PROGRESS IN PERTURBATIVE QCD

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Abstract

We briefly summarize some recent theoretical developments in perturbative QCD, emphasizing new ideas which have led to widening the domain of applicability of perturbation theory. In particular, it is now possible to calculate efficiently processes with many partons; the high order behavior of perturbation theory can be at least partly understood by going beyond leading twist accuracy; factorization with more than one hard scale (such as in DIS with heavy quarks) can be made consistent with the renormalization group; and large infrared logs can be resummed beyond the renormalization group. The use of the renormalization group to resum large longitudinal scales may allow the use of perturbation theory even in the absence of a large transverse scale.

Summary of the theory working group

at \textbf{DIS 97}, Chicago, April 1997

To be published in the proceedings

June 1997

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Perturbative QCD beyond leading order

It is common knowledge that within the last decade the status of perturbative QCD has changed from being a part of the standard model which could be only qualitatively tested to that of a firmly established theory which has been tested to high accuracy. The theory can now be used to perform the detailed calculations required, for instance, in order to determine the background to new physics — as it is the case for the recent HERA high $Q^2$ events, where QCD uncertainties can be traced and quantitatively assessed in detail. From the theoretical side, this is due to the fact that a set of standard perturbative results and techniques is now available in order to describe within a unified framework numerous phenomena, ranging from the completely inclusive (such as scaling violations of structure functions) all the way to observables related to completely exclusive parton production processes (such as the momentum distribution of large $p_T$ jets).

Broadly speaking, perturbative QCD computations are made possible by factorizing the cross section $\sigma(P_1, \ldots, P_n)$ for a process with $n$ (one or two, in physically relevant cases) hadrons in the initial state in terms of parton distributions $f_{i_k}$ for partons $i$ in the $k$-th hadron, and a hard partonic cross section $\hat{\sigma}_{i_1, \ldots, i_n}$:

$$\sigma(P_1, \ldots, P_n) = \sum_{i_1, \ldots, i_n} \int dx_1 \ldots dx_n f_{i_1}(x_1, \mu^2) \ldots f_{i_n}(x_n, \mu^2) \hat{\sigma}_{i_1, \ldots, i_n}(x_1 P_1, \ldots, x_n P_n; \alpha(\mu^2)).$$ (1)

Standard techniques involve the determination of the hard cross section from perturbative calculation of Feynman diagrams in powers of $\alpha(\mu^2)$, factorization of collinear singularities in the parton distributions, and renormalization-group improvement of the dependence of the latter on the factorization scale $\mu^2$ by solving the QCD evolution equations. Setting the otherwise arbitrary scale $\mu^2$ equal to a physical scale of the process then ensures that the perturbative expansion of $\hat{\sigma}$ is free of large logarithms. However, the dependence on the arbitrary scale $\mu$ can only be reduced by performing these computations to high enough order. The state of the art in most precision tests of QCD is based on next-to-leading order (NLO) computations.

A class of processes where this program has been successfully completed recently to NLO for a large number of observables is heavy quark production, where the mass of the quark provides the hard scale which justifies the applicability of perturbation theory. A comparison with experiment is possible both for photoproduction (at HERA) and hadroproduction (at the Tevatron), and quantities computed at NLO include total cross sections, single-inclusive $p_T$ distributions, and $Q\bar{Q}$ correlations. A general feature of these results is a considerable improvement of the NLO description of the data when compared to the leading order one, even though there still remain several instances in which the agreement is marred by large uncertainties. Typical examples are the normalization of the total $c\bar{c}$ photoproduction cross section or of the single-inclusive $p_T$ distribution for $b$ production at the Tevatron (which only agrees with the theory for extreme choices of the scale $\mu$).

Progress in these computations has been achieved not only because of the determination of the hard matrix elements for a large number of processes, but also thanks to the development of efficient techniques in order to go from the amplitudes to the physical cross sections which correspond to physical observables. The main technical obstacle here is that the infrared singularities due to the emission of massless particles which plague the individual amplitudes
only cancel when combined into a physical cross section. Whereas in practice the cross section must be computed numerically in order to compare to experiment, this cancellation has to be carried out analytically before the numerical calculation is feasible, by either excluding the singular regions from the numerical integrations (slicing method) or subtracting the analytically computed singularity before numerical integration in order to make the latter finite (subtraction method). The main disadvantage of the latter method, namely that it becomes cumbersome in processes with many partons because of the large number of individual cancellations, has been recently overcome thanks to a systematic organization of the cancellations at the level of the cross section [4].

We are thus reaching a stage where further progress requires new theoretical ideas. A representative selection of the problems we face, and the sort of solutions we are after, is the following: (a) we need more effective techniques to avoid the proliferation of Feynman diagrams and terms in each diagram when calculating amplitudes with many partons and loops; (b) we have to cope with the divergent nature of the perturbative expansion in $\alpha$ and consequently go beyond purely logarithmic accuracy; (c) we must deal with processes with more than one hard scale; (d) we must resum logs, such as infrared logs, even when the renormalization group doesn’t help; (e) we would like to extend the applicability of leading log resummation to cases, such as logs of $1/x$, where the light-cone expansion is not available.

In the sequel we will give a brief overview of some of the techniques recently proposed to deal with all these issues.

Perturbation theory beyond Feynman diagrams

The determination of matrix elements with many partons is an important ingredient in the computation of physically relevant QCD processes: for example, the cross section for the production of six jets or five jets and a vector boson in $pp$ collisions is a significant background to top production. However the number of Feynman diagrams, and, more importantly, the number of terms corresponding to each diagram grow very rapidly with the number of partons involved.

For example, because the three-gluon vertex in QCD is the sum of six terms, a typical four gluon one-loop diagram (such as that where all gluons attach to a gluon loop) will have about $6^4$ terms. Each of these corresponds to a loop integral which, once evaluated, results by itself in a lengthy expression, so the full starting amplitude consists of about $10^4$ terms. Yet when the full computation is carried through, the resulting amplitude [5] fits in less than a page and is remarkably simple in structure. This large amount of cancellation, together with the observation that each diagram does not respect the symmetries of the final result, suggests that Feynman diagrams are not an effective way of organizing the computation.

This also suggests that string theory might instead provide a better way of organizing such calculations [6]. Field theory can be obtained from string theory by taking the limit of infinite string tension, in which case all excited modes of the string decouple so that only a field theory survives; by suitably choosing the string theory one can make sure that the desired field theory is obtained in the limit. The main reason why this actually simplifies the calculation is that in string theory at each order of perturbation theory there is only one diagram which encompasses
the full set of Feynman diagrams of the underlying field theory: many cancellations thus take place at an early stage of the computation. On top of this, string amplitudes have several additional useful properties, such as, for instance, the fact that it is possible to switch from bosons to fermions in the loop by changing world-sheet boundary conditions, which allows using information from one calculation to perform the other.

A significant obstacle which hampered progress in this direction was the need to use a fully consistent string theory with the correct low-energy limit, such as the heterotic string. However, once computational rules have been derived, the string theory can be dispensed with. In fact, it can be checked explicitly that the same rules can be obtained from simpler albeit inconsistent theories such as the bosonic string. One is then left with a set of rules which can be used directly to compute generic one-loop helicity amplitudes. These rules optimize and extend to the one-loop case a set of ideas which had been developed (without resorting to string techniques) to cope with the complexity of tree level amplitudes with many partons, such as spinor helicity methods to treat gluon polarizations, color decompositions, supersymmetric identities and recursion relations between amplitudes with different numbers of partons [7].

The rules can be further improved by use of unitarity methods. The basic idea here is that the absorptive part of the one-loop amplitudes can be determined by dispersion relations from the dispersive part, which in turn is related to a product of tree amplitudes. The loop amplitudes are thus computed by sewing tree amplitudes, rather than tree diagrams, so that one can take advantage of the full set of cancellations which have already occurred at the level of the computation of the tree diagrams themselves.

While tree-level computations are now a closed chapter[7], since it is possible to determine numerically and even (for certain helicities) analytically the amplitudes for any number of particles, progress is made in one-loop computations. Specifically, while the one-loop four-parton cross sections were calculated using standard techniques [5], the corresponding helicity amplitudes have been first determined exploiting string and unitarity methods, which have subsequently allowed the determination of five-gluon amplitudes as well. The computation of amplitudes with four partons and a vector boson is in progress [6]. Also, with these techniques it has been possible to determine the one-loop $2 \rightarrow N$ gluon amplitude at the level of next-to-leading logs of the center-of-mass energy of the gluon pair [8].

Going beyond one loop is an interesting challenge, especially when one realizes that no results beyond NLO are available for any process depending on more than one kinematic variable: for instance, anomalous dimensions for some specific operators are known beyond NLO, but no splitting function is known beyond NLO. However, going beyond one loop requires going back to the string theory as a guidance to formulate suitable computational rules [9, 10]. At present some results in $N=4$ supersymmetric QCD are available [9], while the two-point Green function, from which the QCD $\beta$ function may be extracted, should be available shortly [10]. Perhaps more interestingly, going to two loops suggests conjectures on the structure of amplitudes which may lead to a determination of some quantities (such as the $\beta$ function itself) to all perturbative orders [9, 10].
High perturbative orders beyond log accuracy

It is a well-known fact that the perturbative expansion of gauge theories diverges. As a consequence, perturbative computations cannot be pushed to indefinitely higher orders. Nevertheless, one may attempt a resummation of the divergent expansion. Even though a full understanding of the diagrammatic structure of QCD is not available, individual towers of diagrams which lead to divergent behavior of perturbation theory may be isolated. Specifically, the so-called renormalon diagrams, which correspond to a vertex correction dressed by a chain of bubble quark self-energy insertions, lead to factorial behavior of the corresponding perturbative coefficients \[11\].

A factorially divergent series can be turned into a geometric series by Borel transformation: if \( C(t) \) admits an expansion of the form \( C = \sum_{n=1}^{\infty} n!t^{-n} \) then its Borel transform \( B(u) \), defined by the relation \( C(t) = \int_{0}^{\infty} e^{-tu} B(u) du \), satisfies \( B = \sum_{n=0}^{\infty} u^n = \frac{1}{1-u} \). If we wish to use this as a means to resum the original series we must invert the transform. We will then encounter a singularity on the path of integration at \( u = u_0 = 1 \) (infrared renormalon pole). If the series had alternating signs the singularity (ultraviolet renormalon) would not be on the path of integration, but the integral would still run outside the radius of convergence of the series; we will not discuss this case further. The prescription chosen to treat the singularity introduces then an ambiguity of order \( e^{-u_0 t} = e^{-t} \). In typical QCD computations the expansion is in powers of \( \frac{1}{t} = \frac{1}{\ln(Q^2/\Lambda^2)} = \beta_0 \frac{2\pi}{\alpha_s} \) (to leading order) so the ambiguity is of order \( \frac{\Lambda^2}{Q^2} \). This means that even though the expansion is in logs of \( Q^2 \), the resummation ends up producing terms which behave like powers of \( Q^2 \), so that a treatment of the high-order behavior of the leading-twist perturbation theory requires going beyond leading twist.

The physics behind this phenomenon has been extensively elucidated \[11\]: the chain of renormalon diagrams is sensitive to the infrared region of the loop momentum integration, which must be matched to the power ultraviolet divergent behavior of higher twist operators which appear in the operator-product expansion. Hence, this is a manifestation of operator mixing upon shifts of the factorization scale beyond log accuracy. The relevance of the infrared momentum integration region is highlighted by realizing that the sum of the same class of diagrams can be obtained equivalently by simply computing a single vertex correction, without the chain of bubble insertions, but with an effective gluon propagator that corresponds to a massive gluon \[11\]; this can also be viewed as the result of introducing a dispersive representation for the strong coupling in terms of an effective coupling which is regular in the infrared \[12\]. The poles in the Borel transform discussed above are then in one-to-one correspondence with non-analytic contributions to the amplitude calculated with the effective propagator. This equivalence is not exact, however, when calculating renormalon corrections to less inclusive processes, in which the chain of bubbles can directly contribute to the final state.

The relation between power-like ambiguities of the perturbative corrections to leading twist operators and their mixing with higher-twist contributions to the OPE suggests that one may actually extract physical information from renormalons. Indeed, any ambiguity related to operator mixing, i.e. ultimately to the choice of factorization, must cancel in physical observables. Thus, the renormalon ambiguity must be matched by an equal and opposite ambiguity in the matrix elements of higher twist operators. We may then conjecture that the \( x \) dependence of the renormalon ambiguity is the same as that of the higher twist contribution which corre-
sponds to the set of these operators. Then, by computing the renormalon diagrams we can predict the shape of the higher twist contribution up to a normalization. This is equivalent to assuming that the matrix elements of the higher twist operators are dominated by their ultraviolet behavior (while the particular subclass of diagrams which corresponds to renormalons does indeed provide the dominant perturbative behavior).

This assumption doesn’t have a solid theoretical foundation: in fact, it can only be approximately true, because the logarithmic scaling violations of the higher twist operators are not the same as those of the renormalons[13]. However, it seems to be phenomenologically rather successful: the shape of the higher twist contributions to structure functions obtained from a fit to the observed scaling violations, for instance, display a good qualitative agreement with those computed with this method[12, 14]. This suggests that indeed the higher twist matrix elements are dominated by their perturbative tail, and thus do not provide any nonperturbative information on hadron structure[11].

On the other hand, it also suggests that we may use the same method to predict the shape of higher twist corrections even in cases where an OPE is not directly available, such as event shapes or jet observables[11, 12, 13]. Performing the computation one arrives at the interesting result that such processes may acquire $\frac{1}{Q^2}$ corrections; this has been recently shown to be the case (in a somewhat more subtle way) for fragmentation functions as well[13]. Going one step further, one may assume that such $\frac{1}{Q^2}$ corrections are universal[13, 12]. This assumption can again be only approximately true; it may be justified in the “dispersive” approach (which, as already mentioned, is no longer exactly equivalent to the Borel transform approach).

Recent progress in this field has involved the computation and classification of such corrections for a large variety of processes. The corresponding phenomenology (and the aforementioned assumptions of perturbative dominance and universality) seem to work much better than they ought to: this poses an interesting theoretical problem. More unconventional recent research directions have involved the study of nonlocal operators as a means to systematically identify the kinematic regions related to the power corrections[11], and the use of lattice methods to compute numerically perturbative corrections to the vacuum expectation value of the gluon condensate to high orders and thus test whether they display the high order behavior expected on the basis of renormalon calculations, thereby verifying directly the dominance of renormalon diagrams on the full perturbative behavior[17]. Interestingly, the observed large order behavior shows good agreement with the leading renormalon estimate, but only after the lattice running strong coupling is redefined in order to eliminate a $\frac{1}{Q^2}$ term which should not be present according to the OPE. The result suggests that “spurious” power corrections may arise if the strong coupling is not properly defined beyond logarithmic accuracy[18]: this may provide a window to the behavior of the running coupling beyond perturbation theory.

**Heavy quarks beyond fixed order**

Resummation to all orders in the coupling of leading and subleading logarithms is routinely performed when solving the QCD evolution equations for structure functions. This allows the resummation to all orders of the logs of the only large scale available when quark masses are neglected, namely $Q^2$. It is clear however, that heavy quark masses cannot be considered to be
negligible in many physically relevant applications: for instance, a large share of the HERA $F_2$ data comes from regions where $Q^2$ is close to the charm or the bottom threshold.

A standard way of including the heavy quark contributions is to neglect them below threshold, while considering the heavy quark as effectively massless above threshold (variable flavor number or VFN scheme). However, above but not very far from the threshold this approximation is not justified, and the threshold behavior will be poorly reproduced by it. An alternative possibility is to include the heavy quarks contributions (via photon-gluon fusion) to the hard coefficient functions, while not including the heavy quarks among the active partons (fixed flavor number or FFN scheme). This, however, fails when $Q^2 \gg m_Q^2$ because then logarithmic terms in $\ln \frac{Q^2}{m_Q^2}$ are not resummed.

The recent more precise structure function data, as well as the availability of direct determinations of the charm parton distribution, call for an improved treatment. This can be achieved by means of $m \neq 0$ factorization theorems [19] (alternative approaches have also been proposed [20]): the basic idea is that one may include in both the photon-gluon fusion contribution to the coefficient function as well as the direct contribution from the evolution of heavy quark partons above threshold, provided a subtraction term is included to avoid double counting. The result then reduces to the VFN scheme when $Q^2 \gg m_Q^2$, and to the FFN scheme when $Q^2 \approx m_Q^2$, so that all relevant large logs are being summed. The shortcoming of the method is its complexity: thus, in practice, the most efficient scheme may depend on the specific observable. The relevant issues, however, are settled from a conceptual point of view, and results can now be used in phenomenological applications.

### Resummation beyond the renormalization group

QCD processes are typically marred by large infrared logarithms. The standard renormalization group methods used to sum ultraviolet logarithms are not applicable, yet resummation is required in order to obtain accurate predictions. For a wide class of processes involving the emission of soft gluons this resummation has been carried through. Consider, for instance, the top production cross section, whose accurate determination has become recently of great phenomenological relevance. In general, emission of soft gluons has a small effect, because it affects the kinematics of the process only slightly. However, when approaching the threshold for production of the $t\bar{t}$ pair emission of a soft gluon may have a large effect, because even the small amount of energy spent in the gluon radiation can subtract a significant fraction of the available energy, thereby suppressing the emission. Because radiation effects are already included in the structure functions $f_i$ in eq. (1) (when solving the QCD evolution equations) the sign of the correction to the partonic cross section $\hat{\sigma}$ due to soft radiation is scheme dependent: in the $\overline{\text{MS}}$ and DIS schemes $\hat{\sigma}$ the suppression is negative, i.e. $\hat{\sigma}$ is actually increased.

The emission of soft gluons is thus logarithmically enhanced: each extra emission produces a contribution of order $\alpha_s \ln^2 (1 - \frac{Q^2}{s})$, where $Q^2$ is the invariant mass of the $t\bar{t}$ pair ($Q^2 = 4m_t^2$ if the pair is produced at rest). Close enough to threshold the log may offset the suppression due to $\alpha_s$, and the soft gluon emission must be resummed to all orders in order to achieve an accurate description of the process. This sort of resummation has been achieved since some time for inclusive processes, such as Drell-Yan or DIS, where it appears as the need to resum
to all orders logs of $1 - x$ in the evolution equation, and it has recently been accomplished to
leading log accuracy in the case of heavy quark production[21,22,23].

The leading log resummation is performed by exponentiation in moment space. Even though
existing computations differ in the treatment of subleading corrections, the results agree within
error. The correction is found to be rather small (the enhancement is smaller than 10% and
compatible with zero within the given uncertainties for all calculations) but grows rapidly
with energy due to the increase in importance of the threshold region. The next-to-leading
log corrections have also been computed in moment space for the $\bar{q}q \rightarrow \bar{Q}Q$ and $gg \rightarrow \bar{Q}Q$
subprocesses[24]. The corresponding moment inversions as well as the computation of the
$gg \rightarrow gg$ amplitude are in progress[24].

In this context, phenomenology is lagging behind theory: the relevant issues are well-
understood theoretically, and detailed analytic results are available, yet no systematic phe-
nomenology is available. Even at the fully inclusive level, despite the increase of interest in
accurate computations in the large $x$ region related to the HERA events, the phenomenological
relevance of resummation of ln$(1 - x)$ corrections to structure functions has not been system-
atically included in available structure function analyses, and, for instance, its impact on the
determination of $\alpha_s$ from DIS is unknown.

The renormalization group beyond large $Q^2$

A different class of large logarithmic corrections to perturbative computations which calls for
a resummation is that related to large center-of-mass energy logs. In DIS these correspond
to logarithmic corrections in $\frac{1}{x}$, which have of course attracted considerable interest since the
availability of DIS data at small $x$ from HERA. The physics behind large energy logs is similar
to the familiar physics of the large logs of $Q^2$ which are summed by the renormalization group:
as the center-of-mass energy or, respectively, the virtuality, increases so does the phase space
for parton radiation, and thus the latter is enhanced by logs of the relevant scale. Specifically,
in DIS the value of $x$ in a given event expresses the longitudinal momentum which is put on
shell by the virtual photon, expressed in units of the incoming proton’s momentum. If the
center-of-mass energy of the $\gamma^*p$ collision $W^2 = Q^2 \frac{1 - x}{x} \approx \frac{Q^2}{x}$ is very large, then a very small
fraction of it is put on shell by the virtual photon. This implies that there is a large phase space
for emission of a parton cascade in which each parton’s longitudinal momentum is very soft as
compared to its parent parton, so that the final (struck) parton carries only a tiny fraction of
the original proton’s longitudinal momentum. Each parton emission has a logarithmic behavior
at the infrared edge of the longitudinal momentum integration — similar to that discussed in
the previous section, but now the soft parton is that which takes part in the cascade, i.e. the
one which further fragments until being eventually struck by the virtual photon, rather than
that which is emitted in the final state. As a consequence, each new emission carries a factor
of $\alpha_s \ln \frac{1}{x}$.

The leading energy log corrections to gluon-gluon scattering have been determined long
ago[25,26], and recent progress has involved making this consistent with the QCD evolution
equations by means of suitable factorization theorems, as well as extending the computation to
the quark sector[27]. In this framework, leading logs of $\frac{1}{x}$ appear as contributions proportional
to $\frac{1}{x}(\ln 1/x)^{k-1}$ to the $k$-loop Altarelli-Parisi gluon splitting functions and the $k+1$-loop quark splitting functions and coefficient functions, whose coefficients are thus exactly determined to all orders in $\alpha_s$. Very recently, significant progress has been made towards the determination of the next to leading corrections in the gluon sector, in that the corrections to the anomalous dimensions have been computed [24]. Interestingly, part of the calculation has been cross-checked by using the string- and unitarity-based methods which we discussed previously [8]. In order for these results to be useful, however, the corresponding coefficient function must still be determined.

The theoretical and phenomenological status of summing large energy logs by including $\ln \frac{1}{x}$ contributions to the splitting functions to all orders in the coupling remains however unsatisfactory. On the one hand, the ensuing treatment of scaling violation is unnaturally asymmetric, since leading logs of $\frac{1}{x}$ are summed by including them by hand in the anomalous dimensions, but the evolution equations themselves resum logs of $Q^2$. On the other hand, the inclusion of these terms in the usual evolution equations is phenomenologically useless. While the agreement between the observed and computed scaling violations is spectacular at the NLO level, it gets actually worse when energy logs are included unless one fine-tunes the factorization scheme in such a way that the energy logs have no detectable effect [28].

It has been suggested recently that both of these problems may be overcome if the renormalization group is directly used to resum energy logs [29]. The basic idea here is to construct a factorization theorem (“energy factorization”) in which, in comparison to the standard mass factorization, the roles of $Q^2$ and $\frac{1}{x}$, i.e. of transverse and longitudinal momentum scales, are interchanged. It is then possible to obtain the resummation of all leading logs of $\frac{1}{x}$ as the result of renormalization group improvement of the energy-factorized cross section, by solving a leading-order evolution equation with a splitting function which depends on transverse momentum, and that can be easily determined at leading order by means of standard Weizsäcker-Williams methods. While the coefficients of the leading $\alpha_s \ln \frac{1}{x}$ contributions to the DIS cross section are then the same as in the standard approach [24, 27], energy factorization allows one to define suitably energy–factorized parton distributions, and to determine the leading order running of the coupling. It is then found that there exists only one energy-factorized parton, which at large energy is asymptotically free. This leads to a universal, perturbatively calculable high-energy behavior of the inelastic scattering cross section, which turns out to be consistent with unitarity bounds. The most striking consequence of this approach is that it seems to imply asymptotic freedom even when $Q^2$ is small provided the center of mass energy is large enough, and thus an unexpected extension of the kinematic domain in which perturbative methods can be applied.

**Beyond perturbative QCD**

A common thread linking the directions of research which we have sketched is that they lead to a widening of the domain of applicability of perturbative methods. The availability of a perturbative approach to problems (such as power corrections or small $x$ effects) which until recently could be exclusively tackled by means of effective models allows a cleaner separation of the perturbative physics from the genuinely nonperturbative soft input. On the other hand, the extension of perturbative results to high orders and their matching to the underlying nonper-
turbative behavior may provide some clues on the structure of the theory beyond perturbation theory. Thus the availability of powerful and deep perturbative results leads us naturally to look beyond perturbation theory itself[30].

Acknowledgements: I thank all the contributors to the working group for their lively participation and for providing the ideas on which this paper is based, E. Levin for a fruitful collaboration in organizing the session, and G. Ridolfi and R. D. Ball for a critical reading of the manuscript.

References

[1] S. Kuhlmann, these proceedings; S. Kuhlmann, H. L. Lai and W. K. Tung hep-ph/9704338.
[2] See e.g. J. C. Collins and D. E. Soper, Ann. Rev. Nucl. Part. Sci., 37, 383 (1987) and ref. therein; G. Sterman, these proceedings.
[3] See S. Frixione, M. L. Mangano, P. Nason and G. Ridolfi, hep-ph/9702287 and ref. therein; G. Ridolfi, these proceedings.
[4] S. Frixione, these proceedings; S. Frixione, Z. Kunszt and A. Signer, Nucl. Phys. B467 399 (1996); S. Catani and M. H. Seymour, Phys. Lett. B378 287 (1996).
[5] R. K. Ellis and J. C. Sexton, Nucl. Phys. B269 445 (1986).
[6] See D. A. Kosower, these proceedings; Z. Bern, L. Dixon and D. A. Kosower, Ann. Rev. Nucl. Part. Sci., 46, 109 (1996) and ref. therein.
[7] See M. Mangano and S. J. Parke, Phys. Rep. 200, 301 (1991) and ref. therein.
[8] V. Del Duca, these proceedings; hep-ph/9605404 and ref. therein.
[9] Z. Bern, these proceedings; Z. Bern, J. S. Rozowsky and B. Yan, hep-ph/9702424.
[10] L. Magnea, these proceedings; P. Di Vecchia, L. Magnea, A. Lerda, R. Marotta and R. Russo, Phys. Lett. B388 65 (1996).
[11] See M. Beneke, these proceedings; hep-ph/9609215 and ref. therein; V. Braun, hep-ph/9505317 and ref. therein.
[12] G. Marchesini, these proceedings; Y. L. Dokshitzer, G. Marchesini and B. R. Webber, Nucl. Phys. B469, 93 (1996).
[13] R. Akhoury, these proceedings; R. Akhoury and V. I. Zakharov, hep-ph/9610492 and ref. therein.
[14] M. Maul, E. Stein, A. Schäfer and L. Mankiewicz, hep-ph/9612300.
[15] M. Beneke, V. M. Braun and L. Magnea, hep-ph/9701309; M. Beneke, these proceedings.
[16] G. Sterman, *these proceedings*.

[17] F. Di Renzo, E. Onofri and G. Marchesini *Nucl.Phys.* B457 202 (1995); G. Marchesini, *these proceedings*.

[18] G. Grunberg, [hep-ph/9705290](http://arxiv.org/abs/hep-ph/9705290), R. Akhoury and V. I. Zakharov, [hep-ph/9705318](http://arxiv.org/abs/hep-ph/9705318).

[19] See W.-K. Tung, *these proceedings* and ref. therein; M. A. G. Aivazis, J. C. Collins, F. I. Olness and W.-K. Tung, *Phys.Rev.* D50 3102 (1994).

[20] R. G. Roberts, *these proceedings*; A. D. Martin, R. G. Roberts, G. Ryskin and W. J. Stirling, [hep-ph/9612449](http://arxiv.org/abs/hep-ph/9612449).

[21] E. Berger, *these proceedings*; [hep-ph/9606421](http://arxiv.org/abs/hep-ph/9606421) and ref. therein.

[22] S. Catani, M. L. Mangano, P. Nason and L. Trentadue, *Nucl. Phys.* B478 273 (1996).

[23] E. Laenen, J. Smith and W. L. van Neerven, *Nucl. Phys.* B369 543 (1992).

[24] N. Kidonakis and G. Sterman, *these proceedings*; [hep-ph/9607222](http://arxiv.org/abs/hep-ph/9607222) and ref. therein.

[25] V. Fadin, *these proceedings*; L. N. Lipatov, [hep-ph/9610276](http://arxiv.org/abs/hep-ph/9610276) and ref. therein.

[26] For a review see V. Del Duca, *these proceedings*; [hep-ph/9503226](http://arxiv.org/abs/hep-ph/9503226) and ref. therein.

[27] S. Catani, [hep-ph/9608310](http://arxiv.org/abs/hep-ph/9608310) and ref. therein; M. Ciafaloni, [hep-th/9510025](http://arxiv.org/abs/hep-th/9510025) and ref. therein.

[28] W. J. Stirling, [hep-ph/9708411](http://arxiv.org/abs/hep-ph/9708411); R. D. Ball and A. De Roeck, [hep-ph/9609309](http://arxiv.org/abs/hep-ph/9609309).

[29] R. D. Ball and S. Forte, *these proceedings*; [hep-ph/9703417](http://arxiv.org/abs/hep-ph/9703417).

[30] E. Levin, *these proceedings*, [hep-ph/9706341](http://arxiv.org/abs/hep-ph/9706341) and ref. therein.