**Abstract**

This paper presents the geometrical representation of the load demand by using orbits diagrams in the Mandelbrot set, to identify changing behaviors during a day period of the real and reactive powers. To perform this, different power combinations were used to represent the fractal diagrams with an algorithm that considers the mathematical model of Mandelbrot set and orbits diagrams. A qualitative analysis of the orbits is performed to identify the fractal graphic patterns with respect to the real and reactive power consumptions. The results show repetitive graphic patterns in the fractal space of the power consumption during the day, which help represent the consumption behavior on a daily load demand curve. The orbit diagrams save form and structure relations during the daily behavior of the power consumption. This work shows a different method of evaluating load demand behavior by using orbit diagrams as a potential tool that will lead to identify load behavior, useful in operational decisions and power system planning.

**Keywords:** Real power, reactive power, fractal geometry, Julia set, Mandelbrot set, behavior patterns, power factor

1 **Introduction**

Mathematician Benoit Mandelbrot has defined the concept of fractals as a semi-geometric element with a repetitive structure at different scales [1], with characteristics of self-similarity as seen in some natural formations such as snowflakes, ferns, peacock feathers, and romanesco broccoli. Fractal theory has been applied to various fields such as biology [2,3], health sciences [4–8], stock markets [9], network communications [10–12], and others. Fractal theory is one of the methods used to analyze data and obtain relevant information in highly complex problems. Thus, it has been used to study the price of highly variable markets, which are not always explainable from classical economic analysis.

For example, in [9], the authors demonstrate that current techniques have some issues to explain the real market operation and a better understanding is achieved by using techniques such as chaos theory and fractals. In their publication, the authors show how to apply fractal behavior to stock markets and refer to multifractal analysis and multifractal topology. The first describes the invariability of scaling properties of time series and the second is a function of

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the Hölder exponents that characterize the degree of irregularity of the signal, and their most significant parameters.

In [13], the authors discuss the basic principle of fractal theory and how to use it to forecast the short-term electricity price. In the first instance, the authors analyze the fractal characteristic of the electricity price, confirming that price data have this property. In the second instance, a fractal model is used to build a forecasting model, which offers a wide application in determining the price of electricity in the markets.

Similarly, the authors of [14] demonstrate that the price of thermal coal has multifractal features by using the concepts introduced by Mandelbrot-Bouchaud. Hence, a quarterly fluctuation index (QFI) for thermal power coal price is proposed to forecast the coal price caused by market fluctuation. This study also provides a useful reference to understand the multifractal fluctuation characteristics in other energy prices.

Fractal geometry analysis has been also applied to study the morphology and population growth of cities, and to electricity demand related to the demography of cities. In [15], a multifractal analysis is used to forecast electricity demand, explaining that two fractals are found that reflect the behavior pattern of load demand. Two concepts linked to fractal geometry are fractal interpolation and extrapolation, which are related to the resolution of a fractal-encoded image. In [16], an algorithm is used to forecast the electric charge in which fractal interpolation and extrapolation are also involved; for the forecast dataset, the average relative errors are only 2.303% and 2.296%, respectively, indicating that the algorithm has advantages in improving forecast accuracy.

In [17] a design method of antenna array consisting of eight microstrip patches modified with Sierpinski fractal curves has been presented and experimentally validated in this paper. Method proposed has enabled the achievement of considerable miniaturization of array length (26%), together with multi-band behavior of the antenna, which proves the attractiveness of presented design methodology and its ability to be implemented in more complex microstrip structures.

In [18] is studied the application of Triangular Prism Method (TPM) algorithm in computer assisted Papanicolaou smears analysis that is useful in cervical cancer screening. The TPM algorithm allows estimation of the FD (fractal dimension) for optical density of cell nuclei. Selection of the local FD for green color channel gives efficient separation between both cell nuclei classes. Proposed algorithm (Tiled TPM) improves separation by the fractal based estimation using larger area of the cell nuclei.

No paper in the extant literature has examined the orbit diagrams in the Mandelbrot set to represent the daily load demand. Therefore, this work focuses on studying the behavior of the different combination of complex numbers corresponding to a daily load demand, which form different orbit diagrams in the Mandelbrot set. Particularly, in this work, the real and reactive powers are used to represent the different orbit diagrams calculated with the Mandelbrot algorithm, perform observations, and analyze qualitatively fractal geometry related to load demand.
For this reason, this paper proposes the real and reactive powers of load demand curve can be characterized by orbit diagrams in the Mandelbrot set. The test focuses on identifying a clear pattern with similarities in the daily load demand. Besides, this method is proposed to obtain a new way of visualizing the behavior of load demand curves, identify the stability of the orbit diagram according to the variation in the electric power consumption, and identify easily loadability of the power system. The rest of this document is organized as follows. Section 2 includes a brief explanation of the theory of the Mandelbrot sets and orbit diagrams. Section 3 presents the results and discusses the most relevant examples of orbits created for the real and reactive powers. Finally, the main conclusions of this research work are summarized.

2 Research method

An algorithm that creates fractal diagrams applied to the typical load demand curve is presented, with the aim of identifying patterns from orbit diagrams in the Mandelbrot set that represent power consumption. Below, this section shows the general procedure and the algorithms implemented to obtain the fractals.

2.1 General procedure

Figure 1 presents a step-by-step procedure applied to graph the fractal diagrams from the load demand with the Mandelbrot and Julia sets. This figure shows that the first step (P1) is to convert the initial data to manage the procedure to the Mandelbrot and Julia algorithms. Next, the Mandelbrot algorithm is programmed according to the mathematical theory (P2) to generate a new data set. Besides, the Julia algorithm is also programmed to perform the generation of the new sets, based on the Mandelbrot set (P3). With these data sets, it is possible to plot the different fractals (P4) which are then analysed to present the different results in this paper (P5) and the corresponding conclusions.
2.2 Load demand curve

The load demand has a strong daily pattern that represents the working days as the data have a very similar demand profile. Thus, the time series is seasonal because it has a regular repetition pattern during the same period of time. The periodic behavior is reflected in parameters such as mean, standard deviation, asymmetry, and autocorrelation asymmetry.

With the purpose to study the orbit diagram of the points related to the evolution function $Z_{t+1} = Z_t^2 + C$, that moves on the set of Mandelbrot, according to the law dictated by the load demand curve. The process begins by reading the typical load demand records of real and reactive powers defined for a 24-hour period (see Table 1). The per unit values of the load demand are calculated with the following expression: $Per\_unit\_value = Actual\_MVA/Base\_MVA$. In this case, the base power is 4000 MVA. These data are used to plot the diagrams.
Table 1: Daily power demand

| Hour  | P   | Q   | $P_{pu}$ | $Q_{pu}$ |
|-------|-----|-----|----------|----------|
| 00:00:00 | 889 | 371 | 0.222   | 0.092   |
| 01:00:00 | 834 | 405 | 0.208   | 0.101   |
| 02:00:00 | 792 | 337 | 0.197   | 0.082   |
| 03:00:00 | 790 | 324 | 0.199   | 0.081   |
| 04:00:00 | 804 | 323 | 0.201   | 0.080   |
| 05:00:00 | 925 | 355 | 0.231   | 0.088   |
| 06:00:00 | 1041| 482 | 0.260   | 0.120   |
| 07:00:00 | 1105| 556 | 0.276   | 0.139   |
| 08:00:00 | 1191| 610 | 0.297   | 0.152   |
| 09:00:00 | 1256| 704 | 0.314   | 0.176   |
| 10:00:00 | 1309| 744 | 0.327   | 0.186   |
| 11:00:00 | 1366| 775 | 0.341   | 0.193   |
| 12:00:00 | 1385| 793 | 0.346   | 0.198   |
| 13:00:00 | 1356| 774 | 0.339   | 0.193   |
| 14:00:00 | 1337| 759 | 0.334   | 0.189   |
| 15:00:00 | 1350| 774 | 0.337   | 0.193   |
| 16:00:00 | 1336| 773 | 0.334   | 0.193   |
| 17:00:00 | 1312| 749 | 0.328   | 0.187   |
| 18:00:00 | 1287| 687 | 0.321   | 0.171   |
| 19:00:00 | 1420| 683 | 0.355   | 0.170   |
| 20:00:00 | 1389| 660 | 0.351   | 0.167   |
| 21:00:00 | 1311| 605 | 0.327   | 0.151   |
| 22:00:00 | 1175| 544 | 0.293   | 0.136   |
| 23:00:00 | 1030| 489 | 0.257   | 0.122   |

with the algorithms, in which the lowest and highest consumption points are considered to evaluate the different fractal diagrams.
2.3 Algorithm to create the Mandelbrot set

Mandelbrot set, denoted as \( M = \{ c \in \mathbb{C} / J_c \} \), represents the sets of complex numbers \( C \) obtained after iterating the from the initial point \( Z_n \) and the selected constant \( C \) as shown \([1]\), the results form a diagram with connected points remaining bounded in an absolute value. One property of \( M \) is that the points are connected, although in some zones of the diagram it seems that the set is fragmented. The iteration of the function generates a set of numbers called orbits. The results of the iteration of those points out of the boundary set tend to infinity.

\[
Z_{n+1} = F(Z_n) = Z_n^2 + C
\]  

(1)

From the term \( C \), a successive recursion is performed with \( Z_0 = 0 \) as the initial term. If this successive recursion is dimensioned, then the term \( C \) belongs to the Mandelbrot set; if not, then they are excluded. Therefore, Figure 2 shows the Mandelbrot set with points in the black zone that are called the prisoners, while the points in other colors are the escapists and they represent the velocity that they escape to infinity.

**Figure 2: Representation of Mandelbrot diagram**

![Mandelbrot Set Diagram](image)
From this figure, the number -1 is inside of the set while the number 1 is outside of the set. In the Mandelbrot set, the fractal is the border and the dimension of Hausdorff is unknown. If the image is enlarged near the edge of the set, many areas the Mandelbrot set are represented in the same form. Besides, different types of Julia sets are distributed in different regions of the Mandelbrot set. Whether a complex number appears with a greater value than 2 in the 0 orbit, then the orbit tends to infinity.

The pseudocode of the algorithm that is used to represent the Mandelbrot set is presented as follows:

Start
For each point $C$ in the complex plane do:
   Fix $Z_0 = 0$
   For $t = 1$ to $t_{\text{max}}$ do:
      Calculate $Z_t = Z_t^2 + C$
      If $|Z_t| > 2$ then
         Break
      End if
      If $t < t_{\text{max}}$ then
         Draw $C$ in white (the point does not belong to the set)
      Else if $t = t_{\text{max}}$ then
         Draw $C$ in black (as the point does belong to the set)
      End if
   End For
End

In this research, the presented algorithm has been used to obtain the Mandelbrot set and the diagram that represent it. Some points related to the real and reactive powers with the respective signs are studied in the Mandelbrot set and related to those points created for the orbits diagrams as explained in the following sections.

2.4 Orbit diagrams and attractors

One way to visualize the state of a system is through the orbit diagram. An orbit is a set points related with the evaluation function of a dynamic system. For discrete dynamics systems the orbits are successions. The basic classifications can be defined as: (a) fix points (b) periodic orbits, and (c) orbits no constants [19]. With respect to the attractor, the main properties are a) compression, b) expansion, and c) folding [20].

If there is an attractor in the complex plane, the orbit associated with a complex number of the form $z = a + bi$ is an orbit of complex numbers, with
the same dynamics. The orbits are a sequence of complex numbers and their characteristics depend fundamentally on the values of the initial point $Z_n$ from which it is split and the selected constant $C$. The pseudocode used to generate orbit diagrams and find the attractor of a complex number inside the $M$ set is described as follows.

**Start**

1. Read $C$
2. Fix $Z_0 = C$
3. **For** $t = 1$ to $t_{MaxNumOrbits}$ **do**:
   - Calculate $Z_t = Z_t^2 + C$
   - If $|Z_t| > 2$ **then**
     - Break
   - **End if**
   - Draw orbits of $Z_t$
   - $Z_t = Z_{t+1}$
4. **End For**

**End**

For each hour the number of orbits and the attractor value is shown in the table 2.

### 2.5 Algorithm to study the orbits of the load demand

In order to obtain the results of the fractal topology patterns that represent the real and reactive power of the load demand curve, the procedure shown in Fig. 3 was followed. The algorithm begins by reading the data of the real and reactive power, in which the comparative curve can be obtained for the different graphs to be made. Then, with these same power values and the reading of the power base, the calculation of the values per unit of power can be made.

A third step corresponds to calculate the $M$ set using the Mandelbrot algorithm and with this the values of real and reactive powers are adjusted within the $M$ set. The fourth step was to graph the respective orbit diagrams associated to the Mandelbrot set. Finally, the fifth step considers the evaluation of the orbit diagrams related to the daily power consumption.
| Hour   | Attractor | Num.orbits |
|--------|-----------|------------|
| 00:00:00 | 0.319     | 5          |
| 01:00:00 | 0.294     | 5          |
| 02:00:00 | 0.274     | 5          |
| 03:00:00 | 0.276     | 5          |
| 04:00:00 | 0.280     | 3          |
| 05:00:00 | 0.305     | 5          |
| 06:00:00 | 0.371     | 9          |
| 07:00:00 | 0.393     | 9          |
| 08:00:00 | 0.415     | 18         |
| 09:00:00 | 0.433     | 30         |
| 10:00:00 | 0.448     | 32         |
| 11:00:00 | 0.463     | 50         |
| 12:00:00 | 0.468     | 53         |
| 13:00:00 | 0.462     | 44         |
| 14:00:00 | 0.457     | 38         |
| 15:00:00 | 0.464     | 41         |
| 16:00:00 | 0.456     | 41         |
| 17:00:00 | 0.451     | 41         |
| 18:00:00 | 0.445     | 41         |
| 19:00:00 | 0.486     | 89         |
| 20:00:00 | 0.482     | 89         |
| 21:00:00 | 0.449     | 41         |
| 22:00:00 | 0.416     | 18         |
| 23:00:00 | 0.365     | 14         |
3 Results and analysis

Figure 4 presents the typical load demand curve plotted with the data of Table 1 and Fig. 5 presents the load demand plotted in the first quadrant of the complex plane. As real and reactive powers are positive, they represent a load consumption related to inductive elements. Under these conditions, the three most interesting values of the power consumption that are selected are the lowest consumption at 3:00, the highest consumption at 19:00, and the approximate average consumption at 09:00. Other hours of the day represent diagrams that are forms between the values as shown in the following results in this section.
Figure 4: Typical load demand in a day

Figure 4 shows a very low power consumption value at 03:00, which then increases as the hours pass in the day.
In addition, it is observed that the curve presents changes during the different hours of the day, due to the combinations of real and reactive powers. In some cases, the curves cross indicating that the complex numbers formed are equal at different times of the day. Due to the number of curves that can be created with orbit diagrams, only those that are marked in this curve have been created to obtain an analysis of them.

Figure 6 shows the orbit diagram generated for each point in Fig. 5. These orbits are created by performing iterations of the complex numbers obtained from the daily load demand (see table 2).
Figure 6: Representation of orbit diagram of load demand in the first quadrant of the complex plane of M set

(a) 1:00  (b) 3:00  (c) 5:00  (d) 7:00  
(e) 9:00  (f) 11:00  (g) 13:00  (h) 15:00  
(i) 17:00  (j) 19:00  (k) 21:00  (l) 23:00

The process of generating the orbit diagram reveals folds with the following properties: from 1:00 to 11:00 (Fig. 6a - Fig. 6f) the orbits expand causing their space to stretch, associated with the progressive increase in the hourly load demand. From 11:00 to 15:00 (Fig. 6f - Fig. 6h) the orbits remain practically
unaltered, corresponding with a constant demand of the hourly electrical power. Then, at 17:00 (Fig. [6]) the orbit is reduced because of the power reduction and after that at 19:00 the maximum consumption with a large orbit is presented (Fig. [6]). From 21:00 to 23:00 (Fig. [6k] – Fig. [6]) the load demand decreases and the orbit diagram also reduces.

Fig. [7] shows the number of points obtained with the orbit diagram algorithm after iterating the complex numbers of real and reactive powers. The orbit diagram in the complex plane of the Mandelbrot set obeys the daily periodic curve profile of the load demand, which is continuously reduced from 20:00 to 5:00. In the period 00:00 – 05:00, the lower load demand is presented, which corresponds to orbit diagrams of 5 points without folding. From 05:00, the load demand begins to increase and the points are greater than 10, expanding the orbits in the complex plane. Besides, the number of points with highest values correspond to the same peaks in the load demand curve at 12:00 and 19:00. A value close to the peak is presented at 20:00, because the consumption remains also in a high value close to that of the peak at 19:00.

This figure shows some clear relations between the power consumption and the number of points created in the orbits. This result helps to understand the system loadability and the increasing number of points when the system is reaching the consumption limits, as the expansions of orbits are presented in the complex plane.

Figure 7: Orbits obtained from the load demand

![Orbits obtained from the load demand](image-url)
Figure 8 shows the values of the attractor for the different points in the load demand curve (see table 2). This figure shows that the attractor moves according to the behavior of the power consumption, which help to identify a clear pattern that repeat during different days in the power consumption. This result also shows that the real and reactive power consumption can be identified based on the position of the attractor, as the method helps to identify the maximum values reached by the power load.

Figure 8: Attractors obtained from the load demand
4 Conclusions

This paper presented the geometrical representation of the load demand by using orbits diagrams in the Mandelbrot set, to identify changing behaviors during a daily load demand. Power combinations were used to represent the orbit diagrams with an algorithm that considers the mathematical model of Mandelbrot set and orbit diagrams. The results allowed to identify a new space of analysis of the discrete dynamic system based on the daily load demand of real and reactive powers, by means of orbit diagrams in the Mandelbrot set. The orbit diagrams in the complex plane of the Mandelbrot set obtained with real and reactive powers of the load demand curve established the behavior of the iterated sequence for given values of \((P_0, Q_0)\), interpreting the orbit geometry obtained, allowing to quickly discern the behavior of the load demand. As the load demand changes constantly in the daily load demand, the orbit diagrams create small number of points because of the small power consumption and create large number of points because of large power consumption. Stability of the periodic orbits appears and disappears with the power variation in the daily demand curve. The density of the folding of the orbits is related to the proximity of the load demand of real and reactive powers to the limits of the Mandelbrot set.

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