Determining the Optimal Location of Vehicle Inspection Facilities Under Uncertainty via New Optimization Approaches

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ABSTRACT This study addresses the problem of optimally locating vehicle inspection facilities under uncertain customer demand and varying velocity considering regional constraints. The objective is to simultaneously minimize the transportation time of all customers and their transportation cost, while ensuring consumers to reach their desired destinations within their expected time and cost. We study two variants of the problem: vehicle inspection location with complete probability distributions of customer demand and vehicle velocity, and that with partial information of customer demand and vehicle velocity, i.e., only the supports and means of the stochastic variables are known. For the former problem, an expected value model and a chance-constrained program are first formulated. Then based on explored problem properties, the expected value program is equivalently reformulated as a deterministic non-linear program. To efficiently deal with the chance-constrained program, a sample average approximation (SAA)-based approach is proposed. For the latter one, we develop a new distribution-free model. Computational results for benchmark examples demonstrate that: i) for the former, the proposed deterministic program reformulation-based approach and SAA algorithm outperform the state-of-the-art approaches; ii) for the latter, the proposed distribution-free model can effectively deal with the problem with partial demand and velocity information.

INDEX TERMS Facility location allocation, stochastic demand, varying velocity, optimization, sample average approximation, distribution-free model.

I. INTRODUCTION
Facility location allocation (FLA) decision has a great and long-term influence on the tactical and operational decisions, since the bad results caused by unreasonable FLA decisions cannot be remedied by strengthening and improving its subsequent management activities. As a result, performing an optimal FLA decision is important and crucial for the success of a city’s economies. Since an FLA problem was proposed by Cooper [1] in 1963, there are numerous important applications in real life, for example, the construction of transportation facilities, emergency service systems, telecommunication networks and public services. From then on, many variants and various solution approaches have been proposed. A variety of mathematical models have been built, i.e., center models [2], [3], median models [4], [5], covering models [6], [7], hub location models [8], and hierarchical location models [9]–[11]. Considerable solution algorithms are developed for solving them, such as simulated annealing, genetic algorithm, harmony search algorithm, and tabu search [12]–[20]. However, most of the existing FLA problems are deterministic by assuming constant input parameters.
FLA decision is a long term strategic planning issue that would be affected by uncertainty. In the literature, various stochastic FLA problems have been studied for decades, e.g., [21]–[29]. One of the FLA applications is the location of vehicle inspection facilities. With the rapid development of economy and high urbanization, the car-ownership in many countries all over the world has increased rapidly. For example, the car-ownership in China has reached 250 million by the end of June, 2019 for the first time. This naturally brings challenges for the economical and sustainable construction and management of vehicle inspection facilities.

In recent years, some scholars have studied the optimal location of vehicle inspection facilities subject to stochastic variables. Tian et al. [30] formulate a stochastic programming model to locate vehicle inspection facilities under a random environment to minimize the total transportation cost of customers. Ling et al. [31] construct a car parking location model to minimize the total transportation cost of customers and solve the model by proposing an iterative search algorithm. Wang et al. [32] deal with the optimal location and arrangement of park-and-ride facilities to maximize their profit and minimize their cost. The works [33], [34] establish fuzzy stochastic programs to optimize the location of vehicle inspection facilities by considering the fuzziness of customer demands. Jia et al. [35] build a stochastic fuzzy chance-constrained program for locating a vehicle inspection facility.

The work [36] presents stochastic programs for an energy-efficient facility location problem by considering carbon emission. To solve the proposed models in [30], [33]–[36], an Monte Carlo sampling or fuzzy simulation-based technique is used to assess uncertain parameters. Tian et al. [37] develop a chance-constrained programming model to determine the optimal location of vehicle inspection facilities and achieve the total transportation cost of customers minimization while ensuring an expected investment profit. Later, the work [38] develops effective models and approaches to solve an extended problem of that in [37]. Recently, Tian et al. [39] construct an expected value model and a chance-constrained program to locate vehicle inspection facilities under stochastic demand and varying velocity to minimize the total transportation cost and time.

In this study, we investigate the optimal location of vehicle inspection facilities under stochastic customer demand and uncertain velocity considering regional constraints, which was studied in [39] only. However, the work [39] has two main deficiencies. First, the stochastic variables are assessed by an Monte Carlo sampling that needs thousands of runs in each iteration. It is very time-consuming, especially for large-scale problems. Second, the stochastic functions and constraints dealt with by Monte Carlo sampling cannot be guaranteed to be optimal. To overcome them, we solve the expected value model and chance-constrained program in [39] by proposing a new equivalent deterministic reformulation-based approach and a new SAA-based method, respectively. Computational results show that the proposed equivalent deterministic program can yield better solutions that reduce 2.00% of the weighted objective function value and the proposed SAA-based approach can reduce 2.25% by spending much less computational time compared with MCS-TLBO [39], for solving the benchmark instances reported in [39].

Most of the existing works assume that complete probability distributions of customer demand and transport velocity are known in advance. Nevertheless, the complete information is not easy to obtain in real life, because it is quite difficult to predict them. For instance, new location issue of vehicle inspection facilities in China occur in new urban areas. It is usually unlikely to acquire the full information [40]. In addition, obtaining complete data is expensive. Consequently, performing an optimal location decision under partial information is necessary and important for decision-makers.

Besides the investigation of the optimal location of vehicle inspection facilities under known complete probability distributions of stochastic variables, we study the optimal location of vehicle inspection facilities under known partial information only. For the problem, we develop a novel distribution-free model. Benchmark instances are solved to show its correctness and effectiveness in comparison with MCS-TLBO under complete demand information.

The remainder of the paper is structured as follows. Section II first introduces the expected value model and chance-constrained program with complete probability distributions of customer demand and vehicle velocity. In Section III, we present the equivalent deterministic program reformulation-based method for the expected value model and the SAA-based approach for the chance-constrained program. In Section IV, the distribution-free model under partial information is developed. Benchmark examples are tested to verify the performance of the proposed methods in Section V. Section VI concludes this research.

II. PROBLEM DESCRIPTION AND FORMULATION

In this section, we first formally describe the studied problem, and then present its formulations.
A. PROBLEM DESCRIPTION

The problem can be briefly described as follows. There are a set of geographically dispersed customers in a geographic space needing vehicle inspection services, denoted by \( D \). The demand quantity at each demand point \( i \in D \) is stochastic. Some regions in the geographic space are not allowed to construct vehicle inspection facilities due to geographical, policy and environmental constraints. Facing the stochastic demands and regional constraints, a set of vehicle inspection facilities, denoted by \( N \), are expected to be optimally opened to meet their needs. Moreover, the velocity of a vehicle from a demand point \( i \in D \) to a vehicle inspection station \( j \in N \) is uncertain. The goal is to minimize the total cost of customers and their total transportation time, simultaneously.

To better define and formulate the problem, the following assumptions are made, as is the case in [39].

i) The distribution condition of customers in a demand region is ignored, i.e., the center of a customer demand region is treated as its coordinated location.

ii) The inspection capabilities of vehicle inspection facilities are ignored, namely vehicle inspection facilities are designed to be big enough to meet the requirements of users.

iii) The transportation cost per kilometre \( c_{ij} \) from demand region \( i \) to vehicle inspection facility \( j \) is considered to be constant and known in advance.

For the sake of brevity, the notations used for formulating the studied problem are listed as follows.

1) INDICES AND PARAMETERS

\( i \): Index of demand points, \( i \in D \).
\( j \): Index of vehicle inspection facilities, \( j \in N \).
\( l_i \): Horizontal coordinate of the location of demand point \( i \).
\( u_i \): Vertical coordinate of the location of demand point \( i \).
\( c_{ij} \): Unit transportation expense from demand point \( i \) to vehicle inspection facility \( j \), with its unit being Yuan.
\( d_{ij} \): Travel distance between demand point \( i \) and vehicle inspection facility \( j \), \( d_{ij} = \sqrt{(x_j - l_i)^2 + (y_j - u_i)^2} \).

2) RANDOM VARIABLES

\( \varepsilon_{ij} \): The number of vehicles from demand point \( i \) to inspection facility \( j \).
\( v_j \): The velocity of a vehicle from demand point \( i \) to inspection facility \( j \), and its unit is m/s.

3) DECISION VARIABLES

\( x_j \): Horizontal coordinate of the location of vehicle inspection facility \( j \).
\( y_j \): Vertical coordinate of the location of vehicle inspection facility \( j \).

B. PROBLEM FORMULATION

In this section, we present the expected value and chance-constrained programs for optimally locating vehicle inspection facilities subject to uncertain customer demand, varying velocity and regional constraints.

1) AN EXPECTED VALUE PROGRAM FOR LOCATING VEHICLE INSPECTION FACILITIES

As pointed out by Tian et al. [39], in the actual process of building vehicle inspection facilities, decision-makers may expect to minimize the average total transportation cost and their average transportation time. Meanwhile, the decision-makers hope that both the cost and time should not exceed given values. Furthermore, as mentioned above, some specific regions cannot be allowed to open vehicle inspection facilities due to various constraints. To handle this issue, we first present the expected value program for locating vehicle inspection facilities. The formulation is given as follows: Model \( P_e \):

\[
\begin{align*}
\min & \quad \theta_1 E (C) + \theta_2 E (T) \quad (1) \\
\text{s.t.} & \quad E \left( \sum_{i \in D} \sum_{j \in N} \varepsilon_{ij} c_{ij} d_{ij} \right) \leq C' \quad (2) \\
& \quad E \left( \sum_{i \in D} \sum_{j \in N} \varepsilon_{ij} d_{ij} y_j \right) \leq T' \quad (3) \\
& \quad d_{ij} = \sqrt{(x_j - l_i)^2 + (y_j - u_i)^2} \quad (4) \\
& \quad h \left( x_j, y_j \right) \leq 0, \quad \forall j \in N \quad (5) \\
& \quad x_j \in (x_l, x_u), \quad \forall j \in N \quad (6) \\
& \quad y_j \in (y_l, y_u), \quad \forall j \in N \quad (7)
\end{align*}
\]

Objective (1) is to minimize the weighted expected total transportation cost and time, where \( \theta_1 \) and \( \theta_2 \) are the weighting factors, respectively; and \( C \) and \( T \) are the average total transportation cost and time, respectively. (2) and (3) ensure that the expected transportation cost and time of serviced customers should be less than the given values \( C' \) and \( T' \), respectively. (4) calculates the distance between a demand region \( i \) and vehicle inspection facility \( j \) to be opened. (5) specifies that the location of any vehicle inspection facility should be within a constrained region. (6) and (7) are the restrictions on decision variables.

2) A CHANCE-CONSTRAINED MODEL FOR LOCATING VEHICLE INSPECTION FACILITIES

For the actual location process of vehicle inspection facilities, one may expect to obtain the minimum transportation cost of customers subject to a specified confidence level. In addition, one hopes to satisfy the total transportation time constraint with a given confidence level. Therefore, we present a chance constrained programming model for locating vehicle inspection facilities under stochastic demand, varying vehicle velocity, and regional constraints.

Model \( P_c \):

\[
\begin{align*}
\min & \quad \theta_1 \bar{C} + \theta_2 \bar{T} \quad (8) \\
\text{s.t.} & \quad P_r \left( \sum_{i \in D} \sum_{j \in N} \varepsilon_{ij} c_{ij} d_{ij} \leq \bar{C} \right) \geq \alpha \quad (9)
\end{align*}
\]
and (4)-(7).

Objective (8) minimizes the weighted value of the minimum total transportation cost and time. \( \alpha \) and \( \beta \) are given confidence levels, (9) and (10) are probability constraints, which \( \bar{C} \) and \( \bar{T} \) are defined as \( \min \{C[P_r | \mathcal{C}(x, y, \xi) \leq \bar{C}] \} \geq \alpha \) and \( \min \{T[P_r | \mathcal{T}(x, y, \xi) \leq \bar{T}] \} \geq \beta \), respectively. (11) and (12) ensure that the minimum total transportation cost and time should be less than given values \( C' \) and \( T' \), respectively.

### III. RESOLUTION APPROACH

In this section, we propose an equivalent deterministic program reformulation-based approach and an SAA-based approach for solving problem \( P_e \) and problem \( P_r \), respectively.

#### A. EQUIVALENT DETERMINISTIC PROGRAM

**REFORMULATION-BASED APPROACH**

Model \( P_e \) is a stochastic expected value program, which has been solved by using the Monte Carlo sampling and TLBO combined algorithm (i.e., MCS-TLBO) [39]. However, as previously mentioned, MCS-TLBO has two main deficiencies: 1) it is time-consuming due to each iteration requiring several thousands of simulation runs. For example, in [39] 3000 simulation cycles are set, i.e., each iteration of MCS-TLBO requires 3000 runs, and 2) MCS-TLBO cannot guarantee that optimal-proved solutions are derived. To overcome these deficiencies, based on explored problem properties, we propose an equivalent deterministic program reformulation-based approach for handling model \( P_e \).

As is the case in the work [39], it is assumed that the customer demand quantity at any demand point \( i \in D \) (i.e., \( e_{ij} \)) follows an independent Normal distribution \( N_{ij}(\mu_{ij}, \sigma_{ij}^2) \). The velocity of vehicle inspection customers follows an Uniform distribution \( U_{ij} \). Before proceeding, we recall the following definition.

**Definition 1:** If \( Z = g(x, y) \) is the function of one-dimensional random variables \( x \) and \( y \), and \( f(x, y) \) is the probability density of two-dimensional random variables \( (x, y) \), then we have

\[
E(Z) = E[g(x, y)] = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y)f(x, y)dxdy \tag{13}
\]

According to the definition of Normal distribution, we have

\[
\int_{-\infty}^{+\infty} \varepsilon_{ij}f(\varepsilon_{ij})d\varepsilon_{ij} = \mu_{ij} \tag{14}
\]

According to (2), we have

\[
E \left( \sum_{i \in D} \sum_{j \in N} e_{ij}d_{ij}c_{ij} \right) = \sum_{i \in D} \sum_{j \in N} d_{ij}c_{ij}E(e_{ij}) \tag{15}
\]

According to (13), the following inequality holds.

\[
\sum_{i \in D} \sum_{j \in N} d_{ij}c_{ij}E(e_{ij}) = \sum_{i \in D} \sum_{j \in N} d_{ij}c_{ij} \int_{-\infty}^{+\infty} \varepsilon_{ij}f(\varepsilon_{ij})d\varepsilon_{ij} \tag{16}
\]

Thus, (2) can be transformed into an equivalent deterministic expression, as shown as follows:

\[
E \left( \sum_{i \in D} \sum_{j \in N} e_{ij}d_{ij}c_{ij} \right) = \sum_{i \in D} \sum_{j \in N} d_{ij}c_{ij}E(e_{ij}) \tag{17}
\]

Similarly, based on (3), we can have

\[
E \left( \sum_{i \in D} \sum_{j \in N} \frac{e_{ij}}{v_{ij}} \right) = \sum_{i \in D} \sum_{j \in N} d_{ij}E\left(\frac{e_{ij}}{v_{ij}}\right) \tag{18}
\]

Furthermore, (18) can be written as follows:

\[
\sum_{i \in D} \sum_{j \in N} d_{ij}E\left(\frac{e_{ij}}{v_{ij}}\right) = \sum_{i \in D} \sum_{j \in N} d_{ij} \int_{-\infty}^{+\infty} e_{ij}f(\varepsilon_{ij})d\varepsilon_{ij} \int_{a_{ij}}^{b_{ij}} \frac{g(v_{ij})}{v_{ij}}dv_{ij} \tag{19}
\]

The velocity of vehicles follows a Uniform distribution \( U_{ij} \), then

\[
\int_{a_{ij}}^{b_{ij}} \frac{g(v_{ij})}{v_{ij}}dv_{ij} = \int_{a_{ij}}^{b_{ij}} \frac{1}{v_{ij}b_{ij} - a_{ij}}dv_{ij} \tag{20}
\]

Thus, (3) can also be transformed into an equivalent deterministic expression shown as follows:

\[
E \left( \sum_{i \in D} \sum_{j \in N} \frac{e_{ij}}{v_{ij}} \right) = \sum_{i \in D} \sum_{j \in N} d_{ij} \mu_{ij} \frac{b_{ij}}{b_{ij} - a_{ij}} \ln \frac{b_{ij}}{a_{ij}} \leq T' \tag{21}
\]

Therefore, with the above analysis, model \( P_e \) can be transformed into the following equivalent deterministic model \( P_e' \):

```
\min \theta_1E(C) + \theta_2E(T)
```

subject to constraints (4)-(7), (17) and (21).

#### B. SAA-BASED APPROACH

As previously mentioned, the existing study [39] efficiently solve the chance-constrained program by an Monte Carlo sampling-based meta-heuristic algorithm (i.e., MCS-TLBO). However, MCS-TLBO is time-consuming due to considerable iterations, whose solution quality can also be improved. Therefore, a new SAA-based approach is proposed to more efficiently tackle the presented chance-constrained program.
An SAA-based approach solves a stochastic program by Monte Carlo sampling and deterministic optimization techniques [41]. Its core idea is to represent the stochastic variables by samples and transform the stochastic programming problem into a deterministic optimization problem. The solutions to the sample mean problem reasonably approximate the original problem.

We first give the definitions of new notations and decision variables, then stochastic program $P^c_e$ with adapting the SAA method is introduced.

**New parameters:**
- $\Omega$: Set of scenarios, i.e., $\Omega = \{1, \ldots, |\Omega|\}$.
- $w$: Index of scenarios, $w \in \Omega$.

Model $P^c_e$:

$$\min \left\{ \frac{1}{|\Omega|} \left( \sum_{w \in \Omega} \theta_1 \hat{C}(w) + \sum_{w \in \Omega} \theta_2 \hat{T}(w) \right) \right\}$$

s.t. $$\sum_{i \in D, j \in N} \varepsilon_{ij}(w) c_{ij} d_{ij} \leq \hat{C}(w)$$

$$\sum_{i \in D, j \in N} \varepsilon_{ij}(w) d_{ij} v_{ij}(w) \leq \hat{T}(w)$$

$$\hat{C}(w) \leq C'$$

$$\hat{T}(w) \leq T'$$

and (4)-(7).

Objective (22) minimizes the weighted expected transportation cost and time under the given set of scenarios. The transportation cost and time (23)-(26) depend on different scenarios. (23) and (24) indicate the transportation cost and time of customers should less than the average transportation cost and time under each scenario $w$. (25) and (26) mean the transportation cost and time under each scenario $w$ should not exceed the given value $C'$ and $T'$, respectively.

### IV. A DISTRIBUTION-FREE FORMULATION UNDER PARTIAL INFORMATION

Existing studies assume that decision-makers can acquire enough historical data to fit complete probability distributions [39]. Nevertheless, it is not easy to obtain the complete probability distributions due to various factors. Estimating the complete probability distributions is very difficult, but it is relatively easy to obtain partial easy-obtainable information, such as the support and mean [42]. Based on the above consideration, we consider the optimal location of vehicle inspection facilities under partial information, i.e., only the support and mean of demands are known, and develop a novel distribution-free model for it.

In order to construct a distribution-free model, we need to reformulate the chance constraints (9) and (10). $[U^L_{ij}, U^U_{ij}]$ and $[a_{ij}, b_{ij}]$ are the support of $\varepsilon_{ij}$ and $v_{ij}$, respectively.

For the sake of convenience, let $\tilde{C} = \sum_{i \in N} c_{ij} d_{ij} \varepsilon_{ij}$ and $\tilde{\mu} = \sum_{i \in N} c_{ij} d_{ij} \mu_{ij}$. Since both $\varepsilon_{ij}$ and $v_{ij}$ are random variables, let $\tau_{ij} = \frac{\varepsilon_{ij}}{v_{ij}}$, then we calculate that the support of $\tau_{ij}$ is $[\frac{U^L_{ij}}{v_{ij}}, \frac{U^U_{ij}}{v_{ij}}]$ and the mean of $\tau_{ij}$ is $\frac{\mu_{ij}}{a_{ij} - a_{ij}} \ln \frac{b_{ij}}{a_{ij}}$.

Thus (10) can be reformulated as follows.

$$\Pr \left( \sum_{i \in D, j \in N} \tau_{ij} d_{ij} \leq \bar{T} \right) \geq \beta \quad (27)$$

Before formulating the distribution-free model, we first recall Hoeffding’s theorem presented as follows:

**Theorem 1:** If random variables $X_1, X_2, \ldots, X_n$ are independent and $a_i \leq X_i \leq b_i$, $i = 1, 2, \ldots, n$, then for a given positive value $t > 0$, we have

$$\Pr \left( \bar{X} - \mu \geq t \right) \leq \exp \left( -\frac{2n^2 t^2}{\sum_{i=1}^{n} (b_i - a_i)^2} \right) \quad (28)$$

where $\bar{X} = \sum_{i=1}^{n} X_i/n$ and $\mu = E(\bar{X})$.

Let $S$ denote the sum of all the variables (i.e., $S = \sum_{i=1}^{n} X_i$). Then the following inequality holds:

$$\Pr \left( S - \bar{X} \geq t \right) \leq \exp \left( -\frac{2n^2 t^2}{\sum_{i=1}^{n} (b_i - a_i)^2} \right) \quad (29)$$

According to (9), we have

$$\Pr \left( \tilde{C} - \tilde{\mu} \geq \bar{C} - \bar{\mu} \right) \geq \alpha \quad (30)$$

Then according to (29), the following inequality holds:

$$\Pr \left( \tilde{C} - \tilde{\mu} \geq \bar{C} - \bar{\mu} \right) \leq \exp \left( -\frac{2 \tilde{C} - \tilde{\mu} \geq (\bar{C} - \bar{\mu})^2}{\sum_{i=1}^{n} (c_{ij} d_{ij} (U^U_{ij} - U^L_{ij}))^2} \right) \quad (32)$$

Then it is easy to know if

$$\exp \left( -\frac{2 \tilde{C} - \tilde{\mu} \geq (\bar{C} - \bar{\mu})^2}{\sum_{i=1}^{n} (c_{ij} d_{ij} (U^U_{ij} - U^L_{ij}))^2} \right) \leq 1 - \alpha \quad (33)$$

holds, then (9) is also satisfied. Consequently, (9) can be reformulated as the following distribution-free expression:

$$\exp \left( -\frac{2 \tilde{C} - \sum_{i \in N} c_{ij} d_{ij} \mu_{ij} \geq (\bar{C} - \bar{\mu})^2}{\sum_{i=1}^{n} (c_{ij} d_{ij} (U^U_{ij} - U^L_{ij}))^2} \right) \leq 1 - \alpha \quad (34)$$

(34) can be rewritten as:

$$\sum_{i \in N} c_{ij} d_{ij} \mu_{ij}$$

$$\sum_{i \in N} \frac{\mu_{ij}}{b_{ij} - a_{ij}} \ln \frac{b_{ij}}{a_{ij}} \leq \tilde{C} \quad (35)$$

Similarly, (10) can be reformulated as the following distribution-free expression:

$$\sum_{i \in N} \frac{\mu_{ij}}{b_{ij} - a_{ij}} \ln \frac{b_{ij}}{a_{ij}} \leq \bar{T} \quad (36)$$
TABLE 1. Parameters of the benchmark examples [39].

| Region | $h_i$ (m) | $v_i$ (m) | $\mu_{ij}$ ( Thousand ) | $\sigma_{ij}$ ( Thousand ) | $c$ (Yuan/km) |
|--------|-----------|-----------|-------------------------|---------------------------|-------------|
| 1      | -19553.93 | -6822.87  | 2.660                   | 0.2                       | 2           |
| 2      | 6818.23   | -2988.68  | 2.894                   | 0.2                       | 2           |
| 3      | -14319.44 | -3175.23  | 1.557                   | 0.2                       | 2           |
| 4      | -3625.74  | -2088.84  | 3.556                   | 0.2                       | 2           |
| 5      | -1109.74  | 285.12    | 5.074                   | 0.2                       | 2           |

TABLE 2. Demand mean and support corresponding to the 95th percentile.

| Region | $\mu_{ij}$ (Thousands) | $U^U_{ij}$ (Thousands) | $U^L_{ij}$ (Yuan/km) |
|--------|------------------------|------------------------|----------------------|
| 1      | 2.660                  | 2.989                  | 2.331                |
| 2      | 2.894                  | 3.223                  | 2.565                |
| 3      | 1.557                  | 1.886                  | 1.228                |
| 4      | 3.556                  | 3.885                  | 3.227                |
| 5      | 5.074                  | 5.403                  | 4.745                |

For the optimal location of vehicle inspection facilities under partial information, the mean of each demand region is set according to those in TABLE 1. For the support, we consider both wide and narrow types. The support of the wide type is set as $\left[ \min \{0, \mu_{ij} - 1.6449\sigma_{ij}\}, \mu_{ij} + 1.6449\sigma_{ij}\right]$, corresponding to the 95th percentile. For the narrow one, the support is set as $\left[ \max \{0, \mu_{ij} - 1.2816\sigma_{ij}\}, \mu_{ij} + 1.2816\sigma_{ij}\right]$, corresponding to the 80th percentile. The detailed values are listed in TABLE 2 and 3, respectively.

Example 1: Decision-makers want to locate vehicle inspection facilities with a minimum total transportation cost and time of serviced customers. Area constraints $x^2 + y^2 \leq 3.5 \times 10^7$ and $x^2 + y^2 \geq 1.8 \times 10^7$ should be satisfied. Meanwhile, the expected transportation cost and transportation time can not exceed $2.4 \times 10^5$ Yuan and $1.6 \times 10^7$ Seconds, respectively. Vehicle velocity $v_{ij}$ follows an uniform distribution, $v_{ij} \sim U_{ij}(5, 20)$ meter/second. According to the proposed approach, this problem can be formulated as the following deterministic program $P_d(E1)$:

$$\min \; \theta_1 \bar{C} + \theta_2 \bar{T}$$

subject to (4)-(7), (11), (12), (35) and (36).

V. CASE STUDY

In this section, we evaluate the performance of the proposed approaches by testing the benchmark examples from the actual location for vehicle inspection facilities in Fushun city, China. There are five vehicle inspection regions in Fushun city, i.e., Development, Dongzhou, Wanghua, Xinfu and Shuncheng districts, as shown in FIGURE 1. The coordinate of the city center is $(41578784.96, 4638592.97)$. To handle the problem conveniently, the coordinates of these five districts are transformed into the following numerical values in TABLE 1 by taking the center coordinate as the coordinate origin. Moreover, the number of vehicles follows normal distributions and the cost per kilometer of each demand regions are listed in TABLE 1. All the proposed models are solved by using solver LINGO to find their globally optimal solutions. The proposed approaches are compared with MCS-TLBO [39] in terms of solution quality and computational efficiency.
time of serviced customers with given confidence levels $\alpha = 0.9$ and $\beta = 0.9$, respectively. Vehicle inspection facilities should be constructed within the area $x^2 + y^2 \leq 3.5 \times 10^7$ and $x^2 + y^2 \geq 1.8 \times 10^7$. Moreover, the total transportation cost and time can not exceed $2.8 \times 10^5$ Yuan and $1.8 \times 10^5$ seconds, respectively. Vehicle velocity $v_{ij}$ follow a uniform distribution $v_{ij} \sim U(5, 20)$ meter/second. This problem can be formulated as the following SAA-based program $P_{\epsilon}(E2)$:

$$
\min \left\{ \frac{1}{|\Omega|} \left( \sum_{w \in \Omega} \theta_1 \tilde{C}(w) + \sum_{w \in \Omega} \theta_2 \tilde{T}(w) \right) \right\}
$$

s.t. $\sum_{i=1}^{5} \sum_{j \in N} \varepsilon_{ij}(w)c_{ij}d_{ij} \leq \tilde{C}(w)$ \hspace{1cm} (44)

$\sum_{i=1}^{5} \sum_{j \in N} \varepsilon_{ij}(w)d_{ij} v_{ij}(w) \leq \tilde{T}(w)$ \hspace{1cm} (45)

$\tilde{C}(w) \leq 2.8 \times 10^5$ \hspace{1cm} (46)

$\tilde{T}(w) \leq 1.8 \times 10^5$ \hspace{1cm} (47)

$x_j^2 + y_j^2 \geq 1.8 \times 10^7$ \hspace{1cm} (48)

$x_j^2 + y_j^2 \leq 3.5 \times 10^7$ \hspace{1cm} (49)

$\bar{x}_j \in (-19553.93, 6818.23)$ \hspace{1cm} (50)

$y_j \in (-6822.87, 285.12)$ \hspace{1cm} (51)

By solving model $P_{\epsilon}(E2)$, we obtain the following results:

$(x, y) = (-3682.58, -2106.79)$

$C = 218463.30$

$T = 10090.59$

The results show that the location coordinate of the vehicle inspection facility is $(-3682.58, -2106.79)$. The minimum average transportation cost and time are 218463.30 Yuan and 10090.59 s, respectively.

In summary, the location coordinates of the vehicle inspection facilities under complete distribution information are $(-3682.58, -2106.80)$ and $(-3682.58, -2106.79)$, respectively. Moreover, the coordinate sketch-map for vehicle inspection facilities under complete information is shown in FIGURE 2.

**Example 3:** Decision-makers hope to locate vehicle inspection facilities with partial customer demand and vehicle velocity information, i.e., only the supports and means are known. The mean of each demand region is listed in TABLE 2 and 3. The area constraints $x^2 + y^2 \leq 3.5 \times 10^7$ and $x^2 + y^2 \geq 1.8 \times 10^7$ should be satisfied. The total transportation cost and time can not exceed $2.8 \times 10^5$ Yuan and $1.8 \times 10^5$ seconds, respectively. The values of $\alpha$ and $\beta$ are 0.9. This problem can be formulated as the following distribution-free model $P_{df}(E3)$:

$$
\min \theta_1 \tilde{C} + \theta_2 \tilde{T}
$$

s.t. $\tilde{C} \leq 2.8 \times 10^5$ \hspace{1cm} (52)

$\tilde{T} \leq 1.8 \times 10^5$ \hspace{1cm} (53)

and constraints (35) and (36).

The results show that the coordinates of the vehicle inspection facilities corresponding to the 95th and 80th percentile are $(-3683.26, -2105.61)$ and $(-3683.77, -2104.72)$, respectively. The coordinate sketch-map for the vehicle inspection facility under partial information is shown in FIGURE 3.

The computational results obtained by the proposed approaches are summarized in TABLE 4, where $\tilde{C}$, $\tilde{T}$, $(x, y)$ denote the $\alpha$-pessimistic value of the total transportation cost, the $\beta$-pessimistic value of the total transportation time, and
TABLE 4. Comparison results for the benchmark examples.

| Information  | Model/Range     | Method          | $(x, y)$          | $\bar{C}$     | $T$     | $0.5 \times \bar{C} + 0.5 \times T$ | CPU Time (s) |
|--------------|-----------------|-----------------|-------------------|----------------|---------|-------------------------------------|--------------|
| Complete     | Expected Value  | MCS-TLBO        | (-3680.16, -2133.55) | 222890.18 | 10299.76 | 116594.97                           | 407          |
|              | Equivalent Reformulation | (-3682.58, -2106.80) | 218440.20 | 10094.12 | 114267.16 | 1                                      |
| Chance-Constrained | MCS-TLBO       | SAA-based approach | (-3707.49, -2142.22) | 223478.55 | 10326.94 | 116902.75                           | 788          |
| Partial      | Narrow          | Distribution-free | (-3683.26, -2105.61) | 239221.40 | 28461.85 | 133841.63                           | 3            |
|              | Wide            | Distribution-free | (-3683.77, -2104.72) | 254877.90 | 26984.25 | 140931.08                           | 3            |

the coordinates of vehicle inspection facilities to be opened, respectively. To show their superiority, we compare them with the best solutions for the expected value model and chance-constrained model obtained by MCS-TLBO [39].

From TABLE 4, we can draw the following conclusions:

For the problems under complete information:

i) The proposed equivalent deterministic reformulation-based approach obtains the optimal solution with the weighted objective value of 114267.16. Compared with the best solution obtained by MCS-TLBO, it reduces 2.00% (i.e., (116594.97-114267.16)/116594.97) of transportation cost and time weighted value, which shows that the equivalent deterministic reformulation-based approach is more effective than MCS-TLBO in solving the expected value model.

ii) The proposed SAA-based approach obtains the transportation cost and time weighted objective value of 114276.95, which improves the quality of the best solution obtained by MCS-TLBO by 2.25% (i.e., (116902.75-114267.95)/116902.75).

iii) The proposed approaches spend much less computational time than MCS-TLBO, which confirms the efficiency of the proposed methods.

For the problems under partial information:

i) The solution obtained by the distribution-free model $P_{df}$ under either wide or narrow case is very close to the solutions obtained by $P_e$ under given complete probability distributions. It indicates that the proposed distribution-free model can effectively deal with the location of vehicle inspection facilities under partial information of customer demand and vehicle velocity.

ii) The objective function value obtained by the proposed model $P_{df}$ under a narrow support is a bit less than that under a wide support. This suggests that the proposed distribution-free model is more effective under a narrow support. This mainly because the narrower the support, the more accurate the demand information.

VI. CONCLUSIONS

This paper introduces two approaches to address the optimal location of vehicle inspection facilities under stochastic demand and varying velocity considering regional constraints. We investigate two variants of the problem: i) with complete information of demand and velocity (i.e., complete probability distributions are known); and ii) with partial information (i.e. only the mean and the support are known). For the former, an expected value model and a chance-constrained program are first formulated. Then based on our analysis results, an equivalent deterministic reformulation-based approach is proposed for the expected value model and an SAA-based approach is devised for the chance-constrained model. For the latter, a new distribution-free model is developed.

The computational results show that the proposed approaches for the problem under complete information are more effective than the state-of-the-art MCS-TLBO [39]. Computational results also confirm the effectiveness and efficiency of the distribution-free model in dealing with stochastic location of vehicle inspection facilities under partial information. Especially, the proposed approaches significantly outperform the existing algorithm MCS-TLBO in terms of computational efficiency.

In our future research, we will expand this study from the following aspects. First, this study considers the condition of regional constraint with round shapes, but in real applications it can be irregular shapes. It is interesting and important to expand the research to consider irregular regional constraints. Second, we will investigate the optimal location of vehicle inspection facilities when less information is available. Last but not least, it is of practical significance to simultaneously take routing decision into account when locating vehicle inspection facilities under uncertainty.

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