Frequent Itemset Mining with Multiple Minimum Supports: a Constraint-based Approach

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Abstract

The problem of discovering frequent itemsets including rare ones has received a great deal of attention. The mining process needs to be flexible enough to extract frequent and rare regularities at once. On the other hand, it has recently been shown that constraint programming is a flexible way to tackle data mining tasks. In this paper, we propose a constraint programming approach for mining itemsets with multiple minimum supports. Our approach provides the user with the possibility to express any kind of constraints on the minimum item supports. An experimental analysis shows the practical effectiveness of our approach compared to the state of the art.

1 Introduction

Discovering relevant patterns for a particular user remains a challenging task in data mining. In real-life applications, relevant patterns may be either frequent or rare ones in the data. In itemset mining, setting the minimum support threshold is a real dilemma (a high value misses rare itemsets, a low value generates a large number of meaningless itemsets). To tackle the rare item problem \cite{8}, several approaches were proposed to mine frequent pattern with multiple minimum supports. In \cite{8}, the problem of mining frequent itemsets with multiple Minimum Item Supports (MIS) was introduced with a first revision of Apriori algorithm (MSApriori). Then, other Apriori-like approaches were proposed like MMS_Cumulate and MMS_Stratify \cite{12}. The well-known FPGrowth was extended with a condensed FP-tree structure to mine frequent itemsets with multiple MIS (CFPGrowth \cite{5}, CFPGrowth ++ \cite{6}). In \cite{3}, FP – ME was proposed based on set-enumeration-tree structure and sorted downward closure property.

The specialized algorithms introduced previously are effective for mining patterns with multiple MIS. However, most of the time the user is interested
in patterns that satisfy some specific properties. For instance, the user may ask for patterns where items are around the same frequency threshold, or patterns of an adaptive size (large frequent patterns and/or concise rare patterns). Looking for patterns with additional user-specified constraints remains a bottleneck. According to Wojciechowski and Zakrzewicz [13], there are three ways to handle the additional user’s constraints. We can use a pre-processing step that restricts the dataset to only transactions that satisfy the constraints. Such a technique cannot be used on all kinds of constraints. We can use a post-processing step to filter out the patterns violating the user’s constraints. Such a brute-force technique can be computationally infeasible when the problem without the user’s constraints has too many solutions. We can finally integrate the filtering of the user’s constraints into the specialized data mining process in order to extract only the patterns satisfying the constraints. Such a technique requires the development of a new algorithm for each new mining problem with user’s constraints.

In a recent line of work [9, 7, 11, 1, 2], constraint programming (CP) has been used as a declarative way to solve some data mining tasks. Such an approach has not competed yet with state of the art data mining algorithms in terms of CPU time for standard data mining queries but the CP approach is competitive as soon as we need to add user’s constraints. In addition, adding constraints is easily done by specifying the constraints directly in the model without the need to revise the solving process. However, the CP approach has not yet been applied to mining frequent itemsets with multiple MIS.

In this paper we introduce a CP model for finding frequent itemsets with multiple MIS. For that, we introduce a new global constraint, FreqRare for mining frequent itemsets with multiple MIS. We provide a propagator for FreqRare and we show that, for a given variable ordering, it is sufficient to mine frequent itemsets with multiple MIS in backtrack-free manner. Our constraint, FreqRare, can be used to express any kind of constraints on the minimum item supports. We show that our CP model can easily be extended for taking into account any kind of user’s constraints. Experiments on several known large-scale datasets show the effectiveness of our CP model.

The paper is organized as follows. Section 2 gives some background material. Section 3 presents our global constraint FreqRare and its propagator. Section 4 presents the possible extensions of our CP model to express constraints on the minimum item supports. Section 5 reports experiments. Finally, we conclude in Section 6.
2 Background

2.1 Itemset mining

Let \( I = \{p_1, \ldots, p_n\} \) be a set of \( n \) distinct objects, called items. An itemset \( P \) is a non-empty subset of \( I \). A transactional dataset \( D \) is a bag of \( m \) itemsets \( t_1, \ldots, t_m \), called transactions. The cover of an itemset \( P \) in \( D \), denoted by \( \text{cover}(P) \), is the bag of transactions from \( D \) containing \( P \).

The frequency of an itemset \( P \) in \( D \), denoted by \( \text{freq}(P) \), is the cardinality of its cover, i.e. \( \text{freq}(P) = |\text{cover}(P)| \). Let \( S = \{s_1, \ldots, s_n\} \) be a set of minimum supports associated to items (i.e., the multiple MIS set), where \( s_i \) is the minimum support of the item \( p_i \). The itemset \( P \) is frequent iff:

\[
\text{freq}(P) \geq \min_{p_i \in P} s_i
\]

For the sake of simplicity, we replace in what follows items \( p_i \) and transactions \( t_j \) with their respective indices \( i \) and \( j \) and we denote the presence of item \( i \) in transaction \( j \) by \( D_{ij} \).

Example 2.1 The \( D \) dataset presented in Table 1 has 4 items and 5 transactions. According to its multiple MIS, \( AB \) is frequent, \( ABC \) is infrequent and \( ABCD \) is frequent.

| D: | Items |
|----|-------|
| \( t_1 \) | A B D |
| \( t_2 \) | A C D |
| \( t_3 \) | A B C D |
| \( t_4 \) | B C |
| \( t_5 \) | A B C |

\[ S : \begin{array}{cccc}
A & B & C & D \\
4 & 3 & 3 & 1
\end{array} \]

2.2 Constraint programming

A Constraint Programming model (or CP model) specifies a set of variables \( X = \{x_1, \ldots, x_n\} \), a set of domains \( \text{dom} = \{\text{dom}(x_1), \ldots, \text{dom}(x_n)\} \), where \( \text{dom}(x_i) \) is the finite set of possible values for \( x_i \), and a set of constraints \( C \) on \( X \). A constraint \( c_j \in C \) is a relation that specifies the allowed combinations of values for its variables \( \text{var}(c_j) \). An assignment on a set \( Y \subseteq X \) of variables is a mapping from variables in \( Y \) to values, and a valid assignment is an assignment where all values belong to the domain of their variable. A solution is an assignment on \( X \) satisfying all constraints. Constraint programming is the art of writing problems as CP models and solving them by finding solutions. Constraint solvers typically use backtracking search to
explore the search space of partial assignments. At each assignment, constraint propagation algorithms (aka, propagators) prune the search space by enforcing local consistency properties such as domain consistency.

**Global constraints** are constraints defined by a relation on a non-fixed number of variables. These constraints allow the solver to better capture the structure of the problem. The constraint `ALLDIFFERENT`, specifying that all its variables must take different values is an example of global constraint (see [10]).

**Example 2.2** Consider the following instance of a CP model. \( X = \{x_1, x_2, x_3\} \), \( \text{dom}(x_1) = \{0, 2\} \), \( \text{dom}(x_2) = \{0, 2, 4\} \), \( \text{dom}(x_3) = \{1, 2, 3, 4\} \), and \( C = \{x_1 \geq x_2, x_1 + x_2 = x_3\} \). Value 4 for \( x_2 \) will be removed by domain consistency because of constraint \( x_1 \geq x_2 \). Values 1 and 3 for \( x_3 \) will be removed by domain consistency because of constraint \( x_1 + x_2 = x_3 \). This instance of CP model admits the two solutions \((x_1 = 2, x_2 = 0, x_3 = 2)\) and \((x_1 = 2, x_2 = 2, x_3 = 4)\).

### 2.3 CP model for itemset mining

In [9], De Readt et al. have introduced CP4IM, a first CP model to solve itemset mining tasks. For mining frequent itemsets, the CP model uses two vectors of Boolean variables \( x \) and \( y \). \( x_i \) represents the presence of item \( p_i \) in the searched itemset. \( y_j \) represents the presence of the searched itemset in the transaction \( t_j \). Bear in mind that \( x \) are decision variables representing the searched itemset, where \( y \) are auxiliary variables representing the cover of the searched itemset. For mining frequent itemsets and given a unique minimum support \( s \), the CP model is expressed using two sets of reified constraints:

\[
\forall j \in \mathcal{D} : (y_j = 1) \leftrightarrow \sum_{i \in \mathcal{I}} x_i (1 - D_{ij}) = 0 \quad (1)
\]

\[
\forall i \in \mathcal{I} : (x_j = 1) \rightarrow \sum_{j \in \mathcal{D}} y_j D_{ij} \geq s \quad (2)
\]

where,

(1) are channelling constraints of arity \((n + 1)\) ensuring the relationship between \( x \) and \( y \).

(2) are constraints of arity \((m + 1)\) ensuring the minimum frequency of the searched itemset wrt a minimum support \( s \).
3 CP for frequent itemsets with multiple MIS

Thanks to the expressiveness of CP, the CP4IM model presented in Section 2.3 can easily be revised to mine frequent itemsets with multiple MIS. All we have to do is to replace the constraints (2) by (3):

\[ \forall i \in I : (x_i = 1) \rightarrow \sum_{j \in D} y_{ij} D_{ij} \geq \min_{k \in I} (s_k : x_k \neq 0) \]

where the minimum operator in (3) returns the corresponding minimum item support \( s_i \) during search. Let us call the revised version of CP4IM for multiple MIS the CP model \texttt{ReifiedMIS}. The \texttt{ReifiedMIS} model represents a straightforward CP encoding of the problem using reified constraints. Such encoding suffers from scalability issue due to the use of auxiliary variables \( y \) and an important number of reified constraints. \texttt{ReifiedMIS} requires \((n + m)\) constraints of arity \((n + 1)\) and \((m + 1)\) to encode the whole dataset.

For an effective and scalable CP model, we propose the \texttt{FreqRare} global constraint encoding the minimum frequency constraint with multiple MIS (equation (3)). \texttt{FreqRare} requires neither reified constraints nor auxiliary variables.

Our global constraint is expressed only on the decision variables \( x \). We will use the following notations:

- \( \mathcal{P} = \{ i \in I \mid \text{dom}(x_i) = \{1\} \} \).
- \( \mathcal{N} = \{ i \in I \mid \text{dom}(x_i) = \{0\} \} \).
- \( \mathcal{U} = \{ i \in I \mid \text{dom}(x_i) = \{0, 1\} \} \).

**Definition 1 (FreqRare Constraint)** Let \( x \) be a vector of Boolean variables. Let \( D \) be a dataset and \( S \) the corresponding set of minimum item supports (MIS). The global constraint \texttt{FreqRare}_{D,S}(x) holds if and only if:

\[ (\mathcal{U} = \emptyset) \land (\text{freq}(\mathcal{P}) \geq \min_{p_i \in \mathcal{P}} s_i) \]

**Example 3.1** Consider the transaction dataset of Table 2. Let \( x = \langle x_A, x_B, x_C, x_D \rangle \) with \( \text{dom}(x_i) = \{0, 1\} \) for \( i \in \{A, B, C, D\} \). Consider the frequent itemset \( CD \) encoded by \( x = (0, 0, 1, 1) \), where \( \mathcal{P} = \{C, D\} \), \( \mathcal{N} = \{A, B\} \) and \( \mathcal{U} = \emptyset \). Here, \texttt{FreqRare}_{D,S}(x) holds because all variables are instantiated (i.e., \( \mathcal{U} = \emptyset \)) and \( [\text{freq}(CD) = 2] \geq [\min(s_C, s_D) = 1] \).

\(^1\)A reified constraint connects a constraint to a boolean variable that catches its truth value \cite{10}.
We present now a propagator algorithm for the global constraint \textsc{FreqRare}.

Algorithm.
Algorithm 1 takes as input the variables \(x\) and the multiple item supports \(S\). We start by computing the cover of the itemset \(P\) and store it in \texttt{cover} (line 4). Then, we compute the minimum item support \(s \in S\) with respect to \(P\) and \(U\) items (line 5). If \(P\) is infrequent with respect to \(s\), then \(P\) cannot be extended to a solution (using \(U\) items) and we return a failure (line 7). Otherwise, we must remove items \(p_i \in U\) that cannot belong to a solution containing \(P\) (lines 8-11).

Algorithm 1: Propagator for \textsc{FreqRare}

\begin{verbatim}
1 In: \(S\): minimum item supports
2 InOut: \(x = \{x_1, \ldots, x_n\}\): Boolean item variables;
3 begin
4     \texttt{cover} ← \texttt{cover}(\textit{P});
5     \(s ← \min_{i \in P \cup U} s_i;\)
6     if |\texttt{cover}| < s then
7         return failure;
8     foreach \(i \in U\) do
9         if |\texttt{cover} \cap \texttt{cover}(\textit{i})| < s then
10            \texttt{dom}(x_i) ← \texttt{dom}(x_i) \setminus \{1\}
11            \(U ← U \setminus \{i\};\)
12            \(N ← N \cup \{i\};\)
\end{verbatim}

Proposition 1 (Backtrack-free search) Enumerating frequent itemsets with multiple MIS using the propagator of \textsc{FreqRare} (Algorithm 1) with an increasing minimum item support as variable ordering heuristic: \texttt{minMis} heuristic, is backtrack-free.

Proof.
We first prove that if \textsc{FreqRare} admits a solution, \(P\) is necessarily one of them. Suppose there is a solution and \(P\) is not one of them. This means that there exits a superset of \(P\), \(P \cup i\), which is frequent. As we use \texttt{minMis} heuristic, we have the guarantee that \(s\), the minimum support computed at line 5 corresponds to an item in \(P \cup U\) (i.e., not in \(N\)). That is,
\[
 s = \min_{j \in P \cup U} (s_j) = \min_{j \in (P \cup \{i\} \cup (U \setminus \{i\}))} (s_j)
\]
Here, if $P \cup i$ is frequent wrt $s$, $P$ is necessary frequent (anti-monotony of the frequency) and thus $P$ is a solution too, which contradicts the assumption.

We now prove that Algorithm 1 returns failure if and only if $\text{FreqRare}$ does not admit any solution. We know that $\text{FreqRare}$ has no solution if and only if whatever the partial instantiation submitted to Algorithm 1, $P$ is not a solution. No solutions means that whatever the item $i$ its frequency is below the corresponding minimum item support $s_i$ (i.e., $\forall i \in I : \text{freq}(i) < s_i$). Here, the search process will prune the value 1 from the domain of all $x_i$ at line 10 and thus, converge on a complete instantiation where $N = I, P = \emptyset$.

We now prove that Algorithm 1 prunes value 1 from $\text{dom}(x_i)$ exactly when $i$ cannot belong to a solution containing $P$. Suppose that value 1 of $x_i$ is pruned by Algorithm 1. This means that the test in line 9 was true, that is, $P \cup \{i\}$ is infrequent. Thus, and by definition, $P \cup \{i\}$ does not belong to any solution. Suppose now that value 1 of $x_i$ is not pruned. From line 9, we deduce that $P \cup \{i\}$ is frequent. Thus $P \cup \{i\}$ is a solution and mining frequent itemsets with multiple MIS is backtrack-free using Algorithm 1 with $\text{minMis}$ heuristic.

**Proposition 2 (Time complexity of Algorithm 1)**

Given a transaction dataset $D$ of $n$ items and $m$ transactions and $S$ the corresponding set of minimum item supports (MIS), Algorithm 1 has an $O(n \times m)$ time complexity.

**Proof.** Computing the size of the cover of an itemset is in $O(n \times m)$ (line 4). The cover of $P \cup \{i\}$ in line 9 is computed in $O(m)$ thanks to $\text{cover}$. The loop in line 8 is called at most $n$ times leading to a time complexity of lines 8-10 in $O(n \times m)$. Thus, Algorithm 1 has an $O(n \times m)$ time complexity.

4 Constrained frequent itemsets with multiple MIS

In this section, we illustrate the power of CP to state diverse queries, while maintaining the declarativeness of our CP model and by taking into account different user-specified constraints.

In addition to mining frequent itemsets with multiple MIS, the user may have more restrictions on the itemsets to mine. In this section we discuss the possible constraints that a user may have on the set of MIS. Let $Q_0$ refers to the basic query: "mining frequent itemsets with multiple MIS".
4.1 Distance between MISs

The user may be interested in itemsets that include items of the same nature. That is, distances between MISs are bounded above by a given value $ub$. For instance, in a sales transactions dataset, the item $\{\text{car}\}$ occurs rarely and should have a low MIS value (e.g., $s_{\text{car}} = 1$). On the other hand, the item $\{\text{bread}\}$ occurs frequently and should have a high MIS value (e.g., $s_{\text{bread}} = 1,000$). In this case, the itemset $\{\text{car, bread}\}$ is frequent wrt $s_{\text{car}}$. To avoid generating such itemset, we can put a restriction on the distance between MIS pairs, for instance, $(|s_i - s_j| \leq 50 : \forall i, j)$, where in such case, the itemset $\{\text{car, bread}\}$ will not be a relevant one to return.

In our CP model, the distance constraint can be expressed as follows:

$$\text{DISTANCEMIS}_{S, ub}(x) \equiv \forall i, j \in I, (|s_i - s_j| x_i x_j \leq ub)$$

The user may ask the following query:

$$Q_1: \text{Given a dataset } D, \text{ an MIS vector } S \text{ and an upper bound } ub, \text{ extract frequent itemsets of MIS distances bounded above by } ub.$$ 

The query $Q_1$ can easily be expressed in CP using $\text{FREQRARE}$ and with the user distance constraints as follows:

$$Q_1(D, S, ub) = \begin{cases} \text{FREQRARE}_{D, S}(x) \\ \text{DISTANCEMIS}_{S, ub}(x) \end{cases}$$

4.2 Cardinality constraint

In addition to the distance constraint, the user may ask to strengthen her query with a restriction on the cardinality of the returned itemsets. For instance, the user may ask for itemsets with a size of at least $c$.

Here a query that the user may ask:

$$Q_2: \text{Given a dataset } D, \text{ an MIS vector } S, \text{ an upper bound } ub \text{ and a lower bound } c, \text{ extract frequent itemsets of MIS distances bounded above by } ub \text{ and of a size of at least } c.$$ 

The query $Q_2$ can be expressed in CP as follows:

$$Q_2(D, S, ub, c) = \begin{cases} Q_1(D, S, ub) \\ \sum_{x \in \bar{L}} x_i \geq c \end{cases}$$
4.3 $k$-patterns mining

A promising road to discover useful patterns is to impose constraints on a set of $k$ related patterns ($k$-pattern sets) \cite{4}. In this setting, the interest of a pattern is evaluated wrt a set of patterns. For instance, the user may be interested in extracting $k$ distinct frequent itemsets with a cardinality and distance constraints on MIS.

Here an example of a particular query that the user may ask:

\[ Q_3: \text{Given a dataset } \mathcal{D}, \text{ an integer } k, \text{ an MIS vector } S, \text{ an upper bound } ub \text{ and a lower bound } c, \text{ extract } k \text{ distinct frequent itemsets of MIS distances bounded above by } ub \text{ and with a sizes of at least } c. \]

The query $Q_3$ can be expressed as follows:

\[
Q_3(\mathcal{D}, S, k, ub, c) = \begin{cases} 
\forall i \in [1, k]: \text{FreqRare}_{\mathcal{D}, S}(x^i) & (1) \\
\forall i \in [1, k]: \text{DistanceMIS}_{S, ub}(x^i) & (2) \\
\forall i \in [1, k]: \sum_{j \in \mathcal{I}} x^i_j \geq c & (3) \\
\forall i, j \in [1, k]: x^i \cap x^j = \emptyset & (4) 
\end{cases}
\]

The role of each type of constraint is the following:

(1) ensures that for every $i$ in $[1, k]$, the itemset $P_i$ is frequent wrt $S$.
(2) ensures that distances between MIS pairs of the $k$ itemsets are not exceeding an upper bound $ub$.
(3) ensures that the $k$ itemsets are of a size of at least $c$.
(4) ensures that the $k$ itemsets are distinct.

5 Experiments

This section describes the experimental settings (including benchmark datasets, protocol and implementation), the experimental results and comparison with the state of the art approach \texttt{CFPGrowth++}.

5.1 Experimental protocol

We selected several real-sized datasets from the FIMI repository\footnote{fimi.ua.ac.be/data/}. These datasets have various characteristics representing different application domains. The first part of Table 2 reports, for each dataset, the number of
transactions $|T|$, the number of items $|I|$, the average size of transactions $\overline{|T|}$ and the density $\rho$ (i.e., $\overline{|T|}/|I|$). The datasets are presented by increasing size $|I|\cdot|T|$. We selected datasets of various size and density. Some datasets, such as Zoo and Chess, are very dense (resp. 44% and 49%). Others are very sparse (e.g., 4% for T40). The sizes of these datasets vary from around 4,000 to more than $10^8$.

For our experiments, we assign the MIS values for items according to their frequencies and using the formula proposed in [8]:

$$s_i = \max(\beta \text{freq}(i), \text{MIS}_{\text{min}})$$

where $\beta \in [0, 1]$ is a parameter, $\text{freq}(i)$ is the frequency of the item $i$ and $\text{MIS}_{\text{min}}$ is the lowest support that an item can have. The second part of Table 2 reports, for each dataset, the selected $\beta$ value, the lowest, the highest and the averaged MIS relative values ($\text{MIS}_{\text{min}}, \text{MIS}_{\text{max}}, \text{MIS}_{\text{avg}}$).

The implementation of our CP model with FREQRARE global constraint, coined CP4MIS, is carried out in the Oscar solver using Scala. The code is publicly available at github.com/CP4MIS/cp4mis. After a few preliminary tests and based on the findings presented in Section 3, we decided to use minMis as variable ordering heuristic and largest value first as value ordering heuristic. We compared our CP approach to the CFPgrowth++ specialized algorithm for extracting frequent itemsets with MIS [6]. We used the implementation of CFPgrowth++ publicly available in the SPMF platform. All experiments were conducted on an Intel core i7, 2.8Ghz with a RAM of 16GB and with a timeout of one hour.

Our evaluation aims to answer the following four research questions:

- **RQ1**: How effective is the use of FREQRARE global constraint comparing to the basic ReifiedMIS CP model?
- **RQ2**: How effective is CP4MIS for mining frequent itemsets with multiple MIS (queries of type $Q_0$) and compared to specialized algorithm?
- **RQ3**: How effective is CP4MIS for mining constrained frequent itemsets with multiple MIS (queries of type $Q_1$ and $Q_2$) and compared to specialized algorithm?
- **RQ4**: How effective is our CP approach for mining $k$ distinct constrained frequent itemsets with multiple MIS (queries of type $Q_3$) and compared to specialized algorithm?

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3bitbucket.org/oscarlib/oscar/

4www.philippe-fournier-viger.com/spmf/
### Table 2: Dataset Characteristics.

| Name   | \(||T||, |I||) | \(\rho\) (%) | \(\beta\) | \(\text{MIS}_{\text{min}}\) (%) | \(\text{MIS}_{\text{max}}\) (%) | \(\text{MIS}_{\text{avg}}\) (%) |
|--------|-------------|-------------|-----------|-------------------------------|-------------------------------|-------------------------------|
| Zoo    | (101, 36)   | 44          | 0.1       | 1                             | 9                             | 4                             |
| Vote   | (435, 48)   | 33          | 0.1       | 0.2                           | 6                             | 3                             |
| Anneal | (812, 89)   | 45          | 0.9       | 12                            | 90                            | 45                            |
| Chess  | (3K, 75)    | 49          | 0.5       | 31                            | 50                            | 36                            |
| Mushroom | (8K, 112) | 19          | 0.1       | 0.1                           | 10                            | 2                             |
| Connect | (68K, 129) | 33          | 0.7       | 30                            | 70                            | 39                            |
| T40    | (100K, 942) | 4           | 0.1       | 0.05                          | 3                             | 0.5                           |
| Pumsb  | (49K, 2K)   | 3           | 0.9       | 41                            | 90                            | 42                            |

### 5.2 Results

Our first experiment compares our CP4MIS model to ReifiedMIS model and to CFPGrowth ++ on queries of type \(Q_0\), where the user is looking for frequent itemsets with multiple MIS. Table 3 reports the result of the comparison with the CPU time given in seconds (s), the memory consumption in megabytes (MB) of the two CP models, and the number of solutions \(#sol\) for each instance.

#### 5.2.1 RQ1: ReifiedMIS vs FreqRare on \(Q_0\)

The main observation when comparing the reified CP model to CP4MIS is that the use of FreqRare global constraint outperforms significantly the basic model. Let us take a closer look to the four first datasets where neither a timeout nor an out of memory are reported. In terms of CPU time, we can observe a speed-up factors of 2, 9, 10 and 98. In terms of memory consumption, we denotes factors of 47, 63, 188 and 270. This is explained by the huge number of reified constraint to propagate and to check at each node of the search tree comparing to a single call per node of FreqRare propagator. We denote two timeout and two out-of-memory instances for ReifiedMIS model. Again, this is explained by the size of the model and the number of the posted reified constraints. For instance, if we take Pumsb dataset, ReifiedMIS contains \(||I|| + ||T||) = 51K variables, \((||T|| = 49K)\) reified constraints to express the channelling constraints on \((|I|| = 2K)\) variables (equation [1]), \((|I|| = 2K)\) reified constraints to express the minimum frequency of itemsets on \((|T|| = 49K)\) variables (equation [2]). This means that the CP solver has to load in memory a CP model of 51K reified constraints expressed on 51K variables. We observe that with a timeout of one hour and a memory of 16GB, Oscar solver is not able to manage CP models of size exceeding \((|I|| \times |T||) \approx 10^5\). Another observation is the experimental validation of Proposition 1.

FreqRare of
Table 3: CFPGrowth + + vs (ReifiedMIS vs CP4MIS) on the query $Q_0$ (Time in seconds and Memory in MB)

| Dataset | (a) Time | (b) Time | Memory | (c) Time | Memory | #sol |
|---------|----------|----------|--------|----------|--------|------|
| Zoo     | 0.81     | 12.00    | 3,760  | 1.34     | 20     | 1,314,983 |
| Vote    | 1.56     | 196.17   | 2,164  | 2.23     | 8      | 2,177,409 |
| Anneal  | 30.91    | 134.74   | 3,095  | 64.82    | 49     | 71,757,451 |
| Chess   | 11.64    | 305.03   | 3,153  | 28.20    | 67     | 22,660,643 |
| Mushroom| 45.53    | TO       | –      | 106.00   | 48     | 105,291,573 |
| Connect | 48.45    | TO       | –      | 854.59   | 218    | 91,740,453 |
| T40     | 409.55   | –        | OOM    | 91.70    | 2,304  | 15,859,400 |
| Pumsb   | 38.60    | –        | OOM    | 115.67   | 916    | 13,507,227 |

(a): CFPGrowth + +, (b): ReifiedMIS, (c): CP4MIS.

CP4MIS combined with minMis heuristic enumerates the whole set of frequent itemsets of the 8 datasets in a backtrack-free manner and without any fail during search (#failures = 0).

5.2.2 RQ2: CFPGrowth + + vs CP4MIS on $Q_0$

As expected, CFPGrowth + + is very efficient in enumerating all frequent itemsets with multiple MIS. CFPGrowth + + is from 2 to 18 times faster than CP4MIS. However, our CP4MIS approach is also reasonable and can enumerate up to millions frequent itemsets in a few seconds. Furthermore, CP4MIS is even better than CFPGrowth + + on one instance i.e. T40.

Interestingly, the less solutions there is to mine the better is our approach CP4MIS. This is illustrated in Figure 1 where we compare CFPGrowth + + to CP4MIS for mining frequent itemsets on Connect with $\beta = 0.8$ and by varying MIS$_{\min}$. We clearly see that our approach CP4MIS is highly correlated with the number of solutions compared to CFPGrowth + +. With an MIS$_{\min}$ of 30%, CFPGrowth + + is able to mine more than 17 millions of itemsets in a few seconds (exactly 10.21 seconds), whereas CP4MIS spends more than 3 minutes to generate the whole set of solutions. However, with MIS$_{\min}$ starting from 74%, CP4MIS starts to be more efficient and it is able to extract the 915 itemsets (MIS$_{\min} = 96\%$) in 2 seconds, an instance on which CFPGrowth + + spends more than one minute. The explanation for this good behavior of CP4MIS is the strength of constraint propagation to rule out inconsistent parts of the search space. On the contrary, on an instance with 17 millions solutions (MIS$_{\min} = 30\%$), the CP solver is almost reduced to an enumerating process.
Figure 1: CFPGrowth ++ vs CP4MIS varying MIS_{\text{min}} on Connect with $\beta = 0.8$.

5.2.3 RQ3: CFPGrowth ++ vs CP4MIS on $Q_1$ and $Q_2$

Our second experiment compares CFPGrowth ++(C) to our approach, CP4MIS, for mining constrained frequent itemsets (queries of type $Q_1$ and $Q_2$).

CFPGrowth ++(C) (C is for checker) is a revised version of CFPGrowth ++ with a checker to filter out itemsets violating the constraints.

In Table 4, we report the time, in seconds, for the two approaches and for each instance for mining frequent itemsets of MIS distances bounded above by $ub$ (queries of type $Q_1$). We also report the number of solutions for each instance: #sol. We selected an $ub$ value for each instance in order to have less than 10K solutions.

The main observation that we can draw from Table 4 is that CP4MIS outperforms CFPGrowth ++(C) on all the instances. Comparing to the results of $Q_0$, the fact that we strengthen the query with the distance constraint makes the CP resolution more effective. For instance, on Connect, CP4MIS needed around 14 minutes to enumerate the 91 millions of solutions corresponding to $Q_0$, where it took only 2 seconds to return the 1,683 solutions corresponding to $Q_1$ query. Thanks to constraint propagation that drastically reduces the search space. On the other hand, CFPGrowth ++(C) has no pruning power during the enumeration, and does not take advantage of
Table 4: CFPGrowth + +(C) vs CP4MIS on the query $Q_1$ (time in seconds)

| Dataset | $ub$ | (a) | (b) | #sol |
|---------|------|-----|-----|------|
| Zoo     | 2    | 0.61| 0.17| 5,765|
| Vote    | 1    | 0.82| 0.18| 2,466|
| Anneal  | 30   | 12.95| 0.25| 1,790|
| Chess   | 80   | 5.96| 0.27| 1,442|
| Mushroom| 50   | 20.20| 0.46| 2,641|
| Connect | 1000 | 21.28| 2.11| 1,683|
| T40     | 100  | 401.93| 107.92| 5,846|
| Pumsb   | 1000 | 26.97| 3.04| 1,287|

(a):CFPGrowth + +(C); (b):CP4MIS.

To strengthen our findings, we conduct an experiment with $Q_2$ query type. In addition to the distance constraint, $Q_2$ considers the cardinality constraint i.e., itemsets should have a size of at least $c$. Table 5 reports the time, in seconds, for the two approaches and for each instance acting on $Q_2$ query type. We also report the number of solutions of each instance: #sol. The parameter $c$ is selected in order to have less than 50 solutions per instance.

Again, CP4MIS is the winner where it is from 4 to 43 faster than CFPGrowth + +(C). This is explained by the additional power added to the propagation process during the resolution to enumerate the few solutions present in the huge search space. Where CFPGrowth + +(C) needs to enumerate the millions of candidates and then filter out the ones violating the user-constraints.

Table 5: CFPGrowth + +(C) vs CP4MIS on the query $Q_2$ (time in seconds)

| Dataset | $ub$ | $c$ | (a) | (b) | #sol |
|---------|------|-----|-----|-----|------|
| Zoo     | 2    | 10  | 0.62| 0.10| 14   |
| Vote    | 1    | 10  | 1.14| 0.13| 12   |
| Anneal  | 30   | 8   | 12.78| 0.18| 7    |
| Chess   | 80   | 8   | 6.49| 0.23| 30   |
| Mushroom| 50   | 8   | 19.19| 0.36| 27   |
| Connect | 1000 | 10  | 20.88| 2.03| 2    |
| T40     | 100  | 6   | 309.80| 54.31| 14   |
| Pumsb   | 1000 | 8   | 27.91| 2.86| 17   |

(a):CFPGrowth + +(C), (b):CP4MIS.

the additional constraints (the distance constraint in this case) to reduce the search space (21 seconds to enumerate the 1,683 solutions).

To strengthen our findings, we conduct an experiment with $Q_2$ query type. In addition to the distance constraint, $Q_2$ considers the cardinality constraint i.e., itemsets should have a size of at least $c$. Table 5 reports the time, in seconds, for the two approaches and for each instance acting on $Q_2$ query type. We also report the number of solutions of each instance: #sol. The parameter $c$ is selected in order to have less than 50 solutions per instance.

Again, CP4MIS is the winner where it is from 4 to 43 faster than CFPGrowth + +(C). This is explained by the additional power added to the propagation process during the resolution to enumerate the few solutions present in the huge search space. Where CFPGrowth + +(C) needs to enumerate the millions of candidates and then filter out the ones violating the user-constraints.
5.2.4 RQ4: CFPGrowth ++ vs CP4MIS on $Q_3$.

For our last experiment, we compares CFPGrowth ++ to CP4MIS on a query of type $Q_3$ in $k$-patterns mining context. Table 6 reports the different instances selected for our experiment. The instances are selected in order to vary the number of solutions from 0 to 100 $k$-patterns solutions.

Using CP, the problem is expressed using $k$ Boolean vectors (see Section 4.3). For such problem, a baseline can be the use of specialized algorithm like CFPGrowth ++ combined with a post-processing step. One can imagine two scenarios:

1. The use of CFPGrowth ++ to mine the total number of frequent itemsets with multiple MIS and a generate-and-test search trying to find distinct itemsets satisfying the user-constraints (distance and cardinality constraints in our case). This baseline is coined CFPGrowth ++(PP) (PP is for post-processing).

2. The use of CFPGrowth ++(C) to mine the total number of frequent itemsets with multiples MIS satisfying the user-constraints and a generate-and-test search trying to find distinct itemsets. This second baseline is coined CFPGrowth ++(CPP) (CPP is for checker + post-processing).

Such baselines can be very expensive. In both cases, the post-processing will generate the possible $k$ combinations of itemsets. Table 6 reports the solutions of $Q_0$ and $Q_2$ that represent the number of candidates on which the $k$ combinations will be generated for, respectively, CFPGrowth ++(PP) and CFPGrowth ++(CPP) baselines.

Table 7 reports the CPU time, in seconds, for each instance and for the three approaches (CFPGrowth ++(PP), CFPGrowth ++(CPP) and CP4MIS) acting on $Q_3$ query type. We also report the number of solutions of each instance: #sol.

The main observation that we can draw from Table 7 is that CP4MIS is able to cope with such complex query and to return the 96 $K$ $k$-patterns solutions in 40 minutes. This demonstrates again the power of propagation using a CP resolution. On the other hand, the CFPGrowth ++(PP) baseline timeouts on all instances and CFPGrowth ++(CPP) on 5 out of the 8 instances. This is an expected result knowing that the baselines have to cope with a massive number of combinations. For instance, on Connect dataset and $k = 6$ we have more than $10^{24}$ candidates.

If we take the instance of Chess with $k = 6$. Here we have an instance without solution and CP4MIS is able to prove it in less than 26 seconds, where the two baselines reach timeout without proving the unsatisfiability of the instance.

Note that the used checker in CFPGrowth ++(CPP) allows the baseline to reduce the possibilities, but not enough to avoid the explosion when $k$ grows.
Table 6: Selected instances for k-patterns mining.

| Dataset  | $\beta$ | $\text{MIS}_{\text{min}}$ | $\text{MIS}_{\text{max}}$ | $\text{MIS}_{\text{avg}}$ | $\#\text{sol}_1$ | $ub$ | $c$ | $\#\text{sol}_2$ |
|----------|---------|-----------------|-----------------|-----------------|-----------------|-----|----|-----------------|
| Chess    | 0.9     | 78%             | 90%             | 80%             | 5,553           | 150 | 3  | 932             |
| Mushroom | 0.9     | 18%             | 90%             | 26%             | 2,977           | 3000| 2  | 138             |
| Connect  | 0.9     | 89%             | 90%             | 89%             | 41,143          | 300 | 3  | 397             |
| Pumsb    | 0.9     | 88%             | 90%             | 88%             | 7,044           | 400 | 3  | 347             |

$\#\text{sol}_1 = \text{solutions of } Q_0$ \hspace{1cm} $\#\text{sol}_2 = \text{solutions of } Q_1$.

Table 7: CFPGrowth + + (PP) vs CFPGrowth + + (CPP) vs CP4MIS on the query $Q_3$ (time in seconds)

| Dataset  | $k$ | (a) | (b) | (c) | #sol |
|----------|-----|-----|-----|-----|------|
| Chess    | 5   | TO  | 61.41 | 19.10 | 480 |
| Chess    | 6   | TO  | TO   | 25.42 | 0   |
| Mushroom | 9   | TO  | TO   | 862.55 | 1,407|
| Mushroom | 10  | TO  | TO   | 1014.49 | 0   |
| Connect  | 6   | TO  | 182.71 | 133.68 | 36,537|
| Connect  | 7   | TO  | TO   | 143.69 | 0   |
| Pumsb    | 6   | TO  | 199.51 | 177.80 | 61,186|
| Pumsb    | 7   | TO  | TO   | 202.49 | 0   |

(a): CFPGrowth + + (PP), (b): CFPGrowth + + (CPP), (c): CP4MIS.

CFPGrowth + + (CPP) first prunes non-solutions using CFPGrowth + + (C) reducing the search space and thus solving the Connect instance when $k = 6$ in 3 minutes. However, with $k = 7$, the search space explodes and CFPGrowth + + (CPP) is not able to prove no solution exist within the time limit.

To strengthen our observations, we compare the three approaches on Mushroom while varying $k$ in Figure 2. We observe that the two baselines CFPGrowth + + (PP) and CFPGrowth + + (CPP) are acting better than CP4MIS within a small $k$ (i.e., $k < 5$). However, CP4MIS scales very well when $k$ grows and it is able to prove that no solution exists on $k = 14$ instance in less than 27 minutes. While the two baselines CFPGrowth + + (PP) and CFPGrowth + + (CPP) follow an exponential scale and they reach the one hour time limit when $k$ is equal to, respectively, 5 and 9.

To sum up, our experimental evaluation shows that a specialized algorithm like CFPGrowth is faster on basic queries (e.g., asking for frequent itemsets), but it cannot cope with complex queries in a huge search space and of few solutions. It would need to think and to propose ad-hoc solutions, whereas CP approach enables a novice DM-user to express his query as constraints.
6 Conclusion

In this paper, we have introduced a constraint programming approach for itemset mining with multiple minimum supports MIS. For this, we have defined a new global constraint \textsc{FreqRare} and provided a filtering algorithm that mine frequent itemsets with multiple MIS in backtrack-free manner, given a variable ordering. We have empirically evaluated our CP approach. The experiments showed the performance of our CP model comparing to the specialized approach, \textsc{CFPGrowth++}. Although \textsc{CFPGrowth++} is very efficient on basic queries like mining frequent itemsets, our CP approach out-performs \textsc{CFPGrowth++} on complex queries with user-constraints and/or with search space explosion in a $k$-patterns mining. Furthermore, our CP approach provides the user with the flexibility to express any kind of constraints including constraints on MIS without the need to revise the solving process.
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