 10\text{KeV} \) and \( 20\text{KeV} \). Note that the quantity \( H_m \) is itself independent of the cutoff except that one should look at the \( u \) relevant to the allowed energy transfer interval.

The curves shown correspond to LSP masses as in the undirectional case. Again due to the normalization of \( R_0 \), the area under the corresponding curve is unity.

The above quantities for \( \lambda = 0.5 \) and 1.0 are shown in Fig. 7, but only for \( Q_{\text{min}}=0 \). Their dependence on the energy transfer cutoff shows behavior similar to that of Fig. 1. In any case for \( Q_{\text{min}}=10, 20\text{KeV} \), the functions \( R_0 \) and \( R_m \) show a behavior similar to that for \( Q_{\text{min}} = 0 \), except that they start from higher energy transfer. We should remind the reader, however, that that in all cases \( R_0 \) represents the relative differential rate, i.e. it is normalized so that the area under the corresponding curve is unity for all LSP masses. One, therefore, should take into account the factor \( t^0 \) of tables 5-7.
Figure 6: The relative differential event rate $R_0$ and the amplitudes for modulation $R_m$ and $H_m$ vs $u$ for the target $^{53}I^{127}$ in the case of symmetric velocity distribution, $\lambda = 0$ (for the definitions see text).
Figure 7: The same as in Fig.2 in the asymmetric case ($\lambda = 0.5$ and $\lambda = 1.0$). Only the case $Q_{\text{min}} = 0$ is exhibited.
Table 6: The same as in the previous table, but for the value of the asymmetry parameter $\lambda = 0.5$.

| Quantity | $Q_{\min}$ | 10   | 30   | 50   | 80   | 100  | 125  | 250  |
|----------|------------|------|------|------|------|------|------|------|
| $i^0$    | 0.0        | 2.309| 1.682| 1.153| 0.737| 0.595| 0.485| 0.288|
| $h_1$    | 0.0        | 0.138| 0.128| 0.117| 0.108| 0.105| 0.103| 0.100|
| $h_2$    | 0.0        | 0.139| 0.137| 0.135| 0.133| 0.133| 0.133| 0.132|
| $h_3$    | 0.0        | 0.175| 0.171| 0.167| 0.165| 0.163| 0.162| 0.162|
| $h_m$    | 0.0        | 0.327| 0.307| 0.284| 0.266| 0.261| 0.257| 0.250|

From the plots shown one can see that $R_m$ can be quite large, about 20% of $R_0$, but it falls slower as a function of the energy transfer. For this reason the modulation amplitude $H_m$ is increasing as a function of $u$. It is interesting to note that the modulation amplitude $H_m$ is increased by more than a factor of two, as the asymmetry parameter $\lambda$ changes from zero to one, for all energy transfers. Thus, even at zero energy transfer, for $\lambda = 1$ the variation in the amplitude due to the earth’s motion can increase by about 40% between the minimum (around December 2) and the maximum (around July 10 or the end of May), a big effect indeed. It can become even larger if one can restrict oneself to only part of the phase space, i.e. if one is satisfied with fewer counts.

In the case of the directional differential rate one clearly needs, in addition to $R_0$, the functions $H_l(u)$, $l=1,2,3$, which are plotted in Fig.4. In the case of $\lambda = 0$ only $H_1$ is plotted, since the other two are in this case constant ($H_2 = 0.117$, $H_3 = 0.135$). We see that in the presence of asymmetry, e.g. $\lambda = 0.5$ and 1.0, all functions, but especially $H_1$ and $H_3$, are substantially increased.
Figure 8: The same as in Fig.3 for the quantities $H_1$, $H_2$ and $H_3$. These quantities do not depend on $Q_{\text{min}}$, except for the fact one should look at $u > u_{\text{min}}$. 

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Table 7: The same as in the previous, but for the value of the asymmetry parameter $\lambda = 1.0$.

| Quantity | $Q_{min}$ | LSP mass in GeV |
|----------|-----------|-----------------|
|          |           | 10   | 30   | 50   | 80   | 100  | 125  | 250  |
| $\rho^0$ | 0.0       | 2.429| 1.825| 1.290| 0.837| 0.678| 0.554| 0.330|
| $h_1$    | 0.0       | 0.192| 0.182| 0.170| 0.159| 0.156| 0.154| 0.150|
| $h_2$    | 0.0       | 0.146| 0.144| 0.141| 0.139| 0.139| 0.138| 0.138|
| $h_3$    | 0.0       | 0.232| 0.222| 0.211| 0.204| 0.202| 0.200| 0.198|
| $h_m$    | 0.0       | 0.456| 0.432| 0.404| 0.382| 0.375| 0.379| 0.361|
| $\rho^0$ | 10.0      | 0.000| 0.354| 0.502| 0.410| 0.349| 0.295| 0.184|
| $h_1$    | 10.0      | 0.000| 0.241| 0.197| 0.174| 0.167| 0.162| 0.154|
| $h_2$    | 10.0      | 0.000| 0.157| 0.146| 0.142| 0.140| 0.140| 0.138|
| $h_3$    | 10.0      | 0.000| 0.273| 0.231| 0.213| 0.208| 0.205| 0.200|
| $h_m$    | 10.0      | 0.000| 0.565| 0.464| 0.413| 0.398| 0.387| 0.370|
| $\rho^0$ | 20.0      | 0.000| 0.047| 0.169| 0.186| 0.170| 0.150| 0.100|
| $h_1$    | 20.0      | 0.000| 0.297| 0.226| 0.190| 0.179| 0.172| 0.159|
| $h_2$    | 20.0      | 0.000| 0.177| 0.153| 0.144| 0.142| 0.141| 0.139|
| $h_3$    | 20.0      | 0.000| 0.349| 0.256| 0.224| 0.216| 0.211| 0.203|
| $h_m$    | 20.0      | 0.000| 0.709| 0.550| 0.448| 0.424| 0.408| 0.380|

7 Conclusions

In the present paper we have expanded the the results obtained in of our recent letter. We have calculated all the parameters, which can describe the modulation of the direct detection rate for supersymmetric dark matter. The differential as well as the total event rates were obtained both for the non directional as well as directional experiments. All components of the earth’s velocity were taken into account, not just its component along the sun’s direction of motion. Realistic axially symmetric velocity distributions, with enhanced dispersion in the galactocentric direction, were considered. The obtained results were compared to the up to now employed Maxwell-Boltzmann distribution.

We presented our results in a suitable fashion so that they do not depend on parameters of supersymmetry other than the the LSP mass. Strickly speaking the obtained results describe the coherent process in the case of $^{127}I$, but we do not expect large changes, if the axial current is considered. Recall that the dependence on supersymmetry is contained in the parameter $\hat{R}$ not discussed in the present paper. The nuclear form factor was taken into account and the effects of the detector energy cut off were also considered.
Our results, in particular the parameters $t$ and $t_0$ (see tables 2-4 and 5-8) indicate that for large reduced mass, the advantage of $\mu_r$ (see Eqs. (14)-(16)) is lost when the nuclear form factor and the convolution with the velocity distribution are taken into account.

In the case of the undirectional total event rates we find that in the symmetric case the modulation amplitude for zero energy cutoff is less than 0.07. It gets substantially increased in the case of asymmetric velocity distribution with largest asymmetry ($\lambda = 1$). It can reach values up to 0.31. In the presence of the detector energy cutoff it can increase even further up to 0.46, but this occurs at the expense of the total number of counts. The modulation amplitude in the case of the differential rate is shifted by the asymmetry at higher energy transfers and, for maximum asymmetry $\lambda = 1$, gets about doubled compared to the symmetric case ($\lambda = 0$). This amplitude does not depend on the energy cutoff, but the lower energy transfers, will, of course, be excluded if such a cutoff exists.

Analogous conclusions can be drawn about the directional differential event rate. The presence of asymmetry more than triples the differential modulation amplitude (from about 10% to about 35%). There exist now regions of the energy transfer such that the modulation amplitude can become as large as 50%.

Finally it is important that one should consider all components of the Earth's motion, not just its velocity along the sun's motion, especially if the directional signals are to be measured.

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