Superstrings from Supergravity

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Abstract

We study the origins of the five ten-dimensional “matrix superstring” theories, supplementing old results with new ones, and find that they all fit into a unified framework. In all cases the matrix definition of the string in the limit of vanishingly small coupling is a trivial 1+1 dimensional infra-red fixed point (an orbifold conformal field theory) characterized uniquely by matrix versions of the appropriate Green–Schwarz action. The Fock space of the matrix string is built out of winding T–dual strings. There is an associated dual supergravity description in terms of the near horizon geometry of the fundamental string solution of those T–dual strings. The singularity at their core is related to the orbifold target space in the matrix theory. At intermediate coupling, for the IIB and SO(32) systems, the matrix string description is in terms of non–trivial 2+1 dimensional fixed points. Their supergravity duals involve Anti de–Sitter space (or an orbifold thereof) and are well–defined everywhere, providing a complete description of the fixed point theory. In the case of the type IIB system, the two extra organizational dimensions normally found in F–theory appear here as well. The fact that they are non–dynamical has a natural interpretation in terms of holography.
1. Introduction and Summary

1.1. Matrix Motivations

One of the recent satisfying products of the duality industry of the last three years has been a significant rephrasing of the properties of string theory. At the very least, we have better understanding of how to characterize many non-perturbative statements about string theory, usually using duality to another theory. Although we have no proof of duality (in the traditional sense), recasting it in terms of being a symmetry of a (yet to be fully specified) parent theory, called “M–theory”, has been shown to be an extremely economical and powerful way to proceed.

A partial specification of M–theory has been given[1] in terms of “Matrix theory”, which captures the physics of certain degrees of freedom of the theory infinitely boosted in one spatial direction. While this infinite momentum frame (IMF) definition of the theory is certainly not the whole story, it has certainly been shown to be robust, surviving many important tests.

One of these tests is simply to understand whether the various duality symmetries of string theory can be recovered upon taking suitable limits. Most of these limits involve compactifying the matrix theory on manifolds and taking geometrical limits of these manifolds. For compactifications to above five dimensions on manifolds breaking no more than one half of the 32 supersymmetries, there has been considerable success[2] in reproducing the known string theories (and some partial success in the case where a quarter is broken[3,4]) and their duality properties.

To be fair, given that the definition of matrix theory involves some of the vital ingredients with which we phrase duality at the outset, we should not be completely surprised to find such a success. There are precise arguments[5,6] which explain the successes of these compactifications (and point to their failure below six dimensions also), using precisely that fact. Nevertheless, progress has occurred, because we have been able to restate the duality results in terms which may generalize beyond the situations in which we originally discovered them. The understanding of the relevance of holography; a simple supersymmetric quantum mechanical statement of the onset of non–commutative geometry at short distance; and the rephrasing of compactified matrix theory and the resulting string dualities in terms of properties of field theories, are all elements of this progress.

Let us turn to the issue of string theory in ten dimensions. We obtain them from some of the simplest compactifications of M–theory, and correspondingly, we should get IMF definitions of string theories by analogous simple compactifications of the matrix theory. These definitions are called “matrix string theories” for obvious reasons.

Although we do not expect any surprises here, there is much to be gained in this exercise. While obtaining an alternative definition of weakly coupled string theories from this procedure, (which may or may not be more useful than the original weakly coupled definitions),
we also have a natural extension of that definition to the theory at not only very strong coupling, but intermediate coupling as well. (By contrast, earlier attempts at extending the more standard perturbative string definition beyond weak coupling were not nearly as successful.) In fact, we will see that sometimes the description at intermediate coupling is in some sense the most natural region of coupling space to which the matrix definition of the string has access. Given that the original duality statements about the structure of string theory away from weak coupling concerned very strong coupling, this is also a bonus of the matrix definition.

Much of the language of the matrix string theory\cite{7} technology has been developed in the context of the type IIA string\cite{8,9} (with extensions to the $E_8 \times E_8$ heterotic string\cite{10,11,12,14}), and less explicitly for the type IIB string. At weak coupling they are defined in terms of trivial orbifold fixed point theories in 1+1 dimensions, and the free string Fock space has a description in terms of winding strings comprising the twisted sectors of the orbifold theory. The type IIB string away from weak coupling has also been described in terms of a fixed point\cite{15,8}, this time a 2+1 dimensional interacting one. This is natural, as it really describes the string at intermediate coupling where the type IIB strings are interacting.

We shall begin by reviewing and refining how the matrix strings arise from the original matrix theory definition. We will then extend the discussion to the remaining string theories (i.e., the $SO(32)$ system), providing a complete description of all five matrix string theories in ten dimensions.

1.2. Summary of Results

- We observe that the matrix strings at weak coupling are all defined in terms of 1+1 dimensional fixed points similar to the original type IIA matrix string theory. (This was already observed for the $E_8 \times E_8$ heterotic string.) The basic structure is simply that the Fock space of the free string is made up of winding strings of a species which is T–dual to the string in question. While this was known for the type IIA and $E_8 \times E_8$ matrix strings, we see that it extends to all of the string theories, by simply following the limits implied by duality and matrix theory. The 1+1 dimensional theories are orbifold conformal field theories.

- The orbifold conformal field theories may each\footnote{Here, our results differ from those presented in ref.\cite{16} for the $SO(32)$ system. We thank T. Banks for pointing out that paper to us after reading an earlier version of this manuscript.} be characterized as the large $N$ limit of a 1+1 dimensional effective theory defined by a Lagrangian which is of the (matrix) Green–Schwarz form for the matrix string in question. The constituent fields are $N \times N$ matrices.

- The limits which define the matrix string theories also define certain supergravity back-
grounds, which can be interpreted as “dual” descriptions in the sense of ref.\[17\]. In the free string limits the supergravity dual is simply the near horizon geometry of a fundamental string in the supergravity associated to the T–dual species of string. The infra–red limit of the 1+1 dimensional field theory defining the free matrix string is associated with the center of the fundamental string solution. This “dual” is therefore not a good description at the core of the string configuration, as it breaks down due to strong curvature corrections precisely at this point, i.e., at the infra–red limit\[2\]. This behaviour is expected from string duality, as will be discussed.

• Although the singularity in the supergravity prevents us from using it as a complete dual definition of the 1+1 dimensional fixed point, this is not a problem, as the orbifold description is simple enough to characterize without further appeal to such a dual. Nevertheless, the supergravity description serves to organize and inform us about the structure of the matrix definition of the weakly coupled string, helping to lead to the description of the free string limit given above.

• Moving away from the weakly coupled limits of the matrix strings, we find that the supergravity description is smooth. Especially in the cases which involve ten dimensional string/string duality (the type IIB and the $SO(32)$ type IB/heterotic pair), there is a complete and concise supergravity dual description of the space in terms of $AdS_4 \times S^7$ for the first case, and $(AdS_4/\mathbb{Z}_2) \times S^7$ for the second case. The latter defines a novel 2+1 dimensional fixed point theory with broken Lorentz invariance. (Such fixed points were conjectured to exist in ref.\[19\]. This $AdS_4/\mathbb{Z}_2$ description is a concrete proposal for their study.)

In the cases of the type IIA and $E_8 \times E_8$ heterotic string cases, the intermediate and strong coupling situations are best described in terms of the original 0+1 dimensional matrix system.

• We notice also that the organizing two extra hidden dimensions of ten dimensional type IIB string theory, which play a role in F–theory, appear here in describing the type IIB matrix string at intermediate coupling. That they are non–dynamical (but of course still important) is seen here to be a consequence of the holographic nature of the AdS/CFT correspondence of ref.\[17\]. So of the apparent twelve dimensions with signature (10,2) involved in defining non–perturbative type IIB, a pair of dimensions with signature (1,1) have no dynamics associated to their size.

The outcome of this investigation is thus a comprehensive characterization of all of the ten dimensional string theories at all values of their coupling, in terms of 1+1 and 2+1 dimensional field theories and quantum mechanics. We find that a supergravity solution is sometimes a complete dual description, and in all cases they highlight some of the key

\[2\] This connection was made for the type IIA system in ref.\[18\]. Here, we point out that this behaviour is natural and necessary.
features of the matrix theory. This framework is appealing\(^3\), and even though it mainly reproduces much that we already know (namely, ten dimensional strings and their duals) it may serve as a vital starting point for defining string theories where we do not have the usual tools available in ten dimensions. With this in mind, we close the paper in section 6.2 with some detailed preliminary remarks concerning such applications.

2. The case of Type IIB

The matrix theory definition of M–theory in the infinite momentum frame (IMF) is given by\(^1\) the \(N=16\) supersymmetric \(U(N)\) quantum mechanics arising from \(N\) coincident D0–branes’ world–volume, in the limit \(\ell_s\to0\) and \(N\to\infty\). The special longitudinal direction, \(x^{10}\), (initially compactified on a circle of radius \(R_{10}\)), is decompactified in the limit also. The type IIA string theory used to define this theory has parameters:

\[
g_{\text{IIA}} = R_{10}^{3/2} R_p^{-3/2}; \quad \ell_s = \ell_p^{3/2} R_{10}^{-1/2},
\]

(2.1)

where \(\ell_s\) is the string length and \(\ell_p\) is the eleven dimensional Planck length.

Consider the matrix definition of (IMF) M–theory compactified on a torus in the directions \(x^8, x^9\). When the torus is small, we should have a description of the type IIB string theory\(^{21,22}\) in the light cone gauge\(^{15,8}\). T–duality from the D0–brane system succinctly gives the definition in terms of \(N\) D2–branes, on whose world–volume there lives 2+1 dimensional \(N=8\) supersymmetric \(U(N)\) Yang–Mills theory. Representing the torus by a pair of circles of radius \(R_9\) and \(R_8\), respectively, the Yang–Mills coupling is computed as:

\[
\frac{1}{g_{\text{YM}}^2} = \frac{\ell_s}{\tilde{g}_{\text{IIA}}} = \frac{R_8 R_9}{R_{10}}.
\]

(2.2)

Here, \(\tilde{g}_{\text{IIA}}\) is the \(T_{89}\)–dual type IIA string coupling, and the D2–branes are wrapped on a dual torus (in directions \(\hat{x}^8, \hat{x}^9\)) of size \(\hat{R}_8 = \ell_s^2 / R_8\) and \(\hat{R}_9 = \ell_s^2 / R_9\).

In the limit where the torus shrinks away (\(R_8, R_9\to0\), with \(N\to\infty\), the dual torus decompactifies, and the strongly coupled \(U(N)\) Yang–Mills theory flows\(^{15,8,23}\) to a non–trivial superconformal infra–red fixed point with an \(SO(8)\) R–symmetry. This \(SO(8)\) is the manifestation\(^{15,8}\) of the spacetime Lorentz symmetry of the lightcone theory thus defined — the type IIB string theory. The coordinates of the ten dimensions are the manifest \(x^1–x^7\) and a new dimension \(\hat{x}^{10}\), in which the \(SO(8)\) acts, while the direction which goes with time \(x^0\) to define the light–cone or IMF (8) acts is a linear combination of \(\hat{x}^8\) and \(\hat{x}^9\), set by the ratio \(R_9/R_0 = g_{\text{IIB}}\), the (matrix) type IIB string coupling.

\(^3\) A description of the relationships among all the strings and their dual theories in terms of field theory fixed points was anticipated quite a while ago in section 5 of ref.\(^{20}\).
2.1. The Role of Eleven Dimensional Supergravity

Notice that equation (2.2) also tells us that in the limit, the T–dual type IIA coupling $\tilde{g}_{\text{IIA}}$ is also infinite (also, $\ell_s \to 0$), and we should be working in eleven dimensional supergravity, with $N$ M2–branes extended in $\hat{x}^8$ and $\hat{x}^9$. In the large $N$ limit, the branes produce a non–trivial gravitational effect on the spacetime in which they are embedded, and this is summarized neatly in terms of the M2–brane supergravity solution. Writing the supergravity solution in the large $N$ limit in terms of $U = r/\ell_s^2$, (where $r$ is the transverse distance from the core of the brane configuration), defining the characteristic energy scale of the gauge theory[17], we may study the renormalization group flow of the theory by moving in the “near horizon” spacetime created by the brane configuration.

In the strongly coupled limit (i.e., the infra–red, $U = 0$), with $N \to \infty$, the complete description is in terms of eleven dimensional supergravity compactified on $\text{AdS}_4 \times S^7$, (with appropriate choices for the three form potential) where the $S^7$ has a fixed radius defined in terms of the radius of the $\text{AdS}_4$.

This supergravity compatification is conjectured[17] to be a complete description of the 2+1 dimensional infra–red fixed point, because of the following features:

- The curvatures of the compactification are small everywhere, and thus supergravity is well–defined.
- The isometries of $\text{AdS}_4$ form the group $SO(3, 2)$, which coincides with the superconformal group of the fixed point.
- The isometries of the $S^7$, the group $SO(8)$, give rise to a Kaluza–Klein gauge symmetry in the $\text{AdS}_4$ spacetime. This in turn gives rise to a global $SO(8)$ R–symmetry of the fixed point theory on the boundary.

The brane construction suggests that the 2+1 dimensional theory living on the boundary of $\text{AdS}_4$ is the fixed point theory. This AdS/CFT correspondence is “holographic” in the sense that the physical degrees of freedom of the AdS supergravity can be described by the theory living on the boundary. This correspondence was made more precise in refs.[24] where a precise dictionary between the supergravity/conformal field theory description was suggested. They gave a precise prescription for the relation between insertions of operators in the boundary conformal field theory and supergravity modes in the bulk. In the case in hand, many entries in the dictionary were verified explicitly in ref.[25].

The matrix definition of the non–perturbative (matrix) type IIB string theory may therefore be regarded as having a dual supergravity description.

We now digress slightly and briefly, to make remarks concerning a connection to another description of the non–perturbative type IIB string.
2.2. Holography and F–Theory

The strength of the type IIB coupling is determined by the shape of the torus. The complete complex type IIB coupling is given in terms of the modular parameter of the torus:

\[ \tau = A^{(0)} + ie^{-\Phi} = A^{(0)} + \frac{i}{g_{\text{IIB}}}, \quad (2.3) \]

where \( A^{(0)} \) is the Ramond–Ramond scalar and \( \Phi \) is the dilaton. In the case in hand, we have \( A^{(0)}=0 \), and \( g_{\text{IIB}}=R_9/R_8 \).

The situation just described above assumed that we had treated both directions \( x^8 \) and \( x^9 \) on the same footing, and so we took \( R_8, R_9 \to 0 \) holding fixed the ratio \( g_{\text{IIB}}=R_9/R_8=1 \), i.e., \( \tau=i \). So the \( \text{AdS}_4 \times S^7 \) limit defines type IIB at the strong/weak coupling self–dual point.

This description of the type IIB string at intermediate coupling is defined globally everywhere, up to an overall \( SL(2, \mathbb{Z}) \) transformation. In general, \( \tau \), the complex structure data of the torus may vary from place to place in the ten dimensional spacetime of the type IIB string, giving a description where the string coupling \( \lambda_{\text{IIB}} \) varies, with variations in \( A^{(0)} \) signaling the presence of D7–branes and O7–planes and their Hodge duals. This is the point of departure for the F–theory description, which describes such vacua of the type IIB string in terms of compactifications on elliptically fibred Calabi–Yau manifolds of a (naively) 12 dimensional theory. Non–perturbative type IIB string theory therefore seems to involve twelve dimensions.

The extra two dimensions of F–theory are, from many points of view, not on the same footing as the other ten of the type IIB theory, however, as they have no independent dynamics associated with their size. The torus of the extra two dimensions is the memory of the complex structure of the torus which was shrunken away in coming from M–theory to type IIB. The signature of the extra two–space is apparently (1,1), giving a complete 12 dimensional spacetime with signature (10,2). The extra two dimensions are regarded as serving an organisational role in the type IIB theory.

As we are describing type IIB non–perturbatively here, we might hope to see a sign of these extra dimensions and indeed we do: The \( \text{SO}(3, 2) \) isometry of the \( \text{AdS}_4 \) space, which becomes the superconformal group of the 2+1 dimensional theory defining the matrix type IIB theory is the Lorentz group of flat space with signature (3, 2). This is the natural space in which \( \text{AdS}_4 \) is defined as a hyperbolic submanifold. Taking the 2+1 dimensional theory as an auxiliary theory, describing the IIB theory in 9+1 dimensions, leaves a two dimensional space with signature (1,1) left over. When combined with the type IIB’s space

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4 Our description here may be thought of as focussing on a local piece of such a general type IIB background, at intermediate coupling.
gives a twelve dimensional spacetime\footnote{Another way to count would be to simply regard the defining $AdS_4 \times S^7$ eleven dimensional supergravity compactification as intrinsically using a $(10,2)$ space with $(1,1)$ holographed away in the construction of matrix type IIB.} of signature $(10,2)$.

The space with signature $(1,1)$ is again non–dynamical, and now we see why: The extra time–like direction is part of the embedding space defining the $AdS_4$, while the extra spatial one is “projected out” by the AdS/CFT holographic relationship.

A matrix definition of the weakly coupled type IIB string can be found by taking the $R_8, R_9 \to 0$ limit, but keeping $g_{\text{IIB}}=R_9/R_8<<1$. This will define a 1+1 dimensional fixed point theory very similar to that which defines the type IIA case. We will describe that theory first, and return to the weakly coupled type IIB string at the end of the next section.

Let us now turn to the case of type IIA.

3. The case of Type IIA

The matrix definition of IMF type IIA string theory arises from that of the IMF M–theory definition in a way similar to above\footnote[27]{\textsuperscript{27}}. Compactifying on a circle of radius $R_9$ results in $N$ coincident D1–branes in type IIB string theory, which have a 1+1 dimensional $U(N)$ with $N = (8, 8)$ symmetry. The gauge coupling is computed to be:

$$\frac{1}{g_{\text{YM}}^2} = \frac{\ell_s^2}{g_{\text{IIB}}^2} = \ell_s^2 \frac{R_9}{R_{10}} = \tilde{\ell}_s^2 g_{\text{IIB}}^2. \quad (3.1)$$

(Here, $\tilde{\ell}_s$ is the S–dual type IIB string length and $g_{\text{IIB}}$ is its coupling.) This theory has an $SO(8)$ R–symmetry, which are the manifest rotations of the spacetime transverse to the branes.

The theory has a definition in terms of a “matrix Green–Schwarz” action for the type IIA string:\footnote[9]{\textsuperscript{9}}

$$S = \frac{1}{2\pi} \int d^2\sigma \text{Tr} \left( (D_\mu X^i)^2 + \theta^T \gamma^\mu D_\mu \theta + \tilde{g}_{\text{IIA}}^2 F_{\mu\nu}^2 - \frac{1}{\tilde{g}_{\text{IIA}}^2} [X^i, X^j]^2 + \frac{1}{\tilde{g}_{\text{IIA}}} \theta^T \Gamma_i [X^i, \theta] \right). \quad (3.2)$$

The dimensionless coupling $\tilde{g}_{\text{IIA}}$ is the matrix type IIA string coupling, equal to $R_9/\ell_s$. The $X^i$ are eight scalar fields, and $\theta$ contains two fermionic fields $\theta_L^a$ and $\theta_R^\alpha$ which respectively transform in the $\mathbf{8}_v$, $\mathbf{8}_s$ and $\mathbf{8}_c$ (vector, spinor and conjugate spinor) representations of the $SO(8)$. They are all $N \times N$ hermitian matrices. The world–volume coordinates are $\sigma^0, \sigma^1$, which are identified with the (rescaled) spacetime directions $x^0/\tilde{R}_9, x^9/\tilde{R}_9$, so that $0 \leq \sigma \leq 2\pi$. The direction $\hat{x}^9$ is a circle of radius $\tilde{R}_9=\ell_s^2/R_9$. 

\footnote[8]{\textsuperscript{8} Another way to count would be to simply regard the defining $AdS_4 \times S^7$ eleven dimensional supergravity compactification as intrinsically using a $(10,2)$ space with $(1,1)$ holographed away in the construction of matrix type IIB.}
In the limit $R_9 \to 0$, $N \to \infty$ defining the matrix type IIA string, the theory flows to an infra-red fixed point, which defines the “matrix type IIA string” with coupling $\tilde{g}_{\text{IIA}} = R_9/\ell_s$. The theory is a trivial orbifold conformal field theory, based on the sigma model with target space $(R^8)^N/S_N$, where $S_N$ is the group of permutations of $N$ identical objects (the D1–branes themselves). The correspondence works roughly as follows:

The $g_{\text{YM}} \to \infty$ long distance limit, defining the infra–red theory has been shown to correspond to the type IIA string theory, where the finite length type IIA strings arise from the twisted sectors of the orbifold. The $X^i$ represent the matrix coordinates of the D1–branes. In the strong coupling limit, lowest energy configurations are obtained when the matrices commute, and may be simultaneously diagonalized, up to the action of the Weyl group, which permutes the eigenvalues of the matrices along the diagonal.

As one goes once around the world sheet’s spatial direction $\sigma^1 = \hat{x}_9/\hat{R}_9$, one can come back to the same configuration up to a permutation of the eigenvalues. One can build up a closed string of length $n$ by acting with a permutation involving $n$ different eigenvalues as one goes around $\sigma^1$, requiring $n$ jumps (windings) of length 1 to return to the starting eigenvalue. This defines a matrix type IIA string with momentum $P_9 = n/R_9$. Long strings which survive the limit are those with $n/N$ finite as $N \to \infty$.

Notice that the strings which wind to build up the Fock space of the string in the free limit are actually type IIB strings. One way to see this is to notice that the matrix coordinates $X^i$ start out initially as D1–brane positions, and so those are the strings which wind, as is manifest from the lagrangian (3.2). But the weakly coupled matrix type IIA string occurs when the type IIB string coupling is infinite, and so our winding strings are really the S–dual fundamental type IIB strings. The supergravity description will make this explicit too.

In order to describe interactions in the theory, a twist operator has to be turned on in the theory, which exchanges eigenvalues at a given point, thus creating the splitting/joining interaction of the strings.

This interaction vertex is identified with the $\mathbb{Z}_2$ twist operator of the conformal field theory. It was shown in ref.[9] to correspond to the type IIA string vertex. It is the leading irrelevant operator in the theory and therefore in order to describe the interaction of the strings, one has to move away from the infra–red limit. It is also worth noting that as its identification as an irrelevant operator is consistent with the fact that the target spacetime singularity $\mathbb{R}^8/\mathbb{Z}_2$ is not able to be smoothly resolved by switching it on.

3.1. The Role of Type IIB Supergravity

As the theory involves a large number ($N$) of D1–branes, in the limit $\ell_s \to 0$, we may also consider the supergravity fields created by them, in an analogous fashion to the case of defining the type IIB string in the previous section.
Again, $U = r/\ell_s^2$ defines an energy scale in the theory, and the solution may be rewritten in terms of this coordinate. We may study the renomalization group flow of the theory in these terms. The $N$ D1–brane solution was written in these coordinates in ref. [18]. Its behaviour is (neglecting many constants for clarity):

$$
\begin{align*}
\text{ds}^2 &= \ell_s^2 \left( \frac{U^3}{g_{YM}N} (dx_0^2 + dx_1^2) + \frac{g_{YM}\sqrt{N}}{U^3} dU^2 + g_{YM} \frac{\sqrt{N}}{U} d\Omega_7^2 \right) \\
\text{e}^{\Phi} &= \left( \frac{g_{YM}^6 N}{U^6} \right)^{\frac{1}{2}}.
\end{align*}
$$

In the low energy limit ($U \to 0$), we approach the core of the configuration where we see that the dilaton (and hence the type IIB string coupling) grows large, infinite in the limit. This is consistent with the field theory analysis above. We use S–duality to transform to a solution where the coupling is small in this region, giving the fundamental string solution:

$$
\begin{align*}
\text{ds}^2 &= \ell_s^2 \left( \frac{U^6}{g_{YM}^4 N} (dx_0^2 + dx_1^2) + \frac{1}{g_{YM}^2} dU^2 + \frac{U^2}{g_{YM}^2} d\Omega_7^2 \right) \\
\text{e}^{\Phi} &= \left( \frac{g_{YM}^6 N}{U^6} \right)^{-\frac{1}{2}}.
\end{align*}
$$

The string coupling vanishes at the core ($U=0$) of this $N$ fundamental IIB string configuration. The curvature diverges there, however, signaling that the IIB supergravity breaks down, just as we approach the infra–red limit, as already observed in ref. [18].

Let us further remark here that this supergravity analysis is perfectly consistent with the matrix string discussion recalled above:

- The strong coupling limit of the Yang–Mills theory is also the strong coupling of the type IIB theory, turning the D1–branes into F1–branes (fundamental strings). This occurs here in the same coupling/energy regime.

- We learned from the field theory analysis that the moduli (target) space contains un–resolvable orbifold singularities. Because of the self–duality of the type IIB theory, we should take the 1+1 dimensional D1–brane field theory lessons seriously for the dual fundamental string also [20]. The curvature divergence at the core of the fundamental string configuration is the supergravity realization of this phenomenon.

- The free string theory is singular from the supergravity perspective. The region where supergravity is valid is away from the infra–red, where the vertex operator representing the string coupling is switched on. So supergravity can be used to give a definition of the matrix type IIA string only away from weak coupling.
3.2. The Weakly Coupled Type IIB String

In the case of the type IIB matrix string definition of the previous section, eleven-dimensional supergravity gave the defining infra-red theory in terms of a compactification on $AdS_4 \times S^7$. In the limit, we took $R_8 \sim R_9 \to 0$ and therefore we have also defined type IIB at intermediate coupling but at a non-trivial infra-red fixed point, by contrast.

We may define a weakly coupled limit of the matrix type IIB string by taking the $R_8, R_9 \to 0$ limit, but keeping $g_{\text{IIB}} = R_9/R_8 \ll 1$, (or its inverse). In this case, the $\hat{x}^9$ direction of our M–theory configuration effectively shrinks away, taking us back to ten dimensional type IIA supergravity (see next subsection). The 2+1 dimensional fixed point theory under discussion becomes effectively 1+1 dimensional\[8\]. It is a 1+1 dimensional fixed point. One might imagine that it is essentially an orbifold theory. This theory must clearly (see the next subsection for confirmation by the supergravity dual) be the matrix Green–Schwarz action (3.2) (at strong coupling) where now the Green–Schwarz fermions have the same chirality. This is the type IIB Green–Schwarz action.

This is the analogue of the matrix type IIA theory, where now the winding strings which make up the Fock space of the weakly coupled matrix type IIB string are fundamental type IIA strings.

3.3. The Role of Type IIA Supergravity

It is easy to see that the supergravity limit bears witness to this description also: the M2–branes wrap the $\hat{x}^9$ circle as it shrinks away and become $N$ fundamental type IIA strings lying along the $\hat{x}^8$ direction. We therefore have a description of the theory in terms of the neighbourhood of the core of the $N$ fundamental type IIA string solution in type IIA supergravity. These are the strings which wind and make up the Fock space of the matrix type IIB string. This solution is singular at the core again. The singularity occurs just as we get to the 1+1 dimensional fixed point.

4. The $E_8 \times E_8$ Heterotic String.

Thus encouraged by the above complementary pictures, showing us how to define the type II matrix string theories, we should expect a sharpening of the matrix string definitions of all of the remaining string theories in ten dimensions.

The $E_8 \times E_8$ heterotic string arises from placing M–theory on a line interval in $x^9$, say. A matrix definition of the heterotic string\[1,1,2,4\] proceeds by using the type IA string theory background, working with the quantum mechanics of $N$ D0–branes in the presence of a collection of 16 D8–branes with two orientifold O8–planes, a distance $\pi R_9$ apart. Eight of the branes are at each orientifold plane. (The matrix $E_8 \times E_8$ heterotic string coupling is $g_{\text{HA}} = R_9/\ell_s.$)
The 0+1 dimensional model has an $O(N)$ gauge symmetry with an $SO(16) \times SO(16)$ global symmetry coming from the background branes. Bound states of the D0–brane system localized on each 8 D8–brane + 1 O8–plane “wall” correspond to spacetime vectors carrying that gauge symmetry. In the limit, each $SO(16)$ is filled out to $E_8$ by 128 additional bound states (a spinor of $SO(16)$) becoming massless\[1\].

The matrix definition of the lightcone $E_8 \times E_8$ heterotic string should arise in the limit $R_9 \to 0$. As before, the description is given in terms of the $T_9$–dual system, which is in this case the type IB $SO(32)$ system. The $N$ D0–branes turn into $N$ D1–branes lying along the direction $x^9$ (with radius $\hat{R}_9$), while the 16 D8–branes + 2 O8–planes turn into the 16 D9–branes, a pure world sheet parity projection $\Omega$, with a Wilson line breaking the $SO(32)$ to $SO(16) \times SO(16)$.

The 1+1 dimensional $O(N)$ Yang–Mills theory has $(0,8)$ supersymmetry and a coupling:

$$\frac{1}{g_{YM}^2} = \ell_s^2 \frac{R_9}{R_{10}} = \ell_s^2 \frac{g_{IB}}{g_{HB}}. \quad (4.1)$$

(Here $\ell_s$ is the heterotic string length and $g_{HB}$ is the $SO(32)$ heterotic string coupling.) Once again, there is an $SO(8)$ R–symmetry coming from the rotations transverse to the D1–branes. The defining action may be thought of a “matrix Green–Schwarz” action for the heterotic string:

$$S = \frac{1}{2\pi} \int d^2 \sigma \text{Tr} \left( (D_\mu X^i)^2 + \theta^T \gamma^\mu D_\mu \theta + g_{HA}^2 F_{\mu\nu}^2 - \frac{1}{g_{HA}} [X^i, X^j]^2 + \frac{1}{g_{HA}} \theta^T \Gamma_i [X^i, \theta] + \chi^A D_L \chi^A \right). \quad (4.2)$$

The fermion $\theta$ now contains two fermions $\theta_L^\alpha, \theta_R^\dot{\alpha}$ which are in the $8_s, 8_c$. The $\theta_L$ are the superpartners of the world sheet gauge field. The $\chi^A$ are 32 real fermions coming from strings stretched between the D9–branes and the D1–branes. The effect of the Wilson line is to make 16 of them periodic and the other 16 antiperiodic. As before $R_9 \to 0, N \to \infty$ defines a weakly coupled matrix string theory, this time of the $E_8 \times E_8$ heterotic string with coupling $g_{HA} = R_9/\ell_s$, as a trivial orbifold conformal field theory limit. The orbifold moduli (target) space obtained in the strong (field theory) coupling limit is\[12\]

$$\mathcal{M} = \frac{(\mathbb{R}^8)^N}{S_N \times (\mathbb{Z}_2)^N}. \quad (4.3)$$

where and the $\mathbb{Z}_2$ acts on the 32 current algebra fermions $\chi^A$. As the type IB coupling is large here, the winding strings which build up the Fock space of the $E_8 \times E_8$ heterotic string are $SO(32)$ heterotic strings (with the Wilson line) in complete analogy with the matrix type IIA case.

The GSO projection assembles $E_8 \times E_8$ from the $SO(16) \times SO(16)$ states in the usual way, combining the $(1, 120) \oplus (120, 1)$ from the periodic–periodic sector with the $(1, 128) \oplus (128, 1)$
from the periodic–antiperiodic sectors, while throwing out the \((16, 16)\). That these are the sectors which survive at large \(N\) should be enforced by the fact that the expression for the momentum of the heterotic strings in the \(x^9\) direction is shifted away from the naive value by the presence of the Wilson line\(^{[11]}\).

4.1. The Role of \(\mathcal{N}=1\) Supergravity + \(SO(32)\) Yang–Mills

The supergravity discussion is similar to that for the matrix type IIA case, now using D1–brane and F1–brane solutions of the \(D=10, \mathcal{N}=1\) supergravity. Of course, this theory is anomalous, and this is cured by adding the \(SO(32)\) \(D=10, \mathcal{N}=1\) super–Yang–Mills theory to it\(^{[8]}\). Type IB/heterotic duality will also come into play at the level of supergravity to exchange the \(N\) D1–branes into \(N\) winding F1–branes as one approaches the core of the solution. These are the winding \(SO(32)\) heterotic strings. Again, at the core, the singularity signals the approach of the free 1+1 fixed point.

5. The \(SO(32)\) Heterotic/Type IB case.

Turning to the \(SO(32)\) system, we know that this should be realized by compactifying M–theory on a cylinder. In other words, we must compactify the system of the previous section on an additional circle, say in the \(x^8\) direction, of radius \(R_8\). We must then take \(R_8, R_9 \to 0\) to find a ten–dimensional theory.

It is easy to see that we recover at finite (but small) \(R_8, R_9\) a description in terms of a D2–brane stretched between two copies of the 8 D8 + 1 O8 system, now pointlike in \(\hat{x}^8\), a distance \(\pi \tilde{R}_8\) apart, where \(\tilde{R}_8 = \ell_s^2 / R_8\). We are in the type IA system again, with coupling

\[
\frac{1}{g^2_{YM}} = \frac{\ell_s}{g_{IIA}} = \frac{R_8 R_9}{R_{10}}. \tag{5.1}
\]

In taking \(R_8, R_9 \to 0\) we will approach the strong type IA coupling limit. This time, with the \(SO(16) \times SO(16)\) Wilson line, there is no choice\(^{[24, 25, 29, 31]}\) but for the system to go to M–theory on a line interval, the D2–branes becoming M2–branes (extended in \(\hat{x}^8, \hat{x}^9\)) while the 8+1 dimensional system at each end of the boundary each become a single “M9–plane” defining the ends of the M–theory line interval in \(\hat{x}^8\). Notice that we recover the full \(SO(8)\) rotations of the transverse directions, in this limit, as we have gained the extra direction \(x^{10}\). This translates into the \(R–symmetry\) of the field

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6 There is of course another choice to fix the anomaly, the system with gauge symmetry \(E_8 \times E_8\). As one might expect, we will see this arise when we consider the matrix theory of the weakly coupled \(SO(32)\) heterotic string.
theory on the M2–brane, which in turn corresponds to the Lorentz group of the \( SO(32) \) matrix string system in the IMF.

There are three distinct versions of the \( R_9, R_8 \rightarrow 0 \) limit:

5.1. \( R_9/R_8 << 1 \). The Weakly Coupled \( SO(32) \) Heterotic String.

Generically, the system defines an effective 1+1 dimensional system, because the \( \hat{x}^9 \) direction decompactifies faster than the interval in \( \hat{x}^8 \). We have an effective dimensional reduction of the theory on the M2–branes, as one of their directions is stretched between the M9–planes. Before taking the limit, the type IA system of D2–branes stretched between the ends of the interval give an \( O(N) \) Yang–Mills theory with coupling

\[
\frac{1}{g_{YM}^2} = \frac{\tilde{R}_8 \ell_s}{g_{\text{IIA}}} = \frac{\tilde{R}_8 R_9 R_8}{R_{10}} = \frac{\ell_s^2 R_9}{R_{10}}. \tag{5.2}
\]

This is a 1+1 dimensional analogue of the gauge theory constructions of Hanany and Witten\(^{30}\), which were generalized to include orthogonal and symplectic groups by adding orientifolds in ref.\(^{31}\).

In the full limit therefore, we get a 1+1 dimensional theory which descends from stretching the M2–branes between M9–planes and taking the limit as the planes approach one another. This defines a 1+1 fixed point theory with \( \mathcal{N} = (0, 8) \) supersymmetry and the required \( SO(8) \) R–symmetry coming from rotations in \( x^1 - x^7, x^{10} \).

This is precisely the reduction we want to describe a weakly coupled matrix \( SO(32) \) heterotic string. The \( SO(32) \) heterotic string coupling is proportional to the size of the original interval, as it should be:

\[
g_{\text{HB}} = \frac{R_9}{R_8} = \frac{\tilde{R}_8}{R_9} << 1. \tag{5.3}
\]

Once again, we can think of this effective theory as arising from the flow from an effective 1+1 dimensional Yang–Mills theory with \( SO(8) \) R–symmetry. We can say precisely what the content of this 1+1 dimensional theory must be. It is of course the matrix Green–Schwarz action for the \( SO(32) \) heterotic string, which is the same action as eqn.\(^{4,2}\), the same effective action as found on the D1–brane in type IB above, but now the winding \( SO(32) \) heterotic strings which built up the Fock space of that theory are replaced by winding \( E_8 \times E_8 \) heterotic strings, as dictated by the limits. Now we have the other GSO projection on the 32 fermions, which throws away the spinors of \( SO(16) \) and allows the vectors \( (16, 16) \) to join the \( (1, 120) \oplus (120, 1) \), filling out the adjoint of \( SO(32) \).

A quick way to see this is to realize that in this limit we have pushed the D8–branes together fast enough that we never need to approach the M–theory limit (see ref.\(^{3}\) for a relevant discussion). T–dualizing along the small \( \hat{x}^8 \) direction, we can work in terms of \( N \)
D1–branes in type IB, with the full $SO(32)$ restored: we have recovered the correct GSO projection.

This is the exact analogue of that which defined the matrix $E_8 \times E_8$ heterotic string, This is the full description of the matrix $SO(32)$ heterotic string at weak coupling.

5.2. The Role of $\mathcal{N}=1$ Supergravity + $E_8 \times E_8$ Yang–Mills

The supergravity observations that we made earlier can now be used to lend support to these facts:

The supergravity solution in this limit arises from eleven dimensional supergravity with $N$ M2–branes stretched between the two M9–planes at the end of the interval, in the limit where the size of the interval shrinks and $N \to \infty$. This is best described in terms of the reduced ten–dimensional theory, which is precisely the $E_8 \times E_8$ $\mathcal{N}=1$ super Yang–Mills + $\mathcal{N}=1$ supergravity, as determined by the anomaly considerations of ref. [28].

So we that the other $\mathcal{N}=1$ supergravity arises naturally in the story as well, as expected. The $N$ M2–branes are now effectively $N$ one dimensional objects in the theory. They are $N$ fundamental $E_8 \times E_8$ heterotic strings. We are looking at the core of the fundamental string solution again. It is singular, signaling the approach of the trivial fixed point describing the free matrix $SO(32)$ string. These $E_8 \times E_8$ fundamental strings build up the Fock space of the free matrix $SO(32)$ strings.

5.3. $R_9/R_8 >>1$. The Weakly Coupled $SO(32)$ Type IB String.

To study this limit, we begin again with the M2–branes stretched between the M9–plane again. This time we see that the $\hat{z}^9$ interval grows more slowly than the $\hat{z}^8$ interval and therefore we obtain an effective 1+1 dimensional system again. This is again a 1+1 dimensional fixed point theory, defining the weakly coupled type IB $SO(32)$ string, with coupling $g_{IB} = R_8/R_9 = \tilde{R}_9/\tilde{R}_8 << 1$.

The 1+1 dimensional theory is a $\mathbb{Z}_2$ orbifold (in $\hat{z}^8$) of the 1+1 dimensional theory which we found defined the weakly coupled limit of the type IIB string. The fixed points of the orbifold are at infinity. The winding strings which build up the Fock space of the the matrix type IB strings are fundamental type IA strings stretched between the ends of the $\hat{z}^8$ interval. These are simply type IIA strings with endpoints on D8–branes at infinity (in the limit).

It is easy to see the nature of the effective 1+1 dimensional theory which flows to the field theory fixed point defining the free string. It comes from a (matrix) Green–Schwarz action for the type IIB string, but with the extra condition that the strings ends are fixed at the ends of the $\hat{z}^8$ interval. This is of course the definition of the (matrix) type IB Green–Schwarz string action!
In contrast to the matrix heterotic string limits, there is no family of 32 heterotic fermions and so the gauge symmetry must arise elsewhere. Instead, we have the additional degree of freedom to choose which D8-brane to end on at each end of the $\hat{x}^8$ interval. This is of course simply the introduction of Chan–Paton factors! It is clearer to count states working with the covering space of the $\hat{x}^8$ interval: We have 16 D8-branes plus an orientifold at each end. We trivially get the adjoint $(1, \mathbf{120}) \oplus (\mathbf{120}, 1)$ of the manifest $SO(16) \times SO(16)$ by considering each end separately. However, we also must include the $(\mathbf{16}, \mathbf{16})$ coming from considering mixed states. This fills out the adjoint of $SO(32)$.

The infra–red limit of this $O(N)$ effective 1+1 dimensional matrix model will define the free matrix $SO(32)$ type IB string as an orbifold fixed point. The Fock space is defined by winding type IA strings.

5.4. The Role of Type IA Supergravity

The supergravity limits support the conclusions immediately above. In the limits which we took, the $N$ M2–branes stretched between the M9–planes become $N$ fundamental type IA strings stretched between the two collections of 8 D8–branes + 1 O8–plane at each end of the $\hat{x}^8$ interval.

The supergravity description of the limit is therefore the $N$ fundamental string solution of the type IIA supergravity, stretched between two domain walls at infinity. In general, this is the type IIA massive supergravity of ref. [32], but the choice of D8–brane arrangements we have here sets the cosmological constant to zero. As there is a one–to–one correspondence between the arrangement of D8–branes and the value of the cosmological constant, fundamental type IIA string solutions in massive type IIA supergravity with other choices of the cosmological constant will correspond to the matrix description of (nearly) free $SO(32)$ type IB strings with specific choices of Wilson line.

(It is amusing to note that in essence, we have constructed a macroscopic version of the type I string. It is difficult to construct it directly as a soliton of the dual $SO(32)$ heterotic string because it is unstable to breaking into smaller pieces in the ten dimensional theory. We have evaded that problem here by stretching the string transverse to the space in which it allowed to break, and sending the ends off to infinity. In effect, we have magnified the region “between the D9–branes” in order to construct a stable $SO(32)$ type I string.)

5.5. $R_9/R_8=1$. The $SO(32)$ System at Intermediate Coupling

In this case, taking this limit $R_9, R_8 \to 0$ and $N \to \infty$ will define an uncompactified M–theory limit, staying in eleven dimensions. Our matrix string definition is the theory on the M2–branes, as both spatial directions are on the same footing.

We have therefore a 2 + 1 dimensional theory with eight supercharges, but still possessing $SO(8)$ R–symmetry. It is the large $N$ limit of a variant of $O(N)$ Yang–Mills theory with
coupling

\[
\frac{1}{g_{\text{YM}}^2} = \frac{\ell_s}{g_{\text{IIA}}^2} = \frac{R_9 R_8}{R_{10}} \tag{5.4}
\]

in the strong coupling limit.

We expect that this defines a fixed point theory with conformal invariance. Furthermore, we expect it to define an interacting fixed point theory, as the matrix strings it defines are not weakly coupled. This theory has regions where it has 16 supercharges, but there are two 1+1 dimensional submanifolds with half that number. There are “twisted sector” degrees of freedom living on those submanifolds. This theory should be an example of an infra–red limit of the type of orbifold Yang–Mills theories studied in ref.[19,33]. The existence of such a fixed point was conjectured in ref.[19].

5.6. A Return to 11D Supergravity

The supergravity intuition we have developed over the course of the paper now helps us again, defining precisely the content of the theory: The non–trivial superconformal fixed point we require is defined by a \( \mathbb{Z}_2 \) orbifold of eleven dimensional supergravity on \( AdS_4 \times S^7 \), where the orbifold symmetry acts on the \( AdS_4 \) factor, leaving the \( S^7 \) (and hence the \( SO(8) \) R–symmetry necessary for matrix string Lorentz invariance) untouched. This breaks the rotation symmetry in one of the \( AdS \) directions and hence we define a subgroup of the \( SO(3,2) \) superconformal symmetry of the 2+1 dimensional fixed point.

The orbifolded \( AdS_4 \) is described by first placing the \( \hat{x}^8 \) direction on a circle, identifying \( \hat{x}^8 \) and \( -\hat{x}^8 \) (which is clearly a symmetry to begin with) and then decompactifying the circle again. This results in two 2+1 dimensional boundaries located at \( \hat{x}^8=0 \) and \( \hat{x}^8=\infty \). Evidently, consistency of the theory will require some treatment of these two boundaries analogous to the treatment of ref.[28] for the \( E_8 \times E_8 \) heterotic string dual. The \( \mathbb{Z}_2 \) action will also act on the fields in the supergravity, and throwing out those odd (and presumably adding appropriate twisted sectors at the boundary) will define via the AdS/CFT correspondence, the spectrum of the 2+1 fixed point theory. The holography will clearly work in both sectors, projecting the bulk of the orbifold AdS to the bulk of the 2+1 dimensional theory, and the fixed point set will also be projected onto the fixed point set of the 2+1 theory.

(It is interesting to note that the geometry of the 9+1 dimensional fixed points, after rescaling using the relation between the ten and eleven dimensional metrics[33],
\[
d_{S_{11}}^2 = e^{4\phi/3} \left[(d\hat{x}_4 + A^\mu dx_\mu)^2 + e^{-2\phi} d_{S_{10}}^2 \right],
\]
is precisely the near–horizon geometry of a fundamental string. (\( A \) is the R–R one–form potential in ten dimensions.))

This orbifold of \( AdS \) is an easily stated supergravity prescription, and therefore merits further study, as a means of describing an unusual type of fixed point.
6. Closing Remarks and Outlook

6.1. Big Superstrings

We have seen that the matrix string description of all of the ten dimensional string theories is qualitatively the same, but only in the neighbourhood of weak coupling, as one might expect on general grounds. To summarize:

- The free matrix string is defined by a 1+1 dimensional fixed point, which is a trivial orbifold conformal field theory. The twisted sectors of the orbifold are made up of winding strings of the T–dual variety. These build the Fock space of the free matrix string.

- In each case, the fixed point can be described by an effective 1+1 dimensional large $N$ gauge theory. This theory is a “matrix Green–Schwarz” action, for the string. The fields are $N \times N$ matrices. (Precisely at the free string limit, the matrices commute, and the action decomposes into $N$ copies of the usual Green–Schwarz action.)

- There is a supergravity “dual” description of the effective gauge theory, which is simply the large $N$ metric of the fundamental string solution associated to the T–dual string. (This solution is in the associated supergravity of the T–dual string, of course). The supergravity description breaks down at the fixed point, which is associated to the center of the fundamental string solution which is singular. This singularity is the dual of the orbifold singularities in the conformal field theory description.

Away from the weak coupling limit, the theories are divided into two classes of behaviour: The type IIA and $E_8 \times E_8$ heterotic are in one class, while the type IIB and the $SO(32)$ strings are in the other.

- For the first class, the fixed point describing the matrix string stays 1+1 dimensional and flows back to the effective gauge theory (it arises from a description in terms of a D1–brane). By time one gets to infinite coupling, the most economical description is in terms of matrix quantum mechanics, as the circle (or line interval) which M–theory was placed upon decompactifies.

- For the second class, the fixed point theory grows an extra dimension and becomes 2+1 dimensional (it lives on a M2–brane). The brane unwraps the circle (or line interval) it was placed upon. As the coupling grows, the other direction that the brane extends in shrinks, and in the infinite coupling limit, we get a new effective 1+1 dimensional theory with an associated trivial fixed point. This is ten dimensional string/string duality.

- At intermediate coupling, the description is fully 2+1 dimensional. The 2+1 dimensional fixed point is characterized by an 11 dimensional supergravity description involving $AdS_4$ (or an orbifold). The latter is a new type of fixed point which deserves further study.

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7 Here, our results differ from those presented in ref.[16] for the $SO(32)$ system. We thank T. Banks for pointing out that paper to us after reading an earlier version of this manuscript.
This complete picture is very satisfying.

6.2. Little Superstrings

Now that we have a complete alternative definition of the familiar ten-dimensional strings at weak coupling, and their extension to arbitrary coupling, a next step is obvious. There have been shown to exist\[35,36,37,38\] consistent string theories in six dimensions, which previously evaded a direct construction by the usual ten-dimensional techniques. A matrix string description may be better suited to characterizing them. It has been argued\[36\] that all of the (big) superstrings in ten dimensions have a (little) six dimensional descendant. So far, these strings have not all been completely described in the matrix manner.

Inspired by the results presented here, we can anticipate some key feature of the little strings’ description:

For the (0, 2) six dimensional strings (“type iia”), a description is found in terms of a fixed point theory derived from a D1–D5 brane system\[39,37,40\]. This fixed point is a non–trivial one, in contrast to the ten dimensional type IIA case.

The interpretation of all of this in the present context is that there should be a smooth supergravity description associated with this fixed point. Indeed, the fixed point is believed to have a description in terms of type IIB supergravity on $AdS_3 \times S^3 \times T^4$, as the brane construction would suggest. This description is smooth. It describes the iia string system at intermediate coupling, which is arguably\[36\] the coupling at which it has its most natural description.

Notice that like the ten dimensional type IIB system at intermediate coupling, this description also has two holographed–out dimensions of signature (1, 1). This might be evidence for the descendant of F–theory, (“f–theory”) whose existence was suggested by ref.\[41\].

For the (0, 1) six dimensional strings, one would expect an heterotic supergravity compactification on $AdS_3 \times S^3 \times T^4$ to be involved in defining the little heterotic systems. Another way to get (0, 1) is of course to compactify further on a manifold which breaks half of the supersymmetry. This suggests that $AdS_3 \times S^3 \times K3$ might be an alternative description\[8\] of the little heterotic system. The $E_8 \times E_8$ intersection cohomology lattice of $K3$ will give the global symmetry which the little string is supposed to have. That this is reasonable follows from (essentially) heterotic/type IIA duality.

It would be very interesting and useful to determine and classify the matrix descriptions of all of the little strings along the lines suggested here.

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8 This idea arose in a conversation with H. Verlinde.
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