Affleck–Dine baryogenesis via mass splitting

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We propose a version of the Affleck–Dine baryogenesis, which uses a decay of two superheavy scalar fields with close masses. These scalars acquire non-zero expectation values during inflation through linear couplings to a function of an inflaton. After the inflaton decay, the model possesses approximate $U(1)$-invariance, explicitly broken by a small mass splitting. This splitting leads to the baryogenesis in the early Universe. The resulting baryon charge density is fully determined by the inflaton dynamics and the Lagrangian parameters, i.e., is independent of initial pre-inflationary conditions for the scalars. As a consequence, baryon perturbations are purely adiabatic. We point out a possible origin of the mass splitting: masses of scalars degenerate at some large energy scale may acquire different loop corrections due to the interaction with the inflaton.

Introduction. The matter-antimatter asymmetry of the Universe is one of the great puzzles in physics. One way to tackle the problem was suggested by Affleck and Dine in Ref. [1]. The Affleck–Dine (AD) mechanism introduces a $U(1)$-charged scalar condensate $\Psi$ in the early Universe. Later on the charge is converted into the observed baryon asymmetry (BA) through a decay of the condensate into quarks, see, e.g., Refs. [2, 3] for the reviews. The natural environment, where the AD mechanism operates, is provided in the supersymmetric extensions of the Standard Model (SM) [4]. In this picture, the scalar condensate is formed along the flat directions of its potential, which are inherent in supersymmetry. However, non-observation of supersymmetry in the collider experiments motivates to look for different realizations of the AD mechanism. One such realization, which does not assume the existence of flat directions, is the subject of the present work.

We consider two scalar fields $\Psi_i$, where $i = 1, 2$, linearly coupled to some function $F(\phi)$ of the inflaton $\phi$, cf. Ref. [5],

$$\propto \Psi_i F(\phi) .$$

These couplings induce non-zero expectation values $\langle \Psi_i \rangle \propto F(\phi)$. We assume that the fields $\Psi_i$ are very heavy, i.e., their masses $M_i \gtrsim H$, where $H$ is the inflationary Hubble parameter. In this case the fields relax to the expectation values $\langle \Psi_i \rangle$ within a few Hubble times independently of their initial values. As the inflaton decays after inflation, $F(\phi) \to 0$, the fields $\Psi_i$ start oscillating around zero with the amplitudes set by their expectation values during inflation. If $M_1 = M_2$, the system possesses $U(1)$-invariance with respect to global rotations of the complex field $\Psi = \Psi_1 + i \Psi_2$. We associate this $U(1)$-invariance with the baryon symmetry. Successful baryogenesis requires baryon number violation. We achieve it by assuming a small mass splitting, $|M_1 - M_2|/|M_1 + M_2| \ll 1$, which slightly breaks $U(1)$-symmetry. The resulting baryon charge is converted into the SM sector through the decay of the condensate $\Psi$ into quarks.

Note that BA generated in this way is fully independent of the initial conditions for the fields $\Psi_i$. This is in contrast with the standard AD mechanism, where the parameter set includes the initial configuration of the fields $\Psi_i$. In conventional AD scenarios one typically overproduces baryon isocurvature perturbations [6, 7] in conflict with the cosmological data [8]. In our version of the model baryon isocurvature perturbations are automatically suppressed, as we deal with superheavy fields, cf. Ref. [9]. Finally, compared to some realizations of the AD mechanism, in our case BA is naturally small, i.e., we do not need to introduce very small coupling constants.

The model. Consider the following action:

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2} |\partial_{\mu} \Psi|^2 - \frac{1}{2} M^2 |\Psi|^2 \right] + S' + S'' .$$

Here $\Psi = \Psi_1 + i \Psi_2$ is a complex scalar with the mass $M$. In the absence of terms $S'$ and $S''$ the model possesses $U(1)$-symmetry, which we identify with the baryon invariance. The symmetry breaking necessary for the generation of BA is encoded in terms $S'$ and $S''$ defined as

$$S' = - \int d^4x \sqrt{-g} \left[ \alpha_1 \Psi_1 + \alpha_2 \Psi_2 \right] F(\phi) ,\quad (1)$$

and

$$S'' = - \int d^4x \sqrt{-g} V(\Psi, \Psi^*) ,$$

respectively. Here $F(\phi)$ is a function of the inflaton $\phi$. We assume that the inflaton $\phi$ is a canonical scalar slowly rolling the slope of its flat potential $U(\phi)$. Note that the mechanism of BA generation we propose works for fairly arbitrary functions $F(\phi)$. We only assume that $F(\phi) \to 0$
as } \phi \to 0\text{. The coupling constants } \alpha_i\text{, } i = 1, 2\text{, which we choose to be dimensionless, measure the strength of interactions between the fields } \Psi_i\text{ and the inflaton. These couplings explicitly break } U(1)\text{-invariance of the model. Nevertheless, the term } S' \text{ alone does not lead to production of the baryon charge, as we show below. Its role is to give non-zero expectation values for the fields } \Psi_i\text{, i.e., } \langle \Psi_i \rangle \neq 0\text{, which is crucial for the AD mechanism. Baryon charge production is naturally triggered by the symmetry breaking potential } V(\Psi, \Psi^*)\text{. Like in the standard AD scenarios it is assumed to be small compared to the other terms in the action.}

For } M \gtrsim H\text{, where } H \text{ is the Hubble parameter during inflation, the fields } \Psi_i\text{ quickly relax to their effective minima, which are offset from zero due to the interaction with the inflaton:}

\begin{equation}
\Psi_i = -\frac{\alpha_i F(\phi)}{M^2}. \tag{2}
\end{equation}

The requirement that the fields } \Psi_i \text{ are spectators (they do not affect dynamics during inflation) is fulfilled if}

\begin{equation}
\frac{\alpha^2 F^2(\phi)}{M^2} \ll U(\phi). \tag{3}
\end{equation}

One remark is in order here. Were the fields } \Psi_i \text{ light, i.e., } M \ll H\text{, the interaction with the inflaton would not be necessary to generate BA. Indeed, the condensate } \langle \Psi_i \rangle \neq 0\text{ is formed automatically because the fields } \Psi_i\text{ starting from generically non-zero values are in the slow roll regime during inflation. However, in this case considered in some details in Appendix A, the evolution of the fields } \Psi_i \text{ and hence the resulting BA strongly depend on their initial conditions set prior to inflation. On the contrary, in our scenario the solutions (2) are the attractors and thus initial conditions for the fields } \Psi_i \text{ are completely irrelevant.}

Despite the explicit breaking of } U(1)\text{-invariance by the interaction with the inflaton condensate, no baryon charge is produced during inflation. This immediately follows from Eq. (2) and the expression for the Noether charge density:}

\begin{equation}
Q = \Psi_1 \dot{\Psi}_2 - \Psi_2 \dot{\Psi}_1. \tag{4}
\end{equation}

In terms of the amplitude } \lambda \text{ and the phase } \varphi \text{ of the complex field, } \Psi = \lambda e^{i\phi}\text{, the Noether (baryon) charge density is given by } Q = \lambda^2 \dot{\varphi}. \text{ Hence, the fact that } Q = 0 \text{ means } \dot{\varphi} = 0\text{, so the relative phase of the fields } \Psi_i\text{ remains frozen with time, } \tan \varphi = \frac{a_{\varphi}}{a}\text{. Note that including the potential } V(\Psi, \Psi^*) \text{ does not alter this conclusion, but leads to inessential shifts of the fields } \Psi_i\text{.}

This behavior is crucially different from that in the conventional AD scenarios, where the phase is not fixed by the model parameters, but depends on the initial configuration of the complex field. In the standard case, the phase field can acquire large baryon isocurvature perturbations [6, 7] in contradiction with the data [8]. This is a rather common outcome of the AD mechanism, albeit not unavoidable. In our case, perturbations of the fields } \Psi_i\text{ are highly adiabatic. Indeed, for } M \gtrsim H\text{, any admixture of isocurvature fluctuations quickly relaxes to zero within a few Hubble times.}

After inflation, the field } \phi \text{ decreases, hence } F(\phi) \to 0\text{, and the expectation values of the fields } \Psi_i\text{ relax to zero; the fields } \Psi_i\text{ start oscillating. In this regime the baryon charge is produced due to the potential } V(\Psi, \Psi^*). \text{ In the standard AD scenarios quartic and higher order potentials } V(\Psi, \Psi^*) \text{ are normally used to produce BA. Here we put forward another mechanism of violating the baryon symmetry, which occurs through the quadratic potential:}

\begin{equation}
V(\Psi, \Psi^*) = \frac{\beta M^2}{4} (\Psi^2 + \Psi^* 2). \tag{5}
\end{equation}

Here } \beta \text{ is a dimensionless constant describing the splitting of the masses of the fields } \Psi_i\text{,}

\begin{equation}
M_i^2 = M^2(1 + \beta), \quad M_2^2 = M^2(1 - \beta). \tag{6}
\end{equation}

We assume } |\beta| \ll 1\text{, so that } \beta \approx (M_1 - M_2)/M.\text{ The mechanism of baryon charge generation is as follows. After inflation, the fields } \Psi_i\text{ evolve as free heavy scalar fields. They undergo rapid oscillations with the amplitudes decreasing with the scale factor } a:\n
\begin{equation}
\Psi_i \approx -\frac{\alpha_i A F_e}{M^2} \cdot \left(\frac{a_{\varphi}}{a}\right)^{3/2} \cdot \cos \left[ M_i \cdot (t - t_e) + \varphi_e \right]. \tag{7}
\end{equation}

Hereafter the subscript ’c’ stands for the end of inflation; } F_e \equiv F(\phi_e)\text{. The coefficient } A \text{ and the phase } \varphi_e \text{ account for the effects of preheating. They are not important for understanding the qualitative picture of baryogenesis. However, the coefficient } A \text{ affects the amount of BA generated. We concretize it below, when evaluating BA, see also Appendix B. Substituting the solutions (5) into Eq. (4), it is straightforward to calculate the Noether charge density:}

\begin{equation}
Q(t) \approx \alpha_1 \alpha_2 \cdot \frac{A^2 F_e^2}{M^3} \cdot \left(\frac{a_{\varphi}}{a}\right)^3 \cdot \sin \left[ \beta M \cdot (t - t_e) \right]. \tag{8}
\end{equation}

Here we omitted the term oscillating with the frequency of order } M \text{ and kept only the contribution characterized by the reduced frequency } \omega = \beta M. \text{ On the time scales } t \simeq \Gamma^{-1}\text{, where } \Gamma \text{ is the scalar field decay rate into quarks, the former undergoes multiple oscillations, and its contribution to BA is washed out. This is not necessarily the case for the term written in Eq. (8). If the mass splitting is small relative to the decay rate } \Gamma\text{, i.e.,}

\begin{equation}
\Gamma \gg \beta M, \tag{9}
\end{equation}

this term changes slowly on the time scales } t \text{ and thus sources BA. Provided that the hierarchy (7) holds, one can replace the sine in Eq. (8) by its argument:}

\begin{equation}
Q(t) \approx \alpha_1 \alpha_2 \cdot \beta \cdot \frac{A^2 F_e^2}{M^3} \cdot \left(\frac{a_{\varphi}}{a}\right)^3 \cdot t. \tag{10}
\end{equation}
Here we also replaced $t - t_\text{e}$ by $t$ assuming $t \gg t_\text{e}$. We see that the baryon charge density linearly grows with time until $t \sim \Gamma^{-1}$, when it gets converted into the standard matter–antimatter asymmetry. In what follows, we do not assume any particular value of $\Gamma$, but it must exceed the decay rate of the fields $\Psi_i$ triggered by the interaction $i$.

**Evaluating baryon asymmetry.** To evaluate the Noether charge density $Q(t)$, we should concretize the function $F(\phi)$. We choose it as follows:

$$F(\phi) = \frac{1}{M_{Pl}} \cdot T_\mu^\mu. \quad (8)$$

Another choice is a power law dependence of $F(\phi)$ considered later. Eq. (8) contains the Planck mass $M_{Pl}$ and the trace of the inflaton energy momentum tensor, $T_\mu^\mu = -(\partial_\mu \phi)^2 + 4U(\phi)$. Note that in the weak coupling regime, $\alpha_i \lesssim 1$, the function (8) satisfies the constraint $\S$ for any $M > H$. The Noether charge density produced with (8) is given by

$$Q(t) \simeq \frac{9\alpha_1 \alpha_2}{4\pi^2} \cdot \frac{A^2 \cdot H_i^4 \cdot M_{Pl}^2}{M^2} \cdot \left( \frac{a_e}{a(t)} \right)^3 \cdot t. \quad (9)$$

We see that the baryon charge is fully described by the inflaton dynamics (encoded in $H_\text{e}$ and $a_e$) and parameters of the Lagrangian of the fields $\Psi_i$. As it follows, BA $\Delta_B \simeq \frac{\alpha_i}{\alpha_i}$ grows linearly with time during the hot stage. This growth is cut off at the time $t \sim \Gamma^{-1}$ corresponding to the decay of the $\Psi$-condensate to quarks. Hence, observed BA can be estimated as $Q/s$ at $t \simeq \Gamma^{-1}$:

$$\Delta_B \simeq \frac{50\alpha_1 \alpha_2}{\pi^4 g_\ast} \cdot \frac{1}{\Gamma} \cdot \frac{\beta M \cdot A^2 \cdot H_i^4 \cdot M_{Pl}^2}{M^3 \cdot T_{\text{reh}}^3} \cdot \left( \frac{a_e}{a_{\text{reh}}} \right)^3. \quad (10)$$

We made use of the equilibrium expression for the entropy density:

$$s = \frac{2\pi^2 g_\ast}{45} \cdot T^3, \quad (10)$$

where $g_\ast$ is the number of ultra-relativistic degrees of freedom. Note that at the times of interest essentially all the SM species are ultra-relativistic, i.e., $g_\ast \gtrsim 100$.

To get an idea of the parameter space in the model, let us first assume the immediate conversion of the inflaton energy density into radiation. In the limit of an instant preheating, so that the inflaton and hence $F(\phi)$ drop to zero abruptly as inflation ends, we have $A \simeq 1$. In a more generic case discussed later, the coefficient $A$ can be substantially smaller. Setting $a_e = a_{\text{reh}}$ we have

$$T = T_{\text{reh}} = \left( \frac{45}{4\pi^4 g_\ast} \right)^{1/4} \cdot (H_\text{e} \cdot M_{Pl})^{1/2}. \quad (11)$$

Thus we obtain

$$\Delta_B \simeq \frac{100\alpha_1 \alpha_2}{\pi^{7/4} g_\ast^{1/4}} \cdot \frac{\beta M \cdot M_{Pl}^{1/2}}{M} \cdot \left( \frac{H_i}{M} \right)^{5/2} \cdot \left( \frac{M_{Pl}}{M} \right)^{1/2}. \quad (12)$$

For instance, taking $M \simeq M_{Pl}$, from Eq. (12) we find that only a tiny amount of BA can be produced in that case, unless $\alpha_i \gg 1$, which would imply a strongly coupled regime. On the contrary, for $M \simeq H_\text{e}$ (the lowest possible value), the required amount of BA can be generated in the weak coupling regime. For the high scale inflation with $H_\text{e} \simeq 10^{13}$ GeV, the set of parameters $\alpha_i \simeq 10^{-5}$ and $\beta M/\Gamma \simeq 10^{-3}$ would do the job.

The mechanism of BA generation may also occur through renormalizable interactions between the fields $\Psi_i$ and the inflaton. Consider the function $F(\phi)$ of the form

$$F(\phi) = \phi^3. \quad (13)$$

The baryon charge generated by the time $t \gtrsim \Gamma^{-1}$ is given by

$$Q = \frac{\alpha_1 \alpha_2 \cdot A^2 \cdot \frac{\beta M}{\Gamma} \cdot \phi_e^6}{M^3} \cdot \left( \frac{a_e}{a_{\text{reh}}} \right)^3. \quad (13)$$

Hence, BA is given by

$$\Delta_B \simeq \frac{45\alpha_1 \alpha_2}{2\pi^2 g_\ast} \cdot \frac{\beta M}{\Gamma} \cdot \frac{\phi_e^6}{M^3} \cdot T_{\text{reh}}^3 \cdot \left( \frac{a_e}{a_{\text{reh}}} \right)^3. \quad (13)$$

Consistency of our discussion requires that the condition $\S$ is obeyed. This sets the upper bound on the coupling constants $\alpha_i$, which cannot be larger than

$$\alpha_{\text{max}} \simeq \frac{M \sqrt{U(\phi_\ast)}}{\phi_\ast^2}. \quad (13)$$

Hence, for the super-Planckian fields $\phi$ characteristic for chaotic inflation, we are always in the weak coupling regime, i.e., $\alpha_{\text{max}} \ll 1$. Here the subscript ‘$\ast$’ denotes the moment of time deep in the inflationary epoch, when $M \sim H$, and the fields $\Psi_i$ start rolling towards their effective minima. If $M \gg H$ throughout inflation, then the moment ‘$\ast$’ coincides with the beginning of inflation. In the instant preheating approximation, resulting BA can be written as follows

$$\Delta_B \simeq \frac{1}{\pi^{3/4} g_\ast^{1/4} \cdot \alpha_1 \alpha_2 \cdot \frac{\beta M}{\Gamma} \cdot \left( \frac{\phi_e}{\phi_\ast} \right)^6 \cdot \frac{H_e^2 \cdot M_{Pl}^{1/2}}{H_\ast^{3/2} \cdot M}. \quad (13)$$

We see that the observed value of BA is easily achieved even for $\alpha_i \simeq \alpha_{\text{max}}$ and $\beta M/\Gamma \simeq 1$, if the ratio $\phi_e/\phi_\ast$ is sufficiently small.

Let us comment on the realistic situation, when the preheating epoch is long. The corresponding effects on the evolution of the fields $\Psi_i$ and resulting BA are encoded in the coefficient $A$. We assume that the function $F(\phi)$ drops as a power law at the times $t > t_\text{e}$:

$$F(\phi) = F_\text{e} \cdot \left( \frac{t}{t_\text{e}} \right)^s. \quad (13)$$

where the power $s$ is model-dependent. For example, for our choice (8) and quadratic inflation, $U(\phi) \propto \phi^2$, one
has $s = 2$. Note that in Eq. (13) we neglected oscillations of the inflaton. We have checked that keeping them gives a sub-dominant contribution to the final result. In Appendix B, we show that in the limit of large $Mt_c \simeq M/H_c$, the coefficient $A$ has the following asymptotic behavior:
\[
A \simeq \frac{H_c}{M}. \tag{14}
\]
Notably, this asymptotics is largely independent of the actual value of $s$ (which can be an integer or fractional number) and the rate of cosmological expansion during preheating. Hence, unless the preheating is instant, BA is additionally suppressed by the factor $H^2_c/M^2$.

**Discussions.** We conclude with two comments. First, in the present work we assume that the mass splitting measured by the constant $\beta$ is an independent model parameter. Let us point out the opportunity that the mass difference could be induced through the loop corrections involving a virtual inflaton. Namely, we have $M_1 = M_2 = M$ at the scale, where the effective interaction with the inflaton is induced, that is the Planck scale $M_{Pl}$ by our assumption. Disregarding quadratic divergences, we can estimate the loop corrections to the mass splitting as
\[
\frac{M_1^2 - M_2^2}{M^2} \sim \frac{\alpha_1^2 - \alpha_2^2}{4\pi^2} \log \frac{|\Psi|}{M_{Pl}}. \tag{15}
\]
Such corrections would follow, e.g., from the couplings $\Psi_i(\partial_\mu \phi)^2/M_{Pl}$. In a particular model of underlying theory at $M_{Pl}$ (or other high-energy scale) the relation between $U(1)$-breaking masses and couplings with the inflaton may be more complicated, and we do not elaborate more on the subject.

Second, as we have pointed out earlier, baryogenesis may occur through the quartic potential:
\[
V(\Psi, \Psi^*) = \frac{\xi}{4} (\Psi^4 + \Psi^{*4}), \tag{16}
\]
where $\xi$ is the dimensionless constant. Such symmetry breaking potentials are commonly utilized in the AD scenarios. Contrary to the case involving the mass splitting considered above, the baryogenesis triggered by the potential takes place right after inflation, when the fields $\Psi_i$ just start oscillating. Second, the production rate of the baryon charge is proportional to $\sin 4\varphi$. The latter vanishes, if the fields $\Psi_i$ are equally coupled to the inflaton, i.e., $\alpha_1 = \alpha_2$. Hence, non-zero BA is possible only for $\alpha_1 \neq \alpha_2$. One can show that, in the instant preheating approximation, the resulting BA is given by
\[
\Delta_B \simeq \frac{80\xi \alpha_1 \alpha_2 (\alpha_1^2 - \alpha_2^2)}{\pi^{15/4} g_*^{1/4}} \cdot \left(\frac{H_c}{M}\right)^{11/2} \cdot \left(\frac{M_{Pl}}{M}\right)^{5/2}.
\]
We have assumed the function $F(\phi)$ as in Eq. (3). As in the case of the AD baryogenesis through the mass splitting, BA is too small for the Planckian masses $M$, unless the coupling constants are large. On the flipside, for $M \ll M_{Pl}$, baryogenesis may occur in the weakly coupled regime. For $H_c \simeq 10^{13}$ GeV and $M \simeq 10^{15}$ GeV, the possible set of parameters is $\alpha_i \simeq 10^{-1}$ and $\xi \simeq 10^{-3}$.

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**APPENDIX A: SMALL MASSES $M \ll H$**

Let us consider the situation, when the fields $\Psi_i$ do not couple to the inflaton. In that situation, they still may have non-zero expectation values $\langle \Psi_i \rangle \neq 0$, if their masses are small relative to the Hubble rate during inflation, $M_i \ll H$. The fields $\Psi_i$ evolve in the slow roll regime starting from some initial values $\Psi_i(0)$. The slow roll terminates at some moment $t_{osc}$ in the hot epoch, when $M_i \simeq H$, and the fields $\Psi_i$ start oscillating about their zero values. Again assuming the small splitting $|\beta| \approx |M_1 - M_2|/M \ll 1$ and following the same steps as in the main part of the text, one gets for the Noether charge density:
\[
Q(t) \simeq \frac{\beta}{\Gamma} \frac{M^2 |\Psi_{i,0}| |\Psi_{2,0}|}{M_{Pl}} \left(\frac{a_{osc}}{a(t)}\right)^3.
\]
The linear growth is cut at the times $t - t_{osc} \approx \Gamma^{-1}$, when the Noether charge $Q(t)$ gets converted to the quark sector. Hence,
\[
\Delta_B \simeq \frac{\beta}{\Gamma} \frac{M^2 |\Psi_{i,0}| |\Psi_{2,0}|}{M_{Pl}} \left(\frac{a_{osc}}{a(t)}\right)^3.
\]

The entropy density is given by Eq. (10), and the temperature is given by Eq. (11), where one should set $H \simeq M$. We end up with the following estimate for BA:
\[
\Delta_B \simeq \frac{4\pi^{1/4} \beta}{g_*^{1/4}} \cdot \frac{\beta}{\Gamma} \cdot \frac{M |\Psi_{i,0}| |\Psi_{2,0}|}{(M_{Pl})^{3/2}}.
\]
We see that BA strongly depends on the initial values of the fields $\Psi_i$. For $\Psi_{i,0}$ of the Planckian order we get
\[
\Delta_B \simeq \frac{4\pi^{1/4} \beta}{g_*^{1/4}} \cdot \frac{\beta}{\Gamma} \cdot \left(\frac{M_{Pl}}{M}\right)^{1/2}.
\]
Hence, the required splitting is estimated as
\[
\beta \simeq 10^{-10} \cdot \frac{\Gamma}{M} \cdot \left(\frac{M}{M_{Pl}}\right)^{1/2}.
\]
This can be used in order to estimate the largest possible mass splitting in this scenario. Substituting $\Gamma \approx M$ and $M \approx 10^{-9} M_{Pl}$, one gets $\beta_{max} \approx 10^{-13}$. We conclude that for the Planckian fields $\Psi_i$, generated BA is large, unless the mass splitting is tiny.

**APPENDIX B: EFFECTS OF LONG PREHEATING**

Here we estimate the constant $A$ entering Eq. (5). Recall that in the approximation of the (almost) instant preheating one has $A \sim 1$. We show that for any longer duration of preheating, the coefficient $A$ may substantially deviate from unity.

In the presence of the coupling function $F(\phi)$, the general solution for the fields $\Psi_i$ is given by (we omit ‘$i$’ below)

$$\Psi = -\frac{\alpha \sin(Mt)}{a^{3/2}} \int dt \frac{\cos(Mt)}{M} F(\phi)a^{3/2}(t) + \frac{\alpha \cos(Mt)}{a^{3/2}} \int dt \frac{\sin(Mt)}{M} F(\phi)a^{3/2}(t).$$

(17)

We assume that the fields $\Psi$ are at rest by the end of inflation, i.e.,

$$\Psi |_{t=t_e} = -\frac{\alpha F_e}{M^2}, \quad \Psi |_{t=t_e} = 0.$$

(18)

For $F(\phi)$ we choose the power law behavior as in Eq. (13). For the sake of concreteness, consider the inflaton with the quadratic potential $U(\phi) \propto \phi^2$ at preheating. In that case we deal with the matter-dominated stage, so that $s=2$, while the scale factor grows as $a(t) \propto t^{2/3}$. The generalization to arbitrary $s$ and different types of cosmological expansion is straightforward, and we comment on it below.

It is convenient to introduce the dimensionless variable $\xi = M(t - t_e)$. Substituting Eq. (13) with $s=2$ into Eq. (17), and using $a(t) \propto t^{2/3}$, one writes down the solution for the fields $\Psi$, which respects initial conditions (18):

$$\Psi = \frac{\alpha F_e \cos \xi}{M_t^2} \cdot \frac{M_t}{M_t + \xi} \cdot \left[ \int_0^\xi d\xi' \frac{M_t \sin \xi'}{M_t + \xi'} - 1 \right]$$

$$- \frac{\alpha F_e \sin \xi}{M^2} \cdot \frac{M_t}{M_t + \xi} \int_0^\xi d\xi' \frac{M_t \cos \xi'}{M_t + \xi'}.$$

(19)

We are interested in the late time behavior of the fields $\Psi$, i.e., $\xi \gg 1$. The integrals entering the expression above have nice converging properties, so that one can replace the upper limits of integration by infinity. In the limit of large $M_t$, the integrals of interest have the following asymptotics (10):

$$\int_0^\infty d\xi' \frac{M_t \sin \xi'}{M_t + \xi'} = 1 - \frac{2}{(M_t)^2} + O \left[ \frac{1}{(M_t)^4} \right],$$

(20)

and

$$\int_0^\infty d\xi' \frac{M_t \cos \xi'}{M_t + \xi'} = \frac{1}{M_t} + O \left[ \frac{1}{(M_t)^3} \right].$$

(21)

Neglecting the terms suppressed by $1/(M_t)^2$, we write the solution for the fields $\Psi$ as follows:

$$\Psi \approx -\frac{\alpha F_e \sin \xi}{(M_t) \cdot M^2} \cdot \frac{M_t}{M_t + \xi}.$$

Comparing the latter with Eq. (5) we justify our estimate (14). We have checked numerically that the same asymptotic behavior as in Eqs. (20) and (21) holds for fairly wide range of $s$ in Eq. (13).

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