Mode-walk-off interferometry for position-resolved optical fiber sensing

Luis Costa
Department of Electrical Engineering, California Institute of Technology, Pasadena, California 91125, USA and
Seismological Laboratory, California Institute of Technology, Pasadena, California 91125, USA

Zhongwen Zhan
Seismological Laboratory, California Institute of Technology, Pasadena, California 91125, USA

Alireza Marandi*
Department of Electrical Engineering, California Institute of Technology, Pasadena, California 91125, USA

Simultaneously sensing and resolving the position of measurands along an optical fiber enables numerous opportunities, especially for application in environments where massive sensor deployment is not feasible. Despite significant progress in techniques based on round-trip time-of-flight measurements, the need for bi-directional propagation imposes fundamental barriers to their deployment in fiber communication links containing non-reciprocal elements. In this work, we break this barrier by introducing a position-resolved sensing technique based on the interference of two weakly-coupled non-degenerate modes of an optical fiber, as they walk-off through each other. We use this mode-walk-off interferometry to experimentally measure and localize physical changes to the fiber under test (axial strain and temperature) without the typical requirement of round-trip time-of-flight measurements. The unidirectional propagation requirement of this method makes it compatible with fiber links incorporating non-reciprocal elements, uncovering a path for multiple sensing applications, including ultra-long range distributed sensing in amplified space-division-multiplexed telecommunication links.

I. INTRODUCTION

By harnessing intrinsic light-matter interactions, distributed fiber sensing methods enable off-the-shelf optical fibers to operate as highly sensitive sensor arrays [1, 2]. Multiple variations of these techniques have been used to measure a diverse range of physical parameters, from the acoustic [3, 4] and optical [5] properties of the fiber’s surroundings to direct measurements of the interrogated fiber’s physical state (e.g., pressure [6, 7], temperature, or axial strain [8, 9]). Distributed acoustic sensing (DAS) [1, 10] encapsulates the subset of distributed sensors oriented towards fast acquisition of the strain profile along a fiber cable, and has traditionally seen application in areas such as surveillance, pipeline monitoring, and vertical seismic profiling [1, 10].

Recently, DAS has captured the attention of the geophysics community for seismic studies [11–14], and considerable research efforts are now aimed at addressing the demands of this new application space. Despite its recent adoption, however, there are plenty of demonstrations of its successful use in geophysical settings: from metropolitan areas [15] to near-shore deployments [13]. Nonetheless, environments demanding access to the full extent of ultra-long-haul telecommunication links are still mostly inaccessible through these technologies, which are limited to ∼100 km ranges despite remarkable efforts at range extension [16, 17].

The deep ocean floor is a particularly relevant example of an environment where traditional geophysical sensing equipment is sparse, costly, and often temporary [18], and where the existence of transoceanic telecommunication fibers presents an exceptional opportunity for fiber-based sensing. So far, attempts at exploring transoceanic fibers for sensing have remained limited in their localization capabilities, either unable to discriminate the origin of each strain contribution [19–21], or able to localize only a few dominant perturbations along the cable through bi-directional measurements [19]. Fully distributed DAS techniques, on the other hand, struggle to meet the criteria for sensing in such long-haul cables. The reasons go beyond the aforementioned range limits, as their fundamental reliance on intrinsic backscattering for localization imposes a barrier to massive range enhancement via in-line amplification (owing to the presence of optical isolators in the amplifiers), and the probe pulse’s characteristically high peak powers (∼200 mW) render these techniques incompatible with co-propagating data channels [14, 22], which is especially limiting given the lack of abundant fiber strands in transoceanic cables.

These fundamental roadblocks to ultra-long range DAS motivate the exploration of novel interrogation methods, capable of surpassing the fundamental challenges of current DAS techniques in specific settings. One possible way to expand the design of distributed sensing methods is to target future telecommunication fiber deployments. At the moment, the ability to transport data traffic is lagging under the ever increasing demands of consumers, limited by the gain band of in-line amplifiers and the available power budget of telecommunication fibers [23]. The current techno-economic landscape suggests that the next iteration of telecommunication fibers will expand capacity through space-division multiplexing (SDM) tech-
niques [24] (either in the form of few-mode [25] or multicore fibers [26]) in order to remain economically competitive [23, 27]. Transoceanic distributed sensing methods are expected to closely follow these developments, opening new opportunities to address existing technical limitations by exploring the new spatial degrees of freedom of a multimode telecommunications backbone.

Existing few-mode fiber sensing demonstrations [28–31] have employed the added modes to tackle cross-sensitivity [32], improve SNR [33], or in the case of multicore fibers, mostly for complementary measurements using multiple independent channels [34, 35]. One aspect that is less targeted is the need for bidirectional propagation in existing distributed sensing techniques: either for roundtrip time-of-flight measurements [10, 36], or to engineer a local parametric interaction between two counter-propagating lightwaves [8, 37].

Multimode platforms enable the generalization of the time-to-position mapping principles to co-propagating designs, provided that the carried modes/supermodes are weakly coupled and possess different group velocities. This mapping can be observed in reports on the characterization of multimode links and devices [38–41], as well as a recent sensing work which proposed the measurement of changes in the local coupling strength as a way to monitor the distribution of transverse stresses applied to the fiber [6, 42]. Yet, more relevant sensing quantities such as strain/temperature remain unexplored, and no sensing demonstration to our knowledge has relied on the baseline coupling and intermode/intercore crosstalk for interrogation - instead, it has been considered as one of the main drawbacks of few-mode fibers (FMF) for sensing purposes [29].

In this work, we demonstrate mode-walk-off interferometry along an optical fiber. This technique enables position-resolved, coherent interrogation of the fiber in a single unidirectional measurement. Through our method, all sections of a weakly-coupled fiber behave as an independent interferometer, whose response is stored at a different time-instant of the temporally broadened output. This enables the recovery of the full profile of common quantities of interest, such as strain and temperature, in a single unidirectional measurement. Our technique employs swept-wavelength interferometry [40, 43], while avoiding common drawbacks such as the range limitations originating from the coherence length of widely tunable sources, and the need for a dedicated, optical path-length-matched, local oscillator fiber for recovery of the fiber transmission response. We expect this work to spearhead future unidirectional, multimode distributed sensing designs, able to benefit from in-line amplification and easily integrable with future data-carrying, ultra-long haul telecommunication links, thus unlocking dense (kilometer-long resolutions) strain and temperature sensing in future transoceanic (and other ultra-long-haul), amplified fiber deployments.

II. RESULTS

Sensing Concept

Our transmission-only sensing method is illustrated in figure 1. A short pulse of light is injected into the fundamental mode (OAM₀, in the Optical Angular Momentum mode basis [44]) of a two-mode fiber. Under the assumption of constant weak coupling between mode groups, while most light remains in the same mode as it was injected, a small fraction is coupled to one of two degenerate higher-order modes (OAM±1) over every position in the fiber. Our assumption of weak coupling is valid for the purposes of telecommunication few-mode operation, and constant coupling can be assumed for our perturbations of interest (axial strain/temperature) for a two-mode fiber [45].

After coupling, the now higher-order mode light experiences both a different propagation constant, β₀(ℓ), and a different group velocity, v₀(ℓ) (ℓ being the topological charge of the corresponding OAM mode) for the remaining length of fiber. The difference in each of these two quantities enables the estimation of measurand amplitude and localization of perturbations along the fiber link (see figure 1).

Localization information is encoded in time as a result of the difference in group velocities between mode groups, since light coupled from the fundamental mode at each position will be displaced from pre-existing light in the higher order modes. As a result, at the fiber output, the OAM±1 signal will display a broad temporal envelope, each time instant mapping to a specific position in the fiber.

The position information can therefore be recovered via a differential time-of-flight measurement on the higher order mode output. The observed temporal walk-off for light undergoing coupling at position z will be a function of the remaining length of fiber (L_{FUT} - z) and the differential mode group delay (DGD = 1/v₀(±1) - 1/v₀(0)) between the two mode groups

$$\Delta t(z) = DGD(L_{FUT} - z).$$  \hspace{1cm} (1)

This method of localization is analogous to the traditional backscattering-based methods in conventional distributed techniques, which rely on roundtrip time-of-flight for position discrimination. The differential nature of the measure, however, yields a temporally compressed optical trace (i.e., the impulse response obtained for a given pair of input/output modes) compared to traditional roundtrip measurements (by a factor $CF \approx \frac{2\pi}{\Delta n(0)}$). The justification and the implications of this compression are described in detail in the supplementary section 1.

Any perturbation to the local optical path of the fiber (induced by strain or temperature, in this case) can be measured by comparison of the recovered optical trace to
FIG. 1. Principle of transmission-only distributed sensing. a. Light is launched as the fundamental mode of a fiber carrying at least a pair of non-degenerate modes, of different phase and group velocities. b. Weak distributed coupling (of coupling strength $k$) converts light from the injected mode to higher order modes as it propagates. The difference in group velocities leads to an effective walk-off between previously coupled light and the injected mode beam (d.), such that the point of coupling is mapped to a specific time-instant at the output. c. The difference in group velocity broadens the higher-order mode output, and the difference in phase velocities leads to interference between the newly coupled and previously coupled light (e.), generating a noise-like broad optical trace at each higher mode output: each time-instant of the OAM$_{\pm 1}$ stores the local interference resulting from light traversing two different optical paths.

a previous acquisition. This is achieved by an interferometric measurement of optical path length difference, similar to Rayleigh scattering based systems [10, 36]. As light couples from the OAM$_{0}$ mode to any of the higher order OAM$_{\pm 1}$ modes, it shall interfere with pre-existing light in the higher order mode with which it overlaps. The optical path difference accumulated due to propagation as different modes over any length of fiber enables us to perceive the fiber as a stack of effective interferometers (sensing points), which we are able to individually access via the aforementioned ability to pinpoint the origin of the coupling by a time-of-flight measurement. The resulting noise-like output of the higher order mode (the optical trace), stores the response of each of the sensing points of the fiber at different time instants.

Recovering the measurand information can then be achieved in one of two ways: by observing the changes to the instantaneous phase evolution along the obtained higher order mode output (analogous to coherent phase-sensitive OTDR interrogation [46]), or by probing the equivalent frequency shift that compensates the change in the intermodal interference due to a perturbation-induced change in local optical path difference (in analogy to frequency-demodulation phase-sensitive OTDR methods [47, 48]). We opt for the latter approach, since it avoids problems resulting from cumulative measurements of phase, such as poor phase estimation at points of fading and ambiguity due to phase unwrapping errors.

Consider the interference happening at position $z$ in the fiber, as newly coupled light interferes after travelling a short length $dz$ (stored at a specific time-instant of the recovered optical trace, given by equation 1). The phase difference accumulated between the two interfering waves due to the difference in propagation constants will be $\Delta \varphi(z) = \Delta \beta dz$ (for $\Delta \beta = \beta_{0}^{(0)} - \beta_{0}^{(\pm 1)}$), which may be re-written as

$$\Delta \varphi(z) = \frac{2\pi}{c_{0}} (\Delta n(z) \cdot dz) \nu_{0}. \tag{2}$$

Equation 2 illustrates the equivalence of altering the optical path difference ($\Delta n(z) \cdot dz$ ($\Delta n = n^{(0)} - n^{(1)}$, $n^{(l)}$ being the effective index of each respective mode of $\ell$ topological charge) and detuning the probe center frequency $\nu_{0}$ by a specific amount. A change in the optical path difference ($\Delta(\Delta n(z) \cdot dz)$) can therefore be adequately compensated by a change in center frequency, such that

$$\frac{\Delta \nu_{0}}{\nu_{0}} = \frac{\Delta(\Delta n(z) \cdot dz)}{(\Delta n(z) \cdot dz)}. \tag{3}$$

A simple strategy for interrogation of all sensing points, then, consists of probing the FUT with multi-center frequencies and reconstructing the frequency response of each effective interferometer formed at every position in the fiber. A perceived shift in the frequency response will therefore be proportional to the optical path difference, according to the relation given in equation 3. Strain and temperature can then be inferred from well known fiber coefficients that take into account the total length change, elasto-optic effect, thermal expansion and thermo-optic effect [48].

Experimental demonstration

To experimentally demonstrate our proposed method, we slightly depart from our previous simplified conceptual description. The main difference is the use of a continuous-wave frequency swept input instead of a pulse for interrogation. This type of interrogation is commonly
The maximum achievable spatial resolution in our methodology $\zeta_{max}$ is proportional to the total bandwidth spanned by the sweep and the DGD, $\zeta_{max} \propto \frac{1}{B_{max} \cdot DGD}$. In our specific implementation, however, each acquired sweep is divided in several sub-sweeps of bandwidth $B_{sub} < B_{max}$, each with a different laser center frequency $\nu_0$. This is the second main difference from the previous conceptual description, and it enables us to reconstruct the frequency response of the fiber from a single swept acquisition, simultaneously probing the fiber with multiple center frequencies at the cost of spatial resolution.

As a result, the calculated spatial resolution $\zeta$ is given by

$$\zeta = \frac{1}{B_{sub} \cdot DGD} \quad (4)$$

and can be determined in post-processing by choosing the number of independent sub-sweeps (or the total bandwidth of each sub-sweep) that the acquired portion of the scan is sliced into. This interrogation method entails a inverse proportionality between the sensor’s measurand resolution, spatial resolution and total bandwidth $B_{max}$, due to the limits of estimation accuracy of the frequency detuning [50] (a discussion of these trade-offs is provided in the supplementary section 3).

Successfully employing SWI with a broadly tunable laser source also demands several post-processing steps, in order to correct non-linearities of the laser sweep and address the variation in sweep rates over acquisitions. Both of our sensing demonstrations used a FUT consisting of a single 2300 meter long step-index SMF-28 fiber (carrying 2 mode groups at 1064 nm). The FUT configurations used in each experiment are represented in figure 2.

**Multipoint Temperature Measurements**

To first evaluate the potential for distributed sensing and validate the principle for localization and discrimination of multiple measurements within the fiber, we performed a multipoint sensing measurement by heating two positions in the fiber. We generated two hotspots by coating two sections of fiber (~10 m long and ~15 m long) separated by more than one spatial resolution (figure 2). Room temperature was measured to be approximately 22 °C.

Each of the fiber coils were heated by hovering a warm object (~35 °C) close to the coils for about 1 minute without touching the fiber, and then removing it and allowing that hot spot to cool down. The spatial resolution was calculated to be 16.1 m according to equation 4 (see the supplementary section 2 for details in the determination of fiber DGD and laser sweep rate).

The results are depicted in figure 3. The spatial separation between both perturbations is clearly evidenced by computing the RMS temperature shift in the dashed

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**Figure 2**

a. Simplified depiction of the interrogation setup. The complete setup is described in the methods section (figure 5). b. Recovered impulse response obtained from one sub-sweep, (for a single mode and polarization) using swept-wavelength interferometry. Amplitude is normalized to the initial peak at time delay 0, occurring from poor demultiplexing at the output (see supplementary section 2 for additional information on the optical traces). c. FUT configuration for the single-point strain measurement and d. for the multipoint temperature measurement.
areas, and plotting them in the bottom part of the figure. Notably, we see that the full-width at half maximum for the perturbation observed for the influence is 22.78 m for the \( \sim 10 \) m long hot spot and 23.65 for the \( \sim 15 \) m long one. These results are reasonably consistent with the estimated lengths for the coils and the calculated spatial resolutions. Nonetheless, they seem to suggest some worsening of spatial resolution, which is expected to occur due to fluctuations in the laser sweep rate over each acquisition (see supplementary section 2).

The temperature is calculated from the apparent effective frequency shift using standard coefficients used for telecommunication step-index fibers [48]

\[
\Delta T \approx -\frac{1}{6.92 \times 10^{-6}} \frac{\Delta \nu_0}{\nu_0}. \tag{5}
\]

We notice the appearance of some residual crosstalk between spatial channels at positions prior to the measurement. While crosstalk effects have been observed for other Rayleigh-based distributed sensing methods, there are clear distinctions when compared to our approach. First, spatial crosstalk normally affects subsequent positions in backscattering-based technologies, and can be calculated from the amplitude of the perturbation that induces it [51]. This is contrasted with what we observe in the transmission setup: the crosstalk affects positions prior to the point of perturbation, and does not seem to scale predictably with the perturbation that induces it. This suggests that the origin might be an indirect effect, onset by changes to the strong coupling between the two degenerate OAM \( \pm 1 \) modes. One possible explanation in this case may be a change in coupling strength from a combination of the fiber coiling and thermal effects. This is supported by the fact that the shorter coil had approximately half of the coiling radius as the longer coil (leading to stronger crosstalk). This may not occur in fiber installations that are not substantially bent or coiled, and may be avoided altogether through the use of a nondegenerate higher order mode for interrogation (thus avoiding any strong coupling effects), in fibers carrying a higher number of modes.

Strain Measurements

To evaluate the potential for strain measurements and assess the linearity of our interrogation process, we coiled a roughly 15 m long section of the fiber around a piezoelectric cylinder, at meter 500. A slow sinusoidal oscillation with \( 100 \) s period was applied to the fiber stretcher, with \( 100 \) V amplitude (\( \sim 100 \) \( \mu \varepsilon / \)V, according to specifications). The strain distribution was recovered over 300 seconds, and is represented in figure 4. The acquired effective frequency shift was converted to strain through the following relation [48]

\[
\varepsilon \approx -\frac{1}{0.78} \frac{\Delta \nu_0}{\nu_0} \tag{6}
\]

Once again, the spatial resolution was selected to be 16.1 m, which was found to maximize the strain SNR for the recovered perturbation. The amplitude of the measured strain sine wave was found to be 16.4 \( \mu \varepsilon \), and the strain resolution (computed as the average of the standard deviations of all points in an undisturbed section of fiber, from meter 700 to 1600) was measured as 1.2 \( \mu \varepsilon \).

III. DISCUSSION

In this work, we introduced and demonstrated a new method to perform distributed sensing of common physical parameters by exploring the weak coupling between spatial modes carried in optical fibers, relying exclusively on unidirectional propagation in the fiber.

As a proof-of-concept, we performed two sensing demonstrations using standard step-index SMF-
potential to output multiple optical traces at every measurement. As such, our method may enjoy other benefits commonly mentioned for multimode-based fiber sensors [29], onset from the ability to access multiple mode outputs with different optical properties. In the presented work, we demonstrated the simplest case, with the minimum possible number of modes in a few-mode circularly symmetric fiber. However, our method generalizes to fibers carrying a higher number of modes or coupled-core multicore fibers, so long as the conditions of having access to a pair of weakly coupled modes (or supermodes) with differing group velocities is fulfilled. The ability to access multiple optical trace outputs also opens new processing possibilities, which may range from simple averaging of incoherent sources of noise (as done in this work), to more advanced processing schemes aimed at preventing the onset of anomalous estimations from cross-correlation [52], and thus increasing the sensing dynamic while mitigating the consequent accumulation of 1/f noise [53].

We also note that our simple design can be readily adapted for different sensing paradigms, such as previously reported distributed transverse stress by the addition of simple processing steps, such as averaging of the optical traces acquired from each subsweep to produce an incoherent measurement of the coupling strength envelope [6]. A demonstration of using our setup for incoherent measurements is presented in supplementary section 4.

Despite the remaining optimization efforts required for a field demonstration, we stress the potential of this design for future seismic sensing, in space-division multiplexed telecommunication links consisting of weakly coupled fiber links. Further investigation should provide answers on the compatibility and total interrogation range achievable with this technique in telecommunication-grade few-mode fibers (using in-line amplifiers), and what performances can be expected when using kilometer-length spatial resolutions. With the aim of ultralong range, kilometer-length spatial resolutions, and strain measurements in mind, a potential roadmap for future designs based on this technology may include the use of a mixed pulsed/swept approach, analogous to some works described in the Rayleigh-backscattering based sensing literature [17, 50]. This would entail limiting the total interrogated frequency range to a single wavelength channel, spanning multiple weakly coupled spatial channels. Our method can be adapted to such an implementation with only a few alterations to the hardware and processing scheme, by replacing the single-sweep approach into a multi-shot interrogation where the center frequency of each pulse is slowly modulated. This type of interrogation would benefit from the potential to co-propagate multiple pulses in the same fiber due to the transmission-based nature of the technique.

28 working in few-mode operation. We successfully localized and demonstrated linear measurements of strain/temperature with inferred measurand resolutions of 1.2 με (0.135 K, in equivalent temperature), and spatial resolutions in the tens of meters, at acquisition rates of 0.2 Hz. While these values are not representative of the ultimate performance limits of the technique and there is ample room for optimization, they serve as strong evidence for the future potential of this sensing principle. In particular, optical SNR is limited by the imperfect correction of the laser sweep nonlinearity and imperfect spatial mode demultiplexing at the fiber output, and the acquisition rate is limited by the total time required to ensure an approximately linear sweep from the laser source. Also, despite the slow acquisition rates, we note that this type of interrogation in fact relaxes the fundamental acquisition rate limit of common backscattering implementations, which require the minimum of full roundtrip time for the whole length of interrogated fiber between acquisitions. Conversely, our method benefits from a much narrower recovered optical trace such that this condition is massively relaxed, and multiple probe pulses can simultaneously coexist in the same FUT.

We also highlight the important distinction of our method from single-mode implementations, due to the
IV. METHODS

Experimental Setup

The full schematic for our setup is depicted in figure 5. The laser source was a Toptica CTL 1050, operating at center wavelength 1064 nm and swept by driving the internal stepper motor with a 0.2 Hz sine wave. 10% of the laser output power is diverted into an imbalanced Mach-Zehnder interferometer (having 20 m delay fiber in one path), which is used to measure the laser sweep rate and to correct nonlinearity in the frequency sweep in post-processing (see supplementary section 2). The remaining 90% (~ 12 dBm) are launched into the FUT. The laser is swept at an average rate of 1.63 THz/s (6.19 nm/s) over each acquisition (consisting of 0.83 s around the point of highest linearity of the positive slope of the sinusoidal modulation).

The fiber under test consisted of 2.3 km of SMF-28 fiber carrying three modes (2 non-degenerate mode groups, OAM$_0$ and OAM$_{±1}$) at the probe wavelength. The DGD was measured to be 1.23 ps/m (see the supplementary section 2 for information on the measurement of DGD). At the input, light is injected exclusively into the OAM$_0$ mode. At the output, the three modes carried by the fiber are separated through a free-space spatial mode demultiplexer: the fiber output is collimated into free space and split into 3 paths. One of the paths is immediately coupled into a single-mode HI1060 fiber, without undergoing any mode conversion, while the other two are sent through spiral phase masks (which add/subtract 1 topological charge) before being coupled to the single-mode HI1060 fiber. The spiral phase masks function as a mode converter, while the single-mode fibers act as a spatial rejection filter that only accepts the portion of light with 0 topological charge. The OAM$_0$ output is then used as the local oscillator of a polarization diversity balanced interferometer (see figure 5), which produces an auxiliary Mach-Zehnder interferometer (see figure 5), which produces an auxiliary signal $s_{aux}$. The instantaneous phase of this signal can be recovered from the Hilbert transform, and used to compute a correction function

$$\phi_{inst}(t) = \mathcal{H}\{s_{aux}(t)\}. \quad (7)$$

This function can then be used to correct the OAM$_{±1}$/OAM$_1$ outputs of the FUT by resampling the acquired data with $\phi_{inst}(t-\tau_{FUT})$, where $\tau_{FUT}$ is the total delay accumulated by propagation through the FUT and fibers on the detection setup.

Quantitative measurand evaluation

For each sensing position, the measurand amplitude (relative to a previous acquisition) can be evaluated by computing the effective detuning of the frequency response of the corresponding effective interferometer. The frequency detuning estimation between m-th and r-th (reference) acquisitions is accomplished through the generalized cross-correlation algorithm

$$\Delta \nu^c_m(z_i) = \arg \max\{R_m(z_i, \Delta \nu)\}, \quad (8)$$

where $\Delta \nu^c_m(z_i)$ is the effective frequency detuning proportional to the applied local perturbation to the fiber, and $R_m(z_i, \Delta \nu)$ is the cross-correlation between the frequency responses acquired at the instant $m$ and $r$, for the $z_i$-th measurement point. Subsample accuracy is achieved through parabolic fitting using the three points surrounding the maximum of the cross-correlation.

The frequency responses of all positions in the fiber can be acquired in a single shot by partitioning each acquired time-series of length $t_{acq}$ into $N_s$ sub-sections of length $t_{sub} < t_{acq}$. Each sub-section is equivalent to probing the fiber with a sweep covering a narrower bandwidth ($B_{sub} \approx \gamma(t) \times t_{sub}$, for an average laser sweep-rate $\gamma(t)$). Since each sub-sweep probes the fiber with a shifted laser center frequency, we can sample the frequency response of all sensing positions of the fiber (over the full sweep’s bandwidth). The frequency responses can be therefore reconstructed if the sensing positions of the fiber (over the full sweep’s bandwidth). The frequency responses can be therefore reconstructed if the Nyquist sampling criterion is satisfied

$$\delta \nu < \frac{B_{sub}}{2}, \quad (9)$$
where \( \delta \nu \) is the center frequency difference of the sweep corresponding to each successive sub-section. As such, after processing, every acquisition yields four measurement matrices (for each pair of modes and polarizations), each comprising the frequency response of every effective sensing position of the fiber (illustrated in the supplementary materials, section 2). At every acquisition, each of the four measurement matrices are independently processed, each producing an estimation of the measurand. The resulting estimations are then combined by averaging.

Drifts and fluctuations in the center frequency of the laser over multiple acquisition time-steps manifest as a common-mode noise component. After recovering the full fiber measurand profile, the perfect spatial correlation of this source of noise facilitates its correction by simply removing the mean strain/temperature obtained along an unperturbed section of fiber (from meter 700 to 1600) [54] (see supplementary section 2 for additional details on the noise due to laser center frequency drifts).

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**Author Contributions** A.M., L.C., and Z.Z. conceived the idea. A.M. and Z.Z. supervised the project. L.C. devised and executed the experiments and data processing. All authors contributed to the writing and discussion of the results.

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**Data availability** All data in the main text or supplementary materials is available from the corresponding author upon reasonable request.

**Supplemental document** See Supplemental Document for supporting content.

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This document provides supplementary information to "Mode-walk-off interferometry for position-resolved optical fiber sensing", including details on the properties of the recovered raw optical traces compared to roundtrip-based methods, post-processing of the optical signals, analysis of the noise scaling with spatial resolution, and potential for incoherent measurements with alterations to the processing stack.

I. TEMPORAL WIDTH OF OPTICAL TRACE

In backscattering-based distributed optical fiber sensing systems, the total temporal width of each obtained optical trace \( W_{bs} \) is calculated from total roundtrip time-of-flight

\[
W_{bs} = 2L_{FUT}/v_g,
\]

where \( L_{FUT} \) is the length of the fiber under test (FUT) and \( v_g \) is the group velocity. In contrast, in our proposed unidirectional scheme, the temporal width \( W_{cp} \) of each acquired optical trace is determined by the differential time-of-flight between the two co-propagating modes,

\[
W_{cp} = DGD \cdot L_{FUT} = \left( \frac{1}{v_g^{(\pm1)}} - \frac{1}{v_g^{(0)}} \right) L_{FUT},
\]

where DGD is the differential mode group delay.

When interrogating the same fiber with both methods, then, one expects that our proposed method will produce a narrower (temporally compressed) optical trace. The compression factor can be calculated as

\[
CF = W_{bs}/W_{cp} = \frac{2L_{FUT}}{v_g} \frac{v_g^{(0)}}{v_g^{(0)} - v_g^{(\pm1)}} \approx -\frac{2v_g^{(0)}}{\Delta v_g}.
\]

A narrower trace has two main consequences. First, there is a natural penalty to the spatial resolution. Second, the fundamental limits of acquisition rate are relaxed. The first is a direct consequence of the narrowing of the optical trace, such that the same temporal resolution in measurement maps to a longer spatial resolution. The second occurs because backscattering-methods demand that a full roundtrip is completed before launching another probe into the fiber, in order to prevent the optical traces originating from two consecutive probe pulses to overlap, thereby preventing time-to-position mapping. This requirement is relaxed in the co-propagating case: two pulses need to be separated by no more than the width of the pulse plus the width of the optical trace, allowing several consecutive pulses or sweeps to co-exist in the fiber.

While in our demonstration the acquisition rate is clearly limited by the laser sweep time, this is not a fundamental limit. Chirped-pulse implementations [1–3], for instance, restricted to one WDM telecommunication channel could benefit from co-propagating multiple probe pulses.

Few-mode fibers typically have DGDs in the order of a few picoseconds per meter, implying a \( CF \) of the order of \( 10^3 \). We note that for transoceanic seismic applications, in particular, these trade-offs are advantageous. The desired spatial resolutions for these applications can be on the order of hundreds of meters to kilometers. Meanwhile, faster sampling enables quasi continuous power to exist in the fiber, translating to better SNR.

\*marandi@caltech.edu
II. METHODS

A. Swept-Wavelength Interferometry

![Diagram showing swept-wavelength interferometry]

FIG. 1. Working principle of swept-wavelength interferometry. A group delay ($\tau$) is mapped into a specific beatnote of frequency $f_b$, proportional to the laser sweep rate. In the event of a nonlinear sweep, this time-to-frequency mapping is compromised, as a single group delay maps to a distribution of beatnotes.

Using swept-wavelength interferometry (SWI), a time-domain impulse response (optical trace) can be reconstructed in the frequency domain due to the time-to-frequency mapping resulting from a linear frequency sweep. A simple visual description is depicted in figure 1, for the simple case of an imbalanced interferometer.

The nonlinearity of the sweep in broadly tunable sources compromises the one-to-one mapping of a group delay to a specific RF beatnote required for proper SWI: instead, a single group delay is mapped to a distribution of beatnotes, blurring the frequency-domain signal obtained (figure 1, right). Frequency sweep nonlinearity is a known and well-studied problem in the literature of swept wavelength techniques [4, 5], which can be addressed through post-processing, provided that the sweep does not deviate extremely from linearity or has sharp discontinuities.

The output of injecting a perfectly linear frequency sweep into an auxiliary imbalanced interferometer would be a single sinusoid $s_{aux}(t) = \sin(2\pi f_{aux} t)$, where $f_{aux}$ (the beatnote frequency measured at the auxiliary interferometer) is given by

$$f_{aux} = \gamma \tau_{aux}.$$ (4)

Here, $\gamma$ is the laser sweep rate, in Hz/s, and $\tau_{aux}$ is the time delay between the two arms of the imbalanced auxiliary MZI. In the case of a nonlinear frequency sweep, then, $\gamma$ is a function of time and the recovered signal is given by

$$s_{aux}(t) = \sin(2\pi \gamma(t) \tau_{aux} t).$$ (5)

Therefore, the instantaneous phase function ($\phi_{inst}$) is not necessarily a linear function of $t$, so the auxiliary signal will produce a broad spectrum instead of a single beatnote, as exemplified in figure 1. However, by resampling the auxiliary signal data in constant phase steps, one is able to correct this broad spectrum to that of the ideal, linear sweep case. This same resampling function $\phi_{inst}(t)$ can then be applied to the signals obtained from the output of the fiber in order to correct the effect of a nonlinear sweep (figure 2).

The sweep non-linearity can then be corrected by computing the instantaneous phase of $s_{aux}(t)$, and therefore recovering a resampling function that reshapes the auxiliary signal $s_{aux}(t)$ into a single beatnote sine wave. This can be obtained by calculating the angle of the Hilbert transform of the interferometer signal $\phi_{inst}(t)$

$$\phi_{inst}(t) = \angle \mathcal{H}\{s_{aux}(t)\} = 2\pi \gamma(t) \tau_{aux} t.$$ (6)

The path length between the laser source and the photodetectors at the fiber output is not necessarily the same as the path length between the laser source and the auxiliary interferometer. Therefore, it is important to appropriately delay the resampling function $\phi_{inst}(t)$ by the total time delay originating from this difference in optical paths, before
FIG. 2. Basic processing applied to the raw data, to correct the sweep nonlinearity. (a) Signal obtained from the auxiliary interferometer (blue), and resampling function (orange). (b) The effects of applying the resampling to the raw measurement data, in the frequency domain. (c) The two obtained optical traces for each spatial mode (for one of the two polarizations).

using it to resample the signals at the FUT output. The function used to correct the outputs of the fiber under test, then, is \( \phi_{\text{inst}}(t - \tau_{\text{FUT}}) \), where \( \tau_{\text{FUT}} \) is the total delay accumulated by propagation through the FUT and fibers on the detection setup. Since the two higher order mode outputs of the FUT propagate through different delay lines after demultiplexing, the optimal delay \( \tau_{\text{FUT}} \) is different for each of them. This means that in order to minimize artifacts in the optical trace from improper compensation of nonlinearity, each corresponding optical trace is acquired after resampling the same signal output with the corresponding optimal \( \tau_{\text{FUT}}^{(\pm1)} \) delay.

In summary, the processing done at every acquisition consists of acquiring \( \phi_{\text{inst}}(t) \) from the auxiliary interferometer, and resampling the FUT outputs (using linear interpolation) with the function \( \phi_{\text{inst}}(t - \tau_{\text{FUT}}^{(\pm1)}) \). The delays are applied to the FUT output signals through convolution with a delayed sinc function, in order to allow for subsample accuracy.

B. Single-sweep interrogation

FIG. 3. Method for single sweep interrogation of the fiber. (a) Concept. The raw data obtained from a subsweep is split into shorter time sections. Each is then processed to obtain the optical trace, which is interpolated to a fixed number of samples, and is then stored in a matrix for optical traces. Each column of this matrix corresponds to one optical trace obtained for a given center frequency detuning, while each row stores the frequency response of the effective interferometer formed at a specific point in the fiber. (b) and (c) Example measured data. (b) Example optical trace. (c) Example trace matrix.

The measurand interrogation is accomplished by estimating an equivalent frequency detuning of the frequency
response of each of the effective sensing positions in the fiber, compared to a previously acquired reference. In order to probe several center frequencies with a single sweep, the (already resampled) output data is partitioned into \( N_s \) subsweeps of bandwidth \( B_{\text{sub}} \). Each of these subsweeps produces a different optical trace, owing to the different center frequency. By combining the optical traces acquired by successive subsweeps of different center frequencies, we can reconstruct the frequency response of all sensing points (see figure 3).

The center frequency shift between two consecutive subsweeps is given by \( \delta \nu \) must be carefully chosen in order to adequately sample each sensing point’s frequency response, according to the Nyquist criterion (\( \delta \nu < \frac{B_{\text{sub}}}{2} \)). This value is determined from the time separation between the start of each subsweeps \( (\Delta t_{ss}) \) and the average laser sweep rate \( \langle \gamma(t) \rangle \) as

\[
\delta \nu = \langle \gamma(t) \rangle \Delta t_{ss}.
\]  

(7)

The average sweep rate of the laser \( \langle \gamma(t) \rangle \) over an acquisition is acquired by measuring the average auxiliary interferometer beatnote over the course of the acquisition and calculating the average sweep-rate using equation 4, from a known \( \tau_{\text{aux}} \) (see figure 4 a).

After computing all the optical traces from all subsweeps, and interpolating them to the same length, a matrix of frequency responses for each effective sensing position in the fiber can be created. This process is depicted in figure 3. The interpolation step is required because fluctuations of the laser sweep rate also lead to misalignment and variations of the width of the obtained optical trace (this problem and its solution are expanded in greater detail in section III C).

The measurand estimation at each position can then be accomplished through the generalized cross-correlation algorithm, by monitoring the frequency response obtained at successive acquisitions for frequency shifts. This can be formally described as

\[
\Delta \nu_m^{\text{eff}} = \arg \max\{ R_m^{(z_i)}(\Delta \nu) \},
\]  

(8)

where \( \Delta \nu_m^{\text{eff}} \) is the effective frequency detuning that compensates a local perturbation to the fiber, and \( R_m^{(z_i)}(\Delta \nu) \) is the cross-correlation function between the frequency responses between the \( m \)-th and reference \( (r \)-th) acquisitions, obtained for the \( z_i \)-th measurement point.

The evaluated frequency shift will be proportional to perturbations acting on the fiber, enabling the determination of local measurand changes relative to an initial acquisition. Subsample accuracy is achieved through parabolic fitting using the three points surrounding the maximum of the cross-correlation.

After the measurand estimation step, a total of 4 references and measurement matrices are independently recovered, one for each output spatial mode at each polarization state. The strain and temperature results of all modes/polarizations are combined by averaging the measurand data. After measurand determination, fluctuations in the center frequency of the laser over the acquisition time-step manifest as a common-mode noise component. The perfect spatial correlation of this source of noise facilitates its correction by simply removing the mean strain/temperature obtained along an unperturbed section of fiber (from meter 700 to 1600) [6].

C. Fluctuations of the laser sweep rate

Changes of the sweep rate over each full sweep will affect the spectral positioning of the optical trace (in the downconverted RF spectrum) as well as its width. The beatnote at which each optical trace will be located in the frequency domain, for the \( i \)-th subsweep, is proportional to the average sweep rate over the subsweep \( (\gamma_i) \) and the time delay added by the delay line used for the respective probe mode \( (\tau_{DL}) \) by

\[
f_i = \gamma_i \tau_{DL}.
\]  

(9)

Additionally, the time-to-frequency mapping is also dependent on the sweep rate, so the frequency range that each impulse response spans is given by

\[
\Delta f_i = \gamma_i \cdot DGD \cdot L_{FUT}.
\]  

(10)

Variations in \( \gamma_i \) for different subsweeps or acquisitions lead to fluctuations in the position of the optical trace in the acquired RF spectrum (equation 9), as well as the width of each optical trace (equation 10). This problem is illustrated in figure 4.
FIG. 4. Estimation of laser sweep rate using the auxiliary interferometer. (a) Top: Spectrogram of the signal obtained from the auxiliary interferometer, for a single acquisition and the calculated average laser sweep rate (red dashed line). Bottom: Raw acquisition of the auxiliary interferometer. (b) Effects of the sweep rate drifts over slow time (between acquisitions). Top: example raw optical trace. Green shading is the actual optical trace, and the red shaded regions correspond the spatial mode crosstalk used to determine the start and end of the fiber. Bottom: optical traces over successive acquisitions. (c) Quantification of laser fluctuations. Top-left: measurement of the sweep rate over different subsweeps. Top-right: laser average sweep rate between acquisitions. Bottom: Laser average center frequency shift between acquisitions.

In order to accurately match positions in the fiber between subsweeps (and between acquisitions), we first determine the start and end of each acquired optical trace obtained from each subsweep, and then interpolate it to a pre-defined number of samples \( Z \), which corresponds to the number of effective sensors we interrogate.

The first step is accomplished by locating the high coupling-strength peaks due to spatial crosstalk at the demultiplexer (see figure 4 b, top), and at the fiber input from the connection between SMF-28 and HI1060 fiber. Both of these result from imperfect multiplexing/demultiplexing of the spatial modes launched/obtained from the fiber, but aid us in setting clear limits to the fiber’s start and end position. We then interpolate the frequency span between these two identified peaks to \( Z \) samples, using cubic splines interpolation.

After processing, all optical traces resulting from a subsweep are stacked in a matrix. Each column of this matrix comprises the optical traces obtained from probing the fiber with a different center frequency, or alternatively, each row stores the frequency response of the effective interferometer at a given position (figure 2).

A measurement, then, consists in estimating an effective frequency shift of each frequency response due to an optical
path change, at every acquisition (as described in the methods section of the main paper). This is done by finding the maximum lag for cross-correlation [1, 3, 7] between an initial acquired fiber state and the one obtained at every full sweep.

Note that the process of determining the start and end of the impulse response is subject to errors of at least one spatial resolution. This may lead to some position mismatching for some of the acquired optical traces for some of the subsweeps, and in turn lead to a small penalty to both measurand resolution and spatial resolution.

D. Estimating Differential Mode Group Delay

The differential mode group delay (DGD) of the fiber is inferred through previous knowledge of the length of fiber and by measuring the total delay at the fiber output between the ballistic OAM_{±1} and OAM_0 components of the optical trace.

By computing the sweep rate for a given subsweep (according to the strategy defined in section IIB) and by finding the width of the impulse response (monitoring the coupling strength peaks occurring at the input and output of the fiber, as described in section IIC), we can calculate the DGD from equation 10, assuming prior knowledge of the fiber length. In our experiments, the length of FUT $L_{FUT}$ was estimated using a commercial phase-sensitive OTDR with 10-meter spatial resolution.

III. MEASURAND RESOLUTION VS. SPATIAL RESOLUTION

![Graph showing the relationship between noise standard deviation and spatial resolution.](image)

FIG. 5. Noise standard deviation (calculated in an unperturbed section of the fiber) as a function of spatial resolution. The log-log slope suggests a relationship of $\sigma \propto (SR)^{-1.05}$, which is close to our predicted CRLB scaling.

The finite bandwidth covered by the total frequency sweep, as well as the presence of additive noise in the optical traces, lead to a fundamental limit to the estimation accuracy of a frequency detuning. The effects of signal properties on frequency detuning estimation are well studied, particularly in the analogous case of time-delay estimation, and therefore enable us to make predictions to the performance scaling of our sensor.

The accuracy limit for frequency detuning (or time-delay) using an unbiased estimator can be determined the Cramér-Rao Lower Bound (CRLB) of the estimation process, and has been widely studied in the context of Radar [8, 9] and time-delay-based distributed sensing systems [1]. A full derivation of the CRLB is beyond the scope of this work, but with some assumptions we can make some predictions on the performance scaling. Assuming a well conditioned signal for cross-correlation, having a sufficient time-bandwidth product, and enough optical SNR (≥10 dB), The lower bound for a time-delay estimation measurement scales as follows
\[
\sigma^2 \propto \frac{1}{SNR} \frac{1}{T} \frac{1}{B^3}.
\]  
(11)

where \(SNR, T\) and \(B\) are the optical SNR, time window length of the measurement, and the bandwidth of the signals being compared [1, 8, 9]. Converting these quantities to the equivalent ones to our frequency domain interrogation, we obtain

\[
\sigma^2 \propto \frac{1}{SNR} \frac{1}{B_{\text{max}}} B_{\text{sub}}^3.
\]  
(12)

where \(B_{\text{max}} = \gamma t\) (\(t\) being the total acquisition time), and \(B_{\text{sub}} = \frac{1}{\frac{1}{DGD \cdot SR}}\), \(SR\) being the inferred spatial resolution of the system. Assuming now that the optical SNR scales linearly with total subsweep time (calculated as a fraction of the total sweep time \(t_{\text{sub}} = t \cdot \frac{B_{\text{sub}}}{B_{\text{max}}\gamma}\)), we can then write the SNR as \(SNR = \frac{t_{\text{sub}}}{C^2} = \frac{1}{C} \cdot t \cdot \frac{1}{DGD \cdot SR \gamma}\), \(C\) being a factor with units of time. Equation 12 can now be written as

\[
\sigma^2 \propto C^2 \left( \frac{1}{DGD \cdot SR} \right)^2,
\]  
(13)

which highlights the inverse scaling between noise standard deviation and spatial resolution

\[
\sigma \propto SR^{-1},
\]  
(14)

We investigated this claim by doing a parameter sweep of the spatial resolution on our strain measurement data, and plotting the results in figure 5. The standard deviation of noise was calculated as the average standard deviation in an unperturbed section of fiber, from meter 700 to 1600. The results show the right scaling of strain resolution according to the predictions.

While preliminary, this analysis suggests that the current strain resolution bottleneck is given by additive noise sources, and hints at the performance scaling. In order to translate this type of analysis to the case of interrogation with several chirped-pulse with different center wavelengths, as proposed in the discussion section in the main text, some corrections have to be made. In particular, the total bandwidth has to be defined independent of time, so the only function of time (pulse width, in that case) is the SNR. Nevertheless, we expect the spatial resolution/strain resolution scaling to remain the same.

IV. INCOHERENT MEASUREMENTS

With a few alterations to the post processing steps, our method can be used to recover incoherent measurements of the relative coupling strength along the fiber. By averaging all obtained impulse responses from all acquired subsweeps, we can mitigate the coherent, noise-like aspect of the optical trace resulting from interference.

This is a fundamentally different paradigm of sensing than the one described in the main text. It draws a closer analogy incoherent Optical Time-Domain Reflectometry (OTDR) [10] than distributed sensing techniques, and it will be sensitive to fundamentally different phenomena (e.g., bending or transverse stress [11, 12]).

An example of an incoherent acquisition (done for the same dataset as the one for the strain measurements in the main text) is presented in figure 6. We note that despite baseline changes in coupling strength from spooling, we see no relevant changes of coupling strength as a result of the perturbation at the position of the PZT. This is expected, as the bending radius is not expected to change meaningfully, and axial strains should not induce intergroup mode coupling [11]. This proves that the observations in the main text are result indeed from changes to the local interference, and not from changes to coupling strength, as previously demonstrated in other works [12].

Notably, since both methodologies (coherent interrogation for optical path measurements, and incoherent interrogation for coupling strength measurements) differ only in the post-processing, they can in principle be implemented simultaneously.

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FIG. 6. Incoherent measurements, obtained by averaging all optical traces obtained from all subsweeps, for a given acquisition. Top: measured relative coupling strength of the fiber, normalized to the median coupling strength. At positions where the fiber is coiled, tightly bent, or suffers microbends from poor spooling, we can see stronger coupling strength. Bottom: measurements of coupling strength changes over time, for the same data as the strain acquisition presented in the main text.

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