SRAR-RISs: Simultaneous Reflecting and Refracting Reconfigurable Intelligent Surfaces

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Abstract—In this letter, simultaneous reflecting and refracting reconfigurable intelligent surfaces (SRAR-RISs) are studied. Compared with RISs that only operate in the reflection mode, the coverage area of SRAR-RISs is extended via simultaneous reflection and refraction. We separately present a hardware model and channel models for the near-field region and the far-field region. The proposed channel models give closed-form expressions of the channel gains in terms of physical parameters of the SRAR-RIS and the geometrical information of the receivers. The differences between the far-field and the near-field channel models are revealed by analytical results and simulations.

Index Terms—Channel modelling, electromagnetics, reconfigurable intelligent surface, simultaneous reflection and refraction.

I. INTRODUCTION

Recently, the concept of reconfigurable intelligent surfaces (RISs) and channel modelling for RISs have received heated discussion in the research community. It is envisioned that the RIS can assist wireless communication networks by intelligently controlling part of the scattering environment [1]. By deploying the RIS, we are able to enhance the spectrum efficiency and the quality of service for future-generation wireless systems. We notice that the existing research contribution assumes the RIS to be operated either in reflection or refraction mode [2]. These two working conditions correspond to the mode [2]. These two working conditions correspond to the

We study the electromagnetic wave response of SRAR-RIS by applying a plane wave excitation. Consider a vertically polarized incident plane wave on the SRAR-RIS. The electrical component of the incident field at element $m$ can be expressed as:

$$\vec{E}_{i}^{m} = \hat{y}E_{0}e^{jk^{1}·r_{m}}, \quad (1)$$

where $\hat{y}$ is the unit vector in the $y$-direction, $E_{0}$ is the amplitude of the incident plane wave, $k^{1}$ is the wave number vector, and $r_{m}$ is the position of the $m$-th element. Assume that the SRAR-RIS produces both reflected and refracted fields with the same polarization. At the $m$-th element, these fields can be expressed as:

$$\vec{E}_{r}^{m} = R_{m}\vec{E}_{i}^{m}, \quad \vec{E}_{t}^{m} = T_{m}\vec{E}_{i}^{m}, \quad (2)$$

where $R_{m}$ and $T_{m}$ are the reflection and refraction coefficients of the $m$-th element, respectively. According to generalized sheet transition conditions (GSTC), these coefficients are directly related to the local surface averaged electric and magnetic impedance $Y_{m}$ and $Z_{m}$:

$$R_{m} = -\frac{2(\eta_{0}^{2}Y_{m} - Z_{m})}{(2 + \eta_{0}^{2}Y_{m})(2\eta_{0} + Z_{m})}, \quad (3)$$

$$T_{m} = \frac{2 - \eta_{0}^{2}Y_{m}}{2 + \eta_{0}^{2}Y_{m}} - R_{m}, \quad (4)$$

$^1$The refraction coefficient is commonly known as the transmission coefficient in physics. For consistency, we use refraction coefficient in this letter.
where $\eta_0$ is the impedance of free space. As illustrated in Fig. 1, the electric impedance $Y_m$ can be configured by tuning the capacitor ($C_m$) and resistor ($R_m$) of the equivalent circuit. If the SRAR-RIS is passive or lossless, the following constraint on the local reflection and refraction coefficients must be satisfied:

$$|R_m|^2 + |T_m|^2 \leq 1.$$  

(5)

Thus, the electromagnetic wave response of the $m$-th element can be characterized by its local reflection and refraction coefficients. In channel models, these coefficients are more commonly expressed as follows:

$$R_m = \sqrt{\beta_{mr}} e^{j\phi_{mr}}, \quad T_m = \sqrt{\beta_{mt}} e^{j\phi_{mt}},$$  

(6)

where $\beta_{mr}, \beta_{mt}$ are real-valued coefficients satisfying $\beta_{mr} + \beta_{mt} \leq 1$, $\forall m \in \{1, 2, \ldots, M\}$, $\phi_{mr}$ and $\phi_{mt}$ are the phase shifts introduced by element $m$ for the reflected and refracted signals.

### III. Channel Model

A downlink SRAR-RIS-assisted multi-user wireless network is considered, where the transmitter and receivers are equipped with single antenna, and the SRAR-RIS consists of $M$ reconfigurable elements. The users are divided into two groups. Users in group $A$ are located on the same side with the transmitter with respect to the SRAR-RIS, and thus, they can only receive signals reflected by SRAR-RIS. Users in group $B$ and the transmitter are located on different side of SRAR-RIS, and thus, these users can only receive signals refracted by SRAR-RIS. At SRAR-RIS, signals are reflected and refracted to these two groups of users simultaneously. We denote the direct link between the transmitter and the $k$-th user in group $A$ by $h_k^A$, and the direct link between the transmitter and the $l$-th user in group $B$ by $h_l^B$. Let $g_k^A$ denote the channel between the transmitter and the $k$-th user in group $A$ reflected by SRAR-RIS. Let $g_l^B$ denote the channel between the transmitter and the $l$-th user in group $B$ refracted through SRAR-RIS. Therefore, the effective channel for users in group $A$ and group $B$ are given by:

$$H_k^A = h_k^A + g_k^A,$$  

(7)

$$H_l^B = h_l^B + g_l^B.$$  

(8)

According to (7) and (8), the overall channels of users in group $A$ or group $B$ consist of two parts. The direct links from the transmitter to the receivers, $h_k^A$ and $h_l^B$, do not involve SRAR-RIS and are usually modelled by Ricean fading channels. The other part is the channel corresponding to the links through SRAR-RIS, namely $g_k^A$ and $g_l^B$. In this section, we focus on modelling these links. Moreover, since receivers in both groups could locate in the far-field or near-field region with respect to SRAR-RIS, we separately present the channel models for such cases.

### A. Far-field channel model

In this case, the receivers are located in the far-field region of the SRAR-RIS. We denote the channel between the transmitter and the SRAR-RIS by $h_{T,S} = (h_{T,S}^T, \ldots, h_{T,S}^M)^T$, where $h_{T,S}^M$ is the channel between the transmitter and the $m$-th element. In addition, let $r_k^A = (r_{k,1}^A, \ldots, r_{k,M}^A)^T$ denote the channel between SRAR-RIS and receiver $k$ in group $A$. Let $r_l^B = (r_{l,1}^B, \ldots, r_{l,M}^B)^T$ denote the channel between SRAR-RIS and receiver $l$ in group $B$. Since all receivers are located in the far-field region of SRAR-RIS, the ray tracing technique can be adopted by studying a number of $M$ geometrical rays, each corresponding to an element. This leads to the channel model as follows:

$$g_k^A = (r_k^A)^H \text{diag}(\sqrt{\beta_{Mr} e^{j\phi_{Mr}}}, \ldots, \sqrt{\beta_{Mr} e^{j\phi_{Mr}}}) h_{T,S}^A,$$  

(9)

$$g_l^B = (r_l^B)^H \text{diag}(\sqrt{\beta_{Mr} e^{j\phi_{Mr}}}, \ldots, \sqrt{\beta_{Mr} e^{j\phi_{Mr}}}) h_{T,S}^B.$$  

(10)

For convenience, we denote $\text{diag}(R_1, R_2, \ldots, R_M)$ by $R$, and $\text{diag}(T_1, T_2, \ldots, T_M)$ by $T$. In addition, in the far-field channel model, $r_k^A$, $r_l^B$, and $h_{T,S}$ can be written in the form of the path loss (large-scale fading) multiplied with the normalized small-scale fading. The large-scale fading depends on the distance between the transmitter, SRAR-RIS and the receivers, while the small-scale fading depends on the scattering environment. Let $d_k^A$ denote the distance between SRAR-RIS and the receivers in group $A$, $d_l^B$ denote the distance between SRAR-RIS and the receivers in group $B$, and $d_0$ denote the distance between the transmitter and SRAR-RIS. The channel gains can be expressed as:

$$|g_k^A| = \frac{1}{(d_k^A)^{\alpha_A} (d_0)^{\alpha_0}} |(\tilde{r}_k^A)^H R h_{T,S}^A|,$$  

(11)

$$|g_l^B| = \frac{1}{(d_l^B)^{\alpha_B} (d_0)^{\alpha_0}} |(\tilde{r}_l^B)^H T h_{T,S}^B|,$$  

(12)

where $\tilde{r}_k^A$, $\tilde{r}_l^B$, and $h_{T,S}$ are the corresponding small-scale fading components. In addition, $\alpha_A$, $\alpha_B$, and $\alpha_0$ are the path loss coefficients of the corresponding channels.

### B. Near-field channel model

In scenarios where the receivers are located within the near-field region of SRAR-RIS, the conventional ray tracing technique based channel models can not be adopted. In this region, it is reasonable to assume that the links between SRAR-RIS and the receivers are not affected by scatters. Base on the Huygens-Fresnel principle, A. Fresnel and Kirchhoff arrived at the analytical result which is known as the Fresnel-Kirchhoff diffraction formula. According to the this formula,
the contribution to the received field at each wave point is proportional to the area on the wavefront, the amplitude of the field at each point on the wavefront, the leaning factor at each point on the wavefront, and the inverse of the distance between each point on the wavefront and the receiver. As illustrated in Fig. [2] the electromagnetic signal at the receiver can be calculated by summing up the contribution of every elements on the wavefront. The wavefront is chosen as the plane \( z = 0 \) where SRAR-RIS is located.

\[
g_k^A = \frac{1}{j\lambda} \int_{(\Sigma)_{\text{hos}}} U^r(Q) F(\theta^A) \frac{e^{2j\pi d_n^A/\lambda}}{d^A} d\Sigma, \quad (13)
\]

\[
g_k^B = \frac{1}{j\lambda} \int_{(\Sigma)_{\text{hos}}} U^t(Q) F(\theta^B) \frac{e^{2j\pi d_n^B/\lambda}}{d^B} d\Sigma, \quad (14)
\]

where \( j \) is the imaginary unit, \( U^r(Q), U^t(Q) \) are the aperture distributions for the reflected and refracted waves at point \( Q \) on SRAR-RIS, \( F(\theta) \) is the leaning factor at point \( Q \), \( d^A \) and \( d^B \) are the distances between SRAR-RIS and the receivers in group \( A \) and group \( B \), respectively, and \( \lambda \) is the free-space wavelength of the signal.

Next, for patch-array based SRAR-RIS with \( M \) elements, the integrals in (13) and (14) can be evaluated by element. Assuming that the aperture distributions \( U^r(Q) \) and \( U^t(Q) \) are uniform within each element, at the \( m \)-th element, we have \( U^r(Q) = R_m h_m^T,S \) and \( U^t(Q) = T_m h_m^T,S \). Thus, (13) and (14) can be expressed as:

\[
g_k^A = \frac{A}{\lambda} \sum_m R_m h_m^T,S F(\theta_m^A) \frac{e^{2j\pi d_n^A/\lambda}}{d^A}, \quad (15)
\]

\[
g_k^B = \frac{A}{\lambda} \sum_m T_m h_m^T,S F(\theta_m^B) \frac{e^{2j\pi d_n^B/\lambda}}{d^B}, \quad (16)
\]

where \( A \) is the area of each element, and \( \theta_m^A \) and \( \theta_m^B \) are the direction of the users in group \( A \) and group \( B \), with respect to the normal direction of SRAR-RIS, as illustrated in Fig. [2].

According to the Fresnel-Kirchhoff diffraction formula, the leaning factor is \( F(\theta_m) = (1 + \cos \theta_m)/2 \), which holds for both group \( A \) and group \( B \). Thus, the channel gains can be expressed as:

\[
|g_k^A| = \frac{A}{\lambda} \sum_m R_m h_m^T,S (1 + \cos \theta_m^A) \frac{e^{2j\pi d_n^A/\lambda}}{2d^A}, \quad (17)
\]

\[
|g_k^B| = \frac{A}{\lambda} \sum_m T_m h_m^T,S (1 + \cos \theta_m^B) \frac{e^{2j\pi d_n^B/\lambda}}{2d^B}. \quad (18)
\]

By comparing these results with the channel gains of the near-field channel model, it can be noticed that the distances between SRAR-RIS elements and the receiver can not be treated the same and be brought outside of the summation. In addition, the contribution of the leaning factor should be explicitly considered in the near-field model.

IV. PHASE-DEPENDENT AMPLITUDE VARIATION

As pointed out in [7], in practical cases, the amplitudes and the phases for reflection and refraction coefficients are correlated. According to \[3 \] and \[4 \], these coefficients depend on local surface impedance \( Y_m \) and \( Z_m \). In principle, for an ideal SRAR-RIS with adjustable \( Y_m \) and \( Z_m \), the phases for reflection and refraction coefficients can take on arbitrary values within \([0, 2\pi]\), independent of their amplitudes.

**Corollary 1:** For patch-array based SRAR-RISs with negligible thickness, the discontinuity of only the electric component of the field is considered, and we have \( Z_m = 0 \) [8]. This means there is no discontinuity for the transverse component of the magnetic field at the surface. By substituting \( Z_m = 0 \) into \[3 \] and \[4 \], we have:

\[
T_m - R_m = 1. \quad (19)
\]

In this case, we arrive at (19) as another fundamental condition on reflection and refraction coefficients for passive or lossless patch-array based SRAR-RISs. The following diagram illustrates these conditions for the amplitudes and phases for \( R_m \) and \( T_m \) on the complex plane. In Fig. [3] the circle is centered at point \( C \) with diameter \( OQ \) of length equals to one. With condition (19), the reflection and refraction coefficients for each element have only two independent scalar values, which correspond to the position of point \( P \) in Fig. [3]. According to \[5 \], if point \( P \) is in Region I (within the circle), \( QP \) and \( OP \) represent legitimate reflection and refraction coefficients for a passive RIS. If point \( P \) is on the circle, the corresponding vectors represent the reflection and refraction coefficients for a lossless element. If point \( P \) is in Region II (outside of the circle), the corresponding reflection and refraction coefficients can only be achieved by an active element.
In addition, Fig. 3 shows that for lossless elements, the phases of reflection and refraction coefficients, namely $\phi^r_m$ and $\phi^i_m$, can not take on arbitrary values and they are correlated with the amplitudes. This will be demonstrated by the numerical results in Section V.

V. NUMERICAL RESULTS

In this section, we numerically demonstrate our proposed hardware model and channel models. According to Section IV we characterize each SRAR-RIS element by its reflection coefficient ($R_m$) and refraction coefficient ($T_m$). Suppose the coefficients follow the conditions in (5) and (19). Fig. 4 illustrates constraints on the reflection and refraction coefficients for the lossless patch-array based SRAR-RISs on the complex plane. It is worth noting that perfect reflection occurs at $\phi^r_m = \pi$, and perfect refraction occurs at $\phi^i_m = \pi/2$ or $3\pi/2$. Moreover, the phase of both reflected and refracted waves has a configurable range of $\pi$. In other words, the $2\pi$ full-range phase shift adjustment can not be achieved by the a lossless patch-array based RIS.

Fig. 5 shows the channel gain of a sixteen-by-sixteen patch-array based SRAR-RIS. The spacing of each element is chosen as $\lambda/2$, and the power ratio between reflection and refraction is set to 3:2. For the horizontal axis, $d$ denotes the distance between the receiver and the center of SRAR-RIS. The signal is reflected to the region where $d > 0$, and refracted to the region where $d < 0$. In the simulation, we assume that the receiver moves on both sides of SRAR-RIS, along a straight line with an angle of $60^\circ$ with respect to the normal direction of SRAR-RIS. As illustrated in Fig. 5 the dash-dotted blue line represents the channel gains calculated using the near-field channel model without considering the leaning factor. In other words, we assume $F(\theta^r_m) = F(\theta^i_m) = 1$ in (15) and (16). The solid red line represents the channel gains calculated using the near-field channel model with leaning factor $F(\theta^A_m) = (1 + \cos \theta^A_m)/2$. The dashed black line represents the channel gains calculated using the far-field channel, according to (11) and (12). It can be observed that results for the near-field model start to approach the far-field model after the receiver and (12). It can be observed that results for the near-field region, the far-field model fails to provide a physically meaningful results as the channel gain tends to infinity when the distance approaches zero. Moreover, it is noticed that the effect of the leaning factors is noticeable in the near-field region.

VI. CONCLUSIONS

In this paper, a physic-compliant model was proposed for SRAR-RIS. Proper constraints on the reflection and refraction coefficients were formulated following the law of energy conservation and the boundary conditions for the electromagnetic field at SRAR-RIS. In addition, channel models were proposed for both the far-field and the near-field regions. Closed-form expressions for channel gains of users receiving the reflected signal as well as the refracted signal were derived. Analytical and numerical results demonstrated the differences between the channel gains in the far-field and the near-field region.

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