Some notes on induced functions and group actions on hyperspaces.

Summary: Let $X$ be a topological space and $\text{CL}(X)$ be the family of all nonempty closed subsets of $X$. In this paper we discuss the problem of when a continuous map between topological spaces induces a continuous function between their respective hyperspaces. As a main result we characterize the continuity of the induced function in the case of the Fell and Attouch-Wets hyperspaces. Additionally we explore the problem of whether a continuous action of a topological group $G$ on a topological space $X$ induces a continuous action on $\text{CL}(X)$. In particular we give sufficient conditions on the topology of $G$ to guarantee that the induced action on $\text{CL}(X)$ is continuous, provided that $\text{CL}(X)$ is equipped with the Hausdorff or the Attouch-Wets metric topology.

MSC:

- 54B20 Hyperspaces in general topology
- 54C05 Continuous maps
- 54C35 Function spaces in general topology
- 54H15 Transformation groups and semigroups (topological aspects)
- 57S05 Topological properties of groups of homeomorphisms or diffeomorphisms

Keywords:

- hyperspaces
- Attouch-Wets metric
- Fell topology
- Vietoris topology
- induced functions
- group action
- group topology

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