Magnetospheric Truncation, Tidal Inspiral, and the Creation of Short-period and Ultra-short-period Planets

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Abstract

Sub-Neptunes around FGKM dwarfs are evenly distributed in log orbital period down to $\sim 10$ days, but dwindle in number at shorter periods. Both the break at $\sim 10$ days and the slope of the occurrence rate down to $\sim 1$ day can be attributed to the truncation of protoplanetary disks by their host star magnetospheres at corotation. We demonstrate this by deriving planet occurrence rate profiles from empirical distributions of pre-main-sequence stellar rotation periods. Observed profiles are better reproduced when planets are distributed randomly in disks—as might be expected if planets formed in situ—rather than piled up near disk edges, as would be the case if they migrated in by disk torques. Planets can be brought from disk edges to ultra-short ($<1$ day) periods by asynchronous equilibrium tides raised on their stars. Tidal migration can account for how ultra-short-period planets are more widely spaced than their longer-period counterparts. Our picture provides a starting point for understanding why the sub-Neptune population drops at $\sim 10$ days regardless of whether the host star is of type FGK or early M. We predict planet occurrence rates around A stars to also break at short periods, but at $\sim 1$ day instead of $\sim 10$ days because A stars rotate faster than stars with lower masses (this prediction presumes that the planetesimal building blocks of planets can drift inside the dust sublimation radius).

Key words: planet–disk interactions – planet–star interactions – planets and satellites: dynamical evolution and stability – planets and satellites: formation – stars: pre-main-sequence – stars: magnetic field

1. Introduction

The Kepler mission, in combination with radial velocity surveys, has revealed that sub-Neptunes, with radii $R < 4R_{\oplus}$, are fairly ubiquitous at orbital periods $P < 100$ days (e.g., Fressin et al. 2013; Dressing & Charbonneau 2015). Microlensing indicates that they occur just as frequently beyond, at least around M stars (e.g., Clanton & Gaudi 2016).

Though commonplace overall, sub-Neptunes (a population including Earths and super-Earths) are more likely to be found at certain orbital periods than others. At longer periods, they appear more or less evenly distributed across logarithmic intervals in $P$. But at shorter periods, sub-Neptunes are less common. The occurrence rate as a function of orbital period follows a broken power law, with a break at $P_{\text{break}} \sim 10$ days (e.g., Youdin 2011; Mulders et al. 2015). Within $\sim 10$ days, the occurrence rate scales approximately as $dN/d\log P \propto P^{1.5}$, while beyond $\sim 10$ days the occurrence rate plateaus. Remarkably, as we show in Figure 1, these power-law slopes characterize planets found around stars with widely varying spectral types, from FGK (Fressin et al. 2013) to early M (Dressing & Charbonneau 2015).$^3$

The lower occurrence rate of planets at $P < P_{\text{break}}$ (hereafter “short-period” planets) may reflect a truncation of their parent disks—perhaps by the magnetospheres of their host stars (e.g., Mulders et al. 2015; see also Plavchan & Bilinski 2013). The theory of “disk locking” posits that the inner disk edge corotates with the host star in equilibrium (e.g., Ghosh & Lamb 1979; Koenigl 1991; Ostriker & Shu 1995; Long et al. 2005; Romanova & Owocki 2016). Disk locking is supported observationally by “dippers,” young low-mass stars with relatively evolved disks that exhibit material lifted out of the disk midplane, presumably by magnetic torques, near the corotation radius (Stauffer et al. 2015; Andell et al. 2016; Bodman et al. 2016).$^4$ The rotation periods $P_f$ of stars that are 1–40 Myr old, displayed in Figure 2, range from 0.2 to 20 days, squarely in the range occupied by short-period planets. Moreover, the distribution of $P_f$ peaks near $\sim 10$ days and falls toward shorter periods, mirroring, at least qualitatively, the decline in the planet occurrence rate at $P < P_{\text{break}}$.$^5$ In this paper we develop, in quantitative detail, this possible connection between disk truncation at corotation and the occurrence rate profile of short-period planets.

Planets that form outside the inner disk truncation radius can be brought to the disk edge, and even transported inside it, by a variety of migration mechanisms. These include disk torques (e.g., migration types I and II; see the review by Kley & Nelson 2012), stellar tidal friction, and planet–planet gravitational interactions. We model the first two processes in this paper but not the third. Below we briefly review migration by planet–planet interactions

$^3$ The Trappist-1 planetary system (Gillon et al. 2017, and references therein) is hosted by a late M dwarf.

$^4$ Stellar magnetic fields measured from Zeeman broadening do not correlate well with field strengths predicted from magnetospheric truncation (see the review by Bouvier 2007 and references therein). However, relaxing the assumption that the stellar field is a pure dipole brings theory into closer agreement with observations (see, e.g., Cauley et al. 2012, and references therein).

$^5$ All stellar rotation periods $P_f$ shown in Figure 2 are measured from periodic light curve variations driven by starspots. Photometric retrieval of $P_f$ is biased against young and especially active T Tauri stars exhibiting irregular variability caused by strong and unsteady disk accretion. This bias is small—Herbst et al. (2002) estimate that their sample of periodic stars is incomplete by 8%–15%—and its effect on our analysis should be still more muted because we take the orbital architecture of planets to be established during the latest stages of disk evolution, when the disk has largely (but not completely) dissipated (see Section 2.2). As for the clusters NGC 2362 and NGC 2547, Irwin et al. (2008a, 2008b) find, respectively, that their surveys are complete down to $\sim 0.2 M_\odot$ and across all measured rotation periods.

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and explain why it cannot produce the bulk of short-period sub-Neptunes.

1. Secular excitation of eccentricities in a multi-planet system, coupled with tidal circularization, can shrink planetary orbits and produce short-period planets (e.g., Wu & Lithwick 2011; Hansen & Murray 2015; Matsakos & Königl 2016). The sub-Neptune 55 Cnc e may have been so nudged onto its current 0.7 day orbit (Hansen & Zink 2015). But general-relativistic precession at short periods usually defeats secular forcing by perturbers less massive than Jupiter. While many short-period sub-Neptunes are members of multi-planet systems (e.g., Pu & Wu 2015; Steffen & Coughlin 2016, and references therein), their nearby companions are not typically gas giants and therefore cannot compete against general-relativistic precession. 55 Cnc e is atypical because its four neighbors range in mass from 0.2 to 5 M\(_J\) (where M\(_J\) is the mass of Jupiter), creating a web of strong secular resonances.

2. Violent planet–planet scatterings (i.e., close encounters) can deliver planets from large stellocentric distances to small ones where their orbits can be circularized by tides. But tidal circularization takes time, on the order of Myr to Gyr for periapse distances between 0.03 and 0.1 au. During that time, planets may collide with others and fail to attain short periods. When bodies are on eccentric and crossing orbits, encounters/mergers are more likely to occur at large rather than small distances because bodies spend less time near their periapsis, and because interactions are less gravitationally focused at small distances. High-eccentricity migration of short-period sub-Neptunes is hard to reconcile with the observation that such planets are commonly found with close exterior neighbors.\(^6\)

\(^6\) By contrast, high-eccentricity migration of hot Jupiters is compatible with the observation that they have distant, not close, exterior companions (Bryan et al. 2016).

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\[ \text{Figure 1. Occurrence rates of sub-Neptunes} (R < 4R_{\oplus}) \text{ orbiting FGK dwarfs} \text{ (Fressin et al. 2013) and early M dwarfs (Dressing & Charbonneau 2015). A distinctive break at } P_{\text{break}} \sim 10 \text{ days divides long-period planets at } P > P_{\text{break}} \text{ from their less common, short-period counterparts at } P < P_{\text{break}} \text{. Strikingly, short-period planets around FGK host stars all appear to be distributed according to } dN/d \log P \propto P^{3.4 \pm 0.3}. \text{ We also distinguish "ultra-short-period" (USP) planets at } P < 1 \text{ day, using data from Sanchis-Ojeda et al. (2014). All planets are labeled according to their host star spectral types. Points without arrows correspond to sub-Neptunes larger than } 0.8R_{\oplus} \text{ for M dwarf hosts, and larger than } 1R_{\oplus} \text{ (M dwarfs) and larger than } 1.25R_{\oplus} \text{ (FGK dwarfs).} \]

\[ \text{Figure 2. Histograms of pre-main-sequence stellar rotation periods } P_{*} \text{ in clusters of various ages: the Orion Nebula Cluster (ONC, Herbst et al. 2002; Rodríguez-Ledesma et al. 2009), NGC 2362 (Irwin et al. 2008a), and NGC 2547 (Irwin et al. 2008b). Rotation periods are measured from periodic light curve variations due to starspots. For higher-mass stars (top panel), the distribution of } P_{*} \text{ for all three clusters peaks at } \sim 10 \text{ days and falls off at shorter periods. Lower-mass stars (bottom panel) tend to rotate faster at the same age. Older clusters harbor more rapidly rotating stars, as expected from stellar contraction. See Section 3.2 for a discussion of these trends.} \]

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In this work we attempt to explain the observed occurrence rate profiles of short-period planets, focusing on reproducing the break period \(P_{\text{break}}\), and the shape of the decline in planet frequency at \(P < P_{\text{break}}\). We take as our starting point circumstellar disks whose inner edges are located at corotation with their host stars, drawing disk truncation periods directly from observed stellar rotation periods (i.e., Figure 2). Outside the disk truncation radius, we consider separately the possibility that sub-Neptunes migrate inward by torques exerted by residual disk gas, and the possibility that disk-driven migration is negligible, i.e., that planets form in situ. In either case, we account for orbital decay driven by asynchronous equilibrium tides raised by planets on their host stars. We examine whether tidal friction can create “ultra-short-period” planets (USPs with \(P < 1 \text{ day} \)) from short-period planets, in proportions similar to those observed. Our Monte Carlo (MC) model incorporating these various ingredients is presented in Section 2. We summarize in Section 3, highlighting those areas that need the most shoring up, and venturing a few predictions.

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2. MC Simulations

We construct a MC model for the occurrence rates of sub-Neptunes versus orbital period. Our goal is to identify those
parameters related to disk truncation, disk-driven planet migration, and tidal inspiral that can reproduce the observed occurrence rate profiles as shown in Figure 1.

The following Sections (2.1–2.3) detail the three steps taken to construct a given MC model.

2.1. Step 1: Draw Disk Truncation Periods from Stellar Rotation Periods

We create \( N_{\text{disk}} = 20,000 \) disks, each truncated at its own innermost edge at period \( P_{\text{in}} \). The disks are assumed to be magnetospherically truncated at corotation with their host stars (i.e., we assume disk locking); each value of \( P_{\text{in}} \) is drawn directly from an empirical distribution of stellar rotation periods \( P_s \). A given MC model utilizes rotation periods measured for one of three stellar clusters: the Orion Nebula Cluster (ONC, age of 1 Myr), NGC 2362 (5 Myr), and NGC 2547 (40 Myr). For MC models of planets around FGK dwarfs, we draw \( P_{\text{in}} = P_s \) using stars with masses \( 0.5 M_\odot < M_s < 1.4 M_\odot \) (upper panel of Figure 2). For MC models of planets around M stars, we do the same for stars with masses \( 0.2 M_\odot < M_s < 0.5 M_\odot \) (lower panel of Figure 2).

2.2. Step 2: Lay Down Planets Following Either “In Situ” Formation or “Disk Migration”

Each disk is taken to contain \( N_{\text{planet}} = 5 \) planets. Their initial periods—initial as in before stellar tides have acted—are decided according to one of two schemes. “In situ” formation models are very simple: the pre-tide planet periods are distributed randomly and uniformly in log period from \( P_{\text{in}} \) to \( P_{\text{out}} = 400 \) days. The logarithmic spacing befits planetary systems that form by in situ, oligarchic accretion, whereby planets are separated by a multiple number of Hill radii (e.g., Kokubo & Ida 2012). Our specific choices of \( N_{\text{planet}} = 5 \) and \( P_{\text{out}} = 400 \) days affect only the normalization of the occurrence rate profile; this normalization is adjusted later to fit the observations (see the captions to Figures 4 through 8). The aim of this paper is to reproduce the breaks and slopes of the observed occurrence rates, not their absolute normalizations.

The alternative “disk migration” models start the same way as the in situ models, but transport planets to shorter pre-tide periods according to the Type I migration rate:

\[
\dot{a} = \frac{2T_L}{M_p \Omega a}
\]

where \( a \) is a given planet’s orbital radius, \( M_p \) is its mass, \( \Omega \) is its orbital angular frequency, and \( T_L \) is the combined Lindblad and corotation torque exerted by the disk on the planet (e.g., Kley & Nelson 2012):

\[
T_L = -\xi \left( \frac{M_p}{M_*} \right)^2 \frac{a^2}{h^2} \Sigma_{\text{gas}} a^4 \Omega^2,
\]

where \( \xi = (1.36 + 0.62 \beta + 0.43 \gamma) \) is an order-unity constant calibrated from simulations (D’Angelo & Lubow 2010), and \( \beta \) and \( \gamma \) characterize an assumed power-law gas disk of surface density

\[
\Sigma_{\text{gas}} = \Sigma_{\text{gas},0} (a/a_0)^{-\beta} \exp(-t/t_{\text{disk}})
\]

and temperature

\[
T = T_0 (a/a_0)^{-\gamma}.
\]

Other variables include the disk scale height \( h = \sqrt{kT/\mu m_H \Omega^2} \), the gas mean molecular weight \( \mu \), the mass of the hydrogen atom \( m_H \), and Boltzmann’s constant \( k \).

For simplicity, regardless of whether the host stars modeled are FGK dwarfs or M dwarfs, we fix \( a_0 = 0.1 \) au, \( T_0 = 1000 \) K, \( \gamma = 1/2 \), \( \mu = 2 \), and \( M_\odot = 5 M_\odot \). We fix \( M_s = 1 M_\odot \) for FGK dwarfs and \( M_s = 0.5 M_\odot \) for M dwarfs. The errors introduced by fixing these variables are subsumed to some extent by large uncertainties in the disk parameters \( \beta, \Sigma_{\text{gas},0}, \) and the dissipation timescale \( t_{\text{disk}} \). We now describe and comment on our choices for these three parameters:

1. For a given MC model, we set \( \beta \) to one of three values: \( \beta \in \{1, 2, 3\} \). If we assume that a disk’s gas traces its solids (i.e., a spatially constant gas-to-solids abundance ratio), then these \( \beta \)-values encompass the range of quoted slopes in the literature, as inferred from observations of exoplanets comprising mostly solids: \( \Sigma_{\text{solid}} \propto a^{-1.5} \) to \( a^{-2.4} \) (e.g., Hansen & Murray 2012; Chiang & Laughlin 2013; Schlichting 2014; Hansen 2015). Note that \( \Sigma_{\text{solid}} \sim dn/d\log P \times M_p/a^2 \propto a^{-2} \) follows immediately from \( dn/d\log P \propto P^0 \) (this flat slope is seen at long periods in Figure 1) if the characteristic planet mass \( M_p \) does not vary much with distance.

2. We experiment with two normalizations for the gas surface density at \( a_0 = 0.1 \) au: \( \Sigma_{\text{gas},0} \in \{40, 400\} \) g cm\(^{-2}\). These choices yield rather gas-poor disks, depleted by \( 3-4 \) orders of magnitude relative to solar-composition “minimum-mass nebulae.” Such highly gas-depleted environments are motivated by the need to defeat gas dynamical friction before sub-Neptune cores can form by the mergers of protocores (Lee & Chiang 2010, their Figure 5). We assume that Type I migration proceeds normally in these gas-poor disks and discuss this assumption in the Appendix.

3. For the gas exponential decay timescale, we try \( t_{\text{disk}} \in \{1, 10\} \) Myr, which overlaps with the range of typical disk lifetimes reported from observations (Mamajek 2009; Alexander et al. 2014; Pfalzner et al. 2014).

Having assigned all parameters for our disk migration model, we then migrate every planet inward by integrating Equation (1) over a time interval of \( 10 \times t_{\text{disk}} \) (10 e-foldings of the disk gas density).

2.2.1. What Happens when Planets Migrate into the Innermost Edge

The first planet to migrate by disk torques toward the disk truncation radius will park in its vicinity—either interior to the disk edge where the last of the principal Lindblad torques peters out (Goldreich & Tremaine 1980; see also Lin et al. 1996); at the disk edge because of principal corotation torques (Tanaka et al. 2002; Masset et al. 2006; Terquem & Papaloizou 2007); or exterior to the disk edge following the reflection of planet-driven waves off the edge (Tsang 2011). In our MC model, we assume for simplicity that the first planet that can migrate by disk torques to arrive at the inner edge does so, and stays there (until step 3, after which it and all other planets move further in by stellar tides). Experiments that do not park this first planet at \( P = P_{\text{in}} \) but instead at \( P = P_{\text{in}}/2 \) (so that the planet is located at the interior 2:1 resonance with the disk edge) change none of our qualitative conclusions.

We have two options for the planets that subsequently follow this first parked planet. In one class of MC model (“resonance lock”; see Figure 3 for an illustration), we stop a planet when it
is the stellar radius and $\dot{\Sigma}_{\text{gas}}$ for $M_{\star}$ exceeded the disk
$\Sigma_{\text{gas}} \sim 10^{-3} \Sigma_{\text{MSN}}$ (gas is depleted by ~4 orders of magnitude
with respect to the solar-composition minimum-mass solar nebula). All 5
planets have equal mass ($5M_\oplus$) and adjacent pairs are initially spaced 14
mutual Hill radii apart (see, e.g., Figure 6 of Pu & Wu 2015; note that in our
actual MC models, we randomize the initial orbital spacings). Each planet’s
trajectory is computed by integrating Equation (1). The innermost planet is
forced to halt once it reaches the disk’s inner edge. Under the “resonance lock”
assumption, planets that follow the first planet are stopped once they migrate,
convergently, into 3:2 resonance. The final state comprises a chain of 3
resonant planets inside 10 days, 1 non-resonant planet that has migrated to just
outside 10 days (about a third of its starting period), and 1 outermost planet that
has hardly moved because its migration timescale $\dot{a}/a$ exceeded the disk e-
folding time $t_{\text{e-fold}}$, assumed for this figure to be 1 Myr.

migrates convergently into mean-motion resonance with its
interior neighbor. For simplicity, we consider only 3:2
resonances (the choice of 3:2 is motivated by the observation
that, of the small subset of Kepler planets located near
resonances, most are situated near 3:2; Fabrycky et al. 2014).
In a second class of MC model (“merged”), planets are allowed to
migrate unimpeded to the innermost disk edge and assumed to
merge there with the first planet. We count the merger product
as a single planet, equivalent to all other planets in our final
tally of planet occurrence rates (this simplification should be
acceptable insofar as observed occurrence rates based on transit
data are typically tallied by planet radius, which at fixed bulk
density does not change appreciably when a planet doubles or
triples its mass).

The intention behind these two flavors of MC model is to
bracket the range of possible planet–planet interactions near
the disk edge. On the one hand, convergently migrating
planets can become trapped in a resonant chain (e.g., Terquem
& Papaloizou 2007) or linger near resonances (Deck & Batygin
2015). Although observed short-period planets are generally not found in resonance (e.g., Sanchis-Ojeda et al. 2014, their Figure 8), orbital instabilities over the Gyr ages
of observed systems can lead to mergers of resonant planets,
transforming them into (lower multiplicity) non-resonant systems.
Numerical $N$-body integrations of high-multiplicity systems show
that resonances are preferentially unstable; non-resonant neighbors
appear to destabilize otherwise stable resonant pairs (Mahajan &
Wu 2014; Pu & Wu 2015, their Figure 5). Our “resonance lock”
model captures mergers of destabilized resonant pairs insofar as
merger products should have the same period distribution,
statistically, as their progenitors.

On the other hand, planets that collect near the inner disk
edge may never lock into resonances and instead collide into each other—hence our “merged” model. Given the potential
complexity of planet–planet and planet–disk interactions near
the disk edge (e.g., Tanaka et al. 2002; Masset et al. 2006;
Tsang 2011; Deck & Batygin 2015; Liu et al. 2017)—
interactions that may be modulated by the looming stellar
magnetosphere (e.g., Romanova & Owocki 2016) and/or disk
magnetic fields (e.g., Terquem 2003)—it would be unsurprising
if planets fail in general to be captured into resonance in
this messy environment.

2.3. Step 3: Apply Tidal Orbital Decay

The asynchronous equilibrium tide raised by a planet on its
host star causes its orbital semimajor axis to decay at a rate

$$\dot{a} = -\frac{9}{2} a^{-11/2} G^{1/2} \frac{M_p}{M_{\star}} \frac{R_{\star}}{Q_i^4}$$

where $R_{\star}$ is the stellar radius and $Q_i$ is the effective tidal
quality factor (Goldreich & Soter 1966; see also Ibgui &
Burrows 2009). For every planet, we integrate Equation (5)
over a time interval of 5 Gyr to calculate its final “post-tide”
orbital period.7 Equation (5) assumes that planets remain on
circular orbits, and that planet orbital periods are shorter than
the stellar rotation period (otherwise planets would migrate
outward). The latter assumption becomes safer as stars age
beyond the zero-age main sequence and slow their spins.
Although the assumption is violated for the planets with the
longest periods (say $\gtrsim 10$ days), such planets hardly migrate by
tides anyway because of the steep dependence of tidal friction
on $a$. We do not consider planet–planet interactions during tidal
migration, as tides act to separate orbits: divergently migrating
planets can cross resonances but do not lock into them, and the
eccentricities that sub-Neptunes impart to one another when
crossing resonances are small (see, e.g., Dermott et al. 1988).

For FGK dwarfs, we fix $R_\star = 1 R_\odot$ and vary $Q_i \in \{10^4, 10^5, 10^6\}$, a range that overlaps with estimates based on
observations and modeling of hot Jupiters (e.g., Matsumura
et al. 2008; Hansen 2010; Schlaufman et al. 2010; Penev
et al. 2012). For M dwarfs, we fix $R_\star = 0.5 R_\odot$ and vary $Q_i \in \{10^4, 10^5, 10^6, 10^7, 10^8\}$. The smaller values of $Q_i$ for M
dwarfs are motivated by the idea that stellar tidal dissipation
mostly occurs in convective zones, which are larger for lower-
mass stars (see, e.g., Hansen 2012, their Figure 3).

Having computed the final periods of all planets in all disks,
we tally them up to calculate model occurrence rates versus
orbital period, using the same period bins as reported by
the observations (Fressin et al. 2013 and Sanchis-Ojeda
et al. 2014 for FGK dwarfs; Dressing & Charbonneau
2015 for M dwarfs). We construct both probability distribution functions
(PDFs) and corresponding cumulative distribution functions
(CDFs) to compare directly with their observational

7 The models shown in this paper assume that the host star radius is fixed in
time and ignore the pre-main-sequence contraction phase during which the star
is still distended and the tidal decay rate (which scales as $R_\star$) is
correspondingly enhanced. We experimented with relaxing this assumption
and found that accounting for the full time evolution of stellar radius changes
negligibly the final planet occurrence rate profiles. The pre-main-sequence
phase lasts only $\sim 20–60$ Myr for $1–5 M_\odot$ stars, and is typically much shorter
than the orbital decay timescales of planets that do not fall onto the star.
counterparts. This comparison is done by eye—we are looking for broad trends and aiming to identify, only in an order-of-magnitude sense, those regions of parameter space that are compatible with the observations. Our choices on how to normalize our PDFs and CDFs are given in the figure captions.

2.4. Results for FGK Host Stars

We begin by reviewing the results of MC models for disk migration, first omitting the effects of tidal orbital decay in Section 2.4.1 and then incorporating them in Section 2.4.2. In situ + tidal models are presented in Section 2.4.3.

2.4.1. Disk Migration without Tidal Friction

Figure 4 shows that despite the many free parameters, our disk-migration-only models give practically the same results: overprediction of the number of short-period planets at 0.5 day\(^{-1}\), a tendency to underpredict longer-period planets at \(P > 10\) days, and a failure to capture ultra-short-period planets at \(P < 0.5\) day. Disk migration scoops out too many planets at large \(P\) to overpopulate small \(P\). If we allow disk migration to \(P = P_m/2\) then the models have the opposite problem of overproducing USPs. See later figures for models that include both disk and tidal migration.

![Figure 4](image_url)

Figure 4. Model occurrence rates (solid lines) including Type I disk migration in a dissipating nebula but excluding tidal migration. Model curves are computed using the same period bins as those reported from observations of sub-Neptunes orbiting FGK stars (Fressin et al. 2013, black points) and GK stars (Sanchis-Ojeda et al. 2014, blue points), and normalized such that their maxima match the maximum observed occurrence rate. The left panels correspond to “merged” models in which planets that migrate to the disk inner edge merge there. The right panels correspond to “resonance lock” models in which planets that migrate convergently into 3:2 resonance are locked into that resonance. At \(P \approx 1\) day, merged models exhibit a pile-up in the occurrence rate, while resonance lock models exhibit a sharper decline. At \(P \lesssim 10\) days, models do not differ much when disk parameters \(\beta, \Sigma_{gas,0}\) and \(t_{disk}\) are varied (see each panel’s annotations). The distribution of disk truncation periods is drawn from the stellar rotation periods in NGC 2362 (age \(\sim 5\) Myr). The model curves shown here omit tidal migration and are unable to produce USPs. If we allow disk migration to \(P = P_m/2\) then the models have the opposite problem of overproducing USPs. See later figures for models that include both disk and tidal migration.
tried relaxing this assumption, allowing planets to migrate to $P_\text{in}/2$ (i.e., parking them at 2:1 resonance with the disk edge; see Section 2.2.1). This succeeds in creating USPs, but merely extends the problem of overproducing short-period planets (relative to the observations) to $P < 0.5$ day (data not shown).

The occurrence rate profiles at $P \lesssim 10$ days appear almost identical over a wide range of disk parameters ($\beta$, $\Sigma_{\text{gas,0}}$, $t_{\text{disk}}$) because “all roads lead to Rome”: various disk migration histories all end with the same outcome of planets converging onto disk inner edges, whose period distribution is fixed by the distribution of stellar rotation periods. The details of the planet distribution in the vicinity of the edge depend on what assumption we adopt: “merged” models pile up planets at $P \sim 0.5$–1 day, while “resonance lock” models spread planets out more evenly in log $P$ from $\sim 1$–3 days (under our assumption that planets lock into 3:2 resonance; in general, for a $j$: $(j - 1)$ resonance, the pile-up sharpens with increasing $j$). At longer $P > 10$ days, there is more variation between disk models. In particular, $\beta = 1$ yields such high disk densities at large orbital radius that all planets at long periods are flushed inward by disk migration.

2.4.2. Disk Migration with Tidal Friction

Incorporating tidal inspiral into our disk migration models can yield improved fits to the occurrence rate of USPs at $P \lesssim 1$ day. In Figure 5 we apply tidal decay to one of the disk-migration-only models shown in Figure 4 ($\Sigma_{\text{gas,0}} = 40$ g cm$^{-2}$, $\beta = 2$, and $t_{\text{disk}} = 1$ Myr). From the grid of models in Figure 5, it appears that using stellar rotation periods measured for a cluster of a certain age: 1 Myr for the Orion Nebula Cluster (ONC), 5 Myr for NGC 2362, and 40 Myr for NGC 2547. Irrespective of whether the innermost planets merge at the truncation radii of their disks (top panels labeled “merged”) or are spaced according to mean-motion resonances (bottom panels labeled “resonance lock”), the observed occurrence rate profile for USPs ($P < 1$ day) can be reproduced by tidal migration, with the best fits corresponding to a stellar tidal friction parameter $Q'_* \sim 10^7$ and an intermediate age (NGC 2362). Note, however, how in all cases disk migration overestimates the frequency of planets with periods between 1 and 10 days.

Figure 5. Occurrence rates of planets that have undergone both disk (Type I) migration in a dissipating nebula, and tidal migration over 5 Gyr. For reference, the black solid lines include only disk migration and exclude tidal migration. As in Figure 4, we overplot the observed occurrence rates of sub-Neptunes around FGK stars with black and blue circles. All model curves utilize the same period bins as the reported observations, and are normalized such that they intersect the observed data point at $P = 39.5$ days. In each column, disk truncation periods are drawn from stellar rotation periods measured for a cluster of a certain age: 1 Myr for the Orion Nebula Cluster (ONC), 5 Myr for NGC 2362, and 40 Myr for NGC 2547. Irrespective of whether the innermost planets merge at the truncation radii of their disks (top panels labeled “merged”) or are spaced according to mean-motion resonances (bottom panels labeled “resonance lock”), the observed occurrence rate profile for USPs ($P < 1$ day) can be reproduced by tidal migration, with the best fits corresponding to a stellar tidal friction parameter $Q'_* \sim 10^7$ and an intermediate age (NGC 2362). Note, however, how in all cases disk migration overestimates the frequency of planets with periods between 1 and 10 days.

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seen in disk-migration-only models (Section 2.4.1). Tidal orbital decay has too short a reach to eliminate this excess.

**2.4.3. In Situ Models with Tidal Friction**

Figure 6 showcases in situ MC models, with and without tidal friction. At $P \gtrsim 3$ days, these models fit the observations remarkably well, performing better than the disk+t-tidal migration models in this period regime. Simply assuming that sub-Neptunes are randomly distributed in their disks (in a uniform $\log P$ sense) avoids the overproduction-at-short-$P$/underproduction-at-long-$P$ problem introduced by disk migration.

Ultra-short-period planets (USPs) can be reproduced, approximately, by tidal orbital decay of planets located near the inner edges of their disks. Drawing the shorter disk truncation periods from older clusters like NGC 2362 and NGC 2547 works best by giving planets a “head start” toward the shortest periods.

Figure 7 zooms in to directly compare one of the better fitting in situ+t models with its disk+t counterpart. With respect to the observations, the in situ+t model does better than the disk+t model at $P \sim 1$–10 days by not overproducing the number of planets. But for that same reason, the disk+t model does better than the in situ+t model at $P \lesssim 1$ day: disk migration generates more USPs by supplying more planets for tidal inspiral, and matches the observations at ultra-short periods more closely. Both classes of model overpredict the number of planets at $P \sim 1$–3 days.

**2.5. Results for M Host Stars**

Figure 8 compares model occurrence rates for planets orbiting M dwarfs against observations. The best agreement is obtained for in situ+t models calculated by drawing $P_\text{in}$ from the ONC and by adopting $Q'_s \sim 10^5$. Using these same assumptions in disk-induced migration models gives worse fits characterized by an excess of planets inside $\sim 10$ days. Appealing to stronger tidal dissipation can remove much of this excess but simultaneously destroys USPs by shuttling them to the stellar surface.

We require a significantly smaller $Q'_s$ to produce a best-fitting in situ+t model for M dwarfs ($Q'_s \sim 10^3$) as compared to FGK dwarfs ($Q'_s \sim 10^7$). M stars tend to rotate faster than FGK stars, so stronger tides are needed to remove the excess of planets near $\sim 1$–2 days. The smaller $Q'_s$ for M dwarfs supports the idea that the bulk of stellar tidal dissipation occurs in convective zones, which are larger for lower-mass stars (e.g., Hansen 2012, and references therein).

Drawing $P_\text{in}$ from clusters older than the ONC overestimates the number of planets inside 10 days, as stars spin up with age.
and truncate their disks at shorter periods. This effect is amplified for M stars over FGK stars because M dwarfs spin up more rapidly as they get older; see in Figure 2 how the peak of the period distribution shifts more dramatically with age for lower-mass stars. The excess population is especially difficult to remove when planets enter into and remain in a “resonance lock” near the disk edge. Resonance lock models distribute planets broadly over this period range, beyond the reach of tidal friction in M stars even for $Q_* = 10^4$.

3. Summary and Discussion

Around FGK and M stars, sub-Neptunes appear evenly distributed at orbital periods $P \sim 10$ days, but become increasingly rare toward shorter periods. Through MC population synthesis calculations, we demonstrated how this orbital architecture arises from disks truncated at corotation with their host star magnetospheres, and from orbital decay driven by stellar tidal dissipation. The dropoff in planet occurrence rate at $P \sim 1$–10 days traces the distribution of planets near the inner edges of their parent disks; in turn, the distribution of disk edges reflects the distribution of stellar rotation periods in young clusters 1–40 Myr old. At the same time, despite disk truncation, planets can migrate inward to ultra-short periods ($P < 1$ day) by the asynchronous equilibrium tides they raise on their host stars.

We found better fits to observed occurrence rates when planets were randomly distributed in log $P$ within their parent gas disks—as might be expected if they formed in situ—rather than brought to disk edges by disk torques. Figure 9 recaps how well our in situ + tidal models reproduce the occurrence rates of planets orbiting FGK dwarfs and early M dwarfs. However, none of our models, predicated either on in situ formation or disk migration, fit the data perfectly; the former do better at large $P$ while the latter do better at short $P$. In situ models might yield improved fits if we had a more accurate theory for tidal orbital decay, or if the distribution of disk inner edges were broader than we have assumed. Such broadening could be achieved by relaxing our assumption that disks truncate exactly at corotation, or by issuing sub-Neptunes from disks with a range of dissipation timescales. For example, the disk truncation period could be drawn from a mix of stellar rotation periods characterizing both the young Orion Nebula Cluster and the older NGC 2547 (but see Section 3.2). It is also
possible that the stellar rotation period distributions we have used are not entirely representative. The cluster Cep OB3b has a similar age (4–5 Myr) as NGC 2362, but its median rotation period for M dwarfs may be ∼1.6 times longer, assuming its low-mass stars are correctly identified as such (Littlefair et al. 2010).

Below we explore how our theory might be extended to bear on other observed properties of short-period sub-Neptunes and their host stars, attempting where possible to make predictions for future observations.

3.1. Orbital Spacings of USPs

We have shown in this paper the essential role tides play in creating USPs. Another line of evidence pointing to the influence of tides can be found by examining planetary orbital
spacings. Because tidal orbital decay generally proceeds more quickly at smaller periods, interplanetary spacings should be stretched out more (in a fractional sense) at small $P$ than at large $P$. Just such a signature is observed: USPs are more widely spaced than their longer-period counterparts (Steffen & Farr 2013). In Figure 10 we demonstrate how orbital spacings widen preferentially at the shortest periods because of tides. Comparison of our model with observations shows only qualitative agreement; the empirical data (gray circles and orange line) suggest that tidal friction in reality is more effective at drawing planets apart than our constant-$Q^*$ theory allows (blue curves). A better theory for tidal friction, one incorporating dynamical tides and the evolution of stellar spin and structure, is needed to better reproduce the data (see, e.g., Bolmont & Mathis 2016, and references therein).

3.2. Trends with Stellar Mass

Mulders et al. (2015) point out how the sub-Neptune occurrence rate breaks at $P_{\text{break}} \sim 10$ days regardless of whether the host star is of spectral type F, G, K, or M. In our theory, the break and dropout at shorter periods reflect the distribution of stellar spin periods $P_*$. We can reproduce the invariance of $P_{\text{break}}$ by drawing $P_*$ from a distribution that does not vary with stellar mass. The Orion Nebula Cluster (ONC) provides such a spin distribution, which peaks at $\sim 10$ days irrespective of whether the stars are $\sim 1 M_\odot$ or $\sim 0.3 M_\odot$ (compare the top and bottom panels of Figure 2). Drawing $P_*$ from the ONC implies that planets emerge from disks that dissipate over the ONC age of $\sim 1$ Myr. One problem with our ONC-based models is that they underestimate the number of USPs, particularly around FGK stars (left column of Figure 6); not enough stars rotate with periods faster than a day. USPs are produced more easily around older, more rapidly rotating stars (middle and right columns of Figure 6), but these have $P_*$ distributions that do vary with stellar mass (Figure 2) and therefore have trouble reproducing the invariance of $P_{\text{break}}$ (note how the model break period in the middle and the right columns of Figure 8 misses the observed break period by a factor of 5). Simultaneously fitting both USPs and their longer-period counterparts near the break is a challenge, requiring improvements in our understanding of disk truncation and tides.

There is also room for improvement for observations: the USP occurrence rates from Sanchis-Ojeda et al. (2014) appear systematically offset from the Fressin et al. (2013) data at longer periods; see how the blue points in Figure 9 appear to be shifted higher than the black points. Whether this offset is real or an artifact of combining two different data sets is unclear; removal of the offset would ameliorate if not solve the problem noted above with our ONC-based models. A uniform analysis is needed to measure planet occurrence rates from $\sim 0.1$ to $> 10$ days, not just for FGK stars but also for M stars.

Detecting and characterizing planets around main-sequence A stars is the next frontier. The invariance of $P_{\text{break}}$ with respect to stellar spectral type may not extend to A stars. Compared to T Tauri stars, Herbig Ae stars are observed to be faster rotators, with typical spin periods of $\sim 1$ day (Hubrig et al. 2009; Alecian et al. 2013). It follows that their gas disks should truncate at shorter periods. We therefore predict the

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8 Herbig Ae stars can have magnetospheres. They are observed to have typical surface magnetic field strengths of $\sim 100$ G (see, e.g., Hubrig et al. 2009, their Figure 10). For disk accretion rates of $10^{-7} - 10^{-5} M_\odot \text{yr}^{-1}$ around a star of mass $2 M_\odot$ and radius $4 R_\odot$, magnetospheric truncation radii range between $\sim 1$ and $4$ stellar radii.
occurrence rate of sub-Neptunes to break at a correspondingly short period $P_{\text{break}} \sim 1$ day around A stars, as compared to $P_{\text{break}} \sim 10$ days for FGKM stars. This shift in $P_{\text{break}}$ between high-mass and low-mass stars is illustrated in Figure 11. The break around A stars might occur at even shorter sub-day periods, as our spin period data for A stars are taken from stellar magnetic field studies whose samples are biased to include more slowly rotating stars (Hubrig et al. 2009; Alecian et al. 2013). Figure 11 also shows that tidal inspiral does not much alter the period break for A stars for $Q'_i \sim 10^{-5} - 10^0$. These larger values of $Q'_i$ are motivated by main-sequence A stars lacking outer convective envelopes (see, e.g., Mathis 2015, their Figure 4). A caveat to our prediction that sub-Neptunes around A stars exhibit a shorter $P_{\text{break}}$ is that we have so far discussed only disks of gas, not disks of solids—and sub-Neptunes are primarily composed of solids. Dust disks around A stars may not extend down to periods of $\sim 1$ day because of sublimation (e.g., Dullemond & Monnier 2010, their Figure 7). Nevertheless, larger solids (planetesimals orders of magnitude larger than dust grains) are more resistant to vaporization and can drift, aerodynamically or otherwise, through gas disks toward their inner edges, assembling into planets there.

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**Appendix**

**Migration in Gas-poor Disks**

In our MC models with disk-induced Type I migration, we chose gas surface densities so low that the planet mass $M_p$ exceeds, by roughly two orders of magnitude, the local disk mass $\Sigma_{\text{gas}}a^2$. For the planet to migrate inward, it must lose its angular momentum to the disk. Because $M_p \gg \Sigma_{\text{gas}}a^2$, one might worry that the need to transfer so much angular momentum from the planet to the local disk would severely perturb the latter, so much so that the planet would cease to undergo normal Type I migration. Perhaps the planet stops migrating altogether; or perhaps it opens a deep and wide gap and migrates at the Type II rate set by disk viscosity.

Even when $M_p \gg \Sigma_{\text{gas}}a^2$, Type I migration might still prevail in disks with large enough viscous stresses to prevent the opening of gaps and to transport angular momentum—both the planet’s and the disk’s—outward to distances of a few au. For our chosen disk parameters, we find that as long as the Shakura–Sunyaev viscosity parameter $\alpha \gtrsim 2 \times 10^{-4}$, super-Earths of mass $5 M_\oplus$ will not open gaps at short periods (Duffell & MacFadyen 2013; Fung et al. 2014). Furthermore, for the disk depletion time to be as long as $t_{\text{disk}} \sim 1$ Myr, the disk must be able to transport mass and momentum across large distances extending well beyond the innermost regions where short-period planets reside. For example, assuming $\alpha \gtrsim 2 \times 10^{-4}$ (as per above), the viscous drainage time of the disk, $t_{\text{visc}} \sim a^2/\nu \sim \Omega a^2 \mu m_\oplus/(\alpha kT)$ (for viscosity $\nu$), is as long as $t_{\text{disk}} \sim 1$ Myr only for $a \gtrsim 2$ au. The mechanism of disk transport remains unclear, but the magneto-rotational instability and magnetized winds are perennial candidates, especially in the hot and well-ionized innermost disk, and perhaps also beyond, under gas-poor conditions (see, e.g., Wang & Goodman 2017).

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9 By contrast, in inviscid disks, the inability of the disk to carry away the planet’s angular momentum presents a serious problem for Type I migration. A “migration feedback” torque can stall planets in low-mass, laminar disks (Ward & Hourigan 1989; Ward 1997; Rafikov 2002; Li et al. 2009; Yu et al. 2010; Fung & Chiang 2017).
Even if migration is not Type I, its exact form should have little bearing on our disk+tide migration models at orbital periods shorter than ~20 days. As we demonstrated in Section 2.4.1, the period distribution of close-in planets is determined largely by the distribution of stellar rotation periods, which sets disk truncation periods. In our disk+tide models, it does not much matter how planets are transported to disk edges, as long as they arrive there. We reiterate that our models, it does not much matter how planets are transported to disk edges, as long as they arrive there. We reiterate that our

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