Interacting Ghost Dark Energy Models with Variable $G$ and $\Lambda$

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Abstract

In this paper we consider several phenomenological models of variable $\Lambda$. Model of a flat Universe with variable $\Lambda$ and $G$ is accepted. It is well known, that varying $G$ and $\Lambda$ gives rise to modified field equations and modified conservation laws, which gives rise to many different manipulations and assumptions in literature. We will consider two component fluid, which parameters will enter to $\Lambda$. Interaction between fluids with energy densities $\rho_1$ and $\rho_2$ assumed as $Q = 3Hb(\rho_1 + \rho_2)$. We have numerical analyze of important cosmological parameters like EoS parameter of the composed fluid and deceleration parameter $q$ of the model.

Introduction

In modern cosmology, despite to hard work and interesting ideas, several crucial question still are open, which makes authors to propose different models and different approaches to find keys for the problems. Modern era in theoretical cosmology starts, when observations of high redshift type SNIa supernovae [1-3] reveal the speeding up expansion of our Universe. Then, other series of observations like to investigation of surveys of clusters of galaxies show that the density of matter is very much less than critical density [4], observations of Cosmic Microwave Background (CMB) anisotropy indicate that the Universe is flat and the total energy density is very close to the critical $\Omega_{tot} \simeq 1$ [5]. Faced with these results we started to find realistic models to explain experimental data concerning to the nature of the accelerated expansion of the Universe and a huge number of hypothesis were proposed. For instance, in general relativity framework, the desirable result could be achieved by so-called dark energy: an exotic and mysterious component of the Universe, with negative pressure (we thought that the energy density is always positive) and with negative EoS parameter $\omega < 0$. Dark energy occupies about 73% of the energy of our Universe, other component (dark matter) about 23%, and usual baryonic matter occupy about 4%. The simplest model for a dark energy is a cosmological constant $\omega_\Lambda = -1$ introduced by Einstein. This model has two famous problems: fine-tuning problem and cosmological coincidence problem. Absence of a fundamental mechanism which sets

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the cosmological constant zero or very small value makes researchers to go deeper and deeper in theories to understand the solution of the problem, because in the framework of quantum field theory, the expectation value of vacuum energy is 123 order of magnitude larger than the observed value [6] of cosmological constant. The second problem asks why are we living in an epoch in which the densities of dark energy and matter are comparable. To alleviate these problems alternative models of dark energy suggest a dynamical form of dark energy, which at least in an effective level, can originate from a variable cosmological constant [7, 8], or from various fields, such as a canonical scalar field [9-14] (quintessence), a phantom field, that is a scalar field with a negative sign of the kinetic term [15-23] or the combination of quintessence and phantom in a unified model named quintom [24-37]. By using some basic of quantum gravitational principles we can formulate several other models for dark energy, and in literature they are known as holographic dark energy paradigm [38-49] and agegraphic dark energy models [50-52]. Interaction between components is proved to be other way which can solve coincidence problem. From observations no piece of evidence has been so far presented against interactions between dark energy and dark matter. From theoretical side we have not any known symmetry which prevents or suppresses a non-minimal coupling between dark energy and dark matter.

Research in theoretical cosmology proposes two possible ways to explain later time accelerated expansion of the Universe. Remember that field equations make connection between geometry and matter content of Universe in a simple way. Therefore, there is two possibilities either we should modify matter content which is coded in energy-stress tensor or we should modify geometrical part including different functions of Ricci scalar etc. Different type of couplings between geometry and matter could give desirable effects as well. Recently, huge number of articles appears, where we are trying to make connection between scalar field and other models of dark energy. In literature, often used idea of fluid despite to other ideas, because over the years we learned that modifications of geometrical part of field equations can be coded in fluid expression. From this point of view an important component becomes to be Equation of State (EoS), which makes connection between energy density and pressure. Well studied examples are barotropic fluid $P = \omega \rho$ with its modifications like $P = \omega(t) \rho^n$. In this contexts other interesting parametrization is a barotropic fluid with more general form (see for instance in [53] and references therein) or Chaplygin gas models [54-59],

$$P = \mu \rho - \frac{B}{\rho^n},$$

where $\mu$ is a positive constant. This model is more appropriate choice to have constant negative pressure at low energy density and high pressure at high energy density. The special case of $\mu = \frac{1}{3}$ is the best fitted value to describe evolution of the universe from radiation regime to the $\Lambda$CDM regime. In [60] one of the authors motivated by a series of works [61-64] proposed a model of varying generalized Chaplygin gas and considered its sign-changeable interaction of the form $Q = q(3Hb\rho + \gamma \dot{\rho})$ with Tachyonic Fluid. We also can consider fluids with more general form of EoS given as,

$$f(\rho, P) = 0.$$  

Today, we do not feel lack of models for DE, which could be seen from the number of references given above. However, all of them are phenomenological models and wait to be proved by observational data. The same can be said for DM, which thought to operate on large scales and be responsible for structure formation, evolution etc.

Apart attempts of fluid modifications in modern cosmology modification of geometrical part also very popular subject of discussions. Examples are $F(R)$, $F(T)$, $F(G)$ etc just to mention a few. Models of this origin however contain some future singularities which can be solved in principle. But, there are models also, that can explain accelerated expansion without any DE, for instance, Cardassian Universe [65-70]. In this model, one need to modify Friedmann equations and therefore having usual matter is enough.

It is well known that Einstein equations of general relativity do not permit any variations in the gravitational constant $G$ and cosmological constant $\Lambda$ because of the fact that the Einstein tensor has zero divergence and energy conservation law is also zero. So, some modifications of Einstein equations are necessary. This is because, if we simply allow $G$ and $\Lambda$ to be a variable in Einstein equations, then energy conservation law
is violated. Therefore, the study of the varying $G$ and $\Lambda$ can be done only through modified field equations and modified conservation laws. It was Dirac who proposed possibility of variation in $G$, which open a door for a lot of works and manipulations. This period in cosmology can be called era of Dirac’s Large Number Hypothesis. For instance, observation of spinning-down rate of pulsar $PSR J2019 + 2425$ provides the result,

$$\left| \frac{\dot{G}}{G} \right| \leq (1.4 - 3.2) \times 10^{-11} \text{yr}^{-1}. \quad (3)$$

Depending on the observations of pulsating white dwarf star $G117 - B15A$, Benvenuto et al. [71] have set up the astroseismological bound as,

$$-2.50 \times 10^{-10} \leq \left| \frac{\dot{G}}{G} \right| \leq 4 \times 10^{-10} \text{yr}^{-1}. \quad (4)$$

For a review to ”Large Number Hypothesis” (LNH) we refer our readers to [72] and references therein.

In this paper we would like to propose different phenomenological models cosmological constant:

1. $\Lambda(t) = \rho_1 + \rho_2 e^{-tH}$,
2. $\Lambda(t) = H^2 + (\rho_1 + \rho_2) e^{-tH}$,
3. $\Lambda(t) = t^{-2} + (\rho_1 - \rho_2) e^{-tH}$.

We will consider two component composed fluid for our Universe and $\rho_1$ and $\rho_2$ referred to the energy densities of the fluid components. We use an interaction between fluids of the $Q = 3Hb(\rho_1 + \rho_2)$ form and analyze important cosmological parameters like EoS parameter of a fluid and deceleration parameter $q$ of the model. This article is organized in following way. Section introduction is devoted to introduce basic ideas and gives some general information related to the research field and our motivation. Next section review FRW Universe with variable $G$ and $\Lambda$. In section ”Interacting Fluids and Model Setup” we recall the basics of origin of the fluids and general settings how problem can be solved. In other sections we consider various models of variable $\Lambda$ and then we give conclusions.

**FRW Universe with Variable $G$ and $\Lambda$**

Flat FRW Universe described by the following metric,

$$ds^2 = -dt^2 + a(t)^2 (dr^2 + r^2 d\Omega^2), \quad (5)$$

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$, and $a(t)$ represents the scale factor. Also, field equations that govern our model with variable $G(t)$ and $\Lambda(t)$ (see for instance [73]) are,

$$R^{ij} - \frac{1}{2} R g^{ij} = -8\pi G(t) \left( T^{ij} - \frac{\Lambda(t)}{8\pi G(t)} g^{ij} \right), \quad (6)$$

where $G(t)$ and $\Lambda(t)$ are function of time. These leads to the following Friedmann equations,

$$H^2 = \frac{\dot{a}^2}{a^2} = \frac{8\pi G(t)\rho}{3} + \frac{\Lambda(t)}{3}, \quad (7)$$

and,

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3} (\rho + 3P) + \frac{\Lambda(t)}{3}. \quad (8)$$

Energy conservation $T_{ij}^{\;\;\;j} = 0$ reads as,

$$\dot{\rho} + 3H (\rho + P) = 0. \quad (9)$$
Combination of the equations (7), (8) and (9) gives the relationship between $G(t)$ and $\dot{\Lambda}(t)$ as the following,

$$G = -\frac{\dot{\Lambda}}{8\pi\rho}.$$  \hfill (10)

Subject of our interest is to consider composed fluids. Basic components thought to be a barotropic fluid with EoS $P_b = \omega(t)\rho_b$, where varying EoS parameter given by $\omega = \omega_0 + \omega_1 \frac{\dot{H}}{H^2}$, and ghost dark energy. Among various models of dark energy, a new model of dark energy called Veneziano ghost dark (GD) energy, which supposed to exist to solve the $U(1)_A$ problem in low-energy effective theory of QCD, and has attracted a lot of interests in recent years [74-86]. Indeed, the contribution of the ghosts field to the vacuum energy in curved space or time-dependent background can be regarded as a possible candidate for the dark energy. It is completely decoupled from the physics sector. Veneziano ghost is unphysical in the QFT formulation in Minkowski space-time, but exhibits important non trivial physical effects in the expanding Universe and these effects give rise to a vacuum energy density $\rho_{GD}$ in Minkowski space-time, but exhibits important non trivial physical effects in the expanding Universe and it can be argued that the form of this behavior can be result of the fact of the very complicated topological structure of strongly coupled QCD. This model has advantage compared to other models of dark energy, which can be explained by standard model and general relativity. Comparison with experimental data, reveal that the current data does not favorite compared to the ΛCDM model, which is not conclusive and future study of the problem is needed. Energy density of ghost dark energy may reads as,

$$\rho_{GD} = \theta H,$$  \hfill (11)

where $H$ is Hubble parameter $H = \dot{a}/a$ and $\theta$ is constant parameter of the model, which should be determined. The relation (11) generalized by the Ref. [87] as the following,

$$\rho_{GD} = \theta H + \vartheta H^2,$$  \hfill (12)

where $\theta$ and $\vartheta$ are constant parameters of the model. Such kind of fluids could be named as a geometrical fluids, because it is clear that it contains information about geometry of the space-time and metric. Recently a model of varying ghost dark energy were proposed in the Ref. [88]. We will assume that components are interacting on Universe with variable $G$ and $\Lambda$. This model is a phenomenological and we are interested by the evolution of the Universe with this setup. As there is an interaction between components, there is not energy conservation for the components separately, but for the whole mixture the energy conservation is hold. The forms of interaction term considered in literature very often are of the following forms: $Q = 3Hb\rho_{dm}$, $Q = 3Hb\rho_{de}$, $Q = 3Hb\rho_{tot}$, where $b$ is a coupling constant and positive $b$ means that dark energy decays into dark matter, while negative $b$ means dark matter decays into dark energy. Other forms for interaction term considered in literature are $Q = \gamma \dot{\rho}_{dm}$, $Q = \gamma \dot{\rho}_{de}$, $Q = \gamma \dot{\rho}_{tot}$, and $Q = 3Hb\gamma \rho_i + \gamma \dot{\rho}_i$, where $i = \{dm, de, tot\}$. These types of interactions are either positive or negative and can not change sign. However, Cai and Su found that the sign of interaction $Q$ in the dark sector changed in the redshift range of $0.45 \leq z \leq 0.9$ and a sign-changeable interaction in the Refs. [64] and [89] introduced as,

$$Q = q(\gamma \dot{\rho} + 3bH\rho),$$  \hfill (13)

where $\gamma$ and $b$ are dimensionless constants, the energy density $\rho$ could be $\rho_m$, $\rho_{de}$, and $\rho_{tot}$. $q$ is the deceleration parameter given by,

$$q = -\frac{\ddot{a}}{H^2 a} = -1 - \frac{\dot{H}}{H^2}.\hfill (14)$$

For $\Lambda$ were considered over years different forms based on phenomenological approach, some of examples are, for instance, $\Lambda \propto \dot{a}/a$, $\Lambda \propto a$ or $\Lambda \propto \rho$ to mention a few. As we are interested by toy models we pay our attention to the problem from a numerical investigation point of view and we believe that after some effort we also can provide exact solutions for the problem, which will be done in other forthcoming articles. In the next section we consider two components fluid Universe with the sign-changeable interaction (13).
Interacting Fluid and Model Setups

Two-component fluid, in our case, will be described by total energy density \( \rho = \rho_b + \rho_{GD} \) and pressure \( P = P_b + P_{GD} \), where \( b \) stands for barotropic fluid. It is well known that in case of an interaction \( Q \) between fluid components we should consider following conservation equations,

\[
\dot{\rho}_{GD} + 3H(\rho_{GD} + P_{GD}) = -Q, \tag{15}
\]

and,

\[
\dot{\rho}_b + 3H(\rho_b + P_b) = Q. \tag{16}
\]

We will use interaction term of the form,

\[
Q = 3Hb(\rho_b + \rho_{GD}) \tag{17}
\]

which introduced in the introduction. Taking into account the equation (16), form of interaction term (17) and GD density (11) we can write,

\[
\dot{\rho}_b + 3H\rho_b(1 - b + \omega(t)) - 3\theta b H^2 = 0. \tag{18}
\]

From the equation (15) we will have pressure for GD energy as the following,

\[
P_{GD} = -b\rho_b - \theta(b + 1)H - \frac{\theta H}{3H}. \tag{19}
\]

One of the cosmological parameters, which we are interested, is EoS parameter of the composed fluid which reads as,

\[
\omega_{tot} = \frac{P_b + P_{GD}}{\rho_b + \rho_{GD}}, \tag{20}
\]

which reduced to the following expression,

\[
\omega_{tot} = \frac{(\omega(t) - b)\rho_b - \theta(1 + b)H - \frac{\theta H}{3H}}{\rho_b + \theta H}, \tag{21}
\]

where we used equations (11) and (18). EoS of GD energy also can be expressed as a function of \( \rho_b \) and other parameters of the models as the following,

\[
\omega_{GD} = -(1 + b) - b \frac{\rho_b}{\theta H} - \frac{\dot{H}}{3H^2}. \tag{22}
\]

In order to obtain cosmological parameters we will use the following models of cosmological constant.

Model 1

In the first model we consider,

\[
\Lambda(t) = \rho_b + \rho_{GD}e^{-tH}. \tag{23}
\]

Using this relation in Friedmann equation together with the results of the previous section we can obtain,

\[
A\dot{H} + \frac{3}{2}H^2 + BH + C = 0, \tag{24}
\]

where \( A, B \) and \( C \) define as the following,

\[
A = 1 + 4\pi G(t)(\omega_1\rho_b - \frac{H}{\theta H}), \tag{25}
\]

\[
B = -\frac{\theta}{2}(e^{-tH} + 8(1 + b)\pi G(t)), \tag{26}
\]

\[
C = \frac{\theta}{2} \frac{\dot{H}}{3H^2}. \tag{27}
\]
and,

\[ C = -\frac{\rho_0}{2} (1 - 8(\omega_0 - b)G(t)). \]  

Therefore, we can obtain the following equation,

\[ \dot{G} = -\frac{\rho_0 + \theta \dot{H} e^{-tH} - \theta H^2 \dot{H} e^{-tH}}{8\pi(\omega(t)\rho_0 + \theta H)}. \]  

We solved this equation numerically and find \( G \) as an increasing function of time (see Fig. 1).

![Figure 1: Model 1](image)

![Figure 2: Model 1](image)

Using the equations (8) and (14) we can investigate deceleration parameter numerically. We draw this parameter in the Fig. 2 and find that \( q \) is increasing function of time. In some cases we see that \( q \rightarrow -1 \). we know that all forms of matter have the condition \( q \geq -1 \) which is satisfied. Also, energy density \( \rho = \rho_0 + \rho_{\Omega} \) illustrated in the Fig. 3 and total EoS parameter drawn in the Fig. 4.
Therefore, we can obtain the following equation,

\[ A(t) = H^2 + (\rho_b + \rho_{GD})e^{-tH}. \]  

Using this relation in Friedmann equation together with the results of the previous section we can obtain the following equation describing dynamics of Hubble parameter,

\[ \dot{A}H + H^2 + BH + D = 0, \]  

where \( A \) and \( B \) are given by the relations (25) and (26), but \( D \) is defined as the following,

\[ D = -\frac{\rho_b}{2} \left( e^{-tH} - 8(\omega_0 - b)\pi G(t) \right). \]  

Therefore, we can obtain the following equation,

\[ G = -\frac{2\dot{H} + (\rho_b + \theta H)e^{-tH} - (\rho_b + \theta H)\dot{H}e^{-tH}}{8\pi(\omega(t)\rho_b + \theta H)}. \]
We can see that \( \rho \) obtain also, energy density \( \rho \) parameter in the Fig. 6, and similar to the previous model, find that using the equations (8) and (14) we can investigate deceleration parameter numerically. We draw this the value of \( \omega \) effect on \( G \) also, similar to the model 1 we see that increasing \( \omega_0 \) decreases \( G \) but increasing \( \omega_1 \) increases the value of \( G \).

Using the equations (8) and (14) we can investigate deceleration parameter numerically. We draw this parameter in the Fig. 6, and similar to the previous model, find that \( q \) is increasing function of time which shows that \( q \geq -1 \) satisfied. We can choose parameters as \( \omega_0 = 0.2, \omega_1 = 2.5, b = 0.02 \) and \( \theta = 2.5 \) to obtain \( q \to -1 \) at the late time.

Also, energy density \( \rho = \rho_b + \rho_{CD} \) illustrated in the Fig. 7 and total EoS parameter drawn in the Fig. 8. We can see that \( \rho \) and \( \omega_{tot} \) are decreasing function of time.
Using this relation in Friedmann equation together with the results of the previous section we can study dynamics of Hubble parameter,\[ A(t) = t^{-2} + (\rho_b - \rho_{GD})e^{-tH}. \] (33)

Using this relation in Friedmann equation together with the results of the previous section we can study dynamics of Hubble parameter, \[ A\dot{H} + \frac{3}{2}H^2 + EH + F = 0. \] (34)

where \( A \) is given by the equation (25), but \( E \) and \( F \) are given by the following relations,
\[ E = \frac{\theta}{2} \left( e^{-tH} - 8(1 + b_1)\pi G(t) \right), \] (35)
and,
\[ F = -\frac{t^{-2}}{2} - \frac{\rho_b}{2} \left( e^{-tH} - 8(\omega_0 - b_1)\pi G(t) \right). \] (36)

**Model 3**

In the third model we consider,
\[ \Lambda(t) = t^{-2} + (\rho_b - \rho_{GD})e^{-tH}. \]
For $\dot{G}$ we can give the following expression,

$$
\dot{G} = \frac{-2t^{-2} - (\dot{\rho}_b + \theta \dot{H})e^{-tH} - (\rho_b - \theta H)HH e^{-tH}}{8\pi(\omega(t)\rho_b + \theta H)}.
$$

(37)

Figure 9: Model 3

Figure 10: Model 3

Equation (37) may be solve numerically to find behavior of $G$ as an increasing function of time (see Fig. 9).

Using the equations (8) and (14), deceleration parameter also drawn in the Fig. 10, which has expected behavior.

Also, energy density $\rho = \rho_b + \rho_{GD}$ illustrated in the Fig. 11 and total EoS parameter drawn in the Fig. 12. We can see that $\rho$ and $\omega_{tot}$ are decreasing function of time.
In the first model, where cosmological constant is of the form of the equation (23), the variable $G$ found increasing function of time, as illustrated in the Fig. 1. The first plot of the Fig. 1 shows that increasing $\omega_0$ decreases the value of $G$. This is opposite for the $\omega_1$ (see second plot of the Fig. 1). Dependence of $G$ to interaction parameters $b$ and $\theta$ illustrated in the third and fourth plots of the Fig. 1. They have shown that both $b$ and $\theta$ increase the value of $G$. We have seen that variation of $G$ is approximately linear for time. For the second model we found similar behavior to the first model (see Fig. 5). This situation has few differences for the third model. the first and second plots of the Fig. 9 show similar behavior to the previous models with variation of $\omega_0$ and $\omega_1$, but there are no linearity in this case, at least initially. The variable $G$ grows rapidly to $t = 5$ and then has approximately linear behavior. Also similar to first and second cases,
increasing $b$ increases $G$, but increasing $\theta$ decreases $G$ which is opposite with the previous cases. Variation of the deceleration parameter with time for all models illustrated in the Fig. 2, 6 and 10. They have shown that the deceleration parameter is decreasing function of time. The first and second plots of these figures have shown increasing with $\omega_0$ and decreasing with $\omega_1$. Also we found that increasing interaction parameters $b$ and $\theta$ decreases the value of $q$. We know that $q \geq -1$ is important condition which satisfied with some values of our parameters. The best selected parameters to have $q \rightarrow -1$ at the late time are $\omega_0 = 0.2, \omega_1 = 2.5, b = 0.02$ and $\theta = 2.5$ which are identical for all models.

Total energy density presented by the Figs. 3, 7 and 11 which shown that increasing $\omega_0$ and $\theta$ increase value of $\rho$, but increasing $\omega_1$ and $b$ decrease value of $\rho$. As expected, energy densities of all models are decreasing function of time.

Total EoS drawn by the Figs. 4, 8 and 12 which shown that increasing $\omega_0$ increase value of $\omega_{tot}$, but increasing $\omega_1, \theta$ and $b$ decrease value of $\omega_{tot}$. We found that the total EoS corresponding to three models are decreasing function of time.

We also studied scale factor, total pressure and barotropic EoS parameter numerically. Plots of the Fig. 13 shown behavior of scale factor in the first model which is increasing function of time. We found that increasing $\omega_1$ and $b$ increase $a$, but increasing $\omega_0$ decreases $a$. The last plot of the Fig. 13 shows that variation of $\theta$ is not important for the scale factor.

Plots of the Fig. 14 show behavior of total pressure in the first model which is increasing function of time. We found that increasing $\omega_1$ and $b$ increase $P = P_b + P_{GD}$, but increasing $\omega_0$ and $\theta$ decrease $P$. $\omega_b$ of the first model plotted in the Fig. 15 which is increasing function of time. The first plot of the Fig. 15 tells that increasing $\omega_0$ decreases the value of $\omega_b$, while increasing $\omega_1, \theta$ and $b$ increase $\omega_b$.

Variation of the scale factor of the second model with respect to $\omega_0, \omega_1$ and $b$ is similar to the first model (see Fig. 16). The last plot of the Fig. 16 tells that increasing $\theta$ increases scale factor for the late time, but has not important effect at the early time.

in the Fig. 17 and Fig. 18 we see that total pressure and barotropic EoS of the second model are similar to the first model.

Fig. 19 and 20 represent time-dependent scale factor and total pressure of the third model, respectively, which are increasing function of time and have similar manner with the previous models.

Finally, Fig. 21 includes plots of $\omega_b$ of the third model and has similar description with previous models. We conclude that all models of cosmological constant which introduced in this paper yields to acceptable behavior of cosmological parameters.

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Figure 13: Model 1

Figure 14: Model 1
Figure 15: Model 1

Figure 16: Model 2
Figure 17: Model 2

Figure 18: Model 2
Figure 19: Model 3

Figure 20: Model 3
Figure 21: Model 3