Pair production of neutral Higgs particles in the B-LSSM

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Abstract

Higgs pair production provides a unique handle for measuring the strength of Higgs self interaction and constraining the shape of the Higgs potential. Including radiative corrections to the trilinear couplings of $CP$-even Higgs, we investigate the cross section of the lightest neutral Higgs pair production in gluon fusion at the Large Hadron Collider in the supersymmetric extensions of the standard model. Numerical results indicate that the correction to the cross section is about 11% in the B-LSSM, while is only about 4% in the MSSM. Considering the constraints of the experimental data of the lightest Higgs, we find that the gauge couplings of $U(1)_{B-L}$ and the ratio of the nonzero vacuum expectation values of two singlets also affect strongly the theoretical evaluations on the production cross section in the B-LSSM.

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The discovery of the Higgs boson at the Large Hadron Collider (LHC) \cite{1, 2} is a great triumph of the standard model (SM). Combining updated data \cite{3–5}, one obtains the measured mass of the Higgs as \cite{6}

\[
m_h = 125.25 \pm 0.17 \text{ GeV.} \tag{1}
\]

The precise electroweak observable of Higgs boson set some stringent constraints on parameter space of the SM and its various extensions. In addition, the SM cannot provide natural explanations of some problems, such as the neutrino masses, the hierarchy problem, the Dark Matter (DM) candidates etc. The aforementioned problems can be accommodated in some new physics extensions of the SM, in which the various supersymmetric extensions of the SM are popular. However, the minimal supersymmetric extension of the SM (MSSM) cannot explain the tiny masses of neutrino naturally. To obtain the lightest $CP$-even Higgs with mass around 125 GeV in the MSSM, we should take into account the large radiative corrections from the third generation quark and their super partners because the tree level mass of the lightest $CP$-even Higgs is less than $m_Z$. Meanwhile, masses of other neutral scalar and charged Higgs are much heavier than the mass of lightest Higgs, namely 125 GeV. This is known as the little hierarchy problem in the MSSM \cite{7}. Gauge group of the supersymmetry extension of the SM with local $B − L$ symmetry (B-LSSM) \cite{8, 9} is $U(1)_Y \otimes SU(2)_L \otimes SU(3)_C \otimes U(1)_{B−L}$, where $B$ and $L$ stand for the baryon number and the lepton number, respectively. The B-LSSM provide an elegant explanation for the tiny masses of the left-handed neutrino through seesaw mechanism, and the B-LSSM also alleviates the little hierarchy problem of the MSSM, because the exotic singlet Higgs and right-handed (s)neutrinos \cite{10–17} release additional parameter space from the LEP, Tevatron and LHC constraints. Moreover, the model can also provide more DM candidates than that of the MSSM \cite{18–21}.

Higgs pair production provides a unique handle for measuring the strength of the Higgs self interaction and constraining the shape of the Higgs potential. In the SM, the Higgs
potential is
\[ V = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 , \]
which is completely specified by two parameters \( \mu \) and \( \lambda \). \( \mu \) and \( \lambda \) can be determined from the vacuum expectation value (VEV) of the Higgs field, and the mass of the Higgs boson, but there is no direct measurement beyond that. The next step in understanding the shape of the Higgs potential is to measure the Higgs trilinear coupling which can be probed by the neutral Higgs boson pair production at the LHC. In scenarios of the SM the Higgs pair production at the LHC proceeds by the parton process \( gg \rightarrow hh \) through the heavy quark induced box diagrams and also through the production of an off-shell Higgs which subsequently splits into two on-shell Higgs [22–24]. If radiative corrections of top quark to the amplitude of the parton process \( gg \rightarrow hh \) from box diagrams are determined through other experimental data, the production cross-section depends on Higgs trilinear self-coupling sensitively [25–33]. At the center of mass energy of 14TeV, the theoretical evaluation of the production cross section is about 17 fb at the leading order (LO), and reaches roughly 35 fb after including the next-to-leading order (NLO) QCD correction [34, 35] in the SM. In the MSSM the radiative corrections of bottom quark to the amplitude of the parton process \( gg \rightarrow hh \) of box diagrams may be enhanced by the ratio \( \tan \beta = v_2/v_1 \) between the nonzero VEVs of Higgs fields [36–38].

In the B-LSSM, the tree level prediction for the mass of the lightest \( CP \)-even Higgs can exceed \( m_z \) since the spectrum includes four \( CP \)-even Higgs. Based on the present experimental data, the lower bound on the mass of the next-to-light neutral Higgs can be set as 135 GeV [39–41]. Thus, the pair production of the lightest Higgs receives additional contributions from the parton process \( gg \rightarrow h_i \rightarrow hh \) with \( h_i \) denoting a \( CP \)-even non-standard Higgs [42–56]. Furthermore, the trilinear-coupling of the lightest Higgs is also modified drastically from the mixing between the doublets and singlets together with radiative corrections to the scalar potential.

The paper is organized as follows. In Sec II we briefly present the features of the B-LSSM and calculate the radiative corrections to the trilinear couplings of \( CP \)-even Higgs. In Sec III we calculate pair production cross section of the lightest Higgs at the NLO approximation.
The numerical analysis is performed in Sec IV, and conclusions are summarized in Sec V. The tedious formulae are collected in Appendices.

II. THE B-LSSM

In this section, we briefly introduce the basic properties of the B-LSSM, and then present the radiative corrections to the pole masses and trilinear couplings of CP-even Higgs, respectively. Finally we present a concise discussion of the possible constraints on model parameters imposed by $\bar{B} \to X_s \gamma$, $B^0_s \to \mu^+ \mu^-$ and LHC data. Here we adopt the concrete model described in Refs. [57–60] to proceed with our analysis, where the chiral superfields and their quantum numbers are listed in Table. II In order to break down the local symmetry $U(1)_{B-L}$ spontaneously, two chiral singlet superfields $\tilde{\eta}_1$, $\tilde{\eta}_2$ are introduced. In addition, the couplings among three generations of right-handed neutrinos and the singlet $\tilde{\eta}_1$ provide the see-saw mechanism to produce the tiny masses of left-handed neutrinos.

The corresponding superpotential of the B-LSSM is written as

$$W = W_{MSSM} + Y_{\nu,ij} \hat{L}_i \hat{H}_2 \tilde{\nu}_j^c - \mu' \tilde{\eta}_1 \tilde{\eta}_2 + Y_{x,ij} \tilde{\nu}_c^i \tilde{\eta}_1 \tilde{\nu}_c^j,$$

where $W_{MSSM}$ is the superpotential of the MSSM, and $i, j$ are generation indices. To break the supersymmetry, the soft breaking terms are generally given by

$$L_{soft} = L_{MSSM} + \left[ -M_{BB'} \tilde{\lambda}_{B'} \tilde{\lambda}_B - \frac{1}{2} M_{B'} \tilde{\lambda}_{B'} \tilde{\lambda}_B - B_{\mu'} \tilde{\eta}_1 \tilde{\eta}_2 + T_{ij} H_2 \tilde{\nu}_i \tilde{L}_j \right] + T_{x,ij} \tilde{\eta}_1 \tilde{\nu}_i \tilde{\nu}_j + h.c - m_{\tilde{\nu},ij}^2 (\tilde{\nu}_i^c)^* \tilde{\nu}_j^c - m_{\tilde{\eta}_1}^2 |\tilde{\eta}_1|^2 - m_{\tilde{\eta}_2}^2 |\tilde{\eta}_2|^2,$$

where $L_{MSSM}$ is the soft breaking terms of the MSSM, and $\tilde{\lambda}_B$, $\tilde{\lambda}_{B'}$ represent the gauginos of $U(1)_Y$, $U(1)_{B-L}$ correspondingly.

The local gauge symmetry $SU(2)_L \otimes U(1)_Y \otimes U(1)_{B-L}$ is broken down to the electro-
TABLE I: Chiral superfields and their quantum numbers in the B-LSSM.

| superfields | Spin0 | Spin1/2 | $U(1)_Y \otimes SU(2)_L \otimes SU(3)_C \otimes U(1)_{B-L}$ |
|-------------|-------|---------|--------------------------------------------------|
| $\tilde{Q}$ | $\tilde{Q}$ | $Q$ | $(1/6, 2, 3, 1/6)$ |
| $\tilde{D}$ | $\tilde{c}^c$ | $c^c$ | $(3/6, 1, 3, -1/6)$ |
| $\tilde{U}$ | $\tilde{c}^c$ | $u^c$ | $(-2/3, 1, 3, -1/6)$ |
| $\tilde{L}$ | $\tilde{L}$ | $L$ | $(-1/2, 2, 1, -1/2)$ |
| $\tilde{E}$ | $\tilde{e}^c$ | $e^c$ | $(1, 1, 1, 1/2)$ |
| $\tilde{\nu}$ | $\tilde{\nu}^c$ | $\nu^c$ | $(0, 1, 1, 1/2)$ |
| $\tilde{H}_1$ | $H_1$ | $\tilde{H}_1$ | $(-1/2, 2, 1, 0)$ |
| $\tilde{H}_2$ | $H_2$ | $\tilde{H}_2$ | $(3/2, 2, 1, 0)$ |
| $\tilde{\eta}_1$ | $\eta_1$ | $\tilde{\eta}_1$ | $(0, 1, 1, -1)$ |
| $\tilde{\eta}_2$ | $\eta_2$ | $\tilde{\eta}_2$ | $(0, 1, 1, 1)$ |

magnetic symmetry $U(1)_{em}$ when the Higgs fields receive nonzero VEVs:

$$
\begin{align*}
H_1^1 &= \frac{1}{\sqrt{2}}(v_1 + \phi_d + \text{Im} H_1^1), \\
H_2^2 &= \frac{1}{\sqrt{2}}(v_2 + \phi_u + \text{Im} H_2^2), \\
\tilde{\eta}_1 &= \frac{1}{\sqrt{2}}(u_1 + \phi_{\tilde{\eta}_1} + \text{Im} \tilde{\eta}_1), \\
\tilde{\eta}_2 &= \frac{1}{\sqrt{2}}(u_2 + \phi_{\tilde{\eta}_2} + \text{Im} \tilde{\eta}_2). 
\end{align*}
$$

(5)

where the mixing among $\phi_d$, $\phi_u$, $\phi_{\tilde{\eta}_1}$, $\phi_{\tilde{\eta}_2}$ produces four $CP$-even neutral eigenstates of Higgs, and the mixing among $\text{Im} H_1^1$, $\text{Im} H_2^2$, $\text{Im} \tilde{\eta}_1$, $\text{Im} \tilde{\eta}_2$ produces two neutral Goldstones and two $CP$-odd neutral eigenstates of Higgs, respectively. For convenience, we define $u^2 = u_1^2 + u_2^2$, $v^2 = v_1^2 + v_2^2$, and $\tan \beta' = u_2/u_1$ in analogy to the ratio of two nonzero VEVs of doublets.

The effective potential can be written as

$$
V_{\text{eff}} = V_0 + \Delta V,
$$

(6)

where $V_0$ denotes the scalar potential at the tree level, and $\Delta V$ denotes the one-loop radiative corrections to the effective potential. The concrete expression of the tree level scalar potential
The concrete expression for the one-loop effective potential $\Delta V$ dominated by the contributions from top quark, scalar top quark, bottom quark and scalar bottom quark is

\[
\Delta V = \left( m_{H_1}^2 + \mu^2 \right) |H_1|^2 + \left( m_{H_2}^2 + \mu^2 \right) |H_2|^2 + \left( m_{\eta_1}^2 + \mu^2 \right) |\eta_1|^2 + \left( m_{\eta_2}^2 + \mu^2 \right) |\eta_2|^2
\]

\[
-2B_H H_1^2 H_2^2 - 2B_u \eta_1 \eta_2 + \frac{1}{8} g^2 (|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g_y^2 (|\eta_1|^2 - |\eta_2|^2)^2
\]

\[
+ \frac{1}{2} g_y g_{Y_B} (|H_1|^2 - |H_2|^2)(|\eta_1|^2 - |\eta_2|^2)
\].

(7)

In the above we have adopted the abbreviation $g^2 = g_t^2 + g_s^2 + g_{Y_B}^2$, where $g_s$ denotes the gauge coupling of $SU(2)_L$, $g_t$ denotes the gauge coupling of $U(1)_Y$, $g_{Y_B}$ denotes the gauge coupling of $U(1)_{B-L}$, and $g_{Y_B}$ denotes the mixing coupling between $U(1)_Y$ and $U(1)_{B-L}$ in the covariant derivative, respectively. The radiative corrections to the effective potential is dominated by the contributions from top quark, scalar top quark, bottom quark and scalar bottom quark. The concrete expression for the one-loop effective potential $\Delta V$ reads

\[
\Delta V = -\frac{3}{64\pi^2} m_t^4 \left[ \ln\left( \frac{m_t^2}{\langle Q^2 \rangle} \right) - \frac{3}{2} \right] - \frac{3}{64\pi^2} m_b^4 \left[ \ln\left( \frac{m_b^2}{\langle Q^2 \rangle} \right) - \frac{3}{2} \right]
\]

\[
+ \frac{3}{128\pi^2} \sum_{i=1}^{2} m_i^4 \left[ \ln\left( \frac{m_i^2}{\langle Q^2 \rangle} \right) - \frac{3}{2} \right] + \frac{3}{128\pi^2} \sum_{i=1}^{2} m_i^4 \left[ \ln\left( \frac{m_i^2}{\langle Q^2 \rangle} \right) - \frac{3}{2} \right].
\]

(8)

In the basis $(\phi_d, \phi_u, \phi_{\eta_1}, \phi_{\eta_2})$ the tree level mass squared matrix for $CP$-even neutral Higgs bosons is given by

\[
\left[ m_h^2 \right]^{(0)} = \begin{pmatrix}
\left[ m_h^2 \right]_{\phi\phi} & \left[ m_h^2 \right]_{\phi\eta}
\left[ m_h^2 \right]_{\phi\eta}^T & \left[ m_h^2 \right]_{\eta\eta}
\end{pmatrix},
\]

(9)

where the superscript $T$ denotes the transpose operation, and the explicit form of the $2 \times 2$ submatrices are given in the appendix A.

As stating in Ref. [62], the nonzero VEVs and parameters in the tree level scalar potential are $\overline{\text{MS}}$ renormalized quantities. Neglecting width effects, one acquires the pole masses of neutral $CP$-even Higgs through the pole equation

\[
\text{Det}\left[ m_h^2 - \frac{\partial^2 V_{\text{eff}}}{\partial \phi_i \partial \phi_j} \right]_{\phi_i = v_i} - \Pi_{\phi_i \phi_j}(m_h^2) = 0,
\]

(10)

where $\Pi_{\phi_i \phi_j}(p^2)$ is the self-energy (two-point) functions of the $CP$-even Higgs with external 4-momentum p. Taking the approach adopted in Ref. [62], we rewrite the radiative corrections
FIG. 1: One-loop diagrams which induce the radiative corrections to the lightest Higgs boson mass, where $f = t, b,$ and $S_i = \tilde{t}_i, \tilde{b}_i$ with $i = 1, 2.$

to the mass squared matrix of CP-even Higgs as

$$\frac{\partial^2 V_{\text{eff}}}{\partial \phi_i \partial \phi_j} \bigg|_{\phi_i = \nu_i} + \Pi_{\phi_i \phi_j} (m^2_h)$$

$$\left( \frac{\partial^2 V_{\text{eff}}}{\partial \phi_i \partial \phi_j} \bigg|_{\phi_i = \nu_i} + \Pi_{\phi_i \phi_j} (0) \right) + \left[ \Pi_{\phi_i \phi_j} (m^2_h) - \Pi_{\phi_i \phi_j} (0) \right]$$

$$= \left[ m^2_h \right]_{\phi_i \phi_j} + \Delta \Pi_{\phi_i \phi_j} (m^2_h),$$

(11)

where the dominant first term can be calculated in term of derivatives of the effective potential $V_{\text{eff}},$ and the second term is ultraviolet finite, which only induces contributions suppressed by the small couplings and loop factor.

In order to get the pole mass of the lightest Higgs consistently, we write the explicit expressions for the self energy diagrams. It is well known that the dominant corrections to the lightest Higgs mass originate from bottom, sbottom, top and stop. The relevant diagrams are plotted in Fig. (1) and the corresponding corrections are given by Eq. (11). Choosing the relevant parameters appropriately in the B-LSSM, we find that the particle spectrum contains not only a neutral Higgs with a mass around 125 GeV, but also a next-to-lightest neutral Higgs with a mass of several hundreds of GeV whose main component originates from two singlets.

Including the radiative corrections, the trilinear couplings among CP-even Higgs can be written similarly as

$$C_{ha_hb_c} = \sum_{\alpha, \beta, \gamma} \left[ S_{\alpha \beta \gamma} \frac{\partial^3 V_{\text{eff}}}{\partial \phi_\alpha \partial \phi_\beta \partial \phi_\gamma} \right] \left( (Z_H)_{a\alpha} (Z_H)_{b\beta} (Z_H)_{c\gamma} + (Z_H)_{a\alpha} (Z_H)_{c\beta} (Z_H)_{b\gamma} + (Z_H)_{b\beta} (Z_H)_{c\gamma} (Z_H)_{a\alpha} + (Z_H)_{b\beta} (Z_H)_{a\alpha} (Z_H)_{c\gamma} + \Delta \Lambda_{abc} (p^2_a, p^2_b, p^2_c) \right),$$

(12)
FIG. 2: The Feynman diagrams which induce the radiative corrections to the trilinear couplings of CP-even Higgs, where \( f = t, b, \) and \( S_i = \bar{t}_i, \bar{b}_i \) with \( i = 1, 2. \)

where \( p \) denotes the external 4-momentum of the CP-even Higgs, and the \( 4 \times 4 \) matrix \( Z_H \) is the mixing between \( \phi_d, \phi_u, \phi_{\tilde{\eta}_1}, \phi_{\tilde{\eta}_2} \). In addition, the symmetry factor \( S_{\alpha\beta\gamma} \) is

\[
S_{\alpha\beta\gamma} = \left\{ \begin{array}{ll}
1, & \alpha \neq \beta \neq \gamma, \\
\frac{1}{2}, & \alpha = \beta \neq \gamma, \quad \alpha = \gamma \neq \beta, \quad \alpha \neq \beta = \gamma, \\
\frac{1}{6}, & \alpha = \beta = \gamma.
\end{array} \right.
\]

(13)

The second term in Eq. (12) is obtained through evaluation of the triangle diagrams among the CP-even Higgs. Feynman diagrams of dominant corrections to the trilinear couplings among CP-even Higgs are plotted in Fig. 2 and the concrete expressions of \( \Delta \Lambda_{h, hh} \) are collected in the appendix B.

The updated average experimental data on the branching ratios of \( \bar{B} \to X_s \gamma \) and \( B_s^0 \to \mu^+ \mu^- \) are [63, 64]

\[
Br(\bar{B} \to X_s \gamma) = (3.49 \pm 0.19) \times 10^{-4},
\]

\[
Br(B_s^0 \to \mu^+ \mu^-) = (2.9^{+0.7}_{-0.6}) \times 10^{-9},
\]

(14)

which set stringent constraints on the parameter space of the new physics extensions of the SM. Generally the theoretical evaluation of the branching ratio of \( \bar{B} \to X_s \gamma \) is given by

\[
Br(\bar{B} \to X_s \gamma) = R(C_{7\gamma}(\mu_b) + N(E_\gamma)) ,
\]

(15)

where the overall factor \( R = 2.47 \times 10^{-3} \), and the nonperturbative contribution \( N(E_\gamma) = (3.6 \pm 0.6)10^{-3} \). The Wilson coefficient \( C_{7\gamma}(\mu_b) \) at hadronic scale is

\[
C_{7\gamma}(\mu_b) = C_{7\gamma, SM}(\mu_b) + C_{7\gamma, NP}(\mu_b) .
\]

(16)
Choosing the hadron scale $\mu_b = 2.5$ GeV, one obtains the SM contribution as $C_{7\gamma,SM}(\mu_b) \approx -0.3689$, and the new physics correction to the Wilson coefficient at hadronic scale is

$$C_{7\gamma,NP}(\mu_b) \approx 0.5696C_{7\gamma,NP}(\mu_{EW}) + 0.1107C_{8\gamma,NP}(\mu_{EW}), \quad (17)$$

where $\mu_{EW}$ denotes the electroweak scale. Assuming that the supersymmetry partners of quarks are heavier, we formulate the dominate corrections to the Wilson coefficients at electroweak scale in the limit of large $\tan \beta$ as

$$C_{7\gamma,NP}(\mu_{EW}) = \frac{m_b^2 m_s^2 t_\beta^2}{12m_w^2} \left[ -2I_1 + I_3 - 2I_4 \right] (x_t, x_{H\pm}) + \cdots,$$

$$C_{8\gamma,NP}(\mu_{EW}) = \frac{m_b^2 m_s^2 t_\beta^2}{12m_w^2} \left[ -I_1 + 2I_3 - I_4 \right] (x_t, x_{H\pm}) + \cdots, \quad (18)$$

where the concrete expressions of the functions $I_1, I_3, I_4$ can be found in literature $[65]$. In the limit of large $\tan \beta$ and heavy supersymmetric particles, the leading corrections to the effective lagrangian is given by the Wilson coefficient of electroweak scale $C_{S}(\mu_{EW})$

$$C_{S}(\mu_{EW}) = \frac{m_t^2 m_b m_s t_\beta^2}{4m_w^2 m_{H^+} s_w^2} \left[ 2(Z_H)^{11} C_{h_{H^+}H^-} s_w \frac{\partial}{\partial x_{H\pm}} \varrho_{1,1} ight.$$

$$+ (Z_H)^{11}(Z_H)^{21} \left( \frac{\partial}{\partial x_t} \varrho_{2,1} + x_t \frac{\partial}{\partial x_t} \varrho_{1,1} \right) \left( x_t, x_{H\pm} \right) + \cdots \quad (19)$$

where the definition of $\varrho_{m,n}(x,y)$ can also be found in literature $[65]$. Here $C_{h_{H^+}H^-}$ is the trilinear coupling between the lightest neutral Higgs and charged Higgs pair, and its expression is presented in appendix $[65]$. In the concrete analyses, we should also include the corrections of two-loop Barr-Zee and rainbow type diagrams, respectively.

The experimental constraints on the $Z'$ mass can be extracted from the Drell-Yan production cross-section of fermion-antifermion pairs in the LHC. In the narrow width approximation (NWA), the Drell-Yan production cross section can be simplified as

$$\sigma_{f\bar{f}} \simeq \frac{1}{3} \sum_{q=u,d} \left( \frac{dL_{q\bar{q}}}{dm_{Z'}} \hat{\sigma}(q\bar{q} \rightarrow Z') \right) Br(Z' \rightarrow f\bar{f}) \quad (20)$$

where $dL_{q\bar{q}}/dm_{Z'}$ denotes the parton luminosity, and $\hat{\sigma}(q\bar{q} \rightarrow Z')$ is the peak cross section

$$\hat{\sigma}(q\bar{q} \rightarrow Z') = \frac{\pi}{12} g_B^2 \left[ \left( g_1^{q} \right)^2 + \left( g_2^{q} \right)^2 \right]. \quad (21)$$
Couplings between massive $Z'$ and the SM fermions are generally written as

$$\frac{g_B}{2} Z'_\mu \bar{f} \gamma^\mu \left( g_L^f P_L + g_R^f P_R \right) f,$$

where $g_{V,A}^f = \pm g_L^f + g_R^f$.

Suppose in the final state, there are only SM fermions. The NNLO Drell-Yan cross section can be written as

$$\sigma^{NNLO}_{l^+l^-} \simeq K_{P \text{DF}}^{NNLO} \sigma^{LO}_{l^+l^-},$$

(23)

where the QCD factor $K_{P \text{DF}}^{NNLO}$ originates from the NNLO QCD corrections to the PDF’s, and the LO cross section is

$$\sigma^{LO}_{l^+l^-} = \frac{\pi}{48 s} \left\{ c_u w_u(s, m_{Z'}^2) + c_d w_d(s, m_{Z'}^2) \right\}.$$

(24)

The coefficients $c_u, d$ are defined as

$$c_u = \frac{g_B^2}{2} \left[ \left( g_{V}^u \right)^2 + \left( g_{A}^u \right)^2 \right] Br(Z' \rightarrow l^+l^-),$$

$$c_d = \frac{g_B^2}{2} \left[ \left( g_{V}^d \right)^2 + \left( g_{A}^d \right)^2 \right] Br(Z' \rightarrow l^+l^-),$$

(25)

and $w_q(s, m_{Z'}^2)$ ($q = u, d$) are determined by the parton luminosities $\frac{dL_{q\bar{q}}}{dm_{Z'}^2}$, and thus only depend on the collider energy $\sqrt{s}$ and $m_{Z'}$.

By comparing the experimental limits of the LHC at 7 GeV to the theoretical predictions of $c_u - c_d$ plane, the authors of Ref. [66], obtained the lower bound on $m_{Z'}/g_B \sim 1730$ GeV in the $U(1)_{B-L}$ extension of the SM. This value is approximately equal to lower bound on $m_{Z'}/g_B$ of the sequential standard model (SSM). Although the authors of [67] did not specify a lower bound on $m_{Z'}/g_B$ in the $U(1)_{B-L}$ extension of the SM with the updated LHC data. It is reasonable to assume that the lower bound on $m_{Z'}/g_B \sim 6$ TeV in the $U(1)_{B-L}$ extension is approximately equal to that in the SSM by the LHC data at 13 TeV. Furthermore, CMS data of the channel $pp \rightarrow Z' \rightarrow e^+e^-$ at LHC of 13 TeV give the lower bound on the $Z'$ mass as $m_{Z'} \geq 4.72$ TeV, that of the channel $pp \rightarrow Z' \rightarrow \mu^+\mu^-$ give $m_{Z'} \geq 4.89$ TeV, and the combination of two channel sets $m_{Z'} \geq 5.15$ TeV [68], respectively. When $Z'$ can decay into the superparticles in the supersymmetry $U(1)_{B-L}$ extension of the SM, the increasing of the total width of $Z'$ decreases the lower bound on the mass of $Z'$. 

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So far the most stringent constraint on the $U(1)_{B-L}$ gauge boson parameters originates from LEP2 results actually, which implies $M_{Z'}/g_B \geq 6$ TeV at 95% C.L. This bound is based on the assumption that $Z'$ dominantly decays to SM fermions and is derived from the limit on the low energy four-fermion contact interactions induced by $Z'$ exchanged diagram. In the literature [69], the authors also take this lower bound for $Z'$ mass to investigate the invisible decay of $Z'$ in the B-LSSM.

### III. MATRIX ELEMENTS AND CROSS SECTION

In this section, we analyze the pair production cross section of the lightest Higgs $\sigma(pp \rightarrow gg \rightarrow hh)$ at $\sqrt{s} = 14$ TeV using the analytical expressions for one-loop amplitudes of $g(p_1)g(p_2) \rightarrow h(p_3)h(p_4)$ in supersymmetric extensions of the SM. Feynman diagrams contributing to the $gg \rightarrow hh$ process at LO are summarized in Fig.3. Some triangular diagrams contain information of trilinear couplings among $CP$-even Higgs. As masses of scalar quarks all exceed TeV scale, the corrections originating from the scalar quarks to the cross section of the Higgs pair production can be ignored safely. Assuming that the Yukawa couplings of top and bottom quarks are determined through other experimental measurements, one can analyze the correlations among the mass of the next-to-lightest neutral Higgs $m_{h_2}$ and the trilinear couplings $C_{hhh}$, $C_{h2hh}$ from the experimental data on the pair production cross section. Here $h_2$ denotes the next-to-lightest neutral Higgs, $C_{hhh}$ denotes the trilinear coupling among the lightest neutral Higgs, and $C_{h2hh}$ denotes the coupling among one next-to-lightest neutral Higgs and two lightest neutral Higgs, respectively. To ensure the correctness of our calculation, we recover the SM results presented in Ref. [36] and the MSSM results presented in Ref. [42]. In order to calculate the polarization cross section, we introduce the explicit polarization vectors of the helicities ($\lambda_1, \lambda_2$) for gluons as follows

\[
\epsilon_1^\mu(p_1, \lambda_1 = \pm 1) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0), \\
\epsilon_2^\mu(p_2, \lambda_2 = \pm 1) = \frac{1}{\sqrt{2}}(0, \pm 1, -i, 0).
\]  

(26)
FIG. 3: Leading order Feynman diagrams for Higgs pair production in the B-LSSM at the LHC.

$h_i$ are CP-even Higgs, $f$ are top and bottom quarks and $S_{i(j)}$ are stop and sbottom quarks.
In the center-of-mass frame the momenta can be written in terms of the beam energy \( E \) and scattering angle \( \theta \)

\[
\begin{align*}
    p_1^\mu &= E(1, 0, 0, -1) , \\
p_2^\mu &= E(1, 0, 0, 1) , \\
p_3^\mu &= E(1, -\sqrt{1 - \frac{4m_h^2}{s}} \sin \theta, 0, -\sqrt{1 - \frac{4m_h^2}{s}} \cos \theta) , \\
p_4^\mu &= E(1, \sqrt{1 - \frac{4m_h^2}{s}} \sin \theta, 0, \sqrt{1 - \frac{4m_h^2}{s}} \cos \theta) .
\end{align*}
\]

(27)

where \( p_{1,2} \) are the incoming momenta of two gluons, and \( p_{3,4} \) are the outgoing momenta of two lightest Higgs, respectively. In addition, the Mandelstam variables are defined as

\[
\begin{align*}
    \hat{s} &= (p_1 + p_2)^2 = (p_3 + p_4)^2 , \\
    \hat{t} &= (p_1 - p_3)^2 = (p_2 - p_4)^2 , \\
    \hat{u} &= (p_1 - p_4)^2 = (p_2 - p_3)^2 .
\end{align*}
\]

(28)

The cross section of the subprocess \( gg \rightarrow hh \) at LO can be written as

\[
\hat{\sigma} = \int_{\hat{t}_{\text{min}}}^{\hat{t}_{\text{max}}} d\hat{t} \frac{1}{4096 \pi s^2} \left| \sum_n M^{(n)}_{++} \right|^2 + \left| \sum_n M^{(n)}_{+-} \right|^2 + \left| \sum_n M^{(n)}_{-+} \right|^2 + \left| \sum_n M^{(n)}_{--} \right|^2 ,
\]

(29)

where

\[
\begin{align*}
    \hat{t}_{\text{min}} &= (m_h^2 - \frac{\hat{s}}{2}) - \frac{1}{2} \sqrt{1 - \frac{4m_h^2}{s}} \hat{s} , \\
    \hat{t}_{\text{max}} &= (m_h^2 - \frac{\hat{s}}{2}) + \frac{1}{2} \sqrt{1 - \frac{4m_h^2}{s}} \hat{s} .
\end{align*}
\]

(30)

Additionally \( M^{(n)}_{\lambda_1 \lambda_2} \) is the helicity amplitude for pair production of the lightest neutral Higgs boson. \( CP \) conservation implies that \( M_{++} = M_{--} , \quad M_{+-} = M_{-+} \), where the subscripts \( \pm \) denote two independent helicities of the initial gluons. The nonzero amplitudes from the Feynman diagrams in Fig. 3 are summarized in Eq. (D1).

The differential cross section of \( pp \rightarrow gg \rightarrow hh \) is

\[
\frac{d\sigma}{d\sqrt{s}} = \frac{2\sqrt{\hat{s}}}{s} \hat{\sigma}(gg - hh) \frac{dL_{gg}}{d\tau} ,
\]

(31)
where $\tau = \hat{s}/s$, and

$$ \frac{dL_{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x} f_g(x, \mu_F) f_g(\frac{x}{\tau}, \mu_F). \quad (32) $$

Here $f_g(x, \mu_F)$ is the parton distribution function of gluons, $x$ is the relative momentum of the radiated gluon, and $\mu_F$ is the factorization scale, respectively.

The total cross section for the lightest neutral Higgs pair production through $gg$ in $pp$ collision can be written as

$$ \sigma_{LO}(pp \rightarrow gg \rightarrow hh) = \int_{\tau_0}^1 d\tau \hat{\sigma}(gg \rightarrow hh) \frac{dL_{gg}}{d\tau}, \quad (33) $$

at the LO approximation. Here $\tau_0 = (2m_h)^2/s$, and the loop integrals are evaluated by the package LoopTools [70].

It is well known that the QCD corrections enhance the theoretical evaluation of the cross section of the lightest Higgs pair production drastically. The NLO QCD corrections [34, 35] to the lightest Higgs pair production via gluon fusion have been computed in the heavy-top limit (HTL) [34]. The NLO results for the gluon-fusion cross section are summarized generally as [34]

$$ \sigma_{NLO} = \sigma_{LO} + \Delta\sigma_{virt} + \Delta\sigma_{gg} + \Delta\sigma_{gq} + \Delta\sigma_{q\bar{q}}, \quad (34) $$

where the LO contribution to the cross section is given by Eq.(33). Other pieces are presented as

$$ \Delta\sigma_{virt} = \left(\frac{\alpha_s(\mu_F)}{\pi}\right) \int_{\tau_0}^1 d\tau \frac{dL_{gg}}{d\tau} \hat{\sigma}_{LO}(\hat{s} = \tau s) C_{virt}(\hat{s}), $$

$$ \Delta\sigma_{ij} = \left(\frac{\alpha_s(\mu_R)}{\pi}\right) \int_{\tau_0}^1 d\tau \frac{dL_{ij}}{d\tau} \int_{\tau_0}^1 \frac{dz}{z} \hat{\sigma}_{LO}(\hat{s} = z\tau s) C_{ij}(\hat{s}, z). \quad (35) $$

where $\frac{dL_{ij}}{d\tau}$ ($i, j = g, q, \bar{q}$) denote the parton-parton luminosities which are defined analogously to $\frac{dL_{gg}}{d\tau}$ in Eq.(32). $q(x, \mu_F)$ is the quark parton distribution function, and $z$ is the ratio between momentum of the radiated quark and that of the parent proton. Furthermore the concrete expressions for $C_{ij}(\hat{s}, z)$ ($ij = gg, gq, q\bar{q}$) are presented in appendix D.
IV. NUMERICAL RESULTS

In the following we present some numerical results of the lightest Higgs pair production cross section for the process $pp \to gg \to hh$. The relevant SM inputs are $m_t = 172.76$ GeV, $m_b = 4.18$ GeV, $m_z = 90.19$ GeV, $m_w = 80.385$ GeV and $\alpha_{em}(m_z) = 1/128.9$, respectively. Choosing the renormalization scale $\mu_R$ as the invariant mass of the lightest Higgs pair, we adopt the CTEQ5 [71] to generate the parton distribution function with the factorization scale $\mu_F$. Furthermore the collision energy of the LHC is fixed to be 14 TeV. Taking $m_h = 125$ GeV, we find that the theoretical prediction for the Higgs pair production cross section in the SM is approximately 33.1 fb, which is consistent with result in [72]. To be consistent with the updated experimental data, we choose $M_{Z'} = 5.2$ TeV in our numerical analysis. Furthermore the result of Refs. [73, 74] indicates a lower bound on the ratio between the $Z'$ mass and its gauge coupling as $M_{Z'}/g_{\mu} \geq 6$ TeV at 99% CL. Under the above assumption on the mass of heavy neutral vector boson, the scope of $g_{\mu}$ is limited to $0 < g_{\mu} \leq 0.8$. In general, the Yukawa coupling $Y_b = \sqrt{2} m_b/(v c_\beta)$ is smaller than one, so the parameter $\tan \beta$ should be approximatively smaller than 40, and the large $\tan \beta$ has been excluded by the experimental data on $Br(\bar{B} \to X_s \gamma)$ and $Br(B^0_s \to \mu^+\mu^-)$ [65]. Additionally LHC experimental data [58] constrains the parameter $\tan \beta' < 1.5$. Considering the constraints of the experiments [6], we appropriately choose $M_{B_B} = M_{B_{B'}} = 600$ GeV, $\mu' = 800$ GeV, and $m_{\tilde{Q}} = m_{\tilde{U}} = m_{\tilde{D}} = \text{diag}(2, 2, 1.8)$ TeV, respectively.

Actually a natural configuration of the B-LSSM includes another relatively light $CP$-even Higgs $h_2$, with mass $m_{h_2} \geq 135$ GeV besides the lightest Higgs state with mass around 125 GeV. This fact was exploited in Refs. [39–41] to explain potential Run I signal for another Higgs bosons, such as $h_2 \to zz^* \to 4\ell$ [75], $h_2 \to rr$ [76, 77] and $h_2 \to \bar{z}r$ [78] decay modes. In the MSSM, two $CP$-even neutral Higgs are obtained through the mixing between the real neutral components of two $SU(2)_L$ doublets. It is well known that those radiative corrections from the third generation quarks and scalar quarks drastically enhance the theoretical prediction on the mass of the lightest Higgs. In the B-LSSM, four $CP$-even neutral Higgs are given through the mixing among the real neutral components of two $SU(2)_L$ doublets and two singlets. For this reason, there is not an upper limit on the mass
of the lightest Higgs at tree level.

In the numerical analysis we choose those parameters so that the corresponding theoretical prediction of mass of the lightest Higgs coincides with the experimental data within 3 standard deviations. In the MSSM, the ratio \( \tan \beta \) between two doublet nonzero VEVs and the soft breaking parameters \( B_\mu, A_t, \) and \( A_b \) strongly affect the theoretical predictions on the masses of \( CP \)-even Higgs together with their mixing. In the B-LSSM, the ratio \( \tan \beta' \) between two singlet nonzero VEVs and additional gauge couplings \( g_{YB} \) and \( g_B \) also affect the theoretical predictions on the masses of \( CP \)-even Higgs besides those aforementioned in the MSSM. The present experimental data of the lightest Higgs mass sets a strong constraint on the parameter space of various extensions of the SM. In order to fit the experimental data of the lightest Higgs, we assume \( B_\mu = 0.5 \text{ TeV}^2, \mu = 800 \text{ GeV}, \) and \( A_t = A_b = 0.6 \text{ TeV}, \) respectively in the MSSM. The lightest Higgs around 125 GeV is mainly composed by component originating from the real part of neutral \( \phi_u, \) and little component originating from the real part of neutral \( \phi_d. \) Under our assumption on the parameter space of the MSSM, we plot the mass of the next-to-lightest (heaviest) \( CP \)-even Higgs and square of absolute value of the mixing elements versus \( \tan \beta \) in Fig.4. The numerical results indicate that the next-to-lightest (heaviest) \( CP \)-even Higgs is given the mass around 1 TeV, and composed mainly by the component originating from the real part of neutral \( \phi_u. \) In the B-LSSM, the lightest Higgs around 125 GeV is also composed dominantly by component originating from the real part of neutral \( \phi_u, \) and little component originating from the real part of neutral \( \phi_d, \) together with that of two singlets \( \phi_{\tilde{\eta}_1}, \phi_{\tilde{\eta}_2}. \) To fit the lightest Higgs with mass around 125 GeV, we take \( B_\mu = B_\mu' = 0.5 \text{ TeV}^2, \mu = 600 \text{ GeV}, \) \( A_t = A_b = 1.6 \text{ TeV}, \) \(-g_{YB} = g_B = 0.4, \mu' = 800 \text{ GeV, and tan } \beta' = 1.2, \) respectively. Adopting the assumption above on the parameter space of the B-LSSM, we plot the mass of the next-to-lightest \( CP \)-even Higgs and square of absolute value of the mixing elements versus \( \tan \beta \) in Fig.5. The numerical results indicate that the next-to-lightest \( CP \)-even Higgs obtains the mass around 180 GeV, and is composed mainly by the components originating from the real part of two singlets \( \phi_{\tilde{\eta}_1}, \phi_{\tilde{\eta}_2}. \)

Besides enhancement of the theoretical prediction for the mass of the lightest \( CP \)-even Higgs, the radiative corrections also modify the trilinear couplings among Higgs drastically.
FIG. 4: The mass of the next-to-lightest $CP$-even Higgs and square of absolute value of the mixing elements versus $\tan \beta$ in the MSSM.

FIG. 5: The mass of the next-to-lightest $CP$-even Higgs and square of absolute value of the mixing elements versus $\tan \beta$ in the B-LSSM.

Taking $B_\mu = 0.5$ TeV, $\mu = 800$ GeV, and $A_t = A_b = 0.6$ TeV in the MSSM, we present the relative radiative corrections to trilinear coupling of the lightest Higgs and that between the lightest Higgs and the next-to-lightest Higgs versus $\tan \beta$ in Fig. 6, respectively. The radiative contributions reduce trilinear coupling of the lightest Higgs about 16%, and reduce that between the lightest Higgs and the next-to-lightest Higgs 10%, separately. Similarly assuming $B_\mu = B_\mu' = 0.5$ TeV, $\mu = 600$ GeV, $\mu' = 800$ GeV, $A_t = A_b = 1.6$ TeV, $-g_{Y B} = g_B = 0.4$, and $\tan \beta' = 1.2$ in the B-LSSM, we draw the relative radiative corrections to trilinear coupling of the lightest Higgs and that between the lightest Higgs and the next-to-lightest Higgs versus $\tan \beta$ in Fig. 7, respectively. Radiative contributions modify trilinear coupling of the lightest Higgs about 30%, and decrease that among the lightest Higgs and the next-to-lightest Higgs 18%, separately.

Including the NLO QCD corrections, we present the cross section of the lightest Higgs pair production as $\sqrt{s} = 14$ TeV at the LHC. In order to present our results transparently, we
FIG. 6: In the MSSM, the red solid and blue dashed lines represent \( \chi = \frac{C_{hhh} - C(0)_{hhh}}{C(0)_{hhh}} \) and \( \chi = \frac{C_{hzhhh} - C(0)_{hzhhh}}{C(0)_{hzhhh}} \) versus \( \tan \beta \), respectively. \( C(0)_{hzhhh} \) denotes the CP-even Higgs trilinear couplings at the tree level. \( C_{hzhhh} \) denotes the CP-even Higgs trilinear couplings including radiative corrections.

FIG. 7: In the B-LSSM, the red solid and blue dashed lines represent \( \chi = \frac{C_{hhh} - C(0)_{hhh}}{C(0)_{hhh}} \) and \( \chi = \frac{C_{hzhhh} - C(0)_{hzhhh}}{C(0)_{hzhhh}} \) versus \( \tan \beta \), respectively. \( C(0)_{hzhhh} \) denotes the CP-even Higgs trilinear couplings at the tree level. \( C_{hzhhh} \) denotes the CP-even Higgs trilinear couplings including radiative corrections.

separately plot the differential cross section according to resonant, non-resonant, interference and total contribution. At the LO, the differential cross section is

\[
\frac{d\sigma}{d\sqrt{s}} = \frac{2\sqrt{s}}{s} \left( \hat{\sigma}_{res} + \hat{\sigma}_{nr} + \hat{\sigma}_{int} \right) \frac{dL_{gg}}{d\tau},
\]  

(36)
FIG. 8: The resonant, non-resonant, interference, and the total differential cross section versus $\sqrt{s}$ in the MSSM. Where the blue dashed line represents the theoretical prediction on the differential cross section including the radiative corrections, while the red solid line does not.

with

$$\hat{\sigma}_{\text{res}} = \frac{1}{4096\pi^2} \int_{t_{\text{min}}}^{t_{\text{max}}} dt \frac{|M_{\text{res}}|^2}{s^2},$$

$$\hat{\sigma}_{\text{nr}} = \frac{1}{4096\pi^2} \int_{t_{\text{min}}}^{t_{\text{max}}} dt \frac{|M_{\text{nr}}|^2}{s^2},$$

$$\hat{\sigma}_{\text{int}} = \frac{1}{4096\pi^2} \int_{t_{\text{min}}}^{t_{\text{max}}} dt \frac{M_{\text{int}}}{s^2}. \quad (37)$$

Here

$$| M_{\text{res}} |^2 = 2(M_{++}^{(1)} + M_{++}^{(5+6)})^2,$$

$$| M_{\text{nr}} |^2 = 2\left(\sum_n |M_{++}^{(n)}|^2 - (M_{++}^{(1)} + M_{++}^{(5+6)})^2\right) + \left|\sum_n M_{++}^{(n)}\right|^2,$$

$$M_{\text{int}} = 2Re(2(M_{++}^{(1)} + M_{++}^{(5+6)}) * 2\left(\sum_n M_{++}^{(n)} - (M_{++}^{(1)} + M_{++}^{(5+6)})^*\right). \quad (38)$$
Actually, the resonant amplitude $M_{res}$ originates from the diagrams (1), (5), (6) in Fig.3 where the contributions of last two diagrams are suppressed by the TeV masses of squarks. In other words, the resonant amplitude $M_{res}$ strongly depends on the trilinear couplings among $CP$-even Higgs. The non-resonant amplitude $M_{nr}$ includes all other Feynman diagrams, and the interference amplitude $M_{int}$ corresponds to the interference between resonant and non-resonant amplitude which also depends on the aforementioned trilinear couplings of Higgs.

At the NLO approximation, the QCD corrections should be allotted among three pieces appropriately. Choosing $\tan \beta = 24$, $B_\mu = 0.5 \text{ TeV}^2$, $\mu = 800 \text{ GeV}$, and $A_t = A_b = 0.6 \text{ TeV}$ in the MSSM, we present the resonant, non-resonant, interference and total contributions to the differential cross sections versus the available centre-of-mass energy $\sqrt{s}$ of the parton-parton in Fig.8(1), Fig.8(2), Fig.8(3), and Fig.8(4), respectively. The numerical results indicate that the radiative corrections to the trilinear couplings of $CP$-even Higgs reduce the absolute values of the resonant and interference contributions of the differential cross sections about 20% as the centre-of-mass energy of the parton-parton $\sqrt{s} \leq 450 \text{ GeV}$, and do not affect the non-resonant piece of the differential cross sections because the non-resonant contribution does not depend on the trilinear couplings of the $CP$-even Higgs.

Taking $\tan \beta = 25$, $\tan \beta' = 1.2$, $g_B = -g_{YB} = 0.4$, $B_\mu = B_\mu' = 0.5 \text{ TeV}^2$, $\mu = 800 \text{ GeV}$, and $A_t = A_b = 1.6 \text{ TeV}$ in the B-LSSM, we present the resonant, non-resonant, interference and total contributions to the differential cross sections versus the available centre-of-mass energy $\sqrt{s}$ of the parton-parton in Fig.9(1), Fig.9(2), Fig.9(3), and Fig.9(4), respectively. The numerical results indicate that the radiative corrections to the trilinear couplings of $CP$-even Higgs reduce the absolute values of the resonant and interference pieces of the differential cross sections about 40% as the centre-of-mass energy of the parton-parton $\sqrt{s} \leq 450 \text{ GeV}$, and do not affect the non-resonant piece of the differential cross sections.

At the NLO approximation, the resonant contribution is smaller than the non-resonant contribution, thus the dominant contributions originate from the non-resonant sector and the destructive interference between the resonant and non-resonant sectors. Including the radiative corrections to the trilinear couplings of $CP$-even Higgs, the non-resonant contribution remains invariantly, the resonant contribution decreases slightly and the interference contribution decreases obviously. Therefore, the correction to the cross section originates mainly
FIG. 9: The resonant, non-resonant, interference, and the total differential cross section versus $\sqrt{s}$ in the B-LSSM. Where the blue dashed line represents the theoretical prediction on the differential cross section including the radiative corrections, while the red solid line does not.

from the interference sector which depends on the trilinear couplings of Higgs sensitively. In addition, the radiative corrections to the $CP$-even Higgs trilinear couplings enhance the cross section.

With the above assumptions on the parameter space of the MSSM and B-LSSM, the relative correction to the differential cross section from the radiative corrections to the trilinear couplings of $CP$-even Higgs in the MSSM is smaller than that in the B-LSSM because the radiative correction to the trilinear couplings of $CP$-even Higgs in the MSSM is significantly smaller than that in the B-LSSM.

Furthermore, we present the differential cross section $\frac{d\sigma}{d\sqrt{s}}$ versus $\sqrt{s}$ in Fig. 10 where the dotted line represents the theoretical evaluation of the SM, the blue dashed line represents that of the MSSM, and the red solid line represents that of the B-LSSM, respectively. Along with the increasing of $\sqrt{s}$ the differential cross section increases as $250 \text{ GeV} < \sqrt{s} <
FIG. 10: The differential cross section \( \frac{d\sigma}{d\sqrt{s}} \) versus \( \sqrt{s} \) in the SM (dotted), MSSM (blue dashed) and B-LSSM (red solid).

400 GeV, and decreases steeply as \( \sqrt{s} > 400 \) GeV. Actually, the peak around \( \sqrt{s} \sim 400 \) GeV is caused by the interference effect of the resonant and non-resonant sectors. Choosing \( m_h = 125 \) GeV, we find that the theoretical prediction on the lightest Higgs pair production cross section is about 33.1 fb in the SM, 37.9 fb in the B-LSSM and 40.5 fb in the MSSM, respectively. The theoretical predictions of Higgs pair production in the B-LSSM and that in the MSSM differ significantly from that in the SM.

Choosing \( \sqrt{s} = 400 \) GeV, \( \tan\beta = 24, B_\mu = 0.5 \) TeV\(^2\), \( \mu = 800 \) GeV, and \( A_t = A_b = 0.6 \) TeV in the MSSM, we present the angle distribution of the differential cross section

FIG. 11: The differential cross section \( \frac{d\sigma}{d\theta} \) versus \( \frac{\theta}{\pi} \) in the MSSM and B-LSSM. The red solid line represents the theoretical prediction on the differential cross section at NLO. The blue dashed line represents the theoretical prediction on the differential cross section including the radiative corrections.
in Fig.11 (1). Similarly taking $\tan \beta = 25$, $\tan \beta' = 1.2$, $g_B = -g_{YB} = 0.4$, $B_\mu = B_{\mu'} = 0.5$ TeV$^2$, $\mu = 800$ GeV, and $A_t = A_b = 1.6$ TeV in the B-LSSM, we present the angle distribution of the differential cross section in Fig.11 (2). Obviously, when $\theta = \frac{\pi}{2}$, the differential cross section reaches its maximum, and when $\theta = 0$ and $\pi$, the differential cross section reaches its minimum.

In order to satisfy the experimental constraints mentioned above, we choose $B_\mu = 0.5$ TeV$^2$, $\mu = 800$ GeV and $A_t = A_b = 0.6$ TeV in the MSSM, and choose $\tan \beta' = 1.2$, $g_B = 0.4$, $g_{YB} = -0.4$, $B_\mu = B_{\mu'} = 0.5$ TeV$^2$, $\mu = 600$ GeV and $A_t = A_b = 1.6$ TeV in the B-LSSM, respectively. With those assumptions on the parameter space, we plot the total cross section of the MSSM versus $\tan \beta$ in Fig.12 (1) and that of the B-LSSM versus $\tan \beta$ in Fig.12 (2), respectively. With the increasing of $\tan \beta$, the total cross section decreases mildly. In the MSSM, the radiative correction to the theoretical prediction of the total cross section is about 4%, while in the B-LSSM the radiative correction exceeds 11% roughly.

Finally we study the influence on the total cross section of the parameters $g_B$, $g_{YB}$ and $\tan \beta'$ in the B-LSSM. Taking $\tan \beta = 25$, $\tan \beta' = 1.2$, $g_{YB} = -0.4$, $B_\mu = B_{\mu'} = 0.5$ TeV$^2$, $\mu = 800$ GeV, and $A_t = A_b = 1.6$ TeV in the B-LSSM, we draw total cross section versus the gauge coupling $g_B$ in Fig.13 (1). With increasing of the coupling $g_B$, the evaluation of total cross section decreases quickly. Similarly selecting $\tan \beta = 25$, $\tan \beta' = 1.2$, $g_B = 0.4$, $g_{YB} = -0.4$, $B_\mu = B_{\mu'} = 0.5$ TeV$^2$, $\mu = 800$ GeV, and $A_t = A_b = 1.6$ TeV in the B-LSSM, we present the angle distribution of the differential cross section in Fig.13 (2).
FIG. 13: The total cross section $\sigma$ versus $g_B$, $g_{YB}$ and $\tan\beta'$, respectively. Where the red solid line represents the theoretical prediction on the total cross section at NLO, the blue dashed line represents the theoretical prediction on the total cross section including the radiative corrections.

$B_\nu = B_\nu' = 0.5$ TeV$^2$, $\mu = 800$ GeV, and $A_t = A_b = 1.6$ TeV, we draw total cross section versus the gauge coupling $g_{YB}$ in Fig. 13 (2). With the increasing of the coupling $g_{YB}$, the total cross section decreases mildly. In Fig. 13 (3), we plot total cross section changing with $\tan\beta'$ as $\tan\beta = 25$, $g_{YB} = -0.4$, $g_B = 0.4$, $B_\nu = B_\nu' = 0.5$ TeV$^2$, $\mu = 800$ GeV, and
$A_t = A_b = 1.6$ TeV. With increasing of $\tan \beta'$, the evaluation of total cross section increases steeply.

V. SUMMARY

In this paper, we discuss the pair production of the lightest neutral Higgs in supersymmetric extensions of the SM. The lightest neutral Higgs pair is produced dominantly in $pp \rightarrow gg \rightarrow hh$ through the loop-induced gluon fusion mechanism. We analyze theoretical evaluations of the lightest neutral Higgs pair production cross section, and investigate the radiative corrections to the trilinear couplings of $CP$-even Higgs. We find that the relatively radiative corrections to the trilinear coupling of the lightest neutral Higgs is about $-30\%$ in the B-LSSM and $-16\%$ in the MSSM. At the NLO approximation, the dominant contributions to the cross section originate from the non-resonant sector and the destructive interference between the resonant and non-resonant sectors. The radiative correction to the trilinear couplings of $CP$-even Higgs modifies the theoretical predictions on the cross section. In the B-LSSM, the relative correction to the cross section is about $11\%$, while in the MSSM, the relative correction to the cross section is only about $4\%$. Furthermore, we also discuss the effect of some new parameters on the cross section in the B-LSSM, such as $g_B, g_{\gamma B}$ and $\tan \beta'$. We find that the theoretical prediction for the production cross section of $pp \rightarrow gg \rightarrow hh$ depends on these parameters sensitively.

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Appendix A: $2 \times 2$ submatrices

The $2 \times 2$ submatrices in Eq. (9) are formulated respectively as

\[
\begin{bmatrix}
  m_h^2 & \frac{1}{4}g^2v^2c_\beta + \text{Re}(B_\mu)t_\beta, \\
  \frac{1}{4}g^2v^2s_\beta c_\beta - \text{Re}(B_\mu), & \frac{1}{4}g^2v^2s_\beta^2 + \text{Re}(B_\mu)/t_\beta
\end{bmatrix},
\]

\[
\begin{bmatrix}
  m_h^2 & \frac{1}{2}g_B g_y u v \begin{pmatrix} c_\beta c_\beta', & -c_\beta s_\beta' \\ c_\beta' s_\beta, & s_\beta' s_\beta \end{pmatrix}, \\
  m_h^2 & \begin{pmatrix} g_B^2 u^2 c_\beta' + \text{Re}(B_\mu')t_\beta', & -g_B^2 u^2 s_\beta c_\beta' - \text{Re}(B_\mu') \\ -g_B^2 u^2 s_\beta' c_\beta' - \text{Re}(B_\mu'), & g_B^2 u^2 s_\beta^2 + \text{Re}(B_\mu')/t_\beta' \end{pmatrix}
\end{bmatrix},
\]

(A1)

here abbreviations are $c_\beta = \cos \beta$, $s_\beta = \sin \beta$, $t_\beta = \tan \beta$, $c_\beta' = \cos \beta'$, $s_\beta' = \sin \beta'$ and $t_{\beta'} = \tan \beta'$.

Appendix B: The radiative corrections $\Delta \Pi_{hh}^{(1)}$ and $\Delta \Lambda_{abc}(p_a^2, p_b^2, p_c^2)$

\[
\Delta \Pi_{hh}^{(1)} = \frac{-6C_{hhf}^2}{(4\pi)^2}[(4m_f^2 - m_h^2)B(m_f, m_f, m_h) - 4m_f^2 \ln \frac{m_f^2}{Q^2}],
\]

\[
\Delta \Pi_{hh}^{(2)} = \frac{3C_{hs}^2}{(4\pi)^2} [B(m_s, m_s, m_h) - B(m_s, m_s, 0)],
\]

(B1)

where $C_{XYZ}$ is the coupling constant among the fields labeled by subscripts $X$, $Y$, $Z$, and the function $B$ is defined as

\[
B(m_1, m_2, m_3) = \int_0^1 dx \ln \frac{(1 - x)m_1^2 + xm_2^2 + x(x - 1)m_3^2}{Q^2}.
\]

(B2)

\[
\Delta \Lambda_{hh}(p_a^2, p_b^2, p_c^2) = \sum_{a=1}^3 \Delta \Lambda_{hh}^{(a)}
\]

(B3)

with

\[
\Delta \Lambda_{hh}^{(1)} = -\frac{12m_fC_{hf}C_{hf}^2}{(4\pi)^2} \left[ B(m_f, m_f, m_h) + 2B(m_f, m_f, m_h) \\
-3B(m_f, m_f, 0) + (4m_f^2 - \frac{m_h^2}{2} - m_h^2)B_1(m_f, m_f, m_f, m_h, m_h) \\
-4m_f^2B_1(m_f, m_f, m_f, 0, 0) \right],
\]

(B4)
with

$$\Delta \Lambda_{h,hh}^{(2)} = \frac{-3C_{h_{11}S_{11}}C_{h_{12}S_{12}}}{(4\pi)^2} \frac{C_{h_{13}S_{13}}C_{h_{14}S_{14}}}{(4\pi)^2} \left[ B_1(m_s, m_s, m_s, m_h, m_h) - B_1(m_s, m_s, m_s, 0, 0) \right],$$

and

$$\Delta \Lambda_{h,hh}^{(3)} = \frac{3C_{h_{11}S_{11}}C_{h_{12}S_{12}}}{(4\pi)^2} \left[ B(m_s, m_s, m_h) - B(m_s, m_s, 0) \right].$$

Where the function $B_1$ is defined as

$$B_1(m_1, m_2, m_3, m_4, m_5) = \int_0^1 dx \int_0^{1-x} dy [(1 - x - y)m_1^2 + x m_2^2 + y m_3^2 + x(x - 1)m_4^2 + y(x + y - 1)m_5^2]^{-1}.$$  \hspace{1cm} (B7)

**Appendix C: Some couplings**

The trilinear couplings between the lightest neutral Higgs and charged Higgs is written as

$$C_{h_{11}h_{12}h_{13}} = \frac{i}{4} \left[ 2(c_\beta^2 - s_\beta^2) (g_t g_{YV} + g_{YV} g_B) \left[ -\nu_\eta(Z_H)_{14} + \nu_\eta(Z_H)_{13} \right] 
- (Z_H)_{11} \left[ (g_1^2 + g_{YB}^2)(s_\beta^2 - c_\beta^2) \nu_d + g_{YB}^2 \nu_d - 2g_{YB}^2 \nu_d s_\beta c_\beta \right] 
+ (Z_H)_{12} \left[ (g_1^2 + g_{YB}^2)(s_\beta^2 - c_\beta^2) \nu_u - g_{YB}^2 \nu_u + 2g_{YB}^2 \nu_u s_\beta c_\beta \right] \right].$$

**C1**

The couplings between the neutral gauge boson $Z'$ and the SM fermions are

$$g^e_L = \frac{1}{2g_B} \left\{ \left[ (g_t + g_{YV}) s_W - g_{YB} c_W \right] s'_W + (g_{YB} + g_B) c'_W \right\},$$

$$g^e_R = -\frac{1}{2g_B} \left\{ \left[ (2g_t + g_{YV}) s_W s'_W + (2g_{YB} + g_B) c'_W \right] \right\},$$

$$g^u_L = \frac{1}{6g_B} \left\{ \left[ (g_t + g_{YV}) s_W - 3g_{YB} c_W \right] s'_W + (g_{YB} + g_B) c'_W \right\},$$

$$g^u_R = -\frac{1}{6g_B} \left\{ \left[ (4g_t + g_{YV}) s_W s'_W + (4g_{YB} + g_B) c'_W \right] \right\},$$

$$g^d_L = -\frac{1}{6g_B} \left\{ \left[ (g_t + g_{YV}) s_W + 3g_{YB} c_W \right] s'_W + (g_{YB} + g_B) c'_W \right\},$$

$$g^d_R = \frac{1}{6g_B} \left\{ \left[ (2g_t - g_{YV}) s_W s'_W + (2g_{YB} - g_B) c'_W \right] \right\}. \hspace{1cm} (C2)$$
The abbreviations $c_w = \cos \theta_w$, $s_w = \sin \theta_w$, $c'_{w} = \cos \theta'_w$, $s'_{w} = \sin \theta'_w$ with $\theta_w$ denoting the Weinberg angle, and

\[ \sin^2 \theta'_w = \frac{(g_1^2 + g_2^2) g_{Y_B}^2 v^2}{16g_{Y_B}^4 v^2}. \]  

\[(C3)\]

Appendix D: Nonzero amplitudes from Fig. 3

Those nonzero amplitudes are summarized as

\[ M^{(1)}_{++} = \frac{-i(\alpha_s(\mu_R))}{\pi} C_{h_{hh}} C_{h_{ff}} F_\Delta^{(1/2)}, \]

\[ M^{(2+3+4)}_{++} = \frac{i(\alpha_s(\mu_R))}{2\pi \hat{s}} C_{h_{ff}} F_\Box^{(1/2)}, \]

\[ M^{(2+3+4)}_{+-} = \frac{-i(\alpha_s(\mu_R))}{2\pi} C_{h_{ff}} G_\Box^{(1/2)}, \]

\[ M^{(5+6)}_{++} = \frac{i(\alpha_s(\mu_R))}{2\pi} C_{h_{hh}} C_{h_{sisi}} F_\Delta^{(0)}, \]

\[ M^{(7+8)}_{++} = \frac{-i(\alpha_s(\mu_R))}{2\pi} C_{h_{sisi}} F_\Delta^{(0)}, \]

\[ M^{(9)}_{++} = \frac{-i(\alpha_s(\mu_R))}{\pi} C_{h_{sisi}} F'^{(0)}_\Delta, \]

\[ M^{(10+11+12)}_{++} = \frac{i(\alpha_s(\mu_R))}{2\pi \hat{s}} C_{h_{sisi}} F'^{(0)}_\Box, \]

\[ M^{(10+11+12)}_{+-} = \frac{i(\alpha_s(\mu_R))}{2\pi} C_{h_{sisi}} G^{(0)}_\Box. \]  

\[(D1)\]

Here $F_\Delta$ and $F'_\Delta$ are form factors associated with triangular diagrams. $F_\Box$ and $G_\Box$ are form factors of box diagrams which, respectively, correspond to the same and opposite polarizations of the incoming gluons. The concrete expressions of the form factors ($F_\Delta, F'_\Delta, F_\Box, G_\Box$) are

\[ F_\Delta^{(1/2)} = 2m_f + (4m_f^2 - \hat{s}) m_f C_0(0, 0, \hat{s}, m_f^2, m_f^2, m_f^2), \]  

\[(D2)\]
\[ F^{(1/2)}_\square = -4\hat{s} - 8m_f^2C_0(0, 0, \hat{s}, m_f^2, m_f^2, m_f^2)\hat{s} \\
- (8m_f^2 - 2m_f^2)[2(m^2_h - \hat{t})C_0(0, m^2_h, \hat{t}, m_f^2, m_f^2) \\
+ 2(m^2_h - \hat{u})C_0(0, m^2_h, \hat{u}, m_f^2, m_f^2) \\
- (m^4_h - \hat{t}\hat{u})D_0(0, m^2_h, 0, m^2_h, \hat{t}, \hat{u}, m_f^2, m_f^2, m_f^2)] \\
- 2m_f^2(8m_f^2 - 2m_f^2 - \hat{s})\hat{s}[D_0(0, m^2_h, 0, m^2_h, \hat{t}, \hat{u}, m_f^2, m_f^2, m_f^2) \\
+ D_0(0, 0, m^2_h, m^2_h, \hat{s}, \hat{t}, m_f^2, m_f^2, m_f^2)] \\
+ D_0(0, 0, m^2_h, m^2_h, \hat{s}, \hat{u}, m_f^2, m_f^2, m_f^2) \] , 

(D3)

\[ G^{(1/2)}_\square = \frac{1}{(m^4_h - \hat{t}\hat{u})} \left\{ [(8m_f^2 - \hat{t} - \hat{u})(2m^4_h - \hat{t}^2 - \hat{u}^2)C_0(m^2_h, m^2_h, \hat{s}, m_f^2, m_f^2, m_f^2) \\
+ (m^4_h - 8m_f^2\hat{t} + \hat{t}^2)[2(m^2_h - \hat{t})C_0(0, m^2_h, \hat{t}, m_f^2, m_f^2) \\
- \hat{s}C_0(0, 0, \hat{s}, m_f^2, m_f^2, m_f^2) + \hat{s}\hat{t}D_0(0, 0, m^2_h, m^2_h, \hat{s}, \hat{t}, m_f^2, m_f^2, m_f^2)] \\
+ (m^4_h - 8m_f^2\hat{u} + \hat{u}^2)[2(m^2_h - \hat{u})C_0(0, m^2_h, \hat{u}, m_f^2, m_f^2) \\
- \hat{s}C_0(0, 0, \hat{s}, m_f^2, m_f^2, m_f^2) + \hat{s}\hat{u}D_0(0, 0, m^2_h, m^2_h, \hat{s}, \hat{u}, m_f^2, m_f^2, m_f^2)] \\
+ 2m_f^2(m^4_h - \hat{t}\hat{u})(8m_f^2 - \hat{t} - \hat{u})[D_0(0, m^2_h, 0, m^2_h, \hat{t}, \hat{u}, m_f^2, m_f^2, m_f^2) \\
+ D_0(0, 0, m^2_h, m^2_h, \hat{s}, \hat{t}, m_f^2, m_f^2, m_f^2)] \\
+ D_0(0, 0, m^2_h, m^2_h, \hat{s}, \hat{u}, m_f^2, m_f^2, m_f^2)] \right\} , 

(D4)

\[ F^{(0)}_\Delta = 1 + 2m^2_sC_0(0, 0, \hat{s}, m^2_s, m^2_s, m^2_s) \] ,

(D5)

\[ F^{(0)}\square = C_0(m^2_h, m^2_h, \hat{s}, m^2_s, m^2_s, m^2_s) \] ,

(D6)

\[ F^{(0)}_\square = (m^2_h - \hat{t})C_0(m^2_h, 0, \hat{t}, m^2_s, m^2_s, m^2_s) + (m^2_h - \hat{u})C_0(m^2_h, 0, \hat{u}, m^2_s, m^2_s, m^2_s) \\
+ (m^2_h - \hat{t})C_0(m^2_h, 0, \hat{t}, m^2_s, m^2_s, m^2_s) + (m^2_h - \hat{u})C_0(m^2_h, 0, \hat{u}, m^2_s, m^2_s, m^2_s) \\
+ 2\hat{s}C_0(m^2_h, m^2_h, \hat{s}, m^2_s, m^2_s, m^2_s) \\
+ [(m^2_s - m^2_s)\hat{s} - (m^4_h - \hat{t}\hat{u})]D_0(m^2_h, 0, m^2_h, 0, \hat{t}, \hat{u}, m^2_s, m^2_s, m^2_s) \\
+ 2\hat{s}m^2_s[D_0(m^2_h, 0, m^2_h, 0, \hat{t}, \hat{u}, m^2_s, m^2_s, m^2_s) \\
+ D_0(m^2_h, m^2_h, 0, 0, \hat{s}, \hat{t}, m^2_s, m^2_s, m^2_s)] \\
+ D_0(m^2_h, m^2_h, 0, 0, \hat{s}, \hat{u}, m^2_s, m^2_s, m^2_s)] \] ,

(D7)
\[ G^{(0)} = \frac{1}{m_h^4 - i\bar{u}} \{ \hat{s}(2m_{S_i}^2 - 2m_{S_j}^2 + \hat{t} + \hat{u})C_0(0, 0, \hat{s}, m_{S_i}^2, m_{S_j}^2, m_{S_i}^2) \\
- \hat{t}[2(m_h^2 - \hat{t})C_0(m_h^2, 0, \hat{t}, m_{S_i}^2, m_{S_j}^2, m_{S_i}^2)] - \hat{u}[2(m_h^2 - \hat{u})C_0(m_h^2, 0, \hat{u}, m_{S_j}^2, m_{S_i}^2, m_{S_j}^2)] \\
- (m_h^2 - \hat{t})(m_{S_i}^2 - m_{S_j}^2)C_0(m_h^2, 0, \hat{t}, m_{S_i}^2, m_{S_j}^2, m_{S_i}^2) + C_0(m_h^2, 0, \hat{t}, m_{S_j}^2, m_{S_i}^2, m_{S_j}^2) \\
- (m_h^2 - \hat{u})(m_{S_i}^2 - m_{S_j}^2)C_0(m_h^2, 0, \hat{u}, m_{S_i}^2, m_{S_j}^2, m_{S_i}^2) + C_0(m_h^2, 0, \hat{u}, m_{S_j}^2, m_{S_i}^2, m_{S_j}^2) \\
+ (2m_h^4 - \hat{t}^2 - \hat{u}^2)C_0(m_h^2, m_h^2, \hat{s}, m_{S_i}^2, m_{S_j}^2, m_{S_i}^2) \\
+ [-\hat{s}(m_{S_i}^2 - m_{S_j}^2)^2 + (m_{S_i}^2 + m_{S_j}^2)(m_h^4 - \hat{t}\hat{u})][D_0(m_h^2, 0, m_h^2, 0, \hat{t}, \hat{u}, m_{S_i}^2, m_{S_j}^2, m_{S_i}^2) \\
+ D_0(m_h^2, m_h^2, 0, 0, \hat{s}, \hat{t}, m_{S_i}^2, m_{S_j}^2, m_{S_i}^2) + D_0(m_h^2, m_h^2, 0, 0, \hat{s}, \hat{u}, m_{S_j}^2, m_{S_i}^2, m_{S_j}^2) \\
+ [-\hat{s}\hat{t}^2 - (m_{S_i}^2 - m_{S_j}^2)(2\hat{s}\hat{t} - (m_h^4 - \hat{t}\hat{u}))][D_0(m_h^2, m_h^2, 0, 0, \hat{s}, \hat{t}, m_{S_i}^2, m_{S_j}^2, m_{S_i}^2) \\
+ [-\hat{s}\hat{u}^2 - (m_{S_i}^2 - m_{S_j}^2)(2\hat{s}\hat{u} - (m_h^4 - \hat{t}\hat{u}))][D_0(m_h^2, m_h^2, 0, 0, \hat{s}, \hat{u}, m_{S_j}^2, m_{S_i}^2, m_{S_j}^2)]} \]  

(C8)

\( C_0 \) and \( D_0 \) are scalar integrals, defined as

\[
C_0(p_1^2, p_2^2, (p_1 + p_2)^2, m_1^2, m_2^2, m_3^2) = \frac{\mu^{4-D}}{i\pi^{D/2} \gamma_r} \int \frac{d^D q}{[q^2 - m_1^2][(q + p_1)^2 - m_2^2][(q + p_1 + p_2)^2 - m_3^2]}
\]

\[
D_0(p_1^2, p_2^2, p_3^2, p_4^2, (p_1 + p_2)^2, (p_3 + p_4)^2, m_1^2, m_2^2, m_3^2, m_4^2) = \frac{\mu^{4-D}}{i\pi^{D/2} \gamma_r} \int \frac{d^D q}{[q^2 - m_1^2][(q + p_1)^2 - m_2^2][(q + p_1 + p_2)^2 - m_3^2][(q + p_1 + p_2 + p_3)^2 - m_4^2]}
\]

(D9)

(D10)

Where

\[
\gamma_r = \frac{\Gamma^2(1 - \varepsilon)\Gamma(1 + \varepsilon)}{\Gamma(1 - 2\varepsilon)}, D = 4 - 2\varepsilon.
\]

(D11)

The integrals are independent of \( \mu \) in the limit \( \varepsilon \to 0 \). Furthermore the functions \( C_{ij}(\hat{s}, \hat{z}) \) \((ij = gg, gq, qq)\) are given as

\[
C_{gg}(\hat{s}, \hat{z}) = -zP_{gg}(z) \log \frac{\mu^2}{\tau_s} \frac{\log(1 - \hat{z})}{1 - \hat{z}} + d_{gg}(\hat{s}, \hat{z}),
\]

\[
C_{gq}(\hat{s}, \hat{z}) = -\frac{z}{2}P_{gq}(z) \log \frac{\mu^2}{\tau_s(1 - \hat{z})^2} + d_{gq}(\hat{s}, \hat{z}),
\]

\[
C_{qq}(\hat{s}, \hat{z}) = d_{qq}(\hat{s}, \hat{z}).
\]

(D12)
Here $P_{ij}(z) (i,j = g, q, \bar{q})$ are the specific Altarelli-Parisi splitting functions [79], and the concrete expressions for the virtual corrections $C_{\text{virt}}(\hat{s})$ and the real corrections $d_{ij}(\hat{s}, z)$ can be found in Ref. [34], respectively.

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