The Threshold Pion-Photoproduction Of Nucleons In The Chiral Quark Model

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Abstract

In this paper, we show that the low energy theorem (LET) of the threshold pion-photoproduction can be fully recovered in the quark model. An essential result of this investigation is that the quark-pion operators are obtained from the effective chiral Lagrangian, and the low energy theorem does not require the constraints on the internal structures of the nucleon. The pseudoscalar quark-pion coupling generates an additional term at order $\mu = m_{\pi}/M$ only in the isospin amplitude $A^{(-)}$. The role of the transitions between the nucleon and the resonance $P_{33}(1232)$ and P-wave baryons are also discussed, we find that the leading contributions to the isospin amplitudes at $O(\mu^2)$ are from the transition between the P-wave baryons and the nucleon and the charge radius of the nucleon. The leading contribution from the P-wave baryons only affects the neutral pion production, and improve the agreement with data significantly. The transition between the resonance $P_{33}(1232)$ and the nucleon only gives an order $\mu^3$ corrections to $A^{(-)}$.

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1. Introduction

The pion-photoproduction $N(\gamma, \pi)N$ of the nucleon is characterized by three independent amplitudes, $A^(-)$, $A^+$ and $A^0$. They are related to the differential cross section for the threshold pion-photoproduction \[d\sigma \over d\Omega = {k_\pi \over k} |E_{0+}|^2\] by

\[E_{0+} = {g_{\pi,N} \over 8\pi M} \left(1 + {\mu \over 2}\right) \chi_f \left(\frac{1}{2}[\tau_j, \tau_0]A^(-) + \tau_j A^0 + \delta_{j,0} A^+\right) \chi_i,\] (2)

and $\chi_i$ and $\chi_f$ are initial and final isospinors respectively. These isospin amplitudes can be expanded \[2\] in terms of the ratio between the masses of pions and nucleons, $\mu = m_\pi / M$, near the threshold, and the leading terms are determined by the low energy theorem (LET) \[3\], which can be written as

\[A^(-) = 1 + O(\mu^2)\]
\[A^+ = A^0 = -{\mu \over 2} + O(\mu^2).\] (3)

This result is model independent, and a direct consequence of the partial conservation of the axial vector current (PCAC) and the soft-pion theorem, therefore, it provides an important test to the models in hadronic physics. Although the LET only depends on the static properties of the nucleon and no distinction is being made between elementary and composite particles, the proof of the LET in the quark models which treat the nucleon as a three quark system is by no means trivial. The investigations in both quark models \[4, 5\] and the relativistic light cone quark model \[6\] show that it is crucial to separate the center of mass from the internal motions. However, the realization of the LET in the nonrelativistic quark model relies on the constraints on the size and the internal transitions on the nucleon, although the problem of separating the center of mass motion does not exist. The focus of this paper is to provide a general proof of the LET in the quark model with no constraint on the internal structure of the nucleon. Similar to the Thomson limit and low energy theorem in the Compton scattering, the LET is determined by the transition operators corresponding to the center of mass motion, thus the separation of the center of mass from the internal motions is essential. More important, the LET is directly related to the chiral symmetry, thus the constituent quark model alone is no longer enough, one should start from the chiral quark model.
proposed by Manohar and Georgi which combines the chiral symmetry and the constituent quarks.

On the other hand, the pion decays of the baryon resonance play an important role in the $N^*$ physics at CEBAF, which have been the subject of the many studies in the framework of the constituent quark model, especially the $^{3}P_{0}$ model. These calculations are based mostly on the phenomenological approach, and few theoretical justification has been provided, the LET in the threshold pion photoproduction might be a crucial test to the quark-pion couplings in these phenomenological models, this is a major motivation of our investigation. The encouraging result from our study is that the essential quark-pion coupling operators for both LET and the $^{3}P_{0}$ model have exactly the same structure, this at least shows that the $^{3}P_{0}$ model is consistent with the LET, which has shown to be quite successful in describing the strong decays in hadron physics.

In section 3, we shall show that the leading terms from the internal structure of the nucleon come from the transitions between the nucleon and the P-wave baryons and the charge radius of the nucleon at order $\mu^2$, while the leading term from the transition between the nucleon and the $\Delta$ resonance, $P_{33}(1232)$, only contributes to the isospin amplitude $A^{(-)}$ at order $\mu^3$. This is because the decay of the $\Delta$ resonance into pion and nucleon is dominated by p-wave which is suppressed by a factor of $\mu$, and P-wave baryons decays are dominated by s-wave. We further find that the leading term in the transitions between the nucleon and the P-wave baryons only contributes to the isospin amplitude $A^{(+)\,}$, it will affect the neutral pion production significantly. This has not been discussed in the literature, while much discussions have been concentrated on the nucleon-$\Delta$ transition. Indeed, we find an excellent agreement with data for $\gamma p \rightarrow \pi_0 p$ and $\gamma n \rightarrow \pi_0 n$ if the contribution from the P-wave baryons is included.

The paper is organized as follows; a general derivation of the LET is given in sections 2, furthermore, we shall show that the pseudoscalar coupling generates an additional term only in $A^{(-)}$ at order $\mu$. The contributions from the internal structure of the nucleon is calculated in the section 3, and the conclusion is given in section 4.

2. The LET for a three quark system

In the chiral quark model, the pions are being treated as Goldstone bosons that generates the spontaneous chiral symmetry breaking at some scale $\Lambda_{\chi_{SB}}$, the QCD Lagrangian transforms into an effective chiral Lagrangian in the low
energy limit, which is\(^4\):

\[ \mathcal{L} = \bar{\psi} \left[ \gamma_\mu (i \partial^\mu + V^\mu + A^\mu \gamma_5) - m \right] \psi + \ldots \] (4)

where the vector and the axial vector currents are

\[ V_\mu = \frac{1}{2} \left( \xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger \right) \]
\[ A_\mu = \frac{1}{2} i \left( \xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger \right), \] (5)

\[ \xi = e^{i\pi/f}, \quad \pi = \sum \phi_i \tau_i \] for the \( SU(2)_L \otimes SU(2)_R \) and \( f_\pi \approx 93 \text{ MeV} \) is the pion decay constant. The constituent quark mass \( m \) in Eq. \( \text{[4]} \) is around 350 MeV due to the chiral symmetry breaking. Unlike the gluon fields, the chiral transformation of the electromagnetic gauge field will generate an additional quark-photon-pion interaction due to the isospin dependence of the quark charge

\[ e_q = \frac{e}{2} \left( \frac{1}{3} + \tau_0 \right), \] (6)

which is

\[ H_{\pi,e} = \sum_j \frac{-ie}{2f_\pi} \left[ \tau^j_0, \tau^j_i \right] \phi_i \bar{\psi}^j \gamma_\mu \gamma_5 \psi^j A^\mu(k, r_j), \] (7)

where \( \tau^j_i \) is Pauli isospin matrix, and \( A^\mu(k, r_j) \) is a electromagnetic field. The pion-quark coupling at the tree level is therefore the standard pseudovector coupling;

\[ H_{\pi} = \sum_j \frac{1}{f_\pi} \bar{\psi}^j \gamma_\mu \gamma_5 \tau^j_i \psi^j \phi_i, \] (8)

and the electromagnetic coupling is

\[ H_e = -\sum_j e_j \gamma^j_\mu A^\mu(k, r_j). \] (9)

Therefore, one can write the transition matrix element as

\[ \mathcal{M}_{fi} = \langle N_f | H_{\pi,e} | N_i \rangle + \sum_j \left\{ \frac{\langle N_f | H_{\pi} | N_j \rangle \langle N_j | H_e | N_i \rangle}{E_i + \omega - E_j} + \frac{\langle N_f | H_e | N_j \rangle \langle N_j | H_{\pi} | N_i \rangle}{E_i - \omega - E_j} \right\}, \] (10)

where \( N_i(N_f) \) is the initial (final) state of the nucleon, and \( \omega(\omega_n) \) represents the energy of incoming (outgoing) photons(pions). The first term in Eq. \( \text{[\text{l}} \)
corresponds to the Seagul diagram, and the second and the third terms are the direct and the crossed intermediate resonance pole diagrams.

The nonrelativistic expansion of the quark-photon-pion interaction gives

\[ H^{nr}_{\pi,e} = i \sum_j \frac{e}{2f_\pi} \left[ \tau^j_0, \tau^j_l \right] \sigma_j \cdot \epsilon, \]  

where \( \epsilon \) is the polarization vector of photons, and notice that

\[ \langle N_f| \sum_j \left[ \tau^j_0, \tau^j_l \right] \sigma_j |N_i \rangle = g_A \langle N_f| \left[ \tau^T_0, \tau^T_l \right] \sigma^T |N_i \rangle, \]  

where \( \tau^T_l \) and \( \sigma^T \) are the total isospin and spin operators of the nucleon, and \( g_A \) is the axial coupling constant of the nucleon, we have the expression for the Seagul diagram

\[ \langle N_f| H^{nr}_{\pi,e} |N_i \rangle = \frac{ig_Ae}{2f_\pi} \langle N_f| \left[ \tau^T_0, \tau^T_l \right] \sigma^T \cdot \epsilon |N_i \rangle, \]  

which only gives the leading term to the isospin amplitudes \( A^{(-)} \).

The calculation of the direct and crossed resonance pole diagrams follows the similar procedure in Compton Scattering \((\gamma N \rightarrow \gamma N)\) in the low energy limit. Rewriting the electromagnetic interaction \( H_e \) as

\[ \langle N_f|H_e|N_i \rangle = i(E_f - E_i - \omega)\langle N_f|g_e|N_i \rangle + i\omega\langle N_f|h_e|N_i \rangle \]  

for the direct pole diagram and

\[ \langle N_f|H_e|N_i \rangle = i(E_f - E_j - \omega)\langle N_f|g_e|N_j \rangle + i\omega\langle N_f|h_e|N_j \rangle \]  

for the crossed pole diagram, where

\[ g_e = \sum_j e_j r_j \cdot \epsilon e^{ikr_j} \]  

and

\[ h_e = \sum_j e_j r_j \cdot \epsilon (1 - \alpha \cdot \hat{k}) e^{ikr_j}, \]  

and \( \hat{k} = \frac{k}{\omega} \), in which we have replaced the spinor \( \bar{\psi} \) by \( \psi^\dagger \) so that the \( \gamma \) matrices are replaced by the matrix \( \alpha \), the second and the third terms in Eq. 10 can be written as

\[ \mathcal{M}'_{fi} = i \langle N_f|[g_e, H_\pi]|N_i \rangle + i\omega \sum_j \left\{ \frac{\langle N_f|H_\pi|N_j \rangle \langle N_j|h_e|N_i \rangle}{E_i + \omega - E_j} + \frac{\langle N_f|h_e|N_j \rangle \langle N_j|H_\pi|N_i \rangle}{E_i - \omega_\pi - E_j} \right\}. \]
The first term in Eq. 18 is also a Seagull term, and only contributes to the amplitudes $A^{(-)}$, the nonrelativistic expansion near the threshold gives

$$\langle N_f|\left[g_A \tau^T_{0}, \tau^T_{l}\right] \frac{\sigma^T \cdot P \cdot R \cdot \epsilon + R \cdot \epsilon \sigma^T \cdot P}{2M}|N_i\rangle,$$

in which we have separated the center of mass motion from the internal motion by

$$\frac{\mathbf{p}_j}{m_j} = \frac{\mathbf{P}}{M} + \frac{\mathbf{p}'_j}{m_j},$$

where $\mathbf{P}$ is the total momentum of the composite system with mass $M$ at the position $\mathbf{R}$, and $\mathbf{p}'_j$ represents the internal momentum whose matrix elements vanish for the initial and final nucleon states.

The nonrelativistic expansion for $h_e$ in Eq. 18 is[11, 12]

$$h_e = \sum_j \left[ e_j \mathbf{r}_j \cdot \mathbf{e} \left( 1 - \frac{\mathbf{p}_j \cdot \mathbf{k}}{m_j \omega} \right) - \frac{e_j}{2m_j} \mathbf{\sigma}_j \cdot (\mathbf{e} \times \hat{\mathbf{k}}) \right],$$

which $h_e$ is only expanded to order $1/M$. The corresponding pion coupling is therefore

$$H_{\pi}^{nr} = -i \sum_j \frac{\tau^T_j \mathbf{r}_j \cdot \mathbf{k}}{\omega_{\pi}} \left[ \mathbf{\sigma}_j \cdot \mathbf{P} - \mathbf{\omega}_{\pi} \frac{\mathbf{\sigma}_j \cdot \mathbf{P}_j}{m_j} \right]$$

where $\mathbf{k}_\pi \approx 0$ and $\omega_{\pi} \approx m_\pi$ near the threshold, thus only second term is needed in our calculation.

In Ref. [13], we shown how to separate the center of mass from the internal motion to obtain the Thomson limit and the low energy theorem in the Compton scattering. Since the incoming photon energy near the threshold is

$$\omega = M\mu \frac{1 + \frac{\mu}{1 + \mu}},$$

the leading terms in the photon energy $\omega$ are also leading terms in $\mu$. Replacing the outgoing photon operator by $H_\pi$ in Eq. 22 of Ref. [13], we have

$$\mathcal{M}'_{f_1} = -\langle N_f | g_A \omega_{\pi} \tau^T_{0}, \tau^T_{l} \frac{\sigma^T \cdot P \cdot R \cdot \epsilon + R \cdot \epsilon \sigma^T \cdot P}{2M} | N_i \rangle + \langle N_f | h_{\pi}^c | N \rangle \langle N | \frac{-i\epsilon \tau^T_{0} P \cdot \epsilon}{M \omega} + h_{\pi}^e | N_i \rangle - \langle N_f | \frac{-i\epsilon \tau^T_{l} P \cdot \epsilon}{M \omega} + h_{\pi}^c | N \rangle \langle N | h_{\pi}^e | N_i \rangle$$

where

$$h_{\pi}^e = i \sum_j \frac{\omega_{\pi} \tau^T_j \mathbf{\sigma}_j \cdot \mathbf{P}}{M}$$
and

\[ h^e_c = e_T \mathbf{R} \cdot \mathbf{e} - \mu_N \sigma^T \cdot (\mathbf{e} \times \hat{k}) \] (26)

correspond to the center of mass motion, and the leading terms for transition operator corresponding to the internal motions are proportional to \( \omega^2 \), which will be discussed later. Therefore, one could use Eq. 12, and then use the closure relation since the resulting transition operator only connects the ground states, we have

\[
\mathcal{M}_{fi} = - \langle N_f | \frac{g_A e \omega}{f_\pi} [\tau_0^T, \tau_l^T] \frac{\sigma^T \cdot \mathbf{P} \cdot \mathbf{R} \cdot \mathbf{e} + \mathbf{R} \cdot \mathbf{e} \sigma^T \cdot \mathbf{P}}{2M} | N_i \rangle \\
- \frac{g_A \omega}{f_\pi} \langle N_f | \left[ \frac{\sigma^T \cdot \mathbf{P}}{M}, e_T \frac{-i \mathbf{P} \cdot \mathbf{e}}{M \omega} + h^e \right] | N_i \rangle. \] (27)

The electromagnetic interaction in Eq. 27 is

\[ e_T \frac{-i \mathbf{P} \cdot \mathbf{e}}{M \omega} + h^e = (h^s + \tau_0^T h^v) e \] (28)

where

\[ h^{s,v} = \frac{1}{2} - i \frac{\mathbf{P} \cdot \mathbf{e}}{M \omega} \pm \frac{1}{2} \mathbf{R} \cdot \mathbf{e} - \mu^{s,v} \sigma^T \cdot (\mathbf{e} \times \hat{k}), \] (29)

with \( \mu^s = \frac{1}{2}(\mu_p + \mu_n) \) and \( \mu^v = \frac{1}{2}(\mu_p - \mu_n) \). Using the relation

\[ [aA, bB] = \frac{1}{2} [a, b] \{A, B\} + \frac{1}{2} \{a, b\} [A, B], \] (30)

we obtain the transition amplitudes

\[
\mathcal{M}_{fi} = \chi_f \left\{ \frac{1}{2} [\tau_l^T, \tau_0^T] \mathcal{M}^{(-)} + \tau_l^T \mathcal{M}^{(0)} + \delta_{l,0} \mathcal{M}^{(+)} \right\} \chi_i, \] (31)

where

\[
\mathcal{M}^{(-)} = \frac{e g_A}{f_\pi} \langle N_f | \left( -i \sigma^T \cdot \mathbf{e} \pm \mu \left\{ \sigma^T \cdot \mathbf{P}, \frac{1}{2} \mathbf{R} \cdot \mathbf{e} - \frac{-i \mathbf{P} \cdot \mathbf{e}}{2M \omega} - h^v \right\} \right) \rangle \langle N_i \rangle, \] (32)

\[
\mathcal{M}^{(+)} = -\frac{e \mu g_A}{f_\pi} \langle N_f | \left[ \sigma^T \cdot \mathbf{P}, \frac{-i \mathbf{P} \cdot \mathbf{e}}{2M \omega} + h^s \right] \rangle \langle N_i \rangle \] (33)

and

\[
\mathcal{M}^{(0)} = \frac{-\mu e g_A}{f_\pi} \langle N_f | \left[ \sigma^T \cdot \mathbf{P}, \frac{-i \mathbf{P} \cdot \mathbf{e}}{2M \omega} + h^s \right] \rangle \langle N_i \rangle. \] (34)
In the center of mass frame for the incoming photon and nucleon, the operator \( P \) equals to
\[
P = \frac{1}{2}(P_i + P_f) = -\frac{1}{2}k,
\]
(35)
it is straightforward to obtain
\[
A^{(-)} = 1 + O(\mu^2),
\]
(36)
\[
A^{(0)} = -\frac{1}{2}\mu + \frac{1}{4}\mu^2 + \frac{1}{4}(\kappa_p + \kappa_n)\mu^2 + O(\mu^3)
\]
(37)
and
\[
A^{(+)} = -\frac{1}{2}\mu + \frac{1}{4}\mu^2 + \frac{1}{4}(\kappa_p - \kappa_n)\mu^2 + O(\mu^3),
\]
(38)
where \( \kappa \) is the anomalous magnetic moment of the nucleon. Therefore, we have fully recovered the LET of the threshold pion-photoproduction. An interesting result here is that the isospin amplitude \( A^{(-)} \) is only determined by the Seagul diagram, while \( A^{(0)} \) and \( A^{(+)} \) are from the direct and crossed resonance pole diagrams. Unlike the conclusions by the previous investigations[4], the LET is determined by the the transition operator corresponding to the center of mass motion, and no further requirement on the internal structure of the nucleon is needed. The difference is that the intermediate states in the low energies are highly collective, thus the propagator should not be replaced by the quark propagator.

The proof of the LET also highlights that it is essential to start from the effective chiral Lagrangian when pion degrees of freedom are involved. The conclusion that the isospin amplitude \( A^{(-)} \) is determined by the Seagul alone is a natural consequence of the chiral transformation, and can not be obtained from the direct gauge transformation of the pseudovector pion-quark coupling operator. Furthermore, there is no free parameter at the tree level; Eqs. 36, 37 and 38 require that the pion nucleon coupling in the low energy region should be
\[
g_{\pi N} = \frac{g_{AM}}{f_\pi},
\]
(39)
which is the Goldberger-Treiman relation[14]. Thus, the Goldberger-Treiman relation and the LET are direct consequences of the chiral symmetry in the chiral quark model.

Another possible quark-pion interaction is the pseudoscalar coupling;
\[
H^{p,s}_\pi = \sum_j g_{q\pi}\tilde{\psi}_j\gamma_5\psi_j\tau^j_l\phi_l
\]
(40)
In this case, there is no direct pion-quark-photon coupling operator, thus, the pion-photoproduction near the threshold is determined by the direct and crossed resonance pole diagrams. Taking the same procedure in Eqs. 14 and 15, Eq. 18 becomes

\[
\mathcal{M} = \langle N_f [g_e, H_{\pi}^{p.s}] | N_i \rangle + \sum_j \left[ \frac{\langle N_f | H_{\pi}^{p.s} | N_j \rangle \langle N_j | h_e | N_i \rangle}{E_i + \omega - E_j} + \frac{\langle N_f | H_{\pi}^{p.s} | N_j \rangle \langle N_j | h_e | N_i \rangle}{E_i + \omega - E_j} \right],
\]

the nonrelativistic expansion of the first term gives

\[
\langle N_f [g_e, H_{\pi}^{p.s}] | N_i \rangle = -igq_{\pi}\frac{1}{2} \langle N_f | \sum_j \left[ \tau^j_0, \tau^j_l \right] - \sigma_j \cdot p_j \epsilon + \tau^j_1 \cdot \epsilon \sigma_j \cdot p_j \right] | N_i \rangle = \frac{g_A g_{\pi}}{4m_q} \langle N_f | [\tau^T_0, \tau^T_i] \sigma_T \cdot \epsilon | N_i \rangle.
\]

which generates the leading term for the isospin amplitudes \( A^{(-)} \). Since the nonrelativistic expansion of the pseudoscalar coupling \( H_{\pi}^{p.s} \) has the same expression as the pseudovector coupling to order \( O(1/M) \), except the pion-quark coupling is replaced by \( \frac{g_{\pi}}{2m_q} \), the second and third terms in Eq. 41 is the same as those in Eq. 24, thus, the only term that will be affected by the pseudoscalar coupling is the isospin amplitude \( A^{(-)} \). Eq. 32 becomes

\[
\mathcal{M}^{(-)} = \frac{eg_{\pi}g_A}{2m_q} \langle N_f | \left( \sigma^T \cdot \epsilon + i\mu \left( \frac{\sigma^T \cdot p_i - \frac{iP \cdot \epsilon}{2M\omega} - \frac{h^0}{2m_q} \right) \right) | N_i \rangle,
\]

while the matrix elements \( \mathcal{M}^{(0)} \) and \( \mathcal{M}^{(+)} \) are the same as those in Eqs. 33 and 34 except the coupling constant. The result isospin amplitude \( A^{(-)} \) becomes

\[
A^{(-)} = 1 + \mu + O(\mu^2).
\]

The difference between the pseudovector and the pseudoscalar couplings is the term \( \frac{\mu}{2} \) in the isospin amplitude \( A^{(-)} \) in chiral quark model. It is well known that the Born terms for the pseudoscalar coupling violates the LET at order \( \mu \), our investigations shows that the pseudoscalar coupling does generate correct isospin amplitudes \( A^{(0)} \) and \( A^{(+)} \) if one extends the calculations beyond the Born approximations. However, the difference in the isospin amplitudes in \( A^{(-)} \) merits a further study, since it has been show that there should be no term proportional to \( \mu \) in \( A^{(-)} \) from the argument of the crossing symmetry. Thus, the neutral pion-photoproduction should not be sensitive to the pseudovector and pseudoscalar couplings.
Even if there is a indeed difference in $A^{(-)}$ for pseudoscalar and pseudovector couplings, there is no significant phenomenological consequence if the appropriate parameters are used. In principle, this difference might be seen in the ratio between the charged and neutral pion productions, however, the both charged and neutral pion productions would affected by the internal structure of the nucleon at $O(\mu^2)$, which makes it impossible to get a very accurate and model independent result.

3. The contributions from the intermediate resonances

The consistent derivation of the LET makes it possible to discuss the contributions from the internal structure of the nucleon in this framework. The most important contributions from the internal structure of the nucleon are the charge radius of the nucleon and the transitions between the nucleon and the P-wave baryons as well as the $\Delta$ resonance, $P_{33}(1232)$. The nucleon charge radius contribution comes from the form factor\cite{13}, $e^{-\omega_\pi^2/2}$, for the photon absorptions near the threshold, expanding this form factor in terms of $\mu$, we get a correction $-\frac{M^2}{6}\alpha^2\mu^2$ to the isospin amplitude $A^{(-)}$, while the corrections to $A^{(0)}$ and $A^{(+)\text{ is of order } \mu^3}$ since the leading term of these amplitudes is $-\frac{\mu}{2}$.

Following the same procedure in discussing the contributions from the internal transitions to the polarizabilities of the nucleon, contributions from the transitions between the nucleon and the excited states can be easily evaluated. Replacing the electromagnetic transition operator $h^*$ in Eq. 21 of Ref. \cite{13}, the amplitude for the transitions between the nucleon and the P-wave baryons becomes

$$\mathcal{M}^p = -\frac{3\omega_\pi}{f_\pi\omega_h}\langle N_f|(h_3^3 - 2h_\pi^2) \cdot h_3 + h_3 \cdot (h_3^3 - 2h_\pi^2)|N_i\rangle$$

$$-\frac{3\omega^2_\pi}{f_\pi^2\omega_h^2}\langle N_f|(h_3^3 - 2h_\pi^2) \cdot h_3 - h_3 \cdot (h_3^3 - 2h_\pi^2)|N_i\rangle,$$  \hspace{1cm} (45)

where the transition operators $h_3$ and $h_3^3 - h_\pi^2$ are

$$h_3 = -\frac{1}{\sqrt{3}}\frac{e_3}{\alpha}\epsilon,$$  \hspace{1cm} (46)

$$h_3^3 - 2h_\pi^2 = -i\frac{1}{\sqrt{3}m_q}(\tau_3^3\sigma^3 - \tau_3^2\sigma^2)$$  \hspace{1cm} (47)

and the parameter $\omega_h$ is the mass difference between the nucleon and the P-wave baryons, which is related to the string constant $\alpha$ by $\omega_h = \frac{\alpha_0^2}{m_q}$ in the
harmonic oscillator wavefunction. Therefore, we can write Eq. 45 as
\[
\mathcal{M}^p = -i \frac{\omega \pi}{\pi \hbar m_q} \langle N_f | \left\{ e^3, \tau_i^3 \right\} \sigma^3 \cdot \epsilon - \left\{ e^3_0, \tau_i^2 \right\} \sigma^2 \cdot \epsilon + \frac{\omega}{\omega} [\tau_i^3, e_3^3] \sigma^3 \cdot \epsilon \rangle | N_i \rangle.
\] (48)

The first two terms in Eq. 48 correspond to the isospin amplitudes \( A^{(0)} \) and \( A^{(+)} \) at order \( \mu^2 \), while the last term corresponds \( A^{(-)} \) and is suppressed by a factor of \( \mu \) relative to the first two terms. In the SU(6) symmetry limit, Eq. 48 gives
\[
\mathcal{M}^{(0)} = 0
\] (49)
which comes from the symmetry arguments,
\[
\mathcal{M}^{(+)} = \frac{4e \mu^2 M^2}{9 \omega \pi m_q}
\] (50)
and
\[
\mathcal{M}^{(-)} = \frac{eg_\Lambda M^3 \mu^3}{3 \pi \hbar \kappa m_q}.
\] (51)

Therefore, the leading term from the transition between the nucleon and the P-wave baryons contributes to the isospin component \( A^{(+)} \), and the contribution to \( A^{(-)} \) is suppressed by a factor of \( \mu \). Since the ratio \( \frac{M_2}{\omega \hbar m_q} \) is quite large, this contribution is quite significant, in particular to the \( \pi_0 \) photo production near the threshold.

For the transition between the nucleon and the resonance \( P_{33}(1232) \), we have
\[
\mathcal{M} = \frac{-\omega \pi}{E_{\Delta}} \left[ \langle N_f | h^e_{\pi} | \Delta \rangle \langle \Delta | h^m | N_i \rangle + \langle N_f | h^m | \Delta \rangle \langle \Delta | h^e_{\pi} | N_i \rangle \right],
\] (52)
where \( h^m = \sum_j \frac{\epsilon_j \sigma_j \cdot (\hat{k} \times \epsilon)}{2m_q} \), and \( E_{\Delta} \) is the mass difference between the nucleon and \( \Delta \) resonance, and the transition operators \( h^e_{\pi} \) is given in Eq. 25. Evaluating Eq. 52 in the SU(6) symmetry limit gives
\[
\mathcal{M} = \frac{-\omega \pi}{E_{\Delta}} \left[ \langle N_f | \{ h^e_{\pi}, h^m \} \rangle - \frac{g A}{f_{\pi} M} \left\{ \mu_\pi \sigma^T \cdot (\hat{k} \times \epsilon), \tau_i \sigma^T \cdot P \right\} | N_i \rangle \right],
\] (53)

Using the relation for the anticommutator
\[
\{aA, bB\} = \frac{1}{2} \{a, b\} \{A, B\} + \frac{1}{2} [a, b][A, B]
\] (54)
and notice that $P = -\frac{k}{2}$ in the center of mass frame, we have

$$M^\Delta = \frac{\omega \omega_\pi}{2 f_\pi E_\Delta M} \langle N_f | \sum_j [\tau_0^j, \tau_l^j] \frac{i}{\sqrt{2m_q}} \sigma^j \cdot (P \times (\hat{k} \times \epsilon)) \langle N_i \rangle \rangle \left[ \frac{1}{2m_q} - (\mu_p - \mu_n) \right],$$

therefore, the transition between the nucleon and the $\Delta$ resonance only contributes to $A^(-)$ and it is at order $O(\mu^3)$, which is

$$M^\Delta = \frac{eg\mu_3 M^2}{f_\pi E_\Delta} \left[ \frac{1}{2m_q} - (\mu_p - \mu_n) \right]$$

$$= \frac{eg\mu_3 M}{2f_\pi E_\Delta} \kappa_n.$$  \hspace{1cm} (56)

It only affects the charged pion production near the threshold. There is also an additional term from the nucleon-$\Delta$ transition similar to the second term in Eq. \ref{eq:55} by expanding the propagator in terms of $\mu$, this term should contribute to the amplitudes $A^{(0)}$ and $A^{(+)}$. However, it also should be suppressed by a factor of $\mu$ relative to the contribution to $A^{(-)}$, thus should be neglected. The dependence of the nucleon-$\Delta$ on $\kappa_n$ is the characteristic of the electromagnetic transition between the nucleon and the $\Delta$ resonance, it is proportional to the magnetic moments of the neutron which is also the anomalous magnetic magnetic moments of protons and neutrons in the $SU(6)$ symmetry limit. The transition between the nucleon and the $\Delta$ resonance only enters at $O(\omega^2)$ in the Compton scattering, which contributes only to the nucleon polarizabilities, the same is also true for the threshold pion-pion process. The $P_{33}(1232)$ into nucleon and pion is in p-wave so that it is of $O(\mu)$ relative to the s-wave. However, since the transition energy $E_\Delta$ is quite small, this contribution should be at order $O(\mu^2)$ in practical calculations.

One could see the parallel between the contributions of intermediate resonance states in Compton scattering $\gamma N \rightarrow \gamma N$ and in the threshold pion-photoproductions; the transition between the nucleon and the excited states only enter at $O(\omega^2)$ in the Compton scattering, which contributes only to the nucleon polarizabilities, the same is also true for the threshold pion-photoproduction, and nucleon-$\Delta$ only contributes to the magnetic polarizabilities which the transition between the nucleon and the P-wave baryons only contributes to the electric polarizabilities of the nucleon in the Compton scattering \ref{eq:55}, while the intermediate $\Delta$ resonance only contributes to the isospin amplitude $A^{(-)}$, and the transition between the P-wave baryons and the nucleon contributes isospin amplitudes $A^{(+)}$ only. The physics is the same, the M1 transition dominates the nucleon-$\Delta$ transition, and the E1 transition dominates the P-wave baryon nucleon transition. Since the threshold
pion-photoproduction is dominated by the E1 transition, it explains why the nucleon-∆ transition only enter at $O(\mu^3)$, and P-wave baryon nucleon transition is of order $O(\mu^2)$. This also highlights that the P-wave baryon intermediate states are important for the threshold pion photoproduction, which has not been discussed fully in the literature.

Since the transition between the nucleon and the P-wave baryons only contributes to isospin amplitudes $A^{(+)}$, it is very important in the neutral pion-photoproduction. For the process $\gamma n \rightarrow \pi^0 n$, the LET predicts that the amplitude $E_{0+}$ is of order $\mu^2$, and the transition between the nucleon and P-wave baryons is also of order $\mu^2$. This will modified the results for the neutral pion production significantly. Indeed, the experimental results\cite{17}, although controversial, differ from the LET prediction by a factor of 2 for neutron targets. However, if the contributions from P-wave intermediate states are included, the results are very different. From the standard isospin relation

$$A(\gamma p \rightarrow \pi^0 p) = A^{(+)} + A^{(0)}$$
$$A(\gamma n \rightarrow \pi^0 n) = A^{(+)} - A^{(0)},$$

we have the total $E_{0+}$ amplitudes for the neutral pion-photoproduction

$$E_{0+}(\gamma p \rightarrow \pi^0 p) = \frac{e g_A}{8 \pi f_\pi (1 + \mu)} \left[ -\mu + \frac{1}{2} (1 + \kappa_p) \mu^2 + \frac{4 M^2}{9 g_A \omega_h m_q} \mu^2 \right]$$
$$E_{0+}(\gamma n \rightarrow \pi^0 n) = \frac{e g_A}{8 \pi f_\pi (1 + \mu)} \left[ -\frac{1}{2} \kappa_n \mu^2 + \frac{4 M^2}{9 g_A \omega_h m_q} \mu^2 \right]$$

(58)

For $\omega_h = 0.6$ GeV, which is the average mass difference between the P-wave baryons and the nucleon, $M/m_q = 2.79$ which is the standard value in quark model, $f_\pi = 93$ MeV, and $g_A = 1.26$, we get

$$E_{0+}(\gamma p \rightarrow \pi^0 p) = -1.7$$
$$E_{0+}(\gamma n \rightarrow \pi^0 n) = 1.0$$

(59)

in the unit of $10^{-3}/m_\pi$, comparing to $-2.4$ for $\gamma p \rightarrow \pi^0 p$ and 0.4 for $\gamma n \rightarrow \pi^0 n$ of the LET predictions. The contributions from the P-wave intermediate states increase $E_{0+}$ for $\gamma n \rightarrow \pi^0 n$ by more than factor of 2, and reduce $E_{0+}$ for $\gamma p \rightarrow \pi^0 n$ quite significantly. These improved results are in excellent agreement with known data, of which the average results are $-2.0$ for $\gamma p \rightarrow \pi^0 n$ and 1.0 for $\gamma n \rightarrow \pi^0 n$\cite{17}, \cite{18}.

For the isospin amplitude $A^{(-)}$, the total result is

$$M^{(-)} = \frac{e g_A}{f_\pi} \left[ 1 - \frac{M^2}{6 \alpha^2} \mu^2 + \left( \frac{M \kappa_n}{2 E_\Delta} + \frac{M^2}{6 \alpha^2} + \frac{M^3}{3 \omega_h^2 m_q} \right) \mu^3 \right].$$

(60)
The contribution from the charge radius of the nucleon is significant, if the parameter \( \alpha^2 = 0.175 \text{ GeV}^2 \) is used, it will reduce the charge pion production by \( 0.5 \times 10^{-3}/m_\pi \), which is larger than the previous estimates\(^{[10]}\) of the contribution from the nucleon-\( \Delta \) transitions. The contributions from \( \Delta \) resonance and the P-wave baryons effectively cancel each other, so the \( \mu^3 \) corrections could be neglected. However, we believe that the order \( \mu^3 \) corrections is more model dependent, thus less reliable because there are many sources that could contribute to the isospin amplitudes at this order.

4. Conclusion and discussion

We have shown how the LET of the threshold pion-photoproduction is realized in the chiral quark model which the nucleon is treated as a composite system of three nonrelativistic quarks; the LET is only determined by the center of mass motion, and it requires a quark-pion coupling from the effective chiral Lagrangian, in particular, the quark-pion-photon coupling that determines the Seagul term, and no constraint on the internal structure of the nucleon is needed. We also show that the pseudoscalar coupling gives an extra term \( \frac{1}{2} \mu \) in \( A^{(-)} \), thus the neutral pion production is not affected.

The LET provides an important test to the phenomenological models of the strong decays in hadron physics. The essential quark-pion operator for the LET is the term that proportional to \( \sigma \cdot p \), it is exactly the same as the operator in the \( ^3P_0 \) model\(^{[9]} \). This shows that \( ^3P_0 \) model is consistent with the LET, thus raises the possibility to calculate the parameters in the \( ^3P_0 \) model from the effective chiral Lagrangian. On the other hand, the quark-pion coupling operator in the \( ^3S_1 \) model has different structure, thus it could not lead to the LET. Since \( ^3S_1 \) model could not reproduce the model independent result, it is theoretically unjustified in this ground. It is also interesting to note that studies of the strong decay of \( b_1 \to \omega \pi \) shows that \( ^3P_0 \) model is preferred\(^{[9]} \).

We show that leading term of the transition between the P-wave baryons and the nucleon contributes to \( A^{(+)} \) at \( O(\mu^2) \), the most crucial test of this term is the neutral pion production, in particular \( \gamma n \to \pi_0 n \), since the leading term in the LET is also of order \( \mu^2 \). Indeed, the result including the transition between the P-wave baryons and the nucleon improves the agreement with the known data significantly for both neutron and proton targets. We show that the nucleon-\( \Delta \) transition only contributes to \( A^{(-)} \) at order \( \mu^3 \), and it is less important than the transition between the nucleon and the P-wave baryons. This is because the decay of the \( \Delta \) resonance into the nucleon and pion is in p-wave, while the P-wave baryons decays are in S-wave, thus although the \( \Delta \) resonance is a very important resonance, it is suppressed by a factor of \( \mu \).
The more important contribution to the isospin amplitude $A^{(-)}$ is the nucleon charge radius, which is of $O(\mu^2)$. We want to emphasize that contributions from the P-wave baryon intermediate states and the nucleon charges radius to isospin amplitudes $A^{(-)}$ and $A^{(+)}$ at order $\mu^2$ is model independent, although the numerical results may vary. There might be also contributions from the pion loops, which remains to be studied. A systematic study of the contributions at $\mu^3$ may be needed, which the corrections might be significant in the charge pion productions. Our results here provide an important challenge to the experiments; an accurate measurement of the neutral pion productions will provide a direct probe to the structure of the nucleon.

Our calculation here also presents a framework to calculate the $\eta$ and $K$ photoproductions near the threshold, which the contributions from the resonances become much more important. The extension of our study to the pion-electroproduction near the threshold is also in progress.

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