Contribution Rate of MWD Survey Error Sources on Well Trajectory Measurement Error

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Abstract. Since the number of error sources is too large and the standard deviations of the error sources are really difficult, it is of great practical significance to study the contribution rate of each error source on the wellbore trajectory measurement error. The error sources of well trajectory measurement using MWD (Measurement While Drilling) are introduced, and the contribution rate calculation model of each error source on well trajectory measurement error is established. The contribution rate of each error source on well trajectory measurement error is analyzed with an example. The research shows that the error sources contribute most to the well trajectory measurement error for each main semi-axes of the error ellipsoid are different; the error sources contribute most to the well trajectory measurement error are different in the vertical section, the inclined section and the horizontal section; the error source associated with the sensors contributes least to the well trajectory measurement error compared with other sources of error. In drilling engineering, it is necessary to give the standard deviation of the main error sources as accurately as possible to better evaluate the well trajectory measurement error.

1. Introduction
The determination of the wellbore position is the basis for well trajectory design, monitoring, and control. The measurement error of the wellhead position, the error of well survey calculation and the wellbore trajectory measurement error will lead to the wellbore position uncertainty, and the borehole trajectory measurement error is the main factor. With the broad application of complex structure wells such as cluster wells, double horizontal wells, connected wells and relief wells, it is increasingly necessary to reduce well trajectory measurement error or describe well trajectory measurement error accurately.

As early as the 1960s and 1970s, some foreign drilling companies began to study the well trajectory measurement error. The well trajectory measurement error calculation models that have been widely used include cone error model, WdW model, SESTEM model and ISCWSA (Industry Steering Committee on Wellbore Survey Accuracy) model[1-6]. The ISCWSA model has been recognized as an international industry standard. The ISCWSA model is a general term for the well trajectory measurement error calculation model introduced by ISCWSA, including models suitable for MWD and GMWD (Gyroscopic MWD). In the model applicable to MWD, it also includes the standard model and the model with axial correction. The number of the error sources in the ISCWSA model is large, and as many as 22 error sources are considered in the paper [2].
At the drilling engineering site, it is not an easy task to accurately get the standard deviation of these error sources. Therefore, it is of great practical significance to study the contribution rate of each error source on the well trajectory measurement error and to clarify the main error sources that need to accurately get the standard deviation.

2. The error sources considered in the ISCWSA-MWD model

2.1. Error sources ignored in the model
Considering the convenience of actual calculation and application and the effect degree of each error source on the well trajectory error, the ISCWSA-MWD model discards the following types of errors:
(1) Accuracy of tool electronics and accuracy of telemetry systems;
(2) Interference from an external magnetic field;
(3) The error caused by the spacing of the measuring points and the well survey calculation;
(4) Uncertainty of the gravity field;
(5) Gross errors.

2.2. Error sources considered in the model
Based on reasonable assumptions and the rejection of error sources, the following kinds of error sources are considered in the standard ISCWSA-MWD model, as shown in Table 1[2].

| Error Type          | Error Source                                  | No. |
|---------------------|----------------------------------------------|-----|
| Sensor Errors       | Accelerometer bias x-axis (ABX)              | 1   |
|                     | Accelerometer bias y-axis (ABY)              | 2   |
|                     | Accelerometer bias z-axis (ABZ)              | 3   |
|                     | Accelerometer scale x-axis (ASX)             | 4   |
|                     | Accelerometer scale y-axis (ASY)             | 5   |
|                     | Accelerometer scale z-axis (ASZ)             | 6   |
|                     | Magnetometer bias x-axis (MBX)               | 7   |
|                     | Magnetometer bias y-axis (MBY)               | 8   |
|                     | Magnetometer bias z-axis (MBZ)               | 9   |
|                     | Magnetometer scale x-axis (MSX)              | 10  |
|                     | Magnetometer scale y-axis (MSY)              | 11  |
|                     | Magnetometer scale z-axis (MSZ)              | 12  |
| Interference Errors | Constant axial magnetic interference (AZ)     | 13  |
|                     | Direction dependant axial magnetic interference (AMID) | 14  |
| Misalignment Errors | BHA sag (SAG)                                | 15  |
|                     | Tool axial misalignment – x-axis (MX)        | 16  |
|                     | Tool axial misalignment – x-axis (MY)        | 17  |
| Reference Field Errors | Constant declination error (AZD)            | 18  |
|                     | Direction dependant axial magnetic interference (DBH) | 19  |
| Depth Errors        | Depth reference random (DREF)                | 20  |
|                     | Depth scale (DSF)                            | 21  |
|                     | Depth stretch (DST)                          | 22  |

3. Calculation of well trajectory measurement error
Generally, in the field of well survey calculation, it is assumed that the well trajectory position coordinates obey a normal distribution. If \( \Delta s \) is used to indicate the position increment of the measurement point on the well trajectory, it is known from mathematical statistics that the distribution probability density of the point position in the three-dimensional space can be expressed as[7-11]: 
It can be obtained from the above formula, and its equal probability density surface is:

\[
f(\Delta s) = e^{-\frac{1}{2}(\Delta s)^T M^{-1}(\Delta s)} (2\pi)^{-\frac{1}{2}} \sqrt{|M|}
\]

(1)

where, \(\lambda\) represents the confidence factor, and here the value of \(\lambda\) is 2.7955 according to the multiple wells collision prevention convention. Eq. (2) represents an ellipsoid, and the error source accumulation matrix \(M\) in the equation is the covariance matrix of the wellbore position in the geographic coordinate system (N, E, H). Thus, the position error at each measurement point on the well trajectory can be described by an ellipsoid.

Assuming that \(\sigma_1, \sigma_2,\) and \(\sigma_3\) represent the eigenvalues of the covariance matrix \(M\), the three main semi-axis lengths \(r_1, r_2,\) and \(r_3\) of the error ellipsoid can be expressed as:

\[
\begin{cases}
  r_1 = \lambda \sqrt{\sigma_1} \\
  r_2 = \lambda \sqrt{\sigma_2} \\
  r_3 = \lambda \sqrt{\sigma_3}
\end{cases}
\]

(3)

The direction of the three major axes of the combined error ellipsoid is also the direction represented by the unit feature vectors \(p_{i1}, p_{i2},\) and \(p_{i3}\) of the covariance matrix \(M\).

4. Contribution rate of each error source on well trajectory measurement error

Since the position uncertainty at a certain measurement point of the wellbore trajectory can be determined by an error ellipsoid, the size of the error ellipsoid is determined by the length of the three semi-axes. Therefore, the contribution rate of the error source to the wellbore trajectory measurement error can be analyzed by calculating the contribution rate of the error source on the three main semi-axis lengths of the error ellipsoid. The contribution rate of each error source on the three main semi-axis length of the error ellipsoid can be expressed as:

\[
C_{ijk} = \frac{r_{ijk} - r_{ijk}}{r_{ijk}} \times 100\% \quad (j = 1, 2, 3; k = 1, 2, 3, \ldots, 22)
\]

(4)

where, \(C_{ijk}\) is the contribution rate of the \(k\)-th error source to the \(j\)-th semi-axis length of the error ellipsoid of the \(i\)-th measurement point; \(r_{ijk}\) is the \(j\)-th semi-axis length of the error ellipsoid of the \(i\)-th measurement point; \(r_{ijk}\) is the \(j\)-th semi-axis length of the error ellipsoid of the \(i\)-th measurement point after the default of the \(k\)-th error source.

The following result can be obtained from Eqs. (3) and (4):

\[
C_{ijk} = \frac{\sqrt{\sigma_{ijk}} - \sqrt{\sigma_{ijk}}}{\sqrt{\sigma_{ijk}}} \times 100\% \quad (j = 1, 2, 3; k = 1, 2, 3, \ldots, 22)
\]

(5)

Where, \(\sigma_{ij}\) is the \(j\)-th eigenvalue of the measurement error covariance matrix of the \(i\)-th measurement point; \(\sigma_{ijk}\) is the \(j\)-th eigenvalue of the measurement error covariance matrix of the \(i\)-th measurement point after the \(k\)-th error source is not considered. It can be seen from Eq. (5) that the contribution rate of each error source on the semi-axis length of the error ellipsoid is independent of \(\lambda\).

It should be noted that the sum of the contribution rates of each error source on a certain semi-axis length is not necessarily equal to 1, since the contribution of each error source is linearly related to the covariance matrix and is not linearly related to the semi-axis of the error ellipsoid. Therefore,
\sum_{k=1}^{22} C_{ijk} = 1, \text{or } \sum_{k=1}^{22} C_{ijk} \neq 1 \tag{6}

5. Case calculation and analysis

The partial survey data of a horizontal well (JY1084HF) obtained by MWD are shown in Table 2. The geomagnetic field intensity at the well location is 50054.5 nT, the geomagnetic dip angle is 46.53°, and the geomagnetic declination is 4.15°. According to the cluster wells anti-collision convention, the value of the confidence factor $\lambda$ is 2.976. Using the Standard ISCWSA MWD Model provided in the paper [2], the well position measurement error of the JY1084HF can be calculated as shown in Fig. 1.

| MD/m | INC/(°) | AZI/(°) | TVD/m  | N/m  | E/m  |
|------|---------|---------|--------|------|------|
| 0.00 | 0.00    | 0.00    | 0.00   | 0.00 | 0.00 |
| 1000.00 | 0.50   | 0.00    | 999.99 | 4.36 | 0.00 |
| 1500.00 | 0.80   | 0.00    | 1499.95| 10.04| 0.00 |
| 1600.00 | 15.80  | 357.68  | 1598.62| 24.42|-0.55 |
| 1715.78 | 33.17  | 357.61  | 1703.59| 72.18|-2.52 |
| 1800.00 | 33.17  | 357.61  | 1774.09| 118.21|-4.44 |
| \vdots | \vdots | \vdots  | \vdots  | \vdots | \vdots |
| 3389.96 | 33.17  | 357.61  | 3105.02| 987.29|-40.65 |
| 3500.00 | 44.70  | 16.64   | 3190.78| 1054.91|-30.75 |
| 3600.00 | 56.98  | 27.82   | 3253.93| 1126.09|-0.94 |
| 3700.00 | 70.01  | 36.16   | 3298.52| 1201.54|46.62 |
| 3800.00 | 83.36  | 43.19   | 3321.53| 1276.11|108.69 |
| 3831.37 | 87.58  | 45.29   | 3324.00| 1298.50|130.50 |
| 3900.00 | 87.58  | 45.29   | 3326.89| 1346.75|179.23 |
| \vdots | \vdots | \vdots  | \vdots  | \vdots | \vdots |
| 5000.00 | 87.58  | 45.29   | 3373.29| 2120.00|960.21 |
| 5253.93 | 87.58  | 45.29   | 3384.00| 2298.50|1140.50 |
| 5290.00 | 87.58  | 45.29   | 3385.52| 2323.85|1166.11 |

Figure 1. The well position measurement error of JY1084HF
The contribution rate of each error source on the three semi-axis lengths of the error ellipsoids of JY1084HF can be calculated (as shown in Figures 2-4). It can be seen from Fig. 2 that the sensor error sources contribute little to the error ellipsoid semi-axis $r_1$, and the error sources SAG, MX, AMID, DREF and DST contribute more to the semi-axis $r_1$ of the error ellipsoid; In the straight section of the JY1084HF, the error sources except MX, MY, DREF, DSF, and DST contribute negligibly to the semi-axis $r_1$ of the error ellipsoid.

It can be seen from Fig. 3 that the error sources MSX, MSY and MSZ contribute little to the semi-axis $r_2$ of the error ellipsoid, and the error sources SAG, MX and MY contribute more to the semi-axis $r_2$ of the error ellipsoid; In the straight section of the JY1084HF, the error sources except ABX, ABY, MX, MY, and DREF contribute negligibly to the semi-axis $r_2$ of the error ellipsoid.

It can be seen from Fig. 4 that the error sources ABZ, ASX and DSF contribute little to the semi-axis $r_3$ of the error ellipsoid, and the error sources AZD, AMID, AZ, SAG and MX contribute more to the semi-axis $r_3$ of the error ellipsoid.

In summary, the sensor error sources contribute less on the JY1084HF measurement error, misalignment errors (SAG, MX, MY), depth errors (DREF, DST, DSF), interference errors (AMID), and geomagnetic dip error (AZD) contribute more to the JY1084HF measurement error.

![Figure 2: Contribution rate of each error source on the semi-axis $r_1$ of the JY1084HF ellipsoid](image)
Figure 3. Contribution rate of each error source on the semi-axis $r_2$ of the JY1084HF ellipsoid

Figure 4. Contribution rate of each error source on the semi-axis $r_3$ of the JY1084HF ellipsoid
6. Conclusion
(1) The model of the contribution rate of each error source on the wellbore trajectory measurement error is simple and has a clear physical meaning. It can objectively reflect the variation of each source's contribution rate on the wellbore trajectory measurement error with the well measurement depth, and the sensitivity of each error source to the wellbore trajectory measurement error.

(2) For each semi-axis of the error ellipsoid, the error sources that play a major role are different. And in the vertical section, the build-up section and the horizontal section, the error sources that play a major role are also different.

(3) Compared with the other error sources, the sensor error sources contribute less to the wellbore trajectory measurement error. In the drilling engineering, it is necessary to get the standard deviations of the error sources except the sensor error sources as accurately as possible.

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