Spin-Flavor Separation and Non-Fermi Liquid Behavior
in the Multichannel Kondo Problem: A Large N Approach

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We consider a $SU(N) \times SU(M)$ generalization of the multichannel single-impurity
Kondo model which we solve analytically in the limit $N \to \infty$, $M \to \infty$, with
$\gamma = M/N$ fixed. Non-Fermi liquid behavior of the single electron Green function
and of the local spin and flavor susceptibilities occurs in both regimes, $N \leq M$ and
$N > M$, with leading critical exponents identical to those found in the conformal
field theory solution for all $N$ and $M$ (with $M \geq 2$). We explain this remarkable
agreement and connect it to “spin-flavor separation”, the essential feature of the
non-Fermi-liquid fixed point of the multichannel Kondo problem.

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In recent years, the multichannel Kondo model, first introduced by Nozières and Blandin [1], has been the focus of intense activity. This model involves a single local moment of spin $S_I$ antiferromagnetically coupled to $M$ identical conduction bands in an $N_s$-site system, as described by the Hamiltonian,

$$\mathcal{H} = \sum_{k\sigma\alpha} \epsilon_k c_{k\sigma\alpha}^\dagger c_{k\sigma\alpha} + (J/N_s) \vec{S}_I \cdot \left[ \sum_{k\sigma,k'\sigma'} \sum_{\alpha} c_{k\sigma\alpha}^\dagger \vec{r}_{\sigma\sigma'} c_{k'\sigma'\alpha} \right],$$

where $c_{k\sigma\alpha}^\dagger$ creates a conduction electron of (radial) wave vector $k$, spin $\sigma = \uparrow, \downarrow$ and channel index $\alpha$ running from 1, ..., $M$; the coupling $J > 0$ is antiferromagnetic. It is now established [2,3] that in the “overcompensated” regime, $2S_I < M$, the screening of the local moment by the conduction electrons drives the metal into a non-Fermi liquid critical state at $T = 0$, fully validating Nozières and Blandin’s original arguments.

The renewed interest in this model is, in part, due to a number of suggestions for realizations of two-channel overcompensated behavior in explicit experimental contexts, most notably: (i) the quadrupolar Kondo effect [4] in heavy fermion alloys [5]; (ii) non-Fermi liquid scattering rates in narrow copper point-contacts [6]; (iii) “marginal” Fermi liquid normal state properties [7] of the high-$T_c$ materials [4,8]. Moreover, it has been argued that the two-channel Kondo model provides a link to exotic superconductivity [3,4,8]. Related non-Fermi liquid behavior and enhanced pairing correlations have been recently found in the mixed-valence regime at low temperatures in certain extended Anderson single-impurity models motivated by the electronic structure of the high-$T_c$ materials [8].

From a theoretical point of view, remarkable progress has been made in understanding the universal properties of the multi-channel Kondo and other impurity models through the use of conformal field theory techniques [3]. This approach provides a clear picture of the separation of spin, channel, and charge excitations at non-Fermi liquid fixed points and describes the subtle recombination of these degrees of freedom required in the Fermi liquid case. In spite of its elegance and power this method only classifies the possible critical behaviors without providing a constructive route to the solution of a particular model.

In this letter we formulate a controlled calculation method which can, in principle, in-
corporate both the complications of real materials and the conceptual insights gained from conformal field theory. Apart from the obvious practical application to dilute impurity systems, this question is important in studying the possibility for non-Fermi liquid behavior in lattice systems, for which the conformal field theory techniques are not immediately applicable. An explicit route for addressing the latter problem, which appears particularly promising, involves constructing mean-field theories for the lattice by solving single-impurity models embedded in a self-consistent medium [10].

Below we concentrate on the $SU(N) \times SU(M)$ generalization of the multichannel Kondo Hamiltonian of Eq. (1), where $N$ and $M$ are the degeneracies in the spin and flavor quantum numbers, respectively. We use a functional integral approach based on the “slave-Boson” representation [11] which explicitly separates the (local) spin and flavor excitations. The limit $N, M \to \infty$ with $\gamma = M/N$ fixed leads to a closed set of coupled self-consistent integral equations which can be solved analytically in the asymptotic low-frequency, zero temperature limit. These are identical to the “Non-Crossing Approximation” (NCA) equations of perturbation theory [11]. Although these equations break down for sufficiently low energies in the Fermi liquid (single channel) case [12], it is the unique feature of our large $N, M$ treatment of the multi-channel Kondo problem that the NCA becomes exact.

In fact, we show that the single electron Green function, and the local spin and flavor susceptibilities display non-Fermi liquid behavior with leading critical exponents identical to those found in the conformal field theory solution [3] for all $N, M \geq 2$. We explain this apparently surprising result by demonstrating that the fluctuations provide no corrections to the leading exponents obtained from NCA to all orders in $1/N, 1/M$ [13]. Our calculation is an explicit realization of the “spin-charge separation” idea emphasized by Anderson in his theory of the high-$T_c$ materials based on the single-band Hubbard model [14]. Ultimately, we hope that the considerations presented here will be useful in treating lattice Fermion systems with non-Fermi liquid ground states.

Our starting point is a path integral treatment of the generalized infinite $U$ Anderson model Hamiltonian,
\[ H = \sum_{k,\sigma,\alpha} \epsilon_k c_{k,\sigma,\alpha}^\dagger c_{k,\sigma,\alpha} + \epsilon_f \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} + \left( \frac{V}{\sqrt{N_s}} \right) \sum_{k,\sigma,\alpha} [f_{\sigma}^\dagger b_{\alpha} c_{k,\sigma,\alpha} + h.c.] \]  

where the Fermion, \( f_{\sigma}^\dagger \), creates a local spin excitation and the Boson, \( b_{\alpha} \), transforms according to the conjugate representation of \( SU(M) \), and annihilates the flavor quantum number of the “vacuum” state produced by destroying a conduction electron. The “completeness” of the local states at the impurity site, represented by the constraint, \( \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} + \sum_{\alpha} b_{\alpha}^\dagger b_{\alpha} = 1 \), is implemented in the usual way \([11]\), by introducing a fictitious field \( \lambda \), which decouples the interaction term, \( \sum_{\sigma} f_{\sigma}^\dagger f_{\sigma} \), \( \sum_{\alpha} b_{\alpha}^\dagger b_{\alpha} \), and annihilates the flavor quantum number of the free-electron gas partition function to the full partition function, \( Z = Z_c Z_{imp} \). By completing squares, the hybridization term is eliminated in favor of a spin-flavor interaction contribution, \( S_{int} = -(V^2/N) \sum_{\sigma,\alpha} \int d\tau d\tau' f_{\sigma}^\dagger(\tau)f_{\sigma}(\tau')G^0(\tau - \tau')b_{\alpha}^\dagger(\tau')b_{\alpha}(\tau) \), where \( G^0(\tau - \tau') = -\sum_k (\partial^2/\partial\tau^2 + \epsilon_k)^{-1}/N_s \) is the non-interacting conduction electron Green function at the impurity site; and, as usual, in order to obtain a nontrivial large \( N, M \) limit, we have defined a rescaled hybridization matrix element, \( \tilde{V} = \sqrt{N}V \), which should be considered of order unity at the end of the calculation. The next step is to introduce two composite fields, non-local in imaginary time, \( \Phi_{f,\sigma}(\tau, \tau') \) and \( \Phi_{b,\alpha}(\tau, \tau') \) (with \( \Phi_{f,\sigma}^\dagger(\tau, \tau') = \Phi_{f,\sigma}(\tau', \tau) \)), which decouple the interaction term, \( S_{int} \), enabling us to write, \( Z_{imp} = \int [Df][Db][D\Phi_f][D\Phi_b] \exp(-\tilde{S}) \). The effective action, \( \tilde{S} \), is then given by

\[ \tilde{S} = \sum_{\sigma} \int_0^\beta d\tau \int_0^\beta d\tau' f_{\sigma}^\dagger(\tau)[(\partial/\partial\tau + \epsilon_f + \lambda) + \tilde{V}^2/N \sum_{\alpha} \Phi_{b,\alpha}(\tau, \tau')G^0(\tau - \tau')]f_{\sigma}(\tau') + \sum_{\alpha} \int_0^\beta d\tau \int_0^\beta d\tau' b_{\alpha}^\dagger(\tau)[\delta(\tau - \tau')(\partial/\partial\tau + \lambda) - \tilde{V}^2/N \sum_{\sigma} \Phi_{f,\sigma}(\tau, \tau')G^0(\tau' - \tau')]b_{\alpha}(\tau') \]  

Finally, the large \( N, M \) calculation proceeds as usual by carrying out the Gaussian integral.
over the local Fermions and Bosons, \( f, b \), to produce an effective action for the composite fields, \( \Phi_{f,b,\alpha} \); this is then evaluated by a saddle point integration \( [15] \). (Hereafter we drop the bar on the channel index, \( \bar{\alpha} \).

In the \( N, M = \infty \) limit with \( \gamma = M/N \) fixed the saddle point approximation becomes exact and leads to time translationally invariant solutions, \( \Phi_{f,b}(\tau - \tau') \). After Fourier transforming in terms of Fermionic and Bosonic imaginary frequencies, \( \omega_n, \nu_n \), the saddle point equations can be written as \( \Phi_f(i\omega_n) = [i\omega_n - \epsilon_f - \Sigma_f(i\omega_n)]^{-1} \) and \( \Phi_b(i\nu_n) = [i\nu_n - \Pi_b(i\nu_n)]^{-1} \). This defines the self energies, \( \Sigma_f \) and \( \Pi_b \), which, on the real frequency axis (\( i\omega_n = \omega + i0^+, i\nu_n = \omega + i0^+ \)), satisfy the self-consistent equations,

\[
\Sigma_f(\omega) = \frac{\gamma \bar{\Gamma}}{\pi} \int d\epsilon f(\epsilon) \Phi_b(\epsilon + \omega) \tag{4}
\]

\[
\Pi_b(\omega) = \frac{\bar{\Gamma}}{\pi} \int d\epsilon f(\epsilon) \Phi_f(\epsilon + \omega). \tag{5}
\]

Here, \( \bar{\Gamma} = \pi \rho \bar{V}^2 \) is the bare hybridization width (which includes a factor of \( N \) introduced by the rescaling), and \( \rho \) is the conduction electron density of states, assumed constant in the energy range of interest. In principle, the functions \( \Sigma_f \) and \( \Pi_b \) elsewhere in the complex plane can be obtained by the appropriate analytic continuation. Also note that in Eq. (5) the \( \lambda \to \infty \) limit has already been taken, and that, due to the invariance under \( SU(N) \times SU(M) \) rotations the spin and flavor indices have dropped out.

In the low-frequency, low-temperature limit, Equations (3) can be easily solved either by direct substitution or by reducing them to differential equations as done for the single channel case by Müller-Hartmann \( [12] \). The relevant analysis can be found in a number of places in the literature \( [11,12] \), and will not be repeated here.

The saddle-point solutions for \( \Phi_f \) and \( \Phi_b \) can be written in terms of the reduced frequency variable, \( \Theta = \frac{\gamma(1 + \gamma)}{(E_0 - \omega)/T_0} \), where \( E_0 \) is the ground state energy and \( T_0 = D(\gamma \bar{\Gamma}/\pi D)^\gamma \exp(\pi \epsilon_f / \bar{\Gamma}) \) is the Kondo scale. The calculation is most easily done for negative frequencies, \( \omega < E_0 \), and the appropriate analytic continuation extends the results to \( \omega > 0 \). At \( T \to 0 \) the resulting spin Fermion and flavor Boson spectral functions, \( A_{f,b}(\omega) = \text{Im} \Phi_{f,b}(\omega - i0^+)/\pi = A_{f,b}^{(+)}(\omega)\theta(\omega - E_0) \) vanish for \( \omega < E_0 \). In the process of
calculating physical quantities we will also need the spectral function for occupied states \((\omega < E_0)\), defined as \(A^{(-)}_{f,b}(\omega) = \lim_{T \to 0}[\text{Im} \Phi_{f,b}(\omega - i0^+)/\pi \exp(\beta[E_0 - \omega])]\). Close to the threshold at \(E_0\), these quantities take the form

\[
A^{(+)}_f(\omega) = \frac{1}{\pi T_0} \sin\left(\frac{\pi \gamma}{1 + \gamma}\right)|\Theta|^{-\gamma}[1 + 4 \frac{\gamma}{2 + \gamma} \cos(\frac{\pi \gamma}{1 + \gamma})]|\Theta| + ... \tag{6}
\]

\[
A^{(+)}_b(\omega) = \frac{1}{\gamma \tilde{\Gamma}} \sin\left(\frac{\pi \gamma}{1 + \gamma}\right)|\Theta|^{-1}[1 - 4 \frac{W_{ch}}{1 + 2\gamma} \cos(\frac{\pi \gamma}{1 + \gamma})]|\Theta|^\gamma + ... \tag{7}
\]

\[
A^{(-)}_f(\omega) = \frac{\tilde{Z}}{\pi T_0} |\Theta|^{-\gamma}[1 - 4 \frac{\gamma}{2 + \gamma} |\Theta| + ...] \tag{8}
\]

\[
A^{(-)}_b(\omega) = \frac{\tilde{Z}}{\gamma \tilde{\Gamma}} |\Theta|^{-1}[1 - 4 \frac{W_{ch}}{1 + 2\gamma} |\Theta|^\gamma + ...]. \tag{9}
\]

Here, \(W_{ch} = \pi T_0/\tilde{\Gamma}\) is the weight of channel fluctuations in the ground state, and the constant, \(\tilde{Z}\) is related to the degeneracy of the impurity ground state per local degree of freedom (i.e., divided by \(N(1 + \gamma)\)) \[12\]. Also, note the explicit breaking of particle-hole symmetry displayed by the positive \((A^{(+)}_{f,b})\) and negative \((A^{(-)}_{f,b})\) contributions, consistent with the non-symmetric form of (2).

The scaling dimensions of the spin and flavor fields, \(\Delta_f = [2(1 + \gamma)]^{-1}\) and \(\Delta_b = \gamma \Delta_f\) can be read off from the frequency dependence, \(|E_0 - \omega|^{2\Delta_{f,b}^{-1}}\), in Eq. (6). It then follows that the spin, channel and physical Fermion fields, all of which are bilinears of \(f\) and \(b\), have scaling dimensions, \(\Delta_{sp} = 2\Delta_f = 1/(1+\gamma), \Delta_{ch} = 2\Delta_b = \gamma/(1+\gamma)\) and \(\Delta_F = \Delta_f + \Delta_b = 1/2\), respectively. The resulting leading frequency dependence of the corresponding correlation function, \(|E_0 - \omega|^{2\Delta_{sp, ch, F}^{-1}}\) is indeed what we observe. Below we summarize some of our explicit results.

**Single Electron Green Function:** The local electron Green function, \(G_{\sigma,\alpha}\), can be calculated as a convolution of the spin and flavor propagators, leading to a local spectral function, \(\rho_{\alpha,\mu}(\omega, 0) = \text{Im} \ G_{\sigma,\alpha} (\omega - i0^+)/\pi\), of the form,

\[
\rho_{\alpha,\mu}(\omega, 0) \approx \pi/[(1 + \gamma)^2 N \tilde{\Gamma}] \left[1 + \theta(\omega)f_+(\tilde{\omega}) + \theta(-\omega)f_-(\tilde{\omega})\right],  \tag{10}
\]

with \(f_{\pm}(\tilde{\omega}) = (a_\pm|\tilde{\omega}|^{\Delta_{sp}} + b_\pm|\tilde{\omega}|^{\Delta_{ch}}), a_- = -[4\gamma/(2 + \gamma)\pi] \sin(\pi \Delta_{ch})B(2\Delta_{sp}, \Delta_{ch}), a_+ = -\cos(\pi \Delta_{ch})a_-, b_+ = -[4W_{ch}/(1 + 2\gamma)\pi] \sin(\pi \Delta_{ch})B(2\Delta_{ch}, \Delta_{sp})\) and \(b_- = \cos(\pi \Delta_{ch})b_+.\)
Here $\tilde{\omega} = [(1 + \gamma)/\gamma](\omega/T_0)$ and $B(x, y)$ is the Beta function. Note that, in the overscreened case, $M \geq N$, the leading frequency dependence is the same as that obtained by Affleck and Ludwig from conformal field theory [3].

Resistivity: From the single-particle Green function we can obtain the resistivity from the standard transport theory formula, $\rho(T) \sim [\int d\epsilon (-\partial f/\partial \epsilon) \tau(\epsilon, T)]^{-1}$, in terms of the scattering rate, $\tau_{\alpha\mu}(\omega, T)^{-1} = -2\text{Im}^{(1)}(\omega + i0^+, T) = (2\tilde{\Gamma}_{\alpha\mu}(\omega, T))/(\rho N)$, where we have used the relation between the conduction electron $t$-matrix and the single-electron Green function, $t^{(1)}_{\alpha\mu}(\omega, T) = V^2 G_{\alpha\mu}(\omega, T)$. The resulting leading behavior, $\rho(T)/\rho(0) \sim [1 - \alpha(T/T_0)^{\min(\Delta_{sp}, \Delta_{ch})} + ...]$ is in agreement with that obtained by Affleck and Ludwig [3]. In particular, for the special case of $N = M$, $\Delta_{sp} = \Delta_{ch} = 1/2$, we obtain a $\sqrt{T}$ correction as does the conformal approach [3]. Estimating the coefficient $\alpha$ requires a knowledge of the temperature dependence of $\rho_{\alpha\mu}$ which is beyond the scope of the present paper. It is gratifying that for $N = 2$ the magnitude of the spin contribution to the rate, $[(\pi \rho)/(2\tau_{\alpha\mu}(0, 0))] = 3\pi^2/[4(2 + M)^2]$, (which is obtained after removing the $1/4$ of the total rate due to potential scattering) agrees with the results of Affleck and Ludwig [3] to within 8% for all $M \geq 2$.

Local Spin and Channel Flavor Dynamical Susceptibilities: The linear response to external fields coupling to the impurity spin and channel quantum numbers can be easily calculated from the bubble diagrams for the spin and channel excitations, $f$ and $b$, respectively. The leading and next-to-leading contributions to the absorptive part of the local spin and flavor susceptibilities (per spin or channel degree of freedom), $\tilde{\chi}_{sp}'' = \text{Im} \chi_{sp}/N$ and $\tilde{\chi}_{ch}'' = \text{Im} \chi_{ch}/M$, are given by:

$$\tilde{\chi}_{sp}''(\omega, 0) \sim \frac{\gamma\Delta_{sp}^2 \sin(\pi \Delta_{ch})}{T_0} \text{sgn}(\omega)|\tilde{\omega}|(\Delta_{sp} - \Delta_{ch}) B(\Delta_{sp}, \Delta_{sp}) \times 
\left[1 - \frac{8\gamma}{2 + \gamma} \sin^2(\frac{\pi \Delta_{ch}}{2}) B(\Delta_{sp}, 2\Delta_{sp})|\tilde{\omega}|\Delta_{sp} + ...\right],$$

(11)

$$\tilde{\chi}_{ch}''(\omega, 0) \sim \frac{W_{ch}^2 \Delta_{sp}^2 \sin(\pi \Delta_{sp})}{T_0} \text{sgn}(\omega)|\tilde{\omega}|(\Delta_{ch} - \Delta_{sp}) B(\Delta_{ch}, \Delta_{ch}) \times 
\left[1 - \frac{8W_{ch}}{1 + 2\gamma} \sin^2(\frac{\pi \Delta_{sp}}{2}) B(\Delta_{ch}, 2\Delta_{ch})|\tilde{\omega}|\Delta_{ch} + ...\right].$$

(12)
Note that for $N = M$ (which includes the important case $N = M = 2$) both susceptibilities reduce to the general form $\tilde{\chi}''(\omega, 0) \approx \text{Asgn}(\omega)[1 - B\sqrt{|\omega|/T_0} + ...]$. This leading step function behavior in $\chi''$ and the associated logarithmic dependence in the real part, $\chi'(\omega, T) \sim -\ln(\max\{\omega, T\}/T_0)$, provides a possible connection with the marginal Fermi-liquid phenomenology of the high-$T_c$ oxides. This step function behavior was first noted in Ref. [4(b)]. For $N > M > 1$ the real part of the spin susceptibility is constant ($\sim 1/T_0$) and the non-Fermi-liquid behavior is dominated by the $|\tilde{\omega}|^{-(\Delta_{sp}-\Delta_{ch})}$ divergence of the channel susceptibility. In the opposite limit, $N < M$, the flavor susceptibility is constant ($\sim W_{ch}^2/T_0$) and a $|\tilde{\omega}|^{-(\Delta_{ch}-\Delta_{sp})}$ divergence occurs in the spin susceptibility. For $N > M$ the system displays two-parameter universality: the channel fluctuations start dominating below a new energy scale, $T_{ch} \sim T_0W_{ch}^{[1/(\Delta_{sp}-\Delta_{ch})]}$, with $T_{ch} \ll T_0$ for $N > M$ in the Kondo limit ($W_{ch} \ll 1$). We note that $T_{ch}$ evolves into the “pathology” temperature below which non-Fermi liquid behavior ensues in the NCA treatment of the one-channel Kondo model.

1/$N$ Fluctuations: The functional integral formulation outlined above gives a natural framework for estimating the effects of fluctuations, and allows us to explain the remarkable fact that the saddlepoint exponents remain unchanged to all orders in 1/$N$. The arguments are very much in the spirit of the argument usually made to justify the fact that perturbation theory gives the exact exponents characterizing the low energy behavior in conventional Fermi liquids. More precisely, all 1/$N$ fluctuations can be incorporated into the renormalization of interaction vertices in all diagrams ($f$ and $b$ self-energies, the single-particle Green function as well as all susceptibilities). We have checked explicitly to order 1/$N^2$ that these vertex renormalizations modify the amplitudes but only give subleading singular contributions. (For example, the sub-leading corrections to $\Sigma_f$ and $\Sigma_b$ behave as, $\text{Im}\delta \Sigma_f(\omega) \sim |\omega - E_0|^{3\Delta_{ch}}$ and $\text{Im}\delta \Sigma_b(\omega) \sim |\omega - E_0|^{3\Delta_{sp}}$, respectively.) In fact, the saddle point gives the exact low-energy singularities to all orders in 1/$N$, as can be seen by considering arbitrary order diagrams in perturbation theory (around the saddle point). The appropriate propagators carry spectral functions which diverge as $|\omega - E_0|^{\Delta_{f,b}}$.
Equations (6-9)]. As an illustration, consider a generic diagram contributing to $\Pi_b$ which contains $L$ loops (thus $L$ independent energy integrations), $L$ fermion propagators, and $L-1$ boson propagators. The most singular contribution behaves as $|\omega - E_0|^{\zeta_b(L)}$ where $\zeta_b(L) = L + L(\Delta_f - 1) + (L - 1)(\Delta_b - 1) = 1 - \Delta_b$ since unitarity of the scattering amplitude requires $\Delta_f + \Delta_b = 1$. This is indeed the behavior found at the saddle point. Similar arguments apply to $\Sigma_f$, the one particle Green’s function and the spin and channel susceptibilities. It is natural to speculate that, in all systems with spin-charge separated, non-Fermi liquid ground states, the correct low energy behavior can be obtained on the basis of an appropriate (self-consistent) perturbation theory involving the “separated” spin and charge degrees of freedom. In analogy with conventional Fermi liquids, such perturbative arguments should be valid even when no obvious small parameters are available, provided no phase transition occurs to a Fermi liquid state through the “binding” of spin and charge.

**Crossover Effects:** A related issue concerns the crossover to the Fermi liquid solution which becomes the correct ground state in the presence of a spin or channel symmetry breaking field, respectively $H_{sp}$ or $H_{ch}$. Our saddle point calculation leads to a crossover away from multi-channel behavior below respective scales, $T_{sp} \sim H_{sp}^{1+1/\gamma}, T_{ch} \sim H_{ch}^{1+\gamma}$. The corresponding crossover exponents, $\phi_{sp} = 1 + 1/\gamma, \phi_{ch} = 1 + \gamma$, are precisely those obtained from conformal field theory [3]. However, the Fermi liquid behavior which sets in well below the crossover scale $T_{sp(ch)}$ is outside the scope of the NCA. This can be traced back to the fact that the Kondo screening characteristic of the Fermi liquid fixed point involves the formation of a singlet bound state between a conduction electron and the local spin excitation, $f_\sigma$ [10]; the residue of the bound state pole plays the role of the slave-boson mean field amplitude in the conventional large $N$ approach to the Kondo problem. In the multichannel problem this Fermi liquid saddle point becomes possible only in the presence of a channel symmetry breaking field.

Above we have shown that the limit $N \to \infty, M \to \infty$ with $M/N = \gamma$ fixed allows us to obtain the *exact* low energy behavior of the multichannel single impurity Kondo problem. It suggests the possibility of a phenomenological approach to the multichannel Kondo problem.
and other impurity models with non-Fermi liquid ground states based on spin-flavor (or spin-charge) separated degrees of freedom. This can be regarded as the logical extension of Nozieres’ classic discussion of the single channel Fermi liquid case to non-Fermi liquid situations. A detailed analysis of fluctuations about the multichannel saddle point, including the crossover to the Fermi liquid case will be described elsewhere. Moreover, this formulation leads to a natural extension to the lattice through the large-$D$ treatment of correlated systems [10].

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Figure Phase diagram in the $N, M$ plane. The NCA is strictly controlled in the large $M, N$ limit for $M \geq 2$, but gives the exact critical exponents for all $N, M \geq 2$. The universality class is the same for all lines of fixed slope $\gamma = M/N$. 