Matheme and mathematics. On the main concepts of the philosophy of Alain Badiou.

Maciej Malicki  
Department of Mathematics and Mathematical Economics,  
Warsaw School of Economics,  
mamalicki@gmail.com  

Jan 1, 2014

Abstract  
In this paper, I present a critical discussion of the mathematical arguments employed in the philosophy of event of Alain Badiou. On the basis of “Being and Event” as well as his other writings, I analyze the main notions of his philosophy such as the indiscernible, the undecidable, and the unnameable. The focus of my analysis is both on their mathematical consistency, and their philosophical consequences. I argue that the mathematical approach developed by Badiou is seriously defective, and, as a result, that it cannot serve as an ontological basis for the concept of event as presented in “Being and Event”.

1 Introduction  
“Being and event” is an ontological theory of the effect of event, it is a science of the possibility of a novelty that breaks with being’s determinants. Its existence can never be decided once and for all, because the occurrence of an event is always an intervention: its ultimate source lies in a decision of the subject who is constituted in this act. As a result, a post-evental truth enters being. It is infinite and indiscernible, which means that it cannot be described in terms of authoritative knowledge. It modifies the situation by overruling some elements of knowledge, and making other veridical. Even though a truth cannot be accessed in its entirety, the subject supporting a procedure of truth can decide that something belongs to its realm. These gestures of fidelity give rise to the subject language, which forces statements describing the state of affairs after the occurrence of an event.

These are the core concepts of Badiou’s philosophy of event in a nutshell. However — and that is perhaps the most striking feature of “Being and event” — all of its fundamental
categories have a mathematical character, and main theses of this book derive from subtle and quite technical considerations based on the area of mathematics called set theory. Eventually, the science of event turns out to be the result of an investigation into set theory, understood as ontology, or rather proved to be ontology, i.e. the theory of “being qua being”.

In this paper, I am mostly concerned with consistency of mathematical aspects of Badiou’s analysis, as well as some of its philosophical consequences in certain strictly defined key aspects. It is an important reservation, as a large part of Badiou’s thought can be presented without referring to any technical terms. Many of its categories — such as event itself, subject, fidelity, the void or the excess — are well rooted in the language of contemporary philosophy.

Taking a broader perspective, I am interested in a postulated deep ontological structure behind four fundamental domains of truth which are science, politics, art and love. Of course, we do not need any special doctrine in order to recognize a groundbreaking character of basic examples of event discussed by Badiou — Mallarme’s poetry, relativistic physics, the communist revolution or dodecaphony, as well as some similarities between them — a break involved in their occurrence, a resulting new paradigm, a peculiar indeterminateness of their status. On the other hand, some of these examples do not seem to have that much in common with others, at least on the surface of things. In order to acknowledge the relevance of Badiou’s position, more is needed than a simple rephrasing of well known ideas in a new language: what is required is a convincing argument showing that the ontological theory of event is an autonomous interpretative tool, independent of specific subject matters. Moreover, it is rather hard to imagine an argument of this kind that would not contain a proof that the mathematical part of Badiou’s thought is consistent, and it agrees with his fundamental philosophical aims. A formalized doctrine presented by Badiou not only allows for a realization of this postulate but it explicitly requires it.

Finally, one could describe this article as a contribution to the reflection on the concept of matheme. In “Being and Event”, the matheme is understood as a philosophical idea subjected to rigors of deduction, and opposed to the pre-platonic poem. However, the very term ‘matheme’ comes from late writings of Lacan, which are an important reference point for Badiou. There it denotes mathematical objects — such as the Boromean knot or the Klein bottle — allowing to grasp the structure of the order of the real which extends beyond the reach of language. Under such understanding, a matheme would be a place in philosophy, where mathematics attains an autonomous status, completing philosophical discourse and generating statements that are binding for it.

My assessment of the method employed in “Being and Event” is negative. After providing the first part of Badiou’s doctrine, that is the identification of ontology with set theory, and outlining mathematical foundations of the theory of event, I present a critical discussion of four key concepts of this theory: the indiscernible, the undecidable, the unnameable, and the evental site. The conclusion is that their mathematical structure and its implications for Badiou’s philosophy turn out not to meet expectations. They are ill-designed, and this
leads to mathematical inconsistencies as well as philosophical consequences that contradict Badiou’s intentions. Far from rejecting the philosophical substance of these categories, I claim that their mathematical content may serve as an inspiring metaphor or analogy, but it has clearly defined bounds of meaningful interpretation.

2 Mathematics as ontology

The edifice of Badiou’s philosophy of “being qua being” is founded on a thesis establishing the identity of mathematics and ontology. In order to accomplish this task, Badiou in the very first words of “Being and Event” goes back to the old Parmenidean dispute on the status of the one and the multiple. He decides it by giving priority to the multiple, while considering the one as an effect of operation of imposing a structure upon being, organizing the ‘primordial’ inconsistent multiplicity. Inconsistent means here exactly this: “without any unifying principle”.

Being such as it presents itself — and “no access to being is offered to us except presentations” [BE, p. 27] — is a consistent multiplicity. It is a multiplicity because “if the one is not reciprocal with being, the multiple, however, is reciprocal with presentation.” [BE, s. 28]; also, it is consistent because a presented being is always the result of a certain organizing principle, certain operation of counting-as-one. The inconsistent multiplicity is a subtractive basis of an already structured presentation — it is possible to be discerned only retroactively, as that ‘something’ on which a count-as-one operated [BE, p. 25]. From the perspective of presentation it is only the void because the effect of structure encompasses everything without exception. In this manner, the inconsistent multiplicity “sutures presentation to being” [BE, p. 55]; the void is the name of being [BE, p. 56]. This establishment of a consistent multiplicity resulting from an operation of counting-as-one, a combination of the earlier inconsistent and later consistent multiplicity is called a situation.

It is the inconsistent multiplicity that forms the domain of ontology. “Ontology can be solely the theory of inconsistent multiplicities as such. ’As such’ means that what is presented in the ontological situation is the multiple without any other predicate than its multiplicity.” [BE, p. 28] How is the presentation of being regarded as multiplicity — the presentation of presentation — possible if “being has no structure” (s. 27)? What requirements does such a situation need to satisfy? Firstly, the multiple without one is the multiple that consists only of multiplicities. Secondly, the operation of counting-as-one of the ontological situation cannot be anything more than a collection of conditions through which a multiple can be recognized as multiple. Finally, ontology must be a theory of “the suture of presentation to being”, that is a theory of the subtractive void of presentation. In other words, it should derive the existence of its multiplicities only out of the void.

The science satisfying these postulates is set theory — a mathematical theory of the relation of belonging ∈, together with appropriate axioms: axiom of extensionality, regularity, pairing, union, infinity, power set, and axiom schema of comprehension and replacement.
First of all, the universe of set theory contains only sets, so set theory presents only multiplicities consisting of multiplicities. Moreover, set theory is an axiomatic theory, and axioms are formal rules that do not explicitly define objects they refer to. Badiou says: “an axiomatic presentation consists, on the basis of non-defined terms, in prescribing the rule for their manipulation.” [BE, p. 29] Sets de facto satisfy axioms of set theory, however the axioms themselves do not form a criterion of being a set. In other words, what axioms determine about sets is that they are multiplicities, nothing more. Lastly, the unpresented subtractive void of situation appears in set theory in the form of the empty set, which is a basic building block of all sets. It must be a set because every element presented in the ontological situation is the effect of its operation of counting-as-one, however in fact it is a “multiple’ which is neither one nor multiple, being the multiple of nothing, and therefore, as far as it is concerned, presenting nothing in the form of the multiple, no more than in the form of the one. This way ontology states that presentation is certainly multiple, but that the being of presentation, the that which is presented, being void, is subtracted from the one/multiple dialectic.” [BE, p. 59]

3 Belonging, inclusion, and the impasse of being

Let us take a closer look at the mathematical structure of key categories of “Being and Event”. Ontologically, situations — as it has been already said — take the form of sets: multiplicities whose one is nothing more than the unity of the elements they consists of. Two basic types of relations between sets are belonging, that is being an element of a set, being presented in a situation, and inclusion, that is, being a subset of a set, or being represented in a situation. Even though the latter relation is formally reducible to the former one — $y$ is a subset of $x$ if and only if every element of $y$ is an element of $x$ — their properties differ considerably The tension between belonging and inclusion is of fundamental type in so far as it is where the “impasse of being” arises, opening up a situation to the interventional occurrence of an event. More light can be shed on this tension with the help of the concept of power set.

The power set $p(x)$ of $x$ — the state of situation $x$ or its metastructure [BE p. 94] — is defined as the set of all subsets of $x$. Now, the basic relations between sets can be expressed as the following relations between sets and their power sets. If for some $x$, every element of $x$ is also a subset of $x$, then $x$ is a subset of $p(x)$, so $x$ can be reduced to its power set. Conversely, if every subset of $x$ is an element of $x$, then $p(x)$ is a subset of $x$, and the power set $p(x)$ can be reduced to $x$. Sets that satisfy the first condition are called transitive. For obvious reasons the empty set is transitive; other examples of transitive sets also can be easily found. However, the second relation never holds. The mathematician Georg Cantor proved that not only $p(x)$ can never be a subset of $x$, but in some fundamental sense it is essentially larger than $x$. On the other hand, axioms of set theory do not determine the extent of this difference. Badiou says that it is an “excess of being”, an excess that at the
same time is its impasse.

In order to explain the mathematical sense of this statement, one needs to recall the notion of **cardinality**, which clarifies and generalizes a common understanding what quantity is. We say that two sets \( x \) and \( y \) have the same cardinality if there exists a function defining a one-to-one correspondence between elements of \( x \) and elements of \( y \). For finite sets, this definition agrees with common intuitions: if a finite set \( y \) has more elements than a finite set \( x \) (say, \( y \) has 10 elements and \( x \) has 7 elements), then regardless of the method of assigning elements of \( y \) to elements of \( x \), something (some 3 elements) will be left over in \( y \) — precisely because it is larger. In particular, if \( y \) contains \( x \) and some other elements, then \( y \) does not have the same cardinality as \( x \). This seemingly trivial fact is not always true outside of the domain of finite sets. To give a simple example, the set of all natural numbers \( \mathbb{N} \) contains quadratic numbers, that is numbers of the form \( n^2 \), as well as some other numbers but these two sets have the same cardinality. A correspondence witnessing this fact assigns to every number \( n \) a unique quadratic number, namely \( n^2 \).

Counting finite sets has been always done via natural numbers \( 0, 1, 2, \ldots \). In set theory, the concept of such a canonical measure can be extended to all infinite sets using the notion of **cardinal number**. Without getting into details of their definition, let us say that the series of cardinal numbers begins with natural numbers, which are directly followed by the number \( \omega_0 \) that measures the size of the set of all natural numbers \( \mathbb{N} \), then by \( \omega_1 \), the first uncountable cardinal numbers, etc. The hierarchy of cardinal numbers has the property that every set \( x \), finite or infinite, has cardinality equal to exactly one cardinal number \( \kappa \). We say then that \( \kappa \) is the cardinality of \( x \).

It is not hard to prove that the cardinality of the power set \( p(x) \) is \( 2^n \) if the cardinality of \( x \) is finite and equal to \( n \). However, something seemingly quite paradoxical happens when infinite sets are taken into account. Even though Cantor’s theorem does state that the cardinality of \( p(x) \) is always larger than \( x \) — similarly as in case of finite sets — axioms of set theory never determine the exact cardinality of \( p(x) \) of an infinite set \( x \). Moreover, one can formally prove that there exists no proof determining the cardinality of the power sets of any infinite set. There is a general method of building models of set theory, discovered by the mathematician Paul Cohen, and called **forcing** that yields models, where — depending on construction details — cardinalities of infinite power sets take different values. Consequently, quantity — “a fetish of objectivity” [BE p. 83] as Badiou calls it — does not define a measure of being but it leads to its impasse instead. It reveals an undetermined gap [BE p. 83], where an event can occur — “that-which-is-not-being-qua-being” [BE p. 184].

### 4 Forcing, truth and a place of the subject

In order to make the exposition more accessible, let us consider only the power set of the set \( \mathbb{N} \) of all natural numbers, which is the smallest infinite set, i.e. a countable infinity.
Simplifying things slightly, the argument proceeds as follows. By a model of set theory, we understand a set, where, if we restrict ourselves to its elements only, all axioms of set theory are satisfied. It follows from Gödel’s completeness theorem that as long as set theory is consistent, no statement valid in a model of set theory can contradict logical consequences of its axioms. If the cardinality of \( p(\mathbb{N}) \) was a consequence of axioms of set theory, there would exist a cardinal number \( \kappa_1 \) such that the sentence ‘the cardinality of \( p(\mathbb{N}) \) is \( \kappa_1 \)’ was true in all the models. However, for every cardinal \( \kappa_1 \) the technique of forcing allows for constructing a model \( M \) where the cardinality of \( p(\mathbb{N}) \) is equal to some cardinal \( \kappa_2 \) that differs from \( \kappa_1 \). Then the sentence ‘the cardinality of \( p(\mathbb{N}) \) is \( \kappa_2 \)’ is true in \( M \), while at the same time it contradicts the sentence ‘the cardinality of \( p(\mathbb{N}) \) is \( \kappa_1 \)’. This is impossible assuming consistency of set theory, so for no \( \kappa_1 \) the sentence ‘the cardinality of \( p(\mathbb{N}) \) is \( \kappa_1 \)’ is a consequence of axioms of set theory. Therefore they do not decide the cardinality of \( p(\mathbb{N}) \).

The starting point of forcing is a model \( M \) of set theory — called the ground model — that is countably infinite and transitive (it is a crucial assumption). As a matter of fact, the existence of such a model cannot be proved but it is known that there exists a countable and transitive model for every finite subset of axioms. As far as the logic of the construction is concerned, in particular decidability of the sentences obtained by forcing, this difference does not play any role.

A characteristic subtlety can be observed here. From the perspective of ‘an inhabitant of the universe’, that is if all the sets are considered, the model \( M \) is only a small part of this universe. It is deficient in almost every respect; for example all of its elements are countable, even though the existence of uncountable sets is a consequence of axioms of set theory. However, from the point of view of an ‘inhabitant of \( M \)’, that is if elements outside of \( M \) are disregarded, everything is in order. Some of the sets that are countable in the universe are actually uncountable in \( M \) because in this model there are no functions establishing a one-to-one correspondence between them and \( \omega_0 \). One could say that \( M \) simulates properties of the whole universe.

The main objective of forcing is to build a new model \( M[G] \) based on \( M \), which contains \( M \), and satisfies certain additional properties. The model \( M[G] \) is called the generic extension of \( M \). In order to accomplish this goal, a particular set is distinguished in \( M \) — its elements are referred to as conditions — which will be used to determine basic properties of the generic extension. In case of the forcing that proves undecidability of the cardinality of \( p(\mathbb{N}) \), the set of conditions codes finite fragments of a function witnessing the correspondence between \( p(\mathbb{N}) \) and a fixed cardinal \( \kappa_2 \).

In the next step, an appropriately chosen set \( G \) is added to \( M \) as well as other sets that are indispensable in order for \( M[G] \) to satisfy axioms of set theory. The set \( G \) — called generic — is a subset of the set of conditions and it always lays outside of \( M \). The construction of \( M[G] \) is exceptional in so far as its key properties can be described and proved using \( M \) only, or even the set of conditions, thus without referring to the generic set. It is possible for three reasons. First of all, every element of \( M[G] \) has a name existing
already in $M$, and built using the conditions. Second of all, based on these names one can design a language called the forcing language or — as Badiou terms it — the subject language that is powerful enough to express every sentence of set theory referring to the generic extension. Finally, it turns out that validity of sentences of the forcing language in the extension $M[G]$ depends on the set of conditions: the conditions force validity of sentences of the forcing language in a precisely specified sense. As it has been already said, the generic set $G$ consists of some of the conditions, so even though $G$ is outside of $M$, its elements are in $M$. Recognizing which of them will end up in $G$ is not possible for an inhabitant of $M$, however in some cases the following can be proved: provided that condition $p$ is an element of $G$, sentence $S$ is true in the generic extension constructed using this generic set $G$. We say then that $p$ forces $S$.

In this way, with the aid of the forcing language one can prove that every generic set of the Cohen forcing codes an entire function defining a one-to-one correspondence between elements of $p(\mathbb{N})$ and a fixed cardinal number — it turns out that all of the conditions force the sentence stating this property of $G$, so regardless of what conditions end up in the generic set, it is always true in the generic extension. On the other hand, the existence of a generic set in the model $M$ cannot follow from axioms of set theory, otherwise they would decide the cardinality of $p(\mathbb{N})$.

The method of forcing is of fundamental importance for Badiou’s philosophy. The event escapes ontology; it is “that-which-is-not-being-qua-being”, so it has no place in set theory or the forcing construction. However, the post-evental truth that enters and modifies the situation is presented by forcing in the form of a generic set that leads to an extension of the ground model. In other words, the situation understood as the ground model $M$ is transformed by a post-evental truth identified with a generic set $G$, and becomes the generic model $M[G]$. Moreover, the knowledge of the situation is interpreted as the language of set theory, serving to discern elements of the situation; and as axioms of set theory, deciding validity of statements about the situation. Knowledge understood in this way does not decide the existence of a generic set in the situation nor can it point its elements (this property of truth will be thoroughly discussed later). A generic set is always undecidable and indiscernible.

Therefore from the perspective of knowledge it is not possible to establish, whether a situation is still the ground-model or it has undergone a generic extension resulting from the occurrence of an event; only the subject can interventionally decide it. And it is only the subject who decides about belonging of particular elements to the generic set. A procedure of truth or procedure of fidelity [BE p. 329] supported in this way gives rise to the subject language. It consists of sentences of set theory, so in this respect it is a part of knowledge, nonetheless veridicity of the subject language originates from decisions of the faithful subject. Consequently, a procedure of fidelity forces statements about the situation as it is after being extended and modified by the operation of truth.
5 Mathemes of the undecidable and of the evental site

According to Badiou, an undecidable truth is located beyond boundaries of authoritative claims of knowledge. At the same time, undecidability indicates that truth has a post-evental character: “the heart of the truth is that the event in which it originates is undecidable” [BE, p. 221]. Badiou explains that, in terms of forcing, undecidability means that the conditions belonging to the generic set force sentences that are not consequences of axioms of set theory. However, one also needs to answer the question about the role axioms play in the structure of historical situations. If in the domains of specific languages (of politics, science, art or love) the effects of event are not visible, “Being and Event” is an empty exercise in abstraction: even science — perhaps excluding some entirely formalized areas of theoretical physics — let alone art or love — cannot for obvious reasons be exhaustively described in terms of the sole relation of belonging. Anyway, it is doubtful that — to consider just one example — the status of the French revolution is different from the status of the absolute monarchy preceding it as far as the axioms of set theory are concerned. In other words, most likely either all historical facts are decidable or none of them is. Both possibilities lead to a trivial notion of the undecidable.

Judging by numerous examples discussed by Badiou, it seems that he distances himself from such a narrow interpretation of the function played by axioms. He rather regards them as collections of basic convictions that organize situations, the conceptual or ideological framework of a historical situation. For example, the nature of politics in Rousseau’s writings is formulated in the following way: “The major axiom is that in order to definitely have the expression of the general will, [there must] be no partial society in the State” [BE, p. 348]. This approach is also indicated in the only part of “Being and Event” which is devoted to that issue in general terms: “Let us agree that a proposition is singular (…) if, within a historically structured mathematical situation, it implies many other significant propositions, yet it cannot itself be deduced from the axioms which organize the situation. (…) Say that $A$ is this proposition. (…) An event, named by an intervention, is then, at the theoretical site indexed by the proposition $A$, a new apparatus, demonstrative or axiomatic, such that $A$ is henceforth clearly admissible as a proposition of the situation.” [BE, p. 246] Accordingly, undecidability of a truth would consist in transcending the theoretical framework of a historical situation or even breaking with it in the sense that the faithful subject accepts beliefs that are impossible to reconcile with the old mode of thinking.

A clear illustration of the effect of event that in Badiou’s opinion results in breaking with the determinants of the old paradigm, rather than just in moving beyond them, is the birth of relativistic physics: “After Einstein’s texts of 1905, if I am faithful to their radical novelty, I cannot continue to practice physics within its classical framework” [Ethics, p. 42]. The novelty of relativistic physics cannot be reduced to a mere substitution of one formulas with others, more precise, because it gives rise to a completely new understanding of fundamental physical categories such as space, time, reference point or motion. For
profound reasons classical mechanics rules out — instead of simply not deciding it — the very possibility of emerging the theory of relativity within its own conceptual framework. Similarly, the French revolution and communism — essential examples of the effect of event — violently rupture the historical order preceding them. The French revolution overthrew the king and established sovereignty of the people. And if something can be said with certainty about any real or imaginary realization of the communist idea, it is that it definitely abolishes the “capital-parliamentarism”.

However, if one consequently identifies the effect of event with the structure of the generic extension, they need to conclude that these historical situations are by no means the effects of event. This is because a crucial property of every generic extension is that axioms of set theory remain valid in it. It is the very core of the method of forcing, stated in the Theorem of the Generic Model [Jech, Th. 14.5]. Without this assumption, the original Cohen’s construction would have no raison d’etre because it would not establish undecidability of the cardinality of infinite power sets. Let us say this once more: every generic extension satisfies axioms of set theory. In reference to historical situations, it must be conceded that a procedure of fidelity may modify a situation by forcing undecidable sentences, nonetheless it never overrules its organizing principles.

From the point of view of the generic theory of truth, some hypothetical type of social-democracy might be considered as the effect of event. It would abolish chaos and inequalities resulting from mechanisms of democratically controlled market economy by the operation of a new idea that would transfigure the nature of these mechanisms from within. As a religious event, transgressing the Law without literally breaching it, one could probably point to Messianic Judaism or Protestantism. Another interesting case is a theory of the Danish astronomer Tycho de Brahe who in XVI century proposed a solution that allowed for keeping empirical advantages of the heliocentric model, while letting the Earth stay in the center of the Universe. In this conception, all the planets revolve around the Sun, except for the Earth, which is encircled by it. In terms of kinetics, that is if the force of gravity — unknown at that time — is disregarded, the model of de Brahe is entirely equivalent to the Copernican one.

Another notion for which any attempt of locating it within the generic theory of truth leads to similarly extreme consequences is evental site. An evental site — an element “on the edge of the void” [BE p. 175] — opens up a situation to the possibility of an event [BE p. 179]. Ontologically, it is defined as “a multiple such that none of its elements are presented in the situation” [BE, p. 175]. In other words, it is a set such that neither itself nor any of its subsets are elements of the state of the situation. As the double meaning of this word indicates, the state (état) in the context of historical situations takes the shape of the State (État) [BE p. 104]. A paradigmatic example of a historical evental site is proletariat — “entirely dispossessed, and absent from the political stage” [Ethics, p. 69].

The existence of an evental site in a situation is a necessary requirement for an event to occur. Badiou is very strict about this point: “we shall posit once and for all that there are no natural events, nor are there neutral events” [BE p. 178] — and it should be clarified
that situations are divided into natural, neutral, and those that contain an evental site. The very matheme of event — its formal definition is of no importance here — is based on the evental site \[\text{Ethics}, \text{p. 179}\]. An event raises the evental site to the surface, makes it represented on the level of the state of the situation. Moreover, even if a novelty has the structure of the generic set but it does not emerge from the void of an evental site, it leads to a \textit{simulacrum of truth} instead of a truth \[\text{Ethics}, \text{p. 72}\], which is one of the figures of Evil \[\text{Ethics}, \text{p. 87}\]. An example of utterly destructive effects of a simulacrum of truth is the Nazis revolution whose source was the “plenitude” of the German people \[\text{Ethics}, \text{p. 73}\].

However, if one takes the mathematical framework of Badiou’s concept of event seriously, it turns out that there is no place for the evental site there — it is forbidden by the assumption of transitivity of the ground model \(M\). This ingredient plays a fundamental role in forcing, and its removal would ruin the whole construction of the generic extension. As it has been already mentioned, transitivity means that if a set belongs to \(M\), then each of its elements also belongs to \(M\). However, an evental site is a set none of whose elements belongs to \(M\). Therefore, contrary to Badiou’s intentions, there cannot exist evental sites in the ground model. Using Badiou’s terminology one can say that forcing may only be the theory of the simulacrum of truth.

6 **The mathemes of indiscernible and unnameable**

“Thought is nothing other than the desire to finish with the exorbitant excess of the state” \[\text{BE}, \text{p. 282}\]. Since Cantor’s theorem implies that this excess cannot be removed or reduced to the situation itself, the only way left is to take control of it. A basic, paradigmatic strategy for achieving this goal is to subject the excess to the power of language. Its essence has been expressed by Leibniz in the form of the principle of indiscernibles: there cannot exist two things whose difference cannot be marked by a describable property \[\text{BE}, \text{p. 283}\]. In this manner, language assumes the role of a “law of being” \[\text{BE}, \text{p. 283}\], postulating identity, if it cannot find a difference. Meanwhile — according to Badiou — the generic truth is indiscernible: there is no property expressible in the language of set theory that characterizes elements of the generic set. Truth is beyond the power of knowledge, only the subject can support a procedure of fidelity by deciding what belongs to a truth. This key thesis is established using purely formal means, so it should be regarded as one of the peak moments of the mathematical method employed by Badiou. In order to assess its grounding and possible limitations, one needs to analyze the matheme of indiscernible as closely as possible.

To some surprise to the reader, Badiou composes the indiscernible out of as many as three different mathematical notions. First of all, he decides that it corresponds to the concept of inconstructible \[\text{BE}, \text{p. 355}\]. Later, he writes though that “a set \(\delta\) is discernible (...) if there exists (...) an explicit formula \(\lambda(x) (...)\) such that ’belong to \(\delta\)’ and ’have the
property expressed by $\lambda(x)$ coincide” [BE, p. 367]. Finally, at the outset of the argument designed to demonstrate indiscernibility of truth he brings in yet another definition: “let us suppose the contrary: the discernibility of $G$. A formula thus exists $\lambda(x, a_1, \ldots, a_n)$ with parameters $a_1, \ldots, a_n$ belonging to $M[G]$ such that for an inhabitant of $M[G]$ it defines the multiple $G$” [BE, p. 386]. Summarizing, the discernibility is understood as:

1. constructibility

2. definability by a formula $F(y)$ with one free variable and no parameters. In this approach, a set $a$ is definable if there exists a formula $F(y)$ such that $b$ is an element of $a$ if and only if $F(b)$ holds.

3. definability by a formula $F(y, z_1, \ldots, z_n)$ with parameters. This time, a set $a$ is definable if there exists a formula $F(y, z_1, \ldots, z_n)$ and sets $a_1, \ldots, a_n$ such that after substituting $z_1 = a_1, \ldots, z_n = a_n$, an element $b$ belongs to $a$ if and only if $F(b, a_1, \ldots, a_n)$ holds.

Even though in “Being and Event”, Badiou does not explain the reasons for this variation, it clearly follows from his other writings (such as [Conditions, p. 135]) that he is simply convinced that these notions are equivalent. It should be emphasized then that this is not true: a set may be discernible in one sense, but indiscernible in another. First of all, the last definition has been included probably by mistake because it is trivial. Every set is discernible in this sense because for every set $a$ the formula $F(y, x)$ defined as ‘$y$ belongs to $x$’ defines $a$ after substituting $x = a$. Accepting this version of indiscernibility would lead to the conclusion that truth is always discernible, while Badiou claims, and, supposedly, demonstrates that it is not. In particular, the proof of indiscernibility of truth presented by Badiou on page 386 of “Being and Event”, which is based on this definition of indiscernibility, is incorrect.

Is it not possible to choose the second option and identify discernibility with definability by a formula with no parameters? After all, this notion is most similar to the original idea of Leibniz — intuitively, the formula $F(y)$ expresses a property characterizing elements of the set defined by it. Unfortunately, this solution does not warranty indiscernibility of the generic set either. In 2005, Eli Enayat ([Enayat]) published a paper, where he proves the possibility of generic extensions all of whose elements — including the generic set — are definable by a formula with no parameters. Therefore, in this approach truth may be seized by knowledge replacing the intervening subject.

As a matter of fact, if we assume that in ontology discernibility corresponds to constructibility, then Badiou is right that the generic set is necessarily indiscernible. However,

---

*There are several flaws in the proof but the most important one is that Badiou wrongly assumes that an element defined by a formula with parameters in the ground model must be an element of the ground model as well. This is not true as the following discussion will show.*
constructibility is a highly technical notion, and its philosophical interpretation seems very problematic. Let us take a closer look at it.

The class of constructible sets — usually denoted by the letter $L$ — forms a hierarchy indexed or ‘numbered’ by ordinal numbers. Without getting into details of the definition of ordinal number — closely related to that of cardinal number — the inductive procedure of constructing the constructible hierarchy goes as follows. The lowest level $L_0$ is simply the empty set. Assuming that some level — let us denote it by $L_\alpha$ — has been already constructed, the next level $L_{\alpha+1}$ is constructed by choosing all subsets of $L_\alpha$ that can be defined by a formula (possibly with parameters) bounded to the lower level $L_\alpha$.

Bounding a formula to $L_\alpha$ means that its parameters must belong to $L_\alpha$ and that its quantifiers are restricted to elements of $L_\alpha$. For instance, the formula 'there exists $z$ such that $z$ is in $y$' simply says that $y$ is not empty. After bounding it to $L_\alpha$ this formula takes the form 'there exists $z$ in $L_\alpha$ such that $z$ is in $y$', so it says that $y$ is not empty, and some element from $L_\alpha$ witnesses it. Accordingly, the set defined by it consists of precisely those sets in $L_\alpha$ that contain an element from $L_\alpha$.

After constructing an infinite sequence of levels (or — strictly speaking — a limit sequence of levels) the level directly above them all is simply the set of all elements constructed so far. For example, the first infinite level $L_\omega$ consists of all elements constructed on levels $L_0, L_1, L_2, \ldots$.

As a result of applying this inductive definition, on each level of the hierarchy all the formulas are used, so two distinct sets may be defined by the same formula. On the other hand, only bounded formulas take part in the construction. The definition of constructibility offers too little (because only bounded formulas are accepted) and too much at the same time (because many sets can be defined by one formula). This technical notion resembles the Leibnizian discernibility only in so far as it refers to formulas. In set theory there are more notions of this type though.

To realize difficulties involved in attempts to philosophically interpret constructibility, one may consider a slight, purely technical, extension of it. Let us also accept sets that can be defined by a formula $F(y, z_1, \ldots, z_n)$ with constructible parameters, that is, parameters coming from $L$. Such a step does not lead further away from the common understanding of Leibniz’s principle than constructibility itself: if parameters coming from lower levels of the hierarchy are admissible when constructing a new set, why not admit others as well, especially since this condition has no philosophical justification?

Actually, one can accept parameters coming from an even more restricted class, e.g., the class of ordinal numbers — and these are known to be constructible. Then we will obtain the notion of definability from ordinal numbers ($OD$). This minor modification of the concept of constructibility — a relaxation of the requirement that the procedure of construction has to be restricted to lower levels of the hierarchy — results in drastic consequences. Kenneth McAloon ([McAloon](#)) proved in 1972 that the generic set as well as all other elements of the generic extension may be definable from ordinal numbers. Therefore, replacing constructibility with a concept very similar to it — one essentially identical from the point of
view of Badiou’s philosophical motivations — leads to an ontology allowing for discernibility of truth.

Another example of such a heterogeneous composition is the matheme of \textit{unnameable}. In “Being and Event” this notion functions as a synonym of the indiscernible, however in later writings of Badiou — especially “Conditions” and “Ethics” — becomes a distinct, autonomous concept. Referring to the Lacanian psychoanalysis, Badiou says: “in the field determined by a situation and the generic becoming of its truth, a real is attested to by a term, a point, and only one, at which the power of truth is suspended. There is only one term in relation to which no anticipatory hypothesis on the generic subset permits us to force a judgment, (…) no naming is appropriate for this term. That is why I call it unnameable” \cite{Conditions} p. 141. Also, the unnameable understood in the same manner — as a “point that the truth cannot force” \cite{Ethics} p. 85 — witnesses the “powerlessness of truth” in Badiou’s ethical thought. A totally powerful truth — its subject language reaching all elements of the situation — leads to a “disaster”, which is another figure of Evil, along with a simulacrum of truth \cite{Ethics} p. 85.

Thus in set theory, the unnameable has a precisely defined form: it is the unique element of the generic extension such that no condition forces a sentence of the forcing language referring to this element. Unfortunately, the mathematical content of this definition turns out to be empty — forcing unequivocally rules out the existence of the unnameable. This is because one of the basic features of the forcing language is \cite{Jech Th. 14. 7 ii a)] that if no condition forces some sentence $S$, then every condition forces the negation of $S$. In the generic extension, the unnameable is not.

Yet Badiou’s position is that mathematics indicates the possibility of the unnameable. He points to a construction due to Furkhen “in which one term exists, and only one, which cannot receive a name in the sense that it cannot be identified by a formula in the language” \cite{Conditions} p. 143. A familiar substitution can be recognized here. Initially, the unnameable is defined in terms of the forcing language, however Furkhen’s construction refers to definability by a formula. It should be emphasized that this object is not even a model of set theory \cite{Conditions} p. 120; it requires a richer language, and it does not satisfy axioms of set theory. Therefore it cannot serve as a ground model nor a generic extension — forcing has simply no use in this framework. Moreover, the unnameable becomes the indiscernible again with an additional requirement of uniqueness. Is a truth a real of the situation then? Such a suggestion never appears in Badiou’s writings. Otherwise, there always exist at least two indiscernible elements — a truth and a real of the situation — even though the unnameable is supposed to be unique. In presence of these overwhelming formal and interpretative difficulties, one should rather conclude that the matheme of unnameable is an ill-conceived concept.

***

The mathematical form of the undecidable implies that a truth never overrules the fundamental conceptual framework of a situation from before the occurrence of an event.
This consequence remains in sharp contrast with basic examples of the effect of event considered by Badiou. An evental site cannot open up a situation to an evental truth because forcing rules out the very existence of an evental site in ground models. As a result, the generic set may possibly describe the structure of the simulacrum of truth — using Badiou’s terminology — but not the structure of truth itself. Two two remaining mathemes — the indiscernible and the unnameable — are built out of mathematical concepts that do not fit together; moreover, their properties are not as Badiou claims.

This list of problems can be extended. The “name of the event” is not “prohibited by being” [BE p. 184] because the axiom of foundation may be consistently replaced with its negation — for example, the Aczel anti-foundation axiom is consistent with the remaining axioms of set theory. Finite sets are not necessarily definable, even though Badiou tries to prove the opposite [Conditions, p. 137], mistakenly identifying finite sets with numerals. In the so called nonstandard models of set theory the standard omega is finite and non-definable. Generic sets are not coextensive with unconstructible sets [Conditions, p. 135] — for example, the Ground Axiom implies it. It seems though that the presented discussion of the mathemes of indiscernible, undecidable, unnameable, and evental site is sufficient to defend the main thesis of this paper. Mathematics — forced to accept compromises going definitely too far — responds with outcomes which are hostile to fundamental philosophical motivations of Badiou’s doctrine. Despite some points of convergence, his generic theory of truth and his philosophy of event can coexist only at a price of selective and instrumental interpretation of the mathematical component. Therefore one has to conclude that “Being and Event” provides no grounding for a deep ontological structure behind the realms of science, art, love and politics, and that the mathematical formulation of the theory of event has no positive content.

References

[BE] A. Badiou, Being and Event, Continuum 2005.

[Ethics] A.Badiou, Ethics:An Essay on the Understanding of Evil, Verso 2002.

[Conditions] A.Badiou, Conditions, Continuum 2009.

[Enayat] A.Enayat, Models of set theory with definable ordinals,Archive for Math. Logic, vol. 44, 3 (2005), pp. 363-385.

[Jech] T.Jech, Set theory, Springer 2006.

[McAloon] K.McAloon, Consistency results about ordinal definability, Annals of Math. Logic, vol. 2 (1971), pp.449-467.