On the relevance of center vortices to QCD

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(October 28, 2018)

In a numerical experiment, we remove center vortices from an ensemble of lattice SU(2) gauge configurations. This removal adds short-range disorder. Nevertheless, we observe long-range order in the modified ensemble: confinement is lost and chiral symmetry is restored (together with trivial topology), proving that center vortices are responsible for both phenomena. As for the Abelian monopoles, they survive but their percolation properties are lost.

PACS numbers: 11.15.Ha, 12.38.Aw, 12.38.Gc, 11.30.Rd

The essential non-perturbative properties of QCD are confinement and chiral symmetry breaking ($\chi_{SB}$). It has been observed through numerical lattice simulations that these two properties persist in the quenched theory up to a critical temperature $T_c \sim 220 MeV$, where confinement is lost; chiral symmetry appears to be simultaneously restored [1]. The disorder which leads to the area law for the Wilson loop thus seems to be somehow tied to the existence of a chiral condensate. Although effective mechanisms have been proposed to explain confinement or $\chi_{SB}$, no successful common explanation is yet available.

Two effective descriptions of QCD have been receiving a lot of attention: one considers instantons as the effective degrees of freedom (d.o.f.), the other chromo-magnetic monopoles. Instantons are natural candidates to explain $\chi_{SB}$: each instanton is associated with a zero mode of the Dirac operator [1], and there must be an accumulation of zero eigenvalues to obtain a quark condensate [2]. Above $T_c$, the vanishing of the condensate must correspond to a qualitative change in the instanton ensemble (see, e.g., [3] for a recent discussion). An attractive mechanism for confinement is dual superconductivity of the QCD vacuum [3]. Considerable evidence for this dual Meissner effect has been accumulated on the lattice, including a disorder parameter demonstrating the condensation of chromo-magnetic monopoles below $T_c$ [3]. This condensation has been observed directly after gauge-fixing to “Maximal Abelian Gauge” [4], and the idea of “Abelian dominance” has emerged, according to which the Abelian d.o.f. of the Yang-Mills field encode all its long-distance (IR) properties. Indeed, $\chi_{SB}$ and its restoration have been removed by flipping the sign of a subset of SU(2) gauge links. This operation introduces a lot of disorder in the gauge field. Nonetheless, these disordered gauge fields now have a trivial, vortex-free center projection, and so according to the credo of center dominance they should not confine. That is, our introduction of short-range disorder should at the same time bring long-range order. To our surprise, this is indeed what happens. One may then ask if the spectral properties of the Dirac operator are not also dominated by the center components of the gauge field. In that case, our modified ensemble should show no sign of $\chi_{SB}$, since its center projection is the trivial (perturbative) vacuum. Indeed, this is what we observe: removal of center vortices causes both loss of confinement and restoration of chiral symmetry.
The next intriguing question regards the fate of Abelian monopoles as center vortices are removed. They do not disappear; on the contrary, the introduction of short-range disorder increases their number. However, we observe the complete disappearance of monopole current loops winding around the periodic lattice: we can thus identify these as the fundamental objects associated with confinement in the Abelian sector, apparently influenced by the underlying center d.o.f.

Finally, we investigated the effect of Gribov copies which caused our initial skepticism. We repeated our experiment on the same $SU(2)$ ensemble, but introduced a systematic tolerance in the gauge condition to be satisfied before identifying the center vortices to be removed. Although the location and number of center vortices we removed varied appreciably, the modified ensemble was always non-confining and chirally symmetric.

The numerical experiment – We start from an ensemble of $SU(2)$ lattice gauge fields obtained by Monte Carlo using the standard Wilson plaquette action. To identify center vortices, we gauge-fix our configurations in order to bring each $SU(2)$ gauge link $U_\mu(x)$ as close as possible to an element of the center $Z_2 = \{+1, -1\}$. We therefore try to iteratively maximize

$$Q\{\{U_\mu\}\} \equiv \sum_{x,\mu} (\text{Tr}U_\mu(x))^2 , \quad (1)$$

as in [13], where this gauge is called the “direct maximal center gauge.” The gauge-fixed $SU(2)$ links, denoted $U^{GF}_\mu(x)$, are then projected to $Z_2$ elements $Z_\mu(x)$ using

$$Z_\mu(x) = \text{sign}[\text{Tr}U^{GF}_\mu(x)] . \quad (2)$$

Plaquettes in the $Z_2$-projected theory with value $-1$ represent defects of the $Z_2$ gauge field called P-vortices [12]. Numerical evidence has been presented [12][13][14] showing that plaquette-like P-vortices signal the presence of macroscopic, physical excitations, called center vortices, in the unprojected original $SU(2)$ configuration.

Consider then the modified $SU(2)$ configuration made of gauge links $U'_\mu(x)$ constructed as

$$U'_\mu(x) \equiv Z_\mu(x) U_\mu(x) . \quad (3)$$

The gauge transformation which maximizes $Q\{\{U_\mu\}\}$ in (4) also gives the same maximum to $Q\{\{U'_\mu\}\}$, so that the modified gauge-fixed links $U^{GF}_\mu(x)$ are simply $Z_\mu(x) U^{GF}_\mu(x)$. Therefore, we instantly know the center projection $Z'_\mu(x)$ of $U'_\mu(x)$:

$$Z'_\mu(x) = \text{sign}[\text{Tr}U^{GF}_\mu(x)] = \text{sign}[\text{Tr}Z_\mu(x) U^{GF}_\mu(x)] = (\text{sign}[\text{Tr}U^{GF}_\mu(x)])^2 = +1 . \quad (4)$$

Every modified configuration $U'$ thus projects onto the trivial $Z_2$ vacuum: all center vortices have been removed.

Our ensemble consists of about 1000 $SU(2)$ configurations, on a $16^4$ lattice at $\beta = 2.4$. To maximize $Q\{\{U_\mu\}\}$ (Eq.1) we use standard overrelaxation, stopping when

$$\epsilon = \sum_{x,\mu} \Delta(\text{Tr}U_\mu(x))^2 < 10^{-6} \quad (5)$$

from one gauge-fixing sweep to the next.

![FIG. 1. Normalized plaquette distribution on the original and modified ensembles ($SU(2)$, $\beta = 2.40$). Center-vortex removal increases short-range disorder.](image)

In Fig.1 we show the distribution of $SU(2)$ plaquette values on the original and modified ensembles. It is apparent that under the sign flip Eq.(3), many $SU(2)$ plaquettes acquire a negative value. The modified ensemble has an increased action, i.e. more short-range disorder.

![FIG. 2. Creutz ratios on the original and modified ensembles. The dashed band is the string tension result of [16].](image)

Results – In Fig.2 we present our results for the Creutz ratios $\chi_{R,R} \equiv -\ln[\langle W_{R,R}\rangle/\langle W_{R-1,R-1}\rangle/\langle W_{R,R-1}\rangle^2]$ constructed from averages $\langle W_{R,T}\rangle$ of $R$ by $T$ Wilson loops on the original and modified ensembles. For large $R$, $\chi_{R,R}$ tends to the string tension $\sigma$. On the modified ensemble, the Creutz ratios clearly decrease and tend to zero. Despite the increased short-range disorder, long-range order has been created and confinement has been lost.
FIG. 3. Wilson loop values $\langle W_{R,T} \rangle$ on the original and modified ensembles. Note the parallel lines for successive $R$ in the latter: upon center-vortex removal, confinement is lost.

This is even clearer if one looks directly at the Wilson loop values. In Fig. 3 we show $-\ln \langle W_{R,T} \rangle$ as a function of $T$. For a fixed $R$, points at successively larger $T$ form a line whose asymptotic slope is the value of the static potential $V(R)$. The lines corresponding to the modified ensemble are parallel, indicating that $V(R)$ does not grow with $R$: the string tension has vanished.

FIG. 4. Quark condensate $\langle \bar{\psi}\psi \rangle(m_q)$ on the original and modified ensembles. The dashed line corresponds to "poor" center-vortex identification (see text). In all cases, center-vortex removal restores chiral symmetry.

Fig. 4 illustrates our study of chiral symmetry on the original and modified ensembles. As is well-known, $\chi_{SB}$ cannot occur on a finite lattice. Therefore, we measure $\langle \bar{\psi}\psi \rangle(m_q) = \langle \text{Tr}(D + m_q)^{-1} \rangle$ for a range of quark masses $m_q$ where finite-size effects are small, and extrapolate to $m_q \to 0$. In the original ensemble, $\langle \bar{\psi}\psi \rangle$ clearly extrapolates to a non-zero value which signals $\chi_{SB}$. In the modified ensemble, the extrapolated value is zero within errors: center-vortex removal restores chiral symmetry. We expect then the instanton content of the Yang-Mills field to be modified also. To check this, we use improved cooling [17] to measure the topological charge of the modified field: the striking result is that the removal of center vortices always leads to the trivial topological sector.

We therefore have clear evidence for "center dominance": in our modified ensemble, where the center-projected field is the trivial vacuum (all links equal to 1), the Yang-Mills field shows the IR properties of the trivial vacuum, i.e., no confinement, no $\chi_{SB}$ and no topology. The IR properties of the Yang-Mills field appear to be determined by its center projection.

On the other hand, a large number of studies now support the alternative scenario of "Abelian dominance." We use our approach of center-vortex removal to directly assess the relationship between these two scenarios.

In a first experiment, we construct the Abelian projection of our original $SU(2)$ ensemble by gauge-fixing to Maximal Abelian Gauge in the usual way [8], then identify and remove center vortices from the Abelian sector. While the original Abelian-projected ensemble shows confinement, with a string tension similar to the non-Abelian one, the modified Abelian-projected configurations do not confine. Therefore, we find no contradiction between "Abelian dominance" and "center dominance." The latter simply appears more fundamental because of the greater reduction of the number of d.o.f.

FIG. 5. Size distribution of monopole clusters on the original and modified ensembles.

In a second experiment, we look at clusters of Abelian monopole currents, whose percolation has been identified as the signal for confinement [18], obtained from the original and modified ensembles. We find that the removal of center vortices changes the distribution of monopole cluster sizes in a crucial way (see Fig. 5): whereas in the original ensemble, each configuration contains typically one very large, percolating monopole cluster and many very small ones, the modified ensemble gives a more homogeneous size distribution, with a handful of large clusters per configuration; these are the remnants of the very large one, broken into pieces by the vortex removal. Some of them still percolate, even though confinement has disappeared. Therefore, we are led to associate confinement with a more specific feature of the monopole clusters: monopole current loops which wind around the periodic lattice. Such loops can be found frequently on the original, confining ensemble, but never on the modified, non-confining one. We conclude that: (i) on a finite lattice confinement manifests itself in the Abelian sec-
tor by the presence of monopole current loops with non-trivial topology; (ii) center-vortex removal, which destroys confinement, always finds the “weak links” of these non-trivial loops and breaks them into trivial pieces.

Now let us consider the issue of gauge-fixing ambiguities, which was the reason for our initial skepticism about the center-vortex idea. These ambiguities come from the structure of $Q$ (Eq.1), which has many local maxima, any of which can be selected by a local iterative maximization algorithm. Each local maximum, or Gribov copy, will have its own set of P-vortices, differing in number and location. The proposal of [12] is that, no matter which Gribov copy one chooses, P-vortices are the traces of physical center vortices and are roughly located at their center. This argument may account for P-vortices differing in location but not in number. To study this question in more detail, we magnified the effect of gauge-fixing ambiguities, by stopping our iterative algorithm early, as soon as $\epsilon$ (Eq.3) $< 10^2$. Thus we not only explore a different basin of attraction of $Q$, but we do not even stop at a local maximum. One effect of this partial gauge-fixing is expected: the density of P-vortices increases from $\rho \approx 5.5\%$ to $\approx 7.4\%$, i.e. shows an increase $\delta\rho \approx 1.9\%$. The string tension measured in the $Z_2$-projected ensemble increases accordingly: whereas the $Z_2$ string tension after “complete” gauge-fixing ($\sigma a^2 \sim 0.075$) is little larger than but compatible with the non-Abelian string tension ($0.0708(11)$ [16], see Fig.2), it jumps to $\approx 0.12$ after partial gauge-fixing. What is remarkable is that this increase $\delta\sigma \approx 0.045$ is similar to that obtained by placing the surplus $\delta\rho$ of P-vortices at random, uncorrelated locations: $\delta\sigma \approx -\ln(1-2\delta\rho)$. This makes plausible that the center projection always captures the core d.o.f. relevant for IR properties, plus a varying amount of unrelated noise [21]. Indeed, it has been argued that gauge-fixing is not even necessary for center projection [19]. In our description, center-gauge-fixing acts as a UV noise-filtering device, with different Gribov copies letting through different noise components. Further evidence for this is obtained by removing from the original $SU(2)$ ensemble the center vortices identified after partial gauge-fixing only. Just as for “complete” gauge-fixing, we observe that confinement is lost, chiral symmetry restored, and the topological trivial. The only difference is that the modified ensemble now has much more short-range disorder.

In conclusion, we have shown that removal of center-vortices from an $SU(2)$ Yang-Mills ensemble causes the loss of confinement and the restoration of chiral symmetry. One may ask about the connection of the modified ensemble $\{U'\}$ to the physics of the original $SU(2)$ theory. Note that only plaquettes of $\{U'\}$ at the locations of P-vortices differ from those of $\{U\}$: hence, as $a \to 0$, their proportion goes to zero as $a^2$, since the density of P-vortices is physical [15][14]. Therefore, rewriting Eq.3 as $U'_\mu(x) \equiv Z'_\mu(x) \times U'_{\mu}(x)$, we see that the original field $\{U\}$ has been factorized into a (maximally) central part $\{Z\}$ and a quotient, $\{U'\}$, whose field strength differs from the original one only on defects of codimension 2. Nevertheless, this small difference alters the physics dramatically: $\{U'\}$ has perturbative properties, so that all the non-perturbative, IR physics must be carried by $\{Z\}$, which by definition encodes the center vortices. It would be desirable, of course, to formulate an effective action for the center-projected theory. Ref. [22] considers an extension of the Nambu-Goto action, where the fundamental d.o.f. are the 2-dimensional random surfaces dual to the P-vortices. Ref. [22] instead proposes to consider center monopoles and their world-lines. We suggest identifying a “minimum spanning tree” of negative $Z_2$ links responsible for the P-vortices: perhaps only a subset of them form the essential d.o.f. governing the IR properties.

Finally, our vortex-removal procedure can be used to study properties of non-confining non-Abelian fields and effects of center-symmetry breaking. For instance, removing time-like center disorder only would be similar to raising the temperature above $T_c$.

We thank A. Di Giacomo, M. Golterman, J. Greensite, T. Kovacs, C. Lang and O. Miyamura for discussions.

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