Measurement of the Transmission Phase through a Quantum Dot Embedded in One Arm of an Electronic Mach-Zehnder Interferometer

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We investigate an electronic Mach-Zehnder interferometer with high visibility in the quantum Hall regime. The superposition of the electrostatic potentials from a quantum point contact (QPC) and the residual disorder potential from doping impurities frequently results in the formation of inadvertent quantum dots (QD) in one arm of the interferometer. This gives rise to resonances in the QPC transmission characteristics. While crossing the QD resonance in energy, the interferometer gains a phase shift of \( \pi \) in the interference pattern.

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I. INTRODUCTION

The conductance \( G \) of an Aharonov-Bohm (AB) ring oscillates with magnetic field \( B \) with a period \( \Delta B = \Phi_0 / A \), where \( \Phi_0 = h/e \) is the flux quantum. The combination of the AB ring with a quantum dot (QD) in one of its arms gives the opportunity to measure the phase of transmission amplitude through the QD\(^1\). The dot, tuned to a transmission resonance, sustains coherent transport all over the width of the resonance peak. So far this was realized in different groups by recording AB oscillations as a function of the magnetic field and tracing their phase with respect to the energy of the resonant state, which was controlled by a plunger gate\(^2,3,4\). Fano resonances were observed in Ref. 4, while other work\(^2,3\) show the phase evolution of AB oscillations while scanning through each Coulomb peak in energy changing the plunger gate. Here a slip by \( \pi \) was seen, what can be explained in a single particle picture. For the observed \( \pi \)-jump between two Coulomb peaks\(^3\), a theory involving many particles is needed. Besides the phase of the transmitted current also the phase of the reflected current was probed\(^5\), which showed similar results. In order to determine the energy-dependence of the phase, it was necessary to record many \( G(B) \)-traces differing in plunger gate, each resulting, in a single point in the phase evolution. The question arises, whether one could directly measure the phase in an interferometer, when the QD crosses a resonance. This has two attractive advantages. First the measurement process speeds up, because information about phase is acquired in a single sweep of the plunger gate. Second, with such a fast measurement process, one could think about detection of the charge state for the QD by measuring its transmission (reflection) phase. The latter can be important for building charge qubits based on double quantum dot system. However in conventional AB interferometers, the small interference contrast (typically 10\%) and signal noise makes this task difficult\(^6\). In this work we report on measurements of QD transmission phase with an electronic Mach-Zehnder interferometer\(^2\). The electronic

![FIG. 1: SEM image of Mach-Zehnder interferometer with the scheme of paths for nonequilibrium current. The transmission of QPC1 and QPC2 is set to 0.5. QPC0 transmits the outer edge channel and reflects the inner one in case the filling factor being more than 1. The modulation gate (MG) is used to shift the phase.](image-url)

Mach-Zehnder interferometer employs edge channels of a two dimensional electron system in the quantum Hall regime and quantum point contacts as beam splitters. The interference contrast can be very large, up to 80 %, at temperatures near 20 mK\(^6\).

II. EXPERIMENTAL DETAILS

The interferometer (see Fig. 1) was fabricated on the basis of a modulation doped GaAs/Ga\(_{0.7}\)Al\(_{0.3}\)As heterostructure containing a two-dimensional electron gas (2DEG) 90 nm below the surface. At 4 K, the unpatterned 2DEG density and mobility were \( n = 2.0 \times 10^{15} \text{ m}^{-2} \) and \( \mu = 206 \text{ m}^2/(\text{V s}) \), respectively. The details of fabrication procedure can be found in Ref. 7. Each arm of the MZI was approximately 9 \( \mu \text{m} \) long and the gap between the tips of quantum point contacts was 400 nm. This interferometer showed a maximum visibility of 56% and an area of 25 \( \mu \text{m}^2 \) between interfering paths, found from the period of Aharonov-Bohm oscillations. A standard lock-in technique \((f \sim 300 \text{ Hz})\) with 1 \( \mu \text{V} \) excitation at
terminal S and detection at terminal D2 was employed (see Fig. 1). All measurements were performed at a temperature below 50 mK.

III. CHARACTERIZATION OF THE QUANTUM POINT CONTACTS

In many experiments, the transmission characteristics of the QPCs, in high perpendicular magnetic field exhibit resonances superimposed on the conductance steps [Fig. 2(a)], see also\cite{8,9,10}. As we will show below, our data suggest that this originates from the Coulomb blockade of a quantum dot formed inadvertently by the disorder potential in the vicinity of the QPCs. The interference contrast was highest for the outer edge channel.

To record the characteristics in Fig. 2(a), (i) the magnetic field was set to 4.6 T (filling factor 1.6), (ii) QPC0 was adjusted to transmit only the outermost channel, and (iii) the gate voltage for QPC2 was swept to negative voltages, while keeping QPC1 open [see also Fig. 3(a)]. A sequence of peaks (marked by letters A, B, C in Fig. 2(a) in the gate characteristics of QPC2 appears at transmissions less than 1. It is of interest to check if any peak structure can be found in the dependence on a magnetic field. The magnetic field dependence of current transmitted through QPC2 is plotted in Fig. 3(c). While sweeping the magnetic field, the data was recorded after adjusting QPC0 to reflect the half-filled upper Landau level, opening QPC1, and tuning the gate voltage of QPC2 at the maximum of peak "A" ($T_{QPC2} = 0.75$) (B=4.6T). The average current in Fig. 3(c) corresponding to $T_{QPC2} = 0.4$ at B=4.8T increases to the left from that point, reaching $T_{QPC2} = 1$ at B=3.8T, and decreases to the right approaching value $T_{QPC2} \approx 0.1$. This occurs because of the change of energy for the lowest Landau subband. In other words the potential barrier adjusted at B=4.8T decreases and disappears to the left from this point and grows to the right as function of B. We find that this current has oscillatory components, which are not expected for a single barrier but could easily appear in a device with two and more barriers. Fourier analysis reveals two frequencies in B, corresponding to periods of 0.27 and 0.18 T, and their higher harmonics. When interpreted as Aharonov-Bohm oscillations ($\Delta B \cdot A = e/h$) these two periods correspond to areas enclosed by circumference with diameter $d = 2\sqrt{A/\pi}$ of 140 and 170 nm. This is well compatible with the lithographic gap between QPC tips (400nm) and with the spatial variation of the disorder potential near 100 nm measured, e.g., in Ref.\cite{11} for 2DEG structure with parameters similar to ours. Aharonov-Bohm oscillations induced by potential fluctuations in single QPC in high magnetic field was reported before\cite{12}. In addition, we found that the resonances in Fig. 2(a) are also dependent on cooling cycle, i.e. the shape of the resonances is unique for each cool down. This indicates that charging of donor atoms in the doping layer plays a role in the resonance formation.
IV. PHASE SHIFTS BY THE QUANTUM DOT

Next, we discuss the response of the MZI interference near the resonances in the transmission of QPC2. To investigate this, first, the QPC1 must be set to half transmission, generating two interfering paths as shown in Fig. 3(b). Second, the interference signal must be measured as a function of the QPC2 gate voltage. Here two possibilities arise depending on the regime for a modulation gate voltage \( V_{\text{MG}} \) which is normally used to observe interference by shifting the phase of one arm with respect to other. These are i) sweeping \( V_{\text{MG}} \) simultaneously with the QPC2 gate voltage; and ii) keeping \( V_{\text{MG}} \) constant. The former allows to determine the interference contrast as function of \( V_{\text{QPC2}} \) (or \( T_{\text{QPC2}} \)) and was demonstrated before [Ref. 10; Fig. 1(c); Fig. 2(a)]. In contrast, the latter is sensitive to any phase gain during a change of the QPC2 gate voltage. We explore both of these opportunities starting from the measurement of the span for AB oscillation vs. \( V_{\text{QPC2}} \). The relative amplitude of oscillations is called visibility \( \nu_1 \), \( \nu_1 = (I_{\text{max}} - I_{\text{min}})/(I_{\text{max}} + I_{\text{min}}) \), and is plotted in Fig. 2(b). Here one sees that the visibility \( \nu_1 \) peaks at the transmission resonances ”A”, ”B” and ”C”. Qualitatively, this is explained by the fact that the visibility has a maximum when both arms carry equal current, i.e. at \( T_{\text{QPC}} = 1/2 \) (a quantitative analysis follows below). Therefore the closer \( T_{\text{QPC}} \) is to 1/2, the higher is the visibility. The phase information does not show itself in this experiment, but it does when the modulation gate voltage \( V_{\text{MG}} \) is kept constant [Fig. 2(c)].

Now we address quantitatively the results in Fig. 2 in the framework of a model assuming noninteracting particles. This model predicts the MZI transmission coefficient \[ T_{\text{SD2}} = |t_1r_2|^2 + |r_1t_2|^2 - 2|t_1t_2r_1r_2| \cos \Delta \phi \, , \] (1)

here \( t_i, r_i \) are transmission and reflection amplitudes of QPCs and \( \Delta \phi \) is the phase difference between the interferometer arms. From this formula one can easily find the expression for the visibility as function of one of the QPC transmission, namely,

\[ \nu_1 = z \cdot 2 \sqrt{T_{\text{QPC2}}(1 - T_{\text{QPC2})}} \, , \]

where the factor \( z < 1 \) accounts for the decoherence at finite temperature. Using this expression and the measured transmission values for \( T_{\text{QPC2}} \) in Fig. 2(a), we calculate the dependence of visibility \( \nu_1(T_{\text{QPC2}}) \) and compare it with the experimental one. The black, thin line in Fig. 2(b) shows the result, which agrees in general well with the experiment. There are two regions of deviation from the simple model of Eq. 2, marked by arrows in Fig. 2(b), and peak ”B” is within one of those. In contrast to this the peak ”C”, with small transmission value, is well described by the model, as well as the region with transmission close to 1 (\( V_{\text{QPC2}} > -651 \text{ mV} \)) and the one on the right wing of the peak ”A” (\( -708 \text{ mV} < V_{\text{QPC2}} < -678 \text{ mV} \)).

Following the noninteracting particle approach from Ref. 2 we add a quantum dot to the above mentioned MZI, whose transmission properties are modelled by the Breit-Wigner formula. We replace transmission and reflection amplitudes of QPC2 by those of the quantum
dot. The dot transmission amplitude has a phase which must be added to the cosine argument in Eq. 1. Then the coherent component in MZI transmission is proportional to $T_{SD2coh} \propto |t_{QD}| |r_{QD}| \cos(\Delta \varphi + \theta(t_{QD}))$ where $t_{QD}$, $r_{QD}$ and $\theta(t_{QD})$ stand for the transmission (reflection) amplitude of QD and the phase of QD respectively. The Breit-Wigner formula for the selected state with the energy $E_n$

$$t_{QD} = |t_{QD}| e^{i \theta} = \frac{(i \Gamma/2)}{(E - E_n + i \Gamma/2)},$$

with $\theta(t_{QD}) = \arctan \left( \frac{2}{\Gamma(E - E_n)} \right)$ and relation $|r_{QD}| = \sqrt{1 - |t_{QD}|^2}$ were applied for the simulation of experimental curves [in Fig. 4].

This model rather well describes the interference resonances "B" and "C". The peak "A" was analyzed below only qualitatively, because it deviates over a wide range from Eq. 3. The resonance "C" was matched well by calculated curves [Fig. 4(a)] with resonance width of $\Gamma_C = 1.9 \text{ mV}$ determined from fit by Lorentzian the peak in the QPC2 transmission in Fig. 2(a). Therefore the only fitting parameter for this interference resonance was the phase shift $\Delta \varphi$. For peak "B" [Fig. 4(b)] an effective transmission resonance, of smaller amplitude and width $[\Gamma_B = 2.5 \text{ mV} \text{ instead } 3.4 \text{ mV} \text{ in Fig. 2(a)}]$, matching the experimental visibility in the Fig. 2(b), was used.

In addition to the good matching of the experimental curves in Fig. 4(a,b) to Eq. 3 the validity of our interpretation is supported by the correlation between the phase set by the modulation gate voltage and that determined from the best fit to experimental data in Fig. 4a and b with Eq. 3. The period of AB oscillations in $V_{MG}$ was found to be 1.6 mV, which corresponds to phase change of $2\pi$. In the Fig. 4(c) we plot the phase found from fitting as function of modulation gate voltage $\Delta \varphi(V_{MG})$ for the peaks "C" (squares) and "B" (circles), and from these graphs extract the slope $a$ of $\Delta \varphi = a V_{MG}$. We find $\Delta \varphi = \pm 2\pi/1.6 \text{ mV}$, which is in perfect agreement with the period $V_{MG}$ extracted from the interference pattern $I_{D2}(V_{MG})$.

The phase shift $\Delta \varphi$ of peak "A" can only be determined qualitatively as mentioned above. From figures 4(a) and (b) one can see that, a peak in the interference resonance corresponds to $\Delta \varphi = 2\pi$, the dip to $\pi$, combination left peak/right dip to $\pi/2$, and left dip/right peak to $3\pi/2$. Comparing this with the shape of the interference resonance of peak "A" we can deduce an approximate phase shift. It is interesting to investigate the direction of the phase evolution in Fig. 4(c) $\Delta \varphi(V_{MG})$ for the neighboring peaks "A" and "B", and peak "C".

As a result, peak "A" shows the same direction of phase evolution as "B" [Fig. 4(c), triangles]. On the other hand, the phase evolution of peak "C" goes into the opposite direction. This discrepancy may originate from the variability of two barriers, since the dot is defined by the single QPC2 gate and the disorder potential. Tuning the gate potential of QPC2 changes simultaneously the two barrier heights of the QD and its well depth.

The largest of the two barriers must be the branching point of the interferometer path. If the barriers interchange their height, the branching point will interchange its location as well. In this case the $\pi$ phase shift from the quantum dot may contribute either to the upper arm or to the lower one [Fig. 3(b)], and depending on this, gain its different sign in the paths phase difference.

V. CONCLUSIONS

In summary we have shown that the frequently observed transmission resonances in quantum point contacts within an electronic Mach-Zehnder interferometer stem from inadvertent quantum dots formed by the disorder potential in high magnetic field and measured the phase of the transmission amplitude through a quantum dot. We propose to utilize this effect for the detection of the state of charge qubits in the vicinity of a Mach-Zehnder interferometer via their reflection phase. Such a dispersive read-out may allow more sensitive and less invasive detection than the currently used quantum point contacts.

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1. A. Yacoby, M. Heiblum, D. Mahalu, and H. Shtrikman, Phys. Rev. Lett. 74, 4047 (1995).
2. R. Schuster, E. Buks, M. Heiblum, D. Mahalu, V. Umansky, H. Shtrikman, Nature 385, 417 (1997).
3. E. Buks, R. Schuster, M. Heiblum, D. Mahalu, V. Umansky, and H. Shtrikman, Phys. Rev. Lett. 77, 4664 (1996).
4. K. Kobayashi, H. Aikawa, S. Katsumoto, and Y. Iye, Phys. Rev. Lett. 88, 256806 (2002).
5. Y. Ji, Y. Chung, D. Sprinzak, M. Heiblum, D. Mahalu, and H. Shtrikman, Nature 422, 415 (2003).
6. I. Neder, N. Ofek, Y. Chung, M. Heiblum, D. Mahalu, V. Umansky, Nature 448, 333 (2007).
7. L. V. Litvin, A. Helzel, H.-P. Tranitz, W. Wegscheider, C. Strunk, Phys. Rev. B 78, 075303 (2008).
8. I. Neder, F. Marquardt, M. Heiblum, D. Mahalu and V. Umansky, Nature Physics 3, 534 (2007).
9. P. Rouleau, F. Portier, D.C. Glattli, P. Roche, A. Cavanna, G. Faini, U. Gennser, D. Mailly, Phys. Rev. B 76, 420401 (2007).
10 L.V. Litvin, A. Helzel, H.-P. Tranitz, W. Wegscheider, and C. Strunk, Physica E 40, 1706 (2008).
11 G.A. Steele, R.C. Ashoori, L.N. Pfeiffer, and K. W. West, Phys. Rev. Lett. 95, 136804 (2005).
12 P. H. M. van Loosdrecht, C. W. J. Beenakker, H. van Houten, J. G. Williamson, B. J. van Wees, J. E. Mooij, C. T. Foxon and J. J. Harris, Phys. Rev. B 38, 10162 (1988).