Calculation of the oil production rate by an average diameter of the bubbles when sampling in the coil

F A Ikhsanova¹, N N Soloviev², I I Gallyamov¹, A A Fatkullin¹, D Sh Nosirov¹ and A A Abdulmanov¹

¹Ufa State Petroleum Technological University, Branch of the University in the City of Oktyabrsky, 54a, Devonskaya St., Oktyabrsky, Republic of Bashkortostan, 452607, Russia
²LLC Gazprom VNIIGAZ
Staraya Basmannaya str., 20, b. 8, Moscow, Russia

E-mail: ichs195@mail.ru

Abstract. Well abandonment is significant under the enormous number of oil deposits with a wide range of reservoir parameters and physicochemical fluid properties. Underground gas storages are a combination of different objects with long life cycles under multicomponent and multifactor uncertainties of external and internal operational, natural and man-made impacts. Therefore, it is difficult to predict the resource of safe and reliable operation of these wells. Functional safety, system reliability and environmental acceptability depend on their abandonment. The article aims to determine numerical characteristics of the diameter of gas bubbles and well production rates based on the normal distribution law.

1. Introduction
The article aims to determine the relationship between the diameter of the gas-liquid bubbles and external factors. The research method is a mathematical test which determines what law the empirical results comply with. Having determined the relationship, one can develop an algorithm for using the method in practice. The empirical approach is the most workable mathematical description of the processes [1-3].

2. Results and discussion
According to the histograms of relative frequencies of the diameter d of gas bubbles in a sample of a gas-liquid mixture obtained with an interval of 16 hours, the empirical distribution function was calculated. Based on this function, the hypothesis of the distribution type was studied. [4]

The histograms correspond to the density distribution curve and can be used for selection of a distribution model (Figures 1, 2).
Figure 1. – The histogram of the frequency of diameters of gas bubbles in microns at the beginning of the experiment

Figure 2. – The histogram of the frequency of diameters of gas bubbles in microns 16 hours after the beginning of the experiment

Table 1. Probabilities at intervals of gas bubble diameters and empirical distribution functions at the beginning and 16 hours after the beginning of the experiment.

| Average diameter, micron | Probability | Cumulative sum |
|--------------------------|-------------|----------------|
| 20.00                    | 0.01        | 0.01           |
| 59.00                    | 0.01        | 0.10           |
| 98.00                    | 0.02        | 0.42           |
| 137.00                   | 0.03        | 0.58           |
| 176.00                   | 0.04        | 0.73           |
| 215.00                   | 0.05        | 0.84           |
| 254.00                   | 0.06        | 0.91           |
| 293.00                   | 0.07        | 0.97           |
| 332.00                   | 0.08        | 0.97           |
| 371.00                   | 0.09        | 0.98           |
| 410.00                   | 0.10        | 0.98           |
| 449.00                   | 0.11        | 0.99           |
| 488.00                   | 0.12        | 0.99           |
| 527.00                   | 0.13        | 0.99           |
| 566.00                   | 0.14        | 1.00           |

(a) – The probability table, values of the distribution function and diameters of gas bubbles in microns at the beginning of the experiment

| Probability | Average diameter, micron | Cumulative sum |
|-------------|--------------------------|----------------|
| 0.00        | 39.00                    | 0.00           |
| 0.13        | 117.00                   | 0.13           |
| 0.31        | 195.00                   | 0.45           |
| 0.19        | 273.00                   | 0.64           |
| 0.16        | 351.00                   | 0.79           |
| 0.09        | 429.00                   | 0.89           |
| 0.02        | 507.00                   | 0.90           |
| 0.02        | 585.00                   | 0.92           |
| 0.02        | 663.00                   | 0.95           |
| 0.01        | 741.00                   | 0.96           |
| 0.02        | 819.00                   | 0.97           |
| 0.01        | 897.00                   | 0.98           |
| 0.01        | 975.00                   | 0.99           |
| 0.00        | 1053.00                  | 0.99           |
| 0.01        | 1131.00                  | 1.00           |

(b) – The probability table, values of the distribution function and diameters of gas bubbles in microns 16 hours after the beginning of the experiment
The most common form of specification of a random variable corresponding to the diameter of gas bubbles is the distribution function [5]. The probability theory says that the distribution function is \( F(x) = P(X < x) \); for each value of \( x \), it determines the probability that \( X \) will take values less than \( x \). \( F(x) \) is accumulated probability. According to the probability table, let us determine the cumulative sum which is an empirical distribution function (Table 1) and build its graphs (Figures 3, 4).

![Distribution function](image1)

**Figure. 3** The diagram of the diameters and distribution function values at the beginning of the experiment

![Distribution function](image2)

**Figure. 4** The diagram of the diameters and distribution function values at the end of the experiment

The calculation in Excel (Table 2) made it possible to determine indicators of the distribution center: weighted averages (sample averages) based on the results of the study at the beginning and 16 hours after the beginning of the experiment \( \bar{x}_1 = 293, \bar{x}_2 = 585 \); corrected standard deviations \( s_1 = 168.5, s_2 = 348.8 \) [6, 7].

The hypothesis of normal distribution of the diameter of the bubbles was verified according to the criterion of Pearson’s agreement \( \chi^2_{ob} = \sum \frac{(n_i - n_i^*)^2}{n_i^*} \), where \( n_i^* \) are theoretical frequencies: \( n_i^* = \frac{n \cdot h}{\sigma} \phi_i \).

Let us compare the empirical and theoretical frequencies. Using a calculation table, let us find the observed value of the Pearson’s agreement criterion (Table 2):

\( K_{obs} = 3.61 \) during the first experiments, \( K_{obs} = 12.68 \) 16 hours after the beginning of the experiment.
Let us determine the boundary of the critical area. Since Pearson statistics measures the difference between empirical and theoretical distributions, the larger its observed value $K_{obs}$, the stronger the argument against the main hypothesis. Therefore, the critical area for this statistics is always right-handed: $[K_{kp}; +\infty)$.

Its border $K_{kp} = \chi^2 (kr-1; \alpha)$ can be found from the table of critical distribution points $\chi^2$ with the number of degrees of freedom $kr-1 = 13$, significance level $\alpha = 0.05$, the number of parameters $r = 2$ $K_{kp}(0.05; 13) = 22.4$. The observed Pearson statistic values do not fall into the critical area: $K_{obs} < K_{kp}$, which demonstrates the validity of the normal distribution hypothesis.

Hypothesis verification based on the Pearson’s agreement criterion showed that there is no reason to reject this hypothesis.

Table 2. The calculation of the numerical characteristics of the diameter of gas-liquid bubbles and the observed value of the Pearson chi-square criterion.

| Diameters $x_i$ | $x_i^2$ | $(x_i-x\bar{})/s$ | Local Laplace function | theoretical frequency | chi-square |
|-----------------|---------|-------------------|------------------------|-----------------------|------------|
| 20.00           | 400.00  | -1.62             | 0.11                   | 0.37                  | 1.06       |
| 59.00           | 3481.00 | -1.39             | 0.15                   | 0.53                  | 0.42       |
| 98.00           | 9604.00 | -1.16             | 0.20                   | 0.71                  | 0.12       |
| 137.00          | 18769.00| -0.93             | 0.26                   | 0.90                  | 0.01       |
| 176.00          | 30976.00| -0.69             | 0.31                   | 1.09                  | 0.01       |
| 215.00          | 46225.00| -0.46             | 0.36                   | 1.24                  | 0.05       |
| 254.00          | 64516.00| -0.23             | 0.39                   | 1.35                  | 0.09       |
| 293.00          | 85849.00| 0.00              | 0.40                   | 1.39                  | 0.11       |
| 332.00          | 110224.00| 0.23              | 0.39                   | 1.35                  | 0.09       |
| 371.00          | 137641.00| 0.46              | 0.36                   | 1.24                  | 0.05       |
| 410.00          | 168100.00| 0.69              | 0.31                   | 1.09                  | 0.01       |
| 449.00          | 201601.00| 0.93              | 0.26                   | 0.90                  | 0.01       |
| 488.00          | 238144.00| 1.16              | 0.20                   | 0.71                  | 0.12       |
| 527.00          | 277729.00| 1.39              | 0.15                   | 0.53                  | 0.42       |
| 566.00          | 320356.00| 1.62              | 0.11                   | 0.37                  | 1.06       |
| 293.00          | 114241.00| 28392.00          | Corrected average deviation square = 168.50 | Observed chi-square = 3.61 |

Critical chi-square = 22.40

The hypothesis is accepted because $3.61156 < 22.4$
Table 3. The calculation of the numerical characteristics of the diameter of gas-liquid bubbles and the observed value of the Pearson chi-square criterion 16 hours after the beginning of the experiment.

| Xi  | Xi^2  | (xi – \(\bar{x}\)) | Local Laplace function | theoretical frequency | Observed chi-square |
|-----|-------|----------------------|------------------------|-----------------------|---------------------|
| 39.00 | 1521.00 | 0.03 | 0.40 | 0.31 | 1.56 |
| 117.00 | 13689.00 | 0.08 | 0.40 | 0.31 | 1.56 |
| 195.00 | 38025.00 | 0.13 | 0.40 | 0.31 | 1.54 |
| 273.00 | 74529.00 | 0.18 | 0.41 | 0.31 | 1.52 |
| 351.00 | 123201.00 | 0.23 | 0.41 | 0.32 | 1.49 |
| 429.00 | 184041.00 | 0.28 | 0.42 | 0.32 | 1.45 |
| 507.00 | 257049.00 | 0.33 | 0.42 | 0.32 | 1.41 |
| 585.00 | 342225.00 | 0.38 | 0.43 | 0.33 | 1.36 |
| 663.00 | 439569.00 | 0.44 | 0.44 | 0.34 | 1.30 |
| 741.00 | 549081.00 | 0.49 | 0.45 | 0.35 | 1.24 |
| 819.00 | 670761.00 | 0.54 | 0.46 | 0.35 | 1.17 |
| 897.00 | 804609.00 | 0.59 | 0.47 | 0.37 | 1.10 |
| 975.00 | 950625.00 | 0.64 | 0.49 | 0.38 | 1.03 |
| 1053.00 | 1108809.00 | 0.69 | 0.51 | 0.39 | 0.95 |
| 1131.00 | 1279161.00 | 0.74 | 0.53 | 0.40 | 0.88 |

Sum Xi=8775

Sum Xi^2/n=455793

Observed chi-square 12.68

Factors influencing the diameter of gas-liquid bubbles

It is known that with an increase in temperature of a liquid, surface tension forces are weakened due to the increase in the distance between molecules of the liquid [8]. The direct relationship between the surface tension force and temperature is described by formula:

\[ \sigma = \sigma_{25} + \left( \frac{d\sigma}{dt} \right) t, \]

where \(\sigma_{25}\) is the surface tension at 25°C.

For free bubbling, when gas moves as free-floating bubbles, the bubble diameter is calculated by formula:

\[ d = \sqrt{\frac{6d_0\sigma}{g(\rho_g - \rho_l)}}, \]

where \(d_0\) is the diameter of the hole in which the bubble are formed,

\(\sigma\) is the surface tension coefficient.

The minimum size of the steam bubble at the moment of nucleation is critical radius \(R_k\). This size corresponds to the size of irregularities on the surface which are the centers of vaporization [9-11]. To form the steam bubble, it is necessary to increase vapor pressure force \(P_1\) in the bubble to the sum of all external forces acting on it, i.e. the pressure forces of the liquid and the surface tension force.

\[ R_k = \frac{2\sigma T_H}{rp(T_g - T_H)^3} \]
where \( r \) is the heat of vaporization, KJ/Kg; \( T_s \) is the saturation temperature, K; \( \sigma \) is the fluid surface tension force, N/m\(^2\); \( \rho_v \) is the vapor density, kg/m\(^3\).

3. Conclusion
The data on the diameters of the bubbles in the coil showed that the distribution of these statistical data is normal. Factors influencing the size of the bubbles (temperature of the liquid, pressure of the surrounding gas, surface tension force of the liquid) were analyzed.

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