Single Transverse-Spin Asymmetry in Hard-Exclusive Meson Electroproduction in the Backward Region

J.P. Lansberg\textsuperscript{1}\textsuperscript{*}, B. Pire\textsuperscript{1}, L. Szymanowski\textsuperscript{2}

\textsuperscript{1}Centre de Physique Théorique, École polytechnique, CNRS, F-91128, Palaiseau, France
\textsuperscript{2}Soltan Institute for Nuclear Studies, Warsaw, Poland

We discuss the relevance of studying single transverse-spin asymmetry in hard-exclusive meson electroproduction in the backward region. Such an asymmetry could help us discriminate between contributions from a soft baryon exchange in the $u$-channel and a hard parton-induced scattering.

Keywords: Exclusive-Backward Reactions, Single Transverse-Spin Asymmetry

1. Introduction

In Ref. 1–3, we introduced the framework to study backward pion electroproduction,

$$\gamma^*(q)N(p_1) \rightarrow N'(p_2)\pi(p_\pi),$$

on a proton (or neutron) target, in the Bjorken regime ($q^2$ large and $q^2/(2p_1.q)$ fixed) in terms of a factorized amplitude (see Fig. 1) where a hard part is convoluted with Transition Distribution Amplitudes (TDAs), as well as the reaction,

$$N(p_1)\bar{N}(p_2) \rightarrow \gamma^*(q)\pi(p_\pi),$$

in the near forward region.\textsuperscript{4–6} This extended the concept of Generalised Parton Distributions (GPDs). Such an extension of the GPD framework has already been advocated in the pioneering work of Ref. 7. The TDAs involved in the description of Deeply-Virtual Compton Scattering (DVCS)

\textsuperscript{*}lansberg@cpht.polytechnique.fr
in the backward kinematics \( \gamma^*(q)N(p_1) \to N'(p_2)\gamma(p_\gamma) \) and the reaction \( N(p_1)\bar{N}(p_2) \to \gamma^*(q)\gamma(p_\gamma) \) in the near forward region were given in Ref. 8.

Recently, a study of the TDAs in the meson-cloud model\(^9\) has become available.\(^{10}\) Yet, more work is needed before being able to proceed to quantitative comparisons between different TDA models and between theory and experiments. For the time being, model independent analyses – looking for scaling or characteristic polarization effects – sound more expedient. In this context, we would like to argue here that the study of target transverse-spin asymmetry (which we will denote SSA) could be used as a test of the dominance of a hard parton-induced scattering in the backward region at large \( Q^2 \) rather than that of a soft baryon exchange in the \( u \)-channel. Such a reaction would only generate phases through final state interactions, expected to decrease for \( W^2 \gg (M + m_\pi)^2 \) and large \( Q^2 \). On the contrary, in reactions at the parton level, one expects an imaginary part to develop and to generate a SSA independently of whether \( W^2 \) and \( Q^2 \) are large or not. This will be explained later on.

\begin{equation}
\begin{aligned}
\end{aligned}
\end{equation}

Fig. 1. Illustration of the factorisation for backward electroproduction of a pion.

\section{Some definitions}

The five-fold differential cross section for the process \( eP \to e'P'\pi^0 \) can be reduced to a two-fold one – expressible in the center-of-mass frame of the \( P'\pi^0 \) pair – multiplied by a flux factor \( \Gamma \),

\[ \frac{d\sigma}{dE_\gamma d\Omega_\ell_1 d\Omega_\ell_3} = \Gamma \frac{d\sigma}{d\Omega_\gamma} \]

where \( \Omega_\gamma \) is...
the differential solid angle for the scattered electron in the lab frame, and
\( \Omega_n^* \) is the differential solid angle for the pion in the \( P' \pi^0 \) center-of-mass
frame, such that \( d\Omega_n^* = d\phi d\cos \theta_n^* \). \( \theta_n^* \) is defined as the polar angle between
the virtual photon and the pion in the latter system (see Fig. 2). \( \phi \) is the
azimuthal angle between the electron plane and the plane of the process
\( \gamma^* P \to P' \pi^0 \) (hadronic plane) \( \phi = 0 \) when the pion is emitted in the half
plane containing the outgoing electron).

In general, we have contributions from different polarisations of the
photon. For that reason, we define four polarised cross sections, \( d^2\sigma_T \),
\( d^2\sigma_L \), \( d^2\sigma_{TL} \) and \( d^2\sigma_{TT} \), which do not depend on \( \phi \) but only on \( W, Q^2 \)
and \( \theta_n^* \). The \( \phi \) dependence is written as
\begin{equation}
\frac{d^2 \sigma}{d\Omega_n^*} = \frac{d^2 \sigma_T}{d\Omega_n^*} + \epsilon \frac{d^2 \sigma_L}{d\Omega_n^*} + \sqrt{2}\epsilon(1+\epsilon) \frac{d^2 \sigma_{TL}}{d\Omega_n^*} \cos \phi + \epsilon \frac{d^2 \sigma_{TT}}{d\Omega_n^*} \cos 2\phi.
\end{equation}

At the leading-twist accuracy, the QCD mechanism considered here contrib-
utes only to \( \frac{d^2\sigma_{TL}}{d\Omega_n^*} \).

In the scaling regime, the amplitude for \( \gamma^* P(p_1) \to P'(p_2)\pi(p_\pi) \) in the
backward kinematics – namely small \( u = (p_\pi - p_1)^2 = \Delta^2 \) or \( \cos \theta_n^* \) close to -1

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\*For \( L, x, y \), the linear polarisations of the virtual photon (for the definition of the \( x \) & \( y \) axis, see Fig. 1), one
defines\([12,13]\): \( d^2\sigma_L \propto M^0(M')^y \), \( d^2\sigma_T \propto 1/2[M^0(M')^y + M^0(M')^y] \), \( d^2\sigma_{TL} \propto M^0(M')^y + M^0(M')^y \) and
\( d^2\sigma_{TT} \propto 1/2[M^0(M')^y - M^0(M')^y] \).

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Fig. 2. Kinematics of electroproduction of a pion and definition of the angles \( \phi \) and \( \phi_S \)
– involves the TDAs $T(x_i, \xi, \Delta^2)$, where $x_i$ ($i = 1, 2, 3$) denote the light-cone-momentum fractions carried by participant quarks and $\xi$ is the skewness parameter such that $2\xi = x_1 + x_2 + x_3$.

The amplitude is then a convolution of the proton DAs, a perturbatively-calculable-hard-scattering amplitude and the TDAs, defined from the Fourier transform of a matrix element of a three-quark-light-cone operator between a proton and a meson state. We have shown that these TDAs obey QCD evolution equations, which follow from the renormalisation-group equation of the three-quark operator. Their $Q^2$ dependence is thus completely under control.

The momenta of the process $\gamma^* P \to P' \pi$ are defined as in Fig. 1 & 2. The $z$-axis is chosen along the initial nucleon and the virtual photon momenta and the $x-z$ plane is identified with the collision or hadronic plane (Fig. 2). Then, we define the light-cone vectors $p$ and $n$ ($p^2 = n^2 = 0$) such that $2p \cdot n = 1$, as well as $P = \frac{1}{2}(p_1 + p_\pi)$, $\Delta = p_\pi - p_1$ and its transverse component $\Delta_T$ ($\Delta_T, \Delta_T = \Delta^2_T < 0$). From those, we define $\xi$ in an usual way as $\xi = -\frac{\Delta n}{2P n}$.

We can then express the momenta of the particles through their Sudakov decomposition and, keeping the first-order corrections in the masses and $\Delta^2_T$, we have:

\begin{align*}
    p_1 &= (1 + \xi) p + \frac{M^2}{1 + \xi} n, \quad q = -2 \xi \left(1 + \frac{\Delta^2_T - M^2}{Q^2}\right) p + \frac{Q^2}{2(1 + \frac{\Delta^2_T - M^2}{Q^2})} n, \\
    p_\pi &= (1 - \xi) p + \frac{m^2_\pi - \Delta^2_T}{1 - \xi} n + \Delta_T, \quad \Delta = -2 \xi p + \left[\frac{m^2_\pi - \Delta^2_T}{1 - \xi} - \frac{M^2}{1 + \xi}\right] n + \Delta_T \\
    p_2 &= -2 \xi \frac{\Delta^2_T - M^2}{Q^2} p + \left[\frac{Q^2}{2(1 + \frac{\Delta^2_T - M^2}{Q^2})}\right] - \frac{m^2_\pi - \Delta^2_T}{1 - \xi} + \frac{M^2}{1 + \xi} n - \Delta_T, \quad (4)
\end{align*}

For $\epsilon_x = (0, 1, 0, 0)$ and $\epsilon_y = (0, 0, 1, 0)$ with the axis definitions of Fig. 2, one may further specify that

\begin{equation}
    \Delta_T = |\Delta_T| \cos \phi \epsilon_x + \sin \phi \epsilon_y \quad \text{and} \quad s_{T,1} = s_1 = \cos \phi_s \epsilon_x + \sin \phi_s \epsilon_y, \quad (5)
\end{equation}

for the transverse spin of the target ($s_1, p_1 = s_1, p = s_1, n = 0$).

In the following, we use the same definition of the $p \to \pi^0$ TDAs as given by Eq. (15) of Ref. [3].
3. Hard-amplitude calculation

At leading order in \( \alpha_s \), the amplitude \( M_{s_1s_2}^{\lambda} \) for \( \gamma^*(q,\lambda)P(p_1,s_1) \to P'(p_2,s_2)\pi^0(p_\pi) \) reads

\[
M_{s_1s_2}^{\lambda} = -i \frac{(4\pi\alpha_s)^2}{54f_\pi} \frac{1}{\langle \bar{u}_2(\lambda) \rangle} \frac{\gamma^5 u_1}{S_{s_1s_2}} \int \left( \sum_{\alpha=1}^{7} T_\alpha + \sum_{\alpha=8}^{14} T_\alpha \right) \int \left( \sum_{\alpha=1}^{7} T'_\alpha + \sum_{\alpha=8}^{14} T'_\alpha \right) \delta(2\xi - x_1 - x_2 - x_3) \delta(1 - y_1 - y_2 - y_3),
\]

where \( \int \equiv \int_{-1+\xi}^{1+\xi} dx_1 dx_2 dx_3 \delta(2\xi - x_1 - x_2 - x_3) \int_{0}^{1} dy_1 dy_2 dy_3 \delta(1 - y_1 - y_2 - y_3) \).

The expression in Eq. (6) is to be compared with the leading-twist amplitude for the baryonic-form factor\(^\text{14}\)

\[
M^{\lambda} \propto -i(\bar{u}_2(\lambda)u_1) \frac{\alpha_s^2 f_\pi^2}{Q^4} \int \left( \sum_{\alpha=1}^{7} T^\alpha(x_i,y_j,\xi,t) + \sum_{\alpha=8}^{14} T'^\alpha(x_i,y_j,\xi,t) \right).
\]

The factors \( T^\alpha \) are very similar to the \( T_\alpha \) obtained here. However, the integration domain is different. In the form factor case

\[
\int \text{stands for } \int_{0}^{1} dx_1 dx_2 dx_3 \delta(1 - \sum_i x_i) \int_{0}^{1} dy_1 dy_2 dy_3 \delta(1 - \sum_i y_i).
\]

Consequently, the integration of denominators in \( T^\alpha \) such as \( 1/(x_i + i\epsilon) \) do not generate any imaginary parts. On the contrary, the integrations of similar denominators in \( T_\alpha \) and \( T'_\alpha \) over the TDA integration domain will generate an imaginary part when passing from the ERBL region (all \( x_i > 0 \)) to one of the DGLAP regions (one \( x_i < 0 \)). This will be the source of the SSA as we will show later on.
4. Single Transverse Spin Asymmetry

In order to study a possible SSA we shall study the quantity $\sigma^{s_i} - \sigma^{-s_i}$ with the definition

$$\sigma^{s_i} = \sum_A \sum_{s_2} (\mathcal{M}_{s_1s_2}^4)(\mathcal{M}_{s_1s_2}^4)^*.$$  \hfill (9)

Since we are interested in the leading twist contribution of this asymmetry, we can sum only over the transverse polarisation of the virtual photon using $\sum_{\lambda=x,y} e(\lambda)\gamma^\mu(e(\lambda))^*= -g^{\mu\nu} + (p^\mu n^\nu + p^\nu n^\mu)/(p.n)$. The sum on the final-proton spin $s_2$ is done using $\sum_{s_2} u_\alpha(p_2, s_2)\bar{u}_\beta(p_2, s_2) = (p_2 + M)_{\alpha\beta}$. As regards the initial-proton spinor, one uses the following relation involving its transverse spin $s_1$, $u_\alpha(p_1, s_1)\bar{u}_\beta(p_1, s_1) = 1/2(1 + \gamma^5 s^1)(\hat{p}_1 + M)_{\alpha\beta}$.

Hence, one has

$$\sum_{\lambda=x,y} \sum_{s_2} (\mathcal{M}_{s_1s_2}^4)(\mathcal{M}_{s_1s_2}^4)^* = \frac{|C|^2}{Q^8} [-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p.n}] \times$$

$$\left[\bar{u}_2 \gamma^\mu \gamma^5 u_1 \mathcal{I} - \bar{u}_2 \gamma^\mu \frac{\Delta_T}{M} \gamma^5 u_1 \mathcal{I'} \right] \times \left[ -\bar{u}_1 \gamma^5 \gamma_\mu u_2 \mathcal{I}^* + \bar{u}_1 \gamma^5 \frac{\Delta_T}{M} \gamma_\mu u_2 \mathcal{I'}^* \right]$$

$$= \frac{|C|^2}{Q^8} [-g^{\mu\nu} + \frac{p^\mu n^\nu + p^\nu n^\mu}{p.n}] \times$$

$$\left[ -\text{Tr}((\hat{p}_2 + M)\gamma^\mu \gamma^5 (\hat{p}_1 + M)\frac{1}{2}(1 + \gamma^5 s^1)\gamma^5 \mathcal{I})\mathcal{I}^* \right.$$

$$+ \text{Tr}((\hat{p}_2 + M)\gamma^\mu \gamma^5 (\hat{p}_1 + M)\frac{1}{2}(1 + \gamma^5 s^1)\gamma^5 \mathcal{I'})\mathcal{I'}^*$$

$$+ \text{Tr}((\hat{p}_2 + M)\gamma^\mu \gamma^5 (\hat{p}_1 + M)\frac{1}{2}(1 + \gamma^5 s^1)\gamma^5 \frac{\Delta_T}{M} \gamma_\mu \mathcal{I})\mathcal{I}^*$$

$$- \text{Tr}((\hat{p}_2 + M)\gamma^\mu \gamma^5 (\hat{p}_1 + M)\frac{1}{2}(1 + \gamma^5 s^1)\gamma^5 \frac{\Delta_T}{M} \gamma_\mu \mathcal{I'})\mathcal{I'}^* \right]$$

(10)

Dropping the contributions proportional to the proton mass, the spin asymmetry reads ($e_{0123} = -e_{0123}^{0123} = +1$)

$$\sigma^{s_i} - \sigma^{-s_i} = 8 \frac{|C|^2}{Q^6} \frac{1 + \xi}{\xi} \frac{\epsilon^{ps_1, \Delta_T}}{M} \mathcal{I}^* (\mathcal{I}^*)^*$$

$$= -4 \frac{|C|^2}{Q^6} \frac{\Delta_T}{M} \frac{1 + \xi}{\xi} \sin(\phi - \phi_S) \mathcal{I}^* (\mathcal{I}^*)^*.$$  \hfill (11)

Comparing with the expressions for the unpolarised cross section obtained in Ref. 3, one concludes that the asymmetry for the hard-parton induced contribution is leading-twist as soon as $\Delta_T \neq 0$ and $I$ or $I'$ are no longer
pure real or pure imaginary numbers. This is precisely what one expects when DGLAP contributions are taken into account.16

5. Discussion and conclusion

Although the knowledge of baryon to meson TDAs has recently improved significantly thanks to a first study in the meson cloud model10 and another one focused on their spectral representations,15 model-independent observables aimed at studying the backward regime of meson electroproduction will still be the bread-and-butter of this field for the months to come.

In this context, we find it particularly relevant to emphasize that the study of the asymmetry of the target transverse spin would reveal unique information on the nature of the particles exchanged in the \( u \) channel, be it a "mere" baryon slightly off-shell, or three perturbative quarks. For non-vanishing transverse momenta (\( \Delta_T \)), one expects in the latter case an asymmetry of the same order as the unpolarised cross section, while, in the former case, they would be most likely decreasing for increasing \( W^2 \) and \( Q^2 \).

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