The Antinomy of the Liar

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In this essay, I present a dialtheic solution to the Antinomy of the Liar and I evaluate the objection that, if the argument in Curry’s paradox is valid, accepting this solution forces us to accept an analogous solution to Curry’s, which I show to be unacceptable. Thus, to accept this solution, we have to reject the argument in Curry’s. I discuss some arguments defending this view.

Consider the sentence

**Sentence L** Sentence $L$ from $[3]$ is false.

Or, writing $F(x)$ for “$x$ is false” and $T(x)$ for “$x$ is true”:

$$L = F(L)$$

Using the T-Schema

$$T(x) \iff x$$

we try to determine the truth-value of $L$

$$L = F(L)$$

Where step 3 comes from $L$ having to be true or false. We get $L$ being both true and false, a contradiction. This is the Antinomy of the Liar.
The *dialetheic* solution to the antinomy defended by Priest is to accept some contradictions to be true and as evidence that is true and false.

One objection to dialetheism is that at least prima facie, if we have a sentence $A$ which is both true and false, or equivalently if we remain within classical logic, both $A$ and its negation $\neg A$ are true, then we can conclude any other sentence $B$ is true. This is the *principle of explosion*:

$$A, \neg A \vdash B$$

and it can be proven like so:

1. $A$ (given)
2. $A \lor B$ (1; disjunction introduction)
3. $\neg A$ (given)
4. $B$ (2, 3; disjunctive syllogism)

However, in step 4, disjunctive syllogism is not justified if we accept dialetheism; it might be (and it is) that $A$, as well as false, is true. Hence, $A, \neg A \vdash B$ has not been proven and it fails to prove dialetheism wrong if we reject disjunctive syllogism.

This also shows that if we accept dialetheism, we have to reject disjunctive syllogism as well as other rules of inference.

It can be argued that even if negation in English is correctly interpreted as a dialetheic negation, where both $A$ and $\neg A$ can be true, we can still define a connective, $\neg$, which cannot be interpreted as such.

Priest gives the example of defining $\neg A$ as $A \rightarrow \bot$ where $\bot$ is some unacceptable conclusion such as $\forall xT(x)$ i.e. everything is true. This is equivalent to classical negation and it satisfies an analogous version of $A, \neg A \vdash B$.

We can define a sentence $C$ to be such that
$C \iff (C \rightarrow \bot)$

which generates Curry’s paradox:

\begin{enumerate}
\item $C \iff (C \rightarrow \bot)$ (given)
\item 1. $C \iff (C \rightarrow \bot)$ (given)
\item 2.1 $C$ (Assumption for conditional proof (CP henceforth))
\item 2.2 $C \rightarrow \bot$ (1, 2.1; MP)
\item 2.3 $\bot$ (2.1, 2.2; MP)
\item 3. $C \rightarrow \bot$ (2.1, 2.3; CP)
\item 4. $C$ (1, 3; MP)
\item 5. $\bot$ (3, 4; MP)
\end{enumerate}

We conclude $\bot$, which is unacceptable.

If we accept (1) as evidence that $L$ is true and false then we should accept (3) as evidence for both $C$ and $\neg C$ being true (since we have $C$ in step 4 and $\neg C$ in step 3). But since accepting $C$ and $\neg C$ entails unacceptable conclusions ($\bot$), we either reject that (1) shows $L$ is both true and false, and hence reject the dialetheic solution, or show the argument in (3) is invalid but the one in (1) is not.

Since (3) only uses MP and CP if we want to reject it we have to reject MP or CP. MP is impeccable but if we reject it, since it is used in (1), we also reject (1).

Beall and Murzi [1] present a solution rejecting CP; we should reject that there is a deduction-theorem link between validity and conditionals, i.e., reject:

$$A \vdash B \text{ if and only if } A \rightarrow B$$

(DT)

Since, if we accept [DT] and MP, Curry’s paradox cannot be solved, and if we reject [DT] line 3 in (3) is not justified so the argument fails.

However, rejecting [DT] does not seem plausible; having $B$ as a consequence of $A$ is equivalent to
having $A$ and $B$ joint by the consequence connective “$\rightarrow$”. To make this more explicit, consider a sentence $\pi$ satisfying

$$\pi \iff \text{Val}(\pi, \bot)$$

where $\text{Val}(A, B)$ means “The argument from $A$ to $B$ is valid”, and an argument is valid if, when the premises are true, so is the conclusion. So, by definition

$$\text{Val}(A, B) \rightarrow (A \rightarrow B). \quad \text{(V0)}$$

Thus we have an analogous paradox:

1. $\pi \iff \text{Val}(\pi, \bot)$ (Given)
2.1 $\pi$ (Assumption for CP’)
2.2 $\text{Val}(\pi, \bot)$ (1, 2.1; MP)
2.3 $\pi \rightarrow \bot$ (2.2; V0)
2.4 $\bot$ (2.1, 2.3; MP)
3. $\text{Val}(\pi, \bot)$ (2.1-2.4; CP’)
4. $\pi \rightarrow \bot$ (3; V0)
5. $\pi$ (1, 3; MP)
6. $\bot$ (4, 5; MP)

This is \textit{v-Curry’s paradox} [1, p. 152]. Line 3 might seem to present the same problems given for line 3 in (3) but this one is justified since by showing $A \vdash B$ we are precisely showing the argument from $A$ to $B$ is valid; if $A \vdash B$ then $\text{Val}(A, B)$ (CP’).

Since Curry’s and v-Curry’s paradoxes are essentially the same we should have a unified solution. But since for v-Curry [V0] and CP’ are justified we cannot reject (4) by rejecting those, so we cannot solve Curry’s by rejecting CP.

Therefore, since we cannot reject CP, MP is used in [11], and since [3] only uses MP and CP, we cannot reject [3] at all, unless we reject [11] too.
Although it seems like (3) only uses MP and CP, Beall and Murzi \cite{1, p. 146} argue it implicitly uses *Structural Contraction*:

If $\Gamma, y, y \vdash b$ then $\Gamma \vdash b$. \hspace{1cm} (SC)

Other arguments for finding the truth-value of $C$ may use the *rule of contraction*:

$$A \rightarrow (A \rightarrow B) \vdash A \rightarrow B$$ \hspace{1cm} (RC)

Thus, to reject the argument in (3) we can reject SC and RC. If we do, (3) is invalid since, by only one assumption of $C$ in 2.1 we discharged $C$ twice, in 2.2 and 2.3. Neither can we discharge $C$ only once in $(C \iff (C \rightarrow \bot))$ to conclude $\bot$ since we rejected RC.

Similarly, in (4), only from one assumption of $\pi$ we discharge $\pi$ twice; in 2.3 and in 2.4. Hence, the argument is not valid if we reject SC.

Therefore, by rejecting RC and SC we avoid Curry’s and v-Curry’s paradox in a unified way, and since neither RC nor SC is used in (1) (except in steps 1.4 and 2.4 which can be omitted) we can still accept the dialetheic solution.

Rejecting SC seems, however, very unintuitive and does not reflect how ordinary English is used. After all, if $A$ is true, everything following from $A$ is true and we can exploit $A$ as much as we like, it does not matter how many times we use $A$ in our arguments.

Beall and Murzi \cite{1, p. 163} argue we are resistant to rejecting SC because we have a view of validity as truth-preservation in all possible situations. But this might be abandoned if we think of premises instead of as partial descriptions of a situation, as resources. Then it is clearly different to have a single-$A$ resource from a double-$A$ resource. Accepting this requires changing our metaphysical account of validity.

Now, we clearly have $A \rightarrow A$ so, from one premise $A$, we can get as many $A$s as we like. But, of course, this is not true if we reject SC when we use our $A$ resource in $A \rightarrow A$ we obtain $A$ but we used one $A$ hence we are left with the same number of $A$ resources.
So, according to this new metaphysical account of validity, one ought to keep track of how many times they state a premise and how many they discharge. If someone had just proven the Riemann hypothesis and was presenting its implications they would have to state the Riemann hypothesis’ truth, as well as any other premise, hundreds of times before presenting all the implications, which everyone, except maybe Beall and Murzi, would find pedantic.

Even if such a conception of validity is conceivable and we cannot from a single-A resource obtain a double-A resource, it can be argued it is possible to define some other concept which behaves like our natural understanding of validity and so satisfies SC.

Beall and Murzi consider this point for the case of connectives contraction. They argue conditionals do not contract as in [RC] and no other connective does either. If there was one which did, say +, Curry’s paradox would arise again for \( a \iff (a + b) \). Thus, to avoid the paradox they reject such a connective exists. They could also argue that any concept satisfying [SC] is meaningless.

To conclude, accepting the dialetheic solution forces us to accept \( \bot \) unless we reject \( \exists \) by rejecting one of the rules of inference used. I ruled out rejecting MP and CP so the only possibility is to reject [SC] and [RC] which forces us to change our metaphysical account of validity and accept that any paradox-free language is pedantic and has no contractible concepts. Even if this is an acceptable price to pay to solve the paradoxes, the Antinomy of the Liar and Curry’s paradox have very similar structures so it should be possible to generalise their solutions to solve both and any of the same kind, since otherwise we risk the appearance of another paradox for which the specific solutions do not apply. Thus, the dialatheic solution is plausible but since it does not allow for a general solution we might want to reject it.
References

[1] Jc Beall and Julien Murzi. “Two Flavors of Curry’s Paradox”. In: Journal of Philosophy 110.3 (2013), pp. 143–165.

[2] Graham Priest. Doubt truth to be a liar. Clarendon, 2009, pp. 541–544.

[3] Helena Jorquera Riera. “The Antinomy of the Liar” (2022), p. 1.