Direct measurement of the pion valence quark momentum
distribution, the pion light-cone wave function squared

Abstract

We present the first direct measurements of the pion valence quark momentum distribution which is related to the square of the pion light-cone wave function.
function. The measurements were carried out using data on diffractive dissociation of 500 GeV/c $\pi^-$ into di-jets from a platinum target at Fermilab experiment E791. The results show that the $|q\bar{q}|$ light-cone asymptotic wave function, which was developed using perturbative QCD methods, describes the data well for $Q^2 \sim 10 \text{ (GeV/c)}^2$ or more. We also measured the transverse momentum distribution of the diffractive di-jets.
The internal momentum distributions of valence quarks in hadrons enter the calculation of a large variety of processes such as electroweak decays, diffractive processes, meson production in $e^+e^-$ and $\gamma\gamma$ annihilation, relativistic heavy ion collisions, and many others [1]. The momentum distribution amplitudes are generated from the valence light-cone wave functions integrated over $k_t < Q^2$, where $k_t$ is the intrinsic transverse momentum of the valence constituents and $Q^2$ is the total momentum transfer squared (Eqn. 5). Because of the close relationship between the two, the distribution amplitudes are often referred to as the light-cone wave functions [2]. Even though these amplitudes were calculated about 20 years ago, there have been no direct measurements until those reported here. Observables which are related to these distributions, such as the pion electromagnetic form factors, are rather insensitive to the light-cone wave functions.

The pion wave function can be expanded in terms of Fock states:

$$\Psi = a_1 |q\bar{q}\rangle + a_2 |q\bar{q}g\rangle + a_3 |q\bar{q}gg\rangle + \cdots. \quad (1)$$

The first (valence) component is dominant at large $Q^2$. The other terms are suppressed by powers of $1/Q^2$ for each additional parton, according to counting rules [2,3]. In contrast, parton distribution functions are inclusive momentum distributions of partons in all Fock states. Here we are concerned with the momentum distribution of only the valence quark-antiquark part.

Two functions have been proposed to describe the momentum distribution amplitude for the quark and antiquark in the $|q\bar{q}\rangle$ configuration. The asymptotic function was calculated using perturbative QCD (pQCD) methods [4–6], and is the solution to the pQCD evolution equation for very large $Q^2$ ($Q^2 \rightarrow \infty$):

$$\phi_{as}(x) = \sqrt{3}x(1-x). \quad (2)$$

$x$ is the fraction of the longitudinal momentum of the pion carried by the quark in the infinite momentum frame. The antiquark carries a fraction $(1-x)$. Using QCD sum rules, Chernyak and Zhitnitsky (CZ) proposed [7] a function that is expected to be correct for low $Q^2$:

$$\phi_{cz}(x) = \frac{5\sqrt{3}x(1-x)(1-2x)^2}{2}. \quad (3)$$

As can be seen from Eqns. 2 and 3, and from Fig. 2, there is a large difference between the two functions. Measurements of the electromagnetic form factors of the pion were considered to be the best way to study these wave functions. A comprehensive summary of the status of these measurements was published recently [2]. Both existing methods of measuring the pion electric form factor suffer from major drawbacks. Those done using pions elastically scattered from atomic electrons measure the pion electric form factor only for very low $Q^2$. The alternative method is to measure the electron-pion quasi-free scattering cross section [4]. The relation between the form factor and the longitudinal cross section is model-dependent as it includes the $p \rightarrow n\pi^+$ matrix element. Finally, the form factor is related to the integral over the wave function and the scattering matrix element, reducing the sensitivity to the wave function. Indeed, as shown in [4] both wave functions
can be made to agree with the experimental data. A similar situation exists for inelastic form factors, decay modes of heavy mesons, etc. Recent model-dependent analyses of CLEO data on meson-photon transition form factors [8,9] are consistent with the asymptotic wave function. The problem is that comparisons to these observables are not sensitive to details of the wave function and thus cannot provide critical tests of their $x$-dependence. Another open question is what can be considered to be high enough $Q^2$ to qualify for perturbative QCD calculations, what is low enough to qualify for a treatment based on QCD sum rules and how to handle the evolution from low to high $Q^2$.

In this work we describe an experimental study that maps the momentum distribution of the $q$ and $\bar{q}$ in the $|q\bar{q}\rangle$ Fock state of the pion. This provides the first direct measurement of the pion light-cone wave function (squared). The concept of the measurement is the following: a high energy pion dissociates diffractively on a heavy nuclear target imparting no energy to the target so that it does not break up. This is a coherent process in which the quark and antiquark in the pion break apart and hadronize into two jets. If in this fragmentation process each quark’s momentum is transferred to a jet, measurement of the jet momenta gives the quark and antiquark momenta. Thus:

$$x_{\text{measured}} = \frac{p_{\text{jet1}}}{p_{\text{jet1}} + p_{\text{jet2}}}.$$  \hspace{2cm} (4)

The diffractive dissociation of high momentum pions into two jets can be described, like the inclusive Deep Inelastic Scattering (DIS) and exclusive vector meson production in DIS, by factoring out the perturbative high momentum transfer process from the soft nonperturbative part [10]. This factorization allows the use of common parameters to describe the three processes. The virtuality of the process, $Q^2$, is given by the mass-squared of the virtual photon in inclusive deep inelastic scattering (DIS) and in exclusive light vector meson production in DIS. In exclusive DIS production of heavy vector mesons this mass is proportional to that of the produced meson (e.g., $J/\psi$). For diffractive dissociation into two jets, the mass-squared of the di-jets plays this role. From simple kinematics and assuming that the masses of the jets are small compared with the mass of the di-jets, the virtuality and mass-squared of the di-jets are given by:

$$Q^2 \sim M_J^2 = \frac{k_t^2}{x(1-x)},$$  \hspace{2cm} (5)

where $k_t$ is the transverse momentum of each jet and reflects the intrinsic transverse momentum of the valence quark or antiquark. By studying the momentum distribution for various $k_t$ bins, one can observe changes in the apparent fractions of asymptotic and Chernyak-Zhitnitsky contributions to the pion wave function.

Fermilab experiment E791 [11] recorded $2 \times 10^{10}$ events from interactions of a 500 GeV/c $\pi^-$ beam with carbon (C) and platinum (Pt) targets. The trigger included a loose requirement on transverse energy deposited in the calorimeters. Precision vertex and tracking information was provided by 23 silicon microstrip detectors (6 upstream and 17 downstream of the targets), ten proportional wire-chamber planes (8 upstream and 2 downstream of the targets), and 35 drift-chamber planes. Momentum was measured using two dipole magnets. Two multicell, threshold Čerenkov counters were used for $\pi$, $K$, and $p$ identification (not
needed for this analysis). Only about 10% of the E791 data was used for the analysis presented here. From these data, we selected interactions which were exclusive two jet events. This focused on the jets as materializations of the valence quark and antiquark in the pion.

The data were analysed by selecting events in which 90% of the beam momentum was carried by charged particles. This reduced the effects of the unobserved neutral particles and allowed for precise measurement of transverse momentum. The selected events were subjected to the JADE jet-finding algorithm \[12\]. The algorithm uses a cut-off parameter \(m_{\text{cut}}\) whose value was optimized for this analysis using Monte Carlo simulation studies in order to optimize the identification of di-jets. The di-jet invariant mass was calculated assuming that all the particles were pions. To insure clean selection of two-jet events, a minimum \(k_t\) of 1.25 GeV/c was required. Furthermore, the di-jet nature of these events was verified by examining their relative azimuthal angle, which for pure di-jets should be 180°. Strong peaking at 180° was observed (FWHM \(\sim\) 5°), and only events within 20° of back-to-back were accepted for this analysis.
Diffractive di-jets were identified through the dependence of their yield ($q_t^2$ is the square of the transverse momentum transferred to the nucleus and $b = \frac{<R^2>}{3}$ where $R$ is the nuclear radius). Fig. 1 shows the $q_t^2$ distributions of di-jet events from platinum and carbon. The different slopes in the low $q_t^2$ coherent region reflect the different nuclear radii. Events in this region come from diffractive dissociation of the pion.

The basic assumption that the momentum carried by the dissociating $q\bar{q}$ is transferred to the di-jets was examined by Monte Carlo (MC) simulation. MC samples with 4 and 6 GeV/$c^2$ mass di-jets were generated with two different $x$ dependences at the quark level. The $x$-distributions were calculated by squaring the asymptotic and the Chernyak-Zhitnitski (CZ) wave functions. One sample was simulated using the asymptotic wave function and the other, the CZ function. The four samples were allowed to hadronize using the LUND PYTHIA-JETSET [13] package and then passed through a simulation of the experimental apparatus to account for the effect of unmeasured neutrals and other experimental distortions.
FIG. 2. Monte Carlo simulations of squares of the two wave functions at the quark level (left) and of the reconstructed distributions of di-jets as detected (right). $\phi_{\text{asy}}^2$ is the asymptotic function (squared) and $\phi_{\text{CZ}}^2$ is the Chernyak-Zhitnitsky function (squared). The di-jet mass used in the simulation is 6 GeV/c$^2$ and the plots are for $1.5 \text{ GeV}/c \leq k_t \leq 2.5 \text{ GeV}/c$.

In Fig. 2 the initial distributions at the quark level are compared with the final distributions of the detected di-jets, including distortions in the hadronization process and influence due to experimental acceptance. As can be seen, the qualitative features of the two distributions are retained. The results of this analysis come from comparing the observed $x$-distribution to a combination of the distributions shown, as examples, on the right of Fig. 2.

For all results in this paper, we used data from the platinum target as it has a sharp diffractive distribution and a relatively low background. It is also expected that due to the color transparency effect [6,14,15] this heavy target will better filter out the high Fock states. We used events with $q_t^2 < 0.015 \text{ GeV}/c^2$. For these events, the value of $x$ was computed from the measured longitudinal momentum of each jet (Eqn. 4). A background, estimated from the $x$ distribution for events with larger $q_t^2$, was subtracted. This analysis was carried out in two windows of $k_t$: $1.25 \text{ GeV}/c \leq k_t \leq 1.5 \text{ GeV}/c$ and $1.5 \text{ GeV}/c \leq k_t \leq 2.5 \text{ GeV}/c$. The experiment data were compared to Monte Carlo simulations of di-jets having a mass of $4 \text{ GeV}/c^2$ for the lower window and $6 \text{ GeV}/c^2$ for the higher window. The resulting $x$ distributions are shown in Fig. 3. In order to get a measure of the correspondence between the experimental results and the calculated light-cone wave functions, we fit the results with a linear combination of squares of the two wave functions. This assumes an incoherent combination of the two wave functions and that the evolution of the CZ function is slow (as stated in [7]). It is hard to justify these two assumptions because it is hard to make model...
independent evolutions and to know the phase between the two amplitudes. We therefore regard this fit as a qualitative indication of how well each function describes the data. We use results of the simulated wave functions (squared) after they were subjected to effects of experimental acceptance (Fig. 2 right).

FIG. 3. The $x$ distribution of diffractive di-jets from the platinum target for $1.25 \leq k_t \leq 1.5$ GeV/c (left) and for $1.5 \leq k_t \leq 2.5$ GeV/c (right). The solid line is a fit to a combination of the asymptotic and CZ wave functions. The dashed line shows the contribution from the asymptotic function and the dotted line that of the CZ function.
TABLE I. Contributions from the asymptotic ($a_{as}$) and CZ ($a_{cz}$) wave functions to a fit to the data.

| $k_t$ bin GeV/c | $a_{as}$ | $\Delta_{a_{as}}^{stat}$ | $\Delta_{a_{as}}^{sys}$ | $\Delta_{a_{as}}$ | $a_{cz}$ | $\Delta_{a_{cz}}^{stat}$ | $\Delta_{a_{cz}}^{sys}$ | $\Delta_{a_{cz}}$ |
|----------------|--------|----------------|----------------|----------------|--------|----------------|----------------|----------------|
| 1.25 - 1.5     | 0.64   | ±0.12          | +0.07 -0.01    | +0.14 -0.12    | 0.36   | ±0.12          | -0.07 +0.01    | -0.14 +0.12 |
| 1.5 - 2.5      | 1.00   | ±0.10          | +0.00 -0.10    | +0.10 -0.14    | 0.00   | ±0.10          | -0.00 +0.10    | -0.10 +0.14 |
The measured $x$ distributions are shown in Fig. 3 with the combinations resulting from the above fits superimposed on the data. The individual contributions from each wave function are shown as well. In addition to the statistical errors of the fit, we considered systematic uncertainties originating from the background subtraction, from the quality of the jets and their identification and from using discrete-mass simulations. The dominant contribution comes from the quality of the jets in the low $k_t$ region and from using discrete-mass MC in the high $k_t$ region. The results of the fits are given in Table I in terms of the coefficients $a_{as}$ and $a_{cz}$ representing the contributions of the asymptotic and CZ functions, respectively. The total errors are obtained by adding the statistical and systematic errors in quadrature.

The values of $\chi^2$/dof were 1.5 and 1.0 for the low and high $k_t$ bins, respectively. The results for the higher $k_t$ window show clearly that the asymptotic wave function describes the data very well. Because of the dominance of the asymptotic wave function, this conclusion does not depend on the assumptions made in fitting the data to a combination of the two functions. The distribution in the lower window is consistent with a significant contribution from the Chernyak-Zhitnitsky wave function. However, as stated above, if neither function is dominant this can only indicate that at low $k_t$ neither function alone describes the data well.

The requirement that $k_t > 1.5$ GeV/c can be translated (Eqn. 5) to $Q^2 \approx 10 (\text{GeV}/c)^2$. This shows that for these $Q^2$ values, the perturbative QCD approach that led to construction of the asymptotic wave function is reasonable.

The $k_t$ dependence of diffractive di-jets depends on the quark distribution amplitude. It was calculated recently by Frankfurt, Miller, and Strikman [14]. They show that the most important terms are those in which the $|q\bar{q}\rangle$ component of the pion interacts with two gluons emitted by the target. The predicted $k_t$ dependence can be seen from the cross section for this process [14]:

$$\frac{d\sigma}{dk_t^2} \propto \left|\alpha_s(k_t^2) x_{Bj} G(x_{Bj}, k_t^2) \right|^2 \left|\frac{\partial^2}{\partial k_t^2} \psi(x, k_t^2) \right|^2$$

(6)

where $\psi$ is the light-cone wave function. Evaluation of the light-cone wave function at large $k_t$ as due to one gluon exchange gives $\psi \sim \frac{\phi}{k_t^2}$ with $\phi$ a slow function of $k_t$ (e.g. the asymptotic function). Given the weak $\alpha_s(k_t^2)$ dependence, differentiating and squaring gives $\sim k_t^{-8}$. Since $\alpha_s(k_t^2)G(x_{Bj}, k_t^2) \sim k_t^{1/2}$ [16] the expected dependence of the cross section on $k_t$ is:

$$\frac{d\sigma}{dk_t} \sim k_t^{-6}.$$  

(7)

This prediction can be compared with the data. We use the MC simulations discussed above for di-jets having masses of 4, 5, and 6 GeV/c$^2$ and the asymptotic wave function to correct for the experiment acceptance of the $k_t$ distribution. The corrected results are shown in Fig. 4(a). Superimposed on the data is a power-law fit $k_t^n$ for $k_t > 1.25$ GeV/c. We find $n = -9.2 \pm 0.4(stat) \pm 0.3(sys)$ with $\chi^2$/dof = 1.0. This slope is significantly larger than expected. We note, however, that above $k_t \sim 1.8$ GeV/c the slope changes (although
the statistical precision there is poor). A power law fit to this region (Fig. 4(b)) results in $n = -6.5 \pm 2.0$ with $\chi^2/dof = 0.8$, consistent with the predictions. This would support the evaluation of the light-cone wave function at large $k_t$ as due to one gluon exchange.

The steep $k_t$-dependence in lower $k_t$ region may be interpreted \[14\] as a manifestation of non-perturbative effects. We try the non-perturbative Gaussian function: $\psi \sim e^{-\beta k_t^2}$ \[17\]. When inserted in Eqn. 6 and using $\alpha_s(k_t^2)G(x_Bj,k_t^2) \sim k_t^{1/2}$, we get:

$$\frac{d\sigma}{dk_t} = C(k_t^2 - 2\beta k_t^4 + \beta^2 k_t^6)e^{-2\beta k_t^2}$$

where $C$ is a normalization factor. In Fig. 4(b) we show a fit to this function in the low $k_t$ range yielding $\beta = 1.78 \pm 0.05(stat) \pm 0.1(sys)$ with $\chi^2/dof = 1.1$. To the best of our knowledge, there is no previous report of a direct measurement of $\beta$. Model-dependent values in the range of 0.9 - 4.0 were used in calculations of the $\pi - \gamma$ transition form factors \[17\]. The present results may indicate that non-perturbative effects are noticeable up to $k_t \sim 1.5$ GeV/c, as is the case for the light-cone wave function which becomes dominated by the asymptotic function only for larger $k_t$ values.

In summary, we have presented results of direct measurements of the valence quark and antiquark momentum distributions in the pion. They show that above $k_t \sim 1.5$ GeV/c ($Q^2 \sim 10$ GeV/c$^2$) the asymptotic distribution amplitude calculated using perturbative QCD is applicable. The measured $k_t$ distribution in this region is also consistent with this conclusion. In the lower $k_t$ region there may be other contributions, such as from the CZ wave function or other nonperturbative effects.
FIG. 4. Comparison of the $k_t$ distribution of acceptance-corrected data with fits to cross section dependence (a) according to a power law, (b) based on a nonperturbative Gaussian wave function for low $k_t$ and a power law, as expected from perturbative calculations, for high $k_t$.

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