Analysis of Experimental Investigations of Self-Similar Intermediate Structures in Zero-Pressure Boundary Layers at Large Reynolds Numbers

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Abstract.
Analysis of the Stockholm group data on zero-pressure-gradient boundary flows, presented in the thesis of J. M. Österlund (http://www.mesh.kth.se/~jens/zpg/) is performed. The results of processing of all 70 mean velocity profiles are presented. It is demonstrated that, properly processed, these data lead to a conclusion opposite from that of the thesis and related papers: they confirm the Reynolds-number-dependent scaling law and disprove the conclusion that the flow in the intermediate ("overlap") region is Reynolds-number-independent.
1 Introduction

Turbulence is the state of vortical fluid flow where velocity, pressure, and other flow field properties vary in time and space sharply and irregularly, and can be assumed to be random. The experimental investigation of individual realizations of such flows is impossible because the results are irreproducible: Experiments repeated under identical external conditions produce a different outcome. Therefore experimental investigations of turbulent flows can only provide their average properties.

Less trivial and not always recognized is the following: What is of greatest interest in these experiments are intermediate asymptotic states of wider classes of flows, i.e., coherent, self-consistent fragments common to many different flows. Other measurements reflect special properties of a set-up, which cannot be reproduced in other experiments. It may sometimes be useful to investigate a particular device (e.g. an atomizer) for an immediate practical purpose, but one should be cautious in transferring the results to different flows.

Typical examples of intermediate-asymptotic flows are shear flows, where the flow is homogeneous in the direction of mean velocity which depends only on the coordinate perpendicular to the mean flow. A well known example of shear flows occurs in smooth cylindrical circular pipes far from the entrance and outlet. Another example is the zero-pressure gradient boundary layer above a smooth flat plate far from its tip. In spite of their apparent simplicity experiments with such flows require high experimental culture and are expensive, and therefore relatively rare. When they are successful, like, for instance, the experiments of Nikuradze [1] with flows in smooth pipes in the range of Reynolds numbers up to $3.24 \cdot 10^6$, performed 70 years ago under the guidance of L.Prandtl, they become milestones in turbulence studies. They are used to check theories based on special hypotheses valid for special classes of flows. Not enough is known now about the solutions of Navier-Stokes equations to avoid such hypotheses.

The responsibility of experimentalists who perform such experiments and process the data is therefore very high. They are to be very careful in their conclusions. We showed, for instance [2,3] that the experiments of Princeton group (A.Smits, M.V.Zagarola) presented in the thesis of M.V.Zagarola [4], which attempted to increase the range of Reynolds numbers achieved by Nikuradze by an order of magnitude, had a flaw. Starting from $Re = 10^6$, well below the upper boundary of Nikuradze experiments, their data were influenced by roughness — insufficient polishing of the pipe walls.

In the present work we analyze the experiments with zero-pressure-gradient boundary layers presented in the thesis of J.M.Österlund [5]. Like the earlier theses of M.V.Zagarola [4] and M.H.Hites [6], the thesis of J.M.Österlund presents the results of long-time work
of a group headed by a senior scientist (in this case A.V. Johansson), using a complicated, expensive and unique facility. The experimental data and their interpretation presented in this thesis might be accepted by some readers, as in the case of the thesis by Zagarola, without precautions. This was the motivation of our analysis of this thesis.

The number of runs (70 measurements of mean velocity profiles) reported in the thesis \cite{5} is larger than in previously reported series, although the range of Reynolds numbers covered is not as wide; less than in the previous thesis of M.H. Hites \cite{6}: $2500 < Re_\theta < 27,500$. In the thesis \cite{5} the authors make very definite statements: they claim that their experiments confirm the classical two-layer theory, in particular the Reynolds number-independent universal logarithmic law, and, exhibiting no Reynolds number-dependence, disagree with the alternative theory based on the Reynolds number-dependent scaling law.

J.M. Österlund presented the data of 70 mean velocity measurements on the Internet (www.mesh.kth.se/~jens/zpg/). We present here the results of the processing of all these data. We demonstrate that, properly processed, these data lead to the opposite conclusion: they confirm the Reynolds number-dependent scaling law and disagree with the conclusion of Reynolds number-independence.

2 Background

According to the classical two-layer theory of wall-bounded turbulent shear flows at large Reynolds numbers, the distribution of average velocity $u$ across the flow in the basic intermediate region adjacent to the viscous sublayer is represented in the form of universal (Reynolds number-independent) von Kármán-Prandtl logarithmic law

$$\phi = \frac{1}{\kappa} \ln \eta + C. \quad (2.1)$$

Here we use classical Nikuradze-Schlichting et al. notations:

$$\phi = \frac{u}{u_*}, \quad u_* = \left(\frac{\tau}{\rho}\right)^{\frac{1}{2}}, \quad \eta = \frac{u_* y}{\nu}, \quad (2.2)$$

where $\tau$ is the shear stress at the wall, $y$ the distance from the wall; $\nu$ and $\rho$ are the fluid’s kinematic viscosity and density. In the thesis more modern notations are used: $\bar{U}^+$ instead of $\phi$, $y^+$ instead of $\eta$. So, the von Kármán-Prandtl universal law (2.1) is represented in the thesis in the form

$$\bar{U}^+ = \frac{1}{\kappa} \ln(y^+) + B. \quad (2.3)$$
Here $\kappa$ (von Kármán constant) and $B$, according to the logic of the derivation, should be universal constants identical in all high quality experiments. It is known from the literature however that various experiments give substantially different values of these constants. Nikuradze \cite{1} determined $\kappa = 0.417$, $B = 5.84$, Monin and Yaglom \cite{7} give the values $\kappa = 0.40$, $B = 5.1$; Schlichting \cite{8} gives the values $\kappa = 0.40$, $B = 5.5$. The difference is substantial, and for many years doubts have accumulated on the validity of the universal logarithmic law.

In our papers (see e.g. \cite{9,10}) the derivation of the universal logarithmic law was reconsidered. It was shown that one of the basic assumptions is not quite correct, and on the basis of an alternative assumption, a different “scaling” (power) law was proposed:

$$\bar{U}^+ = (C_1 \ln Re + C_2)(y^+)^{c/\ln Re}$$

(2.4)

where the constants $C_1, C_2, c$ should be universal and Reynolds number-independent. Comparison with the experimental data of Nikuradze has given the following values of the constants:

$$C_1 = \frac{1}{\sqrt{3}}, \quad C_2 = \frac{5}{2}, \quad c = \frac{3}{2},$$

(2.5)

so that the law (2.4) is presented in the form

$$\phi = \left(\frac{\sqrt{3} + 5\alpha}{2\alpha}\right)\eta^\alpha, \quad \alpha = \frac{3}{2 \ln Re}$$

(2.6)

or, using the notation of Österlund’s thesis \cite{5}

$$\bar{U}^+ = \left(\frac{1}{\sqrt{3}} \ln Re + \frac{5}{2}\right)(y^+)\frac{1}{2 \ln Re}.$$  

(2.7)

Asymptotically, at $Re \to \infty$, the specific choice of $Re$ is of no importance: $Re$ can be replaced by $Re' = \text{Const} \cdot Re$, and the asymptotic form of (2.7) will remain the same. However, for practical large, but not too large, values of $Re$, its actual expression is significant. It should be remembered that in our comparison with Nikuradze’s data \cite{1} we used his definition of the Reynolds number: $Re = \bar{u} \frac{d}{\nu}$, where $\bar{u}$ is the mean flow velocity (bulk discharge rate divided by cross-section area), and $d$ is the diameter of the pipe. Furthermore, it was recognized from the beginning \cite{11} that the law (2.7) is asymptotic (in the parameter $1/\ln Re$). It should be valid at large $Re$, but at lesser values higher terms in the expansion of the coefficients could be significant.
The experimental data of the Stockholm group (J.M. Österlund, A.V. Johansson) are presented in the form of graphs in the \( \ln y^+, \bar{U}^+ \) plane suggested by the universal logarithmic law \((2.3)\) (see Figure 5 on page 43 and Figures 13, 14 on pages 152–153 of the thesis \([5]\)). They became available on the Internet (www.mesh.kth.se/~jens/zpg/) in complete digital form. The data are presented in our table, parameterized by the authors by the parameter \( \text{Re}_\theta = U\theta/\nu \). Here \( U \) is the free stream velocity, \( \theta \) is the momentum thickness, a quantity measurable \textit{a posteriori}, after the run is performed.

3 Processing of the mean velocity data

The first question to be answered was as follows: are the mean velocity data presented on the Internet compatible with some scaling law, not necessarily the law \((2.7)\). Therefore the data were plotted in the double-logarithmic coordinates \((\lg y^+, \lg \bar{U}^+)\). The result was instructive: the data outside the viscous sublayer form a characteristic shape of a broken line (see Figures 1–70). This shape is similar to the shape obtained for the experiments of the first group according to our classification \([12]\) where there was no influence neither of the external turbulence of the free stream nor of roughness. The Stockholm authors place the lower boundary of the intermediate region at \( y^+ = 200 \). We found this value generally too high, and the standard value \( y^+ = 70 \div 100 \) seems to be more appropriate, however we marked the line \( y^+ = 200 \) on all Figures 1–70.

Thus, all experiments revealed two straight lines forming a broken line in the \( \lg y^+, \lg U^+ \) plane. These straight lines correspond to the scaling laws

\[
\text{(I) } \bar{U}^+ = A(y^+)^\alpha \tag{3.1}
\]

(in the intermediate region adjacent to the viscous sublayer), and

\[
\text{(II) } \bar{U}^+ = B(y^+)^\beta \tag{3.2}
\]

(in the intermediate region adjacent to the free stream). The coefficients \( A, \alpha, B, \beta \) were obtained by standard statistical processing of data (see the table). The coefficients \( A, \alpha \) of the straight line \((3.1)\) representing the scaling law for the mean velocity distribution in the basic intermediate region adjacent to the viscous sublayer are obviously Reynolds-number-dependent: For us this was not unexpected, because previous processing of all available experimental data for a much wider range of Reynolds number led us to the same conclusion (see paper \([12]\) which was known to the Stockholm group and referenced by them). Therefore we conclude that the validity of some Reynolds-number-dependent scaling law for mean
velocity distribution is unquestionably confirmed by the experiments of the Stockholm group as well.

Note that because the Reynolds number range covered by the Stockholm group was not large, substantially less than the range covered by the other groups, in particular the Chicago group \[^6\], there would be a danger that they would not notice the Reynolds-number-dependence because the governing parameter is \( \ln \text{Re} \), not \( \text{Re} \) itself. This is not the case: The \( \text{Re} \)-dependence of the Stockholm experimental data is sufficiently strong to be revealed by proper processing.

By the way, the authors could notice that their values \( \kappa = 0.38 \) and \( \beta = 4.1 \) are substantially less than those presented in the literature as standard. However, if the law is universal Reynolds-number-independent, these parameters should be identical for all experiments of sufficient quality!

The argument against the power law used by the authors (see paper \[^{13}\] reproduced in the thesis) is the following. They introduce the “diagnostic function”

\[
\Gamma = \frac{y^+}{U^+} \frac{dU^+}{dy^+}.
\]

Their statement, “The function \( \Gamma \) should be a constant in a region governed by a power law” is correct for a fixed Reynolds number. However, this is not true for the “diagnostic function averaged for KTH data”, which is shown in their Figure 6.

We invite the reader to look at any of the Figures 1–70. It is clear that for each run \( \Gamma \) is a constant — look at the straight lines in the first intermediate region! However, this constant is different for different runs because the slopes of straight lines is \( \text{Re} \)-dependent! Indeed, the slope in the first region decays with growing Reynolds number. It is clear why \( \Gamma \) obtained by the authors is decreasing: the runs with larger Reynolds number and smaller slopes contribute more to larger \( y^+ \).

Now, when the validity of some Reynolds-number-dependent scaling law for the experiments of the Stockholm group is unquestionably established, we have to investigate whether this scaling law can be represented in the same form (2.7) obtained for flows in pipes. But what is \( \text{Re} \)? We cannot assume it arbitrarily to be equal to \( \text{Re}_\theta \).

This effective Reynolds number \( \text{Re} \) should have the form \( \text{Re} = U\Lambda/\nu \), where \( U \) is the free stream velocity, \( \nu \) the kinematic viscosity, and \( \Lambda \) a length scale which we cannot à \emph{a priori} identify with the momentum thickness \( \theta \), as there is no rationale for such identification. So, the basic question is whether one can find for each run such length scale \( \Lambda \) so that the scaling law (2.7) will be valid for the mean velocity distribution in the first intermediate region. A priori the very existence of such a length scale is under question, but if it does
exist, this means that the law (2.7) is not specific to flows in pipes but can be a general law for wall-bounded shear flows at large Reynolds numbers.

To answer this question one has to take the values $A$ and $\alpha$ for each run, obtained by standard statistical processing of the experimental data in the first intermediate scaling region, and then calculate two values $\ln Re_1$ and $\ln Re_2$ by solving two equations suggested by the scaling law (2.7),

$$\frac{1}{\sqrt{3}} \ln Re_1 + \frac{5}{2} = A , \quad \frac{3}{2 \ln Re_2} = \alpha .\quad (3.3)$$

If the values $\ln Re_1$ and $\ln Re_2$ obtained by solving two different equations (3.3) are close, i.e. if they coincide within experimental accuracy, it will mean that the unique length scale $\Lambda$ can be determined so that the experimental scaling law in the region (3.1), whose existence was proved before, coincides with the basic scaling law (2.7). The table shows that indeed these values are close — for all $Re_\theta > 10,000$, the difference $\ln Re_1 - \ln Re_2$ does not exceed 3%. This allows one to introduce for large Reynolds numbers the effective Reynolds number $Re$ according to the relation

$$\ln Re_1 = \frac{1}{2} (\ln Re_1 + \ln Re_2), \quad \text{or} \quad Re = \sqrt{Re_1 Re_2} ,\quad (3.4)$$

i.e., the geometric mean of $Re_1$ and $Re_2$. This Reynolds number allows the definition of the effective length scale $\Lambda$, which plays a similar role in the scaling law for the boundary layer flow as does the pipe diameter for flow in pipes. We remind the reader that the momentum thickness is calculated by integration of the velocity profile obtained experimentally: the calculation of the length scale on the basis of the measured velocity profile is not more complicated. Naturally the ratio of two length scales $\theta/\Lambda$ is different for different runs: both these quantities depend upon the details of flows, in particular they can depend in principle upon the distance between the tip of the plate and the point of observation.

4 Checking universality

The scaling law (2.7) can be reduced to a universal form

$$\psi = \frac{1}{\alpha} \ln \left( \frac{2 \alpha U^+}{\sqrt{3 + 5 \alpha}} \right) = \ln y^+ \quad (4.1)$$

where $\alpha = \frac{3}{2 \ln Re}$. This formula gives another way to check the applicability of the Reynolds-number-dependent scaling law (2.7) in the intermediate region (3.1). Indeed, according to
in the coordinates \( \ln y^+, \psi \), all experimental points should collapse onto the bisectrix of the first quadrant. Figure 71 shows that all data for large Reynolds numbers \( R_{e\theta} > 15,000 \), 24 runs) presented on the Internet collapse onto the bisectrix with accuracy sufficient to give an additional confirmation to the Reynolds-number-dependent scaling law (2.7). For lesser values of \( R_{e\theta} \) a systematic parallel shift is observed (Figures 72–74). Apparently in this case the choice of \( Re \) according to \( (3.4) \) is insufficient because at small Reynolds numbers the higher terms of the expansion could have some influence (see the paper by Radhakrishnan Srinivasan [13]).

5 Conclusion

The thesis of J. Österlund contains the following statements:

1. (p.22 of thesis), “The classical two layer theory was confirmed and constant values of the slope of the logarithmic overlap region (i.e. von Kármán constant) and additive constants were found and estimated to \( \kappa = 0.38 \), \( B = 4.1 \), and \( B_1 = 3.6 \).”

2. (p.29 of thesis), in fact the Introduction to the paper [14]: “Contrary to the conclusions of some earlier publications, careful analysis of the data reveals no significant Reynolds number dependence for the parameters describing the overlap region using the classical logarithmic relation.”

These statements are not correct. Our careful analysis of experimental data presented in the thesis performed in the present work leads to the opposite conclusions.

1’. The results contradict the classical two-layer theory. The estimates of the constants obtained by the authors are substantially different from the standard values and this reason alone is enough to reject the assumption of universality, the cornerstone of the classical theory.

2’. In full agreement with our earlier publications, careful analysis of the data reveals significant Reynolds number dependence for the parameters describing the “overlap” region and confirms the Reynolds-number-dependent scaling law.

The thesis of J. Österlund was not the first investigation of this kind, many similar experimental investigations were performed earlier covering a much larger range of Reynolds number. (In the thesis only one decade of \( R_{e\theta} \) was covered: \( 2,500 < R_{e\theta} < 27,000 \); in previous investigations, in particular in those reflected in the instructive review [15], the
range $1,000 < Re_\theta < 200,000$ was covered). However, as we showed above, the accuracy of experimental data is sufficient to reveal Reynolds number dependence and correspondence of their data to the Reynolds-number-dependent scaling law (2.7) proposed by us.

The experiments of Österlund et al. allow an additional confirmation of the separation of the basic part of the flow into two self-similar regions (I) and (II) governed by the laws (3.1) and (3.2). It is important that these two self-similar regions cover the whole boundary layer and not a small ($1/6!$) part of it where the universal law is expected by the authors to be valid. These experiments reveal a weak $Re$-number dependence of the parameter $\beta$ (Figure 75): it decreased with growing $Re$. The data are not sufficient to come to a final decision, but they are in approximate agreement with the correlation

$$\beta = \frac{2}{\ln Re} + 0.01 . \quad (5.1)$$

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Figures 1, 2, 3
Figures 4, 5, 6.
Figures 7, 8, 9.
$Re_\theta = 26,612$

$Re_\theta = 23,870$

$Re_\theta = 21,099$

Figures 10, 11, 12.
$\log(\eta) \quad \log(\phi) \\
\text{Re}_\theta = 23,119$

$\log(\eta) \quad \log(\phi) \\
\text{Re}_\theta = 18,479$

$\log(\eta) \quad \log(\phi) \\
\text{Re}_\theta = 16,422$

Figures 13,14,15.
Figures 16, 17, 18.
$Re_\theta = 6.765$

$Re_\theta = 8.298$

$Re_\theta = 6.662$

Figures 19, 20, 21
Figures 22, 23, 24.
$\log(\eta)$

$\log(\phi)$

$Re_\theta = 15,164$

$Re_\theta = 17,813$

$Re_\theta = 20,562$

Figures 25, 26, 27.
Figures 28, 29, 30.
Figures 31, 32, 33.
$Re_\theta = 10,368$

$Re_\theta = 12,886$

$Re_\theta = 14,972$

Figures 34, 35, 36.
\[ \lg(\eta) \]

Figures 37, 38, 39.
$\text{Figures 40, 41, 42.}$

\[
\lg(\phi) \quad \lg(\eta)
\]

- $\text{Re}_\theta = 22,579$
- $\text{Re}_\theta = 4,704$
- $\text{Re}_\theta = 5,747$
Figures 43, 44, 45.
$Re_\theta = 12,239$

$Re_\theta = 13,878$

$Re_\theta = 15,512$

Figures 46, 47, 48.
Figures 49, 50, 51
Figures 52, 53, 54
Figures 55, 56, 57
Figures 58, 59, 60.
$Re_{\theta} = 2,532$

$Re_{\theta} = 3,057$

$Re_{\theta} = 3,651$

Figures 61, 62, 63
\[ \lg(\eta) \]

\[ \lg(\phi) \]

\[ \Re_{\theta} = 4,613 \]

\[ \Re_{\theta} = 5,485 \]

\[ \Re_{\theta} = 6,263 \]

Figures 64, 65, 66
$Re_\theta = 7,147$

$Re_\theta = 7,980$

$Re_\theta = 8,873$

$Re_\theta = 10,161$

Figures 67, 68, 69, 70
\begin{align*}
\text{Re}_\theta &> 15,000 \\
\text{Re}_\theta &< 15,000
\end{align*}

Figures 71, 72.
$6,000 < \text{Re}_\theta < 10,000$

$\ln(\eta)$

$\text{Re}_\theta < 6,000$

$\ln(\eta)$

Figures 73, 74.
\[ y = \frac{2}{\ln(Re)} + 0.01 \]

Figure 75.
| Re  | Alfa | A     | Beta | B     | ln(Re) | ln(Re) | ln(Re) | %    |
|-----|------|-------|------|-------|--------|--------|--------|------|
| 12  | 0.137| 8.61  | 0.209| 5.16  | 10.59  | 10.96  | 10.77  | 3.4% |
| 9   | 0.141| 8.46  | 0.209| 5.29  | 10.32  | 10.64  | 10.48  | 3.0% |
| 8   | 0.142| 8.44  | 0.211| 5.30  | 10.28  | 10.55  | 10.42  | 2.6% |
| 6   | 0.138| 8.68  | 0.211| 5.39  | 10.70  | 10.90  | 10.80  | 1.9% |
| 15  | 0.130| 9.03  | 0.199| 5.45  | 11.32  | 11.53  | 11.42  | 1.9% |
| 17  | 0.135| 8.75  | 0.196| 5.51  | 10.82  | 11.12  | 10.97  | 2.7% |
| 20  | 0.127| 9.25  | 0.188| 5.81  | 11.69  | 11.80  | 11.75  | 0.9% |
| 22  | 0.126| 9.34  | 0.184| 5.95  | 11.86  | 11.94  | 11.90  | 0.7% |
| 25  | 0.125| 9.30  | 0.187| 5.74  | 11.79  | 11.96  | 11.87  | 1.5% |
| 26  | 0.120| 9.74  | 0.177| 6.24  | 12.54  | 12.48  | 12.51  | 0.5% |
| 23  | 0.121| 9.70  | 0.177| 6.30  | 12.46  | 12.42  | 12.44  | 0.4% |
| 21  | 0.125| 9.42  | 0.180| 6.17  | 11.98  | 12.00  | 11.99  | 0.1% |
| 23  | 0.123| 9.52  | 0.177| 6.28  | 12.16  | 12.15  | 12.15  | 0.1% |
| 18  | 0.127| 9.38  | 0.178| 6.35  | 11.91  | 11.85  | 11.88  | 0.5% |
| 16  | 0.131| 9.08  | 0.185| 6.05  | 11.39  | 11.43  | 11.41  | 0.3% |
| 14  | 0.132| 9.01  | 0.191| 5.87  | 11.28  | 11.39  | 11.33  | 1.0% |
| 12  | 0.140| 8.48  | 0.199| 5.56  | 10.36  | 10.69  | 10.53  | 3.1% |
| 9   | 0.144| 8.26  | 0.208| 5.31  | 9.98   | 10.41  | 10.20  | 4.2% |
| 6   | 0.147| 8.05  | 0.219| 5.09  | 9.61   | 10.17  | 9.89   | 5.7% |
| 8   | 0.143| 8.32  | 0.213| 5.24  | 10.08  | 10.49  | 10.29  | 4.0% |
| 6   | 0.146| 8.21  | 0.218| 5.16  | 9.90   | 10.25  | 10.08  | 3.5% |
| 8   | 0.140| 8.49  | 0.214| 5.20  | 10.38  | 10.73  | 10.56  | 3.3% |
| 9   | 0.142| 8.39  | 0.207| 5.36  | 10.20  | 10.56  | 10.38  | 3.5% |
| 12  | 0.137| 8.67  | 0.202| 5.46  | 10.69  | 10.98  | 10.83  | 2.6% |
| 15  | 0.134| 8.80  | 0.199| 5.45  | 10.91  | 11.19  | 11.05  | 2.6% |
| 17  | 0.129| 9.11  | 0.191| 5.73  | 11.45  | 11.63  | 11.54  | 1.6% |
| 20  | 0.130| 9.08  | 0.186| 5.93  | 11.40  | 11.57  | 11.48  | 1.5% |
| 23  | 0.129| 9.11  | 0.184| 5.93  | 11.44  | 11.64  | 11.54  | 1.7% |
| 25  | 0.124| 9.42  | 0.181| 6.04  | 11.99  | 12.08  | 12.04  | 0.7% |
| 27  | 0.124| 9.54  | 0.173| 6.42  | 12.20  | 12.13  | 12.17  | 0.6% |
| 5   | 0.145| 8.29  | 0.216| 5.30  | 10.03  | 10.35  | 10.19  | 3.1% |
| 6   | 0.143| 8.38  | 0.209| 5.39  | 10.19  | 10.52  | 10.36  | 3.2% |
| 8   | 0.143| 8.35  | 0.211| 5.37  | 10.13  | 10.47  | 10.30  | 3.4% |
| 10  | 0.139| 8.58  | 0.204| 5.45  | 10.53  | 10.81  | 10.67  | 2.6% |
| 12  | 0.137| 8.65  | 0.200| 5.52  | 10.65  | 10.95  | 10.80  | 2.7% |
|   |    |    |    |    |    |    |    |    |   |
|---|----|----|----|----|----|----|----|----|---|
| 14| 972| 0.134| 8.81| 0.198| 5.51| 10.92| 11.16| 11.04| 2.2% |
| 17| 102| 0.134| 8.87| 0.188| 5.91| 11.04| 11.23| 11.13| 1.7% |
| 19| 235| 0.126| 9.33| 0.187| 5.89| 11.83| 11.89| 11.86| 0.5% |
| 20| 958| 0.125| 9.44| 0.180| 6.16| 12.02| 12.03| 12.02| 0.1% |
| 22| 579| 0.123| 9.54| 0.179| 6.19| 12.19| 12.16| 12.18| 0.3% |
| 4 | 704| 0.144| 8.26| 0.226| 5.05| 9.97| 10.39| 10.18| 4.1% |
| 5 | 747| 0.146| 8.15| 0.226| 4.94| 9.78| 10.26| 10.02| 4.7% |
| 6 | 768| 0.148| 8.09| 0.221| 5.01| 9.69| 10.17| 9.93| 4.8% |
| 8 | 634| 0.146| 8.18| 0.213| 5.19| 9.83| 10.30| 10.07| 4.7% |
| 10| 502| 0.141| 8.38| 0.204| 5.44| 10.18| 10.62| 10.40| 4.2% |
| 12| 239| 0.139| 8.55| 0.200| 5.50| 10.47| 10.79| 10.63| 3.0% |
| 13| 878| 0.137| 8.69| 0.196| 5.63| 10.73| 10.99| 10.86| 2.4% |
| 15| 512| 0.129| 9.14| 0.189| 5.91| 11.50| 11.61| 11.56| 1.0% |
| 17| 279| 0.129| 9.15| 0.187| 5.95| 11.52| 11.62| 11.57| 0.8% |
| 18| 720| 0.126| 9.34| 0.183| 6.08| 11.85| 11.87| 11.86| 0.2% |
| 3 | 653| 0.150| 8.05| 0.228| 5.10| 9.61| 10.00| 9.81| 4.0% |
| 4 | 312| 0.150| 8.00| 0.225| 5.09| 9.52| 10.00| 9.76| 4.9% |
| 5 | 156| 0.151| 7.96| 0.223| 5.10| 9.45| 9.94| 9.70| 5.1% |
| 6 | 510| 0.149| 8.01| 0.216| 5.20| 9.54| 10.05| 9.80| 5.2% |
| 7 | 969| 0.146| 8.15| 0.210| 5.33| 9.78| 10.24| 10.01| 4.6% |
| 9 | 111| 0.141| 8.42| 0.205| 5.47| 10.25| 10.62| 10.43| 3.6% |
| 10| 313| 0.140| 8.51| 0.202| 5.53| 10.41| 10.74| 10.58| 3.0% |
| 11| 733| 0.139| 8.56| 0.199| 5.57| 10.50| 10.77| 10.63| 2.5% |
| 12| 866| 0.134| 8.88| 0.193| 5.81| 11.05| 11.22| 11.13| 1.5% |
| 14| 289| 0.132| 9.02| 0.188| 5.98| 11.29| 11.39| 11.34| 0.9% |
| 2 | 532| 0.157| 7.84| 0.226| 5.32| 9.24| 9.57| 9.40| 3.4% |
| 3 | 057| 0.157| 7.79| 0.228| 5.17| 9.17| 9.55| 9.36| 4.1% |
| 3 | 651| 0.153| 7.92| 0.223| 5.26| 9.39| 9.83| 9.61| 4.5% |
| 4 | 613| 0.151| 7.94| 0.221| 5.19| 9.43| 9.93| 9.68| 5.2% |
| 5 | 485| 0.148| 8.06| 0.217| 5.23| 9.63| 10.13| 9.88| 5.0% |
| 6 | 263| 0.148| 8.07| 0.213| 5.31| 9.66| 10.13| 9.89| 4.8% |
| 7 | 147| 0.145| 8.23| 0.210| 5.35| 9.92| 10.34| 10.13| 4.2% |
| 7 | 980| 0.146| 8.21| 0.206| 5.47| 9.90| 10.30| 10.10| 4.0% |
| 8 | 873| 0.143| 8.34| 0.202| 5.56| 10.11| 10.46| 10.29| 3.4% |
| 10 | 161| 0.142| 8.43| 0.196| 5.82| 10.27| 10.54| 10.41| 2.5% |