Development of a method for calculating the ultimate resistance force of a sand pad with a variable reinforcement pitch

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Abstract. The paper presents the numerical simulation results of reinforced sand pads under load. During the simulation, the parameters of the soil base, the pad material, as well as the reinforcing materials varied. Based on the results of the study, a formula for determining the ultimate resistance force of a reinforced sand pad was set.

1. Introduction
Based on the conducted investigations, as well as the works of other authors [1-4], it was established that the main effect of soil reinforcement occurs after reaching the initial critical load on the base and moving to the maximum critical load. The problem of determining the load-bearing capacity (ultimate resistance) of the reinforced sand pad structure will be important. Calculation of the load-bearing capacity of reinforced bases is also a compulsory requirement of Code of Regulations 22.13330.2016 Foundations of buildings and structures. Since modern engineering methods do not allow calculating reinforced soil structures with sufficient accuracy at the elastic-plastic stage of work, numerical methods implemented in various software packages can cope with this task with a fairly good convergence.

2. Experiment
To assess the reliability of the numerical simulation results, the calculation was made in the Plaxis 2D software package on the conditions of previously performed field plate load tests of a sand pad at a variable pitch of geosynthetic reinforcement [2].

The calculation was made under conditions of flat deformation. In order to use the results of the field experiment, well-known approaches which consider the dependence of the stress-strain state of the soil mass on different loading options, including area, shape, and stiffness, were accepted. These investigations are described in the works of M. I. Gorbunov-Posadov, O. Ya. Shechter, K. E. Egorov, and other scientists [5-7]. The problem under consideration was solved using the Mohr-Coulomb model [8, 9]. The engineering and geological conditions of the site were taken in accordance with previous studies [2, 9]. The pad base soil is represented by a highly deformable soft-plastic loam, the pad material is sand of medium size. The characteristics are presented in table 1 and 2.
Table 1. Physical and mechanical characteristics of the experimental site soils.

| Soil                  | Thickness m | $\gamma$, kN/m$^3$ | $\varphi$, degree | $c$, kPa | $E$, MPa | $\nu$ |
|-----------------------|-------------|---------------------|-------------------|---------|---------|------|
| High plastic loam     | 6.2         | 19.1                | 16                | 14      | 8.7     | 0.35 |
| Pad material          | medium-sized sand | 16              | 30                | 4       | 29.5    | 0.3  |

Table 2. Mechanical characteristics of the reinforcing material.

| Material   | Linear stiffness (kN/m) | Coefficient of friction ($f_g(\sigma)$)$^a$ |
|------------|-------------------------|---------------------------------------------|
| Geogrid    | 1122/392$^a$            | 0.846                                       |

$^a$ To show the real geogrid-pad material interaction, interface elements with a $R_{ef}$ coefficient equal to the coefficient of friction of the geosynthetic material on the soil were introduced when calculating in the Plaxis 2D PC.

To correlate the results of the field experiment and numerical simulation, graphs of the settlement-pressure dependence were constructed for testing the natural non-reinforced base soil and the reinforced sand pad. The graphs are shown in figures 1 and 2.

![Graph showing settlement-pressure relationship](image-url)

**Figure 1.** Comparison of the field and numerical experiments results of natural base plate bearing tests.
The analysis of these graphs allows us to draw the following conclusions: the results of numerical modeling describe with sufficient accuracy the results of the field plate bearing tests of a natural non-reinforced base and of the reinforced sand pad. The deviations in the results are in the range of 8–9 % for the natural base and 9-10 % for the reinforced sand pad. The values of the final settlement values in the numerical simulation correlate with the experimental data quite well. On the settlement–pressure graphs in figures 1 and 2, typical patterns of base soil compaction and those of shifts development are highlighted. In the case of a reinforced sand pad, the pattern of soil compaction (the section of the base linear work) is longer, and the transition to the shear phase occurs at a pressure of approximately 233 kPa while the natural non-reinforced base enters the shear phase at pressures close to 93 kPa. Thus, the possibility of transferring large loads to the base represented by clay soil increases by up to 2.5 times in the case of reinforced sand pads.

In order to apply the results of the calculation in the Plaxis 2D PC for the design of reinforced soil structures, the vertical stress measurements [10] determined during field tests of reinforced sand pads were compared with the results of the calculation in the Plaxis 2D PC. Graphs of the comparison of the experimental vertical stresses values with the values calculated in the Plaxis 2D PC are shown in figures 3.
Figure 3. Comparison of vertical stresses of: (a) is the natural non-reinforced base at a pressure of 100 kPa on the stamp; (b) is the natural non-reinforced base at a pressure of 170 kPa on the stamp; (c) is the base with a reinforced sand pad at a pressure of 100 kPa on the stamp; (d) is the base with a reinforced sand pad at a pressure of 300 kPa on the stamp.

The obtained data of comparison of the calculated and experimental vertical stress values over the depth of the soil mass with a sand-reinforced pad and the natural (non-reinforced) base demonstrate that the dependences are similar. When calculating in the Plaxis 2D PC, the vertical stresses are underestimated in the range of 8-10 % compared to the experimental data. For large loads above a pressure of 170 kPa, the difference increases to 15 %.

The analysis confirms that the mathematical apparatus presented in the Plaxis 2D program accurately reflects the results of the actual behaviour of reinforced sand pads under load. Thus, in order to develop a method for calculating the ultimate resistance of the reinforced sand pads base, it is possible to use the Plaxis 2D software package with the Mohr-Coulomb model for calculation under conditions of plane deformation.
Before making calculations, the boundary conditions for the applicability of this method were formulated:

- Shallow strip foundations are considered as foundations;
- Geosynthetic materials are used as reinforcing elements;
- The stiffness of geosynthetic materials is set by the values typical for the materials used as reinforcing layers, – 100, 500, 1000, 2000, 3000 kN/m.
- The coefficient of friction is set using the interface elements. It varies in three values - the minimum, equal to \( f_g = 0.5\tan\varphi \), where \( \varphi \) is the angle of internal friction of the pad soil; the maximum, equal to 1, and the average value typical for materials with accepted rigidity according to Recommendation for Design and Analysis of Earth Structures using Geosynthetic Reinforcements EBGEO.
- The pad under consideration is reinforced with horizontally oriented geosynthetic elements. The reinforcement pitch is taken on the basis of the previous investigations described in \[9\], with the variable \( \Delta h = (n - 1)100 + 200 \), where \( n \) is the number of an reinforcing layer. The length of the reinforcement extending beyond the foundation contour (anchorage length) is structurally assumed to be equal to the half of the foundation width (0.5 \( b \)) according to Recommendation for Design and Analysis of Earth Structures using Geosynthetic Reinforcements EBGEO, to satisfy the condition for preventing the reinforcement pulling out;
- A conventional weak clay soil with non-variable parameters: modulus of deformation and specific gravity is considered to be the base soil.
- The angle of internal friction of the base soil is set in the range of 5\(^\circ\)-25\(^\circ\) with a pitch of 5\(^\circ\);
- The specific adhesion of the base soil is set in the range of 5-30 kPa with a pitch of 5 kPa;
- Medium-sized and coarse sand with non-deformable parameters (modulus of deformation and specific gravity) is considered as the pad filler soil;
- The internal friction angle of the pad filler soil is set in the range of 30\(^\circ\)-40\(^\circ\) with a pitch of 2\(^\circ\);
- The specific adhesion of the base soil is set in the range of 0-4 kPa with a pitch of 1 kPa;

The dimensions of the sand pad are given in terms of width-height ratio \( b_n/h_n \). In the works of various authors [11-13], the main methods for determining the pad height and width are given. The method of N. A. Tsytovich was adopted as the basic one [12]:

\[
b_n = b + 2h_n\tan \alpha.
\]

\[
h_n = bK_1.
\]

Consider the formula that will be obtained with ratio \( b_n/h_n \):

\[
\frac{b_n}{h_n} = \frac{b + 2h_n\tan \alpha}{bK_1} = \frac{1+2K_1\tan \alpha}{K_1}
\]

The optimal angle of the pad (\( \alpha \)) is 30\(^\circ\) [11, 12]. Thus, the formula takes the form:

\[
\frac{b_n}{h_n} = \frac{1+1.4K_1}{K_1}
\]

Based on the method described in [12], the coefficient \( K_1 \) depends on the ratio of the calculated resistances of the pad soil to the base soil, and for strip foundations it varies in the range from 0.3 to 1.5;

- The width of the foundation model is assumed on the basis of the specified dimensions of the soil pad. The height of the foundation is assumed to be 1 m.

The matrix view illustrations for calculating the base ultimate resistance force under the accepted assumptions is presented in table 3.
Table 3. General view of the matrix for calculating the ultimate resistance force.

| Angle of internal friction of the base weak soil φ, degree | Base weak soil adhesion C, kPa | Pad soil adhesion C, kPa | Angle of internal friction of the pad soil φ, degree | Ratio b_p/h_n | Pad width / height (b_n/h_n) |
|----------------------------------------------------------|-------------------------------|-------------------------|---------------------------------------------------|----------------|----------------------------|
|                                                          |                               |                         |                                                   |                | 3.000 2.000 1.000 500 100 |
| 6                                                       |                               |                         |                                                   |                | Stiffness of the reinforcing layer, kN/m |
| 4.5                                                     | 4.5/1                         | 3.6/0.8                 | 2.4/0.6                                           | 1.6/0.4        | 0.29–0.84–1 |
| 3.0                                                     | 3/1                           | 2.4/0.8                 | 1.8/0.6                                           | 1.2/0.4        |
| 2.4                                                     | 2.4/1                         | 1.92/0.8                | 1.44/0.6                                          | 0.96/0.4       |
| 5–25 (step 5)                                           | 5–30 (step 5)                 | 0–4 (step 1)            | 30–40 (step 2)                                    |                |
| 2.1                                                     | 2.1/1                         | 1.7/0.8                 | 1.26/0.6                                          | 0.84/0.4       |
| 1.9                                                     | 1.9/1                         | 1.5/0.8                 | 1.14/0.6                                          | 0.76/0.4       |
| 1.8                                                     | 1.8/1                         | 1.44/0.8                | 1.08/0.6                                          | 0.72/0.4       |

According to the results of the simulation, the following regularities were revealed:

- Increase in stiffness above 1000 kN/m does not bring a significant effect in increasing the base ultimate resistance force. The increase is within 2 %.

- If the width-to-height ratio (b_n/h_n) of the sand pad exceeds 3, a significant increase in the ultimate resistance force (over 400 kN/m) is obtained. The use of sand pads with these geometric parameters entails a significant increase in the volume of earthworks and material consumption, which reduces the efficiency of the structures use.

- If the width-to-height ratio (b_n/h_n) of the sand pad is less than 2.1, the base maximum resistance force is less than 200 kN/m, which is only applicable for low-load foundations.

- Increase in the specific adhesion of the base weak clay soil above 15 kPa gives a significant increase in the base ultimate resistance force, which does not allow us to consider the base soil as weak. The use of reinforced sand pads in this case is irrational.

- * Increase in the internal friction angle of the pad filler soil higher than 34 ° slightly increases the base ultimate resistance force.

Taking into account the obtained results and the revealed regularities, some experimental parameters were adjusted, in particular: the stiffness of geosynthetic materials was given by the values of 100, 500, 1000 kN/m; the specific adhesion of the base soil was set in the range of 5-15 kPa with a pitch of 5 kPa; the angle of internal friction of the pad filler soil was set in the range of 30 ° - 34 ° with
a pitch of $2^\circ$; the width-to-height ratio ($bn/hn$) of the sand pad was set by values 3; 2.4; 2.1. The other factors of the experiment were unchangeable.

Thus, a refined matrix was formed for calculating the base ultimate resistance force [14]. The value of the ultimate resistance force of a sand pad with a variable reinforcement pitch ($F_u^{\text{reinforced}}$) is proposed to consider in the following form:

$$F_u^{\text{reinforced}} = F_u^{\text{reinforced int.}} + F_u^{\text{reinforced mech}}, \quad (5)$$

where $F_u^{\text{reinforced int.}}$ is the base ultimate resistance force, which takes into account the effect of reinforcement (the interaction of reinforcement with the soil of the sand pad); $F_u^{\text{reinforced mech}}$ is the increase in the ultimate resistance force due to the mechanical characteristics of horizontal geosynthetic materials.

It was determined experimentally that the base ultimate resistance force, which takes into account the reinforcement effect ($F_u^{\text{reinforced int.}}$), will depend on: the interaction of the reinforcing geosynthetic materials with the sand pad body, the base soil characteristics, the characteristics of the sand as the pad filler, the sand pad size. And the parameter ($F_u^{\text{reinforced mech}}$) will depend on the mechanical characteristics of geosynthetic materials.

The calculation scheme for determining the base ultimate resistance force, improved by a reinforced sand pad, will generally look like the following (figure 4).

![Figure 4](image_url) Design scheme for determining the base ultimate resistance force ($F_u^{\text{reinforced}}$)

For shallow strip foundations (flat problem), the value of the vertical component of the base ultimate resistance force was first determined using the results of K. Terzaghi, in which there was an assumption about the occurrence of a homogeneous base soil under the foundation. In the problem under study, a base with a reinforced sand pad is considered, which requires an adjustment of the accepted already known solution [7].

In order to show the actual behaviour of the soil mass with a reinforced sand pad in order to determining the base ultimate resistance force in the solution mentioned above, correction coefficients $k_\gamma$, $k_q$, $k_c$ were introduced. They show that the ultimate sliding surface does not go through a homogenous soil base [15], but, on the one hand, through a reinforced sand pad with higher strength parameters, and on the other hand, through a weak soil base (see figure 4).

Thus, the formula for determining the base ultimate resistance force, taking into account the effect of reinforcement, for a strip foundation with coefficients $\xi_\gamma$, $\xi_q$, $\xi_c$, assumed to be equal to 1, will have the form:
where \( b', l' \) are the given width and length of the foundation, respectively; \( q \) is the lateral load; \( c' \) is the specific adhesion of the pad base soil; \( \gamma_k \) is the specific weight of the pad base soil; \( N'_{\gamma}, N'_q, N'_c \) are the load-bearing capacity coefficients according to Code of Regulations 22.13330.2016 Foundations of buildings and structures.

Based on the results of numerical modeling, the graphs were constructed to determine the values of correction coefficients \( k_{\gamma}, k_q, k_c \) (figures 5 (a), 5 (b)), depending on the ratio of the angle of internal friction of the pad base soil\( (\phi'_k) \) to the angle of internal friction of the pad filler soil\( (\phi'_F) \).

**Figure 5.** Correction coefficients, a is \( k_q, k_c \), b is \( k_{\gamma} \).

In the case where the load on the reinforced sand pad is not transmitted vertically, it is necessary to take into account the angle of load application. Then the correction coefficients are determined by the following formulas:

\[
\begin{align*}
 k_{\gamma}' &= 1 + (k_{\gamma} - 1)(h_n/h_{n,\delta}), \\
 k_q' &= 1 + (k_q - 1)(h_n/h_{n,\delta}), \\
 k_c' &= 1 + (k_c - 1)(h_n/h_{n,\delta}),
\end{align*}
\]

(7) (8) (9)

where \( h_{n,\delta} \) is the theoretical height of the pad at the load angle of \( \delta \neq 0 \); \( h_n \) is the theoretical height of the pad at the load angle of \( \delta = 0 \). The value \( h_n \) is determined at the initial stage.

The height of the sand pad is determined by the equation

\[
0.5m \leq h_n = \sum_{i=1}^{n}(n_i - 1)100 + 200 \leq h_{n,\text{max}} = \frac{b}{2} \cdot \tan \left( 45^\circ + \frac{\phi_k}{2} \right).
\]

Value \( h_{n,\delta} \) is determined by formula:

\[
\theta_{n,\delta} = \frac{\sin \theta_{n,\delta} \cos (\theta_{n,\delta} - \phi'_F)}{\cos \phi'_F} b,
\]

(10)

\[
\theta_{n,\delta} = \arctan \left( \frac{\tan \phi'_F - \tan \phi'_F}{\tan \phi'_F + \tan \phi'_F} \right) - \tan \phi'_F.
\]

(11)

where \( \theta_{n,\delta} \) is the angle of the sliding destruction wedge surface.

For the convenience of determining the parameter of increasing the ultimate resistance force with respect to mechanical characteristics of horizontal geosynthetic elements \( F_u^{\text{arm. mech.}} \) and taking into account a variable reinforcement pitch, nomograms for the values of the geosynthetic material stiffness of \( G = 100, 500, 1000 \text{ kN/m} \) and the friction coefficients \( f_u = 0.3; 0.6; 0.9 \) were obtained. An example of a nomogram with the stiffness value of the geosynthetic material \( G = 500 \text{ kN/m} \) is shown in figure 6.
Figure 6. Nomogram for determining the increase in the ultimate resistance force with respect to reinforcement application at $G = 500$ kN/m and $f_{fr} = 0.3\,; 0.6\,; 0.9$

To determine the parameter of increasing the ultimate resistance force with respect to the mechanical characteristics of horizontal geosynthetic materials, it is necessary to know the thickness of the sand pad (the number of reinforcing layers) and the mechanical parameters of the reinforcing geosynthetics. Intermediate values can be determined by linear interpolation.

5. Conclusions
1. The numerical simulation methods implemented in the Plaxis 2D PC reflected with sufficient accuracy the behaviour of reinforced sand pads under load when using the elastic-plastic Mohr-Coulomb model and under conditions of plane deformation.
2. Geosynthetic materials with stiffness values in the range of 100-1000 kN/m were the most effective as reinforcing elements. A further increase in the stiffness increased the base ultimate resistance force by 2 %, on average.
3. The best results were demonstrated by sand as the pad filler with an internal friction angle in the range from 30 ° to 34°. The increase in the internal friction angle of the pad filler soil higher than 34 ° caused a slight increase in the base ultimate resistance force.
4. Based on the results of numerical modeling, a refined formula was proposed for determining the base ultimate resistance force with a sand reinforced pad, which allows us to take into account that the ultimate sliding surface goes through a sand pad with higher strength parameters, on the one hand, and through a weak soil base, on the other.

References
[1] Kleveko V I 2014 Investigation of reinforced clay bases work PNRPU Bulletin. Construction and Architecture 4 101–10
[2] Tatiannikov D A and Ponomaryov A B 2017 In-situ stamp testing of reinforced foundation pads PNRPU Bulletin. Construction and architecture 8 (3) 97–105
[3] Timofeeva L M 1991 Reinforcement of soils (theory and practice of application) (Perm: Perm Polytechnic Institute)
[4] Nuzhdin L V and Kuznetsov A A 2000 Reinforcement of foundation soils with vertical rods Proc. of the Int. Seminar on Soil Mechanics, Foundation Construction and Transport Structures pp 204–6
[5] Ter-Martirosyan Z G 2005 Soil Mechanics (Moscow: ASV)
[6] Glushkov A V 2016 *Influence of the Foundation Sole Shapes and Sizes on the Base Stress-Strain State* (Tyumen: Tyumen State Architecture and Construction Academy)

[7] Tsytovich N A 1963 *Soil Mechanics* (Moscow: Goststroizdat)

[8] Shenkman R I and Ponomaryov A B 2013 Selection of geosynthetic shell for soil piles and the effectiveness of their application in the geological conditions of Perm *Bulletin of Civil Engineers* 1 (36) 82–9

[9] Tatiannikov D A and Ponomaryov A B 2019 Forecast bearing capacity of soil cushions with variable reinforcement spacing *Proc. of the Conf. on Geotechnics Fundamentals and Applications in Construction: New Materials, Structures, Technologies and Calculations* (St. Petersburg) pp 378–83

[10] Mirsayapov I T and Popov A O 2008 Experimental and theoretical investigations of reinforced soil mass work *Izvestiya KGASU* 2(10) 75–80

[11] Mangushev R A et al 2012 Methods of artificial bases preparation and arrangement (St. Petersburg: ASV)

[12] Tsytovich N A, Berezantsev V G, Dalmatov B I and Abelev M Yu 1970 Bases and foundations (Moscow: Higher school)

[13] Bai V F and Kraev A N 2014 Investigation of a sand-reinforced cushion work with a curved bottom in weak clay soils *Bulletin of Civil Engineers* 3 107–11

[14] Ponomaryov A B and Tatiyannikov D A 2020 Analysis of the performance of sand cushions reinforced with horizontal geosynthetic elements *Soil Mechanics and Foundation Engineering* 6 371–7

[15] Ofrichter V G, Ponomaryov A B, Kleveko V I and Reshetnikova K V 2010 Methods of reinforced soil structures construction (Perm: Publishing House of Perm State Technical University)