Analysis of flexible fabric structures for large-scale subsea compressed air energy storage

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Abstract. The idea of storing compressed air in submerged flexible fabric structures anchored to the seabed is being investigated for its potential to be a clean, economically-attractive means of energy storage which could integrate well with offshore renewable energy conversion. In this paper a simple axisymmetric model of an inextensional pressurised bag is presented, along with its implementation in a constrained multidimensional optimization used to minimise the cost of the bag materials per unit of stored energy. Base pressure difference and circumferential stress are included in the optimization, and the effect of hanging ballast masses from the inside of the bag is also considered. Results are given for a zero pressure natural shape bag, a zero pressure bag with circumferential stress and hanging masses, and a nonzero pressure bag with circumferential stress and hanging masses.

1. Introduction
The Integrated Compressed Air Renewable Energy System (ICARES) is a system of harvesting and storing renewable energy in development at the University of Nottingham. ICARES is based on the idea of directly compressing air to around 50bar using excess supply of offshore renewable energy and storing that compressed air in inexpensive flexible fabric structures (“energy bags” or simply “bags”) anchored to the seabed at depths of around 500m until demand outweighs supply and it is necessary to retrieve the energy stored in the compressed air. Due to the large hydrostatic pressure of the water column above, high energy densities are possible at such depths (approximately 20MJ/m\textsuperscript{3} at 500m) without the need for a strong, expensive container; the water itself acts as the container. Great value is placed upon the importance of energy storage for an efficient and economical power grid and, as penetration of offshore renewable energy grows, a viable means of storing the energy will become increasingly necessary. ICARES is ideally placed to fill this need.

It is anticipated that energy bags will have lobes and/or annuli so that the patches of membrane between the cusps are small enough to react to the pressure difference without rupturing, and to increase the curvature of the membrane and transmit the forces through reinforcements along the cusps. Masses may be suspended between the surface of the bag and the seabed on cables that hang from the inside of the bag so that as it is inflated, progressively larger masses are lifted off the seabed, providing reaction for some of the differential pressure load. In this paper we consider the shape of axisymmetric bags without any annuli, and the lobed bags with annuli will be modelled in future work using a finite element model.
While storage of thermal energy and compression/expansion machinery will be dealt with elsewhere, in the work presented here we develop and numerically integrate a system of ordinary differential equations to find the shape that a fully-inflated bag will take, and optimise the shape and other parameters to minimise the cost of storing energy. The energy bags are modelled as inextensional axisymmetric membranes in the same way that large scientific balloons have been modelled over the last half century (see [1] and [2]), and a tiered line search optimization routine is used to optimise for pressure difference at the base of the bag, the bag’s centre height, the meridional and circumferential stresses in the membrane, and hanging mass.

While the idea of using wind turbines to compress air is not new (see [3] and www.generalcompression.com), no previous research on subsea pressurised membranes has been found and so the field of scientific ballooning is the main area of relevance. No previous work has been found on shape optimisation using axisymmetric balloon models. Much work on compressed air energy storage (CAES) was carried out in the 1970s and 1980s but the only storage vessels considered were caves, disused mines, aquifers, and rigid pressure vessels. CAES is a proven technology and two dedicated plants already exist (in Huntorf, Germany and McIntosh, USA), salt caverns being used as the stores in both plants. Many more CAES plants are currently in the design phase. [3]

2. Derivation of the equilibrium equations

Figure 2 shows a section view of an infinitesimal patch of membrane.
The patch has meridional length $2\Delta s$, covers a circumferential angle of $2\Delta \theta$, is at radius $r$ and height $z$, and covers a distance $2\Delta r$ in the radial direction and $2\Delta z$ in the vertical direction. The meridional stress $T$ is in N.rad$^{-1}$, the circumferential stress $\sigma_c$ is in N.m$^{-1}$ (force per unit meridional length of membrane), and the hanging load $\sigma_w$ is in N.m$^{-2}$ (force per unit of projected area, not per unit of membrane area).

The change in radius $r$ with distance $s$ along the membrane is easily derived.

$$\frac{dr}{ds} = \cos \alpha$$

(1)

The change in height is found in the same way.

$$\frac{dh}{ds} = -\sin \alpha$$

(2)

Subjected to a pressure difference $p$ between inside and outside, the applied force on the patch is

$$F = pr2\Delta \theta 2\Delta s.$$  

(3)

For force equilibrium this must be equal to the sum of reaction forces in the opposite direction:

$$(T + \Delta T + T - \Delta T)2\Delta \theta \sin \Delta \alpha + \sigma_c 2\Delta s 2\sin \Delta \theta \sin \alpha + \sigma_w r 2\Delta \theta 2\Delta s \cos \alpha \cos \alpha.$$  

(4)

So the force balance in the direction of the normal, after small angle approximations, is

$$pr2\Delta \theta 2\Delta s = 2T2\Delta \theta \Delta \alpha + \sigma_c 2\Delta s 2\Delta \theta \sin \alpha + \sigma_w r 2\Delta \theta 2\Delta s \cos^2 \alpha.$$  

(5)

Rearranging for $\Delta \alpha$ and dividing through by $4T\Delta \theta$,

$$\Delta \alpha = \frac{pr - \sigma_c \sin \alpha - \sigma_w r \cos^2 \alpha}{T} DS.$$  

(6)

Taking the limit as $\Delta s$ goes to zero gives

$$\frac{d\alpha}{ds} = \frac{pr - \sigma_c \sin \alpha - \sigma_w r \cos^2 \alpha}{T}.$$  

(7)

A force balance in the direction of the tangent to the patch moving from top to bottom, yields

$$(T + \Delta T)2\Delta \theta \cos \Delta \alpha + \sigma_w r 2\Delta \theta 2\Delta s \cos \alpha \sin \alpha$$

$$= (T - \Delta T)2\Delta \theta \cos \Delta \alpha + \sigma_c 2\Delta s 2\sin \Delta \theta \cos \alpha.$$  

(8)

Taking small angle approximations, rearranging for $\Delta T$, and dividing through by $4\Delta \theta$,
\[ \Delta T = \Delta s (\sigma_e - \sigma_w r \sin \alpha) \cos \alpha. \] (9)

Taking the limit as \( \Delta s \) goes to zero,

\[ \frac{dT}{ds} = (\sigma_e - \sigma_w r \sin \alpha) \cos \alpha. \] (10)

The shape of the bag is found by solving equations (1), (2), (7), and (10). In carrying out optimization studies it is also necessary to calculate some other quantities that are used in the objective function. The stored energy in the bag depends upon the volume of air contained \( (V) \). For simplicity we assume that the cost of carrying a force over a certain distance is proportional to the force-distance product, which we call strength \( (\gamma) \). An explanation of this simplification follows.

Cost of materials equals the product of volume of material required \( (V_{reqd}) \) and price of material (per unit volume).

\[ \text{Cost} = \text{price} \cdot V_{reqd} \] (11)

\( V_{reqd} \) is the distance over which the force is transmitted \( (x) \) multiplied by the required cross-sectional area \( (A_{reqd}) \), and \( A_{reqd} \) is the force transmitted \( (F) \) divided by the material’s yield stress \( (\sigma_y) \) upon the factor of safety \( (\text{FoS}) \).

\[ \text{Cost} = \text{price} \cdot x A_{reqd} = \text{price} \frac{xF \cdot \text{FoS}}{\sigma_y} \] (12)

Assuming that price per unit volume scales linearly with yield stress, then

\[ \text{Cost} \propto xF. \] (13)

The cost of the bag materials depends upon the surface area \( (A) \), and the required strengths (meridional, circumferential, and vertical). These are found by simultaneously solving five more differential equations and taking the final values.

\[ \frac{dV}{ds} = 2 \pi rh \cos \alpha \] (14)

\[ \frac{dA}{ds} = 2 \pi r \] (15)

\[ \frac{d\gamma_m}{ds} = 2 \pi T \] (16)

\[ \frac{d\gamma_c}{ds} = 2 \pi \sigma_c \] (17)

\[ \frac{d\gamma_w}{ds} = \pi rh \sigma_w \cos \alpha \] (18)
In equation (18), the strength in the vertical direction ($\gamma_v$) has been divided by 2: this is done because, in suspending a series of masses along a cable hanging between the membrane and the base of the bag, the average tension in the cable is only half the tension at the very top of the cable (assuming an equal distribution of mass along the cable).

Equations (1), (2), (7), (10), and (14)-(18) are integrated numerically, in this case as an initial value problem. We always set $V_0, A_0, \gamma_m, \gamma_c, \gamma_w$ to zero (subscript 0 indicating an initial value at the top of the bag). An end fitting at the top of the bag requires nonzero $r_0$ (to represent the radius of the end fitting) and nonzero $\alpha_0$ (so the weight of the end fitting is reacted by the tension in the membrane), but no end fittings are used here and so $r_0$ and $\alpha_0$ are also set to zero. The average density of seawater, $\rho_{water} = 1025 \text{ kg/m}^3$. The density of the compressed air at absolute pressure $P$ is calculated using the equation of state for an ideal gas.

$$\rho = \frac{MP}{RT}$$  \hspace{1cm} (19)

The molar mass of air, $M_{air} = 0.02897 \text{ kg.mol}^{-1}$, and the universal gas constant, $R = 8.314472 \text{ J.K}^{-1}.\text{mol}^{-1}$. It is assumed that the air is stored at 5°C (so $T = 278.15 \text{ K}$).

3. Optimization

3.1. Objective function

We seek to minimise the objective function, $f(x) =$ cost of a bag/stored energy, subject to inequality and equality constraints of the form $g(x) \leq 0$ and $h(x) = 0$. While the total cost of a bag will depend upon the cost of manufacture and deployment, these costs are not taken into account in this study; only the cost of materials is included. This cost breaks down into the sum of the cost of reinforcement, the cost of surface, and the cost of ballast. The cost of reinforcement is the sum of each of the meridional, circumferential, and vertical strengths multiplied by the cost per unit strength of each. The cost of surface is the surface area multiplied by the cost of surface per unit area, and the cost of ballast is the difference between the mass of water and the mass of the air multiplied by the cost of ballast per unit mass.

The energy available in the compressed air store depends upon how the compressed air is expanded. It is conservatively assumed that the air is an ideal gas (the compressibility factor for air at 300K and 60bar is 0.9901 which is very close to 1, the compressibility factor of an ideal gas [4]) and will be expanded isothermally. The work done in the isothermal expansion from stored volume $V_A$ (with absolute pressure $P_A$) to volume $V_B$ (with absolute pressure $P_B$ – atmospheric pressure) is

$$W_{A\rightarrow B} = \int_{V_A}^{V_B} PdV = \int_{V_A}^{V_B} \frac{nRT}{V} dV = nRT \ln \frac{V_B}{V_A}. \hspace{1cm} (20)$$

For an ideal gas, the product $PV$ remains unchanged in an isothermal process, so $\frac{V_B}{V_A} = \frac{P_A}{P_B}$ and

$$W_{A\rightarrow B} = P_A V_A \ln \frac{P_A}{P_B}. \hspace{1cm} (21)$$
3.2. Optimization algorithm

Exterior penalty functions \[5\] are used to transform the constrained problem into a single unconstrained problem. Instead of minimising \( f(x) \) subject to the constraints \( g_i(x) \leq 0 \) for \( i = 1, \ldots, m \) and \( h_i(x) = 0 \) for \( i = 1, \ldots, l \), we minimise the auxiliary function \( f(x) + \mu \alpha(x) \), where \( \mu \) is a large positive penalty parameter and \( \alpha \) is a penalty function that is zero for feasible points and increasingly positive for increasingly infeasible points. A suitable form for \( \alpha \) is

\[
\alpha(x) = \sum_{i=1}^{m} \left[ \max(0, g_i(x)) \right]^2 + \sum_{i=1}^{l} |h_i(x)|^2.
\]  

(22)

By squaring both terms, differentiability is ensured at both \( g(x) = 0 \) and \( h(x) = 0 \). It is possible to get arbitrarily close to a minimum of \( f(x) \) by minimising the auxiliary function for a sufficiently large \( \mu \). The following effects are penalised: an upward turn of the meridian and compressive stresses. An upward turn of the meridian leads to undesired looping and very often to a solution which does not reach the seabed (i.e. does not reach \( h = 0 \)).

For a given base pressure difference, energy density increases slightly more than linearly with depth while the forces on the bag (and so material cost) remain almost unchanged. Therefore the objective function (material costs only) will decrease almost linearly with depth. It would be hard to put costs to the extra problems that installing bags at great depths would bring (e.g. installation and piping) without further research, so the effect of depth is not studied here.

The objective function is minimised over \( h_0, V, \sigma_c, \sigma_w, \) and \( p_{base} \) using a tiered line search. This is a multidimensional optimization procedure in which a local stationary point is found by simply tiering line searches, so minimising a series of minima. As an example, in minimising the objective function \( f(x_1, x_2) \), we would perform a line search in \( x_1 \) for a set value of \( x_2 \) to find the minimising \( x_1 \) (call this minimising value \( \bar{x}_1 \)) for the certain \( x_2 \). \( \bar{x}_1, \bar{x}_2 \) is found by performing a line search adjusting \( x_2 \) (and finding \( \bar{x}_1 \) for each value of \( x_2 \) tried) until the minimum of \( f(\bar{x}_1, x_2) \) is found. \( \bar{x}_1, \bar{x}_2 \) is then a local minimum of \( f(x_1, x_2) \). To minimise for more variables, another tier of the same procedure is added. Though \( V \) cannot be set directly, the meridional stress at the top of the bag, \( T_0 \), which gives a shape that encloses a certain required volume, \( V_{reqd} \), can be found using a root finding algorithm. All of the one-dimensional line searches are carried out using Brent’s method: a combination of the golden section search and parabolic interpolation. \[6\] Other multidimensional search procedures which could be used include Rosenbrock’s method and methods that use derivatives in determining the search direction, such as the method of steepest descent and the method of feasible directions \[7\], as used by Pagitz and Pellegrino in their cutting pattern optimization of lobed superpressure (“pumpkin”) balloons. \[8\]

3.3. Results of the optimization study

In the following analyses, all of the meridional, circumferential, and vertical (hanging cable) stresses are carried through steel at its yield strength (250MPa), with an estimate at the cost of steel of £0.5/kg. The cost of surface is estimated at £2/m\(^2\) and the cost of ballast estimated at £4/tonne. Of course all of the optimum shapes are sensitive to these costs. The optima are all found for a depth of 500m.

3.3.1. Zero pressure natural shape. An optimum zero pressure natural shape (ZPNS) bag, which has zero pressure difference at the base and purely meridional stresses, was found and is shown in figure 3. It stores 289m\(^3\) of air compressed to an absolute pressure of 51.28bar, which is 1.62MWh (5.83GJ)
of energy. The value of the objective function at this optimum is £1104 per MWh. As mentioned before, this figure only accounts for the costs of reinforcement, surface, and ballast materials.

![Graph](image)

**Figure 3.** Optimum zero pressure natural shape bag at 500m depth.

It is interesting to note that the optimum ZPNS shape has quite a wide base radius. This is because the pressure difference at the top of a wide, low bag is not as great as it would be at the top of a taller bag with the same base pressure difference. The sides of the bag meet the seabed at angles close to 90° because the smallest possible meridional tension for a given stored volume is that which gives an entry angle of 90°. At angles away from 90°, larger meridional tension is necessary to balance the buoyancy force \( (bV) \).

### 3.3.2. Zero pressure, nonzero circumferential stress and hanging ballast

The effects of constant nonzero circumferential stress and constant nonzero hanging ballast on a zero pressure bag are now included in the optimization, and the optimum bag is shown in figure 4. This has zero circumferential stress and hanging ballast of 32kN/m², and costs £962 per MWh; a 13% reduction in cost when compared to the ZPNS bag with no hanging masses. It stores 1314m³ of air compressed to an absolute pressure of 51.28bar, which is 7.36MWh (26.50GJ) of energy. The inclusion of positive circumferential stress increases the value of the objective function and so as compressive circumferential stress is not an option, the optimum value is zero.

![Graph](image)

**Figure 4.** Optimum zero pressure bag with nonzero hanging mass at 500m depth.

It is clear that the inclusion of hanging masses has led to an optimum bag shape which has a greater stored volume and lower profile than the ZPNS bag. Hanging mass counteracts the pressure difference force, particularly in horizontal sections (low \( \alpha \)) like at the top of the bag, and so the required net restoring force is reduced. This net restoring force is balanced by curvature of the
membrane, and so the required curvature is reduced, and the bag can be wider and store more energy for a given centre height. Once again the sides of the bag meet the seabed at approximately right-angles. It is anticipated that hanging masses will flatten the P-V curve as the bag is inflated and deflated.

3.3.4. *Nonzero pressure, nonzero circumferential stress and hanging ballast.* Now the effect of nonzero base pressure difference is included. Base pressure difference may be positive (like a superpressure balloon) or negative (subpressure). Subpressure balloons have only been considered when studying partial inflation and ascent shapes [9],[10] and underpressurisation of the base of a balloon at float altitude would be hard to achieve, however underpressurizing the base of an energy bag could be achieved by simply pumping in air at pressures lower than the hydrostatic pressure at the base of the bag. The optimum bag is shown in figure 5. This is a subpressure bag with base pressure difference of -32000Pa, zero circumferential stress, and hanging ballast of 10kN/m$^2$, and costs £907 per MWh. It stores 2249m$^3$ of air compressed to an absolute pressure of 50.96bar, which is 12.52MWh (45.07GJ) of energy.

![Figure 5. Optimum nonzero pressure bag with nonzero hanging mass at 500m depth.](image)

With the inclusion of nonzero base pressure difference, the base of the optimum bag no longer meets the seabed at right angles because it must take on negative curvature at the base to balance the negative pressure gradient whilst maintaining a fairly low centre height (and so maximum pressure difference).

4. Conclusion
The optimum shape of an inextensible, axisymmetric fabric bag used to store compressed air underwater has been found by modelling the bag with membrane equations and using tiered line searches, Brent’s method, and penalty functions to solve the constrained multidimensional optimization problem. The optimum axisymmetric energy bag only has stresses in the meridional direction (so zero circumferential stress), a wide base radius, masses hanging from the inside, and is underpressurised at the base. It costs less than £1000 per MWh though estimated costs have so far only been attributed to materials. Further work with lobed bag models and costs for manufacture and installation will reveal more about the economics of subsea compressed air energy storage.

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