A note on Gauss–Bonnet holographic superconductors

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Received 4 April 2011
Published 3 May 2011
Online at stacks.iop.org/CQG/28/127001

Abstract
We present an analytic treatment near the phase transition for the critical
temperature of (3+1)-dimensional holographic superconductors in Einstein–
Gauss–Bonnet gravity with backreaction. We find that the backreaction makes
the critical temperature of the superconductor decrease and condensation
harder. This is consistent with previous numerical results.

Communicated by Misao Sasaki

PACS numbers: 11.25.Tq, 04.70.Bw, 74.20.–z

1. Introduction

Gauge/gravity duality stems from string theory which provides a rich tool for analyzing
strongly coupled field theory [1]. Especially, the duality provides an established method for
calculating correlation functions in a strongly interacting field theory using a dual classical
gravity description [2, 3]. Holographic superconductors established in [4, 5] are remarkable
examples where the gauge/gravity duality plays an important role. There, a superconducting
phase transition is described by black hole physics according to the duality.

Many previous works on the holographic superconductors were performed in the probe
limit, where the backreaction of matter fields on the spacetime metric is neglected. However,
we found that the effect of the Gauss–Bonnet coupling lowers the critical temperature of
holographic superconductors in previous work [6]. Backreaction becomes important when
considering lower temperatures of black holes in AdS spacetime, because lower Hawking
temperatures of black holes means smaller black holes, i.e., larger Coulomb energy of the
matter fields near the black hole horizon.

The critical temperature was obtained numerically both with and without the backreaction
[4, 5, 7]. As an analytic approach for deriving the critical temperature, an approximate analytic
formula was proposed in the probe limit by a matching method [6]. An alternative analytic
method using the expansion around the critical point where the phase transition occurs was also
proposed [8]. Some other analytic approaches were also proposed in the probe limit [9–12].
In this note, we derive analytically the critical temperature of the Gauss–Bonnet holographic
superconductors with backreaction by combining the small backreaction approximation and the matching method near the critical point.

2. Gauss–Bonnet black holes

We begin with the action for a Maxwell field and a charged complex scalar field coupled to the Einstein–Gauss–Bonnet:

\[ S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} \left[ R + \frac{12}{L^2} + \frac{\alpha}{2} (R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4 R^{\mu\nu} R_{\mu\nu} + R^2) \right] + \int d^5x \sqrt{-g} \left[ -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} - |\nabla \psi - i q A \psi|^2 - m^2 |\psi|^2 \right], \]

where \( g \) is the determinant of a metric \( g_{\mu\nu} \) and \( R_{\mu\nu\rho\sigma} \), \( R_{\mu\nu} \) and \( R \) are the Riemann curvature tensor, Ricci tensor, and the Ricci scalar, respectively. \( q \) is the charge of the scalar field. We take the Gauss–Bonnet coupling constant \( \alpha \) to be positive. Here, the negative cosmological constant term \(-6/L^2\) is also introduced. We look for electrically charged plane-symmetric hairy black hole solutions taking the metric ansatz

\[ ds^2 = -f(r) e^{2\nu(r)} dt^2 + \frac{dr^2}{f(r)} + \frac{r^2}{L^2} (dx^2 + dy^2 + dz^2) \]

(2)

together with a static ansatz for the fields,

\[ A_\mu = (\phi(r), 0, 0, 0), \quad \psi = \psi(r). \]

(3)

Here, without loss of generality \( \psi \) can be taken to be real. First, we look for the solutions in normal phase, \( \psi = 0 \). We find that \( \nu \) is constant and

\[ \phi = \mu - \rho \frac{r}{r^2}. \]

(4)

Here, \( \mu \) and \( \rho \) are interpreted as a chemical potential and charge density of the dual theory on the boundary, respectively. We also find

\[ f(r) = \frac{r^2}{2\alpha} \left[ 1 - \sqrt{1 - \frac{4\alpha}{L^2} \left( 1 - \frac{r^4}{r_h^4} \right) + 2\kappa^2 \frac{4\alpha \rho^2}{3r^2} \left( 1 - \frac{r^2}{r_h^2} \right)} \right], \]

(5)

where we chose the minus sign of the solutions so that we have a solution in the Einstein limit (\( \alpha \to 0 \)). The horizon radius \( r_h \) is defined through the requirement that \( f(r_h) = 0 \), so we set a constant of integration by imposing this condition at the horizon. We see that the above solution becomes the Reissner–Nordström–AdS solution in the Einstein limit. In order to avoid a naked singularity, we need to restrict the parameter range as \( \alpha \leq L^2/4 \). For general \( \alpha \), solution (5) behaves as

\[ f(r) \sim \frac{r^2}{2\alpha} \left[ 1 - \sqrt{1 - \frac{4\alpha}{L^2}} \right], \]

(6)
in the asymptotic region. Hence, we define the effective asymptotic AdS scale by

\[ L_c^2 = \frac{2\alpha}{1 - \sqrt{1 - \frac{4\alpha}{L^2}}} \]

(7)

where \( L_c^2 \) is the maximum value of \( \alpha \) is called the Chern–Simons limit. The Hawking temperature is given by

\[ T_H = \frac{1}{4\pi} f'(r) e^{\nu(r)} \bigg|_{r=r_h}. \]

(8)
where a prime denotes derivative with respect to $r$. This will be interpreted as the temperature of the holographic superconductors.

3. Gauss–Bonnet holographic superconductors

In order to obtain the solutions in superconducting phase, $\psi \neq 0$, we need to take into account the boundary conditions at the horizon and the AdS boundary. The position of the horizon, $r_*$, is defined through $f(r_*) = 0$. We can set $v(r_*) = 0$ there by rescaling the time coordinate. Then the Einstein equations give

$$v'(r_*) = \frac{2\kappa^2}{3} r_* \left( \psi'(r_*)^2 + \frac{g^2 \phi'(r_*)^2 \psi(r_*)^2}{f'(r_*)^2} \right),$$

$$f'(r_*) = \frac{4}{L^2 r_*} - \frac{2\kappa^2}{3} r_* \left( \frac{\phi'(r_*)^2}{2 e^{2\nu(r_*)}} + m^2 \psi(r_*)^2 \right).$$

- Regularity at the horizon for $\phi$ and $\psi$ gives two conditions:

$$\phi(r_*) = 0, \quad \psi(r_*) = \frac{m^2}{f'(r_*)} \psi(r_*).$$

As we want the spacetime to be asymptotically AdS, we look for a solution with

$$v(r) = \text{const}, \quad f(r) = \frac{r^2}{L^2},$$

at the AdS boundary.

- Asymptotically ($r \to \infty$) the solutions of $\phi$ and $\psi$ are found to be

$$\phi(r) \sim \mu - \frac{\rho}{r^2}, \quad \psi \sim \frac{C_-}{r^{\Delta_-}} + \frac{C_+}{r^{\Delta_+}},$$

where $\Delta_\pm = 2 \pm \sqrt{4 + m^2 L^2}$. Note that these are not entirely free parameters, as there is a scaling degree of freedom in the equations of motion. As in [4], we impose that $\rho$ is fixed, which determines the scale of the system. For $\psi$, in order to have a normalizable solution we take $C_- = 0$.

According to the gauge/gravity duality [1–3], we can interpret $\langle O_{\Delta_+} \rangle \equiv C_+$, where $O_{\Delta_+}$ is the operator with the conformal dimension $\Delta_+$ dual to the scalar field. Thus, we are going to calculate the condensate $\langle O_{\Delta_+} \rangle$ for fixed charge density. We note that since $C_-$ is regarded as the source term of the operator, we put it zero at the end by using the gauge/gravity dictionary [2, 3], which is consistent with taking $C_- = 0$.

Let us change the coordinate and set $z = r_*/r$. Under this transformation, the Einstein, Maxwell and the scalar equations become

$$\left( 1 - 2\alpha \frac{z^2}{r_*^2} f \right) \nu' = - \frac{2\kappa^2}{3} \frac{r_*^2}{z^3} \left( \frac{g^2 \phi^2 \psi^2}{f^2 e^{2\nu}} + \frac{z^4}{r_*^2} \psi^2 \right),$$

$$\left( 1 - 2\alpha \frac{z^2}{r_*^2} f \right) f' - \frac{2}{z} f + \frac{4r_*^2}{L^2 z^3} = \frac{2\kappa^2}{3} \frac{r_*^2}{z^3} \left[ \frac{g^2 \phi^2 \psi^2}{f^2 e^{2\nu}} + m^2 \psi^2 + f \left( \frac{g^2 \phi^2 \psi^2}{f^2 e^{2\nu}} + \frac{z^4}{r_*^2} \psi^2 \right) \right],$$

$$\phi'' = \frac{1}{z} + \nu' \phi' - 2 \frac{2r_*^2 \psi^2}{e^4 f} \phi = 0,$$

$$\psi'' = \frac{1}{z} - \nu' - \frac{f'}{f} \psi' + \frac{r_*^2}{e^4 f} \left( \frac{\phi^2}{f^2 e^{2\nu}} - \frac{m^2}{f} \right) \psi = 0.$$
where the prime now denotes a derivative with respect to $z$. The region $r_* < r < \infty$ now corresponds to $0 < z < 1$. If one sets $\phi = \phi/q$, $\psi = \psi/q$ in the action (1), the Maxwell and the scalar equations remain unchanged, while the gravitational coupling of the Einstein equations changes $\kappa^2 \to \kappa^2/q^2$. If one takes the limit $q \to \infty$, the matter sources drop out of the Einstein equations and this is the probe limit. To go beyond the probe limit, we can take either finite $q$ with setting $2\kappa^2 = 1$, or finite $2\kappa^2$ with setting $q = 1$. Paper [5] took the former choice to consider the effects of backreaction of the spacetime metric, but we will take the latter choice in the following.

In order to solve these equations, we focus on near the critical point as in [8, 13], around which the stability was confirmed [14]. It is convenient to introduce a scalar operator as an expansion parameter:

$$\epsilon \equiv \langle O_{\Delta_s} \rangle.$$  \hspace{1cm} (18)

As $\psi$ is small near the critical point, we expand $\psi$ from the first order. From equation (16), $\phi$ and $\psi$ are expanded by $\epsilon^2$ subsequently as follows:

$$\phi = \phi_0 + \epsilon^2 \phi_2 + \epsilon^4 \phi_4 + \cdots,$$  \hspace{1cm} (19)

$$\psi = \epsilon \psi_1 + \epsilon^3 \psi_3 + \epsilon^5 \psi_5 + \cdots,$$  \hspace{1cm} (20)

and in this situation where starting from the normal phase, the background can be expanded around the Reissner–Nordström–AdS spacetime:

$$f = f_0 + \epsilon^2 f_2 + \epsilon^4 f_4 + \cdots,$$  \hspace{1cm} (21)

$$\nu = \epsilon^2 \nu_2 + \epsilon^4 \nu_4 + \cdots,$$  \hspace{1cm} (22)

where $\epsilon \ll 1$.

While $\mu$ is also expanded by the order parameter as

$$\mu = \mu_0 + \epsilon^2 \delta \mu_2,$$  \hspace{1cm} (23)

where $\delta \mu_2$ is also positive. So, we find the order parameter as a function of the chemical potential, which is expressed by

$$\epsilon = \langle O_{\Delta_s} \rangle = \left( \frac{\mu - \mu_0}{\delta \mu_2} \right)^{1/2}.$$  \hspace{1cm} (24)

The exponent $1/2$ is consistent with the Ginzburg–Landau mean field theory for phase transitions. When $\mu = \mu_0$, the order parameter becomes zero, which means the critical value of $\mu$ is defined by $\mu_c = \mu_0$. Now we begin to solve equations order by order.

At zeroth order, we impose $\phi_0(1) = 0$ at the horizon, $z = 1$, and $\phi_0(0) = \mu_0 = \text{const}$ at the boundary, $z = 0$. Then the solution of equation (16) is given by

$$\phi_0(z) = \mu_0 (1 - z^2).$$  \hspace{1cm} (25)

This gives a relation $\rho = \mu_0 \rho^2$ by the coordinate transformation. Inserting this solution into equation (15), we obtain

$$f_0(z) = \frac{r_*^2}{2\alpha} \frac{1}{z^2} \left[ 1 - \sqrt{1 - \frac{4\alpha}{L^2} (1 - z^2) + 2\kappa^4 \frac{4\alpha \mu_0^2}{3r_*^2} z^4 (1 - z^2)} \right],$$  \hspace{1cm} (26)

where we chose the minus sign of the solutions so that we have a solution in the Einstein limit. We also used $f_0(1) = 0$ at the horizon as we did in equation (5). We find that the above solution becomes the Reissner–Nordström–AdS solution in the Einstein limit.
Even at first order, it is difficult to deal with both the backreaction on the spacetime metric and the Gauss–Bonnet term. Here we will appeal to the matching method used in [6]. Before proceeding, let us check the regularity condition at the horizon at this order:

\[
\psi_1'(1) = \frac{r_1^2m^2}{f_0'(1)} \psi_1(1).
\] (27)

The behavior of \(\psi\) at the boundary is given by

\[
\psi_1 \sim D_- z^{\Delta-} + D_+ z^{\Delta+},
\] (28)

where \(\Delta_{\pm}\) is the same as in equation (13). For the boundary conditions, we take \(D_-\) to be zero as we did in equation (13).

Now we find the solution of \(\psi_1\) under these conditions by the matching method and derive the critical temperature. We can expand \(\psi_1\) in a Taylor series near the horizon as

\[
\psi_1(z) = \psi_1(1) - \psi_1'(1)(1-z) + \frac{1}{2} \psi_1''(1)(1-z)^2 + \cdots
\] (29)

Here, we have equation (27) for the first-order coefficients of \(\psi_1\), and without loss of generality we can take \(\psi_1(1) > 0\) to have \(\psi_1(z)\) positive. The second-order coefficients of \(\psi_1\) are computed by using equation (17). Then we find the second derivative at the horizon is expressed by

\[
\psi_1''(1) = \frac{1}{2} \left( -3 - \frac{f_0''(1)}{f_0'(1)} + \frac{r_1^2m^2}{f_0'(1)} \right) \psi_1(1) - \frac{r_1^2 \phi_0'(1)^2}{2 f_0'(1)} \psi_1(1).
\] (30)

After eliminating \(\psi_1'(1)\) from the above equation by using equation (27), an approximate solution near the horizon is given by

\[
\psi_1(z) = \psi_1(1) - \frac{r_1^2m^2}{f_0'(1)} \psi_1(1)(1-z)
\]

\[
+ \left[ - \frac{r_1^2m^2}{4 f_0'(1)} \left( 3 + \frac{f_0''(1)}{f_0'(1)} - \frac{r_1^2m^2}{f_0'(1)} \right) - \frac{r_1^2 \phi_0'(1)^2}{4 f_0'(1)^2} \right] \psi_1(1)(1-z)^2 + \cdots
\] (31)

Here, \(\psi_1(1)\) is still unknown.

On the other hand, from (28), \(\psi_1\) in the asymptotic region are given by

\[
\psi_1(z) \sim D_+ z^{\Delta+}.
\] (32)

We have set \(D_- = 0\) from the boundary condition. Here \(D_+\) is another unknown constant.

Now we try to connect the solutions (31) and (32) smoothly at \(z_m\) in order to obtain \(\psi_1(1)\) and \(D_+\). As we shall see below, the choice of \(z_m\) does not change the qualitative features of solutions. In order to connect those solutions smoothly, we require the following two conditions:

\[
z_m^\Delta_+ D_+ = \psi_1(1) - \frac{r_1^2m^2}{f_0'(1)} (1-z_m) \psi_1(1)
\]

\[
+ \left[ - \frac{r_1^2m^2}{4 f_0'(1)} \left( 3 + \frac{f_0''(1)}{f_0'(1)} - \frac{r_1^2m^2}{f_0'(1)} \right) - \frac{r_1^2 \phi_0'(1)^2}{4 f_0'(1)^2} \right] (1-z_m)^2 \psi_1(1),
\] (33)

\[
\Delta_+ z_m^{\Delta_+ - 1} D_+ = \frac{r_1^2m^2}{f_0'(1)} \psi_1(1)
\]

\[
- 2 \left[ - \frac{r_1^2m^2}{4 f_0'(1)} \left( 3 + \frac{f_0''(1)}{f_0'(1)} - \frac{r_1^2m^2}{f_0'(1)} \right) - \frac{r_1^2 \phi_0'(1)^2}{4 f_0'(1)^2} \right] (1-z_m) \psi_1(1).
\] (34)
From equations (33) and (34), we find the relation between \( \psi_1(1) \) and \( D_+ \),
\[
D_+ = \frac{2z_m}{2z_m + (1 - z_m)\Delta_+} \psi_1(1) \left( 1 - \frac{1 - z_m}{2} \frac{r^2_m m^2}{f'_0(1)} \right).
\]
(35)

Plugging this back into equation (34), we find the following relation in order to get a non-trivial solution, \( \psi_1(1) \neq 0 \),
\[
\frac{2\Delta_+}{2z_m + (1 - z_m)\Delta_+} = -\left( \frac{1 - z_m}{2z_m + (1 - z_m)\Delta_+} + \frac{5 - 3z_m}{2} \right) \frac{r^2_m m^2}{f'_0(1)} \]
\[
- \left( \frac{1 - z_m}{2z_m + (1 - z_m)\Delta_+} \right) \frac{L^4}{f'_0(1)^2} + \frac{1}{2} \frac{1 - z_m}{f'_0(1)^2} \frac{r^4_m m^4}{2} - \left( \frac{1 - z_m}{2z_m + (1 - z_m)\Delta_+} \right) \frac{f'_0(1)^2}{2} = 0.
\]
(36)

Similarly, plugging equation (35) back in equation (33) we can get the solution of \( \psi_1(1) \) and then \( D_+ \) as well in principle. But what we want to know here is the critical temperature rather than deriving the solution of \( \psi \) up to this order, so we are going to focus on it in the following. For this purpose, putting the values of \( f'_0(1) \), \( f''_0(1) \) and \( \phi'_0(1) \) in the above relation, equation (36) yields the equation for \( \mu_0 \):
\[
4\kappa^4 \frac{L^4}{36r^4} \left[ \frac{2\Delta_+}{2z_m + (1 - z_m)\Delta_+} - (1 - z_m)m^2\alpha \right] \mu^4_0
\]
\[
- \left( \frac{1 - z_m}{2z_m + (1 - z_m)\Delta_+} \right) \frac{L^4}{8r^2} \left[ 1 + 2\kappa^2 \left( \frac{5 - 3z_m}{6(1 - z_m)} + \frac{5}{6} \right) m^2 - \frac{8m^2}{3L^2\alpha} \right] \right] \mu^4_0
+ \left( \frac{1 - z_m}{2z_m + (1 - z_m)\Delta_+} \right) \left( \frac{2}{1 - z_m} + \frac{m^2L^4}{4} + \frac{2 - z_m m^2 L^2}{4} \right)
+ \frac{1}{32} m^4 L^4 - (1 - z_m)m^2\alpha = 0.
\]
(37)

In order to solve the above equation with respect to \( \mu_0 \), we assume \( \kappa^2 \ll 1 \) in the following. This means that all functions are expanded by \( \kappa^2 \) as well. That is, solutions near the phase transition are obtained in the small backreaction approximation together with the matching method. As the first term disappears from the above equation, we can solve it easily and we get
\[
\mu_0 = \sqrt{\frac{8}{1 - z_m} \frac{r_+}{L^2} \left[ \frac{1 - z_m}{2z_m + (1 - z_m)\Delta_+} \left( \frac{2}{1 - z_m} + \frac{m^2L^2}{4} \right) \right]^{1/2}
\]
\[
+ \frac{2 - z_m}{4} m^2 L^2 + \frac{1 - z_m}{32} m^4 L^4 - (1 - z_m)m^2\alpha \right]^{1/2}
\times \left[ 1 - 2\kappa^2 \left( \frac{5 - 3z_m}{6(1 - z_m)} + \frac{5}{6} \right) \left( \frac{m^2}{3} \frac{1}{3L^2\alpha} + \frac{\Delta_+}{4z_m + 2(1 - z_m)\Delta_+} \right) \right],
\]
(38)
where \( \mu_0 \) is positive. Combining equation (38) with the relation \( \mu_0 = \rho/r_s^2 \), which is given under equation (25), we find that \( r_s \) is given by

\[
r_s = \frac{\rho^{1/3}}{\pi L^{3/5}} \left( \frac{1 - z_m}{8} \right)^{1/6} \left[ \frac{8 \Delta_+ + (1 - z_m) \Delta_+ m^2 L^2}{8 z_m + 4(1 - z_m) \Delta_+} + \frac{2 - z_m}{4} m^2 L^2 \right. \\
\left. + \frac{1 - z_m}{32} m^4 L^4 - (1 - z_m)m^2 \alpha \right]^{1/6} \\
\times \left[ 1 + \frac{2 \kappa^2}{L^2} \left\{ \frac{\Delta_+}{12 z_m + 6(1 - z_m) \Delta_+} \left( \frac{m^2 L^2}{3} + \frac{16}{3(1 - z_m)} \right) \right. \\
\left. + \frac{5 - 4 z_m m^2 L^2}{18} - \frac{4 m^4 \alpha}{9 - \alpha} \right\} \right]. \tag{39}
\]

The Hawking temperature, equation (8), up to this order becomes

\[
T_H = -\frac{f_0(1) e^{\mu(1)}}{4\pi r_+} = \frac{r_s}{\pi L^2} \left( 1 - 2\kappa^2 L^2 \frac{\mu_0^2}{6 r_s^2} \right). \tag{40}
\]

The critical temperature is defined at the point where the order parameter becomes zero, which leads to \( T_H = T_c \) at \( \mu_0 = \mu_c \). Eliminating \( \mu_0(=\mu_c) \) from the above temperature \( T_H \) using equation (38), we get the critical temperature

\[
T_c = \frac{r_s}{\pi L^2} \left[ 1 - \frac{2 \kappa^2}{L^2} \frac{4 \Delta_+}{6 z_m + 3(1 - z_m) \Delta_+} \left( \frac{2}{1 - z_m} + \frac{m^2 L^2}{4} \right) \right. \\
\left. + \frac{2 - z_m}{3(1 - z_m)} m^2 L^2 + \frac{m^4 L^4}{24} - \frac{4 m^4 \alpha}{3} \right]. \tag{41}
\]

Substituting \( r_s \) with equation (39), we obtain the critical temperature of the form

\[
T_c = T_1 \left( 1 - \frac{2 \kappa^2}{L^2} T_2 \right), \tag{42}
\]

where

\[
T_1 = \frac{\rho^{1/3}}{\pi L^{3/5}} \left( \frac{1 - z_m}{8} \right)^{1/6} \left[ \frac{8 \Delta_+ + (1 - z_m) \Delta_+ m^2 L^2}{8 z_m + 4(1 - z_m) \Delta_+} \right. \\
\left. + \frac{2 - z_m}{4} m^2 L^2 + \frac{1 - z_m}{32} m^4 L^4 - (1 - z_m)m^2 \alpha \right]^{-1/6}, \tag{43}
\]

\[
T_2 = \frac{2 \Delta_+}{2 z_m + (1 - z_m) \Delta_+} \left( \frac{5 m^2 L^2}{36} + \frac{8}{9(1 - z_m)} \right) + \frac{7}{1 - z_m} + \frac{m^4 L^4}{24} - \frac{8 m^2 \alpha}{9}. \tag{44}
\]

4. Conclusion

We presented an analytic treatment near the phase transition for the critical temperature of \((3 + 1)\)-dimensional holographic superconductors in Einstein–Gauss–Bonnet gravity with backreaction by matching the solution expanded from infinity with that expanded from the horizon. In this method, there is an ambiguity in the choice of the matching radius. However, the result turns out to be fairly insensitive to the choice of it. The result reproduces our previous analytic calculation for \( m^2 = -3/L^2 \), \( \kappa^2 = 0 \) with the choice \( z_m = 1/2 \), which was in good agreement with the numerical result [6]. It also agrees with [8] for \( m^2 = -4/L^2 \), \( \kappa^2 = \alpha = 0 \) with the choice \( z_m = 1/2 \). It may be noted that the choice \( z_m \sim 0.5 \) is roughly equal to \( z = \sqrt{r_s/L} \), corresponding to the geometrical mean of the horizon radius and the AdS
scale, $r = \sqrt{r + L}$. We found that the coefficient of the corrections due to backreaction, $T_2$, is positive definite irrespective of the value of $\alpha$. This means the effects of the backreaction make condensation harder. This result also agrees with the numerical results obtained in [15–17].

Acknowledgments

I would like to thank Jiro Soda for useful and stimulating discussions and Misao Sasaki for helpful comments and suggestions. I would also like to thank Luke Barclay, Ruth Gregory and Paul Sutcliffe for previous collaboration and useful discussions. I was a member of the CPT group at the Department of Mathematical Sciences, Durham University, and was supported by an STFC rolling grant while part of this work was carried out. This work is supported in part by grant PHY-0855447 from the National Science Foundation.

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