The Study of Imperfect Maintenance Strategy based on Improved Age Reduction Factor

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Abstract. This paper is going to study the maintenance strategy optimization of repairable equipment in the imperfect maintenance mode. By introducing age reduction factor, repairable equipment can effectively describe the effects of imperfect maintenance. For the current age reduction factor is usually given by industry experts without a clear quantitative calculation formula, this paper offers an improved formula on the basis of its definition. The new formula takes not only traditional factors like repair times, costs, etc. but also subjective factors like equipment importance into account. Based on this and NSGA II (the second generation of non-dominated sorting genetic algorithm), the traditional scheduling maintenance strategy tries to solve its model whose objective function includes the maximum reliability, maximum availability, minimum total costs. Finally, by taking the radio relay equipment of mobile communication vehicle as an example to calculate and demonstrate, its validity was proved.

1. Introduction

When building a maintenance model for some equipment, it is usually assumed that after a preventive maintenance period the equipment will be repaired “as good as new” or “as good as old”. These two states were defined according to the equipment’s failure rate. “As good as new” is also called perfect maintenance, which refers to restoring the equipment’s failure rate to default setting ($\lambda_0$); “as good as old” is also known as the minimum maintenance, which means restoring its failure rate to pre-maintained state ($\lambda_i$). Obviously, these two assumptions are designed for describing and calculating the maintenance model. But in actual situation, most equipment’s failure rate ($\lambda_k$) is between the two after maintenance[1], that is $\lambda_i > \lambda_k > \lambda_0$, and this is called imperfect maintenance. It is more reasonable, because after i-1 times of preventive maintenance, the equipment will be whole-update or restore the failure rate to default setting at high cost at the i-th time.

In the early, Brown et al[2] proposed a model to describe imperfect maintenance, that is to say, the failure rate can attain the perfect maintenance state according to the probability of P, therefore, the minimum maintenance can be achieved at the probability of (1-P); Pham and Wang[3] summarized various types of imperfect maintenance model such as the (p,q) rule, the improved factor method, the ($\alpha,\beta$) rule and the virtual age method, etc.; F.G.Bad and M.D.Berrade discussed the costs of signal imperfect maintenance[4]; Wee Meng Yeo, Xue and Ming Yuan discussed the relationship between costs and maintenance effects[5]; Qu Yuxiang stated the role of reduction factor[6]; Tzong-Ru Tsai et al putted forward that there is a liner relationship between costs and maintenance effects[7]; Ye Penfan
used Markov Chain to describe deterioration process relying on the multi-stage deterioration model[8]. In this essay, starts from the maintenance effect, introduces the parameters, the author tries to describe achieved degree after each imperfect maintenance.

2. Improved age reduction factor

At present, the method of improved factor and age reduction factor is the main method which introduced parameters to describe the effect of imperfect maintenance. Age reduction factor was proposed by Dedopoulos and Smeers[9], and its basic idea is still to describe the maintenance effect by introducing parameters. Differ from improved factor, reduction factor describes the state after maintenance at t point according to (t-α). It is more intuitive by the “regression” on time dimension. The quantitative reduction factor is generally given by industry experts after calculating and combined with experience. The method is simple, but exists subjective effects and numeric value differs widely from different experts. Supposing age reduction factor α∈[0,1),the equipment undergoes N-1 periods of preventive maintenance (the last period’s node is not counted), and its failure rate at each maintenance node can be described as equation(2-1):

\[ λ_i = λ(t) \]

\[ \ldots \]

\[ λ_N = λ(t - \sum_{i=1}^{N-1} α_i T) \]

(2-1)

the corresponding node’s reliability degree \( R_n \) can be described as equation(2-2):

\[ R_N = R(t - \sum_{i=1}^{N-1} α_i T) \]

(2-2)

After getting equations of the failure rate and degree of reliability at each node of preventative maintenance period, what matters is to confirm the age reduction factor α[10].

Combined with the preventative maintenance costs, times and equipment importance, this paper presents the equation of age reduction factor as follow:

\[ α_i = \begin{cases} β(kC_{p_i})^{b_{i}}, & i = 1,2, ..., N - 1 \\ 1, & i = N \end{cases} \]

(2-3)

The meaning of each parameter is: \( C_{p_i} \) is the cost of preventative maintenance at the i-th time, k is the cost regulatory factor and satisfies the condition \( 0 < k < 1/C_{p_i} \); b is the time regulatory factor and satisfies the condition \( 0 < b < 1;β \) is the equipment importance factor and satisfies the condition \( 0 < β < 1 \). At each maintenance, the more cost is putted in, the bigger α is, the better the maintenance effect is, therefore \( α_i \) is proportional to \( C_{p_i} \). At the same time, the more times the equipment is maintained, reflecting equipment’s degradation accelerate to the end of working life, therefore \( α_i \) is inversely proportional to i. Each participant in the maintenance dominates the process and should not be ignored, hence taking β as the equipment importance factor. For different equipment, the value of β reflects which equipment is more important; for one equipment, the value of β reflects which stage is more important. Taking weaponry as an example, its importance degree is obviously different in peacetime and in combat readiness. The bigger β is, the equipment or the stage, the more valued. The maintenance effect is obviously better under the condition that cost and times remain the same, so β reflects the subjective factor of maintenance participants. Meanwhile, the equipment will be replaced instead of being maintained at the last node of design lifetime.

From equation(2-3), calculating equations of reliability degree \( R_n \) and failure rate \( λ_n \) are as follows:

\[ R_N = \begin{cases} R[(i - \sum_{j=1}^{N-1} α_j)T], & i = 1,2, ..., N - 1 \\ R[(N - \sum_{j=1}^{N-1} α_j)T], & i = N \end{cases} \]

(2-4)
Since maintenance nodes are made according to the planing period, the degradation will accelerate and the cost will increase over time. Dedopoulos and Smeers proposed that the maintenance cost, time and times satisfy the equation: $C_{p_1} = C_{f} + C_{v_1}(T_y)$; YCh.R. H and Lo.H.C also proved the rationality of the preventative maintenance cost as a liner function. Therefore, this paper uses the the equation (2-6) to describe the preventative maintenance cost. And in it, $i$ is the maintenance times, $C_f$ is the fixed cost of each maintenance, $C_{v_1}$ is the variable cost. According to equations (2-3), (2-4), (2-5) and (2-6), the failure rate and the reliability degree of each node can be calculated.

$$C_{p_i} = C_f + iC_{v_1}, i = 1, 2, ..., N$$ \hspace{1cm} (2-6)

### 3. The regular planned-maintenance strategy and construction of its multi-objective optimization model

Taking maintenance activities according to regular planned-maintenance strategy as in figure 1[11]: according to planning period time $T$, $2T$, ...(N-1)T, taking $T_y$ hours’ preventative maintenance each time, breakdown maintenance will be used if failure occurs within the period; when failure occurred after $T_1$ point, after the maintenance of $T_x$ hours, the equipment will continue to run for $T_2$ hours until the coming of the next planned maintenance node. At this period, the maintenance at each node is imperfect maintenance. When arriving at the NT-th period node, the equipment will be replaced directly rather than taking preventative maintenance. The most important point of regular planned-maintenance strategy is the determination of planned-period T. This paper takes the period T as the decision variable basing on the regular planned-maintenance strategy, constructs the multi-objective preventative maintenance optimization model whose objective function is equipment availability, total cost and reliability according to the imperfect maintenance method.

![Figure 1 The regular planned-maintenance strategy](image)

#### 3.1. Taking availability degree as objective function

As shown in Figure 1, $T_y$ is the average time of preventative maintenance, $T_x$ is the average maintenance time after failure, $T_r$ is the replacing time, $T_0$ means that the equipment newly puts into use at zero point, and has no preventative maintenance. MDF(Mean Down Failure) in N periods $T$ can be described as:

$$MDT = (N - 1)T_y + \sum_{i=1}^{N} T_x \int^{T_i}_{T_{i-1}} \lambda_i(t) \, dt + T_r$$ \hspace{1cm} (3-1)

MUT(Mean Up Failure) can be described as:

$$MUT = NT - \sum_{i=1}^{N} T_x \int^{T_i}_{T_{i-1}} \lambda_i(t) \, dt$$ \hspace{1cm} (3-2)

Supposing A is availability degree, with its definition, A in N periods can be described as:

$$A = \frac{MUT}{MUT + MDT} = \frac{NT - \sum_{i=1}^{N} T_x \int^{T_i}_{T_{i-1}} \lambda_i(t) \, dt}{NT + (N-1)T_y + T_r}$$ \hspace{1cm} (3-3)
3.2. Taking the total cost as objective function
Supposing \( C(T)_i \) as the i-th period cost, which represents the cost from the ending of \( T_{i-1} \) to the end of \( T_i \); supposing \( C_{pi} \) as the cost of preventative maintenance while \( C_s \) the cost of breakdown maintenance, which includes practical maintenance cost and downtime lose, hence coming the equation:

\[
C(T)_i = C_{pi} + C_s \int_{T_{i-1}}^{T_i} \lambda(t) \, dt
\]  

(3-4)

3.3. Taking the average reliability degree as objective function
From equations (2-5) and (2-6), the reliability degree function \( R_i \) of each planned-maintenance node can be calculated, therefore the average reliability degree can be calculated according to equation (3-5):

\[
\bar{R} = \frac{\sum_{i=1}^{N} R_i}{N}
\]  

(3-5)

Taking the regular planned-maintenance period \( T \) as the decision variable, basing on equations (2-5),(2-6),(2-7),(2-8),(3-3),(3-4),(3-5),under the regular planned-maintenance strategy, constructing the optimization model with the objective function which has the maximum availability, the lowest average cost and the maximum average reliability degree as in equation (3-6):

\[
\begin{cases}
A_{\text{max}} = \frac{NT - \sum_{i=1}^{N} \int_{T_{i-1}}^{T_i} \lambda(t) \, dt}{NT + (N - 1)T_y + T_r} \\
C_{\text{min}} = \sum_{i=1}^{N} (C_r + iC_v) + \sum_{i=1}^{N} C_s \int_{T_{i-1}}^{T_i} \lambda(t) \, dt \\
\bar{R}_{\text{max}} = \frac{\sum_{i=1}^{N} \int\left[ i - \beta \sum_{j=1}^{N-1} k^{b_j} (C_r + jC_v)^{b_j} \right] T_j + \beta \left[ N - \beta \sum_{i=1}^{N-1} k^{b_i} (C_r + jC_v)^{b_i} \right] T_j}{N}
\end{cases}
\]  

(3-6)

Converted into minimum form, the model above equals to:

\[
\begin{cases}
A_{\text{min}} = 1 - \frac{NT - \sum_{i=1}^{N} \int_{T_{i-1}}^{T_i} \lambda(t) \, dt}{NT + (N - 1)T_y + T_r} \\
C_{\text{min}} = \sum_{i=1}^{N-1} (C_r + iC_v) + \sum_{i=1}^{N} C_s \int_{T_{i-1}}^{T_i} \lambda(t) \, dt \\
\bar{R}_{\text{min}} = 1 - \frac{\sum_{i=1}^{N} \int\left[ i - \beta \sum_{j=1}^{N-1} k^{b_j} (C_r + jC_v)^{b_j} \right] T_j + \beta \left[ N - \beta \sum_{i=1}^{N-1} k^{b_i} (C_r + jC_v)^{b_i} \right] T_j}{N}
\end{cases}
\]  

(3-6)

4. Solving multi-objective optimization problem by NSGAI
Genetic algorithm can generate multiple points, which search towards multiple directions at the same time, and one search can obtain several Pareto solution sets. Therefore, genetic algorithm was seen as an important method to solve the multi-objective optimization problem. In 2002, Deb et al. improved non-dominated sorting genetic algorithm (NSGA) proposed by them and got the second generation of NSGA, which was called NSGAII. Compared to the first generation, the second has following improvements: ① dominating sorting quickly. The process of classifying N individuals will continue until all the fronts have completed (deleting individual F1, then classifying solutions’ number dominated by these individuals in order that minus one each time), recycling degree is \( O(N^2) \) at the worst time. ② using “congestion distance” to distinguish diversity. In NSGAII, calculating the congestion distance by ranking the individual and neighboring individual, and this replaced the first generation of \( \alpha \) share Parameters. The smaller the individual distance \( D_i \) is , the greater the congestion degree is. ③ using elitist strategy. In the selection of offspring, if the individual group is smaller than the parent group, select them in order (\( F_i < F_j \)) and fulfill them; if the individual group is greater than the parent group, rank according to the congestion degree and select (\( D_i > D_j \)), that is:
if \((F_i < F_j)\) or \((F_i = F_j)\) and \((D_i > D_j)\)
then \(i < j\) /there \(i,j\) are selecting orders

5. Example Analysis
Now studying on the signal transmission system of node communication vehicle, as supposed in the third part, related parameters of current transmission system are shown in Table 1. this system includes switches, relays, power supplies, multiplex switching equipment, routers, etc. It mainly switch and transmit the radio-frequency signal. The relay is the most important, and other equipment’s analysis process is similar, so the following study takes the relay as the object.

5.1. Analysis of an incomplete maintenance strategy for a life cycle
Firstly, the relay machine is analyzed using the above-mentioned incomplete maintenance strategy model within a lifetime. This section mainly analyzes the failure rate, reliability, and the average reliability, availability, and cost of a service life.

Part parameters of the relay are shown in Table 2.

| System life | Preventative maintenance Period | The average time of preventative maintenance | Downtime lose | Regulator factor \(k\) | Regulatory factor \(b\) | Importance factor \(\beta\) |
|-------------|---------------------------------|-----------------------------------------------|---------------|-----------------------|------------------------|-----------------------|
| 30 (year)   | 1.9 (year)                      | 0.1 (year)                                   | 95 (ten thousand) | 1                     | 0.005                  | 0.5                   |

Table 1. Part parameters of transmission system

| System Life | Fixed cost of preventative maintenance | Variable cost of preventative maintenance | Average replacing Time | Replacing cost | Actual maintenance costs | m   | t  |
|-------------|---------------------------------------|------------------------------------------|-------------------------|---------------|--------------------------|-----|----|
| 30(year)    | 3 (ten thousand)                      | 0.2 (ten thousand)                       | 0.1(year)               | 50 (ten thousand) | 5 (ten thousand)        | 38  | 1.2 |

Table 2. Part parameters of the relay

\(M\) is the proportional parameter and \(t\) is the shape parameter. For Weibull distribution failure rate, when \(m\) is greater, the value of the failure rate is smaller and faster, and when \(t\) is greater, the failure rate rises faster. When \(t\) is greater than 1, the failure rate increases with time. When \(t\) is less than 1, the failure rate decreases with time. For Weibull Distribution Reliability, the smaller the \(m\) is, the faster the reliability declines. When \(t\) is greater, the reliability function is more fluctuated and the fall is faster. The Weibull distribution simulates the reliability and failure rate of actual weapon equipment. The value of \(M, t\) reflects the performance of different equipments, where \(m\) takes 38 and \(t\) takes 1.2. Set a preventive maintenance period of 2 years.

The system life is 30 years, the preventative maintenance period is 2 years, so the number of preventative maintenance is at most 15 times. Observe changes in age-related retirement factors in Table 3.
Table 3. The changing of age reduction factors

| N  | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   |
|----|-----|-----|-----|-----|-----|-----|-----|-----|
| $C_{P_i}$ | 3.2 | 3.4 | 3.6 | 3.8 | 4.0 | 4.2 | 4.4 | 4.6 |
| $\alpha_j$ | 0.503 | 0.506 | 0.510 | 0.514 | 0.518 | 0.522 | 0.527 | 0.531 |

The fallback factor is 15 in the 15th cycle, that is, the equipment is replaced in the 15th cycle so that the equipment reaches the latest non-destructive state. The site where the equipment was installed was inspected for 40 years, and the failure rate and reliability were observed as the number of cycles changed.

Figure 2. The changing of failure rate along with the period

By observing the year-to-year changes in failure rate in Figure 2, it can be seen that every two years, that is, the maintenance cycle, will return to a previous state after maintenance, and this retreat effort depends on the improved retirement age factor proposed in this paper. When the equipment life is reached and the equipment is replaced, the failure rate returns to the initial state. The overall trend of failure rate change depends on Weibull distribution parameters, and parameters are set by experience based on different equipment.

Figure 3. The changing of reliability degree along with the period
Similarly, in Figure 3, it can be seen that the reliability changes with the year, and the reliability returns to a certain state in the previous cycle after maintenance at the maintenance node. Here, when the reliability drops to 50 percent, replace the new parts.

The changing of availability degree and average reliability degree in Figure 4 and the changing of total cost in Figure 5.

![Figure 4](image)

**Figure 4.** The changing of availability degree and average reliability degree along with the period

By observing the average reliability and availability at the maintenance cycle nodes in Figure 4, it can be seen that the availability remains basically unchanged, which means that the equipment can guarantee a higher utilization rate. The average reliability decreases steadily with time.

![Figure 5](image)

**Figure 5.** The changing of cost along with the period

Similarly, observing the change in the total cost with the number of cycles in Figure 5, we can see that the total cost has risen steadily, and there has been a jump in the cost of replacement at the end of its life.

5.2. *Analysis of optimal maintenance cycle under incomplete maintenance strategy*

Setting different maintenance cycles will result in different failure rates and reliability curves, so that the average reliability, availability, and cost indicators are different when the equipment is used up. So we need to find the optimal maintenance cycle so that the above indicators achieve optimal at the same time.

Since the availability degree almost unchanged along with the changing of period, the optimal maintenance period should be seek aiming at cost and reliability.
In Figure 6, the occasional increase in cost was due to the replacement of the equipment, which resulted in the increase of the total cost. Meanwhile, when the maintenance period lengthen to 15 years, the average reliability degree rises at the maintenance node, because the maintenance year is the replacement year.

In order to observe the reliability and cost change trend with the maintenance cycle better, we first normalize the data. X is the data set, and x is the original data. The normalized result is as (5-1).

$$\tilde{x} = \frac{x - \min(X)}{\max(x - \min(X))}$$  \hspace{1cm} (5-1)
Figure 8. The changing of the normalized 1-reliability degree and normalized cost along with the period.

The reliability and cost are stretched over the maintenance cycle curve so that the data is tiled at the beginning and end of the axis as shown in Figure 8.

The scatter chart of the average reliability degree R, the availability degree A and the total cost C are as in Figure 9, in which the marked triangle point is the optimal Pareto set.

Figure 9. The value of the average reliability degree R, the availability degree A and the total cost C

From the scatter chart of the average reliability and cost, it can be seen that the point at the lower left is the desired scheme, that the cost and (1-reliability) are simultaneously minimized when the scheme is on the convex hull and both of the cost and (1-reliability) have the advantages. , that is, the pareto set, the same as the optimal set.

Base on the actual decision, weighing the importance degree between the cost and reliability degree, selecting appropriate maintenance period as shown in Table 4.

Table 4. The optimal Pareto set

| Maintenance period | Cost  | The Average Reliability |
|--------------------|-------|-------------------------|
| 2                  | 185.12| 0.784                   |
| 4                  | 180.00| 0.780                   |

Through experiments, we have found that the optimal maintenance time of the relay under a given parameter is 2 or 4 years in Table 4. The final solution is determined by weighing the importance of actual cost and average reliability.
6. Conclusion

As seen from the verification results, the introducing of improved age reduction factor can better present the changes of equipment failure rate and reliability degree along with the changing times of the period during the imperfect maintenance process, and verify the feasibility of its degradation mechanism. For the enumerable set, the method of fast dominating sorting in NSGA II is used to find the front of Pareto set and then determine its optimal solution, which have certain guiding significance for the formulation of equipment decision-making.

References

[1] Badía F G, Berrade M D. Optimum Maintenance Policy of a Periodically Inspected System under Imperfect Repair. Advances in Operations Research, 2009,691203:1-13.
[2] M Brown, F Proschan. Imperfect Repair. Application Probability, 1983(20):851-859.
[3] Pham H, Wang H Z. Imperfect maintenance. European Journal of Operational Research, 1996,94(3):425-438.
[4] Badía F G, Berrade M D. Optimum Maintenance Policy of a Periodically Inspected System under Imperfect Repair. Advances in Operations Research, 2009,691203:1-13.
[5] Yeo W M, Yuan X. Optimal warranty policies for systems with imperfect repair. European Journal of Operational Research, 2009,199(1):187-197.
[6] Qu Yuxiang. Research on Maintenance Decision of Deteriorating System Based on Incomplete Maintenance Theory[D]. Tsinghua University, 2011.
[7] Tsai T, Liu P, Lio Y L. Optimal maintenance time for imperfect maintenance actions on repairable product. Computers & Industrial Engineering, 2011,60(4):744-749.
[8] Ye Peifan. Study on optimization model of state maintenance strategy based on incomplete maintenance[D]. Tsinghua University 2011.
[9] Dedopoulos I T, Smeers Y. An age reduction approach for finite horizon optimization of preventive maintenance for single units subject to random failures. [J]. Computer & Industrial engineering, 1998,34(3):643-654.
[10] Zhao Yongqiang, Liang Gongqian. Research of Periodic Preventive Maintenance under Reliability and Dynamic Maintenance Cost[J]. Aeronautical Manufacturing Technology, 2013(7).
[11] Wang Shiping, Zhu Minbo. Reliability and maintainability of electronic machinery. Beijing: Tsinghua University Press, 2000.100~120.