ULTRARELATIVISTIC POSITRONIUM PRODUCTION
IN COLLISIONS OF HIGH ENERGY ELECTRONS
AND LASER PHOTONS

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Abstract
We consider the production of ultrarelativistic parapositronium and orthopositronium atoms in collisions of high energy electrons \((E_e > 0.5 \text{ TeV})\) and laser photons. Our results demonstrate the opportunity of intensive positronium beam formation with Lorentz-factor \(\gamma \sim 10^6\) using laser beam conversion on high energy electrons at the future \(e^+e^-\) Linear Accelerators.

Introduction
Formation of ultrarelativistic positronium beams is of immediate interest as a test of main principles of quantum electrodynamics (QED) [1] and theory of relativistic bound states [2]. In particular, the formation of ultrarelativistic parapositronium \((^1S_0 - \text{state})\) and P-wave positronium beams with very large Lorentz–factors \(\gamma\) gives a chance to measure with high accuracy their decay widths. It is needed because of theoretical predictions at present time are more precise than experimental data [3]. At very large Lorentz–factor \(\gamma \sim 10^6\) the positritronium formation length is about several sm for ground states \((n = 1, ^3S_1 \text{ or } ^1S_0)\) [4] and it is reality to check directly the exponential law for the \(e^+e^-\) bound state formation in vacuum and matter.

At present time it is discussed different approach for obtaining relativistic positronium beams. First of all, this is positronium production in \(\pi^0\)-meson decays at Proton Accelerators, which have been tested experimentally some years ago [5].

Relativistic positronium beams may be formed in high energy photon [6] or electron [7] interaction with matter. Recently it was predicted positronium production rate in collisions of heavy ions [8].

Here we consider the formation of ultrarelativistic positronium beams \((\gamma \simeq 10^6)\) via laser photon conversion on high energy electrons. This method may be realized at future Linear \(e^+e^-\) Accelerators at the energy range \(E_e \approx 1 \text{ TeV}\).

The idea of our calculation is the same one which has been discussed previously [9] in the case of realizing of \(\gamma e^-\) and \(\gamma\gamma\) – collisions on the basis of Linear \(e^+e^-\) Collider. The
high density laser beam (approximately $10^{20}$ photons per impulse with the energy $\omega = 1$ eV) backscatters on high energy electron beam of Linear Accelerator and converts into high energy photon beam with probability equal to unity. At the laser power about 10 J and frequency of laser impulse divisible to Linear Collider frequency ($f \sim 10 \div 100$ s$^{-1}$) it will be so many laser photons that almost high energy electron transfers energy to laser photon.

It is obviously that at conditions discussed above, the ratio (1):

$$K = \frac{\sigma(\gamma + e \rightarrow Ps + e)}{\sigma(\gamma + e \rightarrow \gamma + e)},$$

fixed the number of positronia per one electron in the initial beam. There will hard energy spectrum of positronia flying in the narrow cone near the initial electron beam direction, as well as in the case of converted photons.

1 The process $\gamma + e \rightarrow Ps + e$

In the nonrelativistic approximation, positronium is considered as a $e^+e^- –$ system with the fixed mass $M = 2m$, zero binding energy and zero relative momentum. To project the pair of free electron and positron on the $^3S_1$ or $^1S_0$ bound states we have used the next projection operators [10]:

$$\hat{P}(^3S_1) = \frac{\Psi(0)}{\sqrt{2m}}\hat{\varepsilon}(\hat{p}/2 + m),$$

$$\hat{P}(^1S_0) = \frac{\Psi(0)}{\sqrt{2m}}\gamma_5(\hat{p}/2 + m),$$

where $\hat{\varepsilon} = \varepsilon^\mu(p)\gamma_\mu$, $\varepsilon^\mu(p)$ is the orthopositronium polarization four–vector, $p$ is the positronium four–momentum, $\Psi(0) = \sqrt{m^3\alpha^3/8\pi}$ is the nonrelativistic positronium ground state wave function at the origin, $m$ is the electron mass.

In the lowest order in $\alpha = e^2/4\pi$ positronium production in the process $\gamma + e \rightarrow Ps + e$ is described by the Feynman diagrams in Fig 1. The nonzero contribution comes from diagrams 1-6 for parapositronium production and from diagrams 1-4, 7, 8 for orthopositronium production:

$$M_1 = e^3\hat{U}(q_2)\gamma^\mu\hat{P}\gamma_\mu(\hat{q}_1 + \hat{k} + m)\hat{\varepsilon}(k)U(q_1)/$$
$$((q_1 - k)^2 - m^2)(p/2 - q_2)^2$$

$$M_2 = e^3\hat{U}(q_2)\gamma^\mu\hat{P}\hat{\varepsilon}(k)(\hat{p}/2 - \hat{k} + m)\gamma_\mu U(q_1)/$$
$$((p/2 - k)^2 - m^2)(q_2 + p/2)^2$$

$$M_3 = e^3\hat{U}(q_2)\hat{\varepsilon}(k)(\hat{q}_2 - \hat{k} + m)\gamma^\mu\hat{P}\gamma_\mu U(q_1)/$$
$$((q_2 - k)^2 - m^2)(q_1 - p/2)^2$$
\[ M_4 = e^3 \tilde{U}(q_2) \gamma^\mu (-\hat{p}/2 + \hat{k} + m) \hat{\varepsilon}(k) \hat{P} \gamma_\mu U(q_1)/
\]
\[ ((p/2 - k)^2 - m^2)(q_1 - p/2)^2 \]  
(7)
\[ M_5 = e^3 \tilde{U}(q_2) \gamma^\mu U(q_1) \text{Tr}[\hat{P} \hat{\varepsilon}(k)(-\hat{k} + \hat{p}/2 + m) \gamma_\mu]/
\]
\[ (q_1 - q_2)^2((p/2 - k)^2 - m^2) \]  
(8)
\[ M_6 = e^3 \tilde{U}(q_2) \gamma^\mu U(q_1) \text{Tr}[\hat{P} \gamma_\mu (-\hat{p}/2 + \hat{k} + m) \hat{\varepsilon}(k)]/
\]
\[ (q_1 - q_2)^2((p/2 - k)^2 - m^2) \]  
(9)
\[ M_7 = e^3 \tilde{U}(q_2) \gamma^\mu (\hat{q}_1 + \hat{k} + m) \hat{\varepsilon}(k) U(q_1) \text{Tr}[\gamma_\mu \hat{P}]/
\]
\[ p^2((q_1 + k)^2 - m^2) \]  
(10)
\[ M_8 = e^3 \tilde{U}(q_2) \varepsilon(k)(-\hat{p} + \hat{q}_1 + m) U(q_1) \text{Tr}[\gamma_\mu \hat{P}]/
\]
\[ p^2((p - q_1)^2 - m^2) \]  
(11)

Let us define the Mandelstam variables for the process $\gamma + e \rightarrow Ps + e$ in the case of $E_e \gg m \gg \omega \sim 1 \text{eV}$:

\[ s = (q_1 + k)^2 \simeq m^2 + 4\omega E_e, \]  
(12)
\[ t = (k - p)^2 \simeq 4m^2 - 2E_e\omega(1 + \cos \theta), \]  
(13)
\[ u = (q_1 - p)^2 \simeq 5m^2 - 2E_e E(1 - \cos \theta), \]  
(14)

where $k = (\omega, 0, 0, -\omega)$ is laser photon four–momentum, $q_1 = (E_e, 0, 0, E_e)$ is the initial electron four–momentum, $q_2$ is the scattered electron four–momentum, $p = (E, 0, E \sin \theta, E \cos \theta)$ is positronium the four–momentum, $E$ is the energy of positronium, $E_e$ is the energy of initial electron, $\theta$ is scattering angle of positronium,

\[ \cos \theta \simeq 1 - \frac{2\omega E_e + 2m^2}{EE_e}. \]  
(15)

The differential cross section for the process $\gamma + e \rightarrow Ps + e$ as a function of $y = E/E_e \simeq (4m^2 - t)/(s - m^2)$ is expressed in terms of $|M|^2$ as follows:

\[ \frac{d\sigma}{dy}(\gamma + e \rightarrow Ps + e) = \frac{|M|^2}{16\pi(s - m^2)}. \]  
(16)

The total cross section $\sigma(\gamma + e \rightarrow Ps + e)$ is obtained from (16) integrating with respect to $y$ in the limits:

\[ y_{\text{max}}^{\text{min}} = \frac{1}{s - m^2} \left[ 2m^2 + \frac{(s + m^2)(s - 3m^2)}{2s} \right. \]
\[ \pm \left. \frac{(s - m^2)}{2s} \sqrt{(s - 9m^2)(s - m^2)} \right] \]  
(17)
2 Results of calculations

First of all, to note that minimal value of invariant $s$ in the process $\gamma + e \rightarrow Ps + e$ is $s_{\text{min}} = 9m^2 \approx 2.35\text{MeV}^2$ and corresponding threshold electron energy is equal to

$$E_{e,\text{min}} = \frac{s_{\text{min}} - m^2}{4\omega} = \frac{2m^2}{\omega}. \quad (18)$$

At the energy of laser photons $\omega = 1\text{ eV}$ we obtain from (18) $E_{e,\text{min}} = 522\text{ GeV}$. On the other hand, at $E_e = 1\text{ TeV}$ (the energy range of future Linear Colliders) $s$ is equal to 4.2 MeV$^2$. Figure 2 shows the calculated total cross sections for orthopositronium (curve 1) and parapositronium (curve 2) production as a function of $s$ at $\omega = 1\text{ eV}$. The orthopositronium cross section has a maximum at $s = 3.2\text{ MeV}^2$ (or $E_e = 735\text{ GeV}$), where one has $\sigma(\gamma + e \rightarrow 3S_1 + e) \approx 237\text{ pb}$ and $\sigma(\gamma + e \rightarrow 1S_0 + e) \approx 325\text{ pb}$.

The orthopositronium production cross section decreases at large $s$ opposite to parapositronium production cross section which increases logarithmically. This behavior comes from diagrams 4 and 5 in Fig 1. At the electron energy $E_e = 1\text{ TeV}$ and $\omega = 1\text{ eV}$ ($s \approx 4.2\text{ MeV}^2$) we have found that $\sigma(\gamma + e \rightarrow 3S_1 + e) \approx 218\text{ pb}$ and $\sigma(\gamma + e \rightarrow 1S_0 + e) \approx 494\text{ pb}$.

The above mentioned values of cross sections correspond to production of positronia in ground states with $n = 1$. The summation over $n$ enhances the obtained results by a factor of $\zeta(3) = \sum_{n=1}^{\infty} 1/n^3 \simeq 1.202$.

Table 2 shows the values of ratio $K$ (1) at different $s$. In such a manner, at $E_e = 1\text{ TeV}$ and at number of electrons per impulse $n = 2 \cdot 10^{11}$ (the project of $e^+e^-$ Linear Collider VLEPP) it will be produced $\sim 1200$ parapositronium and $\sim 500$ orthopositronium per impulse. At the accelerator frequency $f = 100\text{ s}^{-1}$ it gives $1.2 \cdot 10^5$ and $0.5 \cdot 10^5$ positronium atoms per second. This result exceeds the number of positronium atoms which may be obtained in the recombination process $e^+ + e^- \rightarrow Ps + \gamma$ at $e^+e^-$ Storage Rings \cite{1} at the same parameters of accelerators. The rough estimation on $P$–wave positronium production rate in the process $\gamma + e \rightarrow Ps + \gamma$ is approximately $10^{-4}$ from the number of $S$–wave positronium production rate, i.e. $\sim 100$ atoms per second. It seems, that so large $P$–wave positronium production rate will enough for precise test of their decay widths.

The positronium spectra $dN/dy(y, s)$ normalized to unity at $s = 3.2\text{ MeV}^2$ are shown in Fig.1. The positronium atoms are produced in the kinematic range $0.375 \leq y \leq 0.869$ with the average values $<y > \approx 0.54$ for parapositroniums and $<y > \approx 0.63$ for orthopositroniums and they have ultrarelativistic Lorenz–factors $0.27 \cdot 10^6 \leq \gamma \leq 0.64 \cdot 10^6$.

In conclusion we note that, the obtained via laser photon conversion on high energy electrons, ultrarelativistic positronium beam will be pure, from the point of view of hadronic background. It follows from the trivial kinematic fact that threshold energy of pair $\pi$–meson production is $E_{\text{min}} \approx 2 \cdot 10^4\text{ TeV}$ at $\omega \approx 1\text{ eV}$. 
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Table 1

\[ \mathcal{K} = \frac{\sigma(\gamma + e \rightarrow Ps + e)}{\sigma(\gamma + e \rightarrow \gamma + e)} \]

| \(s, \text{MeV}^2\) | \(^3S_1\) | \(^1S_0\) |
|-------------------|------------|------------|
| 3.2               | 2.35 \cdot 10^{-9} | 3.23 \cdot 10^{-9} |
| 4.2               | 2.61 \cdot 10^{-9}  | 5.88 \cdot 10^{-9}  |

Figure captions

1. Diagrams used for the process \(\gamma + e \rightarrow Ps + e\).

2. Cross section for the process \(\sigma(\gamma + e \rightarrow Ps + e)\) as a function of \(s\) at laser photon energy \(\omega = 1\) eV. Here \(Ps\) is the orthopositronium (curve 1) and parapositronium (curve 2).

3. The positronium spectra \(dN/dy\) normalized to unity in the process \(\gamma + e \rightarrow Ps + e\), where \(y = E/E_e\). Curve 1 is the orthopositronium spectrum, curve 2 is the parapositronium spectrum.
