Scaffolding in problem based learning to increase students’ achievements in linear algebra

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Abstract. The aim of this research is to improve the undergraduate students’ learning achievements in Linear Algebra course. This research was a classroom action research which has two cycles, each consist of four stages, i.e.: planning, implementation, observation/evaluation, and reflection. The subject of the present study was 23 second semester students of Mathematics Education Department, Universitas Pendidikan Ganesha, in academic year 2015/2016. The data about students’ learning achievement was collected from students’ written work in solving Linear Algebra test. The gathered data was analyzed descriptively. The results of this research showed that the implementation of scaffolding in problem based learning are able to improve students’ learning achievement.

1. Introduction
Linear Algebra is one of the basic course in mathematics education department, which is appear in the second semester for the first year university’s students. The course is generally about generalization of a formal theory. Most of the materials of the course are actually discussed in the high school. Therefore the students will be able to understand the content of the course if they mastered the mathematical concepts in senior high school. Unfortunately, the students were struggling in this prerequisite knowledge, for instance in the use of symbols and in the mathematical problems which asking for proving something.

Stewart & Thomas [1] said that the linear algebra is one of demanding course because it is based on a mathematical theory and included many definitions which is served in a very frequent. Hence, the students hardly see the connection between their prior knowledge about mathematical objects in high school with the materials of linear algebra.

To deal with the condition of the students, the lecturer frequently ask the students to learn before they enter the class. Also, to enhance students’ participation, the lecturer give score rewards for the active students. By doing those strategies, it is hope that the students’ understanding in the material will be increase.

Another strategy needed to increase students’ understanding in linear algebra is by implementing teaching and learning which stimulate creativity and reasoning ability. Therefore in this study, we implemented Problem Based Learning (PBL) with scaffolding. PBL had been chosen to solve the problem, because it is predicted that this model can be very helpful to guide the students from basic to abstract concept. It helps the students to be familiar with the mathematical problems and be confidence
to solve it. As is stated in Dolmans, De Grave, Wolfhagen & van der Vleuten [2], PBL is a prospective learning model for the next generation as it is useful to develop the students’ concept construction abilities, self-directed learning, cooperative works and contextualize mathematics based on the real problem encountered in society.

The scaffolding also become a support to enhance students’ successful in learning without making them stop thinking. In other words, we can say that the combination of PBL and scaffolding can motivate the students to achieve their best.

PBL is a constructivism based learning model. It means, this model encourage the learner to construct their own concept by their own exploration through the given situational and authentic problem [3]. The major characteristic of this model is the use of problems and questions related to the upcoming discussed materials to start the lesson. When the students cannot understand the problem, lecturer will indirectly explain it by using clues and cross questions. There are five steps described in PBL [4], including: (1) posing problems, (2) clarifying and investigating problems, (3) solving problems, (4) organizing report, and (5) presenting the work. From those steps, it can be seen that PBL is a student centered learning which give chance to construct their knowledge.

H.S Barrows (in [5]) stated that Problem Based Learning is a learning model which based on the principal that problem is the start to integrate or to find a new knowledge. Nowadays, PBL has been used in various field of education. In university level, PBL is a learning model in which the students can solve an authentic problem to construct their own knowledge, develop their inquiry skill, train their higher order thinking, and to empowering their self-confidence and self-regulated learning. In this case, the lecturer should provide non-routine problems with various solving methods and approaches for the students. It helps them to think critically and creatively to solve the mathematical problems. Hence, it can be seen that PBL is a student-centered learning model which is in line with the constructivism point of view about learning.

As in constructivism, a new knowledge constructed based on the students’ previous experiences [6]. In line with that it is reasonable to acknowledge the students’ prior knowledge will affect their learning [7]. Therefore, the lecturer should connect the students’ prior concept with the current discussed topic. By that, we can argue that mathematics learning is about building connection and understanding about mathematical knowledge.

However in many cases, PBL alone will be difficult to stimulate the students to learn, especially when the students were not used to think creatively in seeing a problem. Hence, the educator needs to provide a clue or limited guidance that can be used by the students to drive their elaboration. In education, this kind of help is known by scaffolding.

Scaffolding plays an important role for the success of the implementation of PBL. By definition, it can be acknowledged as a support for someone, given by classmate or adult who is more competent or capable in certain materials. Scaffolding means provide certain amount of support for children in the early steps and after that reduce it step by step to give them more opportunities to take over more responsibilities to finish their assignments [8]. In its interaction with PBL, we can explain that the students who get difficulties in construct certain concept will be given certain support until they able to master it. In this study, the support can be given by lecturer or from student to student.

Scaffolding is important especially in the beginning year of higher education, because the students are having an adjustment or transition from how they used to learn in school and how they expect to learn in university. The transition in the earlier stage of students’ life in university plays important role that will determine the success of students’ academic for the rest of study [9]. Therefore, they suggest to take care the idea of giving enough support for the students to have an optimal transition.

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The practical values of this research can be described as follows: (a) the participant of the current study will get learning experience by using PBL with scaffolding which hopefully able to increase
their learning achievement, and (b) the result of the study can be used as a consideration to improve the quality of linear algebra teaching and learning and also in different courses.

Reflecting to the aforementioned reason, the question of the research is how can the use of scaffolding in Problem Based Learning help the students to increase their achievement in linear algebra?

2. Methods
This study was an action research study, which aimed to improve the quality of learning process in linear algebra for the mathematics students of Universitas Pendidikan Ganesha. Kemmis & Tanggart [10] stated that “action research is a form of collective self-reflective enquiry undertaken by participants in social situations in order to improve the rationality and justice of their own social or educational practices, as well as their understanding of these practices and the situations in which these practices are carried out” (p.5).

Hine [11] argued that action research can be benefit for the improvement of the teaching and learning in the classroom because it has a genuine setting as the students are keep in their nature and the teacher also serve them as a teacher who also become a researcher in the same time. Hence, it is clear that the teacher can understand her/his class better and can design many improvements based on the findings during the research.

There were 2 cycles employed in this study, each with four steps, including: planning, implementing, observing/evaluating, and reflecting. The design can be seen as in the following Figure 1.

![Figure 1. The steps of action research](image)

The subject of this study was 23 students of B-class mathematics education department. From those, only four students who actively engaged in discussion. The rest of the students were passive during the learning activities. In general, the students were not preparing themselves before they enter the class, for instance, by reading the upcoming discussed topic. They also afraid in the type of problems which asking about mathematical proof.

The result of the preliminary reflection before we conducted the study, found that the mean of the students’ score in achievement test was 49.09 (from 100). One of the difficult problems according to the students’ answer sheets was to determine the determinant of matrix with size $4 \times 4$. It was known
that there only 6 students whose answer this problem with correctness value 50%, while 6 others answered with correctness value less than 50%, 5 students gained score 0 and the rest incorrectly chose a strategy to solve the problem. Specifically, the students who used incorrect strategy was using strategy in finding determinant for $3 \times 3$ matrix to solve the determinant for $4 \times 4$ matrix. Hence, it can be stated that they had a misconception in this topic.

To solve the aforementioned problem, this study employed the PBL model with the support of scaffolding to conduct a lesson in linear algebra. The PBL was employed by started the class by giving an authentic problem and asked the students to use their prior knowledge to elaborate it. For instance, when deal with the topic of linear equation system, a problem of price of some foods were given where the students need to create the mathematical model of it afterwards. From their experience in the high school, they were familiar with the linear equation with two and three variables, and were able to solve it by using elimination, substitution, or graphical methods. Started with that knowledge, the designed lesson challenged the students to think the general step of it. When the students lost, the scaffolding helped by asking students several questions, such as: “what is actually happened when the same variable are eliminated? Why can they be substituted? What is the fact showed by the graph?”

As mentioned before, we employed four steps in this classroom action study. The first step is Planning. There are three activities in this step, including: (a) fixing the schedule for face-to-face meeting, (b) preparing the students’ worksheet, (c) preparing research instrument which is achievement test in linear algebra in form of essay test.

The second step is Implementation. There are four activities in this step, including: (a) distributing the students’ worksheet, (b) group discussion, (c) presenting the result of the discussion in front of the class, and (d) making conclusion. During the Implementation stage, the researcher observe implementation of PBL with scaffolding and how the participation and activity of the students during the teaching and learning process.

The third step is Observation/Evaluation which will be done by checking the students’ responses in answering the prepared achievement. The fourth step, Reflection, will be conducted in the end of the First Cycle by evaluating the students’ difficulties in following linear algebra lesson. The result of Reflection of the First Cycle will be used to improve the lesson plan for the Second Cycle. The Second Cycle has the same steps as the first one, with improvement in the activity during the lesson. The gathered data will be analyzed by calculating the mean of the achievement test’ score and comparing with the previous mean of the achievement test’ score.

3. Results and Discussion

Before conducted the research, a preliminary reflection was done by giving the students a short explanation, continued by problem example and then an exercise task. In the end of the preliminary reflection, the students were given an achievement test using essay form. Based on the result of the test, it is found that the mean of the students’ score was 49.09 (from 100). Specifically, there are 13 students (56.52%) whose score below 55 which means they failed in the test.

During the implementation data were gathered by using the research instrument and in the end of the cycle, a test about students’ achievement in linear algebra was given. After the implementation of PBL with scaffolding, it is found that the mean of the students’ score in achievement test in the first cycle was 65.17. From those, 8 students was gained A (34.78%), 3 students gained B (13.04%), 4 students gained C (17.39), 3 students was gained D (13.04%) and 5 students was gained E (21.74%).

In the second cycle, the mean score of linear algebra achievement test of the students was 74.74. The details can be described as follows: 12 students achieved A (52.17%), 2 students achieved B (8.70%), 4 students achieved C (17.39%), 2 students achieved D (8.70%) and 2 students achieved E (8.70%). The comparison chart of the students’ range of scores during the first and second cycles can be seen in the following Figure 2.
Figure 2. Comparison of students’ score in the first and second cycle

Based on the evaluation of the achievement test conducted in preliminary reflection, cycle I and cycle II, it can be seen that there is an increase of the students’ achievement in learning linear algebra, even though not really high. The improvement of students’ achievement is caused by the implementation of PBL with scaffolding.

In the first cycle, even though the students’ score increased, the minimum criteria of students’ achievement was not fulfilled. It happened due to the not optimal implementation of PBL with scaffolding. Also, the lesson in the first cycle was dominated by certain group, especially the best one. In the cycle II, this factor was improved to support all students equally. This result confirm another finding which stated that in the learning process, cognitive aspect is not enough, PBL should be implemented because the students need to develop their metacognitive abilities and also their motivation in learning [12].

The analysis of the cycle II indicated that the students’ achievement in learning linear algebra improved and the minimum score targeted was fulfilled. This is happened because the optimal implementation of PBL and effective scaffolding which give the students more opportunities to think and work cooperatively with their classmate in solving the given problems.

In general, the scaffolding provides support for the students in term of posing a hint question that can drive them to new connection in their knowledge construction. For instance, in learning Elementary Row Operation, the students got difficulty to understanding the process. The common strategy they used were remembering the steps as it given in the textbook. Without enough understanding, the written steps become inefficient. It is happened because the students ignored the actual goal of elementary row operation is to find the value of each variable and only focused on multiplying numbers. When this happened, the lecturer helped by reminding the students their process of finding the solution of linear equation in two or three variables. For instance, “what was the aim of eliminating these variable?” (pointing at the same variable) and after the students gave the correct responses, the question was continued by “so, what should you do to have the similar answer as you do elimination? It just more variable, but the key idea has no different”.

Another important scaffolding was in helping students to differentiate and in the same time, seeing the similarities, between Gauss Elimination and Gauss-Jordan Elimination. Both were started in the same method and aim to find the solution of linear equation systems as well. Therefore, the students should not be confused why it looks similar. The different only that the Gauss-Jordan have the last step which is finishing the matrix in the reduced-row echelon form. Meanwhile it seems simple case, but from the observation, many students found it unclear, because they unsure whether the steps were same or not. To help the students grasping the method, the lecture ask them to create table of comparison between the steps of Gauss Elimination and Gauss-Jordan Elimination. By trying it
themselves and not merely relying by the given procedures on the book, they were able to see that until certain steps, the process is same. They also concluded that the Gauss-Jordan Elimination was developed from Gauss Elimination itself and both are lead to the same answer.

Based on the improvement of students’ score in linear algebra achievement test during the research, it can be seen that the implementation of PBL with scaffolding is able to support the students in learning. Besides that, it is powerful to manage more dynamic classroom, where the students actively discuss and construct their meaningful understanding, and the lecturer no longer as the center of the study. The previous study was comparing the effect of PBL and traditional method to the students’ achievement in mathematics. It was found that the students who learn with PBL were performing better because in that class the students feel challenged to study and willing to discuss with their group when some obstacles emerged [13].

Moreover, the students who learn mathematics through PBL model also known to be more confidence, because they can making sense the problem and the topic they learned with the reality they have or know previously [14]. It becomes a powerful start for the students to be motivated in learning and solving mathematical problems and later improve the students’ mathematical achievement.

The result of this study found that when the students have more chance to manage their own learning construction, they performed better (see the improvement of students in cycle 2 from cycle 1). This improvement can be preserved in future time [15]. The author argue that PBL is able to help students’ to achieve their best by enabling the students to develop the ability of solving problem efficiently and develop the intrinsic motivation of the students. Therefore, the students will have a skill to life-long learning which will be meaningful for them.

This is in line with the argument saying knowledge is built in students’ knowledge based on their prior knowledge [6]. Besides that, the students can construct their own knowledge through solving a problem which is started from what they already know into what they are going to learn. Therefore, it is necessary for the educator to connect the students’ preconception towards a learning material before move to the current material.

4. Conclusion
Based on the results and discussion, it can be concluded that the implementation of PBL with scaffolding can improve the students’ achievement in linear algebra. It can be seen from the increase of the mean of students’ test score from 49.09 in the preliminary reflection to 65.17 in the first cycle and 74.74 in second cycle. Therefore, it is recommended to the lecturer to use PBL model with scaffolding in teaching linear algebra. To enhance the students’ application ability, it also recommend to use authentic problem or at least a related to the real-world-mathematical problems.

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