Non Zero Spin Boundary Conditional Vector of porous sphere flow for fixed micropolar fluids

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Abstract. This paper presents the non zero spin boundary conditional vector of porous sphere flow for fixed micropolar fluids using brinkman and stokes equation. The fluid flows in and out words of the porous sphere for formulating the stream functions. The evaluation and knowledge of the flow of disparity material parameters are considered for validating and getting the results by using previous case. This present research paper proposes to examine the analytical study of porous sphere flow fixed in micropolar fluid with non zero boundary conditional vector. The brinkman and stokes equation are utilized for flowing in and out wards of the porous sphere and also in the formulations of stream function. The flow is knowledgeable by porous sphere is to evaluate and its disparity with material parameter is considered. Results are validating by means of previous known cases

Keywords: Drag force, Bessel functions, Micropolar fluid, Gegenbauer functions, Brinkman equation, Stokes equation, Ferro fluids

1. INTRODUCTION

Micropolar fluids are only, a fluid with microstructure. Micropolar fluids can likewise personate fluids comprising of inflexible, irregular arranged particles are suspended in a gooey medium. Eringen’s¹¹, Eringen’s²² and Straughan²⁴ model of micro fluids, understanding with a class of fluids which shows un dicey minute impacts which rises from the nearby development and miniaturized scale movements of the fluid particles. Miniaturized scale polar fluids go for a bolster link pressure and body link. In this manner, unmistakable particle of composite fluids involve polymeric suspension, blood of creature, fluid precious stones, ferro fluids, blood streams and bubbly fluids. Stokes force derived from classical hydrodynamics Landau and Lifshitz¹¹) as similar effects of other authors found such a formula in the context of parallel micro- rotation parameters. Hayakawa¹²) allows us to measure the effect of small rotations on rigid bodies movements in micropolar fluids of various boundary conditions set by variables that define small rotations.

In the literature, problems with solid polar fluid flow are widely considered. Rao & Rao³) studied the slow flow in a small polar fluid passing a solid surface and found that gravity in the field is greater than in the Newtonian fluid case. A similar practice has also been reported by Ramsissoon and Majumadar⁵), Aero et al.²) and Stokes⁴). The flow of a small polar fluid past the Newtonian fluid sphere was studied by Ramkissoon⁷) and found that the fluid area felt less gravity than the solid sphere. Gupta & Deo¹⁹) and O’Neill and Ranger⁶) also observed similar behaviors in the fluid field. Stokes’ constant flow of micropolar fluid exceeded the hypothesis studied by Iyengar &
Srinivasacharya and Rajyalakshmi studied a small polar fluid past the framed area and reported that the porous sphere is also affected by gravity than in the Newtonian case but less than the solid sphere case. Ramskissoon and Majumdar studied a small polar fluid past the slightly deformed fluid field. Hayakawa discussed the slow, constant flow of small polar fluid flows around the area and a cylinder. Ramskissoon and Majumdar examined axially axisymmetric drainage in the Stokes movement of micropolar fluid and found a good formula for gravity found in the axially symmetric body. Palaniappan and Ramkissoon also discovered drag formula previously discovered by Ramamssison and Majumdar using a more sophisticated method of calculation. Hoffmann et al. were the first to address the area’s constant flow of small polar fluids using a non-zero boundary state of the micro-rotation vector by producing a small rotation fraction according to the spin \( \tau \) parameter. They evaluated the drag variability due to the spin parameter and found that the drag decreases as \( \tau \) increases and tends to the Newtonian case as the spin parameter approaches 1. Using a non-zero boundary condition for micro rotation, the slow flow of micropolar fluid over the wing area and the fluid field studied by Gupta & Deo and Deo & Shukla respectively.

The issue of Stokes course through a multitude of porous around spheroidal particles by applying limit condition explicitly kuwabara, on the telephone surface have been discussed by Deo and Gupta. Gupta and Deo handled sensibly the issue of Stokes stream of scaled down scale polar liquid past a penetrable hover with non zero cutoff situation for littler scope turns by tolerating the stream is uni-structure. Defer hover in littler scope polar liquids with non zero breaking point conditions for scaled down scale turns has been grasped by Hoffmann et al. He found that restriction power experienced on a solid hover moving with uniform speed by virtue of scaled down scale polar liquids. Qin and Kaloni investigated the eventual outcome of Brinkman's condition Cartesian tensor in the porous media and by applying this course of action they got that a penetrable circle applied the intensity of hydrodynamic. Salami et al. used the hydrodynamic aspect of the liquid metal divertor is simulated using SPH. Miyawaki et al. introduced the Smoothed Particle Hydrodynamics formalism and then it was shown how it could be used to discretize the Navier Stokes equations to simulate hydro dynamical problems. Phanny and Todo used the continuous porous structures of bioactive ceramics such as hydroxyapatite (HA) as scaffolds in bone tissue engineering. It was found the compressive mechanical properties were greatly improved with increasing sintering time. Li and Ito propose a new decontamination system based on forced convection, called the wind decontamination system (WDCS). The fundamental performance of a WDCS prototype is evaluated by using computational fluid dynamics (CFD).

This paper concerns the issue of a logical examination of wet covering stream of an incompressible miniaturized scale polar liquid past a porous hover with non-homogeneous breaking point condition for littler scope turn vector. The Brinkman condition and Stokes condition are used for the stream inside and outside the porous hover independently in their stream work plans. The drag experienced by porous circle is evaluated and its assortment with respect to material boundary is inspected. Results are endorsed with past known cases.

2. MATHEMATICAL FUNCTIONS

Consider with an issue of an incompressible miniaturized scale polar liquid past a permeable circle having a sweep of ‘a’ with unbounded medium having starting point at focus of circle and having uniform speed ‘U’ moves along the positive z axis. The indication of exterior and interior region of porous sphere is given by “i = 1 and 2” correspondingly. Now the equation of motion for porous sphere external region is given by

\[
div \mathbf{v}^{(1)} = 0 \quad \text{(1)}
\]

\[
-\nabla \bar{p}^{(1)} + k \nabla \times \bar{w}^{(1)} - (\mu_t + k) \nabla \times \nabla \times \bar{v}^{(1)} = 0 \quad \text{(2)}
\]

\[
-2k \bar{w}^{(1)} + k \nabla \times \bar{g}^{(1)} - \gamma \nabla \times \nabla \times \bar{w}^{(1)} + (\alpha + \beta + \gamma) \nabla (\nabla \bar{w}^{(1)}) = 0 \quad \text{(3)}
\]

The equation of motions for inside region of porous sphere is governed by “Brinkman equation” as follows
\[ \mu_1 \nabla^2 v^{(2)} - \frac{\mu_2}{k} v^{(2)} = \nabla p^{(2)} \]  
\[ \text{div} \ v^{(2)} = 0 \]  
\[ (4) \]

Here \( \vec{v}^{(1)} \) indicates velocity vector, \( \vec{u}^{(1)} \) indicates micro rotation vector, \( \vec{f}^{(1)} \) indicates fluid pressure, \( k \) indicates permeability of porous where \( \mu_1, k \) are constants indicates “viscosity coefficients and \( \alpha, \beta, \gamma \) are gyro viscosity coefficients” as given

\[ 3\alpha + \beta + \gamma \geq 0, 2\mu_1 + k \geq 0, \gamma \geq \beta, k \geq 0, \gamma \geq 0 \]  
\[ (6) \]

3. **BOUNDARY CONDITIONS**

By applying the boundary conditions applicable for interface of kinematic viscosity of mutual impenetrability of surface \( r = a \) is taken as

\[ \psi^{(1)}(r, \zeta) = \psi^{(2)}(r, \zeta) \]  
\[ (13) \]

Tangential velocity continuity equation of the sphere is taken as

\[ \frac{\partial \psi^{(1)}(r, \zeta)}{\partial r} \bigg|_{r=a} = \frac{\partial \psi^{(2)}(r, \zeta)}{\partial r} \bigg|_{r=a} \]  
\[ (14) \]

Tangential stress continuity equation of the sphere external region is taken as

\[ \tau^{(1)}_{r\theta} = \tau^{(2)}_{r\theta} \bigg|_{r=a} \]  
\[ (15) \]

Here we use the non-zero spin for the micro rotation on the boundary \( \omega^{(1)} = \frac{\tau}{2} \text{curl} \vec{v}^{(1)} \)

By simplification we get,

\[ \psi^{(1)} = -\frac{\tau}{2r \sin \theta} E^2 \psi^{(1)} \bigg|_{r=a} \]  
\[ (16) \]

Pressure continuity equation of the sphere boundary is taken as

\[ p^{(1)}(r, \zeta) = p^{(2)}(r, \zeta) \bigg|_{r=a} \]  
\[ (17) \]

4. **DETERMINATION OF DRAG FORCE**

As the drag force qualifyed by porous sphere is taken by utilizing the given formula as
\[ F = \pi \mu_1 U \int_0^\pi \tilde{\omega}^3 \frac{\partial}{\partial r} \left( \frac{E^2 \phi^{(1)}}{\tilde{\omega}^3} \right) r d\theta \]  
\hspace{1cm} \text{......... (18)}

Here \( \tilde{\omega} = r \sin \theta \)

After integrating (18), we find that
\[ F = 2 \pi a (2 \mu_1 + k) U D_1 \]  
\hspace{1cm} \text{......... (19)}

Also the non dimensional drag is given by
\[ D_N = \frac{F}{F} \]  
\hspace{1cm} \text{......... (20)}

\textbf{Article I. Special Case}

\textbf{Condition 1: Drag for without spin on boundary (}r = 0\text{), the values of drag force for zero spin turn out as}
\[ F = \frac{2 \pi (2 \mu_1 + k)}{\Delta^\prime} \frac{12(1+m)(-1+N)a^2(-2+2+a^2)\lambda}{(2\mu_1+k)(\cosh[a] - \sinh[a])} \]  
\hspace{1cm} \text{......... (21)}

Where
\[ \Delta^\prime = (-(-2 + N)(-3 + 2(-1 + N)a^2) + m(-6 - 4a^2 + N(3 + 4a^2))(-2 + (2 + a^2)\lambda)\cosh[a] + (-(2 + N)(6 + a^2(6 + 4N(-1 + \lambda) - 9\lambda) + 2(-1 + N)a^4(-1 + \lambda) + m(2N^2a^4 - 2(6(-1 + \lambda) + 2a^4(-1 + \lambda) + a^2(-6 + 9\lambda)) + N(6(-1 + \lambda) + a^4(-6 + 4\lambda) + a^2(-10 + 13\lambda)) \sinh[a] \]  
\hspace{1cm} \text{......... (22)}

Here \( \lambda = \frac{\mu_2}{\mu_1} \)

\textbf{Condition 2: Drag on a porous sphere embedded in fluid sphere}

If \( k \to 0 \) i.e. \( m \to 0, N \to 0, \lambda \to 1 \) and \( a^2 \to \eta^2 \), then the micropolar fluid changes into Newtonian fluid. The drag force reduces to
\[ F = \frac{-12\pi \mu_1 a \eta^4 (\sinh \eta - \cosh \eta)}{\eta(3+2\eta^2) \cosh \eta - 3 \sinh \eta} \]  
\hspace{1cm} \text{......... (23)}

This result agrees from earlier derived Qin and Kaloni \(5\) result.

\textbf{Condition 3: A solid sphere in an unbounded medium}

If \( \alpha \to \infty \) then the porous sphere becomes solid sphere. In this case the drag force comes out as
\[ F = -6\pi \mu_1 a U \]  
\hspace{1cm} \text{......... (24)}

\textbf{5. RESULTS AND DISCUSSION}

\textbf{Figure 1. Variation of }D_N\text{ versus }\eta^2\text{ for different}
Figure 2. Variation of DN versus $\eta^2$ for different $\eta^2$ parameters

Figure 1 speaks to the variety for non-dimensional drag DN with admiration to permeability parameter corrosive parameter $\eta^2$ for different values about coupling number $n$ and $m = 5$. It is recognized that drag expands for build from permeability parameter $\eta^2$ in as much as drag decreases same time expanding for coupling number $n$. Also it is found that drag will be less on account of micropolar liquid for turns conditions over that for Newton liquid the event. Variation of non-dimensional drag DN against permeability parameter $\eta^2$ will be demonstrated in figure 2. It is reasonable that there is build in the drag as the micropolar parameter $m$ will be expanding.

Figure 3. Variation of DN versus $\tau$ for different values

Figure 3 demonstrates that variety for non-dimensional drag DN with admiration to parameter $\tau$. It will be apparent that drag is diminishing as turn parameter $\tau$ may be expanding. Also increments for expanding qualities about permeability parameter. Also effect about turn parameter $\tau$ will be exceedingly vast around drag. Impact for permeability parameter $\eta^2$ with respect to non-dimensional drag DN to different values for $m$ with non-zero and zero turn limit condition will be indicated clinched alongside figure 4. It will be fascinating will note that qualities about non-dimensional drag DN may be less on account about non-zero micro revolution over that from claiming zero micro revolution situation.
6. CONCLUSION

The given useful arrangement is acquired for the diagnostic investigation of permeable circle stream fixed in miniaturized scale polar liquid with non zero turn limit restrictive vector. The drag power acquired by the permeable circle is inserted in smaller scale polar liquid is given in the above conditions. The proposed model diminishes the past notable outcomes from the writing review by restricting the case conditions. Here it is seen that the impact of small scale polar boundary \( m \), porousness boundary \( \eta_2 \), coupling number \( N \) and turn boundary \( \tau \) control the drag power of zero smaller scale rotational vector is more prominent contrasted with non zero miniaturized scale rotational vector.

Nomenclature

- \( \vec{v}^{(1)} \): Velocity vector
- \( \vec{\omega}^{(1)} \): Micro rotation vector
- \( \vec{p}^{(1)} \): Fluid pressure
- \( k \): Permeability of porous
- \( G_2(\xi) \): Gegenbauer function
- \( F \): Drag force

Greek symbols

- \( \alpha, \beta, \gamma \): Gyro viscosity coefficients

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