Dependence of the Number of Dealers in a Stochastic Dealer Model

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Abstract. We numerically analyze an artificial market model consisted of $N$ dealers with time dependent stochastic strategy. Observing the change of market price statistics for different values of $N$, it is shown that the statistical properties are almost same when the dealer number is larger than about 30.

1. Introduction

Through intensive analysis of high frequency data, stylized facts have been established about fluctuation of financial markets especially for the foreign exchange markets which are the largest free markets in the world and open 24 hours [1]-[7].

♯ 1 The power law distributions of volatility, or the absolute value of price changes.

♯ 2 The autocorrelation function of price changes which takes a negative value at the first tick and otherwise it is close to 0.

♯ 3 Long time correlation of volatility.

♯ 4 Diffusion properties: Abnormal diffusion in short time scale and normal diffusion in the large time scale.

♯ 5 Non-Poissonian transaction intervals characterized by a non-exponential distribution.

From ♯1 to ♯4 in the above list, these properties are derived by analyzing time series of market prices while ♯5 comes from analysis of transaction intervals. There are many market models which satisfy some of the above empirical laws[8]-[23].
For the purpose of clarifying the relation between the above statistical properties and dealers’ action in the market, we already introduced deterministic dealer models and a stochastic dealer model [24]-[27]. In these series of study of market model construction, we started with the most basic model which produces non-correlated fluctuations both for price changes and transaction intervals. This model, named as "model 1", can be recognized as a random number generator in the market, however, this simple model does not satisfy distribution of transaction intervals and volatility.

In order to realize the fat-tail property of transaction intervals, it is important to introduce a kind of modulation of dealers’ activity called, "self modulation", to be proportional to the market’s activity which is defined by the number density of transactions. The model with this revision is named as "model 2”.

As for power law distribution of price changes, we need to add a term describing the effect of following trends, which are defined by mean gradients of latest price changes, and we call the revised model as "model 3”. In the model-4, we combine model 2 and model 3 and we show model 4 fulfills all major statistical properties.

The goal of this paper is to observe the effect of number of dealers in these models and to reconstruct market stylized facts. In the next section we construct our market model following the style of former model construction, namely, we start with “model 1”, then introduce “model 2” and “model 3”, and finally combining all terms in the subsection “model 4”.

2. Construction of the dealer models

2.1. Model 1

We construct an artificial market consisted of N dealers modeling foreign exchange or stock markets. Every dealer shows his limit prices of bid and ask. A bid price is the price to buy while an ask price is the price to sell. It is natural that the ask price is always higher than a bid price for each dealer because each dealer wants to get some margin by transactions. In the model, we assume that the spread, which is defined by the ask price minus the bid price, to be constant, L, for all the dealers for simplicity. We define the ith dealer’s price at time t, $p_i(t)$, by the mean value between the bid and ask prices, namely, the bid and ask prices are given by $p_i(t) - L/2$ and $p_i(t) + L/2$, respectively. When the ith dealer’s bid price, $p_i(t) - L/2$, is larger or equal to the jth dealer’s ask price, $p_j(t) + L/2$, then a transaction occurs between these dealers. This condition is represented by an inequality, $|p_i(t) - p_j(t)| \geq L$. The market’s deal price $P(t)$ is then given by $(p_i(t) + p_j(t))/2$ and after the transaction we assume that both dealers’ prices are set to be equal to this market’s deal price.

The time evolution of each dealers’ price is given by the following rule:

$$p_i(t + \Delta t) = p_i(t) + cf_i(t) \tag{1}$$

$$f_i(t) = \begin{cases} +\Delta p & \text{(prob. 1/2)} \\ -\Delta p & \text{(prob. 1/2)} \end{cases} \quad i = 1, 2, \cdots, N,$$

where $\Delta t$ is the time step, $c$ is a constant representing the step size of random motion of each dealer, $f_i(t)$ takes either $\Delta p$ or $-\Delta p$ randomly with equal probability, and $\Delta p$ is a positive constant.

We show market price fluctuations and transaction intervals for some values of N in Fig.1 simulated with the following parameters; $L = 0.1$, $c = 0.01$, $\Delta p = 0.01$ and $\Delta t = (\Delta p)^2$. For larger number of dealers, fluctuations of both transaction intervals and price changes become smaller as clearly seen from these figures.

In Fig.2, we show the averages of transaction intervals and volatility as a function of N keeping other parameters constants. An average value of transaction intervals is proportional to $N^{-1}$ and the volatility is approximately proportional to $N^{-0.5}$. The figures of Fig.3 are
statistical properties of model 1 and the number from (i) to (v) in figure is correspond to the number of statistical property in introduction. In these simulation we use following parameters; $c = 0.01$, $\Delta p = 0.01$ and $\Delta t = (\Delta p)^2$. When $N$ equals 2, $L = 0.015$ while the value of $L$ for $N \neq 2$ is determined by the relation $\sqrt{N}/L \sim 6.67$, which is derived from two results; one is the relation between transaction intervals and number of dealers as seen in Fig.2, and the other is that average of transaction interval is proportional to $L^2$. Note that we use same parameters in model 2, 3 and 4.

From the results of Fig.3 (i) and (v), the distribution of volatility and transaction intervals does not have long correlation unlike real market. And autocorrelation function of volatility also does not have long correlation(Fig. 3 (iii)). However the autocorrelation function of price changes takes a non-trivial negative value at one tick in the case of $N \geq 3$, and its value converges for dealer number larger than 32 as shown in Fig.3 (ii). And this negative correlation cause abnormal diffusion in short time scale as shown in Fig.3 (iv). In the following section we compare these statistical properties of each model by using same form.

![Figure 1. Simulation results of model 1 for three cases of N in the same axis scale. The upper figures are transaction intervals and lower figures are market prices.](image1)

![Figure 2. Average values of transaction intervals and volatility as functions of N.](image2)
Figure 3. Statistical properties of model 1. (i): Cumulative distribution of volatility in semi-log plot. (ii): Autocorrelation function of price changes. (iii): Autocorrelation function of volatility in log-log plot. (iv): Diffusion properties. (v): Cumulative distribution of transaction intervals.
model 2

In this subsection we focus on transaction intervals. It is known that transaction intervals in real markets do not follow simple Poisson process and its statistics is characterized by a fat-tailed distribution [7]. Here, we generalize the coefficient $c$ in Eq.(1) to be time dependent as follows:

\begin{equation}
\begin{aligned}
\tau_{i}(t + \Delta t) &= p_i(t) + c(n)f_i(t) \quad i = 1, 2, \cdots, N, \\
f_i(t) &= \begin{cases} 
+\Delta p & \text{(prob. 1/2)} \\
-\Delta p & \text{(prob. 1/2)}
\end{cases}
\end{aligned}
\end{equation}

where $n$ denotes the tick number which counts the number of deals, and

\begin{equation}
c(n) = c_1 \sqrt{\frac{\langle I \rangle_{\tau = c_1}}{\langle I \rangle_{\tau}}},
\end{equation}

where $c_1$ is a positive constant, $\langle I \rangle_{\tau = c_1}$ is the mean value of transaction intervals of model 1 in the case when the parameter $c$ equals to a constant $c_1$, and $\langle I \rangle_{\tau}$ is the moving average of transaction intervals averaged over the latest $\tau$ seconds defined as $\langle I \rangle_{\tau} = \frac{1}{K} \sum_{k=0}^{K-1} I(n - k)$, with $K$ being the number of deals within the time interval $[t - \tau, t]$, and $I(n)$ denotes the $n$th transaction interval, and in the case that $I(n) > \tau$, we set $\langle I \rangle_{\tau} = I(n)$. It is known from Eq.(3) that when $\langle I \rangle_{\tau = c_1}$ over $\langle I \rangle_{\tau}$ is large, the value of $c(n)$ is large, and the dealers tend to make large changes in their prices. As a result there is a tendency that occurrence of transactions becomes higher when there were more transactions in the latest time interval $[t - \tau, t]$ which belongs to a kind of self-modulation effect[28]. By this effect the distribution of transaction intervals in the model 2 is modified to have fat-tails as shown in Fig.4 (v) for various $N$. While the statistical properties of price as shown in from (i) to (v) is unchanged from the model 1. In this simulation, we set parameters as following: $c_1 = 0.01, \Delta p = 0.01, \Delta t = (\Delta p)^2$ and $\tau = 150$. And we set $L=0.015$ when $N = 2$ and $L = \sqrt{N}/66.7$ when $N \neq 2$.

model 3

In this section we add so-called the trend-follow effect to the model 1. We modify the dynamics as follows;

\begin{equation}
\begin{aligned}
\tau_{i}(t + \Delta t) &= p_i(t) + d_i(n) < \Delta P >_M \Delta t + cf_i(t), \\
f_i(t) &= \begin{cases} 
+\Delta p & \text{(prob. 1/2)} \\
-\Delta p & \text{(prob. 1/2)}
\end{cases} \quad i = 1, 2, \cdots, N.
\end{aligned}
\end{equation}

where the parameter $d_i(n)$ characterizes the $i$th dealer’s strategy to be explained in more detail, $< \Delta P >_M$ is the moving average with a linearly decaying weight which is defined by the following equation,

\begin{equation}
< \Delta P >_M = \frac{2}{M(M+1)} \sum_{k=0}^{M-1} (M-k)\Delta P(n - k),
\end{equation}

where $\Delta P(n) = P(n) - P(n - 1)$ is the price change between the $n-1$th tick and the $n$th tick.

A dealer with a positive $d_i$ is called the trend-follower who predicts market prices will go up (or down) when the moving average value Eq.(5) is positive (or negative). On the other hand a dealer with a negative $d_i$ is called the contrarian who forecasts that the market prices will turn and go down (or up) when the moving average value is positive (or negative). The strength
Figure 4. Statistical properties of model 2. (i): Cumulative distribution of volatility in semi-log plot. (ii): Autocorrelation function of price changes. (iii): Autocorrelation function of volatility in log-log plot. (iv): Diffusion properties. (v): Cumulative distribution of transaction intervals.

of such trend dependence is characterized by the absolute value of $d_i$. We assume that each dealers’ strategy is changing at each market deal by the following stochastic process:

$$d_i(n) = \bar{d}(n) + \Delta d_i(n), \quad (6)$$

$$\bar{d}(n+1) = (1 - e_0) \cdot \bar{d}(n) + \phi(n), \quad (7)$$

$$\phi(n) = \begin{cases} +0.01 & \text{(prob. } 1/2) \\ -0.01 & \text{(prob. } 1/2) \end{cases},$$

where $\bar{d}(n)$ gives the mean strategy of traders at the $n$th tick, $e_0$ is a positive constant less than 1, $\Delta d_i$ are random numbers following a normal distribution with 0 mean and the standard deviation 0.1.

In our former study we proved in the special case of $N = 2$ with a constant $d_i = d$ that the dealer model with the trend-follow effect produces time series of prices which are characterized by a market potential force, and the quadratic potential’s coefficient $b$ in the theory of PUCK
analysis is linearly related to the parameter of the dealers’ strategy $d_i(n)$ in Eq.(4)[27]:

$$< b > = -d \left( \frac{L}{c} \right)^2 .$$  \hfill (8)

This result implies that when dealers are trend follower ($d > 0$), the market is unstable ($b < 0$) and when dealers are contrarians ($d < 0$), the market is stable ($b > 0$). As Eq.(7) is an Auto Regressive process around $d(n) = 0$, the corresponding market potential function also follows an Auto Regressive process around $b = 0$[29].

Fig.5 shows simulation results in the case of $N = 2$. Here, the model’s parameters are given as follows; $L = 0.015, M = 1, \Delta p = 0.01, \Delta t = (\Delta p)^2, e_0 = 8.0 \times 10^{-4}$. Fig.5(a) shows the fluctuation of $\tilde{d}$. When $\tilde{d}$ is positive the market is dominated by the trend followers while when $\tilde{d}$ is negative the market is dominated by contrarians. Fig.5(b) shows the corresponding market prices and (c) represents the price changes of market prices.

Fig.6 shows statistical properties of simulation results for different number of dealers by the model 3, we can find power law distribution of price changes (Fig.6 (i)), a quick decay of autocorrelation function of price changes (Fig.6 (ii)), long time correlation of volatility (Fig.6 (iii)), and abnormal diffusion in the short time scale and normal diffusion corresponding to simple random walk in the large time scale (Fig.6 (iv)). However, distribution of transaction intervals in Fig.6 (v) is the same as model 1 because we do not add self-modulation effect to the model 3.

![Simulation results](image_url)

**Figure 5.** Simulation results in the case of $N = 2$. (a): Fluctuation of $\tilde{d}$. (b): An example of market price evolution. (c): Corresponding price changes.
Figure 6. Statistical properties of model 3. (i): Cumulative distribution of volatility in log-log plot. (ii): Autocorrelation function of price changes. (iii): Autocorrelation function of volatility in log-log plot. (iv): Diffusion properties. (v): Cumulative distribution of transaction intervals.
2.4. **model 4**

Here, we combine the time modulation effect of model 2 and the trend-follow effect of model 3. The time evolution equation is given as follows;

\[
p_i(t + \Delta t) = p_i(t) + d_i(n) < \Delta P > \frac{c(n)}{c_1} \Delta t + c(n)f_i(t),
\]

\[
f_i(t) = \begin{cases} 
+\Delta p & \text{(prob. 1/2)} \\
-\Delta p & \text{(prob. 1/2)} 
\end{cases} \quad i = 1, 2, \ldots, N.
\]

\[
d_i(n), < \Delta P > M, \text{ and } c(n) \text{ are defined by Eq.(6), Eq.(5) and Eq.(3) respectively. And we set parameters as following; } c_1 = 0.01, \Delta p = 0.01 \text{ and } \Delta t = (\Delta p)^2, M = 1, e_0 = 8.0 \times 10^{-4} \text{ and } \tau = 150. \text{ And we set } L = 0.015 \text{ when } N = 2 \text{ and } L = \sqrt{N}/66.7 \text{ when } N \neq 2. \text{ By this time evolution rule all empirical findings, } \#1 - \#5, \text{ are fulfilled as seen in Fig.7.}
\]

![Cumulative distribution](image1)

![Autocorrelation](image2)

![Autocorrelation](image3)

![Diffusion properties](image4)

![Cumulative distribution](image5)

**Figure 7.** Statistical properties of model 4. (i): Cumulative distribution of volatility in log-log plot. (ii): Autocorrelation function of price changes. (iii): Autocorrelation function of volatility in log-log plot. (iv): Diffusion properties. (v): Cumulative distribution of transaction intervals.

3. **Concluding Discussion**

We studied 4 types of dealer models with various numbers of dealers. In the model 1 we confirm that the fluctuations of transaction intervals decreases for larger number of dealers proportional...
to $N^{-1}$, and the volatility shrinks proportional to $N^{-0.5}$, respectively. We found that there is a non-trivial autocorrelation in price changes at short time when $N$ is equal to or greater than 3. In the model 2, we reproduce transaction distribution with a fat-tail by adding feedback effect of past transaction intervals for any number of dealers. In the model 3, we added dealers’ trend follow effect to model 1 and reproduced other statistical properties, the power law distribution of price changes, the long time autocorrelation of volatility and the abnormal diffusion property. By combining the model 2 and model 3 to make the model 4, all empirical statistical properties are fulfilled automatically.

In the real foreign exchange markets almost all dealers belong to large international banks and the total number of active dealers is expected to be not so large, about the order of hundreds even in the case of Dollar-Euro markets which is the largest financial market in the world. The number of active dealers in other markets may be smaller, however, if the number is of order of tens we may expect that the basic statistical properties will almost be the same as those of larger markets.

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