Polarization morphology of SiO masers in the circumstellar envelope of the asymptotic giant branch star R Cassiopeiae

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ABSTRACT

Silicon monoxide maser emission has been detected in the circumstellar envelopes of many evolved stars in various vibrationally excited rotational transitions. It is considered a good tracer of the wind dynamics close to the photosphere of the star. We have investigated the polarization morphology in the circumstellar envelope of an asymptotic giant branch star, R Cassiopeiae. We mapped the linear and circular polarization of SiO masers in the \( v = 1, J = 1–0 \) transition. The linear polarization is typically a few tens of per cent, while the circular polarization is a few per cent. The fractional polarization tends to be higher for emission of lower total intensity. We found that, in some isolated features, the fractional linear polarization appears to exceed 100 per cent. We found the Faraday rotation is not negligible but is \( \sim 15^\circ \), which could produce small-scale structure in polarized emission, whilst the total intensity is smoother and partly resolved out. The polarization angles vary considerably from feature to feature, but there is a tendency to favour the directions parallel or perpendicular to the radial direction with respect to the star. In some features, the polarization angle abruptly flips 90°.

We found that our data are in the regime \( gg/\Omega_1 \gg R \gg /\Gamma_1 \), which indicates that the model of Goldreich, Keeley & Kwan can be applied and the polarization angle flip is caused when the magnetic field is at close to 55° to the line of sight. The polarization angle configuration is consistent with a radial magnetic field, although other configurations are not excluded.

Key words: magnetic fields – masers – polarization – stars: AGB and post-AGB – stars: individual: R Cassiopeiae – stars: late-type.

1 INTRODUCTION

The circumstellar envelope (CSE) in an asymptotic giant branch (AGB) star is a very active region. Any AGB star will lose mass in the form of a slow wind at a rate that will significantly affect the mass of the star as well as enriching the interstellar medium with nuclear processed material. This produces a CSE of escaping dust and gas particles. In the zone just above the photosphere, out to 5\( R_\ast \) (stellar radius), the extended atmosphere experiences periodic shocks, so each period is divided into intervals of mass outflow and infall. In this zone, the cycle-averaged temperature drops as the wind flows away from the star, from about 3100 K at the photosphere to \( \approx 750 \) K at 5.5\( R_\ast \) (Gray et al. 2009); cooling takes place by line radiation from various molecules, especially H\(_2\)O. SiO masers provide sub-mas images of this region.

Some molecular species are formed in an equilibrium process deep in the atmosphere and are destroyed in the outer parts of the outflow by interstellar ultraviolet radiation (H\(_2\), CO, H\(_2\)O). Other species, for example, SiO, are depleted due to condensation on dust particles at a few stellar radii (Habing 1996).

The SiO maser zone is thought to be shaped by a combination of shocks from stellar pulsations, gravity and possibly magnetic effects, with radiation pressure on dust becoming possible towards the outer region. SiO masers are seen in rings, with faint or no emission along the line of sight to the star (Diamond & Kemball 2003; Cotton et al. 2006). This is due to tangential beaming from an accelerating, approximately spherical wind, where masers have the deepest amplification paths if they are at similar velocities and in a similar plane of the sky to the star.

Studying the magnetic field properties is done by investigating the polarization morphology of the maser emission. The maser emission is significantly linearly polarized but circular polarization is weaker, as expected for a non-paramagnetic molecule (Herpin et al. 2006). Fractional circular polarization of SiO masers in late-type stars from
single-dish observations is \( m_c \leq 0.5 \) per cent (Habing 1996), while the degree of linear polarization is \( m_c \sim 15-30 \) per cent (Troland et al. 1979).

There are two explanations suggested for the linear polarization of SiO masers. Polarization can be produced by the magnetic field (Goldreich, Keeley & Kwan 1973; Elitzur 1992). Alternatively, in the absence of a magnetic field, anisotropic pumping can produce strongly polarized maser emission (Western & Watson 1983). This mechanism has been proposed as the cause of the tangentially polarized maser emission seen in the map of SiO masers observed by the Very Large Bolometric Array (VLBA; Desmurs et al. 2000).

If it is magnetic in origin, the linear polarization position angle provides information about the structure of the magnetic field in CSEs. The theoretical relationship between the polarization position angle and the projected magnetic field direction depends on \( \theta_V \), the angle between the magnetic field direction and the maser propagation direction (our line of sight). When \( \theta_V < 55^\circ \), the linear polarization vectors are parallel to the magnetic field lines and when \( \theta_V > 55^\circ \) the vectors are perpendicular to the field lines. When \( \theta_V \approx 55^\circ \) the linear polarization vectors can flip within a single feature. At this value of \( \theta_V \), the Van Vleck angle of \( 55^\circ \), the masing region maximum intensity and the fractional linear polarization approaches zero (Goldreich et al. 1973; Elitzur 1992).

R Cassiopeia (R Cas) is an oxygen-rich AGB star which is classified as an M-type Mira variable. Its optical brightness varies from magnitude \(+4.7\) to \(+13.5\) with a period of 430 d and its mass is about 1.2 M\(_\odot\). Vlemmings et al. (2003) used astrometric very long baseline interferometry (VLBI) to measure a distance of 176 ± 6 pc with a proper motion of \((85.5 \pm 0.8, 17.5 \pm 0.7)\) mas yr\(^{-1}\) in RA and Dec., respectively. Various estimates of the stellar velocity \( v_* \) appear in the literature; we have adopted the value of 24 ± 2 km s\(^{-1}\).

We monitored the 43 GHz SiO masers of R Cas for almost two stellar cycles. The total intensity results were discussed in Assaf et al. (2011). Here, we describe the polarization observation in Section 2. We present the polarization results in Section 3, and analyse these results in Section 4, drawing conclusions in Section 5.

## 2 OBSERVATIONS AND DATA REDUCTION

The SiO masers around R Cas were observed as part of a more extensive programme of VLBA\(^1\) monitoring of other stars. In total, we have 23 epochs of data for R Cas. Data were recorded at each VLBA antenna in dual-circular polarization in two 4 MHz windows, each digitally sampled at the full Nyquist rate of 8 Mbps in 1-bit quantization. The lower spectral window was centred at a fixed frequency corresponding to the \( v = 1, J = 0-0 \) SiO transition, at an assumed rest frequency of 43.12207 GHz and a systemic velocity \( V_{LSR} = +24 \) km s\(^{-1}\). R Cas was observed for three 45-minute periods evenly spread over the 8 h duration of the run. Adjacent to each R Cas observation, 5 min was spent observing the continuum calibrator 0359+509 at the same frequency as R Cas. The data were correlated at the VLBA correlator in Socorro, New Mexico. The correlator accumulation interval was set to 2.88 s. All polarization correlation products (RR, RL, LR, LL) were formed. This configuration produced auto- and cross-power spectra in each 4 MHz baseband with a nominal velocity spacing of \( \sim 0.2 \) km s\(^{-1}\).

\(^1\)The Very Long Baseline Array (VLBA) is operated by the National Radio Astronomy Observatory, a facility of the National Science Foundation operated under a cooperative agreement by Associated Universities, Inc.

We reduced the data using the standard approach to VLBI spectroscopy within the NRAO AIPS package (http://www.aips.nrao.edu/cook.html). We processed the visibility data using the semi-automated spectral-line polarization calibration pipeline which was originally written by Diamond in 1998, based on the formalism in Kemball, Diamond & Cotton (1995) and developed further by Kemball & Diamond (1997) (for more details, see Assaf et al. 2011).

After the pipeline processing, we noticed that there were systematic offsets in flux density of a few to 15 per cent between LL and RR spectra for each epoch. We assumed that the net circular polarization of SiO across the band is zero and the total intensity calibration is correct. This enabled us to calculate corrections to align the flux scales for LL and RR and obtain realistic values of Stokes \( V \), albeit with large uncertainties.

The absolute electric vector polarization angle (EVPA) for any linearly polarized emission is unknown because there is no instrumental measurement of absolute R—L phase difference in the reference antenna of the VLBA and in the Very Large Array (VLA), but the VLA can provide absolute astronomical calibration of EVPA relative to a small subset of primary astronomical calibrators, given the lower spatial resolution of the array. We used the VLA to transfer absolute polarization calibration to VLBA data via the compact secondary polarization calibrator (0359+509) which was observed by both arrays on 1999 December 19. The same reference antenna KP was used in the reduction of all epochs of VLBA data, and the polarization angle stability is likely to be better than 35° (C. Walker, private communication). Hence, we were able to estimate the polarization angle of 0359+509 as 30° ± 10°.

We fitted 2D Gaussian components to each patch of total intensity (Stokes \( I \)) maser emission brighter than 5\( \sigma_{rms} \), to measure the position and flux density, as described in Assaf et al. (2011). We measured the \( Q, U, V \) flux densities at the position of the peak of the \( I \) component, as described in Kemball et al. (2009). We use the peak (rather than integrated) intensity measurements of Stokes \( I \) for comparison with the other Stokes parameter measurements. We estimated the uncertainties based on (beam size)/(signal-to-noise ratio), as appropriate for a sparse array (Condon et al. 1998; Richards, Yates & Cohen 1999).

We formed the linearly polarized intensity

\[
P = (Q^2 + U^2)^{1/2},
\]

the fractional circular polarization

\[
m_c = \frac{V}{T},
\]

the fractional linear polarization

\[
m_l = \frac{P}{T}.
\]

and

\[
EVPA = 0.5 \arctan \frac{U}{Q}
\]

(e.g. as defined by Heiles 2002; Elitzur 1992).

We measured the angular full width at half-maximum (FWHM) (\( s \)) of the total intensity components by deconvolving the restoring beam from the measured size. This was used to estimate the brightness temperature \( T_B \) from the integrated intensity. A series of components form features and we estimated the average positions, angular size \( L \), angular FWHM \( d_{\ell} \), peak flux densities and other properties of each feature.
The new shell radius [New (R)], the error in R, the shell thickness (dr) and the absolute difference [Abs(diff)] between the old and the new values, all in mas.

| Epoch | New (R) | Error | dr | Abs(diff) |
|-------|---------|-------|----|-----------|
| BD62A | 25.93   | 0.67  | 8.72 | 1.16      |
| BD62B | 25.49   | 0.05  | 9.05 | 1.59      |
| BD62C | 24.72   | 1.77  | 8.21 | 1.33      |
| BD62D | 24.24   | 1.22  | 8.60 | 1.81      |
| BD62E | 24.86   | 1.59  | 7.87 | 2.24      |
| BD62F | 25.45   | 1.98  | 5.80 | 2.98      |
| BD62G | 25.96   | 2.11  | 6.20 | 1.45      |
| BD62H | 30.34   | 0.50  | 7.92 | 2.25      |
| BD62I | 27.50   | 0.53  | 5.38 | 0.99      |
| BD69A | 26.77   | 1.64  | 7.52 | 0.52      |
| BD69B | 27.97   | 0.37  | 6.19 | 0.26      |
| BD69C | 27.55   | 1.50  | 5.68 | 0.45      |
| BD69D | 28.09   | 0.16  | 5.51 | 0.11      |
| BD69E | 28.02   | 0.50  | 6.07 | 0.52      |
| BD69F | 28.32   | 1.71  | 6.11 | 0.17      |
| BD69G | 24.24   | 2.50  | 8.23 | 4.28      |
| BD69H | 29.99   | 0.26  | 9.62 | 0.83      |
| BD69I | 24.90   | 1.77  | 7.68 | 2.25      |
| BD69J | 24.60   | 0.63  | 14.05| 2.23      |
| BD69K | 21.41   | 3.11  | 13.51| 3.70      |
| BD69L | 18.17   | 1.22  | 7.82 | 3.79      |
| BD69M | 17.27   | 3.83  | 6.07 | 0.52      |
| BD69N | 21.97   | 1.68  | 11.21| 4.85      |

The largest angular separation between components making up a feature, L, measures the actual angular size of the emission detected. The separation of the components with flux density closest to half the peak, d_p, represents the beamed size of the feature, as explained in Richards, Elitzur & Yates (2011), with reference to Elitzur (1992). The beaming angle dΩ = (d_p L)^2. We note that d_p and L may be underestimated, as figs B1–B3 in Assaf et al. (2011) show that a significant amount of the total intensity flux is resolved out. There is no clear, systematic difference between the effects on brighter or fainter emission, so we suggest that the main effect is to increase the uncertainty in dΩ, but it could be somewhat overestimated if the fainter emission is more diffuse, leading to L being more underestimated.

An improved method of finding the centre of emission has led to some minor changes with respect to Assaf et al. (2011), which do not affect those results significantly. Table 1 shows the updated values of the average shell radius and the shell thickness, and the difference from the old values. The differences are close to the uncertainties (given in Assaf et al. 2011), which are higher for the last few epochs due to the fainter emission and large gaps in the maser shell.

3 RESULTS

We made Stokes I, Q, U and V image cubes of R Cas, at each epoch, at spatial and spectral resolutions of approximately (40 × 20) μas^2, 0.2 km s\(^{-1}\).

The polarization detection threshold for individual components is 5σ_{rms} and m_r > 5 per cent or m_l > 15 per cent; lower thresholds are possible when averaging over larger spectral or spatial regions (including the correction for Ricean bias). Each feature is made up of many components, and every feature contains more than 0.5 Jy beam\(^{-1}\) summed total intensity. Quiet channels have σ_{rms} = 0.03–0.04 Jy beam\(^{-1}\), so any linear polarization with m_r > 20 per cent will also have a signal-to-noise ratio ≥ 5.

3.1 Circular polarization

The mean degree of circular polarization for each epoch was estimated using equation (2).

We found the fractional circular polarization is m_c ∼ 0.4–6 per cent. We attempted to fit the expression given by Elitzur (1996) to the first derivative of Stokes V. This failed due to the weakness of Stokes V intensity and the large channel separation which is greater than the Zeeman splitting.

3.2 Linear polarization

Figs 1–3 show the polarization morphology of SiO masers in R Cas for 23 epochs. The linear polarization vectors are superimposed on the map of the total intensity. The orientations of the vectors define the electric field plane, and the length of the vectors is proportional to the linearly polarized intensity. The mean degree of circular polarization for each epoch was estimated using equation (2). We found that the fractional circular polarization is m_c ∼ 0.4–6 per cent. We attempted to fit the expression given by Elitzur (1996) to the first derivative of Stokes V. This failed due to the weakness of Stokes V intensity and the large channel separation which is greater than the Zeeman splitting.

We investigated the relationship between the EVPA and the radial direction with respect to the star, defined by the position angle in the plane of the sky. The polarization structures during the first stellar cycle can be summarized as a bimodal distribution. Most of the linear polarization vectors are either radial (parallel to the position angle of the location of the emission in the projected shell) or tangential (perpendicular). However, the polarization in the inner part of the shell is somewhat less ordered. The later parts of the second stellar cycle do not show any clear pattern, but the emission generally was noisier with fewer significantly polarized features.
Figure 1. Polarization morphology of R Cas SiO masers. Each pane is labelled with the stellar phase. Each symbol represents an individual, fitted maser component. The symbol area in mas$^2$ represents one-eighth of the total intensity in Jy. The vectors show the orientation of the EVPA and 5 mas in length represents $P = 1$ Jy beam$^{-1}$. The black circles show the weighted shell radii given in Table 1. Epochs 1–7.
4 DISCUSSION

We noted in Section 1 that there are two competing models put forward to explain SiO polarization. The adopted model for the transfer of the polarized maser emission in a non-paramagnetic molecular transition plays an important role in the interpretation of the polarization measurements (Elitzur 1996).

First, we made an independent estimate of the magnetic field strength to test the feasibility of the magnetic model. The bulk kinetic energy density \( E_{\text{bulk}} = \frac{1}{2} \rho v^2 \) is \( \lesssim 2.1 \times 10^{-3} \text{ J m}^{-3} \), where \( \rho \) is the volume density of SiO, taken as \( 1.42 \times 10^{-13} \text{ g cm}^{-3} \) (Gray et al. 2009), and \( v \) is the expansion velocity which is up to 5.5 km s\(^{-1}\) (Assaf et al. 2011). The thermal energy density \( E_{\text{thermal}} = \frac{1}{2} n_{\text{H}_2} k T \) is \( 0.832 \times 10^{-3} \text{ J m}^{-3} \), where \( n_{\text{H}_2} \) is the molecular hydrogen density, \( k \) is Boltzmann’s constant and \( T \) is the effective temperature, 1330 K (Gray et al. 2009). A first estimate of \( B \) can be found by comparing the energy densities. The magnetic energy density \( E_B \) is given by

\[
E_B = \frac{B^2}{2 \mu_0} \sim 4 \times 10^{-3} \left( \frac{B}{G} \right) \text{ J m}^{-3} .
\]
By equating these energy densities, we found the first estimate of $B \sim 0.725$ G. Herpin et al. (2006) measured a magnetic field strength in the range 0.9–2.8 G from single-dish observations of R Cas at 86 GHz, slightly higher than our estimate.

### 4.1 Application of the Zeeman interpretation

We can decide which interpretation to apply by comparing three parameters: the stimulated rate emission $R$, the Zeeman splitting rate $g\Omega$ and the decay rate $\Gamma$.

We calculated the maser beaming angle $d\Omega$ and the maser brightness temperature $T_B$ from our data. We found the error-weighted beaming angle (Table 2) averaged over 23 epochs is $d\Omega \sim 0.047\,\text{sr}$ and the mean brightness temperature $T_B \sim 2.76 \times 10^8\,\text{K}$. The stimulated emission rate $R$ for saturated masers in this transition is given by Kemball et al. (2009):

$$R = 23\left(\frac{T_B}{2 \times 10^{10}\,\text{K}}\right)\left(\frac{d\Omega}{10^{-2}\,\text{sr}}\right)\,\text{s}^{-1}$$

(6)

giving $R \sim 15\,\text{s}^{-1}$.

Figure 3. Same as Fig. 1 but for epochs 15–23.
similar conclusion has also been reached by Asensio Ramos, Landi Degl’Innocenti & Trujillo Bueno (2005).

The ratio of \( \frac{n_e}{T_b} \) indicates the saturation level of the maser. Equation (8.6.2) of Elitzur (1992) shows that, for our SiO transition, the masers are saturated when the brightness temperature \( T_b \sim 10^9 \) K, confirming that most of the R Cas SiO masers are saturated. We therefore interpret our results using the magnetic field model for the origin of linear polarization.

Faraday rotation is proportional to the square of the wavelength (\( \lambda^2 \)), and so it is smaller at higher frequencies. However, Faraday rotation is also proportional to the electron density, which is higher at the region close to the star, than at greater distances where the longer wavelength masers originate. Previously, it has been presumed that Faraday rotation is negligible for wavelength 7 mm as in our case.

We used the relation below from Garica-Barreto et al. (1988):

\[
\psi_F = 0.5 \times \left( \frac{n_e}{\text{cm}^{-3}} \right) \left( \frac{B}{\text{mG}} \right) \left( \frac{L}{10^{15} \text{cm}} \right) \left( \frac{\lambda}{\text{18 cm}} \right)^2,
\]

where \( n_e \) is the electron density and \( L \) is the path length in the maser region (maser shell thickness). From Reid & Menten (1997) we found that the electron density in the SiO maser region \( \sim 1500 \) cm\(^{-3}\); hence, the Faraday rotation is about 16° for \( L \sim 2 \times 10^{15} \text{cm} \) for a magnetic field of \( B \sim 750 \) mG. This estimate is in fact slightly smaller than the estimates by Herpin et al. (2006), so Faraday rotation could be somewhat higher. It is still within the uncertainties of our EVPA measurements.

4.2 Linear polarization approaching or exceeding the total intensity

We found that in some isolated features, the percentage of linear polarization is greater than 100 per cent at three epochs \( \psi = 0.744, 1.158 \) and 1.305, when it reaches 445 per cent. This is likely due to resolving out of the total intensity Stokes \( I \) even on the shortest
Figure 6. Stokes I, Q and U flux density as a function of baseline length for channels averaged from 26.3 to 27.4 km s$^{-1}$, $\phi = 1.579$.

VLBA baselines. Emission on scales greater than $\sim 5$ mas cannot be imaged by the VLBA. Comparison of the auto- and cross-correlation spectra (Assaf et al. 2011, figs B1–B3) shows that 10–90 per cent of the total intensity emission of R Cas is resolved out. Fig. 6 compares the flux density as a function of baseline length for Stokes I, Q and U for the spectral range including the feature with $m_\ell > 100$ per cent at $\phi = 1.305$. Other features in the same channels have lower $m_\ell$, but the average visibility amplitudes still show that the total intensity emission rises on large scales (shortest baselines) to a much greater extent than polarized emission. The continuum calibration source 0359+509 does not show any such effect.

More intense polarization than the total intensity has been observed before, in low-frequency continuum observations using WSRT due to Faraday rotation creating smaller-scale structure in the polarized emission (Haverkorn, Katgert & de Bruyn 2003). In R Cas, this could be due either to fluctuations in the magnetic field or to inhomogeneities in the ionization fraction, if these are on smaller scales than turbulence in the neutral medium.

We examined the relationship between auto- and cross-correlation spectra for other epochs of high fractional polarization, for example, $m_\ell \sim 70$–80 per cent around 27 km s$^{-1}$ at $\phi = 1.027–1.432$. This suggests that the ‘true’ fractional polarization could be no greater than 30–40 per cent (including uncertainties). A more exact comparison is not possible due to lack of spatial resolution in auto-correlation data.

4.3 90° change in polarization angle

The linear polarization image of the north-east component in Fig. 7 shows an abrupt transition in EVPA of approximately $\frac{\pi}{2}$ near $V_{\text{LSR}} \sim 26.78$ km s$^{-1}$. At this point, the linearly polarized intensity is near its minimum. The orientation of the polarization vectors is tangential to the projected ring in the part of the feature closer to the centre of expansion, whilst they are radial farther out. Fig. 8 shows the fractional linear polarization for this feature as a function of distance from the centre of expansion. Note the expanded velocity scale. We ordered the components by increasing radial distance from the centre of expansion $r$ and found the error-weighted mean linearly polarized and the total flux densities in 10 bins in order to calculate the fractional polarization, plotted as a function of $r$. Each point is labelled with the mean EVPA and the symbol size is proportional to the mean polarized intensity in each bin, from a minimum of 3 Jy around $r = 25.5$ mas to 11 Jy at $r = 23.8$ and 26.2 mas. The symbols are labelled with the error-weighted mean polarization angle. Note the expanded velocity scale.

Figure 7. The north-east maser feature seen at $\phi = 0.818$ in Fig. 2 and enlarged in Fig. 4. The mapped emission was summed over $V_{\text{LSR}} = 25.67$–27.43 km s$^{-1}$. The contours are at ($-1$, $1$, $2$, $4$, $8$) $\times$ 1.7 Jy beam$^{-1}$. The vectors show the EVPA, plotted every two pixels (0.1 mas), length = 1 mas = 0.8 Jy beam$^{-1}$ linearly polarized intensity.

Figure 8. The feature shown in Fig. 7. The error-weighted mean percentage polarization is plotted for 10 increments in radial distance from the centre of expansion $r$. The symbol size is proportional to the mean polarized intensity in each bin, from a minimum of 3 Jy around $r = 25.5$ mas to 11 Jy at $r = 23.8$ and 26.2 mas. The symbols are labelled with the error-weighted mean polarization angle. Note the expanded velocity scale.
linearly polarized intensity is lower in this range, compared with most emission at more extreme EVPA values.

This is consistent with the prediction of Goldreich et al. (1973) when the angle \( \theta_i \) between the magnetic field and the line of sight changes from \(<55^\circ\) to \(>55^\circ\), if the magnetic field is radial with respect to the star. A similar phenomenon in TX Cam was described by Kemball et al. (2011) and in W43A by Vlemmings & Diamond (2006).

4.4 Time variability of polarization

Fig. 9 shows the relationship between linear polarization and total intensity in the whole maser shell and in the inner ring (radius 25 mas). There is an anticorrelation between the total intensity peak and the fractional polarization during the first optical cycle. The percentage flux density is similar or slightly higher in the whole shell compared with the inner shell up to \( \phi = 0.595 \). The highest fractional polarization, at \( \phi = 0.744 \), comes mainly from the inner shell. The fractional polarization is also higher in the inner shell from \( \phi = 0.873 \) to 1.234. At later epochs, the whole shell has higher polarization but at epoch 1.432 and later, most of the emission comes from the inner shell.

We investigated the variation of mean linear polarization as a function of stellar pulsation phase in Fig. 10. We calculated these values by averaging over all components in each epoch. The error bars represent the standard deviation of average fractional linear polarization. Over the whole shell, the mean linear polarization rises from \( \phi = 0.158 \) until \( \phi = 0.744 \) when it reaches the maximum value. The mean fractional polarization then drops abruptly from the maximum to reach its minimum at \( \phi = 0.873 \). After that, it tends to increase again till \( \phi = 1.783 \).

The total intensity, polarized intensity and fractional polarization for the inner ring (25 mas radius), and for the whole shell, were plotted with Hanning smoothing over five channels, giving \( \sim 1 \text{ km s}^{-1} \) effective resolution.

(i) \( \text{Fig. 11 (epochs } 0.158-0.452\text{)} \). The ratio of the fractional polarization in the inner shell to that in the whole shell barely exceeds 1. The overall flux density is high at these epochs and the fractional polarization is mostly \(<50\%\) per cent, higher at redshifted velocities where the emission is almost entirely from the inner shell. However, at blueshifted velocities, where there is more extended emission, this is usually more polarized. At epoch 0.310 and later, the fractional polarization in the inner shell is slightly higher than in the whole shell, at a few central velocities.

(ii) \( \text{Fig. 12 (epochs } 1.374-1.783\text{)} \). The overall flux density decreases at all epochs except 0.818. The fractional polarization is quite variable, and at epoch 0.744 the fractional polarization from the inner shell exceeds 100 per cent. The ratio of the fractional polarization in the inner shell to that in the whole shell also exceeds 1, and although the uncertainties are high the excess is significant, mostly at intermediate velocities.

(iii) \( \text{Fig. 13 (epochs } 0.873-1.305\text{)} \). The trend continues, with higher fractional polarization from the inner shell at a range of velocities. The total intensity from the whole shell increases, but with the fractional polarization mostly \(<30\%\) per cent, whereas from the inner shell it exceeds 50 per cent at most epochs at some velocities.

(iv) \( \text{Fig. 14 (epochs } 1.374-1.783\text{)} \). The total intensity decreases, especially at redshifted velocities, and the emission is mostly from the inner shell, so the ratio of the polarized intensities is close to 1, although it exceeds 1 at epochs 1.374 and 1.783, when there is more emission from the whole shell but it is on average less polarized than from the inner shell.

Fig. 9 shows that the fractional polarization tends to be greater when the total intensity is weaker. Figs 11–14 show that this is often due to one or two narrow spectral regions, usually near the centre of the spectra, which have high fractional polarization. These are associated with the inner shell (see \( \phi = 0.675, 0.744 \) and 0.956 in the first cycle). A similar behaviour is seen in the second cycle at several epochs.

Thus, there is a distinct tendency for higher fractional polarized intensity to be associated with weaker total intensity and with the inner shell.

The emission at more extreme velocities tends to come from the inner shell, as projected on the sky. This is not surprising since, for a spherical shell with a radial velocity field, we would expect to see the most redshifted and blueshifted emission close to the line of sight to the star. Note that the redshifted emission within the innermost 25 mas (\( R_* \), Weigelt et al 2000) must be on the near side of the star, seen in infall.

There are no obviously cyclic repeating trends in the second cycle compared with the first. The first half of the first cycle shows
lower fractional polarization in the inner shell for most spectral features, whilst the inner shell dominates the polarization in the second half of the cycle. In the second cycle, most of the polarization comes from the inner shell, but there is no clear dependence on phase.

What does this imply? If a magnetic field is responsible for polarization, this would be expected to be stronger nearer the star, and might also be enhanced by shock compression. Since the shock crossing time would not necessarily be an exact multiple of the stellar period, this could produce the out-of-phase enhancement. If, on the other hand, the mechanism were radiative, which could also be strongest in the inner shell, it would be expected to follow the stellar phase, so this is less likely.
4.5 Alfvén speed

If we use the estimate of $B$ based on equating the thermal energy density with the energy density in the magnetic field, we expect the Alfvén speed $v_A$ and sound speed $v_s$ to be similar.

We calculated the sound speed by using the relation

$$v_s = \sqrt{\frac{\gamma kT}{m}},$$

where $\gamma$ is the adiabatic constant, $k$ is the Boltzmann constant and $T$ is the effective temperature in the masing region. We found the sound speed $v_s = 2.77$ km s$^{-1}$. The total random energy in the gas is likely to be slightly higher due to other turbulence.

The Alfvén speed $v_A$ is given by

$$v_A = \frac{B}{\sqrt{\mu \rho}},$$

where $\mu$ is the magnetic permeability.
where $\rho$ is the total mass density for a well-coupled fluid and it is given by

$$\rho = \sum n_i m_i,$$

(10)

where $\mu_0$ is the permeability of vacuum $(4\pi \times 10^{-7})$ H m$^{-1}$. From Reid & Menten (1997) we found the total density is $0.4 \times 10^{15}$ cm$^{-3}$ at the mid-point in the maser shell $(7.3 \times 10^{13}$ cm). We found $v_A = 3.5$ km s$^{-1}$ using $B = 750$ mG, or higher if the values of $B$ from Herpin et al. (2006) are considered. Thus, in the SiO maser clumps, $v_A$ and $v_r$ are similar. In a more diffuse, highly ionized region, with lower density and poorer coupling, $v_A$ could be considerably higher.

5 CONCLUSIONS

The SiO maser emission is significantly linearly polarized. Most of the linear polarization vectors are either perpendicular or tangential to the projected shell, but other angles can be seen. During the first cycle, up to phase 1.158, two-thirds of epochs have the majority of the emission with polarization angles parallel to the radial direction, i.e. parallel to the direction of outflow of the first cycle. Emission with perpendicular position angles is commonest in the other five epochs. Emission with intermediate polarization position angles dominates the remaining epochs, but the intensity tends to be lower leading to greater uncertainties.

The bimodal distribution of EVPA is consistent with that reported by Cotton et al. (2006) in VLBA observations of SiO masers in Mira variable stars at 7 mm. They reported that for some features the EVPA are tangential, while for the others are radial. The linear polarization EVPA is predominantly tangential to the projected intensity shell in the VLBA observation towards TX Cam (Kemball et al. 2009). This pattern has been confirmed in further observations towards TX Cam and IRC+10011 (Desmurs et al. 2000). However, Cotton et al. (2004) found there was no prevalent pattern of EVPA in any observed stars.

Our data are in the limit $\Omega \gg R \gg \Gamma$, which indicates that the Goldreich et al. (1973, hereafter GKK) solutions for the linear polarization are applicable, i.e. the standard interpretation for the radially and tangentially EVPA orientations relative to the projected magnetic field is applicable. This result is consistent with that reported by Kemball et al. (2009) towards TX Cam. The high brightness temperatures observed indicate that the R Cas masers are predominantly saturated so we consider the GKK model for saturated emission. We used the GKK model to estimate the depth, $l$, above the mid-plane of the shell, of the feature showing an abrupt 90° polarization angle change (see Fig. 15). At the point where the (dashed) line of sight crosses the vertical magnetic field line, in the plane of the sky, the angle between the magnetic field direction and the line of sight, $\theta_l$, is $90^\circ$ and $0^\circ \leq EVPA \leq 35^\circ$ with respect to an axis perpendicular to the local magnetic field direction. When $\theta_l$ reaches $55^\circ$ (the Van Vleck angle), the EVPA becomes parallel to the direction of the magnetic field as projected against the plane of the sky. At this point, equation (11) gives $l$ equivalent to $\sim 15$ mas or 2.6 au.

$$\cos 55^\circ = \frac{l}{R_{shell}}$$

(11)

If the magnetic field is radial, then the preponderance of EVPA parallel to the radial direction (during the first cycle, with the best quality data, Section 3.2) suggests that a substantial fraction of the masers which we detect come from a region at least $\pm 2.6$ au deep with respect to the plane of the sky containing the star. This suggests a shell thickness of $\geq 4.6$ mas or 0.8 au, which is indeed similar to or less than the values of $dr$ given in Table 1. The shell thickness varies from about $1/5$ to $2/3$ of the shell radius, error-weighted mean of $1/3$. The next most common orientation of EVPA is tangential, suggesting that it is close to perpendicular to the magnetic field, when emanating from masers closer to the plane of the sky. The magnetic field lines must close at larger distances or orientations.

The percentage of the linear polarization is $m_t \sim 11–58$ per cent averaged over features for each epoch. The highest fractional polarization, from the inner shell, is seen in some features in epochs where the total intensity is lower. There is a tendency at all epochs for the fractional polarization in the inner shell to be greater at the line peaks. Figs 11–14 do not show any other relationships between fractional polarization and velocity. Neither is there any discernible correlation between $\theta_l$ and velocity.

The changes in the fractional linear polarization strength are probably due to fluctuations in the maser amplification process rather than the magnetic field strength. The changes in polarization vector orientation are likely to be due to small variations in the magnetic field direction which are possibly caused by local turbulence (see Section 4.1). Our estimates of the magnetic field strength in the maser region make pumping models based on radiation-driven anisotropies in the magnetic sublevel populations highly unlikely.

A few isolated features with fractional polarization apparently exceeding 100 per cent can be explained if there is structure in the ionized fraction or the magnetic field on scales $< 5$ mas, but the total intensity emission is smooth on larger scales and a greater proportion is resolved out by the VLBA. Since the Alfvén speed is unlikely to exceed the sound speed significantly, and disturbances at either velocity would take $\sim 1$ yr to cross a 5-mas clump, intrinsic variability is unlikely. However, Faraday rotation is not negligible at 7 mm wavelength. We found that it is $\sim 15^\circ$ for masers propagating through the depth of the SiO shell, using our estimate of the magnetic field strength $B = 0.725$ G based on the energy balance, or slightly higher if considering the values of $B$ measured by Herpin et al. (2006). If there are small-scale inhomogeneities in the ionized fraction of the stellar wind, this will affect the propagation of the polarized emission. The small-scale structure seems to be imposed on the maser polarization between the emitting material and the observer, but the effect must occur close to the star where the magnetic field is strong enough. This mechanism is also likely to affect other maser features with high observed fractional linear polarization, such that although the measured $m_t = 11–58$ per cent, the higher values might be approximately halved if all the emission was measured by the interferometer. We are therefore probably
within the limit of the GKK model, wherein the mean fractional linear polarization can be up to 33 per cent.

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Polarization morphology of SiO masers

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