Research on cross-well pseudorandom electromagnetic detection method and extraction of response characteristics

Xijin Song | Moe Momayez | Xuelong Wang | Tao Chen | Feng Dang

Abstract

A method is proposed that uses the metal casing of an oil and gas production well to construct a long dipole emission line source and applies pseudorandom multifrequency excitation signals to it to identify reservoir characteristics of interwell reservoirs. This method can significantly improve the work efficiency and precision of electrical prospecting. The cross-well pseudorandom response signal contains multiple main frequency components, as well as various measuring frequency points. Targeting the frequency components with different amplitudes and initial phases in the pseudorandom signal, the digital coherent detection method and its noise resistance are studied. Using this method can effectively extract the amplitude and phase information of each frequency component in different noise environments and has a strong ability to suppress signals of other frequencies. In the heterogeneous interwell reservoir detection model, the amplitude characteristics of each frequency component of the pseudorandom electromagnetic response at the measuring line with different well spacings are extracted. Numerical results show that the curve of equal-frequency measurement appears downwardly "concave" at the interface of the low-resistivity interlayer, and upwardly " convex" at the interface of the high-resistivity interlayer. With the increase in the resistivity of the low-resistivity interlayer, the dynamic range of the extraction component of the same frequency decreases gradually, and the family of equal-frequency response curves overlap with each other. By contrast, for the high-resistivity interlayer, as its resistivity gradually increases, the dynamic range of the extraction component of same frequency also increases gradually. Moreover, as well spacing varies, the response characteristics of the equal-frequency curve family on different measuring lines are also obviously different. The digital coherent detection method significantly simplifies the receiving circuit system. In practical application, response information of different discrete frequency components can be extracted according to requirements. This thus provides a theoretical basis for the extraction of resistivity and characteristic identification of interwell heterogeneous oil and gas reservoirs.

KEYWORDS
cross-well, digital coherent detection, feature extraction, pseudorandom electromagnetic method
INTRODUCTION

As an important means of geophysical survey, the cross-well electromagnetic method can be used to detect the distribution of remaining oil and find oil and gas enrichment zones, to improve the success rate of drilling and enhance recovery efficiency.\(^1\) In the past 20 years, cross-well detection technology has gradually developed into two kinds of detection methods with different physical mechanisms: cross-well seismic and cross-well electromagnetic detection.\(^5\) The results show that, compared with the cross-well seismic method, the cross-well electromagnetic detection method is more sensitive to the changes in cross-well formation characteristics and fluid properties, and can directly provide the resistivity distribution information describing the reservoir cross-well fluid spatial distribution.

Based on the differences in conductivity, permeability, and dielectric properties of underground media, the ground surface electromagnetic method uses the principle of electromagnetic induction to observe and study the distribution characteristics of electromagnetic fields, and then solve various geological problems. By this method, the transmitting and receiving devices are laid on the ground. The working device can be an electric dipole source, magnetic dipole source, line source, or central loop line source.

The cross-well electromagnetic method is a geophysical method that utilizes electromagnetic wave signals, which are transmitted and received, respectively, in two (or more) boreholes, to image and detect physical parameters between them. Because the transmitter and receiver can be placed in deep boreholes, this method features a large detection depth. Therefore, it is widely used in the fields of mineral exploration, petroleum exploration, and engineering environment geophysical prospecting. Meanwhile, this method can offset the limitations of prospecting depth and resolution in the practical application of conventional surface electromagnetic methods; thus, it has wide application potential in the fields of oil and gas reservoir exploration and development.\(^10\)\(^-\)\(^12\)

A pseudorandom signal contains several multifrequency signals whose amplitudes are close and whose frequencies are evenly distributed logarithmically, making it an ideal field source for electrical prospecting.\(^13\)\(^-\)\(^16\) Based on the resistivity differences of rocks and minerals, the pseudorandom frequency-domain electromagnetic method can simultaneously measure signals of multiple frequencies and obtain multiple resistivity information of different depths by a single power supply, thus increasing the measuring speed by several times to more than ten times. Compared with other frequency-domain electromagnetic methods, it has the advantages of high speed, high precision, and large detection depth.\(^17\)\(^-\)\(^19\) Moreover, this method can measure the total field while supplying power, which overcomes the shortcoming of the time-domain electromagnetic method, in which the received signal is weak due to secondary field measurement, but increasing power supply current contradicts the need for lighter equipment.\(^20\)\(^,\)\(^21\) The practical results in recent years have proved that the pseudorandom frequency-domain electromagnetic method has achieved large depth detection under the premise of ensuring high precision and efficiency. Good results have been achieved in geophysical prospecting of deep-seated metallic minerals, conventional oil and gas, shale gas, engineering geology, coal mine water disasters, and other areas.\(^21\)\(^-\)\(^24\)

At present, the transmission power of the pseudorandom signal transmitter is 100 kW and 30 kW in the fields of medium and shallow metal mineral resources detection, coal field water damage detection, and other fields. For deep exploration, such as searching for volcanic reservoirs, the resistivity of the oil area is very low, generally ranging from a few ohms to a dozen ohms, with the reservoir generally below 4 km. Currently, it is necessary for the pseudorandom transmitter to have sufficient transmission power and a sufficiently low operating frequency. The power of the transmitter needs to reach 200 kW, the maximum output current reaches 200 A, and the minimum operating frequency reaches 0.01 Hz.\(^25\)\(^,\)\(^26\) However, this method is mainly applied to surface exploration at present, and further research is needed on how the advantages of this method can be used for cross-well reservoir identification.

Metal casings are often found in a single well, either the emitting well or the receiving well, or in both wells in the cross-well electromagnetic measurement. In signal processing and inversion interpretation of cross-well measurement data, a large part of the effort is devoted to eliminating the influence of the casing on response signals.\(^27\)\(^-\)\(^29\) As the actual borehole of oil and gas production wells is very small in size, the diameter of the magnetic dipole source in wells is limited, and the transmitting power of a signal source is further limited. As a result, the detection range is limited, making it difficult to effectively detect interwell oil and gas reservoirs. Therefore, this study proposes a method to prospect the remaining oil enrichment zones between production wells and in well patterns. In this method, a long dipole emission line source is constructed using the metal casing of oil and gas production wells to which pseudorandom multifrequency signals are applied. Converting a magnetic dipole into an electric dipole increases the transmission power and thereby increases the exploration range. Using the metal casing to construct electric dipoles not only eliminates the shielding effect of the casing on electromagnetic signals, but also takes advantage of the good conductivity of the metal casing. Using it as a long line source of dipole emission by directly applying emission current to it helps realize long-distance interwell detection.

The cross-well pseudorandom response signal contains multiple main frequency components, as well as various...
measuring frequency points. The conventional method mostly adopts simulated frequency selection. By simulating narrowband filtering, each frequency component is extracted, with the amplitude and phase information of each effective frequency measured through wave demodulation and integration.\textsuperscript{30-32} This method features low noise and stable measurement, suitable for detecting a small number of frequency points. However, as cross-well pseudorandom electromagnetic signals contain multiple main frequency components, excessive measuring frequency points will necessitate complex and bulky receiver hardware. This brings difficulties to instrument design and debugging, and makes it difficult to ensure the consistency of multiple receivers. Therefore, this study adopts the digital coherent detection method, which extracts the amplitude and phase of each frequency component from the cross-well pseudorandom electromagnetic response, and has a strong ability to suppress signals of other frequencies. This method can significantly simplify the hardware circuit design and easily ensure the consistency between multiple instruments. By extracting the pseudorandom electromagnetic response characteristics of the low-resistivity interlayer and high-resistivity interlayer at the measuring line with different well spacing, a good foundation is then laid for identifying the characteristics of interwell heterogeneous oil and gas reservoirs.

2 | CROSS-井 WELL PSEUDORANDOM ELECTROMAGNETIC DETECTION METHOD

In this study, the pseudorandom frequency-domain electromagnetic method is introduced into the field of oil and cross-well gas reservoirs prospecting. In this method, the metal casing in one production well of the well pattern is used to construct an electric dipole emission source, with pseudorandom emission signals applied to it, with the receiving dipoles arranged separately in the remaining wells. The cross-well pseudorandom electromagnetic detection model is shown in Figure 1. In the three layers of the heterogeneous formation, there are four oil and gas wells from left to right, of which Well 0 is a cased well, and Wells 1, 2, and 3 are uncased wells. Line #4 is a horizontal measuring line on the ground surface. In this method, the casing of Well 0 is used to construct an electric dipole emission source, in which two electric dipoles (A and B) are set on the casing wall through contacts to control the length of the electric dipole source. Receiving electrodes are set in Wells 1, 2, and 3; meanwhile, the #4 horizontal measuring line and receiving electrodes are laid on the ground. The interwell formation model is 800 m × 800 m × 400 m, with the upper and lower layers 120 m thick and the middle layer 160 m thick. The distances between the measuring lines of #1, #2, and #3 are 160 m, 200 m, and 280 m, respectively. Each measuring line is 320 m long, and a measuring point is arranged at each 20 m interval, with a total of 17 measuring points. The #4 measuring line is 180 m long, with a total of 10 measuring points.

When operating, pseudorandom multifrequency excitation signals are transmitted once, and the pseudorandom electromagnetic field responses at the receiving electrode at different positions of each measuring line are recorded simultaneously. The amplitude and phase information of different frequency components at each measuring point are extracted by the digital coherent detection method to identify the distribution characteristics of interwell oil and gas reservoirs.

Electromagnetic prospecting is achieved using $2^n$ sequence pseudorandom signals, which are ideal field sources. The author discusses in detail the coding principle, frequency distribution, and energy distribution of $2^n$ sequence pseudorandom signals in reference.\textsuperscript{33} The results show that the $2^n$ sequence pseudorandom code contains a total of $n$ principal components with equal amplitudes and is evenly distributed logarithmically. The energy of the transmitted signal is more concentrated on the main frequency component when $n$ is an odd number. Therefore, the detection depth distribution of pseudorandom coded waveforms is more reasonable over the traditional electromagnetic method. In addition, multiple depths of underground information can be obtained by one survey, greatly improving the efficiency of geophysical exploration. The $2^n$ sequence pseudorandom transmission controller can send single-frequency, three-frequency, five-frequency, and seven-frequency waves. The frequencies range from 0.015625 Hz ($2^{-6}$ Hz) to 8192 Hz ($2^{13}$ Hz) and can be combined according to different frequencies. Considering the parameters of the detection model shown in Figure 1, the excitation signal in this study adopts the frequency combination of three-frequency waves, specifically including three-frequency components of 1 Hz, 8 Hz, and 64 Hz ($2^0$, $2^3$, and $2^6$).
For the continuous pseudorandom multifrequency response signal, the amplitude and phase information of the extracted frequency response signal can be obtained by multiplying the sine or cosine function of the frequency component to be extracted and integrating the product. The derivation process of the specific extraction method is shown in Appendix A. If the response signal is digitized, and then the amplitude and phase information of each discrete frequency is extracted through operation processing, not only the receiving circuit system can be greatly simplified, but also the information of each frequency component can be collected and extracted simultaneously, making it easier to ensure the consistency of multiple instruments. When digitizing the pseudorandom response signal, the digitized sampling frequency is set as \( f_s \), and then, the number of sampling points within the period \( T_1 \) is as follows: \( N = T_1 / f_s \).

Based on the extraction method of different frequency components of continuous pseudorandom signals, we let the digitized sequence of the pseudorandom response signal be \( x(n) \). When the frequency of the extracted signal is \( \omega_k = 2^{k-1} \omega_p \), the cosine component coefficient of the frequency signal is as follows:

\[
a'_{k1} = \left[ \frac{2}{T_K} \sum_{n=0}^{N} x(n) \cos(2\pi n f_s) \right] / \frac{T_1}{T_K} \tag{1}
\]

Similarly, the sinusoidal component coefficients of this frequency signal are follows:

\[
b'_{k1} = \left[ \frac{2}{T_K} \sum_{n=0}^{N} x(n) \sin(2\pi n f_s) \right] / \frac{T_1}{T_K} \tag{2}
\]

where \( T_K = 2\pi / \omega_k \).

Therefore, the amplitude and phase information of each frequency component can be extracted from the digital sequence of multifrequency pseudorandom responses, which is the digital coherent detection method.

Targeting the pseudorandom multifrequency excitation signals adopted in the numerical calculation in this study, after extracting the various frequency components of pseudorandom response signals, amplitude and frequency parameters are introduced to analyze the character variation of pseudorandom electromagnetic response amplitudes at different measuring point positions and with different frequency components, which is defined as

\[
F_{s1}(\%) = \frac{(E_l - E_m)}{E_h} \times 100\% \tag{3}
\]

\[
F_{s2}(\%) = \frac{(E_l - E_m)}{E_m} \times 100\% \tag{4}
\]

where \( E_l \) is the electric field response amplitude corresponding to the 1 Hz frequency component, \( E_m \) is the electric field response amplitude corresponding to the 8 Hz frequency component, and \( E_h \) is the electric field response amplitude corresponding to the 64 Hz frequency component. Then, the frequency difference between the high frequency and low frequency expressed in Equation (3) is 64 times, while that between the high frequency and low frequency expressed in Equation (4) is 8 times.

### 3 | NOISE RESISTANCE OF DIGITAL COHERENT DETECTION METHOD

The pseudorandom excitation signal adopted in this study contains three-frequency components of 1 Hz, 8 Hz, and 64 Hz, with the initial amplitude of each frequency component set as 1 (A) and the initial phase as 0°. Its waveform is shown in Figure 2. When 10% Gaussian white noise is added into the transmitted signal, the amplitude and phase of each frequency component are extracted by means of the digital coherent detection method at different sampling periods as shown in Table 1. At the initial increase stage of the sampling period number, the errors of amplitude and phase extraction results of each frequency component have significant downward trends. However, with the further increase in the number of sampling periods, the errors of the extraction results decrease slowly and oscillate.

The amplitude of each frequency component of the excitation signal is set as 1 (A) and the initial phase as 0°. Considering the extraction error and computational efficiency of coherent detection, the number of sampling periods is set as 32. Gaussian white noises of 2%, 5%, 10%, and 20% are added to the emission signals, respectively, as shown in Figure 3A,B,C,D. With the reduction in SNR (Signal to Noise Ratio), the excitation signal amplitude presents a gradually
TABLE 1  Extraction results of different sampling periods

| Number of sampling periods | 1  | 4  | 8  | 16 | 32 | 48 | 64 |
|----------------------------|----|----|----|----|----|----|----|
| **1 Hz**                   |    |    |    |    |    |    |    |
| Amplitude                  | 0.9977 | 0.9965 | 0.9989 | 1.0002 | 1.0020 | 1.0006 | 1.0000 |
| Phase                      | −0.1505 | −0.1098 | 0.1479 | 0.0233 | 0.0643 | 0.0652 | −0.0229 |
| **8 Hz**                   |    |    |    |    |    |    |    |
| Amplitude                  | 1.0029 | 1.0075 | 0.9974 | 1.0007 | 0.9986 | 1.0005 | 1.0013 |
| Phase                      | −0.0771 | 0.1288 | −0.0341 | 0.0209 | −0.0668 | 0.0638 | 0.0781 |
| **64 Hz**                  |    |    |    |    |    |    |    |
| Amplitude                  | 1.0072 | 0.9970 | 0.9986 | 1.0006 | 1.0003 | 0.9983 | 0.9990 |
| Phase                      | 0.1259 | −0.4355 | −0.2534 | −0.0740 | 0.1211 | 0.0192 | −0.0882 |

FIGURE 3  Excitation signals with noises of different intensities added
increasing trend due to the influence of noise. The amplitude and phase of each frequency component are extracted using the digital coherent detection method, and the results are shown in Table 2. It is easy to calculate that the maximum relative errors of the amplitude extraction from the 1 Hz, 8 Hz, and 64 Hz frequency components are 0.15%, 0.08%, and 0.12%, respectively.

The initial phase of the amplitude of each frequency component is set as 0°, and the number of sampling periods is 32. Let the amplitude of the 1 Hz, 8 Hz, and 64 Hz frequency components be 5 (A), 2 (A), and 1 (A), respectively. Then, the waveform is shown in Figure 4. Gaussian white noises of 2%, 5%, 10%, and 20% are added to the emission signal. The amplitude and phase of each frequency component are extracted using the digital coherent detection method, and the results are shown in Table 3. It can be calculated that the maximum relative errors of the amplitude extractions from the 1 Hz, 8 Hz, and 64 Hz frequency components are 0.04%, 0.13%, and 0.18%, respectively.

When each frequency component of the pseudorandom signal has different initial amplitude and initial phase, the amplitude and phase information of each frequency component are extracted using the digital coherent detection method after adding in Gaussian white noises of different intensities. According to Tables 2 and 3, the initial phase of each

| SNR | 2% | 5% | 10% | 20% | Maxi. relative error (%) |
|-----|----|----|-----|-----|--------------------------|
| 1 Hz |     |    |     |     |                          |
| Amplitude | 1.0007 | 1.0001 | 1.0015 | 0.9988 | 0.15 |
| Phase | 0.0686 | −0.0409 | 0.1002 | −0.0094 | - |
| 8 Hz |     |    |     |     |                          |
| Amplitude | 0.9996 | 0.9993 | 1.0004 | 0.9992 | 0.08 |
| Phase | 0.0055 | 0.0419 | 0.0259 | 0.0360 | - |
| 64 Hz |     |    |     |     |                          |
| Amplitude | 0.9992 | 1.0006 | 1.0006 | 1.0012 | 0.12 |
| Phase | 0.0545 | 0.0585 | −0.0551 | 0.0576 | - |

**FIGURE 4** Excitation signals with different initial amplitudes

**FIGURE 5** Excitation signals with different initial phases
frequency component is 0°. When the initial amplitude of each frequency component is different, the maximum relative error of the extraction result is 0.15%. According to Table 4, when the initial amplitude and initial phase of each frequency component are different, the maximum relative error of the amplitude extraction of each component is 0.18%, and the maximum relative error of the phase extraction of each component is 0.39%. The results fully prove that the digital coherent detection method can effectively extract the required amplitude and phase information of the frequency component and has a strong ability to suppress signals of other frequencies.

4 | EXTRACTION OF CROSS-WELL PSEUDORANDOM ELECTROMAGNETIC RESPONSE CHARACTERISTICS

4.1 | Low-resistivity interlayer

For the cross-well pseudorandom electromagnetic detection model shown in Figure 1, pseudorandom excitation signals are applied to the emitting electric dipole (as shown in Figure 2). Setting the resistivity of the upper and lower media as 100 (Ω m), when the resistivity of the middle layer is a low-resistivity interlayer with a resistivity of 1 (Ω m), the pseudorandom response of each receiving electrode is numerically calculated by the finite element method. Additionally, the electromagnetic field responses of different frequency components on different measuring lines are extracted using the digital coherent detection method.

The 2^n sequence pseudorandom transmission signal is generated by software. By importing it into the finite element numerical calculation program, we can obtain the pseudorandom electromagnetic field response of different measuring lines. The pseudorandom electromagnetic method is a frequency-domain electromagnetic method. The electromagnetic response calculated by the above method is the total field, that is, the superposition of the primary field and the secondary field. Subtracting the background field (the pseudorandom response of the uniform stratigraphic model with resistivity 100 (Ω m)) from the total field, the pure secondary field response under geoelectric conditions shown in Figure 1 can be obtained. Figure 6A shows the curves of electric field response amplitude at different measuring points on the #1 measuring line. As seen in the figure, the

| SNR | 2%  | 5%  | 10% | 20% | Maxi. relative error (%) |
|-----|-----|-----|-----|-----|--------------------------|
| 1 Hz|     |     |     |     |                          |
| Amplitude | 5.0004 | 4.9991 | 5.0020 | 4.9993 | 0.04                      |
| Phase   | 0.0083 | −0.0035 | 0.0069 | −0.0089 | -                         |
| 8 Hz   |     |     |     |     |                          |
| Amplitude | 1.9999 | 1.9997 | 2.0030 | 2.0003 | 0.15                      |
| Phase   | 0.0074 | −0.0258 | −0.0613 | 0.0054 | -                         |
| 64 Hz  |     |     |     |     |                          |
| Amplitude | 0.9991 | 1.0009 | 1.0000 | 0.9997 | 0.09                      |
| Phase   | 0.0281 | −0.0069 | 0.0568 | −0.0731 | -                         |

| SNR | 2%  | 5%  | 10% | 20% | Maxi. relative error (%) |
|-----|-----|-----|-----|-----|--------------------------|
| 1 Hz|     |     |     |     |                          |
| Amplitude | 5.0002 | 5.0010 | 5.0008 | 5.0014 | 0.028                    |
| Phase   | 15.0063 | 15.0112 | 15.0052 | 15.0097 | 0.075                   |
| 8 Hz   |     |     |     |     |                          |
| Amplitude | 1.9997 | 2.0013 | 1.9977 | 1.9974 | 0.13                     |
| Phase   | 20.0203 | 19.9951 | 20.0082 | 19.9219 | 0.39                    |
| 64 Hz  |     |     |     |     |                          |
| Amplitude | 0.9996 | 0.9982 | 0.9988 | 1.0017 | 0.18                     |
| Phase   | 60.0389 | 59.9135 | 60.0589 | 59.8052 | 0.325                   |
response curves of the three-frequency components all appear downwardly concave at the junction between different resistivities, and the response amplitude of the electric field response in the low-resistivity medium decreases. Among the response curves of the three-frequency components separated by the digital coherent detection method, the response amplitude of the 1 Hz frequency component is the highest, while that of 64 Hz frequency component is the lowest, and that of a low-frequency response signal is greater than that of a high-frequency response signal. In Figure 6B, two amplitude frequency parameters (Equations 3 and 4) of the electric field response of the #1 measuring line are drawn in a double-ordinate graph. As can be seen, the amplitude of amplitude frequency 1 is significantly greater than that of amplitude frequency 2, which is determined by the difference between the high frequency and low frequency. Figure 7A shows the electric field response curve at different measuring points on the #2 measuring line, and Figure 7B shows the two amplitude frequency parameters of the electric field response on the #2 measuring line. Figure 8A is the electric field response curve at different measuring points on the #3 measuring line, and Figure 8B is the two amplitude

FIGURE 6 Extraction results of the #1 measuring line (the resistivity of the middle layer is 1 (Ω m))

FIGURE 7 Extraction results of the #2 measuring line (the resistivity of the middle layer is 1 (Ω m))
frequency parameters on the electric field response on the #3 measuring line. According to the comparison between Figures 6A, 7A, and 8A, with the increase in well spacing, the response signal amplitude of each frequency component gradually decreases and moves toward the left of the coordinate axis. On the semilogarithmic coordinate, the concave phenomenon of the response curve to the low-resistivity interlayer is increasingly obvious.

Figure 9A shows the extraction result of each frequency component of the pseudorandom response on the #4 horizontal measuring line located on the ground. As seen in the figure, the amplitude of the response signal gradually decreases with the increase in measuring point serial number, namely the transmitter-receiver separation. The low-frequency response curve is at the top, while the high-frequency response curve is at the bottom. The amplitude of the low-frequency (1 Hz) response signal is larger than that of the high-frequency (64 Hz) response signal. Figure 9B shows two amplitude frequency parameters of the electric field response on the #4 measuring line. Similarly, the amplitude of amplitude frequency 1 is obviously greater than that of amplitude frequency 2.
With other parameters the same as above, when the middle layer is a low-resistivity medium with the resistivity of 10 (Ω m), the response curves of each frequency component at different measuring points on the measuring lines #1, #2, #3, and #4 are extracted as shown in Figure 10. According to Figure 10A-C, the low-frequency response amplitude is greater than the high-frequency response amplitude. With the increase in well spacing, the amplitude of the response curves of the three-frequency components in the pseudorandom electromagnetic response decreases gradually and moves to the left of the coordinate axis. By comparing Figure 9A with Figure 10D, it is found that, as the resistivity of the interwell low-resistivity interlayer increases, the amplitude of the electric field response on the #4 horizontal measuring line decreases.

Similarly, for the cross-well pseudorandom electromagnetic detection model shown in Figure 1, the electromagnetic field responses when the resistivities of the middle layer are 20 (Ω m) and 50 (Ω m) are also calculated, and the response curves of different frequency components are extracted using the digital coherent detection method. To compare the change characteristics of the electromagnetic response at different well spacings, the response curves of equal frequencies on the measuring lines #1, #2, and #3 are drawn in one figure. The results are shown in Figures 11-13. Figure 11 shows the extraction results of the 1 Hz frequency component when the

![Figure 10](image-url)
resistivities of the middle layer are set as 1 (Ω m), 10 (Ω m), 20 (Ω m), and 50 (Ω m). Figure 11 shows that the #1 measuring line has the smallest well spacing and that its electric field response is seriously affected by the field source effect, leading to a large error in judging different resistivity sections between wells. With increased well spacing, the response curves of measuring lines #2 and #3 are smooth. Among them, the electric field response curve on the #2 measuring line increases slightly and then decreases at the junction where resistivity changes; however, the resistivity stratification can be clearly judged from the whole curve. The response curve of the electric field on the #3 measuring line is comparatively ideal and identifies well the interwell formation sections with different resistivities. This is related
to the emitting dipole length, emission signal amplitude, and the resistivity of the heterogeneous reservoir. By comparing Figure 11A-D, it is found that with the increase in the resistivity of the middle layer, the dynamic range of the response curves on the three measuring lines gradually decreases. For example, for the middle layer with a resistivity of 1 (Ω m) (Figure 11A), the dynamic range of the three response curves is 0-1.5 × 10^{-9} (V/m). When the resistivity of the middle layer increases to 50 (Ω m) (Figure 11D), the dynamic range of the three response curves narrows to 0-8 × 10^{-10} (V/m).
Furthermore, with the increase in the resistivity of the middle layer, the amplitude of the response curve of each line also decreases.

Figure 12 shows the extraction results of the 8 Hz frequency component when the resistivities of the middle layer are set as 1 (Ω m), 10 (Ω m), 20 (Ω m), and 50 (Ω m), and

Figure 13 Extraction results of the 64 Hz frequency component
Figure 13 shows the extraction results of the 64 Hz frequency component. By comparing Figures 11A, 12A, and 13A, it is found that, like for the 1 (Ω m) low-resistivity middle layer, the dynamic ranges of the 1 Hz, 8 Hz, and 64 Hz frequency components in the pseudorandom electric field response of the three measuring lines are 0-1.5 × 10⁻⁹ (V/m), 0-1.0 × 10⁻⁹ (V/m), and 0-1.0 × 10⁻¹⁰ (V/m), respectively. It indicates that for the cross-well pseudorandom electromagnetic response, with the increase in the extraction frequency component, the amplitude of the response component decreases gradually.

To examine the change characteristics of the equal-frequency extraction results of the middle layer with different resistivity, when the resistivities of the middle layer are set as 1 (Ω m), 10 (Ω m), 20 (Ω m), and 50 (Ω m), the results of the equal-frequency extraction of the cross-well pseudorandom electromagnetic response are drawn in one figure. Figure 14 shows the extraction results of the equal-frequency components in the electric field response on the #2 measuring line. Among them, Figure 14A, B, C are the extraction results of the 1 Hz, 8 Hz, and 64 Hz frequency components, respectively. As can be seen, the dynamic range of the response

![Figure 14](image-url)
curve decreases with the increase in the resistivity of the middle layer.

As shown in Figure 14A, for the extraction results of the 1 Hz frequency component, when the middle layer resistivity is 1 (Ω m), the response curve has the largest amplitude, located at the outermost position among the family of equal-frequency curves. By contrast, when the middle layer resistivity is 50 (Ω m), the response curve has the smallest amplitude, located at the innermost position among the family of equal-frequency curves. Moreover, a comparison of 14A, 14B, and 14C shows that the overall dynamic range of each equal-frequency curve family decreases with the increase in the extraction frequency. Figure 14A shows the overall dynamic range of the curve family with the low frequency (1 Hz), which is $2-6 \times 10^{-10}$ (V/m), and Figure 14C shows the overall dynamic range of the curve family with the high frequency (64 Hz), which is $2-6 \times 10^{-11}$ (V/m), decreasing by an order of magnitude.

Figure 15 shows the extraction results of the equal-frequency component in the electrical field response on the #3 measuring line.
measuring line when the resistivities of the middle layer are set as 1 (Ωm), 10 (Ωm), 20 (Ωm), and 50 (Ωm). Among them, Figure 15A,B,C is the extraction results of the 1 Hz, 8 Hz, and 64 Hz frequency components, respectively. As seen in the figures, for the middle layer with different resistivities, the extraction result of each frequency component has good distinguishability. Similar to the extraction result of the #2 measuring line, the response amplitude of each frequency component to a low-resistivity interlayer is greater than that to a high-resistivity interlayer. As shown in Figure 15B, for the extraction results of the 8 Hz frequency component, when the middle layer resistivity is 1 (Ωm), the response curve has the largest amplitude, located at the outermost position among the family of equal-frequency curves. By contrast, when the middle layer resistivity is 50 (Ωm), the response curve has the smallest amplitude, located at the innermost position among the family of equal-frequency curves. A comparison of 15A, 15B, and 15C shows that the overall dynamic range of each equal-frequency curve family decreases with the increase in extraction frequency. Figure 15A shows the overall dynamic range of the curve family with the low frequency (1 Hz), which is $0.2 \times 10^{-10} \text{ (V/m)}$, and Figure 15C shows the overall dynamic range of the curve family with the high frequency (64 Hz), which is $0.2 \times 10^{-11} \text{ (V/m)}$, decreasing by an order of magnitude. Moreover, compared with the extraction results on the #2 measuring line, the dynamic range of the extraction results on the #3 measuring line happens to fall into another interval, and there is no overlap between the two. The dynamic range of the family of equal-frequency curves (#3 measuring line), shown in Figure 15A, is $2 \times 10^{-10} \text{ (V/m)}$, while the dynamic range of the family of equal-frequency curves (#3 measuring line), shown in Figure 15A, is $0.2 \times 10^{-10} \text{ (V/m)}$. The two ranges do not overlap, which helps to distinguish the electromagnetic response on different measuring lines.

4.2 High-resistivity interlayer

For the detection model shown in Figure 1, the extraction results of electromagnetic field responses on different measuring lines when the middle layer is a high-resistivity interlayer was analyzed as follows. With other parameters remaining unchanged, when the resistivity of the middle layer is 200 (Ωm), the extraction results of the electric field responses of the three-frequency components at different measuring points on the #2 measuring line are shown in Figure 16. As can be seen, for the high-resistivity interlayer, the pseudorandom response curve appears obviously upwardly convex at the junction between different resistivity sections. The electromagnetic response of pseudorandom excitation signal to a high-resistivity medium is greater than than to a low-resistivity medium. Furthermore, the amplitude of the low-frequency component (1 Hz) is greater than that of the high-frequency component (64 Hz). Figure 16B shows two amplitude frequency parameters of the electric field response on the #2 measuring line (Equations 3 and 4). As seen in Figure 16B, the amplitude frequency 1 (the difference between the high and low frequency is 64 times) is far greater than that of frequency 2 (the difference between the high and low frequency is 8 times).

Similarly, for the middle layer with a resistivity of 200 (Ωm), Figure 17A shows the extraction results of electric field responses at different measuring points on the #3 measuring line. The comparison between Figures 16A and

FIGURE 16 Extraction results of the #2 measuring line (the resistivity of the middle layer is 200 (Ωm))
FIGURE 17 Extraction results of the #3 measuring line (the resistivity of the middle layer is 200 (Ω m))

17A shows that with the increase in well spacing, the amplitude of pseudorandom response decreases. Compared with the extraction results on the #2 measuring line, the three equal-frequency response curves all move to the left of the coordinate axis. Figure 17B shows two amplitude frequency parameters of the electric field response on the #3 measuring line. Figure 18A shows the extraction results of the electric field responses at different measuring points on the #4 measuring line located on the ground surface. As can be seen, the response amplitude of the three-frequency components decreases gradually with the increase in the measuring point serial number. The response curve of the 1 Hz component is at the top, the response curve of the 64 Hz component is at the bottom, and the amplitude of the high-frequency component is smaller than that of the low-frequency component. Figure 18B shows the two amplitude frequency parameters of the electric field response on the #4 measuring line.

With other parameters the same as above, when the resistivity of the middle layer is 500 (Ω m), the response curves of each frequency component at different measuring points in the electric field response on measuring lines #2, #3, and #4 are extracted as shown in Figure 19. A comparison of Figure 19A with B shows that at the junction between the high-resistivity interlayer and the upper and lower formation media, the response curves of various frequency components on measuring line #2 and #3 all have an obvious reflection.

FIGURE 18 Extraction results of the #4 measuring line (the resistivity of the middle layer is 200 (Ω m))
Among them, the amplitude of the 1 Hz component is highest and that of the 64 Hz component is lowest. With the increase in well spacing, the response amplitude of the electric field on the #3 measuring line decreases compared with that on the #2 measuring line. A comparison of Figure 16A and Figure 19A shows that when the resistivity of the high-resistivity interlayer increases from 200 (Ω m) to 500 (Ω m), the amplitude of each frequency component on the #2 measuring line decreases and moves toward the left of the coordinate axis. However, on the semilogarithmic coordinate, the amplitude of the abnormal response to the high-resistivity interlayer (eg, the change between the amplitude at the #5 measuring point and that at the #4 measuring point) increases significantly. By comparing Figure 17A with Figure 19B, it is found that the extraction result of each frequency component on the #3 measuring line also has the same characteristics. Figure 19C shows the extraction result of the electric field response on the #4 horizontal measuring line. A comparison of Figure 18A with Figure 19C shows that, with the increase in the resistivity of the high-resistivity interlayer, the response amplitude of each frequency component of the #4 horizontal measuring line also shows a decreasing trend.

Figure 20 shows the extraction results of the equal-frequency component in the electrical field response on the #2 measuring line when the resistivities of the middle layer are set as 200 (Ω m), 500 (Ω m), 1000 (Ω m), and 2000 (Ω m).
Among them, Figure 20A-C is the extraction results of the 1 Hz, 8 Hz, and 64 Hz frequency components, respectively. It can be seen that with the increase in resistivity of the intermediate high-resistivity interlayer, the response amplitude of each frequency component to different interlayers decreases gradually; however, the dynamic range of the response curve increases. As shown in Figure 20A, the response curve of the middle layer with a resistivity of 2000 (Ω m) is located at the far left of the curve family, with the smallest amplitude, but the whole curve has the largest dynamic range. By contrast, the response curve of the middle layer with a resistivity of 200 (Ω m) is located at the far right of the curve family, with the largest amplitude, but the whole curve has the smallest dynamic range. Moreover, a comparison of Figure 20A-C shows that the dynamic range of electric field response gradually decreases with the increase in extraction frequency component. The dynamic ranges of the extraction results of the 1 Hz, 8 Hz, and 64 Hz frequency components are 0-5 × 10^{-10} (V/m), 0-4 × 10^{-10} (V/m), and 0-4 × 10^{-11} (V/m), decreasing by an order of magnitude.
Figure 21 shows the extraction results of the equal-frequency component in the electrical field response on the #3 measuring line when the resistivities of the middle layer are set as 200 (Ω m), 500 (Ω m), 1000 (Ω m), and 2000 (Ω m). As can be seen, for the high-resistivity interlayer, the extraction results of the #3 and #2 measuring lines have different response characteristics. Unlike the response curve in Figure 20A, in Figure 21A, the response curve of the middle layer with a resistivity of 200 (Ω m) is located at the innermost position of the curve family, with the smallest amplitude, and the whole curve has the smallest dynamic range. With the increase in the resistivity of the high-resistivity interlayer, the amplitude of the response curve increases gradually. When the resistivity of the middle layer is 2000 (Ω m), the response curve is located at the outermost position of the curve family, with the largest amplitude, and the whole curve has the largest dynamic range. Moreover, for the high-resistivity interlayers with different resistivities, the family of electric field response curves on the #2 measuring line does not overlap. With the increase in the resistivity of the high-resistivity interlayer, the amplitude of the response curve decreases gradually. However, the electric field response curves on the #3
measuring line overlap obviously. With the increase in the resistivity of the high-resistivity interlayer, the dynamic range of the response curve increases obviously. The response amplitude for the high-resistivity interlayer increases gradually, while that for the upper and lower formation media decreases gradually. It indicates that the response characteristics of the high-resistivity interlayer on different measuring lines vary with the variation in well spacing.

5 | CONCLUSIONS

The pseudorandom frequency-domain electromagnetic method has made significant achievements in conventional prospecting fields of oil and gas reservoir, shale gas, metal ore, and coalfield water disaster; however, these applications are concentrated in the field of ground surface prospecting. Interwell areas are the main areas where the remaining oil is distributed and are also key areas that need to be prospected to tap potential remaining oil and improve oil recovery. In this study, the pseudorandom frequency-domain electromagnetic method is introduced into the field of interwell oil and gas reservoir detection, with a detection method proposed. This method uses metal casings of oil and gas production wells to construct a long dipole emission line source and applies pseudorandom multifrequency excitation signals to it, providing a new idea for interwell oil and gas reservoir identification. This method can transmit multiple frequency components at one time, which can greatly improve the efficiency of the cross-well frequency electromagnetic method.

The cross-well pseudorandom response signal contains multiple main frequency components, as well as various measuring frequency points. Using the traditional simulative frequency selection technology will make the receiver hardware circuit complex and bulky. Targeting the frequency components with different amplitudes and initial phases in the pseudorandom signal, the digital coherent detection method and its noise resistance are studied. The results show that for the three-frequency wave excitation signal selected in this study, the maximum relative error of the amplitude extraction result of each component is 0.18%, and the maximum relative error of the phase extraction result of each component is 0.39%. The results prove that this method can effectively extract the amplitude and phase information of each frequency component in different noise environments and has a strong ability to suppress signals of other frequencies.

A model for detecting interwell heterogeneous reservoirs is established, and the pseudorandom electromagnetic response at each measuring point on different measuring lines with different well spacing is numerically calculated using the finite element method. The amplitude characteristics of each frequency component are extracted from the pseudorandom response using the digital coherent detection method, and the curves of the response amplitude of the equal-frequency electric field and the amplitude frequency parameter on different measuring lines are drawn. The numerical results show that the curves of equal-frequency measurement all have obvious abnormality at the junction between media of different resistivities.

For the homogeneous formation with a resistivity of 100 (Ω m), when the resistivities of the low-resistivity interlayer are 1 (Ω m), 10 (Ω m), 20 (Ω m), and 50 (Ω m), the family of equal-frequency curves all appear downwardly "concave" at the interface between different media. With the increase in the resistivity of the low-resistivity interlayer, the dynamic range of the extraction component of same frequency decreases gradually, and the family of the equal-frequency response curves overlap. However, for the same uniform formation, when the resistivity of high-resistivity interlayer is 200 (Ω m), 500 (Ω m), 1000 (Ω m), and 2000 (Ω m), the phenomenon of "convexity" appears in the family of equal-frequency curves. As the resistivity of the high-resistivity interlayer increases, the dynamic range of the extraction component of same frequency also increases gradually. Moreover, for different well spacings, the response characteristics of the equal-frequency curve family on each measuring line are also obviously different. This is related to the length of the emitting dipole, the amplitude of the transmitting signal, and the resistivity of the heterogeneous interwell reservoir.

CONFLICT OF INTEREST

None declared.

ORCID

Xijin Song https://orcid.org/0000-0002-4820-3224

REFERENCES

1. Mishra S, Shrivastava C, Ojha A, Miotti F. Waterflood surveillance by calibrating streamline-based simulation with crosswell electromagnetic data. In: International Petroleum Technology Conference; 2019.
2. Zeng S, Dong Q, Chen J. A novel casing antenna system for crosswell electromagnetic telemetry in pad drilling. In: Unconventional Resources Technology Conference (URTEC); 2017.
3. Wilson GA, Donderici B. U.S. Patent Application No. 15/531,384; 2017.
4. Kim J, Um ES, Moridis GJ. Integrated simulation of vertical fracture propagation induced by water injection and its borehole electromagnetic responses in shale gas systems. J Petrol Sci Eng. 2018;165:13-27.
5. Ling Y, Zhang G, Jin Y, et al. Reservoir interwell dynamic connectivity inversion method with considering external fluid influx. In: IOP Conference Series: Earth and Environmental Science, Vol. 252, No. 5:52104). IOP Publishing.
6. Gupta RK, Agrawal M, Pal SK, Kumar R, Srivastava S. Site characterization through combined analysis of seismic and electrical resistivity data at a site of Dhanbad, Jharkhand, India. Environ Earth Sci. 2019;78(6):226.

7. Lan T, Liu N, Han F, Liu QH. Joint petrophysical and structural inversion of electromagnetic and seismic data based on volume integral equation method. IEEE Trans Geosci Remote Sens. 2018;57(4):2075-2086.

8. Vogel EE, Saravia G, Pastén D, Muñoz V. Time-series analysis of earthquake sequences by means of information recognizer. Tectonophysics. 2017;712:723-728.

9. El-Awawdeh R, Zhang J, Yose L, Modavi A. Learnings and challenges of carbonate reservoir geophysical fluid monitoring from giant onshore and offshore oil fields in Middle East. EAGE Workshop on 4D Seismic and Reservoir Monitoring: Bridge from Known to Unknown; 2018.

10. Schmidt HK, Servin JMF, Ellis ES. Experimental verification of a new approach to long-range EM imaging. In: SPE Kingdom of Saudi Arabia Annual Technical Symposium and Exhibition. Society of Petroleum Engineers; 2018.

11. Zhang Y, Hoteit I. Efficient assimilation of crosswell electromagnetic data using ensemble-based history-matching framework. SPE J. 2019;25(01):119-138.

12. Servin F, Manuel J. Monitoring water flood front movement by propagating high frequency pulses through subsurface transmission lines. In: SPE Asia Pacific Oil and Gas Conference and Exhibition. Society of Petroleum Engineers; 2018.

13. Song X, Wang X, Dong Z, Zhao X, Feng X. Analysis of pseudo-random sequence correlation identification parameters and anti-noise performance. Energies. 2018;11(10):2586.

14. Li F, Li Z, Li H, Yang Z, Gao P, Zhou W. Application of pseudo random correlation identification method based on LabView electromagnetic prospecting. In: 2018 37th Chinese Control Conference (CCC). IEEE; 2018:10289-10293.

15. Liu L, Wang P. The application of pseudo-random geoelectrical sounding method in urban archaeological exploration. In: International Geophysical Conference, Qingdao, China, 17–20 April 2017. Society of Exploration Geophysicists and Chinese Petroleum Society; 2017:1012-1013.

16. Mo D, Jiang Q, Li D, Chen C, Zhang B, Liu J. Controlled-source electromagnetic data processing based on gray system theory and robust estimation. Appl Geophys. 2017;14(4):570-580.

17. Lin J, Kang L, Liu C, et al. The frequency-domain airborne electromagnetic method with a grounded electrical source. Geophysics. 2019;84(4):1-43.

18. Morton EJ, Peyton AJ. U.S. Patent Application No. 15/859,777; 2019.

19. Qu X, Li Y, Fang G, Yin H. A portable frequency domain electromagnetic system for shallow metal targets detection. Prog Electromagn Res. 2017;53:167-175.

20. Zhu X, Fu Z, Su X, Qin S. Frequency-domain analysis for pulse current sources in transient electromagnetic method. Near Surface Geophys. 2017;15(2):155-162.

21. Di QY, Xue GQ, Lei D, et al. Geophysical survey over molybdenum mines using the newly developed M-TEM system. J Appl Geophys. 2018;158:65-70.

22. Ziolkowski A, Slob E. Introduction to Controlled-source Electromagnetic Methods: Detecting Subsurface Fluids. Cambridge University Press; 2019.

23. Wang X, Deng J. The study of anti-noise immunity for electromagnetic method based on m pseudo-random sequence. In: Technology and Application of Environmental and Engineering Geophysics. Singapore: Springer; 2017:91-96.

24. Li H, Xue G, Zhao P, Zhong H, Zhou N. The extraction of TEM response from pseudo random binary sequence source EM data. In: Technology and Application of Environmental and Engineering Geophysics. Singapore: Springer; 2017:167-174.

25. Hu YF, Li DQ, Yuan B, Suo GY, Liu ZJ. Application of pseudo-random frequency domain electromagnetic method in mining areas with strong interferences. Trans Nonferrous Metals Soc Chin. 2020;30(3):774-788.

26. Wang M, Jin S, Wei W, Deng M. The technique analysis and achievement of the high power borehole-ground electromagnetic synchronous transmitter system. Chin J Geophys. 2019;10:21.

27. Wu Y, Guo B, Zhang J. Analysis and simulation of metal casing effect on induction logging. Optik. 2017;138:302-313.

28. Chen M, Wang H, Liu Y, Ma L, Wu D, Wang S. Corrosion behavior study of oil casing steel on alternate injection air and foam liquid in air-foam flooding for enhance oil recovery. J Petrol Sci Eng. 2018;165:970-977.

29. Wu D, Bittar MS. U.S. Patent No. 9,651,705. Washington, DC: U.S. Patent and Trademark Office; 2017.

30. Eidsvik J, Dutta G, Mukerji T, Bhattacharjya D. Simulation-regression approximations for value of information analysis of geophysical data. Math Geosci. 2017;49(4):467-0491.

31. Chen Y, Hill J, Lei W, et al. Automated time-window selection based on machine learning for full-waveform inversion. In: SEG Technical Program Expanded Abstracts 2017. Society of Exploration Geophysicists; 2017:1604-1609.

32. Huang Y, Schuster GT. Full-waveform inversion with multisource frequency selection of marine streamer data. Geophys Prospect. 2018;66(7):1243-1257.

33. Song X, Wang X, Li P. Electromagnetic response characteristics of local conductors with pseudo-random coded waveforms. In: International Field Exploration and Development Conference. Singapore: Springer; 2018:1684-1705.

How to cite this article: Song X, Momayez M, Wang X, Chen T, Dang F. Research on cross-well pseudorandom electromagnetic detection method and extraction of response characteristics. Energy Sci Eng. 2020;8:3602–3626. https://doi.org/10.1002/ese3.767
APPENDIX A

Fourier series representation of the periodic signal is as follows:

\[ x(t) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos n\omega_0 t + b_n \sin n\omega_0 t \right) \quad \text{(A1)} \]

The pseudorandom signal is a composite periodic signal containing multiple frequency components, which can be expressed as

\[ x(t) = x_1(t) + x_2(t) + \cdots + x_N(t) \]

\[ = \left( a_{10} + \sum_{n=1}^{\infty} a_{1n} \cos n\omega_0 t + b_{1n} \sin n\omega_0 t \right) + \left( a_{20} + \sum_{n=1}^{\infty} a_{2n} \cos 2n\omega_0 t + b_{2n} \sin 2n\omega_0 t \right) \]

\[ + \cdots + \left( a_{N0} + \sum_{n=1}^{\infty} a_{Nn} \cos Nn\omega_0 t + b_{Nn} \sin Nn\omega_0 t \right) \quad \text{(A2)} \]

For a 2^n sequence pseudorandom signal, it contains n different frequency components with the frequency of 2^i and increasing in a binary relation, and the ratio of adjacent frequencies is 2. Hence:

\[ \omega_1 = 2^{1-1}, \omega_0 = \omega_0, \quad \omega_2 = 2^{2-1}, \omega_0 = 2\omega_0, \quad \omega_3 = 2^{3-1}, \omega_0 = 4\omega_0, \quad \ldots, \omega_N = 2^{N-1}, \omega_0 = \omega_0 \quad \text{(A3)} \]

Substituting Equation (A3) into Equation (A2):

\[ x(t) = x_1(t) + x_2(t) + \cdots + x_N(t) \]

\[ = \left( a_{10} + \sum_{n=1}^{\infty} a_{1n} \cos n\omega_0 t + b_{1n} \sin n\omega_0 t \right) + \left( a_{20} + \sum_{n=1}^{\infty} a_{2n} \cos 2n\omega_0 t + b_{2n} \sin 2n\omega_0 t \right) + \cdots \]

\[ + \left( a_{N0} + \sum_{n=1}^{\infty} a_{Nn} \cos Nn\omega_0 t + b_{Nn} \sin Nn\omega_0 t \right) \quad \text{(A4)} \]

By merging the equal-frequency components in the above equation, we can obtain:

\[ x(t) = (a_{10} + a_{11} \cos \omega_0 t + b_{11} \sin \omega_0 t) + [a_{20} + (a_{12} + a_{21}) \cos 2\omega_0 t + (b_{12} + b_{21}) \sin 2\omega_0 t] + \cdots \]

\[ + [a_{N0} + (a_{1N} + a_{2N} + \cdots + a_{N2N-1}) \cos 2^{N-1}\omega_0 t + (b_{1N} + b_{2N} + \cdots + b_{N2N-1}) \sin 2^{N-1}\omega_0 t] \]

\[ + \sum_{n=1}^{\infty} a_{1n} \cos n\omega_0 t + b_{1n} \sin n\omega_0 t + \sum_{n=N+1}^{\infty} a_{Nn} \cos Nn\omega_0 t + b_{Nn} \sin Nn\omega_0 t \]

\[ + \sum_{n=3}^{\infty} a_{2n} \cos 2n\omega_0 t + b_{2n} \sin 2n\omega_0 t + \sum_{n=N/2+1}^{\infty} a_{N2n} \cos 2Nn\omega_0 t + b_{N2n} \sin 2Nn\omega_0 t + \cdots + \sum_{n=N/N+1}^{\infty} a_{Nn} \cos 2^{N-1}n\omega_0 t + b_{Nn} \sin 2^{N-1}n\omega_0 t \quad \text{(A5)} \]

where

\[ a'_{21} = a_{12} + a_{21}, \quad b'_{21} = b_{12} + b_{21}, \]

\[ a'_{N1} = a_{1N} + a_{2N} + \cdots + a_{N2N-1}, \quad b'_{N1} = b_{1N} + b_{2N} + \cdots + b_{N2N-1} \quad \text{(A6)} \]

Substituting Equation (A6) into Equation (A5), we obtain:
Multiply both sides of Equation (A7) by $\cos \omega_k t$ and integrate low-frequency periodic signals, then:

when $m = 1, \omega_K = 2^{K-1} \omega_0$, in the above equation, except $a_{N1}' \int_{T_1} \cos 2^{K-1} \omega_0 t \cos \omega_k tdt$, all the other terms are 0, then Equation

$$x(t) = (a_{N0} + a_{N1} \cos \omega_0 t + b_{N1} \sin \omega_0 t) + [a_{2N0} + a_{2N1} \cos 2^{N-1} \omega_0 t + b_{2N1} \sin 2^{N-1} \omega_0 t] + \ldots$$

(A7) reduces to:

$$\int_{T_1} x(t) \cos \omega_k tdt = \int_{T_1} a_{N0} \cos \omega_k tdt + \left( \int_{T_1} a_{N1} \cos \omega_0 t \cos \omega_k tdt + \int_{T_1} b_{N1} \sin \omega_0 t \cos \omega_k tdt \right) + \ldots$$

$$+ \left( \int_{T_1} a_{2N1} \cos 2^{N-1} \omega_0 t \cos \omega_k tdt + \int_{T_1} b_{2N1} \sin 2^{N-1} \omega_0 t \cos \omega_k tdt \right) + \ldots$$

(A8) reduces to:

$$\boxed{\int_{T_1} x(t) \cos \omega_k tdt = a_{K1}' \int_{T_1} \cos 2^{K-1} \omega_0 t \cos \omega_k tdt}$$

$$= \frac{a_{K1}'}{T_K} \int_{T_1} \cos 2^{K-1} \omega_0 t \cos 2^{K-1-1} \omega_0 tdt$$

(A9)

Thus, it can be calculated that:

$$a_{K1}' = \left( \frac{2}{T_K} \int_{T_1} x(t) \cos \omega_k tdt \right) / \frac{T_1}{T_K}$$

(A10)

Similarly, we can obtain:

$$b_{K1}' = \left( \frac{2}{T_K} \int_{T_1} x(t) \sin \omega_k tdt \right) / \frac{T_1}{T_K}$$

(A11)
Thus, the amplitude and phase of the frequency ($\omega_K$) signal are extracted, respectively, as follows:

\[ c'_{K1} = \sqrt{(d'_{K1})^2 + (b'_{K1})^2} \]  
(A12)

\[ \theta'_{K1} = \arctan\left(\frac{-b'_{K1}}{d'_{K1}}\right) \]  
(A13)