An Integrative Analysis of the HD 219134 Planetary System and the Inner solar system: Extending DYNAMITE with Enhanced Orbital Dynamical Stability Criteria

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Abstract

Planetary architectures remain unexplored for the vast majority of exoplanetary systems, even among the closest ones, with potentially hundreds of planets still “hidden” from our knowledge. DYNAMITE is a powerful software package that can predict the presence and properties of these yet-undiscovered planets. We have significantly expanded the integrative capabilities of DYNAMITE, which now allows for (i) planets of unknown inclinations alongside planets of known inclinations, (ii) population statistics and model distributions for the eccentricity of planetary orbits, and (iii) three different dynamical stability criteria. We demonstrate the new capabilities with a study of the HD 219134 exoplanet system consisting of four confirmed planets and two likely candidates, where five of the likely planets and candidates are Neptune-sized or below with orbital periods less than 100 days. By integrating the known data for the HD 219134 planetary system with contextual and statistical exoplanet population information, we tested different system architecture hypotheses to determine their likely dynamical stability. Our results provide support for the planet candidates, and we predict at least two additional planets in this system. We also deploy DYNAMITE on analogs of the inner solar system by excluding Venus or Earth from the input parameters to test DYNAMITE’s predictive power. Our analysis finds that the system remains stable while also recovering the excluded planets, demonstrating the increasing capability of DYNAMITE to accurately and precisely model the parameters of additional planets in multiplanet systems.

Unified Astronomy Thesaurus concepts: Exoplanets (498); Astrostatistics (1882)

1. Introduction

Understanding the stability of planetary systems and searching for a form of order in their dynamics has long been a path of study for astronomers, first for our own solar system and then in other systems discovered over the past three decades. While we know much about exoplanets as a population, thanks to the transit survey conducted by the Kepler Space Telescope (Borucki et al. 2010) and various ground-based radial velocity (RV) surveys, certain questions regarding the dynamical properties of exoplanet systems still remain to this day. The underlying physics is complex and time-consuming to compute mathematically, and astronomers often have to make a compromise between accuracy and speed when modeling these interactions. Still, previous studies have been able to ascertain simple empirical rules for orbital dynamical stability that fit the known population statistics fairly well (e.g., Pu & Wu 2015; Gilbert & Fabrycky 2020) and detailed computational models that sacrifice speed for more accuracy via N-body integrations, which work well for the analysis of a single, dynamically complicated system (Volk & Malhotra 2020).

The DYNAMITE3 (Dietrich & Apai 2020) software package provides a unique capability for integrated analysis to combine specific (but often uncertain and incomplete) data of an exoplanet system with population-level statistical information and an empirical rule or other criterion for orbital dynamical stability. This analysis then predicts the presence and parameters of additional “hidden” planets in these planetary systems, which provides enhanced statistical understanding of exoplanet populations. Multiple studies have used DYNAMITE to predict the positions, orbits, and properties (size, nature) of additional planets in dozens of multiplanet systems (e.g., Dietrich & Apai 2020, 2021; Hardegree-Ullman et al. 2021; Basant et al. 2021, submitted). In addition, predictions from DYNAMITE have matched known planets excluded from the analysis, as well as new planets found at the time the integrated analysis was performed (Peterson et al. 2021, submitted).

In previous versions of DYNAMITE, we adopted a simple dynamical stability criterion from He et al. (2019), which was guided by N-body simulations of multiplanet dynamical stability to find that most planet pairs are stable only when their mutual Hill radius is are $\gtrsim 8$. However, this criterion, while useful, does not account for some important details. How would the deviation of an orbit from circular along the plane of the system affect the stability? What about orbital resonances with other planets in the system? Which methods to assess stability provide the best mix of accuracy and speed? This paper explores these questions and evaluates three different models of the dynamical stability of multiplanet systems.

The HD 219134 system, which is the closest star to the solar system with at least six planets or planet candidates and the closest system with transiting planets (Motalebi et al. 2015, hereafter M15; Vogt et al. 2015, hereafter V15; Gillon et al. 2017a, hereafter G17), provides a complex, high-multiplicity system on which the various stability analyses can be tested with DYNAMITE. With two planets confirmed to transit by the Transiting Exoplanet Survey Satelitte (TESS; Ricker et al. 2015) and two of the other RV signals not fully confirmed by

3 https://github.com/JeremyDietrich/dynamite

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various studies, we can also qualitatively assess support for the candidate planets, along with searching for additional planets that are still unknown. The HD 219134 system will likely be a prime target for direct imaging missions and next-generation ground-based telescopes in the near future, so having a more complete understanding of the planetary system through the predictions we can generate from our DyNAMITE analysis can help guide those observations and instruments.

In addition, the solar system has provided a great laboratory for analyzing dynamical stability models, including the effects of general relativity on Mercury’s orbit. Thus, as a benchmark, we also deploy DyNAMITE to analyze the results from the stability criteria on the inner solar system (up to and including Jupiter) as an analog for exoplanet systems, including assessing the system with the exclusion of one of the inner planets. The level of understanding of the solar system will give strong support to the robustness of our stability measures, as well as continuing to develop the knowledge of how unique the solar system’s architecture truly is (e.g., Mulders et al. 2018).

This paper is organized as follows. Section 2 discusses the current knowledge of the HD 219134 system, the closest system with at least five planets. Section 3 introduces the new features of the DyNAMITE software package. The results of the new analysis on the HD 219134 system are shown in Section 4. Finally, Sections 5–7 discuss the effects of our model assumptions and different parameter tests performed on the HD 219134 system, the dynamical stability criteria evaluated and their different use cases, and a comparison case via modeling the inner solar system, including Jupiter.

2. The HD 219134 System

Object HD 219134 (also known as Gliese 892 or HR 8832) is a main-sequence early K dwarf, approximately 80% the size of the Sun and about four times less luminous (Gray et al. 2003; Takeda et al. 2007; Boyajian et al. 2012). It is located at a distance of 6.532 ± 0.004 pc from us (Gaia Collaboration et al. 2018), making it one of the 30 closest known planet-hosting stars to our solar system and the closest to host at least five planets. It is known to be an impressively old star, with derived age estimates of 11.0 ± 2.2 (G17) and 12.46 ± 0.5 (Takeda et al. 2007) Gyr from various stellar evolution models. The rotation period of the star, via $v \sin i$ and stellar activity measurements taken over 12 yr, is likely either $P_{\text{rot}} \approx 22.8$ or ≈42.3 days, relatively short for such an aged star (V15; Johnson et al. 2016; G17). The stellar parameters for HD 219134 can be found in Table 1.

The known system architecture of HD 219134 is illustrated in Figure 1. The first study to detect the presence of planets around HD 219134 was done by M15, who found four planets in the system using RV observations. Three were sub-Neptunian planets located close to the host star with orbital periods of 3.094 (b), 6.767 (c), and 46.66 (d) days, whereas the fourth (e) was a Saturnian or Jovian planet much further out with an orbital period of 1842 days. Shortly thereafter, another RV study by V15 found evidence for six planets in the system. Notably, they confirmed the presence of planets b–d and discovered evidence for two more sub-Neptunian planets in the inner system at orbital periods of 22.81 (f) and 94.2 (g) days but did not find planet e; however, they found a similarly massive planet (h) in the outer system but at a noticeably longer period of 2247 days.

Another study by Johnson et al. (2016), who gathered stellar activity data over a 12 yr baseline, found evidence for a rotation period of 22.8 days while providing no confirmation of the presence of planet f at a very similar orbital period to the rotational/stellar activity period. However, they did confirm planets d and h, stating that planet e was likely a misidentification of planet h. In addition to their RV observations, M15 also found that planet b transited HD 219134 using photometry from the Spitzer Space Telescope. A follow-up transit photometry analysis by G17 also using the Spitzer Space Telescope discovered that planet c also transited the host star. This study determined that by geometric probability alone, planet c only had a 5.4% chance of transiting the host star, but after confirmation of the transits of planets b and d, the transit probability of planet c increased to 21% before it too was found to transit. The observations of planet c transiting, therefore, increase the transit probability of planets f and d to 13% (from 2.5%) and 8.1% (from 1.5%), respectively, as the additional transiting planet means the system inclination is likely closer to edge-on.

TESS observed the HD 219134 system in Sectors 17 and 24 as TOI 1469 and confirmed the transit events for planets b and c but did not detect transits for any other planet.4 Thus, it is likely that any planets in the inner system with an orbital period within 30 days would either have an inclination far enough away from edge-on such that they would not transit HD 219134 or would be smaller than any known planet in the system. However, due to TESS’s observational strategy, the parameter constraints are not tight for planets with orbital periods longer than TESS’s single-sector observing window of ~27 days. Thus, the presence of planets b–d and e/h have been confirmed in multiple studies, with planet e from the original M15 study found to likely be another manifestation of planet h from V15. The validity of planets f and g (both found by V15) has not yet been established. A recent multidecade analysis of RV data for the HD 219134 system by Rosenthal et al. (2021) confirmed a signal at 22.795 ± 0.005 days that would correspond to planet f, with a slightly smaller $m \sin i$ value. However, they did not report a signal near the originally given period of planet g at ~94 days but analyzed signals near 2× and 4× that period and determined them to be false positives due to annual/instrumental systematics (see Section 5.3).

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4https://exofop.ipac.caltech.edu/tess/target.php?id=283722336

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Table 1
Stellar Parameters for HD 219134

| Parameter Name | Value          | Ref. |
|---------------|----------------|------|
| Spectral type | K3V            | (a)  |
| Mass ($M_\odot$) | 0.81 ± 0.03   | (b)  |
| Radius ($R_\odot$) | 0.778 ± 0.004 | (c)  |
| Luminosity ($L_\odot$) | 0.265 ± 0.002 | (c)  |
| Temperature (K) | 4699 ± 16     | (c)  |
| Distance (pc) | 6.532 ± 0.004 | (d)  |
| Rotation period (days) | 42.3 ± 0.1 | (e)  |
| Age (Gyr) | 11.0 ± 2.2    | (b)  |
|              | 12.46 ± 0.5   | (g)  |

Notes: (a) Gray et al. (2003), (b) G17, (c) Boyajian et al. (2012), (d) Gaia Collaboration et al. (2018), (e) M15, (f) Johnson et al. (2016), (g) Takeda et al. (2007).
In our analysis of the HD 219134 system, we do not take the outer gas giant planet \( e/h \) \((\text{M15; V15})\) into account, as it is beyond the limits of our current population statistics. Moreover, its existence and properties are inconsistently reported in different studies, as it is not mentioned in any other studies besides these two discovery papers, which have a relatively small but significant inconsistency in the reported values. However, future studies should consider its effects on the inner system and its dynamical stability.

We gathered our information on HD 219134 from the NASA Exoplanet Archive\(^5\) and the associated references above. Specifically, the specific data we input to DyNAMITE are only the planet parameters (orbital periods, radii/m sin \( i \) values, inclination, and eccentricity, as well as their uncertainties) as they are given from the previous studies; we do not reanalyze any of their data sets, as there is no reason to do so. A detailed summary of the exoplanet population statistical information is detailed in Dietrich & Apai \(2020, 2021\), and we only note here that the key trends follow those reported in Fabrycky et al. \(2014\), Mulders et al. \(2018\), and He et al. \(2019, 2020\). In order to provide a complete analysis despite the presence of planets \( f \) and \( g \) not being well established in the previous data sets, we also explore the different configurations of the system if planets \( f \) and/or \( g \) are considered “unconfirmed” and not included in the system architecture.

\(^5\) [https://exoplanetarchive.ipac.caltech.edu/](https://exoplanetarchive.ipac.caltech.edu)
priors, such as higher precision for data or additional demographic information (e.g., occurrence rate by spectral type, stellar rotational period, etc.), will continue to refine the assumptions in our model (or the specific hypotheses tested) and adjust our predictions accordingly.

DYNAMITE requires as input the orbital periods and radii or minimum masses and their uncertainties (depending on availability from different types of observations) of the planets, as well as the radius, mass, and luminosity of the host star. Additional parameters—where unknown values are allowed—include the inclination and eccentricity of the planets. DYNAMITE then calculates the statistical likelihood distributions given the population models (of which there are multiple modules for the different parameters that can be switched and tested separately) and runs a Monte Carlo sampling chain on them. Every set of parameters is tested for dynamical stability and discarded if the system would be considered dynamically unstable (see Section 3.4 for our new implementation on this consideration).

For ranges in the orbital period distribution that are stable, the likelihood is determined based on the orbital period module chosen. Currently, the two available modules are “equal period ratios,” where planets are most likely to be found when their pairwise period ratios are similar (Mulders et al. 2018), and “clustered periods,” where planets are most likely to be found in clusters with potential gaps in the system (He et al. 2019). We label peaks in the orbital period distribution as significant when ≥90% of injections interior to the outermost planet are found in a specific location in the space. We also note positions where multiple planets could exist if we were testing for the presence of more than one additional planet. The output data of DYNAMITE are testable predictions in the form of orbital period, transit depth/ probability, RV semiamplitude, and direct imaging separation and contrast for the planet parameters with the highest likelihood.

The method was originally tested on known Kepler systems with planets removed; such planets were recovered with a good degree of accuracy on the parameters. DYNAMITE was developed on the known TESS multiplanet systems and refined further for analyzing the τ Ceti planetary system, where we predict that at least one possibly rocky habitable-zone planet is likely to exist but has not yet been discovered (Dietrich & Apai 2021). Additional studies have utilized DYNAMITE analyses for the K2-138 system with at least six planets (Harder-P-Ulman et al. 2021), as well as both Mars in the solar system and the e Eridani (also known as HD 20794; not to be confused with e Eridani) system of three known planets and three planet candidates (Basart et al. 2021, submitted). Here we will briefly discuss the objectives of DYNAMITE in the context of the significant extension of functionalities we added for the present study.

Our goal is to utilize the combined statistical knowledge gathered across all exoplanet systems to determine likelihoods for the presence and properties of planets in specific systems. Our analysis is, therefore, reliant on the accuracy and precision of the data we can gather for a system, as well as our ability to model system parameters and orbital stability. We run our analysis in multiple modes with these different modules of data models to assess the accuracy of these models for future analyses and provide testable hypotheses for models with uncertain levels of knowledge. DYNAMITE has different models for the orbital periods, planet radii, mass–radius relationship, orbital inclination, and orbital eccentricity, and each combination of models provides a different set of predicted planet injections with different likelihoods. The choices made by our analysis are shown in a flowchart in Figure 2. Our motivation to upgrade DYNAMITE and expand its capabilities was to determine the fine structure in dynamical orbital stability, as our previous simple stability model was sufficient for first-approximation results but is likely not a precise model. Here we discuss the updated models and parameter decisions added to DYNAMITE.

### 3.2. Planet Existence Hypotheses

We test multiple different hypotheses of the system architecture to determine how incorporating the presence of not-yet-confirmed or uncertain planets and planet candidates changes our understanding of the system architecture and planet properties. Each of these hypotheses is a “branch” on the DYNAMITE data decision tree (see Figure 3), and we explore how these decisions change the results.

In the HD 219134 system, there are two planets with some degree of uncertainty about their existence. Planet f has an orbital period of 22.7 days, which is very close to a measured rotational period for its host star of 22.8 days (Johnson et al. 2016). However, it was included as a known planet for the purpose of updating transit probabilities from the Spitzer follow-up transit study by G17, and a signal with the same period was found over long-term RV data by Rosenthal et al. (2021). Planet g, reported by V15, has no other measurements from any other study; M15 and G17 did not reference it anywhere in their analysis, while Johnson et al. (2016) briefly mentioned it but did not provide any additional analysis to confirm or reject its existence. The parameter values from the analysis by V15 are much less precise than the other inner planets, but there is no reason to specifically doubt its presence in the system.

For our analysis, we have decided to explore four different system hypotheses in which we include and exclude planet f and planet g from the system architecture and exclude the outer gas giant planet as stated above. The four hypotheses are defined as follows.

1. **Hypothesis H[a].** All five planets, b, c, d, f, and g, exist at their current parameter values.
2. **Hypothesis H[b].** The four planets b, c, d, and f exist, while planet g does not.
3. **Hypothesis H[c].** The four planets b, c, d, and g exist, while planet f does not.
4. **Hypothesis H[d].** The three planets b, c, and d exist, while planets f and g do not.

We treat the excluded planets as “unconfirmed” in our analysis, and they are not known by the injection algorithm. If the Monte Carlo analysis returns a high relative likelihood at the same orbital periods, we consider this as providing support to the existence of these planets.

### 3.3. Inclination and Eccentricity Distributions

We expanded the parameter space available for analysis and refined our modeling by incorporating planets with unknown inclinations and testing two different models for the inclination distribution. For systems where planetary inclinations are unconstrained, our analysis now assumes an isotropic distribution for the system inclination. For systems where only some planets have inclination constraints, we fit the known values to the requisite distribution of the system inclination for each model described below.
After setting a system inclination (which varies in the isotropic case for each iteration), we distribute the mutual inclinations in two ways. The first model comes from the Exoplanet Populations Observation Simulator (EPOS; Mulders et al. 2018) and assumes that a fraction of systems have isotropically distributed inclinations, while the rest have a narrow ($\sigma_i \sim 2$) Rayleigh distribution of mutual inclinations. The second model comes from the Exoplanets Systems Simulator (SysSim; He et al. 2019, 2020) and assumes a lognormal distribution with parameters dependent on the multiplicity of the system, where more planets in the system tends to lead to a more coplanar system.

We also included eccentricity distributions, both as another parameter for which we could assess probabilities and as a necessity for the $N$-body integrations for the dynamical stability criterion (via orbit crossing determinations and angular momentum deficit, AMD, measurements). A large fraction of the eccentricities in the Kepler multiplanet system population are $\lesssim 0.1$ (e.g., Xie et al. 2016; Mills et al. 2019; Van Eylen et al. 2019). Our first eccentricity distribution is a Rayleigh distribution with parameter $\sigma_e = 0.02$ (see, e.g., Fabrycky et al. 2014), which is not dependent on any other parameter in the system. We also used a multiplicity-dependent lognormal distribution from SysSim, and as with the inclination model, more planets in the system implies more circular orbits for the planets. For planets without known eccentricities, we sample them from the corresponding distribution for the multiplicity in the system, given that we have injected one planet into the system.

3.4. Dynamical Stability

Our goal is to assess the precision of our dynamical stability model and compare the trade-offs required between complexity and speed. The dynamical stability criterion determines the likelihood that the parameters of the predicted planets injected into the system by DyNAMITE do not affect the long-term orbital stability of the other planets in the system and the injected planet itself. This is an important limiting factor on the injection likelihoods, but it can be a complex and time-consuming process to model. In particular, orbit crossings (i.e., the inner planet of a planet pair having an apastron larger than the periastron of the outer planet in the pair) are not always an indicator of dynamical instability (e.g., in the solar system, the orbits of Neptune and Pluto cross). Here we make the simplifying assumption that orbital crossings are unstable for planets of similar masses on timescales longer than 1 Gyr, since the reported age of the HD 219134 system is $>10$ Gyr. We further discuss the effect of this assumption in Section 6.

The dynamical stability criterion implemented in our previous DyNAMITE analyses was derived from a study by He et al. (2019), who determined that a simple model where planet
pairs within 8 mutual Hill radii were unstable and planet pairs outside that value were stable was a good empirical match to current data. However, this criterion did not catch finer details relating to orbital resonances and secular chaos in systems. Therefore, in this study, we compare the results from the simple mutual Hill radii cutoff model previously used with two other more detailed analyses involving $N$-body integrations from Tamayo et al. (2020) and Volk & Malhotra (2020).

First, we test the Stability of Planetary Orbital Configurations Klassifier (SPOCK; Tamayo et al. 2020). SPOCK utilizes a machine-learning algorithm that trains on 100,000 data sets of planet parameters (both stable and unstable) from $N$-body

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**Figure 3.** Decision tree of data for DyNAMITE, showing the different hypotheses that can be chosen for the HD 219134 system and updated for any given exoplanet system.
integrations and predicts stability using physically motivated summary statistics measured in short $N$-body integrations ($\sim 10^6$ orbits of the innermost planet). This provides a multiple order-of-magnitude speed-up over full $N$-body integrations and thus can be performed within an hour on a single laptop or desktop machine. SPOCK provides both a simple predictor based on a single set of summary features and a deep regression analysis that uses thousands of realizations to better characterize the dynamics of the planetary system. These predictors look at the stability of a system as a probabilistic measure instead of the definitive result achieved at the end of the full $N$-body integration. The level for which a system becomes unstable is where the predictor’s false-positive rate hits 10%, which occurs at a stability threshold of 0.34. Tamayo et al. (2020) reported that SPOCK provides $\geq 90\%$ accuracy in stability analyses for both predictors, with full $N$-body simulations being essentially ground truth, and is more accurate in comparison to other stability prediction methods, such as the mutual Hill radius cutoff and the Mean Exponential Growth factor of Nearby Orbits chaos indicator (Cincotta et al. 2003).

Second, we test the spectral fraction method from Volk & Malhotra (2020). This analysis performs a medium-length ($\sim 5 \times 10^5$ orbits of the innermost planet) $N$-body integration of the system to determine the likelihood of stability over long $N$-body integrations ($\sim 5 \times 10^9$ orbits of the innermost planet). The spectral fraction is determined by taking the power spectrum of the AMD (a measure of the difference in the orbits from circular and coplanar; see, e.g., Laskar & Petit 2017). If a significant fraction of frequencies ($\geq 1\%$) exist at a relatively high percentage of the main peak in the AMD power spectrum ($\geq 5\%$ of the peak), then the system is likely unstable over long ($\sim 5 \times 10^9$ orbits of the innermost planet) integrations; otherwise, the system is considered stable.

4. Results

We report the properties of the planets in the HD 219134 system and our predictions for each of our architecture hypotheses, dynamical stability analyses, and statistical model choices in Tables 2 and 3. The figures showing the relative likelihoods for each hypothesis in two dynamical stability tests are shown in the Appendix, with the mutual Hill radius criterion injections in blue (Figure A1) and the $N$-body integration injections with the spectral fraction analysis in green (Figure A2). The planet radii are unlikely to be measured via transits for planets external to planet c due to the inclination of the known planets and the increasingly narrow range of transit inclinations as the semimajor axis increases. The statistical model choices for the planet radius, inclination, and eccentricity have little effect, whereas the results from each orbital period model (equal period ratios from EPOS, Mulders et al. 2018, and clustered periods from SysSim, He et al. 2019) and dynamical stability criterion chosen are similar but do show some significant differences. We chose to utilize the equal period ratio orbital period model for our analysis.

4.1. Inclination and Eccentricity

We added a new inclination model to test the predictions between them and also incorporated two different eccentricity models for the dynamical stability analysis that required eccentricities in the $N$-body integrations. The results from the analysis on the HD 219134 system using the different statistical models are shown in Figure 4. For these results, the other population modules were held constant (equal period ratios, “volatile-rich” mass–radius relation, and mutual Hill radius stability criterion) to highlight the specific effect of the inclination and eccentricity choices.

The original inclination model, which fits the known mutual inclinations of the system to a Rayleigh distribution with a parameter of 2° around a central system inclination, found that the transiting planets were likely to be on the “high” side of this mutual inclination distribution, since the remaining planets have not been known to transit. Thus, the system inclination was fit to $\sim 82°$ (where 0° is face-on in the plane of the sky and 90° is edge-on), with the inclination likelihood therefore peaking at 80° and 84°. The new inclination model, which fits the known mutual inclinations of the system to a multiplicity-dependent lognormal distribution, unsurprisingly found that the system inclination was likely to be near the inclination of the two known transiting planets. This is still consistent with the likelihood of the other known planets not transiting, as the limiting inclination for transits approaches 90° as the orbital period increases. Until additional information is gathered from the system, it is unlikely we will be able to further constrain these parameters.

For the simple model of planet eccentricities, there is no difference between any of the system architecture hypotheses, so the predictions follow the Rayleigh distribution, with some slight deviations likely due to injections that are rejected due to other parameters. For the multiplicity-dependent lognormal, we find that higher eccentricities can be expected, in general, than the Rayleigh distribution, and the mean of the distribution is higher in hypothesis H[d] ($\mu = 0.046$) than in hypothesis H[a] ($\mu = 0.023$). However, none of the models show a high likelihood for having one or multiple planets with moderate eccentricities, as seen in the HD 219134 system.

4.2. Additional Planets

Hypothesis H[a]. We find two peaks in the orbital period relative likelihood at $\sim 12$ (PxP=–1 H[a]) and $\sim 174$ (PxP=–2 H[a]) days. The first peak lies between planet c and planet candidate f, and it can be used to determine the likelihood of planet candidate f being a genuine planet in hypotheses H[c] and/or H[d] (see Figure 5). The second peak lies beyond planet candidate g and inside the habitable zone of HD 219134, near the inner edge (e.g., Kopparapu et al. 2013). However, as this second peak is an extrapolation beyond the last available data point, the predictions are less constraining in the space beyond the outermost known planet.

Hypothesis H[b]. We again find two peaks in the orbital period likelihood. The first, at 12 days, corresponds with PxP=–1 H[a], which we now call PxP=–1 H[a], H[b]). The second peak, however, moves inward from the peak in hypothesis H[a], to $\sim 90$ days. Because planet candidate g has been excluded from the analysis, we now find that the peak for PxP=–2 H[b] likely corresponds to planet candidate g.

Hypothesis H[c]. Here the peak around 12 days in the previous hypotheses has been flattened and widened out with the removal of planet candidate f from the analysis. Similar behavior was seen in our past studies (Dietrich & Apai 2020, 2021), where we found evidence that this shape indicates that not one but two planets are present. When PxP=–1 (H[a], H[b]) is added to hypothesis H[c], we do recover a sharp peak very near the orbital period of planet candidate f, as in Figure 5. Thus, we
| Name          | Period (days) | Radius ($R_\odot$) | Mass ($M_\odot$) | Note     | Origin/Reference |
|---------------|---------------|--------------------|-----------------|----------|-----------------|
| HD 219134 b   | 3.093 ± 0.00001 | 1.602 ± 0.055      | 4.74 ± 0.19     | Planet   | M15; G17        |
| HD 219134 c   | 6.765 ± 0.0003 | 1.511 ± 0.047      | 4.36 ± 0.22     | Planet   | M15; G17        |
| H[a] PnP–1    | 12.4 [10.9, 14.7] | 1.69 [1.25, 2.27]  | 3.99 [1.99, 6.36] | Predicted | Ratios, "volatile," $\Delta = 8$ |
|              | 12.4 [10.9, 14.5] | 1.79 [1.33, 2.41]  | 6.71 [2.41, 18.7] | Ratios, "rocky," $\Delta = 8$ |
|              | 13.3 [10.8, 15.4] | 1.68 [1.25, 2.26]  | 3.95 [1.94, 6.31] | Clustered, "volatile," $\Delta = 8$ |
|              | 13.1 [10.7, 15.3] | 1.77 [1.32, 2.39]  | 6.45 [2.34, 18.2] | Clustered, "rocky," $\Delta = 8$ |
| H[b] PnP–1    | 12.4 [10.7, 13.7] | 1.69 [1.26, 2.28]  | 3.99 [2.00, 6.40] | Ratios, "volatile," $N$-body |
|              | 12.4 [10.7, 13.5] | 1.79 [1.34, 2.42]  | 6.71 [2.42, 18.7] | Ratios, "rocky," $N$-body |
|              | 13.3 [10.6, 15.2] | 1.68 [1.26, 2.27]  | 3.95 [1.95, 6.35] | Clustered, "volatile," $N$-body |
|              | 13.1 [10.5, 15.1] | 1.77 [1.33, 2.40]  | 6.45 [2.35, 18.2] | Clustered, "rocky," $N$-body |
| H[c] PnP–1    | 18.1 [14.0, 25.7] | 2.13 [1.30, 3.82]  | 5.75 [2.23, 14.4] | Predicted | Ratios, "volatile," $\Delta = 8$ |
|              | 18.1 [13.9, 25.7] | 1.77 [1.32, 2.39]  | 6.45 [2.34, 19.2] | Ratios, "rocky," $\Delta = 8$ |
|              | 4.58 [4.38, 4.88] | 1.91 [1.25, 3.62]  | 4.84 [1.95, 13.3] | Clustered, "volatile," $\Delta = 8$ |
|              |                | 1.70 [1.28, 2.30]  | 5.61 [2.10, 15.9] | Clustered, "rocky," $\Delta = 8$ |
|              | 18.1 [14.2, 25.2] | 2.14 [1.30, 3.84]  | 5.79 [2.23, 14.6] | Ratios, "volatile," $N$-body |
|              | 18.1 [14.1, 25.2] | 1.78 [1.32, 2.41]  | 6.49 [2.34, 19.4] | Ratios, "rocky," $N$-body |
|              | 4.58 [4.38, 4.88] | 1.92 [1.25, 3.63]  | 4.88 [1.95, 13.5] | Clustered, "volatile," $N$-body |
|              |                | 1.71 [1.28, 2.32]  | 5.65 [2.10, 16.0] | Clustered, "rocky," $N$-body |
| H[d] PnP–1    | 18.1 [14.4, 25.9] | 2.05 [1.29, 4.04]  | 5.41 [2.17, 15.8] | Predicted | Ratios, "volatile," $\Delta = 8$ |
|              | 18.1 [14.5, 25.7] | 1.65 [1.22, 2.22]  | 5.06 [1.83, 14.1] | Ratios, "rocky," $\Delta = 8$ |
|              | 4.58 [4.38, 4.88] | 1.82 [1.23, 3.66]  | 4.48 [1.84, 13.5] | Clustered, "volatile," $\Delta = 8$ |
|              |                | 1.59 [1.20, 2.09]  | 4.46 [1.69, 11.5] | Clustered, "rocky," $\Delta = 8$ |
|              | 18.1 [14.2, 25.4] | 2.06 [1.29, 4.04]  | 5.45 [2.17, 15.8] | Ratios, "volatile," $N$-body |
|              | 18.1 [14.3, 25.2] | 1.66 [1.22, 2.22]  | 5.10 [1.83, 14.1] | Ratios, "rocky," $N$-body |
|              | 4.58 [4.38, 4.88] | 1.83 [1.23, 3.66]  | 4.52 [1.84, 13.5] | Clustered, "volatile," $N$-body |
|              |                | 1.60 [1.20, 2.09]  | 4.50 [1.69, 11.5] | Clustered, "rocky," $N$-body |
| HD 219134 f   | 22.72 ± 0.015  | Unknown            | 7.30 ± 0.40     | Planet?  | M15; G17        |
| HD 219134 d   | 46.86 ± 0.028  | Unknown            | 16.2 ± 0.64     | Planet   | M15; G17        |
| HD 219134 g   | 94.2 ± 0.2     | Unknown            | 11 ± 1          | Planet?  | V15             |
| H[a] PnP–2    | 174 [169, 385]  | 1.69 [1.25, 2.27]  | 3.99 [1.95, 6.36] | Predicted | Ratios, "volatile," $\Delta = 8$ |
|              | 174 [168, 384]  | 1.79 [1.33, 2.41]  | 6.71 [2.41, 18.7] | Ratios, "rocky," $\Delta = 8$ |
|              | 145 [145, 399]  | 1.68 [1.25, 2.26]  | 3.95 [1.94, 6.31] | Clustered, "volatile," $\Delta = 8$ |
|              | 145 [145, 393]  | 1.77 [1.32, 2.39]  | 6.45 [2.34, 18.2] | Clustered, "rocky," $\Delta = 8$ |
|              | 174 [169, 365]  | 1.69 [1.26, 2.28]  | 3.99 [2.00, 6.40] | Ratios, "volatile," $N$-body |
|              | 174 [169, 364]  | 1.79 [1.34, 2.42]  | 6.71 [2.45, 18.7] | Ratios, "rocky," $N$-body |
|              | 145 [145, 381]  | 1.68 [1.26, 2.27]  | 3.95 [1.98, 6.35] | Clustered, "volatile," $N$-body |
|              | 145 [145, 376]  | 1.77 [1.33, 2.41]  | 6.45 [2.04, 6.43] | Clustered, "rocky," $N$-body |
| H[b] PnP–2    | 86.3 [84.8, 206] | 1.65 [1.23, 2.23]  | 3.84 [1.84, 6.18] | Predicted | Ratios, "volatile," $\Delta = 8$ |
|              | 86.3 [85.4, 204] | 1.72 [1.28, 2.31]  | 5.85 [2.11, 16.2] | Ratios, "rocky," $\Delta = 8$ |
find that this gap could likely hide two planets. The outer peak from hypothesis H[a], which we denoted as PxP–2 H[a], is also visible here, so we update its classification to PxP–2 (H[a], H[c]).

**Hypothesis H[d].** We see here the same spread-out pattern between 10 and 30 days indicative of two planets from hypothesis H[c]. We also find that the outer peak has again shifted as in hypothesis H[b] to a similar position as PxP–2 H[b], so PxP–2 (H[b], H[d]) does provide support for planet candidate g.

### 4.3. N-body Integrations

Our results from the dynamical stability criteria that utilized N-body integrations differed for each of the planet hypotheses. The spectral fraction analysis showed very similar results to the mutual Hill radius, even in areas near the mutual Hill radius boundary of 8 (see Figure A2). There were some small yet significant differences, including in areas near orbital period resonances, but most of the difference was found to be random noise generated from the Monte Carlo iterations.

SPOCK’s feature classifier finds that the system architectures for hypotheses H[a]–H[d] would be stable without injecting planets, as the simple stability predictor returns 0.479 ± 0.028, 0.648 ± 0.036, 0.636 ± 0.022, and 0.651 ± 0.023, respectively, for each of the hypotheses. However, when additional planets are injected via Monte Carlo methods, the system under hypothesis H[a] is pushed just past the stability limit of 0.34 from the predictor (set by the 10% false-positive rate), down to 0.328 ± 0.092. The remaining hypotheses, H[b]–H[d], have simple stability predictors of 0.868 ± 0.116, 0.510 ± 0.047, and 0.910 ± 0.052, which are all considered stable.

SPOCK’s deep regressor also shows that all four hypotheses are stable as they stand, with the stability predictor having values of 0.407 ± 0.021, 0.520 ± 0.030, 0.635 ± 0.020, and 0.824 ± 0.019. Adding in another planet to hypothesis H[a], however, causes the stability predictor to drop to 0.135 ± 0.025, and no additional planet was considered stable long-term. In addition, injecting a planet into hypothesis H[b] also drops the stability predictor down to 0.246 ± 0.046, again with no additional planet found to be stable. For hypothesis H[c], the stability predictor was 0.349 ± 0.059 over the injection tests, with 60% of the injections accepted as long-term stable. Hypothesis H[d] was stable with respect to injections with a stability predictor value of 0.473 ± 0.080.

The N-body integrations did find dips in the relative likelihood near resonances, a feature that is missed by the mutual Hill radius criterion, which is focused more on the space closer to planets. Figure 6 shows the difference between the spectral fraction analysis and the mutual Hill radius for the predicted planets between planets c and f in hypothesis H[a].

| Name | Period (days) | Radius ($R_\oplus$) | Mass ($M_\oplus$) | Note | Origin/Reference |
|------|--------------|---------------------|------------------|------|-----------------|
| 73.9 [73.9, 167] | 1.65 [1.22, 2.22] | 3.84 [1.79, 6.09] | Clustered, “volatile,” $\Delta = 8$ |
| 74.1 [74.1, 169] | 1.70 [1.27, 2.28] | 5.61 [2.05, 15.5] | Clustered, “rocky,” $\Delta = 8$ |
| 86.3 [84.6, 205] | 1.65 [1.23, 2.23] | 3.84 [1.84, 6.18] | Ratios, “volatile,” N-body |
| 86.3 [85.2, 203] | 1.72 [1.28, 2.31] | 5.85 [2.11, 16.2] | Ratios, “rocky,” N-body |
| 73.9 [73.9, 166] | 1.65 [1.22, 2.22] | 3.84 [1.79, 6.09] | Clustered, “volatile,” N-body |
| 74.1 [74.1, 168] | 1.70 [1.277, 2.28] | 5.61 [2.05, 15.5] | Clustered, “rocky,” N-body |
| H[c] PxP–2 | 174 [169, 386] | 2.13 [1.30, 3.81] | Predicted | Ratios, “volatile,” $\Delta = 8$ |
| | | 1.77 [1.32, 2.38] | Ratios, “rocky,” $\Delta = 8$ |
| | 141 [140, 389] | 1.91 [1.25, 3.62] | | Clustered, “volatile,” $\Delta = 8$ |
| | | 1.70 [1.28, 2.30] | | Clustered, “rocky,” $\Delta = 8$ |
| | 174 [169, 416] | 2.14 [1.30, 3.84] | | Ratios, “volatile,” N-body |
| | | 1.78 [1.32, 2.40] | | Ratios, “rocky,” N-body |
| | 141 [140, 414] | 1.92 [1.25, 3.63] | Clustered, “volatile,” N-body |
| | | 1.71 [1.28, 2.32] | Clustered, “rocky,” N-body |
| H[d] PxP–2 | 86.3 [85.2, 206] | 2.05 [1.29, 4.04] | Predicted | Ratios, “volatile,” $\Delta = 8$ |
| | | 1.65 [1.23, 2.22] | Ratios, “rocky,” $\Delta = 8$ |
| | 70.2 [69.3, 80.2] | 1.82 [1.23, 3.66] | Clustered, “volatile,” $\Delta = 8$ |
| | | 1.59 [1.20, 2.09] | Clustered, “rocky,” $\Delta = 8$ |
| | 86.3 [84.9, 205] | 2.06 [1.29, 4.04] | Ratios, “volatile,” N-body |
| | | 1.66 [1.23, 2.23] | Ratios, “rocky,” N-body |
| | 70.2 [69.3, 80.1] | 1.83 [1.23, 3.66] | Clustered, “volatile,” N-body |
| | | 1.60 [1.20, 2.09] | Clustered, “rocky,” N-body |

Note: In the PxP naming scheme, H[.] are the different hypotheses as in Section 3.2. Radius is measured for planets b and c, predicted from the mass–radius relation for the remaining planets and predictions. A mass value is given for transit-verified planets b and c, as well as predicted planets, whereas an $m \sin i$ value is given for RV-verified planets d, f, and g with unknown inclination. Planets denoted “Planet?” are the two planets that we tested removing from the system. The specific dynamite period model, mass–radius relation, and dynamical stability criterion used are noted in the last column for the predicted planets. The inclination and eccentricity models were fixed as the multiplicity-dependent lognormals from SysSim (He et al. 2020).
The low-order resonances 2:1 and 3:2 were located at dips in the injections in period space, and additional higher-order resonances 5:3 and 11:6 were added as they coincided with either the same resonance for both planets (11:6) or the 2:1 resonance for the other planet (5:3). The mutual Hill radius criterion does not see this effect.
5. Discussion: DyNAMITE Predictions

5.1. Inclination and Eccentricity Distributions

For planets without inclination measurements and $m \sin i$ values measured by RV observations, the true masses can vary widely due to the assumption of system inclination isotropy, which widens the resulting predicted distribution for the predicted planet mass. For isotropically distributed inclinations, the potentially large difference in inclination between neighboring planets also affects their dynamical stability in the $N$-body integrations, decreasing the available space for injecting stable planets between them. Allowing for unknown inclinations provides the most accurate prediction of the likelihood for planet injections, especially when utilizing $N$-body integrations.

Introducing the eccentricity to the planet parameters for which DyNAMITE tracks in its analysis provides an additional measure in the dynamical stability. While the mutual Hill radius criterion is independent of the eccentricity of the planets, it is easy to determine that a highly eccentric planet would strongly affect the dynamical stability in a way that the mutual

Figure 4. Histograms for the inclinations (top) and eccentricities (bottom) of the predicted injections from DyNAMITE for hypothesis H[a]; the inclinations for planets b–d are unknown and therefore not known to the DyNAMITE algorithm or included in the analysis. The left figures use the Rayleigh + isotropic fraction model for inclinations and the Rayleigh model for eccentricities, while the right ones use the multiplicity-dependent lognormals from the analysis by He et al. (2020). All of the above use the equal ratios period model, the “volatile-rich” mass–radius relation, and the mutual Hill radius dynamical stability criterion.

Figure 5. Relative likelihood assuming hypothesis H[c] for the known planets, but the position of PnP–1 comes from the peak in relative likelihood from hypothesis H[a]. This shows that the lower and more spread out likelihood of injections seen in hypotheses H[c] and H[d] is indicative of two potential planets.
Hill radius criterion would be blind to. While it is somewhat unlikely to have highly eccentric planets in higher-multiplicity systems (e.g., He et al. 2020), our ability to include the eccentricities of the known planets in our predictions helps increase the accuracy of our dynamical stability models and constrain the available parameter space for additional planets.

5.2. Support for Planets f and g

Planet f. Based on their RV measurements, V15 detailed the presence of planet f at 22.81 days (not found in the analysis by M15) and noted the closeness to the rotational period gathered from $v \sin i$ measurements. By testing different segments of the RV curve for rotational spot modulation

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**Figure 6.** The 2:1, 3:2, 5:3, and 11:6 resonances plotted for an additional planet along with planets c and f in hypothesis H[a]. The 2:1 and 3:2 were chosen as low-order strong resonances, whereas 5:3 and 11:6 are shown because they overlapped a 2:1 resonance or the 11:6 resonance of the other known planet. Top: injections (in green) via the $\sim 10^6$ orbit N-body integration dynamical stability criterion, which shows lower probabilities of injection near the resonances. Bottom: injections (in blue) via the mutual Hill radius dynamical stability criterion, which shows no such consistent effect. The difference in the y-scale is due to finding fewer planets overall in this gap, as the likelihood gets normalized to 1 across all injections.
residual signals, V15 determined that the 22.8 day signal was likely to be real and not a result of the stellar rotation. However, a follow-up analysis by Johnson et al. (2016) narrowed the stellar activity indicators to a period of 22.83 ± 0.03 days, very similar to the reported period of planet f. Thus, planet f is reported as “controversial” in the NASA Exoplanet Archive, even though a follow-up analysis of the transiting planets by G17 indicates that this planet is extremely computationally expensive, both of our new integrations to the standard of 10^9 orbits of the innermost planet in the Habitable Zone remain true for all of the implementations of the dynamical stability criterion, as the given and expected eccentricities of planet g and PnP–2 are low, which lessens any effect of the difference between the implementations. We find that an additional planet of ∼4 M⊕ (near the median of the predicted planet mass distribution) could have an orbital period as close as ∼140 days and still be stable with respect to planet g, whereas a planet of ∼11 M⊕ (similar to the masses of the two most exterior planets) could only be located exterior to 150 days and still be stable with respect to planet g.

Rosenthal et al. (2021) reported three false-positive RV signals in the HD 219134 system that they attributed to annual and/or instrumental systematics. One of these signals has a period of ∼28 yr, which is close to the entire length of their legacy survey. However, the other two signals are at 192.06 ± 0.49 and 364.3 ± 1.3 days. While it is likely that these are false positives from an annual and semiannual systematic from Earth-based measurements, these signals are relatively close to where we would expect additional planets to exist in this system. The reported respective semi-amplitudes of 2.0 ± 0.21 and 1.66 ± 0.35 m s⁻¹ would correspond to m sin i values of 13.0 ± 2.4 and 15.7 ± 1.5 M⊕, which would lie in between the reported values for planets d and g, the two most massive planets in the inner system and the two furthest out (disregarding the gas giant in the outer system). These values would be either outside or just inside the 16th–84th percentile range for the predicted mass using the volatile-rich mass–radius relationship (depending on which hypothesis of the system architecture is utilized for the prediction) but would likely still lie within that range for the rocky mass–radius relationship for all system architectures.

### 6. Discussion: Dynamical Stability

Our new dynamical stability criteria utilize N-body integrations to determine the level of stability in a system to a much higher degree of accuracy. However, as using N-body integrations to the standard of 10^9 orbits of the innermost planet is extremely computationally expensive, both of our new models run for shorter integrations (10^4 orbits for SPOCK and 10^6 orbits for the spectral fraction analysis) and predict the
likelihood of stability extended out to $10^9$ orbits from the results of the short integrations. Whereas the mutual Hill radius measure is essentially instantaneous and thousands of Monte Carlo realizations can be run per second on a standard laptop without utilizing multiprocess threads, the $N$-body integrations need thousands of CPU hours on multiple nodes of a high-performance computer cluster to be run efficiently.

For the $N$-body analyses, we count Monte Carlo realizations of systems with injections as unstable if they do not survive the initial integration times. SPOCK simply returns a Boolean value for the system survival (i.e., without hyperbolic orbits) in the first $10^4$ orbits of the innermost planet, but for the spectral fraction analysis, we manually reject crossing orbits, as well as ejected planets. Figure 8 shows the integration times at which different Monte Carlo realizations of HD 219134 under hypothesis H[a] went unstable in the $N$-body integration, as well as showing the spectral fraction analysis for those that survived the initial $10^6$ orbits of the innermost planet.

Not all orbital crossings result in prompt planetary collisions and/or ejections (e.g., Neptune’s and Pluto’s orbits cross). In particular, pairs of similar-mass planets can actually persist in such an unstable configuration for timescales of tens to hundreds of megayears if the mutual inclinations between planets are large enough ($\gtrsim 1^\circ$; Rice et al. 2018). However, for the HD 219134 system, the relatively large eccentricities and (likely) mutual inclinations of order $\gtrsim 1^\circ$ would likely cause increasing instability of the crossing orbits, and for the $\sim 10$ Gyr age of this system, we would expect most crossing orbits to become unstable on this timescale.

6.1. Mutual Hill Radius

The mutual Hill radius measure of dynamical closeness has been a predictor of stability for decades (see, e.g., Gladman 1993). If a pair of neighboring planets’ semimajor axes are within a certain distance of each other, based on their mass and measured by their Hill spheres of influence, then they will eventually cause the planets to become unstable. This measure provides a simple and fast prediction for any pair of neighboring planets, as the only calculations it requires are basic arithmetic.

The theoretical limit for the stability of two-planet systems is that the difference of their semimajor axes (in units of their mutual Hill radius) should not be smaller than $\Delta = 2\sqrt{3}/\approx 3.46$. However, there tends to be a distribution in the mutual Hill radius for exoplanet pairs instead of a sharp cutoff (e.g., Malhotra 2015). In practice, it is found that systems of three or more planets require significantly larger orbital separations of neighboring planets; Pu & Wu (2015) reported $\Delta \gtrsim 12$ for the long-term stability of Kepler multis, while Gilbert & Fabrycky (2020) reported that a large percentage of
The utilization of machine learning and training on hundreds of thousands of planetary systems provides a marked improvement in speed over full N-body integrations, as well as accuracy over the simpler near-instantaneous methods. SPOCK’s simple FeatureClassifier and DeepRegressor both perform much faster than the ground-truth full N-body simulations on which they are trained, as well as the spectral fraction analysis, which is performed on integrations 100× longer than SPOCK. SPOCK provides a probabilistic measure of stability for a single system architecture based on the data sets from which it has learned. However, while the training sets attempt to cover as many system architectures as possible, they can cover only a small subset of the uncountable infinity of possible planetary architectures; therefore, they may not provide as accurate a stability analysis as a full N-body integration.

In our study, SPOCK shows a high level of instability for the more complicated system architecture hypotheses. SPOCK’s FeatureClassifier finds that hypotheses including planet g are shown to be less stable than those where it is excluded, as hypotheses H[α] and H[c] have stability predictors of ≤0.5, while those of H[b] and H[d] are ≥0.85. This suggests that the presence of planet g (and any additional planets beyond it) might negatively affect the stability of the other planets interior to planet g. In addition, the hypotheses including planet f tend to have a higher scatter in the likelihood of stability, as H[a] and H[b] have stability predictor standard deviations of 0.092 and 0.116 as compared to 0.047 and 0.052 for H[c] and H[d]. This is likely a consequence of the moderate reported eccentricity of planet f and the relatively high probability of injecting a planet near 12 days between planets c and f.

SPOCK’s DeepRegressor provides additional samples of the short N-body integration to determine the probability of stability at a more in-depth level while sacrificing some of its speed. The DeepRegressor showed similar results, with all four system architecture hypotheses stable without any additional planets, albeit with a lower stability predictor than the corresponding FeatureClassifier value. However, adding in another planet to both hypotheses H[α] and H[b] causes their stability predictors to drop well below the stability threshold, while hypothesis H[c] remains barely above the value. Hypothesis H[d] is still stable with injections, but its stability predictor is lower than all of the values without additional planets, except for the five-planet architecture in hypothesis H[a]. In other words, the DeepRegressor finds that the HD 219134 system is relatively near the stability limit even with only the three confirmed planets b–d.

In general, if test planets are injected into hypothesis H[a] anywhere or into hypotheses H[b]–H[d] at places that do not specifically correspond to the period of the planet candidates excluded from the analysis of the system, then SPOCK finds that the injections would likely cause the system to become unstable (and are therefore rejected). The SPOCK analysis differs from the mutual Hill radius measure and the spectral fraction analysis of HD 219134 in this way, as the other two do not have as large a fraction of rejected injections. In general, SPOCK predicts that the currently known three to five planets in the HD 219134 system with orbital periods within 100 days would place the system on the brink of long-term orbital dynamical stability, with little to no room for additional planets.

6.3. Spectral Fraction

The spectral fraction analysis is closer to the ground truth of the full N-body integrations with longer test runs but still provides a usable speed for a system given enough computing resources. This analysis requires ~1 CPU hr per 10 Monte Carlo iterations, so the 10^5 iterations used in this analysis took multiple hours on almost 1000 cores of a high-performance computing cluster.

The spectral fraction analysis found results very similar to those from the mutual Hill radius measure, as the injection acceptance rate for each method was 1%–2%. In addition, a very small number of iterations of the system with an additional planet (≤5 × 10^{-4}) were actually found to be unstable via the empirically determined threshold values for the spectral fraction from the analysis by Volk & Malhotra (2020). This could likely be a feature of our crossing orbit limit; if a system survives through 10^9 orbits but two planets have crossing orbits that are unstable from the spectral fraction, the results count this as unstable via crossing orbits and not from the spectral fraction analysis. If the empirical threshold (1% of the total power spectrum is within 5% of the peak value) is loosened slightly (i.e., 1% of the total power spectrum is within 1% of the peak value, as a majority of the power spectrum is multiple orders of magnitude below the peak), we see a modest increase in the fraction of injections found to be unstable via the spectral fraction to ~2 × 10^{-5}. Therefore, it is likely that very few injections into the HD 219134 system would allow for secular chaos to cause instabilities that would be noticeable in the spectral fraction analysis, even for the highest-multiplicity system architecture hypothesis H[a].

However, one significant quantitative difference is the effects of resonances. In particular, in the results from hypotheses H[a] and H[c] that include planet f, the N-body simulations and spectral fraction analysis find notable drops in the period relative likelihood at both the 2:1 and 3:2 resonances external to planet c and internal to planet f, as well as the overlapping 5:3 (with 2:1) and 11:6 (with itself) resonances. This is consistent with the known pattern in the exoplanet population (e.g., Petrovich et al. 2013; Ramos et al. 2017). However, there are notable examples of multiplanet resonant chains that are stable (e.g., Gillon et al. 2017b; Hardege-Ullman et al. 2021).

6.4. Lessons Learned on Dynamical Stability Analysis

The mutual Hill radius dynamical stability criterion seems to be a good blend of accuracy and speed. The results are similar to the N-body integration spectral fraction analysis within 0.5% for a very large majority of cases where Δ ≥ 10. Furthermore,
even running $10^5$ Monte Carlo iterations only takes a few seconds on a standard eight-core desktop computer. Thus, in general, it would be the likely method of choice for many multiplanet systems.

However, if the system architecture near the resonances is being tested, or if there are tightly packed planets where an injection is still deemed likely by the mutual Hill radius criterion, then an $N$-body integration method would likely be more useful. SPOCK can be utilized to gain a quick understanding of the likely stability of the system before any additional planets are injected. If the value SPOCK reports is near the stability threshold, then the spectral fraction analysis can be performed to determine the stability at a deeper level than the simple model; this requires more computational resources to be available.

For future studies, we will create a hybrid model that chooses which criterion to utilize for a given Monte Carlo iteration of the injected planet parameters. If the distance between planets is within a certain value of the mutual Hill radius (larger than the current limit) or near a low-order resonance, we would apply a method with $N$-body integrations. For the majority of Monte Carlo iterations (those that lay outside of the mutual Hill radius and resonance constraints), we would use the simple stability criterion of the mutual Hill radius when adding in the additional planet.

7. Discussion: Solar System Comparison

To compare the results from our different stability criteria for the best-known system, we tested our analysis on a few variants of the inner solar system to check the performance. We analyzed systems consisting of the terrestrial planets plus Jupiter and tested excluding Venus or Earth from this group of planets. As for our analysis of HD 219134, we used the period ratio model for the orbital periods, the volatile-rich power law for the mass–radius relationship, and the SysSim multiplicity-dependent lognormals for the inclination and eccentricity models. Figure 9 shows the results from this comparison.

We again find in this case that there is very little difference in the mutual Hill radius measure versus the $N$-body integration methods, as both SPOCK and the spectral fraction analysis agree that the system would be long-term stable even with an additional planet injected. All methods agree that an additional planet would likely be found interior to Mercury (around 35–40
days, given that we are utilizing the equal period ratio model with a constraint on the innermost planet in a system likely being near a 12 day orbit; Mulders et al. 2018), as well as between Mars and Jupiter at ∼1800 days. Of course, this second peak in the relative likelihood roughly corresponds to the location of the asteroid belt in the solar system, which provides another line of questioning that is beyond the scope of this study: could the location of a predicted planet actually harbor a debris disk or ring, and would it be visible in infrared imaging of the system?

When Venus is excluded from the analysis of the inner solar system, there is an additional large peak in the relative likelihood at ∼190 days, slightly interior to the actual orbit of Venus at 224 days. When Earth is excluded, there is a large peak at ∼390 days, slightly exterior to the actual orbit of Earth. Interestingly, in the case where Earth is excluded, all dynamical stability methods predict a small but noticeable probability of injecting a planet between Mercury and Venus at ∼140 days. However, given the eccentricity of Mercury, it is likely that any planet positioned at this spot in period space could cause Mercury and/or the other planet to become unstable. DyNAMITE also finds a significant likelihood of another planet interior to the orbit of Mercury, although less likely than at the orbital period of the planet excluded from the analysis. We do know that there are no planets interior to Mercury, but it is well known that in this aspect, the solar system is atypical (e.g., Mulders et al. 2018); observers from another planetary system who have determined similar exoplanet demographic data to us would likely be surprised to not find a planet within 88 days in the solar system.

The distribution for the predicted planet radius for additional planets in the inner solar system is shown in Figure 10. We find it is well matched to the terrestrial planets, since Jupiter is much larger than the rest of the planets and therefore would not be in the same “cluster” of planet radius via the clustered radius model from SysSim (He et al. 2019). DyNAMITE does find that there would be some probability of finding a planet with a comparatively large physical size, but the main peak in the planet radius distribution is centered on the size of the terrestrial planets.

In conclusion, our analysis is quite successful at predicting the presence of whichever planet is excluded from the inner solar system, as well as the position of the asteroid belt corresponding to a potential additional planet. While the solar system is atypical as compared to the systems in the Kepler population on which our analysis is based (e.g., Mulders et al. 2018), the accurate predictions from DyNAMITE on the solar system offer another validation of our analysis.

8. Summary

We present an integrated analysis of the HD 219134 planetary system, the closest known six-planet system with up to six planets and planet candidates, using an updated version of DyNAMITE. The key updates and findings of our study are as follows.

1. We update and expand the DyNAMITE software by allowing for unknown inclinations in the planet parameter distributions and including measurements of planet eccentricities.
2. We present advanced dynamical stability criteria (including the inclinations and eccentricities) that utilize N-body integrations. One method uses machine learning to predict stability, and the other performs a “spectral fraction” analysis of the AMD in the system.

3. Utilizing both the simple mutual Hill radius dynamical stability criterion and the updated N-body integration spectral fraction analysis, in hypotheses H[a] and H[c] (where planet f is considered to be real), we find evidence for predicted planet P×P=1 with an orbital period of ~12 days (in between current planets c and f), which would dynamically pack the inner system out to planet g.

4. Utilizing the same two dynamical stability criteria, in hypotheses H[b] and H[d] (where planet f is considered to be uncertain), we find a large integrated probability spread out between the peak for P×P=1 in hypotheses H[a] and H[c] and the expected position of planet f. When P×P=1 is injected at 12 days and we run DyNAMITE again, we find that the new injections recover the period of planet f almost exactly.

5. Using the SPOCK machine-learning predictions, we find that all hypotheses are considered stable without injections, and SPOCK tends to find that including planet g makes the system more unstable in general, whereas including planet f increases the scatter in the likely stability.

6. Using the SPOCK simple feature classifier, we find that the dynamical architecture of hypothesis H[a] is likely fully packed and unstable with any additional planets, while the other hypotheses allow room for additional planets. Using the SPOCK deep regressor predictor, we find that hypotheses H[a] and H[b] are unstable with an additional planet, hypothesis H[c] is very near the stability limit when injecting a planet, and hypothesis H[d] is stable with injections.

7. We find support for both “controversial” planets, planet f with an orbital period of 22.7 days (very near the likely rotation period of its host star) and planet g with an orbital period of 94.2 days (only found in one study of the system through this year and with much larger uncertainties).

8. We also find support for a predicted planet P×P=2 external to the current inner system that would have an orbital period of ~174 days, on the inner edge of the habitable zone, with a likely planet mass of ~4M⊕. Assuming dynamical packing and continuing outward from there, another planet could be found at an orbital period of ~321 days, in the outer parts of the habitable zone. Long-term signals have been found in RV data at ~192 and ~364 days but are found to be more likely to be aliases from Earth’s orbital period (Rosenthal et al. 2021).

9. We apply this integrated analysis to the inner solar system up to and including Jupiter and test excluding one of the terrestrial planets from the input parameters. Our predictions from the analysis are excellent matches to the known values from the solar system planets, validating our results.

Our study demonstrates how additional information on dynamical stability can influence the understanding of system architectures and the possibility of finding additional “hidden” planets in these systems. The HD 219134 system, being the closest system to our Sun with up to six planets and planet candidates, was a prime case on which to test our DyNAMITE algorithm’s integrated analysis. We find that dynamically complex systems (i.e., systems with high multiplicity, high eccentricity, etc.) would still allow the presence of another planet without losing dynamical stability. Our study also provides support for the controversial planets/candidates in the HD 219134 system and predictions for additional planets, one of which would lie in the inner part of the habitable zone. This would make HD 219134 an even more enticing target for direct imaging studies both from space and on the ground in the near future.

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Appendix

DyNAMITE Injection Figures

As stated in Section 4, our results from the Monte Carlo injections are found here, with the mutual Hill radius dynamical stability criterion used in Figure A1 and the N-body spectral fraction analysis used in Figure A2.
Figure A1. DyNAMITE analysis utilizing the simple dynamical stability criterion requiring neighboring planets’ separation in semimajor axis to exceed 8 mutual Hill radii. Top left: analysis assuming Hypothesis H[a], the full five-planet system. Top right: analysis assuming Hypothesis H[b], with no planet f (orbital period near stellar rotation period). Bottom left: analysis assuming Hypothesis H[c], with no planet g (not found in all studies). Bottom right: analysis assuming Hypothesis H[d], with no planet f or g. Excluding planet f causes the predictions to spread out in period space, indicative of two planets possibly missing. Excluding planet g moves the exterior injection space to center almost directly on the period of planet g.
Figure A2. DyNAMITE analysis utilizing the $N$-body simulations and testing orbit crossing and the AMD spectral fraction. Top left: analysis assuming a full five-planet system. Top right: analysis assuming no planet f (orbital period near stellar rotation period). Bottom left: analysis assuming no planet g (not found in all studies). Bottom right: analysis assuming no planet f or g. The results here are similar to the simple dynamical stability criterion, only with tighter restraints near the $\Delta = 8$ limit and at orbital resonances.

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