Heralded Control of Mechanical motion by Single Spins

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We propose a method to achieve high degree control of nanomechanical oscillators by coupling their mechanical motion to single spins. By manipulating the spin alone and measuring its quantum state heralds the cooling or squeezing of the oscillator even for weak spin-oscillator couplings. We analytically show that the asymptotic behavior of the oscillator is determined by a spin-induced thermal filter function whose overlap with the initial thermal distribution of the oscillator determines its cooling, heating or squeezing. Counterintuitively, the rate of cooling dependence on the instantaneous thermal occupancy of the oscillator renders robust cooling or squeezing even for high initial temperatures and damping rates. We further estimate how the proposed scheme can be used to control the motion of a thin diamond cantilever by coupling it to its defect centers at low temperature.

Over the past decade ground state cooling of mechanical oscillators have paralleled the laser cooling methods for atoms, thus allowing to realize quantum mechanical effects even at microscopic length scales [1]. With this ability these devices can be used in a variety of applications ranging from quantum metrology to quantum information processing [2]. Cooling (micro) nanomechanical oscillators (NMO) by laser induced radiation pressure force has received huge attention both theoretically and experimentally which lead to the observation of ground state cooling of mechanical motion [3], quantum back action [4] and other nonclassical effects like squeezing [5]. Recent advances in solid-state materials hosting single localized spins (emitters) are also shown to be promising candidates both for controlling the dynamics of the oscillator and also induce interaction (entanglement) between distant spin by the dynamical back-action of the NMO [6].

Solid state spins in diamond are robust in terms of their good spin coherence properties, high degree spin control [7] and a well-resolved optical spectrum at low temperatures [8] that allows optical excitation to various levels that can either initialize or readout the electronic spin state with fidelities exceeding 98% [9]. On the other hand, they suffer from weak coupling to external spins, incidents photons [10] and phonons [11]. Even with such weak couplings, projective readout techniques, can be used achieve deterministic nuclear spin state preparation [12], entanglement between solid state qubits mediated by photons [13], transfer and storage of single photon states in nuclear spins [14]. In this work we employ these post-selection techniques to achieve robust control of mechanical modes of a NMO that are weakly coupled to the host spins.

Coupling single spins to the mechanical motion of a NMO has already been demonstrated [11,15,17]. Due to the large thermal occupancy of these modes even at low temperatures, realizing gates between distant spins coupled to a common mechanical mode is a nontrivial task [6]. Hence one needs to cool down or squeeze these modes to a high degree to observe any coherent effects or perform gates between distant spins that are coupled to a common mode of NMO. However, with the extremely small spin-phonon coupling achieving such tasks could be challenging. In other words, to achieve coherent coupling between a spin and a phonon the spin cooperativitity $C_S = g^2 T_2 / \gamma$, should be be greater than unity [18]. This can be possible only if the inverse spin coherence time, $1/T_2$, and the dissipation rate of the oscillator $\gamma$ are smaller than the spin-phonon coupling $g$. In this work we will show that even when $g \leq \gamma$, $T_2$, repetitive projections of the spin to a specific state allows a filtering of unwanted thermal occupancies of the oscillator, thereby allowing us to cool down, heat or squeeze its motional degrees of freedom.

The proposed scheme is based on the conditional evolution of the NMO by repetitively post-selecting the spin state dynamics to which it is coupled. When coupling quantum systems of different Hilbert space dimensions, i.e., a two-level spin with an $N$-level oscillator, a short time evolution that looks like a perturbation on the spin state observables can lead to a dramatic change on the oscillator dynamics. Though, repeatedly finding the spin in a given state appears like an effective freezing of its dynamics (Zeno effect), on the contrary can drive the other system quite far from its equilibrium state [19,20]. In this work we will exactly solve the dynamics of the NMO conditioned on the repetitive measurement of a solid state spin to which it is coupled, and extract a measurement induced nonlinear cooling rate that leads to a rapid near ground state cooling of the oscillator modes.

We shall consider the geometry shown in Fig.1 (a), i.e., a one-sided clamped microcantilever with single spins implanted in it. In the presence of a large magnetic field gradient, these spins (two-level systems) couple to a position-dependent magnetic field i.e., the spins experience a phase shift of their energy states that depends on the position of the cantilever. The coupling between single spin and a given mechanical mode of frequency $\omega_m$ is determined by the zero point motion of the oscillator and the magnetic field gradient [21]. The Hamiltonian describing the dispersive interaction between the spin and the NMO (in units of $\hbar = 1$) is given by

$$ H = \sum_m [\omega_m a_m^\dagger a_m + g_m S^z (a_m + a_m^\dagger)] + \Omega(t) S^x, \quad (1) $$

where $g_m$ is the coupling between the spin $(S)$ and the $m^{th}$ mode of the oscillator $(a_m)$ that has a frequency $\omega_m$. The stroboscopic control of the spin is determined by the time-dependent function $\Omega(t)$. In addition to this unitary coupling...
the spin and the oscillator modes suffer nonunitary decay processes which we will consider in the later part of this paper. Under evolution governed by the above Hamiltonian with a stroboscopic spin control i.e., \( \Omega(t) = \pi \sum_k \delta(t - \tau_k) \), the time-evolution operator takes the simple form

\[
U(t) = D_+(t)|1\rangle\langle 1| + D_-(t)|0\rangle\langle 0|,
\]

where \(|1(0)\rangle\) are the eigenstates of the spin operator \( S^z \) and the multimode displacement operators \( D_{\pm} = \exp\left( \pm \sqrt{\omega} \int \varphi \left( \omega \right) \omega \right) \). The (mode) filter function, \( F_m(t) \), is induced by the applied control which for equally spaced \( \pi \)-pulses intervals (i.e., \( \tau_k = \tau \equiv \pi/\omega \) ) takes the well-known form, \( F_\tau(n_e, \omega) = 4 \tan^2[\omega t/(2n_e + 2)] \cos^2(\omega t/2) \). In the limit of large number of \( \pi \)-pulses, \( n_e \), the filter function \( F_m(t) \approx 2n_\epsilon \delta(\omega - \pi/\tau) \), i.e., coupling to single mode (or to modes with a frequency that are integral multiples of \( \omega \) are only relevant). The total time is now expressed in units of \( \tau \) and the dynamics will be determined by a single dimensionless parameter \( \lambda = 2gn_e/\omega \).

The pulse sequence for implementing the proposed scheme is shown in Fig. 1 (c). Starting from the initial spin state \(|0\rangle\), we prepare a superposition state using the first \( \pi \)-pulse and allow it to evolve for a time \( t = n_e \tau \) with stroboscopic interruptions at intervals \( \tau \). The spin is then projected back to the energy basis \(|1(0)\rangle\) with the final \( \pi/2 \) pulse. We now read-out the state of the spin optically and note the measurement result. Owing to a successful measurement result i.e., finding the spin in a given state (\(|1(0)\rangle\)), the procedure is repeated \( M \) times, as shown in Fig. 1. Since the time \( t \) is arbitrary the probability of obtaining a successful measurement is always below unity. Given this conditional spin state dynamics the evolution of the NMO that is initially in thermal equilibrium is greatly effected which we analyze below.

For an oscillator in its thermal state \( \rho_B = \sum_n e^{-\beta n} \sum \varphi_m a_n^\dagger a_n |n\rangle\langle n| \) with an initial thermal occupancy \( n_\omega = \langle n_|a|^2|n\rangle = \text{Tr}[\rho_B a^\dagger a] \), the effect of the spin-phonon coupling would lead to random displacements governed by the operators \( D \) as shown in Eq. (2). As we are dealing with the displacement operators, it is instructive to work in the coherent basis where the above-mentioned thermal state can be rewritten as

\[
\rho_B(0) = \int P_0(\alpha) |\alpha\rangle \langle \alpha| d^2 \alpha, \quad P_0(\alpha) = \frac{1}{\pi n_\omega} e^{-|\alpha|^2/n_\omega}.
\]

Upon obtaining a successful spin state measurement, the oscillator is projected onto the state

\[
\rho_B(t) = \frac{V \rho_B(0) V^\dagger}{\text{Tr}[V \rho_B(0) V^\dagger]},
\]

where \( V(t) = \frac{1}{2}(D_+ (t) + D_-(t)) \). The success probability for this projection is then simply given by \( \text{Tr}[V \rho_B(0) V^\dagger] \). Now \( \rho_B(t) \) would be the initial state of the oscillator for the next repetition and evolving again for a time \( t \), a successful measurement of the spin will project the oscillator onto the state \( \rho_B(2t) = \frac{V \rho_B(0) V^\dagger}{\text{Tr}[V \rho_B(0) V^\dagger]} \). After \( M \)-successful projections of the spin to its desired state the oscillator can again be expressed in the coherent basis as given in Eq. (3), where the \( \mathcal{P} \)-function is now modified as

\[
\mathcal{P}_M(\alpha) = \mathcal{P}_0(\alpha) G_M^\alpha(\epsilon, \lambda),
\]

where

\[
G_M^\alpha(\epsilon, \lambda) = C_M \prod_{k=1}^N \text{Re} \left[ \exp \left( \frac{\lambda(\epsilon^k - 1)}{\epsilon} \right) \left\{ \alpha e^{ik\epsilon} + \alpha^* e^{-ik\epsilon} \right\} \right].
\]

acts like the spin-induced thermal filter function (see Suppl. Info) by suppressing unwanted excitations in the phase space of the oscillator. In the above equation \( \epsilon \) determines the slight off-resonant driving of the spin i.e., the evolution of the spin is interrupted stroboscopically at intervals of \( \tau = \pi/(\omega - \epsilon) \). and \( C_M \) is determined from the normalization condition of the oscillator state i.e., \( \text{Tr}[\rho_B(M\{t\})] = 1 \). In Fig. 2 (a) - (b) we plot the \( \mathcal{P} \)-function in the complex space of \( \alpha \) which show the initial thermal distribution, the squeezed distribution for resonant driving and a narrowed thermal distribution for off-resonant driving of the spin. These confirm the above described picture of the role of a spin-induced filtering in the oscillator’s Hilbert space to achieve a given task. We also confirm the above analysis by performing exact numerical diagonalization of the dynamics generated by the Hamiltonian given in Eq. (1). These results are shown in Fig. 2 (d) - (f).

The key aspect of this measurement control is to obtain a high probability for finding the spin in a given state \(|\langle 0|\rangle\). To
obtain such high probability, the effective spin-oscillator coupling should satisfy the condition $\lambda^2 n_\omega < 1$. With this condition it is guaranteed that probability of finding the spin close to its initial state is much higher than \(50\%\) during the first evolution cycle i.e., over a time \(t\). This in turn also indicates that for higher temperatures it is better to have smaller effective couplings \(\lambda\) (which is possible by adjusting the number of control pulses \(n_c\)). After the first successful projection of the spin, if the oscillator has also been projected onto a state with smaller thermal occupancy then the above condition guarantees that the success probability for the next heralding event will be much higher than the previous one. As the oscillator gets gradually cooled each heralding event increases the probability for the next one, thus resulting in a cascading behavior as shown in Fig. 2d where an exponential convergence to unity will be observed with either cooling or squeezing the oscillator.

The physical reason for this comes from the preferential basis chosen by the conditional dynamics on the phase space of the oscillator. For resonant driving the displacement operator, \(V(t) = \cos[\lambda(a + a^\dagger)]\), is diagonal in the position basis of the oscillator and hence repetitive projections of the oscillator onto this basis will stabilize the system in one of the eigenstates of the position operator, thereby squeezing it maximally (see Fig. 2b, 2e). On the other hand, with slight off-resonant driving, the displacement operator
\[V(t) \approx \text{Re}(\exp[\lambda(c_0(a + c_0^* a^\dagger))])\]
(which is possible by adjusting the number of control pulses) has both position and momentum components, and this does not allow the oscillator to stabilize in either of the basis. Therefore the system continues to remain diagonal in the energy basis with thermal occupancy decreasing for increasing \(M\) (see Fig. 2c, 2f).

As the initial decay of the thermal occupancy is similar for both resonant and off-resonant driving (see Fig. 2f) we will obtain the ground state cooling rate by analyzing the resonant case as it has a comparatively simpler form for the operator \(V(t) = \cos[\lambda(a + a^\dagger)]\). Using this, and for \(\lambda^2 n_\omega < 1\), one can evaluate \(n_\omega(t) = \text{Tr}[V(t)a^* a V(t) \rho_B(t)] = n_\omega(0)[1 - \lambda^2(2n_\omega(0) + 1)]\). The cooling behavior for \(M\) measurements can then be approximated by (see Suppl. Info)
\[n_\omega((M + 1)t) \approx n_\omega(Mt)e^{-2\lambda^2 n_\omega(Mt)}.\] (7)

From the above equation, one can the effective cooling rate
\[\gamma_M \approx \left(\frac{\lambda^2}{2}\right)\langle a^* a \rangle_M.\] Surprisingly, the cooling rate itself depends on the instantaneous thermal occupancy of the oscillator i.e., higher the initial thermal occupancy higher is the decay (cooling rate). This cooling rate decreases with increasing \(M\), eventually reaching to the single phonon limit, which is \(\frac{\lambda^2}{\omega}\). We further find that the mode occupancy can be reduced on an average by approximately \(70\%\) in each repetition cycle, hence the quantum speed limit of cooling the oscillator to its ground state is possible by \(M \approx 2 \log_2(n_\omega)\) successful spin projections.

The resolved side-band cooling methods generally employed for cooling the NMO [21] will only reduce the oscillators thermal occupancy in unit steps i.e., \(|0\rangle|n\rangle \rightarrow |1\rangle|n - 1\rangle\) over a time-scale of \(\sim 1/g\), indicating that more optical cooling cycles for the spin have to be employed to reduce the oscillator thermal occupancy to half its initial value and \(g > \Gamma\) to ensure cooling. On the contrary, the current method importantly shows that one successful projection of the spin can reduce the thermal occupancy drastically and its success probability is bounded by \(0.5\).

As the spin is measured frequently at time intervals of \(t\), the spin dephasing/decay processes are only relevant for time \(t\). At low (\(\sim 4K\)) temperatures and with periodic \(\pi\)-pulses used in our scheme (Fig. 1c), the non-unitary spin processes become less important. On the other hand, the dissipation of the oscillator plays a prominent role in the cooling procedure. To include the damping effects of the oscillator, \(\Gamma \ll \omega\), into the analysis all we need is to replace \(\epsilon\) in Eq. (5) with \(\epsilon + i\Gamma\) (see Suppl. Info). Under this assumption there are two competitive effects: (i) the spin-induced cooling and (ii) thermalization of
To experimentally implement the above described heralded control of mechanical motion, we will consider a single-side clamped diamond cantilever of dimensions \((l, w, t) = (20, 1, 0.2) \mu m\), with a fundamental frequency of \(\sim 10 \text{ MHz}\) [23], and the zero point fluctuation of \(10^{-14}m\). A magnetic (AFM) tip held at a distance of 10nm from the surface of the diamond producing a field gradient of \(2 \text{ G/nm}\), will result in the spin-phonon coupling of \(g \sim 100 \text{Hz}\) [21]. A Nitrogen Vacancy center in diamond has a spin one (three-level) ground state of which the two energetically low spin states form the two-level spin \(S\). These centers are found at a depth \(>10nm\) and can have long \(T_2^* \sim \mu s\). With dynamical decoupling sequences for example the one shown in Fig. 1(c), the spin coherence time can be extended to \(T_2 > 10\text{ms}\) at cryogenic temperatures (4K). At such low temperatures and in high vacuum environment where the experiments are performed the cantilever can have quality factors \(\sim 10^{9}\) [16].

In conclusion we show here that by post-selecting the spin dynamics the evolution of the NMO coupled to it could be steered far from its equilibrium either towards a cooled or a squeezed state. For ultra weak couplings i.e., \(g < T_2 \Gamma\), where the spin cooperativity is close to zero the heralded spin control method proposed here offers an alternative for cooling the NMO via the single spin. The successive spin state measurements heralds the cooling/squeezing of the NMO. Though probabilistic, the event rate to achieve near ground state cooling is quite high due to the dependence of the cooling rate on the instantaneous thermal occupancy. These methods can be further used to achieve heralded entanglement between distant spins (or perform gates) that are weakly coupled to a common mode of NMO’s.

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