Simulation of internal forces of a cantilevered beam with a moving load

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Abstract. The simulation of cantilevered beam internal forces in any beam section with a moving load is presented. The processes of analytical and computer modeling of the structural analysis are provided depending on the location of the moving load. The cantilevered beam in this article can be a schematic for a gantry crane or an overhead crane with a moving trolley. These calculations are not only very useful for cranes, but also all kinds of bridges and trusses with a moving load. The mathematical model is provided and numerically solved in Mathcad software. As a result of the study, the program effectively determines the internal forces in any beam section depending on the location of the moving load. The proposed procedures will reduce the time and cost of conceptual and detailed design very easily when the beam parameters change, which is useful for optimization studies.

1. Introduction
Structural analysis is a very important part of a beam design and should be done before any dynamic modeling. Static and dynamic analysis of a beam with a moving load is an interesting topic [1, 2].

The article describes the internal forces computer modeling process of a cantilevered beam under the action of a moving load at any point (cross-section) for a freely supported beam in the Mathcad system [3].

The analysis can be presented in analytical, numerical, and graphical forms (in the form of influence lines). Influence lines are used to calculate many structures under the moving loads (beams and trusses of a bridge and gantry cranes; tower cranes booms; mobile and railway bridges, etc.). The internal forces modeling makes it possible to determine any load positions, at which the value of the calculated forces is more than permissible.

The purpose of the study is to create a set of analytical expressions, and programs for the internal forces automated calculation in any cross-section of the beam. The process of influence lines computer modeling is considered by a simple beam example.

2. Definitions
The general scheme of the cantilevered beam on two supports with the load on it in the form of a single mobile force $P = 1$ N is shown in Figure 1.
Figure 1. General scheme of the cantilevered beam on two supports with a mobile single load: the considered section a) before the moving load on the console and b) after the moving load.

3. Basics of the theory
We consider the process of determining internal forces on the sample flat-cantilevered beam on two supports with moving a single load of \( P = 1 \) N. So we have:

3.1. The influence lines of the support reactions:
- support reaction lines in the reference points \( A \) and \( B \) are being determined from the moments relative to the support point, depending on the location of the single load.

\[
\sum M_A(x) = P \cdot x - B_y(x) \cdot L = 0, \quad \text{where} \quad B_y(x) = \frac{P \cdot x}{L}.
\]  
\[
\sum M_B(x) = P \cdot (L - x) - A_y(x) \cdot L = 0, \quad \text{where} \quad A_y(x) = \frac{P \cdot (L - x)}{L}.
\]  
\[
\sum P_x = 0, \quad \text{where} \quad A_x = 0, \quad B_y(x) = \frac{P \cdot x}{L}.
\]  

3.2. The influence line of the bending moment:
Two variants should be considered to determine the influence line of internal forces on the console (the \( s \) section):
- when the single load is on the left and right of the section \( s \). If the single load is located to the left of the \( s \) section on the console:

\[
M^L_s(x) = -A_y(x) \cdot s - B_y(x) \cdot (s - L) + P \cdot (s - x) =
\]
\[
= -\frac{P \cdot (L - x) \cdot s}{L} - \frac{P \cdot x}{L} \cdot (s - L) + P \cdot (s - x) =
\]
\[
= -P \cdot s + P \cdot \frac{x \cdot s}{L} - P \cdot \frac{x \cdot s}{L} + P \cdot x + P \cdot s - P \cdot x = 0.
\]  

If the single load is located to the right of the \( s \) section on the console (\( x > s \)):

\[
M^R_s(x) = -A_y(x) \cdot s - B_y(x) \cdot (s - L) =
\]
\[
= -\frac{P \cdot (L - x) \cdot s}{L} - \frac{P \cdot x}{L} \cdot (s - L) =
\]
\[
= -P \cdot s + P \cdot \frac{x \cdot s}{L} - P \cdot \frac{x \cdot s}{L} + P \cdot x = P \cdot (x - s),
\]  
when, \( x = s \) \( M^R_s(x) = P \cdot (x - s) = P \cdot (s - s) = 0. \)

So, for the console:

\[
M_s(x) = \begin{cases} 
0, & \text{for} \ x \leq s \\
P \cdot (x - s), & \text{for} \ x > s 
\end{cases}
\]  

(6)
3.3. The shear forces:
If a single load is located to the left of the s section on the console \((x \leq s)\):

\[
\sum P_{yi}^L = A_y + B_y - P = \frac{P \cdot (L - x)}{L} + \frac{P \cdot x}{L} - P = 0,
\]

and for \((x > s)\):

\[
\sum P_{yi}^R = A_y + B_y = \frac{P \cdot (L - x)}{L} + \frac{P \cdot x}{L} = P,
\]

\(Q_s(x) = \begin{cases} 0, & \text{for } x \leq s \\ P, & \text{for } x > s \end{cases} \).

4. Computer simulation

4.1. Source data:
- amount of the mobile load, \(kN\)
- distance between the supports, the length of the console, \(m\)
- length of the beam (m) and the number of sections under consideration:

\[ a_0 = 6 \quad a_1 = 2 \quad a_2 = 1 \quad a_3 = 1 \quad a_4 = 1 \quad a_5 = 1 \quad a_6 = 1 \quad a_7 = 1 \quad a_8 = 1 \]

\[ \frac{l}{NS} = 10 = 80. \]

4.2. Algorithm of calculation:
1. Determining the reactions forces of the beam at different coordinates for the single force \(P\): By \((x)\) – in the reference point \(B\) and \(A\) \((x)\) – in the reference point \(A\):

- analytical

\[
B_y(x) = \frac{P \cdot x}{a_0} \quad A_y(x) = \frac{P \cdot (a_0 - x)}{a_0}
\]

\[
\Delta x := \frac{l}{NS} \quad i := 0..NS \quad x_i := i \cdot \Delta x \quad B_{yi} := \frac{P \cdot x_i}{a_0} \quad A_{yi} := \frac{P \cdot (a_0 - x_i)}{a_0} \quad x0 := 0
\]

- graphical (Figure 2).

\[ \max(B_y) = 1.333 \quad \min(B_y) = 0 \]

\[ \max(A_y) = 1 \quad \min(A_y) = -0.333 \]

**Figure 2.** Reaction forces at different coordinates for the single force in points \(B\) and \(A\).
2. Determining the influence line of the bending moment in any beam section \( s \): Program 1 (Figure 3).

\[
M(s) := \begin{cases} 
  \prod_{i} & \text{for } i \in 1..NS \\
  \prod_{i} & \text{if } x_i \leq s \leq a_0 \\
  \prod_{i} & \text{if } x_i \geq s \leq a_0 \\
  0 & \text{if } x_i \leq s \geq a_0 \\
  P \cdot (a_i - s) & \text{if } x_i \geq s \geq a_0 
\end{cases}
\]

**Program 1.** Determining the influence line of the bending moment in any section

- An influence line for section \( s = 4.4 \) in graphical kind

![Figure 3. Determining the influence line of the bending moment for section \( s = 4.4 \)](image)

3. Determining the influence line of the shear force in the section — \( s \): Program 2; Program 3; Program 4 (Figure 4):

\[
Q_1(s) := \begin{cases} 
  \prod_{i} & \text{for } i \in 0..NS \\
  \prod_{i} & \text{if } 0 \leq x_i \leq s \leq a_1 + a_0 \\
  0 & \text{if } s \geq a_0 
\end{cases}
\]

**Program 2.** Determining the influence line of the shear force between the supports at the left of the section:

\[
Q_2(s) := \begin{cases} 
  \prod_{i} & \text{for } i \in 0..NS \\
  \prod_{i} & \text{if } 0 \leq x_i \geq s \leq a_1 + a_0 \\
  0 & \text{if } s \geq a_0 
\end{cases}
\]

**Program 3.** Determining the influence line of the shear force between the supports at the right of the section
\[ Q_3(s) = \left\{ \begin{array}{ll}
Q_i & \text{if } x_i \leq s \leq a_0 \\
Q_i & \text{if } x_i \geq s \geq a_0 \\
Q & \text{otherwise}
\end{array} \right. \]

**Program 4.** Determining the influence line of the shear force on the console at the right and left of the section

For example, \( s = 3.5 \)

![Figure 4](image)

**Figure 4.** Determining the influence line of the shear force, for example, \( s = 3.5 \)

5. **Conclusion**

The simulation of cantilevered beam internal forces in any beam section with a moving load is presented. The processes of analytical and computer modeling of the structural analysis were provided depending on the location of the moving load. The mathematical model was provided and numerically solved in Mathcad software. The proposed procedures of modeling of internal efforts in any cross-section of a cantilevered beam from the moving load allow one to reduce time and cost for the performance of such calculations at least ten times and to carry out effectively the research when the beam parameters, moving load parameters, and the supports position is changing.

**References**

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