Matrix elements of four-fermion operators in the HQET.

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The $B$-$\bar{B}$ mixing, $B$-meson lifetimes, the $B_s$-$\bar{B}_s$ lifetime difference and SUSY FCNC effects in $\Delta B = 2$ processes are very important measurable quantities in $B$-meson phenomenology whose theoretical predictions depend on unknown matrix elements of several four-fermion operators. We present preliminary results for the matrix elements of the relevant four-fermion operators computed on a sample of 600 lattices of size $24^3 \times 40$ at $\beta = 6.0$, using the SW-Clover action for light quarks with rotated light quark propagators and the lattice version of the HQET for heavy quarks. As a necessary ingredient of the calculation, we also present results for the next-to-leading order matching of the full theory to the lattice HQET (one- and two-loop anomalous dimensions, one-loop QCD-HQET matching coefficients and one-loop continuum-lattice HQET matching coefficients).

1. Introduction and Motivation.

$B$–hadron decays are a very important source of information on the physics of the standard model (SM) and beyond. In many important cases, long distance strong contributions to these processes can be separated into matrix elements of local operators. Lattice QCD can then be used to compute these non–perturbative parameters from first principles. A list of some four–fermion operators relevant to $B$–meson phenomenology is the following:

- the operator $O_{LL} = \bar{b} \gamma^\mu (1 - \gamma_5) q \bar{b} \gamma_\mu (1 - \gamma_5) q$ ( $q$ denotes a light quark $u$, $d$ or $s$), which determines the theoretical prediction of the $B^0$-$\bar{B}^0$ mixing in the SM, together with $O_{LL}^{S}$, $O_{RR}^{S}$, and $O_{LR}^{S}$, with obvious notation, parametrize SUSY effects in $\Delta B = 2$ transitions [1]. Furthermore, $O_{LL}$ and $O_{LL}^{S}$ determine the $B_s$ width difference $\Delta \Gamma_{B_s}$ [3];

- $Q_{LL} = \bar{b} \gamma^\mu (1 - \gamma_5) q \bar{q} \gamma_\mu (1 - \gamma_5) b$, together with $Q_{LR}^{S}$, parametrize spectator effects in the $\tau_B$ and $\tau_{B_s}$ lifetimes [3];

- finally, the corresponding operators with a $t^a$ (the generator of the SU(3) group) insertion: $O_{LL}^{S}$, $O_{LR}^{S}$ and $O_{LR}^{S}$ contribute to SUSY effects in $\Delta B = 2$ transitions [1], and $Q_{LL}^{S}$ and $Q_{LR}^{S}$ to spectator effects in the $B$-meson lifetimes [3].

Our aim is to non-perturbatively evaluate the matrix elements between $B$–meson states of all the operators relevant to phenomenology using lattice simulations of the HQET for the $b$–quark. The procedure to perform the transition from QCD to lattice HQET, a combination of analytic and numerical calculations, consists in the four steps described in detail below.

2. First step: the continuum QCD – HQET matching.

The starting point is to express the QCD operators as linear combinations of the HQET ones in the continuum at a given scale, say, $\mu = m_b$. To do so, some amplitudes of the relevant operators between appropriate external $B$-meson states are evaluated to one–loop order both in QCD and HQET. Once the amplitudes have been renormalized using the same scheme in both theories, the QCD amplitude is expanded in powers of $1/m_b$ to lowest order. Finally, the coefficients of the matching can be obtained by subtraction at the scale $\mu$. It should be stressed that the matching is renormalization scheme dependent. Notice also that new effective operators can be generated.

For $O_{LL}$, the matching was determined by
Fierz symmetry. We skip all details and refer the reader to ref.\cite{6} for details.

3. Second step: running down to $\mu = a^{-1}$ in the HQET.

The HQET operators obtained in step 1 at the high scale $\mu = m_b$ are evolved down to a lower scale $\mu = a^{-1}$, appropriate for lattice simulations, using the HQET renormalization group equations. The running is governed by the corresponding anomalous dimension matrices. At NLO, one has to calculate both the one- and two-loop anomalous dimension matrices of the relevant effective operators. The former are renormalization scheme independent but the latter are not. The scheme dependence of the two-loop anomalous dimension matrices cancels with the scheme dependence of the matching in step 1.

The calculation of the one-loop anomalous dimension matrix is not difficult

$$\tilde{\gamma}^{(0)} = \begin{bmatrix} -8 & 0 & 0 \\ 4/3 & -8/3 & 0 \\ 0 & 0 & -7 \\ 0 & 0 & 3/2 & -7 \end{bmatrix}$$

in the basis $\{\hat{O}_{ll}, \hat{O}_{ll}^S, \hat{O}_{lr}, \hat{O}_{rl}^S\}$.

However, only the two-loop anomalous dimension of the operator $\hat{O}_{ll}$ is known. It was determined by Gimenez \cite{5} (see also ref.\cite{7}) using NDR and DRED. Now we have extended the calculation to all effective operators in the basis of eq.\cite{1}. Our results for $\hat{O}_{ll}$ agree with ref.\cite{6} but disagree with ref.\cite{8} in the sign of the contribution proportional to $\hat{O}_{latt}^N$ (see below) of the group of Feynman diagrams with radiative corrections between light quark legs only. We refer the reader to ref.\cite{6} for details.

4. Third step: continuum–lattice HQET matching.

Having obtained the continuum HQET operators at the scale $\mu = a^{-1}$, they are expressed as a linear combination of lattice HQET operators at this scale. The procedure is very similar to step 1: one matches two amplitudes at one-loop order, one in the continuum HQET and the other in the lattice HQET. However, some subtle points to remember are the following:

1. the matching has to be calculated using the same action used in the numerical simulation; the matching coefficients depend on the action used in the simulation.

2. Due to the breaking of chiral symmetry induced by the Wilson term for light quarks, the original operators can mix with new lattice operators.

3. The matching, as in step 1, depends on the renormalization procedure used to define the operators in the continuum HQET, which cancels with the remaining scheme dependence of the running in step 2.

The lattice counterpart of the operator $\hat{O}_{ll}$ is already known. It was determined by Flynn et al\cite{4} for the Wilson action and by Borrelli and Pittori\cite{9} for the SW-Clover action. Again, we have extended the calculation to all the HQET operators in the basis of eq.\cite{1}. Our results for $\hat{O}_{ll}$ agree with ref.\cite{4} but disagree with ref.\cite{9} in the sign of the contribution proportional to $\hat{O}_{latt}^N$ (see below) of the group of Feynman diagrams with radiative corrections between light quark legs only. We refer the reader to ref.\cite{9} for details.

As we said before, new lattice operators arise

$$\begin{align*}
O_{latt}^N &= O_{latt}^{LR} + O_{latt}^{RL} + 2 (O_{latt}^{LR}^S + O_{latt}^{RL}^S) \\
O_{latt}^M &= 3/2 (O_{latt}^{ll} + O_{latt}^{ll}^S) \\
&+ 4 (O_{latt}^{ll}^S + O_{latt}^{rr}^S) \\
O_{latt}^P &= O_{latt}^{lr} + O_{latt}^{rl} + 6 (O_{latt}^{lr}^S + O_{latt}^{rl}^S) \\
O_{latt}^Q &= O_{latt}^{ll} + O_{latt}^{rr} + 8 (O_{latt}^{ll}^S + O_{latt}^{rr}^S)
\end{align*}$$

\text{(2)}
We found that $O_{\text{lat}}^{\text{ll}}$ mixes with $O_{\text{ll}}^{\text{rr}}$ and $O_{\text{ll}}^{\text{mn}}$, $O_{\text{ll}}^{\text{lr}}$ with $O_{\text{rl}}^{\text{lr}}$ and $O_{\text{ll}}^{\text{rl}}$, $O_{\text{ll}}^{\text{lr}}$ and $O_{\text{ll}}^{\text{rl}}$ and finally $O_{\text{rl}}^{\text{lr}}$ with $O_{\text{ll}}^{\text{rr}}$, $O_{\text{ll}}^{\text{rr}}$ and $O_{\text{rl}}^{\text{pl}}$ and finally $O_{\text{rl}}^{\text{lr}}$ mixes with $O_{\text{ll}}^{\text{rl}}$ and $O_{\text{ll}}^{\text{rl}}$. We refer the reader to ref. [9] for details.

5. Fourth step: lattice computation of the matrix elements.

In order to obtain the $B$–parameters of the relevant operators, we simulate on the lattice the ratio (see, for example, ref. [10])

$$R_{O_i}(t_1, t_2) = \frac{C_{O_i}(t_1, t_2)}{8/3 C(-t_1) C(t_2)}$$

where $C_{O_i}(t_1, t_2)$ is the three-point correlation function with an insertion of the lattice operator $O_i$ and $C(t)$ is the two-point correlation function for the $B$–meson. To extract the $B$–parameter, we search for a plateau in $t_1$ at fixed (and large) $t_2$. We observe, see figs. 1 and 2, good plateaux over large time–distances for all operators. This makes us confident that the lightest meson state has been isolated. Furthermore, our results are almost independent of the light quark mass, therefore they can be safely extrapolated to the chiral limit. From figs. 1 and 2, we see that the correction given by the subleading operators may be important and cannot be neglected. Using the results from steps 1 to 4, we can evaluate the $B$–parameters of all operators relevant to phenomenology. For example, for the renormalization scale independent $B$–parameter of $O_{\text{ll}}$ we found $\hat{B}_{B_d} = 1.14 \pm 0.16$ and $\hat{B}_{B_s} / \hat{B}_{B_d} = 1.01 \pm 0.01$ [10]. The calculation of the other $B$–parameters is in progress and will be published elsewhere [9].

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