Correlators of currents corresponding to the massive $p$-form fields in AdS/CFT correspondence

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Abstract

By solving the equations of motion of massive $p$-form potential in Anti-de-Sitter space and using the $AdS/CFT$ correspondence of Maldacena, the generating functional of two-point correlation functions of the currents is obtained. When the mass parameter vanishes the result agrees with the known massless case.

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1 Introduction

The fascinating proposal of Maldacena that the bulk action of fields in Anti-de-Sitter space (AdS) may be used as the generating functional of correlators of corresponding conformal currents in the boundary spacetime [1] opened a new way of understanding conformal field theory in real spacetime. Manifold novel physical applications are presented with new insights [2, 3, 4, 5].

For the proposal itself a lot of tests are reported in the direction of confirming it. Various correlators for scalar field case are investigated [6, 7, 8, 9, 10, 11, 13, 14]. Vector fields [7, 15, 16], spinors [18, 19, 15], gravitinos [20, 21], and gravitons [12, 13, 23] are also investigated. The next natural step is the consideration of p-form potential. For massless p-form case it is already obtained by computing first the boundary-to-bulk Green’s function [17]. Since p-form potentials of supergravity actions or branes may acquire masses from the internal excitations of some extra compact dimensions, it is necessary to investigate the AdS/CFT correspondence for the case of massive p-form fields.

Although it is rather easy to determine the boundary-to-bulk Green’s function for massless p-form fields, it is not so for massive cases. The reason is that for massless cases p-form potentials vanish when one goes to the boundary of AdS, thus simplifying the Ward-identity-preserving process of taking $x_0 \to 0$ limit. But for massive cases the story become different. In this case one should be careful taking the limit. The most natural way of obtaining the boundary-to-bulk Green’s function is solving the equations of motion of massive p-form fields. When one uses of the coordinate-space holographic projection technique [16] to the solution of equations of motion one may obtain the desired AdS action which serves as the generating functional of correlators of conformal currents.

In this paper we utilize this idea. In section 2, the classical equations of motion of massive p-form fields in $AdS_{d+1}$ with Poincare metric is solved. The consistency condition, which is in fact the Bianchi identity, greatly simplifies the process of solving the equations.
of motion. In section 3, the coordinate-space holographic projection technique is used to
determine the desired action in terms of boundary fields.

2 The equations of motion of massive \( p \)-form fields

For our purpose it is sufficient to employ the Euclidean \( \text{AdS}_{d+1} \) which is characterized
by the Lobachevsky space \( \mathbb{R}^{1+d}_+ = \{(x_0, \mathbf{x}) \in \mathbb{R}^{1+d} \mid x_0 > 0\} \) with the following Poincaré metric
\[
   ds^2 = \frac{1}{x_0^2} \left[ (dx^0)^2 + \sum_{i=1}^{d}(dx^i)^2 \right].
\]
The \( x_0 \to \infty \) point and the \( x_0 \to 0 \) region consist the boundary of \( \text{AdS} \). To simplify the
notation we sometimes confuse \( x^0 \) with \( x_0 \) unless it is stated explicitly. Roman characters
such as \( i \) and \( j \) are used to denote the boundary spacetime indices \( 1, \ldots, d \), and Greek
characters such as \( \mu \) and \( \nu \) are preserved to denote whole indices of \( \text{AdS} \) including 0.

Consider a massive \( p \)-form potential \( \mathcal{A} = \frac{1}{p!} A_{\mu_1 \ldots \mu_p} dx^{\mu_1} \ldots dx^{\mu_p} \) of \( \text{AdS}_{d+1} \). The free
action of \( \mathcal{A} \) is given by
\[
   I = \frac{1}{2} \int_{\text{AdS}_{d+1}} \left( \mathcal{F} \wedge \ast \mathcal{F} + m^2 \mathcal{A} \wedge \ast \mathcal{A} \right),
\]
where \( \mathcal{F} = d\mathcal{A} \) is the field strength \( p + 1 \) form. According to the Maldacena’s \( \text{AdS}/\text{CFT} \)
proposal the generating functional of the correlation functions of conformal currents is
given by the following relation
\[
   \langle \exp \frac{1}{p!} \int d^d x A_{i_1 \ldots i_p} (\mathbf{x}) J_{i_1 \ldots i_p} (\mathbf{x}) \rangle_{\text{CFT}} = \mathcal{Z}_{\text{AdS}_{d+1}} [\mathcal{A}],
\]
where \( \mathcal{Z}_{\text{AdS}_{d+1}} [\mathcal{A}] \) is the partition function of \( \mathcal{A} \) in \( \text{AdS}_{d+1} \) computed under the condition
that \( \mathcal{A} \) is related to the boundary \( p \)-form \( A \) through the boundary-to-bulk Green’s func-
tion. When ones use the classical approximation this partition function can be written
as
\[
   \mathcal{Z}_{\text{AdS}_{d+1}} [\mathcal{A}] \simeq \exp(-I[A_{i_1 \ldots i_p}]),
\]
where \( I[A_{i_1...i_p}] \) is the corresponding classical action. We use this classical approximation to compute the two-point correlation function of the conformal current \( J_{i_1...i_p} \) corresponding to the massive \( p \)-form field \( A_{i_1...i_p} \).

The classical equation of motion of \( A \) which can be obtained from (2) is

\[
(-)^p d^*dA - m^2 \ast A = 0. \tag{5}
\]

\( A \) also satisfies the following additional consistency condition

\[
d^*A = 0. \tag{6}
\]

Using the metric (1) the equation of motion (5) can be written as

\[
\left[ x_0^2 \partial^2_\mu - (d + 1 - 2p)x_0 \partial_0 + (d + 1 + 2p - m^2) \right] A_{0i_2...i_p} = 0, \tag{7}
\]

\[
\left[ x_0^2 \partial^2_\mu - (d - 1 - 2p)x_0 \partial_0 - m^2 \right] A_{i_1...i_p}
= 2x_0 \left( \partial_{i_1} \omega_{0i_2...i_p} + (-)^{p-1} \partial_{i_2} \omega_{0i_3...i_{p}i_1} + \cdots \right). \tag{8}
\]

By means of the vielbein \( e^\mu_a = x_0 \delta_\mu^a \) we introduce fields with flat indices

\[
A_{0i_2...i_p} = x_0^{p-1} A_{0i_2...i_p}, \quad A_{i_1...i_p} = x_0^p A_{i_1...i_p}. \tag{9}
\]

The equations of motion written in terms of these fields are

\[
\left[ x_0^2 \partial^2_\mu - (d - 1)x_0 \partial_0 - (m^2 + p^2 - dp) \right] A_{0i_2...i_p} = 0, \tag{10}
\]

\[
\left[ x_0^2 \partial^2_\mu - (d - 1)x_0 \partial_0 - (m^2 + p^2 - dp) \right] A_{i_1...i_p}
= 2x_0 \left( \partial_{i_1} \omega_{0i_2...i_p} + (-)^{p-1} \partial_{i_2} \omega_{0i_3...i_{p}i_1} + \cdots \right). \tag{11}
\]

Similarly, the consistency condition (3) becomes

\[
\partial_i A_{i_1...i_{p-1}0} = 0, \tag{12}
\]

\[
\partial_i A_{i_1...i_p} + x_0 \partial_0 A_{0i_2...i_p} = (d - p) A_{0i_2...i_p}. \tag{13}
\]

\footnote{The minimal prerequisites for handling differential forms are presented in appendix.}
Now we solve the equation of motion (10) of $A_{0i_2...i_p}$. In the holographic projection of massive field we need the field component which diverges as

$$A_{0i_2...i_p} \sim x_0^{-\lambda}$$

as $x_0$ goes to 0. Substituting this in (10) we have

$$\lambda(\lambda + d) = m^2 + p^2 - dp.$$  \hspace{1cm} (15)

To solve this we introduce $\nu$ such as

$$\nu = \lambda + \frac{d}{2}.$$  \hspace{1cm} (16)

In terms of $\nu$ we have following two roots of (15),

$$\nu = \pm \sqrt{m^2 + (\frac{d}{2} - p)^2}.$$  \hspace{1cm} (17)

From now on $\nu$ denotes only the positive one of it. This shows that as $x_0$ goes to 0, $A_{0i_2...i_p}$ diverges as

$$A_{0i_2...i_p} \sim x_0^{\frac{d}{2}}(c_{\nu}x_0^{-\nu} + c_{-\nu}x_0^{\nu})$$

where $c_{\nu}$ and $c_{-\nu}$ are constants. We know that the modified Bessel function $K_{\nu}(\xi)$ which satisfies

$$[\xi^2 \frac{d^2}{d\xi^2} + \xi \frac{d}{d\xi} - (\nu^2 + \xi^2)]K_{\nu} = 0$$

has following expansion

$$K_{\nu}(\xi) = \frac{1}{2} \left[ \Gamma(\nu) \left( \frac{\xi}{2} \right)^{-\nu} + \Gamma(-\nu) \left( \frac{\xi}{2} \right)^{\nu} + \cdots \right]$$

as $\xi \to 0$. On the other hand, since (10) does not contain $x^i$, we may Fourier transform $A_{0i_2...i_p}$ in the following way

$$A_{0i_2...i_p}(x_0, x) = \int \frac{d^d k}{(2\pi)^d} e^{-ikx} a_{0i_2...i_p}(k) K_{\nu}(|k|x_0).$$  \hspace{1cm} (21)

It can be shown that it is in fact the desired solution. The consistency condition (12) becomes

$$k_{i_2} a_{0i_2...i_p}(k) = 0.$$  \hspace{1cm} (22)
Now we determine $A_{i_1 \ldots i_p}$. To solve (11) we decompose this into

$$A_{i_1 \ldots i_p} = A_{i_1 \ldots i_p}^{(g)} + A_{i_1 \ldots i_p}^{(p)}, \quad (23)$$

where $A_{i_1 \ldots i_p}^{(g)}$ is the general solution of the homogeneous part of (11), and $A_{i_1 \ldots i_p}^{(p)}$ is a particular solution of the same equation. It is clear that $A_{i_1 \ldots i_p}^{(g)}$ has a similar structure as $A_{0i_2 \ldots i_p}$, such as

$$A_{i_1 \ldots i_p}^{(g)}(x_0, x) = x_0^d \int \frac{d^d k}{(2\pi)^d} e^{-ik\cdot x} a_{i_1 \ldots i_p}(k) K_\nu(|k| x_0). \quad (24)$$

For $A_{i_1 \ldots i_p}^{(p)}$ we assume following form

$$A_{i_1 \ldots i_p}^{(p)} = -i2x_0^d \int \frac{d^d k}{(2\pi)^d} e^{-ik\cdot x} \left( k_{i_1} a_{0i_2 \ldots i_p} + (-)^{p-1}k_{i_2} a_{0i_3 \ldots i_p i_1} + \cdots \right) \frac{1}{|k|^2} H(|k| x_0). \quad (25)$$

Inserting this into (11) one finds that it is in fact a solution if $H(|k| x_0)$ satisfies

$$[\xi^2 \frac{d^2}{d\xi^2} + \xi \frac{d}{d\xi} - (\nu^2 + \xi^2)] H = \xi^2 K_\nu. \quad (26)$$

Rather than dealing this equation directly we solve the remaining consistency condition (13). Under the decomposition (23) it becomes

$$\partial_i A_{i_1 \ldots i_p}^{(g)} + \partial_i A_{i_1 \ldots i_p}^{(p)} + x_0 \partial_0 A_{0i_2 \ldots i_p} = (d - p) A_{0i_2 \ldots i_p}. \quad (27)$$

Since $A_{i_1 \ldots i_p}^{(g)}$ is independent of $A_{0i_2 \ldots i_p}$, we may assume that

$$\partial_i A_{i_1 \ldots i_p}^{(g)} = 0. \quad (28)$$

This, in terms of Fourier component, is

$$k_i a_{i_1 \ldots i_p}(k) = 0. \quad (29)$$

On the other hand $A_{i_1 \ldots i_p}^{(p)}$ satisfies

$$\partial_i A_{i_1 \ldots i_p}^{(p)} + x_0 \partial_0 A_{0i_2 \ldots i_p} = (d - p) A_{0i_2 \ldots i_p}. \quad (30)$$

When one uses (24) and (25) this equation implies the following specific form of $H$,

$$H(\xi) = \frac{1}{2} \left[ \xi \frac{dK_\nu}{d\xi} + (p - \frac{d}{2}) K_\nu \right]. \quad (31)$$
It is easy to check that this in fact satisfies (26). The final form of $A_{i_1...i_p}(x_0, x)$, when we combine (24) and (25), is

$$A_{i_1...i_p}(x_0, x) = \frac{4 \pi}{d!} \int \frac{d^d k}{(2\pi)^d} e^{-i k \cdot x} \left[a_{i_1...i_p} K_\nu(\xi) - \frac{i}{|k|^2} \left(k_{i_2} a_{0i_2...i_p} + (-)^{p-1} k_{i_2} a_{0i_3...i_p,i_1} + \cdots \right) \left(\xi \frac{d K_\nu}{d \xi} + (p - \frac{d}{2}) K_\nu \right)\right],$$

where $\xi = |k|x_0$.

### 3 Holographic projection of fields and the action

In this section we calculate the classical action of massive $p$-form potentials using the explicit forms of fields derived in the preceding section. The action (2), under the equation of motion, is

$$I = \frac{1}{2} \int_{\partial AdS^{d+1}} A \wedge *F = -\frac{1}{2} \lim_{\epsilon \to 0} \int_{x_0 = \epsilon} A \wedge *F. \quad (33)$$

Using the relations (9) it can be written in terms of fields with flat indices in the following way

$$I = \lim_{\epsilon \to 0} \left(\frac{p}{2} \sum_{i_1 < ... < i_p} \int_{x_0 = \epsilon} d^d x \ x_0^{-d} A_{i_1...i_p}^2 - \frac{1}{2} \sum_{i_1 < ... < i_p} \int_{x_0 = \epsilon} d^d x \ x_0^{-d+1} A_{i_1...i_p} \left[ \partial_0 A_{i_1...i_p} + (-)^p \partial_i A_{i_2...i_p,0} + \cdots \right] \right). \quad (34)$$

To compute this we utilize the Ward-identity-preserving coordinate-space holographic projection of fields introduced in Ref [16]. Since both $A_{0i_2...i_p}$ and $A_{i_1...i_p}$ diverge in the power of $x_0^{-\lambda}$ as $x_0 \to 0$ we define the following $\epsilon$-boundary fields such as

$$A_{0i_2...i_p}^h(\epsilon, x) = \epsilon^\lambda A_{0i_2...i_p}(\epsilon, x), \quad A_{i_1...i_p}^h(\epsilon, x) = \epsilon^\lambda A_{i_1...i_p}(\epsilon, x). \quad (35)$$

Using these it can be shown that first term of (33) is proportional to $\epsilon^{-2\nu} (A_{i_1...i_p}^h)^2$ which, in the $\epsilon \to 0$ limit, becomes an unimportant contact term. Similar reasoning shows that terms with $A_{0i_2...i_p}$ do not contribute any meaningful value. The only term left is

$$I = -\frac{1}{2} \lim_{\epsilon \to 0} \sum_{i_1 < ... < i_p} \int_{x_0 = \epsilon} d^d x \ x_0^{-d+1} A_{i_1...i_p} \partial_0 A_{i_1...i_p}. \quad (36)$$
As it is discussed in Ref. [16] the integrand of this has following holographic projection,

\[
A_{i_1 \ldots i_p}(x_0, x) \partial_0 A_{i_1 \ldots i_p}(x_0, x) \bigg|_{x_0 = \epsilon} \to \frac{1}{2} \epsilon^{2\lambda} \partial_0 A_{i_1 \ldots i_p}(\epsilon, x)^2. \tag{37}
\]

Then the action can be written as

\[
I = -\frac{1}{4} \lim_{\epsilon \to 0} \sum_{i_1 < \ldots < i_p} \int d^d x \, \epsilon^{2\nu} \partial_0 \left( A_{i_1 \ldots i_p}^h(\epsilon, x) \right)^2. \tag{38}
\]

When we use (32) we have the \( \epsilon \)-boundary field,

\[
A_{i_1 \ldots i_p}^h(\epsilon, x) = \int \frac{d^d k}{(2\pi)^d} \left[ \frac{1}{2} \Gamma(\nu) \left( \frac{|k|}{2} \right)^{-\nu} \left( a_{i_1 \ldots i_p} - i \frac{\beta}{|k|^2} \left\{ k_i a_{0i_2 \ldots i_p} + (-)^{p-1} k_{i_2} a_{0i_3 \ldots i_p i_1} + \cdots \right\} \right) 
+ \frac{1}{2} \Gamma(-\nu) \left( \frac{|k|}{2} \right)^\nu \left( a_{i_1 \ldots i_p} - i \frac{\bar{\beta}}{|k|^2} \left\{ k_i a_{0i_2 \ldots i_p} + (-)^{p-1} k_{i_2} a_{0i_3 \ldots i_p i_1} + \cdots \right\} \right) \right] e^{2\nu} + \cdots \right] e^{-ik \cdot x},
\]

where

\[
\beta = p - \frac{d}{2} - \nu, \quad \bar{\beta} = p - \frac{d}{2} + \nu. \tag{40}
\]

We define the boundary field by

\[
A_{i_1 \ldots i_p}(x) = \lim_{\epsilon \to 0} A_{i_1 \ldots i_p}^h(\epsilon, x). \tag{41}
\]

The Fourier component of it is

\[
\tilde{A}_{i_1 \ldots i_p}(k) = \frac{1}{2} \Gamma(\nu) \left( \frac{|k|}{2} \right)^{-\nu} \left( a_{i_1 \ldots i_p} - i \frac{\beta}{|k|^2} \left\{ k_i a_{0i_2 \ldots i_p} + (-)^{p-1} k_{i_2} a_{0i_3 \ldots i_p i_1} + \cdots \right\} \right) . \tag{42}
\]

Using the consistency relations (22) and (29) we are able to solve \( a_{0i_2 \ldots i_p}(k) \) and \( a_{i_1 \ldots i_p}(k) \) in terms of \( \tilde{A}_{i_1 \ldots i_p}(k) \),

\[
a_{0i_2 \ldots i_p} = i \frac{2}{\beta\Gamma(\nu)} \left( \frac{|k|}{2} \right)^\nu k_i \tilde{A}_{i_1 \ldots i_p}, \tag{43}
\]

\[
a_{i_1 \ldots i_p} = \frac{2}{\Gamma(\nu)} \left( \frac{|k|}{2} \right)^\nu \left( \tilde{A}_{i_1 \ldots i_p} - k_i \tilde{A}_{i_1 \ldots i_p} \right) \left\{ k_i a_{0i_2 \ldots i_p} + (-)^{p-1} k_{i_2} a_{0i_3 \ldots i_p i_1} + \cdots \right\} \right]. \tag{44}
\]

The action which can be read out of (38) and (39) is

\[
I = -\frac{\nu}{4} \Gamma(\nu)\Gamma(-\nu) \int \frac{d^d k}{(2\pi)^d} \left( \sum_{i_1 < \ldots < i_p} a_{i_1 \ldots i_p}(k) a_{i_1 \ldots i_p}(-k) + \sum_{i_2 < \ldots < i_p} \frac{\beta\bar{\beta}}{|k|^2} a_{0i_2 \ldots i_p}(k) a_{0i_2 \ldots i_p}(-k) \right). \tag{45}
\]
On the other hand, using (43) and (44) we have
\[
\sum_{i_2 < \ldots < i_p} a_{0i_2 \ldots i_p}(k) a_{0i_2 \ldots i_p}(-k) = \frac{4}{\beta^2 \Gamma(\nu)^2} \left(\frac{|k|}{2}\right)^{2\nu} \sum_{ij} \sum_{i_2 < \ldots < i_p} \tilde{A}_{ij_2 \ldots i_p}(k) \tilde{A}_{ij_2 \ldots i_p}(-k) k_i k_j, \tag{46}
\]
\[
\sum_{i_1 < \ldots < i_p} a_{i_1 \ldots i_p}(k) a_{i_1 \ldots i_p}(-k) = \frac{4}{\Gamma(\nu)^2} \left(\frac{|k|}{2}\right)^{2\nu} \sum_{ij} \sum_{i_2 < \ldots < i_p} \tilde{A}_{ij_2 \ldots i_p}(k) \tilde{A}_{ij_2 \ldots i_p}(-k) \left(\frac{1}{p} \delta_{ij} - \frac{k_i k_j}{|k|^2}\right). \tag{47}
\]
Inserting this into (45) we have
\[
I = -\nu \Gamma(-\nu) \sum_{ij} \sum_{i_2 < \ldots < i_p} \int \frac{d^d k}{(2\pi)^d} \tilde{A}_{ij_2 \ldots i_p}(k) \tilde{A}_{ij_2 \ldots i_p}(-k) \left(\frac{|k|}{2}\right)^{2\nu} \times \left(\frac{1}{p} \delta_{ij} - \frac{k_i k_j}{|k|^2} + \frac{\beta k_i k_j}{|k|^2}\right). \tag{48}
\]
When we use the relation
\[
\int \frac{d^d k}{(2\pi)^d} e^{ik \cdot x} |k|^{2\nu} \left(\frac{1}{p} \delta_{ij} + \frac{2\nu}{p - \frac{d}{2} - \nu} \frac{k_i k_j}{|k|^2}\right) = \frac{2^{2\nu} \Gamma\left(\frac{d}{2} + \nu\right)}{\pi^{\frac{d}{2}} \Gamma(-\nu)} \frac{d}{2} + \nu - p \left(\frac{1}{p} \delta_{ij} - 2 \frac{x_i x_j}{|x|^2}\right) \frac{1}{|x|^{d+2\nu}}, \tag{49}
\]
which comes from the equations given in the appendix of Ref. [16] it is easy to show that
\[
I[A_{i_1 \ldots i_p}] = -c \left(\sum_{i_1 < \ldots < i_p} \int d^d x d^d x' \frac{A_{i_1 \ldots i_p}(x) A_{i_1 \ldots i_p}(x')}{|x - x'|^{2\Delta}}\right) \tag{50}
\]
\[
-2 \sum_{ij} \sum_{i_2 < \ldots < i_p} \int d^d x d^d x' (x_i - x'_i)(x_j - x'_j) A_{ij_2 \ldots i_p}(x) A_{ij_2 \ldots i_p}(x') |x - x'|^{2\Delta + 2},
\]
where
\[
c = \frac{\Delta(\Delta - \frac{d}{2}) \Gamma(\Delta)}{\pi^{\frac{d}{2}}(\Delta - p)} \Gamma\left(\Delta - \frac{d}{2}\right). \tag{51}
\]
The conformal dimension \(\Delta\) is given by
\[
\Delta = \lambda + d = \nu + \frac{d}{2}. \tag{52}
\]
The results agree with the known values for massless \(p\)-form case when \(m\) vanishes [17]. For massive vectors [15, 16] and antisymmetric tensors [22] these are also in good agreement with the known results.
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Appendix

Consider a $D$-dimensional (pseudo-)Riemannian space with a metric tensor $g_{ij}$. For a given differential $p$-form $\omega = \frac{1}{p!} \omega_{i_1 \ldots i_p} dx^{i_1} \cdots dx^{i_p}$ and one form $\eta = \eta_i dx^i$ we define the contraction of $\omega$ by $\eta$ in the following way

$$\eta \langle \omega = \frac{1}{p!} \eta_{i_1 \ldots i_p} g^{i_1 j_1} dx^{j_1} \cdots dx^{j_p} - \frac{1}{p!} \eta_{i_1 \ldots i_p} g^{i_2 j_2} dx^{i_1} dx^{j_1} \cdots dx^{j_p} + \cdots. \quad (A.1)$$

Similarly we define

$$\omega \langle \eta = \frac{1}{p!} \omega_{i_1 \ldots i_p} \eta^{i_1} dx^{j_1} \cdots dx^{j_p} - \frac{1}{p!} \omega_{i_1 \ldots i_p} \eta^{i_2} dx^{i_1} dx^{j_1} \cdots dx^{j_p} + \cdots. \quad (A.2)$$

The dual $^*\omega$ of $\omega$ is a $D - p$ form defined by

$$^*\omega = \frac{1}{p!} \omega_{i_1 \ldots i_p} \eta^{i_1} dx^{i_2} \cdots dx^{i_p} - \frac{1}{p!} \omega_{i_1 \ldots i_p} \eta^{i_2} dx^{i_1} dx^{i_3} \cdots dx^{i_p} + \cdots. \quad (A.3)$$

where $v$ is the volume $D$-form given by

$$v = \sqrt{|g|} dx^1 \cdots dx^D. \quad (A.4)$$

It is easy to show that

$$^*(dx^{i_1} \cdots dx^{i_p}) = \sum_{j_{p+1} < \ldots < j_D} g^{i_1 j_1} \ldots g^{i_p j_p} \epsilon_{j_1 \ldots j_{p+1} \ldots j_D} dx^{j_{p+1}} \cdots dx^{j_D}, \quad (A.5)$$

where $\epsilon_{j_1 \ldots j_D}$ is the usual $D$-dimensional permutation symbol defined by $\epsilon_{1 \ldots D} = 1$. Another useful relation is

$$dx^i \wedge ^*(dx^{i_1} \cdots dx^{i_p}) = ^*(dx^{i_1} \cdots dx^{i_p}) \langle dx^i). \quad (A.6)$$
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