A design method for a passive reflectionless transmission-line model based on the cochlea through parameter optimization techniques

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Abstract: A passive reflectionless transmission-line model is able to reproduce the physiological characteristics of the cochlea via parameter tuning. In this paper, we propose a quantitative design method for the passive reflectionless transmission-line model using parameter optimization techniques. The proposed design can achieve the overall physiological characteristics of the cochlea.

Key Words: cochlea, passive reflectionless transmission-line model, optimization technique, peak frequency

1. Introduction

The cochlea is a peripheral organ in the inner ear and has excellent frequency analysis capability. In particular, the basilar membrane characteristics strongly contribute to its frequency responses. In order to widen the dynamic range of frequency analysis (in humans, from 20 Hz to 20 kHz), the frequency response with respect to the distance of the basilar membrane changes exponentially. In addition, since the cochlea has a large dynamic range for amplitude (from the minimum value to the maximum value is about 1,000,000 times), various engineering applications are expected. As an example, it is used as a preprocessing filter for speech recognition with a recursive neural network, wherein it shows better performance than other preprocessing filters [1]. It is used not only for engineering applications but also for reproducing the results of physiological experiments [2].

Oono and Kohda proposed the passive reflectionless transmission-line model of the basilar membrane in the cochlea (hereinafter referred to as the “passive model”) based on an ideal distributed constant circuit [3]. The passive model can reproduce passive properties of the physiological cochlea [4] by adjusting the values of the model parameters. This model has a smaller number of parameters than other transmission-line models based on the cochlea and is considered to be relatively easy to design, so various engineering applications are expected.

Methods for qualitatively determining the parameter values of the passive model have been pro-
posed \[3, 5, 6\]. However, we cannot quantitatively design the passive model through these methods. Therefore, we proposed an improved quantitative design method \[7\]. With this method, we succeeded in reproducing the desired characteristics at some positions in the cochlea. However, it is difficult for the improved method to reproduce the overall characteristics of the cochlea.

Therefore, in this paper, we propose a method for quantitatively determining the parameter values of the passive model that can reproduce the desired overall characteristics. We use the Greenwood function \[8\] as a reference, which gives good approximation for the mammalian peak frequency perception. Next, we introduce parameter optimization methods based on the downhill simplex (DHS) \[10\], particle swarm optimization (PSO) \[11\], and genetic algorithm (GA) \[12\]. In addition, we introduce two objective functions for these optimizations. Finally, we present design examples to evaluate the validity of the proposed methods.

2. Passive model

A minute part of the passive model of the cochlea at distance \(x\) from the input terminal is shown in Fig. 1 \[3\]. The parallel impedance \(Z_p(x, \omega)\) in Fig. 1 is given by

\[
Z_p(x, \omega) = j\omega L_p(x) + R_p(x) + \frac{1}{j\omega C_p(x)},
\]

where \(\omega\) is the angular frequency of the input voltage \(V_{in}(\omega)\). If the value of each circuit element in \(Z_p(x, \omega)\) is assumed to change exponentially with distance, the circuit elements can be written as

\[
R_p(x) = R_0 e^{-ax},
\]

\[
L_p(x) = L_0 e^{ax},
\]

\[
C_p(x) = C_0 e^{ax},
\]

where \(R_0\), \(L_0\), and \(C_0\) are the values of the circuit elements when \(x = 0\) mm, and \(a\) is a positive constant.

If we assume that the characteristic impedance \(Z_0\) is independent of the distance and frequency \[13\], Eq. (5) holds:

\[
Z_{in} = Z_{out} = \sqrt{Z_s(x, \omega)Z_p(x, \omega)} = Z_0.
\]

As a result, the series impedance \(Z_s(x, \omega)\) in Fig. 1 is given by

\[
Z_s(x, \omega) = \frac{Z_0^2}{Z_p(x, \omega)}.
\]
Therefore, the propagation constant $\gamma(x, \omega)$ of the passive model is given by

$$
\gamma(x, \omega) = \sqrt{\frac{Z_s(x, \omega)}{Z_p(x, \omega)}} = \frac{Z_0}{Z_p(x, \omega)}.
$$

(7)

Taking the integral of the propagation constant with respect to $x$ gives

$$
\Gamma(x, \omega) = \int_0^x \gamma(y, \omega) \, dy
$$

$$\begin{align*}
&= \frac{jZ_0}{2a \sqrt{L_0/C_0 + jR_0 L_0 \omega}} \\
&\times \left\{ \ln \left( \frac{\sqrt{1 + jR_0 C_0 \omega} + \sqrt{L_0 C_0 e^{ax \omega}}}{\sqrt{1 + jR_0 C_0 \omega} - \sqrt{L_0 C_0 e^{ax \omega}}} \right) \\
&- \ln \left( \frac{\sqrt{1 + jR_0 C_0 \omega} + \sqrt{L_0 C_0 e^{-ax \omega}}}{\sqrt{1 + jR_0 C_0 \omega} - \sqrt{L_0 C_0 e^{-ax \omega}}} \right) \right\}.
\end{align*}
$$

(8)

With Eq. (8), the voltage $V_p(x, \omega)$ at distance $x$ in Fig. 1 is given by

$$
V_p(x, \omega) = V_p(0, \omega) e^{-\Gamma(x, \omega)}.
$$

(9)

Now, we define the transfer function between the input voltage to the shunt current at distance $x$ as

$$
F(x, \omega) = \frac{I_p(x, \omega)}{V_p(0, \omega)} = \frac{1}{Z_p(x, \omega)} e^{-\Gamma(x, \omega)}.
$$

(10)

In addition, the gain at distance $x$, $g(x, \omega)$, can be defined as

$$
g(x, \omega) = 20 \log |F(x, \omega)|.
$$

(11)

The gain characteristics of the passive model are numerically simulated with the parameter values, $a = 0.288 \text{ mm}^{-1}$, $L_0 = 2.385 \times 10^{-7} \text{ H}$, $C_0 = 2.132 \times 10^{-7} \text{ F}$, $R_0 = 1.5 \Omega$, and $Z_0 = 5 \Omega$ in [14], as shown in Fig. 2. Note that these parameter values were determined using the results of physiological experiments [4] to match the gain characteristics at $f = 200 \text{ Hz}$.

As shown in Fig. 2, the gain characteristic has a peak, i.e., the maximum gain, at a particular distance. The frequency at which the gain peaks for a particular distance is referred to as the “peak

Fig. 2. Gain characteristics with respect to distance $x$. The solid, dotted, and dashed lines represent gain characteristics when the input voltage frequency $f$ ($\omega = 2\pi f$) is $10^2$, $10^3$, and $10^4 \text{ Hz}$, respectively. The triangle, circle, and square symbols mark the peaks of each gain.
frequency.” Beyond the peak frequency, the gain rapidly decreases. The results in Fig. 2 also show that the passive model gain peaks at a high frequency when the distance \( x \) is small, and the gain peaks at a low frequency when \( x \) is large. Moreover, the characteristic of the peak frequency with respect to the distance shows the relationship between the cochlea length and audible range. In humans, the total length of the cochlea is approximately 35 mm, and the audible range is 20 Hz to 20 kHz.

3. Formulation of feature values in the passive model

To reproduce the desired peak frequency characteristics in the passive model, the peak frequency of the passive model must be formulated [16]. From [16], it can be derived as

\[
\frac{\partial |F(x, \omega)|}{\partial x} \bigg|_{\omega=2\pi f_{\text{peak}}(x)} = 0. \tag{12}
\]

From Eq. (12), the peak frequency can be written as

\[
f_{\text{peak}}(x) = \mu(x)f_{\text{res}}(x). \tag{13}
\]

In Eq. (13), the resonance frequency \( f_{\text{res}}(x) \) and \( \mu(x) \) are given as

\[
f_{\text{res}}(x) = \frac{1}{2\pi \sqrt{L_0 C_0}} e^{-ax}, \tag{14}
\]

\[
\mu(x) = \sqrt{-\zeta(x) + \sqrt{\zeta(x)^2 + 1}}, \tag{15}
\]

where \( \zeta(x) > 0 \), and \( \zeta(x) \) is

\[
\zeta(x) = \frac{R_p(x)C_p(x)}{2L_p(x)} \left( \frac{Z_0}{a} - R_p(x) \right). \tag{16}
\]

Figure 3 compares the characteristics of the peak frequency obtained by Eq. (13) (red solid line) to that obtained from the Greenwood function \( f_G(x) \) (Eq. (17)) (blue dashed line):

\[
f_G(x) = A \cdot 10^{B(1-x/x_{\text{max}})} - K, \tag{17}
\]

where \( A, B, \) and \( K \) are constants, and \( x_{\text{max}} \) is the total length of the cochlea, as shown in Fig. 1. In Fig. 3, we used \( A = 165.4 \) Hz, \( B = 2.1 \), \( K = 145.552 \) Hz, and \( x_{\text{max}} = 35 \) mm, as obtained

![Figure 3. Peak frequency obtained from the passive model (red line: Eq. (13)) and that from the Greenwood function (blue line: Eq. (17)) with respect to distance \( x \). The triangle, circle, and square symbols show the corresponding points with the same symbols in Fig. 2.](image-url)
from [8]. These parameter values are determined by psychoacoustic experiments based on [4], and the Greenwood function is employed to design a cochlear implant because it can reproduce the peak frequency characteristic of human hearing. Therefore, we use the Greenwood function as the desired reference characteristic. The results in Fig. 3 show that the peak frequencies of the passive model and Greenwood function are not well matched because the peak frequency characteristic of the passive model aims to match the gain characteristic at \( f = 200 \, \text{Hz} \). Therefore, we determine the parameter values such that the peak frequency characteristic with respect to distance \( x \) of the passive model match those of the Greenwood function.

4. Simply determining parameter values

We propose a simple method for determining the parameter values of the passive model. Based on the feature values derived in Sec. 3, each parameter value is determined so as to match peak frequency characteristic with respect to the cochlear distance \( x \) obtained from the Greenwood function. In order to determine the parameter values simply, we assume that \( \zeta(x) \ll 1 \), so that \( \mu(x) \approx 1 \) from Eq. (15). Then, we can rewrite Eq. (13) as \( f_{\text{peak}}(x) \approx f_{\text{res}}(x) \).

First, the values of parameters \( L_0 \) and \( C_0 \) are determined so that peak frequency \( f_{\text{peak}}(x) \) of the passive model at distance \( x = 0 \, \text{mm} \) matches the highest frequency from the Greenwood function \( f_{G}(0) = 20 \times 10^3 \, \text{Hz} \).

\[
f_{\text{peak}}(0) = \frac{1}{2\pi \sqrt{L_0 C_0}} = 20 \times 10^3, \tag{18}
\]

where \( L_0 = C_0 \) for simplicity, and the values of parameters \( L_0 \) and \( C_0 \) are

\[
L_0 = C_0 = \frac{1}{2\pi \cdot 20 \times 10^3} = 7.958 \times 10^{-6}. \tag{19}
\]

Next, the value of parameter \( a \) is determined. Peak frequency \( f_{\text{peak}}(x_{\text{max}}) \) of the passive model at distance \( x_{\text{max}} = 35 \, \text{mm} \) matches the lowest frequency from the Greenwood function \( f_{G}(35) = 20 \, \text{Hz} \).

\[
f_{\text{peak}}(x_{\text{max}}) = \frac{1}{2\pi \sqrt{L_0 C_0}} \left( 1 - a x_{\text{max}} \right) = 20, \tag{20}
\]

\[
a = -\frac{1}{x_{\text{max}}} \ln \left( \frac{f_{\text{res}}(x_{\text{max}})}{f_{\text{res}}(0)} \right) = -\frac{1}{35} \ln \frac{20}{20 \times 10^3} = 0.1974. \tag{21}
\]

Finally, the parameters \( R_0 \) and \( Z_0 \) are adjusted to satisfy \( \zeta(x) > 0 \) and assumption \( \zeta(x) \ll 1 \). From Eq. (16), we assume that \( Z_0 / a = 10 R_0 \) because \( \zeta(x) > 0 \). Then, we set \( \zeta(0) = 0.1 \) so that the maximum value of \( \zeta(x) \) is smaller than 1.

\[
\zeta(0) = \frac{R_0 C_0}{2 L_0} \left( \frac{Z_0}{a} - R_0 \right) = \frac{9}{2} R_0^2 = 0.1. \tag{22}
\]

Therefore, parameters \( R_0 \) and \( Z_0 \) are \( R_0 = 0.149 \, \Omega \) and \( Z_0 = 0.2941 \, \Omega \), respectively. We compare these parameter values with those obtained from the results of physiological experiments [9]. The results are shown in Table I. Note that, we do not compare the parameter values of the characteristic impedance \( Z_0 \) because each experiment assumed the characteristic impedance differently. From Table I, we confirm that the value of parameter \( a \) obtained from the proposed method is close to physiological experiment results. The values of parameters \( L_0 \) and \( R_0 \) are smaller than the values obtained from physiological experiment results. Conversely, the value of parameter \( C_0 \) is larger than the value obtained from physiological experiment results.

Using the parameter values \( a = 0.1974 \, \text{mm}^{-1} \), \( R_0 = 0.149 \, \Omega \), \( L_0 = 7.958 \times 10^{-6} \, \text{H} \), \( C_0 = 7.958 \times 10^{-6} \, \text{F} \), and \( Z_0 = 0.2941 \, \Omega \) from the above results, we compare the peak frequencies of the passive model and Greenwood function. This result is shown in Fig. 4. In Fig. 4, we find that \( f_{\text{peak}}(0) \) and \( f_{\text{peak}}(35) \) are well matched to the frequency characteristic of the Greenwood function with respect to distance \( x \). However, the overall frequency is not matched to the Greenwood function. Hence, we use an optimization technique to determine the parameter values of the passive model so that the peak frequency characteristics for the overall range of distances approximate the characteristics of the Greenwood function.
Table I. Comparison of parameter values obtained from physiological experiment results [9] with those obtained by the proposed method.

|      | Bogert | Fletcher | Wansdronk | Sakai | Zwischenki | Proposed method in this paper |
|------|--------|----------|-----------|-------|------------|-------------------------------|
| \(a\) | [0.1, 0.2] | [0.0382, 0.293] | [0.0773, 0.41] | [0.0773, 0.41] | 0.15 | 0.1974 |
| \(L_0\) | 0.143 | \(2.48 \times 10^{-2}\) | 4.48 | 1.16 | 2.1 \(\times 10^{-2}\) | 7.958 \(\times 10^{-6}\) |
| \(R_0\) | 6.737 \(\times 10^3\) | 1.95 \(\times 10^3\) | 2.94 \(\times 10^5\) | 8.97 \(\times 10^4\) | 1.5 \(\times 10^2\) | 1.49 \(\times 10^{-1}\) |
| \(C_0\) | 5.81 \(\times 10^{-10}\) | 1.71 \(\times 10^{-9}\) | 1.29 \(\times 10^{-11}\) | 3.6 \(\times 10^{-12}\) | 6.7 \(\times 10^{-9}\) | 7.958 \(\times 10^{-6}\) |

Fig. 4. Peak frequency obtained from the passive model (red line: Eq. (13)) using parameter values \(a = 0.1974\) mm\(^{-1}\), \(R_0 = 0.149\) \(\Omega\), \(L_0 = 7.958 \times 10^{-6}\) H, \(C_0 = 7.958 \times 10^{-6}\) F, and \(Z_0 = 0.2941\) \(\Omega\), and that from the Greenwood function (blue line: Eq. (17)) with respect to distance \(x\).

5. Optimization of the parameter values

5.1 Objective function for peak frequency

To obtain parameter values that provide a better match between the passive model and the Greenwood function, we employ the optimization techniques detailed below.

First, we define the objective function \(E_f\) as

\[
E_f = \sum_{n=0}^{N} \left| \frac{f_{\text{peak}}(n\Delta x) - f_G(n\Delta x)}{f_G(n\Delta x)} \right|,
\]

where \(\Delta x = x_{\text{max}}/N\). \(N\) is a constant of division of the total length of the cochlea.

The design parameters of this function are \(a\), \(R_0\), \(L_0\), \(C_0\), and \(Z_0\). We use the initial values by adding a uniform random number to the parameter values obtained from Sec. 4. Here, the maximum and minimum initial values of each parameter are \(0.1 \leq a \leq 0.4\), \(10^{-3} \leq R_0 \leq 10^3\), \(10^{-9} \leq L_0 \leq 10^{-3}\), \(10^{-12} \leq C_0 \leq 10^{-3}\), and \(10^{-3} \leq Z_0 \leq 10^6\), respectively.

Next, we employ the DHS, PSO, and GA methods to determine the parameter values. We are unable to use a gradient method because we cannot differentiate the objective function \(E_f\) (Eq. (23)) with respect to the parameters \((a, R_0, L_0, C_0, \text{and} \ Z_0)\). DHS, PSO, and GA do not depend on the gradient of an objective function.

DHS is a deterministic optimization method. If we set the design parameters to the same initial values, the same function value will be obtained whenever we perform the simulation. DHS is suitable when the objective function is unimodal.

In contrast, GA is one of the most popular stochastic optimization methods. When the object
function is multimodal, it is difficult for it to be trapped by the local minimum. GA is also suitable for problems with many design parameters. PSO is also one of the most-used stochastic optimization methods. It shares the properties of the DHS and GA.

In the following simulations, the total number of trials is 100 with 10,000 iterations in each trial. In addition, the numbers of simplexes, particles, and individuals in DHS, PSO, and GA, respectively, are equal to 30. Table II shows the specifications of the computer used for the simulations. Simulation results obtained with the object function $E_f$ are summarized in the left half of Table III. In the table, “ave” and “best” show the average and the best values of $E_f$, respectively, and $t_{ave}$ gives the average CPU time. As shown in the Table III, GA obtains a good $E_f$(ave), and DHS obtains the minimum $E_f$(best) with the fastest CPU time. PSO does not exhibit fast CPU time, and its $E_f$(ave) and $E_f$(best) are the worst among the three methods.

Figure 5 shows the peak frequency characteristics and gain characteristics obtained from the optimized parameter values through DHS, PSO, and GA. These parameter values are shown in the left half of Table IV. The results in Fig. 5 confirm that the peak frequency obtained with the optimized parameter values match well with those obtained from the Greenwood function regardless of the optimization method. In contrast, the gain characteristic curves obtained through the above optimizations are all different from those shown in Fig. 2. From Figs. 5 and 2, we suspect that inappropriate $Q$ value is the cause of these differences.

Therefore, we need a definition of $Q$ for the gain characteristics. Figure 6 shows an example of the gain characteristics with respect to the frequency at distance $x = 23$ mm. In the figure, $g_{\text{max}}(x)$ is the maximum value of $g(x, \omega)$ at $f_{\text{peak}}(x)$. We introduce new frequencies $f_L(x)$ and $f_H(x)$ at which $g(x, \omega) = g_{\text{max}}(x) - 10$ dB, and $f_L(x) < f_H(x)$ as shown in Fig. 6 to define $Q_{10}(x)$ as

$$Q_{10}(x) = 20 \log_{10} \left( \frac{f_H(x) - f_L(x)}{f_{\text{peak}}(x)} \right).$$

Table II. Specifications of the computer used for the simulations.

| OS            | CentOS release 6.10 (Final) |
|---------------|-----------------------------|
| Kernal        | Linux version 2.6.32-754.3.5.el6.centos.plus.x86_64 |
| Compiler      | gcc version 4.4.7 20120313 (Red Hat 4.4.7-23) (GCC) |
| CPU           | Intel(R) Xeon(R) CPU E31280 @ 3.50GHz |
| Memory        | 1642092 KB |

Table III. Average (ave) and the best (best) values of the object functions $E_f$ and $E_{\text{sum}}$, and the average CPU time $t_{\text{ave}}$ obtained by the downhill simplex (DHS), particle swarm optimization (PSO), and genetic algorithm (GA) methods. $Q$ value at distance $x = 23$ mm ($Q_{10}(23)$) were estimated from the gain characteristics when the object functions $E_f = E_f$(best) and $E_{\text{sum}} = E_{\text{sum}}$(best).

|                         | Results with $E_f$ (Eq. (23)) | Results with $E_{\text{sum}}$ (Eq. (26)) |
|-------------------------|--------------------------------|---------------------------------|
|                         | without constraints on parameter values. | with constraints as $R_0 \leq 1$, $C_0 \leq 10^{-7}$, and $Z_0 \geq 10$. |
| $E_f$(ave)               | $E_f$(best) | $t_{\text{ave}}$ | $Q_{10}(23)$ | $E_{\text{sum}}$(ave) | $E_{\text{sum}}$(best) | $t_{\text{ave}}$ | $Q_{10}(23)$ |
| DHS                     | 4.598      | 2.193       | 0.678       | 0.6517      | 12.53            | 12.53            | 0.566       | 148.7       |
| PSO                     | 18.9       | 5.617       | 1.23        | 1.305       | 30.65            | 27.5            | 1.31        | 1360        |
| GA                      | 2.355      | 2.194       | 2.06        | 0.6511      | 12.55            | 12.53            | 2.06        | 158.3       |

Table IV. Parameter values of the passive model.

|                         | Without sharpness constraint | With sharpness constraint |
|-------------------------|-----------------------------|----------------------------|
|                         | DHS                      | PSO                   | GA                     | DHS                | PSO                | GA                     |
| $a$ 1/mm                | 0.288        | 0.231      | 0.288      | 0.1802          | 0.3151             | 0.2878                |
| $R_0$ $\Omega$          | 249.6        | 1.137 $\times 10^{-2}$ | 0.2454 | 0.1884          | 0.22363            | 0.5342                |
| $L_0$ $H$               | 5.287 $\times 10^{-5}$  | 1.17 $\times 10^{-8}$   | 1.573 $\times 10^{-8}$ | 6.615 $\times 10^{-4}$ | 5.455 $\times 10^{-7}$ | 1.452 $\times 10^{-6}$ |
| $C_0$ $F$               | 4.74 $\times 10^{-10}$  | 6.755 $\times 10^{-5}$   | 1.588 $\times 10^{-8}$ | 2.246 $\times 10^{-8}$ | 1.708 $\times 10^{-8}$ | 1.588 $\times 10^{-8}$ |
| $Z_0$ $\Omega$          | 2.891 $\times 10^{5}$   | 0.4264     | 26.2       | 1.58 $\times 10^{5}$ | 1.58 $\times 10^{5}$ | 1.204 $\times 10^{5}$ |
Fig. 5. Peak frequency and gain characteristics with respect to distance $x$ with the model parameters resulting from the optimizations with $E_f$. The red lines in (a), (b), and (c) show the characteristics obtained by DHS, PSO, and GA optimizations, respectively. Blue lines are those from the Greenwood function. (d), (e), and (f) show the gain characteristics corresponding to (a), (b), and (c), respectively.

$$Q_{10}(x) = \frac{f_{\text{peak}}(x)}{f_H(x) - f_L(x)}. \quad (24)$$

To calculate $Q_{10}(x)$ when $E_f = E_f(\text{best})$, we estimated $f_L(x)$ and $f_H(x)$ by numerical simulations for the frequency vs. gain characteristics shown in Fig. 6. As a result, the $Q$ values at $x = 23$ mm, $Q_{10}(23)$, are 0.6517, 1.305, and 0.6511 for DHS, PSO, and GA, respectively, as summarized in the fifth column of Table III. There is a large discrepancy between these $Q$ values and $Q_{10}(23) = 18$ obtained from [14]. Therefore, we need to further improve the optimization method to guarantee a large value in $Q_{10}(x)$ while maintaining the distance vs. peak frequency characteristics.

From the observations depicted in Fig. 6, we assume that $Q_{10}(x)$ is high when $f_{\text{peak}}(x) \approx f_{\text{res}}(x)$. This implies, from Eq. (13), that $\mu(x)$ has a certain relationship with $Q_{10}(x)$. To discover the relationship, we constructed log $Q_{10}(x)$ vs. $\mu(x)$ plots as shown in Figs. 7(a), (b), and (c).
Fig. 6. Example of frequency vs. gain characteristic showing definitions of the low frequency $f_L(x)$ and the high frequency $f_H(x)$ used in Eq. (24).

Fig. 7. Sets of values of log $Q_{10}(x)$ and $\mu(x)$ obtained from 100 optimization trials with different initial conditions for (a) DHS, (b) PSO, and (c) GA methods with $E_f$. The dashed lines show Eq. (25). The red square symbols in the figures depict the points that do not follow Eq. (25). (a′), (b′), and (c′) are obtained when $E_f$ is used with the following constraints on parameter values: $R_0 \leq 1$, $C_0 \leq 10^{-7}$, and $Z_0 \geq 10$.

However, the object function considering only the peak frequency characteristics with respect to the distance shows that the sharpness characteristics are not satisfactory. Therefore, to reproduce the desired characteristics while maintaining high sharpness values, the object function is constrained by the function $\mu(x)$, which is expected to increase the sharpness of the passive model as it approaches 1. As a result, as can be seen from Fig. 7, it is found that the $Q_{10}(x)$ value increases around $\mu(x) = 1$. From the figures, we obtained the following regression equation:

$$\mu(x) \cong -0.2(\log Q_{10}(x) + 1)^{-4} + 1,$$

(25)
which is drawn with blue dashed lines in Figs. 7(a), (b), and (c). When we obtained Eq. (25), we neglected the points shown with red squares in Figs. 7(a), (b), and (c). The red square points are undesirable for explicitly formulating the objective function based on \( \mu(x) \); therefore, we should avoid them. Therefore, we evaluated the objective function \( E_{\text{sum}} \) with \( \mu(x) = 0.9 \).

To exclude these points from the optimizations, we consider imposing restrictions on the parameter values. Figure 8 shows histograms of the parameter values of \( a, R_0, L_0, C_0, \) and \( Z_0 \). The parameter counts that produce undesirable points are shown with red bars, while the counts that produce preferable points are shown with gray bars. From Figs. 8(b), (d), and (e), we find that we can separate undesirable parameter values from preferable ones, as shown by the dashed lines with arrows for \( R_0, C_0, \) and \( Z_0 \). Therefore, during the optimizations, we impose the following constraints on these parameter values: \( R_0 \leq 1, C_0 \leq 10^{-7}, \) and \( Z_0 \geq 10 \). On the other hand, we cannot use the same strategy for imposing constraints on the parameters \( a \) and \( L_0 \), as shown in Figs. 8(a) and (c), respectively.

### 5.2 Object function with sharpness constraint

On the basis of the above results, we propose an improved object function \( E_{\text{sum}} \), which considers \( f_{\text{peak}}(x) \) by \( E_f \) as well as \( Q_{10}(x) \) through \( \mu(x) \):

\[
E_{\text{sum}} = \sum_{n=0}^{N} \left( E_f + \left| \frac{\mu(n\Delta x) - 0.9}{0.9} \right| \right) .
\]  

We determine the parameter values again by using \( E_{\text{sum}} \) with DHS, PSO, and GA, imposing constraints on the values of \( R_0, C_0, \) and \( Z_0 \) as mentioned above. The results are summarized in the right half of Table III. From the table, we conclude that DHS is the most preferable in terms of accuracy and speed. We confirmed the effect of the parameter value constraints as shown in Figs. 7(a’), (b’), and (c’), where all the points are on the regression line. In addition, Fig. 9 confirms that the peak frequency matches well with that obtained from the Greenwood function. These parameter values are show in the right half of Table IV. Furthermore, Figs. 9(d), (e), and (f) show great improvement in the gain characteristics obtained by using \( E_{\text{sum}} \) compared with those obtained by using \( E_f \).

![Fig. 8. Histograms of parameter values resulting from 100 trials of DHS, PSO, and GA (a total of 300 trials). Model parameters are (a) \( a \), (b) \( R_0 \), (c) \( L_0 \), (d) \( C_0 \), and (e) \( Z_0 \). The red bars count the parameter values resulting from the red squares (undesirable) in Fig. 7, while the gray bars count those for the plus symbols (preferable). The dashed lines with arrows in (b), (d), and (e) determine the parameter regions where the red square plots in Fig. 7 are not included.](image-url)
6. Conclusions

We proposed a design method to determine the parameter values of the passive model on the basis of parameter optimizations. We used DHS, PSO, and GA for the optimizations. For these optimizations, we proposed an effective objective function that considered both $f_{\text{peak}}(x)$ and $Q_{10}(x)$. We also imposed constraints on some parameter values to validate the formula for $\mu(x)$ used in the objective function $E_{\text{sum}}$. Finally, we demonstrated the effectiveness of the proposed method through design examples. From the simulations, we found that DHS was the most preferable method.

In the future, we will improve the peak frequency characteristics at low frequency and the maximum gain characteristics [7] by further improvements to the objective function.

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