R-Matrix Analysis of Structures in Economic Indices: from Nuclear Reactions to High-Frequency Trading

Frank W K Firk
The Henry Koerner Center for Emeritus Faculty, Yale University, 149 Elm St. New Haven, CT 06520–8368, USA
E-mail: fwkfirk@aol.com

Abstract. It is shown that the R-matrix theory of nuclear reactions is a viable mathematical theory for the description of the fine, intermediate and gross structure observed in the time-dependence of economic indices in general, and the daily Dow Jones Industrial Average in particular. A Lorentzian approximation to R-matrix theory is used to analyze the complex structures observed in the Dow Jones Industrial Average on a typical trading day. Resonant structures in excited nuclei are characterized by the values of their fundamental strength function, (average total width of the states)/(average spacing between adjacent states). Here, values of the ratios (average lifetime of individual states of a given component of the daily Dow Jones Industrial Average)/(average interval between the adjacent states) are determined. The ratios for the observed fine and intermediate structure of the index are found to be essentially constant throughout the trading day. These quantitative findings are characteristic of the highly statistical nature of many-body, strongly interacting systems, typified by daily trading. It is therefore proposed that the values of these ratios, determined in the first hour-or-so of trading, be used to provide valuable information concerning the likely performance of the fine and intermediate components of the index for the remainder of the trading day.

1. Introduction
A basic parameter in systems that exhibit energy- or time-dependent resonant-like structures, both quantum and classical, is the ratio (average width of the individual states)/(average spacing of the states), denoted by <Γ>/<D> (in energy units) or the ratio (average lifetime of the individual states)/(average time interval between states), here denoted by <Δt>/<T> (in time-units). If <Γ>/<D> < 1, the parameters that characterize the individual states can be determined. In quantum mechanical systems, resonant structures are analyzed using the R-matrix theory of Wigner and Eisenbud [1]; it is an exact theory with parameters that are understood at a fundamental level. In the present analysis, the (classical) structures observed in the daily Dow Jones Industrial Average (DJIA) are analyzed using Lorentzian forms as good approximations to the complex R-matrix forms.

As discussed previously by the author [2], a striking similarity is observed between the forms of the fine, intermediate, and gross structure of photonuclear states in the Giant
Electric Dipole Resonance region of medium-mass nuclei, and the fine, intermediate, and gross structure observed in the fluctuations (states) of daily economic indices.

In nuclei, the gross, intermediate and fine structures are attributed to the evolution of nuclear reactions from the underlying single-particle states to few-particle-few-hole states to many-particle-many-hole compound-nuclear states, respectively. In the fluctuations of an economic index, the three structural forms are associated with long, medium, and short-range time-correlations in the complex trading options that take place among the innumerable elements that contribute to the index. The fine structure is a characteristic feature of High-Frequency Trading (HFT).

Traditional trading strategies lack a dynamical model of the mathematical structure associated with a typical economic index; the lifetime of each state is of critical importance.

2. A Time-Dependent Lorentzian: the Breit-Wigner resonance form

In a seminal paper, Thomas [3] developed the exact R-matrix theory of resonant quantum states using arguments that were of practical importance in the analysis of observed spectra. His method has formed the basis of a great deal of the subsequent work in the field of nuclear spectroscopy.

The Thomas R-function is

\[ R_{c'c''} = \sum_{\lambda} \gamma_{\lambda} \gamma_{\lambda}^* / (E - E_{\lambda} - i\Gamma_{\lambda}^e/2), \]

where the \( \gamma \)'s are reduced width amplitudes for the levels \( \lambda \) and the channels \( c', c'' \) and \( \Gamma_{\lambda}^e \) is the width of the “eliminated” channels (in general, the unknown reaction channels) . At sufficiently low neutron energies, equation (1) leads to the following concise form for the neutron total cross section in the region of a single level \( \lambda \),

\[ \sigma_{\text{tot}} = \sigma'/(1 + x^2) + 2ka\sigma x/(1 + x^2) + 4\pi a^2 \] (the Breit-Wigner equation),

where \( x = (2/\Gamma)(E - E_{\lambda}) \), \( E \) is the incident neutron energy, \( E_{\lambda} \) is the resonance energy, \( \Gamma \) is the total width of the resonance (\( \Gamma = \Gamma_{\lambda} + \Gamma_{\lambda}^\gamma(= \Gamma_{\lambda}^e) \)), the sum of the elastic scattering width and the radiation width, \( \sigma \) (the peak cross section when on resonance), \( k \) is the neutron wave number and \( a \) is the effective nuclear radius.

The structures of typical daily economic indices, reported in 10-second intervals, do not have the richness of form observed in high-resolution studies of low-energy neutron-nucleus interactions. An approximate form of the R-matrix theory is therefore justified in the analysis of an economic index such as the daily DJIA.

The neutron total cross section given in equation (2) has a leading term \( \sigma'/(1 + x^2) \); it is a Lorentzian function in the energy domain. To model the structure in the daily DJIA the following time-dependent form is used:

\[ lzn(X) = M_0/(1 + X^2) \]

where \( X = (2/\Delta t)(t - t_0) \), \( t \) is the time variable, \( t_0 \) is the central time of the symmetric function, \( M_0 \) is the maximum value of the function, and \( \Delta t \) is the full width at half maximum of the function.

3. An Analysis of the DJIA data on April 3, 2012

We are concerned with an analysis of the DJIA reported on April 3, 2012; the analysis requires an understanding of the basic components of the index; they are:

1. Gross structure with a characteristic time-scale of many hours,
2. Intermediate structure that appears in two distinct patterns: I, with an average adjacent interval of about 60 minutes and II, with an average adjacent interval of about 10 minutes,
3. Fine structure with a characteristic time-scale of 1- to 2-minutes. It is this component that is associated with HFT.

In a limited interval, the two intermediate and the fine structure components are illustrated in figure 1:
Figure 1. Intermediate components and the fine structure component required to analyze the DJIA. Data are shown in 30-second intervals. The average intervals of the states are $<T>_I \approx 55$ min, $<T>_II \approx 12$ min, and $<T>_F \approx 110$ sec.

3.1. Intermediate structure I
An iterative, least-squares method is used to analyze the overlapping Lorentzian states. The procedure is continued until a satisfactory fit is obtained. The value of $<\Delta t>/<T>_I \approx 0.6$. The small number of intermediate I states during the trading day limits the accuracy of this value.

3.2. Intermediate structure II
The value of $<T>_II$ for the intermediate II structure is obtained by averaging over the entire period of the trading day; it is $<T>_II = 12$ minutes. An iterative fit to the data gives a value of $<\Delta t>/<T>_II \approx 0.6$.

3.3. Fine structure
The fit to the fine structure in the time interval 2:45 to 3:45 p.m. is shown in figure 2.

![Fine structure fit](image)

Figure 2. The thin line illustrates the fit to the fine structure data from 2:45 to 3:45 p.m.
The value of the strength function for these states is \( \langle \Delta t \rangle / \langle T \rangle = 0.52 \).

4. Conclusions
An analysis of the DJIA on April 3, 2012 has been carried out using multi-state Lorentzian functions to model the index. The HFT states are excellent examples of discrete states with spacing and lifetime distributions that are analogous to the statistical distributions that characterize low energy neutron resonance reactions (Porter [4]). The parameters of the defining Lorentzians are used to obtain the fundamental ratios \( \langle \Delta t \rangle / \langle T \rangle \) of the basic components of the index; they are found to be remarkably constant throughout a given trading day. These characteristic constants are associated with the statistical nature of daily trading. The HFT strength function is largely unaffected by the uncontrollable, time-dependent market forces that can have such dramatic effects on the value of an economic index when studied over periods of many hours or days. The value of the fine structure strength function depends upon the average load, bandwidth, processing capacity, and delays in buy and sell orders (the latency) associated with HFT on a given day.

It is proposed that the values of \( \langle \Delta t \rangle / \langle T \rangle \) for the fine and intermediate structure of the states of the DJIA, on a given day, be obtained by analyzing the data in the first one- to two-hours of trading, and that these values be used as predictors of their values for the remaining trading time of the day. This procedure assumes that there is sufficient statistical inertia in the fine and intermediate structure associated with trades on the given day; the present analysis provides support for this assumption.

References
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