Earthquake Scenario Reduction
by Symmetry Reasoning

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Abstract. A recently identified problem is that of finding an optimal investment plan for a transportation network, given that a disaster such as an earthquake may destroy links in the network. The aim is to strengthen key links to preserve the expected network connectivity. A network based on the Istanbul highway system has thirty links and therefore a billion scenarios, but it has been estimated that sampling a million scenarios gives reasonable accuracy. In this paper we use symmetry reasoning to reduce the number of scenarios to a much smaller number, making sampling unnecessary. This result can be used to facilitate metaheuristic and exact approaches to the problem.

1 Introduction

We consider a known problem in pre-disaster planning: forming an investment plan for a transportation network, with the aim of facilitating rescue operations in the case of earthquakes. Multi-stage stochastic problems occur in many real-life situations and are often tackled by Stochastic Programming (SP) methods based on Integer or Mathematical Programming. These methods are guaranteed to find an optimal solutions, but because of the complexity of the problems they may only be practicable for small instances.

The use of metaheuristics such as Tabu Search, Simulated Annealing, Genetic Algorithms and Ant Colony Optimisation is another promising approach to such problems. Though not guaranteed to find optimal solutions, metaheuristics can often find near-optimal solutions in a reasonable time. But applying metaheuristics requires the computation of an objective function (or fitness), which can be prohibitively expensive for stochastic problems. A common way of reducing the computational effort is approximation by scenario sampling.

We propose an alternative method to sampling for the earthquake problem, which does not involve approximation but which makes the fitness computation tractable. The paper is organised as follows. Section 2 describes the problem and an existing approach to solving it. Section 3 outlines a standard metaheuristic approach and points out its impracticality. Section 4 describes our method. Section 5 concludes the paper.

2 A disaster pre-planning problem

The problem is taken from Peeta et al. [12]. Consider the transportation network in Figure 1 (not drawn to scale), with nodes numbered 1–25 and arcs numbered 1–30,
each of whose arcs (which we shall refer to as links) may fail with some probability. This network is modelled on the Istanbul highway network.

![Fig. 1. Istanbul road network](image)

The failure probability of a link can be reduced by investing money in it, and we have a budget limiting the total investment. We aim to minimise the expected shortest path between a specified source and sink node in the network. More generally, we aim to minimise a weighted sum of shortest path lengths between several source-sink pairs, chosen (for example) to represent paths between hospitals and areas of high population. This is an example of pre-disaster planning, where a decision maker aims to maximise the robustness of a transportation network with respect to possible disasters in order to facilitate rescue operations.

First some notation. Represent the network as an undirected graph $G = (V, E)$ with nodes $V$ and arcs or links $E$. For each link $e \in E$ define a binary decision variable $y_e$ which is 1 if we invest in that link and 0 otherwise. Define a binary random variable $r_e$ which is 1 if link $e$ survives and 0 if it fails. Denote the survival (non-failure) probability of link $e$ by $p_e$ without investment and $q_e$ with, the investment required for link $e$ by $c_e$, the length of link $e$ by $t_e$, and the budget by $B$. If source and sink are unconnected then the path length is taken to be a fixed number $M$ representing (for example) the cost of using a helicopter. Actually, if they are only connected by long paths then they are considered to be unconnected, as in practice rescuers would resort to alternatives such as rescue by helicopter or sea. So Peeta et al. only consider a few shortest paths for each source-sink pair, as shown in Table I. We shall refer to these as the allowed paths. In each case $M$ is chosen to be the smallest integer that is greater than the longest allowed path length. They also consider a larger value of $M = 120$ that places a greater importance on connectivity. Let us replace $M$ by 2 new constants: $M_a$ is the length
below which a path is allowed, while $M_p$ is the penalty imposed when no allowed path exists.

| pair | links                                      | length | $M_p$ |
|------|--------------------------------------------|--------|-------|
| 14–20| 21 22 25                                   | 6.65   |       |
|      | 21 22 26 29 30 28                         | 20.41  |       |
|      | 20 17 18 23 24 26 25                      | 29.20  |       |
|      | 20 17 18 23 24 29 30 28                   | 30.27  | 31    |
| 14–7 | 20 16 10                                   | 11.14  |       |
|      | 20 17 14 13 10                            | 20.09  |       |
|      | 20 17 14 11 12 9                          | 25.48  |       |
|      | 20 16 13 11 12 9                          | 26.58  |       |
|      | 20 17 14 11 6 7 9                         | 29.08  |       |
|      | 20 16 13 11 6 7 9                         | 30.17  | 31    |
| 12–18| 17 20 21 22                                | 9.86   |       |
|      | 14 13 16 20 21 22                         | 20.05  |       |
|      | 18 23 24 26                                | 20.24  |       |
|      | 18 23 24 29 30 28 25                      | 27.06  | 28    |
| 9–7  | 13 10                                      | 9.46   |       |
|      | 11 12 9                                    | 14.85  |       |
|      | 14 17 16 10                               | 16.88  |       |
|      | 11 6 7 0                                   | 18.45  | 19    |
| 4–8  | 3 4 6                                      | 14.00  |       |
|      | 5 8 9 12                                   | 17.91  |       |
|      | 3 4 7 12                                   | 18.79  |       |
|      | 5 8 9 7 6                                  | 21.51  |       |
|      | 5 8 10 13 11                               | 26.73  |       |
|      | 5 8 10 16 17 14 11                        | 34.15  | 35    |

Table 1. Allowed paths

In SP terms this is a 2-stage problem. In the first stage we must decide which links to invest in, then link failures occur randomly. In the second stage we must choose a shortest path between the source and sink (the recourse action), given the surviving links. If the source and sink are no longer connected by an allowed path then a fixed penalty $M_p$ is imposed. Peeta et al. point out that, though a natural approach is to strengthen the weakest links, this does not necessarily lead to the best results.

This is a challenging problem because each of the 30 links might independently be affected by earthquakes, giving $2^{30}$ scenarios. Though optimisation time is not critical in pre-disaster planning, over 1 billion scenarios is too many to be tractable. Another source of difficulty is that the problem has endogenous uncertainty: the decisions (which links to invest in) affects the probabilities of the random events (the link failures). Relatively little work has been done on such problems but they are usually much harder to solve by SP methods. For a survey on problems with endogenous uncertainty see [7], which mentions applications including network design and interdiction, server
selection, facility location, and gas reservoir development. Other examples include clinical trial planning [4] and portfolio optimisation [15].

3 A metaheuristic approach

Peeta et al. sample a million scenarios, and approximate the objective function by a monotonic multilinear function. They show that their method gives optimal or near-optimal results on smaller instances. We are interested in applying standard metaheuristics such as genetic algorithms to this problem. As noted in a recent survey of metaheuristic approaches to stochastic problems [1], most research is on continuous problems and problems with noisy or time-varying fitness, and less work has been done on metaheuristics for multi-stage problems.

An obvious approach to the earthquake problem is to use a population of chromosomes, each with 30 binary genes corresponding to \( y_1 \ldots y_{30} \). Thus each chromosome is a direct representation of an investment plan in which values of 1 indicate investment and 0 no investment. Standard genetic operators (selection, recombination and mutation) can be applied to this model. To compute the fitness (objective function) of a chromosome we check every scenario, each representing a network realisation. For a given network realisation we compute the length of shortest paths between source-sink pairs using (for example) Dijkstra’s algorithm, taking value \( M_p \) if there is no path. From all the scenarios we can compute expected path lengths and hence the chromosome fitness.

A slight complication is the budget constraint: a chromosome might contain too many 1-values, corresponding to overspend. There are 3 ways of handling constraints in genetic algorithms [5]:

- Penalise constraint violation by adding a penalty function to the fitness.
- Repair the chromosome so that it no longer violates any constraints.
- Use a decoder to generate a feasible solution from the chromosome, which is treated not as a solution but as a set of instructions on how to construct one.

Any of these approaches can be applied to this problem, and we propose a simple decoder: consider the genes in a fixed order (the numerical order of links 1–30) and treat any 1-value that would violate the budget constraint as a 0-value. The endogenous uncertainty is not a problem here: given an investment plan we can immediately deduce the survival probability for each link, which enables us to compute each scenario probability and hence the path length expectations.

Unfortunately, this straightforward approach is impractical because we must consider a billion scenarios to compute the fitness of each chromosome. In fact a major issue when solving stochastic problems by metaheuristics is the fitness computation, and there are 3 common approaches [1]:

- Use a closed-form expression to compute exact fitness.
- Use a fast approximation to an expensive closed-form expression.
- Estimate fitness by sampling scenarios, as in the field of Simulation Optimisation.

We propose an alternative approach: compute fitness exactly by exploiting symmetries between scenarios, in order to bundle many of them together so that they can be considered simultaneously.
4 Exploiting symmetries between scenarios

We shall illustrate our method using the simple example in Figure 2. The links $e = 1 \ldots 4$ have lengths $t_e \equiv 1$, $p_e \equiv 0.8$, $q_e \equiv 1$, $c_e \equiv 1$, $B = 1$ and $M_a = M_p = 3.5$ so that both possible paths between nodes 1–4 are allowed. We must choose 1 link to invest in, to minimise the expected shortest path length between nodes 1–4. There are 16 scenarios, and the optimal policy is to invest in link 1, giving an expected shortest path length of 2.236.

4.1 Merging scenarios

Some scenarios can be considered together instead of separately. For example consider two scenarios 1001 (in which links 1 and 4 survive but 2 and 3 do not) and 1101 (identical except that link 2 survives). These scenarios have probabilities $0.8 \times 0.2 \times 0.2 \times 0.8 = 0.0256$ and $0.8 \times 0.8 \times 0.2 \times 0.8 = 0.1024$ respectively. As link 3 does not survive, it is irrelevant whether or not link 2 survives because it cannot be part of a shortest path (or any path). We can therefore merge these two scenarios into one, which we shall write as 1i01 where “i” denotes interchangeability: the values 0 and 1 for link 2 are interchangeable. We shall refer to a combined scenario such as 1i01 as a multiscenario. As “i” includes both the 0 and 1 values, the probability associated with the multiscenario is $0.8 \times 1.0 \times 0.2 \times 0.8 = 0.128$.

However, it is impractical to enumerate a billion scenarios then look for ways of merging some of them. Instead suppose we enumerate scenarios by tree search on the random variables. Consider a node in the tree at which links $1 \ldots i$ have been realised, so that random variables $s_1 \ldots s_{i-1}$ have been assigned values, and we are about to assign a value to $s_i$ corresponding to link $e$. Denote by $\ell_e$ the shortest source-sink path length including $e$, under the assumption that all unrealised links survive; and denote by $\ell \bar{e}$ the shortest source-sink path length not including $e$, under the assumption that all unrealised links fail (using $M_p$ when no path exists). So $\ell_e$ is the minimum shortest path length including $e$ in all scenarios below this policy tree node, while $\ell \bar{e}$ is the maximum shortest path length not including $e$ in the same scenarios. They can easily be computed by temporarily assigning $s_{i-1} \ldots s_n$ (where $|E| = n$) to 1 or 0 respectively, and applying a shortest path algorithm. Now if $\ell_e \geq \ell \bar{e}$ then the value assigned to $s_i$ is irrelevant: the shortest path length in each scenario under this tree node is independent of the value of $s_i$, so the values are interchangeable.
It is important to note that the order in which we assign the \( s \) variables affects the size of the multiscenario set. Three multiscenario sets for the example are shown in Table 2 where \( p \) is the multiscenario probability, 0 and 1 are the values of the random variables corresponding to the links, and “i” denotes interchangeable values: given the assignments to the left, the objective value is independent of whether the random variable is set to 0 or 1. The set of size 7 corresponds to the permutation corresponding to the numerical link ordering 1234, the set of size 10 is the largest possible, and the set of size 5 is the smallest possible. Having derived the multiscenarios, we can replace the “i” entries by arbitrary values, for example 0.

| links | \( p \) |
|-------|--------|
| 3 2 4 1 | 0.0400 |
| 0 1 0 1 | 0.0320 |
| 0 1 1 0 | 0.1280 |
| 1 0 0 0 | 0.0320 |
| 1 0 1 0 | 0.0256 |
| 1 0 1 1 | 0.1024 |
| 1 1 0 0 | 0.0256 |
| 1 1 0 1 | 0.1024 |
| 1 1 1 0 | 0.4096 |

| links | \( p \) |
|-------|--------|
| 1 3 2 4 | 0.2000 |
| 0 i 1 1 | 0.2000 |
| 1 0 i 0 | 0.0320 |
| 1 0 i 1 | 0.1280 |
| 1 1 0 0 | 0.0256 |
| 1 1 0 1 | 0.1024 |
| 1 1 1 0 | 0.1024 |
| 1 1 1 1 | 0.4096 |

| links | \( p \) |
|-------|--------|
| 1 4 2 3 | 0.2000 |
| 0 i 1 1 | 0.2000 |
| 1 0 i 0 | 0.0320 |
| 1 0 i 1 | 0.1280 |
| 1 1 i i | 0.6400 |

Table 2. Three multiscenario sets for the small example

4.2 Stochastic dominance, symmetry and network reliability

The above ideas have parallels in several literatures. Firstly, one way of viewing this form of reasoning is as stochastic dominance [9], a concept from the Decision Theory literature: the objective function associated with one choice (0 or 1) is at least as good as with another choice (1 or 0). Because this holds in every scenario, it is the simplest form of stochastic dominance: statewise (or zeroth order) dominance. However, this is usually defined as a strict dominance by adding an extra condition: that one choice is strictly better than the other in at least one state (or scenario). In our case neither value is better so this is a weak dominance. In fact we have two values that each weakly dominate the other, a relationship that can be viewed as a symmetry: the tree is exactly the same whichever value we use for a link. But there does not seem to be an accepted term such as “stochastic symmetry” for this phenomenon.

However, in the Constraint Programming and Artificial Intelligence literatures this type of symmetry is often used to reduce search tree sizes (for non-stochastic problems): see Chapter 10 of [14] for a survey of such techniques. Because the symmetry only occurs under certain assignments to some other variables, it is a conditional symmetry, the condition being that certain other assignments have occurred. And because it is a
symmetry on values in the domain of a variable it is also a *value interchangeability*, a form of symmetry first investigated in [6] and since developed in many ways [8]. More specifically, it is a form called *full dynamic interchangeability* [8,13], the word *dynamic* having a similar meaning to *conditional* here. Though interchangeability has been the subject of considerable research, a drawback is that it does not seem to occur in many real applications [2,11]; we believe that it will occur more often in stochastic settings such as this one.

The Network Reliability literature describes methods for evaluating and approximating the reliability of a network. These include ways of pruning irrelevant parts of a network that have connections to our approach, though we have not found a direct parallel. For a discussion of these ideas see [3].

### 4.3 Application to the earthquake problem: single path

For the earthquake problem we used a simple hill-climbing algorithm to find a good permutation, based on 2- and 3-exchange moves, and accepting moves that improve or leave unchanged the number of multiscenarios. The results are given in Table 3 for each source-sink pair considered separately, and took several minutes each to compute. The table shows the instances numbered 1–5, the source and sink, the chosen constant $M_a$, the best link permutation found, and the size of the corresponding multiscenario set.

| instance | source-sink pair | link permutation | multiscenarios |
|----------|------------------|------------------|---------------|
| 1        | 14–20            | 20 22 2 21 3 17 18 6 25 15 1 23 4 24 11 | 69            |
| 2        | 14–7             | 20 16 10 25 17 14 1 28 9 2 13 3 8 11 4 | 45            |
| 3        | 12–18            | 18 22 23 9 21 20 2 12 24 8 7 11 26 17 13 | 79            |
| 4        | 9–7              | 1 3 28 29 10 13 20 18 8 11 9 27 12 16 17 | 26            |
| 5        | 4–8              | 6 12 24 5 8 18 4 19 9 3 21 23 28 7 10 | 124           |

Note that the choice of permutation has a significant effect. For the 5 instances, if we use the numerical link ordering we obtain multiscenario sets of sizes 4944, 4154, 5268, 87 and 1488 respectively, but we might be more unlucky: we sampled 10 random permutations per instance and found worst-case multiscenario set sizes 31124, 115760, 21200, 994 and 7408 respectively. These are much better than 1 billion but considerably worse than the best sets we found, showing the advantage of searching for a good permutation.

Table 4 shows in full the multiscenario set for the 4th instance, which has the smallest set. The table shows that the expected shortest path length from 9–7 is independent
of the survival or failure of links 1, 2, 3, 4, 5, 8, 15, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29 and 30; it depends only on the remaining 10 links 6, 7, 9, 10, 11, 12, 13, 14, 16 and 17. Therefore the multiscenario set for this instance should be no larger than $2^{10} = 1024$. But among these 10 links there are interchangeabilities: for example if link 10 fails then the expected length is independent of the survival or failure of link 13, but if link 10 survives then what happens to link 13 is important; if link 13 also survives (last line) then no other link matters because the 9–7 shortest path is available.

| 1 | 3 | 28 | 29 | 10 | 13 | 20 | 18 | 8 | 11 | 9 | 27 | 12 | 16 | 17 | 21 | 30 | 19 | 25 | 24 | 2 | 4 | 14 | 22 | 5 | 23 | 26 | 7 | 6 | 15 |
|---|---|----|----|----|----|----|----|---|----|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| i | i | i | i | 0 | i | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | i | 1 | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | 0 | i | i | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 1 | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |
| i | i | i | 0 | i | i | i | 1 | 0 | i | i | i | i | i | i | i | i | i | i | i | 0 | i | i | i | i | i | i | i |

Table 4. Multiscenario set for instance 4

### 4.4 Extension to multiple paths

We aim to minimise the expected weighted sum of shortest path lengths $\ell_i$ between several source-sink pairs:

$$\text{Minimise } z = E\left\{ \sum_i w_i \ell_i \right\}$$
for weights $w_i$. Unfortunately, there is likely to be little interchangeability in this problem, especially if (as we would expect) the pairs are chosen to cover most of the network: for a given link to be irrelevant to the lengths of several paths is much less likely than for one path. But we can avoid this drawback by rewriting the objective function as:

$$\text{Minimise } z = \sum_i w_i E\{\ell_i\}$$

so that each path is treated separately, and can be evaluated using its own link permutation. If we do this using the permutations shown in Table 3, the total number of multiscenarios is 343, so to evaluate an investment plan we need consider only this many multiscenarios: a very tractable. Note that we compute fitness exactly, so the optimal investment plan is guaranteed to occur in the search space of the genetic algorithm (though the algorithm is not guaranteed to find it).

### 4.5 Other risk measures

Note that it is easy to change the objective function in a genetic algorithm. SP researchers have recently explored risk-averse disaster planning including transportation networks [10], and we can use risk-averse objective functions such as conditional value-at-risk (CVaR) for a single source-sink pair. For multiple pairs we can take a weighted sum of CVaRs. For our method to work well, we must optimise some function of statistical parameters computed on each pair separately.

### 5 Conclusion

In future work we shall implement metaheuristic and exact algorithms to solve the reduced problem. We shall also apply the reduction technique to other problems.

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