Using Game Theory in Investing

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Abstract — In this paper, we analyzed the developed methodology for the optimal distribution of public and private investment research in order to obtain the maximum economic effect in a particular block of the industrial cluster. By industrial cluster blocks, we define: block 1 - “R & D”; block 2 - “Procurement and Financial Support”; block 3 - “Production and Technological Activities”; block 4 - “Staffing Support”; block 5 - “Realization of Production equipment”. In this article, we offered methodology for the distribution of investment in blocks of an industrial cluster using game theory. In order to determine the investment strategy, we built a payment matrix. In order to confirm the hypothesis to determine the best solutions, we used the classical and derived conformity criteria: Bayesia, Laplace, Sauvage, Gurviz, Hodge-Lehmann. As a result, we obtain the most optimal investment strategy, which shows the effective distribution of public and private investments in the industrial cluster blocks.

Keywords — industrial cluster, optimal investment strategy, payment matrix

I. INTRODUCTION

Famous scientists proved the advantage of the merger of industrial enterprises in the region. Therefore, it was decided to form a model that would reflect the effect of combining the tangible and intangible resources of industrial enterprises and research centers in order to implement import substitution, in particular, food engineering. Scientists have determined that the joint activity of machine-building enterprises is an important component in determining the development of import-substituting and export-oriented production [1, 2]. The principle of combining a group of enterprises into a cluster will make it possible to form complete production and technological chains in the territory of the Russian Federation [3]. Business combinations will support domestic suppliers of raw materials. We propose to form blocks of the industrial cluster: block 1 - “R & D”; block 2 - “Procurement and Financial Support”; block 3 - “Production and Technological Activities”; block 4 - “Staffing Support”; block 5 - “Realization of Production equipment”. We compared investment strategies and blocks of the industrial cluster. The optimal strategy will be developed on the basis of the obtained economic effects in the cluster blocks [4].

II. RESEARCH METHODS

Today, scientists have developed a huge number of methods and models of resource allocation of enterprises. Such methods are dynamic modeling, time series analysis, correlation and regression analysis, brainstorming, game theory. Each of them has its advantages and disadvantages. We decided to expand the use of game theory methods. We considered this area of research insufficiently studied. In the course of the study, we relied on the works of such scientists as J.V. Neumann, M.E. Porter, L.V. Kantorovich [5–7]. We tried to use several matching criteria in the face of un-certainty. The relevance of the research is related to a weakly developed cluster structure in Russia.

III. RESEARCH BASE

In the process of research, we used game theory. For the effective distribution of public and private investment in order to form the mechanism of import substitution, the authors propose a method of game theory [8]. The study of games is connected with building a payment matrix. In practice, this is the most time-consuming step in the process of preparing to make a certain decision [9].

| Strategies investing (i) | Y1 | Y2 | Y3 | Y4 | Y5 |
|--------------------------|----|----|----|----|----|
| X1                       | 332| 325| 123| 98 | 101|
| X2                       | 211| 197| 198| 201| 199|
| X3                       | 193| 182| 175| 163| 156|
| X4                       | 152| 163| 169| 176| 183|
| X5                       | 120| 111| 124| 135| 149|
| X6                       | 115| 105| 99 | 93 | 89 |
| X7                       | 145| 175| 204| 215| 230|
| X8                       | 119| 125| 146| 168| 236|
| X9                       | 101| 95 | 118| 189| 332|

TABLE I. INVESTMENT DISTRIBUTION MATRIX.

Displayed equations are centered and set on a separate line.

When constructing Table I, we will assume that the industrial cluster should strive to maximize the effect in the target area Y6, and minimize the share of borrowed funds from the state and potential investors X9 [10]. We have designated: Y1 – economic effect in block 1 “R & D”; Y2 – economic effect in block 2 - “Procurement and Financial Support”; Y3 – economic effect in block 3 - “Production and Technological
Activities”; $Y_4$ – economic effect in block 4 - “Staffing Support”; $Y_5$ – economic effect in block 5 - “Realization of Production equipment”.

To solve the payment matrix, the authors proposed to calculate the optimal investment strategy by the criteria of the maximum expected gain [11]. To obtain more reliable information on the adoption of a rational managerial decision, it is necessary to conduct an inspection according to classical criteria (Bayes, Laplace, Savage criterion) [12] and derived criteria (Hodge-Lehmann) [13]. According to the calculated values, you should choose the strategy that will match the values of the criteria. In the case of the repetition of 2 strategies according to different criteria, it is necessary to form the payment matrix again, taking into account the opinions of experts [14].

**Bayes criterion.** By Bayesian criterion, that strategy (pure) is taken as optimal. $X_i$ at which the average gain is maximized $a$ minimizes the average risk $r$ [15]. Let us calculate values $\sum(a_{ij} \cdot p_j)$

$$\sum(a_{1,j} \cdot p_j) = 143 \cdot 0.2 + 114 \cdot 0.2 + 94 \cdot 0.2 + 84 \cdot 0.2 + 65 \cdot 0.2 = 100$$

$$\sum(a_{2,j} \cdot p_j) = 128 \cdot 0.2 + 110 \cdot 0.2 + 98 \cdot 0.2 + 92 \cdot 0.2 + 72 \cdot 0.2 = 100$$

$$\sum(a_{3,j} \cdot p_j) = 119 \cdot 0.2 + 100 \cdot 0.2 + 102 \cdot 0.2 + 98 \cdot 0.2 + 81 \cdot 0.2 = 100$$

$$\sum(a_{4,j} \cdot p_j) = 109 \cdot 0.2 + 98 \cdot 0.2 + 100 \cdot 0.2 + 104 \cdot 0.2 + 89 \cdot 0.2 = 100$$

$$\sum(a_{5,j} \cdot p_j) = 97 \cdot 0.2 + 91 \cdot 0.2 + 107 \cdot 0.2 + 110 \cdot 0.2 + 95 \cdot 0.2 = 100$$

$$\sum(a_{6,j} \cdot p_j) = 86 \cdot 0.2 + 89 \cdot 0.2 + 109 \cdot 0.2 + 114 \cdot 0.2 + 102 \cdot 0.2 = 100$$

$$\sum(a_{7,j} \cdot p_j) = 78 \cdot 0.2 + 85 \cdot 0.2 + 114 \cdot 0.2 + 117 \cdot 0.2 + 106 \cdot 0.2 = 100$$

$$\sum(a_{8,j} \cdot p_j) = 70 \cdot 0.2 + 84 \cdot 0.2 + 118 \cdot 0.2 + 120 \cdot 0.2 + 108 \cdot 0.2 = 100$$

$$\sum(a_{9,j} \cdot p_j) = 61 \cdot 0.2 + 79 \cdot 0.2 + 105 \cdot 0.2 + 113 \cdot 0.2 + 142 \cdot 0.2 = 100$$

On the basis of the data, we construct Table II.

We choose from $(100; 100; 100; 100; 100; 100; 100; 100; 100)$ maximum element, max = 100.

Conclusion: we choose a strategy, $i = 2$.

**TABLE II.** **MATRIX OF INVESTMENT STRATEGY SELECTION BY BAYES CRITERION**

| Strategies investing (i) | Effect in cluster units (j) | $\sum(a_{ij} \cdot p_j)$ |
|------------------------|-----------------------------|--------------------------|
| $X_1$                  | 28.6                        | 22.8                      |
| $X_2$                  | 25.6                        | 22                        |
| $X_3$                  | 23.8                        | 20                        |
| $X_4$                  | 21.8                        | 19.6                      |
| $X_5$                  | 19.4                        | 18.3                      |
| $X_6$                  | 17.2                        | 17.8                      |
| $X_7$                  | 15.6                        | 17.8                      |
| $X_8$                  | 14                          | 16.8                      |
| $X_9$                  | 12.2                        | 15.8                      |
| Probability $p_j$      | 0.2                         | 0.2                       |

**Laplace criterion.** If the probabilities of states of nature are plausible, they are estimated using the principle of the insufficient basis of Laplace, according to which all states of nature are assumed to be equally probable (1), (2):

$$q_1 = q_2 = \cdots = q_n = 1/n$$

$$q_i = 1/5. \quad (2)$$

We construct table III, in which we reflect the sum of the obtained values.

We choose from $(100; 100; 100; 100; 100; 100; 100; 100; 100)$ maximum element, max = 100.

Conclusion: choose a strategy, $i = 2$.

**TABLE III.** **MATRIX OF INVESTMENT STRATEGY SELECTION ACCORDING TO THE LAPLACE CRITERION**

| Strategies investing (i) | Effect in cluster units (j) | $\sum(a_{ij})$ |
|------------------------|-----------------------------|---------------|
| $X_1$                  | 28.6                        | 18.8          |
| $X_2$                  | 25.6                        | 19.6          |
| $X_3$                  | 23.8                        | 20.4          |
| $X_4$                  | 21.8                        | 19.6          |
| $X_5$                  | 19.4                        | 21.4          |
| $X_6$                  | 17.2                        | 21.8          |
| $X_7$                  | 15.6                        | 22.8          |
| $X_8$                  | 14                          | 23.6          |
| $X_9$                  | 12.2                        | 22.6          |
| Probability $p_j$      | 0.2                         | 0.2           |

**Savage Criterion.** Minimum risk criterion Savage recommends choosing as the optimal strategy the one in which the magnitude of the maximum risk is minimized in the worst conditions, i.e. the condition is fulfilled:

$$a = \min(\max(r_{ij})). \quad (3)$$

The Savage Criterion focuses statistics on the most adverse states of nature, i.e. this criterion expresses a pessimistic assessment of the situation. Find the risk matrix, i.e. a measure of inconsistency between the various possible outcomes of adopting certain strategies [13]. Maximum gain in $i$-st column $b_i = \max(a_{ij})$ characterizes the auspiciousness of the state of nature.
1. We calculate the 1st column of the risk matrix.

\[ r_{11} = 143 - 143 = 0; \ r_{21} = 143 - 128 = 15; \ r_{31} = 143 - 119 = 24; \ r_{41} = 143 - 109 = 34; \ r_{51} = 143 - 97 = 46; \ r_{61} = 143 - 86 = 57; \ r_{71} = 143 - 78 = 65; \ r_{81} = 143 - 70 = 73; \ r_{91} = 143 - 61 = 82; \]

2. We calculate the 2nd column of the risk matrix.

\[ r_{12} = 114 - 114 = 0; \ r_{22} = 114 - 110 = 4; \ r_{32} = 114 - 100 = 14; \ r_{42} = 114 - 98 = 16; \ r_{52} = 114 - 91 = 23; \ r_{62} = 114 - 89 = 25; \ r_{72} = 114 - 85 = 29; \ r_{82} = 114 - 84 = 30; \ r_{92} = 114 - 79 = 35; \]

3. We calculate the 3rd column of the risk matrix.

\[ r_{13} = 118 - 94 = 24; \ r_{23} = 118 - 98 = 20; \ r_{33} = 118 - 102 = 16; \ r_{43} = 118 - 100 = 18; \ r_{53} = 118 - 107 = 11; \ r_{63} = 118 - 109 = 9; \ r_{73} = 118 - 114 = 4; \ r_{83} = 118 - 118 = 0; \ r_{93} = 118 - 105 = 13; \]

4. Let us calculate the 4th column of the risk matrix.

\[ r_{14} = 120 - 84 = 36; \ r_{24} = 120 - 92 = 28; \ r_{34} = 120 - 98 = 22; \ r_{44} = 120 - 104 = 16; \ r_{54} = 120 - 110 = 10; \ r_{64} = 120 - 114 = 6; \ r_{74} = 120 - 117 = 3; \ r_{84} = 120 - 120 = 0; \ r_{94} = 120 - 113 = 7; \]

5. Let us calculate the 5th column of the risk matrix.

\[ r_{15} = 142 - 65 = 77; \ r_{25} = 142 - 72 = 70; \ r_{35} = 142 - 81 = 61; \ r_{45} = 142 - 89 = 53; \ r_{55} = 142 - 95 = 47; \ r_{65} = 142 - 102 = 40; \ r_{75} = 142 - 106 = 36; \ r_{85} = 142 - 108 = 34; \ r_{95} = 142 - 142 = 0; \]

### TABLE IV. MATRIX OF INVESTMENT DISTRIBUTION ACCORDING TO THE SAVAGE CRITERION

| Strategies investing (i) | Y_1 | Y_2 | Y_3 | Y_4 | Y_5 |
|-------------------------|-----|-----|-----|-----|-----|
| X_1                     | 0   | 0   | 24  | 36  | 77  |
| X_2                     | 15  | 4   | 20  | 28  | 70  |
| X_3                     | 24  | 14  | 16  | 22  | 61  |
| X_4                     | 34  | 16  | 18  | 16  | 53  |
| X_5                     | 46  | 23  | 11  | 10  | 47  |
| X_6                     | 57  | 29  | 9   | 6   | 40  |
| X_7                     | 65  | 30  | 0   | 34  | 70  |
| X_8                     | 73  | 35  | 13  | 7   | 61  |
| X_9                     | 82  | 35  | 13  | 7   | 61  |

The results of the calculations are arranged in the form of table V.

### TABLE V. MATRIX OF INVESTMENT DISTRIBUTION ACCORDING TO THE SAVAGE CRITERION

| Effect in cluster units (j) | Y_1 | Y_2 | Y_3 | Y_4 | Y_5 | max(a_{ij}) |
|-----------------------------|-----|-----|-----|-----|-----|--------------|
| X_1                         | 0   | 0   | 24  | 36  | 77  | 77           |
| X_2                         | 15  | 4   | 20  | 28  | 70  | 70           |
| X_3                         | 24  | 14  | 16  | 22  | 61  | 61           |
| X_4                         | 34  | 16  | 18  | 16  | 53  | 53           |
| X_5                         | 46  | 23  | 11  | 10  | 47  | 47           |
| X_6                         | 57  | 29  | 9   | 6   | 40  | 40           |
| X_7                         | 65  | 30  | 0   | 34  | 70  | 70           |
| X_8                         | 73  | 35  | 13  | 7   | 61  | 61           |
| X_9                         | 82  | 35  | 13  | 7   | 61  | 61           |

We choose from (77; 70; 61; 53; 47; 57; 65; 73; 82) minimal element, min = 47. Conclusion: we choose a strategy, \( i = 5 \).

**Hodge-Lehmann criterion.** For each line, we calculate the value of the criterion using the following in (4):

\[ W_i = \mu \cdot \sum_{j=1}^{n} a_{ij} \cdot p_j + (1 - \mu) \cdot \min(a_{ij}). \quad (4) \]

Next, we calculate \( W_i \) for each line:

\[ W_1 = 0.5 \cdot 100 + (1 - 0.5) \cdot 65 = 82.5 \]
\[ W_2 = 0.5 \cdot 100 + (1 - 0.5) \cdot 72 = 86 \]
\[ W_3 = 0.5 \cdot 100 + (1 - 0.5) \cdot 81 = 90.5 \]
\[ W_4 = 0.5 \cdot 100 + (1 - 0.5) \cdot 89 = 94.5 \]
\[ W_5 = 0.5 \cdot 100 + (1 - 0.5) \cdot 91 = 95.5 \]
\[ W_6 = 0.5 \cdot 100 + (1 - 0.5) \cdot 86 = 93 \]
\[ W_7 = 0.5 \cdot 100 + (1 - 0.5) \cdot 78 = 89 \]
\[ W_8 = 0.5 \cdot 100 + (1 - 0.5) \cdot 70 = 85 \]
\[ W_9 = 0.5 \cdot 100 + (1 - 0.5) \cdot 61 = 80.5 \]

The resulting values distributed in columns \( \sum(a_{ij}p_j), \min(a_{ij}), W_i \) in Table VI.

### TABLE VI. MATRIX CALCULATIONS ACCORDING TO THE HODGE-LEHMANN CRITERION

| Effect in cluster units (j) | \( \sum(a_{ij}) \) | \( \min(a_{ij}) \) | \( W_i \) |
|-----------------------------|---------------------|-------------------|---------|
| X_1                         | 0 0 24 36 77        | 100               | 65 82.5 |
| X_2                         | 15 4 20 28 70       | 100               | 72 86   |
| X_3                         | 24 14 16 22 61      | 100               | 81 90.5 |
| X_4                         | 34 16 18 16 53      | 100               | 89 94.5 |
| X_5                         | 46 23 11 10 47      | 100               | 91 95.5 |
| X_6                         | 57 25 9 6 40        | 100               | 86 93   |
| X_7                         | 65 29 4 3 36        | 100               | 78 89   |
| X_8                         | 73 30 0 34 70       | 100               | 70 85   |
| X_9                         | 82 35 13 7 61       | 100               | 61 80.5 |

| probabilitiess | 0.2 | 0.2 | 0.2 | 0.2 | 0.2 |
We choose from (82.5; 86; 90.5; 94.5; 95.5; 93; 89; 85; 80.5) maximum element, max = 95.5.

According to the calculations for this criterion, it is necessary to consider the optimal investment strategy, \( i = 5 \).

**Generalized Hurwitz criterion.** This criterion is a certain generalization of the criteria of extreme pessimism and extreme optimism, and is also a special case of the generalized Hurwitz criterion regarding winnings under the following assumption:

\[
\delta_1 = 1 - \delta, \quad \delta_2 = \delta_3 = \ldots = \delta_{n-1} = 0, \delta_n = \delta, \quad \text{where} \quad 0 \leq \delta \leq 1.
\]

Then the indicator of efficiency of the investment strategy \( i \) according to Hurwitz is:

\[
G_i = (1 - \delta) \cdot \min(a_{ij}) + \delta \cdot \max(a_{ij}).
\]  
(5)

The optimal strategy \( i \) will be the strategy with the maximum value of the performance indicator.

Let us build auxiliary matrix VII. Pessimist approach. \( \delta \) is selected from the condition of non-increasing average:

\[
\delta = \frac{b_2}{b_1 + b_3} = \frac{693}{693 + 1102} = 0.386.
\]  
(6)

Therefore, we calculate \( G_i \):

\[
G_1 = 0.386 \cdot 65 + (1 - 0.386) \cdot 143 = 112.886
\]

\[
G_2 = 0.386 \cdot 72 + (1 - 0.386) \cdot 128 = 106.38
\]

\[
G_3 = 0.386 \cdot 81 + (1 - 0.386) \cdot 119 = 104.329
\]

\[
G_4 = 0.386 \cdot 89 + (1 - 0.386) \cdot 109 = 101.279
\]

\[
G_5 = 0.386 \cdot 91 + (1 - 0.386) \cdot 110 = 102.665
\]

\[
G_6 = 0.386 \cdot 86 + (1 - 0.386) \cdot 114 = 103.19
\]

\[
G_7 = 0.386 \cdot 78 + (1 - 0.386) \cdot 117 = 101.943
\]

\[
G_8 = 0.386 \cdot 70 + (1 - 0.386) \cdot 120 = 100.696
\]

\[
G_9 = 0.386 \cdot 61 + (1 - 0.386) \cdot 142 = 110.728
\]

Optimistic approach. \( \delta \) is selected from the condition of non-decreasing average:

\[
\delta = \frac{b_3}{b_1 + b_3} = \frac{1102}{693 + 1102} = 0.614.
\]

Therefore, we calculate \( G_i \):

\[
G_1 = 0.614 \cdot 65 + (1 - 0.614) \cdot 143 = 95.114
\]

\[
G_2 = 0.614 \cdot 72 + (1 - 0.614) \cdot 128 = 93.62
\]

\[
G_3 = 0.614 \cdot 81 + (1 - 0.614) \cdot 119 = 95.671
\]

\[
G_4 = 0.614 \cdot 89 + (1 - 0.614) \cdot 109 = 96.721
\]

\[
G_5 = 0.614 \cdot 91 + (1 - 0.614) \cdot 110 = 98.335
\]

\[
G_6 = 0.614 \cdot 86 + (1 - 0.614) \cdot 114 = 96.81
\]

\[
G_7 = 0.614 \cdot 78 + (1 - 0.614) \cdot 117 = 93.057
\]

\[
G_8 = 0.614 \cdot 70 + (1 - 0.614) \cdot 120 = 89.304
\]

\[
G_9 = 0.614 \cdot 61 + (1 - 0.614) \cdot 142 = 92.272
\]

We choose from (112.886; 106.38; 104.329; 101.279; 102.665; 103.19; 101.943; 100.696; 110.728) max element, max = 112.89.

**TABLE VII. MATRIX OF CALCULATIONS BY GENERALIZED CRITERION OF HURWITZ**

| Strategies investing (i) | Effect in cluster units (\( \frac{1}{b_i} \)) | \( \min(a_{ij}) \) | \( \max(a_{ij}) \) | Pessimist approach | Optimist approach |
|--------------------------|--------------------------------|----------------|----------------|------------------|------------------|
| \( X_1 \) | 54 | 85 | 104 | 141 | 65 | 143 | 112.88 | 95.11 |
| \( X_2 \) | 72 | 92 | 98 | 110 | 128 | 128 | 106.37 | 93.62 |
| \( X_3 \) | 81 | 96 | 100 | 102 | 119 | 81 | 104.32 | 95.67 |
| \( X_4 \) | 83 | 96 | 100 | 103 | 109 | 89 | 101.27 | 96.72 |
| \( X_5 \) | 91 | 95 | 97 | 107 | 110 | 91 | 102.66 | 98.33 |
| \( X_6 \) | 86 | 89 | 102 | 109 | 114 | 86 | 103.18 | 98.81 |
| \( X_7 \) | 78 | 85 | 106 | 114 | 117 | 78 | 101.94 | 93.05 |
| \( X_8 \) | 70 | 84 | 108 | 118 | 120 | 70 | 100.69 | 89.3 |
| \( X_9 \) | 61 | 79 | 105 | 113 | 142 | 61 | 110.72 | 92.27 |

Conclusion: We choose the strategy \( i = 1 \).

IV. CONCLUSION

Finally, the result research of solving a statistical game according to various criteria, the strategy was more often recommended, \( i = 5 \).

From the obtained values, we see that with a specific amount of investment with the main goal to outperform the market in scientific and technological terms, the presented model allows you to distribute money evenly across all blocks to achieve goals.

The formed investment vector must be spread by investment to include it in the management income statement.

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