We discuss the role of unfavoured light quark/antiquark into $D$ meson fragmentation. The unknown parameters of fragmentation process are adjusted to describe the asymmetry for $D^+$ and $D^-$ production measured by the LHCb. Predictions for similar asymmetry for neutral $D$ mesons are presented. The predicted asymmetry at large rapidity (or $x_F$) are very large which is related to the valence-quark contribution. As a result, prompt atmospheric neutrino flux at high neutrino energies can be much larger than for the conventional $c \rightarrow D$ fragmentation. We predict large rapidity-dependent $D^+/D^-$ and $D^0/\bar{D}^0$ asymmetries for low ($\sqrt{s} = 20$–100 GeV) energies. The $q/\bar{q} \rightarrow D$ fragmentation leads to enhanced production of $D$ mesons at low energies. Predictions for $p+^4\text{He}$ collisions relevant for a fixed target LHCb experiment are discussed.

1. Introduction

It is believed that the high-energy neutrinos observed by the IceCube Collaboration (see e.g. [1]) are of extraterrestrial origin. Another important component comes from semileptonic decays of $D$ mesons produced in the atmosphere by the collision of cosmic rays (mostly protons) with the atmosphere (mostly $^{14}\text{N}$). The flux of cosmic rays (charged particles) is relatively well known as measured e.g. by the Auger experiment. It is well-known that the dominant mechanism of charm production at high energies is $gg \rightarrow c\bar{c}$ partonic subprocess.

Recently, we have performed a critical analysis of uncertainties in the high-energy production of charm ($D$ mesons) [2]. The following conclusions were obtained in [2]. The high-energy neutrinos are produced mostly in very high
high-energy proton–proton collisions (larger than at the LHC). The region of $x_F > 0.3$ is crucial for high-energy neutrinos, however not accessible at the LHC. Both very small and very large longitudinal momentum fractions of gluons are important. These regions are not well-known.

Recently, the LHCb Collaboration observed $D^+$ and $D^-$ asymmetry at forward directions [3]. In the literature, one routinely assumes that $D$ mesons are produced from $c$ or $\bar{c}$ fragmentation. This gives no asymmetry, at least in leading-order approach.

Recently, we have considered also subleading unfavoured fragmentation [4]. It is known that the unfavoured fragmentation leads to asymmetry in $K^+$ and $K^-$ production (SPS, RHIC/BRAHMS). Also $\pi^+\pi^-$ asymmetry was observed but there both quark and antiquark fragmentation functions are (assumed) the same.

2. A sketch of our approach

2.1. Unfavoured fragmentation

The dominant at large $x_F$ high-energy processes: $ug \to ug$, $dg \to dg$, $\bar{u}g \to \bar{u}g$ and $\bar{d}g \to \bar{d}g$ and subsequent light quark/antiquark to $D$ meson fragmentation and/or decays are calculated in the leading-order (LO) collinear factorization approach with a special treatment of minijets at low transverse momenta, as adopted in Pythia, by multiplying standard cross section by a somewhat arbitrary suppression factor

$$F_{\text{sup}}(p_T) = \frac{p_T^4}{(p_T^0)^2 + p_T^2} \theta(p_T - p_T,\text{cut}).$$

(1)

In Fig. 1, we show distributions of quarks/antiquarks produced in such mechanisms. In this leading-order calculation, we have used the regulator given by Eq. (1). We can observe much larger cross section than for $c/\bar{c}$ production in the region of large $x_F$.

To get distributions of mesons, we have to include $u, \bar{u}, d, \bar{d} \to D^i$ parton fragmentation. The corresponding fragmentation functions fulfill the following flavour symmetry conditions:

$$D_{d \to D^-}(z) = D_{\bar{d} \to D^+}(z) = D^{(0)}(z).$$

(2)

Similar symmetry relations hold for fragmentation of $u$ and $\bar{u}$ to $D^0$ and $\bar{D}^0$ mesons. However, $D_{q \to D^0}(z) \neq D_{\bar{q} \to D^+}(z)$ which is caused by the contributions from decays of vector $D^*$ mesons. Furthermore, we assume

$$D_{\bar{u} \to D^\pm}(z) = D_{u \to D^\pm}(z) = 0$$

(3)

for doubly suppressed fragmentations.
We limit in the following to a phenomenological approach and ignore possible DGLAP evolution effects important at somewhat larger transverse momenta. We parametrize the unfavoured fragmentation functions in the low-$p_T$ phase space region as

$$D_{q\rightarrow D}(z) = A_\alpha (1-z)^\alpha. \quad (4)$$

Instead of fixing the unknown $A_\alpha$, we will operate rather with the fragmentation probability

$$P_{q\rightarrow D} = \int dz \ A_\alpha (1-z)^\alpha \quad (5)$$

and calculate corresponding $A_\alpha$ for a fixed $P_{q\rightarrow D}$ and $\alpha$. In our approach, we have only two free parameters.

Another simple option we considered in [4] is

$$D_{qf\rightarrow D}(z) = P_{qf\rightarrow D} \ D_{Peterson}(1-z). \quad (6)$$

For heavy quark fragmentation ($c \rightarrow D$), the Peterson fragmentation function is peaked at large $z$. The light quark/antiquark fragmentation is expected to be dominant at small $z$. This is the case of Peterson fragmentation function reflected with respect to $z = 1/2$. We used such a purely phenomenological function as another example to test uncertainties related to the shape of the a priori unknown function.

In addition to the direct fragmentation (given by $D^{(0)}(z)$), there are also contributions with intermediate vector $D^*$ mesons. Then the chain of production of charged $D$ mesons is as follows:

$$\bar{u} \rightarrow D^{*,0} \rightarrow D^+ \ (forbidden),$$
\[ u \rightarrow \bar{D}^{*0} \rightarrow D^- \text{ (forbidden)}, \]
\[ d \rightarrow D^{*+} \rightarrow D^+ \text{ (allowed)}, \]
\[ d \rightarrow D^{*-} \rightarrow D^- \text{ (allowed)}. \]  
\( (7) \)

Including both direct and feed-down contributions, the combined fragmentation function of light quarks/antiquarks to charged \( D \) mesons can be written as
\[
 D_{d/\bar{d}\rightarrow D^\pm}(z) = D_{d/\bar{d}\rightarrow D^\pm}(z) + P_{\mp\rightarrow\mp} D_{d/\bar{d}\rightarrow D^{*\pm}}(z). 
\]  
\( (8) \)

Similar formula can be written for neutral \( D \) mesons [4]. We assume flavour symmetry of fragmentation functions also for vector \( D \) meson production. In our calculations in [4] we assumed in addition
\[
 D^{(0)}(z) \approx D^{(1)}(z) 
\]  
\( (9) \)

which can be easily modified if needed.

2.2. Production asymmetry

The flavour asymmetry in production is defined as
\[
 A_{D^+/D^-}(\xi) = \frac{d\sigma_{D^-}(\xi)}{d\xi} - \frac{d\sigma_{D^+}(\xi)}{d\xi}, 
\]  
\( (10) \)

where \( \xi = x_F, y, p_T, (y, p_T) \). In [4], we have considered several examples.

3. Results

3.1. LHCb asymmetry

In the top panels of Fig. 2, we show results for the asymmetry for \( P_{q\rightarrow D} \) adjusted to the LHCb data. In this calculation, we have fixed \( \alpha = 1 \) in formula (4). In the left panel, we show \( A_{D^+/D^-}(\eta) \) for \( p_{T,D} \in (2,18) \text{ GeV} \) and in the right panel, we show \( A_{D^+/D^-}(p_T) \) for \( 2.2 < \eta < 4.75 \). We find that \( P_{q\rightarrow D} = 0.005 \pm 0.001 \) for triangle fragmentation function and \( P_{q\rightarrow D} = 0.007 \pm 0.001 \) for Peterson \((1 - z)\) is consistent with the main trends of the LHCb data. These are rather small numbers compared to \( c/\bar{c} \rightarrow D/\bar{D} \) fragmentation which happens with probability of the order of 50\%. The results only weakly depend on transverse momentum cut \( p_T^0 \), since the LHCb kinematics excludes the uncertain region of very small meson transverse momenta. In the bottom panels, we show our predictions for \( \sqrt{s} = 13 \text{ TeV} \).

In [4], we showed also our predictions for \( D^0-\bar{D}^0 \) asymmetry.
**Fig. 2.** $A_{D^+}/D^-$ production asymmetry measured by the LHCb Collaboration at $\sqrt{s}=7$ TeV as a function of $D$ meson pseudorapidity (left top panel) and $D$ meson transverse momentum (right top panel). The corresponding predictions for $\sqrt{s}=13$ TeV are shown in the bottom panels.

### 3.2. Low energies

The discussed by us mechanisms of subleading fragmentation of $D$ mesons lead to enhanced production of $D$ mesons at lower energies. In Table I, we show different contributions to the production of $D^+/D^-$ mesons. The dominant at high-energy $gg \rightarrow c\bar{c}$ mechanism gives only 13% and 18% for $\sqrt{s}=27$ and 39 GeV, respectively, and strongly underestimates the NA27 [8] and E743 [9] experimental data. Inclusion of the “subleading” contributions brings theoretical calculations much closer to the experimental data. We predict sizeable $D^+/D^-$ asymmetries at these low energies.

In Fig. 3, we show the lowest energy data for charged $D$ mesons in proton–proton collisions [8, 9]. We show results of conventional calculation in the $k_T$-factorization as well as results obtained with the code FONLL. Both the $k_T$-factorization as well as FONLL results are below experimental data extrapolated to the full phase space. Including also theoretical uncertainties, this leaves room for our subleading fragmentation contribution. In our paper, it was obtained by extrapolating our results, assuming some parametrizations of the subleading fragmentation function, to low en-
ergies based on the asymmetry measured by the LHCb Collaboration. Of course our estimate of the LHCb asymmetry as well as extrapolation to other corners of the phase space cannot be too precise. Clearly, better data for intermediate and low energies are needed to constrain the subleading fragmentation.

TABLE I

Different contributions to the cross sections (in microbarns) for $D^+ + D^-$ production at low energies. The results presented here were obtained with $p_T^0 = 1.5$ GeV.

| Process | $\sqrt{s} = 27$ GeV | $\sqrt{s} = 39$ GeV |
|---------|---------------------|---------------------|
| $g^*g^* \rightarrow c\bar{c}$ ($c/\bar{c} \rightarrow D^\pm$) | 1.52 | 4.58 |
| $q^*q^* \rightarrow c\bar{c}$ ($c/\bar{c} \rightarrow D^\pm$) | 0.08 | 0.19 |
| $gd \rightarrow gd$ ($d \rightarrow D^-$) | 9.53 | 13.89 |
| $gd \rightarrow g\bar{d}$ ($d \rightarrow D^+$) | 3.03 | 4.78 |
| $\bar{d}d \rightarrow \bar{d}d$ ($d \rightarrow D^-$) | 3.07 | 4.29 |
| $\bar{d}d \rightarrow \bar{d}d$ ($d \rightarrow D^+$) | 0.29 | 0.49 |
| $\bar{u}d \rightarrow \bar{u}d$ ($d \rightarrow D^-$) | 0.58 | 0.88 |
| $\bar{u}d \rightarrow \bar{u}d$ ($d \rightarrow D^+$) | 0.58 | 0.88 |
| $u\bar{d} \rightarrow ud$ ($d \rightarrow D^-$) | 2.76 | 3.72 |
| $u\bar{d} \rightarrow ud$ ($d \rightarrow D^+$) | 0.12 | 0.19 |
| Theory predictions | 22.93 | 35.94 |

Experiment NA27: $11.9 \pm 1.5$ E743: $26 \pm 4 \pm 25\%$

Fig. 3. Total cross section for $D^+ + D^-$ production. The experimental data are from Refs. [8] and [9]. The details of different calculations are explained in the figure.
The LHCb Collaboration has good experience in measuring the asymmetry in $D^+$ and $D^-$ production. Such an analysis can be done e.g. for fixed target experiment $p + ^4\text{He}$ with gaseous target. The nuclear effects for $^4\text{He}$ are rather small. Neglecting the nuclear effects, the differential cross section for production of $q/\bar{q}$ (particle 1) and associated parton (particle 2) can be written in the collinear factorization approach as

$$\frac{d\sigma_{p^4\text{He}}}{dy_1dy_2dp_T} = 2 \frac{d\sigma_{pp}}{dy_1dy_2dp_T} + 2 \frac{d\sigma_{pn}}{dy_1dy_2dp_T}. \quad (11)$$

In Fig. 4 we present the relevant predictions for the LHCb experiment. Rather large asymmetries are predicted which could be addressed in the expected analysis of the fixed target experiment.

Fig. 4. $A_{D^+D^-}(y)$ production asymmetry for the fixed target $p + ^4\text{He}$ reaction for $\sqrt{s} = 87 \text{ GeV}$.

In the traditional pQCD approach (production of $c/\bar{c}$ and only $c/\bar{c} \rightarrow D/\bar{D}$ fragmentation), the ratio defined as

$$R_{c/n} \equiv \frac{D^+ + D^-}{D^0 + \bar{D}^0} \quad (12)$$

is a constant, independent of collision energy and rapidity (or $x_F$). Inclusion of the unfavoured contribution changes the situation. In Fig. 5, we show the ratio as a function of meson pseudorapidity $\eta$ for LHC energies (left panel) and meson rapidity $y$ for $\sqrt{s} = 100 \text{ GeV}$ (right panel), taking into account the subleading contribution. At the LHC energies a very small, difficult to measure, effect is found for the LHCb transverse momentum and pseudorapidity range. At $\sqrt{s} = 100 \text{ GeV}$, we predict a strong rapidity dependence of the $R_{c/n}$ ratio. We think that fixed target experiments at the LHCb could address the issue.
Fig. 5. The $R_{c/n}$ ratio as a function of meson pseudorapidity for $\sqrt{s} = 7$ and 13 TeV for the LHCb kinematics (left panel) and as a function of meson rapidity for $\sqrt{s} = 100$ GeV in the full phase space (right panel). Only quark–gluon subleading components are included here.

Fig. 6. Distribution in $x_F$ for charged $D^+ + D^-$ (left panel) and neutral $D^0 + \bar{D}^0$ (right panel) $D$ mesons from conventional (solid lines) and subleading (shaded bands) mechanisms. The top panels are for $\sqrt{s} = 7$ TeV and the bottom panels are for $\sqrt{s} = 43$ TeV.
3.3. High energies

In this short subsection, we wish to show results relevant for high-energy prompt atmospheric neutrinos. As discussed recently in Ref. [2], a rather large $x_F \sim 0.5$ region is important in this context. The $d\sigma/dx_F$ distribution of mesons is the most appropriate distribution in this context.

In Fig. 6, we compare the conventional contribution corresponding to $c \to D$ fragmentation and the subleading one corresponding to $q \to D$ fragmentation, for the sum of $D^+ + D^-$ (left panels) and $D^0 + \bar{D}^0$ (right panels) mesons. While at small $x_F$ the conventional contribution dominates, at large $x_F$ the situation is reversed.

4. Conclusions

We have discussed asymmetry in production of $D^+$ and $D^-$ mesons in proton–proton collisions as described in our recent original paper [4], where for a first time we tried to understand whether the asymmetry observed by the LHCb Collaboration can be understood within parton fragmentation picture, including light quark and antiquark fragmentation functions.

Very small unfavoured fragmentation functions are sufficient to describe the LHCb data. The details depend however on functional form used. The corresponding fragmentation probability for $q/\bar{q} \to D$ is of the order of a fraction of 1%. In [4], we showed predictions for similar asymmetry for neutral $D$ mesons.

We predicted large contribution of the light quark/antiquark fragmentation to $D$ mesons at large $x_F$, which significantly exceeds the conventional $c/\bar{c} \to D$ contribution.

We calculated also the asymmetries for much lower energies ($\sqrt{s} = 20$–100 GeV), relevant for possible measurements in a near future. Much larger asymmetries were predicted, compared to those measured by the LHCb Collaboration. The asymmetries are associated with an increased production of charm in the $q/\bar{q}$ initiated hadronization. We quantified this effect by discussing corresponding asymmetries and rapidity distributions. The corresponding measurements at the fixed target LHCb, RHIC, and at SPS (NA61-SHINE) [10] would allow to pin down the discussed here mechanisms. Especially, the SPS experiment could/should observe an enhanced production of $D$ mesons.

Systematic studies of $D/\bar{D}$ asymmetries at low energies may be paradoxically important to understand the high-energy prompt component of the atmospheric neutrino flux. The predicted large contributions of $D$ mesons at large $x_F$ may have important consequences for prompt neutrino flux at large neutrino energies, relevant for the IceCube measurements. We found
that the contribution of the unfavoured fragmentation may be more important than the conventional one for large neutrino/antineutrino energies $E_\nu > 10^5$ GeV.

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