Fluctuations in the vicinity of the phase transition line for two flavor QCD

S. Ejiri, C.R. Allton, M. Döring, S.J. Hands, O. Kaczmarek, F. Karsch, E. Laermann, and K. Redlich

Fakultät für Physik, Universität Bielefeld, D-33615 Bielefeld, Germany

Department of Physics, University of Wales Swansea, Singleton Park, Swansea, SA2 8PP, U.K.

Institute of Theoretical Physics, University of Wroclaw, PL-50204 Wroclaw, Poland

We study the susceptibilities of quark number, isospin number and electric charge in numerical simulations of lattice QCD at high temperature and density. We discuss the equation of state for 2 flavor QCD at non-zero temperature and density. Derivatives of \( \ln Z \) with respect to quark chemical potential \( \mu_q \) are calculated up to sixth order. From this Taylor series, the susceptibilities are estimated as functions of temperature and \( \mu_q \). Moreover, we comment on the hadron resonance gas model, which explains well our simulation results below \( T_c \).

Thermal fluctuations near the critical temperature \( (T_c) \) provide important information for the understanding of the properties of the QCD phase transition at high temperature and density. Baryon number fluctuations are expected to diverge at the endpoint of the first order chiral phase transition line. The electric charge fluctuation in heavy-ion collisions is one of the most promising experimental observables to identify the critical endpoint. The fluctuations are estimated by the susceptibilities of quark number \( (\chi_q) \) and isospin number \( (\chi_I) \), which are given by \( \chi_q = \frac{\partial^2 p}{\partial \mu_u^2} \) and \( \chi_I = \frac{\partial^2 p}{\partial \mu_d^2} \), where \( \mu_q = (\mu_u + \mu_d)/2 \), \( \mu_I = (\mu_u - \mu_d)/2 \). \( \mu_{u|d} \) is the chemical potential for the \( u|d \) quark. The charge fluctuation is \( \chi_C = \chi_q/36 + \chi_I/4 \) for \( \mu_u = \mu_d \).

The simulation of QCD at non-zero \( \mu_q \) is known to be difficult. However, studies based on a Taylor expansion with respect to \( \mu_q \) at \( \mu_q = 0 \) turned out to be an efficient technique to investigate the low density regime, interesting for heavy-ion collisions. We discussed in [1] the relation between the line of constant pressure (energy density) and the phase transition line calculating the second derivative of pressure. From the study of the fourth derivatives \([2]\), we observed strong enhancement of \( \chi_q \) near \( T_c \), suggesting the presence of the critical point at finite \( \mu_q \). In this study, we take the calculation of \( p \) to \( O(\mu_q^6) \), and that of \( \chi_q \) \( (\chi_I) \) to \( O(\mu_q^4) \). This enables us to estimate the shift of the peak position of \( \chi_q \) in the \((T, \mu_q)\) plane. Also, by calculating the ratio of the Taylor expansion coefficients, the application range of the hadron resonance gas model \([3]\) is discussed.

Quark gluon gas, hadron resonance gas

We define the expansion coefficients \( c_n \) and \( c'_n \) by \( \frac{p}{T^4} = \langle \ln Z \rangle/(VT^3) = \sum_{n=0}^{\infty} c_n(\mu_q/T)^n \) and \( \chi_q(\chi_I)/T^2 = \sum_{n=2}^{\infty} n(n-1)c_n c'_n(\mu_q/T)^{n-2} \) for \( \mu_u = \mu_d \). We expect the equation of state to approach that of a free quark-gluon gas (Stefan-Boltzmann (SB) gas) in the high temperature limit. The coefficients in the SB limit for \( N_f = 2 \), \( \mu_I = 0 \) are well known as \( c_2 = c'_2 = 1 \), \( c_4 = c'_4 = 1/(2\pi^2) \) and \( c_n = c'_n = 0 \) for \( n \geq 6 \).

On the other hand, in the low temperature phase QCD is well modelled by a hadron resonance gas. If the interaction between these hadrons can be neglected, the pressure is obtained by summing over the contributions from all resonance states of hadrons. The contribution to
resonance gas provides a good approximation.

\[ \chi \text{ the mesonic component for } p/T \]

Similarly, we obtain

\[ \chi_{\mu} / T^4 = \frac{1}{2\pi^2} \left( \frac{m_{\mu}}{T} \right)^2 \sum_{l=1}^{\infty} \frac{\eta^{l+1}}{l^2} K_2 \left( \frac{m_{\mu}}{T} \right) z', \] (1)

where \( z = \exp \left[ (3B_i \mu_{\mu} + 2I_{3i} \mu_{I}) / T \right] \), \( K_2 \) is the modified Bessel function, and \( \eta \) is 1 for mesons and -1 for baryons. Moreover, since \( m_{\mu}/T \gg 1 \) for all baryons and \( K_2(x) \approx \sqrt{\pi/2x} \exp(-x) \), the first term of \( l \) is dominant in the baryon sector. Therefore, the pressure can be written as \( p/T^4 = G(T) + F(T) \cosh(3\mu_{\mu}/T) \) for \( \mu_{\mu} = 0 \), where \( F(T) \) is the baryonic component of \( p/T^4 \) at \( \mu_{\mu} = 0 \) and \( G(T) = G_{\text{sing}}(T) + G_{\text{trip}}(T) \) is the mesonic part which has isosinglet and isotriplet components.

Similarly, we obtain \( \chi_T / T^2 = 9F(T) \cosh(3\mu_{\mu}/T) \) and \( \chi_I / T^2 = G^T(T) + F(T) \cosh(3\mu_{\mu}/T) \). Here, the mesonic component for \( \chi_T \) is zero because mesons \( (B_i = 0) \) are independent of \( \mu_{\mu} \). Therefore, \( c_4/c_2 = 3/4, c_6/c_4 = 3/10 \) and \( c_8/c_4 = 3/10 \) in the region where the non-interacting hadron resonance gas provides a good approximation.

Mesonic and baryonic contributions

We investigate these coefficients. Simulations are performed on a \( 16^3 \times 4 \) lattice for 2 flavor QCD. The \( p4 \) improved action is employed at \( ma = 0.1 \), which gives \( m_{\rho}/m_{\pi} = 0.7 \). The number of configurations is 1000-5000 for each \( T \).

We use the random noise method. The results for \( c_{n+2}/c_n \) and \( c_{n+2}/c_n^{\text{sing}} \) are shown in Fig. 1. We find that these results are consistent with the prediction from the hadron resonance gas model for \( T/T_c \leq 0.96 \) and approach the SB values, i.e. \( c_4/c_2 = c_4^{\text{sing}}/c_2^{\text{sing}} = 1/2\pi^2, c_6/c_4 = c_6^{\text{sing}}/c_4^{\text{sing}} = 0 \), in the high temperature limit. These results suggest that the models of free quark-gluon gas and hadron gas seem to explain the behavior of thermodynamical quantities well except in the narrow regime near \( T_c \).

Figure 1. The ratio of Taylor expansion coefficients for \( \chi_q \) (upper) and \( \chi_I \) (lower).

We find that these results are consistent with the prediction from the hadron resonance gas model for \( T/T_c \leq 0.96 \) and approach the SB values, i.e., \( c_4/c_2 = c_4^{\text{sing}}/c_2^{\text{sing}} = 1/2\pi^2, c_6/c_4 = c_6^{\text{sing}}/c_4^{\text{sing}} = 0 \), in the high temperature limit. These results suggest that the models of free quark-gluon gas and hadron gas seem to explain the behavior of thermodynamical quantities well except in the narrow regime near \( T_c \).

Figure 2. Pressure at \( \mu = 0 \) as a function of \( T \).
$F + G_{\text{trip}}$ with the total pressure. In Fig. 2 we plot the baryon component $F = 2c_2/9$ (square), the triplet meson component $G_{\text{trip}} = 3c_2'/4 - c_4'$ (circle) and the total (diamond). The solid line is the pressure obtained with the integral method in [1]. The dashed lines in Fig. 2 represent the predictions from the hadron resonance gas model for the mass parameter of our simulations. The resonance states for this quark mass are adjusted by the method described in [3]. The simulation results for $p/T^4$ obtained with the integral method, together with the triplet meson and baryon contributions, are surprisingly well reproduced by this model calculation. The result for the singlet meson contribution may explains the difference in pressure obtained from the integral method and that from the Taylor expansion which neglects the singlet part.

Susceptibilities at $\mu_q \neq 0$

Next, we calculate quark number and isovector susceptibilities, using three methods, in a range of $0 \leq \mu_q/T \leq 1$. The data connected by solid lines in Fig. 3 are obtained by $\chi_q/T^2 = 2c_2 + 12c_4(\mu_q/T)^2 + 30c_6(\mu_q/T)^4$ and the corresponding equation for $\chi^i_T$. Dot-dashed lines are from the hadron resonance gas model, using $F$, $F^I$, and $G^I$. This turned out to be a good approximation for $T/T_c \approx 0.96$. Dashed lines are calculated by the reweighting method with an approximation in [1]. Here the truncation error is $O(\mu^6_q)$ but the effect from higher order terms of $\mu_q$ is partially included. The sign problem in the reweighting method is serious when the complex phase fluctuations of the quark determinant are large at large $\mu_q/T$. We omitted data for which the standard deviation of the complex phase is larger than $\pi/2$. The difference among the three results is caused by the approximation in the higher order terms of $\mu_q/T$.

Since the statistical error of $c_6$ is still large near $T_c$, the peak of $\chi_q$ seems to lose its statistical significance. However, as seen in Fig. 4 $c_6$ changes its sign at $T_c$. This means the peak position of $\chi_q$ moves left, which is corresponding to the change of $T_c$ as a function of $\mu_q$. $T_c(\mu_q/T = 1)/T_c(\mu_q/T = 0)$ in [1] is about 0.93. Moreover, $\chi_q$ increases with higher orders of the expansion for $T \lesssim T_c$ which confirms the existence of a peak. This suggests the presence of a critical endpoint in the $(T, \mu_q)$ plane. On the other hand, $\chi_T$ in Fig. 4 does not show any singular behavior. This is consistent with the sigma model prediction that only isosinglet degrees of freedom become massless at the critical endpoint.

REFERENCES

1. C.R. Allton et al., Phys. Rev. D66 (2002) 074507.
2. C.R. Allton et al., Phys. Rev. D68 (2003) 014507.
3. F. Karsch, K. Redlich and A. Tawfik, Eur. Phys. J. C29 (2003) 549; Phys. Lett. B571 (2003) 67.
4. U.M. Heller, F. Karsch, and B. Sturm, Phys. Rev. D60 (1999) 114502.
5. F. Karsch, E. Laermann and A. Peikert, Phys. Lett. B478 (2000) 447.