Distributed coherent manipulation of qutrits by virtual excitation processes

Zhen-Biao Yang, Sai-Yun Ye, Alessio Serafini and Shi-Biao Zheng

1 Department of Physics and State Key Laboratory Breeding Base of Photocatalysis, Fuzhou University, Fuzhou 350002, People’s Republic of China
2 Department of Physics & Astronomy, University College London, Gower Street, London WC1E 6BT, UK

E-mail: sbzheng@pub5.fz.fj.cn

Received 19 November 2009, in final form 25 February 2010
Published 5 April 2010
Online at stacks.iop.org/JPhysB/43/085506

Abstract
We propose a scheme for the deterministic coherent manipulation of two atomic qutrits, trapped in separate cavities coupled through a short optical fibre or optical resonator. We study such a system in the regime of dispersive atom–field interactions, where the dynamics of atoms, cavities and fibre operates through virtual population of both the atomic excited states and photonic states in the cavities and fibre. We show that the resulting effective dynamics allows for the creation of robust qutrit entanglement, and thoroughly investigate the influence of imperfections and dissipation, due to atomic spontaneous emission and photon leakage, on the entanglement of the two-qutrit state.

(Some figures in this article are in colour only in the electronic version)

1. Introduction
One of the crucial ingredients in the upcoming area of quantum technologies will be the capability of coherently manipulating quantum systems at a distance, such that entanglement (i.e. quantum correlations) can be created between different nodes of a global quantum system.

Entanglement is one of the most peculiar features of quantum mechanics and the most distinct signature of quantum coherence. Entangled states of two or more particles not only play an important role in tests of quantum nonlocality [1–3] but also lie at the heart of quantum information processing and quantum computing [4]. Entangled quantum states come in many flavours, such as Bell, Einstein–Podolsky–Rosen [1], Greenberger–Horne–Zeilinger [3] or W states [5], generally depending on the dimensionality and tensor product structure of the Hilbert spaces involved. All these states have different qualities and are suitable for different roles in quantum information protocols [3, 5]. In this context, entangled states of multiple systems with Hilbert spaces of dimension $d$ (i.e. of ‘qubits’, with $d > 2$) offer their specific advantages over the—archetypal and most commonly considered—entangled states of two-dimensional systems (of ‘qubits’). For instance, entangled states of two qubits violate local realism more strongly than entangled states of two qubits, and their entanglement is more resilient to noise [6]. Also, quantum cryptographic protocols where qubits are replaced with qudits are both more secure and faster (in that more information may be sent, on average, per sent particle) [7].

Entangled states can currently be generated in a variety of physical systems, such as trapped ions [8], quantum electrodynamics cavities (QED) [9], superconducting circuits [10], semiconductor quantum dots [11], linear optical systems [12] and impurity spins in solids [13]. Cavity QED [14], which concerns the interaction of atoms and photons inside optical cavities, provides experimentalists with a very favourable setting for the generation of entanglement. Atomic systems are qualified to act as qubits or, more generally, qudits, as appropriate internal electronic states can coherently store information over relatively long time scales. At the same time, in such systems photons are suitable for the transfer of information between distant nodes. High-finesse cavities can provide good insulation against the environment and can thus have long coherence times [15]. Two-atom Bell states, and three-particle (two atoms plus one photon) GHZ entangled states have been experimentally demonstrated with Rydberg atoms passing through a superconducting microwave cavity [16, 17].
Schemes for the generation of entangled states of two ‘qutrits’ (i.e. of two three-level quantum systems) for two atoms via a single nonresonant cavity have also been proposed [18]. However, in order to be used for quantum communication protocols [19], such an entanglement should be generated between distant atoms, such as atoms trapped in different cavities. ‘Distributed’ atomic entanglement requires a way to coherently mediate the interaction between the two atoms. One way to establish this interaction is through coincidence detection of photons leaking out of the cavities [20] (recently extended to high-dimensional Hilbert spaces [21]); in this way, entangled states are only probabilistically generated and the success probability is dependent on the efficiency of photon detectors. The other possibility is to directly link the cavities with an optical fibre, waveguide or by a third mediating cavity, entangled states can be deterministically generated in such a way [22, 23]. Both types of quantum connectivity essentially allow for the distribution of entanglement across a quantum network [24].

In this paper, we shall present a method, based on the proper choice of atomic levels’ structure and operating regimes, to engineer a deterministic coherent interaction between two qutrits embodied by atoms trapped in distant cavities, linked by a third optical resonator. We will then move on to study the entanglement that can be generated by such an interaction, as well as its resilience to imperfections and quantum noise.

Before proceeding, let us first review some previous schemes for the deterministic generation of entangled states via such a type of connected cavities [23]. Several schemes have been proposed for the deterministic generation of several diverse kinds of entangled states [25–29], including Bell states [25], W states [29], GHZ states [27, 29] and also qutrit entangled states [26, 28]. In the schemes of [26, 27], the adiabatic passage along dark states is employed. These schemes are based on accurately tailored sequences of pulses and thus require a considerable degree of control. In other schemes [23, 25], the Rabi oscillations of the whole system composed of the atoms, cavity modes and fibre modes are utilized: the entangled states are generated through the exchange of excitation numbers for the atoms and photons. Hence, such schemes [23, 25] are bound to be rather sensitive to atomic spontaneous emission and photon losses. In other schemes [28, 29], the population of the atomic excited states can be effectively suppressed by virtue of dispersive atom–field interactions, but the photonic states in the cavities or fibre are still populated, so that the whole system is still sensitive to photon losses.

The scheme we propose here is different from all such previous schemes [23, 25, 28, 29], even from those adopting the adiabatic passage [26, 27]. The scheme is inspired by a previous idea for virtual-photon-induced phase gates between two distant atoms [30]. In the scheme, the entanglement is generated through virtual population of not only the atomic excited states but also the photonic states in the cavities and fibre. Therefore, our scheme will turn out to be well shielded from both atomic spontaneous emission and photon losses.

The paper is organized as follows. In section 2, we specify our conditions on the physical parameters and derive the effective Hamiltonian for the system. In section 3, we discuss the generation of two-qutrit entangled state via the effective Hamiltonian and study the reliability of the entangled state in the presence of mismatches in the system’s parameters. In section 4, we discuss the influence of atomic spontaneous emission and photon leakage on the entangled state. Section 5 contains some concluding remarks.

2. The model

The schematic of our setup is shown in figure 1. Two distant atoms are individually trapped in two double-mode cavities (A and B), which are connected by a third optical resonator, as shown in figure 1(a). The linking resonator can be either a third cavity coupling the two distant cavities (like in a photonic crystal) or a ‘short’ (in a sense which will be specified shortly) optical fibre. For simplicity, we will henceforth refer to the linking resonator as to the ‘fibre’.

The coupling of the fibre modes to the modes of the cavities in Schrödinger interaction picture may be modelled by the Hamiltonian

\[ H_{\text{fi}}^\text{cf} = \sum_{k=L,R} \Delta_{n,k} b_{n,k}^\dagger b_{n,k} + \nu_{n,k} \left( b_{n,k}^\dagger a_{A,k}^\dagger (-1)^s e^{i\sigma} a_{B,k}^\dagger \right) + \text{H.c.} \] (1)

In this paper, the state of the photon modes for cavity A (B) is of the form \( |\psi_\text{A} (\text{fibre})\rangle\rightarrow|\psi_\text{B} (\text{fibre})\rangle\rightarrow|\psi_\text{B} (\text{ground})\rangle\rightarrow|\psi_\text{A} (\text{ground})\rangle\). The coupling of the fibre modes to the modes of the cavities in Schrödinger interaction picture may be modelled by the Hamiltonian

\[ H_{\text{fi}}^\text{cf} = \sum_{k=L,R} \Delta_{n,k} b_{n,k}^\dagger b_{n,k} + \nu_{n,k} \left( b_{n,k}^\dagger a_{A,k}^\dagger (-1)^s e^{i\sigma} a_{B,k}^\dagger \right) + \text{H.c.} \] (1)

The coupling of the fibre modes to the modes of the cavities in Schrödinger interaction picture may be modelled by the Hamiltonian

\[ H_{\text{fi}}^\text{cf} = \sum_{k=L,R} \Delta_{n,k} b_{n,k}^\dagger b_{n,k} + \nu_{n,k} \left( b_{n,k}^\dagger a_{A,k}^\dagger (-1)^s e^{i\sigma} a_{B,k}^\dagger \right) + \text{H.c.} \] (1)

In this paper, the state of the photon modes for cavity A (B) is of the form \( |\psi_\text{A} (\text{fibre})\rangle\rightarrow|\psi_\text{B} (\text{fibre})\rangle\rightarrow|\psi_\text{B} (\text{ground})\rangle\rightarrow|\psi_\text{A} (\text{ground})\rangle\).

The atoms have three excited states (\( |e_L\rangle, |e_0\rangle\) and \( |e_R\rangle\)) and three ground states (\( |g_L\rangle, |g_0\rangle\) and \( |g_R\rangle\)), which could be the Zeeman sublevels of alkali atoms in the excited- and ground-state manifold, respectively. To fix ideas and portrait a case of practical interest, we consider here a possible implementation with \( ^{87}\text{Rb} \), whose relevant atomic levels are shown in figure 1(b). We only illustrate the involved state transitions by starting from the initial state \( |g_0\rangle |g_0\rangle |g_0\rangle \) for the atoms. Each atom is assumed to be coupled (off-resonantly) to an external π-polarized classical field and both \( \sigma^+\) - and \( \sigma^-\)-polarized photon modes of the local cavity.

We first describe the involved transitions for each atom in its local cavity. In cavity A, the transitions \( |g_0\rangle \rightarrow |e_0\rangle \) and \( |e_0\rangle \rightarrow |g_L\rangle \) (\( |g_R\rangle\)) are coupled to the π-polarized classical field and the \( \sigma^+=\) (\( \sigma^=\)) polarized cavity mode, respectively. In cavity B, the transitions \( |g_0\rangle \rightarrow |e_L\rangle \) and \( |e_L\rangle \rightarrow |g_0\rangle \) (\( |g_0\rangle\)) are coupled to the π-polarized classical field and the \( \sigma^+=\) (\( \sigma^=\)) polarized cavity mode, respectively.
\(|\psi_{AB}(t)\rangle = |\varphi_{A}(t)|_{A}\rangle |\gamma_{B}(t)|_{B}\rangle.\) Using this result, we immediately obtain the temporal evolution of the two atomic states. We will show this in detail by studying the generation of qutrit entanglement between the two distant atoms.

### 3. Generation of qutrit entanglement

We now show that the effective Hamiltonian (3) allows one to generate a qutrit–qutrit entangled state between two atoms \(A\) and \(B.\) Initially, the two cavities and the fibre are in the vacuum state while the two atoms are initialized in \(|\psi_{AB}(0)\rangle = |\varphi_{A}(0)|_{A}\rangle |\gamma_{B}(0)|_{B}\rangle.\) From the effective Hamiltonian (3), we immediately obtain the temporal evolution of the two atoms as follows:

\[
|\psi_{AB}(t)\rangle = e^{-it\chi} \left[ \cos(\sqrt{2}v t)|\varphi_{A}(t)|_{A}\rangle |\gamma_{B}(t)|_{B}\rangle + i e^{-it\lambda} \sin(\sqrt{2}v t) (|\varphi_{A}(t)|_{A}\rangle |\gamma_{B}(t)|_{B}\rangle + |\gamma_{A}(t)|_{A}\rangle |\varphi_{B}(t)|_{B}\rangle) \right].
\]
Setting $\chi t = \arctan(\sqrt{3}t\Omega x/\Delta x)$ ($m = 0, 1, 2, \ldots$), we get a qutrit–qutrit maximally entangled state (in the sense that the local Von Neumann entropy is maximal)

$$|\psi^{3D}_{AB}\rangle = \frac{e^{-i\mu t}}{\sqrt{2}}[|g_0\rangle_A|g_0\rangle_B + i|g_R\rangle_A|g_L\rangle_B].$$

The scheme is deterministic and viable for a rather wide range of system parameters. It should be noted that the process for the generation of the qutrit–qutrit entangled state in the present scheme is at variance with the previous strategies adopting dispersive interactions [28] because here the occupation of both the atomic excited states and of the photonic states in the cavities and fibre are negligible. Moreover, the external control in the present scheme is less demanding as the preparatory step to put the atom in a specific superposition of two ground states [26, 28] or the local manipulation of one of the atoms (by yet another classical field) during the temporal evolution of the whole system [26] is not required here.

In the above analysis, the exact knowledge of the system parameters is assumed. But, in general, there could be various errors in the parameters due to the imperfect characterization of the system. The potential errors include:

- the mismatch of the coupling rate $g_{x,k}$ and $\Omega_x$ ($x = A, B; k = L, R$) for the atoms with the local cavity and classical fields as $g_{x,k}$ and $\Omega_x$ are dependent on the atomic position and might fluctuate;
- the mismatch of the phase $\phi_x$ due to noise in the phases of the classical fields;
- the mismatch of the detuning $\Delta_{x,yz}$ ($x = A, B; y = \epsilon_0, \epsilon_L, \epsilon_R$ and $z = g_0, g_L, g_R$) between atoms and fields due to possibly imprecise control;
- the mismatch of the coupling rate $v_k$ for the cavity and fibre modes as $v_k$ is decided by the manufacture technology and might be imprecise;
- timing errors, due to the finite switching rates of the interactions and the limited precisions of the interaction times ($t_A$ and $t_B$ will be set for each atom interacting with the local fields);
- polarization errors due to unstable magnetic fields, which lead to mismatches in two parameters related to polarization, such as $g_{x,L(R)}, v_{L(R)}$ and $\Delta_{x,L(R)}$ ($\Delta_{x,L(R)}$ here denotes the detuning for $\pi - \sigma^z$); and
- errors in the atom–light coupling strengths in the two cavities and fibre coupling strengths induce different Stark shifts (and hence have different effects) depending on their signs ($f$), while errors in the atom–light coupling strengths in the two different polarizations have, more intriguingly, approximately ‘rotationally symmetric’ effects ($b$). Let us now quantitatively discuss the influence of the various kinds of errors on the fidelity with the entangled ‘reference state’.

From figure 2(c), it is apparent that the fidelity is, on the other hand, very sensitive to imperfections in $\Omega_x$, mainly dependent on the Stark shifts induced by the classical fields (see equation (3)). However, the influences of imperfect $\Omega_x$ through such Stark shifts can be eliminated as one can apply a second classical field to produce offsetting ac-Stark shifts on both atoms [28]. If this is done, then the effect of the errors $|\Omega_x|_0$ on $\Omega_x$ will be analogous to the effect of the deviation $|\delta g_{x,k}|_1$ in $g_{x,k}$, which have already been shown to be very slight. Thus, this simple countermeasure would make the entanglement fidelity robust also against possible errors in $\Omega_x$.

When deriving the effective Hamiltonian (3), we set the condition $\phi_A = \phi_B$ to eliminate the phases of the classical fields in equation (3) (see appendix A). However, such phases are affected by noise and fluctuations and might be slightly different in practical instances. Nevertheless, the fidelity is only marginally degraded by the possible errors in the parameter $\phi_A$; a deviation $|\delta\phi_1| = 3\%\phi_1$ will cause only 2% reduction of the fidelity. In practice, using only one classical field to illuminate both atoms that are distributed in the two cavities would help in keeping the phase fluctuations under control [32].

We remind the reader that in our scheme, the cavity and the classical fields are detuned from the corresponding atomic transitions by specific values. It can be seen from figure 2(g) that the fidelity is highly dependent on the parameter $\Delta_{x,yz}$; a small deviation in $\Delta_{x,yz}$ leads to large oscillation in the fidelity. This is mainly due to the detuning-dependent Stark shifts induced by the cavity and classical fields, besides the possible occurrence of a phase ($\Lambda = \Lambda_{A,\epsilon_0} - \Lambda_{A,\epsilon_R} - \Lambda_{B,\epsilon_0} + \Lambda_{B,\epsilon_R}$) in the exponential factor $e^{\imath \Delta t}$ in equation (3). Though this requirement is strict, it is not a major problem as a reference. Under such conditions maximal qutrit–qutrit entanglement is obtained at the reference time $t = T_0$ (in the notation of the previous section, only the case $m = 0$ is considered, i.e. $T_0 = 0.6755$).

We then set the errors involved in the parameters $g_{x,k}, \Omega_x, \Delta_{x,yz}, v_k, \phi_x$ and $t_k$ to be $\delta g_{x,k}, \delta \Omega_x, \delta \Delta_{x,yz}, \delta v_k, \delta \phi_x$ and $\delta t_k$, respectively. In figure 2, the fidelity is plotted versus the different kinds of errors. Note that these fidelity plots display a number of symmetries. The mirror symmetries about the line bisecting the axes trivially reflect the choices of the error parameters and the symmetry of the system under exchange of the two atoms and cavities. Some of the symmetries are instead more interesting; for instance, errors in the detunings induce additional phases and have clearly oscillatory effects (g), errors in the cavity–fibre coupling strengths induce different Stark shifts (and hence have different effects) depending on their signs (f), while errors in the atom–light coupling strengths in the two different polarizations have, more intriguingly, approximately ‘rotationally symmetric’ effects (b). Let us now quantitatively discuss the influence of the various kinds of errors on the fidelity with the entangled ‘reference state’.

It can be seen from figures 2(a), (b), (e) and (f) that the fidelity $F$ is very robust against errors in the parameters $g_{x,k}, v_k$ and $t_k$. A deviation $|\delta g_{x,k}| \approx 10\% g_{x,k}, |\delta v_k| \approx 10\% v_k$, or $|\delta t_k| \approx 10\% T_0$ will cause only a reduction smaller than $10^{-2}$ in the fidelity.

In order to check how the mentioned errors influence the generation of the entangled state, we define the following fidelity as a measure of the reliability of the qutrit–qutrit maximally entangled state:

$$F = |\langle \psi^{3D}_{AB} | T_{\epsilon_1,f_{c_1}}(\rho(t)) | \psi^{3D}_{AB}\rangle|^2,$$

where $\rho(t)$ is the state of the entire system at arbitrary time (governed by equation (9), where neither decoherence nor errors are accounted for) and $T_{\epsilon_1,f_{c_2}}$ denotes the partial trace over the field degrees of freedom.

We first assume ‘perfect’ interactions, considering the case $g_{x,k} = g, \Omega_x = \Omega \equiv g, \Delta_{A,\epsilon_0} = \Delta_{B,\epsilon_0} = 20g, \Delta_{A,\epsilon_R} = \Delta_{B,\epsilon_R} = 21g, \phi_A = \phi_B$ and $v_k = v = \sqrt{2}g$. In this case, the notation of the previous section, only the case $m = 0$ is considered, i.e. $T_0 = 0.6755$. We then set the errors involved in the parameters $g_{x,k}, \Omega_x, \Delta_{x,yz}$, $v_k, \phi_x$ and $t_k$ to be $\delta g_{x,k}, \delta \Omega_x, \delta \Delta_{x,yz}, \delta v_k, \delta \phi_x$ and $\delta t_k$, respectively.
with the currently developed laser technology in cavity QED experiments; the necessary stabilization of the fields’ frequencies can be achieved by means of acousto-optic modulators [33]. In figure 2(h), we check the stability of the fidelity versus the detunings for the two polarization (π − σ† or π − σ−) Raman channels. Ideally, the conditions $\Delta_{A,L} = \Delta_{A,R}$ and $\Delta_{B,L} = \Delta_{B,R}$ are required. But, in real experiments, this requirement may not be perfectly satisfied due to the fact that the magnetic field can break the degeneracy between the atomic ground states. Our investigation, reported in figure 2(h), shows that the fidelity is only slightly affected by errors in the detunings for both polarization channels; the fidelity with the entangled state of reference will still be larger than 0.96 even when a deviation $|\delta / \Delta_{A(B),k}| \simeq 3\%$ occurs.

4. Influence of spontaneous emission and photon leakage

In all the above arguments, we have assumed that the entire system is ideally isolated from the environment and have not considered any dissipation. In this section, we take into...
account the dissipation due to atomic spontaneous emission and photon leakage from the cavities and fibre. The master equation for the density matrix of the entire system can be expressed as

\[
\dot{\rho} = -i[H_{\text{full}}, \rho] + \frac{\kappa}{2} \sum_{k=L,R} \sum_{x=A,B} (2a_{x,k}^\dagger \rho a_{x,k} - a_{x,k}^\dagger a_{x,k} \rho - \rho a_{x,k}^\dagger a_{x,k} \rho) \\
- \rho a_{x,A,L}^\dagger a_{x,A,R} \rho + \frac{\beta}{2} (2b_{1} \rho b_{1}^\dagger - b_{1}^\dagger b_{1} \rho - \rho b_{1}^\dagger b_{1} \rho) \\
+ \frac{\gamma}{2} \sum_{x=A,B} \sum_{\sigma=L,R,\pi} (2A_{x,\sigma} \rho A_{x,\sigma}^\dagger - A_{x,\sigma}^\dagger A_{x,\sigma} \rho - \rho A_{x,\sigma}^\dagger A_{x,\sigma} \rho),
\]

where \(H_{\text{full}}\) can be seen in appendix B, i.e. equation (B.1); \(A_{x,\sigma} = \sum_{\tau \in \{eL, eR\}} |\tau\rangle \langle \tau| \chi_{\sigma}(\tau) \) is the atomic lowering operator, with \(\chi_\sigma(\tau)\) being the Clebsch–Gordan coefficient (i.e. \(C_{m,m'}\)) for the dipole transition \(|e\rangle \leftrightarrow |g\rangle\) with polarization \(\sigma = L, R, \pi\); \(\kappa, \beta\) and \(\gamma\) stand, respectively, for the spontaneous emission rate and for the fibre and cavity decay rates (assumed for simplicity to be equal for the two cavities and for the two polarized modes). The contribution of the thermal photons has been neglected, as it is possible at optical frequencies.

The master equation (9) has been numerically solved in the subspace \(\Gamma \in \{\Gamma_{\text{full}}, |g_L\rangle_A |g\rangle_B, |g_0\rangle_A |0\rangle_B, |0\rangle_A |g_0\rangle_B, |g R\rangle_A |g_0\rangle_B |0\rangle_A |0\rangle_B, |0\rangle_A |0\rangle_B |g_0\rangle_B, |g_0\rangle_A |0\rangle_B |g_0\rangle_B\}\) (see appendix B). In figure 3(a), the fidelity of the maximal qutrit–qutrit entangled state is plotted versus the dimensionless parameters \(\chi t, \kappa/g\) (\(\kappa = \beta = \gamma\) is set). In figures 3(b), (c) and (d), the fidelity is plotted versus each pair of the three dimensionless parameters \(\kappa/g, \beta/g\) and \(\gamma/g\) (the remaining one is set to be \(10^{-3}\)). In the calculations, we still set \(\Omega_A = \Omega_B = g_{A,k} = g_{B,k} \equiv g\), \(\Delta_{A,\epsilon_0} = 20g\), \(\Delta_{A,\epsilon_\pi} = 20g\) and \(\nu_k = \sqrt{2}g\).

From figure 3(a), we note that the fidelity is almost unaffected by the three decay rates \(\kappa, \beta, \gamma\) when \(\kappa = \beta = \gamma = 10^{-3}g\). Even when \(\kappa = \beta = \gamma = 10^{-2}g\), the fidelity is close to 0.97, which is much larger than the one (\(<0.87\)) obtained in [28]. This improvement is of course due to the suppression of the excited states’ population of the fields, as well as of the atoms. From figures 3(b), (c) and (d), it can be seen that a decay rate of \(10^{-2}g\) for either \(\kappa, \beta\) or \(\gamma\) alone (with the other two parameters set to zero), leads to a fidelity larger than 0.98. Note the previous scheme through the adiabatic passage [26]; a decay rate \(\kappa = 10^{-2}g\) alone degraded the fidelity down to 0.95.

Let us now consider fidelities and operating times under realistic cavity QED conditions. In the case of Fabry–Perot cavities with 6S_{1/2}, F = 4 \rightarrow 6P_{1/2}, F' = 4 transition of Cs atoms coupled to the cavity mode at the wavelength of 852 nm [34], one has \(g = 2\pi \times 110\) MHz, \(\gamma = 2\pi \times 2.6\) MHz and \(\kappa = 2\pi \times 14.2\) MHz. The fibre’s loss factor at the 852 nm wavelength is 2.2 dB km^{-1} [35], corresponding to a decay rate \(\beta = 0.152\) MHz. In this case, the time needed to perform our entangling operation on the atoms is about 0.61\,\mu s, yielding a fidelity with the maximally entangled state of 0.8835.

Considering a microsphere cavity, instead, with \(g = 2\pi \times 280\) MHz, \(\gamma = 2\pi \times 2.6\) MHz and \(\kappa = 2\pi \times 47\) kHz [34], the operating time becomes 0.24\,\mu s and the fidelity increases to a very promising 0.9842.

5. Conclusion

In summary, we have proposed a scheme of atomic levels (with an explicit possible realization in Zeeman sublevels of Cs atoms) that allows for the preparation of the three-photon maximally entangled state, which is a key resource for the implementation of quantum information processing tasks. The fidelity and operating times are found to be very promising for both realistic cavity QED conditions and microcavity realizations.
alkali atoms), where qutrit quantum information can be stored in three ground states and, most importantly, manipulated globally between distant nodes through the virtual excitation of excited atomic levels and mediating bosonic fields like, typically, light.

Our scheme is different from any previously proposed ones in that this choice of atomic levels allows for the whole coherent evolution of the global system—involving the two atoms and the linking bosonic modes—to be driven by the virtual excitation of both the atomic excited levels and the intervening fields.

This feature renders our scheme remarkably more robust than any other previously proposed in the face of decoherence, whose main sources in these settings are photon loss and spontaneous emission from excited levels. Also, our scheme—being based, essentially, on the proper choice of atomic levels and operating regimes—requires very modest control and proves to be rather resilient against experimental imperfections as well. All these qualitative remarks have been substantiated in this work by a very thorough quantitative analysis of such unwanted effects.

Clearly, a price has to be paid for improved robustness; the use of exclusively virtual excitations makes these coherent manipulations very slow if compared to schemes adopting resonant couplings [36, 37]. Ultimately, the choice between faster, resonant schemes and more robust, virtual ones should depend on the use one intends to make of them. Of course, speed would be paramount in applications directly related to quantum computation. However, as pointed out in the introduction, qutrit systems are mainly interesting for quantum communication purposes, where the most delicate task to accomplish is precisely the robust distribution of entanglement between distant nodes of a network, and speed, while very desirable, is not as crucial; fully virtual schemes, like the one presented here, would respond precisely to this need. It should be pointed out that, since our study relies on the ‘short-fibre limit’, the length across which the distributed entanglement can be generated is constrained, assuming a cavity–ﬁbre coupling around 1 MHz (which still allows, as we showed in section 4, for an operating time around 1 μs); the length of the fibre should be much less than 1 km for the short-fibre condition to be met. Longer fibres can be employed in this regime only at the price of longer operating times. Nevertheless, our study shows that atomic systems hold considerable promise for the encoding and coherent, distributed manipulation of multidimensional quantum alphabet for quantum communication purposes.

Acknowledgments

This work is supported by National Natural Science Foundation of China under grant nos 10674025 and 10974028, the Fujian Natural Science Foundation of China under grant no 2009J06002, Doctoral Foundation of the Ministry of Education of China under grant nos 20070386002 and 20093514110009, funds from State Key Laboratory Breeding Base of Photocatalysis, Fuzhou University, and funds from Education Department of Fujian Province of China under grant no JB08010. ZBY is also supported by Science and Technology Innovation Special Fund for PhD students, Fuzhou University. SYY is supported by a KC Wong Scholarship. AS thanks the Central Research Fund of the University of London for financial support.

Appendix A. Derivation of the effective Hamiltonian

Under the condition of large detuning, i.e. for \( \Delta_{A,\gamma} \gg g_{A,k}, \Omega_{A} \), and as long as the atoms are initialized in the ground states, the probability that the atomic excited states are populated is virtual. Thus, the atomic excited states are negligible during the time evolution of the entire system. In this case, we can adopt the time-averaging method [38] to obtain an effective Hamiltonian as follows [39]:

\[
H^\text{eff} = -\frac{i}{\hbar} \int H^\text{eff}(t') \, dt'
\]

\[
= \sum_{k=L,R} [ \mu_{\Omega A} \langle g|g\rangle_A + \mu_{\Omega B} \langle g|g\rangle_B ] + \mu_{\Omega A} a^\dagger_A a_A a_A + a_B a_B a_B + \lambda_{\Omega A} (a_A^\dagger a_A - i \Delta_{\Omega A,\gamma}^r) |g\rangle_A \langle g| + \text{H.c.} + \lambda_{\Omega B} (a_B^\dagger a_B - i \Delta_{\Omega B,\gamma}^r) |g\rangle_B \langle g| + \text{H.c.}],
\]

\[A.1\]

where \( \mu_{\Omega A} = \frac{\Omega_{A,k}}{\Delta_{\Omega A,\gamma}} \), \( \lambda_{\Omega A} = \frac{g_{A,k}}{\Delta_{\Omega A,\gamma}} \), \( \mu_{\Omega B} = \frac{\Omega_{B,k}}{\Delta_{\Omega B,\gamma}} \), \( \lambda_{\Omega B} = \frac{g_{B,k}}{\Delta_{\Omega B,\gamma}} \), and \( \Delta_{\Omega k} = \Delta_{\Omega k}^{\prime} - \Delta_{\Omega k}^{\prime \prime} \).

For \( H^\text{eff} \) in equation (A.1), the first (second) and third (fourth) terms describe the Stark shifts for the states \( |g\rangle_A \) (\( |g\rangle_B \)) and \( |g\rangle_B \) (\( |g\rangle_A \)) of the atom in cavity A (B), induced by the classical and cavity fields, respectively; the fifth (sixth) term describes the Raman coupling between the states \( |g\rangle_A \) and \( |g\rangle_B \) for the atom in cavity A (B), induced collectively by the classical and cavity fields.

Hence, the effective Hamiltonian of the entire system is given by \( H^\text{eff} = H^\text{eff}_R + H^\text{eff}_L \). Let us now introduce three normal modes \( c_k \) and \( e_{\pm} \) by applying the canonical transformation \( c_k = \frac{1}{\sqrt{2}} (a_{A,k} - e^{-i \Delta_{\Omega A,\gamma}^r} a_{B,k}) \) and \( e_\pm = \frac{1}{\sqrt{2}} (a_{A,k} + e^{i \Delta_{\Omega A,\gamma}^r} a_{B,k} \pm \sqrt{2} d_{B,k}) [23] \). Then, we switch to a rotating frame by the unitary transformation \( R = e^{-i H^\text{eff} R / \hbar} \), i.e. \( H^\text{eff}_R = R^\dagger H^\text{eff}_R R - i R^\dagger \frac{dH^\text{eff}_R}{dt} \), and obtain

\[
H^\text{eff}_R = \sum_{k=L,R} [ \mu_{\Omega A} \langle g|g\rangle_A + \mu_{\Omega B} \langle g|g\rangle_B ] + \frac{\mu_{\Omega A}}{4} (c_k^\dagger c_k + c_{\pm}^\dagger c_{\pm} - 2 c_k^\dagger c_k) |g\rangle_A \langle g| + \frac{\mu_{\Omega B}}{4} (c_k^\dagger c_k + c_{\pm}^\dagger c_{\pm} + 2 c_k^\dagger c_k) |g\rangle_B \langle g| + \frac{\mu_{\Omega A}}{4} (c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} - \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r} + \sqrt{2} c_k^\dagger c_{\pm} e^{i \Delta_{\Omega A,\gamma}^r}),
\]
with S. J. Phys. B: At. Mol. Opt. Phys. are conserved quantities during the interaction as all of them
\[ e^{-i\phi_{S-p}} |g_{B}\rangle |g_{0}\rangle + \text{H.c.} \].

For simplicity, we now set \( \mu_{1} = \mu_{0} = \mu_{0}, \mu_{2} = \mu_{0}, \lambda = \lambda_{0}, \Delta_{A} = \Delta_{B,k}, \phi_{A} = \phi_{B} \), and \( v_{k} \). Under the condition \( \sqrt{2}v_{k}/\Delta - \sqrt{2}v_{k}, \Delta + \sqrt{2}v_{k} \) and \( \mu_{2}^{2}/\Delta \), the energy exchange between the bosonic modes and the atoms as well as between the bosonic modes themselves is virtual. The virtual excitation of the bosonic modes leads to the Stark shifts and coupling between the atoms. Then \( H_{e}^{''} \) reduces to [30]

\[ H_{e}^{''} = \sum_{k=L,R} \left\{ \mu_{1}(|g_{0}\rangle A |g_{0}\rangle B |g_{k}\rangle - |g_{k}\rangle A |g_{0}\rangle B |g_{0}\rangle) \right. \]

\[ + \frac{\mu_{2}}{4} (c_{+}^{k}c_{-} + c_{-}^{k}c_{+} + 2c_{k}^{k}c_{k}^{\dagger}) (|g_{k}\rangle A |g_{0}\rangle B |g_{0}\rangle B |g_{0}\rangle) \]

\[ + \frac{\mu_{2}^{2}}{8\sqrt{2}v_{k}} \left( c_{+}^{k}c_{-} + c_{-}^{k}c_{+} - 2c_{k}^{k}c_{k}^{\dagger} \right) (|g_{k}\rangle A |g_{0}\rangle B |g_{0}\rangle B |g_{0}\rangle)^{2} \]

\[ + \frac{\lambda^{2}}{4} \left[ \left( \frac{1}{\Delta - \sqrt{2}v_{k}} c_{+}^{k}c_{k}^{\dagger} \right) + \frac{2}{\Delta} c_{k}^{k}c_{k}^{\dagger} \right] \]

\[ \times (|g_{0}\rangle A |g_{0}\rangle B |g_{0}\rangle B |g_{0}\rangle) \]

\[ - \left( \frac{1}{\Delta - \sqrt{2}v_{k}} c_{+}^{k}c_{k}^{\dagger} + \frac{2}{\Delta} c_{k}^{k}c_{k}^{\dagger} \right) \]

\[ \times (|g_{k}\rangle A |g_{0}\rangle B |g_{0}\rangle B |g_{0}\rangle) \]

\[ - \left( \frac{1}{\Delta - \sqrt{2}v_{k}} c_{-}^{k}c_{k}^{\dagger} + \frac{2}{\Delta} c_{k}^{k}c_{k}^{\dagger} \right) \]

\[ \times (|g_{0}\rangle A |g_{0}\rangle B |g_{0}\rangle B |g_{0}\rangle) \]

\[ \times \left( e^{-i\omega_{S-p}^{a} S_{A,k}^{a} S_{B,k}^{b} + \text{H.c.}} \right) \].

\[ H_{e}^{''} = \sum_{k=L,R} \left\{ \eta_{i}(|g_{0}\rangle A |g_{0}\rangle B |g_{k}\rangle + |g_{k}\rangle A |g_{0}\rangle B |g_{0}\rangle) \right. \]

\[ - \chi (e^{-i\omega_{S-p}^{a} S_{A,k}^{a} S_{B,k}^{b} + \text{H.c.}}) \],

where

\[ \eta = \mu_{1} + \eta' \],

\[ \eta' = \frac{\lambda^{2}}{4} \left[ \left( \frac{1}{\Delta - \sqrt{2}v_{k}} + \frac{1}{\Delta + \sqrt{2}v_{k}} + \frac{2}{\Delta} \right) \right] \],

and

\[ \chi = \frac{\lambda^{2}}{4} \left[ - \frac{1}{\Delta - \sqrt{2}v_{k}} - \frac{1}{\Delta + \sqrt{2}v_{k}} + \frac{2}{\Delta} \right] \].

It should be noted that we have neglected the term \( \eta' |g_{0}\rangle A |g_{0}\rangle \) to maintain the symmetry in the effective Hamiltonian (A.4).

In practice, this term can be compensated by an additional ac-Stark shift for the state \( |g_{0}\rangle \) of atom A [28].

**Appendix B. Validity of the effective dynamics**

We now turn back to the full Hamiltonian of the system and check how accurate is the description of the system through the effective Hamiltonian (3). We take the energy level \( |F = 2\rangle \) of 52S1/2 to be the zero energy reference point, and write down the full Hamiltonian for the entire system as follows:

\[ H_{\text{full}} = \sum_{k=L,R} \left( \omega_{0,k} |b_{k}\rangle \langle b_{k}| + \omega_{a,k} |a_{L,k}^{a} \rangle \langle a_{L,k}^{a}| + \omega_{a,k} |a_{B,k}^{a} \rangle \langle a_{B,k}^{a}| \right) \]

\[ + \omega_{A,k} |g_{A}\rangle |g_{A}\rangle \langle g_{A}| + \omega_{B,k} |g_{B}\rangle |g_{B}\rangle \langle g_{B}| \]

\[ + \omega_{A,0} |g_{A}\rangle |g_{A}\rangle \langle g_{0}| + \omega_{B,0} |g_{B}\rangle |g_{B}\rangle \langle g_{0}| \]

\[ + \Omega_{A} e^{-i\omega_{a}^{A} \mu_{1}} \langle |e_{0}\rangle \langle e_{0}| \]

\[ + \Omega_{B} e^{-i\omega_{a}^{B} \mu_{1}} \langle |e_{0}\rangle \langle e_{0}| \]

\[ + \sum_{k=L,R} \nu_{k} \left( |b_{k}\rangle \langle a_{L,k}^{a}| + e^{i\omega_{a}^{B} \mu_{1}} |b_{k}\rangle \langle a_{B,k}^{a}| + \text{H.c.} \right) \].

\[ \text{(B.1)} \]

where \( \omega_{0,k} \) and \( \omega_{a,k} \) (\( \omega_{a,k} \)) are the energy levels for the polarized photons in the fibre and cavity A (B), respectively. \( \omega_{\lambda1} (\lambda = A, B) \) denotes the energy for the \( \pi \)-polarized classical field \( \lambda \), and \( \omega_{A,0} (m = g_{A}, g_{B}, e_{0}, e_{k}) \) is the energy level for the atomic state \( |m\rangle \).

Taking the initial state \( |\psi(0)\rangle = |g_{0}\rangle A |g_{0}\rangle B |00\rangle_{c_{1}} |00\rangle_{c_{2}} \) and considering all possible states of the system in evolution, we express the state of the system at time \( t \) as \( |\psi(t)\rangle = \sum |c_{i}(t)\rangle |\phi_{i}(t)\rangle \) (\( c_{i}(t) \) being time-dependent amplitudes) within the subspace \( \Gamma_{\text{full}} \) spanned by the vectors \( \{|\phi_{1}\rangle, |\phi_{2}\rangle, |\phi_{3}\rangle\} \):

\[ \Gamma_{\text{full}} = \{ |g_{0}\rangle A |g_{0}\rangle B, |g_{0}\rangle A |g_{L}\rangle B, |g_{0}\rangle A |g_{R}\rangle B, |g_{L}\rangle A |e_{0}\rangle B, |g_{R}\rangle A |e_{0}\rangle B \]

\[ \otimes |00\rangle_{c_{1}} |00\rangle_{c_{2}}, |g_{A}\rangle A |g_{0}\rangle B \]

\[ \otimes |10\rangle_{c_{1}} |00\rangle_{c_{2}}, |00\rangle_{c_{1}} |10\rangle_{c_{2}}, |00\rangle_{c_{1}} |01\rangle_{c_{2}}, |00\rangle_{c_{1}} |11\rangle_{c_{2}} \]

\[ \otimes |01\rangle_{c_{1}} |00\rangle_{c_{2}}, |00\rangle_{c_{1}} |01\rangle_{c_{2}} \].

The occupation probability for each state vector \( |\phi_{i}\rangle \) during the evolution is \( P_{i}(t) = |c_{i}(t)|^{2} \) and satisfies \( \sum_{i=1}^{12} P_{i}(t) = 1 \). Thus, the occupation probability of the atomic excited states and the photonic states are \( P_{e}(t) = \sum_{i=2,5,6}^{12} P_{i} \) and \( P_{p}(t) = \sum_{i=7}^{12} P_{i} \), respectively. The validity of the effective Hamiltonian implies that both the occupation probability \( P_{e}(t) \) and \( P_{p}(t) \) should be small enough; thus, they can be negligible during the time evolution of the entire system. We focus here on quantum state transfer, i.e. on the variation of the occupation probability \( P_{e}(t) \) and \( P_{p}(t) \). For the numerical verification of the effective dynamics, which is portrayed in figure B1. Figures B1(a1) and (a2) are obtained through the solution of the Schrödinger equation \( \frac{\partial |\psi(t)\rangle}{\partial t} = H_{e}^{''} |\psi_{\text{eff}}(t)\rangle \) in the subspace \( \Gamma_{e} \in \{ |\phi_{1}\rangle, |\phi_{2}\rangle, |\phi_{3}\rangle\} \). The two figures display
perfect Rabi oscillations, which indicate ideal state transfer between the states \(|\phi_1\rangle\) and \(1/\sqrt{2}(|\phi_1\rangle + |\phi_2\rangle)\).

Figures B1(b1), (c1), (d1), (b2), (c2) and (d2) are obtained through the solution of the Schrödinger equation \(i\partial \psi /\partial t = \mathcal{H}_{\text{full}} |\psi_{\text{full}}(t)\rangle\) in the subspace \(\mathcal{H}_{\text{full}}\). It can be seen from figures B1(b1) and (b2) that state transfer via the effective Hamiltonian (3) is almost perfect (for figure B1(b2), this is especially apparent), indicating that numerical results obtained from the effective and full Hamiltonians would be equivalent. Figures B1(c1) and (c2) plot the variation of the occupation probability of the atomic excited states, while figures B1(d1) and (d2) plot the variation of the occupation probability of the photonic states in the cavities and fibre. It is apparent that both \(P_p\) and \(P_e\) are very small during the evolution of the entire system. The analysis above verifies the validity of the effective Hamiltonian (3).

Let us now review the physical conditions given in the description of our system, to reveal some insight about and from the perfect state transfer, which are more noticeable in figure B1(b1) than in figure B1(b2).

Let us stress once again that the obtained effective Hamiltonian (3) is indeed valid as the occupations of the atomic excited states and the photonic states have been shown to be strongly suppressed.

References

[1] Einstein A, Podolsky B and Rosen N 1935 Phys. Rev. 47 777
[2] Bell J S 1965 Physics (Long Island City, NY) 1 195
[3] Greenberger D M, Horne M, Shimony A and Zeilinger A 1990 Am. J. Phys. 58 1131
[4] Nielsen M A and Chuang I L 2000 Quantum Computation and Quantum Information (Cambridge: Cambridge University Press)
[5] Dür W, Vidal G and Cirac J I 2000 Phys. Rev. A 62 062314
[6] Kaszlikowski D, Gacinski P, Zukowski M, Miklaszewski W and Zeilinger A 2000 Phys. Rev. Lett. 85 4418
[7] Bechmann-Pasquinucci H and Titel W 2000 Phys. Rev. A 61 062308
[8] Bourennane M, Karlsson A and Bjork G 2001 Phys. Rev. A 64 012306
[9] Walborn S P, Lemelle D S, Almeida M P and Souto Ribeiro P H 2006 Phys. Rev. Lett. 96 090501
[10] Cirac J I and Zoller P 1995 Phys. Rev. Lett. 74 4091
[11] Monroe C, Meekhof D M, King B E, Itano W M and Wineland D J 1995 Phys. Rev. Lett. 75 4714
[12] Kielpinski D, Monroe C and Wineland D J 2002 Nature (London) 417 709
[13] Berman P (ed) 1994 Cavity Quantum Electrodynamics (New York: Academic)
Haroche S and Raimond J M 2006 *Exploring the Quantum* (Oxford: Oxford University Press)

[10] Esteve D 2004 Superconducting qubits *Proc. of the Les Houches 2003 Summer School on Quantum Entanglement and Information Processing* ed D Esteve and J M Raimond (New York: Elsevier)

[11] Loss D and DiVincenzo D P 1998 *Phys. Rev. A* **57** 120

[12] Knill E, Laflamme R and Milburn G J 2001 *Nature (London)* **409** 46

Rausendorf R and Briegel H J 2001 *Phys. Rev. Lett.* **86** 5188

[13] Kane B 1998 *Nature (London)* **393** 133

[14] Raimond J M, Brune M and Haroche S 2001 *Rev. Mod. Phys.* **73** 565

Miller R, Northup T E, Birnbaum K M, Boca A, Boozer A D and Kimble H J 2005 *J. Phys. B: At. Mol. Opt. Phys.* **38** S551

Walther H, Varcoe B T H, Englert B G and Becker T 2006 *Rep. Prog. Phys.* **69** 1325

[15] Kuhr S *et al* 2007 *Appl. Phys. Lett.* **90** 164101

[16] Hagley E, Maitre X, Nogues G, Wunderlich C, Brune M, Raimond J M and Haroche S 1997 *Phys. Rev. Lett.* **79** 1

Osnaghi S, Bertet P, Auffeves A, Maioli P, Brune M, Raimond J M and Haroche S 2001 *Phys. Rev. Lett.* **87** 037902

[17] Rausschenbeutel A, Nogues G, Osnaghi S, Bertet P, Brune M, Raimond J M and Haroche S 2000 *Science* **288** 2024

[18] Zou X B, Pahlke K and Mathis W 2003 *Phys. Rev. A* **67** 044301

Zheng S B 2003 *Phys. Rev. A* **68** 035801

[19] DiVincenzo D P 2000 *Fortschr. Phys.* **48** 771

[20] Cabrillo C, Cirac J I, Garcia-Fernández P and Zoller P 1999 *Phys. Rev. A* **59** 001025

Bose S, Knight P L, Plenio M B and Vedral V 1999 *Phys. Rev. Lett.* **83** 5158

Duan L M and Kimble H J 2003 *Phys. Rev. Lett.* **90** 253601

Feng X L, Zhang Z M, Li X D, Gong S Q and Xu Z Z 2003 *Phys. Rev. Lett.* **90** 217902

[21] Lamata L, García-Ripoll J J and Cirac J I 2007 *Phys. Rev. Lett.* **98** 010502

[22] Cirac J I, Zoller P, Kimble H J and Mabuchi H 1997 *Phys. Rev. Lett.* **78** 3221

Pelizzari T 1997 *Phys. Rev. Lett.* **79** 5242

van Enk S J, Kimble H J, Cirac J I and Zoller P 1998 *Phys. Rev. A* **59** 2659

[23] Serafini A, Mancini S and Bose S 2006 *Phys. Rev. Lett.* **96** 010503

[24] Kimble H J 2008 *Nature (London)* **453** 1023

[25] Yin Z Q and Li F L 2007 *Phys. Rev. A* **75** 012324

[26] Ye S Y, Zhong Z R and Zheng S B 2008 *Phys. Rev. A* **77** 014303

[27] Zheng S B 2009 *Eur. Phys. J. D* **54** 1434

[28] Lü X Y, Liu J B, Ding C L and Li J H 2008 *Phys. Rev. A* **78** 032305

[29] Lü X Y, Si L G, Hao X Y and Yang X 2009 *Phys. Rev. A* **79** 052330

[30] Zheng S B 2009 *Appl. Phys. Lett.* **94** 154101

Zheng S B 2010 *Chin. Phys. B* (at press)

[31] Wilk T, Webster S C, Kuhn A and Rempe G 2007 *Science* **317** 488

[32] Talab M A, Guérin S, Sanguaud N and Jauslin H R 2005 *Phys. Rev. A* **71** 023805

[33] Hennrich M, Legero T, Kuhn A and Rempe G 2000 *Phys. Rev. Lett.* **83** 4872

[34] Spillane S M, Kippenberg T J, Vahala K J, Goh K W, Wilcut E and Kimble H J 2005 *Phys. Rev. A* **71** 013817

[35] Gordon K I, Fernandez V, Townsend P D and Buller G S 2004 *IEEE J. Quantum Electron.* **40** 900

[36] Yang Z B and Zheng S B 2010 submitted

[37] Ogden C D, Irish E K and Kim M S 2008 *Phys. Rev. A* **78** 063805

[38] James D F V 2000 *Fortschr. Phys.* **48** 823

[39] Wu Y 1996 *Phys. Rev. A* **54** 1586

Wu Y and Yang X 2005 *Phys. Rev. A* **71** 053806

James D F V and Jerke J 2007 *Can. J. Phys.* **85** 625