Universal scores for accessibility and inequalities in urban areas

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By the end of this century, most of the world population will be living in cities. The unprecedented level of urban interactions and interconnectedness represents a big challenge to manage the unavoidable growth while aiming at sustainability and inclusiveness. This situation calls for a big effort of the scientific community to come up with engaging and meaningful visualizations and friendly scenario simulation engines. This paper gives a first contribution in this direction by providing a new perspective on how to evaluate accessibility in cities based on data of public transportation. By means of the notion of isochrones, coupled with the multi-layered nature of the public transport system and the spatial distribution of population density, we introduce universal scores quantifying and ranking cities according to their overall accessibility. We highlight great inequalities in the access to good public transport services across the population. The striking similarities in the patterns observed in very different cities suggest the existence of a common mechanism behind the organization of public transportation. The ensemble of the results is released through the interactive platform: www.citychrone.org, aimed at providing the community at large with an effective tool for awareness and decision-making.

Keywords
Accessibility, Science of city, Isochrone, Public Transports, Urban Planning

Introduction

The intrinsic complexity of the emerging challenges human beings collectively face requires a deep comprehension of the underlying phenomena in order to plan effective strategies and sustainable solutions: from the planning of urban infrastructures to containment strategies for pandemics, from the impact of political campaigns to measures against information pollution and misinformation. In this framework, cities stand as a paramount example of how a complex interplay of infrastructures, technologies and human behaviors may lead to outcomes and patterns very far from the usual cause-effect scheme \cite{1}. The science of cities is a relatively new research area that greatly benefited, in the last decades, of the digital revolution \cite{2}. Nowadays, the capillary deployment of Information and Communication Technologies \cite{3} and the
consequent availability of an unprecedented wealth of data, is opening new opportunities for a scientific approach to the complexity of urban environments. In this way, studies are fostered, aimed, at identifying, on the one hand the patterns of co-evolution of human and social behaviors [4, 5, 6, 7] and, on the other hand, the innovation at the level of infrastructures and services [8, 9, 10, 11, 12].

This paper aims at contributing to the ongoing debate about the future of our cities and the way forward to combine growth [13] with efficiency and inclusiveness. To this end we focus on a specific aspect of cities, namely the topic of accessibility, i.e., the capacity of cities to allow people to move efficiently by guaranteeing equity and equal access to personal and professional opportunities. From this perspective, accessibility does not mean only the overall capacity of urban transit: it also needs to be inflected as accessibility of specific areas, for whom and for which purposes. Are not rare the examples where the spearhead of public transit benefits only a tiny fraction of the population and the average traveling conditions are way behind that. It is thus important to be able to quantify accessibility in a way that closely represents the experience of citizens. To this end we start from a relatively minoritarian point of view, namely that of measuring cities in terms of traveling times rather than geographical distances. We believe this perspective is closer to the actual perception of citizens and better represents the mindset citizens adopt in planning their mobility. The key mathematical notion to quantify traveling times will be that of isochronic maps, i.e., maps showing areas related to isochrones between different points. In other words, given a geographical point, its isochronic map will be composed by isochronic contours marking regions reachable in a given timespan, using different transportation systems. Isochronic maps exist since 1881, when Sir Francis Galton published the first isochronic map in the Proceedings of the Royal Geographical Society [14], showing travel times in days from London to different parts of the world. Nowadays, the availability of data related to mobility allows for the compilation of very accurate isochronic maps for different locations, different geographical areas, different social communities and different transportation systems. Since these maps represent spaces based on travel times, they are closer to our individual perceptions related to mobility. One often decides to embark in a trip based on the expected travel time rather than on the physical distances (from Galton’s times the average duration of an around-the-world-trip dropped of almost two orders of magnitude).

Though the notion of isochrone is well defined, its computation depends on the transportation system adopted. Here we focus on public transportation and we compute traveling times and isochrones through a routing approach that exploits multi-modality. This implies that the best route from two points A and B in the city can be realized through a combination of several transportation means (walking, buses, metro lines, trains). For sake of simplicity and without loss of generality, here we only consider the official schedule of public transports for many cities in North America, Europe and Australia. Following a recent interesting trend in the scientific research [15], we developed visualizations on maps of this body of information, as well as several metrics for accessibility, through the open CityChrone platform (http://www.citychrone.org). Data about real-time passages of public transports or about other public or private transportations means can be easily integrated in the platform as well.

Usually the studies of public transport analyze the networks of transport as static graphs, where the nodes represent stops and the edges represent the routes connecting them [16, 17, 18, 19, 20]. Very few study have instead incorporated, in a systematic way, the “temporal” features of these systems [5, 9], i.e., the way in which users navigate through urban networks to reach their destinations. Here we focus specifically on the dynamical aspects of mobility and, based on the notion of isochronic map, we introduce two universal metrics for accessibility of cities: a Velocity Score, quantifying the overall velocity of access to a specific area of the city, and a Sociality Score that exploits data about local population densities and quantifies how social is a specific area of the city, i.e., how many people one can possibly meet from
there. Finally the dependence of the Sociality Score on the total population of a city can be reduced by rescaling it with this quantity. In this way, we define a third accessibility metrics called Cohesion Score that quantifies the fraction of the total population that can be met with a typical trip starting from the considered point. The proposed metrics allow for a capillary study of the level of accessibility of urban areas, a concept formulated several decades ago \cite{21, 22, 23, 24} to quantify the performance of transportation systems in relations to various aspects of individuals’ lives.

Despite its importance, there is not a unique possible definition of accessibility. The term accessibility could refer, depending on the contexts, to the availability of services for disabled or disadvantaged people \cite{25}, the capability of reaching workplaces for common citizens \cite{22}, the possibility of attending to certain activities at given times during the day \cite{26}. Similarly, accessibility can be focused on the traveling times using all or several modes of public or private transport or can just rely on the spatial distribution of commodities and venues \cite{27}. This proliferation of definitions is definitely not helping in reaching a unifying view about cities and their dynamical aspects, contributing instead to a dispersion of scientific efforts in diverging directions. Lacking a comprehensive definition of accessibility prevents policy makers from using it in a operational way and scholars from comparing different approaches and methodologies. Our aim here is that of contributing with a unified point of view whose core is represented by the temporal dimension, which we judge closer to the human perception about spaces and traveling times.

Our metrics are designed to be efficiently computed, thanks to state-of-art routing algorithms, in relatively short times (less than one minute for medium-sized cities), opening the possibility to explore different scenarios in nearly real-time. In addition our metrics are well suited for being shared and easily visualized on maps, making their fruition much easier and intuitive and opening as well the possibility to evaluate the impact of different scenarios, on a quantitative basis.

The quantification of inequality in accessibility has been proven to be an important tool to assess economic and social inequalities at an extra-urban scale \cite{28}. It is worth to mention that the local nature of our metrics allows to evaluate and visualize the geographical fluctuations of the velocity and sociality scores, giving in this way a precise quantification of the inequalities of the accessibility patterns and of the corresponding personal or professional opportunities within each city. In particular, we show that while the distributions of the accessibility metrics seems to have higher values for high-density areas, yet only a small fraction of the population have access to high-performing public transport means. Moreover, the performances of public transport systems decrease in a exponential-like way for all the observed cities, as the temporal distance from the city center increases. These results exhibit strongly similar patterns among all the observed cities, suggesting the existence of a universal mechanism behind their emergence.

Finally, despite the local character of the proposed metrics, their aggregation at a urban scale allows for a quantification of the global level of the performances of public services of a city. In this way, the aggregated Velocity Score, the “City Velocity”, represents the overall velocity allowed by the public transportation services; on the other hand the aggregated Sociality Score, the “City Sociality”, quantifies the amount of people possibly met in a standard trip in a given city; the aggregated Cohesion Score, the “City Cohesion”, roughly indicates how well connected is a random pair of individuals in a given city. We adopt these aggregations to rank cities according to public transport performance, finding that, while in general there are correlations between the city positions in the different rankings, there are also interesting fluctuations due to the complex interplay between public transport and the population density.

The outline of the paper is as follows. In the section Methods we illustrate the main tools we adopt throughout the paper. Here the main notion is that of isochronic maps. We review its definition and we describe the way in which it is adopted in this paper, including the
data and the algorithms to compute it. Based on the computation of the isochronic maps, we introduce several accessibility metrics to quantify the efficiency of the public transportation systems and the opportunities provided to the citizens in terms of mobility. The Results section describes several synthetic scores to allow a universal ranking of cities according to their accessibility patterns. Besides an overall evaluation, we focus in particular on the inhomogeneities of accessibility patterns in cities along as their space-time distribution. Finally we draw some conclusions and highlight interesting future directions.

Methods

Isochronic maps

The accessibility metrics proposed in this paper all rely on the notion of isochronic map. An isochronic map is composed by a set of isochrones centered in a given location $\lambda$. The isochrone $I(\tau, \lambda)$ is the contour of the area reachable from $\lambda$ in at most a time $\tau$ and the ensemble of the isochrones obtained for different values of $\tau$ compose the isochronic map of the location $\lambda$. A more complete definition includes not only the travel-time $\tau$ but also the absolute starting time of the trip. In this way one has $I(\tau, (\lambda, t_0))$ as the contour of the area reachable from $\lambda$ in at most a time $\tau$ starting at time $t_0$. Though the notion of isochrones is explored at a quantitative level since long time [29], nowadays it is possible to compute them massively and very efficiently, opening in this way the possibility for insightful studies. The computation of isochrones is based on the computation of the traveling times between any pair of locations in a city using a multi-modal approach that integrates the adoption of all the available public transportation systems alternated with walking paths. In order to achieve a coarse-grained representation of each city, we adopted an hexagonal tessellation that allows for an exhaustive representation of the public transportation services, while keeping the computational times low. We constructed, in particular, an hexagonal grid with side of hexagons of 0,2,km. It is worth noticing that not the whole area of a city is covered by hexagons. We cover with hexagons all locations of a city containing at least a stop of the public service and all areas reachable from any stop of the public service with walking paths not longer than 15 minutes. In order to compute the walking paths between stops of the public service and the hexagonal grid, we use the back-end version of the Open Source Routing Machine (OSRM) [30]. The OSRM allows for the computation of shortest walking paths on the urban networks of each city, using the corresponding OpenStreetMap network. As for the schedules of public transit we relied on data released by public transports companies. Google created the GTFS standard file (https://developers.google.com/transit/gtfs/) to encourage public transport companies to release their data in uniform way in order to be included in its map platform. It is nowadays possible to find hundreds of companies having released their data, and there are portals where these data are collected and exposed[32]. The databases of public transportation systems are strongly heterogeneous across cities. In some cases some transportation means could be missing while other extra-urban ones could be included. For instance for Berlin and London the GTFS (General Transit Format System) data include all regional trains [32]. In order to adopt a unique and general criterion about the inclusion of areas and transportation means, we adopted the OECD/EU definition of urban areas as functional economic units [33]. The OECD/EU definition exploits the population density to identify a urban core (city core) and travel-to-work flows to identify the hinterland whose labour market is highly integrated with the core (commuting zones). With this definition in mind, we filtered out from our tessellations, for each city, services lying outside both the cores and the hinterlands regions. In addition to the

1We remark that a satisfactory definition of city and its extension is still lacking [2]
accessibility metrics

In this section we introduce two universal scores for accessibility that allow for an easy comparison of different areas of the same city and different cities among them. Interactive representations of those metrics for a large number of cities are available at citychrone.org.

velocity score

The first metrics for accessibility we introduce is named velocity score and aims at giving a synthetic representation of the information encoded in all the isochronic maps computed from all the points of a city. To this end we imagine the isochronic map as a spreading process from a starting point and we are interested in the velocity of expansion of the front of the isochrone as a function of time. More concretely, let us consider the isochrone centered at the hexagon \( \lambda \) at time \( t_0 \) corresponding to a travel-time \( \tau \), \( I(\tau, (\lambda, t_0)) \). In particular the covered area, \( A(\tau, (\lambda, t_0)) \) of the isochrone at time \( \tau \). Assuming that the perimeter of the isochrone is a circle, \( \bar{r} \), i.e. the average travelled distance taking a random direction from the starting point \( p_0 \) is given by:

\[
\bar{r}(\tau, (\lambda, t_0)) = \sqrt{A(\tau, (\lambda, t_0))/\pi},
\]
and dividing by the time $\tau$ we obtain a “circular” velocity:

$$\bar{v}(\tau, (\lambda, t_0)) = \bar{r}(\tau, (\lambda, t_0))/\tau.$$  \hspace{1cm} (2)

The interpretation of $\bar{v}(\tau, (\lambda, t_0))$ is the velocity of expansion of a circular isochrone with the same area of the real one (see Fig.1). This quantity can be though, approximatively, like the average velocity choosing a random direction of a journey of duration $\tau$. On the other hand, this quantity is proportional to the amount of area it is possible to explore in a time interval $\tau$ from the hexagon $\lambda$. We chose to consider the square root of the area instead of the area itself to have a more direct interpretation of it in terms of transportation velocity. This “circular” velocity is defined for every hexagon $\lambda$ and any starting time $t_0$ and travel-time $\tau$. The velocity score is obtained by averaging over both the starting time $t_0$ and the travel-time $\tau$, as:

$$v(\lambda) = \frac{\sum_{t_0=6am}^{10pm} \int_0^{\infty} v(\tau, (\lambda, t_0))f(\tau)d\tau}{\sum_{t_0=6am}^{10pm} 1},$$ \hspace{1cm} (3)

where several starting times have been considered, from 6am to 10pm, every 2h. The average over $\tau$ is performed by weighing each travel-time with a empirical travel-time distribution, $f(\tau)$. The distribution adopted is taken from a survey of the daily budget times spent on bus by UK citizen [40]. Fig.2 (panel A) shows the velocity scores of six different cities. For interactive explorations of the maps and other cities we refer the reader to the platform citychrone.org.

In the Section A of the Appendix it is shown how the accessibility measures proposed are robust against reasonable choices of travel time distributions and the results presented do not sensibly change by choosing different distributions.

**Sociality score**

The velocity score introduced above represents an indicator of how good is the public service in allowing a fast exploration of the urban space. At this stage this score does not take into account the population density distribution. We know instead that there is a strong interplay and feedback loop between the efficiency of the public service and the population density distribution. Therefore, we introduce a new score, the sociality score, that depends on both the velocity score and the population density distribution.
Figure 2: Maps of the velocity score and the sociality score In the six maps of panel A (B) we report the velocity score in km/h (sociality score in millions of inhabitants) for six different cities: Paris, New York, Madrid, Montreal, Sidney, Boston. The values of the velocity score range from brown less than 2 km/h of velocity score up to more than 17 km/h (purple) whereas for sociality score ranges from less 0.05 millions of inhabitants reachable up to more than 3 millions of individuals. The great variability of the colors reveals a strong dissimilarity of performances of the public transports across cities.

density. While it is normal to strengthen the service in highly populated areas, regions with a low population density risks to be poorly served by public transit. In order to quantify this interplay we introduce a second metrics that quantifies the performance of public transit in connecting people. Let us now define $P(\tau, (\lambda, t_0))$ as the amount of population residing within the isochrone $I(\tau, (\lambda, t_0))$. Similarly to what we did for the velocity score, we can average $P(\tau, (\lambda, t_0))$ over the travel time $\tau$ (with the same distribution of daily budget times $f(\tau)$) and over different starting times $t_0$, obtaining the sociality score as:

$$s(\lambda) = \frac{\sum_{t_0=6am}^{10pm} \int_0^\infty P(\tau, (\lambda, t_0)) f(\tau) d\tau}{\sum_{t_0=6am}^{10pm} 1}, \quad (4)$$

While the Velocity Score provides with a measure of the extension of the area that it is possible to explore in a typical working day starting from a hexagon $\lambda$, the Sociality Score measures the amount of people it is possible to meet in a typical working day starting from a hexagon $\lambda$. Fig. 2 (Panel B) shows the Sociality Scores for the same cities considered for the velocity score.
Results

City rankings

The scores introduced above allows us to rank cities according to the overall performances of their public transport system in allowing for a fast and effective mobility for the largest fraction of inhabitants. To this end we introduce the City Velocity indicator as the the average Velocity Score, weighted over the population density. The City Velocity is a measure of the how fast a typical inhabitant can visit the city with a typical trip. The second indicator we introduce is the City Sociality, defined as the average Sociality Score, weighted over the population density in a city. The City Sociality is a measure of the how many distinct people it is possible to meet in a given city with a typical trip. Finally we introduce the City Cohesion indicator, which measures the easiness for two randomly picked individuals to meet within a city. The larger this indicator is, the more the city is cohesive and favors social interactions among its citizens.

City velocity

For each hexagon, \( \lambda \), we have both the number of people living there, \( \text{pop}(\lambda) \), as well as the average velocity of their trips with public transports starting from the considered hexagon, \( v(\lambda) \) (Eq. 3). In this way we can compute the average velocity per person of the whole city, representing the average amount of different places a typical person living in the city can easily access with public transit. In particular we define the City Velocity as the average velocity per person:

\[
v_{\text{city}} = \frac{\sum_{\lambda \in \text{city}} v(\lambda) \times \text{pop}(\lambda)}{\text{pop(\text{city})}}
\]  

(5)

where \( \text{pop}(\lambda) \) is the population in the hexagon \( \lambda \) and we sum over all the hexagons in the city weighted with the population residing in that hexagon, and we divide by the total of the population of the city (including the core and the commuting zones described above), \( \text{pop(\text{city})} \). Notice that we assign zero velocity to all the areas of the city not covered by hexagons, i.e., the areas more than 15 minutes away from any stop. Fig.3 reports the ranking of several cities according to their City velocity. The highest ranked cities are Berlin and Paris with values 20% higher than any other city. This means that typically a citizen of Berlin and Paris can explore the space around at least 20% faster than the others. Copenhagen, Helsinki, Athens, Prague, London and New York features good performance. On the other hand of the spectrum, Mexico City, San Diego and others U.S. cities have a large fraction of the population badly or no served at all by public transports.

City sociality

The City Sociality is defined as:

\[
s_{\text{city}} = \frac{\sum_{\lambda \in \text{city}} s(\lambda) \times \text{pop}(\lambda)}{\text{pop(\text{city})}}
\]  

(6)

As for the City Velocity, we average over the population distribution, and the areas of the city not served by public transport are considered with zero Sociality Score. The City Sociality is the typical number of people that a person living in the city can potentially meet within a typical daily trip. The ranking of cities according to the City Sociality, reported in Fig.4, features some differences with respect to the corresponding ranking obtained with the City Velocity. In this case Paris gains the first position thanks to its high population density in the city core and its efficient and capillary public transit system. Paris is the only city, among the set of
Figure 3: **Ranking of cities according to the City Velocity** defined in Eq. 5. Cities are displayed with circles whose size is proportional to the total population and whose saturation of the filling color is proportional to the overall population density.

cities considered, where on average a person can potentially meet over one million of people in a typical daily trip. Scrolling the ranking, the City Sociality decreases initially quickly, with the most populated cities in the first positions, to eventually decrease very slowly for smaller cities.

Figure 4: **Ranking of cities according to the City Sociality** defined in Eq. 6. Cities are displayed with circles whose size is proportional to the total population and whose saturation of the filling color is proportional to the overall population density.

**City cohesion**

By rescaling the *City Sociality* with the total population of a city, we obtain the *City Cohesion*:

\[
c_{city} = \frac{s_{city}}{pop(city)}.
\]
The *City Cohesion* gives an estimate of the fraction of population that can be reached with a typical trip of a typical inhabitant of the city. Fig. 5 shows the ranking of cities according to the *City Cohesion*. The first city is Athens, thanks to a good public transportation system and a very high density population concentrated in the core of the city. In second and third positions there are Berlin and Copenhagen, which also feature very high *Velocity Score*. Then we find Turin and Florence, which, despite relatively low *City Velocity* and *City Sociality*, feature a good balance between the population distribution and the efficiency of the public transportation system. A large part of US cities have a low City Cohesion score, resulting from the low population density in the city core, making those cities very dispersive.

**Figure 5:** Ranking of cities according to the City Cohesion defined in Eq. 7. Cities are displayed with circles whose size is proportional to the total population and whose saturation of the filling color is proportional to the overall population density.

### Inequalities in urban accessibility patterns

In this section we focus on a particular aspect of accessibility, namely its democratic and inclusive character. A high position of a city in the overall ranking for any of the scores presented above does not imply *per se* that the same accessibility patterns and corresponding opportunities are granted to all citizens. In order to investigate dis-homogeneities in the accessibility patterns one needs to take a closer perspective and look at the accessibility metrics at a more fine grained scale within cities. In order to do this we focus, without lack of generality, to a subset of cities, namely the same cities we focused on in the section devoted to Accessibility Metrics: Paris, New York, Madrid, Montreal, Sydney, Boston.

An interesting way to represent the Velocity and Sociality score is through a violin plot, as reported in Fig. 6. Panels A and B refer to the distributions of the Velocity and Sociality scores, respectively. The way in which one reads these plots is the following. For each city one plot the distribution of areas and population as a function of the Velocity or Sociality score. For instance panel A refers to the Velocity score. For each city, we plot on light green the normalized distribution of areas (hexagons) as a function of the Velocity score, i.e., the fraction of hexagons featuring a specific value of the Velocity score. A very efficient city has this distribution peaked around high values of the Velocity score. From this perspective New York appears to have the most balanced distribution of Velocity scores across its whole area. On the other hand, represented in dark green is the distribution of the population density as a
function of the Velocity score, i.e., the fraction of the population associated to a specific value of the Velocity score. A very inclusive city has this distribution peaked around high values of the Velocity score. From this perspective Paris, New York and Madrid appear to be more inclusive than Montreal, Sydney and Boston. Panel B reports the same information as panel A (in light and dark blue) for the Sociality score. It is striking the difference between Paris, New York and Madrid, on the one hand and Montreal, Sydney and Boston, on the other, in terms of the range of sociality score both for areas and population. Paris and New York appear to feature the widest distribution of Sociality scores across their citizens. It is evident, both

![Figure 6: Distributions of the Velocity (panel A) and the Sociality (panel B) scores and velocity score vs. the population density (panel C) Panel A (panel B): Distribution of the velocity score (sociality score). The light green (blue) area represents the distribution of the areas featuring a given value of the velocity score (sociality score). The dark green (blue) area represents the distribution of population enjoying a given value of the velocity score (sociality score). As expected the distribution for the population density displays higher values than that related to the hexagons, signalling the fact that denser areas are associated, on average, to better public transit systems. We do not report here the results for the Cohesion metrics since, in this representation it would give the same information of the Sociality Score. Panel C. Average value of the velocity score in hexagons with a given population for the six cities of Paris, New York, Madrid, Montreal, Sydney and Boston. In all cases one observes an increasing trend. The shadows around the average values curve are the standard deviation.

in panels A and in B, that (light green and blue areas) a large number of hexagons within the city borders display low values of the accessibility scores. However, when the population density is taken into account (dark green and blue areas), the peaks of the distribution shift towards high-populated areas. This result is somehow unsurprising considering that the public transportation systems are mainly designed to serve the largest amount of citizen as possible as allowed by the limited financial resources. This picture is confirmed by Fig. 6 (panel C) where it is evident the growing trend of the average values of the Velocity scores at fixed population density with the population density for the six cities considered above. The trends reported above, correlating denser populated areas to higher (on average) accessibility scores, do not imply that the planning of public transportation systems succeeds in reducing inequalities in the accessibility patterns. The spread of the distributions is still very high and very few people
(or areas) have access to high quality public transport services compared to the rest of the populations (or areas). This is true for all the accessibility scores introduced above. In order to better quantify the large variability of urban accessibility patterns, we divide urban areas (hexagons) and population in two classes. Hexagons and people featuring the top 1% of values of the Velocity and the Sociality scores and the remaining 99%. For each of the two classes we compute the average values of the Velocity and the Sociality scores and we compare them. The results are reported in Fig. 7: panels A and B for the Velocity score and panels C and D for the Sociality score, panels A and C for the distribution of hexagons and panels B and D for the population densities.

The striking, though perhaps not surprising, result confirms the strong level of inequalities observed for all the cities considered. The ratio between the average values of the scores of the two classes is always larger than two. This implies that, focusing for instance on the velocity score, the top 1% of the hexagons (populations) features values of the velocity score double than the remaining 99%. In other words, 1% of the city areas allows to perform daily trips at least twice as faster than the rest of the city, and 1% of population can move around at least twice as faster than the rest of the population. Similar considerations holds for the sociality scores, which implies that 1% of the population has potentially access at twice the number of opportunities than the rest of the population. The ratio between the values of the top 1% compared to the remaining 99% is similar across all considered cities (see Appendix, Figure G and Figure F), as witnessed by the error bars reported in Fig. 7. It is also interesting to observe that almost all the ratios between values of the average scores computed for the two 1% and 99% classes lie between 2 and 4, suggesting the existence of universal patterns of organization across very different cities and urban environments.

**Space-time distribution of inequality in accessibility patterns**

The quantitative assessment of the strong inequalities observed in the accessibility patterns reported above can be further clarified by looking at the spatial distribution of the accessibility metrics. The maps shown in Fig. 2 highlight the fact that the best performing public transport areas are typically clustered in the center of the cities, while many other areas experience poorly performing public transport. To better quantify this effect we display the behaviour of the velocity and of the sociality scores (Fig. 8) as a function of the travel-time from the center of each city. Here the center is defined for each city as the hexagon with the highest score (velocity and sociality, respectively). Both the velocity and the sociality scores decay fast as a function of the travel-time from the city center. This decay is well described by the exponential function:

\[ f(t) = \sigma_0 e^{-\frac{t}{\tau}} + \sigma_\infty, \]

where \( \tau \) represents the typical decay time, \( \sigma_\infty \) is the lower bound of the velocity score for each city and \( \sigma_0 \) represents the average velocity score of areas (hexagons) nearby the best performing one. We performed the best fit of the data on the binned data \( t \) in order biases coming from the better sampled temporal distances. The value \( \sigma_\infty \) represents the value of the score (either velocity or sociality) acquired in hexagons at the temporal edge of the city itself, i.e., for the farthest (in travel-time) hexagons from the city center. Hence we estimated it as the average velocity score (sociality score) of the 5% least accessible hexagons from the city center. The parameters \( \tau \) and \( \sigma_0 \) are obtained through a linear regression of the quantity \( \log(f(t) - \sigma_\infty) \), which depends linearly on \( t \). The decay of the mean values of the velocity score are well fitted by the curve, the average value among all 32 cities analyzed of R-square is \( R^2 = 0.92 \) (see Appendix, Figures G and G). The average value of the characteristic time \( \tau \) is 0.86 hours, ranging from 0.4 hours of Santiago to the 1.6h of Los Angeles. The dependence of the sociality score on the temporal distance from the best performing hexagon of the city is
Figure 7: Inequalities in accessibility patterns. Panel A: Average values of the velocity scores among the hexagons featuring the top 1% (light green) of the values of the velocity score as compared to the remaining 99% (dark green) for the six selected cities. The last columns report the same values averaged over all 32 cities analysed. Red star marks, on the right $y$-axis, are the ratios between the average values of top 1% and the other 99%. Panel B: Average values of the velocity scores among the population with the highest top 1% (light green) of the values of the velocity score as compared to the remaining 99% (dark green) for the six selected cities. The last columns report the same values averaged over all the 32 cities analysed. Red stars mark, on the right $y$-axis, the are the ratios between the average values of top 1% and the other 99%. Panel C: Same as Panel A for the Sociality score. Panel D: Same as Panel B for the Sociality score.

again well-described by the equation (8), with value of R-square $R^2 = 0.95$ higher with respect to those of the velocity score and an average value of $\tau = 0.55h$. The smaller characteristic time is due to the convolution of the decay of the velocity score with the well-know decay of the population density from the city center, which is again exponential\cite{11, 12, 13}. In Section A of the Appendix we show that these results are again independent of the travel-time distribution of trips used for the calculation of the accessibility metrics. The ensemble of the results confirms that inequalities in the accessibility patterns allowed by public transport favor a small portion of the total city area and a small fraction of the population, typically clustered around a certain area. Moving in space and time away from such area will lead to experiencing generally bad public transport services. This behavior and the stability inequality patterns of Fig. (7) strengthen the hypothesis of a general mechanism behind the emergence of the decay of public transport performances that will be investigated in future works.
Figure 8: Exponential decay of the velocity score and the sociality score with travel-times for the city center. Average values of velocity scores (green points) and sociality scores (blue points) at a given travel-time distance from the hexagon with the highest score in each of the six selected city. The lines are the best fit of the data with the function \((8)\). In the legend for each figure there are the parameters \(\tau\) of equation \((8)\) found for the decay of velocity score and the sociality score.

Discussion

The study of accessibility in urban contexts represents a multifaceted topic whose relevance transcends the mere, though already very complex, problem of optimising transportation systems, to impact the level of opportunities available in a city, the democratic access to them, the level of inclusion of minorities, etc. The meanings the notion of accessibility can take are indeed very many: from the planning of better and more efficient urban environments, to the improvement of quality of life in rural areas, from the definition of real estate market prices, to the definition of new business models of mobility and so on. The extreme generality of the term accessibility also depends on the specific aspects one could be interested in: the availability of jobs in a certain area, the quality of the schools in a neighborhood, the possibility to take part in leisure activities depending on the time of the day. Despite a long tradition of scientific studies on these subjects, no consensus still emerged on how to quantify the notion of accessibility in a universal way, i.e., through metrics applicable in very many situations and very different urban contexts. Many different metrics of accessibility have been proposed, whose scope of application is often restricted to the specific context for which they have been introduced. The main aim of this paper is that of proposing a unifying, simple and general framework through which it is possible to derive a few universal metrics for accessibility that allow for a quantitative comparison of different cities and different areas of the same city. To this end we took a specific angle by looking at the city and at the paths within it from the point of view of traveling-times, instead of that of distances. We contend that this simple change of perspective allows to map the city in a way much closer to individuals’ perception. The cornerstone of this
new approach is the computation of isochronic maps and, based on them, the introduction of several scores that take into account both the opportunities offered the public transportation systems and the demand of mobility of the citizens. The main outcome is a set of universal scores that quantify how well served is a city from the public transit and how well a specific area of the city is connected to the rest of the city. We show how these scores allow to compare the performances of public transport systems of different cities around the world, pointing out the differences in their ability to expand the range of opportunities and enhance social interactions. A very interesting opportunity open by the new scores concerns the possibility to quantify the level of inequalities within a city, i.e., the fluctuations of the accessibility scores among different locations and the corresponding fluctuations on the opportunities offered to citizens. This series of analyses reveal a general pattern observed in all cities considered, namely that 1% of area of a city features accessibility scores with average values at least double than those of the remaining 99% of area. The same patterns is observed by looking at the number of people enjoying specific values of the accessibility scores: also in this case the top 1% of the population is able to move at least twice as faster than the remaining 99% of the population. This very uneven distribution of performances of the public transport within a urban environment is explained in terms of the the rapid decay of the accessibility scores as a function of the temporal distance from the city center. The stability and universality of the ensemble of the results for the space-time distribution of the accessibility scores suggests the existence of a universal mechanisms in place in the planning and organization of public transport systems, leading to these effects. The availability of universal scores for accessibility and inequalities could be a first step towards a more systematic evaluation of the present in urban contexts and a careful planning of future scenarios. The citychrome.org platform is a first example in that direction, already allowing for both the visualization of all the accessibility metrics introduced here and the conception of new scenarios for improved mobility and accessibility.

Data, code and materials

All the data used in this work can be freely accessed from public repositories [33, 32, 34, 36]. Python source code used to compute the accessibility quantities and for the analysis performed are freely downloadable from the online public-transport-analysis GitHub repository [DOI:10.5281/zenodo.1309835][44]. The hexagons tessellation and the related accessibility quantities computed can be download from the online openData GitHub repository [DOI: 10.5281/zenodo.1309927][45]. In the same repository there is also a CSV file (agency.csv) with the list of public transports agency used for each city.

Appendix

A Robustness of the accessibility metrics definitions with the time distributions

In the definitions of the accessibility metrics, see for instance eq.(3) in the main text, we use an empirical daily budget time distribution in order to average over travel time. Such empirical law has been derived in [40] and is the following:

\[ f_{DBT}(t) = N \times \exp(-\alpha T_{bus}/t - \beta t/T_{Bus}) \] (9)

where \( N \) is a normalization constant, and \( \alpha = 0.2 \), \( \beta = 0.7 \) and \( T_{bus} = 67 \) min are obtained by the best fit on surveys data about public transport habits [40]. The choice of the distribution
used to average travel times is somehow arbitrary. However, we checked the robustness of
our results considering also other travel time distributions: the the single trip travel times
distribution \((f_{TTD})\) extracted from Oyster card journeys on bus, Tube, Docklands Light Railway
and London Overground \([9,1]\) and a normalized flat distribution \(f_{1h}\), between 0 and 1 hour. The
distributions are shown in fig. A (panel - A). Since the daily budget time distribution represents
the distribution of the total amount of time a person can spend travelling during the day. Hence,
\(f_{DBT}(t)\) is compute at time \(2t\) in order to consider also the amount of time a person need to
return to the starting point. The other two distributions represent travel times distribution
of trips and hence are evaluated at time \(t\). In fig. A (panel B) we show the velocity score of
all the points of every city in our dataset computed with the \(f_{TTD}\) (red) and \(f_{1h}\) (green)
distributions as functions of the velocity score computed with the distribution \(f_{TTD}\) used in
the main text. We can see from this picture that the similarities between the velocity scores
are quite high and hence the calculation of this score depends only slightly on the choice of the
travel time distribution. We check that the velocity score aggregated in each city used in the
main text to built the city rankings is again only slightly affected by the choice of the travel
time distribution. In Fig. A, panel C (panel D) there we show the scatter plot of the city velocity
(city sociality) computed with the \(f_{TTD}\) (red) and \(f_{1h}\) (green) distributions as functions of the
city velocity computed with the distribution \(f_{TTD}\). Also in this case the correlations between
values is very high keeping in most cases the same rank order and relative distances between

cities.

B Fast and efficient routing algorithm for public transport networks

The accessibility measures computed in the present work are based on the computations of the
minimum travel time between each point in the cities using public transport. Due to the fact
that the number of minimum travel times to be computed is of the order \(10^9\) for each city, using
an efficient routing algorithms is indeed mandatory. Driving times over road network can be
computed efficiently in few milliseconds or less at continental scale, but this is not the case of
travel times with public transport [46]. This is due to the fact that the approaches and speedup
techniques used for road network routing algorithms fail [46] or are not so effective on public
transport networks. In the last years different approaches have emerged in literature where the
most promising ones are the RAPTOR algorithm [47] and the CSA Algorithm [39]. Both these
algorithms takes a different approach by not looking at the graph structure of the problem.
However, these algorithms have a significant limitation when considering footpaths between
public transport stops in order to change means of transportation, reducing the performances in
urban systems. The algorithms we used is based on the CSA algorithm, but with an important
modification than allows to use it in urban systems considering realistic footpaths to move
between stops on foot and change mean of transport. We first describe the CSA algorithms
and then the modified one, the Intransitive Connection Scan Algorithm (ICSA).

B.1 Connections Scan Algorithm (CSA)

A public transit network is defined by its timetable. A timetable consists of a set of stops,
a set of connections and a set of footpaths. The stops are the points on the map where a
traveler can enter or exit from a vehicle. A connection \(c\) represents a vehicle departing from the
stop \(s_{dep}(c)\) at time \(t_{dep}(c)\) and arriving at the stop \(s_{arr}(c)\) at time \(t_{arr}(c)\) without intermediate
halt. Movements between two stops performed on foot are called footpaths and are treaded
separately with respect to connections. Given the timetable it is possible to compute the time
Figure A: Robustness of the accessibility measure definition. Panel A. Plots of the three different travel time probability distributions. The curve $f_{DBT}(t)$ is taken from a survey on the daily use of public transports\cite{40}, the $f_{TTD}$ is extracted from Oyster card journeys of London \cite{9,4}, and the $f_{1h}$ is a flat distribution between 0 and 1 hour. Panel B. Scatter plot of the velocity score computed in all cities with three different travel time distributions. On x-axis is the value computed with the $f_{DBT}(t)$ distribution. Panel C(D). Scatter plot of the city velocity(city sociality) computed with three different travel time distributions. On x-axis there is the values computed with the $f_{DBT}(t)$ distribution.

needed to reach all the other stops given a starting time $t_0$ at the stop $s_{start}$. We label each stop $s_i$ with its arrival time $\tau[s_i]$ and we set them all at starting to infinity $\tau[s_i] = \infty$ except for the starting stop $\tau[s_{start}] = t_0$ that we set to its starting time. Then we build an array containing the connections, ordered by their $t_{dep}(c)$. A connection is defined as reachable if the time $t_{dep}(c)$ of starting stop $s_{dep}(c)$ of the connection $c$ is equal or larger than the time $\tau[s_{dep}(c)]$ of the stop $s_{dep}(c)$. The Connections Scan Algorithm (CSA) scans the ordered array of connections $\{c\}$, testing if each $c$ is reachable. If $c$ can be reached and if the arrival time $\tau[s_{arr}(c)]$ is larger of the arrival time $t_{arr}(c)$ of the connection, the connection is relaxed, meaning that the $\tau[s_{arr}(c)]$ is update to the earlier arrival time $t_{arr}(c)$. After the entire array of connections is scanned the labels $\tau$ contain the earliest arrival time for each stop starting from $s_{start}$. In the above description we do not handle footpaths. Hence, in order to consider them each time the algorithm relaxes a connection, it checks all the outgoing footpath $f[s_{arr}]$ of $s_{arr}(c)$ and updates the time of the neighbors accordingly. The algorithm requires footpaths to be closed under transitivity to ensure correctness. This means that if there is a footpath from stop $s_a$ to stop $s_b$ and a footpath from the stop $s_b$ to the stop $s_c$ there must be a footpath between $s_a$ and $s_c$. So for every connected component of stops connected by footpaths we need all the footpaths connecting them. Since the number of footpaths grows quadratically with the number of stops, it is computationally infeasible to consider them all. In order to reduce computational time
for all stops $s$ do $\tau[s] \leftarrow \infty$;
$\tau[s_{\text{start}}] \leftarrow t_0$;
for all connections $c$ increasing by $t_{\text{dep}}(c)$ do
\hspace{1em} if $\tau[s_{\text{dep}}(c)] \leq t_{\text{dep}}(c)$ then
\hspace{2em} if $\tau[s_{\text{arr}}(c)] > t_{\text{arr}}(c)$ then
\hspace{3em} $\tau[s_{\text{arr}}(c)] \leftarrow t_{\text{arr}}(c)$;
\hspace{3em} for all footpaths $f$ from $s_{\text{arr}}(c)$ do
\hspace{4em} $\tau[f_{\text{arr}}] \leftarrow \min\{\tau[f_{\text{arr}}], \tau[s_{\text{arr}}(c)] + f_{\text{dur}}\}$;
end
end
end

Figure B: Connection Scan algorithm.

and yet considering realistic footpaths, we allowed up to 15 minutes of walk between stops. Despite with this choice all the stops of a each considered city belong to the same connected component, the fact that some footpaths are not consider might lead to an overestimation of the minimum travel time between two locations due to the lack of closeness. In the next section we describe a new version of the CSA algorithms that solves this problem. A pseudo-code of the CSA algorithm is shown in Fig. B.

B.2 Intransitive Connections Scan Algorithm (ICSA)

The variant of the CSA we propose here is able to correctly solve the earliest arrival time problem on public transport network considering footpaths between stops without impose the closeness under transitivity. Let’s consider the case where closeness is not enforce and all the footpaths lasting more than 15 minutes are removed. For each stop $s_i$ consider a subsets of stops $\{s_j\}$ reachable with these footpaths. Thus, the journeys computed by the CSA algorithm could be incorrect due to missing travel time updates that should have been performed through the removed footpaths. Consider the case, we have just relaxed a connection $c$, i.e. the arrival time $\tau[s_i(c)]$ of the arrival stop $s_i(c)$ is updated, see Fig. C. Then the stops $\{s_j\}$, reachable by footpaths from $s_i(c)$, are checked and the arrival time $\tau[s_j]$ of a neighbour stop $s_j$ is updated. $s_j$ is also connected by footpaths to other stops $\{s_k\}$ (see Fig. C), which under closeness should have been updated through footpaths connecting them directly with $s_i$. However, despite they could be still updated through the footpaths starting from $s_j$, this does not happen because the updating of arrival time ends to the first set of neighbour stops. In worst cases it could also happen that the remaining connections that arrive on $s_j$ never relax, because all of them arrive after the time $\tau[s_j]$. Hence, no one of its neighbors will be updated through footpaths connecting them to $s_j$. However, there could be journeys passing through $s_j$ that do not update $\tau[s_j]$ directly, but they might update some of its neighbours $\{s_k\}$ through footpaths. The CSA algorithms is not able to consider this. The Intransitive Connections Scan algorithms (ICSA) overcomes this problem with a small modification of the original CSA algorithm, without increasing the running time and preserving its simplicity. The key is to consider two label $\tau$ and $\tau^f$ for the arrival time for each stops. One represents the arrival time to the stop after the relaxation of one connection and the other due to the updating of the arrival time by footpaths. The ICSA algorithm is the same of CSA except for three modifications:

1. a connection is considered reachable if its starting time $t_{\text{dep}}(c)$ is larger or equal either to arrival time of the stops $\tau[s_{\text{dep}}(c)]$ due to connection updating or to the arrival time
Figure C: In this cartoon shows the updating process of the CSA algorithm and the error when
the closeness by transitivity of the footpaths between stops is not considered. First a connection
$c$ arriving at the stop $s_i$ relaxes, i.e. update the arrival time $\tau[\!s_i\!]$ of $s_i$. Then the arrival time
$\tau[\!s_i^*\!]$ of the neighbour stop $s_i^*$ is update through the footpath. Then other connections $\{c_j\}$
arriving to $s_i^*$ do not relax and the neighbour $\{s_k\}$ are never update by footpaths from $s_i^*$
causing possible errors.

$$\tau_f[\! s_{dep}(c)\!]$$ due to the footpaths update.

2. Both the CSA and ICSA update the arrival time of the stops in two ways: by direct
relaxation of the connections or by footpaths. The ICSA algorithm updates the arrival
time $\tau[\!s\!]$ of the stop $s$ when it is updated by connections, and the arrival time $\tau_f[\!s\!]$ when
it is updated by footpaths.

3. After the complete scanning of the connections array, the arrival time taken for each stops
$s$ is the best arrival time between the two labels $\tau u[s], \tau u f[s]$.

A pseudo-code implementation scheme is shown in fig D

These modifications allow to correctly solve the problem of finding the earliest arrival time
to stops, given a set of connections and footpaths connecting them, without the constraint of
the closeness under transitivity of footpaths.
for all stops \( s \) do \( \tau[s] \leftarrow \infty \);
for all stops \( s \) do \( \tau^f[s] \leftarrow \infty \);
\( \tau[s_{\text{start}}] \leftarrow t_0 \);
for all connections \( c \) increasing by \( t_{\text{dep}}(c) \) do
  if \( \tau[s_{\text{dep}}(c)] \leq t_{\text{dep}}(c) \) or \( \tau^f[s_{\text{dep}}(c)] \leq t_{\text{dep}}(c) \) then
    if \( \tau[s_{\text{arr}}(c)] > t_{\text{arr}}(c) \) then
      \( \tau[s_{\text{arr}}(c)] \leftarrow t_{\text{arr}}(c) \);
    for all footpaths \( f \) from \( s_{\text{arr}}(c) \) do
      \( \tau^f[f_{\text{arr}}] \leftarrow \min\{ \tau^f[f_{\text{arr}}], \tau[s_{\text{arr}}(c)] + f_{\text{dur}} \} \);
  end
end
for all stops \( s \) do \( \tau[s] \leftarrow \min(\tau[s], \tau^f[s]) \);

Figure D: Intransitive Connection Scan Algorithm

Figure E: Panel A: Average values of the velocity scores among the hexagons featuring the top 1% (light green) of the values of the velocity score as compared to the remaining 99% (dark green) for all cities analysed. The last columns report the same values averaged over all cities. Red stars mark, on the right \( y \)-axis, the are the ratios between the average values of top 1% and the other 99%. Panel B: Average values of the velocity scores among the population with the highest top 1% (light green) of the values of the velocity score as compared to the remaining 99% (dark green) for the six selected cities. The last columns report the same values averaged over all the six cities. Red stars mark, on the right \( y \)-axis, the are the ratios between the average values of top 1% and the other 99%.
Figure F: **Panel C**: Same as Panel A of Fig. E for the Sociality score. **Panel D**: Same as Panel B of Fig. E for the Sociality score.
Figure G: Exponential decay of the velocity score with travel-times for the city center. Velocity scores of hexagons at a given travel-time distance from the hexagon with the highest velocity score in each of the six selected city (blue dots). The orange points report a binning of the blue dots. The green line is the best fit of the data with the function 8 in the main text.
Figure H: **Exponential decay of the sociality score with travel-times for the city center.** Sociality scores of hexagons at a given travel-time distance from the hexagon with the highest sociality score in each of the six selected city (blue dots). The orange points report a binning of the blue dots. The green line is the best fit of the data with the function 8 in the main text.
References

[1] of Economic D, Social Affairs PD. World Urbanization Prospects: The 2014 Revision, Highlights (ST/ESA/SER.A/352). Population Division, United Nations. 2014;Available from: http://www.un.org/en/development/desa/news/population/world-urbanization-prospects-2014.html.

[2] Batty M. The new science of cities. Mit Press; 2013.

[3] Mayer-Schönberger V, Cukier K. Big data: A revolution that will transform how we live, work, and think. Houghton Mifflin Harcourt; 2013.

[4] Gallotti R, Bazzani A, Rambaldi S, Barthelemy M. A stochastic model of randomly accelerated walkers for human mobility. Nature Communications. 2016 08;7:12600. Available from: http://dx.doi.org/10.1038/ncomms12600.

[5] Alessandretti L, Karsai M, Gauvin L. User-based representation of time-resolved multimodal public transportation networks. Royal Society Open Science. 2016;3(7). Available from: http://rsos.royalsocietypublishing.org/content/3/7/160156 doi:10.1098/rsos.160156.

[6] Kölbl R, Helbing D. Energy laws in human travel behaviour. New Journal of Physics. 2003;5(1):48. Available from: http://stacks.iop.org/1367-2630/5/i=1/a=348.

[7] Mastroianni P, Monechi B, Liberto C, Valenti G, Servedio VD, Loreto V. Local Optimization Strategies in Urban Vehicular Mobility. PloS one. 2015;10(12):e0143799.

[8] Fleurquin P, Ramasco JJ, Eguíluz VM. Characterization of delay propagation in the US air-transportation network. Transportation journal. 2014;53(3):330–344.

[9] Gallotti R, Bazzani A, Rambaldi S. Understanding the variability of daily travel-time expenditures using GPS trajectory data. EPJ Data Science. 2015;4(1):18. Available from: http://dx.doi.org/10.1140/epjds/s13688-015-0055-z doi:10.1140/epjds/s13688-015-0055-z.

[10] Monechi B, Servedio VD, Loreto V. Congestion transition in air traffic networks. PloS one. 2015;10(5):e0125546.

[11] Sen P, Dasgupta S, Chatterjee A, Sreeram P, Mukherjee G, Manna S. Small-world properties of the Indian railway network. Physical Review E. 2003;67(3):036106.

[12] Guimerà R, Mossa S, Turtschi A, Amaral LN. The worldwide air transportation network: Anomalous centrality, community structure, and cities’ global roles. Proceedings of the National Academy of Sciences. 2005;102(22):7794–7799.

[13] United Nations Secretariat. World Urbanization Prospects: The 2014 Revision. Department of Economic and Social Affairs; 2014.

[14] Galton F. On the construction of isochronic passage-charts. In: Proceedings of the Royal Geographical Society and Monthly Record of Geography. vol. 3. JSTOR; 1881. p. 657–658.

[15] Zastrow M. Data visualization: Science on the map. Nature News. 2015;519(7541):119.

[16] Banavar JR, Maritan A, Rinaldo A. Size and form in efficient transportation networks. Nature. 1999 may;399(6732):130–132. Available from: http://www.nature.com/articles/20144 doi:10.1038/20144.
[17] Louf R, Roth C, Barthelemy M. Scaling in Transportation Networks. PLoS ONE. 2014 jul;9(7):e102007. Available from: http://dx.plos.org/10.1371/journal.pone.0102007. doi:10.1371/journal.pone.0102007.

[18] Masucci AP, Arcaute E, Hatna E, Stanilov K, Batty M. On the problem of boundaries and scaling for urban street networks. Journal of the Royal Society, Interface. 2015 oct;12(111):20150763. Available from: http://www.ncbi.nlm.nih.gov/pubmed/26468071http://www.pubmedcentral.nih.gov/articlerender.fcgi?artid=PMC4614511. doi:10.1098/rsif.2015.0763.

[19] Arcaute E, Molinero C, Hatna E, Murcio R, Vargas-Ruiz C, Masucci AP, et al. Cities and regions in Britain through hierarchical percolation. Royal Society Open Science. 2016 apr;3(4):150691. Available from: http://rsos.royalsocietypublishing.org/lookup/doi/10.1098/rsos.150691. doi:10.1098/rsos.150691.

[20] Li R, Dong L, Zhang J, Wang X, Wang WX, Di Z, et al. Simple spatial scaling rules behind complex cities. Nature Communications. 2017 dec;8(1):1841. Available from: http://www.nature.com/articles/s41467-017-01882-w. doi:10.1038/s41467-017-01882-w.

[21] Hansen WG. How accessibility shapes land use. Journal of the American Institute of planners. 1959;25(2):73–76.

[22] Black J, Conroy M. Accessibility measures and the social evaluation of urban structure. Environment and Planning A. 1977;9(9):1013–1031.

[23] Páez A, Scott DM, Morency C. Measuring accessibility: positive and normative implementations of various accessibility indicators. Journal of Transport Geography. 2012;25:141–153.

[24] Bok J, Kwon Y. Comparable Measures of Accessibility to Public Transport Using the General Transit Feed Specification. Sustainability. 2016;8(3):224.

[25] Schmöcker JD, Quddus MA, Noland RB, Bell MG. Mode choice of older and disabled people: a case study of shopping trips in London. Journal of Transport Geography. 2008;16(4):257–267.

[26] Miller HJ. Measuring space-time accessibility benefits within transportation networks: basic theory and computational procedures. Geographical analysis. 1999;31(1):1–26.

[27] Geurs KT, Van Wee B. Accessibility evaluation of land-use and transport strategies: review and research directions. Journal of Transport geography. 2004;12(2):127–140.

[28] Weiss D, Nelson A, Gibson H, Temperley W, Peedell S, Lieber A, et al. A global map of travel time to cities to assess inequalities in accessibility in 2015. Nature. 2018;553(7688):333.

[29] Forbes J. Mapping accessibility. Scottish Geographical Magazine. 1964 apr;80(1):12–21. doi:10.1080/00369226408735915.

[30] Luxen D, Vetter C. Real-time routing with OpenStreetMap data. In: Proceedings of the 19th ACM SIGSPATIAL International Conference on Advances in Geographic Information Systems. GIS ’11. New York, NY, USA: ACM; 2011. p. 513–516. Available from: http://doi.acm.org/10.1145/2093973.2094062. doi:10.1145/2093973.2094062.

[31] OpenStreetMap; Available at https://www.openstreetmap.org/
[32] TransitFeeds - Public transit feeds from around the world. Accessed on November 2017. https://transitfeeds.com/. Available from: https://transitfeeds.com/.

[33] OECD. Redefining Urban. OECD Publishing; 2012. Available from: https://www.oecd-ilibrary.org/content/publication/9789264174108-en. doi:https://doi.org/https://doi.org/10.1787/9789264174108-en.

[34] Eurostat population grid. Available at http://ec.europa.eu/eurostat/web/gisco/geodata/reference-data/population-distribution-demography/geostat.

[35] Statistics of European Cities. Available at http://ec.europa.eu/eurostat/statistics-explained/index.php/Statistics_on_European_cities.

[36] for International Earth Science Information Network CIESIN Columbia University C. Grid-ded Population of the World, Version 4 (GPWv4): Population Count. Palisades, NY: NASA Socioeconomic Data and Applications Center (SEDAC); 2016. Available from: http://dx.doi.org/10.7927/H4X63JVC.

[37] Delling D, Sanders P, Schultes D, Wagner D. Engineering route planning algorithms. In: Algorithmics of large and complex networks. Springer; 2009. p. 117–139.

[38] Disser Y, Müller-Hannemann M, Schnee M. Multi-criteria shortest paths in time-dependent train networks. In: International Workshop on Experimental and Efficient Algorithms. Springer; 2008. p. 347–361.

[39] Dibbelt J, Pajor T, Strasser B, Wagner D. Intriguingly simple and fast transit routing. In: International Symposium on Experimental Algorithms. Springer; 2013. p. 43–54.

[40] Kölbl R, Helbing D. Energy laws in human travel behaviour. New Journal of Physics. 2003;5(1):48.

[41] Clark C. Urban population densities. Journal of the Royal Statistical Society. 1951;114:490–496.

[42] Muth R. Cities and Housing. University of Chicago Press, Chicago; 1969.

[43] Mills ES. Urban density functions. Urban Studies. 1970;7:5–20.

[44] Biazzo I, Monechi B, Loreto V. public-transport-analysis; 2018. Open source code. https://github.com/CityChrone/public-transport-analysis. doi:10.5281/zenodo.1309835.

[45] Biazzo I, Monechi B, Loreto V. openData; 2018. Open data available at https://github.com/CityChrone/openData. doi:10.5281/zenodo.1309927.

[46] Bast H, Delling D, Goldberg A, Miller-Hannemann M, Pajor T, Sanders P, et al. Route Planning in Transportation Networks. In: Algorithm Engineering. Springer International Publishing; 2016. p. 19–80.

[47] Delling D, Pajor T, Werneck RF. Round-based public transit routing. Transportation Science. 2014;49(3):591–604.