DISCOVERY OF A TIGHT CORRELATION FOR GAMMA-RAY BURST AFTERGLOWS WITH “CANONICAL” LIGHT CURVES

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ABSTRACT

Gamma-ray bursts (GRBs) observed up to redshifts $z > 8$ are fascinating objects to study due to their still unexplained relativistic outburst mechanisms and their possible use to test cosmological models. Our analysis of 77 GRB afterglows with known redshifts revealed a physical subsample of long GRBs with the canonical plateau breaking to power-law light curves with a significant luminosity $L_X^\ast$–break time $T_a^\ast$ correlation in the GRB rest frame. This subsample forms approximately the upper envelope of the studied distribution. We have also found a similar relation for a small sample of GRB afterglows that belong to the intermediate class between the short and the long ones. It proves that within the full sample of afterglows there exist physical subclasses revealed here by tight correlations of their afterglow properties. The afterglows with regular (“canonical”) light curves obey not only the mentioned tight physical scaling, but—for a given $T_a^\ast$—the more regular progenitor explosions lead to preferentially brighter afterglows.

Key words: cosmological parameters – gamma-ray burst: general – radiation mechanisms: non-thermal

1. INTRODUCTION

The detection of gamma-ray burst (GRB) up to high redshifts ($z = 8.2$; Salvaterra et al. 2009; Tanvir et al. 2009), larger than Type I supernovae Ia (SNeIa; $z_{\text{max}} = 1.77$; Riess et al. 2007), makes objects appealing for possible use in cosmology. The problem is that GRBs seem not to be standard candles, with their energetics spanning over seven orders of magnitude. Anyway, several GRB luminosity indicators (Amati et al. 2008; Fenimore & Ramirez-Ruiz 2000; Norris et al. 2000; Ghirlanda et al. 2004; Liang & Zhang 2005, 2006; Ghirlanda et al. 2006; Nousek et al. 2006) and their use to constrain cosmological parameters (Firmani et al. 2006; Liang & Zhang 2005; Liang et al. 2009; Qi et al. 2009; Izzo et al. 2009) have been proposed. Furthermore, Cardone et al. (2009) have derived an updated GRB Hubble diagram using the log $L_X^\ast$–log $T_a^\ast$ (“LT”)$^6$ correlation with five other two-dimensional GRB correlations used by Schaefer (2007). However, the problem of large data scatters in the considered luminosity relations (Butler et al. 2009; Yu et al. 2009) and a possible impact of detector thresholds on cosmological standard candles (Shahmoradi & Nemiroff 2009) have been discussed controversially (Cabrera et al. 2009). Among these attempts, Dainotti et al. (2008) have proposed a way to standardize GRBs as distance indicator with the discovery of the LT anti-correlation, confirmed by Ghisellini et al. (2009) and Yamazaki (2009). The fitted power-law relation is $\log L_X^\ast = \log a + b \cdot \log T_a^\ast$, the constants $a$ and $b$ are determined using the D’Agostini (2005) method.

In this Letter, we study the LT correlation using the extended GRB data set and demonstrate the existence of a physical LT scaling for “canonical” light curves in the GRB rest frame.

The regular light curve afterglows conform rather tightly to this scaling, while the more irregular ones are systematically fainter. A similar correlation is revealed for a subsample of GRB afterglows that belong to the intermediate class (IC). Revealing these physical correlations can help the (still unclear) interpretation of the physical mechanisms responsible for the GRB X-ray afterglow emission and can infer important information about the nature of the emitting source.

2. DATA SELECTION AND ANALYSIS

We have analyzed a sample of all afterglows with known redshifts detected by Swift from 2005 January up to 2009 April, for which the light curves include early X-ray Telescope (XRT) data and therefore can be fitted by a Willingale’s phenomenological model (Willingale et al. 2007). The redshifts $z$ are taken from the Greiner’s Web site http://www.mpe.mpg.de/~jcg/grb.html. We have compared these redshifts with the values reported by Butler et al. (2007) and find that they agree well apart from two cases of GRB 050801 and 060814, but N. R. Butler (2010, private communication) suggested that we should use the Greiner redshifts for those two cases. For original references providing the redshift data see Butler et al. (2007, 2010). Our data analysis, including derivation of $T_a^\ast$ and $L_X^\ast$ (in units of (s) and (erg s$^{-1}$)), respectively for each afterglow, follows Dainotti et al. (2008) and Willingale et al. (2007). The source rest-frame luminosity in the Swift XRT bandpass, $(E_{\text{min}}, E_{\text{max}}) = (0.3, 10)$ keV, is computed from the equation

\[ L_X^\ast(E_{\text{min}}, E_{\text{max}}, t) = 4\pi D_L^2(z) F_X(E_{\text{min}}, E_{\text{max}}, t) \cdot K, \]

where $D_L(z)$ is the GRB luminosity distance for the redshift $z$, computed assuming a flat ΛCDM cosmological model with $\Omega_m = 0.291$ and $h = 0.697$, $F_X$ is the measured X-ray energy flux (in erg cm$^{-2}$ s$^{-1}$) and $K$ is the K-correction for cosmic expansion. Using the Bloom et al. (2001) expression for the

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$^6$ We use the index “$\ast$” to indicate quantities measured in the GRB rest frame in which $L_X^\ast = L_X^\ast(T_a^\ast)$ is an isotropic X-ray luminosity in the time $T_a^\ast$, the transition time separating the afterglow plateau and the power-law decay phases (Dainotti et al. 2008).
K-correction and with \( f(t) \) being the Swift XRT light curve, we have the relation

\[
K F_X(E_{\text{min}}, E_{\text{max}}, t) = f(t) \times \frac{\int_{E_{\text{min}}}^{E_{\text{max}}/(1+z)} E \Phi(E) dE}{\int_{E_{\text{min}}}^{E_{\text{max}}/(1+z)} E \Phi(E) dE},
\]

(2)

where \( \Phi(E) \) is a usual differential photon spectrum assumed to be \( \propto E^{-\gamma_c} = E^{-\beta_c + 1} \), where \( \gamma_c \) and \( \beta_c \) are the photon index and the spectral index, respectively. Willingale et al. (2007) proposed a functional form for \( f(t) \):

\[
f(t) = f_p(t) + f_a(t),
\]

(3)

where the first term accounts for the prompt (the index “p”) \( \gamma \)-ray emission and the initial X-ray decay, while the second one describes the afterglow (the index “a”). Both components are modeled with the same functional form:

\[
f(t) = \begin{cases} 
F_c \exp \left( \frac{t - t_c}{\tau_c} \right) \exp \left( - \frac{t_c}{t} \right) & \text{for } t < T_c \\
F_a \left( \frac{t}{\tau_a} \right)^{-\alpha_a} \exp \left( - \frac{t_c}{t} \right) & \text{for } t \geq T_c 
\end{cases}
\]

(4)

where \( c = p \) or \( a \). The transition from the exponential to the power law occurs at the point \( (T_c, F_c) \), where the two functional sections have the same value and gradient. The parameter \( \alpha_c \) is the temporal power-law decay index and the time \( t_c \) is the initial rise timescale (for further details see Willingale et al. 2007).

For the afterglow part of the light curve, we have computed values \( L_X^a \) (Equation (1)) at the time \( T_a \), which marks the end of the plateau phase and the beginning of the last power-law decay phase. We have considered the following approximation which takes into account the functional form, \( f_a \), of the afterglow component only:

\[
f(T_a) \approx f_a(T_a) = F_a \exp \left( - \frac{T_p}{T_a} \right), \text{ for } t = T_a,
\]

(5)

where we put the time of initial rise, \( t_a = T_p \), because in most cases the afterglow component is fixed at the transition time of the prompt emission, \( T_p \) (for details see Willingale et al. 2007). Then, by applying Equations (5) and (2) in Equation (1), one obtains

\[
L_X^a = \frac{4\pi D_L^2(z) F_X}{(1+z)^{1-\beta_a}}
\]

(6)

where \( F_X = F_a \exp(-\frac{T_p}{T_a}) \) is the observed flux at the time \( T_a \). We have derived a spectral index \( \beta_a \) for each GRB afterglow using the Evans’s Web site (http://www.swift.ac.uk/xrt_curves; Evans et al. 2009) setting a filter time as \( T_a = \pm \sigma_{T_a} \); the \( T_a \) values together with their error bars, \( \sigma_{T_a} \), are derived in the fitting procedure used by Willingale et al. (2007). As mentioned above, the power-law spectrum, \( \Phi(E) \propto E^{-\beta_c + 1} \), was fitted with the model “phabs*phabs*pow” providing the X-ray spectral index, \( \beta_a \). The first absorption component is frozen at the Galactic column density value obtained with the NH FTOOL\textsuperscript{7} and the second is the “phabs” component with the redshift frozen at the value reported in the literature. For further details of the spectral fitting procedure see Evans et al. (2009). The light curves used for the analysis are the same as used in Evans et al. (2009), but binned by us in a different way.

\textsuperscript{7} http://heasarc.gsfc.nasa.gov/heasoft/tools

For some of the derived points \( (L_X^a, T_p^a) \), the error bars are large, indicating that the canonical light curve does not fit the observed light curve well. We have decided to include such cases in the analysis to treat the whole sample in a homogeneous way. Even if points with the largest—a few orders of magnitude—error bars have no physical meaning, they carry information about the light curve irregularity (deviation from the considered model) or an insufficient amount of observational data for precise fitting.

A choice of the Willingale model as a representation for the X-ray GRB light curves allows us to use a homogeneous sample of events to study physical correlation in a statistical way. Let us point out that the fitting procedure can yield values of \( T_a \) within the gap between the end of the Burst Alert Telescope (BAT) and the beginning of XRT observations, like in the case of GRB 050318. One can note that several authors fit the afterglow part of the light curve without modeling the prompt emission light curve. Thus, they can obtain a nearly perfect power-law fits in cases where the Willingale model fitting finds a short afterglow plateau phase.

To analyze how the accuracy of fitting the canonical light curve (Equations (3) and (4)) to the data influences the studied correlations, we use the respective logarithmic error bars, \( \sigma_{L_X^a} \) and \( \sigma_{T_a^a} \), to formally define a fit-error parameter \( u \equiv \sqrt{\sigma_{L_X^a}^2 + \sigma_{T_a^a}^2} \), as measured in the burst rest frame. This definition is used to distinguish the canonical-shaped light curves from the more irregular ones, perturbed by “secondary” flares and various non-uniformities. The symmetric error bars quoted in the Letter are computed with the method of D’Agostini (2005) that takes into account the hidden errors and thus gives greater error estimates than the ones obtained with the Marquardt–Levenberg algorithm (Marquardt 1963).

Our analyzed sample of 77 GRBs from the redshift range 0.08–8.26 includes afterglows of 66 long GRBs and 11 GRBs whose nature is debated. The IC between long and short GRBs described by Norris et al. (2006) as an apparent (sub)class of bursts with a short initial pulse followed by an extended low-intensity emission phase. Our long GRB sample also includes eight X-ray flashes (XRFs; 060108, 051016B, 050315, 050319, 060121B, 080330, 080916C, 090518B). XRFs are scattered within the long GRB distribution in Figure 1, providing further support to a hypothesis that both these phenomena have the same progenitors (Ioka & Nakamura 2001). To study physically homogeneous samples, we decided here to analyze the subsamples of 66 long GRBs (including XRFs) and of 11 IC ones separately.

3. THE RESULTS

The obtained “\( L_X^a \) versus \( T_p^a \)” distributions for long GRBs (Figure 1\textsuperscript{8}) and for a smaller sample of IC GRBs (Figure 2) clearly demonstrate the existence of significant LT correlations, characterized in this Letter by the Spearman correlation coefficient, \( \rho \), a non-parametric measure of statistical dependence between two variables (Spearman 1904). From a visual inspection of Figure 1 and the analysis discussed later in Figure 3, one can note that the lowest error events concentrate in the upper part of the distribution, forming a highly correlated subsample of the full distribution. To visualize this effect, we decided to arbitrarily select eight points with the smallest errors to define

\textsuperscript{8} See the data table for all long and IC GRBs at http://www.oa.uj.edu.pl/M.Dainotti/GRB2010.
our limiting upper envelope subsample, \( u < 0.095 \), see the inset panel in Figure 1.

For the full sample of 66 long GRBs, one obtains \( \rho_{LT} = \rho(\log L_x, \log T_a) = -0.68 \) and a probability of occurring such correlation by chance within the uncorrelated sample \( P = 7.60 \times 10^{-9} \) (cf. Bevington & Robinson 2003). If we remove a few large error points by imposing a constraint \( u < 4 \), we have a limited sample of 62 long GRBs presented in Figure 1, with \( \rho_{LT} = -0.76 \), \( P = 1.85 \times 10^{-11} \), and the fitted correlation line parameters \( a = 51.06 \pm 1.02 \) and \( b = -1.06^{+0.27}_{-0.28} \), while for the upper envelope sample we obtain, respectively, \( \rho_{LT} = -0.93 \), \( P = 1.7 \times 10^{-2} \), \( a = 51.39 \pm 0.90 \), and \( b = -1.05^{+0.19}_{-0.20} \).

In Figure 2, we present data for all IC GRBs. One may note that their afterglows are characterized with the values of \( T_a^* \) in the upper times range of the long GRBs. The IC GRBs in the \( u < 4 \) sample—050724, 051221A, 060614, 060502, 070810, 070809, 070714 (Norris & Bonnell 2010), 060912A (Levan et al. 2007)—follow a similar LT relation as the long ones. A formally computed correlation coefficient for these eight GRBs is \( \rho_{LT} = -0.66 \). This result suggests the existence of another steeper LT correlation for IC GRBs as the one obtained for long GRBs, with different fitted parameters \( a = 52.57 \pm 1.04 \) and \( b = -1.72^{+0.22}_{-0.21} \). The plotted fit line is below the analogous \( u < 4 \) sample of long GRBs showing that the IC GRBs and the normal long ones behave differently, but a limited number of available IC GRBs inhibit us to make a strong statement in that matter.

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\footnote{One may note that the presented fitted slope is different from the slope range quoted in Dainotti et al. (2008), because in the previous paper, in an attempt to reduce the intrinsic scatter in the correlation, the authors limited the sample to the GRBs with \( \log L_x^* > 10^{45} \) erg s\(^{-1}\) and time parameter \( \log T^*_a < 5 \), with a possible resulting bias in the fitted correlation parameters.}

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Figure 1. \( L_x^* \) vs. \( T_a^* \) distribution for the sample of 62 long afterglows with \( u < 4 \), with the fitted correlation line in black. The upper red line, fitted to the eight lowest error (red) points, forms approximately an upper envelope of the full distribution. The upper envelope points with the fitted line are separately presented in an inset panel.

Figure 2. \( L_x^* \) vs. \( T_a^* \) distribution for the sample of 11 IC GRBs. For the picture clarity the three points with very high errors bars (\( u > 4 \)) are shown without error bars. The fit dashed line is presented for the eight points with indicated error bars, for \( u < 4 \). Additionally, both fit lines for long GRBs from the Figure 1 are provided for a reference.

To study the fit error systematic of GRB afterglows, we show below, in Figure 3, how the limiting upper value for \( u \) in the analyzed sample, i.e., how selecting the afterglows with increasing precision of \( L_x^* \) and \( T^*_a \) fits, influences the LT correlation. We present changes of the \( \rho_{LT} \) converging—with decreasing \( u \)—toward a nearly linear LT relation, as observed for our upper envelope sample. In the figure, e.g., we have 62 long GRBs for \( u = 4 \), 33 GRBs for \( u = 0.3 \), 19 GRBs for \( u = 0.15 \), 13 GRBs for \( u = 0.12 \), and 8 GRBs left for our limiting \( u = 0.095 \). A presented accompanying systematic shift upward of the fitted correlation—as measured in the middle
of the distribution as \( \log a - 3.0 \cdot b \) (the fitted correlation line at an arbitrarily selected \( \log T_{u}^* = 3.0 \))—with decreasing \( u \), proves that the limiting \( u < 0.095 \) subsample forms the upper part, the brightest afterglows in the LT distribution. This regular trend allows us to conclude that the subclass of all long GRBs with “canonical” afterglows forms a well-defined physical class of sources exhibiting high correlation of their afterglow parameters. Presence of GRBs with light curves deviating from the Willingale et al. (2007) model increases the scatter in the LT distribution, with larger error points distributed preferentially below the small error ones.

We expect that having a tight LT correlation for canonical light curves, suggesting a precise physical scaling to exist between GRBs with different luminosities, should be accompanied by a regular change of the fitted spectral indices \( \beta_a \), if any such changes occur beside a scatter due to measurement errors. In fact, we find the existence of \( \beta_a - \log T_{u}^* \) (and, of course, \( \beta_a - \log L_X^* \)) correlation, see Figure 4, for a sample of small \( u < 0.095 \) points, where \( \rho_{\beta_a T} \equiv \rho(\beta_a, \log T_{u}^*) = 0.74 \), but a non-negligible probability to occur by chance from an uncorrelated distribution, \( P = 1.0 \times 10^{-4} \). It suggests that the \( \beta_a \) tends to increase for larger \( T_u^* \): the fitted relation reads \( \beta_a = 0.43 + 0.19 \times \log T_{u}^* \). For large limiting \( u = 4 \), this correlation becomes weak with \( \rho_{\beta_a T} \sim 0.16 \). A warning should be considered. Since we determine the value of \( \beta_a \) with a filter time \( T_a \pm \sigma_T, \) the measurement systematics could be introduced to the data if \( \beta \) varies with time in real objects. To evaluate such a possibility, we computed correlations \( \rho(\beta_a, \log T_u) \) using the observer measured times \( T_u \), without a \( z \)-correction, the ones used in real \( \beta_a \) measurements. However—having a large scatter of \( \beta_a \) values at all \( T_u \)—we do not observe any increase or systematic change of correlations as compared to the ones using \( T_u^* \) times. This test neither excludes nor supports systematics in the data, leaving the possibility that the observed effect could be real.

To preliminarily verify if any redshift systematics exists in the GRB afterglow distribution, we have also studied the correlation \( \rho_{Lz} \equiv \rho(\log L_X^*, z) \). We note a positive coupling, \( \rho_{Lz} = 0.53 \), for the full long GRB sample, but—in contrast to the \( \rho_{\beta T} \) distribution discussed above—the \( \rho_{Lz} \) values do not have an increasing trend, fluctuating between 0.36 and 0.61 for smaller \( u \) subsamples. For our upper envelope sample \( \rho_{Lz} = 0.55 \), but it is accompanied by a large random scatter of \( (\log L_X^*, z) \) points. These features do not support a clear significant redshift evolution of the GRB afterglow luminosity distribution, but the issue should be studied in more detail (M. G. Dainotti et al. 2010, in preparation). Let us also note that our limiting upper envelope subsample includes GRBs with redshifts reaching a maximum value of “only” 2.75, while the most distant GRB with \( z = 8.26 \) disappears from the analyzed sample after decreasing \( u \) below 0.25.

On the other hand, to verify if the observed effect of higher LT correlation in the upper envelope is not a systematic effect due to higher photon statistics in brightest afterglows, we compared distribution of the observed afterglow fluxes, \( F_X \), for the upper envelope sample with respect to the fully analyzed GRB sample, including also the IC GRBs (Figure 5). We find that the upper envelope observed flux values span more than three orders of magnitude and are mixed in this range with other points, in the upper part of the flux distribution. The IC GRBs have the same behavior as the long GRBs, but they are on average less luminous. Thus, the \( L_X^*(T_u^*) \) and \( T_u^* \) small errors result preferentially from the smooth light curve shapes, allowing precise fitting to the considered “canonical” shape, not due to a higher observed photon statistics. We note in the considered distribution that the upper envelope points are preferentially in its lower part, with the observed systematics resulting from
the LT anti-correlation (for these GRBs, fluxes are measured at larger fitted $T_\alpha^*$), but possibly also influenced by the mentioned weak $L \sim z$ correlation in the sample.

4. SUMMARY

In the presented analysis, we discovered that the afterglow light curves that are smooth and well fitted by the considered canonical model belong to most luminous GRBs forming the well-correlated upper part of the $(\log T_\alpha^*, \log L_X^*)$ distribution. The GRB cases with appearing flares or non-uniformities of the light curves exhibit a trend to have lower luminosities for any given $T_\alpha^*$. We also noted the possible correlation of the X-ray spectral index $\beta_\alpha$ and the time $T_\alpha^*$, which, together with the LT correlation, provide new constraints for the physical model of the GRB explosion mechanism. Let us also note that the revealed tight LT correlation, if supported with larger statistics, could be a basis for a new independent cosmological test (Cardone et al. 2010).

An LT correlation for the independently analyzed (small) sample of IC GRBs is also revealed. It is different from the long GRBs, with a higher inclination of the fitted correlation lines and its luminosity normalization below the one for long GRBs. It provides a new argument for a separate physical reality of the postulated IC GRB subclass. Any future attempt to study relations between various GRB properties should involve, in our opinion, a separation of the IC GRBs from the long ones, to limit analysis to physically homogeneous subsamples (like our upper envelope one). A simple increase in the studied GRB sample with a mixed content may smooth out any existing relation.

We do not intend to discuss here the consequences of these findings for GRB physical models. Let us simply note that the LT relation is predicted by the models of Cannizzo & Gehrels (2009), Dall’Osso et al. (2010), and Ghisellini et al. (2009), proposed for the physical GRB evolution in the time $T_\alpha$. The Cannizzo & Gehrels (2009) model predicts a steeper correlation slope (3/2) than the observed one ($\approx 1$), which on the other hand is in a good agreement with the model of Yamazaki (2009).

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