Evaluation of melt-water premixture formation due to Rayleigh-Taylor instabilities

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Abstract. A linear analysis of the stability of the three-layer stratified hydrodynamic system "water (top) – steam – melt (bottom)" has been performed. Kinematic and dynamic conditions on the "water – steam" and "steam – melt" interfaces are formulated, and on their basis a dispersion equation is obtained that relates the circular frequency of perturbations to the wavenumber. Analysis of this equation made it possible to determine the region of instability of this system and to find the wavenumber of the most fastly growing harmonics. The results obtained were used to estimate the size of bubbles formed at the interface between steam and water due to the development of the Rayleigh-Taylor instability. The obtained theoretical results are consistent with experimental observations in such systems. The heights of the melt splashes into water due to the collapse of the formed steam bubble were estimated. The obtained estimations demonstrate possibilities of the formation of melt-water mixture region during the spreading of the melt under a water layer, in which a strong steam explosion can occur.

1. Introduction

In the course of development of the severe stage of the beyond-design-basis accident of the light water reactor VVER/PWR, molten materials of the reactor core (corium) can directly contact water (in the reactor pressure vessel or barrel, or in the core melt localization system – molten core catcher). Under certain conditions the interaction of high-temperature melt with water can lead to explosive growth of pressure (steam explosion), which represents direct hazard to the containment integrity and the subsequent release of radioactive materials into environment [1].

For a long time specialists studying a steam explosion considered the melt jet pouring into the pool with liquid coolant (water or other low-temperature boiling liquid) as the initial state for steam explosion. It was assumed that in this case melt jet fragmentation into separate droplets takes place. These droplets mix with the water, thus forming the mixture capable to produce the steam explosions. Reviews of studies of this direction are presented in [1-9].

Alternative option of the stratified initial state of the melt and coolant, when the melt layer is on the bottom of the pool filled with the coolant was analyzed just in several studies. Experiments in this case were carried out at not-high temperatures of the melt [10-17]. For example, water was considered as the melt, and Freon – as the coolant. In the course of these studies it was proposed a concept of stratified steam explosions [18]. According to this concept mixing of the melt with the coolant occurs during fast propagation of explosive wave that is why it cannot get sufficient, thus limiting the explosion force. On
this basis the conclusion is made in [6] that stratified steam explosions do not represent hazards to safety of NPPs.

Still, this standpoint [6] was doubted on the basis of the results of experimental studies carried out by scientists of the Royal Institute of Technology in Stockholm, they studied stratified steam explosions of high-temperature melts, temperature of which was as high as 1800°K. Final results of those studies were published in 2017 [19]. In those experiments they obtained strong steam explosions in the systems, where binary oxide corium simulators (Bi₃O₅-WO₃ etc.) were used as the melt, and water was used as the coolant. During explosions pressure was increased from the initial one atmosphere to 40 and even higher.

To obtain such explosions it is necessary to have sufficiently big volume of the melt-water mixture, which contradicts the model [18]. In [19] they put forward a hypothesis for formation of such mixture of the melt with water, when they initially are in the stratified location. Unlike the model [18], which postulates formation of the mixture during the explosion itself, it is assumed in the hypothesis [19] that such mixture is formed at the stage of the melt spreading out under the layer of water. In this case thin steam layer is formed between the melt and water. On the upper surface of this steam layer steam “domes” are formed, they rend off the layer and become steam bubbles. Being inside the subcooled water, these bubbles are fastly condensed and collapsed. In the process of collapse, high-velocity water stream directed to the melt is formed in the water. This water stream knocks out of the melt droplets, which fly into the water. As a result we observe a process of a particular mixing of the melt with water, when water streams knock out melt droplets upward into the water (melt splashing); melt droplets fly upward, then under gravity they relocate downwards back into the melt layer. But the new streams also knock out next droplets of the melt, etc. As a result, during spreading out of the melt under the layer of water, dynamically existing layer of the mixture of the melt with water capable of generating steam explosion, is formed close to the surface of their separation.

Hypothesis [19] was partially confirmed in [20], where it was obtained that the collapse of the highly superheated steam bubble (due to the contact with the high-temperature melt) inside subcooled water takes place as quickly as the collapse of the cavitation bubble, i.e. the bubble the pressure in which is equal to the saturation pressure, which corresponds to the surrounding water temperature. That is why the collapse of the bubble of the highly superheated steam leads to the formation of the high-velocity water stream directed to the melt. This water stream is capable of knocking out droplets of melt into the water (melt splashing) for the height of several centimeters.

Action force (impact) of the high-velocity water stream onto the melt surface depends greatly on the initial dimensions of the collapsing bubble. The larger the bubble, the more significant is the flow of water in the region, which becomes empty after collapse of the bubble. This water flow is transformed into the high-velocity stream of water. In [20] initial dimension of the bubble was assessed on the basis of Rayleigh-Taylor theory for instability of “steam – water” hydrodynamic system in the gravity field. The present study generalizes the assessment [20] by analyzing stability of the three-layer “melt – steam – water” system.

2. Problem formulation

As the steam film separating the melt and water is much thinner than the layer of melt under it and the layer of water above it, we shall assume that both of these layers are of infinite length in the cross direction, i.e. let’s refer external boundaries of the region under consideration to the “infinite”. We shall also pay no regard to the velocity of the melt spreading along the pool bottom. We shall consider melt, steam and water as ideal incompressible fluids and the flows of them – as potential.

As the initial non-disturbed steady-state of this hydrodynamic system, let’s assume three-layer configuration: horizontal layer of motionless steam of δ thickness, under it there is a semi-infinite layer of motionless melt, and above it - a semi-infinite layer of motionless water.

Let’s use the rectangular coordinate system, axis x is directed along the melt and steam boundary plane, and axis y – is directed upward perpendicular to this plane. Let’s superpose small-scale harmonic
disturbances (Figure 1) with the wavenumber $k$ onto two boundary surfaces “melt-steam” and “steam-water”:

$$\xi = \delta + \xi_0 e^{ikx+\omega t}, \quad \eta = \eta_0 e^{ikx+\omega t}$$

(1)

where $\xi$ and $\eta$ – vertical coordinates of the boundary surfaces, $\xi_0$ and $\eta_0$ – unknown constants, $\omega$ – circular frequency of superposed disturbances, $t$ – time, $i$ – unit imaginary unit.

**Figure 1.** Problem formulation.

Flows occurring due to superposed disturbances can be of damped nature, which leads to return of the considered hydrodynamic system into the initial state, or on the contrary, they can grow and transfer the initial system into some different dynamic state. Interface between these modes can be identified by solving this hydrodynamic problem using the linear analysis. In this case we assume that the parameters of the emerging flows are small, thus we neglect nonlinear terms in hydrodynamic equations and in boundary conditions at the media interfaces (“water – steam” and “steam – melt”).

3. Derivation of the dispersion equation

For potential flows of incompressible fluids equation of continuity reduces to Laplace equation for velocity potential:

$$\Delta \varphi \equiv \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = 0,$$

(2)

where velocity potential $\varphi$ determines fluid velocity $\mathbf{v} = \text{grad}(\varphi)$.

For potential flow of incompressible fluid in the field of gravity from the Euler equation for velocity

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \nabla)\mathbf{v} = -\frac{\text{grad}(p)}{\rho} + \mathbf{g}$$

(3)

($p$ – pressure, $\rho$ – density, $\mathbf{g}$ – gravity acceleration vector)

Cauchy–Lagrange integral is derived:

$$\frac{\partial \varphi}{\partial t} + \frac{\mathbf{v}^2}{2} + \frac{p}{\rho} + gy = 0,$$

(4)

which we will use in the analysis.

At the “steam – melt” interface Laplace formula describes equality of forces acting normally to the surface taking into account the surface tension force:

$$p_2 - p_0 = \frac{\sigma_2}{R}$$

(5)
Here \( p_1 \) and \( p_2 \) – pressures of steam and melt at the interface, \( \sigma_2 \) – surface tension coefficient of the “steam-melt” system, \( R \) – radius of the interface surface curvature in the considered point of the interface. In case of small deviations of the surface curvature from the plane, curvature radius is calculated through the second derivative of the curved surface:

\[
\frac{1}{R} = -\frac{\partial^2 \eta}{\partial x^2}
\]

Applying the formula (6) to Laplace formula (5), we shall get:

\[
p_2 = p_0 - \sigma_2 \frac{\partial^2 \eta}{\partial x^2}
\]

Determining pressures \( p_0 \) and \( p_2 \) from Cauchy–Lagrange integral (4) and neglecting the small terms of the second order, we shall obtain equality condition for the “steam-melt” interface:

\[
\rho_0 \frac{\partial \varphi_0}{\partial t} + \sigma_2 \frac{\partial^2 \eta}{\partial x^2} = \rho_2 \frac{\partial \varphi_2}{\partial t} + (\rho_2 - \rho_0) g \eta
\]

Here \( \rho_0 \) and \( \rho_2 \) – densities of steam and melt, \( \varphi_0 \) and \( \varphi_2 \) – velocity potentials of the steam and melt.

With the help of similar operations we get the condition of equality of forces for the “water-steam” interface:

\[
\rho_1 \frac{\partial \varphi_1}{\partial t} + \sigma_1 \frac{\partial^2 \xi}{\partial x^2} = \rho_0 \frac{\partial \varphi_0}{\partial t} + (\rho_0 - \rho_1) g (\xi - \delta)
\]

Here \( \rho_1 \) – water density, \( \varphi_1 \) – water velocity potential, \( \sigma_1 \) – water surface tension coefficient.

Kinematic equations at the interface surfaces express equality of the media vertical velocities on these surfaces, and vertical velocities of the surfaces themselves:

\[
\frac{\partial \eta}{\partial t} = (v_{0,y})_{y=0} = \left( \frac{\partial \varphi_0}{\partial y} \right)_{y=0}, \quad \frac{\partial \eta}{\partial t} = (v_{2,y})_{y=0} = \left( \frac{\partial \varphi_2}{\partial y} \right)_{y=0}
\]

\[
\frac{\partial \xi}{\partial t} = (v_{1,y})_{y=\delta} = \left( \frac{\partial \varphi_1}{\partial y} \right)_{y=\delta}, \quad \frac{\partial \xi}{\partial t} = (v_{0,y})_{y=\delta} = \left( \frac{\partial \varphi_0}{\partial y} \right)_{y=\delta}
\]

Solutions of Laplace equation (2) in each of the three considered zones, which take into account the velocity potential attenuation at infinity are given by:

\[
\varphi_1 = Ae^{-ky}e^{ikx+i\omega t},
\]

\[
\varphi_0 = (Be^{ky} + Ce^{-ky})e^{ikx+i\omega t},
\]

\[
\varphi_2 = De^{ky}e^{ikx+i\omega t}.
\]

By putting these expressions into Laplace equation (2) we can confirm that these are the solutions of the equation. Values \( A, B, C, D \) in (12) – (14) are the unknown constants.

Let’s put expressions (12) – (14) into conditions for interfaces (8) – (11) and we shall get the system of linear equations in respect to the unknown \( A, B, C, D, \xi_0 \) and \( \eta_0 \):

\[
ke^{-k\delta}A + \omega \xi_0 = 0
\]

\[
ke^{-k\delta}B - ke^{-k\delta}C - \omega \xi_0 = 0
\]

\[
kB - kC - \omega \eta_0 = 0
\]

\[
kD - \omega \eta_0 = 0
\]

\[
\rho_0 \omega B + \rho_0 \omega C - \rho_2 \omega D - (\rho_2 - \rho_0) g + \sigma_2 k^2 \xi_0 = 0
\]

\[
\rho_1 \omega e^{-k\delta}A - \rho_0 \omega e^{-k\delta}B - \rho_0 \omega e^{-k\delta}C + [(\rho_1 - \rho_0) g - \sigma_1 k^2] \xi_0 = 0
\]

The system of linear equations (15) – (20) with zero right side has a non-trivial solution, if its determinant is equal to zero. From this condition after the complicated mathematical transformations we get the dispersion equation:

\[
\omega^4 [\rho_0^2 th(k\delta) + \rho_0 (\rho_1 + \rho_2) + \rho_1 \rho_2 th(k\delta)] + \omega^2 [k\Delta_1 (\rho_0 + \rho_2 th(k\delta)) + k\Delta_2 (\rho_0 + \rho_1 th(k\delta))] + k^2 \Delta_1 \Delta_2 th(k\delta) = 0,
\]

where we introduced the following definitions:

\[
\Delta_1 = (\rho_0 - \rho_2) g + k^2 \sigma_1, \quad \Delta_2 = (\rho_2 - \rho_0) g + k^2 \sigma_2.
\]
Let’s consider the limit case with the steam film of small thickness \((\delta \to 0)\), in this case \(th(k\delta) \to 0\). From the equation (21) we get

\[
\omega^2 = \frac{(\rho_1 - \rho_2)gk - (\sigma_1 + \sigma_2)k^3}{\rho_1 + \rho_2}
\]  

(23)

Formula (23) is a well-known dispersion equation for two-fluid system [21], where the surface tension coefficient is the summary of such coefficients for the melt and water. As the upper liquid (water) is weighing less than the lower liquid (melt), then (23) indicates that \(\omega\) is purely imaginary value, thus such two-fluid system is stable.

Dispersion equation (21) is a biquadratic equation in respect to \(\omega\) value, \(\omega(k)\) dependence can be obtained from this equation. If \(\omega\) has positive real part, then out of (1) it goes that superposed disturbances of the media interfaces will increase exponentially with no limit (in the framework of the applied linear approach), which indicates instability of the analyzed system. In this case \(\omega\) shall be interpreted not as the circular frequency, but as the decrement of the disturbance growth.

Positive real values of the disturbance growth decrement identify wavenumbers (and the corresponding wave lengths), which will grow exponentially and lead to instability of interfaces and the subsequent formation of bubbles. It is evident that the most rapidly increasing disturbance (maximum value of \(\omega\)) defines the dominant size of the formed bubbles.

4. Estimation of the formed bubbles size

In the analysis we used the data of E6 experiment, PULiMS [19] series, which was considered in [20]. Table 1 lists these data.

| Parameter                  | Water | Melt Bi\(_2\)O\(_3\)-WO\(_3\) | Steam |
|----------------------------|-------|-------------------------------|-------|
| Pressure, bar              | 1     | 1                             | 1     |
| Temperature, K             | 348   | 1049                          | 698.5 |
| Density, kg/m\(^3\)        | 974.9 | 7811                          | 0.3105|
| Surface tension coefficient, N/m | 5.9·10\(^{-2}\) | 0.45                         | –     |

Uncertainty parameter of the analyzed system is thickness of the steam layer. It was revealed in [19] that steam layer was thin. Let’s assume that its thickness is 1 mm. Analysis of the dispersion equation (21) has demonstrated with such test parameters (Table 1), positive values of the decrement of the disturbance growth are in the range of the following wavenumbers \(0 < k < 400\) m\(^{-1}\), and the most rapidly growing disturbance is realized for the harmonic with the wavenumber 232.5 m\(^{-1}\), which is shown in Figure 2.
If we accept that the size of the bubble formed out of interface disturbance, is equal to half of the length of the wave of the most rapidly growing disturbance, or \( \pi/\lambda \), then the diameter of this bubble \( d_0 \) will be approximately 1.35 cm, which corresponds to the situation observed in the experiments PULiMS [19].

In [20] it was obtained the evaluation formula for the rise height of the melt droplet under melt splashing depending on the size of the collapsed steam bubble:

\[
\Delta h = \left(0.29 \frac{d_0}{d_2}\right)^6 \left(\frac{\rho_2}{\rho_1}\right)^2 \left(\frac{P_0-P_{sat}}{\rho_1 g}\right)
\]

Here \( d_2 \) – is the diameter of the flying droplet of melt, \( P_0 \) – pressure in the system, \( P_{sat} \) – saturation pressure at the water temperature. With the help of the formula (24) it is possible to assess the rise height of the melt droplet with the diameter of \( d_2 \), \( \Delta h(d_2) \) dependence is shown in Figure 3 for the parameters of E6 experiment (water temperature was 348 K, corresponding saturation pressure was 0.38 bar).
Since the steam layer thickness is an uncertain parameter, the value of this parameter was varied in the range from 0.01 mm to 1 cm. Figure 4 shows the dependence of the wavenumber of the most rapidly growing disturbance on the steam layer thickness. It follows from the Figure 4 that in the considered range, which covers the values of this parameter observed in [19], the value of the wavenumber of the most fastly growing disturbance, and hence the size of the formed bubble, changes insignificantly. It should be noted that the time scale of the disturbance growth, which is estimated as $1/\omega$, increases significantly with decreasing of steam layer thickness. Thus, with very thin steam layers, heavy neighboring liquids (water and melt) limit the rate the disturbance growth and essentially stabilize the system.

![Figure 4](image)

**Figure 4.** Dependence of the wavenumber of the most fastly growing disturbance on the steam layer thickness.

It should also be emphasized that, at a steam layer thickness of 1 mm and above, the wavenumber of the most fastly growing disturbance becomes approximately equal to the value obtained from a linear analysis of the two-fluid “water – steam” system. This indicates that the main factor influencing the stability of the considered three-fluid system is the Rayleigh-Taylor instability of the “water (top) – steam (bottom)” system; the melt under steam has practically no effect on the development of instability.

5. **Conclusions**

On the basis of a linear analysis of the stability of the three-fluid stratified system "water (top) - steam - melt (bottom)", the size of steam bubbles formed at the water - steam interface is theoretically estimated. The obtained results coincidences with the values observed in experiments [19]. It is shown that for the parameters of the considered experiments, the main factor determining the interface instability and the subsequent bubble formation is the Rayleigh-Taylor instability of the two-layer system "water (top) – steam (bottom)", the effect of the melt on this process is insignificant.

The application of the results obtained to an estimation of the rise height of the melt droplet of several millimeters in size under the collapse of bubbles in subcooled water was performed. It was obtained a range of such heights from 2-3 cm to 20 cm, depending on the size of the melt droplets. It is obvious that the superposition of such events during the melt spreading under a layer of water is capable to form melt-water premixture, which can produce strong steam explosions.
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