\( \alpha_s \) review (2016)

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The current world-average of the strong coupling at the Z pole mass, \( \alpha_s(m_Z^2) = 0.1181 \pm 0.0013 \), is obtained from a comparison of perturbative QCD calculations computed, at least, at next-to-next-to-leading-order accuracy, to a set of 6 groups of experimental observables: (i) lattice QCD “data”, (ii) \( \tau \) hadronic decays, (iii) proton structure functions, (iv) event shapes and jet rates in \( e^+e^- \) collisions, (v) Z boson hadronic decays, and (vi) top-quark cross sections in p-p collisions. In addition, at least 8 other \( \alpha_s \) extractions, usually with a lower level of theoretical and/or experimental accuracy today, have been proposed: pion, \( \Upsilon \), W hadronic decays; soft and hard fragmentation functions; jets cross sections in pp, e-p and \( \gamma \)-p collisions; and photon \( F_2 \) structure function in \( \gamma\gamma \) collisions. These 14 \( \alpha_s \) determinations are reviewed, and the perspectives of reduction of their present uncertainties are discussed.

1 Introduction

The strong coupling \( \alpha_s \), one of the fundamental parameters of the Standard Model, sets the scale of the strength of the strong interaction between quarks and gluons, theoretically described by Quantum Chromodynamics (QCD). Its current value at the reference Z pole mass amounts to \( \alpha_s(m_Z^2) = 0.1186 \pm 0.0013 \), with a \( \delta \alpha_s(m_Z^2)/\alpha_s(m_Z^2) \approx 1\% \) uncertainty—orders of magnitude larger than that of the gravitational (\( \delta G/G \approx 10^{-5} \)), Fermi (\( \delta G_F/G_F \approx 10^{-8} \)), and QED (\( \delta \alpha/\alpha \approx 10^{-10} \)) couplings, making \( \alpha_s \) the least precisely known of all fundamental constants in nature. Improving our knowledge of \( \alpha_s \) is a prerequisite to reduce the theoretical uncertainties in the calculations of all high-precision perturbative QCD (pQCD) observables whose cross sections or decay rates depend on higher-order powers of \( \alpha_s \), as is the case for virtually all those accessible at the LHC. Chiefly, in the Higgs sector, the \( \alpha_s \) uncertainty is currently the second major contributor (after the bottom mass) to the parametric uncertainties of its dominant \( H \to b\bar{b} \) partial decay, and it’s the leading one for the \( H \to \gamma\gamma \), g g branching fractions. The \( \alpha_s \) running impacts also our understanding of physics approaching the Planck scale, e.g. the stability of the electroweak vacuum or the scale at which the interaction couplings unify.

The latest update of the Particle-Data-Group (PDG) world-average \( \alpha_s(m_Z^2) \), obtained from a comparison of next-to-next-to-leading-order (NNLO) pQCD calculations to a set of 6 groups of experimental observables, has resulted in a factor of two increase in the \( \alpha_s \) uncertainty, compared to the previous (2014) PDG value. This fact calls for new independent approaches to determine \( \alpha_s \) from the data, with experimental and theoretical uncertainties different from those of the methods currently used, in order to reduce the overall uncertainty of the \( \alpha_s \) world-average. These proceedings provide a summary of all the \( \alpha_s \) determination methods described in detail in refs. where more complete lists of references can be found.

2 Current world \( \alpha_s(m_Z^2) \) average

The six methods used in the latest global \( \alpha_s(m_Z^2) \) extraction are shown in Fig. 1 (left, and top-right) roughly listed by increasing energy scale.
1. The comparison of NNLO pQCD predictions to computational lattice QCD “data” (Wilson loops, quark potentials, vacuum polarization,...) yields $\alpha_s(m_Z^2) = 0.1187 \pm 0.0012$, and provides the most precise $\alpha_s$ extraction today. Its $\delta\alpha_s(m_Z^2)/\alpha_s(m_Z^2) = 1\%$ uncertainty (dominated by finite lattice spacing and statistics) has, however, doubled since the previous PDG pre-average due to a new calculation of the QCD static energy $^4$ which is lower than the rest of lattice-QCD analyses. The expected improvements in computing power over the next 10 years would reduce the $\alpha_s$ uncertainty down to 0.3%. Further reduction to the $\sim 0.1\%$ level requires the computation of 4-th order pQCD corrections.

2. The ratio of hadronic to leptonic tau decays, known experimentally to within $\pm 0.23\%$ ($R_{\tau,\text{exp}} = 3.4697 \pm 0.0080$), compared to pQCD at next-to-NNLO (N$^3$LO) accuracy, yields $\alpha_s(m_{\tau}^2) = 0.1192 \pm 0.0018$, i.e. $\delta\alpha_s(m_{\tau}^2)/\alpha_s(m_{\tau}^2) = 1.5\%$. This uncertainty has slightly increased (from $\pm 1.3\%$) compared to the previous PDG revision to cover the different results obtained by various pQCD approaches (FOPT vs. CIPT, with different treatments of non-pQCD corrections)$^5$. High-statistics $\tau$ spectral functions (e.g. from B-factories, or ILC/FCC-ee in the future) and solving CIPT–FOPT discrepancies (and/or N$^4$LO calculations, within a $\sim 10$ years time scale) are needed to bring $\alpha_s$ uncertainties below $\sim 1\%$.

3. The QCD coupling has been obtained from various analyses of proton structure functions (including N$^3$LO fits of $F_2(x,Q^2)$, $F_L(x,Q^2)$, as well as global (approximately) NNLO fits of PDFs) yielding a central value lower than the rest of methods: $\alpha_s(m^2_{\text{Z}}) = 0.1156 \pm 0.0023$, with a moderate precision $\delta\alpha_s(m^2_{\text{Z}})/\alpha_s(m^2_{\text{Z}}) = 2\%$ (slightly increased from the previous $\pm 1.7\%$, driven by the spread of different theoretical extractions). Resolving the differences among fits, and/or full-NNLO global fits of DIS+hadronic data (including consistent treatment of heavy-quark masses) would yield an $\alpha_s$ extraction with $\sim 1\%$ uncertainty. Ultimate uncertainties in the $\delta\alpha_s(m^2_{\text{Z}})/\alpha_s(m^2_{\text{Z}}) \approx 0.15\%$ range require large-statistics studies at a future DIS machine (such as LHeC or FCC-eh)$^6$.

4. Combining the LEP data on $e^+e^-$ event shapes and rates (thrust, C-parameter, N-jet cross sections) with N$^2$D$^3$LO computations (matched, in some cases, with soft and collinear resummations at N$^2$LL accuracy), one obtains $\alpha_s(m^2_{\text{Z}}) = 0.1169 \pm 0.0034$. The $\delta\alpha_s(m^2_{\text{Z}})/\alpha_s(m^2_{\text{Z}}) = 2.9\%$ uncertainty is mostly driven by the span of individual extractions which use different (Monte Carlo or more analytical) approaches to correct for hadronization effects. Reduction of the non-pQCD uncertainties, e.g. through new $e^+e^-$ jet data at lower (higher) $\sqrt{s}$ for the event shapes (jet rates), plus jet cross sections with improved resummation (beyond NLL), are needed to reach $\alpha_s$ uncertainties below 1%.

5. Three closely-related $Z$ hadronic decays observables measured at LEP ($R^0_\ell = \Gamma_{\text{had}}/\Gamma_\ell$, $\sigma_0^{\text{had}} = 12\pi/m_Z \cdot \Gamma_e \Gamma_\text{had}/\Gamma_Z^2$, and $\Gamma$) compared to N$^3$LO calculations, yield $^7$ $\alpha_s(m_Z^2) = 0.1196\pm0.0030$ with $\delta\alpha_s(m_Z^2)/\alpha_s(m_Z^2) \approx 2.5\%$. Uncertainties at the permil level will require high-precision and large-statistics measurements accessible e.g. with $10^{12}$ $Z$ bosons at the FCC-ee $^8$ (and associated 5-loop calculations, with reduced parametric uncertainties).

6. Top-pair cross sections, theoretically known at NNLO+NNLL, are the first hadron collider measurements that constrain $\alpha_s$ at NNLO accuracy. From the comparison of CMS data to pQCD, one obtains $\alpha_s(m^2_W) = 0.1151 \pm 0.0028$ with a $\delta\alpha_s(m^2_W)/\alpha_s(m^2_W) = 2.5\%$ uncertainty (mostly dominated by the gluon PDF uncertainties)$^9$. Preliminary combination of all $t\bar{t}$ measurements at LHC and Tevatron increases its value to $\alpha_s(m^2_Z) = 0.1186\pm0.0033$.

The $\chi^2$-average of the unweighted values for these 6 subgroups of observables (dashed lines and shaded (yellow) bands in Fig. 1 left) is $\alpha_s(m^2_Z) = 0.1181 \pm 0.0013$, with a $\delta\alpha_s(m^2_Z)/\alpha_s(m^2_Z) = 1.1\%$ uncertainty (dotted line and grey band in Fig. 1 left, and top-right panels)$^2$. 
3 Other $\alpha_s$ extractions

There exist at least 8 other classes of observables, often computed at a lower accuracy (NLO, or approximately-NNLO, aka. NNLO*), used to determine the QCD coupling (Fig. 1 right, bottom), but not yet included in the world-average. Ordered by their energy scale, those are:

- The **pion decay factor** $F_\pi$, with $\alpha_s(m_Z^2) = 0.1174 \pm 0.0017$, although the calculation is (“optimized”) NNLO, the low scales involved challenge the validity of the pQCD approach.

- The jet-energy dependence of the **soft** (low-$z$) parton-to-hadron fragmentation functions (FF), provides $\alpha_s(m_Z^2) = 0.1205 \pm 0.0022$ at NNLO*+NNLL accuracy, with a $\sim 2\%$ uncertainty, which could be halved including full-NNLO corrections.

- $\gamma\gamma$ measurements of the **photon structure function** $F_2^\gamma(x,Q^2)$ have been used to obtain $\alpha_s(m_Z^2) = 0.1198 \pm 0.0054$ at NLO, with $\delta\alpha_s(m_Z^2)/\alpha_s(m_Z^2) \approx 4.5\%$. Extension to NNLO (and inclusion of new B-factories data) would reduce this uncertainty to $\sim 2\%$.

- The $\Upsilon$ decay ratio $R_\gamma \equiv \Gamma(\Upsilon(1S) \rightarrow \gamma X)/\Gamma(\Upsilon(1S) \rightarrow X)$ (with $X =$ light hadrons) has been computed at NLO accuracy in the NRQCD framework. From the CLEO data one obtains $\alpha_s(m_Z^2) = 0.119 \pm 0.007$, with a $\sim 6\%$, uncertainty shared equally by experimental and theoretical systematics. NNLO corrections with improved long-distance matrix elements, and more precise measurements of the $\gamma$ spectrum (and of the parton-to-photon FF) would allow for an extraction with $\delta\alpha_s(m_Z^2)/\alpha_s(m_Z^2) \approx 2\%$ in a few years from now.

- From the scaling violations of the **hard** (high-$z$) parton-to-hadron FFs one extracts $\alpha_s(m_Z^2) = 0.1176 \pm 0.0055$ at NLO, with $\sim 5\%$ uncertainties, mostly of experimental ori-
gin\textsuperscript{13}. Extension of the global FF fits at NNLO accuracy, and inclusion of new datasets (already available at B-factories) would allow reaching $\delta_a(m_Z^2)/a_s(m_Z^2) \approx 2\%$.

- The NNLO\textsuperscript{*} calculation of jet cross sections in DIS and photoproduction provides $a_s(m_Z^2) = 0.120 \pm 0.004$ with $\delta a(m_Z^2)/a_s(m_Z^2) \approx 3\%$ precision today\textsuperscript{14}. Upcoming full-NNLO analyses\textsuperscript{15} could reduce this uncertainty to the $\sim 1.5\%$ level, whereas a future DIS machine (such as LHeC or FCC-ee) would further bring it below $1\%$.

- Measurements of W hadronic decays, although computed at N\textsuperscript{3}LO, provide today a very imprecise $a_s(m_Z^2) = 0.117 \pm 0.030$ with $\pm 25\%$ uncertainty, due to the poor LEP data\textsuperscript{17}. A competitive $a_s$ extraction requires statistical samples of $10^8$ W, available at FCC-ee, which (combined with N\textsuperscript{4}LO corrections) can ultimately yield $\delta a(m_Z^2)/a_s(m_Z^2) \approx 0.1\%$.

- Various jet observables at hadron colliders (ratio of 3- to 2-jets, 3-jet mass, inclusive cross sections) have tested asymptotic freedom at TeV scales. Combining those, one obtains $a_s(m_Z^2) = 0.1179 \pm 0.0023$ at NLO accuracy, with $\delta a(m_Z^2)/a_s(m_Z^2) \approx 2\%$ dominated by theoretical uncertainties. The imminent incorporation of NNLO corrections\textsuperscript{18} and a consistent combination (including correlations) of the multiple datasets available at Tevatron and LHC, may reduce the $a_s$ uncertainties to the $1.5\%$ level in the upcoming years.

Assuming all 14 extraction methods discussed here are computed at NNLO (or above) accuracy, and provided that they yield consistent $a_s$ results, a simple weighted-average would have an uncertainty of $\delta a(m_Z^2)/a_s(m_Z^2) \approx 0.35\%$, $\sim 3$ times better than the present value. A permil-level $a_s$ uncertainty requires high-precision future $e^+e^-$ colliders with very large Z and W samples, complemented with 4th-order pQCD corrections, and improved parametric uncertainties.

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