Phase shift effects on the second moment and skewness of the field profiles obtained by the muon spin relaxation technique

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Abstract. Recent high transverse field muon spin relaxation (TF-µSR) experiments performed on Bi2212 single crystals show that the phase parameter extracted from individual histogram, an indication of the angle between the muon’s initial polarization direction and the detector direction, is temperature dependent. This phase shift effect, probably due to the electronic or initial polarization instability at a time interval comparable with the muon spin precession period, will affect the second moment and skewness of the field profiles. The proper data analysis procedure is discussed to correct this phase shift effect, which is important on the quantitative analysis of the first order transition or melting transition in the Bi2212 mixed state.

Keywords: Phase shift; Muon spin relaxation; Second moment; Skewness

I. INTRODUCTION

In a µSR experiment the spin-polarized muons are assumed to stop at random positions within the sample and precess along the local (internal) magnetic fields \( \vec{B}(\vec{r}) \). The precession frequency is \( \gamma_\mu B \), where \( \gamma_\mu = 135.5 \, MHz/T \) is the gyromagnetic ratio of the muon. If the local fields which the muons feel are not unique i.e. there is a distribution of magnetic field \( f(B) \), consequently the muons will precess with different frequencies, which causes a depolarization of the muons ensemble’s spins. The second moment of the field distribution \( f(B) \) indicates how fast the muons depolarize thus is called the relaxation rate (\( \sigma \) or \( \lambda \)). There are several ways to extract the relaxation rate: we can assume the proper form of the depolarization function \( P(t) \) to fit the µSR spectra (histogram or asymmetry) in the time domain to extract the relaxation rate parameter in the fitting procedure; or we can calculate the second moment of the field distribution \( f(B) \) to get the relaxation rate \( \sigma \) since the Cosine Fourier Transformation (CFT) of µSR spectra indicates the magnetic field distribution inside the sample. The second moment calculated from the discretized field distribution data depends slightly on the field channel width \( \Delta \), there exists the following relations:

\[
\sigma^2 = \frac{\int b^2 f(b) \, db}{\int f(b) \, db} - \left( \frac{\int b f(b) \, db}{\int f(b) \, db} \right)^2 \\
\approx \frac{\sum b^2 f(b) \, db}{\sum f(b) \, db} - \left( \frac{\sum b f(b) \, db}{\sum f(b) \, db} \right)^2 - \frac{\Delta^2}{12} \\
= \sigma_{\text{data}}^2 - \frac{\Delta^2}{12} \tag{1}
\]

where \( \sigma \) is the true relaxation rate and \( \sigma_{\text{data}} \) is the relaxation rate calculated from the discretized field profile. Simulation shows that as long as the channel width \( \Delta \) is much smaller than the second moment of the field profile, \( \sigma_{\text{data}} \) is a good estimate of \( \sigma \). For a µSR spectra in a 10 µs time window, the ideal minimum relaxation rate we can extract from data is 0.628 \( \mu s^{-1} \). If we consider the channel width effect mentioned above, the resolution of the relaxation rate will still be larger.

The local fields \( f(B) \) can either be intrinsic, as they are for ordered magnets and spin glasses, or induced by an external field, as for the vortex lattice of a type II superconductor formed in the external magnetic fields. Usually the distribution \( f(B) \) is dependent on the flux distribution associated with a single vortex line, as well as the arrangement and dynamics of the vortex lines. Note for single vortex, the magnetic field has a component normal to the vortex axis, however, the average transverse component vanishes due to the large amount of contributions from the entire FLL. For general calculation of the magnetic field distribution, see the monograph of Greer and Kossler.

II. PHASE-SHIFTED µSR SPECTRA

In most cases, there is no difficulty to obtain the true field profiles through CFT from low transverse field µSR data. However, when the applied magnetic field is so high (several Tesla) that one precession period is only several times of the channel width, we need to study the effect of time-shifted or phase-shifted \( P(t) \) (due to the instabilities of electronics or initial polarization) on the interpretation of \( f(B) \). Assume \( P(t) = G_T(t) \cos(\omega t) \) where \( G_T(t) \) is the non-oscillating depolarization function part and can be exponential, stretched exponential, Gaussian or other types. Use the notations

\[
P_c(\omega) = \int G_T(t) \cos(\omega t) \, dt \tag{2}
\]

\[
P_s(\omega) = \int G_T(t) \sin(\omega t) \, dt \tag{3}
\]

We can easily derive the Fourier Transform (FT) of the time-shifted (or phase-shifted) \( P(t) \) as follows:
\[ P_{sc}(\omega) = \int G_T(t + \delta t) \cos[\omega_0(t + \delta t)] \cos(\omega t) \, dt \]
\[ = \cos(\omega_0 \delta t) P_c(\omega_0 - \omega) - \sin(\omega_0 \delta t) \frac{1}{2} P_s(\omega_0 - \omega) \quad (4) \]
\[ P_{ss}(\omega) = -\frac{\sin(\omega_0 \delta t)}{2} P_c(\omega_0 - \omega) - \frac{\cos(\omega_0 \delta t)}{2} P_s(\omega_0 - \omega) \quad (5) \]

Note in the expression of \( P_{sc}(\omega) \), the 1st term is symmetric about \( \omega_0 \), however, there is a sign change in the 2nd term when \( \omega \) goes across \( \omega_0 \). This sign change was seen in our Bi2212 high transverse field \( \mu \)SR data [2] at various temperatures.

To construct the true field profile \( P_c(\omega_0 - \omega) \) from the experimental data \( P_{sc}(\omega) \) and \( P_{ss}(\omega) \), we can do the following transformation,

\[ P_{sc}(\omega) \cos(\omega_0 \delta t) - P_{ss}(\omega) \sin(\omega_0 \delta t) = \frac{1}{2} P_c(\omega_0 - \omega) \quad (6) \]

The result of this phase correction on one of the field profiles in Bi2212 is shown in figure 1 where we can see the proper field profile is obtained.

The measurement of the internal magnetic field distribu-

underlying changes of the vortex structure (the one-to-
one mapping still needs to be studied further), is defined from the 3rd and 2nd moments of the field line shape as follows:

\[ \alpha = \frac{\langle (B - \overline{B})^3 \rangle^{\frac{1}{3}}}{\langle (B - \overline{B})^2 \rangle^{\frac{1}{2}}} \]
\[ = \frac{(M_3 - 3M_1 M_2 + 2M_1^2)}{(M_2 - M_1^2)^{\frac{3}{2}}} \quad (8) \]

where \( M_n = \int B^n f(B) dB \), and is the nth moment of \( f(B) \).

Normally we obtain the field profile either from asymmetry plot or from individual histogram then calculate the second moment and skewness of the field profile to probe the possible phase transitions of the vortex matters. In the analysis of high transverse field \( \mu \)SR data on Bi2212, we often find that the phase parameter in the depolarization function changes with temperature and thus needs to be corrected precisely.

**III. CALCULATION RESULTS**

To evaluate the effects of the phase shift \( \phi \) on the second moment and skewness calculations of the field profile, we generate a depolarization function (asymmetry plot) \( P(t) = e^{-\frac{t}{\sigma^2}} \cos(\omega t + \phi) \) where \( \sigma = 2 \mu s^{-1} \), \( \omega = 20 \) Mrad/s (\( \omega \) can be assumed much higher, the final results are similar). These parameters are chosen so that there are no evident visual changes in the field profiles when the phase shift \( \phi \) varies from 0 to 0.5 (~30°). The calculated \( \phi \) dependent second moment (ideally the second moment should be 2 \( \mu \)s~1) and skewness plot is shown in the figure 2.

We can see clearly that the second moment and skewness are very sensitive on the phase shift \( \phi \). A change of initial phase from 0 to 0.5 (~30°) causes about 50 percent change on the relaxation rate and an increase of skewness from 0 to 2. This result requests us to extract the phase parameter accurately after fitting the \( \mu \)SR asymmetry plot. To correct this phase-shift effect, we can follow the modified CFT procedure as discussed in the previous section to obtain the true field profiles, which is also the case when we analyze the individual histogram.

We now study the asymmetry \( A(t) \) which is used to obtain the field profile. We know \( A(t) \) is usually composed by two histograms coming from two oppositely (\( \pi \) out of phase) located positron detectors. However, in reality, especially for high precession frequency data, the two histograms are usually found not exactly \( \pi \) out of phase. To consider this phase deviation \( \phi \) from \( \pi \), we can do the following analytical analysis. As usual, we assume the phase-shifted histograms are (we assume Gaussian depolarization here, we can also use other depolarization function instead)
In order to obtain the proper field profiles through the Cosine Fourier Transform (CFT) on the \( \mu \)SR data and characterize them quantitatively and correctly, we need to extract the initial phase parameter precisely from individual histogram or asymmetry plot and correct the possible phase shift effect through a proposed data analysis procedure in order to obtain the correct second moment (or relaxation rate) and skewness of the field profile.

IV. CONCLUSION

In order to obtain the proper field profiles through the Cosine Fourier Transform (CFT) on the \( \mu \)SR data and characterize them quantitatively and correctly, we need to extract the initial phase parameter precisely from individual histogram or asymmetry plot and correct the possible phase shift effect through a proposed data analysis procedure in order to obtain the correct second moment (or relaxation rate) and skewness of the field profile.

[1] J.H. Brewer, *Encyclopedia of Applied Physics*, v. 11 (VCH Publishers, 1994); E.B. Karlsson, *Solid State Phenomena as seen by Muons, Protons and Excited Nuclei* (Clarendon, Oxford, 1995); A. Schenck and F.N. Gygax, *Handbook on Magnetic Materials* v. 9 (Elsevier, Amsterdam, 1995); P. Dalmas de Reotier et al., J. Phys.: Condens. Matter 9 (1997) 9113.

[2] S.L. Lee et al., Phys. Rev. Lett. 71 (1993) 3862; Phys. Rev. Lett. 75 (1995) 922; Phys. Rev. B 55 (1997) 5666.

[3] X. Wan, unpublished (2000).

[4] V.G. Kogan and J.R. Clem, Phys. Rev. B 24 (1981) 2497.

[5] A.J. Greer and W.J. Kossler, *Low Magnetic Fields in Anisotropic Superconductors, Lecture Notes in Physics* (Springer, Berlin, 1995)

[6] E.H. Brandt, J. Low Temp. Phys. 73 (1988) 355; Phys. Rev. Lett. 66 (1991) 3213.

[7] C. Bernhard et al., Phys. Rev. B 52 (1995) 10488 and R7050.