De Sitter Breaking in Field Theory

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ABSTRACT

I argue against the widespread notion that manifest de Sitter invariance on the full de Sitter manifold is either useful or even attainable in gauge theories. Green’s functions and propagators computed in a de Sitter invariant gauge are generally more complicated than in some noninvariant gauges. What is worse, solving the gauge-fixed field equations in a de Sitter invariant gauge generally leads to violations of the original, gauge invariant field equations. The most interesting free quantum field theories possess no normalizable, de Sitter invariant states. This precludes the existence of de Sitter invariant propagators. Even had such propagators existed, infrared divergent processes would still break de Sitter invariance.

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1 Introduction

Stanley Deser has been my mentor for over two decades. One of the many things he taught me is that theoretical physics is characterized by long periods of stagnation, punctuated by bursts of activity after some insight or technical advance makes progress possible. When this happens one has to push forward as far and as fast as possible because these opportunities don’t arise often. Stanley’s career has exemplified this, starting in the late 50’s with the canonical formulation of gravity that his work with Arnowitt and Misner made possible [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13]. Another fine example is the way Stanley and various collaborators exploited the newly developed technology of dimensional regularization and the background field formalism in the mid 70’s to analyze the one loop divergences of gravity combined with other theories [14, 15, 16, 17].

Much of my recent work has dealt with exploiting a technical advance that has made it possible to get interesting results from quantum field theory during inflation. The advance is the development of relatively simple propagators for massless fields on a locally de Sitter background of arbitrary spacetime dimension. This has made it possible to use dimensional regularization to go beyond the coincidence limits of one loop stress tensors — the technology for which had been codified before I graduated [18]. One can now get at the deeply nonlocal, ultraviolet finite parts of quantum processes during inflation.

Section 2 of this article reviews what has been done. Section 3 explains why some of my methods offend the aesthetic prejudices of the mathematically minded. However much more attractive the formalism might seem their way, it would be neither practical, nor physically correct, nor would its most interesting predictions be free of the unaesthetic properties of my techniques. The various problems of practicality and of principle are described in section 4. Section 5 summarizes my conclusions.

2 Quantum Field Theory during Inflation

I model inflation using a portion of the full de Sitter manifold known as the open conformal coordinate patch. If the D-dimensional cosmological
constant is \( \Lambda \equiv (D-1)H^2 \), the invariant element is,

\[
    ds^2 = a^2 \left( -d\eta^2 + d\vec{x} \cdot d\vec{x} \right)
    \quad \text{where} \quad a(\eta) = -\frac{1}{H\eta}.
\]  

(1)

The conformal time \( \eta \) runs from \(-\infty\) to zero. The various propagators have simple expressions in terms of the following function of the invariant length \( \ell(x; x') \) between \( x^\mu \) and \( x'^\mu \),

\[
    y(x; x') \equiv 4 \sin^2 \left( \frac{1}{2} H \ell(x; x') \right) = a a' H^2 \left( \|\vec{x} - \vec{x}'\|^2 - (|\eta - \eta'| - i\delta)^2 \right). \tag{2}
\]

One might expect that the inflationary expansion of this spacetime makes quantum effects stronger by allowing virtual particles to persist longer than in flat space. Indeed, it is simple to see that any sufficiently long wavelength \( (\lambda > 1/H) \) virtual particle which is massless on the Hubble scale can exist forever \[19\]. However, one must also consider the rate at which virtual particles emerge from the vacuum. Classical conformal invariance causes this rate to fall off exponentially, so any long wavelength virtual particles which emerge become real, but very few emerge \[19\]. To get enhanced quantum effects during inflation requires quanta which are effectively massless and also not conformally invariant. Even one such particle can catalyze processes involving conformally invariant particles.

It has long been known how to write the propagator for a massless, conformally coupled scalar in arbitrary dimension \[18\],

\[
    i\Delta_{\text{cf}}(x; x') = \frac{H^{D-2}}{(4\pi)^{D/2}} \Gamma \left( \frac{D}{2} - 1 \right) \left( \frac{4}{y} \right)^{D/2 - 1}. \tag{3}
\]

Massless fermions are also conformally invariant in any dimension and their propagator is closely related,

\[
    i \left[ iS \right] (x; x') = (aa')^{\frac{1-D}{2}} i \partial_\mu \gamma_\mu \left[ (aa')^{D/2 - 1} i\Delta_{\text{cf}}(x; x') \right]. \tag{4}
\]

One can compute with these propagators but the results are not much different from flat space on account of conformal invariance.

The advance that has made interesting quantum effects computable is explicit expressions for the propagators of particles which are massless and not conformally invariant. The first of these is the minimally coupled scalar,
\[i \Delta_A(x; x') = i \Delta_{cf}(x; x') + H_D - \frac{2}{(4\pi)^{D/2}} \Gamma\left(\frac{D-1}{2}\right) \left\{ \frac{\Gamma\left(\frac{D}{2}\right)}{\Gamma\left(\frac{D}{2} - 1\right)} \left(\frac{y}{4}\right)^{D/2 - 2} - \frac{\pi}{2} \cot\left(\frac{\pi}{2} \frac{D}{2}\right) + \ln(aa') \right\} + \frac{H_{D-2}}{(4\pi)^{D/2}} \sum_{n=1}^{\infty} \left\{ \frac{1}{n} \frac{\Gamma(n+D-1)}{\Gamma(n+\frac{D}{2})} \left(\frac{y}{4}\right)^n - \frac{1}{n-\frac{D}{2}+2} \frac{\Gamma(n+\frac{D}{2}+1)}{\Gamma(n+2)} \left(\frac{y}{4}\right)^{n-\frac{D}{2}+2} \right\}. \quad (5)\]

This might seem a daunting expression but it isn’t so bad because the infinite sum on the final line vanishes in \(D = 4\), and each term in the series goes like positive powers of \(y(x; x')\). This means the infinite sum can only contribute when multiplied by a divergent term, and even then only a small number of terms can contribute.

Fascinating physics has been revealed by endowing such a scalar with different sorts of interactions. When a quartic self-interaction is present one can compute the VEV of the stress tensor \([20, 21]\) and the scalar self-mass-squared \([22]\) at one and two loop orders. The resulting model shows a violation of the weak energy condition — on cosmological scales! — in which inflationary particle production drives the scalar up its potential and induces a curious sort of time-dependent mass. When a complex scalar of this type is coupled to electromagnetism is has been possible to compute the one loop vacuum polarization \([23, 24]\) and use the result to solve the quantum corrected Maxwell equations \([25]\). Although photon creation is suppressed during inflation, this model shows a vast enhancement of the 0-point energy of super-horizon photons which may serve to seed cosmological magnetic fields \([26, 27, 28]\). Finally, when a real scalar of this type is Yukawa coupled to a massless Dirac fermion it has been possible to compute the one loop fermion self-energy and use it to solve the quantum corrected Dirac equation \([29]\). The resulting model shows explosive creation of fermions which should make inflation end with the super-horizon modes in a degenerate Fermi gas!

Electromagnetism is a special case, being conformally invariant in \(D = 4\) but not generally. My favorite gauge fixing term is an analogue of the one introduced by Feynmann in flat space,

\[L_{GF} = -\frac{1}{2} a^{D-4} \left( \eta^\mu_\nu A_\mu,\nu - (D-4) H a A_0 \right)^2. \quad (6)\]

Because space and time components are treated differently it is useful to have
an expression for the purely spatial part of the Minkowski metric,
\[ \bar{\eta}_{\mu\nu} \equiv \eta_{\mu\nu} + \delta^0_\mu \delta^0_\nu. \] (7)
In this gauge the photon propagator takes the form,
\[ i[\mu \Delta_\nu](x; x') = \bar{\eta}_{\mu\nu} a a' i \Delta_B(x; x') - \delta^0_\mu \delta^0_\nu a a' i \Delta_C(x; x'). \] (8)
The B-type and C-type propagators are,
\[ i \Delta_B(x; x') = i \Delta_{ct}(x; x') - \frac{H^{D-2}}{(4\pi)^2} \sum_{n=0}^{\infty} \left\{ \frac{\Gamma(n+D-2)}{\Gamma(n+\frac{D}{2})} \left( \frac{y}{4} \right)^n \right. \]
\[ -\frac{\Gamma(n+\frac{D}{2})}{\Gamma(n+2)} \left( \frac{y}{4} \right)^{n-\frac{D}{2}+2} \} \] (9)
\[ i \Delta_C(x; x') = i \Delta_{ct}(x; x') + \frac{H^{D-2}}{(4\pi)^2} \sum_{n=0}^{\infty} \left\{ (n+1) \frac{\Gamma(n+D-3)}{\Gamma(n+\frac{D}{2})} \left( \frac{y}{4} \right)^n \right. \]
\[ -\left( n-\frac{D}{2} + 3 \right) \frac{\Gamma(n+\frac{D}{2}-1)}{\Gamma(n+2)} \left( \frac{y}{4} \right)^{n-\frac{D}{2}+2} \}. \] (10)
As with the A-type propagator (5), the infinite sums in (9) and (10) vanish in \( D = 4 \). In fact the B-type and C type propagators agree in \( D = 4 \), and the photon propagator is the same for \( D = 4 \) as it is in flat space!

No results have been published using the photon propagator but L. D. Duffy and I are computing the one loop scalar self-mass-squared in scalar QED. We expect its secular growth to eventually choke off the inflationary particle production that so enhances the one loop vacuum polarization [25]. Of course this can’t eliminate scalars which have already been ripped out of the vacuum, or the vacuum polarization they induce. A similar computation of the scalar self-mass-squared of the Yukawa scalar fails to show any secular growth at one loop order [30], implying that the scalar-catalyzed production of super-horizon fermions goes to completion [29].

Gravitons are also massless without conformal invariance. I define the graviton field \( \psi_{\mu\nu}(x) \) as follows,
\[ g_{\mu\nu}(x) \equiv a^2 (\eta_{\mu\nu} + \kappa \psi_{\mu\nu}(x)) \quad \text{where} \quad \kappa^2 \equiv 16\pi G. \] (11)
My favorite gauge fixing term is an analogue of the de Donder term used in flat space [31],
\[ \mathcal{L}_{GF} = -\frac{1}{2} a^{D-2} \eta^{\mu\nu} F_{\mu} F_{\nu}, \quad F_{\mu} \equiv \eta^{\rho\sigma} \left( \psi_{\mu\rho,\sigma} - \frac{1}{2} \psi_{\rho\sigma,\mu} + (D-2) H a \psi_{\mu\rho} \delta^0_\sigma \right). \] (12)
With these definitions the graviton propagator takes the form of a sum of three constant index factors times the three scalar propagators,

\[ i [\mu \nu \Delta_{\rho \sigma}] (x; x') = \sum_{I=A,B,C} [\mu \nu T^I_{\rho \sigma}] i \Delta_I (x; x') . \]  

(13)

The index factors are,

\[ [\mu \nu T^A_{\rho \sigma}] = 2 \eta_{\mu \rho} \eta_{\nu \sigma} - \frac{2}{D-3} \eta_{\mu \nu} \eta_{\rho \sigma} , \]  

(14)

\[ [\mu \nu T^B_{\rho \sigma}] = -4 \delta_{(\mu} \eta_{\nu)} \delta_{(\rho} \delta_{\sigma)} , \]  

(15)

\[ [\mu \nu T^C_{\rho \sigma}] = \frac{2}{(D-2)(D-3)} [(D-3)\delta^0_\mu \delta^0_\nu + \eta_{\mu \nu}] [(D-3)\delta^0_\rho \delta^0_\sigma + \eta_{\rho \sigma}] . \]  

(16)

The full power of the dimensionally regulated graviton propagator has not so far been exploited in published work. However, the one loop graviton self-energy [32] has been computed using a \( D = 4 \) cutoff. The expectation value of the invariant element has also been obtained at two loop order [33]. These results indicate that the back-reaction from graviton production slows inflation by an amount which eventually becomes nonperturbatively large [34]. N. C. Tsamis and I have used the dimensionally regulated formalism to compute the expectation value of the metric at one loop order. E. O. Kahya and I are also using it to compute the one loop scalar self-mass-squared induced by graviton exchange. This might have important consequences for models which inflate for a very large number of e-foldings.

### 3 What Bothers People

Despite all the results that have been obtained, and the ones which are attainable, the response of the theoretical physics community has been — underwhelming. Different segments of the community have different reasons for ignoring my work. Many inflationary cosmologists feel that causality precludes interesting quantum field theoretic effects. Some of them even seem to have forgotten that the density perturbations which figure so prominently in recent observation [35, 36] are driven by precisely the same inflationary particle production [37, 38] that underlies each of the effects reported in the previous section! String theorists are not much interested in physics that doesn’t make essential use of their candidate for a theory of everything. They
also flirt with the notion that there are no observables in de Sitter, which requires them to disbelieve that quantum corrections to the field equations mean anything. Loop space gravity people have trouble achieving correspondence with most forms of perturbation theory, including mine. And phenomenologists seek to work out the consequences of popular theories, so the fact that few people pay attention to my work serves to justify continuing to ignore it!

There isn’t much I can do about this. But I could converse with one segment of the community if only it was possible to overcome the distaste its members have for the methods I use. I refer to the mathematical relativists. They are prepared to accept that quantum field theory might have interesting effects during inflation, and that these can be quantified in a reliable way. They are even willing to let me use Minkowski-signature perturbation theory starting with a prepared initial state! However, they are strongly attracted by the analogy between Minkowski space and de Sitter space, the maximally symmetric solutions of Einstein’s equations for $\Lambda = 0$ and $\Lambda > 0$, respectively. They feel that manifest de Sitter invariance on the full de Sitter manifold should be as powerful an organizing principle for quantum field theory with $\Lambda > 0$ as Poincaré invariance has been for $\Lambda = 0$. So it bothers them that my open conformal coordinate patch (1) does not cover the full de Sitter manifold and that the gauge fixing terms I use — (6) and (12) — are not de Sitter invariant.

4 You Can’t Always Get What You Want

Mick Jagger and Keith Richards are not my favorite authorities on much of anything, but one of their songs seems relevant here. I will argue that it isn’t necessary, convenient or even possible to impose de Sitter invariant gauges and work on the full manifold. Nor would doing so lead to de Sitter invariant results for the most interesting processes if it were possible.

Necessity is the simplest issue. Everyone understands that it isn’t necessary to use de Sitter invariant gauges, just as it isn’t necessary to use Poincaré invariant gauges in flat space. Nor is there any logical problem with restricting physics to the open conformal coordinate patch (1), especially if one contemplates releasing a prepared state from a finite initial time. The condition $\eta = \text{constant}$ defines a perfectly good Cauchy surface. Information from the rest of the full de Sitter manifold can only propagate to the future of such
a surface by passing through it as part of the initial condition. Indeed, the case for restricting to (1) can be put much more strongly if one imagines — as I do — the local de Sitter background as merely a model for the more complicated geometry of the inflating epoch of cosmology. I am not interested in quantum field theory on perfect de Sitter space but rather in potentially observable quantum phenomena from the epoch of primordial inflation. In that case the relevant symmetries are homogeneity and isotropy, not the full de Sitter group, and the conformal coordinate patch — with arbitrary $a(\eta)$ — is the coordinate system in which these symmetries are manifest.

The reason people typically prefer to maintain manifest Poincaré invariance in flat space is that it makes things simpler. That this is not true for de Sitter can be seen by comparing propagators in my gauges with those in the simplest de Sitter invariant gauges. It will sharpen the distinction if we take $D = 4$. In that case the photon propagator in my gauge [12] is the same function of conformal coordinates as it is in flat space,

$$i \left[ \mu \Delta_{\nu} \right] (x; x') \big|_{D=4} = \frac{\eta_{\mu\nu}}{4\pi^2 \Delta x^2},$$

where $\Delta x^2 \equiv \|x - x'|^2 - (|\eta - \eta'| - i\delta)^2$. It is worth pointing out that this expression applies to any homogeneous and isotropic geometry in conformal coordinates, not just the special case of de Sitter.

The simplest de Sitter invariant photon propagator of which I know was obtained by Allen and Jacobson [39] with the gauge fixing term,

$$\mathcal{L}_{\text{inv}} = -\frac{1}{2} \left( g^{\mu\nu} A_{\mu\nu} \right)^2 \sqrt{-g} = -\frac{1}{2} a^{D-4} \left( \eta^{\mu\nu} A_{\mu\nu} - (D-2) H a A_0 \right)^2. \tag{18}$$

Their propagator takes the form,

$$i \left[ \mu \Delta_{\nu} \right] (x; x') \big|_{\text{inv}} = \alpha(y) \left[ \mu g_{\nu} \right] (x; x') + \beta(y) \left[ \mu n_{\nu} \right] (x; x') \left[ n_{\nu} \right] (x; x'), \tag{19}$$

where $y(x; x') \equiv \alpha a' H^2 \Delta x^2$, $\left[ \mu g_{\nu} \right] (x; x')$ is the parallel transport matrix and $\left[ \mu n_{\nu} \right] (x; x')$ and $\left[ n_{\nu} \right] (x; x')$ are the gradients with respect to $x^\mu$ and $x'^\nu$ of the geodesic length. In $D = 4$ the coefficient functions are,

$$\alpha(y) = \frac{H^2}{4\pi^2} \left\{ \frac{1}{y} + \frac{1}{4-y} + \frac{(4-2y)}{(4-y)^2} \ln \left( \frac{4}{y} \right) \right\}, \tag{20}$$

$$\beta(y) = \frac{H^2}{4\pi^2} \left\{ -\frac{5}{4-y} - \frac{2y}{(4-y)^2} \ln \left( \frac{4}{y} \right) \right\}. \tag{21}$$
The 3+1 decomposition of the parallel transport matrix is,

\[
\begin{pmatrix}
  \mu \\
n
\end{pmatrix} = a a' \begin{pmatrix}
  1 & 0 \\
  0 & \delta_{mn}
\end{pmatrix} + \frac{2}{4-y} \begin{pmatrix}
  -(a+a')^2 & (a+a')aa' H \Delta x_n \\
a^2a'^2H^2 \Delta x_m \Delta x_n
\end{pmatrix},
\]  

(22)

The other tensor has the following 3+1 decomposition,

\[
\begin{pmatrix}
  \mu \\
n
\end{pmatrix} = -\frac{1}{y} \begin{pmatrix}
  aa'y+2a^2+2a'^2 & -2a^2a'H\Delta x_n \\
2aa'^2H\Delta x_m & 0
\end{pmatrix}
\]

\[
+ \frac{4}{y(4-y)} \begin{pmatrix}
  (a+a')^2 & -(a+a')aa' H \Delta x_n \\
(a+a')aa' H \Delta x_m & a^2a'^2H^2 \Delta x_m \Delta x_n
\end{pmatrix}.
\]  

(23)

However much one may admire manifest de Sitter invariance, I hope we can all agree that it doesn’t simplify propagators.

But suppose you are fanatical about de Sitter invariance and you prefer to compute on the full de Sitter manifold in a gauge which is manifestly de Sitter invariant, no matter how much harder it is. In that case you risk violating the invariant equations of motion! The problem arises from combining the causal properties of de Sitter with the constraint equations of any gauge theory. Before gauge fixing the constraint equations are elliptic, and they typically result in a nonzero response to sources throughout the de Sitter manifold, even in regions which are not future-related to the source. But gauge fixing in a de Sitter invariant manner results in hyperbolic equations for which the response to sources is zero for regions which are not future-related to the source. As far as I know this problem was first noted by Penrose \[40\] in 1963. Tsamis and I encountered it for gravity in 1994 \[31\] and recent studies for electromagnetism have been conducted by Bičák and Krtouš \[41\].

To better understand the problem let us adopt the standard closed coordinatization of the full de Sitter manifold,

\[
ds^2 = -dt^2 + H^{-2} \cosh^2(Ht) \left( d\chi^2 + \sin^2(\chi) d\theta^2 + \sin^2(\chi) \sin^2(\theta) d\phi^2 \right).
\]  

(24)

Consider the invariant Maxwell equations for a pair of oppositely charged point particles,

\[
\partial_\mu \left( \sqrt{-g} g^{\mu\rho} g^{\nu\sigma} F_{\rho\sigma} \right) = q \int d\tau \left[ \dot{z}_+^\nu(\tau) \delta^4 \left( x-z_+(\tau) \right) - \dot{z}_-^\nu(\tau) \delta^4 \left( x-z_-(\tau) \right) \right].
\]  

(25)

When the +q charge is stationary at \( \chi = 0 \) and the -q charge is stationary at \( \chi = \pi \) a perfectly good solution exists,

\[
A_\mu = \delta_\mu^0 A_0(t, \chi) = \frac{qH}{4\pi} \text{sech}(Ht) \cot(\chi).
\]  

(26)
Suppose we try to find a gauge parameter $\theta(x)$ such that the transformed field, $A'_\mu = A_\mu - \partial_\mu \theta$ obeys the de Sitter invariant condition $A'^\mu;_\mu = 0$,

$$\partial_\mu(\sqrt{-g}g^{\mu\nu} \partial_\nu \theta) = \partial_\mu(\sqrt{-g}g^{\mu\nu} A_\nu) = \frac{-q}{4\pi H} \sinh(2Ht)\frac{\sin^3(\chi)}{\cos(\chi)} \sin(\theta) \equiv S(x). \quad (27)$$

You might think this is easy with a Green’s function,

$$\partial_\mu(\sqrt{-g}g^{\mu\nu} \partial_\nu G(x; x')) = \delta^4(x - x') \implies \theta(x) = \int d^4x' G(x; x') S(x'). \quad (28)$$

However, the retarded Green’s function,

$$G^\text{ret}(x; x') = \frac{H^2\theta(\Delta t)}{4\pi} \left[ 2\delta(y(x; x')) + \theta(-y(x; x')) \right], \quad (29)$$

contains a $\theta$-function tail term which is nonzero throughout the volume of the past light-cone. Because the source (27) actually grows as $t \to -\infty$, the integral (28), and even its gradient, fail to converge.

Note that the electric field of (29) points from the $+q$ to the $-q$ charge and is nonzero throughout the full de Sitter manifold manifold,

$$F^{x^0} = \frac{qH^3}{4\pi} \text{sech}^3(Ht) \csc^2(\chi). \quad (30)$$

This isn’t at all what one gets by integrating the photon retarded Green’s function against the current density in a de Sitter invariant gauge,

$$A^\text{ret}_\mu(x) = \int d^4x' \left[ \mu G^\text{ret}_\nu(x; x') J^\nu(x') \right]. \quad (31)$$

One can recover the retarded Green’s function from the Allen-Jacobson propagator (19) by simply taking the imaginary part and multiplying by $-2\theta(t-t')$,

$$\left[ \mu G^\text{ret}_\nu \right](x; x') = \frac{H^2\theta(\Delta t)}{4\pi} \left\{ \delta(y) - \left( \frac{8}{3} \frac{y}{(4-y)^2} \right) \theta(-y) \right\} \left[ \mu g_\nu \right](x; x')$$

$$+ \frac{H^2\theta(\Delta t)}{4\pi} \left\{ \frac{4}{(4-y)^2} \theta(-y) \right\} \left[ \mu n_\nu \right](x; x') \left[ n_\nu \right](x; x'). \quad (32)$$

The retarded Green’s function is causal, so the response from it vanishes in the vast region of the full de Sitter manifold which is not future-related to either of the source world lines.
It turns out that the Allen-Jacobson Green’s function does give the correct response within the open conformal coordinate patch, so a de Sitter invariant gauge can at least be imposed locally in electromagnetism. (I thank A. O. Barvinsky for correcting me about this, and I apologize to Allen and Jacobson for having said otherwise at the Deserfest.) The same does not seem to be true in gravity. Antoniadis and Mottola have shown that de Sitter invariant graviton propagators — which are also much more complicated than the one in my favorite gauge (12) — lead to local violations of the linearized Einstein equations [42]. These violations are not present when using my non-invariant propagator (13) [31].

Note that the problem reconciling causality and the constraints is classical. I advance for your consideration the folly of working much harder to quantize a formalism that doesn’t even correctly reflect classical physics. When confronted with the causality obstacle de Sitter fanatics sometimes respond that the problem arises from the constraints not having been imposed throughout the initial value surface. When this is done the full system can be evolved just fine. I don’t dispute this but it misses the point. The issue is not whether physics can be done on the full de Sitter manifold. There was never any doubt about that: (26) is the instantaneous Coulomb potential of Coulomb gauge. The issue is rather whether or not physics can be done maintaining manifest de Sitter invariance. The answer is no. Imposing the constraints can always be subsumed into adding a surface gauge condition that breaks manifest de Sitter invariance.

Moving from classical to quantum field theory, recall that the condition for getting enhanced quantum effects during inflation is massless particles which are not classically conformally invariant. There are two such particles: the massless, minimally coupled scalar and the graviton. Consideration of these particles is the only phenomenological justification for studying quantum field theory during inflation, so we cannot dismiss them if they happen to violate aesthetic prejudices. As it happens, the free quantum field theory of neither system possesses a normalizable, de Sitter invariant wave functional. This was proved long ago for the massless, minimally coupled scalar by Allen and Folacci [43]. It can be seen for my graviton propagator (13) by simply performing a naive de Sitter transformation coupled with the compensating gauge transformation needed to restore my noninvariant gauge (12) [44]. Contrary assertions for gravitons are always based upon using de Sitter invariant gauges on the full manifold, which I have just shown to be incorrect.

The fact that the most interesting free quantum field theories have no
de Sitter invariant states means that the propagators of these fields must break de Sitter invariance, not just through the gauge fixing function but in a fundamental way. One can see this in the factor of $\ln(aa')$ on the second line of expression (5) for $i\Delta_A(x; x')$. There is no sense complicating a marginally tractable formalism to respect a symmetry which is not there.

My final point is that even if manifestly de Sitter invariant propagators had existed, the most interesting interactions would still break de Sitter invariance. I don’t mean the interaction vertices would be noninvariant. They are manifestly invariant. What I mean instead is that higher order processes can involve integrals over interaction vertices. De Sitter invariant propagators would make the integrands invariant but would not guarantee that the integrals were invariant. Consider the invariant volume of the past light-cone from some observation point $x^\mu$ back to the initial value surface (IVS) on which the prepared state was released,

$$V(x) \equiv \int_{IVS} d^4x' \sqrt{-g(x')} \theta(\Delta t) \theta(-y(x; x')).$$

The integrand is manifestly de Sitter invariant — one inside the past light-cone and zero outside — but the integral grows as the observation point is taken later and later after the state was released. One can see from the scalar retarded Green’s function (29) that this example is not artificial. Un-suppressed integrals over the volume of the past light-cone occur in many of these computations [20]. They give factors of $\ln(a)$ every bit as important as the explicit ones from the de Sitter breaking terms of $i\Delta_A(x; x')$. It would not be far wrong to say that extracting these secular logarithms is the whole point of studying quantum field theory during inflation.

5 Conclusions

Massless particles which are not conformally invariant can mediate interesting quantum effects during inflation. Even a single non-conformal massless particle can catalyze surprising processes which would otherwise not go [19]. It is now possible to study this by modeling inflation as the open coordinate patch of de Sitter space, and by exploiting simple gauge fixing terms.

This bothers de Sitter fanatics, who would prefer to work on the entire de Sitter manifold and to employ only de Sitter invariant gauge conditions. That would not be easy because de Sitter invariant gauges complicate propagators.
It is also incorrect physically because a de Sitter invariant gauge converts elliptic constraint equations into hyperbolic evolution equations. Whereas the former require a nonzero response throughout the manifold to a sufficiently distributed source, the later give zero response in the vast regions of de Sitter space which are not future-related to a source worldline. In some cases the use of a de Sitter invariant gauge even leads to violations of the original, gauge invariant field equations within the region which is future-related to the a source worldline [42].

The free quantum field theories of the two massless particles which are not conformally invariant admit no de Sitter invariant states. This means that the propagators of these fields cannot be de Sitter invariant [M 11]. Even had all propagators been de Sitter invariant, interactions would still break this invariance. The infrared logarithms which signal this breaking are at the heart of what makes quantum field theory during inflation potentially observable. I think we’d all rather have interesting quantum dynamics without symmetry than sterile dynamics with a beautiful symmetry. As Mick and Keith put it: “You can’t always get what you want. But if you try sometime you find, you get what you need!”

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