Planck constraints on neutrino isocurvature density perturbations

Eleonora Di Valentino and Alessandro Melchiorri

1 Physics Department and INFN, Università di Roma “La Sapienza”, Ple. Aldo Moro 2, 00185, Rome, Italy

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The recent Cosmic Microwave Background anisotropies provided by the Planck experiment have drastically improved our knowledge about the inflationary paradigm (see e.g., [1]). In particular, several inflationary models have been ruled out and the overall picture presented by Planck is perfectly consistent with purely adiabatic and gaussian primordial perturbations.

On the other hand, the recent Planck data is also showing some interesting anomaly or tension that, albeit at small confidence level, is clearly worthwhile of further investigation.

For example, the Planck data is well compatible with a larger value for the number of relativistic degrees of freedom at recombination than what is commonly expected in the standard scenario (1).

Let us remind here that the energy density of relativistic particles in cosmology at the epoch of recombination is given by:

\[ \rho_c = (1 + N_{\text{eff}}) \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} \rho_\gamma, \]

where \( N_{\text{eff}} \) is the effective number of neutrinos and \( \rho_\gamma \) is the CMB photon energy density.

It is worthwhile to point out that not only additional relativistic species would affect the value for \( N_{\text{eff}} \) but also other different neutrino properties, as a non-zero chemical potential, would change it from the standard value of \( N_{\text{eff}} = 3.046 \) (see [2]).

In practice, the \( N_{\text{eff}} \) effective parameter covers a wide range of physical phenomena and it is therefore extremely important to check for its consistency with the standard expectation.

Interestingly enough, the recent Planck data does show some indication for a non standard \( N_{\text{eff}} \). For example, in the analysis of [2], a value of \( N_{\text{eff}} = 3.71 \pm 0.40 \) at 68% c.l. from the Planck CMB data alone is reported. More importantly, when the Planck data is combined with the measurements of the Hubble constant from [3] the constraint becomes \( N_{\text{eff}} = 3.63 \pm 0.27 \) at 68% c.l., i.e. an indication for a non standard value at more than 95% c.l.

The main question that we want to address in this brief paper is if this anomaly can be connected with a non-standard inflationary process.

As pointed in previous papers (see, for example, [3] and references therein), a non zero chemical potential, and, therefore, a larger value for \( N_{\text{eff}} \) could arise in the curvaton scenario, proposed by [6,7].

In this model, while the exponential expansion is driven by the inflaton field, the primordial fluctuations are generated by a different field called “curvaton”. After the inflaton decay, the isocurvature perturbation produced initially by the curvaton is converted in an adiabatic component. In this model some residual isocurvature perturbation is therefore expected in the cosmological fluids (cold dark matter, baryons and neutrinos) (see, for example, [8–13]). In case of a non-vanishing neutrino chemical potential, neutrino density isocurvature perturbations are expected.

In few words, probing neutrino isocurvature density perturbation (NID hereafter), in the curvaton scenario is complementary to constrain the lepton number in the neutrino sector. It is therefore extremely timely to investigate the current CMB bounds on NID perturbation component, allowing at the same time a variation in the neutrino effective number \( N_{\text{eff}} \).

Bounds on neutrino isocurvature perturbations have been presented in the past in [14] and [15]. The Planck collaboration has also provided new and stringent bounds on NID, but fixing \( N_{\text{eff}} \) to the standard value of 3.046.

In this paper we present, for the first time, a combined analysis for \( N_{\text{eff}} \) and NID from the Planck data, considering also the possibility of other datasets as the recent Hubble constant measurements.

The paper is organized as follows. In Section II we review the NID perturbations which are generated in the curvaton scenario, in Section III we describe our analysis method, while in Section IV we present the corresponding results. Our conclusions are reported in Sec. V.
II. NEUTRINO ISOCURVATURE PERTURBATIONS

Let us remind the description of density perturbations in terms of the gauge invariant variable $\zeta$ that describes the curvature perturbation on slices of uniform total density [15]:

$$\zeta = -\psi - H \frac{\delta \rho}{\rho} ,$$  \hspace{1cm} (2)

where the dot denotes derivatives with respect to the cosmological time $t$, $H$ is the Hubble parameter, $\psi$ is the (gauge-dependent) curvature perturbation, and $\rho$ the total energy density.

In the case of multiple fluids, it is possible to define the quantities $\zeta_i$ for each of the $i$-th energy component

$$\zeta_i = -\psi - H \frac{\delta \rho_i}{\rho_i} .$$  \hspace{1cm} (3)

For an adiabatic mode the ratios $\delta \rho_i/\rho_i$ are all the same, so that $\zeta = \zeta_i$ for all components. At the same time, an isocurvature fluctuation $S_i$ in the $i$-th energy component is given by the relative entropy fluctuation with respect to photons:

$$S_i \equiv 3(\zeta_i - \zeta) .$$  \hspace{1cm} (4)

The relativistic neutrinos will follow an equilibrium distribution function as

$$f_i(E) = \left[ \exp(E/T_\nu \mp \xi_i) \right]^{-1} ,$$  \hspace{1cm} (5)

where $T_\nu$ is their temperature, $\xi_i = \mu_i/T_\nu$ with $\mu_i$ as the chemical potential, the index $i$ runs over the three neutrino families, $i = e, \mu, \tau$, and the minus (plus) sign is for neutrinos (antineutrinos). It is important to note that NID perturbations necessarily implies a non zero lepton asymmetry for the neutrino, $n_\nu \equiv n_\nu - n_\bar{\nu}$, unless there is an exact cancellation of the asymmetries in the three flavours.

Given the distribution function Eq. (5), the energy density $\rho_i \equiv \rho_\nu + \rho_\bar{\nu}$ in the high-temperature limit $T_\nu \gg m_\nu$ is given by [19]:

$$\rho_i = \frac{7\pi^2}{120} A_i T_\nu^4 = \frac{7}{8} A_i \left( \frac{T_\nu}{T_\gamma} \right)^4 \rho_\gamma ,$$  \hspace{1cm} (6)

where

$$A_i \equiv \left[ 1 + \frac{30}{7} \left( \frac{\xi_i}{\pi} \right)^2 + \frac{15}{7} \left( \frac{\xi_i}{\pi} \right)^4 \right] .$$  \hspace{1cm} (7)

From our above definition, we have that $N_{\text{eff}} = \sum_i A_i$. We can thus relate the isocurvature perturbation in the total neutrino density to the fluctuations $\delta N_{\text{eff}}^{(i)}$ (see [22]):

$$S_\nu = 3(\zeta_\nu - \zeta) \approx \sum_i \delta N_{\text{eff}}^{(i)} .$$  \hspace{1cm} (8)

In summary, a NID component is naturally connected to a non-standard value for $N_{\text{eff}}$. In the next section we will therefore perform an analysis allowing both components to vary.

III. ANALYSIS METHOD

Our analysis method is based on the Boltzmann CAMB code [20] and a Monte Carlo Markov Chain (MCMC) analysis based on the MCMC package cosmomc [21].

We sample the following set of parameters:

$$\{\omega_b, \omega_c, \Theta_s, \tau, n_s, \log[10^{10} A_s], N_{\text{eff}}, \alpha^{NID} \} ,$$  \hspace{1cm} (9)

$\omega_b \equiv \Omega_b h^2$ and $\omega_c \equiv \Omega_c h^2$ being the physical baryon and cold dark matter energy densities, $\Theta_s$ the ratio between the sound horizon and the angular diameter distance at decoupling, $\tau$ is the reionization optical depth, $n_s$ the scalar spectral index, $A_s$ the amplitude of the primordial spectrum, $N_{\text{eff}}$ the effective neutrino number and $\alpha^{NID}$ is the NID amplitude defined such that the total CMB power spectrum is given by:

$$C_\ell = (1 - \alpha^{\text{NID}})C_\ell^{\text{ad}} + \alpha^{\text{NID}}C_\ell^{\text{NID}} +$$

$$+ 2\text{sign}(\alpha^{\text{NID}})\sqrt{\alpha^{\text{NID}}(1 - \alpha^{\text{NID}})}C_\ell^{\text{corr}},$$  \hspace{1cm} (10)

where $C_\ell^{\text{ad}}$ is the adiabatic component, $C_\ell^{\text{NID}}$ is the neutrino isocurvature density component and $C_\ell^{\text{corr}}$ is the correlated spectrum. With this convention, when $\alpha^{\text{NID}} < 0$ the spectra are totally anti-correlated.

These theoretical power spectra are then compared with the recent CMB measurements made by the Planck experiment. For the Planck data, we add the high-$\ell$ and low-$\ell$ TT likelihoods and we also add the low-$\ell$ TE, EE, BB WMAP likelihood, see Ref. [1] for details. This corresponds exactly to the Planck+WP case presented in Ref. [1]. Moreover, we have marginalized over all foregrounds parameters, using the same procedure and priors presented in Ref. [1]. We also consider the HST constraint on the Hubble constant from [4].

IV. RESULTS

The results of our analysis are reported in Table 1 and Figure 1, in the case of the Planck+WP and the Planck+WP+HST datasets. As we can see, the Planck+WP data does not show any indication for NID or for a larger value for $N_{\text{eff}}$. In practice, a cosmological degeneracy exists along the $\alpha^{\text{NID}}$, $N_{\text{eff}}$ direction and
TABLE I. Constraints at 68% confidence level on $N_{\text{eff}}$, $\alpha^{NID}$ and the main 6 cosmological parameters from the Planck+WP and Planck+WP+HST cases.

| Parameter | Planck+WP | Planck+WP+HST |
|-----------|-----------|---------------|
| $\Omega h^2$ | 0.02215 ± 0.00050 | 0.02260 ± 0.00033 |
| $\Omega_c h^2$ | 0.1222 ± 0.0068 | 0.1273 ± 0.0056 |
| $\theta$ | 1.0405 ± 0.0010 | 1.0408 ± 0.0011 |
| $\tau$ | 0.094 ± 0.015 | 0.099 ± 0.015 |
| $n_s$ | 0.966 ± 0.021 | 0.987 ± 0.012 |
| $\log[10^{10} A_s]$ | 3.115 ± 0.035 | 3.122 ± 0.037 |
| $H_0 [\text{km/s/Mpc}]$ | 68.7 ± 3.9 | 72.5 ± 2.2 |
| $N_{\text{eff}}$ | 3.26 ± 0.48 | 3.70 ± 0.30 |
| $\alpha^{NID}$ | −0.0031 ± 0.0053 | 0.0002 ± 0.0031 |

models with smaller values for $N_{\text{eff}}$ are more consistent with the CMB observations when $\alpha^{NID} < 0$. The current Planck+WP data alone does not show any supporting evidence for NID when variations in $N_{\text{eff}}$ are considered. Moreover, the standard value of $N_{\text{eff}} = 3.046$ is more consistent with Planck observations, due to the larger error on this parameter when NID are considered.

The conclusion is slightly different when also the HST dataset is included. As we can see, including HST reduces the error bars on the NID component while providing an indication for a non-standard value for $N_{\text{eff}}$ at more than two standard deviations. Again, this is consistent with the anti-correlation between $N_{\text{eff}}$ and $\alpha^{NID}$, mentioned above (see Figure 1).

Since, as discussed in the previous section, a positive or a negative value for $\alpha^{NID}$ discriminates between very different physical mechanisms for this NID component, it is interesting to repeat the analysis but imposing each time the $\alpha^{NID} > 0$ or $\alpha^{NID} < 0$ prior. The results for this analysis are reported in Table 2, for the two datasets: Planck+WP and Planck+WP+HST.

As we can see, the interesting aspect is that when a $\alpha^{NID} < 0$ prior is imposed, the Planck+WP case provide a value for the Hubble constant that is in tension with current HST determinations, even if the $N_{\text{eff}}$ parameter is allowed to vary. It is clear from this that a NID component with $\alpha^{NID} < 0$ can’t resolve the current tension on the values of $H_0$ between the Planck data and the HST constraint. On the other hand, the HST prior is clearly against a $\alpha^{NID} < 0$ component, since including it drastically improves the constraint on this parameter.

In the case of $\alpha^{NID} > 0$, on the contrary, the constraint on the NID component are practically left unaffected by the inclusion of a HST prior. This is evident from Figure 2, where we report the 2-D constraints on the $H_0$ vs $\alpha^{NID}$ in the case of $\alpha^{NID} < 0$ (Top Panel) and $\alpha^{NID} > 0$ (Bottom Panel) for the Planck+WP and Planck+WP+HST datasets.

V. CONCLUSIONS

The recent Cosmic Microwave Background data from the Planck satellite experiment, when combined with HST determinations of the Hubble constant, are compatible with a larger, non-standard, number of relativistic degrees of freedom at recombination, parametrized by the neutrino effective number $N_{\text{eff}}$. In the curvaton scenario, a larger value for $N_{\text{eff}}$ could arise from a non-zero neutrino chemical potential connected to residual isocurvature perturbations after the decay of the curvaton field, which component is parametrized by the amplitude $\alpha^{NID}$. Here we present constraints on a joint analysis of $N_{\text{eff}}$ and $\alpha^{NID}$. We found that the Planck+WP dataset does not show any indication for a neutrino isocurvature component and that current indications for a non-standard $N_{\text{eff}}$ component are further relaxed. When the HST prior on the Hubble constant is included, an anticor-
TABLE II. Constraints at 68% confidence level on $N_{eff}$, $\alpha_{NID}$ and the main 6 cosmological parameters from the Planck+WP and Planck+WP+HST cases. The two cases $\alpha_{NID} > 0$ and $\alpha_{NID} < 0$ are considered separately.

| Parameter | Planck+WP $\alpha_{NID} > 0$ | Planck+WP+HST $\alpha_{NID} > 0$ | Planck+WP $\alpha_{NID} < 0$ | Planck+WP+HST $\alpha_{NID} < 0$ |
|-----------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| $\Omega_b h^2$ | 0.02260 ± 0.00043 | 0.02271 ± 0.00031 | 0.02198 ± 0.00043 | 0.02249 ± 0.00031 |
| $\Omega_c h^2$ | 0.1287 ± 0.0059 | 0.1295 ± 0.0050 | 0.1196 ± 0.0056 | 0.1248 ± 0.0049 |
| $\theta$ | 1.04149 ± 0.00082 | 1.04149 ± 0.00082 | 1.04012 ± 0.00085 | 1.04003 ± 0.00080 |
| $\tau$ | 0.095 ± 0.014 | 0.096 ± 0.014 | 0.093 ± 0.014 | 0.102 ± 0.015 |
| $n_s$ | 0.987 ± 0.017 | 0.992 ± 0.011 | 0.958 ± 0.018 | 0.982 ± 0.011 |
| $\log[10^{10} A_s]$ | 3.100 ± 0.033 | 3.104 ± 0.031 | 3.119 ± 0.034 | 3.145 ± 0.033 |
| $H_0$ [km/s/Mpc] | 72.4 ± 3.4 | 73.3 ± 2.0 | 67.3 ± 3.3 | 71.8 ± 2.0 |
| $N_{eff}$ | 3.71 ± 0.42 | 3.81 ± 0.27 | 3.08 ± 0.40 | 3.59 ± 0.27 |
| $\alpha_{NID}$ | < 0.0023 | < 0.0025 | > -0.0056 | > -0.0023 |

related, $\alpha_{NID} < 0$, neutrino isocurvature density component is severely constrained, while the combined analysis suggests a value for $N_{eff}$ larger than the standard expectations at more than two standard deviations.

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FIG. 2. 68% and 95% c.l. likelihood contours for Planck+WP and Planck+WP+HST.