COMMON ENVELOPE: ON THE MASS AND THE FATE OF THE REMNANT

N. Ivanova
Department of Physics, University of Alberta, 11322-89 Avenue, Edmonton, AB, T6G 2E7, Canada

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ABSTRACT

One of the most important and uncertain stages in binary evolution is the common envelope (CE) event. Significant attention has been devoted in the literature so far to the energy balance expected to determine the outcome during the CE event. However, this question is intrinsically coupled with the problem of what is left from the donor star after the CE and its immediate evolution. In this paper we argue that an important stage has been overlooked: the post-CE remnant thermal readjustment (TR) phase. We propose a methodology for unambiguously defining the post-CE remnant mass after it has been thermally readjusted, namely, by calling the core boundary the radius in the hydrogen shell corresponding to the local maximum of the sonic velocity. We argue that the important consequences of the TR phase are (1) a change in the energy budget requirement for the CE binaries and (2) a companion spin-up and chemical enrichment, as a result of the mass transfer (MT) that occurs during the remnant TR. More CE binaries are expected to merge. If the companion is a neutron star (NS), it will be mildly recycled during the TR phase. The MT during the TR phase is much stronger than the accretion rate during the CE, and therefore satisfies the condition for hypercritical accretion better. We also argue that the TR phase is responsible for the production of mildly recycled pulsars in double NSs.

Key words: binaries: close – pulsars: general – stars: evolution – X-rays: binaries

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1. UNCERTAINTY IN THE COMMON ENVELOPE THEORY

In the standard treatment of common envelope (CE) outcomes via “energy formalism” (Webbink 1984), the final separation of the binary is determined by equating the binding energy of the (shunned) envelope E\textsubscript{bind} to the decrease in the orbital energy E\textsubscript{orb}:

\[ E_{\text{bind}} = E_{\text{orb},i} - E_{\text{orb},f} = -\frac{G m_1 m_2}{2 a_i} + \frac{G m_2 m_3}{2 a_i}. \]  

(1)

Here \( a_i \) and \( a_f \) are the initial and final binary separations, \( m_1 \) and \( m_2 \) are the initial star masses, and \( m_c \) is the final mass of the star that lost its envelope.

\( E_{\text{bind}} \) is considered to be the sum of the potential energy of the envelope and its internal energy, and can be found directly from the stellar structure for any accepted core mass (there are also modifications for \( E_{\text{bind}} \), where ionization energy or enhanced winds are taken into account; e.g., Han et al. 1995, 2002; Soker 2004):

\[ E_{\text{bind}} = \int_{\text{core}}^{\text{surface}} \epsilon(m) dm = \frac{G m_1 m_c}{\lambda R_1}. \]  

(2)

Here \( \lambda \) is a parameter introduced to fit \( E_{\text{bind}} \); it characterizes the donor envelope central concentration. \( m_c \) is the mass of the removed giant envelope and is commonly assumed to be \( m_c = m_1 - m_{\text{H}} \); \( R_1 \) is the radius of the giant star at the onset of CE, and \( \epsilon \) is the sum of the specific internal and potential energies.

For the final balance of energy, one more parameter is introduced, \( \alpha_{\text{CE}} \), to measure the energy transfer efficiency from the orbital energy into envelope expansion:

\[ \alpha_{\text{CE}} \left( \frac{G m_1 m_2}{2 a_i} - \frac{G m_1 m_3}{2 a_i} \right) = \frac{G m_1 m_c}{R_1}. \]  

(3)

We anticipate that introduction of the two parameters introduced accordingly two uncertainties. It is common to remove these uncertainties at the same time, considering the product of \( \alpha_{\text{CE}} \) and \( \lambda \), by means of comparison of observations with the binary population synthesis calculations, where the product of the two parameters is varied to match the observations. However, this approach has shown inconsistencies with the observations, which are especially large for the formation rates of black hole low-mass X-ray binaries (LMXBs; Podsiadlowski et al. 2003; Justham et al. 2006). In particular, for LMXBs this required \( \alpha_{\text{CE}} \lambda \gtrsim 2 \) (Yungelson et al. 2006), although in massive giants \( \lambda \ll 0.1 \) (Podsiadlowski et al. 2003) and \( \alpha_{\text{CE}} \) is bound to be \( \leq 1 \).

The other way to reduce uncertainties is to consider them separately, e.g., one can try to determine an “accurate” value of \( \lambda \) from stellar structure calculations. It is then crucial to be precise about the definition of the core—should only the hydrogen envelope be removed, or together with the H-burning shell, and so on (Tauris & Dewi 2001). Without knowing what exactly counts as the core and which material ought to be ejected, the inferred \( \lambda \) can vary by a factor of several from this uncertainty alone.

The physical reason for this variation is that in giants, within the hydrogen shell, the potential is strongly increasing toward the core. The uncertainty increases as the mass of the donor increases, and changes from about a factor of 2 in intermediate-mass stars at early giant stage to a factor of 20 and more for well-evolved massive stars (Tauris & Dewi 2001).

We stress that neither observations nor theory provide now a strong constraint on what post-CE remnant mass should be at the moment when the dynamical phase of the ejection ends. It does not have to be the same as the mass of the remnant that we observe now, e.g., in double white dwarf (WD) systems or in subdwarf B stars: some remaining post-CE hydrogen-rich material can easily be removed through strong winds similar to those on the horizontal branch or asymptotic giant branch, or in Wolf-Rayet stars, etc. Between the dynamical phase and long-term evolution, the core will readjust itself on a thermal timescale,
and this has not been addressed. Here, we address the problem of what the post-ejection mass could be, different regimes in which a post-CE remnant can shed its remaining hydrogen-rich mass, and the consequences for a companion due to post-CE mass transfer (MT).

2. THE POST-EJECTION REMNANT

2.1. The Divergence Point

In smoothed particle hydrodynamic simulations of physical collisions between a red giant (RG) and a neutron star (NS), it has been found that not all hydrogen material is ejected along with the envelope—a tiny layer of hydrogen, from the H-burning shell, remains (Lombardi et al. 2006). This event is not directly comparable to a typical CE event in a binary as at the time of initial approach, at periastron, the H-burning shell of the donor could have been in the immediate Roche lobe of the intruder. This magnifies the mass loss from the H-burning shell and as such can decrease the mass of the post-CE remnant compared to a typical CE, where this shell might never be in the Roche lobe of the spiraling-in companion. This example makes clear that even in a dynamical CE some hydrogen-rich material always remains.

On the other hand, studies of the evolution of stripped cores of low-mass RGs have shown that there is a minimum “envelope” mass \( m_{\text{e,min}} \) that has to be left on the core in order for the star to reexpand; if less mass is left on the core, the star will contract and become a WD (Deinzer & von Sengbusch 1970). This expansion or contraction of the remaining shell occurs on the thermal timescale of the remaining layer, \( \tau_{\text{th}} \).

It is plausible therefore to suppose that there is a unique “divergence” point \( m_d \) inside the hydrogen-burning shell, such that if a post-CE star has any mass above this point, the star will continue to expand on \( \tau_{\text{th}} \). If its final mass is less than \( m_d \), the star will shrink, also on its \( \tau_{\text{th}} \). We recognize that \( \tau_{\text{th}} \) might mean different values in the case of a degenerate core (applicable only to the remaining shell) or a non-degenerate core (where it is likely to depend on the core conditions). We expect that the material above the divergence point, if left, will expand, in order to obtain thermal equilibrium, but is not required to escape to infinity without an additional energy source (such as the orbital energy). During this thermal readjustment (TR), it may also fill its Roche lobe.

2.2. Calculations

We tested this idea of “divergence” point on giants of several initial masses (1, 2, 10, 20, 30 \( M_\odot \)). The stars were evolved using the stellar code and input physics described in Ivanova & Taam (2004). This code is capable of performing both hydrostatic and hydrodynamic stellar evolution calculations. For a Roche lobe overflow evolution in binaries, it finds mass-loss rates implicitly. Massive stars, where wind losses are important, were evolved with wind-loss rates according to Vink et al. (2001) or, where Vink rates are not applicable, according to Kudritzki & Reimers (1978).

For each initial mass, we chose 2–4 evolutionary states within the giant stage with different hydrogen-exhausted core masses \( m_X \). As during the advanced evolution stages stars can shrink, we ensured that the chosen giants had expanded to their current radius for the first time. On these giants, we imposed a very fast (“adiabatic”) mass loss, \( 1 \ M_\odot \text{yr}^{-1} \). Such a timescale for the mass loss \( \tau_{\text{ML}} \) is comparable to a fast CE event. The lower bound on \( \tau_{\text{ML}} \) should be as a CE event has to happen over at least one binary period at the initial Roche lobe overflow.

We do not imply that a CE ejection features a constant mass loss and also do not study the reaction of the outer (convective) envelope. We are interested in the reaction of the inner layers, which are most likely to remain after the envelope ejection has occurred.

For each mass coordinate, let us compare \( \tau_{\text{ML}} \) with the local thermal timescale \( \tau_{\text{TH}}(m) = E_{\text{hri}}(m)/L(m) \) and the local dynamical timescale \( \tau_{\text{dyn}}(m) \): the mass loss and the star evolution will be adiabatic if \( \tau_{\text{ML}} \ll \tau_{\text{TH}}(m) \). The evolution can be described by hydrostatic approximation if \( \tau_{\text{ML}} \gg \tau_{\text{dyn}}(m) \), as a star will always acquire its hydrostatic equilibrium within a dynamical time, and its state at the hydrostatic equilibrium is defined by its thermal structure.

For most stars, \( \tau_{\text{ML}} \) is much shorter than any local thermal timescale (see Figure 1). We note however that in the inner layers that are close to the cores of our most massive stars (20 and 30 \( M_\odot \)), the complete mass-loss sequence can take up to 10% of the local thermal timescale of a few hundred years. Thus, even such a fast mass loss produces only an approximately adiabatic evolution: some thermal evolution proceeds and is expected to be responsible, in particular, for some expansion of inner non-degenerate layers during the CE phase. As a sanity check, we calculated additional mass-loss sequences for massive stars, with faster and slower mass-loss rates, and found only minor differences in the region of interest between the runs with 0.1, 1, and 10 \( M_\odot \text{yr}^{-1} \).

On the other hand, local dynamical timescales are longest at the surface and significantly shorter for the innermost layers (Figure 1). \( \tau_{\text{dyn}}(m) \) is comparable by order of magnitude to \( \tau_{\text{ML}} \) in the outer layer of the massive giants. \( \tau_{\text{dyn}}(m) \) is however by three or more orders of magnitude smaller than \( \tau_{\text{ML}} \) in the helium-rich layers, and, closer to the hydrogen-exhausted core, it is \( \sim 10^{-5} \tau_{\text{ML}} \), even in our most massive considered stars. Therefore, although the evolution of the outer layers is indeed dependent on the inclusion of hydrodynamical terms, the inner layers always have enough time to regain hydrostatic equilibrium and are therefore insensitive under the adopted mass-loss rate. In summary, we find that for studies of the thermal reaction of the inner layers that will form the remnant after the fast envelope ejection, the hydrostatic version of the code is sufficient.

As a result of mass-loss evolution, we obtained sequences of (post-CE) remnants with different initial (post-CE) masses, each of which then was evolved for several \( \tau_{\text{th}} \) to check if this post-CE star is expanding or contracting. We note that in our code the value of the post-CE mass could be resolved no better than the pre-CE resolution in hydrogen shell; this is specifically important for low-mass giants (e.g., we have about 50 mesh points per 0.02 \( M_\odot \) H-shell in 2 \( M_\odot \) RG with an \( m_X = 0.52 \ M_\odot \)).

To summarize, we separate the CE event into two stages: one resulting in the envelope ejection and the subsequent TR of the remnant. The latter part is a distinct phase unless the spiral-in (including the ejection of the envelope) takes place on a timescale comparable to the shortest thermal timescale—several hundred years—and has not been heretofore treated in the literature.

Finally, an admonishment is in order: several estimates exist for the CE duration, though none of them can boast conclusive observational evidence or indeed self-consistency. For example, a “slow” CE could last for 100 years and longer (Podsiadlowski 2001). We cannot justify which CE evolution timescale is more...
appropriate, and this is not the purpose of this paper. We concentrate on the “fast” event, however we see no reason why our results should not be applicable in the “slow” case: the core reaction will be similar, albeit lagging by the time the ejection takes. We also note that a 10 times slower loss rate did not produce a significant difference in our calculations.

2.3. Degenerate Cores

Indeed, as in previous studies, we found that every low-mass giant with a degenerate core has a unique divergence point $m_{\text{div}}$, such that if the post-CE mass is less than $m_{\text{div}}$, it contracts on $\tau_{\text{th}}$. All post-CE remnants with masses above $m_{\text{div}}$, expand, create new outer convective zones, and keep expanding even after $\tau_{\text{th}}$.

After locating $m_{\text{div}}$, we analyzed pre-CE giants structure to find what characteristics these points had in initial giants, before the stripping began. For this, all giants, including massive ones, were used. We noticed that among all the giants, $m_{\text{div}}$ could have initially a wide range of hydrogen content, $X = 0.08$–0.58, and so a criterion involving the specific constant value of hydrogen abundance could not be satisfactory. Similarly, another criterion discussed in the literature—the location where the energy generation rate is maximum (Tauris & Dewi 2001)—does not coincide with the divergence point. We found that in the considered models $m_{\text{div}}$ is close to the “maximal compression point” $m_{\text{cp}}$, which is the mass zone with the maximum value of “compression” $P/\rho$ in the hydrogen shell.

2.4. Non-degenerate Cores

For massive giants, as previously, a post-CE remnant also has a divergence point that corresponds to the minimum post-CE expansion of the remnant, although the overall response is different from the case of the giants with degenerate cores.

1. For all possible remnant masses with $m \lesssim m_{\text{cp}}$, the core slightly adiabatically expands during the fast adiabatic mass loss. Once we stop the mass loss, it can very slightly (a few per cent) expand and then shrink dramatically, becoming smaller than it was before the CE.

2. For larger remnant masses, as previously, the convective envelope is re-formed, and the star remains as an extended giant for a while. The envelope can become larger than the post-CE star.

3. For the intermediate range of remnant masses, above the $m_{\text{cp}}$, but below the boundary where the convective zone redevelops, the core experiences a pulse on $\sim \tau_{\text{th}}$, being able to expand by up to a few hundred times more than this mass had as a radius coordinate before the CE. After the pulse, the post-CE star shrinks significantly, also becoming smaller than prior to the CE.

To illustrate these three types of responses, in Figure 2 we show the typical case of a 9.75 $M_\odot$ (zero-age main sequence (ZAMS) mass 10 $M_\odot$) star, taken when it had a radius of 300 $R_\odot$. For comparison, we show an 18.5 $M_\odot$ (ZAMS mass 20 $M_\odot$) star with a radius of 750 $R_\odot$ (see Figure 3). Even though this giant has a hydrogen profile qualitatively different from the 10 $M_\odot$ star considered above, it shows a similar behavior. The main difference with a 10 $M_\odot$ star is that there is a more contrasting response between the inside $m_{\text{cp}}$ and outside it. In a 30 $M_\odot$ star...
Thus more binaries will merge (this result is in Figure 3 will easily survive a CE event with an NS of 1 to the claim in Deloye & Taam 2010). As an example, the giant α (with the giant, this difference becomes even stronger as Figure 3).

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2.5. The Adiabatic Response

We recognize that the response we discuss above is non-adiabatic, but is the equilibrium response (the one that a star experiences in order to obtain its thermal equilibrium). Another important response to consider is the reaction of a star when the dynamical event ends—true adiabatic response. It is established that the adiabatic response of the surface layers depends on whether they are radiative or convective (Hjellming & Webbink 1987; Soberman et al. 1997). In particular, radiative layers on dynamical timescale tend to shrink and convective layers remain the same or expand. We checked for all considered models the location of the divergence point and found that all of them are located within initially (pre-CE) radiative layers. We conclude that the immediate response for mass removal to the divergence point is always a shrinkage and therefore does not affect our conclusions based on the thermal response.

3. CONSEQUENCES FOR THE ENERGY BUDGET

As the core expands during semi-adiabatic mass loss, a surviving binary must be wider when core expansion is taken into account. Thus more binaries will merge (this result is opposite to the claim in Deloye & Taam 2010). As an example, the giant in Figure 3 will easily survive a CE event with an NS of 1.4 M⊙ (with αCE = 1) if its core did not expand. Setting the core mass \( \lesssim 7.44 \ M_\odot \) will satisfy the energy budget to create a compact binary. However, if one takes into account the core expansion, the binary will merge: for all core masses above \( m_{\text{cp}} \), the remnant will expand significantly and overfill its Roche lobe. A minimum companion mass, for which both energy budget and post-CE core size are taken into account, will be \( \sim 1.82 \ M_\odot \) and the giant core in this case should have been removed to at least \( m_{\text{cp}} \) (see Figure 4). It can be seen from this figure that for a fixed companion mass, there is a unique solution where available orbital energy can exceed binding energy if the remnant expanded after mass loss, whereas a non-expanded remnant gives a wide range of possible core masses, from 5.95 to 7.5 M⊙.

Let us introduce “the energy expense,” the difference between the required energy \( E_{\text{bind}}(m) \) and the available orbital energy \( \Delta E_{\text{orb}} \), normalized per \( E_{\text{bind}}(m) \):

\[
\delta_e = \frac{E_{\text{bind}}(m) - (E_{\text{orb},i} + \Delta E_{\text{orb},i})}{E_{\text{bind}}(m)}.
\]

It is the (normalized) excess energy available to the envelope after all the matter above the given mass coordinate has been removed. Of course, a positive \( \delta_e \) signals that the removal process is not possible from the energy considerations. Figure 5 shows the distribution of \( \delta_e \) as a function of the mass coordinate. For these calculations, \( E_{\text{orb},f} \) was assumed to be at the Roche lobe limited orbit for the post-CE remnant (semi-adiabatic expansion is taken into account), so that available orbital energy is at its maximum.

Note that \( m_{\text{cp}} \) is the energetically optimal position to remove the envelope down to: it is the equilibrium point of the generalized force \( \partial m (E_{\text{bind}} + E_{\text{orb}}) \). The shape and the location of the minimum of \( \delta_e \) only depend on the donor’s energy profile up to the Roche radius and do not depend on the companion mass. The latter only determines the magnitude of the energy excess (see Figure 5).

Thus, it is easy to determine the minimum mass of a companion which allows the survival of the binary: as can be seen from Figure 5, it is such that it will result in the removal of mass to \( m_{\text{cp}} \) precisely.

We verified that the coincidence of the minimum of the energy expense with shedding the envelope to about \( m_{\text{cp}} \) for minimum
likely companion mass holds for many of the studied giants, though does not hold for giants that are early on the giant branch which only recently develop convective envelopes. For example, in an early $10 M_{\odot}$ we observe one more energy minimum, at a higher core mass $\sim 2.45 M_{\odot}$; the local energy minimum at $m_{\text{cp}}$ nonetheless holds. In more massive early giants, the energy expense minimum is between $m_{\text{cp}}$ and $m_{\chi}$ (Figure 6), its location is closer to the location of $m_{\text{cp}}$; the star still has the same reaction on expansion or contraction with respect to $m_{\text{cp}}$ as other stars.

4. POST-CE MASS TRANSFER

4.1. Consequences for the Final Post-CE Mass

Let us consider what happens if the companion was massive enough so that during CE not all mass to $m_{\text{cp}}$ had to be removed. In this case, a CE would end with a binary separation such that Roche lobe overflow for a post-CE remnant during its TR will follow. To consider this, we took a post-CE model showed in Figure 3, considering its remnant with $m_{\text{post-CE}} = 6.84 M_{\odot}$ (larger than its $m_{\text{cp}} = 6.47 M_{\odot}$). This post-CE remnant is then placed in a contact binary with an arbitrary companion of $3 M_{\odot}$; this mass self-consistently satisfies the energy required to shed the mass above $6.84 M_{\odot}$. For the MT we can choose a fully conservative or a fully non-conservative mode.2

As expected, the post-CE remnant rapidly expanded and started the MT; initially at a very high rate $(\sim 1-5) \times 10^{-2} M_{\odot}\text{yr}^{-1}$, in accordance to $\tau_{\text{sh}}$ of the remaining hydrogen-rich layer. After removing most of the layer above $m_{\text{cp}}$, it slowed down to $\tau_{\text{sh}}$ of the core $(\sim 10^{-4} M_{\odot}\text{yr}^{-1})$. The MT continued until the MT rates become comparable to the TR timescale so that the core can shrink faster than it expands due to mass loss. In this particular example the core reached almost exactly the divergence point, shedding $\sim 0.4 M_{\odot}$ during the MT so that the final mass was $m_{\text{cp}}$. We also performed an MT calculation in the fully non-conservative regime. In this case, the final mass of the core after the TR phase is the same as in the conservative calculations.

We also considered the case when the companion is an NS. We note that with a $20 M_{\odot}$ donor then, even for $\alpha_{\text{CE}} = 1$, the energy requirements for envelope ejection would not be satisfied if the core expands as much as we find after our fast mass loss (i.e., the binary would merge). The final mass of the remnant of the massive giant is the same, $m_{\text{cp}}$, as for the $3 M_{\odot}$ companion. If the MT is fully conservative, the NS is presumably spun up, as it would accumulate $0.34 M_{\odot}$. Again, in the case of a fully non-conservative regime, we find that the final mass of the post-CE remnant is $m_{\text{cp}}$.

Next, we considered a system with a $10 M_{\odot}$ giant (same as shown in Figure 2), considering as the post-CE core $2.54 M_{\odot}$ ($m_{\text{cp}} = 2.04$). In our MT simulations with a NS companion, less material above $m_{\text{cp}}$ has been transferred, only $0.24 M_{\odot}$, and the final remnant mass is $2.3 M_{\odot}$. It might be connected to the fact that a $10 M_{\odot}$ star does not have such a sharp profile as a $20 M_{\odot}$ in the post-CE thermal pulse zone, and its post-CE expansion is more flatter until about this mass (see Figure 2). With a smaller companion mass, more of the post-CE remnant mass is stripped off.

4.2. Consequences for the Companion

The MT rates that we encounter in the post-CE TR phase are highly super-Eddington and we face the obvious question whether the MT is approximately conservative or almost non-conservative, as this is crucial for a companion. The question what happens if the mass accretion rate on an NS exceeds the Eddington limit has been discussed extensively in the literature, in particular, the regime in which it exceeds $M_{\text{Edd}}$ by many orders of magnitude.

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2 Partial conservation limited to the Eddington rates can be considered as well, but with the MT rates that we find and describe, this case does not differ much with the fully non-conservative case.
Begelman (1979) showed that if the accretion rate is extremely high, few times $10^{-3} M_\odot$yr$^{-1}$, then within some volume (the “trapping radius,” e.g., King & Begelman 1999) around the star the diffusion of photons outward cannot overcome the advection of photons inward. While a black hole can swallow all the material in this case, if the accretor is an NS, radiation pressure near the NS’s surface resists inflow in excess of the Eddington limit, likely leading to the creation of a Thorne–Zytkov object. Blondin (1986) has also found that when MT rates exceed the Eddington rate by $10^3 \times L_{\text{Edd}}/c^2$ or more, the accretion proceeds in a hypercritical regime.

Hypercritical accretion was then argued to be responsible for such an efficient material accumulation during a CE event that an NS is likely to convert to a black hole (Chevalier 1989). Brown (1995) used this argument to understand double NS formation. He showed that indeed in a CE event the Bondi–Hoyle–Lyttleton accretion rate is about $10^3 \times M_{\text{Edd}}$ and an NS can accumulate up to $1 M_\odot$. He argued that in this case, considering that a number of the discovered double NS have masses closer to the lowest possible NS mass limit, a double NS can be formed only from a binary with almost similar initial masses, evolving then via a double CE event, before either of the NSs was formed.

Houck & Chevalier (1991) considered neutrino losses during accretion on an NS. They studied in detail the regimes of the mass accretion $10^{-3} M_\odot$yr$^{-1} \lesssim M \lesssim 10^4 M_\odot$yr$^{-1}$ and found that radiation diffusion becomes important when the accretion rate falls below $10^{-3} M_\odot$yr$^{-1}$; for smaller rates the radiation pressure cannot support an envelope around NS surface and cannot cease the infall of the material. We note that the accretion rate that separates the hypercritical accretion from the accretion when the radiation diffusion dominates in this case ($10^{-3} M_\odot$yr$^{-1}$) is higher than the one found to work during a CE event ($2 \times 10^{-4} M_\odot$yr$^{-1}$).

If the latter estimate is more realistic than estimates from the studies listed above, then it is possible that a CE hyper-accretion, having too low a mass accretion rate, does not lead to a significant accumulation of the material. Post-CE core expansion, however, in either case can lead to a hyper-accretion regime, as during thermal pulse it provides a much higher mass accretion rate. This can lead to an NS spin-up. In our calculations, MT rates exceeded $10^{-3} M_\odot$yr$^{-1}$ long enough to accrete in a hypercritical regime on an NS 0.29 $M_\odot$ in the case of a 20 $M_\odot$ giant and 0.09 $M_\odot$ in the case of a 10 $M_\odot$.

We note that the observed double NSs are generally mildly recycled, having periods of 0.024–2.7 s (Stairs 2004). The mass distribution of those with mass measurement errors $\lesssim 0.02 M_\odot$ are such that the difference between the masses of the NSs is $\lesssim 0.1 M_\odot$ (e.g., see data in Stairs 2004; Kiziltan et al. 2010). Their location on the $P$–$P$ diagram for galactic field NSs is also intermediate between the millisecond pulsars and non-recycled ones (Arzoumanian et al. 1999), and the post-accretion period is likely to be not millisecond (Lorimer et al. 2005). It could be that the very rapid MT followed the CE and, preceding the second NS formation, is not capable to fully spin up and efficiently reduce the magnetic field on an NS that was formed first. Same mechanism can lead to the formation of mildly recycled binary pulsars with low-mass WD companions (Li 2002; Deloye 2008).

5. CONCLUSIONS

We analyzed a set of giant models with respect to their likely post-CE response. Although our set was not exhaustive, it did exhibit a clear trend that allowed us to conclude that (1) every giant has a well-defined post-CE remnant after it has been thermally readjusted, most likely given by the divergence point (see discussion below) and (2) the divergence points, at the current resolution, are best approximated by the point in the hydrogen-burning shell that had maximal compression (local sonic velocity) $m_{\text{cp}}$ prior to CE. This definition allows us to find quickly a post-CE core mass for any giant without performing mass-loss calculations.

We remark that this divergence point does not necessarily mark the final mass of the remnant (e.g., the stellar wind in He-rich stars could quickly and effectively remove the remaining hydrogen-rich envelope) or immediate post-ejection mass; however, it marks the mass after the thermal core readjustment.

The post-TR core, defined by $m_{\text{cp}}$, most likely coincides with the post-ejection mass in low-mass giants, where re-establishing of a convective envelope for masses above the divergence point happens on a few dynamical timescales, leading to another (likely unstable) MT event. As the $E_{\text{band}}$ of remaining shells during this period has not been changed much compared to the pre-CE state, the whole sequence of events can be considered as one CE event with the final core being $m_{\text{cp}}$.

For giants with non-degenerate cores the situation is more complicated. It is not possible to claim that the divergence point defines the post-CE core (dynamical phase) uniquely from the energetic budget point of view. However, the divergence point does appear to define the remaining post-TR core, if the post-CE configuration allows Roche lobe overflow for the post-CE remnant during the core TR. During this period, (stable) MT can proceed, resulting in the enrichment of a companion star with material from the hydrogen-burning shell.

In all cases, the post-CE remnant of a giant with a non-degenerate core has expanded by a few times by the end of TR. Remnant masses greater than $m_{\text{cp}}$ lead to greater expansion; for each particular giant star, the minimum possible size change is for a remnant mass $\sim m_{\text{cp}}$. The expansion of the remnant means that less orbital energy is available to eject the envelope than if there was no expansion, since the surviving binary must be correspondingly wider. This leads to a reduction in the number of binaries which can survive CE. So, fast CE allows more binaries to survive the end of the dynamical phases. Slower, self-regulating CE leads to more mergers.

We note that for both types of giants, $m_{\text{cp}}$ firmly represents only a maximum post-TR core mass, as we cannot fully rule out that the dynamical phase will not already have removed mass below $m_{\text{cp}}$. However, we argue that such extra mass loss is not likely to happen if, during the final stages of the spiral-in, the characteristic orbital evolution time is comparable to the core response time near $m_{\text{cp}}$ point, which is as short as 10–100 years.

We also find that in most evolved giants, the energy required to shed the envelope down to $m_{\text{cp}}$ is the minimum energy expense: per total $E_{\text{bind}}$ unit, it requires more orbital energy to remove either less or more of the mass from the expanded core. It is fully reasonable to remove less of the envelope (and have a bigger post-CE mass) once $E_{\text{bind}} < \alpha E_{\text{orb}}$; and the TR phase will then remove mass down to $\sim m_{\text{cp}}$. However, it is not plausible to remove the envelope deeper than to $\sim m_{\text{cp}}$; for any remnant mass less than $m_{\text{cp}}$, the difference between $E_{\text{band}}$ and $\alpha E_{\text{orb}}$ increases compared to their value at $m_{\text{cp}}$.

We therefore suggest that a divergence point uniquely defines the core in the post-TR phase, and then a slow wind-loss phase.
follows the CE with almost no core evolution. A companion can be spun up during the MT effected by the remnant’s TR. Roche lobe overflow phase is short; however, it can lead to at least mild recycling. If the companion is an NS, its recycling will depend on whether the conservative MT (due to the hypercritical accretion) is possible or not. We find that hypercritical accretion is more likely during this TR phase than during a CE, as the mass accretion rates are significantly higher. It also leads to a smaller mass accumulation than is found to occur during a CE, corroborating the mass distributions of the observed double NSs. We conclude that the post-CE TR phase can be responsible for the formation of mildly recycled pulsars in post-CE binaries and specifically in double NSs.

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3 We do not include here evolution on nuclear timescale which will follow as usual, e.g., a core of a small enough mass can again become a giant, as it is customary for low-mass He stars.