Implications of a New Particle from the HyperCP Data on $\Sigma^+ \to p\mu^+\mu^-$

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Abstract

The HyperCP collaboration has recently reported the observation of three events for the decay $\Sigma^+ \to p\mu^+\mu^-$ with an invariant mass $m_{\mu^+\mu^-}$ for the muon-antimuon pair of $\sim 214$ MeV. They suggest that a new particle state $X$ may be needed to explain the observed $m_{\mu^+\mu^-}$ distribution. Motivated by this result, we study the properties of such a hypothetical particle. We first use $K^+ \to \pi^+\mu^+\mu^-$ data to conclude that $X$ cannot be a scalar or vector particle. We then collect existing constraints on a pseudoscalar or axial-vector $X$ and find that these possibilities are still allowed as explanations for the HyperCP data. Finally we assume that the HyperCP data is indeed explained by a new pseudoscalar or axial-vector particle and use this to predict enhanced rates for $K_L \to \pi\pi X \to \pi\pi\mu^+\mu^-$ and $\Omega^- \to \Xi^- X \to \Xi^-\mu^+\mu^-$. 

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I. INTRODUCTION

Three events for the decay mode $\Sigma^+ \rightarrow p\mu^+\mu^-$ with an invariant mass of $214.3 \pm 0.5$ MeV for the muon-antimuon pair have been recently observed by the HyperCP collaboration [1]. The branching ratio is obtained to be $[8.6^{-6.6}_{+7.3}(\text{stat}) \pm 5.5(\text{syst})] \times 10^{-8}$ [1]. The central value is considerably larger than the short-distance contribution in the standard model [2]. When long-distance contributions are properly included, it is possible to account for the total branching ratio [2, 3]. However, the clustering of the events whose contribution to the branching ratio is $(3.1^{+2.4}_{-1.9} \pm 1.5) \times 10^{-8}$ around 214 MeV cannot be explained. If this result stands future experimental scrutiny, it is most likely to be due to a particle state $X$ having a mass of 214 MeV.

In this paper we study the properties of such a particle assuming its existence.

The mass 214 MeV of this hypothetical particle is close to, but higher than, the sum of the masses of two muons. It is tempting to identify it as a muonium bound-state. However, the S-wave bound-state has a mass below the sum of the two muon masses. Therefore, the state $X$ cannot be an S-wave muonium state. Radial excitations can yield larger masses, but it is unlikely that the electromagnetic interaction binding the muon and antimuon together can raise the mass by the 3 MeV needed. The $X$ particle, if exists, is likely a new state beyond the standard model (SM). There are theories where such light states naturally exist, for example, the super-partner of the goldstino particle in spontaneously local super-symmetry breaking theories as discussed in Ref. [4]. These particles, pseudoscalar and scalar ones, can have masses lower than a few GeV or even in the MeV range.

In our study we will not attempt to construct models which predict such particles. Instead, we will assume the existence of the new particles and study the implications from the HyperCP data. To be consistent with observations, we follow HyperCP and assume that the hypothetical particles have small widths, are short-lived (they decay inside the detector), and do not interact strongly [1].

II. EFFECTIVE INTERACTIONS

In our study, we will try to be as model independent as possible by parameterizing the interactions of this new particle with known particles. In the HyperCP hypothesis, this particle is produced in the decay of $\Sigma$ to $p$ and subsequently decays into a muon-antimuon pair. At the quark level, the particle $X$ must then couple to $\bar{d}s$ (and of course to $\mu^+\mu^-$ as well). A priori, the state $X$ can be a scalar, pseudoscalar, vector, axial-vector, or even a tensor particle. We will consider four possibilities: scalar ($X_S$), pseudoscalar ($X_P$), vector ($X_V$), and axial vector ($X_A$).

Assuming that the hypothetical new particles have definite parity, do not carry electric or color charge, and are their own anti-particles, we can write their couplings to $\bar{d}s$ and $\mu^+\mu^-$ as

$$\mathcal{L}_S = (-g_{S\bar{d}s} \bar{d}s + \text{H.c.}) X_S + g_{S\mu \bar{\mu} \mu} X_S,$$

$$\mathcal{L}_P = (-ig_{P\bar{d}s} \bar{d}\gamma_5 s + \text{H.c.}) X_P + ig_{P\mu \bar{\mu} \gamma_5 \mu} X_P,$$

$$\mathcal{L}_V = (-g_{V\bar{d}s} \bar{d}\gamma_\mu s + \text{H.c.}) X^\mu_V + g_{V\gamma_\mu \mu} X^\mu_V,$$

$$\mathcal{L}_A = (g_{A\bar{d}s} \bar{d}\gamma_5 \gamma_\mu s + \text{H.c.}) X^\mu_A + g_{A\gamma_\mu \gamma_5 \mu} X^\mu_A.$$

If the particle does not have a definite parity, our results should be interpreted as applying to the parity-even or -odd coupling as appropriate.
A condition that the couplings in the above equations must satisfy is that they must be able to produce the observed branching ratio in $\Sigma^+ \to p\mu^+\mu^-$. To carry out such a fit, one must know how the $X$ couples to the hadron states $\Sigma^+$ and $p$ from the above quark-level couplings. To this end, we employ chiral perturbation theory to obtain the couplings. Our task is simplified by the assumption that the hypothetical particles do not interact strongly as they can then be readily identified with the scalar, pseudoscalar, vector, and axial-vector external sources in the standard-model Lagrangians. With the flavor properties assumed in Eq. (11), the appropriate Lagrangians are then

$$\mathcal{L}_{SB\phi} = b_D \left\langle B \left\{ \xi^\dagger h_S^X \xi + \xi h_S^X, B \right\} \right\rangle + b_F \left\langle B \left\{ \xi^\dagger h_S^X \xi + \xi h_S^X, B \right\} \right\rangle + b_0 \left\langle h_S \left( \Sigma^\dagger + \Sigma \right) \right\rangle \left\langle BB \right\rangle + \frac{1}{2} f^2 b_0 \left\langle h_S \left( \Sigma^\dagger + \Sigma \right) \right\rangle + \text{H.c.}, \quad (2a)$$

$$\mathcal{L}_{PB\phi} = ib_D \left\langle B \left\{ \xi^\dagger h_P^X \xi - \xi h_P^X, B \right\} \right\rangle + ib_F \left\langle B \left\{ \xi^\dagger h_P^X \xi - \xi h_P^X, B \right\} \right\rangle + ib_0 \left\langle h_P \left( \Sigma^\dagger - \Sigma \right) \right\rangle \left\langle BB \right\rangle + \frac{i}{2} f^2 b_0 \left\langle h_P \left( \Sigma^\dagger - \Sigma \right) \right\rangle + \text{H.c.}, \quad (2b)$$

$$\mathcal{L}_{VB\phi} = \frac{1}{2} \left\langle B \gamma_\mu \left[ B, \xi^\dagger \xi h_V^0 \xi + \xi h_V^0 \xi \right] \right\rangle + \frac{1}{2} D \left\langle B \gamma_\mu \gamma_5 \left( \xi^\dagger h_V^0 \xi - \xi h_V^0 \xi \right) B \right\rangle + \frac{1}{2} F \left\langle B \gamma_\mu \gamma_5 \left[ \xi^\dagger h_V^0 \xi - \xi h_V^0 \xi \right] B \right\rangle + \frac{1}{2} C \left[ T_\mu \left( \xi^\dagger h_V^0 \xi - \xi h_V^0 \xi \right) B + B \left( \xi^\dagger h_V^0 \xi - \xi h_V^0 \xi \right) T_\mu \right] - \frac{i}{2} f^2 \left\langle h_V^0 \left( \partial_\mu \Sigma \Sigma^\dagger - \Sigma^\dagger \partial_\mu \Sigma \right) \right\rangle + \text{H.c.}, \quad (2c)$$

$$\mathcal{L}_{AB\phi} = \frac{1}{2} \left\langle B \gamma_\mu \left[ B, \xi^\dagger \xi h_A^0 \xi - \xi h_A^0 \xi \right] \right\rangle + \frac{1}{2} D \left\langle B \gamma_\mu \gamma_5 \left( \xi^\dagger h_A^0 \xi + \xi h_A^0 \xi \right) B \right\rangle + \frac{1}{2} F \left\langle B \gamma_\mu \gamma_5 \left[ \xi^\dagger h_A^0 \xi + \xi h_A^0 \xi \right] B \right\rangle + \frac{1}{2} C \left[ T_\mu \left( \xi^\dagger h_A^0 \xi + \xi h_A^0 \xi \right) B + B \left( \xi^\dagger h_A^0 \xi + \xi h_A^0 \xi \right) T_\mu \right] - \frac{i}{2} f^2 \left\langle h_A^0 \left( \partial_\mu \Sigma \Sigma^\dagger + \Sigma^\dagger \partial_\mu \Sigma \right) \right\rangle + \text{H.c.}, \quad (2d)$$

where we have shown only the terms relevant for this paper, and used the notation $(h_Y)_{kl} = X_Y g_{Yq} h_{kl}$ for $Y = S, P$, and $(h_A^\mu)_{kl} = X_A^\mu g_{Yq} h_{kl}$ for $Y = V, A$, with $h_{kl} = (T_0 + iT_7)_{kl} = \delta_{k2} \delta_{3l}$. The notation and parameter values that we employ here are explained in Appendix A.

With the above effective Lagrangians, we can obtain constraints on the couplings $g_{Yq}$ from other low-energy processes.

### III. RULING OUT THE SCALAR AND VECTOR AS CANDIDATE PARTICLES

With the assumption that the new particles are short-lived and narrow, their contribution to the branching ratio of $\Sigma^+ \to p\mu^+\mu^-$ is given by $\mathcal{B}(\Sigma \to pX)\mathcal{B}(X \to \mu^+\mu^-)$. Using the effective
Lagrangians in Eq. (2), we find the matrix elements for $\Sigma^+ \to pX$ to be

$$
\mathcal{M}(\Sigma^+ \to pX_S) = -2g_{S_q}(b_D-b_F)\bar{p}\Sigma^+ ,
$$
$$
\mathcal{M}(\Sigma^+ \to pX_P) = g_{P_q}B_0(D-F)\frac{m_{\Sigma^+}+m_p}{m_K^2-m_p^2}\bar{p}\gamma_5\Sigma^+ ,
$$
$$
\mathcal{M}(\Sigma^+ \to pX_V) = -g_{V_q}\bar{p}\gamma_\mu\Sigma^+ e_\mu ,
$$
$$
\mathcal{M}(\Sigma^+ \to pX_A) = -g_{A_q}(D-F)\bar{p}\gamma_\mu\gamma_5\Sigma^+ e_\mu .
$$

These expressions follow from a kaon-pole diagram for the pseudoscalar, and from a direct vertex from Eq. (2) for the rest. For the branching ratios, it then follows that

$$
\mathcal{B}(\Sigma^+ \to pX_S \to p\mu^+\mu^-) = 9.0 \times 10^{-12} |g_{S_q}|^2 \mathcal{B}(X_S \to \mu^+\mu^-) ,
$$
$$
\mathcal{B}(\Sigma^+ \to pX_P \to p\mu^+\mu^-) = 3.7 \times 10^{-11} |g_{P_q}|^2 \mathcal{B}(X_P \to \mu^+\mu^-) ,
$$
$$
\mathcal{B}(\Sigma^+ \to pX_V \to p\mu^+\mu^-) = 7.0 \times 10^{-11} |g_{V_q}|^2 \mathcal{B}(X_V \to \mu^+\mu^-) ,
$$
$$
\mathcal{B}(\Sigma^+ \to pX_A \to p\mu^+\mu^-) = 7.0 \times 10^{-11} |g_{A_q}|^2 \mathcal{B}(X_A \to \mu^+\mu^-) .
$$

For the scalar and vector particles, there are severe constraints from $K^\pm \to \pi^\pm\mu^+\mu^-$. The branching ratio of $K^\pm \to \pi^\pm\mu^+\mu^-$ has been measured to be $\mathcal{B} = (8.1 \pm 1.4) \times 10^{-8}$ [5]. The $X$-particle contribution to these decays can again be factorized as $\mathcal{B}(K^\pm \to \pi^\pm X)\mathcal{B}(X \to \mu^+\mu^-)$. Using the effective Lagrangians in Eq. (2), we have the matrix elements for $K^\pm \to \pi^\pm X$

$$
\mathcal{M}(K^\pm \to \pi^\pm X_S) = g_{S_q}B_0 ,
$$
$$
\mathcal{M}(K^\pm \to \pi^\pm X_V) = g_{V_q}(p_K+p_\pi) \cdot \sigma ,
$$

We have assumed $CP$ conservation for simplicity, and so taken the couplings $g_{(S,V)\ell}$ to be real.

The decay modes $K^\pm \to \pi^\pm\mu^+\mu^-$ are long-distance dominated in the SM [3] and the measured spectra agree reasonably well with the predictions [4,5]. In particular, there is no apparent bump in the $m_{\mu^+\mu^-} = 214$ MeV region [3]. In view of this, we require that any contribution from the hypothetical new particles to these rates be below the experimental error, that is [5]

$$
\mathcal{B}(K^\pm \to \pi^\pm\mu^+\mu^-)_X \leq 1.4 \times 10^{-8} .
$$

This leads to the constraints

$$
|g_{S_q}|^2 \mathcal{B}(X_S \to \mu^+\mu^-) < 6.5 \times 10^{-24} ,\quad |g_{V_q}|^2 \mathcal{B}(X_V \to \mu^+\mu^-) < 4.3 \times 10^{-23} .
$$

Combining these limits with Eq. (4), we find

$$
\mathcal{B}(\Sigma^+ \to pX_S \to p\mu^+\mu^-) < 6 \times 10^{-11} ,\quad \mathcal{B}(\Sigma^+ \to pX_V \to p\mu^+\mu^-) < 3 \times 10^{-11} .
$$

These results indicate that $K^\pm \to \pi^\pm\mu^+\mu^-$ data rule out both a scalar particle and a vector particle as explanations for the HyperCP result. Notice that this conclusion still holds if we relax Eq. (4) and allow the new contribution to be as large as the full experimental rate.

The decays $K^\pm \to \pi^\pm X_{P,A}$ are not allowed, as we have assumed $X_{P,A}$ to have no parity-odd couplings. Therefore, there are no constraints from $K \to \pi\mu^+\mu^-$ on the pseudoscalar and axial-vector couplings of the hypothetical particles to quarks.
IV. SOME CONSTRAINTS ON PSEUDOSCALAR AND AXIAL-VECTOR COUPLINGS

We now consider other possible constraints on the couplings involving the pseudoscalar and axial-vector particles. We begin by ignoring CP violation so that \( g_{Pq} \) and \( g_{Aq} \) are real. A strong constraint on flavor-changing neutral currents (FCNC) of this type comes from \( K^0 \rightarrow \bar{K}^0 \) mixing. The mixing parameter \( M_{12} = \mathcal{M}(K^0 \rightarrow X \rightarrow \bar{K}^0)/2m_K \) from an intermediate \( X \)-state in \( K^0 \rightarrow X \rightarrow \bar{K}^0 \) is given by

\[
\mathcal{M}(K^0 \rightarrow X_P \rightarrow \bar{K}^0) = \frac{2B_0^2 f^2 g_{Pq}^2}{m_P^2 - m_K^2},
\]
\[
\mathcal{M}(K^0 \rightarrow X_A \rightarrow \bar{K}^0) = \frac{2f^2 g_{Aq}^2 m_K^2}{m_A^2}.
\]

The measured value of \( \Delta M_{K^0-L^0} = 3.483 \times 10^{-12} \text{MeV} \) can be accommodated in the SM, but its calculation suffers from hadronic uncertainties due to long-distance contributions. To be conservative, we will thus require that any new physics contribution be smaller than the experimental value, namely

\[
(\Delta M_{K^0-L^0})_X = 2(\text{Re} M_{12})_X < 3.483 \times 10^{-12} \text{ MeV}.
\]

With matrix elements from Eqs. (2a) and (2d), but using \( f_K \sim 1.23 f \), instead of \( f \), in Eq. (9) for the kaon decay constant, this results in

\[
g_{Pq}^2 < 3.3 \times 10^{-15},
\]
\[
g_{Aq}^2 < 1.3 \times 10^{-14}.
\]

When we substitute these bounds into Eq. (10), we find

\[
\frac{B(\Sigma^+ \rightarrow pX_P \rightarrow p\mu^+\mu^-)}{B(X_P \rightarrow \mu^+\mu^-)} < 1.2 \times 10^{-3},
\]
\[
\frac{B(\Sigma^+ \rightarrow pX_A \rightarrow p\mu^+\mu^-)}{B(X_A \rightarrow \mu^+\mu^-)} < 9.1 \times 10^{-3}.
\]

These constraints are so weak that \( X_P \) and \( X_A \) are allowed candidates to explain the HyperCP result, provided their branching ratios into muon pairs are at least \( B(X_P \rightarrow \mu^+\mu^-) \geq 2.5 \times 10^{-5} \) and \( B(X_A \rightarrow \mu^+\mu^-) \geq 3.4 \times 10^{-6} \), respectively.

If we allow for CP violation in the \( g_{Pq} \) and \( g_{Aq} \) couplings, the constraints from \( \Delta M_{K^0-L^0} \) become

\[
|\text{Re} g_{Pq}|^2 - |\text{Im} g_{Pq}|^2 < 3.3 \times 10^{-15},
\]
\[
|\text{Re} g_{Pq}| < 3.2 \times 10^{-18},
\]
\[
|\text{Re} g_{Aq}|^2 - |\text{Im} g_{Aq}|^2 < 1.3 \times 10^{-14},
\]
\[
|\text{Re} g_{Aq}| < 1.2 \times 10^{-17}.
\]

The two additional constraints arise from the new-particle contribution to the parameter \( \epsilon_K = \text{Im} M_{12}/(\sqrt{2} \Delta M_{K^0-L^0}) \). This parameter can be calculated more reliably than \( \Delta M_{K^0-L^0} \) in the
standard model and is in good agreement with the result $|\epsilon_K| = 2.284 \times 10^{-3}$. In view of this, we required the new-physics contribution to be less than 30% of the experimental value, which is about the size of the theoretical uncertainty in the SM calculation.

New pseudoscalar and axial-vector particles also contribute to the rare decay $K_L \to \mu^+\mu^-$ via the pole diagram $K_L \to X \to \mu^+\mu^-$. From Eqs. (11) and (18), we obtain from Eq. 15

\[ \mathcal{M}(K_L \to X_P \to \mu^+\mu^-) = \frac{-2i B_0 f_{P\mu} g_{P\mu}}{m_K^2 - m_P^2} g_{P\mu} \mu \gamma_5 \mu, \]
\[ \mathcal{M}(K_L \to X_A \to \mu^+\mu^-) = \frac{4i f_{A\mu} m_{\mu}}{m_A^2} g_{A\mu} \mu \gamma_5 \mu. \] (14)

These matrix elements imply

\[ \mathcal{B}(K_L \to X_P \to \mu^+\mu^-) = 5.6 \times 10^{18} \text{ GeV}^{-1} g_{P\mu}^2 \Gamma(X_P \to \mu^+\mu^-), \]
\[ \mathcal{B}(K_L \to X_A \to \mu^+\mu^-) = 1.2 \times 10^{18} \text{ GeV}^{-1} g_{A\mu}^2 \Gamma(X_A \to \mu^+\mu^-). \] (15)

If we allow for CP violation it is possible to obtain additional, weaker, constraints from considering the mode $K_S \to \mu^+\mu^-$. To be useful, the equations above must be combined with additional information on the couplings of the hypothetical new particles to muons. Partial information can be obtained from considering their contribution to the anomalous magnetic moment of the muon, $a_\mu$.

At one-loop level, the contributions of the new pseudoscalar and axial-vector to $a_\mu$ are given respectively by

\[ a_\mu(P) = -\frac{|g_{P\mu}|^2 m_{\mu}^2}{8\pi^2 m_P^2} f_P(m_{\mu}^2/m_P^2) = -2.28 \times 10^{-3} |g_{P\mu}|^2, \]
\[ a_\mu(A) = \frac{|g_{A\mu}|^2 m_{\mu}^2}{4\pi^2 m_A^2} f_A(m_{\mu}^2/m_A^2) = -8.97 \times 10^{-3} |g_{A\mu}|^2. \] (16)

Here

\[ f_P(r) = \int_0^1 dx \frac{x^3}{1 - x + r x^2}, \]
\[ f_A(r) = \int_0^1 dx \frac{4(x - 1)x + x^2(1 - x) - 2rx^3}{1 - x + r x^2}. \] (17)

At present there is a discrepancy of 2.4σ between the SM prediction and data \[10\], $\Delta a_\mu = a_\mu(exp) - a_\mu(SM) = (23.9 \pm 10) \times 10^{-10}$ with $a_\mu(exp) = (11659208 \pm 6) \times 10^{-10}$. We note that the new contributions reduce the value of $a_\mu$, making the comparison with experiment worse. In view of this, we place a conservative constraint on $g_{X\mu}^2$ by requiring that the new contribution to $a_\mu$ not exceed the experimental error. This results in

\[ |g_{P\mu}|^2 < 2.6 \times 10^{-7}, \quad \Gamma(X_P \to \mu^+\mu^-) < 3.7 \times 10^{-10} \text{ GeV}, \]
\[ |g_{A\mu}|^2 < 6.7 \times 10^{-8}, \quad \Gamma(X_A \to \mu^+\mu^-) < 5.2 \times 10^{-12} \text{ GeV}. \] (18)

Combining the constraints in Eqs. (11) and (18), we obtain from Eq. (15)

\[ \mathcal{B}(K_L \to X_P \to \mu^+\mu^-) < 6.8 \times 10^{-6}, \]
\[ \mathcal{B}(K_L \to X_A \to \mu^+\mu^-) < 8.1 \times 10^{-8}. \] (19)
The measured branching ratio for this mode, $\mathcal{B}(K_L \to \mu^+\mu^-) = (6.87 \pm 0.12) \times 10^{-9}$, is almost completely saturated by the two-photon intermediate state, the absorptive part of this contribution being $\mathcal{B}(K_L \to \gamma\gamma \to \mu^+\mu^-)_{\text{abs}} = (6.63 \pm 0.07) \times 10^{-9}$ (this is referred to as the unitarity bound). This leaves little room for a direct new-physics contribution. Here we assume that a possible new-physics contribution is at most equal to the difference between the measured rate and the unitarity bound plus one standard deviation,

$$\mathcal{B}(K_L \to \mu^+\mu^-)_X \leq 3.6 \times 10^{-10}. \quad (20)$$

Using this as a constraint improves the bounds of Eqs. (11) and (18):

$$|g_{Pq}|^2 \Gamma(X_P \to \mu^+\mu^-) < 6.4 \times 10^{-29} \text{ GeV},$$

$$|g_{Aq}|^2 \Gamma(X_A \to \mu^+\mu^-) < 3.0 \times 10^{-28} \text{ GeV}. \quad (21)$$

If $\Gamma(X_{P,A} \to \mu^+\mu^-)$ are allowed to saturate the bounds of Eq. (18), the above equations imply that

$$|g_{Pq}|^2 < 1.7 \times 10^{-19} \text{ and therefore } \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) < 6.3 \times 10^{-8} \mathcal{B}(X_P \to \mu^+\mu^-),$$

$$|g_{Aq}|^2 < 5.8 \times 10^{-17} \text{ and therefore } \mathcal{B}(\Sigma^+ \to p\mu^+\mu^-) < 4.0 \times 10^{-5} \mathcal{B}(X_A \to \mu^+\mu^-). \quad (22)$$

This in turn means that both the pseudoscalar and axial-vector particles remain viable candidates to explain the HyperCP data after combining the existing bounds from $\Delta M_{K_L-K_S}$, $a_\mu$, and $K_L \to \mu^+\mu^-$. In the case of the pseudo-scalar, these combined bounds require that it decay almost exclusively into a $\mu^+\mu^-$ pair.

V. PREDICTIONS

We now turn the argument around and assume that the HyperCP data is indeed explained by the hypothetical new pseudoscalar or axial-vector particle. This implies that

$$|g_{Pq}|^2 \mathcal{B}(X_P \to \mu^+\mu^-) = (8.4^{+6.5}_{-5.1} \pm 4.1) \times 10^{-20},$$

$$|g_{Aq}|^2 \mathcal{B}(X_A \to \mu^+\mu^-) = (4.4^{+3.4}_{-2.7} \pm 2.1) \times 10^{-20}. \quad (23)$$

These can then be used to predict the contributions of the new particles to other decay modes such as $K \to \pi\pi X_{P,A} \to \pi\pi\mu^+\mu^-$ and $\Omega^- \to \Xi^- X_{P,A} \to \Xi^- \mu^+\mu^-$. We first consider $K \to \pi\pi X_{P,A} \to \pi\pi\mu^+\mu^-$. Employing the Lagrangians in Eqs. (2) and (A1), we derive\(^1\)

$$\mathcal{M}(K^0 \to \pi^+\pi^-X_P) = \frac{B_0 g_{Pq}}{\sqrt{2} f} \left( \frac{m_K^2 + m_\pi^2 - m_{X_{\pi^-}}^2 - m_{X_{\pi^+}}^2}{m_K^2 - m_P^2} \right),$$

$$\mathcal{M}(K^0 \to \pi^0\pi^0X_P) = \frac{B_0 g_{Pq}}{2\sqrt{2} f} \left( \frac{m_K^2 - m_\pi^2 - m_{X_{\pi^0}}^2}{m_K^2 - m_P^2} \right),$$

$$\mathcal{M}(K^+ \to \pi^+\pi^0X_P) = \frac{B_0 g_{Pq}}{2 f} \left( \frac{m_{X_{\pi^0}}^2 - m_{X_{\pi^0}}^0}{m_K^2 - m_P^2} \right). \quad (24)$$

\(^1\) We note that each of the $K^0 \to \pi\pi X_P$ amplitudes receives contributions from both contact and kaon-pole diagrams. The pole terms seem to be missing in Ref. [4].
\begin{align*}
\mathcal{M}(K^0 \to \pi^+\pi^- X_A) &= -\frac{i\sqrt{2}g_{Aq}}{f} p_{\pi^+} \cdot e^* , \\
\mathcal{M}(\bar{K}^0 \to \pi^0\pi^0 X_A) &= -\frac{g_{Aq}}{\sqrt{2}f} p_{K} \cdot e^* , \\
\mathcal{M}(K^+ \to \pi^+\pi^0 X_A) &= -\frac{g_{Aq}}{f}(p_{e^0} - p_{\pi^+}) \cdot e^* , \tag{25}
\end{align*}

where \(m_{ij}^2 = (p_i + p_j)^2\).

Adding the errors in Eq. (23) in quadrature, we obtain the predictions

\begin{align*}
\mathcal{B}(K_L \to \pi^+\pi^- X_P \to \pi^+\pi^- \mu^+\mu^-) &= (1.8^{+1.6}_{-1.4}) \times 10^{-9} , \\
\mathcal{B}(K_L \to \pi^0\pi^0 X_P \to \pi^0\pi^0 \mu^+\mu^-) &= (8.3^{+7.5}_{-6.6}) \times 10^{-9} , \tag{26}
\end{align*}

\begin{align*}
\mathcal{B}(K_L \to \pi^+\pi^- X_A \to \pi^+\pi^- \mu^+\mu^-) &= (7.3^{+6.6}_{-5.7}) \times 10^{-12} , \\
\mathcal{B}(K_L \to \pi^0\pi^0 X_A \to \pi^0\pi^0 \mu^+\mu^-) &= (1.0^{+0.9}_{-0.8}) \times 10^{-10} . \tag{27}
\end{align*}

Notice that these decay modes are highly suppressed by phase space, but that at the \(10^{-8}\)-\(10^{-9}\) level they are comparable to existing limits on other rare \(K_L\) decay modes. The rates for the \(K^+\) decay modes are quite sensitive to isospin-breaking effects, but we find them to be at most at the \(10^{-12}\) level, much less promising than the \(K_L\) modes.

We now consider the modes \(\Omega^- \to \Xi^- X_P A \to \Xi^- \mu^+\mu^-\). For the pseudoscalar particle, a kaon-pole diagram with vertices from Eqs. (A11) and (21) leads to

\begin{align*}
\mathcal{M}(\Omega^- \to \Xi^- X_P) &= -\frac{iB_0 C}{m_K^2 - m_P^2} g_{Pq} \bar{u}_\Xi q_\mu v_\Omega^\mu , \tag{28}
\end{align*}

and for the axial-vector particle a direct vertex from Eq. (2d) gives

\begin{align*}
\mathcal{M}(\Omega^- \to \Xi^- X_A) &= -iC g_{Aq} \bar{u}_\Xi u_\Omega^\mu \epsilon_\mu^* . \tag{29}
\end{align*}

The resulting branching ratios for \((\Omega^- \to \Xi^- X \to \Xi^- \mu^+\mu^-)\) are

\begin{align*}
\mathcal{B}(\Omega^- \to \Xi^- X_P \to \Xi^- \mu^+\mu^-) &= \frac{1}{\Gamma_{\Omega^-}} \frac{|P_{\Xi}|^3}{12\pi m_\Omega} \frac{B_0^2 C^2 |g_{Pq}|^2}{(m_K^2 - m_P^2)^2} (E_\Xi + m_\Xi) \mathcal{B}(X_P \to \mu^+\mu^-) \\
&= 2.4 \times 10^{13} |g_{Pq}|^2 \mathcal{B}(P \to \mu^+\mu^-) , \tag{30}
\end{align*}

\begin{align*}
\mathcal{B}(\Omega^- \to \Xi^- X_A \to \Xi^- \mu^+\mu^-) &= \frac{1}{\Gamma_{\Omega^-}} \frac{|P_{\Xi}|}{12\pi m_\Omega} C^2 |g_{Aq}|^2 (E_\Xi + m_\Xi) \left(3 + \frac{P_{\Xi}^2}{m_A^2}\right) \mathcal{B}(X_A \to \mu^+\mu^-) \\
&= 1.6 \times 10^{13} |g_{Aq}|^2 \mathcal{B}(X_A \to \mu^+\mu^-) . \tag{31}
\end{align*}

Consequently, the HyperCP data implies

\begin{align*}
\mathcal{B}(\Omega^- \to \Xi^- X_P \to \Xi^- \mu^+\mu^-) &= (2.0^{+1.6}_{-1.2} \pm 1.0) \times 10^{-6} , \\
\mathcal{B}(\Omega^- \to \Xi^- X_A \to \Xi^- \mu^+\mu^-) &= (0.73^{+0.56}_{-0.45} \pm 0.35) \times 10^{-6} . \tag{32}
\end{align*}

These numbers represent a substantial enhancement over the existing standard-model prediction \(\mathcal{B}(\Omega^- \to \Xi^- \mu^+\mu^-) = 6.6 \times 10^{-8}\) [11].
VI. SUMMARY

We have studied the hypothesis that a new particle of mass 214.3 ± 0.5 MeV is responsible for the invariant-mass \( m_{\mu^+\mu^-} \) distribution observed by HyperCP in \( \Sigma^+ \rightarrow \mu^+\mu^- \). We find that existing data on \( K^+ \rightarrow \pi^+\mu^+\mu^- \) rule out a scalar particle and a vector particle as possible explanations. We explore all the existing constraints on pseudoscalar and axial-vector particles, and conclude that these possibilities are still allowed. If either one of them is indeed responsible for the HyperCP data, we predict enhanced rates for \( K_L \rightarrow \pi\pi X \rightarrow \pi\pi\mu^+\mu^- \) and \( \Omega^- \rightarrow \Xi^- X \rightarrow \Xi^-\mu^+\mu^- \).

**Note added** After the completion of our paper, the work of Deshpande, Eilam and Jiang appeared. They reach similar conclusions to ours.

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APPENDIX A: DERIVATION OF EFFECTIVE LAGRANGIANS

The chiral Lagrangian that describes the interactions of the lowest-lying mesons and baryons is written down in terms of the lightest meson-octet, baryon-octet, and baryon-decuplet fields \[12, 13, 14\]. The meson and baryon octets are collected into 3 \( \times \) 3 matrices \( \varphi \) and \( B \), respectively, and the decuplet fields are represented by the Rarita-Schwinger tensor \( T^\mu_{abc} \), which is completely symmetric in its SU(3) indices \((a, b, c)\). The octet mesons enter through the exponential \( \Sigma = \xi^2 = \exp(i\varphi/f) \), where \( f = f_\pi = 92.4 \) MeV is the pion-decay constant.

We write the strong chiral Lagrangian at leading order in the derivative and \( m_s \) expansions as

\[
\mathcal{L}_s = \langle B i\gamma^\mu (\partial_\mu B + [\mathcal{V}_\mu, B]) \rangle - m_0 \langle BB \rangle + D \langle B\gamma^\mu\gamma_5 \{A_\mu, B\} \rangle + F \langle B\gamma^\mu\gamma_5 [A_\mu, B] \rangle - \bar{T}^\mu i\varphi T_\mu + m_T \bar{T}^\mu T_\mu + c(\bar{T}^\mu A_\mu B + B A_\mu T^\mu) + \mathcal{H} \bar{T}^\mu A\gamma_5 T_\mu + \frac{b_D}{2B_0} \langle \bar{B} \{X^+, B\} \rangle + \frac{b_F}{2B_0} \langle \bar{B} [X^+, B] \rangle + \frac{b_0}{2B_0} \langle \chi^+ \rangle \langle \bar{B} B \rangle + \frac{c_0}{2B_0} \langle \chi^+ \rangle \bar{T}^\mu T_\mu + \frac{1}{4} f^2 \langle D^\mu \Sigma^\dagger D^\nu \Sigma \rangle + \frac{1}{2} f^2 \langle \chi^+ \rangle ,
\]

(A1)

where \( \langle \cdots \rangle \equiv \text{Tr}(\cdots) \) in flavor-SU(3) space, \( m_0 \) and \( m_T \) are the octet-baryon and decuplet-baryon masses in the chiral limit, respectively, \( \mathcal{V}_\mu = \frac{1}{2}(\xi \partial_\mu \xi^\dagger + \xi^\dagger \partial_\mu \xi) + \frac{i}{2}(\xi \ell^\mu \xi + \xi^\dagger \ell^\mu \xi^\dagger) \), \( \mathcal{A}_\mu = i(\partial_\mu \xi^\dagger - \xi^\dagger \partial_\mu \xi) + \frac{i}{2}(\xi \ell^\mu \xi - \xi^\dagger \ell^\mu \xi^\dagger) \), \( D^\mu T_{kln}^\nu = \partial^\mu T_{kln}^\nu + \gamma_\mu T_{kln}^\nu + \gamma_\mu T_{kln}^\nu + \gamma_\mu T_{kln}^\nu \), \( D^\mu \Sigma = \partial^\mu \Sigma + i\ell^\mu \Sigma - i\Sigma \ell^\mu \), \( \chi^+ = \xi^\dagger \chi^\dagger + \xi \chi \xi^\dagger \), with \( \ell^\mu = \frac{1}{2}\lambda_a \ell^\mu_a = v^\mu + a_\mu \), \( r^\mu = \frac{1}{2}\lambda_a r^\mu_a = v^\mu - a^\mu \), and \( \chi = s + ip \) containing external sources. In the absence of external sources, \( \chi \) reduces to the mass matrix \( \chi = 2B_0 \text{diag}(\hat{m}, \hat{m}, m_s) = \text{diag}(m^2_\pi, m^2_\pi, 2m^2_K - m^2_\pi) \) in the isospin-symmetric limit.
\( m_u = m_d = \hat{m} \). The constants \( D, F, C, H, B_0, b_{D,F,0}, c, \) and \( c_0 \) are free parameters which can be fixed from data.

To extract the couplings of the new particles \( X \) from the above Lagrangian, we identify the external sources with

\[
\begin{align*}
s &= g_{Sq} X_S (T_6 + iT_7) + \text{H.c.} , \\
p &= g_{Pq} X_P (T_6 + iT_7) + \text{H.c.} , \\
u^\mu &= g_{Vq} X_V^\mu (T_6 + iT_7) + \text{H.c.} , \\
a^\mu &= g_{Ag} X_A^\mu (T_6 + iT_7) + \text{H.c.} .
\end{align*}
\]

The Lagrangians in Eq. (2) then follow.

Numerically, we adopt the tree-level values \( D = 0.80 \) and \( F = 0.46 \), extracted from hyperon semileptonic decays, as well as \( |C| = 1.7 \), from the strong decays \( T \to B \phi \). Furthermore, using \( \hat{m} + m_s = 121 \text{ MeV} \) and isospin-symmetric values of the baryon and meson masses, we have

\[
\begin{align*}
b_D &= 0.270 , \\
b_F &= -0.849 , \\
B_0 &= 2031 \text{ MeV} ,
\end{align*}
\]

the other parameters being irrelevant to our calculations.

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