Abstract:
The diquark structure in baryons is commonly accepted as a reasonable approximation which can much simplify the picture of baryons and reduce the length of calculations. However, a diquark by no means is a point-like particle, even though it is treated as a whole object. Therefore, to apply the diquark picture to phenomenological calculations, at the effective vertices for the diquark-gauge boson interactions, suitable form factors must be introduced to compensate the effects caused by the inner structure of the diquark. It is crucial to derive the appropriate form factors for various interactions. In this work, we use the Bethe-Salpeter equation to derive such form factors and numerically evaluate their magnitudes.

1 Introduction

In the quark model, regular baryons are composed of three valence quarks. Compared with the case for meson which contains a quark and an antiquark, the physical picture for baryon is much more difficult to deal with because a three-body system is terribly more complicated than a two-body system. It is believed that the correct description for a baryon which contains three quarks is the Faddeev equation [1] whereas the Bethe-Salpeter equation properly describes the meson structure.

The concept of diquark was raised even at the epoch of the birth of the quark model [2]. However, it is still in dispute that diquark is a substantial structure of color-anti-
triplet or just a mathematical decomposition of the $3 \otimes 3 \otimes 3$ representation of $SU_c(3)$ into $3 \otimes 3$. For the baryons which are composed of two heavy quarks (b,c) and a light quark, it is believed that the two heavy quarks can constitute a more stable diquark with smaller size, but for the baryons with one-heavy-two-light-quark structure or even three-light-quark structure, the diquark-picture is dubious. Analyzing the Faddeev equation, one finds that the diquark picture is an approximation where two quarks are supposed to constitute a more stable subsystem and the interaction with the other quark can be treated as a perturbation which is not strong enough to break the diquark binding.

For a long time, the concept of diquarks has been applied to study the processes where baryons are involved [3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. With the diquark-quark structure, the physical picture of baryons is simplified and the theoretical calculations become much easier. However, one would ask what is missing in the simplified version. In other words, the diquark is by no means a real point-like particle, but has inner structure. For lower energy, the transfer momentum is small, then the inner structure may not manifest itself and the diquark can be nicely treated as a point. On the contrary, as the energy scale in the problem is larger, simply treating it as a structureless particle would bring up large errors to the theoretical calculations. Thus one needs to involve such effects which reflect the inner structure of diquarks. One introduces form factors at each effective vertices where diquarks interact with gauge bosons, such as $W^\pm$, $Z$, $\gamma$ and gluons, to take into account the effects caused by the inner structure. Since diquark is generally in color anti-triplet, and not a physical object, the form factors cannot be experimentally measured, thus one needs to use sort of theoretical models to derive them. The phenomenological expressions of such form factors were introduced by many authors [16, 17, 18, 19]. The authors of Refs. [20, 21] used the Dyson-Schwinger equation to evaluate the electromagnetic form factors of diquarks. Guo et al. [22] used the heavy quark effective theory to derive the form factor for the diquark-gluon coupling and Ebert et al. [23] employed the relativistic quark model to obtain these form factors. Dai et al. [24] calculated baryon transitions. Ahlig et al. [25] derived the wavefunction for the baryon-diquark-quark couplings.

Indeed, as the diquark picture is accepted, it is necessary to derive the form factors at the effective vertices for diquark-gauge-boson couplings or even diquark-mesons couplings, because diquarks are not point-like particles and the effects of the inner structure of diquarks must manifest themselves through the form factors. In this work, we try to derive the form factors at the effective vertices for diquark-gauge-boson couplings in a more general framework and the form factors may be applied to phenomenological calculations of decay rates or production processes where baryons, especially heavy baryons are involved.

The framework we are going to adopt is the Bethe-Salpeter equation (BSE). As discussed above the BSE may be a simplified version of the complicated Faddeev equation which seems to well describe the baryon structure. The BSE established on the quantum field theory is considered to be a feasible approach to study the relativistic two-body bound states. Salpeter [26] adopted the instantaneous approximation to simplify the BSE which can be applied to deal with practical phenomenological problems. Namely, to determine the form factors, we can first obtain the diquark spectra and wavefunctions [27] and then with them as inputs we adopt the instantaneous BSE method developed by Chang et al. [28] to derive the form factors at the effective vertices of diquarks coupling to gauge bosons: gluon, photon, $W^\pm$ and $Z^0$. Then we also try to extend the method to obtain the effective vertex of diquark coupling to pseudoscalar mesons, such as pions.
In this work, after the introduction, we derive all the form factors at the effective vertices in terms of the BSE in Sec.II. In Sec.III the numerical results are presented while the input parameters are given explicitly, and then the last section is devoted to the summary and discussions.

2 Formulation

2.1 The form factors of diquark coupling to gluons

The effective vertex of diquark coupling to a gauge boson ($g$, $\gamma$, $Z^0$) can be written in a form as

$$V_{DGD}^{\mu} = \Gamma_{DGD}(V_{qGq}) \cdot \epsilon_\mu,$$

where $\epsilon_\mu$ is the polarization vector of the gauge boson, $D$, $G$ refer to the diquark and the gauge boson, respectively. Obviously, $\Gamma_{DGD}^{\mu}$ is a unique function of the quark-gauge-boson vertex $V_{qGq}$ which is given in the fundamental theories. Here we only concern the standard model (SM). Below, we are going to derive all the currents $\Gamma_{DGD}^{\mu}$ in terms of the BSE.

1. The effective current of scalar diquark coupling to gluons can be written as:

$$\Gamma_{SGS}^{\mu,a} = -i g_s \lambda^a \frac{2}{\Lambda} G(Q^2) (P_f + P_i)^\mu,$$

$$G(Q^2) = \frac{P_x \mu \{ M_1^{\mu} + M_2^{\mu} \}}{(P_f + P_i) \cdot P_x},$$

where $G(Q^2)$ is the form factor, $P_i$ and $P_f$ are the momenta of the initial and final diquarks respectively and $Q^2 = (P_f - P_i)^2$ is the momentum transfer, $P_x$ is an arbitrary non-zero auxiliary four-vector. The physical picture of a diquark coupling to a gauge boson is depicted in Fig. 1, and the sum of Fig.1 (a) and (b) makes the net contribution to the form factor.

![Figure 1: The Feynman diagram for diquark coupling to a gauge boson](image)

The effective current corresponding to Fig. 1 (a) is

$$M_1^{\mu} = - \int \frac{d^4q d^4q'}{(2\pi)^4} \{ \delta^4(p_2 - p_1) Tr \left[ \bar{\chi}_{P_f}(q') \gamma^\mu \chi_{P_i}(q) S_F^{-1}(p_2) \right] \}$$
\[
= - \int \frac{d^4 q}{(2\pi)^4} Tr \left[ \nabla_{\mu} \chi_{P_j}(q) S_{P_j}^{-1}(p_{2}) \right].
\]

Here \( p_2 = \alpha_2 P_i - q, \) \( p'_2 = \alpha_2 P_f - q', \) and by conservation of momentum one has
\[
q' = \alpha_2 (P_f - P_i) + q,
\]
and the current corresponding to Fig.1(b) is
\[
M_2^\mu = - \int \frac{d^4 q d^4 q'}{(2\pi)^4} \left\{ \delta^4(p_1 - p'_1) Tr \left[ \nabla_{\mu} \chi_{P_j}(q') S_{P_j}^{-1}(p_1) \chi_{P_i}(q) \gamma^\mu \right] \right\}
\]
\[
= - \int \frac{d^4 q}{(2\pi)^4} Tr \left[ \nabla_{\mu} \chi_{P_j}(q') S_{P_j}^{-1}(p_1) \chi_{P_i}(q) \gamma^\mu \right],
\]
where \( p_1 = \alpha_1 P_i + q, \) \( p'_1 = \alpha_1 P_f + q' \) and
\[
q' = \alpha_2 (P_f - P_i) + q.
\]
By the BSE and under the instantaneous approximation, we obtain,
\[
\chi_{P_i}(q) = \frac{1}{p_1 - \eta_{P_i}} \int d^3 k_{P_T} \left[ V(|q_{P_-} - k_{P_-}|) \varphi_{P_i}(k_{P_-}) \right] \frac{1}{p_2 + m_2}
\]
\[
= \frac{1}{p_1 - \eta_{P_i}} \frac{1}{p_2 + m_2},
\]
and
\[
\nabla_{\mu} \chi_{P_j}(q') = \frac{1}{p'_2 + m_2} \int d^3 k_{P_T} \left[ V(|q'_{P_-} - k_{P_-}|) \varphi_{P_j}(k_{P_-}) \right] \frac{1}{p'_1 - \eta_{P_j}}
\]
\[
= \frac{1}{p'_2 + m_2} \frac{1}{p'_1 - \eta_{P_j}},
\]
where \( P \) is the momentum of the baryon which contains the diquark. In fact, as one only discusses the diquark system, the baryon momentum is irrelevant. Here we set the baryon momentum \( P \) as a reference momentum and then we can properly specify other momenta, \( q_{P_-}^\mu = \frac{q.P}{M^2} P^\mu, \) \( q_{P_-}'^\mu = d^\mu - q_{P_-}^\mu, \) \( q = \frac{q.P}{M} \) and \( q_{P_T} = \sqrt{q_{P_-}^2 - q^2} = \sqrt{-q_{P_-}'^2} \) are the projections of the inner momentum \( q \) of quarks inside the diquark on the directions parallel and perpendicular to \( P \) and corresponding invariants respectively. \( \varphi_{P_i(P_j)} \) in Eqs. (8) are defined as
\[
\varphi_{P_i(P_j)}(q_{P_-}) = \int d q_{P_T} \chi_{P_i(P_j)}(q_{P}, q_{P_-}).
\]
The definitions of the subscripts are obvious. \( S_{P_j}^{-1}(p_1) \) is the inverse of a fermion propagator \( \frac{1}{p_1 - \eta_{P_-}} \), and from Eq. (5), one can note that on the other leg connecting to the kernel, there should be another fermion propagator corresponding to \( p_2 \). However, as we properly convert the BS wavefunction into a \( 4 \times 4 \) matrix form from a \( 16 \times 1 \) matrix [29], it automatically turns into its charge conjugation which is equivalently expressed as \( \frac{1}{p_2 + m_2} \).
Then we can re-write Eqs. (4) and (6) as
\[
M_1^\mu = - i \int \frac{d^4 q}{(2\pi)^4} Tr \left[ \frac{1}{p'_2 + m_2} \pi_{P_j}(q_{P_-}') \frac{1}{p'_1 - \eta_{P_j}} \gamma^\mu \frac{1}{p_1 - \eta_{P_i}} \right].
\]
\[ M_2^\mu = -i \int \frac{d^4q}{(2\pi)^4} \text{Tr} \left[ \frac{1}{p_2 + m_2} \eta P_f(p_{\perp}) \frac{1}{p_1 - m_1} \eta P_i(q_{\perp}) \frac{1}{p_2 + m_2} \gamma^\mu \right]. \]  

(12)

Following the commonly adopted method, we also decompose the propagators as

\[ \frac{1}{p_1 - m_1} = \frac{\Lambda_1^+}{\alpha_1 P_i + q_\perp - \omega_1 + i\epsilon} + \frac{\Lambda_1^-}{\alpha_1 P_i + q_\perp + \omega_1 - i\epsilon}, \]  

(13)

\[ \frac{1}{p_2 + m_2} = \frac{\Lambda_2^+}{\alpha_2 P_i - q_\perp - \omega_2 + i\epsilon} + \frac{\Lambda_2^-}{\alpha_2 P_i - q_\perp + \omega_2 - i\epsilon}, \]  

(14)

\[ \frac{1}{p'_1 - m_1} = \frac{\Lambda'_1^+}{\alpha_1 P_{f\perp} + q_{\perp}' - \omega'_1 + i\epsilon} + \frac{\Lambda'_1^-}{\alpha_1 P_{f\perp} + q_{\perp}' + \omega'_1 - i\epsilon}, \]  

(15)

\[ \frac{1}{p'_2 + m_2} = \frac{\Lambda'_2^+}{\alpha_2 P_{f\perp} - q_{\perp}' - \omega'_2 + i\epsilon} + \frac{\Lambda'_2^-}{\alpha_2 P_{f\perp} - q_{\perp}' + \omega'_2 - i\epsilon}, \]  

(16)

where \( \alpha_i \equiv \frac{m_i}{m_1 + m_2} \) (\( i = 1, 2 \)) and

\[ \omega_1 = \sqrt{m_1^2 - (\alpha_1 P_i P_\perp + q_{\perp})^2}, \]  

(17)

\[ \omega_2 = \sqrt{m_2^2 - (\alpha_2 P_i P_\perp - q_{\perp})^2}, \]  

(18)

\[ \Lambda_1^+ = \frac{p M \omega_1 \pm (m_1 + q_{\perp} + \alpha_1 P_i P_\perp)}{2\omega_1}, \]  

(19)

\[ \Lambda_2^+ = \frac{p M \omega_2 \pm (m_2 + q_{\perp} - \alpha_2 P_i P_\perp)}{2\omega_2}, \]  

(20)

\[ \omega'_1 = \sqrt{m_1^2 - (\alpha_1 P_{f\perp} + q'_{\perp})^2}, \]  

(21)

\[ \omega'_2 = \sqrt{m_2^2 - (\alpha_2 P_{f\perp} - q'_{\perp})^2}, \]  

(22)

\[ \Lambda'_1^+ = \frac{p M \omega'_1 \pm (m_1 + q'_{\perp} + \alpha_1 P_{f\perp})}{2\omega'_1}, \]  

(23)

\[ \Lambda'_2^+ = \frac{p M \omega'_2 \pm (m_2 + q'_{\perp} - \alpha_2 P_{f\perp})}{2\omega'_2}. \]  

(24)
Substituting these equations into the expressions of $M_1^\mu$ and $M_2^\mu$ in Eqs. (11, 12), writing $d^4q$ in the covariant form as $dq_p d^3q_{p_\perp}$, and completing the contour integration with respect to $dq_p$, we obtain the following equation,

\[
M_1^\mu = \int \frac{d^3q_{p_\perp}}{(2\pi)^3} Tr \left\{ - \frac{P}{M} \left[ \overline{\psi}_{P_f}^{++}(q_{P_f}) \varphi_{p_i}^{++}(q_{p_i}) + \overline{\psi}_{P_f}^{+-}(q_{P_f}) \varphi_{p_i}^{+-}(q_{p_i}) + \overline{\psi}_{P_f}^{--}(q_{P_f}) \varphi_{p_i}^{--}(q_{p_i}) \right] \right\}.
\]

(25)

For convenience, we re-define

\[
\psi_{1P_i}^{--}(q_{p_\perp}) = \frac{\Lambda_1^- \eta_{P_i}(q_{p_\perp})}{E + \omega_1 - E'},
\]

(26)

\[
\psi_{1P_i}^{+-}(q_{p_\perp}) = \frac{\Lambda_1^+ \eta_{P_i}(q_{p_\perp})}{E + \omega_1 - E'},
\]

(27)

\[
\overline{\psi}_{1P_f}^{+-}(q_{p_\perp}) = \frac{\Lambda_1^- \eta_P'(q_{p_\perp})}{E + \omega_1 - E'},
\]

(28)

\[
\overline{\psi}_{1P_f}^{--}(q_{p_\perp}) = \frac{\Lambda_1^+ \eta_P'(q_{p_\perp})}{E + \omega_1 - E'},
\]

(29)

where $E$ and $E'$ are the energies of the initial and final diquarks in the center-of-mass frame of the baryon whose momentum is $P$. Similarly,

\[
M_2^\mu = \int \frac{d^3q_{p_\perp}}{(2\pi)^3} Tr \left\{ - \frac{P}{M} \left[ \overline{\psi}_{P_f}^{++}(q_{P_f}) \varphi_{p_i}^{++}(q_{p_i}) + \overline{\psi}_{P_f}^{+-}(q_{P_f}) \varphi_{p_i}^{+-}(q_{p_i}) + \overline{\psi}_{P_f}^{--}(q_{P_f}) \varphi_{p_i}^{--}(q_{p_i}) \right] \right\}.
\]

(30)

For convenience, we have also re-defined

\[
\psi_{2P_i}^{++}(q_{p_\perp}) = \frac{\Lambda_1^- \eta_{P_i}(q_{p_\perp})}{E + \omega_1 - E'},
\]

(31)

\[
\psi_{2P_i}^{+-}(q_{p_\perp}) = \frac{\Lambda_1^+ \eta_{P_i}(q_{p_\perp})}{E + \omega_1 - E'},
\]

(32)

\[
\overline{\psi}_{2P_f}^{+-}(q_{p_\perp}) = \frac{\Lambda_1^- \eta_P'(q_{p_\perp})}{E + \omega_1 - E'},
\]

(33)

\[
\overline{\psi}_{2P_f}^{--}(q_{p_\perp}) = \frac{\Lambda_1^+ \eta_P'(q_{p_\perp})}{E + \omega_1 - E'},
\]

(34)
and the wavefunctions of the initial and final $0^+$ diquarks are respectively

$$\varphi_{p_i}(q_{p_\perp}) = \frac{q_{p_\perp}^2 (m_1 + m_2) (\omega_1 - \omega_2) \gamma_0 b_4(q_{p_\perp})}{(\omega_1 + \omega_2)(q_{p_\perp}^2 - m_1 m_2 - \omega_1 \omega_2)}$$

$$+ \frac{q_{p_\perp}^2 (m_1 + m_2) b_3(q_{p_\perp})}{q_{p_\perp}^2 - m_1 m_2 - \omega_1 \omega_2} + \gamma_0 b_3(q_{p_\perp}) + \gamma_0 \gamma_5 b_4(q_{p_\perp}),$$

(35)

$$\overline{\varphi}_{p_f}(q_{p_\perp}) = -\frac{q_{p_\perp}^2 (m_1 + m_2)(\omega_1' - \omega_2') \gamma_0 b_4(q_{p_\perp}')}{(\omega_1' + \omega_2')(q_{p_\perp}'^2 - m_1 m_2 - \omega_1' \omega_2')}$$

$$- \frac{q_{p_\perp}^2 (m_1 + m_2) b_3(q_{p_\perp}')}{q_{p_\perp}'^2 - m_1 m_2 - \omega_1' \omega_2'} - \gamma_0 b_3(q_{p_\perp}') + \gamma_0 \gamma_5 b_4(q_{p_\perp}').$$

(36)

2. The form factors at the effective vertex for vector diquarks coupling to gluons.

This effective coupling has been discussed by some authors [16, 17] and the effective vertex has the following form

$$\Gamma_{AgA}^{\alpha \alpha \beta} = -ig_s \frac{\lambda^\alpha}{2} \left[ G_1(Q^2)(P_f+P_i)^\mu g^{\alpha \beta} - G_2(Q^2)(P_f^\alpha g^{\mu \beta} + P_i^\beta g^{\mu \alpha}) + G_3(Q^2)(P_f+P_i)^\mu P_f^\alpha P_i^\beta \right],$$

(37)

which in the BSE approach can be further expressed as

$$\Gamma_{AgA}^{\alpha \alpha \beta} = -ig_s \frac{\lambda^\alpha}{2} (M_1^{\alpha \beta} + M_2^{\alpha \beta}),$$

(38)

where

$$M_1^{\alpha \beta} = -\int \frac{d^4q}{(2\pi)^4} Tr \left[ \overline{\chi}_{P_f}(q') \gamma^\mu \chi_{P_i}(q) S_F^{-1}(p_2) \right]$$

(39)

$$M_2^{\alpha \beta} = -\int \frac{d^4q}{(2\pi)^4} Tr \left[ \overline{\chi}_{P_f}(q') S_F^{-1}(p_1) \chi_{P_i}(q) \gamma^\mu \right]$$

(40)

and

$$\chi_{P_i}(q) \gamma^\alpha = \chi_{P_i}(q).$$

(41)

where $\epsilon^\lambda$ is the polarization vector of the vector diquark. In analog to the method given in last section, we can simplify $M^{\alpha \mu \beta}$ as follows:

$$M^{\alpha \mu \beta}(\Gamma_{qq}) = \int \frac{d^4q_{p_{\perp}}}{(2\pi)^3} Tr \left\{ -\frac{P_M}{M} \left[ \varphi_{p_{\perp}}^{3)(+)(\alpha_P P_{f_{\perp}} - \alpha_P P_{i_{\perp}} + q_{p_{\perp}})\gamma^\mu \varphi_{p'}^{\alpha \beta\alpha}(q_{p_{\perp}}) \right. \right.$$  

$$+ \varphi_{p_{\perp}}^{3)(+)(\alpha_P P_{f_{\perp}} - \alpha_P P_{i_{\perp}} + q_{p_{\perp}})\gamma^\mu \varphi_{p'}^{\alpha \beta\alpha}(q_{p_{\perp}}) - \varphi_{p_{\perp}}^{3)(-)(\alpha_P P_{f_{\perp}} - \alpha_P P_{i_{\perp}} + q_{p_{\perp}})\gamma^\mu \varphi_{p'}^{\alpha \beta\alpha}(q_{p_{\perp}}) \right.$$  

(42)
where

\[
\varphi_{p_{I}}(q_{p_{I}}) = e_{\perp\mu} \left\{ q_{p_{I}}^\mu \left[ f_{1}(q_{p_{I}}) + \gamma_{0} \frac{\omega_{2} - \omega_{1}}{\omega_{1} + \omega_{2}} f_{8}(q_{p_{I}}) + \gamma_{0} \frac{q_{p_{I}}^{2} - (m_{1} m_{2} - \omega_{1} \omega_{2})}{M(m_{1} + m_{2})} f_{4}(q_{p_{I}}) \right] \right. \\
\left. + \gamma_{0} \frac{q_{p_{I}}^{2}}{q_{p_{I}}^{\perp} (\omega_{1} + \omega_{2})} f_{4}(q_{p_{I}}) \right\} + M \gamma_{\mu} f_{5}(q_{p_{I}}) + M \gamma_{\mu} \gamma_{0} \frac{m_{1} \omega_{2} - m_{2} \omega_{1}}{M(w_{1} + w_{2})} f_{8}(q_{p_{I}}) - \frac{q_{p_{I}}^{2}}{q_{p_{I}}^{\perp} (\omega_{1} + \omega_{2})} f_{4}(q_{p_{I}}) + \gamma_{0} (q_{p_{I}}^{\mu} - \gamma_{\mu} q_{p_{I}}^{\perp}) f_{8}(q_{p_{I}}) \right\} \gamma_{5} = \varphi_{p_{I}}(q_{p_{I}}) e_{\perp\mu},
\]

(43)

and

\[
\overline{\varphi}_{p_{J}}(q_{p_{J}}) = e_{\perp\mu} \left\{ q_{p_{J}}^\mu \left[ f_{1}(q_{p_{J}}) - \gamma_{0} \frac{\omega_{2} - \omega_{1}}{\omega_{1} + \omega_{2}} f_{8}(q_{p_{J}}) - \gamma_{0} \frac{q_{p_{J}}^{2} - (m_{1} m_{2} - \omega_{1} \omega_{2})}{M(m_{1} + m_{2})} f_{4}(q_{p_{J}}) \right] \right. \\
\left. - \gamma_{0} \frac{q_{p_{J}}^{2}}{q_{p_{J}}^{\perp} (\omega_{1} + \omega_{2})} f_{4}(q_{p_{J}}) \right\} + M \gamma_{\mu} f_{5}(q_{p_{J}}) + M \gamma_{\mu} \gamma_{0} \frac{m_{1} \omega_{2} - m_{2} \omega_{1}}{M(w_{1} + w_{2})} f_{8}(q_{p_{J}}) + \frac{q_{p_{J}}^{2}}{q_{p_{J}}^{\perp} (\omega_{1} + \omega_{2})} f_{4}(q_{p_{J}}) + \gamma_{0} (q_{p_{J}}^{\mu} - \gamma_{\mu} q_{p_{J}}^{\perp}) f_{8}(q_{p_{J}}) \right\} \gamma_{5} = \overline{\varphi}_{p_{J}}(q_{p_{J}}) e_{\perp\mu}.
\]

(44)

2.2 The form factors at the effective vertices of diquark coupling to $\gamma$, $Z^{0}$, $W^{\pm}$

1. The effective current for a scalar diquark coupling to a photon can be written as,

\[
\Gamma_{\mu}^{s\gamma_{8}} = -ieG(Q^{2})(P_{f} + P_{f})^{\mu} = -ie(e_{1}M_{1}^{\mu} + e_{2}M_{2}^{\mu}),
\]

(45)

where $e_{1}$ and $e_{2}$ are the charges of the quarks in the diquark with momenta $p_{1}$ and $p_{2}$ respectively.

2. The effective current for a vector diquark coupling to a photon is

\[
\Gamma_{A\gamma_{A}}^{\alpha\beta} = -ie(2G_{2}(Q^{2})(P_{f} + P_{f})^{\mu}g_{\alpha\beta} + G_{3}(Q^{2})(P_{f} + P_{f})^{\mu}P_{f}^{\alpha}P_{f}^{\beta}) = -ie(e_{1}M_{1}^{\alpha\beta} + e_{2}M_{2}^{\alpha\beta}).
\]

(46)

3. The effective current for a scalar diquark coupling to $Z^{0}$ can be written as

\[
\Gamma_{sZ^{0}}^{\mu} = -igG(Q^{2})(P_{f} + P_{f})^{\mu} = -ig(M_{1}^{\mu} + M_{2}^{\mu}).
\]

(47)

4. The effective current for a vector diquark coupling to $Z^{0}$ is
\[ \Gamma_{AZA}^{\alpha\mu\beta} = -ig\left[ G_1(Q^2)(P_f + P_i)^\mu g^{\alpha\beta} - G_2(Q^2)(P_f^\alpha g^{\mu\beta} + P_i^\beta g^{\mu\alpha}) + G_3(Q^2)(P_f + P_i)^\mu P_f^\alpha P_i^\beta \\
- iG_4(Q^2)\epsilon^{\alpha\mu\beta\sigma}(P_f + P_i)_\sigma - iG_5(Q^2)\epsilon^{\alpha\mu\beta\sigma}(P_f - P_i)_\sigma - iG_6(Q^2)\epsilon^{\alpha\beta\rho\sigma} P_f^\rho P_i^\sigma - iG_7(Q^2)\epsilon^{\alpha\beta\rho\sigma} P_f^\rho P_i^\sigma (P_f - P_i)^\mu \\
- iG_8(Q^2)(\epsilon^{\mu\beta\rho\sigma} P_f^\rho P_i^\sigma P_f^\alpha + \epsilon^{\alpha\mu\sigma\rho} P_f^\rho P_i^\sigma P_i^\beta) - iG_9(Q^2)(\epsilon^{\mu\beta\rho\sigma} P_f^\rho P_i^\sigma P_f^\alpha - \epsilon^{\alpha\mu\sigma\rho} P_f^\rho P_i^\sigma P_i^\beta) \right] \\
= -ig(M_1^{\alpha\mu\beta} + M_2^{\alpha\mu\beta}). \quad (48) \]

5. The effective current for a scalar diquark coupling to \( W^\pm \) is

\[ \Gamma_{sW_\pm}^\mu = -ig\left[ G_1(Q^2)(P_f + P_i)^\mu + G_2(Q^2)(P_f - P_i)^\mu \right] \\
= -ig(M_1^\mu + M_2^\mu). \quad (49) \]

6. The effective current for a vector diquark coupling to \( W^\pm \) is

\[ \Gamma_{AVW_\pm}^{\alpha\mu\beta} = -ig\left[ G_1(Q^2)(P_f + P_i)^\mu g^{\alpha\beta} - G_2(Q^2)(P_f^\alpha g^{\mu\beta} + P_i^\beta g^{\mu\alpha}) - G_3(Q^2)(P_f^\alpha g^{\mu\beta} - P_i^\beta g^{\mu\alpha}) + G_4(Q^2)(P_f + P_i)^\mu P_f^\alpha P_i^\beta \\
+ G_5(Q^2)(P_f - P_i)^\mu g^{\alpha\beta} + G_6(Q^2)(P_f - P_i)^\mu P_f^\alpha P_i^\beta \\
+ G_7(Q^2)\epsilon^{\alpha\mu\beta\sigma}(P_f + P_i)_\sigma - iG_8(Q^2)\epsilon^{\alpha\mu\beta\sigma}(P_f - P_i)_\sigma - iG_9(Q^2)\epsilon^{\alpha\beta\rho\sigma} P_f^\rho P_i^\sigma (P_f + P_i)^\mu \\
- iG_{10}(Q^2)\epsilon^{\alpha\beta\rho\sigma} P_f^\rho P_i^\sigma (P_f - P_i)^\mu - iG_{11}(Q^2)(\epsilon^{\mu\beta\rho\sigma} P_f^\rho P_i^\sigma P_f^\alpha + \epsilon^{\alpha\mu\sigma\rho} P_f^\rho P_i^\sigma P_i^\beta) \\
- iG_{12}(Q^2)(\epsilon^{\mu\beta\rho\sigma} P_f^\rho P_i^\sigma P_f^\alpha - \epsilon^{\alpha\mu\sigma\rho} P_f^\rho P_i^\sigma P_i^\beta) \right] \\
= -ig(M_1^{\alpha\mu\beta} + M_2^{\alpha\mu\beta}). \quad (50) \]

7. The effective current for a vector diquark-\( W^\pm \)-scalar diquark coupling is written as

\[ \Gamma_{AVWs}^{\alpha\mu} = -ig\left[ G_1(Q^2)P_f^\alpha P_i^\mu + G_2(Q^2)g^{\alpha\mu} - iG_3(Q^2)\epsilon^{\alpha\mu\rho\sigma} P_f^\rho P_i^\sigma \right] \\
= -ig(M_1^{\alpha\mu} + M_2^{\alpha\mu}), \quad (51) \]

where

\[ M_1^{\alpha\mu} = -\int \frac{d^4q}{(2\pi)^4} Tr\left[ \bar{\chi}_{P_f} (q') V_{q_i W_q}^\alpha \chi_{P_i} (q) S_F^{-1} (p_2) \right] \quad (52) \]
\[ M_2^{\alpha\mu} = -\int \frac{d^4q}{(2\pi)^4} Tr\left[ \bar{\chi}_{P_f} (q') S_F^{-1} (p_1) \chi_{P_i} (q) V_{q_2 W_q'}^\mu \right]. \quad (53) \]

### 2.3 The form factors at the effective vertices of diquark coupling to \( \pi \) mesons

To complete the picture, we also discuss the effective couplings of scalar or vector diquarks and \( \pi \) mesons in terms of the chiral Lagrangian, which has been widely adopted in the studies of the effective quark-meson-quark couplings.
The effective coupling of quark and \( \pi^- \) meson is of the form \([30]\):
\[
\Gamma_{q\pi q} = g_q \frac{f_{\pi}}{f_{\pi}} \gamma_5 k, \tag{54}
\]
where \( k \) is the momentum of the pion. We obtain the effective coupling vertex of \( 1^+ \) diquark and \( \pi^- \) meson as following:
\[
\Gamma_{A\pi A} = \epsilon_1^\alpha \epsilon_2^\beta \Gamma_{A\pi A}^{\alpha\beta} \tag{55}
\]
where \( \epsilon_1, \epsilon_2 \) are the polarization vectors of the two axial-vector diquarks and
\[
\Gamma_{A\pi A}^{\alpha\beta} = i g_q \frac{f_{\pi}}{f_{\pi}} \left[ M_1^{\alpha\beta} + M_2^{\alpha\beta} \right] \tag{56}
\]
where
\[
M_1^{\alpha\beta} = - \int \frac{d^4 q}{(2\pi)^4} Tr \left[ \bar{\chi}^\beta_{P_f} (q') \Gamma_{\pi\pi q} \chi^\alpha_{P_i} (q) S_F^{-1} (p_2) \right] \tag{57}
\]
\[
M_2^{\alpha\beta} = - \int \frac{d^4 q}{(2\pi)^4} Tr \left[ \bar{\chi}^\beta_{P_f} (q') S_F^{-1} (p_1) \chi^\alpha_{P_i} (q) \Gamma_{\pi\pi q} \right], \tag{58}
\]

3 numerical results

The formulas derived above are for form factors at the effective vertices of any diquark which couples to gauge bosons. However, as indicated in the introduction, the diquark picture only works without any doubt for the heavy diquarks. For light diquark, or heavy-light diquark, the relativistic effects may be crucial, therefore in this work, to avoid any ambiguity, we only numerically evaluate the form factors of the \( bc \)–diquark coupling to gauge bosons. There are both scalar and axial vector \( bc \) diquark (here we do not concern the orbital excited states), whereas there is only axial vector diquarks for \( bb \) and \( cc \).

For applying the form factors under consideration to transition processes, where baryons are involved, we need to present the numerical values which are computed in terms of the programs developed by Chang et al. The input parameters are \([27, 31]\): \( m_c = 1.7553 \) GeV, \( m_b = 5.224 \) GeV, \( \lambda = 0.20 \) GeV\(^2\), \( \Lambda_{QCD} = 0.26 \) GeV, \( a = 2.71828 \), \( \alpha = 0.06 \) GeV, \( \beta = 0.5 \), \( V_0 = -0.3 \) GeV.

We plot the dependence of the form factors for scalar diquark coupling to gluon, photon and \( Z^0 \) on \( Q^2 \) in Figs. 2, 3 and 4 respectively. It is noted that the three independent form factors for the effective vertex of a vector diquark coupling to a gluon are simply attributed into only one form factor by reasonable physical considerations in \([32]\). For a comparison, to extract the form factor, in our calculations, we employ the relations given in the reference and adopt the corresponding spin-functions of baryons, then we plot the form factors at the effective vertex of vector diquark coupling to gluon in Fig. 5.

Moreover, since W-emission is accompanied by a flavor change, the initial diquark \( bc \) should transit into \( cc + W^- \) or \( bs + W^+ \). The situation is slightly more complicated. Therefore, for only illustration of the behavior of the diquark form factors, we do not numerically evaluate \( \Gamma_{DWD'} \) where \( D \) and \( D' \) have different flavors and in our following work, we will present them along with other form factors such as \( \Gamma_{AGS} \) etc.
Figure 2: The form factor for the effective vertex of scalar diquark coupling to gluons. The solid line is the result calculated in terms of the BSE and the dashed-line corresponds to the form factor which is phenomenologically introduced by the authors of Ref. [16].

Figure 3: The form factor for the effective vertex of scalar diquark coupling to photon.
Figure 4: The form factor for the effective vertex of scalar diquark coupling to $Z^0$.

Figure 5: The form factors for the effective vertex of vector diquark coupling to gluon. The solid line corresponds to the result obtained by the BSE method and the dashed line corresponds to the phenomenological form factor given in Ref. [16].
4 Summary and discussions

In this work we derive the form factors for the effective vertices of scalar and vector diquarks coupling to gauge bosons, $g, \gamma, W^\pm$ and $Z^0$, as well as to the $\pi$–meson. We carry out our derivations in the framework of the Bethe-Salpeter equation. Even though the BSE is established on the quantum field theory and its validity is not dubious, there still exist some uncertainties when it is applied to deal with practical problems. First the kernel in the equation is not derived based on the fundamental principles, namely because the non-perturbative QCD effects are taken into account, corresponding interaction must be phenomenologically introduced. In this work, we adopt the simple Cornell potential as the kernel. Then for solving the equation, one needs to adopt the instantaneous approximation, and then the original Lorentz invariance is lost. Therefore, for the systems where the relativistic corrections are important, the approximation would bring up large errors. However, for the system especially the systems where only heavy quarks are involved, the results are more reliable. In our earlier works about the spectra of diquarks, it was indicated that if there are light quark constituents, one may make certain modifications. One efficient way is to consider the BSE and the Dyson-Schwinger equation simultaneously[33]. This approach might alleviate the severity of the error, but cannot finally eliminate all shortcomings in the framework. In our later work, we are going to deal with the light diquarks and then estimate the errors. So far, even though we know the origin of the uncertainties, we cannot quantitatively estimate their magnitudes.

On the other side, even though the framework has some problems, it is applicable to the processes where baryons, especially heavy baryons are concerned. Since the diquark picture greatly simplifies the whole calculation and also has achieved remarkable success in phenomenology, one has reason to believe that the diquark picture is suitable for dealing with the baryon production or decay processes. Diquark is definitely not a point-like particle, therefore a form factor(s) at the effective vertex of diquark coupling to gauge bosons and even pions can partly compensate the effect of the inner structure of diquarks. We employ the BSE to derive the form factors. Fortunately, the recent high energy experiments provide more and more accurate information about the baryon structure, and we can wait for more data to test our derivation and find the applicability of this approach.

For a demonstration, we would like to compare the asymptotic behavior of the form factor $F(Q^2)$, which is derived in this work with its phenomenological form given by the authors of Ref.[16] in Fig. 2. In Fig. 2, we compare the result obtained in terms of the BSE which is represented by the solid line, and the dashed line corresponds to the phenomenological form factor $F(Q^2) = \frac{Q_0^2}{Q_0^2 + Q^2}$. It is noted that the form factor introduced in Ref.[16] is for an $ud$–scalar diquark. Since the form factor is introduced phenomenologically by fitting data, the relativistic effects are included in the parameters. QCD is flavor blind, so that we believe that the form of the form factor for $bc$ and $ud$ must be similar except that the parameter $Q_0^2$ which is related to the constituents of the diquark may be different, at least their tendency behavior must be similar. Therefore this comparison is qualitatively significant, but small deviations would be expected.

$F(Q^2)$ decreases monotonically as $Q^2$ becomes large and approaches to zero rather quickly. The authors of Ref.[16] introduced a phenomenological form factor as $F(Q^2) =$
where $Q_0$ is a parameter determined as $Q_0^2 \sim 3.2 \text{ GeV}^2$ by fitting data [16]. The form is obviously understandable. The form factors should be normalized to unity as $Q^2 \to 0$, i.e. as one looks at the diquark from a far distance, the form factor becomes a unity, whereas as $Q^2 \to \infty$, the inspector then penetrates into the diquark, so that he would see the individual quarks instead of the whole and the diquark picture no longer holds and mathematically it is required to approach zero as $Q^2 \to \infty$. The form factor obtained in terms of the BSE generally coincides with the picture.

Since diquark is a boson of color-anti-triplet, it cannot exist as a physical object, but a constituent in baryon, just like a quark in meson. Besides, it resides in a bound state, therefore must be off-shell, but for a not-very-tight bound state, it can be treated as a physical object which is approximately on its mass shell. Thus one can use the wavefunctions of the diquark for calculating the form factors, but obviously certain errors may be caused. All the form factors obtained in this work cannot be directly tested because diquark does not exist as an individual. To test their validity, one needs to apply them into the practical processes where baryons are concerned. Therefore, in our next work, we will calculate the production and decay rates of the processes where baryons are involved, in terms of the form factors derived here and let data confirm or negate this picture, if conclusion is positive, the accuracy degree will also be determined by the data.

As a conclusion, the diquark picture is reasonable and can be applied to study the processes where baryons are involved, especially for the baryons with two heavy quarks, as long as suitable form factors are included. The form factors derived in terms of the BSE are consistent with that obtained by fitting data, namely, they are applicable in practical calculations. However, for the diquark including two light quarks or that including one light and one heavy quarks, the errors in the calculations may be large. For achieving form factors for diquarks which are composed of only light quarks or one light and one heavy quarks should be studied in a more complicated framework which would be the goal of our next work.

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