Construction of Mathematical Model between HIV-AIDS and Lesbian, Gay, Bisexual, and Transgender (LGBT) Transmission in a Population

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Abstract. Human Immunodeficiency Virus (HIV) is a virus that attacks and damages the immune system of the human body so that it is easily attacked by various diseases. A collection of diseases that attack the body is called AIDS. HIV-AIDS are very contagious and deadly. Until now, there is no medicine, serum, vaccine that can cure sufferers of the HIV. HIV-AIDS infection is transmitted through 3 ways: vertically (from mother to child), sexual relations (homosexual, heterosexual, and transsexual), and horizontally through inter-blood contact (sharing of contaminated needles, transfusion of contaminated blood, etc.). Sexual intercourse is the biggest cause of transmission. This research discusses construction of mathematical model of HIV-AIDS and LGBT transmission based on the interaction between infected and susceptible individuals in a population.

1. Introduction
HIV is a virus that attacks and damages the human immune system so that humans are not able to withstand various diseases. Collection of symptoms of diseases that attack the human body is called AIDS. Humans who have reached the stage of AIDS are called people with HIV-AIDS. HIV-AIDS is very contagious and deadly. Until now, there has been no drug, serum, vaccine that can cure sufferers of the HIV. HIV-AIDS infection is transmitted through 3 ways namely vertically (from mother to child), sexual relations (homosexual, heterosexual, and transsexual), and horizontally through blood contact (sharing of contaminated needles, transfusion of contaminated blood, etc.).

Among the 3 causes of HIV-AIDS transmission, sexual intercourse is the biggest cause of transmission. West Sumatra Province in 2002-2016 found 1,831 cases of HIV and 1506 cases of AIDS. Most of the 55% is caused by heterosexual relations, 35% is caused by the Lesbian, Gay, Bisexual, and Transgender (LGBT) groups, and the remaining 12% due to other things [1]. By looking at the total number of individuals in each group, it is clear that the LGBT group is the group most at risk of contracting HIV-AIDS (physical problems). This is caused by their unhealthy sexual lifestyle (changing partners) which triggers HIV-AIDS transmission. Not only physical problems that are owned by LGBT groups but also have problems with their mental health. Researchers call it "syndemic", which two health conditions that occur together in a population.

In this study, a study was conducted to determine the relationship between increasing the number of LGBT with the number of HIV-AIDS cases in a population. One area of science that plays...
a role and helps in efforts to solve these problems is the mathematical model. Mathematical models can be used to represent and explain the physical systems of a problem into mathematical symbols.

Mathematical modeling of HIV-AIDS has been done quite a lot by researchers, including those conducted by [2, 3, and 4]. Mathematical models [2 and 4] divide the population into 3 groups Susceptible (S), Infected (I), and AIDS Cases (A). In model [2] it is assumed that sub-population I will increase due to the influence of transmission rate of sub-population I to sub-population S. Model [4] is a modification of model [2] by increasing transmission rates of individuals in sub-population A of individuals in the S sub-population. In model [3] consider the presence of gay groups infected with HIV-AIDS, so that the model consists of 5 sub-populations consisting of Susceptible groups other than gay (S), Infected in addition to gay (I), AIDS Cases other than gay (A), gay Susceptible (Sg), and Infected gay (Ig).

However, according to the Ministry of Health of the Republic of Indonesia (2019), stages from HIV infection to AIDS, there are stages without symptoms, with symptoms, and declared AIDS. At the asymptomatic stage, a person is infected with HIV, but looks healthy because he is still able to move like a healthy human being. Even though there are no symptoms, if someone has been infected with HIV, then he can pass it on to others. If symptoms of prolonged fever appear, weight loss, diarrhea continues for no apparent reason, coughing, and shortness of breath more than one month continuously, itchy skin and bluish red spots appear then this has entered the stage with symptoms. These symptoms indicate there has been damage to the immune system. The last stage has already been declared AIDS. At this stage the body's immunity has decreased greatly, so it has various diseases, such as pneumonia (tuberculosis), inflammation of the fungus in the mouth and esophagus, nerve disorders (toxoplasmosis), skin cancer, intestinal infections, and other infections.

In addition, HIV-AIDS transmission is not only in the gay group but also in groups other than gay (lesbian, bisexual, and transgender) who are called LGBT. Therefore, in this study a model was developed about the spread of HIV-AIDS by modifying the model [2, 3, and 4] by considering the stage of HIV becoming AIDS based on the Ministry of Health of the Republic of Indonesia (2019) and the LGBT community.

2. Discussion
In this study, the population is divided into two: straight community and LGBT community. Furthermore, each population has an individual infected with HIV. Based on the ministry of health of the Republic of Indonesia, there is a stage of change from individuals infected with HIV to AIDS, so that each population is divided into 4 groups. For the straight community consists of susceptible human group to HIV-AIDS (S1), groups of people who have been infected with HIV but have no symptoms and can transmit the HIV virus to others (I1), groups of people infected with HIV with these symptoms and symptoms indicate there has been damage to the immune system (P1), and groups of people who have been declared infected with HIV-AIDS, in this condition the immune system has greatly decreased so that it has various diseases (A1). For the LGBT community, it was divided into 4 groups: the susceptible LGBT group to HIV-AIDS (S2), the LGBT group who had been infected with HIV but had no symptoms and were able to transmit the HIV virus to others (I2), the LGBT group who was infected with HIV with symptoms and shows there has been damage to the immune system (P2), and the LGBT group that has been declared infected with HIV-AIDS which at this stage the immune system has greatly decreased so that it has various diseases (A2).

It is assumed that transmission of HIV-AIDS in both populations is caused by direct contact (sexual relations) between susceptible individuals and HIV-infected individuals in the straight community and bisexual LGBT community that marked by dotted lines; the increase of the straight susceptible community comes from individuals that born from susceptible group in total populations and individuals that born from other groups are ignored; the increase in susceptible LGBT community comes from susceptible straight community who are recruited by the LGBT community due to various factors such as environmental, family, and genetic factors; there is the deaths of disease complications in the AIDS group; and some of infective groups join the group with symptoms depending on the viral
counts, with a rate $\sigma_k \gamma_k$ where $\gamma_k$ is the rate of movement from asymptomatic infection group and $\sigma_k$ is the fraction of $\gamma_k$ joining the infection group with symptoms and $k = 1, 2$.

In addition, the parameters used in this study are given in Table 1 below.

| Parameters | Description |
|------------|-------------|
| $B_1$      | Recruitment rate into straight susceptible community |
| $B_2$      | Recruitment rate into susceptible LGBT community |
| $\beta_1$  | Spread rate from infected group to straight susceptible community via sexual contact |
| $\beta_2$  | Spread rate from infected group to susceptible LGBT community via sexual contact |
| $c_1$      | The average number of sexual partners from $S_1$ to $I_1$ and $I_2$ per unit time |
| $c_2$      | The average number of sexual partners from $S_2$ to $I_1$ and $I_2$ per unit time |
| $\sigma_1$ | The fraction of $\gamma_1$ joining the infected group with symptoms in straight community |
| $\sigma_2$ | The fraction of $\gamma_2$ joining the infected LGBT community with symptoms |
| $\gamma_1$ | Transition rate from infected group without symptoms into straight infected community with symptoms in straight community |
| $\gamma_2$ | Transition rate from infected LGBT community without symptoms into infected LGBT community with symptoms |
| $\alpha_1$ | Transition rate from straight infected community with symptoms into straight infected community with HIV-AIDS complication |
| $\alpha_2$ | Transition rate from infected LGBT community with symptoms into infected LGBT community with HIV-AIDS complication |
| $\mu$      | Natural death rate |
| $\delta$   | Death rate caused by AIDS ($\delta > \mu$) |

Based on the description above, a mathematical model of HIV-AIDS and LGBT transmission is then constructed. An incoming arrow means addition to a population group that is marked with a positive sign in the model, and an arrow out means a reduction in a population group that is marked by a negative sign in the model. For more details presented in the flow chart as follows.

**Figure 1.** Flow diagram of the model

Furthermore, the mathematical model of HIV virus transmission in straight and LGBT community is given by the following system of non linear ordinary differential equations:
\[
\begin{align*}
\frac{dS_1}{dt} &= B_1 N - \beta_1 c_1 \left( \frac{s_1 l_1}{N} + \frac{s_1 l_2}{N} \right) - \mu S_1 \\
\frac{dI_1}{dt} &= \beta_1 c_1 \left( \frac{s_1 l_1}{N} + \frac{s_1 l_2}{N} \right) - \mu I_1 - \sigma_1 y_1 I_1 - (1 - \sigma_1) y_1 I_1 \\
\frac{dP_1}{dt} &= \sigma_1 y_1 I_1 - \mu P_1 - \alpha_1 P_1 \\
\frac{dA_1}{dt} &= \sigma_1 y_1 I_1 - (\mu + \delta) A_1 \\
\frac{dS_2}{dt} &= B_2 N - \beta_2 c_2 \left( \frac{s_2 l_2}{N} + \frac{s_2 l_1}{N} \right) - \mu S_2 \\
\frac{dI_2}{dt} &= \beta_2 c_2 \left( \frac{s_2 l_2}{N} + \frac{s_2 l_1}{N} \right) - \mu I_2 - \sigma_2 y_2 I_2 - (1 - \sigma_2) y_2 I_2 \\
\frac{dP_2}{dt} &= \sigma_2 y_2 I_2 - \mu P_2 - \alpha_2 P_2 \\
\frac{dA_2}{dt} &= \sigma_2 y_2 I_2 - (\mu + \delta) A_2
\end{align*}
\] (1)

The initial conditions are taken as
\[
S_1(0) = S_0, I_1(0) = I_0, P_1(0) = P_0, A_1(0) = A_0, \quad \text{where } k = 1,2.
\]

The total population \(N\) at any time \(t\) is given by
\[
N(t) = S_k(t) + I_k(t) + P_k(t) + A_k(t)
\]
where \(k = 1,2\). This gives
\[
\frac{dN}{dt} = (B_1 + B_2)N - \mu N - \delta(A_1 + A_2)
\]
Let \(B_1 + B_2 = B\) then
\[
\frac{dN}{dt} = (B - \mu)N - \delta(A_1 + A_2)
\]

We note that in the absence of the diseases and infective: the total population size \(N\) is stationary for \(B = \mu\), declines for \(B < \mu\), and grows exponentially for \(B > \mu\). So, we shall assume that natural mortality rate \(\mu\) will be a function of state variable. To simplify the model, the variables in model can be normalized by setting \(s_k = \frac{s_k}{N}, i_k = \frac{i_k}{N}, p_k = \frac{p_k}{N}, a_k = \frac{a_k}{N}\). The normalized of system (1) become to be system (2) below
\[
\begin{align*}
\frac{dS_k}{dt} &= B_k - \beta_k c_k s_k (i_1 + i_2) - \mu s_k \\
\frac{dI_k}{dt} &= \beta_k c_k s_k (i_1 + i_2) - \mu i_k - \gamma_k i_k \\
\frac{dP_k}{dt} &= \gamma_k i_k - (\mu + \alpha_k) P_k
\end{align*}
\] (2)
\[
\frac{da_k}{dt} = a_k p_k + (1 - \sigma_k) i_k - (\mu + \delta) a_k
\]

where

\[s_k(t) + i_k(t) + p_k(t) + a_k(t) = 1, i = 1,2\]

and

\[s_k(t) > 0, \quad i_k(t) \geq 0, \quad p_k(t) \geq 0, \quad a_k(t) \geq 0, \quad \forall t \geq 0\]

3. Conclusion

The initial research obtained a mathematical model of HIV virus spread in a population. Population is consists of two populations. They are straight and LGBT community. Each of community divides into 4 groups. First, susceptible group, described as a group who might be infected by HIV virus via sexual contact with infected person. Second, infected group but have no symptoms, described as a group who got infection by HIV, but they don’t realize it and they can transmit the virus to others. Third, infected group with symptoms, described symptoms indicate there has been damage to the immune system. Fourth, infected group with HIV-AIDS complication and in this condition the immune system has greatly decreased so that it has various diseases like pneumonia (tuberculosis), inflammation of the fungus in the mouth and esophagus, nerve disorders (toxoplasmosis), skin cancer, intestinal infections, and other infections.

References

[1] Alfitri, Khaterina Welong, Yulfira Media, dan Wawan Wahyudi. 2018. Laporan Penelitian Studi Pemetaan Perilaku Lesbian, Gay, Bisexual, dan Transgender (LGBT) di Provinsi Sumatera Barat. Pemerintah provinsi SUMBAR Badan Penelitian dan Pengembangan.

[2] F. Baryarama J.Y.T. Mugisha dan L. S. Luboobi. 2006. Mathematical Model for HIV-AIDS with Complacency in a Population with Declining Prevalence. Computational and Mathematics Methods in Medicine, Vol.7, No.1, March 2006, hal 27-35. Taylor and Francis Group.

[3] R. Setiawaty, R. Ratianingsih, A. I. Jaya. 2013. Analisis Kestabilan pada Model Penyebaran HIV AIDS di Kota Palu. JIMT Vol 10, No 1 Juni 2013 (hal 74-82), ISSN: 2450-766x

[4] Dwi Haryanto, Nilamsari Kusumastuti, Bayu Prihandono. 2015. Pemodelan Matematika dan Analisis Kestabilan Model pada penyebaran HIV-AIDS. Buletin Ilmiah Matematika Statistika dan Terapannya, vol 04, no. 02 (2015), hal 101-110

[5] Abdallah S. Waziri, Estomih S. Massawe, and Oluwole Daniel Makinde. 2012. Mathematical Modelling of HIV/AIDS Dynamics with Treatment and Vertical Transmission. Applied Mathematics 2012. 2(3): 77-89, DOI: 10.5923/j.am.20120203.06

[6] Maimunah and Dipo Aldila. 2018. Mathematical Model for HIV Spreads Control Program with ART Treatment. IOP Conf. Series: Journal of Physics: Conf. Series 974 (2018) 012035, doi: 10.1088/1742-6596/974/1/012035

[7] R Sriningsih, M Subhan, M L Nasution. 2017. Mathematical Model of Cytomegalovirus (CMV) Disease, IOP Conf. Series: Materials Science and Engineering 335 (2018) 011001 doi:10.1088/1757-899X/335/1/011001, hal 325-328.

[8] http://www.kemkes.go.id/development/site/depkes/pdf.php?id=1-17042500008