Macroscopic Body Motion in Terms of Quantum Evolution

A. Yu. Samarin

Samara Technical State University, 443100 Samara, Russia

Abstract

Macroscopic body is considered as a system, consisting of an infinite number of quantum particles. Mechanical motion of the system center of mass in physical space is described, using Feynman’s representation of the quantum evolution. It is shown that the center of mass of the system moves accordingly with the minimum action principle if action functional is much higher than Planck’s constant.

Keywords: quantum system center of mass, physical space, matter field, continuous medium, mechanical motion, unobservable motion characteristics.

1. Introduction

Since in 1935 Schrödinger formulated the problem of macroscopic body motion description firstly [1], it has become one of the most widely discussed problems in quantum mechanics. Besides the difficulties of interpretation, this problem does not allow to describe, in principle, the reduction of the wave function by means of conventional quantum mechanics. Really, the wave function collapse is a part of the measuring process, and this process requires availability of the macroscopic measuring instrument. Any macroscopic object obeys the laws of classical mechanics, formulated for physical space. Measuring instrument contains the particles that interact with the quantum object. Measuring process includes amplification of this interaction up to macroscopic level. Therefore, such specific description is necessary that could describe the system, having a variable number of particles. Besides this description has to operate in physical space, but this is impossible within conventional quantum mechanics.

In order to introduce into the consideration the notion of a macroscopic body as a mathematical object, using laws of quantum mechanics, first of all, it is necessary to determine the macroscopic body notion by suitable manner. Macroscopic body representation in the form of a mass point (or mass points collection) is impossible. Really, any quantum particle is represented by the wave function and can be considered as a mass point only immediately collapse. Besides, except for quantum particles with high energy, classical mechanical objects consist of a large number of quantum particle. Generally these particles have the quantum properties too. Therefore quantum system can possess the macroscopic properties only in the form of a unified object. Taking into account all this, following assertion can be made: the macroscopic body is a system of quantum particles, center of mass which moves in accordance with the laws of classical mechanics.

Mechanical motion of the center of mass in physical space is the subject of consideration in this paper.

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[1] For the space-time description of quantum evolution is necessary to use basis of the Hilbert space in the coordinate representation. If the number of particles in the system is changed, then this basis changes and corresponding description is impossible.

[2] In classical mechanics of continua the notion of individual particle is the mathematical abstraction. It is the mass point, having zero mass, but obeying Newton’s laws.
2. The Law of Dynamics

First of all let us determine the notion of a quantum particle. The quantum particle is the distributed in physical space indivisible quantum object, that is transformed into a mass point as a result of the collapse. Let the wave functions of the particles be nondegenerate \(^3\). In this case these particles are the matter fields. Mechanical motion of these matter fields (continuums) can not be described in terms of the Hilbert space: basis vectors collection of the Hilbert space is countable, whereas coordinates set of physical space has cardinality of continuum. Then we have to reject the axiomatics of observables and to consider the wave equation as the dynamical law of mechanical motion \(^2\). The object of this analysis is the path of the center of mass. Therefore the wave equation in the integral form is more suitable than the differential wave equation.

Consider evolution of the system, which consist of \(n\) quantum particle, in the form of one-dimensional mechanical motion of these particle. Suppose the dynamic law of this motion is the integral wave equation \(^3\)

\[
\Psi_{t_2}(x_2^1, \ldots, x_2^n) = \int \cdots \int K_{t_2,t_1}(x_2^1, \ldots, x_2^n, x_1^1, \ldots, x_1^n) \Psi_{t_1}(x_1^1, \ldots, x_1^n) \, dx_1^1 \cdots dx_1^n. \tag{1}
\]

where \(\Psi_{t_2}(x_2^1, \ldots, x_2^n), \Psi_{t_1}(x_1^1, \ldots, x_1^n)\) are the wave functions of the system at the time \(t_2\) and the initial time \(t_1 < t_2\) correspondingly; \(K_{t_2,t_1}(x_2^1, \ldots, x_2^n, x_1^1, \ldots, x_1^n)\) is the kernel of the integral evolution operator \(^3\). The superscript of the spatial variable denotes the number of the particle, the subscript — the time. The spatial variables is interpreted as space coordinates of the individual particles of corresponding continuums. The wave function and the kernel of the integral evolution operator depend on time parametrically. Further, instead of the last term, the term transition amplitude \(^4\) will be used.

Let the configuration space has the axes, corresponding to the particles coordinates. Then the transition amplitude has the form of continual integral \(^4\) in this configuration space. Denote by \(\Gamma\) a virtual path in this space. Then, we have

\[
K_{t_2,t_1}(x_2^1, \ldots, x_2^n, x_1^1, \ldots, x_1^n) = \int \exp \frac{i}{\hbar} S_{1,2}[\Gamma] \, [d\Gamma].
\]

The subscript of the action functional denotes the space-time positions of the system. Let \(\gamma\) be a virtual path of a quantum particle in physical space. Substituting \(\gamma\) for \(\Gamma\) in the last expression we get

\[
K_{t_2,t_1}(x_2^1, \ldots, x_2^n, x_1^1, \ldots, x_1^n) = \int \cdots \int \exp \frac{i}{\hbar} S^{\Sigma}_{1,2}[\gamma^1, \ldots, \gamma^n] \, [d\gamma^1] \cdots [d\gamma^n], \tag{2}
\]

where

\[
S^{\Sigma}_{1,2}[\gamma^1, \ldots, \gamma^n] = \sum_{j=1}^{n} \int_{t_1}^{t_2} \left( \frac{m^j (v^j)^2}{2} - U^j(x^j) - \sum_{k=1, k \neq j}^{n} U^{jk}(x^j, x^k) \right) \, dt \tag{3}
\]

is the sum of all action functionals of the particles, that are form the system. The sum is over all particles of the system. In the last expression \(\frac{m^j (v^j)^2}{2}\) — the kinetic energy of the individual particle \(^5\); \(U^j(x^j)\) — the potential energy of the particle \(j\) in external field; \(\sum_{k=1, k \neq j}^{n} U^{jk}(x^j, x^k)\) — the interaction energy of the particles of the system.

3. Motion of the Center of Mass

\(^3\)This assumption does not lead to a loss of generality of the consideration due to superposition principle \(^2\).

\(^4\)This amplitude formally corresponds to the transition between the states, that have spatial localization.

\(^5\)The individual particle is an element of continuous medium.
Consider the motion of the center of mass. Every virtual path of the system in the configuration space determines a unique virtual path of the center of mass in the form

$$X_\Gamma(t) = \sum_{j=1}^{n} \frac{m_j x_j^\Gamma(t)}{\sum_{j=1}^{n} m_j}.$$  

Here the individual particle path $x_j^\Gamma(t)$ corresponds to the path $\Gamma$. Then the total functional (3) is the sum of the functionals corresponding to motion of the center of mass and relative motion of the system particles:

$$S^{\Sigma}_{12} = S^{C}_{12} + S^{R}_{12},$$

where

$$S^{C}_{1,2} = \int_{t_1}^{t_2} \left( \frac{M(V)^2}{2} - \sum_{j=1}^{n} U_j(X, \xi_j^1) \right) dt;$$

$$S^{R}_{1,2} = \sum_{j=1}^{n} \int_{t_1}^{t_2} \left( \frac{m_j(\dot{\xi}_j)^2}{2} - \sum_{k=1, k\neq j}^{n} U_{jk}(\xi_j^1, \xi_k^1) \right) dt;$$

$V$ — the center of mass velocity, $\xi^j$ — coordinate of an individual particle of the quantum particle $j$ relative to center of mass; $M$ — the mass of the system. Using these notation, in return for (2) we get:

$$K_{t_2, t_1}(X_2, X_1) = \exp \left( \frac{i}{\hbar} \int_{t_1}^{t_2} \left( T(V(t)) - U(X(t)) \right) dt \right) [dX(t)].$$

In order for the path of the macroscopic body, it is necessary to estimate the deposits of different virtual paths in the last continual integral value.

4. THE MACROSCOPIC BODY PATH

There is the formal mathematical procedure, which transforms the complex path integral in the real form [5]. Accordingly this procedure the time variable is transformed in the complex form $t = \tau \exp i\varphi$. Then the path integral is considered for the imaginary negative time ($\varphi = -\frac{\pi}{2} \Rightarrow t = -i\tau$):

$$K_{t_2, t_1}(X_2, X_1) = \exp \left( -\frac{1}{\hbar} \int_{t_1}^{t_2} \left( T(V(\tau)) + U(X(\tau)) \right) d\tau \right) [dX(\tau)].$$

In order to review the path of the macroscopic body, it is necessary to estimate the deposits of different virtual paths in the last continual integral value.
where $T(V(\tau)) = \frac{M}{2} \left(\frac{dX(\tau)}{d\tau}\right)^2$ — The kinetic energy expressed as a function of time module. Using the last expression, we obtain

$$\Psi_{\tau_2}(X_2) = C \int \left(\int \exp\left(-\frac{i}{\hbar} S_{12}^E[X(\tau)]\right)[X(\tau)]\right) \Psi_1(X_1) \, dX_1.$$  

Here $S_{12}^E[X(\tau)]$ is the Euclidean action functional, $C$ — the normalization factor. Denote by $S_{12}^E[X_1, X_2]$ the action functionals, that have assigned initial and final positions of the center of mass. Let $S_{12}^{E,\text{min}}(X_1, X_2)$ be the least of these action functional. Then we have

$$\Psi_{\tau_2}(X_2) =$$

$$= C \int \exp\left(-\frac{i}{\hbar} S_{12}^{E,\text{min}}(X_1, X_2)\right) \left(\int \exp\left(-\frac{i}{\hbar} \Delta S_{12}^E[X(\tau)]\right)[X(\tau)]\right) \Psi_1(X_1) \, dX_1.$$  

Here $\Delta S_{12}^E[X(\tau)] = S_{12}^E[X(\tau)] - S_{12}^{E,\text{min}}(X_1, X_2)$. Let $s_{12}^{E,\text{min}}$ be the least action functional $S_{12}^{E,\text{min}}(X_1, X_2)$. Then, we obtain

$$\Psi_{\tau_2}(X_2) = C \exp\left(-\frac{i}{\hbar} s_{12}^{E,\text{min}}\right) \times$$

$$\times \left(\int \exp\left(-\frac{i}{\hbar} \Delta s_{12}^{E,\text{min}}(X_1, X_2)\right) \left(\int \exp\left(-\frac{i}{\hbar} \Delta S_{12}^E[X(\tau)]\right)[X(\tau)]\right) \Psi_1(X_1) \, dX_1.$$  

Here $\Delta S_{12}^{E,\text{min}}(X_2, X_1) = S_{12}^{E,\text{min}}(X_2, X_1) - s_{12}^{E,\text{min}}$. The exponential factor $\exp\left(-\frac{i}{\hbar} s_{12}^{E,\text{min}}\right)$ does not depend on space coordinates and can be include in the normalization factor $C'$. Then

$$\Psi_{\tau_2}(X_2) = C' \times$$

$$\times \left(\int \exp\left(-\frac{i}{\hbar} \Delta s_{12}^{E,\text{min}}(X_1, X_2)\right) \left(\int \exp\left(-\frac{i}{\hbar} \Delta S_{12}^E[X(\tau)]\right)[X(\tau)]\right) \Psi_1(X_1) \, dX_1.$$  

Macroscopic body contains infinite number of quantum particles. Therefore the center of mass of such mechanical system has the Euclidian action (for any finitesimal time interval) such that $s_{12} >> \hbar$. This situation can be formally expressed as $\hbar \to 0$. In this case we have a unique path, that determines the last path integral$^6$. This path $X^{\text{min}}(\tau)$ corresponds to the least Euclidian action $s_{12}^{E,\text{min}}$. If to go back to the real time, using the analytic continuation, we obtain usual principle of least action. The spacial part of the wave function $\Psi_{\tau_2}(X_2)$ is the delta-function $\delta(X_2 - X^{\text{min}}_2)$.  

4. CONCLUSION

Thus in order to change motion state of a macroscopic body, it is necessary the macroscopic external influence$^7$. Taking into account this we can state that in the famous situation of the Schrodinger cat [1] neither evolution nor even the reduction of the decaying nucleus wave function are not immediate causes of the cat death. The death of the cat is directly determined by the macroscopic process of poisoning. Quantum evolution of the entire system results in the macroscopic registering process in the measuring instrument. Dynamics of this phenomenon can be described only by the integral wave equation$^6$. This process determines directly both the collapse result and the dynamics of the cartridge with poison.

$^6$The integral measure of all other sets of the virtual paths is equal to zero because of $\lim_{\hbar \to 0} \exp\left(-\frac{i}{\hbar} \Delta S_{12}^{E,\text{min}}(X_1, X_2)\right) = 0$ and $\lim_{\hbar \to 0} \exp\left(-\frac{i}{\hbar} \Delta S_{12}^E[X(\tau)]\right) = 0$.

$^7$Motion of a macroscopic body is determined by the value of its center of mass or the centers of mass of this body macroscopic parts. In any case these functionals are macroscopic quantities.
References

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