Experimental study of the magnetic field distribution and shape of domains near the surface of type-I superconductors in the intermediate state.

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The importance of accounting for the inhomogeneity of the magnetic field distribution and roundness of domain walls near the surface of type-I superconductors in the Intermediate State (IS) for forming the equilibrium flux structure was demonstrated by Landau eight decades ago. Further studies confirmed this prediction and extended it to all equilibrium properties of the IS. Here we report on direct measurements of the field distribution and shape of domains near the surface of high-purity type-I (indium) films in perpendicular field using Low-Energy muon Spin Rotation spectroscopy. We found that at low applied fields (in about half of the IS field range) the field distribution and domains’ shape agrees with that proposed by Tinkham. However for high fields our data suggest that reality can differ from theoretical expectations. In particular, the width of the superconducting laminae can expand near the surface leading to formation of a maximum in the static magnetic field in the current-free space outside the sample. We speculate that the apparent contradiction of our observations with classical electrodynamics is due to the inapplicability of the standard boundary conditions to the vicinity of an “active” superconductor.

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INTRODUCTION.

The intermediate state (IS) in type-I superconductors is a classical example of a thermodynamically equilibrium system with spatially modulated phases, where a continuous medium is split for domains of different phases. Alike structures with strikingly similar domain patterns are known in a broad variety of physical-chemical systems, in which the pattern forms due to competition between various energy contributions in the system free energy [1]. Relative simplicity in tuning the domain separation (period of the domain structure) in the IS by varying the applied magnetic field and/or temperature, makes the IS a unique and very interesting object for studies of such systems. Recently, being attracted by the beauty of domain patterns and long-standing challenges of the IS physics [2–4], some of us revisited this state experimentally, what resulted in the development of a new theoretical model for the first time consistently addressing all properties of the IS in samples of a planar geometry [5, 6].

Simultaneously, this study made clear an important role of surface related properties for forming all equilibrium characteristics of the IS. Specifically, the role of the field distribution and domain shape (FDDS) near the surfaces through which the flux enters and leaves the sample. Competition between the energy contributions arising from these properties (favoring to a fine domain structure) on one side and those arising from superconducting (S)-normal (N) interphase boundaries in the sample bulk (favoring to a coarse structure) on the other, optimizes and stabilizes the period of the domain structure, volume fractions of the N and S phases, induction $B$ in the N domains, the critical field of the IS/N transition $H_{c1}$, etc.. Being addressed theoretically, these surface related properties of the IS have never been studied experimentally. Here we report on direct measurements of the FDDS near the surface of high-purity type-I (indium) films in the magnetic field perpendicular to the film surface using Low-Energy muon Spin Rotation (LE-$\mu$SR) spectroscopy.

The near-surface properties of the IS were for the first time considered by Landau in 1937 [7]. Cross sectional view of Landau’s field/domain configuration for an infinite slab in perpendicular field is shown in Fig. 1A. Assuming that the boundary of a cross section of the S lamina is the line of induction $B$ all the way including the S/N and the S/V (V stands for vacuum) interfaces with magnitude equal to the thermodynamic critical field $H_c$ at the S/N boundary, Landau calculated the shape of rounded corners of the S laminae near the sample surface. Interestingly, to meet this condition, Landau splits a central field line into two lines (ocd and oba in Fig. 1A), hence challenging the classical electrodynamics [8, 9]. Soon thereafter Landau abandoned this model in favor of a so called branching model [10] (see also [2, 11]), in which N laminae near the surface split into many thin branches so that the flux emerges from the sample uniformly over the whole surface. This branching model was disproved by Meshkovskii and Shalnikov in 1947 [2, 12].

Ten years later Sharvin [13] for the first time observed a regular laminar domain pattern in a slab subjected to a tilted field. Since this pattern resembled that expected in the original (non-branching) Landau model, Sharvin used the latter for interpretation of his results. Ever since, in spite of criticism of Sharvin’s interpretation [14], the re-
field inhomogeneities extending over a “healing length” contribution in the surface related properties comes from
sion of FDDS. Tinkham [3] proposed that the dominant face of samples in the IS [3, 11, 15].
results of Landau calculations of 1937 [7] were considered as an accurate representation of the FDDS near the surface of samples in the IS [3, 11, 15].
There are two simplified modifications of Landau’s version of FDDS. Tinkham [3] proposed that the dominant contribution in the surface related properties comes from field inhomogeneities extending over a “healing length” \( L_h \) outside the sample. \( L_h = (D_n^{-1} + D_s^{-1})^{-1} \), where \( D_n \) and \( D_s \) are the widths of the normal and superconducting laminae, respectively. Correspondingly, Tinkham neglects the roundness of the laminae corners (\( b \) and \( c \) in Fig. 1A). Tinkham’s configuration of the FDDS is shown in Fig. 1B. This configuration is consistent with images of the IS flux structure (see, e.g. [5, 12, 16]) and therefore it is used in the aforementioned model [5, 6]. It turned out that Tinkham’s version of FDDS works surprisingly well, although it apparently violates basics of magnetostatics [4] by allowing the existence of field-free regions near the sample. Note, in Landau’s scenario the field fills entire outside space, as it should [9], since a static magnetic field, as well as a static electric field, cannot make voids in free space.

Abrikosov [15] simplified Landau’s picture in an opposite way. He assumed that the major contribution in the surface related properties is due to the roundness of laminae corners and therefore neglects the field inhomogeneity outside the sample. However, the latter means that the field near the surface is uniform and therefore this scenario is inconsistent with images of the IS flux structure. Abrikosov’s configuration of the FDDS is shown in Fig. 1C, where size of the corners \( c \) is the same as \( L_h \) in Tinkham’s scenario.

An interesting result for a possible domain shapes was obtained by Marchenko [17]. Like Landau [7], Marchenko used conformal mapping to calculate the domain shape in an infinite slab, but in a tilted field. It was found that in a strongly tilted field the curvature of the corners can change the sign as shown in Fig. 1D. We note that in this case the field lines should leave the N-domains converging instead of diverging as in Figs. 1A-C, because bending over a sharp corner (marked \( a \) in Fig. 1D) would take enormous energy [4]. Therefore, the density of the lines (and therefore the field magnitude) should pass through maximum somewhere in the free space above the N-lamina, hence potentially conflicting with the theorem of potential of the classical electrodynamics for time-independent fields [4].

To conclude this brief review of theoretical results, we note that none of them is fully consistent simultaneously with classical electrodynamics and experimental images of the flux structure. To find out the real equilibrium FDDS near the surface of samples in the IS was the goal of our study, which results are presented below.

**EXPERIMENTAL TECHNIQUE AND SAMPLES**

Magnetic properties inside and outside a sample near its surface can be probed using LE-\( \mu \)SR spectroscopy, where polarized positive muons \( \mu^+ \) of tunable energy act as local magnetic microprobes [18–20]. Being embedded inside or stopped outside the sample in a site with an average field (i.e. the induction) \( B \), the muon precesses with frequency \( \gamma_B \), where \( \gamma_B \) is the muon gyromagnetic ratio. The muon is a radioactive particle with the lifetime 2.2 \( \mu s \). It decays into a positron and two neutrinos. The former is preferably emitted in the direction of the muon’s spin at the decay instant (with asymmetry close to 30%). By time-ensemble averaging of \( 10^6 \rightarrow 10^7 \) positrons (events of muons’ decay), precessing muon asymmetry signals reflecting measured \( B \) are recorded by positron counters. On the other hand, muon stopped in a site where \( B = 0 \) does not precess and shoots positron preferably in the direction of its initial spin direction \( P(0) \), hence producing non-precessing asymmetry.
FIG. 3: Data for magnetic moment of a sample with the In-C film measured at increasing (arrow up for 2.0 K) and decreasing (arrow down) parallel field at indicated temperatures. Inset shows the data obtained in perpendicular field; green (orange) points represent the data measured at the increasing (decreasing) field; \( H_c \) is the thermodynamic critical field (measured in the parallel field), and \( H_{ci} \) is the critical field of the transition from the IS to the N state in the perpendicular field.

Both precessing and non-precessing asymmetry signals decay with time due to depolarization of the muon spin ensemble caused by (a) microscopic currents and nuclear spins near the muons’ sites and (b) a possible gradient of the induction \( B \) in the range of muons’ stopping distance [21, 22]. The irregular character of the formers leads to a Gaussian distribution of the probing \( B \). Contrarily, in the latter case the field distribution is non-Gaussian and an adequate theoretical model is required for quantitative interpretation of the spectral data (see, e.g., [18, 23]. However even at very large field gradients (like those in the penetration depth of the extreme type-I materials) the Gaussian approach yields consistent qualitative results [18].

In case of a two-component medium consisting of domains/regions with zero and non-zero \( B \), both precessing and non-precessing asymmetry signals can be recorded at the same time. The initial amplitudes of these signals are proportional to the number of muons stopped in each of these domains/regions and therefore proportional to their volume fraction at a specific depth below or height above the surface. Thus, \( \mu \)SR spectroscopy allows one to measure simultaneously both \( B \) in domains/regions where it is non zero and the volume fraction of these regions. Using LE-\( \mu \)SR, these characteristics can be measured versus distance on both sides from the sample surface by changing the implantation depth (via tuning the muon energy) inside and the height at which muons are stopped outside the sample.

Schematics of the LEM setup, depicting sample, applied field, muon spin and positron counters are shown in Fig. 2. If the measured magnetic field distribution is Gaussian, the asymmetry spectra recorded with the Top and Bottom (TB) and Left and Right (LR) counters with a sample in the IS have the form

\[
A_0 P(t)_{TB} = A_{TB} e^{-\frac{\sigma_{TB}}{2} t} \cos(\gamma \mu B t + \phi)
\]

(1)

\[
A_0 P(t)_{LR} = A_0 P(t)_{TB} + A_{LR} \left[ \frac{1}{3} + \frac{2}{3} \left( 1 - \left[ \sigma_{LR} t \right]^2 \right) e^{-\frac{\sigma_{LR} t}{2}} \right] \cos \phi,
\]

(2)

where \( A_0 P(t)_{TB} \) is the asymmetry recorded vs time \( t \) by the Top and Bottom counters; this asymmetry is caused only by precessing muons (i.e. muons stopped in domains/regions with non-zero \( B \)), and \( A_{TB} \) is its initial...
FIG. 5: Induction $B$ inside (a) and outside (b) of the In-A sample measured vs applied field $H$ at indicated distances from the surface. Negative distances (in (a)) are the depths below the surface; positive distances (in b) are the heights above the surface. $H_{ci}$ is the critical field of the IS/NS transition; dashed-dotted line marked NS is the graph $B(H)$ for the Normal State, where $B = H$.

amplitude. $A_0P(t)_{LR}$ is the asymmetry recorded by the Left and Right counters; it is caused by both precessing and non-precessing muons, and $A_L$ is the initial amplitude of the asymmetry related to non-precessing muons, i.e. to muons stopped in domains/regions with $B = 0$. $\sigma_T$ and $\sigma_N$ are rates of depolarization of precessing and non-precessing muons, respectively; and $\phi$ is the initial phase of each counter.

For spectra measured inside the sample normalized asymmetries $A_{LR}/(A_{TB} + A_{LR})$ and $A_{TB}/(A_{TB} + A_{LR})$ represent volume fractions of the S-component $\rho_s = \frac{w_s}{w}$ and the N-component $\rho_n = \frac{w_n}{w}$, respectively. Here $w_s$ and $w_n$ are, correspondingly, volumes of superconducting and normal phases in a slice parallel to the film surface and having the thickness equal to the width of the stopping distances distribution of the implanted muons of given energy; and $w = w_s + w_n$ is volume of the entire slice. When measured outside the sample, $\rho_s$ and $\rho_n$ are the volume fractions of the regions with zero and non-zero induction, respectively.

For samples in the N-state asymmetries recorded on all counters have the form of Eq. (1), i.e. they differ from each other by the initial phase only.

The field inside the sample at different distances (depths) from the surface was probed in the standard LE-$\mu$SR way, i.e. by implanting muons accelerated to different energies in the range from 3 to 25 keV. Corresponding average stopping distances for In are from 20 to 120 nm, respectively.

To stop muons outside the sample we used a layer of nitrogen deposited on the sample surface from the vapor phase; muons were implanted and stopped in this layer. The rate of N$_2$ deposition is determined by the sample temperature and pressure of nitrogen gas filling the cryostat. In our case the rate was close to 50 nm/min. Then the thickness of the N$_2$ layer is determined by the deposition time, i.e. by the time during which the cryostat is filled with nitrogen. Upon completing measurements with one layer, it was removed by heating the sample to $\sim$30 K. Then the sample was cooled back to the original temperature and a new nitrogen layer was deposited. In all these "outside" measurements, energy of the muons was 14 keV; the average muon stopping depth in the N$_2$ layer was 170 nm, as calculated with the program TRIM.SP [21, 22]. Muons stopping in solid nitrogen may capture an electron to form the hydrogen-like muonium state. The precession frequency of muonium is about hundred times faster compared to Larmor precession of the muon, and cannot be observed in the field range of the experiment. In the deposited N$_2$ layer the fraction of muons precessing at its Larmor frequency is between 40% and 50% [24], causing a corresponding reduction of the amplitude $A_{TB}$ of the precession signal.

We used two indium film samples In-A and In-C. Each film was deposited on a polished sapphire disc of 60 mm in diameter. Simultaneously a few smaller size samples were fabricated for the film characterization. Thickness (residual resistivity ratio) of the In-A and In-C films is

FIG. 6: The same as in Fig. 5 data for $B$ vs $H$ shown with the shifted vertical scales as indicated by arrows.
Low Energy Muons) beamline of the Swiss Muon Source based on the London model for the mixed state. To maintain consistency of the measured spectra with calculations, the Tinkham's formula mentioned above was used to interpret the obtained spectra. It was found that the use of this formula leads to a good agreement with the measured spectra.

The experiments were performed at different beam cycles with an interval of two years. The number of positrons (i.e., number of implanted muons) collected at each experimental point was $4 \times 10^6$. LE-$\mu$SR experiments with In-A and In-C films were performed at different beam cycles with interval of two years.

Fig. 4 shows typical time spectra produced by muons stopped outside the sample (shown are the spectra taken for the In-A film with a 500 nm thick N$_2$ layer) recorded at Left and Right (upper panel) and Top and Bottom (lower panel) counters at an applied field 68 Oe. For comparison, the insert (a) shows the spectra at Right detectors when the sample is in the N state at the same temperature, i.e., at the field (98 Oe) exceeding the critical field of the IS/NS transition $H_c(= 85$ Oe). The insert (b) shows Fourier transforms of the time spectra at the Top and Bottom counters representing the B-spectrum in the N-domains. One can see that (i) initial asymmetry $A_{LB} \neq 0$, indicating the presence of regions with $B = 0$ outside the sample; and (ii) the B-spectrum (shown in Fig. 4b) is fairly close to Gaussian. Spectra recorded on the Left and Right counters inside the sample were similar to those shown in [27], Fig. 4b. The B-spectra in “inside” measurements were also close to Gaussian. This justifies the use of Eqs. (1) and (2) to fit the measured spectra.

Fig. 5 shows data for $B$ vs $H$ obtained at fixed distances $z$ below (Fig. 5a, $z < 0$) and above (Fig. 5b, $z > 0$) the surface for the In-A film. The muon energies at the “inside” measurements were 5, 16 and 24 keV; corresponding average depths of the implanted muons were 27, 75 and 115 nm, respectively. Measurements at positive $z$ were conducted using four N$_2$ layers with thickness 250, 375, 500 and 1000 nm; distances from the sample surface were 80, 200, 330 and 830 nm, respectively. For clarity, in Fig. 6 the data shown in Fig. 5 are presented with shifted vertical axis.

The spectra with 1-μm thick N$_2$ layer were measured both at increasing field after cooling the sample at $H = 0$ and at decreasing field starting from the normal state. Data points for $B$ obtained from these spectra are shown in Figs. 5b and 6b as green solid squares for increasing $H$ and as green stars for decreasing field. As one can see, (a) the data obtained are well reversible and (b) there is a deep supercooling at decreasing field. These results confirm that the sample was essentially pinning-free in the field range studied. We note that similar supercooling, testifying that IS/NS transition is a phase transition of the first order, was reported in other studies of the IS performed with high purity samples, e.g., in measurements of electrical resistance [28], magnetization [6, 29] and $\mu$SR spectra [30].

As seen from Figs. 5 and 6, in the field range $0.6 < H < 3.86$ μm (610) and 2.88 μm (570), respectively. The film In-A was the same film which was used as In-A sample in [6] (i.e., samples used in [6] and in this work were deposited simultaneously). Details for the films fabrication and a typical image of the film surface are available in [18].

Before describing experimental results we note that a problem similar to that we discuss here was addressed in [25] for the mixed state in an extreme type-II superconductor (YBCO film) in perpendicular field. It was a first application of the LE-$\mu$SR technique to superconductivity, targeted to demonstrate the rich capabilities of the new technique. The experiment was performed at a single temperature (20K) and field (104 Oe) by changing the energy of muons implanted in the film and in a thin silver layer deposited on an identical film in order to stop muons outside the sample. The Tinkham’s formula mentioned above was used to interpret the obtained $\mu$SR spectra. It was found that the use of this formula leads to consistency of the measured spectra with calculations based on the London model for the mixed state.

**EXPERIMENTAL RESULTS**

The LE-$\mu$SR experiments were performed at the LEM (Low Energy Muons) beamline of the Swiss Muon Source at the Paul Scherrer Institute [26]. In all measurements the cryostat was kept at a base (the lowest) temperature. Temperature of each sample was determined in situ using the sample’s phase diagram obtained from the magnetization data. It was 2.47 K (2.24 K) for the In-A (In-C) film. In all but one runs the samples were cooled at zero applied field. The number of positrons (i.e., number of implanted muons) collected at each experimental point was $4 \times 10^6$. LE-$\mu$SR experiments with In-A and In-C films were performed at different beam cycles with interval of two years.

FIG. 7: Volume fractions of the superconducting (S) and the normal (N) components vs applied field at fixed muon energies 16 and 24 keV corresponding to average distances 75 and 115 nm, respectively, below the surface of the In-A film. $H_{ci}$ is the critical field of the IS/NS transition.
$H/H_{ci} \lesssim 1$ (marked by a dashed rectangle in Figs. 6a and 6b) a slight irregularity in $B(H)$ inside the sample develops into a strong anomaly outside. Induction $B$ measured with 250-nm N$_2$ layer ($z = 80$ nm) monotonically decreases with increasing $H$ in a similar way as it takes place inside the sample. This signals that the flux pattern at this distance is about the same as that for the applied field, where $B = H$ as shown by the dash-dotted line in Figs. 5 and 6.

In all theoretical scenarios for the transverse field (see Figs. 1A to 1C) induction $B(z)$ above the N-domains decreases with increasing $z$ starting from $z = 0$. However, according to experimental data obtained at the high field ($\gtrsim 0.6H_{ci}$), $B(z)$ first increases before it gradually decreases to the value of the applied field $H$ far away from the sample (i.e. at $z$ on the order of microns). Therefore, a function $B(z)$ outside the sample passes through maximum, indicating that probably the field lines exit the N-domains converging, as it can be expected if the cross-sectional domain shape is similar to that shown in Fig. 1D.

Now we turn to the volume fractions of components/regions. Note, that bending of the field lines on both sides from the surface may effect the amplitude of asymmetries and hence values of $\rho_n$ and $\rho_s$. However, this effect is small [25, 31] and does not exceed the error bars for these quantities.

Fig. 7 shows graphs for $\rho_n$ and $\rho_s$ vs $H$ extracted from the spectra measured with muons of 16 and 24 keV corresponding to the average distances $z = -75$ and -115 nm, respectively. We see that the graphs $\rho_n(H)$ [$\rho_s(H)$] are close to linear and their extrapolation to $H = 0$ passes through the origin (unity), hence satisfying the limiting cases $\rho_n = 0$ [$\rho_s = 1$] at $H = 0$. A linear dependence of the initial amplitude of $\mu$SR asymmetry vs applied field measured deeply inside a single crystal tin sample was also reported in [30]. The linearity of these graphs is consistent with the linear dependence of resistance $R$ vs $H$ measured in a spherical sample in direction perpendicular to $H$ [32] and in cylindrical samples in perpendicular $H$ [28]. In a film sample in tilted field $R$ is also a linear function of $H_\perp$ (perpendicular component of $H$) [5]. The resistance of samples in the IS is due to the presence of the N component, directly measured in this work. Therefore the graphs in Fig. 7 confirm that
In particular, the graphs of the muon stopping distances are small due to effects of surface roughness and the finite width of distribution of the muon stopping distances. In particular, the graphs $\rho_n(H)$ and $\rho_s(H)$ obtained with 5.1 keV muons are very noisy, which led to increased error bars for the data obtained with such ultra-low energy muons ($\lesssim 5$ keV).

In three panels of Fig. 8 volume fractions of the N and S components in the In-A film are shown vs muon energy $E$. Corresponding average distances $z$ are given at the upper scale of each panel. Data points depicted as solid circles were extracted from the spectra measured at fixed $H$ and varying $E$; open circles are the data obtained from measurements at fixed energies and varying $H$. Fig. 9 shows fractions of regions with $B = 0$ outside the film at different distances $z$ from the surface.

As one can see from Fig. 8, at a low field ($H = 44$ Oe, the upper panel) experimental points randomly scatter near constant $\rho_n$ and $\rho_s$ close to 0.5. Taking into account that, as seen from Fig. 9, the same fraction of regions with zero $B$ is present outside the film at this $H$, we conclude that the field lines inside the sample approach the surface being close to parallel and therefore the FDDS at low fields is consistent with the Tinkham’s scenario depicted in Fig. 1B. This explains the successful application of the Tinkham’s formula for $L_h$ in aforementioned works [5, 25], where calculations were performed at relatively low fields.

However at higher field (in the area approximately outlined by rectangles in Figs. 6a and 6b) we see that the observed enhancement of $B$ outside the film is accompanied by a decrease of $\rho_n$ near the surface inside it. This leads us to suggest that for the high fields the cross-sectional domain shape is similar to that shown in Fig. 1D and therefore FDDS in this range of perpendicular fields qualitatively looks as shown in Fig. 10.

To verify the results for FDDS obtained with the In-A film, a similar (but less detailed) experiment was conducted with the In-C film. Data for $B$ vs $H$ extracted from the spectra measured at fixed distances outside the film are shown in Fig. 11. An example of $B$ vs $z$ at fixed $H (\approx 77$ Oe) is shown in the insert. As one can see, alike for the In-A film, there is also a region in the upper half of the field range of the IS, were induction $B(z)$ first passes through maximum before it relaxes down to the applied field far away from the sample. Therefore we conclude that results obtained for the In-A film reflect a general feature taking place near the surface of superconductors in the IS.

**DISCUSSION**

Observed the near-surface widening of the S-laminae and corresponding narrowing of their N-counterparts at high applied field testifies that gain in the sample free energy due to the former exceeds losses due to the latter and the losses due to the increasing inhomogeneity of the outside field. But then a legitimate question arises: if it happens at high fields, why it does not take place at lower field values?

A possible answer is as follows. As known [9], the magnetic field $B$ in N domain exerts pressure on the N/S interface equal to $B^2/8\pi$. A maximum pressure withholding by the N/S interface in type-I superconductors is reached when $B = H_c$ [4]. It is also known [5, 30], that in the IS $B$ in N-domains decreases with increasing applied field from $H_c$ down to $H_{CI}$. Therefore, since narrowing of an N-lamina leads to increasing $B$ due to flux conservation, at low $H$ (when $B \leq H_c$) there is no room to make the narrowing profitable. However such a room does appear at high applied field at which $B$ can be significantly (for more than 50%) less than $H_c$ [5]. Therefore maximum saving of the condensation energy (proportional to the volume of the S phase) can be reached with the flat interface boundary (as in Tinkham’s scenario in Fig. 1B) at low field and with widening S-laminae (as in Fig. 10) at high field.

Another principal question, associated with the FDDS scenario shown in Fig. 10, is related to the potential theorem [4], also referred as Earnshaw’s theorem [8]. As known (see, e.g. [32]) in current-free regions at steady-state conditions the Maxwell equations for the magnetic field are identical to those for static electric field and...
therefore the magnetic field can be described using a magnetic scalar potential $\Phi(r)$ ($r$ is a spatial coordinate) for which the Laplace equation $\nabla^2 \Phi = 0$ holds. This means that $\Phi(r)$ can reach an extremal (maximum or minimum) value and therefore $B = \mu H = -\mu \nabla \Phi$ can be zero ($\mu$ of the free space is 1) only at boundaries of a region where there is the field [4]. Therefore it looks like the field configuration depicted in Fig. 10 conflicts with this theorem.

This can be resolved as follows. The Laplace equation is a matter of boundary conditions, which do not include "active" superconductors [34]. Specifically, it does not hold in the immediate neighborhood of a body, where the effects of molecular currents are significant and therefore the standard transition from always valid Maxwell equations for microscopic magnetic field $\mathbf{h}$ to the average field $\mathbf{B}$ as $\mathbf{h} = \mathbf{B}$ is no longer valid [4]. Molecular currents are persistent currents producing the field in a space range of the order of molecular size and therefore for normal materials the potential theorem is correct outside of this very small distances. Contrarily, in superconductors in the IS we deal with persistent currents of much greater spatial scale, i.e. on the order of a period of the flux structure, which can easily reach a few sample thicknesses [5, 7]. This explains the apparent "violation" of the potential theorem at this range of distances from the sample.

**SUMMARY AND OUTLOOK**

Eight decades ago Landau for the first time has shown the determinative role of the near-surface field distribution and of the domain shape for forming the flux structure of the intermediate state in type-I superconductors. In this work these properties were for the first time measured by low-energy muon spin rotation spectroscopy on pure-limit type-I indium films. It was found that the field-domain configuration proposed by Tinkham is consistent with our experimental results at low values of the applied field. However at higher fields our observations suggest that the cross-sectional width of the superconducting domains near the sample surface is widening, instead of the expected narrowing. Then the field lines emerge from the normal domains converging, and the field outside the sample passes through a maximum before it relaxes to a uniform applied field far away from the sample. There is no reason to believe that similar field/domain configurations are not possible near the surface of type-II superconductors in the mixed state, however details can be different. Verification of these near-surface properties in type-II superconductors constitutes an interesting problem of fundamental superconductivity which is important for a better understanding of the properties of the mixed state, especially in thin films.

In this work the near-surface properties inside and outside superconductors with an inhomogeneous flux distribution were measured applying a large scale $\mu$SR facility. As of today, this is the only technique appropriate for such kind of measurements inside the sample. Unfortunately, the main anomalies inside appear very close to the surface, where the accuracy of the $\mu$SR data reduces due to effects associated with the surface roughness and with the depth distribution of muon stopping distances. However, on the outer side of the sample more detailed and potentially more accurate measurements both with type-I and type-II superconductors can be performed using non-invasive scanning techniques, such as those based on the Hall microprobe, squid-on-tip, or electronic spin resonance of a single nitrogen vacancy center in diamond. We are looking forward to seeing results of such measurement and are ready to share necessary high purity samples.

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