Gravitational lensing by $p$-branes

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Abstract

The scattering of R-R gauge bosons off of Dirichlet $p$-branes is computed to leading order in the string coupling. The results are qualitatively similar to those found in the scattering of massless NS-NS bosons: all $p$-branes with $p \geq 0$ exhibit stringy properties, in particular the Regge behavior. Both the R-R and NS-NS scattering amplitudes agree in the limit of small momentum transfer with scattering off the extremal R-R charged $p$-brane solutions found in the low-energy supergravities. We interpret this as evidence that Dirichlet-branes are an exact world-sheet description of the extremal $p$-branes. The $-1$-brane (D-instanton) is a special object which, unlike all other Dirichlet-branes, exhibits point-like behavior. We show that the field theoretic scattering off of the R-R charged instanton solution to type IIB supergravity exactly reproduces the full stringy calculation. As an aside, we include a discussion of the entropy of non-extremal black holes in ten dimensions, produced by exciting the 0-brane.

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1 Introduction

Recent exciting developments in string theory suggest that it has a variety of non-perturbative symmetries [1, 2, 3, 4, 5, 6, 7, 8, 9]. In general these duality symmetries interchange fundamental strings with so-called $p$-branes, objects with $p$ intrinsic dimensions. In a weakly coupled theory the $p$-branes are heavy objects which may be thought of as solitonic solutions to the equations describing the fundamental strings [10, 11, 12, 13]. When extrapolated to strong coupling, however, the $p$-branes become light and should therefore be regarded as the fundamental degrees of freedom in the strongly coupled theory.

An unusual feature of the $p$-branes we are discussing is that their masses scale as $1/g$, where $g$ is the string coupling, rather than the familiar $1/g^2$ found for solitons in field theories. This unusual dependence on $g$ follows from the fact that the $p$-branes are stabilized by their R-R charges, and the R-R sector of string theory couples to the dilaton differently from the NS-NS sector [7]. The absence of the “sphere term” in the $p$-brane mass suggests that it comes from the disk topology. This reveals a subtle connection of $p$-branes with open strings, even in the type II theory which has closed string excitations only. We will, of course, make use of this relation to open strings in our calculations.

In the weak coupling limit the low-energy effective field theory description of strings is well known, and its various $p$-brane solutions were found years ago [10, 11, 12, 13]. Because the importance of $p$-branes in the non-perturbative dynamics has become certain, it became important to advance their understanding beyond the narrow confines of the low-energy supergravity. A breakthrough in this direction took place a few months ago when, building on earlier work [14, 15], Polchinski showed [16] that the Dirichlet-branes (D-branes) carry precisely the values of the R-R charges needed for the string duality symmetries. Polchinski’s work strongly suggests that the D-branes are the exact stringy versions of the extremal black $p$-brane solutions of the effective supergravity. The conformal field theory description of the D-branes is extremely simple: one introduces auxiliary open strings with Dirichlet boundary conditions in some directions replacing the conventional Neumann boundary conditions [14, 15]. Thus, these open strings are not free to propagate everywhere, but are instead glued to a $p + 1$ dimensional hyperplane which is the world history of the $p$-brane. This is how the hidden connection with open strings alluded to previously is accomplished in type II theory. A nice physical picture for this, suggested by Witten [17], is that the auxiliary open string is in reality a closed string, part of which is hidden behind a black $p$-brane horizon.

The simplicity of the D-brane approach to the R-R charged string solitons has originated a flurry of activity. Many $p$-brane properties that were anticipated based on supersymmetry and duality have been confirmed by explicit D-brane calculations [18, 19, 20, 21]. Furthermore, D-branes have made it possible to formulate new conjectures and concepts [22, 23, 24, 25, 26, 27, 28]. The stringy construction of $p$-branes has made it possible to study those properties that do not follow simply from BPS saturation arguments. A few such studies of D-brane dynamics have appeared in the literature [21, 23, 30], and many more are undoubtedly possible. One interesting physical question that was studied early
concerns the size of p-branes. The effective field theory has little to say about this, but luckily D-branes do provide some stringy information. For instance, they may be probed by scattering of massless string states. In \cite{29} the lowest order scattering amplitudes for massless NS-NS states off D-branes were calculated explicitly. The resulting formulae revealed that, although classically all p-branes appear point-like, the stringy quantum effects endow all the D-branes with $p > -1$ with a size of order the string scale, $\sqrt{\alpha'}$. Furthermore, the effective size grows indefinitely with increasing energy of the probe, exhibiting the Regge behavior well-known in the high-energy scattering of fundamental strings \cite{31}. This suggests that an instantaneous “snapshot” of a p-brane will show it smeared all over space, similar to the “snapshots” of fundamental strings \cite{32}.

While the graviton scattering off D-branes revealed that their gravitational radius exhibits the Regge behavior, it was not explicitly shown that this also holds for the R-R charge radius. In this paper we continue the program started in \cite{29} by calculating the leading order scattering amplitude for two massless R-R states off D-branes\footnote{In scattering off D-branes we have also found non-vanishing two-point functions mixing one R-R and one NS-NS massless states. We do not discuss them in this paper, but hope to present their calculations in the near future.}. While the calculations with the R-R vertex operators \textit{a priori} seem technically challenging, we were surprised to find that they are, in some ways, simpler than the corresponding NS-NS calculations. We obtain a compact general formula for the scattering of any R-R $n$-form field off any Dirichlet $p$-brane. This formula explicitly shows that the Regge behavior and the exponential fall-off at fixed angle found in NS-NS scattering also hold for R-R scattering. The scattering amplitude exhibits an infinite series of $t$-channel poles corresponding to exchange of closed string states with the $p$-brane. The pole at $t = 0$ corresponds to graviton and dilaton exchange and should therefore agree with the amplitude found in the effective field theory. To check this, we study the propagation of the $n$-form fields in the backgrounds of the extremal black $p$-brane solutions to the type II supergravity equations \cite{13}. We find that the residue of the $1/t$ pole in these field theoretic calculations matches precisely with that found in the stringy D-brane calculations. This interesting result provides a good check on the polarization dependence of the scattering off the D-branes and gives a physical explanation of many of its features\footnote{Our check on polarization dependences is similar in spirit to the work in \cite{33} where it was checked that appropriate generalizations of the Dabholkar-Harvey solitons \cite{10} approximately describe long fundamental strings.}. More importantly, it strengthens the claim that the D-branes are the stringy objects which, at large distances, are approximated by the extremal black $p$-branes of the effective supergravity.

The D-instanton ($-1$-brane) deserves special discussion because it appears point-like even in the full stringy calculations \cite{15,29,34,35,36,37}: instead of an infinite sequence of $t$-channel poles there is only one pole at $t = 0$. To gain a better understanding of this phenomenon, we study a solution of the type IIB supergravity equations which describes an instanton carrying R-R “charge” and verify that the field theoretic scattering off the D-
instanton exactly reproduces the full string amplitude. It is remarkable to find an object in string theory that seems to be exactly described by the effective field theory.

The organization of the paper is as follows. In section 2 we briefly summarize some of the amplitudes found previously for NS-NS scattering off of p-branes. In section 3 we present our main technical result, namely the evaluation of all R-R scattering amplitudes off of p-branes. In section 4 we show how the low momentum transfer limit of the results of the previous two sections can be reproduced via scattering off of the classical field of extremal black p-branes. We conclude in section 5 with a discussion of how the general form of the scattering amplitudes elucidates the relation of D-branes to black holes.

2 Review of NS-NS scattering off D-branes

In [29], D-branes were probed by scattering of gravitons and NS-NS antisymmetric tensor particles. For simplicity, their polarization tensors were taken to be non-vanishing only in the directions perpendicular to the p+1 dimensional world volume, i.e. the following conditions were imposed:

\[ \varepsilon^{(1)}_{AB} = \varepsilon^{(1)}_{Ai} = \varepsilon^{(2)}_{AB} = \varepsilon^{(2)}_{Ai} = 0. \]  

Our conventions are to let capital indices run over the longitudinal directions, \( A = 0, \ldots, p \), while lower-case indices run over the transverse directions, \( i = p+1, \ldots, 9 \), and Greek indices refer to all ten directions. The graviton two-point function on the disk was found to be

\[ F_g = -\varepsilon^{(1)}_{ij} \varepsilon^{(2)}_{il} k^2 \prod_{A=0}^{p} \delta(k^A + p^A), \]  

where \( k \) and \( p \) are the momenta of the gravitons, and

\[ k^2 = k_A k^A = p_A p^A. \]

The corresponding calculation for two anti-symmetric tensor particles gave the following result:

\[ F_B = \left[ 2\varepsilon^{(1)}_{ij} \varepsilon^{(2)}_{il} k^k k^l - \varepsilon^{(1)}_{ij} \varepsilon^{(2)}_{ij} k_i k_j \right] \prod_{A=0}^{p} \delta(k^A + p^A). \]

These formulae apply both to the ten-dimensional theory and to its toroidal compactifications, where the \( p \)-brane may be wrapped around some of the cycles. In the latter case, the momenta in the compact directions are, of course, quantized, and the corresponding Dirac delta functions are understood to be replaced by Krönecker delta functions.

The manifestly gauge invariant form of (4) is

\[ F_B = \frac{1}{3} H^{(1)}_{ijl} H^{(2)}_{ijkl} \prod_{A=0}^{p} \delta(k^A + p^A), \]  

\[ 3 \text{In the compactified theories one may also consider scattering of winding states, for which the results are different from (3), (4), but are also explicitly computable.} \]
where

\[ H_{\alpha\beta\gamma}^{(1)} = i(k_\alpha \epsilon^{(1)}_{\beta\gamma} + k_\beta \epsilon^{(1)}_{\gamma\alpha} + k_\gamma \epsilon^{(1)}_{\alpha\beta}). \]  

Note that the term containing \( H_{Ajl}^{(1)} H_{Ajlt}^{(2)} \), which could contribute to the amplitude for transverse polarizations, is “mysteriously” absent. In section 4 this feature of the polarization dependence will be reproduced for scattering off extremal p-branes in the low-energy effective field theory. For R-R scattering we will also find that field theory provides a valuable check on the polarization dependence of the exact string amplitude. This check will also strengthen our confidence in the fact that the p-brane solutions in supergravity are simply low-energy approximations to D-branes.

The D-instanton \((-1\text{-brane})\) is a special case where the low-energy approximation becomes exact. For \( p > -1 \), (4) and (5) exhibit an infinite sequence of poles in the t-channel, and correspondingly in the s-channel. These characteristically stringy properties of the scattering amplitudes guarantee the Regge behavior and the exponential decrease with energy at a fixed angle [29]. For \( p = -1 \), however, these stringy features disappear because \( k^2_{\parallel} = 0 \). The graviton two-point function now vanishes, while that for the antisymmetric tensor simplifies down to

\[ \frac{1}{3k \cdot p} H_{\alpha\beta\gamma}^{(1)} H_{\alpha\beta\gamma}^{(2)} \]  

In section 4 we will be able to reproduce this answer \textit{exactly} by scattering antisymmetric tensor particles off of the instanton solution in supergravity.

### 3 R-R scattering in string theory

This will be entirely devoted to world-sheet computations, so we begin by reviewing some basic facts about the vertex operators for the massless states in the R-R sector. The vertex operator in the canonical ghost picture for an R-R gauge boson with an m-form field strength polarization \( F_{(m)} \) and momentum \( k \) is [16, 38]

\[ V(z, \bar{z}) =: e^{-\frac{i}{2} \phi(z)} S_\alpha(z) e^{ik \cdot X(z)} : F_{(m)}^{\alpha \beta} : e^{-\frac{i}{2} \phi(\bar{z})} \bar{S}_\beta(\bar{z}) e^{ik \cdot \bar{X}(\bar{z})} : \]  

(type IIA)

\[ V(z, \bar{z}) =: e^{-\frac{i}{2} \phi(z)} S_\alpha(z) e^{ik \cdot X(z)} : F_{(m)}^{\alpha \beta} : e^{-\frac{i}{2} \phi(\bar{z})} \bar{S}_\beta(\bar{z}) e^{ik \cdot \bar{X}(\bar{z})} : \]  

(type IIB) (8)

where we have defined

\[ F_{(m)}^{\alpha \beta} = \frac{1}{m!} F_{\mu_1 \ldots \mu_m} \gamma^{\mu_1} \ldots \gamma^{\mu_m}. \]  

(9)

We will use a representation in which the 32 \times 32 Dirac gamma matrices are off-diagonal:

\[ \gamma^\mu = \begin{pmatrix} 0 & \gamma^{\mu\alpha\beta} \\ \gamma_{\alpha\beta} & 0 \end{pmatrix}. \]  

(10)

We normalize the \( \gamma^\mu \) so that \( \{\gamma^\mu, \gamma^\nu\} = -2\eta^{\mu\nu} \), and we pick our representation so that
Figure 1: Scattering of R-R gauge bosons off a p-brane. The low momentum transfer limit is mediated as shown by t-channel exchange of a massless mode.

$$\gamma_{11} = \gamma^0 \cdots \gamma^9 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{11}$$

The leading order contribution to the scattering of an R-R boson off a p-brane, as illustrated in Fig. 1, comes from the two point function on the disk. Suppose the incoming and outgoing bosons have field strength polarizations $F_1^{(1)}$ and $F_2^{(2)}$ and momenta $k$ and $-p$. Then the amplitude is

$$F = \int_\Delta d^2z \langle V^{(1)}(z, \bar{z})V^{(2)}(0, 0) \rangle. \tag{12}$$

The computation of $F$ is possible in the canonical ghost picture because the constraint on the superghost background charge is that the total sum of the charges over both right and left moving sectors is $-2$. Because of the way we will treat spin fields, it is convenient to evaluate the two point function on the half-plane $\mathbb{H}$ and then conformally map the result to the disk to obtain the integrand of $\langle 12 \rangle$.

$^4$Similar disk calculations with the conventional Neumann boundary conditions, and their comparison with field theory, were attempted a long time ago $^3$.\hfill
A convenient method for dealing with the mixed boundary conditions is to extend the holomorphic fields to the whole complex plane, \( \mathbb{C} \), in such a way that their OPE’s on \( \mathbb{C} \) reproduce all the OPEs among holomorphic and antiholomorphic fields on \( \mathbb{H} \). For example, to obtain Dirichlet boundary conditions on all of the \( \psi^\mu \) and \( \tilde{\psi}^\mu \), we extend \( \psi^\mu \) to \( \mathbb{C} \) so that

\[
\tilde{\psi}^\mu(\bar{z}) = -\psi^\mu(\bar{z}) \quad \text{for } z \in \mathbb{H}.
\] (13)

Then the OPE

\[
\psi^\mu(z)\psi^\nu(w) = \frac{-\eta^{\mu\nu}}{z - w} + \text{regular}
\] (14)

reproduces the desired OPEs for \( \psi^\mu \) and \( \tilde{\psi}^\mu \), the interesting one of which is

\[
\psi^\mu(z)\tilde{\psi}^\nu(\bar{w}) = \frac{\eta^{\mu\nu}}{z - \bar{w}} + \text{regular}.
\] (15)

Given the boundary conditions appropriate for a \( p \)-brane, that is Neumann boundary conditions for \( \mu = 0, \ldots, p \) and Dirichlet boundary conditions for \( \mu = p + 1, \ldots, 9 \), it is clear (from bosonization, for example) that the analog of (13) for spin fields is of the form

\[
\tilde{S}_\alpha(\bar{z}) = M_{\alpha\beta} S_\beta(\bar{z}) \quad \text{(type IIA)}
\]

\[
\tilde{S}_\alpha(\bar{z}) = M_{\alpha\beta} S_\beta(\bar{z}) \quad \text{(type IIB)}.
\] (16)

Determining the matrix \( M \) seems somewhat nontrivial, but the solution is almost inevitable given the symmetries of the problem:

\[
M = \gamma^0 \ldots \gamma^p
\] (17)

up to an overall power of i which is immaterial for our calculations. To see how this result comes about for 1-branes, note that if we bosonize using

\[
\frac{i}{\sqrt{2}} \left( \psi^{2i-1}(z) \pm i \psi^{2i}(z) \right) =: e^{\pm i \phi_i(z)}:
\] (18)

where \( \psi^{10} = i\psi^0 \), then the desired boundary conditions on the \( \psi^\mu \) are reproduced if we set

\[
\tilde{\phi}_i(\bar{z}) = \phi_i(\bar{z}) + \pi \quad \text{for } i = 1, \ldots, 4
\]

\[
\tilde{\phi}_5(\bar{z}) = \phi_5(\bar{z}).
\] (19)

Now it is easy to see that

\[
\tilde{S}_\alpha(\bar{z}) =: e^{i\lambda_\alpha \tilde{\phi}_\alpha(\bar{z})} := (\text{sgn } \lambda_{\alpha_5}) S_\alpha(\bar{z}) = (\gamma^0 \gamma^1)_{\alpha\beta} S_\beta(\bar{z})
\] (20)

in this representation where \( \gamma^0 \gamma^1 \) is diagonal. Similar arguments go through in the case of general \( p \).

Using (17) we can bring the two point function into a manageable form: for type IIB,
\[
\langle V^{(1)}(z, \bar{z})V^{(2)}(w, \bar{w}) \rangle = (z - \bar{z})^{-1/4 + 2k_\parallel^2}(w - \bar{w})^{-1/4 + 2k_\parallel^2} \\
\cdot |z - w|^{2(1/4 + k_\parallel p)}|z - \bar{w}|^{2(1/4 + k_\parallel p)} \\
\cdot F^{(1)}_\alpha \gamma^\mu M_{\rho}^\beta F^{(2)}_\gamma \gamma^\sigma M_{\sigma}^\delta \langle S_\alpha(z)S_\beta(\bar{z})S_\gamma(w)S_\delta(\bar{w}) \rangle \\
\cdot \prod_{A=0}^P \delta(k^A + p^A).
\]
(21)

The four point function of spin fields has been calculated [10], and for \(\gamma_{\alpha \beta}^\mu\) symmetric in \(\alpha, \beta\) it reads

\[
\langle S_\alpha(z_1)S_\beta(z_2)S_\gamma(z_3)S_\delta(z_4) \rangle = \frac{z_{14}z_{23}\gamma_{\alpha \beta}^\mu \gamma_{\gamma \delta}^\mu - z_{12}z_{34}\gamma_{\alpha \delta}^\mu \gamma_{\mu \beta}^\gamma}{2(z_{12}z_{13}z_{23}z_{24}z_{34})^{3/4}}
\]
(22)

(the factor of 2 in the denominator compensates for our normalization of the \(\gamma^\mu\), which is different from [10]). The final result for the two point function on the half plane is

\[
\langle V^{(1)}(z, \bar{z})V^{(2)}(w, \bar{w}) \rangle = (z - \bar{z})^{-1/4 + 2k_\parallel^2}(w - \bar{w})^{-1/4 + 2k_\parallel^2} \\
\cdot |z - w|^{2(1/4 + k_\parallel p)}|z - \bar{w}|^{2(1/4 + k_\parallel p)} \\
\cdot \frac{1}{2} \left( |z - \bar{w}|^2 P_1 - (z - \bar{z})(w - \bar{w}) P_2 \right) \\
\cdot \prod_{A=0}^P \delta(k^A + p^A)
\]
(23)

where

\[
P_1 = \left( F^{(1)}_\alpha M_{\rho}^\beta F^{(2)}_\gamma \gamma^\sigma M_{\sigma}^\delta \gamma_{\delta \alpha} \right) = \operatorname{tr} \left( \frac{1 + \gamma_{11}}{2} F^{(1)}_\alpha M_{\rho}^\beta \gamma^\mu \right) \operatorname{tr} \left( \frac{1 + \gamma_{11}}{2} F^{(2)}_\gamma M_{\sigma}^\delta \gamma_{\delta \alpha} \right) \\
P_2 = \left( F^{(1)}_\alpha M_{\rho}^\beta F^{(2)}_\gamma \gamma^\sigma M_{\sigma}^\delta \gamma_{\delta \alpha} \right) = \operatorname{tr} \left( \frac{1 + \gamma_{11}}{2} F^{(1)}_\alpha M_{\rho}^\beta \gamma^\mu \right) \operatorname{tr} \left( \frac{1 + \gamma_{11}}{2} F^{(2)}_\gamma M_{\sigma}^\delta \gamma_{\delta \alpha} \right).
\]
(24)

Writing \(P_1\) and \(P_2\) in terms of 32 × 32 gamma matrices rather than 16 × 16 blocks is convenient because these expressions are correct both for type IIA and type IIB, whereas the index structure for the 16 × 16 block expressions is different in the two cases. The factors of \((1 - \gamma_{11})/2\) enforce the sums over the correct type of indices, upper or lower.

Mapping back to the unit disk and setting \(x = r^2\), where \(r\) is the radial coordinate, we find that the scattering amplitude is

\[
F = \int_0^1 dx (1 - x)^{-1/4 + 2k_\parallel^2}x^{-1/4 + k_\parallel p} \left( P_1 + (1 - x) P_2 \right) \prod_{A=0}^P \delta(k^A + p^A)
\]
\[
= \left[ \frac{\Gamma(1 + 2k_\parallel^2)\Gamma(k \cdot p)}{\Gamma(1 + 2k_\parallel^2 + k \cdot p)} \right] \left[ \frac{2k_\parallel^2 + k \cdot p}{2k_\parallel^2} \right] P_1 + P_2 \prod_{A=0}^P \delta(k^A + p^A)
\]
\[
= \left[ \frac{\Gamma(1 - 2s)\Gamma(-t/2)}{\Gamma(1 - 2s - t/2)} \right] \left[ \frac{s + t/4}{s} \right] P_1 + P_2 \prod_{A=0}^P \delta(k^A + p^A),
\]
(25)
where we have defined \( s = -k_{\parallel}^2 \) and \( t = -(p + k)^2 \). This is our main result: the two-point function of any R-R n-form fields off of a Dirichlet p-brane is reduced to evaluating traces of gamma matrices in ten dimensions.\(^5\) The dependence on polarizations is contained in \( P_1 \) and \( P_2 \), and all amplitudes are multiplied by the universal prefactor

\[
\frac{\Gamma(1 + 2k_{\parallel}^2)\Gamma(k \cdot p)}{\Gamma(k \cdot p + 2k_{\parallel}^2 + 1)}.
\]

This prefactor is the same as in the NS-NS scattering amplitudes, (2)-(5). For \( p > -1 \) this prefactor contains an infinite series of poles and thus guarantees the Regge behavior of D-branes. For \( p = -1, k_{\parallel}^2 = 0 \) and the prefactor collapses to a single pole, \( 1/(p \cdot k) \), which implies a field theoretic structure of the scattering in the D-instanton background (the amplitude, in fact, diverges if \( P_1 \neq 0 \), which is the case for the scattering of R-R scalars only). In summary, as anticipated in [23] all the qualitative features of the D-branes found from NS-NS scattering apply to R-R scattering as well.

The traces in (24) are straightforward but tedious to evaluate. The main tool one uses is the general (anti)-commutator of anti-symmetrized gamma matrices:

\[
\left[ \gamma^{\mu_1} \cdots \gamma^{\mu_m}, \gamma_{\nu_1} \cdots \gamma_{\nu_n} \right]_{(-1)^{m+n+1}} = \\
\sum_{j=1}^{m} (-1)^{1+m_j+j(j+1)/2} \binom{m}{j} \binom{n}{j} 2^j j! \delta_{\mu_1}^{\nu_1} \cdots \delta_{\mu_j}^{\nu_j} \gamma^{\mu_{j+1} \cdots \mu_m} \gamma_{\nu_{j+1} \cdots \nu_n},
\]

which is essentially Wick’s theorem for gamma matrices. The result is

\[
P_1 = -256(p + 2) \delta_{m-p-2} F_{(m)}^{(1)} + (-1)^{m(m+1)/2} \delta_{m+p-8} F_{(m)}^{(1)}
\]

\[
\delta_{n-p} F_{(n)}^{(2)} + (-1)^{n(n+1)/2} \delta_{n+p-8} F_{(n)}^{(2)} \bigg|_{p+2}^{p+1}
\]

\[
-256 \left( \delta_{m-p} F_{(m)}^{(1)} + (-1)^{m(m+1)/2} \delta_{m+p-10} F_{(m)}^{(1)} \right)
\]

\[
\delta_{n-p} F_{(n)}^{(2)} + (-1)^{n(n+1)/2} \delta_{n+p-10} F_{(n)}^{(2)} \bigg|_{p}^{p}
\]

\[
P_2 = (-1)^{mp+m(m+1)/2+p(p+1)/2} 32 \sum_{j=0}^{\min(p+1,n)} (-1)^{j} 2^j (4 - n - p + j) \binom{n}{j}
\]

\[
\delta_{m-n} F_{(m)}^{(1)} + (-1)^{m(m+1)/2} \delta_{m+n-10} F_{(m)}^{(1)} F_{(n)}^{(2)} \bigg|_{j}^{n}.
\]

The notation requires some explanation. If \( U_{(n)} \) and \( V_{(n)} \) are n-forms, then

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\(^5\)Equation (25) is correct only up to an overall factor, which includes not only the trivial factors of 2 and \( \pi \) which we have neglected, but also factors related to the overall normalization of disk amplitudes as compared to sphere amplitudes. These latter factors are necessary in order to make a completely rigorous comparison with field theory based on the effective action for type II theories. We will be satisfied in the following section to demonstrate agreement in the infrared between string theory and field theory up to such overall factors.
\[ \langle U_{(n)}, V_{(n)} \rangle_j = \frac{1}{n!} \eta^{A_1B_1} \ldots \eta^{A_jB_j} \eta^{\mu_{j+1}\nu_{j+1}} \ldots \eta^{\mu_{n}\nu_{n}} U_{A_1\ldots A_j \mu_{j+1} \ldots \mu_n} V_{B_1\ldots B_j \nu_{j+1} \ldots \nu_n} \]  

(29)

where as usual the \( A_i \) and \( B_i \) run only from 0 to \( p \) while the \( \mu_i \) and \( \nu_i \) from 0 to 9. Hodge duals are defined by

\[ (\ast U_{(n)})_{\mu_1 \ldots \mu_{10} - n} = \frac{1}{n!} \varepsilon_{\mu_1 \ldots \mu_{10}} U^{\mu_{11} \ldots \mu_{10}} \]  

(30)

In the following section we write out the polarization dependences explicitly for some special cases and find a simple physical meaning of these formulae.

### 4 Scattering off of black \( p \)-branes

Now, as promised, we will indicate how the \( t = 0 \) pole of the scattering amplitudes (25) can be reproduced by the low-energy effective field theory. For \( p > -1 \), the \( t = 0 \) pole is

\[ F_{\text{string theory}} = -\frac{2}{t} (P_1 + P_2) \]  

(31)

In this equation and in the rest of this section, we suppress the momentum conserving delta functions in (25). In principle, one should be able to reproduce (31) by directly evaluating the graphs corresponding to the \( t \)-channel exchange of a massless particle, as in Fig. 1. However, it is simpler to consider an equivalent description of the phenomena: scattering at large impact parameter of R-R bosons by the classical gravitational field of extremal black \( p \)-branes. To put it in astrophysical terms, we want to look at gravitational lensing of R-R light by \( p \)-branes.

Because \( p \)-branes are BPS saturated, we expect their infrared properties to be described by extreme R-R charged \( p \)-brane solutions to the low-energy effective action:

\[
\begin{align*}
\text{d}s^2 &= A^{-1/2} (-\text{d}t^2 + \text{d}x_1^2 + \ldots + \text{d}x_p^2) + A^{1/2} \left( \text{d}y^2 + y^2 \text{d}\Omega_{8-p}^2 \right) \\
\text{e}^{-2\phi} &= A^{(p-3)/2} \\
F_{(p+2)} &= \frac{Q}{y^{8-p}} A^{-2} \text{d}t \wedge \text{d}x_1 \wedge \ldots \wedge \text{d}x_p \wedge \text{d}y 
\end{align*}
\]

(32)

where \( F_{(p+2)} \) is the R-R field strength coupling to the brane, and

\[ A = 1 + \frac{2}{t - p y^{7-p}} . \]

(33)

For \( p > -1 \) these solutions may be written down simply by a coordinate transformation of the extremal R-R charged \( p \)-branes found in [13]. While [13] discussed only \( 0 \leq p \leq 6 \),

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6As explained in [13], the 3-brane is a special case because it couples to the self-dual 5-form field strength; therefore, the third line of (12) is modified.
solutions (32), (33) may be extrapolated in an obvious way to \( p = 7 \) \[41\] and \( p = 8 \). For the 7-brane the solution is (32) with

\[ A = 1 + 2Q \ln(y/y_0) , \] (34)

while for the 8-brane,

\[ A = 1 + 2Qy = 1 + 2Q|x_9| . \] (35)

A new feature we find for \( p = 7 \) and \( 8 \) is that \( A \) grows with the distance from the \( p \)-brane. Thus, the geometry is not asymptotically flat.

The D-instanton \((p = -1)\) is a special case which we are going to discuss in some detail. The D-instanton is a source of the R-R scalar, which is essentially a ten-dimensional axion. In order to make its axionic properties manifest, we are going to use its dual, 8-form, description. Our goal, therefore, is to find an instanton solution to the following euclidean action,

\[
S = \int d^{10}x \sqrt{G} \left[ e^{-2\phi}(-R + 4(\partial \phi)^2) + \frac{1}{2 \cdot 9!} F_{(9)}^2 \right]
\] (36)

It is not hard to verify that the following is a solution\[7\]

\[
\begin{align*}
ds^2 &= A^{1/2} (dy^2 + y^2 d\Omega_9^2) \\
e^{-2\phi} &= A^{-2} \\
F_{(9)} &= \frac{Q}{y^7} A^{-2} \ast dy
\end{align*}
\] (37)

where

\[ A = 1 + \frac{1}{4} \frac{Q}{y^8} . \] (38)

This solution has the following interesting feature: the Einstein metric,

\[ g_{ij} = G_{ij} e^{-\phi/2} = \delta_{ij} , \] (39)

is flat! Since the Einstein metric describes the physical gravitational field, we conclude that the R-R charged instanton in 10 dimensions emits a dilaton, but no gravitational field.

Large impact parameter scattering can be computed by expanding in powers of

\[
\frac{1}{7 - p} \frac{Q}{y^{7-p}} = \frac{2\pi^{(9-p)/2}}{\Gamma((9-p)/2)} \int \frac{d^{9-p}q}{(2\pi)^{9-p}} e^{iy \cdot q} \frac{Q}{q^2} .
\] (40)

Let us consider scattering of \( n \)-form field strength bosons. Because the gauge fields are abelian, they have no self-interactions and are therefore insensitive to their own background values. The relevant piece of the effective action is

\[ \text{After we independently found this solution we were informed by M. Green that it was originally obtained in [41].} \]
\[
S = \int d^{10}x \sqrt{G} \frac{1}{2 \cdot n!} F_{(n)}^2 \\
= \int d^{10}x \frac{1}{2} \left\langle F_{(n)} , F_{(n)} \right\rangle_0^n \\
+ \int d^{10}x \frac{1}{2} \frac{Q}{T - p} \left[ (4 - p - n) \left\langle F_{(n)} , F_{(n)} \right\rangle_0^n + 2n \left\langle F_{(n)} , F_{(n)} \right\rangle_1^n \right] \\
+ \ldots 
\]

where in the second and third lines we have expanded up to the one graviton contribution.

The angle bracket notation still indicates contraction of indices with the metric \( \eta^{\mu \nu} \), as in (29). Setting \( q = p + k \), one can see from the right hand sides of (40) and (41) that the single graviton contribution determines the \( 1/t \) pole of the scattering amplitude. Neglecting normalization factors as usual, the \( 1/t \) pole is

\[
F_{\text{field theory}} = \frac{1}{t} \left[ (4 - p - n) \left\langle F_{(1)}^{(1)} , F_{(2)}^{(2)} \right\rangle_0^n + 2n \left\langle F_{(n)}^{(1)} , F_{(n)}^{(2)} \right\rangle_1^n \right] .
\]

The test of whether Dirichlet \( p \)-branes bend R-R light like the extremal black \( p \)-branes (at low momentum transfer, of course) is to compare the polarization dependence of (31) and (42). Let us look at some examples:

- \( p = 0, n = 2 \). This is the most physically familiar case: gravitational lensing of vector gauge bosons by a black hole. It is not hard to work out \( P_1 \) and \( P_2 \) directly from (24):

\[
P_1 = -512 \left\langle F_{(2)}^{(1)} , F_{(2)}^{(2)} \right\rangle_1^2 \\
P_2 = -64 \left\langle F_{(2)}^{(1)} , F_{(2)}^{(2)} \right\rangle_0^2 + 384 \left\langle F_{(2)}^{(1)} , F_{(2)}^{(2)} \right\rangle_1^2 \\
F_{\text{string theory}} = \frac{128}{t} \left[ \left\langle F_{(2)}^{(1)} , F_{(2)}^{(2)} \right\rangle_0^2 + 2 \left\langle F_{(2)}^{(1)} , F_{(2)}^{(2)} \right\rangle_1^2 \right] \\
\sim \frac{1}{t} \left[ F_{ij}^{(1)} F_{ij}^{(2)} + 4 F_{Aij}^{(1)} F_{Aij}^{(2)} + 3 F_{AB}^{(1)} F_{AB}^{(2)} \right].
\]

This matches the field theory result

\[
F_{\text{field theory}} = \frac{2}{t} \left[ \left\langle F_{(2)}^{(1)} , F_{(2)}^{(2)} \right\rangle_0^2 + 2 \left\langle F_{(2)}^{(1)} , F_{(2)}^{(2)} \right\rangle_1^2 \right] .
\]

- \( p = 1, n = 3 \).

\[
P_1 = 768 \left\langle F_{(3)}^{(1)} , F_{(3)}^{(2)} \right\rangle_2^3 \\
P_2 = 192 \left\langle F_{(3)}^{(1)} , F_{(3)}^{(2)} \right\rangle_1^3 - 768 \left\langle F_{(3)}^{(1)} , F_{(3)}^{(2)} \right\rangle_2^3
\]
\[ F_{\text{string theory}} = -\frac{384}{t} \langle F^{(1)}(3), F^{(2)}(3) \rangle^3 \]
\[ \sim \frac{1}{t} \left[ F^{(1)}_{Aij} F^{(2)}_{Aij} + 2 F^{(1)}_{ABj} F^{(2)}_{ABj} + F^{(1)}_{ABC} F^{(2)}_{ABC} \right] \] (45)

again matching field theory:
\[ F_{\text{field theory}} = \frac{6}{t} \langle F^{(1)}(3), F^{(2)}(3) \rangle^3 . \] (46)

- \( p = 4, n = 4 \). This is a more complex example: scattering of a 4-form field strength off a 4-brane. Here (31) yields

\[ F_{\text{string theory}} = -\frac{256}{t} \left[ \langle F^{(1)}(4), F^{(2)}(4) \rangle^4 + 6 \langle *F^{(1)}(4), *F^{(2)}(4) \rangle^6 \right] + 3 \sum_{j=0}^{3} \frac{(-1)^j 2^j}{j!(3-j)!} \langle F^{(1)}(4), F^{(2)}(4) \rangle^4 \]
\[ \sim \frac{1}{t} \left[ F^{(1)}_{ijkl} F^{(2)}_{ijkl} + 2 F^{(1)}_{Aijk} F^{(2)}_{Aijk} - 2 F^{(1)}_{ABcj} F^{(2)}_{ABcj} - F^{(1)}_{ABCD} F^{(2)}_{ABCD} \right] . \] (47)

The field theory calculation gives

\[ F_{\text{field theory}} = -\frac{4}{t} \left[ \langle F^{(1)}(4), F^{(2)}(4) \rangle^4 - 2 \langle F^{(1)}(4), F^{(2)}(4) \rangle^4 \right] \]
\[ \sim \frac{1}{t} \left[ F^{(1)}_{ijkl} F^{(2)}_{ijkl} + 2 F^{(1)}_{Aijk} F^{(2)}_{Aijk} - 2 F^{(1)}_{ABcj} F^{(2)}_{ABcj} - F^{(1)}_{ABCD} F^{(2)}_{ABCD} \right] . \] (48)

once again reproducing the correct relative factors between terms. As this case exemplifies, terms of the form \( \langle *F^{(1)}(n), *F^{(2)}(n) \rangle^{10-n} \) appear in the string theory expressions for \( n + p = 8 \) or 10. The bracket notation then becomes inconvenient and it is necessary to expand out all expressions into \( SO(1,p) \times SO(9-p) \) invariants \( F^{(1)}_{A_1...A_{i_1+1}...i_n} F^{(2)}_{A_1...A_{i_1+1}...i_n} \) before a comparison with field theory can be made.

- \( p = -1 \). In this case, the kinematics changes slightly: because there are no directions parallel to the brane, \( s = 0 \). As a result, the beta function prefactor simplifies. For \( n = 3 \), the full result (25) reduces to

\[ F_{\text{string theory}} = \frac{128}{t} \langle F^{(1)}(n), F^{(2)}(n) \rangle^n \]
\[ \sim \frac{1}{t} F^{(1)}_{\mu_1...\mu_n} F^{(2)}_{\mu_1...\mu_n} \] (49)

which is reproduced exactly by the one-graviton field theory calculation using the solution (37):
\[ F_{\text{field theory}} = \frac{2}{t} \left\langle F^{(1)}_{(n)}, F^{(2)}_{(n)} \right\rangle_0 \]  

(50)

For \( n = 1 \), \( P_1 \neq 0 \), and so the amplitude (25) diverges. However, because of the double trace form of \( P_1 \), we suspect that the divergent term is canceled by the disk–cylinder contribution, which is of the same order in \( g \). In this world sheet configuration one R-R scalar vertex operator is inserted on a disk and another on a cylinder. If the divergence cancels, then the string theory and one-graviton field theory calculations again agree exactly.

- \( n = 5 \). We find that the scattering amplitudes for the self-dual 5-forms off of any \( p \)-brane \textit{vanish} both in string theory and in field theory. Here one encounters an interesting subtlety: it is impossible to write down a covariant action for the self-dual 5-form in type IIB supergravity. The procedure that seems to work, however, is to calculate the two-point function from the standard quadratic action for a general 5-form field,

\[ S = \int d^{10}x \sqrt{G} \frac{1}{2 \cdot 5!} F_{(5)}^2, \]  

(51)

and to impose the self-duality constraint on the answer. This constraint makes the residue of the \( t = 0 \) pole, (12), vanish for all \( p \).

Altogether there are 25 different scattering processes where our field theory analysis serves as a check on the string theory computation: \( n = 1, 3, 5 \) for \( p \) odd and \( n = 2, 4 \) for \( p \) even, with \(-1 \leq p \leq 8\). We have checked the agreement between string theory and field theory in all of the 25 possible cases, some of them involving quite nontrivial cancelations between \( P_1 \) and \( P_2 \). The consistent agreement in the polarization dependence seems to us strong evidence for regarding extreme black \( p \)-branes as the low-energy description of Dirichlet \( p \)-branes.

Another interesting check on our results is provided by performing a T-duality transformation on all of the coordinates. This transformation interchanges the Neumann with the Dirichlet boundary conditions, which means that the \( p + 1 \) directions along the brane world volume are interchanged with the \( 9 - p \) transverse directions. Thus, the T-duality relates scattering of \( n \)-forms off \( p \)-branes to scattering of \( n \)-forms off \( 8 - p \)-branes: one simply interchanges the transverse (lower-case) with the longitudinal (capital) indices in the polarization dependence. For instance, from the polarization dependence (13) for \( p = 1, n = 3 \) we infer the polarization dependence for \( p = 7, n = 3 \):

\[ \sim \left[ F^{(1)}_{ijk} F^{(2)}_{ijk} + 2 F^{(1)}_{Aij} F^{(2)}_{Aij} + F^{(1)}_{ABI} F^{(2)}_{ABI} \right]. \]  

(52)

This is easily confirmed by explicit calculations both in string theory and in field theory.

\[ ^8 \text{Such disconnected world sheets are well-known to contribute to D-instanton processes}} \]
In conclusion, let us note that the field theory also explains the $1/t$ pole in the string NS-NS amplitudes. Consider, for instance, the scattering of NS-NS antisymmetric tensors off a $p$-brane. The relevant part of the effective action is

$$\int d^{10}x \sqrt{G}e^{-2\phi} \frac{1}{12} H_{\mu\nu\lambda}^2.$$  (53)

Substituting, for instance, the 0-brane solution we find that small angle scattering is described by

$$\frac{1}{t} H_{ijl}^{(1)} H_{ijl}^{(2)}.$$  (54)

Just as in the string amplitude (5), the term containing $H_{0ijl}^{(1)} H_{0ijl}^{(2)}$ is absent! Similar checks of the polarization dependence work out for the scattering of gravitons.

In the D-instanton case we find that the field theory amplitude reproduces the complete string amplitude (7). This illustrates the special point-like nature of this object. The field theory results are further valuable as a guide towards future string calculations. For example, in scattering NS-NS antisymmetric tensors with arbitrary polarizations off $p$-branes we find the following universal formula:

$$\frac{1}{t} \left[ H_{ijl}^{(1)} H_{ijl}^{(2)} - 3H_{ABl}^{(1)} H_{ABl}^{(2)} - 2H_{ABC}^{(1)} H_{ABC}^{(2)} \right].$$  (55)

This pole at $t = 0$ should be reproduced by the string calculation with arbitrary polarizations, which we leave for future work.

5 Discussion

The low-energy supergravity includes massless fields only and is, therefore, only capable of reproducing the pole at $t = 0$ in the exact scattering amplitude ($t = 2p \cdot k$). The poles at negative integer values of $t$ come from the fact that the $p$-brane also excites an infinite set of massive background fields. This could be anticipated on general grounds. One feature of the exact D-brane scattering that could hardly be anticipated, however, is the infinite sequence of s-channel poles at integer values of $\alpha' k_\parallel^2$. These poles reveal a discrete spectrum of D-brane excitations associated with the massive states of the auxiliary open strings. Since these open strings are in reality closed strings partly trapped behind a horizon, they implement a stringy version of the horizon dynamics anticipated by the “stretched horizon” ideas [42]. It is thus plausible to identify $D$-branes containing such horizon excitations with non-extremal $p$-branes.

Let us review the particularly interesting example of the 0-brane in type IIA theory, which was discussed in detail in [29]. The unexcited Dirichlet 0-brane describes an extremal ten-dimensional dilatonic black hole, carrying a basic unit of the R-R electric charge. Thus, the 0-brane gives us a new insight into black holes in $d = 10$ (black holes in $d < 10$ could be studied by wrapping fundamental strings around the cycles of tori [38]). The mass of
the extremal black hole is $M_0 = 1/(g\sqrt{\alpha'})$. Excited, non-extremal states are obtained by attaching the auxiliary open strings to the 0-brane. The masses of such states are

$$M = \frac{1}{g\sqrt{\alpha'}} + \sum_{i=1}^{k} \sqrt{\frac{n_i}{\alpha'}} + O(g) ,$$  \hspace{1cm} (56)

where we have $k$ open strings with excitation levels $n_i$. We believe that these excited states should be interpreted as non-extremal black holes. Thus, the black hole spectrum is discrete, but becomes continuous in the limit of high excitation numbers (we should also keep in mind that all the $n_i > 0$ states have finite widths). It is remarkable that the gaps in the spectrum do not vanish even in the weak coupling limit. \footnote{We are grateful to C. G. Callan for pointing this out to us.} Perhaps this points to an 11-dimensional interpretation of the discrete spectrum (the connection of the type IIA theory with some unknown theory in 11 dimensions was first proposed in [6, 7]).

The degeneracy of black holes of mass $M$ is given by the number of states of the open string gas with total energy $E = M - M_0$. The number of open string states at oscillator level $n$, $d_n$, is well known to grow exponentially for large $n$:

$$d_n \sim n^{-11/4} \exp \left( \pi \sqrt{8n} \right) .$$  \hspace{1cm} (57)

This number includes states of spin $J$ ranging from 0 to $n + 1$. If we restrict ourselves to non-rotating black holes, we need to isolate the number of $J = 0$ open string states at level $n$. This has been calculated in [43] and turns out to be suppressed only by a factor $1/\sqrt{n}$ compared to $d_n$:

$$d_n^{(J=0)} \sim n^{-13/4} \exp \left( \pi \sqrt{8n} \right) .$$  \hspace{1cm} (58)

If the total energy is

$$E = \sqrt{\frac{N}{\alpha'}} = \sum_{i=1}^{k} \sqrt{\frac{n_i}{\alpha'}}$$  \hspace{1cm} (59)

then the degeneracy is

$$d_{\{n_i\}} = \prod_i d_{n_i} \sim d_N$$  \hspace{1cm} (60)

where we have ignored the slowly varying prefactors multiplying the exponentials. Thus, each multiple-string configuration contributes with roughly the same weight to the degeneracy. This means that the degeneracy is $d_N$ times the number of ways the total energy may be divided among different open strings. This number, $\Omega_N$, grows exponentially, but slower than $d_N$,

$$\Omega_N \sim \exp \left( \frac{cN^{1/3}}{N} \right) .$$  \hspace{1cm} (61)

Thus, the entropy of the ten-dimensional black holes, which are non-extremal excitations of the extremal R-R charged black hole, is given by

$$S = \ln(\Omega_N d_N) = \pi \sqrt{8\alpha'} (M - M_0) + O\left( (M - M_0)^{2/3} \right) .$$  \hspace{1cm} (62)
It is interesting to compare the Dirichlet 0-brane result with the Bekenstein-Hawking entropy of the near-extremal RR-charged black hole in 10 dimensions. The relevant supergravity solution was found in [13],

\[ ds^2 = -\Delta_+ \Delta_-^{-1/2} dt^2 + \Delta_+^{-1} \Delta_-^{-17/14} dr^2 + r^2 \Delta_-^{-3/14} d\Omega_8^2 \]

\[ e^{-2\phi} = \Delta_-^{3/2} \]

where

\[ \Delta_\pm(r) = 1 - \frac{r^7_\pm}{r^7_0} \]

Transforming the above to the Einstein metric, we find that the “area” of the horizon is

\[ A = \omega_8 r^8_+ \left[ 1 - \frac{r^7_+}{r^7_0} \right]^{9/14} \]

where \( \omega_8 \) is the “area” of a unit sphere in 8 dimensions. The ADM mass is given by

\[ M = \frac{8r_+^7 - r_0^7}{2\kappa^2} \]

For nearly extremal black holes, \( r_\pm = r_0 \pm \delta r \). In this regime, the Bekenstein Hawking entropy, \( 2\pi A/\kappa^2 \), scales as

\[ S_{BH} \sim M_0^{1/2} (\delta M)^{9/14} \]

This is clearly different from the scaling \( S \sim \delta M \) found using the D-brane counting of states. Notice however that in equation (12) we have assumed that we have just a single zero brane. If we consider \( n_0 \) zero branes, the second term in (62) is more important and gives a contribution that, in the simplest approximation (disregarding completely the first term), behaves as \( S \sim M_0^{2/3} (\delta M)^{2/3} \) which is indeed closer to (67). The discrepancy may be attributed to the fact that the string coupling is very strong near the horizon. In view of this result, 10-dimensional black holes deserve a more detailed study in which the 0-brane description will undoubtedly be a useful tool.

In general, D-branes are an ideal black hole laboratory because they naturally create heavy black holes in the weak coupling limit, with masses of order \( 1/g \) (for comparison, in order to make a black hole out of a long fundamental string, one needs to wrap it many times around a non-contractible cycle [33]). In order to study black holes in four dimensions we may toroidally compactify the ten-dimensional theory. A Dirichlet 6-brane wrapped around the 6-torus is then a 4-dimensional black hole, whose properties may be studied by scattering massless states off of it. We postpone a detailed discussion of these fascinating issues for future work.

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