A Robust Static State Estimation Considering Slow Power System Dynamics’ Inequality Constraints

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Abstract—This paper summarizes the static state estimation that previously has been used in power systems and also discusses the robustness of the aforementioned estimation by putting some temporal inequality constraints (TIC) on the optimization weighted least square (WLS) problem. Static state estimation (SSE) traditionally tries to linearize power systems' measurement functions in a simple Gauss-Newton method optimization problem to obtain the best estimation of our system states. In addition, when SSE is confronted with constraints, a Lagrangian function would be defined for our system model to reach the best solution in the presence of either equality or inequality constraints. A set of TICs and a proper formulation along with a 14-bus IEEE power system example are provided to show the robustness results in the estimation.

Index Terms—State Estimation, Power Systems, Temporal Inequality Constraints (TIC), Robustness.

I. INTRODUCTION

Access to highly reliable electricity is one of the most crowd-pleasing matters in the future digital world. Along the same line, certain methods can enhance the robustness of power system reliability to a considerable degree. For example, the author in [1] has reinforced the self-healing characteristic of the system by a novel method of optimally allocating control and protective devices in the system, which drastically improved the reliability of the system. Furthermore, state estimation can lead to improving the reliability of the power systems by another method. In other words, control of the power system parameters needs to estimate the system states as accurately as possible [2]. Brand-new challenges for both planning and operating power systems that electrical engineers are facing include:

1- Conventional static state estimation (SSE) cannot absorb the fast and stochastic changes [3] in the transmission and distribution power system [4].

2- Current measurement tools are not fast enough to capture and detect the aforementioned stochastic changes [3].

To solve the first issue, we should use either a dynamic state estimation (DSE) or a robust to SSE [5]. However, in the second issue it is stated that in practice, it is not possible to have measurements that are fast enough to deploy DSE [6]. Practically, a robust SSE could be proposed to obtain an accurate enough estimation of our system states.

Conventionally in SSE, we might hire a simple WLS optimization to minimize the difference between the actual measurements and the system measurement functions value (mainly power equations using voltage angle and magnitude [7]) by estimating the closest set of state variable vectors through linearization progress. However, the main problems with SSE are that it may not consider state transitions [8], there are neither memories of previous states nor prediction of future states [9], and it may only consider the spatial aspect of the system [10].

To robust the estimation, we can add some temporal aspects to this kind of estimation. In fact, measurement with slow dynamics would be perfect to add some constraints [11]. The Lagrangian multipliers have been previously used to produce the object function and solve the optimization problem [12]. However, the constraints are mostly inequalities and would be much harder to implement [13].

To come up with an appropriate solution, some slack variables are considered to make the implementation of inequality constraints easier for the Lagrangian function [14]. Consequently, some kind of Newton-Raphson method is hired to minimize the error through iterations [15]. The result of this robustness would be effective for bad data detection, detecting the bad data injection [16], any kind of cyber-attack [17], or any sort of metering and communication error.

This paper tries to robust SSE by adding some TICs for power systems measurement functions and then examining the proposed method of solving an optimization problem using Lagrangian multipliers [14]. An actual 14-bus IEEE power system would be treated with these inequality constraints and the results of the simulation would prove that this robustness may help the control of power systems in terms of confronting fast and stochastic change [18]. This result can be used for cyber security purposes [17] as well.

Furthermore, the rest of the paper is dedicated to as the following: section II contributes to a conceptual framework for SSE optimization problem. The mathematical and computational formulation is presented in section III. The solution approach is discussed in section IV. A case study simulation is presented in section V, and lastly, conclusions are drawn in section VI.

II. STATIC STATE ESTIMATION CONCEPTUAL FRAMEWORK

In an electrical power system, we may not know some of the buses’ voltage magnitudes (due to lack of measurement in some buses) or in reality, we cannot measure the voltage angle of any buses without phasor measurement units (PMU) [19]. In other words, our state variables are voltage angles and magnitudes that we wish to estimate as well as make it practical to real-world applications.

Mainly in static estate estimation, our goal is to compute the approximated amount of voltage angle and magnitude vector of all system buses [20].
and stochastic changes in power-flow estimation, the dynamic sort of estimation has recently received much attention [6]. In order to accomplish this in practice, we do need to have both fast and digital measurements along with control and processing units with high speed processing and communication rates in parallel [21], [22]. However, many kinds of these dynamic estimations are not applicable in real-world power systems due to a lack of required measurement and processing units.

An SSE approach with acceptable robustness can somehow satisfy our accuracy in estimation [23]. Here is the formulation for a simple SSE without any constraints in the power system:

\[
\begin{align*}
\mathbf{z} &= \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_m \end{bmatrix} = \begin{bmatrix} h_1(x_1, x_2, \ldots, x_n) \\ h_2(x_1, x_2, \ldots, x_n) \\ \vdots \\ h_m(x_1, x_2, \ldots, x_n) \end{bmatrix} + \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_m \end{bmatrix} = h(x) + e
\end{align*}
\]

where:
- \( h^T = [h_1(x), h_2(x), \ldots, h_m(x)] \)
- \( h_i(x) \) is the nonlinear function relating measurement \( i \) to the state vector \( x \).
- \( x^T = [x_1, x_2, \ldots, x_n] \) is the system state vector.
- \( e^T = [e_1, e_2, \ldots, e_m] \) is the vector of measurement errors.

The problem of optimal static estimation is formulated in this section in order to have a minimum amount of residual error. The objective function of this problem is shown in (2):

\[
J(x) = \sum_{i=1}^{m} \frac{(z_i - h_i(x))^2}{R_{ii}} = [z - h(x)]^T R^{-1} [z - h(x)]
\]

(2)

The standard deviation \( \sigma_i \) of each measurement \( i \).

If we consider the X vector as voltage magnitudes and angles and any \( h(x) \) as a power-flow equation, the optimization can be solved through (3):

\[
x^{k+1} = x^k - [G(x^k)^{-1}] \cdot g(x^k)
\]

(3)

where
- \( k \) is the iteration index.
- \( x^k \) is the solution vector at iteration \( k \).
- \( G(x^k) = \frac{\partial g(x^k)}{\partial x} = H^T(x^k) \cdot R^{-1} \cdot H(x^k) \)
- \( g(x^k) = -H^T(x^k) \cdot R^{-1} \cdot (z - h(x^k)) \).

Where \( H(x) = \frac{\partial h(x)}{\partial x} \)

\[
x^T = [\theta_2, \theta_3, \ldots, V_1, V_2, \ldots]
\]
To update the X vector in each iteration, we must use (4):

$$\Delta x^{k+1} = H^T (x^k) R^{-1} [z - h(x^k)] [G(x^k)]^{-1}$$

where

$$\Delta x^{k+1} = x^{k+1} - x^k$$

For more details on the SSE, reference [23]. We want to expand this formulation with some constraints and analyze possible ways to solve or optimize the problem in the presence of TICs.

**Constraints of the Problem**

If it has been assumed that the slow dynamic in internal system features such as the rotor angle, generator speed, and power generations have temporal limits to change, respectively, certain inequality constraints are imposed on the problem as shown in (5):

$$z_t = h(x_t) + e_t$$

$$l(x_{t-1}) \leq h(x_t) - h(x_{t-1}) \leq u(x_{t-1})$$

where $l$ and $u$ are lower and upper bounds for each measurement function change in comparison to its previous time amount, respectively. Moreover, this shows that in a stable system like power transmission or distribution systems, slow dynamic features like load change rate make serious bounds on each measurement function change.

Furthermore, the new objective function can be formulated as shown in (6):

$$\text{Minimize } \alpha f_{1,t} + (1 - \alpha) f_{2,t}$$

Where $f_{1,t} = \sum \frac{1}{q_t} [z_{1t} - h_1(x_t)]^2$

$$f_{2,t} = \sum \frac{1}{q_t} [g_j(x_t)]^2$$

$$g_j(x_t) = 0,$$

if $l_j(x_{t-1}) \leq h_j(x_t) - h_j(x_{t-1}) \leq u_j(x_{t-1})$

$$g_j(x_t) = h_j(x_t) - h_j(x_{t-1}) - u_j(x_{t-1})$$

if $h_j(x_t) - h_j(x_{t-1}) > u_j(x_{t-1})$

$$g_j(x_t) = l_j(x_{t-1}) - (h_j(x_t) - h_j(x_{t-1})),$$

if $h_j(x_t) - h_j(x_{t-1}) < l_j(x_{t-1})$

where $\alpha$ is the coefficient for considering the temporal aspect in our optimization to minimize the weighted square of errors between measurements and the nonlinear measurement functions at time $t$ and minimize the weighted square of deviations between the estimated measurement function amount at time $t$ and $t-1$.

Now we have a new objective function that cannot be treated like a simple WLS problem. To come up with a solution, Lagrangian multipliers can be used properly [14]. First, we should define an appropriate Lagrangian function that considers our TICs in it as well. This objective function is stated in (7):

Minimize $\frac{1}{2} r^T R^{-1} r$

Subject to: $f(x) + s = 0$

$$r - z + h(x) = 0$$

$s \geq 0$

where $s$ is a vector of slack variables used to convert the inequality constraint to an equality constraint and inequality constraints are considered in $f(x)$. Taking into account the KKT conditions along with our Lagrangian function, the equation can be explained as shown in (8):

$$\ell_\mu = \frac{1}{2} r^T R^{-1} r - \lambda^T [f(x) + s] - \pi^T [r - z + h(x)]$$

$$\nabla_x \ell = -\mu S^{-1} e - \lambda = 0$$

$$\nabla_\lambda \ell = -f(x) - s = 0$$

$$\nabla_\pi \ell = r - z - h(x) = 0$$

$$\nabla_r \ell = R^{-1} r - \pi = 0$$

$$s \geq 0$$

This is Lagrangian multiplier for TICs and our objective function, respectively. By using the Gauss-Newton method for iteratively solving nonlinear equations, some linearization approximations would result in a system of equations as shown in (9):

$$\begin{bmatrix} D & 0 & F \\ 0 & R & H \\ F^T & H^T & 0 \end{bmatrix} \begin{bmatrix} \lambda \\ \pi \\ \Delta x \end{bmatrix} = \begin{bmatrix} f(x^k) \\ z - h(x^k) \end{bmatrix}$$

where

$$D = \frac{1}{\mu} (s^k)^2$$

For more details on the computations see [15].

**IV. PROBLEM METHODOLOGY**

The main purpose of this paper is to optimize the new residual function for minimizing total errors in the power system. Consequently, there are temporal inequality constraints imposed on the problem that have been taken into consideration. In this work, in order to analyze the voltage magnitude and angle of each bus, there is a need to run the program iteratively which is very popular for distribution and
power transmission systems. As formulated in section III, the optimization problem in this paper is a non-linear problem which has been solved using Newton-Raphson linearizing optimization method. In addition, solutions via the particle network (PN) [24] and particle swarm optimization (PSO) methods and the heuristic [25],[26] and genetic algorithms are also examined [27],[28].

V. CASE STUDY SIMULATION

A. Case Study

In order to understand on the static state estimation with inequality constraints on the measurement function temporal differences, we implement and run power flow from an IEEE 14 bus test power system. Moreover, a set of one snapshot memory is considered in the external iteration to consider the temporal aspects. In this paper, power lines’ data and buses’ data are presented in tables I and II, respectively. In fact, a set of power injections along with 3 different time snapshots of our power system are shown in Fig. 3 and 4 to illustrate our robustness in power state estimation.

Table I: Parameters of Lines Data

| From Bus | To Bus | R pu | X pu | B/2 pu |
|----------|--------|------|------|--------|
| 1        | 2      | 0.01938 | 0.05917 | 0.0264 |
| 1        | 5      | 0.05403 | 0.22304 | 0.0246 |
| 2        | 3      | 0.04699 | 0.19797 | 0.0219 |
| 2        | 4      | 0.05811 | 0.17632 | 0.0170 |
| 2        | 5      | 0.05695 | 0.17388 | 0.0173 |
| 3        | 4      | 0.06701 | 0.17103 | 0.0064 |
| 4        | 5      | 0.01335 | 0.04211 | 0.0    |
| 4        | 7      | 0.0    | 0.20912 | 0.0    |
| 4        | 9      | 0.0    | 0.55618 | 0.0    |
| 5        | 6      | 0.0    | 0.25202 | 0.0    |
| 6        | 11     | 0.09498 | 0.19890 | 0.0    |
| 6        | 12     | 0.12291 | 0.25581 | 0.0    |
| 6        | 13     | 0.06615 | 0.13027 | 0.0    |
| 7        | 8      | 0.0    | 0.17615 | 0.0    |
| 7        | 9      | 0.0    | 0.11001 | 0.0    |
| 9        | 10     | 0.03181 | 0.08450 | 0.0    |
| 9        | 14     | 0.12711 | 0.27038 | 0.0    |
| 10       | 11     | 0.08205 | 0.19207 | 0.0    |
| 12       | 13     | 0.22092 | 0.19988 | 0.0    |
| 13       | 14     | 0.17093 | 0.34802 | 0.0    |

Table II: Parameters of Buses Data

| Bus | Vsp | PGI | QGi | PLi | QLi |
|-----|-----|-----|-----|-----|-----|
| 1   | 1.060 | 0   | 0   | 0   | 0   |
| 2   | 1.045 | 40  | 42.4 | 21.7 | 12.7 |
| 3   | 1.010 | 0   | 23.4 | 94.2 | 19.0 |
| 4   | 1.0  | 0   | 0   | 47.8 | -3.9 |
| 5   | 1.0  | 0   | 0   | 7.6  | 1.6  |
| 6   | 1.070 | 0   | 12.2 | 11.2 | 7.5  |
| 7   | 1.0  | 0   | 0   | 0.0  | 0.0  |

Table III: Parameters of the System State Estimation in Times t and t-1 Snapshot

| Bus No. | Vpu | Angle (deg) | Vpu | Angle (deg) |
|---------|-----|-------------|-----|-------------|
| 1       | 1.0068 | 0.0000     | 1.0182 | 0.0000     |
| 2       | 0.9899 | -5.5265    | 1.0012 | -5.5008    |
| 3       | 0.9518 | -14.2039   | 0.9628 | -14.1358   |
| 4       | 0.9579 | -11.4146   | 0.9689 | -11.3602   |
| 5       | 0.9615 | -9.7583    | 0.9725 | -9.7121    |
| 6       | 1.0185 | -16.0798   | 1.0299 | -16.0042   |
| 7       | 0.9919 | -14.7510   | 1.0032 | -14.6808   |
| 8       | 1.0287 | -14.7500   | 1.0401 | -14.6798   |
| 9       | 0.9763 | -16.5125   | 0.9874 | -16.4340   |
| 10      | 0.9758 | -16.7476   | 0.9869 | -16.6683   |
| 11      | 0.9932 | -16.5397   | 1.0044 | -16.4617   |
| 12      | 1.0009 | -17.0203   | 1.0121 | -16.9405   |
| 13      | 0.9940 | -17.0583   | 1.0052 | -16.9783   |
| 14      | 0.9647 | -17.8967   | 0.9757 | -17.8121   |

Fig. 3 Voltage Magnitude and Reactive Power Injection
VI. CONCLUSIONS

In this paper, the robustness of static state estimation (SSE) is analyzed by putting some temporal inequality constraints (TICs) on the differential amount of each power measurement function between times of t and t-1. Lagrangian multipliers are hired to model this kind of constraints in the objective function of minimizing the difference between measurement data and the function amount. Some linearization, along with nonlinear functions, are deployed to solve the estimation problem robustness. The results of this robust estimation are shown in comparing diagrams that show the ability to obtain accurate results. Although SSE might have some deficiencies in the process and control of the power system, it is still a good method for practical purposes. Definitely, robustness in this method of power system estimation can be one of the best solutions to obtain meticulous estimation.

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