The Sliding Singlet Mechanism with Gauge Mediated Supersymmetry Breaking

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Abstract

The sliding singlet mechanism is one of the most interesting solutions of the triplet-doublet splitting problem. We analyze this mechanism in the gauge mediated supersymmetry breaking scenario. We show that the sliding singlet mechanism does not work in the naive gauge mediation scenario because of the singlet linear terms derived from gravity, although $F$ term is much smaller than the one in the gravity mediation scenario. We also consider the extension in order for the sliding singlet mechanism to work.
1. Introduction

Supersymmetric theories now stand as the most promising candidates for a theory beyond the standard model. The minimal supersymmetric extension of the standard model naturally solves the gauge hierarchy problem and makes the three gauge couplings unify at the scale of $O(10^{16})$ GeV. Therefore, it suggests to us the idea of a grand unified theory (GUT). However, if we consider GUT, a new fine-tuning problem, the so-called triplet-doublet splitting problem, appears. Therefore the colored triplet Higgs must be superheavy to avoid the rapid proton decay, while the doublet Higgs must have the mass of weak scale. Several ideas to solve this serious problem have been proposed. These are, for examples, the sliding singlet mechanism [1, 2], the missing partner mechanism [3], the Dimopoulos-Wilczek mechanism [4], and the GIFT mechanism [5]. The sliding singlet mechanism is the simplest idea in which triplet-doublet splitting is realized dynamically. When the singlet shifts to the potential minimum, the triplet-doublet splitting is realized automatically. The linear term of the singlet can produce the suitable hierarchy between the weak and the GUT scale in the supersymmetric limit [2]. Since the electro-weak symmetry breaking occurs at the tree level, this model is not the so-called radiative electro-weak symmetry breaking scenario [6].

How does the situation change when supersymmetry breaking is switched on? If the supersymmetry breaking occurs at high energy, such as in the gravity mediation scenario, the radiative corrections of Kähler potential induce the doublet Higgs scalar mass, which is the so-called $B$ term of $O(\langle F \rangle M_{GUT}/M_P)$. It destroys the Higgs mass hierarchy [4]. One approach to avoid this difficulty is to extend the gauge symmetry from $SU(5)$ to $SU(6)$ [8]. Another approach is to consider the low energy supersymmetry breaking [3, 4]. The authors of Ref. [10, 11] predicted that the sliding singlet mechanism may work in the gauge mediation scenario. In this paper we analyze whether the sliding singlet mechanism can really work in the gauge mediation scenario or not.

Through the Kähler potential, the supersymmetry breaking effects induce the singlet linear terms both in the superpotential and in the soft supersymmetry breaking interactions [12]. We show that the sliding singlet mechanism does not work in the naive gauge mediation scenario because of these singlet linear terms, although the $F$ term is much smaller than the one in the gravity mediation scenario. In
order for the sliding singlet mechanism to work, additional extensions are needed. One of the extensions considered is the introduction of the additional strong gauge dynamics as will be shown. Even if we introduce these extensions, we also need one more additional mechanism that induces the sliding singlet soft breaking mass of the order of soft breaking masses of Higgs for the electro-weak vacuum stability.

This paper is organized as follows: In section 2, we review the sliding singlet mechanism. Next, we estimate the linear terms of the singlet, which are induced by the gravitational interactions. Section 3 is devoted to the analysis of the Higgs potential both at the GUT scale and at the messenger scale. In section 4 we give summary and discussions of these results.

2. The Sliding Singlet Mechanism

In this section, we review the sliding singlet mechanism \[1, 2\] and present our framework. In GUT, we have to introduce colored Higgs triplets, \(H_C\) and \(\bar{H}_C\), to embed the Higgs doublets in \(SU(5)\) fundamental representations, \(H = (H_C, H_u)\) and \(\bar{H} = (\bar{H}_C, \varepsilon H_d^T)\). However, the colored Higgs cannot be light because we do not want to have a proton decay that is too fast or to spoil the successful unification of gauge couplings. Thus, we need to split the Higgs doublets and triplets. There are various attempts to solve the splitting problem \[1, 2, 3, 4, 5\]. The sliding singlet mechanism is one of these attempts.

One considers a superpotential for the Higgs fields and an adjoint field \(\Sigma\) of the following form,

\[
W = m_H \bar{H} H + \lambda' \bar{H} \Sigma H.
\]

The adjoint field \(\Sigma\) breaks \(SU(5)\) down to \(SU(3) \times SU(2) \times U(1)\). So, we choose the desired vacuum state,

\[
\langle \Sigma \rangle = \sigma \begin{pmatrix} 2 \\ 2 \\ -3 \\ -3 \end{pmatrix}.
\]
Here $\sigma$ has a value of the order of GUT scale. To split the doublets and triplets, we have to tune the mass parameter to

$$m_H = 3\lambda'\sigma. \quad (2.3)$$

This fine-tuning is an unattractive feature of the minimal $SU(5)$ GUT. To avoid this fine-tuning, it was suggested that one replace the mass parameter by a singlet $S$,

$$W = \lambda S\bar{H}H + \lambda'\bar{H}\Sigma H. \quad (2.4)$$

The vacuum expectation value of the singlet will slide to a GUT scale, because of the $F$-flat conditions,

$$\frac{\partial W}{\partial H} = \bar{H}(\lambda S + \lambda'\Sigma) = 0, \quad (2.5)$$

$$\frac{\partial W}{\partial \bar{H}} = (\lambda S + \lambda'\Sigma)H = 0. \quad (2.6)$$

A question arises whether this device is stable $[2, 7]$. It is necessary that $H$ and $\bar{H}$ have non-zero vacuum expectation values at the GUT scale for successful sliding. We know that the doublet Higgs will have vacuum expectation values at the weak scale, for example, via a radiative electro-weak symmetry breaking scenario $[2]$. However, one cannot assure that the doublet Higgs have the vacuum expectation values of weak scale in the context of the sliding singlet mechanism $[2]$. Nemeschansky $[4]$ discussed the models which have a linear term of the sliding singlet in the superpotential $(2.4)$,

$$W = \lambda S\bar{H}H + \lambda'\bar{H}\Sigma H - LS. \quad (2.7)$$

Then we obtain the supersymmetric vacuum as follows,

$$\langle S \rangle = 3\frac{\lambda'}{\lambda}\sigma, \quad (2.8)$$

$$\langle \bar{H} \rangle = ^T\langle H \rangle = (0, 0, 0, 0, \frac{v}{\sqrt{2}}), \quad v^2 = 2L/\lambda. \quad (2.9)$$

The colored Higgs mass is

$$M_{H_C} = 5\lambda'\sigma, \quad (2.10)$$

and the doublet Higgs mass is just zero.

In general, however, quadratically divergent tadpole terms associated with singlets will arise in softly broken supersymmetric theory $[12]$, even if the Kähler potential is minimal. The tadpole terms arise due to supergravity corrections from
operators suppressed by the Planck mass. The $\ell$-loop induced tadpole term is written in the following typical form,\cite{12}

$$\mathcal{L} \sim \frac{N}{(16\pi^2)^\ell} \frac{\Lambda^2}{M_P} \int d^4\theta e^{K/M_P^2} (S + S^\dagger),$$  \hspace{1cm} (2.11)

where $N$ is the number of light chiral superfields that appear in the loops and $\Lambda$ is the cutoff for the quadratic divergence. We make the reasonable assumption that $\Lambda \sim M_P$. The sliding singlet communicates to the supersymmetry breaking sector due to the superspace density, $e^{K/M_P^2}$ in Eq. (2.11). The superspace density has the expansion,

$$e^{K/M_P^2} = 1 + \frac{1}{M_P^2} \left\{ \theta^2 K^i F_i + \bar{\theta}^2 \bar{K}^i \bar{F}_i + \theta \bar{\theta}^2 (K_{ij} + \frac{K_i K_j}{M_P^2}) F^i F^j \right\},$$  \hspace{1cm} (2.12)

where $K_i$ is the derivative of the Kähler potential with respect to a supersymmetry breaking spurious superfield $Z_i$, which has vacuum expectation values in the scalar and the $F$ component,

$$Z = \langle Z \rangle + \theta^2 \langle F_Z \rangle.$$  \hspace{1cm} (2.13)

We can omit the angles as long as there are no ambiguities. The Lagrangian (2.11) is then

$$\mathcal{L} \sim \frac{N}{(16\pi^2)^\ell} \left\{ \int d^2\theta \frac{K^i F^i}{M_P} S + \frac{K_{ij}}{M_P^2} F^i F^j S + h.c. \right\}.$$  \hspace{1cm} (2.14)

The tadpole term seen in Eq. (2.14) can spoil the weak scale hierarchy \cite{7, 11}. In gravity mediated supersymmetry breaking, we should choose the $F$ component of the spurious fields to be $F_Z = M_W M_P$ to obtain the gravitino mass in the order of weak scale $M_W$. Therefore, the coefficient of the sliding singlet in the equation (2.14) is proportional to $F_Z^2/M_P = M_W^2 M_P$. This is much larger than the weak scale, thus the sliding singlet mechanism is not stable in the gravity mediation model. On the other hand, in the gauge mediation model low energy supersymmetry breaking parameters are given by the ratio $F_X/X \equiv \Lambda_{mes}$, which is about $10^4$-$10^5$ GeV. The chiral superfield $X$ is the spurious field in the messenger sector. Though the determination of $F_X$ in the messenger sector originating from $F_Z$ depends on models, $F_Z$ can be in general much smaller than the gravity mediation model. Thus the sliding singlet mechanism may work in gauge mediation models. It is worth studying the sliding singlet mechanism in the context of gauge mediation models in detail.
In the gauge mediation model where the $F$ component arises radiatively originating from the $F$ component ($F_Z$) in the supersymmetry breaking sector, we find that $F_X = O(10^{-4} - 10^{-5}) F_Z$ and that $X \sim \sqrt{F_X} \sim 10^4 - 10^5 \text{GeV}$, namely, $F_Z = O(10^{12} - 10^{15}) \text{GeV}^2$. Requiring the coefficient of the sliding singlet in equation (2.14) to be less than the weak scale for the case of $\ell = 0$ ($\ell = 2$), we find $F_Z$ to be less than $10^{12} \text{GeV}^2$ ($10^{15} \text{GeV}^2$). In the next section, we find another constraint in aspect of the minimization of scalar potential.

3. Feasibility of the Sliding Singlet Mechanism with Gauge Mediation

In this section, we study the feasibility of implementing the gauge mediated supersymmetry breaking scenario in the framework of the sliding singlet mechanism. We perform the minimization analysis above the messenger scale in the Wilsonian scheme. The supersymmetry breaking parameters coming from the messenger loops are highly suppressed, and are neglected in the analysis. However, supergravity induced breaking parameters will appear. The scalar potential is then written as

$$V = \sum m_i^2 |\phi_i|^2 + |\lambda HH - L|^2 + |(\lambda S + \lambda' \Sigma)H|^2 + |H(\lambda S + \lambda' \Sigma)|^2 + (-\rho S + h.c.) + D\text{-terms} + A, B\text{-terms}.$$  

In the scalar potential, the first term represents the supergravity induced breaking mass term for the scalar fields $\phi_i = \{S, H, \bar{H}\}$, which are relevant in this analysis. The breaking masses $m_i^2$ are nearly equal to the gravitino mass. The final terms are scalar trilinear terms and bilinear terms. The gravitino mass and the parameter $\rho$ is given by the full supersymmetry breaking order parameter $F_Z$ of the complete theory. The parameter $\rho$ is given as

$$\rho \sim \frac{N}{(16\pi^2)\ell} M_P m_3^{2/3}. \hspace{1cm} (3.2)$$

The parameter $L$ includes an expectation value of spurious fields in the supersymmetry breaking sector, $Z$,

$$L \sim -\frac{N}{(16\pi^2)\ell} Z m_3^{2/3}. \hspace{1cm} (3.3)$$
We define the ratio \( B \equiv \rho/L \). This parameterization is convenient especially in the direct gauge mediation model, in which \( B \) equals \( \Lambda_{\text{mes}} \).

The linear term in the scalar potential slides the vacuum expectation values of the sliding singlet and induces the so-called \( \mu \) term, which is a doublet Higgs mass parameter in the superpotential. Denoting the vacuum expectation values of the Higgs fields in the same way as the Eqs. (2.9), we obtain the scalar potential as\

\[
V(S, v) = |\lambda S - 3\lambda'\sigma|^2|v|^2 + \frac{1}{2}v^2 - L|S|^2 + m_{3/2}^2(|S|^2 + |v|^2) + (-\rho S + \text{c.c.}).
\]

(3.4)

The extremization conditions are\

\[
\frac{\partial V}{\partial S} = 2(\lambda S - 3\lambda'\sigma)|v|^2 + 2m_{3/2}^2 S - 2\rho = 0, \tag{3.5}
\]

\[
\frac{\partial V}{\partial v} = v \left( 2|\lambda S - 3\lambda'\sigma|^2 + \lambda(|v|^2 - 2L) + 2m_{3/2}^2 \right) = 0. \tag{3.6}
\]

Note that it is necessary for successful sliding that the Higgs fields have non-zero vacuum expectation values. We obtain the \( \mu \) parameter as\

\[
\mu = \lambda S - 3\lambda'\sigma = \frac{\rho}{\lambda v^2} - \frac{3\lambda' m_{3/2}^2 \sigma}{2v^2} \quad (v \neq 0). \tag{3.7}
\]

Substituting \( \mu \) into equation (3.6), we obtain the following cubic equation with respect to \( v^2 \),\

\[
(\lambda^2 v^2)^3 - 2\lambda L(\lambda^2 v^2)^2 + 2(\lambda\rho - 3\lambda' m_{3/2}^2 \sigma)^2 = 0. \tag{3.8}
\]

This equation yields a minimization solution for \( v \neq 0 \) only when the following condition is satisfied\

\[
(\rho - 3\frac{\lambda'}{\lambda} m_{3/2}^2 \sigma)^2 < \frac{16\lambda}{27} L^3. \tag{3.9}
\]

The minimization solution lies in the range\

\[
\frac{4L}{3\lambda} < v^2 < \frac{2L}{\lambda}. \tag{3.10}
\]

Substituting Eqs. (3.3) and (3.2), we find\

\[
m_{3/2}^2 \gtrsim \frac{B^3}{M_P}, \quad v \gtrsim B. \tag{3.11}
\]

*We neglect \( A, B \) terms because they do not change our main result substantially.

† The extremization condition has another solution, but this solution is on the saddle point.
In models where $F_X$ arises at the messenger level directly from a O’Raifeartaigh mechanism (a direct gauge mediation model), the ratio $B$ is determined as $F_X/X (= \Lambda_{mes})$, and the vacuum expectation values of the Higgs field are larger than 10 TeV. If the condition that the gravitino mass is larger than 1 MeV is not satisfied, the sliding singlet mechanism does not work. Further the doublet Higgs becomes of the order of the GUT scale mass and are integrated out.

It is disastrous that the vacuum expectation value of Higgs is larger than 10 TeV. One may consider that the vacuum expectation value will be modified at lower energy. However, the consequence will not change drastically. The running of the parameter $L$ will be negligibly small‡. As another origin, a non-gravitational tadpole term arises with three-loops in which colored Higgs circulate. We estimate the contribution as

$$\frac{\alpha^2}{(16\pi^2)^2} \frac{1}{M_H} \int d^4\theta SXX^\dagger.$$ (3.13)

However, it is unnatural that this operator just cancels out the parameter $L$ coming from the gravitational contribution.

Below the messenger scale, additional supersymmetry breaking terms will arise due to the messenger loops, but they are less than the order of 100 GeV and are negligible compared to 10 TeV. The mass of the sliding singlet is of the order of the vacuum expectation value of the Higgs field. Thus, the sliding singlet survives to lower energy. After GUT particles decouple, the superpotential can be written as§

$$W = (\mu + \lambda \tilde{S})H_dH_u - L\tilde{S},$$ (3.14)

where $\tilde{S} = S - \langle S \rangle$. The supersymmetry breaking terms are

$$V_{soft} = m^2_{H_d}|H_d|^2 + m^2_{H_u}|H_u|^2 + m^2_{\tilde{S}}|\tilde{S}|^2 + (\lambda A_\lambda \tilde{S}H_dH_u + B_\mu \mu H_dH_u - \rho \tilde{S} + h.c.).$$ (3.15)

We denote the vacuum expectation values of the Higgs doublets by $v_d$ and $v_u$, and the value of the singlet $\tilde{S}$ by $x$. As a function of these vacuum expectation

‡ The renormalization group equation with respect to $L$ is

$$\frac{dL}{d(\log \mu_r)} = \frac{\lambda^2}{8\pi^2} L,$$ (3.12)

where $\mu_r$ is the renormalization scale.

§ Note that we should consider the constant term, $\langle W \rangle \sim -L\langle S \rangle$, in the framework of supergravity. One need another sector to wipe out the cosmological constant.
values, the scalar potential has the form
\begin{align}
V &= |\lambda v_d v_u - L|^2 + |\mu + \lambda x|^2 (|v_d|^2 + |v_u|^2) \\
&\quad + m_{H_d}^2 |v_d|^2 + m_{H_u}^2 |v_u|^2 + m_S^2 |x|^2 \\
&\quad + (\lambda A \lambda v_d v_u x + B \mu v_d v_u - \rho x + h.c.) + \bar{g}^2 (|v_d|^2 - |v_u|^2)^2,
\end{align}
(3.16)
where \(\bar{g}^2 = g^2 + g'^2\).

The extremization conditions can be written as
\begin{align}
\mu'^2 + \frac{\bar{g}^2}{4} v^2 &= \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} \equiv M^2 \\
\sin 2\beta &= \frac{\lambda L - \lambda A \lambda x - B \mu \mu'}{m_{H_d}^2 + m_{H_u}^2 + 2\mu'^2 + \lambda^2 v^2} \\
\mu' &= \frac{\lambda \rho - \lambda^2 A \lambda v_d v_u}{\lambda^2 v^2 + m_S^2},
\end{align}
(3.17, 3.18, 3.19)
where \(\tan \beta\) is defined as \(\tan \beta = v_u/v_d\). Note that the value of \(\mu\) is redefined as \(\mu' = \mu + \lambda x\).

Eqs. (3.18) and (3.19) lead to a cubic equation with respect to \(v^2\),
\begin{align}
(\lambda^2 v^2)^3 - \left(\frac{2\lambda L}{\sin 2\beta} - m_{H_d}^2 - m_{H_u}^2\right) (\lambda^2 v^2)^2 + 2(\lambda \rho)^2 &= 0,
\end{align}
(3.20)
similarly to Eq. (3.8), where we neglect \(A_\lambda, B_\mu\) and \(m_S\)\footnote{The parameter \(A_\lambda, B_\mu\) and \(m_S\) does not change the result as long as they are not comparable with the scale 10 TeV.}. The vacuum expectation value of the Higgs field is
\begin{align}
v^2 &\simeq \frac{2L}{\lambda \sin 2\beta}.
\end{align}
(3.21)
Thus, the value for \(v\) is larger than 10 TeV as a result.

We cannot adopt the models where \(F_X\) arises at the messenger level directly. We, therefore, adopt the models where the \(F\) component in the messenger sector arises radiatively. It is possible that the Higgs field has a vacuum expectation value of weak scale if the ratio \(B = \rho/L\) is smaller than 100 GeV. Suppose that the spurious superfield \(Z\) in the supersymmetry breaking sector dominates the parameter \(L\), then we find the ratio \(B\) to be \(F_Z/Z\), the lower bound for the expectation value of \(Z\). On
the other hand, demanding the parameter $L$ to be less than the weak scale, we find the upper bound for $Z$. The parameter region to satisfy both boundaries exists.

One possibility to avoid the disappointing result of Eq. (3.11) is to introduce an additional linear term in the superpotential. The linear term can arise dynamically in $N_c = N_f$ Supersymmetric QCD,

$$W = S \text{tr}M + \bar{\mu} (\det M - BB - \Lambda^{2N_c}), \quad (3.22)$$

where $\bar{\mu}$ is a Lagrange multiplier. Integrating out the meson field $M$, we obtain the linear term of $S$,

$$W = S\Lambda^2. \quad (3.23)$$

When

$$\sqrt{L} < \Lambda < 100 \text{ GeV}, \quad (3.24)$$

it is possible that Higgs has an vacuum expectation value less than 100 GeV. In this case, the gravitino mass has to be less than 10 keV.

In any case, however, it is difficult to construct a phenomenologically viable model. The main reason is that the gauge mediation models produce large $\mu$ term at low energy.

The most stringent phenomenological constraint on the gauge mediation models is derived from the lower bound of the right-handed scalar electron mass, that is, 80 GeV \cite{13}. This constraint gives the lower bound of the parameter $N_m\Lambda_{mes}^2$ because the supersymmetry breaking scalar mass squared is proportional to $N_m(\alpha_i/4\pi)^2\Lambda_{mes}^2$ at the messenger scale \cite{9}, where $N_m$ is the number of messenger quarks or leptons. If the messenger scale is not so large (e.g. smaller than $10^6$ GeV or so), this parameter $N_m\Lambda_{mes}^2$ has to be greater than about $(40\text{TeV})^2$. As such, $N_m\Lambda_{mes}^2$ makes soft supersymmetry breaking right-handed scalar top mass squared larger than about $(430\text{GeV})^2$. Furthermore, this large soft supersymmetry breaking scalar top mass squared drives $m_{\tilde{t}}^2$ lower than typically minus $(200 \text{ GeV})^2$ through renormalization group equations\cite{14, 15}. In order to yield Z boson mass, $\tilde{g}v/\sqrt{2}$, of the magnitude 91 GeV, the $\mu'$ parameter should be larger than 190 GeV from Eq. (3.17). Even in

\footnote{The RGE correction is dependent on the messenger scale. The right-handed scalar electron mass grows up as raising the messenger scale. However, the messenger scale can not be larger than $10^{10}$ GeV in our scheme because the parameters $L$ and $\rho$ must not be larger than the weak scale. Such a low messenger scale does not change the condition that the parameter $N_m\Lambda_{mes}^2$ should be larger than about $(40\text{TeV})^2$.}
the larger messenger scale, the lower bound of \( \mu' \) parameter is conservatively about 160 GeV \([14]\).

This large \( \mu' \) makes the extremization solution unstable because it is on a saddle point. Neglecting \( A_\lambda \) and \( B_\mu \), we obtain the scalar mass squared matrix \( M^2 \),

\[
M^2 = \frac{1}{2} \frac{\partial^2 V}{\partial v_i \partial v_j} = \frac{1}{2} \begin{pmatrix}
2\lambda^2 v^2 + 2m_Z^2 & 4\lambda \mu' v_d & 4\lambda \mu' v_u \\
4\lambda \mu' v_d & g^2 v_d^2 + 2\lambda \tan \beta v_d v_u - 2\lambda L & (4\lambda^2 - g^2) v_d v_u - 2\lambda L \\
4\lambda \mu' v_u & (4\lambda^2 - g^2) v_d v_u - 2\lambda L & g^2 v_u^2 + 2\lambda \cot \beta
\end{pmatrix},
\]

where we define \( v_1 = x \), \( v_2 = v_d \) and \( v_3 = v_u \). Determinant of this matrix is

\[
\det M^2 = 4v^2 \left\{ 2 \left( m_{H_u}^2 + m_{H_d}^2 + 2\mu'^2 \right) \left( g^2 \cos^2 2\beta + 2\lambda^2 \sin^2 2\beta \right) + g^2 \lambda^2 v^2 \right\} m_S^2
- \lambda^2 \left[ 2(2\mu'^2 + m_{H_d}^2 + m_{H_u}^2 + M_Z^2)(4\mu'^2 - \lambda^2 v^2)
+ (4\lambda^2 - g^2)v^2(6\mu'^2 + m_{H_d}^2 + m_{H_u}^2) \cos^2 2\beta \right].
\]

One can easily find that the expression inside the second square brackets is positive provided \( g^2 v^2, \lambda^2 v^2 \ll \mu'^2 \) and \( |m_{H_u}^2| \). Actually, this expression is positive even when \( \mu' \sim \lambda v \) and \( \mu' \sim g v \). Thus, the phenomenologically viable solution can not lie on the minimum point unless the supersymmetry breaking mass squared of the singlet, \( m_S^2 \), is larger than at least the order of \( \mu'^2 \). However, the gauge mediation model does not make such a large breaking mass of the gauge singlet. We need some extra mechanism for making such a large \( m_S^2 \).

Though we find that such a mechanism yields a phenomenologically viable solution, we need fine-tuning between \( m_S^2 \) and \( \rho \). In the case that \( m_S^2 \) is large, from Eqs. \((3.17)\) and \((3.19)\), we study the cubic equation for \( v^2 \) to find what fine-tuned parameters are needed. Neglecting \( A_\lambda \) and \( B_\mu \), we obtain

\[
X^3 + \left( \frac{g^2 m_S^2}{2\lambda^2 M^2} - 1 \right) X^2 + \frac{g^2 m_S^2}{4\lambda^2 M^2} \left( \frac{g^2 m_S^2}{4\lambda^2 M^2} - 2 \right) X + \left( \frac{g^2 \rho}{4\lambda M^3} \right)^2 - \left( \frac{g^2 m_S^2}{4\lambda^2 M^2} \right)^2 = 0,
\]

where we define a dimensionless variable \( X \equiv \frac{g^2 v^2}{(4M^2)} \). The phenomenologically viable solution is then

\[
X < \frac{(91 \text{ GeV})^2}{2 \times (200 \text{ GeV})^2} \sim 0.1.
\]
We need still a fine-tuned parameter such as $m_S^2 \sim \lambda \rho / M$ for such a small $X$ solution, even if we find an extra mechanism for making large $m_S^2$.

4. Conclusion

In this paper, we analyze the sliding singlet mechanism proposed to solve the fine tuning problem associated with the triplet-doublet splitting problem in the gauge mediation scenario. The supersymmetry breaking effects induce the singlet linear terms both in the superpotential and in the soft supersymmetry breaking interactions. We show that these terms change the potential minimum drastically, which cause the weak scale to be of $O(10)$ TeV in the direct gauge mediation model. Even in the other models, we should choose rather extreme parameters, $F_Z/Z < O(10^2)$ GeV in the supersymmetry breaking sector, and $F_X/X > 10^4$ GeV in the messenger sector. We analyzed the minimal effects derived from the supersymmetry breaking in the messenger sector, which always exist in all messenger models. Our analysis was applied to all gauge mediation models. From this analysis, it was found that the sliding singlet mechanism in the gauge mediation scenario is extremely constrained unless the model is extended. We can avoid the difficulty by, for example, introducing the additional gauge group whose non-perturbative effects induce the linear term in the superpotential but do not induce the soft breaking linear term. However, even if we extend the model, we also need one more additional mechanism that induces such a sliding singlet soft breaking mass as $m_S^2 \sim O(\mu^2)$. Moreover, this $m_S^2$ must be fine-tuned as $m_S^2 \sim \lambda \rho / M$ to obtain the correct weak scale. This corresponds to fine-tuning between the $\mu$ term and soft masses of Higgs scalars. However, this fine-tuning is not special to our model, but appears even in the minimal supersymmetric standard model [14].

Note added in proof

The careful reader may point out the possibility that a new minimum appears under the messenger scale through the quantum effect. Renormalization group equation changes scalar potential at the point $\lambda S = 3\lambda' \sigma$, on which the masses of Higgs
doublets are zero. Correct minimum of minimal supersymmetric standard models (MSSM) appears at least as the local minimum. On the other hand, true minimum of the scalar potential above the messenger scale is on the point \( S = \rho/m_{3/2}^2 \) and \( v = 0 \). Thus, the condition for which the MSSM minimum becomes a true minimum is

\[
V(\lambda S = 3\lambda' \sigma, v = 0) - V(\lambda S = \rho/m_{3/2}^2, v = 0) < (10^2 \text{GeV})^4,
\]

where \( V \) is the same \( V \) as in Eq. (3.4). This condition demands the gravitino mass to be less than \( 10^{-3} \text{ eV} \). Such a small gravitino mass requires the SUSY breaking order parameter \( \sqrt{F} < 10^3 \text{ GeV} \). This is incompatible with gauge mediation models.

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