Simple and Efficient Realization of Transverse Linear Dark Beams by Azimuthal Phase-Shifted Square Zone Plate

Shahrbanoo Asghari  
Urmia University

Arash Sabatyan (✉️ a.sabatyan@urmia.ac.ir)  
Urmia University  https://orcid.org/0000-0002-4447-0376

Research Article

Keywords: linear, azimuthal, ASZP, FZP

DOI: https://doi.org/10.21203/rs.3.rs-714847/v1

License: ☑️  This work is licensed under a Creative Commons Attribution 4.0 International License.  
Read Full License
Simple and efficient realization of transverse linear dark beams by azimuthal phase-shifted square zone plate

Shahrbanoo Asghari · Arash Sabatyan

Abstract Herein, we are about to introduce a novel, reliable and straightforward method to create controllable dark lines employing an azimuthal square zone plate. As a matter of fact, this diffractive element is a square zone plate whose zones are phase-shifted azimuthally. As we illustrate, one way to construct it is combining a square zone plate and radial grating having period m. Considering its focusing behavior, we came to the result that a dark line surrounded by linear bright zones is generated for odd m, as well as a cross-like dark zone is produced when m is even. Furthermore, we illustrate that the length of the dark lines depends on the grating period. Finally, the simulation predictions are verified by experimental results.

1 Introduction

Fresnel zone plate (FZP) is the well-known diffractive optical element that is circularly symmetric and composed of alternating dark and light zones arranged in a quasi-periodic way. Needless to say that it has found many applications in science and technology, for example, X-ray applications [1], structuring the light [2], metrology [3], vision optics [4], terahertz imaging [5], and beam shaping [6]. Undoubtedly, FZP and FZP-based elements have opened up new, simple, cost-effective, reliable, and impressive techniques for manipulating light by modifying its phase profile within the scalar diffraction regime, as we consider here.

Based on it, a variety of zone plates in different geometries such as fractal [7], square [8], polygonal [9], elliptical [10], cross-like [11], spiral zone plate...
Shahrbanoo Asghari, Arash Sabatyan

Among them, a square zone plate (SZP) is considered over which circular zones are interchanged with square ones. Although circular FZP has a more desirable focusing performance than SZP, being more robust to fabricate SZP concerning fabricating imperfections than an FZP, make them a severe and substantial alternative to FZPs for their integration in optoelectronics, integrated optical circuits, micro and nano-optics [14,15].

Hence, some techniques were proposed to improve its efficiency [16,17]. Furthermore, there have been reported some applications of SZP in, for example, infrared detectors [18], Cassegrain concentrator [19], solar energy systems [20], rectangle multi-array focusing[21], and unique rectangle array of structured beam [22].

Azimuthal modulated diffractive elements have been demonstrated play paramount role in beam shaping and structuring [23,24]. As a consequence, it was examined that imposing the modulation on 1D FZP enable it to generate lateral linear structured beams carrying fractional topological charge [24]. Based on it, we aim to present and study azimuthal modulated square zone plate (ASZP) composed of angular regions having $\pi$ phase difference than their immediate neighbors. At the same time, it is shown that the element is built utilizing a square zone plate and radial grating. Thereafter, scrutinizing its focusing behavior revealed that a single horizontal or double perpendicular dark line is generated at the focus provided that the azimuthal period of the grating is odd or even, respectively. Further meticulous analysis indicates that the length of the line is associated directly with the grating period. On the other hand, the length of dark line diminishes by increasing the period. Last but not least, we deem this method highly influential in improving precision alignment procedures [25–27] and also may conveniently be implemented in guiding cold atoms and particle trapping [28,29].

2 Theoretical Discussions and Computational Result

Having introduced the azimuthal square zone plate, we are about to describe it mathematically. So, one way to define its binary transmittance may be given as

$$t_s(x',y') = 0.5 \left( 1 + \text{Sgn}\left\{ \sin\left[ -\frac{i\pi \text{max}(x'^2, y'^2)}{\lambda f} \right]\right\} \right)$$  

(1)

where $\lambda$, $f$, and $r$ are the wavelength of the incident light, focal length, and the radius of ASZP, respectively. Moreover, $(x',y')$ denotes Cartesian coordinates at the element plane, and $\text{Sgn}$ is the sign function. At the same time, the transmittance of a radial binary grating is proposed as

$$t_g(x',y') = 0.5 \left\{ 1 + \text{Sgn}\left[ \sin(-im\theta) \right] \right\}$$  

(2)
in which \( m \) is an integer. Herein, \( \theta = \tan^{-1}(y'/x') \) is azimuthal angle. Finally, the transmission function of the azimuthal square zone plate may be expressed as

\[
T(x', y') = \text{mod}(t_s + t_g + 1, 2)
\]

where \( \text{mod} \) is the modulo operation.

To visualize the structure of ASZP, some samples of SZP, and ASZPs having constructed with different \( m \) is displayed in the first row of Fig. 1. As a matter of fact, these representations are to figure out that the azimuthal modulation transforms an SZP into angular multi-section element in which the subsequent sections have a \( \pi \) phase difference.

Numerical computations rely on the FFT-based (fast Fourier transform) method\[19\] . To this end, one may rewrite Fresnel–Kirchhoff integral using the FFT convolution theorem as the following \[30\]

\[
I(x, y; z) = \left| F^{-1}\{F[T(x, y)]\}\left\{F\left[e^{i\frac{k}{2}(x'^2 + y'^2)}\right]\right\}\right|^2
\]

where \( F \) and \( F^{-1} \) denote for fast Fourier transform and an inverse fast Fourier transform, respectively. Moreover, \( k = \frac{2\pi}{\lambda} \), and \((x', y')\) is Cartesian coordinate at the ASZP plane. Obviously, the intensity distribution at a distance \( z \) from an ASZP is written as

\[
I(x, y; z) \propto |U(x, y; z)|^2
\]

Until now, we have mathematically introduced the idea of the ASZP, following that, surveying diffractive properties of the element is carried out for samples having side length 6 mm and focal length 500 mm, which are under the illumination of a plane beam with wavelength \( \lambda = 632.8 \text{ nm} \). So, the first study is allocated to the three samples showed in Fig. 1. Invoking sequential regions having \( \pi \) phase difference and the same area, we expect to observe single dark lines for odd \( m \) and two orthogonal dark lines when \( m \) is even, as demonstrated in the second row of Fig. 1. In other words, these dark lines are surrounded by bright areas thus, they may called otherwise lateral optical corridors.

In order to verify the simulation predictions, the corresponding samples in simulations were photolithographically printed as a high-resolution binary mask and installed in the midway of the spatial filter-based telescope set-up lightened by He-Ne laser with the central wavelength 632.8 nm, as depicted in Fig. 2 to verify the simulation results by experiment. Accordingly, the focused intensity of the regarded samples in Fig. 1 was recorded and shown in the third row of Fig. 1 which, resembles the simulation ones.

On top of all that, the effect of the azimuthal parameter (\( m \)) on the produced patterns is studied. To this end, some samples of ASZP modulated by \( m=4, 8, 16 \), and \( m=5, 11, 15 \) are considered. Accordingly, their point spread
function (PSF) is examined theoretically and by experiment. As a result, scrutinizing the structure of ASZP depicted in the first row of Figs. 3 and 4 clarify the point that the number of delicate structures increases. Hence, we expect severe diffractions, which leads to shortening the length of the lateral dark lines. This is clearly shown in the second and third row of Figs. 3 and 4, where the focused intensity for the considered m is displayed. Resultantly, the larger m, the shortened dark lines are created.

In summary, we have demonstrated that imposing azimuthal modulation through a radial grating on a square zone plate enable us to create lateral 1D dark beams. Accordingly, these dark lines were shown to be a single or two orthogonal ones depending on the phase shift period is odd or even, respectively. Further delving into ASZP behavior revealed the point that the great

---

**Fig. 1** Comparison of SFZP with ASFZP. The first row, respectively from left to right, the SFZP ($n = 0$), ASFZP with angular divisions $m = 1$ and $m = 2$. The second row, the transverse intensity distributions corresponding to these zones.
period of the grating causes the finer structures are produced, therefore the
length of the dark lines are reduced owing to severe diffraction from the finer
portions.

3 Declarations

We wish to confirm that there are no known conflicts of interest associated
with this publication and there has been no significant financial support for
this work that could have influenced its outcome.

References

1. W. B. Yun and M. R. Howells, “High-resolution Fresnel zone plates for x-ray applications
by spatial-frequency multiplication,” J. Opt. Soc. Am. A 4, 34–40 (1987).
2. N. R. Heckenberg, R. McDuff, C. P. Smith, and A. G. White, “Generation of optical
phase singularities by computer-generated holograms,” Opt. Lett. 17, 221–223 (1992).
3. C. Pati, G. S. Chen, C. G. Konkola, P. T. Heilmann, R. K. Schattenburg, M. L. Liddle,
and A. Anderson, “Precision fringemetrology using a Fresnel zone plate,” J. Vac. Sci.
Technol. B. 20, 2617–2679 (2002).
4. A. Davison and M. J. Simpson, “History and development of the apodized diffractive
intraocular lens,” J. Cataract. Refract. Surg. 32, 849–858 (2006).
5. S. Wang and X. Zhang, “Terahertz technology: terahertz tomographic imaging with a
Fresnel lens,” Opt. Photonics. News 13(12), 58 (2002).
6. A. Sabatyan and B. Meshginjalam, “Generation of annular beam by a novel class of
Fresnel zone plate,” Appl. Opt. 53, 5995–6000 (2014).
7. G. Saavedra, W. D. Furlan, and J. A. Monsoriu, “Fractal zone plates,” Opt. Lett. 28,
971–973 (2003).
8. J. Alda, J. M. Rico-Garca, F. J. Salgado-Remacha, and L. M. Sanchez-Brea, “Diffractive
performance of square Fresnel zone plates,” Opt. Commun. 282, 3402–3407 (2009).
Fig. 3 The first row reveals the structure of some samples of ASFZP modulated with various $m=4, 8, \text{ and } 16$ from left to right, respectively. The second and third show the corresponding simulation and experimental recorded intensity at the focal plane, respectively.

9. J. Alda and F. J. Gonzalez, “Polygonal Fresnel zone plates,” J. Opt. A 11, 085707 (2009).
10. C. Gomez-Reino, J. M. Cuadrado, and M. V. Perez, “Elliptical and hyperbolic zone plates,” Appl. Opt. 19, 1541–1545 (1980).
11. A. Sabatyan and J. Rafighdoost, “Focusing specification of cross-like Fresnel zone plate,” Optik 126, 4796–4799 (2015).
12. Y. Liang, E. Wang, Y. Hua, C. Xie, and T. Ye, “Single-focus spiral zone plates,” Opt. Lett. 42, 2663–2666 (2017).
13. A. Sabatyan and M. Golbandi, "Petal-like zone plate: long depth bifocal diffractive lens and star-like beam generator," J. Opt. Soc. Am. A 35, 1243-1253 (2018).
14. V. Minin, O. V. Minin, A. Petosa, S. Thirakuone, "Improved zoning rule for designing square Fresnel zone plate lenses," Microwave Opt. Technol. Lett. 49, 276 - 278 (2007).
15. J. Alda, G. Boreman, "Optimization of polygonal Fresnel zone plates," Microwave Opt. Technol. Lett. 50, 536 - 541 (2008).
16. A. Calatayud, V. Ferrando, F. Gimenez, W. D. Furlan, G. Saavedra, J. A. Monsoriu, "Fractal square zone plates," Opt. Commun 286, 42 - 45 (2013).
Fig. 4 The first row reveals the structure of some samples of ASFZP modulated with various $m=5, 11,$ and $15$ from left to right, respectively. The second and third show the corresponding simulation and experimental recorded intensity at the focal plane, respectively.

17. A. Sabatyan, H. Besharatfar, "Focusing behavior of azimuthally structured square zone plate," Opt. Quant. Electron. 48, 383 (2016).
18. F. J. Gonzalez, J. Alda, B. Ilic, and G. D. Boreman, "Infrared antennas coupled to lithographic Fresnel zone plate lenses," Appl. Opt. 43, 33 (2004).
19. S. Li, H. Yang, P. Jiang, W. Caiyang, F. Xu, "Application of square-stacked Fresnel zone plate in Cassegrain concentrator," J. Optoelectron. and Adv. Mat. 20, 575-580 (2018).
20. Y. T. Chen, T. H. Ho, "Design method of non-imaging secondary (NIS) for CPV usage," Solar energy 93, 32 (2013).
21. A. Sabatyan, S. Mortaza Taheri Balanoji, S. Mojtaba Taheri Balanoji, "Exploring focusing aspects of multi-regioned square zone plate," Opt. Quant. Electron. 48, 437 (2016).
22. A. Sabatyan, S. Mojtaba Taheri Balanoji, S. Mortaza Taheri Balanoji, "Square array of optical vortices generated by multiregion spiral square zone plate," J. Opt. Soc. Am. A 33, 1793-1797 (2016).
23. A. Sabatyan, and J. Rafighdoost, "Azimuthal phase-shifted zone plates to produce petal-like beams and ring-lattice structures," J. Opt. Soc. Am. B 34, 919-923 (2017).
24. A. sabatyan, and M. Fatehi, "Azimuthal-segmented linear zone plate: 1D beam structuring and topological charge detecting," J. Opt. Soc. Am. B. 35, 1747-1753 (2018).
25. W. B. Herrmannsfeldt, M. J. Lee, J. J. Spranza, and K. R. Trigger, "Precision Alignment Using a System of Large Rectangular Fresnel Lenses," Appl. Opt. 7, 995-1005 (1968).
26. T. Suwada, M. Satoh, S. Telada, and K. Minoshima. "Experimental investigation on focusing characteristics of a He-Ne laser using circular Fresnel zone plate for high-precision alignment of linear accelerators," Rev. Sci. Instrum. 83, 053301 (2012).
27. L. E. Coyle, M. B. Dubin, J. H. Burge, "Design and analysis of an alignment procedure using computer-generated holograms," Opt. Eng. 52, 084104 (2013).
28. X. Xu, K. Kim, W. Jhe, N. Kwon, "Efficient optical guiding of trapped cold atoms by a hollow laser beam," Phys. Rev. A 63, 063401 (2001).
29. H. Bo, C. Xuemei, Z. Hui, C. Haowei, Z. Qian, R. Zhaoyu, D. Shan, and B. Jintao, "Particle trapping and manipulation using hollow beam with tunable size generated by thermal nonlinear optical effect," Appl. Phys. Express 11, 052501 (2018).
30. A. Sabatyan and L. Elahi, "FFT-based convolution algorithm for fast and precise numerical evaluating diffracted field by photon sieve," Optik 124, 4960–4962 (2013).