An Analytical Model of Radial Dust Trapping in Protoplanetary Disks

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Abstract

We study dust concentration in axisymmetric gas rings in protoplanetary disks. Given the gas surface density, we derived an analytical total dust surface density by taking into account the differential concentration of all grain sizes. This model allows us to predict the local dust-to-gas mass ratio and the slope of the particle size distribution, as a function of radius. We test this analytical model by comparing it with a 3D magnetohydrodynamical simulation of dust evolution in an accretion disk. The model is also applied to the disk around HD 169142. By fitting the disk continuum observations simultaneously at λ = 0.87, 1.3, and 3.0 mm, we obtain a global dust-to-gas mass ratio $f_{\text{global}} = 1.05 \times 10^{-2}$ and a viscosity coefficient $\alpha = 1.35 \times 10^{-2}$. This model can be easily implemented in numerical simulations of accretion disks.

Key words: accretion, accretion disks -- protoplanetary disks -- radiative transfer -- scattering -- stars: individual (HD 169142)

1. Introduction

In the last few years, high-quality (sub)millimeter observations of disks at high angular resolution have shown that a significant fraction of them host one or more concentric rings and gaps (e.g., ALMA Partnership et al. 2015; Andrews et al. 2018). These radial structures have been seen in both gas and dust (e.g., Isella et al. 2016). Although their origin is still under debate (see, e.g., Carrasco-González et al. 2016 and references therein), these structures must have a strong impact on the evolution of the dust and gas. Ultimately, understanding this evolution is fundamental to figure out how the formation of planetary systems takes place.

Dust grains migrate radially in protoplanetary disks, due to their interaction with the gas molecules: the radial pressure gradient felt by the gas is responsible for the shear in the angular velocity between the dust grains and the gas molecules, which causes an angular momentum interchange between both components via a drag force. The dust radial velocity is proportional to the radial pressure gradient (e.g., Whipple 1972; Weidenschilling 1977). For a disk with decreasing density and temperature radial profiles, the pressure gradient points radially inward and causes the radial migration of the dust grains toward the central star. In addition, the magnitude of the velocity depends on the size of the dust grains (e.g., Takeuchi & Lin 2002). Small dust grains ($\sim \mu$m) are well coupled with the gas, and large grains with sizes less than $\sim 1$ m feel a strong drag force and are expected to accrete to the star in a timescale much shorter than the lifetime of the gaseous disk; this problem is known as the radial drift barrier, first discussed by Weidenschilling (1977).

If the gas pressure has local maxima, as in the case of vortices (e.g., Barge & Sommeria 1995) or rings (e.g., Pinilla et al. 2012), the dust grains could be trapped, avoiding or retarding their accretion toward the star, promoting the appropriate conditions to build planetesimals.

Pohl et al. (2017) recently studied the disk around HD 169142, where two main gaps, probably created by Jupiter-mass planets, perturb the local gas and trap dust grains in the outer zone of the gaps, where the gas pressure has a maximum. Using their dust evolution model, which includes migration and grain growth, they can explain the 1.3 mm continuum emission of the disk observed by Fedele et al. (2017), showing that the millimeter grain sizes have had important evolution (migration + growth) within the disk. Pohl et al. also modeled the population of small dust grains in order to explain the dust scattering at near-IR wavelengths. This small dust population is well mixed with the gas.

In this paper, we study from an analytic point of view how the dust grains could be trapped in axisymmetric rings where the gas pressure has local maxima, and apply this model to the disk around HD 169142. In this approach, the dust grains radially migrate toward the pressure maxima and reach an equilibrium with the turbulent mixing, which depends on the grain size, but they do not grow. We do not address what creates the gas gaps (which in turn create the gas pressure maxima), as we are only interested in the redistribution of dust grains given the gas surface properties, i.e., the dust dynamics is only influenced by the drag force and turbulent mixing. Possible gravitational interactions with planets within the gaps are also ignored.

The structure of this paper is as follows. Section 2 describes the analytical model of dust concentration within the gas pressure maxima based on the dust dynamics within protoplanetary disks. Section 3 tests the analytical dust model by comparing it with recent dust and gas disk simulations. This model is then used to describe the dust emission of the disk around the star HD 169142 (Section 4). The observational properties of the disk are summarized in Section 4.1. In Section 4.2, we derive a model for the HD 169142 gas surface density, which is used together with the radiative transfer solution described in Section 4.3 in order to obtain the dust surface density from a best-fit model by comparing with the disk emission at multiwavelength millimeter observations (Section 4.4). In Section 5, we discuss the degeneracy between the dust properties that can produce the same observed value of the spectral index. Conclusions are presented in Section 6.
2. Analytical Model

Dust grains can be described as a pressureless fluid, so their evolution in a thin disk can be followed using the advection-diffusion equation,

$$\frac{\partial \Sigma_d}{\partial t} = \nabla \cdot (\Sigma_d \mathbf{v}) - \nabla \cdot \left[ D \Sigma_d \nabla \left( \frac{\Sigma_d}{\Sigma_g} \right) \right],$$

(1)

where $D$ [cm$^2$ s$^{-1}$] is the dust diffusion coefficient, $\Sigma_d$ and $\Sigma_g$ are the dust and gas surface densities, respectively, and $\mathbf{v} = (u_\phi, v_z)$ is the dust velocity in cylindrical coordinates ($\varpi$, $\phi$). A steady-state solution can be reached if the flux of dust grains toward pressure maxima is balanced by diffusion, in contrast with disks, where the pressure monotonically decreases with the radius and where the dust grains always migrate radially inward. In an axisymmetric disk with local pressure maxima, the steady-state solution of Equation (1) is given by

$$0 = \Sigma_d v_z - D \Sigma_g \frac{d}{d\varpi} \left( \frac{\Sigma_d}{\Sigma_g} \right)$$

(2)

This equation assumes that the dust follows the gas, i.e., that there is no backreaction of the dust on the gas. This effect can be ignored if the dust-to-gas mass ratio is less than 1 (e.g., Taki et al. 2016).

In the Epstein regime (where the radii of the dust grains are smaller than 4/9 of the mean free path of the gas), the radial dust velocity is given by (e.g., Windmark et al. 2012)

$$v_z = \left( \frac{S_t}{1 + S_t^2} \right) \frac{1}{\Omega \Sigma_g} \frac{dP}{d\varpi},$$

(3)

where

$$S_t = \frac{\pi \rho_m a}{2 \Omega \Sigma_g}$$

(4)

is the Stokes number of a dust grain with radius $a$ and material density $\rho_m$, and $P = \Sigma_g c_s^2$ is the gas pressure, where $c_s$ is the sound speed. Assuming that the dust diffusion coefficient is $D = D_g/(1 + S_t^2)$ (Youdin & Lithwick 2007), where $D_g = \alpha c_s^2/\Omega$ is the gas diffusion coefficient, $\Omega$ the angular speed, and $\alpha$ the viscosity coefficient, then, Equation (2) can be written as

$$0 = \frac{d}{d\varpi} \left( \frac{\Sigma_d}{\Sigma_g} \right) - \frac{S_t}{\alpha c_s^2 \Sigma_g} \frac{dP}{d\varpi} \left( \frac{\Sigma_d}{\Sigma_g} \right).$$

(5)

Ignoring the thermal gradients with respect to the density gradients, the solution of the above equation is

$$\frac{\Sigma_d(\varpi)}{\Sigma_g(\varpi)} = \frac{\Sigma_d(\varpi_0)}{\Sigma_g(\varpi_0)} \exp \left[ \int_{\varpi_0}^{\varpi} \alpha \frac{d \ln \left( \Sigma_g(\varpi) \right)}{d\varpi} d\varpi \right],$$

(6)

where $\Sigma_d(\varpi_0)$, $\Sigma_g(\varpi_0)$ are the dust and gas densities at a reference radius $\varpi_0$, respectively. Finally, the dust surface density can be written as

$$\Sigma_d(\varpi, a) = \epsilon_0 \Sigma_g(\varpi) \exp \{-ka\},$$

(7)

where

$$k = \frac{\pi \rho_m}{2 \alpha} \left( \frac{1}{\Sigma_g(\varpi)} - \frac{1}{\Sigma_g(\varpi_0)} \right)$$

(8)

and $\epsilon_0$ is the dust-to-gas mass ratio at the reference radius $\varpi_0$. Figure 1 shows the normalized dust surface density for grains of different sizes $a$ [cm] = $10^{-4}$, $10^{-3}$, $10^{-2}$, $10^{-1}$, $10^0$, $10^1$, and a gas surface density with two local maxima at $\varpi = 24$ and 60 au (black dashed line). This profile corresponds to the gas surface density profile of HD 169142 discussed in Section 4.1 below. Note that the largest dust grains are more concentrated around the first maxima. As the radii of the grains decrease, the dust particles tend to become well mixed with the gas. The grains with a size smaller than $10^{-3}$ cm (yellow dashed line) trace the gas surface density.

Equation (7) can also be written as

$$\epsilon(\varpi, a)/\epsilon_0 \propto \exp\{-1/\Sigma_g(\varpi)\},$$

(9)

where $\epsilon(\varpi, a)$ is the local dust-to-gas mass ratio. Therefore, the local dust-to-gas mass ratio follows the gas extrema (maxima or minima). Equation (7) also predicts that the gaps in the dust surface density are deeper than in the gas surface density. Note that, because the dust mass is conserved, the maxima are enhanced, due to the redistribution of the dust grains.

The total dust surface density is given by the sum of the dust densities for each grain size. For a dust size distribution where $n(a) da \propto a^2 da$ is the number of dust grains per unit volume with a radius between $a$ and $a+da$, with minimum and maximum grain sizes, $a_{\text{min}}$ and $a_{\text{max}}$, respectively, the total dust surface density is

$$\Sigma_d(\varpi) = \int_{a_{\text{min}}}^{a_{\text{max}}} \frac{a^2 n(a) da}{\int_{a_{\text{min}}}^{a_{\text{max}}} a^2 n(a) da}.$$

(10)

In this equation, the factor $a^2 n(a) da$ weights the amount of mass associated with each dust grain size, which is assumed to be constant because the dust grains are only redistributed in the...
disk without coagulation or fragmentation. In addition, the redistribution of the dust grains is taken into account by the factor $\Sigma_d(\varpi, a)$, which includes the dust size differential migration; see Figure 1.\textsuperscript{6} Thus, the integral can be computed using the global values of the disk $(a_{\text{max}}, p)$.

In particular, if the original value of the particle size distribution is $p = 3.5$ and $a_{\text{max}} \gg a_{\text{min}}$, the integral is

$$
\Sigma_d(\varpi) = \frac{\epsilon_0 \Sigma_g}{2 \sqrt{k a_{\text{max}}}} \text{erf}(\sqrt{k a_{\text{max}}}),
$$

where erf is the error function. Furthermore, $\epsilon_0$ is constrained by the total dust mass in the disk. If $\epsilon_{\text{global}}$ is the global dust-to-gas mass ratio (typically $\sim 1/100$), then

$$
\epsilon_{\text{global}} = \int \frac{\Sigma_d(\varpi) dA}{\int \Sigma_g(\varpi) dA} = \int \frac{\Sigma_d(\varpi) dA}{M_g},
$$

where $M_g$ is the total gas mass. Therefore, using Equation (11), one obtains

$$
\epsilon_0 = \epsilon_{\text{global}} M_g \left[ \pi \int_0^R \frac{\Sigma_d(\varpi) \text{erf}(\sqrt{k a_{\text{max}}})}{\sqrt{k a_{\text{max}}} \varpi d\varpi} \right]^{-1},
$$

where $R$ is the disk radius. Finally, replacing Equation (13) in Equation (11), one obtains the total dust surface density as a function of the gas surface density, the maximum grain size, and the global dust-to-gas mass ratio as

$$
\Sigma_d(\varpi) = \frac{\epsilon_{\text{global}} M_g}{2 \pi} \left[ \pi \int_0^R \frac{\Sigma_g \text{erf}(\sqrt{k a_{\text{max}}})}{\sqrt{k a_{\text{max}}} \varpi d\varpi} \right].
$$

This equation gives the dust surface density when the gas surface density in asymmetric rings is known. Because the gas surface density evolves in a diffusion timescale that is much longer than the advection timescale of the dust (see Appendix B), the dust will always concentrate following the gas maxima.

Recently, Dullemond et al. (2018) proposed a dust model that assumes that the gas pressure maxima are given by Gaussian functions. To obtain the dust surface density of grains with a single size, they assume a locally maximum gas surface density given by the Toomre stability criterion. Instead, our model uses a gas pressure maximum. In this section, we compare the dust surface density from the simulation for each grain size with that predicted by our analytic dust model (Equation (7)). Then, the gas surface density that appears in the dust model equation is given by the azimuthally averaged gas density profile from the simulation.

We choose a snapshot of the simulation at the time of 400 inner orbits of the disk. At this time, the gas surface density has developed a ring structure without vortices,\textsuperscript{7} and the dust grains have had enough time to concentrate in the ring following the drag force of the gas. The gas density profile has a local maximum (ring) centered at 65 au. We consider the gas density profile between 55 and 85 au, where the radial drift has stopped, and the dust has achieved a steady state.

The concentration of the dust surface density depends on the assumed value of the viscosity coefficient $\alpha$ (Equation (8)), which is found from the best-fit model by minimizing the function

$$
\sigma^2 = \frac{1}{N_p} \sum_a \sum_\varpi \left( \Sigma_{d,\text{norm}}(\varpi, a) - \Sigma_{d,\text{norm}}(\varpi, a) \right)^2,
$$

where $N_p$ is the number of dust particle sizes, $N_p$ is the number of radial points sampled, $\Sigma_{d,\text{norm}}$ is the normalized dust surface density profile of the model, and $\Sigma_{d,\text{norm}}(\varpi, a)$ is the normalized azimuthally averaged dust density profile of the simulation.

The sum over $\varpi$ compares the radial profiles, and the sum over $a$ takes into account all grain sizes, which vary from 50 $\mu$m to 1.0 cm in 10 logarithmically equally spaced bins. Figure 2 shows $\sigma^2$ as a function of $\alpha$. The minimum is obtained for $\alpha_0 = 1.3 \times 10^{-3}$, which is of the same order of magnitude as the averaged value, $3 \times 10^{-3}$, found in the simulation. The value of dust-to-gas mass ratio $\epsilon_0$ is not fitted because the dust particles included in the simulation are only a representative sample of the total dust mass; that is the reason why we compare the normalized dust surface densities.

Figure 3 shows the normalized surface densities from the model (solid lines) and the simulation (dotted lines) using the viscosity coefficient $\alpha_0$ and for three representative dust grain sizes: $a = 0.05$ cm (red line), 0.09 cm (green line), and 1.0 cm (blue line). In all cases, the width of the model profiles match the simulation profiles.

\textsuperscript{6} The local changes in the particle size distribution are discussed in Section (5).

\textsuperscript{7} The azimuthal fluctuations of the gas surface density in the ring with respect to the azimuthal average are smaller than 6%.
4. Application of the Model to HD 169142

In this section, we applied the analytical model (Equation (14)) that gives the dust surface density \( \Sigma_d(\alpha) \) to simultaneously explain the millimeter dust emission of the disk around HD 169142 at different wavelengths \( \lambda = 0.87, 1.3, \) and 3.0 mm, using the observed gas properties.

Section 4.1 summarizes the observed properties of the central star and the disk in this source. In Section 4.2, we derive the disk gas surface density and excitation temperature from the \(^{12}\)CO and \(^{13}\)CO maps. These disk gas properties are used in the analytical model to predict the dust surface density in order to determine the best-fit values for the global dust-to-gas mass ratio \( (\epsilon_{\text{global}}) \) and the viscosity coefficient \( (\alpha) \). We choose the parameters that minimize the reduced chi-squared of the dust continuum emission of the observed dust maps and the emission of a grid of models. Section 4.3 describes how to compute the emergent specific intensity of the models, and Section 4.4 presents the results for the best values of \( \epsilon_{\text{global}} \) and \( \alpha \).

4.1. HD 169142

HD 169142 is a Herbig Ae star (The et al. 1994) in the Sagittarius constellation with an age of 10 Myr (Pohl et al. 2017), a mass of \( M_*=1.65 M_\odot \) (Blondel & Dijé 2006) with an effective temperature \( T_*=8100\ K \), a star radius of \( R_*=2.2 R_\odot \) (Osorio et al. 2014), and at a distance of \( d=117 \pm 4\ pc \) (Gaia Collaboration et al. 2016). The disk around HD 169142 is seen almost face-on, with an inclination angle of \( \sim14^\circ \) \( (\cos i=0.974) \), and with its major axis along a position angle \( \text{P.A.}=5^\circ \) (Raman et al. 2006) on the plane of the sky.

IR-polarized scattered-light images (Quanz et al. 2013; Bertrang et al. 2018) revealed that the disk has a central cavity, surrounded by a bright rim of radius \( \sim0.021\ \text{(} \sim25\ \text{au}\) at 117 pc), and an annular gap ranging from \( \sim0.028-0.048\ \text{(} \sim33-56\ \text{au}\) in radius). An unresolved IR source was detected inside the central cavity (Biller et al. 2014; Reggiani et al. 2014) at a radius of \( \sim0.36\ \text{(} \text{19}\ \text{au}\) and was interpreted as a substellar or planetary companion candidate. The dust thermal emission of the disk was first imaged with the VLA at 7 mm (Osorio et al. 2014; Macías et al. 2017). These 7 mm observations confirmed the IR results and revealed that the ring of radius 25 au is indeed narrow and azimuthally symmetric, with a bright knot at \( \text{P.A.}=-40^\circ \). These 7 mm images also suggest the presence of a possible new gap at a radius of \( \sim85\ au \), located very close to the CO snowline, as imaged from DCO\(^+\) ALMA data (Macías et al. 2017). Finally, the VLA observations revealed a compact source of ionized material near the center of the cavity that could be tracing a weak radio jet, a photoionized inhomogeneous region of the inner disk, or an independent orbiting object.

The HD 169142 disk has been observed with ALMA at \( \lambda = 0.87\ mm, 1.3\ mm, \) and 3 mm (archival data from program 2012.1.00799.S; Fedele et al. 2017 and E. Macías et al. 2019, in preparation). The submillimeter and millimeter continuum ALMA maps of the dust nicely confirmed the ringed structure of the disk initially revealed by the infrared polarized images and the 7 mm observations. The line emission of the gas has been observed with ALMA using \(^{12}\)CO, \(^{13}\)CO, and \(^{13}\)CO \( J:2 \rightarrow 1 \) rotational transitions by Fedele et al. (2017).

In all cases, the contrast between the rings and the gaps in the line emission maps is weaker than that in the continuum maps. For \(^{12}\)CO, this could be a consequence of the line emission being optically thick. However, the \(^{13}\)CO emission, which is usually optically thin, also shows a smaller contrast compared to the dust emission; this behavior is similar to the dust model described in Section 2. Also, the line emission extends to a radius of \( \sim240\ au \), significantly larger than the radius inferred from the dust continuum emission, which only extends to \( \sim100\ au \). This difference could be explained by the radial dust migration (e.g., Brauer et al. 2008).

Figure 4 shows the normalized azimuthally averaged specific intensity of the dust continuum emission at \( \lambda = 870\ \mu m \) (red solid line), 1.3 mm (yellow dashed–dotted line), and 3 mm (green dashed line). It also shows the normalized azimuthally averaged profiles of the \(^{13}\)CO \( J:2 \rightarrow 1 \) (cyan solid line) and the \(^{12}\)CO \( J:2 \rightarrow 1 \) (blue dashed–dotted line) line emission. All of the dust maps were convolved to the same circular beam of 40 au.

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8 Table 2 shows the parameters of the images at each wavelength.
0.20 arcsec, while the gas maps have a circular beam of 0.16 arcsec. These beams are equivalent to 23.4 and 18.7 au at the assumed distance of 117 pc, respectively. Note that, because the dust analytical model depends on the gas surface density (Equation (14)), the model can only resolve dust structures with the angular resolution of the gas maps; this is the reason why we choose a higher angular resolution in the gas. However, prior to comparing the dust properties from the observations and the analytic model, we convolve the analytic dust model such that the final beam coincides with the observations.

4.2. Gas Surface Density

The gas surface density can be inferred using both an optically thin and an optically thick line. We use the $J_{2} \rightarrow 1$ lines and assume that $^{12}$CO is optically thick and $^{13}$CO is optically thin. The $^{13}$CO column density for the $J_{2} \rightarrow 1$ rotational transition (e.g., Estalella & Anglada 1994) is given by

$$N^{13\text{CO}} = c_{0} \tau_{0}^{13} T_{ex} \Delta v \frac{\exp[3h\nu/(2k_{B}T_{ex})]}{\exp[h\nu/(k_{B}T_{ex})] - 1},$$

where $c_{0} = 16\pi k_{B} \nu^{2}/[(2J + 1)h^{3}A_{J,J-1}]$, $h$ and $k_{B}$ are the Planck and Boltzmann constants, $\Delta v$ is the line width and $\nu = 230$ GHz, and $A_{2,1} = 3.25 \times 10^{-7}$ s$^{-1}$ are the frequency and the Einstein coefficient for this transition, respectively. The excitation temperature $T_{ex}$ is given by

$$T_{ex} = \frac{hv/k_{B}}{\ln \left(1 + \frac{hv/k_{B}}{T_{0}^{12} + \mathcal{J}^{13}(2.7 \text{K})}\right)},$$

where $T_{0}^{12}$ is the brightness temperature of the $^{12}$CO $J_{2} \rightarrow 1$ line, $\mathcal{J}(\nu) = (hv/k)/(\exp(hv/kT_{\text{bg}}) - 1)$ is the intensity in units of temperature, and $\mathcal{J}^{13}(2.7 \text{K})$ is the intensity at the frequency of the $^{13}$CO $J_{2} \rightarrow 1$ line evaluated at the background temperature. Finally, the optical depth of the $^{13}$CO $J_{2} \rightarrow 1$ line ($\tau_{0}^{13}$) is given by

$$\tau_{0}^{13} = -\ln \left[1 - \frac{T_{0}^{13}}{\mathcal{J}^{13}(T_{ex}) - \mathcal{J}^{13}(2.7 \text{K})}\right],$$

where $T_{0}^{13}$ is the brightness temperature of the $^{13}$CO $J_{2} \rightarrow 1$ line, and $\mathcal{J}^{13}(T_{ex})$ is the intensity at the excitation temperature. We find that $\tau_{0}^{13} < 0.8$ throughout the disk.

To obtain the gas surface density at each radius, one needs an abundance factor between $^{13}$CO and $^{12}$CO (which dominates the gas mass), such that $\Sigma_{g} = m_{H_{2}}/H_{2}(^{13}\text{CO}) \times N^{13\text{CO}}$. The abundance is obtained by normalizing the gas surface density with the total mass of the disk $M_{\text{disk}} = 2\pi \int_{0}^{R} \Sigma_{g} \, d\varpi$. For a disk mass of $M_{\text{disk}} = 0.019 M_{\odot}$ (Fedele et al. 2017), the abundance between the $^{13}$CO and the $^{12}$CO molecules is $^{13}\text{CO}/^{12}\text{CO} = 1.0 \times 10^{-5}$, similar to values found in the ISM (e.g., Dickman 1978).

The top panel of Figure 5 shows the gas surface density model as a function of the radius. The width of the line gives the uncertainty in the gas surface density, due to the noise of the CO maps and the error propagation. In the next section, we use this gas surface density to model the dust surface density using Equation (14).

![Figure 5](image_url)

4.3. Dust Properties and Radiative Transfer Model

The emergent specific intensity is given by

$$I_{\nu} = I_{\nu}^{bg} e^{-\tau_{\nu}} + \int_{0}^{\tau_{\nu}} S_{\nu}(T) e^{-\chi_{\nu}} \, d\chi,$$

where $I_{\nu}^{bg}$ is the background intensity, and $\tau_{\nu} = (\kappa_{\nu} + \sigma_{\nu}) \Sigma_{d}$ is the optical depth at the frequency $\nu$, where $\kappa_{\nu}$ is the mass absorption coefficient and $\sigma_{\nu}$ is the mass scattering coefficient. The source function $S_{\nu}(T)$ is given by (Mihalas 1978)

$$S_{\nu}(T) = \omega_{\nu} J_{\nu} + (1 - \omega_{\nu}) B_{\nu}(T),$$

where the albedo is $\omega_{\nu} = \sigma_{\nu}/(\kappa_{\nu} + \sigma_{\nu})$, $B_{\nu}(T)$ is the Planck function, and $J_{\nu}$ is the local mean intensity. We approximate $J_{\nu}$ by the analytical solution found by Miyake & Nakagawa (1993, hereafter MI93) for a vertically isothermal slab. Then, the source function can be written as

$$S_{\nu}(T) = B_{\nu}(T)[1 + \omega_{\nu} f(t, \tau_{\nu}, \omega_{\nu})],$$

where

$$f(t, \tau_{\nu}, \omega_{\nu}) = \frac{\exp(-\sqrt{3} \epsilon_{\nu} t) + \exp(\sqrt{3} \epsilon_{\nu} t - \tau_{\nu})}{\exp(-\sqrt{3} \epsilon_{\nu} \tau_{\nu})(\epsilon_{\nu} - 1) - (\epsilon_{\nu} + 1)}.$$

$\tau_{\nu}$ is the total optical depth and $t$ is the variable optical depth, both measured perpendicular to the disk midplane, and

$$\epsilon_{\nu} = \sqrt{1 - \omega_{\nu}}.$$

For a face-on and vertically isothermal disk, the solution of Equation (19) using Equations (21)–(23) and setting $I_{\nu}^{bg} = 0$ is

$$I_{\nu} = B_{\nu}(T)[1 - \exp(-\tau_{\nu}) + \omega_{\nu} F(\tau_{\nu}, \omega_{\nu})],$$

where $F(\tau_{\nu}, \omega_{\nu})$ is the solution of Equation (24) with the appropriate boundary conditions.
Then, in the optically thin regime, the emergent intensity is

\[ I_\nu^{\text{thin}} = B_\nu(T) \tau_\nu (1 - \omega_\nu) = B_\nu(T) \kappa_\nu \Sigma_\text{d}, \]

independent of the albedo (see MI93). In the optically thick regime, the emergent intensity is

\[ I_\nu^{\text{thick}} = B_\nu(T) \left[ 1 - \frac{\omega_\nu}{(\epsilon_\nu + 1)(\sqrt{3} \epsilon_\nu + 1)} \right], \]

consistent with the discussion in D’Alessio et al. (2001, hereafter DA01).

We assume a power-law dust temperature

\[ T_d = T_0 \left( \frac{\nu}{\nu_0} \right)^{-q}, \]

where \( T_0 = 500 \text{ K} \) at \( \nu_0 = 1 \text{ au} \) and \( q = 1/2 \). This slope is estimated by assuming an equilibrium between the star incident radiation and the dust grain emission, such that, in the gray opacity approximation, \( T_d^4 \approx T_0^4 W(\omega) \), where \( W(\omega) \) is the dilution factor given by \( W(\omega) = 1/2 (1 - (1 - (R_d/\omega^2)^{1/2})^2) \), and \( T_\ast \) and \( R_\ast \) are the effective temperature and radius of the star respectively (Section 4.1). Because a large dust central hole has been previously reported in the disk around HD 169142 (e.g., Fedele et al. 2017; Macías et al. 2017), the inner and outer dust radii are set to be 15 au and 90 au, respectively.

The monochromatic opacity \( \kappa_\nu = \kappa_\nu + \sigma_\nu \) and albedo are computed with the code of DA01 for a dust composition of silicates, organics, and ice with the relative abundances described by Pollack et al. (1994). In this code, the total dust opacity is given by

\[ \chi_\nu = \frac{\int_{a_{\text{min}}}^{a_{\text{max}}} \kappa_\nu(a) n(a) da}{\int_{a_{\text{min}}}^{a_{\text{max}}} n(a) da}, \]

where \( \kappa_\nu(a) \) is the opacity of each dust grain size at frequency \( \nu \), and the particle size distribution has \( n(a) da \propto a^{-\rho} da \) with \( \rho = 3.5 \), the minimum grain size is \( a_{\text{min}} = 0.05 \mu \text{m} \), and the maximum grain size \( a_{\text{max}} \) depends on the disk radius. The opacity and albedo curves as a function of the wavelength for different dust size distributions are shown in Figures 9 and 10 of Sierra et al. (2017).

To determine the maximum grain size, \( a_{\text{max}}(\omega) \), one has to fit a power law to the observed intensity \( I_\nu \). This comes from the following considerations: in the optically thin limit and including only the mass absorption coefficient \( \kappa_\nu \), \( I_\nu \propto \kappa_\nu B_\nu(T) \). Also, at low frequencies, the Planck function can be approximated as \( B_\nu(T) = \nu^2 k T / 2 \epsilon^2 \) (Rayleigh–Jeans regime), and the opacity is given by a power law of the frequency \( \kappa_\nu \propto \nu^\beta \) (Beckwith et al. 1990). Then, the specific intensity will follow a power law of the frequency as

\[ I_\nu \propto \nu^{2+\beta}. \]

In general, \( \beta \) will be a function of radius. Because, for a given slope \( p \) of the dust size distribution, \( \beta \) depends on the maximum grain radius (e.g., Ossenkopf & Henning 1994; Pollack et al. 1994), one can find \( a_{\text{max}}(\omega) \) at each radius.\(^9\) Nevertheless, one can avoid the assumptions of optically thin emission and the Rayleigh–Jeans regime, and include the scattering mass opacity \( \sigma_\nu \) in the total opacity, \( \chi_\nu = \kappa_\nu + \sigma_\nu \). In this case, the parameter \( \beta \) can be derived from the optical depth that appears in Equation (24), \( \tau_\nu = \tau_0 (\nu/\nu_0)^\beta \), where \( \tau_0 \) is the optical depth at a reference frequency \( \nu_0 \).

The middle panel of Figure 5 shows the \( \beta \) derived from the dust continuum observations following the latter procedure, without assuming optically thin emission nor the Rayleigh–Jeans approximation, and including the mass scattering opacity. The width of the curve represents the error in the fit. The error is negligible for \( \omega < 80 \) au, where the signal-to-noise ratio in the dust emission maps is high. The bottom panel of the same figure shows the optical depth at \( \lambda = 870 \mu \text{m} \) (magenta solid line), 1.3 mm (green dashed–dotted line), and 3.0 mm (red dashed line): the disk is optically thin at \( \lambda = 1.3 \text{ mm} \) and 3.0 mm; it is only marginally thick close to the two maxima at \( \lambda = 870 \mu \text{m} \).

Figure 6 shows the maximum grain size, \( a_{\text{max}}(\omega) \), as a function of the disk radius assuming a slope of the particle size distribution of \( p = 2.5 \) (magenta solid line), 3.0 (green dashed–dotted line), and 3.5 (red dashed line); see more details in Appendix A). In the following, we assume \( p = 3.5 \) and in Section 5 we discuss this assumption.

We find that \( a_{\text{max}}(\omega) \) is very large at the position of the first maximum and decreases with the disk radius, reaching a value of \( \sim 2 \) mm at \( \omega = 90 \) au. This behavior could be due to differential radial migration of the dust grains toward the gas pressure maxima and/or dust growth.

### 4.4. Best-fit Dust Model

The dust-to-gas mass ratio \( \epsilon_\text{global} \) and the \( \alpha \) parameter that appear in Equations (8) and (14) are the unknown parameters of the dust model.

\(^9\) See the discussion on the degeneracy between \( a_{\text{max}} \) and the slope \( p \) in Section 5.
These parameters are fitted by creating a grid of models and comparing their radial intensity profiles ($I_{\lambda}^{\text{mod}}(\varpi)$) with the observed azimuthally averaged intensity profiles ($I_{\lambda}^{\text{obs}}(\varpi)$) at each wavelength. We vary $\alpha$ from $10^{-5}$ to $10^{-1}$, while $\epsilon_{\text{global}}$ varies from $10^{-4}$ to $10^{-1}$. The number of models for each parameter is 50 (the total number of models is $50 \times 50$), and the values are logarithmically equally spaced. The top panels of Figure 7 show the reduced chi-squared, 

$$\chi_{r,\lambda}^2 = \frac{1}{N_p} \sum_{\varpi} \left( \frac{I_{\lambda}^{\text{mod}}(\varpi) - I_{\lambda}^{\text{obs}}(\varpi)}{\sigma_{\lambda}^{\text{obs}}(\varpi)} \right)^2,$$

of the dust models at $\lambda = 870 \mu m$ (left panel), 1.3 mm (middle panel), and 3.0 mm (right panel), where $N_p$ is the number of radii and $\sigma_{\lambda}^{\text{obs}}(\varpi)$ is the uncertainty of the observed intensity at each radius. The latter is given by

$$\sigma_{\lambda}^{\text{obs}}(\varpi) = \text{rms}_\lambda / \sqrt{n},$$

where rms$_\lambda$ is the root mean square noise of the observed map, and $n$ is the number of beams within the area of each ring. The bottom panels show the intensity profiles of the best dust models (red solid line), the optical depth models (green dashed line), and the observed intensity profiles (blue line) whose error is given by the width of the line.

Figure 8 plots the isocontours where the value of the reduced chi-squared is 1.5 times the minimum value of $\chi_{r,\lambda}^2$ for each wavelength $\lambda = 870 \mu m$ (red solid line), 1.3 mm (green dashed line), and 3.0 mm (blue dashed–dotted line). The best parameters at each wavelength are summarized in Table 1. To obtain a global model, it is required that the parameters do not depend on wavelength. The best values for $\epsilon_{\text{global}}$ are the same, and the best values of $\alpha$ slightly differ with wavelength. Therefore, the best averaged parameters that describe the dust concentration in the disk of HD 169142 are $\epsilon_{\text{global}} = 1.05 \times 10^{-2}$ and $\alpha = 1.35 \times 10^{-2}$; this means that the total dust mass is $2.0 \times 10^{-4} M_\odot$.

Note that the fitted value of $\alpha$ depends on the resolution of the gas structure. For example, if in higher angular resolution observations a ring fragments into thinner rings, that would require a smaller value of $\alpha$ to concentrate the dust in these rings.

Figure 9 shows the disk maps for different wavelengths; the left panels are the observed maps, the middle panels show the emission of the dust model with the best averaged parameters, and the right panels are the maps of absolute differences.

Figure 7. Exploration of the parameter space for the dust continuum emission at $\lambda = 870 \mu m$ (left panels), 1.3 mm (middle panels), and 3.0 mm (right panels). Top panels: reduced chi-squared ($\chi_{r,\lambda}^2$) for different dust models in the ($\epsilon_{\text{global}}, \alpha$) space. Bottom panels: best model at each wavelength; the blue line is the observational dust continuum emission, the red solid line is the best emission model, and the green dashed line is the best optical depth model.

Figure 8. Isocontours where the reduced chi-squared is equal to 1.5 times the minimum value of $\chi_{r,\lambda}^2$ for each wavelength $\lambda = 870 \mu m$ (red solid line), 1.3 mm (green dashed line), and 3.0 mm (blue dashed–dotted line).
The wavelength increases from top to bottom ($\lambda = 870 \mu m$, 1.3 mm, 3 mm, respectively). Note that the main differences between the observations and model are the non-axisymmetric structures, which are not taken into account in the dust model.

For the best averaged parameters, Figure 10 shows the dust surface density (the red solid line), the local dust-to-gas mass ratio (green dashed line), the global dust-to-gas mass ratio ($\epsilon_{\text{global}}$ green dotted line), and the gas surface density scaled by the global dust-to-gas mass ratio (blue dashed–dotted line). The latter would mimic the dust surface density if there is no dust migration, which is not the case in HD 169142. The local dust-to-gas mass ratio in the first maximum increases by a factor of 6 compared with the global dust-to-gas mass ratio, while in the second local maximum it is below the average value. In addition, this figure shows that the maximum dust-to-gas mass ratio is less than 0.06; therefore, one can ignore the backreaction of the dust on the gas.

The amplitude of the dust surface density in the second maximum ($\varpi = 60 \text{ au}$) is much smaller than that in the first maximum ($\varpi = 24 \text{ au}$), in contrast with the behavior of the intensity profiles (bottom panels of Figure 7); this occurs because of opacity effects associated with the maximum grain size at each radius. In the first maximum, the grains have a maximum size larger than 10 cm; such grains do not have a large opacity at millimeter wavelengths. However, in the second maximum, the grains have a maximum size of $\sim 2 \text{ mm}$, which have a large opacity at millimeter wavelengths. For this reason, the contrast in the maxima of the dust surface density is larger than expected. Appendix A explains the effect of the maximum grain size in the opacity for different slopes of the particle size distribution.

![Figure 9. Maps of the thermal dust emission at $\lambda = 870 \mu m$ (first row), 1.3 mm (second row), and 3.0 mm (third row). The left, middle, and right panels are the observational, model, and difference maps.](image)

| Table 1 | Best Physical Parameters |
|--------|--------------------------|
| $\lambda$ (mm) | $\alpha/10^{-2}$ | $\epsilon_{\text{global}}/10^{-2}$ |
| 0.87 | 1.26 | 1.05 |
| 1.3 | 1.53 | 1.05 |
| 3.0 | 1.26 | 1.05 |
In this section, we study the local changes of the particle size distribution, due to dust size differential migration. This effect has been already found by, e.g., Pinte et al. (2016) and Sierra et al. (2017).

The model to obtain the slope $p$ as a function of the disk radius based on the results of the analytical model (Section 2) is as follows: if the total dust surface density is given by Equation (10), the term $\Sigma_d(\tau, a)n(a)da$ can be interpreted to be proportional to the local particle size distribution after dust concentration. If $n_2(a)da$ is this new distribution and $c_1$ the proportionality factor, then,

$$n_2(a)da = c_1 \frac{A}{\int_A \Sigma_d(\tau, a)da} n(a)da.$$  \hfill (32)

The number of particles per unit volume must be the same as the original when averaged in all the disk area $A$; then,

$$\int_A n_2(a)da dA = \int_A n(a)da dA.$$  \hfill (33)

Then, substituting Equations (7) and (33) into Equation (32), one obtains

$$\begin{align*}
\alpha^{-p_2} &\propto \frac{A \Sigma_d(\tau) \exp[-ka]}{\int \Sigma_d(\tau) \exp[-ka] da} n(a)da, \\
\alpha^{-p_2} \exp[-ka] &\propto \frac{\exp[-ka]}{\int \Sigma_d(\tau) \exp[-ka] da} a^{-p}.
\end{align*}$$  \hfill (34)

Equation (34) only depends on the viscosity coefficient $\alpha$ via the factor $k$ defined in Equation (8). One expects that, if the viscosity coefficient $\alpha \to \infty$ (i.e., $k \to 0$), there is no dust differential migration, and the dust particle size distribution does not change, i.e., $p_2 = p$.

For the gas surface density $\Sigma_g(\tau)$ in the top panel of Figure 5, we fit the term within the brackets of Equation (34) with a power law in order to obtain the value of $p_2$ for different values of $\alpha$. The top panel of Figure 11 shows $p_2$ as a function of radius for three values of $\alpha$, assuming that the original distribution has a slope $p = 3.5$. The blue solid line represents a relatively low value of $\alpha$, the green dashed line of an intermediate value, and the red dashed–dotted line of a high value. One recovers the original value of $p$ for the large value of $\alpha$ as expected. For intermediate and low values of $\alpha$, the slope $p_2$ decreases from its original value ($p$) in the inner disk and increases in the outer disk.

The bottom panel of Figure 11 shows the maximum grain size $a_{\text{max}}$ needed to explain the value of $\beta$ shown in the middle panel of Figure 5, given the $p_2$ curves shown in the top panel (see Appendix A). Note that in these cases, the maximum grain size $a_{\text{max}}$ increases with $\alpha$. Therefore, any physical process that changes the slope $p \to p_2$ will alter the inferred value of $a_{\text{max}}$.

### 6. Conclusions

We present an analytic model of the dust concentration in gas pressure maxima in disk rings. This model assumes steady state and only considers the dust radial redistribution, without fragmentation or coagulation. In addition, the model does not consider the backreaction of the dust on the gas, which is negligible if the local dust-to-gas mass ratio is smaller than 1. In the case of HD 169142, this condition is satisfied.

The inward dust migration can be stopped (or retarded) by axisymmetric gas pressure maxima, which act as dust traps. The dust grains concentrate around the gas pressure maxima and change the local dust properties (the particle size distribution, the opacity, and the dust-to-gas mass ratio). The dust concentration depends on the grain size: while the small grains are well coupled with the gas, the large grains tend to decouple and concentrate more around the pressure maxima, due to the drag force. Diffusion prevents a strong concentration of dust grains around gas maxima. If the viscosity coefficient $\alpha$ is large, then the dust grains of all sizes tend to follow the gas distribution. If the viscosity coefficient $\alpha$ is small, the large dust grains (those with $St/\alpha \gg 1$) strongly concentrate around the pressure maxima, while the small grains (those with $St/\alpha \ll 1$) are well mixed with the...
gas (e.g., Birnstiel et al. 2013; Lyra & Lin 2013; Dullemond et al. 2018). This migration has important consequences for the emergent intensity and the maps of the disks. To model the disk emission, we have considered spherical grains with a global particle size distribution \( n(a) da \propto a^{-3.5} da \) and the composition given by Pollack et al. (1994), which determine the values of the opacity exponent \( \beta \) in Figure 12 used to interpret the local physical conditions of the grains. With this in mind, we summarize below our main conclusions.

1. Our analytical model can explain the behavior of the dust grains trapped in a gas ring in the simulation performed by Flock et al. (2015) and Ruge et al. (2016). It can also explain the main characteristics of the thermal dust emission of the disk around the HD 169142 star. Because in both cases the gas surface density has local maxima, an equilibrium between the drag force and diffusion can be reached at these maxima, allowing the accumulation of dust and preventing its migration toward the central star.

2. This model can be easily implemented in numerical simulations that follow only the gas in protoplanetary disks to account for the dust concentration, saving computational time.

3. The redistribution of dust grains effectively increases the dust-to-gas mass ratio and the thermal dust emission within the gas pressure maxima. However, in the case of large grains (\( a_{\text{max}} > 1 \) cm for millimeter observations), even though they provide most of the mass, the emergent intensity does not increase linearly with the dust-to-gas mass ratio because these grains have a low opacity at millimeter wavelengths.

4. The small spectral index derived from the ALMA observations in the inner ring of HD 169142 (10 \( \lesssim r \lesssim 40 \) au) can be explained by the presence of large grains. The maximum dust size in the center of the inner ring (\( \sim 27 \) au) is \( a_{\text{max}} > 10 \) cm. This value monotonically decreases, reaching a value of \( \sim 2 \) mm in the outer region of the disk.

5. The best parameters that describe the dust dynamics in the disk around HD 169142 according to the analytical model described in Section 2 are a global dust-to-gas mass ratio \( f_{\text{global}} = 1.05 \times 10^{-2} \) (similar to the ISM) and a viscosity coefficient \( \alpha = 1.35 \times 10^{-2} \). These parameters can simultaneously explain the dust emission profiles at \( \lambda = 870 \mu m, 1.3 \) mm, and 3.0 mm.

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Software: CASA (v 4.7.0) (McMullin et al. 2007).

Appendix A

Spectral Index \( \beta \) and Dust Opacity

The spectral index \( \beta \) between 870 \( \mu m \) and 7 mm is computed from the opacities obtained from the DA01 code using the Pollack et al. (1994) abundances, in a grid varying the maximum grain size (\( a_{\text{max}} \)) and slope \( p \) of the particle size distribution \( n(a) \propto a^{-p} \). The minimum grain size in all cases is set to 0.05 \( \mu m \); this value is relevant only in the case when the mass of the particle size distribution is dominated by small grains, which occurs when \( p > 4 \).

The left panel of Figure 12 shows the map of \( \beta \) as a function of \( a_{\text{max}} \) and \( p \). Note that in the region of small dust grains with \( a_{\text{max}} \approx 60 \mu m \) or high slope \( p \gtrsim 4 \), the value is \( \beta \sim 1.8 \), which is
characteristic of the interstellar medium (Draine 2006). The region with \( \beta < 1 \) (blue) can only be explained by large dust grains \( a_{\text{max}} \gtrsim 2 \text{ nm} \) and \( p \lesssim 4 \). The black dashed line is the isocountour where \( \beta = 0.5 \). The slope \( \beta \) shows a local maximum for intermediate particles (60 \( \mu \text{m} < a_{\text{max}} < 2 \text{ nm} \) and \( p \lesssim 4 \)), which has been previously reported in the literature (e.g., Ricci et al. 2010).

The right panel of Figure 12 shows as an example the map of the total dust opacity \( \chi \) at \( \lambda = 3 \text{ mm} \) as a function of \( a_{\text{max}} \) and \( p \); the units of \( \chi_{3 \text{ mm}} \) are \( \text{cm}^2 \) per gram of dust. The dust opacity has a local maximum for \( a_{\text{max}} \sim 0.13 \text{ cm} \) and low \( p \); this occurs because when \( a_{\text{max}} \gg \lambda \), the largest dust grains do not contribute to the effective cross section but they dominate the dust mass. In general, when \( a_{\text{max}} \gg \lambda \), the opacity decreases as \( \chi_{\nu} \propto a_{\text{max}}^{-p} \) for \( 3 < p < 4 \) (DA01).

Also, the dust opacity is low for large values of the slope of the particle size distribution (\( p \gtrsim 4 \)); this occurs because for \( p > 4 \), the mass is dominated by the small grains. However, they do not have a large effective cross section at millimeter wavelengths. For small grains \( \log(a_{\text{max}}) < 1.5 \), the opacity at \( \lambda = 3 \text{ mm} \) is also small for all values of \( p \), as their effective cross section at millimeter wavelengths is small.

The behavior of the dust opacity at other millimeter wavelengths is qualitatively the same; the difference is the value of \( a_{\text{max}} \), where the opacity is maximum. We find that total opacity has a maximum at \( a_{\text{max}} \sim \lambda/\pi \). The maps of Figure 12 can be used to find the opacity at other wavelengths between \( \lambda = 870 \mu\text{m} \) and 7 mm via \( \chi_{\nu} = \chi_{3\text{mm}}(\nu/\nu_{3\text{mm}})^{1.0} \).

Appendix B
Gas and Dust Timescales

The dynamics of the gas and dust grains is different, due to the drag force. In particular, the advection timescale of the dust grains is

\[
T_{\text{adv,d}} = \frac{L}{v_\infty},
\]

where \( L \) is a characteristic length scale and \( v_\infty \) is the magnitude of the dust velocity relative to the gas (Equation (3)). The diffusion timescale of the gas and the dust grains are

\[
T_{\text{diff,g}} = \frac{L^2}{D_g},
\]

\[
T_{\text{diff,d}} = \frac{L^2}{D_g}(1 + St^2),
\]

where \( D_g \) is the gas diffusion coefficient and \( St \) the Stokes number (Youdin & Lithwick 2007). Then, the ratio between the diffusion and advection timescales of the dust is given by

\[
\frac{T_{\text{diff,d}}}{T_{\text{adv,d}}} = \frac{\alpha}{\omega} \left\| \frac{\omega d\Sigma_g}{\Sigma_g d\omega} \right\|.
\]

If this ratio is much larger than 1, the dust concentrates in pressure maxima. The ratio between the diffusion timescale of the gas and the dust advection timescale is

\[
\frac{T_{\text{diff,g}}}{T_{\text{adv,d}}} = \frac{\alpha}{\omega} \left( \frac{1}{1 + St^2} \right) \left\| \frac{\omega d\Sigma_g}{\Sigma_g d\omega} \right\|.
\]

If this ratio is much larger than 1, the gas quickly adjusts to the gas density structure and concentrates in gas pressure maxima.

In general, the Stokes number for millimeter and centimeter dust grains \( St(1 \text{ mm}) \sim 0.1 \) and \( St(1 \text{ cm}) \sim 1 \), respectively; also the logarithmic density gradient in accretion disks (\( (\partial \Sigma_g/\partial \omega)(d\Sigma_g/d\omega) \)) is in general, of the order of 1, but in pressure maxima, it is much larger than 1. For example, consider a Gaussian function at a radius \( \omega_0 \) and a total width \( 2\sigma \), \( \Sigma_g \propto \exp\left(-\frac{1}{2}(\omega - \omega_0)^2/\sigma^2\right) \). Then, the logarithmic gradient is \( \Sigma_g(\omega - \omega_0)/\sigma^2 \). For narrow rings such that \( \epsilon = 2\sigma/\omega_0 < 1 \), the logarithmic gradient is larger than 1, except in the neighborhood of \( \omega_0 \), where the dust grains are already close to the equilibrium point. Then, because the viscosity coefficient \( \alpha \) is much smaller than 1, the advection timescale of the dust grains is smaller than the diffusion timescale of both the gas and dust, allowing a fast concentration of the dust grains in the gas pressure maxima.

Figure 13 shows the ratio between the diffusion and advection timescales for the best-fit model of the HD 169142 disk. The different lines show these ratios for dust sizes of 1 cm and 1 mm. Because the Stokes number for 1 mm particles is small, the ratio of the dust diffusion to the advection timescales and the ratio of the gas diffusion to the dust advection timescales are the same. Note that the ratios between the timescales are larger than 1 throughout the disk, with local minima at the position of the gas maxima, where the dust grains concentrate. Thus, a fast trapping scenario and the steady-state assumption are feasible in the HD 169142 disk.

Appendix C
Imaging Parameters

Continuum images were created using CASA (v 4.7.0). Table 2 shows the relevant imaging parameters used for each wavelength.

\(^{11}\) CASA, the Common Astronomy Software Applications package, is a software developed to support data processing of radio astronomical telescopes.
Table 2
Parameters of Continuum and Line Images

| Band | $\lambda$ (mm) | $\nu$ (GHz) | Clean Weighting | Original Synthesized Beam Size | rms Noise Image |
|------|----------------|-------------|-----------------|--------------------------------|----------------|
|      |                |             |                 | Major $\times$ Minor; P.A. | Original $\times$ Convolved |
| dust | 7              | 0.87        | 330             | Briggs; robust = 0.5         | 0.19 $\times$ 0.12; 84.97 | 170 | 196 |
| dust | 6              | 1.3         | 230             | Natural weighting            | 0.19 $\times$ 0.13; 63.42 | 100 | 91  |
| dust | 3              | 3.0         | 100             | Briggs; robust = $-1$        | 0.17 $\times$ 0.07; $-92.22$ | 25  | 23  |
| $^{13}$CO | 6           | 1.3         | 220             | Natural weighting            | 0.16 $\times$ 0.16, 0.0 | 3650$^b$ |   |
| $^{12}$CO | 6           | 1.3         | 220             | Natural weighting            | 0.16 $\times$ 0.16, 0.0 | 2961 |   |

Notes.
$^a$ All dust continuum images were finally convolved to the same circular beam of 0.20 arcsec.
$^b$ The noise of the CO maps corresponds to the rms of the zeroth-moment maps.

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