Reliability-based design optimization of steel frames using direct design

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Abstract. This paper presents an effective method for reliability-based design optimization (RBDO) of steel frames by combining direct design using nonlinear inelastic analysis, an improved importance sampling technique for structural failure probability analysis, and an improved differential evolution (DE) algorithm for optimization. The nonlinear inelastic analysis using the beam-column approach is used to capture the second-order effects and the inelastic behavior of structures. An improved importance sampling technique based on nonlinear inelastic analysis of the structure is employed that significantly reduces the number of structural analyses required for calculating the structural failure probability. An improved DE algorithm, which can effectively eliminate the redundant structural analyses for objective function evaluation, is utilized. A six-story space frame is studied to demonstrate the computational efficiency of the proposed method.

1. Introduction
The development of probabilistic analysis and structural optimization methods during the past decades has been attracting the interest of the reliability-based design optimization (RBDO). This concern comes from the fact that the RBDO approach allows creating a “good” design with the balance between the total cost and the reliability of the structure. The RBDO of space steel frames is often a discrete optimization in which the database of element sections is given by institutes and/or steel factories and the constraints include the deterministic and probabilistic corresponding to conditions of the strength and serviceability load combinations and the structural failure probability. In light of this, RBDO is a highly nonlinear optimization problem, so the meta-heuristic algorithms are often used to solve it. Some efficient meta-heuristic algorithms for structural optimization are harmony search (HS) \cite{1}, differential evolution (DE) \cite{2}, genetic algorithm \cite{3}, etc. Unfortunately, when using a meta-heuristic optimization algorithm, both the probabilistic and optimization processes require a large number of structural analyses to evaluate the constraints, so the computational time is often excessive. To overcome this limitation, an effective RBDO procedure for nonlinear inelastic steel frames should include: (1) a fast and accurate analysis scheme of finite element (FE) model, (2) an effective reliability analysis method which requires less FE model calls, and (3) an appropriate computational implementation.

The RBDO of space steel frames is often solved by using the double-loop method, namely the outer loop for the deterministic optimization and the inner loop for the structural failure probability
analysis [4-5]. From the fact that time-consuming of the inner-loop exceeds 80% of total computational time, most methods for solving RBDO problems seek to develop an effective structural failure probability analysis method, which can be categorized into three method groups: (1) design point (DP) – based, (2) approximate integration, and (3) simulation. DP – based approaches such as the first and the second-order reliability methods estimate structural reliability using the limit state function at design points and the mean and variance values of the random variables [6], while the approximate integration methods allow estimating the probability density function directly through numerical integration [7]. Design point (DP) – based and approximate integration methods can significantly save computational efforts but the error of their results is quite large especially for reliability analysis problems with high nonlinearity. In contrast, simulation approaches like the Monte Carlo simulation (MCS) are very simple to use and highly accurate for reliability analysis of nonlinear large-scale systems [8]. However, due to a great number of samples required, simulation methods often spend the excessive computational time so they are rarely used for practical designs. To overcome this limitation, the development of an effective variance reduction technique that could significantly reduce the number of analyses, the coefficient of variation (COV), and computational time is necessary.

In the current study, an effective procedure for RBDO of steel frames is developed using nonlinear inelastic analysis, an importance sampling (IS) technique of MCS, and the DE method. The objective function of the RBDO problem is the total structural weight, and both deterministic (load-carrying capacity and deflection conditions) and probabilistic (the failure probability of the whole frame) constraints are considered. A direct analysis using the beam-column approach is used to predict the structural nonlinear inelastic behaviors. An effective IS technique is proposed to reduce the number of FE analyses and the COV of the failure probability. The DE algorithm, which in most cases is more robust and captures better global optimum designs, is utilized. To evaluate the proposed program, a six-story space frame is studied.

2. Direct design of steel frames
Most methods considering the nonlinear inelastic analysis of steel frame structures presented in literature can be classified into two categories: finite element and beam-column types. Interpolation functions and plastic zone models are used in finite element methods to capture second-order effects and nonlinear behaviors spreading along with structural elements, respectively [9-10]. Although considered as ‘accurate’ techniques, finite element methods are not widely used for practical design purposes because of their excessive computational times. In the beam-column method, stability functions are employed to estimate second-order effects of a structure, while a refined plastic hinge model, where a structural element is modeled as a linear elastic element with two plastic hinges at the ends, is used to predict structural inelastic behaviors [1-5]. The advantage of this method is that the modeling of a member requires only one or two elements, and hence, the computation cost is considerably saved. In light of this, in this study, steel frames are analyzed using a beam-column method. The columns and beams are simulated as plastic-hinge beam-column elements [11]. The inelastic behaviors are carried out by using the stability functions and the Column Research Council (CRC) concept [12-13]. The partial plastification and transverse shear deformation effects are captured by using the gradual stiffness degradation model [14] and the shear deformation stiffness matrix [15], respectively. Details on this approach are provided in [11].

3. Formulation of RBDO of steel frames using direct design
Generally, RBDO of steel frames using nonlinear inelastic analysis can be formulated as follows:
Minimize \( W(X,Y) = \sum_{i=1}^{nm} \alpha_i A_i L_i \)

Subjected to

\[
1 - \frac{R_j}{S_j} \leq 0 \quad j = 1, \ldots, nstr
\]

\[
\left| \frac{d_{kl}}{d_{k,l}^n} \right| - 1 \leq 0 \quad k = 1, \ldots, nser; l = 1, \ldots, nn
\]

\[
\frac{P_{t,j}(X)}{P_{t,a}} - 1 \leq 0 \quad t = 1, \ldots, npro
\]

\( A_i \subset S_i \)

where \( i, j, k, \) and \( t \) are denoted for the ordinal number of the element, strength load combination, serviceability load combination and load combination for failure probabilistic evaluation, respectively; \( \rho, A \) and \( L \) are the material density, cross-sectional area, and length of the element; \( S \) is the list of cross-sectional areas used for the element; \( Y = (y_1, y_2, \ldots, y_n) \) is the vector of design variables which are the integer values representing the sequence numbers of the cross-section types used for the beams and column in the variable space; \( X = (x_1, x_2, \ldots, x_n) \) is the vector of random variables; \( R \) and \( S \) are the structural load-carrying capacity and the factored loads, respectively; \( d_i \) and \( d_i^n \) are the inner-story drift displacement and its allowable value of the story \( l \), respectively; \( nm \) and \( nn \) are the numbers of frame elements and stories of the structure, respectively; \( nstr \) and \( nser \) are the numbers of strength and serviceability load combinations considered for the deterministic constraint, respectively; \( P_j(X) \) and \( P_a \) are the structural failure probability and its allowable value, respectively; \( npro \) is the numbers of load combinations considered for the probability constraint.

The objective function in Eq. (1) is transformed to unconstrained one as

\[
W_{uncstr}(X,Y) = \left(1 + \sum_{j=1}^{nstr} \alpha_{str,j} \beta_{1,j} \right) + \sum_{k=1}^{nser} \alpha_{disp,k} \beta_{2,k} + \sum_{t=1}^{npro} \alpha_{pro,t} \beta_{3,t} \times \left( \sum_{i=1}^{nm} \alpha_i A_i L_i \right)
\]

with

\[
\beta_{1,j} = \max \left(1 - \frac{R_j}{S_j}, 0 \right) ; j = 1, \ldots, nstr
\]

\[
\beta_{2,k} = \sum_{t=1}^{nm} \max \left( \left| \frac{d_{kl}}{d_{k,l}^n} \right| - 1, 0 \right) ; k = 1, \ldots, nser
\]

\[
\beta_{3,t} = \sum_{i=1}^{npro} \max \left( \frac{P_{t,j}(X)}{P_{t,a}} - 1, 0 \right)
\]

in which \( \alpha_{str,j}, \alpha_{disp,k}, \) and \( \alpha_{pro,t} \) are the penalty factors corresponding to the strength load combination \( j^{th} \), the serviceability load combination \( k^{th} \), and the load combination \( t^{th} \) for probability constraint, respectively.
4. Improve importance sampling method for structural failure probability

When using nonlinear inelastic analysis, the load-carrying capacity of the structure is directly calculated. The structural limit state for the load combination $t^{th}$ is popularly written as

$$G(R,S) = \frac{R}{S} - 1$$  \hspace{1cm} (4)$$

Based using Eq. (4), simulation methods such as MCS can be applied to calculate the structural failure probability $P_f$. However, since $P_f$ of a structure is very small (for example, for a steel frame, $P_f$ is about 0.1%), a great number of samples are required for using the simulation methods. To reduce the number of samples, an alternative limit state of the structure is used in this study such as

$$G(X) = \frac{R_{xm}(x_1, x_2, \ldots, x_{m-1})}{x_m} - 1$$  \hspace{1cm} (5)$$

where $x_m$ is an applied load and $R_{xm}$ is defined as the structural load-carrying capacity in terms of $x_m$. $R_{xm}$ is calculated the following two-step procedure. Firstly, the deformed configuration of the structure subjected to 100% of all applied loads except $x_m$ is calculated. From this structural configuration, the load-carrying capacity of the structure corresponding to $x_m$ is calculated. It is defined as $R_{xm}$. Note that, structural responses subjected to live loads are analyzed from structural deformed configuration subjected to dead loads, so Eq. (5) presents more accurate behaviors of the structure. Based on Eq. (5) the structural failure probability is calculated by using the IS method as follows:

$$P_f = P[G(X) \leq 0] = \int_{G(X) \leq 0} f_{R_{xm}}(r) g_{x_m}(x) \frac{f_{x_m}(x)}{g_{x_m}(x)} drdx$$

$$= \frac{1}{N} \sum_{i=1}^{N} I(x_m^i)$$  \hspace{1cm} (6)$$

where

$$I(x_m^i) = \left\{ \begin{array}{ll}
    \frac{f_{x_m}(x_m^i)}{g_{x_m}(x_m^i)} & \text{if } R_{xm} = R_{xm}(x_1^i, x_2^i, \ldots, x_{m-1}^i) \leq x_m^i \\
    0 & \text{if } R_{xm} = R_{xm}(x_1^i, x_2^i, \ldots, x_{m-1}^i) > x_m^i
    \end{array} \right. \hspace{1cm} (7)$$

in which $N$ is the sample size; $f_{R_{xm}}(r)$ and $f_{x_m}(x)$ are the densities of $R_{xm}$ and $x_m$, respectively; $g_{x_m}(x)$ is the IS distribution function of $x_m$; $x_m^i$ is the sample $i^{th}$ of $x_m$ using $g_{x_m}(x)$. The variance of $P_f^{IS}$ is then reduced using the effective IS method (EIS) [16] in which the generation of $N$ samples for $x_m$ is repeated $k$ - times as follows:

$$P_f^{EIS} = \frac{1}{k} \sum_{j=1}^{k} P_f^{IS,j}$$  \hspace{1cm} (8)$$

where $P_f^{IS}$ is the structural failure probability by using EIS. To improve the performance of the EIS method, the “Latinized” Partially Stratified Sampling (LPSS) [18], a new and efficient sampling
method, is used for generating samples. The new method for structural failure probability analysis is then named as LPSS-EIS. Details of LPSS-EIS is given in [17].

5. Improved differential evolution algorithm
Storn and Price first introduced the DE algorithm in 1997 [19]. Up to now, this method has attracted lots of interest in researchers since its efficiency for various optimization problems in different fields. The authors and the colleagues have developed an improve DE method recently, named EpDE, for the optimization of steel frames with deterministic constraints [2]. In EpDE, a new mutation technique is developed using the p-best method where $p$ for $k^{th}$ generation is determined as

$$p(k) = A \times NP^\left(\frac{k-1}{total\_\_generation-1}\right)$$  \hspace{1cm} (9)

where $A$ and $B$ are given coefficients; $NP$ is the population size; $total\_\_generation$ is the maximum generations. The trial individual, $U = (u_1, u_2, \ldots, u_{nd})$, is then generated using a two-step procedure as follows:

**Mutation step:**

$$V = X_{pbest} + F \times (X_{r1} - X_{r2})$$  \hspace{1cm} (10)

**Crossover step:**

$$u_j = \begin{cases} v_j & \text{if } (\text{rand}(0,1) < CR) \text{ or } (j = I) \\ x_q & \text{otherwise} \end{cases}$$  \hspace{1cm} (11)

in which $X_{pbest}$ is selected randomly in the top 100$p\%$ current individuals of the population; $I$, $r_1$, and $r_2$ are random integers in $[1, ND]$ with $i \neq r_1 \neq r_2$; $F$ and $CR$ are the scale factor and constant crossover rate, respectively, which are user-defined parameters. As to be obtained from Eq. (9), the value of $p$ will be decreased when $k$ increases. This means that in the early phase of optimization process when the population still diverges, trial individuals are generated based on more individuals of the population to maintain the diversity of the population. In the late phase when the population is converged, trial individuals are created using a few individuals to accelerate the convergence rate of optimization process. To improve the performance of EpDE, the Multi-Comparison Technique (MCT) is used [5]. In MCT, the deterministic and probabilistic constraints are evaluated consequently. After one constraint evaluated, the unconstrained objective function of the trial individual is updated. If it is bigger than one of the target individual, the trial individual is abandoned without any further constraints evaluated. In this way, many redundant structural analyses are eliminated. This method is more meaningful since the probabilistic constraints require so many structural analyses. The details on EpDE can be found in [2].

6. Case study
In this section, a six-story space frame with the shape presented in Figure 1 is studied. Dead (D) and live (L) loads are distributed on the beams while wind loads (W) are simulated by equivalent concentrated loads in the Y-direction at every beam-column joint. There are 15 random variables considered in this case with the information given in Table 1. The density of the material is 7850 (kg/m$^3$). Beams and columns are classified into 5 groups as presented in Figure 1. 267 sections from W10–W44 of AISC-LRFD are used for the beams, while 158 sections of W12-W14 and W18-W27 are used for columns. Two strength ($1.2D + 0.5L + 1.6W$ and $1.2D + 1.6L$) and one serviceability ($1.0D + 0.7W + 0.5L$) load combinations are considered. The allowable inter-story drift is $\frac{h}{400}$.
where \( h \) is the frame story height. The probabilistic constraint is defined as the structural failure probability for the load combination \( 1.2D + 0.5L + 1.6W \) is smaller 0.1%.

![Six-story space frame](image)

**Figure 1. Six-story space frame.**

**Table 1. Information for random variables**

| Properties   | Variables | Nominal | Mean/nominal | COV  | Distribution   |
|--------------|-----------|---------|--------------|------|----------------|
| Material     | \( E \)   | 200 (GPa) | 1.00         | 0.04 | Lognormal      |
|              | \( F_y \) | 248 (MPa) | 1.10         | 0.06 | Lognormal      |
| Cross-section| \( A_i \) | -       | 1.00         | 0.05 | Normal         |
|              | \( I_i \) | -       | 1.00         | 0.05 | Normal         |
| Loading      | \( D \)   | 20.5 (kN/m) | 1.05       | 0.10 | Normal         |
|              | \( L \)   | 10.5 (kN/m) | 1.00       | 0.25 | Lognormal      |
|              | \( W \)   | 20 (kN)  | 0.92         | 0.37 | Gumbel         |

Since the numerical results in [2] show that EpDE produces very good optimum designs of steel frames through the comparison with several new and efficient optimization methods, this study only presents the robustness of the proposed method in saving computational efforts. The parameters used for EpDE are \( NP = 20; \ ND = 5; \) Maximum value of generations = 400; scale factor \( F = 0.7; \) crossover
factor CR = random in (0;1); A = B = 1.0. Wind load is chosen to perform LPSS-EIS. The parameter k is equal to 2000. To choose a reasonable number of samples for LPSS-EIS, a frame with member groups of (W24x84, W14x159, W14x99, W16x67, W24x68), which has a failure probability of 0.098%, is studied first. As be presented in Figure 2, the COV of structural failure probability using LPSS-EIS is much smaller than using MCS. Specifically, with the number of samples is 256, COV of $P_f$ using LPSS-EIS is 5.48%. Meanwhile, COV of $P_f$ using MCS is 204.6% with 256 samples and it is still very large (about 30%) with 10000 samples. This means that LPSS-EIS is so much better than MCS. For the optimization, the number of samples used is chosen as 256.

![Figure 2. Structural failure probability with member groups of (W24x84, W14x159, W14x99, W16x67, W24x68).](image)

| Table 2. Optimum results |
|--------------------------|
| Element design group number (mm$^2$) | Best design | Worst design |
| 1 | W27x102 | W24x117 |
| 2 | W18x50 | W21x48 |
| 3 | W24x68 | W24x68 |
| 4 | W24x68 | W24x62 |
| 5 | W12x16 | W12x14 |
| Weight (kg) | 27582.3 | 27950.0 |
| Average weight (kg) | 27654.234 |
| Std. weight (kg) | 160.267 |
| Average number of structural analyses for deterministic constraints evaluation | 5.248 |
| Average number of structural analyses for probabilistic constraints evaluation | 387.456 |

Table 2 presents the optimization results with ten different program runs. The best optimum design has a total weight of 27582.3 (kg), while the weight of the worst one is 27950 (kg). The average weight of the optimum design is 27654.234 (kg) and the standard deviation of the optimum weight is only 160.267 (kg) corresponding to the COV value of 0.57%. This implies the proposed method is stable. Furthermore, the average number of structural analyses required for an optimization process is
only 5.248 and 387.456 corresponding to the deterministic and probabilistic constraints. Notes that, if the MCT technique is not used, an optimization process requires about 15,000 and 1,280,000 structural analyses used for evaluation of deterministic and probabilistic constraints, respectively. This means that the proposed method saves about 69.67% of computational efforts. In addition, Figure 3 shows the best optimum design convergence history.

![Figure 3](image.png)

**Figure 3.** Convergence history of the best optimum design.

7. **Case study**

An efficient method for the RBDO of steel frames is developed. The objective function is the total weight of beams and columns. The constraints include deterministic (load-carrying capacity and displacement limitations) and probabilistic (the overall failure probability of the structure) constraints. Direct analysis using the beam-column method is used for deterministic constraint evaluation. An effective importance sampling technique is employed to calculate the structural failure probability based on nonlinear inelastic analysis which efficiently reduces the required structural analyses. The numerical results of the six-story space frame show that the developed RBDO procedure produces good optimum designs of the structure with a saving of about 70% required structural analyses.

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