Estimation of Parameters for Non-Equilibrium Phase Transitions during the Flow of Oil and Gas Mixtures

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Abstract. Problems of unsteady gas-liquid flows in wellbores with phase transitions are common for hydrocarbon production and well drilling. Typically, flow in wellbores is sufficiently fast which requires taking into account the non-equilibrium nature of phase transitions. The set of equations describing conservation of mass for components and momentum conservation for the mixture is supplemented by the relaxation equation for concentration of gas dissolved in the liquid. It is known that the characteristic times of gas evolution from the liquid and gas dissolution in the liquid differ significantly and depend on fluid properties, thermobaric conditions, and the flow pattern. Therefore, obtaining estimates of relaxation times for real flows turns to be a difficult problem. In this study, we propose a method for estimating the characteristic relaxation times for gas evolution and dissolution during the gas-liquid flow in a well. The method combines the solution of the convective diffusion equation for the growth / collapse of a vapor bubble in a liquid with the current parameters of the gas-liquid flow. Estimates of the relaxation times for the bubbly flow (small bubbles, gas evolution predominates) and slug flow (large bubbles, dissolution predominates), as functions of time from the beginning of the process, are obtained for the parameters of a real flow in a wellbore. It is shown that the obtained dependencies are consistent with the typical scaling of the diffusion process with respect to the size of bubbles. The results are important for mathematical simulation of gas-liquid flows in wellbores during hydrocarbon production, as well as for calculation of the gas-kick process during well drilling with oil-based muds.

1. Introduction

During production of hydrocarbons, gas-liquid flows take place in the wellbores. The flow can be multiphase either in the entire system, if several reservoir fluids inflow to the bottom of the well and are present in the mixture up to the wellhead, or in some part of it, if gas dissolution in the liquid or, conversely, free gas evolution from the liquid occurs in the wellbore. In general, one should talk about three phases: hydrocarbon gas, hydrocarbon liquid (oil), and water [1]. For the goal of this paper, we will address the case of two hydrocarbon phases: free gas and liquid (oil), taking into account dissolved gas in the liquid phase.

When drilling wells, oil-based drilling muds are used in some cases. Their specific is the ability to dissolve gas. If during drilling a gas-saturated formation with high pressure is penetrated, then gas may inflow to the bottom of the well. In this case, multiphase flow also occurs. When gas enters the bottomhole area of the wellbore, it can fully dissolve in liquid, and the flow would become single
phase. But as the mixture approaches the wellhead and the pressure decreases, the dissolved gas starts to separate from the liquid [2].

Typically, multiphase flow in wellbores is sufficiently fast which requires taking into account the non-equilibrium nature of phase transitions in the flowing oil and gas mixture. The non-equilibrium nature of phase transitions is associated with the relaxation time. During the transition from the single-phase to the two-phase state, the gas is released into a separate phase (free gas) by formation of bubbles in the volume of the liquid. This type of phase transitions features relaxation times of the order of first seconds [3]. The reverse process of gas dissolution in the liquid is determined by the rate of components’ diffusion through the formed phase boundary. The characteristic dissolution time depends on the flow pattern and regime and can take values several orders of magnitude higher [4].

The characteristic relaxation time affects the deviation of the dissolved gas concentration (mass fraction) in the liquid in a non-equilibrium (actual) state from its value in the equilibrium (theoretical) state. With an increase in the relaxation time, at fixed flow parameters, the non-equilibrium mass fraction of dissolved gas will become farther from the equilibrium one.

Following [5,2], let us write down a set of conservation equations for the multiphase flow model. Consider a mixture of two components – degassed (dead) oil and gas, which can exist in the form of free gas and gas dissolved in the oil (liquid phase). Let \( \alpha_l \) and \( \alpha_g \) be the local volume fractions of the liquid phase (oil) and gas phase (free gas), respectively, so that the normalization holds:

\[ \alpha_l + \alpha_g = 1. \tag{1} \]

Mass conservation (continuity) equation for the dead oil is:

\[ \frac{\partial}{\partial t} [\alpha_l \rho_l (1 - x_{dg})] + \frac{\partial}{\partial s} [\alpha_l \rho_l (1 - x_{dg}) v_l] = 0, \tag{2} \]

for the gas component is:

\[ \frac{\partial}{\partial t} (A \alpha_g \rho_g + A \alpha_l x_{dg} \rho_l) + \frac{\partial}{\partial s} (A \alpha_g \rho_g v_g + A \alpha_l x_{dg} \rho_l v_l) = 0, \tag{3} \]

and momentum conservation equation for the mixture is:

\[ \frac{\partial}{\partial t} (A \alpha_l \rho_l v_l + A \alpha_g \rho_g v_g) + \frac{\partial}{\partial s} (A \alpha_l \rho_l v_l^2 + A \alpha_g \rho_g v_g^2) + A \frac{\partial P}{\partial s} = - A (f + \rho g \cos \theta), \tag{4} \]

where \( A \) is the cross-sectional area of the pipe or the annulus of the well through which the flow occurs; \( P \) is the pressure; \( \rho_l \) and \( \rho_g \) are the liquid (oil) and gas densities; \( v_l \) and \( v_g \) are the liquid (oil) and gas velocities; \( x_{dg} \) is the mass fraction (concentration) of the dissolved gas in the liquid phase; \( g \) is the acceleration of gravity; \( f \) is the hydraulic losses; \( \theta \) is the local inclination angle of the wellbore relative to the vertical; \( s \) is the coordinate along the wellbore (the measured depth); \( \rho \) is the mean density of the mixture; \( \rho = \alpha_l \rho_l + \alpha_g \rho_g \).

Constitutive relations, including equations of state for the liquid and gas phases, the Zuber-Findlay (drift-flux) relation for velocities and a model for hydraulic losses are added to the system (1)-(4) [2].

To account for non-equilibrium phase transitions, relaxation equation for \( x_{dg} \) can be introduced [6,2]. In addition to the current non-equilibrium value of the dissolved gas concentration \( x_{dg} \), let us consider the corresponding equilibrium concentration \( x_{dgE} \). During relaxation, the system tries to pass from the non-equilibrium state to the equilibrium one:

\[ x_{dg} \rightarrow x_{dgE}. \tag{5} \]

Let \( \Delta x = x_{dg} - x_{dgE} \). As a first approximation, it can be assumed that the rate of change in the dissolved gas concentration \( \Delta x \) at time \( t \) is proportional (with opposite sign) to the current value of \( \Delta x \), that is, to the deviation of the current dissolved gas concentration from the equilibrium one:

\[ \frac{d\Delta x}{dt} \sim -\Delta x, \tag{6} \]
or

\[ \frac{d \Delta x}{dt} = c \Delta x, \tag{7} \]

where \( \frac{1}{\tau} \) is the characteristic relaxation time.

Hence, we obtain a linear relaxation equation for the concentration of dissolved gas in the liquid:

\[ \frac{d x_{dg}}{dt} + \frac{x_{dg} - x_{dg0}}{\tau} = 0. \tag{8} \]

The system (1)-(8) with the closing relations can be used to describe gas-oil flows with non-equilibrium phase transitions in wellbores during hydrocarbon production or well drilling [2,7]. However, as yet mentioned, the values of characteristic relaxation time for gas evolution from the liquid and gas dissolution in the liquid differ significantly and depend on fluid properties, thermobaric conditions, and flow pattern. Therefore, obtaining estimates of relaxation times for real flows is an urgent and difficult problem. In this study, we present a method for estimating the characteristic relaxation times for gas evolution and dissolution during multiphase flow in a wellbore based on the solution of the convective diffusion equation for growth/collapse of a vapor bubble in a liquid.

2. Diffusional growth or collapse of gas bubbles

Let us pose the problem of estimating the characteristic relaxation time using the solution of the convective diffusion equation presented in [8] for growth or collapse of a vapor bubble in a liquid.

A specific feature of the processes considered is the difference between the relaxation times for gas evolution and dissolution. We will take this factor into account by constructing estimates for two different flow regimes (bubble sizes): one with predominating gas evolution and the other with predominating gas dissolution. In order to make estimates for the relaxation times, we will first consider the solution for an individual bubble.

In [8], a single-component spherical gas (vapor) bubble in a flowing liquid is considered. To be specific, we will describe the bubble collapse (dissolution) case. Solution for the bubble growth case is the same, with different values of concentrations in initial and boundary conditions.

It is assumed that at time \( t = 0 \) a bubble with the initial radius \( R_0 \) begins to dissolve in the surrounding liquid, the volume of which is much larger than the volume of the bubble. This situation may happen when pressure is changed in a previously equilibrated system consisting of a gas bubble in a liquid, or when a gas bubble is introduced into a partially degassed (undersaturated) liquid.

The distribution of the dissolved gas concentration in a homogeneous continuous liquid phase around the bubble satisfies the convective diffusion equation, where the concentration is transferred inside a physical system due to two processes: diffusion and convection. Assuming axisymmetric flow around the bubble and negligible molecular diffusion tangent to the bubble compared to convective diffusion in that direction, and making some simplifications [8], the equation takes the following form:

\[ \frac{\partial C}{\partial t} + v_r \frac{\partial C}{\partial r} + \frac{v_\theta}{r} \frac{\partial C}{\partial \theta} = D \frac{\partial^2 C}{\partial r^2}, \tag{9} \]

where \( r \) is the radial coordinate (m); \( \theta \) is the angle (radians); \( t \) is time (sec); \( C \) is the dissolved gas concentration in the liquid \( \text{mol} \text{m}^{-3} \); \( v_r \) is the radial velocity \( \text{m sec}^{-1} \); \( v_\theta \) is the tangential velocity \( \text{m sec}^{-1} \); \( D \) is the diffusion coefficient \( \text{m}^2 \text{sec}^{-1} \); \( R \) is the current radius of the bubble (m).

For equation (9), the boundary and initial conditions are:

- dissolved gas concentration away from the bubble:
  \[ C = C_0 \text{ as } r \to \infty \tag{10} \]
- equilibrium concentration at the bubble surface:
  \[ C = C^* \text{ at } r = R \text{ for } t > 0 \tag{11} \]
- initial dissolved gas concentration:
  \[ C = C_0 \text{ at } t = 0 \text{ for all } r \tag{12} \]

The rate of change in the number of gas moles in the bubble due to dissolution at the surface is:
\[ \frac{dm}{dt} = d \left( \frac{\bar{\rho}_g}{3} \pi R^3 \right) = \frac{\bar{\rho}_g}{3} 4 \pi R^2 \frac{dR}{dt}, \]  

where \( \bar{\rho}_g \) is the molar density of the gas phase (mole/m³).

For the simplified case when the bubble is stationary relative to the liquid, that is, the dissolved gas concentration around the bubble is uniformly distributed, an analytical solution to the problem gives the relation for the bubble growth in time [8]:

\[ R = R_0 + \left( C_0 - C^* \right) \frac{12D}{\bar{\rho}_g} \sqrt{\frac{12D}{\pi} t}, \]

and the gas transfer rate at the bubble surface is computed via (13) accordingly.

For \( C_0 < C^* \), the solution corresponds to gas dissolution in the liquid (bubble collapse), and for \( C_0 > C^* \) – to gas evolution from the liquid (bubble growth).

The initial size of the bubble differs significantly for the two flow regimes of interest – the bubbly flow (small gas bubbles in the volume of the liquid, Fig. 1a), and the slug flow (large individual bubbles – slugs, Fig. 1b). For the process of gas inflow to the well (gas kick) during drilling with an oil-based mud, the slug mode occurs when a gas bubble (slug) enters the wellbore and moves upwards with dissolution. And the bubbly mode occurs when the dissolved gas evolves from the liquid due to decreasing pressure near the wellhead. Thus, an estimate of the characteristic relaxation time for the bubbly mode corresponds to the gas evolution and growth of the bubbles, and in for the slug mode – to the gas dissolution with a decrease in the size of the bubbles (slugs).

![Figure 1](image_url)  
Figure 1. The gas-liquid flow regimes in vertical pipes: a) the bubbly flow; b) the slug flow

3. Calculations for real flow conditions in the well
As the gas-liquid mixture moves along the wellbore towards the wellhead, the pressure and temperature change significantly. As a result, the size of a gas bubble in the stream will increase during gas evolution, or decrease during dissolution. In this section we show how to compute these dependencies based on the solution from [8] presented above.

To compute oil and gas properties for a two-phase flow in the wellbore, we take the relationships from [2] as an example. The temperature versus depth is:

\[ T = 293.16 + 0.01974h \text{ (K)}, \]

where \( h \) is the vertical depth (m).

In petroleum applications, the dissolved gas content in the oil phase is usually expressed in terms of solution gas-oil ratio (sGOR). For equilibrium sGOR, we use the following correlation:
\[ r_E = 0.102 \left( \frac{P}{0.1255} + 1.4 \right) \times 10^{0.648} - 0.73 \left( \frac{m^3}{m^3} \right), \]

where \( P \) is the pressure (MPa); \( u = 0.4375 - 0.00163(T - 255.37) \).

The mass concentration of dissolved gas in the liquid phase is related to sGOR by:

\[ x_{dg} = \frac{\rho g^s}{\rho l + \rho g^s}, \]

where \( x_{dg} = x_{dgE} \) and \( r = r_E \) at equilibrium, \( \rho g^s \left( \frac{kg}{m^3} \right) \) is the dead oil density at standard conditions; \( \rho g^s = 1.3 \gamma g \left( \frac{kg}{m^3} \right) \) – is the gas density at standard conditions, \( \gamma g \) is the gas specific gravity (relative density of gas to air).

To calculate the oil volume factor, we use the Standing correlation [9]:

\[ B_l = 0.00012[4.6r + 2.25(T - 255.37)]^{1.2} + 0.9713 \left( \frac{m^3}{m^3} \right), \]

and for the mass density of the liquid, the following relation is valid:

\[ \rho l = \frac{\rho l^i + \rho g^s}{B_l}. \]

The dissolved gas concentration in the molar-volumetric units, as used in [8] and (9)-(12), is related to the mass concentration by:

\[ C = \frac{x_{dg}}{m_g} \rho l \left( \frac{mole}{m^3} \right), \]

where \( m_g \) is the molar mass of gas, \( m_g = \gamma g m_a \); \( m_a \) is the air molar mass \( \left( \frac{kg}{mole} \right) \).

The mass density of gas is expressed from the equation of state:

\[ \rho g = \frac{\gamma g m_a p}{z R T} \left( \frac{kg}{m^3} \right), \]

where \( R = 8.31 \left( \frac{J}{mol \cdot K} \right) \) is the universal gas constant, and for the \( z \)-factor we accept the approximation of the well-known Standing-Katz chart [9]:

\[ z = 1 - \frac{3.52 P_r}{10^{0.9137 T_r}} + 0.274 \frac{P_r^2}{10^{0.8157 T_r}}, \]

where \( P_r = \frac{P}{P_c} \) is the reduced pressure; \( T_r = \frac{T}{T_c} \) is the reduced temperature; \( P_c \) (MPa) and \( T_c \) (K) are the gas pseudo-critical pressure and temperature, respectively.

For the bubbly flow, the maximum bubble diameter can be determined by [1]:

\[ D_{max} = \frac{N_{we} \sigma \rho u^2}{\rho l^2} \left( \frac{m}{m} \right), \]

where \( N_{we} \) is the Weber number; \( \sigma \) is the gas-liquid surface tension \( \left( \frac{N}{m} \right) \); and \( u \) is the bubble-rise velocity [10]:

\[ u = k \sqrt{c \left( \frac{1}{\rho l} - \frac{1}{\rho g} \right)} \]

with \( m(\theta) = 1 \) for vertical wellbores and [2]:

\[ k = 1.53 + 1.67 \cdot [1 - e^{-2b \cdot a}]b, \]

\[ v_c = \left[ \frac{a g (\rho l - \rho g)}{\rho l} \right]^{\frac{1}{2}}, \]
where \( g \left( \frac{m}{s^2} \right) \) is the acceleration of gravity.

The volume fraction of the gas phase \( \alpha_g \) is taken equal to \( \alpha_g = 0.2 \) for the bubbly flow.

The initial bubble diameter for the bubbly flow is assumed equal to the average bubble size:

\[
D_0 = \frac{D_{\text{max}}}{2},
\]

and so the initial bubble radius is:

\[
R_0 = \frac{D_0}{2}.
\]

For the slug flow, we assume that the initial diameter of the "bubble" (slug) \( D_0 \) can be calculated based on the hydraulic diameter of the annulus. Let \( D_1 \) be the tubing diameter, and \( D_2 \) – the borehole diameter. Then the cross-sectional area of the annulus in which the flow of the gas-oil mixture takes place is:

\[
S = S_2 - S_1 = \frac{\pi D_2^2}{4} - \frac{\pi D_1^2}{4},
\]

and on the other hand:

\[
\tilde{S} = \frac{\pi D_0^2}{4},
\]

which gives the hydraulic diameter

\[
\tilde{D}_0 = \sqrt{\frac{4S}{\pi}},
\]

and the initial radius of the "bubble" for the slug flow is taken 10% less than the hydraulic radius of the annulus:

\[
\tilde{R}_0 = 0.9 \frac{\tilde{D}_0}{2} \text{ (m)}.
\]

For the slug flow, we also assume that the volume fraction of the gas phase is \( \tilde{\alpha}_g = 0.8 \).

For the goal of this study, we nominally assume that the growth or collapse of a bubble is caused by a change in pressure. Away from the bubble, the equilibrium concentration \( C_0 \) corresponds to the pressure \( P \). To find the equilibrium concentration at the bubble boundary \( C^* \), we consider the following two cases. The first case is the bubble growth, which corresponds to the decreasing pressure, so that \( C^* = C_1 \) is the equilibrium concentration at pressure \( P_1 = 0.9P \). The second case is the collapse of the bubble with increasing pressure, and \( C^* = C_2 \) is the equilibrium concentration at pressure \( P_2 = 1.1P \).

Finally, the dependence of the relative bubble radius on time is computed from (13) as

- for the bubbly flow:
  \[
  \tilde{R}(t) = \left( \frac{R_0 + \frac{(C_0 - C_1)}{\rho_g} \sqrt{\frac{12\tilde{D}t}{\pi}}} {R_0} \right) - R_0,
  \]

- for the slug flow:
  \[
  \tilde{R}(t) = \left( \frac{R_0 + \frac{(C_0 - C_2)}{\rho_g} \sqrt{\frac{12\tilde{D}t}{\pi}}} {R_0} \right) - R_0,
  \]

and the relative time derivative of the number of gas moles in the bubble is (14):

- for the bubbly flow:
  \[
  m(t) = \frac{dm}{dt} = \frac{d}{dt} \frac{\rho_g^2 4\pi R_0^3}{\rho_g^2 4\pi R_0^3} = 3\tilde{R}^2 \frac{d\tilde{R}}{dt},
  \]

- for the slug flow:
  \[
  m(t) = \frac{dm}{dt} = \frac{d}{dt} \frac{\rho_g^2 4\pi \tilde{R}_0^3}{\rho_g^2 4\pi \tilde{R}_0^3} = 3\tilde{R}^2 \frac{d\tilde{R}}{dt}.
  \]
4. Calculation of gas mole rate at the bubble surface with the relaxation equation

Consider an element of the gas-liquid stream in a wellbore which contains $N$ gas bubbles. To estimate the characteristic relaxation time $\tau$, we establish a relationship between the solution of the relaxation equation (8) and the solution of the convective diffusion equation (13)-(14). To begin with, we express the total change in the number of moles of the dissolved gas in the liquid using the relaxation equation. According to the solution of (8), the mass concentration of the dissolved gas in the relaxation process is:

$$x_{dg} = x_{dgE} + e^{-\frac{t}{\tau}}(x_{dg0} - x_{dgE}).$$  \hfill (37)

On the other hand, the mass concentration of gas dissolved in the liquid is:

$$x_{dg} = \frac{M_{dg}}{M_l} = \frac{M_{dg}}{M_{dg} + M_{dl}}$$  \hfill (38)

where $M_{dg}$ is the mass of the dissolved gas, $M_{dl}$ is the mass of the dead oil, and $M_l$ is the total mass of the liquid phase within the element.

Expressing $M_{dg}$ from (38) and (37), and differentiating with respect to time, we obtain the rate of change in the total mass of the dissolved gas in the element as:

$$\frac{dM_{dg}}{dt} = -\frac{M_{dl}(x_{dg} - x_{dgE})}{\tau(1 - x_{dg})^2}.$$  \hfill (39)

Since there are $N$ bubbles in the element under consideration, and the relaxation of $x_{dg}$ corresponds to the gas exchange between the bubbles and the liquid, the expression (39) can be related to the rate of change in the number of gas moles in an individual bubble:

$$\frac{dM_{dg}}{dt} = -\frac{dm}{dt}N m_g.$$  \hfill (40)

Now we determine the mass of the dead oil in (39) using the volume fraction of the gas phase:

$$V_g = \frac{4}{3}\pi R_0^3 N;$$  \hfill (41)

$$V_l = \frac{M_l}{\rho_l};$$  \hfill (42)

$$\alpha_g = \frac{V_g}{V_g + V_l};$$  \hfill (43)

$$M_{dl} = (1 - x_{dg})M_l = \frac{4}{3}\pi R_0^3 N \rho_l (1 - x_{dg}) \left(\frac{1 - \alpha_g}{\alpha_g}\right).$$  \hfill (44)

After performing several transformations, we obtain an expression for the relative rate of change for the number of gas moles in a bubble:

$$S(t, \tau) = \frac{\frac{dm}{dt}}{m_0} = \frac{(1 - \alpha_g)\rho e^{-\frac{t}{\tau}}(x_{dg0} - x_{dgE})}{\tau \rho_g \alpha_g m_g (1 - x_{dg})} \approx \frac{(1 - \alpha_g)\rho e^{-\frac{t}{\tau}}(x_{dg0} - x_{dgE})}{\tau \rho_g \alpha_g m_g (1 - x_{dg})}. $$  \hfill (45)

5. Estimation of the characteristic relaxation time with the least squares method

Both the expressions (35)-(36), obtained from the convective diffusion equation, and (45), obtained from the relaxation equation, correspond to the same quantity – the relative rate of change for the number of gas moles in a bubble. Thus, it is possible to find the most suitable value for the characteristic relaxation time by matching (45) to (35)-(36) using the least squares (LS) method.

Since (45) is nearly exponential, it is more convenient to apply LS to the log-transformed values. The LS criterion is:

$$Q = \sum_{i=1}^{n} \left(ln(M(t_i)) - ln(S(t_i, \tau))\right)^2 \rightarrow \min$$  \hfill (46)
where \( \ln(M(t_i)) \) and \( \ln(S(t_i, \tau)) \) are evaluated by (35)-(36) and (45), respectively, for the values of
time \( t_i, i = 1, n. \)

To find the minimum, we use the necessary condition:

\[
\frac{\partial \varrho}{\partial \tau} = 0.
\]  

(47)

After finding the optimal value of \( \tau \), it is substituted into (45) to obtain the relaxation dependence
of the relative changing rate for the number of gas moles in a bubble and compare it to the convective
diffusion dependence (35)-(36).

The described procedure was implemented in Matlab environment for the two flow modes (bubbly
flow and slug flow) and several time intervals.

6. Input data
As the input data, we use a set of depths and pressures presented in Table 1.

| Table 1. Depths and pressures |
|-----------------------------|
| \( h \) (m) | 100 | 300 | 500 | 1000 | 2000 | 3000 |
| \( P \) (10^-1 MPa) | 10 | 30 | 50 | 100 | 200 | 300 |

Other input data are as follows:
\( \gamma_g = 0.573; \ m_a = 0.028966(\text{kg/mole}); \ R = 8.314(\text{J/mole K}); \ \rho_i^g = 850(\text{kg/m}^3); \ N_{we} = 10; \ \sigma = 0.026(\text{N/m}); \)
\( g = 9.8(\text{m/sec}^2); \ D = 2 \cdot 10^{-8}(\text{m}^2 \text{sec}^{-1}); \ D_1 = 0.0763(\text{mm}); \ D_2 = 0.127(\text{mm}); \ t = 0 \ldots 10(\text{sec}). \)

7. Results
Table 2 shows calculated initial bubble sizes for the bubbly flow \( R_0 \) and for the slug flow \( \bar{R}_0. \)

We divide the time range under consideration into several intervals and estimate the characteristic
relaxation time on each of them for the bubbly and slug flow regimes using the least squares method.
As an example, we present calculation results for the depth of 3000 m. To illustrate the procedure of
finding the minimum of (46), Fig. 2 shows a graph for the sum of squared deviations for the time
interval from 1 to 5 seconds for the bubbly flow, and Fig. 3 – for the slug flow.

Table 3 summarizes the obtained values of the characteristic relaxation time for the two flow
regimes and each of the time intervals.

| Table 2. Calculated initial bubble sizes \( R_0, \bar{R}_0 \) (mm) |
|-----------------------------|
| \( \text{Depth (m)} \) |
| \( 100 \) | 0.526 | 0.552 | 0.576 | 0.632 | 0.737 | 0.818 |
| \( 300 \) | \( \text{Bubbly flow (} R_0 \text{)} \) | \( 1 \) | 0.2 | 1 | 5 | 25 |
| \( 500 \) | \( \text{Slug flow (} \bar{R}_0 \text{)} \) | 45.8 |

| Table 3. LS estimates of the characteristic relaxation time \( \tau \) (sec) |
|-----------------------------|
| \( \text{Time interval (sec)} \) |
| \( 0.01 – 0.04 \) | 5.229 | 11.253 | 24.698 | 52.869 | 105.95 |
| \( 0.04 – 0.2 \) | 19.957 | 43.23 | 96.31 | 213.572 | 468.527 |
Knowing the values of the characteristic relaxation time during gas evolution (bubbly flow) and dissolution (slug flow) for each time interval, we substitute them into (45) to obtain the interval-wise approximations to the relative changing rate of the number of gas moles in a bubble. They are compared in Fig. 4-5 to the original graphs corresponding to the solution of the convective diffusion equation for the bubbly flow (35) and for the slug flow (36).

Fig. 6-7 show the values of characteristic relaxation time from Table 3 versus time for the gas evolution (bubbly flow) and gas dissolution (slug flow), respectively. The positions of the points along the horizontal axis are referred to the midpoints of the time intervals for which the estimates were obtained.

To approximate the dependences of Fig. 6-7, we use the power-law function:

\[ \tau = c t^n \]  

(48)

with \( c > 0 \), \( 0 < n < 1 \).

Taking logarithm of the both sides of (48), we obtain a linear relation:

\[ \ln(\tau) = \ln(c) + n \ln(t). \]  

(49)
Figure 4. Interval-wise approximation of the relative changing rate of the number of gas moles in a bubble for the bubbly flow

Figure 5. Interval-wise approximation of the relative changing rate of the number of gas moles in a bubble for the slug flow

Figure 6. The characteristic relaxation time versus time for the bubbly flow
Fig. 7. The characteristic relaxation time versus time for the slug flow

Fig. 8-9 show the straight-line approximations of the points from Fig. 6-7 in logarithmic coordinates. The resulting power-law functions $\tau(t)$ for the two flow regimes in the original coordinates are plotted in Fig. 10-11.

Fig. 8. Straight-line approximation of the characteristic relaxation time graph in logarithmic coordinates for the bubbly flow

Fig. 9. Straight-line approximation of the characteristic relaxation time graph in logarithmic coordinates for the slug flow
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From Fig. 10-11 it can be seen that the approximate dependences closely follow the original points. This means that the power-law dependences $\tau(t)$ correctly reflect variations in the characteristic relaxation time with time for the processes of gas evolution/dissolution for the considered conditions of the gas-oil mixture flow in a well.

8. Scalability of the process
Since we assume the same physical nature of the gas evolution/dissolution process for the bubbly and slug flow, the obtained estimates of the characteristic relaxation times should demonstrate scaling.

Table 4 presents the ratios of the characteristic relaxation times for the two flow regimes at different time intervals. The values are reasonably close, though ~15% deviation at larger times may be noted. It can be attributed to less estimation accuracy at wider time intervals.

Table 4. The ratios of the characteristic relaxation times for the two flow regimes

| Time interval (sec) | $\tau_B$ | $\tau_S$ |
|---------------------|---------|---------|
| 0.01 – 0.04         | 0.262   | 0.26    |
| 0.04 – 0.2          | 0.26    | 0.256   |
| 0.2 – 1             | 0.248   | 0.226   |
| 1 – 5               |         |         |
| 5 – 25              |         |         |
Note that the data of Table 4 is valid only for the accepted parameters of the wellbore, fluids and flow regimes. A more general relation can be derived from dimensional analysis.

9. Conclusions
In this paper, we presented a method for estimating the characteristic relaxation time for non-equilibrium gas evolution and dissolution during the gas-liquid flow in a wellbore. The method is based on matching a solution of the convective diffusion equation for growth/collapse of a bubble in a liquid with a solution of the relaxation equation.

For this purpose, an expression was derived for the rate of change in the number of gas moles in a bubble in a gas-liquid stream due to relaxation of dissolved gas concentration. The LS method was used to match the two solutions at different time intervals, and interval-wise estimates of the characteristic relaxation time were obtained for realistic parameters of the bubbly flow and slug flow regimes, corresponding to the gas evolution and gas dissolution processes respectively.

It was confirmed that the characteristic relaxation times for gas dissolution and evolution differ significantly. Also, it was shown that the estimates of the characteristic relaxation time by the proposed method exhibit a power-law dependence on time and demonstrate expected scalability.

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References
[1] Brill J P and Mukherjee H 1999 Multiphase Flow in Wells Monograph Series 17 (Richardson: Society of Petroleum Engineers)
[2] Tikhonov V S, Indrupskiy I M and Bukashkina O S 2020 Effects of Phase Transitions on Nonstationary Liquid-Gas Flow in a Well During Gas Kicks Journal of Petroleum Science and Engineering 184 106526 https://doi.org/10.1016/j.petrol.2019
[3] Downar-Zapolski P, Bilicki Z, Bolle L and Franco J 1996 The Non-equilibrium Relaxation Model for One-Dimensional Flashing Liquid Flow. Int J. of Multiphase Flow 22 3 p 473-83
[4] Indrupskiy I M, Lobanova O A and Zubov V R 2017 Non-Equilibrium Phase Behavior of Hydrocarbons in Compositional Simulations and Upscaling Computational Geosciences 21 5 p 1173-88
[5] Aadnoy B S, Cooper I, Miska S Z, Mitchell R F and Payne M L (Eds.) 2009 Advanced Drilling and Well Technology (Society of Petroleum Engineers) p 888
[6] Bilicki Z and Kestin J 1990 Physical Aspects of the Relaxation Model in Two-Phase Flow Proc. Royal Soc. Of London, Series A, Mathematical and Physical Sciences 428 p 379-97
[7] Indrupskiy I M and Makhno M A 2020 Modeling of Hydrocarbon Mixture Stationary Flow in a Well with Non-Equilibrium Phase Transitions Automation, Telemechanization and Communication in Oil Industry 4 p 61-8 (in Russian)
[8] Ruckenstein E and James Davis E 1970 Diffusion-Controlled Growth or Collapse of Moving and Stationary Fluid Spheres Journal of Colloid and Interface Science 34 1 p 142-58
[9] Whitson C H and Brule M R 2000 Phase behavior Monograph Series. 20 (Richardson: Society of Petroleum Engineers) p 233
[10] Shi H, Holmes J, Durlofsky L, Aziz K, Diaz L, Alkaya B and Oddie G 2005 Drift-Flux Modeling of Two-Phase Flow in Wellbores. SPE Journal 10 p 24-33