Progress report on hadron spectroscopy with improved actions

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Quenched light hadron masses are measured on blocked lattices, using 6 different lattice discretizations of the Dirac operator. Results are compared with those of unblocked lattices, allowing for a “ranking” of the Dirac discretizations.

1. INTRODUCTION

Two years ago we reported on tadpole-improved fermion actions on blocked lattices [1]. We showed that tadpole improvement was necessary for the clover improvement to give accurate results (see also [2]). Since then many groups have tested the same action and confirmed this conclusion [3].

Here, we include other improved fermion actions with and without tadpole improvement. The aim is to assess the importance of tadpole renormalizations for those actions and to identify the “best” discretization(s) of the Dirac operator.

We consider improvements of the Wilson fermion action [4] which has $O(a)$ leading discretization errors. For improvements of the pure-gauge Wilson action we refer to [2]. They are less important since the discretization error is $O(a^2)$ in the gauge sector. We use quenched blocked configurations of [5] for which the discretization errors are even smaller. By measuring the string tension on the blocked lattices, we observed the restoration of the rotational symmetry. For further details see [6].

One of the first proposals for improved fermion actions was the Eguchi-Kawamoto action [7]:

$$D^{(EK)}_{ij} = \delta_{ij} - \kappa \sum_{\mu=1}^4 [(r - \gamma_\mu) U_{\mu i} \delta_{i+\mu,j} + (r + \gamma_\mu) U^\dagger_{\mu i} \delta_{i-\mu,j}]$$

(1)

with $r' = 2r$ and $\kappa_c = 1/6$. We set here $r = 1$. It cancels the $O(a^2)$ errors of the naive discretization and breaks the chiral symmetry by $O(a^3)$ terms. Therefore, it has $O(a^3)$ discretization errors. Computations with this action can be found also in [8].

The Clover-improved Wilson action is constructed to remove the $O(a)$ errors of the Wilson action by adding the term [9]:

$$\Delta W^{(clover)} = \frac{c_i}{2} \sum_{\mu
u,i} \sigma_{\mu\nu} P_{\mu\nu,i}$$

(2)

with $c = 1$, $\sigma_{\mu\nu} = \frac{i}{2} [\gamma_\mu, \gamma_\nu]$ and $P_{\mu\nu,i}$ the antisymmetric and antihermitian lattice operator that discretizes the field strength tensor $F_{\mu\nu}$ by the nearest-neighbor plaquette terms at lattice site $i$ in the $\mu\nu$ plane. $\Delta W^{(clover)}$ has $O(a)$ discretization errors.

Note that the clover action can be obtained from the Eguchi-Kawamoto action by the isospectral transformation [10]:

$$\psi \rightarrow \psi - \frac{1}{2} D \psi$$
$$\psi \rightarrow \psi + \frac{1}{2} \bar{\psi} D^\dagger$$

(3)

after dropping $O(a^2)$ terms (and higher) and proper normalization, where $D$ is the (massless) naive lattice Dirac operator.

1.1. Tadpole improvement

All the actions above have additional $O(g^2a)$ errors at the one-loop level in perturbation theory (see for example [11]). These errors cannot be removed perturbatively for the usual lattice cutoffs of the order 1 GeV. Therefore, non-perturbative
treatments of this problem have recently been proposed.

Tadpole improvement is a mean field prescription which rescales gauge fields $U_{\mu,i} \to U_{\mu,i}/u_0$ with $u_0 = P^{1/4}$, $P$ being the average plaquette. Another alternative is to tune the clover coefficient $c$ non-perturbatively so that $O(a)$ errors disappear. However, tadpole improvement is trivial to implement and cancels most of the $O(a)$ errors in our application.

Another improved fermion lattice action is the D234 action. It can be written in the form

$$D234 = D^{EK}(r' = r = 1) + \frac{1}{2}\Delta W^{(clover)}$$

with tadpole improved gauge fields. Like the EK action it has $O(a^3)$ classical discretization errors. The free $D234$ action has $\kappa_c = 1/7$.

2. LIGHT HADRON SPECTRUM WITH IMPROVED ACTIONS

We have calculated the light hadron masses in quenched QCD for the following fermion actions:

1) Wilson (W)
2) Wilson - Clover (C)
3) Wilson - Clover - Tadpole (C_LM)
4) Eguchi - Kawamoto (EK)
5) Eguchi - Kawamoto - Tadpole (E_LM)
6) D234

on $8^3 \times 16$ configurations obtained from $32^3 \times 64$ lattices at $\beta = 6$ after 2 blocking steps. We compare our results with those on the original unblocked lattice with Wilson fermions taken from [4]; we denote them by $G$. In all cases a sample of 100 configurations is considered.

The string tension measured on blocked lattices is $a'^2K = 0.93(5)$. Comparison with the experimental value $K = (440 \text{ MeV})^2$ gives $a'^{-1} = 456(12) \text{ MeV}$ or $a' = 0.432(11) \text{ fm}$. By matching this string tension with that of the Wilson gauge action, we found that $a' = 0.432(11) \text{ fm}$ corresponds to $\beta \approx 5.1$. The string tension on the fine lattice is taken from [7], $a^2K = 0.0513(25)$ corresponding to $a = 0.101(2) \text{ fm}$.

In Table 1 we compare the additive quark mass renormalization for different quark actions at $\kappa = \kappa_c$, i.e. $\Delta m = 1/2(1/\kappa_c - 1/\kappa_c^{\text{free}})$ for $W, C, C_LM$ and $\Delta m = 2/3(1/\kappa_c - 1/\kappa_c^{\text{free}})$ for $E_K, E_{LM}, D234$, where $\kappa_c^{\text{free}}$ is the free $\kappa_c$.

| Action | $\Delta m$ |
|--------|------------|
| $W$    | -1.50(2)   |
| $C$    | -1.06(2)   |
| $C_{LM}$ | 0.17(1)   |
| $E_K$   | -1.10(4)   |
| $E_{LM}$ | -0.49(1)  |
| D234   | -0.222(4)  |

In Table 2 we compare the $\rho, N$ and $\Delta$ masses extrapolated to the chiral limit by using the quadratic fit: $m_0 + b(\kappa m_c)^2$. We see that the results on the coarse lattices approach monotonically those on the fine lattices (with the Wilson action) if we order the improved quark actions as: $W, C, E_K, E_{LM}, D234, C_{LM}$.

| Action | $m_0/\sqrt{\kappa}$ | $m_N/\sqrt{\kappa}$ | $m_\Delta/\sqrt{\kappa}$ |
|--------|----------------------|----------------------|--------------------------|
| $W$    | 0.95(1)              | 1.79(3)              | 1.93(1)                  |
| $C$    | 1.13(2)              | 1.85(2)              | 2.12(2)                  |
| $C_{LM}$ | 1.42(1)              | 2.18(4)              | 2.53(2)                  |
| $E_K$   | 1.22(1)              | 1.89(5)              | 2.19(1)                  |
| $E_{LM}$ | 1.35(1)              | 1.92(6)              | 2.37(1)                  |
| D234   | 1.39(1)              | 2.03(5)              | 2.48(1)                  |
| $G$    | 1.48(1)              | 2.17(2)              | 2.56(1)                  |

In Table 3 the hadron masses are extrapolated by adding a cubic term in the fit. In this case the above ordering remains, except the $D234$ and $C_{LM}$ actions exchange places.

| Action | $m_0/\sqrt{\kappa}$ | $m_N/\sqrt{\kappa}$ | $m_\Delta/\sqrt{\kappa}$ |
|--------|----------------------|----------------------|--------------------------|
| $W$    | 0.883(3)             | 1.59(2)              | 1.82(2)                  |
| $C$    | 1.12(1)              | 1.64(1)              | 1.96(2)                  |
| $C_{LM}$ | 1.42(6)              | 1.75(5)              | 2.38(2)                  |
| $E_K$   | 1.22(3)              | 1.652(1)             | 2.24(3)                  |
| $E_{LM}$ | 1.30(4)              | 1.73(23)             | 2.3255                   |
| D234   | 1.41(2)              | 1.82(4)              | 2.41(4)                  |
| $G$    | 1.44(1)              | 2.05(2)              | 2.62(1)                  |
These results confirm our earlier calculations that tadpole improvement is crucial to approach the continuum limit faster [1]. On the other hand tadpole effects for the \( EK \) action are about half of those for the clover action. This is consistent with the expectation that the higher the order of gauge fields in a given operator, the higher the tadpole renormalizations. The \( EK \) action will be therefore less sensitive to the precise value of the tadpole coefficient \( u_0 \).

In Figures 1, 2, 3 we compare the \( \rho \), \( N \) and \( \Delta \) masses respectively for the different fermionic actions. All of them are compared to the results obtained by the Wilson action on the fine lattice. They show that the \( D234 \) action approaches better the results of fine lattices. In Figure 4 we show an Edinburgh plot; it favors the tadpole-improved clover action.

To summarize, we find that the tadpole-improved clover action, the tadpole-improved Eguchi-Kawamoto action and the \( D234 \) action are much better discretizations of the continuum...
Figure 4. Edinburgh plot for the Wilson and the two most improved actions, $D234$ and $C_{LM}$. The dotted line was obtained with Wilson fermions on a fine lattice ($\beta = 6, 32^3 \times 64$) [16]. The rest of the results comes from a twice-blocked lattice ($\beta \approx 5.1, 8^3 \times 16$). The octagons are the physical ratio and infinite quark mass limit.

Dirac operator than the Wilson action, allowing for an increase of the lattice spacing by a factor $\sim 4$. Among these 3 actions, the greater simplicity and the higher locality of the tadpole-improved clover action make it the most appealing.

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