A generic unitary black-hole evaporation model based on first principles

Kuan-Yu Chen, 1, 2, † Pisin Chen, 1, 2, 3, 4, § Hsu-Wen Chiang, 1, 2 ‡ and Dong-Han Yeom 5, 6, §

1 Leung Center for Cosmology and Particle Astrophysics, National Taiwan University, Taipei 10617, Taiwan, R.O.C.
2 Department of Physics and Center for Theoretical Physics, National Taiwan University, Taipei 10617, Taiwan, R.O.C.
3 Graduate Institute of Astrophysics, National Taiwan University, Taipei 10617, Taiwan, R.O.C.
4 Kavli Institute for Particle Astrophysics and Cosmology, SLAC National Accelerator Laboratory, Stanford University, Stanford, CA 94305, U.S.A.
5 Department of Physics Education, Pusan National University, Busan 46241, Republic of Korea
6 Research Center for Dielectric and Advanced Matter Physics, Pusan National University, Busan 46241, Republic of Korea

Based on the discretized horizon picture, we introduce a macroscopic effective model of the horizon area quanta that encapsulates the features necessary for black holes to evaporate consistently. The price to pay is the introduction of a “hidden sector” that represents our lack of knowledge about the final destination of the black hole entropy. We focus on the peculiar form of the interaction between this hidden sector and the black hole enforced by the self-consistency. Despite the expressive power of the model, we arrive at several qualitative statements. Furthermore, we identify these statements as features inside the microscopic density of states of the horizon quanta, with the dimension of the configuration space being associated with the area per quanta in Planck unit, a UV cutoff proportional to the amount of excess entropy relative to Bekenstein’s law at the end of evaporation, and a zero-frequency-pole-like structure corresponding to, similarly, the amount of excess entropy at IR limit. We then relate this nearly-zero-frequency structure to the soft hairs proposed by Strominger et al., and argue that we should consider deviating away from the zero frequency limit for soft hairs to participate in the black hole evaporation.

Introduction. As an inevitable consequence of the semi-classical analysis of black hole (BH), the information loss paradox is often considered a gateway to quantum gravity. The paradox suggests that while derived from quantum field theory in curved spacetime, the Hawking evaporation process inevitably leads to an enormous amount of entanglement entropy between an evaporated BH and the radiation it released, thus violating one or more of the following fundamental assumptions in modern physics: unitarity, locality, and general covariance. Many authors have attempted to solve the paradox by various alternations to the classical BH picture. The final resolution evidently is yet to come. In this work, instead of providing yet another specific solution, we take the minimalistic approach and aim at establishing a generic mathematical structure with consequential physical phenomena necessary for the resolution of the paradox. For simplicity, we adopt the natural unit.

At the center of the paradox are three “principles” derived from well-tested theories of general relativity and quantum field theory: 1) The no-hair theorem [17], which suggests the mass M as the only classical scale for a BH. For brevity, the charge and the angular momentum of BH are neglected. 2) The 1st law of BH thermodynamics [18], which relates the observed mass change ∆M with the change of BH horizon area ∆A = 8πκ⁻¹DM where κ is the BH surface gravity. 3) The existence of Hawking radiation process [3], where a BH with a surface gravity κ in the Unruh vacua (particle-less in-vacua) emits thermal radiation of temperature $T_H = κ/(2π)$.

Together, these principles paint the picture of a featureless BH that evaporates into a tremendous amount of radiation. This deduction alone is sound, as objects do radiate when burnt. The horizon even appears discretized, akin to Planck’s oscillator, according to the 1st law, $ΔM = T_H ΔA/4 = T_H ΔS_H$, where $S_H$ is the thermal entropy of BH deduced from the Hawking radiation. Each quantum with the entropy of one Hawking radiation particle occupies four units of Planck area on the horizon. By integrating the first law, we arrive at the Bekenstein-Hawking bound, $S_H ≤ A/4$ [20], a.k.a. Bekenstein’s law if the bound is saturated. However, unlike ordinary thermal radiations whose entropy originates from the incomplete knowledge about the environment, Hawking radiation remains thermal regardless of the initial condition. Therefore the emitted radiation must be maximally entangled with whatever is inside BH. Unless the horizon is somehow leaky, the entanglement entropy piles up and leads to a Planck-size BH that retains a similar amount of entropy as the original one [21]. This conclusion is in direct conflict with Bekenstein’s law, and the storage of the excess entanglement entropy in a form other than the horizon quanta, is necessary.

Notice that for observers outside BH, this new ingredient must be gravitationally inert for it to slip through the classical analysis. More precisely BH should evaporate at a rate well approximated by the Hawking process.
While this “hidden sector” can represent various mechanisms, including the non-unitary process \cite{14, 22}, intra-Hawking-radiation entanglement \cite{6, 8, 10, 23}, and physical entities that carry away the entropy \cite{11, 13}, we reiterate that our interest lies in the generic mathematical constraints laid down by those three principles. We will first focus on how the hidden sector interacts with BH to escort entropy transfer, and then on whether Hawking temperature could be the statistical temperature of BH. From now on, unless stated otherwise, “entropy” refers to the entanglement entropy between BH and the exterior, and is thus positive-definite and additive intra-BH.

Setup. A question arises when analyzing the hidden sector: what entity does it interact with? Dictated by the three principles, the only choice would be the horizon area quantum. We therefore invoke it as the dynamical variable of our effective model. With that in mind, we introduce a generic \((n + 3)\)-species macroscopic model, as depicted in Fig. 1 that consists of four major components: the Hawking radiation \(R\), the external source \(J\), the horizon quanta, and the hidden sector. The horizon quanta are further categorized into \(n\) species of indistinguishable quanta, labeled by the subscript \(i = 1 \cdots n\). These species enable our model to encompass scenarios other than those \cite{7, 24} strictly following the Bekenstein’s law. However, we must stress that at this stage they are just abstract constructs that characterize the model, analogous to the effective degrees of freedom in the mean-field theory. For simplicity, the amounts of quanta and entropy that belong to a species \(X\) \((\text{or } i)\), both non-negative, are denoted as \(N_X\) and \(S_X\). In particular, \(N_R\) serves as our clock. We further repurpose \(\Delta\) as the succeeding quantity’s rate of change, e.g., \(\Delta N_R = 1\).

The only ad hoc condition in the present work is that the interaction strength is independent of the entropy of each species, leaving the inclusion of entropic gravity \cite{25} as future work. Specifically, the generating function of the model depends only on \(N_i\). This assumption stems from the Weinberg-Witten theorem \cite{25}, which suggests the non-compositeness of gravity that supposedly governs the BH microscopic degrees of freedom. As a result, any transfer of entropy originated from a species \(X\) can be associated with that of quanta, as \(N_X \Delta S_X = S_X \Delta N_X\). To ensure the unitarity, the total amount of quanta, including those transferred to the hidden sector, must be conserved. Consequently, the average entropy per quantum of BH forever lies between the entropy per quantum of each horizon quanta species at an instance, the Hawking radiation, and the external source. Surprisingly, even if the BH entropy in principle should be related to the quantum nature of BH and thus lies beyond our grasp, it is bounded by a quantity proportional to the total amount of quanta \(S^*\), which we utilize as the tracer. This confirms that \cite{27} for BH, the “number of degrees of freedom” still bounds its entropy. However, an additional assumption hides within the argument above. A priori, there is no constraint on the content of the external source. The implicit assumption that the ratio is bounded, i.e., the holographic principle, provides a bridge between the entanglement entropy and the thermal entropy of the 1st law. Notice that the bound emerges naturally if the transfer of quanta happens discretely.

Let us now delve into the functionality of individual interactions in the system. First, a controllable external source \(J = 1 + \Delta A/4\) represents the change to \(M\) other than that due to the Hawking radiation, e.g., accretion. We only consider cases where entropy in the external source is homogeneous. To wit, it feeds into one particular species of quanta, though more complicated models are also possible. The second source defines the effect of the Hawking radiation on BH, contributing \(-T_H\) and 1 to \(M\) and \(S^*\) respectively. This is a manifestation of our obsession with energy conservation and unitarity. Naïvely these two sources are the only observables of BH. But as argued before, there must exist a covert channel between BH and the hidden sector for the excess entropy to sink. First introduced in \cite{28}, this “hidden sink” drains a total amount of \(F\) quanta from BH to the hidden sector. We will focus on it and neglect the intra-horizon-quanta interactions for the moment despite their necessity for the complete determination of the system evolution.

**Quanta counting and the Bekenstein-Hawking bound.** To describe the resolution to the information loss paradox, one must keep track of the horizon area and the total amount of quanta \(S^* = \sum_i N_i\). First, the area \(A\) must be a function of \(N_i\) by construction. In conjunction with the inert condition of the hidden sector, the definition above leads to the evolution equation

$$J - 1 = \Delta A/4 = \sum_i \partial N_i A \Delta N_i/4. \quad (1)$$

Regarding \(S^*\), it is conserved, as argued before. Only the external source \(J\), the addition of a single quantum per tick due to the Hawking process, and the hidden sink \(F\) are capable of modifying \(S^*\), resulting in \(\Delta S^* = c_j^{-1} J + 1 - F\), where \(c_j\) is the area per quantum of the external source. Bekenstein-Hawking bound \(S^* \propto S_{EE} \leq S_H \leq A/4\) with a finite proportionality then

![Diagram of effective model](image-url)
can be expressed as the boundedness of \( S^*/A \), with

\[
\frac{d(S^*/A)}{d\ln A} = \frac{c^2}{4(J - 1)} - \frac{S^*}{A} + \frac{F}{4} - \frac{S^*}{A} - \frac{\partial A}{\partial N}.
\]  

(2)

We now realize that the introduction of the hidden sink in [28] is not a coincidence, but rather out of necessity from the evolution of an entropy-density-bounding quantity. Without it, \( S^*/A \) would diverge as \( A \) approaches nullity, leading to the information loss paradox.

Several conclusions can be drawn from the equation. First, the original Bekenstein-Hawking bound [20, 27] \( S^* \propto A \) is satisfied only if \( F \geq 1 + 4S^*/A \) at a specific entropy density \( S^*/A \geq 1/4 \). The factor \( 1 + 4S^*/A \) coincides with simpler models [21, 27, 28] and represents both the excess entropy directly due to the Hawking radiation and indirectly due to the associated area reduction. Unfortunately, this constraint corresponds to a separatrix, which does not forbid BHs that are already on the other side from further deviating away, and most extremely, from forming remnants: stable, nearly fully evaporated BHs with a huge amount of residue entropy. Notice that remnants in our analysis are classical [11, 27, 29] rather than of quantum gravity origin [29]. Those Planck-size remnants form only if quantum gravity corrections are introduced, which is beyond the semi-classical analysis here.

**Dimensional analysis and the extended Bekenstein’s law** Let us switch gears, and perform the dimensional analysis on the previously dropped intra-horizon-quanta interactions (represented by \( \Delta N_j \)). We categorize the interactions into marginal ones, whose effects on the area \( \partial N_i, A \Delta N_i \) are roughly independent of \( A \), and (ir)relevant ones that are (suppressed)enhanced. For the moment, we turn off the external source \( J \). Since the only other scale of the system is the Planck area, irrelevant interactions matter only in the quantum gravity regime [11, 12, 29]. For the relevant ones, the colossal difference between the Planck area and \( A \) implies that some parameters must reach equilibrium and decouple from the rest rapidly.

A typical choice of parameters is the relative contribution of each species to the horizon area \( z_i \), defined as

\[
\Delta z_i \equiv (\partial N_i, A \Delta N_i - z_i \Delta A) / A.
\]

(3)

\( z_i \) has the benefit of being dimensionless, but in general one may choose whatever suitable. For example, to characterize the asymptotic behavior of \( S^* \) at the end of BH evaporation, the right-hand side of eq. (2) can be parametrized as \( f(z_i) A^k \sim f(S^*/A) A^k \), given the monotonicity of the bounding function. The boundary between complete evaporation (\( \ln S^* \to -\infty \)) and potential remnant formation (\( \ln S^* \) remains finite) happens at

\[
f = -O\left(\frac{S^*/A \ln(S^*/A)}{p} \right)
\]

with \( p > 0 \) if \( k = 0 \) and even weaker if \( k > 0 \). Thus we arrive at a sufficient condition that the entropy \( S \) vanishes if asymptotically the hidden sink drains at a rate

\[
F \geq 4S^*/A.
\]

(4)

FIG. 2: \( Q - A \) flow diagrams with \( \Delta Q \equiv -(Q/A)^{1+p} A^k \) and \( \Delta A = -4 \). The origin is labelled with 0. Each diagram corresponds to a different set of parameters, with \( p > 0 \) and \( k \leq 0 \) meet our criteria.

In the case of [28], the interaction is of the form \( 4\alpha S^*/A \) with \( \alpha \geq 1 \), and remnants never form.

Clearly, our model is governed by the relation between \( A \) and \( N_i \). To illustrate this, let us apply the routine laid down previously to the properties of a species \( j (N_j \) or the area contribution \( A z_j ) \). The evolution equation of a property \( Q \) is quantified as \( \Delta Q \equiv -(Q/A)^{1+p} A^k \), and the trajectories, as demonstrated in Fig. 2, behave wildly differently depending on \( p \) and \( k \). When \( p < 0 \), \( Q \) diverges toward \( -\infty \), and if \( k > 0 \), \( Q \) is quickly attracted toward a pure function of \( A \), and is no longer an independent parameter. In addition, for \( A z_j \), since the overall evolution \( \Delta A = 4 \) is marginal, a species with \( k < 0 \) must be accompanied by yet another species with \( k < 0 \), such that the net area contribution remains marginal. Thus we may treat \( k \sim 0 \) for \( A z_j \), and categorize species according to \( k \) of \( N_j \) alone: \( k < 0 \) (final burst), and \( k = 0 \) (marginal), where \( \Delta N_j \) and \( \Delta A \) are effectively dimensionless, leading to \( \partial A/\partial N_j = c_j \) where \( c_j \) are constants, i.e.,

\[
A/4 = \sum_i c_i N_i.
\]

(5)

This “extended” Bekenstein’s law allows us to apply the horizon quanta picture to marginal species, each with a different area per quantum \( c_i \). Furthermore, \( c_i \) also indicates the existence of a minimal quanta area density \( S^*/A \), i.e., the inverse of the maximally possible area per quanta \( 1/(4 \max_i c_i) \). As will be demonstrated later, this lower bound is related to the dimensionality of the micro-
scopic degrees of freedom through the statistical analysis.

**Generating function, density of states, and soft hairs**

By construction, all observables in our model are functions of $N_i$. Namely, the connected generating function of BH can be expressed as $\ln Z_B(N_i) \equiv \ln Z_B(\beta, N_i|\beta)$, where $\beta = T_H^{-1}$ and $N_i|\beta$ is $N_i$ under the constraint of $\beta = \sqrt{4\pi A(N_i)}$. While the full expression of $\ln Z_B$ requires quantum gravity, we may qualitatively determine the part related to $\beta$ given our knowledge about the rest of the system. In fact, both the external source and the hidden sector are trivial in terms of the energy content as the former is external and the latter is gravitationally inert. The Hawking radiation alone suggests $\beta$ as the conjugate variable of the microscopic total energy $E$, with an effective action $\Gamma_B(E) = \max_\beta \{\beta E + \ln Z_B(\beta)\}$.

The origin of conjugation is beyond us. We consider it as the thermodynamic conjugate [28], but other explanations are also plausible [23, 30]. $Z_B$ then becomes the ensemble average $\int e^{\Gamma_B(E) - \beta E} dE$, with $\ln Z_B \sim -\beta^2/(16\pi)$ from $\bar{M} \sim \langle E \rangle = -\partial_\beta \ln Z_B$. The Gaussian form of $Z_B$ suggests BH as an effective single-particle state [23, 31].

However, as an assumption of our model, and given its entropic nature, BH should be composite. The missing piece is that the Hawking process not only modifies $M$ but also the amount of quanta $N$, rendering $Z_B$ unattainable again. The only exception happens when a particular energy measurement becomes independent of $N$. For a marginal species $i$ satisfying eq. (5), $\partial N_i M \propto M^{-1} \partial M_i A$ depends only on $\beta$, indicating that the average of the single-quantum energy $\omega_i$ can be utilized to extract the single-quantum density of state $g_i$ through

$$\langle \omega_i \rangle = \frac{A z_i}{2N_i \beta} = \frac{2c_i}{\beta} \equiv -\partial_\beta \int_0^\infty g_i(\omega) e^{-\beta \omega} d\omega. \quad (6)$$

We will restrain ourselves from solving the associated chemical potential $\mu_i$ as we care about is its existence inherited from the conservation of quanta set up before, and thus the capability of expressing most forms of $N_i(\beta)$. By reversing the relation between $c_i$ and $g_i$, we arrive at $g_i = \omega_i^{2c_i-1}$. This is typical for relativistic hairs inside a $2c_i$-dimensional configuration space. For BHs following the original Bekenstein’s law $S^* = A/4$ the observation implies that the BH quanta live on the horizon, a picture usually depicted as the salient feature of BH.

For $k < 0$ (final burst) species, it is not apparent how one would factorize the generating function. Nevertheless, we may still apply eq. (6). By the initial value theorem, there would be a pole at zero frequency inside the density of states. However, we must emphasize that given the non-vanishing area contribution, this zero-frequency pole is just an illusion, and its emergence should be considered as an appeal against an arbitrary factorization. Interestingly, soft hairs [10, 32, 33] are often considered a non-energetic entropy storage, capable of memorizing the evaporation history, as does the zero-frequency pole. Another resemblance between the two is that both of them are non-trivial around zero frequency [23, 34]. In fact, our analysis suggests the non-trivial time dependence as the key to the soft hair proposal. Whether this structure has implication on soft hairs requires further analysis.

Given our blunt force of eq. (6) upon $k < 0$ species, one wonders if we may simply consider the entire BH, with $\langle \omega \rangle = A/(2S^* \beta)$. It surely can be done. Since $S^*/A$ does not increase once it reaches the separatrix (equilibrium), $A/S^*$ should be a non-increasing function of $\beta$, which, as a consequence, leads to a system at most as energetic as the one with dimension $d \leq \max\{|A/S^*|/2 = 2\max c_i$. Furthermore, the constraint $S^*/A \geq 1/4$ at the separatrix ensures that $d \leq 2$. The addition of species for the resolution of the information loss paradox thus reduce $\langle \omega \rangle$, or increase $S^*$ at fixed $\beta$, as expected.

**Non-commutativity of the configuration space**

While the analysis above is tempting, it is dangerous since to accommodate the effect of the Hawking evaporation process, there would be at least one species whose spectrum is unbounded from below. To avoid the runaway situation, the net energy transfer from these species to the others must be finite. Effectively, we have two well-separated subsystems, one containing all negative-mass quanta with total mass $M_n$ and the other containing the rest. As a consequence, after a while, the energy transfer rate from the negative-mass subsystem to the rest (excluding Hawking radiation), denoted as $\rho_n$, has to be negative-definite. By parametrizing $Az_i = 4c_i N_i^{1-x}$, with $x_i < 1$ [40] and $c_i \neq 0$,

$$-\Delta M_n = -\sum_i (4c_i)^{-1} N_i^{x_i} \{\partial N_i M_n(N_i) \Delta(Az_i)/(Az_i)\} = \rho_n + T_H \leq T_H = (16\pi c_i N_i^{1-x}/z_i)^{-1/2}, \quad (7)$$

can be transformed into an inner product between a vector independent of $N_i$ and another one independent of $z_i$. One then applies the separation of variables and obtains the relation between the upper and lower bounds of two vectors along variables $z_i$ as

$$\begin{array}{l}
1/\max \left(\sqrt{\pi c_i/z_i c_i^{-1} \Delta(Az_i)}\right) \geq \sqrt{N_i^{1+x_i}} \partial N_i M_n,
\quad \\
1/\min \left(\sqrt{\pi c_i/z_i c_i^{-1} \Delta(Az_i)}\right) \leq \sqrt{N_i^{1+x_i}} \partial N_i M_n. \quad (8)
\end{array}$$

The lower bound indicates $\partial M_n/\partial N_i \geq 0$ for a system satisfying eq. (4), i.e., without remnants, while the upper bound suggests $\partial M_n/\partial N_i \leq 0$ if $c_i^{-1} \Delta(Az_i) > 0$ at $z_i = 0$. Notice that $\Delta(Az_i) = 0$ at $z_i = 0$ is a trivial fixed point where $N_i$ vanishes indefinitely. For species with $c_i = 0$, they decouple from the external observers and become spurious by Occam’s razor. $\partial M_n/\partial N_i$ thus is positive definite unconditionally. Together, we assert that $\partial M_n/\partial N_i = 0$, i.e., $M_n = 0$ for BHs that would not evolve into remnants.

$M_n = 0$ forbids the negative-mass subsystem from sourcing $A$, and is thus held at the trivial fixed point $z = 0$. We therefore have no choice but to consider $x = 1$ for the negative-mass species, i.e., a constant area contribution $A_n$ of the order of the Planck area. Equipped with that knowledge, we may derive the density of states
for the other half of BH
\[ g(\omega) \propto \omega^{d/2} j_{d/2-1}(\sqrt{-\lambda} \omega), \tag{9} \]
where \(j\) is the spherical Bessel of the 1st kind and \(d\) is, as defined previously, the dimension of the configuration space for the positive-mass subsystem. This is typical for fields in a non-commutative space \[35-38\], which hints at the existence of a microscopically discrete horizon \[39\] \[40\], despite the semi-classical nature of our analysis.

**Remark**. The concept and the framework laid down in this work is extremely general. One may even apply the "hidden sector" concept to the analog black hole model \[11\] \[41\] \[42\], and consider the interaction between the microscopic degrees of freedom in the analog horizon and the analog Hawking radiation as a probe to either the horizon, the radiation, or the expected Hawking radiation partner after detecting the other two. Furthermore, by specializing in a particular mode, more information can be extracted. For example, by applying the analysis of the last paragraph to the model in \[28\], one realized that it only makes sense if the energy spectrum is bounded from both below and above, thus incapable of describing a perfect thermal radiation. This is also what prompted us to consider a more generic model in the first place. In contrast, the additional species in \[14\] \[43\] occupies a fixed area, and is compatible with our analysis. One may also consider our model from the information-theoretic point of view \[7\], by treating quanta as qubits. We have to, unfortunately, leave it as a future work.

DY is supported by the National Research Foundation of Korea (Grant no. 2021R1C1C1008622, 2021R1A4A5031460). HW and KY are supported by Ministry of Science and Technology of Taiwan and the Leung Center for Cosmology and Particle Astrophysics (LeCosPA) of National Taiwan University. HW would also like to appreciate the discussions with Masahiro Hotta, Feng-Li Lin, Wei-Hsiang Shao, and Naoki Watanura.

---

[1] S. D. Mathur, Class. Quant. Grav. **26**, 224001 (2009), 0909.1038.
[2] S. W. Hawking, Phys. Rev. D **14**, 2460 (1976).
[3] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975), [167(1975)].
[4] L. Susskind, L. Thorlacius, and J. Uglum, Phys. Rev. D **48**, 3743 (1993), hep-th/9306069.
[5] R. M. Wald, Phys. Rev. D **48**, R3427 (1993), gr-qc/9307038.
[6] M. K. Parikh, Int. J. Mod. Phys. D **13**, 2351 (2004), hep-th/0405160.
[7] P. Hayden and J. Preskill, JHEP **09**, 120 (2007), 0708.4025.
[8] A. Almheiri, D. Marolf, J. Polchinski, and J. Sully, JHEP **02**, 062 (2013), 1207.3123.
[9] J. Maldacena and L. Susskind, Fortsch. Phys. **61**, 781 (2013), 1306.0533.
[10] A. Strominger, JHEP **07**, 152 (2014), 1312.2229.
[11] P. Chen, Y.-C. Ong, and D.-h. Yeom, Phys. Rept. **603**, 1 (2015), 1412.8366.
[12] M. Hotta, R. Schützhold, and W. G. Unruh, Phys. Rev. D **91**, 124060 (2015), 1503.06109.
[13] D. Carney, L. Chaurette, D. Neuenfeld, and G. W. Semenoff, Phys. Rev. Lett. **119**, 180502 (2017), 1706.03782.
[14] W. G. Unruh and R. M. Wald, Rept. Prog. Phys. **80**, 092002 (2017), 1703.02140.
[15] A. Almheiri, N. Engelhardt, D. Marolf, and H. Maxfield, JHEP **12**, 063 (2019), 1905.08762.
[16] S. Pasterski and H. Verlinde (2020), 2012.03850.
[17] P. T. Chrusciel, J. Lopes Costa, and M. Heusler, Living Rev. Rel. **15**, 7 (2012), 1205.6112.
[18] J. M. Bardeen, B. Carter, and S. W. Hawking, Commun. Math. Phys. **31**, 161 (1973).
[19] W. G. Unruh, Phys. Rev. D **14**, 870 (1976).
[20] J. D. Bekenstein, Phys. Rev. D **7**, 2333 (1973).
[21] J. Hwang, D. S. Lee, D. Nho, J. Oh, H. Park, D.-h. Yeom, and H. Zoe, Class. Quant. Grav. **34**, 145004 (2017), 1608.03391.
[22] W. G. Unruh and R. M. Wald, Phys. Rev. D **52**, 2176 (1995), hep-th/9503024.
[23] H.-W. Chiang, Y.-H. Kung, and P. Chen (2020), 2004.05045.
[24] G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2752 (1977).
[25] E. P. Verlinde, JHEP **04**, 029 (2011), 1001.0785.
[26] S. Weinberg and E. Witten, Phys. Lett. B **96**, 59 (1980).
[27] D. N. Page, Phys. Rev. Lett. **71**, 3743 (1993), hep-th/9306083.
[28] M. Hotta, Y. Nambu, and K. Yamaguchi, Phys. Rev. Lett. **120**, 181301 (2018), 1706.07520.
[29] R. J. Adler, P. Chen, and D. I. Santiago, Gen. Rel. Grav. **33**, 2101 (2001), gr-qc/0106080.
[30] J. S. Cotler, E. E. Flanagan, and R. M. Wald, Phys. Rev. D **93**, 124060 (2016), 1503.06109.
[31] H.-W. Chiang, Y.-C. Hu, and P. Chen, Phys. Rev. Lett. **130**, 151206 (2018), 1811.09175.
[32] E. E. Flanagan, Phys. Rev. Lett. **127**, 041301 (2021), 2012.04930.
[33] A. Connes, Commun. Math. Phys. **182**, 155 (1996), hep-th/9603053.
[34] S. Doplicher, K. Fredenhagen, and J. E. Roberts, Commun. Math. Phys. **172**, 187 (1995), hep-th/0303037.
[35] E. Batista and S. Majid, J. Math. Phys. **44**, 107 (2003), hep-th/0205128.
[36] H.-W. Chiang, Y.-C. Hu, and P. Chen, Phys. Rev. D **93**, 084043 (2016), 1512.03157.
[37] A. Hashimoto and N. Itzhaki, Phys. Lett. B **465**, 142 (1999), hep-th/9907166.
[38] S. Ryu and T. Takayanagi, Phys. Rev. Lett. **96**, 181602 (2006), hep-th/0603001.
[41] W. G. Unruh, Phys. Rev. Lett. 46, 1351 (1981).
[42] J. Steinhauer, Nature Phys. 10, 864 (2014), 1409.6550.
[43] P. Chen and G. Mourou, Phys. Rev. Lett. 118, 045001 (2017), 1512.04064.
[44] A. Almheiri, R. Mahajan, J. Maldacena, and Y. Zhao, JHEP 03, 149 (2020), 1908.10996.
[45] G. Penington, S. H. Shenker, D. Stanford, and Z. Yang (2019), 1911.11977.
[46] Through some manipulations, $x > 1$ maps to $p < 0$, which is forbidden.