Electric Field Induced Kondo Tunneling Through Double Quantum Dot

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It is shown that the resonance Kondo tunneling through a double quantum dot (DQD) with even occupation and singlet ground state may arise at a strong bias, which compensates the energy of singlet/triplet excitation. Using the renormalization group technique we derive scaling equations and calculate the differential conductance as a function of an auxiliary dc-bias for parallel DQD.

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Many fascinating collective effects, which exist in strongly correlated electron systems (metallic compounds containing transition and rare-earth elements) may be observed also in artificial nanosize devices (quantum wells, quantum dots, etc). Moreover, fabricated nanoobjects provide unique possibility to create such conditions for observation of many-particle phenomena, which by no means may be reached in "natural" conditions. Kondo effect (KE) is one of such phenomena. It was found theoretically\textsuperscript{4} and observed experimentally\textsuperscript{5} that the charge-spin separation in low-energy excitation spectrum of quantum dots under strong Coulomb blockade manifests itself as a resonance Kondo-type tunneling through a dot with odd electron occupation \( N \) (one unpaired spin \( 1/2 \)). This resonance tunneling through a quantum dot connecting two metallic reservoirs (leads) is an analog of resonance spin scattering in metals with magnetic impurities. A Kondo-type tunneling may be observed under conditions which do not exist in conventional metallic compounds. In particular, Kondo effect survives in essentially non-equilibrium state when the strong bias \( eV \gg T_K \) is applied between the leads\textsuperscript{6} (\( T_K \) is the Kondo temperature which determines the energy scale of low-energy spin excitations in a quantum dot). The KE may be observed as a dynamical phenomenon in strong time dependent electric field\textsuperscript{7}, it may arise at finite frequency under light illumination\textsuperscript{8}. Even the net zero spin of isolated quantum dot (even \( N \)) is not the obstacle for the resonance Kondo tunneling. In this case it may be observed in specific types of double quantum dots (DQD)\textsuperscript{9} or induced by strong magnetic field\textsuperscript{10} whereas in conventional metals magnetic field only suppresses the Kondo scattering. The latter effect was also observed experimentally\textsuperscript{11}.

As was noticed in\textsuperscript{12}, quantum dots with even \( N \) possess the dynamical symmetry \( SO(4) \) of spin rotator in the Kondo tunneling regime, provided the low-energy part of its spectrum is formed by a singlet-triplet (ST) pair, and all other excitations are separated from the ST manifold by a gap noticeably exceeding the tunneling rate \( \gamma \). A DQD with even \( N \) in a side-bound configuration where two wells are coupled by the tunneling \( V \) and only one of them (say, \( l \)) is coupled to metallic leads \( (L,R) \) is a simplest system satisfying this condition\textsuperscript{13}. Such system was realized experimentally in Ref.\textsuperscript{14}.

In the present paper one more unusual property of DQDs with even \( N \) is revealed. It is shown that in the case when the ground state is singlet \( |S\rangle \) and the ST gap \( \delta \gg T_K \), a Kondo resonance channel arises under a strong bias \( eV \) comparable with \( \delta \). The channel opens at \( |eV|\sim\delta < T_K \), and the Kondo temperature is determined by the non-diagonal component \( J_{ST} = \langle T | J | S \rangle \) of effective exchange induced by the electron tunneling through DQD (Fig. 1b).

The basic properties of symmetric DQD occupied by even number of electrons \( N = 2n \) under strong Coulomb blockade in each well are manifested already in the simplest case \( n = 1 \), which is considered below. Such DQD is an artificial analog of a hydrogen molecule \( H_2 \). If the inter-well Coulomb blockade \( Q \) is strong enough, one has \( N = n_l + n_r \), \( n_l = n_r = 1 \), the lowest states of DQD are singlet and triplet and the next levels are separated from ST pair by a charge transfer gap \( \sim Q \). We assume that both wells are neutral at \( n_{l,r} = 1 \). Then the effective inter-well exchange responsible for the singlet-triplet splitting arises because of tunneling \( V \) between two wells, \( J = v^2 / Q = \delta \). It is convenient to write the effective spin Hamiltonian of isolated DQD in the form

\[
H_d = E_S |S\rangle \langle S| + \sum_\eta E_T |T_\eta\rangle \langle T_\eta| \equiv \sum_{A=S,T} E_A X^{\Lambda A} \tag{1}
\]

where \( X^{\Lambda A} = |\Lambda\rangle \langle A| \) is a Hubbard configuration change operator (see, e.g,\textsuperscript{14}), \( E_T = E_S + \delta, \eta = \pm, 0 \) are three
The vector state is involved in spin scattering via the components of the SW transformation being applied to a spin rotator results in independent subsystems. As is shown in Refs. [6,9] the transformation [11], where both leads are considered as tunneling amplitude known as the Schrieffer-Wolff (SW) transformation, where both leads are considered as independent subsystems. As is shown in Refs. [6,9] the SW transformation being applied to a spin rotator results in the following effective spin Hamiltonian

$$H_{\text{int}} = \sum_{\alpha \alpha'} \left[ (J_{\alpha \alpha'}^{ST} S + J_{\alpha \alpha'}^{ST} P) \cdot s_{\alpha \alpha'} \right] + J_{\alpha \alpha'}^{SS} X^{SS} n_{\alpha \alpha'}$$

(3)

Here \( s_{\alpha \alpha'} = \sum_{kk'} c_{\alpha \alpha' \sigma}^{\dagger} \hat{c}^{kk'} c_{k' \alpha' \sigma} \), \( n_{\alpha \alpha'} = \sum_{kk'} c_{\alpha \alpha' \sigma}^{\dagger} c_{k' \alpha' \sigma} \), \( \hat{c} \), \( \hat{1} \) are the Pauli matrices and unity matrix respectively. The effective exchange constants are

$$J_{\alpha \alpha'}^{\Lambda \Lambda} \approx \frac{W^2}{2} \left( \frac{1}{\epsilon_{F \alpha} - (E_S/2 + \delta)} + \frac{1}{\epsilon_{F \alpha'} - (E_S/2 + \delta)} \right)$$

Two vectors \( S \) and \( P \) with spherical components

$$S^+ = \sqrt{2} (X^{10} + X^{01}), \quad S^- = \sqrt{2} (X^{01} + X^{10}),$$

$$S_z = X^{11} - X^{-11}, \quad P_z = -(X^{05} + X^{50}),$$

$$P^+ = \sqrt{2} (X^{1S} - X^{-1S}), \quad P^- = \sqrt{2} (X^{S1} - X^{-S1}).$$

(4)

obey the commutation relations of \( a_1 \) algebra

$$[S_j, S_k] = i \epsilon_{jkl} S_l, \quad [P_j, P_k] = i \epsilon_{jkl} P_l, \quad [P_j, S_k] = i \epsilon_{jkl} S_l$$

\((j, k, l \) are Cartesian coordinates, \( \epsilon_{jkl} \) is a Levi-Chivita tensor). These vectors are orthogonal, \( S \cdot P = 0 \), and the Casimir operator is \( S^2 + P^2 = 3 \). Thus, the singlet state is involved in spin scattering via the components of the vector \( P \).

We use \( SU(2) \)-like semi-fermionic representation for \( S \) operators [12,13]

$$S^+ = \sqrt{2} (f_0^1 f_{-1} + f_{-1}^1 f_0), \quad S^- = \sqrt{2} (f_{-1}^1 f_0 + f_0^1 f_{-1}),$$

$$S_z = f_0^1 f_1 - f_{-1}^1 f_{-1},$$

(5)

where \( f_0^\pm \) are creation operators for fermions with spin “up” and “down” respectively, whereas \( f_0 \) stands for spinless fermion [12,13]. This representation can be generalized for \( SO(4) \) group by introducing another spinless fermion \( f_s \) to take into consideration the singlet state. As a result, the \( P \)-operators are given by the following equations:

$$P^+ = \sqrt{2} (f_0^1 f_s - f_{-1}^1 f_0), \quad P^- = \sqrt{2} (f_{-1}^1 f_1 - f_0^1 f_{-1}),$$

$$P^z = -(f_0^1 f_s + f_{-1}^1 f_0).$$

(6)

The Casimir operator \( S^2 + P^2 = 3 \) transforms to the local constraint \( \sum_{\Lambda, \alpha} f_0^{\Lambda} f_{-1}^{\Lambda} = 1 \).

The spin Hamiltonian is now given by

$$H_{\text{int}} = \sum_{kk', \alpha \alpha' = L, R} J_{\alpha \alpha'}^{SS} f_0^{\alpha \alpha'} f_0^{\alpha' \alpha} c_{\alpha \alpha' \sigma}^{\dagger} c_{k' \alpha' \sigma}$$

(7)

$$+ \sum_{kk', \alpha \alpha' \Lambda \Lambda'} \left( J_{\alpha \alpha'}^{ST} S_{\Lambda \Lambda'}^{\dagger} + J_{\alpha \alpha'}^{ST} P_{\Lambda \Lambda'}^{\dagger} \right) \tau_{\sigma \sigma'}^{\dagger} c_{\alpha \alpha' \sigma} c_{k' \alpha' \sigma}^{\dagger} f_0^{\Lambda} f_{-1}^{\Lambda'}$$

where \( S_{\alpha \alpha'}^{\dagger} \) and \( P_{\alpha \alpha'}^{\dagger} \) are 4 × 4 matrices defined by relations (4) - (6) and \( J_{\alpha \alpha'}^{SS}, J_{\alpha \alpha'}^{ST} \) are singlet, triplet and singlet-triplet coupling SW constants, respectively.

To develop the perturbative approach for \( T > T_K \) we introduce the temperature Green’s functions (GF) for electrons in a dot, \( G_\Lambda (\tau) = -(T_r f_\Lambda (\tau) f_{-1}^{\Lambda} (0)) \), and GF of left (L) and right (R) electrons in the leads \( G_{L, R} (k, \tau) = -(T_r c_{L, R} (k, \tau) c_{L, R}^{\dagger} (k, 0)) \). Performing a Fourier transformation in imaginary time for bare GF’s, we come to following expressions:

$$G^0_{k\alpha} (\epsilon_n) = (i \epsilon_n - \epsilon_k + \mu_{L, R})^{-1},$$

$$G^0_{\eta} (\omega_m) = (i \omega_m - E_T)^{-1}, \quad \eta = -1, 0, 1$$

$$G^0_{s} (\epsilon_n) = (i \epsilon_n - E_S)^{-1},$$

(8)

with \( \epsilon_n = 2 \pi T (n + 1/2) \) and \( \omega_m = 2 \pi T (m + 1/3) \) [12,13]. The first leading and next to leading parquet diagrams are shown on Fig.2.

In equilibrium state \( eV = 0 \) the elastic Kondo tunneling arises only provided \( T_K \gg \delta \) in accordance with the theory of two-impurity Kondo effect [11,14]. Now we will show that in the opposite limit \( T_K \ll \delta \) the elastic channel emerges at \( eV \approx \delta \).

Corrections to the singlet vertex \( \Gamma (\omega, 0; \omega, eV) \) are calculated using an analytical continuation of GF’s to the real axis \( \omega \) and taking into account the shift of the chemical potential in the left lead. In a weak coupling regime \( T > T_K \) the leading non-Born contributions to the tunnel current are determined by the diagrams of Fig. 2 b-e.
The equations for LL co-tunneling are:

\[
\Gamma_{LR}^{(2b)}(\omega) = J_{LL}^{ST} J_{LR}^{ST} \sum_{k} \frac{1-f(\epsilon_{kL}-eV)}{\epsilon_{kL} + \mu_{L} - \delta}
\]  

(9)

Changing the variable \(\epsilon_{kL}\) for \(\epsilon_{kL} - eV\) one finds that \(\Gamma_{LR}^{(2b)}(\omega) \sim J_{LL}^{ST} J_{LR}^{ST} \nu \ln(D/\max(\omega, (eV-\delta), T))\). Here \(D \sim \epsilon_F\) is a cutoff energy determining effective bandwidth, \(\nu\) is a density of states on a Fermi level and \(f(\epsilon)\) is the Fermi function. Therefore, under condition \(|eV-\delta| \ll \max|eV, \delta|\) this correction does not depend on \(eV\) and becomes quasielastic.

Unlike the diagram Fig. 2b, its "parquet counterpart" term Fig. 2c contains \(eV + \delta\) in the argument of the Kondo logarithm:

\[
\Gamma_{LR}^{(2c)}(\omega) = J_{LL}^{ST} J_{LR}^{ST} \sum_{k} \frac{f(\epsilon_{kL} - eV)}{\epsilon_{kL} + \mu_{L} + \delta}
\]  

(10)

At \(eV \sim \delta \gg T, \omega\) this contribution is estimated as \(\Gamma_{LR}^{(2c)}(\omega) \sim J_{LL}^{ST} J_{LR}^{ST} \nu \ln(D/\max(\omega, (eV + \delta), T))\) \(\ll \Gamma_{LR}^{(2b)}(\omega)\).

Similar estimates for diagrams Fig. 2d and 2e give

\[
\Gamma_{LR}^{(2d)}(\omega) \sim J_{LL}^{ST} J_{LR}^{ST} \nu^2 \ln(D/\max(\omega, (eV - \delta), T))
\]

\[
\Gamma_{LR}^{(2e)}(\omega) \sim J_{LL}^{ST} J_{LR}^{ST} \nu^2 \ln(D/\max(\omega, (eV - \delta), T)) \times 
\]

\[
\times \ln(D/\max(\omega, eV, T))
\]  

(11)

Then \(\Gamma_{LR}^{(2c)}(\omega) \ll \Gamma_{LR}^{(2d)}(\omega)\) at \(eV \to \delta\).

Thus, the Kondo singularity is restored in strongly non-equilibrium conditions when the energy loss \(\delta\) in a singlet-triplet excitation is compensated by the external voltage applied to the lead, but the leading sequence of most divergent diagrams degenerates in this case from a parquet to a ladder series.

Following the poor man’s scaling approach, we derive the system of coupled renormalization group (RG) equations for \(J\). The equations for LL co-tunneling are:

\[
\frac{dJ_{LL}^{T}}{d \ln D} = -\nu (J_{LL}^{T})^2, \quad \frac{dJ_{LL}^{ST}}{d \ln D} = -\nu J_{LL}^{ST} J_{LL}^{T},
\]  

(12)

The scaling equations for \(J_{LR}^{T}\) are as follows:

\[
\frac{dJ_{LR}^{T}}{d \ln D} = -\nu J_{LR}^{T} J_{LR}^{T}, \quad \frac{dJ_{LR}^{ST}}{d \ln D} = -\nu J_{LR}^{ST} J_{LR}^{T},
\]  

\[
\frac{dJ_{LR}^{S}}{d \ln D} = \frac{1}{2} \nu \left( J_{LL}^{ST} J_{LR}^{T} + J_{LR}^{ST} J_{LL}^{T} + J_{LL}^{ST} J_{LR}^{T} \right).
\]  

(13)

One-loop diagrams corresponding to the poor man’s scaling procedure are shown in Fig. 3. To derive these equations we collected only terms \(\sim (J_{LL}^{T})^n \ln^{n+1}(D/T)\) neglecting contributions containing \(\ln[D/(eV)]\). The analysis of RG equations beyond the one loop approximation will be published elsewhere.

The solution of the system \[13\] reads as follows:

\[
J_{\alpha, \alpha'} = \frac{J_{\alpha, \alpha'}^T}{1 - \nu J_{\alpha, \alpha'}^T \ln(D/T)}, \quad J_{\alpha, \alpha'}^S = \frac{J_{\alpha, \alpha'}^S}{1 - \nu J_{\alpha, \alpha'}^S \ln(D/T)},
\]  

(14)

Here \(\alpha = L, \alpha' = R\). The Kondo temperature is determined by triplet-triplet processes only. It is given by \(T_K = D \exp[-1/(\nu J_{0}^{T})]\).

The differential conductance \(G(eV, T)/G_0\) is the universal function of two parameters \(T/T_K\) and \(V/T_K\) (see Fig. 4), \(G_0 = e^2/\pi h\):

\[
G/G_0 \sim \ln^{-2} \max[|eV - \delta|, T]/T_K
\]  

(15)

Finite decoherence rate \(h/\tau_d\) effects discussed in \[16\] in a context of strongly nonequilibrium transport through QD with \(S = 1/2\) do not arise in our case. According to the Non-Crossing Approximation (NCA) description, the origin of \(\tau_d\) is inelastic spin relaxation of Kondo state.

In the problem considered the ground state is \(S = 0\) singlet and the spin-relaxation is absent. A Kondo-channel
arises only in *virtual* states of L-R co-tunneling. Repopulation effects at $eV > \delta$ should result in asymmetry of Kondo-peak $\text{[I]}$, but this effect is beyond our quasi equilibrium approach.

Thus, we have shown that the tunneling through single DQDs with $\delta \gg T_K$ exhibits a peak in differential conductance at $eV \approx \delta$ instead of the usual zero bias Kondo anomaly (see Fig. 4) which arises in the opposite limit, $\delta < T_K$. Therefore, in this case the Kondo effect in DQD is induced by a strong external bias. The scaling equations (13), (14) can also be derived in Schwinger-Keldysh formalism (see [13] and also [16]) by applying the “poor man’s scaling” approach directly to the dot conductance $\text{[I]}$. The detailed analysis of the model $\text{[I]}$ in a real-time formalism will be presented elsewhere.

We discuss yet another possible experimental realization of resonance Kondo tunneling driven by external electric field. Applying the alternate field $V = V_{ac} \cos(\omega t)$ to the parallel DQD, one takes into consideration two effects, namely (i) enhancement of Kondo conductance by tuning the amplitude of ac-voltage to satisfy the condition $|eV_{ac} - \delta| \ll T_K$ and (ii) spin decoherence effects due to finite decoherence rate $\text{[I]}$. One can expect that if the decoherence rate $h/\tau \gg T_K$, 

$$G_{\text{peak}}/G_0 \sim \ln^{-2} (h/\tau T_K)$$  \hspace{1cm} (16)$$

whereas in the opposite limit $h/\tau \ll T_K$, 

$$G_{\text{peak}} = \overline{G(V_{ac} \cos[\omega t])}$$  \hspace{1cm} (17)$$

is averaged over a period of variation of ac bias. In this case the estimate $\text{[I]}$ is also valid.

In conclusion, we have provided the first example of Kondo effect, which exists only in non-equilibrium conditions. It is driven by external electric field in tunneling through a quantum dot with even number of electrons, when the low-lying states are those of spin rotator. This is not too exotic situation because as a rule, a singlet ground state implies a triplet excitation. If the ST pair is separated by a gap from other excitons, then tuning the dc-bias in such a way that applied voltage compensates the energy of triplet excitation, one reaches the regime of Kondo peak in conductance. This theoretically predicted effect can be observed in dc- and ac-biased double quantum dots in parallel geometry.

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![FIG. 4. The Kondo conductance as a function of dc-bias $eV/T_K$ and $T/T_K$. The singlet-triplet splitting $\delta/T_K = 10$.](cond-mat/0202404)