Numerical analysis of gas leakage in the piston-cylinder clearance of reciprocating compressors considering compressibility effects

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Abstract. Leakage is a major source of inefficiency in low-capacity reciprocating compressors. Not only does it lower the mass flow rate provided by the compressor, reducing its volumetric efficiency, but also gives rise to outflux of energy that decreases the isentropic efficiency. Leakage in the piston-cylinder clearance of reciprocating compressors is driven by the piston motion and pressure difference between the compression chamber and the shell internal environment. In compressors adopted for domestic refrigeration, such a clearance is usually filled by a mixture of refrigerant and lubricating oil. Besides its lubricating function, the oil also acts as sealing element for the piston-cylinder clearance, and hence leakage is expected to be more detrimental to oil-free compressors. This paper presents a model based on the Reynolds equation for compressible fluid flow to predict leakage in oil-free reciprocating compressors. The model is solved throughout the compression cycle so as to assess the effect of the clearance geometry and piston velocity on leakage and compressor efficiency. The results show that compressible fluid flow formulation must be considered for predictions of gas leakage in the cylinder-piston clearance.

1. Introduction
Gas leakage is a major source of inefficiencies in refrigeration compressors. Not only does it lower the mass flow rate provided by the compressor, reducing its volumetric efficiency, but also gives rise to outflux of energy due to gas leakage, which decreases the isentropic efficiency. According to Silva and Deschamps [1], a gap of 1µm between the valve and its seat can reduce the volumetric and isentropic efficiencies of small reciprocating compressors by 2.7% and 4.4%, respectively. In fact, with respect to reciprocating compressors, the two main sources of gas leakage are: (i) the gap between the valve and the valve seat and (ii) the piston-cylinder clearance.

Two aspects bring about the flow in the piston-cylinder clearance: (i) the difference of pressure between the compression chamber and the internal environment of the compressor shell, (ii) the motion of the piston. In most reciprocating compressors, such clearance is filled by a mixture of refrigerant gas and lubricating oil. Besides its lubricating function, the oil also acts as a sealing element for the piston-cylinder clearance. Therefore, the leakage in the piston-cylinder clearance is more detrimental to oil-free compressors. Moreover, leakage is more critical in low-capacity compressors, since it may be comparable to the mass flow rate of such compressors. The reduction of the clearance results in higher manufacturing costs and may considerably increase the energy loss due to friction in the clearance.
Leakage in the clearance between the cylinder and the piston has been the focus of several studies. Zuk and Smith [2] proposed an analytical model to estimate leakage in small gaps between parallel plates by applying the Reynolds equation for compressible flow with static boundaries. Ferreira and Lilie [3] presented an analytical model to assess the leakage in the piston-cylinder gap with a moving piston, by assuming an incompressible flow formulation and disregarding inertia forces. Another analytical model was proposed by Yuan et al [4], which considers both inertial and viscous forces as well as gas compressibility effects, but neglects the piston velocity effects. The authors observed that inertia effects are negligible for clearances smaller than 6 \( \mu m \).

Rigola et al. [5] applied a numerical model based on the two-dimensional incompressible Reynolds equation including the piston velocity. They reported an exponential relationship between the size of the gap and the leakage rate, and analysed different compressors and refrigerant fluids. Lohn and Pereira [6] developed a three-dimensional numerical model for predictions of leakage in the piston-cylinder clearance, which were compared with the results of two analytical models: (i) Zuk and Smith [2] and (ii) Ferreira and Lilie [3]. Lohn and Pereira [6] verified that the mass flow rates obtained via a compressible flow formulation were considerably different from those calculated with the incompressible flow formulation, pointing out that the compressible formulation is more suitable for this type of flow. They also showed that the piston velocity significantly affects the gas leakage rate when the pressure difference is kept constant.

Although recent studies have shed some light on the effects of piston velocity on the gas leakage rate, the models available in the literature are either accurate and computationally too expensive, such as two and three-dimensional numerical models, or too simple analytical models that do not take into account compressibility effects or the piston velocity. The present paper reports a numerical model based on the one-dimensional compressible Reynolds equation, including the piston motion. The model considers only the refrigerant fluid in the clearance and can predict the gas leakage at a low computational cost, allowing its use in a coupled manner with a simulation model for the compression cycle. Thus, the leakage in the piston-cylinder clearance is calculated for different piston positions and pressure differences during the compression cycle. The role of the piston motion on the gas leakage rate is investigated throughout the entire compression cycle.

2. Numerical model and solution procedure

The flow geometry addressed herein is illustrated in figure 1. The region between the piston and the cylinder of length \( L \) and clearance \( \delta \) represents the solution domain. The pressure in the compression chamber is higher than the pressure in the compressor internal environment for most of the compression cycle. This pressure difference and the piston motion bring about the flow in the piston-cylinder clearance. When the piston moves towards the cylinder head, gas is dragged into the compression chamber and vice-versa when the piston moves in the other direction.

The following hypotheses are assumed for the flow in the flow: i) inertial effects are negligible; ii) the pressure varies only in the \( x \) direction; iii) the piston-cylinder clearance \( \delta \) is much smaller than the piston and cylinder diameters, so the annular gap can be treated as a flat channel; iv) the gas behaves as an ideal gas; v) dynamic viscosity does not vary with \( x \) (although it varies from one crank angle to the next); vi) isothermal flow and vii) piston and cylinder are concentric, so \( \delta \) is constant; viii) subsonic flow condition.

As shown by Hamrock et al. [7], the governing equation for compressible flow in such a geometry can be modelled by equation (1), which is a simplified form of the Reynolds equation, where \( p \) is the static pressure, \( \mu \) is the fluid dynamic viscosity, \( \rho \) is the fluid density, and \( V_p \) the piston velocity.

\[
\frac{d}{dx} \left( \frac{\rho \delta^3}{12\mu} \frac{dp}{dx} \right) = \frac{d}{dx} \left( \frac{\rho \delta V_p}{2} \right)
\]  

(1)
If the Couette term on the right-hand side of equation (2) is eliminated, an analytical solution can be obtained, as indicated by Zuk and Smith [2]. Fluid refrigerants do not behave as an ideal gas in typical operating conditions found in household refrigeration. However, this hypothesis was adopted herein for simplicity since the main objective of our study is to analyze the effect of gas compressibility on leakage. Naturally, a formulation of real gas must be adopted if the interest is to accurately predict the impact of leakage on compressor performance.

Equation (2) is a differential equation that cannot be solved analytically and, for this reason, we applied the finite volume method (FVM) for the numerical solution. The solution domain is divided into equally spaced control volumes to form a computational mesh. The differential equation is discretized in each control volume, resulting a system of algebraic equations.

The pressures at the boundaries of the volumes are estimated via arithmetic mean of the pressures at the neighboring points, allowing the pressure field to be calculated by solving the system of algebraic equations. An iterative procedure is then applied, with the coefficients being updated and the calculations repeated until the convergence criterion in achieved.

Once the pressure field is known, the leakage mass flow rate

\[ \dot{m}_l = \pi D \left( -\frac{\rho \delta^3}{12 \mu} \frac{dp}{dx} + \frac{\rho V_p \delta}{2} \right), \]  

associated with this Couette-Poiseuille flow can be obtained at any position of the solution domain, but we chose the cross section adjacent to the compression chamber, i.e., \( x = 0 \). The first and second terms inside the parentheses are the contributions to leakage due to the pressure difference and piston motion. The pressure derivative in equation (3) was obtained via a second order approximation.

Two pressure boundary conditions are needed in the numerical model. The first one is the pressure in the compressor internal environment, which is constant throughout the compression cycle. The second is the pressure at \( x = 0 \), which is evaluated in the compression chamber \( (p_{cc}) \) at each crank angle (wt) with a simulation model described by Link and Deschamps [9]. This simulation model also evaluates the piston velocity, as well as the dynamic viscosity and density that are required in equation (3) for \( x = 0 \).
0 at each crank angle. In turn, the model for the flow in the piston-cylinder clearance gives the leakage required to evaluate the compression cycle. Figure 2 illustrates the coupling between these two models.

![Figure 2. Schematic of the solution procedure.](image)

3. Results and discussion

The developed model was used to investigate the effects of several parameters on gas leakage, such as compressor operating conditions, compressor speed (represented by the drive frequency) and clearance dimensions. Additionally, compressibility effects were also assessed by comparing the results obtained with formulations of compressible and incompressible flows.

The results presented hereafter were obtained for a low-capacity reciprocating compressor with displacement of 3 cm$^3$ operating with isobutane (R600a). The compression cycle was solved for small increments of crank angle ($\Delta \omega t = 0.001^\circ$). A total of 250 control volumes were used to discretize the solution domain, with a difference less than 0.01% between predictions and estimates via the Richardson extrapolation. It should be mentioned that the model employed to simulate the compression cycle does not consider valve dynamics and heat transfer at the walls of the compression chamber. This simplification is adopted for convenience, since the study is an assessment of compressibility effects on leakage rather than simulation of compressor performance.

The model was initially validated through comparisons between its predictions and the results of a three-dimensional model developed with a CFD commercial code (Ansys FLUENT v.14), considering a single piston position with fixed pressure difference $\Delta p = 7$ bar, diametric clearance $c = 8 \mu m$ and different instantaneous piston velocities. The 3-D model employed the same assumptions used in the simplified model, i.e., isothermal, compressible, laminar flow of an ideal gas, subjected to the same boundary conditions. The computational mesh had approximately 8 and 250,000 volumes in the transversal and longitudinal directions. Figure 3 shows the results for the leakage mass flow rate ($\dot{m}_{\text{leak}}$); it reveals that both models are in good agreement with differences within 1%. Positive velocities represent that the piston is moving towards the bottom dead center, and the opposite for negative values.

The compressor operating conditions are defined by the evaporating and condensing temperatures of the refrigeration cycle, which defines the compressor suction and discharge pressures. Reference operating conditions used to test refrigeration compressors are the low-back pressure (LBP), medium-back pressure (MBP) and high-back pressure (HBP), as shown in table 1. The effect of such operating conditions on gas leakage is also investigated.

The results for $c = 9 \mu m$ and $f = 60$ Hz shown in figure 4 reveal that the leakage mass flow rate in the compressor increases with the evaporating pressure ($p_{\text{evap}}$). This may seem unexpected since the pressure difference decreases as the evaporating pressure increases, keeping the same condensing pressure (7.62 bar). Nevertheless, as indicated by equation (3), the leakage mass flow rate also depends on the gas density, which is highest for the HBP condition, as shown in figure 5, because the gas temperature in the discharge process reaches its lowest value. This aspect compensates for the smaller pressure difference of the HBP condition. The relative leakage mass flow rate ($\dot{m}_{\text{leak}}^*$) is defined as the
leakage mass flow divided by the compressor mass flow rate disregarding the leakage through the piston-cylinder clearance. As opposed to the leakage mass rate ($\dot{m}_l$), the relative leakage ($\dot{m}_l^*$) decreases with evaporating pressure, which shows that the relative influence of the gas leakage is more significant for the LBP case.

![Figure 3. Comparison between simplified and 3-D models.](image)

![Figure 4. Gas leakage for different compressor operating conditions.](image)

![Figure 5. Gas density (a) and temperature (b) throughout the compression cycle.](image)

**Table 1.** Compressor operating conditions.

| Denomination | Evaporating temperature | Evaporating pressure | Condensing temperature | Condensing pressure |
|--------------|-------------------------|----------------------|------------------------|---------------------|
| LBP          | -23.3°C                 | 0.629 bar            | 54.4°C                 | 7.62 bar            |
| MBP          | -6.7°C                  | 1.229 bar            |                        |                     |
| HBP          | 7.2°C                   | 2.011 bar            |                        |                     |

The piston velocity is a direct consequence of the drive frequency. The model was applied with different values of frequency in order to investigate its effects on gas leakage throughout the compression cycle. Figure 6 shows the results of simulations for $c = 5 \, \mu$m and LBP condition. The ‘No velocity’ curve represents the results of equation (3) without the Couette term. Figure 6 shows that the
The instantaneous leakage rate is greatly affected by piston velocity, being reduced in the compression and discharge processes and decreased in the expansion and suction processes as the piston velocity is increased. Positive values of \( \dot{m}_l \) mean that the gas is leaving the compression chamber, whereas negative values imply that gas is actually being admitted to the compression chamber. In the compression process, the piston is moving towards the top dead center and therefore it carries gas to the compression chamber through viscous effects, reducing the instantaneous gas leakage rate. The opposite effect occurs when the piston moves towards the bottom dead center.

The most significant difference between the results of leakage for the three velocity conditions occurs at the crank angle around 135°, when the Couette term reaches its maximum value, acting towards leakage reduction. The total mass of leakage in the cycle, obtained by integrating the curves in figure 6 over time, is reduced by roughly 5% when \( f = 90 \) Hz.

The instantaneous gas leakage rate for two diametric clearances (\( c \)) is shown in figure 7. The simulations were carried out with a frequency of 60 Hz and LBP condition. As can be seen, the influence of the piston velocity represented by the Couette term is more significant in the smaller clearance. The results also suggest that the pressure term in equation (3) becomes increasingly important as the clearance is increased.

![Figure 6](image6.png)

**Figure 6.** Instantaneous gas leakage rate during the compression cycle for different compressor speeds; \( c = 5 \) µm and LBP condition.

![Figure 7](image7.png)

**Figure 7.** Instantaneous gas leakage rate for different clearances; \( f = 60 \) Hz, LBP condition: (a) \( c = 5 \) µm; (b) \( c = 13 \) µm.
In order to investigate the effects of compressibility on gas leakage, the model was compared to the incompressible fluid flow model presented by Ferreira and Lilie [3]. The calculations considered different drive frequencies, two clearances and LBP condition. Figure 8 shows the relative leakage mass flow rate in each case. The results indicate that, although both curves present the same shape, the incompressible fluid model curve overestimates leakage in every case. For low frequencies the leakage predicted by the incompressible fluid model is twice as big as the value predicted by the compressible fluid model, reaching more than 70% of the compressor mass flow rate when $c = 13 \, \mu$m.

![Figure 8](image1)

**Figure 8.** Comparison of leakage prediction between the compressible model and Ferreira and Lilie’s incompressible model [3] for different clearances: (a) $c = 5 \, \mu$m; (b) $c = 13 \, \mu$m.

The volumetric efficiency and isentropic efficiency are two parameters commonly used to evaluate the compressor performance. The first is defined as the ratio between the actual mass flow and the ideal mass flow rate, as indicated in equation (4). The ideal mass flow rate is obtained for the hypothetical case with no gas leakage, cylinder clearance volume, flow restriction and backflow in valves and suction gas superheating.

$$
\eta_v = \frac{\dot{m}}{\dot{m}_{th}} \tag{4}
$$

The isentropic efficiency is defined as the ratio between the compression power required by an isentropic process and the actual indicated power, i.e.:

$$
\eta_s = \frac{W_{th}}{W} \tag{5}
$$

Figure 9 shows the results of efficiency reduction due to leakage, considering different drive frequencies and clearances for the LBP condition. The efficiency reduction is evaluated as the difference between the compressor efficiencies with and without gas leakage (subscript ‘nl’), as indicated by equation (6). Figure 9 reveals that gas leakage is more detrimental to both efficiencies for smaller drive frequencies and larger clearances. In the worst case, the efficiency reduction can reach more than 8% for the volumetric efficiency and more than 22% for the isentropic efficiency.

$$
\Delta \eta_v = |\eta_v - \eta_{v, nl}|, \quad \Delta \eta_s = |\eta_s - \eta_{s, nl}| \tag{6}
$$
4. Conclusions
A simulation model based on the compressible fluid flow formulation of the Reynolds equation was developed to predict gas leakage in the piston-cylinder clearance of small reciprocating compressors adopted for domestic refrigeration. The model was validated by comparing its predictions with results of a three-dimensional model. The model was then coupled with a simulation model of the compression cycle to allow predictions of leakage throughout the compression cycle for different parameters, such as operating conditions, compressor speed and piston-cylinder clearance. The results show that leakage increases with the evaporating pressure when the condensing pressure is kept constant. Nonetheless, leakage becomes more detrimental as the evaporating pressure is decreased because the compressor mass flow rate decreases to a greater extent when the evaporating pressure is lowered. We found that the piston motion plays a minor role on leakage, becoming negligible in comparison with the effect of pressure difference as the clearance is increased. The present study also showed that compressible fluid flow formulation must be considered to accurately predict gas leakage in the piston-cylinder clearance.

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