THE DUST SCATTERING HALO OF CYGNUS X-3
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ABSTRACT

Dust grains scatter X-ray light through small angles, producing a diffuse ‘halo’ image around bright X-ray point sources situated behind a large amount of interstellar material. We present analytic solutions to the integral for the dust scattering intensity, which allow for a Bayesian analysis of the scattering halo around Cygnus X-3. Fitting the halo surface brightness profile yields the dust grain size and spatial distribution. We assume a power law distribution of grain sizes \( n \propto a^{-p} \) and fit for \( p \), the grain radius cut-off \( a_{\text{max}} \), and dust mass column. A model where dust is distributed uniformly along the line of sight to Cyg X-3 fits the halo profile well, with \( p = 3.6 \) and \( a_{\text{max}} = 0.18 \mu m \). We also attempt a model consisting of dust screens, representative of a foreground spiral arm and star forming complex Cyg OB2. This requires a minimum of two dust screens: the closest containing 80% of the total dust mass, and the furthest being within 1 kpc of Cyg X-3. The best two-screen fit parameters yield \( p = 4.8 \) and \( a_{\text{max}} = 0.3 \mu m \). Regardless of which model was used, we found \( \tau_{\text{sca}} \sim 0.8 \ E_{\text{keV}}^{2/3} \). X-ray spectroscopy yields a total ISM column \( N_p \approx 7 \times 10^{22} \text{ cm}^{-2} \), which is higher than previous estimates. We combine this information with the dust mass column to calculate a dust-to-gas mass ratio. The uniform (two-screen) fit yields a ratio that is a fraction of (on the order of) that typically assumed for the Milky Way. By comparing halo profiles in different energy bins, we find hints that large dust grains may be contributing to the absorption of \( E < 2.5 \text{ keV} \) X-rays from Cyg X-3.

1. INTRODUCTION

Dust grains are a vital part of the interstellar medium (ISM), aiding in gas cooling for star formation, providing a site for chemical reactions, and acting as the seeds for planetesimal growth. ISM dust is typically observed in absorption over the UV and optical or emission in the infrared. High energy studies of interstellar dust grains compliment information at other wavelengths for several reasons. First, the dust grain scattering cross-section in the X-ray is highly sensitive to grain size. Thus X-ray scattering probes the large end of the size distribution; such grains are ‘grey’ and thereby affect the normalization of the optical extinction curves. Infrared emission \( (\lambda \lesssim 50 \mu m) \) also tends to be dominated by the smallest carbonaceous grains (PAHs); the larger 0.1 \( \mu m \) grains will glow at these wavelengths only when subjected to intense radiation \( (\text{Draine & Li 2007}) \). Second, dust grains are relatively transparent to X-rays. As a consequence, X-rays probe the full abundance of interstellar elements (gas plus dust) and are also able to probe higher ISM column densities than UV or optical studies.

We focus here on dust scattering of X-rays over small angles, which produces a diffuse halo image around bright X-ray point sources \( (\text{Overbeck 1965}) \). The first X-ray scattering halo was imaged with the Einstein Observatory around 4U1658 – 48 \( (\text{Roll 1983}) \). Since then, X-ray halos have been observed around Galactic X-ray sources \( (\text{e.g. Witt et al. 2001, Smith 2008}) \), anomalous X-ray pulsars \( (\text{Tiengo et al. 2010}) \), and gamma-ray bursts that pass through dusty regions of the Milky Way \( (\text{Vaughan et al. 2006}) \). We examine one of the brightest X-ray scattering halos available in the Chandra archive, associated with the high mass X-ray binary (HMXB) Cygnus X-3. We describe the dust scattering physics and foreground ISM environment below. The observation and PSF subtraction method is described in Section 2. We take the Bayesian approach to fitting the dust grain size distribution in Section 3 using two models for the spatial distribution of dust along the line of sight. In one case we assume dust is uniformly distributed; in the other we model the halo with two infinitesimally thin dust screens. We calculate the ISM dust-to-gas mass ratio from X-ray spectroscopic data in Section 4. In Section 5 we discuss the implications of each model fit, examine the hints that large dust grains are contributing to a decreased halo brightness at \( E < 2.5 \text{ keV} \), and compare our results to other papers that study ISM on the Cyg X-3 sight line. Finally, the Appendix contains the analytic solution for the halo intensity in the case of a power law distribution of dust grain sizes. These solutions allow for fast computation of the halo surface brightness profile, making Bayesian analysis feasible.

1.1. Dust scattering physics

Because Cyg X-3 is situated behind a very large column of gas and dust, it is completely absorbed for \( E < 1 \text{ keV} \). Our calculations for the dust scattering cross-section are safely in the Rayleigh-Gans (RG) regime of \( E_{\text{keV}} \gtrsim a_{\mu m} \), where \( a \) is the dust grain radius \( (\text{Smith & Dwek 1998}) \). We follow \( \text{Smith & Dwek (1998)} \) in applying the Drude approximation for the complex index of refraction, which
treats each dust grain as a sphere of free electrons. Assuming that there are on average two baryons in the atomic nucleus for every electron, the total dust scattering cross-section is

\[
\sigma_{\text{sca}} \approx 6.2 \times 10^{-7} \rho_3^2 a_{\mu m}^3 E_{\text{keV}}^{-2} \text{cm}^2
\]

(1)

where \( \rho_3 = \rho/(3 \text{ g cm}^{-3}) \), \( a_{\mu m} = a/(1 \text{ \mu m}) \), and \( E_{\text{keV}} = E/(1 \text{ keV}) \) are typical values.

The RG differential scattering cross-section contains a first order Bessel function that can be approximated by integrating the scattering cross-section along the line of sight, and requires that there are on average two baryons in the medium, with an X-ray point source at distance \( S \) away from the X-ray source at \( S' \) will see a flux

\[
L_S = F_\text{sca} = \int_{\text{abs}} \int_{\text{abs}} \sigma_{\text{sca}} \, dx \, da
\]

(4)

The effect of absorption should be considered carefully because the scattered light takes a longer path:

\[
\delta x = \frac{\alpha^2 (1 - x)}{2 x}
\]

(5)

Dust scattered light will thus be subject to an additional distance \( \delta \tau_{\text{abs}} = \delta \tau_{\text{abs}} \delta x \), where \( \tau_{\text{abs}} \) is the total absorption along distance \( D \). By the nature of small angle scattering, the observer at \( O \) will mostly view dust that is at intermediate distances: \( x \sim 1/2 \) and \( \delta x \sim \alpha^2/2 \). The largest observation angle available from ObsId 6601 is \( \alpha \approx 100'' \), resulting in \( \delta x \lesssim 10^{-7} \). This is negligible even for ISM columns where \( N_\text{HI} \gtrsim 10^{22} \text{ cm}^{-2} \), which have \( \tau_{\text{abs}} \gtrsim 1 \) \cite{Wilms2000}. Thus from here on forward we combine the absorption term with \( F_\text{sca} \) so that it represents the source flux as it is incident on the observer:

\[
F_\text{obs} = F_\text{sca} e^{-\tau_{\text{abs}}/\pi D^2}
\]

We consider two cases: a uniform distribution of dust grains along the line of sight, \( \xi(x) = 1 \), and an infinitesimally thin screen of dust grains at position \( x_s \), so that \( \xi(x) = \delta(x - x_s) \).

1.2. ISM column

Cyggnus X-3 is located in the galactic plane at \((l,b) = (79.8, +0.7)\). At a distance 7-13 kpc from the Sun \cite{Dickey1985,Predehl2000}, the HMXB is located behind one or two spiral arms (Perseus and the outer arm, e.g. \cite{Russell2003}). Furthermore, the nearby star forming Cygnus X region offers a complex laboratory for ISM study along the line of sight to Cyg X-3.

Radio surveys give a lower limit to the neutral hydrogen column near the Galactic plane, \( N_\text{HI} \gtrsim 10^{22} \text{ cm}^{-2} \) \cite{Kalberla2002}. Cyg X-3 is also situated behind the massive star cluster Cyg OB2 and its associated molecular clouds \cite{Knodel2003}. The Milky Way CO survey shows that the Cyg X-3 sightline is particularly patchy, but contains a total proton density \( N_\text{H} \approx 10^{22} \text{ cm}^{-2} \) in the form of molecular hydrogen \cite{Dame2001}. So in total we expect \( N_\text{HI} > 2 \times 10^{22} \text{ cm}^{-2} \) from radio observations.

Finally, an estimate of the total ISM column can be obtained from X-ray spectroscopy. Absorption by neutral hydrogen via the photoelectric effect dominates below 1 keV, while the metal content of the ISM accounts for a considerable fraction of the total absorption above 1 keV \cite{Wilms2000,Predehl1995} estimated \( N_\text{HI} \sim 3 - 4 \times 10^{22} \text{ cm}^{-2} \) from ROSAT observations, which is consistent with the above radio surveys.

2. OBSERVATION

With 0.5'' per pixel resolution and low background, \textit{Chandra} is the best X-ray observatory available for imaging dust scattering halos. ObsId 6601 is the longest observation (50 ks) of Cygnus X-3 taken with the High Energy Transmission Grating (HETG), which also provides 1-2 eV spectral resolution. The High Resolution Mirror Assembly (HRMA) on \textit{Chandra} focuses about 90% of the X-ray light into a few pixel (2'') region. The point spread function (PSF) is composed of two parts: the core, where the majority of light is focused, and the wings, where light is spread diffusely due to scattering off of fine surface features in the mirror.

Correctly subtracting the PSF from the image is of utmost importance for determining halo brightness, which is on the order of the PSF wing brightness. The PSF wings can be simulated with \textit{Chandra} calibration tools (CharT and MARX), but experimental verification from ground based calibration efforts is limited. We will use another bright X-ray point source as a PSF template, described below.
Bright sources can easily saturate the CCD detector on Chandra (ACIS), which makes it difficult to normalize the PSF correctly. When more than one X-ray photon hits a pixel before the CCD is read out, the resulting electron cloud will be interpreted as either a single photon of larger energy, or may be flagged as a cosmic ray and thrown out all together. This effect is known as pileup, and occurs for any source incident upon ACIS with a brightness on the order of one photon per readout time (typically 3 seconds). Cygnus X-3 is one of the brightest X-ray objects in the sky, with a flux around $6 \times 10^{-9}$ erg s$^{-1}$ cm$^{-2}$, suggesting 14 zeroth order counts per second in ObsId 6601.

Moderate amounts of pileup will flatten the PSF core and alter the CCD extracted spectrum by a factor of two. Heavy pileup is evident in ObsId 6601 – the point source core is hollowed out with zero counts in the center. Fortunately, we can mitigate the effects of pileup for point sources imaged with the HETG in place. The spectrum extracted from the dispersed light, which is rarely subject to pileup, can be used to correctly scale a PSF template.

To get an accurate shape of the PSF wings we must choose a bright X-ray point source with a very low dust column. 3C 273, a quasar observed above the Galactic plane, has a smooth power law spectrum and very low ISM column ($N_H \sim 10^{20}$ cm$^{-2}$, inferred from Schlegel et al. 1998). It is also bright, $\sim 10^{-10}$ erg s$^{-1}$ cm$^{-2}$, or 0.8 zeroth order counts per second. The degree of pileup for this object is closer to that of Cyg X-3 when compared to two other candidate template sources (QSO B1028+511 and Her X-1). Since pileup affects the shape of the PSF core, choosing the brightest calibration object will yield the shape of the core-wing transition that is closer to the true PSF of Cyg X-3.

We chose to build the PSF template with ObsId 459, a 39 ks exposure of 3C 273 imaged with HETG. A surface brightness profile of the zeroth order image was made with 0.5 keV wide bins between 1-6 keV. Each profile was background subtracted, then normalized by the flux as measured from the grating dispersed light. To construct the PSF for Cyg X-3, each template was scaled by the grating dispersed flux of Cyg X-3 for the respective bin.

There is a great deal of systematic error in the PSF core, because the zeroth order image of Cyg X-3 is much more piled up than 3C 273, which does not have a hollowed out core. We threw out data points from the core, where the PSF template exceeded the observed surface brightness, and only kept data points with a signal to noise $> 3$. The 1-6 keV residual surface brightness profile shows the dust scattering halo from 7 – 200 pixels, or 3.5 – 100″ (Figure 2).

The typical background level for a 50 ks observation is 0.02 counts pix$^{-2}$, two orders of magnitude below the scattering halo brightness. For this reason, we ignore the X-ray background in ObsId 6601.

There is a Bok globule located 16″ from Cyg X-3, observed from X-ray scattering (McCallough et al. 2013). We found that a 3″ by 4.5″ region covering the globule contained 7120 counts, which accounts for 20% of the total brightness at that radius. We removed the globule from the measurement shown in Figure 2 because our halo model assumes azimuthal symmetry.

The scattering halo measured from the zeroth order image might also be contaminated from the first order halo dispersed by the HETG. To test this, we extracted a radial surface brightness profile from a rectangular region oriented in the MEG dispersion direction. We found significant contamination from the MEG first order scattering halo in regions $> 100$ pixels away from the point source center. There is plenty of signal in the outer edges of the halo, which are covered by annuli of larger surface area. We thus confine our surface brightness profiles to a rectangular region perpendicular to both the MEG and HEG arms. Utilizing this region reduced the outermost profile data point by 4% and significantly altered some of our fit results (e.g. $a_{\text{max}}$ in Table 1 increased by 50%). This demonstrates how remarkably sensitive X-ray scattering is to grain size distributions.

### 3. FIT TO HALO PROFILE

It has been shown that a power law distribution ($N_d \propto a^{-p}$ with $p \approx 3.5$) of graphite and silicate grains reproduce extinction curves in the UV and optical (Mathis et al. 1977, hereafter MRN). For the RG-Drude scattering regime, the composition of the grains is not very important because they are approximated as spheres of free electrons. We use the average grain density $\rho = 3$ g cm$^{-3}$ (Draine 2011).

Updated grain size distributions beyond the simple power law regime have been developed to better reproduce infrared and microwave emission features (e.g. Weingartner & Draine 2001). However, this mainly affects the lower end of the grain size distribution. Since X-ray scattering is more sensitive to large grains, we choose the simpler power law size distribution. We fix the low end of the distribution at 0.005 $\mu$m and fit for $a_{\text{max}}$ and $p$.

Using $N_d \propto a^{-p}$ also allows for an analytic solution...
to the integral in Equation [4]. For dust distributed uniformly along the line of sight ($\xi = 1$),

$$I_h(\alpha, E) = \frac{F_o}{\sqrt{8\pi}} \frac{\tau_{sca}}{G_o(a, p, \alpha, E)} G_u(a, p, \alpha, E)$$

(6)

where $\tilde{\sigma}_0 = 1.04^{-1} E_{keV}^{-1}$, $G_p$ is an integral over a power law, and $G_u$ is a function of erf and incomplete gamma functions as defined in the Appendix. For an infinitesimally thin dust screen, where $\xi = \delta(x - x_\ast)$,

$$I_h(\alpha, E) = \frac{F_o}{x_\ast} \frac{\tau_{sca}}{2\pi \tilde{\sigma}_0(E)} G_u(a, p, \alpha, x_\ast, E)$$

(7)

which is also described in the Appendix. Screens produce a flat surface brightness profile, and a uniform distribution produces a cuspy profile.

We use the publicly available MCMC code emcee to explore the parameter space in a Bayesian analysis of possible halo fits [Foreman-Mackey et al. 2013]. At a fixed energy, the dust grain distribution parameters $a_{\text{max}}$ and $p$ mainly affect the scattering halo shape. The optical depth to scattering controls the halo normalization, which scales with the total dust mass column, $\tau_{sca} = \kappa_{sca}(a, p)M_d$.

### Table 1

**Uniform fit to dust scattering halo [1-6 keV]**

| Parameter       | Median | 1σ C.I.       | Units        |
|-----------------|--------|---------------|--------------|
| Mass column     | $M_d$  | $2.6 [1.8, 5.2]$ | $10^{-4}$ g cm$^{-2}$ |
| Size cut-off    | $a_{\text{max}}$ | $0.18 [0.16, 0.20]$ | $\mu$m |
| Power law       | $p$   | $3.6 [3.2, 4.1]$ |              |
| Optical depth   | $\tau_{sca}$ | $0.8 [0.7, 0.9]$ | keV$^2$ |

### Table 2

**Two screen fit to dust scattering halo**

| Parameter       | Median | 1σ C.I.       | Units        |
|-----------------|--------|---------------|--------------|
| Screen 1        | Mass column | $M_{d1} = 6.8 [1.5, 23]$ | $10^{-4}$ g cm$^{-2}$ |
| Position        | $x_1$ | $0.55 [0.41, 0.79]$ |              |
| Mass column     | $M_{d2}$ | $1.3 [0.3, 5.0]$ | $10^{-4}$ g cm$^{-2}$ |
| Position        | $x_2$ | $0.08 [0.05, 0.13]$ |              |
| Dust distribution | $a_{\text{max}}$ | $0.33 [0.23, 0.44]$ | $\mu$m |
| Power law       | $p$   | $4.5 [3.9, 5.1]$ |              |
| Total column    | $M_d$  | $8.1 [1.8, 28]$ | $10^{-4}$ g cm$^{-2}$ |
| Optical depth   | $\tau_{sca}E^2$ | $0.8 [0.6, 1.1]$ | keV$^2$ |

### Figure 3

The population distribution of likely fits are plotted in two-dimensional histograms comparing each pair of free parameters ($M_d$, $p$, and $a_{\text{max}}$) and each fit’s $\tau_{sca}$. There is particularly strong covariance between power-law exponent $p$ and dust mass $M_d$, making $\tau_{sca}$ covariant with both these parameters. The vertical dashed lines in each one-dimensional histogram mark the median and 1-σ confidence interval for each parameter.

### 3.1 Uniform fit

We start with the uniformly distributed dust because it has the least number of free parameters and will most likely match the shape of the halo profile, which is cuspy. Guess values of $M_d = 5 \times 10^{-4}$ g cm$^{-2}$, $a_{\text{max}} = 0.1 \mu$m, and $p = 3.5$ yield a reasonable fit by eye.$^4$

We assigned uniform priors to log($M_d$) from $-11.8$ to $-1.8$ cm$^2$ g$^{-1}$ and $a_{\text{max}}$ from $0.01$ to $0.5 \mu$m. We found that, if $p$ was allowed to take on large values, the halo fit solution becomes highly degenerate for grain sizes above $0.3 \mu$m. Larger $a_{\text{max}}$ values require increasingly large dust column and $p > 5$, so that the small end of the size distribution dominates. To suppress the uninformative degenerate solutions, we assigned a Gaussian prior to $p$ with mean 3.5 and standard deviation 0.5.

We used 100 walkers in emcee to obtain 900 independent samples from the population of possible fits. Figure 2 plots the model halo obtained from the median fit (grey dashed line) over the residual surface brightness profile. Figure 3 plots a histogram for each parameter against the others, illustrating that they are highly covariant. The total scattering optical depth was calculated for each sample point (bottom row). The vertical dashed lines indicate the 16th, 50th, and 84th quantiles – corresponding to the median and 1σ confidence interval. These values are listed in Table 1.

### 3.2 Scattering from Dust Screens

X-ray scattering can probe galactic structure in the direction of Cyg X-3, which might include features such as those associated with Cyg OB2 or Galactic spiral arms [12]. When applying the infinitesimally thin screen model, it should be kept in mind that while the total integrated halo flux will be fixed according to the optical depth of the screen, the surface brightness profile will vary according to the screen’s position. For screens closer to the observer (large $x$, see Figure 1), the scattered flux will be spread over a large area, reducing the overall surface brightness profile. When a screen is close

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$^4$ The initial dust mass column was estimated using $N_H = 3.5 \times 10^{22}$ cm$^{-2}$ [Predehl & Schmitt 1995] and assuming a dust-to-gas mass ratio typical of the Milky Way (0.009 [Draine 2011]).

$^5$ Corresponding approximately to $14 \leq \log (N_H) \leq 24$. 

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to the X-ray source (small $x$), the halo surface brightness profile will be more compact and thus brighter close to the point source.

Taking 9 kpc as the best estimate for the distance to Cyg X-3, we expect the Perseus arm, about 5 kpc away, to correspond to a screen with $x_s \approx 0.4$. The outer spiral arm, if in the foreground of Cyg X-3, will be at $x_s \leq 0.1$ and would therefore contribute most to the surface brightness profile. The Cyg OB2 association, 1.7 kpc away (Knodlseder 2000), corresponds to $x_s \approx 0.8$ and would contribute to the outermost portion of the profile.

We attempt to fit a model halo with the least number of dust screens, in this case two. In the optically thin regime, the intensity of the halo from each screen can simply be added to get the total observed halo. We ran emcee with a six parameter model containing two screen positions ($x_1$, $x_2$) and their respective dust mass ($M_{d1}$, $M_{d2}$), assuming the same dust grain distribution for both screens ($a_{\text{max}}$, $p$). The screen positions were given a uniform prior, with the first screen confined to regions close to the observer ($0.3 \leq x_1 \leq 1$) and the second screen close to Cyg X-3 ($0.01 \leq x_2 \leq 0.3$). The priors on $M_d$ and $p$ are the same as those in the uniform fit.

Table 2 lists the median fit parameters and 1σ confidence intervals for the two screen fit, including the total dust mass and scattering optical depth. Figure 2 shows the median fit screens from emcee (dotted lines) and their sum (solid grey).

### 4. DUST-TO-GAS MASS RATIO

The X-ray point source spectrum of Cyg X-3 can be used to fit the hydrogen column, assuming a near solar mix of ISM metals, which contribute significantly to the absorption of X-rays above 1 keV compared to neutral hydrogen by itself (Wilms et al. 2000). Along particularly dusty sight lines, it is also necessary to include the contribution of dust scattering to the total point source extinction. We fit the high resolution spectrum of Cyg X-3 with a three component model: (i) X-ray emission in the form of a power-law, (ii) ISM absorption using the metal abundances outlined in Wilms et al. (2000), and (iii) fixed extinction of the form $\tau_{\text{sc}} E_{\text{sc}}^{-2}$, with optical depths pulled from the results of emcee (Tables 1 and 2).

Figure 4 shows an example X-ray spectral fit with $\tau_{\text{sc}} = 0.8$. A particularly large column density ($N_H = 7 \times 10^{22}$ cm$^{-2}$) is required to produce the shape of the spectrum. Since a large fraction of the X-ray absorption above 1 keV is accounted for by interstellar metals, the $N_H$ value measured from the X-ray continuum should more closely follow the abundance of metals (and dust) than hydrogen. This may explain why $A_V$ per $N_H$ as measured by X-ray spectroscopy is well fit by a power-law even at large column densities (Predehl & Schmitt 1995, Gier & Ozel 2009). According to these studies, the ISM column $N_H \approx 7 \times 10^{22}$ cm$^{-2}$ suggests a large extinction, $A_V \approx 30 – 40$. This value is unusually high, given the average Galactic disk extinction rate of $A_V = 1.8$ mag/kpc (Whittet 1992), implying $A_V \approx 13 – 23$ for Cyg X-3.

The unique environment in the foreground of Cyg X-3 may explain the extinction implied by the rather large ISM column. The Cyg OB2 association exhibits a mean color excess of $E(B-V) \approx 1.2$ (i.e. $A_V \approx 3.7$ for typical Milky Way dust). The three cluster members closest to Cyg X-3 in angular distance, Cyg OB2 No 12, 3, and 9 (17', 18' and 20' away) are significantly more extincted: $E(B-V) = 3.2, 1.6$, and 1.7 implying $A_V \approx 10, 5, and 6$ (Johnson 1965, Nicolet 1978). This accounts for an additional 3-7 magnitudes of extinction beyond average and implies that the sight line to Cyg X-3 is unique.

A final caveat, however, is that the ISM column fit is deeply sensitive to the choice of X-ray emission model. The power law does not produce a particularly good fit ($\chi^2 = 4.1$), partly due to the strong emission lines distributed across the spectrum. The XSPEC model diskpn, a post-Newtonian accretion disk emission model appropriate for HMXBs such as Cyg X-3 (Gierlinski et al. 1999), yields a lower ISM column ($N_H = 6.4 \times 10^{22}$ cm$^{-2}$) at the expense of a slightly worse fit ($\chi^2 = 4.3$). Either way, absorption over the entire Chandra energy range is required to match the general shape of the spectrum. A multi-component model more accurately describes the X-ray emission of Cyg X-3, for which disk reflection tends to dominate the medium/high component around 6 keV (e.g. Szostek et al. 2008, Zdziarski et al. 2010). Since the soft model component is apparently dominated by absorption, we are unfortunately left in the dark regarding the true nature of the soft X-ray spectrum. In the future, X-ray scattering physics could be applied to less obscured X-ray binaries within our Galaxy to yield more accurate dust-to-gas mass ratios along different ISM sight lines.

After fitting for the ISM column, we calculate the dust-to-gas mass ratio ($R_{d_g}$) using the dust mass column ($M_d$) from the scattering halo fit.

$$R_{d_g} \equiv M_d/N_H m_p$$

Table 3 shows the ISM column fit and resulting mass ratio with 1σ confidence intervals. Both dust distribution models suggest similar hydrogen columns because the total scattering optical depth is about the same. The

**Figure 4.** High resolution spectrum of Cyg X-3 obtained from the grating dispersed light in Chandra ObsID 6601. The spectrum is modeled with an X-ray emission power law (light grey), modified by ISM gas absorption (medium grey line) and finally dust scattering (dark grey). The optical depth of the dust scattering component is 0.8 at 1 keV.
dust-to-gas mass ratio obtained from the uniform fit is particularly low, about 1/4 the typical value assumed for the Milky Way ($R_{dg} \approx 0.009$, Draine 2011). The ratio obtained from the two screen fit is typically higher, because it requires more dust by mass. However the two screen fit has a much broader uncertainty, from 1/4 to a few times Galactic. The lower bounds for the hydrogen density along this sight line, $N_H \geq 2 \times 10^{22}$ cm$^{-2}$ from radio observations (12), implies $R_{dg} \geq 0.007$.

5. DISCUSSION

The halo profile goodness of fit as measured by $\chi^2$ is strongly sensitive to minor variations ($\leq 5\%$) in the input parameters, due to the strong degeneracy. Thus it is possible to get $\chi^2 \sim 1$ for both fits. The uniform fit suggests that an MRN distribution is appropriate for describing the Milky Way dust population. This is somewhat surprising considering the large amount of molecular material that lies along the line of sight Cyg X-3. The cold dense regions of molecular cloud cores are likely to be a location of grain growth (Draine 2003). It is possible that Cyg X-3 lies coincidentally in a window of the Cygnus X region that was cleared out by the recent star formation that created the Cyg OB2 cluster (e.g. Fig 1 of Motte et al. 2007).

The two-screen fit requires a total dust mass that is a factor of three times greater than the uniform fit, despite using the same number of six parameters, which still allows a large uncertainty in the results. The uniform fit gives the more likely solution of the two.

5.1. Multiple scattering

Our surface brightness profile models assume an optically thin ISM so that photons only scatter once, but our fits show that the ISM is nearly optically thick to dust scattering. Since the spectral energy distribution (SED) of Cyg X-3 is highly absorbed, the point source flux peaks above 2 keV, where $\tau_{sca} \leq 0.2$. Weighted with the SED, we expect 20% of the total scattered light to come from $E < 2$ keV.

Our model halo flux is calculated from the observed point source flux, described by the following relation.

$$\frac{F_h}{F_{ps}} \equiv \frac{F_a (1 - e^{-\tau_{sca}})}{F_a e^{-\tau_{sca}}} = e^{\tau_{sca}} - 1 \quad (9)$$

The fraction of photons that scatter twice can thus be estimated with $\tau_{sca}^2/2$. A more detailed analysis of the effects of multiple scattering upon dust scattering halos is performed by Mathis & Lee (1991). Less than 10% of photons with $E > 1.5$ keV will undergo multiple scattering events. Weighted with the Cyg X-3 SED, less than 3% of the scattered photons are below 1.5 keV, for which double scattering should be noticeable. Since this is a tiny fraction of the total halo, we conclude that our single-scattering model is reliable for this particular object.

5.2. Hints at Larger Dust Grain Sizes

To test the viability of both fits presented in we split the image of Cyg X-3 into three energy bins. Since its spectrum is highly absorbed below 2 keV, the dust scattering halo is brightest at moderate energies. The energy bins expected to exhibit comparable levels of scattering halo brightness are 1-2.5 keV, 2.5-4 keV, and 4-6 keV. For both fits, the 1-2.5 keV scattering halo is dimmer than expected by a factor of 2-3 in the region $r < 50$ pixels and 30-50% dimmer for $r > 50$ pixels (Figure 5). This represents a $3-10\sigma$ offset, which is largest at the profile edges; the 2.5-6 keV halos are well fit within 1 $\sigma$. The low energy offset implies that there might be micron-sized grains in the foreground of Cyg X-3.

Throughout this study we apply the RG approximation, which holds for grains smaller than the photon energy such that $a_{\mu m} \lesssim E_{keV}$ (Smith & Dwek 1998). When grains become large enough to violate the RG approximation, the Mie scattering solution must be applied, which greatly decreases the scattering cross-section due to absorption by grain material. Unfortunately, calculating

| Parameter                | Median                  | 1σ C.L.                  | Units  |
|-------------------------|-------------------------|--------------------------|--------|
| Uniform Fit             |                         |                          |        |
| ISM column              | $N_H : 7.03$            | [6.97, 7.07]             | 10$^{22}$ cm$^{-2}$ |
| Dust-to-gas mass ratio  | $R_{dg} : 0.22$         | [0.15, 0.39]             | 10$^{-2}$ |
| Two Screen Fit          |                         |                          |        |
| ISM column              | $N_H : 7.0$             | [6.8, 7.1]               | 10$^{22}$ cm$^{-2}$ |
| Dust-to-gas mass ratio  | $R_{dg} : 0.7$          | [0.2, 2.5]               | 10$^{-2}$ |

Fig. 5.— The scattering halo of Cyg X-3 in two different energy bins, 1-2.5 keV (grey) and 2.5-6 keV (black). The uniform (dashed line) and two screen (solid line) fits from agree well with the high energy halo, but over predict the brightness of the low energy halo by a factor $\sim 2$. 

Hinting at Larger Dust Grain Sizes
the halo intensity with Mie scattering is computationally intensive, preventing a Bayesian analysis of the low energy scattering halo at this time.

Increasing to \( \sigma_{\text{max}} \sim 2 \mu \text{m} \) causes the model scattering halo to become more compact by a factor \( \sim 10 \). We found that the inner portion of the halo \( (r \lesssim 40 \text{ pixels}) \) can be fit with a uniform or screen model with \( p = 4 \) and \( \sigma_{\text{max}} = 2.5 \mu \text{m} \). This brings the Mie calculated scattering halo into closer agreement \((1 - \sigma)\) with the data points in the inner portion of the 1-2.5 keV halo. However the outer portion of the halo requires an extra contribution from small grains, which cannot account for the \( 10\sigma \) offset between model and data.

The most likely place to find large dust grains is in the region of the Cyg OB2 cluster. The scattering halo can be fit with a single dust screen associated with the cluster, only if large dust grains \((a \sim 5 \mu \text{m})\) are balanced with a large power-law exponent \((p = 5.7)\). The small dust grains contribute to the outer edge of the profile while the less abundant large dust grains contribute to the inner portion. However, this scattering model overpredicts the low energy halo intensity by an even larger amount \((a \text{ factor of } 5-10)\). The power-law slope is so large that small grains account for the vast majority of the dust mass. In this case the scattering opacity does not depart significantly from the RG approximation at low energies.

We have also evaluated our data for systematic errors regarding the source flux and production of the PSF template. Energy ranges with underestimated photon fluxes would cause us to overestimate the residual scattering halo. We used diskpn emission with ISM absorption and dust scattering to model the point source photon flux used to scale the PSF. Our flux model closely matches the shape of the continuum except in the region 3-5 keV. In this energy range, there is a discrepancy between the HEG and MEG extracted spectrum due to pileup, which occurs more easily for the MEG arm. However the photon flux in the HEG arm is only 10% brighter than the flux calculated with our model. If the HEG spectrum is moderately piled, the true photon flux would be a factor \( \sim 2 \) larger than the observed flux. This would reduce the moderate energy scattering profiles by \( \sim 50\% \) at \( r = 40 \) pixels and \( \sim 25\% \) at \( r = 100 \) pixels. Though pileup is a likely culprit, it does not completely account for the decreased brightness of the 1-2.5 keV halo relative to 2.5-6 keV.

A final option is that the magnitude of X-ray absorption by small grains is underestimated, indicating a problem with our theoretical understanding of X-ray scattering for \( E \lesssim 2.5 \text{ keV} \).

5.3. Comparison to other X-ray halo studies

The highly degenerate nature of X-ray scattering through dust makes the Bayesian approach to data analysis a powerful tool. The analytic solutions for the scattering halo surface brightness profile \( (\text{presented in the Appendix}) \) allowed for fast computation and Bayesian analysis that so far has not been applied in this field. The discussion presented here, however, is one in a line of several differing conclusions regarding the nature of dust along the line of sight to Cygnus X-3.

**Predehl & Schmitt (1995, hereafter PS95)** included Cyg X-3 in a study of X-ray scattering halos visible with ROSAT. The halo around Cyg X-3, being 40% as bright as the central point source, had the largest optical depth in the study. They fit the surface brightness profile by integrating Equation 4 with dust uniformly distributed between some \( x_{\text{min}} \) and \( x_{\text{max}} \) values, which are not reported in the text. Since the ROSAT PSF is much broader, only the outer part of the halo \((a > 100\% \text{ kpc})\) was visible, leading them to be more sensitive to dust close to the observer. Regardless, they got similar values for \( \sigma_{\text{max}} \) \((0.20 \mu \text{m})\) and \( p \) \((3.8)\) in comparison to our uniform fit.

Our fit to the ISM column differs dramatically from PS95, because *Chandra* covers a broader energy range than ROSAT, which is sensitive to 0.3-2.4 keV X-rays. Since the power law slope is expected to match the SED slope near the high energy side, PS95 measured \( N_{\text{HI}} = 3.3 - 4.1 \times 10^{22} \text{ cm}^{-2} \). It is apparent from Figure 4 that the slope you would get from examining the 2-3 keV spectrum is shallower than that from the 5-7 keV region, thus requiring a large amount of ISM absorption to account for the dramatic turn over around 3 keV.

The scattering halo of Cyg X-3 was also included in a study by Xiang et al. (2004). Using ObsID 425, they extracted the halo image from the 1-d projection of the halo data along the direction of MEG arm \( (\text{see also Yao et al. 2003}) \). This method of extraction produced surface brightness profiles that are steep in the inner 5” \((10 \text{ pixels})\) and less steep in the outer regions. This led them to conclude that all of the objects in their study were embedded in molecular clouds, because all of the dust mass needed to be located very close to the X-ray point source. In the case of Cyg X-3, this required \( N_{\text{HI}} \sim 4 \times 10^{22} \text{ cm}^{-2} \) within 100 pc of the HMXB. Our study does not resolve the scattering halo within 5 pixels of the point source, but it is possible that the particularly indirect method they used to resolve the scattering halo led to a systematic error in the inner regions of the halo. We measure the scattering halo directly from the zeroth order image of Cyg X-3 instead.

Cyg X-3 is an eclipsing binary with a 4.8 hour period \( (\text{Brinkman et al. 1972}) \), making it a good object for studying scattering in the time domain. Since the scattered light takes a longer path, there is a time delay between non-scattered and scattered light. Predehl et al. (2000) used the first *Chandra* observations of Cyg X-3 to determine a geometric distance of \( 9^{+2}_{-3} \) kpc without requiring many assumptions regarding the dust distribution. Ling et al. (2009) made a more in depth cross-correlation timing analysis of Cyg X-3. They found that dust associated with Cyg OB2 could account for time lags in the \( \alpha > 65^\circ \) halo, but that only accounted for 7% of the total dust along the line of sight. The remainder of the data was well fit with uniformly distributed dust. We do not perform a uniform plus screen fit because the screen contribution would be similarly negligible.

Depending on the Cyg OB2 distance, Ling et al. (2009) measured 3.4, 7.2, and 9.3 kpc to Cyg X-3. Our two-screen fit also yields interesting distant results. If the first screen is associated with Cyg OB2 at 1.7 kpc, then the true distance to Cyg X-3 would have to be about 3.7 kpc, consistent with one measurement by Ling et al. (2009). If the closest screen is instead associated with the
Perseus arm 5.4 kpc away, Cyg X-3 is 12 kpc away, consistent with the measurement by Predehl et al. (2000). The wide confidence interval on $x_1$ (Table 2) leads to a great uncertainty, preventing our method from uniquely measuring the distance to Cyg X-3.

ObsId 6601 covers three cycles of the Cyg X-3 eclipse. We used all of the light under the assumption that the average brightness of the point source describes the average brightness of the scattering halo. However, there is temporal variation in the scattering halo surface brightness for $\alpha \lesssim 15''$ (30 pixels). The error bars in these regions could be larger due to uncertainty from the light curve, lending more plausibility to our two screen fit. A full timing analysis is beyond the scope of this paper.

6. CONCLUSION

In the Appendix we present an analytic solution for the dust scattering halo intensity from a power law distribution of grain sizes in the RG regime. The solutions are in the form of erf and incomplete gamma functions, which are included in many common software packages such as scipy and Mathematica. This allows for a probabilistic approach to fitting the data, which required computing $\sim 10^5$ halos for each of our dust distribution models. The Bayesian approach is powerful because it allows us to adjust for degenerate solutions and to inject prior knowledge about the model (for example, the approximate positions of the dust screens).

We take this approach to analyze one of the brightest dust scattering halos available in the Chandra archive, that associated with Cyg X-3. We find that a uniform distribution of dust along the line of sight fits the scattering halo profile very well for the region $\alpha < 100''$ (200 pixels). This suggests an MRN type grain size distribution with a cut off at 0.18 $\mu$m. As an alternative, the scattering halo can also be fit with two infinitesimally thin dust screens placed in the foreground of Cyg X-3. About 80% of the dust would be close to the observer, perhaps associated with the Cyg OB2 stellar complex. The remainder would have to be within 0.5 - 1 kpc of Cyg X-3. The grain size distribution suggested by the two-screen fit has a similar cut off to MRN (0.3 $\mu$m), but a much steeper power law ($p=4.8$). Our results are fairly consistent with other published conclusions regarding the distance to Cyg X-3 and the dust grain distribution along its sight line.

We fit the continuum of the Cyg X-3 spectrum for ISM column ($N_H \approx 7 \times 10^{22}$ cm$^{-2}$) and compared it with the dust mass measurement from the scattering halo to yield a dust-to-gas mass ratio. Both of our scattering halo fits imply a departure from Milky Way average, but the uncertainties are large. The abnormally large extinction of Cyg OB2 cluster members (No. 12, 3, and 9) in the angular vicinity of Cyg X-3 emphasizes the unique nature of this particular region.

The nature of the 1-2.5 keV scattering halo, which is dimmer than expected from the fit model halos, might be explained by extra absorption of $E < 2.5$ keV X-rays by micron sized grains. We find that a grain distribution with $\theta_{\text{max}} = 2.5$ $\mu$m and $p = 4$ can resolve the problem only for inner portion of the halo. A full Bayesian analysis with Mie scattering is too computationally intensive to undertake for now.

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APPENDIX

The integral for the scattering halo intensity can be evaluated analytically under a few simple conditions. First, we assume that the grain size distribution is a power law function of grain size:

$$N_d \propto a^{-p}$$

(1)

Second, we use the Rayleigh-Gans differential cross section as described in the text (Equation 2):

$$\frac{d\sigma}{d\Omega} \propto a^6 \exp\left(\frac{-\alpha^2 a^2}{2\bar{\sigma}_0 x^2}\right)$$

(2)

where $\bar{\sigma}_0$ is the characteristic scattering angle for 1 $\mu$m size grains, such that $\bar{\sigma}_0 = 1.04' E_{\text{keV}}^{-1}$.

Finally, we assume that the medium along the line of sight is optically thin to dust scattering. The single-scattering halo intensity, integrated over solid angle, is

$$\int I_o d\Omega = F_a \tau_{\text{sca}}$$

(3)

For more information on second or third order scattering see Mathis \\& Lee (1991).

We can integrate Equation 4 over solid angle to solve for one of our normalization factors. We will use $A$ as a normalization constant that combines the dust grain size distribution and differential cross-section proportionality described above.

$$F_a A \int a^{6-p} \int x^{-2} \xi(x) \int_0^\infty \exp\left(\frac{-\alpha^2 a^2}{2\bar{\sigma}_0 x^2}\right) 2\pi \alpha \ da \ dx \ da = F_a \tau_{\text{sca}}$$

(4)
Integrating over $\alpha$ first will contribute an $(x/a)^2$ term, and we will drop the $\xi(x)$ term for now. This yields

$$A = \frac{\tau_{\text{sca}}}{2\pi \sigma_0^2 G_p(a, p)}$$  \hspace{1cm} (5)

where $G_p$ is a constant:

$$G_p(a, p) \equiv \int_{a_{\text{min}}}^{a_{\text{max}}} a^{4-p} \, da$$  \hspace{1cm} (6)

Under the above simplifying notation, Equation $\text{[4]}$ becomes

$$I_h(\alpha) = \frac{F_a \tau_{\text{sca}}}{2\pi \sigma_0^2} \int a^{6-p} \int x^{-2} \xi(x) \exp \left( -\frac{\alpha^2 a^2}{2\sigma_0^2 x^2} \right) \, dx \, da$$  \hspace{1cm} (7)

**SCREEN CASE**

In the case of an infinitesimally thin screen at position $x_s$, Equation $\text{[7]}$ becomes

$$I_h(\alpha) = \frac{F_a}{x_s^2} \frac{\tau_{\text{sca}}}{2\pi \sigma_0^2} \int a^{6-p} \exp \left( -\frac{\alpha^2 a^2}{2\sigma_0^2 x_s^2} \right) \, da$$  \hspace{1cm} (8)

which produces the solution

$$I_h(\alpha) = \frac{F_a}{x_s^2} \frac{\tau_{\text{sca}}}{2\pi \sigma_0^2} G_s(a, p, \alpha, x_s) \frac{G_s(a, p, \alpha, x_s)}{G_p(a, p)}$$  \hspace{1cm} (9)

where

$$G_s(a, p, \alpha, x_s) = -\frac{1}{2} \left[ \left( \frac{\alpha^2}{2\sigma_0^2 x_s^2} \right) \frac{a^2}{p^2} \left( 1 - \frac{x_s^2}{2} \right) \frac{\alpha^2}{\sigma_0^2} \right]^{a_{\text{max}} a_{\text{min}}}$$  \hspace{1cm} (10)

**UNIFORM CASE**

In the case that the dust grains are uniformly distributed along the line of sight, Equation $\text{[7]}$ becomes

$$I_h(\alpha) = \frac{F_a \tau_{\text{sca}}}{2\pi \sigma_0^2} \int a^{6-p} \int x^{-2} \exp \left( -\frac{\alpha^2 a^2}{2\sigma_0^2 x^2} \right) \, dx \, da$$  \hspace{1cm} (11)

The $x$ term of the integral evaluates to

$$\sqrt{\frac{\pi}{2}} \frac{\sigma_0}{\alpha^2} \left[ 1 - \text{erf} \left( \frac{\alpha a}{\sigma_0 \sqrt{2}} \right) \right]$$  \hspace{1cm} (12)

Plugging this in, we get

$$I_h(\alpha) = \frac{F_a \tau_{\text{sca}}}{8\pi \sigma_0^2} \int a^{5-p} \left[ 1 - \text{erf} \left( \frac{\alpha a}{\sigma_0 \sqrt{2}} \right) \right] \, da$$  \hspace{1cm} (13)

The solution is

$$I_h = \frac{F_a \tau_{\text{sca}}}{8\pi \sigma_0^2} \frac{G_u(a, p, \alpha)}{G_p(a, p)}$$  \hspace{1cm} (14)

where

$$G_u(a, p, \alpha) = \frac{1}{6-p} \left[ a^{6-p} \left( 1 - \text{erf} \left( \frac{\alpha a}{\sigma_0 \sqrt{2}} \right) \right) \right]^{a_{\text{max}} a_{\text{min}}}$$  \hspace{1cm} (15)

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