Conformally invariant Inert Higgs doublet model: an unified model for Inflation and Dark matter

Moumita Das\textsuperscript{1} and Subhendra Mohanty\textsuperscript{2}

Physical Research Laboratory, Ahmedabad 380009, India

E-mail: \textsuperscript{1}moumita@prl.res.in, \textsuperscript{2}mohanty@prl.res.in

Abstract. Motivation of our present study is the searching for an unified model which can describe both the inflation as well as dark matter. From particle physics point of view, Higgs can be the most interesting candidate for the scalar field inflation. Conformal coupling of the inflaton with the gravity can generate the density perturbation and we use this idea in a realistic inert Higgs doublet model. We study the loop corrections of this conformally coupled system and in present era there is electroweak symmetry breaking to provide the mass of the particles. Study of the mass spectrum in present era reveals the scalar dark matter with mass 33.7 GeV and lightest Higgs at 125.6 GeV.

1. Introduction

Scalar field inflation is most successful model of inflation. Higgs can be the most favorable candidate for this kind of model. Introduction of Standard Model Higgs as inflaton was done by F. Bezrukov and M. Shaposhnikov [1]. Generating inflation from the potential of the form $\lambda \phi^4$ demands that $\lambda$ should be $O(10^{-12})$. For standard model Higgs, $\lambda$ is approximately $\sim 1$ and the Higgs can not be used as inflaton. They solved this issue by considering the large coupling ($\xi \sim 10^4$) between Ricci scalar and the Higgs [1]. The problem of unitarity of graviton-scalar scattering [2] causes due to the presence of large coupling $\xi$. However there are some attempts [3] present in the literature to solve this issue.

Rubakov and collaborators have shown different approach for the generation of scale invariant density perturbations [4]. The idea was that a conformally coupled field rolling down a quartic potential can generate scale invariant density perturbation. These perturbations can become superhorizon in an inflationary era or in a ekpyrotic scenario. We implement this idea in a realistic inert Higgs doublet model. The study of electroweak symmetry generate the mass spectrum in present era. The requirement of scale invariance at high energy scale and electroweak symmetry breaking at low energies fixes the coupling constants of the theory. We find that there is a set of benchmark parameters of this model where the mass of the Higgs boson is $m_h = 125.6$ GeV and the mass of the dark matter (which is the lightest neutral component of the inert doublet) $m_{A_0} = 33.7$ GeV, which can be tested at the LHC and in direct detection experiments. The Higgs mass is consistent with the $2.8\sigma$ signal of a 125 GeV higgs decay into $\gamma \gamma$ and $4\ell$ observed by the ATLAS [5] and CMS [6] collaborations.
2. Inert Doublet Model and Coleman-Weinberg loop correction

Inert Doublet Model (IDM) respect the $Z_2$ symmetry, under which all Standard model particles including the SM Higgs $H_1$ are even and an extra scalar doublet $H_2$ is odd. Due to $Z_2$ symmetry, the cubic term and Yukawa term for $H_2$ doublet are forbidden and its neutral component can be a candidate for dark matter. The most general renormalizable potential will be,

$$V_{\text{tree}} = V_c + \mu_1^2 |H_1|^2 + \mu_2^2 |H_2|^2 + \lambda_1 |H_1|^4 + \lambda_2 |H_2|^4 + \lambda_3 |H_1|^2 |H_2|^2 + \lambda_4 |H_1^\dagger H_2|^2 + \frac{\lambda_5}{2} \left[ (H_1^\dagger H_2)^2 + \text{h.c.} \right]$$

(1)

where the two Higgs doublets $H_1$ and $H_2$ are, $H_1 = \left( \frac{h^+}{\sqrt{2}} \right)$ and $H_2 = \left( \frac{H_0 + iA_0}{\sqrt{2}} \right)$

We consider the conformal case where $\mu_1 = \mu_2 = 0$ and the constant potential, $V_c = 4 \times 10^7 \text{ GeV}^4$ such that the minimum of the total potential becomes zero at present era. We derive the one-loop correction to the potential (1) following Coleman-Weinberg formalism [7] and the effective potential becomes,

$$V_{\text{eff}}(H_0, h, \mu) = V_{\text{tree}} + \frac{1}{64\pi^2} \sum_i n_i m_i^4 \ln \left( \frac{m_i^2}{\mu^2} + 1 \right)$$

(2)

where $n_i$ is the degree of freedom and $m_i$ are tree level masses. In this case we have also considered the loop corrections from the top quark and gauge bosons. However the gauge boson loop has negligible effect on the results.

To get the correct electro-weak symmetry breaking in the present era and the scale invariant density perturbation in the early era, we have chosen a set of $\lambda_i = \{-0.14, -11, 2.8, -1.52, -1.52\}$ in present epoch. Now we have to study the running of couplings $\lambda_i$, where $\{i = 1 \text{ to } 5\}$ using the one-loop renormalization group equation for the inert doublet model [8] and we find the $\lambda_i = \{0.49, -0.5, 2.1, 0.84, -2.2\}$ in the early era, $\mu \approx 10^5 \text{ GeV}$.

3. Generation of the scale invariant density perturbation

We now turn to the question of the generation of density perturbations in the early era when $V_{\text{eff}}$ (2) simplifies to the form $V_{\text{inf}} \sim V_c + \frac{\lambda_2}{2} H_0^4$ shown in Fig. (1), where $V_c = 4 \times 10^7 \text{ GeV}^4$ and $\lambda_2 = -0.5$. We take the inert Higgs doublet to be conformally coupled to gravity and the

![Figure 1. Effective potential in the early universe](image-url)
action for this field can be written as,

\[ S = \int d^4x \sqrt{-g} \left[ g^{\mu\nu} \partial_\mu H_2 \partial_\nu H_2 - \frac{R}{6} H_2^2 - V_{inf} \right] \]  

where \( H_2 \) contains the neutral part of the inert doublet i.e. \( H_2 = (H_0 + i A_0) / \sqrt{2} \) and \( R \) is the scalar curvature, which conformally coupled with the field \( H_2 \). Defining \( H_2 = \chi H_2 / a \), we can simplify the equation of the field \( H_2 \) as

\[ \dot{\chi}''_{H_2} + k^2 \chi H_2 + a^3 \frac{\partial V_{inf}}{\partial H_2} = 0 \]  

(4)

The conserved current will be of the form \( \frac{d}{d\eta}(\rho \theta') = 0 \), after expressing, \( \chi_{H_2} = \rho \exp(i \theta) \). Hence, the field rolls along the radial direction while the phase \( \theta \) remains constant with the increase of \( \rho \). Without loss of generality we can choose the fixed phase such that the field \( \chi_{H_2} \) has only real and neutral component \( \chi_{H_0} \). The perturbations of \( H_2 \) will be along the imaginary axis and we can denote the full \( H_2 \) with the perturbations as, \( \chi_{H_2} = \chi_{H_0} + i \delta \chi_{A_0} \). From Eq (4), the equation of motion of \( \chi_{H_0} \) will be,

\[ \chi_{H_0}'' k^2 \chi_{H_0} - \frac{\lambda_2}{2} \chi_{H_0}^3 = 0 \]  

(5)

Considering \( k \ll 1/\eta \) at late time, the solution will be, \( \chi_{H_0} \approx 1 / (\sqrt{-\lambda_2}(\eta - \eta_0)) \), where \( \sqrt{-\lambda_2} \) is a real quantity as \( \lambda_2 \) is negative and \( \eta_0 \) is a constant of integration. At the end of inflation, when \( \mu \ll 10^5 \), the shape of the potential changes to Fig. (2) and \( H_0 \) starts rolling back to zero.

The equation of motion of the perturbation, \( \delta \chi_{A_0} \) is given by,

\[ \delta \chi_{A_0}'' + k^2 \delta \chi_{A_0} + \frac{\lambda_2}{2} \chi_{H_0}^2 \chi_{A_0} = 0 \]  

(6)

and at later times, when \( (k(\eta_0 - \eta) \ll 1) \), solution will be \( \delta \chi_{A_0} \sim 1 / \left( k^{3/2}(\eta_0 - \eta) \right) \). Hence, the super-horizon perturbations of the phase can be defined as \( \delta \theta = \delta \chi_{A_0} / \chi_{H_0} \). The power spectrum of \( \delta \theta \) is scale invariant as follows,

\[ P_{\delta \theta} = \frac{k^3}{2\pi^2} |\delta \theta|^2 = -\frac{\lambda_2}{2\pi^2} \text{ where } \delta \theta = \frac{\sqrt{-\lambda_2}}{k^{3/2}} \]  

(7)

If one considers the \( k \) dependence of the equation of motion of \( H_0 \) as discussed in [9], there will be a deviation from the scale free power spectrum (7) and it will give rise to a non-zero spectral index, \( n_s - 1 = \frac{3\lambda_2}{4k^2} = -0.04 \), which is consistent with the WMAP observation of \( n_s = 0.967 \pm 0.014 \) [10]. The perturbations of the phase \( \delta \theta = \delta A_0 / H_0 \) can be converted to adiabatic perturbation by the decay of the \( A_0 \) field into standard model fields as in the curvaton mechanism [11]. The amplitude of adiabatic perturbation is related to the phase perturbation as

\[ P_\zeta = r^2 P_{\delta \theta} \]  

where \( r \) is the ratio of the energy density in the \( A_0 \) field oscillations to the total energy density. Taking the unperturbed phase to be \( \theta_c \sim \pi/2 \), and with \( \lambda_2 = -0.5 \), we see that \( r = 2 \times 10^{-4} \) is needed to give the required \( P_\zeta = 10^{-10} \).
4. Scalar mass spectrum
As the field $H_2$ has a zero vev in the present universe, the lightest neutral components of $H_2$ will be stable and can be the candidates for dark matter. We study the masses of the fields in present universe from the effective potential taking $<H_1>=246\text{ GeV}$ and $<H_2>=0\text{ GeV}$. We see that the field $A_0$ can be a candidate for light dark matter with mass $M_{A0}=33.7$ and the predicted Higgs mass is $M_h=125.6\text{ GeV}$ which matches with the current observation of Atlas [5] and CMS [6] collaborations. The mass of the other scalars $H^0$ and $H^\pm$ are 273.6 and 433.5 respectively.

5. Conclusions
The inert Higgs doublet model gives a natural extension of the standard model and can be used for explaining the electroweak symmetry breaking by loop corrections [12] starting from a scale invariant tree level potential. We find that the the quartic coupling of the inert doublet, $\lambda_2=-0.5$ at $\mu=10^5\text{ GeV}$, predicts correct spectral index of the power spectrum of the perturbations. The amplitude of the power spectrum $P_\zeta$ also can be tuned to be consistent with the observations by choosing a suitable curvaton mechanism. Finally our model works for the chosen parameters $\lambda_i$ to give the Higgs mass at $m_h=125.6\text{ GeV}$ matches with the observations of ATLAS [5] and CMS [6]. The same set of $\lambda_i$ also offers a candidate for dark matter with a mass $M_{A0}=33.7\text{ GeV}$ which may possibly be observed in direct detection experiment.

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