Comments on No-Hair Theorems and Stability of Blackholes

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ABSTRACT

In the light of recent blackhole solutions inspired by string theory, we review some old statements on field theoretic hair on blackholes. We also discuss some stability issues. In particular we argue that the two dimensional string blackhole solution is semi-classically stable while the naked singularity is unstable to tachyon fluctuations. Finally we comment on the relation between the linear dilaton theory and the 2d blackhole solution.

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No-hair theorems for blackholes seem to be of two types. The earlier theo-
rems\[1\] referred to the absence of macroscopic characteristics such as the higher
moments of the mass distribution in a blackhole. In fact these theorems more or
less established that a blackhole is in many respects like an elementary particle.
On the other hand the second set of no-hair theorems \[2\],\[3\] seemed to show that
blackholes do not have many of the features of elementary particles either. In par-
ticular they cannot have baryon number and more generally it was believed (though
never rigorously established) that in classical field theoretic blackhole solutions all
fields except the static gravitational, electric, and magnetic, fields go to zero (or
unobservable constant) values outside the horizon. In the case of the physically
interesting spontaneous breakdown of continuous gauge symmetry, arguments were
given to the effect that (at least in the abelian case) the gauge field vanished and
the Higgs field went to a constant value everywhere outside the horizon.

In this paper the assumptions underlying the field theoretic no-hair theorems
are examined. This is motivated in part by the appearance of several recent articles
\[4\],\[5\],\[6\]|\[7\],\[8\],\[9\], \[10\] (largely motivated by string theory) which seem to indicate
that various types of (classical) hair (other than the ones permitted by the no-
hair theorems) are possible. In fact it has been conjectured that string theoretic
blackholes may carry an infinite variety of hair \[8\],\[11\],\[10\], \[12\]. The way in which
the classical no-hair theorems are voided in these theories is thus of some interest.
In particular the key role played by the dilaton is elucidated. Next the question of
the stability of some of these black hole solutions is addressed briefly. In particular
the stability of the Adler-Pearson black hole for the spontaneously broken abelian
Higgs model and the two dimensional stringy blackhole of \[8\],\[9\], is discussed.

Let us first review Bekenstein’s argument for static blackholes. The horizon
of a blackhole is a null-surface  $F(x^i) = 0$ \(^\dagger\) and the normal to it is $n_\mu = \partial_\mu F(x^i)$
with the surface element being $dS_\mu = \sigma n_\mu$ with $\sigma$ being an invariant. If $\mathcal{L}(\phi_i)$ is
a Lagrangian for a set $F$ of fields coupled to gravity, $\mathcal{M}$ the region outside the

\(^\dagger\) $\mu = 0,1,2,\ldots,d-1$  \( i = 1,2,\ldots,d-1 \) the signature is $(-,+,+,+,...)$ and the definitions of
curvature tensors is as in MTW \[1\].
horizon with $\partial M$ being its boundary, then the following is easily established from the equations of motion.

$$\sum_{i\in F} \int_{F \subset \mathcal{M}} \sqrt{g} d^d x \left[ \partial_\mu \phi_i \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_i} + \phi_i \frac{\partial \mathcal{L}}{\partial \phi_i} \right] = \int_{\partial M} b_{\mu} dS^\mu, \quad (1)$$

where $b^\mu = \sum_{i} \epsilon_{F} \phi_i \frac{\partial \mathcal{L}}{\partial \partial_\mu \phi_i}$. As Bekenstein points out $g_{ij}$ is a positive definite metric; so it is possible to show, if $g_{ij} b^i b^j$ is bounded on the horizon, that $g_{ij} dS^i b^j = 0$ at the horizon. If in addition $b_0$ is zero at the horizon then the contribution of the latter to the RHS of (1) is zero. At time-like infinity the contribution to the RHS is automatically zero and at space-like infinity it is zero if $\phi$ tend to zero at least as fast as $\frac{1}{r}$, $r$ being the coordinate distance from the blackhole. If these conditions are satisfied then the RHS of (1) is zero and if in addition the terms in the integrand are positive definite then a no-hair theorem is obtained.

In applications of the above, the constraints on $b$ at the horizon are obtained by the requirement that physical scalars are bounded there. Clearly the trace of the stress tensor or any other invariant constructed out of the stress tensor should be bounded on the horizon. Let us look first at scalar field theory with the Lagrangian

$$\mathcal{L} = -\frac{1}{2} (g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi))$$

The stress tensor is

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} (\partial_\alpha \phi \partial^\alpha \phi + V(\phi))$$

$$T^\mu_\mu = \frac{2}{d} \partial_\alpha \phi \partial^\alpha \phi - \frac{d}{2} V(\phi)$$

\[\d \sqrt{g} = \sqrt{\det g_{\mu\nu}}.\]
In the static case $\partial_0 \phi = 0$, and $-T_\mu = \frac{d-2}{2}g^{ij}\partial_i \phi \partial_j \phi + \frac{d}{2}V(\phi)$, and $g^{ij}$ is positive definite, so for a (polynomial) potential whose highest power in $\phi$ is positive, the finiteness of $T$ implies that $\phi$ is bounded.

It is important to note here that if a scalar field has no (polynomial) potential as is the case for the Goldstone boson field, the axion, and the dilaton then the above argument does not apply. However we may still argue that the scalar fields are physical quantities which ought to be bounded at the horizon. In the case of the dilaton for instance, its exponential is the coupling constant, and one would certainly require the latter to be neither zero nor infinite at the horizon. We will consider below the case of the axion and the dilaton blackhole. Solutions having both types of hair have recently been constructed [4], [5], [6], [7], [8], [9], and it is instructive to see how precisely they avoid the no-hair theorem.

The matter action for the axion case is

$$S_A = \int \sqrt{g}d^4x[\partial_\mu a \partial^\mu a + a F_{\mu\nu} \tilde{F}^{\mu\nu} + ...],$$

(2)

where the ellipses denote $a$ independent terms. From the requirement that the trace of the stress tensor is bounded and the positive definiteness of the spatial metric, we have the result that $g^{ij}\partial_i a \partial_j a$ is bounded at the horizon. Then assuming that $a$ is a physical scalar which should also be bounded there, we have from (1) the relation

$$\int d^4x \sqrt{g}(g^{ij}\partial_i a \partial_j a + a F \tilde{F}) = 0.$$

Since the second term is not positive definite we cannot conclude that the field $a$ vanishes outside the horizon. However if the either the electric or the magnetic charge of the black hole were to be zero then we would have a no-hair theorem for the axionic field. This is consistent with the explicit solution of [5] which is indeed a dyonic blackhole.
Let us now consider blackhole solutions coming from the low energy effective action derived from (classical) string theory [13]. The latter takes the following form (keeping just the gauge field and axion terms in addition to the metric and dilaton) in \( d \) dimensional space-time.

\[
S = \int d^d x \sqrt{g} e^{-2\Phi} [R + 4(\nabla \Phi)^2 - \frac{1}{4} F^2 - \frac{1}{12} H^2 - \frac{\dd - 26(10)}{3} + \ldots] \quad (3)
\]

\( \dd = d + \delta c \) where \( \delta c \) is the central charge of a compact unitary cft which may be tensored with the \( d \) target space dimensional sigma model. For a critical string \( \dd = 26 \) (or 10 for the superstring). Note that the dilaton kinetic term in this action has the wrong sign. For \( d > 2 \) we can do a Weyl transformation \( g_{\mu \nu} \to e^{4\Phi/(d-2)} g_{\mu \nu} \) to get the action in the canonical form,

\[
S = \int d^d x \sqrt{g} [R - \frac{4}{d-2} (\nabla \Phi)^2 - \frac{1}{4} e^{-\frac{4\Phi}{d-2}} F^2 - \frac{1}{12} e^{-\frac{8\Phi}{d-2}} H^2
- e^{-\frac{2\Phi}{d-2}} \frac{\dd - 26(10)}{3}] \quad (4)
\]

Then from (1) (taking \( \phi_i = \Phi \) one gets

\[
\int_{\mathcal{M}} \sqrt{g} d^d x \left[ -\frac{8}{d-2} g^{ij} \partial_i \Phi \partial_j \Phi + \frac{\Phi}{d-2} (e^{-\frac{4\Phi}{d-2}} F^2 + \frac{2}{3} e^{-\frac{8\Phi}{d-2}} H^2
+ 2de^{-\frac{2\Phi}{d-2}} \frac{\dd - 26(10)}{3}) \right] = 0. \quad (5)
\]

Thus dilatonic hair may exist only if the blackhole carries electromagnetic and/or axionic charge (or any other type of hair that the stringy blackhole may allow) or if the corresponding string theory is non-critical. In two dimensions however

\* Since the dilaton kinetic energy has the wrong sign the so-called string metric (the natural metric in the sigma model is probably not the physical metric. In fact to get the correct dilaton vertex operator around the flat background one should put \( G = e^{4\Phi/(d-2)} \eta \approx \eta + \frac{4d}{(d-2)^2} \Phi \) in the sigma model metric [14].
a separate discussion is necessary since the above mentioned Weyl transformation cannot be carried out. In this case we have only the non-canonical form (3) but the wrong sign of the dilaton kinetic term does not give rise to a ghost since there is no propagating on mass shell dilaton field in two dimensions. We may treat the metric which comes from solving the classical equations of motion corresponding to (3) as the physical metric and ask whether there may be blackhole solutions with dilatonic hair. Thus from (3) for \( d = 2 \) (1) gives

\[
\int_{\mathcal{M}} \sqrt{g} d^2 x e^{-2\Phi} \left\{ 8(1 + \Phi) g^{ij} \partial_i \Phi \partial_j \Phi - 2\Phi (R + \ldots) \right\}
\]

Even in the absence of gauge or axion fields there is no positive definiteness so we may have pure dilatonic hair in two dimensions as is of course well known now from the solutions given in [8],[9].

Let us now discuss some questions related to the stability of blackholes. First we will consider the blackhole in the abelian Higgs model [3]. The model has the Lagrangian

\[
\mathcal{L} = -\frac{1}{4} F^2 - |d_\mu|^2 - V(\phi)
\]

where \( V(\phi) = \lambda (|\phi|^2 - \mu^2)^2 \) and \( d_\mu = (\partial_\mu - i e A_\mu) \phi \). The metric is written in the form

\[
ds^2 = -e^{2\alpha} dt^2 + e^{2\beta} dr^2 + r^2 d\Omega^2
\]

with \( e^{2\alpha} \simeq r - r_H \), and \( e^{2\beta} \simeq (r - r_H)^{-1} \) near the horizon \( r = r_H \). Following Bekenstein [2] Adler and Pearson [3] choose the gauge \( A_i = 0, \partial_0 A_0 = 0 \) and \( \phi \) real, for the static case. Using the boundedness of the stress tensor in an orthonormal frame to get boundary conditions on the horizon it is argued in [3] that

\[†\] This is the closest to the standard model that we can find in the literature.

\[‡\] Actually it is not clear whether this is a valid choice if one wishes to preserve the asymptotic conditions at spatial infinity which are necessary to put the integral over that surface to zero in the RHS of (1).
$A_0$ is zero everywhere outside the horizon. In the symmetric case ($\mu^2 < 0, \phi \to 0$ asymptotically) on the other hand one gets $\phi = 0$ outside while $A_0 \neq 0$. As in the case of flat space for the $\mu^2 \geq 0$ case it is possible to show that the $\phi = \mu A_0 = 0$ (outside the blackhole) solution is stable under scalar field fluctuations. The field equations are

\[ G_{\mu\nu} = T_{\mu\nu}, \quad \nabla^\mu F_{\mu\nu} = j_\nu, \quad \nabla^2 A \phi = V'(\phi), \]

where

\[ T_{\mu\nu} = \frac{1}{2} F_{\mu\lambda} F^\lambda_{\nu} - \frac{1}{8} g_{\mu\nu} F^2 + d_{(\mu} d^{*}_{\nu)} - \frac{1}{2} g_{\mu\nu} (d_{\lambda} d^{\lambda}_{*} - V(\phi_0)) \]

and

\[ j_\lambda = -ie(\phi \partial_\lambda \phi^* - \phi^* \partial_\lambda \phi) + 2e^2 A_\lambda \phi \phi^*. \]

We need to look at fluctuations around the background $A = 0$, $\phi = \mu$, with the metric given by (7). It is then seen from the above equations that for real fluctuations of $\phi$ the fluctuations of the metric and the gauge field are such that $\delta g \simeq \delta A \simeq O((\delta \phi)^2)$, Thus in analysing the stability, at least under real fluctuations of $\phi$, we are not constrained to take into account the fluctuations of $A$ and $g$. This simplifies the problem considerably. The fluctuations must also satisfy the condition at the horizon coming from the requirement that physical scalars should not blow up there. This implies that $\delta \phi$ should vanish at least as fast as $(r - r_H)^{\frac{1}{2}}$ as $r \to r_H$. In addition of course the fluctuations must satisfy the boundary condition at infinity, namely fall off at least as fast as $\frac{1}{r}$. It is sufficient to consider s-wave fluctuations since the angular momentum barrier contributes a repulsive potential. Thus putting $\delta \phi = e^{\omega t} f(r)$ and using the background values of the metric gauge and scalar fields we have the following equations for the linearized fluctuations:
\(-\omega^2 f = -\frac{e^{\alpha-\beta}}{r^2} \partial_r (r^2 e^{\alpha-\beta} \partial_r f(r)) + 4e^{2\alpha} h \mu^2 f(r)\).

Changing variable to \(x\) such that \(\frac{dx}{dr} = \frac{e^{\beta-\alpha}}{r^2}\) one gets after multiplying by \(r(x)^4\), a Schrodinger operator on the RHS. Given the boundary conditions on the fluctuations and the positive definiteness of the potential it is easily seen that this operator is positive definite. Thus there are no bound states and the blackhole is stable under such perturbations. On the other hand (as in flat space) the \(\phi \rightarrow 0, A \neq 0\), solution is unstable.

Similar arguments can be used to show that the 2d string theoretic blackholes are stable under tachyonic perturbations. Since the tachyon is the only physical propagating field in two dimensional string theory this is the only perturbation we need to consider. The relevant equations (for large black holes so that we can use the leading order equations in \(\alpha'\) outside the horizon) are

\[
\begin{align*}
0 &= R_{\mu\nu} + 2\nabla_\mu \nabla_\nu \phi - \nabla_\mu T \nabla_\nu T + \ldots \\
0 &= -R + 4(\nabla \Phi)^2 - 4\nabla^2 \Phi + (\nabla T)^2 - 2T^2 - 8 + \ldots \\
0 &= -2\nabla^2 T + 4\nabla \Phi \nabla T + V'(T) + \ldots
\end{align*}
\] (8)

where the ellipses indicate higher derivative terms, higher powers of the tachyon, etc. As shown by Mandal et al [9] for \(T = 0\) the general solution of this set of equations may be put in the form \((X^\mu = (t, \phi))\)

\[
ds^2 \simeq -(1 - ae^{-2\sqrt{2}\phi})dt^2 + (1 - ae^{-2\sqrt{2}\phi})^{-1}d\phi^2
\] (9)

with \(\Phi = \sqrt{2}\phi\) and \(a\) is the black hole mass up to a positive constant. This is of course the low energy metric obtained from Witten’s \(\frac{SL(2,R)}{U(1)}\) theory. From (8) it is easy to see that this solution is stable under linearized fluctuations. Firstly from the first two equations of (8)we see that \(T\) affects the dilaton and graviton only at second order so that at linear order we can ignore the metric and dilaton
fluctuations. In terms of the physical tachyon $S = e^{-\Phi T}$ the tachyon equation becomes

$$\nabla^2 S - \frac{R}{4} S = 0$$

Now for the above metric $R = 8ae^{-2\sqrt{2}\phi}$ so for positive $a$ (the blackhole case) we have from the same type of argument as we made earlier, the result that the solution (9) is stable. On the other hand for $a < 0$, the case of a naked singularity, the solution is unstable.

To analyze the quantum stability we need the Euclidean effective action for the tachyon fluctuations. Since the two-dimensional blackhole is a particular background for two-dimensional closed string field theory we may obtain it from the background independent formulation [15] of the Das-Jevicki theory [16] since the latter at least in the flat space case seems to be a string field theory [17]. Hence it is reasonable to expect that the usual arguments* for analyzing quantum stability under small fluctuations are valid when applied to the background independent form of the Das-Jevicki theory. The quadratic (in $S$) piece of the tachyon action is (from equation (4) of [15])

$$\int d^2 x \sqrt{g} (\nabla S \cdot \nabla S + \frac{1}{4} RS^2)$$

and by the usual argument semi-classical (meta)stability depends on the spectrum of the euclidean signature operator

$$-\nabla^2_E + R.$$

If the above operator has no negative eigenvalues then the background is semi-classically stable at least under tachyon fluctuations. Since $-\nabla^2_E$ is a positive definite operator we have the same result as in the classical case: stability for the

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* see for instance [18].
blackhole (since $R > 0$) and instability for the naked singularity. It should be noted that the quadratic tachyon action above is equivalent to the quadratic part of the usual tachyon action for the sigma model tachyon

$$
\int d^2xe^{-2\Phi}\sqrt{g}((\nabla T)^2 - 2T^2)
$$

where $T = e^{\Phi}S$ provided we use the lowest order (in $\alpha'$) beta function equation for the dilaton (second equation in (8)). In this latter form the quadratic action for tachyons is not positive definite and indeed has negative eigenvalues. However the corresponding eigenfunctions $S$ are not square integrable; For instance the eigenfunction $T = 1/\sqrt{V}$ where $V = \int \sqrt{g}$ corresponds to $S = \cosh r/\sqrt{V}$ for $S$. Our point of view is that $S$ is the physical tachyon and it is also the field which comes naturally in the Das-Jevicki action and the string field theory functional measure should be derived from the metric $\|\delta S\|^2 = \int \sqrt{g}\delta S^2$. In other words fluctuations of the sigma model tachyon $T$ should be normalized using the metric $\|\delta T\| = \int \sqrt{g}\delta T^2$. With this interpretation of the string field theory the black-hole solution is semi-classically stable.\(^\dagger\) On the other hand the naked singularity is unstable.\(^\ddagger\)

Is there a possibility that some stringy effect destabilizes the blackhole. The argument in [15] on the existence of the $RS^2$ term in the effective action relied in part on lowest order in $\alpha'$ expressions. In principal even if the Das-Jevicki theory contains all tachyon-dilaton terms which are at least linear in the tachyon (this may well be the case since the theory corresponds to a string moving in a flat background with a linear dilaton) it certainly cannot say anything about curvature tachyon terms. There is evidence (from stringy beta-function calculations [20]) also for a $R^2S$ term in the tachyon beta function. This would then mean that

\(^\dagger\) The stability of the two dimensional blackhole under tachyonic fluctuations has also been established by a somewhat different argument in [19]

\(^\ddagger\) Of course we are assuming here that the existence of negative eigenvalues in the Euclidean space operator governing quadratic fluctuations makes the corresponding Minkowski space configuration unstable.
at non-leading order in $\alpha'$ the sigma model interpretation of the $SL(2, R)/U(1)$ blackhole necessarily contains a non-zero tachyon background [10]. Indeed general arguments [10] seem to indicate that the string theoretic blackhole excites all higher spin modes as well.

The above argument of course does not mean that the blackhole is stable. First of all there is the integral over dilaton and metric fluctuations. As usual in Euclidean gravity this may be problematic. In addition one knows that it must decay by Hawking radiation. The standard interpretation of the Euclidean blackhole solution is that it corresponds to a blackhole in equilibrium with a heat bath [21] at the Hawking temperature (in this case $T_H = 2\sqrt{2}/4\pi$). This interpretation however seems to require that the vacuum solution (or metric dilaton solution in the 2d case) remains valid even in a heat bath. Also being an equilibrium picture it does not shed light on the actual decay of the hole. In particular the nature of the end point is not resolved.

In the 2d case Witten [8] has conjectured that the end point of decay is the linear dilaton theory. In fact it is easy to see that the Euclidean action is a monotonically decreasing function of the blackhole mass (provided the infrared divergence is cutoff). Substituting the solution (9) and $\Phi = 2\sqrt{2}\phi$ in the Euclidean effective action (with $T = 0$)

$$S_E = -\int e^{-2\Phi}(R + 4(\nabla\Phi)^2 + 8)$$

and noting that for this euclidean (cigar) solution the coordinate range for $\phi$ is $[-\infty, -\ln a/2\sqrt{2}]$ we have

$$S_E^{BH} = -16 \int_{-\infty}^{-\ln a/2\sqrt{2}} e^{-2\sqrt{2}\phi}$$

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§ In conformal gauge the (non-diagonal) metric for kinetic terms of the dilaton and Liouville mode is not positive definite.
Cutting off the lower end of the integral to define it we see that action decreases with the blackhole mass ($M \simeq a$) with the lowest action corresponding to the $a = 0$ case i.e. the linear dilaton theory. Now one might ask why the system doesn’t simply slide down from non-zero $a$ to zero. As pointed out by Shenker and Seiberg [19] the reason is that the measure on the space of small metric deformations which change $a$, $\delta g = \frac{\partial g}{\partial a}$, is not defined. The corresponding metric is given by

$$||\delta g||^2 = \int \sqrt{g} (g^{\alpha \gamma} g^{\beta \delta} + g^{\alpha \delta} g^{\gamma \beta}) \delta g_{\alpha \beta} \delta g_{\gamma \delta} \simeq -\ln a/2\sqrt{2} \int e^{4\sqrt{2}\phi} \frac{d\phi d\tau}{(1 - ae^{2\sqrt{2}\phi})^2},$$

and clearly diverges at the tip of the cigar.† Thus there are no (square integrable) metric perturbations which can change the mass of the blackhole so that the black hole cannot decay perturbatively. The effect of this is to erect a sort of potential barrier (or perturbative superselection rule? [19]) between different values of $a$ and hawking decay of the blackhole must take place as a tunneling process. The quantum-mechanical wave function given by the path integral must be a superposition of undecayed and decayed components and its path integral representation clearly must include an instruction to integrate over all classical solutions, i.e. in this case all values of $a$. Thus in order to discuss the decay of one blackhole of a given mass in this formalism one seems to require a solution to the problem of the collapse of the wave function to one classical solution. At least in this two dimensional model of quantum gravity one knows the whole space of classical solutions [9] and so it is perhaps a good laboratory to study these questions.

† It should be noted however that this is not the case if the black hole metric is defined in the (Euclidean conformal gauge $ds^2 = \frac{1}{2z^2 + a} dz d\tau$. In that case one gets $||\delta g||^2 = 4 \int \frac{d\phi d\tau}{(2z^2 + a)^2}$ which is finite for $a \neq 0$. It does blow up for $a = 0$ though, so that the linear dilaton theory is still (perturbatively) isolated from the space of blackhole configurations.
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