Stability Investigation of a Stiffened Plate by Using Numerical Methods

G A Manuylov, S B Kosytsyn and I E Grudsyna

Russian University of Transport, 127994, 15 Obraztsova Street, GSP-4, Moscow, Russian Federation

E-mail: kositsyn-s@yandex.ru, grudsyna_ira90@mail.ru

Abstract: the influence of initial geometric imperfections of a stiffened plate under longitudinal compression on the double critical buckling loads had been studied in this paper. An appropriate bifurcation diagram and bifurcation set were plotted. The results obtained showed that sensitivity of critical load to any initial geometric imperfections is actually insignificant (max 22% loss of the critical load) with specific geometric plate parameters and stiffeners selected.

1. Introduction

Stiffened plates are commonly used as structural components applicable in the engineering, aviation, construction and automotive industries. The apparent simplicity conceals complicated interaction between the different modes of buckling of some structural components under compression: bending of a fully compressed plate, like a wide Euler’s loaded strut with asymmetrical cross-section, to cause the bending moment that produces secondary compression stress in a plate, thereby inducing loss of plate stability in the form of surface undulations. It is considered that a stiffened plate subjected to the double critical load shows its maximum sensitivity to initial geometric imperfections when the critical buckling load of the plate under compression, like an Euler’s strut, becomes equal (or almost equal) to the critical buckling undulation-caused thin plate load. With a stiffened plate subjected to the double critical loads caused by the imperfection undulation amplitude, V. Tvergaard [7] could obtain loss of the critical load reproducing it nearly twice. Need to say that there were other results; particularly, A. Van der Neut [9] stated that the critical load could be reduced insignificantly due to influence of the imperfections even in relation to any multiple loads. Therefore, the purpose of this paper is to study influence of initial geometric imperfections on the behavior of a stiffened plate subjected to equal critical buckling Euler’s loads and undulations in the plate ($P_{Euler} = P_{plate}$). W. T Koiter and M. Pignataro [4] could validate fast stiffened plate response to initial imperfections. These authors applied the Koiter’s asymptotic method for rating the critical load. Rather interesting and important results were obtained by A. Van der Neut [8] and published in a number of papers. He studied plates subjected to multiple and nearly multiple critical loads. Though, A. Van der Neut [9] made the conclusion that there were some situations when sensitivity of the critical loads to initial imperfections could not be too large. The papers published by V. Tvergaard [7] are of special importance. As for an infinitely wide stiffened plate, he showed that the worst relation is produced between a thin plate and the same plate subjected to the double critical load within the range of 0.93 – 1.0. A stiffened plate is
slightly sensitive to initial imperfections, provided that it is spaced right. But the last statement is seemed insufficiently justified. There is paper authored A. Grimaldi, M. Pignataro [2] in which they described the compressed bar and its stable/ unstable post-bifurcation behavior against a number of its cross-sectional symmetry axes. As for stiffened axially loaded plates (theory and experiments), R. Maquoi and Ch. Massonnet [5] found that no local loss of stability occurs when a specific relation between width and thickness of a plate is maintained. The above authors stated that a stiffened plate panel was collapsed under the stress to have been less than the Euler’s load and less than the plate undulation load. V. Fok at al [1] took into account the matter that not only a thin plate but also stringers could lose their stability. Actually, these are very interesting findings. J. Hunt [3] believed that such plate could be mathematically described by a double anticlinic bifurcation point. Figure 1 shows partial forms of loss stability of stiffened plate under compression.

Figure 1. Stiffened plate under compression; partial forms of loss of flexible stiffened plate stability

a) stiffened plate under compression;
b) loss of Euler’s form combined buckling stability;
c) loss of plate undulation stability

2. Finite-element model and its verification
The problem has been solved by using the finite-element method. A stiffened plate is represented by a 86cm×36cm finite-element short-side hinged model with free longitudinal edges that have four longitudinal stingers (Figures 2 and 3). The finite-element model has been simulated using MSC PATRAN - NASTRAN calculation tool and shell-type finite elements (2378 elements). The material concerned displayed its absolutely elastic properties ($E=2\cdot10^6$ kg/cm², $\mu = 0.3$). The compressed load is applied to the centroidal principal cross-section axis. For the stiffened plate cross section, refer Figure 2.

Figure 2. Stiffened plate cross section
where $b=12\text{cm}$, $\delta=0.175\text{cm}$, $e=1.043\text{cm}$, $b_p=1.2\text{cm}$, $h_p=3\text{cm}$, $J=20.6769\text{cm}^4$.

Figure 3: Stiffened plate finite-element model

The finite-element model is verified by means of comparing of values of the first five critical loads for a longitudinally compressed $12\text{cm} \times 86\text{cm}$ edge-hinged plate-strip. The results have been obtained by using the form of linear stability (buckling) and compared with the results obtained from following well-known form (1) [10].

$$P_{cr} = \frac{\pi^2 E \delta^3}{12(1-\mu^2)b^2} \left( \frac{\xi}{n} + \frac{n}{\xi} \right)^2; \quad (1)$$

where $\mu=0.3$, $\delta=0.175\text{cm}$, $\frac{a}{b} = 7.16$, $b=12\text{cm}$, $n=1,2,3…$

After comparing the critical loads against the first five modes of buckling, it has been found that the critical load values calculated by the analytical and numerical 3 figure method match each other; as a matter of fact, the model selected is capable to produce rather reliable results after using geometrically nonlinear analysis for calculating the post-critical behavior of a stiffened plate.

3. Numerical analysis of the stiffened plate stability to the double critical load

It is assumed that a stiffened plate subjected to the double critical loads is the most interesting item applicable for analyzing and calculating the post-critical influence of the initial geometric imperfections. According to V. Tvergaard [7], the very plate is the one that can demonstrate its maximum sensitivity to the initial geometric imperfections in the form of surface undulations produced in plates-strips. The considered stiffened plate of $\delta=0.175\text{cm}$ thickness was subjected to the double load. Critical load values (i.e. $P_{Euler}=53760\text{kg}$ and $P_{plate}=53940\text{kg}$) were calculated by using the MSC PATRAN - NASTRAN (buckling) linear calculation tool. Subsequently, the pre-critical and post-critical equilibrium parameters were calculated taking into account of geometric nonlinearity using nonlinear static options. Forced loading was used. A single $P_{cr}=48700\text{kg}$ double critical load was calculated in scope of the geometric nonlinearity as that inducing buckling of a stiffened plate, like an Euler’s strut, with the plate simultaneously losing its stability in the form of surface undulations (Figure 4). Initially, in the post-bifurcation equilibrium state, the undulation effect occurs in the middle of the plate (Figure 5).
Figure 4. Initial undulation and plate buckling like Euler’s strut
On running over the bifurcation point, an undulation effect propagates along the entire stiffened plate surface. The undulation load value specified at the point of propagation along the stiffened plate is $P=42090$ kg (Figure 5).

Figure 5. Undulation effect propagation along a plate
The bend curve specified against the compression load value is given in Figure 6. The diagram shows drop of the load vs. growing bending force. The load drops due to unstable equilibrium in the semi-symmetric bifurcation point. The bending force induces development of the bending moment, thereby significantly increasing compression stress in the plate. This, in turn, results in growing undulation amplitudes and in relieving the stiffened plate, like an Euler’s strut, owing to reduction of flexural stiffness. Such interference of forms is explained by the fact that each form in relation to another form can be specified as imperfection that becomes evident beginning with the bifurcation point.

Figure 6. Bend curve vs. compression load
4. Semi-symmetrical double bifurcation point

The double semi-symmetric bifurcation is a semi-symmetric critical point since post-critical equilibrium path symmetry can be suitable for the form of plate undulation only when odd half-waves are produced on each plate half-section (over the length); buckling of an asymmetric cross-section stiffened plate, like an Euler’s strut, is asymmetric deformation: such strut always buckles in such a way that stringers find their place on a convex side.

Figure 7. Homeoclinical bifurcation point

$q_1$ – plate undulation form reference coordinate; $q_2$ – strut buckling form reference coordinate; $\lambda$ – load parameter ($P-P_{cr}$). Point 1 – semi-symmetric double bifurcation point; Line 1-2 – uncoupled equilibrium line matching a respective amplitude having the stiffened plate buckling form like an Euler’s strut; Lines 1-3 and 1-4 – coupled equilibrium lines matching a respective amplitude having the plate undulation form and the strut buckling form.

A semi-symmetric bifurcation point is potentially symmetrical to $q_1$ and asymmetrical to $q_2$. Such potential values are sourced from a typical regular cube for two variables by means of casting out those terms that fail to satisfy specific symmetry conditions by coordinate $q_1$:

$$V(q_1, q_2, \varepsilon_j) = \frac{1}{6} V_{222}^{cr} q_2^3 + \frac{1}{2} V_{112}^{cr} q_1^2 q_2 + \frac{1}{2} \lambda (V_{112}^{cr} q_1^2 + V_{222}^{cr} q_2^2) + V_{1\varepsilon_j}^{cr} q_1 + V_{2\varepsilon_j}^{cr} q_2 \quad [6];$$

(2)
Then, the equilibrium equations for a plate with imperfections are expressed as follows:

\[
\frac{\partial V}{\partial q_1} = V_{112}^c q_1 + \lambda V_{11}^c q_1 + V_{1e}^c e_1 = 0; \tag{3}
\]

\[
\frac{\partial V}{\partial q_2} = \frac{1}{2} V_{222}^c q_2^2 + \frac{1}{2} V_{112}^c q_1^2 + \lambda V_{222}^c q_2 + V_{2e}^c e_2 = 0; \tag{4}
\]

Where: 
\( e_1 \) is a plate undulation form imperfection;  
\( e_2 \) is an Euler’s strut like stiffened plate buckling imperfection;  
For the purpose of an ideal system at \( e_1 = e_2 = 0 \), the following equilibrium equation is expressed:

\[
V_{112}^c q_1 q_2 + \lambda V_{11}^c q_1 = 0; \tag{5}
\]

\[
\frac{1}{2} V_{222}^c q_2^2 + \frac{1}{2} V_{112}^c q_1^2 + \lambda V_{222}^c q_2 = 0 \tag{6}
\]

All the full potential energy derivatives are taken in a bifurcation point. With these equation solved, the following three variants may be picked out:

- \( q_1 = q_2 = 0 \) – initial / trivial equilibrium of an unbent stiffened plate;
- solution at \( q_1 = 0, q_2 \neq 0 \) meets a stiffened plate bend like an Euler’s strut. Then, let’s produce the following uncoupled post-bifurcation line equation (Figure 7) from the second equilibrium equation (6) with allowance for \( q_1 = 0 \):

\[
q_2 = -\frac{2\lambda V_{222}^c}{V_{222}^c} \quad \text{или} \quad \frac{\lambda}{q_2} = \frac{V_{222}^c}{2V_{222}^c} \tag{7}
\]

- \( q_2 \neq 0, q_1 \neq 0 \). Then the following equation will be derived from the first one:

\[
\lambda = -\frac{V_{112}^c q_2}{V_{11}^c} \tag{8}
\]

Substituting an expression for \( \lambda \) in the second equilibrium equation:

\[
\frac{1}{2} V_{222}^c q_2^2 + \frac{1}{2} V_{112}^c q_1^2 - \frac{2V_{222}^c q_2}{V_{222}^c} \frac{V_{11}^c}{V_{112}^c} = 0 \tag{9}
\]

the relation is as follows:

\[
\frac{q_1}{q_2} = \pm \left( \frac{2V_{222}^c}{V_{11}^c} - \frac{V_{222}^c}{V_{112}^c} \right)^{\frac{1}{2}} \tag{10}
\]

After that we get two post bifurcation lines:

\( \lambda = kq_2 \) - uncoupled post-bifurcation line (an Euler’s strut form);  
\( \lambda = \pm k (q_1, q_2) \) - coupled post-bifurcation line (the plate undulation form and the strut buckling form).  
If the expression in brackets is negative, no new solutions can be obtained. And the post-bifurcation solution will be determined by the uncoupled equilibrium line (1-2). This is a monoclinic double bifurcation point. If the expression in bracket is positive, and if relation characters \( \frac{2V_{222}^c}{V_{11}^c} \) and \( \frac{V_{222}^c}{V_{112}^c} \) additionally match, we can obtain two coupled lines 1-3 and 1-4 (Figure 7) that “fall” the same direction like the uncoupled equilibrium line. As for a similar half-symmetric double bifurcation point, it is called a homeoclinic point (Figure 7). Let’s point out two coupled equilibrium lines as those producing a flat triangular face of “coupled” equilibriums 4-3-1. If this face is crossed by an unbound imperfective equilibrium curve in plane \( \lambda - q_3 \), a secondary unstable symmetric bifurcation is produced in such plane. A specific secondary critical point can appear at particular “triangular plane”
slope angles and valued by initial imperfection $q_2$, provided that a limit point of the imperfective equilibrium curve will be crossed (so called “hill-top branching”). This kind of effect is caused in the events when the imperfective system bend approaches its maximum and, simultaneously, the bend curve crosses coupled equilibrium triangular 1-3-4 at this peak point and undulation begin.

5. Influence of initial geometric imperfections on the stiffened plate critical load
Let’s consider for the initial geometric imperfections looking through:
- plate undulation form, $\varepsilon_1$;
- strut buckling form, $\varepsilon_2$;
- including the combined imperfection looked through plate undulation and strut buckling forms, $\varepsilon_1+\varepsilon_2$ (amplitude values are summed up).

Initial geometric imperfection values are rated to thickness of a stiffened plate ($0.1\delta$, $0.25\delta$, $0.5\delta$, $1\delta$, $2\delta$). Figure 8 shows the characteristic curve of bends vs. the compression load for a stiffened plate with amplitude-variable undulation-caused imperfection ($0.1\delta, 0.25\delta, 0.5\delta, 1\delta, 2\delta$). As shown by the diagram, each curve “falls” on approaching the limit point at the reduced load and after plate bends go up. All curves asymptotically seek a single unstable equilibrium coupled post-bifurcation path line (line 1-4 in Figure 7). Figure 9 shows the characteristic curve of bends vs. the compression load for a stiffened plate with amplitude-variable Euler’s strut like buckling imperfection ($0.1\delta, 0.25\delta, 0.5\delta, 1\delta, 2\delta$). All imperfective equilibrium paths run over plane $q_2\lambda$ and asymptotically seek another 1-2 line on passing a limit point. A bifurcation point is always produced when the Euler’s strut like imperfective equilibrium curve crosses combined equilibrium plane 2-3-1, thereby generating unstable symmetric undulations in a plate. As shown by Figure 9, bifurcation points can occur on curves both before and after passing the limit point; if they occur in the limit point, they are specified as a special type of the bifurcation homoclinic point – hill-top branching (see above). Figure 10 shows the characteristic curve of bends vs. the compression load for a stiffened plate with amplitude-variable Euler’s strut like undulation and buckling combined imperfection ($0.1\delta+0.1\delta, 0.25\delta+0.25\delta, 0.5\delta+0.5\delta, 1\delta+1\delta, 2\delta+2\delta$). The curves shown in Figure 10 are specified as combined equilibrium paths and, therefore, they seek line 1-3.

Figure 8. Characteristic curve of the bend vs. the compression load for a stiffened plate with amplitude-variable undulation-caused imperfections ($0.1\delta, 0.25\delta, 0.5\delta, 1\delta, 2\delta$)
Figure 9. Characteristic curve of bends vs. the compression load for a stiffened plate with amplitude-variable Euler’s strut like buckling imperfection (0.1δ, 0.25δ, 0.5δ, 1δ, 2δ)

Figure 10. Characteristic curve of bends vs. the compression load for a stiffened plate with amplitude-variable Euler’s strut like undulation and buckling combined imperfection (0.1δ+0.1δ, 0.25δ+0.25δ, 0.5δ+0.5δ, 1δ+1δ, 2δ+2δ)

The purpose of this study is to plot a bifurcation set that is a range of critical points of the imperfective and perfective systems (Figure 11). Every point satisfies equilibrium conditions and determinant matrix Hesse equal to zero. As compared with the resultant data obtained by G. Hunt [3] and V. Tvergaard [7], this solution can be used for deriving the surface that is numerically more inclined; and, therefore, the maximum loss of the critical load with amplitude-variable Euler’s strut like undulation and buckling combined imperfection 2δ+2δ is rated at 21.6% that is less (according to V. Tvergaard) than that produced by G. Hunt. The results obtained prove the A. Van der Neut’s [9] statement that some stiffened plates have the load drops to be relatively insignificant by nature. For the bifurcation set plotting data, refer to Table 1.

Table 1. Critical loads for the bifurcation set plotting

| $\varepsilon_2$ | $\varepsilon_1$ | 0 | 0.1δ | 0.25δ | 0.5δ | 1δ | 2δ |
|-----------------|-----------------|---|-----|------|-----|---|----|
| 0               |                 | 48700 | 44838 | 42008 | 41589 | 40982 | 39664 |
| 0.1δ            |                 | 48307 | 43995 | - | 42221 | - | - |
| 0.25δ           |                 | 43912 | - | 42331 | - | 41074 | - |
| 0.5δ            |                 | 42109 | 41680 | - | 41193 | - | 39943 |
| 1δ              |                 | 40676 | - | 40433 | - | 39789 | - |
| 2δ              |                 | 38454 | - | 38923 | - | 38184 | - |
6. Conclusions

- the finite-element model of the stiffened plate subjected to the double critical loads has been simulated and verified using MSC PATRAN - NASTRAN calculation tool;
- the perfect stiffened plate subjected to the double critical loads has been established, and equilibrium in the semi-symmetric bifurcation point was unstable. The reason of that was interaction between partial forms of loss stability;
- the stiffened plate has been analyzed with initial geometrically imperfections (plate undulation forms, strut buckling forms and combined imperfections);
- the bifurcation set for the stiffened plate subjected to the double critical loads has been plotted, and the stiffened plate has been have less sensitivity to the initial geometrically imperfections than V. Tvergaard’s infinitely wide plate.

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