INFERRING THE PALEO-LONGITUDE DIRECTLY FROM THE PALEO-GEOMAGNETIC DATA

RONG QIANG WEI

ABSTRACT. Knowledge of the ancient geology and tectonics of the Earth owes much to paleo-magnetism. However, it is thought that paleo-magnetism has the incapability in providing paleo-longitude. To obtain this important location parameter many other indirect methods have been developed based on different assumptions. Here we present a direct method to derive the paleo-longitude from the usual paleo-magnetic measurements. This method takes into account the contributions to the Earth’s magnetic potential from additional dipoles with their axes in the equatorial plane, which were omitted by the traditional paleo-magnetism. Two explicit formulae for inferring the longitude are obtained. It is shown in synthetic experiments that longitudes can be perfectly derived through these formulae, from the measured components of magnetic field \(B_x, B_y, B_z\), or declination \(D\) and inclination \(I\). The errors are mainly from \(B_x, B_z\) or \(I\). Thus these two explicit formulae can be used in paleo-magnetism as an alternative approach to estimate the paleo-longitude. By this direct approach, we infer the synthetic paleo-longitudes for the reference points in the continents of North America, Eurasia, and India for the last 200 Ma and a reference paleo-longitude for Indo-Asia collision based on paleo-magnetic data. The synthetic trajectories of these reference points, which determined by synthetic paleo-longitudes and paleo-latitudes, show the movements of these three continents had their own respective properties. All of them included different both translation and rotation in four phases. The paleo-longitude for India-Asia collision was at \(132.6^{\pm 27.2}_{\pm 14.7}^\circ\)E.

Keywords: paleo-longitude; paleo-magnetism; paleo-magnetic data; plate tectonic reconstructions

1. Introduction

Obtaining quantitatively the paleo-position of continents is essential to the plate tectonic reconstructions. The paleo-position of the continents includes basically the paleo-longitude \(\lambda\) and the paleo-latitude \(\phi\) in the past. The \(\phi\) of the continents can be traditionally inferred from the paleo-magnetic data of inclination \(I\), i.e., \(\tan \phi = \frac{1}{2} \tan I\) (e.g., Turcotte and Schubert, 2014). This inferring is based on the assumption that the Earth’s paleo-magnetic filed at some geo-time (or geo-time-averaged field) can be approximated as a dipole field. Such a dipole field can be modeled through a Gaussian spherical harmonic expansion with \(n = 1\) and the Gauss coefficient \(g_1^0\) and \(h_1^1\) equal to zero (see details in the section 2). In this case, the paleo-magnetic field is axis symmetric and can not provide any information on the paleo-longitude.

To obtain the paleo-longitude of the continents, many methods other than paleo-magnetism were developed, and different reference frames were constructed. Some authors established
the hot spots absolute plate motion reference frame (e.g., Müllcr et al., 1993; O’Neill et al., 2005; Torsvik et al., 2008a; Doubrovine et al., 2012), for the motion of the lithospheric plates may be reflected by the track geometry of the hot spots (Morgan, 1971). A global hybrid reference frame, by correlating large igneous provinces and deep mantle heterogeneities at the core-mantle boundary, was established by Torsvik et al. (2008b). This reference frame assumed zero longitudinal motion of Africa before 100 Ma. By modeling of plume motions, these reference frames provided compatible reconstructions of plates with geologic and geophysical data (e.g., Doubrovine et al., 2012). Besides, van der Meer et al. (2010) linked the lower mantle slab remnants with the global orogenic belts reconstructions, and established a sinking slab remnant reference frame. This reference frame assumed a vertical slab sinking at an average rate of 12 ± 3mm/yr.

On the other hand, researchers attempted to estimate paleo-longitude from the data associated with paleo-magnetism, especially the data of polar wander path (PWP). For example, Mitchell et al. (2012) traced the moving trajectory of supercontinents centers in the deep geologic history and presented a true PWP derived reference frame, in which they assumed the stable geoid highs. Wu and Kravchinsky (2014) and Wu et al. (2015) presented a synthesized method to derive paleo-longitude by geometrically parametrizing apparent PWP. The method restores the absolute motion history for the reference geometries from the Euler parameters extracted from the apparent PWP. In this method, a paleo-colatitude correction to the reconstructions was introduced in order to keep the restored paleo-latitudes compatible with the paleo-magnetic prediction.

Although based on different assumptions, these work above are helpful to estimate the paleo-position of the continents, and to understand how the fragments of the outer shell of the Earth have moved relative to a reference system over geological timescales. Here we present another alternative method to infer the paleo-longitudes from the usual paleo-magnetic measurements. The related theory will be given in section 2. Because the inferring the paleo-longitude based on this theory is a nonlinear problem, we present a simple approach to solve it. Then we test this method with synthesized data, and discuss the error from different sources in the section 3. In the section 4 we provide synthetic paleo-longitude for the continents of North America, Eurasia, and India for the last 200 Ma based on the synthetic data; And estimate a reference paleo-longitude for Indo-Asia collision. Finally we give a short discussion in the section 5 and draw some conclusions in the section 6.

2. Theory and Methodology

2.1. Theory. We start from the well-known Gauss’s spherical expression for the potential of the geomagnetic field,

\[ V = a \sum_{n=1}^{\infty} \sum_{m=0}^{n} \left( \frac{a}{r} \right)^{n+1} P_n^m(\theta) (g_n^m \cos m\lambda + h_n^m \sin m\lambda) \]

(1)
where $a$ is the Earth’s radius, $P^m_n(\theta)$ Schmidt polynomials which are related to the associated Legendre polynomials, $g^m_n$ and $h^m_n$ Gauss coefficients of order $n$ and degree $m$, $r$ the distance from the Earth’s center, $\theta$ colatitude and $\lambda$ longitude.

Fig. 1: Longitudes derived from Equation (7) and (9) vs. the setting longitudes. (a) Longitudes are derived with Equation (7); (b), (c), (d), Longitudes are from Equation (9). $B_x, B_y, B_z, D, I$ are from International Geomagnetic Reference Field (IGRF) model.

Generally $V_1 = V|_{n=1}$ is taken as the potential of the centered dipole field, which is a first order approximation but the most important part of the geomagnetic field.

\begin{equation}
V_1 = \frac{a^3}{r^2}(g^1_1 \cos \theta + g^1_1 \sin \theta \cos \lambda + h^1_1 \sin \theta \sin \lambda)
\end{equation}

where the term $g^0_1$ is the strongest component of the field. It describes a magnetic dipole at the center of the Earth and aligned with the Earth’s rotation axis. The terms $g^1_1$ and $h^1_1$ are the next strongest parts. They describe contributions to the magnetic potential from additional dipoles with their axes in the equatorial plane.

One can obtain the components of the geomagnetic dipole field $B_x, B_y, B_z$ at the surface as the following,

\begin{equation}
\begin{aligned}
B_x &= -\frac{1}{r} \frac{\partial V_1}{\partial \theta} \bigg|_{r=a} \\ &\approx \frac{-1}{r} \frac{\partial V_1}{\partial \theta} \bigg|_{r=a} \\ &= \frac{g^0_1}{r} \sin \theta - (g^1_1 \cos \lambda + h^1_1 \sin \lambda) \cos \theta \\
B_y &= -\frac{1}{r \sin \theta} \frac{\partial V_1}{\partial \lambda} \bigg|_{r=a} \\ &\approx -\frac{1}{r \sin \theta} \frac{\partial V_1}{\partial \lambda} \bigg|_{r=a} \\ &= \frac{g^1_1}{r} \sin \lambda - h^1_1 \cos \lambda \\
B_z &= -\frac{1}{r} \frac{\partial V_1}{\partial r} \bigg|_{r=a} \\ &\approx -\frac{1}{r} \frac{\partial V_1}{\partial r} \bigg|_{r=a} \\ &= 2\left[ g^0_1 \cos \theta - (g^1_1 \cos \lambda + h^1_1 \sin \lambda) \sin \theta \right]
\end{aligned}
\end{equation}
From equation (3), the longitude $\lambda$ can be possibly inferred from the measured $B_x$, $B_y$, $B_z$ if $g_1^0$, $g_1^1$ and $h_1^1$ are known. However, $g_1^0$, $g_1^1$ and $h_1^1$ are generally difficult to be obtained. If let $g_1^1 = h_1^1 = 0$ in equation (3), we can immediately get $B_y = 0$, and

\[
\tan I = \frac{B_z}{\sqrt{B_x^2 + B_y^2}} = \frac{B_z}{B_x} = 2 \cot \theta
\]

where $I$ is the magnetic inclination. Equation (4) is the foundational equation of the usual paleo-magnetism. It can be seen that only $\theta$ (then the latitude) appears.

Fig. 2: Longitudes derived from Equation (9) vs. the setting longitudes when $B_x$ is add a perturbation of -5% (a), -10% (b), 5% (c), and 10% (d). $B_x, B_y, B_z$ at the surface and $D, I$ are from IGRF model on Oct. 1, 2005 and at the latitude of 40° N.

Clearly $g_1^1$ and $h_1^1$ can not be omitted in equation (3) to infer $\lambda$. $g_1^1$ and $h_1^1$ will be taken into account in the following approach we adopt. According to the central dipole model (e.g., Hurwitz, 1960; Alldredge and Hurwitz, 1964; Lanza and Meloni, 2006), Equation (3) can be rewritten as,

\[
\begin{align*}
B_x &= K_p [\cos \theta \sin \theta - \sin \theta \cos \theta \cos (\lambda - \lambda_p)] \\
B_y &= K_p \sin \theta \sin (\lambda - \lambda_p) \\
B_z &= 2K_p [\cos \theta \cos \theta + \sin \theta \sin \theta \cos (\lambda - \lambda_p)]
\end{align*}
\]
where \( K_p = M_p/a^3 \), and \( M_p \) is the magnetic moment for the central radial dipole at colatitude \( \theta_p \) and east longitude \( \lambda_p \). It is easy to demonstrate that 

\[
g_1^1 = K_p \sin \theta_p \cos \lambda_p, \quad h_1^1 = K_p \sin \theta_p \sin \lambda_p, \quad \text{and} \quad B_x^2 + B_y^2 + (B_z/2)^2 = K_p^2. \]

Therefore, equation (5) is equivalent to equation (3), and \( g_1^1 \) and \( h_1^1 \) are naturally included in the equation (5).

From equation (5), the longitude \( \lambda \), even \( \theta \), and \( K_p \), can be possibly inferred if \( B_x, B_y, B_z, \theta_p, \) and \( \lambda_p \) are known, because these quantities can be measured or estimated relatively easily.

2.2. Methodology. It is not easy to infer \( \lambda \), even \( \theta \), and \( K_p \) simultaneously from Equation (5), because it is nonlinear. However, if \( \theta \) can be estimated firstly in other ways, \( \lambda \) (even \( K_p \)) can be easily obtained with the known \( B_x, B_y, B_z, \theta_p, \) and \( \lambda_p \).

From Equation (5), one can get,

\[
\begin{align*}
\cos(\lambda - \lambda_p) &= \frac{\cos \theta_p (\sin \theta - B_x/0.5B_z \cos \theta)}{\sin \theta_p (\cos \theta + 0.5B_x \sin \theta)} \\
\sin(\lambda - \lambda_p) &= \frac{B_y}{K_p \sin \theta_p} \\
K_p &= \pm \sqrt{B_x^2 + B_y^2 + (B_z/2)^2} \frac{1}{2}
\end{align*}
\]

Further, one can infer \( \lambda \) from Equation (6) as the following,

\[
\lambda = \lambda_p + \tan^{-1}\left\{ \pm \frac{B_y \sec \theta_p}{B_x^2 + B_y^2 + (B_z/2)^2} \frac{1}{2} \cdot \frac{\cos \theta \cos \theta_p + B_y/0.5B_z \sin \theta}{\sin \theta \cos \theta_p - B_x/0.5B_z \cos \theta} \right\}
\]

One can also get \( \lambda \) with the known declination \( (D) \) and inclination\( (I) \), \( \theta_p \), and \( \lambda_p \), because the following equations hold (e.g., Kono and Tanaka, 1995).

\[
\begin{align*}
\frac{B_y}{B_x} &= \tan D \\
\frac{B_y}{B_z} &= 2 \cot I \cos D \\
\tan I &= 2 \cot \theta \\
\sin \theta &= \pm \sqrt{\frac{B_x^2 + B_y^2}{B_x^2 + B_y^2 + (B_z/2)^2}}
\end{align*}
\]

From Equation (7) and (8), one can obtain,

\[
\lambda = \lambda_p + \tan^{-1}\left( \pm \frac{\sin D \sec \theta_p \cos \theta + 2 \cos D \cot I \sin \theta}{\sin \theta - 2 \cos D \cot I \cos \theta} \right)
\]

Explicit equation (7) and (9) can be used to infer the \( \lambda \) when \( \theta_p \), and \( \lambda_p \) are known and \( B_x, B_y, B_z \) (or \( D, I \)) are measured.
2.3. Case of the Paleomagnetism. In paleo-magnetic study, if the remanent magnetization \( J_r \) of the rocks is assumed to be proportional to \( B \) as the following (equation (10)), we can also infer the paleo-longitude \( \lambda \) with the similar procedure above, like equation (7) and (9).

\[
\begin{align*}
J_{rx} & \approx k \frac{B_x}{\mu_0} = k' B_x \\
J_{ry} & \approx k \frac{B_y}{\mu_0} = k' B_y \\
J_{rz} & \approx k \frac{B_z}{\mu_0} = k' B_z
\end{align*}
\]

where \( \mu_0 \) is permeability constant \((= 4\pi \times 10^{-7} \text{NA}^{-2})\), \( k \) the proportionality constant.

In this case, if \( \theta \) is firstly estimated from equation (4), and the site longitude \( \lambda \), site latitude \( \phi \), magnetic colatitude \( \theta \), and \( D \) are known, \( \theta_p \) \((= \frac{\pi}{2} - \phi_p)\) and \( \lambda_p \) can be estimated from the well-known formulae as follows,

\[
\begin{align*}
\sin \phi_p &= \sin \phi \cos \theta + \cos \phi \sin \theta \cos D \\
\sin(\lambda_p - \lambda) &= \frac{\sin \theta \sin D}{\cos \phi_p} \quad \text{if } \cos \theta > \sin \phi \sin \phi_p \\
\sin(\pi - \lambda_p + \lambda) &= \frac{\sin \theta \sin D}{\cos \phi_p} \quad \text{if } \cos \theta < \sin \phi \sin \phi_p
\end{align*}
\]

Remark. If \( \phi_p \) derived from Equation (11) is negative, it means that the geomagnetic north pole in the northern hemisphere corresponds to a magnetic south pole for the dipole at the Earths center. Then the fraction in the bracket in Equation (7) and (9) is negative, otherwise, positive.

It should be pointed out that the paleo-longitude \( \lambda \) estimated in this way might have errors because the \( \theta, \theta_p \) and \( \lambda_p \) from equation (11) are rough. We will discuss this with synthetic data in section 3.

3. Synthetic experiments

3.1. Inferring longitude from synthetic data. We test Equation (7) and (9) with a series of synthetic experiments at random co-latitudes \( \theta \) \( (= \frac{\pi}{2} - \phi) \). The synthetic data, namely, the components of the Earth’s magnetic field at the surface, \( B_x, B_y, B_z \) for Equation (7), and \( D, I \) for Equation (9), are calculated from International Geomagnetic Reference Field (IGRF) model (International Association of Geomagnetism and Aeronomy, Working Group V-MOD, 2010) when \( n = 1 \), respectively. The time is also given randomly. The setting longitudes are from 0° to 360° at an interval of 5°. \( \lambda_p \) and \( \theta_p \) are calculated from Gaussian coefficients \( g_1^0, g_1^1, h_1^1 \) with the following equation (12) (eg., Lanza and Meloni, 2006),

\[
\begin{align*}
\lambda_p &= \tan^{-1} \left( \frac{h_1^1}{g_1^1} \right) \\
\theta_p &= \cot^{-1} \left( \frac{g_1^0}{\sqrt{(g_1^0)^2 + (h_1^1)^2}} \right)
\end{align*}
\]
Fig. 3: The errors of longitude caused by a perturbation of $B_x$ when $\theta$, $\lambda_p$ and $\theta_p$ can be obtained or given by other methods accurately. $B_x, B_y, B_z$ at the surface are from IGRF model on Oct. 1, 2005 and at the latitude of 40° N.

Fig. 4: The errors of longitude caused by a perturbation of $B_x$ when $\theta$, $\lambda_p$ and $\theta_p$ can be derived from $B_x, B_y, B_z$ (or $D, I$). $B_x, B_y, B_z$ at the surface are from IGRF model on Oct. 1, 2005 and at the latitude of 40° N.
Figure 1 shows the comparison of longitudes derived from Equation (7) and (9) with those setting longitudes for three examples (To see clearly we plot only the longitudes derived at an interval of 15°). It can be seen that the setting longitudes are derived perfectly from both two equations. The numerical results from Equation (7) are equivalent to those from (9). It should be pointed out that \( \lambda_p \) and \( \theta_p \) can also be derived from equation (11); The results from equation (12) are the same to those from equation (11).

3.2. Error analysis. The quantities, \( B_x, B_y, B_z, D, I, \theta, \lambda_p \) and \( \theta_p \) in Equation (7) and (9), must have errors when they are actually measured or calculated, especially in the paleo-magnetic data. In this subsection we check the errors from these sources, by adding some positive or negative perturbations. Here \( B_x, B_y, B_z \) at the surface are from IGRF model on Oct. 1, 2005 and at the latitude of 40° N. For simplicity, the setting longitudes are from 30° E to 330° E, out of which may have nominal great errors. For example, the setting longitude is 0°, but the derived is 359°. The true absolute error is 1° but the nominal one is 358°. Furthermore, we adopt the algorithm of the root-mean-square error (RMSE) to calculate a nominal RMSE for errors from these perturbations. This nominal RMSE will be utilized to represent the "mean" error between the setting and the derived longitudes.

Two cases are considered here. The first is that \( \theta, \lambda_p \) and \( \theta_p \) can be obtained or given by other methods accurately, rather than derived from \( B_x, B_y, B_z \) (or \( D, I \)). We add different perturbations to \( B_x, B_y, B_z, D, I, \lambda_p \) and \( \theta_p \) as shown in Table 1. The corresponding maximum and minimum error, absolute error, and nominal RMSE, respectively, are also shown in Table 1.

We take \( B_x \) as an example to illustrate. Figure 2 shows the comparison of the longitudes derived from Equation (7) or (9) with the setting longitudes when \( B_x \) is added a perturbation of -5%, -10%, 5% and 10%, respectively. It can be found that the setting longitudes are derived but with small errors at the perturbation of ±5%, and larger errors at the perturbation of ±10%. Figure 3 shows the errors vary with the setting longitudes. The errors in this case are different with different setting longitudes and fluctuate clearly: For the perturbation of -5%, the maximum negative errors are at two endpoint longitudes; Between them, the error curve is like a parabola going downwards, with a maximum error at about 188°E and the minimum errors at about 108°E and 288°E, respectively. From Table 1 it can be seen that the maximum error is ±8.1°, the minimum error 0.2°, and the nominal RMSE 5.5°. The error curve for the perturbation of -10% is similar to that of -5%, while those error curves for the positive perturbation have the properties of mirror symmetry with those for the negative perturbations.

From Table 1 it is found that the errors caused by the perturbation of \( B_x \) is the largest, then \( B_z \), then \( \lambda_p \), then \( I \), then \( D \), then \( B_y \), and the last is the \( \theta_p \). The simplest errors, which are constant, are from \( \lambda_p \). This can be inferred from equation (7) and (9).

In the end, we add simultaneously some equal perturbations to \( B_x, B_y, B_z, D, I, \lambda_p \) and \( \theta_p \). It is found that the maximum negative error is -1.5°, maximum error 8.7°, the minimum absolute error 0.0°, and the nominal RMSE 4.9° for the perturbation of -5%. More details can be seen in Table 1.
Table 1: Errors caused by $B_x$, $B_y$, $B_z$, etc., when $\theta$, $\lambda_p$ and $\theta_p$ can be obtained or given by other methods rather than derived from $B_x$, $B_y$, $B_z$ (or $D$, $I$).

| Variable (Perturbation) | $e_{\text{min}}$ ($^\circ E$) | $e_{\text{max}}$ ($^\circ E$) | $|e|_{\text{min}}$ ($^\circ E$) | $|e|_{\text{max}}$ ($^\circ E$) | nRMSE ($^\circ E$) |
|-------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|------------------|
| $B_x$ (-5%)             | -8.1                          | 8.1                           | 0.2                           | 8.1                           | 5.5              |
| $B_x$ (-10%)            | -16.7                         | 16.7                          | 0.4                           | 16.7                          | 11.4             |
| $B_y$ (5%)              | -7.8                          | 7.6                           | 0.3                           | 7.8                           | 5.2              |
| $B_x$ (10%)             | -15.4                         | 14.6                          | 0.5                           | 15.4                          | 10.2             |
| $B_y$ (-5%)             | -1.4                          | 1.4                           | 0.1                           | 1.4                           | 1.0              |
| $B_y$ (-30%)            | -10.0                         | 10.0                          | 0.6                           | 10.0                          | 7.1              |
| $B_y$ (5%)              | -1.4                          | 1.4                           | 0.1                           | 1.4                           | 1.0              |
| $B_y$ (30%)             | -10.0                         | 10.0                          | 0.4                           | 7.4                           | 5.3              |
| $B_y$ (-5%)             | -8.3                          | 7.1                           | 0.2                           | 8.3                           | 5.3              |
| $B_y$ (-10%)            | -17.2                         | 15.6                          | 0.3                           | 17.2                          | 10.9             |
| $B_z$ (5%)              | -7.5                          | 7.8                           | 0.2                           | 7.8                           | 5.1              |
| $B_z$ (10%)             | -15.1                         | 15.3                          | 0.5                           | 15.3                          | 10.2             |
| $D$ (-5%)               | -1.6                          | 1.6                           | 0.1                           | 1.6                           | 1.1              |
| $D$ (-20%)              | -6.9                          | 6.9                           | 0.3                           | 6.9                           | 4.6              |
| $D$ (5%)                | -1.5                          | 1.5                           | 0.1                           | 1.5                           | 1.0              |
| $D$ (20%)               | -5.7                          | 5.7                           | 0.3                           | 5.7                           | 3.8              |
| $I$ (-5%)               | -6.2                          | 6.2                           | 0.2                           | 6.2                           | 4.2              |
| $I$ (-10%)              | -10.2                         | 10.2                          | 0.4                           | 10.2                          | 7.4              |
| $I$ (5%)                | -6.5                          | 6.6                           | 0.2                           | 6.6                           | 3.4              |
| $I$ (10%)               | -12.4                         | 12.6                          | 0.2                           | 12.6                          | 6.9              |
| $\lambda_p$ (-5%)       | 3.6                           | 3.6                           | 3.6                           | 3.6                           | 3.6              |
| $\lambda_p$ (-10%)      | 7.2                           | 7.2                           | 7.2                           | 7.2                           | 7.2              |
| $\lambda_p$ (5%)        | -3.6                          | -3.6                          | 3.6                           | 3.6                           | 3.6              |
| $\lambda_p$ (10%)       | -7.2                          | -7.2                          | 7.2                           | 7.2                           | 7.2              |
| $\theta_p$ (-5%)        | -0.4                          | 0.4                           | 0                             | 0.4                           | 0.3              |
| $\theta_p$ (-20%)       | -2.6                          | 2.6                           | 0.2                           | 2.6                           | 1.9              |
| $\theta_p$ (5%)         | -0.3                          | 0.3                           | 0                             | 0.3                           | 0.2              |
| $\theta_p$ (12%)        | -0.5                          | 0.5                           | 0                             | 0.5                           | 0.3              |
| C (-5%)                 | -1.5                          | 8.7                           | 0                             | 8.7                           | 4.9              |
| C (-10%)                | -0.9                          | 15.3                          | 0.1                           | 15.3                          | 9.0              |
| C (5%)                  | -9.2                          | 2.1                           | 0.2                           | 9.2                           | 4.0              |
| C (10%)                 | -17.8                         | 3.6                           | 0.3                           | 17.8                          | 8.1              |

Note: $e_{\text{min}}$: minimum error; $e_{\text{max}}$: maximum error; $|e|_{\text{min}}$: minimum absolute error; $|e|_{\text{max}}$: maximum absolute error; nRMSE: nominal RMSE. The part in the bracket is the perturbation.

C: The perturbation is added in all of the $B_x$, $B_y$, $B_z$, $D$, $I$, $\lambda_p$, $\theta_p$. $B_x$, $B_y$, $B_z$ at the surface, and $D$, $I$ are from IGRF model on Oct. 1, 2005 and at the latitude of 40° N.

The second case, on the other hand, is that $\theta$, $\lambda_p$ and $\theta_p$ can be derived from $B_x$, $B_y$, $B_z$ (or $D$, $I$), eg., through equation (11). This may occur in the paleo-magnetism. Similar to the first case, we add different perturbations to each quantity above as shown in Table...
The corresponding maximum and minimum error, absolute error, and nominal RMSE, respectively, are also shown in Table 2.

We still take $B_x$ as an example to illustrate. The error curves are shown in Figure 4. It can be seen that Figure 4 is similar to Figure 3 but the maximum error, the minimum error, and the nominal RMSE are less. From Table 2 it is known that these errors are about $-5.5^\circ$, $0.2^\circ$, and $3.7^\circ$ for the perturbation of -5%, respectively.

From Table 2 it is also found that the errors caused by $I$ is the largest, whose nRMSE is $13.9^\circ$ even at the perturbation of -5%; then $B_z$, then $B_x$, and the last is $D$. For $D$, the nRMSE is less than $2.0^\circ$ even at the perturbation of ±50%. Therefore reasonable longitude inversion demands more accurate $I$.

Furthermore, in the calculation in the second case, it is found that when we add simultaneously some equal perturbations to $B_x$, $B_y$ and $B_z$, the corresponding errors are almost the same to those without adding perturbations. In fact, this is a clear result because such a case is equivalent to equation (10). Therefore the longitudes derived with equation (7) should have smaller errors than those from (9), if the conditions above hold.

4. REFERENCE PALEO-LONGITUDE OF THREE CONTINENTS AND INDO-ASIA COLLISION

In this section, we give two simple applications of the method in the section 2 to the paleo-magnetic data. We firstly estimate the synthetic paleo-longitudes of three continents for the last 200 Ma based on the synthetic data (Schettino and Scotese, 2005). And then we infer the paleo-longitude Indo-Asia collision based on the paleo-magnetic data (Dupont-Nivet et al., 2010).

4.1. Synthetic paleo-longitude of three continents. Schettino and Scotese (2005) presented a new technique to analyse paleo-magnetic data and produced smoothed curves of paleo-latitude, declination and paleo-pole for seven major continents. These curves fit the available paleo-magnetic data. We use directly the curves to estimate the synthetic paleo-longitudes for the continents of North America, Eurasia and India. That is, the regression value of paleo-latitude and declination, synthetic paleopole coordinates for the reference points in these three continents, are adopted here. These references points are: Southwest Hudson Bay ($55^\circ$N, $270^\circ$E) in North America, West Siberian lowlands ($60^\circ$N, $70^\circ$E) in Eurasia, and ($24^\circ$N, $77^\circ$E) in India. It should be noted firstly that synthetic paleo-latitudes here are optimization solutions, so do the synthetic paleo-longitudes; And secondly the errors from the regression value of paleo-latitude and declination, synthetic paleopole coordinates are not taken into account.

Figure 5-7 show the synthetic paleo-positions for North America, Eurasia and India, respectively. In each figure, subfigure a shows the variation of synthetic paleo-longitudes with the geological time; subfigure b the synthetic paleo-latitudes; subfigure c the synthetic paleo-longitude vs. synthetic paleo-latitude; subfigure d the space and time evolution of the corresponding continent. Table 3 shows the synthetic paleo-positions of the reference sites for these three continents from -200 Ma to present at 20-Ma intervals. In the following we illustrate briefly one by one, and omit the word of "synthetic" (eg., synthetic paleo-longitude just referred to as paleo-longitude).
Table 2: Errors caused by $B_x$, $B_y$, $B_z$, etc., when $\theta$, $\lambda_p$ and $\theta_p$ can be derived from $B_x$, $B_y$, $B_z$ (or $D$, $I$).

| Variable (Perturbation) | $e_{\text{min}}$ ($^\circ$E) | $e_{\text{max}}$ ($^\circ$E) | $|e|_{\text{min}}$ ($^\circ$E) | $|e|_{\text{max}}$ ($^\circ$E) | nRMSE ($^\circ$E) |
|-------------------------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|------------------|
| $B_x$, $B_y$, $B_z$ (0%) | -2.1                          | 2.1                          | 0                             | 2.1                          | 1.4              |
| $B_x$ (-5%)             | -5.5                          | 5.4                          | 0.2                           | 5.5                          | 3.7              |
| $B_x$ (-10%)            | -12.9                         | 12.9                         | 0.4                           | 12.9                         | 9.1              |
| $B_x$ (5%)              | -9.1                          | 9.3                          | 0.3                           | 9.3                          | 6.2              |
| $B_x$ (10%)             | -15.5                         | 16.7                         | 0.5                           | 16.7                         | 10.9             |
| $B_y$ (-5%)             | -2.0                          | 2.0                          | 0                             | 2.0                          | 1.3              |
| $B_y$ (-50%)            | -0.9                          | 0.9                          | 0                             | 0.9                          | 0.4              |
| $B_y$ (5%)              | -2.2                          | 2.2                          | 0                             | 2.2                          | 1.5              |
| $B_y$ (50%)             | -3.5                          | 3.5                          | 0.2                           | 3.5                          | 2.6              |
| $B_z$ (-5%)             | -9.2                          | 9.4                          | 0.3                           | 9.4                          | 6.4              |
| $B_z$ (-10%)            | -15.9                         | 17.0                         | 0.6                           | 17.0                         | 11.7             |
| $B_z$ (5%)              | -5.6                          | 5.5                          | 0.2                           | 5.6                          | 3.7              |
| $B_z$ (10%)             | -13.1                         | 13.1                         | 0.3                           | 13.1                         | 8.7              |
| $D$ (-5%)               | -2.2                          | 2.2                          | 0                             | 2.2                          | 1.4              |
| $D$ (-50%)              | -3.6                          | 3.9                          | 0                             | 3.9                          | 1.8              |
| $D$ (5%)                | -2.1                          | 2.1                          | 0                             | 2.1                          | 1.4              |
| $D$ (50%)               | -1.8                          | 1.8                          | 0                             | 1.8                          | 1.2              |
| $I$ (-5%)               | -17.9                         | 18.9                         | 0.7                           | 18.9                         | 13.9             |
| $I$ (-10%)              | -45.5                         | 45.3                         | 1.2                           | 45.5                         | 30.0             |
| $I$ (5%)                | -17.9                         | 17.9                         | 0.3                           | 17.9                         | 11.3             |
| $I$ (10%)               | -41.0                         | 41.1                         | 0.4                           | 41.1                         | 25.6             |
| $D/I$ (-5%)             | -18.3                         | 19.9                         | 0.7                           | 19.9                         | 14.1             |
| $D/I$ (10%)             | -46.8                         | 46.9                         | 1.1                           | 46.9                         | 31.3             |
| $D/I$ (5%)              | -17.1                         | 17.0                         | 0.4                           | 17.1                         | 11.0             |
| $D/I$ (10%)             | -37.8                         | 37.8                         | 0.5                           | 37.8                         | 24.2             |

Note: $e_{\text{min}}$: minimum error; $e_{\text{max}}$: maximum error; $|e|_{\text{min}}$: minimum absolute error; $|e|_{\text{max}}$: maximum absolute error; nRMSE: nominal RMSE. The part in the bracket is the perturbation. $D/I$: The perturbation is add in both $D$ and $I$. $B_x$, $B_y$, $B_z$ at the surface, and $D$, $I$ are from IGRF model on Oct. 1, 2005 and at the latitude of 40° N.

North America. Comparing Figure 5a with Figure 5b shows that both paleo-longitudes and paleo-latitudes of the reference point had a similar evolution with geological time like an oblique inversely U-shape. The paleo-longitudes had a change of nearly 90° from -200 Ma to about -100 Ma. After -100 Ma, paleo-longitudes changed a little between -100 Ma and -1.0Ma. Reflected in Figure 5c, four phases of movement and rotation are clearly distinguishable. During the first phase, the reference point was located in the southwest of the map at -200 Ma, and then it moved to the northeast till -100 Ma. A slight clockwise rotation from -200 Ma to -160 Ma accompanied this northeastward movement, followed by a slight counterclockwise rotation till -100 Ma. In the second phase, the reference point rotated and moved to the southwest again from -100 Ma to -40 Ma. Then it moved westward
Fig. 5: Synthetic paleo-position of a reference point in North America at (55° N, 270° E) during the last 200 Ma. a. Synthetic paleo-longitudes vs. geological time; b. Synthetic paleo-latitudes vs. geological time (Data is from Schettino and Scotese (2005)); c. Synthetic paleo-latitudes vs. Synthetic paleo-longitudes; d. Synthetic paleo-latitudes and synthetic paleo-longitudes vs. geological time. The red cross is the reference point. (The same to hereinafter.)

till -1.0 Ma during the third phase along a path to an inversely "S", but soon it moved eastward during the last phase to its present position. The paleo-longitude and paleo-latitude had little variation from the second to the fourth phase, indicating the North America is relatively stable during these three phases.

**Eurasia.** Comparing Figure 6a with Figure 6b shows that paleo-longitudes of the reference point had a nearly antisymmetry evolution with the paleo-latitude from -200 Ma to -20 Ma. The paleo-longitudes had a change of almost 60° from -200 Ma to about -130 Ma. After -120 Ma, paleo-longitudes changed a little between -120 Ma and -20 Ma, then increased by 30°. Reflected in Figure 6c; the trajectory can also be divided into four phases. During the first phase, the reference point was located in the northwest of the map at -200 Ma, and then it moved to the southeast till -130 Ma. A slight counterclockwise rotation accompanied this southeastward movement. In the second phase, the reference point moved northwestward first from -120 Ma to -100 Ma, then southeastward till -60 Ma, finally northwestward again till -20 Ma. During the second phase, the trajectory made a knot. After this, the reference point moved eastward till -1.0 Ma during the third phase, but
Table 3: Synthetic paleo-position for the reference points in North America, Eurasia, and India from -200 Ma to present.

| Age(Ma) | North America | Eurasia | India |
|---------|---------------|---------|-------|
|         | \(\lambda^\circ\)E | \(\phi^\circ\)N | \(\lambda^\circ\)E | \(\phi^\circ\)N | \(\lambda^\circ\)E | \(\phi^\circ\)N |
| 0.0     | 270.0         | 55.0    | 75.0  | 60.0  | 77.0  | 24.0 |
| -1.0    | 245.7         | 54.7    | 123.1 | 59.4  | 139.9 | 23.0 |
| -20.0   | 252.7         | 52.8    | 90.5  | 58.9  | 351.0(?) | 16.5 |
| -40.0   | 256.3         | 52.2    | 92.5  | 57.7  | 0.9   | 4.8  |
| -60.0   | 263.8         | 53.5    | 97.0  | 56.1  | 8.6   | -13.6|
| -80.0   | 268.9         | 54.9    | 95.5  | 56.3  | 18.0  | -30.8|
| -100.0  | 273.6         | 55.8    | 92.0  | 57.3  | 25.7  | -39.2|
| -120.0  | 272.6         | 55.7    | 103.1 | 55.1  | 29.1  | -42.1|
| -140.0  | 262.8         | 52.7    | 100.3 | 54.3  | 33.0  | -39.5|
| -160.0  | 237.8         | 46.5    | 74.0  | 60.3  | 39.2  | -34.2|
| -180.0  | 197.4         | 38.8    | 55.3  | 68.5  | 45.2  | -29.1|
| -200.0  | 182.8         | 30.4    | 52.1  | 72.9  | 48.1  | -25.2|

Note: Data for \(\phi\) and Age are all from Schettino and Scotese (2005).

soon it moved westward during the last phase to its present position. The paleo-longitude and paleo-latitude also had little variation from the second to the fourth phase, indicating the Eurasia is also relatively stable during these three phases.

**India.** Before the illustration, it should be pointed out we modify a paleo-longitude at -20 Ma for the reference point from 351\(^\circ\)E to 9\(^\circ\)E (See also in Table 3). Otherwise, India had to move nearly 360\(^\circ\) during 20 Ma which may be impossible. And we plot the Figure 7 with the data in Table 3 rather than smooth data like Figure 5-6.

Comparing Figure 7a with Figure 7b shows that both paleo-longitudes and paleo-latitudes of the reference point had a similar evolution with geological time like an oblique U-shape. The paleo-longitudes decreased almost linearly from -200 Ma to about -20 Ma. After -20 Ma, the paleo-longitude increased considerably. Reflected in Figure 7, four phases of movement and rotation are clearly distinguishable, but on the whole the trajectory was simple. The reference point in India rotated clockwise from -200 Ma to -1.0 Ma. During the first phase, the reference point moved slowly southwestward till -120 Ma, followed by a rotation and moved northwestward till -40 Ma in the second phase. In the third phase, the reference point rotated and moved eastward -40 Ma to 1.0 Ma. Then it moved westward during the last phase to its present position.

Also, we think the paleo-longitude at -1.0 Ma may be problematic because the reference point had to move about 130\(^\circ\) in about 20 Ma. We infer it should be located between 9\(^\circ\)E and 77\(^\circ\)E. This problem is also remarked with a question mark in the Figure 7a, b and d.

4.2. **Reference paleo-longitude of Indo-Asia collision.** The Indo-Asia continental collision was one of the most profound tectonic events in Cenozoic time. Where and when this event occurred is crucial for continental dynamic models. Scientists of paleo-magnetism
thought that the collision range of paleo-latitude was from 5°N to 30°N at 40-60 Ma (e.g., Westphal and Pozzi 1983; Chen et al. 1993; Liebke et al. 2010). Based on new data, Dupont-Nivet et al. (2010) suggested that India-Asia collision was at 22.8 ± 4.2°N and 46 ± 8 Ma. Here we use further their data to estimate the paleo-longitude for this collision. The corresponding results are listed in Table 4.

Table 4: Paleo-position for Lhasa terrane and the Tethyan Himalaya

| Age(Ma) | φs(°N) | λs(°E) | φ(°N) | λ(°E) | φp(°N) | λp(°E) |
|---------|--------|--------|-------|-------|--------|--------|
| L       | 50.5   | 30.0   | 91.1  | 24.1±4.2 | 132.9±27.6 | 81.2 | 221.4 |
|         | 87.5   | 30.0   | 91.1  | 22.0±2.5 | 54.8±9.9 | 76.7 | 327.1 |
|         | 95.0   | 30.7   | 91.2  | 22.7±6.0 | 126.3±25.4 | 76.3 | 213.8 |
| T       | 59.0   | 28.3   | 88.5  | 6.7±3.1 | 6.9±1.3 | 68.2 | 277.1 |
|         | 68.0   | 28.3   | 88.5  | -5.7±3.5 | 279.5±26.2 | 55.8 | 261.6 |
| S       | 45.7   | 29.0   | 88.0  | 22.8±4.2 | 132.6±27.2 | 81.2 | 221.4 |

Note: φs, λs: site latitude, site longitude; φ, λ: paleo-latitude, paleo-longitude; φp, λp: paleopole latitude, paleopole longitude. L: Lhasa terrane; T: Tethyan Himalaya; S: Suture. All data except λ are from Dupont-Nivet et al. (2010).
Fig. 7: Synthetic paleo-position of a reference point in India at (24°, 77°E) during the last 200 Ma.

From Table 4, it can be seen that the paleo-position for India-Asia collision was at 22.8 ± 4.2°N and 132.6°−27.2°E, if this collision occurred at the suture here.

On the other hand, assuming India-Asia collision can be understood as the "intersection" between the trajectory of the Lhasa terrane and Tethyan Himalaya in a (2+1)-dimensional space (2-dimensional space + 1-dimensional time) as shown in Figure 8, rather than that in the usual 2-dimensional space of paleo-latitude vs. geological time, we can get another paleo-position and time of this collision at 12.8°±3.9°N and 101.7°±12°E and −63.6°+0.6°−0.3 Ma with the data in Table 4. It should be pointed out that this "intersection" is in least squares sense, and this result is only a conjecture that must be constrained or demonstrated by other data.

5. Discussions

Here we provide an approach to estimate the paleo-position, especially the paleo-longitude, through paleo-magnetic measurements, as shown in equation (5) in section 2. In theory, we should solve this equation to obtain $\lambda$, $\theta$, and $K_p$ simultaneously, through the measurements of $B_x$, $B_y$, $B_z$ and the known $\theta_p$, and $\lambda_p$. This involves two problems. The first is how to get accurately the $\theta_p$, and $\lambda_p$. Except the method we adopted in this paper, constructing the reference poles for a certain geological age may be an alternative good way. This demands a large amount of accurate paleo-magnetic measurements and close
cooperation of scientists of paleo-magnetism. At present, there is a lot of databases can be used for this purpose, such as IAGA paleo-magnetic databases (McElhinny and Lock, 1996; McElhinny and McFadden, 1997; McElhinny et al., 1998), IAGA paleointensity database (Perrin et al., 1998; Perrin and Schnepp, 2004), Absolute Palaeointensity (PINT) Database (Biggin et al., 2010; Veikkolainen et al., 2017), Magnetics Information Consortium (MagIC) (Jarboe et al., 2012), Precambrian database (PALEOMAGIA) (Veikkolainen et al., 2017), and so on.

The second problem is how to solve the nonlinear equation (5), even if $B_x, B_y, B_z$ are measured and $\theta_p, \lambda_p$ are known. We have tried many methods, such as homotopy continuation method (Decarolis et al., 2005), artificial neural network inversion tool (Ružek, 2008), a globally optimal iterative algorithm using the best descent vector (Liu and Atluri, 2012), and so on. However, in most cases the results derived are not those we want. Therefore, it is necessary to develop efficient algorithms for solving the nonlinear equation (5), if the first problem is solved.

The approach (equation (17), (19)) used in this paper, which has been shown in section 2, can overcome the difficulties above at a certain extent. If $\theta$ (I), $\lambda_p$, and $\theta_p$ are accurate, the longitude can be derived perfectly. Experiments on synthetic data in the section 3 demonstrate this success. This method demands mainly accurate $B_x, B_z$, and/or $I$. When this approach is used in paleo-magnetism as shown in section 2.3 and 4, there is a defect in theory that $g_1^1$ and $h_1^1$ are omitted when we estimate $\theta$, $\theta_p$, and $\lambda_p$ by the traditional way (equation (4) and (11)), but they are taken into account in equation (3) and (12). In this
paper we cannot eliminate this defect, and new approaches should be developed. However, as shown in the section 3, if the error in $\theta$ $(I)$ can be controlled in a reasonable range, the paleo-longitudes estimated by our approach should be acceptable, especially in the case that $B_x (J_x), B_y (J_y)$ and $B_z (J_z)$ have the same errors.

6. Conclusions

We associate $B_x, B_y, B_z$ with $K_p, \theta_p$ and $\lambda_p$ through equation (5), in which the strongest component ($g^0_1$) of the geomagnetic field and the next strongest term ($g^1_1, h^1_1$) are all taken into account. With this equation, longitude $\lambda$ can be derived in theory.

Based on equation (5), we obtain two explicit formulae (equation (7) and (9)) to estimate $\lambda$ through the measured magnetic data for the given $\theta$, $\theta_p$ and $\lambda_p$. With synthetic data of $B_x, B_y, B_z$ or declination $(D)$ and inclination $(I)$ from IGRF model, we derived the setting longitudes perfectly through these two formulae.

Errors in these two explicit formulae caused by $B_x (e_{B_x})$ is greater than those by $B_z (e_{B_z})$, then $e_{B_x} > e_{\lambda_p} > e_I > e_D > e_{B_y} > e_{\theta_p}$ when these quantities are independent and can be obtained or given by other methods; And $e_I > e_{B_x} > e_{B_z} > e_D$ when $\theta, \lambda_p$ and $\theta_p$ can be derived from $B_x, B_y, B_z$ (or $D, I$). Therefore, the errors are mainly from $B_x, B_z$ or $I$.

For paleo-magnetic data, paleo-longitudes from these two explicit formulae, for example, those in the section 4 could be used as reference, because of the defect in theory when we estimate $\theta, \theta_p$ and $\lambda_p$, and the inaccuracy in traditional paleo-magnetic way. To get more accurate results, constraints from other geo-methods are necessary.

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