Generalized parton distributions of hadrons with composite constituents

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A method is described for calculating the Generalized Parton Distributions (GPDs) of spin 1/2 hadrons made of composite constituents, in an Impulse Approximation framework. GPDs are obtained from the convolution between the light cone non-diagonal momentum distribution of the hadron and the GPD of the constituent. DIS structure functions and electromagnetic form factors are consistently recovered with the proposed formalism. Results are presented for the nucleon and for the $^3$He nucleus. For a nucleon assumed to be made of composite constituent quarks, the proposed scheme permits to study the so-called Efremov-Radyushkin-Brodsky-Lepage (ERBL) region, difficult to access in model calculations. Results are presented for both helicity-independent and helicity-dependent GPDs. For $^3$He, the calculation has been performed by evaluating a non-diagonal spectral function within the AV18 interaction. It turns out that a measurement of GPDs for $^3$He could shed new light on the short-range nuclear structure at the quark level.

Generalized Parton Distributions (GPDs) [1] represent one of the most relevant issues in nowadays hadronic Physics (for recent reviews, see, e.g., Ref. [2]). GPDs parameterize the long-distance dominated part of exclusive lepton Deep Inelastic Scattering (DIS) off hadrons and can be measured in Deeply Virtual Compton Scattering (DVCS), i.e. the process $eH \rightarrow e'H'\gamma$ when $Q^2 \gg m_H^2$, permits to access GPDs (here and in the following, $Q^2$ is the momentum transfer between the leptons $e$ and $e'$, and $\Delta^2$ the one between the hadrons $H$ and $H'$) so that relevant experimental DVCS programs are taking place. The issue of measuring GPDs for nuclei is also being addressed [3], following an observation firstly discussed in [4]. As a matter of facts, the knowledge of GPDs would permit the study of the short light-like distance structure of nuclei, and thus the interplay of nucleon and parton degrees of freedom in the nuclear wave function. In DIS off a nucleus with four-momentum $P_A$ and $A$ nucleons of mass $M$, this information can be accessed in the region where $Ax_{Bj} \simeq Q^2/(2M\nu) > 1$, being $x_{Bj} = Q^2/(2P_A \cdot q)$ and $\nu$ the energy transfer in the laboratory system. In this kinematical region measurements are difficult, because of the vanishing of the cross-sections. As explained in Ref. [4], the same physics can be accessed in DVCS at lower values of $x_{Bj}$, as it will be clear later [5].

In this talk, a method is reviewed for calculating the GPDs of spin 1/2 hadrons made of composite constituents, in an Impulse Approximation (IA) framework. In this scheme, GPDs are given by the convolution between the light cone non-diagonal momentum distribution of the hadron and the GPD of the constituent. Results are presented for the
Figure 1. Left panel: The helicity-independent Non-Singlet GPD $H(x, \xi, \Delta^2)$ for the
flavor $u$, at the hadronic scale $\mu_0^2 = 0.34 \text{ GeV}^2$, for $\Delta^2 = -0.15 \text{ GeV}^2$ and the allowed
values of $\xi$. Right panel: the same is shown for the helicity-dependent GPD $\tilde{H}_u(x, \xi, \Delta^2)$.

nucleon and for the $^3$He nucleus. Details for the description of the nucleon target have to
be found in Refs. [6, 7]. Results are shown here in a new kinematical region. For the $^3$He
nucleus, details can be found in Ref. [8]. Part of the discussion of the $^3$He case has been
presented also in [9] although the kinematical region presented in that paper is extended
here to higher values of the momentum transfer.

If one thinks to a spin 1/2 hadron target, with initial (final) momentum and helicity
$P(P')$ and $s(s')$, respectively, two GPDs $H_q(x, \xi, \Delta^2)$ and $E_q(x, \xi, \Delta^2)$, occur. If one
works in a system of coordinates where the photon 4-momentum, $q_\mu = (q_0, \vec{q})$, and $\bar{P} =
(P + P')/2$ are collinear along $z$, $\xi$ is the so called “skewedness”, defined by the relation
$\xi = -n \cdot \Delta/2 = -\Delta^+ / (2 P_i - x_{Bj}/(2 - x_{Bj} + O(\Delta^2/Q^2))$, where $n$ is a light-like 4-vector
satisfying the condition $n \cdot P = 1$. One should notice that the variable $\xi$ can be completely
fixed experimentally. The well known constraints of $H_q(x, \xi, \Delta^2)$ are:

i) the so called “forward” limit, $P' = P$, i.e., $\Delta^2 = \xi = 0$, where one recovers the usual
parton distribution $H_q(x, 0, 0) = q(x)$ ;

ii) the integration over $x$, giving the contribution of the quark of flavor $q$ to the Dirac
form factor (f.f.) of the target: $\int dx H_q(x, \xi, \Delta^2) = F^q(\Delta^2)$ ;

iii) the polynomiality property, involving higher moments of GPDs, according to which
the $x$-integrals of $x^n H_q$ and of $x^n E^q$ are polynomials in $\xi$ of order $n + 1$.

In Ref. [6], an IA expression for $H_q(x, \xi, \Delta^2)$ of a given hadron target $A$ has been
obtained. Assuming that the interacting parton belongs to a bound constituent $N$ of the
target with momentum $p$ and removal energy $E$, for small values of $\xi^2$ and $\Delta^2 \ll Q^2, M^2$,
it reads:
\( H_q^A(x, \xi, \Delta^2) = \sum_N \int dE \int d\vec{p} P_A^N(\vec{p}, \vec{p} + \Delta, E) \sum_{\xi'} H_N^q(x', \xi', \Delta^2). \) (1)

In the above equation, the kinetic energies of the recoiling residual system and of the recoiling target have been neglected, \( P_A^N(\vec{p}, \vec{p} + \Delta, E) \) is the one-body off-diagonal spectral function for the constituent \( N \) in the target \( A \), the quantity \( H_N^q(x', \xi', \Delta^2) \) is the GPD of the bound constituent \( N \) up to terms of order \( O(\xi^2) \), and in the equation (1) use has been made of the relations \( \xi' = -\Delta^2 / 2p^+ \), and \( x' = (\xi'/\xi)x \). Eq. (1) can be written in the form

\( H_q^A(x, \xi, \Delta^2) = \sum_{z} \frac{dz}{x} h_N^A(z, \xi, \Delta^2) H_q^N \left( \frac{x}{z}, \frac{\xi}{z}, \Delta^2 \right), \) (2)

where \( h_N^A(z, \xi, \Delta^2) = \int dE \int d\vec{p} P_A^N(\vec{p}, \vec{p} + \Delta) \delta \left( z + \xi - p^+ / \bar{P}^+ \right) \).

In Ref. [6, 7], it is discussed that Eq. (1) fulfills the constraints \( i) - iii \) listed above.

The above formalism has been applied in Ref. [6, 7] to the nucleon target. The spectral function of the composite constituent quarks has been approximated by a momentum distribution, calculated within the Isgur and Karl model [10], as shown in Ref. [11], convoluted with the GPDs of the constituent quarks themselves. The latter are modeled by using the structure functions of the constituent quark, obtained generalizing to the GPDs case the approach of [12] which is, in turn, built following the idea of Ref. [13], the double distribution representation on GPDs (see, i.e., [2, 14]), and a recently proposed phenomenological constituent quark form factor [15]. Results have been discussed in Ref. [6] for the helicity-independent GPD \( H(x, \xi, \Delta^2) \). The model has been built to be valid at the so-called hadronic scale, \( \mu_0^2 = 0.34 \text{ GeV}^2 \), and in Ref. [6] also the NLO QCD evolution of the results up to typical experimental values has been discussed and shown. In Ref. [7] everything has been extended to study the helicity-dependent GPD \( \tilde{H}(x, \xi, \Delta^2) \), getting a convolution involving helicity-dependent momentum distributions. Typical results are shown in Fig.1 for the two cases under investigation, in a kinematical scenario which extends the one discussed in Refs. [6, 7].

This phenomenological approach permits to access, in a simple and physical way, also the so-called ERBL region, difficult to study within constituent quark model calculations. Recently, another model approach has been proposed, adding a meson cloud to a light-front quark model scenario introduced in a series of previous papers, starting with Ref. [16]. Such an approach leads to convolution formulas for the GPDs and the ERBL region is also accessed through the meson cloud. In the latter framework the helicity-independent GPDs have been calculated [17].

The study of GPDs for \(^3\text{He}\) is interesting for many aspects. First of all, \(^3\text{He}\) is a well known nucleus, for which realistic studies are possible, so that conventional effects can be calculated and the exotic ones can be distinguished. Besides, \(^3\text{He}\) is widely used as an effective polarized free neutron target [18] and it will be the first candidate for experiments aimed at the study of GPDs of the free neutron, to unveil its angular momentum content.

In what follows, the results of an impulse approximation (IA) calculation [8] of the unpolarized GPD \( H_q^3 \) for the quark of flavor \( q \) of \(^3\text{He}\) will be reviewed. A convolution
formula is discussed and evaluated using a realistic non-diagonal spectral function, so that Fermi motion and binding effects are rigorously estimated. The proposed scheme is valid for $\Delta^2 \ll Q^2, M^2$ and despite of this it permits to calculate GPDs in the kinematical range relevant to the coherent channel of DVCS off $^3$He. In fact, the latter channel is the most interesting one for its theoretical implications, but it can be hardly seen at large $\Delta^2$, due to the vanishing cross section. The main result of this investigation is not the size and shape of the obtained $H_q^3$ for $^3$He, but the size and nature of nuclear effects on it. This permits to test, for the $^3$He target, the accuracy of prescriptions which have been proposed to estimate nuclear GPDs [5].

$H_q^A(x, \xi, \Delta^2)$ for $A=^3$He, Eq. (1), has been evaluated in the nuclear Breit Frame. The non-diagonal spectral function appearing in Eq. (1) has been calculated along the lines of Ref. [19], by means of realistic wave functions evaluated using the AV18 interaction and taking into account the Coulomb repulsion between the protons. The one-body off-diagonal spectral function for the nucleon $N$ in $^3$He reads

$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) = \frac{1}{(2\pi)^3} \frac{1}{2} \sum_M \sum_{R,s} \langle \vec{P}^M | (\vec{P} - \vec{p}) S_{R,s} (\vec{p} + \vec{\Delta}) s \rangle \langle (\vec{P} - \vec{p}) S_{R,s} | \vec{P}^M \rangle \times \\
\delta(E - E_{\text{min}} - E^*_R).$$

(3)

The delta function in Eq (3) defines $E$, the so called removal energy, in terms of $E_{\text{min}} = |E_{3\text{He}}| - |E_{2\text{H}}| = 5.5$ MeV and $E^*_R$, the excitation energy of the two-body recoiling system.
The main quantity appearing in the definition Eq. (3) is the overlap integral
\[
\langle \vec{P} M | \vec{P} R S, \vec{P} S \rangle = \int d\vec{y} \exp(i \vec{p} \cdot \vec{y}) \langle x, \Psi_S^R(\vec{x}) | \Psi_S(\vec{x}, \vec{y}) \rangle ,
\]
(4)
between the eigenfunction \( \Psi_S^M \) of the ground state of \(^3\)He, with eigenvalue \( E_{3\text{He}} \) and third component of the total angular momentum \( M \), and the eigenfunction \( \Psi_S^R \), with eigenvalue \( E_R = E_2 + E_R^* \) of the state \( R \) of the intrinsic Hamiltonian pertaining to the system of two interacting nucleons. Since the set of the states \( R \) also includes continuum states of the recoiling system, the summation over \( R \) involves the deuteron channel and the integral over the continuum states.

The other ingredient in Eq. (1), i.e. the nucleon GPD \( H^N_q \), has been modelled in agreement with the Double Distribution representation [14]. In this model, whose details are summarized in Ref. [8], the \( \Delta^2 \)-dependence of \( H^N_q \) is given by \( F_q(\Delta^2) \), i.e. the contribution of the quark of flavor \( q \) to the nucleon form factor. Now the numerical results will be discussed. If one considers the forward limit of the ratio
\[
R_q(x, \xi, \Delta^2) = H^3_q(x, \xi, \Delta^2)/(2H^p_q(x, \xi, \Delta^2) + H^n_q(x, \xi, \Delta^2)) ,
\]
(5)
where the denominator clearly represents the distribution of the quarks of flavor \( q \) in \(^3\)He if nuclear effects are completely disregarded, the behavior which is found, shown in Ref. [8], is typically EMC–like, so that, in the forward limit, well-known results are recovered. In Ref. [8] it is also shown that the \( x \) integral of the nuclear GPD gives a good description of ff data of \(^3\)He, in the relevant kinematical region, \(-\Delta^2 \leq 0.25 \text{ GeV}^2\). Let us now discuss the nuclear effects. The full result for the GPD \( H^q_3 \), Eq. (1), will be compared with a prescription based on the assumptions that nuclear effects are neglected and the \( \Delta^2 \) dependence can be described by the f.f. of \(^3\)He:
\[
H^3_{q}(x, \xi, \Delta^2) = 2H^3_p(x, \xi, \Delta^2) + H^3_n(x, \xi, \Delta^2) ,
\]
(6)
where \( H^3_{q,N}(x, \xi, \Delta^2) = \bar{H}^N_q(x, \xi)F^3_q(\Delta^2) \) represents the effective GPD corresponding to the flavour \( q \) of the bound nucleon \( N = n, p \) in \(^3\)He. Its \( x \) and \( \xi \) dependences, given by the function \( \bar{H}^N_q(x, \xi) \), is the same of the GPD of the free nucleon \( N \), while its \( \Delta^2 \) dependence is governed by the contribution of the quark of flavor \( q \) to the \(^3\)He f.f., \( F^3_q(\Delta^2) \).

The effect of Fermi motion and binding can be emphasized showing the ratio
\[
R^{(0)}_q(x, \xi, \Delta^2) = H^3_q(x, \xi, \Delta^2)/H^{3(0)}_q(x, \xi, \Delta^2)
\]
(7)
i.e. the ratio of the full result, Eq. (1), to the approximation Eq. (6). The latter is evaluated by means of the model nucleon GPDs used as input in the calculation, and taking \( F^3_q(\Delta^2) = \frac{1}{3} F^3_q(\Delta^2) \), \( F^3_d(\Delta^2) = -\frac{1}{3} F^3_d(\Delta^2) \). The choice of calculating the ratio Eq. (7) to show nuclear effects is a very natural one. As a matter of facts, the forward limit of the ratio Eq. (7) is the same of the ratio Eq. (5), yielding the EMC-like ratio for the parton distribution \( q \) and, if \(^3\)He were made of free nucleons at rest, the ratio Eq. (7) would be one. This latter fact can be immediately realized by observing that the prescription Eq. (6) is exactly obtained by placing \( z = 1 \), i.e. imposing no Fermi motion effects and no convolution, into Eq. (2). Typical results are shown in Fig. 2, where the ratio Eq. (7) is shown for \( \Delta^2 = -0.25 \text{ GeV}^2 \) as a function of \( x_3 = 3x \), for three different
values of $\xi_3 = 3\xi$, for the flavors $u$ and $d$. Some general trends of the results are apparent: i) nuclear effects, for $x_3 \leq 0.7$, are as large as 15% at most; ii) Fermi motion and binding have their main effect for $x_3 \leq 0.3$, at variance with what happens in the forward limit; iii) nuclear effects increase with increasing $\xi$ and $\Delta^2$, for $x_3 \leq 0.3$; iv) nuclear effects for the $d$ flavor are larger than for the $u$ flavor. The behaviour described above is discussed and explained in Ref. [8]. In general, it is found that the realistic calculation yields a rather different result with respect to a simple parameterizations of nuclear GPDs, as some of the ones proposed in Ref. [5]. In Ref. [9], where a part of the material discussed here has been presented for the $^3$He target, it is shown that nuclear effects are found to depend also on the choice of the NN potential, at variance with what happens in the forward case. The study of nuclear GPDs turns out therefore to be very fruitful, being able to detect relevant details of nuclear structure at short light-cone distances. The obtained $^3$He GPDs are being used to estimate cross-sections in order to establish the feasibility of experiments. A natural extension of the proposed formalism and analysis is the investigation of hadron helicity-flip GPDs, which allows to study the possibility of unveiling the quark orbital angular momentum contribution to the free neutron spin.

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