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Pitching stability analysis of half-rotating wing air vehicle

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Abstract. Half-Rotating Wing (HRW) is a new power wing which had been developed by our work team using rotating-type flapping instead of oscillating-type flapping. Half-Rotating Wing Air Vehicle (HRWAV) is similar as Bionic Flapping Wing Air Vehicle (BFWAV). It is necessary to guarantee pitching stability of HRWAV to maintain flight stability. The working principle of HRW was firstly introduced in this paper. The rule of motion indicated that the fuselage of HRWAV without empennage would overturn forward as it generated increased pitching movement. Therefore, the empennage was added on the tail of HRWAV to balance the additional moment generated by aerodynamic force during flight. The stability analysis further shows that empennage could weaken rapidly the pitching disturbance on HRWAV and a new balance of fuselage could be achieved in a short time. Case study using numerical analysis verified correctness and validity of research results mentioned above, which could provide theoretical guidance to design and control HRWAV.

1. Introduction

Flight stability must be carefully considered in the design of air vehicle to ensure safety and comfort of flight. With the development of various types of air vehicle, lots of researchers have carried out a series of studies on the stability of the aircraft. Young attempted to add tail antenna to increase the rolled stability[1]. Nakata proposed a suitable spanwise length to improve the flight stability[2]. Zhang investigated single-DOF pitching angular motion of an orbital reentry vehicle around static trim angle of attack[3].

Due to its excellent flight mobility and wide application fields, the research and development of Bionic Flapping Wing Air Vehicle (BFWAV) has become a new hot spot in recent years. The attitude of Microbat by California Institute of Technology was controlled through the flat tail and vertical tail[4]. Nano Hummingbird could achieve hovering, tumbling and other difficult movements without empennage[5]. However, there have been few studies focused on the flight stability of BFWAV. As an important factor of flight stability, pitching stability of BFWAV was also rarely mentioned in the existing literature.

Half-Rotating Mechanism(HRM) is a new type of bionic mechanism developed by our work team[6,7]. In order to meet the requirements of the bionic flying, the simplified HRM was further designed by adding a local constraint[8,9] to reduce weight. The weis-fogh effect of Half-Rotating Wing(HRW) based on HRM has been also discovered, which was helpful to generate larger lift for HRW[10]. Moreover, based on analyzing the lift of the flapping wing flight, Wang and Dong proposed an estimation method of lift of HRW[11]. Half-Rotating Wing Air Vehicle (HRWAV) is similar as
BFWAV, which used rotating-type flapping instead of oscillating-type flapping. For HRWA V, the flight stability was obviously an unavoidable problem. Based on introducing the working principle of HRW, the study on HRWA V in this paper was to focus on how to restrain the disturbance and improve the pitching stability, which was helpful to design and control HRWA V.

2. Working principle of HRW

The core component of HRWA V is HRM.

HRM as shown in figure 1 is a new kind of bionic motion mechanism as mentioned before. AB and OC respectively represent the bar of HRM and crank, while point C is the position of spindle. The bar of HRM would rotate the angle of $\frac{\varphi}{2}$ with angular velocity of $\frac{\omega}{2}$ counterclockwise when the crank rotated the angle of $\varphi$ with angular velocity of $\omega$ counterclockwise from the initial position.

A wing panel was fixed on the bar of HRM, which was called HRW in figure 2. In the period of rotation, different acting forces acted by fluid on the wing of HRM resulted from the changing velocities of the bar at various positions. At this moment, the acting force on the wing of HRM could be simplified to the point $M$. $F_R$ was the principal vector which was upward and vertical to the wing surface, while $M_C$ represented the principal moment which was rotating around the spindle $C$.

\[
F_R = \frac{1}{6} C_D \rho h R \omega^2 \sin \frac{\varphi}{2} (4R^2 \sin \varphi \frac{\varphi}{2} + 3a^2)
\]  

where $\rho$ was air density, $C_D$ was the resistance coefficient of the wing panel affected by air. $F_R^\prime$ was the component force of $F_R$ in perpendicular direction.

Due to the varied direction and size of principal vector with the rotation of the bar, HRW could be symmetrically arranged in the right position. As shown in figure 3, the crank on the right rotated clockwise as well as the crank on the left rotated counterclockwise with the same angular velocity, so that the component force of both sides counteracted in horizontal direction and doubled in perpendicular direction. $F_z^\prime$ was twice as $F_R^\prime$.

3. Movement rule of HRWA V without empennage

3.1. HRWA V without empennage and stress simplifying

It is necessary to keep the HRWA V staying a pitching angle for generating component forces both in perpendicular and horizontal direction in order to rise and go ahead.
HRWA V is composed of empennage and fuselage that contains HRW. Figure 4 denoted the status of HRWA V without empennage in the air. Point $O'$ was the location of the center of mass of HRWA V, point $D_0$ and point $H$ respectively represented the midpoint of connecting line of the geometrical center on the left and right wing and the geometrical center of the head of fuselage. If the air resistance resulted from airflow speed being neglected, the force acting on HRWA V without empennage essentially contained gravity $P_f$ and the principal vector of aerodynamic force $F_z'$. It could be seen clearly that, the fuselage of HRWA V would pitch resulted from the torque on point $O'$ by $F_z'$. 

![Figure 4](image)

**Figure 4.** The body of HRWA V without empennage and its force.

$F_z'$ was moved to point $D$ along the longitudinal axis of fuselage and became $F_z$, which generated $M_y$ simultaneously. $F_z'$ and $M_y$ were deduced as equation (2) and equation (3):

$$F_z = \frac{1}{3} C_\alpha \rho h R \omega^2 \sin^2 \phi \left( \frac{4}{2} R^2 \sin^2 \phi + 3a^2 \right)$$  \hspace{1cm} (2)$$

$$M_y = \frac{1}{2} F_z h$$  \hspace{1cm} (3)

Equation (2) can be written as equation (4):

$$F_z = \frac{1}{6} C_\alpha \rho h R \omega^2 [R^2 \cos 2\phi - (4R^2 + 3a^2) \cos \phi + 3(R^2 + a^2)]$$  \hspace{1cm} (4)

3.2. Pitching motion analysis of HRWA V without empennage

The force of HRWA V without empennage in the longitudinal symmetry plane was shown in figure 5.

![Figure 5](image)

**Figure 5.** Force of HRWA V without empennage in the longitudinal symmetry plane.

Based on the analysis of rigid body motion, the differential equations of motion were expressed as equation (5), (6) and (7):

$$m_f \ddot{x} = F_z \sin \alpha$$  \hspace{1cm} (5)$$

$$m_f \ddot{z} = F_z \cos \alpha - m_f g$$  \hspace{1cm} (6)$$

$$I, \ddot{\alpha} = M_y - F_z (l_0 - l_0)$$  \hspace{1cm} (7)$$

where $m_f$ represented the mass of fuselage, $I$ was the moment of inertia of fuselage and $\ddot{\alpha}$ was the acceleration of pitching angle.

$\ddot{\alpha}$ was written into the function of $\phi$ and $M_y$ in equation (7) was substituted by equation (3), then equation (7) could be got as equation (8):
\[ I, \alpha^2 \alpha^* = F_\alpha \left[ \frac{h}{2} - (l_{\alpha} - l_{\rho}) \right] \]  

Equation (4) was put into equation (7) and \( \alpha \) could be worked out in equation (9):

\[ \alpha = -\frac{CR^2}{4I_y} \cos 2\phi + \frac{C}{I_y} (4R^2 + 3a^2) \cos \phi + \frac{3C(R^2 + a^2)}{2I_y} \phi^2 + D_1 \phi + D_2 \]  

Equation (9) was a generalized expression about pitching angle \( \alpha \), which was calculated as equation (10):

\[ \alpha = -\frac{CR^2}{4I_y} \cos 2\phi + \frac{C}{I_y} (4R^2 + 3a^2) \cos \phi + \frac{3C(R^2 + a^2)}{2I_y} \phi^2 - \frac{3C}{4I_y} (5R^2 + 4a^2) \]  

The third item on the right of equation (9) was changed when \( \phi \) was growing over time, which caused the increase of pitching angle \( \alpha \) that make HRWAV without empennage turn over from the initial static state. Hence, it was the prerequisite that was adding other devices on HRWAV for restraining enlargement of pitching angle \( \alpha \).

4. Analytical solution of pitching stability of HRWAV with empennage

It was considerable to add torque whose direction was on the opposite of \( M_y \) for avoiding the enlargement of pitching angle of HRWAV without empennage with wings rotating. The torque increased automatically with the change of pitching angle of fuselage would be an optimal plan.

4.1. Effect of empennage on pitching stability

A rectangle thin plate was set as the simplified model of empennage, which was shown in figure 6. The figure on the left was the space stereogram. It illustrated the tail surface which was acted by the aerodynamic resistance force because of the increasing pitching angle of fuselage when wings of HRWAV with empennage rotate.

\[ dF_0 = \frac{1}{2} C_{\rho \theta}(V_0 \sin \alpha)^2 \, dA \]  

It could be seen in figure 7 that \( \dot{\alpha} \) and \( \ddot{\alpha} \) were resulted in when HRWAV got instantaneous
disturbance in the longitudinal symmetry plane (it was assumed that pitching angle and airflow speed were constant). \( P = (m_f + m_e)\rho g \) was the gravity of HRWAV with empennage, \( M_F \) was the moment of resistance of empennage, \( l_f \) was the tail arm and \( c \) was the length of empennage, \( V_m = V_0 \sin \alpha + l_f \dot{\alpha} \) was the component velocity acted vertically on the surface of empennage, thus the pressure element of empennage by air could be got as equation (14):

\[
dF = \frac{1}{2} C_D \rho (V_0 \sin \alpha + l_f \dot{\alpha})^2 b dx
\]

The item with \( \alpha^2 \) could be rounded off when disturbance was small, thus \( dF \) could be linearly simplified as equation (15):

\[
dF_l = \frac{1}{2} C_D \rho \left[ (V_0 \sin \alpha)^2 + 2V_0 \sin \alpha l_f \dot{\alpha} \right] b dx
\]

The moment of resistance of empennage by instantaneous disturbance was in equation (16):

\[
M_{F_l} = M_{F_0} + C_D \rho V_0 \sin \alpha \dot{\alpha} \int_0^c l_f^2 dx
\]

4.3. Stability analysis of HRWAV with empennage when disturbed

\( \ddot{\alpha} = 0 \), when HRWAV with empennage was in equilibrium state, then equation (11) could be calculated as equation (17):

\[
M_{F_0} = F_z \left[ \frac{h}{2} - (l_0 - l_D) \right]
\]

\( \ddot{\alpha} \neq 0 \), when HRWAV with empennage was disturbed, then equation (11) could be expressed as equation (18):

\[
I_\alpha \ddot{\alpha} = F_z \left[ \frac{h}{2} - (l_0 - l_D) \right] - M_{F_l}
\]

Therefore, the perturbation equation of pitching angle for HRWAV with empennage was in equation (19):

\[
I_\alpha \ddot{\alpha} + C_D \rho V_0 \sin \alpha \dot{\alpha} \int_0^c l_f^2 dx = 0
\]

In reference to section 3.2, the pitching angle and airflow speed were constant as soon as HRWAV got instantaneous disturbance. Thus equation (19) could be expressed as equation (20):

\[
I_\alpha \ddot{\alpha} + C_1 \dot{\alpha} = 0
\]

Under the variable substitution from equation (10) to equation (11), it could be got as equation (21):

\[
\dot{\alpha} = D_\alpha \exp \left( \frac{-C_1}{I_\alpha \omega} \varphi \right)
\]

Equation (21) could be solved when \( C_1 \) was substituted in, which could be equation (22):

\[
\dot{\alpha} = \dot{\alpha}_{\varphi=0} \exp \left( -\frac{C_D \rho V_0 \sin \alpha \int_0^c l_f^2 dx}{I_\alpha \omega} \varphi \right)
\]

From equation (22), \( \ddot{\alpha} \) was declined as \( \varphi \) grew, which meant that the variation of pitching angle
of HRWAV with empennage would go down and resume to equilibrium state in the end. Therefore, the pitching stability of HRWAV with empennage after being disturbed was reliable.

The equation $\varphi = \alpha t$ was substituted into equation (21), which could be solved as $\alpha = -\frac{D_d I_y}{C_1} \exp\left(-\frac{C_1}{I_y} t\right) + D_4$. The initial condition was involved, and that was equation (23):

$$\alpha = \frac{I_y}{C_1} \alpha_{t=0} \left[ 1 - \exp\left(-\frac{C_1}{I_y} t\right) \right] + \alpha_{t=0}$$

Eventually, it could be got as equation (24):

$$\alpha = \frac{I_y}{C_1} \alpha_{t=0} \left[ 1 - \exp\left(-\frac{C_D \rho V_0}{I_y} \int_{x=0}^{x=x} \sin \alpha \frac{dx}{I_y} \right) \right] + \alpha_{t=0}$$

It was obvious that $\alpha$ was close to $\alpha_{t=0} + \frac{I_y}{C_1} \alpha_{t=0}$, which meant that HRWAV with empennage would get a new equilibrium position compared with the initial equilibrium state.

Stability could be arrived when got instantaneous disturbance by adding empennage on HRWAV. However, theoretical derivation is mainly used for qualitative analysis, which adopted the assumption that airflow speed and pitching angle of aircraft were instantaneously invariable when being disturbed. In actual flight, the influence of change of pitching angle and airflow on the pitching stability could not be ignored. Therefore, the next analysis would be more detailed by numerical method.

5. Numerical analysis of pitching stability of HRWAV with empennage

Considering that the variation of pitching angle and airflow speed when disturbed had effect on the pitching stability of HRWAV, the influence could be calculated by combining equation (19) and equation (5) on the basis of S-Function in MATLAB/SIMULINK, among which $m_f$ was replaced by $m_i(m_i=m_f+m_e)$. Figure 8 showed the model established by S-Function.

**Figure 8.** Simulation model of HRWAV with empennage when disturbed.

There were two initial conditions in function sfuncbzy set as:

1. The initial pitching angle of HRWAV with empennage in equilibrium position was $\alpha = 10$, and then pitching angle was increased by $\alpha = 0.0175 \text{ rad/s}$ acted by instantaneous disturbance. Figure 9 denoted the change law of the pitching angle and pitching angular velocity.

2. The pitching angle was decreased by $\alpha = 0.0175 \text{ rad/s}$ acted by instantaneous disturbance while other initial conditions were constant. Simulation was shown in figure 10.

**Figure 9.** Curves of pitching angle (a) and pitching angular velocity (b) in No.1 initial condition simulation.
Figure 10. Curves of pitching angle (a) and pitching angular velocity (b) in No.2 initial condition simulation.

It was apparent that the pitching angle and pitching angular velocity would both drop to zero as the instantaneous disturbance came to HRWAV with empennage from equilibrium position in a short time. The simulation results were basically consistent with the analysis of section 3.3 in the condition that the instantaneous disturbance was not large. Consequently, the pitching stability of HRWAV with empennage after the instantaneous disturbance was proved credible by numerical analysis.

6. Conclusion
HRWAV is a new flapping-imitating wing air vehicle. The pitching stability is an important part of flight stability for HRWAV. The research results in this paper are listed as follows.

1) The pitching angle of HRWAV without empennage would increase with the rotation of wings, by which HRWAV would eventually lose its stability, even overturn and fall. Therefore, an empennage on HRWAV is essential to guarantee pitching stability.

2) When the vertical disturbance acted on the HRWAV flying in balance, the resistance moment of empennage would rapidly weaken the pitching disturbance. HRWAV could take a new equilbrious pitching posture in a short time.

References
[1] Young L A, Briggs G A, Derby M R and Aiken E W 2002 Use of Vertical Lift Planetary Aerial Vehicles for the Exploration of Mars Engineering Studies into Vertical Lift Planetary Aerial Vehicles
[2] Nakata T, Liu H, Tanaka Y, Nishihashi N, Wang X and Sato A 2011 Aerodynamics of A Bio-Inspired Flexible Flapping-Wing Micro Air Vehicle Bioinspiration & biomimetics 6(4) 045002
[3] Zhang H X, Yuan X X, Ye Y D and Xie Y F 2002 Research on the Dynamic Stability of an Orbital Reentry Vehicle in Pitching Acta Aerodynamica Sinica 20(3) pp 247-59 (in Chinese)
[4] Keennon M and Grasmeyer J 2003 Development of Two MAVs and Vision of the Future of MAV Design AIAA International Air and Space Symposium and Exposition: The Next 100 Years p 2901
[5] Keennon M, Klingebiel K and Won H 2012 Development of the Nano Hummingbird: A Taillless Flapping Wing Micro Air Vehicle 50th AIAA Aerospace Sciences Meeting including the New Horizons Forum and Aerospace Exposition p 588
[6] Qiu Z Z 2011 Half-rotating Mechanism: Structure, Characteristics and Application (Hefei: Press of University of Science and Technology of China) (in Chinese)
[7] Qiu H and Wang X Y 2011 Half-rotating Mechanism: A Biomimetic Mechanism of Animal Motion and Its Basic Motion Characteristics, Mechanical Science and Technology for Aerospace Engineering 30(2) pp 600-4 (in Chinese)
[8] Wang X Y, Zhang Y Q, Wang H X, Dong Y P, Qiu Z Z and Chen F Q 2016 Research of Local Constraint for Simplified Half-rotating Mechanism Applied Mechanics and Materials 826 pp 35-9
[9] Wang H X, Wang X Y, Qiu H, Chen F Q, Qiu Z Z and Shan J H 2016 Effect of Local Constraint on Driving Torque of Driving Mechanism for Half-rotating Wing *International Conference on Design Engineering and Science (ICDES 2016)* DOI: 10.1051/matecconf/20165202005.

[10] Wang X Y, Shao C Y, Chen F Q and Qiu Z Z 2014 Numerical Analysis of Weis-Fogh Effect for Half-Rotating Wing Based on Fluent *Proceedings of the 5th International Conference on Mechanical Engineering and Mechanics* pp 614-7

[11] Wang X Y, Dong Y P, Qiu Z Z, Zhang Y Q and Shan J H 2016 Lift Estimation of Half-Rotating Wing in Hovering Flight *2nd International Conference on Mechanical Engineering and Automation Science (ICMEAS 2016)* DOI:10.1088/1757-899X/157/1/012001

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