Polarization of inclusively produced $\Lambda_c$ in a QCD based hybrid model

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29 July 1999

Abstract

A hybrid model is presented for hadron polarization that is based on perturbative QCD subprocesses and the recombination of polarized quarks to form polarized hadrons. The model, originally applied to polarized $\Lambda$'s that were inclusively produced by proton beams, is extended to include pion beams and polarized $\Lambda_c$'s. The resulting polarizations are calculated as functions of $x_F$ and $p_T$ for high energies and are found to be in fair agreement with recent experiments.

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*This work is supported in part by funds provided by the U.S. Department of Energy (D.O.E.) #DE-FG02-92ER40702.
1 Introduction

It has long been known that inclusively produced strange hyperons can have sizeable polarization [1] over a wide range of energies. Since the initial discovery of this phenomenon, many theoretical models have been proposed to explain various aspects of that polarization data, with varying success [2, 3, 4, 5]. Because the hyperon data is in the region of relatively small transverse momentum ($p_T \sim 1$ GeV/c), soft QCD effects play a major role in any theoretical explanation. Several years ago W.G.D. Dharmaratna and G.R. Goldstein developed a hybrid model for hyperon polarization in inclusive reactions [3]. The model involves hard scattering at the parton level, gluon fusion, to produce a polarized $s$-quark which then undergoes a soft recombination that, in turn, enhances the polarization of the hyperon. This scheme provided an explanation for the characteristic kinematic dependences of the polarization in $p + p \rightarrow \Lambda + X$. The use of perturbative QCD to produce the initial polarization for strange quarks, with their low current or constituent quark mass (compared to $\Lambda_{QCD}$) made the application of perturbation theory marginal. And the magnitude of the polarization from the gluon fusion subprocesses was quite small.

More recently some information on polarization of $c$-quark baryons is emerging [6, 7]. In this heavy quark realm the perturbative contribution is more reliable. Given these circumstances, I have modified the hybrid model to apply to heavy flavor baryons produced inclusively from either proton or pion beams. The results are encouraging, as the following will show.

2 The hybrid model

The original hybrid model incorporates the order $\alpha_s^2$ QCD perturbative calculation of strange quark polarization. The interference between tree level and one loop diagrams gives rise to significant polarization [8]. Of particular importance for describing existing data on $p + p \rightarrow \Lambda \uparrow + X$ are one loop diagrams leading to strange quark pair production initiated by gluon fusion

$$g + g \rightarrow s \uparrow + \bar{s}, \quad (1)$$
or quark-antiquark annihilation

\[ q + \bar{q} \rightarrow s \uparrow + \bar{s}. \] (2)

The cross sections for polarized s-quarks (polarized normal to the production plane) must then be convoluted with the relevant structure functions for the hadronic beam and target. The inclusive cross section for hadron + hadron → s(polarized up or down) + X is obtained thereby. For protons on protons gluon fusion is the more significant subprocess.

The hadronization process, by which the polarized s-quark recombines with a (ud) diquark system to form a Λ, is crucial for understanding the subsequent hadron polarization. A simple prescription is introduced to “pull” or accelerate the negatively polarized, relatively slow s-quark along with a fast moving diquark (resulting from a pp collision) to form the hadron with particular \( x_F \) while preserving the s-quark’s \( p_T \) value. Letting \( x_Q \) be the Feynman x for the s-quark, the simple form

\[ x_F = a + bx_Q \] (3)

is used. Naively, if the s-quark has 1/3 of the final hyperon momentum (in its infinite momentum frame) and the diquark carries 2/3 of that momentum, then \( a = 2/3 \) and \( b = 1 \). To fit the \( pp \rightarrow \Lambda + X \) data (that existed in 1990) at one \( x_F \) value, the parameters in eqn. 3 were chosen to be \( a = 0.86 \) and \( b = 0.70 \), not far from the naive expectation. This recombination prescription is similar to the classical dynamical mechanism used in the “Thomas precession” model \[4\], which posits that the s-quark needs to be accelerated by a confining potential or via a “flux tube” \[5\] at an angle to its initial momentum in order to join with the diquark to form the hyperon. The skewed acceleration gives rise to a spin precession for the s-quark. That non-zero spin component gives the hyperon its polarization, since the diquark in the Λ hyperon is in a spin 0 state, i.e. all of the polarization is carried by the s-quark. In the Dharmaratna and Goldstein hybrid model, however, the s-quark has acquired negative polarization already from the hard subprocess before it is accelerated in the hadronic recombination process. That “seed” polarization gets enhanced by a multiplicative factor \( A \approx 2\pi \) that simulates the Thomas precession. The hybrid model combines hard perturbative QCD with a simple model for non-perturbative recombination.

The linear form of eqn. 3 maps the s-quark Feynman x region \([-1, (1 - a)/b]\) into the Λ \( x_F \) region \([(a - b), +1]\). The p+\( p \rightarrow s \)-quark differential cross
section, $d^2\sigma/dx_Q dp_T$ is mapped correspondingly into the p+p→Λ cross section $d^2\sigma/dx_F dp_T$. The measured cross sections for the latter are known to fall with positive $x_F$ and to fall precipitously with $p_T$, roughly as

$$(1 - x_F)^\alpha e^{-\beta p_T^2}$$

overall [6], where $\alpha$ and $\beta$ are greater than 1.0 (for $\pi+p \rightarrow \Lambda+X$ $\alpha, \beta \approx 3.0$). However, the directly computed lowest order p+p→s-quark cross section grows with $x_Q$ in the region (-1,0) and it falls more gradually with $p_T$ than the exponential in eqn. 4. Hence the more complete recombination scheme would have to temper the $x_F$ dependence and narrow the $p_T$ distribution. This will not affect the polarization calculation, though, since the individual up or down polarized cross sections will be altered in the same way. For a more thorough calculation this should be done, and work is underway on this point. The polarization results are the focus of this work.

The dominant gluon fusion contribution alone, along with the simple recombination model, accounted for the contemporaneous polarization data on $p + p \rightarrow \Lambda + X$ [9, 10] as well as subsequent data [11]. Those data were determined for a range of energies and $x_F$ and $p_T$ values and the predicted kinematic dependence was confirmed, along with the nearly negligible overall energy dependence of the polarization as fig. 1 shows. Note that the annihilation subprocess eqn. 2 has been included, although it makes little difference for the p+p reaction. The calculated kinematic dependence is quite unique and was noticed to be characteristic of hyperon polarization [1].

It is noteworthy that extensive data has been collected on Λ polarization in several exclusive reactions [12], for which a simple form, $P = (-0.443 \pm 0.037)x_F p_T$, approximates all the polarization data at $p_{lab} = 27.5$ GeV/c. That form provides lower bracketting values for the inclusive polarization, as fig. 1 indicates. In the hybrid model all the final states other than the Λ arise from the hadronization of the $\bar{s}$-quark and the remains of the incoming baryons. Therefore, in the hybrid model it would be anticipated that as the beam energy increases and/or more final states are included in the determination of the Λ polarization, more complicated final states will be accompanied by much lower polarization as $p_T$ increases beyond 1 GeV/c.

Since the initial application of the hybrid model, more extensive data on the polarization of other inclusively produced hyperons have accumulated [13]. For many of the reactions some of the same simple features remain,
Figure 1: Hybrid model Λ polarization in $p + p \rightarrow \Lambda + X$ as a function of $p_T$ for various values of $x_F$. The data at 12 GeV [9], 400 GeV [10], 800 GeV [11] are shown. Exclusive data at 27.5 GeV/c [12] is approximated by the straight line from the origin to $p_T \approx 1$ GeV/c.
including the kinematic dependences, while for others there is a greater complexity. The ratios of overall polarization for different hyperons are roughly given by the ratios of their expected SU(6) wavefunctions \[4\]. The best measured of these, \(\Sigma^+\) polarization \([14]\), does give a larger result than \(-1/3\) (the SU(6) factor) times the \(\Lambda\) polarization. Comparing the \(\Sigma^+\) polarization data of ref. \([14]\) to the \(x_F = 0.5\) graph in fig. 1 shows that the \(\Sigma^+\) data is close to \(-2/3\) times the \(\Lambda\) data for all the measured \(p_T\) values. At 800 GeV though, the \(\Sigma\) polarization has fallen from its values at lower energies, suggesting an approach to the lower value expected from SU(6), but with an energy dependence that is more pronounced than the \(\Lambda\) data.

In any case, it may be that the PQCD based hybrid model is best tested in the production of heavy flavor hadrons, wherein the heavy quark needs to be produced at large energies compared to the \(\Lambda_{QCD}\) scale. The polarization in the QCD subprocesses was calculated \([15]\) to order \(\alpha_s^2\) for all flavors and it was found that, indeed, the polarization increases substantially with constituent mass; the peak polarization goes roughly as the mass, as fig. 2 shows.

The question then arises, how should the heavy quark recombine with light quarks or diquarks to form the heavy flavor hadron? The simplest course to take is to adopt the same algebraic scheme as for the \(\Lambda\) hybrid model. The heavy quark again needs to be accelerated by a string or flux tube potential and polarization is enhanced via the Thomas precession. In heavy quark effective theory, such a polarization enhancement would arise from the excitation of light degrees of freedom which contain orbital angular momentum. In any case, the heavy quark is already polarized when it gets accelerated, so that its “seed” polarization, which grows with mass, is enhanced. This interpretation predicts an increasing polarization with increasing quark mass for a given class of final baryons.

I will now apply the hybrid model to \(\Lambda_c\) polarization. All of the above reasoning was developed with proton-proton scattering in mind. It is known that \(\pi\) induced inclusive strange \(\Lambda\) production also produces significant polarization of \(-28 \pm 10\%\) for positive \(x_F\) and \(p_T\) values near and above 1 GeV/c \([3]\). The same experiment found polarized \(\Lambda_c\), with a rough average value over the full range of kinematics of \(\approx -60\%\). Because the beam is a pion, there is a much higher probability for finding light antiquarks in the beam than for the proton case. So the quark-antiquark annihilation will be a more important QCD subprocess for the pion beam than for the proton beam. Both annihilation and gluon fusion will have to be included. Then in the recombination the
Figure 2: Polarization for the QCD subprocess of gluon fusion to quark pairs. The curves are for d, s, c, b quarks.
ud-diquark system that combines with the strange or heavy flavor polarized quark must be pulled from the target proton or the pion sea. In both cases the diquark could have lower average $x_F$ than in the proton beam situation. This might imply a different form for the Thomas precession enhancement. Furthermore, it should be anticipated that the mass of the heavy quark will play a role in enhancing the polarization further. Various possibilities were considered in determining the polarization for $\Lambda_c$ via $\pi$ production, where new data exists [7]. Without a thorough exploration of the recombination parameterization a preliminary calculation of the pion induced reactions can be made by adopting the same recombination prescription eqn. 3 used for the $p + p \rightarrow \Lambda + X$.

To summarize, the application of the hybrid model to estimate the expected flavor $Q$ baryon $\Lambda_Q$ polarization involves several steps. First assume $g(x_1)g(x_2)$ and $q(x_1)\bar{q}(x_2) \rightarrow Q(x_Q, p_T)\bar{Q}$ dominate the subprocesses that give rise to $Q$-quark polarization and evaluate the polarized cross sections to order $\alpha_s^2$, $d^2\sigma(\uparrow \text{ and } \downarrow)/dx_Q dp_T$. The lengthy expressions for these subprocess polarizations can be found in ref. [15] (an alternative derivation is given in ref. [16]). Then convolute the polarized subprocess cross sections with the gluon, quark and antiquark structure functions for the proton and pion [17], $g^{p,\pi}(x), q^{p,\pi}(x), \bar{q}^{p,\pi}(x)$, or generically $f^{p,\pi}_i(x)$ leading to

$$d^2\sigma(\uparrow \text{ and } \downarrow)/dx_Q dp_T = \sum_{i,j} \int_0^1 dx_1 \int_0^1 dx_2 f^{p,\pi}_i(x_1)f^{p,\pi}_j(x_2)d^2\sigma(\uparrow \text{ and } \downarrow)/dx_Q dp_T.$$  \hspace{1cm} (5)

Next the recombination formula, eqn. 3, is applied to obtain the corresponding $\Lambda_Q$ polarized cross section at $x_F = a + bx_Q$ and $p_T$. The polarization is obtained via

$$P_{\Lambda_Q}(x_F, p_T) = A \frac{d^2\sigma(\uparrow)}{d^2\sigma(\uparrow) + d^2\sigma(\downarrow)} - \frac{d^2\sigma(\downarrow)}{d^2\sigma(\uparrow) + d^2\sigma(\downarrow)},$$  \hspace{1cm} (6)

in an obvious notation. The parameters $a$ and $b$ are given the same values as the original pp scattering case. Regarding $A$ it should be noted, however, that in the $\Lambda$ data, for which these parameters were obtained, 20% to 30% of the $\Lambda$’s came from radiative decays of the $\Sigma$. Hence the actual direct $\Lambda$ polarization should be increased by about 14%, which increases the value of $A$ correspondingly to 7.9.

When this procedure is applied to $\pi^− + p \rightarrow \Lambda_c + X$ the resulting polarization $P(x_F, p_T)$ is obtained. To compare with preliminary data from Fermilab
Figure 3: Estimate of $\Lambda_c$ polarization from $\pi^- p \rightarrow \Lambda_c + X$. The larger polarization includes heavy mass enhancements. The preliminary data [7] is from E791.
Figure 4: $\Lambda_c$ polarization in $\pi^- + p \to \Lambda_c + X$ as a function of $p_T$ for various values of $x_F$. Multiplying these polarizations by $m(\Lambda_c)/m(\Lambda)$ will incorporate the hadron mass enhancements as in fig. 3.

E791 [7], $P$ is integrated over $x_F$ values from $-0.2$ to $+0.6$ [18]. The resulting $P(p_T)$ is shown as the smaller polarization curve in the fig. 3. To account for the possible increased enhancement in the Thomas mechanism for the heavier quark to be accelerated to form the hadron, an additional factor of $m_{\Lambda_c}/m_{\Lambda} \simeq 2$ can be included as suggested by formal studies of the scale dependence on hadronic mass of relevant correlation functions [19]. With this latter factor, the larger polarization curve is obtained. This provides a better approximation to the new data. It also is consistent with the results for strange $\Lambda$ production when scaled down appropriately. The full dependence on both variables $x_F, p_T$ is shown in fig. 4. The polarization corresponds to the smaller polarization in fig. 3. It will be of considerable interest for the hybrid model to see how well this detailed behavior will be confirmed when more data are available.

In conclusion, these results are encouraging for the hybrid model. The
Thomas enhanced gluon fusion model has been modified to include quark-anti-quark annihilation, which should be more prominent for heavy baryon polarization in pion induced reactions, like the above $\pi^- + p \rightarrow \Lambda_c + X$. Experimental data can be analyzed into $x_F$ as well as $p_T$ bins, so the predictions from the hybrid model can be checked in detail. The somewhat ad hoc prescription for the recombination is being studied further in order to accommodate both the polarization and the cross section behavior with the kinematic variables. Furthermore, an investigation of other reactions and observables is underway.

**Acknowledgments**

This work was supported, in part by a grant from the US Department of Energy. The author thanks Austin Napier for rekindling a long term interest in hyperon polarization. He also appreciates correspondence with members of E791, particularly M.V. Purohit, G.F. Fox and J.A. Appel.

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