“Prehawking” radiation

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Using the 2-D quantum energy momentum tensor expectation value near a black hole, the value near a collapsing shell which stops collapsing just outside the putative horizon is calculated and shown not to have anevidence of preHawking radiation.

Recently the issue of what has been called "prehawking" radiation[1] has come to be of interest in a number of papers[2], especially whether or not the quantum emission of radiation from a collapse which never forms a horizon could be sufficient to explain that lack of a horizon. This seems to have begun by a series of papers by Barcelo et al [1] where they raise the possibility of Hawking radiation being produced in a collapse in which the horizon never forms.

Let us use the model in Chen et al[3] in which one has the collapse of a shell of matter with the metric

$$ds^2 = \left\{ \begin{array}{ll}
  du^2 + 2dudr - r^2d\Omega^2 & r < R(u) \\
  (1 - \frac{2m(u)}{R(u)})U'(u)^2du^2 + 2U'(u)dudr - r^2d\Omega^2 & r > R(u)
\end{array} \right. \quad (1)$$

Where we demand that the metric be continuous at $r = R(u)$ the path of the shell. $U = U(u)$ is the usual Eddington-Finkelstein null Schwartzschild parameter outside the shell. The continuity of the metric across the surface $r = R(u)$ gives

$$R'(u) = \frac{U'(u)^2 \left( 1 - \frac{2m(u)}{R(u)} \right) - 1}{2U'(u) - 1} \quad (2)$$

In the spirit of Barcelo et al[1] let us take $m(u)$ to be a constant, and the the shell to be a null shell so that $R(u) = 2m + \epsilon - \frac{u}{2}$ until $R(u)$ is within $\epsilon$ of 2m (ie, at $u=0$). $\epsilon$ is assumed to be very small ($\ll 2m$). At that point we stop the shell so $R(u) = 2m + \epsilon$ thereafter. As Barcelo et al indicated, the Bogoliubov coefficients indicate that this metric will radiate Hawking radiation until $u=0$. from about $u=-m$ to $u=0$. During this time,

$$U' = \frac{1}{(1 - \frac{2m}{2m+\epsilon-u/2})} = \frac{2m + \epsilon - u/2}{\epsilon - u/2} \approx \frac{2m}{\epsilon - u/2} \quad (3)$$

or

$$U(u) \approx 4m \ln \left( \frac{\epsilon - u/2}{m} \right) \quad (4)$$

To get an estimate of the Energy Momentum tensor expectation value, we use the two dimensional calculation of Davies, Fulling and Unruh (DFU) [4] of the energy momentum tensor of a massless scalar field in 2 spacetime dimension. This calculation is easy, because the massless scalar field is conformally invariant in 2 dimensions, and a null left going wave remains a null left going wave no matter how wild the spacetime becomes. There is no back-scattering by the field. Writing the metric in the form

$$ds^2 = e^{2g(u,v)}dudv \quad (5)$$

which is always possible in two dimensions, and assuming that for large negative $u, v$ that the modes which go as $e^{iu}$ or $e^{iv}$ for $\omega > 0$ are the modes associated with the annihilation operators of the incoming vacuum state, then

$$T_{uu} = \frac{1}{48\pi} \left[ 2\xi_{,uu} - (\xi_{,u})^2 \right] \quad (6)$$

$$T_{vv} = \frac{1}{48\pi} \left[ 2\xi_{,vv} - (\xi_{,v})^2 \right] \quad (7)$$

$$T_{uv} = \frac{1}{48\pi} R_{g_{uv}} = -\frac{1}{24\pi} \xi_{,uv} \quad (8)$$
where $\mathcal{R}$ is the scalar curvature. This energy momentum tensor is conserved. The off diagonal term is the so called conformal anomaly. Choosing the above 2-D metric

$$
\begin{align*}
\text{d}s^2 &= \begin{cases}
\text{d}u^2 + 2\text{d}u\text{d}r & r > R(u) \\
U'(u)^2(1 - \frac{2m}{r})\text{d}u^2 + 2U'(u)\text{d}u\text{d}r & r < R(u)
\end{cases}
\end{align*}
$$

we rewrite this in the double null form

$$
\begin{align*}
\text{d}s^2 &= \begin{cases}
U'(u)e^{(U(u)+v)}/4m \left( \frac{e^{(U(u)+v)}/4m}{r^{(U(u)+v)/4m}} \right) \text{d}u\text{d}V & v(V) - u > R(u) \\
v'(V)\text{d}u\text{d}V & v(V) - u < 2R
\end{cases}
\end{align*}
$$

where $r(V - U(u))$ is defined by

$$
\begin{align*}
\frac{r(V, U(u)) + 2m\ln(\frac{r(V, U(u)) - 2m}{2m})}{V} &= \frac{V - U(u)}{2}
\end{align*}
$$

As shown in Chen et al paper, using the condition that the metric across the shell is continuous, we have

$$
\begin{align*}
R'(u) &= -\frac{U'^2(1 - \frac{2m}{R(u)})}{2(U' - 1)} - 1
\end{align*}
$$

which for a null shell, where $R'(u) = -1/2$, gives

$$
U' = \frac{1}{1 - \frac{2m}{R(u)}}
$$

Since it is in the $u, V$ coordinates outside the shell that we have the standard positive norm meaning positive frequency definition of the vacuum state, the DFU conformal factor is

$$
\begin{align*}
e^\xi &= \frac{1 - \frac{2m}{r}}{1 - \frac{2m}{R(u)}}
\end{align*}
$$

we see that along the shell, where $r = R(u)$, $\xi(u, 0) = 0$. Thus along the shell, $T_{uu} = 0$ There is no quantum emission from the shell into the region outside the shell.

For $v < 0$, we have $V = v$. For $v > 0$, the $v$ coordinates cross the shell after it has stopped, and $v(V) = \frac{1}{2m}V$ In both cases there is no flux of radiation inwards beyond the shell although there is at $v = 0$.

If $V > 0, u < 0$ and $r > \epsilon$ this is precisely the calculation in DFU et al [4]. Ie, for $u < 0$, the energy momentum tensor is exactly what one would expect for a black hole. There is a positive flux of energy travelling out to infinity, a negative energy flux heading toward the shell, and the conformal anomaly acts as if it is the source of both of these fluxes. Ie, the outward flux dies off for $r$ less than about $3m$ and the negative ingoing flux dies of for $r$ greater than around $3m$. These fluxes are all very very small (or order $1/m^2$ in Planck units– ie, about one photon of frequency the inverse black-hole time, emitted per black hole timescale. They have a completely negligible effect on the metric, except over very very long time scales (of order $m^3$ in Planck units, or $10^5$3 ages of the current universe for a solar mass black hole. They certainly have no effect on the behaviour of the shell.

For $u > 0, v > 0$, when the shell has stopped, but $u < u_0$ where $u_0 > 0$ is the value of $u$ where the null line ingoing line from the turnaround reflects off $r = 0$, the zero ingoing ($T_{uv}$) radiation produces zero outgoing radiation through the shell. However, $T_{vv}$ is negative, and that negative ingoing flux produces a negative outgoing $T_{uu}$ for $u > u_0$, equal to the negative ingoing $T_{vv}$ This is exactly the situation for the quantum energy momentum tensor in the eternal Schwarzschild spacetime as was shown in DFU. In the usual Schwartzschild coordinates, they showed that this gave an energy momentum tensor with negative energy density near $r = 2m$ (the negative energy density of the Boulware vacuum). Ie, there is no prehawking radiation. The radiation is all just the Hawking radiation for $u < u_0$ and the static Schwartzschild radition for $u > u_0$. These are both tiny (total energy radiated going as $1/m^2$ in Planck units.).

However in order to make the shell behave in this way, one needs a horrendous transverse stress within the shell. One can imagine what is needed by noting that one has an infalling shell travelling at the velocity of light, and on a scale far less than the Planck scale the shell must come to a stop. This would require a transverse pressure far far larger than the energy in the shell(drastically violate all of the energy limits). In order to have the DFU wedge be a reasonable size– eg more than than say a second for a solar mass black hole, between the infalling shell and the outgoing $u=0$ surface, at a radius of a few Schwartzschild radii, $\epsilon$ would have to be of the order of $e^{-5000}$ of the Planck time or distance.
FIG. 1: This figure, in Eddington Finkelstein advanced coordinates (ingoing null rays are at 45 degrees), shows the various regions of the spacetime. The green lines designate the ingoing null matter which comes to a halt a distance $\epsilon$ from the "horizon" ($r = 2m$). The blue lines designate null surfaces. The first outgoing null line is the outgoing null line which connects to the point where the shell first comes to rest. The second outgoing blue line designates the outgoing curve which connects, via reflection from the origin, to the ingoing effect of that stopping point. The region labeled DFU is the region where the quantum energy momentum tensor is what was calculated in the DFU paper, and is identical to what it would be if the collapse had actually produced a black hole. The area labeled Flat is flat spacetime, where the energy momentum tensor expectation value would be expected to be 0. The region labeled Schwarzschild is where the energy momentum tensor is what it would be in the Boulware vacuum of a black hole. Note that we would expect these regions to also be similar in the case of a 4-D system. The region labeled NULL is where the ingoing null flux impinging on the surface of the shell travels into the flat spacetime inside the shell, reflects from $r=0$ and goes out again producing a null outgoing flux outside the shell.

Ie, if one tries to rely on the quantum emission to affect the trajectory of the shell, even in this case where no horizon forms, the quantum emission is simply far too small to produce any effect. One thus needs unphysical internal pressures, with no physical origin, to stop the collapse. If one stops the collapse well outside the surface $r = 2m$ internal pressures can clearly do so. The collapse of matter to form the earth was stopped well outside the horizon by internal pressures. To stop it within some tiny, sub-Planck distance from the horizon is however another matter entirely.

[1] C. Barcelo, S. Liberati, S. Sonego and M. Visser, Phys. Rev. Lett. 97, 171301 (2006).
C. Barcelo, S. Liberati, S. Sonego and M. Visser, Phys. Rev. D 83, 041501 (2011)
C. Barcelo, S. Liberati, S. Sonego and M. Visser, JHEP 1102, 003 (2011)
[2] H. Kawai, Y. Matsuo and Y. Yokokura, Int. J. Mod. Phys. A **28**, 1350050 (2013).
H. Kawai and Y. Yokokura, Phys. Rev. D **93**, no. 4, 044011 (2016)
P. M. Ho, JHEP **1508**, 096 (2015)
P. M. Ho, Nucl. Phys. B **909**, 394 (2016)
V. Baccetti, R. B. Mann and D. R. Terno, arXiv:1610.07839 [gr-qc].
V. Baccetti, R. B. Mann and D. R. Terno, arXiv:1703.09369 [gr-qc];
V. Baccetti, R. B. Mann and D. R. Terno, arXiv:1706.01180 [gr-qc].

[3] P. Chen, W. G. Unruh, C. H. Wu and D. H. Yeom, arXiv:1710.01533 [gr-qc].

[4] P. C. W. Davies, S. A. Fulling, and W. G. Unruh Phys. Rev. D **13**, 2720