Nonlinear Conductance for the Two Channel Anderson Model

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Abstract

Using the integral equations of the Noncrossing Approximation, the differential conductance is computed as a function of voltage for scattering from a two channel Kondo impurity in a point contact. The results compare well to experimental data on Cu point contacts by Ralph and Buhrman. They support a recently proposed scaling hypothesis, and also show finite temperature corrections to scaling in agreement with experiment. The conductance signal is predicted to be asymmetric in the bias when the impurity is not equally coupled to left and right moving electrons.

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The two channel Kondo model \cite{1}, or equivalently the Kondo limit of the two channel Anderson model \cite{2}, has been applied to a wide variety of interesting physical systems: heavy fermion compounds \cite{3,4}, high $T_c$ superconductors \cite{5}, and two level systems (TLS) in metals \cite{6–9}. Although this model contains many of the salient features of the experiments, e.g. marginal-Fermi-liquid behavior \cite{10,5}, the experimental proof for the physical existence of two channel Kondo impurities is far from certain.

One of the strongest experimental cases for the existence of two channel Kondo behavior is an experiment by Ralph and Buhrman \cite{8} on clean Cu point contacts. At low temperatures their samples show the correct temperature, magnetic field, and voltage dependence for a two channel Kondo impurity such as a TLS with electron assisted tunneling. Recently \cite{9}, the data has also been shown to be consistent with a scaling ansatz motivated by the equilibrium Conformal Field Theory (CFT) solution of the problem \cite{11}. In this paper we compute the differential conductance for a two channel Kondo impurity in a point contact and find excellent quantitative agreement with the above experiment, lending strong evidence for the existence of two channel Kondo impurities in this system.

The two channel Kondo model consists of two kinds or channels of mutually noninteracting conduction electrons which are coupled to the same impurity spin via an exchange interaction \cite{1}. For the physical realization of interest here \cite{8} a TLS plays the role of the impurity spin. The states of the TLS correspond to the impurity spin-up and spin-down states. The electrons scattering from the impurity may be characterized by parity. The different parity states play the role of spin-up and spin-down electrons in the conventional Kondo problem. Thus, parity plays the role of the active degree of freedom altered by the interaction, while the physical spin is a spectator degree of freedom, which is conserved by the interaction and which constitutes the two channels of the model \cite{1}.

It is convenient to represent this system by an Anderson hamiltonian of a particle on an impurity level far below the Fermi surface hybridizing with the two channels of conduction electrons. We compute the differential conductance within the Non Crossing Approximation (NCA) for the infinite $U$ Anderson model in the Kondo limit \cite{12–14}. The NCA has been
very successful in describing the one channel Kondo problem except for the appearance of spurious nonanalytic behavior at a temperature far below the Kondo temperature $T_K$. These spurious low-T properties are due to the fact that the NCA neglects vertex corrections responsible for restoring the low temperature Fermi liquid behavior of the one channel model \[15\]. However, it has recently been shown \[16\] that for the two channel problem, where the complications of the appearance of a Fermi liquid fixed point are not present, the NCA does give the exact low-frequency power law behavior of the impurity spectral function $A_d(\omega)$ at zero temperature. Therefore, we expect to achieve a correct description for quantities involving $A_d$ (like the conductance).

In the NCA approach the electron on the impurity is represented in terms of a pseudo Fermion operator, $f$, and a slave Boson operator, $b$ \[17\]. In order to calculate the conductance at finite bias, the NCA must be generalized using nonequilibrium Green functions \[18\]. One solves for both the retarded Green functions for these operators, $G_f^r$ and $G_b^r$, and for the ‘lesser’ Green functions $G_f^<$ and $G_b^<$, which contain information about the nonequilibrium distribution function. The integral equations for these four functions have been solved for the one channel case by Meir, Wingreen, and Lee \[19\]. However, our numerical implementation \[20\] is different from theirs, and we are able to go more than two orders of magnitude lower in temperature, deep into the low temperature scaling regime described below.

In the Anderson model the characteristic energy scale, the Kondo temperature, $T_K$, depends on the width, $\Gamma$, and position, $E_d \ll -\Gamma$, of the bare Anderson impurity level \[13\]. Besides the voltage, $V$, and temperature, $T$, we can also vary the couplings of the impurity to left and right moving electrons $\Gamma_L$ and $\Gamma_R$, normalized to the width $\Gamma$ ($\Gamma_L + \Gamma_R = 1$). Except for Fig. 4 all the numerical results have $\Gamma_L = \Gamma_R$.

Letting $F_{\text{eff}}(\omega) = \Gamma_L F(\omega + eV/2) + \Gamma_R F(\omega - eV/2)$, where $F(\omega) = 1/(1 + e^{\beta\omega})$, and using the conventions of Müller–Hartmann \[13\] for the spectral functions, $A(\omega) = -ImG_f^r(\omega)/\pi$, $B(\omega) = -ImG_b^r(\omega)/\pi$, and the ‘lesser’ Green functions, $a(\omega) = G_f^<(\omega)/2\pi$ and $b(\omega) = G_b^<(\omega)/2\pi$, the nonequilibrium NCA equations for the $N = 2$ spin, $M = 2$ channel Anderson model are \[21\].
\[
\frac{B(\omega)}{|G^b_b(\omega)|^2} = \Gamma N \int \frac{d\epsilon}{\pi} A(\omega + \epsilon) F_{eff}(\epsilon)
\]

(1)

\[
\frac{A(\omega)}{|G^f_f(\omega)|^2} = \Gamma M \int \frac{d\epsilon}{\pi} B(\omega - \epsilon) [1 - F_{eff}(\epsilon)]
\]

(2)

\[
\frac{b(\omega)}{|G^b_b(\omega)|^2} = \Gamma N \int \frac{d\epsilon}{\pi} a(\omega + \epsilon) [1 - F_{eff}(\epsilon)]
\]

(3)

\[
\frac{a(\omega)}{|G^f_f(\omega)|^2} = \Gamma M \int \frac{d\epsilon}{\pi} b(\omega - \epsilon) F_{eff}(\epsilon)
\]

(4)

The true impurity spectral function, \(A_d\), is computed from the slave Green functions via the convolution

\[
A_d(\omega) = \int \frac{d\epsilon}{\pi} [a(\epsilon) B(\epsilon - \omega) + A(\epsilon) b(\epsilon - \omega)].
\]

(5)

A point contact consists of two leads joined by a small constriction. Any additional scattering in the vicinity of the constriction should cause a decrease in the conductance because it impedes the flow of electrons. On the other hand, in a tunnel junction, tunneling may be assisted by electrons hopping on impurities in the junction, increasing the conductance. Thus, it is not surprising that when we generalize earlier calculations for the nonlinear current through a tunnel junction \[22,23\] to the case of a point contact, we find a similar expression for the current, except for an overall minus sign:

\[
I - I_0 \propto - \int d\omega A_d(\omega) [F(\omega - V/2) - F(\omega + V/2)],
\]

(6)

where \(I\) and \(I_0\) are the currents with and without the impurity. In deriving Eq. 6 we have assumed that the point contact is clean, namely the transmission coefficients are close to unity, and that all hopping matrix elements are slowly varying on the scale of the Kondo temperature. The conductance, \(G(V, T)\), is computed by taking a numerical derivative, \(dI/dV\). We will assume that the background conductance \(dI_0/dV\) is ohmic.

In Figure 1 we show the zero bias conductance, \(G(0, T)\), computed in this manner (\(\Gamma_L = \Gamma_R\)). As expected \[11\], the conductance shows a \(T^{1/2}\) dependence at low temperatures with deviations starting at about 1/4 \(T_K\). The Kondo temperature is determined by the width at half maximum of the zero bias impurity spectral function, \(A_d\), at the lowest calculated temperatures (see inset). The slope of the \(T^{1/2}\) behavior defines the constant \(B_{Z}\):
\[ G(0, T) - G(0, 0) = B_\Sigma T^{1/2}. \]  

(7)

The experimental data also show a \( T^{1/2} \)-dependence, but it is difficult to deduce an accurate estimate of \( T_K \) by looking at the deviations from \( T^{1/2} \) behavior (assuming that they would occur at \( 1/4 \ T_K \)). An educated guess gives \( T_K \) to be about 8 Kelvin for sample 1 and 2 and significantly less for sample 3 of Ref. [9].

Recently, it has been proposed from a CFT solution of the problem in equilibrium that the experimental data show scaling of the conductance \( G \) as a function of voltage bias \( V \) and temperature \( T \) of the form [9]

\[ G(V, T) - G(0, T) = B_\Sigma T^{1/2} H((A eV / k_B T)), \]  

(8)

where \( H \) is a universal scaling function (\( H(0) = 0 \) and \( H(x) \sim x^{1/2} \) for \( x >> 1 \)) and \( B_\Sigma \) and \( A \) are nonuniversal constants. In order to examine whether this ansatz is correct, in Fig. 2 the rescaled conductance is plotted as a function of \((eV/k_B T)^{1/2}\) for the numerical data (a) and the experimental data (b, for the best sample (1)). Motivated by the symmetry of the experimental conductance-voltage curves we choose equal coupling to left and right moving electrons, \( \Gamma_L = \Gamma_R \) (see also Fig. 4). Considering that after fixing \( B_\Sigma \) using Eq. (7) there are no adjustable parameters, the agreement is extraordinary. The collapse of the various \( T \) curves at low bias and the linear behavior for the low \( T \) curves in the range of \( 2 < (eV/k_B T)^{1/2} < 4 \) is in agreement with the proposed scaling ansatz Eq. [8]. However, the slope of the linear part shows temperature dependence for both the experimental and the numerical data. This is not contradictory to the scaling ansatz, but it does show that there are significant temperature dependent corrections to scaling within the present approach.

To analyze the scaling plots in more detail, the low bias portion is replotted in Fig. 3(a). The conductance follows an approximate \((eV/k_B T)^2\)-behavior even for \((eV/k_B T) > 1\) before it levels off and enters the \((eV/k_B T)^{1/2}\)-region at higher bias. The prefactor of the quadratic dependence shows no observable temperature dependence until approximately \(0.1T_K\) and consequently obeys the scaling ansatz.
In Fig. 3(b) the slopes of the straight line fits of the linear regions in Fig. 2 are plotted as a function of temperature. Both the numerical data and the experiment show clear temperature dependence. Although $B_{Σ}$ may be determined directly from the zero bias conductance, the Kondo temperature is more difficult to determine experimentally. In Fig. 3(b) we have chosen values for the Kondo temperature which are consistent with the estimates from the deviation of the zero bias conductance from $T^{1/2}$ behavior. The resulting curves are in good quantitative agreement. In order to show that the experimental and numerical curves indeed coincide (for a given $T/T_K$) we also compare the intercepts of the straight line fits for the same Kondo temperatures (inset Fig. 3 (b)). The numerical data do fall right in the middle of the scatter from the three different samples [24]. Note that our theory does not have the additional parameter, $A$, of Eq. 8, but adjusting the Kondo temperature has some of the same effect as adjusting $A$.

Finally, we have also studied the conductance in cases where the impurity is asymmetrically coupled to left and right moving electrons. The NCA equations have no symmetry for $Γ_L ↔ Γ_R$; however, they are symmetric under the parity operation $Γ_L ↔ Γ_R$ and $V ↔ −V$. Figure 4 shows the conductance signal one should observe in an asymmetrically coupled sample. The conductance is clearly a nonuniversal function of the couplings $Γ_L$ and $Γ_R$. The experimental data are symmetric to within experimental error, indicating that left and right moving electrons are equally coupled to the TLS. We think this is reasonable for a point contact where the impurity has no particular asymmetry with respect to left and right moving electrons, but not for a tunnel junction, where one would expect exponential dependence of the couplings on the location of the impurity.

In conclusion, we have performed numerical evaluations of the NCA integral equations for the two channel Anderson model out of equilibrium. We find outstanding agreement with data from an experiment on Cu point contacts [9]. Scaling of the conductance at low bias ($eV < k_B T$) and temperatures is verified. As $V$ and $T$ are increased, the calculation exhibits finite-$T$ corrections to scaling, again in agreement with experiment. Asymmetric conductance signals are predicted for impurities asymmetrically coupled to the leads. Thus,
this work lends strong support to the existence of two channel Kondo impurities in the Cu
point contacts of Ralph and Buhrman.

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FIGURES

FIG. 1. Temperature dependence of the zero bias conductance ($\Gamma_L = \Gamma_R$). The zero bias conductance has $T^{1/2}$- dependence for $T < T_K/4$. This can be used to roughly extract $T_K$ from the experimental data. Inset: The impurity spectral function $A_d(\omega)$ for several voltages. The width at half maximum of the zero bias spectral function (dotted curve) determines $T_K$. As the voltage is increased to $eV = k_BT_K$ the Kondo resonance is reduced (solid curve). At very large bias the resonance shows a shoulder and eventually two peaks. In this paper we compare theory and experiment in the scaling regime, $T \ll T_K$.

FIG. 2. Scaling plots of the conductance for (a) theory and (b) experiment [13]. With $\Gamma_L = \Gamma_R$ and $B_\Sigma$ determined from the zero bias conductance (see Fig. 1), there are no adjustable parameters. There are roughly two regimes in these plots. For $(eV/k_BT)^{1/2} < 1.5$ the curves collapse onto a single curve and the rescaled conductance is proportional to $(eV/k_BT)^2$. For $2 < (eV/k_BT)^{1/2} < 4$ the rescaled conductance is linear on these plots. There are substantial corrections to scaling even at temperatures small compared to $T_K$ (see figure 3 (b)). At even larger biases this linear behavior rounds off, indicating the breakdown of scaling. The temperatures in the theory and experimental plots are in units of $T_K$ and Kelvin, respectively.

FIG. 3. Quantitative analysis of the scaling plots at (a) low bias and (b) high bias. (a) The low bias rescaled conductance as a function of $(eV/k_BT)^2$ (curves are offset). Both theory and experiment (sample 1 of Ref. [9]) show quadratic behavior at low bias. The symbols correspond to the temperatures shown in Fig. 2. (b) The slope of the straight line fits of the linear region in Fig. 2 as a function of temperature for sample 1 (○,+), sample 2 (◇,□), sample 3 (×,*), (for $V > 0$, $V < 0$) and the numerical data (△). The Kondo temperature for the experimental curves is consistent with the deviation of the zero bias conductance from $T^{1/2}$- behavior. All curves drop with increasing temperature even at temperatures small compared to the Kondo temperature. The inset shows the behavior of the intercepts of the fits.
FIG. 4. Dependence of the conductance on the coupling of the impurity to left (right) moving electrons, $\Gamma_{L(R)}$, with $\Gamma_L + \Gamma_R = 1$. In Figs. 1-3 $\Gamma_L$ equals $\Gamma_R$. For a typical low temperature ($T = 0.01T_K$) one observes an asymmetric signal, $G(V, T) \neq G(-V, T)$ for $\Gamma_L \neq \Gamma_R$. Note that $B_\Sigma$, which depends on $\Gamma_L$, has been divided out. These curves show that the scaled conductance is not universal if one allows for asymmetric coupling of the impurity to left and right moving electrons.