Generalized rotating-wave approximation to the two-qubit and cavity coupling system

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(Dated: October 9, 2014)

The generalized rotating-wave approximation (GRWA) is presented for the two-qubit and cavity coupling system. The analytical expressions in the zeroth order approximation recover the previous adiabatic ones. The counterrotating-wave terms can be eliminated by performing the first order corrections. An effective solvable Hamiltonian with the same form as the ordinary RWA one are then obtained, giving a significantly accurate eigenvalues and eigenstates. Energy levels in the present GRWA are in accordant with the numerical exact diagonalization ones in the a wide range of coupling strength. The atomic population inversion in the GRWA is in quantitative agreement with the numerical results for different detunings in the ultrastrong coupling regime.

PACS numbers:

I. INTRODUCTION

Recent experimental progress related to qubit-oscillator systems using superconducting qubit circuits has made it possible to achieve the so-called ultrastrong-coupling regime, where the coupling strength between a single qubit and a single oscillator is comparable to the bare frequencies of the two constituents. In this regime, the ubiquitous rotating-wave approximation (RWA) is expected to break down, leading to a mass of unexplored physics and giving rise to fascinating quantum phenomena, such as the asymmetry of vacuum Rabi splitting, collapse and revival dynamics, a Bloch-Siegert shift, super-radiance transition, and radiation processes based on virtual photons. It is highly desirable to understand the behavior of the qubit-oscillator in the whole coupling regime.

Since the Hamiltonian of a qubit-oscillator system contains counter rotating-wave terms, the total bosonic number is not conserved, it is very challenging to obtain the analytical solutions in the ultrastrong-coupling regime. There is much on-going interest in this field. The Rabi model describing a single qubit interacting with a quantum harmonic oscillator has been studied extensively beyond RWA with various analytical methods in the recent years. Two or more qubits coupled to a common harmonic oscillator in the ultrastrong-coupling regime has more potential applications in quantum information processing than that in the single-qubit Rabi model, such as the implement quantum-information protocols with the oscillator transferring information coherently between qubits, and quantum entanglement of multiqubit properties, and superradiance phase transition in the Dicke model describing two-level atom ensemble in a cavity. We investigate the Tavis-Cummings model beyond the RWA, in which a quantum harmonic oscillator interacts with two identical qubits symmetrically. One of the motivations lies in the absence of extensively study of the two- and more-qubit in the ultrastrong-coupling regime. Recently, an adiabatic approximation functions well when the qubit frequency is much smaller than the oscillator frequency, and the variational treatment reasonably captures the properties of the ground state in the Tavis-Cummings model.

II. HAMILTONIAN AND ZERO ORDER APPROXIMATION

The Hamiltonian of the Tavis-Cummings model, where two identical qubits couple to a harmonic oscillator with the counter rotating-wave interaction, is $\hbar = 1$

$$H = \Delta J_x + \omega a^\dagger a + g(a^\dagger + a)J_z,$$  \hspace{1cm} (1)

where $a$ and $a^\dagger$ are, respectively, the annihilation and creation operators of the harmonic oscillator with frequency $\omega$. The collective spin-1 angular momentum operators $J_z = \frac{1}{2}(\sigma^z_1 + \sigma^z_2)$ and $J_x = \frac{1}{2}(\sigma^x_1 + \sigma^x_2)$. Physically, the spin-1 system can be formed by the two identical qubits in their triplet space. $\Delta$ is the atomic transition frequency, and $g$ denotes the collective qubit-oscillator coupling strength.

To begin with, a brief review of the standard RWA is given in order to establish the arguments used in deriving the generalized approximation. The first step is to...
rewrite Eq. (1) in the form

\[ H = -\Delta J_z + \omega a^\dagger a + \frac{g}{2}(a^\dagger a)(J_+ + J_-), \tag{2} \]

where \( J_\pm \) are the collective atomic raising and lowering operators of a spin-1 system. In the basis \(|j_z = 1, n-1\rangle, |j_z = 0, n\rangle\) and \(|j_z = -1, n+1\rangle\), which is the eigenstates of the noninteracting Hamiltonian \(-\Delta J_z + \omega a^\dagger a\), the interaction term \(a^\dagger J_+ + a J_-\) couples the states \(|j_z = 1, n-1\rangle\) with \(|j_z = 0, n\rangle\), and \(|j_z = 0, n\rangle\) with \(|j_z = -1, n+1\rangle\), where energy is conserved. On the other hand, the counter rotating-wave terms \(a^\dagger J_+ + a J_-\) couples the off-resonant states, such as \(|j_z = 0, n\rangle\) with \(|j_z = 1, n-1\rangle\) and \(|j_z = -1, n+1\rangle\), where energy is non-conserved. To eliminate the counter rotating-wave terms, the RWA Hamiltonian \(H_{RWA} = -\Delta J_z + \omega a^\dagger a + \frac{g}{2}(a^\dagger J_+ + a J_-)\) can be written a matrix form

\[ H_{RWA} = \begin{pmatrix} \omega(n+1) - \Delta & \frac{g}{2} \sqrt{n} & 0 \\ \frac{g}{2} \sqrt{n} & \omega n & \frac{g}{2} \sqrt{n+1} \\ 0 & \frac{g}{2} \sqrt{n+1} & \omega(n+1) + \Delta \end{pmatrix}. \tag{3} \]

In the RWA, one can diagonalize the above Hamiltonian easily.

Including the counter rotating-wave terms, the total photonic number is not conserved, the above subspace related to \(n\) is not closed, rendering the complication of the solution. Here, we present a treatment to the Hamiltonian (1) based on the unitary transformation \(23\): \(H' = \exp(U)H\exp(-U)\) with the following displaced operator

\[ U = \exp \left[ \frac{g}{\omega} J_z (a^\dagger - a) \right]. \tag{4} \]

The transformed Hamiltonian is

\[ H' = H_0 + H_1 + H_2, \tag{5} \]

\[ H_0 = \omega a^\dagger a - \frac{g^2}{\omega J_z^2}, \tag{6} \]

\[ H_1 = \Delta J_x G_0 (a^\dagger a) + i J_y \Delta F_1 (a^\dagger a) (a^\dagger - a), \tag{7} \]

\[ H_2 = \Delta J_x \{ \cosh \left[ \frac{g}{\omega} (a^\dagger - a) \right] - G_0 (a^\dagger a) \} + i J_y \Delta \{ \sinh \left[ \frac{g}{\omega} (a^\dagger - a) \right] - F_1 (a^\dagger a) (a^\dagger - a) \}, \tag{8} \]

where \(G_0 (a^\dagger a)\) denotes zero excitation of photon in state \(|n\rangle\)

\[ \langle n | G_0 (a^\dagger a) | n \rangle = \langle n | \cosh \left[ \frac{g}{\omega} (a^\dagger - a) \right] | n \rangle = e^{-\frac{g^2}{\omega^2}} L_n \left( \frac{g^2}{\omega^2} \right), \tag{9} \]

with the Laguerre polynomials \(L_n^{m-n}(x) = \sum_{m=0}^{\min(m,n)} (-1)^{n-m} \frac{m!}{(m-n)! (n-m)! x^m} \). Note that \(F_1 (a^\dagger a) (a^\dagger - a)\) plays a role of creating and eliminating a single photon. Since \(a F_1 (a^\dagger a)\) only has value in \(|n|+1\rangle\) and the term \(F_1 (a^\dagger a) a^\dagger\) only has value in \(|n+1|n\rangle\), so we have the following overlap

\[ \langle n+1 | F_1 (a^\dagger a) a^\dagger | n \rangle = \langle n+1 | \sinh \left[ \frac{g}{\omega} (a^\dagger - a) \right] | n \rangle = \frac{1}{\sqrt{n+1}} \frac{g}{\omega} e^{-\frac{g^2}{\omega^2}} L_n \left( \frac{g^2}{\omega^2} \right). \tag{10} \]

Since \(\cosh \left[ \frac{g}{\omega} (a^\dagger a) \right]\) and \(\sinh \left[ \frac{g}{\omega} (a^\dagger a) \right]\) contain powers of the number operator \(a^\dagger a\) with even and odd functions respectively, they appear in \(H_2\), as \(\cosh \left[ \frac{g}{\omega} (a^\dagger a) \right] = G_0 (a^\dagger a) + O(\frac{g^2}{\omega})\) and \(\sinh \left[ \frac{g}{\omega} (a^\dagger a) \right] = F_1 (a^\dagger a) (a^\dagger - a) + O(\frac{g^2}{\omega})\), where higher terms describing the double and multi-photon transition processes are neglected. Thus we have, \(H' = H_0 + H_1\), similar to the approximation performed in the single-qubit Rabi model \(23\).

As the zeroth-order approximation, we neglect the terms \(F_1 (a^\dagger a) (a^\dagger - a)\) involving creating and eliminating a single photon, the Hamiltonian is then approximated as

\[ H' = \omega a^\dagger a - \frac{g^2}{\omega J_z^2} + \Delta J_x G_0 (a^\dagger a). \tag{11} \]

In the spin and photonic basis of \(|1, n\rangle, |0, n\rangle\) and \(|-1, n\rangle\), we have

\[ H' = \begin{pmatrix} \omega n - \frac{g^2}{\omega} & \frac{\Delta}{\sqrt{2}} G_0(n) & 0 \\ \frac{\Delta}{\sqrt{2}} G_0(n) & \omega n & \frac{\Delta}{\sqrt{2}} G_0(n) \\ 0 & \frac{\Delta}{\sqrt{2}} G_0(n) & \omega n - \frac{g^2}{\omega} \end{pmatrix}. \]

The corresponding eigenvalues and eigenfunctions are straightforwardly given by

\[ \varepsilon_{\pm,n} = \frac{\omega}{2} (2n - \frac{g^2}{\omega^2}) \pm \sqrt{\left( \frac{\Delta}{\omega} \right)^2 + 4 \left( \frac{G_0(n)}{\omega} \right)^2}, \]

\[ \varepsilon_{0,n} = \omega n - \frac{g^2}{\omega}, \tag{12} \]

and

\[ |\varepsilon_{0,n}\rangle = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, |\varepsilon_{\pm,n}\rangle = \begin{pmatrix} \frac{1}{\sqrt{2}} \pm \sqrt{\frac{8}{\chi_n^2} + \chi_n^2} \\ \frac{1}{\sqrt{2}} \pm \sqrt{\frac{8}{\chi_n^2} + \chi_n^2} \end{pmatrix}, \tag{13} \]

where \(\chi_n = \frac{\sqrt{\Delta^2}}{\omega G_0(n)}\). Interestingly, the eigenvalues and eigenstates obtained in this way are exactly the same as those obtained by the adiabatic approximation \(23\). The zeroth-order energy spectrum is plotted in Fig. 1 with blue dashed lines. For comparison, the energies obtained from numerical exact diagonalization and in the RWA are also given with black solid lines and green dashed lines. The ground-state energy and low excited energies agree well with the numerical results for \(\Delta/\omega = 0.5\). It is obvious that the RWA results become worse in the
strong coupling regime. The adiabatic approximate results also deviate from the numerical ones in the ultra-strong coupling regime, and this situation becomes more serious with increasing atomic transition frequency. Neglecting the term $i J y F_1 (a^\dagger a) (a^\dagger - a)$ in the zeroth order approximation, there only exist transition between states with the same values of oscillator excitation, $|0,n\rangle$ and $|\pm 1,n\rangle$. Hence, the validity of the adiabatic approximation is restricted to the large detuning regime $\Delta \ll \omega$. The transitions between various states with different values of oscillator excitation for large value of $\Delta$ will be considered in the next section.

III. GENERALIZED ROTATING-WAVE APPROXIMATION

As the first-order approximation, the term $i J y F_1 (a^\dagger a) (a^\dagger - a)$ will be included, so the Hamiltonian now consists of two parts

\[ H_0' = \omega a^\dagger a - g^2 \omega J_z^2 + \Delta J_z, \]
\[ H_1' = \Delta J_x [G_0 (a^\dagger a) - \beta] + \frac{\Delta}{2} F_1 (a^\dagger a) (a^\dagger - a) (J_+ - J_-,) \]

where $\beta = G_0 (0) = e^{-\frac{\omega^2}{2\mu_+^2}}$. Obviously, the spin and photons in $H_0'$ are decoupled and its spin part can be diagonalized in the spin basis of $| -1 \rangle, |0 \rangle$ and $|1 \rangle$ by a unitary matrix $S$ as

\[ S = \begin{pmatrix} 1/\lambda_+ & 1/\sqrt{2} & 1/\lambda_+ \\ \mu_- / \lambda_- & 0 & \mu_+ / \lambda_+ \\ 1/\lambda_- & -1/\sqrt{2} & 1/\lambda_+ \end{pmatrix}, \]

where $\mu_\pm = \frac{\chi_0}{2} \pm \frac{\chi_0}{2}$, $\chi_0 = \frac{\lambda^2}{\Delta \gamma^2}$, $\lambda_\pm = \sqrt{2 + \mu_\pm^2}$. The corresponding eigenvalues are $\epsilon_\pm = \frac{\Delta}{2\sqrt{2}} (\pm \chi_0 \pm \sqrt{\chi_0^2 + 8})$ and $\epsilon_0 = -\frac{\chi_0}{\omega}$.

Therefore the diagonal $H_0'$ takes the form

\[ \tilde{H}_0 = \begin{pmatrix} \omega n + \epsilon_- & 0 & 0 \\ 0 & \omega n + \epsilon_0 & 0 \\ 0 & 0 & \omega n + \epsilon_+ \end{pmatrix}. \]

The first order term $H_1'$ is transformed by the unitary matrix

\[ \tilde{H}_1 = S^+ H_1' S \]

\[ = \begin{pmatrix} 2\sqrt{2} \mu_+ / \lambda_+ & 0 & \sqrt{2} (\mu_+ + \mu_-) / \lambda_+ \lambda_- \\ 0 & 0 & 0 \\ \sqrt{2} (\mu_+ + \mu_-) / \lambda_+ \lambda_- & 0 & 2 \sqrt{2} \mu_- / \lambda_+ \lambda_- \end{pmatrix} \Delta [G_0 (a^\dagger a) - \beta] \\
+ \begin{pmatrix} 0 & \frac{\mu_-}{\lambda_-} & 0 \\ \frac{\mu_+}{\lambda_+} & 0 & \frac{\mu_+}{\lambda_+} \\ 0 & -\frac{\mu_-}{\lambda_-} & 0 \end{pmatrix} \Delta F_1 (a^\dagger a) (a^\dagger - a). \]
Neglecting the counter rotating-wave terms $a^\dagger J_+ + aJ_-$ and the remote matrix elements $\frac{\sqrt{2}(\mu^+ + \mu^-)}{\lambda_+ - \lambda_-}$, we give the total Hamiltonian as

$$H_{\text{GRWA}} = \omega a^\dagger a + \{\varepsilon_+ + \frac{2\sqrt{2}\mu^+ \Delta}{\lambda_+}[G_0(a^\dagger a - \beta)]\}|1\rangle\langle1|$$

$$+ \{\varepsilon_- + \frac{2\sqrt{2}\mu^- \Delta}{\lambda_-}[G_0(a^\dagger a - \beta)]\} - 1\rangle\langle -1|$$

$$+ \varepsilon_0|0\rangle\langle0| + \frac{\mu^+ \Delta}{\lambda_+}F_0(a^\dagger a)(a^\dagger a)|1\rangle\langle0| + a^\dagger a|0\rangle\langle1|$$

$$- \frac{\mu^- \Delta}{\lambda_-}F_1(a^\dagger a)(a^\dagger a)|-1\rangle\langle -1| + a^\dagger a|0\rangle\langle0| \} \right), \quad (19)$$

where $\xi_{+,n-1} = \varepsilon_+ + \frac{2\sqrt{2}\mu^+ \Delta [G_0(n-1) - \beta]}{\lambda_+}$, $\xi_{-,n+1} = \varepsilon_- + \frac{2\sqrt{2}\mu^- \Delta [G_0(n-1) - \beta]}{\lambda_-}$, and $R_{n,n+1} = \Delta \langle n|F_0(a^\dagger a)a|n+1\rangle$, $R_{n-1,n} = \Delta \langle n - 1|F_1(a^\dagger a)a|n\rangle$. Similar to the usual RWA Hamiltonian (2), the eigenstates $|\phi_n\rangle$ and eigenvalues $E_n$ of the GRWA can be easily obtained.

For $n = 0$, in the basis $|-1,1\rangle$ and $|0,0\rangle$, we have

$$E_{1,\pm} = \frac{\varepsilon_0 + \omega + \xi_-}{2} \pm \frac{1}{2} \sqrt{\left(\varepsilon_0 - \omega - \xi_-\right)^2 + 4 \left(\frac{\mu^- \Delta R_{0,1}}{\lambda_-}\right)^2}. \quad (23)$$

and eigenstates $|\phi\rangle_{\pm} = \{\frac{\lambda_-}{2\mu^- \Delta R_{0,1}}\}(|\varepsilon_0 - \omega - \xi_-| \pm \sqrt{\left(\varepsilon_0 - \omega - \xi_-\right)^2 + 4 \left(\frac{\mu^- \Delta R_{0,1}}{\lambda_-}\right)^2}] - 1,1\rangle + |0,0\rangle$.

The ground state energy for the state $|-1,0\rangle$ is

$$E_0 = \frac{\Delta \beta}{2\sqrt{2}} \left(-\chi \pm \sqrt{\chi^2 + 8}\right). \quad (24)$$

where there is only the energy-conserving term is $|a\rangle\langle 0| + h.c., a^\dagger| -1\rangle\langle 0| + h.c.$ with renormalized coefficients $\frac{\mu^+ \Delta}{\lambda_+}F_1(a^\dagger a)$ and $\frac{\mu^- \Delta}{\lambda_-}F_1(a^\dagger a)$. So it is exactly same as the Tavis-Cummings model with a renormalized parameters in the RWA form. In this sense, we can also call the first-order approximation as GRWA. The effect of the counter rotating-wave interaction in the original model, which play a role in the ultrastrong coupling regime, now is absorbed in $iJ_yF_1(a^\dagger a)(a^\dagger - a)$. The collapse and revival behavior for a single-qubit case was studied (11, 28, 29) and we explore the atomic population inversion in the two-qubit cavity system. Here we apply the eigenvalues and eigenstates obtained by GRWA to investigate the problem in all coupling regimes. To study the population dynamics, we need the eigenstates for the original Hamiltonian (1) with counter rotating-wave terms, which can be obtained using a uni-

$$H_{\text{GRWA}} = \begin{pmatrix}
\omega(n+1) + \xi_{-,n+1} & -\frac{\mu^+}{\lambda_+}R_{n,n+1}\sqrt{n+1} & 0 \\
-\frac{\mu^+}{\lambda_+}R_{n,n+1}\sqrt{n} & \omega n + \varepsilon_0 & \frac{\mu^-}{\lambda_-}R_{n-1,n}\sqrt{n} \\
0 & \frac{\mu^-}{\lambda_-}R_{n-1,n}\sqrt{n} & \omega(n-1) + \xi_{+,n-1}
\end{pmatrix}, \quad (20)$$

IV. POPULATION DYNAMICS

The collapse and revival behavior for a single-qubit system was studied (11, 28, 29) and we explore the atomic population inversion in the two-qubit cavity system. Here we apply the eigenvalues and eigenstates obtained by GRWA to investigate the problem in all coupling regimes. To study the population dynamics, we need the eigenstates for the original Hamiltonian (1) with counter rotating-wave terms, which can be obtained using a uni-

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tary transformation in zero order approximation as
\[
\varphi_0^0 = U^\dagger |\varepsilon_{0,n}\rangle = \begin{pmatrix} |n\rangle_{1} \\
0 \\
|n\rangle_{-1}
\end{pmatrix},
\]
\[
\varphi_{\pm,n}^0 = U^\dagger |\varepsilon_{\pm,n}\rangle = \left( \chi \pm \sqrt{8 + \chi^2} \right) / 2 |n\rangle_0.
\]
where the oscillator states |n\rangle_j = \exp[i\omega(a^\dagger - a)]|n\rangle, j = 0, \pm 1 are called extended coherent states. Similarly, under the first-order approximation the original eigenstates are evaluated as |\varphi_n^1\rangle = U^\dagger S^\dagger |\phi_n\rangle.

The initial state is set |\varphi(0)\rangle = |\pm 1\rangle |\alpha_{-1}\rangle with |\alpha_{-1}\rangle = e^{g/\omega(a^\dagger - a)} |\alpha\rangle. The wavefunction evolves as |\varphi(t)\rangle = e^{-iHt} |\varphi(0)\rangle, which can be expanded by the eigenvalues and eigenstates for the original Hamiltonian under the zeroth- and first-order approximation.

The population for the qubits remain in the state |1, -1\rangle is expressed as
\[
P_{1,-1}(t) = |\langle -1| Tr_{ph} |\varphi(t)\rangle \langle \varphi(t) | - 1\rangle|^2.
\]

This expectation value with zeroth-order approximation and the GRWA method are plotted respectively in Fig. 2 for coupling strength g/\omega = 0.1, 0.3 with different \Delta/\omega = 1 and 0.5. For comparison, the results from the RWA and numerical exact diagonalization are also collected. Obviously, the population inversion results of the GRWA agree well with the numerical ones. And there is substantial improvements over those obtained by the zeroth order approximation in the ultrastrong coupling regime. It is ascribe to the counterrotating-wave interaction in the first order correction, including the states transition with different oscillator excitations, demonstrating the validity of the eigenstates and eigenvalues in the ultrastrong coupling regime by the GRWA.
V. CONCLUSION

In summary, the effective solvable Hamiltonian for the two-qubit Tavis-Cummings model beyond RWA is derived by a unitary transformation, which can in turn gives accurate eigenvalues and eigenstates. The zeroth-order approximation produce the analytical eigenvalues and eigenstates of the adiabatic approximation completely. The first-order approximation, called GRWA are mainly performed, where the rotating-wave interacting coupling strength is renormalized and a counter rotating-wave interactions are including the renormalized coefficients. In the GRWA, the mathematical simplicity of the ordinary RWA is retained, which facilitate the further study. The obtained energy spectrum are in good agreement with the numerical exact diagonalization ones in a wide range of coupling strength, much better than the previous adiabatic approximation. The population inversion obtained using GRWA is also quantitative agreement with the numerical ones, indicating the valid eigenstates and eigenvalues in the ultrastrong coupling regime for different detuning regime. By the analytical eigensolutions, all properties for this two-qubit cavity coupling system can be easily explored. Our approach can be extended to the multi-qubit case, such as the Dicke model.

Acknowledgments

This work was supported by National Natural Science Foundation of China (Grants No. 11174254 and No. 11104363), and Research Fund for the Central Universities (No. CQDXWL-2013-Z014 and No. CQDXWL-2012-Z005).

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