Anisotropic flow of charged and identified hadrons in the quark-gluon string model for Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV

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The pseudorapidity behaviour of the azimuthal anisotropy parameters $v_1$ and $v_2$ of inclusive charged $(h^\pm)$ hadrons and their dependence on the centrality has been studied in Au+Au collisions at full RHIC energy of $\sqrt{s_{NN}} = 200$ GeV within the microscopic quark-gluon string model. The QGSM simulation results for the directed flow $v_1$ show antiflow alignment within the pseudorapidity range $|\eta| \leq 2$ in a fair agreement with the experimental $v_1(\eta)$ data, but cannot reproduce the further development of the antiflow up to $|\eta| \approx 3.5$. The $\eta$ dependence of the elliptic flow $v_2$ extracted from the simulations agrees well with the experimental data in the whole pseudorapidity range for different centrality classes. The centrality dependence of the integrated elliptic flow of charged hadrons in the QGSM almost coincides with the PHOBOS experimental distribution. The transverse momentum dependence of the elliptic flow of identified ($\pi^\pm$, $K^\pm$, $p$, $\bar{p}$) and inclusive charged hadrons is studied also. The model reproduces quantitatively the low-$p_T$ part of the distributions rather good, but underestimates the measured elliptic flow for transverse momenta $p_T > 1$ GeV/$c$. Qualitatively, however, the model is able to reproduce the saturation of the $v_2(p_T)$ spectra with rising $p_T$ as well as the crossing of the elliptic flow for mesons and baryons.

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I. INTRODUCTION

Ultra-relativistic heavy ion collisions (URHICs) offer a unique opportunity to study the nuclear phase diagram at high temperatures and densities. The matter under such extreme conditions has probably existed in the early universe within the first few fm/$c$'s after the Big Bang. Therefore, it is very tempting to investigate the properties of the Little Big Bang in the laboratory, and to search for a new state of matter, predicted by the fundamental theory of strong interactions (Quantum Chromodynamics – QCD), namely, a plasma of deconfined quarks and gluons (QGP).

Among the various experimental studies of URHICs operates the Relativistic Heavy Ion Collider (RHIC) at BNL since 2000 to investigate gold on gold collisions up to $\sqrt{s_{NN}} = 200$ GeV. After four years of operation strong experimental evidence has been accumulated that at RHIC energies indeed a new state of matter is created which is qualitatively different from a hadron gas (see references therein). This state seems, however, not to behave like a weakly interacting parton gas - as could have been naively expected - but rather like a strongly coupled plasma. One argument towards such a scenario is the large elliptic flow observed at RHIC. The development of strong elliptic flow requires short equilibration times and large pressure gradients to drive the dynamics, both being characteristic features of a strongly interacting system.

An independent argument for such a scenario is provided by lattice QCD, though still on a qualitative basis. While lattice QCD predicts undoubtedly a QGP phase transition around a critical temperature of $T_c \sim 150 \div 180$ MeV the properties of such a state are not yet so well understood. As a striking fact lattice calculations do not reach the Stefan-Boltzmann limit for the pressure of a free parton gas $p \propto T^4$ even at $T \geq 5T_c$ but saturate around 80% of the Stefan-Boltzmann limit. Taking this fact seriously and not as a lattice artefact, one may argue that the QGP at temperatures well above the critical one is still a strongly interacting system.

In line with these arguments goes the success of hydrodynamics in describing the collective flow at RHIC. Only at incident energies of $\sqrt{s_{NN}} \geq 130$ GeV (which corresponds to an initial energy deposit of about 10 GeV/fm$^3$ in the central reaction zone) the hydrodynamic limit for the elliptic flow seems to be reached. Within the limits of tuning of the equation of state (including incorporation of a QGP phase transition) hydrodynamic calculations are able to describe the bulk properties of the collective flow rather reasonable. However, when turning to more differential observables such as centrality dependence, mass dependence or $p_T$ dependence of the elliptic flow, hydrodynamic calculations have also some problems to match the data.

In this context it is also important to obtain an understanding of the reaction dynamics in terms of microscopic models. One class of microscopic models which have been very successfully applied at CERN SPS energies and below are string models. The main assumption of string models is that hadrons are produced as a result of excitation and decay of open strings with different
quarks or diquarks at their ends. Generally, all models are formulated as Monte Carlo event generators allowing to perform a careful analysis of the measurable quantities by introducing all necessary experimental cuts. There are numerous versions of the two basic string-motivated phenomenological approaches: the FRITIOF model and the Dual Parton Model (DPM). These two approaches use different mechanisms of string excitation. In the first approach, relied on relativistic classical string dynamics, the string masses arise from momentum transfer. The second one is based on the Gribov-Regge theory (GRT) in the framework of relativistic quantum mechanics, the string masses are extracted from the cylinder-type diagrams. The s-channel discontinuities of the diagrams, representing the exchange by n-Pomerons, are related to the process of 2k (k \leq n) string production. If the contributions of all n-Pomerons exchanges to the forward elastic scattering amplitude are known, the Abramovskii-Gribov-Kancheli (AGK) cutting rules enable one to determine the cross sections for 2k-strings. Hard gluon-gluon scattering and semi-hard processes with quark and gluon interactions are also incorporated in the model. The inclusive spectra in the QGSM have automatically the correct triple-Regge limit for the Feynman variable x → 1, double-Regge limit for x → 0, and satisfy all conservation laws. The particular stages of the collision model, namely (i) initialization of interacting projectile and target nuclei, (ii) string formation via inelastic nucleon-nucleon (hadron-hadron) interaction, (iii) string fragmentation, i.e. hadronization, and (iv) hadron-hadron rescattering, are solved basically by Monte Carlo simulation techniques.

III. ANISOTROPIC FLOW OF INCLUSIVE CHARGED AND IDENTIFIED HADRONS

Among the main signals, which can help to reveal the formation of the QGP in the experiment, are collective flow phenomena. Flow is directly linked to the equation of state of the excited matter. Generally, an effective EOS extracted from the model studies shows “softness” during the early stages of the collision, but it remains unclear whether the observed softness is (i) due to the proximity of the QCD phase transition or (ii) due to non-equilibrium phenomena, such as the formation and fragmentation of strings, or (iii) due to nuclear shadowing. The transverse collective flow can be subdivided into isotropic and anisotropic flow. Two types of anisotropic flow, that are characterized by the first and the second harmonic coefficients of the Fourier decomposed invarient azimuthal distribution in momentum space, are called directed and elliptic flow. Here, pt = (p_x^2 + p_y^2)^{1/2} is the transverse momentum, y the rapidity and \phi the azimuthal angle between the particle’s momentum and the reaction plane. While the elliptic flow v_2 = 〈cos(2\phi)〉 = 〈(p_x/pt)^2 − (p_y/pt)^2〉 is assumed to be
particularly sensitive to the pressure at the early stages of the collisions [10, 17, 18], the directed flow appears to develop until the late stage of the heavy-ion reaction [10, 17, 18]. On the other hand, the directed flow of particles with high transverse momentum, which are emitted at the onset of the collective expansion, can carry information about the EOS of the dense matter phase from the initial conditions. A lot of measurements of the $v_2$ parameter in Au+Au collisions at RHIC have been performed for charged and identified hadrons, see [3, 4, 5, 6] and references therein, whereas experimental results for the directed flow $v_1$ at RHIC has been reported only quite recently [37, 38].

### A. Directed flow of inclusive charged hadrons

Figure 1 depicts the QGSM simulation result for the pseudorapidity dependence of the directed flow $v_1(\eta)$ of charged hadrons compared to the experimental data from the PHOBOS Collaboration [36] for 6% to 55% central Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV (QGSM predictions for the directed flow of identified particles at lower RHIC energy, $\sqrt{s_{NN}} = 130$ GeV, can be found in [37]). One can see that the model reproduces the $v_1$ data quite well both qualitatively and quantitatively, although the maxima of the directed flow around $|\eta| \approx 2$ are shifted to lower pseudorapidities compared to the experimental data. The $v_1(\eta)$ result show a characteristic wiggle structure with a clear antiflow\(^1\) component in the middle $|\eta|$ region. It was pointed out in [38] (see also [17, 39]) that the phenomenon leading to the formation of a wiggle structure for the directed flow of nucleons is caused by dense baryon-rich matter shadowing, which plays a decisive role in the competition between normal flow and antiflow in noncentral nuclear collisions at ultra-relativistic energies. Within microscopic string model calculations such deviations from the straight line behaviour of the nucleonic flow were first observed in very peripheral Au+Au collisions at AGS energy [31]. Experimentally the wiggle structure of $v_1$ for protons in peripheral Pb+Pb collisions at SPS has been observed by the NA49 Collaboration [40, 41]. However, the QGSM distributions of $v_1(\eta)$ at these lower energies also have peaks which are shifted by approximately one unit of rapidity toward $\eta = 0$ [17, 18] compared to the experimental data. So, this shift, which also leads to a steeper slope in the mid-rapidity region, seems to be a sort of model ‘artefact’ occurring at higher collision energies as well. The question about the origin of the directed flow’s rapidity dependence obtained within the QGSM is not so easily to answer. One reason might be the lack of heavy resonances in the model compared, e.g., to the Ultra-Relativistic Quantum Molecular Dynamics (UrQMD) model [42] or the multi-phase transport (AMPT) model [38]. These resonances are mostly concentrated in the rapidity regions of flying away spectators which exhibit normal flow. Lighter particles coming from the decays of the heavy resonances, each moving in the normal direction and having nearly the same rapidity, may significantly enhance the normal component of the directed flow in a pseudorapidity range closer to the fragmentation region. The total multiplicity of particles with pseudorapidity $|\eta| > 3.5$ is quite low in the present version of the QGSM. The complex connection between the model dynamics of the QGSM and the characteristics of the resulting directed flow requires further lucid investigations to understand in detail the origin of the change in sign, and in particular the strong antiflow behaviour of $v_1(\eta)$ in the mid-rapidity region.

Nevertheless, our picture is a bit different compared to that provided by the microscopic models based on the FRITIOF routine. For instance, the relativistic quantum molecular dynamics (RQMD) model favours weak but still normal flow for pions [33] even for more peripheral topologies with $b = 5–10$ fm, corresponding to a centrality range $\sigma/\sigma_{geo} = 15–60\%$. Both the UrQMD model and the AMPT model show a very flat and essentially zero directed flow [42, 43] in a broad range $|\eta| \leq 2.5$.

\(^1\) Conventionally, the type of flow with positive slope $dv_1/d\eta$ is called normal flow, in contrast to antiflow for which $dv_1/d\eta$ is negative.
FIG. 2: Directed flow $v_1$ for nucleons and charged pions from $\sqrt{s_{NN}} = 200$ GeV Au+Au reactions as a function of pseudorapidity $\eta$ for six different impact parameters $b$ going from central ($b = 2$ fm) to peripheral ($b = 12$ fm) collisions. Only statistical errors are shown.

peripheral, events. – Although the data seem to indicate antiflow behaviour for the directed flow of charged particles with the possible flatness at $|\eta| \leq 1.5$, the measured signal is quite weak, – the magnitude of the flow is less than 1% at $|\eta| \leq 2$. Therefore, relatively large systematic error bars do not permit to disentangle between the different models.

Similar antiflow alignment can be obtained also within the multi module model (MMM) [44], which is based on fluid dynamics coupled to the formation of colour ropes. In this model the effect of the tilted initial state, responsible for the antiflow formation, reaches its maximum for the impact parameter $b \approx 0.5(R_A + R_B)$, i.e. for the centrality $\sigma/\sigma_{geo} \approx 25\%$ in case of a symmetric system of colliding nuclei. To check the centrality dependence of the directed flow at full RHIC energy, the pseudorapidity distributions $v_1(\eta)$ of nucleons and charged pions in Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are displayed in Fig. 2 for six different impact parameters $b$ going from central (upper left panel) to very peripheral (lower right panel) configurations. Regardless of the centrality range, directed flow of pions has negative slope, i.e. antiflow, in the mid-rapidity range. For nucleons the azimuthal anisotropy parameter $v_1$ is small for central collisions ($b = 2$ fm). In accord with our previous studies and conclusions [16, 17], for Au+Au collisions at full RHIC energy deviations of nucleonic flow from a normal flow behaviour occur already at quite small impact parameters. This means that the effect is indeed shifted to more central configurations. With the increase of $b$ these deviations, representing the “wiggle” structure of the flow, appear more distinctly. However, the QGSM simulations for both nucleonic and pionic flow at mid-rapidity $|\eta| \leq 1$ are consistent with the zero flow signal $v_1 = 0$. The other features which should be mentioned here are the broadening of the antiflow region and the increase of its strength as the reaction becomes more peripheral. Therefore, one can disentangle between two processes of different origin employed for the description of the third-flow component: If the formation of nucleonic antiflow is dominated by the creation of QGP, the flow maximum is reached at $b \leq 6$ fm [44], whereas for the shadowing scenario the strong antiflow should be observed also in very peripheral events with $b \approx 10$ fm.

B. Elliptic flow of charged and identified hadrons

Here we investigate the pseudorapidity dependence of the elliptic flow of charged hadrons. The QGSM simulation result for minimum bias Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV are compared in Fig. 3 with the experimental data of the PHOBOS Collaboration [5]. The elliptic flow displays a strong in-plane alignment in accordance with the predictions of Ref. [34]. At mid-rapidity $|\eta| < 1.0$ the elliptic flow is almost constant. Then it rises up slightly and drops rapidly at $|\eta| > 2.0$ with increasing pseudorapidity. The mean value of $v_2^p(|\eta| \leq 2.5)$ is practically as large as the value measured by the PHOBOS Collaboration at mid-rapidity. The experimental data, which indicate a steady decrease in $v_2$ with increasing $|\eta|$, are slightly overestimated by the model only at $|\eta| \approx 2.0$ as a consequence of a double hump structure in the theoretical result. This difference in shape close to $\eta = 0$ still rankles somewhat, although within the error bars the pseudorapidity dependence of the elliptic flow of charged hadrons obtained within the QGSM shows a really fair agreement with the PHOBOS data [5] in the whole $\eta$ range.

However, following the idea of longitudinal boost in-
variance\(^2\) of the expanding hot and dense matter and the common interpretation of elliptic flow as a consequence of secondary particle collisions, one would expect no or at least a weak pseudorapidity dependence of \(v_2\) similar to that of the multiplicity density \(dN/d\eta\) over a large rapidity range. The experimentally observed multiplicity stays approximately constant within three units of pseudorapidity, while the elliptic flow data show a pronounced peak at mid-rapidity. This is somehow in contradiction with the assumption of longitudinal boost invariance over a broad region of rapidity in RHIC collisions. Most of the hydrodynamics calculations reported in the literature are based on boost invariant models. Therefore the results obtained within such approaches are independent of rapidity and one is limited to discuss only the transverse behaviour. Hydrodynamics results for the pseudorapidity dependence of \(v_2\) are scarce. The shape of \(v_2(\eta)\) in Au+Au collisions at \(\sqrt{s_{NN}} = 130\) GeV studied within a full three-dimensional hydrodynamic model by Hirano et al. shows a bump structure in forward and backward rapidity regions, when the initial energy density profile depends strongly on the space-time rapidity \(\eta\). The bumps appear at the same rapidity regions where the initial configuration is not boost invariant anymore. In the case of an almost \(\eta\) independent deformation of the initial energy density, the resultant \(v_2(\eta)\) has no bumps. Thus it was shown in that the pseudorapidity dependence of \(v_2\) is highly sensitive to the parametrization of the initial energy density profile in the longitudinal direction. The initial energy density profile in microscopic transport models is not explicitly parametrized, but it is implicitly fixed by the initial conditions of the projectile and target nucleus. The elliptic flow studied within the UrQMD model in the cascade mode shows also a prominent dip at central rapidities for all inspected hadrons. There it is argued that this rapidity behaviour of \(v_2\) indicates a region of small interaction strength (or low ‘pressure’) because of the direct connection between the strength of the anisotropic flow and the mean free path of the particles forming the hot mid-rapidity region. Therefore, the appearance of the dip in \(v_2\) is linked to a feature of the model dynamics in the early stage, namely the pre-equilibrium string dynamics and interactions on the parton level. Also in the QGSM, the emergence of the double bump structure in \(v_2(\eta)\) is strongly connected with the model dynamics, as one can clearly see in the time evolution of the elliptic flow and its rapidity dependence. The normalized number of binary particle collisions per pseudorapidity interval throughout the evolution of the system as a function of the pseudorapidity is depicted in Fig. 4 for two impact parameters \(b = 2\) fm and \(b = 8\) fm to show the strong correlation between the particle interactions and the flow strength mainly produced by secondary collisions. First of all, this number versus the pseudorapidity difference \(\Delta\eta = \eta_1 - \eta_2\) of the two colliding particles is plotted in the left panel of Fig. 4. The peak around \(\Delta\eta = 0\) indicates unambiguously that independently of the impact parameter the particles with equal or at least very similar rapidities interact most likely. The plot on the right hand side of Fig. 4 where the number of binary collisions normalized to its maximum is shown as a function of the mean pseudorapidity \(\bar{\eta} = (\eta_1 + \eta_2)/2\) of the two colliding particles, is even more instructive. Here, a clear double peak structure appears in the same rapidity region as seen in Fig. 4 for the \(\eta\) dependence of the elliptic flow. The dip in the number of collisions around \(\bar{\eta} = 0\) becomes more pronounced with decreasing centrality. In addition, the two peaks for \(b = 8\) fm are slightly shifted to higher pseudorapidities compared with the result for \(b = 2\) fm. This is nicely consistent with the centrality dependence of \(v_2(\eta)\) obtained within the QGSM as it will be discussed in the following.

Figure 5 presents the pseudorapidity distribution of the elliptic flow for charged hadrons in gold-gold collisions at full RHIC energy for three different centrality classes, ranging from central (bottom panel) via mid-central (middle panel) to peripheral (top panel) in accordance with the definitions in \(E\). The results from the QGSM simulation and the PHOBOS analysis (combined data from the hit- and track-based methods) are overlaid. The model is able to describe the magnitude and shape of \(v_2^{ch}(\eta)\) quite well across all of the three centrality bins within the given systematic and statistical uncertainties.
FIG. 4: The number of binary particle collisions per pseudorapidity interval normalized to its maximum as function of the pseudorapidity difference $\Delta \eta = \eta_1 - \eta_2$ (left) and the mean pseudorapidity $\bar{\eta} = (\eta_1 + \eta_2)/2$ (right) of the two colliding particles. The QGSM results are obtained for simulated Au+Au reactions at $\sqrt{s_{NN}} = 200$ GeV with two different impact parameters $b$.

FIG. 5: Elliptic flow $v_2$ for inclusive charged hadrons from $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions as a function of pseudorapidity $\eta$ for the three centrality classes according to the PHOBOS analysis (combined hit- and track-based results) [5]. Again, the systematic errors of the experimental data are shown as gray boxes together with the statistical error bars, respectively.

The overall shape of the $\eta$ distribution changes only very little with centrality and shows a behaviour very similar to that depicted in Fig. 4 for the minimum bias events. The two-peak structure of $v_2^{ch}(\eta)$ in the interval $|\eta| \leq 2.5$, which is clearly seen for simulated peripheral collisions but ‘washed out’ in the central bin, is not attributed solely to QGSM, but arises also in e.g. UrQMD calculations [42] as discussed in detail above. However, the effect is small, and the mean value of the $v_2^{ch}$ parameter over the aforementioned range increases from central to peripheral collisions in good quantitative agreement with the experimental data. This is a not so trivial result, because neither the pseudorapidity nor the centrality dependence of the elliptic flow, which is discussed below, is reproduced correctly at RHIC so far by the UrQMD calculations and hydrodynamic models.

The next observable is the transverse momentum dependence of $v_2$ for identified hadrons, namely, combined $\pi^+ + \pi^-$, $K^+ + K^-$, as well as $p + \bar{p}$ spectra in minimum bias Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The QGSM simulation results are depicted in Figs. 6 and 7 in comparison to the PHENIX [6] and STAR data [52], respectively. The agreement with the experimental data at least for protons/antiprotons and kaons is rather good in the range of low transverse momenta $0.0 \leq p_T \leq 1.0$ GeV/c, as illustrated in Fig. 6. The steady increase of the elliptic flow with rising $p_T$ and the larger $v_2$ parameter at a given $p_T$ for the lighter mass hadrons compared to the heavier ones is well reproduced by the model calculation. For pions the latter statement is only valid for the transverse momenta below 0.75 GeV/c. The elliptic flow of charged pions starts already to saturate at $p_T > 0.5$ GeV/c. This early saturation of the pionic $v_2$ continues with a slight decreasing at increasing $p_T$ and entails the peculiar result to be smaller than the kaon flow for transverse momenta between 1.0 GeV/c and 2.5 GeV/c, as shown for the overall $p_T$ range in Fig. 7.

This deviation compared to the experimental findings is remarkable because it was shown in [21] that the magnitude of the pionic flow in the QGSM calculations is
FIG. 6: Transverse momentum dependence of $v_2$ for identified hadrons $\pi^+ + \pi^-, K^+ + K^-, p + \bar{p}$ in the low-$p_T$ range $0.0 \leq p_T \leq 1.0$ GeV/c for minimum bias Au+Au collisions at $\sqrt{s_{NN}} = 200$ GeV. The experimental data are PHENIX [6] and STAR [52] results. Only statistical errors are shown.

FIG. 7: The same like in Figure 6 but in the overall $p_T$ range $0.0 \leq p_T \leq 3.5$ GeV/c. The PHENIX data are taken from [6]. Again, only statistical errors are shown.

already twice as large as obtained, e.g. in the RQMD ones. It has been elaborated in [20] that the contributions of hard processes and multi-Pomeron exchanges are very important to reach the reported magnitude of $v_2$ and to reproduce the particle multiplicities correctly. To confirm the latter statement, the transverse mass spectra of positively and negatively charged pions and kaons for $pp$ collisions at $\sqrt{s_{NN}} = 200$ GeV are presented in Fig. 8. The $m_T$ spectra obtained within the full version of the QGSM, which incorporates hard and multichain contributions, are in good agreement with the experimental spectra reported by the STAR Collaboration [53].

Coming back to the $p_T$ behaviour of the elliptic flow depicted in Fig. 4 it is important to stress at this point another striking feature of the QGSM. Namely, this model is able to reproduce at least qualitatively the experimental evidence of the crossing of the elliptic flow for mesons and baryons at $p_T \approx 1.7$ GeV/c [6]. The simulation data show that at $p_T < 1.4$ GeV/c the $v_2$ parameter for kaons is larger than for (anti)protons. At higher transverse momenta $p_T > 1.4$ GeV/c the situation is completely changed. Here, the elliptic flow of protons and antiprotons becomes larger than $v_2$ for charged kaons and pions.

This behaviour is also described within the quark recombination models [54, 55, 56, 57], which assume the statistical coalescence of two or three quarks into a hadron. In contrast, the hydrodynamic picture shows the same mass-ordering for the elliptic flow of different particles at all transverse momenta [11]. We will come to this point after the study of $v_2(p_T)$ for inclusive charged hadrons. The QGSM distributions are shown in Fig. 4 in comparison to the experimental data of the PHOBOS and PHENIX Collaborations [5, 6]. To demonstrate the importance of rescattering processes for the elliptic flow formation the $v_2^{ch}$ extracted from a QGSM simulation run for minimum bias Au+Au collisions without the hadronic cascade is additionally plotted in this figure. It is quite obvious that the model without subsequent secondary interactions of produced hadrons creates zero elliptic flow, because the azimuthal distributions of secondaries in elementary hadron-hadron collisions are isotropic. The anisotropic flow, i.e. the azimuthal anisotropies of the number of produced particles, results from the spatial asymmetry of the collision zone and subsequent rescattering processes, which are crucial to convert the initial spatial anisotropy into the final momentum anisotropy. Hadronic rescattering including hard and multichain con-
tributions creates an elliptic flow, which rises almost linearly according to the experimental data within the interval $0.0 \leq p_T \leq 1.0$ GeV/c, but saturates already for transverse momenta above 1.0 GeV/c at a level of $v_2^{ch} \approx 6\%$ whereas the experimentally measured flow increases further up to $v_2^{ch} \approx 14 - 16\%$ &e it declines for $p_T \geq 3.0$ GeV/c. Although the model seems to describe

the $p_T$ dependence qualitatively well, it underestimates the experimental elliptic flow of charged particles with transverse momenta above 1.0 GeV/c roughly by up to 50\%. What is the reason of these deviations? Recall that hadrons in the QGSM gain the transverse momentum due to (i) transverse motion of the constituent quarks and (ii) transverse momentum of constituents acquired in the course of string fragmentation. The parameters of these two processes are fixed by comparison with the available hadronic data. Other sources of the transverse motion are (iii) the transverse Fermi motion of nucleons in the colliding nuclei and (iv) rescattering of produced particles in the hot and dense nuclear medium. It was already mentioned that the latter process is the most crucial for the development of the elliptic flow in nuclear reactions. However, secondary hadrons with high transverse momenta experience in average less collisions than their low-momentum counterparts because of the large formation time (which originates from the uncertainty principle). As was shown in [42], in the limit of vanishing formation time the elliptic flow increases drastically. But all parameters linked to the formation time of produced particles in the QGSM are also fixed by comparison with experimental data on hadronic interactions. Therefore, the presented elliptic flow can be considered as an upper limit obtained within the hadronic cascade scenario. It clearly indicates on new physical effects not taken into account by the microscopic model. Jets are among the most likely candidates for these processes. Indeed, as was discussed in [58], the non-uniform dependence of the energy loss on the azimuthal angle results in azimuthal anisotropy of jet spectra in non-central nuclear collisions. This leads to a significant increase of the elliptic flow of high-$p_T$ particles. Another possible explanation is, e.g. a dramatic increase of all $s$-channel transition rates in the vicinity of the chiral phase transition [59], which causes a critical opacity and fast thermalization in the system.

Finally, the centrality dependence of the total, i.e. $p_T$ and $p_T$ integrated $v_2$ for charged hadrons is depicted in the middle and right panel of Figure 11. Since the centrality of collision events in the experimental measurements [6] is characterised by the mean number of participants $N_{part}$ (seen in the middle panel), we show in addition to this signal the original impact parameter dependence of the elliptic flow (right panel). The relation between the number of participants $N_{part}$ in the Au+Au reactions and the impact parameter $b$ of the simulated collisions is shown in the left panel of Figure 10. The number of nucleons participating in inelastic scatterings, i.e. $N_{part}$, is directly available in the QGSM. This number depends mainly on the nucleon-nucleon cross section, which for the given initial energy of $\sqrt{s_{NN}} = 200$ GeV is calculated in the framework of GRT. Apart from the cross section and its energy dependence, the number of participants varies with the impact parameter $b$ and with the central energy loss on the azimuthal angle results in azimuthal anisotropy in non-central collisions. Therefore, the pseudorapidity dependence of the total, i.e. $p_T$ and $p_T$ integrated $v_2$ for charged hadrons in the QGSM simulations is more than twice as large as created by the RQMD ones [60].

**IV. SUMMARY AND CONCLUSIONS**

In summary, the pseudorapidity distributions of the azimuthal anisotropy parameters $v_2(\eta)$ and $v_2(\eta)$ of inclusive charged hadrons and their centrality dependence has been studied in Au+Au collisions at full RHIC energy of $\sqrt{s_{NN}} = 200$ GeV within the microscopic quark-gluon string model. The QGSM simulation results for the
FIG. 10: Centrality dependence of the elliptic flow $v_2$ for charged hadrons compared with the PHOBOS data [5] (middle panel), and $v_2$ of inclusive as well as identified charged hadrons as a function of the impact parameter $b$ (right panel). The $b$ dependence of the number of participants $N_{\text{part}}$ is shown in the left panel (the STAR data are taken from [52]). The errors are statistical.

directed flow show antiflow alignment within the pseudorapidity range $|\eta| \leq 2$ in a fair agreement with the experimental $v_1(\eta)$ data, but cannot reproduce the further development of the antiflow up to $|\eta| \approx 3.5$. In a broad pseudorapidity region the model generates a wiggle structure for the directed flow of nucleons $v_1^N$. At mid-rapidity $|\eta| \leq 1$, however, the generated flow is quite weak and consistent with a zero-flow behaviour reported by the STAR and PHOBOS collaborations. The $\eta$ dependence of the elliptic flow $v_2$ extracted from our simulation agrees well with the experimental results in the whole $\eta$ range for minimum bias as well as for central, mid-central and peripheral collisions. The transverse momentum dependence of the elliptic flow $v_2(p_T)$ of identified ($\pi^\pm, K^\pm, p, \bar{p}$) and inclusive charged hadrons has been investigated within the QGSM also. The description by the quark-gluon string model is fairly good in the low $p_T$ range. Here, it was shown that for identified and charged hadrons the $v_2$ parameter rises with increasing $p_T$ according to the experimental data. For higher transverse momenta $p_T > 1$ GeV/c it starts rapidly to saturate already on a level, which is at the largest transverse momenta roughly 50% smaller than the experimentally measured $v_2$. On the other hand, the qualitative behaviour of the elliptic flow in Au+Au collisions in the overall $p_T$ range is well reproduced by the model. In particular, a striking feature of the QGSM is that it is able to describe qualitatively the different $p_T$ dependence of the mesonic and baryonic elliptic flow and reproduces a crossing of the elliptic flow for mesons and baryons at $p_T > 1.4$ GeV observed in the PHENIX experiment at RHIC. The centrality dependence of the integrated elliptic flow of charged hadrons in the QGSM agrees almost perfectly with the PHOBOS experimental data. This fact reflects the dominance of low $p_T$ hadrons.

In conclusion, the microscopic quark-gluon string cascade model based on the colour exchange mechanism for string formation is able to describe qualitatively and quantitatively well the bulk properties of the directed and elliptic flow measured in $\sqrt{s_{NN}} = 200$ GeV Au+Au collisions at RHIC. The limitations of the model should not permit a perfect agreement for all observables. The only signal which is really underestimated by the QGSM is the elliptic flow of hadrons with high transverse momenta $p_T > 1$ GeV/c, although the $v_2$ of hadrons obtained within this model is already stronger than that of string models based on the FRITIOF scenario of excitation of longitudinal strings. The most plausible explanation of this discrepancy is an anisotropic character of jet absorption in a hot and dense asymmetric partonic medium. This effect, colloquially known as jet quenching, is intensively studied now [1]. Other processes related to the quark-hadron phase transition can be considered as well. The collective flow of hadrons appears to have a complex multi-component structure caused by rescattering and absorption processes in a spatially anisotropic medium. Therefore, further detailed investigations of the freeze-out scenario for hadrons are very important to understand properly the flow formation and its evolution.

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