Analysis of the normalization theory of intracavity anti-Stokes laser

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Abstract. Anti-Stokes laser generation is a frequency up conversion. It is a four-wave mixing process of two fundamental photons, one Stokes photon, and one anti-Stokes photon. So, the conversion efficiency of the anti-Stokes laser is low. Therefore, it is particularly important to guide experiments through theoretical optimizations. In this paper, the rate equations under plane wave approximation of intracavity anti-Stokes lasers are normalized. By numerically solving the normalized equations, a group of adaptable curves are obtained to describe the operation of the intracavity anti-Stokes lasers. The influences of the composite variables on the performance of the lasers are analysed.

1. Introduction
With the deepening of research on crystal materials and the growing maturity of preparation technology, crystal Raman laser is one of the most important methods to expand the wavelength of laser, which has been a hot spot of research in solid lasers in recent years [1-7]. The use of anti-Stokes Raman scattering can achieve frequency up conversion, so the crystalline anti-Stokes laser is one important way to further expand the range of the existing laser spectrum [8-12].

Using accurate optimization theory to guide experiments can effectively improve the conversion efficiency of lasers. Rate equation is a simple, commonly used and effective theoretical tool to analyze the characteristics of laser operation. In this paper, the rate equation of the intracavity anti-Stokes laser is normalized under the plane wave approximation. A general theoretical model describing the operation of the intracavity anti-Stokes laser is obtained.

2. Rate equation
Ignoring the effect of spontaneous radiation in the laser medium, the spontaneous Raman scattering in the Raman medium and the effect of the lower level life in the laser medium on the laser operation. The plane wave approximation is adopted to describe the rate equations of the intracavity anti-Stokes laser [12]:

\[
\frac{d \phi_l}{dt} = \frac{2 \sigma n \phi_l}{t_{el}} - \frac{2 g h v_c \phi_l \phi_{rl}}{A_{lr}} - \frac{\phi_l}{t_{el}} L_l + \ln \left( \frac{1}{R_l} \right) \frac{d \phi_l(t)}{dt}_{FWM},
\]

\[
\frac{d \phi_s}{dt} = \frac{2 g h v_c \phi_l \phi_{rl}}{A_{lr}} - \frac{\phi_s}{t_{rs}} L_s + \ln \left( \frac{1}{R_s} \right) \frac{d \phi_s(t)}{dt}_{FWM},
\]

\[
\frac{dn}{dt} = -\gamma \sigma n \phi_l,
\]
where, \( \phi_l \) and \( \phi_s \) are the fundamental photon density in the gain medium and the Stokes photon intensity in the Raman medium, respectively, \( n \) is the population inversion density, \( \nu \) is the frequency of the Stokes laser, \( g \) is the Raman gain coefficient for the fundamental wave, \( c \) is the light speed in vacuum, \( l \) and \( l_R \) are the lengths of the gain medium and the Raman crystal, respectively; \( \gamma \) is the inversion reduction factor of the gain medium, \( \sigma \) is the stimulated emission cross section of the gain medium, \( \tau_{\text{rl}} \) and \( \tau_{\text{rs}} \) are the round-trip transit time of the fundamental wave, and the first and the second Stokes waves; \( n_s \) and \( n_{as} \) are the refractive indexes of the anti-Stokes wave and anti-Stokes wave in the Raman medium; \( L_l \) and \( L_s \) are the intrinsic losses of the fundamental and the first Stokes waves; \( R_l \) and \( R_s \) are the reflectivities of the output coupler at the fundamental and Stokes wavelengths; \( A_{lg} \) and \( A_{lr} \) are the beam areas of the fundamental wave in the gain medium and Raman medium.

Anti-Stokes scattering is four-wave mixing (FWM) process between two fundamental photons, one Stokes photon, and one anti-Stokes photon. In equation (1), \( \frac{d\phi_l}{dt}_{\text{FWM}} \) is the loss rate of the fundamental photon density caused by the FWM, \( \frac{d\phi_s}{dt}_{\text{FWM}} \) is the generation rate of the first Stokes photon density caused by the same process. The fundamental, Stokes and anti-Stokes waves satisfy the phase-matching condition in the Raman medium

\[
\frac{1}{2} \times \frac{l_{cl}}{l_{cs}} \frac{d\phi_l}{dt}_{\text{FWM}} = \frac{d\phi_s}{dt}_{\text{FWM}} = \frac{9h^2 c^3 \nu_l^2}{64 \pi^2 n_s^2 \nu_s l_{cs}} \phi_l^2 \phi_s, \tag{2}
\]

where, \( l_{cl} \) and \( l_{cs} \) are the optical lengths of the fundamental cavity and the Raman cavity, respectively; \( \nu_l \) and \( \nu_{as} \) are the frequencies of the fundamental and anti-Stokes waves, respectively.

By setting \( \phi_s=0, \frac{d\phi_l}{dt}=0, \frac{d\phi_s}{dt}=0, t=0 \) in equation (1), the threshold of the initial conversion population density of the laser medium is obtained

\[
n_{\text{th}} = \frac{L_t + \ln \left( \frac{1}{R_s} \right)}{2 \sigma l}. \tag{3}
\]

The normalized time \( \tau \), normalized fundamental photon densities \( \Phi_l \), normalized Stokes photon densities \( \Phi_s \), normalized initial population inversion density \( N \) are introduced

\[
\tau = \frac{t}{\tau_{\text{cl}}} \left[ \ln \left( \frac{1}{R_s} \right) + L_l \right], \quad \Phi_l = \frac{\phi_l}{L_t + \ln \left( \frac{1}{R_s} \right)}, \quad \Phi_s = \frac{\phi_s}{L_s + \ln \left( \frac{1}{R_s} \right)}, \quad N = \frac{n}{n_{\text{th}}}. \tag{4}
\]

Substituting equations (2)-(4) into equation (1), the normalized equation group of the intracavity anti-Stokes laser under plane wave approximation is yielded

\[
\frac{d\Phi_l}{d\tau} = N \Phi_l - M \Phi_l \Phi_s - \Phi_l - H \Phi_l^2 \Phi_s, \tag{5a}
\]

\[
\frac{d\Phi_s}{d\tau} = M \Phi_l \Phi_s - K \Phi_s + H' \Phi_l^2 \Phi_s, \tag{5b}
\]

\[
\frac{dN}{d\tau} = -N \Phi_l, \tag{5c}
\]

where, \( M \) is the normalize Raman gain; \( H \) and \( H' \) are the parameters reflecting the four-wave mixing capacity of Raman crystals; \( K \) is the ratio between the Stokes loss and the fundamental loss in the cavity.
\[ M = \frac{A_{00} g \hbar \nu_{c} c l_{0}}{A_{0}} L_{R} \gamma \sigma l_{s}, \]

\[ H = \frac{9 \hbar c^{2} \nu_{c} n_{2} g^{2} l_{0}^{2}}{64 \pi n_{2} \nu_{c} \gamma \sigma l_{s} l_{t}} \left[ L_{s} + \ln \left( \frac{1}{R_{l}} \right) \right] \frac{H'}{H} = \frac{L_{d} + \ln \left( \frac{1}{R_{l}} \right)}{2l_{s}}, K = \frac{L_{s} + \ln \left( \frac{1}{R_{l}} \right)}{L_{s} + \ln \left( \frac{1}{R_{l}} \right)}. \]

From equation (5c),

\[ N = N_{0} \exp \left[ -\int_{0}^{t} \Phi_{s} d\tau \right], \]

where, \( N_{0} = n_{0}/n_{th} \) is the normalized initial inversion population density. Substitute equation (7) into equation (5)

\[ \frac{d\Phi}{d\tau} = N_{0} \Phi \exp \left[ -\int_{0}^{t} \Phi_{s} d\tau \right] - M \Phi \Phi_{s} - \Phi_{s} - H \Phi_{s}^{2}, \]  

\[ \frac{d\Phi_{s}}{d\tau} = M \Phi^{2} \Phi_{s} - K \Phi_{s} + H' \Phi_{s}^{2}. \]  

3. Solution of the rate equation

The dependences of \( \Psi_{\text{max}} \) (the maximum of \( \Psi_{\text{as}} \)) and \( \Psi_{\text{integ}} \) on \( N_{0} \) with \( H = 0.5, H'/H = 2, \) and \( K = 0.5 \) for different \( M \) is shown in figure 1. When \( M = \) constant, \( \Psi_{\text{max}} \) and \( \Psi_{\text{integ}} \) increase with the increase of \( N_{0} \). \( M \) stands for Raman gain. That is to say, the larger the \( M \), the stronger the Stokes light. From figure 1, we can see that not the larger the \( M \) is, the stronger the output anti-Stokes is. When \( N \) is large, \( \Psi_{\text{max}} \left( N > 12 \right) \) and \( \Psi_{\text{integ}} \left( N > 9 \right) \) decrease with the increase of \( M \).

![Figure 1](image_url)
the conversion from the fundamental laser to the Stokes laser, but also reduces the photon density of the intracavity fundamental laser. So, the anti-Stokes light will not increase with the increase of the $M$. When the $N_0$ is large, the fundamental laser in the cavity is very strong. With the increase of $M$, the conversion efficiency of Stokes is increased. The fundamental laser which participated in FWM that generates anti-Stokes laser is weaker, and the output of the anti-Stokes light is also weaker.

![Figure 2. The typical pulse shapes of the fundamental wave, Stokes wave, and anti-Stokes wave](image)

The typical pulse shapes of the fundamental, Stokes and anti-Stokes pulses of the anti-Stokes laser are shown in figure 2. It is obvious that the anti-Stokes pulse is generated in the overlapped region of the fundamental and Stokes pulses. The rising time of the anti-Stokes pulse is mainly determined by the rising time of the Stokes pulse, and the falling time of the anti-Stokes pulse is mainly determined by the falling time of the fundamental frequency pulse. When the fundamental laser intensity reaches the threshold of the Stokes generating, the fundamental wave is rapidly converted to Stokes wave. With the increase of $N_0$, the rising edge of Stokes pulse and the descending edge of the fundamental laser pulse are steeped, which determines the rising and the falling times of the anti-Stokes pulse are shortened.

The dependences of $\Psi_{\text{max}}$ and $\Psi_{\text{integ}}$ on $H$ with $N_0=10$, $H'/H=2$, and $K=0.5$ for different $M$ are shown in figure 3. The $H$ factor represents the ability of the four-wave mixing. $\Psi_{\text{max}}$ and $\Psi_{\text{integ}}$ increase monotonously with the increase of $H$ firstly. However, the larger $H$, the more fundamental wave is consumed in the four-wave mixing effect, so when $H$ grows to a certain extent, $\Psi_{\text{max}}$ and $\Psi_{\text{integ}}$ will not increase quickly, but tend to be stable.

![Figure 3. Dependences of (a) $\Psi_{\text{max}}$ and (b) $\Psi_{\text{integ}}$ on $H$ with $N_0=10$, $H'/H=2$, and $K=0.5$ for different $M$](image)

The dependences of $\Psi_{\text{max}}$ and $\Psi_{\text{integ}}$ on $K$ with $N_0=10$, $H=0.5$, and $H'/H=2$ for different $M$ are shown in figure 4. Only when $M$ is small, the value of $K$ has a great influence on the output parameters of the anti-Stokes laser. Usually, for the lasers that realize nonlinear effect in the cavity, the intrinsic loss and output loss of the resonant cavity of the fundamental wave should be reduced as much as possible, that
is to make \( L + \ln \left( \frac{V_R}{R} \right) \) as small as possible to ensure that the power density of the fundamental wave in the cavity is strong enough to improve the conversion efficiency of the nonlinear effect. As can be seen from figure 6, for the intracavity anti-Stokes laser, when \( M \) is small, the loss of Stokes wave should be reduced as far as possible. For example, the light pass surfaces of the Raman medium are plated with anti-reflection coating of the Stokes wave and the resonator mirrors are plated with high reflection coating of the Stokes wave.

4. Conclusion

Based on the rate equation, the normalized equations of an intracavity anti-Stokes laser with plane wave approximation are derived. The relationship between the normalized laser output parameters and the normalized laser parameters is obtained by solving the equations numerically. This set of curves can fully reflect the operating characteristics of the intracavity anti-Stokes laser. The peak power, single pulse energy and pulse width of the output anti-Stokes laser can be simply estimated by the normalization theory. The normalization theory proposed in this paper can be used as an assistant tool for the design of intracavity anti-Stokes lasers, and can effectively improve the conversion efficiency of anti-Stokes lasers.

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