To the possibility of using clocks in gravitational antenna network

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Abstract. When gravitational waves (GW) spread in space-time, we usually say that spatial distances between reference points are distorted. The LISA project was developed to measure distortion in space. Simultaneously with the GW arrival, time intervals are also distorted. For this purpose, the pulsar timing method was developed. This article proposes to use a network of stabilized atomic clocks installed on deep-space spacecraft to measure time interval. A preliminary estimate of the required accuracy of clocks when registering GW is offered. It is indicated that a network of clocks installed on LISA-type spacecraft can serve as an additional and independent GW detection channel.

1. Introduction
According to the basic principles of general relativity (GR), space and time are combined to form “space-time”, which is bent in the presence of a massive body. The curvature of spacetime depends on the body mass and determines the trajectory of a small body’s mass trajectory. With the motion of massive bodies, the spacetime curvature changes, and with their periodic motion a periodic change in the curvature of the space of time appears. Periodic changes in the curvature propagating through the universe are known as gravitational waves (GW).

Multiple detections of the gravitational-wave signals in laboratories pose the question of the systematic and detailed study of the gravitational radiation from cosmic sources. New types of detectors should broaden the scope of the search for gravitational waves.

From a theoretical point of view, the direct detection of GWs is crucial for several reasons. The most important and fundamental is the goal to ultimately determine if GR is a correct theory of gravity or it is simply the best that we have now. In the latter case, we can hope to answer the question when and under what circumstances GR may be wrong. It can also allow us to investigate whether there are other theories unconceivable before that are superior to general relativity in describing everything: from Milky Way and other galaxies evolution before formation and Universe expansion in time. Since GW can be generated and can propagate at any frequency, there is a large variety of possible sources of GW. Moreover, the questions about GW amplitude and its polarization can have answers in alternative theories. This is also the reason why we can investigate some processes inside the massive objects.

GW can penetrate into regions that are dense and opaque to electromagnetic waves. The observations of the interior of black holes and other exotic objects will become possible. As it was noted above, GW from accelerating masses distort space-time. The distortions compress or stretch the time it takes the signal to reach the Earth. How can it be measured? It is known [1] that we can use the
field approach for a zone far from the source, so we can determine the effect of gravitational signals on measuring instruments. It is also known that the propagation of the gravitational potential leads to a change in the position of small masses.

It is a challenge to carry out an experimental measurement of a position in a laboratory in the presence of low-frequency noises (< $10^{-3}$ ÷ $10^{-3}$ Hz) (LIGO and VIRGO advanced antenna’s operating range is 10÷100 Hz), so new measurements in space seem an attractive idea. Moreover there are plenty of the sources in this range of frequencies.

A well-known method of measuring a shift of mass positions in space caused by a gravitational wave is interferometry [2]. From another point of view, the change of mass positions leads to a variation of its velocity. Therefore, there is a possibility of separating the gravitational signal by measuring a deviation of the pulsar velocity [3], or by analyzing on the Earth how the difference between the rates of the pulsar impulses (operating clock frequency) varies with time (pulsar timing method). A layout of a spacecraft equipped with atomic clocks along with an interferometry unit or a radio antenna for two channel detecting is proposed.

We give some estimates concerning the accuracy for registration of a test mass velocity and a frequency drift due to Doppler effect. The scheme of the tracking was outlined firstly by V.B. Braginsky, M.E. Gertzenstein [4], then by papers of researchers from JPL (USA) followed.

For a distance between a satellite and the receiver on the Earth $\Delta l \sim 10^9$m, a GW amplitude $h \sim 10^{-17}$ ÷ $10^{-19}$, a GW frequency $\omega \sim 0.6$ ÷ 60 rad/s ($\nu \sim 10^{-4}$ ÷ 10$^{-1}$Hz) we have $\lambda_g=3\cdot(10^7$ ÷ $10^9)$ m, so $\Delta\nu=\hbar\nu l \sim 3\cdot(10^{-11}$ ÷ $10^{-7})$ m/s; $\Delta l/\nu = 10^{-19} \div 10^{-15}$.

2. The principals of pulsar timing method

The study of a certain class of double stars confirms their existence. In this class of millisecond pulsars (MSP), one of the stars is visible as a radio pulsar (a star that radiates a pulsating narrow beam of radio emission along its magnetic axis). It is a rapidly rotating, magnetized, compact and massive object known as a neutron star (pulsar mass ~ 1.4 Sun mass is concentrated in a ball about 10 km in diameter, magnetic field ~ $10^{10}$ ÷ $10^{12}$ T, angular velocity of rotation 17 ÷ 643.3 rps) [5]. A pulsar can be observed when it moves in the curved space of a companion star. Since this movement creates GW, the spreading ripples carry energy away, so that the size of the binary system decreases, and the distance from the pulsar to its satellite decreases. The rate of this reduction was measured for a few of binary pulsars and, as it turned out, is consistent with GR predictions [6]. Thus, these observations were the indirect evidence of the GW existence.

Our interest in pulsars is caused by the overstability of their rotation and the high coherence of their radiation. When a pulsar rotates, it acts as a cosmic radio beacon, and the pulsar radiation directed towards the Earth may be observed once per rotation creating a narrow impulse. The form of an impulse is varied, but the speed of rotation and repetition of pulsars impulses are practically unviolated: the stability of frequency is up to 10$^{-19}$. These peculiarities motivate us to use pulsars as natural precise cosmic clocks. Their use as instruments is enabled by the synchronization of pulsars. Pulsar time is the regular monitoring of the rotation of a neutron star by tracking (almost exactly) the arrival time of radio pulses. The key point for using pulsar synchronization is that the time of the pulsar impulse arrival unambiguously takes into account the fact that a neutron star has been in place for long time intervals (from years to decades). This unique and very accurate tracking allows us to explore precise astronomical measurements and even to test gravity theory in a strong field case.

In the practically inertial reference frame of the solar system barycenter, the rotation period of the pulsar is almost constant, therefore the time-dependent phase $\phi(t)$ of the pulsar can be approximated by Taylor series

$$\phi(t)=\phi_0+f(t-t_0)+1/2 f^2(t-t_0)^2+1/6 f^3(t-t_0)^3,$$

where $\phi_0$, $t_0$ are arbitrary phase and time references for each pulsar, $f = d\phi/dt$ is the rotation frequency of the pulsar. The last term is six orders less than the third and is often omitted since it is not distinguished from the noise [A. Avramenko Pulsar 2017].
To synchronize pulsars, astronomers “add” radio data modulo an instantaneous pulse period $P$ or pulse frequency $f = 1/P = d\phi/dt$. Averaging over many pulses gives a high average impulse signal-to-noise profile. Although the individual pulse shapes vary considerably, the shape of the middle profile is fairly stable. As a rule, the average profile correlates with the template or model profile so that the phase shift can be determined.

For example, for the initial millisecond pulsar B1937 + 21, the TOA (time of arrival) accuracy is approximately 1 $\mu$s, which is a partial error in the phase of about $6 \times 10^{-4}$ rotations for more than 25 years: $\Delta f \sim 8 \times 10^{-13}$ Hz.

However, in order to measure $\phi(t)$ in this formula, numerous corrections must be made for the observed TOA. For this, terrestrial radio telescopes data are corrected to time $t$ in the barycentric reference system, and then corrections (seven or more) are introduced, including corrections for the ground clock of the observatory, the electromagnetic signal delay inside the solar system and a binary star system, the electron content of the interstellar medium along the propagation path.

Deviations of any of these parameters, as well as other parameters, such as $f$, $f'$ and parameters of self-movement, give us very specific systematic signs on the plots of the time residuals, which are simply phase differences between the observed TOA and the predicted TOA time based on the current model parameters.

As an example, we present estimates of the determination of the position of a MSP millisecond pulsar longitude $\Delta \lambda$ and latitude $\Delta \beta$ in a barycentric reference system

$$\Delta \lambda \sim 1 \times 10^{-3} \text{rad} \quad \text{and} \quad \Delta \beta \sim 2 \times 10^{-3} \text{rad},$$

when the pulsar is located near the ecliptic plane.

3. **MSP as gravitational wave detectors**

The idea of direct detection of GW was proposed by M.V. Sazhin and S. Detweiler [7]. In their experiment, the positions of the observed pulsars were to be calculated with high accuracy, i.e. the synchronization of pulses on the Earth would be recorded accurately and compared with the rotational “timekeeping” model. Minor deviations from the expected TOA would be interpreted as significant "time residues", which may be considered as free masses position displacement, due to reactions on the changes in the spacetime metric. Consequently, the disturbance produced by the GW would slightly displace both the pulsars and the Earth. This would lead to temporary discrepancies that depend on the amplitude of the GW and the total observation time. The sensitivity of GW detection using pulsars was determined directly by the magnitudes of the time residues — these are the quality indicators used to determine the accuracy of the position of the pulsars.

Recently the international pulsar timing array (IPTA) was created. The aim of IPTA is to detect gravitational waves using an array of about 30 pulsars. Researchers are trying to find GW by comparing the deviations seen between TOA from several different pulsars, forming what is known as a pulsar timing array [8].

4. **A scheme of a synchronized atomic clocks network**

Pulsars or modern laboratory atomic clocks with $10^{-16}$-$10^{-18}$ accuracy can be used as a reference. The system of space-based detectors whose operating region covers a large spatial area in the Earth’s neighborhood will form a network. The registered variation of the clock rate will indicate the amplitude of the gravitational signal, the frequency of the variation will indicate the frequency of the gravitational wave. Being filtered from the known time lags, the delay of signals from different clocks of the network can be associated with the direction of a GW front and its polarization. A functioning space-based clock network will play the role of a gravity antenna.
The proposed experiment with operating clocks in remote orbits around the Earth (figure 1) is similar to the terrestrial interferometric detectors, such as LIGO and VIRGO, in which the time of passage of a laser beam is measured along a specific trajectory and compared with the time of passage along an orthogonal trajectory. Instead of the time of passage of the laser beam, the chronometric space network (HSN) measures the time interval that it takes to travel for an electromagnetic pulse in collation with TOA of another pulse. But in comparison with 4-kilometer shoulders (as in the case of LIGO), the HSN arms are much larger, they are the order ~ 1ae (1.4960×10^8 km) (the distance between the space-based atomic clock unit and the Earth). The total number of HSN blocks (and, therefore, the number of "hands" in the detector) is approximately from two to twelve. Each block of the HSN measures the time shift of its pulse from one to eleven pulses coming from the other blocks of the HSN during two to three months. Blocks may be combined into groups comprising from two to twelve operating hours and exchanging asynchronies data with the center. These temporary discrepancies between the data from the HSN units and the ground reference clocks will allow us to investigate a completely different frequency range of GW, their amplitude, polarization, direction (azimuth and declination) and, therefore, another source category. Taking into account that ground detectors are sensitive to 10÷100 Hz, the HSN is sensitive to the GW range of 10÷20 mHz. The main source of gravitational waves are supermassive binary black holes (billions of solar masses), which are supposed to exist in the Universe at the centers of galaxies as the result of previous mergers of these galaxies.

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