Waveform Design for MIMO Radar based on improved genetic algorithm

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Abstract. Multiple-input-multiple-output (MIMO) radar can transmit and receive different waveforms at the same time, and its waveform design has great freedom. Under the constraints of constant modulus, similarity, integrated sidelobe level (ISL) and peak sidelobe level (PSL), it is a complex nonlinear constraint problem to suppress clutter so as to maximize the signal-to-interference-plus-noise ratio (SINR) in the optimal design of matched output waveform. In this paper, an improved genetic algorithm (IGA) with strong global and local searching ability is obtained by using chaotic map discretization to obtain initialized population, simulated annealing selection operator and adaptive crossover and mutation operator. Simulation examples demonstrate that the proposed algorithm has the advantage of lower computational complexity compared with the existing cyclic algorithms (CA) and primal dual type algorithm (PDT) for MIMO radar waveform design. The optimal waveform obtained has better performance.

1. Introduction

In recent years, MIMO radar has been paid more and more attention by scholars because of its unique advantages, and has been widely used in various aspects. Compared with traditional single-input single-output radars and Phased array radars, MIMO radar can transmit multiple waveforms simultaneously and use multiple antennas to receive echo signals. It has better parameter identification, waveform design with greater freedom and signal processing capability [1, 2]. In practical application, MIMO waveform design needs to follow many principles, constant modulus constraint, similarity constraint, peak-to-average ratio (PAR) constraint [3-5], etc. Constant modulus constraints and PAR constraints are necessary because the radar transmitter amplifier electronic system needs to reach saturation conditions, so the signal voltage characteristics must be stabilized. In addition, the waveform with good similarity can make the designed waveform with good ambiguity pattern, that is, with higher detection performance. In order to solve this unimodular quadratic convex optimization problem, literature [4, 6] used FFT, IFFT and singular value decomposition to cyclically optimize the phase coding sequence, and proposed algorithms such as CA (Cyclic Algorithmic), CAN (CA-new), WE-CAN (Weight-CAN). The length of phase coding obtained by this algorithm is limited, and the computational complexity is very large for long coding sequences. Literature [3] proposed a primal dual type algorithm (PDT) to solve the problem of waveform design by constructing augmented Lagrange function. Compared with CAN algorithm (using matched filtering), the cross-correlation sidelobe peak value of the designed waveform is reduced by about 4-5dB, but the calculation time is greatly prolonged. The IGA is proposed to optimize the MIMO radar waveform design by intelligent search method. The computation time is greatly reduced and the waveform obtained has good correlation characteristics.
2. MIMO radar waveform design principle

2.1. Signal model

Supposing a single-station MIMO radar has $N_T$ transmitting antennas and $N_R$ receiving antennas, as shown in Fig. 1.

![target](image)

**Figure 1. Schematic diagram of single-station MIMO radar**

Let $s_n(t)$ represent the transmitting waveform of the $n$th antenna. Assuming that the transmitting period of the waveform is $T_r$ and the number of transmitting pulses is $M$, then the transmitting waveform of the $n$th antenna can be expressed as:

$$\tilde{s}_n(t) = \sum_{m=1}^{M} s_n(t - mT_r) \quad (1)$$

Let $y$ represent the received signal, then it can be expressed as:

$$y = y_t + y_c + N \quad (2)$$

Where $y_t$ represents the target signal; $y_c$ represents clutter signal; $N$ stands for noise signal. The following is to analyze each signal model in turn.

2.1.1. The target signal. Assuming that the carrier frequency of the waveform signal is $f_0$, the azimuth angle of the target is $\theta_0$, and the distance is $r_0$, then the echo after detecting the target is:

$$y_t = \alpha_0 \mathbf{a}_r(\theta_0) \mathbf{a}_t(\theta_0)^T \tilde{s}_n(-\tau_t) e^{j2\pi(f_0 + f_d)(t - \tau_t)} \quad (3)$$

where $\alpha_0$ is the complex amplitude related to the target Radar Cross Section (RCS), $E(|\alpha_0|^2 = \sigma_0^2)$; $\tau_t$ is the delay time of bidirectional transmission; $f_d = \frac{2\nu_t}{\lambda}$ is the Doppler frequency shift caused by the relative motion of the target and the radar, $\lambda$ is the wavelength of the emission waveform; $\mathbf{a}_r(\theta_0) \in \mathbb{C}^{N_r \times 1}$ and $\mathbf{a}_t(\theta_0) \in \mathbb{C}^{N_t \times 1}$ represent the azimuth angle $\theta_0$ transmit and receive direction vector, respectively. For the convenience of description, both transmitting and receiving antennas are assumed to be uniform linear arrays (ULAS) with a distance of half wavelength, then the steering vector can be expressed as:

$$\mathbf{a}_t(\theta_0) = \frac{1}{\sqrt{N_T}} [1, e^{-j\pi 1 \sin \theta}, \ldots, e^{-j\pi (N_T - 1) \sin \theta}]^T \quad (4)$$

$$\mathbf{a}_r(\theta_0) = \frac{1}{\sqrt{N_R}} [1, e^{-j\pi 1 \sin \theta}, \ldots, e^{-j\pi (N_R - 1) \sin \theta}]^T \quad (5)$$
When the signal is converted by down conversion, and the fixed phase term is simplified to the amplitude coefficient, the Doppler frequency shift in the pulse is ignored, the signal can be simplified as:

\[ y_1 = y_0 e^{-j \omega_0 f_0} = \alpha_0 e^{j 2 \pi f d} a_r(\theta_0) a_t(\theta_0)^T S(n) \]  

(6)

where \( S \) = \([s_1, ..., s_{N_T}] \in \mathbb{C}^{N_T \times L} \) is the emission waveform matrix, and \( L \) is the phase encoding length.

2.1.2. Clutter signal and noise signal. In radar system, echo inevitably includes ground clutter, sea clutter, etc., which is caused by echo reflected from objects around the target. Assuming that an airborne MIMO radar is detecting, and its clutter is distributed at a specific distance and angle, as shown in Fig. 2.

![Figure 2. Schematic diagram of MIMO radar detection](image)

Because MIMO radar has the characteristic of simultaneously transmitting signals, the received signals not only have the interference between the echoes of different waveforms from the same detection unit, but also the interference between the echoes generated by adjacent detection units. This is shown in Fig. 3.

![Figure 3. Diagram of received signal](image)

Suppose the position of the clutter is \( (r_k, \theta_k) \), \( r_k \in \{0, 1, ..., N\} \), \( \theta_k \in \{0, 1, ..., D\} \), where \( D \) represents the number of sector partitions in the probe area, Then the clutter signal can be expressed as:

\[ y_c = \sum_{k=1}^{K} \alpha_k e^{j 2 \pi f d} a_r(\theta_k) a_t(\theta_k)^T S(n - r_k) \]  

(7)

According to the above analysis, the received signal can be expressed as:
\[ y = y_t + y_c + N = \alpha_0 A(r_0, \theta_0)s + \sum_{k=1}^{K} \alpha_k A(r_k, \theta_k)s + N \]  

where,

\[ A(r_k, \theta_k) = [\text{Diag}(p(f_d))] \otimes (a_r(\theta_k)a_t(\theta_k)^T)] \]

\[ J_{rk}(n, l) = \begin{cases} 
1, & n - l = N_T \times r_k \\
0, & n - l \neq N_T \times r_k 
\end{cases} 
(n, l) \in \{1, \ldots, N_T L\}^2 \]

The noise signal $N$ is a Gaussian distribution with a mean of 0 and a variance of $\sigma_N^2$. The Doppler frequency and interference in the detection region are assumed to vary slowly or remain substantially unchanged.

### 2.2. The objective function

The purpose of waveform design is to suppress noise and interference and achieve better detection effect. Therefore, we need to maximize the signal-to-interference-plus-noise ratio (SINR) under the waveform design criteria of reducing integrated sidelobe level ($\text{ISL}_C$) and peak sidelobe level ($\text{PSL}_C$) of the cross-correlation function. Let $\mathbf{w} = [\omega_1, \ldots, \omega_{N_T L}]^T \in \mathbb{C}^{N_T L \times 1}$ represent the filter at the receiving end, and the output signal of the filter is,

\[ y_{out} = \mathbf{w}^H y = \alpha_s \mathbf{w}^H A(r_0, \theta_0)s + \mathbf{w}^H \sum_{k=1}^{K} \alpha_k A(r_k, \theta_k)s + \mathbf{w}^H N \]  

The SINR can be expressed as:

\[ \text{SINR} = \frac{\sigma_0^2 |\mathbf{w}^H A(r_0, \theta_0)s|^2}{\mathbf{w}^H (\sum_{k=1}^{K} \sigma_k^2 A(r_k, \theta_k)s) s^H A^H(r_k, \theta_k) \mathbf{w} + \sigma_N^2 \mathbf{w}^H \mathbf{w}} \]  

The $\text{ISL}_C$ and $\text{PSL}_C$ can be expressed as:

\[ \text{ISL}_C = \sum_{i=N_T+1}^{N_T L-1} |r_c(i)|^2 \]  

\[ \text{PSL}_C = \max \{ |r_c(k)|^2 \}_{k=N_T+1}^{N_T L-1} \]  

According to the above analysis, we can obtain the MIMO radar waveform design objective function of maximizing SINR under the constraints of constant mode, reducing $\text{ISL}_C$ and $\text{PSL}_C$:

\[ \begin{array}{l}
\text{max} \quad \text{SINR} \\
\text{s. t.} \quad ||s||^2 = 1, \min \quad \text{ISL}_C \quad \text{&} \quad \text{PSL}_C
\end{array} \]  

### 2.3. Problem analysis

At the receiving end, a matched filter is used to separate the signals, let $\mathbf{w} = s^*$. Substituting it into Eq. (17):

\[ \text{max} \quad \text{SINR} = \frac{\sigma_0^2 |s^H A(r_0, \theta_0)s|^2}{s^H (\sum_{k=1}^{K} \sigma_k^2 A(r_k, \theta_k)s) s^H A^H(r_k, \theta_k)s^* + \sigma_N^2 s^H s^*} \Rightarrow \text{max} \quad \text{PSL}_d \quad \text{&} \quad \min \quad \text{ISL}_d \]
where, $ISL_a$ and $PSL_a$ represent the side lobe peak value and integral side lobe of the autocorrelation function, respectively. The expressions are as follows:

\[ ISL_a = \sum_{n=0}^{N_T} |r_a(n)|^2, \]  
\[ PSL_a = \max\{|r_a(n)|^2\} \]  
\[ r_a(n) = s_n s_n^* \quad n \in \{1, \ldots, N_T\} \]  

Then the objective function can be expressed as:

\[ \text{max } PSL_a \quad \text{&} \quad \text{min } ISL_a \]

\[ \text{s.t. } ||s||^2 = 1, \text{min } ISL_c \quad \text{&} \quad PSL_c \]  

3. Waveform design of MIMO radar based on IGA

3.1. Basic GA

GA — first introduced by John Holland in the early seventies and based on Darwin evolution and Mendel theory — is a global random search method based on the evolution of survival of the fittest, natural selection, and population genetic evolution in nature; it is supported by a large amount of experimental data in biology, suitable for any explicit and non-explicit optimization problem\cite{7, 8}. It demonstrates strong self-organization and adaptive ability, strong robustness, simple basic logic, parallel computing, and is easy to realize. It is widely used in the fields of automatic control, pattern recognition, engineering design, and intelligent fault diagnosis to solve complex nonlinear and multidimensional optimization problems\cite{9-11}. The specific process is described in Section 3.2.

3.2. IGA

3.2.1. Initial Population Generation. In the basic GA, the initial population is generated randomly, resulting in countless possibilities of subsequent convergence, which can easily fall into local optima. In particular, when the phase coding sequence is long, it is difficult to guarantee the direction and efficiency of evolution. To reduce the evolution time of the GA, we propose using chaotic sequence mapping to generate the initial population. Chaos is a deterministic and stochastic dynamic phenomenon with initial sensitivity and white noise. We use the commonly used quadratic sequence and its mapping equation can be expressed as:

\[ x_{k+1} = \mu - 4x^2(k), k = 0, 1, \ldots, n. \]  

where $k$ is the iteration time step and $\mu$ is the system parameters. When $\mu \in [0.351, 0.5]$, the sequence is unordered and unpredictable. According to the Perron–Frobenius equation, the probability density function of the chaotic sequence is

\[ p(x) = \sum_{x_{i}=f^{-1}(x)} \frac{p(x_i)}{|f'(x_i)|} \]  

According to the stochastic process theory, the formula for calculating the autocorrelation function of a chaotic sequence is as shown in (24), where $x(n)$ is chaotic sequence.

\[ R_s(k) = E[x(n)x(n + k)] = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x(n)x(n + k)p(x(n), x(n + k))dx(n)dx(n + k) \]  

It can be inferred that $R_s(k) = 0$, when $k \neq 0$. The autocorrelation function of quadratic sequence can be represented as:
It can be seen that the chaotic sequence exhibits a good uncorrelated property; when the continuous-discrete sequence is transformed into a four-phase coded sequence (as in (25)), this property is destroyed. Moreover, the autocorrelation characteristics of the phase-coding sequences obtained need to be further enhanced.

\[ R_x(k) = \begin{cases} \sigma^2, & k = 0 \\ 0, & k \neq 0 \end{cases} \]  

(24)

3.2.2. Fitness value function. According to the analysis in Section 2.2, it can be concluded that the objective function is actually composed of four sub-objective functions; this is a multi-objective problem and we need to balance it according to actual needs. The main idea is that according to the importance of sub-objective function, it can be weighted and integrated into one objective function, so that each evolution can be optimized at the same time, which greatly reduces the solving time of the algorithm. According to the specific tasks of the radar, such as focusing on the detection range accuracy, Doppler sensitivity and clutter suppression, the weighted value can be changed adaptively through simple machine learning. Thus, the objective function can be expressed as:

\[ OF = W \cdot F = w_1 \cdot PSL_a + w_2 \cdot ISL_a + w_3 \cdot PSL_c + w_4 \cdot ISL_c \]  

(26)

3.2.3. Selection Operators. In the GA, the selection operator uses a roulette to select the parent. The selection probability of individuals is obtained by the ratio of their own fitness to all individuals in the population. The optimal gene is easily destroyed by the randomness of heredity and the convergence speed is slow, especially when the local search ability is weak. To overcome these shortcomings, optimal individual preservation and simulated annealing selection are combined to sort the parent. In this paper, a simulated annealing selection operator is proposed, which combines the local searching ability of the simulated annealing algorithm (GA), and makes the algorithm have stronger "climbing" ability and faster convergence speed. The simulated annealing algorithm uses a simulated solid annealing process in thermodynamics. The cooling process of the solid follows the Metropolis criterion, which can be expressed as:

\[ P = \begin{cases} 1 & E_{x_{\text{new}}} < E_{x_{\text{old}}} \\ e^{-\frac{E_{x_{\text{new}}}-E_{x_{\text{old}}}}{kT}} & E_{x_{\text{new}}} \geq E_{x_{\text{old}}} \end{cases} \]  

(27)

where \( E \) is the internal energy of the object at T temperature, \( E_{x_{\text{old}}} \) is the current state, \( E_{x_{\text{new}}} \) is the new state, and \( K \) is the Boltzmann constant.

The procedure for the simulated annealing selecting operator is as follows:

Step 1: Initialization. The initial temperature is \( N \) progeny, with higher fitness in each generation. The length of the Markov chain is set as \( L_m \) and the cooling function is set as the fitness function.

Step 2: Add a random perturbation and obtain an adjacent feasible solution.

Step 3: Use the Metropolis criterion to determine whether to accept the new solution.

Step 4: Repeat steps 2 and 3 \( L_m \) times to obtain the optimal solution under the Markov process with chain length \( L \) and the optimal solution will act as the parent.

Because the objective function has been fixed, the length of the Markov chain affects the convergence speed and stability of the algorithm; therefore, the selection should be made according to the length and the number of phases of the coded sequences.

3.2.4. Fuzzy adaptive crossover and mutation operator. The probability of crossover and mutation has an important influence on the outcome of evolution. If the probability is too large, the excellent individual will be destroyed; if the probability is small, the ability to jump out of the local optimal will be reduced, which will reduce the evolution speed. A fuzzy adaptive operator can be used to select the
crossover probability \( P_c \) and mutation probability \( P_m \) according to the difference between the population and individual, which not only maintains the diversity of the population but also the convergence of the algorithm. The population differences are expressed as follows:

\[
E_1 = \frac{f_{\text{max}} - f_{\text{avg}}}{f_{\text{max}}}
\]

where \( f_{\text{max}} \) is the maximum adaptation value of the current population and \( f_{\text{avg}} \) is the average adaptation value of the current population. Large \( E_1 \) and \( E_2 \) indicate that the population difference is large, and vice versa. The individual differences are expressed as follows:

\[
E_2 = \frac{f_{\text{cur}} - f_{\text{avg}}}{f_{\text{max}}}
\]

where \( f_{\text{cur}} \) is the current individual adaptation value. Through simple training, classification and recognition learning can be carried out to achieve the effect of adaptively changing the probability of crossover and mutation. In the following simulation experiments, the crossover operation is selected as a two-point crossover and the mutation operation is selected as a random generation \([0, 3]\), which constitutes a new parent.

4. Simulation

Assuming that the number of transmitting antennas and receiving antennas are \( N_T = N_R = 4 \), The coding length is chromosome length \( L = 1024 \), GA and IGA were used to optimize the waveform. The initial population of GA is generated randomly, the crossover probability \( P_c \) is 0.95 and the mutation probability \( P_m \) is 0.05; all are fixed. The selection operator utilized roulette selection. And IGA Use the chaos principle to generate the initial population, simulated annealing selection operators, and fuzzy adaptive crossover and mutation operators, the length of the Markov chain is 15. The simulation results are presented in Fig.4.

![Figure 4. Comparison of IGA and basic GA](image)

It can be seen from Fig.4 that the average fitness value and the optimal fitness value of the initial population of IGA were 500 lower than that of GA in the initial state due to the chaotic sequence discretization. GA fell into the local optimal trap from about the 10th generation. Although it jumped out of the "trap" in the 140th generation, it immediately fell into a new local optimal, and its local searching ability was weak. In the whole process of evolution, IGA has been constantly evolving forward with strong "climbing" ability. Finally, the optimal individual was obtained in about 80 generations, and the fitness of the optimal individual was 5000 less than that of GA. It shows that the IGA has a strong global search ability, and the local search ability has been greatly improved.
Assuming that the target is distributed in $K \in (0 \sim 20)$, which satisfies $K \leq \frac{N-N_T}{N_T}$, it is necessary to inhibit the signal output of the distance unit $K \in [-K - 1] \cup [1 K]$. The above optimized waveform is used for simulation experiment, and the results are shown in Fig. 5:

![Figure 5. Range sidelobe suppression amplitude](image)

The variation trend of the restraining effect of distance sidelobe is consistent with the variation trend of the fitness of the most individual in IGA. In the range where the target is located, the range sidelobe is basically suppressed at about -30dB, which has good detection performance.

![Figure 6. The autocorrelation function of optimized waveform obtained by IGA](image)
It can be seen from Fig. 6 and Fig. 7 that, compared with the CAN algorithm in literature\(^{[12]}\), the peak of the side lobe of the optimized waveform autocorrelation function is reduced by 2-3 dB, and the peak of the cross-correlation side lobe is reduced by 5-6 dB, showing better orthogonality and autocorrelation, that is, better clutter suppression and detection effect. It can be seen from Table 1 that, compared with the algorithm in literature\(^{[3]}\), the autocorrelation side lobe is increased by about 1-2 dB and the cross-correlation side lobe is reduced by about 1 dB, but the computational complexity of the algorithm is significantly reduced. For multi-waveform optimization design, the algorithm still has strong search ability and considerable search time.

5. Conclusion
This paper analyzes the goal and principle of MIMO radar waveform design, and proposes an improved GA which uses chaotic mapping discretization to get the initial population, simulated annealing selection operator and fuzzy adaptive crossover and mutation operator. By comparing the results of the basic GA and the existing optimization algorithm, the algorithm has lower computational complexity and the optimal waveform has better performance. The proposed algorithm reduces the complexity of

| Waveform size | Autocorrelation sidelobe peak (dB) | Cross correlation sidelobe peak (dB) | Computation time |
|---------------|-----------------------------------|-------------------------------------|-----------------|
| \(N_T = N_R\) | \(L\) IGA CAN PDT IGA CAN PDT IGA CAN PDT | | |
| 4             | 1024 -25.43 -22.78 -27.76 -28.55 -23.18 -27.76 60s 50s 25min |
| 10            | 1024 -26.60 -21.70 -26.89 -27.72 -21.17 -26.89 72s —— —— |
| 128           | 1024 -21.82 -19.81 -21.98 -22.13 -17.49 -21.98 182s —— —— |

Table 1 The comparison of IGA with CAN and PDT
waveform design and makes the joint design problem more efficient. The adaptive method adopted in this algorithm is simple machine learning, and the next step is to collect a large amount of training data for training learning according to the actual application, so that the algorithm has a better convergence rate and the performance is further improved.

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References
[1] YU X, ALHUJAILI K, CUI G, et al. MIMO Radar Waveform Design in the Presence of Multiple Targets and Practical Constraints [J]. IEEE Transactions on Signal Processing, 2020, PP(99): 1-1.
[2] TANG B, TUCK J, STOICA P. Polyphase Waveform Design for MIMO Radar Space Time Adaptive Processing [J]. IEEE Transactions on Signal Processing, 2020, PP(99): 1-1.
[3] LIN Z, PU W, Luo Z-Q. Minimax Design of Constant Modulus MIMO Waveforms for Active Sensing [J]. IEEE Signal Processing Letters, 2019, 26(10): 1531-5.
[4] CHENG Z, HE Z, LIAO B, et al. MIMO Radar Waveform Design With PAPR and Similarity Constraints [J]. IEEE Transactions on Signal Processing, 2018, 66(4): 968-81.
[5] CUI G, LI H, RANGASWAMY M. MIMO Radar Waveform Design With Constant Modulus and Similarity Constraints [J]. IEEE Transactions on Signal Processing, 2014, 62(2): 343-53.
[6] SOLTANALIAN M, TANG B, LI J, et al. Joint Design of the Receive Filter and Transmit Sequence for Active Sensing [J]. IEEE Signal Processing Letters, 2013, 20(5): 423-6.
[7] CHOI K, JANG D H, KANG S I, et al. Hybrid Algorithm Combining Genetic Algorithm With Evolution Strategy for Antenna Design [J]. IEEE Transactions on Magnetics, 2015.
[8] B., V., HA, et al. Modified Compact Genetic Algorithm for Thinned Array Synthesis [J]. IEEE Antennas & Wireless Propagation Letters, 2015.
[9] CHENG Y F, SHAO W, ZHANG S J, et al. An Improved Multi-Objective Genetic Algorithm for Large Planar Array Thinning [J]. IEEE Transactions on Magnetics, 2016, 52(3): 1-4.
[10] GAO J, DAI L, ZHANG W. Improved Genetic Optimization Algorithm with Subdomain Model for Multi-objective Optimal Design of SPMSM [J]. CES Transactions on Electrical Machines and Systems, 2018, 2(1): 160-5.
[11] YONGJIN L, XIHONG C, YU Z. Joint synchronization estimation based on genetic algorithm for OFDM/OQAM systems [J]. Journal of Systems Engineering and Electronics, 2020, 31(4): 657-65.
[12] HAO H, STOICA P, JIAN L. Designing Unimodular Sequence Sets With Good Correlations—Including an Application to MIMO Radar [J]. IEEE Transactions on Signal Processing, 2009, 57(11): 4391-405.