Attenuation of $\phi$ mesons in $\gamma A$ reactions

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We present a theoretical analysis of inclusive photoproduction of $\phi$ mesons in nuclei. In particular the dependence of the total $\phi$ meson yield on the target mass number is investigated. The calculations are done using the semi-classical BUU transport approach that combines the initial state interaction of the incoming photon with the coupled-channel dynamics of the final state particles. The conditions of the calculations are chosen such as to match the set up of a recent experiment performed at SPring8/Osaka. Whereas the observables prove to be rather sensitive to the $\phi$ self energy in the medium, the attribution of deviations from the standard scenario to a particular in-medium effect seems to be impossible.

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The question of how the properties of the light vector mesons change once they are put into a strongly interacting environment provides a field of very active research. In particular the theoretical and experimental determination of the vector and isoscalar spectral densities in systems with either finite temperature or baryon density has attracted much attention [1]. In particular the $\phi$ meson provides a well suited probe as it is a very sharp resonance in vacuum. One could thus expect to observe even smallest deviations of its spectral distribution in the nuclear medium from its properties in vacuum. Numerous authors have studied the medium modifications of the $\phi$ properties in different approaches as effective Lagrangians and QCD sum rules [2, 3, 4, 5, 6]. As a general picture from these studies emerges a quite small shift of the $\phi$ meson pole mass, but a sizeable renormalization of its width as temperature and/or density increase. Particular numbers for these changes at nuclear saturation density and vanishing temperature are for instance a width of 28 MeV and a mass shift of $-6$ MeV found in the studies of Refs. [2, 3] and 40 MeV/$-10$ MeV obtained by the authors of Ref. [2, 3]. The origin of the $\phi$ medium modifications has been attributed to either $\phi$-nucleon and/or $\phi$-meson collisions and a change of the $K \bar{K}$ self energy caused by the renormalization of the kaon and antikaon properties inside the nuclear many-body system.

The most direct approach to observe the $\phi$ in-medium properties would be a reconstruction of its mass distribution from the dilepton decay channel since this final state is essentially free of any final state interactions. $\phi$ mesons have indeed been seen in such an experiment on cold nuclear matter at JLAB; its evaluation is presently underway [7]. In a hot nuclear environment the dilepton line has been seen in an ultrarelativistic heavy-ion experiment at the SPS [8]. Exploiting the dominant decay branch of the $\phi$ would be a measurement of the $K \bar{K}$ invariant mass spectrum. The possibility to study modifications of the $\phi$ meson in photon-nucleus reactions has first been considered by the authors of Ref. [2]. These authors have proposed to measure the $K^+K^-$ mass spectrum from $\phi$ mesons produced in finite nuclei with momenta smaller than $100 - 150$ MeV. The condition of small $\phi$ momenta ensures that a large fraction of events in the data sample stem from $\phi$ decays inside the nucleus, hence carrying information about the in-medium $\phi$ spectrum to the detectors. However, later we have shown [10] that such a measurement is not without problems: First, due to the strongly forward peaked angular distribution, the photon-nucleus cross section including such a restrictive momentum cutoff is extremely small. Second, the small kaon mean free path on the one hand reduces the nuclear densities probed due to $K^-$ absorption and on the other hand distorts the $K^+K^-$ invariant mass spectrum due to quasi elastic $K^+N$ and $K^-N$ scattering processes. Moreover, even the nuclear Coulomb potential, that is much longer ranged than the strong interaction phenomenon to be probed, makes it impossible to gather any valuable information about the in-medium $\phi$ properties. Consequences of these effects have been investigated in great detail in Ref. [10].

Despite this somewhat discouraging situation the authors of Ref. [6] have pointed out that there is a possibility to study at least the imaginary part of the $\phi$ self energy in nuclei by an attenuation measurement of the $\phi$ flux in nuclear $\phi$ photoproduction. This is a method that has been used already in early experiments on the $\rho$ meson properties in nuclei [11] and has led there to the first extraction of the $\rho N$ cross section from photoproduction experiments. For the $\phi$ meson this suggestion has been taken up by a nuclear $\phi$ photoproduction experiment at SPring8/Osaka that is optimized to measure $\phi$ mesons under forward angles by detecting the $K^+K^-$ decay channel [12]. The observable in this experiment is the so-called nuclear transparency ratio

\[ T_A = \frac{\sigma_{\gamma A \rightarrow \phi X}}{A \sigma_{\gamma N \rightarrow \phi X}}, \]  

i.e. the ratio of the nuclear $\phi$ production cross section divided by $A$ times the cross section on a free nucleon. It can be interpreted as the momentum and position space...
averaged probability of a $\phi$ meson to get out of the nucleus. The dependence of the loss of flux on the target mass number is related to the absorptive part of the $\phi$-nucleus potential and thus to the $\phi$ width in the nuclear medium.

In a simple Glauber approximation, neglecting Fermi motion, Pauli blocking, coupled-channel effects, nuclear shadowing and quasi elastic scattering processes, the nuclear cross section for $K^+K^-$ photoproduction via the exclusive incoherent production of $\phi$ mesons can be written as

$$\sigma_{\gamma A} = \int d\Omega \int d^3r \rho(r) \frac{d\sigma_{\gamma N}}{d\Omega} \exp \left[ -\sigma_{\phi N}^{\text{inel}} \int_0^{\Delta} dl \rho(r') \right]$$

$$\times F_{\text{abs}}^K(r + \Delta)$$

with

$$r' = r + \frac{q}{|q|} \quad \text{and}$$

$$\Delta = \frac{v}{\gamma} \frac{1}{\Gamma_{\phi}^{\text{phot}}} |q| .$$

$F_{\text{abs}}^K$ is a $K^-$ absorption factor that is obtained by integrating the $K^-$ absorption probability along the $K^-$ trajectory from the decay vertex of the $\phi$ at $r + \Delta$ to infinity and averaging over the possible $K^-$ directions. The Lorentz factor $\gamma$ transforms the $\phi$ width from the $\phi$ rest frame to the $\phi$ moving frame. In the limit of $\Gamma_{\phi}^{\text{dec}} \to 0$ and a vanishing $\phi$ absorption cross section $\sigma_{\phi N}^{\text{inel}} \to 0$ the exponential as well as the $K^-$ absorption factor become equal to unity. This also implies a nuclear transparency ratio of unity as can be seen from the definition Eq. (4).

Using now the low-density theorem that relates the total $\phi N$ cross section to the $\phi$ collision width and the relation

$$\frac{\text{Im} \Pi_{\phi}}{\omega} = \gamma \Gamma_{\phi}^{\text{coll}} = \rho \sigma v_{\phi}$$

the exponential in Eq. (2) can be rewritten as

$$\exp \left[ \frac{1}{q} \int_0^{\Delta} \text{Im} \Pi(q, \rho(r')) dl \right].$$

$\Gamma_{\phi}^{\text{coll}}$ is the in-medium $\phi$ collisional width corresponding to nuclear quasi elastic and absorption channels. The $K^+K^-$ decay channel is omitted from the self energy in Eq. (4) since $\phi$ mesons decaying to this channel will be detected and, hence, do not lead to a loss of flux.

From this expression one can now read off how two different mechanisms will affect the nuclear transparency ratio: First, if the $\phi$ collision width becomes large because of the opening of inelastic nuclear channels, the nuclear cross section and the transparency ratio will become smaller because of the exponential suppression factor. Second, if the $\phi$ decay width to the $K^+K^-$ channel increases e.g. due to kaon self energies in the medium, the $\phi$ decay vertex at $r + \Delta$ lies with higher probability inside the radius of the target nucleus. Antikaons produced inside a strongly attractive nuclear potential will be confined for a longer time to the nuclear volume or may even be bound. Due to the large imaginary part of the nuclear $K^-$ potential these antikaons will be absorbed to inelastic nuclear channels.

In the very same spirit as in Ref. [11] for the case of $\rho$ photoproduction in nuclei the authors of Ref. [12] used a simple Glauber formula in order to extract the total $\phi N$ cross section from the measured nuclear $\phi$ photoproduction cross sections. To this end nuclear effects as Fermi motion, Pauli blocking, quasi elastic scattering, etc. have been neglected. A similar approach has been used in Ref. [6] but including correction factors in order to account for the effects of Pauli blocking and Fermi motion. Moreover, a sophisticated model for the $\phi$ nucleon interaction based on chiral SU(3) dynamics has been incorporated. Nevertheless the experimental attenuation effect has been underestimated by almost a factor of two. In the present paper we aim at a careful treatment of all nuclear effects – i.e. Fermi motion, Pauli blocking, shadowing, elastic and inelastic scattering including sidefeeding and regeneration of all produced particles, decays of unstable particles, collisional broadening, kaon self energies – in order to verify any modification of the $\phi N$ interaction going beyond the standard cross sections estimated from vacuum processes.

In order to calculate the production and propagation of $\phi$ mesons in finite nuclei we use the semi-classical BUU transport theory. This theory describes incoherent photon-nucleon reactions. It has been applied previously to the study of medium modifications of the $\rho$, $\omega$ and $\phi$ mesons by means of dilepton [13], $\pi^0\gamma$ [14] and $K^+K^-$ [10] photoproduction in nuclei. All these reactions are modeled as a two-step process: In the first step the incoming (and potentially shadowed) photon interacts with a single nucleon in the target nucleus and produces one or several particles. In the second step the products of this elementary interaction are propagated through the nuclear many-body system by means of the semi-classical transport equations. During the reaction nuclear effects such as Fermi motion, Pauli blocking, binding energies and shadowing are taken into account. Cross sections for the elementary photon-nucleon interaction are constrained by experimental data as far as possible; for details on the cross sections used for these primary processes see Ref. [11]. Inclusive processes that have not been determined experimentally are modeled by means of the Lund model FRITIOF. During the propagation we include all kinds of final state interactions such as absorption and elastic and inelastic scattering of the pro-
duced particles from the target nucleons. The cross sections for these processes are obtained either from vacuum matrix elements that have been fixed using experimental data or again within the Lund model. A more detailed presentation of the BUU model can be found in Refs. [10, 13, 14, 15, 16, 17] and references therein.

In order to explore the $\phi$ nucleon interaction in the nuclear medium we use a very basic model. For the total $\phi N$ cross section we adopt the parametrization from Refs. [18, 19] that has been obtained using Regge theory and an additive quark model:

$$\sigma_{\phi p} \simeq \sigma_{K^+ p} + \sigma_{K^- p} - \sigma_{\pi^+ p}$$

$$\simeq (10.01s^2 - 1.52s^{-\eta}) \text{ mb}$$

with $s$ in GeV$^2$ and the Regge intercept parameters $\epsilon = 0.0808$ and $\eta = 0.4525$. Because of isospin symmetry the cross section on protons and neutrons are the same. We note that this quark model estimate is in line with the more refined calculations of [5, 6].

For the elastic channel one can estimate the total cross section within the strict vector meson dominance model to amount to roughly 10 mb. Throughout our transport simulations we use the parameterization from Ref. [20]:

$$\sigma_{\phi N}^e = \frac{10}{1 + |q|} \text{ mb}$$

with $q$ in GeV. For invariant energies above 2.2 GeV we rely on the FRITIOF routine to simulate the inelastic $\phi N$ scattering events. For lower energies we treat the inelasticity $\sigma_{\phi N}^{\text{inel}} = \sigma_{\phi N}^{\text{tot}} - \sigma_{\phi N}^{\text{el}}$ as being due to $\phi$ meson absorption. The process $\phi N \to \pi N$ is constrained by its inverse that has been fitted to experimental data in Ref. [21]. The remaining inelasticity we put entirely into the channel $\phi N \to 2\pi N$. This is justified because for inclusive observables as the total nuclear cross section coupled channel effects involving sidefeeding of the $\phi$ meson channel are expected to play a minor role.

The collisional part of the self energy of the $\phi$ meson we obtain via the low density theorem taking now the Fermi-distribution into account

$$\text{Im}\Pi(q_0) = \sqrt{m_\phi^2 + q_0^2}, q_0 = -4 \int \frac{d^3p}{(2\pi)^3} \Theta(|p| - p_F)$$

$$\times \frac{p\sqrt{s}}{E_N(p)} \sigma_{\phi N}^{\text{tot}}(s)$$

with $p$ being the center-of-mass momentum of nucleon and $\phi$, $E_N$ the nucleon on-shell energy in the laboratory frame and $p_F$ the local Fermi momentum. The cross section $\sigma_{\phi N}^{\text{tot}}$ is the total $\phi$ nucleon cross section containing all quasi elastic and absorption channels. For $\phi$ mesons at rest we find a collision width of 18 MeV at normal nuclear matter density, in agreement with the more refined calculations of [22, 23].

The total $\phi$ meson yield has been measured at SPring8 from Li, C, Al and Cu nuclei at photon energies of $E_\gamma = 1.5 - 2.4$ GeV. We start our calculations with including only collisional broadening of the $\phi$ meson as an in-medium effect while maintaining the vacuum properties for all other mesons (in particular the kaons). Results for the transparency ratio are shown in Fig. 1 by the open triangles. The deviation of the transparency ratio from unity is mainly due to two effects: First, the $\phi$ meson or its decay products (if the $\phi$ decays inside the nucleus) are absorbed on their way out of the nucleus. Second, the transparency ratio is reduced by Pauli blocking. Due to the diffractive production mechanism the $\phi$ photoproduction cross section is peaked in forward direction. Hence the momentum transfer to the hit nucleon in general is quite small. As a consequence, the momentum of the final state of the nucleon lies with high probability again within the Fermi sphere, leading to Pauli blocking of this reaction. In the energy regime considered here the coherence length of the $\phi$ component in the incoming photon is still small as compared to the $\phi$ mean free path. Thus, nuclear shadowing leads only to a marginal correction of the nuclear cross section.

The results depicted by the open circles show a further reduction of the transparency ratio due to the limited acceptance of the LEPS spectrometer that can detect kaons and antikaons only under forward angles [24, 25]. The nonzero transverse momentum of the hit target nucleon due to Fermi motion leads to a broadening of the angular distributions of the produced $\phi$ mesons for finite nuclear targets as compared to the reaction on a free nucleon at rest that is forward peaked. Hence, more of the produced mesons do not fall in the acceptance window, leading to the observed reduction of the nuclear cross section when the geometrical acceptance constraint is turned on. This reduction is thus not connected to the absorptive part of the $\phi$ meson selfenergy as some of the $\phi$ mesons just go
into another direction that is not covered by the forward detector setup.

So far, we have assumed that the branching ratio for the decay of the \( \phi \) meson into \( K^+K^- \) is in medium the same as in vacuum. However, in the nuclear environment kaons and antikaons experience strong potentials due to the kaon nucleon interaction \[24, 25, 26, 27, 28, 29\]. For the antikaon these potentials lead to a considerable renormalization of its mass whereas for the kaon a cancellation of scalar and vector potentials results in an at most slight repulsive mass shift. To explore the effect of these potentials we adopt the approach of Ref. [30] where the nuclear kaon dynamics on the antikaon these potentials lead to a considerable renormalization of its mass whereas for the kaon a cancellation of scalar and vector potentials results in an at most slight repulsive mass shift. To explore the effect of these potentials we adopt the approach of Ref. [31], where the influence of a relativistically correct implementation of the nuclear kaon dynamics on the \( K^+ \) flow in heavy ion collisions has been investigated. Relying on the chiral Lagrangian set up by Kaplan and Nelson and applying the mean field approximation the following kaon dispersion relation has been obtained:

\[
E_{K^\pm} = \sqrt{m_{K^\pm}^2 + (p \pm V)^2} \pm V_0
\]

with the effective kaon mass

\[
m_{K^\pm} = \sqrt{m_{K^\pm}^2 - \frac{\Sigma_{KN}}{f_\pi^2} \rho_s + V_\mu V^\mu}
\]

and the vector potential

\[
V_\mu = \frac{3}{8 f_\pi^2} j_\mu,
\]

where \( j_\mu \) is the baryon four vector current. In the case at hand the spatial components of this current vanish since the target nucleus stays close to its ground state during the photonuclear reaction. For the kaon nucleon sigma term we adopt the value of \( \Sigma_{KN} = 450 \text{ MeV} \) from the mean field approach of Ref. [31]. Further parameters of the model are the vacuum pion decay constant \( f_\pi = 93 \text{ MeV} \) and the in-medium pion decay constant at normal nuclear matter density \( f_{\pi}^* = \sqrt{0.7 f_\pi} \).

The relativistic kaon potentials lead to a considerable reduction of the in-medium \( K^- \) mass whereas the \( K^+ \) mass increases only slightly. This leads to a significantly larger phase space for the \( \phi \to K^+K^- \) decay. The \( \phi \) decay width corresponding to the \( K^+K^- \) self energy diagram is proportional to \( p^3 \), where \( p \) is the kaon/antikaon momentum in the \( \phi \) rest frame. At normal nuclear matter density this width can become as large as 20 MeV (2.18 MeV in vacuum). Note, that taking together the effects of the kaon renormalization as well as \( \phi \) nucleon collisions the in-medium \( \phi \) width can add up to values of about 40 MeV at saturation density.

As a consequence of the larger decay width, more of the produced \( \phi \) mesons decay inside the target nucleus. The antikaons produced inside the strongly attractive potential can be confined to the nuclear volume leading to an increase of \( K^- \) absorption. Alternatively, their trajectories are distorted due to the propagation through the nonzero potential gradients. These \( K^+K^- \) pairs are likely to be found outside the experimentally imposed acceptance window. The results for the nuclear transparency ratio including the kaon/antikaon dispersion relation from Eq. (10) are shown in Fig. 1 by the open squares. Indeed the expected reduction is observed. We illustrate the effect of the antikaon potential in Fig. 2. In one curve shown there the antikaons with negative total energy are subtracted from the data sample, whereas for the second curve also these not detectable particles are counted to obtain the total antikaon yield.

However, even after including all these in-medium effects, the strength of the experimentally measured attenuation can still not be reproduced, see Fig. 1. This discrepancy implies an additional in-medium effect. A further reduction of the nuclear cross section could in principle be caused by either a modification of the \( K^+K^- \) decay width going beyond our simple approach, a modified \( \phi N \) absorption cross section or even a change of the primary production processes. Moreover, the renormalization of the kaon properties in the medium could also cause a modification of the \( K^+N \) and \( K^-N \) cross sections. Such effects have, for instance, been studied in Ref. [32, 33]. However, for the time being we disregard the effects of such additional medium corrections and consider the whole attenuation effect as being due to \( \phi \) meson absorption. The comparison to the experimental data then fixes a value for the total \( \phi N \) absorption cross section. Possible changes of the involved initial and final state processes, as discussed, introduce considerable ambiguities in the extraction of this quantity.

In order to fit the \( \phi N \) cross section to the experimental transparency data we make the following ansatz: We multiply the total \( \phi N \) cross section given by Eq. (7) with
a constant normalization $K$-factor

$$\bar{\sigma}_{\phi N} = K \cdot \sigma_{\phi N}. \quad (13)$$

In doing so we keep the partial channels $\phi N \rightarrow \phi N$ and $\phi N \rightarrow \pi N$, which are at least roughly constrained by experimental data, untouched. Thus, the modification of the $\phi$ in-medium cross section is entirely moved into the absorptive channel $\phi N \rightarrow 2\pi N$. The results of these calculations are shown in Fig. 3. Best agreement is obtained with a $K$-factor of $K = 2.6$. The momentum spectrum of $\phi$ mesons in the simulated data sample inside the acceptance window is almost symmetrically distributed around a mean value of about 1.3 GeV. This leads to a total $\phi N$ cross section of $\bar{\sigma}_{\phi N} \approx 27$ mb indicated by the data.

A further quantity extracted from the experimental data in Ref. 12 is the $A$-dependence of the total $\phi$ meson yield. A scaling close to $\sigma \sim A^{2/3}$ implies strong absorption as the total cross section scales with the size of the nuclear surface. On the other hand a scaling close to $\sigma \sim A$ implies weak absorption as all nucleons of the target contribute to the total cross section. Considering only the incoherent events the authors of 12 have fitted the total yield with the ansatz

$$\sigma(A) \propto A^{\alpha} \quad (14)$$

with a value of $\alpha = 0.72 \pm 0.07$. The nuclear cross section as function of the target mass number from our BUU calculations that agree best with the experimental transparency ratio ($K = 2.6$) is shown in Fig. 4. Fitting the ansatz 13 to all calculated nuclei we find a value of $\alpha \approx 0.65$. Omitting the lead target in the fit, that also has not been considered experimentally, we obtain the value $\alpha \approx 0.72$. This is in perfect agreement with experiment. Anyway, both values clearly show that the production of $\phi$ mesons on nuclei is surface dominated.

In summary we find that the change of the kaon self-energy in medium has a significant effect on the observed attenuation of the $\phi$ meson yield from finite nuclei. However, even with the inclusion of this in-medium change the measured transparency cannot be fully explained. In this respect we confirm one of the main conclusions from 5, 12. Considering all additional medium effects as being due to $\phi$ meson absorption, including a proper relativistic treatment of the kaon/antikaon potentials in our BUU simulations, we find that a value of $\sigma_{\phi N} \approx 27$ mb is needed to obtain agreement with the data. This value is considerably higher than usual quark model estimates for the $\phi N$ cross section in vacuum. The nuclear transparency ratio as well as the $A$-dependence of the total $\phi$ meson yield can be reproduced with high accuracy by adopting this value for the $\phi N$ cross section.

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