Interaction of movable operating elements of cutting machines with material being cut

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Abstract. The article covers analysis of the force interaction of knife and material being cut; the components of cutting force on the main operating elements of cutting tool are obtained. The mathematical analysis of the value of “clean” cutting component is given, and the cutting ability of the blade with purpose of improving the cut quality is determined.

1. Introduction
Plunge cutting of food materials is currently being studied mainly from the point of establishing empirical dependences of main indicators of equipment operation (productivity, energy consumption, amount of waste, etc.) on factors conditioned by the type of material being cut, processing mode and cutting tool. This direction of research is important, as it allows, within the studied area of the factor space, to more objectively approach the issues of choosing rational cutting modes, characteristics of the existing cutting tool, as well as design parameters of cutting machines. However, the available empirical dependencies do not always give a satisfactory solution in terms of radical improvement of cutting machines, since they do not sufficiently reveal the essence of the interaction of the blade with the material being cut, the mechanism of destruction and the features of the accompanying phenomena.

The analysis of the power circuits of the interaction of the object of processing and the cutting tool is much more promising, which allows considering the mutual influence of all factors based on the study of cutting mechanics. Taking into account the peculiarities of sliding cutting and the relatively weak consistency of the formed macaroni tubes, it is also necessary in this section to investigate such an important characteristic of knives as their cutting ability.

2. Methods
Throughout the development of the science of cutting, starting with the works of I.A. Time [7] and V.P. Goryachkin [3], the initial and basic task was the problem of forces arising during the mechanical processing of various materials. With the known material to be processed, and the initially selected cutting conditions and mode, only the subsequent force pattern arising on working planes of cutting tool in the end determines the quantitative and qualitative indicators of the cutting equipment operation [1, 2]

When introducing a knife into the cut material, it is customary to distinguish the following successive phases (stages): preliminary deformation, formation of a new surface, friction of material on the working surfaces of the knife [5]. Cutting raw macaroni products also begins with the preliminary deformation of the half-finished product.
Dependence between stress and deformations at the initial section is described by the following expression:

\[
\sigma = \frac{C}{A}(e^{A\varepsilon} - 1)
\]

where: \( C \) and \( A \) are experiment coefficients.

At the stage of preliminary compression of half-finished product, two phases can be distinguished:

- material interacts only with the cutting edge;
- material interacts with edge and facet of the knife.

In the first phase, value of the angle of inclination \( \nu \) of the surface of the deformed material does not reach the value of the sharpening angle \( \beta \) (figure 1-a). The force at the cutting edge is determined by integrating the expression \( \sigma \varepsilon \) \( d_x \), where \( \sigma \varepsilon \) is the stress on the \( d_x \) section with \( X \) abscissa.

Deformation of the material by the knife blade \( \varepsilon = h/H \) (figure 1-a).

Then effort on the blade is equal to:

\[
P = \int_0^l \sigma \varepsilon \, d_x = \int_0^l \left[ e^{A\left(e^{-\frac{ctg \nu}{H}}\right)} - 1 \right] \, d_x.
\]

The upper limit \( l \) is the length of section where deformation propagates. On figure 1 it can be seen that \( l = h \tan \nu \). After integration, we obtain the value of the effort on the blade for the first phase:

\[
P = \frac{CH}{A^2 \, ctg \nu} \left( e^{A\varepsilon} - A\varepsilon - 1 \right).
\]

The second phase is a further compression of the material with a knife, when its facet is in contact with the material, and deformation occurs in two sections, i.e. under the facet and in section adjacent to the left. The nature of deformation in section to the right of the blade corresponds to the first stage and is therefore not considered here.

**Figure 1.** The stage of the preliminary compression of the material. a) the first phase; b) the second phase.
3. Results
Deformation zone spreads farther from the blade as the knife penetrates, and the absolute and relative deformation decreasing as the sections move away from the blade. The deformation of the material under the facet in section with the $X_2$ abscissa is:

$$\varepsilon_x = \frac{h_x}{H} = \varepsilon - \frac{\cot \beta}{H} x.$$  \hspace{1cm} (4)

Then the corresponding compression stress is

$$\sigma_x = \frac{C}{A} \left[ e^{A(\varepsilon - \frac{\cot \beta}{H}) x} - 1 \right].$$  \hspace{1cm} (5)

To determine the values of the effort $P$ of compression of the material over the entire width of the facet $\delta$, it is necessary to sum up the forces $P_x = \sigma_x d_x$, or for $\Delta X \to 0$ we obtain:

$$P_{ch} = \frac{\delta C}{A} \left[ e^{A(\varepsilon - \frac{\cot \beta}{H}) x} - 1 \right] d_x.$$  \hspace{1cm} (6)

After integration:

$$P_{ch} = \frac{C}{A} \left[ \frac{H e^{A\varepsilon}}{A \cot \beta} (1 - e^{-\frac{A \cot \beta}{H}}) - \delta \right].$$  \hspace{1cm} (7)

Deformation of the material out of the wedge for an elementary section with a base $\Delta X$, located at a distance $x_2$ from the cutting edge, is equal to:

$$\varepsilon_x = \frac{h_x}{H} = \frac{h - \delta \cot \beta}{H} - \frac{\cot \beta}{H} x_2.$$  \hspace{1cm} (8)

Where $x$ is absciss of the section $\Delta x$, counting from the extreme point of the facet of the knife.

Let us denote $\frac{h - \delta \cot \beta}{H} = \varepsilon_1$, where $\varepsilon_1$ is the relative deformation of the material under the extreme point of the facet. Then the value

$$\varepsilon_x = \varepsilon_1 - \frac{\cot \beta}{H} x_2,$$  \hspace{1cm} (9)

and the corresponding compression stress is

$$\sigma_x = \frac{C}{A} \left[ e^{\frac{A(\varepsilon_1 - \cot \beta \frac{x_2}{H})}{H}} - 1 \right].$$  \hspace{1cm} (10)

Integrating $\sigma_x d_x$ in the limits from 0 to $(h - \delta \cot \beta) \cot \beta v_2$, we obtain

$$P_M = \frac{CH}{A^2 \cot \beta v_2} \left( e^{A\varepsilon_1} - A\varepsilon_1 - 1 \right).$$  \hspace{1cm} (11)
Values of the angles $\nu_1, \nu_2$ are related to $\varepsilon$ with the correlation of the form:

$$\text{ctg } \nu = K\varepsilon^2.$$  \hspace{1cm} (12)

The critical preliminary compression force is determined by summing:

$$P_c = P + P_{ch} + P_m.$$  \hspace{1cm} (13)

Using expressions (3), (7), (11), we have

$$P_c = \frac{CH}{A_2K\varepsilon^2} (e^{A_\varepsilon} - A\varepsilon - 1) + \frac{C}{A} \left[ \frac{He^{A_\varepsilon}}{Actg\beta} (1 - e^{-\frac{A_\varepsilon}{H} e^{Actg\beta} - \delta}) + \right. \left. \frac{CH}{A_2K\varepsilon^2} (e^{A_\varepsilon} - A\varepsilon - 1) \right].$$  \hspace{1cm} (14)

At a certain, quite definite value $\varepsilon = \varepsilon_{cr}$, the penetration of the knife into the material, i.e. cutting begins. The compression force corresponding to the beginning of cutting is generally determined from (14) by substituting $\varepsilon_{cr}$ instead of $\varepsilon$.

The force of the so-called “clean” cutting, producing the actual formation of a new surface, depends on the stress $\sigma_{max}$, acting on the contact surface of the blade:

$$R_0 = \sigma_{max} a \cdot \eta \cdot K_b$$  \hspace{1cm} (15)

Where $K_b$ is coefficient taking into account the mutual influence of the tubes; $a$ is the width of the cutting edge; $\eta$ is the parameter of the curved support surface.

To simplify the design scheme, we will assume that the knife is stationary, and the macaroni tube moves towards it with a speed $u$ (figure 2). The speed vector $\vec{u}$ before impact is directed perpendicular to the blade line.

The presence of the tangential component of the speed, characteristic of sliding cutting, will be taken into account by the value of $\eta$, which is equal to the ratio between the actual and nominal contact area of the blade and, thereby, depending on the sliding coefficient $K_c$.

Let us denote the force arising at moment of impact by $R$. The action of this force leads to a change in the speed of the tube in accordance with the equation [8]:

$$m \frac{du}{dt} + R = 0$$  \hspace{1cm} (16)

where: $m$ is the mass of unit of tube length,

$$m = \frac{\pi(D_1^2 - D_2^2)}{4} \rho;$$  \hspace{1cm} (17)

where: $D_1, D_2$ are the outer and inner diameters of the tube, respectively; $\rho$ is the density of the dough.

Let us denote by $S$ the distance to which the center of the tube approaches the blade because of local compression at point 0. The speed of this approach is $u = \dot{S}$, and the acceleration is

$$- \frac{du}{dt} = \ddot{S}.$$  From formula (6) we find $\dot{S}$

$$\dot{S} = R/m$$  \hspace{1cm} (18)
To solve equation (8), it is necessary to know the relationship between the force $R$ and the approach $S$. This relationship can be found if we use the results of the problem of collision in the theory of plasticity [6]. When a cylinder is in contact with a flat surface, the contact width is determined by the formula:

$$\theta = 2\sqrt{2R(K_1 + K_2)D_1},$$

where $K_1 = 1 - \mu_C^2 / \pi E_C$; $K_2 = 1 - \mu_T^2 / \pi E_T$.

$\mu_C$ and $\mu_T$ are Poisson coefficients for steel and dough, respectively; it is possible to take $\mu_C = \mu_T = 0.3$; $E_C$ and $E_T$ are moduli of elasticity of steel and dough in compression; $E_C = 2.1 \cdot 10^{11}$ Pa; $E_T = 6.2 \cdot 10^3$ Pa; $D$ is the outer diameter of the tube, m.

Dependence between the width of the contact $\theta$ and the displacement of the center of the tube at moment of impact is presented in the form $\theta^2 = SD_1$. Then expression (9) is transformed to the form:

$$S = bR(K_1 + K_2)$$

**Figure 2.** Calculation scheme for the impact interaction of macaroni tube with a knife.

Substituting the values of $R$ from this expression into equation (18), we obtain

$$\dot{S} = -\frac{1}{8(K_1 + K_2)m} S$$

Multiplying both sides of this equation by $d$ and performing the necessary transformations, we obtain

$$\frac{1}{2}d(\dot{S})^2 = -\frac{1}{8(K_1 + K_2)m} SdS$$

Integration of the last equation gives the following result:

$$\frac{1}{2}(S^2 - u_0^2) = \frac{1}{16(K_1 + K_2)m} S^2;$$

**Figure 3.** Cutting edge model in cross section.
where \( u_0 \) is the speed of the tube before it hits the knife.

The closest approach \( S_{\text{max}} \) occurs at moment when the tube stops, i.e. its speed \( u = \dot{S} = 0 \), then from the previous formula we have:

\[
S_{\text{max}} = 2u_0 \sqrt{2(K_1 + K_2)m}.
\]  

(24)

Knowing \( S_{\text{max}} \), it is possible, using equation (10), to calculate the maximum effort \( R_{\text{max}} \) per unit length of the blade

\[
R_{\text{max}} = \frac{u_0}{2} \sqrt{\frac{m}{K_1 + K_2}}.
\]  

(25)

Maximum stress in contact zone of the blade and the material being cut is determined by the ratio [6]:

\[
\sigma_{\text{max}} = \frac{4R_{\text{max}}}{\pi \beta}.
\]  

(26)

Alternatively, taking into account (17), (19) and (25):

\[
\sigma_{\text{max}} = \frac{2}{\pi^{0.75}} \cdot \frac{u^{0.5} \cdot \rho^{0.25}}{(K_1 + K_0)^{0.75}}.
\]  

(27)

Thus, the force of “clean” cutting can be determined from the ratio:

\[
R_0 = \frac{2 \cdot a \cdot \eta \cdot u^{0.5} \cdot \rho^{0.25} \cdot K_{\beta}}{\pi^{0.75} \cdot (K_1 + K_0)^{0.75}}.
\]  

(28)

The last component of the total cutting force is the frictional force of the material on the surface of the knife. Taking into account the pronounced adhesive properties of half-finished product, the value of this component can be considered proportional to the contact surface area [4]:

\[
F_{cf} = F_H \cdot \tau.
\]  

(29)

where \( \tau \) is the friction stress; \( F_H \) is nominal contact surface area.

Thus, the total cutting force is

\[
R = R_0 + P_c + F_{cf}.
\]  

(30)

Taking to simplify the calculations \( \varepsilon_1 = \varepsilon_{kp} \) and using expressions (14), (28), (29), we obtain the equation for calculating the total force:

\[
R = \frac{2CH}{A^2 K_{\varepsilon_{kp}}^2} (e^{A\varepsilon_{kp}} - A \varepsilon - 1) + \frac{C}{A} \left[ \frac{He^{A\varepsilon_{kp}}}{Actg \beta} \left( 1 - e^{-\frac{A}{H} \cdot \varepsilon_{kp} \cdot \beta} \right) - \delta \right] + \\
+ \frac{2a \eta \cdot u^{0.5} \cdot \rho^{0.25} \cdot K_0}{\pi^{0.75} (K_1 + K_2)^{0.75}} + F_H \cdot \tau
\]  

(31)
4. Discussion
Analysis of formula (16) shows that from a large number of factors influencing the value of the total cutting force, the action of the parameters of the blade microgeometry \((a, \eta)\), indicators of deformation, frictional and strength characteristics of the cutting object \((\varepsilon_{kp}, A, C, \rho, \tau)\), geometric parameters of the cutting tool \(= (\beta, \delta)\) and molded half-finished product \((H)\) are very influential.

Practical use of formula (16) is associated with certain difficulties, the main of which are the absence of numerical values for many of the above characteristics. However, from the obtained formula for calculating the total force \(R\), it follows that the improvement in the cut quality is primarily associated with an increase in the cutting ability of the blades and a decrease in the influence of accompanying cutting processes in the form of material compression and friction along the contact surfaces.

5. Conclusions
Analysis of the force scheme of the interaction between the knife and the material being cut allows obtaining all the components of the total cutting force on the main operating elements of the cutting tool, i.e. the blade, facets and side surfaces. The value of “clean” cutting component \(R_0\) is associated with the formation of a new surface and is determined by the cutting ability of the blade, which, in turn, depends on the microgeometry parameters.

During designing a cutting tool and choosing cutting modes, it is advisable to strive to reduce the total cutting force and, in particular, its components associated with the preliminary deformation of macaroni tube layer and friction of the knife on the material being cut.

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