Search for dark photon and dark matter signatures around electron-positron colliders

Xin Chen\textsuperscript{a,b,c}, Zhen Hu\textsuperscript{a,b}, Yongcheng Wu\textsuperscript{d}

\textsuperscript{a}Department of Physics, Tsinghua University, Beijing 100084, China
\textsuperscript{b}Center for High Energy Physics, Tsinghua University, Beijing 100084, China
\textsuperscript{c}Center for High Energy Physics, Peking University, Beijing 100084, China
\textsuperscript{d}Ottawa-Carleton Institute for Physics, Carleton University, Ottawa, Ontario K1S 5B6, Canada

Abstract

The search for a dark photon produced at $e^+e^-$ colliders in the forward region, which subsequently decays into an excited and a ground state dark matter, is discussed. The excited dark matter decays into a pair of visible charged particles and a normal dark matter after traveling some distance. The visible decay products can be recorded by an emulsion detector, placed around the main $e^+e^-$ detector. This setup can not only explore new parameter regions not reached before, but also re-open some regions thought to be excluded by previous experimental data. The physics potential of such a detector around BESIII and Belle-II is presented.

Keywords: Dark photon, Excited dark matter, Long-lived particle

1. Introduction

The dark matter is well motivated by astrophysical observations, and it is believed to interact weakly with the Standard Model (SM) particle through some mediators, e.g., the dark photon. A number of fixed-target as well as collider-based experiments have searched for a dark photon, where it is produced from charged particles with a tiny coupling strength. The dark photon can decay back into charged SM particles, after flying some distance due to the small coupling and hence small decay width, which is the model of many fixed-target experiments (see \cite{cite1} and references therein). When the dark photon mass is large and the coupling is not so small, it can decay promptly into a pair of charged or neutral dark matter particles, which has been studied at the electron-positron colliders such as Babar \cite{cite2, cite3}.

We here propose a new experimental setup for searching for a dark photon that promptly decaying into a pair of dark matter particles at the $e^+e^-$ colliders. Complementary to the direct search for such a particle in the central detector region, we propose an emulsion detector in the forward region where it is more abundantly produced, and out of the detector’s acceptance. One of the two dark matter particle is in an excited state, so it can further decay into the ground state dark matter and a charged particle pair. By detecting the charged pair, the existence of a dark matter sector with a dark photon mediator can be inferred.

2. Theoretical framework

At low energy, we assume an effective toy model with a dark photon mediator $X$ and complex scalar $\phi$ as the dark matter particle. The relevant Lagrangian reads

$$\mathcal{L} = \mathcal{L}_{SM} + \frac{1}{4} X_{\mu\nu} X^{\mu\nu} + \frac{1}{2} m_\phi^2 \phi^\dagger \phi - \mu \phi^\dagger \phi - \frac{1}{2} \rho^2 (\phi^\dagger \phi + \phi \phi^\dagger), \quad (1)$$

where $X$ is the dark photon, $f$ is the SM fermion with charge $Q_f$, $\epsilon$ is the mixing parameter between the $U(1)_Y$ and $U(1)_D$ gauge fields, $D_\mu = \partial_\mu + ig_0 X_\mu$ is the covariant derivative with dark coupling parameter $g_0$, $\phi = (\phi_1 + i\phi_2)/\sqrt{2}$ is the complex scalar dark matter. The last term in Eq. (1) is a “Majorana”-like mass term that violates the $U(1)_D$ symmetry. Substituting $\phi_{1,2}$ for $\phi$, we have

$$\mathcal{L} \supset \frac{1}{2} \partial_\mu \phi_1 \partial^\mu \phi_1 + \frac{1}{2} \partial_\mu \phi_2 \partial^\mu \phi_2 - \frac{1}{2} (\mu^2 + \rho^2) \phi_1^2 - \frac{1}{2} (\mu^2 - \rho^2) \phi_2^2 - g_D X_\mu (\partial_\mu \phi_1 - \phi_1 \partial_\mu \phi_2) + \frac{1}{2} \rho^2 X_\mu X^\mu (\phi_1^2 + \phi_2^2). \quad (2)$$

It is evident that the presence of the small $\rho$-term in Eq. (2) causes a mass splitting between the two real scalars $\phi_1$ and $\phi_2$, with mass $m_1 = \sqrt{\mu^2 + \rho^2}$ and $m_2 = \sqrt{\mu^2 - \rho^2}$, respectively. It is also possible to achieve a similar mass splitting for a fermionic dark matter by the presence of Majorana mass terms \cite{cite4}, but the scalar model we are studying will be general enough to cover similar kinematics.

3. Dark photon production at $e^+e^-$ colliders

The main production diagrams of a dark photon through $e^+e^- \to \gamma X$ at an $e^+e^-$ collider are shown in Fig. 1.

The Born level differential cross section of the process $e^+e^- \to X\gamma$ with respect to $\cos \theta$, where $\theta$ is the polar angle...
between the outgoing $X$ and the electron beam axis, can be expressed as $^{5}$

$$\frac{d\sigma}{d\cos\theta} = 2\pi e^2\alpha^2 \frac{(s + m_X^2)}{s} \frac{\left(s - m_X^2\right)^2 \cos^2\theta}{s \left(s - m_X^2\right) \left(s \sin^2\theta + 4m^2\right)}, \tag{3}$$

where $s$ is the center of mass (CM) energy squared of the $e^+e^-$ system, and $m_e$ the electron mass. The cross section as a function of $\cos\theta$ is shown in Fig. 2(a), from which it is seen that the dark photon is primarily produced in the very forward directions of a $e^+e^-$ detector system. The total production cross section for different CM energy is shown in Fig. 2(b), from which we see that the dark photon is more favorably produced at lower energy $e^+e^-$ collisions such as BESIII (the cross section for $m_X = 1$ GeV at BESIII is about 7.53 times at Belle-II).

4. Dark photon decay

After $X$ is produced, it then promptly decays into charged SM fermion pair, or an excited dark scalar $\phi_1$ and a ground state dark scalar $\phi_2$. The spin averaged $X$ decay width to each final state can be expressed as

$$\Gamma(X \to f\bar{f}) = \frac{1}{3} e^2 Q_f^2 m_X \left(1 + \frac{2m_f^2}{m_X^2}\right) \left(1 - \frac{4m_f^2}{m_X^2}\right)^{1/2}, \tag{4}$$

$$\Gamma(X \to \phi_1\phi_2) = \frac{\pi g^2}{48\pi} m_X \left[1 - \frac{2(m_f^2 + m_{\phi_2}^2)}{m_X^2} + \frac{m_f^4 + m_{\phi_2}^4 - 2m_f^2m_{\phi_2}^2}{m_X^4}\right] \frac{\left[1 - \frac{(m_f + m_{\phi_2})^2}{m_X^2}\right]^{1/2} \left[1 - \frac{(m_f - m_{\phi_2})^2}{m_X^2}\right]^{1/2}}{m_{\phi_1}}, \tag{5}$$

where $m_f$ denotes the charged fermion mass ($f = e, \mu, u, d, s$), and $\alpha$ is the fine structure constant. The partial decay width into quarks can be collectively named $\Gamma_{\text{had}}$, and can be effectively evaluated using the experimental $R$ data $^{6}$, which is defined as

$$R(\sqrt{s}) = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}, \tag{6}$$

This experimental data input starts from $\sqrt{s} = 0.3$ GeV (just above the $\pi\pi$ mass threshold), with which $\Gamma_{\text{had}}$ can be expressed as

$$\Gamma_{\text{had}} = \Gamma_{\text{had}} \cdot R(\sqrt{s} = m_X). \tag{7}$$

When $g_D$ is orders of magnitude larger than $\epsilon$, $X$ will predominantly decay into the dark final state $\phi_1\phi_2$. Namely, the branching ratio (BR) of $X \to \phi_1\phi_2$ will be large.

5. Excited dark matter decay

The $\phi_1$ will fly for some distance, and then decays into $\phi_2$ and a $f\bar{f}$ pair through an off-shell $X^*$, as shown in Fig. 3

$$\phi_1 \to \phi_2 f\bar{f}$$

Figure 1: The production diagrams of an on-shell dark photon $X$ at a $e^+e^-$ collider.

Figure 2: The differential cross section of the $e^+e^- \to X\gamma$ process as a function of $\cos\theta$ (a), and its total cross section as a function of $\sqrt{s}$ (b), with the chosen parameters indicated in the plots. The CM energy of BESIII and Belle-II are indicated by a solid dot and box, respectively.

Figure 3: The decay diagram of an excited dark scalar $\phi_1$ into the dark matter $\phi_2$ and a $f\bar{f}$ pair through an off-shell $X^*$. 
can be expressed as
\[
d\Gamma(\phi_1 \rightarrow \phi_2 f \bar{f}) = \frac{e^2 Q_1^2 Q_2^2}{8\pi^2 m_1 (m_{f \bar{f}} - m_X^2)^2} [(m_{f \bar{f}} - m_{\phi_2} - m_{\phi_1})^2 (m_{f \bar{f}} + m_{\phi_1})^2 + m_{f \bar{f}}^2 - m_{\phi_2}^2 - m_{\phi_1}^2 + m_1^2 + m_2^2 + m_{f \bar{f}}^2 - m_{\phi_2}^2 - m_{\phi_1}^2 - m_{f \bar{f}}^2 + m_1^2 + m_2^2] \, dm_{f \bar{f}} dm_{\phi_1} \, \phi_{f \bar{f}} \, \phi_{\phi_1} \tag{8}
\]
where \(m_{f \bar{f}}\) denotes the \(f \bar{f}\) pair invariant mass, and \(m_{\phi_2}\) the invariant mass of \(f \) and \(\phi_2\). After integrating out \(m_{\phi_1}\), the partial decay width becomes
\[
d\Gamma_{\phi_1} = \frac{e^2 Q_1^2 Q_2^2}{8\pi^2 m_1 (m_{f \bar{f}} - m_X^2)^2} [(m_{f \bar{f}} - m_{\phi_2} - m_{\phi_1})^2 (m_{f \bar{f}} + m_{\phi_1})^2 + m_{f \bar{f}}^2 - m_{\phi_2}^2 - m_{\phi_1}^2 + m_1^2 + m_2^2 + m_{f \bar{f}}^2 - m_{\phi_2}^2 - m_{\phi_1}^2 - m_{f \bar{f}}^2 + m_1^2 + m_2^2] \, dm_{f \bar{f}} \, \phi_{f \bar{f}} \, \phi_{\phi_1} \tag{9}
\]
and the allowed range for \(m_{f \bar{f}}\) is \(2m_f \leq m_{f \bar{f}} \leq (m_1 - m_2)\). When \(f = e\), the differential decay width of \(\phi_1\) is shown in Fig. 4. It is evident that the \(ee\) pair mass has a broad spectrum spanning the whole allowed mass range. For the hadronic three-body decays of \(\phi_1\), as for the case of \(X\), the \(R\) input is used.

![Figure 4: The differential decay width of \(\phi_1 \rightarrow \phi_2 e^+ e^-\) as a function of \(m_{ee}\), with the chosen parameters indicated in the plot.](image)

Since it is produced from the \(X\) decay, the flight length \(d\) of \(\phi_1\) is determined by its Lorentz boost factor \(\gamma_1\) and its total decay width by
\[
d = D \frac{\gamma_1 \beta_1}{\Sigma_f \Gamma_{\phi_1 \rightarrow f \bar{f}}} \tag{10}
\]
where \(\beta_1\) is the speed of \(\phi_1\). The \(\phi_1\) boost factor varies from event to event, but an average value can be obtained by averaging over the angle \(\theta_1\) in Eq. [12] introduced in the next section. The \(d\) distributions as a function of \(m_1\) and \(m_2\) thus calculated for two different values of \(g_D\) are shown in Fig. 5 in the setting of BESIII. Only parameter spaces with \(2m < d < 50m\) are shown. It is evident that as \(g_D\) increases, the mass \(m_1\) has to decrease for \(d\) to be in an appropriate range.

![Figure 5: The distribution of \(\phi_1\) average flight length (color codes in the unit of meter) as a function of \(m_1\) vs. \(m_2\), in the range \(2m < d < 50m\), with the chosen parameters indicated in the plot. The \(\phi_1\) Lorentz boost factor is based on Eq. [12] but averaged over \(\theta_1\).](image)

6. Excited dark matter distributions in the lab frame

The double differential cross section for the \(\phi_1\) production, in the narrow width approximation and with full spin correlation between \(e^+ e^- \rightarrow \gamma X\) and \(X \rightarrow \phi_1 \phi_2\), is proportional to
\[
\frac{d\sigma}{d\Omega d\Omega_1} \propto \frac{1}{s} \left( s + m_X^2 \right)^2 \left( s - m_X^2 \right)^2 \cos^2 \theta \\
- \left( s - m_X^2 \right)^2 \cos^2 \theta \sin^2 \theta_1 \\
+ 4m_X \sqrt{s} \left( s + m_X^2 \right) \sin \theta_1 \cos \theta_1 \sin \phi_1 \sin \theta \cos \theta \\
- 4m_X^2 \sin^2 \theta_1 \cos^2 \phi_1 \sin^2 \theta \right), \tag{11}
\]
where \(\Omega (\Omega_1)\) is the solid angle of \(X (\phi_1)\) in the \(e^+ e^-\) lab (\(X\) rest) frame, and \(\theta_1\) and \(\phi_1\) are the polar and azimuthal angles of \(\Omega_1\), with the flight direction of \(X\) in the \(e^+ e^-\) lab frame as the polar axis for \(\Omega_1\).

Contrary to Eq. [3] where \(X\) is produced primarily in the forward (\(\theta = 0\) or \(\pi\) region, it is seem from Eq. [11] that when

\[1\text{Note that } \phi_1^*\text{ is also the angle between the plane formed by the beam axis and outgoing } X, \text{ and the plane formed by the } X \text{ decay products.}\]
\( \theta \to 0 \) or \( \pi \), and \( \theta' \to 0 \), the cross section vanishes, which creates a "hole" in the very forward region. With \( \Omega \) and \( \Omega' \) specified, the energy of \( \phi_1 \) in the \( e^+e^- \) lab frame can be expressed as

\[
E_1 = \frac{s + m_X^2}{4\sqrt{s}} \left( 1 + \frac{\theta_1^2}{m_X^2} \right) + \frac{s - m_X^2}{4\sqrt{s}} \left( \left( 1 + \frac{m_X^2 - m_1^2}{m_X^2} \right) - \frac{4m_1^2}{m_X^2} \right) \cos \theta_1', \hspace{1cm} (12)
\]

and the angle between its flight direction and the electron beam axis in the same frame (\( \theta_1 \)) can be determined via

\[
\cos \theta_1 = \left[ \left( s - m_X^2 + (s + m_X^2)\beta_1^2 \cos \theta_1' \right) \cos \theta - 2\sqrt{m_X}\beta_1^2 \sin \theta_1' \cos \phi_1' \sin \theta_1' \right] \cdot \left[ 4s\beta_1^2 \sin^2 \theta_1' + m_X^2 \beta_1^2 \cos \phi_1' \cos \theta_1' \right] \cdot \left( s - m_X^2 + (s + m_X^2)\beta_1^2 \cos \theta_1' \right)^{\frac{1}{2}}, \hspace{1cm} (13)
\]

where \( \beta_1^2 \) is a constant defined as

\[
\beta_1^2 = \left[ 1 - \frac{4m_1^2}{(m_X^2 + m_1^2 - m_2^2)} \right] \frac{1}{2}. \hspace{1cm} (14)
\]

Based on Eq. [11][14], the 2-D probability density distribution of \( \phi_1 \) Lorentz boost factor \( \gamma_1 \) and its \( \cos(\theta_1) \) in the lab frame for two different \( X \) masses are shown in Fig. [7][8], and the differential cross section of \( \phi_1 \) as a function of \( \cos(\theta_1) \) is shown in Fig. [9] in the vicinity of \( \theta_1 = 0 \).

7. Dark matter decay detector

To detect the visible decay products of \( \phi_1 \), we propose to use the emulsion detector, which has been used in Opera [7].

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.70 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 0.5 \text{ GeV} \]
\[ m_1 = 0.35 \text{ GeV} \]
\[ m_2 = 0.1 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 0.5 \text{ GeV} \]
\[ m_1 = 0.25 \text{ GeV} \]
\[ m_2 = 0.10 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.70 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.50 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 0.5 \text{ GeV} \]
\[ m_1 = 0.35 \text{ GeV} \]
\[ m_2 = 0.1 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.70 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.50 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 0.5 \text{ GeV} \]
\[ m_1 = 0.25 \text{ GeV} \]
\[ m_2 = 0.10 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.70 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.50 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 0.5 \text{ GeV} \]
\[ m_1 = 0.25 \text{ GeV} \]
\[ m_2 = 0.10 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.70 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

\[ \gamma = 3.78 \text{ GeV} \]
\[ m_X = 1.0 \text{ GeV} \]
\[ m_1 = 0.50 \text{ GeV} \]
\[ m_2 = 0.20 \text{ GeV} \]

DsTau [8] and FASER\nu [9] experiments. The emulsion detector is composed of 1000 layers, with each layer consists of a 1 mm thick tungsten plate and 0.3 mm thick emulsion layer. Each emulsion layer consists of a 200 \( \mu \)m thick base (made of, e.g., cellulose acetate), sandwiched between two emulsion films of 50 \( \mu \)m thickness, as illustrated in Fig. [10]. The emulsion consists mainly of AgBr (about 66\%) and gelatin material (34\%). The silver bromide crystals are sensitive to ionization by charged particles passing through the emulsion (with an energy band gap of 2.5 eV), and have a typical size of 0.2 \( \mu \)m. Therefore, the emulsion can have a position measurement of tracks with a precision below 1 \( \mu \)m, which makes it ideal for our purpose of detecting a pair of charged particles (ee, \( \mu \mu \) or \( \pi \pi \)) with an energy of a few hundred MeV and a small opening angle. With the detector length \( L = 1.3 \) m, we further proposed...
a cross section area of 20 cm in width and 30 cm in height. After particle events are recorded by the emulsion detector for some period, the films will be developed and the AgBr grains positions will be read out by dedicated microscopes. At the reconstruction level, sequences of aligned grains will be recognized and form tracks of charged as well as neutral particles.

We propose to place the detector near a $e^+e^-$ collider such as BESIII and Belle-II. One possible placement around BESIII is illustrated in Fig. 11. The dark matter detector is facing toward the interaction point (IP) of BESIII. The distance between the IP and its front area is 6 m, and the perpendicular distance between the front area center and the BESIII barrel axis is 0.84 m. With the detector positioned as in Fig. 11, the probability distribution of polar vs. azimuthal angle of particle $\phi$ in the $e^+e^-$ lab frame is shown in Fig. 12, where the area covered by the detector is indicated by a white rectangular box. The signal acceptance efficiencies ($\epsilon_s$) of the detector for two different mass parameter settings are given in Tab. 1.

The tracks from $\phi_1$ decay inside the detector are simulated by Geant4 [10] with the Physics List FTFP_BERT, and a few examples can be found in Fig. 13(a-c). The signal is characterized by two tracks sharing a common vertex inside the detector, which is highly distinguishable from background particles (mainly muons) with single tracks going through the whole detector, as illustrated by Fig. 13(d). The electrons give rise to electromagnetic showers, and are attenuated quickly inside the detector. They deposit larger amount of energy in a few emulsion layers within a short path. On the other hand, muons of a few hundred MeV travel much longer than electrons, but steadily deposit similar amount of energy in each layer. The path’s length for pions is between the electrons and muons, but usually ends up with several neutral particles consisting mainly of neutrons. The energy deposition of neutrons in each layer is rather intermittent according to the simulation, and may not form continuous tracks in the reconstruction stage. It wanders in the detector in a long and convoluted way before it is attenuated. The cosmic muons consist of the major background particles to this search. It is characterized by a continuous and smooth track linking one side of the detector to another side, and can be therefore identified and removed at the reconstruction stage.

8. Background events

With electrons, photons and hadrons absorbed by upstream detector and beam pipe related infrastructures, the main background particles entering the dark matter detector are muons. They consist of cosmic muons and muons produced in $e^+e^-$ collisions, which are described in the following subsections. A summary of the contributions from different background sources is given in Tab. 2.

### Table 2: The estimated number of muons per cm$^2$ from BESIII (at $\sqrt{s} = 3.78$ GeV) or cosmic sources onto the dark matter detector for a given integrated luminosity or period.

| Lumi. or period | 10 fb$^{-1}$ | 4.5 fb$^{-1}$ | 6 months |
|----------------|-------------|-------------|-----------|
| muons/cm$^2$  | 20.2        | 74.4        | 0.28 x 10$^6$ |

8.1. Collider muons

Non-resonant $\mu^+\mu^-$ events are constantly produced in the $e^+e^-$ collisions, whose differential cross section is expressed as

$$\frac{d\sigma}{d\cos \theta} = \frac{\pi\alpha^2}{2s} (1 + \cos^2 \theta). \quad (15)$$

On the other hand, $\mu^+\mu^-$ events can be also produced from hadron resonances such as $J/\psi$ and $\phi(2S)$. Because they are vectors, their differential cross sections have the same angular dependence as in Eq. 15. These collider muons have very small efficiency of acceptance in the dark matter detector, and contribute far less background events than the cosmic source, as shown in Tab. 2.

8.2. Cosmic muons

If the proposed emulsion detector is built above ground (BESIII) or near ground (Belle-II), cosmic-ray muons could be a main background. At sea level, assuming a flat Earth, the muon flux $\Phi$ can be approximately defined as

$$\Phi(\theta) = I_0 \cos^{n-1} \theta, \quad (16)$$

where the zenith angle $\theta$ is the angle between the muons momentum and the normal of the Earths surface, and $I_0$ is the vertical ($\theta = 0$) muon flux integrated over energy. With parameters $n \approx 3$ and $I_0 \approx 85.6 \pm 2.4$ m$^{-2}$sr$^{-1}$s$^{-1}$, obtained by fitting the previous cosmic-ray muon measurements, the muon flux roughly...
Figure 11: The schematic views of the emulsion detector position relative to the BESIII detector (only the muon identifier is shown). It is placed 6 m away from the interaction point, and 0.84 m off from the BESIII axis. From top left to bottom right: the side, front, top and perspective views.

Figure 12: The probability distribution of polar vs. azimuthal angle of particle $\phi_1$ in the $e^+e^-$ lab frame, with the chosen parameters indicated in the plot. The cross sectional area of the dark matter detector is indicated by the white box ($0.2 \times 0.3 \text{ m}^2$), assuming that it is placed as shown in Fig. 11. The absolute color scale is arbitrary.

follows a $\cos^2 \theta$ distribution \[12, 13\]. The integrated muon flux can be obtained as

$$\Phi = \int_{\theta=0}^{\pi/2} \int_{\phi=-\pi}^{\pi} I_0 \cos^2 \theta d\theta d\phi = 179.3 \text{ Hz/m}^2.$$ \[17\]

To control the track occupancy under $10^6$/cm$^2$ \[14\] for the post-processing, the emulsion films have to be replaced before this limit is reached. With the estimated rate in Eq. \[16\] after 6 months’ data taking, the cosmic occupancy will reach $0.28 \times 10^6$/cm$^2$. Therefore, replacement of the emulsion films in every 6 months is good enough. To reduce the cosmic muon occupancy, lead shielding can be used, since the BESIII detector is placed above the ground. Lead with a thickness of approximately 100 cm shields about 50% of all incoming muons \[15\]. On the other hand, the Belle-II detector is placed 11 meters underground. Since concrete material with 100 cm of thickness can shield 33% of cosmic muons \[15\], lead shielding is not needed in this case.

9. Signal sensitivity

The expected number of signal events in the dark matter detector can be calculated as:

$$N_{\text{sig}} = L \sigma(e^+e^- \rightarrow \gamma X) BR(X \rightarrow \phi_1 \phi_2) \epsilon_A (e^{-\frac{2d}{\sigma}} - e^{-\frac{d}{\sigma}}).$$ \[18\]

where $L$ is the integrated luminosity of data, $L = 1.3 \text{ m}$, and $d$ is calculated as in Eq. \[10\] \[3\]This relies on the optimistic scenario with top-up injection and beam current upgrade, as proposed in \[10\].

The parameter spaces in $\epsilon$ vs. $g_D$ for two specific choices of matter parameters with at least one signal event expected in the emulsion detector around BESIII are shown in Fig. 14 \[15\] based on 100 fb$^{-1}$ of data. The gray area has been excluded by
BaBar $X \rightarrow ll$ ($l = e, \mu$) \[2\], or $X \rightarrow$ invisible search \[3\]. When $\epsilon$ is large and $g_D$ is small, the BR($X \rightarrow ll$) might be two large and would have been excluded by \[2\]. This is the case when

$$
\epsilon > \frac{\epsilon_1}{\sqrt{\text{BR}(X \rightarrow ll)}},
$$

where $\epsilon_1$ is the limit on $\epsilon$ taken from \[2\]. On the other hand, the search for an invisible $X$ is based on mono-photon events at BaBar. If $\phi_1$ decays inside the BaBar detector with visible energy depositions, this event will not be categorized into mono-photon any more. Considering the BaBar detector size, we roughly require that $\phi_1$ has to fly by at least 2 m away from IP for the event to be a mono-photon candidate. This means that events satisfying the following condition would have been excluded by BaBar $X \rightarrow$ invisible search:

$$
\epsilon > \frac{\epsilon_2}{\epsilon_1 g_D d},
$$

where $\epsilon_2$ is the limit on $\epsilon$ taken from \[3\]. The combination of Eq. \[19\] and \[20\] forms the grey area in Fig. \[14\] and \[15\]. The cyan regions in these plots would otherwise be excluded by \[3\], but are allowed according to Eq. \[20\]. The yellow regions have a $\phi_1$ with $d < 2$ m, which may leave visible traces inside the BESIII detector as well.

Similar sensitive parameter spaces in the Belle-II setting can be found in Fig. \[16\] and \[17\]. Although the dark photon production cross section at higher collision energy is lower, Belle-II can overtake BESIII with its much larger data size (50 ab$^{-1}$). Compared to BESIII, it can probe higher dark photon masses, and sensitive to $\epsilon$ values lower by an order of magnitude.

10. Conclusion

If dark photon is a mediator between the SM and dark sector, it can be searched for directly at the current or future $e^+e^-$ colliders. If there is a splitting in the dark matter mass, excited dark matter decay can be recorded by a dedicated new small detector around the $e^+e^-$ colliders. The acceptance is complementary to the direct dark photon search in the main $e^+e^-$ detector, since the new detector is in a very froward region out of the main detector’s reach. This new detector can search for parameter spaces not reached by previous experiments with the new BESIII and Belle-II data, and can also re-open some parameter spaces that have been excluded by previous data derived with simple assumptions. Such a detector around BESIII can probe the dark photon coupling parameter $\epsilon$ down to $2 \times 10^{-3}$, and if
it is around Belle-II, can further reach down to $2 \times 10^{-5}$. Such a detector will open a new window for searching for new physics related to a dark sector.

Acknowledgments

X. Chen and Z. Hu are supported by Tsinghua University Initiative Scientific Research Program. Y. Wu is supported by the Natural Sciences and Engineering Research Council of Canada (NSERC).

References

[1] A. R. S. Andreas, C. Niebuhr, New limits on hidden photons from past electron beam dumps, Phys. Rev. D86 (2012) 095019. arXiv:1209.6083 doi:10.1103/PhysRevD.86.095019
[2] J. P. Lees, et al., Search for a dark photon in $\mu^+\mu^-$ collisions at BABAR, Phys. Rev. Lett. 113 (2014) 201801. arXiv:1406.2980 doi:10.1103/PhysRevLett.113.201801
[3] J. Lees, et al., Search for invisible decays of a dark photon produced in $\mu^+\mu^-$ collisions at Babar, Phys. Rev. Lett. 119 (2017) 131804. arXiv:1702.03327 doi:10.1103/PhysRevLett.119.131804
[4] E. Izaguirre, et al., Discovering Inelastic Thermal-Relic Dark Matter at Colliders, Phys. Rev. D93 (2016) 063523. arXiv:1508.03050 doi:10.1103/PhysRevD.93.063523
[5] C.-F. Q. J. Jiang, H. Yang, Exploring Bosonic Mediator of Interaction at BESIII, Euro. Phys. J. C79 (2019) 404. arXiv:1810.05790 doi:10.1140/epjc/s10052-019-6912-3
[6] M. Tanabashi, et al., Review of particle physics, Phys. Rev. D98 (2018) 030001. doi:10.1103/PhysRevD.98.030001 URL http://pdg.lbl.gov/2019/hadronic-xsections/hadron.html
[7] M. Komatsu, OPERA experiment, Nucl. Instrum. Meth. A503 (2003) 124. doi:10.1016/S0168-9002(03)01368-8 URL http://cds.cern.ch/record/642973
[8] S. Aoki, et al., DsTau: study of tau neutrino production with 400 GeV electron beam dumps, Phys. Rev. D86 (2012) 095019. arXiv:1209.6083 doi:10.1103/PhysRevD.86.095019
[9] H. Abreu, et al., Detecting and Studying High-Energy Collider Neutrinos with FASER at the LHC, arXiv:1906.02310
[10] S. Agostinelli, et al., Geant4: a simulation toolkit, Nucl. Instrum. Meth. A506 (2003) 250. doi:10.1016/S0168-9002(03)01368-8 URL http://cds.cern.ch/record/66083
[11] M. Ablikim, et al., Design and construction of the besiii detector, Nucl. Instrum. Meth. A614 (2010) 345. arXiv:0911.4960 doi:10.1016/j.nima.2009.12.050
[12] K. Altenmuller, et al., Muon-induced background in the kattrin main spectrometer, Astropart. Phys. 108 (2019) 40. doi:10.1016/j.astropartphys.2019.01.003
[13] P. Shakla, S. Sankrith, Energy and angular distributions of atmospheric muons at the earth, arXiv:1606.06907
[14] T. Fukuda, et al., The analysis of interface emulsion detector for the opero experiment in japan scanning facility, J. of Instrum. 5 (2010) 04009. doi:10.1088/1748-0221/5/04/P04009
[15] E. Aguayo, et al., Cosmic ray interactions in shielding materials, PNNL 20693.
[16] M. Ablikim, et al., White paper on the future physics programme of be-

URL: https://www.pnnl.gov/main/publications/external/technical_reports/PNNL-20693.pdf