Muon anomalous magnetic moment
and the heavy photon in a little Higgs model

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Abstract

In the Littlest Higgs model, we comprehensively study the phenomenology of the heavy photon $A_H$ which is lightest, in most of the parameter space, among newly introduced heavy gauge bosons and top-like vector quark. Unexpected behavior is that lighter $A_H$ suppresses the corrections to the electroweak precision observables. For the global symmetry breaking scale $f \lesssim 3$ TeV, the heavy photon can be light enough to be produced at the 500 GeV linear collider. Through the calculation of the one-loop correction to the muon anomalous magnetic moment, we show that even the light $A_H$ with mass around 200 GeV results in the negligible contribution. This is consistent with the current inconclusive status of the theoretical calculation of the $(g-2)_\mu$ in the SM. The effects of the Littlest Higgs model on the process $e^+e^- \to \mu^+\mu^-$ are also studied, which is one of the most efficient signals to probe the $A_H$.

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I. INTRODUCTION

The standard model (SM) of particle physics has provided an excellent effective field theory of high energy phenomena up to the energies of order 100 GeV. A direct and important question is what is the cutoff scale of this effective description? The Higgs mass of the SM may have a key because of its quadratic sensitivity to heavy physics. The naturalness argument suggests that the cutoff scale of the SM cannot be too higher than the electroweak scale: New physics will appear around TeV energies. A weak scale supersymmetry (SUSY) model is one of the best motivated candidates for new physics as the cutoff scale is naturally replaced by the soft SUSY breaking scale. Even though various supersymmetry models have been thoroughly studied by enormous number of authors, no piece of experimental data has been discovered. Brane world scenarios with large or warped extra dimensions have been suggested to understand the hierarchy problem as a geometrical stabilization problem. However those theories are not weakly coupled at TeV scale.

Recently, new models, dubbed the “little Higgs” models, have drawn a lot of interest, which remain weakly coupled at TeV sale with the one-loop stabilized Higgs potential. The original idea dates back to 1970’s, such that the lightness of the Higgs boson is attributed to its being a pseudo Goldstone boson [1]. The problem was then the remaining quadratic divergence of the radiative correction to the Higgs mass. A new ingredient, the collective symmetry breaking, was discovered via dimensional deconstruction [2, 3]. It ensures that the Higgs mass is radiatively generated at two loops [29]. Phenomenologically the quadratic divergences due to the SM gauge bosons (top quark) are cancelled by those due to new heavy gauge bosons (fermions): The cancellation occurs at one-loop level between particles with the same statistics, unlike the cancellations in supersymmetric theories; it is due to the exactly opposite coupling of the new particles compared to the SM particles. The heavy gauge bosons come from the extended gauge sector which fixes the new gauge coupling structures. In addition, the requirement of cancellation of quadratic divergence fully determines the Yukawa sector of the top quark. Later the idea has been realized in other simple nonlinear sigma models [7, 8, 9, 10, 11, 12, 13, 30].

In this paper, we concentrate on “the Littlest Higgs” model, described by the global symmetry breaking pattern $SU(5)/SO(5)$ [8]. As one of the simplest realizations of the little Higgs idea, it is the smallest extension of the SM to date which stabilizes the electroweak
scale and remains weakly coupled at TeV scale. The model predicts the presence of new heavy gauge bosons ($W_H, Z_H$ and $A_H$) and a new heavy top-like vector quark $T$ and their couplings. The minimality of the Littlest Higgs model would leave characteristic signatures at the present and future collider experiments. Since the tree level corrections of the Littlest Higgs model to electroweak precision data constrain the heavy particles as massive as a few TeV, a 500 GeV linear collider (LC) has not been expected to efficiently test the model. In literatures, Large Hadronic Collider (LHC) of the CERN is shown to have a potential to detect the new particles [16, 17, 18]. In the Littlest Higgs model, however, we find that the global symmetry structure $SU(5)/SO(5)$ yields substantially light $A_H$, light enough to be produced on-shell at a 500 GeV LC. Moreover, as shall been shown below, the $A_H$ becomes lighter in the parameter space where the corrections to the electroweak precision measurements are minimized.

The presence of a few hundreds GeV heavy photon can be dangerous to other low energy observables. We study its one-loop contributions to a well measured observable, the muon anomalous magnetic moment. Another issue here is the collider signatures of the $A_H$. The process of $e^+e^- \rightarrow \mu^+\mu^-$ is to be discussed, which is one of the most effective processes to probe the model, as the branching ratios (BR) of the heavy photon suggests.

The paper is organized as follows. In Sec. II we briefly review the Littlest Higgs model. We point out the preferred parameter space by considering some tree level relations of the SM gauge boson masses and couplings. In Sec. III physical properties of the heavy photon is studied, focused on its mass and decay patterns. In Sec. IV the one-loop corrections of the new gauge bosons to the muon anomalous magnetic moment are calculated. Numerical value is to be compared with the latest experimental data. In Sec. V we study the effects of the Littlest Higgs model on the process $e^+e^- \rightarrow \mu^+\mu^-$, of which the dominant contribution is from the heavy photon. We summarize our results in Sec. VI.

II. LITTLEST HIGGS MODEL

At TeV scale, the Littlest Higgs model is embedded into a non-linear $\sigma$-model with the coset space of $SU(5)/SO(5)$. The leading two-derivative term for the sigma field $\Sigma$ is

$$\mathcal{L}_\Sigma = \frac{1}{2} f^2 \frac{1}{4} \text{Tr} |D_\mu \Sigma|^2. \quad (1)$$
The local gauge symmetries \([SU(2) \otimes U(1)]^2\) is also assumed, which is clear from the following covariant derivative of the sigma field:

\[
\mathcal{D}_\mu \Sigma = \partial_\mu \Sigma - i \sum_{j=1}^{2} (g_j W^a_j (Q_j a \Sigma + \Sigma Q_j a^T) + g'_j B_j (Y_j \Sigma + \Sigma Y_j^T)).
\] (2)

The generators of two \(SU(2)\)’s are

\[
Q_1^a = \left( \begin{array}{cc}
\frac{\sigma^a}{2} \\
0_{3 \times 3}
\end{array} \right), \quad Q_2^a = \left( \begin{array}{cc}
0_{3 \times 3} \\
-\frac{\sigma^a^*}{2}
\end{array} \right),
\] (3)

and two \(U(1)\) generators are

\[
Y_1 = \text{diag}(-3, -3, 2, 2)/10, \quad Y_2 = \text{diag}(-2, -2, -2, 3, 3)/10.
\] (4)

At the scale \(\Lambda_S \sim 4\pi f\), a symmetric tensor of the \(SU(5)\) global symmetry develops an order \(f\) vacuum expectation value (VEV) of which the direction is into the \(\Sigma_0\) given by

\[
\Sigma_0 = \left( \begin{array}{cc}
1_{2 \times 2} \\
1_{2 \times 2}
\end{array} \right).
\] (5)

Now the following two symmetry breakings occur:

- The global \(SU(5)\) symmetry is broken into \(SO(5)\), which leaves 14 massless Goldstone bosons: They transform under the electroweak gauge group as a real singlet \(1_0\), a real triplet \(3_0\), a complex doublet \(2_{\pm 1}^\pm\), and a complex triplet \(3_{\pm 1}\).

- The assumed gauge symmetry \([SU(2) \otimes U(1)]^2\) is also broken into its diagonal subgroup \(SU(2)_L \otimes U(1)_Y\), identified as the SM gauge group. The gauge fields \(\tilde{W}'\mu\) and \(B'\mu\) associated with the broken gauge symmetries become massive by eating the Goldstone bosons of \(1_0\) and \(3_0\).

The non-linear sigma fields are then parameterized by the Goldstone fluctuations:

\[
\Sigma = \Sigma_0 + \frac{2i}{f} \left( \begin{array}{c}
\phi^+ \\
\frac{h^+}{\sqrt{2}} \\
0_{2 \times 2} \frac{h}{\sqrt{2}} \\
\frac{h}{\sqrt{2}} \Phi \end{array} \right) + \mathcal{O}(\frac{1}{f^2}),
\] (6)
where $h$ is a doublet and $\phi$ is a triplet under the unbroken $SU(2)$. A brief comment is that this Higgs triplet, developing a non-zero VEV, may explain neutrino mass terms through its Yukawa coupling with leptons in a SM gauge invariant way \[19\]. Note that the lepton Yukawa coupling has some freedom since it is insensitive to the quadratic divergence of the Higgs if the cutoff scale is around 10 TeV.

The gauge fields $\mathbf{W}'$ and $B'$ associated with the broken gauge symmetries are related with the SM gauge fields by

$$
W = sW_1 + cW_2, \quad W' = -cW_1 + sW_2, \\
B = s'B_1 + c'B_2, \quad B' = -c'B_1 + s'B_2,
$$

(7)

with the mixing angles of

$$
c = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad c' = \frac{g'_1}{\sqrt{g'_1^2 + g'_2^2}}.
$$

(8)

The SM gauge couplings are then $g = g_1s = g_2c$ and $g' = g'_1s' = g'_2c'$. At the scale $f$, the SM gauge fields remain massless, and the heavy gauge bosons are massive:

$$
m_{W'} = \frac{g}{2sc}f, \quad m_{B'} = \frac{g'}{2\sqrt{5}s'c'}f.
$$

(9)

As shall be discussed in detail, the presence of $\sqrt{5}$ in the denominator of $m_{B'}$ implies relatively light new neutral gauge boson. It is to be compared with the $SU(6)/Sp(6)$ case of $m_{B'} = g'f/(2\sqrt{2}s'c')$.

Even though the Higgs boson at tree level remains massless as a Goldstone boson, its mass is radiatively generated because any non-linearly realized symmetry is broken by the gauge, Yukawa, and self interactions of the Higgs field. Early attempts in constructing a pseudo Goldstone Higgs boson suffered from the same quadratic divergence as in the SM. Recent little Higgs models introduce a collective symmetry breaking: Only when multiple gauge symmetries are broken, the Higgs mass is radiatively generated; naturally the Higgs mass loop correction occurs at least at two loop level. In the phenomenological point of view, the cancellation of the SM contributions at one-loop level occurs as in the supersymmetry model. For example, the quadratic divergence due to the SM gauge boson $B^\mu$ is cancelled by that of the $B'^\mu$ field, as can be seen from

$$
\mathcal{L}_5(B \cdot B) \supset g'^2 B^\mu B'^\nu \text{Tr} \left[ \frac{1}{4} h^\dagger h \right] - g'^2 B'^\mu B'^\nu \text{Tr} \left[ \frac{1}{4} h^\dagger h \right].
$$

(10)
It is clear that the cancellations in little Higgs models are due to the exactly opposite coupling strength, which is provided by a larger symmetry structure. It is to be compared with supersymmetry models where the cancellation occurs due to opposite spin-statistics between the SM particle and its super-partner.

Since more severe quadratic divergence of the quantum correction to the Higgs mass comes from the top quark loop, another top-quark-like fermion is also required. In addition, this new fermion is naturally expected to be heavy with mass of order $f$. We introduce a vector-like fermion pair $\tilde{t}$ and $\tilde{t}^c$ with the SM quantum numbers $(3, 1)_Y$ and $(\bar{3}, 1)_{-Y}$. With $\chi_i = (b_3, t_3, \tilde{t})$ and antisymmetric tensors of $\epsilon_{ijk}$ and $\epsilon_{xy}$, the following Yukawa interaction is chosen in the Littlest Higgs model:

$$L_Y = \frac{1}{2} \lambda_1 f \sum_{i,j,k=1}^{3} \sum_{x,y=4}^{5} \epsilon_{ijk} \epsilon_{xy} \chi_i \Sigma_{jx} \Sigma_{ky} u^c_3 + \lambda_2 f \tilde{t}^c + \text{h.c.}$$

\[ \sum -i\lambda_1 (\sqrt{2} h^0 t_3 + i f \tilde{t} - \frac{i}{f} h^0 h^0 \tilde{t}) u^c_3 + \text{h.c.} \]  

As Eq. (11) shows, the quadratic divergence due to the heavy top quark cancels that due to the SM top quark. And this cancellation is stable from radiative corrections.

Electroweak symmetry breaking is induced by the remaining Goldstone bosons $h$ and $\phi$. Through radiative corrections, the gauge, Yukawa, and self-interaction of the Higgs field generate a Higgs potential [20]. As discussed before, the one-loop quadratic divergence of the Higgs mass coefficient $-\mu^2$ vanishes due to the cancellations between the new gauge boson (top quark) contributions and the SM contributions. Since the $\mu^2$ has the log-divergent one-loop and quadratically divergent two-loop contributions suppressed by a loop factor $1/16\pi^2$, it is to be treated as a free parameter of order 100 GeV. For positive $\mu^2$, the $h$ and $\phi$ fields can develop VEVs of $\langle h^0 \rangle = v/\sqrt{2}$ and $\langle \phi^0 \rangle = v'$, which trigger the electroweak symmetry breaking. Now the SM $W$ and $Z$ bosons acquire masses of order $v$, and small (of order $v^2/f^2$) mixing between $W$ and $W'$ ($Z$ and $Z'$) occurs. In the following, we denote the mass eigenstates of the SM gauge fields by $W_L$ and $Z_L$.

Some discussions on phenomenological points are in order here. First, the requirement of positive mass squared of the Higgs triplet constrains $v'$ to be $v'/v < v/(4f)$. Second, since the final $U(1)_{\text{QED}}$ symmetry remains intact, the mass and couplings of photon are the same as in the SM. For the Yukawa interaction of the other light SM fermions, we assume that
Eq. (11) is valid for all SM fermions including leptons, except that their corresponding extra vector-like fermions are absent. Then the gauge invariance of Eq. (11) with the anomaly free condition fixes the $U(1)_{1,2}$ charges of the SM fermions. For example, the lepton doublet and singlet have the following $U(1)_{1,2}$ charges:

$$L: Y_1 = -\frac{3}{10}, \quad Y_2 = -\frac{1}{5}, \quad e^c : Y_1 = \frac{3}{5}, \quad Y_2 = \frac{2}{5}. \quad (12)$$

The question of the corrections to the electroweak precision data merits some discussions. The absence of custodial $SU(2)$ global symmetry in this model yields weak isospin violating contributions to the electroweak precision observables. In the early study, global fits to the experimental data put rather severe constraint on the $f > 4$ TeV at 95% C.L. [21, 22]. However, their analyses are based on a simple assumption that the SM fermions are charged only under one $U(1)$. If all the SM fermions have common Yukawa couplings with anomaly-free condition as in Eqs. (11) and (12), the bounds become relaxed: Substantial parameter space allows $f \simeq 1 - 2$ TeV [23, 24]. The experimental constraints can be more loosed, i.e., by gauging only $U(1)_Y$ from the beginning [15, 23]. Even though the abandonment of the one-loop quadratic divergences to the Higgs mass from $U(1)_Y$ is obviously a theoretical drawback, the resulting fine-tuning, about 50% for $\Lambda \sim 10$ TeV, is tolerable.

To illustrate the preferred parameter space consistent with the low energy data, we present the SM $Z$ boson mass:

$$M_{Z_L}^2 = m_z^2 \left[ 1 + \Delta \left( \frac{1}{4} + c^2(1 - c^2) - \frac{5}{4}(c^2 - s^2)^2 \right) + 8\Delta' \right], \quad (13)$$

where $m_z = g v/(2c_W)$, $\Delta = v^2/f^2 \ll 1$, $\Delta' = v^2/v^2 \ll 1$, $t_W = s_W/c_W$ and

$$x_H = gg' \frac{s c' (c^2s^2 + s^2c^2)}{2g^2s^2c^2 - 2g^2s^2c^2/5}. \quad (14)$$

And the gauge couplings of the $Z_L$ with the charged leptons, in the form of $\gamma^\mu (g_L^Z P_L + g_R^Z P_R)$ with $P_{R,L} = (1 \pm \gamma^5)/2$, are

$$g_R^Z = \frac{e}{s_W c_W} \left[ -\frac{1}{2} + s_W^2 + \Delta \left\{ \frac{c^2}{2}(c^2 - \frac{1}{2}) - \frac{5}{4}(c^2 - s^2)(c^2 - \frac{2}{5}) \right\} \right], \quad (14)$$

$$g_L^Z = \frac{e}{s_W c_W} \left[ s_W^2 + \frac{5}{2}\Delta(c^2 - s^2)(c^2 - \frac{2}{5}) \right].$$

Here the QED bare coupling $e^2$ is the running coupling at the $Z$-pole, and the bare value of $s_W^2$ is related with the measure value of $s_0^2$ by

$$\frac{1}{s_W c_W} = \frac{1}{s_0 c_0} \left[ 1 - \frac{\Delta}{2} \left\{ c^2s^2 - \frac{5}{4}(c^2 - s^2)^2 \right\} - 2\Delta' \right]. \quad (15)$$
Since the main corrections to low energy observables in Eqs. (13)-(15) are proportional to $c^2$ or $(c'^2 - s'^2)$, the parameter space around $c \ll 1$ and $c' = 1/\sqrt{2}$ suppresses new contributions: The $f$ bound about 2 TeV is allowed in the region around $c \in [0, 0.5]$ and $c' \in [0.62, 0.73]$.

III. PROPERTIES OF THE HEAVY PHOTON

Among various little Higgs models such as the $SU(6)/Sp(6)$ model, the $SU(4)^4/SU(3)^3$ model, the $SO(5)^8/ SO(5)^4$ model and so on, the Littlest Higgs model can be distinguished by the presence of a relatively light $A_H$. From Eq. (9), the mass ratio of the heavy photon $A_H$ to the $Z_H$ is

$$\frac{M_{A_H}^2}{M_{Z_H}^2} = \frac{s_W^2 s'^2}{c_W^2 c'^2} + O \left( \frac{v^2}{f^2} \right) \sim 0.06 \left( \frac{s^2 c^2}{s'^2 c'^2} \right).$$

Even for the case of $c \simeq c'$, the $A_H$ is substantially lighter than the $Z_H(W_H)$. In addition, the electroweak precision data prefer the parameter space of $c \ll 1$ and $c' \sim 1/\sqrt{2}$, which much more suppresses the ratio $M_{A_H}^2 / M_{Z_H}^2$.

In Fig. 1 we present the $M_{A_H}$ and $M_{Z_H}$ as a function of $c'$ with the fixed $f = 1$ TeV (the $M_{A_H}$ and $M_{Z_H}$ increase linearly with $f$). In most of the parameter space, the $A_H$ is much lighter than the $Z_H$: Around $c' = 1/\sqrt{2}$ and $c = 0$, where the corrections to the electroweak

![Graph](https://via.placeholder.com/150)
precision data are minimized, the mass difference is maximized. Since the heavy photon is mainly the \( B' \), its mass depends weakly on the value of \( c \), the mixing parameter between two \( SU(2) \) gauge bosons. The \( M_{A_H} \) for \( c = 0.3 \) is practically identical with that for \( c = 0.1 \). On the contrary, the \( M_{Z_H} \) is sensitive to \( c \), while almost insensitive to \( c' \). In particular, small value of \( c \) enhances the mass of \( Z_H \): For \( c < 0.1 \), the \( Z_H \) becomes too heavy to be sufficiently produced at LHC.

Note that in the parameter space consistent with the low energy observables, the heavy photon becomes light enough to be produced at the future linear collider. Figure 2 illustrates contours for \( M_{A_H} = 200, 300, 500 \) GeV in the parameter space of \((c', f)\). The value of \( c \), which little affects \( M_{A_H} \), is set to be 0.1. In particular, the region around \( c' = 1/\sqrt{2} \) allows the on-shell production of the heavy photon at 500 GeV linear colliders for \( f \lesssim 3 \) TeV.

Next the gauge couplings of heavy neutral gauge bosons are

\[
\mathcal{L} = -(c'^2 Y_1 - s'^2 Y_2) \bar{f} \gamma_\mu f A_\mu^H + \frac{g c}{s} Q_L \gamma_\mu T^3 Q_L Z^\mu_H. \tag{17}
\]

In principle, if \( s'^2/c'^2 = Y_1/Y_2 \), which is allowed only when the anomaly-free condition is violated, the vertex of \( f - \bar{f} - A_H \) vanishes. Another crucial point is that the right-handed top quark coupling with the \( A_H \) has the additional term:

\[
g_{R}^{A_H-t-i} = \frac{g'}{s' c'} \left( \frac{4}{3} - \frac{5}{6} c'^2 - \frac{1}{5} \lambda_1^2 - \lambda_2^2 \right), \tag{18}
\]
while the left-handed top quark does not have, as shown by
\[ g_{L}^{A_{H} \to t^{-i}} = \frac{g'}{s' c'} \left( \frac{1}{15} - \frac{1}{6} c'^{2} \right). \]  
(19)

This is attributed to the proposed top Yukawa coupling in Eq. (11). The physical mass eigenstates of $SU(2)$-singlet top quark $t_{R}^{c}$ and heavy top quark $T_{R}$ are the mixtures of weak eigenstates, $u_{3}^{c}$ and $\tilde{t}^{c}$:
\[ t_{R}^{c} = \frac{1}{\sqrt{\lambda_{1}^{2} + \lambda_{2}^{2}}} (-\lambda_{1} \tilde{t}^{c} + \lambda_{2} u_{3}^{c}) , \quad T_{R} = \frac{1}{\sqrt{\lambda_{1}^{2} + \lambda_{2}^{2}}} (-\lambda_{1} \tilde{t}^{c} + \lambda_{2} u_{3}^{c}) , \]  
(20)

which explain the second terms of the $g_{R}^{A_{H} \to t^{-i}}$ in Eq. (16). Even in the special cases of the suppressed $A_{H}$ couplings, the top quark can substantially interact with the $A_{H}$.

Now let us discuss about the decay of the $A_{H}$ into a fermion pair and $Z - h$. Decay into a SM $W_{L}$ pair is suppressed by a factor of $(v/f)^{4}$. Partial decay rates are
\[ \Gamma(A_{H} \to f \bar{f}) = \frac{N_{c}}{12\pi} \left[ (g_{v}^{A_{H}})^{2}(1 + 2r_f) + (g_{a}^{A_{H}})^{2}(1 - 4r_f) \right] \sqrt{1 - 4r_f M_{A_{H}}} , \]  
\[ \Gamma(A_{H} \to Z h) = \frac{g'^{2}(c'^{2} - s'^{2})}{384\pi c's'} \lambda^{1/2}[(1 + r_{Z} - r_{h})^{2} + 8r_{Z}] M_{A_{H}} , \]

where $N_{c}$ is the color factor, $r_{i} = m_{i}^{2}/M_{A_{H}}^{2}$, and $\lambda = 1 + r_{Z}^{2} + r_{h}^{2} + 2r_{Z} + 2r_{h} + 2r_{Z}r_{h}$. Note that if $c' = 1/\sqrt{2}$, the $A_{H}$ decay into $Z h$ is prohibited. We refer the reader for the full expressions of the $g_{v,a}^{A_{H}}$ to Ref. [25]. In Fig. 3, we show the branching ratios of the $A_{H}$ as a function of $M_{A_{H}}$ for $c' = 0.4$ and $c' = 1/\sqrt{2}$. Two top quark Yukawa couplings $\lambda_{1}$ and $\lambda_{2}$ in Eq. (11) are assumed to be equal to each other. Except for the narrow region around $c' = 1/\sqrt{2}$, the BR patterns are almost the same: The decay into a charged lepton pair is dominant. If $c' = 1/\sqrt{2}$, the $A_{H} - Z - h$ coupling vanishes and the $A_{H}$ gauge couplings with a lepton pair is suppressed since then $s'^{2}/c'^{2} = 1$ is close to $Y_{1}/Y_{2} = 3/2$. In this case, the decay into top quark become dominant if kinematically allowed.

In Fig. 4 we show the total decay rate of the $A_{H}$ as a function of $M_{A_{H}}$ for $c' = 0.4, 1/\sqrt{2}, 0.9$. In particular, the $c' = 1/\sqrt{2}$ case yields a very narrow resonance peak, raising a possibility that the resonance peak might be missed.

The next question is whether this light $A_{H}$ is consistent with the recent measurement of muon anomalous magnetic moment [26] (see also recent review in Ref. [27]). This issue shall be answered in the following section.
FIG. 3: The branch ratios of the $A_H$ as a function of $M_{A_H}$ for $c' = 0.4$ and $c' = 1/\sqrt{2}$. The Higgs mass is 120 GeV.

IV. ANOMALOUS MAGNETIC MOMENT OF MUON AND LITTLE HIGGS

In this section, we study the one-loop level contribution of the Littlest Higgs model by calculating its effects on the anomalous magnetic moment of muon. The present status of the theoretical evaluation of the $(g - 2)_\mu$ in the SM is not conclusive because of the inconsistent values between the hadronic vacuum polarizations based on $e^+e^-$ and $\tau$ data. Comparison
FIG. 4: The total decay rate of the $A_H$ as a function of $M_{A_H}$ for $c' = 0.4$ and $c' = 1/\sqrt{2}$. The Higgs mass is 120 GeV.

with the experimental value implies

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}}(e^+e^-) = (35.5 \pm 11.7) \times 10^{-10} \quad [3\sigma],$$

$$a_\mu^{\text{exp}} - a_\mu^{\text{SM}}(\tau) = (10.3 \pm 10.7) \times 10^{-10} \quad [1\sigma].$$

FIG. 5: Feynman diagrams for one-loop contribution of heavy gauge bosons. (a) shows the contributions from $A_H, Z_H$ and (b) shows the contribution from $W_H$.

In the Littlest Higgs model, one-loop corrections to the muon anomalous magnetic moment come from the Feynman diagrams mediated by the heavy photon $A_H$, the new heavy neutral weak boson $Z_H$ and the new heavy charged weak boson $W_H$ as depicted in Fig 5.

Since each contribution to $\Delta a_\mu$ is inversely proportional to the gauge boson mass squared
and the $M_{A_H}^2$ is smaller than the $M_{Z_{H,W_H}}^2$ at least by an order of magnitude, we consider only the $A_H$ contribution of \[28\]

$$
\Delta a_{A_H} = \frac{1}{12\pi^2} \left( \frac{m_\mu}{M_{A_H}^2} \right)^2 \left[ (g_{v}^{A_H})^2 - 5(g_{a}^{A_H})^2 \right].
$$

(23)

\[f > 1 \text{ TeV}\]

FIG. 6: The contribution to the muon anomalous magnetic moments due to the heavy photon as a function of $c'$ for $M_{A_H} = 200, 300, 500 \text{ GeV}$. Since we require $f > 1 \text{ TeV}$, limited space of $c'$ according to the $M_{A_H}$ is presented. The contribution to the $\Delta a_\mu$ increases as the $M_{A_H}$ decreases and the $c'$ deviates from the value of $s'^2/c'^2 = Y_1/Y_2$. In the whole parameter space, the $\Delta a_{A_H}$ is quite safe from the recent experimental data in Eq. (22). Therefore, it is concluded that the light $A_H$ is not inconsistent with the current status of the theoretical and experimental value of the muon anomalous magnetic moment.

V. $e^+e^- \rightarrow \mu^+\mu^-$

In the Littlest Higgs model the heavy photon mass $M_{A_H}$ depends on the global symmetry breaking scale $f$, two mixing angles of $c$ and $c'$ between light and heavy gauge bosons. In Sec. IIII we have shown that as the model parameters are arranged to suppress the contributions to the electroweak precision data, this $M_{A_H}$ reduces, which is contrary to
the usual case where the heavier new particle suppresses the corrections to the low energy observables. And in a major portion of parameter space, the dominant decay mode is shown to be into a charged lepton pair. Therefore, one of the most effective signals to probe the model can come from the process $e^+e^- \rightarrow \mu^+\mu^-$. The process $e^+e^- \rightarrow \mu^+\mu^-$ has two SM s-channel Feynman diagrams mediated by the photon and $Z$ boson. In the Littlest Higgs model, it has two additional s-channel diagrams mediated by the $A_H$ and $Z_H$. The corresponding helicity amplitude $\mathcal{M}_{\lambda_e\lambda_e\lambda_\mu\lambda_\mu}$, where the $\lambda_l$ and $\lambda_\lambda$ are respectively the polarization of $l^-$ and $l^+$, can be simplified by $\mathcal{M}_{\lambda_e\lambda_\mu}$ since $\lambda_l = -\lambda_\lambda$ with the lepton mass neglected. We have

$$\mathcal{M}_{\lambda_e\lambda_\mu} = -(1 + \lambda_e\lambda_\mu \cos \theta) \sum_{V_j} g_{\lambda_e}^V g_{\lambda_\mu}^V D_{V_j}, \quad (24)$$

where $\theta$ is the scattering angle of the muon with respect to the electron beam, $V_j = A, Z, A_H, Z_H$, and the $D_{V_j}$ is the propagation factor of

$$D_{V_j} = \frac{s}{s - M_{V_j}^2 + iM_{V_j}\Gamma_{V_j}}. \quad (25)$$

And the $g_{\lambda_e}^V \left( = g_{\lambda_e}^{V_{-l^+l^-}} \right)$’s are

$$g_{R_H}^{Z_H} = -\frac{g_{V}^c}{2s}, \quad g_{L_H}^{Z_H} = 0$$
$$g_{R_H}^{A_H} = \frac{g_{V}^f}{s'c'} \left( -\frac{1}{5} + \frac{c'^2}{2} \right), \quad g_{L_H}^{A_H} = \frac{g_{V}^f}{s'c'} \left( -\frac{2}{5} + c'^2 \right). \quad (26)$$

In Fig. 7 we present the total cross section as a function of $\sqrt{s}$ for $c' = 0.4$ and $c' = 1/\sqrt{2}$. We set $M_{A_H} = 400$ GeV and $c = 0.1$. If the $c'$ sizably deviates from the critical point of $s^2/c'^2 = Y_1/Y_2$ (see $c' = 0.4$ case), the coupling $A_H - l^+ - l^-$ is large enough to yield substantial deviation from the SM results even outside the resonance peak. In the parameter region of the suppressed $A_H - l^+ - l^-$ coupling (see $c' = 1/\sqrt{2}$ case), only around the resonance peak can produce significant new signal.

**VI. SUMMARY AND CONCLUSION**

The Littlest Higgs model could be an alternative model for new physics beyond the Standard Model which solves the little hierarchy problem. From the extension in the gauge sector, we expect a new set of gauge bosons. The heavy photon is shown to be lightest of all
FIG. 7: The total cross section of the process as a function of $\sqrt{s}$ with $M_{A_H} = 400$ GeV and $c = 0.1$. We consider two cases of $c' = 0.4$ and $c' = 1/\sqrt{2}$.

and, moreover, lighter in the preferred parameter space by electroweak precision measurements. We checked the consistency of the parameter space by calculating one-loop induced anomalous magnetic moment of muon. Its numerical value is $\Delta a_\mu \leq 0.1 \times 10^{-10}$ in the whole parameter region. Then we study the on-shell production and decay of the heavy photon in the future linear collider. The heavy photon mainly decays to lepton pairs but for $c' = 1/\sqrt{2}$ it could mainly decay to a top-quark pair if the kinematics allows. The high energy process of $e^+e^- \rightarrow \mu^+\mu^-$ is explicitly considered and the resonance structure of the heavy photon production is shown for various parameters of the model. In conclusion, the heavy photon of the Littlest Higgs induces negligible corrections to the anomalous magnetic moment of muon but still can be produced in the future linear collider.

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