Acoustic properties of overheated liquid with gas nuclei during temperature increasing

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Abstract. The characteristics of the reflection and refraction of harmonic waves at its oblique incidence on an interface between a “pure” liquid and liquid with bubbles filled with a vapor–gas mixture have been studied. For the considered problem, we have obtained the dispersion equation and carried out a numerical analysis of the effect of the perturbation frequencies in the range $10^2$–$10^6$ s$^{-1}$ on the dependence of the angle of refraction on that of incidence for three equilibrium temperatures $T_0$. The dependence of the critical angle of incidence on the parameters of a two-phase system and the perturbation frequencies has been studied for the same reflection.

1. Introduction
Bubble media and the wave propagation in them are a promising research area, since these problems have a wide range of applications in biotechnology, gas and oil transportation, chemical and biological industries, as well as in ensuring technosphere safety. Research interest in these problems is known for a long time [1–6].

In works [7, 8] the interaction of a bubble with a body, in particular, with a biomaterial [8] placed in a liquid, was considered, whereas in [9], the behavior of a vapor envelope around a heated solid particle in a variable pressure field was analyzed. Worthy of note is also work [10], in which collapse of several successively located gas bubbles is studied, and the influence of the sizes of bubbles and the order of their collapse and location relative to one another on the collapse process has been analyzed.

In works [11–13] the dynamics of weak acoustic waves in multifractional mixtures of a liquid with vapor–gas and gas bubbles of different sizes and compositions with phase transformations was studied. It was shown that the dispersion and dissipation of acoustic waves depend in many respects on the presence of bubbles of different fractions in the composition of the dispersed phase.

The oblique incidence of an acoustic signal to the interface between a vapor–gas–drop medium and air was considered in [14]. Based on the calculations, the authors found that, if a wave is incident to the interface of the vapor–gas–drop mixture, there is a critical incidence angle at which the wave is totally reflected. In addition, the same authors considered the propagation of sound in a fog [15]. The authors of [16] studied the problem of wave reflection and transmission to the interface between bubbly and pure liquids, when a liquid is “cold”, i.e., there is only gas
in the bubbles. The results of the studies made it possible to find the critical incidence angles at which the total reflection of a wave from the interface is possible.

The authors of [17] considered the propagation of a wave pulse in a bubbly liquid. The suppression criteria of a wave pulse by a bubbly screen were revealed in terms of the initial system parameters. It was established that a bubbly layer with a quite low volume of bubbles could totally suppress a wave signal.

The authors of [18] considered the propagation of an initial transverse-localized wave pulse in a uniform bubbly mixture and a medium between two plane-parallel walls, which was piecewise-nonuniform throughout the bubble content. For the case of a mixture with a nonuniform distribution of a gas volume content, the propagation of a pulse in a region that is piecewise-nonuniform throughout the bubble content is accomplished by the formation of pressure profiles in the transverse direction that have peaks near the layer interfaces. This is caused by the difference in the wave speed in layers with various gas volume contents.

The characteristics of the shock and isentropic action on gas–liquid media were studied in [19]. The parameters of the incident and reflected shock waves in a gas–liquid medium were obtained on the basis of numerical calculations for the cases of isothermal, adiabatic, and shock compression of a gas component. The calculation results were compared with the experimental data on acoustic and shock wave propagation and reflection from an obstacle in vapor–gas–liquid mixtures.

The present work is a continuation of the studies described in [20]. We analyzed the effect of the equilibrium temperature and dispersion degree of the volume content of bubbly phase on the dynamics of the reflection and transmission of an acoustic wave at an oblique incidence on the interface between bubbly and “pure” liquids. Here, we use the terms and notations accepted in [20].

2. Governing equations and dispersive analysis

Let us consider two-dimensional acoustic waves. Let us turn the vertical axis 0x perpendicular to the interface plane between single- and two-phase media to a “pure” liquid and set the origin of coordinates x = 0 on the interface. The horizontal axis 0y is directed along the interface (Figure 1).

Let us assume that a liquid in region x > 0 has temperature $T_0$ and pressure $p_0$, and a liquid in region x < 0 at the same temperature and pressure contains spherical bubbles with radius $a_0$, which in turn contain vapor and a gas insoluble in a liquid phase. This system is described by equations (1)–(12) from [21]. It should be noted, that, if a liquid state is far enough from critical, conditions $p_{\text{cr}} = p_s(T_0)$ are fulfilled [22].

Let a plane harmonic wave be incident at some angle on a flat interface between a “pure” liquid and a bubbly liquid (Figure 1).
and gas-saturated liquid (Figure 1). We assume that the reflected and refracted waves are plane harmonic, as in the case of typical single-phase media [23]. Small perturbations in the water are then the sum of two harmonic waves; in the bubbly liquid, they are one harmonic wave. Let us denote by superscripts \((0), (r)\) and \((s)\) the perturbations corresponding to the incident, reflected, and refracted wave, respectively. In the frames of the accepted model of the bubbly liquid, the viscous and heat exchange processes are only taken into account upon interphase interactions; the mixture is one-speed; the stress tensor is spherical; the liquid is isothermal. Hence, similar to the single-phase media, at interface \(x = 0\), we can apply two boundary conditions, the continuity conditions of the pressure and normal velocity:

\[
p^{(0)} + p^{(r)} = p^{(s)}, \quad v^{(0)} \cos \theta^{(0)} - v^{(r)} \cos \theta^{(r)} = v^{(s)} \cos \theta^{(s)}. \tag{1}
\]

In (1) \(\theta^{(0)}, \theta^{(r)}\) and \(\theta^{(s)}\) are the angles of incidence; \(v^{(0)}, v^{(r)}\) and \(v^{(s)}\) are perturbations of the speed corresponding to the plane incident, reflected, and refracted waves, respectively.

The main calculation procedure is presented in [21]. Let us note its key points.

Wavenumbers \(K^{(0)}\) and \(K^{(r)}\) (from [21]) are related as \(K^{(0)} = K^{(r)} = \omega / C_l\). Wavenumber \(K^{(s)}\) is determined from the dispersion equation

\[
\frac{K^2}{\omega^2} = \frac{(1 - \alpha_{g0})^2}{C_l^2} + 3 \frac{\rho_0 \alpha_{g0} (1 - \alpha_{g0})}{\psi}, \tag{2}
\]

\[
\psi = \frac{3 \gamma \rho_0}{Q} - \frac{\rho_0 ^2 \omega^2 a_0^2}{\xi} - 4 i \rho_0 \nu^{(1)}(\nu) \omega - \frac{2 \sigma}{a_0},
\]

\[
p_{g0} = p_0 - \frac{2 \sigma}{a_0}, \quad \xi = 1 - i \omega t_A, \quad t_A = \frac{a_0}{\sqrt{\gamma \alpha_{g0} C_l}},
\]

\[
Q = 1 + \left( \frac{\gamma - 1}{k_0} H_a k h(y_0) + \frac{\gamma}{1 - k_0} H_a k h(z) \right) \left( \frac{H_a}{k_0} + \frac{\gamma k h(z)}{(1 - k_0) \beta \sinh v(y)} \right)^{-1},
\]

\[
k h(x) = 3 (x \coth x - 1) x^{-2},
\]

\[
\sinh v(x) = 3 (1 + x (A_0 x \tanh x (A_0 - 1) - 1) (A_0 x - \tanh x (A_0 - 1))^{-1} x^{-2},
\]

or

\[
\sinh v(x) = 3 (1 + x) x^{-2},
\]

\[
\eta = \frac{\rho_0 ^2 B_l}{\rho_0 ^2 C_l}, \quad \chi = \frac{\omega_T}{\nu}, \quad H_s = B_l / B_0, \quad H_a = B_a / B_0, \quad H = H_v - H_a.
\]

There \(K\) is a wave vector; \(\omega\) is a frequency of disturbances of the medium; \(\alpha_{g0}, \alpha_{v0}\) are volume content of gas and vapour in a mixture; \(\rho_0 ^2\) is a true density of the \(i\)th phase; \(\gamma\) is a heat capacity ratio; \(C_l\) is frozen velocity of sound in water; \(\sigma\) is a coefficient of surface tension of water; \(c_i\) is a heat capacity at constant pressure of the \(i\)th phase; \(D\) is a diffusion coefficient; \(\nu^{(1)}(\nu)\), \(\nu^{(T)}(\nu)\) are kinematic viscosity and thermal diffusivity of liquid; \(l\) is a specific heat of water evaporation; \(B_i\) is reduced gas constant at \(i\)th phase; indices \(i = l, g, v\) are for liquid, gas and vapour phases.

The derivation of dispersion equation (2) is described in detail in [21].

\[
S = 2 \left( \frac{1 + \frac{C_l K^{(s)}(s) \cos \theta^{(s)}}{\cos \theta^{(0)}}}{\omega} \right), \quad R = S - 1.
\]

Wavenumber \(K^{(0)}\) is determined from dispersion equation (2). In this case, the refraction and reflection coefficients are

\[
S = 2 \left( \frac{1 + \frac{C_l K^{(0)}(s) \cos \theta^{(s)}}{\cos \theta^{(0)}}}{\omega} \right), \quad R = S - 1.
\]
3. Numerical results
As mentioned in [21], the experimental works on the considered topic are extremely complicated. Hence, to confirm the obtained results, we made comparison with those presented in [16]. As values of physical and thermophysical parameters data from the book [24] were used. Figures 2–3 show a comparison of the results of the present work obtained with the use of initial data from [16]. For the calculations, we took an initial bubble radius of $a_0 = 10^{-3}$ m. The dashed curves in the mentioned figures correspond to the data from [16], and the solid curves show our results obtained with the model described above.

**Figure 2.** Influence of the initial volume content of bubbles $\alpha_{g0}$ on modulus ((1) and (2)) and arguments $\phi$ and $\psi$ ((3) and (4)) on the reflection and refraction coefficients for the “oblique” incidence of a wave from the side of “clean” liquid to the interface: $\alpha_{g0} = 10^{-2}$ ((1), (3)) and $\alpha_{g0} = 10^{-3}$ ((2), (4)). The dashed curves in the mentioned figures correspond to the data from [16], and the solid curves show our results obtained with the model described above.

**Figure 3.** Influence of the initial volume content of bubbles $\alpha_{g0}$ on modulus ((1) and (2)) and arguments $\phi$ and $\psi$ ((3) and (4)) on the reflection and refraction coefficients for the “oblique” incidence of a wave from the side of the bubble liquid to the interface: $\alpha_{g0} = 10^{-2}$ ((1), (3)) and $\alpha_{g0} = 10^{-3}$ ((2), (4)). The dashed curves in the mentioned figures correspond to the data from [16], and the solid curves show our results obtained with the model described above.

Figures 2 and 3 show a comparison of the dependences of moduli (curves (1) and (2)) and arguments $\psi$ and $\phi$ (curves (3) and (4)) of the reflection and refraction coefficients on the angular frequency of harmonic waves incident from “pure” liquid on the bubbly mixture (Figure 2) and from the bubbly liquid on “pure” liquid (Figure 3). Clearly, a satisfactory agreement with the previous results is observed. Hence, we believe that the use of the fundamental laws of mechanics and good agreement with the previous data makes it possible to speak of the validity of the results presented in our work.
4. Conclusion

We showed that a low-frequency acoustic wave ($\omega < \omega_R$) incident from a bubbly liquid to the interface at angles greater than the critical one is totally reflected from the interface for the considered initial temperatures $T_0 = 300$ and 353 K. For example, if the angle of incidence exceeds $\theta(0) = 10^\circ$, the angle of refraction asymptotically approaches $\theta(s) = 90^\circ$. We have found that a bubbly layer in a “pure” liquid in this case exhibits the properties of a sound channel.

At the same time, a different pattern is observed at an initial medium temperature $T_0 = 373$ K. The maximum angle of refraction reaches $\theta(s) \approx 70^\circ$. Thus, no total internal reflection formally occurs in a system at temperature 373 K. However, the wave penetrating a “pure” liquid in this case is totally attenuated near the interface due to the large attenuation coefficient and low phase speed, which is indicated in the imaginary part of the value of $n_y(s)$ or $\cos \theta(s)$. Therefore, the total internal reflection is practically realized.

We showed that a wave incident to the interface from a “pure” liquid passes into a bubbly liquid for any angle of incidence (see also [21]).

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