THE PROPAGATION OF THE WAVES IN THE CTO S.A. TOWING TANK

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ABSTRACT

The paper presents the results of research focused on the wave propagation in the CTO S.A. deepwater towing tank. In the scope of paper, the wavemaker transfer function was determined for regular waves, based on the Bièsel Transfer Function and further for irregular waves, based on Hasselman model of nonlinear energy transfer. The phenomena: wave damping, wave breakdown and wave reflection, were measured, analyzed and mathematically modeled. Developed mathematical models allow to calculate the impact of mentioned phenomena on the wave propagation and furthermore to calculate the wave characteristics along the whole measurement area in the CTO S.A. deepwater towing tank, based on wavemaker flap motion control.

Keywords: waves, propagation, nonlinear transfer function, towing tank

INTRODUCTION

During the hydromechanical tests with a model of the object in a sea wave, such as the tests carried out in the CTO S.A. deepwater towing tank (Fig. 1), it is crucial to obtain the spectrum of the generated wave consistent with the modeled sea wave in a reduced scale. The wave should be properly modeled along whole measurement area. The transfer function of the wavemaker and moreover the damping, breakdown and reflection phenomena, have impact on modeling the marine environment conditions.

The knowledge of the above factors is important for realization of the model tests. In the scope of this paper, mentioned factors have been analyzed. The analysis was carried out on the basis of measurements made in CTO S.A.
TOWING TANK

The CTO S.A. deepwater towing tank is a research object with 270 m length, 12 m width and 6 m depth, as shown in Fig. 2. Investigations carried out in the towing tank consist in testing the properties of objects floating freely (vessels), anchored (oil rigs) or permanently fixed to the bottom (offshore wind turbines). The tests are conducted for models of objects in a reduced scale. The models are tested in the uniform flow (simulated by motion of the carriage at constant speed) or subjected to the impact of the waves for modeling the impact of the marine environment.

WAVEMAKER

The waves corresponding to ocean waves in a reduced scale are generated by the rigid flap wavemaker with single articulation above channel bed (and below water level). The flap is an actuator with 4.15 m height and 12.00 m width, articulated 2.30 m above channel bed, as it shown in Fig. 2.

SURVEY OF RELATED WORKS

In 1951, F. Biéssel and F. Suquet published formulas known as Biéssel Transfer Functions $BTF$. The $BTF$ allow to calculate the wave height in a towing tank for several types of wavemakers, also for wavemaker with rigid flap and single articulation above channel bed, like in CTO S.A. [1].

The research carried out in 1972, in the Laboratório Nacional de Engenharia Civil (Lisbon, Portugal) has shown that $BTF$ is proper for regular waves. However, for irregular waves, the measured transfer function is inconsistent with theoretical $BTF$. [2] One possible reason indicated by the authors was the lack of linearity and energy losses.

In 1985, S. Hasselmann and K. Hasselmann described a mechanism of the formation of the JONSWAP spectrum and proposed a model of the nonlinear energy transfer in wind waves [3]. The model has two parameters proposed by the authors. Proper parameterization of the model allows to calculate an energy stream transferred from higher frequency bands to lower frequency bands of the wind wave spectrum.

Van Vledder in 2012 discussed the ambiguity of selection of values of the $C$ and $\lambda$ parameters [4]. In 2014 group of people from Kyushu University (Japan), carried out simulations and study for the method mentioned and for a few others methods of wave model implementation [5].

The control system of the wavemaker in the CTO S.A. deepwater towing tank was modernized in 2015 – an analog control system was replaced by a new digital control system [6]. In 2017 research was conducted on optimization of the digital control system [7].

Research described in this paper is intended to develop mentioned works to enable the prediction of wave characteristics in CTO S.A. deepwater towing tank.

OBJECTIVES AND SCOPE OF THE WORK

The objectives of research were to determine the way of wave propagation and to develop the method of prediction of the wave characteristics in the CTO S.A. deepwater towing tank. For this purpose, transfer function for the wavemaker and further the impact of the damping, breakdown and reflection were investigated. The scope of research included the measurements of the waves along whole measurement area and analysis of registered signals.

SOLUTION

MEASUREMENTS

The measurements were made along whole measurement area of the towing tank, between 1st end on the beach side and 2nd end on the wavemaker side. The 1st end is in 205 m distance from the wavemaker. The 2nd end is 60 m distance from the wavemaker. The waves were measured during generation of the waves: regular and irregular. The regular wave is a harmonic wave. The irregular wave is a synthesis of harmonic waves corresponding to the determined spectrum. Parameters of the generated and measured waves are shown in Tab. 1.

The waves were measured by the resistance probe of the water level (Fig. 3), manufactured by CTO S.A. The resistance probe was mounted to the towing carriage in the towing tank axis, i.e. on the actual track of the models tested in waves. During wave generation, the displacement of the wavemaker flap was measured by linear displacement sensor WDS-1000-P60-SR-U manufactured by MicroEpsilon.
Table 1. Parameters of the generated and measured waves

| No. | Type of the wave | Height of the wave $HW$ [cm] | Frequency of the wave $f$ [Hz] |
|-----|-----------------|-----------------------------|-------------------------------|
| 1   | regular         | 2.5                         | 0.3                           |
| 2   | regular         | 2.5                         | 0.9                           |
| 3   | regular         | 2.5                         | 1.0                           |
| 4   | regular         | 2.5                         | 1.1                           |
| 5   | regular         | 2.5                         | 1.2                           |
| 6   | regular         | 5.0                         | 0.3                           |
| 7   | regular         | 5.0                         | 0.4                           |
| 8   | regular         | 5.0                         | 0.5                           |
| 9   | regular         | 5.0                         | 0.6                           |
| 10  | regular         | 5.0                         | 0.7                           |
| 11  | regular         | 5.0                         | 0.8                           |
| 12  | regular         | 5.0                         | 0.9                           |
| 13  | regular         | 5.0                         | 1.0                           |
| 14  | regular         | 5.0                         | 1.1                           |
| 15  | regular         | 5.0                         | 1.2                           |
| 16  | regular         | 10.0                        | 0.3                           |
| 17  | regular         | 10.0                        | 0.4                           |
| 18  | regular         | 10.0                        | 0.5                           |
| 19  | regular         | 10.0                        | 0.6                           |
| 20  | regular         | 10.0                        | 0.7                           |
| 21  | regular         | 10.0                        | 0.8                           |
| 22  | regular         | 10.0                        | 0.9                           |
| 23  | regular         | 10.0                        | 1.0                           |
| 24  | regular         | 10.0                        | 1.1                           |
| 25  | regular         | 10.0                        | 1.2                           |
| 26  | irregular       | $H_{1/3}^{\text{cr}}=9.5$   | $f_{ao}^{\text{cr}}=0.75$     |
| 27  | irregular       | $H_{1/3}^{\text{cr}}=12.9$  | $f_{ao}^{\text{cr}}=0.78$     |
| 28  | irregular       | $H_{1/3}^{\text{cr}}=14.1$  | $f_{ao}^{\text{cr}}=0.69$     |

(* significant height of the irregular wave
(** average frequency of the irregular wave (corresponding to average period of the irregular wave $T_{01}$)

Fig. 3. Resistance probe of the water level manufactured by CTO S.A.

TRANSFER FUNCTION

For rigid flap wavemaker with single articulation above channel bed, the wave height $HW$ is dependent on flap displacement $a$, wave frequency $f$, water depth $h$ and articulation height above channel bed $h_0$ [1]. The formulas representing this dependence are given as (1) and (2). The Biéssel Transfer Function $BTF$ is understood as the relation (3). The $BTF$ for mentioned type wavemaker was experimentally checked for two types of waves – regular and irregular, in the scope consistent with Tab. 1. For this purpose, two signals were measured: the wave $HW$ – measured at the 2nd end of the measurement area using resistance probe and the flap displacement $a$ – measured using the displacement sensor.

$$HW = \frac{\sinh(hk) \cdot (h - h_0) \cdot k \cdot \sinh(kh) - \cosh(kh) + \cosh(kh_0))}{\sinh(kh) \cdot \cosh(kh) + kh}$$

(1)

$$k = \frac{2\pi \cdot f^2}{1.56}$$

(2)

$$BTF = \frac{HW}{a}$$

(3)

Regular waves

The $BTF$ for regular waves in the CTO S.A. deepwater towing tank with the rigid flap wavemaker was checked. The theoretical $BTF$ was corrected as it shown in (4). Implementation of the correction factor $Cr=0.8$ was necessary to obtain the agreement of the calculations with the results of measurements. As shown in Fig. 4, the results of the measurements are consistent with the calculated $BTF_{corr}$ with satisfactory accuracy.

$$BTF_{corr} = Cr \cdot BTF$$

(4)

Irregular waves

The transfer function for irregular waves $TF$ was obtained, based on the FFT analysis of the signal $HW$ and signal $a$. The signals were measured for irregular waves no. 26-28 (Tab. 1). As it shown in Fig. 5, the measured $TF$ is consistent with the calculated $BTF_{corr}$ in scope from 0.3 Hz to 0.6 Hz. For frequencies higher than 0.6-0.7 Hz, $TF$ is significantly increasing and further rapidly falling below $BTF_{corr}$ characteristic.

The rapidly falling characteristics below the theoretical transfer function has been also observed in other hydrodynamics laboratory [2]. It is mentioned there that this phenomenon may be due to lack of linearity and probably
caused by energy losses as effect of the variations of velocity and acceleration in irregular waves.

Other works devoted to gravity waves, like [3] and further [4] and [5], describes and proves that in the wind waves there are nonlinear interactions that cause the nonlinear energy transfer. It occurs in this way that the energy stream $S_{nl}$ is transferred from higher frequency bands to lower frequency bands of the wind wave spectrum. A similar phenomenon can occur in irregular waves generated in the towing tank. It can be seen in Fig. 5.

These papers ([3], [4], [5]) describe the physics of the phenomenon and the methods of calculation $S_{nl}$ for wind waves spectra.

The mathematical model of nonlinear energy transfer proposed in [3] is described by formula (5), where: $\delta S_{nl}$ relates to the frequency $f$, $\delta S_{nl}^+$ relates to the frequency (6) and $\delta S_{nl}^-$ relates to the frequency (7). Furthermore, $C$ is the strength of the nonlinear interaction parameter [3] and $\lambda$ is the Hasselmann’s shape parameter [8].

$$S_{nl} = \sum_{f=0.1 \text{Hz}}^{f=1.5 \text{Hz}} \frac{\delta S_{nl}^+}{\delta S_{nl}^-} = \left[ \frac{2}{1+\lambda} \right] \times C \cdot g^{-4} \cdot f^{-11} \cdot \left[ (\frac{F_+}{(1+\lambda)^4} + \frac{F_-}{(1-\lambda)^4}) - 2 \cdot \frac{F \cdot F_+ \cdot F_-}{(1-\lambda)^4} \right]$$

(5)

$$f^+ = f \cdot (1 + \lambda)$$

(6)

$$f^- = f \cdot (1 - \lambda)$$

(7)

Values of mentioned parameters were proposed as $\lambda=0.25$ and $C=3 \times 10^6$. The values have been chosen empirically as optimal. Another proposition was given as $\lambda=0.15$ and $C=3.75 \times 10^5$ [4].

The ambiguity of selecting the parameters may indicate necessity of individual optimization for given towing tank or even for given spectrum.

Following this path, the $\lambda$ and $C$ were chosen individually for each spectrum of the waves no. 26-28 (Tab.1), using binary search algorithm to obtain the best compatibility of the nonlinear transfer function $NTF$ (8) with the measured $TF$. Chosen parameters are listed in Tab. 2. The results are shown in Fig. 6-8. The $tnl$ (9) is the time of nonlinear transfer of $S_{nl}$ at the distance $xnl=60$ m between wavemaker flap and resistance probe.

As can be seen in Fig. 6-8 the results of the calculated $NTF$ are consistent with the measured $TF$ with satisfactory accuracy in the frequency scope up to 1.0 Hz.

$$NTF(f) = \frac{BTFCorr(f) \cdot a(f) + S_{nl}(f) \cdot tnl(f)}{a}$$

(8)

$$tnl(f) = \frac{xnl \cdot f}{1.56}$$

(9)
The wave damping was investigated on the axis of the towing tank along the whole measurement area. For this purpose, the $H_W$ for waves no. 6-15 (Tab. 1) was measured at the 2nd end ($H_{W2nd}$ in Tab. 3) and then at the 1st end ($H_{W1st}$ in Tab. 3). The measurements were made while the towing carriage was stationary.

Relative damping factor $DF$, understood as (10) allows to calculate the $DF$ as function $DF(x)$, where $x=145$ m is the distance traveled by generated wave from 2nd end to 1st end. Dependence of $DF$ on $f$, shown in Fig. 9, was linearly approximated (11). Finally, the resulting height of the wave under damping conditions as function $H_{W26, f, x}$ can be calculated as (12).

\[
DF = \frac{H_{W2nd} - H_{W1st}}{H_{W2nd} \cdot x} \quad (10)
\]

\[
DF(f) = 0.0079 \cdot f - 0.0029 \quad (11)
\]

\[
H_{W26, f, x} = H_{W2nd} \cdot [1 - (0.0079 \cdot f - 0.0029)] \quad (12)
\]

The wave breakdown was investigated on the axis of the towing tank between 1st and 2nd ends of the measurement area. The goal of investigation was to determine the content of the power spectral density of harmonic wave at main frequency in the generated waves along whole measurement area. By the main frequency, we mean the frequency generated by the wavemaker. The $H_W$ was measured on five sections between 1st and 2nd end during generation of the regular waves listed in Tab. 1. Each section had length $dx=29$ m.

Based on the spectrum in frequency domain, obtained from measured $H_W$ using FFT, the percentage content of the power spectral density of harmonic wave at main frequency is $P_\%$ defined as (13).

\[
P_\% = \frac{H_{W0.0-1.5Hz}(f)}{H_{W0.0-1.5Hz}(f)} \quad (13)
\]

Characteristics of $P_\%$ as function of $x$ are shown in Fig. 10. As shown, there are a three groups of regular waves due to the dynamics of $P_\%$ changes:

- group A: the waves with frequencies from 0.3 Hz to 0.9 Hz – approximated CP1A as (14),
- group B: the waves with frequencies from 1.0 Hz to 1.1 Hz – approximated CP1B as (15),
- group C: the waves with frequencies from 1.1 Hz to 1.2 Hz – approximated CP1C as (16).

As shown in Fig. 10, the $P_\%$ for waves from the A group is relatively constant. For waves from B group, the $P_\%$
decreases according to S-characteristic and in midway of the measuring area falls below 50%. For waves from C group, the P% decreases according to J-characteristic and in midway of the measuring area falls to about 20%.

\[ P_\% = \frac{HW_M(f)}{HW_{0.0-1.5t_b}(f)} \cdot 100\% \]  

\[ CP1A = 0.0095 \cdot x + 81.629 \]  

CP1B = \(-5 \cdot 10^{-3} \cdot x^3 + 0.0104 \cdot x^2 - 0.1872 \cdot x + 21.424\)  

\[ CP1C = -5 \cdot 10^{-5} \cdot x^3 - 0.0059 \cdot x^2 + 0.2065 \cdot x + 15.153 \]  

**WAVE REFLECTION**

The wave reflection was investigated on the axis of the towing tank between 1st end and 2nd end of the measurement area. The goal of investigation was to determine the reflections arising from the wave reflections from the artificial beach (Fig. 1.) For this purpose, two signals were measured: \( HW \) and velocity of the driving towing carriage \( V \). The towing carriage was driven from the beach to the wavemaker.

The Doppler effect for measured waves, when towing carriage was driven, allows to extract the mean height of the generated waves \( HW_g \) and the mean height of the reflected waves \( HW_r \) from the wave spectrum in frequency domain.

The frequency bands were appropriately shifted depending on measured \( V \).

The mean reflection coefficient \( R \), understood as \( (17) \), calculated for measured \( HW_g \) and \( HW_r \), is listed in Tab. 4. The numbers in the first column of Tab. 4 are consistent with Tab. 1.

Characteristic of \( R \) as function of \( f \) is shown in Fig. 11. As shown, \( R \) ranges from below 10% up to 25%, depending on the \( f \). This dependency was approximated as \( (18) \).

**Table 4. Reflection coefficient based on generated and reflected waves.**

| No. | Height of the generated wave \( HW_g \) [cm] | Height of the reflected wave \( HW_r \) [cm] | Reflection coefficient \( R \) [-] |
|-----|---------------------------------|---------------------------------|---------------------------------|
| 1   | 3.88                            | 0.94                            | 0.24                            |
| 2   | 3.56                            | 0.54                            | 0.15                            |
| 3   | 2.72                            | 0.58                            | 0.21                            |
| 4   | 2.34                            | 0.52                            | 0.22                            |
| 5   | 1.7                             | 0.32                            | 0.19                            |
| 6   | 4.44                            | 0.4                             | 0.09                            |
| 7   | 4.7                             | 0.74                            | 0.16                            |
| 8   | 4.44                            | 0.68                            | 0.15                            |
| 9   | 4.76                            | 1.18                            | 0.25                            |
| 10  | 4.36                            | 0.66                            | 0.15                            |
| 11  | 3.16                            | 0.64                            | 0.20                            |
| 12  | 2.12                            | 0.54                            | 0.25                            |
| 13  | 8.82                            | 0.66                            | 0.07                            |
| 14  | 9.4                             | 1.32                            | 0.14                            |
| 15  | 9.26                            | 0.88                            | 0.09                            |
| 16  | 7.64                            | 1.08                            | 0.14                            |
| 17  | 4.02                            | 0.92                            | 0.23                            |

![Fig. 10. The content of the main frequency P% as function of distance x.](image)
The reflection coefficient for regular waves $R$ as function of frequency $f$.

$$ R = \frac{HW_r}{HW_g} \quad (17) $$

$$ R(f) = 0.1536 \cdot \ln(f) + 0.2076 \quad (18) $$

**CONCLUSIONS**

In the range of waves listed in Tab. 1, it is valid to use $BTF$ for prediction of $HW$ of regular waves in CTO S.A. towing tank. Moreover, it is possible to use $BTF$ and properly parameterized $Snl$ model for prediction $HW$ of irregular waves.

The wave damping phenomenon is dependent on the $f$. As $f$ increases, $DF$ will increases. It is possible to predict $HW$ on given $x$ and for given $f$ basing on the $HW^{2nd}$.

The wave breakdown phenomenon is dependent on the $f$. $P_B$ is constant along the whole measurement area for $f$ up to 0.9 Hz. From $f$ equal to 1.0 Hz the phenomenon is noticeable and for $f$ above 1.1 Hz the phenomenon is significant and waves are rapidly collapsing.

The wave reflection phenomenon arising from reflections from the artificial beach is dependent on $f$. As $f$ increases from 0.5 Hz to 1.2 Hz, $R$ increases from about 10% to 25%.

Using the developed models of the phenomena: wave damping, wave breakdown, wave reflection, nonlinear energy transfer and the transfer function, it will possible to predict how the wave will propagate along the measurement area in the CTO S.A. towing tank, basing on wavemaker flap motion control.

Finally, hitherto the wavemaker control system will be developed in CTO S.A. to the adaptive wave control system.

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