Dynamical assessment of altitudinal trend of drop size distributions in a tropical region of Nigeria.

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Abstract: A comparative assessment of different statistical drop size distributions in a tropical location in Nigeria is presented in this paper. The wet season months of the year 2014 rain data, measured using vertically-pointing Micro Rain Radar (MRR) installed at the Department of Physics, The Federal University of Technology Akure, Nigeria was considered. The data were archived at the Communication Research Group, Department of Physics, The Federal University of Technology Akure. The drop sizes were estimated from the fall velocity. The stratiform rainfall type was considered in this study due to its preponderance of occurrence. Three heights (160 m, 1440 m, and 2240 m) were reviewed for this study. These heights were given due consideration because of the density of the data. All the statistical distributions considered which include Lognormal, Gamma, and Weibull distributions performed better at the 160 m height compared to other heights. It was however observed that the Gamma and Weibull distribution performed better than the Lognormal distribution at this height. The results obtained in this study will be fundamental for the application of remote sensing and the propagation field in this area. The results will be useful to radio communication engineers to properly design a good fade margin in a radio communication system in this region.

Keywords: Drop size distribution; Tropical region; MRR; Height, Fall velocity

1. Introduction

Radio engineers and researchers have recently requested more consideration on drop size distribution (DSD) in the past years. One of the reasons could be its shape of distribution [1]. Considering signal transmission for frequency greater than 10 GHz, raindrop size distribution (DSD) is a fundamental factor that causes attenuation [2]. The DSD and rain rate distribution are the major parameters for the evaluation of rainfall attenuation centimetric and millimetric wavebands. Although several DSD models have been proposed and reported in the literature [3-7], many of these models in use in different regions have been employed for rain scattering and propagation. The familiar ones are the Laws and Parsons, Marshall and Palmer, Joss et al, Weibull, Lognormal, Gamma Distribution. Of all, the Lognormal, the Gamma distribution, and Weibull distributions have been known to perform better in the tropical region [8]. This paper compared these statistical distributions with height to assess the height at which these statistical distributions would perform best. Below are the equations of the statistical distributions considered in this study.

1.1 Lognormal distribution

\[ N(D) = \frac{N}{\sigma \sqrt{2\pi}} \exp\left[\frac{-\ln(D)^2}{2\sigma^2}\right] \quad (m^{-1} \text{mm}^{-1}) \]  

(1)

where \( \mu \) is the mean of \( \ln(D) \), \( \sigma \) is the standard deviation which determines the width of the distribution, and \( N \) (concentration of rainfall drops) is a function of climates, geographical location of measurement, and rainfall type.
1.2 Weibull distribution

\[ N(D) = N_0 \frac{D^\eta}{\gamma_0} \exp \left( -\frac{D}{\gamma_0} \right) \]

Where \( D \) is the diameter in millimeters and \( \eta \) and \( \gamma \) are functions of the precipitation rate \( R \) in millimeters per hour.

1.3 Gamma distribution

\[ N(D) = N_0 D^\mu e^{-\lambda D} \]

Where \( N(D) \) is the number of drops per unit volume per drop diameter interval in m\(^{-3}\) mm\(^{-1}\).

\( N_0 \) is the intercept parameter of the distribution m\(^{-3}\) mm\(^{-1}\), \( D \) is diameter of rain drop in mm, \( \lambda \) is the slope parameter in mm\(^{-1}\), \( \mu \) is the shape of the Drop Size Distribution [8].

2. Data Collection and Analysis

Data for rain season months for the year 2014 were collected from the Communication Physics Research Group of the Department of Physics, the Federal University of Technology Akure (FUTA). The data collected was measured using Micro Rain Radar. The parameters needed for this study which are the rain rate (RR), fall velocity (W) and Number of drops (N) were sorted out.

The classification of rain DSD into different rain regimes was done using the rain rate parameter. The classification was following [14], as stratiform \((0 < RR < 10)\) and convective rain type \((RR \geq 10)\). It is to be noted that the stratiform rain type was only considered for this study due to the numerous data points that occurred with this rain type.

However, due to the lack of physical measurement of the drop diameters, an estimation approach presented by Roberto et al (2015) was used. This was calculated from the drop velocities \( v(D) \) as;

\[ v(D) = (9.65 - 10.3 \exp(-0.6D))dv(h)(m/s) \]

For \( 0.109 \leq D \leq 6 \) mm

Making \( D \) subject of the formula, we have,

\[ D(mm) = \frac{1}{6} \ln \left( \frac{10.3}{9.65 - \left( \frac{v(D)(\frac{m}{s})}{dv(h)(m)} \right)} \right) \]

The correction factor due to change in air density, \( dv(h) \), is approximated as a second-order polynomial [9] given in (7) as:

\[ dv(h) = [1 + 3.68 \times 10^{-5}h + 1.71 \times 10^{-7}h^2] \]

It is worthy to note that from [10], measurement at height closer to the zero-degree isotherm height could be erroneous due to the change of phase water at this region. The study carried out by[11] at the same location with the same instrument showed that the zero-degree isotherm height for the region considered is above 4800 m height. To be on the safer side, our analysis limits the height considered to 3200 m. Chances of attenuation of the electromagnetic radiation during higher rain rates and at higher altitudes are possible. But stable algorithms have been used to correct the error due to such attenuation while deriving rain DSD [12]. This work however seeks to compare the different statistical distributions in the region.

3. Results and discussion

The occurrences of rain at all heights for the wet season months are presented in Figure 1. This was presented to estimate the heights with the highest number of rain occurrences. From figure 1 presented, heights 160m, 1440 m, and 2240 m have the highest number of rain occurrences. Their values are presented in Table 1. Different statistical distributions of Lognormal, Weibull and gamma distributions against the drop sizes were assessed for the heights considered. These distributions were considered due to their peculiarity to the tropical region.
From the results presented in Tables 1-3, it is observed that the estimated parameters of all the statistical distribution assessed, performed better at the lowest bin height which is at the height of 160m. These estimated parameters were computed using the method of moment on XLSTAT. XLSTAT is an advanced statistical feature on excel which is used for preparing data, take decisions and predict. The method of moment has been known to fit proportionally to the moment of integral rain parameters [1]. It is worth noting that the parameters obtained at the lowest bin height for gamma distribution are consistent with those found by [13]. As can be seen from the statistics indicators in Table 3, low positive skewness values have been observed for all DSD parameters for height 160 m, except for the Lognormal distribution which indicates a high positive skewed value. For Weibull and Gamma distributions, the low positive skewed values show that most of the parameter values are distorted to the left of the mean values with more variance for the Lognormal distribution tend to be distributed to the left of the mean values. The reasonable kurtosis values of the gamma distribution and Weibull distribution model parameters show that its datasets were nearly normally distributed. However leptokurtic values observed in the lognormal distribution, overestimated the datasets indicating profusion of outliers.

Table 2: Estimated parameters for Weibull distribution

| Parameter | Height (160 m) Value | Standard Error | Height (1440 m) Value | Standard Error | Height (2240 m) Value | Standard Error |
|-----------|---------------------|----------------|----------------------|----------------|----------------------|----------------|
| Beta      | 2.300               | 0.00           | 1.109                | 0.010          | 1.331                | 0.003          |
| Gamma     | 2.405               | 0.00           | 1.034                | 0.052          | 1.075                | 0.009          |
| (Miu)     | 0.045               | 0.00           | 0.061                | 0.011          | 0.058                | 0.003          |

Table 3: Statistics estimated on the input data and computed using the estimated parameters of the Weibull (3) distribution:

| Statistic | Height (160m) Data | Parameters | Height (1440 m) Data | Parameters | Height (2240 m) Data | Parameters |
|-----------|--------------------|-----------|----------------------|-----------|----------------------|-----------|
| Mean      | 2.185              | 1.055     | 1.055                | 1.055     | 1.055                | 1.055     |
Variance 1.092 0.965 0.945 0.806 0.945 0.945
Skewness (Pearson) 0.039 0.455 1.194 1.712 1.194 1.843
Kurtosis (Pearson) 0.331 -0.034 1.303 4.237 1.303 5.093

Table 4: Estimated parameters for Lognormal distribution

| Parameter | Height (160 m) | Value | Standard error | Height (1440 m) | Value | Standard error | Height (2240 m) | Value | Standard error |
|-----------|----------------|-------|----------------|----------------|-------|----------------|----------------|-------|----------------|
| μ         | 0.546          | 0.24  | 0.054          | 0.492          | 0.054 | -0.463         | 0.054          | 1.207 |
| Sigma     | 0.897          | 0.017 | 0.038          | 1.162          | 0.038 | 1.140          | 0.040          | 3.889 |

Table 5: Statistics estimated on the input data and computed using the estimated parameters of the Lognormal distribution:

| Statistic | Height (160 m) | Data | Parameters |
|-----------|----------------|------|------------|
| Mean      | 2.185          | 2.582 | 1.055      | 1.201 | 1.046 | 1.207 |
| Variance  | 1.092          | 8.251 | 0.945      | 4.178 | 0.777 | 3.889 |
| Skewness  (Pearson) | 0.039 | 4.714 | 1.194      | 9.893 | 0.816 | 9.269 |
| Kurtosis  (Pearson) | 0.331 | 56.480 | 1.303      | 373.983 | 0.370 | 315.043 |

Table 6: Estimated parameters for Gamma distribution

| Parameter | Height (160 m) | Value | Standard error | Height (1440 m) | Value | Standard error | Height (2240 m) | Value | Standard error |
|-----------|----------------|-------|----------------|----------------|-------|----------------|----------------|-------|----------------|
| K         | 4.372          | 0.164 | 0.070          | 1.409          | 0.087 | 9.269          |
| Beta      | 0.500          | 0.020 | 0.067          | 0.743          | 0.055 | 3.889          |

Table 7: Statistics estimated on the input data and computed using the estimated parameters of the Gamma (2) distribution:

| Statistic | Height (160 m) | Data | Parameters |
|-----------|----------------|------|------------|
| Mean      | 2.185          | 2.185 | 1.055      | 2.185 | 2.185 |
| Variance  | 1.092          | 1.092 | 0.945      | 1.092 | 1.092 |
| Skewness  (Pearson) | 0.039 | 0.957 | 1.194      | 0.957 | 0.039 |
| Kurtosis  (Pearson) | 0.331 | 1.372 | 1.303      | 5.093 | 0.331 | 1.372 |
Figure 2: Probability density function against drop sizes for (a) Lognormal fit (b) Weibull (c) Gamma at height 160 m

Figure 3: Probability density function against drop sizes for (a) Lognormal fit (b) Weibull (c) Gamma at height 1440 m

Figure 4: Probability density function against drop sizes for (a) Lognormal fit (b) Weibull (c) Gamma at height 2240 m

4. Conclusion

In this paper, rainfall measurements during the wet season of the year 2014 have been analyzed to compare the statistical distribution with respect to different heights that best fits the data. The height that performs best considering all the statistical distribution assessed is the 160 m height. The three statistical distributions considered performed well at this height with Weibull distribution performing better. The
results however could be fundamental regarding the application of remote sensing and the propagation field in this region.

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