We present a Neutrino-Factory-based setup with three detectors of different kind in principle capable to solve the eightfold-degeneracy in the simultaneous measurement of $\theta_{13}$ and $\delta$, for $\theta_{13} \geq 1^\circ$ ($\sin^2(2\theta_{13}) \geq 10^{-3}$). Our setup includes a Superbeam-driven water Cherenkov (the Superbeam conceived as the first stage of the Neutrino Factory); two muon-storage-ring-driven detectors (namely, a large magnetized iron calorimeter and an emulsion cloud chamber) to take advantage of both the so-called “golden” ($\nu_e \rightarrow \nu_\mu$) and “silver” ($\nu_e \rightarrow \nu_\tau$) channels.

The planned long baseline experiments [1] will improve the measurement of $\Delta m_{\text{atm}}^2$ and of $\theta_{23}$ and measure or increase the bound on $\theta_{13}$ [2, 3] (see also [4]). This new generation of experiments, however, is only the first step of a long-lasting experimental program including the development of some “superbeam” facilities (whose combination can strongly improve our knowledge on $\theta_{13}$, see [5]) and, eventually, of a “Neutrino Factory” [6, 7]. One of the main goals of the Neutrino Factory program (see for example [8, 9] and refs. therein) would be the discovery of leptonic CP violation and, possibly, its study [10]-[11].

The transition probabilities $\nu_e \rightarrow \nu_\mu$ and $\nu_\mu \rightarrow \nu_e$ are extremely sensitive to $\theta_{13}$ and $\delta$: this is what is called the “golden measurement at the Neutrino Factory” [11] and can be easily studied by searching for wrong-sign muons, provided the considered detector has a good muon charge identification capability. The determination of $(\theta_{13}, \delta)$ from this channel is not at all free of ambiguities: in [12] it was shown that, for a given physical input parameter pair $(\bar{\theta}_{13}, \bar{\delta})$, measuring the oscillation probability for $\nu_e \rightarrow \nu_\mu$ and $\bar{\nu}_e \rightarrow \bar{\nu}_\mu$ will generally result in two allowed regions of the parameter space. The first one contains the physical input parameter pair and the second, the “intrinsic ambiguity”, is located elsewhere. Worse than that, new degeneracies have later been noticed [13, 14], resulting from our ignorance of the sign of the $\Delta m_{\text{atm}}^2$ squared mass difference and from the approximate $[\theta_{23}, \pi/2 - \theta_{23}]$ symmetry for the atmospheric angle. In general, for each physical input pair, the measure of $P(\nu_e \rightarrow \nu_\mu)$ and $P(\bar{\nu}_e \rightarrow \bar{\nu}_\mu)$ will result in eight allowed regions of the parameter space, the eightfold-degeneracy [14].

From what learned in the previous papers [15]-[17] we conclude that the optimal combination to deal with the eightfold-degeneracy consists in taking advantage of all the neutrino beams produced in a Neutrino Factory Complex (i.e. factory plus detectors). The Neutrino Factory Complex that we consider consists of a SPL-like superbeam [18] and a 50 GeV muon storage ring [9], plus a network of three detectors of different

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technology:

1. a 40 Kton Magnetized Iron Detector (MID) at $L = 2810$ Km, [19];
2. a 4 Kton Emulsion Cloud Chamber (ECC) at $L = 732, 2810$ Km, [17];
3. a 400 Kton Water Cherenkov (WC) at $L = 130$ Km, [20].

This proposal, resulting from the combination of [12, 16] and [17], corresponds to the design of a possible CERN-based Neutrino Factory Complex, with detectors located at the Frejus (the WC), at Gran Sasso (the ECC) and at a third site to be defined (the MID and possibly the ECC). Each one of these detectors is especially optimized to look for a particular signal: $\nu_\mu \rightarrow \nu_e$ oscillations for the 400 Kton WC, $\nu_e \rightarrow \nu_\mu$ for the 40 Kton MID and $\nu_e \rightarrow \nu_\tau$ for the 4 Kton ECC.

The physical parameters to be measured at the Neutrino Factory Complex are, in the worst case (i.e. if the planned experiment are not able to measure some of them earlier), the two PMNS mixing matrix parameters $\theta_{13}$ and $\delta$, the sign of $\Delta_{atm}$, $\bar{s}_{atm}$, and the $\theta_{23}$-octant, $\bar{s}_{oct}$, where

$$ \bar{s}_{atm} = \text{sign}[\Delta_{atm}^2] ; \quad \bar{s}_{oct} = \text{sign}[\tan(2\theta_{23})]. $$

Both discrete variables can assume the values $\pm 1$, depending on the physical assignments of the sign of $\Delta_{atm}^2$ and of the $\theta_{23}$-octant ($s_{oct} = 1$ for $\theta_{23} < \pi/4$ and $s_{oct} = -1$ for $\theta_{23} > \pi/4$). The other parameters have been considered as fixed quantities, supposed to be known with good precision by the time when the Neutrino Factory will be operational. In particular: $\theta_{12} = 35^\circ$ and $\Delta m_{atm}^2 = 7 \times 10^{-5}$ eV$^2$; $\theta_{23} = 40^\circ, 50^\circ$ (a generic value in the allowed region $\sin^2(2\theta_{23}) > 0.9$ with both possible octant choices) and $|\Delta m_{atm}^2| = 2.9 \times 10^{-3}$ eV$^2$; $A = 1.1 \times 10^{-4}$ eV$^2$/GeV.

The experimental information consists of the number of muons in the detector with charge opposite to that of the muons circulating in the storage ring. We group the events in bins of the final muon energy $E_\mu$ and call $N^{\text{g.s}}(\theta_{13}, \delta)$ the number of “golden” or “silver” in the i-th energy bin for the input pair $(\theta_{13}, \delta)$ [11]. In the case of the Superbeam, $N$ represents the number of electrons in the water Cherenkov, grouped in one single bin. For a given energy bin and fixed input parameters $(\theta_{13}, \delta)$, we can draw a set of curves of equal number of events [17] in the $(\theta_{13}, \delta)$ plane,

$$ N^i_{\mu^+}(\theta_{13}, \delta; \bar{s}_{atm}, \bar{s}_{oct}) = N^i_{\mu^+}(\theta_{13}, \delta; \bar{s}_{atm}, \bar{s}_{oct} = \bar{s}_{oct}) , $$
$$ N^i_{\mu^-}(\theta_{13}, \delta; \bar{s}_{atm}, \bar{s}_{oct}) = N^i_{\mu^-}(\theta_{13}, \delta; \bar{s}_{atm} = -\bar{s}_{atm}, \bar{s}_{oct} = \bar{s}_{oct}) , $$
$$ N^i_{\mu^+}(\bar{\theta}_{13}, \delta; \bar{s}_{atm}, \bar{s}_{oct}) = N^i_{\mu^+}(\theta_{13}, \delta; \bar{s}_{atm} = \bar{s}_{atm}, \bar{s}_{oct} = -\bar{s}_{oct}) , $$
$$ N^i_{\mu^-}(\bar{\theta}_{13}, \delta; \bar{s}_{atm}, \bar{s}_{oct}) = N^i_{\mu^-}(\theta_{13}, \delta; \bar{s}_{atm} = -\bar{s}_{atm}, \bar{s}_{oct} = -\bar{s}_{oct}) . $$

Following the procedure outlined in [12, 17] we can numerically solve eqs. (2)-(5) and found the theoretical location of the clones in the $(\theta_{13}, \delta)$ plane. We present in Fig.1 the outcome of this procedure for the different degeneracies with fixed $\delta = 90^\circ$ and changing $\theta_{13} \in [0.1^\circ, 10^\circ]$. Apart from some exceptional abrupt change (remind $2\pi$-periodicity in the $\delta$ axis), a small change in the input parameter $\theta_{13}$ results in a small shift of the clone location. (Almost) continuous geometrical regions where degeneracies lie are defined.
for a given interval in $\theta_{13}$, illustrating how the clones move due to a change in the input parameters: we will call this the “clone flow”. In Fig. 1(left) we plotted the intrinsic clone flow for a set of different experiments and channels; in Fig. 1(right) the clone flows for the eightfold-degeneracy are presented, for the NF-golden channel only.

**FIGURE 1.** (left) Intrinsic clone flows for the SPL and the NuMI Off-Axis superbeams and for the NF golden and silver channels; (right) The eightfold-degeneracy for the NF golden channel. The thick dot is the true solution, the thin dots the clones location for changing $\bar{\theta}_{13} \in [0.1^\circ, 10^\circ]$ and fixed $\bar{\delta} = 90^\circ$.

Fig. 1(left) shows that the combination of any two facilities solves the intrinsic degeneracy [16, 17]. More difficult is the case when all the degeneracies are treated on equal footing, Fig. 1(right), where the need of the combination of (at least) three facilities is manifest. This is exemplified in Fig. 2, where we present the outcome of combined $\chi^2$ fits performed as in [11] for different combinations of the three detectors, for a fixed input pair $\bar{\theta}_{13} = 2^\circ, \bar{\delta} = 90^\circ$. In Fig. 2(a) four degeneracies can be seen when using the 40 Kton MID only; in Fig. 2(b) and Fig. 2(c) we notice how two of the degeneracies disappear when combining the 40 Kton MID with the 400 Kton WC or the 4 Kton ECC, respectively; eventually, in Fig. 2(d) the combination of the three detectors solve all the degeneracies reconstructing with a good precision the physical input values. CL contours up to 4 sigma are plotted.

Two comments are in order: first, the physical input pair $\bar{\theta}_{13} = 2^\circ, \bar{\delta} = 90^\circ$ is generic and similar results are obtained for different input parameters for $\bar{\theta}_{13} > 1^\circ$; second, these results, although promising, are still preliminary and a new study where particular care is devoted to systematics in the three detectors is currently underway [21].

**REFERENCES**

1. F. Arneodo et al. [ICARUS Coll.], ICARUS-TM/2001-08 LNGS-EXP 13/89 add.2/01; M. Guler et al., OPERA Collaboration, CERN/SPSC 2000-028, SPSC/P318, LNGS P25/2000; E. Ables et al. [MINOS Coll.], FERMILAB-PROPOSAL-P-875; Y. Itow et al., KEK-REPORT-2001-4, arXiv:hep-ex/0106019.
2. M. Komatsu, P. Migliozzi and F. Terranova, J. Phys. G 29 (2003) 443 [arXiv:hep-ph/0210043].
3. M. Diwan et al., NuMI-NOTE-SIM-0714.
4. P. Migliozzi and F. Terranova, arXiv:hep-ph/0302274.
5. P. Huber et al., Nucl. Phys. B 654 (2003) 3; W. Winter, arXiv:hep-ph/0308227.
6. S. Geer, Phys. Rev. D 57 (1998) 6989 [Erratum-ibid. D 59 (1998) 039903].
7. A. De Rujula, M. B. Gavela and P. Hernandez, Nucl. Phys. B 547 (1999) 21.
FIGURE 2. The results of a $\chi^2$ fit for $\bar{\theta}_{13} = 2^\circ$; $\bar{\delta} = 90^\circ$. Four different combinations of experimental data are presented: a) MID; b) MID plus WC; c) MID plus ECC; d) The three detectors together.

8. A. Blondel et al., Nucl. Instrum. Meth. A 451 (2000) 102.
9. M. Apollonio et al., arXiv:hep-ph/0210192.
10. K. Dick et al., Nucl. Phys. B 562 (1999) 29; V. Barger, S. Geer and K. Whisnant, Phys. Rev. D 61 (2000) 053004; A. Bueno, M. Campanelli and A. Rubbia, Nucl. Phys. B 573 (2000) 27.
11. A. Cervera et al., Nucl. Phys. B 579 (2000) 17 [Erratum-ibid. B 593 (2001) 731].
12. J. Burguet-Castell et al., Nucl. Phys. B 608 (2001) 301.
13. H. Minakata and H. Nunokawa, JHEP 0110 (2001) 001.
14. V. Barger, D. Marfatia and K. Whisnant, Phys. Rev. D 65, 073023 (2002).
15. H. Minakata et al., Phys. Rev. D 68 (2003) 013010; H. Minakata et al., Phys. Rev. D 68 (2003) 033017; M. Freund et al., Nucl. Phys. B 615 (2001) 331; A. Rubbia, arXiv:hep-ph/0106088; A. Bueno et al., Nucl. Phys. B 631 (2002) 239; T. Kajita et al., Phys. Lett. B 528 (2002) 245.
16. J. Burguet-Castell et al., Nucl. Phys. B 646 (2002) 301.
17. A. Donini, D. Meloni and P. Migliozzi, Nucl. Phys. B 646 (2002) 321; arXiv:hep-ph/0209240; D. Autiero et al., arXiv:hep-ph/0305185.
18. J. J. Gomez-Cadenas et al, arXiv:hep-ph/0105297.
19. A. Cervera, F. Dydak and J. Gomez Cadenas, Nucl. Instrum. Meth. A 451 (2000) 123.
20. A. Blondel et al., Nucl. Instrum. Meth. A 503 (2001) 173.
21. J. Burguet-Castell et al., in preparation.