PATH INTEGRAL FOR COMPOSITE FERMIONS IN THE HALF-FILLED LOWEST LANDAU LEVEL

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We consider electrons in two dimensions in a strong magnetic field at half filling of the lowest Landau level using the Chern-Simons approach. Starting from a lattice Hamiltonian for the electrons, we derive a path integral (PI) formulation for the composite fermions (CF) which respects the order of the operators. We use a time lattice with intermediate times in order to have a PI expressed in density fluctuations. This formulation reveals that there is no infrared (IR) singularity in the grand-canonical potential in lowest order perturbation theory.

1 Introduction

The Chern-Simons (CS) approach to two-dimensional electrons in a strong perpendicular magnetic field at (or close to) half filling of the lowest Landau level, as pioneered by Halperin, Lee and Read \cite{r1}, is well supported by experiments. The interpretation of these experiments suggests that at half filling, the composite fermions, each consisting of an electron with two attached flux quanta, form a Fermi liquid in zero magnetic field. Theoretically, this approach suffers from IR singularities appearing as \( \ln F \) and \( \ln |\omega| \) (here \( F \) is the area of the square sample and \( \omega \) the frequency of, for example, the self energy). In this contribution, we want to reconsider the \( \ln F \)-singularities.

Recently, we studied the grand-canonical potential \( \tilde{\Phi} \) of the CF in the random-phase approximation (RPA) using the CF interaction in the form

\[
H^{\text{int}} = \frac{1}{2F} \sum_{k_1, k_2, (\neq 0)} W_{k_1, k_2} \hat{\psi}_{k_1}^{\dagger} \hat{\psi}_{k_2}^{\dagger} \hat{\psi}_{k_2} \hat{\psi}_{k_1},
\]

\[
W_{k_1, k_2} = \frac{(2\pi \hbar \tilde{\phi})^2}{m_b} \frac{N}{F} \frac{1}{k^2} + \frac{4\pi \hbar^2 \tilde{\phi}}{m_b} \left( \frac{k_1 \times \frac{k}{k^2}}{i} \right) e_z + \frac{2\pi e^2}{k}.
\]

Here, \( \hat{\psi}_k \) annihilates a (CF) with wave vector \( k \), the filling factor is \( 1/\tilde{\phi} = 1/2 \), \( m_b \) is the electron band mass. The interaction \( \tilde{\Phi} \) corresponds to the standard
PI formulation in the time continuum. In the interaction $W$, the first term results from the square of the CS vector potential $\mathbf{A}$, the second from the linear coupling to $\mathbf{A} (e_z$ is the unit vector in $z$ direction, orthogonal to the system), and the last term is the Coulomb interaction ($\epsilon^2 = e^2/(4\pi\varepsilon_r\varepsilon_0)$, $\varepsilon_r$ is the dielectric constant). In formulating the electron Hamiltonian in terms of the CF operators, the usual approximation replaces in the $A^2$-term the external pair of operators, $\hat{\psi}^\dagger \ldots \hat{\psi}$, by the average electron density $\rho_{el} = N/F$ ($N$ is the mean electron number). That led to the first term in $W$.

The random-phase approximation gave the following main results:

1. The first order (exchange) diagram for the grand-canonical potential $\Omega$ diverges in the RPA as
   $$-\beta \Omega_{ex} \sim (N\beta\mu\phi^2/2) \ln(k_F/k_{min}) + \text{const},$$
   where $k_{min} \propto 1/\sqrt{F}$, $k_F$ is the Fermi momentum, and $\mu$ the chemical potential.

2. The first-order contribution of the Coulomb interaction to the ground state energy is
   $$E^{(1)}_{\text{RPA}}/N = -0.6 \epsilon^2/l_B$$
   ($l_B$ is the magnetic length). This compares reasonably well with $E^{(1)}_{\text{sim}}/N = -0.466 \epsilon^2/l_B$ obtained in numerical simulations in the spherical geometry by Morf and d’Ambrumenil.

3. The number of magnetoplasmon oscillators is finite, i.e. no ultraviolet (UV) cut off is necessary.

   The IR divergency in the grand-canonical potential (for $T = 0$ equal to the ground state energy) led us to re-analyse the derivation of the CS theory.

2 Formulation of the Chern–Simons Theory

2.1 Hamiltonian

We start with the Hamiltonian for the electrons on the physical lattice; the lattice formulation allows a rigorous implementation of the CS transformation.

$$H = -\frac{\hbar^2}{2m_\alpha^2} \sum_m \hat{\psi}^{(e)}(m) \left[ U_{m \leftarrow m+e_j} \hat{\psi}^{(e)}(m + e_j) - \hat{\psi}^{(e)}(m) \right]$$

$$+ \frac{1}{2} \sum_{m_1,m_2} \hat{\psi}^{(e)}(m_1) \hat{\psi}^{(e)}(m_2) \frac{\epsilon^2}{|r_{m_1} - r_{m_2}|} \hat{\psi}^{(e)}(m_2) \hat{\psi}^{(e)}(m_1)$$

$$- (\text{Contribution from positive background}) - \mu \sum_m \hat{\psi}^{(e)}(m) \hat{\psi}^{(e)}(m),$$

$$U_{m \leftarrow m+e_j} = \exp \left\{ \frac{i\epsilon|\alpha}{\hbar} A_j(m + e_j/2) \right\}.$$

$\alpha$ is the lattice constant, $A_j(m + e_j/2)$ the external vector potential; $e_j$ are the vectors to the nearest neighbors on the square lattice ($m$). The electron field operator $\hat{\psi}^{(e)}$ satisfies magnetic quasi-periodic boundary conditions.
The CS transformation relates the electron operator to the CF operator:

\[ \hat{\psi}^{(e)}(m) = \exp \left\{ \frac{i|e|}{\hbar} \sum_n \left[ f^{(np)}(m) \rho_{el} + f^{(p)}(m - n)(\hat{n}_n - \rho_{el}) \right] \right\} \hat{\psi}(m) . \quad (3) \]

Here, \( f^{(np)} \) is nonperiodic; it leads to periodic boundary conditions for \( \hat{\psi} \), and after insertion into \( (2) \), it compensates the external magnetic vector potential. The \( f^{(p)} \) is periodic, and the second term in the exponent corresponds to a compensated CS flux. We omit explicit formulae for the \( f \). The particle number for the electrons or CF is \( \hat{n}_n \). Because the bases for the Hilbert spaces of \( \hat{\psi}^{(e)} \) and \( \hat{\psi} \) differ only by phase factors, the QM trace remains the same.

Because we consider a diluted system \((l_B \gg \alpha)\), we go with \( (2) \) to the continuum approximation by expanding the \( \hat{\psi}(m + e_j) \) and the difference of the exponents from \( (3) \). In the second order in the difference of the \( f^{(p)} \), we obtain an interaction Hamiltonian with six \( \hat{\psi} \)-operators in the order \( \hat{\psi}^\dagger \hat{n} \hat{\psi} \). In the following, we retain this term’s original form, which is not normally ordered, for further approximations by expansion in powers of the density fluctuations. Further, we refrain from approximating the external pair of operators, \( \hat{\psi}^\dagger \ldots \hat{\psi} \), by the average electron density \( \rho_{el} \).

### 2.2 Path Integral Formulation

We now formulate the theory by means of the coherent state Graßmann PI. For the imaginary time, we use a lattice with a lattice constant \( \Delta t \to 0, N\Delta t = -i\hbar \beta \). In order to avoid normal ordering of the \( \hat{\psi}^\dagger \hat{n} \hat{\psi} \) term, we use in the PI besides the time points \( t_\nu, \nu = 0, 1, \ldots, N - 1 \) also intermediate times, i.e. use \( t_\nu, \bar{\nu} = 0, 1/2, 1, \ldots, N - 1/2 \). The partition function is

\[
Z = \int \prod_{k, \bar{\nu}} \{ d\psi^*_\nu(k) d\psi_\nu(k) \} \exp \left\{ S^{(0)} + S^{(AA)} + S^{(jA)} + S^{(Coul)} \right\} \quad (4)
\]

\[
S^{(0)} = \sum_{\nu, k} \psi^*_\nu(k) \{ \psi_{\nu-1/2}(k) - \psi_{\nu}(k) \}
- \frac{i\Delta t}{\hbar} \sum_{\nu, k} \{ E^0(k) - \mu \} \psi^*_{\nu-1/2}(k) \psi_{\nu-1}(k) ,
\]

\[
S^{(AA)} = - \frac{i\Delta t}{\hbar} \frac{\hbar^2 \tilde{g}^2}{2m_0 F^2} \sum_{\nu, k, k', k''} \psi^*_{\nu}(k_1) \psi_{\nu}(k') \psi_{\nu-1/2}(k''') \psi_{\nu-1}(k_1 - k' - k''') \ast \kappa(k') \kappa(k''),
\]

3
\[ S^{(j,A)} = -\frac{i\Delta t}{\hbar} \hat{\mathcal{H}} \frac{\varphi}{m_B F} \sum_{\nu,k_1(k \neq 0)} \psi_{\nu-1/2}(k_1) \psi_{\nu-1}(k_1 - k) \varrho_{\nu-1/2}(k) \kappa(k), \]

\[ S^{(Coul)} = -\frac{i\Delta t}{\hbar} \frac{1}{2F} \sum_{\nu,k_1(k \neq 0)} \psi_{\nu-1/2}(k_1) \psi_{\nu-1}(k_1 - k) \varrho_{\nu-1/2}(k) \frac{2\pi e^2}{k}, \]

\[ \varrho_{\nu}(k) = \sum_{k_1} \psi_{\nu}^*(k_1) \psi_{\nu-1/2}(k_1 + k), \quad \kappa(k) = -2\pi i (e_z \times \frac{k}{k^2}) J_0(kR_0). \]

\( J_0 \) is the Bessel function, \( R_0 \) a cut off (\( \alpha \ll R_0 \ll l_B \)) formally used in the continuum approximation; for the diluted system distances between the CF of order \( R_0 \) are thermodynamically negligible.

Evaluation of the partition function (4) in low order perturbation theory leads to the following results:

1. The first order diagram for the grand-canonical potential is now convergent. The absence of the IR divergency can be traced back to the order of the operators \( \hat{\psi} \hat{n} \hat{\psi} \). Approximation of the external \( \hat{\psi} \hat{n} \hat{\psi} \) by \( \rho_{\text{el}} \) would lead to an UV divergency.

2. Unlike the grand-canonical potential, the CF self energy shows an IR divergency in first order in the interaction:

\[ \Sigma^{(1)}(k) \sim \mp \ln \frac{k_F}{k_{\text{min}}} \quad \text{for} \quad k < k_F \quad \text{or} \quad k > k_F, \quad \text{respectively.} \quad (5) \]

The above considerations show that a PI approach, respecting the order of the operators in the original Hamiltonian, provides a promising starting point for treating the IR divergencies in higher order perturbation theory in the interaction.

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