Chapter 1

Heavy quark skyrmions

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The description of the heavy baryons as heavy-meson–soliton bound systems is reviewed. We outline how such bound systems arise from effective lagrangians that respect both chiral symmetry and heavy quark symmetry. Effects due to finite heavy quark masses are also discussed, and the resulting heavy baryon spectra are compared with existing quark model and empirical results. Finally, we address some issues related to a possible connection between the usual bound state approach to strange hyperons and that for heavier baryons.

1.1. Introduction

During the last quarter of a century it has become clear that the applicability of the Skyrme’s topological soliton model for light baryon structure\textsuperscript{1,2} goes far beyond all the original expectations. In fact, as described in other chapters of this book the underlying ideas have found applications in other areas of physics, notably in the physics of complex nuclei and dense matter, condensed matter physics and gauge/string duality. The purpose of the present contribution is to summarize the work done on the extension of the skyrmion picture to the study of the structure of baryons containing heavy quarks. In this scheme, such baryons are described as bound systems of heavy mesons and a soliton. This so-called "bound state approach" was first developed to describe strange hyperons\textsuperscript{3,4} and was later shown\textsuperscript{5} to be applicable to baryons containing one or more charm (c) and bottom (b) quarks. In these early works only pseudoscalar fields were taken as explicit degrees of freedom with their interactions given by a flavor symmetric Skyrme lagrangian supplemented by explicit flavor symmetric terms to account for the effect of the heavy quark mass. The results on the mass spectra\textsuperscript{6} and magnetic moments\textsuperscript{7} for charm baryons were found to be strikingly close to the quark model description which is expected to work better as the heavy quark involved becomes heavier. However, it was then realized that this description in terms of only pseudoscalar fields was at odds with the heavy quark symmetry\textsuperscript{8} which states that in the heavy
quark limit the heavy pseudoscalar and vector fields become degenerate and, thus, should be treated on an equal footing. This difficulty was resolved in Ref.9 where it was proposed to apply the bound state approach to the heavy meson effective lagrangian10–13 which simultaneously incorporates chiral symmetry and heavy quark symmetry. Such observation led to a quite important number of works where various properties of heavy baryons have been studied within this framework. Here, we present a short review of those studies pointing out their main results as well as the relationship between some different approaches used in the literature. Some still remaining open questions are also mentioned.

This contribution is organized as follows. In Sec.1.2 we outline how heavy baryons can be described within soliton models in the heavy quark limit. In particular, in Subsec.1.2.1 we introduce the type of lagrangian that describes the interactions between light and heavy mesons, and which simultaneously respect chiral and heavy quark symmetries, while in Subsec.1.2.2 we show how bound states of a soliton and heavy mesons are obtained and the system quantized. In Sec.1.3 we show how departures from the heavy quark limit can be taken into account. In Sec.1.4 we discuss some issues related to the connection between the usual bound state approach to strange hyperons with that for heavier baryons given in the previous section. Finally, in Sec.1.5 a summary with some conclusions is given.

1.2. Heavy Baryons as Skyrmions in the Heavy Quark Limit

In this section we outline how a heavy baryon can be described within topological soliton models in the limit in which the heavy quarks are assumed to be infinitely heavy. Corrections due to finite heavy quark masses will be discussed in the following section. In Subsec.1.2.1 we introduce a type of lagrangian for a system of Goldstone bosons and the heavy mesons, which possesses both chiral symmetry and heavy quark symmetry. Next, in Subsec.1.2.2 we show how a heavy-meson–soliton bound state can arise at the classical level, and the way in which such bound system can be quantized.

1.2.1. Effective chiral lagrangians and heavy quark symmetry

For the light sector, the simplest lagrangian that supports stable soliton configuration is the Skyrme model lagrangian

\[ \mathcal{L}_{Sk}^{l} = \frac{f_{\pi}^{2}}{4} \text{Tr} \left[ \partial_{\mu}U^{\dagger} \partial^{\mu}U \right] + \frac{1}{32\epsilon^{2}} \text{Tr} \left[ \left[ U^{\dagger} \partial_{\mu}U, U^{\dagger} \partial_{\nu}U \right]^{2} \right], \]  

(1.1)

where \( f_{\pi} \) is the pion decay constant (\( \approx 93 \) MeV empirically) and \( U \) is an \( SU(2) \) matrix of the chiral field, i.e.

\[ U = \exp \left[ iM/f_{\pi} \right], \]  

(1.2)
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with $M$ being a $2 \times 2$ matrix of the pion triplet

$$M = \vec{\tau} \cdot \vec{\pi} = \begin{pmatrix} \pi^0 & \sqrt{2} \pi^+ \\ \sqrt{2} \pi^- & -\pi^0 \end{pmatrix}. \quad (1.3)$$

Here, the chiral $SU(2)_L \times SU(2)_R$ symmetry is realized nonlinearly under the transformation of $U$

$$U \longrightarrow L \, U \, R^\dagger, \quad (1.4)$$

with $L \in SU(2)_L$ and $R \in SU(2)_R$. Due to the presence of the Skyrme term with the Skyrme parameter $e$, the lagrangian $\mathcal{L}_{Sk}^U$ supports stable soliton solutions.

When discussing the interaction of the Goldstone fields $M(x)$ with other fields it is convenient to introduce $\xi(x)$ such that

$$U = \xi^2, \quad (1.5)$$

and which transforms under the $SU(2)_L \times SU(2)_R$ as

$$\xi \rightarrow \xi' = L \, \xi \, \vartheta^\dagger = \vartheta \, \xi \, R^\dagger, \quad (1.6)$$

where $\vartheta$ is a local unitary matrix depending on $L$, $R$, and $M(x)$.

Consider now heavy mesons containing a heavy quark $Q$ and a light antiquark $\bar{q}$. Here, the light antiquark in a heavy meson is assumed to form a point-like object with the heavy quark, endowing it with appropriate color, flavor, angular momentum and parity. Let $\Phi$ and $\Phi^*$ be the field operators that annihilate $j^\pi = 0^-$ and $1^-$ heavy mesons with $C = +1$ or $B = -1$. They form $SU(2)$ antidoublets: for example, when the heavy quark constituent is the $c$-quark,

$$\Phi = (D^0, D^+) \quad , \quad \Phi^* = (D^{*0}, D^{*+}). \quad (1.7)$$

In the limit of infinite heavy quark mass, the heavy quark symmetry implies that the dynamics of the heavy mesons depends trivially on their spin and mass. Such a trivial dependence can be eliminated by introducing a redefined $4 \times 4$ matrix field $H(x)$ as

$$H = \frac{1 + \not{v}}{2} \left( \Phi_\nu \gamma_5 - \Phi^*_\nu \gamma^\mu \right). \quad (1.8)$$

Here, we use the conventional Dirac $\gamma$-matrices and $\not{v}$ denotes $v_\mu \gamma^\mu$. The fields $\Phi_\nu$ and $\Phi^*_\nu$, respectively, represent the heavy pseudoscalar field and heavy vector fields in the moving frame with a four velocity $v_\mu$. They are related to the $\Phi$ and $\Phi^*$ as

$$\Phi = e^{-i v \cdot x} \Phi_\nu \frac{\gamma^\nu}{\sqrt{2m_\Phi}}, \quad \Phi^* = e^{-i v \cdot x} \Phi^*_\nu \frac{\gamma^\nu}{\sqrt{2m_\Phi}}. \quad (1.9)$$

Under $SU(2)_L \times SU(2)_R$ chiral symmetry operations $H$ transforms as

$$H \rightarrow H \, \vartheta, \quad (1.10)$$

while under the heavy quark spin rotation,

$$H \rightarrow S \, H, \quad (1.11)$$
with $S \in SU(2)_v$, i.e. the heavy quark spin symmetry group boosted by the velocity $v$. Taking this into account it is possible to write down a lagrangian that describes the interactions of heavy mesons and Goldstone bosons, and which possesses both chiral symmetry and heavy quark symmetry. To leading order in derivatives acting on the Goldstone fields, the most general form of such lagrangian is given by

$$L_{lh} = -i v_\mu \text{Tr} \left[ D^\mu H \bar{H} \right] - g \text{Tr} \left( H \gamma_5 A_\mu \gamma^\mu H \right), \quad (1.12)$$

where $\bar{H} = \gamma_0 H^\dagger \gamma_0$, and

$$V_\mu = \frac{1}{2} (\xi^1 \partial_\mu \xi + \xi \partial_\mu \xi^1), \quad A_\mu = \frac{i}{2} (\xi^1 \partial_\mu \xi - \xi \partial_\mu \xi^1). \quad (1.13)$$

Here, $g$ is a universal coupling constant for the $\Phi \Phi^* \pi$ and $\Phi^* \Phi^* \pi$ interactions. The nonrelativistic quark model provides the naive estimation

$$g \approx -\frac{3}{4}. \quad (1.12)$$

On the other hand, for the case of the $D^* \rightarrow \pi D$ decay this lagrangian leads to a width given by

$$\Gamma(D^* \rightarrow D^0 \pi^+) = \frac{1}{6\pi} \frac{g^2 f_\pi}{f_\pi} |\vec{p}_\pi|^3. \quad (1.14)$$

Recent experimental results for this width imply $|g|^2 \approx 0.36. \quad (1.14)$

1.2.2. Heavy-Meson–Soliton Bound States in the Heavy Quark Limit and their Collective Quantization

Following the discussions in the previous subsection we consider here the chiral and heavy quark symmetric effective lagrangian given by

$$\mathcal{L} = \mathcal{L}_{Sk}^S + \mathcal{L}_{lh}, \quad (1.15)$$

where $\mathcal{L}_{Sk}^S$ and $\mathcal{L}_{lh}$ are given by Eqs. (1.1) and (1.12), respectively.

In what follows we will discuss how to obtain heavy baryons following a procedure in which a heavy-meson–soliton bound state is first found and then quantized by rotating the whole system in the collective coordinate quantization scheme. An alternative method will be briefly discussed at the end of this subsection.

The non-linear lagrangian $\mathcal{L}_{Sk}^S$ supports a classical soliton solution

$$U_0(\vec{r}) = \exp \left[ i \vec{r} \cdot \hat{r} F(r) \right], \quad (1.16)$$

with the boundary conditions

$$F(0) = \pi \quad \text{and} \quad F(\infty) = 0, \quad (1.17)$$

which, due to its nontrivial topological structure, carries a winding number identified as the baryon number $B = 1$. It also has a finite mass $M_{sol}$ whose explicit expression in terms of the soliton profile function $F(r)$ can be found in e.g. Refs.\textsuperscript{1,2}.

In order to look for possible heavy-meson–soliton bound states we have to find the eigenstates of the heavy meson fields interacting with the static potentials

$$V^\mu = \left( 0, \vec{V} \right) = \left( 0, i v(r) \hat{r} \times \hat{r} \right),$$

$$A^\mu = \left( 0, \vec{A} \right) = \left( 0, \frac{1}{2} a_1(r) \hat{r} + \frac{1}{2} a_2(r) \hat{r} \cdot \hat{r} \right). \quad (1.18)$$
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where

\[ v(r) = \frac{\sin^2(F/2)}{r} , \quad a_1(r) = \frac{\sin F}{r} , \quad a_2(r) = F' - \frac{\sin F}{r} . \] (1.19)

These expressions result from the soliton configuration \(1.16\) sitting at the origin. In the rest frame \(v_\mu = (1, 0, 0, 0)\), it follows from Eq.\(1.18\) that \(H(x)\) can be expressed in terms of \(2 \times 2\) blocks as

\[ H(x) = \begin{pmatrix} 0 & h(x) \\ \bar{h}(x) & 0 \end{pmatrix} . \] (1.20)

Here we have used that, in that frame, \(\Phi^* v, 0\) is identically zero due to the condition \(v \cdot \Phi^* = 0\). Thus, the lagrangian Eq.\(1.12\) takes the form

\[ L_0 = -M_{\text{sol}} + \int d^3r \left( -i \text{Tr} \left[ \dot{h} \bar{h} \right] + g \text{Tr} \left[ h \bar{A} \cdot \bar{\sigma} \bar{h} \right] \right) , \] (1.21)

where \(\bar{h} = -h^\dagger\). The corresponding equation of motion for the \(h\)-field is\(^{17,18}\)

\[ i \dot{h} = g h \bar{A} \cdot \bar{\sigma} . \] (1.22)

In the “hedgehog” configuration \(1.16\), and consequently in the static potentials \(1.18\), the isospin and the angular momentum are correlated in such a way that neither of them is separately a good quantum number, but their sum (the so-called “grand spin”) \(\vec{K}\) is. Here

\[ \vec{K} = \vec{J} + \vec{I} \equiv (\vec{L} + \vec{S}) + \vec{I} . \] (1.23)

Thus, the equation of motion Eq.\(1.22\) is invariant under rotations in \(K\)-space, and the wavefunctions of the heavy meson eigenmodes can be written as the product of a radial function and the eigenfunction of the grand spin \(K_{kk_3}^{(a)}(\hat{r})\). Namely,

\[ h(\vec{r}, t) = \sum_a \alpha_a h_k^{(a)}(r) K_{kk_3}^{(a)}(\hat{r}) e^{-i\epsilon t} , \] (1.24)

where the sum over \(a\) accounts for the possible ways of constructing the eigenstates of the same grand spin and parity by combining the eigenstates of the spin, isospin and orbital angular momentum, and the expansion coefficients \(\alpha_a\) are normalized by \(\sum_a |\alpha_a|^2 = 1\). Since we are assuming here that both the soliton and the heavy mesons are infinitely heavy in the lowest energy state they should be sitting one on top of the other at the same spatial point, just propagating in time. That is, the radial functions \(h_k^{(a)}(r)\) of the lowest energy eigenstate can be approximated by a delta-function-like one, say \(f(r)\), which is strongly peaked at the origin and normalized as \(\int dr r^2 |f(r)|^2 = 1\). Thus, using orthonormalized eigenfunctions \(K_{kk_3}^{(a)}(\hat{r})\) of the grand spin which satisfy

\[ \int d\Omega \text{Tr} \left[ K_{kk_3}^{(a)}(\hat{r}) \bar{K}_{kk_3}^{(a')}(\hat{r}) \right] = -\delta_{aa'} \delta_{kk_3} \delta_{kk_3} \] , \quad (1.25)

the field \(h\) is normalized as

\[ -\int d^3r \text{Tr}[\bar{h}h] = 1 . \] (1.26)
Replacing Eq. (1.24) in Eq. (1.22) and integrating out the radial part, we obtain

\[ \varepsilon_k \mathcal{K}_{kk3}(\hat{r}) = \frac{gF(0)}{2} \mathcal{K}_{kk3}(\hat{r}) (2\hat{\sigma} \cdot \hat{r} \hat{\tau} \cdot \hat{r} - \hat{\sigma} \cdot \hat{r}) , \]

with \( \mathcal{K}_{kk3} = \sum_a \alpha_a \mathcal{K}^{(a)}_{kk3} \). Here, we have used that, near the origin, \( F(r) \sim \pi + F'(0)r \) and consequently \( \tilde{A} \cdot \hat{\sigma} \sim \frac{1}{2} F'(0) (2\hat{\sigma} \cdot \hat{r} \hat{\tau} \cdot \hat{r} - \hat{\sigma} \cdot \hat{r}) \).

Thus, our problem is reduced to finding \( \mathcal{K}_{kk3} \). For this purpose it is convenient to construct the grand spin eigenstates \( \mathcal{K}^{(a)}_{kk3}(\hat{r}) \) by combining the eigenstates of the spin, isospin and orbital angular momentum. Here, we construct first the eigenfunctions of \( \tilde{\Lambda} = \tilde{L} + \tilde{I} \) by combining orbital angular momentum and isospin eigenstates, and then couple the resulting states to the spin eigenstates. Since we are interested here in the lowest energy eigenmode of positive parity, we can restrict the angular momentum \( \ell \) to be 1. This statement requires some explanation. In general, when departures from a delta-like behavior are considered the differential equations for the heavy meson radial functions have a centrifugal term with a singularity \( \ell_{\text{eff}}(\ell_{\text{eff}} + 1)/r^2 \) near the origin. Here, \( \ell_{\text{eff}} \) is the “effective” angular momentum given by \( \ell_{\text{eff}} = \ell \pm 1 \) if \( \lambda = \ell \pm 1/2 \). That behavior is due to the presence of a vector potential from the soliton configuration \( \tilde{V} \sim i(\hat{r} \times \hat{r})/r \) near the origin, which alters the singular structure of \( \tilde{D}^2 = (\nabla - \hat{V})^2 \) from \( \ell(\ell + 1)/r^2 \) of the usual \( \tilde{V}^2 \) to \( \ell_{\text{eff}}(\ell_{\text{eff}} + 1)/r^2 \). Thus, the state with \( \ell_{\text{eff}} = 0 \) can have most strongly peaked radial function and become the lowest eigenstate. Note that \( \ell_{\text{eff}} = 0 \) can be achieved only when \( \ell = 1 \). It is important to notice that combining the negative parity resulting from this orbital wavefunction with the heavy meson intrinsic negative parity we obtain that ground state heavy baryons have positive parity, as expected. For \( \ell = 1 \) two values of \( \lambda, \frac{1}{2} \) and \( \frac{3}{2} \), are possible. Moreover, from the experience of the bound-state approach to strange hyperons, where a similar situation arises\(^3\), the lowest energy state is expected to correspond to the lowest possible value of \( \kappa \), i.e. \( \kappa = \frac{1}{2} \). Since we have \( s = 0, 1 \) and \( \lambda = \frac{1}{2}, \frac{3}{2}, \frac{5}{2} \), we can construct three different grand spin states of \( \kappa = \frac{1}{2} \). Explicitly,\(^{17}\)

\[
\mathcal{K}^{(1)}_{\frac{1}{2} \pm \frac{1}{2}}(\hat{r}) = \frac{1}{\sqrt{\delta_\pi}} \chi_\pm \hat{\tau} \cdot \hat{r} , \\
\mathcal{K}^{(2)}_{\frac{1}{2} \pm \frac{3}{2}}(\hat{r}) = \frac{1}{\sqrt{24\pi}} \chi_\pm \hat{\sigma} \cdot \hat{\tau} \hat{\tau} \cdot \hat{r} , \\
\mathcal{K}^{(3)}_{\frac{1}{2} \pm \frac{5}{2}}(\hat{r}) = \frac{1}{\sqrt{48\pi}} \chi_\pm (\hat{\sigma} \cdot \hat{\tau} \hat{\tau} \cdot \hat{r} - 3 \hat{\sigma} \cdot \hat{r}) .
\]

Here, \( \chi_+ = (0, -1) \) and \( \chi_- = (+1, 0) \) are the isospin states corresponding to \( \bar{u} \) and \( \bar{d} \), respectively. The eigenstates \( \mathcal{K}_{\frac{1}{2} \pm \frac{1}{2}}(\hat{r}) \) of Eq. (1.27) can be expanded in terms of these states

\[ \mathcal{K}_{\frac{1}{2} \pm \frac{1}{2}}(\hat{r}) = \sum_{a=1}^{3} \alpha_a \mathcal{K}^{(a)}_{\frac{1}{2} \pm \frac{1}{2}}(\hat{r}) , \]
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with the expansion coefficients given by the solution of the secular equation

$$\sum_{b=1}^{3} \mathcal{M}_{ab} \alpha_b = -\varepsilon \alpha_a ,$$

(1.30)

where the matrix elements $\mathcal{M}_{ab}$ are defined by

$$\mathcal{M}_{ab} = \frac{gF'(0)}{2} \int d\Omega \text{Tr} \left[ \mathcal{K}^{(a)}(\hat{r}) \left( 2 \hat{\sigma} \cdot \hat{r} \hat{\tau} \cdot \hat{r} - \hat{\sigma} \cdot \hat{r} \right) \mathcal{K}^{(b)}(\hat{r}) \right] .$$

(1.31)

Note that the minus sign in Eq. (1.30) is due to the fact that the basis states $\mathcal{K}^{(a)}(\hat{r})$ are normalized as indicated in Eq. (1.25). With the explicit form of $\mathcal{K}^{(a)}(\hat{r})$ given by Eq. (1.29), these matrix elements can be easily calculated. The secular equation (1.29) yields three eigenstates. Since $g < 0$ and $F'(0) < 0$ (in the case of the baryon-number-1 soliton solution), there is a heavy-meson–soliton bound state of binding energy $-\frac{3}{2}gF'(0)$. The two unbound eigenstates with positive eigenenergy $+\frac{1}{2}gF'(0)$ are not consistent with the strongly peaked radial functions. They are improper solutions of Eq. (1.27).

In terms of the eigenmodes, the Hamiltonian of the system in the body fixed (i.e. soliton) frame has the diagonal form

$$H_{bf} = M_{sol} + \sum_{nkk^3} \varepsilon_{nk} a_{nkk^3} a_{nkk^3}^\dagger =$$

$$= M_{sol} + \varepsilon_{bs} \left( a_{+1/2}^\dagger a_{+1/2} + a_{-1/2}^\dagger a_{-1/2} \right) + \ldots ,$$

(1.32)

where $n$ represents the extra quantum numbers needed to completely specify a given eigenstate. Moreover, $a_{nkk^3}$ ($a_{nkk^3}^\dagger$) are the usual meson annihilation (creation) operators. In the second line of Eq. (1.32) we have explicitly written the contribution of the bound state with $\varepsilon_{gs} = -\frac{3}{2}gF'(0)$ found above, using the subscript $\pm 1/2$ to indicate the grand spin projection $k_3$.

What we have obtained so far is the heavy-meson–soliton bound state which carries a baryon number and a heavy flavor. Therefore, up to order $O(m_Q^0 N_c^0)$ baryons containing a heavy quark such as $\Lambda_Q$, $\Sigma_Q$ and $\Sigma^*_Q$ are degenerate in mass. However, to extract physical heavy baryons of correct spin and isospin, we have to go to the next order in $1/N_c$, while remaining in the same order in $m_Q$, i.e. $O(m_Q^0 N_c^{-1})$. This can be done by introducing time dependent $SU(2)$ collective variables $C(t)$ associated with the degeneracy under simultaneous $SU(2)$ rotation of the soliton configuration and the heavy meson fields

$$\xi(\hat{r}, t) = C(t) \xi_{bf}(\hat{r}) C^\dagger(t) \quad \text{and} \quad h(\hat{r}, t) = h_{bf}(\hat{r}, t) C^\dagger(t) ,$$

(1.33)

where $\xi_{bf}^2 = U_0$, and then performing the quantization by elevating them to the corresponding quantum mechanical operators. In Eq. (1.33) and in what follows, $h_{bf}$ refers to the heavy meson field in the (isospin) soliton frame, while $h$ refers to that in the laboratory frame, i.e., the heavy quark rest frame. Inserting Eq. (1.33) in
Eq. (1.15) we obtain an extra collective contribution of $O(m_Q^0 N_c^{-1})$ to the lagrangian

$$L_{\text{coll}} = \frac{1}{2} \mathcal{I} \omega^2 + \vec{Q} \cdot \vec{\omega},$$

where the “angular velocity” $\vec{\omega}$ of the collective rotation is defined by

$$C^i \dot{C} \equiv i \frac{\hbar}{2} \vec{\tau} \cdot \vec{\omega},$$

$I$ is the moment of inertia of the rotating soliton, whose explicit expression in terms of the soliton profile function $F(r)$ can be found in e.g. Refs. 1, 2, and

$$\vec{Q} = -\frac{1}{4} \int d^3r \text{Tr} \left[ h_{bf} \left( \xi_{bf} \xi_{bf}^T + \xi_{bf}^T \xi_{bf} \right) \right].$$

Taking the Legendre transform of the lagrangian we obtain the collective hamiltonian as

$$H_{\text{coll}} = \frac{1}{2\mathcal{I}} \left( \vec{R} - \vec{Q} \right)^2,$$

where $\vec{R}$ is the spin of the rotor given by $\vec{R} = \mathcal{I} \vec{\omega} + \vec{Q}$.

With the collective variable introduced as in Eq. (1.33), the isospin of the fields $U(x)$ and $h(x)$ is entirely shifted to $C(t)$. To see this, consider the isospin rotation

$$U \rightarrow \mathcal{A} U \mathcal{A}^\dagger, \quad h \rightarrow h \mathcal{A}^\dagger,$$

with $\mathcal{A} \in SU(2)_V$, under which the collective variables and fields in the soliton frame transform as

$$C(t) \rightarrow \mathcal{A} C(t), \quad h_{bf}(x) \rightarrow h_{bf}(x).$$

Thus, the $h$-field is isospin-blind in the (isospin) soliton frame. The conventional Noether construction gives the isospin of the system,

$$I^a = \frac{1}{2} \text{Tr} \left[ a^a C \tau^b C^\dagger \right] \left( \mathcal{I} \omega^b + \vec{Q}^b \right) = D^{ab}(C) R^b,$$

where $D^{ab}(C)$ is the adjoint representation of the $SU(2)$ transformation associated with the collective variables $C(t)$.

The eigenfunctions of the rotor-spin operator are the usual Wigner $D$-functions. In terms of these eigenfunctions and the heavy meson bound states $| \pm 1/2 \rangle_{bs}$, the heavy baryon state of isospin $i_3$ and spin $s_3$ containing a heavy quark can be constructed as

$$| i; i_3, s_3 \rangle = \sqrt{2i + 1} \sum_{m = \pm 1/2} (i, s_3 - m, 1/2, m|1/2, s_3 \rangle D^{(i)}_{i_3 - s_3 + m}(C) |m\rangle_{bs},$$

where $i = 0$ for $\Lambda_Q$ and $i = 1$ for $\Sigma_Q$ and $\Sigma^*_Q$.

Treating the collective Hamiltonian (1.37) to first order in perturbation theory we obtain

$$m_i = M_{\text{sol}} + \varepsilon_{bs} + \frac{1}{2\mathcal{I}} \left( i(i + 1) + 3/4 \right).$$
Here, we have used that explicit evaluation shows
\[
bs\langle m|\bar{Q}|m\rangle_{bs} = 0 , \tag{1.43}
\]
\[
bs\langle m|\bar{Q}^2|m\rangle_{bs} = 3/4 . \tag{1.44}
\]
These two results deserve some comments. First we note that general use of the Wigner-Eckart theorem implies
\[
<n, k, k_3|\bar{Q}|n, k, k'_3> = c_{nk} < n, k, k_3|\bar{Q}|n, k, k'_3> . \tag{1.45}
\]
The constants \(c_{nk}\) are usually called "hyperfine splitting" constants. Eq.(1.43) implies that for the ground state \(c_{gs} = 0\) in the heavy quark limit. As a consequence of this, the Hamiltonian depends only on the rotor-spin so that \(\Sigma_Q\) and \(\Sigma_Q^*\) become degenerate as expected from the heavy quark symmetry. It is clear that corrections that imply departures from heavy quark limit could lead to non-vanishing values of \(c_{gs}\). It is also important to notice that to obtain the result Eq.(1.44) one should take into account all possible intermediate states.

In order to compare the results with experimental heavy baryon masses, we have to add the heavy meson masses subtracted so far from the eigenenergies. The mass formulas to be compared with data are

\[
m_{\Lambda_Q} = M_{sol} + \overline{m}_\Phi - \frac{3}{2}gF'(0) + \frac{3}{8I} ,
\]
\[
m_{\Sigma_Q} = m_{\Sigma_Q^*} = M_{sol} + \overline{m}_\Phi - \frac{3}{2}gF'(0) + \frac{11}{8I} , \tag{1.46}
\]
where \(\overline{m}_\Phi\) is the weighted average mass of the heavy meson multiplets, \(\overline{m}_\Phi = (3m_{\Phi^*} + m_\Phi)/4\). In the case of \(Q = c\), we have \(\overline{m}_\Phi = 1973\) MeV while for \(Q = b\), \(\overline{m}_\Phi = 5314\) MeV. The \(SU(2)\) quantities \(M_{sol}\) and \(I\) are obtained from the nucleon and \(\Delta\) masses
\[
M_{sol} = 866\text{ MeV}, \quad \text{and} \quad 1/I = 195\text{ MeV}. \tag{1.47}
\]
Finally, the unknown value of \(gF'(0)\) can be adjusted to fit the observed value of the \(\Lambda_c\) mass,
\[
m_{\Lambda_c} = 2286\text{ MeV} = M_{sol} + \overline{m}_\Phi - \frac{3}{2}gF'(0) + \frac{3}{8I} , \tag{1.48}
\]
which implies that
\[
gF'(0) = 417\text{ MeV}. \tag{1.49}
\]
This leads to a prediction on the \(\Lambda_b\) mass and the average masses of the \(\Sigma_Q^{*}\Sigma_Q\) multiplets, \(\overline{m}_{\Sigma_Q} = \frac{1}{3}(2m_{\Sigma_Q} + m_{\Sigma_Q^*})\),
\[
m_{\Lambda_b} = M_{sol} + \overline{m}_B - \frac{3}{2}gF'(0) + 3/8I = 5627\text{ MeV} , \tag{1.50}
\]
\[
\overline{m}_{\Sigma_c} = M_{sol} + \overline{m}_D - \frac{3}{2}gF'(0) + 11/8I = 2481\text{ MeV} , \tag{1.51}
\]
\[
\overline{m}_{\Sigma_b} = M_{sol} + \overline{m}_B - \frac{3}{2}gF'(0) + 11/8I = 5822\text{ MeV} . \tag{1.52}
\]
These are comparable with the experimental masses\textsuperscript{19} of $\Lambda_b$ (5620 MeV), $\Sigma_c$ (2454 MeV), $\Sigma_c^*$ (2518 MeV), $\Sigma_b$ (5811 MeV) and $\Sigma_b^*$ (5833 MeV). Furthermore, with the Skyrme lagrangian (with the quartic term for stabilization), the wavefunction has a slope $F'(0) \sim -2ef_s \approx -700$ MeV near the origin, which implies $g \sim -0.6$. This is also consistent with the values given at the end of the previous subsection.

The role of light vector mesons in the description of the heavy-meson–soliton system was analyzed in Ref.\textsuperscript{16}. In fact, using effective heavy quark symmetric lagrangians that incorporate light vector mesons,\textsuperscript{21,22} it was shown that the effect of these light degrees of freedom could be relevant. Within this scheme the extension of the light flavor group to $SU(3)$ was also considered.\textsuperscript{23}

Up to now, we have discussed how one can obtain the heavy baryon states containing a heavy quark, $\Sigma_Q$, $\Sigma_Q^*$ and $\Lambda_Q$, as heavy-meson–soliton bound states treated in the standard way: a heavy-meson–soliton bound state is first found and then quantized by rotating the whole system in the collective coordinate quantization scheme. This amounts to proceeding systematically in a decreasing order in $N_c$; i.e.: in the first step only terms up to $N_c^0$ order are considered, in the next step terms of order $1/N_c$ are also taken into account, etc. In this way of proceeding, the heavy mesons first lose their quantum numbers (such as the spin and isospin), with only the grand spin preserved. The good quantum numbers are recovered when the whole system is quantized properly. An alternative approach was adopted in Ref.\textsuperscript{9} In this approach, the soliton is first quantized to produce the light baryon states such as nucleons and $\Delta$’s with correct quantum numbers. Then, the heavy mesons with explicit spin and isospin are coupled to the light baryons to form heavy baryons as a bound state. Compared with the traditional one which is a “soliton body-fixed” approach, this approach may be interpreted as a “laboratory-frame” approach. It has been shown\textsuperscript{17}, however, that both approaches lead to the same results in the heavy quark limit.

It should be stressed that in the heavy quark limit discussed so far one cannot account for the experimentally observed hyperfine splittings, like e.g. the $\Sigma_c^*$-$\Sigma_c$ mass difference. Another consequence of taking such limit is the existence of parity doublets in the spectrum of the low-lying excited states.\textsuperscript{18,20} This follows from the fact that in the heavy quark limit the centrifugal barrier that would affect states with $\ell_{eff} > 0$ plays no role. It is clear that finite heavy quark mass corrections have to be taken into account in order to have a more realistic description of the heavy baryon properties in the present topological soliton framework. How to account for such corrections will be discussed in the following section.

1.3. Beyond the Heavy Quark Limit

In the previous section, we have limited ourselves to the heavy quark limit. Thus, heavy baryon masses have been computed to leading order in $1/m_Q$, that is to
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\[ O(m_Q^0). \] Here, we will consider the corrections implied by the use of finite heavy quark masses.

The \( \Sigma^*_Q-\Sigma_Q \) mass difference due to the leading heavy quark symmetry breaking was first computed in Ref.\textsuperscript{24} using the alternative method mentioned at the end of Subsec.1.2.2. As an illustration of the equivalence of the two approaches, we briefly discuss how the corresponding results can be obtained using the soliton body fixed approach described at length in that subsection. The leading order lagrangian in the derivative expansion that breaks the heavy quark symmetry is\textsuperscript{10}

\[ \mathcal{L}_1 = \frac{\lambda_2}{m_Q} \text{Tr} \left[ \sigma^{\mu\nu} H \sigma_{\mu\nu} \bar{H} \right], \]

which leads to a \( \Phi^*-\Phi \) mass difference

\[ m_{\Phi^*} - m_\Phi = -\frac{8\lambda_2}{m_Q}. \]

Assuming as in Subsec.1.2.2 that the radial functions are peaked strongly at the origin, the inclusion of this heavy quark symmetry breaking lagrangian implies that the equation of motion Eq.(1.22) gets an additional term. Namely, one obtains

\[ i \dot{h} = g h \vec{A} \cdot \vec{\sigma} + \frac{2\lambda_2}{m_Q} \vec{\sigma} \cdot (h \vec{\sigma}). \]

One can now consider the last term as a perturbation and compute its effect on the \( k = 1/2 \) bound state. Since \( \mathcal{L}_1 \) breaks only the heavy quark spin symmetry the grand spin is still a good symmetry of the equation of motion. Thus, the eigenstates can be classified by the corresponding quantum numbers. Expanding in terms of the three possible basis states \( K^{(a)}_{\frac{1}{2}k_3} \) given in Eq.(1.29) the problem reduces to finding the solution of the secular equation

\[ \sum_{b=1}^{3} (M_{ab} + \delta M_{ab}) \alpha_b = -\varepsilon \alpha_a, \]

with \( M_{ab} \) given by Eq.(1.31) and

\[ \delta M_{ab} = -\frac{2\lambda_2}{m_Q} \int d\Omega \text{Tr} \left[ \vec{\sigma} \cdot \left( K^{(a)}_{\frac{1}{2}k_3} \vec{\sigma} \right) K^{(b)}_{\frac{1}{2}k_3} \right]. \]

It turns out that up to first order in perturbation, the bound state energy remains unchanged while the corresponding eigenstate \( K^{(a)}_{\frac{1}{2}k_3} \) is perturbed to

\[ K^{(a)}_{\frac{1}{2}k_3} = \frac{1}{2} (1 + 3 \kappa) K^{(1)}_{\frac{1}{2}k_3} - \frac{\sqrt{3}}{2} (1 - \kappa) K^{(2)}_{\frac{1}{2}k_3} , \]

with

\[ \kappa = -\frac{\lambda_2}{m_Q g F'(0)}. \]

The heavy baryons can be obtained by quantizing the heavy-meson–soliton bound state in the same way as explained in Subsec.1.2.2. It leads to the heavy
baryon states of Eq. (1.41) with $|m\rangle_{bs}$ replaced by the perturbed state of Eq. (1.58).

Due to the perturbation, the expectation value of $\bar{Q}$ defined by Eq. (1.36) with respect to the bound states does not vanish. In fact, one gets that the hyperfine constant is given by

$$c = 2\epsilon = -\frac{2\lambda_2}{m_Q g F'(0)}.$$  \hspace{1cm} (1.60)

With the help of Eq. (1.45), one can compute the expectation value of the collective hamiltonian (1.37)

$$m_{i,j} = M_{sol} + \varepsilon_{bs} + \frac{1}{2I} \left( (1-c)i(i+1) + cj(j+1) - ck(k+1) + \frac{3}{4} \right).$$  \hspace{1cm} (1.61)

Thus, the $\Sigma_c^* - \Sigma_c$ mass difference is obtained as

$$m_{\Sigma_c^*} - m_{\Sigma_c} = \frac{3c}{2I} = \frac{(m_\Delta - m_N)(m_{\Phi^*} - m_{\Phi})}{4g F'(0)},$$  \hspace{1cm} (1.62)

where Eqs. (1.54) and (1.60) together with the resulting expression for the $\Delta$-$N$ mass splitting in terms of the moment of inertia $I$ have been used. Note that the mass splittings have the dependence on $m_Q$ and $N_c$ that agrees with the constituent quark model. The $\Phi^* -$ $\Phi$ mass difference is of order $1/m_Q$ and the $\Delta$-$N$ mass difference is of order $1/N_c$. This implies that the $\Sigma_c^* -$ $\Sigma_c$ mass difference is of order $1/(m_Q N_c)$.

Substituting $g F'(0) = 417$ MeV, we obtain

$$m_{\Sigma_c^*} - m_{\Sigma_c} = 25 \text{ MeV} \quad \text{and} \quad m_{\Sigma_b^*} - m_{\Sigma_b} = 8 \text{ MeV}.$$  \hspace{1cm} (1.63)

The experimentally measured $\Sigma_c^* -$ $\Sigma_c$ mass difference $\sim 64$ MeV is about three times larger than this Skyrme model prediction. Something similar happens in the case of the $\Sigma_b^* -$ $\Sigma_b$ mass difference, the empirical value of which is $\sim 21$ MeV.

This failure to reproduce the observed hyperfine splittings naturally suggests the need for including additional heavy-spin violating terms, of higher order in derivatives. However, since there are many possible terms with unknown coefficients such a systematic perturbative approach turns out not to be very predictive. To overcome this problem some relativistic lagrangian models written in terms of the ordinary pseudoscalar and vector fields (rather than the heavy fluctuation field multiplet Eq. (1.8)) have been used. A typical model of this type which only includes pseudoscalar fields in the light sector is given by

$$L = L_i^{sk} + D_{\mu} \Phi (D^\mu \Phi)^\dagger - m_\Phi \Phi \Phi^\dagger - \frac{1}{2} \Phi^{*\mu\nu} \Phi_{\mu\nu}^{*\dagger} + m_{\Phi^*} \Phi^* \Phi^{*\dagger} + f_\phi (\Phi^\mu A^\mu \Phi^{*\dagger} + \Phi_{\mu}^{*\dagger} A^\mu \Phi) + \frac{g_\phi}{2} \varepsilon^{\mu\nu\lambda\rho} (\Phi_{\mu\nu} A^\lambda \Phi^*_{\rho} + \Phi_{\mu}^{*\dagger} A^\lambda \Phi^*_{\nu})$$  \hspace{1cm} (1.64)

where $D_{\mu} \Phi = \partial_{\mu} \Phi - \Phi V_{\mu}$, $\varepsilon_{0123} = +1$, and $f_\phi$ and $g_\phi$ are the $\Phi^* \Phi M$ and $\Phi^* \Phi^* M$ coupling constants, respectively. The field strength tensor is defined in terms of the covariant derivative $D_{\mu} \Phi_{\nu}^* = \partial_{\mu} \Phi^*_{\nu} - \Phi^*_{\nu} V_{\mu}$ as

$$\Phi_{\mu\nu}^* = D_{\mu} \Phi_{\nu}^* - D_{\nu} \Phi^*_{\mu},$$  \hspace{1cm} (1.65)
and the vector $V_\mu$ and axial vector $A_\mu$ have been defined in Eq. (1.13). In principle, Eq. (1.64) has two independent coupling constants $f_Q$ and $g_Q$. However, in order to respect heavy quark symmetry they should be related to each other as

$$\lim_{m_Q \to \infty} f_Q / 2m_\Phi = \lim_{m_Q \to \infty} g_Q = g ,$$

where $g$ is the universal coupling constant appearing in Eq. (1.12). It should be noted that even to order $1/m_Q$, Eq. (1.64) leads to extra contributions to the hyperfine splittings.

The interacting heavy-meson–soliton system described by the lagrangian Eq. (1.64) can be treated following a procedure similar to the one described at length in Subsec 1.2.2. It should be noted, however, that the need to treat the finite mass corrections non-perturbatively implies that departures from a $\delta$-like behaviour of the heavy meson radial wavefunctions should be taken into account. Thus, the equations of motions which describe the dynamics of the heavy mesons moving in the static soliton background field should be solved numerically. It turns out that, for a given value of $g$, the binding energies are somewhat smaller than the ones obtained in the heavy quark limit. Concerning the hyperfine splittings, although the use of the effective lagrangian Eq. (1.64) leads to some improvement, it is not still sufficient to bring the predicted $\Sigma_Q^* - \Sigma_Q$ mass splitting into agreement with experiment. The prediction for such a splitting is actually correlated to those for the $\Sigma_Q - \Lambda_Q$ and $\Delta - N$ splittings according to

$$m_{\Sigma_Q^*} - m_{\Sigma_Q} = m_\Delta - m_N - \frac{3}{2} (m_{\Sigma_Q} - m_{\Lambda_Q}) .$$

This formula follows from Eq. (1.61), and depends only on the collective quantization procedure being used rather than on the detailed structure of the model. If $m_\Delta - m_N$ and $m_{\Sigma_Q} - m_{\Lambda_Q}$ are taken to agree with their empirical value, Eq. (1.67) predicts 42 MeV rather than the empirical value 64 MeV. In the case of the bottom baryons one gets 6 MeV to be compared to the empirical value 21 MeV. This means that, within the present quantization framework, it is not possible to exactly predict all the three mass differences appearing in Eq. (1.67). Thus, the goodness of the approach must be judged by looking at the overall predictions for the heavy baryon masses.

In this context, the study of possible excited states turns out to be of great interest. As already mentioned, in the heavy quark limit degenerate doublets of excited states are obtained. However, such limit implies that both the soliton and the heavy mesons are infinitely heavy sitting one on top of the other. It is evident that, due to the ignorance of any kinetic effects, this approximation is not expected to work well for the orbitally and/or radially excited states. In Ref. 27, the kinetic effects due to the finite heavy meson masses were estimated by approximating their static potentials by a quadratic form with the curvature determined at the origin. Such a harmonic oscillator approximation is valid only when the heavy mesons are sufficiently massive so that their motions are restricted to a very small range. The
situation was somewhat improved in Ref.\textsuperscript{20} by solving an approximate Schrödinger-like equation and incorporating the light vector mesons. In the context of the model defined by Eq.\textsuperscript{(1.64)}, in which only pseudoscalar degrees of freedom are present in the light sector, the exact solution of the equations of motion of the heavy meson bound states were first obtained in Ref.\textsuperscript{28} and their collective coordinate quantization performed in Ref.\textsuperscript{29} The typical resulting excitation spectra for the low-lying charm and bottom baryons obtained from these calculations (SM) are shown in Figs.\textsuperscript{11} and \textsuperscript{12} respectively.

For comparison, we also include in these figures the results of the quark model (QM) calculation of Ref.\textsuperscript{30} (more recent quark model calculations\textsuperscript{31} lead to qualitatively similar results), those resulting from naive extension\textsuperscript{6} of the bound state approach to the strangeness (NSM) and the empirically known values\textsuperscript{19} (EXP).

Fig. 1.1. Excitation spectra of charm baryons in soliton models as compared to the results of the quark model (QM) of Ref.\textsuperscript{30} and the present empirical data\textsuperscript{19} (EXP). NSM corresponds to the soliton model calculation of Ref.\textsuperscript{6} where heavy quark symmetry has not been explicitly implemented. SM and VMM refer to soliton models which incorporate heavy quark symmetry. SM corresponds to a calculation\textsuperscript{29} where only pseudoscalars have taken into account in the light sector, while VMM to the calculation of Ref.\textsuperscript{32} where light vector mesons have been also explicitly included. The numbers above the lowest $\Lambda_c$ state correspond to the absolute masses (in MeV) of this state.
Note that the excitation energies are taken with respect to the mass (also indicated in the figures) of the lowest $\Lambda_c$ and $\Lambda_b$, respectively. Finally, in order to see the impact of including the light vector mesons in the effective lagrangian, the excitation spectra resulting from the calculations of Ref.\textsuperscript{32} (VMM) are also displayed.

In the case of the charm sector, we observe that the predictions for the absolute values of the ground state $\Lambda_c$ mass are similar in all soliton models calculations, and are in reasonable agreement with its empirical value and the QM prediction. As for the low lying spectra, we see that they are all qualitatively similar. From a more quantitative point of view, the SM version of the skyrmion models seems to provide a more accurate description of the splitting between the two lowest lying negative parity excited $\Lambda_c$ baryons, although the corresponding centroid is somewhat underestimated as compared with present experimental results. In any case, for these particular states the soliton models based on heavy quark symmetry certainly do better than the QM of Ref.\textsuperscript{30} and the soliton calculation NSM. For the $\Sigma_c$ baryons, the predictions of the SM and VMM results are very similar with the main difference, with respect to the QM and NSM predictions, being the position of the second $1/2^-$ state. Concerning the bottom sector, looking at the absolute value of the ground state $\Lambda_b$, we clearly see that the NSM tends to grossly overestimate

![Fig. 1.2. Excitation spectra of bottom baryons. Notation as in Fig.1.1.](image)
the bottom meson binding energy. In this sense, although as discussed below the
inclusion of other effects might still be required, the soliton models based in heavy
quark symmetry (SM and VMM) lead to predictions which are in much better
agreement with the empirical values. As for the excitation spectra, we see that all
the models predict a similar ordering of low-lying states. However, the only two
excitation energies that can be compared with existing empirical data, i.e. those
corresponding to the $\Sigma_b$ and $\Sigma_b^*$, are also much better reproduced by the SM and
VMM results. It should be noticed that those models also predict rather small
excitation energies ($\approx 200$ MeV) for the lowest lying negative $1/2^-$ and $3/2^-$ states
as compared with the QM prediction (above $300$ MeV).

Another kinetic correction that has to be taken into account is related to the
recoil effects due to the finite soliton mass. This type of effect has been considered
in several works. As expected, they tend to decrease the heavy-meson–
soliton binding energies leading to predictions which, particularly in the case of
bottom baryons, are in better agreement with empirical data.

It should be mentioned that in the combined heavy quark and large $N_c$ limit a
dynamical symmetry connecting excited heavy baryon states with the correspond-
ing ground states exists. Assuming that such symmetry holds as an approximate
symmetry at finite values of $m_Q$ and $N_c$ one can develop an effective theory formu-
lated in terms of the expansion parameter $\lambda \sim 1/m_Q, 1/N_c$. Within such scheme,
up to next-to-leading order an average excitation energy of $\sim 300$ MeV is obtained
for the first negative parity $\Lambda_b$ excited states. Such value is somewhat larger than
the one obtained within heavy-meson–soliton bound state models, as it can be seen
from Fig.1.2.

We conclude this section by mentioning that, in addition to the masses, other
heavy baryon properties have been studied using the heavy-meson–soliton bound
state picture. For example, magnetic moments have been analyzed in the heavy
quark limit and beyond it. The radiative decays of excited $\Lambda_Q$ have been also
considered. Finally, the possible existence of multibaryons with heavy flavors and
other exotic states have also been investigated.

1.4. Relation with the bound state approach to strangeness

Thus far, we have discussed in detail a description of heavy baryons in which one
begins from the heavy quark symmetry limit and then consider deviations from
such a limit which start with order $1/m_Q$ corrections. However, as mentioned in
the introduction, the picture proposed in Ref. in which the heavy quark regime
is approached from below, i.e. starting form a chiral invariant lagrangian and ac-
counting for the heavy meson mass effects by the inclusion of suitable symmetry
breaking terms, also turns out to be, at least qualitatively, successful. Therefore, it
is interesting to see whether it is possible to find a dynamical scheme which allows
to go continuously from the chiral regime to the heavy quark regime.
Suppose that one starts with three massless quarks, assuming the spontaneous breaking of chiral $SU(3)_L \times SU(3)_R$ down to the $SU(3)_V$ vector symmetry. The chiral field can be written as

$$U = \exp \left[ \frac{i}{f_\pi} \begin{pmatrix} \phi^0 + \frac{1}{\sqrt{3}} \psi & \sqrt{2} \phi^+ \sqrt{\pi} & \phi^- \\ \sqrt{2} \pi & -\phi^0 + \frac{1}{\sqrt{3}} \psi & \phi^0 \\ \phi^0 - \frac{2}{\sqrt{3}} \psi & \phi^0 & -\frac{2}{\sqrt{3}} \psi \end{pmatrix} \right] .$$

Here, $\Phi^+$, $\Phi^0$, $\Phi^-$, $\bar{\Phi}^0$ and $\Psi$ denote the mesons with the quantum numbers of $\bar{h}\gamma_5 u$, $\bar{h}\gamma_5 d$, $\bar{u}\gamma_5 h$ and $\bar{d}\gamma_5 h$, respectively. For example, if $h = s$, they correspond to $K^+$, $K^0$, $K^-$, $\bar{K}^0$ and $\eta_8$. The effective action can be obtained by adding the Wess-Zumino term to the lagrangian for interactions among the Goldstone bosons given by generalizing Eq. (1.1) to three flavors. Namely,

$$\Gamma = \int d^4 x \mathcal{L}_s^S + \Gamma_{WZ} .$$

The Wess-Zumino term cannot be written as a local lagrangian density in $(3 + 1)$ dimensions. However, it can be expressed as a local action in five-dimensions,

$$\Gamma_{WZ} = -\frac{in_f}{240\pi^2} \int_{M_5} d^5 x \varepsilon^{\mu\nu\rho\sigma\lambda} \text{Tr} \left[ U^\dagger \partial_\mu U U^\dagger \partial_\nu U U^\dagger \partial_\rho U U^\dagger \partial_\sigma U U^\dagger \partial_\lambda U \right] ,$$

where the integration is over a five-dimensional disk whose boundary is the ordinary space-time $M_4$ and $U$ is extended so that $U(\vec{r}, t, s = 0) = 1$ and $U(\vec{r}, t, s = 1) = U(\vec{r}, t)$. This term is non-vanishing for $N_f \geq 3$. When the soliton is built in $SU(2)$ space, this term does not contribute. However, we shall be considering $(2+1)$ flavors where one flavor can be heavy, in which case the dynamics can be influenced by the Wess-Zumino term as in the Callan-Klebanov (CK) model. What we are interested in is the situation where the symmetry $SU(3)_L \times SU(3)_R$ is explicitly broken to $SU(2)_L \times SU(2)_R \times U(1)$ by an $h$-quark mass, thereby making the $\Phi$-meson massive and its decay constant $f_\Phi$ different from that of the pion. These two symmetry breaking effects can be effectively incorporated into the lagrangian by a term of the form

$$\mathcal{L}_{sb} = \frac{1}{6} \left( f_\phi^2 - m_\phi^2 \right) \text{Tr} \left[ (1 - \sqrt{3} \lambda_8) (U + U^\dagger - 2) \right] + \frac{1}{12} \left( f_\phi^2 - f_\pi^2 \right) \text{Tr} \left[ (1 - \sqrt{3} \lambda_8) (U \partial_\mu U^\dagger \partial^\mu U + U^\dagger \partial_\mu U \partial^\mu U^\dagger) \right] ,$$

where, for simplicity, we turn off the light quark masses. The appropriate ansatz for the chiral field is the CK-type which we shall take in the form

$$U = N_\pi N_\Phi N_\pi ,$$

where $N_\pi = \text{diag}(\xi, 1)$, with the $SU(2)$ matrix $\xi$ defined by Eq. (1.5), and

$$N_\Phi = \exp \left[ \frac{1}{f_\phi} \begin{pmatrix} \sqrt{2} & \Phi^1 \\ 0 & \Phi^0 \end{pmatrix} \right] .$$
with the Φ-meson anti-doublets Φ = (Φ−, ¯Φ0) and doublets Φ† = (Φ+, Φ0)†.

Substituting the CK ansatz (1.72) into the action (1.69) with the symmetry breaking term (1.71) and expanding up to second order in the Φ-meson field, we obtain

$$L = L^{Sk}_k + D_\mu \Phi (D^\mu \Phi)^\dagger - M^2_\Phi \Phi \Phi^\dagger - \Phi A_\mu A^\mu \Phi^\dagger - \frac{iN_c}{4f_\pi} B_\mu \left( D^\mu \Phi \Phi^\dagger - \Phi (D^\mu \Phi)^\dagger \right),$$

(1.74)

where we have rescaled the Φ-meson fields as Φ/κ with κ = f_/f_π. The covariant derivative (D_μΦ)^† is (∂_μ + V_μ)Φ^†, the vector field V_μ and the axial-vector field A_μ are the same as in the lagrangian (1.64), and B_μ is the topological current

$$B^\mu = \frac{1}{24\pi^2} \epsilon^{\mu\nu\lambda\rho} \text{Tr} \left[ U^\dagger \partial_\nu U U^\dagger \partial_\lambda U U^\dagger \partial_\rho U \right],$$

(1.75)

which is the baryon number current in the Skyrme model.

With the identification Φ = K, the lagrangian Eq.(1.74) has been successfully used in the strange sector. In fact, using the empirical values for m_K and the f_K/f_π ratio this lagrangian leads to a kaon-soliton bound state which allows for a very good description of the strange hyperon spectrum, once an SU(2) collective quantization similar to the one described in Subsec.1.2.2 is performed. Moreover, the existence of an excited ℓ = 0 state provides a natural explanation for the negative parity Λ(1405) hyperon.3,4,6

The results displayed at the end of Sec.1.3 (those labelled NSM in Figs.1 and 2) show that the straightforward extension of this approach5,6 leads to reasonable results in the charm sector, while it certainly fails to provide a quantitative good description of the bottom baryons. This clearly indicates that new explicit degrees of freedom have to be included in the effective lagrangian in order to have the correct heavy quark limit.

To proceed it is important to observe that, to the lowest order in derivatives on the Goldstone boson fields, Eq. (1.74) is the same as the lagrangian Eq. (1.64) when only the heavy pseudoscalars are considered. Furthermore, as argued in Refs.37-49, as the h quark mass increases above the chiral scale Λ_χ, the Wess-Zumino term is expected to vanish, thereby turning off the last term of (1.74). Thus, the two lagrangians are indeed equivalent as far as the pseudoscalars are concerned. However, as discussed in the previous sections, in order to have the correct heavy quark limit one should explicitly take into account the heavy vector degrees of freedom, which become degenerate with the pseudoscalars as one approaches that limit. From an effective lagrangian point of view, the vector mesons can be viewed as “matter fields”. There are several ways of introducing vector matter fields. Here we follow the hidden gauge symmetry (HGS) approach50 in which case the non-anomalous effective lagrangian is

$$L_0 = -\frac{f_\pi^2}{4} \text{Tr} \left[ D_\nu \xi_L \xi_L^\dagger - D_\nu \xi_R \xi_R^\dagger \right]^2 - \alpha \frac{f_\pi^2}{4} \text{Tr} \left[ D_\nu \xi_L \xi_L^\dagger + D_\nu \xi_R \xi_R^\dagger \right]^2 - \frac{1}{2} \text{Tr} (F_{\mu\nu} F^{\mu\nu}).$$

(1.76)
Here, $D_\mu = \partial_\mu + ig_\ast U_\mu$ with
\[
U_\mu = \frac{1}{2} \left( \omega_\mu + \rho_\mu \sqrt{2} \Phi_\mu^* \Psi_\mu^* \right),
\]
(1.77)
and $g_\ast$ is a gauge coupling constant to be specified later. The field strength tensor of the vector mesons is $F_{\mu\nu} = D_\mu U_\nu - D_\nu U_\mu$, and the fields $\xi_L$ and $\xi_R$ are related to the chiral field by $U(x) = \xi_L^\dagger \xi_R$. The vector meson mass $M_{\rho,\omega}$, and the $\rho\pi\pi$ coupling constant can be read off from the lagrangian,
\[
M_{\rho,\omega}^2 = a g_\ast^2 f_\pi^2 \quad ; \quad g_{\rho\pi\pi} = \frac{a}{2} g_\ast.
\]
(1.78)

The usual KSRF relation $m_\rho^2 = 2g_\ast^2 f_\pi^2$, and the universality of the vector-meson coupling $g_{\rho\pi\pi} = g_\ast$, can be used to fix the arbitrary parameter $a$ to 2.

The effective action should satisfy the same anomalous Ward identities as does the underlying fundamental theory, QCD. In the presence of vector mesons $A_{L,R}$ associated with the external (e.g. electroweak) gauge transformations, the general form of the anomalous lagrangian is given by a special solution of the anomaly equation plus general solutions of the homogeneous equation. The former is the so-called gauged Wess-Zumino action $\Gamma_{GWZ}$ (see e.g. Ref. for details) and the latter, the anomaly free terms, can be made of four independent blocks $L_i$ whose explicit forms can be found in Ref. Thus, for the anomalous processes we have
\[
\Gamma_{an} = \Gamma_{GWZ}^0[\xi_L^\dagger \xi_R, A_L, A_R] + \sum_{i=1}^4 \gamma_i \int_{M_4} d^4 x \, L_i,
\]
(1.79)
with four arbitrary constants $\gamma_i$, which are determined by experimental data. Vector meson dominance (VMD) in processes like $\pi^0 \rightarrow 2\gamma$ and $\gamma \rightarrow 3\pi$ is very useful in determining these constants.

As for the symmetry breaking one can take the form
\[
L_{sb} = -\frac{f_\pi^2}{4} \text{Tr} \left\{ (D_\mu \xi_L \xi_L^\dagger - D_\mu \xi_R \xi_R^\dagger) (\xi_R \varepsilon A_L \xi_L^\dagger + \xi_L \varepsilon A_R \xi_R^\dagger) \right\}
-\frac{a f_\pi^2}{4} \text{Tr} \left\{ (D_\mu \xi_L \xi_L^\dagger + D_\mu \xi_R \xi_R) - \xi_R \varepsilon V \xi_L^\dagger + \xi_L \varepsilon V \xi_R^\dagger) \right\}.
\]
(1.80)
The matrix $\varepsilon_{A(V)}$ is taken to be $\varepsilon_{A(V)} = \text{diag}(0,0,c_{A(V)})$, where $c_{A(V)}$ are the SU(3)-breaking real parameters to be determined. In terms of them one obtains
\[
m_{\Phi*}^2 = (1 + c_V) m_{\rho,\omega}^2 \quad ; \quad f_\Phi^2 = (1 + c_A) f_\pi^2.
\]
(1.81)

Finally, we substitute the CK ansatz Eq. (1.72), (that is, $\xi_L^\dagger = N_\pi \sqrt{U_\Phi}$ and $\xi_R = \sqrt{U_\Phi} N_\pi$) into the total effective action
\[
\Gamma = \Gamma_0 + \Gamma_{an} + \Gamma_{sb},
\]
(1.82)
where $\Gamma_0$ and $\Gamma_{sb}$ are obtained from the lagrangians Eq. (1.76) and Eq. (1.80), respectively, and the action $\Gamma_{an}$ is given in Eq. (1.79). One may check that the
resulting lagrangian contains all the terms of Eq. (1.64). Explicitly, one gets

\[
\mathcal{L} = \mathcal{L}_I^{Sk} + D_\mu \Phi D_\mu \Phi^\dagger - m_\Phi^2 \Phi \Phi^\dagger - \frac{1}{2} \Phi^{\mu\nu} \Phi^{\dagger \mu\nu} + m_\Phi^2 \Phi^\dagger \Phi^\dagger + \sqrt{2} m_\Phi \Phi^\dagger (\Phi A^{\mu} \Phi^\dagger + \Phi^* A^{\mu} \Phi^\dagger) + \ldots ,
\]

(1.83)

where the light vector meson fields \( \rho_\mu \) and \( \omega_\mu \) have been replaced by \( 2i V_\mu / g_* \) and \( (c_1 - c_2) i 6\pi^2 B_\mu / g_* f_\pi^2 \), respectively, and terms with higher derivatives acting on the pion fields have not been explicitly written. Comparing Eq. (1.83) with Eq. (1.64), we obtain two relations

\[
f_Q = -\sqrt{2} m_\Phi^* , \quad \text{and} \quad g_Q = i \gamma_4 g_\pi^2 .
\]

(1.84)

The first relation implies that

\[
\frac{f_Q}{2m_\Phi^*} = -\frac{1}{\sqrt{2}} ,
\]

(1.85)

which is quite close to the expected heavy quark limit result Eq.(1.66) with \( g = -0.75 \) evaluated with the NRQM in Sec. 2. Using this relation and assuming that the VMD works in the heavy meson sector, in which case \( \gamma_4 = i N_c / 16\pi^2 \), one obtains \( g_* \) in the heavy quark limit, i.e.

\[
g_* = \sqrt{\frac{16\pi^2}{\sqrt{2} N_c}} \simeq 6 \quad \text{(with } N_c=3 \text{)} .
\]

(1.86)

which is close to \( g_* = g_{\rho\pi\pi} \) found in the light sector. These results seem to indicate that, in principle, it might be possible to construct an effective soliton model which could be used to describe both the strange sector and the heavier sectors. Of course, further work is definitely required in order to test in detail the feasibility of this ambitious program.

To conclude this section, we note that there is an alternative method\(^{54}\) to describe strange hyperons within topological soliton models (for reviews see e.g. Ref.\(^{55}\)). That method is based on treating strange degrees of freedom as light and, thus, on the introduction of rotational \( SU(3) \) collective quantization. It is clear that this treatment becomes better the closer one is to the limit \( m_K \to 0 \). It has been suggested,\(^{56}\) however, that even in such a limit the bound state picture is applicable. In the present context this brings in the very interesting question concerning the possibility of having a unified framework that may allow to smoothly interpolate between the chiral symmetry limit and the heavy quark limit.

1.5. Summary and conclusions

Heavy baryons represent an extremely interesting problem since they combine the dynamics of the heavy and light sectors of the strong interactions. In this contribution we have reviewed the work done on the description of heavy baryons as
heavy-meson–soliton bound systems. We have first discussed how these bound systems can be obtained in the infinite heavy quark limit using effective lagrangians that respect both chiral symmetry and heavy quark symmetry. Next, we have shown how the effects due to finite heavy quark masses can be accounted for, and compared the resulting heavy baryon spectra with existing quark model and empirical results. This comparison indicates that, even though room for improvement is certainly left, the bound heavy-meson–soliton models are reasonably successful in reproducing those results. Finally, we have addressed some issues related to a possible connection between the usual bound state approach to strange hyperons and that for heavier baryons. We have shown that there are some indications that it might be possible to construct an effective soliton model which could be used to describe baryons formed by quarks of any flavor. Of course, further work is definitely required in order to test in detail the feasibility of this ambitious program. We finish by recalling that, although in recent years there has been an enormous progress in both the theoretical and experimental aspects of the heavy baryon physics, many problems still remain to be resolved. For example, most of the $J^P$ quantum numbers of the heavy baryons have not been yet determined experimentally, but are assigned on the basis of quark model predictions. In this sense, the insight obtained from alternative models such as the bound state soliton model discussed in the present contribution might be particularly useful.

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