Coherent control of three-spin states in a triple quantum dot

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Spin qubits involving individual spins in single quantum dots or coupled spins in double quantum dots have emerged as potential building blocks for quantum information processing applications. It has been suggested that triple quantum dots may provide additional tools and functionalities. These include encoding information either to obtain protection from decoherence or to permit all-electrical operation, efficient spin busing across a quantum circuit, and to enable quantum error correction using the three-spin Greenberger-Horn-Zeilinger quantum state.

Towards these goals we demonstrate coherent manipulation of two interacting three-spin states. We employ the Landau-Zener-Stückelberg approach for creating and manipulating coherent superpositions of quantum states. We confirm that we are able to maintain coherence when decreasing the exchange coupling of one spin with another while simultaneously increasing its coupling with the third. Such control of pairwise exchange is a requirement of most spin qubit architectures, but has not been previously demonstrated.

Following the spin qubit proposal by Loss and DiVincenzo and the electrostatic isolation of single spins in quantum dots (QDs) and double quantum dots (DQDs), coherent manipulation was demonstrated in two-level systems based on single-spin up and down states as well as two-spin singlet and triplet states. Here we demonstrate coherent manipulation of a two-level system based on three-spin states.

We employ the triple quantum dot (TQD) device layout shown in Fig. 1a, consisting of multiple metallic gates on a GaAs/AlGaAs heterostructure. The gates are used to electrostatically define three QDs in series within a two-dimensional electron gas 110 nm below the surface. The QDs are surrounded by two quantum point contact charge detectors (QPCs). The QPC conductance identifies the number of electrons in each QD and its derivative with respect to a relevant gate voltage maps out the device configuration stability diagram. We tune the device to the qubit operating electronic configuration, \((N_\text{L}, N_\text{C}, N_\text{R}) = (1,1,1)\), between two spin-to-charge conversion regimes \((1,0,2)\) and \((2,0,1)\), where \(L, C\) and \(R\) refer to the left, centre and right QDs, respectively. The detuning, \(\varepsilon\), controls the energy difference between configurations \((1,0,2)\), \((1,1,1)\) and \((2,0,1)\). The exchange coupling, \(J\), depends on \(\varepsilon\) and the tunnel couplings.

In this paper we concentrate on two scenarios. In the first scenario, at each point in the stability diagram the exchange coupling to the centre spin from one or both of the edge spins is minimal (that is, one edge spin resembles a passive spectator). This configuration is used as a control to confirm that our device maps onto two-spin results in this limit. In the second scenario a true three-interacting-spin regime is achieved. (Results from a third intermediate regime are shown in the Supplementary Information.)

The energy level spectrum of a TQD (ref. 14) consists of quadruplets \(Q\) with total spin \(S = 3/2\) separated by the Zeeman energy in a magnetic field and doublets \(\Delta'\) and \(\Delta\) with \(S = 1/2\). The two states of our qubit consist of one of the quadruplets, \(Q_{3/2}\), and one of the doublets, \(\Delta'_{1/2}\), where

\[
|Q_{3/2}\rangle = |\uparrow\uparrow\uparrow\rangle
\]

\[
|\Delta'_{1/2}\rangle = \frac{(-\hbar c + \hbar c + D)(\uparrow\downarrow\downarrow) - (\downarrow\downarrow\uparrow)}{\sqrt{4D^2 + 2(\hbar c + D)^2}}
\]

with \(\Omega = \sqrt{J_{LC} + J_{RC}} - J_{LC}J_{RC}\) and where \(J_{LC}\) and \(J_{RC}\) is the exchange coupling between the left (right) and centre spins. (Other three-spin states are described in more detail in the Supplementary Information.)

Figure 1b illustrates the three-spin energy spectrum as a function of detuning (zero detuning is defined as the centre of the \((1,1,1)\) regime as shown). Experimentally we can tune the \((1,1,1)\) region size by using gate \(C\) primarily.

The hyperfine interaction couples the state \(\Delta'_{1/2}\) to the state \(Q_{3/2}(Q_{1/2})\) at their anticrossing (asymptotic approach), see Fig. 1c. \((Q_{1/2}\) and \(\Delta_{1/2}\) are also hyperfine coupled.) Figure 1c also illustrates the two types of experiment we describe in this paper. With the single anticrossing (SA) pulse, based on the methodology in ref. 9, the system starts in the \(\Delta_{1/2}\) state in the \((2,0,1)\) (or \((1,0,2)\)) regime, and then a pulse is applied to reach the \((1,1,1)\) regime. The pulse rise time (see Supplementary Information) ensures that Landau-Zener passage through the anticrossing. After a state evolution time, \(\tau\), the pulse steps down, completing the spin interferometer on the return passage through the anticrossing. The probability of the \(\Delta'_{1/2}\) state occupation, \(P_{\Delta'_{1/2}}\), is directly obtained by this projection back into the \((2,0,1)\) state (or \((1,0,2)\)) regime, where the required spin-to-charge information conversion is achieved by the Pauli blockade of the \(Q_{3/2}\) state. An experiment with a double anticrossing (DA) pulse is also illustrated in Fig. 1c. The sequence is similar, with the important distinction that a larger pulse enables LZ tunnelling processes through both anticrossings before again projecting back in the \((2,0,1)\) regime having passed through both anticrossings.

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twice. Important calibration information is obtained if the pulse time is longer than the coherence time (that is, $\tau > T_1^*$) where the mixing at the $\Delta_{1/2} - Q_{3/2}$ anticrossing is detected independently of coherence effects. Figure 1d plots this against magnetic field for a 9-mV-wide $(1,1,1)$ regime midway between the narrow and wide $(1,1,1)$ regimes. The two anticrossings form a ‘spin arch’ which is used to extract the coupling parameters for the model.

The distinction between our two regimes is now clear. In the case of a wide $(1,1,1)$ region, close to zero detuning, both $J_{1C}$ and $J_{BC} \sim 0$, so $E_{\Delta_{1/2}} \approx E_{Q_{3/2}} \approx E_{Q_{1/2}}$. Away from zero detuning only two of the spins are coupled: right–centre (left–centre) at negative (positive) detuning. Experiments using DA pulses in this regime involve coupling to not only $Q_{1/2}$ but also to $Q_{1/2}$. Thus this regime is not suitable for a two-level system involving three interacting spins. As a control experiment, however, in Fig. 2 we plot the coherent Landau–Zener–Stückelberg (LZS) oscillations obtained in this regime for both positive and negative detuning with a SA pulse. These compare to the first LZS experimental results with DQDs from ref. 9, later described theoretically in refs 18,19. The degree of LZ tunnelling, that is, the relative size of $A$ and $B$ in the coherent $A[\Delta_{1/2}^*] + Be^{i\phi}[Q_{1/2}]$ state, depends on the speed, $\nu$, through the anticrossing: $P_{\text{LZ}} = e^{-2\nu\Delta/\hbar}$, where $2\Delta$ is the energy splitting at the anticrossing. The visibility of the oscillations is a balance between this speed and $T_2^*$. For an infinite $T_2^*$, a rise time $\sim 0.2 \mu$s would produce a 50/50 superposition (see also ref. 9). Experimentally it is found that a $6.6 \, \text{ns}$ pulse rise time (or $3.3 \, \text{ns}$ Gaussian time constant) leads to oscillations with the highest visibility. The value of $T_2^*$, obtained from a single parameter fit to the data, ranges from 5 to 18 ns, consistent with previous DQD experiments where $T_2^*$ was limited by fluctuations in the nuclear field environment1.

In Figs 3 and 4 we show results for experiments with DA pulses in a narrow $(1,1,1)$ regime, where $J_{1C}$ and $J_{BC}$ are finite throughout and two well-defined qubit states exist between the two anticrossings (that is, simulations based on experimentally extracted parameters confirm that $\Delta_{1/2}$ has moved far enough below the $Q_{1/2}$ state such that no experimental features are related to interactions with the $Q_{1/2}$ state). The energy level diagrams for this regime are shown
Figure 2 | LZS oscillations from the two $\Delta_{1/2} - Q_{3/2}$ qubits for a wide (1,1,1) region. a,b. Numerical derivative of the conductance with respect to detuning showing LZS oscillations versus pulse duration $\tau$. Black is low, red is medium and yellow is high. Panel a shows measurements with the right QPC for $|\epsilon_{+} - \epsilon_{-}| = 27$ mV along $V_1$; the pulse goes across the (1,0,2) to (1,1,1) charge transfer line at $B = 60$ mT. Both $V_2$ and $V_1$ are swept to detune parallel to the pulse direction in the $V_1 - V_2$ plane. Panel b shows measurements with the left QPC for $|\epsilon_{+} - \epsilon_{-}| = 41.5$ mV along $V_2$; the pulse goes across the (2,0,1) to (1,1,1) charge transfer line at $B = 60$ mT. c,d. Probability of ending in the $\Delta_{1/2}$ state as a function of $\tau$ with fits for $T_{j}^*$. For the right QPC (c) the pulse goes from (1,0,2) to (1,1,1) and $|\epsilon_{+} - \epsilon_{-}| \sim 50$ mV along $V_1$. For the left QPC (d) the pulse goes from (2,0,1) to (1,1,1) and $|\epsilon_{+} - \epsilon_{-}| = 27$ mV along $V_2$. The experimental data are shown as points, whereas the theoretical fits are shown as red lines. The values of $T_{j}^*$ extracted from the single parameter fit to the LZS model are indicated.

Figure 3 | Coherent three-spin state manipulation with a narrow (1,1,1) region a. Stability diagram in the presence of a pulse (drawn as a white line for a given $(V_1, V_2)$), showing coherent LZS oscillations in the (2,0,1) region with features parallel to both charge transfer lines. The colour map (black is low, red is medium and yellow is high) corresponds to the numerical derivative of the left QPC conductance with respect to $V_2$ in the presence of a pulse across the charge transfer line between (2,0,1) and (1,1,1). The (1,1,1) region is tuned to a width of $\sim 5$ mV with gate C. $B = 25$ mT. The stability diagram also shows LZS oscillations involving (2,0,2) and (1,1,2). b. Calculated $\Delta_{3/2}^*/\Delta_{2/2}^*$ map zooming mainly into the (2,0,1) region of the stability diagram from a. The dashed line shows where addition line is expected, although it is not part of the calculation. $B = 40$ mT. c,d. Traces of $\Delta_{3/2}^*/\Delta_{2/2}^*$ versus $\tau$. For c the data points are extracted from Fig. 4b (40 mT, white line) at $V_2 = -1.0, 751$ V. For d the data points are extracted from Fig. 4b (40 mT, blue line) at $V_2 = -1.074$ V. The fits (red lines) use $B = 60$ mT. The values of $T_{j}^*$ extracted from the fits are indicated.
Finally, we note the direct observation of tracking the resonance across the maximum dotted lines) that the resonances double back on themselves. This is corresponding to LZS oscillations. It can be seen (for example, curved dotted lines) that coherence is maintained as the regime between the two anticrossings, while the resonances above 25 mT. The region between the boundaries corresponds to oscillations at different magnetic fields. Two boundaries, marked with horizontal white dashed lines, can be observed at fields above 25 mT. The region between the boundaries corresponds to the regime between the two anticrossings, while the resonances correspond to LZS oscillations. It can be seen (for example, curved dotted lines) that the resonances double back on themselves. This is a direct observation of tracking the resonance across the maximum in the Δ_{1/2} versus detuning curve (see Figs 4a and 1b). We speculate that operating at this spot may provide more protection from charge noise, as the energy levels become locally flat versus detuning.

Although the frequency of coherent oscillations grows with field, owing to the increased spacing between the two qubit levels, it seems as if the experiment and theory differ by 20 mT for experimental data at 40 mT and by 15 mT for data at 25 mT. We attribute this to a dynamic nuclear polarization effect (DNP; ref. 20). To make this quantitative we extract horizontal slices in Fig. 4b at 40 mT (blue and white lines) and fit them to obtain T_{1/2}^* (25 and 40 mT). It is found that a DNP ~20 mT is required to properly describe the period of oscillations. This is why the stability diagram in Fig. 3b is calculated at 40 mT rather than 25 mT, and why the fits in Fig. 3c,d are calculated at 60 mT instead of 40 mT.

Figure 4 | Magnetic field dependence of coherent three-spin state manipulation with a narrow (1,1,1) region. a. Energy spectra for the three-spin states for different magnetic fields. The colour code for the states is the same as in Fig. 1b. From left to right we have: B = 5 mT and |ε_1 − ε_2| = 3.9 mV; B = 25 mT and |ε_1 − ε_2| = 5.1 mV; B = 40 mT and |ε_1 − ε_2| = 5.6 mV; and B = 60 mT and |ε_1 − ε_2| = 4.6 mV. b. Coherent oscillations shown in the τ = V_2 plane. c. Calculated dP_{Δ_{1/2}}/dV_2 maps (black is low; red is medium and yellow is high) in the ε plane for the same experimental settings as for b.

To gain further insight, Fig. 4b and c show experimental and theoretical plots of the pulse duration dependence of LZS oscillations at different magnetic fields. Two boundaries, marked with horizontal white dashed lines, can be observed at fields above 25 mT. The region between the boundaries corresponds to the regime between the two anticrossings, while the resonances correspond to LZS oscillations. It can be seen (for example, curved dotted lines) that the resonances double back on themselves. This is a direct observation of tracking the resonance across the maximum in the Δ_{1/2} versus detuning curve (see Figs 4a and 1b). We speculate that operating at this spot may provide more protection from charge noise, as the energy levels become locally flat versus detuning.
marked with a white spot in Fig. 4b. This is a non-trivial feature corresponding to a resonance condition of two interacting spin interferometers, one between the two anticrossings and a second, beyond the second anticrossing.

In conclusion, we have demonstrated coherent control of a qubit based on three-interacting-spin states. We have confirmed that there is no detectable change in the coherence time in the three-spin experiments compared with the two-spin experiments. We have realized the pairwise control of exchange for a three-spin system by pulsing the detuning energy of a triple quantum dot. The same technique should carry over when more quantum dots are added in series to increase the number of qubits. Pairwise control of exchange, as demonstrated here, will then be useful for building complex quantum algorithms based on electron spin qubits in quantum dots.

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Author contributions

Z.R.W. developed and grew the 2DEG heterostructure free of telegraphic noise; A.K. fabricated the triple quantum dot device capable of reaching the few-electron regime; P.Z., L.G. and S.A.S. and M.P-L. designed and built the high-frequency lines up to 50 GHz at millikelvin temperatures; P.Z., L.G. and S.A.S. performed the experiments compared with the two-spin experiments. We have realized the pairwise control of exchange for a three-spin system by pulsing the detuning energy of a triple quantum dot. The same technique should carry over when more quantum dots are added in series to increase the number of qubits. Pairwise control of exchange, as demonstrated here, will then be useful for building complex quantum algorithms based on electron spin qubits in quantum dots.

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