Does the QCD plasma contain propagating gluons?

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Comparison of two appropriately chosen screening masses of colour singlet operators in the pure glue QCD plasma indicates that at sufficiently high temperature it contains a weakly-interacting massive quasi-particle with the quantum numbers of the electric gluon. Still in the deconfined phase, but closer to \( T_c \), the same mass ratio is similar to that at zero temperature, indicating that the propagating modes are more glueball-like, albeit with a lower scale for the masses. We observe a continuity between these two regimes.

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With the RHIC fully operational and busy taking data, certain questions about the treatment of the QCD plasma have become urgent. One of the most basic is about the modes of excitation in a plasma: does it contain weakly interacting quark and gluon quasi-particles, or some more complicated collective excitations? We show here that lattice computations yield detailed answers to this question.

The Debye screening mass, \( m_D \), in pure gauge QCD, at temperature \( T > T_c \) has an expansion in the strong coupling \( g \) of the form:

\[
\frac{m_D}{T} = g - \frac{3}{4\pi} g^2 \log g + b g^2 + c g^4 + \mathcal{O}(g^4)
\]

where the leading term \( g \) is well known, the second term has been extracted in perturbation theory and the non-perturbative coefficients \( b = 2.46 \pm 0.15 \) and \( c = -0.49 \pm 0.15 \) have been computed in a lattice simulation of dimensionally reduced QCD. The QCD coupling \( g \) has to be evaluated at the scale \( 6.742T \). Since \( T_c/\Lambda_{MS} = 1.15 \pm 0.05 \), \( g = \mathcal{O}(1) \) for \( T/T_c \simeq 3 \) or less, and the error due to the neglect of the \( g^4 \) term is about 35\%, where \( d \) is the coefficient of this term. As a result, it becomes difficult to validate perturbation theory by comparing this screening mass to lattice data.

In this work we address a prior question: at temperatures of interest to current and near-future experiments what mediates the longest correlations in the plasma? We examine this by comparing screening masses, \( m_i \), obtained from correlations of two gauge invariant operators with different symmetry properties. They are chosen in such a way that one would be obtained by the exchange of two electric gluons while the other would need three, if indeed such gluons are the lightest excitations in the plasma. As a result, the two screening masses would be roughly in the ratio 3/2.

Since screening masses involve the transfer matrix in a spatial direction, they are classified by the symmetry group of the lattice sliced perpendicular to a spatial direction. For the thermodynamics of a 3+1 dimensional field theory realised on a hypercubic Euclidean lattice, since the Euclidean time direction is distinguished from the spatial directions, this is the group \( D_4 \times Z_2(T) \times Z_2(C) \), where \( D_4 \) is the tetragonal group, \( Z_2(C) \) is the charge-conjugation symmetry of the fields, and the \( Z_2(T) \) factor arises from the symmetry \( t \leftrightarrow -t \). The transfer matrix can be block diagonalised in irreps of this group.

Extensive thermodynamic quantities depend only on the lowest eigenvalue of the transfer matrix, and the phase structure is determined by the degeneracies and symmetries of the corresponding eigenvectors. In high temperature QCD, these are the scalar, \( A_1^{++} \) irrep. In the confined phase, the unique ground state is scalar under the \( Z_3 \) symmetry of the center of the colour \( SU(3) \) group. In the deconfined phase the ground state is three-fold degenerate and corresponds to the three irreps of \( Z_3 \). On any finite lattice the degeneracy is lifted by an exponentially small quantity due to tunnelings between these states. In the \( A_1^{++} \) sector, therefore, an unphysical small “tunneling” mass, \( m_T \), may dominate the screening. We show later that we can control this and obtain the correct physical screening mass.

Since the link variable, \( U \sim \exp[\int dx i g A] \sim \exp[i a g A] \) (here \( a \) is the lattice spacing), one can turn this around and define a lattice gluon field of momentum \( k \) by the relation

\[
A_\mu(k) = i \sum_x e^{ik \cdot x} \left[ U_\mu(x) - U_\mu^+(x) - \text{Im} \, \text{Tr} U_\mu(x) \right],
\]

where the sum is over all lattice sites in a slice, the components of \( k \) run over the set \( 2\pi l/N \) where \( 1 \leq l \leq N \) and \( N \) is \( N_t \) for the temporal momenta and \( N_s \) for the spatial momenta (we assume an \( N_t \times N_s \) lattice). This definition gives an element of the \( SU(N) \) algebra which goes over into the continuum definition of the colour octet gluon.
field in the limit of zero lattice spacing. The electric gluon $A_1(0)$ is in the $A_2^-$ irrep. Since $A_2^- \otimes A_2^- = A_1^{++}$ and $A_2^- \otimes A_2^- \otimes A_2^- = A_2^-$, screening correlators in the colour singlet $A_1^{++}$ and $A_2^-$ sectors would be dominated by two and three electric gluon exchange respectively. We discuss the influence of the magnetic sector later.

In this work we use two classes of $A_1^{++}$ operators—Wilson loops, specifically several linear combinations of the plaquette and the planar 6-link loop (sometimes called the fenster), and the trace of the real part of the Wilson line. For the $A_2^-$ operator we use the imaginary part of the trace of the Wilson line. The zero momentum projection is obtained as usual by summing over all sites in the slice. Since the Wilson lines are non-trivial under $Z_3$ transformations, they are evaluated in the phase where the expectation value is real.

Due to the $Z_3$ symmetry of the vacua, there are only two tunneling masses of relevance—one in the vicinity of $T_c$ due to tunneling between the disordered state and any of the ordered states, and the other for all $T > T_c$ due to tunneling between any two of the ordered states (the latter are relevant only to operators which are non-trivial under $Z_3$). Each mass has very specific dependence on the volume, $V = a^3N_s^2N_t$, of a slice orthogonal to the direction of propagation—

$$m_T(V) = \left(\frac{C}{V}\right) \exp(-\sigma V), \quad (3)$$

where $C$, $\alpha$ and $\sigma$ are constants. In a $d$-dimensional scalar theory a one-loop computation gives $\alpha = d/2$ [11]. Tunneling arises when, on a finite system, simulations start exploring the non-Gaussian part of the free energy away from local minima. Clearly, an appropriate correlation function can give $m_T$ only when the order parameter distribution shows multiple peaks. A consequence of eq. (3) is that one can perform a finite size scaling study to check whether the lowest screening mass obtained is a tunneling mass.

Details of our runs with the pure gauge Wilson $SU(3)$ action are summarized in Table I. The critical coupling, $\beta_c$, and its shift on finite lattices, is known for $N_t = 4$ with high precision [11, 3]. Since the finite-size shift of $\beta_c$ on the smallest lattice, $N_s = 16$, is less than 2 parts in $10^3$, the temperature scale is known with high precision at $T_c$. It is also known at similar precision at 1.5$T_c$, 2$T_c$ and 3$T_c$ from measurements with $N_t = 6, 8$ and 12. At other points the temperature scale is interpolated through the QCD beta function and has errors, which are indicated in the table. Loop operators are measured at five levels of single link fuzzing [6]. Cross correlations between all loops in the same irrep are measured and the lowest screening mass obtained by a variational procedure. Other details of measurements and analysis remain as in [6].

At $T = T_c$, the statistics collected are large enough that tunnelings between the deconfined and confined phases occur many times, as do those between different deconfined phases. As a result, it is only to be expected that the lowest screening mass that we can extract in the $A_1^{++}$ sector is the tunneling mass. Evidence for this is the good fit to eq. (3) shown in Figure 1; the fit gave $\chi^2 = 1.005$ per degree of freedom.

To stabilise the pure phases we move away from $T_c$. The distance from criticality, $\Delta \beta = |\beta - \beta_c|$, needed to

| $N_s$ | $\beta$ | $T/T_c$ | statistics |
|------|---------|---------|------------|
| 24   | 5.6500  | 0.89 (1)| 11370      |
| 24   | 5.6800  | 0.97 (1)| 12080      |
| 16   | 5.6908  | 1.00    | 20000      |
| 20   | 5.6918  | 1.00    | 20000      |
| 24   | 5.6920  | 1.00    | 20000      |
| 16   | 5.7010  | 1.02 (1)| 10000      |
| 20   | 5.7010  | 1.02 (1)| 10000      |
| 24   | 5.7010  | 1.02 (1)| 20002      |
| 24   | 5.7100  | 1.04 (1)| 10160      |
| 24   | 5.7200  | 1.07 (1)| 18010      |
| 24   | 5.8000  | 1.27 (2)| 32220      |
| 24   | 5.8941  | 1.50    | 15130      |
| 24   | 6.0625  | 2.00    | 31350      |
| 24   | 6.3500  | 3.00    | 31630      |

FIG. 1: $A_1^{++}$ screening masses at $T_c$ and their dependence on the lattice size. The lowest mass (unfilled circles) is a tunneling mass, as evidenced by the good fit to the form in eq. (3). Also shown are estimates of the second variational mass (filled circles) and a comparison with a measurement of the lowest mass at $T/T_c = 0.97$ on a $4 \times 24^3$ lattice (horizontal band). Data for $N_s = 8$ is from [6].
remain in a single phase for arbitrarily long runs depends on \( N_s \): \( \Delta \beta \) can decrease exponentially with \( N_s \). In the confined phase we have estimated \( m(T_c) \) by a measurement with \( \Delta \beta = 0.0125 \), corresponding to a \( 3 \pm 1\% \) shift of \( T \) below \( T_c \). A \( 4 \times 24^3 \) lattice was observed to stay in the confined phase throughout the run. The second variational level was seen to correspond to the screening mass in this phase. Our measurement of the \( A_{1}^{++} \) screening mass at \( T/T_c = 0.97 \) gives \( m(T)/T_c = 3.41 \pm 0.08 \) (at \( T/T_c = 0.89 \) we have \( m(T)/T_c = 3.52 \pm 0.07 \)).

In comparison, a zero temperature measurement of the scalar glueball mass at a similar lattice spacing gives \( m(T = 0)/T_c = 3.93 \pm 0.05 \) \cite{13}. At \( T = 0 \), glueball masses depend strongly on the lattice spacing, whereas mass ratios are less sensitive. In this phase, we may then expect that the ratio \( m(T)/m(T = 0) \) may scale better than the ratio \( m(T)/T \) as the lattice spacing goes to zero.

Our measurements show that \( m(T)/m(T = 0) \approx 0.8 \) in the scalar channel near \( T_c \), in agreement with recent measurements from temporal correlators \cite{14}.

Another method of extracting the physical correlation length is to pick out the configurations in which the whole lattice is in a single phase. We do this through the distribution of the action density, which has peaks corresponding to each of the phases. By restricting measurements of correlation functions to configurations with some \( S > S_{\text{cut}} \), suitably chosen, we can isolate the deconfined phase and measure the physical screening mass. This procedure on the largest lattice gave \( m(T_c)/T_c \approx 1 \) in the deconfined phase. However with increasing \( S_{\text{cut}} \) statistics become poorer; consequently, errors increase rapidly, and it is hard to quote a more precise value.

We worked in the deconfined phase by taking \( \Delta \beta \geq 0.0085 \) for \( N_s = 16, 20 \) and 24, corresponding to moving off from \( T_c \) by \( 2 \pm 1\% \) or more. None of our \( T > T_c \) runs showed any tunnelings between the different ordered states except the run at \( T/T_c = 1.02 \) on the \( 4 \times 16^3 \) lattice, where we found one tunneling event between two deconfined vacua (which forced us to use only the correlations of loop operators here). Within the precision of our measurement, the screening masses at \( T/T_c = 1.02 \) are independent of the lattice size: an indication that they are not tunneling masses. For all our measurements of the \( A_{1}^{++} \), shown in Fig. 3, the masses extracted from the Wilson loops and those from the Wilson line agree at the 95% confidence level.

The rough agreement between the measurement at \( 1.02T_c \) and that at \( T_c \) in the deconfined phase, displayed in Figure 2, should also be noted. We also draw attention to the feature that the screening masses, when expressed in units of \( T \), dip near \( T_c \). If the dip persists, then it cannot be understood in the context of the perturbation theory leading to eq. (1). An attempt to capture this effect in a model has been made in \cite{6}. We plan a more detailed study in both phases of the region near \( T_c \).

In \cite{2} screening masses have been extracted from the free energy change due to the addition of a static quark pair in the plasma, i.e., the logarithm of the point-to-point Wilson-line correlation. Our measurements of \( m(T)/T \) for the scalar in the deconfined phase are completely compatible with their results. This agreement is non-trivial since we extract the screening mass from zero-momentum screening correlators.

![FIG. 2: \( A_{1}^{++} \) and \( A_{2}^{-} \) screening masses as a function of temperature on \( 4 \times 24^3 \) lattices. Note the discontinuity in the \( A_{1}^{++} \) screening mass at \( T_c \).](image)

![FIG. 3: Ratio of the screening masses in the \( A_{2}^{-} \) and \( A_{1}^{++} \) sectors as a function of temperature.](image)

Our results for the \( A_{2}^{-} \) screening mass are also shown in Figure 3 \cite{2}. In Figure 3 we have displayed the same data in the form of the ratio of the two screening masses. Clearly, already at temperatures a little above \( 1.25T_c \), the ratio is close to the perturbative value of \( 3/2 \), approaching it from above. It has been shown earlier \cite{14} that no mass assignment for magnetic gluons simultaneously satisfies the \( B_{1}^{++} \) and \( B_{2}^{++} \) screening masses, both of which should arise due to the exchange of two such gluons. If these colour singlet channels are treated as collective excitations, then their large screening mass prevents...
them from contributing through a pair exchange to the $A_{1+}^+$ correlators. Even if we choose to disregard this argument, and insist on the existence of magnetic gluons, their screening mass is very high (in this temperature range) simply because of the large screening mass of the $B_{1-}^-$. The simple fact that the $A_{1+}^+$ has the smallest screening mass protects the argument of electric gluon exchange when the ratio $m(A_{2-}^{-})/m(A_{1+}^+)$ is seen to be 3/2 as in Figure 1.

We have not extended our measurement of the $A_{2-}^-$ screening mass below $T_c$ since previous studies have shown that in the confined phase the degeneracies of the screening mass are those expected from the $T = 0$ symmetries of the transfer matrix $\mathcal{M}$. At $T = 0$ the $A_{2-}^-$ is just one component of a very heavy vector glueball ($T_1^-$ in the usual $T = 0$ notation), and the measurement of its screening mass would require large numbers of operators and a significantly larger data set. It is interesting, though, that $T = 0$ measurements, made over a broad range of lattice spacings, show that the ratio of this vector mass to the scalar is roughly 3 $[17]$. The ratio $m(A_{1+}^+)/m(A_{2-}^-)$ for $T > T_c$ seems to interpolate smoothly between the $T = 0$ and the infinite temperature values, crossing over rapidly from one regime to another in the temperature range between $T_c$ and 1.25$T_c$.

It is interesting to recall that the entropy density of the plasma is relatively low for $T/T_c < 1.1$, and begins to saturate only for $T/T_c > 1.25$ $[14]$. While thermodynamics is more sensitive to short distance modes and screening to long-distance modes in the plasma, the two observations can nevertheless be due to a single cause, if the partition function for $T/T_c < 1.1$ were essentially saturated by the colour singlet modes that we have seen. At sufficiently high temperatures, the propagating electric gluons can contribute significantly to the entropy $[13]$. In summary, we have shown that the screening in the scalar ($A_{1+}^+$) sector is discontinuous across the pure QCD transition at $T_c$ and falls abruptly by a factor of nearly 3 in going from the confined to the deconfined phase. At larger temperatures, this screening mass rises faster than linearly in $T$. Above $T_c$, the ratio of the $A_{2-}^-$ to the scalar screening mass interpolates smoothly from a value close to the $T = 0$ ratio to the value 3/2 expected when these correlations are saturated by electric gluon exchange. The agreement with the latter value for $T > 1.25T_c$ indicates the presence of weakly coupled, massive quasi-particles with the quantum numbers of the electric gluon as the lightest excitations of the QCD plasma. Closer to $T_c$, the mass ratio is more similar to its $T = 0$ value. Taken together with the smallness of the entropy density, this indicates that excitations are more nearly glueball-like.

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