Algorithm to optimize the accuracy of the metering devices for obtaining loose material mixture of a given quality

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Abstract. The paper shows the relevance of solving the problem of ensuring the accuracy of the metering devices. The problem of obtaining a minimum dose of a multicomponent mixture containing blending components with a given accuracy is considered. The dose consists of $N$ cells of mixture micro-components supplied by the automated batchmeters to the batcher. The problem is solved in the framework of the probability theory of normally distributed independent random variables. An exact estimation for the value $N$ with the given probabilistic characteristics of the batchmeters operation is found. It is shown how to arrange the work of batchmeters to provide the required number of cells in the dose.

Keywords: the degree of mixture homogeneity, the accuracy of mixture supplying, the size and number of dosing cells, micro volumes of components in the mixture, mathematical modeling of the processes to obtain loose material mixtures.

1. Introduction

The quality of loose material mixtures obtained by a continuous process depends mainly on the technological parameters providing metering and feeding devices [1-8]. This especially relates to mixing equipment implementing the technology of deterministic formation of mixture homogeneity [9, 10].

There is a large number of theoretical and practical works in this field [1, 4-8]. A wide range of batchmeters and loose material feeders used in many industries has been developed. Technical characteristics of this equipment are adapted to many technological processes. However, the really specified controlled quality of the finished mixed products is possible only with the use of high-tech equipment providing an unlikely formation of mixture homogeneity. This, in turn, imposes higher requirements on the parameters and characteristics of the metering and feeding equipment and tooling, the main of which are the mass of the dosing component micro-volumes and the accuracy of their supply. The quality of the resulting mixtures depends on these process parameters.

The novelty of the development is in the combining of probabilistic indicators of the batchmeters accuracy for loose materials with the output quality indicators of the finished mixture at a given level. Therefore, the authors proposed to derive a detailed correlation between them, using both theoretical [11-13] and empirical data [3-5].

The main assumption and condition in this work is to consider the micro volumes of components obtained by automated feeders as normally distributed random variables, which is typical for most of the known devices.
Then, a new scheme for the formation of a mixture from single cells carrying all the components to be mixed in accordance with the specified proportions-ratios is proposed. Further calculations are carried out using the machinery of probability theory. The incoming parameters of the metering devices are mathematically and technologically related to the output parameters of the quality resulting mixture indicators. This allows one in practice to control the quality of the mixture at the early stages of its synthesis, thus ensuring high quality of the mixed products and its multidirectional monitoring. Practical automated implementation of the proposed technology largely confirms the theoretical one [14].

One of the research results was the creation of a logical visual nomogram to use and develop existing and newly designed metering equipment for loose materials based on the normalized parametric series.

2. Problem statement

The mixture is produced in the deterministic formation of homogeneity. Loose component dosing is carried out gravitationally in volume per unit time by automatic rotary or sluice feeders. From the total volume of the components in the hopper, the feeders cut off the micro volumes of each, according to a certain law and in the order provided by the design of the feeders. They order them at the bottom of the mixture tank in the mixer, thus providing a controlled structure of the mixture, with quality characteristics directly depending on the accuracy of the metering devices [9,10]. After that, the technological relationship between the quality characteristics of the finished mixture and the accuracy of the mechanical batchmeters forming this mixture is analyzed.

The mixture consists of $S$ components (nutrients $A_1, A_2, ... A_S$). The mixture is composed as follows.

First, the batchmeters form the first cell of the mixture, consisting of the sum

$$
\xi_1 + \xi_2 + ... + \xi_{S1}
$$

of microdoses $\xi_1 + \xi_2 + ... + \xi_S$ components $A_1, A_2, ... A_S$ by mass $m_1, m_2, ..., m_S$, respectively.

Then the batchmeters form the second mixture cell

$$
\xi_{12} + \xi_{12} + ... + \xi_{S2}
$$

etc., where $\xi_{ij}$ is the value (mass or volume) of the microdose $\xi_i$ in the $j$ mixture cell.

Each microdose $\xi_i$ is a random variable with known mathematical expectation $m_i$ and dispersion $D_i$ determined empirically.

All microdoses are independent random variables. $N$ mixture cells form a dose of the mixture (Fig.1). It is required to find or estimate the number $N$ at which the mass ratio of mixture components...
in a dose with high probability and accuracy is close to the ideal mass ratio
\[ m_1 : m_2 : \ldots : m_S \]
of these components in dose.

3. Problem solving algorithm

Suppose: all random variables \( \xi_i \) are normally distributed random ones \(^{[11]}\), for which the probability density is given by the formula
\[
p(x)_i = \frac{1}{\sqrt{2\pi} \sigma_i} e^{-\frac{(x - \mu_i)^2}{2\sigma_i^2}},
\]
where \( \sigma_i = \frac{D_i}{m_i} \) is the standard deviation.

First, for each \( i = 1, \ldots, S \) find the value \( N \) at which the probability of the mass deviation absolute value \( \mu_i \) for the component \( A_i \) in the dose from the ideal value \( Nm_i \) does not exceed \( \varepsilon Nm_i \) and is equal to \( 1 - \alpha \), that is
\[
P(\left| \mu_i - Nm_i \right| \leq \varepsilon Nm_i) = 1 - \alpha,
\]
where \( \varepsilon, \alpha \) are given positive numbers close to or relatively close to zero. The smaller \( \varepsilon \), the higher the quality of the mixture. And the smaller \( \alpha \), the higher the probability with the specified mixture quality.

Random variable
\[
\mu_i = \sum_{j=1}^{N} \xi_{ij}
\]
is normally distributed with mathematical expectation \( Nm_i \) and mean square deviation \( \sqrt{N}\sigma_i \) \(^{[12]}\). Its probability density is determined by the formula
\[
p(x)_{\mu_i} = \frac{1}{\sqrt{2\pi} \sqrt{N}\sigma_i} e^{-\frac{(x - Nm_i)^2}{2\sigma_i^2}}.
\]

Therefore,
\[
P(\left| \mu_i - Nm_i \right| \leq \varepsilon Nm_i) = P(1 - \varepsilon \leq \frac{\mu_i}{Nm_i} \leq 1 + \varepsilon) =
\]
\[
\int_{(1-\varepsilon)Nm_i}^{(1+\varepsilon)Nm_i} \frac{1}{\sqrt{2\pi} \sqrt{N}\sigma_i} e^{-\frac{(x - Nm_i)^2}{2\sigma_i^2}} dx
\]

Make a replacement
\[
\frac{x - Nm_i}{\sqrt{N}\sigma_i} = z
\]
in the last integral, the result is
\[
P(1 - \varepsilon \leq \frac{\mu_i}{Nm_i} \leq 1 + \varepsilon) = \frac{\varepsilon \sqrt{N}}{q_i} \int_{-\frac{\varepsilon \sqrt{N}}{q_i}}^{\frac{\varepsilon \sqrt{N}}{q_i}} e^{-\frac{z^2}{q_i^2}} dz = 2\Phi\left( \frac{\varepsilon}{q_i \sqrt{q_i}} \right),
\]
where \( q_i = \frac{\sigma_i}{m_i} \), \( \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{y^2}{2}} dy \).
$\Phi(x)$ is Laplace function, the values table of which is in all textbooks on probability theory, including the cited work [12].

From (1) we obtain the equality

$$\Phi\left(\frac{e}{q_i} \sqrt{N}\right) = \frac{1}{2}(1 - \alpha),$$

from which according to the known right side and the function table $\Phi(x)$ we find the left side value $Z$ of the equality and come to

$$\frac{e}{q_i} \sqrt{N} = Z.$$

Denote

$$Q_i = \left(\frac{Z q_i}{e}\right)^2.$$

Taking into account that $N$ is a whole number, we derive

$$N = Q_i,$$

if $Q_i$ is a whole number and $N = [Q_i] + 1$, if $Q_i$ is not a whole one,

where $[Q_i]$ is an integral part of the number $Q_i$.

The received value $N$ is accurate and improves the estimation of this number obtained in [12].

Denote the received values $N$ for $i = 1, ..., S$ by $N_1, ... N_S$ respectively and find the largest one of these numbers

$$N = \max(N_1, ..., N_S) \quad (2)$$

According to the results obtained above, we have

$$P\left(1 - e \leq \frac{\mu_i}{Nm_i} \leq 1 + e\right) > 1 - \alpha$$

for components $A_i$ where $N_i < N$

$$P\left(1 - e \leq \frac{\mu_i}{Nm_i} \leq 1 + e\right) = 1 - \alpha$$

and for components where $N_i = N$.

Therefore, for all $i = 1, ..., S$

$$P\left(1 - e \leq \frac{\mu_i}{Nm_i} \leq 1 + e\right) \geq 1 - \alpha.$$

Now consider the inequations with $i \neq j$

$$1 - e \leq \frac{\mu_i}{Nm_i} \leq 1 + e, \quad 1 - e \leq \frac{\mu_j}{Nm_j} \leq 1 + e,$$

from which we derive

$$\frac{1 - e}{1 + e} \leq \frac{\mu_j}{Nm_j} \leq \frac{\mu_i}{Nm_i} \leq \frac{1 + e}{1 + e}.$$

Accurate to size of order $e^2$ this inequation will be written over in the form

$$(1 - 2e) \frac{m_i}{m_j} \leq \frac{\mu_i}{\mu_j} \leq (1 + 2e) \frac{m_i}{m_j}.$$

The obtained inequality is considered as the definition of $2e$ - the proximity of the masses ratio $(\mu_1 : \mu_2 : ... : \mu_S)$ to the ideal one $(m_1 : m_2 : ... : m_S)$. The fulfillment of inequation (1) guarantees $2e$ - proximity $(\mu_1 : \mu_2 : ... : \mu_S)$ to $(m_1 : m_2 : ... : m_S)$ with probability $1 - \alpha$.

The number $N$ defined by the equation (2) is the exact solution of the stated problem. The value $N$ depends on $e, \alpha$ and ratios.
\[ q_i = \frac{\sigma_i}{m_i}, \quad i = 1, \ldots, S. \]

Numbers \( q_i \) are measures of the batchmeters operation accuracy. The smaller \( q_i \), the more accurate the batchmeter operates, the smaller numbers \( N_i \) and, respectively, \( N \) are.

If the parameter \( \alpha \) is set within \( 0,01 \leq \alpha \leq 0,1 \), the number \( Z \) (the same for \( i = 1, \ldots, S \)) lies within \( 1,65 < Z < 2,58 \), where it is seen that the parameter \( \alpha \) does not greatly influence the value of the numbers \( N_i \).

Also note, that \( N \) does not depend on the ratio value \( \frac{m_i}{m_j} \) in the mixture. As in the case when \( m_1, \ldots, m_S \) are values of the same order. And in the case when \( m_1, \ldots, m_S \) significantly differ from each other (for example \( m_1 > m_2 > \ldots > m_S \)), the number \( N \) will be the same, if only \( q_i \) is the same for both cases.

### 4. Results discussion

Let's study two examples of solving the stated problem, which show how the number \( N \) depends on the accuracy of the batchmeters \( q_i \). In general examples, the mixture consists of three components \( A_1, A_2, A_3 \) with the following values

\[ \varepsilon = 0,02, \quad \alpha = 0,1. \]

Example 1. \( q_1 = 0,07, q_2 = 0,06, q_3 = 0,09 \).

According to these values \( \varepsilon, \alpha, q_i \), we consistently find:

\[ \Phi(Z) = \frac{1}{2}(1-\alpha) = 0,45 \implies Z = 1,65; \]

\[ Q_1 = \left( \frac{Zq_1}{\varepsilon} \right)^2 = 33,392, \quad Q_2 = \left( \frac{Zq_2}{\varepsilon} \right)^2 = 24,502, \quad Q_3 = \left( \frac{Zq_3}{\varepsilon} \right)^2 = 55,132; \]

\[ N_1 = 34, \quad N_2 = 25, \quad N_3 = 56, \]

\[ N = \max(34,25,56) = 56. \]

Thus, in the first example, the dose consists of 56 cells.

Example 1. \( q_1 = 0,15, q_2 = 0,14, q_3 = 0,17 \).

Here \( Z = 1,65 \) again.

\[ Q_1 = 153,140, \quad Q_2 = 133,402, \quad Q_3 = 196,701; \]

\[ N_1 = 154, \quad N_2 = 134, \quad N_3 = 197, \implies N = 197. \]

We present the table of values \( q \) (\( q \) is the ratio of the mean square deviation to the mathematical expectation), providing the value \( N = 1 \) at the specified parameters \( \varepsilon, \alpha \).

| \( \varepsilon \) \( \backslash \) \( \alpha \) | 0.02   | 0.04   | 0.06   | 0.08   | 0.1   |
|----------------|-------|-------|-------|-------|-------|
| 0.01           | 0.0043| 0.0048| 0.0053| 0.0057| 0.0060|
| 0.02           | 0.0086| 0.0097| 0.0106| 0.0114| 0.0121|
| 0.03           | 0.0129| 0.0145| 0.0159| 0.0171| 0.0181|
| 0.04           | 0.0172| 0.0194| 0.0212| 0.0228| 0.0242|
| 0.05           | 0.0215| 0.0242| 0.0265| 0.0285| 0.0303|
To get the values $q$ leading to $N = 2, 3, ..., n$, one should increase the values $q$ from the above table in $\sqrt{2}, \sqrt{3}, ..., \sqrt{n}$ times, respectively.

Below there is a generalizing nomogram describing possible cases in designing new batchmeters and using existing ones for loose materials. All types of metering devices are factored into standard series, with the main series parameters - the microdose mass (volume) and its supply accuracy. Here $M_i$ is the expected (projected) dosing accuracy of feeding devices in a discrete parametric series in accordance with [13], due to the range of produced batchmeters (the number of "working" ones is technologically limited $n = 2...4$), with extrapolation to the projected ($n = 0; 1; 5; 6$ is earlier or later members), with higher and lower productivity and accuracy of obtained microdoses, respectively (Figure 2):

![Nomogram of Batchmeters for Loose Materials](image)

**Figure 2.** Fields of using batchmeters for loose materials in accordance with the parametric range of their production.

"Working field of application" combines the existing designs of metering devices with the average performance used in modern industrial enterprises. "Area of development" shows the main directions of improving these devices, objectively increasing and reducing productivity and, thus, the accuracy of mixture components. This pattern is fully characterized by the relationship between given indicators and the quality indicators of the finished mixture, developed and presented above. "Innovation" allows one to bring the estimated parameters of the whole system "batcher-mixer-mixture" to obtaining the new, higher-quality mixtures.

5. **Conclusion**

It was established, that in the process of manufacturing high-quality multicomponent mixtures it is necessary that the number of cells $N$ in the effective mixture dose is minimal. The choice of a reasonable, optimal number $N$ is provided only by the high accuracy of the batchmeters with streams of components microdoses making up the final mixture. To design new batchmeters and use existing
ones for loose materials, the generalizing nomogram, allowing one to increase accuracy of giving and
to predict development of essentially new designs of the metering devices, was recommended.

In the following works, the authors propose to present the software implementation of the above
results for the creation of applied engineering design techniques and the use of newly created and
modernized dosing and mixing equipment.

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