CP Phases, LFV, RpV & all that

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Abstract

Most phenomenological analyses of searches for supersymmetric particles have been performed within the MSSM with real SUSY parameters and conserved R-parity and lepton flavour. Here we summarize recent results obtained in the (s)lepton sector when one of the above assumptions is relaxed.

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Abstract

Most phenomenological analyses of searches for supersymmetric particles have been performed within the MSSM with real SUSY parameters and conserved R-parity and lepton flavour. Here we summarize recent results obtained in the (s)lepton sector when one of the above assumptions is relaxed.

Since supersymmetry must be broken at low energy, and the mechanism of its breaking is still unknown, even the minimal supersymmetric model (MSSM) introduces more than 100 new parameters. The MSSM is understood as an effective low energy model defined by a) minimal particle content, b) R-parity conservation, c) most general soft supersymmetry breaking terms. The number of parameters can be further enlarged by relaxing a) or b), or reduced by constraining c) with additional assumptions on SUSY breaking parameters. So far most phenomenological studies on supersymmetric particle searches have been performed within the MSSM with drastically reduced number of parameters by assuming e.g. that all SUSY parameters are real, lepton flavour is conserved, universality at high scale holds etc.
However, current experimental limits on the SUSY parameter space admit many of the above assumptions to be relaxed. We briefly present some phenomenological consequences in the (s)lepton sector of i) complex phases, ii) lepton flavour violation, iii) R-parity violation.

1 CP phases

The assumption of real SUSY parameters has partly been justified by the experimental limits on the electric dipole moments (EDM) of the electron, neutron and mercury atom. However, the EDM constraints can be avoided assuming masses of the first and second generation sfermions large (above the TeV scale), or arranging cancellations between the different SUSY contributions to the EDMs \[1\]. As a result, the complex phase of the Higgsino mass parameter $\mu$ is much less restricted than previously assumed, whereas the complex phases of the soft-breaking trilinear scalar coupling parameters $A_f$ are practically unconstrained.

Recently an analysis of production and decay rates of $\tilde{\tau}_1$, $\tilde{\tau}_2$ and $\tilde{\nu}_\tau$ at an $e^+e^-$ linear collider with a CMS energy $\sqrt{s} = 0.5 - 1.2$ TeV with complex $\mu$, $A_\tau$ and $M_1$ ($M_1$ is the $U(1)$ gaugino mass parameter) has been performed \[2\]. Explicit CP violation in the Higgs sector induced by stop and sbottom loops with complex parameters \[3\] has also been included, and the scalar mass matrices and trilinear scalar coupling parameters have been taken flavor diagonal.

The stau mass matrix in the interaction basis $(\tilde{\tau}_L, \tilde{\tau}_R)$ reads:

$$
\mathcal{M}^2_{\tilde{\tau}} = \begin{pmatrix}
m^2_{\tilde{\tau}_L} & a_\tau^* m_\tau \\
 a_\tau m_\tau & m^2_{\tilde{\tau}_R}
\end{pmatrix},
$$

$$m^2_{\tilde{\tau}_L} = M^2_L + m^2_Z \cos 2\beta \left( \sin^2 \theta_W - \frac{1}{2} \right) + m^2_\tau,$$

$$m^2_{\tilde{\tau}_R} = M^2_E - m^2_Z \cos 2\beta \sin^2 \theta_W + m^2_\tau,$$

$$a_\tau m_\tau = (A_\tau - \mu^* \tan \beta) m_\tau = |a_\tau m_\tau| e^{i\phi}. \tag{4}$$

where $M_L, E$ and $A_\tau$ are slepton soft SUSY–breaking parameters, with $A_\tau = |A_\tau| e^{i\phi_A\tau}$ and $\mu = |\mu| e^{i\phi_\mu}$. The mass eigenstates are defined as

$$
\begin{pmatrix}
\tilde{\tau}_1 \\
\tilde{\tau}_2
\end{pmatrix} = \begin{pmatrix}
e^{i\phi_{\tilde{\tau}}} \cos \theta_{\tilde{\tau}} & \sin \theta_{\tilde{\tau}} \\
- \sin \theta_{\tilde{\tau}} & e^{-i\phi_{\tilde{\tau}}} \cos \theta_{\tilde{\tau}}
\end{pmatrix}\begin{pmatrix}
\tilde{\tau}_L \\
\tilde{\tau}_R
\end{pmatrix}. \tag{5}
$$
In principle, the imaginary parts of the complex parameters involved could most directly and unambiguously be determined by measuring suitable CP violating observables. However, in the $\tilde{\tau}$-system this is not straightforward, because the $\tilde{\tau}$ are spinless and their main decay modes are two–body decays. On the other hand, also the $\bar{\tilde{\tau}}$–$\tilde{\tau}$ decay branching ratios depend in a characteristic way on the complex phases. This is illustrated in Fig. 1.

The masses $m_{\tilde{\tau}_{1,2}}^2$ and mixing angle $\theta_{\tilde{\tau}}$ depend on the phases only through a term $m_{\tau}^2 |A_\tau| \mu |\tan \beta \cos(\varphi_{A_\tau} + \varphi_\mu)|$. Therefore $m_{\tilde{\tau}_{1,2}}^2$ are essentially independent of the phases because $m_\tau$ is small, whereas the phase dependence of $\theta_{\tilde{\tau}}$ is strongest if $|A_\tau| \approx |\mu| |\tan \beta|$ and $|m_{\tilde{\tau}_{1}}^2 - m_{\tilde{\tau}_{2}}^2| \lesssim |a_\tau m_\tau|$. Since the $Z \tilde{\tau}_i \tilde{\tau}_j$ couplings are real, the production cross sections do not explicitly depend on the phases (although $Z \tilde{\tau}_1 \tilde{\tau}_2$ coupling is complex, for $\tilde{\tau}_1 \tilde{\tau}_2$ production only $Z$ exchange contributes). However, the various $\tilde{\tau}$ decay branching ratios depend in a characteristic way on the complex phases. This is illustrated in Fig. 1. The fit to the simulated experimental data with 2 ab$^{-1}$ at a collider like TESLA shows that $3mA_\tau$ and $ReA_\tau$ can be determined with an error of order 10%.

Figure 1: Branching ratios of $\tilde{\tau}_1$ and $\tilde{\tau}_2$ as indicated. Left: for $m_{\tilde{\tau}_1} = 240$, $m_{\tilde{\nu}_\tau} = 233$, 238, 243 (from bottom to top), and $\varphi_\mu = \varphi_{U(1)} = 0$, $|\mu| = 300$, $|A_\tau| = 1000$, $\tan \beta = 3$, and $M_2 = 200$. Center: for $\varphi_{U(1)} = \varphi_{A_\tau} = 0$, $m_{\tilde{\tau}_1} = 240$, $m_{\tilde{\tau}_2} = 500$, $M_2 = 280$, $|\mu| = 150$, $\tan \beta = 3$, and $|A_\tau| = 1000$, assuming $M_\tilde{L} < M_\tilde{E}$. Right: for $\varphi_\mu = 0$ $m_{\tilde{\tau}_1} = 240$, $m_{\tilde{\tau}_2} = 500$, $M_2 = 450$, $|\mu| = 600$, $M_2 = 450$, $\varphi_{U(1)} = 0$, $|A_\tau| = 900$, assuming $M_\tilde{L} > M_\tilde{E}$. All mass parameters are in GeV.
2 Lepton flavour violation

Neutrino oscillation experiments have established the existence of lepton flavour violation (LFV) with \( \tan^2 \theta_{\text{Atm}} \simeq 1, \tan^2 \theta_{\odot} = 0.24 - 0.89 \) and \( \sin^2(2\theta_{13}) \lesssim 0.1 \) \cite{4}. On the other hand, there are stringent constraints on LFV in the charged lepton sector, the strongest being \( BR(\mu^- \to e^-\gamma) < 1.2 \times 10^{-11} \) \cite{5}.

In supersymmetric models gauge and Lorentz invariance does not enforce total lepton number \( L = L_e + L_\mu + L_\tau \) or individual lepton flavour \( L_e, L_\mu \) or \( L_\tau \) to be conserved. One usually invokes R-parity symmetry, which forces total lepton number conservation but still allows the violation of individual lepton number, e.g. due to loop effects in \( \mu^- \to e^-\gamma \) \cite{6}. Moreover, in the MSSM a large \( \nu_\mu-\nu_\tau \) mixing can lead to a large \( \tilde{\nu}_\mu-\tilde{\nu}_\tau \) mixing via renormalisation group equations. Therefore one can expect clear LFV signals in slepton and sneutrino production and in the decays of neutralinos and charginos into sleptons and sneutrinos at the LHC and at future lepton colliders.

In ref. \cite{7} the consequences of LFV assuming the most general mass matrices for sleptons and sneutrinos have been studied. The charged slepton mass matrix, generalized to include flavour mixing as well as left-right mixing, is given by:

\[
M^2_L = \begin{pmatrix}
M^2_{L,ij} + \frac{1}{2}v^2_dA_{ij}Y_{\nu E}^* + D_L\delta_{ij} & -\frac{1}{\sqrt{2}}(v_dA_{ji} - \mu v_uY_{\tau E}^*) \\
-\frac{1}{\sqrt{2}}(v_dA_{ji} - \mu v_uY_{\tau E}^*) & M^2_{R,ij} + \frac{1}{2}v^2_dY_{\nu E}^*Y_{\tau E}^* - D_R\delta_{ij}
\end{pmatrix},
\]

with \( D_L = \frac{1}{8}(g^2 - g^2)(v_d^2 - v_u^2) \) and \( D_R = \frac{1}{8}g^2(v_d^2 - v_u^2) \), and the indices \( i, j, k = 1, 2, 3 \) counting flavors \( e, \mu, \tau \). \( M^2_L \) and \( M^2_R \) are the soft SUSY breaking mass matrices for left and right sleptons, respectively. \( A_{ij} \) are the trilinear soft SUSY breaking couplings of the sleptons and Higgs bosons, and \( Y_{\nu E}^* \) are charged lepton Yukawa couplings. Similarly, one finds for the sneutrinos

\[
M^2_{\tilde{\nu},ij} = M^2_{L,ij} + \frac{1}{8}
(\begin{pmatrix}
0 & 2g^2 \\
g^2 & 0
\end{pmatrix} - v_d^2 - v_u^2)\delta_{ij}.
\]

For the numerical analysis the SPS1 reference point \cite{8} (defined by \( M_{1/2} = 250 \text{ GeV}, M_0 = 100 \text{ GeV}, A_0 = -100 \text{ GeV}, \tan\beta = 10 \) and \( \text{sign}(\mu) = + \) at the GUT scale) has been chosen, with the following slepton mass parameters at the electroweak scale: \( M_{R_{11}} = 138.7 \text{ GeV}, M_{R_{33}} = 136.3 \text{ GeV}, M_{L_{11}} = 202.3 \text{ GeV}, M_{L_{33}} = 201.5 \text{ GeV} \) and \( A_{33}/Y^*_{33} = -257.3 \text{ GeV} \). With these parameters fixed, a scan over the nondiagonal entries of \( M^2_L, M^2_R \) and \( A \) shows that values for \( |M^2_{R,ij}| \) up to \( 8 \times 10^3 \text{ GeV}^2 \), \( |M^2_{L,ij}| \) up to \( 6 \times 10^3 \text{ GeV}^2 \) and \( |A_{ij}v_d| \) up to 650 GeV$^2$ are compatible
with the current experimental constraints. In most cases, one of the mass squared parameters is at least one order of magnitude larger than all the others. However, there is a sizable part in parameters where at least two of the off-diagonal parameters have the same order of magnitude.

Possible LFV signals at an $e^+e^-$ collider include $e\mu E_T$, $e\tau E_T$, $\mu\tau E_T$ in the final state plus a possibility of additional jets. Varying the parameters randomly on a logarithmic scale: $10^{-8} \leq |A_{ij}| \leq 50$ GeV, $10^{-8} \leq M_{ij}^2 \leq 10^4$ GeV$^2$, 8000 points consistent with the experimental data have been generated. In Fig. 2 the cross section of $e^+e^- \rightarrow e^\pm\tau^\mp E_T$ and the corresponding ratio signal over square root of background ($S/\sqrt{B}$) are shown as a function $BR(\tau \rightarrow e\gamma)$ assuming an integrated luminosity of 100 fb$^{-1}$ at $\sqrt{s} = 500$ GeV. All possible SUSY and Higgs cascade decays have been included together with ISR- and SUSY-QCD corrections for the production cross sections, while the background comes from all possible SUSY cascade decays faking the signal and the SM $W$, $t$-quark and $\tau$-lepton pair production processes. Although no cuts have been applied, there is in most cases a spectacular signal. The accumulation of points in Fig. 2 along a band is due to a large $\tilde{e}_R\tilde{\tau}_R$ mixing which is less constraint by $\tau^- \rightarrow e^-\gamma$ than the corresponding left-left or left-right mixing.

Note that the collider LFV signals can be very competitive to those from rare charged lepton decay, like $\tau \rightarrow \mu\gamma$. This is illustrated in Fig. 3 [9], where for simplicity the flavour mixing has been restricted to the 2-3 generation subspace of
sneutrinos with the mixing angle $\theta_{23}$ and $\Delta m_{23} = |m_{\tilde{\nu}_2} - m_{\tilde{\nu}_3}|$ as free, independent parameters.

Figure 3: $3\sigma$ significance contours in the $\theta_{23}$ and $\Delta m_{23}$ plane for $\sqrt{s} = 500$ GeV LC and for luminosity 500 fb$^{-1}$ (A), and 1000 fb$^{-1}$ (B). Line C: $\tilde{\nu}\tilde{\nu}^*$ contribution with luminosity 500 fb$^{-1}$. Dotted lines: $\text{BR}(\tau \to \mu\gamma) = 10^{-7}, 10^{-8}, 10^{-9}$.

3 R-parity violation

Supersymmetric models with explicit bilinear breaking of R-parity (RpV) \cite{10} provide a simple and calculable framework for neutrino masses and mixing angles in agreement with the experimental data. The simplest bilinear RpV model, studied in ref. \cite{11}, is characterized by three additional terms $\epsilon_i$ in the superpotential

$$W = W_{\text{MSSM}} + \epsilon_i \hat{L}_i \hat{H}_u,$$

and the corresponding terms in the soft SUSY breaking part of the Lagrangian,

$$\mathcal{L}_{\text{soft}} = \mathcal{L}^{\text{MSSM}}_{\text{soft}} + B_i \epsilon_i \hat{L}_i H_u.$$

$W_{\text{MSSM}}$ is the ordinary superpotential of the MSSM and $i = e, \mu, \tau$. As a result of eq. (9), the scalar neutrinos develop non-zero vacuum expectation $v_i = \langle \tilde{\nu}_i \rangle$ in addition to the VEVs $v_u$ and $v_d$ of the MSSM Higgs fields $H_u^0$ and $H_d^0$. Together with the bilinear parameters $\epsilon_i$ the $v_i$ induce mixing between particles distinguished (only) by lepton number (or R–parity): charged leptons mix with charginos, neutrinos with neutralinos, and Higgs bosons with sleptons. Mixing between the neutrinos and the neutralinos generates a non-zero mass for one specific linear superposition of the three neutrino flavour states of the model at tree-level; the remaining two masses are generated at 1-loop.
Charged scalar leptons lighter than all other supersymmetric particles will decay through R-parity violating couplings. Possible final states are either $l_j\nu_k$ or $q\bar{q}'$. For right-handed charged sleptons ($\tilde{l}_R^i$) the former by far dominate over the hadronic decay mode. In the limit $(m_{f_j}, m_{\nu_k}) \ll m_{\tilde{f}_i}$ the two-body decay width for $\tilde{f}_i \rightarrow f_j + \Sigma_k \nu_k$ for $i \neq j$ scales as

$$\Gamma = \frac{m_{\tilde{f}_i}}{16\pi} (\cos \theta_{\tilde{l}_i} Y_{\tilde{l}_i} \frac{e_j}{\mu})^2,$$

which implies that the decay length $\sim$ Yukawa$^{-2}$. The numerical calculations were performed in the mSUGRA version of the MSSM by scanning the parameters in the following ranges: $M_2 \in (0, 1.2)$ TeV, $|\mu| \in (0, 2.5)$ TeV, $m_0 \in (0, 0.5)$ TeV, $A_0/m_0$ and $B_0/m_0 \in (-3, 3)$ and $\tan \beta \in (2.5, 10)$. All randomly generated points were subsequently tested for consistency with the minimization (tadpole) conditions of the Higgs potential as well as for phenomenological constraints from supersymmetric particle searches. In addition, points in which at least one of the charged sleptons was lighter than the lightest neutralino, and thus the LSP, were selected. This latter requirement prefers strongly $m_0 \ll M_2$. The R-parity violating parameters were chosen in such a way that the neutrino masses and mixing angles are approximately consistent with the experimental data.

Figure 4: Left: Charged slepton decay length as a function of $m_{\tilde{l}_i}$ at a linear collider with 0.8 TeV c.m.s. energy. From top to bottom: $\tilde{e}$ (dark, on color printers blue), $\tilde{\mu}$ (light shaded, green) and $\tilde{\tau}$ (dark shaded, red). Right: Ratios of branching ratios for scalar tau decays.

As seen in Fig. 4 sleptons, including the LSP $\tilde{\tau}_R$, decay within the detector. The
three generations of sleptons decay with quite different decay lengths and thus it should be possible to separate the different generations experimentally at a future linear collider. Note that the ratio of the decay lengths $L(\tilde{\tau})/L(\tilde{\mu})$ is approximately given by $(h_{\mu}/h_{\tau})^2$. Ratios of branching ratios of various charged slepton decays contain rather precise information on ratios of the bilinear parameters $\epsilon_i$, Fig. 4.

**Summary:** Relaxing constraints on the MSSM parameter space can lead to a variety of striking signals. We are still far from understanding all possible facets of the MSSM, not to mention non-minimal supersymmetric models. Nevertheless, future $e^+e^-$ colliders will serve as a powerful tool to unravel the underlying theory.

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