suppressing the Kibble-Zurek mechanism by a symmetry-violating bias

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The formation of topological defects in continuous phase transitions is driven by the Kibble-Zurek mechanism. Here we study the formation of single- and half-quantum vortices during transition to the polar phase of 3He in the presence of a symmetry-breaking bias provided by the applied magnetic field. We find that vortex formation is suppressed exponentially when the length scale associated with the bias field becomes smaller than the Kibble-Zurek length. We thus demonstrate an experimentally feasible shortcut to adiabaticity – an important aspect for further understanding of phase transitions as well as for engineering applications such as quantum computers or simulators.

In continuous phase transitions, random local choice of the symmetry-breaking order parameter leads to the formation of topological defects, such as quantized vortices. Originally a speculation in high-energy physics and cosmology [1], this mechanism, known as the Kibble-Zurek mechanism (KZM), [2–6], is now a cornerstone of out-of-equilibrium condensed matter physics. KZM has been observed in a range of systems such as superfluids, superconductors, and Bose condensates [7, 8]. In the KZM scenario the transition takes place independently in various regions with the characteristic size depending on the transition rate. Each region inherits a random realization of the broken-symmetry feature of the new phase, such as the phase of the order parameter in a superfluid transition. When the expanding regions merge, topological defects, such as quantized vortices, are formed. The predicted power-law dependence of the defect density on the quench rate has been confirmed in superfluid helium [9, 10] as well as in other systems (see e.g. [7, 8, 11]).

In the theory of broken-symmetry phase transitions, a symmetry-violating bias field plays an important role, initiating the choice between the different degenerate states [12]. Bias can in particular be applied to non-adiabatic thermodynamic [1, 2] or quantum [5] phase transitions that result in the formation of topological defects via the KZM. It has been proposed that if the applied symmetry-breaking bias is sufficiently large, the adiabatic (defect-free) regime is restored [13]. The crossover from the Kibble-Zurek regime to the adiabatic regime occurs at the characteristic value of the bias defined by the quench rate. Such crossover has been analyzed theoretically in a quantum phase transition in the Ising chain [13, 14] and in its classical counterpart [15]. Generally speaking, the KZM is expected to be modified in the presence of external factors such as inhomogeneities [16], or a propagating front of the phase transition [17]. Applying a bias allows for the external control of the magnitude of the KZM directly. Controlled restoration of the adiabatic regime by a symmetry-breaking bias can be utilized in applications requiring delicate and fast control of engineered quantum systems [13, 18].

In this Letter we probe experimentally the use of an external bias for suppressing the formation of single quantum vortices (SQV) and half-quantum vortices (HQV) [9, 19, 20] produced by the KZM in the phase transition from normal 3He to the superfluid polar phase [21]. We report three central observations: (i) For HQVs the threshold bias for the onset of suppression is set by matching the characteristic length of the applied symmetry-breaking field to the Kibble-Zurek length, set by the quench rate. (ii) Beyond the onset, the suppression takes over exponentially, with the onset threshold normalizing the bias field in the exponent. (iii) The creation of SQVs is similarly suppressed for increasing bias fields while the threshold value is different from that for HQVs.

The spectrum of topological defects in the polar phase, and the bias fields one can apply, are understood in terms of the order parameter of the polar phase

\[ A_{j\beta} = \Delta_P \hat{d}_j \hat{m}_\beta e^{i\Phi}. \]

Here \( \Delta_P \) is the maximum gap in the quasiparticle energy spectrum and \( \Phi \) is the superfluid phase. The unit vector \( \hat{d} \) determines the direction of the easy plane of the magnetic anisotropy and \( \hat{m} \) that of the orbital anisotropy. The anisotropy originates from p-wave Cooper pairing with the orbital momentum and spin of a pair equal to one. In the p-wave superfluid, confinement modifies the resulting order parameter [21, 23–28]. The polar phase is stabilized within the confining nanomaterial, which consists of nearly parallel solid strands, and \( \hat{m} \) is pinned along the strand direction, Fig. 1(a). The direction of \( \hat{d} \) is set by the competition between the magnetic anisotropy energy \( \chi(\hat{d} \cdot \mathbf{H})^2/2 \) in the magnetic field \( \mathbf{H} \), and the spin-orbit interaction energy \( g_{so}(\hat{d} \cdot \mathbf{m})^2 \), where \( \chi \) is the magnetic susceptibility and \( g_{so} \) is the spin-orbit coupling.

In large magnetic fields \( H^2 \gg H^2_{so} = g_{so}\chi^{-1} \), the \( \hat{d} \) vector is kept in the plane perpendicular to \( \mathbf{H} \) and the spin-orbit interaction takes the form

\[ F_{so} = g_{so} \sin^2 \theta \sin^2 \alpha, \]

where \( \alpha \) and \( \theta \) are polar and azimuthal angles in the spin-orbit plane, respectively.
Here $G$ describes the symmetries of normal $^3\text{He}$, $U(1)$ is the symmetry under the phase transformation, and $SO(2)$ is the symmetry under rotation in spin space about the axis of the magnetic field. $\Upsilon$ denotes the symmetry of the polar phase order parameter, where $Z_2$ is the spin rotation by $\pi$ (corresponding to the change $\hat{d} \rightarrow -\hat{d}$) accompanied by the phase change by $\pi$. Since the homotopy group $\pi_1(G/\Upsilon) = Z \times Z \times Z_2$, this symmetry-breaking scheme leads to three types of topological defects: SQVs in the superfluid phase ($\Phi$-field), spin vortices in the orientation of the spin anisotropy vector ($\alpha$-field), and HQVs, where both the superfluid phase $\Phi$ and the angle of the spin anisotropy vector $\alpha$ change by $\pi$.

When the spin-orbit interaction is turned on ($\mu \neq 0$), the $SO(2)$ symmetry in Eq.(3) is explicitly violated, and one obtains the following symmetry-breaking scheme:

\[
\tilde{G} = U(1) \to \tilde{\Upsilon} = 1 .
\]

Now the homotopy group is $\pi_1(\tilde{G}/\tilde{\Upsilon}) = Z$, which means that only SQVs remain stable, since they are not influenced by the spin-orbit interaction. Spin vortices and HQVs become termination lines of topological solitons [9, 19, 29–31], illustrated in Fig. 1(b). Assuming only HQVs are present, $\hat{d}$-solitons connect pairs of HQVs of the opposite $\hat{d}$-winding.

The presence of the solitons can be detected and their total volume in the sample measured using the nuclear magnetic resonance (NMR) techniques. The bulk of the sample forms the main peak in the continuous-wave NMR spectrum at the frequency $\omega_{\text{main}}$; see Fig. S1 in Ref. [15]. The $\hat{d}$-soliton provides a trapping potential for standing spin waves, seen as a satellite peak in the NMR spectrum at the frequency $\omega_{\text{sat}}$ [9]. The relative sizes of the main peak and the satellite are determined by the volume occupied by the $\hat{d}$-solitons in the sample. We note that the vortices created by the KZM are randomly oriented, but in our case the vortex density is low and thus the soliton volume connecting two HQVs is simply defined by the inter-vortex distance [15]. Measuring the initial density of KZ defects has traditionally been difficult due to the fast annihilation of non-equilibrium defects at temperatures close to the phase transition [3, 4, 10, 32]. In our experiments the confining strands pin vortices in place [9, 19, 30], providing the observer a frozen window to the out-of-equilibrium physics of the phase transition and a direct measurement of the KZ vortex density.

We calibrate the size of the satellite peak by preparing a state by a very slow cooldown through the critical temperature $T_c$ at $H = 0$ while the sample is in rotation. This way we create HQVs with aerial density $n_v = 4\Omega\kappa^{-1}$, where $\Omega$ is the angular velocity, $\kappa = h/(2m_3)$ is the quantum of circulation, $h$ is the Planck constant, and $m_3$ is the $^3\text{He}$ atom mass. The calibration gives the relative satellite size $I_{\text{sat}} = I_0/\sqrt{\Omega}$, where $I_0 = 0.090$ s$^{1/2}$ rad$^{-1/2}$ [15]. The inter-vortex distance assuming a square lattice is

\[
L = n_v^{-1/2} = \frac{1}{2} \sqrt{\kappa} I_0 / I_{\text{sat}} .
\]

We use this relation to calculate the HQV density and inter-vortex distance also for HQVs created purely by the
KZM (i.e. for Ω = 0). The combined effect of rotation and KZM is discussed in Ref. [15].

We control the spin-orbit bias by applying a fixed magnetic field of $H = 11\,\text{mT}$ with transverse component $H_\perp = H\sin\mu$. Open squares ($\tau_Q \approx 6.0 \cdot 10^2\,\text{s}$) correspond to measurements with zero axial field component, $H_\perp = H$. Vortex density is constant for $H_\perp < H_\perp$ and suppressed for higher bias fields. The suppression starts when the characteristic length scale of the bias field $\xi_{bias}(H_\perp)$ becomes smaller than the relevant Kibble-Zurek length. Solid lines correspond to theoretical model, see text for details. The dashed line shows where the intervortex distance becomes comparable with the container size. The inset shows the extracted threshold bias length as a function of $H_\perp$. We suggest that the threshold field $H_{\perp}^\text{bias}$, given by the thickness of the gray shaded region, which correspond to the spin-orbit bias with similar cooldown rates, Fig. 2. We observe a constant satellite size for small $H_\perp$ and its gradual suppression for larger values of $H_\perp$. We suggest that the threshold field $H_{\perp}^\text{bias}$ where the suppression of the formation of HQVs starts is determined by comparing the Kibble-Zurek length $l_{KZ} = a_0\tau_Q/\tau_0^{1/4}$ with the characteristic length of the bias, $\xi_{bias}$, given by the thickness of the d solitons. Here $a \sim 1$ fixes the exact length scale for the defect formation (in our measurements $a \approx 2.3$ [15]), the quench rate is $\tau_Q^{-1} = -d(T/T_c)/d|T-T_c|$, $T$ is temperature, $t$ is time, $\tau_0$ is the superfluid coherence length at low temperature, $\tau_0 = \xi_0v_F^{-1} \sim 1\,\text{ns}$ is the order parameter relaxation time, $v_F$ is the Fermi velocity, $\xi_{bias} \sim \xi_{so}/\sin\mu$, and $\xi_{so} = 17\,\mu\text{m}$ is the dipole length [9].

Equating $\xi_{KZ}$ with $\xi_{bias}$ gives the following threshold bias for the suppression of HQV creation

$$H_{\perp} = \frac{\xi_{so}}{l_{KZ}} H.$$  \hfill (6)

In the spirit of Ref. [13] we propose that the defect density $\alpha I_{\text{sat}}^2$ decays exponentially after the transition field. In terms of the satellite intensity, this reads

$$I_{\text{sat}} = \begin{cases} I_{\text{sat}0} & \text{for } H_{\perp} < H_{\perp}^t, \\ I_{\text{sat}0}\exp(1 - H_{\perp}/H_{\perp}^t) & \text{for } H_{\perp} \geq H_{\perp}^t, \end{cases}$$  \hfill (7)

where $I_{\text{sat}0}$ is the initial satellite intensity. We note that for this model $\int_0^{\infty} I_{\text{sat}} dH_{\perp} = 2I_{\text{sat}0} H_{\perp}^t$ and the numerical integral of the measured $I_{\text{sat}}$ can be used to determine $H_{\perp}^t$ without fitting.

Our experiments, Fig. 2, confirm the validity of the model (7). We use the zero-bias inter-vortex distance $L|_{H_{\perp}=0}$, Eq. (5), as the measured value of $l_{KZ}$ [9, 32, 33]. We emphasize that the threshold field $H_{\perp}^t$, which also normalizes the exponent, is determined by integration of the experimental data without a fitting procedure. The result agrees well with the conjecture $\xi_t \equiv \xi_{bias}(H_{\perp}^t) = l_{KZ}$. The result for the slowest quench rate deviates, however, from this dependence. In the presence of a thermal gradient, the phase transition proceeds via a propagating front, where the ordering of the low-temperature phase lags behind the temperature front where $T = T_c$ by distance $l_\perp$. The KZM operates in the band of width $l_\perp$ and is modified in comparison to the homogeneous cooling scenario [17, 34, 35]. As $\tau_Q$ increases, $l_\perp$ decreases and $l_{KZ}$ increases. We suggest that the smaller of the two characteristic lengths, $l_{KZ}$ and $l_\perp$, determines the threshold bias $\xi_t$. We estimate that in our measurement $l_\perp < l_{KZ}$ only for the slowest quench rate (black diamonds in Fig. 2) for which $l_\perp \sim 210\,\mu\text{m}$ (gray patterned diamond in Fig. 2 inset), matching the observed value of $\xi_t$ [15].

Alternatively we can apply a direct field bias with a weak magnetic field oriented perpendicular to $\hat{m}$. The small magnetic field $H_{\perp} < H_{so}$ violates the symmetry under rotation about $\hat{m}$, which leads to the formation of solitons, absent at zero magnetic field, with the soliton thickness now determined by the magnetic field directly, $\xi_{bias} = \xi_t = \xi_{so} H_{so}/H$. Equating $\xi_{bias}$ with $l_{KZ}$ yields a criterion for the threshold field similar to that in Eq. (6) but with $H$ replaced by $H_{so}$. The expected decrease of the threshold field in this case is $H_{so}/H = \Omega_F/\omega_{\text{main}} \approx \sqrt{2(1 - \omega_{\text{sat}}/\omega_{\text{main}})} \approx 0.17$ [9], where $\Omega_F$ is the polar phase Leggett frequency. It is confirmed experimentally by the blue squares in Fig. 2.

Here the ratio of the threshold field relative to the red circles, which correspond to the spin-orbit bias with similar quench rate, is 0.16.
anisotropy may play the role of the symmetry-violating field. Due to the absence of the topological solitons one would naïvely expect that the bias has no effect on the KZM for SQVs. Without the solitons, SQVs are not seen in continuous-wave NMR, but they were found to increase the relaxation rate of a magnon BEC [30, 36]. Independent measurements with SQVs created by rotation indicate that the BEC relaxation rate increases monotonically when the SQV density grows [30]. In our measurements we create both HQVs and SQVs by the KZM and subtract the effect of HQVs by using a calibration array of HQVs created by rotating the sample has no effect on the KZM even when the characteristic length scale of the added lattice becomes smaller than the Kibble-Zurek length. We also note that HQVs are composite defects, whose KZM formation is rarely studied experimentally, and that they are analogs of Alice strings [40–42]. The KZM formation of HQVs studied here may shed light to defect formation across phase transitions in theories considering such systems.

Our results can be generalized to the bias-induced restoration of adiabaticity in various phase transitions including quantum phase transitions, which could provide applications for technologies such as quantum simulators and computers [13, 18]. On a more speculative note, it is not excluded that the bias plays a role in the so-called collapse of the wave function in quantum mechanics. In principle, the latter can be seen as “phase transition” occurring in the continuous spectrum of an infinite system [43–45]. One of the many quantum states participating in a given quantum superposition is perhaps then selected by the infinitesimal bias unavoidably present in any experiment.

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