Nucleon polarizability contribution to the hydrogen
Lamb shift and hydrogen – deuterium isotope shift

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Abstract

The correction to the hydrogen Lamb shift due to the proton electric and magnetic
polarizabilities is expressed analytically through their static values, which are known
from experiment. The numerical value of the correction to the hydrogen 1S state is
$-71 \pm 11 \pm 7$ Hz. Correction to the H-D 1S-2S – isotope shift due to the proton and
neutron polarizabilities is estimated as $53 \pm 9 \pm 11$ Hz.

1. High experimental precision attained in the hydrogen and deuterium spectroscopy
(see, e.g., [1, 2]) stimulates considerable theoretical activity in this field. In particular,
the deuteron polarizability contribution to the Lamb shift in deuterium was calculated in
\cite{3}–\cite{10}. A special feature of these corrections is that they contain logarithm of the ratio
of a typical nuclear excitation energy to the electron mass, $\ln \bar{E}/m_e$.

In the present note we consider the problem of the proton polarizability correction to
the Lamb shift in hydrogen. The typical excitation energy for the proton $\bar{E}_p \sim 300$ MeV is
large as compared to other nuclei (to say nothing of the deuteron). So, $\ln \bar{E}_p/m_e$ is not just
a mere theoretical parameter, it is truly large, about $6 – 7$, which makes the logarithmic
approximation quite meaningful quantitatively.

In our calculation we follow closely the approach of \cite{8}. In particular, we use the gauge
$A_0 = 0$ for virtual photons, so that the only nonvanishing components of the photon
propagator are $D_{im} = d_{im}/k^2$, $d_{im} = \delta_{im} - k_i k_m/\omega^2$ ($i, m = 1, 2, 3$). The electron-proton
forward scattering amplitude, we are interested in, is

$$T = 4\pi i \alpha \int \frac{d^4 k}{(2\pi)^4} D_{im} D_{jn} \gamma_i (\hat{l} - \hat{k} + m_e) \gamma_j \frac{\gamma_i (\hat{l} - \hat{k} + m_e) \gamma_j}{k^2 - 2l_k} M_{mn}. \tag{1}$$

Here $l_\mu = (m_e, 0, 0, 0)$ is the electron momentum. The nuclear-spin independent Compton
forward scattering amplitude, which is of interest to us, can be written as

$$M = \bar{\alpha}(\omega^2, k^2) E^* E + \bar{\beta}(\omega^2, k^2) B^* B = M_{mn} e_m e_n^*, \tag{2}$$

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where $\bar{\alpha}$ and $\bar{\beta}$ are the nuclear electric and magnetic polarizabilities, respectively. The structure $\gamma_i(l - \hat{k} + m_e)\gamma_j$ in (4) reduces to $-\omega \delta_{ij}$. Perhaps, the most convenient succession of integrating expression (4) is as follows: the Wick rotation; transforming the integral over the Euclidean $\omega$ to the interval $(0, \infty)$; the substitution $k \to k \omega$. Then the integration over $\omega$ is easily performed with the logarithmic accuracy:

$$\int_0^\infty \frac{d\omega^2}{\omega^2 + 4m_e^2/(1+k^2)^2} \left[ (3 + 2k^2 + k^4)\alpha(-\omega^2, -\omega^2k^2) - 2k^2\beta(-\omega^2, -\omega^2k^2) \right] \left[ (3 + 2k^2 + k^4)\bar{\alpha}(0) - 2k^2\bar{\beta}(0) \right] \ln \frac{\bar{E}^2}{m_e^2}.$$  

(3)

The crucial point is that, within the logarithmic approximation, both polarizabilities $\bar{\alpha}$ and $\bar{\beta}$ in the lhs can be taken at $\omega = 0$, $k^2 = 0$. The final integration over $d^3k$ is trivial.

The resulting effective operator of the electron-proton interaction (equal to $-T$) can be written in the coordinate representation as

$$V = -\alpha m_e \left[ 5\bar{\alpha}(0) - \bar{\beta}(0) \right] \ln \frac{\bar{E}}{m_e} \delta(r).$$  

(4)

This expression applies within the logarithmic accuracy for arbitrary nuclei. It should be mentioned that a similar relation for hydrogen was obtained in [11], our numerical result agrees with theirs. On the other hand, the formula derived in [12] for an arbitrary nucleus differs from ours (4) by the absence of the magnetic polarizability $\bar{\beta}(0)$ only. The corresponding estimate presented in [3] differs from our result by the factor at $\bar{\alpha}_p(0)$ (2 instead of 5) and by the absence of $\bar{\beta}_p(0)$. Besides, the authors of [3] (and of [11]) choose the inverse nucleon radius, instead of the excitation energy, for the logarithmic cut-off in the corresponding formulae.

The experimental data on the proton polarizabilities, which follow from the Compton scattering, can be summarized as follows [13]:

$$\bar{\alpha}_p(0) + \bar{\beta}_p(0) = (14.2 \pm 0.5) \times 10^{-4} \text{ fm}^3;$$

$$\bar{\alpha}_p(0) - \bar{\beta}_p(0) = (10.0 \pm 1.5 \pm 0.9) \times 10^{-4} \text{ fm}^3.$$  

(5)

Now,

$$5 \bar{\alpha}_p(0) - \bar{\beta}_p(0) = 2 \left[ \bar{\alpha}_p(0) + \bar{\beta}_p(0) \right] + 3 \left[ \bar{\alpha}_p(0) - \bar{\beta}_p(0) \right] = (58.4 \pm 5.3) \times 10^{-4} \text{ fm}^3.$$  

(6)

The errors are added in quadratures.

Finally, at $\bar{E}_p \sim 300$ MeV the proton polarizability correction to the hydrogen 1S state is

$$- 71 \pm 11 \pm 7 \text{ Hz}.$$  

(7)

Here the first error is that of the logarithmic approximation, which we estimate as 15%. The second one originates from the values of the polarizabilities.

2. Though being calculated rather accurately from the theoretical point of view, the correction (6) to the hydrogen Lamb shift is too small to be detected experimentally.
However, the corresponding effect in the H-D 1S-2S isotope shift is comparable with the experimental accuracy (150 Hz) attained for it [2]. This effect is comparable also with the theoretical precision (70 Hz) for the contribution of the deuteron polarizability due to relative motion of the proton and neutron to the deuterium Lamb shift [10].

The deuteron is a weakly bound system. Then it is natural to assume that deuteron polarizability is the sum of the polarizability due to relative motion of the nucleons and the internal polarizabilities of the nucleons. Simple physical arguments, supported by model estimates, demonstrate that nucleon polarizabilities in deuteron coincide with polarizabilities for free nucleons, well within the accuracy of our logarithmic approximation. Therefore, in the corresponding effect in the H-D isotope shift the proton contributions cancel, and we are left with that of a neutron (with opposite sign)

\[
\delta V_{\text{H-D}} = \alpha_m e \left( 5\alpha_n(0) - \beta_n(0) \right) \ln \frac{E_n}{m_e} \delta(r).
\]  

(8)

The neutron electric polarizability is [14]

\[
\alpha_n(0) = (9.8^{+1.9}_{-2.3}) \times 10^{-4} \text{ fm}^3;
\]

Its magnetic polarizability \(\beta_n\) is not known. Under the assumption that \(\beta_n\) does not change the result considerably, this contribution to the difference between the Lamb shifts of the ground states of hydrogen and deuterium is

\[
61 \pm 10 \pm 12 \text{ Hz}. \tag{9}
\]

The corresponding contribution to the isotope shift between hydrogen and deuterium 1S-2S transition due to the internal polarizabilities of the nucleons can be estimated as

\[
53 \pm 9 \pm 11 \text{ Hz}. \tag{10}
\]

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