Bus schedule modelling in Bandung, Indonesia, using max-plus algebra

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Abstract. Congestion has been a big problem in many big cities in the developing countries. One of the main factors causing traffic congestion is the lack of use of public transportations. The network problems related to synchronization problems, e.g. scheduling of public transportation, can be modeled and solved using the Max-plus algebra. Bandung as the capital and the biggest city in the West Java has also faced such a problem, that is congestion. In this paper, the scheduling of DAMRI bus, one of public transportation in Bandung will be modelled using the Max-plus algebra. We choose two DAMRI routes in Bandung, those are Dipatiukur - Leuwi Panjang and Ledeng - Leuwi Panjang. Both routes are transformed into a directed graph with the bus stops as the nodes. The Max-plus algebra is applied to develop the bus scheduling. In the developed model, the eigenvalues and eigenvectors of the system are calculated. The eigenvalues represent the period of departure at each bus stops and the eigenvectors represent the initial time of departure of the DAMRI bus. The results show that for every 3.7492308 minutes a bus will depart at each bus stop. If the DAMRI bus operation starts at 05.00, then the first departure should be from the bus stop in Pasir Kaliki to the bus stop in Hasan Sadikin Hospital.

1. Introduction
Many big cities in the developing countries, such as Indonesia, has a bad quality of public transportations. It causes the lack of the use of public transportations and implies serious congestion, which is also happened in Bandung. Based on the survey conducted by the Bandung government, at least 66% of residents of Bandung using private vehicles consisting of 50% motorcycles and 16% of the car. Moreover, the increasing of population implying the number of private vehicles and small public transportations such as Angkot and online public transportations causes many roads in Bandung are now too narrow to accommodate those vehicles. The government of Bandung try to increase the quality and quantity of the public transportations such as DAMRI bus to reduce the congestion. For example, every bus DAMRI is now equipped with AC. However, the good scheduling of bus DAMRI has also very important to improve the service quality of DAMRI bus. Looking at to the developed countries, i.e. Japan, most people used the public transportations and using the private vehicles are less popular. The most important reason for that choice is the accuracy and certainty of bus arrivals and they avoid from the congestion. All efforts are done in order to increase the use of public transportations, especially the use of DAMRI bus.

This paper offers a study concerning the scheduling of bus DAMRI in Bandung. Our study is based on the Max-plus algebra theory. The 90s Max-Plus algebra theory can be used in modelling, analysis and controlling in transportation networks, manufacturing, communication networks and systems...
computers [1-8]. For developing the DAMRI bus schedule in Bandung, we choose two DAMRI bus routes those are Dipatiukur - Leuwi Panjang line and the Ledeng - Leuwi Panjang line. Those routes are transformed into the graph. Here the nodes of graph represent the bus stops and the arcs of graph represent the travel time between the bus stops. In the developed model, eigenvalues and eigenvectors are calculated and became the main elements in the bus DAMRI scheduling model. The eigenvalues and eigenvectors are calculated using the Scilab 5.3.3 and Max-Plus Toolbox Algebra.

2. Method

In ordinary algebra two operations are introduced, those are "+" and "x" which are known as addition and multiplication. In Max-plus algebra two operations are also introduced, those are $\oplus$ and $\otimes$ which are known as maximum and addition. Domain and those two operations in the Max-plus algebra are defined as follows [9].

Domain of the Algebra Max-Plus is $\mathbb{R} \cup \{-\infty\}$ and is equipped operation $\oplus$ and $\otimes$.

$(\mathbb{R}, \oplus, \otimes)$ is called Max-Plus Algebra if $x \oplus y = \max (x, y)$ and $x \otimes y = x + y$, $\forall x, y \in \mathbb{R}$.

Operations $\oplus$ and $\otimes$ on $\mathbb{R} \cup \{-\infty\}$ can be expanded to operations in the matrix as follows:

Given $R_{\max} = \{ A = (A_{ij}), B = (B_{ij}) | A_{ij}, B_{ij} \in R_{\max} = 1, 2, \ldots, n \text{ and } j = 1, 2, \ldots, m\}$

- $(A \oplus B)_{ij} = A_{ij} \oplus B_{ij}$
- $(\alpha \otimes A)_{ij} = \alpha \otimes A_{ij}, \forall \alpha \in R_{\max}$
- If $A \in R_{\max}^{nxn}, B \in R_{\max}^{nxn}$, then $((A \otimes B)_{ij} = \otimes_{k=1}^{p} A_{ik} \otimes B_{kj}$
- $\text{Trace}(A) = \max(A_{11}, A_{22}, \ldots, A_{nn}), \forall A \in R_{\max}^{nxn}$

Defining eigenvalues and eigenvectors in Max-Plus algebra is different from ordinary algebra. The definition of eigenvalues and eigenvectors in Max-Plus Algebra is as follows [4]:

Given $A \in R_{\max}^{nxn}, V = (v_1, v_2, \ldots, v_n)^T \in R_{\max}^n$ with $V \neq (-\infty, -\infty, \ldots, -\infty)^T$. $\lambda$ is called the max-plus eigenvalue of matrix $A$ if: $A \otimes V = \lambda \otimes V$.

**Theorem 2.1**: $\lambda \in R_{\max}$ is called the Max-Plus Algebra eigenvalue of the matrix $A$ if

$$\lambda = \otimes_{k=1}^{n} \frac{1}{k} (\text{Trace} (A \otimes^k))$$

With $A \otimes^k = A \otimes A \otimes \ldots \otimes A$ $k$ times and $\text{trace}(A \otimes^k)$ is the maximum value of the matrix diagonal $A \otimes^k$ [10].

3. Results and discussion

3.1. Determination graf of routes bus DAMRI in Bandung

In this paper, we choose two DAMRI bus routes in Bandung. Those two bus DAMRI routes are:

- **Line 1** (Dipatiukur - Leuwi Panjang). Departure: Dipatiukur - Ir. H. Juanda - Merdeka - Braga - Suniaraja - Pasar Baru - Otista - Jl. Peta - Kopo - Leuwi Panjang. Arrival: Leuwi Panjang - Kopo - Pasir Koja - Astana Anyar - Gardu Jati - Kebon Jati - Train Station - Wastu Kencana - Ir. H. Juanda – Dipatiukur.

- **Line 2** (Ledeng - Leuwi Panjang). Departure: Ledeng - Setiabudhi - Sukajadi – Perintis Kemerdekaan - Braga - Suniaraja - Pasar Baru - Otista - Jl. Peta - Kopo - Leuwi Panjang. Arrival: Leuwi Panjang - Kopo – Pasir Koja - Astana Anyar - Gardu Jati – Pasir Kaliki - Sukajadi - Setiabudhi – Ledeng.

The two DAMRI bus routes will go through or go to several stops where the passengers can go up and go down, and also depart or stop at the the final detination or terminal. In this study, we consider 16 bus stops (shelters) and 3 terminals (final points), although in practice it is difficult to distinguish the bus stops and the terminals because bus DAMRI can pick up or drop off the passangers every where.
The bus stops are Ledeng Terminal (LDG), Stopping Point in front of the NHI Tourism College (NHI), Paris Van Java Stop (PVJ), Hasan Sadikin Hospital (RSH) Stop, Stop on Perintis Kemerdekaan (PRS), Stop on Jalan Merdeka (MRD), Stop Point in front of Superindo Dago (SPR), Stop Point in front of SMAN 1 Dago (SM1), Final Stop in Dipatiukur (DU), Stop Point at Jalan Pasir Kaliki (PK), Stop Point near East Station (STT), Stop in front of Bank Indonesia (BI), Stop Point opposite SMA N 4 Bandung (SM4), Stop Point opposite Pasar Baru (PSB), Stop Point on Astana Anyar Street (AAN), Bus Stop on Map (PT), Bus Stop opposite Immanuel Hospital (IMN), Stopping Point in front of Babakan Tarogong SDN (SDN), and Leuwi Panjang Terminal (LWP). Those routes are represented in a graph.

In the graph, 19 bus stops and terminals are represented as nodes as shown in figure 1. From the two DAMRI bus routes, there are several meeting points allowing passengers to change lines/routes, those are LWP, SDN, PT, IMN, PSB, AAN, SM4, BI, PRS. The passengers are in those meeting points can move from one line to another line.

In the developed graph, the travel time data between nodes (vertices) are obtained by doing of the field surveys. The travel time written in table 1 (in minutes), is the average travel time obtained from five surveys calculations. Allocation or the number of bus DAMRI operated on those routes are obtained from internet.

| No. | Variable | From    | To     | Time  | Number of Buses |
|-----|----------|---------|--------|-------|-----------------|
| 1   | $x_1$    | DU      | SM1    | 3.28  | 1               |
| 2   | $x_2$    | SM1     | SPR    | 5.08  | 1               |
| 3   | $x_3$    | SPR     | MRD    | 4.63  | 1               |
| 4   | $x_4$    | MRD     | BI     | 1.07  | 1               |
| 5   | $x_5$    | BI      | PSB    | 4.13  | 2               |
| 6   | $x_6$    | PSB     | PT     | 13.10 | 2               |
| 7   | $x_7$    | PT      | SDN    | 4.38  | 2               |
| 8   | $x_8$    | SDN     | LWP    | 1.72  | 4               |
| 9   | $x_9$    | LWP     | SDN    | 3.57  | 4               |
| 10  | $x_{10}$ | SDN     | IMN    | 3.22  | 4               |
| 11  | $x_{11}$ | IMN     | AAN    | 7.62  | 2               |
| 12  | $x_{12}$ | AAN     | SM4    | 3.13  | 2               |
| 13  | $x_{13}$ | SM4     | STT    | 7.05  | 2               |
| 14  | $x_{14}$ | STT     | PRS    | 2.38  | 1               |
| 15  | $x_{15}$ | PRS     | SPR    | 6.30  | 1               |
| 16  | $x_{16}$ | SPR     | SM1    | 3.13  | 1               |
| 17  | $x_{17}$ | SM1     | DU     | 5.57  | 1               |
| 18  | $x_{18}$ | LDG     | NHI    | 6.77  | 1               |
| 19  | $x_{19}$ | NHI     | PVJ    | 10.08 | 1               |
| 20  | $x_{20}$ | PVJ     | RSH    | 12.32 | 1               |
| 21  | $x_{21}$ | RSH     | PRS    | 12.15 | 1               |
| 22  | $x_{22}$ | PRS     | BI     | 1.03  | 2               |
| 23  | $x_{23}$ | BI      | PSB    | 3.68  | 2               |
| 24  | $x_{24}$ | PSB     | PT     | 13.52 | 2               |
| 25  | $x_{25}$ | PT      | SDN    | 3.65  | 2               |
| 26  | $x_{26}$ | SDN     | LWP    | 2.02  | 4               |
| 27  | $x_{27}$ | LWP     | SDN    | 3.30  | 4               |
| 28  | $x_{28}$ | SDN     | IMN    | 3.62  | 4               |
| 29  | $x_{29}$ | IMN     | AAN    | 8.72  | 2               |
| 30  | $x_{30}$ | AAN     | SM4    | 2.87  | 2               |
| 31  | $x_{31}$ | SM4     | PK     | 2.87  | 2               |
| 32  | $x_{32}$ | PK      | RSH    | 5.55  | 1               |
| 33  | $x_{33}$ | RSH     | PVJ    | 5.45  | 1               |
| 34  | $x_{34}$ | PVJ     | NHI    | 13.07 | 1               |
| 35  | $x_{35}$ | NHI     | LDG    | 6.07  | 1               |

To determine the Eigen values and Eigen vectors, synchronization rules are first applied. It is important to ensure that passengers can move from one route to another route as soon as possible.
3.2. Model formation

Based on the synchronization rules that have been written above, it can be written into the Max-Plus Algebra model as follows:

- $x_1(k + 1) = [5.57 \otimes x_1(k)]$
- $x_2(k + 1) = [3.28 \otimes x_1(k)]$
- $x_3(k + 1) = [5.08 \otimes x_2(k)] \oplus [6.30 \otimes x_1(k)]$
- $x_4(k + 1) = [4.63 \otimes x_3(k)]$
- $x_5(k + 1) = [1.07 \otimes x_4(k)] \oplus [1.03 \otimes x_2(k - 1)]$
- $x_6(k + 1) = [4.13 \otimes x_5(k - 1)] \oplus [3.68 \otimes x_3(k - 1)]$
- $x_7(k + 1) = [13.10 \otimes x_6(k - 1)] \oplus [13.52 \otimes x_4(k - 1)]$
- $x_8(k + 1) = [4.38 \otimes x_7(k - 1)] \oplus [3.65 \otimes x_5(k - 1)]$
- $x_9(k + 1) = [1.72 \otimes x_8(k - 3)] \oplus [2.02 \otimes x_6(k - 3)]$
- $x_{10}(k + 1) = [3.57 \otimes x_9(k - 3)] \oplus [3.30 \otimes x_7(k - 3)]$
- $x_{11}(k + 1) = [3.22 \otimes x_{10}(k - 3)] \oplus [3.62 \otimes x_8(k - 3)]$
- $x_{12}(k + 1) = [7.62 \otimes x_{11}(k - 1)] \oplus [8.72 \otimes x_9(k - 1)]$
- $x_{13}(k + 1) = [3.13 \otimes x_{12}(k - 1)]$
- $x_{14}(k + 1) = [7.05 \otimes x_{13}(k - 1)]$
- $x_{15}(k + 1) = [2.38 \otimes x_{14}(k)] \oplus [12.15 \otimes x_2(k)]$
- $x_{16}(k + 1) = [6.30 \otimes x_{15}(k)]$
- $x_{17}(k + 1) = [3.13 \otimes x_{16}(k)]$
- $x_{18}(k + 1) = [6.07 \otimes x_{35}(k)]$
- $x_{19}(k + 1) = [6.77 \otimes x_{18}(k)]$
- $x_{20}(k + 1) = [10.08 \otimes x_{35}(k)]$
- $x_{21}(k + 1) = [12.32 \otimes x_{20}(k)] \oplus [5.55 \otimes x_{32}(k)]$
- $x_{22}(k + 1) = [12.15 \otimes x_{21}(k)] \oplus [1.80 \otimes x_{14}(k)]$
- $x_{23}(k + 1) = [1.07 \otimes x_{4}(k)] \oplus [1.03 \otimes x_2(k - 1)]$
- $x_{24}(k + 1) = [4.13 \otimes x_{25}(k - 1)] \oplus [3.68 \otimes x_3(k - 1)]$
- $x_{25}(k + 1) = [13.10 \otimes x_6(k - 1)] \oplus [13.52 \otimes x_4(k - 1)]$
- $x_{26}(k + 1) = [4.38 \otimes x_7(k - 1)] \oplus [3.65 \otimes x_5(k - 1)]$
- $x_{27}(k + 1) = [1.72 \otimes x_8(k - 3)] \oplus [2.02 \otimes x_6(k - 3)]$
- $x_{28}(k + 1) = [3.57 \otimes x_9(k - 3)] \oplus [3.30 \otimes x_7(k - 3)]$
- $x_{29}(k + 1) = [3.22 \otimes x_{10}(k - 3)] \oplus [3.62 \otimes x_8(k - 3)]$
- $x_{30}(k + 1) = [7.62 \otimes x_{11}(k - 1)] \oplus [8.72 \otimes x_9(k - 1)]$
- $x_{31}(k + 1) = [2.87 \otimes x_{30}(k - 1)]$
- $x_{32}(k + 1) = [2.87 \otimes x_{31}(k - 1)]$
- $x_{33}(k + 1) = [5.55 \otimes x_{32}(k - 1)]$
- $x_{34}(k + 1) = [5.45 \otimes x_{33}(k - 1)]$
- $x_{35}(k + 1) = [13.07 \otimes x_{34}(k - 1)]$

From the above equation, the model can be expressed in the general form of the max-plus algebra model as follows: $x(k + 1) = \bigoplus_{p=1}^{M} (A_p \otimes x(k + 1 - p))$. In the Max-plus algebra model above, $A_p$ is a matrices $n \times n$, $n$ is the number of departure variables. Matrices $A_p$ is matrices that correspond to $x(k + 1 - p)$, $M$ is the maximum number of bus allocations per departure path variable. In the DAMRI case in Bandung, the departure variable used is 35. The model can be stated as follows: $x(k + 1) = (A_1 \otimes x(k)) \oplus (A_2 \otimes x(k - 1)) \oplus (A_3 \otimes x(k - 2)) \oplus (A_4 \otimes x(k - 3))$. The above model can be simplified again into the general form of Algebra Max-Plus as follows: $x(k + 1) = \tilde{A} \oplus \tilde{x}(k)$.
3.3. Scheduling design
In developing of the bus DAMRI bus schedule, it is important to find the eigenvalues and the Max-plus algebra eigenvector vectors from the matrix. The eigen values and Eigen vectors of a Max-plus matrix are calculated by the procedures as explained in section 2.

![Graphic image of two DAMRI Routes in Bandung.](image)

However, because the matrix is quite large, which is 14, which is quite difficult to do manually. Then the determination eigenvalue and eigenvector of the matrix will use Scilab which is supported by Algebra Max-Plus Toolbox. By performing the algorithm performed with the help of Scilab and Max-Plus Toolbox Algebra, a max-algebra eigenvector vector is obtained, and the Max-plus algebra eigenvalue for the matrix is 3,7492308.
The Max-Plus Algebra eigenvalue shows the performance of the bus departure scheduling system, which means that every 3,7492308 minutes departs at each stop. Value Eigen also be periodic values for each departure point of dismissal / stop in the design of the scheduling. This value can be added by waiting time to raise and lower passengers at each shelter or bus stop if needed for optimal scheduling accuracy.

As for the Max-plus Algebra eigenvector vector, the results listed in the Scilab application need to be reduced to vectors with the help of Scilab as well. The results of the eigenvectors are as follows:

\[ \begin{align*} 
\| \mathbf{x}(k+1) & = \mathbf{A}^{(k+1)} \otimes \mathbf{v}. 
\end{align*} \]

Because bus departures are periodic and pre-departure times have been determined, DAMRI bus departures can be arranged for subsequent departures, with evolution: \( \mathbf{x}(k+1) = \mathbf{A}^{(k+1)} \otimes \mathbf{v} \).

If the operation of the DAMRI bus starts at 05.00, it means the first departing on the network is \( \mathbf{x}_{32} \) or the departure from PK to RSH, which is at 5:00. Departure \( \mathbf{x}_{1}(0) \) is 05: 45.2185, departure \( \mathbf{x}_{2}(0) \) is 05: 44.7493, and so on.

4. Conclusions
The conclusions from the results of modelling of bus DAMRI bus in Bandung is the eigen value \( \lambda = 3,7492308 \), indicates the periodic departure of the bus each stopping point / stop / terminal. It means that the performance of the scheduling system is quite good, that is every 3.7492308 minutes. Notes that the eigenvector shows the initial departure time. It is should be noted that in this paper, the terminal or the final stop for the bus route is the Ledeng Terminal, Leuwii Panjang and Dipatiukur Terminal.

Suggestion to the Department of Transportation of Bandung that it should be discipline at each DAMRI bus stop or terminal, i.e. picking up and dropping off the passengers are only in the bus stops in order to guarantee the accuracy and certainty of bus arrival.

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