Position estimation method considering directivity of a transmitter

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**Abstract:** In this paper, we study an estimation method of the position of an unknown transmitter based on received signal strength. The original method considers only an omni-directional antenna pattern of the transmitter, so we extended the estimation method to take into account the directivity of the transmitting antenna. The estimation performance of the method is discussed by a computer simulation and an experiment. We showed that, when the transmitter has directivity, the accuracy of the position estimation was improved by considering the directivity.

**Keywords:** Position estimation, Received signal strength, Directivity

**Classification:** Sensing

**References**

[1] A. J. Weiss, “On the accuracy of a cellular location system based on RSS measurements,” IEEE Transactions on Vehicular Technology, vol. 52, No. 6, pp. 1508-1518, Nov. 2003.

[2] B. Wang, S. Zhou, W. Liu and Y. Mo, “Indoor localization based on curve fitting and location search using received signal strength,” IEEE Transactions on Industrial Electronics, vol. 62, No. 1, pp. 572-582, Jan. 2015

[3] M. Bshara, F. Gustafsson and L. V. Biesen, “Fingerprinting localization in wireless networks based on received-signal-strength measurements: A case study on WiMAX networks,” IEEE Transactions on Vehicular Technology, vol. 59, No. 1, pp. 283-294, Jan. 2010.

[4] P. Zuo, T. Peng, H. Wu, K. You, H. Jing, W. Guo and W. Wang, “Directional source localization based on RSS-AOA combined measurements,” China Communication, pp. 182-193, Nov. 2020.

[5] S. Shimizu, T. Kurihara, K. Yano and Y. Suzuki, “Location estimation method for unknown signal source,” IEICE Communications Express, Vol.9, No. 8, pp. 389-393, Aug. 2020.
1 Introduction
Various position estimation methods have been proposed until now. For example, a lateration-based estimation method using received signal strength (RSS) has been proposed [1, 2]. These methods need to first obtain estimated distances from a mobile device to multiple access points (APs). Given an observed RSS, the distance to one AP can be estimated. In addition, an estimation method using RSS fingerprinting technique [2, 3] and a method based on RSS and angle-of-arrival (AOA) measured at base stations [4] have been proposed.

Among estimation methods using RSS, we focused on the method proposed in [2, 5]. In the method, a model of spatial distribution of received power is assumed and the distribution is optimally fitted to RSS measured at multiple points to estimate the position of the transmitter. The original estimation method considered only omni-directional patterns of the transmitters. In order to estimate the source position not only in the case of omni-directional transmitting antenna, we have established a new extended version of the estimation method by which directivity of the transmitter can be considered [6]. In this paper, first, we showed the extended method. Next, we have performed a simulation and a measurement to confirm the validity of the method. Moreover, we showed the performance of the position estimation was improved by considering the directivity appropriately.

2 Estimation method based on received signal strength and extension by considering directivity of a transmit antenna
In the position estimation method proposed in [5], a large amount of RSS is measured and a spatial distribution model of received power is fitted to the measured RSS to estimate the position of the transmitter.

In an actual position estimation, the position estimation in three-dimensional region is required, but in this paper the position estimation in two-dimensional region is assumed in order to simplify the discussion.

The coordinates of the unknown position of a transmitter and the transmit power are expressed as \((x, y, p)\). Similarly, the unknown coordinates and the measured RSS at the \(i\)-th position are expressed as \((x_i, y_i, r_i)\). In the estimation, the following formula is assumed to obtain the estimated RSS \(\hat{r}_i\), given by the receiver gain \(G_r\), the transmitter gain \(G_s\), the wavelength \(\lambda\) and the estimated distance \(\hat{d}_i\). In the formula, the free space propagation is assumed and the propagation constant \(a\) is 2.

[6] S. Okamoto, T. Akiyama, R. Goto, Y. Kawai, S. Shimizu and H. Iwai, “Team Radio Catcher Challengers,” B-03, ISAP2020 Student design contest, Category B. Localization of RF Sources, Jan. 2021.
\[ \hat{r}_i = pG_sG_r \left( \frac{\lambda}{4\pi d_i} \right)^2 \]  

(1)

where \( d_i \) is expressed as

\[ d_i = \sqrt{(x-x_i)^2 + (y-y_i)^2} \]  

(2)

In [5], an antenna pattern of the transmitter is omni-directional, and \( G_s \) is assumed to be a constant. In the case, an error function \( f(x, y, p) \) is defined by the following expression.

\[ f(x, y, p) = \sum_{i=1}^{N} w_i (r_i - \hat{r}_i)^2 \]  

(3)

where \( N \) is the number of the measurement points of RSS. The coefficient \( w_i \) is a weighting factor to consider the reliability of the measured RSS adopted in [5]. In this paper, \( w_i \) is set to unity for all \( i \) considering simplicity. Minimizing the error function provides the optimal positions of the transmitter, which becomes a nonlinear optimization problem. The steepest descent method for the optimization is shown in the following iteration process.

\[
\begin{bmatrix}
x_{k+1} \\
y_{k+1} \\
p_{k+1}
\end{bmatrix} = \begin{bmatrix}
x_k \\
y_k \\
p_k
\end{bmatrix} - \alpha \nabla f(x_k, y_k, p_k)
\]  

(4)

where \( \alpha \) is the step size, and \( k \) is the iteration number. The estimation performance is expected to depend on the initial values of \((x, y, p)\), and \( \alpha \) needs to be chosen appropriately along with initial values. \( \alpha \) is \( 10^{-7} \) in [5]. The update vector in Eq. (4) is represented by the following expression.

\[
\begin{bmatrix}
\frac{\partial f(x_k, y_k, p_k)}{\partial x_k} \\
\frac{\partial f(x_k, y_k, p_k)}{\partial y_k} \\
\frac{\partial f(x_k, y_k, p_k)}{\partial p_k}
\end{bmatrix} = \begin{bmatrix}
4p_kG_sG_r \sum_{i=1}^{N} w_i \left( \frac{\lambda}{4\pi d_i} \right)^2 \frac{(x_k-x_i)^2}{d_i^2} (r_i - \hat{r}_i) \\
4p_kG_sG_r \sum_{i=1}^{N} w_i \left( \frac{\lambda}{4\pi d_i} \right)^2 \frac{(y_k-y_i)^2}{d_i^2} (r_i - \hat{r}_i) \\
-2G_r \sum_{i=1}^{N} w_i \left( \frac{\lambda}{4\pi d_i} \right)^2 (r_i - \hat{r}_i)
\end{bmatrix}
\]  

(5)

Here we extend the estimation method to consider the directivity of a transmitter. We assume a pattern of the transmitter is known, but the beam direction is unknown. Therefore, the gain of the transmitter \( G_s \) is not a constant, but a function of the angle. Since the beam direction \( \theta \) is unknown and is required to be optimized, we extend the aforementioned error function to \( f(x, y, p, \theta) \) and optimize the four variables in the same way as the original method. In the case, the update vector \( \nabla f(x, y, p, \theta) \) is replaced by the following expression.
\[
\frac{\partial f(x_k, y_k, p_k, \theta_k)}{\partial x_k} = \frac{2p_k G_r \left( \frac{\lambda}{4\pi} \right)^2}{4\pi d_i} \left( G_r^2 \sum_{i=1}^{N} w_i \left( \frac{\lambda}{4\pi d_i} \right)^2 G_r(\theta_k)(r_i - \hat{r}_i) \right)
\]

The following equation gives the directional gain of a dipole antenna which we used in the simulation and the experiment.

\[
G_r(\theta) = 1.64 \frac{\cos^2 \left( \frac{\pi}{2} \cos(\theta - \hat{\theta}) \right)}{\sin^2(\theta - \hat{\theta})}
\]

Then, the differential of \(G_r(\theta), G_r'(\theta)\) is given as follows.

\[
G_r'(\theta) = 1.64 \left( -\pi \sin \left( \frac{\pi}{2} \cos(\theta - \hat{\theta}) \right) \cos \left( \frac{\pi}{2} \cos(\theta - \hat{\theta}) \right) \right)
\]

This allows us to estimate the coordinates of the unknown transmitter, the transmit power, and the beam direction of the transmitter.

### 3 Evaluation by simulation and experiment

We evaluated the performance of the position estimation by computer simulation and an experiment. In the experiment, the frequency is 320 MHz, and we used a module of a CW oscillator as a transmitter and measured RSS using a spectrum analyzer. The transmission power is 0 dBm. The experiment was carried out in an anechoic chamber. The transmitter and the receiver equip with a horizontally polarized dipole antenna. As for the receiver, we turned the direction of the receiving antenna to look for the directional peak and measure the RSS, so we assume the antenna gain of the receiver is a constant.

The configuration of the simulation and the measurement is identical and shown in Fig. 1. In the figure, RSSs at the measurement positions are indicated as squares \((N=54)\). Due to the limitation of the possible positions of the measurement in the anechoic chamber, we placed the measurement positions as in the figure. The
position of the transmitter is illustrated by the circle in the red color, and the beam direction is expressed by the arrows.

![Fig. 1. Position of transmit antenna and spatial distribution of RSS.](image)

In order to evaluate the performance improvement of the position estimation by considering directivity, we compared the position estimation using Eq. (6) with that using Eq. (5).

### 3.1 Estimation results

The results of the position estimation are summarized in Table I. In the simulation and the experiment, we set the initial position at the origin (0, 0) of Fig. 1 and the value of \( \alpha \) as \( 10^7 \).

| Directivity of transmitter | Actual position [m] | Actual direction [deg.] | Estimated position [m] | Estimated direction [deg.] | Error distance [m] |
|----------------------------|---------------------|-------------------------|------------------------|---------------------------|------------------|
| Considered                 | (-1.70, 0.00)       | 90                      | (-1.06, -0.16)         | -0.66                     |
| Not considered             |                     |                         | (-1.07, 0.00)          | (-0.97, 0.14)             | 0.74             |

In the measurement considering the directivity of the transmitter, the direction of the transmitter was well estimated. The estimation error was 0.66 m when we didn’t consider the directivity, but the error reduced to 0.12 m by considering the directivity. From these results, we achieved the more accurate position estimation by considering the directivity of the transmitter.

### 3.2 Dependence on propagation constant

In the estimation process, the propagation constant \( \alpha \) is assumed 2. However, it is probable that the propagation constant differs from the value in an actual propagation environment. Therefore, we evaluated the dependence of the estimation performance on the value of \( \alpha \) by simulation. The result is shown in Fig. 2. It shows the estimation error for the variation of \( \alpha \). The estimation error is defined as the distance between the estimated result and the actual position.
In the simulation we changed the position of the transmitter in order to consider the effect of the position. When $\alpha$ is 2, the position of the transmitter can be estimated accurately wherever the transmitter locates. When $\alpha$ is not 2, the estimation error increases. In the case, the error distance depends on the position of the transmitter. It is also seen that, when the position of the transmitter is the right side in Fig. 1 (4.0, 0.0), the estimation error is large. It is possibly due to less number of measurement positions at the side.

4 Conclusion

We discussed a method using the received signal strength measured at multiple points to estimate the position of the transmitter. We extended the estimation method to take into account the directivity of the transmitter. We achieved a position estimation considering directivity of the transmitter in the simulation and the experiment to show more accurate position estimation method.

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