A Poincaré-Covariant Parton Cascade Model for Ultrarelativistic Heavy-Ion Reactions

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Abstract

We present a new cascade-type microscopic simulation of Nucleus-Nucleus Collisions at RHIC energies. The basic elements are partons (quarks and gluons) moving in 8N-dimensional phase space according to Poincaré-covariant dynamics. The parton-parton scattering cross sections used in the model are computed within perturbative QCD in the tree-level approximation. The $Q^2$ dependence of the structure functions is included by an implementation of the DGLAP mechanism suitable for a cascade, so that the number of partons is not static, but varies in space and time as the collision of two nuclei evolves. The resulting parton distributions are presented and compared to distributions obtained with the VNI model.

1 Talk given by V. Börchers in Quark Matter ’99, May 13, 1999, Torino, Italy. For the transparencies, see http://www.qm99.to.infn.it/program/qmprogram.html
1 Introduction

For a long time nuclear reactions have been described by phenomenological hadronic models quite successfully. At nuclear collision energies of the order of $10\text{GeV} \cdot \text{A}$ it is already clear that partonic degrees of freedom will not be negligible; thus pure hadron cascades do not suffice in the SPS regime.

Parton cascade models on the other hand provide a microscopic description of high energy collisions in terms of partons. The QCD-model of hard scatterings, the theoretical basis of parton cascades, limits them to reactions with high transferred momentum. The large number of involved particles (nucleons) in heavy ion collisions lets one assume that not all single nucleon-nucleon collisions will be hard. However, for sufficiently high energies of the incident nuclei one can expect that hard collisions are dominant.

In the framework of a parton cascade non-perturbative effects can be either completely neglected or treated phenomenologically. For the model to be presented here we have chosen the first alternative, although this makes the model not applicable in the SPS energy regime.

The first parton cascade model for heavy ion collisions, HIJING [1], was formulated in momentum space only, and thus did not include a space-time description of the cascade evolution. A full phase-space description was first provided by Klaus Geiger’s VNI [2, 3], but this model is not manifestly Poincaré-invariant.

A description of the space-time evolution of the system is imperative if one aims to include effects due to the high density produced in the $A - A$ collision.

2 Parton cascades and Poincaré-covariance

The obvious way to enhance a parton cascade with a space-time description would be to choose an observer Lorentz frame and then demand that two particles $i$ and $j$ interact instantaneously as seen in this frame. It is clear that such a procedure makes the ordering of the interactions frame-dependent. This has been recognized as a problem of cascades for more than 15 years: it was found in hadronic cascades even at low energy ($\approx 1 \text{GeV} \cdot \text{A}$) that the collision sequences and the resulting distributions depend noticeably on the reference frame (LAB or CMS) in which the simulation is run [4, 5].

The frame-dependence gets worse if one also makes the interaction criterion dependent on the spatial distance of particles\(^2\).

If, on the other hand, we insist on Poincaré-covariance we run into trouble because of the No Interaction Theorem [6], which asserts that interacting particles cannot be described Poincaré-covariantly within a classical 6N-dimensional Hamiltonian theory of point particles.

2.1 Poincaré-Covariant Dynamics

The solution of this problem chosen in the present work is the Poincaré-Covariant Dynamics (PCD) approach described previously [7]. In this approach the phase space is extended to

\(^2\text{It may be worthwhile to point out an additional frame dependence inherent in any description involving partons: the parton concept itself is frame-dependent, since it is well-defined in the infinite-momentum frame only.}\)
8N dimensions (N is the number of particles). Position and momentum vectors, \( r_i^\mu(s) \) and \( p_i^\mu(s) \) are parametrized by one Lorentz-invariant parameter \( s \) (which has no obvious physical meaning).

The particle motion and interactions are determined by the Lorentz-scalar ‘Hamiltonian’

\[
H = \sum_{i=1}^{N} \frac{m_i^2 - p_i^2}{2m_i} + V,
\]

where \( V \) is a Lorentz-scalar pseudo-potential. The equations of motion are

\[
\frac{d}{ds} r_i(s) = \{H, r_i \} = -\frac{\partial H}{\partial p_i},
\]

\[
\frac{d}{ds} p_i(s) = \{H, p_i \} = +\frac{\partial H}{\partial r_i}.
\]

Since the evolution proceeds in the invariant parameter \( s \), every particle carries its own time \( t_i = r_i^0(s) \). Note that in this formalism particles are classically off-shell when within the range of the quasi-potential \( V \).

### 3 Dynamics of the model

Applying the concept of PCD to the Poincaré Covariant Parton Cascade (PCPC), we simplify the dynamics by the following prescriptions:

- Partons are free between collisions.
- Binary interactions between partons \( i \) and \( j \) can occur when their 4-distance \( d_{ij} \) defined as

\[
d_{ij} := \sqrt{-\left( r - \frac{r \cdot p}{p^2} p \right)^2} \quad \text{with} \quad r := r_i - r_j, \quad p := p_i + p_j
\]

has a minimum and will occur if this distance is smaller than the interaction distance \( \sqrt{\sigma_{\text{tot}}/\pi} \). Note that \( d_{ij} \) is an Lorentz-invariant way of writing the spatial (3-)distance in the center of momentum frame.

Since the interaction criterion depends on Lorentz scalars only, the ordering of interactions is frame-independent.

### 4 Initialization

For setting up an initial state of the cascade, we define initial parton distributions in two steps, successively resolving the nuclei into nucleons and the nucleons into partons. First nucleonic Woods-Saxon distributions (in momentum and coordinate space) are used, and
the nucleons are boosted to the rapidities appropriate for the particular collision. Then the flavor and momentum of each parton are determined as follows: the parton momenta are

\[ p' = \left( \sqrt{(xP_z^{\text{nuc1}})^2 + p_{\perp}^2 + \mu^2}, p_{\perp}, xP_z^{\text{nuc1}} \right). \]

The longitudinal momenta \( p_z = xP_z^{\text{nuc1}} \) of the partons are determined, together with their flavors, from the parton distribution functions \( f(x, Q^2) \) in the GRV94LO \(^8\) parametrization. The distribution functions are evaluated at a fixed (and small) \( Q_0^2 \), i.e. at a fixed scale. The parton transverse momenta \( |p_{\perp}| \) are Gauss-distributed (mean value 0, width 0.3 GeV).

The one free quantity remaining is the parton’s off-shell mass \( \mu = \sqrt{m^2 - q^2} \). This is fixed by the fact \( |\vec{\beta}_{\text{parton}}| = |\vec{\beta}_{\text{nucleon}}| \). This feature guarantees that partons originating from the same mother nucleon move together initially, thus modeling the constraint that initially partons are confined in the nucleons. Note how well PCD with its concept of potential-dependent masses is suited to describe such field-theoretic features in the classical framework of a cascade model.

The parton spatial coordinates \( r_i \) are distributed exponentially in the rest system of the nucleon. The boosted distributions are thus Lorentz-contracted automatically, and ad-hoc modifications of the spatial parton distributions (viz. “Distributed Lorentz Contraction” or “boost invariant sea parton distributions”) that were proposed to enhance the interaction rate are not needed in this model. Due to the covariant formulation of our model they do not significantly influence the cascade evolution.

5 Parton Interactions

As stated in (Sec. 3), partons \( i \) and \( j \) may interact at \( s = s_{ij} \) if their invariant distance \( d_{ij}(s) \) as defined in (1) has a minimum value. At this moment the interaction scale \( Q^2 \) (taken to be \( -(t+u) \)), and from that the relevant total cross section \( \sigma_{ij} \) for the two partons is determined. \( Q^2 \) is not only the scale at which the running QCD coupling constant is evaluated but also decides if the scattering is “hard”. Then the total cross section is determined from QCD in tree level approximation with inclusion of massive quarks. Only if then \( d_{ij} \leq \sqrt{\sigma_{ij}/\pi} \), the scattering actually occurs.

Although our model is based essentially on binary hard scatterings, some non-perturbative effects are included in the case in which the interaction scale \( Q_h^2 \) of two partons is larger than the scale \( Q_0^2 \) at which the partons were initially resolved. In that case the scale is adjusted via a “DGLAP scale evolution” \(^8\). This way \( 2 \to n \) events are effectively incorporated. In the picture below the scale evolution of parton \( a \) from the initial scale \( Q_0^2 \) up to the interaction scale \( Q_h^2 \) is depicted. The resolution scale is increased by successively radiated secondary partons, and it is the final parton \( b \) which is subjected to the binary scattering process.\(^3\)

\(^3\)Note the numbering: A larger scale means higher resolution, i.e. more partons.
6 Results

As numerical results we show (Fig. 2.) rapidity and transverse momentum distributions for central \( \bar{p}-p \) and \( Au-Au \) collisions at RHIC energies (at \( \sqrt{s} = 200\text{GeV}\cdot\text{A} \)). They were calculated with a Monte Carlo simulation based on the described model. Since the model in its present version does not include a hadronization mechanism, we do not compare our results with experimental data, but rather with a different parton cascade model. We have chosen the above-mentioned VNI code by Klaus Geiger \[14\], version 4.12 with hadronization disabled (partons only), and soft interactions switched off.

In the overall features VNI and PCPC agree reasonably well. For \( \bar{p}-p \) both models show a marked dip at midrapidity (as is to be expected). For \( Au-Au \) this dip is smeared out in our results, but, surprisingly, in VNI it remains essentially unchanged. Note also the significant differences in the transverse momentum distributions at low \( p_\perp \).

In comparing these results, it should be noted that in VNI the hadron distributions obtained in runs with the hadronization mechanism switched on, are markedly different from the parton distributions shown here.

7 Summary

The Poincaré-Covariant Parton Cascade Model (PCPC) as presented here is still incomplete. In this stage, it incorporates the following features:

- new cascade-type microscopic simulation model of Nucleus-Nucleus Collisions at RHIC energies
- basic quantities are partons (quarks and gluons) moving in 8N-dimensional phase space according to Poincaré-Covariant Dynamics (PCD)
- Interactions: hard parton-parton scatterings with cross sections determined from perturbative QCD with a DGLAP scale evolution.

What is left to do next is:

- to include a hadronization mechanism
- to add a thermodynamic analysis of results in space and time
- to include shadowing effects, e.g. by using the EKS98 parametrization of parton distribution functions \[17\].
These are currently under development.

The PCPC code [in C++] is obtainable from
http://hix.physik.uni-bremen.de/tpk.

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Figure 1: Comparison of the PCPC and VNI parton distributions for central collisions at $\sqrt{s} = 200$ GeV·A: $\bar{p}-p$ (top panels) and $Au-Au$ (bottom panels). The transverse momentum distributions $\frac{1}{N_{p\perp}} \frac{dN}{dp_{\perp}}$ are displayed on the left (cut: $|y| < 1$), rapidity distributions $\frac{1}{N} \frac{dN}{d|y|}$ on the right.