How a spin-glass remembers. Memory and rejuvenation from intermittency data: an analysis of temperature shifts.

1,2 Paolı Sibani* and 2Henrik Jeldtof Jensen
1 Theoretical Physics, Oxford University, 1 Keble Rd, Oxford OX1 3NP, UK
2 Imperial College London, South Kensington Campus, SW7 2AZ, UK
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The memory and rejuvenation properties of intermittent heat transport are explored theoretically and by numerical simulation for Ising spin glasses with short-ranged interactions. The theoretical part develops a picture of non-equilibrium glassy dynamics recently introduced by the authors, which links irreversible ‘intermittent’ events, or ‘quakes’ to thermal fluctuations of record magnitude. The pivotal idea is that the largest energy barrier $b(t_w, T)$ surmounted prior to $t_w$ at temperature $T$ determines the rate $r_q \propto 1/t_w$ of the intermittent events near $t_w$. The idea implies that the same rate after a negative temperature shift should be given by $r_q \propto 1/t_w^{q^*/w}$. The ‘effective age’ $t_{eff}^{q/w} \geq t_w$ has an algebraic dependence on $t_w$, whose exponent contains the temperatures before and after the shift. This analytical expression is confirmed by numerical simulations. Marginal stability suggests an asymmetry between cooling and heating, i.e. a positive temperature shift $T \rightarrow T'$ should erase the memory $b(t_w, T)$. This is confirmed by the simulations, which show a rate $r_q \propto 1/t_w(w)$ controlled by the barrier $b(t_w, T') \geq b(t_w, T)$. Additional ‘rejuvenation’ effects are also identified in the intermittency data for shifts of both signs.

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I. INTRODUCTION

Many fascinating aspects of non-equilibrium glassy dynamics are revealed by temperature changes applied while the system ages, both experimentally, see e.g. [1, 2, 3, 4] and in numerical simulations [5, 6, 7]. Spin-glass experiments often measure the out of phase AC susceptibility $\chi''(\omega, t_w)$, or the (related) derivative of the ZFC magnetization with respect to the logarithm of time $S(t, t_w)$. In the protocol introduced in ref. [3] the temperature is decreased at a constant rate, except for one temperature $T_0$ which is maintained for e.g. hours. During this aging stage, $\chi''$ decreases, revealing the characteristic dynamical stiffening generally present in glassy systems. As cooling is resumed, the susceptibility returns to the $T$ dependence of an un-aged system. The time spent aging at one temperature is thus of little avail at a lower temperature, and the $\chi''$ versus $T$ curve features a dip centered at $T_0$. That this dip is reproduced when re-heating at a constant rate through $T_0$ demonstrates the persistence of the memory of the configurations visited. A later extension of this work [8] shows that two dips can be produced and retrieved by aging at different temperatures. More importantly, the memory can be erased by overheating, i.e. in a second cooling phases the dip is no longer present, provided that the intervening temperature has been sufficiently high.

In temperature shifts experiments, the system is aged for a time $t_w$, and a small magnetic field is then turned on to probe the dynamics. Under isothermal conditions, $S(t, t_w)$ has a maximum at $t = t_w$. If a small temperature shift of either sign is imposed at $t_w$, this maximum moves [9]. The position of the maximum on the $t$ axis defines an effective age, which is studied as a function of various parameters. Ref. [4] is particularly concerned with the issue of the invertibility of the relation between age and effective age, an issue which we return to in the discussion. Numerical simulations are also able to follow the time dependence of the energy density $e(t)$ after a $T$ shift: the so called ‘Kovacs’ effect is best observed when shifting the temperature from $T_1$ to $T_2$ at a late time, for which the energy is already close to its equilibrium value at $T_2$: After the shift, $e(t)$ approaches the isothermal energy density at $T_2$ in a non-monotonic way. This effect can be used to define an effective age [5, 7] as a time shift allowing one to superimpose the the $T$ shifted $e(t)$ onto its isothermal counterpart at the final temperature.

The present investigations give only indirect insight on configurational changes and how these changes are memorized by the aging spin-glass. They do, however directly shed light on a related issue, i.e. how the size of the dynamical barriers surmounted in an aging system is memorized. The data allow a parameter-free theoretical interpretation by a recent model of non-equilibrium relaxation [8], which provides a unified description of pure aging by attributing a pivotal role to energy fluctuations of record size. The simulations combine temperature shift techniques with a statistical analysis of heat transfer [10] motivated by the intermittency studies of refs. [11, 12, 13] in soft-condensed matter systems and glasses. In this way, many important aspects of aging dynamics can be extracted from a one-time quantity, the energy. This is potentially useful whenever eliciting a linear response is difficult or impossible. The definitions used for the effective age are based on the aging behavior of the rate of intermittent events, and differ from the definitions used

*Permanent address: Fysisk Institut, SDU, Odense, DK
in studies of linear response effects and energy density, as discussed in detail at the end of the paper. Nevertheless, whenever a comparison is possible, concurring physical conclusion can be drawn.

The Probability Density Function (PDF) of the heat flow data shows that two types of events occur: reversible fluctuations, which are described by the Gaussian part of the PDF, and large, irreversible events, dubbed quakes, through which the excess energy initially trapped in the sample is transferred to the heat bath. These events appear as an asymmetric exponential tail of the PDF. The general form of the PDF is easily understood: thermal fluctuations occur independently and simultaneously in different part of the systems, whence their sum is Gaussianly distributed. Occasionally, a thermal fluctuation induces a large configurational rearrangement associated with a correspondingly large energy release. Such a quake is arguably composed of a series of consecutive ‘downhill’ flips of contiguous spins, and its size is exponentially distributed if each movement can be the last one with equal probability.

A closer inspection of the data reveals that the rate \( r_q(t_w) \) of the quakes: \( i \) is temperature independent \[10\], which is striking for activated dynamics, and \( ii \): decreases, under isothermal conditions, as the reciprocal of the age \( 1/t_w \). The effective age \( t^{eff}_w \) of a sample, possibly subject to \( T \) shifts, can hence be defined as \( 1/r_q \), and its behavior can be empirically investigated under different conditions. The theoretical idea used to explain these findings is that a glassy system ‘remembers’ the largest or ‘record’ energy barrier \( b(t_w, T) \) surmounted during its (iso)thermal history up to time \( t_w \). Specifically, only fluctuations exceeding \( b \) are able to produce the intermittent events on each time scale. For isothermal aging, record dynamics explains the decelerating rate of events \( r_q(t_w) \propto 1/t_w \) and accounts for the temperature independence of \( r_q \). Within this framework, a mathematically consistent explanation is only possible if the dynamically available configurations are distributed in energy according to an exponential ‘local density of states’ (LDOS), a form which is widely observed in complex systems.

In this paper, we draw a further consequence of these ideas, by predicting the effect of a temperature shift imposed at \( t_w \) on the rate \( r_q(t_w, T, T') \) of intermittent events. Unlike the isothermal case, this rate could depend, beside \( t_w \), on the temperatures \( T \) and \( T' \) before and after the shift. For negative temperature shifts, the form of \( t^{eff}_w \) is derived analytically under the assumption that the memory of the extremal barrier \( b(t_w, T) \) established prior to the shift persists and that its value still determines the evolution at the lower temperature \( T' \). A positive shift allows stronger energy fluctuations which very quickly overcome \( b(t_w, T) \) and reach \( b(t_w, T') \), the extremal barrier for isothermal relaxation at the final temperature \( T' \). Hence, the intermittent energy release continues at a rate \( r_q \propto 1/t_w \). These properties are fully confirmed by the simulations.

A second observation is that the extremal barrier does not account for the proportionality constant between \( r_q \) and \( 1/t^{eff}_w \) (or \( 1/t_w \)). This constant is larger (i.e. stronger intermittency) after shifts of either sign than for the isothermal relaxation. In this sense, the shifts make the system appear younger, and induce what could be called a rejuvenation effect.

In the next section we summarize the theoretical ideas linking non-equilibrium drift to record fluctuations. In section \( \text{III} \) we derive the formula for the effective age, which is tested in Section \( \text{IV} \) where the numerical results are also given. Section \( \text{V} \) provides technical details on the simulations and on the data handling. and section \( \text{VI} \) is a brief summary followed by a discussion of how the present results relate to previous investigations.

II. WHY RECORD FLUCTUATIONS COULD BE IMPORTANT IN GLASSY SYSTEMS

Extremal fluctuations are largely irrelevant in equilibrium, but may have pivotal effects in metastable situations, i.e. when exceeding a threshold can push the system into a completely different dynamical path. Consider first, as an example, an artifact, e.g. a skyscraper, which collapses if hit by seismic events above a certain magnitude. We assume that, against good engineering practice,
a ‘marginally better’ skyscraper is built following any collapse, which is able to resist events up to the size causing the latest break-down, but unable to resist any—ever slightly—stronger event. In the sequence of gradually stronger skyscrapers built according to this prescription, each element collapses if hit by an earthquake stronger than all its predecessors. The magnitude is irrelevant, and the statistics of record-sized earthquakes describes in this case physical events.

Consider now a sequence of random numbers drawn independently from the same continuous distribution. The distribution of the number of records, i.e. numbers larger than all previous extractions, is independent of the distribution from which the data are extracted \[15\]. An approximate statistical description of the records \[9, 15\] is the log-Poisson distribution

\[
P_n(t_1, t_2) = \frac{(\alpha \log(t_2/t_1))^n}{n!}(t_2/t_1)^{-\alpha}, \quad \alpha \leq 1, \quad t_2 \geq t_1
\]

where \(P_n\) is the probability that \(n\) records will occur between trials \(t_1\), and where the parameter \(\alpha\) describes that the records can occur in \(\alpha\) independent processes running ‘in parallel’. We shall later think of these trials as a time sequence, and of the random numbers as energy values produced by thermal fluctuations. Furthermore we will mainly need the formula in the limit where \(t_2 \approx t_1\). Glossing over the discreteness of these parameters, and with \(t_2 = t_w\) the rate of events near \(t_w\) falls off as \(r_q \propto 1/t_w\).

Marginal stability provides a physical mechanism by which the rank, and not the magnitude of the fluctuations determines the non-equilibrium dynamics associated with changes of attractor. Because the temporal statistics of record events is system independent, the concept is potentially useful whenever metastability is important. Its physical relevance was first noticed in a simple model of Charge Density Waves dynamics \[10\], later extended to describe the glassy dynamics of various systems with multiple metastable configurations \[15, 17, 18\], and more recently to thermal aging \[9, 17, 18\] of glasses. The numerical investigations of ref. \[14\] show explicitly how selected energy records characterize the attractors visited during unperturbed thermal aging after a quench.

A priori, marginal stability can play a dynamical rôle whenever: i) Numerous attractors, or ‘intrinsic states’ exists, with different degrees of stability, the latter gauged by a typical exit time or, equivalently, exit energy barrier. A marginal increase of the degree of stability of the attractor selected can arise from an entropic effect, i.e. simply reflect an overwhelming predominance of shallow attractors.

ii) the initial deep quench typically produces configurations with a large energy excess. Similar to the gravitational energy of a building released irreversibly during its collapse, this energy can be released in relative large and (hence) irreversible bursts \[9, 10\].

As in the skyscraper example, if the increase in attractor stability is marginal, the non-equilibrium dynamics is driven by fluctuation extrema. The number of quakes occurring in the interval \([1, t_w]\) is then described by the log-Poisson distribution given in Eq. \[11\] with \(t_1 = 1\) and \(t_2 = t_w\).

To reach quantitative predictions, some basic aspects of real-space morphology must also be considered: Aging in spin-glasses with short-ranged interactions is characterized by a slowly growing length scale, the thermal correlation length, which remains shorter than a few lattice spacings \[2, 10\] for accessible time scales. It is therefore reasonable to assume that in an extended system with short range interactions many thermalized sub-domains exist, whose linear size matches the thermal correlation length and which are separated by frozen degrees of freedom. Their reversible thermal fluctuations are statistically independent and are described, through the usual tools of statistical mechanics, by a ‘local’ energy landscape comprising, for each domain, the configurations available.

The model assumption explored in the following is that an extremal fluctuation within each sub-domain is able to trigger a quake \[9, 10\]. This has two consequences: Firstly, the total number of quakes is a sum of independent Poisson processes, and hence itself a Poisson process with average \(\alpha > 1\). This average increases with the system size and attains the smallest possible value \(\alpha = 1\) only if a single sub-domain is present. Secondly, as we argue below, combining the thermal nature of the extremal fluctuations with the properties of record statistics means that the sub-domains must have a nearly exponential local density of states LDOS, of the type observed numerically in a variety of small glassy systems \[18, 20, 21\].

Ignoring for simplicity any difference between the sub-domains, we write this LDOS as

\[
D(\epsilon) \propto \exp(\epsilon/\epsilon_0),
\]

and note that the condition \(T < \epsilon_0\) is required in order to ensure the thermal metastability of any attractor to which this LDOS applies.

Importantly, a log-Poisson description is only possible as long as a sharp distinction between local reversible fluctuations and irreversible changes is meaningful. The limitation does not seem unduly restrictive in deeply quenched glassy system, which for many decades of time remain far from true thermal equilibrium. The statistical independence of fluctuations which is required by the theory, is approximately fulfilled at low temperature, as trajectories mostly dwell at the bottom of local wells.

The above ideas can be directly tested in intermittency measurements, where different parts of the same data set provide the required information \[14\]. E.g. the left panel of Fig. \[1\] shows the Probability Density Function (PDF) of the heat transferred over a small time intervall \(\delta t\) under isothermal conditions. The Gaussian part of the PDF describes the reversible fluctuations and is found to be independent of \(t_w\) \[10\], indicating that the attractors successively explored are similar with respect to their equilibrium energy fluctuations, as already im-
The temperature is

\[ T = 0.20 \]

Upper data sets: One additional cooling step takes the system from \( T = 0.25 \) to \( T = 0.20 \). For the \( i \)'th data set, \( i = 1, 2, \ldots 5 \) the change occurs at age \( t_{w,i} \), determined such that the corresponding effective ages are \( t_{w,i}^{eff} = 2000, 4000, 8000, 16000 \) and \( 32000 \). The corresponding sampling intervals are \( \delta t = t_{w,i}^{eff}/100 \) and \( t = t_{w,i}^{eff}/2 \). Keeping the ratio \( \delta t/t_{w,i}^{eff} \) constant collapses the data, whence the effective age plays the same role as the actual age does in the isothermal case. The insert shows that the skewness of the PDF’s falls into two distinct groups, with only a small variation within each group.

The exponential tail describes the intermittent events, which are rare but far more frequent than Gaussian fluctuations lending quantitative support to this assumption [10]. Figure 2 shows the quality of the data collapse, of the sort shown in Fig. 2 deteriorates as the condition of constant \( \delta t/t_{w,i}^{eff} \) is violated. The effect is achieved by changing the value of \( \delta t \), which amounts to varying \( t_{w,i}^{eff} \) around its theoretical value given by Eq. 2. The abscissa is the relative deviation, and zero thus corresponds to the theoretical prediction. The following negative \( T \) jumps are considered \( 0.25 \rightarrow 0.20 \) (circles), \( 0.35 \rightarrow 0.30 \) (squares), \( 0.45 \rightarrow 0.40 \) (diamonds) and \( 0.55 \rightarrow 0.50 \) (stars). Lines are guides to the eye. Note that in all cases the best collapse is obtained at or very near the predicted value of the effective age.

III. HOW A SPIN-GLASS REMEMBERS

If uncorrelated energy records trigger the quakes, the time intervals between these quakes, i.e. the residence time \( t_r \) spent in metastable states, grows in proportion to the age \( \ref{eq:scaling} \) and is independent of the temperature. This so-called ‘pure scaling’ behavior is well known and analytically predicted for log-Poisson statistics. We presently choose to ignore the usually small but always systematic deviations from pure aging, sub- or super-aging, which appear in both numerical and experimental results.

A second expression for \( t_r \) can be derived using that the system is thermally equilibrated in the metastable states between the quakes. In this case, hopping over the extremal barrier \( b(t_{w}, T) \) (see below) is mediated by equilibrium thermal fluctuations. Hence, the residence time in a metastable state entered on a time scale \( t_{w} \) is proportional to the Arrhenius scale \( t_r = \tau_A \propto \exp(b(t_{w}, T)/T) \). Consistency with our previous argument requires \( \tau_A \approx t_{w} \) which is mathematically possible only if the LDOS from which the energy fluctuations are drawn has the exponential form given by Eq. 2.

Equation 2 describes the distribution in energy of the configurations accessible to an arbitrary thermalized subdomain, with zero defined as the lowest available energy. The probability of visiting states of energy larger than \( \epsilon \) without exiting the attractor is then

\[ P_{eq}(\epsilon) = \exp(-a(T)), \]

where the reduced inverse temperature

\[ a(T) = (1/T - 1/\epsilon_0) \]

describes, for temperatures well below \( \epsilon_0 \), the entropic effect of the exponential divergence of the LDOS. This so-called 'pure scaling' behavior is well known and analytically predicted for log-Poisson statistics. We presently choose to ignore the usually small but always systematic deviations from pure aging, sub- or super-aging, which appear in both numerical and experimental results.
out, the value $\epsilon_0 = 0.86$ is used. This value, which is close to the critical temperature of the 3d Edwards-Anderson spin-glass [22], was obtained in ref. [10] by fitting the theoretical form predicted by Eq. 4 to the heat capacity data inferred from the Gaussian heat fluctuations.

The largest fluctuation occurring during $(0, t_w)$ is the largest of $O(t_w)$ energy values drawn independently from the exponential equilibrium distribution $\propto \exp(-\beta E)$. According to a standard result of extremal statistics [23] (NB: this result only requires that the tail of the Boltzmann distribution be exponentially distributed) its average value grows with the age as

$$b(t_w, T) = \ln(t_w) / a(T) + \text{const.},$$

whence the Arrhenius time $\tau_A = \exp(a(T)b(t_w, T))$ remains equal to $t_w$, as anticipated.

The above argument is now extended to the situation where the temperature is instantaneously lowered from $T$ to $T'$ at age $t_w$: the extremal barrier $b(t_w, T)$ previously established remains the same, but must now be overcome at a lower temperature in order to induce a quake. The relevant Arrhenius time is then given by

$$\tau_A = t_w^{eff} \propto \exp(a(T)b(t_w, T)) \propto t_w^{a(t_w) / T} \quad T' \leq T,$$

where Eq. 5 is used for the right-most equality. The rate of intermittent events after the shift is $r_q \propto 1/t_w^{eff}$. Note that the argument ignores the decay of the rate occurring while the statistics is collected. In the cases considered, the drift remains small since the observation time is well below the effective age.

Equation 3 implies that the PDF’s of the heat transfer obtained for various $\delta t$ and $t_w$ values collapse if the ratio $\delta t/t_w^{eff}$ is kept constant. This is demonstrated in Figs. 2 and 3 which show that $t_w^{eff}$ plays a role analogous to $t_w$ in isothermal aging experiments (See Figs. 2 and 3 and ref. [10]). As anticipated, the step has an additional effect: the collapse leads to a master curve with a considerably stronger exponential tail than the curve correspondingly obtained from isothermal data.

In summary, a temperature drop has two effects: $a$: It effectively increases the size of all barriers, leading to an effective age greater than the age. In this respect the system looks older. $b$: The amount of heat delivered by the intermittent events is higher, whence the system looks in this respect younger. As later discussed, a similar duality is seen in refs. 4, 6. A positive temperature shift qualitatively differs from a negative shift. The extremal barrier $b(t_w, T)$ still sets the time scale for the first intermittent event after the step. Since this barrier is now strongly reduced relative to the typical size of the thermal fluctuations, its crossing is neither rare nor necessarily irreversible. The additional time needed to reach the extremal barrier $b(t_w, T')$ is, clearly, bounded by $t_w$, whence we can expect the data to be scaled by keeping $\delta t/t_w$ constant. This is confirmed by Fig. 4 showing an excellent data collapse after the positive shift. Again, comparing the master curve with its isothermal counterpart shows a stronger exponential tail. Hence, a positive step rejuvenates the dynamics in the same sense as discussed in item b above.

IV. SIMULATION RESULTS

The simulation results are now described in more detail. As in ref. [10], the statistics of the amount of heat exchanged over a time interval of length $\delta t < t_w$ is collected for $n$ contiguous intervals, with $n$ chosen such that $n\delta t = t$. The $k$'th value sampled is thus

$$H_k = E(t_w + (k + 1)\delta t) - E(t_w + k\delta t), \quad k \leq n,$$

where $E$ is the energy of the system. Since the statistical properties of the heat transfer depend on $\delta t$, $t$ and $t_w$ only, the process itself is denoted by $H(\delta t, t, t_w)$. The drift toward lower energies is described by an asymmetric exponential tail, and the fluctuations by the Gaussian part of the PDF. For fixed $t$ and $\delta t$, the PDF’s approach a Gaussian shape as $t_w$ increases and the skewness (third central moment) correspondingly approaches zero from below. This is seen in the left frame of Fig. 4 and in the corresponding insert. The three PDF’s are taken at $t_w = 4400, 8800$ and $17600$, all with with $t = 1200$, $\delta t = 22$ and $T = 0.20$. The data shown in the right hand panel of Fig. 4 are obtained with the same parameters, except that the temperature is instantaneously decreased from $T = 0.25$ to $T = 0.20$ at $t_w = 2140$. The Gaussian fluctuations are not affected by the step. The tails, however, are less pronounced than in the isothermal case, and thus qualitatively similar to those of an isothermally aged but older system.

Figure 2 contains twelve data sets, which fall into two distinct groups of equal size. Data with the most pronounced intermittent tails are taken after instantaneously decreasing the temperature from $T = 0.25$ to $0.20$ at ages $t_w = 275, 460, 770, 1283, 2143$ and 3578. According to Eq. 6 these values correspond to the effective ages $t_w^{eff} = 2000, 4000, \ldots, 64000$. The data collection starts immediately after the step and continues in each case up to the age corresponding to $\langle 3/2 \rangle t_w^{eff}$. The energy differences in the r.h.s. of Eq. 4 are taken over a time interval $\delta t = t_w^{eff}/100$. For each set of physical parameters, 6000 independent runs were performed. The collapse demonstrates that the rate of intermittent events is proportional to the theoretical prediction $1/t_w^{eff}$. For comparison, six data sets were collected isothermally, from age $t_w$ to age $(3/2)t_w$, with $\delta t = t_w/100$ and $t_w = 2000, 4000, 8000, \ldots, 64000$. They collapse among themselves but, even though the $t_w$ values pairwise correspond to the $t_w^{eff}$ values for the temperature step, the two groups of data remain clearly distinct. This is the dual effect previously mentioned.

Very similar results were obtained for other temperature steps, i.e. $0.55 \rightarrow 0.50, 0.45 \rightarrow 0.40, 0.35 \rightarrow 0.30$. Figure 3 shows that the quality of the results is highly
sensitive to the value chosen for $t_{w}^{eff}$, with the best results obtained at or near the theoretical value. The ordinate empirically quantifies the quality of the data collapse of two PDF’s, say $a$ and $b$, and is obtained as follows. First, the extreme tails, which are mostly affected by statistical error, are cut off. We then let $p(a)$ and $p(b)$ denote the probabilities carried by the rest of the tails. The error is defined as $e(a, b) = |(p(a) - p(b))| |(p(a)p(b))^{1/2}$. For each data set, $p$ is calculated over an interval extending over 8 units of energy, i.e. from $-4$ to $-12$ for the $0.25 \rightarrow 0.20$ data. This interval is gradually moved to the left (more negative values) for $0.35 \rightarrow 0.25$ etc., to match the increasing width of the Gaussian part of the curves as the temperature increases.

The upper data set in Fig. 4 shows the data collapse obtained after a positive temperature shift. As was the case for the negative shift, the Gaussian parts are unaffected, while the intermittent tails are strongly affected. The set, comprises six different PDF’s from systems of different ages, $t_{w} = 2000, 4000, \ldots 32000$, with the ratio $\delta t/t_{w} = 1/100$. Data are collected from $t_{w}$ to $(3/2)t_{w}$, and 2000 independent runs were performed for each set of parameters. Since, as the data collapse demonstrates, the rate of intermittent events remains proportional to $1/t_{w}$, the stronger fluctuations available after the shift are able to quickly overcome all barriers in the range between $b(t_{w}, T)$ and $b(t_{w}, T')$, the extremal barrier which would have been reached isothermally at the higher temperature. The latter barrier hence sets the rate for the intermittent events. The lower data sets comprise five isothermal PDF’s obtained under the same conditions, but for fixed temperature $T = 0.25$. The presence of two distinct master curves shows, again, the presence of an additional rejuvenation aspect.

Finally, Fig. 5 explores how the quality of the collapse of PDF’s obtained after a positive temperature declines, as the criterion $\delta t/t_{w} =$constant is systematically violated. These results, obtained for initial temperatures $T = 0.20, 0.30 \ldots 0.50$, upward shifted in each case by 0.05, show that the conclusion drawn from Fig. 4 remains valid for a range of temperatures.

V. TECHNICAL DETAILS

All simulations are performed for a $10^3$ Edwards-Anderson spin-glass model with nearest neighbor Gaussian couplings of unit variance and zero mean. The Boltzmann constant is set to one.

We employ an event driven simulation technique, the Waiting Time Method (WTM), also used in the landscape exploration. The ‘intrinsic’ or current time of the system is initially zero and corresponds for large system sizes to the usual Monte Carlo sweep. A slight modification of the WTM procedure described in 24 is required to deal with temperature changes beyond the initial instantaneous quench at $t_{w} = 0$, if the mathematical equivalence of the WTM to the standard Metropolis algorithm is to be maintained. This is discussed below together with the main points of the algorithm.

Let $\delta E_{i}$ be the energy difference produced by flipping the $i$th spin. Initially, random waiting times are drawn independently from exponential distributions with average $\tau_{i} = \max(\exp(\delta E_{i}/T), 1)$. These random quantities
also define, for each spin \( i \), its flipping time. This is the time at which the spin would flip, were \( \delta E_i \) to remain constant in the meanwhile. The simulation iterates the following two steps: \( i \) the current time is set to the earliest flipping time, and the corresponding spin, e.g. number \( k \), is flipped; \( ii \) the waiting times of spin \( k \) and its nearest neighbors are re-calculated taking the new \( \delta E_i/T_s \) into account. Adding these waiting times to the current time yields the updated flipping times of spin \( k \) and its neighbors. All other spins are unaffected by the move and need not be updated, as they are engaged in a memory-less (Poisson) waiting process. The modification alluded to is that, whenever \( T \) changes, all \( \delta E_i/T_s \) values are affected and all the waiting and flipping times are re-calculated.

The data shown are obtained, for each choice of physical parameters, from 2000 to 6000 independent runs. Each run typically samples 50 values of \( H \) (this figure varies slightly according to the parameter settings). PDFs are thus estimated as the empirical frequencies of \( N = 1.5 \cdot 10^6 \) data points for 150 bins covering the range of the observations, and normalized for convenience to cover a unit area. The statistics of the Gaussian part of the data is excellent and does not need further discussion. The statistical error on the exponential tail is, unfortunately, not completely negligible. Consider a data bin whose theoretical probability is \( p \), with \( p << 1 \). The number \( n \) of independent measurements which out of \( N > 1 \) trials fall into this bin is well approximated by a Poisson variable, with average and variance both equal to \( pN \). The estimate \( n/N \) for \( p \) has mean \( p \) and standard deviation \( (p/N)^{1/2} \). The relative error on a data point with theoretical probability \( p \) is the ratio of the standard deviation to the mean and equals \( (1/pN)^{1/2} \). The slow convergence with \( N \) prevents a perfect collapse of the tail data. A relative error of 10\%, which is still graphically evident in a semi-logarithmic plot, is obtained for our range of \( N \) for values of \( p \) above \( 10^{-3} \).

To skewness (third central moments), which is used as a measure of the deviation from the equilibrium-like Gaussian shape, amplifies the noise in the poorly sampled extreme tail of the PDF. To mitigate the effect, we estimated the skewness using PDFs curtailed at \( 10^{-3} \). This appears to be sufficiently accurate for comparing PDF’s describing different physical situations.

VI. SUMMARY AND DISCUSSION

Statistical analysis of mesoscopic noise is a powerful probe of aging dynamics in a variety of glassy systems, as reversible equilibrium fluctuations and irreversible drift appear as distinct components of the signal’s Probability Distribution Function (PDF): the former are described by a Gaussian of zero average, and the latter by an ‘intermittent’ exponential tail. Our investigations utilize the intermittent tail to describe the aging behavior of intermittent events, or quakes, showing how their rate decays with the age and how the decay is modified by temperature shifts. A unified explanation of both these effects and of pure aging behavior is achieved by postulating that quakes occur whenever a record sized thermal fluctuation crosses the extremal barrier established in the course of the preceding thermal history: a spin glass is thus claimed to remember a sequence of ever growing extremal barriers. This information is erased by a positive \( T \) shift but preserved by a negative shift. The mechanism suggests the presence of a strong entropic bias favoring shallow attractors, i.e. ensuring that the least stable attractor is selected with high probability after each quake.

The rate of intermittent events following a \( T \) shift is thus a sensitive probe of the size of the barriers surmounted by an aging system before the shift, but carries no information on e.g. the energy released through the quakes. The latter crucially affects the energy density \( c(t) \) as a function of the age, whose behavior under \( T \) shifts has been thoroughly investigated: At temperature shifts, \( c(t) \) approaches the isothermal energy density curve obtained at the final temperature. The approach is non-monotonic for positive shifts, a feature generally called a Kovacs effect. As the energy density is closely related to the heat transport PDF a comparison is very instructive.

The effective ages as presently defined depend on other dynamical properties than those extracted from \( c(t) \), i.e. on the rate of intermittent events rather than the full shape of the energy decay. In particular, they do not fully describe the dynamical effect of the \( T \) shift, e.g. the shape of the collapsed PDFs differs for \( T \) shifted and isothermally aged systems. By contrast, in energy density studies the effective age is constructed to make the \( T \)-shifted and isothermally aged curves coincide. Furthermore, our definition most directly relates to the barriers scaled prior to the step and to the effect of the step itself, unlike the effective age extracted from \( c(t) \) curves which most directly gauges the behavior after the thermal step.

In an aging system one expects the configurational memory to vanish on a time scale equal to the system age. For a negative \( T \) step we expect the memory of the step to vanish on a time scale of the order of \( t_{sf}^u \), i.e. \( t_{wf} \), which qualitatively agrees with the results of refs. [5, 7]: This reasoning does not apply to a positive step: the intermittency rates immediately adjust to those of an isothermally aged system, while the energy density shoots up and then approaches the isothermal curve on a time scale which is shorter than \( t_{wf} \) but differs from zero. Clearly, an abrupt change in the barrier structure does not logically imply an equally abrupt change in energy or configuration, and the data tell us that the two types of change happen on different scales. In summary, while a naive correspondence cannot be established, a substantial physical similarity emerges when notational and methodological differences are properly accounted for.

The same applies to the effect of \( T \) shifts other phys-
logical properties, i.e. the AC susceptibility $\chi$ of Anderson spin-glasses in four and three spatial dimensions studied in simulations by Berthier and Bouchaud \([5]\) and Takayama and Hukushima \([6]\). These authors parameterize their results with effective ages extracted in a way similar to the energy density analysis just discussed. As in our findings, negative shift have a double effect: the apparent rejuvenation consisting in a fast decay of $\chi''$ immediately after the shift and resembling the evolution right after the initial quench. Secondly, the curve 'undershoots' the isothermal data at the final temperature, which makes the system look older. In ref. \([6]\), the relation between age and effective age is nearly a power-law, i.e. similar to Eq. 6. The exponent is however given as $T''=T'$, and not our $a(T'')/a(T)$, where $a(T)$ is given in Eq. 4. A positive shift is there described by an effective age shorter than the age, while our effective age simply remains equal to $t_w$. This difference corresponds to the difference already discussed in connection with the $e(t)$ data, but the asymmetry between positive and negative shift found in ref. \([6]\) is similar to our findings.

Comparing simulation studies with experimental results, as we shall do next, requires an element of faith, particularly so for dynamical effects: simulation algorithms such as the Metropolis algorithm and the WTM presently utilized are designed to sample equilibrium distributions, but the time scales of e.g. partial equilibrations are algorithm dependent and cannot be meaningfully connected to experimental time scales. A large empirical evidence nevertheless points to the relevance for dynamical phenomena of the information gained from simulations of glassy systems, perhaps unsurprisingly, given that the dynamical features experimentally defining glassy behavior are similar across a wide range of microscopically very different systems, and that aging dynamics is, broadly speaking, controlled by time scales ratios, rather than magnitudes.

The temperature and age dependence studies of the imaginary part of the AC susceptibility $\chi''$, show how the memory of configurations explored at a certain temperature survives cooling and is retrieved by re-heating. This information is not directly available from heat transfer intermittency data. However, the related property that the memory is erased by a sufficiently strong over-heating \([3]\) is qualitatively similar to the asymmetry between positive and negative step already discussed.

The relationship between effective and actual age under positive and negative temperature shifts is investigated experimentally in ref. \([4]\). For sufficiently large $T$ shifts, a lack of reversibility of the $T$ step, called a ‘non-accumulative aging effect’ is present. This is qualitatively similar to our findings, where the shifts applied are of the order of $5 \%$ of $T_g$, i.e. large compared to those used in ref. \([4]\). We can thus confirm our conclusion that the physical insight obtained from the intermittency analysis broadly concurs with the results of other types of investigation.

As intermittent heat transfer says only little about con-figuration changes, information regarding the high temperature sensitivity of the thermal correlation length, so called ‘chaos’ \([20]\), which is often linked to rejuvenation effects, can only be gained indirectly. We note however that since memory and rejuvenation effects appear in systems of very modest size, properties of large scale thermal objects, e.g. droplet excitations out of a ground state \([27]\) are largely immaterial for the present discussion.

As recently shown \([14]\), geometrical properties of the energy landscape of a spin-glass depend in a systematic way on the extremal energy barrier crossed. More generally, a linear relationship between the average energy barrier crossed and the configurational changes induced by the crossing has been measured experimentally \([2]\) and numerically, in different types of spin-glass models \([14, 28]\) and using different exploration techniques \([29]\).

Due to the strong connection between barriers overcome and configuration changes we can expect that erasing the memory of the extremal barrier would also lead to some, more gradual, loss of configuration memory.

A linear relationship between barriers and Hamming distances is included in descriptions of spin-glass dynamics as a master equation on a set of hierarchically organized states \([30, 31, 32, 33, 34]\). The tree models used in these descriptions are energy landscape descriptions of local regions of a complex system, and are able to reproduce salient features of aging, including $t/t_w$ scaling, and the effects of temperature shifts based on thermal hopping. rejuvenation effects have also been found \([35]\) in the so-called LS tree \([32]\) based on the idea that a kinetic bias exist favoring shallow attractors. The record induced dynamics scenario presently advocated also requires the presence of a hierarchy of inequivalent dynamical attractors. However, unlike the case of a static tree model, these attractors are not given in advance, but are dynamically selected by record energy fluctuations.

Unlike other methods, the intermittency probe of aging dynamics has a direct analytical interpretation, and, at least for spin-glasses, appears to offer a different age and temperature window where it can conveniently be applied. I.e. intermittency rates can be obtained for all ages, whether isothermal evolution is fast or slow, but are most conspicuous at low temperatures (below $T \approx 0.6$ for the present system) where Gaussian fluctuations are small \([10]\).

Secondly, since intermittent events entail large configurational rearrangements, they expectedly influence in similar ways different physical properties. E.g. studies of the electrical voltage fluctuations in Laponite, a gel, and in a polycarbonate glass \([37]\) show that the intensity of the intermittent signal decreases while the system ages (as in our fig. \([1]\)). In spin-glasses, where the spin contribution to the energy is not easily separated out, one could study the intermittency property of the magnetic noise or, in metallic spin-glasses, the electrical fluctuations associated with spin flips \([38]\). The method presented can in principle be applied to analyze intermittency rates of any relevant and experimentally available
quantity. Measuring the rate of intermittent events as a function of the age would be extremely interesting from a theoretical point of view, but also extremely challenging. Some changes of the PDF’s cause by temperature shifts, see e.g. Fig. [1] might be observable even in the lack of precise rate measurements.

VII. ACKNOWLEDGMENTS

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