Why the Singularity Cannot Happen

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Abstract

The concept of a Singularity as described in Ray Kurzweil’s book cannot happen for a number of reasons. One reason is that all natural growth processes that follow exponential patterns eventually reveal themselves to be following S-curves thus excluding runaway situations. The remaining growth potential from Kurzweil’s “knee”, which could be approximated as the moment when an S-curve pattern begins deviating from the corresponding exponential, is a factor of only one order of magnitude greater than the growth already achieved. A second reason is that there is already evidence of a slowdown in some important trends. The growth pattern of the U.S. GDP is no longer exponential. Had Kurzweil been more rigorous in his fitting procedures, he would have recognized it. Moore’s law and the Microsoft Windows operating systems are both approaching end-of-life limits. The Internet rush has also ended — for the time being — as the number of users stopped growing; in the western world because of saturation and in the underdeveloped countries because infrastructures, education, and the standard of living there are not yet up to speed. A third reason is that society is capable of auto-regulating runaway trends as was the case with deadly car accidents, the AIDS threat, and rampant overpopulation. This control goes beyond government decisions and conscious intervention. Environmentalists who fought nuclear energy in the 1980s, may have been reacting only to nuclear energy’s excessive rate of growth, not nuclear energy per se, which is making a comeback now.

What may happen instead of a Singularity is that the rate of change soon begins slowing down. The exponential pattern of change witnessed up to now dictates more milestone events during year 2025 than witnessed throughout the entire 20th century! But such events are already overdue today. If, on the other hand, the change growth pattern has indeed been following an S-curve, then the rate of change is about to enter a declining trajectory; the baby boom generation will have witnessed more change during their lives than anyone else before or after them.
1. Background

In 2005 together with four other members of the editorial board of Technological Forecasting & Social Change I was asked to review Ray Kurzweil’s book The Singularity Is Near. The task dragged me back into a subject that I was familiar with. In fact, ten years earlier I had thought I was the first to have discovered it only to find out later that a whole cult with increasing number of followers was growing around it. I took my distance from them because at the time they sounded nonscientific. I published on my own adhering to a strictly scientific approach (Modis 2002a and Modis 2003). But to my surprise the respected BBC television show HORIZON that became interested in making a program around this subject found even my publications “too speculative”. In any case, for the BBC scientists the word singularity is reserved for mathematical functions and phenomena such as the big bang.

Kurzweil’s book constitutes a most exhaustive compilation of “singularitarian” arguments and one of the most serious publications on the subject. And yet to me it still sounds nonscientific. Granted, the names of many renowned scientists appear prominently throughout the book, but they are generally quoted on some fundamental truth other than the direct endorsement of the so-called singularity. For example, Douglas Hofstadter is quoted to have mused that “it could be simply an accident of fate that our brains are too weak to understand themselves.” Not exactly what Kurzweil says. Even what seems to give direct support to Kurzweil’s thesis, the following quote by the celebrated information theorist John von Neumann “the ever accelerating process of technology…gives the appearance of approaching some essential singularity” is significantly different from saying “the singularity is near”. Neumann’s comment strongly hints at an illusion whereas Kurzweil’s presents a far-fetched forecast as a fact.

What I want to say is that Kurzweil and the singularitarians are indulging in some sort of pseudo-science, which differs from real science in matters of methodology and rigor. They tend to overlook rigorous scientific practices such as focusing on natural laws, giving precise definitions, verifying the data meticulously, and estimating the uncertainties.

The work I present here uses a number of science-based approaches to argue against the possibility that the kind of singularity described by Kurzweil in his book will take place during the 21st century. I will concentrate on the near future because horizons of several hundred years permit and are more appropriate for fantasy scenarios that tend to satisfy the writer’s urge for sci-fi prose.

2. There Are No Exponentials. There Are only S-Curves

2.1 Exponential vs. S-Curve

The law describing natural growth has been put into mathematical equations called growth functions. The simplest mathematical growth function is the so-called a logistic. It is derived from the law, which states that the rate of growth is proportional to both the amount of growth already accomplished and the amount of growth remaining to be accomplished. If either one of these quantities is small, the rate of growth will be small. This is the case at the beginning and at the end of the process. The rate is greatest in the middle, where both the growth accomplished and the growth remaining are sizable. This natural law described with words here has been cast in a differential equation (the logistic equation) the solution of which gives rise to an S-shaped pattern (S-curve). In Equation (1) below $M$ is the value of the final ceiling, $t_o$ is the time of the midpoint and $\alpha$ reflects the steepness of the rising slope.
It is easy to see that for $t$ large and positive the population $X(t)$ tends to $M$. Similarly for $t$ large and negative the expression reduces to a simple exponential.

It is a remarkably simple and fundamental law. It has been used by biologists to describe the growth in competition of a species population, for example, the number of rabbits in a fenced off grass field. It has also been used in medicine to describe the diffusion of epidemic diseases. J. C. Fisher and R. H. Pry refer to the logistic function as a diffusion model and use it to quantify the spreading of new technologies into society (Fisher and Pry 1971). One can immediately see how ideas or rumors may spread according to this law. Whether it is ideas, rumors, technologies, or diseases, the rate of new occurrences will always be proportional to how many people have it and to how many don’t yet have it.

The analogy has also been pushed to include the competitive growth of inanimate populations such as the sales of a new successful product. In the early phases, sales go up in proportion to the number of units already sold. As the word spreads, each unit sold brings in more new customers. Sales grow exponentially. It is this early exponential growth which gives rise to the first bend of the S-curve. Business looks good. Growth is the same percentage every year — exponential growth — and hasty planners prepare their budgets that way. Growth, however, cannot be exponential. Explosions are exponential. Natural growth follows S-curves, but all S-curves behave like exponentials early on. Kurzeil himself addresses this issue, which he calls “The Criticism from Malthus” toward the end of his book. He admits that the exponential patterns he publicizes will eventually turn into S-curves, but dismisses the fact because, as he claims, this will happen so far into the future — after the Singularity — that for all practical purposes it becomes irrelevant. (Elsewhere, Kurzeil acknowledges that there are smaller S-curves that saturate early, but argues that they are replaced by other small S-curves thus cascading indefinitely. He does not seem to be aware of the fact that there is a fractal aspect to such cascades and constituent S-curves are bound by envelope S-curves, which themselves saturate with time, see discussion in Section 3.3 below.)

Let us see whether it is possible to estimate when exponential trends can be expected to turn into S-curves. First let us see at what time the S-curve deviates from the exponential pattern in a significant way, see Figure 1. Table I below quantifies the deviation between a logistic and the corresponding exponential pattern as a fraction of the S-curve’s final ceiling. By “corresponding” exponential I mean the limit of Equation (1) as $t \to -\infty$.

$$X(t) = \frac{M}{1 + e^{-\alpha(t-t_0)}} \quad (1)$$
Figure 1. The construction of a theoretical S-curve (gray line) and the exponential (thin black line) it reduces to as time goes backward. The formulae used are shown in the graph.

Table I: The Deviation between Exponential and S-Curve Patterns as a Function of how much the S-Curve Has Proceeded to Completion

| Deviation | Penetration |
|-----------|-------------|
| 11.1%     | 10.0%       |
| 12.2%     | 10.9%       |
| 13.5%     | 11.9%       |
| 15.0%     | 13.0%       |
| 16.5%     | 14.2%       |
| 18.3%     | 15.4%       |
| 20.2%     | 16.8%       |
| 22.3%     | 18.2%       |
| 24.7%     | 19.8%       |
| 27.3%     | 21.4%       |
| 30.1%     | 23.1%       |
| 33.3%     | 25.0%       |
| 36.8%     | 26.9%       |
| 40.7%     | 28.9%       |
| 44.9%     | 31.0%       |
| 49.7%     | 33.2%       |
| 54.9%     | 35.4%       |
| 60.7%     | 37.8%       |
| 67.0%     | 40.1%       |
| 74.1%     | 42.6%       |
| 81.9%     | 45.0%       |
| 90.5%     | 47.5%       |
| 100.0%    | 50.0%       |

In Table I we appreciate the size of deviation between exponential and S-curve patterns as a function of how much the S-curve has proceeded to completion. Obviously, beyond a certain point the difference becomes flagrant. When exactly this happens maybe subject to judgment so Table I is there to quantitatively help readers make up their mind. Most readers will agree that a 15% deviation between exponential and S-curve patterns is significant because it makes it clear that the two processes can no longer be confused. This happens when the S-curve that
corresponds to the exponential has reached about 13% of its ceiling level. In other words, the
future ceiling that caps this growth process is about 7 times today’s level.

This moment when an exponential pattern begins deviating significantly from an S-curve also
defines an upper limit for Kurzweil’s so called “exponential knee”. Of course, exponential
patterns do not have “knees” (this can be trivially demonstrated with a logarithmic plot where
the exponential pattern becomes a straight line from \( t = -\infty \) to \( t = +\infty \)). What Kurzweil sees as
the moment an exponential “abruptly” rises will move toward the future (or the past) as we
increase (or decrease) the vertical scale of a linear plot. But if we interpret Kurzweil’s knee as
the moment when a growth process still following an exponential pattern begins having very
serious impact on society — a subjective definition carrying large uncertainties — then there
will be at least a 7-fold increase remaining before the process stops growing.

It has been theoretically demonstrated that fluctuations of a chaotic nature begin making their
appearance as we approach the ceiling of an S-curve. This is evidence of the intimate
relationship that exists between growth and chaos (the logistic equation in discrete form
becomes the chaos equation) (Modis 2007). In an intuitive way these fluctuations can be seen
as the stochastic search of the system for equilibrium around a homeostatic level.

But it has been argued that fluctuations of a chaotic nature may also precede the steep rising
phase of the logistic (Modis and Debecker 1992). The intuitive understanding of these
fluctuations is “infant” mortality. In all species survival is uncertain during the early phases of
the life cycle.

Infant mortality and common sense can help us establish a lower limit for Kurzweil’s knee. Any
natural growth process that has achieved less than 10% of its final growth potential cannot
possibly have a serious impact on society. In fact 10% growth is usually taken as the limit of
infant mortality. A tree seedling of height less than 10% of the tree’s final size is vulnerable to
rabbits and other herbivores or simply to be stepped on by a bigger animal.

Below we look at some real cases. They all corroborate a lower limit of the order of 10%
below which the impact on society cannot be considered very serious.

2.2 US Oil Production

A real case is the production of oil in the United States, shown in Figure 2. Serious oil
production in the United States began in the second half of the 19th century. During the first one
hundred years or so cumulative oil production followed an exponential pattern. But soon it
became clear that the process followed a logistic growth pattern — S-curve — and rather
closely, see Figure 2. If we try to fit an exponential function to the data, we obtain a good fit —
comparable in quality with the logistic fit — only on the range 1859-1951. As we stretch this
period beyond 1951 the quality of the exponential fit progressively deteriorates. S-curve and
exponential begin diverging from the 1951 onward, which corresponds to around 20% penetration level of the S-curve.1

If we were to position a “knee” a la Kurzweil on this exponential pattern, it could by some
thinking be in the early 1930s time by which almost all horses had been replaced by cars
maximizing the demand for oil. At that time the penetration level of the S-curve was around
7%. Consequently on this growth process, which seemed exponential for the better part of one
hundred years, there is a ceiling waiting at about 14 times the knee’s production level.

1 The fitted exponential here is not the same as the exponential that the S-curve tends to as \( t \to -\infty \)
2.3 Moore’s Law

The celebrated Moore’s Law is a growth process that has been evolving along an exponential growth pattern for four decades. The number of transistors in Intel microprocessors has doubled every two years since the early 1970s. But it is now unanimously expected that this growth pattern will eventually turn into an S-curve and reach a ceiling. On page 63 of his book Kurzweil claims that Moore’s law is one of the many technological exponential trends whose knee we are approaching. But he also agrees that Moore’s law will reach the end of its S-curve before 2020. Moore himself agrees, “no physical thing can continue to change exponentially forever,” he says and positions an end for this phenomenon around 2015. But he still expects three more generations (we should mention that in 1995 Moore had consistently expected 5 more generations.)

It must be pointed out — particularly for those who claim that every time people thought a limit was reached in the past new ways were found to cram more transistors together — that in the very first formulation of Moore’s law in 1965 the doubling was every year. David House, an Intel executive, raised it to 18 months, and later in 1975 Moore himself raised it to two years. These successive adjustments may not constitute proof but the fact that we are dealing with an eventual S-curve cannot be disputed. Given that we are dealing with an S-curve, the slowing down must be gradual so that three generations may bring an overall increase with respect to today’s numbers by a factor smaller than $2^3 = 8$. But even if the factor is 8, today’s level (which Kurzweil argues is the exponential’s “knee”) corresponds to around 12.5% of the S-curve’s ceiling.

Figure 3 shows an S-curve fit on the data, which has been constrained to reach a ceiling by the late 2020s (a conservative constraint). The corresponding exponential is also shown as was done in Figure 1. The expected announcement by Intel of the Poulson processor in 2012 argues in favor of the S-curve and against the exponential trend.
2.4 World Population

The evolution of the world population during the 20th century followed an S-curve in an exemplary way. Figure 4 shows an agreement between data and curve that is astonishing if we consider the variety of birth rates and death rates around the world, the multitude of stochastic processes that impact the evolution of the world population such as epidemics, catastrophes, wars (twice at world level), important climate changes, etc., and on the other hand the simplicity of the curve’s description, namely only three parameters (plus a parameter for a pedestal in this case).
And yet again, the evolution of the world population has often been likened to an explosion following an exponential trend. Where could a “knee” for this exponential be positioned? Looking at Figure 4, any “knee” would have to be positioned before the 1980s by which time the trend significantly deviated from an exponential pattern.

By some historians the population explosion began in the West, around the middle of the 19th century. The number of people in the world had grown from about 150 million at the time of Christ to somewhere around 700 million in the middle of the 17th century. But then the rate of growth increased dramatically to reach 1.2 billion by 1850. In this case the exponential “knee” would have occurred when world population reached 8% of its final ceiling.

In the above three examples — Sections 2.2 to 2.4 — we have seen that the “knee” of the exponential curve tends to occur at a threshold situated between 7% and 13% of the ceiling of the corresponding S-curve, which translates to a factor of at most 14 between the level of the “knee” and the level of the final ceiling. This factor has been estimated rather conservatively and corroborates a corollary of natural-growth studies: infant mortality.

Such growth potential on any of the variables purported to contribute to a singularity by mid 21st century would fall short of becoming alarming.

3. There Is already Some Evidence for Saturation

3.1 The U.S. GDP

The evolution of the gross domestic product (GDP) in America in constant dollars had followed an exponential growth pattern for a while but has deviated from it some time ago and now has almost reached the midpoint of an S-curve. Kurzveil was too quick to pronounce its evolution as exponential. On logarithmic scale his straight wide band accommodated the gentle curving of the time-series data and his criterion of correlation — coefficient $r^2$ — was close enough to 1.0. But high correlation between two curves does not mean one is a good representation of the other. Just think of two straight lines, one almost vertical the other almost horizontal; they will be 100% correlated (correlation coefficient 1.0) and yet one will be a very poor representation of the other. A closer examination reveals that a logistic function fits better the evolution of the U.S. GDP than an exponential one, see Figure 5. And if we judge the fits by
Figure 5. Exponential (thin black line) and logistic (thick gray line) fits on the evolution of the real U.S. GDP. This graph can be directly compared with the one on Page 98 from Kurzweil’s *The Singularity Is Near*. 

Data source: U.S. Department of Commerce, Bureau of Economic Analysis.

their Chi Square ($\chi^2$), a more appropriate criterion than correlation coefficients, the logistic fit comes out three times better than the exponential one ($\chi^2$ of 1112 instead of 3250). Conclusion: the U.S. GDP will certainly not contribute to the building up toward a singularity event around 2045 because from 2013 onward its rate of growth will progressively slow down.

3.2 The End of the Internet Rush

One of the “explosive” variables in Kurzweil book is that of the diffusion of the Internet, but the graph on Page 79 of his book shows clear evidence that we are dealing with an S-curve developed about half-way to its ceiling. What is being witnessed instead is the end of the Internet rush (Modis 2005). In an article published in 2005 I demonstrate that the number of Internet users will not grow much in near future. In the US the ceiling of the S-curve has already been reached at 72% of the population, and in the E.U. it should not rise much above today’s 67%. In the rest of the world today’s 18% will grow to a ceiling of 33% in ten years.

It would be unreasonable to expect the percentage of the rest of the world to remain at this low level forever. The rest of the world includes such countries as Japan, Korea, Honk Kong, and Australia where the number of Internet users is already practically at maximum. But the rest of the world also includes Africa, China, and India, where one can be certain that the number of Internet users will eventually grow by a large factor. However, it will be some time before the necessary infrastructures are put in place there to permit large-scale Internet diffusion.

For the time being one may infer that the boom we have been witnessing in Internet expansion is over. The parts of the world that were ready for it have practically filled their niches whereas the parts of the world that were not ready for it need much preparatory work (infrastructures, nourishment, education, etc.) and will therefore grow slowly.

The final percentage of Internet users may also reflect cultural differences. A percentage of 72% in the US compared to 67% in the E.U. might partially reflect missing infrastructures in
some of the lesser-developed E.U. countries but most likely also reflects the different life styles. European society admits less change than American society. For example, there are fewer cars per inhabitant in Europe, and the Europeans never went to the moon. They will probably end up using the Internet somewhat less than the Americans.

We can make a rough estimate of when a follow-up Internet growth phase should be expected. To a first approximation logistics that cascade harmoniously show periods of low and high growth of comparable duration (Modis 2007). Accordingly, and given that Internet has had a decade of rapid growth, a decade of low growth can reasonably be expected before a new S-curve begins. Contrary to the image of an explosive uncontrollable growth process we are witnessing piecemeal growth with stagnating periods in between that offer fertile ground for control and adaptability.

In the next section we quantify the cascade of S-curves in a natural succession and the relationship between their life cycles.

3.3 Fractal Aspects of Natural Growth

Sustained growth is not a steady and uniform process. It consists of successive S-shaped steps, each of which represents a well-defined amount of growth. A new S-curve begins where the previous one left off. Every step is associated with a niche that opened following some fundamental change (a mutation, a major innovation, a technological break-through, etc.).

Successive growth stages depicted by cascading logistic curves may outline an overall growth process that is itself amenable to a logistic description. Two such examples from published work are the discovery of stable chemical elements, which spanned three centuries and can be broken down into four rather distinct regularly spaced growth phases (de Solla Price 1936), and the diffusion of Italian railways which came in three waves (Marchetti 1986).

The graph of Figure 6 has been constructed in a rigorous quantitative way (Modis 1994). Notice that the step size and the associated time span (life cycle) of the constituent S-curves first progressively increases but then progressively decreases going over a maximum step and longest life cycle around the middle of the envelope S-curve. Life cycles become longer during the high-growth period and shorter during the low-growth periods.

The Fractal Nature of an S-Curve

![Figure 6. A large S-curve decomposed into smaller ones.](image)
Table II: The Relation between the Shortening of Life Cycles and Saturation

| Life-cycle length (relative to longest) | Level of saturation (percent of ceiling) |
|----------------------------------------|------------------------------------------|
| 0.17                                   | 3.1                                      |
| 0.19                                   | 4.0                                      |
| 0.20                                   | 5.2                                      |
| 0.22                                   | 6.9                                      |
| 0.24                                   | 9.1                                      |
| 0.30                                   | 12.8                                     |
| 0.41                                   | 20.0                                     |
| 0.70                                   | 31.4                                     |
| 1.00                                   | 50.0                                     |
| 0.70                                   | 68.6                                     |
| 0.41                                   | 80.0                                     |
| 0.30                                   | 87.2                                     |
| 0.24                                   | 90.9                                     |
| 0.22                                   | 93.1                                     |
| 0.20                                   | 94.8                                     |
| 0.19                                   | 96.0                                     |
| 0.17                                   | 96.9                                     |

The phenomenon of shrinking product life cycles, an important concern of marketers, can be quantitatively linked to the saturation of the enveloping process. Table II relates the shortening of product life cycles to the level of saturation reached.

The idea that a growth process can, on closer examination, reveal similar but smaller cascading growth processes, suggests a fractal nature for the logistic curve. The implication is that further “zooming-in” may reveal an even finer structure of logistic cascades.

The fractal nature of logistics permits one to estimate the level of overall saturation from observing life cycle trends. This is a powerful approach. Table II, first published in Modis 1994, can be used by anyone to determine the position on the envelope curve from observations on constituent curves. For example, a non-specialist, such as a laborer loading boxes into trucks off the dock of a computer manufacturer, may notice that the names on the boxes change three times as frequently as they used to and deduce that the technology behind these products is more than 80% exhausted. This image may be naive, but the approach offers valuable insights for those situations in which the tracking of the overall envelope is rather imponderable, for example, the evolution of the Windows operating systems.

3.4 Windows Operating Systems

Microsoft has been regularly releasing new operating systems. New software developments triggered by hardware improvements make it necessary to introduce major changes to the operating systems. This is a typical characteristic of all new industries like microchips; they are mutational. But as the industry matures, “mutations” become rare and life cycles become longer. Figure 7 shows the evolution of the life cycle of Microsoft operating system as defined by the time to the next launching announcement.
Figure 7. Lifetimes of Windows Operating Systems as defined by the time period to the next release. Labels highlight the three longest-lived ones.

Data sources: Microsoft and Wikipedia.

Windows XP was the operating system with the longest life cycle. Vista’s life cycle was 57% of that of Windows XP and the life cycle of Windows 7 will be 53% if Windows 8 comes in 2012. From Table II we can estimate that the Windows technology will be between 68.6% and 80% exhausted by that time.

With Windows XP at the center of the envelope S-curve for Windows operating systems and the life cycle being symmetric to a first approximation, we can reasonably expect an end for this growth process by the late 2020s. This coincides with our previous estimate for the end of Moore’s law.

Just as with Internet users, here again we are facing an upcoming lull in the rate of growth of such “explosive” processes as microchip and PC operating system improvements. This lull should also last about 20 years, duration comparable to the duration of the rapid-growth phase of the envelope S-curve. From then (ca 2030) onward a new sequence of cascading S-curves may slowly enter the scene as per Figure 6. But once again, we have a hard time accommodating singular events due to “explosive” trends like these by mid 21st century.

4. Society Auto-Regulates Itself

There have been many documented cases where society has demonstrated a wisdom and control unsuspected by its members. In this section we will see four such examples portraying society as a super species capable of auto-regulating and safeguarding itself from runaway trends.

4.1 Car Accidents

Car accidents have received much attention and at times provoked emotional reactions. In the 1960s cars had been compared to murder weapons. To better understand the mechanisms at play
we must look at the history of car accidents. We need accurate data and the appropriate indicator. Deaths are better defined and recorded than lesser accidents. Moreover, the car as a public menace is a threat to society, which may “feel” the pain and try to keep them under control. Consequently, the number of deaths per one hundred thousand inhabitants per year becomes a better indicator than accidents per mile, or per car, or per hour of driving (Marchetti 1983).

The data shown in Figure 8 are for the United States starting at the beginning of the 20th century. What we observe is that deaths caused by car accidents grew along an exponential trend that led into an S-curve that reached a ceiling in the mid 1920s, when deaths reached twenty-four per one hundred thousand per year. From then onward car accidents have stabilized even though the number of cars continued to grow. A homeostatic mechanism seems to enter into action when this limit is reached, resulting in an oscillating pattern around the equilibrium position. The peaks may have produced public outcries for safety, while the valleys could have contributed to the relaxation of speed limits and safety regulations. What is remarkable is that for over sixty years there has been a persistent auto-regulation on car safety in spite of tremendous variations in car numbers and performance, speed limits, safety technology, driving legislation, and education.

Why the number of deaths is maintained constant and how society can detect excursions away from this level? Is it conceivable that someday car safety will improve to the point of reducing the level of automobile accidents to practically null the way it was before cars were invented? American society has tolerated this level of accidents for the better part of a century. A Rand analyst has described it as follows: “I am sure that there is, in effect, a desirable level of automobile accidents — desirable, that is, from a broad point of view, in the sense that it is a necessary concomitant of things of greater value to society.” (Williams 1958). Abolishing cars from the roads would certainly eliminate car accidents, but at the same time it would introduce more serious hardship to citizens.

Car Accidents in the US

![Car Accidents in the US](image)

Figure 8. The annual number of deaths from motor vehicle accidents per 100,000 population has been fluctuating around 24 since the mid 1920s. The peak in the late 1960s provoked a public outcry that resulted in legislation making seat belts mandatory. Data after 1980 show a decline in the number of deadly car accidents, but this is due the fact that travelers have been replacing the automobile by other means of transportation and in particular the airplane, see discussion in Modis 1992 and Modis 2002b.

Data source: Statistical Abstract of the United States.
A homeostatic equilibrium represents a state of well-being. It has its roots in nature, which develops ways of maintaining it. Individuals may come forward from time to time as advocates of an apparently well-justified cause. What they do not suspect is that they may be acting as unwitting agents to deeply rooted necessities for maintaining the existing balance, which would have been maintained in any case. An example is Ralph Nader’s crusade for car safety, *Unsafe at Any Speed*, published in the 1960s, by which time the number of fatal car accidents had already demonstrated a forty-year-long period of relative stability. But examining Figure 8 more closely, we see that the late 1960s show a relative peak in accidents, which must have been what prompted Nader to blow the whistle. Had he not done it, someone else would have. Alternatively, a timely social mechanism might have produced the same result; for example, an “accidental” discovery of an effective new car-safety feature.

Another such example of society’s ability to auto-regulate and safeguard itself is the spreading of AIDS in the United States.

4.2 The AIDS Niche

At the time of the writing of my first book *Predictions*, AIDS was diffusing exponentially claiming a progressively bigger share of the total number of deaths every year, and forecasts ranged from pessimistic to catastrophic. Alarmists worried about the survival of the human species. But finally the AIDS “niche” in the U.S. turned out to be far smaller than that feared by most people. In this case the variable studied was death victims from AIDS as a percentage of all deaths.

The S-curve I fitted on the data up to and including 1988 had indicated a growth process that would be almost complete by 1994. The ceiling for the disease’s growth, projected as 2.3% of all deaths was projected to be reached in the late 1990s, see Figure 9. In other words my conclusion at that time was that a place had been reserved for AIDS in American society just above the 2% level of all deaths.

The little circles in the figure confirm the S-curve trend and the completion of this microniche by 1995. By the late 1990s questions were being raised why forecasts had overestimated the AIDS threat by so much.

There seems to have been a mechanism that limited AIDS in a natural way even in the absence of adequate medication. As if there were other, more important causes of death. This mechanism may have reflected the control exercised by American society through subconscious collective concern. The natural-growth pattern that the disease followed from its beginning correctly anticipated that AIDS would not spread uncontrollably. Eventually of course there would be effective medication for the disease and the number of victims would decline. Those who had predicted imminent doom in the absence of a miracle drug in the 1980s had failed to take into account the natural competitive mechanisms which regulate the split of the overall death rate among the different causes of death, safeguarding all along an optimum survival for society.

After 1995 the number of deaths from AIDS progressively declined in what could be described as another natural process — a downward S-curve — reflecting the development of progressively effective medication. What Figure 9 spells out as shown is society’s ability to safeguard its wellbeing in the absence of effective medication and miracle drugs.

A more subtle example of society’s ability to auto-regulate and safeguard itself is primary-energy substitution and the advent of nuclear energy.
The AIDS Niche in the US

Figure 9. Deaths from AIDS in the United States. The ceiling of the S-curve fitted on data up to and including 1988 (black dots) is 2.3. Data from years 1989–1998 (open circles) confirm that the AIDS niche in the United States was essentially completed by 1995 (Modis 1992 and Modis 2002b). 

Data source: HIV/AIDS Surveillance, Centers for Disease Control, U.S. Department of Health and Human Services, Atlanta, GA.

4.3 Nuclear Energy

During the last one hundred years, wood, coal, natural gas, and nuclear energy have been the main protagonists in supplying the world with energy. More than one energy source is present at any time, but the leading role passes from one to the other. Other sources of energy (such as hydro, wind, etc.) have been left out because they command too small a market share.

In the early nineteenth century and before, most of the world’s energy needs were satisfied through wood burning and to a lesser extent animal power not considered here, see Figure 10. The substitution process shows that the major energy source between 1870 and 1950 was coal. Oil became the dominant player from 1940 onward, as the automobile matured, together with petrochemical and other oil-based industries. The vertical scale of Figure 10 is logistic transforming S-curves into straight lines. All straight sections in this figure would show up as S-shaped on a graph with a linear vertical scale.

It becomes evident from this picture that a century-long history of an energy source can be described quite well — thin smooth lines — with only two constants, those required to define a straight line. (The curved sections are calculated by subtracting the straight lines from 100%.) The destiny of an energy source, then, seems to be cast during its early stages of phasing-in, as soon as the two constants describing the straight line can be determined.

A detailed description and the many ramifications of Figure 10 are discussed in detail in the literature (Marchetti 1987, Modis 1992, Modis 2009). Here I want to draw the reader’s attention to the subtle ways with which society imposes its will. This graph was originally put together in 1988 and indicated that natural gas would progressively replace oil. Its update twenty years later (little circles) shows that the persistent consumption of coal has been at the expense of natural
Figure 10. Data, fits, and projections for the shares of different primary energy sources consumed worldwide. For nuclear, the smooth straight line is not a fit but a trajectory suggested by analogy. The futuristic source labeled “Solfus” may involve solar energy and thermonuclear fusion. The small circles show how things evolved since 1982 when this graph was first put together.

gas. This may not only be due to aggressively developing countries such as China who use coal ravenously. Developed countries such as the UK and the US have also proven reluctant to give up coal. Whoever the culprit, the widening gap between the persistent level of coal use and coal’s naturally declining trajectory becomes a source of pressure to the system, which is likely to manifest itself through the voice of environmentalists.

Environmentalists in any way have been very vocal in their support of natural gas. I wonder, however, what has really been their role in the greening of natural gas. The importance of gas in the world market has been growing steadily for the last ninety years, independent of latter-day environmental concerns. The voice of environmentalists resembles Ralph Nader’s crusade in the 1960s for car safety, while the number of deaths from car accidents had already been pinned around twenty-four annually per one hundred thousand population for more than forty years.

Environmentalists have also taken a vehement stand on the issue of nuclear energy. This primary energy source entered the world market in the mid 1970s when it reached more than 1% share. The rate of growth during the first decade seems disproportionately rapid, however, compared to the entry and exit slopes of wood, coal, oil and natural gas, all of which conform closely to a more gradual rate. At the same time, the opposition to nuclear energy also seems out of proportion when compared to other environmental issues. As a consequence of the intense criticism, nuclear energy growth has slowed considerably, and it is not surprising to see the little circles in Figure 10 approach the straight line proposed by the model. One may ask what the prime mover here was — the environmental concerns that succeeded in slowing the rate of growth or the nuclear energy craze that forced environmentalists to react?

The coming to life of such a craze is understandable. Nuclear energy made a world-shaking appearance in the closing act of World War II by demonstrating the human ability to access superhuman powers. I use the word superhuman because the bases of nuclear reactions are the mechanisms through which stars generate their energy. Humans for the first time possessed the sources of power that feed our sun, which was often considered as a god in the past. At the same time mankind acquired independence; nuclear is the only energy source that would remain indefinitely at our disposal if the sun went out.
The worldwide diffusion of nuclear energy during the 1970s followed a rate that could be considered excessive, compared to previous energy sources. The market share of nuclear energy grew along a trajectory much steeper than the earlier natural ones of oil, natural gas, and coal. This abnormally rapid growth — possibly responsible for a number of major nuclear accidents — may have been what triggered the vehement reaction of environmentalists, who succeeded in slowing it down. Appropriately, as the technology matured, the number of major nuclear accidents was drastically reduced. However, environmentalists are far from having stopped nuclear energy. Ironically, under pressing concerns of CO₂ pollution, their opposition to nuclear energy had considerably weakened until the accident at Fukushima nuclear plant. I believe they will again reduce their opposition as public opinion cools off and better safety measures are put in place.

The changeable behavior of environmentalists suggests that there are other more fundamental forces at play while environmentalists behave more like puppets. These forces do not involve governments and their policies that usually become shaped after the fact in response to public outcries.

4.4 World Population – The Big Picture

Another example of society’s wise and subtle ways of controlling human behavior is the slowing down in the rate of growth of the world population during the 20th century. The phenomenon has sometimes been erroneously attributed to people having become aware of the perils of Earth’s overpopulation and reacted accordingly with adequate birth control. But it is only China that has imposed nationwide birth controls via legislation and that accounts for only 20% of the world’s population. The main reason the world population has slowed down is that rising standards of living offer people more highly preferred things to do than having children. The flattening of the S-curve shown earlier in Figure 4 is very little a consequence of top-down conscious decision-making. It is mostly a consequence of subconscious bottom-up forces shaping the patterns of our lives.

But let us zoom back and consider a much greater historical window. Figure 11 shows world population data since the time of Christ (before that time estimates are rather unreliable). A dramatic exponential pattern belongs to an S-curve penetrated only to 23% with an eventual ceiling of 1,750,000,000,000 by year 2700. Obviously the uncertainties involved on the level of the ceiling estimated from data that cover only the beginning of the S-curve are very large. From a detailed Monte-Carlo study on error estimation we obtain up to ±75% uncertainty on this number with confidence level of 90% (Debecker and Modis 1994). The year 2700 is a far-fetched horizon date for making forecasts and such statements are more appropriate to fiction than to scientific writing, but can there be a grain of truth? At first glance such a conclusion may seem absurd by today’s standards and in view of Section 2.4 above. But is it really absurd? Could the S-curve of Figure 4 be followed up by other S-curves in the paradigm of Section 3.3?

Altogether possible, claims Cesare Marchetti, who calculated that Earth’s carrying capacity is around 10¹² people. His is not a forecast but scientific calculations taking into account availability of resources, energy, housing, and the environment (Marchetti 1979). If he is right that there is no fundamental law violated by reaching such a number, you can be sure that this will eventually happen (no niche in nature that could be filled to completion was ever left occupied only partially). But as we saw earlier, people’s subconscious behavior during the 20th century assures us that such a thing would take place slowly, controllably, and avoiding catastrophe.
5. More than Just Cerebral Intelligence

This section addresses the claim that intelligent machines will eventually take over as a new species — posthumans — reducing humans to the equivalent of monkeys for us today.

Intelligence according to the singularitarians is measured by the speed of calculation. I believe that the astronomical numbers of FLOPS (floating-point operations per second) forecasted by Kurzweil as the ultimate computing power, namely $10^{50}$ and beyond, will fall short by a large factor, at least 25 orders of magnitude, mostly because such computing power will no longer be desired. You can get too much of a good thing, for example there is no longer demand for cars to go faster, or for pocket calculators to become thinner/smaller, something that would have never crossed the mind of early-car and early-calculator users.

But assuming unfathomable computing power becomes available, this would only be competition to our cerebral intelligence. Humans also possess physical intelligence responsible for our reproduction, growth, self-healing, and disease-fighting capabilities. Human understanding of the body’s intelligence is to say the least inadequate. For example, the enteric part of our autonomic nervous system has more memory capability than the spinal cord. A mouse will function brainless to an impressive extent. Such phenomena are far from having been thoroughly investigated and understood. Where will this knowledge come from?

In any case there is a catch. If we humans were to provide the superhumans with all the knowledge, we would certainly refrain from giving them the power to overtake us, or build in mechanisms to prevent such an eventuality. If on the other hand, superhumans were to obtain themselves the missing knowledge, then they would need to do the studying themselves. But before superhuman machines begin dissecting us and putting us under microscopes — as we do with monkeys — they will first need physical bodies themselves, which they should be able to fabricate and maintain. One cannot argue that advance robotics will produce machines that can do that because there is a catch. In order for these robots to be able to move around, gather resources, and carry out research to acquire the missing knowledge they would first need to have in their system the vey knowledge they set out to obtain through studies.
On the other hand, intellectual power all by itself would not achieve much. Besides some evidence for occasional correlation, it is well known that in general among the most intelligent people you are not likely to find: the richest, the happiest, the most normal (by definition!), the best-adjusted, the most good-natured, the most trustworthy, the most creative, the best artists, the most powerful, the most popular, or the most famous. In short, fast thinking is not the ultimate desirable quality, and thinking faster is not necessarily better in an absolute sense.

All in all, superhumans enabled to develop thanks to supercomputing power would certainly not pose a threat to humans by mid 21st century. In fact, I wouldn’t hesitate to extend this reassuring message to the far-fetched horizon date of 2700 that we mentioned earlier.

6. What May Happen Instead

A central graph early in Kurzweil’s book, which he uses as a platform to launch the whole Singularity development, displays a set of data I had painstakingly collected earlier for my own publications (Modis 2002a and 2003). The set of data basically consists of thirteen independent timelines for the most significant turning points — milestones — in the evolution of the Universe. The emerging overall trend displays an unambiguous crowding of milestones in recent times. The thinking behind my article was that the spacing of the most important milestones could serve to quantify the evolution of change and complexity and therefore enabling us to forecast it.

It is reasonable to assume that the greater the change associated with a given milestone, or the longer the ensuing stasis, the greater the milestone's importance will be.

\[
\text{Importance} = (\text{change introduced}) \times (\text{duration to the next milestone})
\]

(1)

Following each milestone there is change introduced in the system. At the next milestone there is a different amount of change introduced. Assuming that milestones are approximately of equal importance, and according to the above definition of importance we can conclude that the change \( \Delta C_i \) introduced by milestone \( i \) of importance \( I \) is

\[
\Delta C_i = \frac{I}{\Delta t_i}
\]

(2)

where \( \Delta t_i \) the time period between milestone \( i \) and milestone \( i+1 \).

My dataset has a number of weaknesses. Only twelve out of the fifteen timelines used were independent. One timeline had been given to me without dates and I introduce them myself; another consisted of my own guesses. Both were heavily biased by the other twelve in my disposal. Moreover, some data were simply weak by their origin (e.g., an assignment post on the Internet by a biology professor for his class, which is no longer accessible today.)

As a matter of fact only one timeline (Sagan’s Cosmic calendar) covers the entire range (big bang to 20th century) with dates. A second complete set (by Nobel Laureate Boyer) was provided to me without dates. All the other timelines coming from various disciplines covered only restricted time windows of the overall timeframe, which results in uneven weights for the importance of the milestones as each specialist focused on his or her discipline.
To the extent that milestones of equal importance appear more frequently, the respective change they introduce increases. The area of each rectangle represents importance and remains constant. The scales of both axes are linear.

A histogram of all milestones with logarithmic time buckets suggested by clusters of milestone dates. The thin black line is superimposed to outline the clusters of milestones that define the dates of a canonical milestones set. On the horizontal axis we read the dates of these canonical milestones. Present is defined as year 2000. The width of each peak becomes a measure of the uncertainty on the date of the canonical milestone (and consequently also the uncertainty on the change it brings).
In any case, the grand total of all milestones came to 302 and in a histogram — Figure 13 — revealed clusters of milestones with peaks that defined a canonical set of milestones. Present time was taken as year 2000. Within all sources of uncertainties mentioned above, I tried to quantitatively study the evolution of complexity and change in the Universe.

Figure 14 shows that the evolution of change with milestone number has so far followed an exponential-growth pattern, which could also be an S-curve (logistic) or its life cycle (first derivative) as all three behave exponentially early on. Given that the data depict change per milestone I fit to the latter shown by the thick gray line. The implication of doing so is that the total amount of change in the Universe will be finite by the time the Universe ends.

Figure 14. Exponential and logistic life-cycle fits to the data of the canonical milestone set. The vertical axis depicting the amount of change per milestone is linear (graph at the top) and logarithm (graph at the bottom). The intermittent vertical line denotes the present. The gray circles on the forecasted trends indicate change from future milestones. The change associated with the most recent milestone, No 28, will not be known before the appearance of milestone No 29. The error bars have been calculated from the spread of entries clustered around each canonical milestone date.
The quality of the fit being a little better and the position of the last point both argue in favor of the logistic life cycle rather than the corresponding exponential. But these are weak arguments. More serious impact have the forecasts for the change expected by future milestones. Table III lists these forecasts for the next five milestones. The logistic fit expects milestones to begin appearing less frequently in the future whereas the exponential fit expects them at an accelerating pace. In particular, the logistic fit has next milestone appearing in 2033 and the one after that in 2078. In contrast, the exponential fit has next milestone in 2009 (remember zero was defined as year 2000 in this study), and the one after that in year 2015. By year 2022 the exponential fit forecasts a new milestones every 6 days and less than a year later infinite change will have taken place! This spells out “Singularity” and brings it forward by 20 years or so, but the uncertainty of this determination could be more than 20 years considering the crudeness of the method and the enormous timescale involved.

The logistic life cycle peaks in the mid 1990s. It indicates that we are presently traversing the only time in the history of the Universe in which 80 calendar years — the lifetime of people born in the 1940s — can witness change in complexity coming from as many as three evolutionary milestones. This positions us presently at the world’s prime!

Coincidentally people who will partake in this phenomenon belong to the mysterious baby boom that creates a bulge on the world population distribution. As if by some divine artifact a larger-than-usual sample of individuals was meant to experience this exceptionally turbulent moment in the evolution of the cosmos.

The large-scale logistic description of Figure 14 indicates that the evolution of change in the Universe has been following a logistic/exponential growth pattern from the very beginning, i.e. from the big bang. This is remarkable considering the vastness of the time scale, and also the fact that change resulted from very different evolutionary processes, for example, planetary, biological, social, and technological. The fitted logistic curve has its inflection point — the time of the highest rate of change — in mid 1990. Considering the symmetry of the logistic-growth pattern, we can thus conclude that the end of the Universe is roughly another 15 billion years away. Such a conclusion is not really at odds with the latest scientific thinking that places the end of our solar system some 5 billion years from now.

### Table III: Forecasts for Change as a Function of Time

| Milestone number | Logistic fit | Exponential fit |
|------------------|--------------|----------------|
|                  | Change* | Year | Change* | Year |
| 29               | 0.0223 | 2033 | 0.1540 | 2009 |
| 30               | 0.0146 | 2078 | 0.3247 | 2015 |
| 31               | 0.0081 | 2146 | 0.6846 | 2018 |
| 32               | 0.0041 | 2270 | 1.4435 | 2020 |
| 33               | 0.0020 | 2515 | 3.0436 | 2021 |

* In the same arbitrary units as Figure 14.

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2 The pattern of a decaying exponential is asymptotic, i.e. it needs infinite time to reach zero, but its definite integral between x and ∞ is finite.

3 It has been often argued that the baby boom was due to soldiers coming back from the fronts of WWII. This is wrong because the phenomenon began well before the war and lasted until the early 1970s. The effect of WWII was only a small and narrow wiggly dent in the population’s evolution.
We have obviously been concerned with an anthropic Universe here because we have to a large extent overlooked how change has been recently evolving in other parts of the Universe. Still, I believe that such an analysis carries more weight than just the elegance and simplicity of its formulation. The celebrated physicist John Wheeler has argued that the very validity of the laws of physics depends on the existence of consciousness. In a way, the human point of view is all that counts! It reminds me of a whimsical writing I once saw on a tee shirt: “One thing is certain, Man invented God; the rest is debatable.”

7. Epilogue

The exponential pattern of milestones in Figure 14, which provides a central argument for the Singularity, resembles — and to some extent is affected by — the patterns of Moore’s law (Figure 3) and world population (Figure 11). All three show many orders of magnitude growth along exponential trends. But Figures 3 and 11 have both avoided the ominously rising trend and have done so in a natural way.

The last two milestones with present defined as year 2000 are:

- 5 years ago: Internet / human genome sequenced
- 50 years ago: DNA / transistor / nuclear energy

The next such world-shaking milestone should be expected — even by common sense — around 2033 rather than of around 2009 because despite a steady stream of significant recent discoveries, there is still no obviously candidate in 2011. That was not the case with the last two milestones: the significance of the Internet became clear simultaneously with its diffusion, and the significance of nuclear energy had become clear long before it was materialized.

AN AFTERTHOUGHT

Playing the Devil’s Advocate

Could it be that on a large scale there may be no acceleration at all? Could it be that the crowding of milestones in Figure 13 is simply a matter of perception? The other day I was told that I should have included Facebook as a milestone. “It is just as important as the Internet,” she told me. Would Thomas Edison have thought so? Will people one thousand years from now, assuming we will survive, think so? Will they know what Facebook was? Will they know what the Internet was?

It is natural that we are more aware of recent events than events far in the past. It is also natural that the farther in the past we search for important events the fewer of them will stick out in society’s collective memory. This by itself would suffice to explain the exponential pattern of our milestones. It could be that as importance fades with the mere distancing from the present it “gives the appearance”, in John von Neumann’s words, that we are “approaching some essential singularity”. But this has nothing to do with year 2045, 2025, today, von Neumann’s time — the 1950s — or any other time in the past or the future for that matter.

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4 John Wheeler was a renowned American theoretical physicist. One of the later collaborators of Albert Einstein, he tried to achieve Einstein's vision of a unified field theory.
APPENDIX – The Canonical Milestones

The dates generally represent an average of clustered events not all of which are mentioned in this table. That is why some events e.g. asteroid collision appears dated too recent. Highlighted in bold is in the most outstanding event in the cluster. Present time is taken as year 2000.

| No. | Milestone                                                                 | Years ago         |
|-----|---------------------------------------------------------------------------|-------------------|
| 1   | **Big Bang** and associated processes                                   | $1.55 \times 10^{10}$ |
| 2   | Origin of Milky Way / first stars                                        | $1.0 \times 10^{10}$ |
| 3   | Origin of life on Earth / formation of the solar system and the Earth / oldest rocks | $4.0 \times 10^{9}$ |
| 4   | **First eukaryots** / invention of sex (by microorganisms) /atmospheric oxygen /oldest photosynthetic plants / plate tectonics established | $2.1 \times 10^{9}$ |
| 5   | First multicellular life (sponges, seaweeds, protozoans)                 | $1.0 \times 10^{9}$ |
| 6   | Cambrian explosion / invertebrates / vertebrates / plants colonize land / first trees, reptiles, insects, amphibians | $4.3 \times 10^{8}$ |
| 7   | First mammals / first birds / first dinosaurs / first use of tools       | $2.1 \times 10^{8}$ |
| 8   | First flowering plants / oldest angiosperm fossil                        | $1.3 \times 10^{8}$ |
| 9   | Asteroid collision / first primates / mass extinction (including dinosaurs) | $5.5 \times 10^{7}$ |
| 10  | First humanoids / first hominids                                         | $2.85 \times 10^{7}$ |
| 11  | First orangutan / origin of proconsul                                     | $1.66 \times 10^{7}$ |
| 12  | Chimpanzees and humans diverge / earliest hominid bipedalism             | $5.1 \times 10^{6}$ |
| 13  | First stone tools / first humans / ice age / homo erectus / origin of spoken language | $2.2 \times 10^{6}$ |
| 14  | Emergence of Homo sapiens                                                 | $5.55 \times 10^{5}$ |
| 15  | Domestication of fire / Homo heidelbergensis                             | $3.25 \times 10^{5}$ |
| 16  | Differentiation of human DNA types                                        | $2.0 \times 10^{5}$ |
| 17  | Emergence of "modern humans" / earliest burial of the dead               | $1.06 \times 10^{5}$ |
| 18  | Rock art / protowriting                                                    | $3.58 \times 10^{4}$ |
| 19  | Invention of agriculture                                                  | $1.92 \times 10^{4}$ |
| 20  | Techniques for starting fire / first cities                               | $1.1 \times 10^{4}$ |
| 21  | Development of the wheel / writing / archaic empires                      | 4907              |
| 22  | Democracy / city states / the Greeks / Buddha                            | 2437              |
| 23  | Zero and decimals invented / Rome falls / Moslem conquest                  | 1440              |
| 24  | Renaissance (printing press)/discovery of new world/the scientific method | 539               |
| 25  | Industrial revolution (steam engine) / political revolutions (French, USA) | 223               |
| 26  | Modern physics / radio / electricity / automobile / airplane               | 100               |
| 27  | DNA / transistor / nuclear energy / W.W.II / cold war / sputnik            | 50                |
| 28  | Internet / human genome sequenced                                         | 5                 |

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