Blue Perturbation Spectra from Hybrid Inflation with Canonical Supergravity

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Abstract

We construct a hybrid inflationary model associated with the superheavy scale $M_X \sim 10^{16}$ GeV of supersymmetric grand unified theories in which the inflaton potential is provided entirely by canonical supergravity. We find that the spectrum of adiabatic density perturbations is characterized by a strongly varying spectral index which is considerably larger than unity. Moreover, the total number of e-foldings is very limited. Implications of our analysis for other hybrid inflationary scenarios are briefly discussed.
The majority of successful inflationary scenarios \cite{1} invoke a very weakly coupled gauge singlet scalar field known as inflaton in order to account for the smallness of the observed temperature fluctuations $\Delta T/T$ in the cosmic background radiation. Recently, Linde \cite{2} proposed a new inflationary model which looks as a hybrid of chaotic inflation with a quadratic potential for the gauge singlet inflaton and the usual theory of spontaneous symmetry breaking involving a possibly gauge non-singlet field. During inflation the non-inflaton field is trapped in a false vacuum state and the universe is dominated by the false vacuum energy density. Inflation ends with a phase transition when the non-inflaton field rolls very rapidly to its true vacuum state ("waterfall"). The beauty of this hybrid model lies in the fact that it does not use small coupling constants in order to produce the observed temperature fluctuations and that it reconnects inflation with phase transitions in grand unified theories (GUTs). Although the original hybrid model is non-supersymmetric, there is a direct adjustment to supersymmetric (SUSY) models \cite{3} with the most commonly used superpotential for symmetry breaking, an inflaton mass of the order of 1 $TeV$, the SUSY breaking scale, and an intermediate scale ($\sim 10^{11} - 10^{12} GeV$) of symmetry breaking. An additional motivation for a SUSY hybrid inflationary scenario is the possibility offered by supersymmetry to naturally forbid large self-couplings of the inflaton through R-symmetries \cite{4}. In connection with supersymmetry it is very desirable to associate hybrid inflation with symmetry breaking scales $\sim 10^{16} GeV$ consistent with the unification of the gauge couplings of the minimal supersymmetric standard model (MSSM) which is favored by LEP data. However, the electroweak mass of the inflaton provided by SUSY breaking is too weak to account for the correct value of $\Delta T/T$ and an appropriate potential for the inflaton has to be found \cite{4}. In particular, a variation of hybrid inflation, smooth hybrid inflation \cite{5}, in which the phase transition takes place gradually during inflation, has been invented to successfully address this issue. As soon as one replaces global by local supersymmetry it is well-known that the potential becomes very steep and inflation becomes, in general, impossible. This is, to a large extent, due to the generation of a mass for the inflaton which is larger than the Hubble constant $H$. For the simple superpotentials used so far in SUSY hybrid inflation such a
mass for the inflaton is not generated provided the canonical form of the Kähler potential of $N = 1$ supergravity is employed. However, even in these cases one has to question the extent to which canonical supergravity affects a successful global SUSY inflationary scenario, especially if during inflation the inflaton takes values close to the supergravity scale $M_P/\sqrt{8\pi} \simeq 2.4355 \times 10^{18} \text{GeV}$ ($M_P \simeq 1.221 \times 10^{19} \text{GeV}$ is the Planck mass). Intermediate scale models are not expected to be seriously affected by the additional terms generated by canonical supergravity whereas the same does not apply to models in which inflation is associated with the superheavy scale $M_X \sim 10^{16} \text{GeV}$ of SUSY GUTs.

A logically distinct possibility is that during inflation supergravity dominates the inflaton potential instead of simply being a perturbation. Our purpose in the present paper is to investigate the above possibility and attempt to construct a hybrid inflationary model associated with the superheavy scale of SUSY GUTs in which the inflaton potential is provided entirely by the terms generated when global supersymmetry is replaced by canonical supergravity. We will see that the most natural implementation of this idea is in the context of SUSY GUTs based on semi-simple gauge groups of rank six. An interesting property of supergravity dominated hybrid inflation is that the spectral index of the adiabatic density perturbations is considerably larger than 1 (blue primordial spectra) and strongly varying.

We consider a SUSY GUT based on a (semi-simple) gauge group $G$ of rank $\geq 5$. $G$ breaks spontaneously directly to the standard model (SM) gauge group $G_S \equiv SU(3)_c \times SU(2)_L \times U(1)_Y$ at a scale $M_X \sim 10^{16} \text{GeV}$. The symmetry breaking of $G$ to $G_S$ is obtained through a superpotential which includes the terms

$$W = S(-\mu^2 + \lambda \Phi \bar{\Phi}).$$  \hfill (1)

Here $\Phi, \bar{\Phi}$ is a conjugate pair of left-handed SM singlet superfields which belong to non-trivial representations of $G$ and reduce its rank by their vacuum expectation values (vevs), $S$ is a gauge singlet left-handed superfield, $\mu$ is a superheavy mass scale related to $M_X$ and $\lambda$ a real and positive coupling constant. The superpotential terms in eq. (1) are the dominant couplings involving the superfields $S, \Phi, \bar{\Phi}$ which are consistent with a continuous
R-symmetry under which $W \rightarrow e^{i\gamma} W$, $S \rightarrow e^{i\gamma} S$, $\Phi \rightarrow \Phi$ and $\bar{\Phi} \rightarrow \bar{\Phi}$. Moreover, we assume that the presence of other $SM$ singlets in the theory does not affect the superpotential in eq. (1). The potential obtained from $W$, in the supersymmetric limit, is

$$V = | -\mu^2 + \lambda \Phi \bar{\Phi} |^2 + | \lambda S |^2 ( | \Phi |^2 + | \bar{\Phi} |^2 ) + D - terms,$$  \hspace{1cm} (2)

where the scalar components of the superfields are denoted by the same symbols as the corresponding superfields. The SUSY vacuum

$$< S > = 0, < \Phi > < \bar{\Phi} > = \mu^2 / \lambda, \quad | < \Phi > | = | < \bar{\Phi} > |$$  \hspace{1cm} (3)

lies on the D-flat direction $\Phi = \bar{\Phi}^*$. By appropriate gauge and R-trasformations on this D-flat direction we can bring the complex $S, \Phi, \bar{\Phi}$ fields on the real axis, i.e. $S \equiv \frac{1}{\sqrt{2}} \sigma$, $\Phi = \bar{\Phi} \equiv \frac{1}{\sqrt{2}} \phi$, where $\sigma$ and $\phi$ are real scalar fields. The potential in eq. (2) then becomes

$$V(\phi, \sigma) = (-\mu^2 + \frac{1}{4} \lambda \phi^2)^2 + \frac{1}{4} \lambda^2 \sigma^2 \phi^2$$  \hspace{1cm} (4)

and the supersymmetric vacuum corresponds to $| < \frac{\phi}{2} > | = \frac{\mu}{\sqrt{\lambda}} = \frac{M_X}{g}$ and $< \sigma > = 0$, where $M_X$ is the mass acquired by the gauge bosons and $g$ is the gauge coupling constant. For any fixed value of $\sigma > \sigma_c$, where $\sigma_c = \sqrt{2} \mu / \sqrt{\lambda} = \sqrt{2} | < \frac{\phi}{2} > |$, $V$ as a function of $\phi$ has a minimum lying at $\phi = 0$. The value of $V$ at this minimum for every value of $\sigma > \sigma_c$ is $\mu^4$.

Adding to $V$ a mass-squared term for $\sigma$ we obtain Linde’s potential with the only difference that in the SUSY model the critical value $\sigma_c$ of $\sigma$, below which the minimum at $\phi = 0$ disappears, becomes very closely connected with the vev of $\phi$. When $\sigma > \sigma_c$ the universe is dominated by the false vacuum energy density $\mu^4$ and expands quasi-exponentially. When $\sigma$ falls below $\sigma_c$ the mass-squared term of $\phi$ becomes negative, the false vacuum state at $\phi = 0$ becomes unstable and $\phi$ rolls rapidly to its true vacuum thereby terminating inflation.

Let us now replace global supersymmetry by $N = 1$ canonical supergravity. From now on we will use the units in which $\frac{M_P}{\sqrt{8\pi}} = 1$. Then, the potential $V(\phi, \sigma)$ becomes

$$V(\phi, \sigma) = [(-\mu^2 + \frac{1}{4} \lambda \phi^2)^2(1 - \frac{\sigma^2}{2} + \frac{\sigma^4}{4}) + \frac{1}{4} \lambda^2 \sigma^2 \phi^2(1 - \frac{\mu^2}{\lambda} + \frac{1}{4} \phi^2)^2] e^{\frac{1}{2}(\sigma^2 + \phi^2)}.$$  \hspace{1cm} (5)
V still has a minimum with \( V = 0 \) at \( |\phi| = \frac{\sigma}{\sqrt{\lambda}} \) and \( \sigma = 0 \) and a critical value \( \sigma_c \) of \( \sigma \) which remains essentially unaltered. The important difference lies in the expression of \( V(\sigma) \) for \( \sigma > \sigma_c \) and \( \phi = 0 \)

\[
V(\sigma) = \mu^4(1 - \frac{\sigma^2}{2} + \frac{\sigma^4}{4})e^{\frac{\sigma^2}{2}},
\]

which now has a non-zero derivative \( V'(\sigma) \) with respect to \( \sigma \)

\[
V'(\sigma) = \frac{1}{2}\mu^4\sigma^3(1 + \frac{\sigma^2}{2})e^{\frac{\sigma^2}{2}}.
\]

Expanding \( V(\sigma) \) in powers of \( \sigma^2 \) and keeping the first non-constant term only we obtain

\[
V(\sigma) \simeq \mu^4 + \frac{1}{8}\mu^4\sigma^4 \quad (\sigma^2 << 1).
\]

We see that no mass-squared term for \( \sigma \) is generated \[3\] and that for \( \sigma^2 << 1 \) the model resembles the original hybrid inflationary model with a quartic, instead of quadratic, inflaton potential in which the quartic coupling takes naturally the very small value \( \frac{1}{2}\mu^4 \). The number of e-foldings \( \Delta N(\sigma_{in}, \sigma_f) \) for the time period that \( \sigma \) varies between the values \( \sigma_{in} \) and \( \sigma_f \) (\( \sigma_{in} > \sigma_f \)) is given, in our approximation, by

\[
\Delta N(\sigma_{in}, \sigma_f) = -\int_{\sigma_{in}}^{\sigma_f} \frac{V}{V'} d\sigma = \sigma_f^{-2} - \sigma_{in}^{-2}.
\]

Also the ratio \( (\frac{\Delta T}{T})_T^2/(\frac{\Delta T}{T})_S^2 \sim \sigma_H^6 << 1 \), where \( (\frac{\Delta T}{T})_T \) and \( (\frac{\Delta T}{T})_S \) are the tensor and scalar components of the quadrupole anisotropy \( \frac{\Delta T}{T} \) respectively and \( \sigma_H \) is the value that the inflaton field had when the scale \( \ell_H \), corresponding to the present horizon, crossed outside the inflationary horizon. Therefore, we can safely ignore \( (\frac{\Delta T}{T})_T \) and obtain \[6\]

\[
\frac{\Delta T}{T} \simeq \frac{1}{4\pi \sqrt{45}} \left(\frac{V^{3/2}}{V'}\right)_{\sigma_H} = \frac{1}{2\pi \sqrt{45}} \frac{\mu^2}{\sigma_H^3} \quad (\sigma_H^2 << 1).
\]

Using the above approximate expressions we are going to investigate the possibility that \( V(\sigma) \) is the inflaton potential. Let us assume first that \( \sigma_H \simeq \sigma_c \). From eq. \( (9) \) this happens only if \( \sigma_H^2 \simeq \sigma_c^2 << N_H^{-1} \), where \( N_H \) (\( \simeq 50 - 60 \)) is the number of e-foldings for the time period that \( \sigma \) varies between \( \sigma_H \) and \( \sigma_c \). Then, for \( \sigma_H \simeq \sigma_c \) and \( \frac{\Delta T}{T} \simeq 6.6 \times 10^{-6} \), eq. \( (10) \) gives
\[\mu \simeq 0.0281\left(\frac{\sigma_c}{\sqrt{2}|<\frac{\phi}{2}>|}\right)^{\frac{3}{2}} |<\frac{\phi}{2}>|^\frac{3}{2} \quad (\sigma_c^2 << N_H^{-1}). \tag{11}\]

If we assume the relation \(\sigma_c \simeq \sqrt{2}|<\frac{\phi}{2}>|\), which holds in our simple model, and use the MSSM values \(M_X = 2 \times 10^{16}\) GeV, \(g = 0.7\) to calculate \(|<\frac{\phi}{2}>| = \frac{M_X}{g} \simeq 0.01173\), we obtain from eq. (11) \(\mu \simeq 3.57 \times 10^{-5}\) and \(\lambda \simeq 9.26 \times 10^{-6}\). We see that, for scales as large as the scale implied by the MSSM, \(\mu << |<\phi_2>|\) and \(\lambda << 1\). For smaller scales the situation becomes even worse because \(\frac{\mu}{|<\phi_2>|} \sim |<\phi_2>|^\frac{1}{2}\). The only essentially different possibility is that \(\sigma_H^2 >> N_H^{-1} \simeq \sigma_c^2\), i.e.

\[|<\frac{\phi}{2}>| \simeq (2N_H)^{-\frac{1}{2}} \left(\frac{\sqrt{2}|<\frac{\phi}{2}>|}{\sigma_c}\right) \quad (\sigma_c \simeq N_H^{-\frac{1}{2}}). \tag{12}\]

Again, if the relation \(\sigma_c \simeq \sqrt{2}|<\frac{\phi}{2}>|\) holds, this possibility is ruled out for the scale of MSSM. However, a scale \(\sim 10^{17}\) GeV, implied by the relation \(N_H^{-\frac{1}{2}} \simeq \sigma_c \simeq \sqrt{2}|<\frac{\phi}{2}>|\), should not be excluded in SUSY GUTs with a spectrum of states different from the spectrum of MSSM. Our subsequent discussion of inflation applies to this case as well.

The above arguments lead to the conclusion that if in the context of MSSM we insist in avoiding \(\mu << |<\phi_2>|\), the choice of \(V(\sigma)\) as the inflaton potential leads to acceptable values of \(\Delta T\) only if the relation \(\sigma_c \simeq \sqrt{2}|<\frac{\phi}{2}>|\) of the simplest supersymmetric model is violated in order to allow \(\sigma_c >> |<\phi_2>|\). This will be achieved by introducing in the discussion a second gauge non-singlet field acquiring a large vev. As a consequence the minimum rank of the gauge group \(G\) must be extended from five to six. We do not regard this extension as a serious restriction, but rather as a different treatment of existing fields in realistic models, since most semi-simple gauge groups used in model building are rank-six subgroups of \(E_6\).

The superpotential responsible for the breaking of \(G\) to \(G_S\) includes now the terms

\[W = \tilde{S}(-\mu_1^2 + \tilde{\lambda}_1 \Phi_1 \bar{\Phi}_1 + \tilde{\lambda}_2 \Phi_2 \bar{\Phi}_2) + \tilde{\bar{S}}(-\mu_2^2 + \tilde{\lambda}_3 \Phi_1 \bar{\Phi}_1 + \tilde{\lambda}_4 \Phi_2 \bar{\Phi}_2). \tag{13}\]

We have two conjugate pairs \(\Phi_1, \bar{\Phi}_1\) and \(\Phi_2, \bar{\Phi}_2\) of left-handed \(SM\) singlet superfields which belong to non-trivial representations of \(G\) and whose vevs reduce its rank by two units, \(\tilde{S}, \tilde{\bar{S}}\) are gauge singlet left-handed superfields, \(\mu_1, \mu_2\) are superheavy masses \(\sim M_X\) and
\( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) real coupling constants. Under the continuous R-symmetry \( W \to e^{i\gamma} W, \bar{S} \to e^{i\gamma} \bar{S}, \) \( \bar{S}' \to e^{i\gamma} \bar{S}' \) with the remaining superfields transforming trivially. Let us define \( \mu^2 \equiv (\mu_1^2 + \mu_2^2)^{\frac{1}{2}}, \cos \theta \equiv \mu_1^2 / \mu^2, \sin \theta \equiv \mu_2 / \mu^2, S \equiv \cos \theta \bar{S} + \sin \theta \bar{S}', S' \equiv -\sin \theta \bar{S} + \cos \theta \bar{S}', \lambda_1 \equiv \cos \theta \lambda_1 + \sin \theta \lambda_3, \lambda_2 \equiv \cos \theta \lambda_2 + \sin \theta \lambda_4, \lambda_3 \equiv -\sin \theta \lambda_1 + \cos \theta \lambda_3 \) and \( \lambda_4 \equiv \sin \theta \lambda_2 - \cos \theta \lambda_4. \) Then, \( W \) becomes

\[
W = S(-\mu^2 + \lambda_1 \phi_1 \Phi_1 + \lambda_2 \Phi_2 \Phi_2) + S'(\lambda_3 \Phi_1 \Phi_1 - \lambda_4 \Phi_2 \Phi_2).
\] (14)

Along the D-flat directions of the potential the symmetries of \( W \) allow us to define real scalar fields \( \phi_1, \phi_2, \sigma, \sigma_1, \sigma_2 \) through the relations \( \Phi_1 = \Phi_1 \equiv \frac{1}{2} \phi_1, \Phi_2 = \Phi_2 \equiv \frac{1}{2} \phi_2, S \equiv \frac{1}{\sqrt{2}} \sigma \) and \( S' \equiv \frac{1}{\sqrt{2}} (\sigma_1 + i \sigma_2). \) The potential obtained from \( W \) is then given by

\[
V(\phi_1, \phi_2, \sigma, \sigma_1, \sigma_2) = (-\mu^2 + \frac{1}{4} \lambda_1 \phi_1^2 + \frac{1}{4} \lambda_2 \phi_2^2)^2 + \frac{1}{16} (\lambda_3 \phi_1^2 - \lambda_4 \phi_2^2)^2
\]

\[+ \frac{1}{4} [(\lambda_1 \sigma + \lambda_3 \sigma_1)^2 + \lambda_2^2 \sigma_2^2] \phi_1^2 + \frac{1}{4} [(\lambda_2 \sigma - \lambda_4 \sigma_1)^2 + \lambda_1^2 \sigma_2^2] \phi_2^2.\] (15)

We assume that \( \lambda_1, \lambda_2, \lambda_3, \lambda_4 \) are real and positive couplings which satisfy the constraints \( \lambda_2 >> \lambda_1 > \lambda_3 = \lambda_4. \) Then \( V \) has a SUSY minimum at \( |\frac{\phi_1}{2}| = |\frac{\phi_2}{2}| = \mu/\sqrt{\lambda_1 + \lambda_2} \simeq \mu/\sqrt{\lambda_2} \) and \( \sigma = \sigma_1 = \sigma_2 = 0. \) Moreover, for fixed \( \sigma >> \sigma_1, \sigma_2, \) \( V \) as a function of \( \phi_1 \) and \( \phi_2 \) has a minimum at \( \phi_1 = \phi_2 = 0 \) provided \( \sigma > \sigma_c \simeq \sqrt{2} \mu/\sqrt{\lambda_1}. \) If we set \( \phi_1 = \phi_2 = 0 \) in \( V \) we find again that \( V = \mu^4. \) Once more there is not a potential neither for \( \sigma \) nor for \( \sigma_1 \) and \( \sigma_2. \)

Replacing global supersymmetry by canonical supergravity and setting \( \phi_1 = \phi_2 = 0 \) the potential \( V(\sigma, \sigma_1, \sigma_2) \) becomes

\[
V(\sigma, \sigma_1, \sigma_2) = \mu^4 [(1 - \sigma^2/2 + \frac{\sigma^4}{4}) + \frac{1}{4} \sigma^2 (\sigma_1^2 + \sigma_2^2)] e^{\frac{1}{2} (\sigma^2 + \sigma_1^2 + \sigma_2^2)}.\] (16)

This potential has now for fixed \( \sigma > \sigma_c \) a minimum with respect to \( \sigma_1 \) and \( \sigma_2 \) at \( \sigma_1 = \sigma_2 = 0 \) with both fields acquiring at this minimum masses larger than \( H. \) Moreover, setting \( \sigma_1 = \sigma_2 = 0 \) in \( V(\sigma, \sigma_1, \sigma_2) \) and keeping \( \sigma > \sigma_c, \) we recover \( V(\sigma) \) of eq. (6). Supergravity, by giving large masses to \( \sigma_1 \) and \( \sigma_2, \) chooses \( \sigma \) as the inflaton and generates a potential for it.

The above heuristic arguments are only meant to illustrate the salient features of the complete supergravity potential \( V \) derived from \( W. \) Careful analysis of this potential reveals
that it possesses an absolute minimum with respect to all fields at $|\phi_1| = |\phi_2| = \mu/\sqrt{\lambda_1 + \lambda_2}$ and $\sigma = \sigma_1 = \sigma_2 = 0$, where $V$ vanishes. Moreover, for fixed $\sigma > \sigma_c \simeq \sqrt{2}\mu/\sqrt{\lambda_1}$, $V$ has a minimum with respect to the remaining fields at $\phi_1 = \phi_2 = \sigma_1 = \sigma_2 = 0$. At this minimum the above fields have masses larger than $H$ and $V(\sigma)$ is given by eq. (6).

We now proceed to a more careful treatment of inflation with inflaton potential $V(\sigma)$ given by eq. (6). Taking as a criterion for the beginning of inflation the effective frequency $\left(\frac{V'}{V}\right)^2$ of $\sigma$ to be less than $\frac{3}{2}H$, we obtain $\frac{V'}{V} \leq \frac{3}{4}\sigma$ or $\sigma^2 \leq \sqrt{61} - 7 \simeq 0.81$, i.e. $\sigma \leq 0.9$. Nevertheless, we only consider in the following values of $\sigma \leq 1$.

The number of e-foldings in the slow roll approximation is given by

$$N(\sigma) \equiv -\int \frac{V'}{V} d\sigma + \frac{1}{3} \ln \left| \frac{V'}{V} \right| = \sigma^{-2} + \frac{3}{2} \ln \left( \frac{\sigma^2}{2} \right) - \frac{5}{6} \ln (1 + \frac{\sigma^2}{2}) - \frac{1}{3} \ln (1 + \frac{\sigma^6}{8}).$$

(17)

The only parameter that $V(\sigma)$ depends on is the mass scale $\mu$ or equivalently the false vacuum energy density $\mu^4$. As a consequence all quantities characterizing inflation, such as $\sigma_H$, $\sigma_c$ and the (average) spectral index $n$, will depend on $\mu$ only. The relation between $\mu$ and $\sigma_H$ is

$$\mu^4 \simeq 720\pi^2 \left( \frac{\Delta T}{T} \right)^2 \left( \frac{V}{\mu^4} \right)^2 + \frac{27}{4} \frac{1}{\sigma_H^2},$$

(18)

where the contribution of both the scalar and the tensor components to the quadrupole anisotropy $\frac{\Delta T}{T}$ is taken into account. Let us denote by $\ell_H$ the scale corresponding to our present horizon and by $\ell_o$ another length scale. Also let $\sigma_o$ be the value that the inflaton field had when $\ell_o$ crossed outside the inflationary horizon. We define the average spectral index $n(\ell_o)$ for scales from $\ell_o$ to $\ell_H$ as

$$n(\ell_o) \equiv 1 + 2\ln \left( \frac{\delta \rho}{\rho} \right)_{\ell_o} \left( \frac{\delta \rho}{\rho} \right)_{\ell_H} / \ln \left( \frac{\ell_H}{\ell_o} \right) = 1 + 2\ln \left( \frac{V^{3/2}}{V'} \right)_{\sigma_o} \left( \frac{V^{3/2}}{V'} \right)_{\sigma_H} / \Delta N(\sigma_H, \sigma_o).$$

(19)
Here \((\delta \rho/\rho)_\ell\) is the amplitude of the energy density fluctuations on the length scale \(\ell\) as this scale crosses inside the postinflationary horizon and \(\Delta N(\sigma_H, \sigma_o) = N(\sigma_o) - N(\sigma_H) = \ln(\ell_H/\ell_o)\). Finally, the value of \(\sigma_c\) is determined by requiring that \(N_H \equiv \Delta N(\sigma_H, \sigma_c) = N(\sigma_c) - N(\sigma_H)\) for some chosen value of \(N_H\).

Table 1 gives the values of \(\sigma_H, \sigma_c, n \equiv n(\ell_1)\) and \(n_{\text{COBE}} \equiv n(\ell_2)\), where \(\ell_1 (\ell_2)\) is the scale that corresponds to 1Mpc (2000Mpc) today, for different values of \(\mu\) assuming that the present horizon size is 12000Mpc, \(\Delta T/T = 6.6 \times 10^{-6}\) and \(N_H = 50\). We see that canonical supergravity leads naturally to very high values of the average spectral index which are higher and more rapidly varying as the false vacuum energy density becomes higher. Another distinctive feature of supergravity dominated hybrid inflation is that the total number of e-foldings \(\sim \sigma_c^{-2}\) is very limited compared to most other inflationary scenarios.

When \(\sigma\) falls below the critical value \(\sigma_c \simeq \sqrt{2\mu/\sqrt{\lambda_1}}\) the mass-squared term of \(\phi_1\) becomes negative and the local minimum of the potential at \(\phi_1 = \phi_2 = \sigma_1 = \sigma_2 = 0\) becomes unstable. The subsequent evolution is quite involved and can only be studied numerically. With a starting value for \(|\phi_1|\) of the order of a quantum fluctuation in de-Sitter space \(|\phi_1|\) becomes \(\sim 2\mu/\sqrt{\lambda_1}\) in a few Hubble times and inflation as described so far is terminated. At the same time \(\sigma_1\) grows. In order for \(|\phi_2|\) to grow as well the coupling \(\lambda_1\) has to be a few times larger than \(\lambda_3\). Then, \(|\phi_2|\) grows and gets stabilized at \(|\phi_2| \simeq 2\mu/\sqrt{\lambda_2}\). At the same time \(|\phi_1|\) leaves the value \(2\mu/\sqrt{\lambda_1}\) and \(\phi_1\) starts oscillating again around zero but now with a much larger amplitude. The same happens with \(\sigma_1\). Finally, after the energy density falls a few orders of magnitude, \(|\phi_1|\) gets stabilized at \(|\phi_1| = |\phi_2| = 2\mu/\sqrt{\lambda_1 + \lambda_2}\) with \(\sigma, \sigma_1, \sigma_2\) oscillating around zero.

In the above discussion we insisted that the vevs of \(\phi_1\) and \(\phi_2\) be equal and have the value suggested by the gauge coupling unification of MSSM. This restriction can be easily relaxed by choosing the value of \(\frac{\lambda_3}{\lambda_3} = \frac{\langle \phi_2^2 \rangle}{\langle \phi_2^2 \rangle}\). Then, \(\langle \phi_2^2 \rangle = 4\mu^2/\lambda_2(1 + \frac{\lambda_1}{\lambda_2} \frac{\lambda_1}{\lambda_3})\) and \(\sigma_c^2 \simeq 2\mu^2/\lambda_1\), always assuming that \(\lambda_2 \gg \lambda_1\). We also insisted that the second non-inflaton field be a gauge non-singlet. This restriction could be relaxed as well in order to allow a non-inflaton field whose large vev breaks a continuous global symmetry.
As a byproduct of our analysis it should be obvious that if in the original non-supersymmetric hybrid inflationary model, where $\sigma_c$ is much less constrained, one replaces the mass-squared term of the inflaton with a suitably small quartic coupling one could easily obtain very large but varying values of the spectral index without having to assume an enormous vev for the non-inflaton field.

Before concluding we would like to point out briefly that our mechanism for raising the value of $\sigma_c$ relative to the symmetry breaking scale could be easily modified to lead to a value of $\sigma_c$ much lower than the vevs of the non-inflaton fields. If we choose $\lambda_1$ in $W$ of eq. (14) to be negative and $\lambda_1 + \lambda_2 << -\lambda_1 < \lambda_2, \lambda_3 = \lambda_4$, the resulting theory has a SUSY vacuum at $\langle S \rangle = \langle S' \rangle = 0, \langle \Phi_1 \rangle = \langle \Phi_1 \rangle = \langle \Phi_2 \rangle = \langle \Phi_2 \rangle = \mu^2/(\lambda_1 + \lambda_2), |\langle \Phi_1 \rangle| = |\langle \Phi_1 \rangle| \text{ and } |\langle \Phi_2 \rangle| = |\langle \Phi_2 \rangle|$, with $\sigma_c^2 \simeq 2\mu^2/|\lambda_1|$. This observation could find applications in other occasions. For example, it could be used in the scenario of ref. [11] to raise the value of the symmetry breaking scale to the MSSM value $M_X \simeq 2 \times 10^{16} GeV$.

We conclude by summarizing our results. We investigated the possibility of using as inflaton potential in SUSY hybrid inflation the one generated when global supersymmetry is replaced by canonical supergravity. We argued that in the context of MSSM this possibility is strongly connected with the relaxation of the very tight relationship between the vev of the non-inflaton field and the critical value of the inflaton field encountered in the simplest SUSY models. We then presented a mechanism involving two non-inflaton fields which accomplishes the relaxation of the unwanted relationship. Finally, we studied the resulting supergravity dominated inflationary scenario and pointed out that it leads naturally to blue primordial spectra for adiabatic density perturbations.

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| $\mu/10^{15} GeV$ | $\sigma_{H}/10^{17} GeV$ | $\sigma_{c}/10^{17} GeV$ | $n$  | $n_{COBE}$ |
|------------------|--------------------------|--------------------------|------|------------|
| 1                | 2.0562                   | 1.7634                   | 1.021| 1.022      |
| 2                | 3.2524                   | 2.3540                   | 1.051| 1.055      |
| 4                | 5.1188                   | 2.8232                   | 1.119| 1.139      |
| 6                | 6.6435                   | 3.0025                   | 1.187| 1.242      |
| 8                | 7.9684                   | 3.0894                   | 1.252| 1.358      |
| 10               | 9.1556                   | 3.1379                   | 1.312| 1.484      |
| 12               | 10.2415                  | 3.1678                   | 1.367| 1.618      |
| 15               | 11.7342                  | 3.1945                   | 1.442| 1.830      |
| 20               | 14.0129                  | 3.2171                   | 1.553| 2.209      |

Table 1. The values of $\sigma_{H}$, $\sigma_{c}$, $n$ and $n_{COBE}$ as a function of $\mu$. 
