Research of a pulse magnetic system for eliminating electron conductivity in an ion diode for the neutron generation

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Abstract. The work presents the results of modeling the process of suppressing an electronic component in a small-sized coaxial ion diode by a pulsed magnetic field of a spiral electrodynamic line. Information was obtained on the features of this process, which is necessary for designing a diode with pulsed magnetic insulation.

1. Introduction
Small-sized pulsed neutron generators based on sealed diode accelerator tubes (AT) are used for neutron logging of oil and gas and ore wells, rapid elemental analysis of substance composition, neutron tomography, detection and identification of hidden dangerous substances and objects and other applications [1-3].

To increase the energy efficiency of neutron generation in AT, various methods of suppressing electron emission from the cathode are used [4-6]. One of them is the method of pulsed magnetic isolation using a spiral cone-shaped electrodynamic line [7, 8]. A schematic section of a CT diode system with similar magnetic insulation (DMI) is shown in Fig. 1.

Figure 1. Schematic section of the DMI model:
1 - insulator; 2 - anode; 3 - laser plasma-forming target; 4 - spiral
5 - cathode with a neutron-forming target
This article is devoted to a theoretical study of its electrodynamic characteristics and features of the implementation of the process of suppressing electronic conductivity. For this, a mathematical experiment was conducted on a computer. His algorithm contained four positions: calculating the inductances and mutual inductances of the elements of the magnetic system for suppressing the electronic conductivity of DMI, calculating the currents excited in them when the storage capacitance is discharged to a spiral, calculating the magnetic fields in the working volume of the DMI and calculating the dynamics of the formation of the electron beam with subsequent analysis of the percentage of hits electrons to the anode electrode.

2. The algorithm for calculating inductances and mutual inductances of the elements of the magnetic system

To calculate the inductance of a spiral line, we represent it in the form of a model combination of K thin annular conductive volume formations with a square cross section with a transverse dimension \( h \):

\[
V_i : \{ r \in [r_{si} - 0.5h, r_{si} + 0.5h], z \in [z_{si}, z_{si} + h] \},
\]

\( r, z \) – cylindrical coordinates, \( hr, hz \) – transverse dimensions of the conductor,

\[
r_{si} = R_i - \frac{R_1 - R_K}{K - 1}, z_{si} = \frac{H_s}{K - 1}, i = 0,..., K - 1, (h \ll H_s, K > 1),
\]

\( R_{1,K}\) - the central radii of the 1st and Kth rings, \( H_s\) – distance between them. Let a constant current \( I_s \) flow through the conductors with a bulk density

\[
\mathbf{j}_s = \frac{e_0 I_s}{h^2}.
\]

Then the line inductance can be determined by performing the integration operation in the well-known formula for the conductor inductance proposed in [9]:

\[
L_s = \frac{\mu_0}{4\pi} \frac{1}{2} \sum_{i=0}^{K-1} \sum_{j=0}^{K-1} \int \frac{dV_i}{V_j} \int dV_i (\mathbf{j}_i(\mathbf{r}), \mathbf{j}_j(\mathbf{r})) |\mathbf{r} - \mathbf{r}_0|^{-1} \cong
\]

\[
\approx \frac{\mu_0}{h^3} \sum_{i=0}^{K-1} \sum_{j=0}^{K-1} \int_0^{r_{si} + 0.5h} \int_0^{z_{si} + h} \int_{r_{si} - 0.5h}^{r_{si} + 0.5h} \int_{z_{si} - 0.5h}^{z_{si} + h} F(r, r_0, z, \theta, j)
\]

where

\[
F(r, r_0, z, \theta, j) = \ln \left( \frac{z - z_{si} + \sqrt{R(r, r_0, \theta)^2 + (z - z_{si})^2}}{z - z_{si} - h + \sqrt{R(r, r_0, \theta)^2 + (z - z_{si} - h)^2}} \right),
\]

\[
R(r, r_0, \theta) = \sqrt{r^2 + r_0^2 - 2rr_0 \cos \theta}.
\]

To take into account the influence of the currents induced in the cathode electrode (metal substrate of a neutron-forming target), we divide it into \( N \) identical separate ring fragments with a rectangular cross section: \( h_t \times \frac{H_t}{N} \):

\[
V_{tn} : \{ r \in [R_t, R_t + h_t], z \in [d + z_{tn}, d + z_m + \frac{H_t}{N - 1}] \},
\]
where \( R_t \) – cathode inner radius, \( h_t \) – cathode electrode thickness,

\[
z_{in} = n \frac{H_t}{N-1}, \quad n = 0, \ldots, N-1, \quad (N>1).
\]

Let a constant current \( I_{tn} \) with a bulk density:

\[
\mathbf{j}_{tn} = e_0 \frac{I_{tn} N}{h_t H_t}.
\]

flow through each ring.

Then, by analogy with the previous consideration, the inductance of each such fragment can, according to [9], be determined by the following approximate formula:

\[
L_n = \frac{\mu_0}{4\pi I_{tn} v_{tn}} \int \int dV dV_0 (\mathbf{j}(\mathbf{r}), \mathbf{j}(\mathbf{r}_0)) |\mathbf{r} - \mathbf{r}_0|^{-1} \approx \\
\approx \frac{\mu_0 R_t}{2\beta \sqrt{2}} \int_0^\pi d\theta \cos \theta \ln \left( \frac{\sqrt{1 + \beta^2 - \cos \theta + \beta}}{\sqrt{1 + \beta^2 - \cos \theta - \beta}} \right); \quad \beta = \frac{H_t}{2\sqrt{2}R_t(N-1)}.
\]

To determine the currents excited in individual fragments of the cathode electrode, it is necessary to know the mutual inductance of the helix and the corresponding fragment, which can also be determined by the formula proposed in [9]:

\[
L_{sn} = \frac{\mu_0}{4\pi I_{tn} v_{tn}} \int \int dV dV_0 (\mathbf{j}_s(\mathbf{r}), \mathbf{j}_{in}(\mathbf{r}_0)) |\mathbf{r} - \mathbf{r}_0|^{-1} \approx \\
\approx \frac{\mu_0 N R_t}{H_t h} \sum_{i=0}^{N-1} \int_0^{\pi/2} d\theta \cos \theta \int_{z_{ni}}^{z_{ni} + h} dz \ln \left( \frac{z - z_{ni} + \sqrt{R(R_t, r_{ni}, \theta)^2 + (z_{ni} - z)^2}}{z_{ni} - z - \frac{H_t}{N} + \sqrt{R(R_t, r_{ni}, \theta)^2 + (z_{ni} - z - \frac{H_t}{N})^2}} \right).
\]

3. Algorithm for calculating currents in elements of a magnetic system for suppressing electronic conductivity

Let capacity \( C \) charged to voltage \( U_0 \) be discharged to the spiral. Temporal dependences of voltage across capacitor plates \( U \) and current \( I_s \) are determined by the following approximate linear system of differential equations:

\[
\begin{align*}
C \frac{dU}{dt} &= -I_s, \\
\left( L_s - \sum_{i=0}^{N-1} L_{sn} \right) \frac{dI_s}{dt} &\approx U - \frac{I_s}{\Sigma_s},
\end{align*}
\]

where:

\[
\Sigma_s = \frac{h^2}{\rho_s \Pi_s}, \quad \Pi_s \approx 2\pi \sum_{i=0}^{N-1} r_{ni}.
\]
ρs- resistivity of the metal of which the spiral is made. System (1) did not take into account the voltage drop across the active resistance of the substrate of the target and the interaction of \(I_n\) currents with each other. The error of such an approximation according to estimates made by the perturbation method does not exceed 5%.

In this approximation, the partial current in the nth fragment of the target substrate will be determined by the expression:

\[
I_{tn}(t) \approx \frac{L_n}{L} I_s(t)
\]

(2)

We introduce the dependence on time \(C\):

\[
q(t) = CU(t).
\]

Then, taking into account the equation

\[
I_s = -\frac{dq}{dt} C,
\]

arising from the first equation of system (1), the second equation of this system can be converted into a second-order differential equation with respect to the dependence \(q(t)\) of the form:

\[
(L_s - \sum_{n=0}^{N-1} \frac{L_n^2}{L_n}) \frac{d^2 q}{dt^2} - \sum_{s=1}^{N} \frac{d q}{dt} - C^{-1} q = 0,
\]

with initial condition: \(q(0) = U_0 C, \dot{q}(0) = 0\), corresponding to an oscillatory circuit with effective inductance

\[
\bar{L} = (L_s - \sum_{n=0}^{N-1} \frac{L_n^2}{L_n}).
\]

Its solution has the form:

\[
q(t) = U_0 C \exp(-\beta t) \cos(\omega t),
\]

(3)

where

\[
\beta = (2\bar{L}\Sigma_s)^{-1}, \quad \omega = [(\bar{L}C)^{-1} - (2\bar{L}\Sigma_s)^{-2}]^{0.5}.
\]

Differentiating expression (3) with respect to time, we find the desired time dependence of the current in the spiral:

\[
I_s(t) = U_0 \sqrt{\frac{C}{\bar{L}}} \exp(-\beta t) \cos(\omega t - \arctg \frac{\omega}{\beta}).
\]

4. The algorithm for calculating the magnetic field induction

From the Bio-Savart law, after integration over the angle, we can obtain the following expression for the magnetic field induction vector in the considered system of suppression of electronic conductivity in DMI:
\[ \mathbf{B}(r, z, t) \approx I(t) \left\{ \sum_{i=1}^{K} b_i(r, r_{si}, z-z_{si}) + \sum_{k=1}^{K} b_k(r, r_{ak}, z-z_{ak}) \right\} - \left\{ \mathbf{e}_r \sum_{n=1}^{N} \frac{L_n}{I_n} b_n(r, R_n, z-n \frac{H}{N-1}) + \mathbf{e}_z \sum_{k=1}^{K} \frac{L_n}{I_n} b_n(r, r_{ak}, z-z_{ak}) \right\} \]

where \( \mathbf{e}_r, \mathbf{e}_z \) - unit vectors of a cylindrical coordinate system,

\[ b_i(a, r, z, t) = \frac{\mu_0}{2\pi} \left\{ E[f(a, r, z)] \frac{a^2 + r^2 + z^2}{(a-r)^2 + z^2} - K[f(a, r, z)] \right\} \frac{z}{\sqrt{r^2 + \delta^2}(a+r)^2 + z^2}, \]

\[ b_k(a, r, z, t) = \frac{\mu_0}{2\pi} \left\{ E[f(a, r, z)] \right\} \frac{a^2 - r^2 - z^2}{(a-r)^2 + z^2} + K[f(a, r, z)] \frac{1}{\sqrt{(a+r)^2 + z^2}} \]

- components of the magnetic field induction vector created by a thin ring of radius \( a \) with a unit current at a point in space with cylindrical coordinates \( r, z \) [9].

\[ f(a, r, z) = \frac{4ar}{(a+r)^2 + z^2}. \]

\( K(x) \) is \( E(x) \)- full elliptic integrals of the first and second kind, respectively, \( r_{ak} \)- the radii of individual rings, \( K \)- the number of turns.

5. Algorithm for calculating the dynamics of the formation of an electron stream

The dynamics of the formation of the electron beam in the considered DMI was calculated by analogy with [10] using the algorithm for computer modeling of plasma systems described in [11]. It allows one to describe in a self-consistent manner the dynamics of charged particles in a plasma simultaneously with the solution of the azimuthally symmetric Poisson equation for the electric field potential \( \varphi(r, z) \) in a cylindrical coordinate system with a fixed external magnetic field, neglecting the magnetic field created by the analyzed particles themselves. The latter is a fraction of a percent of the magnet field, and therefore may not be taken into account in the process of mathematical modeling. In this case, the voltage at the diode gap of the DMI exceeded 150 kV.

In such a computational model, real particles are combined into large particles-clouds in the form of toroidal formations, the number of which is many orders of magnitude lower than the number of real particles. Their dynamics is described by the system of Newton-Lorentz equations:

\[ \begin{aligned}
\frac{dp_k}{dt} &= -q_k \nabla \varphi(r_k) + \frac{q_k c}{\sqrt{p_k^2 + m_k^2 c^2}} [p_k, \mathbf{B}(r_k, t)] \\
\frac{dr_k}{dt} &= \frac{p_k c}{\sqrt{p_k^2 + m_k^2 c^2}}
\end{aligned} \]

where \( q_k, m_k, r_k = r_k + e_z z_k, \) \( p_k, r_k, z_k \) - respectively, the charge, mass, radius vector, momentum and cylindrical coordinates of the enlarged particle with number \( k \); \( c \) - is the speed of light.

6. Results of computer simulation of suppression of electronic conduction

As a result of a computer experiment using the algorithms described above, the effectiveness of the proposed pulsed magnetic system for suppressing the electronic conductivity of DMI
was shown. This is well illustrated by the characteristic pattern of the electron distribution in the DMI, realized in the presence of a discharge current in the spiral and shown in Fig. 2.

![Figure 2. Electron distribution in the accelerating gap DMI](image)

The left rectangle in the figure indicates the anode electrode, the right is the cathode. The fraction of electrons of the total number emitted by the cathode is a fraction of a percent.

7. Conclusion
In the process of computer simulation, the possibility of the effective implementation of the investigated system of suppressing electronic conductivity in DMI with transverse and longitudinal dimensions <0.1 m and the amplitude of the accelerating pulse> 150 kV was established. Such parameters are characteristic of small-sized UT used in applied physics. This indicates the prospects of using DMI with the considered system of suppressing electronic conductivity in small-sized pulsed neutron generators of a new generation.

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