Hints of higher twist effects in the slope of the proton structure function

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Abstract: We critically analyse the data available on the reduced cross-section in deeply inelastic $e\, p$ scattering from the H1 collaboration at HERA. We use available data on the longitudinal structure function to deduce the nature of $\partial F_2 / \partial \ln Q^2$ at different $Q^2$ for fixed values of $x$ near $x \sim 10^{-4}$. We present the results in a manner which effectively isolates possible higher twist effects in the structure function $F_2$.

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Deeply inelastic scattering (DIS) at the $e\, p$ collider at HERA has provided precision data on the proton structure functions over the last few years. Data on the slope of the structure function, $F_2(x, Q^2)$, was presented for the first time last year [1, 2]. This showed a surprising dip in the slope, $\partial F_2 / \partial \ln Q^2$, below $x \sim 10^{-4}$ (see Fig. 1), although $F_2$ itself continued to show a rise towards smaller $x$ down to the kinematical limit. Such a dip was not anticipated or predicted by available parametrisations at that time [3]. There have since been intensive discussions on this effect using leading twist as well as higher twist contributions within a perturbative framework. In fact, there now exist new parametrisations [4, 5], which attempt to incorporate this effect, albeit in a leading twist analysis. The case for higher twist effects is not yet overwhelming [6] although they occur naturally in an operator product approach to DIS [7]. In this context, it is relevant to ask whether there exist other data or methods which, when combined with existing data on $\partial F_2 / \partial \ln Q^2$, can clearly indicate the presence of higher twist terms, which have long been poorly understood in DIS. This letter addresses this issue. In particular, we show that the slope of the reduced DIS cross-section, $\sigma_r$, defined in eq. (1) below, is very sensitive to the slope of $F_2$. Along with available data on the longitudinal structure function, $F_L(x, Q^2)$, we show that the slope of $\sigma_r$ yields information on the nature of the higher twist content of $\partial F_2 / \partial \ln Q^2$ and hence that of $F_2$.

Recently, the H1 collaboration [8] at HERA has measured the reduced cross-
In order to do this, we shall separately analyse the small $y$ related. This is because, from eq. (1), we have
\[ \sigma_r(x, Q^2) = \left[ F_2(x, Q^2) - \frac{y^2}{Y_+} F_L(x, Q^2) \right], \]
which is a clear indicator of the size of the longitudinal structure function, $F_L$, compared to that of $F_2$, in the measured deep inelastic $e p$ cross-section. Here $y$ is determined by $Q^2 = sxy$ with $Y_+ = 1 + (1 - y)^2$, where $s$ is the square of the total cm energy (which is constant at HERA) and the two kinematical variables, $Q^2$ and $x$, are as usual the momentum transfer and Bjorken scaling variable respectively.

The quantity $\sigma_r$ has been independently measured as a function of both $x$ and $Q^2$. Hence, the slope of $\sigma_r$ with respect to either $x$ or $Q^2$, with the other variable kept fixed, can be determined. Furthermore, since $y$ lies between 0 and 1, it is clear that $F_L$ contributes significantly to $\sigma_r$ only at fairly large values of $y$. For example, even when $y = 0.5$, the quantity multiplying $F_L$ in $\sigma_r$ is only $f_2 \equiv y^2/Y_+ = 0.2$. Finally, $F_L = 0$ exactly, at leading order, due to the Callan-Gross relation and is non-zero only at next-to-leading order. Hence, $F_L$ is suppressed (by at least one power of $\alpha_S$) compared to $F_2$. As a consequence, at low $y$, $\sigma_r$ is essentially determined by $F_2$; conversely, any study of $F_L$ must therefore be made in the large $y$ region for maximum sensitivity.

The data on $\partial F_2/\partial \ln Q^2$ is averaged over $Q^2$, with the average $\langle Q^2 \rangle$ increasing with increasing $x$ in such a way that all the data corresponds essentially to low $y \lesssim 0.3$; hence, it is relevant to ask whether the behaviour shown in Fig. 1 persists at all $Q^2$. In short, what is the $Q^2$ dependence of $\partial F_2/\partial \ln Q^2$ at fixed $x$ values?

We analyse the H1 $\sigma_r$ [8] data with a view to extracting such a $Q^2$ dependence. In order to do this, we shall separately analyse the small $y$ and large $y$ $\sigma_r$ data. To begin with, we recognise that the $Q^2$ dependences of $F_L$ and $\partial F_2/\partial \ln Q^2$ are related. This is because, from eq. (1), we have
\begin{align*}
S_x \equiv \left. \frac{\partial \sigma_r}{\partial \ln y} \right|_x &= \frac{\partial F_2}{\partial \ln Q^2} - f_1 F_L - f_2 \frac{\partial F_L}{\partial \ln Q^2}; \\
S_q \equiv \left. \frac{\partial \sigma_r}{\partial \ln Q^2} \right|_{Q^2} &= -\frac{\partial F_2}{\partial \ln x} - f_1 F_L + f_2 \frac{\partial F_L}{\partial \ln x}.
\end{align*}
Here the factors $f_2 = y^2/Y_+$ and $f_1 = 2y^2(2-y)/Y_+^2$ are significant only for large $y$. In particular, $f_1 \sim 1$ when $y = 0.7$; furthermore, $f_1 \sim 2f_2$ over the entire $y$ range. It is observed that $\sigma_r$ is a fairly linear function of $\ln y$ at all $y$ as can be seen from Fig. 2, where $\sigma_r$ has been plotted as a function of $y$ for some selected $x$ values ranging from $x = 10^{-4}$ to $10^{-2}$. Hence the slope $S_x$ can be obtained (at these different $x$ values) from straight line fits to the $\sigma_r$ data.

**The small $y$ data**: The $y$-derivative of $\sigma_r$ (whether at constant $x$ or at constant $Q^2$) is insensitive to $F_L$ or its slope when $y$ is small. In other words,
the behaviour of $S_x$ at low $y$ values (and hence low $Q^2$ for a given $x$ value) directly constrains the slope of $F_2$. The resulting slopes, $\partial F_2/\partial \ln Q^2$, are shown in comparison with those extracted differently by ZEUS [4] in Fig. 3. Note that in our calculation we have obtained the slope at fixed $Q^2$, equal to the average $Q^2$ of the corresponding ZEUS data for different values of $x$. All the points have $y \lesssim 0.3$; in fact the large $x$ ($x > 10^{-3}$) data have $y \lesssim 0.15$. The error bars in our extraction of the slope are due only to the errors arising from the straight–line fit to the $\sigma_r$ data while the ZEUS data include both statistical and systematic errors. We see that there is good agreement between the two data sets, leading us to conclude that $f_1 F_L \ll \partial F_2/\partial \ln Q^2$, so that $S_x$ in eq. (2) in indeed saturated by $\partial F_2/\partial \ln Q^2$ at small $y$. The specific values of the slope, $\partial F_2/\partial \ln Q^2$ for certain $x$ and $Q^2$ values have been shown in Table 1 labelled as “small $y$” data; we shall need this later on in our analysis.

The large $y$ data: In the large $y$ region, the $F_L$ contribution can no longer be neglected. We therefore use the value of $F_L$ determined by the H1 collaboration [3] at various $x$ values, for fixed $Q^2$ (corresponding to large $y = 0.7$) to analyse the large $y S_x$ data. Hence the large $y$ analysis will be restricted to $y = 0.7$. Since $F_L$ has contributions only at NLO, its slope ($\partial F_L/\partial \ln Q^2$) is rather small; this term is further suppressed by $f_2$ ($\sim 0.45$ at $y = 0.7$). Hence, we neglect the contribution of the slope of $F_L$ in what follows. We shall comment on the validity of this approximation later on. Then $S_x$ in eq. (2), along with the data on $F_L$, yields a value for $\partial F_2/\partial \ln Q^2|_x$ at large $y = 0.7$ within this approximation. Sparse data is available for $F_L$ in the region $1 \times 10^{-4} \lesssim x \lesssim 6 \times 10^{-4}$, hence we

| $x$ | $Q^2$ | $y$ | $\partial F_2/\partial \ln Q^2$ ($\equiv \partial \sigma_r/\partial \ln y|_x$) |
|-----|-------|-----|-------------------------------------------------|
| 1.4 $10^{-4}$ | 3.85 | 0.305 | 0.371 ± 0.017 |
| 2.4 $10^{-4}$ | 5.24 | 0.242 | 0.376 ± 0.010 |
| 4.0 $10^{-4}$ | 8.76 | 0.243 | 0.345 ± 0.005 |
| 6.2 $10^{-4}$ | 10.63 | 0.190 | 0.338 ± 0.010 |

| $x$ | $Q^2$ | $F_L$ | $\partial F_2/\partial \ln Q^2$ ($\equiv \partial \sigma_r/\partial \ln y|_x + f_1 F_L$) |
|-----|-------|-------|-------------------------------------------------|
| 1.4 $10^{-4}$ | 8.84 | 0.51 ± 0.3 | 0.921 ± 0.32 |
| 2.4 $10^{-4}$ | 15.2 | 0.35 ± 0.3 | 0.756 ± 0.32 |
| 4.0 $10^{-4}$ | 25.3 | 0.33 ± 0.27 | 0.675 ± 0.29 |
| 6.2 $10^{-4}$ | 34.7 | 0.39 ± 0.28 | 0.758 ± 0.30 |

Table 1: The slope $\partial F_2/\partial \ln Q^2$ evaluated using eq. 2 for different (small) $x$ values for the small and large $y$ ($= 0.7$) data. Note that $\partial \sigma_r/\partial \ln y|_x$ is constant for all $y$ (see Fig. 2) and directly equals $\partial F_2/\partial \ln Q^2$ at small $y$. The data on $F_L$ are taken from the H1 Collab [3].
can extract $\partial F_2/\partial \ln Q^2$ only at these $x$ values. These values (which we refer to as “large $y$” data) at different $Q^2$, but at the same $x$ values as the “small $y$” data, are shown in Table 1. The “large $y$” sample obviously corresponds to a larger $Q^2$ than the “small $y$” data at a given $x$; however, note that the average $Q^2$ in the sample we have analysed increases with $x$. The large error bars (much larger than that of the small $y$ data) are essentially due to large errors in the $F_L$ data.

We have therefore extracted $\partial F_2/\partial \ln Q^2$ as a function of $x$ in the range $1–6 \times 10^{-4}$. These values are listed in Table 1. At each $x$ value, we have obtained $\partial F_2/\partial \ln Q^2$ at two different $Q^2$ values corresponding to small and large $y$ data as discussed above. Note that in all cases $Q^2 \gtrsim 4$ GeV$^2$, which corresponds to a fairly stable perturbative regime.

We use the NLO GRV (1994) [11], GRV (1998) [4] and MRS (1998) [5] parametrisations as typical indicators of the theoretical expectation based on purely twist–two perturbative DGLAP [12] evolution equations. These predict a primarily logarithmic dependence of $F_2$ on $Q^2$ (along with a small $1/Q^2$ piece from the heavy quark contributions). This implies that the slope, $\partial F_2/\partial \ln Q^2$, at fixed $x$ is essentially flat, with a small (positive) slope due to the charm quark contribution.

In Fig. 4 we show the extracted large and small $y$ data samples as a function of $x$ along with the NLO fits from the different parametrisation sets at the same $(x, Q^2)$ values as the data. Since $\partial F_2/\partial \ln Q^2$ increases with $Q^2$, the upper points correspond to the large $y$ (and hence larger $Q^2$) sample (See Table 1). The bigger error bars on these points are due to the larger uncertainties in $F_L$. The errors on the small $y$ sample arise from the errors on our fits to the slope of $\sigma_r$ and are much smaller. (The error bars on the $\sigma_r$ data are very small except at the edges of the kinematically accessible regions; we have included them in the fits to the slopes but not in the error estimates of the slope). We have also plotted the values for $\partial F_2/\partial \ln Q^2$ (averaged over $Q^2$) obtained by ZEUS [1] at the same $x$ values, at the same average $Q^2$ as the small $y$ data, for comparison.

We see that all parametrisations are consistent (to within 1σ) with the large $y$ data. In the case of the small $y$ data, which also corresponds to the smaller $Q^2$ sample, the requirements are more stringent due to the smaller error bars. The 1998 fits are in general a better fit than the 1994 one. However, it is clear that if the error bars decrease due to more and better data being made available, while the values remain near the current central ones, a pure change in normalisation will not suffice to fit both the data sets. This is because the central values of the two data sets differ by more than 100%, while the parametrisations differ by less than 50%. It therefore appears that the small $y$ (or equivalently, small $Q^2$) data are suppressed relatively more than the large $y$ data. In particular, the

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Footnote 3: The number of $x$ values is restricted by $F_L$ data. There exists more data on $F_L$ from H1 [10] that has been extracted using data on $S_q$. This data is consistent with the existing ones in the region of overlap, but we shall not use them here since the data are at slightly different $y$ values.
separation between the data at the smallest $x$ value ($= 1.4 \times 10^{-4}$) clearly indicates a substantial evolution of $\partial F_2/\partial \ln Q^2$ from $Q^2 \sim 4$ to 9 GeV$^2$ (See Table 1). Such a behaviour can be attributed to higher twist effects. To elaborate further, $F_2$ can be expressed in terms of a leading twist and higher twist part, as

$$F_2 = C_{LT} \ln Q^2 + \frac{C_{HT}}{Q^2},$$

where the coefficients are in general functions of $x$ and we have included only an additional twist–4 piece. This results in a $Q^2$ dependence of $\partial F_2/\partial \ln Q^2$ of the form

$$\frac{\partial F_2}{\partial \ln Q^2} = C_{LT} - \frac{C_{HT}}{Q^2}.$$  \hspace{1cm} (3)

That $\partial F_2/\partial \ln Q^2$ evaluated from the parametrisations at a given $x$ decreases with $Q^2$ is due to threshold effects from terms like $m_c^2/Q^2$ occurring in the charm contribution. Any further suppression of $\partial F_2/\partial \ln Q^2$ with decreasing $Q^2$ then arises due to higher twist effects coming from the light quark sector and can explain the trend of the data shown in Fig. 4. Such an effect will be particularly visible for smaller $Q^2$ data, such as that corresponding to the smallest $x$ value in Fig. 4: this results in the small $y$ data being suppressed more than the large $y$ data. This explains the observed agreement of the large $y$ data with existing twist–2 parametrisations while the small $y$ data at the same $x$ value disagree by as much as $2\sigma$. This disagreement disappears with increasing $x$ for both large $y$ and small $y$ data since $Q^2$ also increases with $x$. It is rather surprising that higher twist effects are visible at such seemingly large $Q^2$. However, it must be remembered that this is the first time that the slope of $F_2$ has been so precisely measured. It thus appears that the $Q^2$ dependence of the slope, $\partial F_2/\partial \ln Q^2$, can provide a more sensitive test of the $Q^2$ dependence of $F_2$ and hence of the elusive higher twist effects in deep inelastic scattering.

Finally, we address the issue of the slope of the longitudinal structure function, $\partial F_L/\partial \ln Q^2$, in eq. (2), which has been neglected in this analysis. We estimate the size of this contribution using the GRV (1994) parametrisation. We find that this quantity does not exceed 0.1 for any of the $(x, Q^2)$ values of interest here. Furthermore, its contribution at small $y$ is suppressed because of $f_2$; $f_2 < 0.1$ for the small $y$ sample, so that this contribution never exceeds a percent. For the large $y$ sample, $f_2$ is larger, $f_2 \sim 0.45$; however, the slope of $F_L$ at these $Q^2$ values is small enough so that the term contributes less than about 5%, which is small compared to the size of the error bars of this data set. Hence it is reasonable and consistent to ignore the contribution from this term.

In conclusion, we have analysed a limited sample of the HERA H1 data on the reduced cross-section, $\sigma_r$, along with available data on the longitudinal structure function, $F_L$, in order to study the $Q^2$ dependence of $\partial F_2/\partial \ln Q^2$ in a perturbative regime. A comparison with available twist–2 NLO parametrisations shows indications of large $Q^2$ dependences in the data for $\partial F_2/\partial \ln Q^2$—larger
than that indicated by purely twist–2 behaviour. This is especially true for small $x$ values ($\sim 10^{-4}$), which also correspond to a smaller $Q^2 (\sim 4 \text{ GeV}^2)$, indicating that substantial higher twist effects may be operative here. While the effect is clearly marked only in the first data point, and the data analysed is obviously limited, the trend of the data is tantalisingly similar to that expected from higher twist effects. We therefore urge a detailed analysis of the $\sigma_r$ data at various $(x, Q^2)$ values in order to shed more light on the role of higher twists in $F_2$. More data (and improved errors on $F_L$) will be needed to refine this observation.

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Figure 1: Preliminary data on $\partial F_2 / \partial \ln Q^2$ as a function of $x$ from ZEUS; the graph is taken from reference [1]. The average $Q^2$ corresponding to each $x$ bin is also shown.
Figure 2: The data for the reduced cross-section, $\sigma_r$, defined in eq. (1), and taken from [8], shown as a function of $y$ along with our straight line fits to $\sigma_r$ as a function of $\ln y$ for various $x$ values.
Figure 3: The slope $\partial F_2/\partial \ln Q^2$ as obtained from $\sigma_r$ data (solid circles) as described in the text (small $y$ sample) shown as a function of $x$ in comparison with ZEUS preliminary data (crosses) [1]. The $Q^2$ of the points equals the average of that of the ZEUS data and is shown in Fig. 1.
Figure 4: The slope $\partial F_2 / \partial \ln Q^2$ (see Table 1) shown as a function of $x$ in comparison with standard parametrisations, as obtained from the data given in Refs. [8, 9] (see text for details). The upper (lower) circles correspond to the large (small) $y$ data with the corresponding parametrisations shown as solid (dashed) lines. The preliminary ZEUS data [1] (at small $y$) are also shown (as crosses) in the figure.