Duality as a Gauge Symmetry and Topology Change

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ABSTRACT

Duality groups as (spontaneously broken) gauge symmetries for toroidal backgrounds, and their role in (∞-dimensional) underlying string gauge algebras are reviewed. For curved backgrounds, it is shown that there is a duality in the moduli space of WZNW sigma-models, that can be interpreted as a broken gauge symmetry. In particular, this duality relates the backgrounds corresponding to axially gauged abelian cosets, $G/U(1)_a$, to vectorially gauged abelian cosets, $G/U(1)_v$. Finally, topology change in the moduli space of WZNW sigma-models is discussed.

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1 Introduction

The new results in this talk are based on work with E. Kiritsis [1]. I will discuss two issues in the moduli space of a WZNW sigma-model:

(a) There is a target space duality that can be interpreted, in the effective action, as a broken gauge symmetry. In particular, this duality relates the backgrounds corresponding to axially gauged abelian cosets to vectorially gauged abelian cosets.

(b) There is a topology change, namely, there are deformations lines in the moduli space of sigma-models along which the topology is changed.

The importance of the interpretation of duality as a gauge symmetry is:

(1) It shows that duality is an exact symmetry in string theory. In particular, it shows that axial-vector duality is an exact symmetry; this has implications in the study of black-hole duality in string theory [2], and duality in cosmological string solutions [3, 4].

(2) Dualities appear to be discrete symmetries (some kind of a “Weyl subgroup”) of an (infinite-dimensional) universal gauge algebra.

In section 2, I will explain point (2) reviewing known results on duality as a gauge symmetry for toroidal backgrounds. In section 3, I will discuss the $J\bar{J}$ deformation of $SU(2)$ (or “$SL(2)$”) WZNW sigma-model, and a duality along this line will be interpreted as a broken gauge symmetry. Finally, in section 4, I will discuss topology change in the moduli space of WZNW sigma-models.

2 Duality as a gauge symmetry and universal gauge algebras

2.1 $R \to 1/R$ circle duality as a gauge symmetry

I will first discuss the simplest case of a single scalar field compactified on a circle with radius $R$. The moduli space of circle compactifications is the positive real line, describing all possible compactification radii $R > 0$. For each compactification radius, the conformal field theory (CFT) has (at least) a $U(1)_L \times U(1)_R$ affine symmetry, generated by the chiral and antichiral currents $J$ and $\bar{J}$. The truly marginal deformation of the CFT, corresponding to a change of the compactification radius from $R$ to $R + \delta R$, is given by adding $\delta S = \delta (R^2) \int d^2z J\bar{J}$ to the worldsheet action.

A striking property of string theory is that there is a “target-space duality” symmetry relating backgrounds with different geometries, that correspond to the same CFT. In this
case, the CFT corresponding to a radius $R$ circle is equivalent to the CFT of a radius $1/R$ circle \[5\].

The conformal field theory at the self-dual point, $R = 1$, has an extended affine symmetry $SU(2)_L \times SU(2)_R$. At this point there are three chiral currents $J^a$, $a = 1, 2, 3$, and three antichiral currents $\bar{J}^b$, generating the affine $SU(2)_L$ and $SU(2)_R$ symmetry, respectively. We shall refer to the point $R = 1$ as the $SU(2)$ point. To deform the theory away from the $SU(2)$ point, one may choose to identify $J \equiv J^3$, $\bar{J} \equiv \bar{J}^3$. But this choice is not unique: any deformation of the type

$$
(\sum_{a=1}^{3} \alpha_a J^a)(\sum_{b=1}^{3} \beta_b \bar{J}^b) \quad (2.1)
$$

is truly marginal. The set of critical points, that can be reached from the $SU(2)$ point by such conformal deformations, span a 5-dimensional surface in the 9-dimensional euclidean space generated by the couplings to $J^a \bar{J}^b$. However, because different truly marginal deformations give rise to the same CFTs, the dimension of the physical moduli space is 1. This can be shown as follows: At the $SU(2)$ point, $SU(2)_L$ and $SU(2)_R$ rotations of the $J^a$'s and $\bar{J}^b$'s are symmetries of the CFT, and as a consequence, any deformation of the type $\begin{align} (2.1) \end{align}$ give rise to the same CFT given for an appropriate $J^3 \bar{J}^3$ deformation.

In particular, the duality transformation $R \rightarrow 1/R$ corresponds to the Weyl transformation in $SU(2)_L$ that takes $J^3$ to $-J^3$ \[8\]. Infinitesimally near the $SU(2)$ point, this corresponds to the identification of the theory given by the deformation $\delta \alpha J^3 \bar{J}^3$ with the theory given by the deformation $-\delta \alpha J^3 \bar{J}^3$.

The discussion, so far, was from the worldsheet point of view. But in string theory, worldsheet properties have a target-space analogue. Indeed, coupling constants to operators in the worldsheet action become space-time fields in the effective action of string theory. In particular, for a string compactified on a circle, we expect to have a scalar field corresponding to the worldsheet coupling to the $J^a \bar{J}^b$ deformation. The vacuum expectation values (VEVs) of the scalar field correspond to the compactification radii. At the particular VEV, corresponding to the $R = 1$ point, there is a gauge symmetry $SU(2)_L \times SU(2)_R$. At this point, extra scalar fields and gauge fields become massless. Changing the VEV of scalar fields away from this point (or, equivalently, changing the compactification radius), the gauge symmetry is spontaneously broken to $U(1)_L \times U(1)_R$, and, in addition, there is a symmetry corresponding to $R \rightarrow 1/R$ duality. In that sense, the target space duality is a discrete symmetry of the spontaneously broken $SU(2)$ gauge algebra. The $Z_2$ duality is the Weyl group of $SU(2)$.

### 2.2 $O(d, d, Z)$ dualities as gauge symmetries

Compactifying the bosonic (heterotic) string on a $d$-dimensional torus, the circle duality is generalized to a duality group isomorphic to $O(d, d, Z)$ ($O(d, d + 16, Z)$) \[8 \ 9\].

\[^{3}\text{I choose } \alpha' = 1, \text{ where } \alpha' \text{ is the inverse string tension.}\]
The duality group may be interpreted (in the effective action) as a residual discrete symmetry group of some spontaneously broken (\(\infty\)-dimensional) universal gauge algebra. Moreover, the \(O(d, d, Z)\) dualities are expected to be some kind of a Weyl subgroup of the underlying algebra. From the worldsheet point of view, this can be shown as follows \([10]\): When \(d > 1\) there is an infinite number of points in the moduli space of \(d\)-tori backgrounds that have extended affine symmetries. For instance, when \(d = 2\) there is an infinite number of points, in the moduli space, where the affine \(U(1)^2\) symmetry is extended to \(SU(2) \times SU(2)\) or to \(SU(3)\). Now, any product of Weyl reflections acting on conformal deformations around points with extended symmetries relate geometrically different toroidal backgrounds that correspond to the same CFT. It turns out that any such “Weyl reflections” product is an element of \(O(d, d, Z)\). Moreover, any element of \(O(d, d, Z)\) correspond to a product of Weyl reflections.\(^4\)

These worldsheet properties are realized in the target space in an intriguing way. In the effective action, one expects to find a (\(\infty\)-dimensional) gauge algebra, that is spontaneously broken for any VEV of the scalar fields to an appropriate (finite-dimensional) gauge group, and residual discrete symmetries generating the duality group. This is the interpretation of target space dualities as the residual discrete symmetries of a spontaneously broken gauge algebra.

Such an effective action was constructed in ref. \([11]\) for the \(d = 4\), \(N = 4\) heterotic string, \(i.e.,\) the toroidal compactification of the heterotic string to four dimensions. The \(\infty\)-dimensional gauge algebra, called the “Duality Invariant String Gauge algebra” (DISG) is isomorphic to

\[
\text{Duality Invariant String Gauge algebra} \sim \text{“LatticeAlg”}(\Gamma^{6,22}), \quad (2.2)
\]

denote, the algebra of dimension 1 operators in the CFT of 28 chiral scalars, compactified on an even-self dual lorentzian lattice with signature \((6,22)\). The infinite-dimensional gauge symmetry is spontaneously broken for \(\text{any VEV of the scalar fields to a finite-dimensional gauge algebra (typically } U(1)^6 \times E_8 \times E_8)\), and a group of residual discrete symmetries: the \(O(6, 22, Z)\) target-space dualities.

This duality group is some kind of a Weyl subgroup of the underlying gauge algebra. The subgroup of the the duality group, that fixes a point in the moduli space of toroidal backgrounds, is related to the Weyl group of the enhanced (finite-dimensional) gauge symmetry at the fixed point. To get points with large enhanced gauge symmetries and large duality subgroups that fix such points, it is useful to compactify time as well.

\[\text{2.3 Compactifying time}\]

By compactifying all the coordinates – including timelike dimensions – on a torus, one recovers points in the moduli space of backgrounds with large gauge symmetries; these

\(^4\)More precisely, every element of \(O(d, d, Z)\) is a product of Weyl reflections in the moduli space of \((d + 1)\)-tori \([11]\).
correspond, in the worldsheet, to large on-shell algebras of dimension (1,0) or (0,1) operators. Some backgrounds are fixed points of large duality subgroups. Compactifying time is, therefore, useful in the search for an underlying universal gauge algebra of string theory.

Such a program was initiated for the critical $N = 2$ string, compactified completely on a torus $T^{2,2}$ [12]. A distinguished point in the moduli space of (2,2) toroidal backgrounds is the one where the Narain lattice $\Gamma^{4,4}$ is the direct sum of a $(2,2)_L$ even self-dual lorentzian lattice of left-movers, and a $(2,2)_R$ even self-dual lorentzian lattice of right-movers

\[ \text{Maximal Extended Symmetry Point : } \Gamma^{4,4}_{MES} = \Gamma^{2,2}_L \oplus \Gamma^{2,2}_R. \]

At this point the extended on-shell gauge algebra is infinite-dimensional: it is generated by the area-preserving diffeomorphisms of null 2-tori in the $(2,2)_L$ (and $(2,2)_R$) torus.

A universal gauge algebra is a background independent algebra that is a minimal Lie algebraic closure which contains all the on-shell algebras. Candidates for universal gauge algebras of the $N = 2$ string were presented in ref. [12]. These are:

1. “Lattice-Algebra” ($\Gamma^{4,4}$).
2. Volume-Preserving-Diffeomorphisms($T^{4,4}$).

Here $T^{4,4} \equiv R^{4,4}/\Gamma^{4,4}$ is the Narain torus defined by the Narain lattice $\Gamma^{4,4}$. The first candidate is the analogue of the DISG for the $N = 2$ string; the duality group $O(4,4,\mathbb{Z})$ is some kind of its Weyl subgroup. The second candidate seems more natural for the $N = 2$ string.

In a recent paper [13], Moore has presented a candidate for a universal symmetry algebra of the bosonic string. One starts by compactifying all coordinates on a (1,25) torus. A distinguished point in the moduli space of toroidal backgrounds is the one where the Narain lattice $\Gamma^{26,26}$ is the direct sum of a $(25,1)_L$ even self-dual lorentzian lattice of left-movers, and a $(1,25)_R$ even self-dual lorentzian lattice of right-movers

\[ \text{MaximalExtended Symmetry Point : } \Gamma^{26,26}_{MES} = \Gamma^{25,1}_L \oplus \Gamma^{1,25}_R. \]

At this point the extended on-shell gauge algebra is infinite-dimensional: it is the direct sum of the “Fake Monster Lie Algebra” ($\sim$ “Lattice-Algebra” ($\Gamma^{1,25}$)) of left-movers and right-movers.

The candidate for a universal gauge symmetry is a modification of the DISG algebra (on a modified Narain lattice, needed to include properly the left-moving enhanced symmetries together with the right-moving enhanced symmetries): Universal Symmetry $\sim D\hat{I}SG \equiv \text{“LatticeAlg”} (\tilde{\Gamma}^{26,26})$. For more details see ref. [13].
3  $J\bar{J}$ deformation of $SU(2)$ (or $SL(2)$) WZNW sigma-model, and a duality as a broken gauge symmetry

The discussion in section 2 is restricted to toroidal backgrounds. What about target space dualities in the moduli space of curved backgrounds? Can they be interpreted as some (spontaneously broken) gauge symmetries? Here I shall discuss, from the worldsheet point of view, a particular element of the duality group in the moduli space of WZNW models. For simplicity, I will discuss the simplest non-trivial case, namely, duality acting on the conformal deformations line of $SU(2)$ (or “$SL(2)$”) WZNW models. In this note I will only present the results; the details appear in [1].

The action for $SU(2)_k$ (in a particular parametrization of the group elements) is given by

$$S[x, \theta, \tilde{\theta}] = \frac{k}{2\pi} \int d^2z (\partial \theta, \tilde{\partial} \tilde{\theta}, \partial x) \left( \begin{array}{ccc} \sin^2 x & \cos^2 x & 0 \\ -\cos^2 x & \cos^2 x & 0 \\ 0 & 0 & 1 \end{array} \right) \left( \begin{array}{c} \tilde{\partial} \tilde{\theta} \\ \tilde{\partial} \bar{\theta} \\ \partial x \end{array} \right)$$

$$- \frac{1}{8\pi} \int d^2z \phi_0 R^{(2)}, \quad (3.1)$$

where $x \in [0, \pi/2)$ and $\theta, \tilde{\theta} \in [0, 2\pi)$. We will refer to the matrix in (3.1) as “the background matrix” $E$.

The action $S$ (3.1) is manifestly invariant under the $U(1)_L \times U(1)_R$ affine symmetry generated by the currents

$$J = k(-\sin^2 x \partial \theta + \cos^2 x \partial \tilde{\theta}) \quad \bar{J} = k(\sin^2 x \bar{\partial} \tilde{\theta} + \cos^2 x \bar{\partial} \bar{\theta}). \quad (3.2)$$

It is possible to deform the action $S$ to new conformal backgrounds by adding to it any marginal deformation. We will focus on the marginal deformation $J\bar{J}$:

$$S \to S + \alpha \int d^2z J\bar{J}. \quad (3.3)$$

This deformation is equivalent to a particular 1-parameter sub-family of $O(2, 2, R)$ rotations acting on the background matrix and dilaton in (3.1). Parametrizing this family by $0 \leq R < \infty$, one finds the line of exact CFTs with background matrices:

$$kE_R = k \left( \begin{array}{ccc} \tan^2 x/\Delta & 1/\Delta & 0 \\ -1/\Delta & R^2/\Delta & 0 \\ 0 & 0 & 1 \end{array} \right), \quad \Delta = 1 + R^2 \tan^2 x. \quad (3.4)$$

The dilaton also transforms under the deformation (see [1]).

Some special points along the $R$-line of deformations are:

(1) At $R = 1$, one recovers the original $SU(2)$ point (3.1).
(2) At \( R \to \infty \), the background corresponds to a direct product of an axially gauged \( SU(2)/U(1)_a \) sigma-model \([13]\), and a non-compact free scalar field.

(3) At \( R \to 0 \), the background corresponds to a direct product of a vectorially gauged \( SU(2)/U(1)_v \) sigma-model, and a non-compact free scalar field.

We now arrive to the important point of this part. The Weyl reflection \( J \to -J \) is given by a continuous group rotation at the WZNW point. This implies that \( J \to -J \) is a symmetry at the WZNW point, and therefore, the infinitesimal deformation \( S_{WZNW} \to S_{WZNW} + \delta \alpha \int d^2z J \bar{J} \) is the same CFT as the one given by the deformation \(-\delta \alpha \int d^2z J \bar{J}\).

The points \( \delta \alpha \) and \(-\delta \alpha \) along the \( \alpha \)–modulus \([13]\) are the same CFT. In string theory we say: \( \delta \alpha \) and \(-\delta \alpha \) are related by a residual \( Z_2 \) symmetry of the broken gauge symmetry (of the enhanced symmetry point); the residual symmetry is the target space duality. This symmetry can be integrated to finite \( \alpha \), giving rise to a \( Z_2 \in O(2,2,\mathbb{Z}) \) duality\(^5\) along the \( R \)–line \([3,4]\). The action of duality on the background \( E_R \) is:

\[
\text{Duality along the } R\text{–line : } E_R \to E_{1/R} \quad \text{i.e.} \quad R \to 1/R. \quad (3.5)
\]

In particular, the boundary points \( R \to 0 \) and \( R \to \infty \) are the same CFT. As a consequence, axial-vector duality of \( SU(2)/U(1) \) and \( SL(2)/U(1) \) is exact, and corresponds to a residual discrete symmetry of the broken gauge algebra.

I shall end this part with few remarks:

(a) In the \( SU(2) \) \((SL(2))\) case, the CFT along the \( R\)-line is a \( Z_k \) orbifoldization of a compact (non-compact) parafermionic theory and a free scalar field with radius \( \sqrt{k}R \).

(b) In the \( SU(2) \) case, the modular invariant \( R \)-dependent genus one partition function can be written.

(c) The results of this section can be extended to any group \( G \) \([1]\).

4 Topology change in the space of WZNW sigma-models

Along the \( 0 < R < \infty \) deformations line of the \( SU(2) \)-WZNW sigma-model, the topology of the background space is of the three sphere. However, at the boundaries \((R = 0, \infty)\), the topology is changed to that of a product of a two-disc (corresponding to the \( SU(2)/U(1) \) conformal background) with a circle whose radius shrinks to 0. In the “\( SL(2) \)” case, the background at the boundary is the direct product of a degenerated

\(^5\) The \( O(d,d,\mathbb{Z}) \) group is a duality (sub-)group also for \textit{curved} backgrounds with \( d \) abelian isometries \([4]\).
circle with a semi-infinite cigar (at $R \to \infty$), or the infinite trumpet (at $R \to 0$); these correspond to the dual pair of the 2-$d$ euclidean black-hole backgrounds.

A topology change at the boundary of moduli space is not surprising. However, the $R$-line of deformations is not the full story in the moduli space of $G$-WZNW sigma-models. In fact, any conformal sigma-model with $d$ abelian isometries can be transformed to a new conformal background by $O(d, d, R)$ rotations. In this bigger moduli space of sigma-models, there are some more interesting deformation lines, along which the topology might change.

For example, in the moduli space of the $SU(2)$–WZNW sigma-model, there is a one-parameter sub-family of $O(2, 2, R)$ rotations that generate backgrounds with metric

$$ds^2(\alpha) = \frac{k}{\Delta(\alpha)} [\sin^2 x d\theta^2 + \cos^2 x d\bar{\theta}^2] + k dx^2,$$

$$\Delta = \cos^2 \alpha \cos^2 x + (\cos \alpha + k \sin \alpha)^2 \sin^2 x.$$  \hspace{1cm} (4.1)

(There are also non-trivial dilaton and torsion along this $\alpha$–line). At the point $\alpha = 0$ the background includes a metric of the $SU(2)_k$ group manifold $S^3$ (as well as an antisymmetric background). Along the line $0 < \alpha < \pi/2$ the background includes the metric (4.1) with the topology of $S^3$ (as well as an antisymmetric background and a dilaton field). At the point $\alpha = \pi/2$ the background metric is

$$ds^2(\alpha = \pi/2) = \frac{1}{k} d\theta^2 + \frac{1}{k} \cot^2 x d\bar{\theta}^2 + k dx^2.$$ \hspace{1cm} (4.2)

At this point the manifold has a topology of $D_2 \times S^1_{1/k}$, where $D_2$ is a two-disc and $S^1_{1/k}$ is a circle with radius $R^2 = 1/k$. One may continue to deform this theory by, for example, changing the compactification radius $R$ of the free scalar field $\theta$.

It is remarkable that (for integer $k$) the neighborhood of the point $\alpha = \pi/2$ is mapped to the neighborhood of the point $\alpha = 0$ by an element of $O(2, 2, Z)$, namely, a target space generalized duality. Therefore, a region in the moduli space, where a topology change occurs, is mapped to a region where there is no topology change at all. A similar phenomenon happens for more complicated examples in the moduli space of Calabi-Yau compactifications.

Finally, let me make few comments:

(a) The sigma-models along the $\alpha$–line (4.1) have conical singularities. Therefore, to make sense of the CFTs along the $\alpha$–line one should understand CFTs corresponding to backgrounds with (non-orbifold) conical singularities.

(b) After the topology is changed at (4.2), by deforming the compactification radius of the free scalar field $\theta$, one does not get rid of the curvature singularity encountered at the point where the topology is changed.\footnote{In the bosonic string, the $O(d, d)$ rotations give only the leading order in $\alpha'$ of the conformal backgrounds; but, there are higher order corrections that make them exact [14, 17].}
(c) To cure both problems, one should look at the moduli space of WZNW sigma-models in higher dimensions; for instance, the sigma-model moduli space discussed for cosmological backgrounds in [4].

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