Dark Energy Evolution and the Curvature of the Universe from Recent Observations

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Abstract

We discuss the constraints on the time-varying equation of state for dark energy and the curvature of the universe using observations of type Ia supernovae from Riess et al. and the most recent Supernova Legacy Survey (SNLS), the baryon acoustic oscillation peak detected in the SDSS luminous red galaxy survey and cosmic microwave background. Due to the degeneracy among the parameters which describe the time dependence of the equation of state and the curvature of the universe, the constraints on them can be weakened when we try to constrain them simultaneously, in particular when we use a single observational data. However, we show that we can obtain relatively severe constraints when we use all data sets from observations above even if we consider the time-varying equation of state and do not assume a flat universe. We also found that the combined data set favors a flat universe even if we consider the time variation of dark energy equation of state.
1 Introduction

Almost all current cosmological observations indicate that the present universe is accelerating. It can be explained by assuming that the universe is dominated by dark energy today. Although many candidates for dark energy have been proposed so far, we still do not know the nature yet. Many studies have been devoted to investigate dark energy assuming or constructing a specific model and then study its consequences on cosmological observations such as cosmic microwave background (CMB), large scale structure (LSS), type Ia supernovae (SNeIa) and so on. On the other hand, many efforts have also been made to study dark energy in phenomenological way, i.e., as model independent as possible. In such approaches, dark energy can be parameterized with its equation of state and constraints on it can be obtained using cosmological observations. Assuming that the equation of state for dark energy \(w_X\) is constant in time, current observations give the constraint as \(w_X \sim -1\) \[1, 2, 3, 4, 5, 6\]. Although one of the most famous models for dark energy is the cosmological constant whose equation of state is \(w_X = -1\), most models proposed so far have time-varying equation of state. Thus, when we study dark energy, we should accommodate such time dependence in some way.

Many recent works on dark energy investigate the time dependence of the dark energy equation of state using simple parameterizations such as \(w_X = w_0 + w_1 (1 - a)\) with \(a\) being the scale factor of the universe \[7, 8\]. Parameterizing the dark energy equation of state in simple ways, the constraints on the time evolution of \(w_X\) have been considered (for recent works on this issue, for example, see Ref. \[9\]). It should be mentioned that, when one studies the equation of state for dark energy, it is usually assumed that the universe is flat. It should be also noted that dark energy is usually assumed to be the cosmological constant when one derives the constraint on the curvature of the universe. However, it has been discussed that, assuming a non-flat universe, the constraints on \(w_X\) and the curvature of the universe can be relaxed to some extent even with the time independent \(w_X\) from the CMB data alone \[10\]. Also, even if we assume a flat universe, there are degeneracies among the parameters which describe the evolution of dark energy, i.e., the time dependence of \(w_X\) when we consider the constraints on dark energy \[11, 12\]. Furthermore it has been also discussed that if we remove the prior of a flat universe, the degeneracies becomes much worse \[13\]. Since it is very important to study the time dependence of \(w_X\) to differentiate the models of dark energy and also the curvature of the universe to test the inflationary paradigm, we should investigate how the prior on the curvature of the universe affects the determination of the time-varying equation of state for dark energy and vice versa. Some works along this line have been done using a specific one-parameter parameterization for the time-varying \(w_X\) \[14\].

In this paper, we study this issue, namely the determination of the evolution of dark energy and the curvature of the universe, in some detail using widely used parameterization of the equation of state for dark energy. We consider the constraints from observations of SNeIa reported in Refs. \[3, 6\], the baryon acoustic oscillation detected in the SDSS luminous red galaxy survey \[15\] and recent CMB observations including WMAP \[1\]. In the
next section, we summarize the analysis method we adopt in this paper. Then we discuss the constraints from above mentioned observations on the curvature of the universe with time-varying equation of state for dark energy followed by the analysis on $w_X$ without assuming a flat universe. The final section is devoted to the summary of the paper.

2 Method

In this section, we briefly summarize the method for constraining the parameters which describe the dark energy evolutions and other cosmological parameters. To study the evolution of dark energy, we use the following parameterization for the time-varying equation of state [7, 8]

$$w_X(z) = w_0 + \frac{z}{1+z}w_1 = w_0 + (1-a)w_1,$$

where $z$ is the redshift. In this parameterization, the equation of state at the present time is $w_X(z = 0) = w_0$ and for the early time it becomes $w_X(z = \infty) = w_0 + w_1$. Since we are interested in the late-time acceleration of the universe due to dark energy, we consider the case where the dark energy dominates the universe only at late time. Thus, in this paper, we assume

$$w_0 + w_1 < 0,$$

in order not to include the possibilities of early-time dark energy domination. With this parameterization, the energy density of dark energy can be written as

$$\rho_X(z) = \rho_{X0}(1+z)^{3(1+w_0+w_1)} \exp \left( \frac{-3w_1z}{1+z} \right),$$

where $\rho_{X0}$ is the energy density of dark energy at present time. The Hubble parameter is given by

$$H^2(z) = H_0^2 \left[ \Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_k(1+z)^2 + \Omega_X(1+z)^3(1+w_0+w_1) \exp \left( \frac{-3w_1z}{1+z} \right) \right],$$

where $H_0$ is the Hubble parameter at the present epoch, $\Omega_i$ is the energy density of a component $i$ normalized by the critical energy density and the subscripts $r, m, k$ and $X$ represent radiation, matter, the curvature of the universe and dark energy, respectively. To consider the constraints on dark energy and other cosmological parameters, we use the data from SNeIa, the baryon acoustic oscillation peak and the CMB.

As for SNeIa data, we use the gold data set given in Ref. [4] and the first year data of the Supernova Legacy Survey (SNLS) released recently [5]. Constraints from SNeIa can be obtained by fitting the distance modulus which is defined as

$$M - m = 5 \log \left( \frac{d_L}{\text{Mpc}} \right) + 25.$$
Here $d_L$ is the luminosity distance which is written as

$$d_L = \frac{1 + z}{\sqrt{|Ω_k|}} S \left( \sqrt{|Ω_k|} \int_0^z \frac{dz'}{H(z')/H_0} \right),$$

(6)

where $S$ is defined as $S(x) = \sin(x)$ for a closed universe, $S(x) = \sinh(x)$ for an open universe and $S(x) = x$ with the factor $\sqrt{|Ω_k|}$ being removed for a flat universe.

We also use the baryon acoustic oscillation peak detected in the SDSS luminous red galaxy survey [15]. To obtain the constraint, we make use of the parameter $A$ which is defined as

$$A = \left[ \frac{1}{(H(z_1)/H_0)^{1/3}} S \left( \sqrt{|Ω_k|} \int_0^{z_1} \frac{dz'}{H(z')/H_0} \right) \right]^{2/3},$$

(7)

where $z_1 = 0.35$ and $A$ is measured to be $A = 0.469 \pm 0.017$ [15].

For the CMB data, we only use the shift parameter $R$ which determines the whole shift of the CMB angular power spectrum [16]. $R$ is given by

$$R = \frac{\sqrt{Ω_m}}{\sqrt{|Ω_k|}} S \left( \sqrt{|Ω_k|} \int_0^{z_2} \frac{dz'}{H(z')/H_0} \right),$$

(8)

where $z_2 = 1089$. It has been discussed that using the CMB shift parameter is a robust way to include the constraints from observations of CMB [17]. From the recent observations of CMB including WMAP, CBI and ACBAR, the shift parameter is constrained to be $R = 1.716 \pm 0.062$ [1, 17].

In this paper, we only consider the effect of the modification of the background evolution by the change in the cosmological parameters. In fact, the properties of dark energy can also modify the evolutions of cosmic density fluctuation. When we consider the effects of dark energy perturbation, we also have to specify the speed of sound of dark energy component. However, such modification can arise at low multipole region of the CMB power spectrum where the errors due to cosmic variance are large. Thus the constraints from CMB on dark energy mostly come from the position of acoustic peaks which can be described by the shift parameter. Hence we do not consider the perturbation of dark energy in this paper#1.

### 3 Constraints on the $Ω_m$ vs. $Ω_X$ plane

Now we discuss the implication of the dark energy evolution on the determination of the curvature of the universe. For this purpose, we derive the constraints from the observations on the $Ω_m$ vs. $Ω_X$ plane. First we consider the case with the constant equation of state for dark energy. However, the speed of sound can be useful to differentiate the models of dark energy. There are some works which discuss the constraint on the speed of sound. For interested reader, we refer Refs. [18, 19, 20].

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Figure 1: Constraints on $\Omega_m$ and $\Omega_X$ from SNeIa (a), the baryon acoustic oscillation peak (b), the CMB shift parameter (c) and all data combined (d). For the constraint from SNeIa data and all data combined, we show the contours obtained from the gold set of Ref. [4] and SNLS [6] separately. Contours of $1\sigma$ (red solid line) and $2\sigma$ (green dashed line) are shown (for SNLS data, $1\sigma$ and $2\sigma$ contours are shown in blue dash-dotted line and purple dotted line, respectively). We assumed the cosmological constant as dark energy and a flat universe here.

dark energy. In Fig. 1 we show the contours of $1\sigma$ and $2\sigma$ constraints from observations of SNeIa (a), the baryon acoustic oscillation peak (b) and the CMB shift parameter (c). We also show the constraint from the combination of all data sets (d). In Fig. 1 we assume the equation of state as $w_X = -1$. For the constraints from SNeIa, we used the gold data set from Ref. [4] and the data from SNLS [6] separately. As we can see, although each constraint from a single observational data has the degeneracy in the $\Omega_m$ vs. $\Omega_X$ plane, when all the data sets are combined, we can obtain a severe constraint as $\Omega_m \sim 0.3$ with $\Omega_k \sim 0$, which is a well-known result.

Next we consider the case where the equation of state for dark energy is allowed to vary, but still we keep $w_X$ constant in time. In Fig. 2 we show the contours of $1\sigma$ and $2\sigma$ constraints after marginalizing over the values of $w_X$. Here we assumed the prior on $w_X$ as $-5 \leq w_X \leq 0$. As seen from the figure, the allowed regions become larger compared to those for the case with $w_X = -1$ when we use a single data set alone. However, by combining all data, we can obtain almost the same constraint as the case with $w_X = -1$.
Figure 2: The same as Fig. 1 except that we marginalized over the value of $w_X$. Here we considered the constant $w_X$.

even though we marginalize over $w_X$. This can be understood by noting that the values of $w_X$ which give the minimum $\chi^2$ for fixed $(\Omega_m, \Omega_X)$ from each data set are different. To see this, we also plot contours of constant $w_X$ which gives minimum values of $\chi^2$ at each point on the $\Omega_m$ vs. $\Omega_X$ plane in Fig. 3. We can clearly see that the favored values of $w_X$ vary for different data sets. Observations we consider here measure some distance scales to certain redshifts which are determined by the energy density of matter $\Omega_m$, dark energy $\Omega_X$ and the equation of state $w_X$. If we vary the value of $w_X$, the density parameters $\Omega_m$ and $\Omega_X$ can have more freedom to be consistent with observations since the fit to the data depends on the combinations of these quantities. For a single observation, when $w_X$ is allowed to vary, larger range of $(\Omega_m, \Omega_X)$ can be consistent with observations. However, when we use all observations, the combinations of $\Omega_m$, $\Omega_X$ and $w_X$ which are consistent with observations become fairly limited. Thus while the allowed regions become larger for each data set, when we combine all the data, the allowed region converges towards the concordance model with $\Omega_m + \Omega_X \sim 1$ and $\Omega_m \sim 0.3$ even if we do not assume the cosmological constant as dark energy.

Next we discuss the case with the time dependent equation of state parameterized as Eq. (1). As it has been already pointed out in the literature, even if we assume a flat universe, there exists a degeneracy among the parameters which describe the evolution of equation of state for dark energy using a single observational data. It is also known that
Figure 3: Contours of constant $w_X$ which gives minimum $\chi^2$ when we marginalize over $w_X$ for the constraints from SNeIa (a), the baryon acoustic oscillation peak (b), the CMB shift parameter (c) and all data combined (d). Contours of $w_X = -2$ (red solid line), $-1$ (green dashed line) and $-0.8$ (blue dotted line) are shown except for the case where all three data are combined (panel (d)). For the panel (d), contours of $w_X = -1.1$ (red solid line), $-1$ (green dashed line) and $-0.9$ (blue dotted line) are shown. For SNeIa, the data from SNLS are used here.

The degeneracy can be removed using more than one observation assuming the flatness. Here we discuss to what extent the evolution of dark energy equation of state can affect the determination of $\Omega_m$ and $\Omega_X$ when we do not assume a flat universe. In Fig. 4 the contours of $1\sigma$ and $2\sigma$ constraints are shown as the same manner as Fig. 3 except that we varied the both values of $w_0$ and $w_1$ which appear in Eq. (1) and marginalized over them to obtain the constraint. We assumed the prior on them as $-5 \leq w_0 \leq 0$ and $-4 \leq w_1 \leq 4$ under the condition of Eq. (2). For the constraint from SNeIa, the allowed regions get larger compared to the case with the constant equation of state discussed above (see Fig. 2). As for the constraint from the baryon acoustic oscillation peak and the CMB shift parameter, the allowed regions are almost the same as the case with a constant equation of state. For observations which measure a single distance scale, the fit to the data does not significantly become better, even if the time evolution of $w_X$ is allowed, due to the degeneracy between $w_0$ and $w_1$. Furthermore, when we use all three different observations together, the allowed region becomes almost the same as that of the case.
with a constant equation of state. This is because each observation favors different values of \( w_X \). Even if we consider the time-varying equation of state, different combinations of \( w_0 \) and \( w_1 \) are chosen to minimize the value of \( \chi^2 \) for each observation. Thus the allowed region from the combined data set is almost unchanged although the constraints from a single observation, in particular from SNeIa, can become weaker.

Here we comment on the effect of the prior on \( w_0 \) and \( w_1 \). Although the constraints from a single data alone somewhat depend on the prior, the allowed region for all data combined is almost unaffected since the combinations of \( w_0 \) and \( w_1 \) favored around the allowed region from all data sets are far from the the edge of the prior we assumed.

4 Constraints on the \( \Omega_m \) vs. \( w_X \) plane

In this section, we consider the constraints on the \( \Omega_m \) vs. \( w_X \) plane. First we show the constraint on \( \Omega_m \) and \( w_0 \) in a flat universe with the equation of state for dark energy being constant. In Fig. 5 contours of 1\( \sigma \) and 2\( \sigma \) constraints from observations of SNeIa (a), the baryon acoustic oscillation peak (b), the CMB shift parameter (c) and all data combined (d) are shown. As is well-known, SNeIa and CMB are complementary for constraining dark energy, which can be seen from the figure. In addition, we can see that the constraint
Figure 5: Constraints on $\Omega_m$ and $w_X$ from observations of SNeIa (a), the baryon acoustic oscillation peak (b), CMB (c) and all data combined (d). We assumed a flat universe and a constant equation of state here.

Next we consider the case with the time-varying equation of state for dark energy. Here we discuss the case with a flat universe. In Fig. 6, the constraints on $\Omega_m$ and $w_0$ are shown marginalizing over $w_1$. When we marginalize the values of $w_1$, we assumed the prior on $w_1$ as $-4 \leq w_1 \leq 4$. Similarly to the situation where we constrain $\Omega_m$ and $\Omega_X$ discussed in the previous section, if we consider the time-varying equation of state for dark energy, the allowed region becomes larger when we use a single data set. However, when we consider all three data sets, we can obtain a relatively severe constraint. Although the allowed region of $w_0$ extends to larger values, $\Omega_m$ is constrained to be $\Omega_m \sim 0.3$ which is the almost same as the case with assuming a constant equation of state. This is because the values of $w_1$ favored by different data sets given $\Omega_m$ and $w_0$ are different. Thus the degeneracy among the parameters describing the time dependence of the dark energy equation of state can be removed drastically by using all three data sets.

Now we discuss the constraints without assuming a flat universe. First we discuss the case with $w_X$ being constant in time. In Fig. 7 we show the constraints from SNeIa (a), the baryon acoustic oscillation (b) and all data combined (c). For the last case, the data from the CMB shift parameter is included. To obtain the constraint, we marginalized
Figure 6: The same as Fig. 5 except that we consider the time dependent equation of state for dark energy as in Eq. (1). To obtain the constraint, we marginalized over $w_1$. Here we assumed a flat universe.

over $\Omega_k$ with the prior $-0.3 \leq \Omega_k \leq 0.3$. Here we do not show the constraint from the CMB shift parameter alone since we cannot obtain a significant constraint from it in the parameter region we consider in Fig. 4. This is because, as it has already been pointed out, the curvature of the universe and $w_X$ are strongly degenerate in the CMB power spectrum [10]. Again, although the constraints from a single data set alone are weakened, if we consider all data sets, we can obtain almost the same constraint as that with the case where a flat universe is assumed. Notice that the constraint on $w_0$ is also not changed much compared to that with flat universe prior. Hence from Fig. 6 we can say that the prior on the curvature does not affect the determination of the equation of state for dark energy much.

We also show the case where we consider the time-varying equation of state without assuming a flat universe. In Fig. 6 the constraints are shown as the same as Fig. 4 except that we marginalized over $\Omega_k$ and $w_1$ in this case. The prior on $w_1$ is taken as $-4 \leq w_1 \leq 4$. The CMB shift parameter cannot give a significant constraint in the parameter range we consider in this case too. As in the previous cases, using each observational data alone, the allowed region becomes significantly larger compared to those with constant equation of state and a flat universe. However, when we use all data sets, the allowed region does not change much. From Figs. 5 and 6 we can conclude that the prior on the curvature
of the universe does not affect the constraint on $\Omega_m$ and $w_0$ much if we use all data sets. In particular, we can obtain the constraint such that $\Omega_m \sim 0.3$ without assuming a flat universe. Moreover, the assumption of the constancy of the equation of state for dark energy also does not affect the constraint on $\Omega_m$. On the constraint on the equation of state, if we consider the time-varying equation of state, the allowed region becomes slightly larger even if all data combined is used.

5 Constraints on the evolution of dark energy

In this section, we consider the constraints on the time dependence of dark energy equation of state, i.e., on $w_0$ and $w_1$ without assuming the prior of a flat universe. However first we discuss the case with a flat universe for the later comparison with the case where a flat universe is not assumed. In Fig. 8 contours of $1\sigma$ and $2\sigma$ allowed regions are shown for the case with a flat universe. Here we fix the energy density of matter as $\Omega_m = 0.28$. Notice that we do not consider the region where $w_0 + w_1 > 0$ in order not to include the possibilities of early-time dark energy domination. As we can see from the figure, the constraint from SNeIa is stringent compared to baryon acoustic oscillation peak and CMB. Since the position of the baryon acoustic oscillation peak measures a single distance scale from $z = 0$ to $z = 0.35$ and the CMB shift parameter also gives a single scale from $z = 0$ to $z = 1089$, there is strong degeneracy among the parameters which describe the equation of state for dark energy. Notice that the distance scales are determined by the integration of the inverse of the Hubble parameter, which can smear out the information on the time dependence of the equation of state. This is the reason why the strong degeneracy exists when only a single scale is considered. However, as for SNeIa data, the degeneracy is
Figure 8: The same as Fig. 7 except that we consider the time dependent equation of state for dark energy and do not assume a flat universe. In this figure, we marginalized over $\Omega_k$ and $w_1$.

removed to some extent since we have the distances from $z = 0$ to various redshifts. Thus observations of SNeIa can mainly constrain the time dependence of the equation of state for dark energy.

Next we show the constraints on $w_0$ and $w_1$ in a flat universe without assuming a particular value for $\Omega_m$. To obtain the constraint, we marginalized over $\Omega_m$ with the prior $0 \leq \Omega_m \leq 0.5$. In Fig. 10, we show the constraints on $w_0$ and $w_1$ from SNeIa (a) and all data combined (b). We do not show the constraints from the baryon acoustic oscillation peak and the CMB shift parameter because we cannot obtain significant constraints from them in the parameter region of $w_0$ and $w_1$ we consider here. Also in this case, such severe degeneracies among the parameters exist because those observations measure a single distance scale. As in the previous case, when we include all data sets in the analysis, we can obtain almost the same constraint as that with $\Omega_m$ being fixed to 0.28. This is partly because the best fit value of $\Omega_m$ is near 0.28 when we marginalized over it, but also because the values of $\Omega_m$ minimizing $\chi^2$ for each observational data set are different. To see this, in Fig. 11, we show contours of constant $\Omega_m$ which minimize $\chi^2$ when we marginalize over $\Omega_m$ for the constraints from SNeIa (a), the baryon acoustic oscillation peak (b), the CMB shift parameter (c) and all data combined (d). As is clear from the figure, preferred values of $\Omega_m$ for each observation are quite different. Thus when all data are used, we can obtain a severe constraint although a single observation cannot constrain the equation of state for dark energy. It should be also mentioned that the prior on $\Omega_m$ does not affect the constraint on the time dependence of the equation of state when we use all data combined.

Now we discuss the case with $\Omega_m$ not being fixed and not assuming a flat universe. In Fig. 12, the allowed regions are shown after marginalizing over $\Omega_m$ and $\Omega_k$. We assumed

\cite{11, 12}.

#2Of course, there is a limitation to determine the time dependence of the equation of state using SNeIa data.\cite{11, 12.}
Figure 9: Constraints on $w_0$ and $w_1$ from SNeIa (a), the baryon acoustic oscillation peak (b), the CMB shift parameter (c) and all data combined (d). We assumed $\Omega_m = 0.28$ with a flat universe.

the prior on these variables as $0 \leq \Omega_m \leq 0.5$ and $-0.3 \leq \Omega_k \leq 0.3$. In this case, the constraint is significantly weakened when we consider a single data set alone. We only report here the constraint from SNeIa (a) and that from all data combined (b) since the baryon acoustic oscillation peak and the CMB shift parameter cannot give meaningful constraints on this plane in this case too. As we can see, when we consider the constraint from all data sets, it significantly becomes severe compared to that from SNeIa alone as seen from the figure. This is because the favored values of $\Omega_m$ and $\Omega_k$ from each data set are different as in the previous cases. Thus we can conclude that the prior on the curvature does not affect much the determination of the equation of state for dark energy if we use all data combined.

6 Summary

We considered the constraint on the curvature of the universe and the equation of state for dark energy from observations of SNeIa, the baryon acoustic oscillation peak and the CMB shift parameter. Usually, when one discusses the curvature of the universe, dark energy is assumed to be the cosmological constant. Moreover, when one considers the constraint on
Figure 10: Constraints on $w_0$ and $w_1$ from observations of SNeIa (a) and all data combined (b). A flat universe is assumed but we marginalized over $\Omega_m$.

the evolution of dark energy, in particular the time dependence of the dark energy equation of state, a flat universe is usually assumed. In this paper, we discussed the constraints on the curvature of the universe without assuming the cosmological constant and also the time dependence of the equation of state for dark energy without assuming a flat universe. We showed that the constraint on the curvature of the universe is significantly relaxed from a single observation when we allow the time dependence in the dark energy equation of state. However, it was also shown that, when we use all data sets, the curvature of the universe or the energy density of matter and dark energy are severely constrained to be $\Omega_m \sim 0.3$ with a flat universe even if we consider the time-varying equation of state for dark energy. Observations we consider here measure some distance scales to certain redshifts which can be determined by the energy density of matter $\Omega_m$, dark energy $\Omega_X$ and the equation of state $w_X$. If we assume a broad range for $w_X$, the energy density of matter $\Omega_m$ and dark energy $\Omega_X$ can have more freedom to be consistent with observations since the fit to the data depends on the combinations of these variables. For a single observation, if $w_X$ is allowed to vary more freely, much larger range of values of $\Omega_m$ and $\Omega_X$ can be consistent with observations. However, when we use all observations, the combinations of $\Omega_m$, $\Omega_X$ and $w_X$ consistent with observations become fairly limited. Thus even if the allowed regions become large for each data set, when we combine all the data, we can obtain a severe constraint, which is interestingly almost the same region as that we obtain assuming the cosmological constant as dark energy.

We also investigated the constraint on the time-varying equation of state for dark energy without assuming a flat universe. Similarly to the situation where we constrained the curvature with the time-varying equation of state, the allowed region for $w_X$ becomes larger when we use a single observational data. However, if we use all data sets considered in this paper, we can obtain almost the same constraint as that in the case where a flat universe is assumed.

Finally, we summarize what we found in this paper. The combination of the current
Figure 11: Contours of constant $\Omega_m$ which gives minimum $\chi^2$ when we marginalize over $\Omega_m$ for the constraints from SNeIa (a), the baryon acoustic oscillation peak (b), CMB shift parameter (c) and all data combined (d). Contours of $\Omega_m = 0.3$ (red solid line), 0.2 (green dashed line) and 0.1 (blue dotted line) are shown except for the case where all three data are combined (panel (d)). For the panel (d), $\Omega_m = 0.29$ (red solid line), 0.28 (green dashed line) and 0.27 (blue dotted line) are shown. For SNeIa, we use the data from SNLS.

observations

- favors a flat universe regardless of the prior on the equation of state for dark energy with or without time evolution.

- favors $\Omega_m \sim 0.3$ regardless of the flatness prior and the prior on the equation of state for dark energy with or without time evolution.

- yields constraints on the time evolution of dark energy equation of state regardless of the prior on $\Omega_m$ and $\Omega_k$.

In future observations, we can obtain much more stringent constraints on the equation of state for dark energy as well as the curvature of the universe regardless of the prior on other cosmological parameters. Hence we can expect that we will be able to have much insight on the nature of dark energy and also the inflationary paradigm.
Figure 12: The same as Fig. 10 except that we marginalized over $\Omega_m$ and $\Omega_k$.

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