DARK ENERGY PERTURBATIONS AND A POSSIBLE SOLUTION TO THE COINCIDENCE PROBLEM

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We analyze some generic properties of the dark energy (DE) perturbations, in the case of a self-conserved DE fluid. We also apply a simple test (the “F-test”) to compare a model to the data on large scale structure (LSS) under the assumption of negligible DE perturbations. We exemplify our discussions by means of the ΛXCDM model, showing that it provides a viable solution to the cosmological coincidence problem.

Keywords: Dark energy; Cosmological perturbations; Renormalization group.

1. Introduction

In recent times Cosmology has become an accurately testable branch of Physics. Theoretical models can now be confronted with a large quantity of high-precision data coming from different sources, including studies of distant supernovae,1 the anisotropies of the CMB2 or the LSS of the Universe.3 All these observations give strong support to the existence of DE, although the ultimate nature of this component remains a complete mystery.

Remarkably enough, the simplest DE candidate, namely a cosmological constant (CC) Λ, gives rise to a model (ΛCDM) in accurate agreement with all the currently available observational data. Moreover, a general prediction of quantum field theory (QFT) is the existence of a vacuum energy which would precisely take the form of a CC in the Einstein equations. However, the value predicted by the theory happens to be many orders of magnitude larger than the observed DE density. This fact, known as the “cosmological constant problem”,4 makes very unlikely the identification of

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DE with a strictly constant vacuum energy density, since that would require an extremely fine-tuned cancellation of the different contributions, unless there is a dynamical mechanism taking care of such adjustment. In a different vein, the CC problem can also be addressed in quantum cosmology models of inflation, through the idea of multiverses and the application of anthropic considerations.

The CC problem could be alleviated if we allow the DE to be dynamical. The most popular models exploiting this idea are undoubtedly the scalar field models (XCDM). These models, although well-motivated from the Particle Physics point of view, present two major drawbacks. First of all, the field should have an extremely tiny mass, \( m_X \sim H_0 \sim 10^{-33} \text{ eV} \), which is even much smaller than the observed value of the mass scale associated to the DE (\( \sqrt{\rho_D} \sim 10^{-3} \text{ eV} \)). And second, in this kind of models one implicitly assumes that the vacuum energy predicted by QFT cancels out for some reason, so the fine-tuning problems associated to the vacuum energy are not solved but simply obviated and traded for those of the scalar field itself. In short, the situation is as follows: on the one hand, from QFT we expect a vacuum energy contribution to the DE in the form of a CC; on the other, we have the popular and well-motivated scalar field models, which may serve to alleviate the CC problem due to their dynamical nature.

Therefore, it seems quite natural to study a more complete model in which the DE combines both ingredients, which we call the \( \Lambda \)XCDM model. The new model presents additional advantages: e.g. \( \Lambda \) need not be constant, but may evolve with a renormalization group (RG) equation, as any other parameter in QFT. The other DE component, the “cosmon” \( X \), need not be a fundamental field either; it could be e.g. an effective representation of dynamical fields of various sorts or even of higher order curvature terms in the action. In fact we do not have to assume anything about the nature of \( X \): its dynamics is completely determined from that of \( \Lambda \) once we have a good ansatz for the latter, due to the fact that both components may exchange energy. In the original \( \Lambda \)XCDM model, the result of these assumptions is a 3-parameter cosmological framework which incorporates the \( \Lambda \)CDM and XCDM models as special cases.

One of the most appealing features of the \( \Lambda \)XCDM model is that it provides a solution to the “cosmological coincidence problem”, i.e. the problem of explaining why the energy densities of matter and DE are currently of the same order. In the standard \( \Lambda \)CDM model, this fact is indeed a coincidence since the evolution of the two components is very different; namely, while the DE density remains constant, the matter density decays
fast with the scale factor as $a^{-3}$. In contrast, in the ΛXCDM model, the ratio $r = \rho_D/\rho_M$ between the DE and matter densities may be bounded and not vary too far away from 1 for a significant fraction of the history of the Universe. It means that, for a very long time, $\rho_D$ and $\rho_M$ stay naturally of the same order as they are nowadays. Recently, a generalized version of the ΛXCDM model has been suggested in Ref. 9 with even more far reaching consequences, namely it is able to relax the value of the DE in the present Universe starting from an arbitrary value in the early epochs, i.e. it constitutes an interesting attempt to solve the old CC problem without using the traditional adjustment mechanisms based on scalar fields.5

Whatever its nature, if the DE is not a strict CC, then, according to cosmological perturbation theory, it should fluctuate. In order to find out its impact on the LSS formation, we will discuss some generic properties of the DE perturbations, exemplifying them by means of the ΛXCDM model. We will also address the question of how the LSS data can be used to constrain a model. The information about LSS is encoded in the galaxy fluctuation power spectrum, $P_{GG}(k)$, which is determined observationally and must be reproduced by the predicted matter power spectrum $P(k) \equiv |\delta_M(k)|^2$ of the theoretical model. A first, and economical, approach to the problem is to simply neglect DE perturbations. Using the fact that the ΛCDM model provides a good fit to the data, we may take it as a reference and impose that the power spectrum of our model does not deviate by more than 10% from the ΛCDM value ("F-test"10,11). As we will see, this simple analysis may serve to strongly restrict the parameter space of a model. Nevertheless, it does not reflect some important features that only come to light when making a full study of the combined system of matter and DE perturbations. We will show that such a study12 is useful not only to check the validity of the previous approach, but it can also help us to further constrain the physical region of the parameter space.

The net result of our analysis of the DE perturbations and its implication on LSS formation is quite rewarding, as we are able to find a sizable region of the ΛXCDM parameter space where the model is in full agreement with LSS data, and other cosmological observations, while providing at the same time a plausible dynamical solution to the cosmological coincidence problem.
2. Dark energy perturbations

In this section, we discuss some general properties of the DE perturbations for models in which both matter and DE are self-conserved:

\[ \rho_n' + \frac{3}{a}(1 + \omega_n)\rho_n = 0 \]  

Here a prime denotes differentiation with respect to the scale factor \( f' \equiv df/da \) and \( n = M, D \) stands for each of the energy components, matter/radiation and DE. We take \( \omega_M = 0 \), since we are interested in studying the perturbations in the matter-dominated (MD) era, and we denote the equation of state (EOS) of the DE component as \( \omega_e \), where the subindex \( "e" \) serves us to remember that the EOS may be an effective one. Let us first introduce the basic equations for the fluctuations, which we derive following the standard approach.\(^{13} \) For the background space-time we adopt the spatially flat FLRW metric, \( ds^2 = dt^2 - a^2 \delta_{ij} dx^i dx^j \). We perturb it

\[ g_{\mu\nu} \rightarrow g_{\mu\nu} + h_{\mu\nu}, \]

keeping only the scalar part of the perturbation. In order to have uniquely defined fluctuations \( h_{\mu\nu} \), a gauge choice is mandatory, i.e. we have to choose a specific coordinate system. Here we adopt the synchronous gauge,\(^{13} \) for which \( h_{00} = h_{0i} = 0 \).

We should also perturb the energy-momentum tensor, considering thus perturbations on the density, pressure and 4-velocity of each fluid:

\[ \rho_n \rightarrow \rho_n + \delta \rho_n, \quad p_n \rightarrow p_n + \delta p_n, \quad U_\mu^\nu \rightarrow U_\mu^\nu + \delta U_\mu^\nu. \]

(3)

The equations for the fluctuations are then obtained by perturbing the 00-component of the Einstein equations, \( R_{\mu\nu} - g_{\mu\nu}R/2 = 8\pi G T_{\mu\nu} \), and the conservation law for the energy momentum-tensor, \( \nabla_\mu T^\mu_\nu = 0 \). At the end we obtain 5 equations depending on the following set of 7 variables:

\[ \tilde{\phi} \equiv \frac{\partial}{\partial t} \left( \frac{h_{ii}}{a^2} \right), \quad \theta_n \equiv \nabla_\mu (\delta U_\mu^\nu) = \nabla_j (\delta U_j^\nu), \quad \delta U^\nu_\mu, \quad \delta \rho_n, \quad \delta p_n \]

(4)

Therefore, in order to solve our system we need to give an expression for \( \delta p_D \) (since for the matter component, indeed \( \delta p_M = 0 \)). In the case of adiabatic perturbations, we simply have \( \delta p_D = c_s^2 \delta \rho_D \), where

\[ c_s^2 = \frac{p_D'}{\rho_D'} = \frac{\omega_e}{3(1 + \omega_e)} \]

(5)

is the adiabatic speed of sound of the DE fluid. In general, however, there could be an entropy contribution to the pressure perturbation. In this case,
the relation between $\delta p_D$ and $\delta \rho_D$ in an arbitrary system of reference is given as follows:  

$$\delta p_D = c_s^2 \delta \rho_D - \frac{a^3}{H} \rho'_D H (c_s^2 - c_a^2) \frac{\theta_D}{k^2},$$  

(6)

where $c_s^2$ is the rest-frame (or effective) speed of sound and $k$ is the wave number, as we have moved to Fourier space. This expression is gauge-invariant, and thus it can be computed in any desired gauge, in particular in the synchronous one. When $\rho_D$ is self-conserved, equation (1) holds for $\rho_D$ and, in such case, (6) takes on the form

$$\delta p_D = c_s^2 \delta \rho_D + 3 H a^2 (1 + \omega_c) \rho_D (c_s^2 - c_a^2) \frac{\theta_D}{k^2},$$  

(7)

Finally, the equations for the perturbations read:

$$\delta_M = -\frac{1}{aH} \left( \theta_M - \frac{\dot{h}}{2} \right)$$  

(9)

$$\theta_M = -\frac{2}{a} \theta_M$$  

(10)

$$\delta_D = -\frac{1 + \omega_c}{aH} \left[ 1 + \frac{9a^2 H^2 (c_s^2 - c_a^2)}{k^2} \right] \theta_D - \frac{\dot{h}}{2} - \frac{3}{a} (c_s^2 - \omega_c) \delta_D$$  

(11)

$$\theta_D = -\frac{1}{a} \left[ (2 - 3c_s^2) \theta_D + \frac{k^2}{a^3 H} \frac{c^2_s \delta_D}{(1 + \omega_c)} \right];$$  

(12)

where $\tilde{\Omega}_n(a) \equiv \Omega_n(a) H_0^2 / H^2$ and $\Omega_n(a) \equiv \rho_n(a)/\rho_c^0 = 8 \pi G \rho_n(a)/(3 H_0^2)$. From these equations we get a second-order differential equation\(^a\) for $\delta_M$:

$$\delta_M''(a) + \frac{3}{2} \left[ 1 - \omega_c(a) \tilde{\Omega}_D(a) \right] \frac{\delta_M'(a)}{a} - \frac{3}{2} \tilde{\Omega}_M(a) \frac{\delta_M(a)}{a^2} =$$

$$= \frac{3}{2} \tilde{\Omega}_D(a) \left[ (1 + 3c_s^2) \frac{\delta_D(a)}{a^2} + 9H(a) (c_s^2 - c_a^2) \frac{\theta_D(a)}{k^2} \right].$$  

(13)

In order to study the properties of DE perturbations, it is useful to write also a second-order differential equation for $\delta_D$. This equation is much simpler if we use differentiation with respect to the conformal time $\eta (dt = ad\eta)$ and work in the comoving gauge. Notice that gauge issues are unimportant for sub-Hubble perturbations, as the ones we study here, so the behavior

\(^a\)In (13) we have corrected a typo that appears in Eq. (50) of Ref. 12.
of the perturbations will not depend on the chosen gauge. Defining the expansion rate in the conformal time \( H = \frac{da}{dn}/a \equiv \dot{a}/a \), the counterpart of Eq. (13) in the comoving gauge reads:

\[
\ddot{\Delta} - \left[ 3 \left( 2\omega_e - c_s^2 \right) - 1 \right] \dot{H} \dot{\Delta} + 3 \left[ \frac{3}{2} \omega_e^2 - 4\omega_e - \frac{1}{2} + 3c_s^2 \right] H^2 + \frac{k^2}{3} c_s^2 \Delta = 0.
\]

(14)

2.1. Generic properties of the DE perturbations

Once we have shown the basic equations, let us discuss some general properties of the DE perturbations. Looking at Eq. (14), we see that the coefficient of \( \Delta \) presents two terms. If it is the second of these terms (the one proportional to \( k^2 \)) that dominates, and forgetting for a moment about the term proportional to \( \dot{\Delta} \), we are left with the equation of a harmonic oscillator. This defines the sound horizon, a “Jeans scale” for the DE,

\[
\lambda_s = \left| \int_0^n c_s d\eta \right|,
\]

(15)
such that for scales well inside it, i.e. \( l \sim k^{-1} \ll \lambda_s \):

\[
\delta_D = C_1 e^{ic_s k \eta} + C_2 e^{-ic_s k \eta},
\]

(16)

where \( C_1 \) and \( C_2 \) are constants, and we have assumed constant \( c_s^2 \) for simplicity. Therefore, we see that:

- If \( c_s^2 < 0 \), the perturbations grow exponentially, situation which is unacceptable for structure formation. As long as \( \omega_e \) is not varying too fast, \( c_s^2 < 0 \) [cf. Eq. (5)], so in general the perturbations cannot be adiabatic.
- If \( c_s^2 > 0 \), the perturbations oscillate. When we take into account the \( \dot{\Delta} \) term, what we have is a damped harmonic oscillator, and thus the oscillations have decaying amplitude. As the matter perturbations grow typically as \( \delta_M \sim a \), this ensures that \( \delta_D/\delta_M \to 0 \), i.e. that DE will be a smooth component, as usually assumed. Nevertheless, the larger the scale \( l \) or the smaller the speed of sound \( c_s^2 \), the more important DE perturbations are, because then \( k^{-1} \ll \lambda_s \) is not such a good approximation.

Now the question is whether the scales relevant for the matter power spectrum are really inside the sound horizon or not. The linear regime of the power spectrum lies in the range \( 0.01h\text{Mpc}^{-1} < k < 0.2h\text{Mpc}^{-1} \) or, equivalently \( (600H_0)^{-1} \lesssim t \lesssim (30H_0)^{-1} \). On the other hand, we expect that at present \( \lambda_s \sim c_s^2 (H_0)^{-1} \). Thus we conclude that (at least for \( c_s^2 \) not
too close to 0), the scales relevant for the observations of LSS are well below the sound horizon, and so the features previously described apply to them.

Inspection of Eqs. (11) and (12) reveals another important property of the DE perturbations: they diverge if the EOS of the DE acquires the value $\omega_e = -1$ (known as the “CC boundary”), i.e. if the DE changes from quintessence-like (QE) behavior ($-1/3 > \omega_e > -1$) to phantom ($\omega_e < -1$) or vice versa. Note that, even though $c_s^2$ diverges at the crossing [cf. (5)], $(1 + \omega_e)c_s^2$ remains finite and, therefore, Eq. (11) is well-behaved. Thus, the problem lies exclusively in the $(1 + \omega_e)$ factor in the denominator of (12) and only disappears for vanishing sound of speed $c_s^2$. One can argue that the physical source of momentum transfer is not $\theta_D$ but $\rho_D V_D$, with

$$V_D = \theta_D(1 + \omega_e),$$

(14) and hope to get rid of the divergence through such a redefinition of variables, but unfortunately this is not the case. Getting around this difficulty is not always possible, and in fact there is no way for a single scalar field (or single fluid) model to cross the CC boundary, (14) and even with two fields some very special conditions should be arranged. In the absence of a mechanism to avoid this singularity, we are forced to restrict our parameter space by removing the points that present such a crossing in the past.

3. The $\Lambda$XCDM model

The properties discussed in the previous section apply in principle to any model in which the DE is self-conserved. The $\Lambda$XCDM model, introduced in Ref. 8 as a possible explanation to the cosmological coincidence problem, constitutes a non-trivial example of these kind of models. In it, the DE is a composite fluid, constituted by a variable CC and another generic component $X$, which can exchange energy with $\Lambda$:

$$\rho_D = \rho_\Lambda + \rho_X.$$  \hspace{1cm} (17)

The evolution of $\Lambda$ can be (as any parameter in QFT) tied to the RG in curved space-time:\textsuperscript{15}

$$\frac{d\rho_\Lambda}{d\ln \mu} = \frac{3\nu}{4\pi} M_P^2 \mu^2 \to \rho_\Lambda = \rho_\Lambda^0 + \frac{3\nu}{8\pi} M_P^2 (H^2 - H_0^2),$$  \hspace{1cm} (18)

where we identified $\mu$ (the energy scale associated to the RG in Cosmology) with the Hubble function $H$ at any epoch and $\nu$ is a free, dimensionless, parameter related to the mass ratio (squared) of the heavy particles contributing to the running versus the Planck mass.\textsuperscript{15} While the ultimate justification for this ansatz is the application of the RG method and the general considerations of covariance of the effective action in QFT in curved
space-time, a more profound study is needed, see Refs. 16 and 12 (section VI of the latter) for a more detailed discussion. Interestingly enough, the evolution law (18) can be tested from different points of view,”17,18 including cosmological perturbations19 (see also Refs. 20 and 21 for related phenomenological studies).

It should be clear that, in spite of the dynamical nature of Λ, its EOS parameter is \( \omega_\Lambda = -1 \), and it is in this sense that we may call it a “cosmological constant”. As for the X component, we assume that it has a constant EOS lying in the range \(-1 - \delta < \omega_X < -1/3 \) (where \( \delta > 0 \) is small). We need not make any assumption about the nature of the cosmon, since its evolution becomes determined by that of the CC through the energy conservation equation:

\[
\rho'_X + \rho'_\Lambda + \frac{3}{a}(1 + \omega_X)\rho_X = 0. \tag{19}
\]

The solution of the model in the MD era can be found from Eqs. (18), (19) and the Friedmann equation:

\[
H^2 = H_0^2[\Omega_M(a) + \Omega_D(a)], \tag{20}
\]

with \( \Omega_M(a) = \Omega_M^0 a^{-3} \). For the normalized DE density, we find:

\[
\Omega_D(a) = \frac{\Omega_\Lambda^0 - \nu}{1 - \nu} + \frac{\epsilon}{w_X - \epsilon} \left[ \frac{1 - \Omega_\Lambda^0}{1 - \nu} - \frac{w_X \Omega_M^0}{w_X - \epsilon} \right] a^{-3(1+w_X-\epsilon)}, \tag{21}
\]

where we have defined \( \epsilon \equiv \nu(1 + w_X) \). Assuming as a prior that \( \Omega_D^0 = \Omega_X^0 + \Omega_\Lambda^0 \simeq 0.7 \), we are left with 3 free parameters: \( \nu \), the parameter that controls the running of \( \Lambda \); \( \omega_X \), the barotropic index of the X component; and \( \Omega_\Lambda^0 \), the current energy density of the CC. Let us note that the model includes as special cases both the ΛCDM \((\nu = 0, \Omega_X^0 = 0)\) and XCDM \((\nu = 0, \Omega_\Lambda^0 = 0)\) models. The effective EOS parameter of the model,

\[
\omega_e(a) = -1 - \frac{a}{3 \Omega_D(a)} \frac{d\Omega_D(a)}{da}, \tag{22}
\]

can present a variety of behaviors8 compatible with \( \omega_e(a_0) \simeq -1 \) (the subindex 0 standing for the present value), as suggested by observations.2

3.1. The coincidence problem

In order to understand why the ΛXCDM model can provide an explanation for the coincidence problem, it is convenient to consider the ratio \( r \) between...
the DE and matter energy densities, which in the standard ΛCDM model reads:

\[ r = \frac{\Omega_D}{\Omega_M} = \frac{\Omega_D^0}{\Omega_M^0} a^3. \] (23)

We see that \( r \) tends to zero in the past and grows unboundedly in the future. Only at the present time we have \( r \simeq 1 \). The unavoidable conclusion seems to be that we live in a very special moment, namely one very close to the time when the expansion of the Universe started to be accelerated.

In contrast, in the ΛXCDM, \( r \) reads as follows:

\[ r = \frac{(\Omega_D^0 - \nu) a^3}{(1 - \nu) \Omega_M^0} + \frac{\epsilon}{\omega_X - \epsilon} + \left[ \frac{1 - \Omega_D^0}{\Omega_M^0(1 - \nu)} - \frac{\omega_X}{\omega_X - \epsilon} \right] a^{-3(\omega_X - \epsilon)}. \] (24)

Such, more complex, structure allows for the existence of a maximum of this ratio in the future, which implies that \( r \) may be bounded and relatively small (not very different from 1), say \( r \leq 10 r_0 \), for a very prolonged stretch of the history of the Universe. In this case, the value \( r \sim 1 \) would no longer be seen as special.

It is important to note that the ability to solve the coincidence problem is a very general feature of the model. In order to show that, let us recall that the solution to the coincidence problem is linked to the existence of a future stopping of the Universe expansion. Now, from the Friedmann equation (20), it is clear that it is necessary that the DE density becomes negative for the expansion to stop. But this condition can be realized even in the simplest setups of the ΛXCDM model. Let us assume e.g. that \( \nu = 0 \), so there is no exchange of energy between the CC and the \( X \) component:

\[ \Omega_D = \Omega_D^0 + \Omega_X^0 a^{-3(1 + \omega_X)}. \] (25)

In this case we have a truly constant \( \Lambda \) and the cosmon behaves effectively as a QE/phantom scalar field; \( \Omega_D \) will eventually become negative if any of the following conditions is fulfilled:

\[ \Omega_A^0 < 0 \quad \text{and} \quad -1 < \omega_X < -1/3 \quad \text{or} \quad \Omega_X^0 < 0 \quad \text{and} \quad \omega_X < -1. \] (26)

Let us also stress that in the ΛXCDM the individual components are not observable, the only thing we can measure is the total \( \Omega_D \). Therefore, there is no problem in having a negative value for \( \Omega_A^0 \) or \( \Omega_X^0 \), as long as \( \Omega_D^0 = 0.7 \). Remember also that \( X \) need not be a real fluid, its nature could be effective.

In Fig. 1a we show that there is a large 3D-volume of the parameter space for which this solution to the coincidence problem is possible (the projections of that volume onto three orthogonal planes are shown as the
shaded regions in Fig. 1b, c, d.) All the points in it present a relatively low maximum of the ratio $r$ ($r_{\text{max}} \leq 10 r_0$), ratio that, in addition, is small enough at the nucleosynthesis epoch ($r_N \lesssim 0.1$, where in this case $\Omega_M$ is the density of radiation and $\Omega_D$ is to be computed in the radiation-dominated era), so as to make sure that the predictions of the Big Bang model are not spoiled.

Fig. 1. (a) 3D volume constituted by the points of the $\Lambda\Omega$CDM parameter space which provide a solution to the coincidence problem, presenting a low maximum of the ratio $r$ (24), $r_{\text{max}} \leq 10 r_0$, and satisfy the nucleosynthesis bound $r_N \lesssim 0.1$ (see the text); (b), (c) and (d) Projections of the 3D volume in (a) onto the perpendicular planes $\nu = 0$, $\omega_e = 0$ and $\Omega_\Lambda = 0$ (all the shaded area). When we ask for the F-test (29) to be fulfilled and the current value of the EOS to be close to -1 (30), we are left with the medium and dark-shaded regions. Finally, by considering DE perturbations, we are forced to exclude the points for which the equations are ill-defined, i.e. those for which the EOS of the DE acquires the value -1 at some point in the past. By doing so we get the final physical parameter space of the $\Lambda\Omega$CDM model (dark-shaded region).
4. Perturbations in the ΛXCDM model

In this section we address the problem of how the parameter space of a model can be constrained by means of LSS data, using as an example the ΛXCDM model. As we saw in Sec. 2.1, at the scales relevant to the linear part of the matter power spectrum the DE perturbations are expected to be negligible as compared to the matter ones. Thus, a reasonable approach is to simply neglect the DE perturbations from Eq. (13):

\[ \delta''_M(a) + \frac{3}{2} \left[ 1 - \omega_e(a) \Omega_D(a) \right] \frac{\delta'_M(a)}{a} - \frac{3}{2} \Omega_M(a) \frac{\delta_M(a)}{a^2} = 0, \tag{27} \]

and study the evolution of the perturbations from some initial scale factor \( a = a_i \) in the MD era (where \( \delta_M \sim a \)) until the present time, \( a_0 = 1 \). As Eq. (27) does not depend on the wave number \( k \), all the scales grow in the same fashion and we can characterize models by means of the “growth factor”:

\[ D(a) = \frac{\delta_M(a)}{\delta_{CDM}(a_0)}, \tag{28} \]

whose present value, \( D(a_0) \), compares the growth of the perturbations in the model considered to the growth in a pure cold dark matter (CDM) model. The parameter that measures the agreement between the observed galaxy distribution power spectrum, \( P_{GG}(k) \), and the matter power spectrum of a model, \( P(k) \equiv |\delta_M(k)|^2 \), is the linear bias, which at the present time is defined as \( b^2(a_0) = P_{GG}/P \). Most remarkably, the LSS data point to the value \( b_\Lambda(a_0) = 1 \), to within a 10\% accuracy, for the ΛCDM model.\(^3\) This suggests that the comparison to the ΛCDM can be a valid and more economical criterion for studying the viability of a model. In particular, we may require that any DE model should pass the following “F-test”\(^10\):

\[ |F| \equiv \left| 1 - \frac{b^2(a_0)}{b^2_\Lambda(a_0)} \right| = \left| 1 - \frac{P_{\Lambda}(a_0)}{P(a_0)} \right| = \left| 1 - \frac{D^2_\Lambda(a_0)}{D^2(a_0)} \right| \leq 0.1. \tag{29} \]

This was done for the ΛXCDM model (and also for the running CC model\(^15\)) in Ref. 11, where, in addition, we imposed that the current value of the EOS parameter of the DE should be close to -1:

\[ |\omega_e(a_0) + 1| \leq 0.3, \tag{30} \]

as suggested by recent observational limits\(^2,b\). As seen in Fig. 1b, c, d, there is still a sizable region of the parameter space (medium and dark-shaded

\(^a\)Let us remark, though, that such limits on the EOS parameter are usually derived under the assumption of a constant \( \omega_e \) and, therefore, do not strictly apply to our model.
regions) where the ΛXCDM satisfies these two new conditions and the nucleosynthesis bound, while still providing a solution to the coincidence problem.

Neglecting DE perturbations provides us therefore with a simple and effective method to constrain the parameter space of a model. Although we expect it to be a reasonable approximation, we cannot be completely sure unless we perform a full analysis in which the DE fluctuations are also included. Such an analysis\textsuperscript{12} implies an immediate and very important consequence. As discussed in Sec. 2.1, if the effective EOS of the model crosses the CC boundary ($\omega_e = -1$) at some point in the past, the perturbation equations will diverge. In the absence of a mechanism to get around this singularity (and indeed we cannot have it without a microscopic definition of the $X$ component, i.e. one that goes beyond a mere conservation law), we are forced to restrict our parameter space by removing the points that present such a crossing. This new constraint knocks off many of the points allowed by the previous simple analysis; in fact, we are left with the dark-shaded region in Fig. 1b, c, d, and so we end up with a rather definite prediction for the values of the parameters of the ΛXCDM model. It is worth noticing that only small (and positive) values of $\nu$ are allowed, $\nu \sim 10^{-2}$ at most, which is in very good agreement with theoretical expectations.\textsuperscript{12} Another interesting consequence of the new constraint is that the effective EOS of the DE can be QE-like only,\textsuperscript{12} i.e. $-1 < \omega_e < -1/3$.

We want to compare the matter power spectrum predicted by the ΛXCDM model, $P_{\Lambda X}(k)$, with the $P_{GG}(k)$\textsuperscript{c} measured by the 2dFGRS collaboration.\textsuperscript{3} The former can be found by evolving the perturbation equations (8)-(12) from $a = a_i$ to $a_0 = 1$, where in this case $a_i \ll 1$ is the scale factor at some time well after recombination. In order to set the initial conditions, we took into account that the DE does not begin to play an important role until very recently, so that the values of the metric and matter perturbations at $a = a_i$ should be the same for our model and for the ΛCDM model – the power spectrum $P_\Lambda(k)$ of the latter being available from standard analytical fits in the literature, see Ref. 12 and references therein. As for the DE perturbations, we assumed that they vanish at $a = a_i$. This is reasonable because, as noticed before, the DE perturbations are expected to be negligible at the scales relevant to the linear part of the matter power spectrum.

The ΛXCDM power spectrum was calculated for two different fiducial

\textsuperscript{c}And also with the ΛCDM spectrum, $P_\Lambda(k)$, which provides a good fit to $P_{GG}(k)$
sound speeds, $c_s^2 = 1$ and $c_s^2 = 0.1$ and several combinations of the parameters $\nu$, $\omega_X$ and $\Omega_0$. For values of the parameters not fulfilling the F-test (even though satisfying all the other conditions stated in Fig. 1) we obtain huge discrepancies, as expected. The discrepancy appears as an approximate global suppression gap (in the entire $k$ range) of the amount of growth with respect to the $\Lambda$CDM model (cf. Fig. 2b). This suppression is typical of the QE-like behavior and occurs even if the DE perturbations are neglected (dotted line), in which case $P_{\Lambda X}(k)$ presents the same shape as $P_{\Lambda}(k)$ (because then the $k$-dependence disappears from the equations, which reduce just to Eq. (27)). The effect of considering DE perturbations is only visible at large scales (small $k$), where they tend to compensate the aforementioned suppression. The smaller the speed of sound or the larger the scale, the more important is the effect of DE perturbations, as expected from the general considerations of Sec. 2.1.

![Fig. 2. The 2dFGRS observed galaxy power spectrum, $P_G(k)$ (points), and the $\Lambda$CDM power spectrum, $P_{\Lambda}(k)$ (dot-dashed line) versus the spectrum predicted by the $\Lambda$CDM, $P_{\Lambda X}(k)$, for DE sound speeds $c_s^2 = 0.1$ (dashed line) and $c_s^2 = 1$ (solid/gray line): (a) for a set of parameters allowed by the analysis of Ref. 11 (in the dark-shaded region of Fig. 1b, c, d), $\Omega_X^0 = 0.8$, $\nu = \nu_0 = 2.6 \times 10^{-3}$ and $w_X = -1.6$; (b) for a set of parameters satisfying all the conditions in that analysis but the F-test, $\Omega_X^0 = +0.35$, $\nu = -0.2$ and $w_X = -0.6$. In this case it is also shown the power spectrum obtained by neglecting DE perturbations (dotted line), which presents the same shape as $P_{\Lambda}(k)$.]

In contrast, in Fig. 2a we see that for values allowed by the F-test (and satisfying all the other constraints as well, i.e. lying in the dark-shaded region in Fig. 1b, c, d), $P_{\Lambda X}(k)$ is very similar to $P_{\Lambda}(k)$, with numerical results in very good agreement with those obtained through the F-method. In particular, their shape is identical, indicating that DE perturbations do not play a role here. Indeed, in Fig. 3a we see that $\delta_D$ oscillates with decreasing amplitude, as predicted in Sec. 2.1. For positive sound speed, the perturbations get stabilized (and therefore the ratio $\delta_D/\delta_M$ becomes negligible).
once the sound horizon (15) is crossed, i.e. when $k\lambda_s = \pi$, as seen in Fig. 3b. Similarly, in the adiabatic case, the perturbations begin their exponential growth once $c_s^2$ (which is negligible in the far past in the ΛXCDM model) eventually takes a sizable negative value. The runaway behavior is triggered by the term proportional to $k^2 c_s^2 < 0$ in (12), or equivalently in (14).

Fig. 3. (a) The ΛXCDM growth of DE perturbations for a small scale $k = 0.2$ (in units of $h\text{Mpc}^{-1}$) and the same set of parameters assumed in Fig. 2a, and for DE sound speed $c_s^2 = 0.1$; (b) Evolution of the DE perturbations $\delta_D$ (black lines) for the same set of parameters as in (a), at the large scale $k = 0.01$ and for three different speeds of sound: $c_s^2 = c_a^2 < 0$ (solid line), $c_s^2 = 0$ (dashed line) and $c_s^2 = 0.1$ (dot-dashed line). The evolution of the ratio $\delta_D/\delta_M$ is also shown (gray lines).

5. Conclusions

We have analyzed the behavior of the DE perturbations in models with self-conserved DE. We have exemplified them by means of the ΛXCDM model, which is a non-trivial model of the cosmic evolution with a number of appealing properties. Unlike other proposed solutions to the coincidence problem (an incomplete list includes tracking scalar fields, interactive QE models, K-essence, Chaplygin gas, etc - see e.g. Ref. 12 and references therein), the ΛXCDM model accounts for the energy of vacuum through a (possibly running) Λ, giving allowance for other dynamical contributions, $X$, of general nature. The comparison of the ΛXCDM power spectrum to the LSS data, first by means of the F-test and then through a full analysis of the DE perturbations, resulted in a strong additional constraint on the parameter space of the model, hence increasing its predictive power and pinpointing a region where the ΛXCDM model provides a realistic solution to the coincidence problem, i.e. fully compatible with present observations.
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