The Electric Dipole Moment of the W and Electron in the Standard Model

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ABSTRACT

I show that the electric dipole moment of the W-boson $d_W$ vanishes to two loop order in the standard Kobayashi-Maskawa Model of $CP$ violation. The argument is a simple generalization of that used to show the vanishing of the quark electric dipole moment. As a consequence, the electron electric dipole moment vanishes to three loop order. Including QCD corrections may give a non-vanishing result; I estimate $d_W \approx 8 \cdot 10^{-31} e\text{cm}$, which induces an electron EDM $d_e \approx 8 \cdot 10^{-41} e\text{cm}$, considerably smaller than a previous calculation.

PACS: 11.30.Er, 12.15.Ji, 13.10.+q, 13.40.Fn

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1 Introduction

Any model of CP violation will in general induce — through loop effects — P and T violating electric dipole moments (EDMs) for elementary particles, including the W-boson. The P and T violating interaction of the W boson with a photon can be described by the effective Lagrangian[1]

\[ \mathcal{L}_{CP}^{W \gamma} = i \lambda_1 \tilde{F}_{\mu\nu} W^\mu W^\nu + i \frac{\lambda_2}{M_W^2} \tilde{F}_{\mu\nu} W^{\mu\sigma} W^{\sigma\nu}. \]  

(1)

Here \( \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \), \( W_{\mu\nu} = \partial_{\mu} W_{\nu} - \partial_{\nu} W_{\mu} \) and \( W_{\nu} \) is the W-boson field[1]. In terms of the Lagrangian eqn (1), the coefficient of the W-boson EDM (WEDM) is then given by \( d_W = (\lambda_1 + \lambda_2/M_W^2)(e/2M_W) \).

Although it is possible in principle to measure the WEDM directly, for example in scattering experiments[2], it will also contribute through loops in lower energy phenomena. It was proposed years ago as an explanation of CP violation in \( K^0 - \bar{K}^0 \) mixing[3]. In most models, the WEDM (and related operators) give the dominant contribution to the electron EDM. This is believed to be the case in the Standard Kobayashi-Maskawa Model (KM model) of CP violation, where the contribution of the WEDM to the electron EDM has been calculated to be \( d_e \sim 10^{-38} \) e cm[4], although this is in conflict with an estimate of \( d_W \)[5] which is itself on the order of \( 10^{-38} \) e cm.

In this note I will show that in fact \( d_W \) vanishes to two-loop order in the KM model. This in turn implies that \( d_e \) vanishes to three-loop order, contradicting the result of ref. (4). The remainder of this paper is organized as follows: in section two I review the mechanics of CP violation in the KM model. In section three I demonstrate the vanishing of \( d_W \) and in the following section I estimate the three-loop QCD contribution to \( d_W \) and \( d_e \). Finally, in section five I conclude.

2 The KM Model

Unitarity of the CKM matrix and rephasing invariance imply that all CP violating effects in the standard model are governed by the quantity \( \Phi \):\[ \Phi_{u_1d_1} = \text{Im}(V_{u_1d_1} V_{u_2d_2}^* V_{u_2d_2}^* V_{u_1d_1}^*). \] Here \( V_{ud} \) is the KM matrix element and \( u_i, d_j \) are arbitrary up and down-type quarks. \( \Phi \) has two important properties[6, 8]: it

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[1] As an aside, note that the first term in eqn. (1) is not SU(2) gauge invariant, so it should be accompanied by operators containing the Higgs field and its coefficient should be proportional to SU(2) breaking terms — it would not be present in an unbroken theory.
is antisymmetric in \((u_1, u_2)\) and \((d_1, d_2)\) and the sum over any index, e.g. \(u_1\), vanishes. For three families there is the particularly simple relation\(^7\)

\[
\Phi_{u_1d_1}^{u_2d_2} = J \sum_{\gamma k} \epsilon_{u_1u_2}\epsilon_{d_1d_2} \gamma
\]

Any \(CP\) violating amplitude is given by summing \(\Phi\) together with the Feynman diagrams \(A_{u_1u_2}^{d_1d_2}\) for each configuration of quarks that contribute to the process in question. \(A\) is a function of quark masses and possibly KM angles (not phases). In the case where the weak interactions occur along a single quark line — as is true for the diagrams which generate \(d_W\) — there is no additional dependence on the KM angles so that one has \(A_{u_1u_2}^{d_1d_2} = A(m_{u_1}^2, m_{u_2}^2, m_{d_1}^2, m_{d_2}^2)\). Since the weak interactions in the KM model are purely left handed, quark masses enter the diagrams quadratically, through the denominators of the rationalized propagators.\(^2\) Because of the antisymmetry of \(\Phi\), only those parts of Feynman diagrams which are not symmetric under the exchange of up or down quark masses will contribute to \(CP\) violation. This anti-symmetrization, which is the GIM mechanism, leads to cancellations between the contributions of different quarks and is responsible for suppressing most \(CP\) violating effects in the KM model.\(^9\)

### 3 The WEDM in the KM Model

The two loop contributions to the WEDM in the KM model are shown in figure 1. An additional set of five is generated by interchanging the roles of the up and down quarks in the loop, for a total of ten diagrams. I will only consider the first set, since the treatment of the second set is exactly parallel.

It is easy to eliminate from further consideration the diagram where the photon attaches to the bottom of the loop in figure 1; the photon momentum \(k\) appears only in the down quark line, so the two up-quark propagators have the same momentum dependence. Consequently, the diagram is symmetric in the up-quark masses and as discussed earlier will not contribute to the WEDM; it will be clear from the renormalized form of the self-energy that this statement remains true after renormalization.

Instead of studying the four remaining diagrams directly, consider a simpler set of diagrams obtained by cutting the \(d_2\) quark line in figure 1. They are shown in figure 2.

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2 Strictly speaking, this is true only in the unitary gauge. For other gauge choices, mass dependence will also enter through the unphysical Higgs vertices. However, it is possible to arrange the calculation so that these masses drop out.
Figure 1: The two-loop contributions to $d_W$.

Figure 2: Vertex sub-graphs of the diagrams of figure 1.
The diagrams of figure 2 combine to give (keeping only the chiral projectors from the W-vertices)

$$A_\mu = L \frac{1}{\not{p} - \frac{\gamma}{2} - m_{u_2}} \Gamma_\mu(p, k) \frac{1}{\not{p} + \frac{\gamma}{2} - m_{u_1}} R$$

(2)

where $\Gamma_\mu$ is the complete vertex function defined by

$$\Gamma_\mu(p, k) = \frac{1}{\not{p} - \frac{\gamma}{2} - m_{u_2}} \Sigma(p - k/2) + \frac{1}{\not{p} + \frac{\gamma}{2} - m_{u_1}} \gamma_\mu + \Gamma^{\text{IR}}_\mu(p, k),$$

(3)

$\Sigma$ is the renormalized self-energy function and $\Gamma^{\text{IR}}$ is the irreducible vertex function. These have been computed many times before [6, 10, 11] and their detailed forms are not required. However, because the results are not generally well known, I will sketch the calculation. Since the EDM is a static effect, I will expand the vertex in powers of the photon momentum $k$, keeping only the first term $3$.

After renormalization, the self-energy $\Sigma$ takes the form

$$\Sigma_{u_2u_1} = (\not{p} - m_{u_2})\Sigma_{u_2u_1}(\not{p} - m_{u_1})$$

(4)

with

$$\Sigma = F_0^{(1)}(p^2)(\not{p} R + m_{u_2} R + m_{u_1} L) + F_0^{(2)}(p^2) \not{p} L.$$  

(5)

Here $F_0^{(1)}$ and $F_0^{(2)}$ are

$$F_0^{(1)} = \frac{p^2 F_0 + m_{u_1} m_{u_2} X_0}{(p^2 - m_{u_1}^2)(p^2 - m_{u_2}^2)},$$

(6)

$$F_0^{(2)} = \frac{m_{u_1} m_{u_2} F_0 + (m_{u_1}^2 + m_{u_2}^2 - p^2) X_0}{(p^2 - m_{u_1}^2)(p^2 - m_{u_2}^2)},$$

(7)

and

$$F_0 = f(p^2) - \frac{m_{u_1}^2 f(m_{u_1}^2) - m_{u_2}^2 f(m_{u_2}^2)}{m_{u_1}^2 - m_{u_2}^2},$$

(8)

$$X_0 = \frac{m_{u_1} m_{u_2}}{m_{u_1}^2 - m_{u_2}^2} f(m_{u_1}^2) - f(m_{u_2}^2).$$

(9)

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3 An equivalent analysis of the vertex function using a different approach has recently been presented in ref. [12]
Finally, \( f(p^2) \) is the function which occurs in the unrenormalized self energy, \( \Sigma_0(p) = f(p^2) \not p L \). Similarly, the irreducible vertex has the form

\[
\Gamma^\text{IR}_\mu(p, k) = -Q_u \frac{\partial}{\partial p_\mu} \Sigma + f_2\{p, \sigma_{\mu\nu} k^\nu\} L, \tag{10}
\]

where \( f_2(p^2) \) does not depend on the down-quark masses.

In terms of the above functions, \( A_\mu \) has the relatively simple form (to \( O(k) \))

\[
A_\mu(p, k) = \{p, \sigma_{\mu\nu} k^\nu\} R \frac{p^2 f_2(p^2) + p^2 F_0^{(1)} - m_{u1} m_{u2} F_0^{(2)}}{(p^2 - m_{u1}^2)(p^2 - m_{u2}^2)} - Q_u \frac{\partial}{\partial p_\mu} L \Sigma(p) R. \tag{11}
\]

Recalling the expression for \( \Sigma \) one sees that \( A_\mu \) is a symmetric function of \( m_{u1} \) and \( m_{u2} \). It is this symmetry which is at the heart of all the vanishing two loop effects in the KM model. It follows that the diagrams of figure 1 combine to produce a symmetric function of the up-quark masses so that as I intended to show, the W-boson EDM vanishes to two loops in the Standard Model.

4 Estimates

In order to obtain a non-zero WEDM, it is necessary to destroy the symmetric way in which the quark propagator enter the diagrams. This can be accomplished by including QCD loop corrections. The resulting 3-loop diagrams are rather difficult to calculate. One way to simplify these calculations would be to perform an effective lagrangian expansion, including gluonic operators, and then integrate out the gluon fields. However, in view of the small expected size of the WEDM in the KM model, a cruder estimate will suffice.

Because the GIM mechanism effectively cuts off the the momentum integrals, it is reasonable to estimate them by their infrared limits. Gluon loops will typically introduce logarithmic mass dependence. This is also true of the functions \( f(p^2) \) and \( f_2(p^2) \). Moreover, the entire diagram has a superficial logarithmic divergence. Consequently I expect the mass dependence of the diagrams contributing to \( d_W \) to be logarithmic. Because of the insensitivity of the logarithm — even for the widely separated scales in the KM model.
— the GIM cancellation will be weak and I can obtain a reasonable (over-) estimate of $d_W$ by setting these logs to 1. I then obtain

$$d_W \approx J \left( \frac{1}{16\pi^2} \right)^2 \left( \frac{g_W^2}{8} \right)^2 \left( \frac{\alpha_s}{4\pi} \right) \left( \frac{e}{2M_W} \right) \simeq 8 \cdot 10^{-30} \text{e cm}. \quad (12)$$

In ref. (5) Chang et al. estimated the vanishing two-loop contribution to be

$$d_W = J \left( \frac{g_W^2}{8\pi^2} \right)^2 \left( \frac{e}{2M_W} \right) \frac{m_4^2 m_2^2 m_c^2}{M_W^3}. \quad (13)$$

However, this estimate is unduly pessimistic because it assumes the quark mass dependence of the diagrams is polynomial, leading to the small mass ratios in their estimate.

In order to estimate the electron EDM $d_e$, I use the analysis of Marciano and Queijeiro[13], who have updated the original study by Salzman and Salzman[3]. They calculate the contribution of $d_W$ to $d_e$ and obtain the relation

$$d_e \approx \left( \frac{g_W^2}{32\pi^2} \right) \left( \frac{m_e}{M_W} \right) \left[ \ln \frac{\Lambda^2}{M_W^2} + O(1) \right] d_W, \quad (14)$$

where $\Lambda$ is a cutoff describing the scale of “new physics”. Setting the term in brackets to 1, I find for the KM model

$$d_e \approx 8 \cdot 10^{-41} \text{e cm}, \quad (15)$$

which is considerably smaller than the value obtained by Hoogeveen [4], whose calculation of $d_e$ contains the graphs of figure 1 as sub-graphs and should thus vanish.

5 Conclusions

The vanishing of the WEDM to two-loop order in the KM model shows that the W-boson is on the same footing as all other fundamental particles in the KM model: none of them possess electric dipole moments to two-loop order. This was shown years ago for the quarks[14] and leptons[14], and more recently it was shown that the chromo-electric dipole moment of the gluon (the Weinberg operators) also vanishes[14].

In order to generate a non-vanishing EDM for these particles, it is necessary to destroy the symmetric way in which the quark propagators enter the diagrams. This can be done by considering higher-order operators, or at the cost of another loop by including QCD corrections. Either way, this leads to an extra suppression of $CP$ violating effects in the standard model.
Note Added

After completing this work, I became aware of the work of Khriplovich and Pospelov (ref. [15]) who also consider this problem. Inspired by the same work (E. P. Shabalin, ref. [6]) we reach the same conclusion about the vanishing $d_W$ and $d_e$. My work can be viewed as an extension of theirs in so far as I obtain actual estimates of the QCD loop contribution to $d_W$ and $d_e$.

Acknowledgements

I am grateful to Scott Willenbrock for bringing reference [6] to my attention. This work was supported in part by DOE grant AC02 80ER 10587.
References

1. K. Hagiwara, R. D. Peccie and D. Zeppenfeld, Nucl. Phys. B 282, 253 (1987).
2. A. Queijeiro, Phys. Lett. B 193, 354 (1987).
3. F. Salzman and G. Salzman, Phys. Lett. 15, 91 (1965); Nuovo. Cim. 41A, 443 (1966).
4. F. Hoogeveen, Nucl. Phys. B 341, 322 (1990).
5. D. Chang, W.-Y. Keung and J. Liu, Nucl. Phys. B 355, 295 (1991).
6. E. P. Shabalin, Sov. J. Nucl. Phys. 28, 75 (1978).
7. C. Jarlskog, Phys. Rev. Lett. 55, 1039 (1985); Z. Phys. C 29, 491 (1985); Dan-di Wu, Phys. Rev. D 33, 860 (1986).
8. I. Dunietz, Ann. Phys. 184, 350 (1988); I. Dunietz, O. W. Greenberg and Dan-di Wu, Phys. Rev. Lett. 55, 2935 (1985).
9. M. J. Booth, R. A. Briere and R. G. Sachs, Phys. Rev. D 41, 177 (1990).
10. E. P. Shabalin, Sov. J. Nucl. Phys. 32, 129 (1980).
11. D. V. Nanopoulos, A. Yildiz and P. H. Cox, Ann. Phys. 127, 126 (1980); N. G. Deshpande and G. Eilam, Phys. Rev. D 26, 2463 (1983); N. G. Deshpande and M. Nazerimonfared, Nucl. Phys. B 213, 390 (1983); S.-P. Chia, Phys. Lett. B 130, 315 (1983).
12. M. J. Booth, “A note on Weinberg Operators in the Standard Model”, preprint EFI-92-13-Rev.
13. W. J. Marciano and A. Queijeiro, Phys. Rev. D 33, 3449 (1986).
14. J. Donoghue, Phys. Rev. D 18, 1632 (1978).
15. I. B. Khriplovich and M. Pospelov, Sov. J. Nucl. Phys. 53, 638 (1991).