Research on the Prediction of Mechanical Response to Concrete under Sulfate Corrosion Based on the Grey Theory

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Abstract: In order to predict the influence on mechanical properties of concrete under sulfate erosion, based on the experimental data and Grey Theory, GM (1,1) model is used to establish models C20, C40, C60, C80 to predict concrete crushing strength in the natural drying process under an erosion of 6.9\% concentration sodium sulfate, and correlation is calculated between concrete strength grade, splitting tensile strength and compressive strength. The calculation analysis shows that (1) for low strength concrete (C20), most strength loss occurs in the initial stage of deterioration, and over time the deterioration rate gradually decreases and then increases again until being damaged; (2) in the wet-dry cycles, at the same strength level, the splitting tensile strength loss is more significant than the compressive strength loss; (3) with the increase of concrete grade, the sulphate resistance capability of concrete is reinforced. Compared with C20, the compressive strength loss of C80 is reduced to 27.4\% and 30\% respectively in 360 and 720 times of wet-dry cycles. The anti-erosion capacity of concretes has been greatly enhanced; (4) the deterioration of high-strength concrete is very slow in the later stage. The compressive strength loss of C80 decreases by 2\% in every 90 times of wet-dry cycles and decreases to 27\% in 1600 times of wet-dry cycles.

1. Introduction
There have been a lot of studies on concrete corroded by sulfate dry-wet circulation, it is still controversial by using dry-wet circulation system such as the high temperature80°C accelerating the dehumidifying \textsuperscript{(1-2)}On the one hand, high-temperature dehumidification will change material’s composition, such behavior is inconsistent with actual environment. Chinese scholar named run-dong Gao took a year's time to research concrete corroded by sulfate dry-wet circulation in normal temperature \textsuperscript{(9)}, exposted the degradation mechanism and macroscopic mechanical response from the micro and macro aspects. What he had done is to put concrete in sodium sulfate solution for 15 days, and remove it to the air environment for drying last 15 days, that called a whole dry-wet circulation. If we predict the concrete corroded situation for 2 or 3 years or more, it is so difficult to do that and we can’t ensure good results. Therefore this paper applies the gray system theory to the deterioration of concrete, uses some experimental data to establish relationship between concrete strength and dry-wet cycles.

2. GM(1,1) Model
Grey Dynamic Model, GM in short, is the basic model of Grey Theory and the basis of Grey Control Theory as well. Based on the grey module, it is a model built with fitting differential method.
GM has the quality of Differential, Difference and compatible with index; the parameter of GM is adjustable and non-unique; the structure of GM is elastic and varied; its tectonic mechanism is grey; GM is constant coefficient in nature and its parameter distribution is grey. These qualities allow us to easily construct a model to reflect the system status.

There are three features about Gray Model:(1) needing less information, usually four data is ok;(2) we don't have to know the distribution of original prior data, we just generated regular sequences by some rules;(3) In the system ,we can get high precision and keep the original characteristics, so GM can reflect the actual condition of the system better.

2.1 The general form of the GM (1, 1) Model
Let the reference sequence be:
\[ \begin{align*}
X^{(0)} &= (x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n))
\end{align*} \]
Relative sequences can be denoted as
\[ \begin{align*}
X^{(i)} &= (x^{(i)}(1), x^{(i)}(2), \cdots, x^{(i)}(n))
\end{align*} \]
\[ x^{(r)}(k) = \sum_{i=1}^{r} x^{(r-i)}(i) \quad k = 1, 2, \cdots n \]
Sequence \( Z^{(i)} \) is taken as the close generating sequence to the Sequence \( X^{(i)} \):
\[ \begin{align*}
Z^{(i)} &= \left( z^{(i)}(2), z^{(i)}(3), \cdots, z^{(i)}(n) \right) \\
z^{(i)}(k) &= \frac{1}{2} \left( x^{(i)}(k) + x^{(i)}(k-1) \right) \quad k = 2, 3, \cdots n
\end{align*} \]
\[ x^{(0)}(k) + a z^{(i)}(k) = b \] for GM (1, 1) can be denoted the general form of the GM (1, 1) Model.

2.2 Distinguishable Algorithm of the Parameters
Assume that \( \hat{a} = [a, b]^T \) is parameter sequence and
\[ Y = \begin{bmatrix} x^{(0)}(2) \\ x^{(0)}(3) \\ \vdots \\ x^{(0)}(n) \end{bmatrix}, \quad B = \begin{bmatrix} -z^{(i)}(2) & 1 \\ -z^{(i)}(3) & 1 \\ \vdots & \vdots \\ -z^{(i)}(n) & 1 \end{bmatrix} \]
then GM(1,1)Model \( x^{(0)}(k) + a z^{(i)}(k) = b \) ,the least squares estimate parameter sequence satisfies that:
\[ \hat{a} = (B^T B)^{-1} B^T Y \]

2.3 Reduction and Prediction of the Model
(1) \( \frac{dx^{(i)}}{dt} + ax^{(i)} = b \), the solution is also called time corresponding functions:
\[ x^{(i)}(t) = \left( x^{(i)}(1) - \frac{b}{a} \right) e^{-at} + \frac{b}{a} \]
(2) GM (1,1) Model \( x^{(0)}(k) + a z^{(i)}(k) = b \),its time corresponding sequence:
\( \hat{x}^{(i)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)e^{-ak} + \frac{b}{a}, \quad k = 1, 2, \ldots, n \)

(3) Reduction value:
\( \hat{x}^{(i)}(k+1) = \hat{x}^{(i)}(k+1) - \hat{x}^{(i)}(k) = \left(x^{(0)}(1) - \frac{b}{a}\right)(1-e^{-u})e^{-ak} + \frac{b}{a}, \quad k = 1, 2, \ldots, n \)

2.4 Applicable conditions of the model
- \( a \) is development coefficient, \( b \) is Grey action coefficient
  (1) when \( -a \leq 0.3 \), \( \text{GM} (1, 1) \) can be used for medium and long-term prediction;
  (2) when \( 0.3 \leq -a \leq 0.5 \), \( \text{GM} (1, 1) \) can be used for short-term prediction;
  (3) when \( 0.5 \leq -a \leq 0.8 \), \( \text{GM} (1, 1) \) is used for short-term prediction unreliable;
  (4) when \( 0.8 \leq -a \leq 1 \), should adopt Residual error correction of the GM (1, 1) model;
  (5) when \( -a > 1 \), \( \text{GM} (1, 1) \) is in vain;

3. Experimental Data
According to the test data of literature [9] to predict concrete dry-wet cycles in 6.8% concentration sulfate environment and the relationship between the compressive strength and the dry-wet cycles. Based on the corresponding prediction model, to predict its strength; And the correlation degree of concrete strength grade with splitting tensile strength and compressive strength are calculated.

4. The Model of concrete compressive strength prediction
4.1 Gray Simulation and Prediction about 0, 90, 180, 90, 360 cycles to C20 concrete
Indicator \( \frac{f_{\text{attack}}}{f_0} \) is sorted out action sequences,
\( X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(5)\right) = (1.000, 0.855, 0.810, 0.755, 0.710) \);

According 1-AGO relative strength accumulation generation sequence data:
\( X^{(i)} = \left(x^{(i)}(1), x^{(i)}(2), \ldots, x^{(i)}(5)\right) = (1.000, 1.855, 2.665, 3.420, 4.130) \);

Generating closed mean value:
\( Z^{(i)} = \left(z^{(i)}(2), z^{(i)}(3), z^{(i)}(4), z^{(i)}(5)\right) = (1.4275, 2.260, 3.0425, 3.775) \);

\[
Y = \begin{bmatrix}
0.855 \\
0.810 \\
0.755 \\
0.710
\end{bmatrix}, \quad B = \begin{bmatrix}
-1.4275 & 1 \\
-2.260 & 1 \\
-3.0425 & 1 \\
-3.775 & 1
\end{bmatrix}
\]

so: parameter \( \hat{a} = \begin{bmatrix} a, b \end{bmatrix}^T = \left(B^T B\right)^{-1}B^T Y = [0.0626, 0.9468] \);

\( a = 0.0626 < 0.3 \), that can be used long term prediction, so the responding model of the compressive strength associates with the dry-wet cycles is (Spacing interval for 90 times):
\[
\hat{x}^{(i)}(k+1) = \left(x^{(0)}(1) - \frac{b}{a}\right)(1-e^{-u})e^{-ak} + \frac{b}{a} = 0.9125e^{-0.0626k} + 15.125 \quad (k = 1, 2, 3 \cdots n)
\]

After reduction of the data:
\[ X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(5)\right) = (1.000, 0.8572, 0.8052, 0.7563, 0.7105) \]

Average relative error \( \bar{\varepsilon} = 0.2733\% \). The error is less than 10\%, accuracy meets the requirements. Through the model forecasts later four steps (450 cycles, 540 cycles, 630 cycles, 720 cycles) relative pressure strength variation:

\[ X^{(0)} = \left(x^{(0)}(6), x^{(0)}(7), x^{(0)}(8), x^{(0)}(9)\right) = (0.6674, 0.6269, 0.5889, 0.5532) \]

In the same way:

Establish C40 concrete strength response relationship model over time:

\[ \tilde{x}^{(0)}(k + 1) = 1.0829e^{-0.0687k} + 16.227 \quad (k = 1, 2, 3 \cdots n) \]

Average relative error \( \bar{\varepsilon} = 2.4340\% \)

Establish C60 concrete strength response relationship model over time:

\[ \tilde{x}^{(0)}(k + 1) = 1.0323e^{-0.0314k} + 33.363 \quad (k = 1, 2, 3 \cdots n) \]

Average relative error \( \bar{\varepsilon} = 0.0494\% \)

Establish C80 concrete strength response relationship model over time:

\[ \tilde{x}^{(0)}(k + 1) = 1.0658e^{-0.0268k} + 40.239 \quad (k = 1, 2, 3 \cdots n) \]

Average relative error \( \bar{\varepsilon} = 1.9332\% \)

The strength grade of concrete is established through the above the degradation model and can be seen that the development coefficient \( a \) is far less than 0.3, and can predict concrete deterioration development for a long term, the strength of concrete at all levels the degradation diagram is shown in figure 1; And the relative error of each level model meets the requirements, residual table is shown in table 1, 2.

### Table 1. Residual table

| Serial number | Time | C20 actual value | C20 simulated value | Residual (%) | C40 actual value | C40 simulated value | Residual (%) |
|---------------|------|-----------------|--------------------|-------------|-----------------|-------------------|-------------|
| \( X_0^{(0)} \) | 0    | 1.000           | 1.000              | 0.00        | 1.000           | 1.000             | 0.00        |
| \( X_1^{(0)} \) | 90   | 0.855           | 0.8572             | 0.26        | 0.990           | 1.0110            | 2.12        |
| \( X_2^{(0)} \) | 180  | 0.810           | 0.8052             | 0.59        | 0.970           | 0.9439            | 2.69        |
| \( X_3^{(0)} \) | 270  | 0.755           | 0.7563             | 0.17        | 0.900           | 0.8813            | 2.08        |
| \( X_4^{(0)} \) | 360  | 0.710           | 0.7105             | 0.07        | 0.800           | 0.8228            | 2.85        |
| \( X_5^{(0)} \) | 450  |                 | 0.6674             |             |                 | 0.7682            |             |
| \( X_6^{(0)} \) | 540  |                 | 0.6269             |             |                 | 0.7172            |             |
| \( X_7^{(0)} \) | 630  |                 | 0.5889             |             |                 | 0.6696            |             |
| \( X_8^{(0)} \) | 720  |                 | 0.5332             |             |                 | 0.6252            |             |

### Table 2. Residual table 2

| Serial number | Time | C60 actual value | C60 simulated value | Residual (%) | C80 actual value | C80 simulated value | Residual (%) |
|---------------|------|-----------------|--------------------|-------------|-----------------|-------------------|-------------|
| \( X_0^{(0)} \) | 0    | 1.000           | 1.000              | 0.00        | 1.000           | 1.000             | 0.00        |
| \( X_1^{(0)} \) | 90   | 1.000           | 1.0004             | 0.040       | 1.060           | 1.0376            | 2.11        |


4.2 Data Analysis
For C20 concrete, because the water cement ratio is bigger, concrete hydration process is relatively sufficient, but poor compactness, gaps inside concrete is abundant, relatively probability of connected gaps appears bigger. Degradation of concrete strength loss faster, strength fell about 15% in 90, 360, 720 times fell 25%, 30% and 45% respectively. Especially at the beginning of the degradation, strength losses is bigger, this is mainly because although hydration is fully, but the gap appears more, pores inside injury concrete plays a dominant factor.

As the level of concrete increases, resistance of concrete sulfate attack enhances, it has a great relationship with compaction rate. As the level of water cement ratio declines, Later the time of concrete hydration increases, concrete strength increases with the production of hydration due to gel material, the more to later, the hydration reaction is more fully, strength steadily increases, that C80 concrete strength losses in 360 was only 2.6%, and losses 14% in 720, while is about 30% lower than C20 concrete strength losses. Thus, reducing the water cement ratio, improving the compactness of concrete, improving concrete strength grade both can take a good role to resist to sulfate degradation.

5. Conclusion
(1) For the lower strength concrete (C20), degradation of concrete strength losses occurs at the initial stage, with time grows, the degradation rate decreases; With the increase of concrete grade level, concrete sulfate attack resistance enhances, C80 contrasts C20, concrete under 360 times, 720 times of the dry-wet circulation, compression strength losses reduce 27.4%, 30%, the erosion resistance increases obviously; later stage of degradation is very slow to high strength concrete, every 90 times of the dry-wet circulation of C80 concrete, compressive strength drops only about 2%, 1600 cycles losses is 27%.

(2) Under sulfate erosion environment, after a certain degradation of dry-wet circulation, with the increase of concrete grade level, the splitting tensile strength losses is more obviously than compressive strength losses, it suggests that it is not a feasible method by only improving strength grade for enhancing the performance of tensile strength.

(3) It is inconvenient to conduct the wetting and drying cycle under room temperature, especially the long-term degradation test, it waste time to operate that. While using the grey theory to get the better degradation model and the experimental data fitting, and the error is very small. It suggests that within the scope of the existing test data, gray prediction model needed less information, convenient operation, high precision, gray prediction model can significantly reduce the workload to test, shorten the test cycle; For engineering and technical personnel, they take more attention to the concrete compressive strength, splitting tensile strength, just like this macro mechanics indexes, the grey system theory take a good role to solve this problem, and to make predictions then be used as engineering reference.

Acknowledgments

| \( X^{(0)}_1 \) | 180 | 0.970 | 0.9694 | 0.060 | 0.980 | 1.0102 | 3.08 |
| \( X^{(0)}_2 \) | 270 | 0.940 | 0.9395 | 0.05 | 0.975 | 0.9835 | 0.87 |
| \( X^{(0)}_3 \) | 360 | 0.910 | 0.9104 | 0.04 | 0.974 | 0.9575 | 1.69 |
| \( X^{(0)}_4 \) | 450 | 0.8823 | | | | 0.9322 |
| \( X^{(0)}_5 \) | 540 | 0.8550 | | | | 0.9075 |
| \( X^{(0)}_6 \) | 630 | 0.8286 | | | | 0.8835 |
| \( X^{(0)}_7 \) | 720 | 0.8029 | | | | 0.8602 |
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Reference
[1] Messad, Carcassès, Linger, and Boutillon. PERFORMANCE APPROACH USING ACCELERATED TEST METHOD FOR EXTERNAL SULFATE ATTACK .[J] 3rd fib International Congress – 2010.
[2] Ferraris C F, Stutzman P E, Snyder K A. Sulfate Resistance of Concrete: A New Approach[R]. PCA, 2006
[3] Ouyang C, Nanni A, Chang WF. Internal and external sources of sulfate ions in Portland cement mortar: two types of chemical attack. Cem Concr Res1988;18:699–709.
[4] Jianming Gao, Zhenxin Yu, Luguang Song, Tingxiu Wang, Sun Wei. Durability of concrete exposed to sulfate attack under flexural loading and drying–wetting cycles. [J]Construction and Building Materials 39 (2013) 33–38.
[5] Julong Deng. Gray control system [M]. Wuhan: Hua zhong University of Science and Technology Press, 1985.
[6] Sifeng Liu etc. The Grey System Theory and its Application [M]. Beijing. Science Press.
[7] Jianmin Du etc. Underground Structure Concrete Sulfate Corrosion Mechanism and Performance Degradation. [M]. Beijing. China Railway Publishing Press.
[8] Rakesh Kumar, B. Bhattacharjee. Porosity, pore size distribution and in situ strength of concrete[J]. Cem and Concr Res, 2003, 33: 155-164.
[9] Rundong Gao etc. Dry-wet circulation under the action of degradation mechanism of concrete sulfate attack experimental study[J]. Journal of civil engineering. 2010 (02).
[10] HuaXi etc. Freeze-thaw concrete constitutive equation based on grey theory study [J]. Journal of disaster prevention and mitigation engineering. 2013 (02).