Realistic low-momentum effective interactions and nuclear structure near double closed shells

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Abstract. We report on a shell-model study of nuclei with four valence nucleons in the $^{132}$Sn and $^{208}$Pb regions. This aims at investigating to what extent the striking similarity existing between the low-energy properties of $^{134}$Sb and $^{210}$Bi persists when adding a pair of identical particles. We employ realistic low-momentum effective interactions derived from the CD-Bonn nucleon-nucleon potential through use of the $V_{\text{low-}k}$ approach. The calculated results are in very good agreement with the available experimental data and emphasize the persistence of a close resemblance between the spectroscopy of the two regions when moving away from the one proton-one neutron systems.

1. Introduction

A fundamental problem of nuclear physics is to understand the properties of nuclei starting from the forces among nucleons. Nowadays, there are two main lines of attack to attain this goal. The first one comprises the so-called \textit{ab initio} calculations in which nuclear properties, such as binding and excitation energies, are calculated directly from first principles of quantum mechanics using an appropriate computational scheme. To this category belong the Green’s function Montecarlo Method [1], the no-core shell-model [2], and the coupled cluster method [3]. Clearly, all \textit{ab initio} calculations need huge amount of computational resources and are therefore currently limited to light nuclei.

The second line of attack consists of using the shell model with two-nucleon effective interactions derived microscopically from the bare nucleon-nucleon ($NN$) potential. In this kind of calculations, which are generally referred to as realistic shell-model calculations, only the valence nucleons are treated as active particles while the core polarization effects are taken into account perturbatively in the derivation of the effective interaction $V_{\text{eff}}$. As we shall discuss later, these effects are of key importance for an accurate description of nuclear properties. Of course, this approach allows to perform calculations for medium- and heavy-mass nuclei which are far beyond the reach of \textit{ab initio} calculations.

The two main ingredients of realistic shell-model calculations are the $NN$ potential $V_{NN}$ and the many-body methods for deriving $V_{\text{eff}}$. As regards the first point, there are several high-quality potentials which fit equally well ($\chi^2$/datum $\approx 1$) the $NN$ scattering data up to the inelastic threshold. This means that their on-shell properties are essentially identical, namely they are phase-shift equivalent. This may raise the question of how much nuclear structure
results may depend on the choice of the \( NN \) potential one starts with. We shall come back to this point in the next section.

As regards the derivation of \( V_{\text{eff}} \), a main difficulty is the strong short-range repulsion contained in all modern \( NN \) potentials. As is well known, the traditional way to circumvent this problem is the Brueckner \( G \)-matrix method. However, a few years ago a new approach to the renormalization of \( V_{NN} \) has been proposed \[4\], which has proved to be an advantageous alternative to the use of the \( G \)-matrix \[5\]. This consists in deriving from \( V_{NN} \) a low-momentum potential, \( V_{\text{low}-k} \), that preserves the deuteron binding energy and scattering phase shifts of \( V_{NN} \) up to a certain cutoff momentum \( \Lambda \). Since \( V_{\text{low}-k} \) is a smooth potential, it can be used directly to derive \( V_{\text{eff}} \).

Since its introduction in 2002, we have used this approach to derive realistic low-momentum effective interactions for shell-model calculations in various mass regions. In this paper, we first give an outline of the theoretical framework in which these calculations have been performed, referring the reader to \[5\] for a detailed description. Then we present and discuss some recent results for nuclei with four valence nucleons in the \( ^{132}\text{Sn} \) and \( ^{208}\text{Pb} \) regions, focusing attention on the similarity between the spectroscopy of these two regions. A short summary is given in the last section.

2. Outline of theoretical framework

The shell-model effective interaction \( V_{\text{eff}} \) is defined, as usual, in the following way. In principle, one should solve a nuclear many-body Schrödinger equation of the form

\[
H \Psi_i = E_i \Psi_i ,
\]

with \( H = T + V_{NN} \), where \( T \) denotes the kinetic energy. This full-space many-body problem is reduced to a smaller model-space problem of the form

\[
P H_{\text{eff}} P \Psi_i = P (H_0 + V_{\text{eff}}) P \Psi_i = E_i P \Psi_i .
\]

Here \( H_0 = T + U \) is the unperturbed Hamiltonian, \( U \) being an auxiliary potential introduced to define a convenient single-particle basis, and \( P \) denotes the projection operator onto the chosen model space,

\[
P = \sum_{i=1}^{d} |\psi_i\rangle \langle \psi_i| ,
\]

\( d \) being the dimension of the model space and \( |\psi_i\rangle \) the eigenfunctions of \( H_0 \). The effective interaction \( V_{\text{eff}} \) operates only within the model space \( P \). In operator form it can be schematically written as

\[
V_{\text{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \int \hat{Q} \int \hat{Q} + \ldots ,
\]

where \( \hat{Q} \), usually referred to as the \( \hat{Q} \)-box, is a sum of irreducible valence-linked diagrams, and the integral sign represents a generalized folding operation. \( \hat{Q}' \) is obtained from \( \hat{Q} \) by removing terms of first order in the interaction. Once the \( \hat{Q} \)-box is calculated, the folded-diagram series of equation (4) can be summed up to all orders by iteration methods, as, for instance, the Lee-Suzuki one \[6\].

As pointed out in the Introduction, we “smooth out” the strong repulsive core contained in \( V_{NN} \) by constructing a low-momentum potential \( V_{\text{low}-k} \). This is achieved by integrating out the high-momentum modes of \( V_{NN} \) down to a cutoff momentum \( \Lambda \). This integration is carried out with the requirement that the deuteron binding energy and phase shifts of \( V_{NN} \) up to \( \Lambda \) are preserved by \( V_{\text{low}-k} \). Once the \( V_{\text{low}-k} \) is obtained, we use it, plus the Coulomb force
for protons, as input interaction for the calculation of the matrix elements of the shell-model effective interaction within the framework of the folded-diagram method outlined above. The calculation of $Q$, which is in principle an infinite sum of irreducible diagrams, can only be made approximately by selecting certain classes of diagrams. In our calculations we have included in $Q$ all the 1-body and 2-body diagrams up to second order in $V_{\text{low-k}}$.

A detailed description of the derivation of $V_{\text{low-k}}$ from $V_{NN}$ as well as a discussion of its main features can be found in [4, 5]. However, we should mention here that the use of $V_{\text{low-k}}$ largely reduces the ambiguity in the choice of $V_{NN}$. In fact, we have verified [5] that shell-model effective interactions derived from phase-shift equivalent $NN$ potentials through the $V_{\text{low-k}}$ approach lead to very similar results. In other words, $V_{\text{low-k}}$ gives an approximately unique representation of the $NN$ potential.

3. Calculations and results

We start by giving some details of our shell-model calculations and then present and discuss the results obtained for the four-valence particle nuclei $^{136}$I, $^{136}$Sb and $^{136}$Sn in $^{132}$Sn region and their counterparts in $^{208}$Pb region, $^{212}$At, $^{212}$Bi and $^{212}$Pb. As mentioned in the Introduction, our interest centers on the close resemblance between the spectroscopy of these two regions. In this connection, it is worth noting that the existence of this resemblance was pointed out long ago in [7], where it was noticed that every single-particle proton or neutron state in $^{132}$Sn region, characterized by quantum numbers $(nlj)$, has its counterpart around $^{208}$Pb with quantum numbers $(nl + 1j + 1)$. It has been, however, the substantial progress in gathering information on neutron-rich nuclei around $^{138}$Sn that has made this issue the focus of recent studies [8, 9, 10, 11].

For the nuclei in $^{132}$Sn region we assume that the valence protons occupy the five levels $0g_{7/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, and $0h_{11/2}$ of the 50-82 shell, while for the neutrons the model space includes the six levels $0h_{9/2}$, $1f_{7/2}$, $1f_{5/2}$, $2p_{3/2}$, $2p_{1/2}$, and $0i_{13/2}$ of the 82-126 shell. Similarly, in $^{208}$Pb region we take as model space for the valence protons the six levels of the 82-126 shell and let the valence neutrons occupy the seven levels $1g_{9/2}$, $0i_{11/2}$, $0j_{15/2}$, $2d_{5/2}$, $3s_{1/2}$, $1g_{7/2}$, and $2d_{3/2}$ of the 126-184 shell. As regards the choice of the single-proton and single-neutron energies, they have been taken from experiment. The adopted values are reported in [12] and [13] for the $^{132}$Sn and $^{208}$Pb regions, respectively.

For both regions the two-body effective interaction is derived from the CD-Bonn $NN$ potential [14] renormalized by means of the $V_{\text{low-k}}$ approach, as outlined in the previous section. As in our previous studies (see [11] and references therein), the cutoff momentum $\Lambda$ is given the value 2.2 fm$^{-1}$. The computation of the diagrams included in the $Q$-box is performed within the harmonic-oscillator basis using intermediate states composed of all possible hole states and particle states restricted to the five proton and neutron shells above the Fermi surface. The oscillator parameter is 7.88 MeV for the $A = 132$ region and 6.88 MeV for the $A = 208$ region, as obtained from the expression $\hbar\omega = 45A^{-1/3} - 25A^{-2/3}$. The shell-model calculations have been performed by using the OXBASH computer code [15].

We now present and discuss the results of our study. To start with, in table 1 we compare the calculated excitation energies with the available experimental ones [16, 17, 18] in $^{136}$Sb, $^{136}$I, $^{212}$Bi and $^{212}$At. We see that the agreement between experiment and theory is very good, the discrepancies being below 50 keV for the large majority of states.

To give emphasis to the resemblance between the spectroscopy of the $^{132}$Sn and $^{208}$Pb regions, we show in figure 1 the calculated energies of $^{136}$Sb and $^{136}$I for the lowest states with $J^g = 0^−$ to $J^g = 7^−$ and in figure 2 the energies of the $J^g = 0^−$ to $J^g = 9^−$ states in $^{212}$Bi and $^{212}$At. These curves may be considered the evolution of the $πg_{7/2}νf_{7/2}$ and $πh_{9/2}νg_{9/2}$ multiplets in the one proton-one neutron nuclei $^{134}$Sb and $^{210}$Bi, respectively, which we also report in figures 1 and 2 for the sake of comparison. In fact, this allows direct visualization of how the structure of proton-neutron multiplets is affected when adding two identical particles. A detailed discussion
Table 1. Experimental and calculated excitation energies (in MeV) for \(^{136}\)Sb, \(^{136}\)I, \(^{212}\)Bi, and \(^{212}\)At.

| Nucleus | \(J^\pi\) | \(E_{\text{expt}}\) | \(E_{\text{calc}}\) |
|---------|---------|----------------|----------------|
| \(^{136}\)Sb | 1\(^-\) | 0.0 | 0.0 |
|          | 2\(^-\) | 0.053 | 0.021 |
|          | 4\(^-\) | 0.226 | 0.179 |
|          | 6\(^-\) | 0.278 | 0.210 |
| \(^{136}\)I | 1\(^-\) | 0.0 | 0.0 |
|          | 2\(^-\) | 0.087 | 0.101 |
|          | 3\(^-\) | 0.222 | 0.203 |
| \(^{212}\)Bi | 0\(^-\) | 0.239 | 0.174 |
|          | 1\(^-\) | 0.0 | 0.0 |
|          | 2\(^-\) | 0.115 | 0.141 |
|          | 3\(^-\) | 0.213 | 0.214 |
|          | 4\(^-\) | 0.251 | 0.258 |
| \(^{212}\)At | 1\(^-\) | 0.0 | 0.0 |
|          | 2\(^-\) | 0.160 | 0.196 |
|          | 3\(^-\) | 0.206 | 0.227 |
|          | 5\(^-\) | 0.275 | 0.296 |
|          | 8\(^-\) | 0.328 | 0.338 |
|          | 9\(^-\) | 0.223 | 0.301 |

of the proton-neutron multiplets in \(^{210}\)Bi and \(^{134}\)Sb can be found in [13, 19], where we have also made an analysis of the various contributions to our effective interaction. This has given evidence of the crucial role played by the core-polarization effects, in particular those arising from 1\(p\)-1\(h\) excitations.

As regards the structure of the states in \(^{136}\)Sb and \(^{136}\)I, they are dominated by the \(\pi g_{7/2}(\nu f_{7/2})^3\) and \(\pi (g_{7/2})^3\nu f_{7/2}\) configurations, respectively. Similarly, for \(^{212}\)Bi and \(^{212}\)At the states reported in figure 2 are dominated by the \(\pi h_{9/2}(\nu g_{9/2})^3\) and \(\pi (h_{9/2})^3\nu g_{9/2}\) configurations, respectively. For all four nuclei, however, there is a significant configuration mixing. This is particularly large for \(^{212}\)Bi, with a percentage of components other than dominant one going from 25 to 52% .

From comparison of figures 1 and 2 we see that the curves for the four-valence-particle nuclei all have a similar shape, but they differ from that of the multiplets in \(^{134}\)Sb and \(^{210}\)Bi. The main new features are an overall flattening of the staggering and a rather sharp energy increase of the 0\(^-\) state with respect to the 1\(^-\) one. These are a manifestation of the dominant role played by pairing correlations, which means that the two additional protons or neutrons in both A = 136 and 212 systems are prevalently coupled to zero angular momentum. However, the wave functions of the states reported in figures 1 and 2 also contain components with the pair coupled to \(J \neq 0\), which have a larger weight when a neutron rather than a proton pair is added. As a matter of fact, it turns out that the proton-proton pairing component of our effective interaction plays a more significant role than that of the neutron-neutron one. As is shown in [20], this is largely due to the 1\(p\)-1\(h\) core excitations.
Figure 1. Proton-neutron multiplets in $^{134}\text{Sb}$ (circles), $^{136}\text{Sb}$ (squares) and $^{136}\text{I}$ (triangles).

Figure 2. Proton-neutron multiplets in $^{210}\text{Bi}$ (circles), $^{212}\text{Bi}$ (squares) and $^{212}\text{At}$ (triangles).

As regards the effects of components with the pair coupled to $J \neq 0$, we have verified that their presence tends to produce staggering with phase opposite to that observed in one proton-one neutron systems. This is evident for $^{136}\text{Sb}$ but not for $^{212}\text{Bi}$. The explanation resides in the fact that in $^{212}\text{Bi}$ this effect is quenched by configuration mixing, which, as mentioned before, we have found to be more relevant for this nucleus. As regards $^{136}\text{I}$ and $^{212}\text{At}$, we don’t find evidence of significant staggering, the curves in figures 1 and 2 being almost flat but for the lowest angular momenta. This is because, as mentioned above, the wave functions of the considered states in these nuclei contain a small percentage of nonzero-coupled proton pair components, giving rise to a staggering that is largely washed out by the configuration mixing.

Let us now turn to the four-valence-neutron nuclei $^{136}\text{Sn}$ and $^{212}\text{Pb}$. In figure 3 we report the calculated and experimental [16] energies of the lowest states in the latter together with
our predictions for the former. We see that the calculated curve for $^{136}\text{Sn}$ shows just the same
behavior as that relative to its counterpart nucleus $^{212}\text{Pb}$, the latter reproducing quite well the
experimental one. For both nuclei, the considered states receive significant contributions from
configurations other than the dominant one. In fact, we find that the percentage of the $(\nu f_{7/2})^4$
configuration for the states of $^{136}\text{Sn}$ and of the $(\nu g_{9/2})^4$ configuration for the states of $^{212}\text{Pb}$
ranges from 65 to 83% and from 51 to 75%, respectively.

![Figure 3](attachment:image.png)

**Figure 3.** Energies of the low-lying states in $^{136}\text{Sn}$ and $^{212}\text{Pb}$. The theoretical results are
represented by open diamonds ($^{212}\text{Pb}$) and open circles ($^{136}\text{Sn}$) while the experimental data for
$^{212}\text{Pb}$ by black diamonds.

4. Summary
We have presented here the results of a comparative shell-model study of four-valence-particle
nuclei in the $^{132}\text{Sn}$ and $^{208}\text{Pb}$ regions. In our calculations we have employed realistic low-
momentum effective interactions derived from the CD-Bonn N.N potential without the use of
any adjustable parameters.

Our results for all the considered nuclei are in very good agreement with the available
experimental data and show evidence that a close resemblance between the spectroscopy of these
two regions persists when moving away from the immediate neighbors of doubly magic $^{132}\text{Sn}$ and
$^{208}\text{Pb}$. This makes us confident in the soundness of our predictions for both the missing states
of the proton-neutron multiplets in $^{136}\text{Sb}$, $^{136}\text{I}$, $^{212}\text{Bi}$ and $^{212}\text{At}$ and the low-energy spectrum of
exotic $^{136}\text{Sn}$. We hope that they may be verified in a not too distant future.

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