Can a spontaneous collapse in flavour oscillations be tested at KLOE?

Kyrylo Simonov$^1$ and Beatrix C. Hiesmayr$^1$

$^1$University of Vienna, Faculty of Physics, Boltzmanngasse 5, 1090 Vienna, Austria.

Why do we never see a table in a superposition of here and there? This problem gets a solution by so called collapse models assuming the collapse as a genuinely physical process. Here we consider two specific collapse models and apply them to systems at high energies, i.e. flavour oscillating neutral meson systems. We find on one hand a potentially new interpretation of the decay rates introduced by hand in the standard formalism and on the other hand that these systems at high energies constrain by experimental data the possible collapse scenarios.

I. INTRODUCTION: THE MEASUREMENT PROBLEM

Quantum mechanics is an exceedingly successful theory which can explain a plethora of experimental results covering physical phenomena on different energy scales. However, in its standard formulation quantum mechanics meets some conceptual problems. For instance, formally a macroscopic object can be in a superposition as the famous example of the Schrödinger’s cat. The interaction of a seemingly macroscopic system such as a measurement apparatus with a seemingly microscopic system, a quantum system, is postulated to force a reduction/collapse of the wave function of the quantum system, i.e. breaking up the superposition. Standard quantum theory neither provides a mechanism how this collapse takes place nor reveals whether it is a real physical process. We have two completely different types of dynamics at hand, i.e.

- a deterministic unitary time evolution ruled by the Schrödinger equation and
- a stochastic and non-unitary reduction process of the wave function caused by a measurement obeying Born’s rule.

Obviously, several questions arise: What defines a system to be microscopic versus macroscopic? The Copenhagen interpretation uses this concept but never defines it. Is the collapse procedure a real physical process? Does the collapse manifest at high energies differently than for systems at lower energies? Do collapse models allow for a different interpretation of the measurable dynamics of flavour oscillating mesons? In turn, how do these systems at high energies restrict the plethora of collapse models?

Dynamical reduction models (or collapse models) claim to solve this issue by modeling the collapse as a genuinely physical process. The aim of this contribution is to investigate collapse models for oscillation systems at high energies such as the K-mesons produced by DAΦNE. The paper is organised by giving a short introduction to collapse models and then applying two popular models to neutral meson systems followed by interpreting the results.

II. COLLAPSE MODELS: A SOLUTION TO THE MEASUREMENT PROBLEM?

Dynamical reduction models provide one of the possible solutions to the measurement problem. In general they propose a new universal dynamics which introduces the collapse of the wave function as an objective physical process. In order to recover the predictions of quantum mechanics this dynamics should fulfill the following properties

- non-linearity: The new dynamics should break superpositions on a macroscopic level, particularly, during a measurement.
- stochasticity: The new dynamics should produce the quantum probabilities obeying Born’s rule.
- no superluminal signaling: The new dynamics should not be in conflict with the special relativity.

Let us mention that it is by no means trivial that a mathematically consistent framework exists with the above properties and simultaneously agrees with all observed data. The first dynamical reduction model was
the GRW model introduced in 1985 by Ghirardi, Rimini and Weber [1]. It proposes a simple mechanism for
the collapse for a system of $N$ particles through spontaneous localisations, i.e.

1. Each particle undergoes a sudden localisation at a random time $t$ (mathematically modelled by a
multiplication with a peaked Gaussian function).

2. Between such localisations the state of the system evolves due to the Schrödinger equation.

In this way the GRW model, and collapse models in general, introduce two new fundamental constants, the
localisation rate $\lambda_{GRW}$, which fixes the mean rate of the localisation process, and the coherence length $r_C$, which
fixes the width of the Gaussian function, i.e. the localisation effect. These two constants allow to
divide the world into a microscopic regime, where the Schrödinger equation rules the dynamics mainly, and
a macroscopic regime where superposition breaks (Schrödinger’s cat is alive). Consequently, taking collapse
models seriously there are two new natural constants ruling our world.

Around 1989 two more realistic dynamical reduction models appeared on the market. These are the
QMUPL model (Quantum Mechanics with Universal Position Localisation) [2] and the CSL model (Continuous Spontaneous Localisation) [3,5]. In contrast to the GRW model these models introduce the spontaneous
collapse as a continuous process. The dynamics is obtained by adding to the Schrödinger equation non-linear
and stochastic terms. For a given Hamiltonian $\hat{H}$ and state $|\psi_t\rangle$ at time $t$ the dynamics is defined by [6]

$$
d|\psi_t\rangle = \left[-i\hat{H}\ dt + \sum_{i=1}^{N} (\hat{A}_i - \langle \hat{A}_i \rangle_t) dW_{i,t} - \frac{\lambda}{2} \sum_{i=1}^{N} (\hat{A}_i - \langle \hat{A}_i \rangle_t)^2 dt\right] |\psi_t\rangle,
$$

where $\hbar = 1$ is taken, $\langle \hat{A}_i \rangle_t = \langle \psi_t | \hat{A}_i | \psi_t \rangle$ is the quantum mechanical expectation value, and $\lambda$ sets the
strength of the collapse (connected to the rate and coherence length). Here $\hat{A}_i$ represent a set of $N$ self-
adjoint commuting operators introducing the collapse, $W_{i,t}$ represent a set of $N$ independent standard Wiener
processes, one for each collapse operator $\hat{A}_i$, and lead to a white noise field $w_{i,t} = \frac{d}{dt} W_{i,t}$ which plays a crucial
role in the model. The two collapse models differ mainly in the choice of the collapse operators $\hat{A}_i$.

Both models have maintained a high interest over the last years being intensively investigated from the
fundamental point of view as well as in experiments. Particularly, the QMUPL model has been investigated by
analysing the spontaneous radiation emission from a free charged particle [7,8] and X-ray emission by an
isolated slab of Germanium [9,11]. For the CSL model experiments with optomechanical cavities [12,13]
and mechanical oscillators [14] were proposed, contributions from cold-atom experiments [15] were also recently
obtained.

In the high energy regime the first computations for neutral meson and neutrino systems can be found in
Refs. [16,17]. These predictions were compared to specific decoherence models [18,20], where the decoherence
strength was computed by KLOE data [21,23] and by data from B-factories [24,26]. Last but not least it
was shown that the collapse models can contribute to an effective cosmological constant [27]. Thus the
dynamical reduction models reveal a rich spectrum of potential experimental tests.

III. NEUTRAL MESONS: A LABORATORY FOR TESTING THE COLLAPSE MODELS

Flavour oscillating systems, in particular the neutral $K$-mesons, have proven to be an exceptional laboratory for exploring quantum foundations, for instance a puzzling connection between the violation of the
$CP$ symmetry and the violation of a Bell inequality [25] has been found (experimentally feasible with the
KLOE detector). Neutral mesons are massive systems with two different mass eigenstates $| M_L \rangle$ and $| M_H \rangle$
which decay with distinct decay rates. The difference between decay rates for $K$-mesons is rather huge, in strong contrast to the other mesons. In what follows we neglect the tiny violation of the $CP$ symmetry
(which means that $\langle M_H | M_L \rangle = 0$). A neutral meson consists of quark-antiquark pair, and both the particle
state $| M^0 \rangle$ and the antiparticle state $| \bar{M}^0 \rangle$ can decay into the same final states. Therefore, neutral mesons
have to be considered as a two-state system and their dynamics is then described by the following effective
Schrödinger equation due to the Wigner–Weisskopf approximation

$$
\frac{d}{dt} |\psi_t\rangle = -i \hat{H} |\psi_t\rangle \implies |\psi_t\rangle = a(t) | M^0 \rangle + b(t) | \bar{M}^0 \rangle,
$$

where $M^0$ and $\bar{M}^0$ are the mass eigenstates of the neutral meson,
where the effective Hamiltonian $\hat{H} = \hat{M} + i \hat{\Gamma}$ is non-hermitian, and $\hat{M}$ is the hermitian mass operator, and $\hat{\Gamma}$ is a hermitian operator which describes the decay. The diagonalised Hamiltonian defines the mass eigenstates

$$\hat{H} |M_n\rangle = \left( m_n + \frac{i}{2} \Gamma_n \right) |M_n\rangle,$$

where $\Gamma_L, \Gamma_H$ are the corresponding decay rates (with $c = 1$). The flavour eigenstates are related to the mass eigenstates by

$$|M^0\rangle = \frac{1}{\sqrt{2}} \left( |M_H\rangle + |M_L\rangle \right),$$

$$|\bar{M}^0\rangle = \frac{1}{\sqrt{2}} \left( |M_H\rangle - |M_L\rangle \right).$$

The general collapse scenario, Eq. (1), has been shown to be in our case equivalent to [29, 30]

$$i \frac{d}{dt} |\psi_t\rangle = \left[ \hat{H} - \sqrt{\lambda} \sum_{i=1}^{N} \hat{A}_i w_{i,t} \right] |\psi_t\rangle := \left[ \hat{H} + \hat{N}(t) \right] |\psi_t\rangle,$$

which has a form of a standard Schrödinger equation with a random perturbation $\hat{N}(t)$. Solving this equation [5] with the help of Dyson series up to second order we obtained after a cumbersome computation the following probabilities [31, 32] for the propagation of the mass eigenstates for the QMUPL and CSL models

$$P^{\text{QMUPL}}_{M_{\mu=L/H} \rightarrow M_{\nu=L/H}}(t) = \delta_{\mu\nu} \left( 1 - \Lambda^{\text{QMUPL}}_{\mu} \cdot t + 3 \cdot \frac{1}{2} \left( \Lambda^{\text{QMUPL}}_{\mu} \right)^2 \cdot t^2 \right) \cdot e^{-\Gamma_{\mu} t},$$

$$P^{\text{CSL}}_{M_{\mu=L/H} \rightarrow M_{\nu=L/H}}(t) = \delta_{\mu\nu} \left( 1 - \Gamma^{\text{CSL}}_{\mu} \cdot t + \frac{1}{2} \left( \Gamma^{\text{CSL}}_{\mu} \right)^2 \cdot t^2 \right) \cdot e^{-\Gamma_{\mu} t} \approx \delta_{\mu\nu} e^{-\left( \Gamma^{\text{CSL}}_{\mu} + \Gamma_{\mu} \right) t},$$
\[ \begin{align*}
P_{QMUPL}^{M^0 \rightarrow M^0/\bar{M}^0} & = \frac{1}{4} \left\{ \sum_{i=H,L} e^{-i\Gamma_{i} t} \left( 1 - \Lambda_{i}^{QMUPL} t + 3 \cdot \frac{1}{2} (\Lambda_{i}^{QMUPL})^2 t^2 \right) \right. \\
 & \quad \pm 2 \cos(\Delta m t) e^{-\frac{r_{H} + r_{L}}{2} t} \left( 1 - \frac{\Delta m^2}{m_0^2} (1 - \theta(0)) \right) \cdot t \\
 & \quad + 3 \cdot \frac{1}{2} \left( \frac{\Delta m^2}{m_0^2} (1 - \theta(0)) + \frac{m_{H} m_{L}}{m_0^2} (1 - 2\theta(0)) \right)^2 t^2 \left\} \right., \\

P_{CSL}^{M^0 \rightarrow M^0/\bar{M}^0} & = \frac{1}{4} \left\{ \sum_{i=H,L} e^{-i\Gamma_{i} t} \left( 1 - \Gamma_{i}^{CSL} t + \frac{1}{2} (\Gamma_{i}^{CSL})^2 t^2 \right) \right. \\
 & \quad \pm 2 \cos(\Delta m t) e^{-\frac{r_{H} + r_{L}}{2} t} \left( 1 - \frac{\gamma}{(4\pi r_C)^d} \left[ \frac{\Delta m^2}{m_0^2} (1 - \theta(0)) \right] + \frac{m_{H} m_{L}}{m_0^2} (1 - 2\theta(0)) \right) \cdot t \\
 & \quad + \frac{1}{2} \left( \frac{\gamma}{(4\pi r_C)^d} \left[ \frac{\Delta m^2}{m_0^2} (1 - \theta(0)) \right] + \frac{m_{H} m_{L}}{m_0^2} (1 - 2\theta(0)) \right)^2 t^2 \left\} \right. \\
 & \approx \frac{1}{4} \left\{ \cos(\Delta m t) e^{-\frac{r_{H} + r_{L}}{2} t} \left[ e^{-(\Gamma_{H}^{CSL} + \Gamma_{L}^{CSL}) t/2} + e^{-(\Gamma_{H}^{CSL} + \Gamma_{L}^{CSL}) t/2} \right] \right. \\
 & \quad \cdot \left. \frac{\cos(\Delta m t)}{\cosh\left(\frac{\gamma}{(4\pi r_C)^d} \right) - \frac{\gamma}{(4\pi r_C)^d} \left( \frac{\Delta m^2}{m_0^2} \right) t^2} \right\} , \end{align*} \]

where we used the abbreviations
\[ \Lambda_{i}^{QMUPL} = \frac{\alpha}{2} \cdot \frac{m_{i}^2}{m_0^2} (1 - 2\theta(0)) \] (10)

and
\[ \Gamma_{i}^{CSL} = \frac{\gamma}{(4\pi r_C)^d} \cdot \frac{m_{i}^2}{m_0^2} (1 - 2\theta(0)). \] (11)

Here \( d \) is the dimensionality, \( \lambda = \frac{\Lambda_{QMUPL}}{2r_{H}} \) is the collapse strength in the QMUPL model, \( \gamma = \gamma_{GRW} \cdot (4\pi r_C)^d \) is the collapse strength in the CSL model, \( m_0 \) is a reference mass which is taken usually to be the nucleon mass (we will use the rest mass of the corresponding neutral meson), \( \Delta m = m_{H} - m_{L} \) is the mass difference, \( \theta(0) \) is the Heaviside function at zero, and \( \sqrt{\alpha} \) represents a width of the wave packet that is assumed to be the initial state of the meson for the QMUPL model.

The obtained probabilities reveal several interesting results, namely:

- For the QMUPL model the probabilities have a non-exponential contribution which makes the collapse effect in principle observable.
- For the CSL model the probabilities can be expanded to exponentials, allowing for new interpretations.
- The collapse dynamics contributes to the transition probabilities by two independent effects, a damping of the oscillation and a contribution to the decay rates.
- While the damping of the oscillation is proportional to the experimentally measurable squared mass difference, the effective decay constants are proportional to the squared absolute masses (which do not show up in the standard quantum mechanical framework as measurable quantities).
Dynamical reduction models provide a possible solution to the measurement problem of standard quantum mechanics by introducing a physical mechanism of the collapse of the wave function. We have investigated two popular collapse models and derived their respective deviations from the standard dynamics of flavour oscillations which are intensively studied at accelerator facilities. We have found that in principle such changes are observable and the dynamics can change the oscillation behaviour and under certain constraints explain the decay behaviour of neutral mesons completely. We have also illustrated how the meson system restricts the plethora of collapse models considerably.

IV. CONCLUSIONS

Dynamical reduction models provide a possible solution to the measurement problem of standard quantum mechanics by introducing a physical mechanism of the collapse of the wave function. We have investigated two popular collapse models and derived their respective deviations from the standard dynamics of flavour oscillations which are intensively studied at accelerator facilities. We have found that in principle such changes are observable and the dynamics can change the oscillation behaviour and under certain constraints explain the decay behaviour of neutral mesons completely. We have also illustrated how the meson system restricts the plethora of collapse models considerably.

Precision data from experiments such as KLOE will allow further to restrict or even rule out certain collapse scenarios when one extends the dynamics to two entangled pairs of mesons or/and includes $CP$ violation.

|       | $\Gamma_{H}^\text{exp} [s^{-1}]$ | $\Delta m_{H}^\text{exp} [hs^{-1}]$ | $m_{L} [hs^{-1}]$ | $m_{H} [hs^{-1}]$ |
|-------|----------------------------------|------------------------------------|-------------------|-------------------|
| $K$-mesons | $1.117 \cdot 10^{-10}$ | $0.529 \cdot 10^{-10}$ | $2.311 \cdot 10^{8}$ | $5.524 \cdot 10^{9}$ |
| $D$-mesons | $2.454 \cdot 10^{-12}$ | $0.950 \cdot 10^{-10}$ | $1.468 \cdot 10^{12}$ | $1.477 \cdot 10^{12}$ |
| $B_{u}$-mesons | $6.582 \cdot 10^{-13}$ | $0.510 \cdot 10^{-12}$ | $1.020 \cdot 10^{13}$ | $1.020 \cdot 10^{13}$ |
| $B_{s}$-mesons | $7.072 \cdot 10^{-13}$ | $1.776 \cdot 10^{13}$ | $2.477 \cdot 10^{14}$ | $2.655 \cdot 10^{14}$ |

TABLE I: Experimental values of the decay rates, the mass difference and the computed values of the absolute masses for $K$-, $D$-, $B_{u}$- and $B_{s}$-mesons. The table is taken from [92].
Acknowledgements: Both authors acknowledge gratefully the Austrian Science Fund (FWF-P26783).

[1] G. C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D 34, 470 (1986).
[2] L. Diosi, Phys. Rev. A 40, 1165 (1989).
[3] P. Pearle, Phys. Rev. A 39, 2277 (1989).
[4] G. C. Ghirardi, P. Pearle and A. Rimini, Phys. Rev. A 42, 78 (1990).
[5] G. C. Ghirardi, R. Grassi and F. Benatti, Found. Phys. 25, 5 (1995).
[6] A. Bassi, D. D"urr and G. Hinrichs, Phys. Rev. Lett. 111, 210401 (2011).
[7] A. Bassi and D. D"urr, J. Phys. A 42, 485302 (2009).
[8] A. Bassi and S. Donadi, Phys. Lett. A 378, 761 (2014).
[9] C. Curceanu, B. C. Hiesmayr and K. Piscicchia, J. Adv. Phys. 4, 263 (2015).
[10] C. Curceanu et al., Found. Phys. 46, 263 (2016).
[11] K. Piscicchia et al., [arXiv:1501.04462] (2015).
[12] M. Bahrami et al., Phys. Rev. Lett. 112, 210404 (2014).
[13] S. Nimmrichter, K. Hornberger and K. Hammerer, Phys. Rev. Lett. 113, 020405 (2014).
[14] L. Diosi, Phys. Rev. Lett. 114, 050403 (2015).
[15] M. Bilardello et al., [arXiv:1605.01801] (2016).
[16] S. Donadi et al., Found. Phys. 43, 813 (2013).
[17] M. Bahrami et al., Sci. Rep. 3, 1952 (2013).
[18] R. A. Bertlmann, W. Grimus, B. C. Hiesmayr, Phys. Rev. D 60, 114032 (1999).
[19] B. C. Hiesmayr, Found. Phys. Lett. 14, 231 (2001).
[20] R. A. Bertlmann, K. Durstberger, B. C. Hiesmayr, Phys. Rev. A 68, 012111 (2003).
[21] F. Ambrosino et al. (KLOE collaboration), Phys. Lett. B 642, 315 (2006).
[22] A. Di Domenico et al. (KLOE Collaboration), J. Phys.: Conf. Ser. 171, 012008 (2008).
[23] A. Di Domenico et al. (KLOE Collaboration), Found. Phys. 40, 852 (2010).
[24] A. Go et al. (Belle Collaboration), Phys. Rev. Lett. 99, 131802 (2007).
[25] G. Richter, “Stability of Nonlocal Quantum Correlations in Neutral B-Meson Systems” (PhD Thesis, Technische Universität Wien, 2008).
[26] B. D. Yabsley, “Quantum entanglement at the ψ(3770) and Upsilon(4S)” (Flavour Physics & CP Violation Conference, Taipei, 2008).
[27] T. Josset, A. Perez and D. Sudarsky, [arXiv:1604.04183] (2016).
[28] B. C. Hiesmayr et al., Eur. Phys. J. C 72, 1896 (2012).
[29] S. L. Adler and A. Bassi, J. Phys. A 40, 15083 (2007).
[30] S. Donadi, “Electromagnetic Radiation Emission and Flavour Oscillations in Collapse Models” (PhD Thesis, Università degli studi di Trieste, 2012).
[31] K. Simonov and B. C. Hiesmayr, Phys. Lett. A 380, 1253 (2016).
[32] K. Simonov and B. C. Hiesmayr, Phys. Rev. A 94, 052128 (2016).
[33] M. Carlesso, A. Bassi, P. Falferi and A. Vinante, Phys. Rev. D 94, 124306 (2016).
[34] S. L. Adler, J. Phys. A: Math. Theor. 40, 2935 (2007).
[35] R. A. Bertlmann, W. Grimus, B. C. Hiesmayr, Phys. Rev. A 73, 054101 (2006).
[36] J. Bernabéu, N. E. Mavromatos and P. Villanueva-Pérez, Phys. Lett. B 724, 269 (2013).