Evaluation of small non-sphericity of converging shock waves in bubbles during collapse in acetone and tetradecane

D Yu Toporkov
Institute of Mechanics and Engineering - Subdivision of the Federal State Budgetary Institution of Science "Kazan Scientific Center of the Russian Academy of Sciences" (IME - Subdivision of FIC KazanSC of RAS) 2/31, Lobachevsky str., Kazan 420111 Russia

E-mail: top.dmtr@gmail.com

Abstract. A strong compression of vapor in cavitation bubbles during their collapse in acetone and tetradecane at a temperature of 293 K and 663 K, respectively, is considered. At the beginning of collapse, the bubbles are at rest, their radius is 500 μm, the vapor in the bubbles is in a state of saturation at a temperature of the surrounding liquid. The pressures of tetradecane and acetone are 50 and 15 bar, respectively. At these pressures, the degrees of strong vapor compression in the cavitation bubbles in tetradecane and acetone are fairly close to one another. An approximate estimate of maximum perturbations of the shock wave sphericity is made. It is shown that at the moment of focusing, the converging shock wave inside the bubble in acetone turns out to be more spherical despite the fact that at the end of collapse, the non-sphericity of the bubble in acetone is much greater.

1. Introduction
The dynamics of bubbles in liquid is of great interest because of the possibility of attaining large inner densities, pressures and temperatures [1, 2]. High degrees of compression of the bubble content are realized in the final stage of convergence of shock waves which are formed in their cavity. In a small central area of a bubble for an extremely short time period, the temperatures and the densities can achieve values exceeding $10^7$ K and 10 g/cm$^3$, respectively [1-4]. A necessary condition for attaining very high degrees of compression of the bubble content by a converging shock wave is the closeness of the wave shape to a spherical one, up to its entering the small central region of the bubble. It is known that the shock wave at the time of its formation has a shape close to that of the bubble surface. During the convergence, the non-sphericity of the shock wave may greatly increase. Therefore, the shape of the shock wave near its focusing depends on the change in the bubble shape during collapse as well as on the evolution of the non-sphericity of the shock wave.

A criterion of the formation of converging shock waves in a bubble during its collapse is presented in [5]. According to that criterion, vapor-gas media with a higher molecular weight $M$ and a smaller adiabatic index $\gamma$ are more suitable for realizing such shock waves. Taking this into account, an approximate mathematical formulation was used in [6] to compare the opportunities of producing the convergent shock waves in vapor bubbles collapsing in acetone ($M = 58$ g/mol, $\gamma = 1.125$) and
tetradecane \((M = 198 \text{ g/mol, } \gamma = 1.0265)\). The possibility of attaining high degrees of vapor compression in the case of acetone was earlier demonstrated in [4]. According to [6], tetradecane is a more promising medium for achieving its strong compression in collapsing bubbles, as compared with acetone. This result was used in [7] to show that close degrees of vapor compression in collapsing bubbles in tetradecane and acetone were achieved when the pressure and the temperature were 50 bar and 663 K in the case of tetradecane and 15 bar and 293 K in the case of acetone. It was also shown in [7] that in the case of acetone, the amplitude of small non-spherical perturbations of the bubble during its collapse grew significantly larger (by more than 12 times for low-frequency perturbations). Along with this, the distance from the place of shock wave formation to the bubble center in the case of tetradecane was 6 times as much. This may result in larger non-sphericity of the shock wave by the time of its focusing at the bubble center in the case of tetradecane. In the present work, the growth in the amplitude of small non-sphericity of the convergent shock waves in bubbles collapsing in acetone and tetradecane is estimated, using approximate analytical formulas derived in [8].

2. Problem statement
The collapse of a weakly nonspherical cavitation bubble in an infinite volume of a stationary liquid (acetone and tetradecane) is considered. The temperature and the pressure of acetone are 293 K and 15 bar, the temperature and the pressure of tetradecane are 663 K and 50 bar. At the beginning of collapse, the radius of the bubbles is \(R_0 = 500 \mu\text{m}\), the vapor pressure in the bubbles is equal to the saturation pressure \(p_s (T_0) = 0.24\text{ bar}\) in the case of acetone and 10.3 bar in the case of tetradecane. Non-sphericity of the bubbles is considered to be small.

The movement of the vapor and liquid and the change of the bubble radius \(R\) is described using hydrodynamic model [4,9], in which the dynamics of vapor and liquid is assumed spherically symmetric, the compressibility of the liquid, the nonstationary heat conductivity of vapor and liquid, the non-equilibrium heat and mass transfer on the bubble surface, the imperfection of vapor and liquid are taken into account, realistic wide-range equations of state are applied [10,11].

To analyze the growth of non-sphericity of a bubble during its collapse, the perturbation \(\delta(t, \theta, \varphi)\) of the spherical shape \(r = R(t)\) of a bubble is taken as

\[
\delta(t, \theta, \varphi) = \varepsilon_{nm}(t) Y_{nm}(\theta, \varphi).
\]

Here \(r, \theta, \varphi\) are the spherical coordinates with the origin in the center of the bubble, \(\varepsilon_{nm}(t) = a_{nm}(t)/R(t)\), \(a_{nm}(t)\) is the amplitude of the bubble shape deviation from the spherical one in the form of spherical harmonic \(Y_{nm}(\theta, \varphi)\) of degree \(n\) and order \(m\), \(|\varepsilon_{nm}| < 1\). The evolution of a sphericity perturbation is simulated by model [9], according to which the perturbation amplitude \(a_{nm}(t)\) is governed by an ordinary differential equation of the second order that takes into account the influence of the surface tension, the liquid viscosity, and the bubble content [12]. In the following, index \(m\) is omitted since the evolution of \(\varepsilon_{nm}\) does not depend on \(m\).

It is known [13,14] that during collapse the value of \(\varepsilon_n\) with relatively small numbers \(n\) (i.e. when the liquid viscosity effect is insignificant) changes in the form of oscillations with an increasing amplitude, and the increase in the amplitude is independent from the harmonic number \(n\). Let the amplitude of oscillations of \(\varepsilon_n\) be \(|\varepsilon_n^*| \approx |\varepsilon_{n,0}|\) with its initial value \(|\varepsilon_{n,0}|\). Then one can write [13,14]

\[
|\varepsilon_n^*| = |\varepsilon_{n,0}| \left(\frac{R}{R_0}\right)^{-1.25},
\]

To estimate the growth of small perturbations of the sphericity of a convergent shock wave in a bubble, approximate analytical expressions are used [8]. According to [8], a non-spherical perturbation \(\delta_{wn}(t, \theta, \varphi)\) of the shock wave front in the form \(Y_{nm}(\theta, \varphi)\) is presented as

\[
\delta_{wn}(t, \theta, \varphi) = \varepsilon_{wn}(t) Y_{nm}(\theta, \varphi)
\]

Here \(\varepsilon_{wn} = R_{wn,0}/R_w\), \(R_{wn,0}\) is a dimensional perturbation of a spherical shape of the shock wave, \(R_w\) is the radius of the shock wave. It was derived in [8] that
where
\[
\lambda_{n} = 1 + \frac{2(\gamma - 1)}{\gamma(\gamma - 1 + 2b_{1})} + \frac{\gamma + 2b_{1}}{\gamma - 1 + 2b_{1}} \left( \frac{2(\gamma - 1)}{\gamma} \right)^{0.5}.
\]

Here, the parameter \( b \) characterizes the rigid core in the Van der Waals equation of state, \( \gamma \) is the adiabatic index, \( \rho_{1} \) is the vapor density at the moment of focusing the shock wave.

According to (2), the value of \( \varepsilon_{w,n} \) (similar to the value of \( \varepsilon_{b,n} \)) executes oscillations with an increasing amplitude, and the growth of the amplitude does not depend on the harmonic number \( n \). Let the amplitude of oscillations of \( \varepsilon_{w,n} \) be \( \varepsilon_{w,n}^{*} \) with its initial value \( \varepsilon_{w,n}^{*0} \). Then it follows from (2) that

\[
\left| \varepsilon_{w,n}^{*} \right| = \left| \varepsilon_{w,n}^{*0} \right| \left( \frac{R_{w,n}}{R_{w,n}^{0}} \right)^{0.5-1/\lambda}.
\]

3. Results

Figure 1a shows the change in the radius of a bubble during its collapse in acetone and tetradecane. In both cases, converging shock waves are formed in the cavity of the bubbles in the final stage of their collapse. During the convergence, their intensity rapidly increases, and at the time of their focusing, the thermodynamic parameters become very high. The radii of bubbles \( R_{\text{extr}} \) at the moment of focusing are very different: \( R_{\text{extr}} = 39 \, \mu m \) in the case of acetone and \( R_{\text{extr}} = 252 \, \mu m \) in the case of tetradecane. The maximum radial velocities of the bubbles are also very different: 600 m/s in the case of acetone and 90 m/s in the case of tetradecane. It should be noted that the range of change in the bubble radius (from \( R_{0} \) to \( R_{\text{extr}} \)) in the case of tetradecane is significantly less than in the case of acetone, whereas the range of change in the radius of the shock waves (from their inception to focusing) is much larger than in the case of acetone.

![Figure 1](image-url)

**Figure 1.** Collapse of bubbles in acetone at \( T_{0} = 293 \, K \) and \( p_{0} = 15 \, \text{bar} \) (curves 1, 3) and tetradecane at \( T_{0} = 663 \, K \) and \( p_{0} = 50 \, \text{bar} \) (curves 2, 4):

(a) the change in the radius of the bubbles (solid lines) and the shock waves from the moment they arise (dashed lines). Squares mark the moments of formation of the shock waves, circles indicate the moments of their focusing.

(b) the spatial distribution of the pressure in the vapor and the surrounding liquid layer at the moment of the shock wave focusing. Circles indicate the values on the bubble surface.

The bubble radii at the moment of the shock waves arise 335.8 and 57.2 \( \mu m \) in the cases of acetone and tetradecane, respectively, and the corresponding radii of the shock waves are 251 in the
case of tetradecane and 40.95 μm in the case of acetone. The values of parameter $b$ in (3) are taken to be 0.001039 and 0.0008334 m$^3$/kg [6]. Following [8], the values of $\rho_1$ in (3) are taken to be equal to the average vapor density at the moment of the shock wave focusing, which are 422.1 kg/m$^3$ in the case of tetradecane and 689.4 kg/m$^3$ in the case of acetone.

Figure 1b shows the pressure distribution at the moment of the shock waves focusing at the center of the bubbles collapsing in acetone and tetradecane. The pressure is presented only outside a small central region with a radius of $r = 0.25$ μm, because in that region, owing to high degrees of vapor compression, there is a significant influence of the processes of dissociation, ionization, and shock wave sphericity distortion, not allowed for in the mathematical model used. Figure 1b shows that, despite the large difference in the kinematic characteristics of bubble collapse in acetone and tetradecane, in both cases the maximum values of the pressure (as well as the temperature and the density not shown in this figure) are comparable with one another under the assumption that the convergent shock waves remain spherical.

In reality, the bubbles and the shock waves always have non-spherical perturbations, which increase during bubble collapse [13, 14]. The perturbations of the sphericity of the shock waves during their convergence to the bubble center also increase (and also in oscillation mode), but in comparison with the bubble perturbations they grow slower [15].

![Figure 2](image)

**Figure 2.** Collapse of bubbles in tetradecane (lines 1, 3) and acetone (lines 2, 4):

the evolution of the bubble non-sphericity amplitude $[\varepsilon_{n10}]$ (lines 1, 2); the corresponding increase in the bubble non-sphericity amplitude $[\varepsilon_{n}^*]$ according to (1) (line 5) and the shock wave non-sphericity amplitude $[\varepsilon_{c}^*]$ according to (3) (lines 3, 4). Circles indicate the values at the moment of the shock wave focusing, squares indicate the values at the moment of the shock wave formation, diamonds indicate the values at the moment the shock wave reach the boundary of the area $r \leq 0.25$ μm.

Figure 2 characterizes the growth of the non-sphericity of the bubble and the shock wave during the bubbles collapse in tetradecane and acetone ($[\varepsilon_{n10}] = |\varepsilon_{10}/\varepsilon_{10,0}|$, $[\varepsilon_{n}^*] = |\varepsilon_{n}^* / \varepsilon_{n,0}^*|$, $[\varepsilon_{c}^*] = |\varepsilon_{c}^* / \varepsilon_{c,0}^*|$). It should be noted that an approximate estimate (1) of the bubble non-sphericity amplitude growth shows satisfactory results only at the initial stage of collapse, where the non-sphericity increases in the form of oscillations, and is not applicable to the stage of the final burst of the non-sphericity. Therefore, the evolution of the bubble non-sphericity is presented in the figure for the harmonic with a fairly high number 10. It can be seen that the increase in the amplitude of the bubble non-sphericity in the case of tetradecane is significantly less than in the case of acetone. In the range $2 \leq n \leq 10$, the difference is more than 12 times.

In this paper, it is assumed that the initial shape of the shock wave, i.e. the shape at the time of its arising, is similar to that of the bubble surface. The initial non-sphericity the shock wave in the bubble in the form of the harmonic with $n = 10$ is equal to $[\varepsilon_{n,0}^*] = 1.467$ in the case of tetradecane and $[\varepsilon_{c,0}^*] = 17.125$ in the case of acetone. The exponent in (3) is equal to $-1.151$ in the case of tetradecane and to $-0.996$ in the case of acetone, both greater than the value $-1.25$ in (1). This means that the non-sphericity of the convergent shock wave grows in the bubble in tetradecane stronger than in the
bubble in acetone. For this reason, by the moment when the shock wave reaches the boundary of the area \( r \leq 0.25 \mu m \), its non-sphericity increases stronger (by approximately 1.5 times) in the case of tetradecane than in the case of acetone, despite the fact that in the case of tetradecane the shock wave is much more spherical at the moment of its formation.

References

[1] Taleyarkhan R P, West C D, Cho J S, Lahey R T (Jr), Nigmatulin R I and Block R C 2002 Science 295 1868
[2] Nigmatulin R I, Lahey R T (Jr), Taleyarkhan R P, West C D and Block R C 2014 Physics-Uspekhi 57 (9) 947
[3] Xu Y and Butt A 2005 Nucl. Eng. Des. 235 1317
[4] Nigmatulin R I, Akhatov I Sh, Topolnikov A S, Bolotnova R Kh, Vakhitova N K, Lahey R T (Jr) and Taleyarkhan R P 2005 Phys. Fluids 17 107106
[5] Nigmatulin R I, Aganin A A, Toporkov D Yu and Ilgamov M A 2014 Dokl. Phys. 59 431
[6] Aganin A A and Toporkov D Yu 2017 Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki 159 (3) 271 (In Russian)
[7] Toporkov D Yu 2018 Multiphase Systems 13 (3) 23 (In Russian)
[8] Evans A K 1996 Phys. Rev. E. 54 (5) 5004
[9] Nigmatulin R I, Aganin A A, Toporkov D Yu and Ilgamov M A 2016 Doklady Physics 61 (3) 138
[10] Nigmatulin R I and Bolotnova R Kh 2007 Doklady Physics 52 (8) 442
[11] Nigmatulin R I and Bolotnova R Kh 2017 High Temperature 55 (2) 199
[12] Lin H, Storey B D and Szeri A J 2002 J. Fluid Mech. 452 145
[13] Plesset M S and Mitchell T P 1956 Quart. Appl. Math. 13 (4) 419
[14] Nigmatulin R I, Aganin A A, Ilgamov M A and Toporkov D Yu 2014 Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki 156 (1) 79
[15] Somogyi Z, Roberts P H 2007 Quart. J. Mech. Appl. Math. 60 289