On the performance of gas-lubricated bearing-rotor system in microengine

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Abstract. This paper reports a comprehensive analysis of the performance for gas-lubricated bearing-rotor system in silicon microengine. The generalized lubrication model and dynamics model are developed and the effects of micro fabrication defects, temperature and viscous friction force on both steady-state and dynamic characteristics of bearing system are investigated. The results can serve as a useful tool for the design of silicon microbearing-rotor systems.

1. Introduction
Gas-lubricated bearings are the best candidates to support the rotating members in Power MEMS devices such as micro-engines, due to their unique advantages of low friction losses, near-zero wear and no contamination through lubricant leakages compared with their rolling-element or oil-lubricated counterparts. However, the implementation of gas bearings encounters challenges because of their characteristics which are different from their conventional ones as follows: the very high rotating speed and temperature to achieve high power density, the super thin gas film between rotors and static structures, and the relative large fabrication uncertainties according to the imperfection of the fabrication technology [1-3]. In fact, as the rotating speed is supper high, the compressibility of the gas film force is very significant, and the gas film force is strongly nonlinear. The forces and moments of the system depend not only on the instantaneous kinematical state but also upon the history of the motion, which results in coupling of the lubrication analysis with the dynamics of the rotor motion. Moreover, tightened by the constraints of the micro fabrication progress (DRIE, etc.), the length-to-diameter ratio (L/D) of the silicon-based micro gas journal bearing is about 0.05~0.1, which is one order of magnitude lower than in conventional gas journal bearings. For the conventional gas journal bearing-rotor systems, the viscous friction force is much lower compared with the gas film force and usually ignored in the research of the system [4]. However, in the ultra-short micro journal bearing-rotor systems, the order of magnitude of the viscous friction force is the same as that of the gas film force. Many researchers have studied the dynamic characteristics of micro gas bearings [5-7]. Unfortunately, the influence of the viscous friction force on the nonlinear dynamic behavior has not been studied yet. This work is undertaken to provide insight into the effects of micro fabrication defects, temperature and viscous friction force on both steady-state and dynamic characteristics.

2. Mathematical analysis

2.1. Lubrication model
Figure 1 is a three-dimensional schematic drawing of a silicon-based microengine in which the rotor is supported axially by a pair of spiral-grooved micro gas hydrodynamic thrust bearings, and radially by a hydrodynamic journal bearing. When operating the film thickness $h$ lies in the range of $0.5\mu m \leq h \leq 20\mu m$, resulting the Knudsen number $Kn$ is about $0.0032 \sim 0.1291$, which means the gas
film is rarefied and the continuous flow model is no longer valid. With the assumptions of laminar and isothermal flow and the ignorance of the inertia item, the non-dimensional governing compressible form of the MGL model is [8]

\[
\frac{\partial}{\partial \theta} (PH\frac{\partial P}{\partial \theta}) + \frac{\partial}{\partial \xi} (PH\frac{\partial P}{\partial \xi}) = A \frac{\partial}{\partial \theta} (PH) + 2A \frac{\partial}{\partial \xi} (PH) \tag{1}
\]

where \( H = 1 - \varepsilon \cos(\theta - \phi) \) is the non-dimensional film thickness, \( P = p/p_a \) is the non-dimensional gas film pressure, \( \tau = \omega t \) is the non-dimensional time, \( A \) is the bearing number, \( Q \) is the non-dimensional flow rate coefficient which is related to the inverse Knudsen number \( D_k \) [8]. In terms of continuous flow model, \( Q \) equals 1 at any film thickness. \( \theta \) and \( \xi \) are the circumferential and axial coordinates, respectively. The non-dimensional gas film force, \( F_g = f_g/(P_a LD) \), and the attitude angle \( \phi \) can be obtained as

\[
\begin{align*}
F_{gh} &= \frac{R}{2L} \int_0^{2\pi} \int_0^\xi P \cos \theta d\theta d\xi \\
F_{g\theta} &= \frac{R}{2L} \int_0^{2\pi} \int_0^\xi P \sin \theta d\theta d\xi \\
F_{g\xi} &= \sqrt{F_{gh}^2 + F_{g\theta}^2} \\
\phi &= \arctan\left(-\frac{F_{g\theta}}{F_{gh}}\right)
\end{align*}
\tag{2}
\]

The sheer stress at the rotor surface arising from the viscosity of the gas is

\[
\tau_k = \frac{Hcp_k}{2R} \frac{\partial P}{\partial \theta} + \frac{\mu \omega R}{cH} \tag{3}
\]

The non-dimensional viscous friction force, \( F = f_f/(P_a LD) \), can then be calculated as

\[
\begin{align*}
F_{fx} &= \frac{R}{2L} \int_0^{2\pi} \int_0^\xi \tau_k \cos \theta d\theta d\xi \\
F_{fy} &= \frac{R}{2L} \int_0^{2\pi} \int_0^\xi \tau_k \sin \theta d\theta d\xi \\
F_f &= \sqrt{F_{fx}^2 + F_{fy}^2}
\end{align*}
\tag{4}
\]

2.2. Rotor dynamic model

The non-dimensional kinematic equations of motion of the rotor is

Figure 1. Schematic of the bearing-rotor system for micro gas turbine engine.
where \( F_e \) stands for non-dimensional external force. Eq. (1) and Eq. (5) are not independent of each other, causing the coupling of the kinematic equation and the lubrication model.

3. Results and discussion

3.1. Effects of microfabrication defects on the performance of microbearings

The pressure distribution comparison among the bearings with different degrees of taper can be seen in Figure 2. As shown, the clearance of the tapered bearing with \( \Delta c \neq 0 \mu m \) is not constant axially, thus the pressure distribution is not symmetrical about the axial mid-plane where \( \xi = 0 \). Besides, the pressure peak value of the tapered bearing drops and the greater effects will happen when \( \Delta c \) becomes larger.

![Figure 2](image)

**Figure 2.** Non-dimensional pressure distribution of bearings with different degrees of taper (a) \( \Delta c = 0 \mu m \) and (b) \( \Delta c = 2 \mu m \)

![Figure 3](image)

**Figure 3.** Comparison of \( M_{st} A \) chart among bearings with different degrees of taper (a) \( \Delta c = 0 \mu m \) and (b) \( \Delta c = 2 \mu m \)

The \( M_{st} A \) charts are denoted in Figure 3, where the dotted lateral lines (i.e., \( M = 0.95 \)) represent the present rotor mass value. When the rotor mass is fixed, the turning point of eccentricity to be stable or unstable is different for bearings with different degrees of taper. For example, when \( M = 0.95, \lambda = 1 \), the transition occurs at about 0.8197 for a no-tapered bearing. However, when clearance of 2\( \mu m \) difference exists, the transitional eccentricity becomes about 0.8791. Thus the tapered bearing needs to achieve a higher eccentricity to realize the stable operation. On the other hand, at the same eccentricity and bearing number, the tapered bearing needs to have a smaller rotor mass to operate stably. For example, to running stably in a range of bearing number from 0.1 to 10 at \( \varepsilon = 0.8 \), the non-dimensional rotor mass has to be less than 0.0378 for a no-tapered bearing, whereas the value is 0.0036 for a
Δc=2μm bearing. In conclusion, a tapered bearing is more liable to contact with the static structure or become unstable.

3.2. Effects of temperature on the performance of microbearings
As discussed above, the increase of the temperature could cause the increase of molecular mean free path and thus cause the rarefaction effect, which might decrease the load capacity, to become more important. However, on the other hand, the gas viscosity may increase with the increase of the temperature, and higher gas viscosity implies larger load capacity. The simulation results show that the temperature variation range from 300K to 1600K could be divided into two parts, as shown in Figure 4. The left part is the viscosity effect domain region (300K~753K) and the other one is the rarefaction effect domain region (753K~1600K). In the first part, the influence of temperature on gas viscosity is dominant and the load capacity becomes larger as temperature increases. In the other part, the influence of temperature on rarefaction effect is more important and the inverse trend can be found as temperature goes high.

![Figure 4](image1)

**Figure 4.** Curves of non-dimensional load capacity versus temperature at different rotational speeds.

![Figure 5](image2)

**Figure 5.** Maximum axial displacement at different values of temperature

The maximum axial displacement in the history of the transient response of the rotor, \(Z_{\text{max}}\), is an important parameter to characterize the dynamic behavior of the micro bearing system since \(h_{\text{min}} = 0.5 - Z_{\text{max}}\) represents the minimum distance between the rotor and the bearing pad. Figure 5 shows the curves of \(Z_{\text{max}}\) versus temperature at different rotational speed. The contour graphs of pressure distribution corresponding to points of a, b, c and d are also displayed at the top of Figure 5. When the rotational speed go beyond the value of \(6 \times 10^5\) rpm, as can be seen, the stability of the thrust bearing system becomes weaker when the operating temperature lies in the range from 700K to 1000K, which is the junction zone between the viscosity domain region and the rarefaction domain region. In this case, the excessive deviation of the rotor position from the original equilibrium position \((Z=0)\) will causes the system to crash.

3.3. Effects of viscous friction on the performance of microbearings
In this section, the nonlinear dynamic characteristics for both conditions of with and without the viscous friction force considered are studied and compared. Here the bearing number is selected as the bifurcation parameter and increases from 2.0 to 14.0, at \(m=10.0\text{mg}, \varepsilon_0=0.85\) and \(L=330\mu\text{m}\). The bifurcation diagrams without or with the consideration of the viscous friction force are shown in Figure 6(a) and (b) respectively. When ignoring the viscous friction force, as the increase of the bearing number, the system performs period-1 synchronous motion firstly when \(\Lambda\) is less than 2.7, and then bifurcates into quasi-periodic motion or multi-periodic motion. The system motion turns back to synchronous when \(\Lambda\) lies between 3.3 and 7.6 and then shifts between quasi-periodic motion and multi-periodic motion once again after \(\Lambda=7.6\). Quite different phenomenon could be observed by taking the viscous friction force into account. As plotted in Figure 6 (b), the system exhibits synchronous motion at low bearing numbers, and loses its stability at 7.1, from which a period-
doubling phenomenon occurs. As $\Lambda$ increases over 9.2, the system shifts back to 1-periodic motion, and then performs quasi-periodic motion over the bearing number interval $10.0 \leq \Lambda < 10.7$. After that, the rotor center motion transits through the following behaviors: $3T$—$T$—$3T$—quasi.

![Figure 6. Bifurcation diagrams (a) ignoring the viscous friction force and (b) considering the viscous friction force.](image1)

![Figure 7. Waterfall plots of the system response in the x-direction (a) ignoring the viscous friction force and (b) considering the viscous friction force.](image2)

From the waterfall plots of the system responses for the two conditions illustrated in Figure 7 (a) and (b), one can see that a low-frequency large-amplitude self-excited whirl motion occurs and dominates as the rotational speed increases when ignoring the viscous friction force. However, for the other case, the synchronous motion prevails in the whole rotational speed range with some narrow extents of whirl motion, and moreover, the amplitude of the motion is rather smaller than the case shown in Figure 7 (a). The reason for this phenomenon is that for ultra short gas journal bearing-rotor systems, the viscous friction force absorbs the energy of the whirl motion and thus decreases the whirling amplitude and a more stable rotor motion then appears.

4. Conclusions
The work presented here differs from previous work in the specific effects of micro fabrication defects, viscous friction force and temperature on both static and dynamic characteristics of gas-lubricated bearing-rotor system in silicon microengine, and the results can serve as a useful tool for design of silicon microbearing-rotor systems.

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