The Sagnac effect on a scale-dependent rotating BTZ black hole background

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Abstract
In this paper, we investigate the Sagnac effect by calculating the difference in travel time and phase shift observed for photon beams counter-propagating in a rotating interferometer on a BTZ black hole solution in the context of scale-dependent gravity, which describes the field around a massive static and rotating object in 2 + 1 gravity.

Keywords: 2 + 1 dimensions, scale-dependence, gravity, Sagnac effect

(Some figures may appear in colour only in the online journal)

1. Introduction

It is very well known that black holes (BHs hereafter) are a generic prediction of Einstein’s general relativity (GR). Nowadays, we know that they are more than just simple solutions of Einstein field equations. Of crucial relevance in the black hole theory was Hawking’s work [1, 2], where it was shown that black holes indeed emit radiation from their horizon, the reason why they are an excellent scenario to study and understand different aspects of gravitational theories.

Thus, as the Hawking radiation lies at the Frontier between GR and quantum field theory (QFT hereafter) [3], a detailed analysis of this and other effects could help us in search of a complete theory of quantum gravity. Despite the fact that Hawking radiation is usually considered as blackbody radiation, it is not entirely true. The so-called greybody factor measures the modification of the original black body radiation (see [4, 5] for early calculations and [6–10] for recent applications).

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Black holes are described by a few parameters: the mass, the angular momentum, and the charge. Besides, they have interesting properties which, after the LIGO direct detections of gravitational waves [11–13], have received considerable attention. This is the case of the quasinormal modes (QNM) of black holes, which encoded invaluable information regarding the solution’s aforementioned parameters. For classical review see [14, 15] and for more recent solutions see [9, 16–18]. Given that BHs collect classical and quantum effects, the research of these kinds of objects could give us insights about the appropriate way to mix gravity with quantum mechanics.

Gravity in (2 + 1) dimensions is a perfect background to investigate in detail several well-known effects. Among other interesting properties, gravity at 2 + 1 dimensions has a deep connection with Chern–Simons theory [19–21], reason why it receives considerable attention. What is more, an extra motivation to investigate black holes in three dimensions is their crucial role in the context of the AdS/CFT correspondence [22–25]. The first black hole solution with negative cosmological constant in 2 + 1 dimensions was the so-called Bañados–Teitelboim–Zanelli (BTZ hereafter) solution, which is characterized by mass, angular momentum and charge [26, 27].

Since its discovery, the BTZ black hole solution has been object of a great amount of investigations based, for example, in its geodesic structure [28, 29], thermodynamics properties [30–32], quasinormal modes [9, 33, 34], stable and regular interior solutions that matches with a BTZ background [35–42], among others. Three-dimensional gravity can also be considered as a theory without a field of gravitation; where no matter is present, space is flat. In such a case, matter is required to produce curvature. As was pointed out by Staruszkiewicz [43], point particles are the most simple material objects to be considered in 3D. Thus, it should be noticed the close relation of (2 + 1) dimensional models with those commonly called ‘spinning point particle’-solutions, widely studied by G ‘t Hooft and collaborators [44, 45]. In this regards, Clément [46] computed a new rotating point-particles solution in 3D gravity. Also, the same author found the charged solution in reference [47]. For more recent applications see [48] and references therein. Finally, in those ‘spinning point particle’-solutions, the cosmological constant is not required, so the models become more symmetric.

In particular, we put our attention in the Sagnac effect, which is, roughly speaking, ‘a comparison of roundtrip times of two light signals traveling in opposite directions along a closed path on a rotating disc’ [49]. Our goal here is to discuss the Sagnac effect on a scale-dependent rotating BTZ black hole background and compare it with the classical counterpart, as well as analyze if some interesting featuring appears after the inclusion of scale-dependent coupling. This article is organized as follows: after this brief introduction, we will discuss the fundamental ingredients of the scale-dependent theory of gravity in section 2 and, after that, in section 3 we will discuss the scale-dependent black hole solution in (2 + 1) dimensions briefly. Then, in section 4 we perform an analysis of two counter-propagating photons to compute the Sagnac effect on this scale-dependent rotating BTZ black hole solution. Finally, in the section 5 we will briefly summarize and discuss the main result of this paper.

2. Scale-dependent theory

In this section we will summarize the equations of motion for the scale-dependent BTZ black hole solution with angular momentum. The idea and notation follows [50–62]. Firstly, the scale-dependent couplings of the theory are two: (i) the Newton’s constant $G_k$ and (ii) the cosmological coupling $\Lambda_k$. Notice that Newton’s coupling can be related to the gravitational coupling via $\kappa_k \equiv 8\pi G_k$. What is more, there are two independent fields; they are: (i) the metric
field $g_{\mu\nu}(x)$ and (ii) the scale field $k(x)$. The effective action is then written as
\[
\Gamma[g_{\mu\nu},k] = \int d^4x\sqrt{-g} \left[ \frac{1}{2\kappa_k} (R - 2\Lambda_k) + \mathcal{L}_M \right],
\]
(1)
where $\mathcal{L}_M$ is the Lagrangian density of the matter fields, and the effective Einstein field equations are obtained by varying the action with respect to the inverse metric field, which produce:
\[
G_{\mu\nu} + \Lambda_k g_{\mu\nu} \equiv \kappa_k T_{\mu\nu}^{\text{effect}}
\]
(2)
where the effective energy–momentum tensor is defined according to
\[
\kappa_k T_{\mu\nu}^{\text{effect}} = \kappa_k T_{\mu\nu}^M - \Delta t_{\mu\nu}.
\]
(3)
Thus, we can understand this effective term as follow: first, the term $T_{\mu\nu}^M$ is the usual contribution given by a matter source, whereas the second term $\Delta t_{\mu\nu}$ encoded the scale-dependent effect, which incorporate any quantum correction. This new tensor is defined as:
\[
\Delta t_{\mu\nu} \equiv G_k \left( g_{\mu\nu}\Box - \nabla_{\mu} \nabla_{\nu} \right) G_k^{-1}.
\]
(4)
In what follows, we will consider a background without any matter source (i.e., $T_{\mu\nu}^M = 0$) to focus our attention on any effects, if it is present, given by the scale-dependent scenario. To complete the set of equations, we can vary the effective action with respect to the scale-field $k(x)$ to produce
\[
\left[ R \frac{\partial}{\partial k} \left( \frac{1}{G_k} \right) - 2 \frac{\partial}{\partial k} \left( \frac{\Lambda_k}{G_k} \right) \right] \cdot \partial k = 0.
\]
(5)
While the above equation closes the system, the implementation of this is a difficult task. Besides, the previous equations are complemented by the relations corresponding to global symmetries of the system. Precisely, for the case of coordinate transformations we have
\[
\nabla^\mu G_{\mu\nu} = 0.
\]
(6)
In the next section, we will briefly discuss a new black hole solution in the context of scale-dependent couplings inspired by quantum gravity recently reported in [58].

3. Black hole solutions

Adopting circular symmetry, and only radial dependence on the metric functions, we have the line element defined in terms of the usual Schwarzschild coordinates ($ct, r, \phi$) according to
\[
dx^2 = -f(r)\,d(ct)^2 + \frac{dr^2}{f(r)} + r^2[N(r)d(ct) + d\phi]^2.
\]
(7)
Also, notice that $f(r)$ and $N(r)$ are the lapse and shift functions, respectively, and they should be obtained with the help of the effective Einstein field equations. Thus, both functions are computed, and therefore, no additional assumptions (beyond they depend on the radial coordinate only) are made. Solving the corresponding effective Einstein field equations, combined with the line element (7), we have the scale-dependent functions given by
\[
G(r) = \frac{G_0}{1 + \epsilon r},
\]
(8)
\[ N(r) = -\frac{4G_0J_0}{c^2r^2} Y(r), \]  
\[ f(r) = -8\frac{G_0M_0}{c^2} Y(r) + \frac{r^2}{\ell_0^2} + \frac{16G_0^2J_0^2}{c^6r^2} Y(r)^2, \]  
\[ \Lambda(r) = -\frac{1}{\ell_0^2} \left( \frac{1 + 3\epsilon r}{1 + \epsilon r} \right) - \frac{N^2(r)}{4} \left( \frac{\ln Y(r)}{Y(r)} \right)^2 + \frac{8M_0G(r)Y(r)}{c^2r^2} \left[ \epsilon r + \frac{1 + 2\epsilon r}{2} \left( \frac{\ln Y(r)}{Y(r)} \right) \right] - N^2(r) \frac{\epsilon r}{1 + \epsilon r} \left( \frac{\ln Y(r)}{Y(r)} \right) \]  
where  
\[ Y(r) \equiv 1 - 2\epsilon r \ln \left( 1 + \frac{1}{r\epsilon} \right). \]  

This solution is parameterized by five constants of integration, which are \{ \( G_0, J_0, M_0, \Lambda_0, \epsilon \) \}. Their physical meaning is directly linked to the classical solution as well as the non-rotating case. On one hand, the parameter \( J_0 \to 0 \) does not appear in the scale-dependent but non-rotating solution \[50\]. Taking this into account, the condition \( J_0 \to 0 \) implies that the solutions (8)–(11) are reduced to the non-rotating scale-dependent black hole (see \[50\]). Also notice that the solution used here differs from \[50\] by a numerical factor 8. The latter can be adjusted conveniently. On the other hand, the rotating classical solution \[26, 27\], should be obtained when the running parameter \( \epsilon \) is taken to be zero, i.e.,  
\[ \lim_{\epsilon \to 0} G(r) = G_0, \]  
\[ \lim_{\epsilon \to 0} N(r) = N_0(r) \equiv -\frac{4G_0J_0}{c^2r^2}, \]  
\[ \lim_{\epsilon \to 0} f(r) = f_0(r) \equiv -8\frac{M_0G_0}{c^2} + \frac{r^2}{\ell_0^2} + \frac{16G_0^2J_0^2}{c^6r^2}, \]  
\[ \lim_{\epsilon \to 0} \Lambda(r) = \Lambda_0. \]  

It could be interesting to analyze the case where \( \epsilon \) or \( J_0 \) is taken to be large. Regarding these possibilities, we have to say that: (i) the scale-dependent parameter cannot be large following the spirit of quantum corrections. The latter means that the potential deviations respect the classical solution are quite small, and (ii) the angular momentum \( J_0 \) cannot be taken very largely. The reason for that is shown in \[58\]. So, the scale-dependent rotating BTZ black hole has a maximum value for the angular momentum, which is \( J_0^{\text{max}} \equiv M_0\ell_0 \), which coincides with the classical one.

As we said, corrections due to quantum scale dependence should be small. Therefore, we expand the functions to first order in the \( \epsilon \) parameter which implies that the metric functions are given by  
\[ G(r) \approx G_0(1 - \epsilon r), \]  
\[ N(r) \approx N_0(r)(1 - 2\epsilon r). \]
Figure 1. Schematic of the Sagnac effect as measured by an observer with a clock rotating with constant angular velocity $\Omega$. The proper time difference between the emission and absorption of the co-propagating (+) and the counter-propagating (−) beam is considered.

\[
f(r) \approx f_0(r)(1 - 4 \epsilon r) - 4 \epsilon r \left[ \frac{4G_0 M_0}{c^2} - \left( \frac{r}{\ell_0} \right)^2 \right], \quad (19)
\]

\[
\Lambda(r) \approx \Lambda_0(1 + 2 \epsilon r). \quad (20)
\]

4. Sagnac effect

To investigate the Sagnac effect, we consider a $2 + 1$ circularly symmetric distribution of radii $r = R_* > r_+$ and classical mass $M_0 = M_* \text{ embedded in an exterior scale-dependent BTZ background, as is shown in figure 1. Rotating and static fluid distributions that match with the classical BTZ can be found, for example, in [35–42]. Assuming that the coordinates associated with the distribution are } (t', r', \phi'), \text{ we perform a coordinate change to another frame fixed in an interferometer that rotates at } \dot{r}' = R > R_. \text{ Thus, the transformation to the frame of the rotating platform}

\[
ct = ct', \quad \phi = \phi' - \Omega t', \quad (21)
\]

where $\Omega$ is the constant angular velocity of the physical frame, yields to the following metric coefficients

\[
g_{tt} = - f(R) + R^2 \left( N(R) + \frac{\Omega}{c} \right)^2, \\
g_{\phi\phi} = R^2, \\
g_{t\phi} = g_{\phi t} = R^2 \left( N(R) + \frac{\Omega}{c} \right). \quad (23)
\]

Therefore, the non-zero components of the unit vector $\gamma^a$ along the trajectory $r = R$ are given by

\[
\gamma' = \frac{1}{\sqrt{g_{tt}}} = \gamma_j, \quad (24)
\]
\[
\gamma_t = -\sqrt{-g_{tt}} = -\gamma_t^{-1},
\]
(25)
\[
\gamma_\phi = g_{\phi t} \gamma_t^t = R^2 \left( N(R) + \frac{\Omega}{c} \right) \gamma_J,
\]
(26)
where
\[
\gamma_J = \left[ f(R) - R^2 \left( N(R) + \frac{\Omega}{c} \right) \right]^{-1/2},
\]
(27)
and thus, the gravito-magnetic vector potential becomes
\[
\vec{A}_G^\phi = c^2 \gamma_\phi \gamma_t = -c^2 R^2 \left( N(R) + \frac{\Omega}{c} \right) \gamma_J^2.
\]
(28)
As was shown in [63–67], it is possible to express the phase shift \(\Delta \Phi\) and time delay \(\Delta \tau\) between light beams detected by a co-moving observer on the interferometer in terms of the gravito-magnetic vector potential employing the expressions
\[
\Delta \Phi = \frac{2 E \gamma_t}{\hbar} \int_{(S)} \vec{A}_G^\phi \cdot d\vec{r},
\]
(29)
and
\[
\Delta \tau = \frac{2 \gamma_t}{c^3} \int_{(S)} \vec{A}_G^\phi \cdot d\vec{r},
\]
(30)
where \(E\) is the relative energy of the photon as measured in the interferometer.

Then, the phase shift turns out to be
\[
\Delta \Phi = \frac{\tau_0 E}{\hbar} \frac{\Omega - \Omega_0}{\sqrt{\Omega_\infty^2 - (\Omega - \Omega_0)^2}},
\]
(31)
and the (dimensionless) time delay becomes
\[
\Delta \tilde{\tau} \equiv \frac{\Delta \tau}{\tau_0} = \frac{\Omega - \Omega_0}{\sqrt{\Omega_\infty^2 - (\Omega - \Omega_0)^2}},
\]
(32)
where \(\tau_0 \equiv \frac{8\pi R}{c}, Y_R \equiv Y(R)\), and
\[
\Omega_\infty = \sqrt{\Omega_\ell^2 + \Omega_0^2 - \frac{8 G_0 M_\ast}{R^2} Y_R},
\]
(33)
\[
\Omega_\ell = \frac{c}{\ell},
\]
(34)
\[
\Omega_0 = 4 \frac{G_0 J_0}{c^2 R^2} Y_R \equiv 4 \Omega_J Y_R.
\]
(35)
Note that \(\Omega_J\) can be interpreted as the precession velocity for the Lense–Thirring effect in the \(2 + 1\) gravity [68, 69]:
Figure 2. LEFT: plot for the dimensionless time delay $\Delta \tilde{\tau}$ as a function of the angular velocity $\Omega$ for different values of the running parameter: solid black line $\epsilon = 10^{-1}$, dashed red line $\epsilon = 10^{-2}$ and dotted blue line $\epsilon = 10^{-3}$ (in arbitrary reciprocal length units) and fixed radii of the interferometer $R = 20$ (in arbitrary length units). MIDDLE: plot for the dimensionless time delay $\Delta \tilde{\tau}$ as a function of the radii of the interferometer $R$ for different values of the running parameter: solid black line $\epsilon = 10^{-1}$, dashed red line $\epsilon = 10^{-2}$ and dotted blue line $\epsilon = 10^{-3}$ (in arbitrary reciprocal length units) and fixed radii of the interferometer $R = 20$ (in arbitrary length units). RIGHT: this plot shows the dimensionless time delay $\Delta \tilde{\tau}$ as a function of the running parameter $\epsilon$ for fixed orbital radii $R = 20$ (in arbitrary length units) and different values of the angular velocity: solid black line $\Omega = 7.5 \times 10^{-2}$, dashed red line $\Omega = 6.5 \times 10^{-2}$ and dotted blue line $\Omega = 5.5 \times 10^{-2}$ (in arbitrary reciprocal time units).

The dimensionless time delay $\Delta \tilde{\tau}$ can be written as

$$\Delta \tilde{\tau} = \frac{\tilde{\Omega}}{\sqrt{1 - \frac{r_+^2}{Y_+} \left(\frac{r_+}{R}\right)^2 - \Omega^2}},$$

where $\tilde{\Omega} \equiv \Omega / \Omega_0$ is the dimensionless angular velocity, $Y_+ \equiv Y(r_+)$, and $r_+$ is the horizon event of the non-rotating scale-dependent BTZ black hole obtained from equation (10) for $J_0 = 0$. As is expected in this case, $\Omega = 0$ implies that no Sagnac effect arise. Also, note that

$$\omega_{LT} = -\Omega_J = -\frac{G_0 J_0}{c^2 R^2},$$

which allows us to identify $\Omega_{LT} = -\Omega_0$ as the precession velocity for the Lense–Thirring effect for the scale-dependent rotating BTZ black hole. Also, by inspection of equation (32), we see that if $\Omega = 0$, a time difference appears, which is due to the rotation of the source. Obviously, if $\Omega = \Omega_0$, there is not Sagnac effect, and an observer measures no time delay.

In the left panel of figure 2 we have plotted the dimensionless time delay $\Delta \tilde{\tau}$ as a function of the angular velocity of the interferometer $\Omega$ for different values of the running parameter $\epsilon$, showing that the increase in $\epsilon$ causes a decrease in the maximum angular velocity that the device can reach to obtain the Sagnac effect. In the middle panel of figure 2 we have plotted the dimensionless time delay $\Delta \tilde{\tau}$ as a function of the orbital radii of the interferometer $R$, for fixed value of the running parameter $\epsilon = 10^{-1}$ (in arbitrary reciprocal length units) and different values of the angular velocity: $\Omega = 7.5 \times 10^{-2}$, $\Omega = 6.5 \times 10^{-2}$ and $\Omega = 5.5 \times 10^{-2}$ (in arbitrary reciprocal time units). This plot shows that there is a minimum for $\tilde{\tau}$, which depends strongly on the running parameter such that the position of this minimum grows directly with $\epsilon$. In the right panel of figure 2 we show the dimensionless time delay $\Delta \tilde{\tau}$ as a function of the running parameter $\epsilon$ for fixed orbital radii $R = 20$ (in arbitrary length units) and different values of the angular velocity: $\Omega = 7.5 \times 10^{-2}$, $\Omega = 6.5 \times 10^{-2}$ and $\Omega = 5.5 \times 10^{-2}$ (in arbitrary reciprocal time units).

On the other hand, for the non-rotating case, $\Omega_J = 0$, the dimensionless time delay can be written as

$$\Delta \tilde{\tau} = \frac{\tilde{\Omega}}{\sqrt{1 - \frac{r_+^2}{Y_+} \left(\frac{r_+}{R}\right)^2 - \Omega^2}},$$

where $\tilde{Y} \equiv Y / \Omega_0$ is the dimensionless angular velocity, $Y_+ \equiv Y(r_+)$, and $r_+$ is the horizon event of the non-rotating scale-dependent BTZ black hole obtained from equation (10) for $J_0 = 0$. As is expected in this case, $\Omega = 0$ implies that no Sagnac effect arise. Also, note that
there is an upper limit for the (dimensionless) angular velocity $\tilde{\Omega}$ given by

$$\tilde{\Omega} < \tilde{\Omega}_R \equiv \sqrt{1 - \frac{Y}{Y^\prime} \left(\frac{r_+}{R}\right)^2}.$$  \hspace{1cm} (38)

In figure 3 we have plotted $\tilde{\Omega}$ as a function of $R$ for three values of the running parameter $\epsilon = 10^{-1}$, $\epsilon = 10^{-2}$ and $\epsilon = 10^{-5}$ in arbitrary reciprocal length units, and shows that the increase in $\epsilon$ causes the maximum angular velocity $\Omega_R$ to be reached faster.

An interesting remark for the last result is that in the classic non-rotating BTZ black hole the previous relation reduces to

$$\tilde{\Omega} < \tilde{\Omega}_R \equiv \sqrt{1 - \left(\frac{r_+}{R}\right)^2}.$$  \hspace{1cm} (39)

Recently, a similar result was obtained by Villanueva et al for a $3+1$ static toroidal topological black hole in conformal Weyl gravity [70], where the term $(r_+ / R)$ has a power index 3 instead the index 2 present in equation (38). This result looks dependent on the spatial dimension of the manifold, but that assertion deserves a deeper study.

Finally, note that for the classical rotating BTZ black hole, the time delay obtained by Raychaudhury [49] contains a sign mistake in his equation (18), since the quantity inside of the square root becomes negative for $r > r_+$. Thus, in the limit $\epsilon \to 0$ ($Y \to 1$), equation (32) reduces to the correct expression for the Sagnac effect for the classical rotating BTZ black hole.

5. Conclusions

In this article, we have studied the Sagnac effect in light of the scale-dependent scenario in a rotating BTZ black hole background. As our formalism incorporates quantum corrections, our solution generalizes the standard case previously reported in [49]. Remarkably, we should recover the classical Sagnac effect when we turn $\epsilon$ off. However, it is not the case. We thus conclude that the Raychaudhuri solution (and not our solution) has a minimal error. More precisely, the discrepancy appears when we analyze the global sign inside the square, resulting
in a non-physical solution for a Sagnac effect because the validity of his expression is for \( r < r_+ \) (equation (18) in his paper). A non-trivial feature is present in our solution; namely, in the scale-dependent scenario, the time delay decreases when the running parameter increases, an effect that can give some insights about the presence of quantum effects. Thus, it provides a new way to check if the corresponding quantum corrections modify the underlying theory. Also, an important remark about the non-rotating case, more precisely inside the square root of equation (39), is that the upper limit of the angular velocity depends on the term \((r_+/R)^2\) instead of \((r_+/R)^3\), which was found by Villanueva et al [70] in the context of the topological toroidal black hole in the 3 + 1 conformal Weyl gravity. This fact looks to depend on the number of spatial coordinates, but that assertion deserves a more in-depth investigation.

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