Bayes metaclassifier and Soft-confusion-matrix classifier in the task of multi-label classification.

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Summary
The aim of this paper was to compare soft confusion matrix approach and Bayes metaclassifier under the multi-label classification framework. Although the methods were successfully applied under the multi-label classification framework, they have not been compared directly thus far. Such comparison is of vital importance because both methods are quite similar as they are both based on the concept of randomized reference classifier. Since both algorithms were designed to deal with single-label problems, they are combined with the problem-transformation approach to multi-label classification. Present study included 29 benchmark datasets and four different base classifiers. The algorithms were compared in terms of 11 quality criteria and the results were subjected to statistical analysis.

KEYWORDS:
multi-label classification, soft confusion matrix, Bayes metaclassifier

1 INTRODUCTION

Under the traditional supervised classification framework, the object is assigned to only one class. However, many real-world datasets contain objects that can be classified into several different categories. All these categories sum up to full description of the object and whenever one of them is missing, the information is incomplete. For example, the same image can be described using various tags, such as sea, beach and sunset. Such classification process is referred to as multi-label (ML) classification.

In the last 15 years, multi-label learning has found a number of practical applications, including text classification, multimedia classification, and bioinformatics, to mention a few.

Multi-label classification algorithms can be generally divided into two main groups, i.e. dataset transformation and algorithm adaptation approaches.

The algorithm adaptation approach represents a generalization of an existing multi-class algorithm, in which the generalized algorithm can be used to solve a multi-label classification problem directly. The best known approaches from this group include multi-label nearest neighbors algorithm, ML Hoeffding trees, structured output support vector machines and deep-learning-based algorithms.

In the dataset transformation approach, a multi-label problem is decomposed into a set of single-label classification tasks. During the inference phase, the outputs of the underlying single-label classifiers are combined into a multi-label prediction. One example of the dataset transformation approach is binary relevance (BR) approach in which a multi-label classification task is decomposed into a set of one-vs-rest binary classification problems. This algorithm is based on the assumption that labels are conditionally independent, which does not occur too often in the case of most real-life recognition problems. Despite this, the BR framework is still one of the most widespread multi-label classification methods, due to its excellent scalability and...
acceptable classification quality. However, the method can be easily outperformed by algorithms that are adjusted for mutual relationships between labels. An alternative technique to decompose a multi-label classification task into a set of binary classifiers is the label-pairwise (LPW) scheme. In this approach, each pair of labels are assigned with a one-vs-one binary classifier. The outcome of the classifier is interpreted as an expression of pairwise preference in a label ranking. Unlike the previously mentioned decomposition technique, the pairwise method is adjusted for paired inter-label dependencies. Contrary to the BR approach, during this type of decomposition a substantially larger number of base classifiers are generated and need to be built. In general, the transformed datasets are less imbalanced than those obtained as a result of one-vs-rest transformation. Moreover, the models created with the base classifiers obtained using this method tend to be simpler than those based on one-vs-rest classifiers.

The primary aim of this study was to compare soft-confusion-matrix approach (SCM) and Bayes metaclassifier (BMC) algorithm under the framework of multi-label classification. Specifically, both techniques were tested for BR and LPW decomposition transformations. Briefly, SCM and BMC algorithms are overlays that can be placed on the top of any base classifier. Moreover, they both derive from the RRC classifier idea introduced by Woloszynski. While the SCM and BMC algorithms are constructed using similar ideas, to the best of our knowledge, they have not been compared directly thus far. Therefore, we searched for potential differences between these two approaches.

The concept of the soft confusion matrix was first introduced in a study aimed at improving the classification quality of systems that recognize hand gestures. In that study, the SCM-based system was used due to its ability to utilize soft class-assignment. Moreover, the system’s potential to improve the response of base classifiers was considered an important argument to use this solution. Also, the soft confusion matrix-based approach was employed under a multi-label classification framework to improve the quality of binary relevance classifiers. While the study confirmed validity of this approach, it also demonstrated its sensitivity in the case of unbalanced class distribution in a binary problem. The algorithm was also used in research on LPW decomposition. Those experiments also showed that the use of the model contributed to a substantial improvement of the outcome of the committee built using the one-vs-one approach.

The concept of Bayes metaclassifier was first introduced in. The authors of that study presented in detail and validated the concept of BMC algorithm. The study demonstrated that the upper bound of the BMC improvement over base classifier was a Bayes error. BMC performance was shown to be directly related to the classification quality of a base classifier. Moreover, the experiments involving benchmark datasets showed that BMC significantly improved base classifier classification, especially whenever the chosen base classifier was far from optimum. Additionally, analysis of the results from each cross-validation phase demonstrated that BMC contributed to a decrease in the classification variance when compared with its basic counterpart results. BMC was shown to perform better with balanced datasets. However, if the a priori probabilities of the BMC design are used to solve imbalanced problems, its decision boundary is moved towards a majority class. Since BMC provides probabilistic interpretation for any base classifier response, this method was also used to address sequential classification problems, and was shown to be a useful tool for constructing multi-classifier systems, especially during classifier fusion.

This paper is organized as follows. Section provides formal notation used throughout the article, and introduces the SCM correction algorithm and Bayes metaclassifier. Section contains a description of the experimental setup. In section the results of the study are presented and discussed, and final conclusions are presented in the section.

2 | METHODS

2.1 | Preliminaries

In this section, a single-label classification is introduced. The scope is reduced to only binary classification because only binary classifiers are considered in this paper. As stated in the Introduction, in the single-label classification approach, a $d$-dimensional object $x = [x_1, x_2, \ldots, x_d] \in \mathbb{X} = \mathbb{R}^d$ is assigned a class $m \in \mathbb{M}$, where $\mathbb{M} = \{0, 1\}$ is an output space (a set of available classes). The single-label, binary classifier $\psi : \mathbb{X} \mapsto \mathbb{M}$ is an approximation of an unknown mapping $f : \mathbb{X} \mapsto \mathbb{M}$ which assigns the classes to the instances. The classification methods analyzed in this paper follow the statistical classification framework. Hence, a feature vector $x$ and its label $m$ are assumed to be realisations of random variables $\mathbf{X}$ and $\mathbf{M}$, respectively. The random variables follow the joint probability distribution $P(\mathbf{X}, \mathbf{M})$. Given the loss function $l : \mathbb{M} \times \mathbb{M} \mapsto \mathbb{R}^+ \cup \{0\}$, assessing the similarity of the objects in the output space, the optimal prediction $\psi^*(x)$ for the object $x$ can be calculated as follows:

$$
\psi^*(x) = \arg\min_{k \in \mathbb{M}} \sum_{m \in \mathbb{M}} l(k, m)P(M = m | X = x).
$$
If the loss function $l$ is the zero-one loss, the optimal decision is made using the maximum a posteriori rule:

$$\psi^*(x) = \arg\max_{k \in \mathcal{M}} P(m = k | x = x),$$

where $P(m = k | x = x)$ is the conditional probability that the object $x$ belongs to class $k$.

In this paper, the so-called soft output of the classifier $v : \mathcal{X} \mapsto [0, 1]^L$ is also defined. The soft output vector $v$ contains values proportional to the conditional probabilities. Consequently, the following conditions need to be satisfied:

$$v_i \approx P(m = i | x = x),$$

$$v_i(x) \in [0, 1],$$

$$\sum_{i=0}^{1} v_i(x) = 1.$$  

In this study, a classifier $\psi$ is built in a supervised learning procedure using the training set $\mathcal{T}$ containing $|\mathcal{T}|$ pairs of feature vectors $x$ and corresponding labels $m$:

$$\mathcal{T} = \{(x^{(1)}, m^{(1)}), (x^{(2)}, m^{(2)}), \ldots, (x^{(|\mathcal{T}|)}, m^{(|\mathcal{T}|)})\},$$

where $x^{(k)} \in \mathcal{X}$ and $m^{(k)} \in \mathcal{M}$. To evaluate the classifier the validation set $\mathcal{V}$ was used as well.

### 2.2 Multi-label Classification

Under the Multi-label formalism, an object $x$ is assigned to a set of labels indicated by a binary vector of length $L$: $y = [y_1, y_2, \ldots, y_L] \in \mathcal{Y} = \{0, 1\}^L$, where $L$ denotes the number of labels. Each element of the binary vector corresponds to a single label. If for some object $x$ the element of the vector $y_i$ is set to 1 (0), this means that the label associated with $i$-th position is relevant (irrelevant) to object $x$. Relevant labels are assigned to instances by an unknown mapping $g : \mathcal{X} \mapsto \mathcal{Y}$. A multi-label classifier $h : \mathcal{X} \mapsto \mathcal{Y}$ is an approximation of the unknown mapping.

The classification process often consists of two steps. During the first step, the classifier produces soft outputs $\omega : \mathcal{X} \mapsto \mathbb{R}^L$. Then the classifier’s outcome is generated using the thresholding procedure:

$$h(x) = \left[\left[\omega_1(x) \geq \Theta_1\right], \left[\omega_2(x) \geq \Theta_2\right], \ldots, \left[\omega_L(x) \geq \Theta_L\right]\right],$$

where $[\cdot]$ is the Iverson bracket and $\Theta_i$ is a label-specific threshold that may be found using various thresholding strategies.

As mentioned above, as a result of BR transformation, a separate binary classifier is obtained for each label. Hence, the BR ensemble consists of $L$ binary classifiers:

$$C_{BR} = \{\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(L)}\}.$$  

The output of the multi-label classifier is obtained as follows:

$$h_1(x) = \psi^{(i)}(x).$$

The label-pairwise (LPW) transformation produces multi-label classifier $h$, using an ensemble of binary classifiers $\Psi$, and then, a single binary classifier is assigned to each pair of labels:

$$C_{LPW} = \{\psi^{(i,j)} | i, j \in \{1, 2, \ldots, L\}, i < j\},$$

$$\psi^{(i,j)}(x) \in \{i, j\},$$

$$|C_{LPW}| = \frac{L(L-1)}{2}. $$

The soft output of the LPW ensemble is obtained by combining the outcomes of the base classifiers:

$$\omega_j(x) = \frac{1}{L-1} \sum_{i \in 1, 2, \ldots, L, i < j} \left[\psi^{(i,j)}(x) = 1\right].$$

The soft output is then converted into a binary response using a thresholding procedure.
2.3 Randomized Reference Classifier

In the hereby presented approaches, the behaviour of a base classifier $\psi$ was modeled using a stochastic classifier defined by a probability distribution over the set of labels $\mathcal{M}$. In this study, the randomized reference classifier (RRC) proposed by Woloszynski and Kurzynski\textsuperscript{13} was used. The RRC is a hypothetical classifier that allows a randomised model of a given deterministic classifier to be built.

We assumed that for a given instance $x$, the randomised classifier $\psi^{(R)}$ generates a vector of class supports $[v_1(x), v_2(x)]$ being observed values of random variables $[\Delta_1(x), \Delta_2(x)]$. The chosen probability distribution of random variables needs to satisfy the following conditions:

\begin{align}
\Delta_1(x), \Delta_2(x) & \in (0, 1), \\
\Delta_1(x) + \Delta_2(x) & = 1, \\
E[\Delta_i(x)] & = v_i(x), \ i \in \{0, 1\},
\end{align}

where $E$ is the expected value operator. Conditions (14) and (15) follow from the normalisation properties of class supports, whereas condition (16) provides the equivalence of the randomized model $\psi^{(R)}$ and base classifier $\psi$. Based on the latter condition, the RRC can be used to provide a randomised model of any classifier that returns a vector of class-specific supports $v(x)$.

The probability of classifying an object $x$ into the class $i$ using the RRC can be calculated from the following formula:

$$P(\Psi = m|X = x) = Pr[\Delta_m(x) > \Delta_{\{0,1\}\setminus m}(x)],$$

where $Pr[\Delta_m(x) > \Delta_{\{0,1\}\setminus m}(x)]$ is the probability that the value obtained by the realisation of random variable $\Delta_m$ is greater than the realisation of random variable $\Delta_{\{0,1\}\setminus m}$. The key step in the modeling process presented above is selection of the probability distributions for random variables $\Delta_i(x), \ i \in \{0, 1\}$ that satisfy the conditions (14)-(16). In this study, in line with the recommendations given in\textsuperscript{13}, the beta distribution with parameters $\lambda_i(x), \mu_i(x), \ i \in \{0, 1\}$ was applied. The parameters were chosen based on the following set of equations:

$$\frac{\lambda_i(x)}{\lambda_i(x) + \mu_i(x)} = v_i(x),$$

$$\lambda_i(x) + \mu_i(x) = 2.$$

The rationale for the choice of beta distribution based on the theory of order statistics can be found in\textsuperscript{13} along with a detailed description of the estimation parameters. For the beta distribution, the following formula for probability (17) was obtained:

$$P^{(R)}(\Psi = m|X = x) = \int_0^1 b(u, \lambda_m(x), \mu_m(x))B(u, \lambda_j(x), \mu_j(x)) \, du, \ j \neq m,$$

where $B(\cdot)$ is a beta cumulative distribution function and $b(\cdot)$ is a beta probability density function. It needs to be stressed that no validation set is required to calculate the probabilities (19), as knowledge of the correct classification of object $x$ is not a must. The MATLAB implementation of the RRC classifier is freely available at\textsuperscript{1}. Implementation for WEKA is also available\textsuperscript{2}.

2.4 Soft-confusion Matrix Classifier

The proposed correction method is based on an assessment of the probability of classifying an object $x$ into the class $s \in \mathcal{M}$ using the binary classifier $\psi$. It also provides an extension of the Bayesian model in which the object’s description $x$ and its true label $m \in \mathcal{M}$ are realizations of random variables $X$ and $M$, respectively. In the SCM approach, classifier $\psi$ predicts randomly based on the probabilities $P(\Psi(x) = s) = P(s|x)\textsuperscript{25}$. Hence, the outcome of the classification $s$ is a realization of the random variable $\Psi(x)$.

According to the extended Bayesian model, the posterior probability $P(m|x)$ of label $m$ can be defined as:

$$P(m|x) = \sum_{s \in \mathcal{M}} P(s|x)P(m|s, x).$$

\textsuperscript{1}http://www.mathworks.com/matlabcentral/fileexchange/28391-a-probabilistic-model-of-classifier-competence
\textsuperscript{2}https://github.com/ptrajdos/rrcBasedClassifiers/tree/develop
TABLE 1 The confusion matrix for a binary classification problem.

| m = 1 | s = 1 | s = 2 |
|-------|-------|-------|
| true  | $\epsilon_{1,1}$ | $\epsilon_{1,2}$ |
| m = 2 | $\epsilon_{2,1}$ | $\epsilon_{2,2}$ |

where $P(m|s, x)$ denotes the probability that an object $x$ belongs to the class $m$ given that $\Psi(x) = s$.

Unfortunately, the assumption that base classifier assigns labels in a stochastic way is rather impractical, since most real-life classifiers are deterministic. This issue was addressed by implementation of deterministic binary classifiers in which their statistical properties were modelled using the RRC procedure, as described in section 2.3.

2.4.1 Confusion Matrix

During the inference phase, the probability $P(m|s, x), s \in \mathcal{M}$ was estimated using a local, soft confusion matrix. An example of such a matrix for a binary classification task is given in Table 1. The rows of the matrix correspond to the ground-truth classes, whereas the columns match the outcome of the classifier. The confusion matrix is considered soft because the decision regions of the random classifier are expressed in terms of fuzzy set formalism. Thus, the membership function of a point $x$ is proportional to the probability of assigning $x$ to a given class using the randomized model of the classifier.

Based on subsets of the validation set that contain object belonging to class $m$, the fuzzy decision region of $\psi$ and the neighborhood of $z$ are defined according to the formulas:

$$
\mathcal{V}_s = \{(x^{(k)}, s^{(k)}, 1) : (x^{(k)}, s^{(k)}) \in \mathcal{V}, s^{(k)} = s\},
$$

$$
\mathcal{D}_s = \{(x^{(k)}, s^{(k)}, \mu_{D_s}(x^{(k)})) : (x^{(k)}, s^{(k)}) \in \mathcal{V}\},
$$

$$
\mathcal{N}(z) = \{(x^{(k)}, s^{(k)}, \mu_{\mathcal{N}_z}(x^{(k)})) : (x^{(k)}, s^{(k)}) \in \mathcal{V}\},
$$

where each triplet $(x^{(k)}, s^{(k)}, \zeta)$ defines the fuzzy membership value $\zeta$ of instance $(x^{(k)}, s^{(k)})$, and $\mu_{D_s}(x) = P(R|s|x)$ indicates the fuzzy decision region of the stochastic classifier. Additionally, $\mu_{\mathcal{N}_z}(x)$ denotes the fuzzy neighbourhood of the instance $z$. The membership function of the neighbourhood is defined using the Gaussian potential function:

$$
\mu_{\mathcal{N}_z}(x^{(k)}) = \exp(-\beta \delta(z, x^{(k)})^2),
$$

where $\beta \in \mathbb{R}_+$ and $\delta(z, x^{(k)})$ is a distance function between two vectors from the input space $\mathcal{X}$.

The fuzzy sets defined above can be then employed to approximate the entries of the local confusion matrix:

$$
\hat{\epsilon}_{m,s}(z) = \frac{|\mathcal{V}_s \cap \mathcal{D}_m \cap \mathcal{N}(z)|}{|\mathcal{N}(z)|},
$$

where $|$ is the cardinality of a fuzzy set. Finally, the approximation of $P(s|m, x)$ is calculated as follows:

$$
P(m|s, x) \approx \frac{\hat{\epsilon}_{m,s}(z)}{\sum_{d \in \mathcal{M}} \hat{\epsilon}_{d,s}(z)}.
$$

2.5 Bayes metaclassifier–BMC

First, the probabilistic model of classification will be introduced. The probability distribution of $(X, M)$ is determined based on a priori class probabilities $p_m = P(M = m)$ and class-conditional density functions $f(x|m) = f_m(x)$.

The Bayes metaclassifier (BMC) $\psi^{BMC}$, represents the probabilistic generalization of any base classifier (27) which has the form of the Bayes scheme built over the classifier $\psi$. Thus, in $\psi^{BMC}$ approach, the decision is based on the maximum a posteriori probability rule:

$$
\psi^{BMC}(\psi(x) = s) = \arg\max_{k \in \mathcal{M}} \{p(k|\psi = s)\}.
$$

A posteriori probabilities $p(i|s) \equiv P(M = i|\psi(x) = s), i \in \mathcal{M}$ derive from the Bayes rule:

$$
P(M = i|\psi = s) = \frac{p_i p(s|i)}{\sum_j p_j p(s|j)},
$$
where probability \( p(s|i) \equiv P(\psi(x) = s | M = i) \) denotes class-dependent probability of erroneous (if \( s \neq i \)) or correct (if \( s = i \)) classification of an object \( x \) by the base classifier \( \psi \).

When the base classifier \( \psi \) is placed in a probabilistic frame defined by the BMC \( \psi^{BMC} \), a common probabilistic interpretation of any response of base classifiers is obtained, regardless of the design paradigm.

The key element in the BMC scheme (27) and (28) is the calculation of probabilities \( P(\psi(x) = s | M = i) \) at point \( x \), i.e. the class-dependent probabilities of correct classification and misclassification with base classifiers. Usually, such probabilities for a base deterministic classifier would be either 0 for misclassification or 1 for correct classification of a given \( x \). However, in this paper, an alternative method for approximating these probabilities was proposed, based on the original concept of a randomized reference classifier (RRC).

The RRC \( \psi^{RRC}(x) \) is a stochastic classifier defined by a probability distribution chosen in such a way, that RRC acts, on average, as a modeled base classifier. Under such assumption, the class-dependent probabilities of correct classification \( P_c(j|x) \) and misclassification \( P_m(j|x) \) can be calculated as an equivalent to the modeled base classifier:

\[
P(\psi(x) = s | i) \approx P(\psi^{RRC}(x) = s | i).	ag{29}
\]

In the computational procedure, the probabilities \( P(\psi(x) = s | i) \approx P(\psi^{RRC}(x) = s | i) \) (denoting that an object \( x \) belongs to class \( i \) given that \( \psi(x) = s \)) are calculated for each validation point included in the validation set. As these values are only known for discrete points from \( V \), to enable dynamic calculation of any new object \( x \) during classification, a neighborhood function is needed to describe how the probabilities at validation points affect the new \( x \). For the BMC algorithm, Gaussian potential function is used as a neighborhood function:

\[
P_c^{RRC}(j|x) = \frac{\sum_{x^{(k)} \in V \setminus \{j\} = j} P_c(j|x) \cdot \exp(-\beta \delta(x, x^{(k)}))^2)}{\sum_{x^{(k)} \in V \setminus \{j\} = j} \exp(-\beta \delta(x, x^{(k)}))^2)},
\[
P_m^{RRC}(j|x) = \frac{\sum_{x^{(k)} \in V \setminus \{j\} \neq j} P_m(j|x) \cdot \exp(-\beta \delta(x, x^{(k)}))^2)}{\sum_{x^{(k)} \in V \setminus \{j\} \neq j} \exp(-\beta \delta(x, x^{(k)}))^2)},
\]

where \( \beta \) value in equation (30) is a scaling factor which should be adjusted independently to classification problem. Similarly, \( a \ priori \) probabilities \( (p_l, l \in M) \) introduced in (28) are estimated using the validation set \( V \).

### 3 | EXPERIMENTAL SETUP

The study included two scenarios using BR transformation and LPW approach, respectively. Regardless the scenario, the following methods were compared:

1. Unmodified base classifiers,
2. Base classifiers combined with the Bayes metaclassifier approach,
3. Base classifiers combined with the SCM approach.

The following single-label classifiers were used during the study:

- J48 (C4.5) classifier,
- SVM classifier with radial kernel,
- Naive Bayes classifier,
- Nearest Neighbour classifier.

All the experimental code was implemented using WEKA. The base classifiers were also obtained from this framework. During the study, the parameters of the J48 algorithm were set to its defaults. For the naive Bayes classifier, the kernel estimator with the Gaussian kernel was used to calculate the probabilities. The parameters of the SVM classifier \( (C \in \{.001, 1, 2, \ldots, 10\}, \gamma \in \{.001, 1, 2, \ldots, 5\}) \) were tuned using grid search and threefold cross-validation. Also the number of the nearest neighbors was tuned using the threefold cross-validation. The number of neighbors was selected from the following values \( K \in \)
The Euclidean distance function was employed to choose the nearest neighbours. The $F_1$ criterion calculated for the minority class was used as the quality criterion for the tuning procedures. Other parameters of the base classifiers were set to their defaults.

The Bayes metaclassifier and the SCM-based algorithms were implemented in JAVA. The source code for the algorithms is available online. The size of the neighborhood, expressed as $\beta$ coefficient, was chosen using a threefold cross-validation procedure and the grid search technique. The search space was defined as follows:

$$\{ \beta = 2 + 0.9 \cdot i, \quad i \in \{0, 1, ..., 10\} \}.$$  

The conversion of soft outputs into binary responses was done using S-Cut algorithm with the number of cross-validation folds set at three. The thresholds and the size of the committee were chosen in a way that provided the best value of the $F_1$ criterion.

To evaluate the proposed methods, the following multi-label classification quality criteria were used:

- Hamming loss,
- Zero-one loss,
- Example based $FDR$, $FNR$, $F_1$,
- Macro-averaged $FDR$, $FNR$, $F_1$,
- Micro-averaged $FDR$, $FNR$, $F_1$.

In line with the recommendations of statistical significance of the results was verified using the two-step procedure. The first step was the Friedman test conducted for each quality criterion separately. Since multiple criteria were employed, the familywise errors (FWER) should be controlled. Thus, the Holm’s procedure was used to control the FWER of the Friedman tests. Whenever the Friedman test demonstrated a significant difference within the group of classifiers, the pairwise comparisons were conducted with the Wilcoxon signed-rank test. To control the FWER of the Wilcoxon tests, the Holm approach was employed. The level of statistical significance for all tests was set at $\alpha = 0.05$.

Table 2 presents the collection of the benchmark sets that were used during the experimental evaluation of the proposed algorithms. The table is organized as follows. The first column contains the names of the datasets. The names under which the datasets are registered in the repositories were used. The second column contains the numbers of the datasets preceded by the number of the table. Further columns contain the set-specific characteristics of the benchmark sets:

- The number of instances in the dataset ($|S|$),
- Dimensionality of the input space ($d$),
- The number of labels ($L$),
- Average number of labels for a single instance ($LC$),
- The number of unique label combinations ($LU$),
- Average between-labels imbalance ratio ($IR$).

The datasets are available online. During the preprocessing stage, the datasets underwent a few transformations. First, all nominal attributes, except binary attributes, were converted into a set of binary variables. This approach is one of the simplest methods to replace nominal variables with binary variables. The transformation is necessary whenever the SVM-based or distance-based algorithms are employed. The features were also normalized to have zero mean value and zero unit variance.

In this study, some datasets that follow multi-instance-multi-label (MIML) framework were employed. In these datasets, each object consists of a bag of instances tagged with a set of labels. To tackle these data, we followed the recommendation of transforming the set into single-instance multi-label data. The multi-target regression sets (datasets: were also harnessed. The datasets were converted into multi-label data, using a simple
thresholding procedure. Specifically, when the value of output variable for a given object was greater than zero, the corresponding label was set to be relevant to this object. The number of labels in the stackex_chess (2.26) dataset was reduced to 15, to reduce the computational burden. Moreover, features were selected using a correlation-based approach before the learning phase.

Both training and testing datasets were extracted using tenfold cross-validation. The validation set was essentially the same as the training set, but the base-classifier responses were obtained using twofold cross-validation.

### RESULTS AND DISCUSSION

To compare multiple algorithms on multiple benchmark sets, the average ranks approach was used. In this approach, the winning algorithm achieves rank equal ‘1’, the second achieves rank equal ‘2’, etc. In the case of ties, the ranks for algorithms that achieve the same results are averaged. The average ranks are visualized on radar plots. Visualization properties of the radar plots are similar to the properties of parallel coordinates plots. In other words, radar plots can be interpreted as parallel coordinates plots drawn in polar coordinate systems. In the plots, the data are visualized in such way that the lowest ranks are closer to the center of the graph. The radar plots illustrating the results of the present experiment are shown in FIGURE 1 and 2.

The numerical results are presented in Tables 3 – 10. Each table has the same structure. The first row contains numbers assigned to algorithms in section. Then the table is divided into eleven sections, each corresponding to a single evaluation criterion. The first row of each section is the name of the analyzed quality criterion. The second row contains p-value for the Friedman test, whereas the average ranks for the algorithms are shown in the third row. Further rows contain p-values for the pairwise Wilcoxon tests. The p-value equal to 0.000 depicts p-values lower than $10^{-3}$ and p-value equal to 0.999 corresponds to p-values greater than 0.999.
Complete results of the study are available online.

### 4.1 Binary Relevance

The results related to the binary relevance transformation are presented in figure 1 and tables 3 – 6. The results seem to be partially inconclusive. While no statistically significant between-method differences in all base classifiers and quality criteria were found on the Friedman tests, the significant method-related differences were observed between some classifiers on the post-hoc Wilcoxon tests. A few trends can be observed in the analyzed data. First, regardless the base classifier, both methods had the same average ranks for macro-averaged FDR, macro and micro-averaged $F_1$ measures. Moreover, for macro-averaged $F_1$ criterion, the post-hoc test demonstrated that for all base classifiers except KNN, the Bayes metaclassifier outperformed the reference approach. Also for macro-averaged FDR, the Bayes metaclassifier significantly outperformed the reference method for three out of four classifiers (except the naive Bayes classifier). These findings imply that the Bayes metaclassifier provided better classification quality for rare labels. Regarding the macro-averaged measures, based on the average ranks, the SCM-based classifier seems to be slightly less conservative than the Bayes metaclassifier. However, the differences between the two approaches were statistically significant only in the case of SVM base classifier.

No consistent conclusions can be formulated from the example-based quality criteria. The order of investigated classifiers (according to their average ranks) varied depending on the base classifier. Also, the results of the post-hoc tests were inconsistent. For these quality criteria, no significant differences were found between the investigated classifiers. Furthermore, no evident trend was observed in the results for the micro-averaged criteria.

### TABLE 3 Binary Relevance transformation. Wilcoxon test for J48 base classifiers – p-values for paired comparisons of investigated methods.

| Nam. | 1     | 2     | 3     | 1     | 2     | 3     | 1     | 2     | 3     | 1     | 2     | 3     |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Frd. | Hamming | Zero-One | ExFDR | ExFNR |
|      | 1.00e+00 | 1.00e+00 | 1.00e+00 | 9.677e-01 |
| Rank | 1.983 | 1.879 | 2.138 | 2.190 | 1.983 | 1.828 | 2.086 | 1.741 | 2.172 | 2.069 | 2.207 | 1.724 |
| 1    | 0.741 | 0.138 | 0.548 | 0.785 | 0.723 | 1.000 | 0.933 | 0.395 |       |       |       |       |
| 2    | 0.087 |       |       | 0.785 |       |       |       |       |       |       |       |       |
| Nam. | ExF1  | MaFDR | MaFNR | MaF1  |
| Frd. | 1.00e+00 | 4.658e-01 | 2.183e-01 | 4.559e-01 |
| Rank | 2.052 | 1.776 | 2.172 | 2.293 | 1.672 | 2.034 | 2.241 | 2.172 | 1.586 | 2.362 | 1.879 | 1.759 |
| 1    | 0.999 | 1.000 | 0.040 | 0.336 | 0.182 | 0.073 | 0.007 | 0.073 |       |       |       |       |
| 2    | 1.000 | 0.287 |       |       |       |       |       |       |       |       |       |       |
| Nam. | MiFDR | MiFNR | MiF1  |
| Frd. | 1.00e+00 | 1.277e-01 | 4.658e-01 |
| Rank | 2.052 | 1.914 | 2.034 | 2.241 | 2.207 | 1.552 | 2.328 | 1.948 | 1.724 |       |       |       |
| 1    | 1.000 | 1.000 | 0.464 | 0.036 | 0.094 | 0.101 |       |       |       |       |       |
| 2    | 1.000 | 0.287 |       |       |       |       |       |       |       |       |       |       |

### 4.2 Label Pairwise

The results for the label-pairwise transformation are shown in figure 2 and tables 7 – 10. The results seem to be consistent; whenever the p-value for the Friedman test was significant, at least one significant between-algorithm difference was found on the post-hoc Wilcoxon test.

The statistical analysis demonstrated that regardless the base classifier, the Bayes metaclassifier and the classifier based on the soft confusion matrix were more conservative than the reference method, i.e. provided significantly better results in terms
Table 4: Binary Relevance transformation. Wilcoxon test for SVM base classifiers – p-values for paired comparisons of investigated methods.

| Nam. | Frd. | 1  | 2  | 3  | 1  | 2  | 3  | 1  | 2  | 3  | 1  | 2  | 3  |
|------|------|----|----|----|----|----|----|----|----|----|----|----|----|
| Hamming | 9.579e-01 | 1.741 | 2.017 | 2.241 | 1.914 | 2.017 | 2.069 | 1.845 | 2.086 | 2.069 | 1.983 | 2.155 | 1.862 |
| Zero-One | 1.000e+00 | 0.108 | 0.005 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.776 | 0.693 | 0.570 |
| ExFDR | 1.000e+00 | 0.005 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.776 | 0.693 | 0.570 |
| ExFNR | 1.000e+00 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.776 | 0.693 | 0.570 |

Table 5: Binary Relevance transformation. Wilcoxon test for Naive Bayes base classifiers – p-values for paired comparisons of investigated methods.

| Nam. | Frd. | 1  | 2  | 3  | 1  | 2  | 3  | 1  | 2  | 3  | 1  | 2  | 3  |
|------|------|----|----|----|----|----|----|----|----|----|----|----|----|
| Hamming | 1.000e+00 | 1.948 | 2.017 | 2.034 | 2.259 | 1.672 | 2.069 | 2.397 | 2.052 | 1.552 | 2.397 | 1.983 | 1.621 |
| Zero-One | 1.000e+00 | 1.000 | 1.000 | 1.000 | 0.020 | 0.405 | 0.017 | 0.017 | 0.004 | 0.023 | 0.004 | 0.023 | 0.029 |
| ExF1 | 1.000e+00 | 0.005 | 1.000 | 1.000 | 0.020 | 0.405 | 0.017 | 0.017 | 0.004 | 0.023 | 0.004 | 0.023 | 0.029 |
| MaFDR | 1.000e+00 | 1.000 | 1.000 | 1.000 | 0.020 | 0.405 | 0.017 | 0.017 | 0.004 | 0.023 | 0.004 | 0.023 | 0.029 |
| ExFNR | 1.000e+00 | 1.000 | 1.000 | 1.000 | 0.020 | 0.405 | 0.017 | 0.017 | 0.004 | 0.023 | 0.004 | 0.023 | 0.029 |
| MaF1 | 1.000e+00 | 1.000 | 1.000 | 1.000 | 0.020 | 0.405 | 0.017 | 0.017 | 0.004 | 0.023 | 0.004 | 0.023 | 0.029 |

of the FDR (precision) criterion and significantly worse outcomes in terms of the FNR (recall) criterion. Consequently, the analyzed methods performed significantly better in terms of the $F_1$ measure. In other words, the proposed methods were more conservative as they identified fewer instances as relevant, but a larger proportion of the identified instances turned out to be truly relevant for the instance under classification. The results are consistent across all example-based, micro and macro-averaged criteria. This means that the results were better from the perspective of the whole label vector and each label separately, whether common or rare in the dataset. Moreover, the analyzed algorithms significantly outperformed the reference method in terms of the Hamming loss and the zero-one loss. The improvement in terms of the zero-one loss seems to be particularly important as this is the most strict quality criterion that can be used for quality assessment of multi-label classifiers. That is to say, the criterion assigns the loss equal to 1 if only a single label is misclassified. The results of our present study clearly show that unlike for the BR-based approach, the analyzed algorithms contributed to a substantial improvement of classification quality.
for the label-pairwise transformation. A reason behind the evident quality improvement for the label-pairwise transformation might be the fact that the datasets obtained by the LPW transformation were markedly less imbalanced than those obtained by the BR transformation. As a result, the neighborhood of a point was predominated by points belonging to the majority class. If the neighborhood is imbalanced, statistical properties of the base classifiers cannot be estimated appropriately.

No statistically significant differences were found between the Bayes metaclassifier and the classifier based on the confusion matrix. This is not so surprising since both analyzed algorithms are based on similar principles, namely, they both use a validation set to provide a probabilistic interpretation of a base classifier.

### TABLE 6 Binary Relevance transformation. Wilcoxon test for KNN base classifiers – p-values for paired comparisons of investigated methods.

| Nam. | Hamming | Zero-One | ExFDR | ExFNR |
|------|---------|----------|-------|-------|
| Frd. | 3.827e-01 | 8.242e-02 | 1.000e+00 | 1.000e+00 |
| Rank | 2.207 | 1.621 | 2.172 | 2.466 | 1.810 | 1.724 | 2.027 | 1.759 | 2.034 | 2.121 | 2.017 | 1.862 |
| 1 | 0.002 | 0.137 | 0.003 | 0.008 | 0.039 | 0.324 |
| 2 | 0.024 | 0.115 | 0.782 | 0.241 | 0.197 |

### TABLE 7 Pairwise transformation. Wilcoxon test for J48 base classifiers – p-values for paired comparisons of investigated methods.

| Nam. | Hamming | Zero-One | ExFDR | ExFNR |
|------|---------|----------|-------|-------|
| Frd. | 5.916e-09 | 7.088e-09 | 5.498e-09 | 4.744e-08 |
| Rank | 3.000 | 1.500 | 1.500 | 2.946 | 1.375 | 1.679 | 3.000 | 1.607 | 1.393 | 1.071 | 2.356 | 2.393 |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.884 | 0.243 | 0.479 | 0.126 |

| Nam. | ExF1 | MaFDR | MaFNR | MaF1 |
|------|------|-------|-------|------|
| Frd. | 5.916e-09 | 7.088e-09 | 5.498e-09 | 4.744e-08 |
| Rank | 3.000 | 1.500 | 1.500 | 2.946 | 1.375 | 1.679 | 3.000 | 1.607 | 1.393 | 1.071 | 2.356 | 2.393 |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.884 | 0.243 | 0.479 | 0.126 |

| Nam. | MiFDR | MiFNR | MiF1 |
|------|------|-------|------|
| Frd. | 5.916e-09 | 7.088e-09 | 5.498e-09 | 4.744e-08 |
| Rank | 3.000 | 1.500 | 1.500 | 2.946 | 1.375 | 1.679 | 3.000 | 1.607 | 1.393 | 1.071 | 2.356 | 2.393 |
| 1 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| 2 | 0.884 | 0.243 | 0.479 | 0.126 |
CONCLUSIONS

In this study, we compared Bayes metaclassifier and soft-confusion-matrix-based classifier. The classifiers were compared with each other and with the reference method under the multi-label classification framework. Specifically, the classifiers were compared using problem-transformation approach to multi-label learning. Two most common transformation methods were used: binary relevance and label-pairwise.

The study showed that:
TABLE 8 Pairwise transformation. Wilcoxon test for SVM base classifiers – p-values for paired comparisons of investigated methods.

|       | 1       | 2       | 3       | 1       | 2       | 3       | 1       | 2       | 3       |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Nam.  | Hamming | Zero-One| ExFDR   | ExFNR   |
| Frd.  | 4.282e-09| 9.299e-09| 6.585e-09| 6.601e-08|
| Rank  | 3.000   | 1.643   | 1.357   | 2.964   | 1.518   | 1.518   | 3.000   | 1.464   | 1.536   |
|       | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   |
|       | 0.438   | 0.576   | 0.814   | 1.071   | 2.464   | 2.464   |
|       | 0.000   | 0.000   | 2.893   | 1.500   | 1.607   |

TABLE 9 Pairwise transformation. Wilcoxon test for Naive Bayes base classifiers – p-values for paired comparisons of investigated methods.

|       | 1       | 2       | 3       | 1       | 2       | 3       | 1       | 2       | 3       |
|-------|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Nam.  | Hamming | Zero-One| ExFDR   | ExFNR   |
| Frd.  | 4.710e-09| 7.641e-09| 4.710e-09| 4.949e-08|
| Rank  | 3.000   | 1.643   | 1.357   | 2.964   | 1.607   | 1.429   | 3.000   | 1.643   | 1.357   |
|       | 0.000   | 0.000   | 0.000   | 0.000   | 0.000   | 0.782   | 0.000   | 0.000   | 0.000   |
|       | 0.508   | 0.782   | 0.386   | 0.920   |
|       | 0.567   | 0.245   | 0.955   | 0.218   |

- For the binary-relevance transformation, virtually no significant differences existed between the compared algorithms. While the Bayes metaclassifier seems to be slightly better than the reference method, no statistically significant differences were found between this classifier and the SCM-based method.

- For the label-pairwise transformation, both the Bayes metaclassifier and the SCM-based classifier significantly outperformed the reference method in terms of all quality criteria. The analyzed algorithms seem to be more conservative than the reference method. No significant differences were found between the Bayes metaclassifier and the SCM-based classifier.
**TABLE 10** Pairwise transformation. Wilcoxon test for KNN base classifiers – p-values for paired comparisons of investigated methods.

|       |      |      |       |      |      |       |      |      |
|-------|------|------|-------|------|------|-------|------|------|
|       | Hamming | Zero-One | ExFDR | ExFNR |
| Nam.  | 7.937e-08 | 6.152e-08 | 9.242e-08 | 2.475e-07 |
| Frd.  |       |       |       |       |
| Rank  | 2.929 1.643 1.429 | 2.911 1.393 1.696 | 2.929 1.500 1.571 | 1.107 2.500 2.393 |
| 1     | 0.000 0.000 | 0.000 0.000 | 0.000 0.000 | 0.000 0.000 |
| 2     | 0.508 | 0.319 | 0.745 | 0.339 |

|       |      |      |       |      |      |       |      |      |
|-------|------|------|-------|------|------|-------|------|------|
|       | ExF1 | MaFDR | MaFNR | MaF1 |
| Nam.  | 2.087e-06 | 8.048e-09 | 6.910e-07 | 2.145e-05 |
| Frd.  |       |       |       |       |
| Rank  | 2.821 1.607 1.571 | 3.000 1.536 1.464 | 1.143 2.357 2.500 | 2.714 1.679 1.607 |
| 1     | 0.000 0.000 | 0.000 0.000 | 0.000 0.000 | 0.000 0.000 |
| 2     | 0.728 | 0.236 | 0.955 | 0.374 |

|       |      |      |       |      |      |       |      |      |
|-------|------|------|-------|------|------|-------|------|------|
|       | MiFDR | MiFNR | MiF1 |
| Nam.  | 6.954e-08 | 9.242e-08 | 9.570e-06 |
| Frd.  |       |       |       |
| Rank  | 2.929 1.679 1.393 | 1.071 2.500 2.429 | 2.750 1.750 1.500 |
| 1     | 0.000 0.000 | 0.000 0.000 | 0.000 0.000 |
| 2     | 0.126 | 0.614 | 0.236 |
FIGURE 2 Average ranks of for Label Pairwise approach.
References

1. Gibaja Eva, Ventura Sebastián. Multi-label learning: A review of the state of the art and ongoing research. WIREs Data Mining Knowl Discov. 2014;4(6):411-444.

2. Jiang Jung-Yi, Tsai Shian-Chi, Lee Shie-Jue. FSKNN: Multi-label text categorization based on fuzzy similarity and k nearest neighbors. Expert Systems with Applications. 2012;39(3):2813-2821.

3. Sanden Chris, Zhang John Z.. Enhancing multi-label music genre classification through ensemble techniques. In: ACM Press; 2011.

4. Wu Jian-Sheng, Huang Sheng-Jun, Zhou Zhi-Hua. Genome-Wide Protein Function Prediction through Multi-Instance Multi-Label Learning. IEEE/ACM Trans. Comput. Biol. and Bioinf.. 2014;11(5):891-902.

5. Read Jesse, Bifet Albert, Holmes Geoff, Pfahringer Bernhard. Scalable and efficient multi-label classification for evolving data streams. Mach Learn. 2012;88(1-2):243-272.

6. Díez Jorge, Luaces Oscar, Coz Juan José, Bahamonde Antonio. Optimizing different loss functions in multilabel classifications. Prog Artif Intell. 2014;3(2):107-118.

7. Wei Yunchao, Xia Wei, Lin Min, et al. HCP: A Flexible CNN Framework for Multi-Label Image Classification. IEEE Trans. Pattern Anal. Mach. Intell.. 2016;38(9):1901-1907.

8. Alvares Cherman Everton, Metz Jean, Monard Maria Carolina. A Simple Approach to Incorporate Label Dependency in Multi-label Classification. In: Springer Berlin Heidelberg 2010 (pp. 33-43).

9. Tsoumakas Grigorios, Katakis Ioannis, Vlahavas Ioannis. Mining Multi-label Data. In: Springer US 2009 (pp. 667-685).

10. Read Jesse, Pfahringer Bernhard, Holmes Geoff, Frank Eibe. Classifier Chains for Multi-label Classification. In: Springer Berlin Heidelberg 2009 (pp. 254-269).

11. Fürnkranz Johannes. Round Robin Classification. Journal of Machine Learning Research. 2002;2(4):721–747.

12. Hüllermeier Eyke, Fürnkranz Johannes. On predictive accuracy and risk minimization in pairwise label ranking. Journal of Computer and System Sciences. 2010;76(1):49-62.

13. Woloszynski Tomasz, Kurzynski Marek. A probabilistic model of classifier competence for dynamic ensemble selection. Pattern Recognition. 2011;44(10-11):2656-2668.

14. Kurzynski Marek, Krysmann Maciej, Trajdos Pawel, Wolczowski Andrzej. Multiclassifier system with hybrid learning applied to the control of bioprosthetic hand. Computers in Biology and Medicine. 2016;69:286-297.

15. Trajdos Pawel, Kurzynski Marek. A dynamic model of classifier competence based on the local fuzzy confusion matrix and the random reference classifier. International Journal of Applied Mathematics and Computer Science. 2016;26(1).

16. Trajdos Pawel, Kurzynski Marek. An Extension of Multi-label Binary Relevance Models Based on Randomized Reference Classifier and Local Fuzzy Confusion Matrix. In: Springer International Publishing 2015 (pp. 69-76).

17. Trajdos Pawel, Kurzynski Marek. A Correction Method of a Binary Classifier Applied to Multi-label Pairwise Models. Int. J. Neur. Syst.. 2017;28(0).

18. Trajdos Pawel, Kurzynski Marek. Weighting scheme for a pairwise multi-label classifier based on the fuzzy confusion matrix. Pattern Recognit. Lett.. 2018;103:60-67.

19. Majak Marcin, Kurzynski Marek. On a New Method for Improving Weak Classifiers Using Bayes Metaclassifier. In: Springer International Publishing; 2018; Cham.

20. Kurzynski Marek, Majak Marcin, Zolnierak Andrzej. Multiclassifier systems applied to the computer-aided sequential medical diagnosis. Journal of Biocybernetics and Biomedical Engineering. 2016;36:619–625.
21. Kurzynski Marek, Majak Marcin. Meta-Bayes Classifier with Markov Model Applied to the Control of Bioprosthetic Hand. In: Springer International Publishing; 2016; Cham.

22. Knuth Donald E.. Two Notes on Notation. The American Mathematical Monthly. 1992;99(5):403.

23. Ioannou Marios, Sakkas George, Tsoumakas Grigoris, Vlahavas Ioannis. Obtaining Bipartitions from Score Vectors for Multi-Label Classification. 2010 22nd IEEE International Conference on Tools with Artificial Intelligence. 2010.

24. Yang Yiming. A study of thresholding strategies for text categorization. In: ACM Press; 2001.

25. Berger James O.. Statistical Decision Theory and Bayesian Analysis. Springer New York; 1985.

26. Zadeh L.A.. Fuzzy sets. Information and Control. 1965;8(3):338-353.

27. Dhar Mamoni. On Cardinality of Fuzzy Sets. IJISA. 2013;5(6):47-52.

28. Quinlan J. R.. C4.5 : Programs for machine learning. San Francisco, CA, USA: Morgan Kaufmann Publishers Inc.; 1993.

29. Cortes Corinna, Vapnik Vladimir. Support-vector networks. Mach Learn. 1995;20(3):273-297.

30. Chang Chih-Chung, Lin Chih-Jen. LIBSVM. TIST. 2011;2(3):1-27.

31. Hand David J., Yu Keming. Idiot’s Bayes: Not So Stupid after All?. International Statistical Review / Revue Internationale de Statistique. 2001;69(3):385.

32. Cover T., Hart P.. Nearest neighbor pattern classification. IEEE Trans. Inform. Theory. 1967;13(1):21-27.

33. Hall Mark, Frank Eibe, Holmes Geoffrey, Pfahringer Bernhard, Reutemann Peter, Witten Ian H.. The WEKA data mining software. SIGKDD Explor. News.. 2009;11(1):10.

34. Rijsbergen C. J. Van. Information Retrieval. Newton, MA, USA: Butterworth-Heinemann; 2nd ed.1979.

35. Luaces Oscar, Díez Jorge, Barranquero José, Coz Juan José, Bahamonde Antonio. Binary relevance efficacy for multilabel classification. Prog Artif Intell. 2012;1(4):303-313.

36. Demšar J.. Statistical comparisons of classifiers over multiple data sets. The Journal of Machine Learning Research. 2006;7:1–30.

37. Garcia Salvador, Herrera Francisco. An extension on“statistical comparisons of classifiers over multiple data sets”for all pairwise comparisons. Journal of Machine Learning Research. 2008;9:2677–2694.

38. Friedman Milton. A Comparison of Alternative Tests of Significance for the Problem of m Rankings. Ann. Math. Statist.. 1940;11(1):86-92.

39. Yekutieli Daniel, Benjamini Yoav. The control of the false discovery rate in multiple testing under dependency. Ann. Statist.. 2001;29(4):1165-1188.

40. Holm Sture. A Simple Sequentially Rejective Multiple Test Procedure. Scandinavian Journal of Statistics. 1979;6(2):65–70.

41. Wilcoxon Frank. Individual Comparisons by Ranking Methods. Biometrics Bulletin. 1945;1(6):80.

42. Tian Yingjie, Deng Naiyang. Support Vector Classification with Nominal Attributes. In: Hao Yue, Liu Jiming, Wang Yuping, et al., eds. Computational Intelligence and Security, Springer Berlin Heidelberg 2005 (pp. 586-591).

43. Zhou Zhi-Hua, Zhang Min-ling. Multi-instance multilabel learning with application to scene classification. In: ; 2007.

44. Zhou Zhi-Hua, Zhang Min-Ling, Huang Sheng-Jun, Li Yu-Feng. Multi-instance multi-label learning. Artificial Intelligence. 2012;176(1):2291-2320.

45. Hall Mark A. Correlation-based feature selection for machine learning. PhD thesisThe University of Waikato1999.

46. Ueda Naonori, Saito Kazumi. Parametric mixture models for multi-labeled text. In: :737–744; 2003.
47. Briggs Forrest, Lakshminarayan Balaji, Neal Lawrence, et al. Acoustic classification of multiple simultaneous bird species: A multi-instance multi-label approach. *The Journal of the Acoustical Society of America*. 2012;131(6):4640-4650.

48. Trohidis Konstantinos, Tsoumakas Grigorios, Kalliris George, Vlahavas Ioannis P.. Multi-Label Classification of Music into Emotions.. In: Bello Juan Pablo, Chew Elaine, Turnbull Douglas, eds. *ISMIR*, :325-330; 2008.

49. Klimt Bryan, Yang Yiming. *Introducing the Enron Corpus*. 2004.

50. Bache K., Lichman M.. *UCI Machine Learning Repository*. 2013.

51. Solar Flare. *Nature*. 1973;245(5426):431-431.

52. Diplaris Sotiris, Tsoumakas Grigorios, Mitkas Pericles A., Vlahavas Ioannis. Protein Classification with Multiple Algorithms. In: Springer Berlin Heidelberg 2005 (pp. 448-456).

53. Xu Jianhua. Fast multi-label core vector machine. *Pattern Recognition*. 2013;46(3):885-898.

54. Kong Xiangnan, Shi Xiaoxiao, Yu Philip S. Multi-label collective classification. In: :618–629SIAM; 2011.

55. Read Jesse, Pfahringer Bernhard, Holmes Geoff, Frank Eibe. Classifier chains for multi-label classification. *Machine Learning*. 2011;85(3):333–359.

56. Pestian John P., Brew Christopher, Matykiewicz Pawel, et al. A shared task involving multi-label classification of clinical free text. In: Association for Computational Linguistics; 2007.

57. Hersh William, Buckley Chris, Leone T. J., Hickam David. OHSUMED: An Interactive Retrieval Evaluation and New Large Test Collection for Research. *SIGIR ’94*. 1994::192–201.

58. Boutell Matthew R., Luo Jiebo, Shen Xipeng, Brown Christopher M.. Learning multi-label scene classification. *Pattern Recognition*. 2004;37(9):1757-1771.

59. Tomás Jimena Torres, Spolaôr Newton, Cherman Everton Alvares, Monard Maria Carolina. A Framework to Generate Synthetic Multi-label Datasets. *Electronic Notes in Theoretical Computer Science*. 2014;302:155-176.

60. Charte Francisco, Rivera Antonio J., Jesus Maria J., Herrera Francisco. QUINTA: A question tagging assistant to improve the answering ratio in electronic forums. In: IEEE; 2015.

61. Srivastava A.N., Zane-Ulman B.. Discovering recurring anomalies in text reports regarding complex space systems. In: IEEE; 2005.

62. Džeroski Sašo, Demšar Damjan, Grbović Jasna. *Applied Intelligence*. 2000;13(1):7-17.

63. Elisseeff André, Weston Jason. A Kernel Method for Multi-Labelled Classification. In: :681–687MIT Press; 2001.

64. Saary M. Joan. Radar plots: a useful way for presenting multivariate health care data. *Journal of Clinical Epidemiology*. 2008;61(4):311–317.

**How to cite this article:** Trajdos Pawel, Majak Marcin (2018), Bayes metaclassifier and Soft-confusion-matrix classifier in the task of multi-label classification, *Computational Intelligence, ?*. 
Bayes metaclassifier and Soft-confusion-matrix classifier in the task of multi-label classification.

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Summary
The aim of this paper is to provide a comparison of soft confusion matrix approach and Bayes metaclassifier under the multi-label classification framework. Although the methods were successfully applied under the multi-label classification framework, there is a lack of direct comparison of these methods. The comparison is vital because both methods are quite similar since they are both based on the concept of the randomised reference classifier. Since both of these algorithms are designed to deal with single-label problems, they are combined with the problem-transformation approach to multi-label classification. The experimental study is conducted using 29 benchmark datasets and four different base classifiers. The algorithms were compared in terms of 11 quality criteria backed up with thorough statistical analysis.

KEYWORDS:
multi-label classification, soft confusion matrix, bayes metaclassifier

1 INTRODUCTION

Under the traditional, supervised classification framework, an object is assigned to only one class. However, many real-world datasets contain objects that are assigned to different categories at the same time. All of these concepts constitute a full description of the object and the omission of one of these tags induces a loss of information. For example, an image may be described using such tags as sea, beach, and sunset. The classification process in which such kind of data is involved is called multi-label (ML) classification. In the last 15 years, multi-label learning has been employed in a wide range of practical applications, including text classification, multimedia classification, and bioinformatics, to name only a few.

Multi-label classification algorithms can be broadly partitioned into two main groups i.e. dataset transformation and algorithm adaptation approaches.

The latter provides a generalisation of an existing multi-class algorithm, where the generalised algorithm is able to solve the multi-label classification problem in a direct way. Among this group, the most known approaches are: multi label Nearest Neighbours algorithm, the ML Hoeffding trees, the structured output support vector machines, or deep-learning-based algorithms.

The methods from the former group decompose a multi-label problem into a set of single-label classification tasks. During the inference phase, outputs of the underlying single-label classifiers are combined in order to create a multi-label prediction. An example of this is the binary relevance (BR) approach that decomposes a multi-label classification task into a set of one-vs-rest binary classification problems. This algorithm assumes that labels are conditionally independent, what is not so common in most real-life recognition problems. BR framework is still considered to be one of the most widespread multi-label classification methods, due to its excellent scalability and acceptable classification quality. However, the method can be easily outperformed
by algorithms that consider dependencies between labels\textsuperscript{7,8}. An alternative technique of decomposing the multi-label classification task into a set of binary classifiers is the label-pairwise (LPW) scheme\textsuperscript{9}. Under this framework, each pair of labels is assigned with a one-vs-one binary classifier. The outcome of the classifier is interpreted as an expression of pairwise preference in a label ranking\textsuperscript{9}. In contrast to the previously mentioned decomposition technique, the pairwise method considers the paired-inter-label dependencies. Contrary to the BR approach, this kind of decomposition produces a significantly larger number of base classifiers that must be built. The transformed datasets are, in general, less imbalanced than datasets produced using one-vs-rest transformation. What is more, the resulting base classifiers tend to produce simpler models than one-vs-rest classifiers\textsuperscript{9}.

The main goal of this paper is to compare soft-confusion-matrix approach (SCM) and Bayes metaclassifier (BMC) algorithm under the framework of multi-label classification. More precisely, the above-mentioned classification techniques are going to be tested for BR and LPW decomposition transformations. In short, SCM and BMC algorithms are overlays that may be placed on the top of any base classifier. What is more, both of them are also based on the idea of the RRC classifier introduced by Woloszynski\textsuperscript{7}. Since the SCM and BMC algorithms are constructed using similar ideas and so far they have not been compared directly, so this paper aims at determining the differences between these two approaches.

The concept of the soft confusion matrix was first introduced in research aimed at improving the classification quality of systems that recognise hand gestures\textsuperscript{7,9}. In the above-mentioned research, the FCM-based system was used because it possesses the ability to utilise soft class-assignment. Its ability to improve the response of base classifiers was also seen to be noteworthy. The fuzzy-confusion matrix-based approach was also employed under a multi-label classification framework\textsuperscript{7}. Namely, it was used to improve the quality of Binary Relevance classifiers. Experiments confirmed the validity of its use, but also showed sensitivity to the unbalanced class distribution in a binary problem. The algorithm was also used in studies related to the LPW decomposition\textsuperscript{7,9}. The conducted experiments also confirmed that the model is able to provide the significant improvement of the outcome of the committee built using the one-vs-one approach.

The concept of Bayes metaclassifier was firstly introduced in\textsuperscript{7} where a detailed BMC algorithm construction was presented and validated. Provided experiments confirmed that the upper bound of BMC improvement over base classifier is Bayes error. BMC performance directly depends on the classification quality of a base classifier. What is more, experiments on benchmark datasets shows that BMC can significantly correct base classifier classification, especially for cases where chosen base classifier is not optimal. Additionally, when analyzing results from each cross-validation phase, it was mentioned that BMC decreases classification variance comparing to its base counterpart results. BMC works better with balanced datasets, while for imbalanced problems usage of a’priori probabilities in BMC design, causes that its decision boundary is moved towards majority class. Since BMC provides probabilistic interpretation for any base classifier response, this method was also directly used in a sequential classification problems and as a useful tool for constructing multi-classifier system, especially during classifier fusion\textsuperscript{7,9}.

This paper is organized as follows. Section\textsuperscript{2} provides the formal notation used throughout the article, and introduces the SCM correction algorithm and Bayes metaclassifier. Section\textsuperscript{3} contains a description of the experimental setup. In section\textsuperscript{4} the experimental results are presented and discussed while section\textsuperscript{5} concludes whole paper.

\section{Methods}

\subsection{Preliminaries}

In this section, a single-label is introduced. The scope is reduced to only binary classification because only binary classifiers are considered in this paper.

As it was stated in the introductory section, according to the single-label classification approach, a \(d\) - dimensional object \(x = [x_1, x_2, \ldots, x_d] \in \mathbb{X} = \mathbb{R}^d\) is assigned a class \(m \in \mathbb{M}\), where \(\mathbb{M} = \{0, 1\}\) is a set of available classes. The single-label, binary classifier \(\psi : \mathbb{X} \mapsto \mathbb{M}\). The single-label, binary classifier \(\psi : \mathbb{X} \mapsto \mathbb{M}\) is an approximation of an unknown mapping \(f : \mathbb{X} \mapsto \mathbb{B}\) which assigns the classes to the instances.

The classification methods investigated in this paper follow the statistical classification framework. As a consequence, vector \(x\) and label \(m\) are assumed to be realisations of multi-dimenstional random variables \(X\) and \(M\), respectively. The random variables are distributed according the joint probability distribution \(P(X, M)\). Given the loss function \(l : \mathbb{M} \times \mathbb{M} \mapsto \mathbb{R}_+ \cup \{0\}\), which assesses the similarity of the objects in the output space, the optimal prediction \(\psi^*(x)\) for the object \(x\) is found in the following manner:

\[
\psi^*(x) = \arg\min_{k \in \mathbb{M}} \sum_{m \in \mathbb{M}} l(k, m) P(M = m|X = x).
\]
When the loss function \( l \) is a zero-one loss, then the optimal decision is taken using the maximum \textit{a posteriori} rule:

\[
\psi^*(x) = \arg \max_{k \in \mathcal{M}} P(M = k|X = x),
\]

where \( P(M = k|X = x) \) is the conditional probability that the object \( x \) belongs to class \( m \).

In this paper, the so-called soft output of the classifier \( v : \mathfrak{X} \mapsto [0,1]^2 \) is also defined. The soft output vector \( v \) contains values proportional to the conditional probabilities. Consequently, the following properties must be met:

\[
\begin{align*}
\nu_i &\approx P(M = i|X = x), \\
\nu_i(x) &\in [0,1], \\
\sum_{i=0}^{1} \nu_i(x) &= 1.
\end{align*}
\]

In this study, classifier \( \psi \) is built in a supervised learning procedure using the training set \( \mathcal{T} \) containing \(|\mathcal{T}| \) pairs of feature vectors \( x \) and corresponding labels \( m \):

\[
\mathcal{T} = \{(x^{(1)}, m^{(1)}), (x^{(2)}, m^{(2)}), \ldots, (x^{(|\mathcal{T}|)}, m^{(|\mathcal{T}|)})\},
\]

where \( x^{(k)} \in \mathfrak{X} \) and \( m^{(k)} \in \mathcal{M} \). To evaluate the classifier the validation set \( \mathcal{V} \) is also used.

### 2.2 Multi-label Classification

Under the Multi-label formalism, an object \( x \) is assigned to a set of labels indicated by a binary vector of length \( L \): \( y = [y_1, y_2, \ldots, y_L] \in \mathcal{Y} = \{0,1\}^L \), where \( L \) denotes the number of labels. Each element of the binary vector corresponds to a single label. If for some object \( x \) the element of the vector \( y_i \) is set to \( 1 \) \((0)\), it means that the label associated with the \( i \)-th position is relevant (irrelevant) to object \( x \). Relevant labels are assigned to instances by an unknown mapping \( g : \mathfrak{X} \mapsto \mathcal{Y} \). A multi-label classifier \( h : \mathfrak{X} \mapsto \mathcal{Y} \) is an approximation of the unknown mapping.

The classification process is often conducted in a two-stage way. During the first step, the classifier produces soft outputs \( \omega : \mathfrak{X} \mapsto \mathbb{R}^L \). Then the classifier outcome is produced using thresholding procedure:

\[
h(x) = \left\lfloor \omega_1(x) \geq \Theta_1 \right\rfloor, \left\lfloor \omega_2(x) \geq \Theta_2 \right\rfloor, \ldots, \left\lfloor \omega_L(x) \geq \Theta_L \right\rfloor,
\]

where \( \left\lfloor . \right\rfloor \) is the Iverson bracket\(^7\) and \( \Theta_i \) is a label-specific threshold that may be found using various thresholding strategies\(^7\).\(^7\).

As it was mentioned, the BR transformation builds a separate binary classifier for each label. As a consequence the BR ensemble consists on \( L \) binary classifiers:

\[
C_{BR} = \{\psi^{(1)}, \psi^{(2)}, \ldots, \psi^{(L)}\}.
\]

The output of the multi-label classifier is produced as follows:

\[
h_i(x) = \psi^{(i)}(x).
\]

The label-pairwise (LPW) transformation builds the multi-label classifier \( h \), using an ensemble of binary classifiers \( \Psi \) and a single binary classifier is assigned to each pair of labels:

\[
C_{LPW} = \{\psi^{(i,j)}|i, j \in \{1,2,\ldots,L\}, i < j\},
\]

\[
\psi^{(i,j)}(x) \in \{i,j\},
\]

\[
|C_{LPW}| = \frac{L(L-1)}{2}.
\]

The soft output of the LPW ensemble is obtained by combining the outcomes of the base classifiers:

\[
\omega_j(x) = \frac{1}{L-1} \sum_{i,j \in \{1,2,\ldots,L\}, \ i < j} \left\lfloor \psi^{(i,j)}(x) = l \right\rfloor.
\]

The soft output is then converted into a binary response using a thresholding procedure\(^7\).
2.3 Soft-confusion Matrix Classifier

The proposed correction method is based on an assessment of the probability of classifying an object \( x \) to the class \( s \in \mathcal{M} \) using the binary classifier \( \psi \). The proposed approach provides an extension of the Bayesian model defined in the previous section. Namely, the above-mentioned Bayesian model requires that the object description \( x \) and its true label \( s \in \mathcal{M} \) are realizations of random variables \( \mathbf{X} \) and \( \mathbf{S} \), respectively. SCM, on the other hand, assumes that the prediction of the classifier \( \psi \) is made in a random way according to the probabilities \( P(\psi(x) = m) = P(m|x) \in [0, 1]^7 \). As a result the classification result \( m \) is a realization of the random variable \( \mathbf{S}(x) \).

The extended Bayesian model allows the posterior probability \( P(s|x) \) of label \( s \) to be defined as:

\[
P(s|x) = \sum_{m \in \mathcal{M}} P(m|x)P(s|m, x). \tag{14}
\]

where \( P(m|x) \) denotes the probability that an object \( x \) belongs to the class \( s \) given that \( \psi(x) = m \).

Unfortunately, assuming that the base classifier assigns labels in a stochastic way is rather impractical, because most real-life classifiers are deterministic. This issue was dealt with by employing deterministic binary classifiers in which their statistical properties were modelled using the RRC procedure\(^7\). The explanation of the RRC model is given in section 2.5.

2.3.1 Confusion Matrix

During the inference phase, the probability \( P(s|m, x) \) is estimated using a local, soft confusion matrix. An example of such a matrix for a binary classification task is given in Table 1. The rows of the matrix correspond to the ground-truth classes, whereas the columns match the outcome of the classifier. The confusion matrix becomes a soft one because the decision regions of the random classifier are expressed in terms of fuzzy set formalism\(^7\). Thus, the membership function of a point is proportional to the probability of assigning the point to a given class by the randomised model of the classifier.

To provide an estimation of \( P(s|m, x) \) that depends on the description of the instance \( x \), a confusion matrix that is built using the concept of the neighbourhood of the instance is defined. The neighbourhood of the instance is also defined using the fuzzy set formalism. The fuzzy neighbourhood is employed in order to utilize all the points included in the validation set.

On the basis of subsets of the validation set containing object belonging to class \( s \), the fuzzy decision region of \( \psi \) and the neighbourhood of \( z \) were defined, respectively:

\[
\mathcal{V}_s = \{(x^k, s^k, 1) : (x^k, s^k) \in \mathcal{V}, s^k = s\}, \tag{15}
\]

\[
\mathcal{D}_z = \{(x^k, s^k, \mu_{D_z}(x^k)) : (x^k, s^k) \in \mathcal{V}\}, \tag{16}
\]

\[
\mathcal{N}(z) = \{(x^k, s^k, \mu_{\mathcal{N}(z)}(x^k)) : (x^k, s^k) \in \mathcal{V}\}, \tag{17}
\]

where each triplet \((x^k, s^k, \zeta)\) defines the fuzzy membership value \( \zeta \) of instance \((x^k, s^k)\), and \( \mu_{D_z}(x) = P(\psi(x) = m|x) \) indicates the fuzzy decision region of the stochastic classifier. Additionally, \( \mu_{\mathcal{N}(z)}(x) \) denotes the fuzzy neighbourhood of the instance \( z \). The membership function of the neighbourhood was defined using the Gaussian potential function:

\[
\mu_{\mathcal{N}(z)}(x^k) = \exp(-\beta \delta(z, x^k)^2), \tag{18}
\]

where \( \beta \in \mathbb{R}_+ \) and \( \delta(z, x^k) \) is a distance function between two vectors from the input space \( \mathcal{X} \).

The above-defined fuzzy sets are employed to approximate \( P(s|m, x) \): The following fuzzy sets are employed to approximate entries of the local confusion matrix:

\[
\hat{\epsilon}_{s,m}(z) = \frac{|\mathcal{V}_s \cap \mathcal{D}_m \cap \mathcal{N}(z)|}{|\mathcal{N}(z)|}, \tag{19}
\]

where \(|.| \) is the cardinality of a fuzzy set\(^7\). Finally, the approximation of \( P(s|m, x) \) is calculated as follows:

\[
P(s|m, x) \approx \frac{\hat{\epsilon}_{s,m}(z)}{\sum_{u \in \mathcal{M}} \hat{\epsilon}_{u,m}(z)}. \tag{20}
\]

2.4 Bayes Metaclassifier–BMC

Firstly, the probabilistic model of classification will be introduced. This model denotes that feature vector \( x \in \mathcal{X} \) and class label \( j \in \mathcal{M} \) are observed values of the pair of random variables \( \mathbf{X} \) and \( \mathbf{J} \), respectively. The probability distribution of \((\mathbf{X}, \mathbf{J})\) is determined by the \textit{a priori} class probabilities \( p_j = P(\mathbf{J} = j) \) and classconditional density functions \( f(x|j) = f_j(x) \).
The Bayes metaclassifier (BMC) $\psi^{BMC}$, which originally was introduced in\textsuperscript{7}, constitutes the probabilistic generalization of any base classifier (21) which has the form of the Bayes scheme built over the classifier $\psi$. This means, that $\psi^{BMC}$ takes the decision according to the maximum \textit{a posteriori} probability rule\textsuperscript{7},\textsuperscript{7}:

$$\psi^{BMC}(\psi(x) = k) = \hat{i} \iff p(\hat{i}|\psi = k) = \arg\max_{i \in \mathcal{M}} \{ p(\hat{i}|\psi = k) \}.$$  \hfill (21)

\textit{A posteriori} probabilities $p(\hat{i}|k)$ $\equiv P(\hat{J} = i|\psi(x) = k), i \in \mathcal{M}$ are given by Bayes rule:

$$P(\hat{J} = i|\psi = k) = \frac{p_i p(k|i)}{\sum_j p_j p(k|j)},$$  \hfill (22)

where probability $p_i(k|i) \equiv P(\psi_i(x) = k_i|i)$ denotes class-dependent probability of error (if $k \neq i$) or correct (if $k \equiv i$) classification of an object $x$ by the base classifier $\psi$.

Placing the base classifier $\psi$ in a probabilistic frame defined by the BMC $\psi^{BMC}$ we get a common probabilistic interpretation of any responses of base classifiers, regardless of their design paradigms.

The key element in the BMC scheme (21) and (22) is the calculation of probabilities $P(\psi_i(x) = k_i|i)$ at point $x$, i.e. class-dependent probabilities of correct and misclassification for base classifiers. Normally, for any base deterministic classifier, such probabilities $P(\psi(x) = k|i)$ would be either 0 for misclassification or 1 for correct classification for given $x$. In this paper, the proposed method for approximating these probabilities is based on the original concept of a hypothetical classifier called a Randomized Reference Classifier (RRC).

The RRC $\psi^{RRC}(x)$ is a stochastic classifier defined by a probability distribution which is chosen in such a way, that RRC acts, on average, as modeled base classifier. In this context, it is possible to calculate class-dependent probabilities of correct classification $P_c(j|x)$ and misclassification $P_e(j|x)$ and furthermore consider them equivalent to the modeled base classifier:

$$P(\psi(x) = k|i) \approx P(\psi^{RRC}(x) = k|i).$$  \hfill (23)

In the computational procedure, probabilities $P(d(x) = k|i) \approx P(\psi^{RRC}(x) = k|i)$ (denoting that an objects $x$ belongs to class $i$ given that $\psi(x) = k$) are calculated for each validation points. These values are only known at discrete points from $\mathcal{Y}$ so to enable dynamic calculation of any new object $x$ during classification we need a neighborhood function describing how probabilities at validation points affects new $x$. For BMC algorithm, Gaussian potential function is used:

$$P^{RRC}_c(j|x) = \frac{\sum_{x_k \in \mathcal{V}, j_k = j} P_c(j|x) \cdot \exp(-\alpha \cdot ||x, x_k||^2)}{\sum_{x_k \in \mathcal{V}, j_k = j} \exp(-\alpha \cdot ||x, x_k||^2)},$$

$$P^{RRC}_e(j|x) = \frac{\sum_{x_k \in \mathcal{V}, j_k \neq j} P_e(j|x) \cdot \exp(-\alpha \cdot ||x, x_k||^2)}{\sum_{x_k \in \mathcal{V}, j_k \neq j} \exp(-\alpha \cdot ||x, x_k||^2)},$$  \hfill (24)

where $\alpha$ value in equation (24) is a scaling factor and should be adjusted independently to classification problem. Similarly, \textit{a priori} probabilities $(p_i, i \in \mathcal{M})$ introduced in (22) are estimated using validation set $\mathcal{V}$.

### 2.5 Randomized Reference Classifier

As was said in the previous section, to calculate $P(\Psi = m|X = x)$ we need a randomised classifier that behaves in a similar way to the base classifier $\psi$. In our approach, the behaviour of a base classifier $\psi$ is modeled using a stochastic classifier defined by a probability distribution over the set of labels $\{0, 1\}$. In this work, the randomized reference classifier (RRC) that was proposed by Woloszynski and Kurzynski\textsuperscript{7} was employed. The RRC is a hypothetical classifier that allows a randomised model of a given deterministic classifier to be built.
We assume, for the given instance \( x \), that the randomised classifier \( \Psi^{(R)} \) generates a vector of class supports \( [v_1(x), v_2(x)] \) which are observed values of random variables \( \Delta_1(x), \Delta_2(x) \). The probability distribution of random variables is chosen in such a way that the following conditions are satisfied:

\[
\Delta_1(x), \Delta_2(x) \in (0, 1) \tag{25}
\]

\[
\Delta_1(x) + \Delta_2(x) = 1 \tag{26}
\]

\[
\mathbb{E}[\Delta_i(x)] = v_i(x), \quad i \in \{0, 1\}, \tag{27}
\]

where \( \mathbb{E} \) is the expected value operator. Conditions (25) and (26) follow from the normalisation properties of class supports, while condition (27) ensures the equivalence of the randomized model \( \Psi^{(R)} \) and base classifier \( \Psi \). This condition also shows that the RRC can be used to provide a randomised model of any classifier that returns a vector of class-specific supports \( v(x) \).

It is clear that the probability of classifying an object \( x \) to the class \( i \) by the RRC is the following:

\[
P(\Psi = m|X = x) = Pr[\Delta_m(x) > \Delta_{\{0,1\}\backslash m}(x)], \tag{28}
\]

where \( Pr[\Delta_m(x) > \Delta_{\{0,1\}\backslash m}(x)] \) is the probability that the value obtained by the realisation of random variable \( \Delta_m \) is greater than the realisation of random variable \( \Delta_{\{0,1\}\backslash m} \).

The crucial step in the modeling presented above is to choose the probability distributions for random variables \( \Delta_i(x), i \in \{0, 1\} \) so that the conditions (25, 27) are met. In this study, according to the recommendations given in\(^7\), the beta distribution with parameters \( \lambda_i(x), \mu_i(x), \quad i \in \{0, 1\} \) is applied. The parameters are chosen so that the following set of equations is met:

\[
\begin{align*}
\frac{\lambda_i(x)}{\lambda_i(x) + \mu_i(x)} &= v_i(x), \\
\lambda_i(x) + \mu_i(x) &= 2,
\end{align*}
\tag{29}
\]

The justification of the choice of the beta distribution resulting from the theory of order statistics and a detailed description of parameters estimation can be found in\(^7\).

For the beta distribution, the following formula for probability (28) is achieved:

\[
P^{(R)}(\Psi = m|Y = x) = \int_0^1 b(u, \lambda_m(x), \mu_m(x)) \times B(u, \lambda_j(x)), \mu_j(x) \; du, \quad j \neq m, \tag{30}
\]

where \( B(\cdot) \) is a beta cumulative distribution function and \( b(\cdot) \) is a beta probability density function. It must be noted that for calculation probabilities (30), validation set is not necessary because it does not need to know the correct classification of the object \( x \). The MATLAB implementation of the RRC classifier is freely available at\(^1\). Implementation for WEKa is also available\(^2\).

### 3.1 Experimental Setup

The experimental study is divided into two main sections. In the first one, utilizes the BR transformation, whereas the other uses the LPW approach. In both scenarios, the following methods are compared:

1. Unmodified base classifiers.
2. Base classifiers combined with the Bayes metaclassifier approach.
3. Base classifiers combined with the SCM approach.

In the conducted experimental study, the following single-label classifiers were utilized:

- J48 (C4.5) classifier\(^7\),

---

1. [http://www.mathworks.com/matlabcentral/fileexchange/28391-a-probabilistic-model-of-classifier-competence](http://www.mathworks.com/matlabcentral/fileexchange/28391-a-probabilistic-model-of-classifier-competence)
2. [https://github.com/ptrajdos/rrcBasedClassifiers/tree/develop](https://github.com/ptrajdos/rrcBasedClassifiers/tree/develop)
• SVM classifier with radial kernel\(^2\),
• Naive Bayes classifier\(^2\),
• Nearest Neighbour classifier\(^2\).

During the experimental study, the parameters of the J48 algorithm were set to its defaults. For the Naive Bayes classifier, the kernel estimator with the Gaussian kernel was used to calculate the probabilities. The parameters of the SVM classifier \((C \in \{0.001, 1, 2, \ldots, 10\}, \gamma \in \{0.001, 1, 2, \ldots, 5\})\) were tuned using grid search and 3-fold cross validation. The number of nearest neighbours was also tuned using 3-fold cross validation. The number of neighbours was chosen among the following values \(K \in \{1, 3, 5, \ldots, 11\}\). The Euclidean distance function was employed to choose the nearest neighbours. The quality criterion used in the tuning procedures was the \(F_1\) criterion calculated for the minority class. The remaining parameters of the base classifiers were set to their defaults.

The Bayes meta-classifier and the SCM-based algorithms were implemented in java. The source code of the algorithms is available online\(^3\).

The size of the neighbourhood, which is determined by coefficient \(\beta\), is chosen using a three-fold cross-validation procedure and the grid search technique. The search space is defined as follows:

\[
\{\beta = 2 + 0.9i | i \in \{0, 1, \ldots, 10\}\}.
\]

The conversion of soft outputs into binary-responses was done using S-Cut algorithm\(^2\) with the number of cross-validation folds also set to three. Thresholds and size of the committee were chosen in such a way to provide the best value of the \(F_1\) criterion\(^2\).

To evaluate the proposed methods the following multi-label classification-quality criteria are used\(^2\):

• Hamming loss,
• Zero-one loss,
• Example based FDR, FNR, \(F_1\),
• Macro-averaged FDR, FNR, \(F_1\),
• Micro-averaged FDR, FNR, \(F_1\).

Following the recommendations of\(^2\) and\(^7\), the statistical significance of the obtained results was assessed using the two-step procedure. The first step is to perform the Friedman test\(^7\) for each quality criterion separately. Since the multiple criteria were employed, the familywise errors (FWER) should be controlled\(^7\). To do so, the Holm’s\(^7\) procedure of controlling FWER of the conducted Friedman tests was employed. When the Friedman test shows that there is a significant difference within the group of classifiers, the pairwise tests using the Wilcoxon signed-rank test\(^7\) test were employed. To control FWER of the Wilcoxon-testing procedure, the Holm approach was employed\(^7\). For all tests the significance level was set to \(\alpha = 0.05\).

Table 2 displays the collection of the benchmark sets that were used during the experimental evaluation of the proposed algorithms. The table is organized as follows. The first column contains the names of the datasets. The names under which the data sets are known in the repositories are used. The second column contains the numbers of the datasets preceded by the number of the table. The remaining ones contain the set-specific characteristics of the benchmark sets.

| Dataset Name | Dataset Name | Dataset Name |
|--------------|--------------|--------------|
| \(|S|\)       | \(d\)        | \(L\)        |
| \(LC\)       | \(LU\)       | \(IR\)       |

[https://github.com/ptrajdos/rrcBasedClassifiers/tree/develop]
The datasets are available online. During the dataset-preprocessing stage, a few transformations on datasets were applied. First and foremost, all nominal attributes, except binary attributes, were converted into a set of binary variables. This approach is one of the simplest methods to replace nominal variables with binary variables. This transformation is necessary if the SVM-based or distance-based algorithms are employed. The features were also normalised to have zero mean and unit variance.

In this work, some datasets that follow multi-instance-multi-label (MIML) (datasets: 2.2, 2.4, 2.5, 2.11, 2.12, 2.20, 2.21,2.17) framework were employed. In these datasets each object consists of a bag of instances tagged with a set of labels. In order to tackle this data, the suggestion made in was followed and the set was transformed to single-instance multi-label data. The multi-target regression sets (datasets: 2.9, 2.28) were also harnessed. The datasets were converted into multi-label data using a simple thresholding procedure. To be more precise, when the value of output variable for a given object is greater than zero, the corresponding label was set to be relevant to this object. The number of labels in the stackex_chess (2.26) dataset was reduced to 15. This is since the computational burden should be reduced. Additionally, before the learning phase, features are selected using a correlation-based approach.

The extraction of training and testing datasets was performed using ten-fold cross-validation. The validation set is sequel to the training set, however base-classifier responses are obtained using two-fold crossvalidation.

### RESULTS AND DISCUSSION

To compare multiple algorithms on multiple benchmark sets the average ranks approach is used. In the approach, the winning algorithm achieves rank equal ’1’, the second achieves rank equal ’2’, and so on. In the case of ties, the ranks of algorithms that...
achieve the same results, are averaged. To provide a visualisation of the average ranks, the radar plots are employed. Radar plot provides similar visualisation properties as parallel coordinates plots. In other words, radar plots can be interpreted as parallel coordinates plots drawn in polar coordinates. In the plots, the data is visualised in such way that the lowest ranks are closer to the centre of the graph. The radar plots related to the results of experiments are given in FIGURE 1 and 2.

The numerical results are given in Tables 3–10. Each table is structured as follows. The first row contains numbers assigned to algorithms in section 3. Then the table is divided into eleven sections – one section is related to a single evaluation criterion. The first row of each section is the name of the quality criterion investigated in the section. The second row shows the p-value of the Friedman test. The third one shows the average ranks achieved by algorithms. The following rows show p-values resulting from pairwise Wilcoxon test. The p-value equal to 0.000 informs that the p-values are lower than 10^{-3} and p-value equal to 1.000 informs that the value is higher than 0.999.

The full results are available online at https://github.com/ptrajdos/MLResults/tree/master/BMAndSCM.

4.1 Binary Relevance

Results related to the Binary Relevance transformation are given in figure 1 and tables 3–6. In the case of the BR transformation, the results of the statistical evaluation are a bit inconsistent. That is, the Friedman test indicates that, for all base classifiers and quality criteria, there are no significant differences between compared methods. However, in some cases, the post-hoc Wilcoxon test shows significant differences between the investigated classifiers. Result analysis shows a few trends in the data. First of all, for macro-averaged FDR, macro and micro-averaged $F_1$ measures the order of investigated methods (according to the average ranks) is the same for each base classifier. What is more, for macro-averaged $F_1$ criterion, the post-hoc test shows that for all base classifiers except for KNN, the Bayes meta-classifier outperforms the reference approach. Additionally, for macro-averaged FDR, the Bayes meta-classifier significantly outperforms the reference method for three out of four classifiers (except the Naive Bayes classifier). These results show that the Bayes meta-classifier achieves better classification quality for rare labels. Staying with macro-averaged measures, the average ranks suggest that SCM-based classifier may be a bit less conservative than Bayes meta-classifier. However, the differences are significant only for SVM base classifier.

No consistent conclusions can be drawn from example-based quality criteria. The order of investigated classifiers (according to their average ranks) varies depending on the base classifier. What is more, no consistent results of the post-hoc test can be observed. For these quality criteria, there are no significant differences between the investigated classifiers.

For the micro-averaged criteria, there is also no clear trend in the results.

4.2 Label Pairwise

Results related to the label Pairwise transformation are given in figure 2 and tables 7–10. The results are pretty consistent. That is, each time the Friedman test shows a significant p-value, the post-hoc Wilcoxon test finds at least one significant difference between algorithms.

The performed statistical analysis shows that for all base classifiers, Bayes meta-classifier and the classifier based on the soft confusion matrix are more conservative than the reference method. In other words, they achieve significantly better results in terms of the FDR (precision) criterion and significantly worse in terms of the FNR (recall). Consequently, the performance of the investigated methods is significantly better in terms of the F1 measure. In other words, the proposed methods are more conservative because they mark fewer instances as relevant, and more of the marked instances are truly relevant for the instance under classification. The results hold for all example-based, micro and macro-averaged criteria. It means that the results are improved from the perspective of the entire label vector and each label separately regardless of whether the label is common or rare in the dataset. What is more, the algorithms investigated in this paper achieve significantly better results than the reference method in terms of the Hamming loss and the zero-one loss. The improvement in terms of the zero-one loss is very important because this criterion is the strictest quality criteria that may be employed for quality assessment of multi-label classifiers. That is to say, the criterion assigns the loss equal to 1 if an only a single label is misclassified. These results clearly show that for the label pairwise transformation, contrary to the BR-based approach, the investigated algorithms offer a great improvement of classification quality. The reason why the quality improvement for the label pairwise transformation is so clear may be the fact that the datasets produced by the LPW transformation are far less imbalanced than the datasets produced by the BR.
TABLE 3 Binary Relevance transformation. Wilcoxon test for J48 base classifiers – p-values for paired comparisons of investigated methods.

|  | 1       | 2       | 3       | 1       | 2       | 3       | 1       | 2       |
|---|---------|---------|---------|---------|---------|---------|---------|---------|
| Nam. | Hamming | Zero-One | ExFDR   | ExFNR   |
| Frd. | 1.000e+00 | 1.000e+00 | 1.000e+00 | 9.677e-01 |
| Rank | 1.983 | 1.879 | 2.138 | 2.190 | 1.983 | 1.828 | 2.086 | 1.741 | 2.172 | 2.069 | 2.207 | 1.724 |
| 1    | 0.741 | 0.138 | 0.087 | 0.548 | 0.785 | 0.785 | 0.723 | 1.000 | 0.933 | 0.395 |
| 2    | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |

|  | 1       | 2       | 3       | 1       | 2       | 3       | 1       | 2       | 3       |
|---|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Nam. | ExF1    | MaFDR   | MaFNR   | MaF1    |
| Frd. | 1.000e+00 | 4.658e-01 | 2.183e-01 | 4.559e-01 |
| Rank | 2.052 | 1.776 | 2.172 | 2.293 | 1.672 | 2.034 | 2.241 | 2.172 | 1.586 | 2.362 | 1.879 | 1.759 |
| 1    | 0.999 | 1.000 | 1.000 | 0.040 | 0.336 | 0.096 | 0.182 | 0.073 | 0.007 | 0.073 |
| 2    | 1.000 | 1.000 | 1.000 | 0.287 | 0.036 | 0.230 | 0.094 | 0.101 | 0.007 | 0.073 |

TABLE 4 Binary Relevance transformation. Wilcoxon test for SVM base classifiers – p-values for paired comparisons of investigated methods.

|  | 1       | 2       | 3       | 1       | 2       | 3       | 1       | 2       | 3       |
|---|---------|---------|---------|---------|---------|---------|---------|---------|---------|
| Nam. | Hamming | Zero-One | ExFDR   | ExFNR   |
| Frd. | 9.579e-01 | 1.000e+00 | 1.000e+00 | 1.000e+00 |
| Rank | 1.741 | 2.017 | 2.241 | 1.914 | 2.017 | 2.069 | 1.845 | 2.086 | 2.069 | 1.983 | 2.155 | 1.862 |
| 1    | 0.108 | 0.005 | 0.005 | 1.000 | 1.000 | 1.000 | 0.999 | 0.999 | 0.999 | 0.776 | 0.693 |
| 2    | 0.005 | 1.000 | 1.000 | 0.005 | 1.000 | 1.000 | 0.020 | 0.405 | 0.017 | 0.017 | 0.004 | 0.023 |

|  | 1       | 2       | 3       | 1       | 2       | 3       | 1       | 2       |
|---|---------|---------|---------|---------|---------|---------|---------|---------|
| Nam. | ExF1    | MaFDR   | MaFNR   | MaF1    |
| Frd. | 1.000e+00 | 5.110e-01 | 5.611e-02 | 1.217e-01 |
| Rank | 1.948 | 2.017 | 2.034 | 2.259 | 1.672 | 2.069 | 2.397 | 2.052 | 1.552 | 2.397 | 1.983 | 1.621 |
| 1    | 1.000 | 1.000 | 1.000 | 0.020 | 0.405 | 0.024 | 0.017 | 0.017 | 0.004 | 0.023 |
| 2    | 1.000 | 1.000 | 1.000 | 0.024 | 0.024 | 0.029 | 0.004 | 0.023 | 0.156 |

|  | 1       | 2       | 3       | 1       | 2       | 3       |
|---|---------|---------|---------|---------|---------|---------|
| Nam. | MiFDR   | MiFNR   | MiF1    |
| Frd. | 2.130e-01 | 1.692e-01 | 1.000e+00 |
| Rank | 1.603 | 2.121 | 2.276 | 2.293 | 2.121 | 1.586 | 2.224 | 2.017 | 1.759 |
| 1    | 0.007 | 0.030 | 0.096 | 0.052 | 0.022 | 0.181 | 0.181 | 0.442 |
| 2    | 0.007 | 0.030 | 0.096 | 0.052 | 0.022 | 0.181 | 0.181 | 0.442 |

transformation. As a consequence, the neighbourhood of a point is dominated by points belonging to the majority class. The imbalanced neighbourhood does not allow providing a proper estimation of the statistical properties of the base classifiers.

On the other hand, when the Bayes meta-classifier and the classifier based on the confusion matrix are compared, there are no significant differences between them. This result is not so surprising since both algorithms investigated in this paper are based on similar principles. They are both using a validation set to provide a probabilistic interpretation of a base classifier.
TABLE 5 Binary Relevance transformation. Wilcoxon test for Naive Bayes base classifiers – p-values for paired comparisons of investigated methods.

|      | 1          | 2          | 3          | 1          | 2          | 3          | 1          | 2          | 3          |
|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Nam. | Hamming    | Zero-One   | ExFDR      | ExFNR      |
| Frd. | 1.000e+00  | 1.000e+00  | 1.000e+00  | 1.000e+00  |
| Rank | 2.052      | 1.845      | 2.103      | 2.224      | 1.845      | 1.931      | 2.121      | 1.707      | 2.172      |
| 1    | 0.741      | 0.299      | 1.000      | 0.999      | 1.000      | 0.859      | 0.426      | 0.966      | 1.000      |
| 2    | 0.217      | 1.000      | 0.859      | 1.000      | 1.000      | 1.000      | 1.000      | 1.000      | 1.000      |

TABLE 6 Binary Relevance transformation. Wilcoxon test for KNN base classifiers – p-values for paired comparisons of investigated methods.

|      | 1          | 2          | 3          | 1          | 2          | 3          | 1          | 2          | 3          |
|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| Nam. | Hamming    | Zero-One   | ExFDR      | ExFNR      |
| Frd. | 3.827e-01  | 8.242e-02  | 1.000e+00  | 1.000e+00  |
| Rank | 2.207      | 1.621      | 2.172      | 2.466      | 1.810      | 1.724      | 2.207      | 1.759      | 2.034      |
| 1    | 0.002      | 0.137      | 0.024      | 0.003      | 0.008      | 0.115      | 0.039      | 0.324      | 0.241      |
| 2    | 0.137      | 0.137      | 0.137      | 0.003      | 0.008      | 0.115      | 0.039      | 0.324      | 0.241      |

5.1 CONCLUSIONS

In this paper, we compared Bayes metaclassifier and soft-confusion-matrix-based classifier. The classifiers were compared with each other and with the reference method under the multi-label classification framework. More precisely, the classifiers were compared using problem-transformation approach to multi-label learning. Two most common transformation methods were employed – binary relevance and label pairwise respectively.

The experimental evaluation showed that:
For the binary-relevance transformation, there are almost no significant differences between the investigated algorithms. The results suggest that the Bayes meta-classifier is slightly better than the reference method. On the other hand, the obtained data shows no sign of a significant difference between Bayes meta-classifier and the SCM-based method.

For the label pairwise transformation, the results show that the Bayes meta-classifier and SCM-based classifier significantly outperformed the reference method for all quality criteria. The results show that the methods investigated in this paper are more conservative than the reference method. On the other hand, there is no significant difference between Bayes meta-classifier and SCM-based classifier.

1. Add acknowledgements
TABLE 7 Pairwise transformation. Wilcoxon test for J48 base classifiers – p-values for paired comparisons of investigated methods.

|     | 1              | 2              | 3              | 1              | 2              | 3              | 1              | 2              | 3              |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Nam.| Hamming        | Zero-One       | ExFDR          | ExFNR          |                 |                 |                 |                 |                 |
| Frd.| 5.916e-09      | 7.088e-09      | 5.498e-09      | 4.744e-08      |                 |                 |                 |                 |                 |
| Rank| 3.000 1.500 1.500 | 2.946 1.375 1.679 | 3.000 1.607 1.393 | 1.071 2.536 2.393 |                 |                 |                 |                 |                 |
| 1   | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    |                 |                 |                 |                 |                 |
| 2   | 0.884          | 0.243          |                 | 0.479          | 0.126          |                 |                 |                 |                 |
| Nam.| ExF1           | MaFDR          | MaFNR          | MaF1           |                 |                 |                 |                 |                 |
| Frd.| 8.157e-05      | 5.916e-09      | 2.801e-08      | 2.392e-03      |                 |                 |                 |                 |                 |
| Rank| 2.679 1.786 1.536 | 3.000 1.571 1.429 | 1.071 2.643 2.286 | 2.536 1.714 1.750 |                 |                 |                 |                 |                 |
| 1   | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    |                 |                 |                 |                 |                 |
| 2   | 0.264          | 0.508          |                 | 0.245          | 0.779          |                 |                 |                 |                 |
| Nam.| MiFDR          | MiFNR          | MiF1           |                 |                 |                 |                 |                 |                 |
| Frd.| 5.916e-09      | 4.710e-09      |                 | 3.969e-06      |                 |                 |                 |                 |                 |
| Rank| 3.000 1.571 1.429 | 1.000 2.643 2.357 | 2.786 1.750 1.464 |                 |                 |                 |                 |                 |                 |
| 1   | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    |                 |                 |                 |                 |                 |                 |
| 2   | 0.779          | 0.236          |                 | 0.255          |                 |                 |                 |                 |                 |

TABLE 8 Pairwise transformation. Wilcoxon test for SVM base classifiers – p-values for paired comparisons of investigated methods.

|     | 1              | 2              | 3              | 1              | 2              | 3              | 1              | 2              | 3              |
|-----|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| Nam.| Hamming        | Zero-One       | ExFDR          | ExFNR          |                 |                 |                 |                 |                 |
| Frd.| 4.282e-09      | 9.299e-09      | 6.585e-09      | 6.601e-08      |                 |                 |                 |                 |                 |
| Rank| 3.000 1.643 1.357 | 2.964 1.518 1.518 | 3.000 1.464 1.536 | 1.071 2.464 2.464 |                 |                 |                 |                 |                 |
| 1   | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    |                 |                 |                 |                 |                 |
| 2   | 0.438          | 0.576          |                 | 0.814          | 0.646          |                 |                 |                 |                 |
| Nam.| ExF1           | MaFDR          | MaFNR          | MaF1           |                 |                 |                 |                 |                 |
| Frd.| 6.601e-08      | 6.601e-08      | 1.727e-07      | 9.899e-08      |                 |                 |                 |                 |                 |
| Rank| 2.929 1.571 1.500 | 2.929 1.536 1.536 | 1.143 2.357 2.500 | 2.893 1.500 1.607 |                 |                 |                 |                 |                 |
| 1   | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    |                 |                 |                 |                 |                 |
| 2   | 1.000          | 0.662          |                 | 0.438          | 0.991          |                 |                 |                 |                 |
| Nam.| MiFDR          | MiFNR          | MiF1           |                 |                 |                 |                 |                 |                 |
| Frd.| 3.415e-09      | 6.585e-09      | 9.631e-09      | 9.631e-09      |                 |                 |                 |                 |                 |
| Rank| 3.000 1.679 1.321 | 1.000 2.464 2.536 | 2.964 1.679 1.357 |                 |                 |                 |                 |                 |                 |
| 1   | 0.000 0.000    | 0.000 0.000    | 0.000 0.000    |                 |                 |                 |                 |                 |                 |
| 2   | 0.056          | 0.630          |                 | 0.412          |                 |                 |                 |                 |                 |
### TABLE 9 Pairwise transformation. Wilcoxon test for Naive Bayes base classifiers – p-values for paired comparisons of investigated methods.

|        | Nam.          | Frd.          | Rank | 1     | 2     | 3     | 1     | 2     | 3     | 1     | 2     | 3     | 1     | 2     | 3     |
|--------|---------------|---------------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|        | Hamming       | 4.710e-09     | 3.000| 1.643 | 1.357 | 2.964 | 1.607 | 1.429 | 3.000 | 1.643 | 1.357 | 1.107 | 2.393 | 2.500 |
|        | Zero-One      | 7.641e-09     | 0.000| 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|        | ExFDR         | 4.710e-09     | 0.508 | 0.782 | 0.036 | 0.864 | 0.920 |
|        | ExFNR         | 4.949e-08     | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

### TABLE 10 Pairwise transformation. Wilcoxon test for KNN base classifiers – p-values for paired comparisons of investigated methods.

|        | Nam.          | Frd.          | Rank | 1     | 2     | 3     | 1     | 2     | 3     | 1     | 2     | 3     | 1     | 2     | 3     |
|--------|---------------|---------------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|        | Hamming       | 7.937e-08     | 2.929 | 1.643 | 1.429 | 2.911 | 1.393 | 1.696 | 2.929 | 1.500 | 1.571 | 1.107 | 2.500 | 2.393 |
|        | Zero-One      | 6.152e-08     | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|        | ExFDR         | 9.242e-08     | 0.508 | 0.319 | 0.745 | 0.339 | 0.236 |
|        | ExFNR         | 2.475e-07     | 0.000 | 0.000 | 0.000 | 0.000 |

|        | Nam.          | Frd.          | Rank | 1     | 2     | 3     | 1     | 2     | 3     | 1     | 2     | 3     | 1     | 2     | 3     |
|--------|---------------|---------------|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
|        | MiFDR         | 4.710e-09     | 3.000 | 1.643 | 1.357 | 1.036 | 2.536 | 2.429 | 3.000 | 1.607 | 1.393 | 1.107 | 2.500 | 2.393 |
|        | MiFNR         | 1.323e-08     | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
|        | MiF1          | 4.710e-09     | 0.316 | 0.630 | 0.630 | 0.316 | 0.630 | 0.630 | 0.316 | 0.630 | 0.630 | 0.316 | 0.630 | 0.630 | 0.316 | 0.630 | 0.630 |
FIGURE 2 Average ranks of for Label Pairwise approach.
How to cite this article: Trajdos Pawel, Majak Marcin (2018), Bayes metaclassifier and Soft-confusion-matrix classifier in the task of multi-label classification, *Computational Intelligence*, ?.