Effective defect mode suppression in a magnetophotonic crystals in the magnetic resonance region

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Abstract. The influence of magnetic field on the reflection spectra of one-dimensional magneto-photonic crystals of two types has been investigated. The first type is a periodically layered defect-free magnetic-dielectric structure, the second one is a magnetically active defect placed between dielectric photonic crystal mirrors. If the frequency of magnetic resonance is close to the central frequency of one of the photonic band gaps or to the frequency of the defect mode, it creates a significant reconstruction of the spectrum in the mentioned frequency domain. Namely, it can bring the suppression of the oscillation spectrum and the defect mode, which makes it possible to effectively control the spectrum of such a structure by an external magnetic field.

1. Introduction
One-dimensional photonic-crystal (PC) structures and layered periodic structures made of various materials have gained much attention of researchers over the last years. Owing to periodic modulation of the refractive index, the photonic spectrum of these structures possesses the forbidden band gaps in which the incident radiation is reflected almost completely [1, 2, 3, 4]. This property is important for various applications of PC structures, in particular, for controlling the optical radiation in laser engineering and data transmission systems. These structures functionality can be significantly extended by controlling their spectral characteristics via the variation of the geometrical or physical parameters of the structure. In particular, the photon spectrum of the structure can be modified by introducing the active magnetic layers into the structure, as a result the spectrum can be controlled by the external magnetic field [5, 6, 7, 8]. A significant spectrum reconstruction can also be caused by a local breaking of the structures periodicity that can lead to a defective transmission miniband in the forbidden band gap [9, 10, 11, 12]. The structure, which has a defect layer placed between two Bragg photonic crystal mirrors (Fabry-Perot microcavity), is of special interest. This layer in such a structure plays the role of an optical microcavity, where the light wave field can be localized, thus greatly enhancing many effects of the light-matter interaction [8, 9, 10]. In such a structure, this layer plays the role of the optical microcavity where the light wave field can be localized and various effects of interaction of radiation with media can be significantly increased [13, 14, 15]. An effective control over the photonic spectrum, by using the external magnetic field, is also possible in microcavity...
structures with a magnetic defect. The resonance response of the magnetic permeability of the defect to the high frequency field of the propagating wave in the region of magnetic resonance, can substantially modify the spectral line of the defect mode up to its complete suppression. In this work, we investigate the modification of reflection spectra of a magnetic-dielectric defect-free PC structure and microcavity structure with a magnetic defect in the external magnetic field in the domain of magnetic resonance, as well as the possibility of suppression of the oscillation spectrum and the defect mode of the spectra concerned.

2. Material parameters of layers and transfer matrices

Let us consider two types of one-dimensional magnetoactive PC structures. The first type is the finite defect-free periodically layered structure consisting of alternating magnetic and dielectric layers. The second structure type is a symmetrical magnetoactive microcavity, in which the magnetic layer is placed between the dielectric PC mirrors.

The structures of the first type include non-magnetic dielectric layers, which is characterized by the thickness $L_d$ and scalar permittivity and permeability (DP and MP) $\varepsilon_d$ and $\mu_d$. Layers of two different dielectrics with thicknesses $L_{1,2}$, permittivities $\varepsilon_{1,2}$ and permeabilities $\mu_{1,2}$ are used in the structures of the second type. In the structures of both types the layers of uniformly magnetized magnetic are characterized by thickness $L_m$ and are described by the scalar DP and tensor MP in the high frequency range:

$$\tilde{\mu}_m = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \mu & -i\mu_a \\ 0 & i\mu_a & \mu \end{pmatrix}. \quad (1)$$

We assume that the external static magnetic field $H_0$ is oriented along the $OX$-axis. Tensor components $\tilde{\mu}_m$ are the following:

$$\mu = 1 + \frac{\omega M(\omega + i\alpha \omega)}{\omega^2}, \quad \mu_a = \frac{\omega M \omega}{(\omega + i\alpha \omega)^2}, \quad (2)$$

where we introduce the parameters $\omega_H = \gamma H_0$, $\omega_M = 4\pi \gamma M_0$, $M_0$ is the saturation magnetization, $\gamma$ is the gyromagnetic ratio, $\alpha = \Delta H/H_0$ is the parameter of the magnetic damping, $\Delta H$ - is the resonance line width [16].

We assume that the waves propagate in the structure along its periodicity axis (the $OY$ axis), and the field $H_0$ is oriented perpendicular to this axis, i.e. lies in the layers plane. The solution of the Maxwells equations, taking into account the propagation direction and the orientation of the bias field, yields two eigenmodes of the structure, the TE and TM modes. Only TE wave with the components of wave field $(E_x, H_y, H_z)$ is controlled by an external magnetic field. TM-type wave with $(H_x, E_y, E_z)$ components practically does not react to changes in the external magnetic field.

Solutions of wave equations for each component in $j$-th layer can be represented as a superposition of the forward and backward waves:

$$F_{\alpha,j}(y) = F_{\alpha,j}^1 \exp [i(\omega t - k_j y)] + F_{\alpha,j}^2 \exp [i(\omega t + k_j y)], \quad (3)$$

where $k_j = k_0 \sqrt{\varepsilon_j \mu_j}$ are the wave numbers in each of the layers, $k_0 = \omega/c$, $c$ is the light speed in vacuum. For dielectric layers $\varepsilon_j = \varepsilon_d$, $\mu_j = \mu_d$. For the magnetic layers in the case of TM wave $\varepsilon_j = \varepsilon_m$, $\mu_j = 1$, and in the case of TE wave

$$\mu_j = \mu_{\perp} = \mu - \frac{\mu_a^2}{\mu} = \frac{(\omega_a + i\alpha \omega)^2 - \omega^2}{\omega^2 - \omega_a^2 + i\alpha \omega(\omega_a + \omega_H)}, \quad (4)$$
where we introduce the magnetic resonance frequency \( \omega_r = \sqrt{\omega_H \omega_a} \) and antiresonance frequency \( \omega_a = \omega_H + \omega_M \) \[16\]. The frequency and field dependences of \( \mu_\perp(\omega, H_0) \) determine in many ways the characteristics of TE wave interaction with the magnetoactive PC structure.

Figure 1 shows the dependences of the real part of the effective permeability on the frequency for the field values \( H_0 = (400; 600) \) Oe (the solid curves 1, 2), and the magnitude of the magnetizing field for the frequency values \( \omega = (1.5; 2.0) \cdot 10^{10}\, s^{-1} \) (the dashed curves 3, 4). The calculations were made for a magnetic with parameters \( 4\pi M_0 = 1780 \) G, \( \alpha = 0.05 \). It can be seen that each value corresponds to the magnetizing field value of resonance frequency, and each frequency corresponds to the resonance value of the magnetic field.

![Figure 1](image)

**Figure 1.** Frequency dependences of the real part of the effective magnetic permeability for the values of the field \( H_0 = (0.4; 0.6) \) kOe (solid curves 1, 2) as well as the value of the magnetizing field for values of the frequencies \( \omega = (1.5; 2.0) \cdot 10^{10}\, s^{-1} \) (dashed curves 3, 4); \( \omega_0 = 10^{10}\, s^{-1} \).

The increase of the field leads to a shift of the resonance line to higher frequencies, and the increase in the frequency to the region of high fields. Let us consider the frequency region \( \omega_a < \omega < \omega_a \), where the real part of the effective permittivity \( \mu_\perp \) is negative (here we introduced the frequency antiresonance \( \omega_a = \omega_H + \omega_M \)). By changing the magnetic field, these features of the functions \( \mu_\perp(\omega, H_0) \) allow to provide proximity magnetic resonance frequency or region of negative effective MP to the center frequency of the photonic band gap, or to the frequency of the defect mode. This, in turn, should lead to a substantial restructuring of the photon spectrum of a magnetophotonic crystal.

Using the boundary conditions for wave fields and the periodicity, we can get the connection of wave fields in planes separated by an arbitrary number of layers. In the case of defect-free periodic structure containing a finite number of periods, this relationship is a matrix \( \hat{G} = (\hat{M})^n \), which is the \( n \)-th power of the matrix of one period. In its turn, the transfer matrix of one period is the product of the transfer matrices of the layers that make up the period, i.e. \( \hat{M} = \hat{N}_1\hat{N}_2 \). In this case, the transfer matrices of each layer are as follows:

\[
\hat{N}_j = \begin{pmatrix}
C_j & \frac{k_0\mu_j}{ik_j} S_j \\
-\frac{ik_j}{k_0\mu_j} S_j & C_j
\end{pmatrix},
\]  

where we introduced the notation \( C_j = \cos(k_j L_j) \), \( S_j = \sin(k_j L_j) \). Using (5) the transfer matrix of one period has the following matrix elements:

\[
\begin{align*}
M_{11} &= C_1 C_2 - \frac{\mu_1 k_2}{\mu_2 k_1} S_1 S_2, & M_{12} &= -\frac{k_0\mu_2}{ik_2} C_1 S_2 - \frac{k_0\mu_1}{ik_1} S_1 C_2, \\
M_{21} &= \frac{ik_2}{k_0\mu_2} C_1 S_2 + \frac{ik_1}{k_0\mu_1} S_1 C_2, & M_{22} &= C_1 C_2 - \frac{\mu_2 k_1}{\mu_1 k_2} S_1 S_2,
\end{align*}
\]  

The symmetric PC resonator structure is intended to include a layer of uniformly magnetized magnetic between the two lateral dielectric photonic-mirrors. We assume that each of the side PC-mirrors consists of the same number of structure periods, and the period is composed of...
two layers of non-absorbing dielectrics. The inversion means changing the order of the layers of one part of the structure against the other one. In terms of defects such structure contains a double defect - the inversion layer and the introduction layer \([12]\). Inverted period corresponds to the transfer matrix \(\hat{M} = \hat{N}_2 \hat{N}_1\), whose matrix elements are related to the elements of the matrix of the normal period by the ratio \(\hat{M}_{\alpha, \beta} = \hat{M}_{3-\alpha, 3-\beta}\), where \(\alpha, \beta = 1, 2\). Transfer matrices of two types of considered resonator structures can be written as \(\hat{G} = (\hat{M})^5 \hat{N}_m (\hat{M})^5\), \(\hat{G} = (\hat{M})^5 \hat{N}_m (\hat{M})^5\).

Energy reflection and transmission coefficients for the investigated PC structures can be represented as follows:

\[
R = \left(\frac{G_{11} + G_{12} - G_{21} - G_{22}}{G_{11} + G_{12} + G_{21} + G_{22}}\right)^2, \quad T = \frac{4}{(G_{11} + G_{12} + G_{21} + G_{22})^2}, \quad (7)
\]

where \(G_{\alpha, \beta}\) are the matrix elements of the transfer matrix structure. These coefficients satisfy the law of conservation of energy \(R + T + A = 1\), where the absorption coefficient \(A\) determines the amount passed into heat energy.

3. Defect-free PC-structure magnetic-dielectric.

Based on the above relations, we performed an analysis of the reflection and transmission spectra of a defect-free periodic structure containing a finite number of periods, and investigated their modification under the influence of an external magnetic field. The transfer matrix of a structure consisting of \(n\) periods is the \(n\) - th power of the transfer matrix of one period, i.e., \(\hat{G} = (\hat{M})^n\). To simulate reflective and selective properties of a PC structure we use the following values of permittivity \(\varepsilon_d = 10\) and permeability \(\mu_d = 1\) of layers (material MCT10) and \(\varepsilon_m = 15.1\), function \(\mu_\perp(\omega; H_0)\) is the same as in the figure 1 (doped yttrium iron garnet 10CH6B) \([17]\).

![Figure 2](image)

**Figure 2.** The reflection spectra of structure \((M)^{20}\) with thicknesses of the layers in \(L_d = 1.49\) cm and \(L_m = 1.21\) cm with and without application of an external field \(H_0 = 0.3\) kOe (a and b); \(\omega_0 = 10^{10}\) s\(^{-1}\).
4. Inverted structure with a magnetic defect

Now let us consider the symmetric PC resonator structure. Each of the side PC dielectric mirrors consists of five periods, each period consists of two layers of non-absorbing dielectric with real constants \( \varepsilon_{1,2}, \mu_{1,2} = 1 \), and thicknesses \( L_{1,2} \). The thickness of the cavity layer of uniformly magnetized magnetic is \( L_m \). Side mirrors are inverted from each other.

To simulate high-frequency properties of this structure we use the following values of permittivity layers of PC-mirrors: \( \varepsilon_1 = 10, \varepsilon_2 = 25 \) (materials MCT10 and MCT25) \cite{17}. We choose doped yttrium iron garnet 10CH6B, whose characteristics are already described above, as the defect layer material. Thickness of layers were chosen to ensure the same optical thickness of the dielectric layers of PC mirrors, i.e. \( \sqrt{\varepsilon_1 L_1} = \sqrt{\varepsilon_2 L_2} = L_0 \), and the optical thickness of the magnetic layer was taken equal to double optical thickness of the dielectric layers, i.e. \( \sqrt{\varepsilon_m < \mu'_\perp} L_m = 2L_0 \), where \( < \mu'_\perp > = 2.57 \) (average permeability in the field of magnetic resonance). The optical thickness \( L_0 \approx 24.4 \text{ mm} \). For the considered structures the position and width of the spectral line of the defect mode depend stronger on the sequence of layers in the PC-mirrors as well as on the value of \( < \mu'_\perp > \).

![Figure 3](image-url)  

**Figure 3.** The reflection and transmission spectra (solid and dashed curves) for the structure \((M)^5 N_m(M)^5\) (a, b) and \((\overline{M})^5 N_m(M)^5\) (c, d) prepared for field values of \( H_0 = 0.2 \text{ kOe} \) (a, c) and \( H_0 = 1.5 \text{ kOe} \) (b, d); \( \omega_0 = 10^{10} \text{ s}^{-1} \).  

Figure 3 shows the reflection and transmission spectra (solid and dashed lines) for the structures \((M)^5 N_m(M)^5\) and \((\overline{M})^5 N_m(M)^5\), obtained for different values of the field \( H_0 = 0.2 \text{ kOe} \) (a, c) and \( H_0 = 1.5 \text{ kOe} \) (b, d), \( < \mu'_\perp > = 2.57 \). At the value of the field \( H_0 = 0.2 \text{ kOe} \) the resonant frequency \( \omega_r = 1.1 \cdot 10^{10} \text{ s}^{-1} \) and the spectra correspond to the case when the photonic band gap coincides with the domain of “metallic reflection”, for which the values are negative. At \( H_0 = 1.5 \text{ kOe} \), \( \omega_r = 3.9 \cdot 10^{10} \text{ s}^{-1} \) the situation is shown which corresponds to the equation \( \mu'_\perp < < \mu'_\perp \), where the defect mode is in the center of the band gap. Further increase of the magnetizing field leads to a shift of the resonant frequency to the optical frequency range, when the value \( \mu'_\perp \) becomes smaller than \( \mu'_\perp \) and the defect mode is shifted to high frequency edge of the band gap.

Figure 4 shows the reflectance spectra of the structure \((\overline{M})^5 N_m(M)^5\) with thicknesses of layers \( L_1 = 7.72 \text{ mm} \), \( L_2 = 4.88 \text{ mm} \) with the same optical thickness \( L_0 = 24.4 \text{ mm} \) (at the frequency \( \omega_0 = 10^{10} \text{ s}^{-1} \)) and with defect thicknesses \( L_m = (0.001; 0.01; 2.0)L_0 \) and \( L_m = (0.001; 0.01; 2.0)\lambda_r \) (a, b; solid, dashed, dotted lines). The spectra were obtained at \( H_0 = 0.5 \text{ kOe} \), the resonant frequency \( \omega_r = 1.88 \cdot 10^{10} \text{ s}^{-1} \) and \( \lambda_r = 2\pi c/\omega_r \approx 10 \text{ cm} \). It reveals the substantial dependence of the spectrum character on the thickness of the defect layer near the defect mode. The thicker it is, the stronger the effect of the magnetic resonance spectrum becomes. If the thickness of the defect layer is the same or greater than the optical thickness, the defect mode is weak or completely suppressed. With a significant decrease in the thickness of the defect layer the defect mode is manifested associated with a defect inversion in the structure.
Figure 4. The reflection spectra of the structure $\left( \frac{M}{\lambda} \right)^5 N_m (\frac{M}{\lambda})^5$ with thicknesses of the layers in $L_1 = 7.72 \text{ mm}$ and $L_2 = 4.88 \text{ mm}$; $L_m = (0.001; 0.01; 2) L_0$ and $L_m = (0.001; 0.01; 2) \lambda_r$ (a, b; solid, dashed, dotted lines); with an external field $H_0 = 0.5 \text{ kOe}$; $\omega_0 = 10^{10} \text{ s}^{-1}$.

5. Conclusion
The analysis carried out in the work suggests the possibility of effective control over reflectivity and transmissivity of a magnetoactive PC structure in the region of magnetic resonance by an external magnetic field. For TE waves we demonstrated almost complete suppression of the oscillations of the reflection coefficient of a defect-free PC and of the defect mode in the case of a defect structure at the coincidence of the magnetic resonance with the frequency doimain of the photonic band gap. In the case of propagation of TM wave in these structures, there is no resonance in the magnetic layers, so the management of its wave characteristics by an external magnetic field is not possible. Thus, the magnetic sensitivity of the defect mode in the case of TE polarization is nonmagnetic sensitive in the case of TM polarization, i.e. the defect mode in the above structure is polarization-sensitive. The detected effects can be used as the basis for the creation of such high-frequency radiation control units, as modulators, filters and switches.

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