Holographic Glueball Decay*

FREDERIC BRÜNNER, DENIS PARGANLIJA AND ANTON REBHAH

Institut für Theoretische Physik, Technische Universität Wien, Wiedner Hauptstraße 8-10, 1040 Vienna, Austria

We announce new results on glueball decay rates in the Sakai-Sugimoto model, a realization of holographic QCD from first principles that has only one coupling constant and an overall mass scale as free parameters. We extend a previous investigation by Hashimoto, Tan, and Terashima who have considered the lowest scalar glueball which arises from a somewhat exotic polarization of supergravity modes and whose mass is uncomfortably small in comparison with lattice results. On the other hand, the scalar glueball dual to the dilaton turns out to have a mass of about twice the mass of the rho meson (1487 MeV), very close to the scalar meson $f_0(1500)$ that is frequently interpreted as predominantly glue. Calculating the decay rate into two pions we find a surprisingly good agreement with experimental data for the $f_0(1500)$. We have also obtained decay widths for tensor and excited scalar glueballs, indicating universal narrowness.

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1. Holographic Glueballs

Glueballs, color-neutral bound states of gluons, are a theoretical prediction of QCD with reliable quantitative results on the mass spectrum available from lattice simulations, in particular for pure Yang-Mills theory [1, 2]. Experimental evidence for the existence of glueballs is, however, still elusive [3, 4], because glueballs are expected to mix with isoscalar quark-antiquark bound states that have identical quantum numbers [5]. In order to identify the lightest (scalar) glueball, whose mass is expected to lie between 1 GeV and 2 GeV, additional information such as decay rates (as opposed to decay constants) would be needed from first-principles approaches, but traditional methods like lattice QCD or chiral perturbation theory lack the ability to perform such calculations.

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A new, promising direction for dealing with strongly coupled gauge theories and thus a possible avenue to a theoretical description of glueballs is given by so-called holographic QCD, which is not restricted to Euclidean spacetime signature. It is based on (or, in phenomenological bottom-up models, motivated by) the AdS/CFT correspondence, a conjectured exact duality between certain four-dimensional quantum field theories and higher-dimensional superstring theories [6, 7], which in the limit of infinite number of colors and 't Hooft-coupling become accessible in the form of weakly coupled supergravity.

Here we shall be interested in so-called top-down models that have a well-defined superstring theory as their basis and which correspondingly have minimal freedom to adjust parameters. A top-down model of low-energy nonsupersymmetric QCD was pioneered by Witten [8], who has formulated a holographic dual of a five-dimensional super-Yang-Mills theory compactified on a circle of radius $R = 1/M_{\text{KK}}$, which breaks supersymmetry and leaves only gauge bosons as low-energy degrees of freedom. In the limit of infinite Kaluza-Klein mass $M_{\text{KK}}$ one obtains pure Yang-Mills theory in four dimensions, however calculations using the supergravity approximation are only possible at finite $M_{\text{KK}}$ and large coupling. This theory allows for a dual description of glueballs, which correspond holographically to fluctuations in the background geometry [9, 10]. A full analysis of these modes was given in [11] and their spectrum was derived, which bears a striking resemblance to the spectrum obtained in lattice gauge theory when one of the glueball states, e.g. the lowest tensor glueball, is matched to the lattice result.

2. Interaction with chiral quarks

A possibility to add chiral quarks to the Witten model described above was found by Sakai and Sugimoto [12, 13] in the form of pairs of flavor D8-D8-branes intersecting the compactification circle at antipodal points. In the Sakai-Sugimoto model chiral symmetry breaking (as well as symmetry restoration at high temperature) has a simple geometric interpretation. It features massless pions and massive vector and axial-vector mesons corresponding to gauge fields on the D8-branes described by a Dirac-Born-Infeld action, which, after integrating out a four-dimensional sphere and expanding to quadratic order in the field strength, is given by

$$S_{\text{DBI}} = \frac{g_{\text{YM}}^2 N_c^2}{216 \pi^3} \int d^4 x \ d z \ Tr \left[ \frac{1}{2} K^{-1/3} F_{\mu \nu}^2 + K F_{\mu z}^2 \right]. \quad (1)$$

Here $z$ is the holographic radial coordinate with $K = 1 + z^2$, $g_{\text{YM}}$ is the four-dimensional Yang-Mills coupling at the scale $M_{\text{KK}}$, and $N_c$ the number of colors. The pions and vector mesons are contained in the nonabelian flavor...
gauge field on the D8-branes, $A_\mu = \psi_1(z)\rho_\mu(x^\nu)$ and $A_z = \phi_0(z)\pi(x^\nu)$, where $\phi_0(z) \propto 1/K$, and $\psi_1(z)$ is the lowest fluctuation mode along the holographic direction. Matching the mass of the rho meson and the pion decay constant $f_\pi$ to their experimental values fixes the only free parameters $g_{\text{YM}}$ and $M_{\text{KK}}$ to

$$\lambda \equiv g_{\text{YM}}^2N_c \approx 16.63, \quad M_{\text{KK}} \approx 949 \text{ MeV}$$

with $N_c = 3$. Doing so reproduces quite nicely the observed spectrum of the next heavier vector and axial-vector mesons in QCD (while recent lattice simulations and extrapolations to the chiral limit and large-$N_c$ show stronger deviations [14]).

The effective Lagrangian for the mesons then also allows one to calculate the decay rate of a rho meson into two pions with the result

$$\frac{\Gamma_{\rho \rightarrow \pi\pi}}{m_\rho} = \frac{7.659}{g_{\text{YM}}^2N_c^2} = 0.1535$$

which is remarkably close to the experimental value $\Gamma_{\rho \rightarrow \pi\pi}/m_\rho \approx 0.19$ [15]. (This decay rate was calculated already in [16], but their final numerical result differs from ours by a factor of 2 due to the error mentioned in footnote [1].)

This result is quite encouraging to also calculate the decay rates of glueballs into pions and other mesons within the Sakai-Sugimoto model. This was already carried out by Hashimoto, Tan and Terashima [16] for the lowest lying glueball in the spectrum obtained in [11] which was compared with the $f_0(1500)$ isoscalar meson that is frequently interpreted as being predominantly glue [17, 18, 19] (for alternative scenarios see e.g. [3, 4, 20]).

However, with the overall mass scale being fixed by the Sakai-Sugimoto model after matching the mass of the rho meson, the lowest scalar glueball mode turns out to be much too light for this identification – at 855 MeV it is only 10% heavier than the rho meson. Of course, the Sakai-Sugimoto model can a priori not be expected to be quantitatively accurate to any degree, but other mass ratios typically come out much better. The resolution of this discrepancy may well be in corrections beyond the leading terms of the supergravity approximation. However, the next scalar mode in the supergravity spectrum would fit almost perfectly given its mass $M \approx 1.567M_{\text{KK}} \approx 1487$

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1 In the original and published version of Ref. [12, 13] an error in the normalization of the D8-brane action for the multi-flavor case led to a different value of $\lambda \approx 8.3$ that was corrected later in the e-print version. Unfortunately, this error has not been corrected in the paper by Hashimoto, Tan and Terashima [16] on glueball decay. Corrected results for the decay rates discussed therein will be presented in Ref. [25].

2 Modifications of the Sakai-Sugimoto model corresponding to a non-maximal separation of D8 and D8 branes would not help, but only aggravate this problem.
MeV (bearing in mind that various $f_0$ states below 2 GeV are considered as lowest scalar glueball candidates). This scalar glueball is coming from the lowest dilaton mode, which in bottom-up AdS/QCD models usually corresponds to the lowest-lying glueball [21, 22, 23]. In fact, in the context of the analogous problem in QCD obtained from a supersymmetry-breaking circle compactification of AdS$_5 \times$ S$^5$, it has been argued that the lowest mode in the spectrum obtained in [11], which is associated to an “exotic” [10] polarization of gravitational modes, should be discarded after all [24]. In the following we shall therefore consider the dilatonic holographic glueball in the Sakai-Sugimoto model and calculate its decay rate.

The ten-dimensional action for the dilaton $\Phi$ corresponding to a scalar glueball is given by

$$S_{\text{Dil}} \propto \int d^{10}x \sqrt{-g} e^{-2\Phi} \left[ R(g_{ab}) + 4 \partial_\mu \Phi \partial^\mu \Phi \right],$$

(4)

where $g_{ab}$ is the metric of the Witten model and $R(g_{ab})$ the Ricci scalar. The equations of motion corresponding to this action allow one to determine the spectrum. (This was carried out in [11] in an 11-dimensional supergravity setting, from which the dilaton mode follows upon dimensional reduction.)

Combining the dilaton fluctuations about the (nontrivial) dilaton background, $\Phi = \Phi_0(z) + \phi$, with the DBI action for pions and mesons, we arrive at an effective four-dimensional interaction Lagrangian. If we assume the dilaton to be of the form $\phi = H(z)G(x^{\nu})$, after integrating out the holographic coordinate the coupling of a single scalar glueball to two pions is given by

$$S_{G \pi \pi} = \text{Tr} \int d^4x \frac{1}{2} \tilde{c}_1 \partial_\mu \pi \partial^\mu \pi G,$$

(5)

where the constant $\tilde{c}_1$ can be numerically determined as

$$\tilde{c}_1 = \frac{14.92}{g_{YM} N_c^{3/2} M_{KK}}$$

(6)

for the lightest dilaton mode (full details will be given in [25]).

With these ingredients, it is straightforward to determine the corresponding decay rate. It is given by

$$\frac{\Gamma_{G \to \pi \pi}}{M} = \frac{3|\tilde{c}_1|^2 M^2}{128\pi} = \frac{4.076}{g_{YM}^2 N_c^3} = 0.027.$$

(7)

Note that compared to the decay rate of the rho meson evaluated above this is parametrically suppressed by an additional factor $1/N_c$. 

The experimental value for the decay of the $f_0(1500)$ isoscalar into two pions has been determined as $\Gamma_{f_0(1500)\rightarrow\pi\pi}/M_{f_0(1500)} = 0.025(3)$ \cite{15}, which matches the holographic result surprisingly well. This nearly perfect agreement is of course somewhat fortuitous. Let us consider one alternative to fixing the coupling constant in the Sakai-Sugimoto model: instead of matching the pion decay constant $f_\pi$ we could also choose to match the rho meson width obtained in Eq. (3), leading to a slightly higher width for the dilatonic glueball, to wit, $\Gamma_{G\rightarrow\pi\pi}/M = 0.034$. This is still sufficiently close to the observed width of the $f_0(1500)$ to be taken as very interesting. It certainly does encourage deeper investigations of glueball properties in the almost parameter-free Sakai-Sugimoto model.

We will further elaborate on glueball decay rates in the Sakai-Sugimoto model in \cite{25}, discussing systematically all of the lowest-lying glueballs and also the decay into four pions. As an apparently generic feature we have found narrow widths also for the tensor glueball and the excited scalar glueballs arising from dilaton modes.

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Erratum

The effective Lagrange density for the coupling of the (predominantly) dilatonic glueball given in Eq. (5) is incomplete, because it did not take into account all metric perturbations that are induced by the dilaton perturbation $\phi$. Eq. (5) should instead read

$$S_{G\pi\pi} = \text{Tr} \int d^4x \frac{1}{2} \tilde{c}_1 \partial_\mu \pi \partial_\nu \pi \left( \eta^{\mu\nu} - \frac{\partial^\mu \partial^\nu}{M^2} \right) G,$$

with $\tilde{c}_1$ now given by

$$\tilde{c}_1 = \frac{17.23}{g_{YM} N_c^{3/2} M_{KK}}.$$

The result for the decay width changes to

$$\frac{\Gamma_{G\rightarrow \pi\pi}}{M} = \frac{3|\tilde{c}_1|^2 M^2}{512\pi} = \frac{1.359}{g_{YM}^2 N_c^3} = 0.009.$$

Alternatively fixing the 't Hooft coupling such that the rho meson width is matched to experimental data gives $\Gamma_{G\rightarrow \pi\pi}/M = 0.011$. Both results are thus below the experimental value of 0.025(3) for the decay of the $f_0(1500)$ isoscalar into two pions. Given that the larger decay width of the $f_0(1500)$ may be due to admixture of less narrow $q\bar{q}$ states, we still take this as an encouraging result of our attempt to describe glueball properties through the almost parameter-free Sakai-Sugimoto model.\(^3\)

\(^3\) In this context we note that the recent paper by S. Janowski, F. Giacosa, and D. Rischke, arXiv:1408.4921, which extends Ref. [19] to the three-flavor case, no longer identifies the $f_0(1500)$ isoscalar as a predominantly glueball state, but instead favors the $f_0(1710)$, which has a smaller decay width into two pions.