CDF $m_W$ and the muon $g - 2$ through the Higgs-phobic light pseudoscalar in type-X two-Higgs-doublet model

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Abstract

The recent measurement of the $W$ boson mass by the CDF collaboration adds an exciting anomaly to the long-standing anomaly of the muon $g - 2$. Type-X in two-Higgs-doublet model provides an attractive solution to two anomalies with light pseudoscalar $A$. But type-X confronts with the exotic Higgs decays of $h \to AA$ and the lepton flavor universality data in the $\tau$ and $Z$ decays, because of the light $M_A$ itself. We propose to adopt the Higgs-phobic $A$ with $\lambda_{hAA} = 0$ in type-X, which is shown to explain the two anomalies as well as the theoretical and experimental constraints including the lepton flavor universality data. Additionally requiring the cutoff scale above 10 TeV through the RGE analysis, we find that the parameter space is severely limited such that $\tan \beta \in [36, 65]$, $M_A \in [11, 38]$ GeV, $M_{H^\pm} \in [283, 338]$ GeV, and $M_H \in [249, 306]$ GeV. Due to sizable mass gaps between $M_A$ and $M_{H,H^\pm}$, originated from the CDF $m_W$ measurement, dominant decay modes of $H$ and $H^\pm$ are bosonic such as $H \to ZA$ and $H^\pm \to W^\pm A$. With $\mathcal{B}(A \to \tau\tau) \simeq 100\%$, this characteristic provides a unique signal of the $4\tau$ states associated with two gauge bosons at the LHC.

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I. INTRODUCTION

The CDF collaboration at Fermilab has come out with the most precise measurement of $W$ boson mass \[1\]

\[ m_{C^{DF}}^{W} = 80.4335 \pm 0.0094 \text{ GeV}, \]

using the data set collected at 8.8 fb$^{-1}$ luminosity. The new mass deviates from the Standard Model (SM) prediction of $m_{W}^{SM} = 80.357 \pm 0.006 \text{ GeV} \ [2]$ by $7\sigma$. Previously the world average of $m_{W}$ measurements \[2\] was only $1.8\sigma$ standard deviation from the SM expectation. The discrepancy of the $W$ mass still needs to be confirmed as there is a tension between the CDF measurement and ATLAS report [3]. However, if we accept the new mass of $W$ boson then the validity of SM will be under serious question, leading to the requirement of new physics beyond the SM (BSM). In this context, the discrepancy of $W$ boson mass is parameterized in terms of the Peskin-Takeuchi oblique parameters $S$, $T$, and $U$ so that new physics can be explored by its contribution to the gauge boson self energies. In most BSM models, the contribution to $U$ is significantly small and hence setting $U = 0$ is usually accepted. Then we have large shift of the central values as $S = 0.15 \pm 0.08$ and $T = 0.27 \pm 0.06$ with the correlation $\rho_{ST} = 0.93 \ [4]$. Recently, various new physics models have been actively studied to explain the new $S$ and $T \ [4-53]$.

Another long standing problem in particle physics is the anomalous magnetic moment of muon ($\muon g - 2$). The combined result of Fermilab National Accelerator Laboratory (FNAL) experiment [54, 55] and the Brookhaven National Laboratory (BNL) experiment [56] has shown clear deviation from the SM prediction by $4.2\sigma$, which is reported to be

\[ \Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = 251(59) \times 10^{-11}. \]
The CDF $W$ mass and muon $g-2$ anomalies hint towards new physics. Several works are done to explain both the anomalies in the context of $U(1)$ gauge extended models with vectorlike leptons [57–59], vector leptoquark [60] along with the explanation to $R_{K,K^*}$ and $R_{D,D^*}$ anomalies, scalar leptoquark model [61, 62] along with $R_{K^*}$ and $R_{D^*}$, Zee model [63], models with vectorlike leptons [64, 65], and NMSSM [66].

In this paper, we focus on type-X or lepton-specific two-Higgs-doublet model (2HDM) to simultaneously explain the CDF $W$ boson mass and muon $g-2$ anomalies. One of the most important characteristic features of type-X 2HDM is the enhanced coupling of the non-SM Higgs bosons (neutral $CP$-even $H$, $CP$-odd $A$, and charged Higgs $H^\pm$) to the leptons by $\tan\beta$, the ratio of the vacuum expectation values of the two Higgs doublets. Through this enhanced leptonic coupling, the type-X 2HDM can explain muon $g-2$ anomaly via two loop Barr-Zee diagram with $\tau$-loop [67]. Sizable positive contribution to $\Delta a_\mu$ is obtained with large $\tan\beta$ and small $M_A$. However, a light pseudoscalar with $M_A < m_h^{SM}/2$ opens up $h_{SM} \to AA$ which is severely constrained by $h_{SM} \to AA \to 4\tau/2\mu 2\tau$ channels [68]. Kinematical solution of $M_A > m_h^{SM}/2$ demands very large $\tan\beta$ for $\Delta a_\mu$, which brings inconsistency with the lepton flavor universality (LFU) data in the $\tau$ and $Z$ decays [69]. This motivates us to consider the pseudoscalar to be Higgs-phobic and to study whether the model can explain the CDF $m_W$ and muon $g-2$ anomalies as well as the theoretical and experimental constraints including the LFU data. There are our main results.

Now the natural question is how the changes of $S$ and $T$ due to the new $W$ mass would affect the parameter space compatible with the muon $g-2$. To investigate that, we do a complete parameter scan based on the old and new sets $S$ and $T$. We divide the parameter scanning into four different steps. In Step-I we impose the theoretical bounds (vacuum stability of the potential, unitarity, and perturbativity) as well as the muon $g-2$ constraints. In Step-II we compute the $S$ and $T$ parameters assuming $U = 0$ and compare the best fit results before the CDF $m_W$ measurement as given by the Particle Data Group [2] (PDG) with those after the CDF. In Step-III, we impose the Higgs precision data and the direct search bounds from LEP, Tevatron, and LHC. In Step-IV, we further restrict the parameter space by the LFU data in the $\tau$ and $Z$ decays which come via loop effects of BSM scalars [70]. In addition to the parameter scan, we study the RGE evolutions of the allowed model parameters to check the stability of the potential. Another essential aspect would be searching for light Higgs-phobic pseudoscalar at the LHC. In our case, 4-$\tau$ states associated with two gauge bosons would be the golden discovery modes at the HL-LHC.

The paper is organized in the following way. In Sec. II, we give a brief overview of type-X 2HDM and the characteristics of the Higgs-phobic pseudoscalar. In Sec. III, we do the parameter scanning both in the framework of old and new sets of $S$ and $T$ values in four steps. In Sec. IV we discuss the RGE evolutions and the cutoff scales. Section V deals with the golden discovery channels of the Higgs-phobic type-X at HL-LHC. Finally we conclude in Sec. VI.
II. TYEP-X 2HDM WITH A HIGGS-PHOBIC PSEUDOSCALAR BOSON

The 2HDM introduces two \( SU(2)_L \) complex scalar doublet fields with hypercharge \( Y = +1 \), \( \Phi_1 \) and \( \Phi_2 \) [71]:

\[
\Phi_i = \begin{pmatrix} w_i^+ \\ v_i + \rho_i + i\eta_i \end{pmatrix}, \quad (i = 1, 2)
\]

where \( v_1 \) and \( v_2 \) are the nonzero vacuum expectation values of \( \Phi_1 \) and \( \Phi_2 \), respectively. The electroweak symmetry is broken by \( v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV} \). The ratio of \( v_2 \) to \( v_1 \) is defined by \( \tan \beta \equiv v_2/v_1 \).

We introduce a discrete \( Z_2 \) symmetry to prevent the tree-level flavor-changing neutral currents (FCNC) [72, 73], under which \( \Phi_1 \rightarrow \Phi_1 \) and \( \Phi_2 \rightarrow -\Phi_2 \). Allowing the softly broken \( Z_2 \) symmetry and retaining the CP invariance, we write the scalar potential as

\[
V_{\Phi} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 (\Phi_1^\dagger \Phi_2 + \text{H.c.}) + \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) + \frac{1}{2} \lambda_5 [(\Phi_1^\dagger \Phi_2)^2 + \text{H.c.}].
\]

The 2HDM accommodates five physical Higgs bosons, the lighter \( CP \)-even scalar \( h \), the heavier \( CP \)-even scalar \( H \), the \( CP \)-odd pseudoscalar \( A \), and a pair of charged Higgs bosons \( H^\pm \). For the relations of the mass eigenstates with the weak eigenstates via two mixing angles of \( \alpha \) and \( \beta \), we refer the reader to Ref. [74]. The SM Higgs boson, \( h_{SM} \), is a linear combination of \( h \) and \( H \), given by

\[
h_{SM} = s_{\beta - \alpha} h + c_{\beta - \alpha} H.
\]

We take \(-\pi/2 < (\beta - \alpha) < \pi/2 \) as in the public codes such as 2HDMC [75], HiggsSignals [76], and HiggsBounds [77]. Two scenarios exist in explaining the observed Higgs boson, the normal scenario where \( h \) is observed and the inverted scenario where \( H \) is observed while \( h \) has been hidden [53, 69, 78]. This work focuses on the normal scenario, i.e., \( m_h = 125 \text{ GeV} \).

The quartic couplings in Eq. (4) play a crucial role in governing the perturbativity, unitarity, and vacuum stability. We write them near the alignment limit [79], which is highly motivated

\[1\] In what follows, we will use the simplified notation of \( s_x = \sin x \), \( c_x = \cos x \), and \( t_x = \tan x \).
by the current Higgs precision data [80–82]:

\[
\begin{align*}
\lambda_1 & \approx \frac{1}{v^2} \left[ m_h^2 + t_\beta^2 \left( M^2_H - M^2 \right) \right], \\
\lambda_2 & \approx \frac{1}{v^2} \left[ m_h^2 + \frac{1}{t_\beta^2} \left( M^2_H - M^2 \right) \right], \\
\lambda_3 & \approx \frac{1}{v^2} \left[ m_h^2 - M^2_H - M^2 + 2M^2_{H^\pm} \right], \\
\lambda_4 & \approx \frac{1}{v^2} \left[ M^2 + M^2_A - 2M^2_{H^\pm} \right], \\
\lambda_5 & \approx \frac{1}{v^2} \left[ M^2 - M^2_A \right],
\end{align*}
\]

where \( M^2 = m_{Z2}/(s_\beta c_\beta) \). Particularly important is the perturbativity of \( \lambda_1 \) when \( t_\beta \) is large [69]. The \( t_\beta^2 \) terms in \( \lambda_1 \) easily break the perturbativity and unitarity, unless \( M^2 \) is almost the same as \( M^2_H \). The perturbativities of \( \lambda_4 \) and \( \lambda_5 \) with \( M^2 \approx M^2_H \) demand \( M_H \) similar to \( M_A \) and \( M_{H^\pm} \):

\[
M \approx M_H \approx M_A \approx M_{H^\pm},
\]

where \( M \equiv \sqrt{M^2} \) because only the positive \( M^2 \) is allowed.

The Yukawa interactions of the SM fermions are described by

\[
\mathcal{L}^{\text{Yuk}} = - \sum_f \left( \frac{m_f}{v} \xi^h_f \tilde{f} f h + \frac{m_f}{v} \xi^H_f \tilde{f} f H - i \frac{m_f}{v} \xi^A_f \tilde{f} \gamma_5 f A \right)
\]

\[
- \left\{ \frac{\sqrt{2}}{v} \left( m_t \xi^A_t P_- + m_b \xi^A_b P_+ \right) bH^+ + \frac{\sqrt{2} m_\tau}{v} \xi^A_\tau \bar{\nu}_\tau P_\tau H^+ + \text{H.c.} \right\},
\]

where \( P_\pm = (1 \pm \gamma^5)/2 \). The Higgs coupling modifiers in type-X are

\[
\xi^h_t = \frac{c_\alpha}{s_\beta}, \quad \xi^h_b = - \frac{s_\alpha}{c_\beta}, \quad \xi^H_t = \frac{s_\alpha}{c_\beta}, \quad \xi^H_b = \frac{c_\alpha}{s_\beta}, \quad \xi^A_\tau = \frac{1}{\xi^A_t} = - \frac{1}{\xi^A_b} = t_\beta.
\]

For the trilinear scalar couplings, we parameterize the Lagrangian as

\[
\mathcal{L}^{\text{tri}} = v \left[ \frac{1}{3!} \sum_{\varphi_0} \hat{\lambda}_{\varphi_0} \varphi_0^3 + \frac{1}{2} \hat{\lambda}_{hhH} hhH + \frac{1}{2} \hat{\lambda}_{hHH} hHH \\
+ \sum_{\varphi_0} \left\{ \frac{1}{2} \hat{\lambda}_{\varphi_0 AA} A^2 \varphi_0 + \lambda_{\varphi_0 H^+H^-} H^+H^- \varphi_0 \right\} \right].
\]

where \( \varphi_0 = h, H \).

Our central concern is the exotic decay of the observed Higgs boson, \( h \rightarrow AA \), which is severely restricted by the current Higgs precision data [80–82]. We need to forbid it. Although heavy \( M_A \) above \( m_h/2 \) kinematically prohibits the decay, the muon \( g-2 \) anomaly calls for a light pseudoscalar boson. An alternative way is to turn off the \( h-A-A \) vertex: the pseudoscalar...
boson becomes Higgs-phobic. So we consider type-X with the Higgs-phobic pseudoscalar boson \( A \), simply called the Higgs-phobic type-X in what follows.

The trilinear coupling for the \( h-A-A \) vertex is

\[
\hat{\lambda}_{hAA} = \frac{1}{4s_\beta c_\beta} \left[ \left( 2M_A^2 - m_h^2 \right) c_{\alpha-3\beta} - \left( 2M_A^2 + 3m_h^2 - 4M^2 \right) c_{\alpha+\beta} \right].
\]  

(11)

Since \( s_{\beta-\alpha} \) and \( c_{\beta-\alpha} \) are useful parameters for the Higgs precision data, we use the identities of

\[
\frac{c_{\alpha-3\beta}}{s_\beta c_\beta} = -2s_{\beta-\alpha} - \left( t_\beta - \frac{1}{t_\beta} \right) c_{\beta-\alpha},
\]

(12)

and \( \frac{c_{\alpha+\beta}}{s_\beta c_\beta} = 2s_{\beta-\alpha} - \left( t_\beta - \frac{1}{t_\beta} \right) c_{\beta-\alpha}, \)

and write \( \hat{\lambda}_{hAA} \) as

\[
\hat{\lambda}_{hAA} = \left( 2M^2 - 2M_A^2 - m_h^2 \right) s_{\beta-\alpha} + \left( m_h^2 - M^2 \right) \left( t_\beta - \frac{1}{t_\beta} \right) c_{\beta-\alpha}.
\]

(13)

Then, the condition of \( \hat{\lambda}_{hAA} = 0 \) accords with

\[
\text{Higgs-phobic } A: \quad \frac{s_{\beta-\alpha}}{c_{\beta-\alpha}} = - \left( t_\beta - \frac{1}{t_\beta} \right) \frac{m_h^2 - M^2}{2M^2 - 2M_A^2 - m_h^2}.
\]

(14)

Note that the exact Higgs alignment is not feasible in the Higgs-phobic \( A \) limit. Since \( s_{\beta-\alpha} \) is determined by \( t_\beta, M^2, \) and \( M_A \), the model has five parameters of

\[
\{ t_\beta, M_A, M_H, M_{H\pm}, m_{12}^2 \}
\]

(15)

An interesting consequence of the Higgs-phobic \( A \) is that the Higgs alignment naturally arises, although not exact. In Fig. 1, we show \( s_{\beta-\alpha} \) as a function of \( M \) satisfying Eq. (14). For \( t_\beta = 100 \), two cases are considered, \( M_A = 70 \text{ GeV} \) (left panel) and \( M_A = 300 \text{ GeV} \) (right panel). In both cases, we have \( |s_{\beta-\alpha}| \approx 1 \) over the whole range of \( M \), except for extremely narrow region of \( M \). If we restrict ourselves to \( M \approx M_A \), as shown by the colored regions corresponding to \( M \in [0.5M_A, 2M_A] \), the preference for the alignment is greater.

III. SCANNING STRATEGIES AND THE RESULTS

Focusing on the Higgs-phobic type-X, we study the implication of the theoretical and experimental constraints, including the muon \( g - 2 \) anomaly as well as the electroweak precision data before and after the CDF \( m_W \) measurement. Over the randomly generated parameters in the ranges of

\[
\begin{align*}
t_\beta & \in [1, 200], \quad m_{12}^2 \in [0, 15000] \text{ GeV}^2, \\
M_H & \in [130, 1000] \text{ GeV}, \quad M_A \in [10, 200] \text{ GeV}, \quad M_{H\pm} \in [80, 1000] \text{ GeV},
\end{align*}
\]

(16)
FIG. 1: $\sin(\beta - \alpha)$ as a function of $M(\equiv \sqrt{M^2})$ with the Higgs-phobic $A$. For $t_\beta = 100$, we consider $M_A = 70$ GeV (left panel) and $M_A = 300$ GeV (right panel). The colored regions correspond to $M \in [0.5M_A, 2M_A]$.

we cumulatively enforce the following steps:\footnote{An important constraint is from flavor physics like $b \to s\gamma$ \cite{83, 84}. In type-X, the region with small $t_\beta$ and the light charged Higgs boson is significantly constrained: $\tan \beta > 2.7 (2.6)$ for $M_{H^0} = 110 (140)$ GeV \cite{83}. But the observed $\Delta a_\mu$ requires large $t_\beta$ above 35. The FCNC processes have negligible effects.}

**Step-I: $\Delta a_\mu + \text{Theory}$**

1. We obtain $s_{\beta - \alpha}$ from the model parameters by using the condition in Eq. (14). For more efficient scanning, we preliminarily demand $0.8 < |s_{\beta - \alpha}| < 1$ at this level, considering the most updated results on the Higgs coupling modifiers of $\kappa_Z > 0.86$ and $\kappa_W > 0.94$ in the category of $\kappa_{W,Z} \leq 1$ at 95% C.L. \cite{80}.

2. We demand the bounded-from-below potential \cite{85}, the unitarity of scalar-scalar scatterings \cite{71, 86}, the perturbativity of Higgs quartic couplings \cite{78}, and the stability of the vacuum \cite{87-89}.

3. We require that the model explains $\Delta a_\mu$ in Eq. (2).

**Step-II: EWPD + Step-I**

We consider the Peskin-Takeuchi electroweak oblique parameters $S$ and $T$ with $U = 0$ before and after the CDF $m_W$ measurement \cite{4}:

\[
\begin{align*}
\text{PDG:} & \quad S = 0.05 \pm 0.08, \quad T = 0.09 \pm 0.07, \quad \rho_{ST} = 0.92, \\
\text{CDF:} & \quad S = 0.15 \pm 0.08, \quad T = 0.27 \pm 0.06, \quad \rho_{ST} = 0.93,
\end{align*}
\]

where $\rho_{ST}$ is correlation between $S$ and $T$. With respect to the results in the 2HDM \cite{90-92}, we perform the $\chi^2$ analysis in the $(S, T)$ plane, requiring $p > 0.05$. \[\]
Step-III: Collider + Step-II

(1) The Higgs precision data are checked via the public code HiggsSignals-v2.6.2 [76] which takes into account 111 Higgs observables [93–100]. Since our model has five parameters, the number of degrees of freedom is 106. Based on the $\chi^2$ value from the HiggsSignals, we demand that the $p$-value should be larger than 0.05.

(2) The direct searches for new Higgs bosons at the LEP, Tevatron, and LHC are examined by using the open code HiggsBounds-v5.10.2 [77]. We exclude a parameter point if any cross section predicted by the model exceeds the observed 95% C.L. upper bound.

Step-IV: LFU + Step-III

We perform a global $\chi^2$ fit of the Higgs-phobic type-X to $\Delta a_\mu$ and the LFU data.

(1) For the $\tau$ decay, we adopt the HFLAV global fit results of [101]

$$\frac{g_\tau}{g_\mu}, \frac{g_\tau}{g_e}, \frac{g_\mu}{g_e}, \begin{pmatrix} g_\tau \\ g_\mu \end{pmatrix}_\pi, \begin{pmatrix} g_\tau \\ g_\mu \end{pmatrix}_K.$$ (19)

One redundant degree of freedom should be removed since it has a zero eigenvalue in the covariance matrix.

(2) We include the Michel parameters [102, 103], based on the energy and angular distribution of $\ell^-$ in the decay of $\tau^- \rightarrow \ell^- \nu \nu_\tau$:

$$\rho_e, (\xi \delta)_e, \xi_e, \eta_\mu, \rho_\mu, (\xi \delta)_\mu, \xi_\mu, \xi_\pi, \xi_\rho, \xi_{a1}.$$ (20)

(3) We also cover the accurate measurement of the leptonic $Z$ decays. Two ratios of the partial decay rates are considered [104]:

$$\frac{\Gamma(Z \rightarrow \mu^+ \mu^-)}{\Gamma(Z \rightarrow e^+ e^-)}, \frac{\Gamma(Z \rightarrow \tau^+ \tau^-)}{\Gamma(Z \rightarrow e^+ e^-)}.$$ (21)

For the theoretical calculations of the LFU observable in type-X and the experimental data, we refer to Ref. [69]. For 17 independent observables in the global fit, we take the number of degrees of freedom to be $N_{dof} = 17$: the model parameters have already been restricted through three steps (Step-I, II, and III). We demand the $p$-value larger than 0.01.

We performed a random scanning over the five-dimensional parameter space in Eq. (16). For the PDG and CDF cases in Eq. (17), we independently obtained $10^6$ parameter points that pass Step-I. Setting Step-I as the reference, we calculate the survival probabilities at each step as

$$\text{PDG: } P_{\text{Step-II}} = 5.47\%, \quad P_{\text{Step-III}} = 3.14\%, \quad P_{\text{Step-IV}} = 0.62\%,$$

$$\text{CDF: } P_{\text{Step-II}} = 1.54\%, \quad P_{\text{Step-III}} = 0.99\%, \quad P_{\text{Step-IV}} = 0.20\%.$$ (22)

---

3 Equation (29) in Ref. [69] has a typo. Correct one is $R_{1,2,3} = 1 + \delta_{\text{loop}}$. 
The Higgs-phobic type-X does have considerable parameter points that explain all the theoretical and experimental constraints, including the muon $g - 2$ and the LFU data. The $W$-mass before or after the CDF measurement does not intrinsically affect the validity of the model, though yielding different magnitude of the survival probabilities. The PDG result has approximately three times greater probability than the CDF result. But we are aware that just because $m_W^{PDG}$ has more viable parameter points does not mean it is a better solution.

As shown in Eq. (22), each step plays a different role in curtailing the parameter space. The exclusion at Step-II by the oblique parameters of $S$ and $T$ is significant, more for the CDF case. Delicate balance among the masses of new Higgs bosons is required to satisfy Eq. (17), which rules out most of the parameter points. The Higgs precision data and direct search bounds at Step-III affect mildly, which approximately maintain the survival probabilities. But the LFU data at the final step is the killer, which eliminates a large portion of the surviving parameters at Step-III.

![Diagram](image)

**FIG. 2:** Allowed regions of $(M_A, \tan \beta)$ at Step-I, with the color code indicating $\Delta a_\mu$.

Now we investigate the allowed model parameters in more detail. First, we present $t_\beta$ versus $M_A$ at Step-I in Fig. 2, which is common for $m_W^{PDG}$ and $m_W^{CDF}$. The color code indicates $\Delta a_\mu$. The observed $\Delta a_\mu$ allows the band shape in $(M_A, t_\beta)$, which gets wider as $M_A$ or $t_\beta$ increases. We need large $t_\beta$ above $\sim 35$ and light $M_A$ below $\sim 170$ GeV. Larger $M_A$ above 170 GeV is also feasible if $t_\beta$ is greater than 200. But we avoid too large $t_\beta$ to retain the perturbativity of the Yukawa coupling of the tau lepton to $H$ and $A$.

For the masses of new Higgs bosons, we show $M_{H^\pm}$ versus $M_A$ in Fig. 3, with the color code of $M_H$. We present the results step by step: at Step-II (left panels), Step-III (middle panels), and Step-IV (right panels). We also compare the PDG (upper panels) with the CDF (lower panels). One of the most important features is that upper bounds exist on the masses of new Higgs bosons, which happens from Step-I. The main reason is that the light $M_A$, which is required to explain $\Delta a_\mu$, brings down $M_H$ and $M_{H^\pm}$ because of Eq. (7). The upper bounds
FIG. 3: $M_{H^\pm}$ versus $M_A$ at Step-II (left panels), Step-III (middle panels), and Step-IV (right panels), with the color code indicating $M_H$. We consider the PDG results (upper panels) and the CDF results (lower panels).

on $M_H$ and $M_{H^\pm}$ remain almost intact to the last step such that $M_{H,H^\pm} \lesssim 600$ GeV.

On the contrary, the lower bound on $M_{H^\pm}$ is different in the PDG and CDF cases: $M_{H^\pm} \gtrsim 250$ GeV for the PDG and $M_{H^\pm} \gtrsim 300$ GeV for the CDF at the final step. The difference begins at Step-II. Let us restrict the discussion to the case with $M_A \lesssim 50$ GeV for the moment. The oblique parameters of $S$ and $T$ from $m_W^{\text{CDF}}$ require a larger mass gap between $M_A$ and $M_{H^\pm}$. That is the reason for a larger lower bound on $M_{H^\pm}$ in the CDF case. At Step-III, the lower bound increases as the region with light $M_A$ and light $M_{H^\pm}$ is excluded. The Higgs precision data curtail the region, mainly from $h \to \tau^+\tau^-$. Although the region with $M_A \gtrsim 120$ GeV allows light $M_{H^\pm}$ around 100 GeV at Step-III, the final Step-IV eliminates all the parameter points with $M_A \gtrsim 38$ GeV. Therefore, $M_{H^\pm} \gtrsim 250$ (300) GeV for the PDG (CDF) applies to the whole parameter space.

The LFU data in the $\tau$ and $Z$ decays have been known challenging to explain, in connection with the muon $g - 2$ anomaly. Neither the SM nor the aligned type-X succeeds: in the global fit to $\Delta a_\mu$ and the LFU data, the SM has $p = 0.003$ and the aligned type-X has $p = 0.02$ [69]. On the other hand, the Higgs-phobic type-X has $\chi^2_{\min} = 27.40$ in the CDF case (corresponding to $p = 0.052$), which is consistent at 95% C.L.: the $p$ value in the PDG case is almost the same. Since the allowed parameter points at 95% C.L. are tiny, we adopt 99% C.L. in our analysis.

The impact of the LFU data on the masses of new Higgs bosons is most profound. The region of $M_A \gtrsim 30$ GeV is eliminated. The exclusion is primarily from the tree-level contributions to
the lepton flavor violating decays of $\tau$, mediated by the charged Higgs boson. The key parameter is [69]

$$\delta_{\text{tree}} = \frac{m_\mu m_\tau t_\beta^2}{M_{H^\pm}^2}. \quad (23)$$

Very large $t_\beta$, which accords with heavy $M_A$ because of $\Delta a_\mu$, blows up the $\chi^2$ value in the global fit. Since the light pseudoscalar boson is feasible in the Higgs-phobic type-X, our model can maintain $\delta_{\text{tree}}$ in an acceptable range. We also observe that within the range of $M_A \lesssim 38$ GeV, there is a correlation between $M_A$ and $M_{H,H^\pm}$: the lighter $M_A$ is, the heavier $M_{H,H^\pm}$ is.

![Graph](image)

FIG. 4: $\tan \beta$ versus $|\sin(\beta - \alpha)|$ with color code of $M_{H^\pm}$ at Step-II (left panels), Step-III (middle panels), and Step-IV (right panels). We compare the PDG (upper panels) with those of the CDF (lower panels). As shown in Fig. 1, the Higgs-phobic type-X generically has the Higgs alignment, although not 100%. This characteristic is evident from Step-I. When imposing the Higgs precision data at Step-III, the tendency toward the alignment is stronger. A dramatic change happens in Step-IV from the LFU data. Large $t_\beta$ above $\sim 65$ is excluded.

Now we show $t_\beta$ versus $|s_{\beta-\alpha}|$ with color code of $M_{H^\pm}$ in Fig. 4. We sequentially present the allowed parameter points at Step-II (left panels), Step-III (middle panels), and Step-IV (right panels). We compare the results of the PDG (upper panels) with those of the CDF (lower panels). As shown in Fig. 1, the Higgs-phobic type-X generically has the Higgs alignment, although not 100%. This characteristic is evident from Step-I. When imposing the Higgs precision data at Step-III, the tendency toward the alignment is stronger. A dramatic change happens in Step-IV from the LFU data. Large $t_\beta$ above $\sim 65$ is excluded.

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4 We set the ranges of $|s_{\beta-\alpha}|$ at Step-III and Step-IV differently from that at Step-II, to show the results more closely.
Fig. 5: Distributions of the cutoff scales of the parameter points at Step-III (left panel) and Step-IV (right panel) in the CDF case.

IV. CUTOFF SCALES VIA THE RGE ANALYSIS

Now that the Higgs-phobic type-X is shown to explain two anomalies of $m_{W}^{\text{CDF}}$ and $\Delta a_{\mu}$, the LFU data, and other theoretical/experimental constraints, a question arises as to what energy scale this model is valid. The answer is different for each parameter point. Therefore, we make each point running via the RGEs, and check three conditions—unitarity, perturbativity, and vacuum stability—as increasing the energy scale. If any condition is broken at a particular energy scale, we stop the evolution and record the energy scale as the cutoff scale $\Lambda_{c}$. An excellent way to present the high energy behavior of all the viable parameter points is to show the distributions of $\Lambda_{c}$.

We use the public code 2HDME [105, 106] to run the following parameters:

\begin{equation}
\begin{gathered}
g_{s}, \quad g, \quad g', \quad \lambda_{1,..,5}, \quad \xi_{f}^{h,H,A}, \quad m_{ij}^{2}, \quad v_{i}, \quad (i = 1, 2). \\
\end{gathered}
\end{equation}

We incorporated the RG running of $v_{1}$ and $v_{2}$, originated from the mixing effects of two scalar doublet fields with equal quantum numbers. The top quark pole mass scale, $m_{t}^{\text{pole}} = 173.4$ GeV, is used to match the 2HDM to the SM parameters. The boundary conditions at $m_{t}^{\text{pole}}$ are referred to Ref. [105]. We convert each physical parameter point of Eq. (15) into the generic parameter point in Eq. (24). And we evolve them into higher energy scale through the one-loop RGEs.\(^5\)

Figure 5 presents the distributions of the cutoff scales in the CDF case.\(^6\) To demonstrate the effect of the LFU data on $\Lambda_{c}$, we compare the $\Lambda_{c}$ distributions of the parameter points

\(^5\) In the $\Lambda_{c}$ distributions, the difference between one-loop and two-loop RGEs are not significant. To save computing time, we take the one-loop RGEs.

\(^6\) The PDG case has similar results as in the CDF case.
that pass Step-III (left panel) with Step-IV (right panel). The “Rate” in the y-axis denotes the ratio \( N_{\Lambda_c}/N_{\text{step}} \): \( N_{\Lambda_c} \) is the number of the parameter points that fail to satisfy unitarity, perturbativity, or stability at the cutoff scale \( \Lambda_c \); \( N_{\text{step}} \) is the total number of the parameter points that pass the corresponding step. For the parameter points that pass Step-III, the Higgs-phobic type-X is stable up to about \( 10^7 \) GeV. Requiring the consistency with the LFU data at Step-IV, however, the model is valid only up to about 100 TeV, which is about hundred times smaller than at Step-III. Although the Higgs-phobic type-X is a viable model at the electroweak scale, it needs an extension at the energy scale not far from the current LHC reach. Future colliders, targeted at \( \sqrt{s} = 100 \) TeV, such as the Future hadron-hadron Circular Collider (FCC-hh) at CERN [107] and the CEPC [108, 109] are expected to find a hint of the next-level new physics model.

The final discussion is on the parameter points allowed by high cutoff scales. If we impose the condition of \( \Lambda_c > 1 \) TeV, the surviving probability is almost halved. If we further strengthen the condition into \( \Lambda_c > 10 \) TeV, the final surviving probability in the CDF case goes down to 0.01%. The parameter space is almost pinned down as

\[
\begin{align*}
\text{for } \Lambda_c > 10 \text{ TeV} : \\
M_A &\in [11, 38] \text{ GeV}, \\
M_H &\in [249, 306] \text{ GeV}, \\
M_{H^\pm} &\in [283, 338] \text{ GeV}, \\
M &\in [249, 306] \text{ GeV}, \\
t_\beta &\in [36.6, 64.7].
\end{align*}
\]

Since the masses of \( H \) and \( H^\pm \) are well within the LHC reach, we expect that the HL-LHC has a high potential to probe the model.

V. GOLDEN DISCOVERY CHANNELS AT THE LHC

In this section we explore the golden channels of the Higgs-phobic type-X at the LHC, the multi-\( \tau \) states [110–113]. Before going into the production modes, we discuss the branching ratios of \( A, H^\pm, \) and \( H \) for the viable parameter points consistent with the CDF \( W \) mass and muon \( g - 2 \). Since the pseudoscalar is light, its decay is restricted only to the fermionic sector. Furthermore, the suppressed couplings of \( A \) to the quark sector by large \( t_\beta \) make \( A \to \tau^+\tau^- \) dominant: its branching ratio is almost 100%. Another interesting decay channel is \( A \to \mu^+\mu^- \), with the branching ratio of about 0.3%, which can be useful in reconstructing the mass of the pseudoscalar [110, 114]. On the other hand, \( H^\pm \) and \( H \) have not only substantially heavy masses but also large mass splitting from \( M_A \). The bosonic decay modes of \( H^\pm \to W^\pm A \) and \( H \to ZA \) are kinematically open. In addition, their partial decay widths are enhanced by a factor of \( (M_{H^\pm}^2/M_W^2)^2 \) and \( (M_H^2/M_Z^2)^2 \). Therefore, the bosonic decays of \( H^\pm \) and \( H \) are dominant.

To illustrate this feature, we present in Fig. 6 the branching ratios of \( H^\pm \) (left panel) and \( H \) (right panel). For \( H^\pm \), the bosonic mode of \( H^\pm \to W^\pm A \) is dominant over the leptonic mode of \( H^\pm \to \tau\nu_\tau \): the minimum of \( \mathcal{B}(H^\pm \to W^\pm A) \) is about 70%. The decay of \( H \) is also dominated by \( H \to ZA \) with its branching ratio above 60%.
After having a clear understanding of the branching ratios, we move on to investigate the possible multi-$\tau$ states through the electroweak processes.$^7$ The $3\tau$ states are through

$$3\tau : \begin{aligned} pp &\rightarrow H^\pm A \rightarrow [\tau^\pm \nu_\tau][\tau^+ \tau^-], \\
pp &\rightarrow H^\pm H \rightarrow [\tau^\pm \nu_\tau][\tau^+ \tau^-]. 
\end{aligned} \quad (26)$$

The $4\tau$ from the decays of new Higgs bosons, including the associated production of gauge bosons, are

$$4\tau : \begin{aligned} pp &\rightarrow HA \rightarrow [\tau^+ \tau^-][\tau^+ \tau^-], \\
pp &\rightarrow H^\pm A \rightarrow [W^\pm A]A \rightarrow [W^\pm \tau^+ \tau^-][\tau^+ \tau^-], \\
pp &\rightarrow HA \rightarrow [ZA]A \rightarrow [Z\tau^+ \tau^-][\tau^+ \tau^-], \\
pp &\rightarrow H^\pm H \rightarrow [W^\pm A][ZA] \rightarrow [W\tau^+ \tau^-][Z\tau^+ \tau^-], \\
pp &\rightarrow H^+ H^- \rightarrow [W^+ A][W^- A] \rightarrow [W^+ \tau^+ \tau^-][W^- \tau^+ \tau^-]. 
\end{aligned} \quad (27)$$

The production of $HA$ ($H^\pm A$) mediated by $Z$ ($W^\pm$) is favored by the Higgs alignment because the vertex of $Z$-$H$-$A$ ($W^\pm$-$H^\pm$-$A$) is proportional to $s_{\beta-\alpha}$.

We implement the type-X 2HDM in FeynRules [115] to obtain the Universal FeynRules Output (UFO) [116]. Interfering the UFO file with MadGraph5-aMC@NLO [117], we compute the cross-sections of $pp \rightarrow H^\pm A/H^\pm H/H A/H^+ H^-$ at 14 TeV LHC using NNPDF31_lo_as_0118 [118] parton distribution function set. The two-body cross-sections are multiplied by relevant

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$^7$ The $2\tau$ state can come through charged Higgs pair creation and its subsequent decay into $\tau\nu_\tau$. However, the charged Higgs pair production via $Z^*/\gamma^*$ is much smaller than the charged current channel via $W^*$. Moreover, the leptonic decay of $H^\pm$ is suppressed. Hence we do not consider the $2\tau$ state.
branching ratios of $A$, $H^\pm$ and $H$, obtained by the 2HDMC \cite{2HDMC}. Figure 7 presents the parton level cross-sections of $3\tau$ and $4\tau$ states in Eqs. (26) and (27). In the left panel, we show the cross-sections of the $3\tau$ and $4\tau$ states via leptonic decays $H^\pm$ and $H$, without accompanying gauge bosons. For the cross section of the $3\tau$, we combine the two signals in Eq. (26). In the middle panel we show the cross-sections for $4\tau + V$ processes where $V = W^\pm, Z$, originated from $pp \rightarrow H^\pm A/H A$. Since the bosonic decays of $H^\pm$ and $H$ are dominant, we clearly see a large cross-sections compared to the $3\tau$ and $4\tau$ states without $V$. The right panel demonstrates the cross-sections of $4\tau + VV'$, which are smaller than those of $4\tau + V$. Since the productions of $4\tau + VV'$ proceed through $pp \rightarrow H^\pm H/H^+ H^-$, the heavy masses of $H^\pm$ and $H$ (see Fig. 3) suppress the production cross section.

Comparing the three plots in Fig. 7, we find that $4\tau + V$ has the largest cross section, maximally about 50 fb. The cross sections of $3\tau/4\tau$ and $4\tau + VV'$ have similar values. But what determines the signal significance is the SM background. For rough estimate, we calculate the parton level cross section of $pp \rightarrow 4\tau + W^\pm$ and $4\tau + ZW^\pm$ in the SM by using the MadGraph5-aMC@NLO \cite{MadGraph5}. We minimally impose the kinematic cuts on $\tau$ as $p_T^\tau > 10$ GeV, $|\eta_\tau| < 2.5$, and $\Delta R(\tau, \tau) > 0.4$. The SM cross sections are $\sigma(pp \rightarrow 4\tau + W^\pm) \simeq 36.8$ fb and $\sigma(pp \rightarrow 4\tau + ZW^\pm) \simeq 0.26$ ab. Reducible backgrounds are the production of four QCD jets plus $W^\pm$ or $ZW^\pm$, where the QCD jets are misidentified as $\tau$. Considering the mistagging rates of $P_{j \rightarrow \tau} = 0.02$ in the one-prong $\tau$ decays and $P_{j \rightarrow \tau} = 0.01$ in the three-prong $\tau$ decays, it is hard for the QCD jet backgrounds to mimic the $4\tau$ states. Other possible reducible backgrounds would be $t\bar{t}$+jets, $V$+jets, and $VV'$+ jets ($V^0 = Z, W^\pm$). The $4\tau$ production associated with double gauge bosons ($Z$ and $W$) which decay into leptonic modes has very small backgrounds.

\footnote{The 2HDM bosons file in the MadGraph misses some important decay modes of new scalar bosons such as $H^\pm \rightarrow cs$ and $A \rightarrow gg$.}
and hence could be the most promising channel at the LHC. We can reduce $V+$jets backgrounds significantly by imposing the selection cuts like $n_\ell \geq 3$ and $n_{\tau_h} \geq 4$, where $\ell = e, \mu$. In addition to that, the $b$-veto will kill the $t\bar{t}+$jets background. Nevertheless, due to the low mass of $A$ and the decay chains involving $W^\pm/Z$, the $\tau$-jets will be soft resulting in a low $\tau$-tagging efficiency and hence low sensitivity. In that situation another alternative would be to consider the mixed state like $2\ell + 2\tau_h$ decay mode of $4\tau$ [110]. Then the final state of $2\ell + 2\tau_h + ZW^\pm$ with leptonic decays of $Z$ and $W^\pm$ results in five $\ell$'s, of which the backgrounds are very small. In the era of new $W$ boson mass, it is worth studying the feasibility of $4\tau +VV'$ states at the high luminosity phase of LHC which we leave for our future study.

VI. CONCLUSIONS

The recent measurement on the $W$ boson mass by the CDF Collaboration calls for new physics beyond the SM, as the Peskin-Takeuchi parameters of $S$ and $T$ significantly deviate from the SM expectation: $S = 0.15 \pm 0.08$ and $T = 0.27 \pm 0.06$ with $U = 0$. Another anomaly from the muon anomalous magnetic moment has been around for quite some time. Type-X in the 2HDM is one of the most attractive solutions for the muon $g - 2$ via a light pseudoscalar boson, but it suffers from the exotic Higgs decay of $h \rightarrow AA$ and the lepton flavor universality data in the $\tau$ and $Z$ decays. Therefore, we have studied the type-X with the Higgs-phobic $A$ by requiring the $h-A-A$ vertex to vanish.

Through random scanning of the model parameters, we impose the theoretical and experimental constraints step by step: Step-I is for the muon $g - 2$ and theoretical stabilities; Step-II is for the oblique parameters before and after the CDF $m_W$ measurement; Step-III applies the Higgs precision data and the direct search bounds at high energy colliders; Step-IV adopts the global $\chi^2$ of the model to $\Delta a_\mu$ and the LFU data. The most important consequence is that the Higgs-phobic type-X can explain not only $m_W^{CDF}$ and $\Delta a_\mu$ anomalies but also all the other experimental data including the LFU data.

As the combination of various constraints highly curtails the parameter space, the characteristics of the finally viable parameter points are strong.

- The muon $g - 2$ anomaly requires light $M_A$ and large $t_\beta$.
- The LFU data plays the most essential role, as eliminating the whole region with $M_A \gtrsim 38$ GeV and $t_\beta \gtrsim 70$.
- The CDF $W$ boson mass does not give critically different result, compared to before. But a large mass gap between $M_A$ and $M_{H^\pm}$ will result in a significant implications on the LHC phenomenology.
- The RGE running of the parameters tell that the model can retain the stability of the scalar potential up to about 100 TeV. Imposing an additional condition of $\Lambda_c > 10$ TeV
narrows the parameters as $M_A \in [11, 38] \text{ GeV}$, $M_{H^\pm} \in [283, 338] \text{ GeV}$, and $M_H \in [249, 306] \text{ GeV}$.

- The light mass of the pseudoscalar with sizable mass gaps from $M_{H^\pm}$ and $M_H$ implicates that the $4\tau$ states associated with two gauge bosons, $ZW$ and $WW$, are the golden search modes at the LHC.

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