Next-to-leading gluonic reggeons in the high-energy effective action

R. Kirschner† and L. Szymanowski†#

†Naturwissenschaftlich-Theoretisches Zentrum
und Institut für Theoretische Physik, Universität Leipzig
Augustusplatz 10, D-04109 Leipzig, Germany

# Soltan Institut for Nuclear Studies, Hoża 69, 00-681 Warsaw, Poland

Abstract: We study the next-to-leading gluon exchange in the high-energy scattering that contributes to the amplitude to order \( s^0 \) up to logarithmic corrections. Similar to the leading gluon exchange these contribution can be described in terms of reggeon exchanges. There are several gluonic reggeons at the next-to-leading level. Some of them transfer parity or gauge group representation different from the leading gluonic reggeon. Unlike the leading one they are sensitive to the helicity and transverse momenta of the scattering partons.

We extend the high-energy effective action and derive from the action of gluodynamics the terms describing the next-to-leading reggeons and their interaction in the multi-Regge approximation.

---

1Supported by Deutsche Forschungsgemeinschaft KI 623/1
and the German-Polish agreement on scientific and technological cooperation N-115-95
1 Introduction

The leading contribution to the high-energy asymptotics of scattering processes in QCD can be described by a reggeon and its interaction. The exchange of two such leading reggeized gluons with interactions summed up in the leading $\ln s$ approximation results in the BFKL pomeron [1], which is by now successfully applied in the phenomenological analysis of semi-hard processes and in particular of deep-inelastic scattering at small values of the Bjorken variable $x$. The systematic improvement of the leading $\ln s$ approximation can be organized using the reggeized gluon concept: The exchange of an even number of leading reggeized gluons with interactions taken in the same approximation as in the BFKL pomeron gives rise to unitarity correction to the latter. The exchange of three reggeized gluons leads to the odderon [2], the actual role of which in phenomenology is still unclear. Unitarity corrections to the odderon result from the exchange of an odd number of reggeized gluons. Further next-to-leading $\ln s$ corrections result in corrections to the scattering and production vertices and in new vertices. These corrections are related to going beyond multi-Regge approximation (to be explained below, sec. 2.1).

Much effort has been applied in the last years to calculate the latter corrections [3] and also to calculate the Regge singularity induced by multiple exchange of leading reggeized gluons [4] as well as by higher reggeon interaction vertices [5]. The reggeon concept is a starting point of the multiparticle unitarity approach to high-energy scattering [6].

The paper is devoted to the study of contribution from gluon exchange suppressed by one power of the cms energy squared $s$ compared to the leading gluon exchange. We propose to apply the reggeon concept also to the non-leading exchanges.

Analysing the high-energy asymptotics it is convenient to consider the Mellin transform of the amplitude with respect to the energy squared $s$. It is essentially the $t$-channel partial wave and the Mellin variable $j$ is the complex angular momentum. The leading gluon exchange induces Regge singularities near $j = 1$, the non-leading gluon exchanges studied here appear as reggeons with poles near $j = 0$.

There are observables in high-energy scattering to which the non-leading exchanges contribute not just as a small correction. Non-vacuum quantum numbers like odd ($C$ or $P$) parity can be transferred by gluons.
Consider the scattering with small momentum transfer of a high-momentum gluon or quark on a source of colour fields. The leading interaction contributes to the amplitude proportional to the first power of the large momentum. It conserves helicity of the high-momentum gluon or quark and it is not sensitive to the details of the colour source like its distribution in the transverse (impact parameter) plane or to its spin structure. However such details are resolved by interactions suppressed by one power of the large momentum compared to the leading one.

The exchange of one leading and one non-leading gluonic reggeon gives a contribution to the small-$x$ asymptotics of the spin structure function $g_1(x)$ of the proton [7] measuring the helicity asymmetry. The exchange of two non-leading gluonic reggeons contributes to the small-$x$ asymptotics of the spin structure function $F_{\gamma}^3(x)$ of the photon (spin - 1 target), measuring the transverse polarization asymmetry of gluons (gluon transversity) [8].

The high-energy effective action provides a technical framework for formulating and analysing the problems of high-energy scattering in gauge theories. It has been proposed originally as a summary of the leading ln $s$ results for the leading gluon exchange in a simple form and a starting point for going beyond this approximation [9]. Then it has been understood that it is indeed an effective action in the sense of Wilson. It can be derived from the original action by separating the fields into modes and integrating over those modes that do not correspond neither to scattering quanta (partons) nor to exchanged quanta [10]. The effective action has been studied up to now in the multi-Regge approximation for describing the leading gluon and leading fermion exchanges. There are results going beyond this approximation [11].

Here we are going to extend the procedure to non-leading gluon exchanges contributing to $O(s^0)$ to the amplitude. We restrict ourselves for simplicity to the case without fermions, i.e. to pure gluodynamics. We stay within the multi-Regge approximation improving the known procedure by keeping terms suppressed by one power of $s$. The experience from the exercise in high-energy scattering in (linearized) gravity [12] helped us to optimize the extensive calculations.

The structure of the effective vertices with fermions included can be obtained from the gluonic vertices by supersymmetry transformations relying on the similarity of QCD to supersymmetric Yang-Mills theory. A short description of our results including a discussion about fermions has been published earlier [13].
Some of the technical steps in our procedure can be justified only in the framework of perturbation theory, the applicability of which is restricted to the semi-hard region. There we have besides of the energy squared a second large momentum scale (momentum transfer or virtuality $Q^2$) which is much smaller than $s$ but still large compared to the hadronic scale. Referring to dominating momentum configurations in this perturbative Regge region, we can give the inverse derivatives appearing in the calculations and in the final result a meaning, since the typical longitudinal momenta are not small and small transverse momenta should not be essential either. Non-local interactions are an essential feature of our effective action.

In the next section we discuss the separation of modes according to the multi-Regge kinematics. This is the first essential step towards the effective action. We start from the Yang-Mills action in the light-like axial gauge with the redundant field components eliminated. In this way we have a direct correspondence between the fields and the physical degrees of freedom. This gauge is convenient but can be avoided. The resulting action does not depend on gauge. We study the impact of the separation of modes on the triple and quartic interaction terms. Thinking of the physical situations of high-energy gluon scattering in an external field and of gluon-gluon quasi-elastic scattering gives us a guideline to collect the most important terms for deriving the vertices of the effective action. We study the quartic terms contributing to high-energy elastic scattering. The ones corresponding to $s$-channel gluon exchange far off-shell are calculated in section 3. Eliminating these "heavy modes" is the second essential step towards the effective action.

The resulting terms describing effectively quasi-elastic scattering are analyzed in section 4. We obtain a sum of terms of the form current times current, where the currents describe the gluons scattering with relatively small momentum transfer. This factorizability is essential for identifying the Regge exchanges. At the end of section 4 we write down the effective action of quasi-elastic scattering introducing pairs of pre-reggeon fields.

It becomes clear that due to parity symmetry, interchanging the incoming particles, only a part of quartic terms is sufficient to obtain the effective action for quasi-elastic scattering. This observation is used to cut short the calculations in section 5 where we study the terms of order 5 describing effectively the inelastic $2 \rightarrow 3$ process in the multi-Regge kinematics. By factorization we obtain the effective production vertices. The high-energy effective action is obtained by adding the production vertices to the effective
action of quasi-elastic scattering. The features of this action are discussed in the last section.

2 Separation of modes

2.1 Multi-Regge kinematics

It is convenient to start from the Yang-Mills action in the light-cone axial gauge $A_- = 0$,

$$
\mathcal{L} = \mathcal{L}^{(2)} + \mathcal{L}^{(3)} + \mathcal{L}^{(4)}
$$

$$
\mathcal{L}^{(2)} = -2 A^a (\partial_+ \partial_- - \partial \partial^*) A^a
$$

$$
\mathcal{L}^{(3)} = -\frac{g}{2} j^a A^a - \frac{g}{2} j^a A^a
$$

$$
\mathcal{L}^{(4)} = \frac{g}{8} J^a \partial_- J^a - \frac{g}{8} j^a j^a
$$

We use light-cone components for the longitudinal part of vectors and complex numbers for the transverse part \([10]\). The space-time derivatives are normalized such that \(\partial_+ x_- = \partial_- x_+ = \partial x = \partial^* x^* = 1\). The gluon field is represented by the transverse gauge potential \(A^a, A^{as}\). It enters the interaction terms (2.1) in the combinations

$$
A^a_+ = \partial_-^{-1} (\partial A^a + \partial^* A^{as}) , \quad A^a = i(\partial A^a - \partial^* A^{as})
$$

and in the currents

$$
J^a_+ = i (A^a A^a) , \quad j^a = (A^a A^a).
$$

In the following we encounter besides of the longitudinal components \(J^a_+\) also the transverse components \(J^a, J^{as}\) of the vector current (obtained by replacing \(\partial_-\) by \(\partial^*\) and \(\partial\), respectively). We use the abbreviation

$$
(AT^a B) = -i f^{abc} A^b B^c
$$

with \(f^{abc}\) the structure constants of \(SU(N)\).

The notations are chosen such that there is a close relation to the leading terms of the effective action \([10]\): The expression \(A^a_+ (2.2)\) describes the
leading gluonic reggeon and the current $J^a$ determines the leading scattering vertex. We shall see that some of the non-leading gluonic reggeons are described by the expression $A'^a$ (2.2) in terms of the original gluon field $A^a$. From the point of view of momentum representation $A^a$ and $A'^a$ represent the projections of the transverse gauge potential $A^a(k)$, the first parallel to the transverse part of its momentum $\kappa^\mu$ and the second orthogonal to $\kappa^\mu$. The non-leading scattering vertices involve besides of the current $j^a$ other currents like $J^a$, $J^a\ast$. Removing the redundant field components in the light-cone gauge is convenient because now the complex field $A^a$ is directly related to the gluonic degrees of freedom and as we have stated already in sec. 1 introducing this gauge is a technical step which can be avoided since the effective action is gauge invariant.

We separate the field into modes $A = A_t + A_s + A_1$. $A_t$ are the momentum modes typical for exchanged gluons, $A_s$ are the modes typical for scattering gluons and $A_1$ are the heavy modes, which do not contribute directly to the scattering or exchange and will be integrated out.

The modes are separated according to the multi-Regge kinematics, i.e. the momentum configuration of a multi-particle $s$-channel (intermediate) state $(p_l, l = 0,1,\ldots,n)$ giving the dominant contribution in the leading $\ln s$ approximation, see Fig. 1. Decomposing the transferred momenta $k_l = p_A - \sum_{i=0}^{l-1} p_i$ with respect to the (almost light-like) momenta of incoming particles $p_A, p_B$

$$k^\mu = \sqrt{\frac{1}{s} (k^\mu_B + k^\mu_A) + \kappa^\mu},$$

the multi-Regge kinematics is characterized by the conditions

$$|k_{+n}| \gg \ldots \gg |k_{+1}|, \quad |k_{-n}| \ll \ldots \ll |k_{-1}|$$

$$|k_{+l}k_{-l}| \ll |\kappa_l|^2, \quad s_l = |k_{-l}k_{+l+1}| \gg |\kappa_l|^2$$

$$\prod_{l=1}^{n} s_l = s \prod_{l=2}^{n} |\kappa_l - \kappa_{l-1}|^2.$$  (2.6)

Here $\kappa$ denotes the transverse (with respect to $p_A, p_B$) part of the momentum $k$. It is represented by a 4-vector in (2.5) and in the following it will be represented by a complex number keeping the same notation. The longitudinal momenta are strongly ordered. The subenergies $s_l$ are large compared to the transferred momenta. The longitudinal contribution to the transferred
momenta squared is small. In loops the main contribution from \(s\)-channel intermediate particles arises from the vicinity of the mass shell. Therefore the modes \(A_t\), \(A_s\), \(A_1\) are characterized by the following conditions

\[
\begin{align*}
A_t & : |k_- | \ll |\kappa|^2 \\
A_s & : | |k_- | - |\kappa|^2 | \ll |\kappa|^2 \\
A_1 & : |k_- | \gg |\kappa|^2 .
\end{align*}
\] (2.7)

We introduce the mode separation into the action (2.1) by substituting \(A\) by \(A_s + A_t + A_1\). The kinetic term decomposes into three, one for each of the modes, which follows immediately from momentum conservation

\[
\mathcal{L}^{(0)}_{\text{kin}} = -2A^a_s (\partial_+ \partial_- - \partial \partial^*) A^a_s + 2A^a_t \partial \partial^* (1 - \frac{\partial_+ \partial_-}{\partial \partial^*}) A^a_t - 2A^a_1 \partial_+ \partial_-(1 - \frac{\partial \partial^*}{\partial_+ \partial_-}) A^a_1 . \] (2.8)

In the kinetic term for \(A_t\) and \(A_1\) the second operator in the brackets will be treated as a small one. In the calculations we have to keep the first order in these corrections.

### 2.2 The triple interaction

Consider now the triple terms \(\mathcal{L}^{(3)}\) of the action (2.1). We introduce the mode decomposition in \(\mathcal{L}^{(3)}\) and obtain many terms. The most important terms for our discussion are those, where two of the fields have longitudinal momenta of the same order and third one carries much larger or much smaller longitudinal momentum. We denote by \(\mathcal{L}^{(3)}_1\) these terms with one of the three fields in the modes \(A_t + A_s = \tilde{A}_t\) and two in the modes \(A_s + A_1 = \tilde{A}_1\) as

\[
\mathcal{L}^{(3)}_1 = -\frac{g}{2} \left\{ i(\tilde{\mathcal{A}}^* T^a \overset{\leftrightarrow}{\partial} \tilde{\mathcal{A}}^a_{+} + (\tilde{\mathcal{A}}^* T^a \tilde{\mathcal{A}}^a_{+}) + i(\tilde{\mathcal{A}}^* T^a \tilde{\mathcal{A}}^a_{-}) + (\tilde{\mathcal{A}}^* T^a \tilde{\mathcal{A}}^a_{+}) \right\} . \] (2.9)

\(\tilde{\mathcal{A}}_{+}, \tilde{\mathcal{A}}_{-}\), and \(\tilde{\mathcal{A}}_{+}, \tilde{\mathcal{A}}_{-}\) are given by the expressions (2.2) with the fields restricted to the modes \(A_t + A_s = \tilde{A}_t\) and \(A_s + A_1 = \tilde{A}_1\), respectively. We rearrange the terms in (2.3) by using the definition (2.4) for the bracket...
\[(A^*T^aA)\) and by performing integration by part in order to put the fields with the mode \(\tilde{A}_t^a\) as the last factor in each term (Fig. 2)

\[
\begin{align*}
\mathcal{L}_1^{(3)} &= -\frac{g}{2} \left\{ i(\tilde{A}^* T^a \tilde{\partial}_{-} \tilde{A})\tilde{A}_{i+} + (\tilde{A}^* T^a \tilde{A})\tilde{A}_{i} - i(\tilde{A}^* T^a \tilde{\partial}^* \tilde{A})\tilde{A}_{i}^a \\
&\quad - i(\tilde{A}^* T^a \tilde{\partial}^* \tilde{A})\tilde{A}_{i} + i(\tilde{A}^* T^a \tilde{A})(\partial\tilde{A}^a_i - \partial^* \tilde{A}^a_i) + 2i(\tilde{A}_+ T^a \tilde{A})\partial \tilde{A}_{i}^a \\
&\quad - 2i(\tilde{A}^* T^a \tilde{A}_+)\partial \tilde{A}_{i}^a \right\}.
\end{align*}
\] (2.10)

We decompose the fields in the \(\tilde{A}_t\) mode in each term into the expressions \(\tilde{A}^a + \tilde{A}'^a\) and \(\partial \tilde{A}_t = \frac{1}{2}(\partial \tilde{A}_+ + \partial \tilde{A}_- - i\tilde{A}_t')\). Writing this field always as the last factor allows in the following to omit the subscript \((t, s, 1)\) and the tilde referring to the range of modes.

We use the currents \(J^a, J^a, J^* a, j^a\) introduced above in (2.3) and furthermore \(J^a_s = (\partial_{-} A T^a A)\), \(J^a_2 = (\partial^* A^* T^a A)\) (2.11) to express the two fields in the \(\tilde{A}\) mode in each term and obtain

\[
\begin{align*}
\mathcal{L}_1^{(3)} &= -\frac{g}{2} \left\{ J^a - \frac{1}{2} \partial_{-} (\partial J^a + \partial^* J^* a) \\
&\quad - i \frac{\partial^2}{\partial \partial^*} (\partial(J^a_s + J^a_2) - \partial^* (J^* a_s + J^* a_2)) \right\} \mathcal{A}_+^a \\
&\quad - \frac{g}{2} \left\{ 2j^a + \frac{i}{2} \partial \partial^* (\partial J^a - \partial^* J^* a) \\
&\quad - \partial_{-} (\partial(J^a_s + J^a_2) + \partial^* (J^* a_s + J^* a_2)) \right\} \mathcal{A}^a. \quad (2.12)
\end{align*}
\]

The separated triple terms (2.12) describe in particular the interaction of a high-energy gluon (modes \(A_s\) involved in the currents) with an external field (described by the \(A_t\) modes in \(\mathcal{A}_+\) and in \(\mathcal{A}'\)). In the case of scattering with a large momentum component \(k_-\) the term with \(J^a_-\) gives the leading contribution of order \(O(k_-)\), the terms with \(J^a, J^* a\) and \(j^a\) contribute to order \(O(k_0^0)\) and the other terms with \(J^a_s, J^a_2\) result in corrections of the order \(O(k_-^1)\). In describing scattering with large \(k_+\) the ordering goes in the reverse direction.

Notice, that the \(J^a_s\) terms contribute to helicity flip whereas all other vertices conserve the helicity of the scattering gluon.
2.3 Quartic interactions and elastic scattering

We introduce the mode separation \( (2.7) \) into the quartic term \( \mathcal{L}^{(4)} \) of the action \( (2.1) \). We pick up the terms with two fields in the modes \( \tilde{A} = A_s + A_t \) and the other two in all modes \( A \) with the additional condition that the first two have a large longitudinal momentum \( k_- \) and the latter two have small \( k_- \) but large \( k_+ \),

\[
\mathcal{L}^{(4)}_{\text{scatt}} = \frac{g^2}{4} \tilde{J}^a \frac{1}{\partial_-} J^a - \frac{g^2}{4} \tilde{j}^a j^a - \frac{g^2}{8} \left\{ (\tilde{A}^* T^a \tilde{\partial}_- A) \frac{1}{\partial_-} (\tilde{A}^* T^a \tilde{\partial}_- A) + \ldots \right\} - \frac{g^2}{8} \left\{ (\tilde{A}^* T^a A) (\tilde{A}^* T^a A) + \ldots \right\} .
\]  

(2.13)

The periods \( \ldots \) stand for the three terms obtained from the explicite ones by shifting the tilde signs to the other fields in each of the brackets. Now we look at the derivatives acting on the fields \( \tilde{A} \) carrying large \( k_- \) modes. We apply approximations like

\[
\frac{1}{\partial_-} \tilde{A}^* T^a A = (\tilde{A}^* T^a A) - \frac{1}{\partial_-} \tilde{A}^* T^a \tilde{\partial}_- A + \ldots
\]  

(2.14)

and keep only terms which, after changing to momentum representation, are of the order \( k_1^1 \) or \( k_0^1 \), and obtain

\[
\mathcal{L}^{(4)}_{\text{scatt}} = \frac{g^2}{4} \tilde{J}^a \frac{1}{\partial_-} J^a - \frac{g^2}{4} \tilde{j}^a j^a - \frac{g^2}{2} (\tilde{A}^* T^a A) (\tilde{A}^* T^a A) + \mathcal{O}(k_-^{-1}) .
\]  

(2.15)

We would like to write also the third term as a product of a factor involving \( \tilde{A} \) only and a second factor involving \( A \). We use the relation for the generators \( T^a \) of the adjoint representation of SU\((N)\)

\[
(T^e)^{ab}(T^e)^{cd} - (T^e)^{ac}(T^e)^{bd} = (T^e)^{ad}(T^e)^{cb}
\]  

(2.16)

and introduce the generators \( D^r \) of the reducible representation arising as the symmetric part in the tensor product of the two adjoint representations of SU\((N)\) in order to write

\[
(T^e)^{ab}(T^e)^{cd} + (T^e)^{ac}(T^e)^{bd} = (D^r)^{ad}(D^r)^{cb} .
\]  

(2.17)
We introduce the current

\[ j_D = (A^* D^r A) . \]  

(2.18)

Using relations (2.16), (2.17) and definition (2.18) we obtain

\[ \mathcal{L}_{\text{scatt}}^{(4)} = \frac{g^2}{4} \tilde{J}^a \frac{1}{\partial^2} J^a - \frac{g^2}{4} \tilde{j}^a j^a - \frac{g^2}{4} \tilde{j}^r j^r. \]  

(2.19)

The separated quartic terms (2.19) contribute to the quasi-elastic scattering of gluons at high-energy and limited momentum transfer.

The triple terms induce further contributions to quasi-elastic scattering: Two vertices from \( \mathcal{L}_1^{(3)} \) (2.12) can be contracted by \( t \)-channel (\( A_t \)) or by \( s \)-channel (\( A_s \)) exchanges. We consider first the contribution of \( t \)-channel exchange. The contribution from heavy modes \( A_1 \) will be discussed in the next section.

In one of the vertices the fields in the currents describe scattering gluons with large \( k_- \). We are going to describe high-energy scattering in the accuracy including terms \( \mathcal{O}(s^0) \), therefore in this vertex we can disregard the contribution of the currents \( J_s \) and \( J_2 \) in (2.12). We restrict the fields in the currents to the modes \( A_s \) which are close to mass shell. Then, up to terms \( \mathcal{O}(k_-^{-1}) \), we can substitute \( \partial J + \partial^* J^* \) by \( \partial_+ J_- \),

\[ \mathcal{L}_1^{(3+)} = -\frac{g}{2}(J^a - \frac{1}{2} \partial_+ \partial^* J^a)A_+^a \]

\[ - \frac{g}{2} \left[ 2j^a + i \frac{1}{2} \partial \partial^* (\partial J^a - \partial^* J^a) \right] A^a . \]  

(2.20)

These approximations do not apply to the other vertex the currents of which describe gluons with small \( k_- \). We transform the currents using the relation

\[ \partial J^a + \partial^* J^* \equiv \partial_- J^a + \partial_+ J_- + 2i(\Box A^* T^a A) - 2i(A^* T^a \Box A) , \]  

(2.21)

which holds on the tree level, and obtain

\[ \mathcal{L}_1^{(3)} = \mathcal{L}_1^{(3+)} + \mathcal{L}_2^{(3-)} \]

\[ \mathcal{L}_1^{(3-)} = \mathcal{L}_1^{(3+)} + \frac{g}{4} \left( \frac{\partial^2}{\partial \partial^*} J_+^a \right) A_+^a \]
\[
\mathcal{L}_2^{(3-)} = \frac{g}{2} \frac{\partial}{\partial \partial^*} \left[ \left( \Box A^T A^a - (A^* T^a \Box A) \right) + \partial_+ (J_s^a + J_2^a) - \partial^* (J_s^{*a} + J_2^{*a}) \right] A^a_+ \\
+ \frac{g}{2} \frac{\partial}{\partial \partial^*} \left[ \partial(J_s^a + J_2^a) + \partial^* (J_s^{*a} + J_2^{*a}) \right] A^a . \tag{2.22}
\]

Notice that the expression for \( \mathcal{L}_1^{(3-)} \) does not change if we restrict the fields in the currents to the scattering modes \( A_s \).

The quartic terms induced by \( t \)-channel exchange are

\[
\langle \mathcal{L}_1^{(3+)} \mathcal{L}_1^{(3-)} \rangle_{\mathcal{A}_t} + \langle \mathcal{L}_1^{(3+)} \mathcal{L}_2^{(3-)} \rangle_{\mathcal{A}_t} . \tag{2.23}
\]

The contraction \( \langle \ldots \rangle_{\mathcal{A}_t} \) simply means substituting in view of the kinetic term (2.8) the product of two exchanged fields according to

\[
\langle A^a_+ A^b_+ \rangle_{\mathcal{A}_t} \rightarrow -\delta_{ab} \left( 1 + \frac{\partial_+ \partial_-}{\partial \partial^*} \right), \quad \langle A^a A^b \rangle_{\mathcal{A}_t} \rightarrow \delta_{ab} . \tag{2.24}
\]

We add the first term in (2.23) to the original quartic terms \( \mathcal{L}_{\text{scatt}}^{(4)} \) (2.19) and obtain (disregarding total derivatives)

\[
\langle \mathcal{L}_1^{(3+)} \mathcal{L}_1^{(3-)} \rangle_{\mathcal{A}_t} + \mathcal{L}_{\text{scatt}}^{(4)} = \\
= \frac{g^2}{8} J_s^a \frac{1}{\partial \partial^*} J_s^a - \frac{g^2}{16} \partial_+ J_s^a \frac{1}{\partial \partial^*} \partial_- J_s^a \\
+ \frac{g^2}{2} j_s^a j_s^a - \frac{g^2}{2} j_s^a j_s^a - \frac{g^2}{4} j_D j_D , \tag{2.25}
\]

where

\[
j_s^a = j^a + i \frac{1}{4 \partial \partial^*} (\partial J^a - \partial^* J^{*a}) . \tag{2.26}
\]

is the current appearing already in \( \mathcal{A}' \) channel in expression (2.12). We adopt the convention that the first current factor involves the fields describing the scattering gluons with large \( k_- \) and the second current the ones with large \( k_+ \). The form of this piece (2.25) coincides, up to the modifications to be discussed, with the effective action for quasi-elastic scattering.

For later reference we write also the second term in expression (2.23) explicitly

\[
\langle \mathcal{L}_1^{(3+)} \mathcal{L}_2^{(3-)} \rangle_{\mathcal{A}_t} = 
\]
\[ L_1^{(3)} = \frac{i g^2}{4} J^a - \frac{1}{2} \frac{1}{\partial^2} \left[ \partial (J^a_s + J^a_2) - \partial^* (J^*_a + J^*_s) \right] \]

\[ + \frac{1}{\partial^-} \left( \left[ (\Box A^a T^a A) - (A^a T^a \Box A) \right] \right) \]

\[ - \frac{g^2}{2} J^a_s \frac{\partial^-}{\partial \theta^a} \left[ (J^a_s + J^a_2) + \partial^* (J^*_a + J^*_s) \right] \]. \quad (2.27)

### 3 Integration over heavy modes

The triple vertices \( L_1^{(3)} \) induce further quartic terms by contracting two of them with an intermediate virtual gluon in the heavy mode \( A_1 \). Clearly also these terms contribute to high-energy quasi-elastic scattering. Assuming as above that two fields in the triple vertices \((2.12)\) carry large \( k^- \) and restricting to the accuracy \( O(k^1) \) we can write these terms as

\[ < L_1^{(3)} L_1^{(3)} >_{A_1} \]. \quad (3.1)

The other terms in \( L_1^{(3)} \) \((2.12)\) contribute only to order \( O(k^-1) \).

The relevant part of the action for obtaining these quartic terms is the kinetic term of the \( A_1 \) modes and the triple terms \( L_1^{(3+)} \) linearized in \( A_1 \)

\[ - 2 A_1^a \partial_+ \partial^- \left( 1 - \frac{\partial^*}{\partial_+ \partial^-} \right) A_1^a + L_1^{(3+)} \]. \quad (3.2)

The result of the approximate integration over \( A_1 \) in the functional integral can be expressed by the value of this action at the saddle point given by the Lagrangian term

\[ 2 A_1^a \partial_+ \partial^- \left( 1 - \frac{\partial^*}{\partial_+ \partial^-} \right) A_1^a \]. \quad (3.3)

Here \( A_{1C} \) is the solution of the linearized equation of motion derived from \((3.2)\)

\[ A_{1C} = \]

\[ = \frac{ig}{2} \left( \frac{1}{\partial_+} \tilde{A}_{1+} T^a A_s \right) \]

\[ - \frac{ig}{2} \left( \frac{\partial^-}{\partial_+} \tilde{A}_{1+} T^a \frac{1}{\partial_-} A_s \right) - \frac{ig}{2} \left( \frac{1}{\partial^2_+} \tilde{A}_{1+} T^a (1 - \frac{\partial^*}{\partial_+ \partial^-}) \partial_+ A_s \right) \]
\[
+ \frac{ig}{2} \left\{ \left( \frac{\partial^2}{\partial^2_+} \tilde{A}_t T^a \frac{1}{\partial_- A_s} \right) + \left( \frac{\partial^2}{\partial^2_-} \tilde{A}_t T^a \frac{\partial}{\partial_- A_s} \right) + \left( \frac{\partial}{\partial^2_+} \tilde{A}_t T^a \frac{\partial^*}{\partial_- A_s} \right) \right\}
+ \frac{g}{2} \left( \frac{1}{\partial_+} \tilde{A}_t T^a \frac{1}{\partial_- A_s} \right)
- \frac{ig}{2} \left( \frac{1}{\partial_+} \tilde{A}_t T^a \frac{\partial^*}{\partial_- A_s} \right) - \frac{ig}{2} \left( \frac{1}{\partial_+} \tilde{A}_t T^a \frac{\partial}{\partial_- A_s} \right).
\]

(3.4)

The leading contribution at large \( k_- \) arises only from the first term. We neglect terms contributing to \( \mathcal{O}(k_-^{-1}) \) and use the free equations of motion \( \Box A_s = 0 \) for the scattering modes.

The two fields in \( A_s \) modes in the result (3.3) can be expressed in terms of the currents introduced above. To illustrate how this works we pick up the terms where the leading term in (3.4) is multiplied with the one involving \( \tilde{A}_t' \)

\[
- \frac{ig^2}{2} \left\{ \left( A_s^* T^a \frac{1}{\partial_+} \tilde{A}_t \right) \left( \tilde{A}_t' T^a A_s \right) \right\}.
\]

(3.5)

We take into account that in the considered part of the mode separation \( \partial_- \tilde{A}_t \) is small compared to \( \partial_- A_s \) but \( \partial_+ \tilde{A}_t \) is large compared to \( \partial_+ A_s \) and obtain

\[
- \frac{ig^2}{2} \left[ \left( A_s^* T^a \frac{1}{\partial_+} \tilde{A}_t \right) \left( \tilde{A}_t' T^a A_s \right) - \left( \frac{1}{\partial_-} A_s^* T^a \frac{1}{\partial_+} \tilde{A}_t' \right) \left( \tilde{A}_t T^a \partial_- A_s \right) \right].
\]

(3.6)

In this approximation we transform the expression (3.6) by partial integration

\[
- \frac{ig^2}{2} \left[ \left( A_s^* T^a \frac{1}{\partial_+} \tilde{A}_t \right) \left( \tilde{A}_t' T^a A_s \right) - \left( A_s^* T^a \tilde{A}_t' \right) \left( \frac{1}{\partial_+} \tilde{A}_t T^a A_s \right) \right].
\]

(3.7)

Using the relation (2.16) for the generators we obtain finally

\[
\frac{ig^2}{2} \left( A_s^* T^a A_s \right) \left( \frac{1}{\partial_+} \tilde{A}_t T^a \tilde{A}_t' \right) = \frac{ig^2}{2} j^a \left( \frac{1}{\partial_+} \tilde{A}_t T^a \tilde{A}_t' \right).
\]

(3.8)

Writing the current involving the large \( k_- \) scattering modes as the first factor we can suppress all signs referring to the modes.
In this way we obtain for the quartic terms induced by the approximate heavy mode integration

\[
\mathcal{L}_4^{(3+)} \mathcal{L}_4^{(3+)} \rangle_{A_t} =
\]

\[
= \frac{ig^2}{8} j^a \left( \frac{1}{\partial_+} A_+ T^a A_+ \right)
\]

\[
+ \frac{g^2}{16} \left( \partial J^a + \partial^* J'^a \right) \left\{ i \left( \frac{1}{\partial_+} A_+ T^a \frac{1}{\partial_+} A_+ \right) + \frac{1}{\partial_+} A_+ T^a \frac{1}{\partial_+} A_+ + c.c. \right\}
\]

\[
+ \frac{ig^2}{2} \left( \frac{1}{\partial_+} A_+ T^a A' \right)
\]

\[
+ \frac{g^2}{16} \left( \partial J^a - \partial^* J'^a \right) \left\{ i \left( \frac{1}{\partial_+} A_+ T^a \frac{1}{\partial_+} A' \right) - \frac{1}{\partial_+} A_+ T^a \frac{1}{\partial_+} A_+ + c.c. \right\}
\]

\[
+ \frac{g^2}{8} \left( \partial A_+ D^r \frac{1}{\partial_+} A_+ \right) - i \left( \frac{1}{\partial_+} A_+ D^r \frac{1}{\partial_+} A' \right)
\]

\[
+ c.c. \right\}. \tag{3.9}
\]

The fact that the two fields in \( A_s \) modes carrying large \( k_- \) can be factorized in terms of the currents is the first sign of factorizability in quasi-elastic scattering which is essential for identifying the perturbative reggeons.

4 Quasi-elastic scattering

4.1 Quartic terms

Summarizing the results of the previous two sections we write down the sum of quartic terms, where two of the fields are in scattering modes with large \( k_- \) and the two others in all modes but with small \( k_- \) (see Fig. 3)

\[
\mathcal{L}_{tot}^{(4)} = \langle \mathcal{L}_1^{(3+)} (\mathcal{L}_1^{(3-)} + \mathcal{L}_2^{(3-)}) \rangle_{A_t}
\]

\[
+ \langle \mathcal{L}_1^{(3+)} \mathcal{L}_1^{(3+)} \rangle_{A_t} + \mathcal{L}_{scatt}^{(4)} \right\}. \tag{4.1}
\]

14
To avoid misunderstanding we say that $L_{\text{tot}}^{(4)}$ does not mean all quartic terms but a set of terms complete in the sense that it allows to extract the vertices of the effective action.

We have observed that the two fields in scattering modes factorize in terms of the currents. We shall first study the terms, where this current is $J^a$ or $(\partial J^a + \partial^* J^a)$. The latter can be replaced by $\partial_+ J^a$ (compare Eq. (2.21)). These terms correspond to the exchange of $A_{t+}$.

The terms with currents $j^a$ or $j^a_s$ (Eq. (2.26)) are related to the exchange of $A_{t}'$. Further there are terms with $j^r_D$ which correspond to the representation symmetric in the colour indices in the $t$-channel, the generators of which we have denoted by $D^r$ (2.18).

In each case we shall consider in particular the contribution to elastic scattering obtained by restricting also the other two fields to the scattering modes. In this way we shall arrive at the effective action $L_{\text{eff,scatt}}$ describing high-energy quasi-elastic scattering.

### 4.2 $A_+$ channel

From Eq. (4.1) supplemented by Eqs. (2.25), (2.27) and (3.9) we obtain disregarding total derivatives

$$L_{\text{tot}}^{(4)}|_{A_+} =$$

$$= g^2 \frac{1}{8} J^a \left\{ \frac{1}{\partial \partial^*} \left( 1 + \frac{1}{2} \frac{\partial_+ \partial_-}{\partial \partial^*} \right) [J^a_+ + 2i(\partial(J^a_s + J^a_2) - \partial^*(J^{sa} + J^{2a}))] 
+ 2i \frac{1}{\partial_-} \left( (\square A^T a A) - (A^T a \square A) \right) + \frac{1}{2} \left( \frac{\partial_+}{\partial \partial^*} \right)^2 J^a_+ + i \left( \frac{1}{\partial_+} A^T a A_+ \right) 
- \frac{1}{2} \frac{\partial_+}{\partial \partial^*} \left[ i \left( \frac{\partial_+}{\partial_+} A^T a \frac{1}{\partial_+} A_+ \right) + i \left( \frac{1}{\partial_+} A^T a \frac{1}{\partial_+} A_+ \right) 
+ \left( \frac{\partial_+}{\partial_+} A^T a \frac{1}{\partial_+} A_+ \right) \right] \right\}. \tag{4.2}$$

If we restrict all fields to the scattering modes, where we have $\square A_s = 0$, a number of terms vanishes obviously. We shall see that the contribution to quasi-elastic scattering is given by the first term only with the substitution of $J^a_+$ by

$$J^a_{R+} = i \left( A^T a \frac{\rightarrow}{\partial_+} A_R \right), \quad A^a_R = - \frac{\partial^r}{\partial_+} A^{ra}. \tag{4.3}$$
We can say that all the other terms just result in dressing the "bare" current to become $J_{R+}^a$. Notice that for $A_s$ modes the relation between $A_R^a$ and $A^a$ is just the gauge transformation which leads from the gauge $A_R^a = 0$, which is used here, to the gauge $A^a = 0$. Clearly this structure of the result is just what should be expected from parity symmetry, which implies the symmetry under the exchange of indices + and − and the gauge transformation $(4.3)$

$$A \rightarrow A_R \ .$$ (4.4)

The contribution to quasi-elastic scattering with $A_+$ exchange can be written as

$$L_{eff,scatt}^{(4)}|_{A_+} = \frac{g^2}{8} J_+^a \frac{1}{\partial^+} J_{R+}^a - \frac{g^2}{16} \left( \partial^+ J_+^a \right) \frac{1}{(\partial^+)^2} \partial^- J_{R+}^a .$$ (4.5)

To prove this assertion it is convenient to use the following "dressing relations", leading to the replacement of the current $J_+$ by the "dressed" one $J_{R+}$,

$$J_+^a + i \partial \partial^* \left( \frac{1}{\partial^+} A_+ T^a A_+ \right) + 2i \left( \partial (J_s^a + J_2^a) - \partial^* (J_s^{*a} + J_2^{*a}) \right)$$

$$= J_{R+}^a - i \partial^+ \left[ (\partial^- A_+ T^a \frac{1}{\partial^+} A_+) - (\frac{\partial}{\partial^*} A T^a A) - \left( \frac{\partial}{\partial^-} A^* T^a A^* \right) \right] .$$ (4.6)

$$\frac{i}{2} \left( \frac{\partial^*}{\partial^-} A_+ T^a \frac{1}{\partial^+} A' \right) - \frac{i}{2} \left( \frac{\partial}{\partial^-} A_+ T^a \frac{1}{\partial^+} A' \right)$$

$$= \left( \frac{\partial}{\partial^*} A T^a A \right) + \left( \frac{\partial}{\partial^*} A^* T^a A^* \right) .$$ (4.7)

$$i \left( \partial^- J_+^a + \partial_+ J_+^a \right) - 2 \partial^- (\partial (J_s^a + J_2^a) - \partial^* (J_s^{*a} + J_2^{*a}))$$

$$+ 2 \partial \partial^* \left( \partial^- A_+ T^a \frac{1}{\partial^+} A_+ \right) = i \left( \partial_+ J_{R-}^a + \partial_- J_{R+}^a \right)$$

$$+ 2 \partial_+ \left[ \partial \left( \partial_+ T^a \frac{1}{\partial^+} A_+ \right) + \partial^* \left( \partial^- A^* T^a \frac{1}{\partial^+} A_+ \right) \right] .$$ (4.8)

The relations hold only if all fields involved are in the $A_s$ modes (which we have not indicated explicitly to simplify the notations). They are derived by straightforward calculations, where the condition $\square A_+ = 0$ is used repeatedly. With these relations applied to $(4.2)$ we obtain $(4.3)$ immediately.
4.3 $A'$ channel

From Eq. (4.1) supplemented by Eqs. (2.25), (2.27) and (3.9) we obtain

$$L^{(4)}_{\text{tot}}|_{A'} =$$

$$= g^2 j_s^a \left\{ j_s^a - \frac{1}{2} \frac{\partial}{\partial \theta^s} (\partial (J_s^a + J_2^a) + \partial^* (J_s^{*a} + J_2^{*a})) + \frac{i}{2} \left( \frac{1}{\partial_+} A_+ T^a A' \right) \right\}$$

$$- \frac{g^2}{2} j^a j^a$$

$$+ \frac{g^2}{4} (j_s^a - j^a) \left\{ i \left( \frac{\partial}{\partial_+} A_+ T^a \frac{1}{\partial} A' \right) - \left( \frac{\partial}{\partial_+} A_+ T^a \frac{1}{\partial_+} A_+ \right) + c.c. \right\}.$$

(4.9)

If we restrict all fields to the scattering modes we obtain a simple expression where also the two fields with small $k_-$ can be expressed in currents $j^a_{R s}$ and $j^a_R$. These currents are given by the expressions for $j_s^a$ and $j^a$ with substituting $A^a$ by $A^a_R$ in analogy to Eq. (4.3).

As in the previous subsection it is convenient to use ”dressing relations” which hold if all fields involved are in the $A_s$ modes, with $\Box A_s = 0$,

$$2j_s^a - j^a + i \left( \frac{1}{\partial_+} A_+ T^a A' \right) - \frac{1}{2} \frac{\partial}{\partial \theta^s} (\partial (J_s^a + J_2^a) + \partial^* (J_s^{*a} + J_2^{*a}))$$

$$= 2j^a_{R s} - j^a_R$$

(4.10)

$$j^a + \frac{i}{2} \left[ \left( \frac{\partial}{\partial_+} A_+ T^a \frac{1}{\partial} A' \right) + c.c. \right] = j^a_R.$$

(4.11)

Applying these relations to (4.9) we obtain that quasi-elastic scattering with $A'$ exchange is described by

$$L_{\text{eff,scatt}}|_{A'} = g^2 j_s^a j^a_{R s} - \frac{g^2}{2} j^a j^a_R.$$

(4.12)

4.4 Symmetric gauge group channel

From Eq. (4.1) supplemented by Eqs. (2.25) and (3.9) we have

$$L^{(4)}_{\text{tot}}|_D = -\frac{g^2}{4} j^a_D \left\{ j^a_D - \frac{1}{2} \left[ \left( \frac{\partial}{\partial_+} A_+ D^a \frac{1}{\partial_+} A_+ \right) \right.$$}

$$\left. - i \left( \frac{\partial}{\partial_+} A_+ D^a \frac{1}{\partial} A' \right) + c.c. \right\}.$$

(4.13)
As in the other cases we study the special configuration where all fields are in the modes $A_s$. Now we have as the ”dressing relation”

$$j^r_D + \frac{1}{2} \left[ i \left( \frac{\partial}{\partial \tau} A + D^r \frac{1}{\partial} A' \right) + c.c. \right] = j^r_{RD} \quad (4.14)$$

and obtain as the symmetric gauge-group channel contribution to quasi-elastic scattering

$$\mathcal{L}_{\text{eff,scatt}} |_D = -\frac{g^2}{4} j^r_D j^r_{RD} \quad (4.15)$$

### 4.5 Effective action for quasi-elastic scattering

The high-energy scattering of gluons at the tree level can be effectively described by the sum of terms (4.5), (4.12), (4.15) up to corrections of order $O(s^{-1})$. It is remarkable that we have a sum of terms factorized with respect to the $t$-channel. Parity symmetry implies that the currents describing the scattering at large $k_-$ have their counterparts in the case of scattering at large $k_+$ obtained from the former by replacing indices $- \rightarrow +$ and by the gauge transformation $A \rightarrow A_R$ (4.4). Due to this symmetry the result has always the bilinear form of products of currents factorizable in the $t$-channel. It is enough to study the triple terms $\mathcal{L}^{(3)}_1$ describing large $k_-$ scattering and the quartic terms of the original action $\mathcal{L}^{(4)}_{\text{scatt}}$ to find out what kind of terms arise in the effective action of high-energy quasi-elastic scattering.

Indeed we see that the quartic terms $< \mathcal{L}^{(3+)}_1 \mathcal{L}^{(3-)}_1 >_{A_+ + \mathcal{L}^{(4)}_{\text{scatt}}} (2.25)$ coincide with the ones in (4.3), (4.12) and (4.15) up to the index $R$ at the second current indicating the substitution $A \rightarrow A_R$. This dressing is the only effect of the contribution from the heavy mode intermediate state and of terms in $\mathcal{L}^{(3)}_1$ negligible in the large $k_-$ scattering. The lengthy explicit expression for the complete quartic terms $\mathcal{L}^{(4)}_{\text{tot}}$ is necessary only in the derivation of the effective vertices of gluon production discussed in the next section.

We introduce 5 pairs of pre-reggeon fields, one pair for each current times current term. In this way the quasi elastic high-energy scattering can be described by the following effective action

$$\mathcal{L}_{\text{eff,scatt}} = \mathcal{L}_{\text{kin}} + \mathcal{L}_{s-} + \mathcal{L}_{s+}$$

$$\mathcal{L}_{\text{kin}} = -2 A^a_+ (\partial_- \partial_+ - \partial_\tau^* \partial^* \partial^a_+) A^a_+ - 2 A^a_+ \partial \partial^a_{+} A^a_+ + A^a_{(+) A^a_{(-)}}$$

18
\[-A'^{(a(+)}_aA'^{(a(-)}_a + 2A'^{(a(+)}_aA'^{(a(-)}_a + B'^{r(+)B'^{r(-)}\]

\[\mathcal{L}_{s-} = -\frac{g}{2} J^a_+ A^a_+ - \frac{g}{4} \left( \frac{\partial_+}{\partial y^a} J^a_+ \right) A^a_+ - g j^a_s A'^{(a(+)}_s A'^{(a(-)}_s - g j^a_s A'^{(a(+)}_s - \frac{g}{2} j^a_D B'^{r(+)} \cdot \]

\[\mathcal{L}_{s+}\] is obtained from \[\mathcal{L}_{s-}\] by replacing the labels \ [+\leftrightarrow -\] and the currents by their partners with label \(R\). This result is checked simply by observing that each of the quartic scattering terms (4.5), (4.12), (4.15) is reproduced from (4.16) by integrating out the corresponding pre-reggeon.

Each pre-reggeon is represented by a pair of fields. The distinction by the labels \(+\) and \(-\) is related to the fact that the reggeon exchange is oriented in rapidity: The pre-reggeon fields with label \(+\) couple to scattering gluons with large \(k_-\) only and vice versa.

The pre-reggeon \((A^{a}_+, A^a_+)\) is the leading one. It contributes as \(s^1\) to the amplitude and it is well known that the exchange of two of them, interacting via emission and absorption of \(s\)-channel gluons, results in the BFKL pomeron. The pre-reggeon \((A^{a}_{(\pm)}, A^a_{(\pm)})\) can be regarded as the non-leading partner of the first one. Both conserve helicity of the scattering gluon and both carry positive \(P\)-parity and negative \(C\)-parity.

In the \(A'\) channel, considered in the sub-section 4.3 we encounter two pre-reggeons at the non-leading level \(O(s^0)\), \(A'^{(a(\pm)}_s A'^{(a(\pm)}_2\). Both conserve helicity but their vertices depend on the sign of the helicity. They carry negative \(P\)- and \(C\)-parities. The difference between them is not obvious. At this point we just notice that the coupling of \(A'^{(a(\pm)}_s\) involves a term sensitive to the transverse momenta of the scattering gluons whereas \(A'^{(a(\pm)}_2\) is insensitive to transverse momenta.

The pre-reggeons discussed so far are in the gauge-group state of the gluon. The \(t\)-channel factorization unavoidably leads also to exchanges in other gauge-group representations, which arise as the symmetric part in the tensor product of two adjoint representations. Among the corresponding pre-reggeons there is a colour singlet one.

5 Inelastic scattering

5.1 Terms of order 5

The essential ingredients of the high-energy effective action beyond the effective action of quasi-elastic scattering are the production vertices. In general
these vertices can be obtained from the effective interaction terms of order 5, where the fields are in scattering modes $A_s$ and their longitudinal momenta correspond to the kinematics of an inelastic $2 \rightarrow 3$ scattering in multi-Regge kinematics \((2.6)\), which we denote by $\mathcal{L}^{(5)}$. The production vertices, triple vertices involving two pre-reggeons and one scattering particle, are obtained by factorizing the scattering vertices $\mathcal{L}_{s^-}$ and $\mathcal{L}_{s^+}$

\[
\mathcal{L}^{(5)} = < \mathcal{L}_{s^-} \mathcal{L}_p \mathcal{L}_{s^+}> .
\]  

Here the brackets stand for contraction of the pre-reggeons by substituting their product by propagators read off from the kinetic terms in $\mathcal{L}_{\text{eff,scatt}}$ \((4.16)\).

A contribution to $\mathcal{L}^{(5)}$ in the desired mode configuration is obtained from $\mathcal{L}_{\text{tot}}^{(4)}$ by contracting with $\mathcal{L}_{1}^{(3-)}$, $< \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_{1}^{(3-)}>_{A_t}$. $\mathcal{L}_{\text{tot}}^{(4)}$ involves two fields in $A_s$ mode with large $k_-$ and $\mathcal{L}_{1}^{(3-)}$ two fields in $A_s$ mode with large $k_+$. One of the two other fields in $\mathcal{L}_{\text{tot}}^{(4)}$ is to be restricted to the scattering mode and the other to the exchange mode.

In this contribution the factorization in the two $t$-channels is immediate, because the pair of fields with large $k_-$ is factorized in $\mathcal{L}_{\text{tot}}^{(4)}$, \((1.2)\), \((1.9)\), \((1.13)\) and the pair of fields with large $k_+$ is factorized in $\mathcal{L}_{1}^{(3-)}$. Therefore the production vertices involving only those pre-reggeons the couplings (currents) of which appear in $\mathcal{L}_{1}^{(3\pm)}$ can be read off directly from $\mathcal{L}_{\text{tot}}^{(4)}$. This does not apply only to one of the odd-parity reggeons $A^{\pm}$ and to the gauge-group symmetric pre-reggeons $B^{\tau(\pm)}$.

Besides of the terms discussed we have more contributions. To order 5 the interaction terms in the mode configuration of interest are given by (Fig. 4)

\[
\mathcal{L}^{(5)} = < \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_{1}^{(3-)}>_{A_t} + < \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_{2}^{(3-)}>_{A_t} + < \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_{1}^{(3+)}>_{A_1} + < \mathcal{L}_{1}^{(3+)} \mathcal{L}_{\text{scatt}}^{(4)}>_{A_t} + < \mathcal{L}_{1}^{(3+)} \mathcal{L}_{\text{scatt}}^{(4)}>_{A_1} .
\]  

In sec.4 we have seen that the heavy mode contributions and the contribution with $\mathcal{L}_{2}^{(3-)}$ merely lead to the ”dressing” of the currents involving the large $k_+$ scattering modes. Analogously, the second and third terms in \((5.2)\) contributes only to dressing the currents in $\mathcal{L}_{s^+}$ \((4.16)\). The structure of the result appears already in the sum of the other terms and the effective production vertices can be obtained without writing the second and the third terms in \((5.2)\) explicitely.
5.2 $A_+ - A_+$ exchanges

As discussed above the production vertices involving the even-parity pre-reggeons $A_{\pm}$ and $A_{(\pm)}$ can be read off directly from the complete quartic terms $L_{tot}^{(4)}|_{A_{+}}$ (12). We recall that $J^a_-$ involves the two scattering fields with large $k_-$. One of the other two fields has to be restricted to the scattering modes too. We may restrict ourselves to the contributions where the inelastic gluon corresponds to the field $A^*_s$. The production vertex with $A_s$ is then obtained by complex conjugation. The remaining field in $L_{tot}^{(4)}$ is to be projected onto $A_+$ in the $t$-channel modes, because the wanted contribution $< L_{tot}^{(4)} L_1^{(3-)} >$ to $L_{p}^{(5)}$ is obtained by contraction with $L_1^{(3-)}|_{A_+} = \frac{g}{4} \left( \frac{\delta^2}{\partial \partial^*} J^a_+ \right) A^a_{t+}$.

For illustration we pick up in $L_{tot}^{(4)}$ (4.2) the term $i \frac{g^3}{8} J^a_- \left( \frac{1}{2} A_+ T^a A_+ \right)$. Its contribution to the inelastic production with $A^*_s$ is given by

$$i \frac{g^3}{32} \left\{ \left( J_+ T^a (1 + \frac{\partial_+ \partial_-}{\partial \partial^*}) \frac{1}{\partial \partial^*} J_+ \right) + \partial_+ \left( J_+ T^a (1 + \frac{\partial_+ \partial_-}{\partial \partial^*}) \frac{1}{\partial \partial^*} J_+ \right) \right\} \frac{1}{\partial} A^*_s \cdot$$

(5.3)

For the leading exchange related to $J_{\pm}$ the correction to the propagator $(1 + \frac{\partial_+ \partial_-}{\partial \partial^*})$ has to be included. Up to terms contributing to order $O(\frac{1}{s})$ we obtain

$$i \frac{g^3}{16} \left\{ \left( J_+ T^a \frac{1}{\partial \partial^*} J_+ \right) + \partial_+ \left( J_+ T^a \frac{\partial_-}{(\partial \partial^*)^2} J_+ \right) + 2 \partial_+ \left( J_+ T^a \frac{\partial_-}{(\partial \partial^*)^2} J_+ \right) \right\} \frac{1}{\partial} A^*_s \cdot$$

(5.4)

The last term can be transformed in the following way

$$\left( \partial_+ J_+ T^a \frac{1}{\partial \partial^*} J_+ \right) \frac{1}{\partial} A^*_s =$$

$$= - \left( \partial_+ J_+ T^a \frac{\partial_-}{\partial \partial^*} J_+ \right) \frac{1}{\partial \partial^*} A^* s + \left( \partial_+^2 J_+ T^a \frac{1}{\partial \partial^*} J_+ \right) \frac{1}{\partial \partial^*} A^*_s \cdot \quad (5.5)$$

The second term gives a small contribution. We have used the condition $\Box A_s = 0$. 

21
Factorizing $L_{s\pm}$ we obtain the corresponding contribution to $L_p$,

$$
ig \left\{ (\partial^s A T^a A^s) - \frac{1}{2} \left( \partial^s A_{(-)} T^a \frac{1}{\partial^s} A_{(+)} \right) \right.
$$

$$
- \partial_+ \left( \partial^s A T^a \frac{1}{\partial^s} A_{(+)} \right) - \frac{1}{2} \frac{\partial}{\partial^s} \left( \partial^s A_{(-)} T^a A_{+} \right) \left( \frac{1}{\partial^s} A_{s}^a \right) \right\} (5.6)
$$

Repeating the outlined procedure with all terms in $L^{(4)}_{tot} (4.2)$ we obtain the effective production vertices in the channels with the pre-reggeons $A_{\pm}$ and their non-leading partners $A_{(-)}$.

$$
\left. L_p |_{A_{\pm}} = \right.
$$

$$
= - \frac{ig}{2} \left\{ \left[ 2 (\partial^s A T^a \partial A_{+}) + \frac{1}{2} \left( A_{(-)} T^a \partial A_{(+)} \right) \right] \left( \frac{1}{\partial^s} A^s_a \right) + \left[ \left( \frac{1}{\partial^s} A_{(-)} T^a \partial A_{+} \right) + \frac{1}{\partial^s} \left( \partial^s A_{(-)} T^a \partial A_{+} \right) \right] \left( \frac{\partial}{\partial^s} A^s_a \right) + \left[ \left( \partial^s A T^a \frac{1}{\partial^s} A_{(+)} \right) + \frac{1}{\partial^s} \left( \partial^s A T^a \partial A_{(+)} \right) \right] \left( \frac{\partial}{\partial^s} A^s_a \right) + c.c. \right\}. (5.7)
$$

5.3 $A' - A'$ exchange

We consider first the contribution in the term $< L^{(4)}_{tot} L^{(3-)} > |_{A'}$ to these channels obtained by combining $L^{(4)}_{tot} |_{A'} (4.9)$ with $L^{(3-)}_{1} |_{A'} = -g j_s A^a, (2.22)$,

$$
< L^{(4)}_{tot} L^{(3-)} > |_{A' - A'} =
$$

$$
= - i g \left\{ \frac{3}{4} \left( 2 \left( \frac{1}{\partial^s} j_s T^a \partial j_s \right) + \left( \partial^s j_s T^a \frac{1}{\partial^s} j_s \right) \right) + \left( \partial^s j_s T^a \frac{\partial}{\partial^s} j_s \right) + \partial^s \left( (j_s - j) T^a \frac{1}{\partial^s} j_s \right) \right\} \left( \frac{1}{\partial^s} A^s_a \right) + c.c. \right\}. (5.8)
$$
Now we calculate the contribution of $<\mathcal{L}^{(3+)}_1\mathcal{L}_{\text{scatt}}^{(4)}>\vert_{A_t}$ using

$$\mathcal{L}^{(3+)}_1\vert_{A'} = -g_j^a A^a, \quad \mathcal{L}_{\text{scatt}}^{(4)}\vert_{A'} = -\frac{g_j^a}{2} j^a.$$

(5.9)

One of the two fields in the first current factor in $\mathcal{L}_{\text{scatt}}^{(4)}\vert_{A'}$ corresponds to the produced particle and the other involves the $t$-channel modes to be contracted with $\mathcal{L}^{(3+)}_1\vert_{A'}$,

$$\mathcal{L}_{\text{scatt}}^{(4)}\vert_{A'} \rightarrow \frac{ig^2}{4} \left( \frac{1}{\partial} A'^a j \right) A'^a + c.c. \quad .$$

(5.10)

This implies

$$<\mathcal{L}^{(3+)}_1\mathcal{L}_{\text{scatt}}^{(4)}>_{A_t} \vert_{A'-A'} = -\frac{ig^3}{4} \left( \frac{1}{\partial} j^a T^a j \right) A'^a + c.c. \quad .$$

(5.11)

The next term to be evaluated is $<\mathcal{L}_{\text{scatt}}^{(4)}\mathcal{L}^{(3+)}_1>_{A_t}$ supplying the bremsstrahlung correction to the original quartic vertex caused by the leading term in $\mathcal{L}^{(3)}_1$ (2.12). In section 3 we have done the calculation for the heavy mode contributions $\mathcal{L}^{(3)}_1\vert_{A'} = -g_j^a A^a$ resulting in Eq. (3.8). The heavy mode contribution involving the quartic vertex $\mathcal{L}_{\text{scatt}}^{(4)}\vert_{A'}$ is given by the analogous expression with the substitution $A'^a \rightarrow \frac{g}{2} j^a$. This gives immediately the result:

$$<\mathcal{L}_{\text{scatt}}^{(4)}\mathcal{L}^{(3+)}_1>_{A_t} \vert_{A'-A'} = -\frac{ig^3}{4} \left( j^a T^a j \right) \frac{1}{\partial} A'^a + c.c. \quad .$$

(5.12)

For the sum of the calculated contributions we have

$$<\mathcal{L}_{\text{scatt}}^{(4)}\mathcal{L}^{(3+)}_1>_{A_t} + <\mathcal{L}_{\text{scatt}}^{(4)}\mathcal{L}^{(3+)}_1>_{A_t} \vert_{A'-A'} = -\frac{ig^3}{4} \left\{ 2 \left( \frac{1}{\partial} j^a T^a \partial j_s \right) + 2 \left( \partial^s j^a T^a \frac{1}{\partial^s j_s} \right) 
+ \left( \frac{\partial^s j^a T^a}{\partial^s j_s} \right) + (j^a T^a j) 
- \left( \frac{1}{\partial} j^a T^a \right) + (j^a T^a j) 
- \left( \partial^s j^a T^a \frac{1}{\partial^s j_s} \right) - (j^a T^a j) \right\} \frac{1}{\partial} A'^a + c.c. \quad .$$

(5.13)

The factorization with respect to $\mathcal{L}_{s\pm}$ amounts to the replacement

$-g j_s^a \rightarrow A_s^a, \quad \frac{g}{2} j^a \rightarrow A'^a$ and we obtain the effective production vertices

$$\mathcal{L}_p\vert_{A'-A'} = \quad$$
\[
\begin{align*}
&= -\frac{ig}{4} \left\{ 2 \left( \frac{1}{\partial s} A_s^{(-)} T^a \partial A_s^{(+)} \right) + 2 \left( \partial^* A_s^{(-)} T^a \frac{1}{\partial^* A_s^{(+)}} \right) \\
&+ \left( \frac{\partial^*}{\partial s} A_s^{(-)} T^a \frac{\partial}{\partial s^* A_s^{(+)}} \right) + \left( A_s^{(-)} T^a A_s^{(+) \rangle} \right) + 4 \left( A_2^{(-)} T^a A_2^{(+) \rangle} \right) \\
&+ 2 \left( \frac{1}{\partial s} A_s^{(-)} T^a \partial A_s^{(+) \rangle} \right) + 2 \left( A_s^{(-)} T^a A_s^{(+) \rangle} \right) \\
&+ 2 \left( \partial^* A_2^{(-)} T^a \frac{1}{\partial^* A_2^{(+) \rangle}} \right) + 2 \left( A_2^{(-)} T^a A_2^{(+) \rangle} \right) \right\} \frac{1}{\partial s} A_s^{a} + c.c. \quad (5.14)
\end{align*}
\]

### 5.4 $A' - A_+$ exchanges

Unlike the previous case the production vertices can be obtained from the quartic terms $\mathcal{L}_{tot}^{(4)}| A'_+ \rangle$. We calculate the part $< \mathcal{L}_{tot}^{(4)} \mathcal{L}_1^{(3-) \rangle}$ obtained by contracting the quartic terms with $\mathcal{L}_1^{(3-)}| A_+ \rangle$,

\[
< \mathcal{L}_{tot}^{(4)} \mathcal{L}_1^{(3-)} > | A'_+ - A_+ \rangle = \]

\[
\frac{g^3}{8} \left\{ \partial_- \left( \frac{1}{\partial s} j_s T^a \frac{1}{\partial s^* J_+} \right) + \frac{\partial}{\partial s} \left( j_s T^a \frac{1}{\partial s^* J_+} \right) \right\} (5.15)
\]

The remaining contributions in (5.12) lead merely to the "dressing" substitution $J_+ \to J_{R+}$. The corresponding contribution to the effective production vertices is obtained factorizing $\mathcal{L}_s^+| A'_+ \rangle$ and $\mathcal{L}_s^-| A_+ \rangle$. The result is obtained from (5.13) by the following substitutions:

\[
g j^a_s \to A^a_s, \quad \frac{g}{2} j^a \to A^a, \quad -\frac{g}{2} \frac{1}{\partial s^+} j^a \to A^a, \quad \frac{g}{2} \frac{1}{\partial s^2} j^a \to A^a, \quad \frac{g}{2} \frac{1}{\partial s^+}, \quad j^a \to A^a_+ \]

\[
\mathcal{L}_p| A'_+ - A_+ \rangle = \]

\[
= -\frac{g}{2} \left\{ -\partial_- \left( \frac{1}{\partial s} A_s^{(-)} T^a \partial A_+ \right) - \partial_- \left( A_s^{(-)} T^a \partial A_+ \right) \\
- 2 \frac{\partial}{\partial s} \left( A_2^{(-)} T^a \partial A_+ \right) + 2 \left( A_2^{(-)} T^a \frac{1}{\partial s^* A_+} \right) \right\} \frac{1}{\partial s} A^a + c.c. \quad (5.16)
\]

The case where the reggeon $A_{\pm}$ couples to the incoming particle with large $k_-$ and $A'$ to the one with large $k_+$ looks more complicated. Similar to the $A' - A'$ case in subsection 5.3 besides of $< \mathcal{L}_{tot}^{(4)} \mathcal{L}_1^{(3-)} >$ more
contributions have to be evaluated explicitly. The result \( \mathcal{L}_{\ell} \mid_{A_+ - A'} \) can be obtained however from the above expression (5.16) by interchanging the labels \( + \leftrightarrow - \) on the reggeon field and longitudinal derivatives and by replacing \( \frac{1}{\sigma} A^a \leftrightarrow - \frac{1}{\sigma} A^a \), which is just the \( P \)-parity transformation (4.4).

### 5.5 Vertices with colour-symmetric pre-reggeons

Analogous as in the previous subsection we consider first the case that the non-octet reggeon \( B^{r(\pm)} \) is in the \( t \)-channel connected to the large \( k_- \) scattering particles. It is not difficult to check that there is no gluon production vertex with the reggeons \( B^{r(\pm)} \) in both \( t \)-channels. The vertex with \( B^{r(+)} \) coupling to large \( k_- \) can be read off from the corresponding terms in \( \langle \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_{\text{tot}}^{(3-)} \rangle \). The relevant projection is (4.13),

\[
\mathcal{L}_{\text{tot}}^{(4)} \mid_{D} = \frac{g^2}{8} j_D \left\{ \left( \frac{\partial^r}{\partial^+} A_+ D^r \frac{1}{\partial^\ast \partial^+} A_+^\ast \right) + i \left( \frac{\partial^r}{\partial^+} A_+ D^r \frac{1}{\partial^\ast} A^\ast \right) + \text{c.c.} \right\} \\
- \frac{g^2}{4} j_D j_D^r . \tag{5.17}
\]

We consider first the case of the reggeons \( A_{\pm} \) or \( A_{(\pm)} \) in the other \( t \)-channel, i.e. we contract

\[
\mathcal{L}_{1}^{(3-)} \mid_{A_+} = \frac{g}{4} \left( \frac{\partial^2}{\partial\partial^\ast J_+^a} \right) A^a_+ \tag{5.18}
\]

and obtain

\[
\langle \mathcal{L}_{\text{tot}}^{(4)} \mathcal{L}_{1}^{(3-)} \rangle_{B^{(-)} - A_+} = \frac{g^3}{32} j_D^r \left\{ 2 \left( \frac{\partial^r}{\partial} A^s_+ D^r \frac{1}{\partial^\ast \partial^+} J_+ \right) + \left( \frac{\partial^r}{\partial} A^s_+ D^r \frac{\partial}{\partial\partial^2} J_+ \right) \right\} + \text{c.c.} . \tag{5.19}
\]

We recall the notation for the bracket with \( D^r \)

\[
(A_1 D^r A_2) B^r = A_1^a A_2^b B^r D_{ab} \tag{5.20}
\]

and introduce the notation

\[
(B D^a A_2) A_1^a = A_1^a A_2^b B^r D_{ab} \tag{5.21}
\]
for the same expression for convenience in order to write
\[
< \mathcal{L}_{tot}^{(4)} \mathcal{L}_{1}^{(3-)} >_{B^{(-)}-A_+} =
\frac{g^3}{32} \left\{ -2 \frac{\partial}{\partial s} \left( j_D \bar{D}^a \frac{1}{\partial^*} J_+ \right) - \partial^* \left( j_D \bar{D}^a \frac{\partial}{\partial \partial^*} J_+ \right) \right\} \frac{1}{\partial} A_s^a + c.c. .
\] (5.22)

The remaining contributions in (5.2) lead to the dressing substitution \( J_+ \rightarrow J_{R+} \). We obtain the effective production vertices by factorising \( \mathcal{L}_{s\pm} \),
\[
\mathcal{L}_p|_{B^{(-)}-A_+} =
= \frac{g}{2} \left\{ \frac{\partial}{\partial s} \left( B^{(-)} \bar{D}^a \partial A_+ \right) - \frac{1}{2} \partial^* \left( B^{(-)} \bar{D}^a \frac{1}{\partial^*} A_{(+)} \right) \right\} \frac{1}{\partial} A_{a^*} + c.c. .
\] (5.24)

The vertex \( \mathcal{L}_p|_{A_- - B^{(+)}_s} \) can be obtained by the substitutions discussed at the end of section 5.4 which are implied by parity symmetry. The calculation directly from (5.2) takes to evaluate more terms.

Now we consider the case of a pre-reggeon \( A'_s \) in the second \( t \)-channel. There is no vertex with \( B^{(2-)}_s \) in one and \( A'^{(+)}_s \) in the other \( t \)-channel. This is easily seen because \( \mathcal{L}_{1}^{(3-)} \) does involve \( j_s \) but not \( j \).

We contract \( \mathcal{L}_{tot}^{(4)} \) with
\[ \mathcal{L}_1^{(3-)} \big|_{A'} = -g j_s^a A'^a \] (5.25)
and obtain
\[
< \mathcal{L}_{tot}^{(4)} \mathcal{L}_1^{(3-)} >|_{B'A'} =
= \frac{ig^3}{8} \partial^* \left( j_D \bar{D}^a \frac{1}{\partial^*} j_s \right) \frac{1}{\partial} A'^{a} + c.c. .
\] (5.26)

The remaining contribution in (5.2) lead to the dressing substitution \( j_s \rightarrow j_{R_s} \). We obtain the effective production vertex by factorizing \( \mathcal{L}_{s\pm} \),
\[
\mathcal{L}_p|_{B' - A'} =
= -\frac{ig}{4} \partial^* \left( B^{(-)} \bar{D}^a \frac{1}{\partial^*} A'^{a(+)}_s \right) \frac{1}{\partial} A'^{a} + c.c. .
\] (5.27)

The vertex \( \mathcal{L}_p|_{A' - B} \) can be obtained by the parity substitution (4.4).
6 The high-energy effective action

The effective action of high-energy scattering is obtained from the effective action of quasi-elastic scattering (4.16) by adding the effective production vertices \( \mathcal{L}_p \)

\[
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{eff,scatt}} + \mathcal{L}_p. \tag{6.1}
\]

The production vertices \( \mathcal{L}_p \) have been obtained for all pre-reggeons appearing in gluodynamics in the vicinity of \( j = 1 \) and \( j = 0 \) in Eqs.(5.7), (5.14), (5.16), (5.27)

\[
\mathcal{L}_p = \mathcal{L}_p|_{A_+ - A_+} + \mathcal{L}_p|_{A_- - A_+} + \mathcal{L}_p|_{A_+ - A_-} + \mathcal{L}_p|_{A_- - A_-} + \mathcal{L}_p|_{B_+ - A_+} + \mathcal{L}_p|_{A_+ - B_+} + \mathcal{L}_p|_{A_- - B_-} + \mathcal{L}_p|_{A_+ - B_-} + \mathcal{L}_p|_{B_+ - A_-} + \mathcal{L}_p|_{B_- - A_+}. \tag{6.2}
\]

The effective action can be written in terms of the complex field \( \phi^a = i \frac{1}{D} A^a \), describing the two helicity states of scattering gluons, and the pre-reggeon fields \( A_\pm, A_{(\pm)}, A_{(\pm)}', A^{(\pm)}, B^{(\pm)} \).

The parity transformation, interchanging the incoming particles, acts on the terms of the action by interchanging labels \(+ \leftrightarrow -\) accompanied by interchanging \( \phi \) and \( \phi^* \) (i.e. inversion of the helicities of scattering gluons). \( \mathcal{L}_{\text{kin}} \) is symmetric under this transformation. The currents representing the scattering vertices \( J_-, J, J^*, j_s, j, j_D \) of large \( k_- \) transform into the corresponding currents of vertices with large \( k_+ \), \( J_{R+}, J_R, J^*_{R}, j_{R}, j_R, j_{RD} \), respectively. This means that \( \mathcal{L}_{s+} \) and \( \mathcal{L}_{s-} \) transform into each other. The production vertices \( \mathcal{L}_p \) are symmetric under the parity transformation. In this way we check that the whole effective action is parity symmetric, as expected.

In the effective action the longitudinal and transverse space-time dimensions are separated to a large extent. Besides of the kinetic term of the scattering fields \( \phi \), all terms are invariant under separate scale transformations in longitudinal and in transverse directions. All terms are invariant with respect to longitudinal Lorentz boosts and with respect to rotations in the transverse plane.

The notations for the pre-reggeon fields have been chosen in such a way that they all are invariant with respect to transverse rotations. The rotation acts on the complex numbers \( x \) describing the impact parameters as

\[
x \rightarrow e^{i\alpha} x. \tag{6.3}
\]
The (transverse) gluon fields $A$ transform in the same way and the derivatives in the opposite way: $\partial \rightarrow e^{-i\alpha} \partial$.

We have written all vertices in $\mathcal{L}_p$ in such a way that no transverse derivatives act on $\phi^a = i \frac{1}{2} A^\alpha_a$ or $\phi^*$, which are invariant under (6.3). We consider the vertices as transitions of a ”left” (coupling to large $k_-$ gluons) to a ”right” (coupling to large $k_+$ gluons) pre-reggeon. Consider for example the leading pre-reggeon vertices

$$- ig \left[ (\partial^\alpha A T^a \partial A) \phi^* + (\partial A T^a \partial^\alpha A) \phi^a \right].$$

(6.4)

In the production term of $\phi^*$ the state related to the left reggeon $A_-$ transforms as $e^{i\alpha}$, i.e. it has transverse angular momentum $n = 1$. One can read this term as a transfer of the angular momentum $n = 1$ from the ”left” to the ”right” pre-reggeon. In the production vertex of $\phi$ the angular momentum $n = -1$ is transferred from the left to the right.

We observe that in general non-negative angular momentum $n \geq 0$ is transferred from left to right in all production vertices of $\phi^*$ whereas in the terms of $\phi$ we have the transfer of $n \leq 0$ only.

In the $\phi^*$ vertices we encounter $n = 1$ in the leading $(A_- , A_+)$ vertex, $n = 0, 1, 2$ in the other vertices.

Notice, in particular, that the transfer of $n = 1$ is absent in the non-leading positive-parity pre-reggeons $A_\pm$ and that there is only zero angular momentum transfer in the $(A_2^{(n=0)} , A_2^{(n=1)})$ vertex.

7 Summary

In this paper we have extended the high-energy effective action to the next-to-leading exchange contributing to $O(s^0)$ to the amplitude. Staying within the multi-Regge approximation, i.e. assuming the $s$-channel multiparticle intermediate states to obey the conditions (2.6), we improve the approximation in all transformations related to the separation of modes, integration over heavy modes and extraction of effective vertices keeping all terms suppressed by one power of $s$ compared to the leading ones. We have introduced pre-reggeon fields describing in the effective action the next-to-leading exchanges. For this the factorizability of the quartic terms for high-energy scattering is essential. We encounter several pre-reggeons at the level $O(s^0)$: Some of them carry the gauge group representation of the gluon and some
of them other representations. Among the first ones we have the next-to-leading partner of the leading reggeized gluon carrying positive $P$-parity and insensitive to the helicity of scattering partons. Two pre-reggeons carry odd $P$-parity and are sensitive to the helicity. The physical difference between the latter is one of the questions to be investigated in more detail. One of them is sensitive to the transverse momenta and the other not. More general, it has to be investigated whether all these pre-reggeons reggeize in the same way as the leading gluon exchange.

The effective vertices describing emission or absorption of a gluon from the exchanged pre-reggeons is an essential part of our result. Whereas the corresponding effective vertices in the case of the leading gluons and quarks [1], [14] where known long before the effective action approach to high-energy scattering was proposed, the effective vertices for the next-to-leading exchanges appear here the first time. To investigate these vertices more closely will be our next task.

The investigations along these lines contribute to a deeper understanding of the high-energy asymptotics of gauge theories. There is the hope that some of the interesting structure uncovered in perturbation theory is significant in general. On the other hand there are problems arising from the present day phenomenology of semi-hard processes to which the next-to-leading high-energy effective action can be applied.

Acknowledgements

L.Sz. would like to acknowledge the warm hospitality extended to him during his stay at University of Leipzig.

References

[1] L.N. Lipatov, Sov.J.Nucl.Phys. 23(1976)338
   V.S. Fadin, E.A. Kuraev and L.N. Lipatov, Phys. Lett. 60B(1975)50; Sov.Phys. JETP 44(1976)443; ibid 45(1977)199
   Y.Y. Balitski and L.N. Lipatov, Sov.J.Nucl.Phys. 28(1978)882
[2] J. Bartels, Nucl. Phys. B175(1980)365
   J. Kwiecinski and M. Praszalowicz, Phys. Lett. 94B(1980)413

[3] V.S. Fadin and L.N. Lipatov, Sov. Phys. JETP Lett. 49(1989)352; Sov.
   J. Nucl. Phys. 50(1989)712;
   S. Catani, M. Ciafaloni and F. Hautmann, Phys. Lett. B242 (1990) 97;
   Nucl. Phys. B366 (1991) 3
   J.C. Collins and R.K. Ellis, Nucl. Phys. B360 (1991) 3
   V.S. Fadin and L.N. Lipatov, Nucl. Phys. B406(1993)259
   V.S. Fadin, R. Fiore and A. Quatarolo, Phys. Rev. D50 (1994) 5893
   V.S. Fadin, Zh. Eksp. Teor. Fiz. Pisma 61 (1995) 342
   M.I. Kotsky and V.S. Fadin, Yadernaya Fizika 59(6) (1996) 1
   V.S. Fadin, R. Fiore and A. Quatarolo, Phys. Rev. D53 (1996) 2729
   V.S. Fadin, R. Fiore and M.I. Kotsky, Phys. Lett. B359 (1995) 181;
   Phys. Lett. B387 (1996) 593; Phys. Lett. B389 (1996) 737
   V.S. Fadin and L.N. Lipatov, Nucl. Phys. B447 (1996) 767
   G. Camici and M. Ciafaloni, Phys. Lett. B386 (1996) 341; Nucl. Phys.
   B496 (1997) 305; Phys. Lett. B412 (1997) 396
   V.S. Fadin, M.I. Kotsky and L.N. Lipatov, [hep-ph/9704267], to appear
   in Phys. Lett. B
   V.S. Fadin, R. Fiore, A. Flachi and M.I. Kotsky, [hep-ph/9711427], to
   appear in Phys. Lett. B
   V.S. Fadin and L.N. Lipatov, [hep-ph/9802290]

[4] L.N. Lipatov, JETP Lett. 59(1994)571
   L.D. Fadeev and G.P. Korchemsky, Phys. Lett. 342B(1994)311

[5] J. Bartels, Zeit. f. Physik C60(1993)471
   J. Bartels and M. Wüsthoff, Zeit. f. Physik C66(1995)155

[6] A. White, Int. J. Mod. Phys. A6(1991)1859; ibid A8(1993)4755

[7] J. Bartels, B.I. Ermolaev and M.G. Ryskin, Zeit. f. Physik
   C70(1996)273, ibid C72(1996)627

[8] B.I. Ermolaev, R. Kirschner and L. Szymanowski, work in progress
   R. Kirschner, [hep-ph/9710253], Proceedings of the workshop "Deep-
   inelastic scattering off polarized targets", DESY-Zeuthen, September
   1997

30
[9] L.N. Lipatov, Nucl. Phys. B365(1991)614

[10] R. Kirschner, L.N. Lipatov and L. Szymanowski, Nucl. Phys. B452(1994)579; Phys. Rev. D51(1995)838

[11] L.N. Lipatov, Nucl. Phys. B452(1995)369

[12] R. Kirschner and L. Szymanowski, Phys. Rev. D52(1995)2333

[13] R. Kirschner and L. Szymanowski, Phys. Lett. B419 (1998) 348

[14] V.S. Fadin and V.E. Sherman, Sov. Phys. JETP 45(1977)861
Figure 1: The inelastic process in the multi-Regge kinematics
Figure 2: The graphical illustration of vertices entering Eqs. (2.10) and (2.12).

Figure 3: The graphical illustration of Eq. (4.1) for the complete quartic terms. Different line forms represent different modes: full line - scattered modes, dotted line - exchange modes, bold line - heavy modes, dashed line - the sum of all modes.
Figure 4: The graphical illustration of the terms describing inelastic scattering Eq. (5.2) and of the factorization leading to the production vertices Eq. (5.1).