Infrared models for the Bjorken sum rule in the APT approach

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Abstract. We consider continuation of the Bjorken sum rule (BSR) in the low $Q^2$ region. QCD coupling is 'frozen' with the help of 'Massive' APT modification. For higher twist corrections we apply spectral representation with several particular spectral physically motivated functions. Analytic restrictions are imposed stemming from Gerasimov-Drell-Hearn sum rule and the finiteness of respective spin-dependent cross-sections. Results are compared with the current experimental data.

1. Introduction

In the framework of the Quantum Chromodynamics (QCD) at the high energy scale observable quantities are being successfully extracted (and experimentally confirmed) using perturbative expansion due to the asymptotic freedom. On the over hand low energy QCD behavior is not well understood yet. The standard perturbative approach fails to describe low energy behavior due to Landau singularity of the coupling $\alpha_S(Q^2)$ with $Q^2$ of the order of $\Lambda_{QCD}^2 \approx 1 \text{ GeV}^2$.

Several QCD modifications has been proposed to improve coupling low energy behavior, one of them is Analytic Perturbation Theory (APT) \cite{1} (for a review see \cite{2, 3}). The APT using analyticity and causality converts power expansion in the perturbative BSR corrections $\Delta_{BSR}(Q^2)$ to the nonpower one. The APT has been successfully applied to describe polarized $\Gamma_{p-n}^{p-n}(Q^2)$ data down to few hundred MeV \cite{4}.

The effective gluon mass as infrared regulator ($Q^2 \to Q^2 + M_{gl}^2$) was introduced by Cornwall \cite{5} and also by Simonov \cite{6}. The 'Massive' APT (MPT) recently proposed by D.V. Shirkov \cite{7, 8} combined it with the APT scheme and extended consideration to the higher orders up to NNNLO.

In this work we apply MPT approach with spectral HT resummation from \cite{9} with the use of Breit-Wigner (BW) distribution \cite{10} to analyze lowest polarized moment $\Gamma_{p-n}^{p-n}(Q^2)$ up to 4-loop level and compare it with the low-$Q^2$ JLab data.

2. Fitting procedure details

We base on non-perturbative HT resummation proposed in \cite{9}:

$$\sum_{n=1}^{\infty} \mu_{2n+2}^{-n} \left( \frac{M_{HT}^2}{Q^2} \right)^n = \int_{-\infty}^{\infty} \frac{f(x)M^2}{Q^2 - xM^2} dx,$$

(1)
Delta-like spectral density was analyzed in [11]. Here we use Breit-Wigner like spectral density instead:

\[
\Delta_{HT}(Q^2) = \int_{-\infty}^{\infty} \frac{1}{\pi} \frac{M_{HT} \sigma}{(y + M_{HT}^2)^2 + M_{HT}^2 \sigma^2} d\mu, \tag{2}
\]

that leads to HT term:

\[
\Delta_{HT}(Q^2) = \frac{M_{HT}^2 (M_{HT}^2 + Q^2)}{(M_{HT}^2 + Q^2)^2 + M_{HT}^2 \sigma^2} \tag{3}
\]

There are 4 free parameters: "glueball mass" \(M_{g^0}^2\), \(M_{HT}^2\), \(\mu_{4\rho}^{p-n}\) and BW width \(\sigma\). By the use of condition:

\[
\Gamma_{1}^{p-n}(Q^2 = 0) = 0, \tag{4}
\]
determined by the finiteness of cross section in the real photon limit one can eliminate \(\mu_{4\rho}^{p-n}\).

Similarly to [12, 13] basing our \(\Gamma_{1}^{p-n}(Q^2)\) IR analysis on Gerasimov-Drell-Hearn [14, 15] and Burkhardt-Cottingham [16] sum rules the equation

\[
\frac{d}{dQ^2} \Gamma_{1}^{p-n}(Q^2 = 0) = \frac{-(\mu_p - 1)^2 + \mu_n^2}{8M^2}, \tag{5}
\]

implies additional restriction and leaves 2 free parameters. If we fix resonance width where will be only one free parameter to fit.

We consider 4 orders in coupling constant from the leading order up to NNNLO in order to trace sensitivity and stability of our numerical calculation. For simplicity we will limit ourselves by the leading order RG equation for higher order analytic couplings calculation.

3. \(\sigma = \rho_0\) width case

One important case motivated by VDM is \(\sigma = \Gamma_{\rho_0}\), width of \(\rho_0\) meson. It is worth to mention that in the LO and NLO model exhibits weak dependence on variable parameters such as \(M_{g^0}^2\) (or in other words has infinite 1 \(\sigma\) confidence interval) as can be seen in the Fig. 1 while the numerical detail are given in the table 1.

In the Fig. 1 one can see fitting results at the each of the four loops respectively. Contour plot represents \(\chi^2\) dependence on the \((M_{g^0}^2 \times m_{HT}^2)\)-plane, thin green line corresponds to the \(m_{HT}^2(M_{g^0}^2)\) dependence as an additional constraint, the red dot on it points to the best fit, while the blue dots show the edges of 1 sigma dispersion.

| Order | \(M_{g^0}^2, \text{GeV}^2\) | \(m_{HT}^2, \text{GeV}^2\) | \(\mu_4, \text{GeV}\) | \(\chi^2\) |
|-------|--------------------------|--------------------------|----------------|--------|
| LO    | 0.789 + 0.354            | 0.694 + 0.088 - 0.011    | -0.180 + 0.012 - 0.010 | 8.133  |
| NLO   | 0.915 + 0.276            | 0.681 + 0.121 - 0.017    | -0.163 + 0.017 - 0.020 | 8.634  |
| \(N^2\)LO | 1.226 + 0.847 - 0.177   | 0.679 + 0.218 - 0.058    | -0.125 + 0.021 - 0.037 | 9.071  |
| \(N^3\)LO | 0.509 + 0.043 - 0.044 | 0.245 + 0.309 - 0.202    | -0.059 + 0.003 - 0.024 | 0.782  |
Figure 1. Contour plots on $\chi^2$ in the LO, NLO, NNLO, NNNLO respectively. Green line - $m^2_{HT}(M^2_{gl})$. Red dot - found fit. Blue dots - edges for 1 $\sigma$ dispersion

4. On the choice of $\sigma$

Let us now return to general evaluation of applicability of the finite width model. On the Fig. 2 the $\chi^2(\sigma)$ dependence for $N^3LO$ order is illustrated. As one can see best fit parameters demonstrate stable values and almost do not change up to $\sigma \approx 0.2$, what at one side admits possibility of the $\rho_0$ meson exchange. On the other side tell us that the Dirac $\delta$ function dispersion approximation for spectral function is also applicable and gives reasonable results. Although it manifests less stable results for $m^2_{HT}$ giving in the $N^2LO$ deviation at the order of magnitude lower. In order to reveal stability of the results and improve precision it is important to derive exact analytic constant in the each order of consideration.

It is worth noting some qualitative characteristics of the $m^2_{HT}(M^2_{gl})$ dependence. The character of the dependence qualitatively changes with the $\sigma$ as one can see in the Fig 1. Nevertheless in the LO, NLO and NNLO it has an independent on $\sigma$ singularity. Which in the
LO for example is determined by

\[ 4g_AM_p^2 + 3(\mu_p - 1)M_p^2\pi \log^2 \left( \frac{M_p^2}{\Lambda_{PTLO}} \right) \beta_0 = 0 \tag{6} \]

which gives \( M_p^2 \approx 0.225 \) in the LO as well as \( M_p^2 \approx 0.432 \, M_p^2 \approx 0.229 \) in the NLO and NNLO respectively.

![Figure 2](image-url)  
**Figure 2.** Best fit dependence on \( \sigma \) in the \( N^3LO \). Vertical red line corresponds to \( \rho_0 \) width.

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