One–pion transitions between heavy baryons in the constituent quark model

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July 1999

Abstract

Single pion transitions of $S$ wave to $S$ wave, $P$ wave to $S$ wave and $P$ wave to $P$ wave heavy baryons are analyzed in the framework of the Heavy Quark Symmetry limit (HQS). We then use a constituent quark model picture for the light diquark system with an underlying $SU(2N_f) \otimes O(3)$ symmetry to reduce the number of the HQS coupling factors required to describe these transitions. A single constituent quark model $p$-wave coupling is necessary to describe transitions among the $S$ wave ground states. One $s$-wave and one $d$-wave coupling factors are required to determine each of the transitions from the orbitally symmetric K-multiplet and from the orbitally antisymmetric k-multiplet down to the ground state. $P$ wave to $P$ wave single pion transitions are described by altogether eight constituent quark model coupling constants. We also estimate decay rates of some single pion transitions between charm baryon states.
1 Introduction

Based on Heavy Quark Symmetry (HQS) and the $SU(2N_f) \times O(3)$ light diquark symmetry, the construction of heavy baryon wave functions in the limit $m_Q \to \infty$, was formulated in [1, 2]. The analysis of the current-induced $S$ wave bottom baryon to both $S$ wave and $P$ wave charm baryon transitions, in the constituent quark model, was reported in [3]. We follow the same procedure to study the single pion transitions between heavy charm or bottom baryons. The physics of the single–pion transitions between heavy baryons is quite simple: the pion is emitted from the light diquark while the heavy quark propagates unaffected by the pion emission process. Since the heavy baryon is infinitely massive it will not recoil when emitting the pion, i.e. the velocity of the heavy quark and, thereby, that of the heavy baryon remains unchanged.

Heavy Quark Symmetry allows us to write the heavy baryon spin wave function for arbitrary orbital angular momentum as [1, 2, 3]

$$
\Psi_{\alpha \beta \gamma}(v, k, K) = \phi_{\alpha \beta}^{\mu_1 \ldots \mu_j}(v, k, K) \psi_{\mu_1 \ldots \mu_j; \gamma},
$$

(1)

where we have neglected flavor factors for the moment. The “superfield” heavy-side baryon spin wave function $\psi_{\mu_1 \ldots \mu_j; \gamma}$ stands for the two spin wave functions corresponding to baryon spin $\{j - 1/2, j + 1/2\}$, where $j$ is the total angular momentum of the light diquark system. The Dirac indices $\alpha$, $\beta$ and $\gamma$ refer to the two light quarks and the heavy quark, respectively, and the $\mu_i$'s are Lorentz indices. Here, $v$ is the velocity of the baryon, while $k = \frac{1}{2}(p_1 - p_2)$ and $K = \frac{1}{2}(p_1 + p_2 - 2p_3)$ are the two independent relative momenta which can be formed from the two light quark momenta $p_1$ and $p_2$ and from the heavy quark momentum $p_3$. Furthermore, the light diquark wave function can be written as

$$
\phi_{\alpha \beta}^{\mu_1 \ldots \mu_j}(v, k, K) = \hat{\phi}_{\alpha \beta}^{\mu_1 \ldots \mu_j}(v, k, K) A_{\alpha \beta}^{\delta \rho},
$$

(2)
Table 1: Spin wave functions (s.w.f.) and flavor wave functions (f.w.f.) of heavy \( \Lambda \)-type and \( \Sigma \)-type \( S \)- and \( P \)-wave heavy baryons.

|                  | light diquark s.w.f. | f.w.f. | heavy diquark s.w.f. | \( J^P \) |
|------------------|----------------------|--------|----------------------|-----------|
| \( S \)-wave states \( (l_K = 0, l_K = 0) \) | \( \hat{\phi}^{\mu_1 \cdots \mu_j} \) | \( T \) | \( \psi^{\mu_1 \cdots \mu_j} \) | \( J^P \) |
| \( \Lambda_Q \)  | \( \chi^0 \) | \( T^{(3^*)} \) | \( 0^+ \) | \( u \) | \( \frac{1}{2}^+ \) |
| \( \Sigma_Q \)   | \( \chi^{1, \mu} \) | \( T^{(6)} \) | \( 1^+ \) | \( \frac{1}{\sqrt{3}} \gamma_5^\perp \gamma_5 u \) | \( \frac{1}{2}^+ \) |
| \( \frac{1}{\sqrt{3}} \gamma_5^\perp \gamma_5 u \) & \( \frac{3}{2}^+ \) |
| \( P \)-wave states \( (l_K = 0, l_K = 1) \) | \( \frac{1}{\sqrt{3}} \gamma_5^\perp \gamma_5 u \) & \( \frac{3}{2}^+ \) |
| \( \Lambda_{QK1} \) | \( \chi^0 K_\mu^\perp \) | \( T^{(3^*)} \) | \( 1^- \) | \( \frac{1}{\sqrt{3}} \gamma_5^\perp \gamma_5 u \) | \( \frac{1}{2}^- \) |
| \( \Sigma_{QK0} \) | \( \frac{1}{\sqrt{3}} \chi^1 \cdot K_\perp \) | \( T^{(6)} \) | \( 0^- \) | \( u \) | \( \frac{1}{2}^- \) |
| \( \Sigma_{QK1} \) | \( \frac{i}{\sqrt{2}} \varepsilon(\mu \chi^1 K_\mu v) \) | \( T^{(6)} \) | \( 1^- \) | \( \frac{1}{\sqrt{3}} \gamma_5^\perp \gamma_5 u \) | \( \frac{1}{2}^- \) |
| \( \Sigma_{QK2} \) | \( \frac{1}{2} \{ \chi^{1, \mu_1 K_\mu_2^\perp} \}_0 \) | \( T^{(6)} \) | \( 2^- \) | \( \frac{1}{\sqrt{10}} \gamma_5^\perp \gamma_5 \{ \mu_1 \mu_2 \} \) | \( \frac{3}{2}^- \) |
| \( P \)-wave states \( (l_K = 1, l_K = 0) \) | \( \frac{1}{\sqrt{10}} \gamma_5^\perp \gamma_5 \{ \mu_1 \mu_2 \} \) & \( \frac{3}{2}^- \) |
| \( \Sigma_{QK1} \) | \( \chi^0 k_\perp^\mu \) | \( T^{(6)} \) | \( 1^- \) | \( \frac{1}{\sqrt{3}} \gamma_5^\perp \gamma_5 u \) | \( \frac{1}{2}^- \) |
| \( \Lambda_{QK0} \) | \( \frac{1}{\sqrt{3}} \chi^1 \cdot k_\perp \) | \( T^{(3^*)} \) | \( 0^- \) | \( u \) | \( \frac{1}{2}^- \) |
| \( \Lambda_{QK1} \) | \( \frac{i}{\sqrt{2}} \varepsilon(\mu \chi^1 k_\mu v) \) | \( T^{(3^*)} \) | \( 1^- \) | \( \frac{1}{\sqrt{3}} \gamma_5^\perp \gamma_5 u \) | \( \frac{1}{2}^- \) |
| \( \Lambda_{QK2} \) | \( \frac{1}{2} \{ \chi^{1, \mu_1 k_\mu_2^\perp} \}_0 \) | \( T^{(3^*)} \) | \( 2^- \) | \( \frac{1}{\sqrt{10}} \gamma_5^\perp \gamma_5 \{ \mu_1 \mu_2 \} \) | \( \frac{3}{2}^- \) |
where the $\hat{\phi}_{\delta p}^{\mu_1\cdots\mu_j}(v,k;K)$ are spin projection operators which project out particular spin and parity states of the diquark from the unknown orbital wave functions $A$. In the following, we shall refer to the $\hat{\phi}_{\delta p}^{\mu_1\cdots\mu_j}(v,k;K)$ as the light diquark spin wave functions (s.w.f.). The spin wave functions for both the heavy-side and light diquark system, for $S$ wave and $P$ wave baryons, are listed in Table 1. In this table, $\chi^0 = \frac{1}{2\sqrt{2}}[(\not{v} + 1)\gamma_5 C]$ and $\chi^{1,\mu} = \frac{1}{2\sqrt{2}}[(\not{v} + 1)\gamma_\mu C]$ with $C$ being the charge conjugation operator. Details of the normalization of the light diquark and heavy-side spin wave functions can be found in [2] and [3].

In Table 1, we have included the symmetric and antisymmetric light diquark flavor wave functions $T_{ab}^{(6)}$ and $T_{ab}^{(3^*)}$ respectively. They transform as the sextet ($\Sigma$-type) and anti-triplet ($\Lambda$-type) representation of flavor $SU(3)$. Explicit representations of these functions, in terms of their quark content, will be given later on. The $\hat{\phi}_{\alpha\beta}^{\mu_1\cdots\mu_j}(v,k;K) \otimes T_{ab}$ are constructed ensuring overall symmetry with respect to \textit{colour} $\otimes$ \textit{flavour} $\otimes$ \textit{spin} $\otimes$ \textit{orbital}. It is not difficult to see from Table I that, including the flavor factors and defining the indices ($A = \alpha, a$; $B = \beta, b$), the $\hat{\phi}_{AB}^{\mu_1\cdots\mu_j} = \hat{\phi}_{\alpha\beta}^{\mu_1\cdots\mu_j}T_{ab}$ are symmetric for $S$ wave states under interchange of the indices $A$ and $B$ whereas they are symmetric (antisymmetric) for $P$ wave states depending on whether they are functions of $K$ ($k$). Since $\alpha$ and $\beta$ are essentially two component indices in $\chi^0$ and $\chi^{1,\mu}$ because of the Bargmann-Wigner equations [4], [5], the light diquark spin-flavor wave functions $\hat{\phi}_{AB}^{\mu_1\cdots\mu_j}$ transform as irreducible representations of $SU(6) \otimes O(3)$ if $N_f = 3$ is the number of light flavors. The three different blocks in Table I correspond to the three $SU(6) \otimes O(3)$ irreducible representations $21 \otimes 1$, $21 \otimes 3$ and $15 \otimes 3$ respectively. This will be an important fact to remember later.

In the next section, we shall present the HQS predictions for single pion transitions among ground states and from $P$ wave to $S$ wave states. Sec. 3 is devoted to derive the constituent quark model relations using the $SU(2N_f) \otimes O(3)$ symmetry of the light degrees of freedom. Various phe-
nomenological predictions for charmed baryons strong decays are presented in Sec. 4. Section 5 contains some concluding remarks. Moreover, the constituent quark model is also employed to reduce the number of heavy quark symmetry coupling constants for \( P \) wave to \( P \) wave transitions in Appendix A. Finally, we use the quantum theory of angular momentum in Appendix B to rewrite our constituent quark model results on the one-pion transitions in terms of a general master formula containing a product of 6j- and 9j-symbols.

2 Heavy Quark Symmetry Relations

In the heavy quark limit, the orbital momenta of the pion relative to the diquark \( l_\pi \) and relative to the baryon \( L_\pi \) are identical (\( l_\pi = L_\pi \)). Using Heavy Quark Symmetry the one–pion transition amplitudes between heavy baryons can then be written as \([2]\)

\[
M^\pi = \langle \pi(\vec{p}), B_{Q2}(v) | T | B_{Q1}(v) \rangle \\
= \psi_2^{\nu_1...\nu_{j_2}}(v)\psi_1^{\mu_1...\mu_{j_1}}(v)\left( \sum_{l_\pi} f_{l_\pi} t_{\mu_1...\mu_{j_1};\nu_1...\nu_{j_2}}^{l_\pi} \right).
\] (3)

The light diquark transition tensors \( t_{\mu_1...\mu_{j_1};\nu_1...\nu_{j_2}}^{l_\pi} \) of rank \((j_1 + j_2)\), built from \( g_{\perp \mu \nu} = g_{\mu \nu} - v_\mu v_\nu \) and \( p_{\perp \mu} = p_\mu - v \cdot \mathbf{p} v_\mu \), should have the correct parity and project out the appropriate partial wave amplitude with amplitude \( f_l \).

From now on, we shall sometimes use \( l \) to refer to the pion momentum \( l_\pi \) if there can be no confusion. In Table 2 we summarize the relevant covariant tensors for the allowed diquark transitions considered in this paper. We have introduced the traceless, symmetric and second rank tensor \( T_{\mu \nu} \), appropriate for d-wave transitions from \( P \) wave to \( S \) wave. It is defined by

\[
T_{\mu_1 \mu_2}(p) = p_{\perp \mu_1} p_{\perp \mu_2} - \frac{p_1^2}{3} g_{\perp \mu_1 \mu_2} ,
\] (4)

note that \( p_1^2 = -|\mathbf{p} \cdot \mathbf{v}| \). One also needs the corresponding third rank and traceless tensor \( T_{\mu \nu \rho} \) appropriate for f-wave transitions among the \( P \) wave
Table 2: Tensor structure of covariant pion couplings for the allowed diquark transitions. The fourth column gives the values of the rate factors $|c(j_1,j_2,l)|^2$ in the rate formula Eq. (10).

| Orbital Wave | Diquark Transition | Covariant Coupling | Rate Factor |
|--------------|--------------------|--------------------|-------------|
| s-wave      | $0^- \rightarrow 0^+$ | scalar             | $1/2$       |
| ($l_\pi = 0$) | $1^- \rightarrow 1^+$ | $g_{\perp \mu_1 \nu_1}$ | $1/2$       |
| p-wave      | $1^\pm \rightarrow 0^\pm$ | $p_{\perp \mu}$   | $1/6$       |
| ($l_\pi = 1$) | $2^- \rightarrow 1^-$ | $i(\mu_1 \nu_1 p \nu)$ | $1/6$       |
| d-wave      | $1^- \rightarrow 1^+$ | $T_{\mu_1 \nu_1}$  | $1/9$       |
| ($l_\pi = 2$) | $2^- \rightarrow 0^+$ | $T_{\mu_1 \mu_2}$  | $1/15$      |
| f-wave      | $2^- \rightarrow 1^-$ | $i(\mu_1 \nu_1 \rho \sigma) \nu^\sigma T_{\mu_1 \nu_1}^{\mu_2}$ | $1/25$      |
| ($l_\pi = 3$) | $2^- \rightarrow 2^-$ | $i(\mu_1 \nu_1 \rho \sigma) \nu^\sigma T_{\mu_1 \nu_1}^{\mu_2}$ | $1/15$      |

states which is given by

$$T_{\mu_1 \mu_2 \mu_3} = p_{\perp \mu_1} p_{\perp \mu_2} p_{\perp \mu_3} - \frac{p_\perp^2}{5} \left( g_{\perp \mu_1 \mu_2 \mu_3} + cycl. (\mu_1 \mu_2 \mu_3) \right), \quad (5)$$

The f-wave tensor $T_{\mu \nu \rho}$ is symmetric with respect to the exchange of any pair of Lorentz indices. The construction of the most general traceless tensor $T^{\mu_1 \mu_2 \cdots \mu_l}$ of rank $l$, necessary to describe $l$-wave transitions can be done in two stages. One begins with projecting $T^{\mu_1 \mu_2 \cdots \mu_l}$ on the space of completely symmetric tensors by symmetrizing it. Then one subtracts appropriate tensors in order that the total trace is null. The general form of such a tensor is given in [1].
A simple $LS$-coupling exercise shows that the number of independent covariants or partial waves contributing to a given transition $j_1^P \rightarrow j_2^P + \pi$ is given by $N = j_{\min} + \frac{1}{2}(1 - n_1 n_2)$ where $j_{\min} = \min\{j_1, j_2\}$. The normality $n$ of a diquark state with quantum numbers $j^P$ is defined by $n = P(-1)^j$. By similar reasoning one finds that the $\epsilon$-tensor should be present when the product of the normalities is even ($n_1 n_2 = +1$).

In Table 3, we list the allowed transitions between the heavy baryon ground states ($S$ wave to $S$ wave) and those from the excited ($P$ wave) state down to the ground state. The $S$ wave to $S$ wave transitions involve two $p$-wave coupling constants while each of the single pion transitions from the K-multiplet and from the $k$-multiplet down to the ground state are determined in terms of seven coupling constants. There are three $s$-wave and four $d$-wave couplings for each.

The HQS single pion transitions of Table 3 are expressed in terms of transition amplitudes. They could have equally well be written down using the language of effective Lagrangians. In this case the heavy baryon spin wave functions would be replaced by the appropriate heavy baryon superfields with derivative couplings to the pion field.

Let us also briefly comment on the relation of our approach to the chiral invariant coupling method used in [4, 5, 6] when the chiral invariant Lagrangian is expanded to first order in the pion field. The chiral formalism implies that all the one pion coupling factors are proportional to the factor $1/f_\pi$ associated with the pion field. For the $s$-wave transitions in the chiral approach there is an extra factor $v \cdot p = E_\pi$, which comes in because the pion field is coupled via the scalar term $v \cdot A$, where $A$ is the nonlinear axial field. Thus, in the rest frame of the heavy baryons the pion field couples through the scalar component of the axial vector current in the $s$-wave transitions. This is in contrast to our effective coupling approach, to be discussed later on, where the coupling is always through the three-vector piece of the axial vector current. Also the $p_{\perp}^2$ factors required in the construction of our
d-wave and f-wave tensors, Table 2 can be written as $E_\pi^2$ in the chiral formalism when terms linear in the light quark masses are neglected (see [6]). In fact, when these terms are kept, one does obtain the correct $p_\perp^2$ factor. Up to the mentioned factors there is a one to one correspondence between our coupling factors and those in the chiral approach.

The decay rates of the various transitions of Table 3 can be calculated using the general rate formula

$$\Gamma = \frac{1}{2J_1 + 1} \frac{|\vec{p}|}{8\pi M_1^2} \sum_{\text{spins}} |M^\pi|^2.$$  \hspace{1cm} (6)

Here, $|\vec{p}|$ is the pion momentum in the heavy baryon’s rest frame $|\vec{p}|^2 = p_{\perp \mu}p_{\perp \nu}g^{\mu\nu}$ and $M^\pi$ are the transition amplitudes listed in Table 3. When calculating decay rates in terms of the covariant expressions of Table 3 it is highly advisable to use the accelerated spin sum algorithm of [7]. For example, the four transitions in $\Lambda_{QK2} \rightarrow \Sigma_Q + \pi$ or $\Sigma_{QK2} \rightarrow \Sigma_Q + \pi$ are given by

$$\Gamma \left( \frac{3}{2}^- \rightarrow \frac{1}{2}^+ + \pi \right) = f_{\text{had}}^2 \frac{|\vec{p}|^5}{20\pi} \frac{M_2}{M_1},$$

$$\Gamma \left( \frac{3}{2}^- \rightarrow \frac{3}{2}^+ + \pi \right) = f_{\text{had}}^2 \frac{|\vec{p}|^5}{20\pi} \frac{M_2}{M_1},$$

$$\Gamma \left( \frac{5}{2}^- \rightarrow \frac{1}{2}^+ + \pi \right) = f_{\text{had}}^2 \frac{1}{45\pi} \frac{M_2}{M_1},$$

$$\Gamma \left( \frac{5}{2}^- \rightarrow \frac{3}{2}^+ + \pi \right) = f_{\text{had}}^2 \frac{7}{90\pi} \frac{M_2}{M_1}.$$  \hspace{1cm} (7)

where the flavor factors have been omitted. It is easy, from these equations, to show that the decay rates for these four one–pion transitions satisfy the following ratios

$$\Gamma_{\frac{3}{2}^- \rightarrow \frac{1}{2}^+ + \pi} : \Gamma_{\frac{3}{2}^- \rightarrow \frac{3}{2}^+ + \pi} : \Gamma_{\frac{5}{2}^- \rightarrow \frac{1}{2}^+ + \pi} : \Gamma_{\frac{5}{2}^- \rightarrow \frac{3}{2}^+ + \pi} = 9 : 9 : 4 : 14.$$  \hspace{1cm} (8)

Moreover, we have

$$\Gamma_{\frac{3}{2}^- \rightarrow \frac{1}{2}^+ + \pi} + \Gamma_{\frac{3}{2}^- \rightarrow \frac{3}{2}^+ + \pi} = \Gamma_{\frac{5}{2}^- \rightarrow \frac{1}{2}^+ + \pi} + \Gamma_{\frac{5}{2}^- \rightarrow \frac{3}{2}^+ + \pi}.$$  \hspace{1cm} (9)
Table 3: Heavy Quark Symmetry (HQS) predictions for the allowed single pion transitions between ground states ($S$ wave) and from excited ($P$ wave) down to the ground state.

| Heavy baryon transitions | Transition amplitudes |
|--------------------------|-----------------------|
| $B_Q' \to B_Q + \pi$    | $M^\pi$               |
| Ground-State             |                       |
| $\Sigma_Q \to \Lambda_Q$| $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{3}} \gamma_\mu \gamma_5 u_1(v) \\ u_1^\mu(v) \end{pmatrix} I_1 f_{1p} p_{\perp \mu}$ |
| $\Sigma_Q \to \Sigma_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{3}} \gamma_\mu \gamma_5 u_1(v) \\ u_1^\mu(v) \end{pmatrix} I_2 f_{2p} i \varepsilon (\mu \nu \rho \nu)$ |
| K-multiplet              |                       |
| $\Lambda_{QK1} \to \Sigma_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{3}} \gamma_\mu \gamma_5 u_1(v) \\ u_1^\mu(v) \end{pmatrix} I_3 \left(f_{1s} g_{\mu \nu} + f_{1d}^{(K)} T_{\mu \nu}\right)$ |
| $\Sigma_{QK0} \to \Lambda_Q$ | $I_1 f_{2s}^{(K)} \bar{u}_2(v) u_1(v)$ |
| $\Sigma_{QK1} \to \Sigma_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{3}} \gamma_\mu \gamma_5 u_1(v) \\ u_1^\mu(v) \end{pmatrix} I_2 \left(f_{3s} g_{\mu \nu} + f_{3d}^{(K)} T_{\mu \nu}\right)$ |
| $\Sigma_{QK2} \to \Lambda_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{10}} \gamma_5 \gamma_\perp^{(\mu_1 \mu_2)}(v) \\ u_1^{(\mu_1 \mu_2)}(v) \end{pmatrix} I_1 f_{4d}^{(K)} T_{\mu_1 \mu_2}$ |
| $\Sigma_{QK2} \to \Sigma_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{10}} \gamma_5 \gamma_\perp^{(\mu_1 \mu_2)}(v) \\ u_1^{(\mu_1 \mu_2)}(v) \end{pmatrix} I_2 f_{5d}^{(K)} i \varepsilon_{\mu_1 \nu \rho \sigma} \rho T_{\mu_2}^\rho$ |
| K-multiplet              |                       |
| $\Sigma_{Qk1} \to \Sigma_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{3}} \gamma_\mu \gamma_5 u_1(v) \\ u_1^\mu(v) \end{pmatrix} I_2 \left(f_{1s}^{(k)} g_{\mu \nu} + f_{1d}^{(k)} T_{\mu \nu}\right)$ |
| $\Lambda_{Qk0} \to \Lambda_Q$ | $\bar{u}_2(v) u_1(v) I_4 f_{2s}^{(k)}$ |
| $\Lambda_{Qk1} \to \Sigma_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{3}} \gamma_\mu \gamma_5 u_1(v) \\ u_1^\mu(v) \end{pmatrix} I_3 \left(f_{3s}^{(k)} g_{\mu \nu} + f_{3d}^{(k)} T_{\mu \nu}\right)$ |
| $\Lambda_{Qk2} \to \Lambda_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{10}} \gamma_5 \gamma_\perp^{(\mu_1 \mu_2)}(v) \\ u_1^{(\mu_1 \mu_2)}(v) \end{pmatrix} I_4 f_{4d}^{(k)} T_{\mu_1 \mu_2}$ |
| $\Lambda_{Qk2} \to \Sigma_Q$ | $\bar{u}_2(v) \begin{pmatrix} \frac{1}{\sqrt{10}} \gamma_5 \gamma_\perp^{(\mu_1 \mu_2)}(v) \\ u_1^{(\mu_1 \mu_2)}(v) \end{pmatrix} I_3 f_{5d}^{(k)} i \varepsilon_{\mu_1 \nu \sigma} \rho T_{\mu_2}^\rho$ |
The result, Eq.(8), and the sum rule, Eq.(9), agree with the conventional approach using Clebsch–Gordan coefficients [8] and also with the more recent and compact procedure using the 6j-symbols [2, 9, 10]. In fact, using the 6j-symbol approach, one can write down a closed form expression for the decay rates of all transitions. One has

\[
\Gamma_i = \frac{1}{\pi} \frac{M_2}{M_1} |\vec{p}|^{2l+1} f_{\delta} |I_i|^2 |c(j_1, j_2, l)|^2
\]

\[
(2j_1 + 1)(2J_2 + 1) \left( \left\{ \begin{array}{ccc} l & j_1 & j_2 \\ s_Q & J_2 & J_1 \end{array} \right\} \right)^2.
\]

(10)

Here, we have included the flavor factors $|I_i|^2$ which depend on the specific flavor channels involved in the transition. Explicit forms for these SU(3) flavor factors are given in Eq.(12). The curly bracket stands for the usual 6j-symbol given in Table 7 of appendix B and $c(j_1, j_2, l)$ is the ratio of the reduced amplitude appearing in the 6j-approach and the invariant coupling factor $f_{\delta}$ of Table 3. This proportionality factor is a function of $j_1, j_2$ and $l$ alone. The rate factors $c(j_1, j_2, l)$ for the transitions discussed in this paper are given in Table 2.

Now, using the standard orthogonality relation for the 6j-symbols

\[
\sum_{J_2} (2j_1 + 1)(2J_2 + 1) \left( \left\{ \begin{array}{ccc} l & j_2 & j_1 \\ s_Q & J_2 & J_1 \end{array} \right\} \right)^2 = 1,
\]

(11)

one can immediately derive Eq.(9). This sum rule shows that the total rate of pionic decays from any of the two HQS doublet states $J_1 = j_1 \pm \frac{1}{2}$ into the HQS doublet states $J_2 = j_2 \pm \frac{1}{2}$ is independent of $J_1$. In a similar manner one concludes that, the rates of transition into a heavy quark singlet state from the two doublet states and vice versa are identical to one another.

Table 3 also contains the appropriate $SU(3)$ factors denoted by

| $I_1$  | $(6 \to 3^* + \pi)$  | $T^{(3^*)ac}T^{(6)}_{bc}M^b_a$ |
|--------|----------------------|--------------------------------|
| $I_2$  | $(6 \to 6 + \pi)$    | $T^{(6)}_{ac}T^{(6)}_{bc}M^b_a$ |
\[ I_3 \ (3^* \to 6 + \pi) = T^{(6)bc} T^{(3^*)}_{bc} \bar{M}^b_a \]  
\[ I_4 \ (3^* \to 3^* + \pi) = T^{(3^*)bc} T^{(3^*)}_{bc} \bar{M}^b_a \]  

where \( T^{(6)}_{ab} \) and \( T^{(3^*)}_{ab} \) are respectively the symmetric and antisymmetric flavor tensors, transforming as sextet and anti-triplet in SU(3) and are built from two light quark states. Explicit expressions for these tensors are given by:

i) \( T^{(6)}_{ab} \)

\[
\begin{align*}
\Sigma^{++}_{c} & : \ uu \\
\Sigma^{+}_{c} & : \frac{1}{\sqrt{2}}(ud + du) \\
\Sigma^{0}_{c} & : \ dd \\
\Xi^{'+}_{c} & : \frac{1}{\sqrt{2}}(us + su) \\
\Xi^{0}_{c} & : \frac{1}{\sqrt{2}}(ds + sd) \\
\Omega^{0}_{c} & : \ ss 
\end{align*}
\]

ii) \( T^{(3^*)}_{ab} \)

\[
\begin{align*}
\Lambda^{+}_{c} & : \frac{1}{\sqrt{2}}(ud - du) \\
\Xi^{+}_{c} & : \frac{1}{\sqrt{2}}(us - su) \\
\Xi^{0}_{c} & : \frac{1}{\sqrt{2}}(ds - sd) 
\end{align*}
\]

Note that we have labeled the members of the sextet and anti-triplet representations according to their charm baryon content. In the bottom sector one would have to make the changes \( c \to b \) and lower the respective charge by 1. \( \bar{M}^b_a \) is the transpose of the 3 \( \times \) 3 pion flavor wave function with the components \( \pi^+ : ud\bar{d}, \pi^0 : \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d}) \) and \( \pi^- : d\bar{u} \).

\(^1\) We normalize the \( T^{(3^*)}_{ab} \) states to unity while in Ref. \(^3\) they are normalized to 2. Therefore, the relations between the \( SU(3) \) factors \( I'_i \) defined in \(^3\) to ours are \( I'_1 = \sqrt{2}I_1, I'_3 = \sqrt{2}I_3 \) and \( I'_4 = 2I_4 \).
3 Constituent Quark Model Relations

To describe the light diquark transitions, we shall use the constituent quark model with its underlying $SU(6) \otimes O(3)$ symmetry to go beyond the Heavy Quark Symmetry predictions. For the $S$ wave to $S$ wave transitions, the light diquark tensors of the matrix elements in Eq. (3), including flavor factors, can be written as

$$ t_{\mu_1...\mu_{j_1};\nu_1...\nu_{j_2}} = \left( \hat{\phi}_{\mu_1...\mu_{j_1}} \right)^{AB}_{A'B'} \left( O^{t_s} \right)_{AB}^{A'B'} \left( \hat{\phi}_{\nu_1...\nu_{j_2}} \right)^{A'B'}_{A'B'} . \quad (15) $$

Here, the light diquark spin wave functions $\hat{\phi}$'s involve either $\chi^{0}_{\alpha\beta} T^{a}_{\alpha\beta}$ for spin zero or $\chi^{1}_{\alpha\beta} T^{a}_{\alpha\beta}$ for spin one and the operator $O$ is given by an overlap integral which we do not attempt to calculate. We have retained the generic representation of the light diquark spin wave functions in terms of $j_1$ and $j_2$ Lorentz indices. For $S$ wave to $S$ wave pion transitions, the partial wave involved is $l_{\pi} = 1$ and hence it is easy to see that $O$ must be a pseudoscalar operator involving one power of the pion momentum $p$. In the constituent quark model the pion is emitted by just one of the light quarks, therefore, the transition operator $O$ must be a one-body operator. In $1/N_C$ [11], two-body emission operators for pion couplings to $S$ wave heavy baryons are non leading and can be neglected in the constituent quark model approach [12, 13]. The $1/N_C$-approach thus provides a justification of neglecting two-body and higher emission operators in the constituent quark model.

Because of the overall symmetry of the light diquark spin-flavor wave function $\left( \hat{\phi}_{\mu_1...\mu_{j_1}} \right)^{A'B'}_{A'B'}$ for $S$ waves, one then has uniquely

$$ (O(p))^{AB}_{A'B'} = \frac{1}{2} \left( (\gamma_\sigma \gamma_5)_A^{A'} \otimes (\not{\mu}_B^{B'}) + (\not{\mu}_A^{A} \otimes (\gamma_\sigma \gamma_5)_B^{B'}) \right) f_p p_\perp \quad (16) $$

with $(\gamma_\sigma \gamma_5)_A^{A'}$ defined as

$$ (\gamma_\sigma \gamma_5)_A^{A'} = (\gamma_\sigma \gamma_5)_A^{A'} M_a^a . \quad (17) $$
When considering excited states, it is convenient to explicitly pull out the relative momentum factor in the light diquark spin wave functions according to
\[
\hat{\phi}^{\mu_1...\mu_j}(v, p) = \hat{\phi}^{\mu_1...\mu_j;\lambda}(v)p_{\perp\lambda}
\]
(18)
where \(p_{\perp\lambda}\) is either \(k_{\perp\lambda}\) or \(K_{\perp\lambda}\) and \(\hat{\phi}^{\mu_1...\mu_j;\lambda}(v)\) is only a function of \(v\). The generalization to higher orbital excitations is straightforward. The relevant matrix elements for \(P\) wave to \(S\) wave transitions are given by
\[
t_{\mu_1...\mu_j\nu_1...\nu_{j_2}}^\pi = \left( \hat{\phi}_{\nu_1...\nu_{j_2}}^{\pi} \right)^{AB} \left( \mathcal{O}^{(l_\pi)}_{\lambda} \right)_{AB}^{A'B'} \left( \hat{\phi}_{\mu_1...\mu_j;\lambda} \right)^{A'B'}_{A'B}.
\]
(19)
Here, the operators \(\mathcal{O}^{(l_\pi)}_{\lambda}\) are responsible for the s-wave \((l_\pi = 0)\) and d-wave \((l_\pi = 2)\) transitions. Neglecting two-body emission contributions, the transition operators from the K-multiplet down to the ground state are given by
\[
\left( \mathcal{O}^{K}_A(p) \right)^{AB}_{A'B'} = \frac{1}{2} \left( (\gamma^\sigma\gamma_5)_{A'} A' \otimes (1)_{B'}^B + (1)_{A'}^A \otimes (\gamma^\sigma\gamma_5)_{B'}^B \right) \left( f_{s}^{(K)} g_{\sigma\lambda} + f_{d}^{(K)} T_{\sigma\lambda} \right).
\]
(20)
For transitions from the k-multiplet the operators instead have the form
\[
\left( \mathcal{O}^{K}_A(p) \right)^{AB}_{A'B'} = \frac{1}{2} \left( (\gamma^\sigma\gamma_5)_{A'} A' \otimes (1)_{B'}^B - (1)_{A'}^A \otimes (\gamma^\sigma\gamma_5)_{B'}^B \right) \left( f_{s}^{(K)} g_{\sigma\lambda} + f_{d}^{(K)} T_{\sigma\lambda} \right).
\]
(21)
The relative minus sign in the effective operator Eq. (21) comes about because the quark-quark operator is inserted between the orbitally antisymmetric \(k\)-states and the orbitally symmetric \(S\)-wave states. In the \(1/N_C\)-approach, the contributions of one- and two-body emission operators are of the same order when \(P\)-wave baryons are involved in pion transitions. However, their contributions are proportional to one another so that one needs to keep only the one-body emission operators in the constituent quark model approach [13].

One can also proceed to construct the operators \(\mathcal{O}^{(l_\pi)}_{\lambda_1\lambda_2}\) for the allowed \(P\) wave to \(P\) wave transitions associated with the p-wave \((l_\pi = 1)\) and f-wave
We write

\[ (O_{\lambda_1 \lambda_2}(p))^{AB}_{A'B'} = \frac{1}{2} \left( (\gamma^\sigma \gamma_5)^A_{A'} \otimes (\mathbb{1}^B_{B'}) \pm (\mathbb{1}^A_{A'} \otimes (\gamma^\sigma \gamma_5)^B_{B'}) \right) \]

\[ \left( \sum_{L=0}^{2} g_{p}^{(L)} P_{\sigma \lambda_1 \lambda_2}^{(L)} + g_{f} T_{\sigma \lambda_1 \lambda_2} \right), \quad (22) \]

with

\[ P_{\sigma \lambda_1 \lambda_2}^{(0)} = p_{\perp \sigma} g_{\lambda_1 \lambda_2} \]

\[ P_{\sigma \lambda_1 \lambda_2}^{(1)} = \frac{1}{2} \left( p_{\perp \lambda_1} g_{\sigma \lambda_2} - p_{\perp \lambda_2} g_{\sigma \lambda_1} \right) \]

\[ P_{\sigma \lambda_1 \lambda_2}^{(2)} = \frac{1}{2} \left( p_{\perp \lambda_1} g_{\sigma \lambda_2} - p_{\perp \lambda_2} g_{\sigma \lambda_1} \right) - \frac{1}{3} p_{\perp \sigma} g_{\lambda_1 \lambda_2} \], \quad (23) \]

and the third rank tensor \( T_{\sigma \lambda_1 \lambda_2} \) is defined in Eq. (8). The \( p \)-wave transition tensors \( P_{\sigma \lambda_1 \lambda_2}^{(L)} \), with \( L = 0, 1, \) and 2 correspond to the three possible angular momenta inducing the transitions between the two orbital \( P \) wave states. The \( p \)-wave and \( f \)-wave coupling constants are represented by \( g_{p}^{(L)} \) and \( g_{f} \), respectively. Note that, from angular momentum conservation, there is only one coupling possibility for the \( f \)-wave transition. In Eq. (22), the (+) sign has to be used for the \( K \rightarrow K \) and \( k \rightarrow k \) pionic transitions, whereas the (−) sign is appropriate for the \( K \rightarrow k \) and the \( k \rightarrow K \) transitions. The constituent quark model analysis of \( P \) wave to \( P \) wave transitions will be presented in Appendix A. The generalization to transitions involving higher orbital excitations is straightforward.

The matrix elements Eq. (15) and Eq. (19) of the operators Eq. (14), Eq. (20) and Eq. (21), can be readily evaluated using the light diquark spin wave functions in Table I. The two ground state to ground state coupling strengths can be seen to be related to the single coupling \( f_{P} \) as

\[ f_{1p} = f_{2p} = f_{P}. \]

\[ (l_\pi = 3). \]
For $P$ wave ($K$-multiplet) to $S$ wave transitions the evaluation of the matrix elements leads to the following relations

\begin{align*}
  f_{1s}^{(K)} &= f_s^{(K)} ; & f_{2s}^{(K)} &= \sqrt{3} f_s^{(K)} ; & f_{3s}^{(K)} &= -\sqrt{2} f_s^{(K)} \\
  f_{1d}^{(K)} &= f_d^{(K)} ; & f_{3d}^{(K)} &= \frac{1}{\sqrt{2}} f_d^{(K)} ; & f_{4d}^{(K)} &= f_d^{(K)} ; & f_{5d}^{(K)} &= -f_d^{(K)}
\end{align*}

(25)

The number of independent coupling constants has been reduced from seven to the two constituent quark model s-wave and d-wave coupling factors $f_s^{(K)}$ and $f_d^{(K)}$.

In a similar way, one reduces the seven decay couplings for pion transitions from the excited $k$-multiplet to the two corresponding constituent quark model coupling factors $f_s^k$ and $f_d^k$. With the appropriate replacement

\begin{align*}
  f_{is, id}^{(K)} &\to f_{is, id}^{(k)} \text{ and } f_{s, d}^{(K)} &\to f_{s, d}^{(k)} ,
\end{align*}

(27)

these relations are identical to those in Eqs. (25) and (26). The constituent quark model predictions for $P$ wave to $S$ wave are in agreement, after taking care of the different normalization of the $\Lambda$-type flavor factors, with corresponding results in the Chiral formalism [6]. We mention that the constituent quark model calculation in [6] has been done using static quark model wave functions and explicit Clebsch-Gordan coefficients as compared to our covariant approach. The two methods are of course equivalent to each other even if they use different calculationl techniques.

In appendix A, we give our results on the $P$ wave to $P$ wave single pion transitions. It is obvious that the $D$ wave to $S$ wave or $D$ wave to $P$ wave transitions, not treated here, can be analyzed following the same procedure.

### 4 Phenomenological Predictions

Pion transitions of charm baryons are interesting from the experimental point of view where data are already published by some laboratories [14, 15].
Therefore, it is important to predict some of these transition rates and to compare them with other theoretical models. The constituent quark model coupling $f_p$ determines the single pion transitions among the $S$ wave heavy baryon states. On the other hand, $f_s^{(K)}$ and $f_d^{(K)}$ are sufficient to predict transitions from the $P$ wave $K$-multiplet to the ground state.

Using PCAC the p-wave coupling constant $f_p$ in Eq. (24) can be related to the quark’s axial vector current coupling strength $g_A$ [16, 17], one obtains

$$f_p = g_A / f_\pi,$$ \hspace{1cm} (28)

with $g_A$ of the order of unity. However, we mention that there is some additional theoretical support for values of $g_A < 1$ from an analysis of the NJL model [18] and the Skyrme model [19]. This result is also obtained if one demands that the experimentally measured $G_A/G_V$ value comes out right in the constituent quark model. An indication about the $f_p$ strength can be obtained from Eq. (28) by taking $g_A \approx 0.75$ [16, 17] and $f_\pi = 0.093$ GeV. One gets

$$f_p = 8.06 \text{ GeV}^{-1}.$$ \hspace{1cm} (29)

The single-pion decay rates can be calculated using the rate formula, Eq. (3), Table 3 and the relations (24) and (25-27). Considering $P$ wave to $S$ wave transitions, the decay rates for the one pion transition of the $\Lambda_{QK1}$ doublet to $\Sigma_c$ are given by

$$\Gamma\left(\frac{1}{2}^- \rightarrow \frac{1}{2}^+ + \pi\right) = f_s^{(K)} I_3^2 \left| \vec{p} \right|^5 \frac{M_2}{M_1^2},$$ \hspace{1cm} (30)

$$\Gamma\left(\frac{3}{2}^- \rightarrow \frac{1}{2}^+ + \pi\right) = f_d^{(K)} I_3^2 \left| \vec{p} \right|^5 \frac{M_2}{18 \pi M_1^2}.$$ \hspace{1cm} (31)

These equations are derived using Eq. (19) and Table 2. Replacing $f_s^{(K)}$ by $f_s^{(k)}$, $f_d^{(K)}$ by $f_d^{(k)}$ and $I_3$ by $I_2$, one can immediately calculate the decay rates for the one pion transition $\Sigma_{QK1} \rightarrow \Sigma_c$ for which there is unfortunately no data available at present. The other interesting pion transitions are those for
\[ \Sigma_{QK2} \rightarrow \Sigma + \pi \text{ and } \Lambda_{Qk2} \rightarrow \Sigma_c + \pi \] which proceed via d-wave transitions. In terms of \( J^P \) quantum numbers one has the transitions \( \{ \frac{3}{2}^-, \frac{5}{2}^- \} \rightarrow \{ \frac{1}{2}^+, \frac{3}{2}^+ \} + \pi \) and their decay rates are given by Eq. (7).

Using \( SU(6) \times O(3) \) symmetry the coupling constants \( f_s^{(K)} \) and \( f_d^{(K)} \) are sufficient to predict transitions from the K-multiplet to the ground state. To estimate the \( f_s^{(K)} \) coupling, we use the recent experimental values for the decay width of \( \Lambda_{cK1}(\frac{1}{2}^-) \) (\( \Gamma_{\Lambda_{cK1}(2593)} = 3.6^{+3.7}_{-3.0} \text{ MeV} \)) reported by CLEO [15]. Assuming that this width is saturated by the strong decay and using Eq. (30) one gets

\[
f_s^{(K)} = 1.05^{+0.54}_{-0.44}
\] (32)

The uncertainty in \( f_s^{(K)} \) is mainly due to the experimental error in \( \Gamma_{\Lambda_{cK1}(\frac{1}{2}^-)} \).

To predict the value of the coupling constant in the Chiral formalism \( h_2 \) defined in [21, 4, 3, 9] we use \( h_2 = f_s^{(K)} \frac{f_{\pi}}{E_{\pi}} \), with \( f_{\pi} = 0.093 \text{ GeV} \), to obtain

\[
h_2 = 0.69^{+0.35}_{-0.29}.
\] (33)

Strong decays involving \( \Xi_{cK1}^+ \) can also be used to determine the coupling \( f_s^{(K)} \). Assuming that 70% of the \( \Xi_{cK1}(2815) \) width is saturated by \( \Xi_{cK1}^+(2815) \rightarrow \Xi_c^0 \pi^+, \) one finds

\[
f_s^{(K)} = 0.48,
\] (34)

here we have taken (\( \Gamma_{\Xi_{cK1}^+(\frac{3}{2}^+)} < 3 \text{ MeV} \)) which is the upper limit set by CLEO [13]. This result suggests that the strength of the \( \Lambda_{cK1} \) single pion coupling to \( \Sigma_c \) is about twice the \( \Xi_{cK1}^+ \) to \( \Xi_c^0 \) coupling. We would like to mention that most our predictions are in agreement with those reported in ref. [22] using a three-quark model.

There is no precise experimental data available which can be used to estimate the \( f_d^{(K)} \) numerical value. However, the upper bound set by CLEO for the transition \( \Lambda_{cK1}(2625) \rightarrow \Sigma_c^0 \pi^+ \) < 0.13 MeV and Eq. (31) can be used to predict the coupling \( f_d^{(K)} \). One obtains

\[
f_d^{(K)} < 27.26 \pm 0.06 \text{ GeV}^{-2}.
\] (35)
One may also use the decay $\Lambda(1520) \rightarrow \Sigma\pi$, reported in [23], and Eq. (34) to get some idea about the strength of this coupling which is very sensitive to the numerical value of the emitted pion momentum. Moreover, one should bear in mind that, since the strange quark is not heavy enough, the $1/m_s$ corrections can be important in this case. Therefore, the predicted rates and couplings should be taken only as rough guesses. Using the published decay rate for this transition, the coupling $f_d^{(K)}$ is estimated to be

$$f_d^{(K)} = 18.62 \pm 0.05 \text{ GeV}^{-2}. \quad (36)$$

Using this result, one predicts the one-pion decay rates for $\Lambda_{cK1}(2625) \rightarrow \Sigma_c$ to be

$$\Gamma_{\Lambda_{cK1}(2625) \rightarrow \Sigma_c\pi} = 0.05 \text{ MeV}. \quad (37)$$

One concludes that the spin-$\left(\frac{3}{2}^+\right)$ member of the $\Lambda_{cK1}$ doublet is constrained to decay to the $\Sigma_c(1^+)$ via d-wave which is suppressed. Its preferred s-wave decay into $\Sigma_c(\frac{3}{2}^+)$ cannot occur because this channel is not accessible kinematically. In fact, the SCAT group reported the first evidences for the $\Sigma_c(\frac{3}{2}^+)$ at a mass of 2530 MeV [21]. This was confirmed later on by the CLEO Collaboration who quote a mass of $\approx 2518$ MeV for the $\Sigma_c(\frac{3}{2}^+)$. The suppression of the $\Lambda_{cK1}(\frac{3}{2}^-) \rightarrow \Sigma_c(\frac{1}{2}^+)$ single pion decay mode is in agreement with theoretical predictions reported in Ref. [5] obtained within the framework of chiral perturbation theory. These predictions have to wait until more experimental data will be available, however, it is still within the range of the most recent measurement published by the CLEO collaboration [15] ($\Gamma_{\Lambda_{cK1}(2625)} < 1.9 \text{ MeV}$).

5 Summary and Conclusion

We have written down the most general one pion coupling structure in the heavy quark symmetry limit guided by Lorentz and flavor invariance. It is not
difficult to see that the same coupling structure emerges when considering the leading order contribution of the corresponding chirally invariant Lagrangians written in [4, 5, 6]. Using a constituent quark model approach we exploited the $SU(2N_f) \times O(3)$ symmetry for the light diquark system to significantly reduce the number of independent coupling factors. The constituent quark model predictions were worked out in a covariant fashion using covariant spin wave functions for the light diquark system. Our constituent quark model predictions agree with the corresponding chiral formalism [6] in which rest frame quark model wave functions and explicit Clebsch-Gordan coefficients were used. This should not be surprising since both calculations are based on the same quark model picture. They, in fact, must be equivalent to each other even though it is not simple to see that at every step of the calculation.

There is, however, a slight difference between our predictions for the $s$-wave decay rates and those obtained by [4] and [20] who used heavy quark and chiral symmetries. This is due to the difference in the interaction of the pion field and the heavy quark in the chiral formalism for S wave transitions. The coupling in the chiral formalism is of scalar type while we use a vector coupling. This leads to the appearance of an extra $(\frac{E_{\pi}}{m})^2$ factor in the $s$-wave decay rate formula.

To conclude, we would like to mention that, the predictive power of the constituent quark model for pion transitions is limited to the heavy baryon sector. When applied to one pion transitions between heavy mesons the constituent quark model provides no predictions that go beyond those of Heavy Quark Symmetry (HQS). In this sense heavy baryons represent an ideal setting for probing the dynamics of a light diquark system as we have tried to emphasize in our analysis.
Appendix A

P wave to P wave one pion transitions

In this appendix we shall analyze single pion transitions among the P wave states. There is a proliferation of possible coupling factors for both the diagonal transitions ($K \rightarrow K$) and ($k \rightarrow k$) as well as for the non diagonal ones ($k \rightarrow K$) or ($K \rightarrow k$). In fact, one counts 8 p-wave plus 3 f-wave couplings for each of the diagonal cases. For the non diagonal case one needs 13 p-wave and 5 f-wave couplings where one should keep in mind that the ($k \rightarrow K$) and ($K \rightarrow k$) couplings are related to one another.

In Table 4 we list the allowed one pion transitions among the P wave states together with their associated coupling factors. The HQS transition amplitudes for the allowed P wave to P wave decays can be easily written down in a manner similar to those quoted in Table 3. In order to save on space, we shall not write down the amplitudes explicitly. They can be constructed using Table 4 for the heavy-side spin wave functions, Table 2 for the Lorentz structure and Eq. (12) for the flavor factors. Table 5 provides three examples for the diagonal and non diagonal single pion transition amplitudes among P wave states. In the examples listed in Table 5 we have also given the appropriate flavour factors according to the $SU(3)$ coupling factors of Eq. (12). Explicitly one has the coupling factors $I_4$ for $\Lambda \rightarrow \Lambda$, $I_3$ for $\Lambda \rightarrow \Sigma$, $I_1$ for $\Sigma \rightarrow \Lambda$ and $I_2$ for $\Sigma \rightarrow \Sigma$ transitions.

To proceed, using the constituent quark model one reduces the full set of coupling factors into just three p-wave and one f-wave coupling in each of the above mentioned cases. The quark model couplings \footnote{In order to fix the normalization of our coupling constants, one has to strictly adhere to the use of the building blocks as prescribed above and the corresponding matrix elements presented in Table 6 where examples for P wave to P wave transitions are provided.} will be labeled as $g_p^{(L)}$, with $L = 0, 1, \text{and } 2$, and $g_f$ for transitions among the K-multiplet, $g_p^{(L)}$ and $g_f'$ for transitions among the k-multiplet and $h_p^{(L)}$ and $h_f$ for the non diagonal...
Table 4: $P$ wave to $P$ wave allowed one pion transitions and coupling factors.

| $K \to K$ transitions | coupling factors | $k \to k$ transitions | coupling factors | $k \to K$ transitions | coupling factors |
|------------------------|-----------------|------------------------|-----------------|------------------------|-----------------|
| $\Lambda_{QK1} \to \Lambda_{QK1}$ | $g_{1p}^{(K)}$ | $\Sigma_{Qk1} \to \Sigma_{Qk1}$ | $g_{1p}^{(k)}$ | $\Sigma_{Qk1} \to \Lambda_{QK1}$ | $h_{1p}$ |
| $\to \Sigma_{QK0}$ | $g_{2p}^{(K)}$ | $\to \Lambda_{QK0}$ | $g_{2p}^{(k)}$ | $\to \Sigma_{QK0}$ | $h_{2p}$ |
| $\to \Sigma_{QK1}$ | $g_{3p}^{(K)}$ | $\to \Lambda_{QK1}$ | $g_{3p}^{(k)}$ | $\to \Sigma_{QK1}$ | $h_{3p}$ |
| $\to \Sigma_{QK2}$ | $g_{4p,4f}^{(K)}$ | $\to \Lambda_{QK2}$ | $g_{4p,4f}^{(k)}$ | $\to \Sigma_{QK2}$ | $h_{4p,4f}$ |
| $\Sigma_{QK0} \to \Sigma_{QK1}$ | $g_{5p}^{(K)}$ | $\Lambda_{Qk0} \to \Lambda_{Qk1}$ | $g_{5p}^{(k)}$ | $\Lambda_{Qk0} \to \Lambda_{QK1}$ | $h_{5p}$ |
| $\Sigma_{QK1} \to \Sigma_{QK1}$ | $g_{6p}^{(K)}$ | $\to \Lambda_{Qk1}$ | $g_{6p}^{(k)}$ | $\to \Sigma_{Qk1}$ | $h_{6p}$ |
| $\to \Sigma_{QK2}$ | $g_{7p,7f}^{(K)}$ | $\to \Lambda_{Qk2}$ | $g_{7p,7f}^{(k)}$ | $\to \Sigma_{QK2}$ | $h_{7p,7f}$ |
| $\Sigma_{QK2} \to \Sigma_{QK2}$ | $g_{8p,8f}^{(K)}$ | $\Lambda_{Qk2} \to \Lambda_{Qk2}$ | $g_{8p,8f}^{(k)}$ | $\Lambda_{Qk2} \to \Lambda_{QK1}$ | $h_{8p,8f}$ |

transitions. They multiply the p-wave tensor $P_{\sigma\lambda_1\lambda_2}^{(L)}$ and the f-wave tensor $T_{\sigma\lambda_1\lambda_2}$, respectively, in the relevant effective single pion transition operator Eq.(22).

The matrix elements for the $P$ wave to $P$ wave single pion transitions are given by

$$
\left(\bar{\phi}_{\nu_1...\nu_j;\lambda_2}\right)^{AB} \left(\mathcal{O}_{\lambda_1\lambda_2}^{(l_\pi)}\right)_{AB}^{A'B'} \left(\phi_{\mu_1...\mu_j;\lambda_1}\right)^{A'B'}.
$$

Finally, using the light diquark spin wave functions in Table 1 and Eq.(23) the heavy quark symmetry coupling factors can be related to those of the constituent quark model. In the chiral formalism, it was shown that the
Table 5: Heavy Quark Symmetry (HQS) matrix elements for some of $P$ wave to $P$ wave single pion transitions.

| $B'_Q \to B_Q + \pi$ | $M^\pi$ |
|------------------------|---------|
| $\Lambda_{K1} \to \Lambda_{K1}$ | $\frac{1}{\sqrt{3}} \bar{u}_2(v) \gamma_5 \gamma_\perp \bar{u}_2'(v)$ | $\frac{1}{\sqrt{3}} \gamma_\perp^\mu \gamma_5 u_1(v)$ | $I_4 g_{1p}^{(K)} i\varepsilon(\mu\nu p\nu)$ |
| $\Sigma_{K1} \to \Sigma_{K1}$ | $\frac{1}{\sqrt{3}} \bar{u}_2(v) \gamma_5 \gamma_\perp' \bar{u}_2'(v)$ | $\frac{1}{\sqrt{3}} \gamma_\perp' \gamma_5 u_1(v)$ | $I_2 g_{1p}^{(K)} i\varepsilon(\mu\nu p\nu)$ |
| $\Sigma_{K1} \to \Lambda_{K1}$ | $\frac{1}{\sqrt{3}} \bar{u}_2(v) \gamma_5 \gamma_\perp' \bar{u}_2'(v)$ | $\frac{1}{\sqrt{3}} \gamma_\perp' \gamma_5 u_1(v)$ | $I_1 h_{1p} i\varepsilon(\mu\nu p\nu)$ |

$L = 1$ and 2 contributions are nominally down by two powers of $|\vec{p}|/m$. Therefore, we shall only list the $L = 0$ case in the constituent quark model coupling factors. If needed the $(L = 1$ and 2) relations can be read off from the corresponding entries in Table 6 for the non diagonal transitions. We simplify the notations by dropping out the $(L = 0)$ superfix and denoting the constituent couplings as $g_p$ and $g_f$, one has the following relations

**Diagonal Transitions:**

\[
g_{1p}^{(K)} = 0 \quad , \quad g_{2p}^{(K)} = \frac{1}{\sqrt{3}} g_p \quad , \quad g_{3p}^{(K)} = -\frac{1}{\sqrt{2}} g_p \quad , \quad g_{4p}^{(K)} = g_p
\]

\[
g_{5p}^{(K)} = \sqrt{\frac{2}{3}} g_p \quad , \quad g_{6p}^{(K)} = -\frac{1}{2} g_p \quad , \quad g_{7p}^{(K)} = \frac{1}{\sqrt{2}} g_p \quad , \quad g_{8p}^{(K)} = g_p \quad (A.1)
\]

and

\[
g_{1f}^{(K)} = g_f \quad , \quad g_{2f}^{(K)} = -\frac{1}{\sqrt{2}} g_f \quad , \quad g_{3f}^{(K)} = g_f \quad (A.2)
\]

Similar relations hold for the $(k \to k)$ transitions with the replacement

\[
g_{il}^{(k)} \to g_{il}^{(k)} \quad , \quad g_p \to g_p' \quad \text{and} \quad g_f \to g_f' \quad (A.3)
\]
This means that diagonal transitions among $P$ wave heavy baryon states are described by only four constituent couplings $g_p, g'_p, g_f$ and $g'_f$. We mention that constituent quark model p-wave transitions have also been worked out in [6], using static quark model wave functions and explicit Clebsch-Gordan Coefficients, which are in agreement with our predictions. However, the results on the $f$-wave transitions are new. For the diagonal p-wave coupling factors, one has the additional PCAC relations

$$g_p = g'_p = \frac{g_A}{f_\pi}.$$  \hspace{1cm} (A.4)

**Non Diagonal Transitions:**

Finally, we work out the non diagonal case where we retain all the three ($L = 0, 1$ and 2) p-wave transitions, which are now all nominally of the order $(|\vec{p}|/m)^2$ since the leading contribution is zero due to the orthogonality of the orbital wave functions in the non diagonal case [6]. The constituent quark model relations are summarized in Table 6, which shows that K-multiplet to k-multiplet and k-multiplet to K-multiplet transitions, among $P$ wave states, are determined by four independent couplings $h_p^{(0)}, h_p^{(1)}, h_p^{(2)}$ and $h_f$. We are in agreement with the results of [6] on the $L = 0$ and $2$ p-wave transitions. Our results on the $f$-wave transitions are new. The contributions from $L = 1$ tensor operator $P^{(2)}_{\sigma\lambda_1\lambda_2}$ were neglected in [6] assuming that the light quark momenta are not changed in the pion emission process.
Table 6: Constituent quark model predictions for non diagonal $P$ wave to $P$ wave one pion transitions.

|     | $h_{p}^{(0)}$ | $h_{p}^{(1)}$ | $h_{p}^{(2)}$ | $h_{f}$ |
|-----|---------------|---------------|---------------|--------|
| $h_{1p}$ | 0             | 0             | 0             | -      |
| $h_{2p}$ | $\frac{1}{\sqrt{3}}$ | $\frac{1}{\sqrt{3}}$ | $\frac{5}{3} \cdot \frac{1}{\sqrt{3}}$ | -      |
| $h_{3p}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{2 \cdot \sqrt{2}}$ | $\frac{5}{6} \cdot \frac{1}{\sqrt{2}}$ | -      |
| $h_{4p,4f}$ | 1             | $-\frac{1}{2}$ | $\frac{1}{6}$ | 1      |
| $h_{5p}$ | $\frac{1}{\sqrt{3}}$ | $-\frac{1}{\sqrt{3}}$ | $\frac{5}{3} \cdot \frac{1}{\sqrt{3}}$ | -      |
| $h_{6p}$ | $\sqrt{\frac{2}{3}}$ | $-\frac{1}{\sqrt{6}}$ | $-\frac{5}{3} \cdot \frac{1}{\sqrt{6}}$ | -      |
| $h_{7p}$ | $-\frac{1}{\sqrt{2}}$ | $\frac{1}{2 \cdot \sqrt{2}}$ | $\frac{5}{6} \cdot \frac{1}{\sqrt{2}}$ | -      |
| $h_{8p}$ | $\sqrt{\frac{2}{3}}$ | $\frac{1}{\sqrt{6}}$ | $-\frac{5}{3} \cdot \frac{1}{\sqrt{6}}$ | -      |
| $h_{9p}$ | $-\frac{1}{2}$ | 0             | $-\frac{5}{6}$ | -      |
| $h_{10p,10f}$ | $\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | $\frac{\sqrt{2}}{3}$ | $-\frac{1}{\sqrt{2}}$ |
| $h_{11p,11f}$ | 1             | $\frac{1}{2}$ | $\frac{1}{6}$ | 1      |
| $h_{12p,12f}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{1}{\sqrt{2}}$ | $-\frac{\sqrt{2}}{3}$ | $-\frac{1}{\sqrt{2}}$ |
| $h_{13p,13f}$ | 1             | 0             | $-\frac{1}{3}$ | 1      |
Appendix B
Recoupling coefficient approach to diquark transitions in the constituent quark model

In this appendix, we shall make use of the quantum theory of angular momentum to describe one-pion transitions between heavy baryon states in the constituent quark model. We show that all one-pion transition matrix elements can be written in a very compact form in terms of a product of a 6j- and 9j-symbols and corresponding reduced matrix elements. This derivation and the results are completely equivalent to the covariant coupling approach used in the main text. We have added the material in this Appendix for those of our readers who are more familiar with the recoupling approach to angular momentum transitions in composite systems than the covariant approach used in the previous sections. We should mention that we will omit flavour factors in our discussion of the one-pion transitions in this appendix. These factors can be obtained from Eq. (12) as described at the end of the second paragraph in Appendix A.

As we have discussed before, in the heavy quark limit the pion is coupled to the light diquark system and the heavy quark does not participate in the transition process. The number of angular momenta involved in the single pion transition between the two light diquark systems total 12 altogether. These angular momenta belong to three different angular momentum spaces, the spin, the orbital angular momentum and the total angular momentum spaces. In the spin space, we have the spectator quark spin $S_{q_s} = \frac{1}{2}$, the initial and final active quark spins $S_{q_1} = \frac{1}{2}$ and $S_{q_2} = \frac{1}{2}$, respectively, and the quark level one-pion transition operator $\mathcal{O}^\sigma$ with spin 1 assuming that the one-pion transition is due to one-body interaction. Also, we have the initial and final diquark spins $S_1 = 0, 1$ and $S_2 = 0, 1$, respectively. In the orbital angular momentum space, one has the diquark initial and final orbital
angular momenta $L_1$ and $L_2$ as well as the orbital transition operator $\mathcal{O}^L$ with $|L_1 - L_2| \leq L \leq |L_1 + L_2|$. And, finally, there are the initial and final diquark total angular momenta $j_1$ and $j_2$, respectively, and the diquark pion emission operator $\mathcal{O}^l$ with angular momentum $l$ which is even or odd depending on the parity of the diquark state. These last three angular momenta operate in the total angular momentum space.

Since there are 12 angular momenta involved in the single pion transition, one would presume, at first sight, that these transitions are described in terms of a 12-j symbol. One notices that the spin and orbital spaces, however, factories when the spin-orbit coupling is neglected. Hence, as we shall demonstrate later on, the one-pion transitions can be written in terms of a product of a 6j- and 9j-symbols and the corresponding reduced matrix elements.

First, let us write down the relevant two coupling schemes in spin space and in orbital angular momentum space as well as the relevant recoupling coefficients between the two respective coupling schemes. In the spin space, we have

**Coupling Scheme I:**
\[
\vec{S}_{qs} + \vec{S}_{q_2} = \vec{S}_2, \quad \vec{S}_2 + \vec{\sigma} = \vec{S}_1
\]

**Coupling Scheme II:**
\[
\vec{S}_{q_2} + \vec{\sigma} = \vec{S}_{q_1}, \quad \vec{S}_{qs} + \vec{S}_{q_1} = \vec{S}_1,
\]
with the recoupling coefficient (6j-symbol) \[ \left\{ \begin{array}{ccc} S_{qs} & S_{q_2} & S_2 \\ \sigma & S_1 & S_{q_1} \end{array} \right\}. \]

In orbital angular momentum space one has

**Coupling Scheme I:**
\[
\vec{L} + \vec{\sigma} = \vec{l}, \quad \vec{L}_2 + \vec{S}_2 = \vec{j}_2, \quad \vec{l} + \vec{j}_2 = \vec{j}_1
\]

**Coupling Scheme II:**
\[
\vec{L} + \vec{L}_2 = \vec{L}_1, \quad \vec{\sigma} + \vec{S}_2 = \vec{S}_1, \quad \vec{L}_1 + \vec{S}_1 = \vec{j}_1,
\]
and the recoupling coefficient (9j-symbol) is given by

\[
\begin{pmatrix}
L & \sigma & l \\
L_2 & S_2 & j_2 \\
L_1 & S_1 & j_1
\end{pmatrix}
\]

A pictorial representation of the 6j- and 9j- symbols both in orbital and spin spaces are shown in Figure 1. In these diagrams, the links represent the orbital (spin) angular momenta while the nodes represent their couplings.

Next, one needs to define reduced matrix elements for the transition amplitudes in spin, orbital angular momentum and total angular momentum space. We use the Wigner-Eckart theorem and the conventions of Ref. [24] to write

\[
\langle S_{q_2} m_{q_2} \mid \mathcal{O}^\sigma(m_\sigma) \mid S_{q_1} m_{q_1} \rangle = C_{S_{q_2} m_{q_2} S_{q_1} m_{q_1} \sigma m_\sigma} \frac{\langle S_{q_2} \parallel \mathcal{O}^\sigma \parallel S_{q_1} \rangle}{\sqrt{2S_{q_2} + 1}} \quad (B.5)
\]

\[
\langle L_{2} m_{L_2} \mid \mathcal{O}^L(m_L) \mid L_{1} m_{L_1} \rangle = C_{L_{2} m_{L_2} L_{1} m_{L_1} L_{m_L}} \frac{\langle L_{2} \parallel \mathcal{O}^L \parallel L_{1} \rangle}{\sqrt{2L_2 + 1}} \quad (B.6)
\]

\[
\langle j_{2} m_{j_2} \mid \mathcal{O}^j(m_l) \mid j_{1} m_{j_1} \rangle = C_{j_{2} m_{j_2} j_{1} m_{j_1} \sigma m_\sigma} \frac{\langle j_{2} \parallel \mathcal{O}^j \parallel j_{1} \rangle}{\sqrt{2j_2 + 1}} \quad , \quad (B.7)
\]

here, \( C_{jm_{j} km_{k}}^{im_{i} m_{i}} \) are Clebsch-Gordan Coefficients (C. G.). The total diquark pion emission operator \( \mathcal{O}^j(m_l) \) can be written as a product of the quark level one-pion transition operator \( \mathcal{O}^\sigma(m_\sigma) \) and the orbital transition operator \( \mathcal{O}^L(m_L) \) according to

\[
\mathcal{O}^j(m_l) = \sum_{m_\sigma, m_L} C_{\sigma m_\sigma L m_L}^{im_l} \mathcal{O}^\sigma(m_\sigma) \mathcal{O}^L(m_L) \quad . \quad (B.8)
\]

One can then relate the reduced matrix element of the diquark transition to the product of the reduced matrix elements of the quark level transition and the orbital transition. This can be done using the relevant identities for 6j- and 9j- symbols [24]. After a little bit of algebra, one obtains

\[
\langle j_{2} \parallel \mathcal{O}^j \parallel j_{1} \rangle = (-1)^{S_{q_2} + S_{q_1} + l + S_1 + j_1 - j_2} \frac{\sqrt{(2l + 1)(2j_1 + 1)(2j_2 + 1)(2S_1 + 1)(2S_2 + 1)}}
\]
Up to a proportionality and phase space factors, the reduced diquark matrix elements of the diquark transition $\langle j_2 || O^i || j_1 \rangle$ corresponds to the HQS coupling factors $f_{ls}$ defined in Eq. (3) and in Table 3. On the other hand, the product of the reduced matrix elements $\langle S_{q_2} || O^\sigma || S_{q_1} \rangle$ and $\langle L_2 || O^l || L_1 \rangle$ corresponds to the coupling factors $f_p, f_{a(K,k)}, f_{d(K,k)}$ etc. defined in Sec. 3.

The relations between the reduced matrix elements Eq. (B.9) correspond to the relations given in Eqs. (24-27) for $S$ wave to $S$ wave and $P$ wave to $S$ wave transitions and the relations given in Appendix (A) for $P$ wave to $P$ wave single pion decays.

For the sake of completeness we shall also give the relation between the reduced matrix elements of the heavy baryon transitions and the light diquark transitions \[9, 10\]. Using the conventions of [24], the reduced matrix elements for the one-pion transition between heavy baryons is defined by

$$\langle J_2 M_2 | O^l (m_l) | J_1 M_1 \rangle = C_{J_2 M_2 J_1 M_1 m_l} \langle J_2 || O^l || J_1 \rangle \sqrt{2 J_2 + 1}. \quad (B.10)$$

In the heavy quark limit one has

$$\langle J_2 || O^l || J_1 \rangle = (-1)^{J_1 + l + j_2 - j_1} \sqrt{(2 J_1 + 1)(2 J_2 + 1)}$$

$$\left\{ \begin{array}{c} \frac{l}{2} \ j_1 \ j_2 \ j_1 \ j_1 \end{array} \right\} \langle j_2 || O^l || j_1 \rangle. \quad (B.11)$$

The pictorial representation of the 6j-symbol appearing in this equation is shown in Figure 2. To close this appendix, we also give values [24] for 6j-symbols required to calculate strong decay rates of Eq. (10) for all transitions among heavy baryon states.
Table 7: Table of values for 6j-symbols necessary to calculate the decay rates in Eq.(10) where \(s = l + j_1 + j_2\).

| \(J_1\) | \(J_2 = j_2 + \frac{1}{2}\) | \(J_2 = j_2 - \frac{1}{2}\) |
|---------|-----------------|-----------------|
| \(J_1 = j_1 + \frac{1}{2}\) | \((-1)^{(s+1)}\frac{1}{2} \left[ \frac{(s+2)(s-2l+1)}{(2j_1+1)(j_1+1)(2j_2+1)(j_2+1)} \right]^{1/2}\) | \((-1)^{(s+1)}\frac{1}{2} \left[ \frac{(s-2j_2+1)(s-2j_1)}{(2j_1+1)(j_1+1)j_2(j_2+1)} \right]^{1/2}\) |
| \(J_1 = j_1 - \frac{1}{2}\) | \((-1)^{(s+1)}\frac{1}{2} \left[ \frac{(s-2j_2)(s-2j_1+1)}{j_1(2j_1+1)(2j_2+1)(j_2+1)} \right]^{1/2}\) | \((-1)^{(s+1)}\frac{1}{2} \left[ \frac{(s+1)(s-2l)}{j_1(2j_1+1)j_2(2j_2+1)} \right]^{1/2}\) |

Acknowledgments

We would like to acknowledge informative discussions with D. Pirjol. We would, also, like to thank A. Ilakovac, U. Kilian and J. Landgraf for their participation in the early stages of this work. S. T. would like to thank Patrick O. J. O’Donnell and the Department of Physics, University of Toronto, Toronto, Canada for the hospitality during the final stages of this work and also for reading the manuscript. J.G.K. acknowledges partial support from the BMBF (Germany) under contract 06MZ865.
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Figure Captions

Fig. 1: Constituent quark model recoupling diagrams representing a 6j- and 9j-symbols a) 6j-symbol acting in spin space b) 9j-symbol acting in orbital angular momentum space. Links represent angular momenta and nodes represent their couplings. The Angular momenta are defined in the text.

Fig. 1: The HQS limit recoupling diagram representing a 6j-symbol acting in total angular momentum space. Links represent angular momenta and nodes represent their couplings. The Angular momenta are defined in the text.
\[ \frac{\sigma}{2} = 1 \]

Figure 1
Figure 2