Predict the Plastic Deformation of Perforated Sheet Metals using Image-based Finite Element Modeling

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Abstract. In this work, numerical prediction of the plastic deformation behaviors of perforated sheet metals, specifically AISI 1018 low carbon steel, C260 Cu-Zn brass, under uniaxial tension was carried out. An image-based mesh generation technique was applied to better represent and discretize the perforated geometry of sheet metal specimens. Digital image correlation measured displacement boundary conditions were also implemented into the finite element modeling. The predicted results of perforated sheet metal under tension as nominal stress-strain curves and localized plastic strain distribution agree well with the experimental observations. Possible error sources and uncertainty of this image-based finite element modeling were also discussed.

1. Introduction
Perforated sheet metal is extensively used in industries, like aircraft, automobiles, space vehicles, etc., because of its high strength-to-weight ratio and excellent energy absorption capability. For an effective application, it is essentially necessary to thoroughly understand its mechanical properties under the influence of such perforations. The equivalent continuum method is traditionally used to analyze the deformation of the perforated sheets. Based on this method, O’Donnell [1] calculated the apparent elastic constants of perforated sheets and proposed a criterion of stress limit for structural design. ASME presented a similar procedure to determine the peak stress multipliers for perforated plates [2].

Nowadays increasing researches have focused on plastic behavior to better apply the perforated sheet metals. Chen [3], Baik [4], Lee [5] analyzed the plastic deformation behavior of perforated sheets with circle perforations in a triangular pattern and proposed yield criteria based on the equivalent continuum concept. Geltmacher [6] numerically examined the flow localization behavior of metal sheet containing a pair of through-thickness circle perforations under uniaxial and biaxial tension. Reinhardt [9] developed a fourth-order anisotropic yield criterion to predict the plastic deformation of perforated sheet metals under biaxial loading. Jia [8, 9] developed numerical methods to predict the strain localization of metal sheets with randomly distributed circle perforation under uniaxial tension, resulting in good agreement with experimental results. Duan [10] studied the local deformation of sheet metal with random perforations from experimental and numerical points, which is significantly affected by the loading conditions (uniaxial and biaxial tension) and the perforation pattern. Monchiet [11] developed an anisotropic yield criterion for porous ductile material considering material anisotropy and perforation shapes. Khatam [12] numerically investigated the plastic yield initiation and strain localization in perforated sheets in plane stress condition.

Numerical analysis, such as finite element (FE) modeling, was increasingly used for mechanical investigation of perforated sheet metals as mentioned above. The numerical simulation, if adequately formulated and accurately realized can fully characterize the mechanical response of the perforated sheet metals. The accuracy of the FE modeling about the mechanical behavior of perforated sheet metals is
strongly dependent on material anisotropy, strain hardening, and loading conditions [6]. Due to the cold-rolling process, the sheet metal usually behaves in an anisotropic manner. Anisotropic yield criterion is thus suggested to use in FE analysis, and proven to be able to predict the onset of localized necking of perforated sheet [6], while the von Mises isotropic yield criterion was applied in refs [3, 4, 8-10, 12]. In turns of the strain hardening behaviors of sheet metals, either the empirical equations [6, 9, 10, 12], or perfect plastic assumption [9, 12] were adopted in numerical analysis, which might diverge from actual material behaviors. Homogeneous displacement boundary conditions were usually assumed in mechanical testing but were found different from some experimental observations [8, 9].

Besides material behaviors, the effectiveness of numerical prediction also relies on the accurate geometric characterization of perforated sheet metals, as the plastic behavior is a highly localized phenomenon influenced by the local geometric disorder [6]. For refs [6, 8-10], the FE models of perforated sheet metals were generated based on perfect perforation patterns. Manufacturing introduced variations including circle centroids, size, and shape were completely excluded, which may greatly affect the strain localization [10, 11]. The image-based meshing technique presents a realistic and accurate process to discretize the perforated sheet metal based on specimen images. This image-based finite element modeling technique is proven to be able to assess the mechanical behaviors of materials and structures fully considering their microarchitectures [13-15].

This work presents a mechanical characterization of perforated sheet metals under uniaxial tension using the image-based FEM method. Specimens with four perforation patterns were first fabricated from anisotropic sheet metals (AISI 1018 low carbon steel, C260 Cu-Zn brass) and mechanically tested. The technique of image-based FE modeling was applied to study the plastic deformation of perforated sheet metals. Geometric characteristics of perforated sheet metals captured by digital images were discretized into FE models using an image-based mesh generation technique [16]. The effects of meshing methods, yield criteria, and displacement boundary conditions were discussed. The numerical prediction including material strength, strain localization was compared with experimental results. Possible uncertainties and enhancements of this image-based finite element modeling were also discussed.

2. Material and Method

2.1. Sample Preparation

Rectangular blank specimens were sheared from rolled thin sheet metals (C260 Cu-Zn brass, AISI 1018 low carbon steel) with a width of about 25.4mm and a total length of 200mm. The axial direction of tensile coupons was aligned with the rolling direction of sheet metals. Four perforation patterns within the central rectangular area of 34.9 mm×25.4 mm were designed, named as regular square array (RGS), regular triangular array (RGT), random square array (RDS) and random triangular array (RDT) with distorted middle column perforation. The diameter of circles d equals 2.5mm. The circles located at least 3.05 mm and 2.7 mm, larger than one diameter, away from the specimen edges of RGS and RDS perforation patterns, respectively. The spacing between columns of RGS and RGT l =5.6mm. The distance between center lines of perforations 2R is 5.6mm for RGS and respectively 5mm for RGT. The nominal area porosity of perforation patterns ρa within the interested area (34.9mm×25.4mm) is calculated as 11.07%. For each of the four patterns of three materials, at least three identical samples were prepared. Samples were designated using the initial of materials (B: brass and S: steel), following the perforation patterns (RGS, RDS, RGT, RDT) and the serial number of tested samples with the same material and perforation pattern (_1, _2, etc). Digital images (1600×1200 pixels) of the perforated region were recorded by Keyence VHX-500 digital microscope.

2.2. Image-Based Finite Element Modeling

An image-based meshing was applied based on specimen images of resolution 1600×1200 pixels. The unstructured mesh generator DistMesh2D based on the Delaunay triangulation and force-equilibrium algorithms [16] were employed to perform the element discretization. An essential implementation of DistMesh2D is to represent the interested geometry using a signed distance function φ(x, y). In
mathematics, the signed distance function of a set \( \Omega \) in a metric space determines the closest distance of a given point \( x \) from the boundary of \( \Omega \), with a negative sign if \( x \) is in \( \Omega \). The signed distance function can be mathematically defined using one or more explicit functions for simple geometries. The function \( \phi(x, y) = \sqrt{x^2 + y^2} - 1 \), for instance, is the signed distance function of a unit circle.

Images are special cases of implicit geometry definitions since the boundaries of objects in the image are not available in an explicit form. A natural approach for image-based meshing is to identify the interested objects from the captured image and form a signed distance function implicitly with a level set to represent the boundaries. An algorithm of Active Contour without Edge [17] was applied to segment the interested object from the captured image which results in a new binary image with a value 1 or 0 to represent the pixel if inside or outside the object domain. The object boundaries in the binary image were traced and the interface evolution of closed boundaries or the unsigned distance function was then calculated by a numerical technique of fast marching method [18]. By comparing this interface evolution with the segmented binary image, the signed distance function \( \phi(x, y) \) was obtained.

Mesh size function \( h(x, y) \) specifying the size of generated elements (the length of element edges) is another key implementation for image-based meshing. It is necessary for the FE model to have dense elements to accurately capture the local mechanical phenomenon. A good mesh size function should be able to resolve the geometry features, including curved boundaries and narrow regions, and also maintain a proper gradient of mesh sizes among neighboring elements. Definition of mesh size function was detailed by Persson [19]. The triangular mesh generation is carried on by inputting the signed distance function \( \phi(x, y) \) and mesh size function \( h(x, y) \) into DistMesh2D once ready. The area porosity of discretized FE model \( \rho_a \) was calculated and compared with the result based on the sample image. Remeshing was possible to maintain the same area porosity of the individual sample by adding a small constant to the signed distance function. An example of image-based meshing of BRDS_3 from the captured image to mesh generation is shown in figure 2.
Nonlinear numerical analysis of all 24 perforated plates under uniaxial tension using a two-dimensional plane stress model was performed using commercial FE code ANSYS v12.1. Besides the generated mesh by the aforementioned image-based meshing technique, the meshing based on the four perforation designs was also input into ANSYS. Four-node structure element PLANE182 (triangular degeneration is allowable) was used with full integration and a large strain effect. The quadratic Hill’s flow function [20] and multi-linear isotropic strain hardening represented by more than 20 discrete data points were used to characterize rolled sheet metals. The true stress-strain curves of sheet metals along the rolling direction were obtained from standard uniaxial tension tests. The post-necking strain hardening behaviors were then calibrated by a local correction factor method [21] and shown in figure 3. The plastic strain ratios $R_{90}$ brass and steel were found to be 0.77, and 1.55, respectively using digital image correlation-based strain measurement. The plastic strain ratios $R_{90}$ calibrated as 1.04 and 1.23 respectively in [22].

The uniaxial tension boundary conditions were applied as follows: at the left edge $x=0$, $u_x=0$ at all nodes; at the right edge $x=34.9\text{mm}$, uniform axial displacement $u_x=d$ at all nodes with constant increment. The numerical simulation was terminated once the maximum effective strain of the element becomes
larger than the input strain hardening behavior. No failure model was considered since the material response of perforated plates far beyond the peak load was not concerned in this work.

Figure 3. The input stress-strain relation of sheet metals.

2.3. Experiment Procedures
To validate the image-based FE modeling, a set of experimental tests was carried out. The tensile tests of perforated plates were conducted using a Model 5582 Instron Universal Materials Testing System (MTS) at a constant cross-head speed of 0.6mm/min for brass, and 1mm/min for steel. Each sample was stretched until the post-necking force level fell down to 30% of its peak load. Before testing, one surface from the interested perforated area of each tensile coupon was decorated with fine black-and-white paint spackles to facilitate the digital image correlation (DIC) based surface strain measurement. Digital images of the tension coupon with the paint speckles were captured during tensile testing by a CCD camera with an acquisition rate of around 2 frame/sec. The %elongation of the interested perforated region over the centered gage length 34.9mm was measured by non-contact DIC-based video extensometer. The obtained %elongation was then synchronized in time with the applied axial load recorded by MTS. The nominal engineering stress was defined by dividing the applied axial load with the initial cross-section area of test coupon assuming with no perforation, and thus the nominal engineering stress-strain relations were obtained.

3. Results and Discussion
3.1. Experimental Results
The typical force-displacement relation of perforated sheet metal is shown in figure 4, also included are the captured images at different displacement loadings. The solid and open circles indicate the peak load and crack initiation points where the crack is detected from captured images. The failure path of samples is shown in figures 5 and 6 on un-deformed designs. It is assumed that the failure path lies upon line segments that connect the center of circles, and the first and last line segments of the path are vertical lines that connect the center of a circle to the edge of the sample [8, 9]. The area porosity of the failure path is defined as

$$\rho_{fr} = \frac{n \cdot D}{L}$$

where \(n\) is the number of circles on the fracture path, \(D\) is the diameter of circle and \(L\) is the total length of fracture length. It is straightforward to understand that fracture occurs on the path of higher area porosity. It is clear to see that for specimens of regular square or triangular perforation, the failure happens (or would happen) in a single vertical column (for triangular perforated specimens, the tests terminated before completely fracture). In this way, the area porosity of the failure path is calculated as \(\rho_{fr}=0.38\). The failure path of randomized square array perforation is identical for all specimens except two steel samples. The area porosity is 0.48 for all failure paths of
randomized square array perforation, making the randomized middle column predominant for ductile fracture compared with the single vertical column.

Figure 4. Force displacement relation of specimen SRDS_2 with captured images during test.

Figure 5. The failure path of perforated brass samples with (a) regular square array, (b) randomized regular square array, (c) regular triangular array, and (d) randomized regular triangular array. Different colors and line types represent different samples.
Figure 6. The failure path of perforated steel samples with (a) regular square array, (b) randomized regular square array, (c) regular triangular array, and (d) randomized regular triangular array. Different colors and line types represent different samples.

3.2. Numerical Results

3.2.1. Effect of mesh generation methods. A mesh sensitivity study was conducted to analyze the influence of meshing methods on mechanical response predicted by FE modeling. Figure 7.a compares the nominal stress-strain relations of sample SRDS_2 using different mesh generation methods. These methods are image-based meshing using images of perforated sheet metals and perforation design, and ANSYS-based meshing using perforation design. The numerical results from FE modeling using specimen image gives the best approximation of experimental results. The numerical results from FE modeling using either image-based or ANSYS-based mesh generation based on pattern design are obviously overestimated (figure 7.a). The predicted ultimate strength from these three modelings are 0.6%, 2.6%, and 2.8% higher than corresponding experimental measurements, respectively, at the strain level of 5.1%. The cross-section area porosity along x (loading) direction $\rho_{ax}$ of sample SRDS_2 (figure 7.b) indicates to be higher than designed porosity. This is the major reason that FE prediction using design-based meshing tends to overestimate the ultimate strength. The predicted strain localization based on different mesh generations shows similar features in locations and adjacent connections, though small variations in strain magnitude exist among FE analyses.

3.2.2. Effect of the mesh size. To examine the element dependency, three different image-based mesh generations of sample SRDS_2 were generated based on the digital image with 12776, 20136, 36188 elements, respectively. The nominal stress-strain relations predicted by FE analysis based on three mesh generations from 2D uniaxial tension simulations were compared in figure 8.a. The results based on fine mesh discretization give a slightly overestimation of nominal stress-strain relation, while the results calculated using coarse and medium meshes predicted even higher stress level. The error of predicted ultimate strength in comparison to experimental ultimate strength is 0.4%, 0.8%, and respectively, 1.3% higher. The accuracy of FE modeling depends on both the element size and the material properties. The fine mesh generation was therefore selected, so the combined effect of element size and material properties is consistent. The image-based mesh generations of all perforated plates were of similar node
and element numbers. The smallest size of the triangular meshes (the edge of a triangle) is around 0.15mm and each round circle is discretized by at least 50 elements. In addition, three-dimensional (3D) finite element modeling was performed by offsetting the 2D image-based mesh with 12 increments (12 elements through-thickness) to generate a full representation of samples BRDT_3 and SRDS_3. The nominal engineering stress-strain relations obtained from 3D FE modeling did not significantly differ from those of the plane stress models within the interested strain range, see figure 8.b. All of the reported results, therefore, are from the plane stress cases.

![Figure 7](image7.png)

**Figure 7.** (a) nominal stress-strain relations of sample SRDS_2 from FE modeling using different methods, (b) cross-section area porosity along x (loading) direction.

![Figure 8](image8.png)

**Figure 8.** Nominal stress-strain relations affected by (a) element size of image-based meshing of sample SRDS_2, (b) 2D and 3D models.

### 3.2.3. Effect of boundary conditions

One problem during experiments was that as strain localization progressed, the specimen might rotate [8, 9]. However, the applied uniaxial tension BCs in FE modeling excluded this rotation effect. To quantify this effect, the actual edge displacement of perforated plates BRDT_3 on the left and right edges of interested area with gage length 34.9mm were measured via DIC between image pairs #63 and #503. The measurements indicate that the bottom edge was stretched more than the top edge (figure 9.a). These actual BCs were then applied in FE modeling node-by-node using image-based meshing. Only the axial elongation was considered since the relative displacement in the vertical direction over 34.9mm gage length is about 3% or less of axial deformation. The predicted strain localization adding uniaxial tension BCs and actual BCs displayed in figures 9.b and c, and compared with the #503 images (the corresponding image at star symbol). This comparison shows that the numerical simulation with the actual BCs could give relatively consistent prediction of strain localization with the experimental results (see the red rectangles in figure 9.b). The global stress-strain response from FEA using actual BCs matches the experimental results very well, while the results based on uniaxial tension BCs tends to slightly underestimate the material strength when strain is larger than...
2.5%. The predicted ultimate strength by applying these actual BCs is 0.2% higher than the experiment, and 0.6% lower if using uniaxial tension BCs at a strain of 3.5%.

![Figure 9](image)

Figure 9. Influence of boundary conditions: (a) the actual boundary condition measured by DIC, (b) strain distribution $\varepsilon_x$ of sample BRDT_3 at peak load point by applying (b) uniaxial tension and (c) the actual BCs, (d) the corresponding specimen image with fracture path.

### 3.2.4. Effect of yield criterion

The predicted stress-strain relations of sample SRDS_2 using von Mises yield criterion and quadratic form of Hill yield criterion were compared in figure 10. The stress-strain relations of steel calibrated by the local correction factor method [21] based on corresponding yield criteria were set as material inputs. The results indicate that Hill’s model gives a slight overestimation of nominal stress-strain relation compared with the experiment, while the von Mises criterion tends to underestimate the mechanical response. The variation in predicted ultimate strength compared with the experimental result are respectively 0.6% and -0.9% from FE analysis using Hill and von Mises criteria. The strain localization is similar in location and connection, though the results using the von Mises criterion gives slightly higher strain magnitude. The Hill yield criterion is found slightly better than the von Mises criterion to describe the mechanical behavior of perforated sheet metal due to its material and geometric anisotropy.

![Figure 10](image)

Figure 10. Numerical simulation of SRDS_2 using Hill and von Mises yield criteria.
3.3. Plastic Deformation

Figure 11 compares the experimental and numerical nominal stress-strain relations of all tested samples (left column), the predicted strain localization of selected 8 (one sample per material and perforation pattern) at the strain level of star symbol in nominal stress-strain relations (middle column), and the corresponding captured images during the experiment (right column). It is generally found the predicted nominal stress-strain relations are in good agreement with the experimental results. The failure path in the perforated sheet metals is the strain localization of adjacent circles corresponding to the regions of high magnitude plastic strain [8, 9]. These regions coincide with the actual fracture path of selected samples (see the solid line in the right column images). As is evident, the image-based FE modeling is able to capture the major features of strain localization of perforated sheet, including the locations and connection between each other, and also effectively predict the global mechanical response of perforated sheet very well until slightly beyond the peak load.

![Figure 11](image)

Figure 11. Nominal stress-strain relations (left), strain localization $\varepsilon_x$ (middle) and corresponding deformed sample images (right) at the star symbol of samples. (a) BRGS_1, (b) BRDS_3, (c) BRGT_1, (d) BRDT_3. The solid lines in right columns are the failure path of selected samples, while the dash and dot lines are for other samples with same the perforation.
Figure 12 (Continued). (e) SRGS_1, (f) SRDS_2, (g) SRGT_1 and (h) SRDT_2.

The predicted ultimate strength of each test of image-based FE modeling agrees well with experimental results. The relative error in predicted ultimate strength of image-based FE modeling based on sample images and FE modeling using perforation pattern designs varies from -1.12%~2.28% and respectively 1.28%~5.56% in comparison with experiments. This indicates that the ignorance of actual microarchitecture variation including location, shape and size of circle perforation would result in a 1%~4% error in the prediction of ultimate strength. Similarly, the predicted strain at ultimate strength, elastic modulus, and yielding stress also show a linear correlation with experimental results.

3.4. Errors and Uncertainties

The level of agreement between numerical and experimental results validates the effectiveness of the image-based FE modeling in the mechanical determination of perforated sheet metals considering the microarchitecture features. The differences between experiment and numerical analysis, for instance in ultimate strength, are -1.12%~2.28%. This is caused by errors or uncertainties in experimental measurement and image-based FE modeling. Experimental errors or uncertainties include measurement of axial load, sample geometry, nominal strain, and manufacture-introduced material damage. In the
numerical modeling aspect, the errors or uncertainties are the material characterizations of sheet metals, microarchitecture representation of samples, and simplification of computational models.

Based on the measurement precision in axial load, specimen geometry, the error in nominal stress is calculated at about 0.4%. The measurement error of axial strain by DIC-based video extensometer was estimated to be better than ±0.005%, and thus is negligible [23]. The effect of residual stress due to mechanical manufacture, however, is hard to quantify. For plastically deformable materials, residual stresses may accelerate or delay the initiation of local yielding, which in turn determines the final strain localization of samples, especially with regular perforation patterns. It was reported that annealing treatment is able to relieve or even remove the residual stress [8], but not applied in this work due to lack of annealing condition information.

Material characterization can greatly affect the accuracy of image-based FE modeling. The strain hardening behaviors of sheet metals were identified to a large strain level by the local correction factor method, with an error level of about 0.5% [21]. The anisotropic parameters of Hill’s yield criterion in ANSYS rely on measurements of plastic strain ratio along rolling direction $R_{45}$, and transverse direction $R_{90}$ of sheet metals. In this work, $R_0$ is directly measured. $R_0=R_{45}$ was initially assumed, and $R_{90}$ was calibrated in ref [22]. On the other hand, the yielding behavior of sheet metals may not follow the quadratic form of Hill’s plastic yield criterion. Non-quadratic form anisotropic yield function has been proved to be able to accurately describe the material behavior of aluminum [24-25]. Parameter calibration and program implementation are, however, needed. A better understanding of the mechanical response of sheet metals is thus necessary.

The microarchitecture discretization of perforated sheet metals using image-based meshing technique also introduces errors or uncertainties into image-based FE modeling. For each step of image-based meshing, like image segmentation, edge detection, calculation of signed distance function and mesh size function, and mesh generation, partial information of samples in the digital image may be lost. Inaccurate input of each step would then generate new errors in the output. The area porosity of sample was guaranteed in mesh generation, the variation of local microarchitecture feature is necessary but difficult to quantify. Meanwhile, modeling strain localization using the FE method introduces an artificial length scale into modeling once strain localization occurs which in turn makes the FE solution mesh dependent [11]. Though the strategy to maintain the similar element size in this work can partially relieve the mesh dependency within interested certain strain range, advanced measures should be applied to relief or even eliminate the element size effect [26].

4. Conclusion

In this work, perforated sheet metals of AISI 1018 low carbon steel, C260 Cu-Zn brass were fabricated with four different perforation patterns and tested under uniaxial tension. An image-based FE modeling technique was employed to numerically investigate the plastic behavior of perforated sheet metals under tension. The effects of mesh generation methods, element size, displacement boundary conditions, and yield criterion on the numerical results were assessed by comparing with the experiment measurement. The predicted nominal stress-strain relations until slightly beyond the ultimate strength as well as the strain localization obtained from image-based finite element modeling closely matched experimental results. The predicted ultimate strength well correlated with the corresponding experimental measurement. This is remarkable given that the applied image-based FEM technique in this work considered accurate characterization of material behavior and realistic representation of microarchitecture of perforated sheet metals. It is of great potential to extend the image-based FE modeling to explore material behavior of porous materials, multi-phase composites of complicated microarchitectures.

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