Hubble Constant from LSST Strong-lens Time Delays with Microlensing Systematics

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Abstract

Strong-lens time delays have been widely used in cosmological studies, especially to infer $H_0$. The upcoming LSST will provide several hundred well-measured time delays from the light curves of lensed quasars. However, due to the inclination of the finite AGN accretion disk and the differential magnification of the coherent temperature fluctuations, the microlensing by the stars can lead to changes in the actual time delay on the light-crossing timescale of the emission region of ∼days. We first study how this would change the uncertainty of $H_0$ in the LSST era, assuming the microlensing time delays can be well estimated. We adopt 1/3, 1, and 3 days respectively as the typical microlensing time-delay uncertainties. The relative uncertainty of $H_0$ will be enlarged to 0.47%, 0.51%, and 0.76%, from the uncertainty without the microlensing, impact 0.45%. Then, due to our lack of understanding of the quasar models and microlensing patterns, we also test the reliability of the results if one neglects this effect in the analysis. The biases of $H_0$ will be 0.12%, 0.22% and 0.70%, suggesting that 1 day is the cutoff for a robust $H_0$ estimate.

Key words: distance scale – gravitational lensing: strong – methods: data analysis

1. Introduction

Cosmology has entered the precision era due to an increasingly huge amount of observational data. The theoretical cosmic models and the parameters therein have been well-studied and constrained. Currently, the most plausible model for describing the universe is the ΛCDM model in which the matter content is dominated by cold dark matter and the flat universe is accelerated by dark energy (Planck Collaboration et al. 2016). Meanwhile, we are facing various tension problems stemming from either the systematics in each measurement or potential new physics. For example, the Hubble constant ($H_0$) measured by distance ladder method is inconsistent with the one from Planck data by 3σ uncertainty (Riess et al. 2016; Freedman 2017). To understand this discrepancy, one method is to measure $H_0$ in a totally independent manner up to the sub-percent level with systematics under control (Weinberg et al. 2013).

Strong gravitational lensing by galaxies (Treu 2010) has been widely applied in astrophysics (Zackrisson & Riehm 2010) and cosmology (Treu & Marshall 2016; Liao et al. 2017b). The typical system consists of a distant quasar lensed by the foreground elliptical galaxy, forming multiple images of the AGN and several pieces of arcs as the imaging of the host galaxy. The time delays between multiple AGN images measured by comparing their light curves (Tewes et al. 2013) can be used to constrain $H_0$ (Refsdal 1964), which is not only an independent but also a one-step method to compare the distance ladder techniques from SNe Ia or the inverse distance ladder techniques from cosmic microwave background (CMB) and baryon acoustic oscillations (BAO) (Komatsu et al. 2011; Hinshaw et al. 2013). Roughly speaking, the time delay between two AGN images $\Delta t \propto H_0^{-1}(1 - \langle \kappa \rangle)$, where $\langle \kappa \rangle$ is the mean convergence or surface density in the annulus between the images (Kochanek 2002). The inferred $H_0$ uncertainty should follow the error propagation formula $\sigma_{H}/H_0 \propto \sigma_{2M}/\Delta t^2 + \sigma_{\text{LM}} + \sigma_{\text{LOS}})/N$ for $N$ observed lensing systems. Here, we split the convergence uncertainty into two parts because they are from independent measurements: the lens galaxy modeling uncertainty $\sigma_{\text{LM}}$ and the fluctuation along the line of sight (LOS) $\sigma_{\text{LOS}}$. The former is determined with the observation of central velocity dispersion of the lens galaxy, the lensed host galaxy, and the AGN image positions (Treu & Koopmans 2002; Suyu et al. 2010; Wong et al. 2017) while the later comes from the measurements of the line of sight mass distribution with spectroscopy and multi-band imaging (Rusu et al. 2017).

One of the current limitations of the strong-lens time-delay cosmology is the small number of lensing systems with well-measured time delays, high-resolution imaging, and spectroscopy measurements, though a few lenses have shown the power to constrain $H_0$ (Bovin et al. 2017; Wong et al. 2017), (see the state-of-art program HOLICOW (Suyu et al. 2017). A 3% precision of $H_0$ has been achieved with only four lenses (Birrer et al. 2018). In the next stage, The Time Delay Challenge (TDC; Liao et al. 2015) shows that the upcoming Large Synoptic Survey Telescope (LSST) will bring us 400 well-measured lenses with average precision of ∼3% and accuracy of ≤1% for the time-delay measurements. With these lenses, we are supposed to achieve an unprecedented precision for $H_0$. However, one must be aware that for any precise measurements, the systematics or bias should be controlled below the statistical uncertainties to get a robust result, especially for the LSST data, which may hit the systematics floor. Concerns must be addressed for all independent observations; for example, the lens modeling systematics may already dominate over $\sigma_{\text{(v)}}$ (Schneider & Sluse 2013; Birrer et al. 2016). The ongoing Time Delay Lens Modelling Challenge (TDLMC; Ding et al. 2018) aims to test the systematics in current algorithms. In this work, we only focus on the time-delay measurements and assume the lens modeling and LOS measurements are accurate.

The TDC concluded that the time-delay measurements from the light curves are accurate enough compared with the precision. However, Tie & Kochanek (2018; hereafter TK18) found that the time delays measured as the shifts of the light-curve pair in the time domain are not the cosmological ones but combinations of cosmological and microlensing time delays. The microlensing time delays are due to the differential magnification of the coherent accretion disk variability of the lensed quasars. This
effect can change the measured time delays by light-crossing timescales of the disk on the order of \( \sim \) days.

Therefore, to get an unbiased result we need to incorporate this into the analysis. Chen et al. (2018; hereafter C18) tried to constrain the microlensing effects on time delays and got a weaker constraint for \( H_0 \), especially for short time delays. They developed a pipeline in a Bayesian framework using time-delay ratios and simulated microlensing time-delay maps as priors. Birrer et al. (2018; hereafter B18) estimated the effect of microlensing time delay and found it to be much smaller than the statistical uncertainty. However, as indicated, problems still exist due to the assumptions in the AGN model, the size of the disk and the local properties of the lens at each image. It is unlikely that these assumptions could also bring extra bias unless further blind analysis tests that idea. Due to these complexities, Bonvin et al. (2018) chose not to include the microlensing time delays in the estimates if there is no evidence showing the time delay changes at different observation epochs (Bonvin et al. 2018).

In this paper, we ignore all these details and study on what level of the microlensing time-delay variations would bias the results in LSST data. In Section 2, we introduce the simulated LSST lenses and the lessons we learned from TDC and H0LiCOW. In Section 3, we introduce the microlensing time delays. The results are shown in Section 4 and we provide the summaries and discussions in Section 5. We assume a flat \( \Lambda \)CDM model with \( \Omega_M = 0.3 \) and \( H_0 = 70 \) km s\(^{-1}\) Mpc\(^{-1}\) as the benchmark in the simulations.

2. Lensing Observation in the LSST Era

The upcoming LSST will monitor \( \sim 10^3 \) strongly lensed quasars during its 10 yr campaign repeatedly monitoring 18,000 deg\(^2\) of the sky (Oguri & Marshall 2010). To understand whether the proposed observation strategy can provide sufficient information on time-delay measurements, a TDC program was conducted (Liao et al. 2015). The challenge “Evil” Team simulated thousands of time-delay light-curve pairs including all anticipated physical and experimental effects, while the community was then invited to extract the time delays from the mock light curves blindly based on their own algorithms as the “Good” Teams. One of the main goals was to test the average precision and the bias to make sure the cosmological parameters could be constrained precisely and also accurately. The TDC claimed that \( \sim 400 \) well-measured time delays would be obtained with precision \( \sim 3\% \) and bias \( \lesssim 1\% \). We call such lenses “Golden lenses.” We adopt this number of “Golden lenses” in our work as the benchmark, making it easier to estimate the cosmological parameter uncertainties scaling as \( 1/\sqrt{N} \) if the number of “Golden lenses” changes in the next TDC2 challenge to make more realistic simulations. On the other hand, to perform cosmology we also need inputs from the \( HST \) and Keck Telescopes such that the quasar imaging and the central velocity dispersion of the lens can be used to perform lens modeling. However, in reality their observing time is a limited resource. It may take a long time for all LSST lenses to undergo cosmology.

In TDC, the selected time delays from the OM10 catalog (Oguri & Marshall 2010) for simulations were between 10 days and 120 days and the image magnitudes were within the limiting magnitude 23.3 in the \( i \) band, totaling \( \sim 2000 \) systems. Though C18 claimed the time-delay ratios from quad systems can be well-constrained by the host imaging, providing the constraining information on the microlensing time delays, there are more doubles in reality. For LSST, only 15% of the lenses are with quad images (Oguri & Marshall 2010). In principle, all three independent time delays in quads are used to infer \( H_0 \) in the Bayesian framework. However, to make it simple, we only take the two images with the largest time delay, which mainly dominate the inference of the cosmology. Another consideration for this is based on the absolute microlensing time-delay error, which inclines to bias smaller time delays (see Section 3); for example, consider the three close images in the cusp configuration. This assumption would result in 350 double-image-like time delays in this analysis. The distributions of the redshifts and time delays from OM10 are plotted in Figures 1 and 2. We note that the lessons in TDC showed that the precision of individual time-delay measurements decreases with time delay itself, which is consistent with the time-delay uncertainty being approximately constant in days (Liao et al. 2015). It was expected that the absolute precision is mainly determined by the cadenced sampling of the
light curves. We take 1 day as the constant uncertainty, so that the average precision is close to the TDC. On the other hand, to infer \(H_0\), we need high-resolution imaging of the lensed host galaxies, good PSFs to model the point images (Chen et al. 2016), and central velocity dispersion measurements to perform the lens modeling and then determine the Fermat potential differences. According to the state-of-the-art H0LiCOW program, the lens modeling uncertainty for each lens can be achieved at percent level for individual systems, with blind analysis controlling the systematics (Bovin et al. 2017; Birrer et al. 2018), so we take a relative uncertainty of 3% as the benchmark in this work.

Finally, all mass along the LOS affects the lens potential that the light passes through. These mass fluctuations can also lead to additional focusing and defocusing of the light rays, which in turn affects the measured time delays. To avoid biasing the result, one can give an estimate by spectroscopic/photometric observations of local galaxy groups and LOS structures in combination with ray-tracing through N-body simulations, or even realistic simulations of lens fields (Collett et al. 2013; Greene et al. 2013; McCully et al. 2014; Treu & Marshall 2016).

For HE 0435-1223, the estimate of external convergency \(\kappa_{\text{ext}}\) would result in 2.5% relative uncertainty on the time-delay distance (Rusu et al. 2017); we refer to this value in our analysis.

### 3. Microlensing Time Delays

The community used to believe that the time delays measured from the light-curve pairs only depend on the large structure of the lenses and can be directly used in constraining cosmological parameters. The systematics only come from the independent microlensing magnification light curves for each image and the algorithms to extract the time delays. However, TK18 questioned the use of measured time delays in cosmology. They proposed that an important physical process had not been considered, that is, due to the finite disk size and the differential magnification of the coherent temperature fluctuations, the microlensing effect changes the time delays on the scale of the light-crossing time of the accretion disk (~days). While the accretion disk moves relative to the lens, the microlensing time delay also changes.

Therefore, to get an unbiased result, one needs to understand the details of all physical processes, which is hardly currently achievable. Although C18 and B18 have considered the microlensing time-delay effects in a Bayesian framework, there still exist many uncertain inputs for the microlensing time-delay priors. First, the assumed “thin-disk” and “lamp-post” model (Cackett et al. 2007) on the accretion disk may not be correct. The size of the accretion disk may be larger than the prediction from the standard thin-disk theory with an Eddington ratio 0.1 (Kollmeier et al. 2006; Morgan et al. 2010; Mosquera & Kochanek 2011; Shappee et al. 2014; Fausnaugh et al. 2016), which would make the systematics severer. Alternative accretion disk models giving different variability are proposed (Dexter & Agol 2011). Second, the adopted fixed best-fit local environments for the images and mass function for the stars can also cause uncertainties (Chen et al. 2018). For different local convergency \(\kappa\), shear \(\gamma\) and star proportion \(f_a\), the standard deviation of the magnification map changes (Liao et al. 2015), and so does the microlensing time-delay map. Third, the inclination, position angle, and especially the size of the accretion disk make the time-delay map different (Tie & Kochanek 2018); C18 has considered a conservative range of these variables. Finally, the microlensing time delay changes with the relative motion of the source, while C18 and B18 considered the time delays from three finite epochs as constants. Whether this is appropriate for the LSST 10 yr light curves requires further testing.

Nevertheless, from the analysis of RXJ 1131-1231, HE 0435-1223, and PG 1115+080 (Bonvin et al. 2018; Chen et al. 2018; Tie & Kochanek 2018), the microlensing time delay for individual images is estimated to be from \(\sim 0.1\) to \(\sim 4\) days, as is the delay between two images, and it is an absolute rather than fractional error. Therefore, to assess the impact of the microlensing on the LSST time-delay cosmology, we assume 1/3, 1, and 3 days as the microlensing time-delay uncertainties between two images for all LSST lenses.

### 4. Methodology and Results

According to strong-lensing theory, the cosmological time delay between two images \(i, j\) is given by

$$\Delta t_{\text{COSM}} = \frac{D_{\text{LSST}}(1 + z_d) \Delta \phi}{c},$$

where \(c\) is the light speed, \(\Delta \phi = [(\theta_i - \theta_j)^2/2 - \psi(\theta_i) - (\theta_i - \theta_j)^2/2 + \psi(\theta_j)]\) is the difference between Fermat potentials (representative of \(\kappa\) mentioned above) at different image angular positions \(\theta_i, \theta_j\), with \(\beta\) denoting the source position, and \(\psi\) being the two-dimensional lensing potential determined by the Poisson equation \(\nabla^2 \psi = 2\kappa\), where \(\kappa\) is the surface mass density of the lens in units of critical density \(\Sigma_{\text{crit}} = c^2D_s/(4\pi GD_dD_\text{LSST})\), \(D_d\), \(D_s\), and \(D_\text{LSST}\) are angular diameter distances to the lens (deflector) located at redshift \(z_d\) to the source, located at redshift \(z_s\) to the source, and between them, respectively.

From an observation perspective, the determination of Fermat potential difference can be split into two independent parts:

$$\Delta \phi = \Delta \phi_{\text{Lens}} + \Delta \phi_{\text{LOS}},$$

where \(\Delta \phi_{\text{Lens}}\) is determined by lens galaxy observations and \(\Delta \phi_{\text{LOS}}\) is determined by the mass distribution along the LOS. People used to only consider the lens structure. When we give the measurements of the LOS environment, we can take it as the statistical uncertainty rather than the systematics. As the same, the time delay also includes two parts:

$$\Delta t_{\text{COSM}} = \Delta t_{\text{LC}} - \Delta t_{\text{ML}},$$

where \(\Delta t_{\text{LC}}\) is from the light curves and \(\Delta t_{\text{ML}}\) now corresponds to the microlensing time delay. If we can understand the detailed physical processes and have high-quality observations to assess \(\Delta t_{\text{ML}}\), we can take it as the statistical uncertainty as C18 and B18 suggested. Otherwise, this would be an important systematic uncertainty source.

In this work we consider three cases that the community may encounter. Case 1: considering the AGN model has not been well understood, there is a scenario in which the microlensing effect is small enough, which could be ignored. We study \(H_0\) constraints based on simulated LSST data. Case 2: we consider the condition that the microlensing time delays can be well estimated as the statistical errors or priors in a Bayesian framework; we also give the constraint results. Case 3: the effects do exist but they can be neglected during analysis, because this would underestimate the uncertainty and result in a
biased $H_0$ estimation. We test how reliable this could be by studying the induced systematics on $H_0$.

To avoid the impact of specific noise realizations, we adopt the minimum $\chi^2$ statistics in the analysis using different noise realizations for the observables. The statistic is expressed as

$$X^2 = \sum_{i=1}^{N} \frac{[D_{\Delta i}^{\text{obs}} - D_{\Delta i}^{\text{th}}(H_0, \Omega_M; \hat{z}_{d,i}, \hat{z}_{i,i})]^2}{\sigma_{D_{\Delta i}}^2},$$

(4)

where

$$\sigma_{D_{\Delta i}}^2 = \frac{c}{1 + \hat{z}_{d,i}} \left( \frac{\Delta t_{i}^2 \sigma_{\Delta t_{i}}^2}{\Delta \phi_i^4} + \frac{\sigma_{\Delta i}^2}{\Delta \phi_i^2} \right) + (D_{\Delta i}^{\text{obs}} * 2.5\% )^2.$$  

(5)

Note that we use $X^2$ rather than the conventional $\chi^2$ to remind readers that the uncertainty given by Equation (5) is not a rigorous Gaussian distribution because it occurs through error propagation from Table 1. However, we still adopt a Gaussian assumption in the analysis. First, rather than inferring an accurate $H_0$ from realistic data, we mainly give an estimate of the impact of microlensing time delays, as the relative impact level is what we are interested in. Second, to correctly assess the bias with non-Gaussian effects, one has to start from the original observations, i.e., the pixel values of the host imaging, the velocity dispersion, the AGN positions, and the time delays taken as Gaussians. However, these observational uncertainty details have not been set up for LSST lenses. Third, the H0LiCOW shows the inferred $D_{\Delta i}$ and $H_0$ approximately follow Gaussian distributions (Wong et al. 2017). Therefore, while the analysis can be presented simply and clearly for readers, we think the main conclusion would not change. This assumption also appears widely in the literature (Coe & Moustakas 2009; Paraficz & Hjorth 2009; Linder 2011). To unearth more details, further study is needed in the ongoing TDC2 (being prepared by TDC team) and TDLMC programs (Ding et al. 2018).

Given the randomly selected 350 lensing systems, we distribute noises to Fermat potential differences, LOS masses, time delays from the light curves, and microlensing time delays as summarized on Table 1. For each noise realization, we do minimization to find the best-fit values of $H_0$ and $\Omega_M$. We repeat this process 3000 times for different noise realizations and take all best-fit values as the constraint inputs; the marginalized histograms of $H_0$ are plotted in Figures 3 and 4. They approximately look like Gaussian distributions. Thus, we simply calculate the standard deviations of the PDFs as $1\sigma$ uncertainty. Furthermore, to avoid the impact of specifically selected systems, we repeatedly select 350 systems from the whole OM10 catalog consisting of $\sim$2000 systems that meet LSST criteria (Liao et al. 2015) 15 times and for each data set, we repeat the above process. Finally, we calculate the mean of all uncertainties as the average constraint power on $H_0$ summarized in Table 2. For case 3, the standard deviations must be larger than case 1, which can be seen as a combination of the statistical and systematic uncertainties. This is due to the microlensing time delays as the systematics that enlarge the variations. We subtract the statistical ones in case 1 and get the systematics/bias of $H_0$; see case 3-1 in Table 2. As we can see from the results, the microlensing time delay matters when it is typically larger than 1 day, when the systematics are comparable to the statistical uncertainty, leading to a biased estimate on $H_0$.
5. Conclusions and Discussions

We have tested the robustness of future LSST strong-lens time-delay cosmology by studying the systematics in the time-delay measurements. We summarize our main conclusions as follows:

1. With the assumption that all current measurements related to strong-lens time-delay cosmology are accurate, including the time delays from the light curves, i.e., the scenario where microlensing time delays can be ignored, $\sigma_{\mu}/H_0$ can be constrained to a 1σ uncertainty of 0.45% for 350 lenses in LSST.

2. Cosmological time-delay measurements may be affected by microlensing effects. If these microlensing time delays can be estimated correctly as extra statistical uncertainties or priors in the Bayesian framework, the constraint on $H_0$ will be weaker, as expected. We assume 1/3, 1, and 3 days as the typical microlensing time-delay uncertainties for all lenses; the 1σ uncertainties of $\sigma_{\mu}/H_0$ are 0.47%, 0.51%, and 0.76%.

3. If the microlensing time delays exist but we ignore them, they could be the systematics, i.e., one would underestimate the observational uncertainties in inferring $H_0$. Taking 0.45% as the expected average statistical uncertainty for $\sigma_{\mu}/H_0$, we show the systematics of 0.12%, 0.22%, 0.70%, respectively inside the $H_0$ estimation for 1/3, 1 and 3 days of the microlensing time-delay uncertainties. Therefore, a microlensing time delay longer than 1 day would strongly bias the result. If the systematics is small enough, the lens modeling and LOS uncertainties would dominate the total uncertainty. For current tension $\sim$7% between strong-lensing $H_0 \sim 72$ km s$^{-1}$ Mpc$^{-1}$ (Birrer et al. 2018) and CMB $H_0 \sim 67$ km s$^{-1}$ Mpc$^{-1}$ (Planck Collaboration et al. 2016), we propose that it may partially stem from the microlensing time-delay errors (depending on how large they are). For example, for a $\sim 14$ day time delay like in HE 0435-1123 or PG 1115+080, 1 day microlensing bias would result in a tension at this level, though for the four quad systems (Birrer et al. 2018), it is not likely to result in an overall 2σ tension.

For future studies, we need the AGN accretion model to have more astrophysics inputs and more precise measurements of local image environments. If we can confirm that the microlensing time delays are typically smaller than 1 day or much smaller than statistical uncertainties, as seen in Birrer et al. (2018), the result could be seen as unaffected. The relative motion of the source and the monitoring time also matter. If the images stay locally on the map, the mean microlensing time delay may be close to zero but still non-negligible. We could use either a constant for the whole 10 yr light curve or the epoch-dependent model for describing the light curve, which requires further discussions. Selecting lenses with larger time delays like SDSS 1206+4332 (Birrer et al. 2018) may make the absolute systematics less important; for example, we have estimated that for 1/4 of the systems with time delays larger than 60 days, the statistical uncertainty of $H_0$ is 0.81% and the systematics are almost the same.

We may also extend the strong-lensing systems using other sources; for example, the supernovae are point sources such that the time delays measured are cosmological (Kelly et al. 2015). Furthermore, transients like gravitational waves (Liao et al. 2017a) can also measure the time delays precisely and accurately.

Finally, we note that we only test the systematics of time delays in this work; in addition, the lens modeling and LOS should also be tested to understand the systematics therein. The results in this work should not be simply seen as the predictions of LSST cosmology. The systematic floor needs to be further studied to determine how powerful the lensing method could be from Earth in future cosmological studies.

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