Non-adiabatic pulsations in ESTER models

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The challenges in interpreting the pulsations of rapidly rotating stars

- theoretical challenges
  - 2D geometry – complicated formulas and numerically demanding
  - no automatic mode classification procedure
- lack of *simple* frequency patterns
  - p-modes: superposition of multiple independent patterns
  - g-modes: varying period separation + numerous inertial modes
- amplitudes are difficult to predict (classical pulsators)
(Deupree et al. 2012)
The benefits of non-adiabatic calculations

- find out which modes are excited
- consistent calculation of $\delta T_{\text{eff}} / T_{\text{eff}}$
  - amplitude ratios
  - phase shifts
  - line profile variations (LPVs)
  - mode identification
Previous 2D non-adiabatic pulsation calculations

| Reference            | Model                          | Pulsations              |
|----------------------|--------------------------------|-------------------------|
| Lee & Baraffe (1995) | Chandrasekhar expansion        | 2 or 3 harmonics        |
| Lee (2001)           | Spherical                      | 10 harmonics            |
| Savonije (2005, 2007)| Spherical                      | 2D calculations         |
| Lee (2008)           | Chandrasekhar expansion        | 4 harmonics             |

Comparisons with traditional approximation

- Savonije (2005, 2007): stabilising effect of Coriolis force
- Lee (2008): stabilising effect of centrifugal deformation
Previous 2D non-adiabatic pulsation calculations

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in all cases, the effects of rotation are approximated
⇒ there is a need for full 2D calculations, with 2D models
Necessary ingredients for 2D non-adiabatic pulsations

- 2D rotating models
  - hydrostatic equilibrium: adiabatic calculations
  - energy conservation equation: non-adiabatic calculations
- a 2D pulsation code which includes non-adiabatic effects
Necessary ingredients for 2D non-adiabatic pulsations

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ESTER (models) + TOP (pulsations)
The ESTER code (a very brief overview)

- ESTER = Evolution STEllaire en Rotation
- fully includes centrifugal deformation
- satisfies energy conservation equation:
  - baroclinic (isobars ≠ isochores ≠ isotherms)
  - self-consistent 2D rotation profile

(Rieutord & Espinosa Lara, 2009)
The TOP pulsation code

- TOP = Two-dimensional Oscillation Program
- fully includes centrifugal deformation
- can handle baroclinic models
- includes non-adiabatic effects

http://johnmannophoto.com/blog/?p=103
Pulsation equations

Continuity equation (conservation of mass)

\[ 0 = \frac{\delta \rho}{\rho_o} + \nabla \cdot \xi \]

Poisson’s equation

\[ 0 = \Delta \psi - 4\pi G \left( \rho_o \frac{\delta \rho}{\rho_o} - \xi \cdot \nabla \rho_o \right) \]

- \( \delta \rho \) = Lagrangian density perturbation
- \( \rho_o \) = equilibrium density profile
- \( \xi \) = Lagrangian displacement
- \( \psi \) = Eulerian perturbation to the gravitational potential
Pulsation equations

Euler’s equations (conservation of momentum)

\[ 0 = \left(\omega + m\Omega\right)^2 \vec{\xi} - 2i\vec{\Omega} \times \left[\omega + m\Omega\right] \vec{\xi} - \vec{\Omega} \times \left(\vec{\Omega} \times \vec{\xi}\right) \]

\[ - \vec{\xi} \cdot \vec{\nabla} \left(\varpi \Omega^2 \vec{e}_{\varpi}\right) - \frac{P_o}{\rho_o} \vec{\nabla} \left(\frac{\delta P}{P_o}\right) + \frac{\vec{\nabla} P_o}{\rho_o} \left(\frac{\delta \rho}{\rho_o} - \frac{\delta P}{P_o}\right) - \vec{\nabla} \Psi \]

\[ + \frac{\vec{\nabla} \left(\frac{\vec{\xi} \cdot \vec{\nabla} P_o}{\rho_o}\right)}{\rho_o^2} + \frac{\left(\vec{\xi} \cdot \vec{\nabla} P_o\right)}{\rho_o} \vec{\nabla} \rho_o - \frac{\left(\vec{\xi} \cdot \vec{\nabla} \rho_o\right)}{\rho_o} \vec{\nabla} P_o \]

\[ \omega = \text{pulsation frequency} \]
\[ m = \text{azimuthal order} \]
\[ \Omega = \text{rotation profile} \]
\[ \varpi = \text{distance to the rotation axis} \]
\[ \delta P = \text{Lagrangian pressure perturbation} \]
Pulsation equations

**Energy conservation equation**

- unperturbed form:

\[
\rho_o T_o \frac{dS_o}{dt} = \epsilon_o \rho_o - \vec{\nabla} \cdot \vec{F}_o
\]

- perturbed form:

\[
i [\omega + m\Omega] \rho_o T_o \delta S = \epsilon_o \rho_o \left( \frac{\delta \epsilon}{\epsilon_o} + \frac{\delta \rho}{\rho_o} \right) - \vec{\nabla} \cdot \delta \vec{F} + \vec{\xi} \cdot \vec{\nabla} \left( \vec{\nabla} \cdot \vec{F}_o \right) - \vec{\nabla} \cdot \left[ \left( \vec{\xi} \cdot \vec{\nabla} \right) \vec{F}_o \right]
\]

- \(\delta \vec{F}\) = Lagrangian perturbation to the energy flux
- \(\delta S\) = Lagrangian entropy perturbation
- \(\delta \epsilon\) = Lagrangian perturbation to the energy production
Pulsation equations

### Energy flux

- **total energy flux**
  \[ \vec{F}_o = \vec{F}_o^R + \vec{F}_o^C \]

- **unperturbed form of radiative energy flux:**
  \[ \vec{F}_o^R = -\frac{4acT_o^3}{3\kappa_o\rho_o} \nabla T_o = -\chi_o \nabla T_o \]

- **perturbed form of radiative energy flux:**
  \[ \delta \vec{F}_o^R = \left[ (1 + \chi T) \frac{\delta T}{T_o} + \chi \rho \frac{\delta \rho}{\rho_o} \right] \vec{F}_o^R \]
  \[ - \chi_o \left[ T_o \nabla \left( \frac{\delta T}{T_o} \right) + \xi \cdot \nabla \left( \nabla T_o \right) - \nabla \left( \xi \cdot \nabla T_o \right) \right] \]

- **frozen convection approximation:**
  \[ \delta \vec{F}_o^C \simeq 0 \]
Pulsation equations

Equation of state, opacities, and nuclear reaction rates

\[
\begin{align*}
\frac{\delta P}{P_o} &= \Gamma_1 \frac{\delta \rho}{\rho_o} + P_T \frac{\delta S}{c_v} = P_{\rho} \frac{\delta \rho}{\rho_o} + P_T \frac{\delta T}{T_o} \\
\frac{\delta T}{T_o} &= \frac{\delta S}{c_v} + (\Gamma_3 - 1) \frac{\delta \rho}{\rho_o} = \frac{\delta S}{c_p} + \nabla_{\text{ad}} \frac{\delta P}{P_o} \\
\frac{\delta \chi}{\chi_o} &= \chi_\rho \frac{\delta \rho}{\rho_o} + \chi_T \frac{\delta T}{T_o} \\
\frac{\delta \epsilon}{\epsilon_o} &= \epsilon_T(\omega) \frac{\delta T}{T_o} + \epsilon_\rho(\omega) \frac{\delta \rho}{\rho_o}
\end{align*}
\]

*In what follows we will neglect \( \delta \epsilon \)*
Boundary conditions

- in the centre: regularity conditions
- at infinity: gravitational potential perturbation goes to zero
- at the surface:

\[
\nabla_{\text{vert.}} \left( \frac{\delta P}{P_o} \right) = 0
\]

\[
4 \frac{\delta T}{T_o} = \frac{\delta F^R}{F^R_o}
\]
Summary

- final result: a system of 10 equations with 10 unknowns:

$$\frac{\delta P}{P_0}, \frac{\delta S}{c_p}, \bar{\delta F}^R, \frac{\delta T}{T_o}, \Psi$$

- although some of these variables can be cancelled algebraically, they are needed to ensure good convergence
it is possible to derive an integral expression for the complex frequencies:

\[ A\omega^2 + 2B\omega + C = 0 \]

where

\[ A = \int_V \rho_0 \xi^2 dV, \]
\[ B = \int_V \rho_0 \left[ m\Omega \xi^2 - i\tilde{\Omega} \cdot \left( \tilde{\xi} \times \tilde{\xi}^* \right) \right] dV, \]
\[ \Re(C) = \text{a complicated expression} \]
\[ \Im(C) = -\int_V \Im \left\{ \frac{\delta P \delta \rho^*}{\rho_0} \right\} dV \]

From this we deduce the excitation rate:

\[ \Im(\omega) = -\frac{\Im(C)}{2(A\Re(\omega) + B)} \]
Comparison with Lee & Baraffe (1995)

| Lee & Baraffe (1995) | Current work |
|----------------------|--------------|
| Model based on       | 2D baroclinic model |
| Chandrasekhar expansion |              |
| Eulerian perturbations | Lagrangian perturbations |
| $\vec{F}^C = 0$      | $\delta \vec{F}^C = 0$ |
Numerical implementation

- explicit expression in spheroidal coordinates
- projection onto spherical harmonics
- radial discretisation using Chebyshev polynomials

| $N_r$ | $N_h$ | Memory (in Gb) | Time (in min) | Num. proc. |
|-------|-------|----------------|---------------|------------|
| 400   | 10    | 3.5            | 5             | 4          |
| 400   | 15    | 7.9            | 10            | 8          |
| 400   | 20    | 13.4           |               | 8          |
| 400   | 29    | 28.0           | 22            | 8          |
| 400   | 40    | 52.7           | 26            | 16         |
| 400   | 50    | 82.3           |               |            |
the problem is stiff: reduced numerical accuracy

estimated accuracy based on variational expression:
  - frequencies: $\sim 10^{-4}$
  - excitation/damping rates: $10^{-2}$ to $10^{-1}$
Model

- 9 M⊙ models
- Ω = 0.0 to 0.8 Ω_K
- z = 0.025
- OPAL opacities

Modes

- β Cep type pulsations
- p and g modes
- excited by iron opacity bump at log(T) = 5.3
Frequencies

0.0 $\Omega/\Omega_K$

Frequency, $\omega/\Omega_K$

Dominant harmonic, $l$

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Frequencies

\[ 0.1 \, \Omega / \Omega_K \]

Frequency, \( \omega / \Omega_K \)

Dominant harmonic, \( l \)

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Frequencies

\[ 0.2 \frac{\Omega}{\Omega_K} \]

Frequency, \( \omega/\Omega_K \)

Dominant harmonic, \( l \)

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Frequencies

0.3 $\Omega/\Omega_K$

Frequency, $\omega/\Omega_K$

Dominant harmonic, $l$
Frequencies

0.4 $\Omega/\Omega_K$

Frequency, $\omega/\Omega_K$

Dominant harmonic, $l$
Frequencies

\[ 0.5 \, \Omega/\Omega_K \]

**Dominant harmonic, \( l \)**

**Frequency, \( \omega/\Omega_K \)**
Frequencies

0.6 $\Omega/\Omega_K$

Frequency, $\omega/\Omega_K$

Dominant harmonic, $l$
Frequencies

$0.7 \Omega / \Omega_K$ - incomplete

\begin{figure}
\centering
\includegraphics[width=\textwidth]{frequencies.png}
\caption{Frequencies for $0.7 \Omega / \Omega_K$ - incomplete.}
\end{figure}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{frequencies_2.png}
\caption{Details of the frequency distribution for $0.7 \Omega / \Omega_K$ - incomplete.}
\end{figure}
Damping rates

Frequency difference, \( \frac{(\omega_{\text{non-ad}} - \omega_{\text{ad}})}{\Omega_K} \)

Damping rate, \( \frac{\tau}{\Omega_K} \)

\( \ell = 0 \)
\( \ell = 1 \)
\( \ell = 2 \)
\( \ell = 3 \)

\( |m| = 0 \)
\( |m| = 1 \)
\( |m| = 2 \)
\( |m| = 3 \)
Island modes
Whispering gallery modes
Mixed modes

- see Ouazzani et al. (2015) for mixed modes in the adiabatic case
Rosette modes

also see Takata & Saio (2015) for non-adiabatic effects on Rosette modes and associated angular momentum transport
prograde modes remain unstable longer
Lee (2008) also found a preference for prograde modes
prograde modes remain unstable longer
Lee (2008) also found a preference for prograde modes
Work integral

- red = driving regions
- blue = damping regions

| Acoustic | Work | Work (vs. log T) |
|----------|------|-----------------|

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Work integral

- obtained by integrating in horizontal direction + vertical anti-derivative
A multiplet

\[ \text{rotation rate} = 0.0 \quad \Omega_K, \, \varepsilon = 0 \]
A multiplet

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rotation rate $= 0.1 \Omega_K, \ \varepsilon = 4.9 \times 10^{-3}$
A multiplet

\[ \text{rotation rate} = 0.2 \Omega_K, \quad \varepsilon = 1.9 \times 10^{-2} \]
A multiplet

rotation rate $= 0.3 \, \Omega_K, \quad \varepsilon = 4.3 \times 10^{-2}$
A multiplet

- rotation rate = 0.4 \( \Omega_K \), \( \varepsilon = 7.4 \times 10^{-2} \)
A multiplet

- rotation rate = 0.5 $\Omega_K$, $\varepsilon = 11.2 \times 10^{-2}$
A multiplet

rotation rate = 0.6 $\Omega_K$, $\varepsilon = 15.5 \times 10^{-2}$
A multiplet

\[ m = 0 \]
Stable

\[ m = -2 \]
Excited

\[ m = 2 \]
Stable

rotation rate = 0.4 \( \Omega_K \), \( \varepsilon = 7.4 \times 10^{-2} \)
# Amplitude ratios

## Previous works

- Daszyńska-Daszkiewicz et al. (2002, 2007), Townsend (2003)
  - non-adiabatic treatment
  - approximate treatment of rotation
- Reese et al. (2013) (see also Lignières et al. 2006, Lignières & Georgeot 2009)
  - full treatment of rotation
  - adiabatic calculations
Equations

- **non-pulsating star:**

  \[ I = \int_{\text{Vis.Surf.}} \int I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs}} \cdot d\vec{S} \]

- **pulsating star:**

  \[ \delta I = \int_{\delta \text{Vis.Surf.}} \int I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs}} \cdot d\vec{S} \]
  \[ + \int_{\text{Vis.Surf.}} \int \delta I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs}} \cdot d\vec{S} \]
  \[ + \int_{\text{Vis.Surf.}} \int I(g_{\text{eff}}, T_{\text{eff}}, \mu) \vec{e}_{\text{obs}} \cdot \delta(d\vec{S}) \]
Equations

First term

\[ \int \int_{\delta S} \ldots \vec{e}_{\text{obs}} \cdot \vec{dS} \propto \xi^2 \Rightarrow \text{negligible} \]

Second term

\[ \delta l = l \cdot \left( \frac{\partial \ln I}{\partial \ln T_{\text{eff}}} \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} + \frac{\partial \ln I}{\partial \ln g_{\text{eff}}} \frac{\delta g_{\text{eff}}}{g_{\text{eff}}} \right) + \frac{\partial l}{\partial \mu} \delta \mu \]

- \( \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} \), \( \frac{\delta g_{\text{eff}}}{g_{\text{eff}}} \), and \( \delta \mu \) are deduced from the pulsation mode
- see next slide for \( I \) and its derivatives

Third term

- \( \delta (d\vec{S}) \) is deduced from the Lagrangian displacement
### Intensities

\[ I(T_{\text{eff}}, g_{\text{eff}}, \mu) = I_0(T_{\text{eff}}, g_{\text{eff}})h(\mu, T_{\text{eff}}, g_{\text{eff}}) \]

- \( I_0(T_{\text{eff}}, g_{\text{eff}}) \) from blackbody spectrum
- \( h(\mu, T_{\text{eff}}, g_{\text{eff}}) \) from Claret (2000)
- bolometric, Strömgren, and Johnson-Cousins photometric bands

#### Diagrams

**Bolometric**

**Strömgren, u**
Various profiles

Amplitude ratios

- Adiabatic
- Non-adiabatic
- Real
- Imag

Colatitude, $\theta$

0.0 0.5 1.0 1.5 2.0 2.5 3.0

Amplitude

$\xi_r$

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Various profiles

\[ \frac{\delta T_{\text{eff}}}{T_{\text{eff}}} \]

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Non-adiabatic pulsations in ESTER models
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)

\[
\begin{align*}
\text{Wavelength (in nm)} & \quad \text{Amplitude ratio} \\
300 & \quad 0.60 \\
400 & \quad 0.55 \\
500 & \quad 0.50 \\
600 & \quad 0.45 \\
700 & \quad 0.40 \\
800 & \quad 0.35 \\
900 & \quad 0.30 \\
\end{align*}
\]

- Stable
- Unstable

$\Omega / \Omega_K = 0.0$
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)

- Stable
- Unstable

$\Omega/\Omega_K = 0.1$
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)

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Non-adiabatic pulsations in ESTER models
Amplitude ratios for an $\ell = 3$ multiplet $(i = 30^\circ)$

\[
\Omega / \Omega_K = 0.3
\]
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)
Amplitude ratios for an $\ell = 3$ multiplet ($i = 30^\circ$)

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Non-adiabatic pulsations in ESTER models
Amplitude ratios for an \((\ell = 2, m = 0)\) mode

Amplitude ratio

\[ \Omega / \Omega_K = 0.0 \]

\( i = 0^\circ \)

\( i = 30^\circ \)

\( i = 60^\circ \)

\( i = 90^\circ \)
Amplitude ratios for an \((\ell = 2, m = 0)\) mode

\[
\begin{align*}
&\text{Wavelength (in nm)} \\
&0.40 \\
&0.45 \\
&0.50 \\
&300 \quad 400 \quad 500 \quad 600 \quad 700 \quad 800 \quad 900 \\
&\text{Amplitude ratio} \\
&i = 0^\circ \\
&i = 30^\circ \\
&i = 60^\circ \\
&i = 90^\circ \\
&\frac{\Omega}{\Omega_K} = 0.1
\end{align*}
\]
Amplitude ratios for an \((\ell = 2, m = 0)\) mode

\[
\begin{array}{c|ccccc}
\text{Wavelength (in nm)} & 300 & 400 & 500 & 600 & 700 & 800 & 900 \\
0.30 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0.35 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0.40 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0.45 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0.50 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0.55 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
0.60 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
\end{array}
\]

Amplitude ratio

\(i = 0^\circ\)

\(i = 30^\circ\)

\(i = 60^\circ\)

\(i = 90^\circ\)

\(\Omega/\Omega_K = 0.2\)
Amplitude ratios for an \((\ell = 2, m = 0)\) mode

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Amplitude ratios for an \((\ell = 2, m = 0)\) mode

\[
\frac{\Omega}{\Omega_K} = 0.4
\]
Amplitude ratios for an \((\ell = 2, m = 0)\) mode

\[
\begin{array}{c|c|c|c|c|c|c|c|c|c|c|c|c}
\hline
\text{Wavelength (in nm)} & 300 & 400 & 500 & 600 & 700 & 800 & 900 \\
\hline
\text{Amplitude ratio} \\
\hline
i = 0^\circ & 0.65 & 0.60 & 0.55 & 0.50 & 0.45 & 0.40 & 0.35 \\
i = 30^\circ & 0.45 & 0.40 & 0.35 & 0.30 & 0.25 & 0.20 & 0.15 \\
i = 60^\circ & 0.25 & 0.20 & 0.15 & 0.10 & 0.05 & 0.00 & 0.05 \\
i = 90^\circ & 0.05 & 0.00 & 0.05 & 0.10 & 0.15 & 0.20 & 0.25 \\
\hline
\end{array}
\]

\(\Omega/\Omega_K = 0.5\)
Line Profile Variations (LPVs)

Previous works
- Clement (1994): 2D calculations
- Townsend (1997): the traditional approximation

Description
- includes Doppler shifts and $\delta (d\tilde{S})$
- $\delta T_{\text{eff}}$ and $\delta g_{\text{eff}}$ neglected
- use of blackbody spectrum (incl. gravity darkening)
- rudimentary description of limb darkening
Increasing rotation rates

\[ \Delta M_e = 5.76 \times 10^{-4} \]
\[ \min(M_e) = 0.1835 \]
\[ \max(M_e) = 0.1845 \]

1st moment
2nd moment (rescentered)

\[ \nu \equiv \frac{176.5 \text{ mHz}}{f_i/l_i} = (2,1,1) \]

\[ i = 45^\circ \]
\[ \Omega = 0.1 \Omega_c \]
\[ V_{eq} = 67.4 \text{ km/s} \]
Increasing rotation rates

\[ \Delta M_i = 5.76 \times 10^{-5} \]
\[ \min(M_i) = 0.1835 \]
\[ \max(M_i) = 0.1845 \]

For the 1st moment and 2nd moment (recentered):

- Fundamental
- 1st harmonic
- 2nd harmonic

\[ \nu = 171.4 \, \mu \text{Hz} \]
\[ (\bar{n}, \bar{l}, \bar{m}) = (2, 1, 1) \]
\[ i = 45^\circ \]
\[ \Omega = 0.2 \, \Omega_c \]
\[ V_{eq} = 134.0 \, \text{km/s} \]
Increasing rotation rates

LPVs

$M_1 = 5.78 \times 10^{-5}$

$\min(M_2) = 0.1835$

$\max(M_2) = 0.1845$

$\nu = 164.8 \, \mu\text{Hz}$

$(\bar{n}, \bar{\ell}, \bar{m}) = (2, 1, 1)$

$i = 45^\circ$

$\Omega = 0.3 \, \Omega_c$

$V_{eq} = 199.0 \, \text{km/s}$

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Increasing rotation rates

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Non-adiabatic pulsations in ESTER models
Increasing rotation rates

\[ \Delta M_t = 5.78 \times 10^{-5} \]
\[ \text{min}(M_t) = 0.1835 \]
\[ \text{max}(M_t) = 0.1845 \]

**1st moment**

**2nd moment**

(reparented)

\[ \nu = 142.3 \mu \text{Hz} \]
\[ (\overline{n}, \overline{i}, m) = (2, 1, 1) \]
\[ i = 45^\circ \]
\[ \Omega = 0.5 \Omega_c \]
\[ V_{eq} = 320.9 \text{ km/s} \]
Increasing $\ell$ value

$\Delta M_\ell = 5.76 \times 10^{-8}$

$\min(M_\ell) = 0.1835$

$\max(M_\ell) = 0.1845$

$\nu = 74.3 \mu\text{Hz}$

$(\tilde{n}, \tilde{l}, m) = (0, 0, 3)$

$i = 60^\circ$

$\Omega = 0.5 \Omega_c$

$V_{eq} = 320.9 \text{ km/s}$
Increasing \( \ell \) value

\[
\Delta M = 5.78 \times 10^{-5} \\
\min(M_\ell) = 0.1835 \\
\max(M_\ell) = 0.1845 \\
\text{1}\st \text{moment} \\
\text{2}\nd \text{moment} \\
\text{(rescutered)}
\]

\[
\nu = 78.6 \ \muHz \\
(\tilde{\nu}, \tilde{\ell}, \tilde{m}) = (1, 0, 3) \\
j = 60^\circ \\
\Omega = 0.5 \ \Omega_c \\
V_{eq} = 320.9 \ \text{km/s}
\]
Increasing $\ell$ value

$\Delta M_i = 5.78 \times 10^{-5}$
$\min(M_i) = 0.1835$
$max(M_i) = 0.1845$

$1^{\text{st}}$ moment
$2^{\text{nd}}$ moment (resattered)

$\nu \equiv 82.6 \mu$Hz
$(\pi, \ell, m) = (0, 1, 3)$

$i = 60^\circ$
$\Omega = 0.5 \Omega_c$
$V_{eq} = 320.9$ km/s
Increasing $\ell$ value

\begin{align*}
\Delta M &= 5.78 \times 10^{-3} \\
\min(M_\ell) &= 0.1835 \\
\max(M_\ell) &= 0.1845
\end{align*}

$\ell^\text{th}$ moment (normalized)

Fundamental, $1^\text{st}$ harmonic, $2^\text{nd}$ harmonic

\begin{align*}
\nu &\equiv 86.6 \mu\text{Hz} \\
(\bar{n}, \ell, m) &= (1, 1, 3)
\end{align*}

$i = 60^\circ$ \\
$\Omega = 0.5 \, \Omega_c$ \\
$V_{eq} = 320.9 \text{ km/s}$
Conclusion

- important step forward:
  - can now predict which modes are unstable
  - can calculate amplitude ratios and LPVs

Prospects

- understand how rotation (de)stabilise modes
- what are the differences between prograde and retrograde modes
- include more realistic atmosphere
- identify modes