The Effects of Toroidal Magnetic Field on the Vertical Structure of Hot Accretion Flows

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Abstract

We solved the set of two-dimensional magnetohydrodynamic (MHD) equations for optically thin black hole accretion flows incorporating the toroidal component of the magnetic field. Following global and local MHD simulations of black hole accretion disks, the magnetic field inside the disk is decomposed into a large-scale field and a fluctuating field. The effects of the fluctuating magnetic field in transferring the angular momentum and dissipating the energy are described through the usual α description. We solved the MHD equations by assuming a steady-state and radially self-similar approximation in the r − θ plane of the spherical coordinate system. We found that as the amount of magnetic field at the equatorial plane increases, the heating by the viscosity decreases. In addition, the maximum amount of the heating by the viscous dissipation is produced at the midplane of the disk, while that of the heating by the magnetic field dissipation is produced at the surface of the disk. Our main conclusion is that in terms of the no-outflow solution, thermal equilibrium still exists for the strong magnetic field at the equatorial plane of the disk.

Key words: accretion, accretion disks – magnetohydrodynamics (MHD)

1. Introduction

It is well known that mass accretion onto a black hole is a common process in the universe and is the power source of many active phenomena, including X-ray binaries (XRBs), active galactic nuclei (AGNs), and gamma-ray bursts. Based on the temperature of the accretion flow, two distinct classes of black hole accretion solutions exist, i.e., cold and hot flows.

The standard thin disk, commonly called an α-disk, is categorized in cold accretion flows (Novikov & Thorne 1973; Shakura & Sunyaev 1973; Lynden-Bell & Pringle 1974). In this model the disk is considered to be in the limit of an optically thick, geometrically thin (H/R ≤ 1, where H is the half-thickness of the disk in the cylindrical radius R), Keplerian rotating, and radially subsonic disk. Moreover, in the α-disk typically global heat transport is neglected and the energy released via viscosity is radiated away locally. Therefore, the accreting flow becomes cool very efficiently and cannot produce a high-energy spectrum. Additionally, the mass accretion rate of the standard disk is mildly low, i.e., \( M \lesssim M_{\text{crit}} \), where \( M_{\text{crit}} \) is the critical mass accretion rate. The high/soft state of black hole binaries, as well as usual luminous AGNs, belongs to this model (see reviews by Pringle 1981; Frank et al. 2002; Kato et al. 2008; Abramowicz & Fragile 2013; Blaes 2014; Lasota 2016, for more details).

When the mass accretion rate is extremely high, i.e., \( M \gtrsim M_{\text{crit}} \), the accreting flow becomes optically thick and the energy released cannot radiate away locally. Consequently, the radiation is trapped and advected with the accreting matter inwardly. This flow with such a high mass accretion rate is called a slim disk (see Abramowicz et al. 1988; see also Katz 1977; Begelman 1979; Begelman & Meier 1982; Eddington et al. 1988; Chen & Taam 1993; Narayan & Popham 1993; Kato et al. 2008, for more details). The slim disk also belongs to the cold class and, like the standard disk, emits blackbody-like radiation. The horizontal pressure gradients, radial velocity, and also advective heat transport of slim disks are not negligible. This class of solutions is applied to systems such as ultraluminous X-ray sources, ultraluminous supersoft X-ray sources (ULSs), luminous quasars, and narrow-line Seyfert 1 galaxies (Fukue 2004; Kato et al. 2008).

In contrast to the cold disk model, a hot accretion solution also exists when the mass accretion rate is very low (\( M \approx a^2 M_{\text{Edd}} \), where \( a \) is the viscous parameter and \( M_{\text{Edd}} \) is the Eddington accretion rate). Early work on hot accretion flows was first initiated by Ichimaru (1977) and Rees et al. (1982) and then rediscovered by Narayan & Yi (1994, 1995a, 1995b) and Abramowicz et al. (1995). In this model, the disk is optically thin, and since the cooling mechanism is inefficient, the gas temperature therefore becomes extremely high (nearly virial). Consequently, the gas pressure acts to puff up the inner region of the accretion disk and the disk becomes geometrically thick, i.e., \( H/R \sim 1 \). The dynamics and radiative properties of the hot accretion solution have been investigated in detail, and this model has widespread applications in various sources, such as the supermassive black hole in our Galactic center, Sagittarius A* (Sgr A*), low-luminosity AGNs (LLAGNs), and black hole X-ray binaries in the hard and quiescent states (for more details see Narayan 2005; Yuan 2007, 2011; Ho 2008; Narayan & McClintock 2008; Yuan & Narayan 2014).

In recent years, many numerical hydrodynamic (HD) and magnetohydrodynamic (MHD) simulations have been performed to study the structure and the dynamics of hot accretion flows (e.g., Igumenshchev & Abramowicz 1999, 2000; Stone et al. 1999; Hawley et al. 2001; De Villiers et al. 2003; Igumenshchev et al. 2003; Yuan & Bu 2010; Pang et al. 2011; Yuan et al. 2012a, 2012b, 2015; Narayan et al. 2012; Bu et al. 2013, 2016a, 2016b; Sadowski et al. 2013). Some of the most important and interesting findings, unlike pioneer analytical works that have been done on the hot accretion flow, are the simulations that revealed that the mass inflow rate is not constant and decreases inwardly, which results in the existence of wind/outflow from the system. It should be noted
here that the properties and dynamics of the hot accretion flows with outflow are beyond the scope of the present work, and we will postpone this hot topic to our future investigations.

Based on the self-similar assumption, many analytical works have also been done to investigate the structure and properties of the hot accretion flow in one dimension (e.g., Blandford & Begelman 1999; Akizuki & Fukue 2006; Abbassi et al. 2008; Zhang & Dai 2008; Bu et al. 2009; Abbassi & Mosallanezhad 2012a, 2012b; Mosallanezhad et al. 2012, 2013) and also in two dimensions (e.g., Narayan & Yi 1995a; Xu & Chen 1997; Blandford & Begelman 2004; Xue & Wang 2005; Tanaka & Menou 2006; Jiao & Wu 2011; Mosallanezhad et al. 2014, 2016; Samadi & Abbassi 2016; Samadi et al. 2017). As mentioned here, since the hot accretion flows are geometrically thick, the height-integrated approximation used in one-dimensional self-similar solutions is not appropriate. This is mainly because in this case the physical variables are not only a function of $r$ but also a function of vertical direction, $\theta$, and therefore solving the hot accretion flow in two dimensions is more reasonable. In all of the above-mentioned works in two dimensions, their results only belong to the simplest case in which the advection parameter, i.e., $f$, was constant, which may not be accurate.

Several theoretical works have recently attempted to answer the question of how the advection parameter varies in the vertical direction (see Gu et al. 2009; Samadi et al. 2014; Gu 2015; Zeraatgari & Abbassi 2015, hereafter ZA15). For instance, ZA15 adopted the polytropic relation in the vertical direction instead of using the energy equation. They also considered the modified $\alpha$ description of viscosity defined by Bisnovatyi-Kogan & Lovelace (2007). By some modifications, they found an analytical solution for hot accretion flows, and their results were in good agreement with those presented in Narayan & Yi (1995a) without any difficulty of solving ordinary differential equations with two boundary conditions.

In almost all of the above works the magnetic field has not been included. It is now well known that the magnetic field must be present and plays a significant role in the structure and the dynamics of the hot accretion flow, such as the angular momentum transfer by magnetorotational instability (MRI; Balbus & Hawley 1998), the convective instability of the accretion flow (Narayan et al. 2012; Yuan et al. 2012b), and the driving mechanism of the wind/outflow (Yuan et al. 2015). Therefore, the aim of this paper is to consider the toroidal component of the magnetic field in comparison to our previous work, i.e., ZA15, and solve the flow equations, including the induction equation. The second change compared to our previous work is adopting the modified $\alpha$ description of viscosity for both the viscosity and the magnetic diffusivity due to the MRI.

The presented paper is structured as follows. In Section 2, we introduce the basic equations and assumptions. The self-similar solutions and boundary conditions are given in Section 3. In Section 4, we present numerical results. In Section 5, we summarize and conclude.

### 2. Basic Equations and Assumptions

In this section, we describe the basic equations for optically thin black hole accretion flows incorporating magnetic fields. We adopt spherical coordinates $(r, \theta, \phi)$. The resistive MHD equations, including conservation of mass, momentum, energy, and the induction equation, are as follows:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0,$$  \hspace{1cm} (1)

$$\rho \left[ \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} \right] = -\rho \nabla \psi - \nabla p + \nabla \cdot \mathbf{F} + \frac{J \times \mathbf{B}}{c},$$  \hspace{1cm} (2)

$$q_{\text{adv}} = q_+ - q_- = f q_+,$$  \hspace{1cm} (3)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left( \mathbf{v} \times \mathbf{B} - \frac{4\pi}{c^2} \eta_m \mathbf{J} \right),$$  \hspace{1cm} (4)

where $\rho$ is the density, $\mathbf{v}$ is the velocity, $p$ is the gas pressure, $\psi$ is the gravitational potential of the central black hole, $\mathbf{T}$ is the viscous stress tensor, $\mathbf{B}$ is the magnetic field, $\mathbf{J} = c \mathbf{\nabla} \times \mathbf{B} / 4\pi$ is the current density, and $\eta_m$ is the magnetic diffusivity. In the energy equation, $q_{\text{adv}}$ is the advective cooling rate, $q_+$ is the heating rate, $q_-$ is the radiative cooling rate, and $f$ represents the advection parameter, which measures the fraction of the advection energy stored as entropy. We decomposed the heating rate into two components,

$$q_+ = q_{\text{res}} + q_{\text{vis}},$$  \hspace{1cm} (5)

where $q_{\text{res}}$ and $q_{\text{vis}}$ show heating by dissipation of the magnetic field and viscosity, respectively. Based on numerical simulation of Stone et al. (1999), we assume that the azimuthal component of the viscous tensor $\mathbf{T}$ is the only nonzero component and is described as

$$T_{\varphi \varphi} = \rho \nu \frac{\partial}{\partial r} \left( \frac{v_\varphi}{r} \right),$$  \hspace{1cm} (6)

where $\nu$ is the kinematic viscosity. Heating by dissipation of the magnetic field and viscosity can be written as

$$q_{\text{res}} = \frac{4\pi}{c^2} \eta_m \mathbf{\omega}^2,$$  \hspace{1cm} (7)

$$q_{\text{vis}} = T_{\varphi \varphi} \frac{\partial}{\partial r} \left( \frac{v_\varphi}{r} \right).$$  \hspace{1cm} (8)

In order to avoid the disparity in terms of the turbulent viscosity and magnetic diffusivity, following Lovelace et al. (2009), we adopt the modified $\alpha$ description of viscosity, which in this model is not constant (see also Penna et al. 2013). We also assume that both the viscosity and the magnetic diffusivity are due to the MRI as

$$\nu = \mathcal{P} \eta_m = \alpha \frac{P}{\rho \Omega_k^2} g(\theta).$$  \hspace{1cm} (9)

Here $\mathcal{P}$ is the magnetic Prandtl number, $\alpha$ is the viscosity parameter, $\Omega_k = (GM/c^3)^{1/2}$ is the Keplerian angular velocity of the disk, and $g(\theta)$ is a dimensionless function equal to unity and zero in the body and surface of the disk, respectively (see Lovelace et al. 2009; ZA15; Habibi et al. 2016). For simplicity, here we consider $g(\theta) = \sin \theta$ in order to satisfy the above-mentioned conditions. Following global and local MHD simulations of black hole accretion disks, the magnetic field inside the disk is decomposed into a large-scale field and a fluctuating field. The effects of the fluctuating magnetic field in transferring the angular momentum and dissipating the energy are described through the usual $\alpha$ description (see $\nabla \cdot \mathbf{F}$ in Equation (2) and $q_{\text{vis}}$ in Equation (5)).

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**Note:** The equations and the text provided are from a scientific paper discussing the dynamics and properties of hot accretion flows, particularly focusing on the role of the magnetic field and the advection parameter. The equations represent the conservation of mass, momentum, and energy, as well as the induction equation, which is crucial for understanding the magnetic field transport in these flows. The paper also references previous works to contextualize its own contributions.
As mentioned above, \( B \) in Equations (2) and (4) corresponds to the large-scale component of the magnetic field, and we consider that the toroidal component is the only nonzero component of the magnetic field, \( B = (0, 0, B_\phi) \). To solve the set of Equations (1)–(4), we assume a steady-state \((\partial / \partial \phi = 0)\) and axisymmetric \((\partial / \partial \phi = 0)\) accretion flow. The gravitational potential of the central black hole is described in terms of the Newtonian potential, \( \psi = - (GM) / r \). Following Narayan & Yi (1995a), we assume \( v_\phi = 0 \), which corresponds to a hydrostatic equilibrium in the vertical direction. However, this assumption may not be so appropriate when we want to investigate the effects of outflow on the dynamics of the accretion flow (see, e.g., Jiao & Wu 2011; Mosallanezhad et al. 2014, 2016, for more details). Therefore, the continuity equation, the three components of the momentum equation, and the induction equation are as follows:

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho v_r \right) = 0, \\
\rho \left[ v_r \frac{\partial v_r}{\partial r} - \frac{\rho v_r^2}{r} \right] = - \frac{\rho GM}{r^2} - \frac{\partial p}{\partial r} + \frac{1}{4\pi} \left( J_\phi B_\phi \right), \\
\frac{\rho v_r^2 \cot \theta}{r} = \frac{1}{\rho} \frac{\partial p}{\partial \theta} + \frac{1}{4\pi} \left( J_\phi B_\phi \right), \\
\rho \left[ v_r \frac{\partial \phi}{\partial r} + \frac{v_r v_\phi}{r} \right] = \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho \phi \right), \\
- \frac{\partial}{\partial r} \left( r v_\phi B_\phi \right) + \frac{\partial}{\partial \theta} (\rho \mu) - \frac{\partial}{\partial r} (\rho n_k) = 0,
\]

where the components of current density, \((J)\), are written as

\[
J_r = \frac{1}{r \sin \theta} \frac{\partial}{\partial \theta} (B_\phi \sin \theta), \\
J_\theta = - \frac{1}{r} \frac{\partial}{\partial r} (r B_\phi), \\
J_\phi = 0.
\]

In this paper, we are interested in investigating the variation of the advection parameter, \( f \), with the polar angle in the case of hot accretion flows. Therefore, following Gu et al. (2009), Gu (2015), and ZA15, instead of using the energy equation, we apply the polytropic relation, \( p = K \rho^\gamma \), in the \( \theta \) direction as in our last equation. We also point out that some simulations revealed that the power index \( \Gamma \) is a constant less than unity (see, e.g., Figure 3 of De Villiers et al. 2005). This means that the time-averaged density drops faster than pressure from the equatorial plane to the rotation axis. Based on these results, we set \( \Gamma \) to be less than 1 throughout this paper.

### 3. Self-similar Solutions and Boundary Conditions

#### 3.1. Self-similar Solutions

To better understand the physics of the hot accretion flow incorporating the toroidal component of the magnetic field, in this section we seek self-similar solutions of the aforementioned equations. Self-similar solutions can describe the physical behavior of the accretion flow in an intermediate region of the disk (far away from inner and outer radial boundaries), and we think that self-similar solutions are still good enough to study the variation of the physical variables within the disk.

In plasma physics, parameter \( \beta \) measures the strength of the magnetic field in the plasma. In this part, we seek self-similar solutions along the vertical direction and represent the results in the case of \( \beta(\theta) \). The standard definition of plasma \( \beta \) is expressed as

\[
\beta = \frac{P_{\text{mag}}}{P_{\text{mag}}},
\]

where \( P_{\text{mag}} \) represents the magnetic pressure, and because the toroidal component is the only component of the magnetic field, the total magnetic pressure is then given as

\[
P_{\text{mag}} = \frac{B_\phi^2}{8\pi}.
\]

The radial self-similar solutions can be written as

\[
\rho(r, \theta) = \rho(\theta) r^{-\beta / 2}, \\
v_r(r, \theta) = r \Omega_k (r) \nu_r(\theta), \\
v_\phi(r, \theta) = r \Omega_k (r) \nu_\phi(\theta) \sin \theta, \\
p(r, \theta) = G M p(\theta) r^{-5 / 2},
\]

Substituting the above self-similar assumptions into the equations of the system and using the polytropic relation in the vertical direction, we can rewrite the main equations as

\[
\rho(\theta) \left[ - \frac{1}{2} \nu_\theta (\theta)^2 - \Omega(\theta)^2 \sin^2 \theta \right] = - \rho(\theta) + \frac{3}{2} p(\theta) + \frac{1}{4} b(\theta)^2, \\
\rho(\theta) \Omega(\theta)^2 \sin \theta \cos \theta \frac{d p(\theta)}{d \theta} = b(\theta) \frac{d b(\theta)}{d \theta} + b(\theta)^2 \cot \theta, \\
\rho(\theta) \nu_r(\theta) = - \frac{3}{2} \nu_\phi(\theta) \sin \theta, \\
\frac{d p(\theta)}{d \theta} = K \Gamma \rho(\theta) \Gamma - 1 \frac{d \rho(\theta)}{d \theta}, \\
\frac{d^2 b(\theta)}{d \theta^2} = \left( \frac{9}{8} p - \frac{3}{16} b(\theta) + b(\theta) \csc^2 \theta - \frac{d b(\theta)}{d \theta} \cot \theta \right) + \left( \frac{d b(\theta)}{d \theta} + b(\theta) \cot \theta \right) \left( \frac{d \ln \rho(\theta)}{d \theta} - \frac{d \ln \rho(\theta)}{d \theta} - \cot \theta \right).
\]

Now, we have a set of ordinary differential equations for given values of \( \alpha, K, \Gamma, \beta, \) and \( P \) that should be solved numerically with the boundary conditions, which will be introduced in the next subsection.

#### 3.2. Boundary Conditions

Equations (25)–(29) are differential equations for three variables: \( \rho(\theta), b(\theta), \) and \( db(\theta) / d\theta \). Actually, other variables such as \( p(\theta), \nu_r(\theta), \) and \( \nu_\phi(\theta) \) \( = \Omega(\theta) \sin \theta \) can be determined by our main three variables. In this work, we assume that the accretion flow is evenly symmetric about the midplane, i.e., \( \rho(\pi - \theta) = \rho(\pi - \theta), p(\pi - \theta) = p(\pi - \theta), \) \( \nu_r(\pi - \theta) = \nu_r(\pi - \theta), \) \( \nu_\phi(\pi - \theta) = \nu_\phi(\pi - \theta), \) and \( b(\pi - \theta) = b(\pi - \theta) \). At the midplane of the disk
we have by the symmetry
\[ \frac{\pi}{2} = \theta \quad \frac{d}{d\theta} \frac{dP}{d\theta} = \frac{dB}{d\theta} = 0. \] 

(30)

In essence, to solve Equations (25)–(29), besides the symmetric boundary conditions at \( \theta = 90^\circ \), it is required to apply appropriate boundary conditions at the rotation axis, i.e., \( \theta = 0^\circ \). Indeed, this is a two-point boundary value problem in which the solutions behave properly in the whole \( r - \theta \) space. We have tried to obtain such a solution but failed. This is a caveat in this work. Here, in contrast to our previous work, i.e., Zeraatgari & Abbassi (2015), we shoot from \( \theta = \pi/2 \) toward the axis and simply stop the integration when we meet an unphysical solution. Therefore, we only require the solution satisfying the boundary condition at \( \theta = \pi/2 \). We think that a solution obtained in this way should still be physically meaningful.

As a boundary condition, we set the density to be \( \rho(\pi/2) = 1.0 \) at the equatorial plane throughout this paper. Under the above boundary conditions and symmetries, Equation (29) can be simplified into the following equation:
\[ \frac{d^2b(\pi/2)}{d\theta^2} = \frac{1}{8} \left[ 9P + \frac{13}{2} \right] b(\pi/2). \] 

(31)

It should be noted here that since the second derivation of the toroidal component of the magnetic field has a positive value at the midplane of the disk, this clearly implies that \( b(\pi/2) \) should be minimum there. We adopt the values of \( b(\pi/2) \) with reference to our previous study (see Mosallanezhad et al. 2014, 2016). We do find that by considering any small value for the toroidal component of the magnetic field at the beginning of the integration, at a certain critical value of \( \theta \), denoted as \( \theta_c \), \( v_0^2(\theta_c) \) begins to vanish and becomes zero. Then, at the region of \( \theta < \theta_c \), \( v_0^2(\theta) \) becomes negative and the solution is no longer physical.

The solution in the region of \( \theta_c < \theta < \pi/2 \) is still physical, mainly because, on the one hand, the solution satisfies the equations and the boundary conditions at \( \pi/2 \) and, on the other hand, the values of all physical quantities of the accretion flow at \( \theta \) are also physical. Therefore, we can reasonably treat these values as boundary conditions at \( \theta_c \). Another caveat that needs to be mentioned here is that we consider constant values for \( K \), \( \rho \), and \( b \) at the equator. As a result, the pressure and sound speed are constants there and would not be self-consistently determined as part of the solution.

4. Numerical Results

We have solved Equations (25)–(29) numerically. In all of the solutions \( \alpha = 0.1 \). The results have been shown in Figures 1–7. Figure 1 shows the effect of the magnetic field on angular profiles of physical variables from the equatorial plane (\( \theta = 90^\circ \)) to the rotation axis (\( \theta = 0^\circ \)), for \( K = 0.3, \Gamma = 0.95, b(90) = 10^{-2} \), and \( \rho(90) = 1.0 \). From left to right and top to bottom we plot radial velocity, \( v_r \); azimuthal velocity, \( v_\theta \); sound speed squared, \( c_s^2 \); density, \( \rho \); pressure, \( P \); and the toroidal component of the magnetic field, \( b \). It is clear that the radial and azimuthal components of velocity decrease toward the rotation axis. As we explained in the previous section, at a certain angle, the azimuthal component of velocity is null. Therefore, we stop the integration there and consider this angle as the surface of the disk. Figure 1 also shows sound speed squared, which shows that the disk temperature increases and reaches a value near virial temperature near the disk.
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Figure 2. Angular profiles of azimuthal velocity for (a) different values of magnetic field strength and (b) different Prandtl numbers. Here $\alpha = 0.1$, $\mathcal{P} = 1.0$, $K = 0.3$, $\Gamma = 0.95$, and $b(90) = 10^{-2}$.

Figure 3. Angular profile of density for given values of $\Gamma$. Here $\alpha = 0.1$, $\mathcal{P} = 1.0$, $K = 0.3$, $b(90) = 10^{-2}$, and $\rho(90) = 1.0$.

surface. Moreover, the density has been normalized to unity at the equatorial plane and decreases toward the rotation axis. This clearly shows that the maximum accretion process happens in a regime near the disk midplane, which is in good agreement with results obtained from numerical simulations. Also, at the equatorial plane the pressure is the maximum and decreases toward the rotation axis. Compared to our previous analytical works done in the presence of the magnetic field, including $b(90)$, which is the value of the toroidal component of the magnetic field at the equatorial plane of the disk and also the magnetic Prandtl number, $\mathcal{P}$. Figure 2 represents the angular profile of the azimuthal velocity for different values of the magnetic field strength at the equatorial plane (panel (a)) and different Prandtl numbers (panel (b)). In panel (a), the dotted, dashed, dot-dashed, and solid lines are for $b(90) = 0.01$, 0.05, 0.1, and 0.5, respectively. It is clear that azimuthal velocity decreases with increasing magnetic field strength at the equatorial plane. For the minimum value of $b(90)$, i.e., $b(90) = 0.01$, the disk surface is around $\sim5^\circ$, which is very close to the rotation axis, while for the higher value of the magnetic field strength, $b(90) = 0.5$, azimuthal velocity becomes null near $\sim53^\circ$. In panel (b), the dotted, dashed, and solid lines are for $\mathcal{P} = 0.5$, 1.0, and 5.0, respectively. It should be noted here that based on Equation (9), since we fix $\alpha = 0.1$ throughout this paper, those values for the Prandtl number correspond to $\eta_m = 0.05$, 0.1, 0.5, respectively. It can be seen clearly that increasing the Prandtl number does reduce the azimuthal velocity. For $\mathcal{P} = 0.5$, the disk surface is around $\sim10^\circ$, and for $\mathcal{P} = 5.0$, the disk surface is approximately $\sim30^\circ$. Therefore, we conclude that in the case of the no-outflow solution, the toroidal component of the magnetic field can decrease the disk surface, and in comparison with nonmagnetized hot accretion disks, magnetized disks might be thinner.

Figure 3 shows the angular profile of density for three values of $\Gamma$. As we explained in previous sections, we set the value of $\Gamma$ to be less than unity. This figure shows the variation of mass density for different values of $\Gamma$, so we can check how this index can affect the density profile. The dotted, dashed, and solid lines are for $\Gamma = 0.55$, 0.75, and 0.95, respectively. The mass density from the equatorial plane toward the rotation axis increases with $\Gamma$ around $\theta = 40^\circ$. As you can see, for $\theta \lesssim 40^\circ$, the density becomes inverse, so it has a decreasing trend. This result clearly shows that the density rapidly decreases for higher values of $\Gamma$ from the equatorial plane toward the disk...
surface. Therefore, hereafter we set $\Gamma$ to be nearly unity for two main reasons. First, numerical simulations of hot accretion flows show that the mass density rapidly decreases near the disk surface (see, e.g., De Villiers et al. 2005; Yuan & Bu 2010), and second, we want to compare these results with our previous HD case (ZA15).

Figure 4 shows angular profiles of magnetic field variables, including $b$, the toroidal component of the magnetic field, $db/d\theta$, the first derivation of the magnetic field, $p_m$, magnetic pressure, and $\beta(\theta)$, plasma beta. All variables are plotted for four values of magnetic field strength at the equatorial plane. Dotted, dashed, dot-dashed, and solid lines are for $b(90) = 0.01, 0.05, 0.1, \text{ and } 0.5$, respectively. Clearly, with increasing strength of the magnetic field at the equatorial plane, $b$ increases all over the flow. Also, as is shown in the top right panel, $db/d\theta$ becomes increasingly negative toward the disk surface and is null at the disk midplane. In the bottom left panel magnetic pressure, $p_m$, has an increasing trend from $\theta = 90^\circ$ toward the disk surface. This is mainly because the dimensionless magnetic pressure is proportional to $b^2$ and an increase in the magnetic field strength at the equatorial plane will cause the magnetic pressure increases. In the bottom right panel, it is clear that plasma beta, $\beta(\theta) = p_{gas}/p_{mag}$, the ratio of gas pressure to magnetic pressure, decreases from the equatorial plane toward the rotation axis, while magnetic pressure, the denominator, increases. In addition, when the magnetic field strength at the equatorial plane increases, $\beta$ decreases.

Figure 5 shows angular profiles of magnetic field variables for given values of the Prandtl number, including $P = 0.5, 0.1$, and $5$. As you can see, as the Prandtl number increases, $b$, $db/d\theta$, and $p_m$ increase, while $\beta$ decreases, the same trend as we discussed in Figure 4. These results are also in good agreement with those obtained from numerical simulations and analytical works (Yuan & Bu 2010; Yuan et al. 2012b; Mosallanezhad et al. 2014).

As we described in the previous section, we applied the polytropic equation of state in the vertical direction instead of the energy equation to answer to the question of how the advection parameter, $f$, varies in the vertical direction. Moreover, since our numerical results are in good agreement with previous works done with a fixed advection parameter ($f = 1.0$), such as Narayan & Yi (1995a), Mosallanezhad et al. (2014, 2016), and Samadi & Abbassi (2016), we conclude that our assumptions and results are reliable. Now, we investigate the variation of the advection parameter with magnetic field parameters. To do so, first we plot the heating/cooling terms along the vertical direction before integrating the heating rates and advective cooling rate in the energy equation with angle $\theta$. The top left and top right panels of Figure 6 show angular profiles of the viscous dissipation heating rate $q_{vis}$ and the magnetic field dissipation heating rate $q_{res}$, respectively. In
the top left panel, viscous dissipation decreases from the equatorial plane of the disk toward the rotation axis. Also, as \( b(90) \) increases, viscous dissipation decreases. In contrast, the angular profile of the magnetic dissipation heating rate increases from the equatorial plane toward the disk surface. Moreover, magnetic field dissipation heating increases with increasing magnetic field strength at the equatorial plane. The reason for these behavior can be easily understood from Equations (7) and (8), where \( q_{\text{vis}} \propto v_0 \) and \( q_{\text{res}} \propto b^2 \) (see Figure 1). The bottom left panel of Figure 6 is the angular profile of advective cooling, \( q_{\text{adv}} \), which decreases from the equatorial plane toward the rotation axis. Also, advective cooling decreases with an increase in the value of magnetic field strength at the equatorial plane of the disk. In addition, with increasing magnetic field strength at the equatorial plane of the disk, the advection parameter will be increased. As can be seen, the solution with the largest values of the magnetic field at the equatorial plane, i.e., \( b(\pi/2) > 0.4 \), is likely unphysical. This is mainly because at some \( \theta \) angles the advection parameter will become greater than unity and it would rule out the solutions with this property.

The vertical integration of the viscous dissipation heating, magnetic field dissipation heating, and advective cooling is as follows:

\[
Q_{\text{vis}} = 2 \int_{\theta_0}^{\pi/2} q_{\text{vis}} r \sin \theta \, d\theta, 
\]

\[
Q_{\text{res}} = 2 \int_{\theta_0}^{\pi/2} q_{\text{res}} r \sin \theta \, d\theta, 
\]

\[
Q_{\text{adv}} = 2 \int_{\theta_0}^{\pi/2} q_{\text{adv}} r \sin \theta \, d\theta. 
\]

Then, the energy advection parameter can be defined as

\[
f_{\text{adv}} = \frac{Q_{\text{adv}}}{Q_+}, 
\]

where \( Q_+ = Q_{\text{vis}} + Q_{\text{res}} \). Figure 7 shows the variation of the energy advection parameter, \( f_{\text{adv}} \), versus the magnetic field strength at the equatorial plane for different values of \( \Gamma \). The dotted, dot-dashed, and solid lines are for \( \Gamma = 0.75, 0.85, \) and \( 0.95 \), respectively. Here \( \alpha = 0.1, \ P = 1.0, \ K = 0.33, \) and \( \rho(90) = 1.0 \). It is seen that the energy advection parameter decreases as \( \Gamma \) increases. Moreover, the energy advection parameter becomes unity for \( 0.4 < b(90) \leq 0.5 \). This result illustrates that advective cooling can balance the total dissipation.
heating rate. Consequently, our results also show that in terms of the no-outflow solution incorporating the toroidal magnetic field, thermal equilibrium still exists for both the strong magnetic field at the equatorial plane ($b(90) \sim 0.5$) and higher values of the $\Gamma$ index.

5. Summary and Conclusions

To summarize, we have solved MHD equations of optically thin, geometrically thick black hole accretion flows in spherical coordinates ($r, \theta, \phi$). The gravitational potential of the central black hole was assumed to be Newtonian. The cooling in the energy equation was considered advective cooling, and the heating rate was decomposed into two components, magnetic field and viscosity dissipations. We assumed that both the viscosity and the magnetic diffusivity are due to MRI and adopted the modified description of viscosity. The azimuthal component of the viscous stress tensor, $T_{\phi\phi}$, was considered as the only nonzero component. We only considered the toroidal component of the magnetic field, $B_\theta$. We did not consider wind/outflow in the flow, so $v_\phi = 0$. Instead of using the energy equation, we applied the polytropic relation in the vertical direction. Following some simulations revealing that the power index $\Gamma$ is a constant less than unity, we set $\Gamma$ to be less than 1 throughout this paper. We used self-similar solutions to solve the MHD equations because it is good for studying the main body of the flow far from the inner and outer radial boundaries.

We could not solve a two-point boundary value problem; therefore, we shifted to the singular boundary value problem and believe that our solution is still physically meaningful. We found that there is a surface for the flow at some angle $\theta_s$ where
the azimuthal velocity, \(v_\phi\), is zero. This result is satisfied by any small value of the toroidal component of the magnetic field at the beginning of the integration.

Our numerical results show that the radial and azimuthal components of velocity decrease toward the rotation axis while the sound speed squared increases. Moreover, other physical variables, such as mass density and gas pressure, drop rapidly toward the disk surface. However, the magnetic pressure has a minimum at the equatorial plane and reaches its maximum value around the disk surface. Consequently, the plasma beta, \(b = \frac{\rho}{\mu B^2}\), decreases toward the rotation axis and approximately reaches unity at the disk surface. In spite of the simplicity of our model in viscosity, magnetic field, and the disk itself, we think that the presented analytical results give us a better understanding of such a complicated system incorporating the magnetic field. It is good to note here that our numerical results are in good agreement with some previous simulations and analytical studies (see Yuan et al. 2012a, 2015; Mosallanezhad et al. 2014, 2016; Samadi & Abbassi 2016).

We found that the viscous dissipation heating decreases from the equatorial plane of the disk toward the rotation axis and decreases for higher amounts of magnetic field strength at the equatorial plane of the disk. In contrast, the angular profile of magnetic dissipation heating increases from the equatorial plane toward the disk surface.

Our main conclusion was that in terms of the no-outflow solution incorporating the toroidal magnetic field, thermal equilibrium still exists for both the strong magnetic field at the equatorial plane of the disk and higher values of the \(I\) index.

As we mentioned in Section 1, the magnetic field can transfer angular momentum by MRI, which is related to the wind production. Hence, in our future work we will study the effect of the magnetic field on the dynamics and the structure of the hot accretion flow extracting the wind from the system. For better understanding of the system, we will also include all components of the velocity and magnetic field in MHD equations to check how the global magnetic field affects the dynamics of hot accretion flows such as radial and angular velocity.

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