Experimental Investigation of the Eccentric Couette Flow with Superimposed Cross Flow

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Abstract. The technical or industrial aspect of the Couette Flow has been the subject of various papers and the effect of eccentricity resulting in a variable height of the gap between outer and inner cylinder has been investigated. Without doubt eccentricity is one dominant parameter characterizing the flow. However, technical applications such as journal bearings have another particular feature, which has an impact on the basic Couette flow. A permanent supply of fluid is introduced through feed holes, which merges with the rotating flow to compensate the losses at the ends or sides of the bearing. Hence, three dimensional flow structures exist in the vicinity of the feed holes. The presented paper gives experimental of a Couette flow with a gap width of less than 10% relative to the inner diameter and rotating inner cylinder. An eccentricity of 50 % in relation to the mean gap is considered. The results are shown in form of velocity profiles, which are obtained experimentally and are compared with analytical data and computed data. Moreover, a refined experimental set-up is presented, which allows velocity measurements with greater precision together with a wider range of adjustment possibilities for eccentricity and feed hole location.

1. Introduction

The similarity between Couette Flow in an annulus and the technical flow in journal bearings has been discussed in literature. An excellent summary is presented by DiPrima and Swinney [1], which addresses the aspects of gap width and eccentricity. In particular, the first instability of the Couette flow i.e. the formation of Taylor vortices is investigated in great detail. This stability limit is described as a function of the radii ratio \( \eta \) of inner and outer cylinder \( R_1 \) and \( R_2 \), respectively and the Taylor number \( Ta \), which are defined as

\[
\eta = \frac{R_1}{R_2}
\]  

(1)
\[ T_a = \frac{U_1 \cdot H_0}{\nu} \sqrt{\frac{H_0}{R_i}} \]  \hspace{1cm} (2)

with \( U_1 \) being the circumferential velocity of the inner cylinder and where the mean gap width \( H_0 \) is given by

\[ H_0 = R_2 - R_1 \]  \hspace{1cm} (3)

Fig. 1 shows a stability diagram for Couette flow with respect to the onset of Taylor vortices for a system where the inner cylinder rotates and the outer one is at rest and where the data between \( \eta = 0.9 \) and \( \eta = 1 \) are interpolated.

\[ \begin{align*}
\text{psi} &= \frac{H_0}{R_1} \\
\text{Re} &= \frac{U_1 \cdot H_0}{\nu}
\end{align*} \]  \hspace{1cm} (4) \hspace{1cm} (5)

Together with the stability limit of the Couette flow according to [1], the working range of typical journal bearings, experimental data based on flow visualization by Hinko & Andereck [2] and Schwarz [3] and numerical results based on DNS by Scurtu et al. [4] are shown in Fig. 2. Both, the experimental and numerical results are in very good agreement with the predicted stability limit.
Figure 2. Stability diagram for Couette flow, limit by DiPrima and Swinney [1], experimental data by Hinko and Andereck [2] and Schwarz [3], numerical results by Scurtu [4].

With the stability boundary confirmed experimentally and numerically it is obvious that the working range of journal bearings lies in the regime of the Couette flow, but it needs to be investigated how a superimposed cross flow changes the structure of the basic Couette flow.

2. Experimental set-up
The experimental set-up is designed similar to a journal bearing and therefore has to incorporate several criteria, which are taken from literature describing design and performance of journal bearings: Bartz et al. [5] and Lang & Steinhilper [6]. Fig. 3 indicates the main parameters.

Geometry: In general, journal bearings have a width B to diameter D1 ratio \( \gamma \) between 0.5 and 1 and for the principle at work a value \( \gamma = 1 \) is chosen.

Gap: As Fig. 2 indicates a normal gap width of journal bearings is around 0.1% in relation to the inner radius. Here, a compromise between true scale and accessibility by means of LDV technique into the gap is found. The gap width \( \Psi \) of the apparatus can be adjusted between 2.5 and 10%.

Eccentricity: The eccentricity \( e \) can be adjusted between 0 and 0.85% in relation to \( H_0 \).

Cross flow: In journal bearings the cross flow has to compensate for oil losses at the ends of the bearing, which are caused by the pressure distribution in the gap [5]. The cross flow rate \( Q_{in} \) can be calculated according to Eq. 6, which is given in several versions in [5].

\[
Q_{in} = \alpha(\gamma,e) \frac{1}{2} U_1 B H_0
\]  

The cross flow coefficient \( \alpha \) is a function of the width B to diameter D1 ratio \( \gamma \) and the normalized eccentricity \( e \). The circumferential or main flow rate \( Q_0 \) is \( \frac{1}{2} U_1 B H_0 \). However, Peeken [7] confirms a more practical approach, which was suggested by Vogelpohl [8] in 1958 that sets \( Q_{in} \) between 0.33 and 0.5 in relation to the circumferential flow \( Q_0 \). The form of the feed hole can be rather complicated, but a simple circular hole with a diameter of 0.1 D1 is quite common.
Figure 3. Main parameters of the experimental set-up, rotating inner cylinder Z1, fixed outer cylinder Z2.

In addition to the above listed criteria the Reynolds similarity has to be considered. At the selected gap width the experiments are carried out at sub-critical Reynolds numbers, which are based on the viscosity of the used fluid. Moreover, the ratio of the Reynolds numbers for the main flow in the gap and the superimposed cross flow are equal in the experiment and the average journal bearing, which yields a relation for the ratio $\Phi$ between mean velocity of the cross flow measured inside the feed hole and $U_1$. This relation takes the different gap widths of experiment and bearing into account.

$$\Phi_E = \frac{\Psi_E}{\Psi_B} \Phi_B$$  (7)

The index E denotes the experiment and the index B the bearing, respectively. It is evident that the experiment with larger gaps calls for higher cross flow velocities to maintain the Reynolds number ratio found in journal bearings.

Early experiments with a first apparatus indicated the technical limits of the design with respect to the achievable accuracy of velocity measurements inside the gap. Hence, a re-design became necessary, which is described in the following. Besides other improvements the main focus was laid on the precision of the

- eccentricity adjustment,
- positioning of measuring volume of the LDV-system.

Fig. 4 shows a drawing of the new apparatus. The eccentricity is adjusted by means of two micro measuring screws fixed in the outer cylinder, which are adjusted to a given value and set the distance between inner and outer cylinder. When the outer cylinder is moved into the proper position, it is secured by clamps at the bottom end onto a turntable. Then the measuring screws are turned back to remove the pin out of the gap avoiding any unnecessary obstruction of the flow. The laser beams of the LDV enter the measuring section through the top cover and the measuring volume can be traversed radially. Therefore, the initial position in relation to the inner cylinder needs to be calibrated accurately. Moreover, instead of traversing the LDV in two independent directions to find other circumferential positions around the inner cylinder only the turntable is rotated in other words, the gap is rotated around the inner cylinder. This is accomplished by means of a stepper motor, which drives the turntable. Hence, the before described initial position of the LDV measuring volume once established can be maintained in relation to the inner cylinder. To reduce the tolerance chain the turntable is mounted with a four-point-bearing on the main drive shaft that supports the inner cylinder, too. Fig. 5 shows a bottom view of the apparatus indicating the turntable drive and the main drive for the inner cylinder.
3. Results
First experiments were carried out with an existing set up. The eccentricity was adjusting by using calibrated gauges, which were used to set the minimum gap. A 3D-traversing unit was used for the
positioning of the LDV measuring volume, which allowed a step size of 0.02 mm in each direction. The LDV was calibrated against the circumferential speed of the inner cylinder.

The errors that undermined the effort to measure velocity profiles inside the gap accurately became evident quite soon. An insufficiently rigid mounting system of the outer cylinder did not allow a precise adjustment of the eccentricity. Hence, the real eccentricity deviated up to 10% from the target value, which resulted in a skewing of the measured velocity profile.

Whereas, the traversing of the measuring volume at the given step size was well within the necessary limits the initial or reference position inside the gap could not be found better than 0.2 mm resulting in an unacceptable offset.

However, these findings lead to an extensive effort to define means to correct the measured velocity profile and to determine the true eccentricity set for the experiment. Fortunately, due to the nature of the flow all velocity profiles in circumferential direction remain parabolic and a comparison with results derived from the Reynolds equation allow a calculation of the three major errors: eccentricity $\Delta \varepsilon$, position $\Delta y$ and velocity $\Delta u/U_1$. Fig. 6 shows a profile of the circumferential velocity $u$ based on corrected measured data in comparison with analytical results based on the Reynolds equation and computed data using a DNS solver. It must be noted that measurements adjacent to the wall are difficult due to reflections. It must be noted additionally that at speeds below 5 mm/s the data rate decreases significantly resulting in poor signal to noise ratios. Nevertheless, the agreement between experiment and theory is well and distinct features of the profile like $u = 0$ and reversed flow are detected correctly.

![Figure 6](image)

**Figure 6.** Velocity profiles in mid section and widest gap, $z = 0$, $\varphi = 0^\circ$, $\Psi = 7.7\%$, $\varepsilon = 50\%$, $Re = 157$ in comparison with numerical data at $Re = 170$ and analytical results of the Reynolds equation, $y = 0$ indicates the rotating inner cylinder and $y = 7.5$ the fixed outer cylinder.

Moreover, these experiments helped to identify those features of the set-up, which needed refinement and is consequently implemented in the improved experimental rig: a twin system of measuring screws to adjust the eccentricity, a calibration device to determine the reference position of the LDV and an integrated turntable, which is needed to traverse the measuring volume circumferentially with better precision. The requirement $\Delta \varepsilon < 2\%$ can be extracted from Fig. 7, if the resulting error for the velocity value should be less than 1%.
Figure 7. Error analysis, effect of misadjusted eccentricity on the velocity profile across the gap.

An analysis of the angular positioning revealed that due the cos-effect of $\phi$ the velocity is rather robust against deviation from the true circumferential position as long as the profiles are measured near 0 and 180°, where maximum and minimum gap are found, respectively. Here the velocity correction factor is less than 1.01, if the positioning is better than 1°, which can be seen in Fig. 8.

Figure 8. Error analysis, correction factor for $u/U_1$ due deviation from true $\phi$-position.
4. Summary
The main flow in the annulus between a rotating inner and a fixed outer cylinder is of the Couette type, if a critical Reynolds number, which is depending on the gap width $\Psi$, is not exceeded. This flow is governed by the circumferential velocity of the inner cylinder, the gap width and the eccentricity. Particular interest is given to the under-critical flow at narrow gap widths to study the flow that is comparable to the technical flow in journal bearings. Measurements of the velocity inside the gap require precise adjustment of these parameters. Moreover, together with a high resolution velocity measurement the positioning of the point of measurement requires close attention. Post-measurement correction of the data can be recommended in addition to the preparation efforts. The project at work shows successful means to measure the flow velocity profiles inside the gap between the cylinders. An error analysis yields margins for core parameters.

An improved test rig is presented, which allows the preparation of the experiment and adjustment of the mechanical components with the required precision. Additionally, the design incorporates means to introduce a cross flow the wall of the outer cylinder. The design parameters are based on a literature research and corresponding experimental parameters reflect the requirements to fulfill the Reynolds similarity.

Consecutive experiments will be focused on the flow structures in the vicinity of the feed hole covering a wide range of variation of the core parameters: Reynolds number, eccentricity and cross flow rate.

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