Possible experiment for determination of the role of microscopic vortex rings in the \( \lambda \)-transition in He-II

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It is suggested that microscopic vortex rings (MVR) play an important role in the \( \lambda \)-transition in helium-II and substantially determine the value of \( T_\lambda \). For very thin films of He-II, with thickness \( d \) less than the size of the smallest MVR, the rings do not fit in and, therefore, do not exist in such films. Consequently, for superfluid films of He-II, a peculiarity in the form of a smoothed-out jump should be observed in the curve \( T_m(d) \) at the values of thickness approximately equal to the size of the smallest MVR, \( d \approx 6 \AA \pm 3 \AA \) (\( T_m \) is the temperature of the maximum of the broad peak on the curve of the dependence of the specific heat on temperature). The absence of a similar peculiarity will be an evidence that MVR do not influence the values of \( T_\lambda \) and \( T_m \), and do not play any key role in the \( \lambda \)-transition. The currently available experimental data are insufficient for revealing the predicted peculiarity.

1 INTRODUCTION

The microscopic nature of the \( \lambda \)-transition in He-II is still not quite clear. Most of the authors believe that the \( \lambda \)-transition is caused by the destruction of ODLRO and is accompanied by the exhaustion of the condensate, which probably has composite nature. The viewpoint according to which the microscopic vortex rings (MVR) play an important role in the \( \lambda \)-transition is also popular enough [1]–[5]. The latter idea was proposed about 50 years ago [1], but a role of MVR in the \( \lambda \)-transition is not clear until now. Here, by microscopic rings we understand vortex rings with radius \( R \leq 10 \AA \) and with quantized circulation \( \kappa = \hbar/m \) [6].

The superfluid (SF) transition in He-II films on disordered substrates is characterized by two temperatures, \( T_{KT} \) and \( T_m \) (always \( T_{KT} < T_m < T_\lambda \)). As the thickness of the SF layer of the film \( d \to \infty \), it is observed that \( T_{KT} \), \( T_m \to T_\lambda \), where \( T_\lambda \) is the temperature of the bulk \( \lambda \)-transition. The peculiarities at \( T \approx T_{KT} \) are caused by the dissociation of big pairs of the Kosterlitz–Thouless (KT) vortices [7]–[10], in particular, a narrow peak is observed at \( T \approx T_{KT} \) in the curve \( C(T) \) of the dependence of the specific heat on temperature [11,12]. \( T_m \) is the temperature of the maximum of the broad peak (BP) in \( C(T) \). This BP may be caused by the dissociation of small pairs of KT-vortices [10], or it may be a finite-size (FS) rounding of the \( \lambda \)-transition [11,13], or both; it depends on the substrate and on the value of \( d \), see below.

For thick films with \( d \gtrsim 21 \AA \), BP correspond to FS-rounded \( \lambda \)-transition [14]–[16]. At \( d \lesssim 21 \AA \) (for Nuclepore [14]), the deviation from the scaling law \( T_m - T_\lambda \sim d^{-1/\Theta} \) is observed (see Fig. 3 in [14]), which indicates the appearance of the contribution from the KT-vortices (VKT) to BP. With a decrease in \( d \), the contribution of VKT to BP increases. The contribution of VKT to the
specific heat, $C_{VKT}$, is proportional to the concentration of VKT, $N_{VKT}$ (as for MVR, see [17]), and $N_{VKT} \sim a^{-3}$ [7], where $a$ is the core radius of VKT. According to [18] [19], $a \approx (\hbar^2/2mU_0n^{(2)})^{1/2}$, where $U_0 = \int drU(r)$, and $U(r)$ includes the potential of the substrate. So we have, roughly,

$$C_{VKT} \sim N_{VKT} \sim (n^{(2)}U_0/d)^{3/2}.$$  

(1)

Apparently, the specific heat of the ensemble of VKT depends strongly on $d$ and on the substrate potential, and can differ by several orders for various substrates and values of $d$. It should be expected that $C_{VKT} \sim a^{-3}$ is highest at $d \approx 1$ a.l. (atomic layer, 3.6 Å), because $a$ is minimal at $d \gtrsim 1$ a.l. [19].

Thus, for various substrates and values of $d$, the following versions of BP in the curve $C(T)$ could be realized:

(I) a single BP caused by dissociation of small pairs of VKT [10]; this is typical for thin films with $d \lesssim 1$ a.l. (e.g., Millipore- and Anopore-films, $d \approx 1$–3 Å [13]; for these films BP decreases (in comparison with the background) with an increase in $d$, at $d > 2$ Å, which signifies the two-dimensional nature of the peak [13];

(II) a single BP due to rotons (R) and, perhaps, MVR, which means that such a BP is an FS-rounded $\lambda$-peak; this case is observed for thick films [14] [15] [20], e.g., for Nuclepore at $d \approx 21$ Å [14];

(III) a single broad peak (bump) resulting from both VKT and R+MVR, possible examples of which are Vycor-films at $d > 2$ Å [11] [21] [22] and Nuclepore at $d \approx 10$–20 Å [14, 23]; in this case BP grows and narrows with an increase in $d$, which manifests that BP is an effect of mainly bulk quasiparticles, rotons, and MVR;

(IV) two different BPs at a given $d$, one being caused by VKT and the other one being the FS-rounded $\lambda$-peak (this version was not observed until now, but it is possible at $d \sim 1$ a.l., perhaps, also for the substrates of [13]).

In our paper, we use the following notation: $T^*_m$ is the temperature of BP caused by VKT [10] [13] (with negligible contribution to BP from rotons and MVR); $T_m$ is the temperature of the maximum of BP resulting from R+MVR or both VKT and R+MVR. Thus, $T_m$ is the temperature of the FS (or FS+VKT) modified $\lambda$-peak [11] [13] [21] [22].

Below, we predict a possible jump, first of all, in the curve $T_m(d)$. A similar peculiarity could also exist in the curve $T^*_m(d)$ at $d \approx d_0^*$ because of the smoothed-out jump of $\rho_s$ at $d \approx d_0$ (see below), but the BP at $T^*_m$ may be undistinguishable at $d \approx d_0$.

2 ON THE POSSIBILITY OF A JUMP IN THE CURVE $T_m(d)$

We suggest that the ensemble of MVR induces the $\lambda$-transition in the bulk He-II and determines the value of $T_\lambda$ (whatever the mechanism is). This suggestion means that, in the absence of an ensemble of MVR in He-II, all other quasiparticles would not cause the $\lambda$-transition at $T = 2.17$ K. To induce the $\lambda$-transition without MVR, the number of other quasiparticles would have to grow to a certain critical value. This means that, in the absence of MVR in the bulk He-II, the value of $T_\lambda$ would be higher. The vortex rings are known to have certain critical size $d_0$ [21]. For thin superfluid films of He-II, with the thickness $d$ of the superfluid layer less than the size $d_0$ of the smallest MVR, the rings do not fit in and, therefore, do not exist. Consequently, the value of $T_m$ for films with thickness $d \approx d_0$ should abruptly increase in comparison with $T_m$ for films with thickness $d$ just larger than $d_0$ (in the last case, the rings still fit in the film). The value of $T_m$ is known to decrease with the decrease of $d$ mainly as a consequence of the finite-size scaling [14] [15], but, as we have shown, a peculiarity should exist in the curve $T_m(d)$ at $d \approx d_0$, similar to that shown in Fig. 1.

Experimentally, it is known [25] [26] that vortex rings with the core radius $d \approx 0.8$–1.5 Å may exist in He-II, and the smallest radius of MVR detected in the experiment is $R \lesssim 5$ Å [26].
Figure 1: The dependence $T_m(d)$ for He-II films with the predicted anomaly at $d \approx d_0 \approx 6 \text{ Å}$; $d$ is the thickness of the superfluid layer of the film. The values of $T_m$ at $d < 2.5 \text{ Å}$ and $d > 10 \text{ Å}$ correspond approximately to the crosses and squares in Fig. 2, respectively, and the dotted line shows $T_\lambda$.

Figure 2: The experimental dependence $T_m(d)$ for He-II films; $d$ is the thickness of the superfluid layer. Stars correspond to the data of Frederikse [20] with jeweller’s rouge substrate; full triangles correspond to the data of Brewer and colleagues [21, 22], Vycor; open triangles correspond to the data of Brewer [22], $N_2$-plated Vycor; squares correspond to the data of Gasparini and colleagues [23], 2000 Å Nuclepore; crosses correspond to the data of Finotello et al. [11], Vycor; circles are the points with $T_m = 1.25$ K and $T_m = 1.75$ K from [21, 22], with $d$ defined more precisely according to [11]; the dotted line is $T_\lambda$. Continuous lines are drawn by the spline method.
Since any vortex should have a core, the size of the smallest circular or elliptical ring should be roughly two to three core diameters, \(d_0 \approx 4a - 6a \approx 3 - 9 \text{ Å}\). For ‘lying’ rings, which are parallel to the substrate, the ‘hight’ of the ring should be a bit smaller, \(d_0 \approx 3a\) (the core plus the minimal layer of the fluid rotating around the core). Such dependence of \(d_0\) on the orientation of the ring should slightly blur the jump in \(T_m(d)\). According to the approximate model [24] describing the circular vortex ring as a solution of the Gross–Pitaevskii equation, \(d_0 \approx 4a\), where \(a\) is the core radius of the large rings.

Let us estimate the value of the possible jump of \(T_m\). We assume that the \(\lambda\)-transition in the bulk He-II is accompanied by complete exhaustion of the one-particle condensate. According to the calculation [27], the fraction of the one-particle condensate is \(n_0 = 0.078\) at \(T = 0\), and \(n_0 = 0.058\) at \(T = T_\lambda = 2.17\) K, i.e., the condensate does not vanish completely at \(T = T_\lambda\), although \(n_0(T_\lambda) \approx 0\) in the experiment. It is suggested in [27] that the one-particle condensate in He-II is exhausted completely \([n_0(T_\lambda) = 0]\) because of vortex rings. The rings were not taken into account in the calculation of [27], and the decrease in \(n_0\) at \(T \rightarrow T_\lambda\) was due to rotons: the number of the atoms pulled out of the condensate was directly proportional to the number of rotons. The concentration of free rotons is known [18]:

\[
n_r = 0.051 \times e^{-\Delta/T} \left( \frac{q_r}{1.925 \text{ Å}^{-1}} \right)^2 \sqrt{\frac{\mu T_K}{0.14 m_4}} \text{ Å}^{-3},
\]

where \(T_K\) is the temperature in Kelvins. In order that rotons provide \(n_0 = 0\), it is necessary that their number be four times greater than that at \(T = 2.17\) K; in this case, we have \(n_0 = 0\) instead of \(n_0 = 0.058\). For this, temperature \(T \approx 3.12\) K is required according to [2]. Thus, if the calculation of \(n_0\) in [27] is correct, the value of \(T_\lambda\) would be higher, then the observed \(2.17K\), by \(\delta T_\lambda = 0.95\) K in the absence of the vortex rings in He-II. This is the upper bound on the value of the possible jump of \(T_m\) for He-II films.

It should be noted that one could expect a small anomaly also in the curve \(T_{K_T}(d)\) at \(d \approx d_0\). Since the rings disappear from the He-II film at \(d < d_0\), the value of \(\rho_s^{3D}\) should grow at \(d \approx d_0\) compared to \(\rho_s^{3D}\) at \(d\) just larger than \(d_0\). So far as \(T_{K_T} \sim d\rho_s^{3D}\), a bump-like peculiarity, similar to the anomaly in the curve \(T_m(d)\), should exist also in the curve \(T_{K_T}(d)\) at \(d \approx d_0\). However, since at \(d \approx d_0\) \(T_{K_T}\) is appreciably smaller than \(T_m\), the number of MVR \(N_{vr}\) and their contribution to \(\rho_s\) should be several times smaller at \(T_{K_T}\) than at \(T_m\), because \(N_{vr}, \rho_s^{vr} \sim \exp(-E_0/kT)\) \((E_0\) is the energy of the smallest MVR; the interaction between rings may also be important and must be included in \(E_0\)). These simple estimates show that the jump in \(T_{K_T}(d)\) must be roughly three to ten times weaker (or even may be negligible, if \(E_0\) is high enough) than the jump in \(T_m(d)\). Thus, one should look for an anomaly, first of all, in the curve \(T_m(d)\). The experimental data [28] do not give clear evidence of the anomaly in \(T_{K_T}(d)\) at \(d < 7\) Å.

Thus, taking into account our estimates and the theory and experiment for vortex rings, one can see that an ensemble of microscopic vortex rings, in which the smallest MVR have size about \(d_0 \approx 6\) Å, should exist in He-II. And, if the \(\lambda\)-transition in the bulk He-II is induced by MVR, then the anomaly should exist in the curve \(T_m(d)\) at \(d \approx d_0 \approx 6\) Å. We have drawn this anomaly approximately in Fig. 1, basing ourselves on the estimates for \(d_0\) and \(\delta T_\lambda\) and taking into account the smoothing of the jump in \(T_m\) caused by the finite-size scaling, by possible heterogeneity of the film thickness and by the circumstance that part of the rings have size greater than \(d_0\).

It is difficult to make exact calculation of the SF-transition in thin helium films, taking into account of all kinds of quasiparticles. But our simple estimates are sufficient for the prediction of the smoothed-out jump in the curve \(T_m(d)\) and for the approximate demonstration of the form and location of the anomaly.

The experimental data on the dependence \(T_m(d)\) for thin films of He-II is shown in Fig. 2. The value of \(T_\lambda\) for films is defined as the temperature of the maximum of the broad peak on the curve
of the dependence of the specific heat on temperature. The crosses in Fig. 2 are obtained using the data of Fig. 1 from [11]. According to [11], the Brewer’s curve (triangles) should be shifted to the left (circles in Fig. 2); in this case, the data by Finotello et al. [11] (crosses) well agrees with that obtained by Brewer and his colleagues [21, 22]. As a whole, as can be seen from Fig. 2, the data of different works do not fully agree with each other, and there is a large dispersion of the data points. The main causes of this disagreement are the imprecise measurement of the film thickness and the difference in the substrates. Using these data, we cannot determine the existence of the predicted peculiarity in the curve $T_m(d)$.

More precise measurements of the dependence $T_m(d)$ are necessary for several substrates, for $d$ in the interval between 1 Å and 20 Å with small step $\Delta d \leq 1$ Å. Vycor glass, Nuclepore and, perhaps, also Mylar and some other substrates can be used in this case (see Introduction). These must be substrates on which the He$^4$ films are superfluid, and KT-effect is observed very well. Ordered substrates with strong attraction, such as graphite substrates, are not suitable. Precise measurements of $T_m^*(d)$ and $T_{KT}(d)$ for $d = 1–20$ Å could also be interesting.

A discovery of such an anomaly would be the first experimental evidence of the existence of an ensemble of MVR as thermal excitations in He-II and could stimulate further theoretical and experimental investigations of vortex rings in He-II.

The presence of the anomaly will indicate that MVR play an important role in the bulk $\lambda$-transition and substantially influence the value of $T_\lambda$, and the absence of the anomaly will be an evidence that MVR do not influence the value of $T_\lambda$ and do not play any key role in the $\lambda$-transition in He-II. In any case, we would obtain information on the nature of the $\lambda$-transition in He-II. Therefore, exact measurement of the dependence $T_m(d)$ for He-II films, in our opinion, is of great interest.

The idea of this work is developed in more detail in [17].

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