PMU-Based Decentralized Mixed Algebraic and Dynamic State Observation in Multi-Machine Power Systems

M. Nicolai L. Lorenz-Meyer, Alexey A. Bobtsov, Senior Member, IEEE, Romeo Ortega, Fellow, IEEE, Nikolay Nikolaev, Member, IEEE, and Johannes Schiffer, Member, IEEE

Abstract—We propose a novel decentralized mixed algebraic and dynamic state observation method for multi-machine power systems equipped with Phasor Measurement Units (PMUs). More specifically, we prove that for the third-order flux-decay model of a synchronous generator, the local PMU measurements give enough information to reconstruct algebraically the load angle and the quadrature-axis internal voltage. Also, we prove that the relative shaft speed can be globally estimated combining a classical Immersion and Invariance (I&I) observer with—the recently introduced—dynamic regressor and mixing (DREM) parameter estimator. This adaptive observer does not require the knowledge of the mechanical subsystem parameters and ensures global convergence under weak excitation assumptions that are verified in applications.

Index Terms—Dynamic state estimation, parameter estimation, power grid monitoring, power system operation, phasor measurements, synchronous generator.

I. INTRODUCTION

A. Motivation and Existing Literature

SITUATIONAL awareness of the transient dynamics of power systems is becoming increasingly important as these systems undergo major changes due to the massive introduction of power-electronics-interfaced components on the generation side, while at the same time novel demand-response technologies and more complex loads are implemented on the customer side [1], [2]. Furthermore, the systems operate more frequently at high loading and thus closer to the stability limits [2], [3]. Additionally, cascaded failures and disconnections can occur due to improper protection and control methods in such stressed conditions [2], [3].

It is, hence, highly relevant for power system operators to reliably monitor the system states in real time, especially during and after disturbances [1], [4]. At the same time, these developments render the steady-state assumptions used in traditional static state estimation questionable. Thus, the development of methods for dynamic state estimation (DSE) is gaining increasing importance for power system control and protection [1]. This is further emphasized, as the dynamics of power systems become faster and more volatile due to the mentioned changes [5].

A key enabler for DSE is the increasing availability of phasor measurement units (PMUs) [1]. Their ability to provide time-stamped, high-frequency measurements of voltages, currents and powers [7], [6] gives rise to modern dynamic observer designs. Yet, as of today, the prevalent algorithms in the literature for DSE in power systems are Extended Kalman Filter (EKF)-based techniques [17], [18], [19], [23], [33], [1]. This fact gives rise to three important issues.

- The main reasoning behind using these linearization-based techniques is that they are—in principle—applicable to nonlinear systems such as power system models. However, all convergence results of the EKF in the deterministic state observer framework rely on the assumption that the initial state estimation error is “sufficiently” small and the system is “very weakly” nonlinear [15], rendering its reliable applicability to the highly nonlinear description of a power system very questionable.

- In all the abovementioned publications the observer convergence is only shown empirically, e.g., via simulations, but no rigorous convergence guarantees are provided. A consequence of this lack of a theoretical analysis is that—in the absence of guiding laws for the choice of the observer gains—the tuning of the observers is a critical issue, usually solved via trial-and-error.

- In the standard theory Kalman filters are designed for noisy systems and a critical aspect is the choice of the matrices representing the Gaussian noise covariances. It is not clear to the authors in which sense a power system can be embedded into a stochastic—moreover, Gaussian—environment. Furthermore, simulated evidence has shown that the choice of these matrices strongly affects the performance of the EKF applied to power systems.

On the other hand, the problem of state observation of nonlinear systems has been extensively studied in the control systems literature, providing answers to many important theoretical questions and proposing effective observer designs that neither rely on linearizations nor artificial stochastic embeddings nor least-squares-based principles. We refer the reader to [10], [11] for a review of the literature. Remarkably, among the 107 references cited in the report of the IEEE Task Force on Power System Dynamic State and Parameter
estimation [1], non of these developments are mentioned. We tend to believe that this unfortunate situation is, in part, due to the fact that there is too little effective interaction between the control and the power systems community. One of the main objectives of this paper is to contribute, if modestly, to redress this situation.

B. Contributions

By exploiting the huge potential provided by PMUs, we address the problem of DSE in a multi-machine power system, in which the synchronous generators are represented by the standard three-dimensional flux-decay model [25], [22], [14]. In this setting, our main contributions are three-fold:

1) We propose a decentralized algebraic observer for the load angle and the quadrature-axis internal voltage using only the measurements provided via PMUs at the terminal buses.

2) A certainty equivalent adaptive observer for the relative shaft speed is designed by combining the classical Immersion and Invariance (I&I) technique [10] with the recently introduced dynamic regressor and mixing (DREM) procedure [9], which has been successfully applied to several engineering problems [12], [13], [26]. This observer is decentralized, does not require any additional prior knowledge on the system and its convergence is ensured via very weak excitation assumptions.

3) The effectiveness of the proposed DSE technique is demonstrated in simulations based on the New England IEEE-39 bus system [32].

The remainder of the paper is structured as follows. The mathematical model of the multi-machine power system is introduced in Section II. An algebraic observer for the load angle and the quadrature-axis internal voltage is derived in Section III. In Section IV a DREM-based I&I adaptive observer for the relative shaft speed is presented. The proposed method is validated in Section V using simulation results. Final remarks and a brief outlook on future work are given in Section VI.

II. MATHEMATICAL MODEL OF A DECENTRALIZED MULTI-MACHINE POWER SYSTEM

The multi-machine power system is comprised of \( N > 1 \) synchronous generators, each described by the well-known third-order flux-decay model [25, Eq. (7.176-7.178)], see also [22, Eq. (4.4)-(4.6)], [16, Eq. (1)]. Thus, the equations for the \( i \)-th machine read

\[
\frac{dx_{1,i}}{dt} = \omega_i - \omega_{t,i} = x_{2,i} - \omega_{t,i} + \omega_s, \tag{2a}
\]

\[
\frac{dx_{2,i}}{dt} = \frac{\omega_s}{2H_i} (T_{m,i} - T_{e,i} - D_i x_{2,i}), \tag{2b}
\]

\[
\frac{dx_{3,i}}{dt} = \frac{1}{T_{d0,i}} (-x_{3,i} - (x_{d,i} - x_{d,i}') I_{d,i} + E_{f,i}), \tag{2c}
\]

where the unknown state is defined as

\[
x_i := [x_{1,i} \ x_{2,i} \ x_{3,i}]^\top = [\delta_i - \theta_{t,i} \ \omega_i - \omega_s \ E_{q,i}'],
\]

with \( \delta_i \) the rotor angle, \( \omega_i \) the shaft speed, \( \omega_{t,i} \) the terminal voltage speed, \( \omega_s \) the nominal synchronous speed, \( E_{q,i}' \) the quadrature-axis internal voltage, \( T_{e,i} \) is the electrical air gap torque, \( E_{f,i} \) the field voltage and \( V_{t,i} \) the terminal voltage magnitude. Furthermore, the constants, which are all assumed unknown, are the damping factor \( D_i \), the inertia constant \( H_i \), \( T_{m,i} \) the mechanical power, the direct-axis transient open-circuit time constant \( T_{d0,i}' \), the direct-axis reactance \( x_{d,i} \) and the direct-axis transient reactance \( x_{d,i}' \). Moreover, \( \theta_{t,i} \) is the terminal voltage phase angle relative to the global \( DQ \)-coordinate system rotating at nominal synchronous speed \( \omega_s \) and defined as

\[
\theta_{t,i} = \theta_{0,i} + \int_0^t (\omega_{t,i} - \omega_s) \, dt,
\]

where \( \theta_{0,i} \) is the initial condition of \( \theta_{t,i} \). Hence, the load angle \( x_1 \) is constant in a synchronized state.

Remark 1. The rotor angle \( \delta_i \) is the angular position of the rotor with respect to the local synchronously rotating reference frame, while the quadrature-axis internal voltage leads the terminal voltage by the load angle \( x_{1,i} = \delta_i - \theta_{t,i} [14] \). To clarify the difference between the introduced angles and angular speeds, the local \( dq \)-coordinate system of the \( i \)-th machine is shown in Figure 1 in relation to the global \( DQ \)-coordinate system.

For the subsequent derivations, we assume the stator resistance to be zero [25] and make the following assumption on the direct-axis transient reactance \( x_{d,i}' \) and the quadrature-axis reactance \( x_{q,i} \).

Assumption 1. \( x_{d,i}' = x_{q,i} \).

With Assumption 1, the stator equations [25, Eq. (7.1780-7.181)] for the \( i \)-th machine, are given by

\[
I_{tq,i} = Y_i V_{t,i} \sin(x_{1,i}), \quad I_{td,i} = Y_i (x_{3,i} - V_{t,i} \cos(x_{1,i})) \tag{2a},
\]

where we have introduced the transient admittance magnitude

\[
Y_i = \frac{1}{x_{d,i}'},
\]
With zero stator resistance, the electrical air gap torque can be approximated by the terminal electrical power, i.e., (see also [17], [19])

\[ T_{e,i} \approx P_{t,i} = E'_{q,i}I_{q,i} = Y_i x_{3,i} V_{t,i} \sin(x_{1,i}), \]

where we have used (2) to obtain the last equality. Hence, by defining the unknown constants

\[
\begin{align*}
  a_{1,i} &= \frac{\omega_s D_i}{2H_i}, & a_{2,i} &= \frac{\omega_s}{2H_i}, \\
  a_{3,i} &= \frac{x_{d,i}}{x'_{d,i}T_{d0,i}}, & a_{4,i} &= \frac{x_{d,i} - x_{q,i}}{x'_{d,i}T_{d0,i}},
\end{align*}
\]

the model (1) for the \(i\)-th machine can be compactly written as

\[
\begin{align*}
  \dot{x}_{1,i} &= x_{2,i} - \omega_{t,i} + \omega_s, \\
  \dot{x}_{2,i} &= -a_{1,i}x_{2,i} + a_{2,i}(T_{m,i} - Y_i V_{t,i} x_{3,i} \sin(x_{1,i})), \tag{4a} \\
  \dot{x}_{3,i} &= -a_{3,i}x_{3,i} + a_{4,i} V_{t,i} \cos(x_{1,i}) + E_{f,i}. \tag{4c}
\end{align*}
\]

The measurements, which are provided via a PMU at the generator terminal, are defined as

\[
\begin{align*}
  y_{1,i} &= V_{t,i}, \tag{5a} \\
  y_{2,i} &= P_{f,i} = Y_i V_{t,i} x_{3,i} \sin(x_{1,i}), \tag{5b} \\
  y_{3,i} &= Q_{f,i} = \Im\{V_{td,i} + j V_{tq,i}(I_{d,i} - j I_{q,i})\} \\
  &= Y_i \{V_{t,i} x_{3,i} \cos(x_{1,i}) - V_{q,i}^2\}, \tag{5c} \\
  y_{4,i}^2 &= I_{d,i}^2 + I_{q,i}^2, \tag{5d} \\
  y_{5,i} &= f_{i,t} = \frac{\omega_{t,i}}{2\pi}, \tag{5e}
\end{align*}
\]

where \(y_{1,i} > 0\) is the terminal voltage, \(y_{2,i}\) is the terminal active power, \(y_{3,i}\) is the terminal reactive power, \(y_{4,i}\) is the terminal current and \(y_{5,i}\) the terminal voltage frequency. The imaginary part is denoted with \(\Im\{\cdot\}\).

In this note we first prove that it is possible to algebraically reconstruct the states \(x_{1,i}\) and \(x_{3,i}\) for the \(i\)-th machine in a decentralized setting from the terminal measurements

\[ y_i = [y_{1,i} \quad y_{2,i} \quad y_{3,i} \quad y_{4,i}]^T. \]

Then, an adaptive observer for the relative shaft speed \(x_{2,i}\) is designed.

### III. AN ALGEBRAIC OBSERVER FOR \(x_{1,i}\) AND \(x_{3,i}\)

As shown in the proposition below some straightforward algebraic operations on the measured signals \(y_i\) allows us to explicitly compute the unmeasurable states \(x_{1,i}\) and \(x_{3,i}\), requiring only the knowledge of the transient admittance magnitude \(Y_i\).

**Proposition 1.** The states \(x_{1,i}\) and \(x_{3,i}\) can be determined uniquely from the PMU measurements (5) via

\[
\begin{align*}
  x_{3,i} &= \sqrt{\frac{y_{4,i}^2 + 2Y_i y_{3,i}}{Y_i^2}} + y_{1,i}^2, \tag{6a} \\
  x_{1,i} &= \arcsin\left(\frac{y_{2,i}}{Y_i y_{1,i} x_{3,i}}\right). \tag{6b}
\end{align*}
\]

**Proof.** From (5c) and (5d) it follows

\[ y_{4,i}^2 + 2Y_i y_{3,i} = Y_i^2 (x_{3,i}^2 - y_{1,i}^2). \]

Thus, \(x_{3,i}\) can be calculated as

\[ x_{3,i} = \sqrt{\frac{y_{4,i}^2 + 2Y_i y_{3,i}}{Y_i^2}} + y_{1,i}^2. \]

With \(x_{3,i}\) known, \(x_{1,i}\) is obtained from (5b) as

\[ x_{1,i} = \arcsin\left(\frac{y_{2,i}}{Y_i y_{1,i} x_{3,i}}\right). \]

\[ \Box \]

**Remark 2.** Proposition 1 clearly reveals that we can obviate the use of dynamic state observers for the reconstruction of the generator load angle and the quadrature-axis internal voltage from the classical single axis flux-decay model—see Section VI for a discussion on the potential extension to the more detailed two-axis model [25]. We underscore the fact that, besides the PMU measurements, the only additional prior knowledge required in Proposition 1 is the transient admittance magnitude \(Y_i\). The DSE problem in power systems is now widely recognized to be of major importance to enhance its awareness and security, see [1], hence the interest of this result.

**Remark 3.** Alternatively to the proposed computation of \(x_{3,i}\) from (5c) and (5d), a reconstruction from (5b) and (5d) is also feasible. By rearranging and squaring of (5d) we get

\[ (Y_i^2 x_{3,i}^2 + Y_i^2 V_{t,i}^2 - y_{4,i}^2)^2 = (Y_i^2 V_{t,i} x_{3,i} \cos(x_{1,i}))^2. \]

Adding the left- and right-hand side to

\[ (2Y_i y_{2,i})^2 = (2Y_i^2 V_{t,i} x_{3,i} \sin(x_{1,i}))^2, \]

which follows from (5b), results after some algebraic manipulations in

\[ \begin{align*}
  0 &= (Y_i^2 x_{3,i}^2 - 2Y_i^2 x_{3,i} (y_{4,i}^2 + y_{2,i}^2)) \\
  &+ 4Y_i^2 y_{2,i} + (Y_i^2 V_{t,i}^2 - y_{4,i}^2)^2,
\end{align*} \]

yielding an equation of the form

\[ a^2 - 2ab + c = 0. \]

Thus, the positive root \(x_{3,i} > 0\) can be calculated uniquely from

\[ x_{3,i} = \sqrt{\frac{y_{4,i}^2 + 2Y_i y_{3,i}}{Y_i^2}} + 2Y_i \sqrt{\frac{y_{4,i}^2 V_{t,i}^2 - y_{2,i}^2}{Y_i^2}}, \]

since \(a > 0\) and \(c > 0\) by definition.

**Remark 4.** In spite of its obvious simplicity the result of Proposition 1 has not—to the best of our knowledge—been reported in the literature. A related reference is [28], where it is shown that the single axis flux-decay model is a differentially flat system whose flat outputs are the network-frame currents. This property is used in [28] for (open-loop) trajectory planning and feedback linearization of the system.
IV. A DREM-BASED I&I ADAPTIVE OBSERVER FOR $x_{2,i}$

In this section we combine the well-established I&I technique [10] for observer design with the recently introduced DREM parameter estimator [9] to design—using $x_{1,i}$ obtained via (6)—an adaptive observer for the rotor angular speed $x_{2,i}$ of the $i$-th machine. For the sake of clarity we divide the presentation of the result in three parts: first, an I&I observer assuming the mechanical parameters $a_{1,i}$, $a_{2,i}$ and $T_{m,i}$ are known. Second, we design a DREM estimator for these parameters. Third, with an ad-hoc application of the certainty equivalent principle, we propose the final DREM-based I&I adaptive observer, replacing the true parameters by their on-line estimates.

A. An I&I observer with known $a_{1,i}$, $a_{2,i}$ and $T_{m,i}$

**Lemma 1.** Consider the mechanical subsystem dynamics given in (4a) and (4b) with $x_{1,i}$ obtained via (6). Define the I&I observer for the $i$-th machine

$$\dot{\hat{x}}_{2,i} = -(a_{1,i} + k)(x_{2,i}^P + kx_{1,i}) + k(\omega_{t,i} - \omega_s),$$

$$\hat{x}_{2,i} = x_{2,i} + kx_{1,i},$$

with $k > 0$ a tuning parameter. Then,

$$\hat{x}_{2,i}(t) = e^{-(a_{1,i} + k)t}\hat{x}_{2,i}(0), \forall t \geq 0,$$

where $\hat{x}_{2,i} = \hat{x}_{2,i} - x_{2,i}$ is the state observation error.

**Proof.** Following the I&I observer design technique [10, Chapter 5] we propose to generate the estimate of $x_{2,i}$ as the sum of a proportional and an integral component, with the former being a function of the measurable signals, in this case of $x_{1,i}$. That is,

$$\dot{\hat{x}}_{2,i} = x_{2,i}^P(x_{1,i}) + \hat{x}_{2,i}^I,$$

with $x_{2,i}^P(x_{1,i})$ a function to be defined. Computing the time derivative of the observation error $\hat{x}_{2,i}$ we get

$$\dot{\hat{x}}_{2,i} = x_{2,i}^P(x_{1,i}) + \hat{x}_{2,i}^I - \dot{x}_{2,i} = \frac{dx_{2,i}^P(x_{1,i})}{dx_{1,i}}(x_{2,i} - \omega_t + \omega_s) + \hat{x}_{2,i}^I - \dot{x}_{2,i}$$

$$= \frac{dx^P_{2,i}(x_{1,i})}{dx_{1,i}}(\hat{x}_{2,i} - \omega_t + \omega_s) + \hat{x}_{2,i}^I - \dot{x}_{2,i}$$

$$= k(\hat{x}_{2,i} - \dot{x}_{2,i} - \omega_t + \omega_s) + \hat{x}_{2,i}^I + a_{1,i}x_{2,i}$$

$$- a_{2,i}(T_{m,i} - y_{2,i})$$

$$= -(a_{1,i} + k)\hat{x}_{2,i},$$

where we have selected $x_{2,i}^P(x_{1,i}) = kx_{1,i}$ in the fourth identity and used the definition of $\hat{x}_{2,i}^I$ to obtain the last one. Solving the last differential equation completes the proof.

**Remark 5.** Notice that the design of the I&I observer does not require the assumption that $T_{m,i}$ is constant, and it may be a time-varying measurable signal.

B. A parameter estimator for $a_{1,i}$, $a_{2,i}$ and $T_{m,i}$

The lemma below proposes a DREM-based estimator for the parameters $a_{1,i}$, $a_{2,i}$ and $T_{m,i}$ of the $i$-th machine that ensures convergence under some suitable excitation conditions.

**Lemma 2.** Consider the mechanical subsystem dynamics given in (4a) and (4b) with $x_{1,i}$ obtained via (6). Define the vector of unknown parameters

$$\theta_i := \text{col}(a_{1,i}, a_{2,i}, T_{m,i}),$$

the signals

$$z_i := \frac{\lambda^2 p^2}{(p + \lambda)^2} [x_{1,i}] + \frac{\lambda^2 p}{(p + \lambda)^2} [\omega_t, i]$$

$$\psi_i := \left[ -\frac{\lambda^2 p}{(p + \lambda)^2} [y_{2,i}] - \frac{\lambda^2}{(p + \lambda)^2} [U_{-1}(t)] \right],$$

with $p := \frac{d}{dt}$, $\lambda > 0$ a tuning parameter and $U_{-1}(t)$ a step signal, the dynamic extension

$$\dot{Z}_i = -\alpha Z_i + \alpha \psi_i z_i$$

$$\dot{\Psi}_i = -\alpha \Psi_i + \alpha \psi_i \psi_i^T,$$

with $\alpha > 0$ a design parameter and the signals

$$Z_i := \text{adj} (\Psi_i) Z_i$$

$$\Delta_i := \text{det} (\Psi_i).$$

The scalar parameter estimators

$$\dot{\hat{\theta}}_i = -\gamma_i \Delta_i (\dot{\hat{\theta}}_i - Z^T_i),$$

with $\gamma_i > 0$ adaptation gains, ensures the parameter estimation error $\hat{\theta}_i = \dot{\theta}_i - \theta_i$ verifies

$$\lim_{t \to \infty} \int_0^t \Delta_i^T(s)ds = \infty.$$

**Proof.** From (4a) and (4b) we get

$$\dot{x}_{1,i} + \omega_t, i = -a_{1,i}(x_{1,i} + \omega_t, i - \omega_s) + a_{2,i}(T_{m,i} - y_{2,i}).$$

Applying the filter $\frac{\lambda^2 p^2}{(p + \lambda)^2}$ to the equation above we get

$$\frac{\lambda^2 p^2}{(p + \lambda)^2} [x_{1,i}] + \frac{\lambda^2 p}{(p + \lambda)^2} [\omega_t, i] = \frac{\lambda^2 p}{(p + \lambda)^2} [x_{1,i}]

- \frac{\lambda^2}{(p + \lambda)^2} [\omega_t, i - \omega_s] - a_{2,i} (\frac{\lambda^2}{(p + \lambda)^2}) [y_{2,i}]$$

$$+ a_{2,i} (\frac{\lambda^2}{(p + \lambda)^2}) [U_{-1}(t)].$$

Using (7) and (8) we can write (13) as a linear regression equation

$$z_i = \psi_i^T \theta_i.$$ (14)
Following the DREM procedure [9], [21] we carry out the next operations
\[ \psi_i z_i = \psi_i \psi_i^\top \theta_i \quad (\Leftarrow \psi_i \times (14)) \]
\[ \frac{\alpha}{p+\alpha} \psi_i z_i = \frac{\alpha}{p+\alpha} [\psi_i \psi_i^\top] \theta_i \quad (\Leftarrow \frac{\alpha}{p+\alpha} [\cdot]) \]
\[ Z_i = \Psi_i \theta_i \quad (\Leftarrow (9)) \]
\[ \text{adj} \{ \Psi_i \} Z_i = \text{adj} \{ \Psi_i \} \Psi_i \theta_i \quad (\Leftarrow \text{adj} \{ \Psi_i \} \times) \]
\[ Z_i^2 = \Delta_i \theta_i, \quad j = 1, 2, 3 \quad (\Leftarrow (10)). \]

where, to obtain the last identity, we have used the fact that for any (possibly singular) \( q \times q \) matrix \( M \) we have \( \text{adj} \{ M \} = \text{det} \{ M \} I_M \), where \( \text{adj} \{ \cdot \} \) is the adjunct (also called “adjugate”) matrix. Replacing the latter equation in (11) yields the error dynamics
\[ \dot{\theta}_i^j = -\gamma_i^j \Delta_i^2 \theta_i^j, \quad j = 1, 2, 3. \]

The proof is completed observing that the solution of the later equations are given by
\[ \dot{\theta}_i^j (t) = e^{-\gamma_i^j \int_0^t \Delta_i^2 (\tau) d\tau} \dot{\theta}_i^j (0), \quad j = 1, 2, 3. \]

Remark 6. It is clear that it is possible to directly apply a classical gradient descent estimator to the vector linear regression equation (14), that is
\[ \dot{\theta}_i = -\Gamma_i \psi_i (\psi_i^\top \hat{\theta}_i - z_i), \quad \Gamma_i > 0, \quad (15) \]
which yields the error equation
\[ \dot{\theta}_i = -\Gamma_i \psi_i \psi_i^\top \hat{\theta}_i. \quad (16) \]

Our motivation to use, instead, the more complicated DREM estimator is to relax the excitation assumptions that guarantee its convergence. Indeed, it is well-known [24, Theorem 2.5.1] that a necessary and sufficient conditions for global (exponential) convergence of the error equation (16) is that the regressor \( \psi_i \) satisfies a stringent persistent excitation requirement [24, Equation 2.5.3]. Some simulation results have shown that this condition is not satisfied in normal operation of the power system. On the other hand, it has been shown in [21] that the non-square-integrability condition (12) is strictly weaker than persistent excitation.

Remark 7. We have presented Lemma 2 for the scenario where the mechanical power \( T_{m,i} \) is constant but unknown. It is clear that it is straightforward to extend it—applying the filter \( \frac{1}{p+\alpha} \Delta_i^2 \) and redefining \( z_i \) and the regressor vector \( \psi_i \)—to the case where \( T_{m,i} \) is time-varying, but measurable.

C. Adaptive I&I observer

Combining the known parameter observer of Lemma 1 with the parameter estimator of Lemma 2 yields the final certainty equivalent adaptive observer
\[ \dot{x}_{2,i}^j = -\left( \hat{\theta}_{1,i} + k \right) (x_{2,i}^j + k x_{1,i}) + k (\omega_{1,i} - \omega_s) \]
\[ - \hat{\theta}_{2,i} y_{2,i} + \hat{\theta}_{3,i} \]
\[ \dot{x}_{2,i} = x_{2,i}^j + k x_{1,i}, \]

with \( \hat{\theta}_i \) defined via (8)-(11). Under the excitation assumption (12), convergence of the adaptive observer is established via standard cascaded systems stability analysis, see e.g., [29].

V. Simulation Results

In this section we present simulation results demonstrating the effectiveness of the proposed methods. We use the well-known New England IEEE 39 bus system shown in Figure 2, with the parameters provided in [32]. All synchronous generators are represented by the third-order flux-decay model (1) and are equipped with automatic voltage regulators and power system stabilizers according to [32]. To monitor the system, we assume that PMUs are installed at the terminal buses of generators 5 and 8. The employed parameters for the DREM-based I&I adaptive observer for \( x_{2,i} \) are given in Table I.

The performance evaluation is undertaken as follows. We demonstrate that \( x_{1,i} \) and \( x_{3,i} \) can be reconstructed instantaneously using (6) in Proposition 1. Then, we show that already minor load variations, as continuously occurring during regular operation of the system, provide sufficient excitation to estimate the unknown parameters following Lemma 2. Thus, in combination with Lemma 1, the second state \( x_{2,i} \) can be reconstructed via the certainty equivalence adaptive observer (17). In addition, we show that the proposed observers (6) and (17) are also capable of capturing the fast transient behavior of the system after a three-phase short circuit.

A. Load Variation

We simulated minor load variations in the system. The resulting frequency variations are within 60 ± 0.025 [Hz]
Algebraic state estimation for $x_{1,5}$ and $x_{3,5}$ of generator 5 in presence of load variations.

Algebraic state estimation for $x_{1,8}$ and $x_{3,8}$ of generator 8 in presence of load variations.

and hence consistent with those during regular operation of transmission grids [34]. As can be seen from Figure 3 and 4 the algebraic observer is capable of reconstructing the states $x_{1,i}$ and $x_{3,i}$ as stated in Proposition 1, $i = \{5, 8\}$.

In Figure 5 and 6, the results of the DREM-based parameter estimation (Lemma 2) and the I&I adaptive observer for $x_{2,i}$ (Lemma 1) are shown. The y-axis of these plots is limited to the most relevant range. It can be seen that the simulated load variations provide enough excitation, so that the DREM-based parameter estimation converges after approximately 20 seconds for both monitored generators. As stated in Lemma 1, the I&I adaptive observer for $x_{2,i}$ depends on the estimated parameters. Thus, the observer error $\hat{x}_{2,i} - x_{2,i}$ only converges towards zero after the parameter estimates $\hat{\theta}_i$ of $\theta_i$ in (7) have converged at $t = 20$ secs.

B. Three-Phase Short Circuit

In this scenario, following [32] we simulated a three-phase short circuit at Bus 16 occurring at $t = 2$ secs and cleared at $t = 2.2$ secs. It is assumed that the DREM-based parameter estimation has already converged during regular operation before the fault. Thus, the parameters are assumed known in this scenario. The results of the algebraic and dynamic state estimation are shown in Figure 7 and 8 for the monitored generators 5 and 8. As is to be expected, the algebraic observer (6) exhibits no estimation error. Furthermore, the observer (17) for the frequency $x_{2,i}$ also performs very satisfactorily, with only a minor estimation error shortly after the fault.

VI. CONCLUSIONS AND FUTURE RESEARCH

A decentralized mixed algebraic and dynamic state observer was presented for DES in multi-machine power systems. It was shown that the load angle and the quadrature-axis internal voltage can be reconstructed algebraically from available PMU
measurement at the terminal bus of a synchronous generator. For observing the relative shaft speed a DREM-based I&I adaptive observer was proposed.

In simulation studies using the New England IEEE 39 bus system the effectiveness of the proposed observer was demonstrated. In particular, the convergence of the DREM-based parameter estimation was shown under regular operation conditions and load variations. Moreover, by simulating a three-phase short circuit the ability of the method to monitor the state evolution during fast transients was demonstrated.

The classical flux-decay model (1) provides a fairly accurate description of the behavior of a synchronous machine to assess transient stability in a multi-machine scenario. However, its precision can be improved by including additional dynamic effects. For instance, it is argued in [20, Chapter 11] that including a second differential equation to account for rotor body effects in the $q$-axis significantly improves the accuracy of the model. This leads to a fourth-order model. Our current research is aimed at extending Proposition 1, i.e., the reconstruction via algebraic operations of (some of) the system state variables from the PMU measurements, to this model. Unfortunately, it is possible to show that the (relevant part) of the mapping $y \mapsto x$ does not satisfy the rank conditions of the Implicit Function Theorem, suggesting that its required injectivity condition is not satisfied. The (possibly negative) results of this research will be reported in the near future.

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M. Nicolai L. Lorenz-Meyer received his M.Sc. in Engineering Science from the Technical University of Berlin in 2019 and his B.Eng. in Business Administration and Engineering for Environment and Sustainability from the Bournemouth University of Applied Sciences and the Berlin School of Economics and Law in 2016.

He is currently pursuing the Ph.D. degree with the chair of Control Systems and Network Control Technology at the Brandenburg University of Technology Cottbus-Senftenberg, Germany. His current research interests include the development of control theory-based methods for on-line dynamics security assessment in power systems.

Alexey A. Bobtsov (SM’10) received the M.S. degree in electrical engineering from ITMO University, St. Petersburg, Russia in 1996, received his PhD in 1999 and the degree of Doctor of Science (habilitation thesis) in 2007 from the same University. From November 1999 to December 2000 he served as Assistant Lecturer of Department of Automation and Remote Control. From December 2000 to May 2007 Dr. Bobtsov served as Associate Professor of Department of Control Systems and Informatics. In May 2007 Dr. Bobtsov was appointed as Professor of Department of Control Systems and Informatics. In September 2008 he was elected as the Dean of Computer Technologies and Control Faculty. He is currently the Dean of School of Computer Science and Control at ITMO University. He is a Senior Member of IEEE since 2010. He is a Member of International Public Association Academy of Navigation and Motion Control. He is co-author of more than 300 journal and conference papers, 5 patents, 15 books and textbooks. His research interests are in fields of nonlinear and adaptive control, control of oscillatory and chaotic systems and computer-aided control systems design with applications to mechanical and robotic systems.

Nikolay Nikolaev received the M.S. degree in electrical engineering from ITMO University, St. Petersburg, Russia in 2003, received his Ph.D in 2006 from the same University. From 2002 until 2013 he worked as Engineer of Department of Control Systems Design for Power Plants at JSC Kirovsky Zavod (Kirov Plant). From 2013 he is an Assistant Professor in Department of Control Systems and Robotics from ITMO University. He is a Member of IEEE since 2006. His research interests are in fields of nonlinear and adaptive control.

Johannes Schiffer received the Diploma degree in engineering cybernetics from the University of Stuttgart, Germany, in 2009 and the Ph.D. degree (Dr.-Ing.) in electrical engineering from Technische Universität (TU) Berlin, Germany, in 2015. He currently holds the chair of Control Systems and Network Control Technology at Brandenburgische Technische Universität Cottbus-Senftenberg, Germany, where he also serves as Deputy of Research. Prior to that, he has held appointments as Lecturer (Assistant Professor) at the School of Electronic and Electrical Engineering, University of Leeds, U.K. and as Research Associate in the Control Systems Group and at the Chair of Sustainable Electric Networks and Sources of Energy both at TU Berlin.

In 2017 he and his co-workers received the Automatica Paper Prize over the years 2014-2016. His current research interests include distributed control and analysis of complex networks with application to microgrids and power systems.

Romeo Ortega (S’81, M’85, SM’98, F’99) was born in Mexico. He obtained his BSc in Electrical and Mechanical Engineering from the National University of Mexico, Master of Engineering from Polytechnical Institute of Leningrad, USSR, and the Doctorate D’Etat from the Polytechnical Institute of Grenoble, France in 1974, 1978 and 1984 respectively.

He then joined the National University of Mexico, where he worked until 1989. He was a Visiting Professor at the University of Illinois in 1987-88 and at the McGill University in 1991-1992, and a Fellow of the Japan Society for Promotion of Science in 1990-1991. He has been a member of the French National Researcher Council (CNRS) since June 1992. Currently he is in the Laboratoire de Signaux et Systemes (SUPELEC) in Gif-sur-Yvette. His research interests are in the fields of nonlinear and adaptive control, with special emphasis on applications.

Dr Ortega has published three books and more than 350 scientific papers in international journals, with an h-index of 83. He has supervised 35 PhD thesis. He has served as chairman in several IFAC and IEEE committees and participated in various editorial boards. Currently, he is Editor in Chief of Int. J. on Adaptive Control and Signal Processing.