A Bus Signal Priority Model at Oversaturated Intersection under Stochastic Demand

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This paper focuses on the optimization of bus signal priority with the consideration of the stochastic traffic demand. Based on a situation of the variability of traffic composition, the phase clearance reliability (PCR) value of each phase and traffic composition ratio is introduced to reflect the traffic condition at intersection. Then, a bus signal priority optimization model is proposed with the purpose of the maximum of the total vehicular departure as the optimization goal. In order to obtain the optimal solution, an improved algorithm is designed by introducing the PCR value search strategy. Finally, two cases’ study is exhibited to demonstrate the reasonability of the model, theory, and algorithm. The result shows that the model can not only clear the queue under the condition of continuous dynamic traffic flow but also reduce the vehicle queuing and passenger delay.

1. Introduction

The acceleration of urbanization and the increasing urban population have led to serious traffic problems. Because of the large capacity of transferring passengers and efficiency, public transportation can reduce traffic congestions [1, 2] and reduce environmental pollution [3]. There are different public transit modes, such as high-speed rail, subway, tram, and bus. All of them have advantages such as large capacity and high reliability. But high-speed rail, subway, and tram require large financial investments, overly long construction times, and significant maintenance costs. In contrast, the investment for creating bus routes is smaller and the operational cost is lower compared to that of other public modes. However, with the increase of the number of vehicles, more and more signalized intersections become oversaturated in rush hours due to the growth of traffic demand, and many oversaturated intersections then form oversaturated arterials [4] and oversaturated networks [5]. As a result, the delay of the bus increases observably at the intersections and leads to a decline in the sharing rate of public transportation. Furthermore, it weakens the role of public transportation in improving traffic congestion and reducing urban environmental pollution [6, 7]. Therefore, the development of intelligent public transport systems is essential to improve the level of public transport service and operational efficiency.

Bus signal priority can reduce bus queuing time and passenger delay and can effectively improve bus operation efficiency, level of service, and reliability. The existing bus signal priority strategies are classified into three categories: passive priority, active priority, and real-time priority. The passive priority method is to configure the intersection signal timing scheme reasonably offline according to the historical traffic flow data without any detection equipment. The advantages of passive priority strategy are of lower cost of implementation and easier operation. Ma et al. [8] proposed a method to bus signal priority based on the theory of passenger capacity, in which both the passive priority of bus signal and the influence of the bus lane setting were considered. Eichler and Daganzo [1] studied the influence of intermittent bus lanes on road capacity and established a bus signal priority model under the condition of intermittent bus lanes. However, passive priority is difficult to adapt to the traffic flow fluctuations, and the effect of the actual application is limited.

Compared with passive priority, active priority is more flexible. Active priority is to detect the traffic flow through detection equipment and to predict the time when the bus
arrives at the stop line at the intersection through the detection data to give priority to the bus is then determined. Christofa and Skabardonis [9] used actuated control to establish an isolated intersection bus priority method with minimum per capita delay as the optimization goal. This method considered both the priority of bus signals and the overall operating benefits of the intersection. Ma and Yang [10] studied the active priority strategy of single point public transportation based on green time extension, red time early break, and phase insertion. The result shows that the red light early break control effect is better than green time extension, and the benefit of the phase insertion control strategy is related to many factors.

With the development of vehicle detection technology, real-time bus priority is proposed based on real-time traffic flow data, whose control model is established based on the real-time traffic flow detection data, which is more flexible than the active priority. Ma et al. [11] proposed a dynamic planning model of bus priority based on the real-time traffic flow data. The model considered the full load rate of the bus, operation schedule, and traffic demand, but it did not discuss the influence of intersection saturation on signal timing. Li et al. [12] proposed an adaptive bus priority model to minimize the weighted sum of transit and nontransit delay and evaluated the control effect of this model under different saturation of intersections. However, the influence of the change of bus arrival rate on the model was not considered. Han et al. [13] presented a prediction model of transit delay and took transit delay and total vehicle delay at intersections as decision variables to build the optimization model of transit signal priority. This model can dynamically adjust the green time and realize the real-time priority of transit signal control. However, the influence of intersection saturation was not discussed on the bus priority model.

The oversaturated state of an intersection means that when the traffic flow is greater than its maximum capacity, the vehicles at the intersection cannot be emptied in one cycle, and there can be secondary or multiple queues. The oversaturated state of the intersection can greatly impact bus signal priority. For bus priority, existing studies assume that the traffic volume of the intersection was saturated or undersaturated, and there were few bus priority methods for the oversaturated intersection. Ma et al. [14] presented the fuzzy control system of bus signal priority with gather-disperse theory at the oversaturated intersection. However, the influence of bus signal priority on nonbus delay was not considered. It is very important to give priority to bus signals in the oversaturated intersections. Liu et al. [15] proposed an optimization model of signal timing in the oversaturated state of intersection by utilizing the idea of reverse modeling, which can quickly and effectively solve the signal timing scheme in the oversaturated state. However, that model did not consider the composition of arriving traffic flow at the intersection. Meanwhile, buses and nonbuses were equally treated, inevitably cause which would lead to the total passenger delays to be larger at the whole intersection.

In order to overcome those aforementioned deflections, the bus signal priority model is proposed in this paper with the consideration of the oversaturated state of the intersection and the composition of arriving traffic flow. The proportion of the arriving traffic at the intersection between bus and nonbus is considered in the model and the clearance reliability of the bus and nonbus phase is introduced. In order to obtain the optimal solution, this paper also improves the algorithm proposed by Zhao et al. [16] by adding the quadratic optimization condition of intersection signal timing based on the phase clearance reliability of buses and nonbuses, which not only guarantees the speediness of the algorithm solution but also ensures the accuracy of the model solution. Compared with the existing research, our proposed model can improve bus operation efficiency and reduce passenger delay.

2. Model Formulation

In this section, the reverse causal-effect modeling approach by Liu et al. [15] is firstly introduced. Then, a bus priority model is proposed based on reverse causal-effect modeling. For the quickness of contrast of the pertinent work, the notation of Liu et al. [15] and Zhao et al. [16] is adopted, as shown in Table 1.

2.1. Signal Timing Model of an Oversaturated Intersection. The signal timing model objective is to minimize the delay for an oversaturated intersection. Figure 1 illustrates the delay and cumulative vehicles during a congested period. \(A(t)\) represents the cumulative number of vehicles on the approach in the \(t\) period, while \(D(t)\) represents the cumulative number of vehicles departing the intersection in the \(t\) period. The area between \(A(t)\) and \(D(t)\) represents the total vehicle delay during the period of congestion.

Liu et al. [15] proposed that traffic flows could be represented by smooth time-dependent functions, considered homogeneous arrivals/departures of traffic flows within each cycle. That is, in each cycle \(k\), \(A(t) = \int_0^t \lambda(t)dt, D(t) = \int_0^t \mu(t)dt\).

Hence, the total vehicle delay in a cycle of an oversaturated period can be written as

\[
D^C = \int_0^C [A(t) - D(t)]dt
\]

\[
= \int_0^C \int_0^t [\lambda(s) - \mu(s)]ds dt
\]

\[
= \int_0^C dt \int_0^t [\lambda(s) - \mu(s)]ds
\]

\[
= \int_0^C (C - s)[\lambda(s) - \mu(s)]ds
\]

\[
= \int_0^C (C - s)[\lambda(k) - \mu(k)]ds
\]

\[
= \left[\frac{C^2}{2} [\lambda(k) - \mu(k)]\right].
\]
Based on equation (1), Liu et al. [15] proposed a model with the minimum delay as the optimization goal. The objective function is

\[
\min \sum_{p \in P} \sum_{m \in M_p} \lambda_m^p(k)C - \mu_m^p(k)C.
\]

(2)

Since \(\lambda_m^p(t)\) and \(C\) are uniform within each cycle, (2) can be modified as

\[
\max \sum_{p \in P} \sum_{m \in M_p} \mu_m^p(k).
\]

(3)

The constraints can be developed as follows.

First, the summation of the green time ratios of all signal phases does not exceed the total admissible green time ratio:

\[
\sum_{p \in P} \eta^p(k) \leq \eta, \quad \forall p \in P.
\]

(4)

Second, the green time of each phase cannot exceed the minimum green time:

\[
\eta^p(k)C \geq g_p, \quad \forall p \in P.
\]

(5)

Third, the outflow rate in each phase does not exceed the average saturation outflow rate:

\[
\mu_m^p(k) \leq \eta^p(k) s_m^p, \quad \forall m \in M_p \forall p \in P.
\]

(6)

Finally, because the number of vehicles arrived in unit time is greater than its capacity at an oversaturated intersection, the queues that are not cleared in the current cycle will move to the next cycle, so the constraint can be written as

\[
\mu_m^p(k) \leq \frac{X_m^p(k)}{C} + \lambda_m^p(k), \quad \forall m \in M.
\]

(7)

Thus, the linear program (LP) for cycle \(k\) can be formulated as

\[
\begin{align*}
\min & \sum_{p \in P} \sum_{m \in M_p} \lambda_m^p(k)C - \mu_m^p(k)C \\
\text{subject to} & \sum_{p \in P} \eta^p(k) \leq \eta, \quad \forall p \in P \\
& \eta^p(k)C \geq g_p, \quad \forall p \in P \\
& \mu_m^p(k) \leq \eta^p(k) s_m^p, \quad \forall m \in M_p \forall p \in P \\
& \mu_m^p(k) \leq \frac{X_m^p(k)}{C} + \lambda_m^p(k), \quad \forall m \in M
\end{align*}
\]
\[
\text{max} \sum_{p \in P} \sum_{m \in M_p} \mu_m^p(k) \\eta^p(k) \geq \eta^p(k)C \geq g_{\min}^p, \quad \forall p \in P
\]

\[
\sum_{p \in P} \eta^p(k)C \geq \eta^p(k) \geq \eta^p(k)C \geq g_{\min}^p, \quad \forall p \in P
\]

\[
\eta^p(k)C \geq \sum_{m \in M_p} \mu_m^p(k) \eta^p(k) s^p_m, \quad \forall m \in M^p
\]

\[
\mu_m^p(k) \leq \frac{X_m(k)}{C} + \lambda_m^p(k), \quad \forall m \in M
\]

\[
\mu_m^p(k) \geq 0, \quad \forall m \in M
\]

\[
\eta^p(k) \geq 0, \quad \forall p \in P.
\]

After solving model (8) and obtaining the signal timing plan in cycle \(k\), the queue length at the beginning of cycle \(k + 1\) can be calculated using equation (9) and the signal timing plan for cycle \(k + 1\) can be also solved:

\[
X_m(k + 1) = X_m(k) + \lambda_m^p(k)C - \mu_m^p(k)C, \quad \forall m \in M.
\]

Following this method, a series of LP models can be recursively solved. Finally, the optimal signal timing plan for the total oversaturated period with numerous cycles can be obtained.

Model (8) (represented by model 1) is a linear program model, and it can be employed standard LP algorithms to solve for the optimal signal timing plan at the oversaturated intersection. It was proven in Li et al. [17] that the obtained signal timing plan is globally optimal. However, when the inflow contains bus flow at each cycle, an adaptive strategy to conform signal timing based on the realized traffic flow over the planning horizon is needed, and the bus priority at the oversaturated intersection should be considered.

### 2.2. Bus Priority Model at the Oversaturated Intersection

The number of arrival vehicles at each intersection approach is undetermined, whereas the arrival rate within a cycle is assumed homogeneous, that is, vehicle arrival rate in one signal cycle is consistent. This research works on stochastic demand. Therefore, it is assumed that traffic arrival rates vary from cycle to cycle, following a certain known distribution. Model 1 proposes an optimized model of signal timing for oversaturated intersection, but the composition of traffic flow and bus priority is not considered. In order to achieve the priority of the bus at the intersection, this paper studied the problem of bus signal priority and proposed a bus priority model at the oversaturated intersection based on model 1. It is assumed that the arrival traffic flow is composed of two parts, namely, bus flow \(\lambda_m^p(k)\) and nonbus flow \(\lambda_{mb}^p(k)\) that will vary from cycle to cycle. In order to unify bus flow and nonbus flow, we convert the bus flow into the nonbus flow based on the carrying capacity. The calculation formula is as follows:

\[
\omega = \frac{O_b^p}{O_p^p}
\]

\[
\lambda_m^p(k) = \lambda_m^p(k) + \lambda_{mb}^p(k) \omega.
\]

Then, equation (7) can be rewritten as

\[
\mu_m^p(k) \leq \frac{X_m(k)}{C} + \lambda_m^p(k) + \lambda_{mb}^p(k) \omega, \quad \forall m \in M.
\]

The phase clearance reliability (PCR) is defined as \(\alpha\); the probability of the green time ratio greater than the actual green signal ratio:

\[
\alpha_p = P_r[\eta^p s \geq \lambda^p].
\]

Bus signal priority can ensure that queues of the bus phase are preferentially emptied as much as possible in each signal cycle, but this inevitably leads to increasing queues and delay for other phases. If the intersection is oversaturated, this situation can be even worse. In order to ensure the benefit of the entire intersection, the bus signal priority should not be an absolute priority. The green time of the bus phase should have some adjustment space for other phases to alleviate the queues and delays.

This paper defines the adjustment space for the bus phase, that is, the PCR value of the bus phase meets a certain level. The constraint can be written as

\[
P_r[\eta^p s \geq \lambda_{mb}^p (t) + \lambda_{mb}^p(t) \cdot \omega] \geq \alpha_p, \quad \forall m \in M.
\]

The previous equation indicates that the probability of the green signal ratio of the bus phase is greater than or equal to the actual required green signal ratio which is greater than or equal to \(\alpha_p \in [0, 1]\) (e.g., if \(\alpha_p = 1\), it means that the PCR value of the bus phase is 100%, which means that the bus phase has absolute priority; if \(\alpha_p = 0.95\), it means that the PCR value of the bus phase is 95%; i.e., the bus phase reserves 5% green signal ratio adjustment space for other phases).

Secondly, in order to coordinate the overall benefit of the intersection and prevent the increasing delay of vehicles in the nonbus phase due to the priority of the bus phase, the PCR value of the nonbus phase should meet a certain level. The following constraints should be added:

\[
P_r[\eta^p s \geq \lambda_{mb}^p] \geq \alpha_p, \quad \forall m \in M, p \in P \{p_b\}.
\]

The previous formula indicates that the probability of the green signal ratio of the nonbus phase is greater than or equal to the actual required green signal ratio which is greater than or equal to \(\alpha_p \in [0, 1]\) (e.g., \(\alpha_p = 0.7\) means the PCR value of the nonbus phase is 70%).
In this way, after considering the PCR level of bus phase and nonbus phase, the model can be written as follows:

\[
\max \sum_{p \in P} \sum_{m \in M_p} \mu_p^m(k)
\]

\[
\sum_{p \in P} \eta_p^m(k) \leq \eta
\]

\[
g_{\min}^p \leq \eta_p^m(k)C \leq g_{\max}^p, \quad \forall p \in P
\]

\[
\mu_p^m(k) \leq \eta_p^m(k) s_p^m, \quad \forall m \in M_p
\]

\[
\mu_p^m(k) \leq \frac{X_m(k-1)}{C} + \lambda_{m}(k) + \lambda_{nb}(k)\omega, \quad \forall m \in M
\]

\[
P_1 \left[ \eta_p^m(k)s \geq \lambda_p^m \right] \geq \alpha_p^m, \quad \forall m \in M, p \in P \setminus \{p_0\}
\]

\[
\text{s.t.} \quad P_1 \left[ \eta_p^m(k)s \geq \lambda_p^m \right] \geq \alpha_p^m, \quad \forall m \in M
\]

\[
\mu_p^m(k) \geq 0, \quad \forall m \in M
\]

\[
\eta_p^m(k) \geq 0, \quad \forall p \in P
\]

\[
\alpha_p^m > \alpha_p, \quad \forall p \in P
\]

\[
0 \leq \alpha_p^m \leq 1
\]

\[
0 \leq \alpha_p \leq 1.
\]

3. Solution Algorithm

Zhao et al. [16] defined \(b_m^p = (\lambda_m + X_m/C)/s_m^p\) as shown in Figure 2(a). That is, when \(\eta_p < b_m^p\), the vehicles of stream \(m\) cannot be absolutely dismissed within the green time (remaining queue \(X' = (\eta_p - b_m^p)\lambda_m C\)); that is, the smoothed departure flow rate is \(\mu_m = \eta_p s_m^p\), which linearly increases in \(\eta_p\). When \(\eta_p > b_m^p\), all the vehicles of stream \(m\) are dispatched, and \(\mu_m\) will remain constant as \(\lambda_m + X_m/C\).

In each phase \(p \in P\), the departure flow rate \(\sum_{m \in M_p} \mu_m^p\) is a nondecreasing piecewise linear concave function, as shown in Figure 2(b).

For model 1,

\[
\max \sum_{p \in P} \sum_{m \in M_p} \mu_p^m(k) = \max \sum_{p \in P} \sum_{m \in M_p} \min \left\{ \eta_p^m s_p^m, \lambda_m + X_m/C \right\}
\]

\[
= \max \sum_{p \in P} \left( \eta_p^m a_m + b_m s_m + \eta_p^m a_p^m \sum_{i=1}^{N_p} b_m s_m + \eta_p^m a_p^m \sum_{i=1}^{N_p} b_m s_m \right),
\]

where \(a_m^p = \sum_{i=1}^{N_p} s_m\) is the slope of piece \(m\) (between \(b_m^p\) and \(b_m^p\)) in the objective function of the \(p\)th signal phase (see Figure 2(b)).

Define \(\Delta \eta_p^m = \eta_p^m - g_{\min}^p/C\); \(\eta = \sum_{p \in P} g_{\min}^p/C\); model 1 can be converted to a special continuous knapsack problem. Thus, Zhao et al. [16] applied the following greedy
search algorithm that guarantees to find the optimal signal timing plan for model 1.

We proposed a heuristic algorithm based on Zhao et al.’s [16] algorithm to solve model (15). We use the algorithm of Zhao et al. [16] to calculate and allocate the green time for each phase and then adjust the assigned green time based on the PCR value of the bus phase and the nonbus phase. The adjustment method is described as follows: first, check the PCR value of the bus phase. If formula (13) does not hold, the algorithm ends; if formula (13) holds, it indicates that the phase clearance reliability of the bus phase meets a certain level, and continue to check the PCR value of the nonbus phase. If the PCR value of the nonbus phase satisfies formula (14), the algorithm ends; otherwise, the green time adjustment is needed. For the bus arrival rate $\lambda_m$ minus unit adjustment $\lambda_0$, the green time of each phase is recalculated. At this time, the green time of the bus phase has been adjusted to the nonbus phase. After multiple iterations, the optimal solution of the model can be obtained. Both our algorithm and Zhao et al.’s [16] algorithm have similar properties with few iterations and fast solution speed, which can solve the model quickly and efficiently. The algorithm pseudocode is shown in Algorithm 1.

4. Case Study

In this section, two tests are presented to scrutinize the control capability of our model. The first test case is to verify the model’s performance of the phase queue length, queue delay, total queue length, and total delay in the same traffic flow scenario and different bus headway conditions. The second test case is to verify the control capability of the overall control effectiveness of the model under the time-dependent traffic flow scenario (for the convenience of expression, the model in this paper is referred to as model 2 in the following).

4.1. Test Case 1

4.1.1. Experiment Design. A hypothetical signalized intersection with three signal phases is shown in Figure 3. The phase configuration and saturation flow rate settings in each phase are shown in Figure 3. The signal timing is set as follows: cycle length $C = 110s$, loss time $L = 10s$, minimum green time $g_{\text{min}} = 15s$, $g_{\text{min}} = 12s$, $g_{\text{min}} = 10s$, maximum green time $g_{\max} = g_{\min} = g_{\max} = 65s$, and bus phase $\alpha^p = 0.95$. The traffic flow of nonbus at each approach of the intersection is shown in Table 2.

It is assumed that there are buses in the left turn direction of the east bound. The average number of passengers on a bus is 30, and the average number of passengers on a nonbus is 2. Use equation (10) to convert the arrival rate of buses in Table 3 to the arrival rate of nonbus; the eastbound left-turn arrival rate is the sum of the nonbus arrival rate and the bus Converted arrival rate (e.g., Table 3 in the case of traffic scenario 1, the left arrival rate at the east bound is $\lambda = 400 + 90 = 490veh/h$).

To compare the three signal timing models, the following performance indicators are used:

1. The total queue length $L(k)$: the sum of the queue lengths in all directions at the end of the cycle $k$, that is, $L(k) = \sum_{m \in P} X_m (k + 1)$.

2. The delay $D_r(k)$: the delay of passengers in the queue during the red light of each cycle, that is, $D_r(k) = O_p \sum_{m \in P} P | \lambda_m (k) - \mu_m (k) | \sum_{p \in P} | p | g_p^f$. $O_p = (Q(k)_p O_p + Q(k)_p O_p + Q(k)_p O_p) / (Q(k)_p + Q(k)_p + Q(k)_p)$ is the average number of passengers in phase $p$.

4.1.2. Result and Analysis. First, we use model 2 to calculate the green time of each phase. Second, we use the following two evaluation indicators $(L(k)D_r(k))$ to verify the effect of model 2. The results of the green time calculation by model 2 are plotted in Figure 4. The x-axis is the bus headway of the eighteen scenarios, whereas the y-axis is their corresponding green time. The series of lines represent the results for different signal phases and the PCR value under consideration. The solid lines show the green time for nonbus phase PCR value is 0 $(\alpha^p = 0)$, that is, absolute priority on bus phase. With the increase of bus arrival rate in phase 2, the green time in this phase shows an increasing trend, while the green time in phase 1 shows a decreasing trend. The reason is...
Read cycle length $C$, number of phase $P$, number of iterations $N$, saturation rate $s_m$, arrival rates $\lambda_m$, a sufficiently large number $M$, initial queue length for each phase $X_m$. Iteration indicator $n=0$, $\alpha_m^n=\sum p s_m$, $\beta_m^n=(\lambda_m+X_m/C)/s_m$, $\eta=p g_{\min}/C$ ($\forall p \in P$), $T_n=\sum p \eta p$. For each phase $p \in P$, sort by $\beta_m^n$ in ascending order ($\forall m \in M^p$); if $\lambda_{mb}^p>0$ then $\alpha_m^n=M$; $\lambda_m^p=\lambda_{mb}^p+\lambda_m^p \cdot \omega$; while $n<N$ do while $A_n<\omega$ do $\alpha_m^n=\max\left\{\alpha_m^n, A_n\right\} \{a_m^n\}$; while $\beta_m^n < g_{\min}$ do $T_{n+1}=T_n$, $\eta_n-\eta_p > \eta$ if $P_r\left[p \eta_s \geq \lambda_{mb}^p(t) \cdot \omega \geq \alpha^p \right]$ then $g_p=\eta ^p C$ and end; else $\lambda_m^p(t)-\lambda_{mb}^p(t) - \lambda_0$ and $n=n+1$; else end; $T_{n+1}=T_{n+1}+\beta_m^n - \eta_n$ and $\eta_n-\eta_p = \beta_m^n$ and $n=n+1$; return $g_p$

**Algorithm 1:** The algorithm pseudocode.

**Figure 3:** The studied intersection and its phase.

**Table 2:** Arrivals of the intersection (veh/h).

| Traffic stream | Left (L) | Though (T) | Right (R) | Total |
|---------------|---------|-----------|----------|-------|
| North         | 300     | 500       | 200      | 1200  |
| South         | 200     | 600       | 200      | 1600  |
| East          | 400     | 1900      | 200      | 2500  |
| West          | 300     | 1800      | 500      | 2600  |

that the green time is preferentially allocated to the bus phase, and the green time in the nonbus phase is compressed. Since the first two phases are allocated preferentially, the green time of phase 3 keeps the minimum green time of this phase unchanged. As shown by the dotted line in Figure 4, three PCR values of the nonbus phase ($\alpha^p = 0.75$, $\alpha^p = 0.8$, $\alpha^p = 0.85$) are selected to display the curve after the green time is adjusted; that is, the PCR values of the nonbus phase are 0.75, 0.8, and 0.85 based on considering the priority of the bus phase. The green time of the nonbus phase will be allocated more than the PCR increases. It can be seen from Figure 4 that the larger the PCR of the nonbus phase, the higher possibility of adjusting the green time.

When $\alpha^p = 0.95$, $\alpha^p = 0.75$, the two indexes ($L(k)$ and $D_r(k)$) of bus and nonbus are calculated by literature [14], model 1, and model 2, respectively. The calculation results are shown in Figure 5. It can be seen in Figures 5(a) and 5(c) that the two indicators ($L(k)$ and $D_r(k)$) of the Ma et al. [14] model and model 2 are significantly lower than model 1. Compared with Ma et al.’s [14] model, the average reduction of the two indicators in model 2 is about 37% and 43%. The model 2 method significantly reduces the delay ($D_r(k)$)
overall intersection, which is 31.1% and 10.26% lower than model 1 and Ma et al. [14] model, but the queue ($L(k)$) overall of the intersection increases by about 61.02% and 21.69%. The reason for this phenomenon is that the priority of the bus phase is compressing the green time of the nonbus phase, which causes the queue length of the nonbus phase to increase. However, due to the large number of passengers carried by bus, the implementation of bus priority can significantly reduce the queue and delay on the bus phase, and the total delay of passengers at intersections is also significantly reduced. It can also be seen from Figures 5(b) and 5(d) that when the bus headway is between 5 min and 10 min, the total queue length of model 2 increases by about 13%–18%, and the total passenger delay reduces by about 25%–34% compared with Ma et al.’s [14] model.

However, when the bus headway is less than 5 minutes, the growth of the total queue length at intersections increases significantly, with an increase of about 28%–47%, and the decrease of the total passenger delay at intersections is less than 17%–23%. This indicates that the bus signal
priority will significantly increase the total queue length at the intersection when the bus arrival rate is large, but the optimization effect of total passenger delay has not been changed significantly. At this time, both bus signal priority and space priority should be considered to improve this phenomenon.

4.2. Test Case II

4.2.1. Experiment Design. The goal of the second case is to scrutinize the control capability of the proposed model 2 by comparing with the Ma et al. [14] model and model 1. A time-dependent traffic flow plan is created to duplicate a total period of congestion including the whole queue evolution process. As shown in Tables 4 and 5, nine traffic demands and four bus headway scenarios are given to each of the turning movements at a time step of 110 s. Set the PCR value of the bus phase to $\alpha_{pb}^{0} = 0.95$, and the nonbus phase to the PCR value of $\alpha_{p}^{0} = 0.75$. The other basic setting conditions are the same as those in Case 1.

4.2.2. Result and Analysis. Use model 1 and model 2 to calculate the green time and queue length of each phase in the case of nine traffic demands and four bus headway scenarios. The calculation results are shown in Table 5 ($H_t$ represents the bus headway).

It is found from Table 5 that the green time calculated by model 1 and model 2 is not the same, but Figure 6 shows that...
the queue clearing time of the two models is almost the same in different traffic scenarios, indicating that model 2 can effectively clear the queue while ensuring the bus priority. Secondly, the total delay from model 2 and Ma et al.’s [14] model is different. The total passenger delay from the proposed model 2 is much less than that of the Ma et al. [14] model. This can be also proved by the cumulative curve diagram in Figure 7. In Figure 7, the area between the curve

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### Table 3: Bus headway (min) and arrival rates (veh/h).

| Plan | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|---|---|---|---|---|---|---|---|---|
| Headway | 10 | 9.5 | 9 | 8.5 | 8 | 7.5 | 7 | 6.5 | 6 |
| Arrival rate | 6 | 6.3 | 6.7 | 7.1 | 7.5 | 7 | 8 | 8.6 | 9.2 | 10 |
| Equivalent arrival rate | 90 | 95 | 100 | 106 | 113 | 120 | 129 | 139 | 150 |
| Plan | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| Headway | 5.5 | 5 | 4.5 | 4 | 3.5 | 3 | 2.5 | 2 | 1.5 |
| Arrival rate | 10.9 | 12 | 13.3 | 15 | 17.1 | 20 | 24 | 30 | 40 |
| Equivalent arrival rate | 164 | 180 | 200 | 225 | 257 | 300 | 360 | 450 | 600 |

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### Table 4: The dynamic traffic plan (veh/h).

| Traffic plan | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|--------------|---|---|---|---|---|---|---|---|---|----|----|----|
| North bound  | L | 100 | 110 | 120 | 90 | 70 | 50 | 30 | 20 | 20 | 20 | 20 |
|              | Th| 400 | 440 | 480 | 360 | 280 | 200 | 120 | 80 | 80 | 80 | 80 |
|              | R | 100 | 110 | 120 | 90 | 70 | 50 | 30 | 80 | 80 | 80 | 80 |
| South bound  | L | 20 | 22 | 24 | 18 | 14 | 10 | 10 | 10 | 10 | 10 | 10 |
|              | Th| 50 | 55 | 60 | 45 | 35 | 25 | 25 | 25 | 25 | 25 | 25 |
|              | R | 30 | 33 | 36 | 27 | 21 | 15 | 15 | 15 | 15 | 15 | 15 |
| East bound   | L | 400 | 440 | 480 | 360 | 280 | 200 | 180 | 180 | 180 | 180 | 180 |
|              | Th| 1800 | 1980 | 2160 | 1620 | 1260 | 900 | 540 | 360 | 360 | 360 | 360 |
|              | R | 200 | 220 | 240 | 180 | 140 | 100 | 60 | 40 | 40 | 40 | 40 |
| West bound   | L | 100 | 110 | 120 | 30 | 70 | 50 | 30 | 20 | 20 | 20 | 20 |
|              | Th| 400 | 440 | 480 | 360 | 280 | 200 | 120 | 80 | 80 | 80 | 80 |
|              | R | 100 | 110 | 120 | 90 | 70 | 50 | 30 | 20 | 20 | 20 | 20 |

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![Figure 6: Total queue length of the intersection.](image-url)
Figure 7: The total intersection delay.

Table 5: The green time and queue of the dynamic plan.

| Traffic plan | Bus headway | $H_t = 8$ min (113 veh/h) | $H_t = 6$ min (150 veh/h) | $H_t = 4$ min (225 veh/h) | $H_t = 2$ min (450 veh/h) |
|--------------|-------------|---------------------------|-----------------------------|-----------------------------|-----------------------------|
|              | P1          | P2           | P3           | Queue | P1          | P2           | P3           | Queue | P1          | P2           | P3           | Queue | P1          | P2           | P3           | Queue |
| 1            | Model 1     | 61           | 12           | 27     | 11          | 61           | 12           | 27     | 15          | 61           | 12           | 27     | 21          |
|              | Model 2     | 52           | 38           | 10     | 20          | 50           | 40           | 10     | 29          | 50           | 38           | 10     | 62          | 39           | 30           | 13     |
| 2            | Model 1     | 67           | 12           | 21     | 29          | 67           | 12           | 21     | 36          | 67           | 12           | 21     | 50          |
|              | Model 2     | 49           | 41           | 10     | 52          | 47           | 43           | 10     | 77          | 43           | 47           | 10     | 101         |
| 3            | Model 1     | 73           | 12           | 15     | 55          | 73           | 12           | 15     | 65          | 73           | 12           | 15     | 101         |
|              | Model 2     | 61           | 38           | 10     | 94          | 55           | 38           | 10     | 115         | 53           | 37           | 10     | 167         |
| 4            | Model 1     | 43           | 18           | 58     | 39          | 43           | 18           | 39     | 67          | 43           | 18           | 39     | 101         |
|              | Model 2     | 30           | 56           | 14     | 36          | 31           | 56           | 14     | 57          | 31           | 56           | 14     | 98          |
| 5            | Model 1     | 21           | 72           | 12     | 23          | 18           | 72           | 10     | 39          | 18           | 72           | 10     | 87          |
|              | Model 2     | 20           | 62           | 23     | 36          | 20           | 62           | 23     | 63          | 43           | 47           | 10     | 178         |
of the total arrival passengers and the curve of the total departure passengers represents the total passenger delay. In Figure 7, the total passenger delay of model 2 is reduced by an average of 67% ($H_t = 8$ min), 64% ($H_t = 6$ min), 57% ($H_t = 4$ min), and 46% ($H_t = 2$ min) compared to model 1; compared with the Ma et al. [14] model, the total passenger delay has been reduced by an average of 16% ($H_t = 8$ min), 19% ($H_t = 6$ min), 23% ($H_t = 4$ min), and 30% ($H_t = 2$ min). It is shown that the model 2 method has good adaptability to time-varying traffic demand, and the model 2 method can effectively reduce the total passenger delay at the intersection.

5. Conclusions

There is an overflow queue at every cycle of the oversaturated intersection. The occurrence of the bus in the overflow queue will cause a series of problems, such as the increase of passenger delay and the decline of bus operation service level. Aiming at optimizing the bus priority parameters at oversaturated intersections, this paper extends the signal timing model 1 at oversaturated intersections. It establishes an optimization model for bus signal priority control at oversaturated intersections; the model solving algorithm based on Zhao et al.’s [16] method is also proposed. The two numerical examples are given in the paper; the results show that the model can make better use of the green time to generate a signal timing scheme. The proposed model can effectively empty the vehicle queue at intersections and significantly reduce the total delay of vehicle queue length and passengers at intersections, which can be applied to implement for the existing urban road network. Because this method requires upgrading and transformation of most intersections, which involves capital costs, land costs, and other issues, the proposed model only considers the bus signal priority at existing urban intersections but did not consider the issue of bus signal priority in combination with the design of bus approach lanes.

Future research should analyze the values of each signal phase PCR value based on considering the short-time random characteristics of bus arrival. A bus signal priority method considering the short-time random (or real-time) characteristics of bus arrival rate and the dynamic PCR value can be established to improve the effectiveness of the model. For the urban road network newly built or reconstructed, we will continue to study the bus priority method that combines bus signal priority and bus approach lanes design.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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