Internal ballistic simulation and optimum design of gas-fired rodless cylinder ejection device

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Abstract. A kind of gas-fired rodless cylinder ejection device which is stoke and weight controlled and suitable for mobile launch was designed . The leakage test of the ejection device is carried out based on the principle prototype of rodless cylinder ejection device and the empirical formula between leakage rate and pressure and travel length was fitted. The mathematical model of gas-fired rodless cylinder ejector internal ballistic with considering the leakage rate is established based on the assumption of zero-dimensional internal ballistic. When comparing the results of internal ballistics considering and not considering the leakage rate ,it can be seen that the influence of leakage rate on the results of internal ballistics can not be ignored. The internal ballistic model which coupled the leakage rate is optimized, taking the shortest design stroke and smallest ejection stability index as design target and the parameters of the grain are used as design variables. The results show that after the optimization , the mass flow rate of gas at the end of ejection is increased by 19%, the stroke required for piston speed reaching the design target is shortened by 13.1% , and the launch stability index is increased by 10.7%.

1. Introduction
With the rapid development of modern space reconnaissance technology, it also puts forward higher requirements for missile’s stealth survival and maneuverability, so ejection has been more and more widely used. Currently, the most widely used ejection methods are gas type, compressed air type, hydraulic type and electromagnetic type, etc [1].

Rui ShouZhen et al. [2] compared the characteristics and interior ballistic performance of different missile ejection power systems.Bai Junhuaet al. [3] put forward the uncooled launching power system, and studied the working process and interior ballistic characteristics of the gas generator and the launching cylinder.Huiweihua et al. [4] established the ballistic and kinematic equations of low-pressure chamber based on the existing test data of high-pressure chamber of gas generator, and realized the analysis of missile gas separation process with the test data taken into account.Shao etal. [5] established the whole energy analysis model of the ejection system, and proposed a structural optimization method based on the maximization of exergy efficiency. Considering the mixing characteristics of combustion gases, the energy loss caused by thermal convection and radiation in the combustion chamber was analyzed. Based on this, the dynamic model of the ejection system was established and solved with the fourth-order Runge-Kutta method.Li Renfeng et al. [6] established the secondary combustion mathematical model of gas ejection considering the obstacles on the wall of the
launching cylinder by using k-w SST turbulence model and finite rate/dissipation model. Cheng Hongjie et al. [7-8] established a two-dimensional axisymmetric numerical model with secondary combustion and trail cover motion for the problem of two pressure wave peak impact of missile’s low combustion temperature ejection, studied the flow field mechanism of the smooth bottom pressure curve of the annular cavity, decoupled and analyzed the influence of its structural parameters on the smooth effect of pressure impact, and put forward the idea of sequence optimization. Ren Rui et al. [9] studied the ejection performance of a multi-stage pneumatic hydraulic ejection device with the oil self buffering structure, which used compressed air as the power source and oil as the transmission medium. Ren Jie, Yang Fengbo et al. [10-13] proposed a high pressure air two-stage three cylinder ejection device. Based on the improved corresponding state virial equation, an interior ballistic model considering the real effect was established. Shao Yajun et al. [14] designed a solid grain with variable cross-section based on the scheme of erecting driven by gas and hydraulic pressure, and established the integrated dynamic model of erecting system. Yao Lin et al. [16] proposed a new type of high-pressure air driven rodless cylinder missile ejection device, which can realize small size, long range and low overload launching. The interior ballistic model of rodless cylinder ejection device considering the real gas effect was established, and the variation laws of thermal parameters and missile motion parameters in the process of Missile Ejection were obtained.

The literature above has studied many different launch modes. For the traditional gas launch mode, the low-pressure chamber is generally located in the rear space of the missile in the launch cylinder, and the high-temperature gas has strong thermal ablation and strong impact effect on the missile body and its adjacent launch units [17]. For the compressed air ejection mode, due to the need of air compressor, high-pressure gas cylinder, pipeline and other supporting equipment, the ejection device is bulky and heavy, which is not suitable for flexible launch. On the basis of Yao Lin's work, this paper presents a kind of gas-fired rodless cylinder ejector, which integrates the low-pressure chamber and rodless cylinder. The catapult working medium does not act on the tail of the missile, so the thermal protection requirements are low. At the same time, the structure of the device is compact, the weight is controllable, the adaptability is good, and it is suitable for mobile and flexible launch.

In this paper, the leakage test of the ejection device was carried out on the principle prototype of the rodless cylinder ejector using high-pressure gas source. The leakage rate of the rodless cylinder ejector under different cylinder pressure and piston stroke was obtained, and the empirical formulas of leakage rate, pressure and stroke were fitted. The mathematical model of internal ballistics of gas-fired rodless cylinder ejector considering leakage was established, and the results of internal ballistics considering leakage and not considering leakage were compared. Finally, the multi-objective optimization of internal ballistics considering leakage was carried out to obtain the optimized results of internal ballistics and provided theorectical support for further engineering design.

2. Structure and working principle of gas-fired rodless cylinder ejector

2.1. Structure of the model
In this paper, a model of gas-fired rodless cylinder ejector is presented. Its structure is shown in figure 1.

The model is mainly composed of gas generator, rodless cylinder, guide rail, cylinder fixing box, bomb holder, piston assembly, sealing belt, ammunition supporting platform, buffer cylinder, sealing belt tensioning mechanism and other components. The gas generator is connected to the air inlet through the pipeline, the piston assembly, as the power output part, is connected to the bomb support platform through the bolt. The piston assembly, the guide rail and the inner wall surface of the cylinder slide together to play a guiding role, ensuring the linear movement of the bomb support platform.
2.2. working principle

The working principle of the gas-fired rodless cylinder ejector is: after the control box sent out the launching command, the igniter of the gas generator immediately responds to ignite the grain. High temperature and high pressure gas flows into the cylinder (low pressure chamber) through the gas transmission pipeline from the gas generator (high pressure chamber) with the nozzle. When the force on the piston is greater than the self weight of the missile, the fuel gas drives the piston assembly, the missile support platform and the missile to accelerate together. After reaching the stroke, the lateral arm of the piston assembly impacted the buffer oil cylinder assembly. Under the action of the hydraulic damping cylinder, the speed of the missile support platform and the piston assembly decreases rapidly. The missile flies away from the support platform and the system completes a launch task.

3. Leakage test of rodless cylinder ejector

The structural characteristics determine that there must be a certain degree of leakage, and the amount of leakage directly affects the prediction accuracy of the internal ballistic model. Based on the prototype of the rodless cylinder, the leakage test of the ejector was carried out. The leakage rate of the rodless cylinder under different cylinder pressure and piston stroke was obtained, and the empirical formula of leakage rate, pressure and stroke was fitted. In reference, it was pointed out that the leakage flow decreased with the increase of the leakage working medium temperature. Considering the cost and feasibility of the test, the high pressure air source was used instead of the gas generator to obtain the upper limit of the leakage. In addition, for safety reasons, the maximum pressure in the rodless cylinder was limited below 6Mpa.

3.1. leakage test plan

As shown in figure 2, the leakage test system was composed of rodless cylinder, steel pipe for limit the position, air compressor, high pressure air source, pneumatic valve, pressure sensor, temperature sensor and data acquisition instrument. The volume of high pressure air source was about 2.5m³, which makes it inconvenient to measure the temperature change. Considering that the pressure change of high pressure air source and the temperature change caused by it were not big during the test, it was assumed that the temperature of high pressure air source was a constant value, and only the temperature changed in the cylinder was measured.

The main steps of leakage test were as follows: ① Limit the piston at different stroke through different length of limit steel pipe. ② Use air compressor to fill the air source with high pressure.
air. ③ Open the pneumatic valve to make the high-pressure air enter into the rodless cylinder and keep the valve open for 1s. ④ Obtain the pressure time history curve of air source, rodless cylinder and the temperature time history curve of the rodless cylinder during the process of keeping the valve opened through data collection system. ⑤ Calculate the density of working medium in air source and rodless cylinder, and convert the leakage quality of working medium in the process of valve opening, so as to obtain the leakage rate in unit time.

![Figure 2. Schematic diagram of leakage test plan.](image)

### 3.2. Leakage rate fitting

The leakage test of the ejection device was carried out. The leakage of the working medium of the ejection device when the cylinder pressure was 3.76 Mpa and the piston stroke was 2.18 m was shown in figure 3. During the process of keeping the valve opened, the pressure-time history curve of the air source and the rodless cylinder were shown in figure 4, and the time history curve of the temperature in the rodless cylinder was shown in figure 5.

![Figure 3. Working fluid leakage process.](image)

![Figure 4. Pressure time history curves.](image)

![Figure 5. Temperature time history curves.](image)

From the data in figure 4, figure 5 and the density conversion formula, we could calculate the air source and the working medium density in the cylinder at the initial time and the final time, and then got the total mass of the air source and the working medium in the cylinder at the initial time and the final time. The difference between them was the leakage amount of the working medium in the
The process of valve opening, so as to obtain the leakage rate in unit time. The density conversion formula was as follows:

\[
\rho_{\text{Pressure}} = \rho_{\text{Actual}} \times \frac{P_{\text{Actual}}}{P_0} \times \frac{273.15}{T_{\text{Absolute}}}
\]  

(1)

The final calculation value of leakage rate of ejection device under different cylinder pressure and piston stroke was shown in table 1.

| Leakage rate under different pressure and stroke. |
|-----------------------------------------------|
|       1.77MPa     |   2.76MPa    |  3.76MPa   |  4.85MPa   |  5.92MPa   |
|-----------------|-------------|-----------|-----------|-----------|
| 0m              | 8.01%       | 7.86%     | 7.71%     | 7.52%     | 7.30%     |
| 1.11m           | 8.28%       | 8.05%     | 7.90%     | 7.71%     | 7.54%     |
| 2.18m           | 8.48%       | 8.31%     | 8.11%     | 7.85%     | 7.66%     |
| 3.24m           | 8.60%       | 8.42%     | 8.28%     | 8.06%     | 9.84%     |
| 4.18m           | 8.80%       | 8.62%     | 8.43%     | 8.20%     | 8.02%     |
| 5.18m           | 8.98%       | 8.77%     | 8.61%     | 8.39%     | 8.20%     |

The fitted relationship between leakage rate, pressure and stroke was as follows:

\[ S_{\text{leak}} = 8.38 - 0.178 \times P_2 + 0.185 \times L - 2.82 \times 10^{-3} \times P_2 \times L - 1.14 \times 10^{-4} \times P_2^2 - 7.41 \times 10^{-4} \times L^2 \]

4. Interior ballistic modeling of gas-fired rodless cylinder ejector

The actual launching process was a complicated physical and chemical process. In order to analyze the interior ballistic process of the ejection system with the theory of kinematics and thermodynamics, it was necessary to distinguish the primary and secondary factors, ignored some secondary factors, and simplified the launching process into a more ideal launching process. According to the needs of modeling, the following assumptions were made:

1) The average axial flow parameters were taken for both the high pressure chamber and the low pressure chamber, regardless of the change of flow parameters along the length.
2) Both gas and air obeyed the ideal gas state equation.
3) The propellant was completely combustion, regardless of the change of combustion product composition and secondary combustion. The combustion temperature was equal to the combustion temperature of the propellant at constant pressure.
4) Neglecting the amount of gas produced by ignition gunpowder, the charge ignited instantaneously and completely under ignition pressure, and the combustion conformed to the law of geometric combustion.
5) The energy coefficient was introduced to correct the energy loss caused by heat transfer, and its value was assumed to be a constant in the whole process.
6) When the displacement of the piston reached the stroke, the air inlet cavity was connected with the air release hole, and the gas in the low-pressure chamber was released in time, ignoring the influence of the residual gas in the air inlet cavity on the piston buffer process.

4.1. Mathematical model of internal ballistics of gas generator

4.1.1. Conservation equation of mass. As the power source of ejection system, gas generator could be regarded as a special solid rocket motor. In the course of its work, according to the principle of conservation of mass [19], we could get:

\[
\frac{dm_1}{dt} = M_b - Q_b
\]

\( dm_1 \) was the gas mass change rate in the high pressure chamber, \( M_b \) was the gas generation rate
generated by gunpowder combustion, $Q_n$ was the flow-per-second of gas from the nozzle outlet to the low pressure chamber.

4.1.2. **Discharge equation** The gas generation rate of high pressure chamber with exponential burning rate $u = \frac{de}{dt} = ap_1^n$ could be expressed as:

$$M_b = \rho_p A_y u$$  \hspace{0.5cm} (3)

In the formula, $u$ was the burning rate of the gunpowder, $e$ was the burning thickness of the gunpowder at a certain time, $a$ was the burning rate coefficient, $n$ was the pressure index, $p_1$ was the pressure of the high pressure chamber, $\rho_p$ was the density of the high pressure chamber, and $A_y$ was the burning area of the high pressure chamber charge.

The mass flow of gas $Q_n$ from the high pressure chamber to the low pressure chamber was determined by the characteristics of the mixed gas

$$Q_n = \begin{cases} \mu_e \rho_p A_y \frac{2k}{k-1} \left( \frac{p_2}{p_1} \right)^{\frac{k+1}{k}} \left( 1 - \left( \frac{p_2}{p_1} \right)^{\frac{k}{k+1}} \right)^{\frac{2}{k+1}} \frac{\mu}{k} & \text{if } \frac{p_2}{p_1} < 1 \\ \mu_e \frac{k}{k+1} \frac{p_2}{p_1} A_y \frac{1}{\sqrt{R_g T_i}} & \text{if } \frac{p_2}{p_1} \leq \left( \frac{2}{k+1} \right)^{\frac{k}{k+1}} \end{cases}$$  \hspace{0.5cm} (4)

In the formula, $\mu_e$ was the flow correction coefficient, $k$ was the adiabatic index of gas, $A_y$ was the sectional area of nozzle, $p_2$ was the pressure of low pressure chamber.

4.1.3. **Equation of state of gas** According to hypothesis (2), gas was considered as an ideal gas, and its state equation was

$$p_1 = \rho_g R_g T_i$$  \hspace{0.5cm} (5)

In the formula, $p_1$ was the gas pressure in the high pressure chamber, $R_g$ was the gas constant, and $T_i$ was the temperature of the gas in the high pressure chamber.

Expand the left side of formula 1 to get

$$\frac{dm_1}{dt} = \frac{d}{dt} (\rho_g V_i) = \rho_g \frac{dV_i}{dt} + V_i \frac{d\rho_g}{dt}$$  \hspace{0.5cm} (6)

In the formula, $V_i$ was the volume occupied by the gas, which was called the free volume of the high pressure chamber. $\rho_g \frac{dV_i}{dt}$ was the mass of the gas filled due to the increase of the free volume of the high pressure chamber in unit time. The increase of the free volume was actually equal to the volume of the propellant burned off, as $dV_i = A_y de$, so as

$$\rho_g \frac{dV_i}{dt} = \rho_e A_y u$$

(7)

It could be obtained from equation (5), $\rho_g = \frac{p_1}{R_g T_i}$, differentiated it, we got

$$\frac{d\rho_g}{dt} = \frac{1}{R_g T_i} \frac{dp_1}{dt} - \frac{p_1}{(R_g T_i)^2} \frac{d(R_g T_i)}{dt}$$

(8)

According to hypothesis (3), during the combustion of propellant, the gas temperature $T_i$ was a constant; the ideal gas component and gas constant $R_g$ were constant, then the second term at the right end of the above formula was zero, as a result.
Substituting equations (3), (4) and (9) into equation (1), we could get

$$\frac{V_1}{R_g T_1} \frac{dp_1}{dt} = (\rho_f - \rho_g) A_p u - Q_m$$

(10)

The above formula was the differential equation for calculating the p1-t curve of the zero dimensional interior ballistics of the high pressure chamber.

4.2. Mathematical model of internal ballistics of gas generator

4.2.1. Conservation equation of mass According to the principle of mass conservation during the operation of low pressure chamber, it could be concluded that:

$$\frac{dm_g}{dt} = -Q_m$$

(11)

$m_g$ was the quality of gas entering into the low pressure room. The mixed gas expanded in the low pressure chamber, pushed the piston to move, and increased the mechanical energy of the system.

4.2.2. Equation of conservation of energy

$$U_1 + Q = U_2 + W$$

(12)

In the formula

$$U_1 + Q = x_c M_g C_{ig} T_1 + M_d C_{ia} T_i$$

$$U_2 = M_g C_{ig} T_1 + M_c C_{ic} T_i$$

$$W = 0.5 M_{Dv}^2 + \int F_z dl$$

(13)

In the formula, $U_1$ represented the initial state energy of the mixed working medium, $Q$ represented the value of external heat dissipation, it was negative. According to hypothesis (6), the internal energy of the gas was corrected by the energy coefficient $x_c$, $U_2$ represented the final state energy of the mixed working medium, $W$ represented the work output by the thermal system, which was equal to the sum of the internal energy obtained by the missile and the work done to overcome the resistance. $M_g$, $M_a$, $M_D$ were gas mass, air mass, ejecta mass respectively, $C_{ig}$ $C_{ia}$ were gas specific heat, air specific heat respectively, $T_1$ was air temperature, $T_i$ was low pressure chamber temperature, $F_z$ was the sum of various resistances, $v$, $l$ were ejection speed and ejection displacement.

4.2.3. Equation of state of gas According to hypothesis (2), the mixed gas in low pressure chamber was considered as ideal gas, and its state equation was

$$p_2 = \frac{x_p (R_g M_g + R_a M_a) T_1}{W_0 + S_i l}$$

(14)

$x_p$ was the pressure coefficient of low pressure chamber, $R_g$ and $R_a$ were the gas constant and air constant respectively, $S_i$ was the thrust area of piston, $W_0$ was the initial volume of low pressure chamber.

4.2.4. Equation of motion of piston

$$M_D \frac{dv}{dt} = (1 + x_h) [(p_2 - p) S_i - F_z]$$

(15)

$x_h$ was the kinetic energy coefficient and $p$ was the atmospheric pressure.

4.2.5. Piston velocity equation
\frac{dl}{dt} = v \quad (16)

4.2.6. Acceleration equation of piston

\frac{dv}{dt} = a \quad (17)

The above equation (11) ~ (17) constituted the working model of the low-pressure chamber, the gas pressure, temperature, acceleration, velocity and displacement of the piston movement in the low pressure chamber could be calculated according to the model.

Set \( X_1 = e \), \( X_2 = p_1 \), \( X_3 = m_\ell \), \( X_4 = T_\ell \), \( X_5 = V \), \( X_6 = l \), \( X_7 = p_2 \), \( X_8 = a \). According to the above analysis, the closed state equations of interior ballistics considering the actual leakage could be established, as shown in equation (18).

\[
\begin{align*}
X_1 &= aX_2^3 \\
\dot{X}_2 &= (X_1 <= e_0) \frac{R_p T_\ell[(\rho_p - X_5/X_2)A_0 X_1 - X_3]}{V_1}, \quad e_0 \text{ is initial thickness of gunpowder} \\
X_3 &= Q_n \\
X_4 &= \frac{(x_p Q_n C_{\ell q} T_\ell - M_\ell X_3 X_6 - F_3 X_4)(X_4 C_{\ell q} + M_\ell C_{\ell q}) - C_{\ell q} Q_n (x_p X_4 C_{\ell q} T_\ell + M_\ell C_{\ell q} T_\ell - 0.5 M_\ell X_5^2 - F_3 X_4)}{(X_4 C_{\ell q} + M_\ell C_{\ell q})^2} \\
\dot{X}_5 &= X_8 \\
\dot{X}_6 &= X_5 \\
\dot{X}_7 &= S_{ww} \left\{ x_p [R_p Q_n X_4 + (R_p X_3 + R_s M_\ell) X_3]\{W_0 + S_0 X_6\} - x_p R_s X_3 + R_s M_\ell X_4 X_5 \right\} \\
\dot{X}_8 &= \frac{(W_0 + S_0 X_6)^2}{M_\ell} \\
S_{ww} &= 8.38 - 0.178 \times P_2 + 0.185 \times L - 2.82 \times 10^{-3} \times P_2 \times L - 1.14 \times 10^{-4} \times P_2^2 - 7.41 \times 10^{-4} \times L^2
\end{align*}
\]

(18)

5. Result analysis and optimization design of internal ballistics

5.1. Interior ballistic results and analysis without considering leakage

During the ejection process, with the movement of the missile, the free volume between the bottom of the missile and the launching cylinder increased. Using the inner hole increased surface combustion gunpowder which both the ends and the outer sides were covered, could ensure the missile to make a acceleration movement in the cylinder. The cross section shape of the grain was shown in figure 6. According to the burning law of gunpowder, the burning area of gunpowder at a certain time was

\[
A_0 = N \pi (d_2 + 2e)L_c
\]

\( A_0 \) was the combustion area of the gunpowder, \( N \) was the quantity of the gunpowder, \( L_c \), \( D_c \), \( d_c \) and \( e \) were respectively the length, outer diameter, inner diameter and the thickness of the gunpowder that had been burned at a certain time, and the total thickness of the burning flesh of the gunpowder was \( e_0 = (D_c - d_c) / 2 \).
Figure 6. Cross section of the grain.

The initial parameters of the 8 independent variables were shown in table 2, and other parameters used in the solution process were shown in table 3.

**Table 2.** The initial value of the variable.

| $X_1$ ($m$) | $X_2$ ($Pa$) | $X_3$ ($kg$) | $X_4$ ($K$) | $X_5$ ($m/s^3$) | $X_6$ ($m$) | $X_7$ ($Pa$) | $X_8$ ($m/s^2$) |
|------------|-------------|-------------|-------------|----------------|-----------|------------|------------|
| 0          | 1e5         | 0           | 30          | 0              | 0         | 1e5        | 0          |

**Table 3.** The relevant parameters of model.

| parameters                                                                 | numerical value |
|---------------------------------------------------------------------------|-----------------|
| Effective thrust area of cylinder $S_t/m$                                  | 0.222           |
| Initial volume of low pressure chamber $W_0/m$                             | 0.222           |
| Isobaric combustion temperature of propellant $T_1/K$                      | 1070            |
| Gas constant $R_g$ / (J/(kg·K))                                           | 417.8           |
| Air gas constant $R_a$ / (J/(kg·K))                                       | 287             |
| Specific heat of gas at constant volume $c_v$ /                           | 1510            |
| Specific heat of air at constant volume $c_{va}$ /                         | 717             |
| Gas adiabatic index $k$                                                   | 1.27            |
| Burning rate coefficient $a$                                               | 2.9e-4          |
| Pressure exponent $n$                                                     | 0.335           |
| Charge density of high pressure chamber $\rho_p$/(Kg·m)                   | 1600            |
| Flow correction factor $\mu$                                              | 0.96            |
| Throat area $A_t/m$                                                       | 7.85e-3         |

Regardless of leakage, the value of $S_{tie}$ in equation (18) was 0, and the numerical calculation results were shown in figure 7-10.
Figure 7 showed the pressure change law of high pressure chamber and low pressure chamber. It could be seen from the figure that the pressure in the low pressure chamber was always less than that in the high pressure chamber. At the initial time, both the high pressure chamber and the low pressure chamber were air under normal temperature and pressure. Ignoring the amount of gas produced by ignition gunpowder, assuming that the charge was self ignited at the same time, with the combustion of the grain, the pressure of the high pressure chamber continued rising, the high temperature and high pressure gas continuously entered into the low pressure chamber through the nozzle, and the pressure of the low pressure chamber rised with the increase of the pressure of the high pressure chamber in the early stage of ejection, and decreased in the middle and later stages gradually. It could be seen from the analysis that in the early stage of ejection, the pressure in the high pressure chamber was small, the piston speed was also small, then the volume growth rate of the low pressure chamber was small, too. Therefore, the pressure difference between the high pressure chamber and the low pressure chamber was very small, the gas mass flowed from the high pressure chamber to the low pressure chamber was small, and the pressure in the high pressure chamber and the low pressure chamber increased with almost similar slopes. In the middle and later stages of ejection, the piston speed was higher, and the volume of the low pressure chamber increased rapidly, but the gas generation rate was not enough to maintain the stability of the pressure of the low pressure chamber, so the pressure of the low pressure chamber was in a decreasing state.

It could be seen from figure 8 that before 0.3s, the gas mass flowed into the low pressure chamber from the high pressure chamber increased approximately linearly with a lower slope, and after 0.3s, the gas mass flowed increased linearly with a slope several times than that before. It could be seen from the analysis that before 0.3s, the combustion area, combustion speed and pressure of the high pressure chamber of the charge are small, the gas generation rate was small, the piston speed was small, and the volume increase rate of the low-pressure chamber was also small, so the pressure difference between the high pressure chamber and the low pressure chamber was very small, and the gas mass flow increase rate was low. After 0.3s, the burning area, burning rate and pressure of high-pressure chamber increased, the gas generation rate increased gradually, and the pressure difference between high pressure chamber and low pressure chamber increased gradually, so the gas mass flow rate increased at a greater rate.

![Figure 7. Pressure curves.](image1)

![Figure 8. Mass flow curves.](image2)
Figure 9 and figure 10 showed the change curve of the kinematic parameters of the missile during the ejection process. It could be seen from figure 9 that after 0.5s, the overload curve of missile decreases at a fast rate, and the overload coefficient decreased to 5 at 0.9s. The reduced overload factor results in a cylinder travel of about 9.8m when the missile reached the target speed.

![Figure 9. Piston overload curves.](image)

![Figure 10. The speed-displacement curves of piston.](image)

In addition, the launch stability $k \ (k = \frac{2a_{\text{max}}L}{V^2})$ was the evaluation index of the launch process. The smaller the $k$ value was, the more stable the launch process was, on the contrary the higher the launch quality was, the lower the launch quality was. Because of the decrease of the pressure in the cylinder at the later stage of the ejection, the overload coefficient decreased, the stability of the missile ejection process decreased, and the launching quality was not high.

In conclusion, without considering the leakage, the pressure in the low pressure chamber and the overload coefficient of the missile decreased rapidly in the middle and later stages of the ejection. This conclusion was consistent with the simulation results in document and the experimental test results.

5.2. Comparative analysis of the results without considering leakage and leakage rate coupling interior ballistics

5.2.1. Coupling algorithm considering leakage According to the test data, the empirical formula of leakage rate was about the relationship between the pressure and the stroke in the cylinder. The leakage rate was determined by the pressure and the stroke in the cylinder, and the pressure in the cylinder was related to the leakage rate and then affected the ejection stroke, and the two affect each other. Therefore, in the numerical calculation, iterative coupling calculation was needed. The specific calculation method is as follows:

1) $t = t_1 + \Delta t$ Calculated the ejection movement process (solved the movement equations).
2) $t = t_1 + \Delta t + \Delta t$ Calculated the leakage rate according to the calculated cylinder pressure and stroke.
3) Updated the leakage rate value into the interior ballistic equation and recalculated the ejection movement.
4) $t = t_1 + \Delta t + \Delta t$ ...3) Until the ejection process is completed.
5.2.2. *Comparison of interior ballistic calculation results without considering leakage and considering leakage* The comparison between the calculation results of main interior ballistics without considering leakage and considering leakage was shown in the figure 11.
5.3. Optimization results of interior ballistics considering leakage based on genetic algorithm

When the inner hole increased surface combustion grain was not optimized, the pressure in the low pressure chamber and the overload coefficient of the missile decreased rapidly, which reduced the stability of missile ejection process. The optimization design of the structural parameters of the propellant grain was carried out. In the early stage of the ejection, the combustion area was small, avoiding the excessive initial impact. In the middle and later stages of the ejection, the combustion area was increased as required, making the gas generated with the increase of the rate of formation, the low pressure chamber could be "aerated" to maintain the pressure stability.

In order to reduce the amount of comparison, the remaining structural parameters of the high pressure chamber were controlled to be consistent with the previous scheme, such as the number of grains, volume of high pressure chamber, nozzle diameter, volume of initial volume chamber, effective thrust area of cylinder and other parameters. Leakage was considered in the calculation. It can be seen from figure 5 that the inner diameter \( d_c \), outer diameter \( D_c \) and length of the grain \( L_c \) were the direct factors affecting the burning surface area. Taking them as the design variables in the optimization process, and the maximum overload coefficient and the cylinder pressure of the missile were the constraints, the travel length \( L \) when the missile reached the speed index, and the launch stability index \( k \) were the objective functions. The shorter the travel \( L \) was, the shorter the equipment length was, the better the equipment adaptability was. The smaller the value \( k \) was, the more stable the launching process was, the higher the launching quality was. After several trials, the range of design variables and constraints were given as shown in the table 4.

| Table 4. Value range of design variable. |
|-----------------------------------------|
| parameter                              | numerical value |
| \( d_c \)                              | 0.01~0.06       |
| \( D_c \)                              | 0.08~0.2        |
| \( L_c \)                              | 0.1~0.8         |
| Maximum overload coefficient of missile| 8               |
| Cylinder pressure /MPa                 | 9               |
The multi-objective optimization problem (MOP) was to optimize multiple objectives at the same time. The above optimization was the multi-objective optimization problem. Most of the optimization problems encountered in practice were multi-objective optimization problems. Generally, the objectives and objectives interacted with each other. In the process of optimization design, multiple objectives needed to be compared and the importance was weighed. The mathematical expression of the multi-objective optimization problem was

\[
\text{Minimize } f_m(x), \ m=1,2,\ldots,M \\
\text{Subject to } g_j(x) \leq 0, \ j=1,2,\ldots,J \\
h_k(x) = 0, \ k=1,2,\ldots,K \\
X_i^{(L)} \leq X_i \leq X_i^{(U)}, \ i=1,2,\ldots,I
\]

\(X_i\) was the i-th design variable, \(n\) was the total number of design variables; \(X_i^{(L)}\) and \(X_i^{(U)}\) were the lower and upper limit of the value range of the i-th design variable. \(f_m(x)\) was the m-th sub-objective function, \(m\) was the total number of sub objective functions; \(g_j(x) \leq 0\) was the j-th inequality constraint condition, \(j\) was the total number of inequality constraints; \(h_k(x)\) was the k-th equality constraint condition, \(k\) was the total number of equality constraints.

There were two kinds of multi-objective optimization methods, normalization method and non-normalization method. In this paper, the typical multi-objective genetic algorithm, NSGA-II algorithm was used.

Figure 12 was the optimization history curve of the optimization target parameters, i.e. missile launch stroke \(L\) and launch stability evaluation parameter \(k\). It could be seen from the figure that after multi-objective optimization, the optimization targets showed convergence trend, the algorithm converges, and the optimal solution of interior ballistic optimization was obtained. The optimized parameters were shown in Table 5.

| parameter | numerical value |
|-----------|----------------|
| \(d_c\)   | 0.3            |
| \(D_c\)   | 0.13           |
| \(L_c\)   | 0.23           |

![Figure 12. Optimization process of different objective parameters.](image)

The interior ballistic optimization results were shown in figure 13-16. Compared with figure 11(a) and figure 13, it can be seen that the maximum pressure of the high pressure chamber before and after the optimization is the same; for the pressure of the low pressure chamber, the pressure of the low pressure chamber after the optimization is significantly increased in the middle and later stages of the ejection. From figure 14, it can be seen that in the optimized scheme, the gas mass flow into the low
pressure chamber is always greater than that before the optimization. Combined with figure 11(c) and figure 15, the decreased value of overload coefficient of missile is obviously smaller, and the ejection process is more stable; when the piston reaches the speed index, the required stroke is reduced from the original 10.7m to 9.3m, the equipment length is shortened, and the adaptability is greatly improved.

Figure 13. Pressure curves.

Figure 14. Mass flow curves.

Figure 15. Piston overload curves.

Figure 16. Speed-displacement curves of piston.
Some parameter values before and after optimization were compared as shown in Table 6.

| Parameters when the piston reaches the speed target | Before optimization | After optimization | Percent change |
|---------------------------------------------------|---------------------|--------------------|----------------|
| $Q_n$(Kg·s)                                       | 105                 | 125                | 19%            |
| Required stroke $L$/m                            | 10.7                | 9.3                | 13.1%          |
| Launch stability index $k$                       | 1.14                | 1.02               | 10.7%          |

6. Conclusion
1) The leakage test of ejection device was designed and carried out to obtain the leakage rate of rodless cylinder under different cylinder pressure and piston stroke, and the relationship between the fitted leakage rate, pressure and stroke was coupled to the ejection interior ballistic model. In this paper, the leakage experiment scheme and the empirical formula fitting method of the leakage rate of the ejection device were proposed to provide a reference for similar structural leakage problems.

2) Based on the assumption of zero dimensional interior ballistics, a mathematical model for the interior ballistics of a gas-fired rodless cylinder catapult with leakage was established and the interior ballistics result with and without leakage were compared. With considering the leakage, the pressure and overload coefficient of the high and low pressure chamber were lower than those without considering the leakage. When the piston speed reaches the design index, the required stroke was longer than that without considering the leakage. The influence of the leakage rate on the interior ballistic results couldn’t be ignored.

3) In order to solve the problems: the pressure of low pressure chamber and overload coefficient of missile in the middle and later stages of ejection decreased rapidly, the launch stability was poor, the required distance of ejection was long, the adaptability of equipment was poor and launch quality was not good when using the increasing combustion scheme before optimization, genetic algorithm was used to optimize the grain shape. The optimization results showed that at the end of the ejection, the maximum gas mass flow rate increased by 19%, the stroke required for the piston speed to reach the design target shortened by 13.1%, the launch stability index increases by 10.7%, and the equipment adaptability and launch quality were greatly improved. The optimization results provided theoretical support for the engineering design of gas-fired rodless cylinder catapult.

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