A Simplified Limit Equilibrium Model for Calculating Limit Support Pressure on Shallow Shield Tunnel face

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Abstract. Shield tunnel excavation face may collapse if the applied support pressure is not enough. Based on an elaborated failure mode of shallow shield tunnel face, the least limit support pressure to maintain stability is analysed by using the limit equilibrium method. The calculated limit support pressure decreases nonlinearly with the increase of friction angle, decreases linearly with the increase of cohesion, and decreases with the ratio of buried depth to tunnel diameter. By comparing the calculated results with existing tests, the theoretical model was shown to be reasonably predict the limit support pressure of tunnel excavation face in sand under drained condition and in clay under undrained condition.

1. Introduction
The key problem of shield tunneling safety is to control the stability of excavation face; it is necessary to determine the limit support pressure. The instability and failure of shallow shield tunnel face are mainly caused by surface overload, soil weight etc. It is difficult to establish a three-dimensional limit equilibrium model because of the great change of soil pressure during shield construction and the complicated failure mode. In this paper, considering the long excavation section, the stability analysis of excavation face is carried out from the perspective of two-dimensional circumferential simplification.

For the research on the stability of tunnel excavation face, model tests can directly reflect the failure mode in circumferential section. Wu et al. [1] analyzed the circumferential instability of single and double-track tunnels in clay. Lee et al. [2] used centrifugal test to simulate the failure mechanism of tunnel face in saturated sand. Compared with the uncertainty of model test error, theoretical models were often used, including limit analysis method [3-8] and limit equilibrium method [9-11]. Davis et al. [3] and Osman et al. [4] obtained the upper and lower bound solutions of the limit support pressure with different failure modes in clay under undrained condition. Yamamoto et al. [5] considered the circumferential failure mode of tunnel face in sand under drained condition, and obtained the variation law of limit support pressure. Yang et al. [6] introduced finite element method into the upper bound analysis, and a more accurate support pressure solution was obtained. Compared with the limit
analysis method, the limit equilibrium method has the advantage of simple calculation and to be easily used in engineering application. Based on plane strain condition, Nomikos et al. [9] analyzed the stability of a circular tunnel roof by limit equilibrium method. Lei et al. [10] combined with nonlinear failure criterion to analyze tunnel face stability and produced an optimized solution.

This paper aims to analyze the stability of shield tunnel face by using a simplified circumferential mode. Following the introduction in first part, the second part is to build a limit equilibrium model and get the expression of limit support pressure. The third part is to analyze the variation law of limit support pressure with friction angle and cohesion, and verifies the rationality of the results by comparing with existing results.

2. Limit equilibrium method

The limit equilibrium method (LEM) needs a pre-assumed failure mode. Combined with the failure modes in literatures [3] and [6], a failure mode with single parameter variable is established, as shown in Figure 1. The failure mode is composed of two sliding blocks ① and ③ or ② and ③. According to the relationship between failure mode and buried depth ratio, it can be determined whether the failure line extends to the surface. When the failure line extends to the surface, the upper sliding body is ①, otherwise it is ②. The figure 1 includes weight forces \( w_1 \) and \( w_2 \), surface overload \( q \), normal stresses \( N \) and \( N_1 \), frictional resistances \( T \) and \( T_1 \), average support pressure \( p \) at the vault and bottom of the tunnel, the ratio of the lateral support pressure to the vertical support pressure of the tunnel \( \lambda \) and the angle \( \alpha \) of the failure surface.

![Figure 1. Failure mode of limit equilibrium method](image)

According to the equilibrium of sliding body ① or ②, the loosen soil pressure \( \sigma_v \) which is transmitted from body ① or ② to the ③ can be obtained by

\[
\begin{align*}
\frac{w_1}{2} + \frac{q}{2} (h + 2h \tan \alpha - 2H \tan \varphi) + N \sin \varphi &= T \cos \varphi + \sigma_v \\
T &= N \tan \varphi + \frac{cH}{\cos \varphi} \\
N \cos \varphi + T \sin \varphi &= K(2q + \gamma H)\frac{H}{2}
\end{align*}
\]

(1)

Where \( K \) is obtained from the results of Terzaghi’s Trapdoor test, and the value is 1.0.

Furthermore, the force equilibrium of the sliding body ③ is carried out

\[
\begin{align*}
w_2 + \sigma_v &= \frac{(1 + \lambda)ph}{4} \cos \alpha + N_1 \sin \alpha + T_1 \cos \alpha \\
T_1 &= N_1 \tan \varphi + c \frac{h(1 + \sin \alpha)}{2 \cos \alpha} \\
\frac{(1 + \lambda)ph}{4} (1 + \sin \alpha) + T_1 \sin \alpha &= N_1 \cos \alpha
\end{align*}
\]

(2)

By solving the equation (2), the support pressure is arranged into the form of

\[
p = cN_v + qN_q + 0.5\gamma hN_y
\]

(3)
By optimizing the fracture angle in equation (3), when the $p$ value of support pressure reaches the minimum, it is the limit equilibrium solution of support pressure.

3. Results and discussion

3.1. Parametric analysis

Based on the established limit equilibrium model, the circumferential failure state of tunnel excavation face and the limit support pressure are analyzed. In the case of sand stratum under drainage condition, the friction angle of soil is $\phi$. Based on the principle of controlled variables; the relationship between limit support pressure and soil cohesion and friction angle is shown in figure 2. With the increase of friction angle, the limit support pressure shows a nonlinear decreasing trend, and when the friction angle is small, the limit support pressure decreases greatly. With the increase of friction angle and cohesion, the limit support pressure can be less than 0, which means the self-stable state of the tunnel without support can be obtained. Furtherly, considering the characteristics of clay under undrained condition, $\phi_u=0$ and $c_u\neq0$, the varying trend of limit support pressure with cohesion is shown in figure 3. The results show that with the increase of cohesion, the limit support pressure approximately decreases linearly.

![Figure 2. Influence of friction angle on limit support pressure](image1)

![Figure 3. Influence of cohesion on limit support pressure](image2)

3.2. Comparison and verification

The calculated limit support pressure was compared to the centrifuge test results [2]. The test material is saturated sand, the maximum and minimum of sand densities measured in dry state were 16.6 kN/m$^3$ and 14.1 kN/m$^3$, the friction angle $\phi=38^\circ$, cohesion $c=0$, and the rubber airbag was used to apply uniform support pressure, so the lateral support pressure coefficient $\lambda=1$. For comparison, the test results are normalized to $P_{lim}/P_0$, where $P_{lim}$ is the limit support pressure and $P_0$ is the initial soil pressure at the tunnel centerline. The results are shown in figure 4. With the increase of $C/D$, the change trend of limit equilibrium solution is basically in accordance with the model test, and the value agrees well with the test results.

The proposed model was used to analyze the limit support pressure on tunnel face in homogeneous clay [1], the soil parameters are $\gamma=18.1$kN/m$^3$, the undrained shear strength $c_u=31$, 37.9, 30.25, 33.3, 32.17kPa, and the lateral support pressure coefficient $\lambda=1$. As shown in figure 5, the results obtained from the limit equilibrium method are close to those from model tests, which shows that the calculated results are accurate. Furthermore, the upper bound solutions of limit analysis in [12] were compared. The upper bound solutions of $\gamma D/c_{ud}=2$, 3 and 4 were compared and analyzed. In order to compare the results more reasonably, the results of limit support pressure $P_{lim}$, surface overload $q$ and undrained shear strength $c_{ud}$ were normalized to dimensionless parameters and are shown in figure 5. When $C/D$ varies from 1 to 5, the results of the limit equilibrium method are close to those values in [12].

![Figure 4. Comparison of limit support pressure](image4)

![Figure 5. Upper bound solutions of limit analysis](image5)
4. Conclusions
Based on limit equilibrium method, the stability of shallow shield tunnel face and limit support pressure are studied, and conclusions can be obtained.

(1) The limit equilibrium model is based on a failure mode with optimized failure angle variable, and the failure mode is mainly affected by the friction angle. The calculated results can accurately predict the limit support pressure of tunnel face in sand and clay stratum.

(2) The calculated limit support pressure decreases nonlinearly with the friction angle, approximately increases linearly with cohesion, and decreases nonlinearly with the ratio of buried depth to diameter of tunnel.

Acknowledgement
The financial support by National Key Research and Development Program of China (through grant No. 2016YFC0800200) is gratefully acknowledged.

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