Recent Progress in Electroweak Baryogenesis

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The status of electroweak baryogenesis in the minimal supersymmetric standard model is reviewed. I discuss the strength of the phase transition, CP violation and transport at the bubble walls, and the possibility of a two stage transition involving charge and color breaking.

1 Electroweak Baryogenesis: Review

The basic ideas underlying electroweak baryogenesis have been reviewed elsewhere, so I will give an abbreviated description here in order to leave room for some of the interesting details. To create the baryon asymmetry of the universe during the electroweak phase transition (EWPT), it is assumed that Sakharov’s out-of-equilibrium requirement for baryogenesis is fulfilled by making the EWPT first order: bubbles of nonzero Higgs field VEV nucleate from the symmetric vacuum and fill up the universe. As these bubbles expand, there is a flux of particles going into the bubbles, but there is also some probability for the particles to bounce off the wall back into the symmetric phase, where their masses are smaller. If there are new CP violating effects in the bubble walls, this reflection probability can be different for left-handed quarks and their antiparticles, for example. This produces a CP-asymmetric flux of particles back into the symmetric phase. Baryon number violating sphaleron interactions are biased by this CP asymmetry into producing a baryon asymmetry in front of the wall. Since the wall is steadily advancing, whereas particles in the vicinity of the wall are diffusing, these baryons make their way inside the bubbles, where the sphaleron interaction rate is much smaller. As long as sphalerons are ineffective in the interior, these baryons survive to become the visible matter of the present universe. The rest is history, one might say.

2 Strength of the Phase Transition

Not only is new physics needed to get enough CP violation, but the hypothesis of a strongly first order electroweak phase transition also requires it. By strong we mean that the Higgs VEV in the broken phase, \( v_c \), at the critical temperature, \( T_c \), must be sufficiently large,

\[
v_c/T_c > 1.
\]

(1)
Otherwise, sphaleron interactions continue to be too fast inside the bubbles and destroy any baryon asymmetry which gets produced. If the top quark and Higgs boson were both light, this condition could be achieved within the SM, but it is now well established not only that the transition is too weak, but it is smooth, with no bubbles at all. To get a first order transition, one needs a negative cubic term in the finite-temperature effective Higgs potential,

\[
V(H) = \frac{1}{2} (-\mu^2 + c_h T^2) H^2 - E T H^3 + \frac{1}{4} \lambda_h H^4.
\]

At the critical temperature, where \( V(H) \) takes the form \( (\lambda/4) H^2 (H - v_c)^2 \), it is easy to show that

\[
\frac{v_c}{T_c} = \frac{2E}{\lambda}.
\]

Thus one needs not only for \( E \) to be large, but also \( \lambda \) must be small, which implies a small mass for the Higgs boson. The cubic term in \( \Box \) arises from the one-loop free energy for bosons, which at high temperatures goes like

\[
V(m(H)) = \frac{1}{24} m^2 T^2 - \frac{1}{12\pi} m^3 T + O(m^4)
\]

per degree of freedom, where \( m(H) \) is the Higgs field dependent boson mass. In the SM, such cubic contributions are provided by the gauge bosons, but they are too small to make \( v_c/T_c > 1 \). However new particles which couple strongly enough to the Higgs field can increase the cubic coupling, provided that they are also sufficiently light.

For those who look to supersymmetry for new physics, the natural candidate for strengthening the transition is the right top squark (stop), since it has the strong top Yukawa coupling \( y^2 |\tilde{t}_R|^2 |H_2|^2 \) to the second of the two Higgs doublets required by SUSY. We focus on \( \tilde{t}_R \) because the left stop tends to make excessive contributions to the electroweak \( \rho \) parameter if it is light. If we write \( H = \sqrt{H_1^2 + H_2^2} \) and \( H_2 = \sin \beta H \), the field-dependent stop mass has the form

\[
m_{\tilde{t}_R}^2 = m_U^2 + c_s T^2 + y'^2 H^2,
\]

where \( y' \) is less than \( y \sin \beta \) when mixing between left and right stops is accounted for. But to get a cubic term \( H^3 \) from \( m_{\tilde{t}_R}^2 \), it is necessary that the soft SUSY-breaking mass \( m_{\tilde{t}_R}^2 \) be negative, so as to approximately cancel the thermal contribution \( c_s T^2 \). At zero temperature, the physical mass of the stop will then be less than \( y' v \), which in turn is less than the top quark mass. To be more precise, \( c_s \simeq 0.5 \) and \( T \simeq 90 \) GeV, so \( m_{\tilde{t}_R} \simeq 160 \) GeV in the no-mixing case, and arbitrarily smaller (down to the experimental limit of 85 GeV ) with mixing between the left and right stops.
It was first pointed out by Espinosa\textsuperscript{7} that two-loop contributions to $V(H)$, such as those shown in figure\textsuperscript{1}, also make a considerable improvement in the strength of the transition. There have now been a number of studies of the phase transition using the two-loop effective potential\textsuperscript{8}−\textsuperscript{13} which obtain similar results for the range of Higgs and stop masses that are compatible with having a strong enough phase transition,

$$85 \text{ GeV} < m_h < 107 - 116 \text{ GeV} ;$$

$$120 \text{ GeV} < m_{\tilde{t}_R} < 172 \text{ GeV} .$$

The range of values for the upper limit of the Higgs mass is due to our freedom to vary the mass ($m_Q$) of the left stop\textsuperscript{12} which radiatively corrects $m_h$. We somewhat arbitrarily considered upper limits of $m_Q < 1 - 2 \text{ TeV}$ to obtain this range. Although even larger values of $m_Q$ are conceivable, it is difficult to explain why there should be such a vast hierarchy between the left and right stop masses.

The fact that two-loop effects are so important, and that the usual loop expansion does not generally converge well at high temperature, are cause for concern as to whether reliable results can be obtained from the two-loop potential. Fortunately we have empirical evidence that the two-loop approximation is good, because lattice gauge theory simulations of the phase transition have been done\textsuperscript{11} which agree reasonably well with the analytic results; the lattice gives values of $v_c/T_c$ about 10% higher than those of the effective potential, for the limited range of MSSM parameters where the comparison could be made.

![Figure 1: Some of the virtual squark diagrams contributing to $V(H)$.

2.1 Studying the EWPT in the MSSM

In this subsection I will summarize the results of ref. [12], whose goal was to make a broad search of the MSSM parameter space for those values giving a strong enough phase transition, in the sense of inequality (1). The main tasks were the following:
1. We generalized the two-loop effective potential of ref. [9] to include mixing between the various Higgs bosons and between the left and right stops and sbottoms. This allowed us to check the cases where the CP-odd Higgs boson $A^0$ or the left stop and sbottom were light, rather than decoupled from the thermal bath.

2. For each set of trial MSSM parameters, we checked the experimental constraints on the light Higgs and squark masses, $\tan \beta$, the $\mu$ parameter (which is the $\mu H_1 H_2$ coupling in the superpotential), the electroweak $\rho$ parameter, and the prohibition against charge- or color-breaking minima due to condensation of the stop field. More will be said about the latter in section 4.

3. For sets that passed the above cuts, we found the critical temperature $T_c$, not from the approximation (3) but by tuning $T$ to get degenerate minima of the full potential. We then found the bubble nucleation temperature, $T_{\text{nuc}} < T_c$, where one bubble starts to appear per Hubble volume and Hubble time; we also computed the latent heat of the phase transition, which allows one to find the reheating temperature $T_r$, somewhere between $T_{\text{nuc}}$ and $T_c$.

4. To accurately find the sphaleron rate, $\Gamma_{\text{sph}}$, we numerically solved for the energy of the sphaleron configuration in the full potential at $T_r$. $\Gamma_{\text{sph}}$ is given by $(v_c/T_r)^7 e^{6.9 - E_{\text{sph}}/T_r}$. This was compared to a critical allowed rate, $\Gamma_{\text{crit}}$, roughly equal to the expansion rate of the universe, rather than using the approximation (4), to determine if the phase transition was strong enough to avoid baryon dilution by sphalerons.

5. For each accepted parameter set, we computed the profile of the bubble wall, i.e., $H_1(r)$ and $H_2(r)$, which enables one to find the spatial variation of $\tan \beta(r) = H_2/H_1$. This quantity was of interest because the efficiency of electroweak baryogenesis is predicted by some to be proportional $\partial \beta / \partial r$.

6. These steps were carried out for approximately 10,000 randomly chosen sets of parameters, where we varied $\tan \beta$, the soft-SUSY-breaking squark masses and mixing parameters, and the mass of the CP-odd Higgs boson, $m_{A^0}$.

In figure 2 we show the distributions of parameters which pass all the cuts mentioned above. The most important of these is the right stop soft-breaking mass parameter, $m_{U_2}^2$. Since it must be negative to get a light enough
stop and strong enough phase transition, we define $\tilde{m}_U = \frac{m_{\tilde{t}}^2}{|m_U|}$. As I will discuss below, too negative a value of $\tilde{m}_U$ gives color breaking vacua, which are excluded. The dependence of the strength of the EWPT on $\tilde{m}_U$, as measured by $v/T$, is shown in figure 3, where the tendency for $v/T$ to increase as $\tilde{m}_U$ becomes more negative in clear. One also sees that $v/T$ is a decreasing function of $\tan \beta$, but that this can always be counteracted by going to more negative $\tilde{m}_U$ values.

Figure 2: Distributions of parameters giving a strong enough electroweak phase transition for baryogenesis in the MSSM. $\tilde{t}$ is the stop mixing angle. $A_t$ and $\mu$ appear as $y(A_t H_1 - \mu H_2)$ in the off-diagonal part of the squark mass matrix. Units are GeV.

Figure 3: Contours of constant $v/T$ (solid lines) and $m_h$ (dashed lines) in the plane of $\tan \beta$ and $\tilde{m}_U$, for two choices of $m_Q$, 100 and 500 GeV.
2.2 Correlations

Perhaps more illuminating are the correlations between parameters. One of these, \( \sin^2(\alpha - \beta) \), measures the alignment between the direction of symmetry breaking in the \( H_1-H_2 \) plane and the direction of the lightest Higgs field. When \( \sin^2(\alpha - \beta) = 1 \), one has recovered the limit of a SM-like (single Higgs field) Higgs sector. Figure 4 shows that most of the accepted points correspond to this regime, which is also the limit where \( m_{A^0} \gg m_h \). Recent results from L3 at LEP have increased the experimental lower limit on the Higgs mass to exclude many of the points shown in figure 4(a): the limit at \( \sin^2(\alpha - \beta) = 1 \) is now \( m_h > 95.5 \text{ GeV} \).

![Figure 4: Higgs field alignment parameter \( \sin^2(\alpha - \beta) \) versus (a) \( m_h \) and (b) \( m_{A^0} \), in GeV.](image)

In figure 5 we show how \( m_h \) and \( \tan \beta \) depend upon the mass of the heavy (left) stop, \( m_Q \). This dependence comes about because of the radiative corrections to \( m_h \), which are roughly proportional to \( y_t^2 m_t^2 \ln(m_{t_R} \sim m_Q/m_t^2) \). To insure that \( m_h \) exceeds the experimental limit, one can go to large \( m_Q \), or alternatively one can make \( \tan \beta \) large, which increases the tree-level contribution to \( m_h \). The resulting upper limit on \( m_h \) or lower limit on \( \tan \beta \) can be expressed in terms of \( \hat{m}_Q \equiv m_Q/(100 \text{ GeV}) \) as

\[
\begin{align*}
m_h &< 85.8 + 9.2 \ln(\hat{m}_Q) \text{ GeV} \\
\tan \beta &> (0.03 + 0.076 \hat{m}_Q - 0.0031 \hat{m}_Q^2)^{-1}
\end{align*}
\]

Although it would seem rather fine-tuned to have \( m_Q \gg m_{t_R} \), this logical possibility provides a means of pushing \( m_h \) just outside of the experimental limits at the present and even after the end of the current LEP2 run.
Another quantity of interest is the deviation of $\tan \beta \equiv H_2/H_1$ from constancy as a function of distance along the bubble wall, since some contributions to the baryon asymmetry are suppressed by the change in $\beta$, $\Delta \beta$. Figure 6(a) shows that in the large $m_{A_0}$ limit, $\Delta \beta$ is suppressed, although two-loop effects do enhance it. Figure 6(b) is the distribution of $\Delta \beta$ values (which we define somewhat differently than ref. [14]) obtained from our Monte Carlo.
3 CP Violation and Transport at the Bubble Wall

Although the strength of the EWPT in the MSSM is well understood, there is less agreement about how to compute the size of the baryon asymmetry. The standard approach is to solve a set of diffusion equations for the density or chemical potential $\mu_L$ of the left-handed fermions which bias sphalerons. The baryon asymmetry is related to the sphaleron rate, the bubble wall velocity, and the integral of this chemical potential in front of the wall, by

$$n_B \sim \frac{\Gamma_{\text{sph}}}{v_w} \int_0^\infty \mu_L(x) \, dx \, .$$  

(8)

The problem is how to compute the CP-violating source term in the diffusion equations for $\mu_L$. Ref. [15] uses a closed-time-path, nonequilibrium quantum field theory method, however ref. [18] makes some approximations that have been questioned. In ref. [17] we instead adapted a formalism in which the semiclassical CP-violating force exerted by the wall on the charginos is taken into account, using the WKB approximation. This gives a force term in the Boltzmann equation which is the leading effect in an expansion in derivatives of the bubble wall.

Both methods give qualitatively similar results: it is the charginos which are primarily responsible for the CP asymmetry, because of the CP violating phase $\text{Im}(m_2 \mu)$ of the parameters in the Wino-Higgsino Dirac mass matrix,

$$M_{\tilde{\chi}} = \begin{pmatrix} m_2 & gH_2/\sqrt{2} \\ gH_1/\sqrt{2} & \mu \end{pmatrix} \, .$$  

(9)

And they both, roughly speaking, predict a larger baryon asymmetry when $m_2 \approx \mu$. But quantitatively the two methods disagree; for example they obtain different dependences on the wall velocity, as demonstrated in figures 7 and 8.

Figure 7: Contours of $\eta_{10}$, the baryon to photon ratio times $10^{10}$, in the $m_2$-$\mu$ plane, for three different wall velocities, using the classical force analysis of ref. [17]. Masses in GeV.
A second discrepancy between references [15] and [17] is that the WKB analysis shows no suppression in the result proportional to $\Delta \beta$ (discussed in the previous section), whereas ref. [15]'s result is proportional to $\Delta \beta$. A third difference is that we do find that $\mu_L$ is suppressed by helicity-flipping interactions of Higgsinos in front of the wall, involving the $\mu$ term of eq. (9), while ref. [15] does not. For the moment these disagreements remain unresolved.

4 Was SU(3)$_\text{color}$ broken just before the EWPT?

We have seen that the right stop must be very light for electroweak baryogenesis to work in the MSSM. This leads to the interesting possibility that the SU(3) gauge group of QCD was temporarily broken before the EWPT. Because the right-stop soft-breaking mass $m_{\tilde{t}_R}^2$ is negative in eq. (9), the symmetric vacuum is unstable toward condensation of the stop field in some random direction in color space. If $m_{\tilde{t}_R}$ is sufficiently negative, it can be energetically preferable to have a period of color breaking before the normal electroweak vacuum state takes over. Then one could have the sequence

$$ (H, \tilde{t}_R) = (0, 0) \rightarrow (0, v_1) \rightarrow (v_h, 0) $$

(10)

of phase transitions, which would change our view of cosmological history in an interesting way. For example, the second transition tends to be very strong, which is favorable for baryogenesis. But there is an energy barrier, shown in figure 9(a), which impedes this second stage of the transition, due to a term in the potential

$$ y^2 |H_2|^2 |\tilde{t}_R|^2 $$

(11)

whose positivity (provided squark mixing is not too strong) is a consequence of supersymmetry. It could happen that the rate of tunneling from the color-
broken to the electroweak phase is so small that it will never happen in the history of the universe. Guy Moore has made the following conjecture: if $m_{\tilde{t}}$ is ever small enough for transition 1 to take place, then the universe gets stuck in the color broken phase and never completes transition 2. Although preliminary studies of this question have been done, it deserves a more careful treatment.

We have undertaken such a study, by constructing the two-loop effective potential $V(H, \tilde{t}_R)$ for the Higgs and stop fields, and computing the nucleation rate for the most likely bubbles interpolating between the color-broken and electroweak phases. This involves finding the path in the $(H, \tilde{t}_R)$ field space along which the bubble evolves, which gives the lowest bubble energy, hence the fastest rate of transitions. The field equations with boundary conditions $(0, v_1)$ and $(v_h, 0)$ at the respective ends must be solved along this path. One needs a value of bubble energy over temperature smaller than $E/T \approx 170$ to get a tunneling rate per unit volume, $\Gamma \sim T^4 e^{-E/T}$, that is competitive with the Hubble rate per Hubble volume, $H^4$. That is, $E/T$ must be less than $4 \ln(T/H)$. However, we find that even when all MSSM parameters are adjusted to the values that are optimal for tunneling, the exponent $E/T$ is too large by a factor of 5. This allows one to rule out color breaking at the EWPT. Any resulting constraints must be made in a high-dimensional space.

\[ \text{Figure 9: (a) The two-loop effective potential } V(H, \tilde{t}_R) \text{ for the Higgs field (right axis) and right stop (left axis), at the temperature } T = 50 \text{ GeV where the tunneling rate from the metastable CCB minimum to the electroweak vacuum is maximized. (b) Lower limit on } m_{\tilde{t}} \text{ to avoid color breaking, as a function of } (\Delta_t - \mu \cot \beta)^2 / m_{\tilde{t}}^2, \text{ for two values of } m_h. \]

\[ ^a\text{and ref. [21], as was brought to our attention} \]
of parameters. As an example we show the lower limit on the right stop mass as a function of the squark mixing parameter $\tilde{A}/m_Q$ in figure 9(b).

It therefore appears that one must go beyond the MSSM in order to make color-breaking a real possibility just before the electroweak phase transition. However it may be possible to change this conclusion in extended models, such as those with $R$-parity breaking, if one can arrange for large negative corrections to the Debye mass of the light stop.

5 References

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