An assessment of some non-gray global radiation models in enclosures

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Abstract. The accuracy of several non-gray global gas/soot radiation models, namely the Wide-Band Correlated-K (WBCK) model, the Spectral Line Weighted-sum-of-gray-gases model with one optimized gray gas (SLW-1), the (non-gray) Weighted-Sum-of-Gray-Gases (WSGG) model with different sets of coefficients (Smith et al., Soufiani and Djavdan, Taylor and Foster) was assessed on several test cases from the literature. Non-isothermal (or isothermal) participating media containing non-homogeneous (or homogeneous) mixtures of water vapor, carbon dioxide and soot in one-dimensional planar enclosures and multi-dimensional rectangular enclosures were investigated. For all the considered test cases, a benchmark solution (LBL or SNB) was used in order to compute the relative error of each model on the predicted radiative source term and the wall net radiative heat flux.

1. Introduction
Accurate modeling of radiative heat transfer is essential for the improvement of the performance of industrial flame furnaces (thermal efficiency and pollutant emissions). The main thermal radiation issues to be addressed are the radiative properties of combustion products (gases and particles) and their modeling, and the Radiative Transfer Equation (RTE) solution methods. The development of computationally efficient and accurate radiative property models for non-gray gas/soot radiation analysis has been an active research subject for decades (see [1–4] and references therein).

Line-By-Line (LBL) calculations yield exact solutions but millions of RTE resolutions are required. The turnaround time is therefore not acceptable for engineering calculations. Gray gas (GG) models, based on total emittance correlations [5, 6], are computationally efficient and thus widely used for engineering calculations but often yield relatively large errors [7].

Between these two extremes, a vast literature has been devoted to the development of spectral models. The Statistical Narrow Band (SNB) model [8], formulated in average transmissivity or absorption coefficients with a correlated \( k \)-distribution method (CK), is the most accurate gas radiative properties model and is considered as a reference model when LBL calculations are not available [9]. Wide Band (WB) models are less accurate than SNB models but more widely used for industrial applications [10]. For combustion applications, global models like the non-gray Weighted-Sum-of-Gray-Gases model (WSGG) [11], the Spectral-Line Weighted-sum-of-gray-gases model (SLW) [12–14] and the Wide Band Correlated-k model (WBCK) [15–17] may yield relatively good predictions in a computationally efficient manner. However, the accuracy assessment highly depends on the radiative transfer problem studied (e.g., type of enclosure,
participating species, distributions of temperature and concentrations) \[7, 18\] and the spectral databases used to compute the LBL solutions \[9, 19\] and the parameters of the models.

The objective of this work is to investigate the performances of several (non-gray) global radiation models by comparison with benchmark solutions available in the literature. The accuracy of the following models was assessed: the Wide-Band Correlated-K model with nineteen gray gases (WBCK-19) \[15–17\], the Spectral Line Weighted-sum-of-gray-gases model with one optimized gray gas (SLW-1) \[20, 21\], the (non-gray) Weighted-Sum-of-Gray-Gases model \[11\] with coefficients from Smith et al. \[22\] (WSGG-Smith), Soufiani and Djavdan \[23\] (WSGG-Soufiani), Taylor and Foster \[24\] (WSGG-Taylor). A gray gas model (GG), based on total emittance correlations from Leckner \[6\], was also included in the study. The test problems were selected based on the following criteria: (i) availability of benchmark solutions (LBL or SNB); (ii) participating media composed of mixtures of water vapor, carbon dioxide and soot; (iii) distributions of temperature and concentrations close to those encountered in flame furnaces. To the best of the author’s knowledge, the evaluation of the aforementioned global models on the same set of test problems has not been carried out in the past.

The present paper is organized as follows. The second section gives a brief overview of the global models evaluated in this study. The test problems are described in the third section. The obtained results are presented and discussed in the fourth section.

2. Description of global radiation models
2.1. Weighted-Sum-of-Gray-Gases (WSGG) model
The WSGG model \[5\] expresses the total gas emittance as a weighted sum of gray gas emittances. The emission weighting factors and the absorption coefficients are obtained from a fit to total emittance measurements of isothermal and homogenous columns. The WSGG model is often used in combustion problems assuming that the medium is gray. In such a case, the absorption coefficient of the medium is calculated from the total gas emittance and the mean beam length. The total gas emittance can be computed using the Hottel’s charts \[5\] or the Leckner’s correlations \[6\]. In the present study, we used the Leckner’s correlations.

Modest \[11\] showed that the WSGG approach may be implemented as a non-gray gas model by solving the RTE for \( N_g \) gray gases plus one transparent gas. The non-gray WSGG model with the coefficients determined by Smith et al. \[22\], Soufiani and Djavdan \[23\] and Taylor and Foster \[24\] was used in the present study.

More recent sets of coefficients, based on HITEMP-2010 \[25\], can be found in \[26, 27\] while recent methodologies were proposed to derive the coefficients for either different molar ratios \[28\] or arbitrary concentrations of water vapor, carbon dioxide and soot \[29\].

2.2. Spectral-Line Weighted-sum-of-gray-gases (SLW) model
The SLW may be regarded as an improved WSGG model, obtained from the line-by-line spectra of the absorbing species \[12–14\]. For a sufficient number of gray gases, the SLW model yields exact solutions of the gas radiation problem \[30, 31\]. In the present study, we used the SLW model with one optimized gray gas (SLW-1) \[20, 21\]. The parameters of SLW-1 were derived from the new correlations for the absorption line blackbody distribution function (ALBDF) of water vapor, carbon dioxide and carbon monoxide developed recently by Pearson et al. \[32\].

2.3. Wide Band Correlated-k (WBCK) model
WB models yield transmittances and cannot thus be easily be coupled to differential solution methods of the RTE. The WBCK model \[15–17\] gives an absorption coefficient for each spectral interval. The weight of each spectral interval is calculated from the blackbody fraction of the interval. Since there are several hundreds of wavenumber intervals for a mixture of water vapor and carbon dioxide, a gray gases formulation is used to further decrease the computational effort.
by gathering intervals with an absorption coefficient within an interval range. In the present study, the WBCK model with nineteen gray gases was used (WBCK-19).

2.4. Soot radiation model
If soot particles are very small so that the Rayleigh theory applies for all particles and relevant wavelengths, the extinction coefficient is described by:

$$\kappa_{\lambda} = C_0 \frac{f_v}{\lambda}$$

where $C_0$ is a constant depending only on the soot index of refraction [3]. The Planck-mean extinction coefficient can be computed as:

$$\kappa_P = 3.83 f_v C_0 T / C_2$$

where $C_2$ is the second Planck function constant.

In the present study, the Planck mean absorption coefficient of soot is added to each gray gas for the GG, WBCK and WSGG models using the index of refraction given by Tien and Lee [33]. In the SLW-1 model, soot is treated as a non-gray gas species for which ALBDF is calculated [20].

3. Description of test problems
The accuracy of WBCK-19, SLW-1, (non-gray) WSGG and GG models was assessed on several test cases from the literature, namely non-isothermal (or isothermal) participating media containing non-homogeneous (or homogeneous) mixtures of water vapor, carbon dioxide and soot in one-dimensional planar enclosures or multi-dimensional rectangular enclosures. All models were not applied to all test cases though (test cases 1 and 2 : GG, WBCK-19, SLW-1 and WSGG-Smith ; test cases 3 and 4 : WSGG-Smith, WSGG-Soufiani and WSGG-Taylor).

Each solution of the GG, WBCK and (non-gray) WSGG models was obtained using the RTE solver (FVM) available in two different commercial CFD codes (GTM-X [34] and ANSYS Fluent [35]). Polar/azimuthal angular quadrature schemes and the step scheme (spatial discretization) are used in both codes. The GG and WBCK models provided in GTM-X were used while the (non-gray) WSGG models were implemented in ANSYS Fluent with user-defined functions.

The exact analytical RTE solution of Solovjov et al. [36] was used for one-dimensional test cases with the SLW-1 model. To the best of the author’s knowledge, the SLW-1 cannot yet be readily used in non-isothermal participating media containing non-homogeneous (or homogeneous) mixtures in multi-dimensional enclosures. Issues related to the choice of the reference state and the optimization procedure have yet to be clarified. Therefore, SLW-1 was not implemented in the CFD codes used for this study.

For all the considered test cases, a benchmark solution (LBL or SNB) was used in order to compute the relative error of each model on the predicted radiative source term and the wall net radiative heat flux. Since different computational codes were used in this study, the computational costs associated with each model could not be compared fairly and are thus not reported here.

3.1. Test case 1 : homogeneous and isothermal mixtures in one-dimensional planar enclosures
The first problem (test case 1) was previously studied by Denison [13, 14]. It consists of a homogeneous and isothermal medium containing a mixture of water vapor (40%), carbon dioxide (20%) and soot at 1250 K in one-dimensional planar enclosures. The total gas pressure is 1 atm. The walls are assumed gray ($\epsilon_w = 0.8$) and at 400 K and 1500 K at $x = 0$ and $x = L$. 
respectively. Four different configurations were investigated by varying the distance between the walls ($L = 0.1$, 1 or 3 m) and the soot volume fraction ($f_v = 10^{-7} \text{ or } 10^{-6} \text{ m}^3/\text{m}^3$).

3.2. Test case 2: non-homogeneous and non-isothermal mixture in a one-dimensional planar enclosure

The second problem (test case 2) was previously studied by Bressloff [37] and other authors (e.g., [21, 38, 39]). It consists of a non-homogeneous and non-isothermal medium containing a mixture of water vapor, carbon dioxide and soot in a one-dimensional planar enclosure ($L = 1 \text{ m}$). The total gas pressure is 1 atm. The walls are assumed black and cold at 0 K. Temperature, gas species (molar fraction) and soot (volume fraction) have the following parabolic distributions:

$$T(x) = 800 + 4000x(L - x)$$
$$X_{CO_2}(x) = 0.06 + 0.4x(L - x)$$
$$X_{H_2O}(x) = 2X_{CO_2}$$
$$f_v(x) = [40x(L - x) + 6] \times f_{v0}$$

where $f_{v0} = 10^{-7} \text{ m}^3/\text{m}^3$ (configuration A, high soot loading) or $10^{-8} \text{ m}^3/\text{m}^3$ (configuration B, intermediate soot loading). Figure 1 represents the distributions of the temperature and concentrations fields along the enclosure.

![Figure 1](image)

Figure 1. Distributions of the normalized temperature and concentrations fields for test case 2 ($T_{max} = 1800 \text{ K}$, $f_{vmax} = 16 f_{v0}$).

3.3. Test case 3: homogeneous and non-isothermal mixture in a two-dimensional rectangular enclosure

The third problem (test case 3) was previously studied by Goutière et al. [18] and other authors (e.g., [15, 40]). It consists of a homogeneous and non-isothermal medium containing a mixture of water vapor (20%) and carbon dioxide (10%) in a two-dimensional rectangular enclosure ($1 \times 0.5 \text{ m}^2$). The total gas pressure is 1 atm. The walls are assumed black and cold at 0 K. The temperature distribution is defined as follows:

$$T(x, y) = \begin{cases} 
800 + (14000x - 400) (1 - 3y_0^2 + 2y_0^3), & \text{if } x \leq 0.1 \text{ m} \\
800 - \frac{10000}{9} (x - 1) (1 - 3y_0^2 + 2y_0^3), & \text{if } x \geq 0.1 \text{ m}
\end{cases}$$
with \( y_0 = |0.25 - y|/0.25 \). The temperature reaches a maximum (1800 K) along the centerline at \( x = 0.1 \) m. Figure 2 represents the distribution of the temperature field in the rectangular enclosure.

![Figure 2. Distribution of the temperature field (in K) for test case 3.](image)

3.4. Test case 4: homogeneous and non-isothermal mixture in a three-dimensional rectangular enclosure

The fourth problem (test case 4) was previously studied by Liu [41] and other authors (e.g., [42–44]). It consists of a homogeneous and non-isothermal medium containing a mixture of water vapor (20%) and carbon dioxide (10%) in a three-dimensional rectangular enclosure \((2 \times 2 \times 4 \) m\(^3\)). The total gas pressure is 1 atm. The walls are assumed black and cold at 300 K. The temperature is non-uniform but symmetrical along the centerline of the enclosure:

\[
T(r) = (T_c - T_e) f (r/R) + T_e \tag{8}
\]

\[
f (r/R) = 1 - 3 (r/R)^2 + 2 (r/R)^3 \tag{9}
\]

where \( T_c, T_e, r, \) and \( R \) are the temperature along the centerline of the enclosure, the exit temperature at \( z = 4 \) m, the distance from the enclosure centerline and the radius of the circular region \((R = 1 \) m\), respectively. The temperature outside the circular region is assumed to be uniform and at the value of the exit temperature. The centerline temperature is assumed to increase linearly from 400 K at the inlet \((z = 0 \) m\) to 1800 K at \( z = 0.375 \) m, then decreases linearly to 800 K at the exit. Figure 3 represents the distribution of the temperature field in the rectangular enclosure.

4. Results and discussion

4.1. Test case 1: homogeneous and isothermal mixtures in one-dimensional planar enclosures

LBL calculations from Denison [13, 14] were used as benchmark solutions. Figure 4 shows the predicted profiles of the radiative source term. The S-shaped profiles obtained with the benchmark solutions (LBL) were well reproduced by all models but the gray gas model. The cold wall net radiative heat flux and the relative error by comparison with LBL benchmark data are reported in Table 1. The results obtained with the models were almost exact (relative errors less than 3%) with the noticeable exception of the gray gas model (relative errors greater than 7.5% for all configurations but one).
Table 1. Results of the cold wall net radiative heat flux for test case 1: \( L = 0.1 \text{ m}, \) no soot (configuration A) ; \( L = 3 \text{ m}, \) no soot (configuration B) ; \( L = 0.1 \text{ m}, f_v = 10^{-7} \text{ m}^3/\text{m}^3 \) (configuration C) and \( L = 1 \text{ m}, f_v = 10^{-6} \text{ m}^3/\text{m}^3 \) (configuration D). Relative errors greater than 5% are shaded.

| Model              | Config. A (kW/m²) | Config. B (kW/m²) | Config. C (kW/m²) | Config. D (kW/m²) |
|--------------------|--------------------|--------------------|--------------------|--------------------|
| LBL (benchmark) [13, 14] | 184.0              | 162.0              | 181.0              | 112.0              |
| WBCK-19            | 183.5 -0.3%        | 160.3 -1.0%        | 181.4 0.2%         | 115.1 2.8%         |
| SLW-1              | 183.2 -0.4%        | 157.5 -2.8%        | 180.3 -0.4%        | 113.8 1.6%         |
| WSGG-Smith         | 181.4 -1.4%        | 163.5 0.9%         | 179.2 -1.0%        | 114.0 1.8%         |
| Leckner            | 169.1 -8.1%        | 135.6 -16.3%       | 167.5 -7.5%        | 113.7 1.5%         |

4.2. Test case 2: non-homogeneous and non-isothermal mixture in a one-dimensional planar enclosure

SNB calculations from Bressloff [37] were used as benchmark solutions. The results obtained by Bressloff with the (non-gray) WSGG model using the coefficients of Truelove [45] are also given. Figure 5 shows the predicted profiles of the radiative source term. The M-shaped profiles obtained with the benchmark solutions (SNB) were well reproduced by all models. The maximum discrepancy occurred at the center and at the vicinity of the cold walls. The relative errors by comparison with SNB benchmark data are reported in Table 2. The relative error was computed as follows:

\[
\delta = \frac{\left| \nabla \cdot \mathbf{q}_{\text{SNB}} - \nabla \cdot \mathbf{q} \right|}{\left| \nabla \cdot \mathbf{q}_{\text{SNB}} \right|_{\text{max}}}
\]  

WSGG-Truelove yielded the most accurate results among the models tested (mean and maximum relative errors less than 3% and 5%, respectively). The results obtained with SLW-1 and GG (Leckner) were in very good agreement with the benchmark solution only for configuration B (mean and maximum relative errors less than 3% and 5%, respectively). The relatively good results obtained with the gray gas model on configuration B were not expected.
and might be a coincidence. For all the models tested, the non-gray behavior of soot seemed to be the main source of error (larger discrepancies with the benchmark solution were found on configuration A vs. configuration B). This aspect should be further investigated.

4.3. Test case 3: homogeneous and non-isothermal mixture in a two-dimensional rectangular enclosure

LBL calculations from Chu et al. [40] were used as benchmark solutions. The results obtained by Chu et al. [40] with SNB-CK and Goutièvre et al. [18] with SNB are also reported. The results presented below were computed on a cartesian uniform mesh composed of 200 \times 100 grid cells and 144 discrete ordinates (N_\theta = N_\phi = 6). The influence of the spatial and angular discretization on the results (radiative source term and wall radiative heat flux) was thoroughly verified [46]. Figure 6 shows the predicted wall net radiative heat flux distributions on the top and right walls of the enclosure calculated using different numbers of discrete ordinates with WSGG-Smith. The location of the maximum on the top wall slightly shifted to the right of the enclosure with finer angular discretizations. The shape of the predicted profile on the right wall was very sensitive to the angular discretization used. With a coarse angular discretization (i.e.,
Figure 5. Predicted profiles of radiative source term compared with the SNB benchmark data [37] for test case 2: (a) $f_{v0} = 10^{-7} \text{ m}^3/\text{m}^3$ (configuration A) and (b) $f_{v0} = 10^{-8} \text{ m}^3/\text{m}^3$ (configuration B).

Table 2. Relative errors by comparison with SNB benchmark data [37] for test case 2: $f_{v0} = 10^{-7} \text{ m}^3/\text{m}^3$ (configuration A) and $f_{v0} = 10^{-8} \text{ m}^3/\text{m}^3$ (configuration B). Mean and maximum relative errors greater than 5% and 10% respectively are shaded.

| Model                  | Config. A | Config. B |
|------------------------|-----------|-----------|
|                        | $\delta$ (%) | $\delta$ (%) | mean | max. | mean | max. |
| WSGG-Truelove [37]     | 3.2       | 4.9       | 2.8  | 3.6  |
| WBCK-19                | 11.6      | 15.9      | 10.6 | 23.4 |
| SLW-1                  | 6.5       | 11.3      | 2.6  | 5.1  |
| WSGG-Smith             | 10.0      | 14.9      | 7.4  | 18.0 |
| Leckner                | 11.0      | 15.6      | 4.6  | 8.0  |

less than 64 discrete ordinates, $N_{\theta} = N_{\phi} = 4$), the profile was M-shaped whereas the converged profile obtained with a fine angular discretization ($N_{\theta} = N_{\phi} = 6$) was U-shaped and rather flat at the center. Similar results were obtained with WSGG-Soufiani and WSGG-Taylor but are not reported here for the sake of brevity. Besides, using a finer mesh composed of 400 × 200 grid cells did not alter the results (not reported here).

Figure 7 shows the predicted radiative source term along the enclosure centerline in the $x$ and $y$ directions. The profiles obtained with the benchmark solutions (LBL) were well reproduced by the WSGG model. The location of the minimum value of the radiative source term along the centerline in the $x$ direction was exact and corresponded to the maximum of emission (maximum of gas temperature). The M-shaped profile along the centerline in the $y$ direction was rather well predicted by the models as well. The maximum discrepancy occurred at the center. Relative errors on the radiative source term along the enclosure centerline in the $x$ direction and $y$ direction are reported in Table 3. The main difference between the models lied in the prediction of the minimum of the radiative source term along the centerline in the $x$ direction. While WSGG-Smith was close to the benchmark data (relative error less than 6%),
Figure 6. Comparison of wall net radiative heat flux distributions on the top, (a), and right, (b), walls of the enclosure calculated using different angular discretizations for test case 3.

Both WSGG-Soufiani and WSGG-Taylor exhibited discrepancies greater than 20%.

Figure 7. Predicted profiles of radiative source term along the enclosure centerline in the $x$ direction, (a), and $y$ direction, (b), compared with the LBL benchmark data [40] for test case 3.

Results of the net wall radiative heat fluxes at the center of the top and right walls and the radiative source term at the center of the enclosure are reported in Table 4. Based on these local values, WSGG-Soufiani appeared to yield the most accurate results among the models tested (relative errors less than 5%). The SNB results of Goutière et al. are close to the benchmark data except for the predicted net radiative heat flux at the center of the top wall (relative error greater than 20%). This unexpected discrepancy might be due to differences in the spatial/angular discretizations used by the authors.
Table 3. Relative errors on the radiative source term along the centerline by comparison with the LBL benchmark data [40] for test case 3. Mean and maximum relative errors greater than 5% and 10% respectively are shaded.

| Model          | $\delta_x$ (%) | $\delta_y$ (%) |
|----------------|----------------|----------------|
|                | mean max.      | mean max.      |
| SNB-CK [40]    | 0.8 1.9        | 1.4 4.1        |
| WSGG-Smith     | 5.7 12.8       | 5.9 14.9       |
| WSGG-Soufiani  | 6.0 22.3       | 3.4 7.0        |
| WSGG-Taylor    | 5.6 21.5       | 3.7 8.8        |

Table 4. Results of the net wall radiative heat fluxes at the center of the right and top walls and the radiative source term at the center of the enclosure compared with the LBL benchmark data [40] for test case 3. Relative errors greater than 10% are shaded.

| Model          | $q$ ($kW/m^2$) | $-\nabla \cdot q$ ($kW/m^3$) |
|----------------|----------------|-----------------------------|
|                | At (1 m, 0.25 m) | At (0.5 m, 0.5 m) | At (0.5 m, 0.25 m) |
| LBL (benchmark) [40] | 13.053 | 17.881 | -233.090 |
| SNB-CK [40]    | 12.572 | -3.7%  | 17.222 | -3.7%  | -223.290 | -4.2%  |
| SNB [18]       | 12.608 | -2.9%  | 21.630 | -21.0% | -226.000 | -3.0%  |
| WSGG-Smith     | 13.509 | 3.5%   | 20.821 | 16.4%  | -259.416 | 11.3%  |
| WSGG-Soufiani  | 13.636 | 4.5%   | 18.572 | 3.9%   | -221.797 | -4.8%  |
| WSGG-Taylor    | 11.834 | -9.3%  | 17.445 | -2.4%  | -240.333 | 3.1%   |

4.4. Test case 4: homogeneous and non-isothermal mixture in a three-dimensional rectangular enclosure

SNB calculations from Liu [41] were used as benchmark solutions. The results presented below were computed on a cartesian uniform mesh composed of $40 \times 40 \times 80$ grid cells and 72 discrete ordinates ($N_\theta = N_\phi = 3$). The influence of the spatial and angular discretization on the results (radiative source term and wall radiative heat flux) was thoroughly verified. Grid independence of the results is demonstrated in Figure 8 by comparing the results of the radiative source term along the centerline ($1 \ m, 1 \ m, z$) and the wall net radiative heat flux along ($2 \ m, 1 \ m, z$) calculated using three different grids with WSGG-Smith. The effect of grid size on the results was found negligible except at the location of the minimum of the radiative source term and the maximum of the wall net radiative heat flux. Unlike test case 3, no influence of the angular discretization was found (not reported here). Similar results were obtained with WSGG-Soufiani and WSGG-Taylor but are not reported here for the sake of brevity.

Figure 9 shows the predicted of radiative source term and wall radiative heat flux. The profiles obtained with the benchmark solutions (SNB) were qualitatively reproduced by the (non-gray) WSGG model. The minimum of the radiative source term along the centerline was well captured although its intensity varied notably with the coefficients used for the predictions. The location of the peak of the net radiative heat flux was not well predicted while its magnitude depended on the coefficients used.

Relative errors by comparison with the SNB benchmark data are reported in Table 5. The relative errors were computed as follows:

$$\delta_\phi = |\Phi_{\text{SNB}} - \Phi| / |\Phi_{\text{SNB}}|_{\text{max}}$$ (11)
where $\Phi = \nabla \cdot q$ or $q$. The mean relative errors were less than 7.5% for all the models tested. The maximum relative errors were between 17% and 30%, indicating that the (non-gray) WSGG model failed to yield accurate local predictions, mostly at the location of the peak of gas temperature.

5. Conclusion
Three non-gray global gas radiation models were employed to solve radiative transfer problems in enclosures. For all the considered test cases, a benchmark solution (LBL or SNB) was used in order to compute the relative error of each model on the predicted radiative source term and the wall net radiative heat flux.
Table 5. Relative errors by comparison with the SNB benchmark data [41] for test case 4. Mean and maximum relative errors greater than 5% and 10% respectively are shaded.

| Model          | $\delta_q$ (%) | $\delta_{\nabla \cdot q}$ (%) |
|----------------|----------------|------------------------------|
|                | mean | max. | mean | max. |
| WSGG-Smith     | 2.7  | 8.0  | 7.2  | 20.3 |
| WSGG-Soufiani  | 5.4  | 7.2  | 6.1  | 29.1 |
| WSGG-Taylor    | 5.2  | 6.5  | 3.8  | 17.2 |

The following conclusions were drawn from the analysis carried out:

1. SLW-1 and WSGG yielded reasonable overall accuracy in one-dimensional planar enclosures. WBCK-19 did not prove to be more accurate than SLW-1 and WSGG on the test problems investigated.

2. WSGG failed to accurately predict local quantities in multi-dimensional enclosures. The optimal set of coefficients depended on the test case investigated and the quantity looked at to compute the relative error with the benchmark data. This was clearly not satisfactory.

3. The non-gray modeling of soot should be further investigated (e.g., the influence of the optical constant of soot on the predictions).

4. Spatial and angular independent solutions were computationally costly with the FVM implemented in the two CFD codes used for this study. This might preclude the use of WSGG-like models with a number of gray gases greater than three or four for practical industrial applications.

Further work may include: (i) additional test cases, namely mixtures with different (constant or variable) molar ratios (e.g., [13]), multi-dimensional enclosures containing soot (e.g., [39]) or non-homogeneous mixtures; (ii) accuracy assessment of the updated coefficients for WSGG and (iii) investigation of the combined use of global models and P-1 in multi-dimensional enclosures.

Acknowledgments

Dr. Luuk Thielen (CelSian Glass & Solar) prepared the special executables of the CFD code used to test the WBCK model. Professor Vladimir Solovjov (Brigham Young University) provided the tabulated data of Bressloff (SNB and WSGG-Truelove) and Denison (LBL and SLW). Professor Solovjov also provided the SLW-1 model (Maple codes).

References

[1] Taine J and Soufiani A 1999 Adv. Heat Transfer 33 295–414
[2] Howell J R, Siegel R and Meneguç M P 2010 Thermal Radiation Heat Transfer 5th ed (Boca Raton, FL: CRC Press, Inc.)
[3] Modest M 2013 Radiative Heat Transfer 3rd ed (New York: Academic Press)
[4] Modest M 2013 J. Heat Transfer 135 061801–1–12
[5] Hottel H and Sarofim A 1967 Radiative Transfer (New York: McGraw-Hill)
[6] Leckner B 1972 Combust. Flame 19 33–48
[7] Goutiere V, Charette A and Kiss L 2002 Numer. Heat Transfer, Part B 41 361–81
[8] Soufiani A and Taine J 1997 Int. J. Heat Mass Transfer 40 987–91
[9] Chu H, Liu F and Zhou H 2011 Int. J. Heat Mass Transfer 54 4736–45
[10] Edwards D 1976 Advances in Heat Transfer 12 115–93
[11] Modest M 1991 J. Heat Transfer 113 650–56
[12] Denison M and Webb B 1993 J. Heat Transfer 115 1004–11
[13] Denison M 1994 *A Spectral Line-based Weighted-sum-of-gray-gases model for arbitrary RTE solvers* Ph.D. thesis Brigham Young University

[14] Denison M and Webb B 1995 *J. Heat Transfer* **117** 359–65

[15] Ströhle J and Coelho P 2002 *Int. J. Heat Mass Transfer* **45** 2129–39

[16] Ströhle, J, Schnell, U and Hein, KRG 2003 Optimisation of correlated-k method for non-gray radiative transfer calculations in multidimensional enclosures *Proc. of Eurotherm 73 on Computational Thermal Radiation in Participating Media*, April 15-17, 2003, Mons, Belgium

[17] Ströhle J 2008 *J. Quant. Spectr. Rad. Transfer* **109** 1622–40

[18] Goutière V, Liu F and Charette A 2000 *J. Quant. Spectrosc. Radiat. Transfer* **64** 299–326

[19] Poitou D and André F 2013 *Int. J. Therm. Sci.* **64** 11–21

[20] Solovjov V, Lemonnier D and Webb B 2011 *J. Quant. Spectrosc. Radiat. Transfer* **112** 1205–12

[21] Solovjov V, Lemonnier D and Webb B 2011 *J. Heat Transfer* **133** 102701–1–9

[22] Smith T, Shen Z and Friedman J 1982 *J. Heat Transfer* **104** 602–8

[23] Soufiani A and Djavdan E 1994 *Combust. Flame* **97** 240–50

[24] Taylor P and Foster P 1975 *Int. J. Heat Mass Transfer* **18** 1331–32

[25] Rothman L, Gordon I, Barber R, Dothe H, Gamache R, Goldman A, Perevalov V, Tashkun S and Tennyson J 2010 *J. Quant. Spectr. Rad. Transfer* **111** 2139–50

[26] Kangwanpongpan T, França F H, Corrêa da Silva R, Schneider P S and Krautz H J 2012 *Int. J. Heat Mass Transfer* **55** 7419–33

[27] Dorigon L J, Duciai G, Brittes R, Cassol F, Galarça M and França F H 2013 *Int. J. Heat Mass Transfer* **64** 863–73

[28] Johansson R, Leckner B, Andersson K and Johnsson F 2011 *Combust. Flame* **158** 893–901

[29] Cassol F, Brittes R, França F H and Ezekoye O A 2014 *Int. J. Heat Mass Transfer* **79** 796–806

[30] Solovjov V P and Webb B W 2011 *J. Heat Transfer* **133** 042701–1–9

[31] Solovjov V P, Lemonnier D and Webb B W 2014 *J. Quant. Spectrosc. Radiat. Transfer* **143** 83–91

[32] Pearson J T, Webb B W, Solovjov V P and Ma J 2014 *J. Quant. Spectrosc. Radiat. Transfer* **138** 82–96

[33] Lee S and Tien C 1981 Optical constants of soot in hydrocarbon flames *Symp. (Int.) on Combustion* vol 18 pp 1159–66

[34] CelSian 2012 *GTM-X User Manual Version v4.4.6* CelSian Glass & Solar BV

[35] Ansys 2009 *ANSYS FLUENT 12.0 Theory Guide* Ansys, Inc.

[36] Solovjov V P and Webb B W 2008 *J. Quant. Spectrosc. Radiat. Transfer* **109** 245–57

[37] Bressloff N 1999 *Int. J. Heat Mass Transfer* **42** 3469–80

[38] Solovjov V and Webb B 2001 *J. Heat Transfer* **123** 450–57

[39] Demarco R, Consalvi J, Fuentes A and Melis S 2011 *Int. J. Therm. Sci.* **50** 1672–84

[40] Chu H, Liu F and Zhou H 2012 *Int. J. Therm. Sci.* **59** 66–74

[41] Liu F 1999 *J. Heat Transfer* **121** 200–3

[42] Coelho P 2002 *J. Quant. Spectr. Rad. Transfer* **74** 307–28

[43] Trivis D N 2004 *Int. J. Heat Mass Transfer* **47** 1367–82

[44] Selçuk N and Doner N 2009 *J. Quant. Spectr. Rad. Transfer* **110** 184–91

[45] Truelove J 1975 *The Zone Method for Radiative Heat Transfer Calculations* AERE-R (Heat Transfer and Fluid Flow Service, AERE)

[46] Roache P 2009 *Fundamentals of Verification and Validation* (Socorro, NM: Hermosa Publishers)