Morphological Analysis of Cryogenic Spray Images

Hrishikesh Ganu and B.N. Raghunandan
Atomization and Sprays Laboratory
Department of Aerospace Engineering
Indian Institute of Science
Bangalore-560012
Contents

1 Introduction ........................................................................................................... 2

2 The Malvern Mastersizer ....................................................................................... 2

3 Need for an Imaging system .................................................................................. 2

4 Method of capturing images .................................................................................. 2

5 Image Processing .................................................................................................. 3

6 Mathematical Morphology ................................................................................... 3
   6.1 The basic morphological operations ................................................................. 3
   6.2 Analysis of Opening and Closing ..................................................................... 8

7 Granulometry ......................................................................................................... 10
   7.1 Sieving and Granulometry on a Cryogenic Spray Photograph ......................... 10
   7.2 Axiomatic development of granulometric mappings ......................................... 14

8 Conclusions ............................................................................................................ 15
1 Introduction

This study gives the development of a new technique for analyzing images of Cryogenic sprays, to estimate the drop-size distribution. It has a sound mathematical basis, in the form of Mathematical Morphology, and we have tried to build up a formulation for a granulometry, starting from the elementary operations of Dilation and Erosion. An axiomatic foundation for granulometry has also been discussed.

We have taken an actual $LN_2$ spray photograph for analysis, to illustrate the use of Morphological operations, culminating in a granulometry.

2 The Malvern Mastersizer

The Malvern mastersizer is based on the principle of laser ensemble light scattering. It is currently, the most widely used instrument for estimating the droplet size-distributions of sprays. Ensemble scattering means that droplets cannot be measured individually; only a group of a large number of droplets can be sized. It is a non-imaging instrument, since sizing is done without forming an image of the object.

A laser beam emerging from the source, is scattered by a collection of droplets in a spray, and the scattered light falls on the receiver. The receiver is an array having several annular detector elements. Each element senses the light scattered by droplets in a particular size-class. The sizes of the droplets are related to the angle of scattering.

3 Need for an Imaging system

In case of the Malvern Mastersizer, sufficient light must fall on the detector for it to sense the amount of scattering. Excessive obscuration, encountered in dense sprays prevents this. Further, in the specific case of a liquid nitrogen spray, injected into the ambient at supercritical temperature, vapour around the evaporating droplet leads to density gradients, which in turn affect the refractive index. This effect is called beam steering.

As it is, Malvern Mastersizer gives an excessively large SMD (typically, $800\mu m$ at 450kPa). We know from the physics of droplet breakup, that this is not possible. Moreover, studies by Chin et.al indicate that the Malvern Mastersizer underpredicts the SMD of dense sprays. Thus the drop size distribution given by Malvern Mastersizer is physically untenable. There is, therefore, a need for an imaging system, which can directly capture the spray on an image, which can then supply information about the drop sizes.

4 Method of capturing images

We have used $LN_2$ as a simulant for cryogenic propellants. The axisymmetric jet issues out of the plain injector, into the ambient at pressure drops between 300kPa and 700kPa. For capturing the images a 1540-P2 strobolume pulse light generator together with a digital camera of resolution 300dpi is used with front-lighting. The width of the pulse is 50 ns at a light power output of 250 mW. The wavelength of emission is 660
nm with a spectral width of 27nm. Images are captured at a high film speed equivalent to ISO 400 on a conventional photographic film.

5 Image Processing

Photographs of the spray yield just a raw image, which can scarcely be of any use. It must be first pre-processed to a form on which Morphological operations can be performed. Pre-processing includes conversion to grayscale, followed by contrast enhancement if required. The grayscale version is then converted to binary, using an appropriate threshold. These are standard image processing operations, and details may be found in [1].

6 Mathematical Morphology

Morphology is a term commonly used in the biological sciences for describing the form or structure of an organism. In image processing, by morphology, we mean the geometrical or textural features of the image.

The techniques of morphology try to duplicate the manner in which a human being perceives an image. While observing a flower for example, we might first observe the colour, then the texture, followed by the geometry and so on, in succession not all at the same time. We then try to look at that aspect, which interests us the most, in greater detail.

In morphological image processing (MIP) too, we select a specific characteristic of interest, and analyze the image with respect to that. Knowledge about the other characteristics is now irrelevant to us and, is off-loaded before we proceed. This is why in our experiments, the binary version has been used, though we could as well have used the grayscale version.

6.1 The basic morphological operations

Here are some basic operations which are extensively used in MIP. The book by Gonzalez and Woods [1] is an excellent introduction to Morphology. Let A be a set in \( \mathbb{R}^2 \) and \( x \in \mathbb{R}^2 \).

1. Translation
   
   \[ A + x = \{ a + x : a \in A \} \]
   
   Which is to say that a point \( z \in A + x \iff \exists \ a \text{ point } a \in A, \text{ such that } z=a+x. \]

2. Minkowski addition
   
   For A and B \( \subset \mathbb{R}^2 \)
   
   \[ A \oplus B = \cup A + x, \text{ with } x \in B. \text{ This leads to:} \]
   
   \[ A \oplus B = \cup \{ x + y \}, \text{ such that } x \in A \text{ and } y \in B. \text{ Several properties immediately follow this definition}: \]
Figure 1: Dilation of an image by a structuring element

(a) $A + \bar{0} = A$.
(b) $A \oplus x = A + x \forall x \in \mathbb{R}^2$.
(c) Commutativity, $A \oplus B = B \oplus A$.
(d) Associativity, $A \oplus (B \oplus C) = (A \oplus B) \oplus C$, and in particular $A \oplus [B + x] = [A \oplus B] + x$.

3. Minkowski subtraction
$A \ominus B = \cap A + x$, such that $x \in B$. With scalar multiplication being defined as $tA = \{tx : x \in A\}$ we get $-A = \{-x : x \in A\}$, which is called the reflection of $A$.

4. Erosion An alternative expression, is:
$A \ominus B = \{x : -B + x \subset A\}$, from which the erosion is defined as:
$A \ominus -B = \mathcal{E}(A, B) = \{x : B + x \subset A\}$.

5. Dilation It is the same as Minkowski addition
$\mathcal{D}(A, B) = A \oplus B$.
Dilation and Erosion obey:

(a) $\mathcal{D}(A, B) = \mathcal{D}(B, A)$
(b) $\mathcal{D}(A, B + x) = \mathcal{D}(A, B) + x$. 
(c) $\mathcal{E}(A, B) = \mathcal{E} A, B + x + x$.

(d) For a given $B$, and $A_1 \subset A_2$,
   i. $\mathcal{D}(A_1, B) \subset \mathcal{D}(A_2, B)$
   ii. $\mathcal{E}(A_1, B) \subset \mathcal{E}(A_2, B)$.

\[ A \ominus B = \{x : -B + x \subset A\} \]

Figure 2: Erosion of an image

(e) **Opening** Opening is a combination of dilation and erosion and is defined as:

   $\mathcal{O}(A, B) = \mathcal{D}[\mathcal{E}(A, B), B]$ 

(f) **Closing** $\mathcal{C}(A, B) = \mathcal{E}[\mathcal{D}(A, -B), -B]$. These operations will now be analyzed in detail, since they were used to analyze cryogenic spray photographs in this exercise.
Figure 3: Dilation of an image

\[ (-B + x) \cap A \neq \emptyset \]
\phi(A, B) = \Phi[\phi(A, B), B]

Figure 4: Opening, as erosion followed by dilation
6.2 Analysis of Opening and Closing

1. $C(A, B)^c = O(A^c, B)$ and

2. $O(A, B)^c = C(A^c, B) \ldots$ Duality.
   Thus, if the complement of the first operand, $A$ is known, we can find the opening, given the closing and vice-versa.

3. $O(A, B) = \bigcup \{B + y : B + y \subseteq A\}$
   which says that the opening is the union of translates of $B$ which are contained within $A$.

4. $C(A, B) = \bigcap \{(B + y)^c : (B + y) \subseteq A^c\}$
   Closing is the intersection of the complements of certain translates of $B$.

5. A point $z$ is $\in C(A, B) \iff (B + y) \cap A \neq \emptyset \forall (B + y)$

6. $O(A, B) \subseteq A$
   The opening is antiextensive.

7. $A_1 \subseteq A_2 \Rightarrow O(A_1, B) \subseteq O(A_2, B)$
   It is increasing.
8. \( \mathcal{O}[\mathcal{O}(A, B), B] = \mathcal{O}(A, B) \)
   and also idempotent.
   The closing is also increasing and idempotent,

9. \( \mathcal{C}(A, B) \supset A \), but is extensive.
   A set \( A \) is \( B \)-closed (or \( B \)-open) when \( \mathcal{C}(A, B) = A \) (or \( \mathcal{O}(A, B) = A \)).

10. \( A \) is \( B \)-closed \( \iff \) \( A^c \) is \( B \)-open.

11. This is connected with the Minkowski, addition and subtraction as:
   A is \( B \)-open \( \iff \) \( \exists \) a set \( E \) such that \( A = E \oplus B \) and
   A is \( B \)-closed \( \iff \) \( \exists \) a set \( E \) such that \( A = E \ominus B \)

12. Generalized laws of idempotency for a set \( A \) which is \( B \)-open, and any other set \( F \):
   \( \mathcal{O}[\mathcal{O}(F, B), A] = \mathcal{O}(F, A) \) and
   \( \mathcal{O}[\mathcal{O}(F, A), B] = \mathcal{O}(F, A) \)

13. If \( r \geq s \), with \( r, s, \in \mathbb{R} \) and \( B \) is convex, then for any \( A \), \( \mathcal{O}(A, rB) \subset \mathcal{O}(A, sB) \)

\* A set is called convex when a line joining any two of its points, lies entirely within the set.
7 Granulometry

Let $A$ be a compact set $\subset \mathbb{R}^2$ and $t \in \mathbb{R}$. We next consider $\Psi$ an image-image transformation (which could be the opening, on a cryogenic spray image, for instance), applied to $A$

$\Psi : A \rightarrow \Psi(A)$. 

If $E$ is a convex, generating structuring element (which is really, an image). Then $\{tE\}$ is a family of structuring elements. The mapping $t \mapsto \partial(A, tE)$ is decreasing. This means that $\partial(A, tE)$ shall be empty for sufficiently large $t$. The mapping $t \mapsto \partial(A, tE)$ is known as a granulometry. Granulometric filtering is analogous to the physical operation of sieving. Intuitively, it is sufficient to understand that this is a kind of filtering in which those sections of the image which are not sufficiently large to hold the structuring element $tE$ are removed from the image.

Though this can be better appreciated, by following the algorithm, with a photograph of an actual cryogenic spray as will be seen in this sub-section, the axiomatic development of granulometries, given in the next sub-section, should not be skipped if one wishes to really understand the process. This study is on Binary images; Morphological operations in the gray-scale domain are discussed in [2].

7.1 Sieving and Granulometry on a Cryogenic Spray Photograph

The algorithm for generating a granulometry is represented by the flowchart shown in Figure 6 below. It accepts the raw image $A$ as the input and delivers a single parameter granulometry $\Phi_t(A)$ with $t = 1, 2, \ldots$ as the parameter. The generating structuring element used, $E$, is a unit square.

Incrementing the value of $t$ after each step yields a sequence of images under the granulometry $t \mapsto \partial(A, tE)$. The process stops when at some stage the opening is empty (as it shall be; see Section 7.1). Figure 7 shows the physical analogy for the operation. A sequence $\Psi_t(A)$ of openings is thus produced. It can be seen that there will be a minimum mesh size at which nothing will be retained on the sieve.

After a sequence of images $\Psi_t(A)$, $t=1, 2, \ldots$ has been generated, their complements w.r.t. the entire image are determined. This sequence is denoted as $\Phi_t(A)$ in Figure 11 which shows how the difference between successive images of this set can be used to get an image of droplets in a particular pixel-size class. These images are denoted as $\Phi_{r, r-1}$, where $r$ denotes the higher size-class. The pixel-size classes are converted to length classes, knowing the resolution at which the image was captured.

Once this is done, we just have to count the number of droplets in each such image $\Phi_{r, r-1}(A)$ and these values can be directly used for plotting the droplet size-distribution, followed by calculations for the SMD and other diameters.

---

1. A compact set is closed and bounded.
2. A closed set contains its own boundary.
3. A bounded set is contained within a disk of finite radius, centred at the origin.
Figure 6: Flowchart for generating a granulometry
Figure 7: Physical Analogy (Sieving) for Opening

Figure 8: Opening by a structuring element of size 4 units $\Psi_4(A)$

Figure 9: Opening by a structuring element of size 8 units $\Psi_8(A)$
Figure 10: Droplets in the *pixel-size class* 7-8

Figure 11: How the image $\Psi(r, r-1)$ is obtained
Figure 8 and Figure 9 are openings, while Figure 10 shows the droplets in the *pixel-size class* 7-8 for an image of an actual cryogenic spray.

### 7.2 Axiomatic development of granulometric mappings

Let $X$ be the collection of Euclidean images $\subseteq \mathbb{R}^2$. Then a *granulometry* on $X$ is a family of mappings

$$
\Psi_t : X \rightarrow X, t > 0,
$$

such that

1. $\Psi_t(A) \subset A \ \forall \ t > 0 \ldots \Psi_t$ is antiextensive.

2. $A \subset B \implies \Psi_t(A) \subset \Psi_t(B) \ldots \Psi_t$ is increasing.

3. $\Psi_t \circ \Psi_i = \Psi_i \circ \Psi_t = \Psi_{\max(t,i)}, \ \forall \ t, \ i > 0$.

   In particular, $\Psi_0(A) = A$. Once we have these axioms, several results can be easily derived. One such result is:

   For a granulometry $\Psi_t$, if $r \geq s$, $\Psi_r(A) \subset \Psi_s(A)$

If we add the following two axioms to the existing three, we get an *Euclidean granulometry* for any $t > 0$:

4. $\Psi_t$ is compatible with translation.

5. For an image $A$, $\Psi_t = t \ \Psi_t(1/t \ A)$
8 Conclusions

In this report, it is seen how Mathematical Morphology can be used in processing the image of a spray to quantify the structures observed in it. Though this technique was developed, considering the need for analyzing Cryogenic sprays, it could be applied to storable propellants, as well.

Morphological opening is however, a shape-distorting operation and therefore, no attempt must be made to relate the shapes of droplets, in the images comprising the granulometry, with those in the actual spray.

References

[1] R. Gonzalez and R. Woods, Digital Image Processing 2nd Edn., Pearson Education (Singapore) Pte. Ltd.

[2] R. Haralick, S. Sternberg and X. Zhuang, IEEE Transactions on Pattern Analysis and Machine Intelligence (PAMI), v9-n4, July 1987 pp 532-549.

[3] Chin, J. S., And Zhang, Y., Experimental Study Of The Effect Of Dense Spray on Drop Size Measurement by Light Scattering Technology., ASME J. Eng. Power, Vol.114, 82-88, January 1992.