Tierkreis: a Dataflow Framework for Hybrid Quantum-Classical Computing

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Abstract—We present Tierkreis, a higher-order dataflow graph program representation and runtime designed for compositional, quantum-classical hybrid algorithms. The design of the system is motivated by the remote nature of quantum computers, the need for hybrid algorithms to involve cloud and distributed computing, and the long-running nature of these algorithms. The graph-based representation reflects how designers reason about and visualise algorithms, and allows automatic parallelism and asynchronicity. A strong, static type system and higher-order semantics allow for high expressivity and composability in the program. The flexible runtime protocol enables third-party developers to add functionality using any language or environment. With Tierkreis, quantum software developers can easily build, visualise, verify, test, and debug complex hybrid workflows, and immediately deploy them to the cloud or a custom distributed environment.

Index Terms—Quantum Computing, Dataflow, Distributed

I. INTRODUCTION

All quantum computing tasks involve the interplay between classical and quantum parts. The VQE algorithm [1] is typical: a classical driver program generates a set of quantum circuits; these circuits are submitted via a web API to a remote quantum computer; after waiting in a queue, the circuits are executed, and the results of these circuits are returned to the driver; when all the circuits complete, the results are the input to a classical optimiser, from which a new batch of circuits is generated, and the loop repeats. Another common pattern, seen in error-mitigation [2], takes quantum circuits as inputs, modifies them, runs the modified circuits on a quantum device, and post-processes these results. Fault-tolerant algorithms also often interleave quantum and classical steps [3]. The back-and-forth between classical and quantum phases, which run on different systems at different times, makes understanding, debugging, and verifying quantum software unnecessarily difficult.

The classical phases of such algorithms may vary widely in their resource requirements. For example, a chemical simulation may require a memory-intensive integration calculation followed by storage of a large Hamiltonian, whilst GPU acceleration might be needed for a machine learning task. Given the differing characteristics of qubit technologies, some quantum devices will be better suited to particular algorithms – or even subroutines – than others. From this perspective, quantum computers are special nodes in a larger heterogeneous distributed computing environment, and quantum software frameworks must allocate the right task to the right system.

Quantum computers are intended to accelerate tasks which are classically infeasible, but that does not imply that the quantum algorithms are objectively fast. On the contrary, when classical methods are impractical we expect that a quantum algorithm will be long-running. Given that demand for quantum processors is likely to continue to exceed supply, we might also expect queuing and batch processing to remain the norm. In this situation, tracking, pausing, and resuming the algorithm are valuable facilities, to monitor progress and recover from runtime failure. Further, the high cost of machine time recommends the use of strong static analysis to detect programming errors before execution. Few of these facilities are offered by current toolkits [4], [5].

Tierkreis is designed to address these needs. The program is represented as a dataflow graph where values flow along the edges, between nodes representing functional units, which potentially execute on different servers. Tierkreis has a strong static type system and supports higher-order functions which allow flexible control flow and compositional structures.

The system is designed to allow easy transition from existing quantum computing workflows, which are predominantly made up of Python libraries and scripts, to a paradigm which naturally mimics algorithm design, allows type-safety based verification, and enables immediate deployment to cloud execution following local testing and debugging.

Section II outlines the programming model of Tierkreis. The runtime system is described in Section III.

Related work

Higher-order dataflow programming has been studied by many authors [6]–[8], albeit with different semantics than Tierkreis, and the dataflow paradigm has been applied to a variety of problem domains. For example, the popular machine-learning library TensorFlow [9], uses dataflow graphs of tensor values for optimised training of machine-learning models; LabVIEW [10] is a widely used graphical environment for lab equipment testing and measurement; Enso [11] is a functional language with both a visual and textual representation, with a focus on data science visualisation and analysis. There are many others. Our contribution in Tierkreis is to define a
semantics and execution model suitable for hybrid quantum-classical computing, and which accommodates existing quantum workflows, in a distributed cloud-native system.

There are other quantum software systems which aim to address similar problems. For instance, QCWare Forge and qBraid offer hosted python environments alongside prebuilt libraries for executing quantum experiments. TensorFlow Quantum [12] allows adding quantum steps to TensorFlow machine-learning workflows. Qiskit Runtime [13] enables offloading of some computation to the IBM cloud, from where a hybrid computation involving IBM quantum devices can execute. Tierkreis and Qiskit Runtime are complementary in design: Qiskit Runtime allows quantum subroutines to be called as RESTful web services, whereas Tierkreis can be used to build the whole program that would call such services.

II. PROGRAMMING MODEL

The program representation is designed with two primary goals: to naturally represent typical algorithms and to serve as a suitable runtime representation for a distributed, asynchronous execution model. It is also intended to be a suitable compilation target for higher-level languages or front-ends, such that they can be executed on the Tierkreis runtime.

A. Dataflow

The representation is a directed graph of nodes connected by edges. The nodes are labelled with function names that identify pure functional primitives: functions which compute a set of outputs from a set of inputs, with no side-effects. Edges connect nodes, from an output port of the source node to an input port of the target. The port names identify the function inputs and outputs. A value produced at an output port flows along the edge to the target input; values are immutable. Table I lists the available value types. This structure is a dataflow graph as it specifies only data dependencies: a partial, rather than total, order of the operations. See Fig. 1a for a visualisation of a simple Tierkreis graph, generated by the python package. This graphical approach is a natural conceptual model for a hybrid algorithm – with data moving between algorithmic units rather than as a sequence of procedural steps.

B. Asynchronous and parallel

The approach also allows the program to be asynchronous and trivially parallelisable without any further input from the programmer. Since nodes correspond to pure functions, they can execute asynchronously whenever their required inputs are ready. Similarly any two nodes whose inputs are available at the same time can execute in parallel, at the discretion of the runtime, and could even execute on different systems. The property that nodes can have multiple outputs (unlike tree-like representations) allows the outputs of one node to trigger multiple parallel nodes.

The error-mitigation routine Digital Zero Noise Extrapolation (ZNE) [14] illustrates the use of parallel nodes. Broadly, this error mitigation scheme runs multiple versions of the input circuit on the quantum device, artificially increasing the noise incurred each time. In this implementation, using functions from the Qermit package [2], noise is increased by appending the inverse of the circuit and then the circuit again, causing an odd numbered multiplicative increase in circuit size (known as folding). The resulting circuit is equivalent to the original, but more noise is incurred due to additional operations. Results from each experiment, typically the expectation value of some

1.0

(a) zexp_to_parity

1.0

(b) initial

Fig. 1: Visualisations of simple Tierkreis graphs. Each box is a node with wires denoting the edges connecting them. Function nodes are labelled with their names, and the names of their input (top) and output (bottom) ports; the inputs and outputs of the graph itself are shown in the same style. Constant nodes are labelled with their value. (a) A graph computing the function \( f(x) = (1-x)/2 \) on floating-point values. The nodes labelled \( \text{fsub} \) and \( \text{fdiv} \) are built-in floating point subtraction and division functions respectively. (b) A graph which takes in a graph computing function \( f \) via its \( \text{run} \) port, and uses an \( \text{eval} \) node to compute \( f(x) \), where \( x \) is the constant \([0, 2] \) at the top. We then use some built-in functions to construct \([[(x, f(x))] \). In Figure 5 we show how this graph can be used used in a bigger program to compute the initial value for a variational optimising loop.
In Tierkreis, graphs are first-class values that can flow as nodes – named inputs and outputs – so the programmer can write complex algorithms in a modular and compositional style. For example, most hybrid algorithms wrap classical processing around an inner loop which executes a quantum circuit. Similarly, most error mitigation methods transform a quantum execution in some way to achieve better results. By providing a graph for evaluating a circuit as a higher-order parameter, for example the runner input in Fig. 2, algorithms can use a basic circuit runner (like in Fig. 3) or an error-mitigated alternative. Indeed the design of the Qermit [2] package is based on composing mitigation schemes in just this manner.

In Tierkreis, values of Graph type are executed at runtime using the built-in `eval` function which, when provided with a graph and suitable values for all its inputs, executes it, and returns the output values. Figure 1b shows an example. Another functional feature present in Tierkreis is partial evaluation, via the partial built-in. Given a graph and some of its inputs, partial returns a new graph constructed by plugging the inputs into the appropriate ports. This feature is used by our Tierkreis front-end to automatically construct closures. These allow input values which are in scope at the definition point of the graph, to be retained at the point of use. When building the program, the user can use values from outer graph scopes freely, and at runtime these values are inserted in to the inner scope using the partial function. Figures 3 and 4 show examples of graphs which capture outer scoped values, marked by the `_c0` input port. Fig. 5 shows how these values are injected by partial nodes.

The graph also allows nesting subgraphs through the use of Box nodes. A Box node contains a subgraph and acts like a standard function node with the same interface as the contained graph, akin to a function invocation on a user-defined function. This allows for simple hierarchical structure and modularity in complex algorithms. Fig. 2 contains the node user-labelled as “Box: make_list(3)” signifying that it contains a subgraph rather than a function.

### E. Control flow

The execution model allows that a node may execute (only) when all of its inputs are ready; this implies that a well-formed Tierkreis graph is acyclic. Despite this restriction, control flow constructs such as iteration, branching, and pattern matching can be accommodated in Tierkreis using special built-in higher-order functions and derived constructs.

Branching is implemented via the switch function, which takes a boolean input and two others (with matching types), and returns one of them based on the value of the boolean. Using the switch with two graph values, and applying `eval` to its return value suffices to implement if-then-else. (See Fig. 4 for an example.) Similarly, pattern matching on variants is implemented using the special Match node, which takes a variant value as input as well a graph to handle each variant of the type. Match chooses the appropriate graph value, and partially evaluates it with the inner value held by the input variant.
run_circuit

_c0 params
params

"a", "b"

circ
params
symbs

pytket/substitute_symbols
value

1000
"AerBackend"

pytket/execute
value
circuit shots backend_name

box: zexp_to_parity
value

Box: zexp_to_parity
value

Fig. 3: Tierkreis graph for calculating the cost function of a variational loop, using functions implemented using pytket. The parameters in a symbolic circuit are set to those input, the circuit is executed using a quantum device simulator, and the results used to calculate the expectation value of the parity of the measured distribution. The sub-graph contained in the zexp_to_parity Box is shown in Fig. 1a. The _c0 input indicates the circuit has been automatically captured from an outer scope. At runtime a partial node will insert the circuit as a constant prior to the graph being executed, as shown in Fig. 5.

Iteration is done via the loop function, and a parameterised Variant type with two tags, break and continue. The loop node accepts a body graph and an initial value, and returns an output. The body graph must accept the initial value as its input, and return a value of the prescribed variant type. If it returns a value tagged continue, this value is used as the next input to body; if it returns a value tagged break, this value is the output of the loop. Our runtime implementations of loop use iteration, rather than recursively unrolling the graph.

Fig. 4 gives an example of a loop body and Fig. 5 shows how it is used with the loop function.

F. Type system

To improve the reliability of programs, and reduce the likelihood of runtime failure in the middle of a potentially costly or long-running computation, the Tierkreis graph has a strong static type system. All values can be assigned a type from Table I, which includes user-definable algebraic data types via Struct and Variant types as well as Graph types for higher-order computation. A graph is well-typed if, prior to runtime, type annotations compatible with the signatures of all the nodes can be assigned to every edge. Tierkreis implements a type inference algorithm which can fully infer the types of even completely unannotated programs, making user-provided annotations unnecessary. Fig. 6 shows the graph from Fig. 1b with edge-annotations inferred by the algorithm. In particular edges connected to the polymorphic functions eval, make_pair, and push are annotated with concrete values if and else are shown expanded inside dotted boxes.
TABLE I: Available value types in Tierkreis

| Type            | Description                                      |
|-----------------|--------------------------------------------------|
| Bool            | True or False boolean.                           |
| Int             | 64-bit signed integer.                           |
| Float           | Double precision float.                         |
| Str             | Utf-8 encoded string.                            |
| Pair(A, B)      | Pair (product type) of types A, B.               |
| Vec(A)          | Vector of element type A, with efficient push and pop operations. |
| Map(A, B)       | Hash map of key type A and value type B.         |
| Struct(R)       | Anonymous compound type with named fields and associated types. |
| Variant(R)      | Tagged sum type. Also known as discriminated unions or enum types. |
| Graph(Γ → Ω)    | A graph which takes inputs Γ and returns outputs Ω. |

Fig. 5: Tierkreis graph of a variational quantum algorithm. The graph values indicated by run_circuit, loop_def and the box initial can be seen in Figures 3, 4 and 1b respectively. Code for generating these graphs is listed in the Appendix.

Types when input constant values constrain them, and type variables when no such constraints exist, making the whole graph polymorphic.

The signature of a function has a polymorphic type which is instantiated for every node that refers to the function. The type inference algorithm then generates unification constraints between the types of input and output node ports which are connected by an edge. After all constraints are collected Tierkreis attempts to find a most general type which satisfies the constraints, and returns the program with explicit type annotations filled in. In the case of a type error Tierkreis provides high quality error messages by discovering a small set of locations which together make the program untypeable.

While the type system can statically guarantee that submitted programs compose operations in a type safe way, external workers (see section III-A) could still produce outputs which do not conform to the type of the operation that they advertise. To detect this early and provide the user with useful diagnostic information, the runtime system checks that the values produced by workers are consistent with the expected types. The provided Python library for workers can additionally produce a Tierkreis type signature from Python type annotations to prevent a mismatch and to assist authors of worker functions in providing type safe operations.

A key component of Tierkreis’ type system is the row type \( \vec{R} \), which is a finite labelled list of types denoted by \((\tau_1, \ldots, \tau_n)\). Rows are used to express the types of the input and output ports of nodes and Graph types, the fields of Struct types as well as the variants of Variant types. Open rows \((\tau_1, \ldots, \tau_n | \vec{R})\) allow for partial specification of a row type, where \(\vec{R}\) is a type variable which stands in for the rest of the row. This is used to give a type to operations such as eval and make_struct (see Table II) which have a flexible set of input and output ports. The labels of a row type are required to be distinct; to ensure that this property is respected for open rows, Tierkreis internally uses lacks constraints \(1/\vec{R}\) which prevent a row variable \(\vec{R}\) from being unified with a row that contains the label \(1\). Lacks constraints are not shown in type signatures as they are implicit. Partition constraints \(\vec{R}_1 \sqcup \vec{R}_2 \sim \vec{R}_3\) can ensure that a row \(\vec{R}_3\) is the disjoint union of rows \(\vec{R}_1\) and \(\vec{R}_2\), which is used to express the type of partial application and parallel composition of graphs.

In future work, rows will be used to tag computations with the resources they expect to be present in their environment, similar to the effect annotations used by Koka.

III. Runtime System

Whereas the Tierkreis graph defines the program to be run, the Tierkreis runtime system is responsible for its execution. The system can be divided into two parts: the runtime, which evaluates the overall computation graph, and the workers which provide implementations of the individual nodes.

Once the client submits the graph and its required inputs, the runtime need not interact with the client again until the output is ready, making this a simple model for remote job submission. While execution must respect the data dependency specified in the graph structure, task management is otherwise up to the implementation of the runtime. In particular, it is
free to execute the individual nodes concurrently and asynchronously.

The runtime maintains an index of all the functions provided by its workers, including the built-in worker. When a node is triggered, the runtime invokes an implementation of matching signature from its workers, passing the input values it received from the predecessor nodes as arguments. Should a node require a function not provided by any of the workers, whether local or remote, the graph cannot be executed. This results in a type-check error before any execution.

The Tierkreis system thus defines two interfaces to allow different parts of the system to communicate: the runtime interface allows graphs to be submitted for execution, and the worker interface allows external function implementations to be called by a runtime managing the execution. Both interfaces are defined using gRPC\(^1\) and all data – both types and values, including graphs themselves – are represented in Protocol Buffer format\(^2\). This provides many choices of implementation language and host platform for Tierkreis clients and servers.

### A. Extensibility via Workers

By implementing new workers, third-party developers can extend Tierkreis with arbitrary functions that can be used as node operations. Workers may be separate processes, communicating over gRPC, or in-process, serving function requests as sub-routines. The worker interface can be asked to:

- list the functions it provides, as a \textit{signature} — a mapping from names to type schemes, as in Table II,
- execute any of those functions given the inputs, returning the outputs.

Workers can be implemented in any language with a gRPC implementation; the gRPC library does most of the work in setting up a server to handle requests, leaving the programmer merely to implement the functions. The tierkreis Python package automatically converts Tierkreis types (Table I) to their Python equivalents, so a worker can be as simple as in Figure 7.

Thus, workers allow the Tierkreis system to execute operations written in any language, and provide a “top down” migration path: if only the outer high-level structure/control flow is ported to a Tierkreis graph, it can call out to leaf functions written in legacy code (perhaps running on separate hardware). A typical migration would involve implementing high-performance reusable algorithmic blocks as worker functions using existing code, and porting the high-level algorithm by wiring these functions together in a graph.

Additionally a worker can call back into the runtime to execute a graph (and inputs) and return the results back into the worker function; this allows a second “bottom up” migration route, where top-level code in legacy languages can call leaf functions that have been ported to Tierkreis.

### B. Deployment

A user uses the runtime interface to submit graphs, typically over a network connection using some front-end client, such as the one provided by our Python package. In the simplest scenario, a client connects to a runtime, which uses a local worker (in the same process) to provide a basic set of built-in functions (some of which are shown in Table II above). Since the state of each program is just the values placed onto edges so far, this lends itself to a serverless implementation where every value set onto an edge is persisted into a store; storing additional checkpoints (the start of each loop iteration, and optionally any currently-executing/pending requests to workers) avoids redundant execution but is not essential. However, a server where the state lives in memory may be

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1[https://grpc.io](https://grpc.io)
2[https://developers.google.com/protocol-buffers/]

| Operation | Type | Description |
|-----------|------|-------------|
| eval      | \(\forall \vec{f}, \vec{O}. \ (\text{thunk} : \text{Graph}(\vec{I} \to \vec{O}) | \vec{I}) \to \vec{O}\) | Evaluates a graph (determined at runtime) given values for all its inputs. |
| partial   | \(\forall \vec{f}, \vec{I}_1, \vec{I}_2, \vec{O}. \ (\vec{I}_1 \cup \vec{I}_2 \sim \vec{I}) \Rightarrow (\text{thunk} : \text{Graph}(\vec{I} \to \vec{O}) | \vec{I}_1) \to (\text{value} : \text{Graph}(\vec{I}_2 \to \vec{O}))\) | Partially evaluates a graph given some inputs, returns a graph taking the remaining inputs. |
| switch    | (\text{pred} : \text{Bool}, \text{if_true} : T, \text{if_false} : T) \to (\text{value} : T) | Combines multiple values into a Vector. |
| push      | \(\forall A. (\text{vec} : \text{Vec}(A), \text{item} : A) \to (\text{vec} : \text{Vec}(A))\) | Pushes an element onto a Vector. |
| make_pair | \(\forall A, B. (\text{first} : A, \text{second} : B) \to (\text{pair} : \text{Pair}(A, B))\) | Combines two elements into a Pair. |
| fsub      | (\text{a} : \text{Float}, \text{b} : \text{Float}) \to (\text{value} : \text{Float}) | Floating point subtraction. |
| fdiv      | (\text{a} : \text{Float}, \text{b} : \text{Float}) \to (\text{value} : \text{Float}) | Floating point division. |
| make_struct | \(\vec{R} \to (\text{struct} : \text{Struct}(\vec{R}))\) | Combines multiple values into a Struct. |
| parallel  | \(\forall \vec{f}, \vec{I}_1, \vec{O}_1, \vec{O}_2. \ (\vec{I}_1 \cup \vec{I}_2 \sim \vec{I}, \vec{O}_1 \cup \vec{O}_2 \sim \vec{O}) \Rightarrow (\text{a} : \text{Graph}(\vec{I}_1 \to \vec{O}_1), \text{b} : \text{Graph}(\vec{I}_2 \to \vec{O}_2)) \to (\text{value} : \text{Graph}(\vec{I} \to \vec{O}))\) | Combines two graphs with disjoint input and output ports into a single graph by composing in parallel. |
| loop      | \(\forall V, R. (\text{body} : \text{Graph}((\text{value} : V) \to (\text{value} : \text{Variant}(\text{continue} : V | \text{break} : R)), \text{value} : V) \) \to (\text{value} : R)\) | Runs body until it returns break; possibly non-terminating. |
Fig. 6: The graph from Fig. 1b with edge types annotated by the type system. The types of static values input to polymorphic functions constrain the inferred types of values on those edges. However, the return type of the input graph value `run` is not constrained, so the whole graph is polymorphic over the type variable `var4`.

At Quantinuum, we have two runtime implementations:

- A Rust runtime, that runs as a gRPC service and supports connecting to arbitrary workers, also over gRPC. Since these workers are separate processes, they may run in independently sand-boxed environments, allowing them to be scaled and replicated as necessary, or run on dedicated hardware (e.g. for performance, functionality, security or IP protection reasons). This runtime is multi-threaded and fully asynchronous.

- A Python runtime, intended for testing and debugging and included in the Python package, that supports only workers written in Python (e.g. that of Fig. 7), and runs them directly in the same process as the runtime and the client code. This avoids the overhead of protobuf (de)serialisation to speed communication, which is particularly effective when large volumes are data are transmitted between worker functions.

The Python runtime allows inspection of intermediate values in the graph via a user-provided callback function. A future extension to the Rust runtime will allow intermediate values to be streamed back to the client as they appear on edges.

C. Tree of runtimes

The runtime interface includes the worker interface, with its signature being the union of the signatures of the workers. Hence, a runtime acting as a client can connect to another acting as a server, just like any other worker. This enables a tree of distributed runtimes, with parents requesting children to execute functions; a runtime receiving such a request forwards it onto an appropriate one of its children. Ongoing work will allow a parent runtime to send a subgraph (marked by a `Box`) for execution on a child runtime, allowing data local to that subgraph to stay on the child runtime for performance. For example, there could be a runtime instance co-located to the quantum device, that runs the variational loop subroutine of a larger algorithm to reduce latency, while the parent runtime in the cloud (with which the client interacts) runs the rest. A schematic of such a setup is in Fig. 8(b).

IV. CONCLUSION

Tierkreis is a powerful system for programming cloud-native quantum-classical algorithms. The dataflow graph allows higher-order algorithms to be represented naturally, and, when combined with the runtime, unlocks automatic parallelism, asynchronous execution and distribution over the cloud. Platform-neutrality (via protobuf) and extensibility via workers allows easy and graduated migration of existing code as well as composition of components from different languages and environments.

The `tierkreis` Python package along with all gRPC and Protocol Buffer definitions are available under an open-source licence\(^3\).

\[^3\]https://github.com/CQCL/tierkreis
individual components can be located on different hosts; for example, co-located with the quantum hardware.

In (a) a local client interacts with a remote runtime, which uses two external workers to evaluate the nodes. In (b) the runtime sends part of the graph to a second specialised worker. Dashed arrows indicate function calls within the same process. Alex Rice for their helpful feedback on earlier drafts. John Children for also kick-starting much of the implementation. We also thank Alex Chernoguzov, William Whistler and Alex Roy for invaluable discussions on the design of Tierkreis, and

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Listing 1: Python 3 source for building a graph for a simple variational algorithm. A symbolic pytket [4] quantum circuit is built and inserted as a static constant. Features demonstrated include decorator-based builder environments, automatic capture of values in closures, and incremental type inference (if the addition of a node fails an incremental type check, a Python exception is raised at the offending line). Functions from the builtin (bi), python_nodes (pn) and pytket (pt) workers are used. Figures 1, 2, 3, 4, and 5 show visualisations of the graphs defined here. Imports and other setup code are not included for brevity.
async def new_params(
    prev_params: list[CandRecord],
) -> list[float]:
    assert len(prev_params) > 0
    if len(prev_params) == 1:
        return [random.random()] * len(prev_params[0])

    first, second = prev_params[-2:]

    ar1 = np.array(first[0])
    ar2 = np.array(second[0])

    diff = ar2 - ar1
    diff[diff == 0.0] = 1e-14
    grad = (second[1] - first[1]) / diff
    ar2 -= 0.1 * grad
    return list(ar2)

async def converged(prev: list[CandRecord]) -> bool:
    n = 5
    if len(prev) >= n:
        last_n = np.array([x[1] for x in prev[-n:]])
        moving_av = sum((last_n - last_n.mean()) ** 2)
        return bool(moving_av < 1e-5)
    return False

Listing 2: Worker definitions of gradient descent functions used in the example, using the worker building tools from the Tierkreis Python package.