BINDING ENERGY OF Λ HYPERNUCLEI FROM REALISTIC YN INTERACTIONS

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s− and p−wave Λ single–particle energies are obtained for a variety of Λ hypernuclei from the relevant self–energy constructed within the framework of a perturbative many–body approach. Results are presented for present realistic hyperon–nucleon interactions such as Jülich B and Nijmegen SC models. Effects of the non–locality and energy–dependence of the self–energy on the bound states are investigated.

1 Introduction

Several approaches have been followed to derive hyperon properties. Traditionally, people have employed Woods–Saxon potentials which reproduce quite well the measured Λ single–particle energies from medium to heavy hypernuclei. Non–localities and density dependent effects have been included in non–relativistic Hartree–Fock calculations with Skyrme YN interactions in order to improve the overall fit to the single–particle energies. Hypernuclei have been studied within a relativistic framework and microscopic hypernuclear structure calculations have also been performed.

Our work follows this last approach and its intention is to test the present YN interactions (i.e. Jülich and Nijmegen models). To this end we will evaluate the s– and p–wave Λ single–particle energies for some Λ hypernuclei. Comparison with experiment may help in further constraining the YN interactions. The starting point of this work is a nuclear matter G–matrix evaluated at a fixed nuclear density and starting energy. This nuclear matter G–matrix is used to construct a finite nucleus G–matrix which will be employed in the evaluation of the hyperon self–energy. Finally, the real part of the hyperon self–energy will be used as a non–local potential in a Schrödinger equation in order to get the single–particle energies for the different orbits.

2 Formalism

In this section we present a method to construct an effective YN interaction in finite nuclei through an expansion in terms of a nuclear matter G–matrix.
The explicit details of this method can be found in Ref. [1]. The first step is to obtain the nuclear matter $G$–matrix in terms of the bare interaction $V$ by solving the Bethe–Goldstone equation

$$G_{NM} = V + V \left( \frac{Q}{e} \right)_{NM} G_{NM} . \quad (1)$$

It is important to note that this is a coupled channel problem because the hyperon of the intermediate state $YN$ can be a $\Lambda$ or a $\Sigma$.

Consider now the corresponding Bethe–Goldstone equation for the finite nucleus case

$$G_{FN} = V + V \left( \frac{Q}{e} \right)_{FN} G_{FN} . \quad (2)$$

Eliminating the bare potential $V$ in both equations it is not difficult to write $G_{FN}$ in terms of $G_{NM}$ through an integral equation which can be truncated at second order because the difference between the finite nucleus and the nuclear matter intermediate propagators is small

$$G_{FN} \simeq G_{NM} + G_{NM} \left[ \left( \frac{Q}{e} \right)_{FN} - \left( \frac{Q}{e} \right)_{NM} \right] G_{NM} . \quad (3)$$

Finally, once we have $G_{FN}$, the hyperon self–energy at Brueckner–Hartree–Fock level reads in schematic form as

$$\Sigma_{HF} = \sum_{N} \langle Y''N|G_{FN}|YN \rangle \simeq \sum_{N} \langle Y''N|G_{NM}|YN \rangle$$

$$+ \sum_{Y'N} \langle Y''N|G_{NM}|Y'N \rangle \left[ \left( \frac{Q}{e} \right)_{FN} - \left( \frac{Q}{e} \right)_{NM} \right] \langle Y'N|G_{NM}|YN \rangle . \quad (4)$$

3 Results and Discussion

In this section we show and discuss some of the results obtained for $\Lambda$ hypernuclei. First, we have tested the stability of our results against variations of the nuclear density and the starting energy used in the calculation of the nuclear matter $G$–matrix. We have found that the $1^{st}$ and $2^{nd}$ order terms, shown in eq. (3), depend quite strongly on these parameters which is an indication that the density dependent effects are important when the finite size of the nucleus is taken into account. Nevertheless, the whole calculation up to $2^{nd}$ order gives results very stable (see tables I-IV of Ref. [1]). In the next table we present the $s$– and $p$–wave $\Lambda$ single–particle energies for a variety of $\Lambda$ hypernuclei. All the results are given only for the Jülich B interaction and no results are shown...
for the Nijmegen SC because they appear clearly underbound with respect the experimental data (e.g. in the case of $^{17}_Λ$O Nijmegen SC gives -7.39 MeV in front of the experimental value -12.5 MeV).

Table 1: $Λ$ binding energies in the $1s_{1/2}$, $1p_{3/2}$ and $1p_{1/2}$ orbits for different hypernuclei.

| Hypernuclei | Orbit     | $1s^2$ | $1s^2 + 2p1h$ | Exp.      |
|------------|-----------|-------|---------------|-----------|
| $^{13}_Λ$C | $1s_{1/2}$| -7.93 | -9.48         | -11.69 ($^{13}_Λ$C) |
|           | $1p_{3/2}$|      |               |           |
|           | $1p_{1/2}$| 0.08  | -1.06         |           |
| $^{17}_Λ$O| $1s_{1/2}$| -10.15| -11.83        | -12.5 ($^{16}_Λ$O) |
|           | $1p_{3/2}$|      | -0.87         | -2.5 (1p) |
|           | $1p_{1/2}$|      | -0.08         |           |
| $^{41}_Λ$Ca| $1s_{1/2}$| -16.85| -19.60        | -20 ($^{40}_Λ$Ca) |
|           | $1p_{3/2}$|      | -6.70         | -12 (1p)  |
|           | $1p_{1/2}$|      | -6.92         |           |
| $^{91}_Λ$Zr| $1s_{1/2}$| -22.24| -25.80        | -23 ($^{89}_Λ$Zr) |
|           | $1p_{3/2}$|      | -14.74        | -16 (1p)  |
|           | $1p_{1/2}$|      | -14.86        |           |
| $^{209}_Λ$Pb| $1s_{1/2}$| -26.28| -31.36        | -27 ($^{208}_Λ$Pb) |
|           | $1p_{3/2}$|      | -21.22        | -22 (1p)  |
|           | $1p_{1/2}$|      | -21.30        |           |

The agreement with experimental data is rather good, especially for the $s$–wave. Note that according to empirical information the spin-orbit splitting is very small and note also that the $1p_{1/2}$ energy is lower than the $1p_{3/2}$ which is tied to the particular spin structure of the Jülich interaction.

Finally, we show in the figure the $1s_{1/2}$ wave function of a $Λ$ in $^{17}_Λ$O obtained from our non–local self–energy (solid line) or from a local Woods–Saxon potential (dashed line) of depth $-30.2$ MeV and radius adjusted to reproduce the same binding energy. Nevertheless, as can be seen our wave function is far more extended and this can have important implications on the mesonic weak decay of hypernuclei. Only if we allow the Woods–Saxon potential to have a shallower depth ($-23.6$ MeV) and a larger radius we can not only reproduce the binding energy but also maximize the overlap of the resulting
wave function (dot–dashed line) with ours.

Figure 1: Wave function in r–space for the $1s_{1/2} \Lambda$ in $^{17}_{\Lambda}$O.

4 Conclusions

We have developed a method to obtain an effective $YN$ interaction in finite nuclei based on an expansion over a $G$–matrix calculated in nuclear matter at fixed nuclear density and starting energy. The truncation of this expansion up to second order gives results very stable against variations of these parameters. A single–particle energies are well reproduced by the Jülich B model but appear clearly underbound by the Nijmegen SC. Future perspectives of this work are the study of scattering states and the study of the implications of our wave functions on the mesonic weak decay of hypernuclei.

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