Recent intensive research on nanoelectromechanical systems (NEMS) was motivated by a variety of physical effects involved and the prospect of practical applications \cite{1}. NEMS have been realized experimentally with molecules \cite{2}, semiconductor beams \cite{3}, and suspended carbon nanotubes \cite{4}. Phenomena observed include negative differential resistance, phonon-assisted transport, and tuning the eigenmodes by the gate voltage. Most of these experiments were performed in the single electron tunneling (SET) device regime \cite{5}.

In this regime, a NEMS is essentially a SET device coupled to a mechanical (harmonic) oscillator. The coupling is provided by a force $F$ \cite{6} acting on the oscillator, the value of the force depending on the charge state of the SET device. It determines the dimensionless coupling parameter, $\lambda = F^2/\hbar M \omega^3$, where $M$ and $\omega$ are the mass and the frequency of the oscillator. It was recognized already \cite{7} that for strong coupling $\lambda \gg 1$, mechanical degrees of freedom strongly influence transport through a SET device, leading, for instance, to polaron physics and Franck-Condon effect. However, the weak-coupling regime $\lambda \ll 1$ is characteristic for most of NEMS and will be considered below.

Naively, the effect of the oscillator on transport current in this regime must be small and proportional to $\lambda$. However, an underdamped oscillator can be swung up to big amplitudes even by a weak random force originating from stochastic electron transfers through the device \cite{8}; this amplitude may provide a strong feedback on the current. A less obvious effect is the extra friction due to electron tunneling \cite{9} which has been erroneously disregarded in \cite{10}. We demonstrate in this Letter that such electron-induced dissipation may become negative, resulting in the generation of mechanical oscillations and in strong mechanical feedback. This takes place if the average charge accumulated in the SET device is a non-monotonous function of gate voltage.

The strong feedback is the most manifest in the current noise. The natural measure of noise in nanostructures is the Poisson value \cite{11}, $S_P = 2eI$. We demonstrate that in the strong feedback regime the noise is always parametrically bigger than $S_P$ due to long-time correlations of oscillator amplitude. If the generation is bistable, we predict a telegraph noise that can be exponentially big. Even if the strong feedback is absent, the noise may still exceed $S_P$. The experimental observation of the enhanced noise thus would provide a strong evidence for mechanical motion.

SET systems are known to exhibit a (quasi) periodic structure of Coulomb diamonds in the plane of bias $V$ and gate $V_g$ voltages. Inside each diamond, the number of extra electrons $n$ is fixed to an integer \cite{12}. We concentrate on the region adjacent to the two neighboring diamonds with $n = 0$ and $n = 1$ where only these two charge states of the SET device participate in transport. In the classical limit, the statistical description of the system is provided by the joint distribution function $P_n(x, v, t)$, with $x$ and $v$ being the position and velocity of the oscillator. This distribution function obeys the following master equation \cite{13}

$$\frac{\partial P_n}{\partial t} + \left\{ v \frac{\partial}{\partial x} + \frac{\partial}{\partial v} F \right\} P_n - \text{St} [P] = 0; \quad (1)$$

$$F = -M \omega^2 x - \frac{M \omega v}{Q} + F_n; \quad (2)$$

$$\text{St} [P] = (2n - 1) \left( \Gamma^+ (x) P_0 - \Gamma^- (x) P_1 \right). \quad (3)$$

Here, the total force $F$ acting on the oscillator is the sum of the elastic force, friction force, and charge-dependent coupling force, respective to the order of terms in Eq. (2). $Q \gg 1$ is the quality factor. We count the position of the oscillator from its equilibrium position in the $n = 0$ state. In this case, $F_n = nF$.

The "collision integral" $\text{St} [P]$ represents SET. There are four tunnel rates, $\Gamma_{L,R}^\pm$, where the subscripts $L$ and $R$ denote tunneling through the left or right junction, and the superscripts $+$ and $-$ correspond to the tunneling to and from the island, respectively. In Eq. (3), $\Gamma^\pm = \Gamma_L^\pm + \Gamma_R^\pm$. It is enough for our purposes to assume that each rate is a function of the corresponding energy cost $\Delta E_{L,R}^\pm$ associated with the addition/removal of an electron to/from the island in the state $n = 0/1$ via left or right junction ($\Delta E_L^+ = -\Delta E_L^-$). Two independent energy differences are determined by electrostatics and
depend linearly on the voltages. Additionally, they are contributed by the shift of the oscillator,
\[ \Delta E_L^+ = -W + W_L - Fx, \quad \Delta E_R^- = -W + W + Fx, \]
where we introduce a convenient parameter \( W \) representing both \( eV \) and \( eV_G \), with \( W_L, W_R \) lying at the boundaries of the diamonds and \( W_L < W < W_R \) in the transport region. The condition of applicability of the classical approach is that the energy differences are much bigger than energy quantum of the oscillator, \( W \gg \hbar \omega \).

To simplify Eq. (1), we implement the separation of the frequency scales: the inverse damping time \( \kappa \) of the oscillator, the oscillator frequency \( \omega \), and the total tunneling rate \( \Gamma_t = \Gamma_t^+ + \Gamma_t^- \), assuming \( \kappa \ll \omega \ll \Gamma_t \). The first condition implies that the mechanical energy hardly changes during an oscillation, while the second condition implies that the coordinate varies so slowly that \( \Gamma(x) \) hardly changes between two successive tunneling events. In this case, we arrive at a Fokker-Planck equation for the distribution function of the slowest variable — mechanical energy \( E \), \( P(E) \). It reads
\[ \frac{\partial P}{\partial t} = \hat{\mathcal{L}} P, \quad \hat{\mathcal{L}} \equiv \frac{\partial}{\partial E} \left( E\kappa(E) + D(E) \frac{\partial}{\partial E} \right). \tag{4} \]
Here \( D(E) \) is the diffusion coefficient in energy space and the inverse damping time is given by \( \kappa(E) = \tilde{\kappa}(E) + \omega / Q \), \( \tilde{\kappa} \) being the SET contribution. It is instructive to express those parameters in terms of the average number of extra electrons in the island, \( \bar{n}(x) \equiv \Gamma^+ / \Gamma_t \),
\[ \left\{ \frac{D(E)}{E} \right\} = \frac{\bar{n}^2}{M} \left\{ \frac{1}{\Gamma_t} \left\{ \tilde{\kappa}(1 - \bar{n}) \right\} \right\}. \tag{5} \]
Here, the angle brackets denote an average over the oscillation period, \( \langle A(x) \rangle = \int (d\theta / \pi) \cos^2 \theta A(x(E) \sin \theta) \), the oscillation amplitude being given by \( x(E) = \sqrt{2E/M} / \omega \).

The SET contribution to the damping \( \tilde{\kappa} \) has been erroneously disregarded in Ref. 8. In fact, as Eq. (5) suggests, the diffusion and damping are closely related. In particular, in the absence of bias (\( W_L = W_R \)) the average number of electrons is determined by the Boltzmann distribution and one proves that \( \bar{n}/dW = \tilde{n}(1 - \bar{n})/k_B T \). In this case, the diffusion coefficient obeys the Einstein relation \( D(E) = k_B T E \kappa(E) \). This, in its own turn, guarantees that Eq. (4) is satisfied with the Boltzmann distribution \( P(E) \propto \exp(-E/k_B T) \). At \( eV > k_B T \), the Einstein relation does not hold anymore. The effective temperature \( E^2k_B / \kappa \) of the oscillator may become of the order of \( eV \). Moreover, we will demonstrate that for energy-dependent tunneling rates, the damping \( \tilde{\kappa} \) can become negative. This signals instability with respect to interaction with the oscillator. To stress the importance of the SET contribution we will disregard other contributions to the damping \( (Q \to \infty) \), so that \( \kappa = \tilde{\kappa} \).

The stationary solution of Eq. (4) apart from a normalization constant reads
\[ P(E) \propto \exp \left( - \int_0^E dE' \kappa(E') / D(E') \right). \tag{6} \]

The current is modified by mechanical motion. At a given mechanical energy \( E \), the current averaged over the oscillation period, \( I_W(E) \), is determined from the dependence of the current on the energy parameter \( I(W) \equiv \Gamma_L^+ W / \Gamma_R^+ W / \Gamma_L^- W / \Gamma_R^- W \) in the absence of oscillations: \( I_W(E) \equiv \int (d\theta / 2\pi) I(W + Fx(E) \sin \theta) \). In the limit of small amplitudes, one has \( I_W(E) \approx I(W) \sinh(\Delta E / \lambda) \). The actual current \( I_W \) is obtained by averaging \( I_W(E) \) over \( E \) with the distribution function \( P(E) \). Zero-frequency current noise in the Fokker-Planck framework is obtained as
\[ S = -4 \int_0^\infty dE \delta I_W(E) \hat{\mathcal{L}}^{-1} \delta I_W(E) P(E), \tag{7} \]
with \( \delta I_W(E) \equiv I_W(E) - I_W \). In our assumptions, the distribution function is sharp at the energy scale of interest. Indeed, the typical mechanical energy needed to modify the rates is determined from the relation \( eV \approx Fx(E) \), yielding \( E \approx M \omega^2 (eV / F)^2 \). If \( \lambda \ll 1 \), this always exceeds the typical energy fluctuation \( eV \). If the damping is positive at all \( E \), \( P \) has a sharp maximum at \( E = 0 \) and the current is very close to \( I_W(0) \). The average amplitude of the oscillations is too small to induce a noticeable mechanical feedback.

The situation changes drastically if \( \kappa(E) \) becomes negative, indicating instability and growing amplitude of the oscillations. Since \( \kappa(E) \) is determined by the tunnel rates only, the amplitude growth can only be stabilized by significant modification of the rates by the amplitude growing. This is the strong mechanical feedback. Positions of probability maxima are determined by the roots of
\[ E \kappa(E) = 0. \tag{8} \]

A non-trivial root \( E_0 \neq 0 \) indicates a generation of mechanical oscillation with almost constant amplitude. This may strongly modify the current that is now given by \( I_W(E_0) \). Our analysis shows that the negative damping can only arise from the energy dependence of the tunnel amplitudes. This dependence is intrinsic for both semiconductor quantum dots and molecules.

To illustrate, we have chosen exponential energy dependence typical for wide tunnel barriers 12, and one electron level in the SET system,
\[
\Gamma_{L,R}^+ = 2 \Gamma_0 L \lambda e^{-a L_R \kappa \Delta E_{L,R}^+} \left( 1 - f_f(-\Delta E_{L,R}^+ \hbar) \right); \\
\Gamma_{L,R}^- = \Gamma_0 L \lambda e^{a L_R \kappa \Delta E_{L,R}^-} f_f(\Delta E_{L,R}^-), \tag{9}
\]
the factor 2 accounting for the spin degeneracy of the state \( n = 1 \). The energy dependence sets a new energy
scale \( W_c \approx 1/\alpha_{L,R} \) which is assumed to be much smaller than the charging energy. For concrete illustration, we choose \( \alpha_L = 0.03/W_c \), \( \alpha_R = 0.75/W_c \), \( k_B T = 0.2 W_c \), \( \Gamma^0_L = \Gamma^0_R \). Fig. 1 presents the regions in gate-bias voltage plane that differ by number and stability of the roots of Eq. 10. The region (i) corresponds to positive damping \( \Gamma \) for all \( E \). We make use of Eq. (7) approximating the noise in the absence of feedback (region (i) in Fig. 1). We illustrate the modification of the current by mechanical motion in Fig. 2. The dashed (solid) lines give the current modified (unmodified) by mechanical motion. The two upper panels demonstrate that the modification is restricted to region (ii) where the generation takes place. The two lower panels illustrate the bistable regions (iii) and (iv). In the left lower panel, the current in the region (iv) gives a jump where the probability of two stable amplitudes are the same. To the right of the jump, the lower amplitude value is more probable. This value decreases and becomes zero at the border of region (iii). Therefore the modification of the current ceases there (see inset). In the right lower panel, the probabilities become the same in the region (iii). Therefore the modification ceases immediately after the jump.

We illustrate the modification of the current by mechanical motion in Fig. 2. The modification is noticeable provided the generation of oscillations takes place. It can be of the same order of magnitude as the unmodified current, and of either sign. The current can even exhibit jumps if there are at least two stable values \( E_1, E_2 \) of the amplitude generated. The position of the jump corresponds to the values of \( W \) at which the probabilities \( P(E_1), P(E_2) \) are equal.

Let us now turn to the current noise. First we evaluate the noise in the absence of feedback (region (i) in Fig. 1). We make use of Eq. 10 approximating \( D(E), \kappa(E), \delta I_W(E) \) by their values at \( E \to 0 \). This yields

\[
S = \frac{F^4}{M^2 \omega^4} \left( \frac{\partial^2 I}{\partial W^2} \right)^2 \frac{D^2(E)}{\kappa^4(E) E^2} \bigg|_{E=0}.
\]

The ratio of the electromechanical noise and the Poisson value \( S_P \) is of the order of

\[
\frac{S}{S_P} \sim \left( \frac{\Gamma^0}{\omega} \right)^2 \frac{\hbar \omega \lambda}{W}. \tag{11}
\]

The small value of the second factor can be compensated by the large value of the first one. In this case, the electromechanical noise, concentrated at frequencies of the order of \( \kappa \), exceeds the Poisson value.

In the region (ii), where the stable generation of the oscillation with \( E_0 \) takes place, the current noise is due to small fluctuations of the oscillation amplitude. These fluctuations occur at a frequency scale of the order of \( \kappa'(E_0) E_0 \). The noise is given by

\[
S = 4 \left( \frac{I'(E_0) D(E_0)}{E_0^3 \kappa^2(E_0)} \right) ; \quad \frac{S}{S_P} \sim \left( \frac{\Gamma^0}{\omega} \right)^2. \tag{12}
\]

That is, it exceeds the Poisson value by a large factor. Our numerical results for regions (i) and (ii) (Fig. 3a,b) show that Eqs. 11, 12 give a scale of the noise rather than a good estimation. The actual values of noise change by three-four orders of magnitude. The reason for that is that the parameters \( I', I'', \kappa, \kappa' \) may become close to zero.

In the regions (iii) and (iv) the oscillation amplitude randomly switches between two values \( E_{1,2} \). Since they
FIG. 3: Mechanical contribution to current noise for different bias voltages. (a) Noise in stable region (i) (Eq. 10) becomes zero at $I'' = 0$. (b) Noise in the region (ii) changes by orders of magnitude approaching zero at $E_0 \to 0$ (Eq. 12). (c,d) Telegraph noise is presented in the bistable regions only. The solid lines indicate the region where $I \equiv 0$. The switching times are therefore exponentially big if the generation is bistable.

To conclude, we analyzed the SET system weakly coupled to a mechanical oscillator and proved the existence of significant modification of the current under condition of strong feedback where generation of mechanical oscillation takes place. The latter is feasible for energy-dependent tunneling amplitudes. The current noise generated by mechanical motion in the strong feedback regime significantly exceeds the Poisson value and may be exponentially big if the generation is bistable. Even if no generation takes place, this extra noise may exceed $S_p$ for sufficiently fast tunneling rates.

This work was supported by the Netherlands Foundation for Fundamental Research on Matter (FOM) and EC FP6 funding (contract no. FP6-2004-IST-003673). This publication reflects the views of the authors and not necessarily those of the EC. The Community is not liable for any use that may be made of the information contained herein.

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