Exchanging Quantum Correlations and Non-Local Information Between Three Qubit-System

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Abstract
The possibility of exchanging the quantum correlations and the non-local information between three qubits interact directly or indirectly via Dzyaloshinskii-Moriya (DM) is discussed. The initial state settings and the interaction strength represent control parameters on the exchanging phenomena. The non-local information that encoded on the different partitions doesn’t exceed the initial one. It is shown that, the ability of DM interaction to generate entanglement is larger than that displayed for the dipole interaction. The possibility of maximizing the quantum correlations between the three qubits increases as one increase the strength of interaction and starting with large initial quantum correlations. The long-lived quantum correlations could be achieved by controlling the strength of the dipole interaction.

Keywords Quantum correlation · Non-local information · Dzyaloshinskii-Moriya interaction · Concurrence

1 Introduction

Entanglement is the core idea of modern theoretical physics [1], where it plays an important role in different applications of quantum information processing (QIP, like quantum cryptography [2], quantum teleportation [3], quantum dense coding [4], and etc. Therefore, there is a need to be generated and quantified, as well as keep it survival for a long time. Indeed, it has been generated between different objects; atoms [5] charged qubits [6], quantum dots [7], etc. Dzyaloshinskii-Moriya (DM), [8] represents one of the most common interaction that has used widely to generate quantum correlation between different systems. For example, the possibility of generating a thermal entanglement between two qubits Heisenberg chain in the presence of the Dzyaloshinskii-Moriya is discussed in [9]. The effect of DM interaction on the dynamics of a two-qutrit system is investigated by Jafarpour and Ashrafpour [10]. Sharma

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and Pandey [11] studied the entanglement dynamics between qubit-qutrit systems via DM interaction. The influence of the anisotropic antisymmetric exchange interaction, on the entanglement of two qubits is examined in [12]. The DM interaction is used to generated entangled quantum network [13]. The orthogonality speed of two-qubit state interacts locally with spin chain in the presence of Dzyaloshinsky–Moriya interaction is investigated by Kahla, et al [14]. Therefore, we are motivated to discuses the phenomena of exchanging the quantum correlations and the non-local information between three qubits, where two of them are either classically or non-classically correlated. It is assumed that, by using DM interaction one of the correlated qubits interacts locally with a control qubit that is initially prepared in a vacuum state. The effect of the initial state settings of the correlated qubits and the strength of DM interaction on the exchange process is investigated.

The outline of the paper is arranged as: In Section 2, we review the most common quantifiers of quantum correlation; concurrence, entanglement of formation and negativity. The qubits system and its evolution is described in Section 3. The exchange of quantum correlations between the marginal states of the three qubits is discussed in Section 4. In Section 5, the behavior of the exchanged non-local information is investigated. Finally, we summarized our results in Section 6.

2 Measure of Entanglement

In this section, we review some important quantifiers of quantum correlations (entanglement). Among of these quantifiers are; concurrence [15], entanglement of formation, and the negativity. It is well known that, a quantum system that consist of two qubits $\rho_{12}$, is called a separable or classically correlated if $\rho_{12} = \rho_1 \otimes \rho_2$, otherwise the system is entangled (quantum correlated). In the following subsections, we review the three measures in details.

2.1 Concurrence

This measure represents one of the most simplest quantifiers of entanglement, it is called Wootteres Concurrence [15, 16], where it satisfies all the criteria of the good measures of entanglement. Mathematically it is defined as,

$$C = \max(0, \sqrt{\lambda_1} - \sqrt{\lambda_2} - \sqrt{\lambda_3} - \sqrt{\lambda_4}).$$

where $\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4$, $\lambda_i$ (i = 1,2,3,4) are the eigenvalues of the density operator $\rho_{12}$, such that

$$\rho_{12} = \rho_{12}(\sigma_y^1 \otimes \sigma_y^2)\rho_{12}^* (\sigma_y^1 \otimes \sigma_y^2).$$

The operator $\rho_{12}^*$ is the complex conjugate of the density operator of the system $\rho_{12}$, and $\sigma_y = i(|0\rangle \langle 1| - |1\rangle \langle 0|)$ is Pauli operator in $y$-direction. For separable states the concurrence $C = 0$ and for the maximum entangled state $C = 1$.

2.2 Entanglement of Formation

One of the meaningful measurement of entanglement is the entanglement of formation (E_F) [17]. For a two qubits system the $E_F$ is defined as a function of the concurrence [15] as,
$$\mathcal{E}_F(\rho) = H \left( \frac{1 + \sqrt{1 - C^2(\rho)}}{2} \right),$$  \hspace{1cm} (3)$$

where $H(x) = -x \log_2 x - (1 - x) \log_2 (1 - x)$ represents the Shannon Entropy function and $0 \leq \mathcal{E}_F \leq 1$.

### 2.3 Negativity

The third measure of entanglement is the negativity, which is based on the eigenvalues of the partial transpose quantum system [18, 19]. However for two qubit system $\rho_{12}$, the negativity $\mathcal{N}$ is defined as,

$$\mathcal{N} = 2 \sum_{i=1}^{4} \max(0, -\mu_i),$$  \hspace{1cm} (4)

where $\mu_i$ are the negative eigenvalues of the partial transpose of the given quantum system, namely $\rho_{12}^{T_b}$. Similarly the negativity $\mathcal{N}(\rho_{12})$ is ranged between 0 and 1, where $\mathcal{N}(\rho)=0$ if the quantum system is separable and $\mathcal{N}(\rho)=1$ if the given quantum system is maximally entangled.

### 3 The Suggested Model

The system composed of two qubits $A$, and $B$, are prepared in a partial entangled state interacts by using Heisenberg XX Spin model [20]. It is assumed that, one of the subsystems, say $A$, interacts with third qubit $C$, as a controller via Dzyaloshinskii-Moriya (DM). The total Hamiltonian which describes this system is given by,

$$\mathcal{H}_{\text{sys}} = \frac{1}{2} \omega (\sigma_A^x \sigma_B^x + \sigma_A^y \sigma_B^y) + D \cdot (\sigma_A \times \sigma_C),$$  \hspace{1cm} (5)

where $D = (D_x, D_y, D_z)$ is the DM interaction, and $D_i x = x, y, z$ are the interaction strength on the three directions. $\omega$ is the coupling parameter between qubits $A$, $B$, and $\sigma_i^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_i^y = \begin{pmatrix} 0 & i \\ -i & 0 \end{pmatrix}$, $i = A, B$ are the Pauli operators for both qubits. In this context, it is important to mention that the Hamilation (5) is of $8 \times 8$ dimensions, where its eigenvalues and eigenvectors are given by,

$$\lambda_i = 0, i = 1..4, \lambda_5 = -\sqrt{4D_z^2 + \omega^2}, \quad \lambda_6 = \lambda_5, \quad \lambda_7 = \lambda_8 = -\lambda_5$$  \hspace{1cm} (6)

, while the corresponding eigenvectors $|\xi_i\rangle$, $i = 1..8$ are,
\[ |\xi_1\rangle = |111\rangle, \quad |\xi_2\rangle = \frac{1}{\sqrt{1+\left(\frac{2\delta}{\omega}\right)^2}} \left( -\frac{2iD}{\omega} |101\rangle + |110\rangle \right), \]
\[ |\xi_3\rangle = \frac{1}{\sqrt{1+\left(\frac{2\delta}{\omega}\right)^2}} \left( -\frac{i\omega}{2Dz} |001\rangle + |010\rangle \right), \quad |\xi_4\rangle = |000\rangle. \]
\[ |\xi_5\rangle = \frac{1}{\sqrt{1+\left(\frac{2\delta}{\omega}\right)^2}} \left( -\frac{i\sqrt{4D^2+w^2}}{2Dz} |011\rangle + \frac{i\omega}{2Dz} |101\rangle + |110\rangle \right), \]
\[ |\xi_6\rangle = \frac{1}{\sqrt{1+\left(\frac{2\delta}{\omega}\right)^2} + \left|\frac{\omega}{\sqrt{4D^2+w^2}}\right|^2} \left( -\frac{2iDz}{\sqrt{4D^2+w^2}} |001\rangle - \frac{\omega}{\sqrt{4D^2+w^2}} |010\rangle + |100\rangle \right), \quad (7) \]
\[ |\xi_7\rangle = \frac{1}{\sqrt{1+\left(\frac{2\delta}{\omega}\right)^2} + \left|\frac{\omega}{\sqrt{4D^2+w^2}}\right|^2} \left( i\sqrt{4D^2+w^2} |011\rangle + \frac{i\omega}{2Dz} |101\rangle + |110\rangle \right), \]
\[ |\xi_8\rangle = \frac{1}{\sqrt{1+\left(\frac{2\delta}{\omega}\right)^2} + \left|\frac{\omega}{\sqrt{4D^2+w^2}}\right|^2} \left( \frac{2iDz}{\sqrt{4D^2+w^2}} |001\rangle + \frac{\omega}{\sqrt{4D^2+w^2}} |010\rangle + |100\rangle \right). \]

In this contribution, we assume that \( \mathbf{D} \) to be polarized on the \( z \) direction, namely, \( \mathbf{D} = (0,0,D_z) \). Let us assume that the initial state of the whole system at time \( t = 0 \) is given by,

\[ \rho_s(0) = \rho_{AB}(0) \otimes \rho_C(0), \quad (8) \]

where

\[ \rho_{AB}(0) = \kappa |\varphi^{ab}\rangle \langle \varphi^{ab}| + \frac{1}{4} (1 - \kappa) I_{4 \times 4}, \quad \rho_C(0) = |\varphi^c\rangle \langle \varphi^c|, \quad (9) \]

with \( |\varphi^{ab}\rangle = \cos(\alpha) |e\rangle + \sin(\alpha) |g\rangle \) and \( |\varphi^c\rangle = \cos(\gamma) |e\rangle + \sin(\gamma) |g\rangle \). \( I_{4 \times 4} \) is the identity operator. It is clear that, at \( \alpha = \frac{\pi}{4} \), the state \( |\varphi^{AB}\rangle \) reduces to Bell state \( |\psi^+\rangle \). At \( t > 0 \), the total density operator of the whole system is given by,

\[ \rho_s(t) = \mathcal{U}(t) \rho_s(0) \mathcal{U}(t)^\dagger, \quad \mathcal{U}(t) = \exp[-i \mathcal{H}_{\text{sys}} t], \quad (10) \]

where the unitary operator \( \mathcal{U}(t) \) takes the form,

\[ \mathcal{U}(t) = \sum_{i=1}^{2} \alpha_i \beta_i, \quad (11) \]

where,

\[ \alpha_1 = \cos(\frac{\omega}{2} t) I_{8 \times 8} - i \sin(\frac{\omega}{2} t) \sigma_A^x \otimes \sigma_B^y \otimes I_{2 \times 2}, \]
\[ \alpha_2 = \cos(D_z t) I_{8 \times 8} - i \sin(D_z t) \sigma_A^y \otimes \sigma_B^x \otimes \sigma_C^z, \]
\[ \beta_1 = \cos(\frac{\omega}{2} t) \cdot I_{8 \times 8} + i \sin(\frac{\omega}{2} t) \sigma_A^x \otimes \sigma_B^y \otimes \sigma_C^z, \]
\[ \beta_2 = \cos(Dt) \cdot I_{8 \times 8} + i \sin(Dt) \sigma_A^y \otimes \sigma_B^x \otimes \sigma_C^x. \quad (12) \]

Then by using the unitary operator (11) and the initial state \( \rho_s(0) \) at any \( t > 0 \). Due to this interaction, there will be new entangled states are generated; \( \rho_{AC}(t) \) is generated between the qubits \( A, C \), and \( \rho_{BC}(t) \) between the qubits \( B \) and \( C \). Moreover, the amount of entanglement of the initial state \( \rho_{AB}(0) \) will be affected by this interaction. In this context, it is important to quantify the amount of entanglement between
the three partitions $AB,AC,$ and $BC,$ which are defined by the final states $\rho_{AB}(t),\rho_{AC}(t)$ and $\rho_{BC}(t),$ respectively.

In the computational basis the three final density operators are given by a matrix of size $4 \times 4.$ The density operators of the three partitions are defined by the non-zero elements $\varrho^{ij},\ ij=ab,ac,bc.$ Mathematically any $\rho_{k\ell}(t),k\ell=AB,AC,BC$ may be written as:

$$\rho_{k\ell}(t) = \begin{pmatrix}
\varrho_{00}^{ij} & 0 & 0 & 0 \\
0 & \varrho_{11}^{ij} & 0 & 0 \\
0 & 0 & \varrho_{22}^{ij} & 0 \\
0 & 0 & 0 & \varrho_{33}^{ij}
\end{pmatrix},$$

(13)

The density operator of the state $\rho_{AB}(t)$ is defined by the following elements:

$$\begin{align*}
\rho_{00}^{ab} &= \frac{1}{4}\kappa \cos^2(2D_z t), \\
\rho_{11}^{ab} &= \frac{1}{8}(2\cos^2(2t)\cos^2(2D_z t) + (3 + 3\kappa - (1 - \kappa) \cos(4D_z t)) \sin^2(2t)), \\
\rho_{22}^{ab} &= \frac{1}{8}(2\sin^2(2t)\cos^2(2D_z t) + (3 + 3\kappa - (1 - \kappa) \cos(4D_z t)) \cos^2(2t)), \\
\rho_{21}^{ab} &= \frac{1}{8}(2\sqrt{3}\kappa \cos(2D_z t) - t \sin(4t)(-\frac{1}{2}\kappa(3 + \cos(4D_z t) - 2\sin^2(2D_z t))), \\
\rho_{33}^{ab} &= \frac{1}{4}(1 - \kappa + \sin^2(2D_z t)), \quad \rho_{12}^{ab} = \rho_{21}^{ab*}.
\end{align*}$$

Similarly the state $\rho_{ac}$ is described by the following non-zero elements,

$$\begin{align*}
\rho_{00}^{ac} &= \frac{1}{4}(1 - \kappa) \sin^2(2t) \sin^2(2D_z t), \\
\rho_{11}^{ac} &= \frac{1}{4}((1 - \kappa + \cos^2(2t)) \cos^2(2D_z t) + (1 + 2\kappa) \sin^2(2t)), \\
\rho_{22}^{ac} &= \frac{1}{8}(2\sqrt{3}\kappa \sin(2t) \sin(2D_z t) + (\kappa - 2) \cos(2t) \sin(4D_z t)), \\
\rho_{21}^{ac} &= -\frac{1}{16}(\kappa - 3 - (1 - \kappa) \cos(4t)) \sin^2(2D_z t), \\
\rho_{33}^{ac} &= \frac{1}{16}(7 + (1 + 4\kappa) \cos(4t) + 8 \cos^2(t) \cos(4D_z t) \sin^2(t)), \quad \rho_{12}^{ac} = \rho_{21}^{ac*}.
\end{align*}$$

Finally the non zero elements of third partition $\rho_{bc}$ are given by,

$$\begin{align*}
\rho_{00}^{bc} &= \frac{1}{4}(1 - \kappa) \cos^2(2t) \sin^2(2Dt), \\
\rho_{11}^{bc} &= \frac{1}{4}((1 - \kappa + \sin^2(2t)) \cos^2(2Dt) + (1 + 2\kappa) \cos^2(2t)), \\
\rho_{22}^{bc} &= \frac{1}{8}\kappa \sin(2D_z t)(-2\sqrt{3}\cos(2t) + t(\sin(2(1 + D_z t)) + \sin(2(1 - D_z t))), \\
\rho_{21}^{bc} &= \frac{1}{8}(1 + (1 - \kappa) \sin^2(2t)) \sin^2(2D_z t), \\
\rho_{33}^{bc} &= \frac{1}{8}(3 - (1 + 2\kappa) \cos(4t) + 2 \cos^2(2t) \cos(2D_z t)), \quad \rho_{12}^{bc} = \rho_{21}^{bc*}.
\end{align*}$$

4 Exchanging Quantum Correlations

In this section we investigate the amount of entanglement that loses from the initial state $\rho_{AB}(0)$ and that exchanged between the terminals of the initial state and the control qubit $C.$ In Fig. 1, we quantify the amount of entanglement on the initial state via the concur-
currence, $C,$ negativity, $N$ and the entanglement of formation $E_F.$ It is it assumed that, the initial state of the system is prepared by setting $\alpha = \pi/3,$ namely $\rho_{AB}(0)$ is a partial entangled state. It is clear that, the quantum correlation between the subsystems “A” and “B” appears at larger values of the weight parameter $k,$ namely $k > 0.37.$ From this figure, it is
clear that, the predicted quantum correlation depends on the used quantifier. However, the concurrence and the negativity predict the entanglement at smaller values of the weight $\kappa$. Moreover, the smallest upper bounds of the quantum correlations are predicted by using the entanglement of formation, $E_F$.

### 4.1 Lose and Gain Quantum Correlations of $\rho_{AB}(t)$

Due to the interaction with the environment which is described by the control qubit $C$, the initial state $\rho_{AB}(0)$ loses some of its quantum correlation. Therefore, it is important to investigate the effect of interaction’s parameters on the quantum correlation of the initial quantum system.

Figure 2 shows the behavior of the three quantifiers for a system is initially prepared in a partial entangled state, where $\alpha = \pi/3$ and different values of the weight parameter $\kappa$.

![Figure 2](image)

**Fig. 2** The dynamics of the quantum correlation loses of the state $\rho_{AB}(t)$ that predicted by using the concurrence (solid), the negativity (dash), and the entanglement of formation (dot-lines). It is assumed that, $\rho_{AB}(0)$ is prepared such that $\alpha = \pi/3$ and control qubit is prepared by setting $\gamma = \pi/2$, $D_z = 0.5$ and $\omega = 2$ (a) $\kappa = 0.3$ and (b) $\kappa = 0.9$.
Moreover, the control qubit is prepared in the state $\rho_C(0) = |0\rangle \langle 0|$, namely we set $\gamma = \pi/2$. From Fig. 2a, it is clear that, at small value of the weight parameter, $\kappa = 0.3$, the quantum correlation is generated at $t \approx 0.9$. However, the quantifiers increase gradually to reach their maximum bounds for the first time at $t = 1.2$ and decrease to vanish completely at $t \approx 1.6$. The largest bounds are predicted for the concurrence $C$, while the smallest bounds of quantum correlations are predicted by the entanglement of formation $\mathcal{E}_F$.

In Fig. 2b, we investigate the behavior of the quantum correlation at large value the weight parameter $\kappa$. It is clear that, at $t = 0$, all the quantifiers predicate the presence of the quantum correlation. However, as the interaction time increases, the predicted quantum correlations flocculated between their lower and upper bounds. Moreover, the lower bounds that displayed by the concurrence $C$ are the largest one, but the maximum values of $C$ and the negativity $N$ are coincide. As it is shown from Fig. 2b, the smallest amount of quantum correlation are quantified by using the $\mathcal{E}_F$.

From Fig. 2, one may conclude that, the weight of the initial state plays a central role on keeping the existence of the quantum correlations. If the weight parameter is small enough such that the initial state behaves as a product sate, the DM interaction has the strength to generate quantum correlation between the subsystems of the initial state. To clarify the role that played by the strength of DM interaction, we investigate in Fig. 3 the behavior of the three quantifiers at larger values of DM interaction, where we set $D_z = 0.9$. The predicted behavior of the three quantifiers shows that, as soon as the interaction is switched on, the quantum correlations that predicted decrease suddenly to reach their minimum values. At further interaction time, the correlation rebirths again to reach their maximum values. The maximum values of the concurrence and the negativity are coincide, while the lower bounds of $C$ are much better than that displayed by the negativity $N$. The minimum and maximum values of the quantum correlations that predicted via the three quantifiers are displayed at the same interaction time.

Figure 4, exhibits the effect of the dipole’s strength interaction where we set $\omega = 0.5, 1$, while all the parameters values are the same as Fig. 2b. The predicted behavior is similar to that displayed in Fig. 3. As it is shown from Fig. 4a, where $\omega = 1$, all the quantifiers predict a long-lived quantum correlations. The phenomena of the sudden death/birth are predicted at the same interaction time for all the quantifiers. Moreover, the quantum correlations

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**Fig. 3** The same as Fig. 2b, but $D_z = 0.9$
vanish at larger interaction time compared with that displayed in Fig. 3. As one increases
the strength of the dipole interaction, \((\omega = 0.5)\) in Fig. 4b, the long-lived behavior of quan-
tum correlation is lost and the oscillation behavior is shown. Moreover, the minimum val-
ues of these oscillations are larger than those displayed in Fig. 3.

From the results that displayed in Fig. 4, one can consider the strength of the dipole
interaction as a control parameter to generated a long-lived quantum correlations between
the terminals of the initial quantum system. Moreover, it could be used to improve the
lower bounds of these correlations.

### 4.2 Lose and Gain Quantum Correlations of \(\rho_{AC}(t)\)

The amount of quantum correlation that is generated between the subsystems \(A\) and \(C\) is
described in Fig. 5, where the same initial state settings are considered. It is clear that, as
soon as the interaction is switched on between the subsystem \(A\) and the control param-
eter \(C\), an entangled state is generated. All the quantifiers \(C, \mathcal{N}\) and \(\mathcal{E}_r\) predict that there
is a quantum correlation is generated between the two qubits. From Figs. 2a and 5a, it is
clear DM interaction generates an entangled state between the subsystems \(A\) and \(C\), while
the qubits \(A\) and \(B\) are disentangled at small values of \(\kappa\). This means that, the ability of

![Fig. 4](image-url) The same as Fig. 2 but (a) \(\omega = 1\), and (b) \(\omega = 0.5\). The initial and the control states are prepared such that \(\alpha = \pi/3\), \(\kappa = 0.9\) and \(\gamma = \pi/2\)

![Fig. 5](image-url) The amount of quantum correlation that is generated between the subsystems of the state \(\rho_{AC}(t)\), \(\kappa = 0.9\), \(\omega = 0.5\), \(\gamma = \pi/2\), \(\alpha = \pi/3\) and (a) \(D_z = 0.5\) (b) \(D_z = 0.9\)
generating entanglement via DM is stronger than that may be generated by using dipole interaction. On the other hand, at large weight, namely \( \kappa = 0.9 \), the quantum correlation is generated between the qubits \( \text{A} \) and \( \text{C} \) on the expanse of the initial quantum correlation between \( \text{A} \) and \( \text{B} \). This phenomena is displayed in Fig. 5b, by investigating the behaviors of the three quantifiers, where as soon as the interaction is switched on, the quantum correlation of the state \( \rho_{\text{AB}}(t) \) decreases, while it increases between the qubits \( \text{A} \) and \( \text{C} \).

In Fig. 6, we investigate the effect of the dipole interaction parameter on the generated entangled state between the terminal (\( \text{A} \)) and the control qubit \( \text{C} \). To get a clear vision on this effect we consider different values of the dipole parameter, where we set \( \omega = 1, 2 \).

It is clear from Fig. 6a, at small values of the dipole interaction, means that less quantum correlation between the initial qubits \( \text{A} \) and \( \text{B} \), a large quantum correlation between the qubits \( \text{A} \) and \( \text{C} \) is predicted. Moreover, small values of \( \omega \) increases the survival time of the quantum correlation of the state \( \rho_{\text{AC}}(t) \). Figure 6b displays the behavior of quantifiers at large values of the dipole interaction, where we set \( \omega = 2 \). The behavior is similar to that displayed in Fig. 6a, but the upper bounds of all the quantifiers are larger than those displayed at \( \omega = 1 \).

### 4.3 Lose and Gain Quantum Correlations of \( \rho_{\text{BC}}(t) \)

As mentioned above, the DM interaction generates an entangled between the qubits \( \text{B} \) and \( \text{C} \) represented by the density operator \( \rho_{\text{BC}}(t) \). The behavior of the three quantifiers of the entanglement is predicted in Fig. 7 at different values of the interaction strength.

The behavior of the three quantifiers for the marginal state \( \rho_{\text{BC}}(t) \) at \( D_z = 0.5 \) is displayed in Fig. 7a, where it displays a similar behavior to that shown in Fig. 5a for the marginal state \( \rho_{\text{AC}}(t) \). However, the generated quantum correlation the depicted by the three quantifiers increases suddenly as soon as the interaction is switched on, while the gradual behavior is displayed for the marginal state \( \rho_{\text{AC}}(t) \). Moreover, the maximum values of the quantum correlations that shown by the three quantifiers are predicted at smaller interaction time compared with that displayed for the marginal state \( \rho_{\text{AC}}(t) \). As it is displayed from Fig. 7b, the maximum bounds of quantum correlations are larger than those displayed in Fig. 7a, where we increase the interaction’ strength. For both marginal states, the sudden birth/death of the quantum correlation are depicted at the same interaction time, meanwhile the maximum quantum correlations are displayed for marginal state \( \rho_{\text{AB}}(t) \).

![Fig. 6](image-url) The same as Fig. 4 but for the state \( \rho_{\text{AC}}(t) \) (a) \( \omega = 1 \), and (b) \( \omega = 2 \)

(Springer)
In Fig. 8, we examine the effect of the dipole interaction on the behavior of quantum correlation for the marginal state $\rho_{BC}(t)$, where we set $\omega=1,2$ for Fig. 8a and b, respectively. It is clear that, the number oscillations of the three quantifiers are larger than those displayed at small value of $\omega$ as displayed in Fig. 5. Similarly, the phenomena of the sudden death/birth are displayed at smaller interaction time compared with those displayed in Fig. 5. Moreover, the amount of the predicted quantum correlation for the marginal state $\rho_{AC}$, which is generated via direct interaction with DM, is larger than that displayed for the state $\rho_{BC}(t)$, which is generated indirectly.

5 Exchanging the Non-local Information

In this section, we investigate the behavior of the non-local information $I_{\text{non}}$ that coded on all the possible partition $\rho_{AB}(t), \rho_{AC}(t)$ and $\rho_{BC}(t)$. There is a possibility of exchanging the non-local information between the three qubits. In Fig. 9a, we quantify the amount $I_{\text{non}}$ for the initial state $\rho_{AB}(0)$, as a function on the weight parameter $\kappa$ and the angle $\alpha$. At fixed value of $\alpha = \pi/3$, the amount of the non-local information increases gradually to reach its maximum value, namely $I_{\text{non}}=2$, at $\kappa = 1$. Due to the interaction with the
control parameter, the density operator which describe the three qubits is given by $\rho_{ABC}(t)$. Figure 9b displays the behavior of the amount of the non-local information, $I_{non}(\rho_{ABC}(t))$, at $Dz = 0.5$ and the dipole strength $\omega = 2$. Again as it exhibited from (9b), the non-local information of the whole system increases gradually as $\kappa$ increases to reach its maximum value $I_{non}(\rho_{ABC}(t)) = 3$ at $\kappa = 1$. The DM strength will exhibit a change on the behavior of information that could be generated or loses between the three qubits.

Figure 10 displays the exchange of the non-local information between the three states, $\rho_{ij}(t), ij = AB, AC$ and $BC$ at different values of the weight parameter $\kappa$, while the strength of DM interaction is fixed such that $Dz = 0.9$. In Fig. 10a, we investigate the behavior of $\rho_{ij}(t)$ where it is assumed that, the weight parameter $\kappa = 0.3$. It is clear that, at $t = 0$, the non-local information that is coded in the state $\rho_{AB}(t)$ increases gradually to reach its maximum bounds. However, as the interaction time $t$ increases the nonlocal information $I_{non}(\rho_{AB}(t))$ decreases gradually to its minimum value. This behavior is repeated periodically at further $t$, where the upper and lower bounds of the non-local information are similar during the time’s interaction. The similar periodic behavior is predicted for $I_{non}(\rho_{AC}(t))$ and $I_{non}(\rho_{BC}(t))$, where as $I_{non}(\rho_{AB}(t))$ increases, the non-local information that coded on $\rho_{AC}(t)$

![Figure 9](image_url) Fig. 9 The non local information that coded on the states (a) $\rho_{AB}(0)$ and (b) $\rho_{ABC}$, where $w = 2, y = \frac{1}{2}, \alpha = \frac{2}{3}, D = 0.5$

![Figure 10](image_url) Fig. 10 The weight effect of the initial state on the generated non-local information, where $Dz = 0.9, w = 0.5, y = \frac{1}{2}, \alpha = \frac{2}{3}$. The solid(black), dash dot(blue), and the dot (red) curves, represent the non-local information for the states $\rho_{AB}(t)$, $\rho_{AC}(t)$, and $\rho_{BC}(t)$, respectively where, (a) $\kappa = 0.3$, and (b) $\kappa = 0.9$
and $\rho_{BC}(t)$ decreases. Whenever, $I_{\text{non}}(\rho_{AB}(t))$ reaches its maximum values, both $I_{\text{non}}(\rho_{AC}(t))$ and $I_{\text{non}}(\rho_{BC}(t))$ are minimum. As an important observation, the maximum values of the non-local information that predicted for $\rho_{AC}(t)$ and $\rho_{BC}(t)$ doesn’t exceed that coded on the initial state $\rho_{AB}(t)$. The amount of non-local information that is coded on the states $\rho_{AC}(t)$ and $\rho_{BC}(t)$ via direct and indirect interaction by DM, respectively, oscillates periodically between their maximum and lower bounds. Their minimum values don’t exceed the minimum values of $\mathcal{L}_{\text{non}}(\rho_{AB}(t))$. In Fig. 10b, we investigate the behavior of the non-local information that coded on the states $\rho_{ij}$ at larger values of the weight parameter, where we set $\kappa = 0.9$. A similar behavior is predicted as that shown Fig. 10a, but the upper and lower bounds of $I_{\text{non}}(\rho_{ij}(t))$ are much better than those shown at $\kappa = 0.3$.

Figure 11a displays the behavior $I_{\text{non}}(\rho_{ij}(t))$ at smaller values of DM strength, where we set $D_{Z} = 0.5$. It is clear that, the for $\rho_{AC}(t)$ (direct interaction) and $\rho_{BC}(t)$ (indirect interaction) the non-local information decreases, while the non-local information on the marginal state $\rho_{AB}(t)$ increases. However, the decreasing rate of $I_{\text{non}}$ that depicted for $\rho_{AC}(t)$ is larger than that displayed in Fig. 10a. Moreover, the crests and the troughs of the marginal states are exchanged between the states $\rho_{AC}(t)$, $\rho_{BC}(t)$ and the initial state $\rho_{AB}(t)$. The effect of larger values of $\omega$ is shown in Fig. 11b, where we et $\omega = 2$. The behavior is similar to that displayed in Fig. 10b. However, the numbers of oscillations are larger, while their amplitudes are smaller, namely the non-local information is better than that shown in Fig. 10b.

## 6 Conclusion

In this manuscript, a system of two qubits is initially prepared in a partial entangled state governed by XX chain. One of its subsystems interacts locally with a control qubit via Dzyaloshinskii-Moriya (DM). Due to these interactions, there well be entangled states are generated between the three qubits. The possibility of exchanging the quantum correlations and the non-local information between all the partitions is discussed. We investigate the effect of the initial state settings and the strengths of the interaction on the this process.

It is shown that, at small values of the weight parameter the ability of DM interaction to generate quantum correlations between the initial two qubits is larger than that may be generated by the dipole interaction. However, large weight parameter is a guarantor for generating long-lived quantum correlation at small strength of DM interaction. This...
behavior is changed if one increases the strength of DM, where the upper bounds are larger while the minimum bounds are smaller than those displayed at small weight parameter. Moreover, the long-lived quantum correlations are displayed as one increases the strength of dipole interaction. The numerical computations exhibit that, the quantum correlations are exchanged between the initial state the two marginal states, where as quantum correlations decrease on the initial system, it increases on the two marginal systems and vice versa. The maximum bounds of the quantum correlations that predicted for the marginal systems never exceed that displayed for the initial system. As soon as quantum correlation of the initial state vanishes, the marginal states exhibit a maximum quantum correlation. However, at large values of interaction strength, one finds that, the quantum correlation of marginal states have different phase due to the direct/indirect interaction with DM.

The phenomena of the exchanging the non-local information between all the three partitions is examined at different values of the interaction’s strength and the weight parameter. Our results display that, the non-local information behaves simultaneously with the quantum correlations, where the large quantum correlations, the large non-local information. One can maximize the amount of the non-local information by increasing the strength of DM interaction and decreasing the dipole interaction strength. However, at large dipole strength, the number of oscillations and their amplitudes increase, and consequently, the lower bounds of the non-local information decreases.

The generated quantum correlations are quantified by using the concurrence, entanglement of formation and negativity. The behavior of the three quantifiers is discussed, where the large amount of quantum correlations are predicted by concurrence and the negativity, while the entanglement of formation displays the smallest values of quantum correlations.

In conclusion, it is possible to exchange the quantum correlations and the non-local information between all marginal states, which may be generated via direct or indirect DM interaction. The maximum amount of these correlations and information do not exceed the initial ones. The interaction parameters are considered as control parameters for generating long-lived quantum correlations. We believe that these results are important for generating entangled quantum network, since one can generate long- live entanglement between distant particles as members of a quantum network and exchanging the non-local information between its members is possible.

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