THE BULK LORENTZ FACTORS OF FERMI-LAT GAMMA RAY BURSTS

XIAO-HONG ZHAO1,2, ZHUO LI3,4, AND JIN-MING BAI1,2
1 National Astronomical Observatories/Yunnan Observatory, Chinese Academy of Sciences, P.O. Box 110, 650011 Kunming, China; zhaoxh@ynao.ac.cn
2 Key Laboratory for the Structure and Evolution of Celestial Bodies, Chinese Academy of Sciences, P.O. Box 110, 650011 Kunming, China
3 Department of Astronomy, Peking University, Beijing 100871, China
4 Kavli Institute for Astronomy and Astrophysics, Peking University, Beijing 100871, China

Received 2010 May 24; accepted 2010 November 1; published 2010 December 17

ABSTRACT

The Lorentz factor (LF) of gamma-ray burst (GRB) ejecta may be constrained by observations of high-energy (HE) spectral attenuation. The recent Fermi-LAT observations of prompt GeV emission from several bright GRBs have led to conclusions of unexpectedly large LFs, $\Gamma > 10^2$. Here we revisit this problem with two main concerns. (1) With a one-zone assumption where all photons are assumed to be generated in the same region (radius) and time, we self-consistently calculate the $\gamma\gamma$ optical depth by adopting a target photon spectrum with an HE cutoff. We find that this might be important when the GRB LF is below a few hundreds. (2) Recent Fermi-LAT observations suggest that the bulk MeV-range and HE ($\gtrsim100$ MeV) emission may arise from different regions. We then consider a two-zone case where HE emission is generated in much larger radii than that of the MeV-range emission. We find that the HE emission may mainly be attenuated by the MeV-range emission and that the attenuated HE spectrum does not show an exponential spectral cutoff but a slight steepening. This suggests that there may be no abrupt cutoff due to $\gamma\gamma$ attenuation if relaxing the one-zone assumption. By studying the spectra of three bright Fermi-LAT GRBs, 080916C, 090510, and 090902B, we show that bulk LFs of $\Gamma \sim 600$ can be consistent with observations in the two-zone case. Even lower LFs can be obtained in the multi-zone case.

Key words: gamma-ray burst: general

1. INTRODUCTION

Relativistic expansion is a key property of gamma-ray bursts (GRBs) and has been confirmed by measurements of radio afterglow sizes, such as the indirect estimation by radio scintillation in GRB 970508 (Waxman et al. 1998) and direct imaging of nearby GRB 030329 (Taylor et al. 2004). These observations revealed mildly relativistic GRB ejecta, $\Gamma < 1$ in a few, in the radio afterglow phase. However, it is believed that GRB ejecta are ultrarelativistic in the beginning—this is required to solve the so-called compactness problem (e.g., Piran 1999). The compact GRB source, as suggested by the rapid variabilities in MeV light curves, and the huge luminosity suggest high, optically thick GRB sources, which are in conflict with the nonthermal and hard GRB spectra. Relativistic expansion of the emission region is introduced to solve this problem. In order for the ~100 MeV photons, as detected by EGRET in several GRBs, to escape from the emission region, avoiding $\gamma\gamma$ attenuation, the bulk Lorentz factor (LF) of the emission region is required to be extremely large, $\Gamma \gtrsim 10^2$ (e.g., Lithwick & Sari 2001; Krolik & Pier 1991; Fenimore et al. 1993; Woods & Loeb 1995; Baring & Harding 1997). Recently, the powerful Fermi satellite revealed, in much more detail, the high-energy (HE) emission from GRBs. Several bright GRBs are reported to show time-integrated spectra extending up to GeV or even tens of GeV, without any signs of spectral cutoff. Assuming the $\gamma\gamma$ optical depths for these HE photons are below unity, these observations have led to even larger bulk LFs, $\Gamma > 10^3$ (Abdo et al. 2009a, 2009b, 2009c). This is putting the theoretical problem of relativistic jet formation to extremes.

In the previous constraints two assumptions are usually made. First, all photons, from low to high energy, are produced in the same region and at the same time. This “one-zone” assumption is not solid, as Fermi observations actually revealed that the onset of HE emission is delayed relative to MeV emission (e.g., Abdo et al. 2009a, 2009b, 2009c); the HE emission lasts longer than MeV emission (e.g., Abdo et al. 2009a, 2009b, 2009c); the bulk emission shifts toward later times as the photon energy increases (Abdo et al. 2009a) and the shift is longer than the variability times in MeV light curves, as pointed out by Li (2010); and some GRBs show obviously distinct HE components with different temporal behaviors (Abdo et al. 2009b, 2009c). All these features may imply that photons with different energies are produced in different regions.

In particular, the bulk $>100$ MeV emission in GRB 080916C shows a ~1 s shifting relative to MeV emission, which is much longer than the MeV variability time, $<100$ ms as revealed by the International Gamma-Ray Astrophysics Laboratory (INTEGRAL; Greiner et al. 2009), strongly implying that $>100$ MeV emission is produced in a region of much larger radii than MeV emission (Li 2010). As pointed out by Li & Waxman (2008), within the framework of the internal shock model, the internal collisions at small radii, which would produce the prompt MeV emission, are expected to lead to “residual” collisions at much larger radii, which would produce low-frequency emission. HE emission could be produced by electrons accelerated in residual collisions at larger radii, inverse-Compton scattering the MeV photons and/or double scattering of the low-frequency photons (Li 2010; Zhao et al. 2010). In this case, the MeV and HE photons are produced in different regions. In the comoving frame of the HE emission region, the MeV photons would be collimated rather than isotropic, thus the $\gamma\gamma$ absorption is angular dependent.

Second, the target photon spectrum is assumed to extend to infinity. As pointed out by Li (2010), the calculation of the $\gamma\gamma$ optical depth using such a target photon field is obviously not self-consistent, because the HE spectral end should be cut off due to absorption considered in the calculation.

In this paper, we revisit the problem of GRB LF constraint by modifying the above-mentioned two assumptions. In Section 2,
we consider a one-zone case where the $\gamma\gamma$ optical depth is calculated self-consistently by assuming a truncated target spectrum, then in Section 3 we consider a simple two-zone case with the anisotropic effect on $\gamma\gamma$ optical depth taken into account. In Section 4, we study the spectra of the three bright Fermi-LAT GRBs and constrain their LFs. In Section 5, we present a discussion and our conclusions. In the following, we assume the concordance universe model with $(\Omega_m, \Omega_{\Lambda}) = (0.27, 0.73)$ and $H_0 = 71$ km s$^{-1}$ Mpc$^{-1}$.

2. ONE-ZONE CASE

Consider a GRB ejecta with bulk LF $\Gamma$ and radius $R$. Assume that the photons in the comoving frame of the ejecta are isotropic, with photon number density per unit photon energy $dn'/d\epsilon'$. Hereafter, unless specified otherwise, quantities with prime signs denote the comoving frame and quantities without prime signs denote the frame of observer on the Earth.

In the (comoving-frame) dynamical time $R/\Gamma c$, a photon travels a path of $R/\Gamma$. For a photon of energy $\epsilon' = \epsilon(1+z)/\Gamma$ (with $z$ the GRB redshift), the optical depth due to $\gamma\gamma$ collisions during a dynamical time is given by Gould & Schréder (1967)

$$\tau(\epsilon') = \frac{R}{2\Gamma} \int_{m_e^2/\epsilon'}^{m_e^2/\epsilon} d\epsilon' \frac{dn'}{d\epsilon'} \int_{-1}^{1} d\mu'(1 - \mu')\sigma(E),$$

where $\mu' = \cos\Theta'$ and $\Theta'$ is the angle between the colliding photon pair. The cross-section is given by

$$\sigma(E) = \frac{3\sigma_T}{16} (1 - \beta_e^2) \left[(3 - \beta_e^2) \ln \frac{1 + \beta_e}{1 - \beta_e} - 2\beta_e (2 - \beta_e^2)\right],$$

where $\beta_e = \sqrt{1 - (m_e^2c^2/E)^2}$ and $E = \sqrt{\epsilon'\epsilon'(1 - \mu'^2)/2}$ are the velocity and energy, respectively, of the generated electron in the center of the momentum frame of the collision. The radius $R$ of the emission region can be related to the angular spreading time $\delta t_{\text{ang}}$, due to a geometric effect, by $R = 2\Gamma^2 c \delta t_{\text{ang}}/(1+z)$. As the angular spreading time is related to the observed variability time $\delta t$ by $\delta t_{\text{ang}} = \delta t$, we have

$$R = 2\Gamma^2 c \frac{\delta t}{1+z}.$$  

For a GRB with the observed photon number per unit time per unit photon energy per unit detector area, denoted by $N(\epsilon)$, the photon number density per unit photon energy in the comoving frame can be given by

$$\frac{dn'}{d\epsilon'} = \left(\frac{d_\epsilon}{R}\right)^2 \frac{N(\epsilon)}{c(1+z)^2},$$

where $d_\epsilon$ is the GRB luminosity distance, and $\epsilon = \Gamma \epsilon'/(1+z)$.

It is important to note a difference from previous works. In Equation (1), we did not take the upper limit of the integration to be infinity but a certain photon energy $\epsilon_{\text{max}}$, because the HE tail is expected to be cut off due to $\gamma\gamma$ absorption. The cutoff energy is just where $\tau(\epsilon_{\text{max}}) = 1$ happens. To self-consistently solve for the cutoff energy $\epsilon_{\text{max}} = \Gamma \epsilon_{\text{max}}/(1+z)$ for given $\Gamma$, we need to take the upper limit of the integration to be $\epsilon_{\text{max}}$, and solve $\tau(\epsilon_{\text{max}}) = 1$ using Equations (1)–(4) and observed GRB spectrum $N(\epsilon)$.

It is well known that the GRB spectrum can be fitted by the Band function (Band et al. 1993)

$$N(\epsilon) = \begin{cases} A \left(\frac{\epsilon}{100 \text{keV}}\right)^{\alpha - 1} \exp\left[-\epsilon/(\epsilon_{\gamma\gamma})\right] & \epsilon < \epsilon_{\gamma\gamma} \\ A \left(\frac{(\alpha - \beta)\epsilon}{(2 + \alpha)100 \text{keV}}\right)^{-\beta} \exp(\beta - \alpha)\left(\frac{\epsilon}{100 \text{keV}}\right)^{\beta} & \epsilon > \epsilon_{\gamma\gamma} \end{cases},$$

where $\epsilon_{\gamma\gamma} = \epsilon_0 (\alpha - \beta)/(2 + \alpha)$, and $A$, $\alpha$, $\beta$ and $\epsilon_0$ are the normalized coefficient, low-energy slope, the HE slope, and the $\nu F_\nu$ peak energy, respectively. In some Fermi-LAT GRBs, an extra spectral component beyond the Band function is claimed to exist, especially in the HE end (Abdo et al. 2009b, 2009c). This extra component can be described as a power law,

$$N(\epsilon) = A_{\text{PL}} \left(\frac{\epsilon}{1 \text{GeV}}\right)^{\beta_{\text{PL}}},$$

with $A_{\text{PL}}$, the normalization at 1 GeV and $\beta_{\text{PL}}$, the spectral index. It is helpful to solve the $\Gamma - \epsilon_{\max}$ relation with some approximations first. Typically, the HE, $\gtrsim 100$ MeV, photons mainly interact with photons above the peak energy. Let us approximate the target photon distribution as a single power-law $N(\epsilon) = N_0 \epsilon^{-s}$ in the following analytical derivation.

In Equation (1), the upper limit of the first integral is usually taken to be $\infty$. This is valid for $\epsilon_{\text{max}} \gg \Gamma^2 m_e^2c^2/[\epsilon_{\text{max}} (1+z)^2]$ and the spectrum slope $s > 1$. In this case, using $\delta$-approximation for the crosssection at target photon energy above the threshold, $\sigma \approx (3/16)\sigma_T$, $\epsilon(\epsilon_{\text{max}}) = 1$ can be solved to give $\Gamma$ as function of $\epsilon_{\text{max}}$.

$$\Gamma \approx \epsilon_{\max}.$$  

However, when $\epsilon_{\max} \gtrsim \Gamma^2 m_e^2c^2/[\epsilon_{\text{max}} (1+z)^2]$, i.e., the energy of annihilated photons is compared with that of target photons, the upper limit cannot be taken as $\infty$ any more. In this case, $\Gamma$ is given by Li (2010)

$$\Gamma \approx \frac{\epsilon_{\max}}{m_e c^2}(1+z).$$

Next, we carry out a numerical calculation to solve $\tau(\epsilon_{\text{max}}) = 1$. For the observations, we take the three bright Fermi-LAT GRBs 080916C, 090510, and 090902B, and consider the same time intervals in the GRBs where the LFs have been constrained by Abdo et al. (2009a, 2009b, 2009c), as well as section “a” in GRB 080916C. The properties of spectra and flux for these GRBs are shown in Table 1. The calculated results are given in Figure 1, where we compare the results of the self-consistent calculation and the previous method using a target photon spectrum without the HE cutoff. We see that the results deviate from each other for $\epsilon_{\text{max}} \lesssim 100$ MeV or $\Gamma \lesssim 10$ hundreds. In the case of section “a” in GRB 080916C, where the maximum observed photon energy is lower (see Figure 1), the LF limit with the self-consistent calculation is much smaller than that with the previous method. Thus, to be self consistent, the upper bound of the integration in Equation (1) should be carefully taken as the maximum photon energy in this case. We also note that the LF constraints using an upper limit of infinity are still valid for those time segments that have been used by Abdo et al. (2009a, 2009b, 2009c).

3. TWO-ZONE CASE

As discussed in the introduction, the Fermi-LAT observations hint that there may be different emission regions of different
radii in the GRB prompt emission. As the HE delay of onset and the shifting of the bulk HE emission are in the seconds scale, whereas the MeV-range variability times, reflecting the dynamical time of the MeV emission region, are in the tens of ms scale, the MeV emission regions have much smaller sizes (radii), by orders of magnitude, than that of the HE emission regions.

Consider that the ejecta expand to radius $R$ where HE emission is being produced. The photons that are emitted in much smaller radii and just arrive at radius $R$ should be produced by those ejecta released from the central engine with a time delay $\Delta t$. Once we observed the HE delay of onset delayed timescale of the HE emission. Thus, once we observed a time delay $\Delta t$ for the HE emission relative to the MeV emission, the HE emission size is implied to be

$$R = 2\sqrt{\zeta} \frac{\Delta t c}{1 + \zeta}. \quad (9)$$

As the MeV emission comes from inner regions with smaller radii, $R_{\text{MeV}} = 2\Gamma_{\text{MeV}} c \Delta t /(1 + \zeta) \ll R$, the MeV photons in the comoving frame of the HE emission region are beamed. Here we also denote the LF of MeV emission region as $\Gamma_{\text{MeV}}$ since it may be different from the LF of the HE emission region, $\Gamma$, in the framework of the internal shock model, and the difference could be small, $\Gamma_{\text{MeV}} \sim \Gamma$.

Consider the geometry plotted in Figure 2. Due to the relativistic beaming effect, the MeV emission beam that illuminates an HE photon produced at $R$ can be approximated as an “MeV photon cone” with a half open angle of $\alpha = R_{\text{MeV}} / R \Gamma_{\text{MeV}}$. Outside of the cone the MeV photon flux can be neglected. In the comoving frame of the HE emission region, the solid angle is then $\Delta \Omega = 2\pi (1 - \cos \alpha')$, with $\cos \alpha' = (\cos \alpha - \beta\Gamma)/(1 - \beta\Gamma \cos \alpha)$ and $\beta\Gamma = \sqrt{\Gamma^2 - 1}/\Gamma$.

The optical depth is not only energy dependent but also angle dependent. Consider an HE photon of $\epsilon'$ traveling with an angle $\theta'$ (\(\mu' = \cos \theta'\)) with respect to the central axis of the target photon beam; then the optical depth corresponding to the

| GRB Name     | Time Interval (s) | $\epsilon_p$ (keV) | $\alpha$ | $\beta$ | $A$ (cm$^{-2}$ s$^{-1}$ keV$^{-1}$) | $A_{\text{PL}}$ (cm$^{-2}$ s$^{-1}$ keV$^{-1}$) | $\delta t$ (ms) | $\epsilon_{\text{highest}}$ (GeV) |
|--------------|------------------|-------------------|----------|--------|-------------------------------|-----------------------------------------------|----------------|-------------------------------|
| GRB 080916C-a | 0.004–3.58       | 440               | -0.58    | -2.63  | 0.055                         | ...                                           | ...             | 4.35                         |
| GRB 080916C-b | 3.58–7.68        | 1170              | -1.02    | -2.21  | 0.035                         | ...                                           | ...             | 4.35                         |
| GRB 090510   | 0.8–0.9          | 1894              | -0.86    | -3.09  | 0.028                         | -1.54                                         | 6.439 x 10^{-9} | 0.903 12                      |
| GRB 090902B  | 9.6–13           | 821               | -0.38    | -5.0   | 0.082                         | -1.98                                         | 4.3 x 10^{-10c} | 1.822 53                      |

Notes.

\(a\) Greiner et al. (2009).

\(b\) The spectral flux of this section at $\epsilon > 20$ MeV is only an upper limit, as shown in the supporting material of Abdo et al. (2009a).

\(c\) Private communication with Francesco de Palma; the other parameters are taken from Abdo et al. (2009a, 2009b, 2009c).

---

**Figure 1.** Relation between the observed maximum photon energy and the lower limit to the bulk LF in the one-zone case for the three bright GRBs. The adopted parameters of the GRBs are shown in Table 1. As marked in the plot, the dashed lines correspond to results using target photons without a spectral cutoff, while the solid lines correspond to our self-consistent calculations using truncated target photon spectra. The stars denote the observed highest energy of photons in the relevant time intervals.

**Figure 2.** Schematic diagram of the geometry when the HE photon produced at large radius $R$ is being illuminated by the MeV photon beam from much smaller radii $R_{\text{MeV}}$. The upper panel is for the observer frame, while the bottom two panels are for the comoving frame of the GeV emission region, with $\theta' < \alpha'$ and $\theta' > \alpha'$, respectively.
distance it travels in a dynamical time is given by

$$\tau(\epsilon', \mu') = \frac{R}{\Gamma} \int_{m'_c/\epsilon'}^{\infty} \frac{d\epsilon'}{\epsilon'} \int_{\Delta \Omega} \frac{d^2n'}{d\epsilon' d\Omega} (1 - \tilde{\mu}') \sigma(E)$$

$$= \frac{2R}{\Gamma \Delta \Omega} \int_{m'_c/\epsilon'}^{\infty} \frac{d\epsilon'}{\epsilon'} \frac{dn'}{d\epsilon' \times}$$

$$\begin{cases} 
\int \cos(\theta' - \alpha') \ d\tilde{\mu}' \phi' (1 - \tilde{\mu}') \sigma(E) & \theta' > \alpha' \\
\int \cos(\theta' + \alpha') \ d\tilde{\mu}' \phi' (1 - \tilde{\mu}') \sigma(E) & \theta' < \alpha'. 
\end{cases} \quad (10)$$

Here $\tilde{\mu}' = \cos \Theta'$ with $\Theta'$ being the angle between the HE photon and the colliding target photon, $d^2n'/d\epsilon' d\Omega$ is the energy distribution of target photons per unit solid angle, with $d\Omega = \sin \Theta d\Theta d\phi$, and $\phi' = \pi$ if $\theta' < \alpha'$ and $\tilde{\mu} > \cos(\alpha' - \theta')$, otherwise

$$\phi' = \arccos \left( \frac{\cos \alpha' - \cos \Theta' \cos \theta'}{\sin \Theta' \sin \theta'} \right). \quad (11)$$

The comoving frame target photon density $dn'/d\epsilon'$ is given by Equation (4) but with $\epsilon = 2\Gamma \epsilon'/(1 + z)$—the factor 2 appears for the highly beamed case of $\alpha' \ll 1$.

In the extreme case when $R_{\text{MeV}}/R \to 0$ or $\Gamma_{\text{MeV}}/\Gamma \to +\infty$ the target photons are totally beamed (hereafter other cases are called partly beamed) in the comoving frame of the HE emission region, then $\Delta \Omega \to 0$ and the optical depth reduces to

$$\tau(\epsilon', \mu') = \frac{R}{\Gamma} \int_{m'_c/\epsilon'}^{\infty} \frac{d\epsilon'}{\epsilon'} \frac{dn'}{d\epsilon'} (1 - \mu') \sigma(E). \quad (12)$$

Consider an area element in the sphere emitting photons which lies at an angle $\theta$ with respect to the line of sight; then, in its comoving frame a photon traveling along the line of sight would have an angle with respect to the central axis of the target photon beam of

$$\mu' = \frac{\mu - \beta \mu}{1 - \beta \mu}. \quad (13)$$

Due to the Doppler effect, the photon energy in the comoving frame is related to the observed photon energy as

$$\epsilon' = \Gamma(1 - \beta \mu) \epsilon(1 + z). \quad (14)$$

Denote $d^3P'/d\Omega d\epsilon' dS$ as the (comoving-frame) emitting power per unit solid angle per unit photon energy by material in unit area of the sphere surface. This emission should be modified by the $\gamma'\gamma$ attenuation factor $e^{-\tau(\epsilon', \mu')}$. The observed "time-averaged" flux is, then, integration over the sphere,

$$F_\epsilon \propto \int dS \frac{1}{\Gamma^2(1 - \beta \mu)^2} \frac{d^3P'}{d\Omega d\epsilon' dS} e^{-\tau(\epsilon', \mu')} \quad (15)$$

Assuming isotropic emission power in the comoving frame, constant emissivity along the sphere, and power-law dependent on photon energy, then

$$\frac{d^3P'}{d\Omega d\epsilon' dS} \propto \epsilon^{-h+1}. \quad (16)$$

Using $dS = 2\pi R^2 d\mu$ and Equations (9), (13), and (14), we have

$$F_\epsilon \propto t^2_0 \Gamma^{-h+3} \epsilon^{-h+1} f(\epsilon; \Gamma), \quad (17)$$

where

$$f(\epsilon; \Gamma) = \int d\mu (1 - \beta \mu)^{-h+1} e^{-\tau(\epsilon', \mu')} \quad (18)$$

Note that $f(\epsilon; \Gamma)$ is the suppression factor of the primary spectrum.

It is useful to analyze this factor analytically for the totally beamed case. As shown in the Appendix, for the single power-law target photon distribution, $N(\epsilon) = N_0 \epsilon^{-\gamma}$ and using the approximation $\sigma(E) \approx \sigma_0 E^{-2}$, the $f$ factor can be approximated as

$$f(\epsilon; \Gamma) = \left\{ \begin{array}{ll}
\frac{\epsilon}{\epsilon_{\text{br}}} \frac{1}{\Gamma} & \epsilon < \epsilon_{\text{br}} \\
1 & \epsilon > \epsilon_{\text{br}} 
\end{array} \right. \quad (19)$$

with the break energy at

$$\epsilon_{\text{br}} = \frac{2}{s(1 + z)^2} \left[ \frac{1 + s}{m^2 c^4} \right]^{\frac{1}{2}} \frac{N_0 \sigma_0 d^2_t}{\Gamma^{\frac{1}{2(1-s)}}}. \quad (20)$$

We carry out the numerical calculation of the $f$ factor and show the result in Figure 3. The analytical result is a good approximation.

Thus, in the totally beamed case the spectrum is not affected until $\epsilon > \epsilon_{\text{br}}$, where the spectrum steepens by a factor of $\Gamma - 1$. Thus, unlike in the case of the isotropic target photons, the spectrum is not cut off exponentially but shows a steepening power law. This can easily be understood in the beam target photon case: the HE photons can always escape if they travel with a small enough angle with respect to the target beam.

The break energy is LF dependent, thus the detection of the break in the spectrum can be used to measure the LF of GRBs. For GRB 090510, the break energy is $\epsilon_{\text{br}} \approx 1$ GeV for $\Gamma = 600$ and $t_d = 0.1$ s.
In Figure 3, we also show the numerical results for partly beamed cases. As can be seen, for the energy range of interests, say, <1 TeV, the partly beamed cases with $\Gamma_{\text{MeV}} = \Gamma$ approach the totally beamed case when $R_{\text{MeV}}/R < 0.1$, which is just the case we are considering because $\delta t \ll t_d$. We also illustrate the small effect of the LF variation by showing the cases of $\Gamma_{\text{MeV}}/\Gamma = 0.5$–2 with $R_{\text{MeV}}/R = 0.01$. Indeed, when $\Gamma_{\text{MeV}} > \Gamma$ as expected, the MeV photons are more strongly beamed and the situation more closely approaches the totally beamed case. Even when $\Gamma_{\text{MeV}}/\Gamma \lesssim 1$ there is only very little effect at very high energy (see Figure 3), since $R_{\text{MeV}}/R \ll 1$ and the MeV photons are still highly beamed. Thus, we will ignore the effect of variation of LFs and only consider $\Gamma_{\text{MeV}} = \Gamma$ in the following calculations.

It should be noted here that in the two-zone case, in addition to the HE absorption due to the inner-emerging beamed MeV photons, the absorption due to interactions with photons locally originated from the HE emission region can also contribute to the total optical depth. This adds an extra attenuation factor $e^{-\tau_{\text{self}}(\epsilon)}$ to the resulting spectrum, where the optical depth $\tau_{\text{self}}(\epsilon)$ is given by Equation (1) with $R$ being the HE emission region radius (Equation (9)) instead. We also consider this absorption in the following case studies.

### 4. CASE STUDIES

In this section, we study the three bright *Fermi*-LAT GRBs 080916C, 090510, and 090902B, and constrain their LFs with assumptions of one-zone or two-zone origins.

#### 4.1. GRB 080916C

This is a bright long GRB, with a duration of $\sim 50$ s and 145 photons detected above 100 MeV, among which 15 are beyond 1 GeV and 1 beyond 10 GeV. The redshift is quite high, $z = 4.35$, so that the isotropic-equivalent energy turns out to be $E_{\text{iso}} = 8.8 \times 10^{54}$ erg, the largest energy measured so far (Abdo et al. 2009a).

The wide energy range spectrum of this GRB is well fitted by a single Band function, which may imply that all radiation originates from one region. Indeed, with the one-zone assumption, the one-component spectrum favors synchrotron origin over IC emission, and the spectral slopes can be understood in the framework of the synchrotron emission model (Wang et al. 2009). Using the time interval 3.58–7.68 s, and under the one-zone assumption, the LF has been constrained to be $\Gamma > 900$ by Abdo et al. (2009a). It should be noted that the constraint is variability time dependent. In this constraint $\delta t = 2$ s is adopted from the Gamma-ray Burst Monitor (GBM) on board *Fermi* light curve. However, INTEGRAL also detected this GRB and showed much shorter variability time in the MeV range, $\delta t < 100$ ms. With this shorter variability time, we constrain the LF to satisfy $\Gamma > 1130$ (Figure 1).

However, the onset of $>100$ MeV emission is $\sim 4$ s delayed relative to the MeV emission; and in time bin “b” the bulk emission shifts toward later time as the photon energy increases, as pointed out by Abdo et al. (2009a), and the shift is $1$ s scale much longer than the variability times in MeV light curves, $\delta t < 100$ ms, as noted by Li (2010). These temporal behaviors suggest that HE emission may have different origins and larger emission regions than the MeV emission. Indeed, the “single” spectral component favors synchrotron over inverse Compton radiation; however, as pointed out by Li (2010), the observed highest energy photon in this GRB cannot be generated by synchrotron radiation, implying a different component/origin for the HE emission.

Here we consider a simple two-zone case, where the ejecta that produce HE emission $>\epsilon_0$ is released with a delay $t_d = 1–4$ s relative to that produce MeV emission. It is hard to determine the threshold energy $\epsilon_0$ currently, but we take $\epsilon_0 \gtrsim 30$ MeV, due to the different temporal behaviors above 30 MeV. We use the observed flux and spectrum to calculate the optical depth due to absorption by inner-emerging beamed photons, and only use that at $>\epsilon_0$ to calculate the optical depth due to self absorption by local-originated photons from the HE emission region. With the sum of these two optical depths we can calculate the suppression factor to modify the original HE emission that is free of absorption. We consider the time interval 3.58–7.68 s following Abdo et al. (2009a) and assume the observed HE spectrum as the original one.

The result is presented in Figure 4. It can be seen that, with $\Gamma = 600$ and $\epsilon_0 = 30$ MeV, the “self” absorption is less important than the “beamed” absorption and the attenuated spectrum does not show a sharp cutoff but a slight steepening, as in Figure 3, in contrast with the one-zone case. We also show that taking $\epsilon_0 \gtrsim 30$ MeV does not change the conclusion much.

Indeed, for a photon of 3 GeV, the highest observed energy in the relevant time bin, and given $\Gamma = 600$ the threshold energy of the $\gamma\gamma$ interaction is $\epsilon_{\text{th}} = \Gamma^2 (m_c c^2)^2/3 \text{GeV}(1+z)^2 \approx 1$ MeV, much smaller than 30 MeV. Moreover, we try different $\Gamma$ values and find that the break energy, where the steepening happens, increases with $\Gamma$. $\Gamma \sim 600$ can be consistent with the observed spectrum in the two-zone case. Finally, it should be noted that the self-absorption becomes important when taking smaller threshold energy and time delay, i.e., $\epsilon_0 = 10$ MeV and $t_d = 1$ s (for $\Gamma = 600$).
4.2. GRB 090510

This is a short GRB with a duration of 2.1 s, but very bright, with 18 photons at >1 GeV detected. Given the redshift $z = 0.903 \pm 0.003$ and the total (0.5–1.0s) energy fluence in the 10 keV–30 GeV band, $(5.02 \pm 0.26) \times 10^{-5}$ erg cm$^{-2}$, the total isotropic-equivalent energy release is $(1.08 \pm 0.06) \times 10^{53}$ erg (Abdo et al. 2009b).

Using the spectrum in the time interval 0.8–0.9 s which includes a highest energy photon of 31 GeV and can be fitted by the Band function plus a power-law component, the LF constraint in the one-zone case is $\Gamma > 1200$ (Abdo et al. 2009b). Under the one-zone assumption we find that $\Gamma > 990$ (Figure 1). The two results are broadly consistent, though our result is a little smaller than that of Abdo et al. (2009b), which could be due to the different definitions of $R$. Our defined $R$ is larger by a factor of 2.

Moreover, there are some distinct features in this GRB: the time-integrated spectrum in the time interval 0.5–1.0 s is best fitted by two spectral components, a Band-function component at low energy plus a power-law component dominating HE emission; this emission above 30 MeV is delayed by $\delta t_d = 248$ ms relative to that below 1 MeV as shown by the data analysis in Abdo et al. (2009b). These suggest that HE may have a different origin and/or emission region. Therefore, we consider the simple two-zone assumption for this GRB again. Since there are two components in the spectrum that may be consistent with the two components in the temporal behavior, we use the Band-function component to calculate the optical depth due to the MeV photon beam and use the power-law component for the calculation of self absorption. Thus, we obtain the $f$ factor to modify the power-law component, assuming the observed best-fit spectrum as the original one without $\gamma \gamma$ attenuation.

The resulting spectra are shown in Figure 5. We find that the LF of $\Gamma \sim 600$ can be still consistent with the observed spectrum. It should be noted that although the power-law component dominates in energy, the Band-function component still dominates in photon number, so the absorption due to beamed MeV photons could be more important. For the parameters used, the self-absorption due to locally originated photons contributes a comparable, though less important, effect; thus, the attenuated spectrum is steeper than the beamed-MeV-photon only case, but still much smoother than the sharp cutoff in the one-zone case.

4.3. GRB 090902B

With a redshift of 1.822, this long, fairly strong GRB has an isotropic-equivalent energy $E_{\text{iso}} = 3.63 \pm 0.05 \times 10^{54}$ erg, comparable with that of the highest-energy one GRB 080916C (Abdo et al. 2009c). The duration in the energy interval 50–300 keV of Fermi (GBM) is 22 s. The highest energy photon (33.4 GeV) in this GRB is detected at 82 s after trigger, while that in the prompt phase is 11.2 GeV and in the interval of 9.6–13 s. Using this time interval and the one-zone assumption, Abdo et al. (2009c) constrain the LF to be $\Gamma > 1000$ (Abdo et al. 2009c), while we get, in Figure 1, $\Gamma > 830$. Similar to GRB 090510, this GRB also has a distinct spectral component fitted with a power law besides the Band function one. A peculiar characteristic of its spectrum is that its power-law component extends to a lower band (<10 keV). Similar to GRB 080916C, there is an obvious delay of a few seconds in the HE onset. Looking at the time bin “b” in Abdo et al. (2009c), the light curve peak also seems to shift toward high energy, with one second delay. Then, we again consider a simple two-zone case taking $\delta t_d = 1–5$ s. Similar to GRB 090510, we use the Band-function component as the beamed MeV target photons of the two-zone absorption, and the power-law component for the self absorption at the HE emission region. The total optical depth will lead to $f$ factor calculation, which will further modify the original spectrum, assumed to be the observed spectrum. The results in Figure 6 suggest that $\Gamma \sim 600$ can still be consistent with observations.

5. DISCUSSION AND CONCLUSIONS

We have revisited in this work the problem of constraining the GRB LFs by HE attenuation. Although this problem has been considered by many previous works, two concerns that have been ignored in those works have been emphasized here. First, we notice that in the one-zone case, in order to self-consistently calculate the $\gamma \gamma$ optical depth, one needs to consider the target photons with an HE spectral cutoff, rather than extending to infinity. This concern is important when the LFs are below a
few hundreds, or when the luminosity of GRBs is low. Second, we relax the one-zone assumption and consider a simple two-zone case where the beaming of target photons in the emission region should be taken into account. Our results show that in the two-zone case, the $\gamma\gamma$ absorption does not lead to an abrupt spectral cutoff but a spectral steepening. If the target photon energy distribution is with a power law with photon index $s$, then the spectral slope is changed by a factor of $\frac{1}{\gamma} - 1$. This also predicts that there should be no spectral cutoff in the GRB spectra if the prompt emission is not produced in one single region.

It should be noted that there have been some attempts by other authors to improve the approximation for the optical depth. Baring (2006) concluded that the pair attenuation signature appears as a broken power law rather than an exponential cutoff by considering the skin effect and introducing an attenuation descriptor of $1/(1 + \tau)$ instead of $e^{-\tau}$. Granot et al. (2008) considered the emission zone as a very thin layer producing impulsive emission. They calculated in detail the opacity evolution during a pulse and claimed that the attenuation signature can be different from that derived from the simple one-zone approximation. Essentially, these two works are still concerned with one-zone problem, with $\Delta R \sim R_{\text{MeV}}$. However, in the two-zone problem that we have considered here, the HE and MeV emission components are emitted at very different radii, with $\Delta R \gg R_{\text{MeV}}$, which leads to much smaller optical depth and hence smaller LF at the HE emission region.

Furthermore, we used our new concerns to analyze the spectra of the three bright GRBs 080916C, 090510, and 090902B and found that in the two-zone case an LF of $\Gamma \sim 600$ can still be consistent with the observed spectra. This relaxes the strict requirement, $\Gamma > 10^3$, in the one-zone assumption.

We note that in the present observational situation where only tens to hundreds of HE photons were detected in one GRB, a slight change of the spectral slope is not easy to identify. A single power law may still fit the HE spectral tail.

We have considered a simple two-zone case here, but the situation can be more complicated. The central engines of GRBs may naturally create variabilities in a wide range of timescales, e.g., from ~1 ms to ~10 s. In the framework of the internal shock model, this will lead to kinetic dissipation in a wide range of radii. Even in the single-timescale case, internal collisions will happen as the ejecta expand until the material is spatially leaves the MeV front, while our integration corresponding to one dynamical time expansion is equivalent to integration up to $2R$, where the generated HE photon doubles its radius. Because both the number density of the target photons and the angle between the traveling directions of the HE photon and the MeV front decrease rapidly with radius, the interaction is strongly dominated by those at small radius, and hence the upper limit of the integration is unimportant—whether the upper limit is $R_{\text{max}}(\gg R)$ or $2R$ the result is practically the same. Furthermore, they use an “averaged” optical depth to constrain the LF, which may not be appropriate since we have shown that no sharp cutoff is expected in the two-zone case. We consider more carefully the spectral profile due to suppression. Finally, they neglect the self absorption in the local region which may contribute a significant effect as we show.

We thank Francesco de Palma for information. X.H.Z. also thanks Z.G. Dai and X.Y. Wang for helpful discussions. This work was supported by the National Natural Science Foundation of China (NSFC; grants 10973034 and 10843007), the 973 Program (grant 2009CB824800), and the Foundation for the Authors of National Excellent Doctoral Dissertations of China.

APPENDIX

DERIVATION OF THE SUPPRESSION FACTOR FOR THE TOTALLY BEAMED CASE

Here we derive the suppression factor $f$ (Equation (19)) in the two-zone case, assuming the target photon distribution as a single power law with $N(\varepsilon) = N_0\varepsilon^{-(\Gamma + 1)}$. In the comoving frame of the HE emission region, the target photon distribution can be given by Equation (4).

The cross-section of $\gamma\gamma$ collisions in the relativistic limit ($E \gg m_e c^2$) is $\sigma(E) \approx \sigma_0 E^{-2}$, where $\sigma_0 = (3/8)\sigma_f (m_e c^2)^2 [2 \ln(2E/m_e c^2) - 1]$ weakly depend on $E$ and can be considered as constant due to roughly a constant of $2 \ln(2E/m_e c^2) - 1$, which is as an approximation taken as $3$. With these approximations the $\gamma\gamma$ optical depth (Equation (12)) can be reduced to

$$\tau = C_1 \Gamma^{-3} \epsilon_q^{-1} (1 - \mu')^2 (1 + z)^{-1},$$

$$C_1 = N_0 \sigma_0 (2m_e c^2)^{-4} d_t^2 (s c^2 t_d)^{-1}.$$  \(\text{(A1)}\)

Using the transformations of Equations (13) and (14), the optical depth further becomes

$$\tau = C_2 \epsilon_q^{-1} (1 - \mu') (1 - \beta \mu)^{-1},$$

$$C_2 = C_1 2^\frac{3}{2}(1 + z)^{2(1 - v - 1)} \Gamma^{-4}.$$  \(\text{(A2)}\)

As $\theta$ increases ($\mu$ decreases) $\tau$ increases. Let us define the minimum $\mu_{\text{min}}$ where $\tau(\epsilon, \mu_{\text{min}}) = 1$, then at $\mu < \mu_{\text{min}}$ the
emission at $\varepsilon$ is significantly absorbed. Using the approximation that $e^{-\tau} = 1$ when $\tau < 1$ and $e^{-\tau} = 0$ when $\tau > 1$, the expression for the $f$ factor is approximated by

$$f(\varepsilon; \Gamma) \approx \int_{\mu_{\min}}^{1} d\mu (1 - \beta \mu)^{-h-1}$$

$$\approx (1 - \beta \mu_{\min})^{-h-1}(1 - \mu_{\min}).$$  \hspace{1cm} (A3)

The second equality holds for $\mu_{\min} \to 1$.

From the definition of $\mu_{\min}$, we can solve from $\mu_{\min}$,

$$1 - \mu_{\min} = C_2^{-\frac{1}{s}} (1 - \beta \mu_{\min})^\frac{1}{s} \varepsilon^{\frac{1}{s} - 1}$$

$$\approx C_2^{-\frac{1}{s}} (1 - \beta \mu_{\min})^\frac{1}{s} \left[ 1 + \frac{\beta \mu_{\min}}{s(1 - \beta \mu_{\min})} \right] \varepsilon^{\frac{1}{s} - 1},$$  \hspace{1cm} (A4)

where the second equality in the above equation, again, holds for $\mu_{\min} \to 1$.

We can see that approximately,

$$1 - \mu_{\min} \approx \begin{cases} \varepsilon^{\frac{1}{s} - 1} & 1 - \mu_{\min} < \frac{s}{\beta \mu_{\min}} (1 - \beta \mu_{\min}), \\ \text{const.} & 1 - \mu_{\min} > \frac{s}{\beta \mu_{\min}} (1 - \beta \mu_{\min}). \end{cases}$$  \hspace{1cm} (A5)

The critical condition

$$1 - \mu_{\min} = \frac{s}{\beta \mu_{\min}} (1 - \beta \mu_{\min})$$  \hspace{1cm} (A6)

corresponds to a “break energy” $\varepsilon_{br}$. Substituting Equation (A6) into both sides of the first equality in Equation (A4), we get

$$\varepsilon_{br} = \frac{2}{s(1 + z)^2} \left[ (1 + s) \left( \frac{m_e^2 c^4}{N_0 \sigma_{\text{d}} d_L^2} \right)^{\frac{1}{1 + s}} \Gamma^{\frac{1 + 1}{1 + s}} \right].$$  \hspace{1cm} (A7)

$\varepsilon > \varepsilon_{br}$ corresponds to $1 - \mu_{\min} < \frac{s}{\beta \mu_{\min}} (1 - \beta \mu_{\min})$ and $\varepsilon < \varepsilon_{br}$ to $1 - \mu_{\min} > \frac{s}{\beta \mu_{\min}} (1 - \beta \mu_{\min})$. So, we can write

$$f(\varepsilon; \Gamma) \propto \begin{cases} \varepsilon^{\frac{1}{s} - 1} & \varepsilon > \varepsilon_{br}, \\ \text{const.} & \varepsilon < \varepsilon_{br}. \end{cases}$$  \hspace{1cm} (A8)

or, after an arbitrary normalization,

$$f(\varepsilon; \Gamma) = \begin{cases} 1 & \varepsilon < \varepsilon_{br} \\ (\varepsilon_{br}^\delta - \varepsilon_{br}^\delta) & \varepsilon > \varepsilon_{br} \end{cases}.$$  \hspace{1cm} (A9)

REFERENCES

Abdo, A. A., et al. 2009a, Science, 323, 1688
Abdo, A. A., et al. 2009b, Nature, 462, 331
Abdo, A. A., et al. 2009c, ApJ, 706, L138
Band, D., et al. 1993, ApJ, 413, 281
Baring, M. G. 2006, ApJ, 650, 1004
Baring, M. G., & Harding, A. K. 1997, ApJ, 491, 663
Cenko, S. B., et al. 2010, arXiv:1004.2900
Fenimore, E. E., Epstein, R. I., & Ho, C. 1993, A&AS, 97, 59
Gould, R. J., & Schréder, G. P. 1967, Phys. Rev., 155, 1404
Granot, J., Cohen-Tanugi, J., & do Couto e Silva, E. 2008, ApJ, 677, 92
Greiner, J., et al. 2009, A&A, 498, 89
Krolik, J. H., & Pier, E. A. 1991, ApJ, 373, 277
Li, Z. 2010, ApJ, 709, 525
Li, Z., & Waxman, E. 2008, ApJ, 674, L65
Lithwick, Y., & Sari, R. 2001, ApJ, 555, 540
Piran, T. 1999, Phys. Rep., 314, 575
Taylor, G. B., Frail, D. A., Berger, E., & Kulkarni, S. R. 2004, ApJ, 609, L1
Tchekhovskoy, A., Narayan, R., & McKinney, J. C. 2010, New Astron., 15, 749
Wang, X.-Y., Li, Z., Dai, Z.-G., & Mészáros, P. 2009, ApJ, 698, L98
Waxman, E., Kulkarni, S. R., & Frail, D. A. 1998, ApJ, 497, 288
Woods, E., & Loeb, A. 1995, ApJ, 453, 583
Zhao, X. H., Dai, Z. G., Liu, T., Bai, J. M., & Peng, Z. Y. 2010, ApJ, 708, 1357
Zou, Y.-C., Fan, Y.-Z., & Piran, T. 2010, arXiv:1008.2253