A DIFFERENTIABLE RANKING METRIC USING RELAXED SORTING OPERATION FOR TOP-K RECOMMENDER SYSTEMS

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ABSTRACT

A recommender system generates personalized recommendations for a user by computing the preference score of items, sorting the items according to the score, and filtering the top-\(K\) items with high scores. While sorting and ranking items are integral for this recommendation procedure, it is nontrivial to incorporate them in the process of end-to-end model training since sorting is non-differentiable and hard to optimize with gradient-based updates. This incurs the inconsistency issue between the existing learning objectives and ranking-based evaluation metrics of recommendation models. In this work, we present DRM (differentiable ranking metric) that mitigates the inconsistency and improves recommendation performance, by employing the differentiable relaxation of ranking-based evaluation metrics. Via experiments with several real-world datasets, we demonstrate that the joint learning of the DRM cost function upon existing factor based recommendation models significantly improves the quality of recommendations, in comparison with other state-of-the-art recommendation methods.

1 Introduction

With the massive growth of online content, it has become common for online content platforms to operate recommender systems that provide personalized recommendations, aiming at facilitating better user experiences and alleviating the dilemma of choices [1]. In general, recommender systems generate the relevance score of items with respect to a user, and recommend top-\(K\) items of high scores. Thus, sorting (or ranking) items serves an important role in such top-\(K\) recommendation tasks.

While learning-based recommenders are popular, they are generally trained upon objectives that are limited in accurately reflecting the ranking nature of top-\(K\) recommendation tasks. It is because sorting operation is non-differentiable, and so incorporating it into the end-to-end model training driven by gradient-based updates is challenging, when commonly used objectives such as mean squared error or log likelihood are used.

As has been noted in several research works [2,3,4], optimizing objectives that are not aware of the ranking nature of top-\(K\) recommendation tasks, does not always guarantee the best performance. Although there exist ranking-oriented objectives: pairwise objectives such as Bayesian personalized pairwise loss [5] and listwise objectives based on Plackett-Luce distribution [6,4], neither objectives fit well with top-\(K\) recommendation tasks. Pairwise objectives consider only the pairwise ranking between a pair of items, while top-\(K\) recommendation tasks intend for generating recommendation lists of size \(K\). On the other hand, listwise objectives consider all the items yet with equal importance regardless of their ranks, while it is natural that top-\(K\) recommendation tasks need to relatively give more weights on the items of higher ranks which are highly likely to be in a recommendation list.

To bridge such inconsistency between the existing learning objectives commonly used for training recommendation models and the ranking nature of top-\(K\) recommendation tasks, we present DRM (differentiable ranking metric), which is a differentiable relaxation scheme of ranking-based evaluation metrics such as Precision@\(K\) or Recall@\(K\). By
employing the differentiable relaxation scheme for sorting operation [7], DRM expedites direct optimization of given ranking-based evaluation metrics for recommendation models.

We first reformulate the ranking-based evaluation metrics in terms of permutation matrix arithmetic forms, and then relax the non-differentiable permutation matrix in the arithmetic forms to a differentiable row-stochastic matrix. This reformulation and relaxation allows us to represent non-differentiable ranking metrics in a differentiable form of DRM. Using DRM as an optimization objective renders end-to-end recommendation model training highly consistent with ranking-based evaluation metrics. Moreover, DRM can be readily incorporated atop of an existing recommendation models via the joint learning with its own objectives without modifying its structure.

For evaluating the effect of DRM upon existing models, we adopt two state-of-the-art factor based recommendation models, WARP [8] and CML [9]. Our experiments demonstrate that the DRM objective significantly improves the performances of top-K recommendations on several real-world datasets in terms of ranking based evaluation metrics, in comparison with several other recommendation models.

2 Preliminaries

Given a set of users \( U = \{1, 2, \ldots, M\} \), a set of items \( I = \{1, 2, \ldots, N\} \), and a set of interactions \( y_{u,i} \) for all users \( u \) in \( U \) and all items \( i \) in \( I \), a recommendation model aims to learn to predict preference, or score \( y_{u,i} \in \mathbb{R} \) of user \( u \) to item \( i \). We use binary implicit feedback \( y_{u,i} \) such that \( y_{u,i} = 1 \) if user \( u \) has interacted with item \( i \), and 0 otherwise. Note that in this work, we only consider this binary feedback format, while our approach can be easily generalized to various implicit feedback settings. We use \( u \) to index a user, and \( i \) and \( j \) to index items, usually \( i \) is for items that user \( u \) has interacted, and \( j \) is items that user \( u \) did not interact. We denote a set of items with which user \( u \) has interacted as \( I_u \).

We also use \( \hat{y}_u \) to represent the items that user \( u \) has interacted, in the bag of words notation, meaning column vector \([\hat{y}_{u,1}, \hat{y}_{u,2}, \ldots, \hat{y}_{u,n}]\). Similarly, we use \( \hat{y}_u \) to the vector of predicted scores of items, meaning \([\hat{y}_{u,1}, \hat{y}_{u,2}, \ldots, \hat{y}_{u,n}]\).

2.1 Objectives of Recommendation Models

Objective for recommendation models are grouped into three categories: pointwise, pairwise, and listwise.

Pointwise objectives maximize the accuracy of predictions independently. Mean squared error and Cross entropy are commonly used pointwise objectives for training machine learning models for recommenders. It is known that pointwise objectives for recommendation have a limitation in that high predictive accuracy does not always lead to high-quality recommendations [10].

Pairwise objectives gained popularity because they are more closely related to the top-K recommendation tasks than pointwise objectives. It enables a recommendation model to learn users’ preferences by viewing the problem as a binary classification, predicting whether user \( u \) prefer item \( i \) to item \( j \). As noted in [11], One of the main concerns on pairwise approaches is that it is formalized to minimize classification errors of item pairs, rather than errors of item rankings.

Listwise objectives minimize errors in the list of sorted items or scores of items. They have been explored by a few prior works [12][13][14][15], yet they are not fully investigated in the recommender systems domain. It is because list operations such as permutation or sorting are hard to be differentiated. One significant drawback of listwise objectives is that they treat all items of rankings with equal importance. However, items at higher ranks can be recommended and are more important to the top-K recommendation.

Our objective overcomes the limitations of the pairwise objectives and current listwise objectives while exploiting both ranking nature and emphasis on items at top ranks of the top-K personalized recommendations.

2.2 Ranking Metrics for Model based Recommendation Models

In practice, performances of trained recommendation models should be validated with respect to its given objectives before being deployed in target services.

In our notation, we represent the list of items ordered by the predicted scores with respect to user \( u \) as \( \pi_u \), and the item at rank \( k \) as \( \pi_u(k) \). In addition, we define the Hit() function that specifies whether the \( k \)-th highest scored item for user \( u \) in the recommendation list is in the validation dataset \( I_u \) that contains all the items interacted by \( u \), i.e.,

\[
\text{Hit}(k) = \mathbb{I}[\pi_u(k) \in I_u]
\]

where \( \mathbb{I}[\text{statement}] \) is the indicator function, yielding 1 if the statement is true and 0 otherwise.

Precision and Recall are two of the most widely used evaluation metrics for top-K recommendation tasks [13]. For each user, \( u \), both metrics are based on how many items in the top-K recommendation are in the validation dataset \( I_u \).
The Precision metric specifies the fraction of hit items in the validation dataset $I_u$ among the items in the top-$K$ recommended list, while the Recall metric specifies the fraction of recommended items among the items in the validation dataset $I_u$. Notice that both metrics emphasize items in high ranks by counting only items having rank $K$ or smaller. They do not distinguish the relative ranking among the items in the top-$K$ rankings.

\[
\text{Precision}@K(u, \pi_u) = \frac{1}{K} \sum_{k=1}^{K} \text{Hit}(k)
\]

\[
\text{Recall}@K(u, \pi_u) = \frac{1}{|I_u|} \sum_{k=1}^{K} \text{Hit}(k)
\]

On the other hand, Truncated Discounted Cumulative Gain (DCG) \[14\] and Truncated Average precision (AP) \[15\] take into account for the relative ranking of items by weighting the impact of Hit$_k$ according to its rank $k$. Furthermore, Normalized DCG (NDCG) specifies a normalized value of DCG@$K$, which is divided by the ideal discount cumulative gain $\text{IDCG}@K = \max_{\pi_u} \text{DCG}@K(u, \pi_u)$.

\[
\text{NDCG}@K(u, \pi_u) = \frac{\text{DCG}@K(u, \pi_u)}{\text{IDCG}@K}
\]

where

\[
\text{DCG}@K(u, \pi_u) = \sum_{k=1}^{K} \frac{\text{Hit}(k)}{\log(k+1)}
\]

The truncated AP is defined as

\[
\text{AP}@K(u, \pi_u) = \frac{1}{|I_u|} \sum_{k=1}^{K} \text{Precision}@k(u, \pi_u) \text{Hit}(k).
\]

AP can be viewed as a weighted sum of Hit for each rank $k$ weighted by Precision@$k$.

Notice that all the metrics above are consistent in a common form of a weighted sum of Hit. Accordingly, we formulate these metrics in a unified way as $O(K)$ conditioned on the weight function $w(k, K)$. For simplicity, we omit the arguments $(u, \pi_u)$ of these metrics without loss of generality.

\[
O(K) = \sum_{k=1}^{K} w(k, K) \text{Hit}(k)
\]

\[
= \begin{cases} 
\text{Precision}@K & \text{if } w(k, K) = 1/K \\
\text{Recall}@K & \text{if } w(k, K) = 1/|I_u| \\
\text{NDCG}@K & \text{if } w(k, K) = \frac{1}{\log(k+1)/\text{IDCG}@K} \\
\text{AP}@K & \text{if } w(k, K) = \frac{\text{PR}@K}{|I_u|} 
\end{cases}
\]

### 3 Proposed Method

In this section, we propose DRM. We begin this section by introducing two working blocks for our method. The first part introduces matrix factorization with weighted hinge loss. Then, we introduce how to represent ranking based evaluation metrics in terms of vector arithmetic and then relax the metrics to be differentiable, which can be optimized by gradient descent. We conclude this section with training procedure of DRM.

#### 3.1 Factor Based Recommenders with Hinge Loss

Factor based recommenders represent users and items in a latent vector space $\mathbb{R}^d$, and then formulate the preference score of user $u$ to item $i$ as a function of two vectors $\alpha_u$ and $\beta_i$ in $d$-dimensional vector space $\mathbb{R}^d$, for users and items. The dot product is one common method for mapping a pair of user and item vectors to a preference score \[5\] \[16\] \[17\]. In \[9\], the collaborative metric learning (CML) embeds users and items in the euclidean metric space and defines its score function as a negative value of L2 distance of two vectors.

\[
\hat{y}_{u,i} = \frac{(\alpha_u^T \beta_i)}{-\|\alpha_u - \beta_i\|^2}
\]

where $\|x\|$ is the L2 norm of the vector $x$. 

3
Our model uses either dot product or L2 distance of user vector \( \alpha_u \) of user \( u \) and item vector \( \beta_i \) of item \( i \) as a score function. Regardless of score functions, we update our model using weighted hinge loss with weight are calculated by an approximated ranking of the positive item \( i \) with respect to user \( u \).

\[
\mathcal{L}_{\text{hinge}} = \sum_{u \in \mathcal{U}} \sum_{i \in I_u} \sum_{j \in I - I_u} \Phi_{ui}[\mu - \hat{y}_{ui} + \hat{y}_{uj}]_+
\]

where \([x]_+ = \max(x, 0)\) is a clamp function, and \( \mu \) is the margin for clamp function. We empirically tune the margin \( \mu \) to be 1. The weight \( \Phi_{ui} \) is defined to have larger values if the rank of positive item \( i \) is estimated to be at lower rank. Similar to sampling procedure in \([9]\), it is defined to be parallel to allow fast computations on GPU. Explicitly, \( \Phi_{ui} \) is

\[
\Phi_{ui} = \log \left( 1 + \left[ -\hat{y}_{ui} + \frac{1}{T_u} \sum_{j \in I_u^{\text{neg}}} \hat{y}_{uj} \right]_+ \right)
\]

With sampling \( I_u^{\text{neg}} \) items for each update from the set of items that the user \( u \) did not interact with. The number of negative sampling \( I_u^{\text{neg}} \) is usually between ten to a few hundreds.

### 3.2 Relaxed Precision

An \( n \)-dimensional permutation \( p = [p_1, p_2, \ldots, p_n]^T \) is a vector of distinct indices from 1 to \( n \). Every permutation \( p \) can be represented using a permutation matrix \( P(p) \in \{0, 1\}^{n \times n} \) and its element can be described as:

\[
P_{i,j} = \begin{cases} 
1 & j = \pi_i \\
0 & \text{otherwise}
\end{cases}
\]

For example, a permutation matrix \( P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \) maps a score vector \( v = [3, 5]^T \) to \( Pv = [5, 3]^T \).

We can represent a sorting by decreasing order with the score vector \( s \) and the permutation matrix \( P(s) \) as follow (Corollary 3 in \([7]\)):

\[
P_{i,j}^{(s)} = \begin{cases} 
1 & j = \text{argmax}\{(n + 1 - 2i)s - A_i1\} \\
0 & \text{otherwise}
\end{cases}
\]

where \( 1 \) refers to the column vector having 1 for all elements and \( A_i \) is the matrix such that \( A_i,j = |s_i - s_j| \). Note that the \( k \)-th row \( P_k \) of the permutation matrix \( P \) is equal to the one-hot vector representation of the item of rank \( k \). Thus we can represent \( \text{Hit}(\text{Eq. (1)}) \) using the dot product of \( y_u \) and \( P_k^{(s)} \).

\[
\text{Hit}(k) = y_u^T P_k^{(s)}
\]

Thus we obtain the representation of ranking metrics Eq. (2) in terms of vector arithmetic.

\[
\mathcal{O}(K, w) = \sum_{k=1}^K w(k, K) y_u^T P_k^{(s)}
\]

In \([7]\), they propose a differentiable generalization of sorting by relaxation of the permutation matrix Eq. (4) into row-stochastic matrix, allowing differentiation operation involving sorting of elements of real values. We can construct this relaxed matrix \( \tilde{P}(s) \) by following equation:

\[
\tilde{P}_k^{(s)} = \text{softmax} \left[ \tau^{-1} \left( (n + 1 - 2k)s - A_k1 \right) \right]
\]

where \( \tau > 0 \) is a temperature parameter. Higher value of \( \tau \) means each row of our relaxed matrix becomes flatter. This relaxation is continuous everywhere and differentiable almost everywhere with respect to the elements of \( s \). As \( \tau \to 0 \), \( \tilde{P}(s) \) reduces to the permutation matrix \( P(s) \).

We can obtain differentiable relaxed objective, which can be used for optimization using gradient-based update, by simply replacing \( P(S_u) \) in Eq. (5) to \( \tilde{P}(S_u) \). Explicitly,

\[
\tilde{\mathcal{O}} = \sum_{k=1}^K w(k, K) y_u^T \tilde{P}_k^{(S_u)}
\]
Since softmax function is differentiable, this value is differentiable now. We empirically find that this is slightly more stable to update model by the equation below:

\[
\mathcal{L}_{\text{neu}} = \|\mathbf{y}_u - \hat{P}_{[1:K]}\|^2
\]  

(7)

where \(\hat{P}_{[1:K]} = \sum_k w(k, K) \hat{P}_k (\mathbf{y}_u)\). Note that minimizing Eq. (7) is equivalent to maximizing Eq. (6).

\[
\hat{O} = \mathbf{y}_u^T \hat{P}_{[1:K]} \\
= \frac{1}{2} \left[ \mathbf{y}_u^T \mathbf{y}_u + \hat{P}_{[1:K]}^T \hat{P}_{[1:K]} \right] - \frac{1}{2} \|\mathbf{y}_u - \hat{P}_{[1:K]}\|^2 \\
\geq -\frac{1}{2} \|\mathbf{y}_u - \hat{P}_{[1:K]}\|^2 = -\frac{1}{2} \mathcal{L}_{\text{neu}}
\]

3.3 Model Update

Once we developed our objective, as explained in above, we incorporate our loss Eq. (7) into the model learning structure Eq. (3). We propose learning from two worlds by joint learning:

\[
\mathcal{L} = \mathcal{L}_{\text{hinge}} + \lambda \mathcal{L}_{\text{neu}} \\
= \sum_{u \in \mathcal{U}} \sum_{i \in \mathcal{I}_u} \sum_{j \in \mathcal{I} \setminus \mathcal{I}_u} \Phi_{ui} \left[ \hat{y}_{u,i} + \hat{y}_{u,j} + \hat{y}_{u,i} \right] + \\
+ \lambda \sum_{u \in \mathcal{U}} \|\mathbf{y}_u - \hat{P}_{[1:K]}\|^2
\]  

(8)

Eq. (8) is the objective of our model. We can also view this objective as regularizing the pairwise ranking objective \(\mathcal{L}_{\text{hinge}}\) (Eq. (5)) upon violations of correct rankings using \(\mathcal{L}_{\text{neu}}\). The effect of \(\mathcal{L}_{\text{neu}}\) is controlled using scaling parameter \(\lambda\).

As we consider factor based models with gradient updates using negative sampling \([5, 9, 18, 19, 20]\) we follow similar sampling procedure with additional positive item sampling. One training sample contains a user \(u\), \(\rho\) positive items that user \(u\) has interacted with, and \(\nu\) negative items that user \(u\) did not interact with. We empirically set \(\nu\) to be 15 times of \(\rho\). We construct a list of items \(\hat{\mathbf{y}}_u\) with \(\rho\) positive item and \(\nu\) negative items sampled to build \(\rho + \nu\) size array where first \(\rho\) elements are all 1 and zero elsewhere. We can construct \(\hat{\mathbf{y}}_u\) similarly

\[
\hat{\mathbf{y}}_u = [\hat{y}_{u,i_1}, \hat{y}_{u,i_2}, \ldots, \hat{y}_{u,i_\rho}, \hat{y}_{u,j_1}, \ldots, \hat{y}_{u,j_\nu}]^T
\]

. The learning procedure for our model is summarized in Alg. [1]
Algorithm 1: Learning Procedure for DRM

| Initialize user factors $\alpha_u$ where $u \in \{1, 2, \ldots, M\}$ |
| Initialize item factors $\beta_i$ where $i \in \{1, 2, \ldots, N\}$ |
| repeat |
| Sample user $u$ from $\mathcal{U}$ |
| Sample $\rho$ items $i_1, i_2, \ldots, i_\rho$ from $\mathcal{I}_u$ |
| Sample $\nu$ items $j_1, j_2, \ldots, j_\nu$ from $\mathcal{I} - \mathcal{I}_u$ |
| $\Delta \alpha_u \leftarrow 0$ |
| $\Delta \beta_i \leftarrow 0$ for $i \in i_1, i_2, \ldots, i_\rho$ |
| $\Delta \beta_j \leftarrow 0$ for $j \in j_1, j_2, \ldots, j_\nu$ |
| Construct $\hat{y}_u \leftarrow [1, 1, \ldots, 1, 0, 0, \ldots, 0]^T$ |
| Construct $\hat{y}_u \leftarrow [\hat{y}_{u,i_1}, \hat{y}_{u,i_2}, \ldots, \hat{y}_{u,i_\rho}, \hat{y}_{u,j_1}, \ldots, \hat{y}_{u,j_\nu}]^T$ |
| Choose one positive item $i$ with smallest score $\hat{y}_{u,i}$ among $i_1, \ldots, i_\rho$ |
| Choose one negative item $j$ with largest score $\hat{y}_{u,j}$ among $j_1, \ldots, j_\nu$ |
| for $\theta \leftarrow \{\alpha_u, \beta_i, \beta_j\}$ do |
| $\Delta \theta \leftarrow \Delta \theta + \nabla L_{\text{hinge}}$ |
| end for |
| for $\theta \leftarrow \{\alpha_u, \beta_i, \beta_j\}$ do |
| $\Delta \theta \leftarrow \Delta \theta + \lambda \nabla L_{\text{neu}}$ |
| end for |
| for $\theta \leftarrow \{\alpha_u, \beta_i, \beta_j\}$ do |
| Update $\theta$ with $\Delta \theta$ using Adagrad Optimizer |
| $\theta \leftarrow \theta / \min(1, \|\theta\|)$ |
| end for |
| until Converged |

4 Related Work

Bayesian Personalized Ranking [5] has proposed pairwise cost function to maximize area under the curve (AUC). This framework gives a method for factor based recommenders and nearest neighbors based recommenders to learn personalized ranking. One of the significant drawbacks of this model is that the AUC does not discriminate between items in higher ranks and those in lower ranks, unlike NDCG and MAP. This property does not fit very well with the top-$k$ recommendation tasks. Our model, unlike BPR, focuses on a few items at higher ranks. This fits more in the actual recommendation tasks where only a small number of items can be recommended at a time, thus resulting in better top-$k$ recommendations.

Cofactor [21] has proposed word2vec-like [22, 23] embedding techniques to embed item co-occurrence information into the matrix factorization model. This is achieved by adding additional objective function to pointwise mean squared error matrix objective. SRRMF [24] claims that merely treating missing ratings to be zeros leads suboptimal behaviors. It proposes smoothing negative feedback to nonzero values according to their approximated ranks. These two works are most similar to ours in that they propose a new objective or view of interpreting data without requiring additional input such as context. However, they are limited in that their objectives cannot be applied to general gradient based models.

Listwise Collaborative Filtering [4] attempts to tackle the misalignment between cost and objective on memory based, $K$-Nearest Neighbors recommenders [25]. It proposed a method to calculate similarity between two lists. Our work is complementary to it because we propose a solution for factor based, or model based recommenders.

5 Empirical Evaluation

In this section, we evaluate our proposed method against various existing recommendation models.

5.1 Experiment Setup

We evaluate our approach and baseline models with four datasets of real-world user-item interactions. Their statistics and characteristics are summarized in Table 1.
Table 1: Dataset statistics. #users and #items denote the number of users and the number of items, respectively; #interactions denotes the number of transactions or clicks; avg. row and avg. col denote the average number of items that each user has interacted with, and the average number of users who have interacted with each item respectively; density denotes the interaction matrix density (i.e., density = #interactions / (#users × #items);

|        | SketchFab | Epinion | ML-20M | Melon |
|--------|-----------|---------|--------|-------|
| #users | 16K       | 20K     | 133K   | 104K  |
| #items | 28K       | 59K     | 15K    | 81K   |
| #interactions | 447K   | 500K    | 8M     | 3.3M  |
| avg. row | 28.74 | 23.59   | 58.45  | 31.39 |
| avg. col | 15.52 | 8.52    | 514.54 | 40.44 |
| density | 0.10%    | 0.04%   | 0.38%  | 0.04% |

- **SketchFab** [26]: This dataset contains user click streams on 3D models. We consider only items whose interaction count is greater than or equal to 5.
- **Epinion** [27]: This dataset has product reviews and five-star rating information from a web commerce site. We view each rating as an user-item interaction signal.
- **ML-20M** [28]: This dataset contains five-star ratings (with a half star) of movies. We interpret each rating as a user-item interaction. We exclude rating lower than four, and treat remainders as binary implicit feedback.
- **Melon** [5]: This dataset contains playlists from a music streaming service. To be consistent with the implicit user feedback setting, we treat each playlist as a user, and songs in a playlist as a list of items a user has interacted with.

**Evaluation Protocol** We randomly split interaction data into training, validation, and test datasets in 70%, 10%, and 20% portions, respectively. We first train models once using training data to find the best hyperparameter settings for each model, evaluating the hyperparameter settings using the validation datasets. We then train models five times with the best hyperparameter settings using both training and validation data, evaluate the models using test data, and report the average of evaluation metrics. We skip evaluating users with fewer than three interactions in the training dataset. We use Recall@50 for model validation. We conduct Welch’s T-test [29] on results and denote results with a $p$ value lower than 0.01 in boldface and ties in italic.

As results, we report mean AP@10 (MAP@10), NDCG@10, Recall@50, and NDCG@50.

**Our Model** We use two variants of our model. One use dot product as a score function and is denoted as DRM$_{dot}$; the other exploits the negative value of the L2 distance of user vector $\alpha_u$ and item vector $\beta_i$ as a score, and it is denoted as DRM$_{L2}$.

**Baselines** We compare our method with the following baselines:

- **SLIM** [20] is a state-of-the-art item based collaborative filtering algorithm in which the item-item similarity matrix is represented as a sparse matrix. It generates item preferences of a user by the weight sum of similarities between items that the user has previously consumed.
- **CDAE** [20, 31] is a factor based recommender, which represents user factors using an encoder, or a multi-layer perceptron, whose input is embeddings of items that the user has consumed.
- **WMF** [17] is a state-of-the-art matrix factorization model that uses pointwise loss and minimize loss using alternating least squares.
- **BPR** [5] is a matrix factorization model that exploits pairwise sigmoid objective which is designed to optimize the AUC of ROC score.
- **WARP** [8] is a matrix factorization model trained using hinge loss with approximated rank based weights.
- **CML** [9] is a factor based recommendation model that models user-item preference as a negative value between the distance of user vector and item vector.
- **SQLRank-MF** [6] is matrix factorization models having cost function based on a permutation probability of list of items.

\[5\] https://arena.kakao.com/c/7
- SRRMF [24] is a state-of-the-art factor based recommendation, which interpolates scores of unobserved feedback to be nonzero, giving different importances on unobserved feedback.

For WMF, and BPR, we used an open-source implementation, Implicit [32]. For WARP, we used an open-source recommender, lightFM. [33]. We used implementations publicly available by the authors for SLIM [34], SQLRank-MF and SRRMF. We implemented CDAE, CML, and DRM in using Pytorch 1.5.0. We run our experiment on a machine with Intel(R) Xeon(R) CPU E5-2698 and NVIDIA Tesla V100 GPU with CUDA 10.1.

Table 2: Recommendation Performances of different methods. the best performing models with

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{Datasets} & \text{Metrics} & \text{SLIM} & \text{CDAE} & \text{BPR} & \text{WMF} & \text{WARP} & \text{CML} & \text{SQL-Rank} & \text{SRRMF} \\
\hline
\text{SketchFab} & \text{MAP@10} & 0.0390 & 0.0351 & 0.0216 & 0.0355 & 0.0365 & 0.0358 & 0.0101 & 0.0200 \\
& \text{NDCG@10} & 0.1163 & 0.1301 & 0.0905 & 0.1257 & 0.1354 & 0.1379 & 0.0417 & 0.0862 \\
& \text{Recall@50} & 0.2696 & 0.2793 & 0.2168 & 0.2862 & 0.2923 & 0.3040 & 0.1422 & 0.1550 \\
& \text{NDCG@50} & 0.1067 & 0.1657 & 0.1218 & 0.1645 & 0.1724 & 0.1778 & 0.0537 & 0.0995 \\
\text{Epinion} & \text{MAP@10} & 0.0086 & 0.0128 & 0.0062 & 0.0123 & 0.0100 & 0.0130 & 0.0036 & 0.0107 \\
& \text{NDCG@10} & 0.0357 & 0.0453 & 0.0238 & 0.0486 & 0.0387 & 0.0493 & 0.0168 & 0.0428 \\
& \text{Recall@50} & 0.1081 & 0.1123 & 0.0661 & 0.1325 & 0.1158 & 0.1347 & 0.0432 & 0.1275 \\
& \text{NDCG@50} & 0.0410 & 0.0646 & 0.0362 & 0.0726 & 0.0610 & 0.0736 & 0.0252 & 0.0680 \\
\text{ML-20M} & \text{MAP@10} & 0.1287 & 0.1369 & 0.0787 & 0.1034 & 0.1030 & 0.1331 & N/A & 0.0987 \\
& \text{NDCG@10} & 0.2761 & 0.3205 & 0.1917 & 0.2561 & 0.2300 & 0.2824 & N/A & 0.2532 \\
& \text{Recall@50} & 0.4874 & 0.4829 & 0.3431 & 0.4676 & 0.4187 & 0.4874 & N/A & 0.4786 \\
& \text{NDCG@50} & 0.2511 & 0.3667 & 0.2394 & 0.3288 & 0.2887 & 0.3416 & N/A & 0.3244 \\
\text{Melon} & \text{MAP@10} & 0.1768 & 0.1901 & 0.0972 & 0.1303 & 0.1217 & 0.1659 & N/A & 0.1324 \\
& \text{NDCG@10} & 0.3415 & 0.1928 & 0.2159 & 0.2537 & 0.2577 & 0.3361 & N/A & 0.2863 \\
& \text{Recall@50} & 0.2206 & 0.1335 & 0.1363 & 0.1716 & 0.1654 & 0.2206 & N/A & 0.1842 \\
\hline
\end{array}
\]

Table 2: Recommendation Performances of different methods. the best performing models with \( p \leq 0.01 \) with paired T test are boldfaced. We describes the values in italic if the performances of two or more models are not statistically significant.

5.2 Alignment between Training Cost and Evaluation Metrics

We conduct an illustrational experiment to show the objective of DRM [9]. Figure 1 describes normalized costs and loss and MAP@10 for WARP is −0.933, The Correlation between loss and MAP@10 for WARP + DRM is −0.990.

Figure 1: Normalized loss per sample and MAP@10 using training data versus training epochs. The correlation between loss and MAP@10 for WARP is −0.933, The Correlation between loss and MAP@10 for WARP + DRM is −0.990.

https://github.com/WuLiWei9278/SQL-Rank/  
https://github.com/KarypisLab/SLIM

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6https://github.com/HERECJ/SRRMF
5.3 Quantitative Results

We cannot train SQLRank-MF with large datasets, ML-20M and Melon, because of the huge training time. It took about a day to train on Epinion dataset, and it is impossible to run on the ML-20m and Melon datasets. Therefore, we record the performances of SQLRank-MF only on the SketchFab and Epinion datasets.

Table 2 shows the performance of various models for four datasets in terms of various ranking metrics. We observe that the proposed methods outperform state-of-the-art models by a large margin for all the datasets we use. We observe that although matrix factorization models (BPR, WMF, and WARP, DRMDot) share the same model formulation, but the differences among their performances is large. For example, WMF achieves the smallest training error using pointwise loss, however, its prediction quality is below other pairwise model such as WARP, CML and our models in many datasets. Note that WARP behaves poorly in SketchFab dataset, however DRMDot achieves best prediction qualities among models we evaluate. They only differ in the additional loss term $L_{\text{neu}}$. These trends are same for all other datasets. We credit this performance gain to the proposed objective, enabling factor based models to be aware of top-K recommendation nature.

5.4 Effects of Hyperparameters

Our objective uses sampling items to sort and rank. Thus, we only sample $\rho$ positive items and $\nu$ negative items to build a training sample, other than using the entire itemset. In figure 2, we conduct an experiment to see the effect of the size of the number of positive items in the sampling. We set $\nu$ to be 15 times and the number of positive items $\rho$ in the training sample. We observe the number of positive items $\rho$ has a positive relation with the recommendation performance. However, their effect varies across datasets, and increasing $\rho$ does not increase the performance further at some points.

5.5 Exploratory Analysis

Figure 3 shows NDCG@10 among different user groups by the number of interactions. The numbers in the parenthesis denote the number of users in each group.

Figure 3 shows NDCG@10 of user groups grouped by the number of interactions in the training datasets. Our loss function consistently improves recommendation performances for all user groups, especially when the number of interactions of users is small. For models using negative L2 distance as a score function, CML and DRML2 significantly improves the quality of recommendations.
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Table 3: Comparison on Melon dataset with joint learning (DRM L2) with pairwise loss only (CML), and DRM only.

|                  | CML    | DRM-only | DRM L2 |
|------------------|--------|----------|--------|
| MAP@10           | 0.0358 | 0.0310   | 0.0390 |
| NDCG@10          | 0.1379 | 0.1234   | 0.1466 |
| Recall@50        | 0.3040 | 0.2802   | 0.3028 |
| NDCG@50          | 0.1778 | 0.1608   | 0.1836 |

In Table 3 we evaluate the model learned using DRM cost only, over two models, one trained CML and one trained only with DRM cost. We find that the model trained only with DRM loss performs worse than the other two models. We conjecture it may come from that the listwise objective does not have enough training samples (only one sample exists for a user).

6 Conclusion

While learning-based recommender systems are popular, their performance in terms of personalized ranking might be suboptimal because they are not directly optimized for top-K recommendation tasks. In this work, we have proposed DRM, a differentiable ranking metric that enables sorting-embedded end-to-end training for factor based recommenders. DRM utilizes the relaxation of sorting to continuous operation, hence leading to the high-performance cost function that can directly maximize metrics such as Precision. Via experiments, we demonstrate that DRM achieves the higher quality of recommendation models in comparison with other state-of-the-art recommender methods on several real-world datasets.

Our future work is to apply the DRM cost function to various recommendation models including AutoEncoder and deep neural network models. It is also interesting to investigate other types of differentiable ranking metrics than the relaxed Precision we explored in this work.

References

[1] Barry Schwartz. The paradox of choice. 2018.
[2] Christopher J. C. Burges, Robert Ragno, and Quoc Viet Le. Learning to rank with nonsmooth cost functions. In Proceedings of the Twentieth Annual Conference on Neural Information Processing Systems, pages 193–200, 2006.
[3] Jun Xu, Tie-Yan Liu, Min Lu, Hang Li, and Wei-Ying Ma. Directly optimizing evaluation measures in learning to rank. In Proceedings of the 31st Annual International ACM SIGIR Conference on Research and Development in Information Retrieval, SIGIR, pages 107–114, 2008.
[4] Shanshan Huang, Shuaiqiang Wang, Tie-Yan Liu, Jun Ma, Zhumin Chen, and Jari Veijalainen. Listwise collaborative filtering. In Proceedings of the 38th International ACM SIGIR Conference on Research and Development in Information Retrieval, pages 343–352, 2015.
[5] Steffen Rendle, Christoph Freudenthaler, Zeno Gantner, and Lars Schmidt-Thieme. BPR: bayesian personalized ranking from implicit feedback. In UAI, Proceedings of the Twenty-Fifth Conference on Uncertainty in Artificial Intelligence, pages 452–461, 2009.
[6] Liwei Wu, Cho-Jui Hsieh, and James Sharpnack. Sql-rank: A listwise approach to collaborative ranking. In Proceedings of the 35th International Conference on Machine Learning., pages 5311–5320, 2018.
[7] Aditya Grover, Eric Wang, Aaron Zweig, and Stefano Ermon. Stochastic optimization of sorting networks via continuous relaxations. In 7th International Conference on Learning Representations, 2019.
[8] Jason Weston, Samy Bengio, and Nicolas Usunier. Wsabie: Scaling up to large vocabulary image annotation. In Toby Walsh, editor, Proceedings of the 22nd International Joint Conference on Artificial Intelligence, pages 2764–2770, 2011.
[9] Cheng-Kang Hsieh, Longqi Yang, Yin Cui, Tsung-Yi Lin, Serge J. Belongie, and Deborah Estrin. Collaborative metric learning. In Proceedings of the 26th International Conference on World Wide Web, pages 193–201, 2017.
[10] Paolo Cremonesi, Yehuda Koren, and Roberto Turrin. Performance of recommender algorithms on top-n recommendation tasks. In Proceedings of the 2010 ACM Conference on Recommender Systems, pages 39–46, 2010.
[11] Zhe Cao, Tao Qin, Tie-Yan Liu, Ming-Feng Tsai, and Hang Li. Learning to rank: from pairwise approach to listwise approach. In Machine Learning, Proceedings of the Twenty-Fourth International Conference, pages 129–136, 2007.

[12] Yue Shi, Martha A. Larson, and Alan Hanjalic. List-wise learning to rank with matrix factorization for collaborative filtering. In Proceedings of the 2010 ACM Conference on Recommender Systems, pages 269–272, 2010.

[13] David C. Blair and M. E. Maron. An evaluation of retrieval effectiveness for a full-text document-retrieval system. Communications of the ACM, 28(3):289–299, 1985.

[14] Kalervo Järvelin and Jaana Kekäläinen. Cumulated gain-based evaluation of IR techniques. ACM Transactions on Information Systems, 20(4):422–446, 2002.

[15] Ricardo A. Baeza-Yates and Berthier A. Ribeiro-Neto. Modern information retrieval, volume 463. 1999.

[16] Ruslan Salakhutdinov and Andriy Mnih. Probabilistic matrix factorization. In Proceedings of the Twenty-First Annual Conference on Neural Information Processing Systems, pages 1257–1264, 2007.

[17] Xiangnan He, Lizi Liao, Hanwang Zhang, Liqiang Nie, Xia Hu, and Tat-Seng Chua. Neural collaborative filtering. In Proceedings of the 26th International Conference on World Wide Web, pages 173–182, 2017.

[18] Dawen Liang, Rahul G. Krishnap, Matthew D. Hoffman, and Tony Jebara. Variational autoencoders for collaborative filtering. In Proceedings of the 2018 World Wide Web Conference, 2018.

[19] Ben Fredrickson. Fast python collaborative filtering for implicit datasets. https://github.com/benfred/implicit, 2017.