High-precision analytical TDoA positioning algorithm for eliminating the ambiguity of coordinates determination

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Abstract. An analytical method is proposed for determining the position of an object based on the analysis of Time Difference of Arrival (TDoA) of signals from the transmitter of the object to the base stations (BS) receivers, which eliminates the ambiguity in determining the coordinates of the object on the plane and improves the accuracy of determining coordinates when the TDoA measurements are out of sync with base stations.

1. Introduction
Accurate remote determination of the coordinates of objects on a plane and in space is necessary in many technical applications, such as driving unmanned vehicles of the “smart city”, robotic production complexes, security systems in the city and in the workplace. The coordinates of the object are determined by receiving a radio signal transmitted by the object by several base stations. In global positioning systems (for example, GPS), the radio signal is received by navigation satellites; it is possible to use specialized microwave radars within the city or a production facility. The main task of positioning the object is to determine the coordinates (position) of the object on the ground, which can be carried out using various algorithms. These include: RSSI (Received Strength Signal Indication) - the distance to the object is estimated by the signal power; AoA (Angle of Arrival) - the location of the object is determined within the triangle area formed by the intersection of the axes of the antenna patterns of the sectors of the three base stations (modified triangulation method); ToF (Time of Flight) - measuring the travel time of an electromagnetic wave from an object transmitter to a base station by using a signal with linear frequency modulation; ToA (Time of Arrival) — measurement of the signal transit time from the mobile terminal to the base station, at which the distance to the object is calculated based on the difference in the time the signal was sent and received; TDoA (Time Difference of Arrival) - measures the difference in the time of arrival of a signal from an object to several base stations; RTT (Round Trip Time) - the base station sends a signal to a mobile device and waits for a response signal, the time it takes for the signal to travel in both directions is determined by the difference in the time the signal is sent and received, and therefore the distance between the objects; LPT (Location Patterning Techniques) - Location determination is performed using pattern recognition of radio signals based on sampling and recording of radio patterns of signal behavior in a specific environment.

In the difficult changing environment of the “smart city” and the production facility, the TDoA method is preferred, since this method is highly accurate and requires time synchronization only at base stations [1-5]. In the TDoA method, microwave radar measures the difference in the time of
arrival of the signal from the transmitter of the object to the receivers of base stations (TDoA). The lines on which the difference in the arrival times of the microwave signal to two BSs is constant, are hyperbolas on the plane (hyperboloids in space), while both BSs are in the foci of the hyperbolas. The coordinates of the object are the intersection point of the entire set of hyperbolic curves on the plane included in the system of equations.

To obtain an estimate of the position at reasonable noise levels, the Taylor-series method [6] is also used. This is an iterative method: it starts with the initial assumption and improves the estimate at each step by defining a local linear least-squares (LS) solution. An initial assumption close to the true solution is necessary to avoid local minima. Choosing such a starting point is not easy in practice. Moreover, the convergence of the iterative process is not guaranteed. It also requires large computational resources, since LS computation is required in each iteration. In [7], it was proposed to use Taylor series expansion and linearization of hyperbolic equations. As a result, the system of equations reduces to linear in matrix form [8].

The “Time of Flight” method, using a linear frequency-modulated microwave radar waveform, is used in smart city automotive applications to automatically control the distance between moving vehicles, to warn about cross-traffic, and help in changing lanes (cross traffic alert and lane change assist), parking and detection of obstacles, pedestrians and blind spots detection (parking aid, obstacle, pedestrian and blind spots detection). In addition, radars use traffic police services to ensure traffic safety using administrative measures. Microwave range automobile radars for the simultaneous measurement of speed and range use the generation and emission of signals of a special shape, which include: frequency-modulated continuous wave (FMCW), frequency shift keying (FSK) continuous wave. A refined mathematical model of the received microwave signal, taking into account a wide range of velocity changes of the target was proposed in [9]. It is shown that the use of phase methods to determine the speed of the target significantly reduces the noise immunity of the radar, and, therefore, the minimum measurable value of the target scattering area (EPR). From this point of view, methods based on processing the amplitude spectrum of the FFT of the received radio signal are more preferable. In [10], the algorithm for estimating the speed and distance to the target as applied to the mmWave automotive radar with linear FMCW using the FFT amplitude spectrum was considered. A specific feature of signal processing algorithms based on an amplitude spectrum estimate is the occurrence of false targets when estimating the radial velocities and distances. The probability of false targets occurring increases dramatically with an increase in the total number of accompanied targets in the monitoring process. In [11, 12] we have been investigated the influence of millimeter-wave radar receiver noise on the probability of unambiguous determination of unmanned vehicles speed and range in the intelligent transportation system of the “smart city”. In [13] the results of synthesis and analysis of the effectiveness of the optimal location-based system for joint detection and estimation of informative parameters of quasi-determined radar signals with frequency modulated continuous wave are presented.

In [14], an exact analytical solution of hyperbolic equations was obtained for the case when the number of TDoA measurements is equal to the number of unknown transmitter coordinates. This solution, however, cannot use TDoA measurements with additional base stations used to improve positioning accuracy. A definite drawback of the analytical method is the increased requirement for the computing power of the processor of the positioning system, since there is a need to solve nonlinear equations. However, with the development of microprocessor technology, this drawback is leveled.

The analytical method is more preferable for optimizing the calculation process and increasing the accuracy of determining the position of an object in the TDoA method. In [15], formulas are given for transforming the coordinates of a hyperbola during a shift and rotation on a plane. Based on the proposed transformations in [16], the effectiveness of two methods of TDOA analytical solution was compared to determine the possible positions of the target - intersection points of hyperboloids. The first method was based on coordinate transformation from initial system to new system, where system of equations solve is simpler, whereas the second one was based on matrix. In [17], the results of an
experimental verification of an analytical method based on coordinate transformation are presented in a positioning system of three receiving antennas at a distance of 15 meters. However, the method [16] at certain positions of the object and base stations is not able to eliminate the ambiguity in determining the coordinates of the object.

The purpose of this article is to develop an analytical method for determining the position of an object based on the analysis of Time Difference of Arrival (TDoA) of microwave radar signals from the transmitter to the BS receivers, which eliminates the ambiguity in determining the coordinates of the object on the plane and improves the accuracy of determining coordinates when the TDoA measurements are out of sync with base stations.

2. The linearization method of hyperbolic equations [7]
In the absence of TDoA measurement errors and reception of direct visibility signals, the real values of the difference in the time of arrival of the TDoA signal between BS, and BS, are determined by the expression

\[ \Delta \tau_{il} = \frac{r_i - r_j}{c_l} = \frac{r_i}{c_l}, \quad i = 2, \ldots, M, \]

where \( M \) is the number of BSs, \( c_l \) is the speed of light, \( r_i \) and \( r_j \) are the distances between the object and BS, \([x_i, y_i]\) and \([x_j, y_j]\) are the coordinates of the object. From the \( \Delta \tau_{il} \) follows \( r_i - r_j = \frac{1}{2} \left( (x_i - x_j)^2 + (y_i - y_j)^2 - (c_l \cdot \Delta \tau_{il})^2 \right) \), which in matrix form has the form \( A \theta = b \), where

\[
A = \begin{bmatrix}
    x_2 - x_1 & y_2 - y_1 & c_l \cdot \Delta \tau_{21} \\
    x_3 - x_1 & y_3 - y_1 & c_l \cdot \Delta \tau_{31} \\
    \vdots & \vdots & \vdots \\
    x_M - x_1 & y_M - y_1 & c_l \cdot \Delta \tau_{M1}
\end{bmatrix}, \quad
b = \frac{1}{2} \begin{bmatrix}
    (x_2 - x_1)^2 + (y_2 - y_1)^2 - (c_l \cdot \Delta \tau_{21})^2 \\
    (x_3 - x_1)^2 + (y_3 - y_1)^2 - (c_l \cdot \Delta \tau_{31})^2 \\
    \vdots & \vdots & \vdots \\
    (x_M - x_1)^2 + (y_M - y_1)^2 - (c_l \cdot \Delta \tau_{M1})^2
\end{bmatrix}, \quad \theta = \begin{bmatrix}
    x - x_1 \\
    y - y_1 \\
    r_i
\end{bmatrix}^T
\]

The top index \( \theta^T \) denotes the transpose of the matrix. Solution of the equation \( A \theta = b \) can be carried out in various ways of solving a system of linear equations. After calculating the values of the vector \( \theta \) the coordinates of the target are determined

\[ x = \theta_1 + x_1, \quad y = \theta_2 + y_1, \]

where \( x_1 \) and \( y_1 \) - known coordinates of BS,.

3. An analytical method for determining the coordinates of an object in a positioning system with 4 BS
The option of constructing a positioning system on a plane of 3 BS is the most cost-effective, but it has the disadvantage that the intersection of two hyperbolas can occur at two points on the plane. In this case, it becomes impossible to reveal the true position of the object. To eliminate the ambiguity of determining the position of the object, it is necessary to include the fourth BS in the positioning system. With the addition of the fourth BS, it becomes possible to solve three systems of equations that have one common root corresponding to the true position of the target. An analytical solution to
the problem of determining the location of an object for a positioning system of 4 BSs was obtained in [16], however, not all coefficient values are given in [16], which complicates the direct use of the algorithm. In addition, the problem of eliminating the ambiguity in determining the coordinates is not resolved.

Consider a positioning system on a plane that includes four microwave signal receivers: BS_C, BS_L, BS_R, BS_U. The position coordinates of the transmitter are solutions of the system of hyperbolic equations:

\[
\begin{align*}
\sqrt{(x-x_L)^2 + (y-y_L)^2} - \sqrt{(x-x_C)^2 + (y-y_C)^2} &= c \cdot \Delta \tau_{LC} \\
\sqrt{(x-x_R)^2 + (y-y_R)^2} - \sqrt{(x-x_C)^2 + (y-y_C)^2} &= c \cdot \Delta \tau_{RC} \\
\sqrt{(x-x_U)^2 + (y-y_U)^2} - \sqrt{(x-x_C)^2 + (y-y_C)^2} &= c \cdot \Delta \tau_{UC}
\end{align*}
\]

(4)

Hence, \(\Delta \tau_{LC}, \Delta \tau_{RC}, \Delta \tau_{UC}\) - the difference in the arrival times of the microwave signal between BS\(_L\), BS\(_R\), BS\(_U\) and reference station BS\(_C\).

After substituting \(X = x - x_C\), \(Y = y - y_C\), the system of equations can be rewritten in the form

\[
\begin{align*}
\sqrt{(X-X_L)^2 + (Y-Y_L)^2} - \sqrt{X^2 + Y^2} &= L \\
\sqrt{(X-X_R)^2 + (Y-Y_R)^2} - \sqrt{X^2 + Y^2} &= R \\
\sqrt{(X-X_U)^2 + (Y-Y_U)^2} - \sqrt{X^2 + Y^2} &= U
\end{align*}
\]

(5)

where \(X_L = x_L - x_C, Y_L = y_L - y_C, L = c \cdot \Delta \tau_{LC}\)

\(X_R = x_R - x_C, Y_R = y_R - y_C, R = c \cdot \Delta \tau_{RC}\)

\(X_U = x_U - x_C, Y_U = y_U - y_C, U = c \cdot \Delta \tau_{UC}\)

Let’s note \(K = \sqrt{X^2 + Y^2}\), where \(K > 0\), thus \(K^2 = X^2 + Y^2\)

(6)

The system of equations (5) can be rewritten in the form

\[
\begin{align*}
\sqrt{(X-X_L)^2 + (Y-Y_L)^2} &= K + L \\
\sqrt{(X-X_R)^2 + (Y-Y_R)^2} &= K + R \\
\sqrt{(X-X_U)^2 + (Y-Y_U)^2} &= K + U
\end{align*}
\]

(7)

The squaring and reduction of the general terms leads to the form

\[
\begin{align*}
-2X_L X - 2Y_L Y &= 2LK + L^2 - X_L^2 - Y_L^2 \\
-2X_R X - 2Y_R Y &= 2RK + R^2 - X_R^2 - Y_R^2 \\
-2X_U X - 2Y_U Y &= 2UK + U^2 - X_U^2 - Y_U^2
\end{align*}
\]

or

\[
\begin{align*}
-2X_L X - 2Y_L Y &= A + BK \\
-2X_R X - 2Y_R Y &= C + DK
\end{align*}
\]

where

\[
\begin{align*}
A &= L^2 - X_L^2 - Y_L^2; \quad B = 2L; \quad C = R^2 - X_R^2 - Y_R^2; \quad D = 2R; \quad E = U^2 - X_U^2 - Y_U^2; \quad F = 2U
\end{align*}
\]

(8)

In matrix form, three systems of equations have the form:
\[
\begin{bmatrix}
-2X_L & -2Y_L \\
-2X_R & -2Y_R
\end{bmatrix}
\begin{bmatrix}
X \\
Y
\end{bmatrix}
= \begin{bmatrix}
A + BK \\
C + DK
\end{bmatrix},
\]
\[
\begin{bmatrix}
-2X_U & -2Y_U \\
-2X_L & -2Y_L
\end{bmatrix}
\begin{bmatrix}
X \\
Y
\end{bmatrix}
= \begin{bmatrix}
E + FK \\
A + BK
\end{bmatrix}.
\]

Solution of the first system of equations
\[
X = \frac{1}{\Delta_1} \begin{bmatrix}
A + BK & -2Y_L \\
C + DK & -2Y_R
\end{bmatrix} = M_{1X} \cdot K + N_{1X}, \text{ где}
\]
\[
M_{1X} = \frac{4}{\Delta_1} \cdot (RY_L - LY_R); \quad N_{1X} = \frac{2}{\Delta_1} \cdot (CY_L - AY_R);
\]
\[
Y = \frac{1}{\Delta_1} \begin{bmatrix}
-A + BK \\
-C + DK
\end{bmatrix} = M_{1Y} \cdot K + N_{1Y}, \text{ где}
\]
\[
M_{1Y} = \frac{4}{\Delta_1} \cdot (LX_R - RX_L); \quad N_{1Y} = \frac{2}{\Delta_1} \cdot (AX_R - CX_L); \quad \Delta_1 = 4 \cdot (X_L Y_R - Y_L X_R).
\]

Solution of the second system of equations
\[
X = \frac{1}{\Delta_2} \begin{bmatrix}
C + DK & -2Y_L \\
E + FK & -2Y_U
\end{bmatrix} = M_{2X} \cdot K + N_{2X}, \text{ where}
\]
\[
M_{2X} = \frac{4}{\Delta_2} \cdot (UY_R - RX_U); \quad N_{2X} = \frac{2}{\Delta_2} \cdot (EY_R - CY_U);
\]
\[
Y = \frac{1}{\Delta_2} \begin{bmatrix}
-A + BK \\
-C + DK
\end{bmatrix} = M_{2Y} \cdot K + N_{2Y}, \text{ где}
\]
\[
M_{2Y} = \frac{4}{\Delta_2} \cdot (RX_U - UX_R); \quad N_{2Y} = \frac{2}{\Delta_2} \cdot (CX_U - EX_R); \quad \Delta_2 = 4 \cdot (X_R Y_U - Y_R X_U).
\]

Solution of the third system of equations
\[
X = \frac{1}{\Delta_3} \begin{bmatrix}
E + FK & -2Y_U \\
A + BK & -2Y_L
\end{bmatrix} = M_{3X} \cdot K + N_{3X}, \text{ where}
\]
\[
M_{3X} = \frac{4}{\Delta_3} \cdot (LY_U - UY_L); \quad N_{3X} = \frac{2}{\Delta_3} \cdot (AY_U - EY_L)
\]
\[
Y = \frac{1}{\Delta_3} \begin{bmatrix}
-A + BK \\
-C + DK
\end{bmatrix} = M_{3Y} \cdot K + N_{3Y}, \text{ where}
\]
\[
M_{3Y} = \frac{4}{\Delta_3} \cdot (UX_L - LX_U); \quad N_{3Y} = \frac{2}{\Delta_3} \cdot (EX_L - AX_U); \quad \Delta_3 = 4 \cdot (X_U Y_L - Y_U X_L).
\]

Substitution (10 ÷ 15) in (6) defines a quadratic equation with respect to the variable \(K\)
\[
a_p K^2 + b_p K + c_p = 0, \text{ where } p \text{ corresponds to the choice of a pair of hyperbolas, } p = 1, 2, 3,
\]
\[
a_p = M_{pX}^2 + M_{pY}^2 - 1; \quad b_p = 2 \left( M_{pX} N_{pX} + M_{pY} N_{pY} \right); \quad c_p = N_{pX}^2 + N_{pY}^2
\]

(16)
The roots of the quadratic equation take values

\[ K_{p_1} = \frac{-b_p + \sqrt{b_p^2 - 4a_p c_p}}{2a_p}, \quad K_{p_2} = \frac{-b_p - \sqrt{b_p^2 - 4a_p c_p}}{2a_p}, \quad \text{moreover} \quad b_p^2 - 4a_p c_p \geq 0 \quad (17) \]

In the event that one of the two roots \(K_{p_1}\) and \(K_{p_2}\) is negative for the same value of \(p\), it can be immediately excluded from the solution, and then the other root of the equation remains in the algorithm. However, it is possible that both roots \(K_{p_1}\) and \(K_{p_2}\) are positive. For example, a solution for one of three systems of equations (that is, for three BS, not four) with the coordinates of the object \([x; y]\) = \([10; 1]\) and the coordinates of BS \(L\) \([x_L; y_L]\)=\([-40; 40]\), BS \(R\) \([x_R; y_R]\)=\([-40; -40]\), BS \(U\) \([x_U; y_U]\)=\([40; -40]\) meters, determines the root values \(K_{p_1}=63.41\) and \(K_{p_2}=54.98\). In this case, it becomes impossible to determine the coordinates of the object. It is for such a fairly common case that you have to use the fourth BS in the positioning system based on the analytical method. It should be noted that the method of linearization of hyperbolic equations with the indicated combination of object coordinates and base stations generally leads to a gross error in determining the coordinates of the object.

Substitution of the roots \(K_{p_1}\) and \(K_{p_2}\) in equations (10 ÷ 15) defines six possible pairs of coordinates of the object.

\[ [x_{p_1} = X_{p_1} + x_C, \quad y_{p_1} = Y_{p_1} + y_C ] \quad \text{и} \quad [x_{p_2} = X_{p_2} + x_C, \quad y_{p_2} = Y_{p_2} + y_C ], \quad \text{где} \]

\[ X_{p_1} = M_{pX} \cdot K_{p_1} + N_{pX}, \quad Y_{p_1} = M_{pY} \cdot K_{p_1} + N_{pY} , \]
\[ X_{p_2} = M_{pX} \cdot K_{p_2} + N_{pX}, \quad Y_{p_2} = M_{pY} \cdot K_{p_2} + N_{pY} \]  
(18)

If there are no TDoA measurement errors from six pairs of calculated coordinates, the three pairs of coordinates will be the same. It is this decision that is true. To identify it, twelve distances \(D\) between the positions of the object determined by (18) are calculated:

\[ D_{p_1,q_1} = \sqrt{(x_{p_1} - x_{q_1})^2 + (y_{p_1} - y_{q_1})^2} \quad (19) \]
\[ D_{p_2,q_2} = \sqrt{(x_{p_2} - x_{q_2})^2 + (y_{p_2} - y_{q_2})^2} \quad (20) \]

where both indices \(p\) and \(q\) take the values 1, 2, 3, and \(p \neq q\).

The next step is to search for the minima (19) and (20) and the corresponding minima of the indices. The values of the minima (19) and (20) are compared, the smallest is chosen. The coordinates of the object \([x; y]\) are selected from the coordinate values (18) by the index of the smallest of (19) and (20) minimum.

4. Comparison of algorithms for inaccurate TDoA measurement

TDoA measurement errors occur mainly due to time out of sync at base stations and are the main reason for the inaccuracy in determining the position of an object. Of the six pairs of coordinates of the proposed algorithm, only one pair under the conditions of noise of time synchronization at the BS is as close as possible to the true coordinates of the object. The remaining pairs are either false, or determine the coordinates with a larger error. The proposed algorithm selects precisely the most accurate pair of coordinates \([x; y]\) of the object.

Using a computer experiment in the LabVIEW environment, we investigated the dependence of the standard deviation (RMS) of the estimated coordinates of the object on the standard deviation of the difference in the arrival times of microwave signals in a positioning radar. For this, a Gaussian white noise with the same standard deviation value for all BSs was added to the TDoA value at the input of
each receiver of the positioning system. To determine the standard deviation of the estimate of the position of the object, 100 computational experiments were carried out for each value of the standard deviation of Gaussian white noise. The standard deviation of the estimate of the coordinates of the object was determined in accordance with the expression:

\[ RMS = \sqrt{\frac{1}{n} \sum_{i=1}^{n} \left( (x_i - x)^2 + (y_i - y)^2 \right)}, \]  

where index \( i \) is the number of the experiment, \([x_i; y_i]\) - estimate of the coordinates of the object in the \( i \)-th experiment, \([x; y]\) are the true values of the coordinates of the object. In Figure 1 shows the dependences of the standard deviation of the estimate of the coordinates of the object on the standard deviation of the Gaussian white noise of the difference in the arrival times of microwave signals in the BS for the matrix linearization algorithm and the proposed analytical algorithm. Dependencies are calculated for the coordinates of the object \([x; y] = [10; 1]\) and coordinates of BS \([x_C; y_C] = [-40; 40]\), BS \([x_L; y_L] = [-40; -40]\), BS \([x_R; y_R] = [40; -40]\), BS \([x_U; y_U] = [40; 40]\) meters.

![Figure 1. The dependence of the standard deviation of the estimate of the coordinates of the object from the standard deviation of the Gaussian white noise of the difference in arrival times](image)

From the simulation results it follows that the proposed algorithm provides an accuracy of determining the coordinates of the object 50 times higher compared to the method of linearization of hyperbolic equations. The proposed method works with high accuracy (5 cm) when the BS is out of sync up to 300 ps, while the linearization method begins to produce gross errors (failures) even when the BS is out of sync in 100 ps. It can be seen from all that the proposed algorithm significantly exceeds the widely used positioning algorithm based on the linearization of hyperbolic equations.

5. Conclusion

An analytical positioning algorithm based on measuring the difference in the arrival times of signals from the transmitter of the object to the base stations eliminates the possibility of the appearance of territorial zones on the plane in which a strict solution of hyperbolic equations leads to ambiguity in determining the coordinates. The proposed algorithm has an accuracy of determining the coordinates ten times higher in comparison with the method of linearization of hyperbolic equations, less sensitive to time out of sync at base stations. Due to these advantages, the proposed algorithm is promising for remote determination of the parameters of unmanned vehicles in the technologies of the “smart city”.

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As future Works we propose make practice implementations to compare the performance with the simulations results. We will also expand the proposed algorithm for determining the coordinates of a target in space (3D model) and apply the method for multi-target mode.

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