Apparent Deviations from Keplerian Acceleration for Stars Around the Supermassive Black Hole at the Galactic Center

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Abstract

We show that the time-dependent Doppler effect should induce measureable deviations of the time history of the projected orbit of a star around the supermassive black hole in the Galactic center (SgrA*) from the expected Keplerian history. In particular, the line-of-sight acceleration of the star generates apparent acceleration of its image along its velocity vector on the sky, even if its actual Keplerian acceleration in this direction vanishes. The excess apparent acceleration simply results from the transformation of time between the reference frames of the observer and the star. Although the excess acceleration averages to zero over a full closed orbit, it could lead to systematic offsets of a few percent in estimates of the dynamical mass or position of the black hole that rely on partially sampled orbits with pericentric distances of \( \sim 10^{15} \) cm. Deviations of this magnitude from apparent Keplerian dynamics of known stars should be detectable by future observations.

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I. INTRODUCTION

The latest infrared snapshots of the inner arcsecond of the Milky Way galaxy show bright stars orbiting around a common center of mass of $\sim 4 \times 10^6 M_\odot$, providing unprecedented evidence for the existence of a supermassive black hole at the location of the radio/infrared source, SgrA* [1–3]. The inferred stellar orbits appear to be Keplerian to within the current measurement errors. Nevertheless, one would expect to see deviations from classical Keplerian orbits due to various general-relativistic effects, that can in turn be used to constrain the mass and spin of the black hole [4,5]. Here we point out that the leading-order correction to the apparent orbit of a star involves a time transformation rather than a physical modification of the orbit, and is of order $\sim |v/c|$, where $v$ is the star velocity and $c$ is the speed of light. Since some stars approach pericenter at a few percent of the speed of light, future observations reaching the percent level of sensitivity should detect this effect. As we show in §2, the naive Keplerian fit to the apparent orbit of the star in these future data sets should reveal an acceleration residual due to the time-dependent Doppler effect. Although this excess acceleration component averages to zero over a full closed orbit (during which the star returns to its original position and velocity), it could lead to systematic inconsistencies in estimates of the dynamical mass and position of the black hole that are based on partially sampled orbits.

II. ACCELERATION RESIDUAL DUE TO THE TIME-DEPENDENT DOPPLER EFFECT

The standard Doppler transformation of infinitesimal time intervals between the reference frames of the observer ($dt_{\text{obs}}$) and the star ($dt$) is given to leading-order in $|v/c|$ by [6],

$$dt_{\text{obs}} = dt \left(1 + \frac{v_\parallel}{c}\right), \quad (1)$$

where $v_\parallel(t)$ is the velocity of the star along the line-of-sight. The observed transverse velocity of the star is therefore different$^1$ from its actual transverse velocity, $v_\perp = (dx_\perp/dt)$,

$$\frac{dx_\perp}{dt_{\text{obs}}} = \left(\frac{dt}{dt_{\text{obs}}}\right) \frac{dx_\perp}{dt} \approx \left(1 - \frac{v_\parallel}{c}\right) v_\perp, \quad (2)$$

where the transverse position vector $x_\perp$ corresponds to angular coordinates on the sky times the distance to the Galactic center ($\approx 8$ kpc [7]). Throughout the paper, the terms parallel or transverse (perpendicular) are relative to the line-of-sight axis that starts at the observer and goes through the star.

Taking the $t_{\text{obs}}$-derivative of both sides of equation (2) and keeping terms to leading-order, we get two Doppler components that contribute to the difference between the observed and Keplerian values of the transverse acceleration of the star on the sky.

$^1$The spatial line traced by the orbit remains unchanged in the two reference frames. We ignore the Lorentz transformation of the spatial coordinates since it corresponds to corrections of order $\sim (v/c)^2$ or higher.
\[ a_{\perp, \text{obs}} = a_{\perp} - \frac{2v_{\parallel}}{c} a_{\perp} - \frac{v_{\perp}}{c} a_{\parallel}, \]  

(3)

where \( a_{\text{obs}} \equiv \left( \frac{d^2 x}{dt^2}_{\text{obs}} \right) \) is the observed acceleration and \( a \equiv \left( \frac{d^2 x}{dt^2} \right) \) is the actual Keplerian acceleration of the star. These Doppler correction terms were not included in past analysis of SgrA* data [1–3]. The last term on the right-hand-side of equation (3) implies that the apparent transverse acceleration gets a contribution from the Keplerian acceleration along the line-of-sight, \( a_{\parallel} \). The image of the star may therefore show apparent acceleration along its velocity vector on the plane of the sky even if its Keplerian transverse acceleration vanishes, \( a_{\perp} = 0 \) (as long as \( v_{\perp} a_{\parallel} \neq 0 \)). Note that \( v_{\perp} \) is the transverse component of the net relative velocity between the star and the observer, \textit{including} the transverse velocity of the Sun around the Galactic center.

A time-independent Doppler effect with \( a_{\parallel} = 0 \), such as expected for a face-on orbit, gives \( a_{\perp, \text{obs}} = \left( 1 - 2v_{\parallel}/c \right) a_{\perp} \), leading to a simple miscalibration of the black hole mass by a constant factor of \( \left( 1 - 2v_{\parallel}/c \right) \). But more generally, the additional effect of the last term in equation (3) cannot be compensated for by a simple fudge factor, as it depends on the particular parameters of the stellar orbit and on the orbital phase.

The two excess acceleration terms on the right-hand-side of equation (3) amount to a fractional correction amplitude of a few percent for the inferred value of \( a_{\perp} \) near pericenter of the closest known stars to SgrA*. These stars reach their pericentric distance of \( \sim 100 \text{ AU} = 1.5 \times 10^{15} \text{ cm} \) with \( (v/c) \lesssim 3 \times 10^{-2} \). Omission of the above Doppler terms could lead to systematic offsets in estimates of the black hole mass and position that are based on partially sampled orbits.

Spectroscopic detection of the time-dependence of Doppler-shifted frequencies, \( \nu_{\text{obs}} \propto \left( 1 - v_{\parallel}/c \right) \), of spectral lines emitted from the atmosphere of the star (such as the Br \( \gamma \) or He I features reported for the star SO-2 in ref. [8]) can be used to measure \( v_{\parallel} \) as a function of \( t_{\text{obs}} \) and thus infer the Keplerian line-of-sight acceleration, \( a_{\parallel} = \left( 1 - v_{\parallel}/c \right) (dv_{\parallel}/dt_{\text{obs}}) \), as a function of intrinsic time, \( t = f dt_{\text{obs}}(1 - v_{\parallel}/c) \). This can, in turn, be used to check for consistency with the astrometric derivation of \( a_{\parallel} \) from equation (3) using time-tagged snapshots of the orbit of the star. However, observers are likely to find it much more challenging to obtain the required spectroscopic data than to improve the precision on existing imaging techniques, for the purpose of recovering \( a_{\parallel} \).

The leading-order Doppler effects in equation (3) can be easily incorporated into a computer program that searches for the best-fit Keplerian orbit under the constraints of a given data set. The \textit{Doppler-corrected} Keplerian fit could provide direct constraints on the radial motion of the star based on its apparent motion on the sky. Such a fit would involve the same number of free parameters as in the standard Keplerian fit.

### III. DISCUSSION

The time-dependent Doppler effect adds two \textit{apparent} acceleration terms to the Keplerian acceleration of a star on the sky in Equation (3). The last term on the right-hand-side of this equation implies that the observer may see the star accelerating along the projection of its velocity on the sky even if its Keplerian acceleration vanishes in this direction. The existence of this term would be particularly intriguing for a star on a highly eccentric (nearly...
radial) orbit along the line-of-sight. The transverse motion of the sun around the Galactic center with \((v_\perp/c) \approx 7 \times 10^{-4}\) would induce a minimum transverse acceleration of the stellar image of \(\sim 7 \times 10^{-4}a_\parallel\) in this case.

Additional special-relativistic or general-relativistic effects are of the order of \(\sim (v/c)^2\) or \(\sim \phi/c^2\), or smaller, where \(\phi\) is the local gravitational potential probed by the star \((\phi \sim v^2)\). These corrections are at least an order of magnitude smaller than the Doppler effect discussed here, for orbits with pericentric distances of \(\gtrsim 10^{15}\) cm.

Deviations from apparent Keplerian orbits may also be caused by gravitational scattering off other stars or black hole companions to SgrA* [9]. The time dependence of the excess acceleration caused by such scattering events can be easily differentiated from that of the periodic and fully-deterministic Doppler effect.

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REFERENCES

[1] A. M. Ghez, et al., Astrophys. J., submitted (2003), astro-ph/0306130; A. M. Ghez, et al., Astrophys. J. Lett., submitted (2003), astro-ph/0309076
[2] R. Genzel, et al., Astrophys. J. 594, 812 (2003)
[3] R. Schödel, et al., Astrophys. J., submitted (2003); astro-ph/0306214
[4] M. Jaroszynski, Acta Astronomica, 48, 653, (1998)
[5] E. Pfahl, & A. Loeb, Astrophys. J., submitted (2003)
[6] G. B. Rybicki, & A. P. Lightman, Radiative Processes in Astrophysics, (New York: Wiley, 1979), p. 111
[7] F. Eisenhauer, et al., Astrophys. J. Lett., submitted (2003); astro-ph/0306220
[8] A. M. Ghez, et al., Astrophys. J., 586, 127 (2003)
[9] K. P. Rauch, & S. Tremaine, New Astronomy, 1, 149 (1996)