Better detection of Multipartite Bound Entanglement with Three-Setting Bell Inequalities

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It was shown in Phys. Rev. Lett. 87, 230402 (2001) that N (N ≥ 4) qubits described by a certain one parameter family F of bound entangled states violate Mermin-Klyshko inequality for N ≥ 8. In this paper we prove that the states from the family F violate Bell inequalities derived in Phys. Rev. A56, R1682 (1997), in which each observer measures three non-commuting sets of orthogonal projectors, for N ≥ 7. We also derive a simple one parameter family of entanglement witnesses that detect entanglement for all the states belonging to F. It is possible that these new entanglement witnesses could be generated by some Bell inequalities.

I. INTRODUCTION

Entanglement has been a very useful resource for information processing. Some form of entangled pure states were required for achieving the desired results in quantum applications like quantum dense coding Ref. [1], quantum teleportation Ref. [2] and so forth. In practical situations, we do not have pure entangled states due to noise from the environment. Consequently, there is a need to distill using only local operations and classical communications (LOCC) some amount of pure state entanglement. While it is true that all pure states with non-zero entanglement can be transformed by LOCC with some probability to maximally entangled states, there is no general criterion for determining if a mixed entangled state can be distilled. Surprisingly, it was shown by Horodecki [3] that there are mixed states which cannot be distilled despite being entangled. Such states are now known as bound entangled states. The discovery of bound entanglement Ref. [4] immediately posed two important questions: Firstly, is bound entanglement useful in quantum information? Secondly, do bound entangled states admit local, realistic description?

The answer to the first question was first given in Ref. [4] where it was shown that one can increase the fidelity of the so-called conclusive teleportation with the aid of bound entanglement. Moreover, the increase in fidelity beyond some threshold cannot be achieved without the use of bound entanglement. Therefore, bound entanglement can be a valuable resource in quantum information processing.

However, there is only a partial answer to the second question. On the one hand it was shown in Ref. [5] that some bound entangled N-qubit states from the family F do not admit local, realistic description for N ≥ 8 and yet on the other hand it was shown numerically that there exists a bound entangled state Ref. [6] in 3 ⊗ 3 Hilbert space which does not violate local realism Ref. [7].

The main difficulty in answering the second question lies with the fact that we do not have analytical tools (usually, Bell inequalities) that provide necessary and sufficient conditions for the existence of a local, realistic description of a bi- or multi-partite quantum systems except for the two-qubit case. In the two-qubit case, one has the Clauser-Horne inequalities but unfortunately no bound entangled states exist. Recently a set of Bell inequalities for higher dimensional bipartite systems Ref. [8, 9] as well as for multipartite qubit systems Ref. [10, 11] were discovered. However, these inequalities have not been proven to be sufficient conditions for the existence of a local realistic description. Indeed, it was shown that the inequalities derived in Ref. [10] are not violated for certain pure non-maximally entangled states Ref. [12]. This means that these inequalities are only necessary condition for a local realistic description. The only available method currently known uses a linear programming algorithm to extract necessary and sufficient conditions for local realism. However, this algorithm is computationally inefficient.

Note that the most versatile tool possible for investigating the existence of a local realistic description of quantum correlations, namely Bell inequalities, has a serious drawback. The reason is that Bell inequalities are generally derived for the situation in which each observer measures two non-commuting observables (i.e. two settings of measuring apparatus). Therefore, even if we have a set of necessary and sufficient Bell inequalities for two settings of measuring apparatus and if these inequalities are not violated for a given state, it may happen that there exists another set of Bell inequalities utilizing more than two settings of measuring apparatus that could be violated by the state. Thus, although there is strong numerical evidence that this is not the case for the two-qubit systems Ref. [13], it was shown in Ref. [14] that one can derive Bell inequalities, in which each observer uses three settings of the measuring apparatus, to obtain a stronger violation of local realism than the inequalities for two settings.

In this paper, we will show that the three-setting Bell inequalities derived in Ref. [15] are violated by all the states...
from the family $\mathcal{F}$ for $N \geq 7$. Note that violation of the two-setting Mermin-Klyshko Ref. \[16\] inequalities requires an additional qubit. From the experimental perspective, the reduction of even one qubit has a distinct advantage in which one knows that it is always easier to produce and control fewer qubits. Moreover, the lowering of the number of qubits using three-setting Bell inequalities may imply that perhaps one can violate local realism even with four qubits using states from the above family and stronger Bell inequalities, perhaps with four or more settings. The family $\mathcal{F}$ is also the first example of entangled states for which $d$-setting ($d > 2$) Bell inequalities reveal entanglement whereas two-setting ones do not.

We have also derived, based on the structure of the one parameter family of entanglement witnesses generated by the three-setting Bell inequalities, a one parameter family of entanglement witnesses that can detect entanglement for all the bound entangled states from the family $\mathcal{F}$ for $N \geq 4$.

II. THREE-SETTING BELL INEQUALITY

In Ref. \[15\], a series of Bell inequalities for $N$ entangled qubits, in which each observer measures three non-commuting observables, was derived. It was shown that the violation of these inequalities by the GHZ state $|\psi\rangle$ of the form

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle^\otimes N + |1\rangle^\otimes N),$$

where the two states $|0\rangle, |1\rangle$ form an orthogonal basis in the Hilbert space of each qubit is stronger than Mermin-Klyshko Ref. \[16\] inequalities for $N \geq 4$.

Let us first obtain the Bell operator generated by the inequalities in Ref. \[15\]. The inequalities have the following structure

$$-2^{N-1}\sqrt{3} \leq \sum_{k_1,k_2,...,k_N=1}^3 c_{k_1,k_2...k_N} E_{k_1,k_2...k_N} \leq 2^{N-1}\sqrt{3}.$$ (2)

The coefficients of the inequality read $c_{k_1,k_2...k_N} = \cos(\phi_{k_1}^1 + \phi_{k_2}^2 + \ldots + \phi_{k_N}^N)$, where $\phi_1^1 = \frac{2\pi}{3}, \phi_2^1 = \frac{\pi}{3}, \phi_3^1 = \frac{5\pi}{6}$ and $\phi_k^n = 0, \phi_2^n = \frac{\pi}{3}, \phi_3^n = \frac{2\pi}{3}$ for $n = 2, 3, \ldots, N$ are the phases associated with three sets of two orthogonal projectors corresponding to the three settings of the measuring apparatus measured by each observer. Here, the superscripts denote the observers and the subscripts enumerate the set of the projectors. We assume that each set of projectors is obtained from the set $P_{0}^k(0) = |0\rangle\langle 0|, P_{0}^k(1) = |1\rangle\langle 1|$ by the rotation using the unitary operators $U(\phi_1^k), U(\phi_2^k, U(\phi_3^k))$ of the form

$$U(\phi_1^k) = \frac{1}{\sqrt{2}} \left( \exp(i\phi_1^k) - \exp(i\phi_1^k) \right),$$ (3)

where $l = 1, 2, 3$. Therefore, the $k$-th observer in the $l$-th experiment measures two projectors $P_{l}^k(0) = U(\phi_1^k)|0\rangle\langle 0| U(-\phi_1^k), P_{l}^k(1) = U(\phi_2^k)|1\rangle\langle 1| U(-\phi_2^k)$. Corresponding to the result of the measurement of the projector $P_{l}^k(0)$, we ascribe the value $-1$ whereas we ascribe the value $+1$ corresponding to the result of the measurement of the projector $P_{l}^k(1)$. $E_{k_1,k_2...k_N}$ is the correlation function defined in the standard way

$$E_{k_1,k_2...k_N} = Tr(\rho \sum_{l_1,l_2,...,l_N=1}^2 (-1)^{l_1+l_2+...+l_N} P_{k_1}(l_1) \otimes P_{k_2}(l_2) \otimes \ldots \otimes P_{k_N}(l_N)),$$ (4)

where $\rho$ is an arbitrary quantum state.

The Bell operator $B_N$ for the inequality reads

$$B_N = \sum_{k_1,k_2,...,k_N=1}^3 c_{k_1,k_2...k_N}$$

$$\sum_{l_1,l_2,...,l_N=1}^2 (-1)^{l_1+l_2+...+l_N} P_{k_1}(l_1) \otimes P_{k_2}(l_2) \otimes \ldots \otimes P_{k_N}(l_N).$$ (5)
It is convenient to write this operator in the matrix form. Using the basis defined by the vectors |00...0⟩, |00...1⟩, |00...10⟩, |00...11⟩, ..., |11...10⟩, |11...11⟩, this operator reads

\[ B_N = \begin{pmatrix} 0 & 0 & \cdots & 0 & (-3)^N \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ (-3)^N & 0 & \cdots & 0 & 0 \end{pmatrix} \]  

(6)

Note that only the matrix elements ⟨00...0|B_N|11...1⟩ and ⟨11...1|B_N|00...0⟩ are non zero.

It is easy to see that for the GHZ state |0⟩ Tr(B_N|ψ⟩⟨ψ|) = (-3)^N 2, i.e., one has a violation of the inequality 3. For N ≥ 4, this violation is stronger than the violation obtained through the Mermin-Klyshko inequality.

It is instructive to note that the Bell operator B_N(α_N), of the form

\[ B_N(α_N) = U(\frac{α_N}{N})^0 B_N U(\frac{−α_N}{N})^N, \]  

(7)

where U(α_N) = |0⟩⟨0| + exp(iα_N)|1⟩⟨1|, is optimal for the violation of the three-setting Bell inequality for the state |ψ(α_N)⟩ = \frac{1}{\sqrt{2}}(|0⟩^N + exp(iα_N)|1⟩^N). To see this, one observes that the state |ψ(α_N)⟩ can be obtained through local rotation by each observer using the unitary transformation U(α_N) on their portion of the state |ψ⟩.

III. VIOLATION OF LOCAL REALISM

It was shown in Ref. 3 that the following one-parameter family F of N-qubit states

\[ \rho_N(α_N) = \frac{1}{(N + 1)}(|ψ(α_N)⟩⟨ψ(α_N)|) + \frac{1}{2} \sum_{k=1}^{N} (P_k + \bar{P}_k), \]  

(8)

where P_k is a projector on the state |00...010...0⟩ with 1 being on the k-th position and \(\bar{P}_k\) is a projector on the state |11...101...11⟩ with 0 being on the k-th position, is a family of states that are entangled but which cannot be distilled if N ≥ 4. By distillation we understand, following Dur, the impossibility of extracting any pure entangled state from the states belonging to the family by means of LOCC. Dur also showed that for \(α_N = \frac{π}{\sqrt{2(N−1)}}\), these states violate Mermin-Klyshko inequality for N ≥ 8. The latter result means that they do not admit a local realistic description. As noted earlier, this is the first known example of bound entangled states violating local realism.

Let us now apply the rotated Bell operator in Eq.(3) to \(\rho_N(α_N)\). A straightforward computation gives

\[ Tr(B_N(α_N)\rho_N(α_N)) = \frac{(-3)^N}{2(N + 1)}. \]  

(9)

To obtain a violation of the inequality 3 we must have \(\frac{(-3)^N}{2(N + 1)} > 2^{N−1}\sqrt{3}\) which happens for N ≥ 7.

Therefore, we have shown that local realism is violated for the bound entangled states from the given family if N ≥ 7 regardless of the parameter α_N.

Moreover, the strength of violation of local realism for the three-setting Bell inequalities presented here is greater than for Mermin-Klyshko inequalities. In our case, the strength of violation is defined as the minimal amount \(V_N(0 ≤ V_N ≤ 1)\) of pure noise \(\rho_{N\text{noise}} = 2^{-N} I ⊗ I ⊗ \cdots ⊗ I\) one has to add to the state \(\rho_N(α_N)\) so that the resulting state \(\sigma_N(α_N) = (1 − V_N)\rho_N(α_N) + V_N\rho_{N\text{noise}}\) does not admit a local realistic description. We have \(Tr(B_N(α_N)\sigma_N(α_N)) = (1 − V_N)\frac{(-3)^N}{2(N + 1)}\). Thus, the violation occurs for \(V_N > \frac{2^{N−2(3N−N)(N+1)}}{\sqrt{3}}\). For instance for N = 7 or 8 one must add around 19% or 39% of noise respectively in order to allow the state \(\rho_{(7\text{or } 8)}(α_{(7\text{or } 8)})\) to have a local realistic description when using the three-setting inequality. It is important to note that the amount of noise necessary for a local realistic description in the case of Mermin-Klyshko inequalities for N = 8 (for N = 7 there is no violation) is 20%.

It is still an open question whether there exist stronger Bell inequalities that are violated by all the states from the family, i.e. for every N ≥ 4 and for any choice of the parameter α_N. We next provide some numerical evidences that for N = 4 one cannot violate local realism if each observer is allowed to measure two or three sets of projectors.
FIG. 1: Function of the amount of noise, $V_N$, as a variation of the number of qubits, $N$, for the three-setting and two-setting inequalities.

As shown in Ref. [14, 17, 18] one can check, using linear programming algorithm, if a given quantum state admits local realistic description. Due to the computational complexity of linear programming one must resort to numerical methods. Let us briefly describe the idea of testing local realism by means of linear programming Ref. [14, 17, 18].

In a Bell experiment involving $N$ observers measuring $M$ sets of projectors (each set consisting of two orthogonal projectors) on a $N$-qubit state, the outcome is a set of $M \times 2^N$ probabilities which we can denote as $p(l_1(k_1)l_2(k_2)\ldots l_N(k_N); \vec{\phi}_{k_1}, \vec{\phi}_{k_2},\ldots, \vec{\phi}_{k_N})$ where $k_i = 1, 2, \ldots, M$ ($i = 1, 2, \ldots, N$). These labels, $k_i = 1, 2, \ldots, M$ ($i = 1, 2, \ldots, N$), tell us which set of projectors is measured by $i$-th observer. The index $l_i(k_i) = 0, 1$ denotes the outcome of the measurement for $i$-th observer should he measure the $k_i$-th set of projectors. The vector $\vec{\phi}_{k_i}$ is a set of real parameters (the vector notation has only a symbolic meaning) defining the $k_i$-th set of projectors (for instance, $\vec{\phi}_{k_i}$ has three components if each observer applies $SU(2)$ transformation to his qubit). The correlations between the outcomes of the local measurements performed by the observers are the only information available according to quantum mechanics. Nevertheless, a local realistic theory tries to go further. In a local realistic theory, the basic assumption is that each particle carries a probabilistic or deterministic set of instructions regarding how to respond to all possible local measurements that it might be subjected to. Therefore local realism assumes the existence of non-negative joint probabilities (summing up to unity) involving all possible observations from which it should be possible to obtain all the quantum predictions as marginals. Let us denote these hypothetical probabilities by $p^{HV}(l_1(1), l_1(2), l_1(M); l_2(1), l_2(2),\ldots, l_2(M);\ldots; l_N(1), l_N(2),\ldots, l_N(M))$. The local realistic probabilities for experimentally observed events are the marginals

$$p^{HV}(l_1(k_1), l_2(k_2),\ldots, l_N(k_N))$$

$$\sum_{l_1(m_1)\neq l_1(k_1)} \sum_{l_2(m_2)\neq l_2(k_2)} \ldots$$

$$\sum_{l_N(m_N)\neq l_N(k_N)} p^{HV}(l_1(1), l_1(2), \ldots, l_1(M); l_2(1), l_2(2), \ldots, l_2(M);\ldots; l_N(1), l_N(2),\ldots, l_N(M)). \quad (10)$$

The $M \times 2^N$ equations in (10) form the complete set of necessary and sufficient conditions for the existence of a local realistic description of the experiment. Thus, if it is possible to find such a joint probability distribution so that

$$p(l_1(k_1), l_2(k_2),\ldots, l_N(k_N); \vec{\phi}_{k_1}, \vec{\phi}_{k_2},\ldots, \vec{\phi}_{k_N}) = p^{HV}(l_1(k_1), l_2(k_2),\ldots, l_N(k_N)) \quad (11)$$
for the given choice of projectors then the quantum probabilities have a local realistic description.

Replacing $p^{HV}(l_1(k_1), \ldots, l_2(k_2), \ldots, l_N(k_N))$ by the right-hand side of (10) and putting it into (11), we get a set of $M \times 2^N$ linear equations with $2^{M+N}$ unknowns (joint probabilities). Therefore, we have more unknowns than equations. Moreover we have a set of linear constraints on the unknowns since these unknowns must be non-negative and they must also sum up to unity.

The linear programming algorithm allows us to check if there is a solution to the above set of equations (for details see Ref. [3]). However, we must remember that the left-hand side of the equations depends on the parameters defining the measured projectors. Therefore even if there is a solution to the equations for some choice of the projectors, we do not know if such a solution exists for some other set of the projectors.

We applied the above method to the bound entangled states from the given family for $N = 4$ with $\alpha_N$ being 0, $\frac{\pi}{2}$, $\pi$ and in which for each choice of $\alpha_N$, each observer can measure two or three sets of projectors. The sets of projectors were obtained by the rotation of the projectors $|0\rangle\langle 0|, |1\rangle\langle 1|$ through unitary operators chosen from the $SU(2)$ group. For each possible case, we picked 1000 randomly chosen sets of projectors for each observer. We found solutions to the appropriate set of equations in all cases. This strongly suggests that one cannot violate local realism with the four qubit states chosen from the family. Perhaps, one should consider wider sets of projectors but this computation lies beyond current capability of the computers at our disposal.

IV. STRONG ENTANGLEMENT WITNESSES

Multipartite entanglement witness $W$ is a hermitian operator with the property that $Tr(W_P \otimes P_2 \otimes \ldots \otimes P_N) \geq 0$ for any projectors $P_k$ ($k = 1, 2, \ldots, N$) and there exist some entangled states $\rho_{ent}$ for which $Tr(W_P \rho_{ent}) < 0$. In the latter case, we say that $W$ detects the entanglement of $\rho_{ent}$.

The family of entanglement witnesses associated with the Bell inequality [3] has the form

$$W_N(\alpha_N) = 2^{N-1} \sqrt{3}I - |B|_N(\alpha_N),$$

where $|B|_N(\alpha_N)$ is the operator from Eq. (3) with the matrix elements replaced by their moduli. It is convenient to “normalize” the operators in the above equation by dividing them with the number $2^{N-1}\sqrt{3}$. Therefore we need only consider the entanglement witnesses of the form

$$W_N = \begin{pmatrix}
1 & 0 & \ldots & 0 & -\frac{3^N}{2^{N-1}\sqrt{3}}
0 & 1 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
-\frac{3^N}{2^{N-1}\sqrt{3}} & 0 & \ldots & 0 & 1
\end{pmatrix},$$

(13)

It is interesting to find a family of entanglement witnesses having the similar structure to the above ones but that can detect entanglement for all the states from the family $\mathcal{F}$, i.e., for $N \geq 4$. To this end let us consider a new family of entanglement witnesses $S_N$ of the form

$$S_N = \begin{pmatrix}
1 & 0 & \ldots & 0 & -|S_N| \\
0 & 1 & \ldots & 0 & 0 \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
-|S_N| & 0 & \ldots & 0 & 1
\end{pmatrix},$$

(14)

where $S_N$ is a real number.

We note that the operators $S_N$ must first be positive on all the product projectors $P_1 \otimes P_2 \otimes \ldots \otimes P_N$. Therefore, we have the following condition for $|S_N|

$$0 \leq Tr(S_N P_1 \otimes P_2 \otimes \ldots \otimes P_N) = 1 + 2|S_N|Re[\prod_{k=1}^{N} \langle 0|P_k|1\rangle]$$

$$= 1 + 2|S_N|Re[\exp(-\sum_{k=1}^{N} \phi_k) \prod_{k=1}^{N} \sin \theta_k \cos \theta_k] = 1 + 2^{1-N}|S_N|\cos(\sum_{k=1}^{N} \phi_k) \prod_{k=1}^{N} \sin 2\theta_k,$$

(15)

where we used the fact that every pure state of a qubit can be written as $\cos(\theta)|0\rangle + \exp(i\phi)\sin(\theta)|1\rangle$. The above equation is positive for arbitrary product projectors if $1 - 2^{1-N}|S_N| \geq 0$, which implies that $|S_N| = \kappa_N 2^{N-1}$ for $0 \leq \kappa_N \leq 1$. 
Let us then check for the range of values of $\kappa_N$ so that the entanglement witnesses can detect entanglement for states within the family and determine the minimal value of $N$. Before doing this let us notice that it is enough to consider the states from $\mathcal{F}$ for which $\alpha_N = 0$. If $\alpha_N \neq 0$ then we simply rotate $S_N$ as done previously for the operator $B_N$.

A straightforward computation gives us $Tr(S_N \rho_N (\alpha_N = 0)) = \frac{1}{1+\kappa_N} (1 - \kappa_N 2^{N-1} + N)$, which is negative if $\kappa_N > \frac{1+N}{2N}$. Therefore, it is enough to put $\kappa_N = 1$, in which case the entanglement witnesses $S_N$ (strictly speaking rotated entanglement witnesses) detect entanglement for all states from the family $\mathcal{F}$, i.e., for $N \geq 4$.

It may be possible that the entanglement witnesses $S_N$ can be obtained from some Bell inequality, perhaps using more than three settings of measuring apparatus.

V. CONCLUSIONS

We have shown that the three-setting Bell inequalities derived in Ref. [15] are better for the detection of multipartite bound entanglement for the family $\mathcal{F}$ presented in Ref. [5]. Application of the three-setting Bell inequalities allows us to detect bound entanglement for seven qubits whereas such a detection is possible for two-setting Bell inequalities only for eight qubits and only for certain choice of the parameter $\alpha_N$. We have also provided some numerical evidence suggesting that bound entanglement of four qubits from the family $\mathcal{F}$ cannot be detected in a Bell experiment in which each observer uses two or three settings of the measuring apparatus.

We have derived a family of entanglement witnesses that detect bound entanglement for all members of the family $\mathcal{F}$, i.e., for four and more qubits. The structure of these new entanglement witnesses resembles the structure of entanglement witnesses generated by two- and three-setting Bell inequalities suggesting that it may be possible to find some Bell inequalities that can detect bound entanglement for all members of the family $\mathcal{F}$.

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[1] C.H. Bennett and S.J. Wiesner, Phys. Rev. Lett. 69, 2881 (1992)
[2] C.H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Peres and W.K. Wootters, Phys. Rev. Lett., 70 1895 (1993).
[3] P. Horodecki, Phys. Lett. A 232, 233 (1997).
[4] P. Horodecki, M. Horodecki and R. Horodecki, Phys. Rev. Lett. 82, 1056 (1999).
[5] W. Dur, Phys. Rev. Lett. 87, 230402 (2001).
[6] C. H. Bennett, D. DiVincenzo, T. Mor, P. Shor, J. Smolin and B. Terhal, Phys. Rev. Lett. 82, 5385 (1999).
[7] D. Kaszlikowski, M. Żukowski, P. Gnacinski, Phys. Rev. A 65, 032107 (2002).
[8] D. Kaszlikowski, L. C. Kwek, Jing-Ling Chen, M. Żukowski and C. H. Oh, Phys. Rev. A 65, 032118 (2002).
[9] D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Phys. Rev. Lett. 88, 040404 (2002).
[10] M. ˙Zukowski, C. Brukner, Phys. Rev. Lett. 88, 210401 (2002).
[11] R. F. Werner and M. M. Wolf, quant-ph/00102024.
[12] M. Żukowski, C. Brukner, W. Laskowski, and M. Wiesniak 88, 210402 (2002).
[13] V. Scarani and N. Gisin, J. Phys. A:Math Gen, 34, 6043 (2001).
[14] M. Żukowski, D. Kaszlikowski, and A. Baturz, quant-ph//9910058.
[15] M. Żukowski, D. Kaszlikowski, Phys. Rev. A 56, R1682-R1685 (1997).
[16] N.D. Mermin, Phys. Rev. Lett. 65, 1838 (1990); A.V. Belinskii and D.N. Klyshko, Phys. Usp., 36, 653 (1993).
[17] D. Kaszlikowski, quant-ph//0008086.
[18] D. Kaszlikowski, P. Gnacinski, M. Żukowski, W. Miklaszewski, and A. Zeilinger, Phys. Rev. Lett. 85, 4418 (2000).