DARK ENERGY AND THE HUBBLE AGE

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ABSTRACT

I point out that an effective upper limit of approximately $20(0.2/\Omega_m)^{1/2}$ Gyr (for a Hubble constant of 72 km s$^{-1}$ Mpc$^{-1}$), or alternatively, $1.49(0.2/\Omega_m)^{1/2}$ for the $H_0$-independent quantity $H_{0b}$, exists on the age of the universe, essentially independent of the equation of state of the dominant dark energy component in the universe. If astrophysical constraints on the age of the universe can convincingly reduce the upper limit to well below this value, a useful lower limit on the equation of state parameter $w$ for this component can be obtained. Direct dating by stars does not provide a useful constraint, but model-dependent cosmological limits from supernovae and the CMB observations can do so.

Subject heading: cosmological parameters — cosmology: theory — equation of state — Galaxy: fundamental parameters — gravitation

1. INTRODUCTION

The realization that some unknown form of energy density associated with otherwise empty space appears to dominate the gravitational dynamics of the universe has changed virtually everything in cosmology. For example, the future evolution of the universe becomes largely independent of its geometry (Krauss & Turner 1999), so a closed universe can expand forever, and an open universe can ultimately collapse.

One of the earliest motivations for assuming the existence of a cosmological constant, the simplest form of dark energy, involved a comparison of the Hubble age of the universe—determined for an assumed cosmological model on the basis of the observed expansion rate today—with a lower limit on the age of the oldest objects in our Galaxy. In order to resolve the paradox when the latter exceeded the former (Janes & Demarque 1983), cosmologists were driven to consider the possibility of exotic cosmological models that might allow an older universe for a fixed value of the Hubble constant today.

An accelerating universe allows for this possibility simply because galaxies that are now located at a certain distance from us and that are moving at some fixed velocity were separating from us at a smaller velocity at earlier times and thus would have required longer to achieve their present separation than would otherwise be required. For a flat universe, in the limit where a cosmological constant dominates the energy density, the Hubble age can approach infinity for any value of the Hubble constant.

We currently have no fundamental understanding of the nature of dark energy (or perhaps more accurately “dark pressure”). It is significant that a lower limit on the age of the universe determined from globular cluster dating techniques now provides independent evidence for the existence of dark energy and puts a limit on the equation of state parameter $w_p/\rho$ (where $w$ is the pressure/energy density) of the dark energy $w < -0.4$ (Krauss & Chaboyer 2003). However, the question also arises: if exotic equations of state for dark energy can increase the Hubble age, can one put useful constraints on such equations of state from an upper limit on the age of globular clusters, or from direct estimates of the Hubble age itself? I investigate these questions here.

2. EXOTIC EQUATIONS OF STATE AND THE HUBBLE AGE IN A FLAT UNIVERSE

Determinations of the distance-redshift relation made using distant Type Ia supernovae (Perlmutter et al. 1999; Schmidt et al. 1998) combined with independent estimates for both the mass density in the universe today and the geometry of the universe from CMB measurements (de Bernardis et al. 2000; Hanany et al. 2000) have definitively established the need for a dominant component to the energy budget that involves a negative pressure.

While there is currently no fundamental understanding of the nature of this dark pressure, one particular value of the equation of state parameter carries special weight. A vacuum energy density is fixed, by Lorentz invariance, to have the form $T_{\mu\nu} = \Lambda g_{\mu\nu}$, and thus $w = -1$. Unfortunately, however, all estimates of the vacuum energy density on the basis of first-principle calculations yield a value that is orders of magnitude too large, and thus it was commonly assumed for many years that some new symmetry mechanism might yield a value for the vacuum energy that is precisely zero. A uniform scalar field, for example, that is slowly rolling down a potential and has not yet achieved its minimum value can mimic vacuum energy. For such a field, $w$ is given by

$$w = \frac{\dot{\phi}^2/2 - V}{\dot{\phi}^2/2 + V}. \quad (1)$$

Since the kinetic energy of the scalar field as it rolls in the potential gives a positive contribution to the pressure, any rolling implies $w > -1$.

Of course, since we do not have any underlying theory for the dark pressure, one must allow for the possibility that $w < -1$ (Caldwell 2002). It is clear that Lagrangian models that have an equation of state of this form will be extremely exotic, implying, for example, a negative kinetic term. Such models will have the remarkably odd feature that the energy density of the dark energy will increase with time! As a result, the Hubble constant itself will continue to increase with time.

If a cosmological constant allows for an older universe for a fixed Hubble constant today, what will be the effect of even...
more exotic forms of dark pressure? If the equation of state parameter remains constant, for a fixed universe, the age-Hubble constant relation is given by

$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)^{\frac{1}{2}}} \left[ \Omega_m (1+z)^3 + \Omega_X (1+z)^{3(1+w)} \right]^{1/2},$$

where \(\Omega_m\) is the fraction of the closure density in matter today and \(\Omega_X\) is the fraction of the closure density in material with an equation of state parameter \(w\).

For varying \(w\), this equation can be usefully rewritten as

$$H_0 t_0 = \int_0^1 dy y^{1/2} \left\{ (\Omega_m) + (\Omega_X) \exp \left[ -3 \int_0^y dy' w(y'/y') \right] \right\}^{1/2}.$$  

It is clear from equations (2) and (3) that as \(\Omega_m\) approaches unity, the age of the universe can approach infinity if \(w \leq -1\). However, we have good estimates of the density of dark matter today, which come from gravitational lensing of clusters (Wittman et al. 2000), X-ray studies of clusters (Evrrard 1997), and studies of large-scale structure (Dodelson et al. 2002; Hawkins et al. 2002), which suggest \(\Omega_m \geq 0.21\). Combining CMB limits with large-scale structure (Spergel et al. 2003) implies \(\Omega_m = 0.29 \pm 0.07\). If we thus assume a conservative minimal value, \(\Omega_m = 0.15\), then the implications of the above relation between age and Hubble constant for exotic forms of energy become quite different than if we allow an arbitrarily high fraction of the closure density to be in dark energy. (Note that as \(\Omega_m\) increases, the upper age limit decreases, so that the age dependence of \(w\) becomes progressively less strong with larger \(\Omega_m\).)

If we normalize to the Hubble Key Project best-fit value of \(H_0 = 72 \pm 7\) km s\(^{-1}\) Mpc\(^{-1}\) (Freedman et al. 2001), we can plot the above relation for age as a function of \(w\), as shown in Figure 1 for two values of \(\Omega_m\): the Wilkinson Microwave Anisotropy Probe (WMAP) midpoint value of 0.29 and the extreme lower value of 0.15. Also shown in this figure is the Hubble-independent product \(H_0 t_0\). As is clearly seen in the figure, the age of the universe is a sharply increasing function of \(-w\) for \(w < 0\), but then it quickly begins to asymptote, so that for \(w < -10\) the age increases by less than 0.5 Gyr for \(w > -30!\)

This behavior is easily understood. As has been described, as \(w\) decreases below \(-1\), the net energy density stored increases with time. Thus, the relative contribution of this exotic energy to the total energy budget of the universe was smaller at earlier times (higher redshifts) than, say, the energy density stored in a cosmological constant. In short, this exotic energy has “just” become important. As a result, the more negative \(w\) is, the less time there has been for it to have an effect, even though the acceleration rate increases during the period in which it is significant. The net result is that, for a fixed fraction of the closure density today in matter, there is effectively a maximum age for the universe, independent of how negative \(w\) is! For \(H_0 > 65\) today, one finds, for example, that for \(w > -600\), \(t_0 < 23\) Gyr. Put in \(H_0\)-independent terms, one finds that \(H_0 t_0 \leq 1.7\).

It is important to note that from the point of view of an upper limit on \(w\) from cosmic ages, any unknown possible variation in \(w\) does not significantly alter the asymptotic shape of the constraint curve. Indeed, one can derive an asymptotic upper bound \(H_0 t_0 < 2/(300^{1/2}) = 1.72\) (for \(\Omega_m = 0.15\))\(^1\) by taking the limit of equations (2) and (3) above for \(H_0 t_0\) as \(w\) approaches negative infinity. The integral in the denominator of equation (3) is dominated simply by the value of \(w\) for \(y \approx 1\) as \(w\) becomes large and negative.

3. LIMITING \(w\) FROM AGE CONSIDERATIONS

From the above analysis, it is clear that in order to derive a robust constraint on negative values of the equation of state parameter \(w\), one must be able to place an upper limit on the age of the universe in the range of 15 Gyr (for \(\Omega_m = 0.29\)) or 21 Gyr (for \(\Omega_m = 0.15\)) for \(H_0 = 72\). The sharp rise of age with negative \(w\) tails off considerably above these values.

Such a direct determination of the upper limit on the age of stellar systems is not possible. Recent Monte Carlo studies of stellar age constraints in old globular clusters yield a 95% upper limit of 16 Gyr (Krauss & Chaboyer 2003). If this were the end of the story, then some useful constraint would be derivable. However, to this upper limit one must add a conservative upper limit on the time between the big bang and the formation of stars in our Galaxy. While the epoch of first star formation is likely to be at \(z > 6\), galaxy formation continues down to redshifts as low as 1–2. This implies that the first stars in our Galaxy could have started forming when the universe was as old as 4–5 Gyr. This is not likely, but it is possible. While new techniques (see, e.g., Chaboyer & Krauss 2002) may allow the upper limit on globular cluster ages to decrease, it is difficult to imagine ways to significantly reduce this latter 5 Gyr uncertainty on the period before the formation of our Galaxy. (Note that the new WMAP detection of reionization at a redshift in the range of 10–20 strongly suggests that globular clusters formed within about 1 Gyr after the big bang; see Krauss 2003).

As a result, the most robust age constraints on \(w\) (or on the time-weighted integral of \(w\), if \(w\) varies over cosmic time) come not from direct lower limits on \(t_0\) itself but from cosmological estimates of \(t_0\) or the combination \(H_0 t_0\), which can be directly probed by redshift versus distance measures (e.g., Riess et al.

\(^1\) This result was suggested for inclusion here by Chiba (for constant \(w\); T. Chiba 2003, private communication) and Repko (who suggested the form of eq. [3] for \(H_0 t_0\) presented here for varying \(w\); W. Repko 2003, private communication) following the posting of the original astro-ph version of this article.

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**Fig. 1** — Age of the universe as a function of the equation of state parameter, \(w\), for constant equation of state. Limits for two different values of \(\Omega_m\) within the range allowed by WMAP are shown.
1998) and CMB experiments, (e.g., Knox et al. 2001). These
depend to some extent on cosmological parameter estimation
and on uncertainty in cosmological models (which become
more uncertain if \( w \) is not constant), but recent estimates al-
dready suggest that the CMB measurements put a limit of
\( H_0 t_0 < 1.05 \). Using the age arguments described here, this
would put a constraint on the constant \( w > -1.2 \) at the 95%
confidence level (Krauss 2003). Note that this is somewhat
more constraining than independent measurements coming
from SN redshift versus distance measures, at present, where
different studies put limits at \( w > -1.29 \) (Knop et al. 2003) and
\( w > -1.48 \) (Tonry et al. 2003).

Equations of state with \( w < -1 \) violate the weak equiva-
ience principle and thus have not been examined theoretically
in great detail. One might hope, therefore, that observational
constraints could provide some significant guidance for theo-
rists in this regard. Unfortunately, direct age determinations
have a residual uncertainty which is unlikely to allow signif-
icient constraints to be derived. Instead, cosmological estimates
using supernovae and CMB data appear to offer the best
possibility for constraining negative values of \( w \).

At the same time, it is significant that cosmology implies an
upper limit on the age of the universe that is essentially in-
dependent of the unknown value of \( w \). This allows globular
cluster ages, at the very least, to provide a direct consistency
test of our fundamental cosmological framework. A direct age
determination in excess of \( \approx 20 \) Gyr would have been in-
consistent with the Hubble age for any cosmic equation of state,
for a Hubble constant of \( 72 \pm 7 \), and for \( \Omega_m > 0.2 \).

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