Amplitude dependent damping from granular viscoelastics

B Darabi* and J A Rongong†

Department of Mechanical Engineering, The University of Sheffield, Sheffield, S1 3JD, UK
*Corresponding author
E-mail: b.darabi@sheffield.ac.uk
†E-mail: j.rongong@sheffield.ac.uk

Abstract. The ability of a granular medium to dissipate vibrational energy is studied at different frequencies and amplitudes. The filler comprises relatively large particles with significant viscoelasticity and is placed in a rectangular box-shaped container and vibrated perpendicular to the direction of gravity. The performance of a model based on wave behaviour that is suitable for very low amplitude vibrations is compared with discrete elements and experimental results. Frequency dependent behaviour for the viscoelastic material is taken into account. The effects of vibration amplitude on performance are considered carefully – especially at the point where particles begin to move relative to each other. One interesting finding is that internal and interface loss mechanisms are closely interrelated – reduction in internal loss increases the mobility of individual particles and therefore more energy dissipation via friction. As a result, the overall effectiveness of the granular medium is less sensitive to material and configurationally parameters than might be expected.

1. Introduction
Levels of vibration damping in hollow structures can be increased significantly by placing granular materials within the cavities. For low amplitude vibrations, the most effective fillers tend to be made from low modulus materials with high loss factor [1]. However, as the vibration amplitude increases, causing the particles to lose contact with one another temporarily, it has been found that the damping level can drop significantly [2]. At these higher amplitudes, interface friction becomes an important loss mechanism [3] allowing the use of harder materials with low internal loss such as metals. It has also been shown that while interface friction and the internal loss both contribute to the overall damping level, increases in one tend to reduce the other and vice versa [4]. This paper considers the effects of amplitude on the damping performance of a granular medium made from viscoelastic polymer particles whose properties change significantly over the frequency range considered. Experimental results are compared with those from two different numerical approaches. The first numerical approach involves Finite Element (FE) analysis to calculate the effect of internal waves in the granular medium while the second approach relies on the Discrete Element Method (DEM) to represent the behaviour of individual particles [5].

2. Granular system
The study was conducted for an open topped rectangular box with rigid walls and cavity area 190×120 mm that was filled to a nominal depth of 35 mm. The granular medium consisted of 15.1 mm diameter spheres made from a synthetic elastomer. The effectiveness of this system in dissipating vibration energy was studied for dynamic forcing applied perpendicular to gravity.
For the physical experiments the container was constructed using blocks of Perspex 30 mm in thickness. This provided high rigidity and good visibility as shown in Figure 1. The container was suspended using nylon line and light metal springs to simulate free boundary conditions. Excitation was provided via an electrodynamic exciter.

![Image](image1.png)

**Figure 1:** Experimental set-up showing container and particles

Initial tests showed that flexible modes of the container were above 500 Hz while the rigid body modes were below 5 Hz. The container was then filled with 260 randomly placed particles and the power dissipation under sinusoidal loading measured at various amplitudes and frequencies. Following the approach used by Yang [6] and Wong [3], the average power transmitted per cycle is,

\[ P_{av} = \frac{1}{2} \sum f_n v_n \cos(\alpha_n - \beta_n) \]  

(1)

where \( f_n \) and \( \alpha_n \) are the magnitude and phase of the force while \( v_n \) and \( \beta_n \) are the magnitude and phase of the velocity. The experiment was repeated with the container empty to estimate the power dissipated by boundary conditions and electronics.

3. Material properties

The material displayed significant viscoelasticity in the temperature and frequency ranges considered. The material was characterised using Dynamic Mechanical Thermal Analysis (DMTA) equipment to generate the master curves that define properties at different temperatures and frequencies. Complex modulus curves used in this work are shown in Figure 2.

![Image](image2.png)

**Figure 2:** Effect of frequency on Young’s modulus and loss factor of elastomer at 20 °C

![Image](image3.png)

**Figure 3:** Hysteresis loop measurement for particle at 2.5 Hz and 20°C with 0.1 mm pre-compression
Theoretically, the axial load-deflection behaviour of an elastic sphere follows Hertz’s law. The hysteresis loop of a single sphere was measured at low frequency – the test rig and result are presented in Figure 3. Note that only the dynamic component of the force is shown in the hysteresis loop. The underlying elastic curve (solid green line in Figure 3) is clearly nonlinear. The loss factor obtained by dividing the area of the hysteresis loop with the peak stored dynamic strain energy was 0.142 which is close to the value of 0.155 obtained directly from the master curve (Figure 2). The density of the elastomer was 1170 kg/m³ and the Poisson’s ratio was assumed to be approximately 0.45.

4. Model for low amplitude vibration of a granular medium
It has been shown that the damping of structural resonances can be increased significantly if standing waves are induced in an attached medium with high energy dissipation capacity [7]. The approach taken involves representing the granular medium as an equivalent solid using FE analysis.

3.1 Equivalent homogenous solid
A granular medium subjected to a confining pressure and vibrating at low amplitude, where particles remain in contact, can be approximated as a homogeneous solid. For uniform spheres with random packing and rough contacts Walton [8] showed that the bulk modulus could be estimated as,

\[ K = \frac{1}{6} \left( \frac{3E^2 \phi^3 n^2 p}{\pi^2 (1 - \nu_{eff}^2)^2} \right)^{\frac{1}{3}} \]

with \( \nu_{eff} = \frac{V}{2(5 - 3\nu)} \) (2)

where \( E \) and \( \nu \) are the dynamic Young’s modulus and the Poisson’s ratio of the sphere material respectively, \( \phi \) is the packing fraction, \( n \) is the number of contact points and \( p \) is the confining pressure. The effective Young’s modulus and density of the equivalent homogeneous medium are,

\[ E_{eff} = 3(1 - 2\nu)K \quad \rho_{eff} = \rho \phi \] (3)

with \( \rho \) the density of the sphere material. The confining pressure in granular medium is related to the container geometry and the distance from the free surface \( y \). For a rectangular box filled with randomly arranged rough spheres Janssen’s model [(9, 2)] gives,

\[ p = \frac{\rho_{eff} g}{0.75S} \left( 1 - e^{-0.75Sy} \right) \quad \text{with} \quad S = \frac{2(b + w)}{bw} \] (4)

For a shallow box Janssen’s model gives the same results as standard hydrostatic pressure. In this work, it was assumed that the structure had an amorphous arrangement with \( \phi = 0.64 \) and \( n = 6 \) [10].

3.2 Standing waves
Solid elements with 20 nodes were used to model the fill medium and the material was assumed to be fixed rigidly at all five contacting walls. Standing wave information (i.e. natural frequency, mode shape and effective mass for horizontal motion) was obtained for the internal cavity using a standard elastic eigenvalue extraction routine assuming an effective Young’s modulus based on properties at 100 Hz. Natural frequencies were adjusted to account for frequency dependence using,

\[ \omega = \omega_{00Hz} \sqrt{\frac{E_{actual}}{E_{00Hz}}} \] (5)
while mode shape and effective mass were assumed not to change. Properties of typical modes are shown in Figure 4. This approach decoupled the effect of the granular system to that of many SDOF units where $k_{i_{\text{eff}}}$ and $m_{i_{\text{eff}}}$ are the effective mass and stiffness of each mode while,

$$c_{i_{\text{eff}}} = \eta \sqrt{k_{i_{\text{eff}}} m_{i_{\text{eff}}}}$$  \tag{6}$$

where $\eta$ is the loss factor of the viscoelastic material at that particular frequency. As the experiment described in Section 2 represents base motion of the system, the relative displacement amplitude (where $z=x-y$ and $\omega$ and $\omega_b$ are the exciting and natural frequencies) is given by,

$$z = \frac{m_{i_{\text{eff}}} \gamma \omega^2}{k_{i_{\text{eff}}} - \omega^2 m_{i_{\text{eff}}} + j \omega c_{i_{\text{eff}}}}$$  \tag{7}$$

The total power dissipated is obtained from the real part of the total complex power \cite{6,3} from all SDOF units and is calculated using \cite{11},

$$\text{Power dissipated} = \frac{1}{2} \omega^2 |R(z)|^2$$  \tag{8}$$

| Mode 1 | Frequency: 75.2 Hz | Effective mass: 0.301 kg |
|--------|-------------------|--------------------------|
| Mode 71 | Frequency: 186 Hz | Effective mass: 0.032 kg |

**Figure 4:** Typical mode shapes with significant horizontal contribution

5. **Discrete element model**

The calculations performed in the Discrete Element Method (DEM) alternate between the application of Newton’s Second Law to the particles and a force-displacement law at the contacts. In this work DEM, is based on PFC3D (Particle Flow Code in 3 Dimensions) software \cite{12}. In Figure 5, the contact parameters considered are shown. These include mass, normal and shear stiffness ($k_n$ and $k_s$), normal and shear damping ($c_n$ and $c_s$) and the friction coefficient. A description of the approach to finding these parameters can be found in the literature \cite{4} while actual values used are presented in Figure 6. Note that stiffness and damping values were modified to reflect the excitation frequency using the same approach as for the modal properties – see Equation 5.

**Figure 5** Contact model of particles, data at 100Hz
6. **Comparison of theoretical models and experiment**

The DEM model has been shown to be accurate at higher amplitude but over a limited frequency range [4]. In this section, a comparison is made between the damping predicted by the two calculation methods and experiments at very low amplitudes. As a large frequency range was of interest, results are compared for displacement amplitudes of $10^{-7}$, $10^{-6}$ and $10^{-5}$ metres depending on the frequency. These values were selected to ensure maximum accelerations remained below 0.3g. Results are presented in Figure 6. The results show that both prediction methods perform well in the frequency range 80 to 400 Hz. At higher frequencies increased damping in the experiment is thought to arise from the presence of container wall resonances. Below 80 Hz, the wave approach does not match the other methods as it only considers standing waves – the lowest of which is near this figure.

![Figure 6: Comparison of equivalent damping for three different displacement amplitude levels](image)

**Figure 6:** Comparison of equivalent damping for three different displacement amplitude levels

7. **Parametric studies**

The sensitivity of damping to excitation amplitude was studied using DEM – see Figure 7.

![Figure 7: Equivalent damping versus excitation amplitude, at 100Hz excitation](image)

**Figure 7:** Equivalent damping versus excitation amplitude, at 100Hz excitation
It can be seen that in the very low amplitude (solid) region where the particles are in contact permanently performance is close to that predicted using the low amplitude theory. When the amplitude increases (accelerations of 0.03g to 0.3g) the particles start to slide over each other and damping increases: the fraction of energy dissipated by friction increases from 1% at 0.03g to 32% at 0.05g. At higher amplitudes still, the particles slide and roll more freely over each other (convection). In this case there is a trade off between damping due to friction and viscous effects [4] resulting in similar damping levels irrespective of the material loss factor. Increasing the amplitude to extremely high levels, the particles separate from each other (gas region). It can be seen the particles with lower damping reach the gas region earlier because they are less sticky than the other type and more collision can happen so although the damping for each individual particle is less but the total damping increases.

8. Conclusions
This paper considered the damping from polymeric granular materials. The simulation approaches are in good agreement with experiment. In a very low amplitude excitation the theory shows fairly good agreement with simulation and experiment particularly higher than the first natural frequency. The equivalent damping is independent of low vibration amplitudes (solid region) and is higher than the convection and gas regions where the particles collide and move with each other. It can be seen that in the convection region the comparison between two different materials - very high and very low loss factor –shows that the overall damping levels are close to each other and less sensitive to material.

9. References
[1] House J R 1990 Vibration Damping Materials World Intellectual Property Organization International Patent Classification F16F7/00,G10k11/16 , International publication Number WO90/01645
[2] Rongong J A and Tomlinson G R 2002 Vibration damping using granular viscoelastic materials proc. Of ISMA 27 Noise and Vibration Engineering Conference( Leuven: Belgium) 431-40
[3] Wong C X, Daniel M C and Rongong J A 2009 Energy dissipation prediction of particle damperes J. Sound and Vibration 319 91-118
[4] Darabi B and Rongong J A 2012 Polymeric particle dampers under steady-state vibrations J. Sound and Vibration 313 3304-16
[5] Cundall P and Strack O 1979 A Distinct Element Model for Granular Assemblies , Géotechnique 29 47–65
[6] Yang M Y 2003 Development of master design curves for particle impact dampers Doctoral Thesis ( The Pennsylvania State University )
[7] Ungar E E and Kerwin E M 1964 Plate damping due to thickness deformations in attached viscoelastic layers J. the Acoustical Society of America 36 (2) 384-92
[8] Walton K 1987 The effective elastic moduli of a random packing of spheres, J.Mech.Phys.Solids 35 213-26
[9] Janssen H A 1895 Experiments on grain pressure in silos Transactions of the Association of German Engineers 39 1045-49
[10] Zamponi F 2008 Packing close and loose Nature 453 606-7
[11] Rao S S and Yap F F 2010 Mechanical vibration (Pearson Education)
[12] Itasca Consulting Group Inc. 2008 PFC 4D V4.0 manual (Minneapolis/Minnesota : USA)