Principles of constructing multi-mode control based on S-shaped sigma functions

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Abstract. For a wide class of multi-mode control objects operating under uncertainty, the principles of synthesis of control actions based on S-shaped sigma functions approximating combinations of single-valued typical nonlinearities have been developed. The use of these functions made it possible to ensure, with a decrease in deviations of the controlled variables from the specified influences, a smooth change of control levels without jumps, which does not require setting the switching times to the ranges of change of the implemented controls, as well as to exclude frequent maximum effects on the actuator and to limit their amplitude when changing the modes of operation of the system, which leads to an increase in performance in transient modes and a decrease in overshoot in steady-state modes. In addition, with practical implementation, a favorable operation of the executive mechanism is ensured. Based on the analysis of the obtained transient processes in systems with relay control and in systems with approximating control, the features of the application of the principles for multi-mode systems operating in transient and steady-state modes are shown.

Keywords: principle of multi-mode control, approximating function, combination of typical nonlinearities

1. Introduction

Based on the tasks solved in the control of underwater technological complexes, including the control of descent submersibles in the open sea, analysis and synthesis of robust control systems for tethered underwater objects, transport marine objects, technical objects with electric drives and technological units of periodic action, taking into account the essential features the functioning of the automatic control systems of these devices when changing modes, as well as the tightening of requirements for ensuring high-quality control in each mode with the achievement of high final accuracy, it seems expedient to distinguish two main modes in the operation of these and many other objects and systems: the first is the starting mode (transient mode), for example, the mode of descent-lifting of the object, the mode of transferring the object from one state to another by cooling or heating to the optimal temperature, and the second – the mode of stabilizing its polo operation or operating parameters (steady-state). Such a decomposition of the modes of operation of objects and systems presupposes the use of the principle of multi-mode control. This principle corresponds to the essence of a systematic approach to the construction of control systems [1]. With the increasing requirements for the quality of control systems, this principle is increasingly recognized and spread in engineering practice. One of the first principles of multi-mode control is based on switching controllers when changing modes, for example, from a relay controller to a linear one, depending on the current dynamic situation. In connection with the provision of diverse processes of the system functioning in one mode or another, requiring different goals, it is necessary to develop two systems: an object control system for a starting (unsteady) mode,
optimal in terms of speed, and a system for stabilizing an object in a given state, optimal in accuracy for a steady-state mode. Following this principle, with the inadmissibility of significant overruns of the controlled variable when changing modes, it is necessary to solve the problem of determining the optimal switching point of the controllers. This problem does not have a universal solution, and therefore switching is implemented in each specific case in its way. Taking this into account, it is relevant to develop an algorithm for controlling the operation of a multi-mode object and, first of all, the main modes: transient and stabilizing [2, 3, 4]. By now, various methods of controlling multi-mode objects are known: for example, changing the structure of a system or a control algorithm; selection of a quality indicator for only one mode and its assessment based on the results of system synthesis; neglecting the presence of uncertainty and, as a consequence, reducing the requirements for the system and optimal switching times, etc. Despite the large number of works in different areas, it seems expedient to base the synthesis of multi-mode control on the principle outlined in [5]: «... from a certain level complexity of the problem, a «good» controller will necessarily be nonlinear».

Taking into account what has been said in this work, for control objects operating in conditions of multimodality and uncertainty, the analysis of the developed principles of approximating control based on a combination of S-shaped sigma functions is carried out. The use of these functions does not require the determination of the moments of switching control algorithms in case of changing modes, excludes breaks during switching and frequent multiple impacts of the maximum amplitude on the actuators of the system.

2. Methodology and experimental research
It is known that in order to obtain the maximum response rate in the transient mode in the absence of restrictions on the variables, the control must be nonlinear, namely, relay control. However, with the help of relay control in a multi-mode ACS, it is impossible to eliminate the contradiction between speed and oscillation during the transition from the starting mode to the stabilization mode, i.e. with sequential change of modes. In confirmation of this, Figure 1 shows transient processes in an ACS with relay control of a dynamic object described by a second-order model with averaged interval values of parameters in different modes in the form of a transfer function:

\[
W_s(p) = \frac{\text{mid}[K_{ob}] e^{-p \cdot \text{mid}[\varepsilon]}}{\text{mid}[T_2] p^2 + \text{mid}[T_1] p + 1},
\]

where parameters matter:

- \(\text{mid}[K_{ob}] = (K_{ob\text{min}} + K_{ob\text{max}})/2 = (0.029 + 0.375)/2 = 0.202\% / \text{m}^3/\text{h};\)
- \(\text{mid}[\varepsilon] = (\tau_{\text{min}} + \tau_{\text{max}})/2 = (2.7 + 7.5)/2 = 5.1\text{ min};\)
- \(\text{mid}[T_1] = (T_{1\text{min}} + T_{1\text{max}})/2 = (14.8 + 10.19)/2 = 12.5\text{ min};\)
- \(\text{mid}[T_2] = (T_{2\text{min}} + T_{2\text{max}})/2 = (6.44 + 10.08)/2 = 8.26\text{ min.}\)

In the ACS, a nonlinear control algorithm of the form:

\[
U(t) = M_1 \cdot \text{sign}[e(t)] + M_2 \cdot \text{sign}[e(t) - a],
\]

where \(M_1, M_2 > 0 – \text{const},\) the value of the control action; \(\text{sign}[e(t)] – \text{sign function (sign function)}; e(t)\) is the deviation \((e(t) = g - x(t))\) between the controlled value \(x(t)\) and the given action \(g;\) \(a – \text{the magnitude of the control delay } M_2.\)
Figure 1. Graphs of the static characteristics of a nonlinear controller (a), an adjustable variable (curve 1) and a control action (curve 2) (b) in an ACS with a law control (1) with parameters \( M_1 = 1 \), \( M_2 = 5.8 \), \( a = 1 \).

Figure 1 shows that the transient process is characterized by a significant stick out — the maximum dynamic deviation (0.295 m.u.) from the steady-state value equal to 0.5 m.u. (m.u. — machine units). To reduce the stick out, the values of \( M_1 \) and \( M_2 \) are increased. As can be seen from Figure 2, the stick out decreased by 27.57\%.

However, the transient does not reach the set value of 5 m.u. Therefore, it is necessary to correct the value of \( M_1 \), providing an output to the specified stabilization mode. For this, we use the method of harmonic linearization of nonlinearities [6].

As it applies to this nonlinear element the parameter of \( M_1 \) is determined from the condition of origin in the system of possible self-excited oscillations with amplitude of \( A \):

\[
W_s(-\xi + j \omega) = - [q(A) + j q'(A)]^{-1},
\]

where \( W_s(-\xi + j \omega) \) is an equivalent transmission function of the linear part of the system; \( q(A) \) and \( q'(A) \) — are coefficients of the harmonic linearizing of nonlinear description (because these descriptions unambiguous, then \( q'(A) = 0 \)); \( \xi \) is a fading index; \( j \omega \) is imaginary frequency.

Setting a value \( A = a = 2 \), accepting \( \xi = 0 \) and using equalization (2) and formulas for the calculation of \( q(A) \) for our nonlinear element, we will define the parameter of \( M_1 \):

\[
M_1 = \frac{\pi \cdot a}{4} \Re \{W_s(-\xi + j \omega_0)\}^{-1},
\]

where the value of \( \omega_0 \) is found from the condition \( \Im \{W_s(-\xi + j \omega_0)\} = 0 \).

By formula (3) we get: \( M_1 = (3.14 \cdot 2) / (4 \cdot 0.157) = 10 \) m.u.
Transient process at $M_1 = 10$ m.u. shown in Figure 3. As can be seen from the graph of the transient process, there are steady self-oscillations, the amplitude of which does not exceed 5% of the set value of the controlled variable. The rise time does not exceed 20 minutes. However, the frequency of the influence of the controller on the object is very high, which is not always realizable (due to the inertia of the actuator and lag) and is not always acceptable in practice (adversely affects the operation of the actuator), in particular in air supply schemes using a frequency-controlled electric drive in compressor installations; at a water station; in ventilation and air conditioning systems, etc. Thus, the variation of the parameters of the relay control law (1) does not allow achieving optimal quality indicators of the transient process when the system operates in each of the modes.

![Graphs of the static characteristics](image)

**Figure 3.** Graphs of the static characteristics of a nonlinear controller (a), an adjustable variable (curve 1) and a control action (curve 2) (b) in an ACS with a control law (1) with parameters $M_1 = M_2 = 10$, $a = 2$.

Ensuring the ability to independently fulfillment of the requirements for each of these indicators in each of the modes, including obtaining the necessary noise immunity in the stabilization mode, remains an unsolved problem in a multi-mode system with relay control. Elimination of this contradiction requires correction of control during the operation of the system in such a way that in the transient mode the correction is determined from the condition of ensuring the speed requirement, and in the stabilization mode – based on the requirement to eliminate the oscillation of the process. Such a change in a correction during the transient process requires a nonlinear characteristic and multilevel control, the amplitude of which changes as a function of the control error $\varepsilon(t)$ during the transient process in the system. A similar characteristic can be obtained by approximating a combination of typical nonlinearities using S-shaped sigma functions. It can be noted that quite often the approximation is estimated by the accuracy of reproducing the real characteristics of the nonlinear link. However, the defining criteria for a successful approximation, as noted in [7], should be considered not a high accuracy of the reproduction of a real characteristic, but the following:

– suitability in the entire investigated area of variation of variables, including sections of starting and steady-state modes;
– continuity, differentiability and absence of discontinuities;
– a relatively simple form of the approximating function;
– correct reproduction of basic physical conformities to law.

When approximating the characteristics of nonlinear elements $f = f(x)$, of all types, polynomials $f = k_1 + k_2x + k_3x^2 + \ldots$, power functions $f = kx^a$, exponents $f = k_1e^{ax} + k_2$ or other simplest transcendental functions, the nature of which corresponds to the form of the approximated dependence [7]. However, the use of such approximating dependences requires restrictions on the output signals of the controller. The levels of these signals, as well as the ranges of the acting disturbances, are not known in advance.
Let us consider transient processes in a system with approximating control laws obtained by approximating the characteristics of nonlinear links and their combinations. The following laws of approximating control are investigated:

\[
U(\varepsilon) = \left[ \frac{M_1}{1 + \exp(-\lambda \cdot \varepsilon)} - \frac{M_1}{1 + \exp(\lambda \cdot \varepsilon)} \right] + \left[ \frac{M_2}{1 + \exp[-(\lambda \cdot (\varepsilon - a \cdot i)]} - \frac{M_2}{1 + \exp[\lambda \cdot (\varepsilon + a \cdot i)]} \right];
\]

\[
U(\varepsilon) = \sum_{i=1}^{2} \left[ \frac{M}{1 + \exp[-\lambda \cdot (\varepsilon - a \cdot i)]} - \frac{M}{1 + \exp[\lambda \cdot (\varepsilon + a \cdot i)]} \right];
\]

where \(M_1, M_2\) – the value of the control action in the dead zone (DZ) and beyond, respectively; \(M\) is the value of the control action; \(\varepsilon\) – control error; \(\lambda\) – tuning parameter; \(2a\) – the magnitude of DZ.

The control law (4) makes it possible to separately correct the coefficients \(M_1\) and \(M_2\), which affects the nature of the approximation in the dead zone and beyond. It can be seen from the graph in Figure 4 that at \(M_1 = 0\), the greatest overshoot of the variable takes place relative to the steady-state value compared to the transient process at \(M_1 = 0.5\) and 1.0 m.u.

![Figure 4](image)

**Figure 4.** Graphs of the static characteristics of a two-level approximating controller (a), an adjustable variable (curve 1, 2, 3) and a control action (curve 1, 2, 3) (b) in an ACS with a control law (5) with parameters \(M_1 = 0; 0.5; 1\) (curves 1, 2, 3, respectively) \(M_2 = 5, \lambda = 6, a = 1\).

Decreasing the stick out gives a multilevel control law (5). From the graph of the transient process in Figure 5, one can see a decrease in the runout by a factor of 2.8 relatives to the steady-state value while maintaining the rise time equal to 20 minutes. However, it has a residual deviation of 0.07 m.u., which is 4.76% of the set value.

![Figure 5](image)

**Figure 5.** Graphs of the static characteristics of a multilevel approximating controller (a), an adjustable variable (curve 1) and a control action (curve 2) (b) in an ACS with a control law (3) with parameters \(M = 10, \lambda = 5, a = 2\).
Improving the quality of the transient process in the stabilization mode is possible due to separate independent adjustment of the values of the coefficients $M_1$, $M_2$ and $\lambda$ of the control law (4). The coefficient $\lambda$ is used to set the slope of the characteristic of the approximating controller in the section from the dead zone to the saturation region. When choosing the value of the coefficient $\lambda = 50$ m.u. the approximating characteristic of the control algorithm is as close as possible to the relay one, which makes it possible to reduce the maximum effect of the controller by reducing the value of $M_1$ from 10 to 6 m.u. compared with the control law (5). Figure 6 shows the graphs of the static characteristics of the controller with approximating control (4) and transient processes at the output of the ACS with new values of the parameters of the control law (4). As a result, a decrease in the residual deviation to 1.3% was obtained. Moreover, the output action of the controller is smooth, which favourably affects the operation of the actuator.

**Figure 6.** Graphs of the static characteristics of a two-level approximating controller (a), an adjustable variable (curve 1) and a control action (curve 1, 2) (b) in an ACS with a control law (4) with parameters $M_1 = 5$, $M_2 = 10$, $\lambda = 50$, $a = 4$.

The analysis of the results obtained allows us to note that the efficiency of nonlinear correction based on the approximation of the relay characteristic in combination with other nonlinearities (for example, with a dead zone, limitation (saturation)), selected for a certain type and magnitude of external influences, can decrease with other unaccounted for influences on the system. Therefore, the wider the range of external influences and uncontrollable uncertainties, the more difficult it is to choose a nonlinear correction. The latter is aggravated by the fact that, unfortunately, there is no general methodology for selecting nonlinear correcting links, and in practice, during the synthesis; one has to resort to the modeling method, using the literature recommendations on the use of individual private methods of nonlinear correction.

The examples considered show that for the purposes of multimode control, it is advisable to use the approximation of nonlinearities in the form of an S-shaped sigma function, which satisfies almost all of the listed criteria for a successful approximation. In addition, this function has the property of amplifying small error signals better than large ones, and prevents saturation from large signals, since they correspond to the regions of the arguments where the character has a gentle slope.

### 3. Results

Analysis of the known approaches to the synthesis of ACS by multi-mode objects allows us to conclude that attempts to integrate a multitude of dissimilar controllers (algorithms) in the system do not allow efficient control. The solution to the problem of choosing the structure of a multi-mode control system, taking into account different criteria for all modes of operation of the object, is possible on the basis of the principle of approximating control. The use of approximating control can significantly improve the quality of control systems, providing the ability to work with several approximated nonlinearities in the control algorithm and make their informed choice taking into account the modes of operation of the system.

It should be noted that for one mode of operation of the system, some parameters of the approximating control law may be preferable, for another mode, other parameters.
4. Conclusion
Recently, the principle of multi-mode control should be considered one of the most active and promising areas of applied research in the field of control. Multimodal control turns out to be especially relevant when there is uncertainty in the description of technical and technological systems, which makes it difficult or even excludes the use of exact quantitative methods and approaches necessary to determine the optimal moments of control switching when changing modes.

In the field of control of technical and technological systems, multi-mode control allows you to get better results compared to the results that are based on the use of traditional analytical models and control algorithms. The range of application of multi-mode control covers areas such as control of underwater and aircraft, lifting and transport mechanisms, industrial robots, as well as non-stationary and non-linear technological units with program-logic algorithms of functioning, for which in practice it is not possible to take into account all internal and external factors, various emerging unforeseen situations and interferences that affect the process of functioning of the object. Using traditional methods of relay and PID control, it is impossible to ensure an acceptable quality of multi-mode control from the beginning of the object's operation mode (for example, start-up, intensive cooling or heating, movement at a given speed) until it stops at a given position or with given parameter values. The interconnection of individual modes of operation of the units leads to the complication of the object control algorithms and, ultimately, to the problem of synthesis of multi-mode control systems, which is effectively solved using the principle of approximating control considered in this work.

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