Nuclear mass parabola and its applications

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Abstract

We propose a method to extract the properties of the isobaric mass parabola based on the total double β decay energies of isobaric nuclei. Two important parameters of the mass parabola, the location of the most β-stable nuclei $Z_A$ and the curvature parameter $b_A$, are obtained for 251 $A$ values based on the total double β decay energies of nuclei compiled in NUBASE2016 database. The advantage of this approach is that one can efficiently remove the pairing energies term $P_A$ caused by odd-even variation, and the mass excess $M(A, Z_A)$ of the most stable nuclide for mass number $A$ in the performance process, which are used in the mass parabolic fitting method. The Coulomb energy coefficient $a_c = 0.6910$ MeV is determined by the mass difference relation of mirror nuclei $0.5b_A(A-2Z_A) = \Delta_{1H-n} + a_c(A^{2/3} - 1.0583c)$, and $c = 1.1914$ MeV. The symmetry energy coefficient is also studied by the relation $a_{sym}(A) = 0.25b_A Z_A$.

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I. INTRODUCTION

The nuclide refers to a nucleus as characterized by the number of protons ($Z$) and neutrons ($N$) that the nucleus contains. A chart of nuclides is formed by listing the known nuclei, both stable and radioactive, in an array on a graph of $Z$ versus $N$. If perpendicular to the plane in which a nuclide is located the atomic mass is plotted as a third dimension, the resulting mass surface has the shape of a valley. An isobaric ($A =$ constant) slice through this mass surface yields roughly a parabola which is usually referred to as a Bohr-Wheeler parabola [1]. The isobars located on the sides of the parabola are unstable to radioactive decay to more stable nuclides lower on the parabola, though usually the most stable nucleus is not located exactly at the minimum $Z_A$ of the parabola. Nuclides on the low $Z$ side of the parabolic minimum $Z_A$ decay by $\beta^-$ emission toward the minimum. Nuclides on the high $Z$ side of the minimum $Z_A$ decay in the opposite direction toward the minimum, the decay being either by $\beta^+$ emission or electron capture. The term “$\beta$ decay” is used here to cover disintegration by emission of $\beta^-$ and $\beta^+$ particles and by capture of electrons. The $\beta$ decay energies is the maximum value in the $\beta$ energy spectrum. Indeed, it follows from the parabolic mass dependence that the $\beta$ decay energy is directly proportional to $Z - Z_A$.

Because of the extra stability associated with pairs of like nucleons an isobaric slice through the mass surface at an even $A$ value tends to yield two parabolas of the same shape but displaced one below the other. On the lower parabola lie the even-even nuclides, on the upper parabola lie the odd-odd nuclides. Consequently the $\beta$ decay energies plotted versus $Z$ lie not on a single straight line but on a pair of parallel straight lines.

Isobaric analysis of $\beta$-decay energies have been made by several authors [1–3], the early work necessarily being based on scanty experimental data. In Ref. [1], Bohr and Wheeler had used the least squares method to determine the parameters of the mass parabola – the location of the most $\beta$-stable nuclei $Z_A$, the curvature parameter $b_A$ and odd-even mass difference $\Delta E$ for 20 $A$ values by three-parameter parabola fitting to the isobaric mass excess. In Ref. [2], J. W. Dewdney had analyzed the isobaric $\beta$ decay energies and used the least squares method to determine the parameters of the mass parabola for 157 $A$ values. The experimental data are taken from the Refs. [4, 5]. Later, X. Y. Li and co-workers [3] updated the three parameters of isobaric mass parabola for 234 $A$ values ($10 \leq A \leq 253$) in the same manner as Dewdney but adopted the different mass table AME1977 [6] including
about 1000 nuclides. In recent decades, with the development of experimental instruments and the progress of science technology, a large number of unstable nuclei can be produced in a single experiment in their ground and/or isomeric states, and their masses can be measured with high precision. The experimental information or recommended values for nuclear and decay properties of 3437 nuclides in their ground and excited isomeric states are compiled in the mass table of the NUBASE2016 [7]. The number and precision of nuclear mass in NUBASE2016 have highly increased in comparison with the results compiled in the AME1977 database [6]. The data now available are so much more extensive that it is possible to analyse them in a statistical way.

Double β decay has been and continues to be a popular topic, which is a rare transition between two nuclei with the same mass number that changes the nuclear charge number by two units. The double β decay is a second-order weak process of the form \( (A, Z) \rightarrow (A, Z+2) \) that has been observed in many nuclei [8, 9]. It has been long recognized as a powerful tool for the study of lepton conservation in general and of neutrino properties in particular. Because the lifetimes of ββ decay are so long greater than 10^{17} years, the experimental study of ββ decay is particularly challenging and has spawned a whole field of experiments requiring very low background. The energy release to be expected in double β decay shows even greater regularity than that of single β-decay as shown in Ref. [10], but detailed studies of the properties of the mass parabola have not been performed by using the total double β-decay energies. In this paper we use more than 2400 total double β decay energies, which are complied in the NUBASE2016 database, to analyze the properties of the Bohr-Wheeler isobaric mass parabola by all modes of double β decay in the theoretical method. The advantage of this approach is that one can efficiently remove the pairing energies term \( P_A \) caused by odd-even variation, and the mass excess \( M(A, Z_A) \) of the most stable nuclide for mass number \( A \) in the performance process, which are used in the mass parabolic fitting method. Furthermore, the over-all simplicity of the double β decay energy pattern may point the way to a convenient empirical mass formula.
II. THEORETICAL FRAMEWORK

Bohr and Wheeler [1] expressed the semi-empirical formula for the mass surface in the form for constant $A$,

$$M(A, Z) = M(A, Z_A) + \frac{1}{2} b_A (Z - Z_A)^2 - P_A - S(N, Z), \quad (1)$$

Where $M$ is the mass excess of the nucleus $(A, Z)$ (nuclidic mass minus mass number), $Z$ is as usual the proton number and $A$ the total number of nucleons in the nucleus. $M(A, Z_A)$ the mass excess of the stable nuclidic $Z = Z_A$ for mass number $A$, $b_A$ a measure of the curvature of the isobaric mass section, $Z_A$ the charge (not necessarily an integer) of the most $\beta$-stable isobar, $P_A$ the pairing energy due to the odd-even variation, and $S(N, Z)$ the shell correction term.

The $Q$ value is defined as the total energy released in a given nuclear decay. The $Q$ value of $\beta$ decay is calculated by the mass difference between the two nuclei $(A, Z)$ and $(A, Z + 1)$, meanwhile, the expressions for $\beta$ decay energies may be derived from empirical mass equation Eq. (1) when shell correction be neglected. If such parabolas actually exist, the $\beta$ decay energies for constant values of odd $A$ will give precisely straight lines when plotted against the proton number. For even $A$ value, the $\beta$ decay energies tends to yield a pair of parallel straight lines. For negative $\beta$ decay and positive $\beta$ decay (or electron capture), one can combine them into one universal description,

$$Q_\beta = M(A, Z) - M(A, Z + 1) = -b_A (Z - Z_A + \frac{1}{2}) + \Delta E. \quad (2)$$

Because the reaction will proceed only when the $Q$ value is positive. Where “$Q_\beta > 0$” for $\beta^-$ decay occurs when the mass of atom $M(A, Z)$ is greater than the mass of atom $M(A, Z + 1)$; “$Q_\beta < 0$” for $\beta^+$ decay occurs when the mass of atom $M(A, Z + 1)$ is greater than that of $M(A, Z)$. The $\Delta E = 2P_A$ for even-even nuclides, the $\Delta E = -2P_A$ is for odd-odd nuclides, and for odd-A nuclides (i.e. even-odd and odd-even) $\Delta E = 0$.

In this paper our aim is propose a very simple empirical formula only depends on the basic parameters $Z_A$ and $b_A$. So we use the total double $\beta$ decay energies to analyze the properties of the mass parabola rather than total single $\beta$ decay energies. The parameters $M(A, Z_A)$ and $\Delta E$ are removed by the mass difference of $(A, Z)$ and $(A, Z + 2)$. We can obtain the universal expression to describing total double $\beta$ decay energies in the same manner above
mentioned, 

\[ Q_{2\beta} = M(A, Z) - M(A, Z + 2) = -2b_A(Z - Z_A + 1), \]  

(3)

where \( Q_{2\beta} > 0 \) for \( \beta^-\beta^- \), and \( Q_{2\beta} < 0 \) for \( \beta^+\beta^+ \). \((Z, A)\) will be called hereafter the “disintegrating atom” no matter whether the decay actually proceeds from \((A, Z)\) to \((A, Z + 2)\) or vice versa. The total double \( \beta \) decay energies are lie on the same lines for odd-odd and even-even nuclide for even-A values, and for adjacent odd-A and even-A values lie approximately on the same lines.

III. THE RESULTS AND DISCUSSIONS

A. The procedure of the calculated values of \( Z_A \) and \( b_A \)

We use the three equations Eqs.(1)-(3) to analyse the corresponding experimental data, i.e. mass excess, total \( \beta \) decay energies and total double \( \beta \) decay energies, both parameters \( Z_A \) and \( b_A \) of the isobaric mass parabola are obtained by using the least-squares fitting procedure. We obtain the optimal values \( Z_A \) and \( b_A \) are list in three attached files (see the attached files table 1-3). The calculated results are compared, and they are almost identical for three different methods to fit the corresponding experimental data.

But the first equation Eq.(1) at least need three parameters \( Z_A, b_A \) and \( M(A, Z_A) \) to fit experimental mass excess for odd-A nuclei. Here, \( P_A =0 \) is taken, in fact the values of \( P_A < 0.3 \) MeV for odd-A nuclei presented in Ref. [2]. For even-A nuclei, an isobaric slice through the mass surface tends to yield two parabolas of the same shape but displaced one below the other. On the lower parabola lie the even nuclides, on the upper parabola lie the odd nuclides. So it is necessary to unify the parameters \( Z_A \) and \( b_A \) for even-even nuclei and odd-odd nuclei, \( b_A = 0.5(b_A^{o-o} + b_A^{e-e}) \) and \( Z_A = 0.5(Z_A^{o-o} + Z_A^{e-e}) \), where \( b_A^{e-e} \) and \( Z_A^{e-e} \) for even-even nuclei, \( b_A^{o-o} \) and \( Z_A^{o-o} \) for odd-odd nuclei, respectively. Finally, the value of \( \Delta E \) is obtained by the difference of \( \Delta E = M^{o-o}(A, Z_A) - M^{e-e}(A, Z_A) \). For Eq. (2), it is similar procedure to that of Eq. (1), the value of \( \Delta E \) equals the half of the difference between two intercepts of a pair of parallel lines.

For the construction of the \( Q_{2\beta} \) formula we start by plotting the total double \( \beta \) decay energies \( Q_{2\beta} \) versus the charge number \( Z \) for 11 odd-\( A \) values \( 7 \leq A \leq 257 \) in Fig.1. The experimental data of \( Q_{2\beta} \) are taken from NUBASE2016 [7]. One can see clearly straight
FIG. 1: (Color online) Total double $\beta$ decay energies $Q_{2\beta}$ as a function of the charge number of $Z$ for 11 odd-$A$ values ($7 \leq A \leq 257$). The experimental data are taken from NUBASE2016 [7].

line relation in this plot, and the slopes $2b_A$ of these lines show the general trend of $Q_{2\beta}$ to decrease slightly with increasing mass number $A$. The similar linear relation is also shown for even-$A$ values in Fig. 2.

Fig. 2 shows two examples of the treatment of experimental data for $A = 131$ and $A = 132$ as a function of the nuclear charge number $Z$. The Fig.2 (a) is a plot of the experimental isobaric mass parabola at $A = 131$. The mass parabola imply the $\beta$ or $2\beta$ decay energies directly below (b). The intercept $Z_A$ on the $\beta$ decay energy graph corresponds to the minimum of the mass parabola. The slope $b_A$ of the $\beta$ energy graph is a measure of the steepness of the mass parabola. The red dashed lines in fig.2 (b) and (d) denote the results of the fitting experimental data $Q_{2\beta}$ by Eq. (3) transformation. The fig.2 (c) and (d) are similar plots but for $A = 132$. Fig. 2 (c) shown mass excesses lie alternately on a pair of parabolas of identical shape. The parabola on which the even masses lie falls a distance $\Delta E$
FIG. 2: (Color online) Two examples of experimental data and their treatment. The mass excesses (a), the $\beta$ decay energies (b) and the curves fitted to them for $A = 131$, (c) and (d) are same as (a) and (b) for $A = 132$, as a function of the nuclear charge number $Z$. The solid curves in Fig. 2 (a) and (c) denote the results of the fitting experimental mass excess by Eq. (1). The solid lines in Fig. 2 (b) and (d) denote the results of the fitting experimental data $Q_\beta$ by Eq. (2), and the dashed lines in Fig. 2 (b) and (d) denote the results of the fitting experimental data $Q_{2\beta}$ by Eq. (3).

below the odd parabola. This implies that $\beta$ decay points lie alternately on a pair of parallel straight lines, while the double $\beta$ decay points lie on a single straight line both odd-$A$ and even-$A$ values.

Fig. 3 presents the mass parabola parameters $Z_A$ (a), $b_A$ (b) and $\Delta E$ (c) as a function of nuclear mass number $A$. The calculated values of $Z_A$ and $b_A$ are shown in Fig. 3 (a) and (b) with the blue solid curves by using Eq. (3). The $\Delta E$ are plotted with blue open circles in Fig. 3 (c) calculated by Eq. (2). The solid curve $\Delta E = 2P_A$ is the result to fit the $\beta$
FIG. 3: (Color online) The mass parabola parameters $Z_A$ (a), $b_A$ (b) and $\Delta E$ (c) as a function of nuclear mass number $A$. The calculated values of them are compared with early results of Bohr and Wheeler [1] (the solid squares), Dewdney [2] (solid circles) and X.Y.Li [3] (solid triangles). The solid curve is the result to fit the $\beta$ decay energies.
decay energies, and the pairing energy term $P_A = 11.50\delta/A^{1/2}$ is obtained, where $\delta$ equals 0 for odd-A, +1 for even-even and -1 for odd-odd nuclides. The calculated values of them are compared with early results of Bohr and Wheeler [1] (the solid squares), Dewdney [2] (solid circles) and X.Y.Li [3] (solid triangles). There are currently known to be 358 beta-decay stable nuclides [11] plotted in Fig. 3(a) with open circles. We also find that Dewdney in Ref. [2] maybe make a mistake on $Z_0=44.912$ for $A=111$, and X. Y. Li et.al [3] make an almost identical mistake on $b_A=0.8401$ for $A=144$. Because the marked discontinuities occur only at the two $A$ values shown in Fig. 3. Our results are $Z_0=47.7566$, $b_A=1.7955$ for $A=111$, and $Z_0=59.9768$, $b_A=1.5102$ for $A=144$.

B. Determination of the Coulomb energy coefficient $a_c$ by the relation of $Z_A$ and $b_A$

In semi-empirical Bethe-Weizsäcker mass formula [12, 13], the binding energy $B(A, Z)$ of a nucleus can be expressed as a function of mass number $A$ and charge number $Z$,

$$ B(A, Z) = a_v A - a_s A^{2/3} - a_c Z^2 A^{1/3} - a_{\text{sym}} \frac{(N-Z)^2}{A} + P_A, \quad (4) $$

with $P_A = a_p \delta A^{-1/2}$, where the “$\delta=+1$” is for even-even nuclides, the “$\delta=-1$” is for odd-odd nuclides, and for odd-A nuclides (i.e. even-odd and odd-even) $\delta = 0$. The $a_v$, $a_s$, $a_c$, $a_{\text{sym}}$ and $a_p$ are the volume, surface, Coulomb, symmetry and pairing energy coefficients, respectively.

In the textbook, the relation between the nuclear mass excess and the binding energy is written as

$$ B(A, Z) = 931.4943 \times (0.008665 A - 0.00084 Z) - M(A, Z), \quad (5) $$

where 0.008665 is the mass excess of neutron and 0.00084 the hydrogen-neutron mass difference in atomic mass unit, and one atomic mass unit is equivalent to 931.4943 MeV.

Mirror nuclei mass relation is deduced based on three assumptions as follows: 1) The difference between the binding energies of mirror nuclei is only due to the Coulomb interaction. It is known that in the absence of Coulomb interactions between the protons, a perfectly charge-symmetric and charge-independent nuclear force would result in the binding energies of mirror nuclei being identical [14–16]; 2) The Coulomb energy difference between a pair of mirror nuclei is proportional to $Y = N - Z$, the same as the assumption used in Ref. [14],

$$ -\Delta B = E_C[A, \frac{1}{2}(A+Y)] - E_C[A, \frac{1}{2}(A-Y)] = b_c Y, \quad (6) $$
in which $b_c$ is the proportionality coefficient; 3) The coefficient of proportionality which indeed depends on $A$, may be considered as independent of $Y$ for a given $A$. In Refs. [14, 17], it was found that the $b_c$ coefficients are roughly constant (see the Tables 3-7 in Ref. [17]) for a given $A$, and Ormand obtained an empirical formula $b_c = 0.710A^{2/3} - 0.946$ MeV by fitting to 116 experimental data with an rms deviation of 102 keV [14].

On the other hand, the mirror mass relations can be obtained by Eq.(1). Firstly, one need to express charge number $Z$ with mass number $A$, that is to say, $Z = \frac{1}{2}(A \pm Y)$ for a pair of mirror nuclei. The difference of mass excess for a pair mirror nuclei for both odd and even values of $A$ with different $Y = 1, 2, 3, ...$ reads

$$\Delta M = M[A, \frac{1}{2}(A + Y)] - M[A, \frac{1}{2}(A - Y)] = \frac{1}{2}b_A(A - 2Z_A)Y. \quad (7)$$

Combining the three equations Eqs. (5)-(7), and taking the hydrogen-neutron mass difference $\Delta(^{1}H-n) = -0.7825$ MeV, we can obtain

$$\frac{1}{2}b_A(A - 2Z_A) - \Delta(^{1}H-n) = b_c. \quad (8)$$

This result is consistent with that in Ref. [18]. The calculation results are presented in Fig. 4, the mass differences of the 95 pairs of mirror nuclei, scaled by the charge difference $Y$, are plotted versus $A^{2/3}$ and are seen to lie on a straight line. The value of $Y$ ranges from 1 (32 cases) to 6 (1 case). The thick solid curve and the dashed curve represent the left hand of Eq. (8) including the shell correction energies and removed the shell correction energies, respectively. The shell correction energies take from Ref. [19]. The thick solid curve shows some oscillations and fluctuations due to the shell effect. When the shell corrections are taken into account, the fluctuations in the extracted $b_c$ are reduced effectively for the dashed curve. The thin solid line is the fitting straight line. We obtain $b_c = 0.691A^{2/3} - 0.8724$ MeV by fitting left hand of Eq.(8) to 251 $A$ values with an rms deviation of 0.384 MeV smaller than that $b_c = 0.710A^{2/3} - 0.946$ MeV with rms deviation of 0.588 MeV. The intercept 0.8724 is the contributions of the Coulomb exchange term and other correction term such as the nuclear surface diffuseness correction. If adopt the Coulomb energy expression $E_c = \frac{a_cZ^2}{A^{1/4}}(1 - cZ^{-2/3})$, in which the contributions of the Coulomb exchange term and other correction terms are taken into account, we obtain $a_c = 0.691$ MeV and $c = 1.1914$ MeV. Previous determinations of Coulomb-energy coefficient from the energy difference for mirror nuclides have been restricted to small values of $A$ [20], since mirror nuclides are not observed
FIG. 4: (Color online) Scaled mass difference (solid squares) of 95 pairs of mirror nuclei in the region $11 \leq A \leq 75$ as a function of $A^{2/3}$. The solid curve and the dashed curve represent the left hand of Eq. (8) including the shell correction energies and removed the shell correction energies, respectively. The shell correction energies are taken from Ref. [19]. The thin solid line is the fitting straight line.

for $A > 75$. However, it is shown that the mirror-nuclide method can be extended to include all values of $A$ in the Eq. (8). The availability of the Coulomb energy coefficient for the complete range of $A$ values should be helpful in a study of the variation of the Coulomb energy with mass number.
C. Determination of the symmetry energy coefficient $a_{\text{sym}}$ by the relation of $Z_A$ and $b_A$

Insert Eq.(4) and Eq.(5) into the expression Eq.(3) and take the double $\beta$ decay Q value of $(A,Z-1)$, we can obtain

$$Q_{2\beta}(A, Z - 1) = M(A, Z - 1) - M(A, Z + 1)$$

$$= B(A, Z + 1) - B(A, Z - 1) + 2\Delta(^1H-n)$$

$$= -\left(\frac{16a_{\text{sym}}}{A} + \frac{4a_c}{A^{1/3}}\right)Z + 8a_{\text{sym}} + 1.5649. \tag{9}$$

Meanwhile, the double $\beta$ decay Q value of $(A,Z-1)$ in Eq.(3) is written as

$$Q_{2\beta}(A, Z - 1) = -2b_A(Z - Z_A), \tag{10}$$

The results of symmetry energy coefficient $a_{\text{sym}}$ are obtained by solving the combination of Eqs.(9) and (10).

$$a_{\text{sym}} = \frac{(2b_AZ_A - 1.5649)}{8} \simeq \frac{b_AZ_A}{4} \tag{11}$$

The mass dependence of the symmetry energy coefficient of nuclei is written by Danielewicz and Lee \cite{21} as $a_{\text{sym}}(A) = S_0/(1 + \kappa A^{-1/3})$, where $S_0$ is the volume symmetry energy coefficient of the nuclei and $\kappa$ is the ratio of the surface symmetry coefficient to the volume symmetry coefficient. The other form for description of the mass dependence of $a_{\text{sym}}(A)$ is frequently used and written as $a_{\text{sym}}(A) = S_0(1 - \kappa A^{-1/3})$ \cite{22–27}. Figure 5 shows the extracted symmetry-energy coefficient $a_{\text{sym}}(A)$ as a function of $A$ from Eq. (11). The open circles denote the results removed the shell corrections from the KTUY model being taken into account. But it also show some odd-even staggerings and fluctuations, they can be caused by nuclear residual pairing interaction and the quantum effect, i.e., simple fluctuations of simple particle levels near the Fermi energy. The blue solid curve and the red dashed curve denote the results of two analytic expressions in which the coefficients are determined by fitting the open circles. By performing a two-parameter fitting to the $a_{\text{sym}}(A)$ obtained previously by the relation Eq.(11), one can obtain the values of $S_0$ and $\kappa$. With 95% confidence intervals, we obtain the values of $S_0 = 26.575 \pm 0.271$ and $\kappa = 0.99 \pm 0.035$, and the corresponding rms deviation is 389 keV, if assuming mass dependence of symmetry energy coefficient $a_{\text{sym}}(A) = S_0(1 - \kappa A^{-1/3})$. When adopting $a_{\text{sym}}(A) = S_0/(1+\kappa A^{-1/3})$, one
FIG. 5: (Color online) Extracted symmetry energy coefficient of finite nuclei $a_{\text{sym}}(A)$ as a function of mass number $A$ from Eq. (11) (open circles), but the shell corrections $S(N,Z)$ from Ref. [19] are removed from the mass excess of nuclei. The blue solid curve and the red dashed curve denote the results of two analytic expressions in which the coefficients are determined by fitting the open circles.

obtains $S_0 = 30.102 \pm 0.741$ MeV and $\kappa = 2.091 \pm 0.166$, respectively, with an rms deviation of 398 keV. The result is shown in Fig.5 by the red dashed curve. The obtained values of $S_0$ and $\kappa$ are in agreement with the range of $S_0 = 31.1 \pm 1.7$ MeV and $\kappa = 2.31 \pm 0.038$ given by Min Liu [28].
IV. SUMMARY

In summary, we have proposed a method to determine the two important parameters of the well-known Bohr-Wheeler mass parabola, the location of the most $\beta$-stable nuclei $Z_A$ and the curvature $b_A$ of the isobars. The linear relation of the total double $\beta$ decay energies is deduced based on the isobaric mass parabola, which only include two parameters of $Z_A$ and $b_A$, their values have been refitted for 251 $A$ values based the latest experimental total double $\beta$ decay energies of nuclei compiled in NUBASE2016 database. The advantage of this approach is that one can efficiently remove the pairing energies term $P_A$ caused by odd-even variation, and the mass excess $M(A, Z_A)$ of the most stable nuclide for mass number $A$ in the performance process, which are used in the mass parabolic fitting method. Based on the obtained two parameters of $Z_A$ and $b_A$, the Coulomb energy coefficient $a_c = 0.691$ MeV is determined by the mass difference relation of mirror nuclei $0.5b_A(A - 2Z_A) = \Delta(e_{H-n}) + a_c(A^{2/3} - 1.0583c)$, and $c = 1.1914$ MeV. Two analytic expressions of symmetry energy coefficient for description of the mass dependence of $a_{\text{sym}}(A)$, $a_{\text{sym}}(A) = \frac{30.102}{(1+2.091A^{-1/3})}$ and $a_{\text{sym}}(A) = 26.575(1-0.99A^{-1/3})$, are also determined by the relation $a_{\text{sym}}(A) = 0.25b_AZ_A$. The obtained values of $S_0$ and $\kappa$ are in agreement with the results in references. It imply the method is reliable to determine Coulomb energy coefficient and the mass dependence of the symmetry energy coefficient by the correspondence relations only including two parameters $Z_A$ and $b_A$. The further work is in progress based several theoretical mass tables and in comparison with the results of the experimental data.

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