Exploring the Application of Mathematical Modeling in Solving Mathematical Application Problems in Junior High School

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Abstract: The idea of mathematical modeling has an important educational function and has a far-reaching impact on students’ learning and growth. Based on this, this paper studies in detail the application of mathematical modeling thought in the teaching of practical problems in junior middle schools, aiming at improving students’ problem-solving efficiency, cultivating students’ mathematical literacy and continuously improving students’ mathematical learning ability.

Keywords: Mathematical Modeling Thought; Junior High School Application Questions; Problem Solving Efficiency

1. Introduction

The new curriculum standard points out that mathematics teaching should pay attention to developing students’ sense of number, symbol consciousness, space concept, geometric intuition, data analysis concept, operation ability, reasoning ability and model thought. The establishment of model thought is the basic way for students to experience and understand the connection between mathematics and the outside world. The process of establishing and solving the model includes abstracting mathematical problems from real life or specific situations, using mathematical symbols to establish equations, inequalities, functions and so on to express the quantitative relations and changing laws in mathematical problems, and obtaining the results. Through this process, students can form the concept of model and improve their interest in learning mathematics and their awareness of application. Common mathematical models in junior middle school include equation model, inequality model, function model, right triangle model, geometric model and statistical model.

Practical problems cover a wide range and are inseparable from people’s daily life and production. They are an examination of students’ ability to apply mathematical knowledge, and they are also a hot issue in the senior high school entrance examination. Mathematical application problems are characterized by lengthy narrative and scattered quantitative relationship, so many students are at a loss when solving problems. When these problems are transformed into mathematical models and then expressed in mathematical language, it is much easier to solve them. It can be seen that the construction of mathematical model is the basis of solving practical problems with mathematical knowledge. Mathematical modeling has a low starting point, and it is simple and quick for students to master, and it is also interesting to a certain extent. In daily teaching, teachers should guide and motivate students to think more, express more and model more, so as to reflect the application value of mathematics and help students master mathematical knowledge and skills flexibly. How to build a mathematical model to guide deep thinking, solve practical problems, and finally generate ability and wisdom? Examples are given below.

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2. Establish an equation model to solve application problems

There are a large number of quantitative relations in actual production and life, including equality relations. Equation model is a mathematical model reflecting this relationship. To establish equation model is to abstract the equality relations in practical problems into equations by using mathematical symbols and language, to obtain the solutions of the equations by solving them, and then to check whether they conform to the meaning of the questions, so as to solve the problems. It can enable us to describe and grasp the real world from the aspect of quantitative relations.

Example 1: Yangzhou Jinfu Company sent 65 personnel to produce A and B products, and it takes one person a day to produce two A products and one B product. After market research, it is found that one A product can get profits from 15 yuan and one B product can get profits from 120 yuan. However, in actual work, it needs to spend a certain amount of other expenses, but it is limited to B products. For each additional B product, each product needs to spend more 2 yuan.

(1) According to the information to fill in the Table 1.

| Production type | Member of workers per day | Production per day/piece | Profit per product/Yuan |
|-----------------|---------------------------|--------------------------|------------------------|
| A               | 65-x                      | x                        | 15                     |
| B               | x                         | x                        |                        |

Table 1

(2) If the profit from producing B products every day is less than that of A products, what is the total profit from producing A and B products every day?

Analysis: (1) If x people are arranged to produce B products every day, (65-x) people are arranged to produce A products every day, and x B products can be produced every day, with a profit of (120-2x) yuan per piece and 2(65-x) A products per day. Therefore, the answer is: 2(65-x), 120-2x.

(2) 15×2(65-x) - (120-2x) x = 650.
Organized: x²-75x+650 = 0.
x₁ = 10, x₂ = 65 (if it doesn’t fit the question, give it up).
15×2(65-x) + (120-2x) x = 2650.
Answer: The total profit that the enterprise can obtain from producing A and B products every day is 2650 yuan.

Commentary: The problem of sales profit is solved by establishing a mathematical model of one-dimensional quadratic equation. The two values of unknowns in the problem are the solutions of the original equation, but one does not conform to the meaning of the problem and must be discarded. This is the difference between the solution of the equation and the solution of the actual problem.

3. Establish an inequality model to solve application problems

Like equality relations, there are a lot of inequality relations in the real world, such as greater than, less than, not equal to, not exceeding, not less than, at least, at most, etc., all of which indicate inequality relations. Inequalities or inequality groups need to be established to solve problems, which play an important role in range estimation, scheme design and investment decision-making.

Example 2: Nanyang municipal company decided to purchase 40 eco-cars of two models, A and B, to clean all roads in the city. One A-type eco-car and two B-type eco-cars are needed to treat 100 tons of garbage every week, and two A-type eco-cars and one B-type eco-car are needed to treat 110 tons of garbage every week.

(1) How much garbage can A-type eco-cars clean up every week? How much garbage can B-type eco-cars clean up every week?

(2) The municipal company is going to allocate 9.1 million yuan to buy eco-cars and clean up at least 1,400 tons of garbage every week. It is known that the price of each A-type eco-car is 250,000 yuan, and the price of each B-type eco-car is 200,000 yuan. Please design a purchase plan that conforms to the meaning of the question and seek the pur-
chase plan with the least cost.

Analysis: (1) Set A and B eco-cars, each of which can treat A tons and B tons of garbage respectively every week. According to the meaning of the question, \(a+2b = 100\), \(2a+b = 110\), and \(a = 40\), \(b = 30\).

A: Type A eco-cars clean up 40 tons of garbage every week, while Type B eco-cars clean up 30 tons of garbage every week.

(2) M A-type eco-cars and 40-m B-type eco-cars shall be purchased, and the required capital is \(Y\) yuan.

\[
25m+20(40-m) \leq 910, 40m+30(40-m) \geq 1400, \text{ and the solution is } 20 \leq m \leq 22.
\]

As \(m\) is an integer, \(m\) can be 20, 21 and 22, and there are three purchase schemes.

| Scheme 1 | Scheme 2 | Scheme 3 |
|----------|----------|----------|
| Type A eco-cars | 20 | 21 | 22 |
| Type B eco-cars | 20 | 19 | 18 |

Table 2

\[
Y = 25m+20(40-m) = 5m+800, \text{ then when } m = 20, y \text{ gets the minimum value, and then } y = 900.
\]

Answer: Scheme 1 needs the least amount of funds, and 9 million yuan is the minimum amount of funds needed.

Commentary: This topic is a typical application of inequality model by establishing inequality group, which is often combined with equation group model. The two models are interrelated and serve to solve practical problems together. The solution set of inequality group listed in this topic contains countless solutions, but for practical problems, it has only three solutions, so the application problem of one-dimensional linear inequality group is to find the non-negative integer solution of inequality group.

4. Establish a functional model to solve application problems

In the real world, there are a large number of phenomena that one quantity changes with another quantity, such as the temperature changes with time, the sales volume changes with the unit price, the graphic area changes with the side length, etc. Such relations are all functional relations in mathematics, in which case a functional model can be established to solve them. The functional models studied in junior middle school include: linear function model, inverse proportional function model, quadratic function, etc., and it is common to solve problems by using its increment and decrement.

Example 3 as shown in the Figure 1: There is an idle land in Nanhai middle school. beside the idle land, there is an old wall MN with a length of a meter. the school league Committee is going to use the old wall and a fence with a length of 100 meters to form a rectangular garden ABCD.

(1) As shown in Figure 1, it is known that one side of the rectangular garden uses the old wall, and \(AD \leq MN\), and \(AD = x\) meters is set.

(1) If \(a = 20\), the area of the enclosed rectangular garden is 450 square meters, and the length of \(AD\) on one side of the rectangular garden (2) the maximum \(ABCD\) area of the rectangular garden;

(2) As shown in Figure 2, if \(a = 20\), make full use of the length of the old wall and fence to form a rectangle with
the largest area. At this time, the maximum ABCD area of the rectangle is ___ square meters.

![Figure 2](image)

Analysis: (1) (a) \(AB = \frac{100-x}{2}\) m.

According to the meaning of the question, \(x = \frac{100-x}{2} = 450\), \(x_1 = 10\) and \(x_2 = 90\).

Since \(90 > 20\), it should be abandoned.

A: The length of AD is 10 meters.

(b) let \(AD = x\) m.

The area of ABCD of rectangular garden is:

\[s = \frac{1}{2}x(100-x) = \frac{1}{2}(x-50)^2 + 1250\]

When \(a \geq 50\), it can be obtained that when \(x = 50\), \(S\) reaches the maximum value of 1250.

When \(0 \leq x \leq a < 50\), it can be obtained that when \(0 < x \leq a\), \(S\) increases with the increase of \(x\), and when \(x = a\), \(S\) reaches the maximum value of \(50a - \frac{1}{2}a^2\).

To sum up, when \(a \geq 50\), the maximum value of \(S\) is 1250 m\(^2\); The maximum value of \(s\) is \((50a - \frac{1}{2}a^2)\) m\(^2\) when \(0 < a < 50\).

(2) Let the area of rectangular ABCD be \(W\) and \(AD = x\), then \(AB = 60-x\).

\[W = x(60-x) = -(x-30)^2 + 900\ (20 < x < 60)\]

When \(x = 30\), the ABCD area of rectangular garden reaches the maximum value of 900 m\(^2\).

Commentary: The mathematical model of quadratic function is established to solve the problem of maximum value of rectangular garden, which mainly uses the maximum value property and increment/decrement of quadratic function. When determining the maximum value, we must pay attention to the value range of independent variables, so that the analytical formula of function and practical problems are meaningful.

5. Establish a right triangle model to solve application problems

In junior high school, we have learned many properties about right triangle, such as Pythagorean theorem, mutual complement of two acute angles, trigonometric function of acute angle, properties of hypotenuse midline, 30-degree angle, etc. If we encounter graphic problems, it will be easier to solve them if we can change them into right triangles by segmentation or supplement.

Example 4: A good sitting posture is beneficial to teenagers’ bone growth and health. In an ideal state, the upper body keeps upright, the head is straight and the neck is straight, the hands on both sides of the body are naturally flat, the legs are shoulder-wide, and stand up. As shown in Figure 3, the eyes in Figure 3 are marked as point A, the abdomen as point B, the nib as point D, and the intersection point between BD and the table edge as point C.
(1) If $\angle ADB = 53^\circ$ and $\angle b = 60^\circ$, calculate the distance from point A to BD and the distance between points C and D (the result is accurate to 1 cm).

(2) The teacher found that Xiaohong’s writing posture was incorrect, and his eyes tilted to point E in Figure 4, which happened to be on the perpendicular bisector of CD, and $\angle BDE = 60^\circ$, so he asked him to correct it to the correct posture and ask for the distance that the eyes should rise (the result is accurate to 1 cm).

Reference data: $\sin 53^\circ \approx 0.80$, $\cos 53^\circ \approx 0.60$, $\tan 53^\circ \approx 1.33$, $\sqrt{2} \approx 1.41$, $\sqrt{3} \approx 1.73$.

Analysis: (1) Let point A be AH $\perp$ BD at point H.

Then $\angle AHD = \angle AHB = 90^\circ$.

With $AD = 30$, $\angle ADB = 53^\circ$, $AH = AD \times \sin 53^\circ \approx 30 \times 0.80 = 24$, $DH = AD \times \cos 53^\circ \approx 30 \times 0.60 = 18$.

From $\angle b = 60^\circ$, then $BH = \frac{AH}{\sqrt{3}} \approx \frac{24}{1.73} \approx 14$,

$BD = BH + DH \approx 32$,

$BC = 12$, then $CD = 32 - 12 = 20$.

Answer: The distance from point A to BD is about 24 cm, and the distance between points C and D is 20 cm.

(2) Crossing point e as EG $\perp$ CD, crossing point a as the extension of AF $\perp$ EG intersection at point F.

The quadrilateral AFGH is rectangular, then $FG = AH = 24$. If the point e is just on the perpendicular bisector of CD, then $DG = \frac{1}{2} CD = 10$. If $\angle EDC = 60^\circ$, then $EG = DG = 10\sqrt{3} \approx 17.3$, then $EF = FG - EG \approx 7$ (cm).

Answer: The distance that the eyes should rise is 7 cm.

Commentary: Firstly, this topic divides the acute triangle into two right triangles, and then divides and supplements a quadrilateral into right triangles and rectangles, thus establishing a right triangle model, and then using the properties of right triangles to solve it.

From the above examples, it can be summarized the general steps of mathematical modeling: (1) Model preparation, understanding the background of the problem, mastering all kinds of information, and describing the problem in mathematical language; (2) Model hypothesis, using mathematical tools to describe the relationship between variables and establish mathematical structure; (3) Solving the model, and calculating the model by using known conditions; (4) Model analysis; (5) Model test, which compares the model results with the actual situation and modifies the assump-
tions; (6) Model application. In junior high school mathematics teaching, students should be trained in mathematical modeling, and form good thinking habits and ability to use mathematics.

6. Conclusion

To sum up, the continuous penetration of mathematical modeling ideas in solving practical problems in junior high school can lead students to turn practical problems into mathematical problems in order to get concise and efficient answers, which can help students better master the basic knowledge and skills of mathematics, understand mathematical thinking methods and enhance students’ application awareness. Therefore, teachers should guide students to use mathematical models skillfully, realize efficient problem solving, and promote the progress of quality education.

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