Depairing currents in the superconductor/ferromagnet proximity system Nb/Fe

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Abstract

We have investigated the behaviour of the depairing current \( J_{dp} \) in ferromagnet/superconductor/ferromagnet (F/S/F) trilayers as function of the thickness \( d_s \) of the superconducting layers. Theoretically, \( J_{dp} \) depends on the superconducting order parameter or the pair density function, which is not homogeneous across the film due to the proximity effect. We use a proximity effect model with two parameters (proximity strength and interface transparency), which can also describe the dependence of the superconducting transition temperature \( T_c \) on \( d_s \). We compare the computations with the experimentally determined zero-field critical current \( J_c(0) \) of small strips (typically 5 µm wide) of Fe/Nb/Fe trilayers with varying thickness \( d_{Nb} \) of the Nb layer. Near \( T_c \) the temperature dependence \( J_c(0)(T) \) is in good agreement with the expected behaviour, which allows extrapolation to \( T = 0 \). Both the absolute values of \( J_c(0)(0) \) and the dependence on \( d_{Nb} \) agree with the expectations for the depairing current. We conclude that \( J_{dp} \) is correctly determined, notwithstanding the fact that the strip width is larger than both the superconducting penetration depth and the superconducting coherence length, and that \( J_{dp}(d_s) \) is correctly described by the model.

I Introduction

A still relatively little explored area of research in non-equilibrium superconductivity concerns phenomena involving spin-polarized quasiparticles. Pioneering work on spin-polarized tunneling in conventional s-wave superconductors was performed by Meservey and Tedrow [1], who studied different ferromagnets (F) in F/Al₂O₃Al tunnel junctions and found that the tunnel current can show varying degrees of spin polarization. More recently, experiments were performed by different groups in order to establish whether superconductivity can be suppressed by injecting spin-polarized quasiparticles [2–4]. In these cases the combinations existed of a d-wave high-\( T_c \) superconductor (XBa₂Cu₃O₇, with \( X = Y, Dy \)) and a fully spin-polarized ferromagnetic manganite (\( A_{0.67}B_{0.33}MnO_3 \) with \( A = La, Nd \) and \( B = Ca, Sr \)), either with or without a barrier of a different oxide; measured was the change in the zero-field critical current density \( J_c(0) \) of the superconducting films upon applying a current bias through the ferromagnet. The results are not fully conclusive, and certainly not quantitative. Although generally a suppression of \( J_c(0) \) was observed, heating effects could not always be fully ruled out since the manganites are highly resistive metals (see the discussion in ref. [4]), and the geometry did not always allow to determine the area of the current injection, and therefore the injected current density. Moreover, since \( J_c(0) \) in high \( T_c \) superconductors generally is not the depairing
current $J_{dp}$ but involves flux motion, $J_{cd}$ is not a direct measure for the amount of depression of the superconducting order parameter. Similar experiments have to our knowledge not been performed with combinations of conventional metals, although that would have some clear advantages. The interpretation of results would not be complicated by e.g. inhomogeneous currents in the ferromagnet or anisotropic gaps in the superconductor; lithographic techniques could be brought to bear in order to have well defined superconducting bridges and injection contacts; and it should be possible to identify the effects of the spin-polarized quasiparticles on $J_{dp}$.

Still, two points deserve special interest. The first is that, in planning such an F/I/S experiment, there is the potential problem of insufficient knowledge of the tunneling process. This was already apparent in the work of Meservey and Tedrow cited above since the experiments always showed a positive sign for the spin polarization, even in the cases of e.g. Co and Ni where a negative sign was expected. Recently, this was explained by demonstrating that the choice of barrier material can strongly influence and even reverse the spin polarization of the tunneling current, with obvious consequences for the interpretation of the injection experiments. It may be advantageous to also contemplate an (F or N)/I/F/S configuration: in this case the barrier is only used to increase the energy of the electrons coming from an N or F contact, while the polarization now takes place in a thin F layer between barrier and superconductor. The disadvantage here is that the F layer in connection with the superconductor will suppress the order parameter and therefore $J_{dp}$ in the S layer. Still, since the proximity effect (which gives rise to the depression) for S/F systems is understood reasonably well, the effect on $J_{dp}$ may also be quantifiable. The second point for consideration is that even in the case of conventional superconductors the determination of $J_{dp}$ need not be straightforward. The difficulty lies in the fact that the superconducting bridge must have a width of no more than both the superconducting penetration depth $\lambda$ and the superconducting coherence length $\xi$. The first is needed to avoid current pile-up near the edges (as a consequence of screening of the self-field), the second is required in order to avoid vortex nucleation and flow, which gives rise to dissipation before $J_{dp}$ is reached. These conditions can be met e.g. for Al, which has a Bardeen-Cooper-Schrieffer (BCS) coherence length $\xi_0$ of about 1.5 $\mu$m, while $\lambda$ can also be made of the order of 1 $\mu$m by making the film thin enough. For Al-bridges of less than 1 $\mu$m wide it was shown by Romijn et al. that the measured $J_{dp}$ agreed very well with the theoretical calculations by Kuprianov and Lukichev based on the Eilenberger equations and therefore valid in the whole temperature regime below $T_c$.

For a material such as Nb, with $\xi_0$ and $\lambda$ of the order of 50 nm, such agreement need not be expected.

In this paper we show that, at least close to $T_c$, the values of the zero-field critical current $J_{cd}$ measured on bridge-structured Nb samples are essentially the values expected for the depairing current. Furthermore, we measure the depression of $J_{cd}$ in trilayers of Fe/Nb/Fe as function of the thickness $d_{Nb}$ of the Nb layer. We compare the behaviour of $J_{cd}(d_{Nb})$ with the behaviour of $T_c(d_{Nb})$, and also with calculations of the proximity effect and the pairbreaking velocity using a two-parameter formalism based on the Usadel equations. We find that $J_{cd}(d_{Nb})$ is well described by the same two parameters which describe the behaviour of $T_c(d_{Nb})$. The conclusion is that the suppression of the depairing current as a consequence of the depression of the order parameter in S/F structures can be well described by proximity effect theory, making (F,N)/I/F/S injection experiments a distinct possibility.

### II Depairing current; theory

Close to $T_c$, the classical Ginzburg-Landau (GL) result for the temperature dependence of the depairing current of a thin film, under the assumption of a homogeneous superconducting order parameter over the film thickness, is given by:

$$J_{dp}^{GL}(t) = J_{dp}^{GL}(0) (1 - t)^{3/2},$$  \hspace{1cm} (1)

with $t = T/T_c$. The prefactor $J_{dp}$ is of the order of $H_c/\lambda$, with $H_c$ the thermodynamic critical field, and will be given more precisely below. For arbitrary temperatures, calculations were performed by Kupriyanov and Lukichev, who essentially solved...
the Eilenberger equations for a superconductor carrying a current, with the velocity of the condensate leading to a phase gradient \[ \nabla \times \mathbf{v} = \gamma (\rho_d) \mathbf{J} \]. Their results recover the GL behaviour near \( T_c \):

\[
J_{dp}^{GL}(t) = \frac{16}{9 \sqrt{\zeta(3)}} \chi(\rho_d) [eN(0) v_F k_BT_c] \times (1 - t)^{3/2}.
\]

(2)

Here, the constants have their usual meaning, \( N(0) \) is the density of states at the Fermi level per spin direction, and \( \chi(\rho_d) \) is the G’orkov function controlled by the ‘dirt parameter’ \( \rho_d = (\hbar v_F)/(2\pi k_BT_c \ell_e) \), with \( \ell_e \) the electronic mean free path. In the dirty limit, \( (\rho_d \to \infty) \chi(\rho_d) \to 1.33 \ell_e/\xi_0 \), this becomes:

\[
J_{dp}^{GL}(0) = 1.26 [eN(0) v_F \Delta(0)] \sqrt{\frac{\ell_e}{\xi_0}}
\]

(3)

which is equivalent to the expression given by Romijn et al. \[ \cite{6} \] :

\[
J_{dp}^{GL}(0) = \frac{16\pi^2}{63\zeta(3)} [eN(0) v_F k_BT_c] \sqrt{\frac{k_BT_c \ell_e}{\hbar v_F}},
\]

(4)

which can also be written in terms of experimental parameters as

\[
J_{dp}^{GL}(0) = 7.84 \left[ \frac{(k_BT_c)^3}{e^2 \hbar v_F (\rho_e/\rho)} \right]^{1/2}.
\]

(5)

This way of writing also emphasizes the proportionality \( J_{dp}^{GL}(0) \propto \sqrt{1/\rho} \), since the product \( \rho_e \ell \) is a materials constant. At low temperatures the value of \( J_{dp} \) saturates, reaching a zero-temperature value of:

\[
J_{dp}(0) = 1.491 eN(0) \sqrt{D/\hbar} \Delta^{3/2}(0) = 0.486 [eN(0) v_F \Delta(0)] \sqrt{\ell_e/\xi_0},
\]

(6)

with \( D = 1/3v_F \ell_e \) the diffusion constant. Comparison with eq. \[ \cite{6} \] shows that the ratio between the saturation value and the GL-extrapolated value equals \( J_{dp}(0)/J_{dp}^{GL}(0) = 0.385 \).

In the case of F/S (or N/S) multilayers, the superconducting order parameter is depressed near the interfaces, and this has to be taken into account in calculating \( J_{dp} \). For this we use the proximity effect model, based on the Usadel equations (dirty limit conditions), which was also used for calculating the depression of \( T_c \) with decreasing thickness of the superconductor \[ \cite{8} \]. Details will be given in Appendix A but here we briefly introduce the main parameters of the theory. In principle, the shape of the order parameter on both sides of the interface depends on the bulk transition temperature \( T_{cb} \), on the coherence lengths \( \xi_{S,F} \), on the normal state resistivities \( \rho_{S,F} \), and on the transparency \( T \) of the interface. From the boundary conditions for the order parameter (see eqs. \[ \cite{4} \]) it follows that, apart from \( T_{cb} \), only two independent parameters are needed, the proximity strength parameter \( \gamma \) and the transparency parameter \( \gamma_b \).

The value of \( \gamma = (\xi_S \rho_S)/(\xi_F \rho_F) \) can be fully determined from experiment; the only free parameter is \( \gamma_b \) (\( 0 \leq \gamma_b \leq \infty \)), which is approximately connected to the transparency \( T \) (with \( 0 \leq T \leq 1 \)) by

\[
T = \frac{1}{1 + \gamma_b}
\]

(7)

As was shown in ref. \[ \cite{8} \], in F/S systems, \( T \) can be quite low for a high magnetic moment in the F-layer, which is presumably due to the suppression of Andreev reflections by the exchange splitting of the spin subbands. Fig. \[ \cite{9} \] gives the results of some typical calculations, performed for the system Fe/Nb/Fe with the appropriate proximity effect parameters \( \gamma = 34.6 \) and \( \gamma_b = 42 \) (see section \[ \cite{4} \]). Shown is \( J_{dp}(t) \) for two different thicknesses \( (d_S/\xi_S = 20, 7.5) \), normalised on the bulk value \( J_{dp}^{bulk}(0) \) as given by eq. \[ \cite{1} \]. Note that this involves a factor \( (T_c/T_{cb})^{3/2} \). The thickness dependence of \( T_c \) and the normalised depairing current at \( T = 0 \) (see the insert of Fig. \[ \cite{9} \]) are quite different, with a much stronger depression of the depairing current at relatively high thickness of the superconductor. This can be qualitatively understood by noting that \( T_c \) is a measure for the maximum value of the superconducting order parameter in the layer, while the depairing current comes from an average over the layer thickness, which also involves lower values of the order parameter.
Figure 1: The temperature dependence of the normalised depairing current $J_{dp}(t)/J_{dp}^{\text{bulk}}(0)$ of an F/S/F trilayer for S-layer thicknesses $d_S/\xi_S = 20$ (upper), 7.5 (lower). Parameters typical for Fe/Nb/Fe were used, namely $\gamma = 34.6$ and $\gamma_b = 42$. Insert: thickness dependences of the normalised depairing current at $T = 0$, and of $T_c$. The black dots are at $d_S/\xi_S = 20, 7.5$.

III Experimental

Samples were grown on Si(100) substrates, by DC sputtering in a system with a base pressure of $10^{-9}$ mbar in an Ar-pressure of $6 \times 10^{-3}$ mbar. Sputtering rates were of the order 0.1 nm/s for Nb and 0.03 nm/s for Fe. One series of samples consisted of trilayers Nb/Fe/Nb with Nb thickness $d_{Nb} = 5$ nm and the Fe thickness $d_{Fe}$ varying between 2 nm and 25 nm. These were used to determine the magnetization $M_{Fe}$ of the Fe layers in the presence of Fe/Nb interfaces with a commercial SQUID-based magnetometer. The behavior of $M_{Fe}$ versus $d_{Fe}$ could be well described with a straight line, yielding a magnetic moment per Fe atom of 2.36 $\mu_B$ ($\mu_B$ being the Bohr magneton), slightly above the bulk value of 2.2 $\mu_B$ and a magnetically dead layer per interface of 0.1 nm. This value is somewhat lower than reported for MBE-grown samples [4,10]. Two other series of samples consisted of trilayers of Fe/Nb/Fe with $d_{Fe} = 5$ nm and varying $d_{Nb}$. One set was structured by Ar-ion etching into strips with a width $w = 100 \mu m$, the other into strips with a width $w = 6 \mu m$, or sometimes 10 $\mu m$ or 20 $\mu m$. In both cases the length between the voltage contacts was 1 mm. The first set (deposited in two different runs) was used for measuring $T_c(d_{Nb})$, the second set for both $T_c(d_{Nb})$ and $J_c(d_{Nb})$. In all cases, the typical width of the resistive transitions to the superconducting state was 50 mK.

Also measured were single films of Fe and Nb with different strip widths in order to establish values for the specific resistivity $\rho_{Fe,Nb}$ (at 10 K), for $T_c$ and for the upper critical field $B_{c2}(T)$. On average, we find $\rho_{Fe} \approx 7.5 \mu \Omega cm$, $\rho_{Nb} \approx 3.7 \mu \Omega cm$, $T_c = 9$ K and $S = -dB_{c2}/dT = 0.24$ T/K, yielding $\xi_{GL}(0) = \sqrt{\Phi_0/(2\pi ST_c)} = 12.2$ nm. This corresponds to $\xi_S = 7.8$ nm. No special precautions were taken to shield residual magnetic fields. The zero-field critical current $I_c$ was determined at different temperatures $T$ by measuring current ($I$)-voltage ($V$) characteristics. For this, a DC current was switched on for the time of the order of 1 s and the voltage recorded, to prevent heating via the contacts. All samples showed a clear transition from the superconducting to the normal state, with a large and almost instantaneous increase in voltage at $I_c$. Upon detecting this rise, the current was also turned off since the sample then started to heat immediately. Most samples also showed a small rise in voltage prior to the major transition, probably due to vortex motion. We shall come back to this point in the discussion. Important for the theoretically expected value of $J_{dp}(0)$ is the value of the resistivity of the superconducting layer (see Eq. 4). This value, $\rho_{Nb}$, was extracted from the normal state resistance $R_n$ at 10 K of the patterned samples by assuming that the Nb layer and the 10 nm thick Fe layer ($\rho_{Fe} = 7.5 \mu \Omega cm$) contribute as parallel resistors. The resulting values for $\rho_{Nb}$ are given in Table I together with the strip width $w$ and $T_c$. The values for the thinner films (around 50 nm) are somewhat larger than what we usually find for single Nb films, and approach that value for the thick films.

IV Results

Fig. 3 shows the measured values for $T_c(d_{Nb})$ for both sample sets, with the the two types of open symbols denoting the two deposition runs for that set, and the solid symbols denoting the samples used for measuring $J_{dp}$. The overall data spread
Table 1: Parameters of the Fe/Nb/Fe samples and the single Nb film used for the critical current measurements. Given are the thickness of the Nb layer $d_{Nb}$, the strip width $w$, the critical temperature $T_c$, the calculated specific resistance of the Nb layer $\rho_{Nb}$ and the Ginzburg-Landau extrapolated critical current at zero temperature $J_{GL}^c(0)$.

| type | $d_{Nb}$ [nm] | $w$ [µm] | $T_c$ [K] | $\rho_{Nb}$ [µΩcm] | $J_{GL}^c(0)$ [10^{11} A/m²] |
|------|---------------|----------|----------|---------------------|-----------------------------|
| F/S/F | 36            | 6        | 3.63     | 5.97                | 0.522                       |
| F/S/F | 40            | 6        | 4.36     | 6.51                | 1.55                        |
| F/S/F | 42            | 10       | 5.07     | 10.4                | 1.58                        |
| F/S/F | 53            | 10       | 5.62     | 8.08                | 2.64                        |
| F/S/F | 60            | 6        | 6.63     | 5.03                | 3.46                        |
| F/S/F | 75            | 6        | 7.34     | 4.95                | 6.14                        |
| F/S/F | 100           | 6        | 8.05     | 4.58                | 6.86                        |
| F/S/F | 150           | 6        | 8.61     | 3.94                | 11.2                        |
| Nb    | 53            | 20       | 9.00     | 7.24                | 15.1                        |

Figure 2: $T_c$ of the different sets of Fe/Nb/Fe trilayers. The solid symbols denote the samples used for the critical current measurements. The line shows the theoretical dependence $T_c(d_s)$ for the parameter values $\gamma = 34.6$ and $\gamma_b = 42$.

Figure 3: Experimentally determined critical current density $J_{c0}$ versus reduced temperature $t = T/T_c$ for the Fe/Nb/Fe trilayers with $d_{Nb} = 42$ nm (triangles), 60 nm (solid diamonds) and 75 nm (open squares). The solid and open symbols for $d_{Nb} = 42$ nm corresponds to measurements with non-pumped and pumped He-bath respectively.
Different strip widths between 0.1 µm and 10 µm of the Nb-layer, as indicated. The drawn line indicates the GL-behavior, the dotted line is the result of the full calculation.

$[J_{\alpha}(t)/J_{\alpha}^{GL}(0)]^{2/3}$ versus $t$ together with the line $1 - t$ (the GL-behavior) and the result of the full theoretical calculation, which is now independent of the parameters. All data collapse on the universal curve above $t = 0.9$. At lower temperatures, the thinnest films ($d_s = 36, 40, 42, 53$ nm) follow the full calculation quite closely, even down to $t \approx 0.6$. The difference between the data of 36 nm and 40 nm is mainly due to the choice of the normalization value, and reflects the accuracy of that determination. For thicker films the first deviation progressively shifts to higher $t$.

V Discussion

The first point to be discussed is whether the measured values of $J_{\alpha}$ agree with the theoretical estimates for $J_{dp}$. The absolute value of $J_{dp}^{GL}(0)$ can be calculated with Eq. [8]. The materials constants for Nb are well documented [1]: we use the values $v_F = 5.6 \times 10^5 \text{ m/s}$ and $\rho \ell = 3.75 \times 10^{-16} \Omega \text{m}^2$. Eq. [8] then yields for the Nb film $J_{dp,Nb}^{GL}(0) = 1.70 \times 10^{12} \text{ A/m}^2$, which is quite close to the experimentally determined value of $J_{\alpha,Nb}^{GL}(0) = 1.5 \times 10^{12} \text{ A/m}^2$ (see Table [1]). It is also in good correspondence with the data presented by Ando et al. [2] on films with a thickness of 100 nm and different strip widths between 0.1 µm and 10 µm, who found a fitted value $J_{dp,Nb}^{GL}(0) = 1.26 \times 10^{12} \text{ A/m}^2$. It appears that the depairing current is directly probed by the measurement of $J_{\alpha}$.

Next we consider the dependence of $J_{\alpha}^{GL}(0)$ on the superconducting Nb layer thickness $d_{Nb}$. As Eq. [9] shows, $J_{dp}(0)$ is proportional to $\sqrt{T_{\alpha}/\rho_{Nb}}$. Since $\rho_{Nb}$ of the samples differs, this leads to some variation in the expected value for $J_{dp}(0)$ which can be taken into account by multiplying $J_{\alpha}^{GL}(0)$ by $\rho_{Nb}^{1/2}$. Normalizing this value to the single Nb film yields the dependence on $d_{Nb}$ as shown in Fig. [9]. $J_{\alpha,Nb}^{GL}(0)$ in the trilayers is clearly reduced with respect to the bulk Nb value and increases with increasing $d_{Nb}$, but much more slowly than $T_{\alpha}$ does. The correspondence with theory is good at low $d_{Nb}$, with some deviations above $d_{Nb} \approx 75$ nm. This coincides with the findings on the temperature dependence of $J_{\alpha}(t)$, shown in Fig. [8]: for small $d_{Nb}$ there are only small deviations in the whole measured temperature regime, for large $d_{Nb}$ the deviations are large below $t = 0.9$. This suggests that at high $d_{Nb}$ the extrapolation for $J_{\alpha,Nb}^{GL}(0)$ leads to somewhat underestimated values. In essence, we conclude that the model used to describe the depression of $T_{\alpha}$ in F/S/S trilayers also adequately describes the behavior of $J_{dp}$.

This leaves one final point to be addressed. In order to determine the depairing current it is usually...
understood that two conditions have to be fulfilled [13]: the current has to be distributed uniformly over the strip, and the width \( w \) should be small enough to preclude vortex formation and motion. In terms of penetration depth \( \lambda(t) \), strip thickness \( d_s \) and Ginzburg-Landau coherence length \( \xi(t) \) this means:

\[
\begin{align*}
  w < \lambda_{eff}(t) &= \lambda(t), & d_s < \lambda_{eff}(t) \\
  \lambda^2(t)/d_s, & & d_s > \lambda_{eff}(t) \\
  w < 4.4\xi(t) &= 4.4\xi(0)/(1-t)^{1/2}.
\end{align*}
\]

(8)

Estimating \( \lambda(0) \) from \( \lambda(0) = 1.05 \times 10^{-3} \sqrt{\rho_0/T_c} \) we find it ranges between 67 and 113 nm. Both conditions mean for all samples \( 1 - t < 10^{-4} \), much smaller than the region where \( J_{d}(t) \propto (1-t)^{3/2} \), (Fig. 3), and the question is valid whether the current is uniform, as has implicitly been assumed in the analysis.

Qualitatively, current is expected to pile up at the edges of the strip in order to minimize the self-field inside. The edge current will then sooner reach the edges, but is kept out of the rest of the sample by the field penetrates over a distance \( d/2 \) from the edges, and is kept out of the rest of the sample by the screening current. Then, the current within \( d/2 \) from the edges can be set equal to \( J_{dGL}(0) \) and beyond \( d/2 \) it decreases according to Eq. [1]. The following calculation can be done for the Nb film.

The transport current \( I_T \) in the screened part of the strip can be calculated from Eq. [8] by using

\[
J(x = W - d/2) = J_{dGL}(0).
\]

(10)

The total current \( I \) including the edges is given by:

\[
I/d = 2d/2J_{dGL}(0) + \int_{-W+d/2}^{W-d/2} \frac{I_T}{\pi d \sqrt{W^2 - x^2}} dx
\]

\[
= dJ_{dGL}(0) \times \left(1 + \frac{w}{2d} \left(\sin^{-1}\frac{x}{W}\right)^{W-d/2} \right)
\]

(11)

The ratio \( I/(wd) / J_{dGL}(0) \), which can be calculated from Eq. [1], gives the fraction of \( J_{dGL}(0) \) which would be actually measured as the critical current under the given current distribution, where the depairing current is reached at the edges. It can be easily seen that it equals 1 when the current is uniform. For the Nb film with \( w \) and \( d \) as given in Table 3, Eq. [1] yields a fraction of 0.11, an order of magnitude below what is actually measured. The conclusion is that \( J(x) \) is much more uniformly distributed than might be expected. The reason is probably that a field and moving vortices exist in the film, indicated by a voltage onset below the jump to the normal state. This breaks up the Meissner state, causes a much more uniform current distribution and allows the correct determination of the depairing current over a much larger region than expected on the basis of the condition \( w < \lambda, \xi \). Still, the deviations of \( J_{d0}(t) \) compared to the theoretical behaviour at higher \( d_{Nb} \) in the Fe/Nb/Fe trilayers may be due to the low value of \( \lambda \) and inhomogeneities in the current distribution at these high thicknesses. At low thicknesses, there are two effects which increase \( \lambda \) above the bulk value. Firstly, for \( d_s < \lambda \) (around \( d_{Nb} \approx 75 \) nm) the effective penetration depth increases according to \( \lambda_{eff} = \lambda(0)^2/d_s \), and can become significantly higher than \( \lambda(0) \). Secondly, the suppression of the order parameter as measured by the decrease of \( T_c/T_{c0} \) results in a higher value for \( \lambda(0) \). From that point of view the full agreement between the measured and calculated values of \( J_{d0}(t) \) at the lowest thicknesses is not surprising.

VI Summary

In this paper we have addressed the question of the value of the superconducting depairing current in F/S/F trilayers with varying \( d_s \), where the superconducting order parameter is inhomogeneously
suppressed by the pair breaking in the F-layers. The same model which is adequately describes the suppression of \(T_c\) with decreasing \(d_s\) with two parameters (proximity strength \(\gamma\) and interface transparency \(\gamma_b\) or \(T\)) can also be used to compute the suppression of the depairing current. Measurements of the zero-field critical current \(J_{c0}\) (as defined by the current where the resistance jumps to the normal state value) in thin strips of Fe/Nb/Fe show that the temperature dependence near \(T_c\) is as expected for the depairing current. Also the absolute value of \(J_{c0}\) of single Nb films is close to the theoretically expected value and the measured suppression of \(J_{c0}\) in the trilayers follows the calculated behaviour. We conclude that the current distribution is homogeneous and that the depairing current is measured, even though the strip widths are larger that the superconducting penetration depth and coherence length. Also, the proximity effect model correctly describes the shape of the order parameter, at least in the superconducting layer. These findings can be of use in experiments on the effect of injecting polarized quasiparticles.

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**Appendix A : calculation of \(J_{dp}\)**

We assume that the dirty limit conditions are fulfilled in both S and F layers, so that the F/S bilayer can be described by the Usadel equations. In the absence of a depairing current in the S layer, and in the regime of large exchange energy in the ferromagnet \(\left(E_{ex} \gg k_BT_c\right)\) these equations were discussed extensively by Buzdin et al. [13] (see also Demler et al. [14]). Here we rewrite these equation in \(\theta\)-parametrization \((F = \sin \theta, G = \cos \theta)\) and include the pair-breaking effects by current along the S film:

\[
\xi_S^2 \frac{d^2}{dz^2} \theta_S(z) - \tilde{\omega} \sin \theta_S(z) + \Delta(z) \cos \theta_S(z) = 0, \quad (A1)
\]

\[
\xi_F^2 \frac{d^2}{dz^2} \theta_F(z) - i \sin \theta_F(z) = 0, \quad (A2)
\]

\[
\Delta \ln(T/T_c) + \pi T \sum_{\omega_n} \left( \frac{\Delta}{|\omega_n| - \sin \theta_S} \right) = 0, \quad (A3)
\]

where \(\omega_n = \pi(2n+1)T/T_c\) is the normalized Matsubara frequency, \(\tilde{\omega} = |\omega_n| + Q^2 \cos \theta(z), \Delta\) is the pair potential in a superconductor normalized to \(\pi T_c\), \(\xi_S = (\hbar D_S/2\pi T_c)^{1/2}\), \(\xi_F = (\hbar D_F/2E_{ex})^{1/2}\) and \(D_{F,S}\) are the coherence lengths and the electronic diffusion coefficients in F and S metals. Moreover, \(Q = \xi_F \partial \chi/\partial x\) is the normalized gradient-invariant superfluid velocity in the \(x\)-direction, with \(\chi\) the phase of the pair potential \(\Delta\). There are two sources of pair-breaking in the problem, the volume one by the current and the surface one by the ferromagnet. The latter is described by the boundary conditions at the FS interface \((z = 0)\):

\[
\xi_S \frac{d}{dz} \theta_S = \gamma \xi_F \frac{d}{dz} \theta_F \quad (A4)
\]

\[
\gamma_b \xi_F \frac{d}{dz} \theta_F = \sin(\theta_S - \theta_F) \quad (A5)
\]

where the parameter \(\gamma = \rho_S \xi_S/\rho_F \xi_F\) describes the strength of the suppression of superconductivity in S by the ferromagnet.

The parameter \(\gamma_b\) describes the effect of boundary transparency (coupling strength) between the layers. In the NS case, when the decoupling is due to the presence of an additional potential barrier at the interface, \(\gamma_b = R_B/\rho_F \xi_F\), with \(R_B\) the normal state resistance of the N/S interface [7]. In the F/S bilayer there is no general microscopic derivation for \(\gamma_b\), combining the effect of exchange splitting and an additional interface barrier. A simple estimate is still possible, when the exchange splitting is the main cause for intransparency [6]. Then \(\gamma_b = (2/3)(l_F/\xi_F) < (1 - T_A)/T_A\) where \(T_A\) is the transmission probability of scattering between the majority and minority spin subbands, i.e. the probability of Andreev reflection. This process
is implicitly described by the boundary condition 
\[ \gamma_b \xi_F \frac{\partial \theta_F}{\partial z} = \sin(\theta_S - \theta_F) \] 
since \( \theta_F \) is off-diagonal in spin indices. Here the brackets \( \langle ... \rangle \) 
denote the Fermi surface averaging, which is generally proportional 
to the overlap area of the projections of different spin subbands onto the contact plane [18,19]. 
As a result, \( T_A \) drops roughly linearly (for spherical Fermi surfaces) as a function of \( E_{ex} \), both for ballisitic and diffusive interfaces [20]. 
The supercurrent density is given by 
\[ J_s(z, Q) = \frac{2\pi \sigma}{e} QT \sum_{\omega_n} \sin^2 \theta_s . \quad (A6) \]

Since the superconducting pair potential \( \Delta \) and the Green’s function \( \theta_s \) are suppressed by the superflow \( Q \), the dependence \( J_s(Q) \) must be found self-consistently. 
In the well known spatially homogeneous case [21] the function \( J_s(Q) \) behaves non-monotonously: the supercurrent \( J_s \) increases with \( Q \) at small \( Q \), then reaches a maximum and finally drops to zero, when \( \Delta \) is fully suppressed by current. 
The depairing current is defined as the maximum of \( J_s(Q) \). A similar situation holds in the spatially inhomogeneous case considered here, 
with the difference that the solutions for \( \theta(z) \) and \( \Delta(z) \) of the proximity effect problem (eqs. A1-A3) should be calculated selfconsistently for a given \( Q \) using the boundary conditions at the FS interface (eqs. A4, A5). 
This problem is solved numerically by the method applied previously to NS bilayers and described in detail in [22]. 
Then the local \( z \)-dependent supercurrent density \( J_s(z, Q) \) is calculated from eq.A6 by summing the solutions \( \sin^2 \theta_s \) over \( \omega_n \). 
Finally the density is averaged over film thickness \( J_s(Q) = d_s^{-1} \int_0^{d_s} J_s(z, Q) dz \) and the depairing current is found from the maximum of the dependence of \( \langle J_s \rangle \) on \( Q \).

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