The mass and spin of the mesons, baryons and leptons

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The rest masses of the stable mesons and baryons and the rest masses of their antiparticles, as well as the rest masses of the $\mu^\pm$ and $\tau^\pm$ leptons can be explained, within 1% accuracy, with the standing wave model, which uses only photons, neutrinos, charge and the weak nuclear force. And we can explain the spin of the stable mesons and baryons and the spin of the $\mu^\pm$ and $\tau^\pm$ leptons without any additional assumption. We can also determine the rest masses of the $e$, $\mu$ and $\tau$ neutrinos.

Introduction

The so-called “Standard Model” of the elementary particles has, until now, not come up with a precise theoretical determination of the masses of either the mesons and baryons or of the leptons, which means that neither the mass of the fundamental electron nor the mass of the fundamental proton have been explained. This is so although the quarks, the foundation of the standard model, have been introduced by Gell-Mann [1] forty years ago. The masses of the quarks which are considered range from zero rest mass to values from 1.5 to 4.5 MeV for e.g. the u-quark, according to the Review of Particle Physics [2], to values on the order of 100 MeV. Suppose one has agreed on definite values of the masses of the various quarks then one stands before the same problem one has faced with the conventional elementary particles, namely one has to explain why the quarks have their particular masses and what they are made of. The other most frequently referred to theory dealing with the elementary particles, the “String Theory” introduced about twenty years ago (Witten [3]), or its successor the superstring theory, have despite their mathematical elegance not led to experimentally verifiable results. There are many other attempts to explain the elementary particles
or only one of the particles, too many to list them here. There have been,
for example, in the last years, the articles of El Naschie on a general theory
for high energy particles and the spectrum of the quarks [4-7]. Our model
and El Naschie’s mechanical model version of his topological theory are not
far apart.

The need for the present investigation has been expressed by Feynman [8]
as follows: “There remains one especially unsatisfactory feature: the observed
masses of the particles, m. There is no theory that adequately explains these
numbers. We use the numbers in all our theories, but we do not understand
them - what they are, or where they come from. I believe that from a
fundamental point of view, this is a very interesting and serious problem”.
Today, twenty years later, we still stand in front of the same problem.

1 The spectrum of the masses of the particles

As we have done before [9] we will focus attention on the so-called “stable”
mesons and baryons whose masses are reproduced with other data in Tables
1 and 2. It is obvious that any attempt to explain the masses of the mesons
and baryons should begin with the particles that are affected by the fewest
parameters. These are certainly the particles without isospin (I = 0) and
without spin (J = 0), but also with strangeness S = 0, and charm C = 0.
Looking at the particles with I,J,S,C = 0 it is startling to find that their
masses are quite close to integer multiples of the mass of the \( \pi^0 \) meson. It
is \( m(\eta) = (1.0140 \pm 0.0003) \cdot 4m(\pi^0) \), and the mass of the resonance \( \eta' \) is
mass \( m(\eta') = (1.0137 \pm 0.00015) \cdot 7m(\pi^0) \). Three particles seem hardly to be
sufficient to establish a rule. However, if we look a little further we find
that \( m(\Lambda) = 1.0332 \cdot 8m(\pi^0) \) or \( m(\Lambda) = 1.0190 \cdot 2m(\eta) \). We note that the
\( \Lambda \) particle has spin 1/2, not spin 0 as the \( \pi^0, \eta \) mesons. Nevertheless, the
mass of \( \Lambda \) is close to \( 8m(\pi^0) \). Furthermore we have \( m(\Sigma^0) = 0.9817 \cdot 9m(\pi^0) \),
\( m(\Xi^0) = 0.9742 \cdot 10m(\pi^0) \), \( m(\Omega^-) = 1.0325 \cdot 12m(\pi^0) = 1.0183 \cdot 3m(\eta) \), (\( \Omega^- \)
is charged and has spin 3/2). Finally the masses of the charmed baryons are
\( m(\Lambda_c^+) = 0.9958 \cdot 17m(\pi^0) = 1.024 \cdot 2m(\Lambda), m(\Sigma_c^0) = 1.0093 \cdot 18m(\pi^0), m(\Xi_c^0) = 1.0167 \cdot 18m(\pi^0), \) and \( m(\Omega_c^0) = 1.0017 \cdot 20m(\pi^0). \)

Now we have enough material to formulate the **integer multiple rule**, according
to which the masses of the \( \eta, \Lambda, \Sigma^0, \Xi^0, \Omega^-, \Lambda_c^+, \Sigma_c^0, \Xi_c^0, \) and \( \Omega_c^0 \)
particles are, in a first approximation, integer multiples of the mass of the
\( \pi^0 \) meson, although some of the particles have spin, and may also have charge
Table 1: The $\gamma$-branch of the particle spectrum

| m/m($\pi^0$) multiples | decays | fraction (%) | spin | mode$^1$ |
|-------------------------|--------|--------------|------|---------|
| $\pi^0$ 1.0000 | $\gamma \gamma$ | 98.798 | 0 | (1.) |
| $\pi^+ e^- \gamma$ | 1.198 |
| $\eta$ 4.0559 | $\gamma \gamma$ | 39.43 | 0 | (2.) |
| $3\pi^0$ | 32.51 |
| $\pi^+ \pi^- \pi^0$ | 22.6 |
| $\pi^+ \pi^- \gamma$ | 4.68 |
| $\Lambda$ 8.26577 | $p\pi^-$ | 63.9 | $\frac{1}{2}$ | 2-(2.) |
| $n\pi^0$ | 35.8 |
| $\Sigma^0$ 8.8352 | $\Lambda \gamma$ | 100 | $\frac{1}{2}$ | 2-(2.) + (1.) |
| $\Xi^0$ 9.7417 | $\Lambda \pi^0$ | 99.52 | $\frac{1}{2}$ | 2-(2.) + 2(1.) |
| $\Omega^-$ 12.390 | $\Lambda K^-$ | 67.8 | $\frac{3}{2}$ | 3-(2.) |
| $\Xi^0 \pi^-$ | 23.6 |
| $\Xi^- \pi^0$ | 8.6 |
| $\Lambda_c^+$ 16.928 | many | $\approx$100 | $\frac{1}{2}$ | $\Lambda_c^+ + \pi^-$ |
| $0.9958 \cdot 17\pi^0$ | |
| $0.9630 \cdot 17\pi^\pm$ | |
| $\Sigma_c^0$ 18.167 | $\Lambda_c^+ \pi^-$ | (seen) | $\frac{1}{2}$ | 2-(3.) |
| $\Xi_c^0$ 18.302 | nine | (seen) | $\frac{1}{2}$ | 2-(3.) |
| $\Omega_c^0$ 20.033 | six | (seen) | $\frac{1}{2}$ | 2-(3.) + 2(1.) |

$^1$The modes apply to neutral particles only. The · sign marks coupled modes.
as well as strangeness and charm. A consequence of the integer multiple rule must be that the ratio of the mass of any meson or baryon listed above divided by the mass of another meson or baryon listed above is equal to the ratio of two integer numbers. And indeed, for example $m(\eta)/m(\pi^0)$ is practically two times (exactly $0.9950 \cdot 2$) the ratio $m(\Lambda)/m(\eta)$. There is also the ratio $m(\Omega^-)/m(\Lambda) = 0.9993 \cdot 3/2$. We have furthermore e.g. the ratios $m(\Lambda)/m(\eta) = 1.019 \cdot 2$, $m(\Omega^-)/m(\eta) = 1.018 \cdot 3$, $m(\Lambda^+)/m(\Lambda) = 1.02399 \cdot 2$, $m(\Sigma^0)/m(\Sigma^0) = 1.0281 \cdot 2$, $m(\Omega^0)/m(\Xi^0) = 1.0282 \cdot 2$, and $m(\Omega^0)/m(\eta) = 0.9857 \cdot 5$.

We will call, for reasons to be explained later, the particles discussed above, which follow in a first approximation the integer multiple rule, the $\gamma$-branch of the particle spectrum. The mass ratios of these particles are in Table 1. The deviation of the mass ratios from exact integer multiples of $m(\pi^0)$ is at most 3.3%, the average of the factors before the integer multiples of $m(\pi^0)$ of the nine $\gamma$-branch particles in Table 1 is $1.0066 \pm 0.0184$. From a least square analysis follows that the masses of the ten particles on Table 1 lie on a straight line given by the formula

$$m(N)/m(\pi^0) = 1.0065 \cdot N - 0.0043 \quad N \geq 1,$$

(1)

where $N$ is the integer number nearest to the actual ratio of the particle mass divided by $m(\pi^0)$. The correlation coefficient in equation (1) has the nearly perfect value $r^2 = 0.999$.

The integer multiple rule applies to more than just the stable mesons and baryons. The integer multiple rule applies also to the $\gamma$-branch baryon resonances which have spin $J = 1/2$ and the meson resonances with $I,J = 0$, listed in [2] or in Tables 2,3 in [9]. The $\Omega^-$ particle will not be considered because it has spin $3/2$ but would not change the following equation significantly. If we combine the particles in Table 1 with the $\gamma$-branch meson and baryon resonances, that means if we consider all mesons and baryons of the $\gamma$-branch, “stable” or unstable, with $I \leq 1, J \leq 1/2$ then we obtain from a least square analysis the formula

$$m(N)/m(\pi^0) = 1.0056 \cdot N + 0.0610 \quad N \geq 1,$$

(2)

with the correlation coefficient 0.9986. The line through the points is shown in Fig.1 which tells that 22 particles of the $\gamma$-branch of different spin and isospin, strangeness and charm; eight $I,J = 0,0$ mesons, thirteen $J = 1/2$
baryons and the $\pi^0$ meson with $I,J = 1,0$, lie on a straight line with slope 1.0056. In other words they approximate the integer multiple rule very well.

![Graph of $y = 1.0056 N + 0.0610$ with $r^2 = 0.9986$](image)

**Fig. 1:** The mass of the mesons and baryons of the $\gamma$-branch with $I \leq 1, J \leq \frac{1}{2}$ in units of $m(\pi^0)$ as a function of the integer $N$. $y = m/m(\pi^0)$.

Searching for what else the $\pi^0, \eta, \Lambda, \Sigma^0, \Xi^0, \Omega^-$ particles have in common, we find that the principal decays (decays with a fraction $> 1\%$) of these particles, as listed in Table 1, involve primarily $\gamma$-rays, the characteristic case is $\pi^0 \rightarrow \gamma\gamma$ (98.8\%). We will later on discuss a possible explanation for the 1.198\% of the decays of $\pi^0$ which do not follow the $\gamma\gamma$ route. After the $\gamma$-rays the next most frequent decay product of the heavier particles of the $\gamma$-branch are $\pi^0$ mesons which again decay into $\gamma\gamma$. To describe the decays in another way, the principal decays of the particles listed above take place *always without the emission of neutrinos*; see Table 1. There the decays and the fractions of the principal decay modes are given, taken from the Review of Particle Physics. We cannot consider decays with fractions $< 1\%$. We will
refer to the particles whose masses are approximately integer multiples of the mass of the $\pi^0$ meson, and which decay without the emission of neutrinos, as the $\gamma$-branch of the particle spectrum.

To summarize the facts concerning the $\gamma$-branch. Within 0.66% on the average the masses of the particles of the $\gamma$-branch are integer multiples (namely 4, 8, 9, 10, 12, and even 17, 18, 20) of the mass of the $\pi^0$ meson. It is improbable that nine particles have masses so close to integer multiples of $m(\pi^0)$ if there is no correlation between them and the $\pi^0$ meson. It has, on the other hand, been argued that the integer multiple rule is a numerical coincidence. But the probability that the mass ratios of the $\gamma$-branch fall by coincidence on integer numbers between 1 and 20 instead on all possible numbers between 1 and 20 with two decimals after the period is smaller than $10^{-20}$, i.e nonexistent. The integer multiple rule is not affected by more than 3% by the spin, the isospin, the strangeness, and by charm. The integer multiple rule seems even to apply to the $\Omega^-$ and $\Lambda^+_c$ particles, although they are charged. In order for the integer multiple rule to be valid the deviation of the ratio $m/m(\pi^0)$ from an integer number must be smaller than $1/2N$, where $N$ is the integer number closest to the actual ratio $m/m(\pi^0)$. That means that the permissible deviation decreases rapidly with increased $N$. All particles of the $\gamma$-branch have deviations smaller than $1/2N$.

The remainder of the stable mesons and baryons are the $\pi^\pm$, $K^\pm$, $p$, $n$, $D^\pm$, $D^\pm_0$, and $D^\pm_s$ particles which make up the $\nu$-branch of the particle spectrum. The ratios of their masses are given in Table 2.

These particles are in general charged, exempting the $K^0$ and $D^0$ mesons and the neutron $n$, in contrast to the particles of the $\gamma$-branch, which are in general neutral. It does not make a significant difference whether one considers the mass of a particular charged or neutral particle. After the $\pi$ mesons, the largest mass difference between charged and neutral particles is that of the $K$ mesons (0.81%), and thereafter all mass differences between charged and neutral particles are < 0.5%. The integer multiple rule does not immediately apply to the masses of the $\nu$-branch particles if $m(\pi^\pm)$ (or $m(\pi^0)$) is used as reference, because $m(K^\pm) = 0.8843 \cdot 4m(\pi^\pm)$. $0.8843 \cdot 4 = 3.537$ is far from integer. Since the masses of the $\pi^0$ meson and the $\pi^\pm$ mesons differ by only 3.4% it has been argued that the $\pi^\pm$ mesons are, but for the isospin, the same particles as the $\pi^0$ meson, and that therefore the $\pi^\pm$ cannot start another particle branch. However, this argument is not supported by the completely different decays of the $\pi^0$ mesons and the $\pi^\pm$ mesons. The $\pi^0$ meson decays almost exclusively into $\gamma\gamma$ (98.8%), whereas the $\pi^\pm$ mesons...
Table 2: The $\nu$-branch of the particle spectrum

|                  | m/m($\pi^\pm$) | multiples   | decays$^2$                      | fraction (%) | spin | mode                      |
|------------------|----------------|-------------|---------------------------------|--------------|------|---------------------------|
| $\pi^\pm$        | 1.0000        | 1.0000 $\cdot \pi^\pm$ | $\mu^+\nu_\mu$                  | 99.9877      | 0    | (1.)                      |
| $K^{\pm,0}$      | 3.53713       | 0.8843 $\cdot 4\pi^\pm$ | $\mu^+\nu_\mu$                  | 63.43        | 0    | (2.) + $\pi^0$            |
|                  |                |             | $\pi^\pm\pi^0$                  | 21.13        |      |                           |
|                  |                |             | $\pi^+\pi^-\pi^+$               | 5.58         |      |                           |
|                  |                |             | $\pi^0 e^+\nu_e (K^+_{e3})$      | 4.87         |      |                           |
|                  |                |             | $\pi^0\mu^+\nu_\mu (K^+_{\mu3})$| 3.27         |      |                           |
| n                | 6.73186        | 0.8415 $\cdot 8\pi^\pm$ | $p e^-\overline{\nu}_e$         | 100.         | $\frac{1}{2}$ | 2-(2.) + 2$\pi^\pm$ |
|                  |                | 0.9516 $\cdot 2K^\pm$ |                                 |              |      |                           |
| $D^{\pm,0}$      | 13.393         | 0.8370 $\cdot 16\pi^\pm$ | $e^+\text{anything}$             | 17.2         | 0    | 2(2-(2.) + 2$\pi^\pm$)   |
|                  |                | 0.9466 $\cdot 4K^\pm$ | $K^-\text{anything}$             | 24.2         |      |                           |
|                  |                | 0.9954 $\cdot (p + \bar{n})$ | $\overline{K}^0\text{anything}$ | < 13         |      |                           |
|                  |                |             | $\overline{K}^0\text{anything}$ + $K^0\text{anything}$ | 59           |      |                           |
|                  |                |             | $\eta\text{anything}$            | < 13         |      |                           |
|                  |                |             | $K^+\text{anything}$             | 5.8          |      |                           |
| $D_s^\pm$        | 14.104         | 0.8296 $\cdot 17\pi^\pm$ | $K^-\text{anything}$             | 13           | 0    | body centered cubic       |
|                  |                | 0.9968 $\cdot 4K^\pm$ | $\overline{K}^0\text{anything}$ + $K^0\text{anything}$ | 39           |      |                           |
|                  |                |             | $K^+\text{anything}$             | 20           |      |                           |
|                  |                |             | $e^+\text{anything}$             | 8            |      |                           |

$^2$The particles with negative charges have conjugate charges of the listed decays. Only the decays of $K^\pm$ and $D^\pm$ are listed. The oscillation modes carry one electric charge.
decay practically exclusively into $\mu$ mesons and neutrinos, as in $\pi^+ \to \mu^+ + \nu_\mu$ (99.9877%). Furthermore, the lifetimes of the $\pi^0$ and the $\pi^\pm$ mesons differ by nine orders of magnitude, being $\tau(\pi^0) = 8.4 \cdot 10^{-17}$ sec versus $\tau(\pi^\pm) = 2.6 \cdot 10^{-8}$ sec.

If we make the $\pi^\pm$ mesons the reference particles of the $\nu$-branch, then we must multiply the mass ratios $m/m(\pi^\pm)$ of the above listed particles with an average factor $0.848 \pm 0.025$, as follows from the mass ratios on Table 2. The integer multiple rule may, however, apply directly if one makes $m(K^\pm)$ the reference for masses larger than $m(K^\pm)$. The mass of the neutron is $0.9516 \cdot 2m(K^\pm)$, which is only a fair approximation to an integer multiple. There are, on the other hand, outright integer multiples in $m(D^\pm) = 0.9954 \cdot (m(p) + m(n))$, and in $m(D^\pm_s) = 0.9968 \cdot 4m(K^\pm)$. A least square analysis of the masses of the $\nu$-branch in Table 2 yields the formula

$$m(N)/0.853m(\pi^\pm) = 1.000 N + 0.00575 \quad N \geq 1,$$

with $r^2 = 0.998$. This means that the particles of the $\nu$-branch are integer multiples of $m(\pi^\pm)$ times the factor 0.853. One must, however, consider that the $\pi^\pm$ mesons are not necessarily the perfect reference for all $\nu$-branch particles, because $\pi^\pm$ has $I = 1$, whereas for example $K^\pm$ has $I = 1/2$ and $S = \pm 1$ and the neutron has also $I = 1/2$. Actually the factor 0.853 in Eq.(3) is only an average. The mass ratios indicate that this factor decreases slowly with increased $m(N)$. The existence of the factor and its decrease will be explained later.

Contrary to the particles of the $\gamma$-branch, the $\nu$-branch particles decay preferentially with the emission of neutrinos, the foremost example is $\pi^\pm \to \mu^\pm + \nu_\mu(\bar{\nu}_\mu)$ with a fraction of 99.9877%. Neutrinos characterize the weak interaction. We will refer to the particles in Table 2 as the *neutrino branch* ($\nu$-branch) of the particle spectrum. We emphasize that a weak decay of the particles of the $\nu$-branch is by no means guaranteed. Although the neutron decays via $n \to p + e^- + \bar{\nu}_e$ in 887 sec (100%), the proton is stable. There are, on the other hand, decays as e.g. $K^+ \to \pi^+\pi^-\pi^0$ (5.59%), but the subsequent decays of the $\pi^\pm$ mesons lead to neutrinos and $e^\pm$. The decays of the particles in the $\nu$-branch follow a mixed rule, either weak or electromagnetic.

To summarize the facts concerning the $\nu$-branch of the mesons and baryons. The masses of these particles seem to follow the integer multiple rule if one uses the $\pi^\pm$ meson as reference, however the mass ratios share a common factor $0.85 \pm 0.025$. 
To summarize what we have learned about the integer multiple rule: In spite of differences in charge, spin, strangeness, and charm the masses of the mesons and baryons of the $\gamma$-branch are integer multiples of the mass of the $\pi^0$ meson within at most 3.3% and on the average within 0.66%. Correspondingly, the masses of the particles of the $\nu$-branch are, after multiplication with a factor $0.85 \pm 0.025$, integer multiples of the mass of the $\pi^\pm$ mesons. The validity of the integer multiple rule can easily be verified with a calculator from the data in the Review of Particle Physics. The integer multiple rule has been anticipated much earlier by Nambu [10], who wrote in 1952 that “some regularity [in the masses of the particles] might be found if the masses were measured in a unit of the order of the $\pi$-meson mass”. A similar suggestion has been made by Fröhlich [11]. The integer multiple rule suggests that the particles are the result of superpositions of modes and higher modes of a wave equation.

2 Standing waves in a cubic lattice and the particles of the $\gamma$-branch

We will now study, as we have done in [12], whether the so-called “stable” particles of the $\gamma$-branch cannot be described by the frequency spectrum of standing waves in a cubic lattice, which can accommodate automatically the Fourier frequency spectrum of an extreme short-time collision by which the particles are created. The investigation of the consequences of lattices for particle theory was initiated by Wilson [13] who studied a cubic fermion lattice. This study has developed over time into lattice QCD.

It will be necessary for the following to outline the most elementary aspects of the theory of lattice oscillations. The classic paper describing lattice oscillations is from Born and v. Karman [14], henceforth referred to as B&K. They looked at first at the oscillations of a one-dimensional chain of points with mass $m$, separated by a constant distance $a$. This is the monatomic case, all lattice points have the same mass. B&K assume that the forces exerted on each point of the chain originate only from the two neighboring points. These forces are opposed to and proportional to the displacements, as with elastic springs (Hooke’s law). The equation of motion is in this case

$$m \ddot{u}_n = \alpha (u_{n+1} - u_n) - \alpha (u_n - u_{n-1}).$$  (4)
The $u_n$ are the displacements of the mass points from their equilibrium position which are apart by the distance $a$. The dots signify, as usual, differentiation with respect to time, $\alpha$ is a constant characterizing the force between the lattice points, and $n$ is an integer number. For $a \to 0$ Eq.(4) becomes the wave equation $c^2 \partial^2 u / \partial x^2 = \partial^2 u / \partial t^2$ (B&K).

In order to solve (4) B&K set

$$u_n = Ae^{i(\omega t + n\phi)},$$

which is obviously a temporally and spatially periodic solution. $n$ is an integer, with $n < N$, where $N$ is the number of points in the chain. $\phi = 0$ is the monochromatic case. We also consider higher modes, by replacing $n\phi$ in Eq.(5) with $\ell n\phi$, with integer $\ell > 1$. The wavelengths are then shorter by $1/\ell$. At $n\phi = \pi/2$ there are nodes, where for all times $t$ the displacements are zero, as with standing waves. If a displacement is repeated after $n$ points we have $na = \lambda$, where $\lambda$ is the wavelength, $a$ the lattice constant, and it must be $n\phi = 2\pi$ according to (5). It follows that

$$\lambda = 2\pi a / \phi. \quad (6)$$

Inserting (5) into (4) one obtains a continuous frequency spectrum given by Eq.(5) of B&K

$$\omega = \pm 2\sqrt{\alpha / m} \sin(\phi / 2). \quad (7)$$

B&K point out that there is not only a continuum of frequencies, but also a maximal frequency which is reached when $\phi = \pi$, or at the minimum of the possible wavelengths $\lambda = 2a$. The boundary conditions are periodic, that means that $u_n = u_{n+N}$, where $N$ is the number of points in the chain. Born referred to the periodic boundary condition as a “mathematical convenience”. The number of normal modes must be equal to the number of particles in the lattice.

Born’s model of the crystals has been verified in great detail by X-ray scattering and even in much more complicated cases by neutron scattering. The theory of lattice oscillations has been pursued in particular by Blackman [15], a summary of his and other studies is in [16]. Comprehensive reviews of the results of linear studies of lattice dynamics have been written by Born and Huang [17], by Maradudin et al. [18], and by Ghatak and Kothari [19].
3 The masses of the $\gamma$-branch particles

We will now assume, as seems to be quite natural, that the particles consist of the same particles into which they decay, directly or ultimately. We know this from atoms, which consist of nuclei and electrons, and from nuclei, which consist of protons and neutrons. Quarks have never appeared among the decay products of elementary particles. For the $\gamma$-branch particles our assumption means that they consist of photons. Photons and $\pi^0$ mesons are the principal decay products of the $\gamma$-branch particles, the characteristic example is $\pi^0 \rightarrow \gamma\gamma$ (98.8%). Table 1 shows that there are decays of the $\gamma$-branch particles which lead to particles of the $\nu$-branch, in particular to pairs of $\pi^+$ and $\pi^-$ mesons. It appears that this has to do with pair production in the $\gamma$-branch particles. Pair production is evident in the decay $\pi^0 \rightarrow e^+ + e^-$ (1.198%) or in the $\pi^0$ meson’s third most frequent decay $\pi^0 \rightarrow e^+e^-e^-e^-$ (3.14·10$^{-3}$%). Pair production requires the presence of electromagnetic waves of high energy. Anyway, the explanation of the $\gamma$-branch particles must begin with the explanation of the most simple example of its kind, the $\pi^0$ meson, which by all means seems to consist of photons. The composition of the particles of the $\gamma$-branch suggested here offers a direct route from the formation of a $\gamma$-branch particle, through its lifetime, to its decay products. Particles that are made of photons are necessarily neutral, as the majority of the particles of the $\gamma$-branch are.

We also base our assumption that the particles of the $\gamma$-branch are made of photons on the circumstances of the formation of the $\gamma$-branch particles. The most simple and straightforward creation of a $\gamma$-branch particle are the reactions $\gamma + p \rightarrow \pi^0 + p$, or in the case that the spins of $\gamma$ and $p$ are parallel $\gamma + p \rightarrow \pi^0 + p + \gamma'$. A photon impinges on a proton and creates a $\pi^0$ meson. The considerations which follow apply as well for other photoproductions such as $\gamma + p \rightarrow \eta + p$ or $\gamma + d \rightarrow \pi^0 + d$, but also for the electroproductions $e^- + p \rightarrow \pi^0 + e^- + p$ or $e^- + d \rightarrow \pi^0 + e^- + d$, see Rekalo et al. [20].

In $\gamma + p \rightarrow \pi^0 + p$ the pulse of the incoming electromagnetic wave is in $10^{-23}$ sec converted into a continuum of electromagnetic waves with frequencies ranging from $10^{23}$ sec$^{-1}$ to $\nu \rightarrow \infty$ according to Fourier analysis. There must be a cutoff frequency, otherwise the energy in the sum of the frequencies would exceed the energy of the incoming electromagnetic wave. The wave packet so created decays, according to experience, after $8.4 \cdot 10^{-17}$ sec into two electromagnetic waves or $\gamma$-rays. It seems to be very unlikely that Fourier
analysis does not hold for the case of an electromagnetic wave impinging on a proton. The question then arises of what happens to the electromagnetic waves in the timespan of $10^{-16}$ seconds between the creation of the wave packet and its decay into two $\gamma$-rays? We will investigate whether the electromagnetic waves cannot continue to exist for the $10^{-16}$ seconds until the wave packet decays.

If the wave packet created by the collision of a $\gamma$-ray with a proton consists of electromagnetic waves, then the waves cannot be progressive because the wave packet must have a rest mass. However standing electromagnetic waves can have a rest mass. Standing electromagnetic waves are equivalent to a lattice and the lattice oscillations can absorb the continuum of frequencies of the Fourier spectrum of the collision. So we assume that the very many photons in the wave packet are held together in a cubic lattice. It is not unprecedented that photons have been considered to be building blocks of the elementary particles. Schwinger [21] has once studied an exact one-dimensional quantum electrodynamical model in which the photon acquired a mass $\sim e^2$. On the other hand, it has been suggested by Sidharth [22] that the $\pi^0$ meson consists of an electron and a positron which circle their center of mass.

We will now investigate the standing waves of a cubic photon lattice. We assume that the lattice is held together by a weak force acting from one lattice point to the next. We assume that the range of this force is $10^{-16}$ cm, because the range of the weak nuclear force is on the order of $10^{-16}$ cm, according to Perkins [23]. The range of the weak force is of the same magnitude as the uncertainty $\Delta x = a/\pi$ of the location of a wavepacket whose energy $E$ is $\approx \pi \hbar \nu_0 / 2 = \pi / 2 \cdot h c / 2\pi a$, where the energy $E$ is the average energy of the photons in a lattice with the lattice constant $a$, as we will see later. For the sake of simplicity we set the sidelength of the lattice at $10^{-13}$ cm, the exact size of the nucleon is given in [24] and will be used later. With $a = 10^{-16}$ cm there are then $10^9$ lattice points. As we will see the ratios of the masses of the photon lattices are independent of the sidelength of the lattice. Because it is the most simple case, we assume that a central force acts between the lattice points. We cannot consider spin, isospin, strangeness or charm of the particles. The frequency equation for the oscillations of an isotropic monatomic cubic lattice with central forces is, in the one-dimensional case, given by Eq.(7). The direction of the oscillation is determined by the direction of the incoming wave.

According to Eq.(13) of B&K the force constant $\alpha$ is
\[ \alpha = a(c_{11} - c_{12} - c_{44}), \]  
(8)

where \( c_{11}, c_{12}, \) and \( c_{44} \) are the elastic constants in continuum mechanics which applies in the limit \( a \to 0 \). If we consider central forces then \( c_{12} = c_{44} \) which is the classical Cauchy relation. Isotropy requires that \( c_{44} = (c_{11} - c_{12})/2 \). The waves are longitudinal. Transverse waves in a cubic lattice with concentric forces are not possible according to \([19]\). All frequencies that solve Eq.(7) come with either a plus or a minus sign which is, as we will see, important. The reference frequency in Eq.(7) is

\[ \nu_0 = \sqrt{\alpha/4\pi^2m}, \]  
(9)

or as we will see, using Eq.(11), \( \nu_0 = c_*/2\pi a \).

The limitation of the group velocity in the photon lattice has now to be considered. The group velocity is given by

\[ c_g = \frac{d\omega}{dk} = a\sqrt{\alpha/m} \cdot \frac{df(\phi)}{d\phi}. \]  
(10)

The group velocity in the photon lattice has to be equal to the velocity of light \( c_* \) throughout the entire frequency spectrum, because photons move with the velocity of light. In order to learn how this requirement affects the frequency distribution we have to know the value of \( \sqrt{\alpha/m} \) in a photon lattice. But we do not have information about what either \( \alpha \) or \( m \) might be in this case. We assume in the following that \( a\sqrt{\alpha/m} = c_* \), which means, since \( a = 10^{-16} \) cm, that \( \sqrt{\alpha/m} = 3 \cdot 10^{26} \) sec\(^{-1} \), or that the corresponding period is \( \tau = 1/3 \cdot 10^{-26} \) sec, which is the time it takes for a wave to travel with the velocity of light over one lattice distance. With

\[ c_* = a\sqrt{\alpha/m} \]  
(11)

the equation for the group velocity is

\[ c_g = c_* \cdot \frac{df}{d\phi}. \]  
(12)

For a photon lattice that means, since \( c_g \) must then always be equal to \( c_* \), that \( df/d\phi = 1 \). This requirement determines the form of the frequency distribution regardless of the order of the mode of oscillation or it means that instead of the sine function in Eq.(7) the frequency is given by
\[ \nu = \pm \nu_0 [\phi + \phi_0]. \] (13)

For the time being we will disregard \( \phi_0 \) in Eq.(13). The frequencies of the corrected spectrum must increase from \( \nu = 0 \) at the origin \( \phi = 0 \) with slope 1 (in units of \( \nu_0 \)) until the maximum is reached at \( \phi = \pi \). The energy contained in the oscillations (Eq.14) must be proportional to the sum of all frequencies. The second mode of the lattice oscillations contains 4 times as much energy as the basic mode, because the frequencies are twice the frequencies of the basic mode, and there are twice as many oscillations. Adding, by superposition, to the second mode different numbers of basic modes or of second modes will give exact integer multiples of the energy of the basic mode. Now we understand the integer multiple rule of the particles of the \( \gamma \)-branch. There is, in the framework of this theory, on account of Eq.(13), no alternative but integer multiples of the basic mode for the energy contained in the frequencies of the different modes or for superpositions of different modes. In other words, the masses of the different particles are integer multiples of the mass of the \( \pi^0 \) meson, assuming that there is no spin, isospin, strangeness or charm.

We remember that the measured masses in Table 1, which incorporate different spins, isospins, strangeness, and charm spell out the integer multiple rule within on the average 0.65% accuracy. It is worth noting that there is no free parameter if one takes the ratio of the energies contained in the frequency distributions of the different modes, because the factor \( \sqrt{\alpha/m} \) in Eq.(7) cancels. This means, in particular, that the ratios of the frequency distributions, or the mass ratios, are independent of the mass of the photons at the lattice points, as well as of the magnitude of the force between the lattice points.

It is obvious that the integer multiples of the frequencies are only a first approximation of the theory of lattice oscillations and of the mass ratios of the particles. The equation of motion in the lattice (4) does not apply in the eight corners of the cube, nor does it apply to the twelve edges nor, in particular, to the six sides of the cube. A cube with \( 10^9 \) lattice points is not correctly described by the periodic boundary condition we have used, but is what is referred to as a microcrystal. A phenomenological theory of the frequency distributions in microcrystals, considering in particular surface waves, can be found in Chapter 6 of Ghatak and Kothari [19]. Surface waves may account for the small deviations of the mass ratios of the mesons and baryons from the integer multiple rule of the oscillations in a cube. However,
it seems to be futile to pursue a more accurate determination of the oscillation frequencies as long as one does not know what the structure of the electron is, whose mass is 0.378% of the mass of the $\pi^0$ meson and hence is a substantial part of the deviation of the mass ratios from the integer multiple rule.

Let us summarize our findings concerning the particles of the $\gamma$-branch. The $\pi^0$ meson is the basic mode of the photon lattice oscillations. The $\eta$ meson corresponds to the second oscillation mode, as is suggested by $m(\eta) \approx 4m(\pi^0)$. The $\Lambda$ particle corresponds to the superposition of two second modes, as is suggested by $m(\Lambda) \approx 2m(\eta)$. This superposition apparently results in the creation of spin 1/2. The two modes would then have to be coupled. The $\Sigma^0$ and $\Xi^0$ baryons are superpositions of one or two basic modes on the $\Lambda$ particle. The $\Omega^-$ particle corresponds to the superposition of three coupled second modes as is suggested by $m(\Omega^-) \approx 3m(\eta)$. This procedure apparently causes spin 3/2. The charmed $\Lambda_c^+$ particle seems to be the first particle incorporating a third oscillation mode. $\Sigma_c^0$ is apparently the superposition of a negatively charged basic mode on $\Lambda_c^+$, as is suggested by the decay of $\Sigma_c^0$. The easiest explanation of $\Xi_c^0$ is that it is the superposition of two coupled third modes. The superposition of two modes of the same type is, as in the case of $\Lambda$, accompanied by spin 1/2. The $\Omega_c^0$ baryon is apparently the superposition of two basic modes on the $\Xi_c^0$ particle. All neutral particles of the $\gamma$-branch are thus accounted for. The modes of the particles are listed in Table 1.

We have also found the $\gamma$-branch antiparticles, which follow from the negative frequencies which solve Eq.(7). Antiparticles have always been associated with negative energies. Following Dirac’s argument for electrons and positrons, we associate the masses with the negative frequency distributions with antiparticles. We emphasize that the existence of antiparticles is an automatic consequence of our theory.

All particles of the $\gamma$-branch are unstable with lifetimes on the order of $10^{-10}$ sec or shorter. Born [25] has shown that the oscillations in cubic lattices held together by central forces are unstable. It seems, however, to be possible that the particles can be unstable for reasons other than the instability of the lattice. For example, pair production seems to make it possible to understand the decay of the $\pi^0$ meson $\pi^0 \rightarrow e^- + e^+ + \gamma$ (1.198%). Since in our model the $\pi^0$ meson consists of a multitude of electromagnetic waves it seems that pair production takes place within the $\pi^0$ meson, and even more so in the higher modes of the $\gamma$-branch where the electrons and positrons created by pair production tend to settle on mesons, as e.g. in $\eta \rightarrow \pi^+ + \pi^- + \pi^0$ (22.6%) or
in the decay $\eta \to \pi^+ + \pi^- + \gamma$ (4.68%), where the origin of the pair of charges is more apparent. Pair production is also evident in the decays $\eta \to e^+ e^- \gamma$ (0.6%) or $\eta \to e^+ e^- e^+ e^-$ (6.9·$10^{-3}$%).

Finally we must explain the reason for which the photon lattice or the $\gamma$-branch particles are limited in size to a particular value of about $10^{-13}$ cm, as the experiments tell. Conventional lattice theory using the periodic boundary condition does not limit the size of a crystal, and in fact very large crystals exist. If, however, the lattice consists of standing electromagnetic waves the size of the lattice is limited by the radiation pressure. The lattice will necessarily break up at the latest when the outward directed radiation pressure is equal to the inward directed elastic force which holds the lattice together. For details we refer to [26].

4 The mass of the $\pi^0$ meson

So far we have studied the ratios of the masses of the particles. We will now determine the mass of the $\pi^0$ meson in order to validate that the mass ratios link with the actual masses of the particles. The energy of the $\pi^0$ meson is

$$E(m(\pi^0)) = 134.9766 \text{ MeV} = 2.1626 \cdot 10^{-4} \text{ erg}.$$ 

For the sum of the energies of the frequencies of all standing one-dimensional waves in $\pi^0$ we use the equation

$$E_\nu = \frac{Nh\nu_0}{2\pi} \int_{-\pi}^{\pi} f(\phi) d\phi. \quad (14)$$

This equation originates from B&K. $N$ is the number of all lattice points. The total energy of the frequencies in a cubic lattice is equal to the number $N$ of the oscillations times the average of the energy of the individual frequencies. In order to arrive at an exact value of Eq.(14) we have to use the correct value of the radius of the proton, which is $r_p = (0.880 \pm 0.015) \cdot 10^{-13}$ cm according to [24] or $r_p = (0.883 \pm 0.014) \cdot 10^{-13}$ cm according to [27]. With $a = 10^{-16}$ cm it follows that the number of all lattice points in the cubic lattice is

$$N = 2.854 \cdot 10^9.$$ 

The radius of the $\pi^\pm$ mesons has also been measured [28] and after further analysis [29] was found to be $0.83 \cdot 10^{-13}$ cm, which means that within the
uncertainty of the radii we have $r_p = r_\pi$. And according to [30] the charge radius of $\Sigma^-$ is $(0.78 \pm 0.10) \cdot 10^{-13}$ cm.

If the oscillations are parallel to an axis, the limitation of the group velocity is taken into account, that means if Eq.(13) applies and the absolute values of the frequencies are taken, then the value of the integral in Eq.(14) is $\pi^2$. With $N = 2.854 \cdot 10^9$ and $\nu_0 = c_\star / 2\pi a$ it follows from Eq.(14) that the sum of the energy of the frequencies corrected for the group velocity limitation of the basic mode is $E_{corr} = 1.418 \cdot 10^9$ erg. That means that the energy is $6.56 \cdot 10^{12}$ times larger than $E(m(\pi^0))$. This discrepancy is inevitable, because the basic frequency of the Fourier spectrum after a collision on the order of $10^{-23}$ sec duration is $\nu = 10^{23}$ sec$^{-1}$, which means, when $E = h\nu$, that one basic frequency alone contains an energy of about $9 m(\pi^0)c^2$.

To eliminate this discrepancy we use, instead of the simple form $E = h\nu$, the complete quantum mechanical energy of a linear oscillator as given by Planck

$$E = \frac{h\nu}{e^{h\nu/kT} - 1}. \quad (15)$$

This equation was already used by B&K for the determination of the specific heat of cubic crystals or solids. Equation (15) calls into question the value of the temperature $T$ in the interior of a particle. We determine $T$ empirically with the formula for the internal energy of solids

$$u = \frac{R\Theta}{e^{\Theta/T} - 1}, \quad (16)$$

which is from Sommerfeld [31]. In this equation $R = 2.854 \cdot 10^9$ k, where $k$ is Boltzmann’s constant, and $\Theta$ is the characteristic temperature introduced by Debye [32] for the explanation of the specific heat of solids. It is $\Theta = h\nu_m/k$, where $\nu_m$ is a maximal frequency. In the case of the oscillations making up the $\pi^0$ meson the maximal frequency is $\nu_m = \pi\nu_0$, therefore $\nu_m = 1.5 \cdot 10^{26}$ sec$^{-1}$, and we find that $\Theta = 7.2 \cdot 10^{15}$ K.

In order to determine $T$ we set the internal energy $u$ equal to $m(\pi^0)c^2$. It then follows from Eq.(16) that $\Theta/T = 30.20$, or $T = 2.38 \cdot 10^{14}$ K. That means that Planck’s formula (15) introduces a factor $1/(e^{\Theta/T} - 1) \approx 1/e^{30.2} = 1/(1.305 \cdot 10^{13})$ into Eq.(14). In other words, if we determine the temperature $T$ of the particle through Eq.(16), and correct Eq.(14) accordingly then we arrive at a sum of the oscillation energies of the $\pi^0$ meson which is $1.0866 \cdot 10^{-4}$ erg = 67.82 MeV. That means that the sum of the energies
of the one-dimensional oscillations consisting of N waves is $0.502E(m(\pi^0))$. We have to double this amount because standing waves consist of two waves traveling in opposite direction with the same absolute value of the frequency. The sum of the energy of the oscillations in the $\pi^0$ meson is therefore

$$E_{\nu}(\pi^0)(\text{theor}) = 2.1732 \cdot 10^{-4} \text{ erg} = 135.64 \text{ MeV} = 1.005E(m(\pi^0))(\text{exp})$$

(17)

if the oscillations are parallel to the $\phi$ axis. The energy in the measured mass of the $\pi^0$ meson and the energy in the sum of the oscillations agree fairly well, considering the uncertainties of the parameters involved.

To sum up: We find that the energy in the rest mass of the $\pi^0$ meson and the other particles of the $\gamma$-branch are correctly given by the sum of the energy of standing electromagnetic waves in a cube, if the energy of the oscillations is determined by Planck’s formula for the energy of a linear oscillator. The $\pi^0$ meson is like an adiabatic, cubic black body filled with standing electromagnetic waves. We know from Bose’s work [33] that Planck’s formula applies to a photon gas as well. For all $\gamma$-branch particles we have found a simple mode of standing waves in a cubic lattice. Since the equation determining the frequency of the standing waves is quadratic it follows automatically that for each positive frequency there is also a negative frequency of the same absolute value, that means that for each particle there exists also an antiparticle. For the explanation of the mesons and baryons of the $\gamma$-branch we use only photons, nothing else. A rather conservative explanation of the $\pi^0$ meson and the $\gamma$-branch particles which does not use hypothetical particles.

From the frequency distributions of the standing waves in the lattice follow the ratios of the masses of the particles which obey the integer multiple rule. It is important to note that in this theory the ratios of the masses of the $\gamma$-branch particles to the mass of the $\pi^0$ meson do not depend on the sidelength of the lattice, and the distance between the lattice points, neither do they depend on the strength of the force between the lattice points nor on the mass of the lattice points. The mass ratios are determined only by the spectra of the frequencies of the standing waves in the lattice.

5 The neutrino branch particles

The masses of the neutrino branch, the $\pi^\pm$, $K^{\pm,0}$, $n$, $D^{\pm,0}$ and $D_s^{\pm}$ particles, are integer multiples of the mass of the $\pi^\pm$ mesons times a factor $0.85 \pm 0.02$ as we
stated before. We assume, as appears to be quite natural, that the $\pi^\pm$ mesons and the other particles of the $\nu$-branch consist of the same particles into which they decay, that means of neutrinos and antineutrinos and of electrons or positrons, particles whose existence is unquestionable. Since the particles of the $\nu$-branch decay through weak decays, we assume, as appears likewise to be natural, that the weak nuclear force holds the particles of the $\nu$-branch together. The existence of the weak nuclear force is also unquestionable. Since the range of the weak interaction is only about a thousandth of the diameter of the particles, the weak force can hold particles together only if the particles have a lattice structure, just as macroscopic crystals are held together by microscopic forces between atoms. In the absence of a force which originates in the center of the particle and affects all neutrinos of the particle the configuration of the particle is not spherical but cubic, reflecting the very short range of the weak nuclear force. We will now investigate the energy which is contained in the oscillations of a cubic lattice consisting of electron and muon neutrinos and their antiparticles, and in their rest masses.

It will be necessary to outline the basic aspects of diatomic lattice oscillations. In diatomic lattices the lattice points have alternately the masses $m$ and $M$, as with the masses of the electron neutrinos $m(\nu_e)$ and muon neutrinos $m(\nu_\mu)$. The classic example of a diatomic lattice is the salt crystal with the masses of the Na and Cl atoms in the lattice points. The theory of diatomic lattice oscillations was started by Born and v. Karman [14]. They first discussed a diatomic chain. The equation of motions in the chain are according to Eq.(22) of B&K

$$m\ddot{u}_{2n} = \alpha(2u_{2n+1} + u_{2n-1} - 2u_{2n}), \quad \text{(18)}$$

$$M\ddot{u}_{2n+1} = \alpha(u_{2n+2} + 2u_{2n} - 2u_{2n+1}), \quad \text{(19)}$$

where the $u_n$ are the displacements, $n$ an integer number and $\alpha$ a constant characterizing the force between the particles. Eqs.(18,19) are solved with

$$u_{2n} = Ae^{i(\omega t + 2n\phi)}, \quad \text{(20)}$$

$$u_{2n+1} = Be^{i(\omega t + (2n+1)\phi)}, \quad \text{(21)}$$

where $A$ and $B$ are constants and $\phi$ is given by $\phi = 2\pi a/\lambda$ as in (6). $a$ is the lattice constant as before and $\lambda$ the wavelength, $\lambda = na$. The solutions
of Eqs.(20,21) are obviously periodic in time and space and describe again standing waves. Using (20,21) to solve (18,19) leads to a secular equation from which according to Eq.(24) of B&K the frequencies of the oscillations of the chain follow from

\[ 4\pi^2\nu^2_z = \frac{\alpha}{M \cdot m} \cdot ((M + m) \pm \sqrt{(M - m)^2 + 4mM \cos^2 \phi}). \]  

(22)

Longitudinal and transverse waves are distinguished by the minus or plus sign in front of the square root in (22).

6 The masses of the \( \nu \)-branch particles

The characteristic case of the neutrino branch particles are the \( \pi^\pm \) mesons which can be created in the process \( \gamma + p \rightarrow \pi^- + \pi^+ + p \). A photon impinges on a proton and is converted in \( 10^{-23} \) sec into a pair of particles of opposite charge. Fourier analysis dictates that a continuum of frequencies must be in the collision products. The waves must be standing waves in order to be part of the rest mass of a particle. The \( \pi^\pm \) mesons decay via \( \pi^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu) \) (99.98770\%) followed by e.g. \( \mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \) (\( \approx 100\% \)). Only \( \mu \) mesons, which decay into charge and neutrinos, and neutrinos result from the decay of the \( \pi^\pm \) mesons. If the particles consist of the particles into which they decay, then the \( \pi^\pm \) mesons are made of neutrinos, antineutrinos and \( e^\pm \). Since neutrinos interact through the weak force which has a range of \( 10^{-16} \) cm according to p.25 of [23], and since the size of the nucleon is on the order of \( 10^{-13} \) cm, the \( \nu \)-branch particles must be held together in a lattice. It is not known with certainty that neutrinos actually have a rest mass as was originally suggested by Bethe [34] and Bahcall [35] and what the values of \( m(\nu_e) \) and \( m(\nu_\mu) \) are. However, the results of the Super-Kamiokande [36] and the Sudbury [37] experiments indicate that the neutrinos have rest masses. The neutrino lattice must be diatomic, meaning that the lattice points have alternately larger (\( m(\nu_\mu) \)) and smaller (\( m(\nu_e) \)) masses. We will retain the traditional term diatomic. The term neutrino lattice will refer to a lattice consisting of neutrinos and antineutrinos. The lattice we consider is shown in Fig. 2. Since the neutrinos have spin 1/2 this is a four-Fermion lattice. The first investigation of cubic Fermion lattices in context with the elementary particles was made by Wilson [13]. A neutrino lattice is electrically neutral. Since we do not know the structure of the electron we cannot consider lattices with a charge.
A neutrino lattice takes care of the continuum of frequencies which must, according to Fourier analysis, be present after the high energy collision which created the particle. We will, for the sake of simplicity, first set the sidelength of the lattice at $10^{-13}$ cm that means approximately equal to the size of the nucleon. The lattice then contains about $10^9$ lattice points, since the lattice constant $a$ is on the order of $10^{-16}$ cm. The sidelength of the lattice does not enter Eq.(22) for the frequencies of the lattice oscillations. The calculation of the ratios of the masses is consequently independent of the size of the lattice, as was the case with the $\gamma$-branch. The size of the lattice can be explained with the pressure which the lattice oscillations exert on a crosssection of the lattice. The pressure cannot exceed Young’s modulus of the lattice. We require that the lattice is isotropic.

From the frequency distribution of the axial diatomic oscillations (Eq.22), shown in Fig. 3, follows the group velocity $d\omega/dk = 2\pi a \frac{d\nu}{d\phi}$ at each point $\phi$. With $\nu = \nu_0 f(\phi)$ and $\nu_0 = \sqrt{\alpha/4\pi^2M} = c_4/2\pi a$ as in Eq.(9) we find

$$c_g = \frac{d\omega}{dk} = a\sqrt{\alpha/M} \cdot \frac{df(\phi)}{d\phi}.$$  \hspace{1cm} (23)

In order to determine the value of $d\omega/dk$ we have to know the value of $\sqrt{\alpha/M}$. From Eq.(8) for $\alpha$ follows that $\alpha = ac_{44}$ in the isotropic case with central forces. The group velocity is therefore
Fig. 3: The frequency distribution $\nu_{-}/\nu_0$ of the basic diatomic mode according to Eq.(22) with $M/m = 100$. The dashed line shows the distribution of the frequencies corrected for the group velocity limitation.

\[
c_g = \sqrt{a^3 c_{44}/M \cdot df/d\phi}.
\]  

(24)

We now set $a\sqrt{\alpha/M} = c_*$ as in Eq.(11), where $c_*$ is the velocity of light. It follows that

\[
c_g = c_* \cdot df/d\phi,
\]  

(25)

as it was with the $\gamma$-branch, only that now on account of the rest masses of the neutrinos the group velocity must be smaller than $c_*$, so the value of $df/d\phi$ is limited to $< 1$, but $c_g \cong c_*$, which is a necessity because the neutrinos in the lattice soon approach the velocity of light as we will see. Equation (25) applies regardless whether we consider $\nu_+$ or $\nu_-$ in Eq.(22). That means that there are no separate transverse oscillations with their theoretically higher frequencies.
The rest mass \( M \) of the heavy neutrino can be determined with lattice theory from Eq. (24) as we have shown in [12]. This involves the inaccurately known compression modulus of the proton. We will, therefore, rather determine the rest mass of the muon neutrino with Eq. (27), which leads to \( m(\nu_\mu) \approx 50 \text{ milli-eV/c}^2 \). It can be verified easily that \( m(\nu_\mu) = 50 \text{ meV/c}^2 \) makes sense. The energy of the rest mass of the \( \pi^\pm \) mesons is 139 MeV, and we have \( N/4 = 0.7135 \cdot 10^9 \) muon neutrinos and the same number of anti-muon neutrinos. It then follows that the energy in the rest masses of all muon and anti-muon neutrinos is 71.35 MeV, that is 51% of the energy of the rest mass of the \( \pi^\pm \) mesons, \( m(\pi^\pm)c^2 = 139.57 \text{ MeV} \). A small part of \( m(\pi^\pm)c^2 \) goes, as we will see, into the electron neutrino masses, the rest goes into the lattice oscillations.

The rest mass of the \( \pi^\pm \) mesons is the sum of the oscillation energies and the sum of the rest masses of the neutrinos. For the sum of the energies of the frequencies we use Eq. (14) with the same \( N \) and \( \nu_0 \) we used for the \( \gamma \)-branch. For the integral in Eq. (14) of the corrected axial diatomic frequencies we find the value \( \pi^2/2 \) as can be easily derived from the plot of the corrected frequencies in Fig. 3. The value of the integral in Eq. (14) for the axial diatomic frequencies \( \nu = \nu_0 \phi \) is \( 1/2 \) of the value \( \pi^2 \) of the same integral in the case of axial monatomic frequencies, because in the latter case the increase of the corrected frequencies continues to \( \phi = \pi \), whereas in the diatomic case the increase of the corrected frequencies ends at \( \pi/2 \), see Fig. 3. We consider \( c_g \) to be so close to \( c_\star \) that it does not change the value of the integral in Eq. (14) significantly. It can be calculated that the time average of the velocity of the electron neutrinos in the \( \pi^\pm \) mesons is \( \bar{v} = 0.99994c_\star \), if \( m(\nu_e) = 0.365 \text{ milli-eV/c}^2 \) as will be shown in Eq. (33). Consequently we find that the sum of the energies of the corrected diatomic neutrino frequencies is \( 0.5433 \cdot 10^{-4} \text{ erg} = 33.91 \text{ MeV} \). We double this amount because we deal with the superposition of two waves of the same energy and find that the energy of the neutrino oscillations in \( \pi^\pm \) is

\[
E_\nu(\pi^\pm) = 67.82 \text{ MeV} = 1/2 \cdot E_\nu(\pi^0) = 0.486 m(\pi^\pm)c_\star^2. \tag{26}
\]

In order to determine the sum of the rest masses of the neutrinos we make use of \( E_\nu(\pi^\pm) \) and obtain an approximate value of the rest mass of the muon neutrino from

\[
m(\pi^\pm)c_\star^2 - E_\nu(\pi^\pm) = \sum [m(\nu_\mu) + m(\bar{\nu}_\mu) + m(\nu_e) + m(\bar{\nu}_e)]c_\star^2 = 71.75 \text{ MeV} . \tag{27}
\]
If \( m(\nu_e) \ll m(\nu_\mu) \) and \( m(\nu_\mu) = m(\bar{\nu}_\mu) \), as we will justify later, we arrive with \( N/2 = 1.427 \times 10^9 \) at

\[
m(\nu_\mu) \approx 50 \text{ milli-eV/c}^2.
\]

The sum of the energy of the rest masses of all neutrinos Eq.(27) plus the oscillation energy Eq.(26) gives the theoretical rest mass of the \( \pi^\pm \) mesons which is, since we used \( m(\pi^\pm) \) in the determination of the neutrino rest masses with Eq.(27), equal to the experimental rest mass of 139.57 MeV.

A cubic lattice and conservation of neutrino numbers during the reaction \( \gamma + p \to \pi^+ + \pi^- + p \) necessitates that the \( \pi^+ \) and \( \pi^- \) lattices contain just as many electron neutrinos as anti-electron neutrinos. If the lattice has six equal sides there must also be a center neutrino in each lattice. Conservation of neutrino numbers requires furthermore that the center neutrino of, say, \( \pi^+ \) is matched by an antineutrino in \( \pi^- \). However, in the decay sequence of (say) the \( \pi^- \) meson \( \pi^- \to \mu^- + \bar{\nu}_\mu \) and \( \mu^- \to e^- + \nu_\mu + \bar{\nu}_e \) an electron neutrino does not appear. But since \( (N-1)/4 \) electron neutrinos \( \nu_e \) must be in the \( \pi^- \) lattice \( (N-1)/4 \) electron neutrinos must go with the electron emitted in the \( \mu^- \) decay. Whether or not this interpretation is correct can be decided only after the explanation of the structure of the electron.

The antiparticle of the \( \pi^+ \) meson is the particle in which all frequencies of the neutrino lattice oscillations have been replaced by frequencies with the opposite sign, all neutrinos replaced by their antiparticles and the positive charge replaced by the negative charge. If, as we will show, the antineutrinos have the same rest mass as the neutrinos it follows that the antiparticle of the \( \pi^+ \) meson has the same mass as \( \pi^+ \) but opposite charge, i.e. is the \( \pi^- \) meson.

The primary decay of the \( K^\pm \) mesons \( K^\pm \to \mu^\pm + \nu_\mu (\bar{\nu}_\mu) \) (63.5%), leads to the same end products as the \( \pi^\pm \) meson decay \( \pi^\pm \to \mu^\pm + \nu_\mu (\bar{\nu}_\mu) \). From this and the composition of the \( \mu \) mesons we learn that the \( K \) mesons must, at least partially, be made of the same four neutrino types as in the \( \pi^\pm \) mesons namely of muon neutrinos, anti-muon neutrinos, electron neutrinos and anti-electron neutrinos and their oscillation energies. However the \( K^\pm \) mesons cannot be solely the second mode of the lattice oscillations of the \( \pi^\pm \) mesons, because the second mode of the \( \pi^\pm \) mesons has an energy of \( 4E_\nu(\pi^\pm) + N/2 \cdot (m(\nu_\mu) + m(\nu_e)) c^2 \approx (271.3 + 71.75) \text{MeV} = 343 \text{MeV} \). The 343 MeV characterize the second or (2.) mode of the \( \pi^\pm \) mesons, which fails \( m(K^\pm)c^2 = 493.7 \text{MeV} \) by a wide margin.

Anyway, the concept of the \( K^\pm \) mesons being alone a higher mode of the \( \pi^\pm \) mesons contradicts our point that the particles consist of the particles into which they decay. The decays \( K^\pm \to \pi^\pm + \pi^0 \) (21.13%), as well as \( K^+ \)
\[ \pi^0 + e^+ + \nu_e \ (4.87\%), \text{ called } K^+_{\mu 3}, \text{ and } K^+ \rightarrow \pi^0 + \mu^+ + \nu_\mu \ (3.27\%), \text{ called } K^+_{\mu 3}, \text{ make up } 29.27\% \text{ of the } K^\pm \text{ meson decays. A } \pi^0 \text{ meson figures in each of these decays. If we add the energy in the rest mass of a } \pi^0 \text{ meson } m(\pi^0)c^2 = 134.97 \text{ MeV to the } 343 \text{ MeV in the second mode of the } \pi^\pm \text{ mesons then we arrive at an energy of } 478 \text{ MeV, which is } 96.8\% \text{ of } m(K^\pm)c^2. \text{ Therefore we conclude that the } K^\pm \text{ mesons consist of the second mode of the } \pi^\pm \text{ mesons plus a } \pi^0 \text{ meson or are the state } (2.) \pi^\pm + \pi^0. \text{ Then it is natural that } \pi^0 \text{ mesons from the } \pi^0 \text{ component in the } K^\pm \text{ mesons are among the decay products of the } K^\pm \text{ mesons.}

We obtain the } K^0 \text{ meson if we superpose onto the second mode of the } \pi^\pm \text{ mesons instead of a } \pi^0 \text{ meson a basic mode of the } \pi^\pm \text{ mesons with a charge opposite to the charge of the second mode of the } \pi^\pm \text{ meson. The } K^0 \text{ and } \overline{K^0} \text{ mesons, or the state } (2.) \pi^\pm + \pi^\mp, \text{ is made of neutrinos and antineutrinos only, without a photon component, because the second mode of } \pi^\pm \text{ as well as the basic mode } \pi^\mp \text{ consist of neutrinos and antineutrinos only. The } K^0 \text{ meson has a measured mean square charge radius } \langle r^2 \rangle = -0.076 \pm 0.021 \text{ fm}^2 \text{ according to } [38], \text{ which can only be if there are two charges of opposite sign within } K^0 \text{ as our model postulates. Since the mass of a } \pi^\pm \text{ meson is by } 4.59 \text{ MeV/c}^2 \text{ larger than the mass of a } \pi^0 \text{ meson the mass of } K^0 \text{ should be larger than } m(K^\pm), \text{ and indeed } m(K^0) - m(K^\pm) = 3.995 \text{ MeV/c}^2 \text{ according to } [2]. \text{ Similar differences occur with } m(D^\pm) - m(D^0) \text{ and } m(\Xi^0) - m(\Xi^\pm). \text{ The decay } K^0_S \rightarrow \pi^+ + \pi^- \ (68.6\%) \text{ creates directly the } \pi^+ \text{ and } \pi^- \text{ mesons which are part of the } (2.) \pi^\pm + \pi^\mp \text{ structure of } K^0 \text{ we have suggested. The decay } K^0_S \rightarrow \pi^0 + \pi^0 \ (31.4\%) \text{ apparently originates from the } 2\gamma \text{ branch of electron positron annihilation. Both decays account for } 100\% \text{ of the decays of } K^0_S. \text{ The decay } K^0_L \rightarrow 3 \pi^0 \ (21.1\%) \text{ apparently comes from the } 3\gamma \text{ branch of electron positron annihilation. The two decays of } K^0_L \text{ called } K^0_{\mu 3} \text{ into } \pi^\pm \mu^\mp \nu_\mu \ (27.18\%) \text{ and } K^0_{e 3} \text{ into } \pi^\pm e^\mp \nu_e \ (38.79\%) \text{ which together make up } 65.95\% \text{ of the } K^0_L \text{ decays apparently originate from the decay of the second mode of the } \pi^\pm \text{ mesons in the } K^0 \text{ structure, either into } \mu^\mp + \nu_\mu \text{ or into } e^\mp + \nu_e. \text{ The same types of decay, apparently tied to the } (2.) \pi^\pm \text{ mode, accompany also the } K^\pm \text{ decays where, however, a } \pi^0 \text{ meson replaces the } \pi^\pm \text{ mesons in the } K^0_L \text{ decay products. Our rule that the particles consist of the particles into which they decay also holds for the } K^0 \text{ and } \overline{K^0} \text{ mesons. The explanation of the } K^0, \overline{K^0} \text{ mesons with the state } (2.) \pi^\pm + \pi^\mp \text{ confirms that the state } (2.) \pi^\pm + \pi^0 \text{ was the correct choice for the explanation of the } K^\pm \text{ mesons. The state } (2.) \pi^\pm + \pi^\mp \text{ is also crucial for the explanation of the absence of spin of the } K^0, \overline{K^0} \text{ mesons, as we will see later.}
The neutron with a mass $\approx 2m(K^\pm)$ is either the superposition of a $K^+$ and a $K^-$ meson or of a $K^0$ meson and a $\bar{K}^0$ meson. As we will see, the spin rules out a neutron consisting of a $K^+$ and a $K^-$ meson. On the other hand, the neutron can consist of a $K^0$ and a $\bar{K}^0$ meson. In this case the neutron lattice contains at each lattice point a $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrino quadrupole plus the second mode of the lattice oscillations and a quadrupole of positive and negative electrical charges. The lattice oscillations in the neutron must be coupled in order for the neutron to have spin 1/2, just as the $\Lambda$ baryon with spin 1/2 is a superposition of two $\eta$ mesons. With $m(K^0)(\text{theor}) = m(K^\pm) + 4\,\text{MeV}/c^2 = 482\,\text{MeV}/c^2$ from above it follows that $m(n)(\text{theor}) \approx 2m(K^0)(\text{theor}) \approx 964\,\text{MeV}/c^2 = 1.026m(n)(\exp)$.

The proton does not decay and does not tell which particles it is made of. However, we learn about the structure of the proton through the decay of the neutron $n \to p + e^- + \bar{\nu}_e$ (100%). One single anti-electron neutrino is emitted when the neutron decays and 1.293 MeV are released. But there is no place for a permanent vacancy of a single missing neutrino and for a small amount of permanently missing oscillation energy in a nuclear lattice. As it appears all anti-electron neutrinos are removed from the structure of the neutron in the neutron decay and converted into the kinetic energy of the decay products. This type of process will be discussed again in the following section. On the other hand, it seems to be certain that the proton consists of a neutrino lattice carrying a net positive electric charge. Actually, in our model the proton contains three charges $e^+e^-e^+$. The concept that the proton carries just one electrical charge has been abandoned a long time ago when it was said that the proton consists of three quarks carrying fractional electrical charges. Each elementary charge in the proton has a magnetic moment, all of them point in the same direction because the spin of the one $e^-$ must be opposite to the spin of the two $e^+$. Each magnetic moment of the elementary charges has a g-factor $\approx 2$. All three electric charges in the proton must then have a g-factor $\approx 6$, whereas the measured $g(p) = 5.585 = 0.936$.

The $D^\pm$ mesons with $m(D^\pm) = 0.9954\,(m(p) + m(\bar{n}))$ are the superposition of a proton and an antineutron of opposite spin or of their antiparticles, whereas the superposition of a proton and a neutron with the same spin creates the deuteron with spin 1 and a mass $m(d) = 0.9988\,(m(p) + m(n))$. In this case the proton and neutron interact with the strong force, nevertheless the deuteron consists of a neutrino lattice with standing waves. The $D_s^\pm$ mesons seem to be made of a body centered cubic lattice as discussed in
The average factor $0.85 \pm 0.025$ in the ratios of the particles of the $\nu$-branch to the $\pi^\pm$ mesons is a consequence of the neutrino rest masses. They make it impossible that the ratios of the particle masses are integer multiples because the particles consist of the energy in the neutrino oscillations plus the neutrino rest masses which are independent of the order of the lattice oscillations. Since the contribution in percent of the neutrino rest masses to the $\nu$-branch particle masses decreases with increased particle mass the factor in front of the mass ratios of the $\nu$-branch particles must decrease with increased particle mass.

Summing up: The characteristic feature of the $\nu$-branch particles is the cubic lattice consisting of $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrinos. The rest masses of the $\nu$-branch particles comes from the sum of the rest masses of the neutrinos and antineutrinos and from the energy in the lattice oscillations. The existence of the neutrino lattice is a necessity if one wants to explain the spin, or the absence of spin, of the $\nu$-branch particles. For the explanation of the $\nu$-branch particles we do not use hypothetical particles either.

7 The rest masses of the leptons

Surprisingly one can also explain the mass of the $\mu^\pm$ mesons with the standing wave model. The $\mu$ mesons are part of the lepton family which is distinguished from the mesons and baryons by the absence of strong interaction with the mesons and baryons. The leptons make up 1/2 of the number of stable elementary particles. The standard model of the particles does not deal with the lepton masses. Barut [40] has given a simple and quite accurate empirical formula relating the masses of the electron, $\mu$ meson and $\tau$ meson, which formula has been extended by Gsponer and Hurni [41] to the quark masses.

The mass of the $\mu$ mesons is $m(\mu^\pm) = 105.658357 \pm 5 \cdot 10^{-6}$ MeV/c$^2$, according to the Review of Particle Physics [2]. The mass of the $\mu$ mesons is usually compared to the mass of the electron and is often said to be $m(\mu) = m(e)(1 + 3/2\alpha_f) = 206.554$ m(e), (a$\alpha_f$ being the fine structure constant), whereas the experimental value is 206.768 m(e). The $\mu$ mesons are “stable”, their lifetime is $\tau(\mu^\pm) = 2.19703 \cdot 10^{-6} \pm 4 \cdot 10^{-11}$ sec, about a hundred times longer than the lifetime of the $\pi^\pm$ mesons, that means longer than the lifetime of any other elementary particle, but for the electrons, protons and neutrons.
Comparing the mass of the $\mu$ mesons to the mass of the $\pi^\pm$ mesons $m(\pi^\pm) = 139.57018 \text{ MeV}/c^2$ we find that $m(\mu^\pm)/m(\pi^\pm) = 0.757027 = 1.00937 \cdot 3/4$ or that the mass of the $\mu^\pm$ mesons is in a good approximation $3/4$ of the mass of the $\pi^\pm$ mesons. We have also $m(\pi^\pm) - m(\mu^\pm) = 33.9118 \text{ MeV}/c^2 = 0.24297 m(\pi^\pm)$ or approximately $1/4 \cdot m(\pi^\pm)$. The mass of the electron is approximately $1/206$ of the mass of the muon, the contribution of $m(e^\pm)$ to $m(\mu^\pm)$ will therefore be neglected in the following. We assume, as we have done before and as appears to be natural, that the particles, including the muons, consist of the particles into which they decay. The $\mu^+$ meson decays via $\mu^+ \rightarrow e^+ + \bar{\nu}_\mu + \nu_e \left( \approx 100\% \right)$. The muons are apparently composed of some of the neutrinos, antineutrinos and their oscillations which are present in the cubic neutrino lattice of the $\pi^\pm$ mesons according to our standing wave model.

In the standing wave model the $\pi^\pm$ mesons are composed of a cubic lattice consisting of $N = 2.854 \cdot 10^9$ neutrinos and antineutrinos. We must now be more specific about $N$, which is an odd number, because a lattice with six sides of equal composition has a center particle, just as the NaCl lattice. In the $\pi^\pm$ mesons there are then $(N - 1)/4$ muon neutrinos $\nu_\mu$ and the same number of anti-muon neutrinos $\bar{\nu}_\mu$, as well as $(N - 1)/4$ electron neutrinos $\nu_e$ and the same number of anti-electron neutrinos $\bar{\nu}_e$, plus a center neutrino or antineutrino. We replace $N - 1$ by $N'$. Since $N'$ differs from $N$ by only one in $10^9$ we have $N' \cong N$. Although the numerical difference between $N$ and $N'$ is negligible we cannot consider e.g. $N/4$ neutrinos because that would mean that there would be fractions of a neutrino. $N'$ is an integer multiple of 4, because of the equal numbers of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ neutrinos.

From Eq.(27) followed that the rest mass of a muon neutrino should be about 50 milli-eV/$c^2$. Provided that the mass of an electron neutrino is small as compared to $m(\nu_\mu)$ we find, with $N = 2.854 \cdot 10^9$, that:

(a) The difference of the rest masses of the $\mu^\pm$ and $\pi^\pm$ mesons is nearly equal to the sum of the rest masses of all muon, respectively anti-muon, neutrinos in the $\pi^\pm$ mesons.

$$m(\pi^\pm) - m(\mu^\pm) = 33.912 \text{ MeV}/c^2 \quad \text{versus} \quad N'/4 \cdot m(\nu_\mu) \approx 35.68 \text{ MeV}/c^2.$$

(b) The energy in the oscillations of all $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ neutrinos in the $\pi^\pm$ mesons is nearly the same as the energy in the oscillations of all $\bar{\nu}_\mu, \nu_e, \bar{\nu}_e$, respectively $\nu_\mu, \nu_e, \bar{\nu}_e$, neutrinos in the $\mu^\pm$ mesons. The oscillation energy is
the rest mass of a particle minus the sum of the rest masses of all neutrinos in the particle as in Eq.(27). So

\[ E_\nu(\pi^\pm) = m(\pi^\pm)c^2_\ast - N'/2 \cdot [m(\nu_\mu) + m(\nu_e)]c^2_\ast = 68.22 \text{ MeV} \quad (28) \]

versus

\[ E_\nu(\mu^\pm) = m(\mu^\pm)c^2_\ast - N'/4 \cdot m(\nu_\mu)c^2_\ast - N'/2 \cdot m(\nu_e)c^2_\ast = 69.98 \text{ MeV} \quad (29) \]

(a) seems to say that the energy of the rest masses of all muon (respectively anti-muon) neutrinos of the lattice is consumed in the \( \pi^\pm \) decay. We attribute the 1.768 MeV difference between the left and right side of (a) to the second order effects which cause the deviations of the masses of the particles from the integer multiple rule. There is also the difference that the left side of (a) deals with two charged particles, whereas the right side deals with neutral particles. (b) seems to say that the oscillation energy of all neutrinos in the \( \pi^\pm \) lattice is conserved in the \( \pi^\pm \) decay, which seems to be necessary because the oscillation frequencies in \( \pi^\pm \) and \( \mu^\pm \) must follow Eq.(13) as dictated by the group velocity limitation. If indeed

\[ E_\nu(\pi^\pm) = E_\nu(\mu^\pm) \quad (30) \]

then it follows from the difference of Eqs.(28) and (29) that

\[ m(\pi^\pm) \text{ or } m(\mu^\pm) = N'/4 \cdot m(\nu_\mu). \]

We should note that in the \( \pi^\pm \) decays only one single muon neutrino is emitted, not \( N'/4 \) of them, but that in the \( \pi^\pm \) decay 33.9 MeV are released. Since according to (b) the oscillation energy of the neutrinos in the \( \pi^\pm \) mesons is conserved in their decay the 33.9 MeV released in the \( \pi^\pm \) decay can come from no other source then from the rest masses of the muon or anti-muon neutrinos in the \( \pi^\pm \) mesons. The energy in the rest masses of these muon neutrinos is used to supply the kinetic energy in the momentum of the single emitted muon neutrino \( (pc_\ast = 30 \text{ MeV}) \) and in the momentum of the emitted \( \mu \) meson \( (pc_\ast = 4 \text{ MeV}) \). The average energy of the neutrinos in the \( \pi^\pm \) lattice is about 50 milli-eV, so it is not possible for a single neutrino in \( \pi^\pm \) to possess an energy of 33.9 MeV. The 33.9 MeV can come only from the sum of the muon neutrino rest masses. However, what happens then to the neutrino numbers? Either conservation of neutrino numbers is violated or the decay energy comes from equal numbers of muon and anti-muon neutrinos. Equal numbers \( N'/8 \) muon and anti-muon neutrinos would then be
in the $\mu^\pm$ mesons. This would not make a difference in either the oscillation energy or in the sum of the rest masses of the neutrinos or in the spin of the $\mu^\pm$ mesons. The sum of the spin vectors of the $N'/4$ muon or anti-muon neutrinos converted into kinetic energy is zero, as will become clear in Section 9.

Inserting $m(\pi^\pm) - m(\mu^\pm) = N'/4m(\nu_\mu)$ from (a) into Eq.(28) we arrive at an equation for the theoretical value of the mass of the $\mu^\pm$ mesons. It is

$$m(\mu^\pm)c^2_\star = \frac{1}{2} \cdot [E_\nu(\pi^\pm) + m(\pi^\pm)c^2_\star + N'm(\nu_e)c^2_\star/2] = 103.95 \text{ MeV} ,$$

(31)

which expresses $m(\mu^\pm)$ through the well-known mass of $\pi^\pm$, the calculated oscillation energy of $\pi^\pm$, and a small contribution (0.4%) of the electron neutrino and anti-electron neutrino masses. A different form of this equation is, with $E_\nu(\pi^\pm) = E_\nu(\mu^\pm)$,

$$m(\mu^\pm) = E_\nu(\mu^\pm)c^2_\star + N'm(\nu_\mu)/4 + N'[m(\nu_e) + m(\bar{\nu}_e)]/4 .$$

(32)

Eq.(31) shows that our explanation of the mass of the $\mu^\pm$ mesons comes close to the experimental value $m(\mu^\pm) = 105.658 \text{ MeV}/c^2_\star$.

Our model of the $\mu^\pm$ mesons means that the $\mu$ mesons have the same size as the $\pi^\pm$ mesons, namely $0.88 \cdot 10^{-13} \text{ cm}$. This contradicts the commonly held belief that the $\mu$ mesons are point particles. However, since in our model the $\mu$ mesons consist of a neutrino lattice plus an electric charge and since neutrinos do not interact, in a good approximation, with charge or mass it will not be possible to determine the size of the $\mu$ meson lattice through conventional scattering experiments. Therefore the $\mu$ mesons will appear to be point particles.

Finally we must address the question for what reason do the muons or leptons not interact strongly with the mesons and baryons? We have shown in [9] that a strong force emanates from the sides of a cubic lattice caused by the unsaturated weak forces of about $10^6$ lattice points at the surface of the lattice of the mesons and baryons. This follows from the study of Born and Stern [42] which dealt with the forces between two parts of a cubic lattice cleaved in vacuum. If the muons have a lattice consisting of one type of muon neutrinos, say, $\bar{\nu}_\mu$ and of $\nu_e$ and $\bar{\nu}_e$ neutrinos their octahedral lattice surface is not the same as the surface of the cubic $\nu_\mu$, $\bar{\nu}_\mu$, $\nu_e$, $\bar{\nu}_e$ lattice of the mesons and baryons in the standing wave model. Therefore the muon lattice does not bond with the cubic lattice of the mesons and baryons.
To summarize what we have learned about the $\mu^\pm$ mesons. Eq.(32) says that the energy in $m(\mu^\pm)c^2$ is the sum of the oscillation energy plus the sum of the energy of the rest masses of the neutrinos and antineutrinos in $m(\mu^\pm)$, similar to the energy in the $\pi^\pm$ mesons. The three neutrino types in the $\mu^\pm$ mesons are the remains of the cubic neutrino lattice in the $\pi^\pm$ mesons. Since all $\nu_\mu$ respectively all $\bar{\nu}_\mu$ neutrinos have been removed from the $\pi^\pm$ lattice in the $\pi^\pm$ decay the rest mass of the $\mu^\pm$ mesons must be $\approx 3/4 \cdot m(\pi^\pm)$, in agreement with the experimental results. The $\mu^\pm$ mesons are not point particles.

The mass of the $\tau^\pm$ mesons follows from the decay of the $D^+_s$ mesons. It can be shown readily that the oscillation energies of the lattices in $D^+_s$ and in $\tau^\pm$ are the same. From that follows that the energy in the rest mass of the $\tau^\pm$ mesons is the sum of the oscillation energy in the $\tau$ meson lattice plus the sum of the energy of the rest masses of all neutrinos and antineutrinos in the $\tau$ meson lattice, just as with the $\mu^\pm$ mesons. We will skip the details. The tau mesons are not point particles either.

If the same principle that applies to the decay of the $\pi^\pm$ mesons, namely that in the decay the oscillation energy of the decaying particle is conserved and that an entire neutrino type supplies the energy released in the decay, also applies to the decay of the neutron $n \to p + e^- + \bar{\nu}_e$, then the mass of the anti-electron neutrino can be determined from the known difference $\Delta = m(n) - m(p) = 1.293332 \text{ MeV/c}^2$. Nearly one half of $\Delta$ comes from the energy lost by the emission of the electron, whose mass is $0.510999 \text{ MeV/c}^2$. N anti-electron neutrinos are in the neutrino quadrupoles in the neutron, one-fourth of them is carried away by the emitted electron. We have seen in the paragraph below Eq.(27) that the decay sequence of the $\pi^\pm$ mesons requires that the electron carries with it $N'/4$ electron neutrinos, if the $\pi^\pm$ mesons consist of a lattice with a center neutrino or antineutrino and equal numbers of $\nu_e, \bar{\nu}_e, \nu_\mu, \bar{\nu}_\mu$ neutrinos as required by conservation of neutrino number during the creation of $\pi^\pm$. The electron can carry $N'/4$ anti-electron neutrinos as well. Since, as we will see shortly, $m(\nu_e) = m(\bar{\nu}_e)$ this does not make a difference energetically but is relevant for the orientation of the spin vector of the emitted electron. After the neutron has lost $N'/4$ anti-electron neutrinos to the electron emitted in the $\beta$-decay the remaining $3/4 \cdot N'$ anti-electron neutrinos in the neutron provide the energy $\Delta - m(e^-)c^2 = 0.782321 \text{ MeV}$ released in the decay of the neutron. After division by $3/4 \cdot N'$ the rest mass
of the anti-electron neutrino is
\[ m(\bar{\nu}_e) = 0.365 \text{ meV} / c^2. \] (33)

Since theoretically the antineutron decays as \( \bar{n} \rightarrow \bar{p} + e^+ + \nu_e \) it follows from the same considerations as with the decay of the neutron that
\[ m(\nu_e) = m(\bar{\nu}_e). \] (34)

We note that
\[ N'/4 \cdot m(\nu_e) = N'/4 \cdot m(\bar{\nu}_e) = 0.51 m(e^\pm). \] (35)

Inserting (34) into Eq.(27) we find that
\[ m(\nu_\mu) = 49.91 \text{ meV} / c^2, \] (36)

Since the same considerations apply for either the \( \pi^+ \) or the \( \pi^- \) meson it follows that
\[ m(\nu_\mu) = m(\bar{\nu}_\mu). \] (37)

Experimental values for the rest masses of the different neutrino types are not available. However it appears that for the \( \nu_\mu \leftrightarrow \nu_e \) oscillation the value for \( \Delta m^2 = m_2^2 - m_1^2 = 3.2 \times 10^{-3} \text{ eV}^2 \) given on p.1565 of [36] can be used to determine \( m_2 = m(\nu_\mu) \) if \( m_1 = m(\nu_e) \) is much smaller than \( m_2 \). We have then \( m(\nu_\mu) \approx 56.56 \text{ milli-eV} / c^2 \), which is compatible with the value of \( m(\nu_\mu) \) given in Eq.(36).

The mass of the \( \tau \) neutrino can be determined from the decay \( D_s^\pm \rightarrow \tau^\pm + \nu_\tau (\bar{\nu}_\tau) \), and the subsequent decay \( \tau^\pm \rightarrow \pi^\pm + \bar{\nu}_\tau (\nu_\tau) \), as discussed in [39]. The appearance of the \( \tau \) meson in the decay of \( D_s^\pm \) and the presence of \( \nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e \) neutrinos in the \( \pi^\pm \) decay product of the \( \tau^\pm \) mesons means that there must be \( \nu_\tau, \bar{\nu}_\tau, \nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e \) neutrinos in the \( D_s^\pm \) lattice. The additional \( \nu_\tau \) and \( \bar{\nu}_\tau \) neutrinos can be accommodated in the \( D_s^\pm \) lattice by a body-centered cubic lattice, in which there is in the center of each cubic cell one particle different from the particles in the eight cell corners. In a body-centered cubic lattice there are \( N'/4 \) cell centers, if \( N' \) is the number of lattice points without the cell centers. If the center particles are tau neutrinos there must be \( N'/8 \) \( \nu_\tau \) and \( N'/8 \) \( \bar{\nu}_\tau \) neutrinos, because of conservation of neutrino numbers. From

\[ m(D_s^\pm) = 1968.5 \text{ MeV} / c^2 \] and \( m(\tau^\pm) = 1777 \text{ MeV} / c^2 \) follows that
\[ m(D_s^\pm) - m(\tau^\pm) = 191.5 \text{ MeV} / c^2 = N'/8 \cdot m(\nu_\tau). \] (38)
The rest mass of the $\tau$ neutrinos is therefore
\[ m(\nu_\tau) = m(\bar{\nu}_\tau) = 0.537 \text{ eV}/c^2. \]  

We can now explain the ratio $m(\mu^\pm)/m(e^\pm)$ as well as $m(\pi^\pm)/m(e^\pm)$ and $m(p)/m(e^-)$. From Eqs. (33,36) we find that

\[ m(\nu_\mu)/m(\nu_e) = 136.74, \]  

which is 99.8% of the inverse of the fine structure constant $\alpha_f = 1/137.036$. It does not seem likely that this is just a coincidence. We set $N'/4\cdot m(\nu_e) = 0.5 \ m(e^\pm)$, not at 0.51 $m(e^\pm)$ as in Eq.(35). Then $m(e^\pm) = N'/2\cdot m(\nu_e)$ or $N'/2\cdot m(\bar{\nu}_e)$. We also set $E_\nu(\pi^\pm) = 0.5 \ m(\pi^\pm)c^2$, not at 0.486 $m(\pi^\pm)c^2$ as in Eq.(26). With $E_\nu(\pi^\pm) = E_\nu(\mu^\pm)$ from Eq.(30) it follows with Eq.(28) that $E_\nu(\mu^\pm) = N'/2\cdot [m(\nu_\mu) + m(\nu_e)]c^2$. From Eq.(32) then follows that

\[ m(\mu^\pm) = 3/4 \cdot N' m(\nu_\mu) + N' m(\nu_e), \]  

and with $m(e^\pm) = N'/2\cdot m(\nu_e)$ from above we have

\[ \frac{m(\mu^\pm)}{m(e^\pm)} = \frac{3}{2} \cdot \frac{m(\nu_\mu)}{m(\nu_e)} + 2, \]  

or with $m(\nu_\mu)/m(\nu_e) \approx 1/\alpha_f$ it turns out that

\[ \frac{m(\mu^\pm)}{m(e^\pm)} \approx \frac{3}{2} \cdot \frac{1}{\alpha_f} + 2 = 207.55. \]  

The mass of the muon is, according to Eq.(42), much larger than the mass of the electron because the mass of the muon neutrino, which is dominant in the muon, is so much larger ($m(\nu_\mu)\alpha_f \approx m(\nu_e)$) than the mass of the electron neutrino. The ratio of the mass of the muon to the mass of the electron is independent of the number $N'$ of the neutrinos of either type in both lattices. The empirical formula for the mass ratio is $m(\mu^\pm)/m(e^\pm) = 3/2\alpha_f + 1 = 206.55$, whereas the actual ratio is 206.768. The empirical formula was given by Barut [43], following an earlier suggestion by Nambu [10] that $m(\mu) \approx m(e) \cdot 3/2\alpha$. We attribute the 0.5% difference between Eq.(43) and the empirical formula for $m(\mu^\pm)/m(e^\pm)$ to the absence of electrical charge on the right hand side of Eq.(42). The $\mu^\pm$ mesons, which have spin and a magnetic moment, have +1 added to the ratio $m(\nu_\mu)/m(\nu_e)$ in Barut’s formula for $m(\mu^\pm)/m(e^\pm)$, whereas the $\pi^\pm$ mesons, which do not have spin and
a magnetic moment, have \(-1\) subtracted from \(m(\nu_\mu)/m(\nu_e)\) in the formula for \(m(\pi^\pm)/m(e^\pm)\) given in the following.

Similarly we obtain

\[
\frac{m(\pi^\pm)}{m(e^\pm)} = 2 \left[ \frac{m(\nu_\mu)}{m(\nu_e)} + 1 \right] \approx \frac{2}{\alpha_f} + 2 = 276.07, \tag{44}
\]

whereas the empirical formula is \(m(\pi^\pm)/m(e^\pm) = 2/\alpha_f - 1 = 273.07\). The experimental ratio is \(m(\pi^\pm)/m(e^\pm) = 273.132 = 1.00022 (2/\alpha_f - 1)\).

In order to determine \(m(n)/m(e^\pm)\) we start with \(K^0 = (2.)\pi^\pm + \pi^\mp\) and \((2.)\pi^\pm = 4E_\nu(\pi^\pm) + N'/2[m(\nu_\mu) + m(\nu_e)]c^2\). Then \(m(K^0) = 7N'/2[m(\nu_\mu) + m(\nu_e)]\), and with \(m(n) = 0.9439 \cdot 2m(K^0)\) it follows that

\[
\frac{m(n)}{m(e^\pm)} = 0.9439 \cdot 14 \left[ \frac{m(\nu_\mu)}{m(\nu_e)} + 1 \right] = 1824.2, \tag{45}
\]

that is 99.2% of the experimental value 1838.68. With \(m(p) = 0.9986 \cdot m(n)\) we have

\[
\frac{m(p)}{m(e^\pm)} = 0.9426 \cdot 14 \left[ \frac{m(\nu_\mu)}{m(\nu_e)} + 1 \right] \approx 0.9426 \left[ \frac{14}{\alpha_f} + 14 \right] = 1821.5, \tag{46}
\]

that is 99.2% of the experimental value 1836.16. The empirical formula for \(m(p)/m(e^\pm)\) is \(m(p)/m(e^\pm) = 14 [1/\alpha_f - 6] = 0.9991 m(p)/m(e^\pm)(\text{exp})\).

To summarize what we have learned about the masses of the leptons: We have found an explanation for the mass of the \(\mu^\pm\) mesons and \(\tau^\pm\) mesons. We have also determined the rest masses of the \(e, \mu, \tau\) neutrinos and antineutrinos. In other words, we have found the masses of all leptons, excepting the electron, a topic which we will deal with later.

8 The spin of the \(\gamma\)-branch particles

It appears to be crucial for the validity of a model of the elementary particles that the model can also explain the spin of the particles without additional assumptions. The spin or the intrinsic angular momentum is, after the mass, the second most important property of the elementary particles. As is well-known the spin of the electron was discovered by Uhlenbeck and Goudsmit [44] more than 75 years ago. Later on it was established that the baryons have spin as well, but not the mesons. We have proposed an explanation of
the spin of the particles in [45]. For current efforts to understand the spin of the nucleon see Jaffe [46] and of the spin structure of the Λ baryon see Göckeler et al. [47]. The explanation of the spin requires an unambiguous answer, the spin must be 0 or 1/2 or integer multiples thereof, nothing else.

The spin of the particles is, of course, the sum of the angular momentum vectors of the oscillations plus the sum of the spin vectors of the neutrinos, antineutrinos and the electric charges in the cubic lattice of the mesons and baryons. It is striking that the particles which, according to the standing wave model, consist of a single oscillation mode do not have spin, as the $\pi^0, \pi^\pm$ and $\eta$ mesons do, see Tables 1 and 2. It is also striking that particles whose mass is approximately twice the mass of a smaller particle have spin 1/2 as is the case with the Λ baryon, $m(\Lambda) \approx 2m(\eta)$, and with the nucleon $m(n) \approx 2m(K^\pm) \approx 2m(K^0)$. The $\Xi^0_c$ baryon which is a doublet of one mode has also spin 1/2. Composite particles which consist of a doublet of one mode plus one or two other single modes have spin 1/2, as the $\Sigma^0, \Xi^0$ and $\Lambda^+_c, \Sigma^0_c, \Omega^0_c$ baryons do. The only particle which seems to be the triplet of a single mode, the $\Omega^-$ baryon with $m(\Omega^-) \approx 3m(\eta)$, has spin 3/2. It appears that the relation between the spin and the oscillation modes of the particles is straightforward.

In the standing wave model the $\pi^0$ and $\eta$ mesons consist of $N = 2.85 \cdot 10^9$ standing electromagnetic waves, each with its own frequency. Their oscillations are longitudinal. The longitudinal oscillations of frequency $\nu_i$ in the $\pi^0$ and $\eta$ mesons do not have angular momentum or $\sum_i j(\nu_i) = 0$, with the index running from 0 to $N$. Longitudinal oscillations cannot cause an intrinsic angular momentum because for longitudinal oscillations $\vec{r} \times \vec{p} = 0$.

Each of the standing waves in the $\pi^0$ and $\eta$ mesons may, on the other hand, have spin $s = 1$ of its own, because circularly polarized electromagnetic waves have an angular momentum as was first suggested by Poynting [48] and verified by, among others, Allen [49]. The creation of the $\pi^0$ meson in the reaction $\gamma + p \rightarrow \pi^0 + p$ and conservation of angular momentum dictates that the sum of the angular momentum vectors of the $N$ electromagnetic waves in the $\pi^0$ meson is zero, $\sum_i j(s_i) = 0$. Either the sum of the spin vectors of the electromagnetic waves in the $\pi^0$ meson is zero, or each electromagnetic wave in the $\pi^0$ meson has zero spin which would mean that they are linearly polarized. Linearly polarized electromagnetic waves are not expected to have angular momentum. That this is actually so was proven by Allen [49]. Since the longitudinal oscillations in the $\pi^0$ and $\eta$ mesons do not have angular momentum and since the sum of the spin vectors $s_i$ of the electromagnetic
waves is zero, the intrinsic angular momentum of the $\pi^0$ and $\eta$ mesons is zero, or

$$\sum_i j(\nu_i) + \sum_i j(s_i) = 0 \quad (0 \leq i \leq N). \quad (47)$$

In the standing wave model the $\pi^0$ and $\eta$ mesons do not have an intrinsic angular momentum or spin, as it must be.

We now consider particles such as the $\Lambda$ baryon which consist of superpositions of two circular oscillations of equal amplitudes and of frequencies $\omega$ and $-\omega$, $|\omega| = \omega$, at each of the $N$ points of the lattice. The oscillations in the particles are coupled what we have marked in Tables 1,2 by the $\cdot$· sign. These particles contain $N$ circular oscillations, each with its own frequency and each having an angular momentum of $\hbar/2$ as we will see.

The superposition of two perpendicular linearly polarized traveling waves of equal amplitudes and frequencies shifted in phase by $\pi/2$ leads to a circular wave with the constant angular momentum $j = \hbar$. The total energy of a traveling wave is the sum of the potential and the kinetic energy. In a traveling wave the kinetic energy is always equal to the potential energy.

From

$$E_{pot} + E_{kin} = E_{tot} = \hbar \omega, \quad (48)$$

follows

$$E_{tot} = 2E_{kin} = 2\frac{\Theta \omega^2}{2} = \hbar \omega, \quad (49)$$

with the moment of inertia $\Theta$. It follows that the angular momentum $j$ is

$$j = \Theta \omega = \hbar. \quad (50)$$

This applies to a traveling wave and corresponds to spin $s = 1$, or to a circularly polarized photon.

We now add to one monochromatic circular oscillation with frequency $\omega$ a second circular oscillation with $-\omega$ of the same absolute value as $\omega$ but shifted in phase by $\pi$, having the same amplitude, as we have done in [45]. Negative frequencies are permitted solutions of the equations for the lattice oscillations. In other words we consider the circular oscillations

$$x(t) = \exp[i\omega t] + \exp[-i(\omega t + \pi)], \quad (51)$$

$$y(t) = \exp[i(\omega t + \pi/2)] + \exp[-i(\omega t + 3\pi/2)]. \quad (52)$$
This can also be written as

\[ x(t) = \exp[i\omega t] - \exp[-i\omega t] , \quad (53) \]

\[ y(t) = i \cdot (\exp[i\omega t] + \exp[-i\omega t]) . \quad (54) \]

If we replace \( i \) in the Eqs. above by \( -i \) we have a circular oscillation turning in opposite direction. The energy of the superposition of the two oscillations is the sum of the energies of both individual oscillations, and since in circular oscillations \( E_{\text{kin}} = E_{\text{pot}} \) we have according to Eq.(49)

\[ 4E_{\text{kin}} = 4\Theta \omega^2 / 2 = E_{\text{tot}} = \hbar \omega , \quad (55) \]

from which follows that the circular oscillation has an angular momentum

\[ j = \Theta \omega = \hbar / 2 . \quad (56) \]

The superposition of two circular monochromatic oscillations of equal amplitudes and frequencies \( \omega \) and \( -\omega \) satisfies the necessary condition for spin \( s = 1/2 \) that the angular momentum is \( j = \hbar / 2 \).

The standing wave model treats the \( \Lambda \) baryon, which has spin \( s = 1/2 \) and a mass \( m(\Lambda) = 1.0190 \cdot 2m(\eta) \), as the superposition of two particles of the same type with \( N \) standing electromagnetic waves. The waves are circular because they are the superposition of two circular waves with the same absolute value of the frequency and the same amplitude. The angular momentum vectors of all circular waves in the lattice cancel, except for the wave at the center of the crystal. Each oscillation with frequency \( \omega \) at \( \phi > 0 \) has at its mirror position \( \phi < 0 \) a wave with the frequency \( -\omega \), which has a negative angular momentum since \( j = mr^2 \omega \) and \( \omega = \omega_0 \phi \). Consequently the angular momentum vectors of both waves cancel. The center of the lattice oscillates, as all other lattice points do, but with the frequency \( \nu(0) \) which is determined by the longest possible wavelength, which is twice the sidelength \( d \) of the lattice, so \( \nu(0) = c / 2d \). As the other circular waves in the lattice the circular wave at the center has the angular momentum \( \hbar / 2 \) according to Eq.(56). The angular momentum of the center wave is the only angular momentum which is not canceled by an oscillation of opposite circulation.

Consequently the net angular momentum of the \( N \) circular oscillations in the lattice which are superpositions of two oscillations reduces to the angular momentum of the center oscillation and is \( \hbar / 2 \). Since the circular lattice
oscillations in the Λ baryon are the only possible contribution to an angular momentun the intrinsic angular momentum of the Λ baryon is $\hbar/2$ or

$$j(\Lambda) = \sum_i j(\nu_i) = j(\nu_0) = \hbar/2.$$  \hspace{1cm} (57)

We have thus explained that the Λ and likewise the Ξ⁰ baryon satisfy the necessary condition that $j = \hbar/2$ for $s = 1/2$. The intrinsic angular momentum of the Λ baryon is the consequence of the superposition of two circular oscillations of the same amplitude and the same absolute value of the frequency.

The other particles of the γ-branch, the Σ⁰, Ξ⁰, Λ⁺, Σ⁺⁰ and Ω⁺⁰ baryons are composites of a baryon with spin 1/2 plus one or two π mesons which do not have spin. Consequently the spin of these particles is 1/2. The spin of all particles of the γ-branch, exempting the spin of the Ω⁻ baryon, has thus been explained. For an explanation of $s(\Sigma^{±,0}) = 1/2$ and of $s(\Xi^{−,0}) = 1/2$, regardless whether the particles are charged or neutral, we refer to [45].

9 The spin of the particles of the ν-branch

The characteristic particles of the neutrino-branch are the π⁺ mesons which have zero spin. At first glance it seems to be odd that the π⁺ mesons do not have spin, because it seems that the π⁺ mesons should have spin 1/2 from the spin of the charges $e^±$ in π⁺. However that is not the case. The solution of this puzzle is in the composition of the π⁺ mesons which are, according to the standing wave model, made of a lattice of neutrinos and antineutrinos (Fig. 2) each having spin 1/2, the lattice oscillations, and an electrical charge.

The longitudinal oscillations in the neutrino lattice of the π⁺ mesons do not cause an angular momentum, $\sum_i j(\nu_i) = 0$, as it was with the π⁰ meson. In the cubic lattice of $N = O(10^9)$ neutrinos and antineutrinos of the π⁺ mesons the spin of nearly all neutrinos and antineutrinos must cancel because conservation of angular momentum during the creation of the particle requires that the total angular momentum around a central axis is $\hbar/2$. In fact the spin vectors of all but the neutrino or antineutrino in the center of the lattice cancel. In order for this to be so the spin vector of any particular neutrino in the lattice has to be opposite to spin vector of the neutrino at its mirror position. As is well-known only left-handed neutrinos and right-handed antineutrinos exist. From $\nu = \nu_0 \phi$ (Eq.13) follows that the direction
of motion of the neutrinos in e.g. the upper right quadrant (\(\phi > 0\)) is opposite to the direction of motion in the lower left quadrant (\(\phi < 0\)). Consequently the spin vectors of all neutrinos or antineutrinos in opposite quadrants are opposite and cancel. The only angular momentum remaining from the spin of the neutrinos of the lattice is the angular momentum of the neutrino or antineutrino at the center of the lattice which does not have a mirror particle. Consequently the electrically neutral neutrino lattice consisting of \(N'/2\) neutrinos and \(N'/2\) antineutrinos and the center particle, each with spin \(s(n_i) = 1/2\), has an intrinsic angular momentum \(j = \sum_i j(n_i) = j(n_0) = \hbar/2\).

But electrons or positrons added to the neutral neutrino lattice have spin 1/2. If the spin of the electron or positron added to the neutrino lattice is opposite to the spin of the neutrino or antineutrino in the center of the lattice then the net spin of the \(\pi^+\) or \(\pi^-\) mesons is zero, or

\[
j(\pi^\pm) = \sum_i j(n_i) + j(e^\pm) = j(n_0) + j(e^\pm) = 0 \quad (0 \leq i \leq N) .
\]

It is important for the understanding of the structure of the \(\pi^\pm\) mesons to realize that \(s(\pi^\pm) = 0\) can only be explained if the \(\pi^\pm\) mesons consist of a \textit{neutrino lattice} to which an electron or positron is added whose spin is opposite to the net spin of the neutrino lattice. \textit{Spin 1/2 of the electric charges can only be canceled by something that has also spin 1/2, and the only conventional choice for that is a single neutrino.}

\textit{The spin, the mass and the decay of \(\pi^\pm\) require that the \(\pi^\pm\) mesons are made of a neutrino lattice and an electrical charge.}

The spin of the \(K^\pm\) mesons is zero. With the spin of the \(K^\pm\) mesons we encounter the same oddity we have just observed with the spin of the \(\pi^\pm\) mesons, namely we have a particle which carries an electrical charge with spin 1/2, and nevertheless the particle does not have spin. The explanation of \(s(K^\pm) = 0\) follows the same lines as the explanation of the spin of the \(\pi^\pm\) mesons. In the standing wave model the \(K^\pm\) mesons are described by the state \((2.)\pi^\pm + \pi^0\), that means by the second mode of the \(\pi^\pm\) mesons plus a \(\pi^0\) meson. The second mode of the longitudinal oscillations of a neutral neutrino lattice does not have a net intrinsic angular momentum \(\sum_i j(\nu_i) = 0\). But the spin of the neutrinos contributes an angular momentum \(\hbar/2\), which originates from the neutrino or antineutrino in the center of the lattice, just as it is with the neutrino lattice in the \(\pi^\pm\) mesons, so \(\sum_i j(n_i) = \hbar/2\). Adding an electric charge with a spin opposite to the net intrinsic angular momentum of the neutrino lattice oscillations creates the charged \((2.)\pi^\pm\)
mode which has zero spin
\[
j((2.)\pi^\pm) = \sum_i j(n_i) + j(e^\pm) = j(n_0) + j(e^\pm) = 0. \tag{59}
\]

As discussed in Section 6 it is necessary to add a \(\pi^0\) meson to the second mode of the \(\pi^\pm\) meson in order to obtain the correct mass and the correct decays of the \(K^\pm\) mesons. Since the \(\pi^0\) meson does not have spin the addition of the \(\pi^0\) meson does not add to the intrinsic angular momentum of the \(K^\pm\) mesons. So \(s(K^\pm) = 0\) as it must be.

The explanation of \(s = 0\) of the \(K^0\) and \(\bar{K}^0\) mesons described by the state \((2.)\pi^+ + \pi^-\) is different. The oscillations of the second mode of \(\pi^\pm\) as well as of the basic \(\pi^\mp\) mode do not create an angular momentum, \(\sum_i j(\nu_i) = 0\). The second mode of the \(\pi^\pm\) mesons, or the \((2.)\pi^\pm\) state, and the basic \(\pi^\mp\) mode each have \(N'/2\) neutrinos and \(N'/2\) antineutrinos plus a center neutrino or antineutrino, so the number of all neutrinos and antineutrinos in the sum of both states, the \(K^0\) meson, is \(2N\). Since the size of the lattice of the \(K^\pm\) mesons and the \(K^0\) mesons is the same it follows that two neutrinos are at each lattice point of the \(K^0\) or \(\bar{K}^0\) mesons. We assume that Pauli’s exclusion principle applies for neutrinos as well. Consequently each neutrino at each lattice point must share its location with an antineutrino. That means that the contribution of the spin of all neutrinos and antineutrinos to the intrinsic angular momentum of the \(K^0\) meson is zero or \(\sum_i j(2n_i) = 0\). The sum of the spin vectors of the two opposite charges in the \(K^0\) and \(\bar{K}^0\) mesons, or in the \((2.)\pi^+ + \pi^-\) state, is also zero. Since neither the lattice oscillations nor the spin of the neutrinos and antineutrinos nor the electric charges contribute an angular momentum
\[
j(K^0) = \sum_i j(2n_i) + j(e^+ + e^-) = 0. \tag{60}
\]

The intrinsic angular momentum of the \(K^0\) and \(\bar{K}^0\) mesons is zero, or \(s(K^0,\bar{K}^0) = 0\), as it must be. In simple terms, since the structure of \(K^0\) is \((2.)\pi^+ + \pi^-\), the spin of \(K^0\) is the sum of the spin of \((2.)\pi^+\) and of \(\pi^-\), both of which do not have spin. It does not seem possible to arrive at \(s(K^0,\bar{K}^0) = 0\) if both particles do not contain the \(N\) pairs of neutrinos and antineutrinos required by the \((2.)\pi^+ + \pi^-\) state which we have suggested in Section 6.

In the case of the neutron one must wonder how it comes about that a particle which seems to be the superposition of two particles without spin ends up with spin 1/2. The neutron, which has a mass \(\approx 2m(K^\pm)\) or \(2m(K^0)\),
is either the superposition of a $K^+$ and a $K^-$ meson or of a $K^0$ and a $\bar{K}^0$ meson. The intrinsic angular momentum of the superposition of $K^+$ and $K^-$ is either 0 or $\hbar$, which means that the neutron cannot be the superposition of $K^+$ and $K^-$. For a proof of this statement we must refer to [45].

On the other hand the neutron seems to be the superposition of a $K^0$ and a $\bar{K}^0$ meson. A significant change in the lattice occurs when a $K^0$ and a $\bar{K}^0$ meson are superposed. Since each $K^0$ meson contains $N$ neutrinos and $N$ antineutrinos, as we discussed before in context with the spin of $K^0$, the number of all neutrinos and antineutrinos in superposed $K^0$ and $\bar{K}^0$ lattices is $4N$. Since the size of the lattice of the proton as well of the neutron is the same as the size of $K^0$ each of the $N$ lattice points of the neutron now contains four neutrinos, a muon neutrino and an anti-muon neutrino as well as an electron neutrino and an anti-electron neutrino. The $\nu_\mu, \bar{\nu}_\mu, \nu_e, \bar{\nu}_e$ quadrupoles oscillate just like individual neutrinos do because we learned from Eq.(7) that the ratios of the oscillation frequencies are independent of the mass as well as of the interaction constant between the lattice points. In the neutrino quadrupoles the spin of the neutrinos and antineutrinos cancels, $\sum_i j(4n_i) = 0$. The superposition of two circular neutrino lattice oscillations, that means the circular oscillations of frequency $\nu_i$, contribute as before the angular momentum of the center circular oscillation, so $\sum_i j(\nu_i) = \hbar/2$. The spin and charge of the four electrical charges hidden in the sum of the $K^0$ and $\bar{K}^0$ mesons cancel. It follows that the intrinsic angular momentum of a neutron created by the superposition of a $K^0$ and a $\bar{K}^0$ meson comes from the circular neutrino lattice oscillations only and is

$$j(n) = \sum_i j(\nu_i) + \sum_i j(4n_i) + j(4e^\pm) = \sum_i j(\nu_i) = j(\nu_0) = \hbar/2,$$

as it must be. In simple terms, the spin of the neutron originates from the superposition of two circular neutrino lattice oscillations with the frequencies $\omega$ and $-\omega$ shifted in phase by $\pi$, which produces the angular momentum $\hbar/2$.

The spin of the proton is $1/2$ and is unambiguously defined by the decay of the neutron $n \rightarrow p + e^- + \bar{\nu}_e$. We have suggested in Section 7 that $3/4 \cdot N'$ anti-electron neutrinos of the neutrino lattice of the neutron are removed in the $\beta$-decay of the neutron and that $N'/4$ anti-electron neutrinos leave with the emitted electron. The intrinsic angular momentum of the proton originates then from the spin of the central $\nu_\mu \bar{\nu}_\mu, \nu_e$ triplet, from the spin of the $e^+e^-e^+$ triplet which is part of the remains of the neutron, and from the angular momentum of the center of the lattice oscillations with the super-
position of two circular oscillations. The spin of the central $\nu_\mu \bar{\nu}_\mu \nu_e$ triplet is canceled by the spin of the $e^+ e^- e^+$ triplet. According to the standing wave model the intrinsic angular momentum of the proton is

$$j(p) = j(\nu_\mu \bar{\nu}_\mu \nu_e)_0 + j(e^+ e^- e^+) + j(\nu_0) = j(\nu_0) = \hbar/2,$$

as it must be.

The other mesons of the neutrino branch, the $D^{\pm,0}$ and $D_s^{\pm}$ mesons, both having zero spin, are superpositions of a proton and an antineutron of opposite spin, or of their antiparticles, or of a neutron and an antineutron of opposite spin in $D^0$. The spin of $D^{\pm}$ and $D^0$ does therefore not pose a new problem. The spin of $D_s^{\pm}$ is explained in [39].

For an explanation of the spin of $\mu^{\pm}$ we refer to [50]. Since all muon or anti-muon neutrinos have been removed from the $\pi^{\pm}$ lattice in the $\pi^{\pm}$ decay it follows that a neutrino vacancy is at the center of the $\mu^{\pm}$ lattice. Without a neutrino in the center of the lattice the sum of the spin vectors of all neutrinos in the $\mu^{\pm}$ lattice is zero. However the $\mu^{\pm}$ mesons consist of the neutrino lattice plus an electric charge whose spin is 1/2. The spin of the $\mu^{\pm}$ mesons originates from the spin of the electric charge carried by the $\mu^{\pm}$ mesons and is consequently $s(\mu^{\pm}) = 1/2$. The same considerations apply for the spin of $\tau^{\pm}$, $s(\tau^{\pm}) = 1/2$.

An explanation of the spin of the mesons and baryons can only be valid if the same explanation also applies to the antiparticles of these particles whose spin is the same as that of the ordinary particles. The antiparticles of the $\gamma$-branch consist of electromagnetic waves whose frequencies differ from the frequencies of the ordinary particles only by their sign. The angular momentum of the superposition of two circular oscillations with $-\omega$ and $\omega$ has the same angular momentum as the superposition of two circular oscillations with frequencies of opposite sign, as in $\Lambda$. Consequently the spin of the antiparticles of the $\gamma$-branch is the same as the spin of the ordinary particles of the $\gamma$-branch. The same considerations apply to the circular neutrino lattice oscillations which cause the spin of the neutron, the only particle of the $\nu$-branch which has spin. In the standing wave model the spin of the neutron and the antineutron are the same.

From the foregoing we arrive at an understanding of the reason for the astonishing fact that the intrinsic angular momentum or spin of the particles is independent of the mass of the particles, as exemplified by the spin $\hbar/2$ of the electron being the same as the spin $\hbar/2$ of the proton, notwithstanding
the fact that the mass of the proton is 1836 times larger than the mass of the electron. However, in our model, the spin of the particles including the electron is determined solely by the angular momentum $\hbar/2$ of the center point of the lattice, the other angular momentum vectors in the particles cancel. Hence the mass of the particles contained in the other $10^9$ lattice points is inconsequential for the intrinsic angular momentum of the particles. That does indeed mean that the spin of the particles is independent of the mass of the particles.

**Conclusions**

We conclude that the standing wave model solves a number of problems for which an answer heretofore has been hard to come by. Only photons, neutrinos, charge and the weak nuclear force are needed to explain the masses of the stable mesons and baryons and of the leptons. We can also explain the spin of the baryons and the absence of spin in the mesons, and the spin of the $\mu^\pm$ and $\tau^\pm$ mesons as well.

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The magnetic moment of the neutron

In the same way we can explain the magnetic moment of the neutron which has a magnetic moment with a g-factor $g(n) = -3.826 = -0.9564$. This magnetic moment must originate from a pair of electric charges since the g-factor of a single electric charge is $\approx 1.00116$. At least two electric charges of opposite sign must be within the neutron to have any magnetic moment at all. However, in the standing wave model four charges are in the neutron, two each of opposite sign. In order to have a magnetic moment twice the moment of either $e^+$ or $e^-$ the spin of two charges must be parallel, whereas the spin of the two other charges must be antiparallel so that their magnetic moments cancel, or there must be two opposite charges with opposite spin. As discussed in the preceding paragraph the magnetic moment of the proton is caused by the parallel spin of two $e^+$ and the opposite spin of one $e^-$, causing a moment three times the moment of one charge. These three charges carry $N/4$ electron neutrinos each, because all anti-electron neutrinos have been removed from the neutron lattice in its decay, as mentioned before. As follows from the decay $n \rightarrow p + e^- + \bar{\nu}_e$ there must be an additional $e^-$ in the neutron as compared to the proton. The additional $e^-$ carries $N/4$ anti-electron neutrinos because of conservation of angular momentum. Consequently this electron has a spin opposite to the spin of the $e^-$ in the proton. The magnetic moments of the two electrons with opposite spin cancel. However two positive charges $e^+$ having the same spin are also in the neutron, as they are in the proton. They create the magnetic moment of the neutron and have a g-factor $\approx 4$.

There seems to be an excess angular momentum caused by the parallel spin of the two $e^+$. However this is taken care of by the spin of the anti-electron neutrino which comes with the neutron decay and by the spin of the $N/4$ electron neutrinos not bound in the electric charges in the neutron. Summing up, the magnetic moment of the neutron is caused by the parallel spin of a pair of the four electrical charges which are in the neutron according to our model. The magnetic moment of the other pair of charges cancel because their spin is antiparallel.