Electromagnetic Fields and Charged Particle Motion Around Magnetized Wormholes

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Abstract We perform a study to describe motion of charged particles under the influence of electromagnetic and gravitational fields of a slowly rotating wormhole with nonvanishing magnetic moment. We present analytic expression for potentials of electromagnetic field for an axially symmetric slowly rotating magnetized wormholes. While addressing important issues regarding the subject, we compare our results of motion around black holes and wormholes in terms of the ratio of radii of event horizons of a black hole and of the throat of a wormhole. It is shown that both radial and circular motions of test bodies in the vicinity of a magnetized wormhole could give rise to a peculiar observational astrophysical phenomenon.

Keywords Magnetized wormholes Electromagnetic fields Charged particle motion

1 Introduction

Astrophysical objects called wormholes link widely separated regions of a Universe or of two different Universes, joining two different spacetimes (Morris & Thorne 1988; Visser 1995).

The subject of strong electromagnetic fields due to highly magnetized rotating neutron stars like pulsars and magnetars is of great relevance for the physics of a Wormhole (WH) and for the particle motion around it, specially around its throat.

In paper (Teo 1998) author in details considered solution for rotating WH and properly described ergoregion, which surrounds the throat at the equator of WH. A wormhole may be a reason for gravitational lensing effects (Dev & Sen 2008) and may also create a black hole through accretion of matter (Kardashev et al. 2007). Thus, it is a subject of astrophysical importance. The motion of particles around a wormhole and a possible dragging of such moving particles towards its vicinity constitute a subject of Physical reality. To make a Lorentzian wormhole traversable and stable, one uses exotic matter, which violates the well-known energy conditions according to the need of the geometrical structure (Morris et al. 1988; Visser et al. 2003; Dadhich et al. 2002). Viable models for such a WH have recently been studied (Kuhfitting 2008; Harko et al. 2008; Lobo 2005a; Sushkov 2003; Lobo 2005b; Böhmer et al. 2008).

The rotation of a magnetized star in vacuum induces electric field (Deutsch 1955). General Relativity (GR) generates additional electric field (See, for example, (Muslimov & Tsygan 1992; Konno & Kojima 2000; Rezzolla et al. 2001a, 2001b)) through its role in the context of dragging of inertial frames and becomes very important in pulsar magnetosphere (Biskin 1990; Muslimov & Tsygan 1990). Under the framework of GR, slowly rotating wormholes have been a subject of study, particularly in the context of stress-energy tensor (Bergliaffa & Hibberd 2000), scalar fields (Kashargin & Sushkov 2008; Kim 2003) and electromagnetic fields (Jamil & Rashid 2008). The exact solutions of the wormhole with classical, minimally coupled, massless scalar field, and electric charge are discussed in the paper (Kim & Lee 2001). They concluded that the addition of electric charge might change the gravitational field of the WH but will not change the spacetime seriously.
Here, we focus on the motion of charged test particles in gravitational and electromagnetic field of slowly rotating wormhole with magnetic dipole momentum. We use Hamilton-Jacobi equation to find influence of both the fields on the effective potential due to radial motion of test particles. In section 2, we calculate potential of electromagnetic field due to axially-symmetric slowly rotating magnetized wormhole.

We then consider the separation of variables in the Hamilton-Jacobi equation and derive the effective potential for the motion of charged particles around slowly rotating wormhole with dipolar electromagnetic field in Section 3. We also calculate stable circular orbits for charged particles in terms of the magnetic moment of the wormhole and offer a table. We present the numerical results for periods of anharmonic oscillations of charged particles. Finally, we conclude our main findings and present some astrophysical applications of our results in section 4. Our present investigation of the motion of charged particles around a slowly rotating magnetized wormhole involving these potentials is carried out with the aim to find astrophysical evidence for the existence of such objects and to explore its possible differences with the other class objects called black holes. Finally, on the contrary to the model (Kardashev et al. 2007), for our WH model which has a magnetic dipole momentum one can obtain the observable difference on circular motion of charged particle around WH and around compact object as stars, BH etc.

Throughout the paper, we use a space-like signature \((- + + + +)\) and a system of units in which \(G = c = 1\) (However, for those expressions with an astrophysical application we have written the speed of light explicitly.). Greek indices are taken to run from 0 to 3 and Latin indices from 1 to 3; covariant derivatives are denoted with a semi-colon and partial derivatives with a comma.

2 Potential of the Electromagnetic Field

Around a Wormhole

We may safely ignore quadratic term of the angular velocity \((\omega)\) of the free falling frame because of the slow rotation of the wormhole. Thus, the metric that describes spacetime around an axially symmetric slowly rotating wormhole, may be written in the following form (Kardashev et al. 2006; Shatskii 2007):

\[
ds^2 = -e^{2\phi(r)} \cdot dt^2 + \left[ 1 - \frac{b(r)}{r} \right]^{-1} dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\varphi^2 \right) - 2\omega(r) r^2 \sin^2 \theta d\varphi dt. \tag{1}
\]

Here, \(r\) is the radial coordinate, \(\phi(r)\) is the so-called lapse function, \(b(r)\) is the shape function. The \(\omega(r) = 2J/r^3\) is also known as Lense-Thirring angular velocity, where \(J\) is the total angular moment of the gravitating object. The neck of the wormhole corresponds to the minimum \(r = r_0 = b(r_0)\), where we have \(\partial b/\partial r|_{r_0} \leq 1\). The presence of a horizon implies \(\phi \rightarrow -\infty\) or \(e^\phi \rightarrow 0\) such that \(\phi\) is finite everywhere.

Solution of the Einstein equations for WH has been compared with Reissner-Nordstrom solution for compact objects with upper limit for magnetic charge in ref. (Kardashev et al. 2006), wherein components of the WH metric (1) have been written as

\[
\exp \phi = \left( 1 - \frac{r_h}{r} \right)^{1+\delta}, \tag{2}
\]

and

\[
b(r) = r_h \left[ 1 + \left( 1 - \frac{r_h}{r} \right)^{1-\delta} \right]. \tag{3}
\]

The quantity \(\delta\) in the above expressions, (2) and (3), may be found from transcendental equation \(b(r_0) = r_0\):

\[
\delta = \frac{\ln \left( \frac{r_b}{r_0} \right)}{\ln \left( \frac{r_b}{r_0} \right) - 1}. \tag{4}
\]

Common to considered model by (Kardashev et al. 2006) is the assumption that the tunnel of magnetic WH is penetrated by an initial magnetic field, which should display a radial structure for an external observer in the spherically symmetrical case; i.e., it should be correspond to a macroscopic magnetic monopole. It means that metric (1) with expressions (2) and (3) correspond to the wormhole mode of the substance which is the monopole magnetic field and dipole electric field in slowly rotating approximation (equations (12) and (25) of the paper (Kardashev et al. 2006)):

\[
B^\hat{\alpha} \approx \frac{q_m}{r^2}, \tag{5}
\]

\[
E^\hat{\alpha} \approx \frac{2ar^2}{r^3} B^\hat{\beta}(r_h) \cos \theta, \tag{6}
\]

\[
E^\hat{\theta} \approx \frac{ar^2}{r^3} B^\hat{\beta}(r_h) \sin \theta, \tag{7}
\]

where \(q_m\) can be interpreted as magnetic monopole of magnetic WH described with metric (1) in addition with expressions (2) and (3). Here \(\hat{\alpha}\) (hat) stands for orthonormal components of the electric and the magnetic fields:

\[
E^\alpha = F_{\alpha\beta} u^\beta, \quad B^\alpha = -\frac{1}{2} \eta^{\alpha\beta\gamma\rho} F_{\beta\gamma} u_\rho, \tag{8}
\]
that are measured by zero angular momentum observer (ZAMO) with four velocity
\[ u^\alpha = e^{-\phi}(1, 0, 0, \omega), \quad u_\alpha = e^{\phi}(1, 0, 0, 0), \]
where
\[ \eta^{\alpha\beta\gamma\rho} = -\frac{1}{\sqrt{-g}}\epsilon_{\alpha\beta\gamma\rho}, \]
\[ \epsilon_{\alpha\beta\gamma\rho} \]
is the Levi-Civita symbol.

Tensor of electromagnetic field in the presence of monopole magnetic charge \( q_m \) can be described as (Cabibbo \\& Ferrari 1962):
\[ F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu} - \eta_{\mu\nu\rho\sigma} \tilde{A}^{\rho\sigma}, \quad (9) \]
\[ \tilde{F}_{\mu\nu} = \tilde{A}_{\nu,\mu} - \tilde{A}_{\mu,\nu} + \eta_{\mu\nu\rho\sigma} \tilde{A}^{\rho\sigma}, \quad (10) \]
which is included not only the 4-vector potential \( A_\mu \), but also a pseudovector \( \tilde{A}_\mu \). Lagrangian of the system can be written as:
\[ L = L_{em} + L_{int} + L_m = -\frac{1}{4} [n^\mu (A_{\nu,\mu} - A_{\mu,\nu})]^2 - \frac{1}{2} [n^\mu (\tilde{A}_{\nu,\mu} - \tilde{A}_{\mu,\nu})]^2 - \frac{1}{4} \xi_{\mu\nu\rho\sigma} n^\mu n^\nu \tilde{A}^{\rho\sigma} n^\alpha \tilde{A}_{\alpha\mu} + \frac{1}{4} \xi_{\mu\nu\rho\sigma} n^\mu n^\nu A^{\rho\sigma} n^\alpha \tilde{A}_{\alpha\mu}, \quad (11) \]
where \( n^\mu \) is an arbitrary fixed unit four-vector. Hamilton-
Jacobi equation corresponding to this system can be written as:
\[ g^{\alpha\beta} \left( \frac{\partial S}{\partial x^\alpha} + e_{\text{em}} \tilde{A}_\alpha \right) \left( \frac{\partial S}{\partial x^\beta} + e_{\text{em}} \tilde{A}_\beta \right) = -m^2. \quad (12) \]
The value of pseudovector \( \tilde{A}_\mu \) being responsible for the electromagnetic field \( \tilde{F}_{\mu\nu} \) can be found using equations \( 8 \) and \( 9 \), \( 10 \) as
\[ \tilde{A}_\alpha = \frac{q_m}{r} \left( 0, 0, 0, \frac{1 - \cos \theta}{\sin \theta} \right), \quad (13) \]
having singularity at \( \theta = \pi \) which can be removed by coordinate transformations.

Using the variable separation technique one can easily find equation of radial motion of charged particle around magnetic monopole:
\[ f(r, \delta) \left( \frac{dr}{d\sigma} \right)^2 = \mathcal{E}^2 - V_{\text{eff}}(q_m, r, \tilde{\mathcal{L}}), \quad (14) \]
where \( \tilde{\mathcal{L}} = \mathcal{L} + eq_m \), \( \mathcal{L} \) is the angular momentum of the charged particle moving around magnetic monopole. \( f(r, \delta) \) is a function which tends to 1/2 when spacetime metric \( \mathbb{H} \) becomes flat one and
\[ V_{\text{eff}} = \frac{\tilde{L}^2}{2m r^2}. \quad (15) \]

Study of the influence of magnetic monopole to the motion of charged particles around WH can be not considered when the massive WH monopolar magnetic field of the magnetized WH is negligible \( (B \lesssim 10^7 G) \) for object with total mass \( M \simeq 10^6 M_\odot \), see table 1 and equation (12) of the paper (Karashev et al. 2006), with compare to the dipolar magnetic field as \( B \sim 10^{12} G \) of magnetized object. This magnetic field can be created by the stellar azimuthal electric currents). It is not difficult to show that the electromagnetic corrections created by the magnetic dipole being proportional to the electromagnetic energy density are rather small in most WHs. Indeed if \( \rho \) is the average rest mass density off WH of total mass \( M \) and radius of the throat \( r_h \) as measured at infinity, these corrections are at most
\[ \frac{B^2}{8\pi \rho_0 c^2} \simeq 6.7 \cdot 10^{-3} \left( \frac{B}{10^{12} \text{ G}} \right)^2 \left( \frac{10^6 M_\odot}{M} \right) \left( \frac{r_h}{2 \cdot 10^6 \text{ km}} \right)^3. \quad (16) \]

The influence of electromagnetic field with WH monopolar configuration on the test particles motion should be taken in account for lower mass WHs, which can be studied in is the future investigations.

We may now consider the general form of Maxwell’s equations written as:
\[ 3F_{[\alpha\beta, \gamma]} = 2(F_{\alpha\beta, \gamma} + F_{\gamma\alpha, \beta} + F_{\beta\gamma, \alpha}) = 0, \quad (17) \]
\[ F^{\alpha\beta ; \beta} = 4\pi J^\alpha, \quad (18) \]
where \( F_{\alpha\beta} = A_{\beta, \alpha} - A_{\alpha, \beta} \) is the electromagnetic field tensor, \( A_\alpha \) is the four potential of the electromagnetic field and \( J^\alpha \) is the four-electric current.

Next, we describe a few assumptions that are going to be used hereafter. First, we assume there is no matter outside the WH so that the conductivity \( \sigma = 0 \) for outside. We also assume that the magnetic moment of the WH does not vary in time by supposing very high conductivity of the WH matter, where magnetic field produced. However the components of the electromagnetic field will change periodically due to misalignment between the direction of magnetic dipole \( \mu \) and axis of rotation

In the presence of the the magnetic dipole momentum of the wormhole, the four potential has two non-
vanishing components only:

\[ A_0 = \frac{\mu \Omega r_0^2}{3r^3} \left[ \cos \chi \left( 3 \cos^2 \theta - 1 \right) + 3 \sin \chi \cos \lambda \sin \theta \cos \theta \right] , \quad (19) \]
\[ A_3 = \frac{2\mu}{r} \left( \cos \chi \sin \theta - \sin \chi \cos \theta \cos \lambda \right) , \quad (20) \]

according to the expressions for the four potentials derived in paper [Rezzolla et al. 2001a] for slowly rotating magnetized neutron star. For the solutions (19)-(20), the exact external solutions of the Maxwell equations (17)-(18), take the following form (Rezzolla et al. 2001a)

\[
E^\hat{r} = -\frac{\mu \Omega r_0^2 e^{-\phi}}{r^4} \sqrt{1 - \frac{b(r)}{r}} \left[ \cos \chi \left( 3 \cos^2 \theta - 1 + \frac{8M}{5r} \sin \theta \right) + \sin \chi \cos \lambda \left( \frac{3}{2} \sin 2\theta - \frac{8M}{5r} \cos \theta \right) \right] , \quad (21)
\]
\[
E^\hat{\theta} = \frac{2\mu \Omega r_0^2 e^{-\phi}}{r^4} \left[ \cos \chi \cos \theta \left( \sin \theta + \frac{4M}{5r} \right) + \sin \chi \cos \lambda \left( 2\cos \theta + \frac{4M}{5r} \sin \theta \right) \right] , \quad (22)
\]
\[
E^\hat{\varphi} = \frac{\mu \Omega e^{-\phi}}{r^2} \left[ \frac{r_0^2}{r^2} + 2 \csc \theta \right] \sin \chi \sin \lambda \cos \theta \quad (23)
\]
\[
B^\hat{r} = \frac{2\mu}{r^3} \left( \sin \chi \cos \lambda + \cos \chi \cot \theta \right) , \quad (24)
\]
\[
B^\hat{\theta} = \frac{2\mu}{r^3} \left( 1 - \frac{b(r)}{r} \sin \chi \cot \theta \cos \lambda - \cos \chi \right) , \quad (25)
\]
\[
B^\hat{\varphi} = \frac{2\mu}{r^3} \sin \chi \sin \lambda \cot \theta , \quad (26)
\]

\( \mu \) is the magnetic moment of the wormhole, \( \Omega \) is the angular velocity, \( M \) is the total mass (see, for example, [Kardashev et al. 2006]), \( \chi \) is the inclination angle of the magnetic moment relative to the rotation axis as \( \chi = 0 \) in order to keep particles in equatorial plane. Indeed under this condition electromagnetic field of wormhole forces charged particle to move in the equatorial plane. This can be seen from solutions (21)-(26): in the equatorial plane the \( \hat{E}^\theta \) component of the electric field disappears and only \( \hat{B}^\theta \) component of the magnetic field is nonvanishing. Then the equation for radial motion of charged particles takes the form

\[
\left( \frac{dr}{d\sigma} \right)^2 = \varepsilon^2 - V_{eff}^2 \left( r, \mu, \Omega, r_h, r_h, \delta, \varepsilon, \mathcal{L} \right) , \quad (30)
\]

where quantity

\[
V_{eff}^2 = \left[ 1 - \frac{r_h}{r} - \frac{r_h}{r} \left( 1 - \frac{r_h}{r} \right)^{-1-\delta} \right] \times \left[ \frac{1}{r^2} \left( \mathcal{L} + \frac{2e\mu}{r^2} \right)^2 - 1 - \left( 1 - \frac{r_h}{r} \right)^{-2(1+\delta)} \times \left( \varepsilon^2 + \frac{r_0^2 \Omega \varepsilon}{15r} \right) \right] \quad (31)
\]

3 Motion of the Charged Particles Around Slowly Rotating Magnetized Wormhole

The Hamilton-Jacobi equation

\[
g^{\mu\nu} \left( \frac{\partial S}{\partial x^\mu} + eA_\mu \right) \left( \frac{\partial S}{\partial x^\nu} + eA_\nu \right) = m^2 , \quad (27)
\]

may be used in investigating the motion of charged particles only when separation of variables may be put to effect.

It was demonstrated by [Dadhich & Turakulov 2002] that for metrics describing axially symmetric space-times, variables in Hamilton-Jacobi equations may be made separated provided action \( S \) is separable as

\[
S = -\varepsilon t + \mathcal{L} \varphi + S_\theta (r, \theta) . \quad (28)
\]

Using expressions (19) and (20), equation (27) may be written in the following form:

\[
\left\{ \left( -\varepsilon + \frac{e\mu r_0^2}{3r^3} \left( \cos \chi \left( 3 \cos^2 \theta - 1 \right) + \frac{3}{2} \sin \chi \cos \lambda \times \sin 2\theta \right) \right)^2 + \frac{8Mr_0 \Omega \varepsilon}{5r^3} \left( 1 - \frac{r_h}{r} \right)^{-2(1+\delta)} \right\} \times \left( \frac{\partial S_\theta}{\partial r} \right)^2 \times \left[ 1 - \frac{r_h}{r} + (1 - \frac{r_h}{r})^{-1+\delta} \right] \left( \frac{\partial S_\theta}{\partial \theta} \right)^2 + \frac{1}{r^2} \left( \frac{\partial S_\theta}{\partial \theta} \right)^2 + \frac{1}{r^2 \sin^2 \theta} \times \left[ \mathcal{L} + \frac{2e\mu}{r} \right] \left( \cos \chi \sin \theta - \sin \chi \cos \theta \cos \lambda \right)^2 = m^2 . \quad (29)
\]

It is not possible to separate variables in this equation for the general case but it may be made possible for the motion in the equatorial plane \( \theta = \pi/2 \). However this is not allowed as there are no particle orbits under the Lorentz force confined to the equatorial plane. For this reason we are forced hereafter to choose the inclination angle of the magnetic moment relative to the rotation axis as \( \chi = 0 \) in order to keep particles in equatorial plane.
particle in the equatorial plane of the slowly rotating magnetized WH for different value of the parameter $\delta$ (a) and the magnetic dipole momenta $\mu$ (b). From this dependence one can obtain radial motion of charged particle in the equatorial plane of the WH. As it is seen from the figure the parameter $\delta$ changes the shape of effective potentials near the object. In the case of far distances from central object influence of parameter is negligible, which means that one can see the difference between WH and black hole (or compact object with non-exotic matter) only near these objects.

Motion of charged particle in the presence of this kind of effective potential can be explained as follows: increasing of the magnitude of magnetic dipole momenta of the WH may make circular objects to be more unstable and let particle go away to infinity. From the potential we can infer the qualitative structure of the particles orbits. As it is seen from the figure the potential carries the repulsive character. It means that the particle coming from infinity and passing by the source will not be captured: it will be reflected and will go to infinity again as it was in the case of black holes. For weak electromagnetic field of the WH particles can follow bound orbits depending on their energy. As magnetic dipole momenta $\mu$ increases following feature arises: the orbits start to be only parabolic or hyperbolic and no more circular or elliptical orbits exist.

From the equation one can easily get equations describing the motion of the test particle what is done below.

Trajectory of the charged particle around slowly rotating magnetized WH can be drawn from the following equation:

$$\left( \frac{dr}{d\phi} \right)^2 = \frac{r h^2 \left[ 1 + \left( 1 - \frac{e \mu}{F} \right)^{1-\delta} - r^2 \right] \left( 1 - \frac{e \mu}{F} \right)^{-2(1+\delta)}}{\left( L + \frac{2 e \mu}{r} - r^2 (E + \frac{e \mu \Omega \Sigma^2}{3 \pi^3}) (1 - \frac{e \mu}{F})^{-2(1+\delta)} \right)^2} \times \left[ \left( L + \frac{2 e \mu}{r} \right)^2 - r^2 \right] \left( 1 - \frac{r h}{r} \right)^{2(1+\delta)} - r^2 \times \left( E + \frac{e \mu \Omega \Sigma^2}{3 \pi^3} \right) \left( \frac{c r \mu \Omega \Sigma^2}{3 \pi^3} + \frac{4 e \mu \omega}{F} + E + 2 \mathcal{L} \omega \right) \right].$$

(32)

It is almost impossible to integrate equation in general form. However one can get shape of the trajectory of the test particle by using basic assumptions and numerical integration. Figure illustrates the shape of

![Fig. 1 Radial dependence of the effective potential of radial motion of charged particle near the magnetized WH (a) for different values of the parameter $\delta$ and (b) for different values of the magnetic dipole momenta $\mu$.](image)
of charged particles starting from sufficiently far distances towards the slowly rotating central object for different values of small parameter $\delta$ and zero momenta of the particle in infinity. From the presented figure one can see that increase of the parameter $\delta$ makes gravitational field of the central object more stronger which forces test particle approach closer to the central object.

Radial motion of the charged particles near the slowly rotating magnetized WH can be described using the following equation derived from (29):

$$\left(\frac{dr}{dt}\right)^2 = r_h \left[ 1 + \left(1 - \frac{r_h}{r}\right)^{1-\delta} - 1 \right] (1 - \frac{r_h}{r})^{2(1+\delta)}$$

$$\left\{ r \left( e^2 \Omega r_0^2 + 2e\mu \omega \right) + r \left[ E + L\omega \right] \right\}^2$$

$$\times \left\{ \left( \mathcal{L} + 2e\mu r \right)^2 - r^2 \right\} \left(1 - \frac{r_h}{r}\right)^{2(1+\delta)} - r^2$$

$$\times \left( E + e^2 \Omega r_0^2 \right) \left( e^2 \Omega r_0^2 + \frac{4e\mu \omega}{r} + E + 2L\omega \right).$$

(33)

In paper Novikov et al. (2007) it was shown from the solution of equation of radial motion of particle in spherical symmetric spacetime of nonmagnetized WH that particles can make radial harmonic oscillations. We obtain here from equation (33) in the case of magnetized slowly rotating WH, that charged particles make radial anharmonic oscillations. The periods of that oscillations are presented in the Table 1, for the different values of the magnetic parameter and the parameter $\delta$.

Finally we study periods of the circulating charged particle around slowly rotating magnetized WH (stability of the circular orbits will be discussed later in the next subsections) by using the following equation derived from (29):

$$\left(\frac{d\varphi}{dt}\right)^2 =$$

$$\left( \mathcal{L} + 2e\mu \right) (1 - \frac{r_h}{r})^{2(1+\delta)} - r^2 \left( E + e^2 \Omega r_0^2 \right) \omega(r) \right) \}^2$$

$$\times r^4 \left\{ \left( e^2 \Omega r_0^2 + E \right) + \left[ 2e\mu \right] \omega(r) \right\}^2$$

(34)

Figure 3 shows the dependence of the period of the circulating particle from the magnetic dipole momenta of the WH for different values of the small parameter $\delta$. The graphs justify that increase of the parameter $\delta$ cause particles to approach closer to the central object.

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**Figure 2** Shape of the trajectory of charged particles around magnetized WH for the different value of the parameter $\delta$.

**Figure 3** Dependence of the period of the motion of charged particles around WH from magnetic dipole momenta of central object for the different values of the parameter $\delta$. 
3.1 Stable Circular Orbits for Charged Particles

Special interest for the accretion theory of test particles around a slowly rotating wormhole with a dipolar electromagnetic field is related to the study of circular orbits which are possible in the equatorial plane $\theta = \pi/2$ when $dr/d\sigma$ is zero. Consequently the right hand side of equation (30) vanishes:

$$\mathcal{E}^2 - V_{\text{eff}}^2 (r, \mu, \Omega, r_0, r_h, \delta, \mathcal{E}, \mathcal{L}) = 0 \quad \text{(35)}$$

along with its first derivative with respect to $r$

$$\frac{\partial V_{\text{eff}}}{\partial r} = 0 \quad \text{(36)}$$

The radius of marginal stability, the associated energy and angular momentum of the circular orbits may be derived from the simultaneous solution of the condition:

$$\frac{\partial^2 V_{\text{eff}}}{\partial r^2} = 0 \quad \text{(37)}$$

From equations (35) and (36), one may find expression for energy

$$\mathcal{E} = e\gamma C_0(1-t)^3 \pm \left( [e\gamma C_0(1-t)^3 + \omega r_h \kappa (1 \pm \alpha)]^2 - \gamma [1 - (1-t)^2\kappa^2 (1 \pm \alpha)^2] \right)^{1/2} + \omega r_h \kappa (1 \pm \alpha) \quad \text{(38)}$$

and expression for momentum

$$\mathcal{L} = -eC_3(1-t) + r_h \kappa (1 \pm \alpha) \quad \text{(39)}$$

of charged test particles. Here we have used the following notations:

$$\alpha = \left( 1 - \frac{\beta r_h}{\kappa C_3} \right)^{1/2}, \quad \kappa = \frac{C_3 \ln t}{1 + \ln t - 3t - 6t \ln t}, \quad \beta = \frac{(1 + 2 \ln t)t - 1 - \ln t}{\ln t}, \quad C_3 = \frac{2e\mu}{r_h}$$

and

$$\gamma^{-1} = 4t^2 \delta \ln t.$$ 

Now, inserting (38) and (39) into equation (37), one may obtain the basic equation

$$\mathcal{L} r^3 \left[ -4C_3\lambda + \mathcal{L}(r - 7\lambda) \frac{r}{r_h} \right]$$

$$+ r \left[ \frac{2C_1}{r_h} \lambda^2 + \frac{3}{r_h} (3\lambda - r) - 4\delta \epsilon \lambda^2 (\mathcal{E} r^3 - 2\eta \mathcal{L}) \right]$$

$$- 2\lambda \left( 2C_3 \left( 4\delta \epsilon \eta \lambda^2 + \mathcal{L}(r - 4\lambda) \frac{r^3}{r_h} \right) - C_3^2 \lambda r^2 \right.$$

$$+ \frac{r}{r_h} \left[ C_1 13\lambda - \frac{5r}{r_h} \lambda - r^6 + \mathcal{L}^2 \frac{6\lambda - 3r}{r_h} r^3 \right.$$

$$+ 2\delta \epsilon \lambda \left( \mathcal{E}(5r - 7\lambda) r^3 + 2\eta \mathcal{L}(13\lambda - 5r) \right) \right) \ln \frac{\lambda}{r}$$

$$+ 4\delta \epsilon \lambda^2 r_h \left[ \frac{2C_3 \eta \lambda (4\lambda - 3r)}{r_h} \right.$$ \n
$$+ 3r \left( \mathcal{E} r^3 \frac{2\lambda - r}{r_h^2} + C_0 \varsigma + 2\eta \mathcal{L} \right) \right] \ln \frac{\lambda}{r} = 0. \quad \text{(40)}$$

In this, we have used following additional notations:

$$r - r_h = \lambda \ , \ \eta = 4M r_0^2 \Omega / 5 \ , \ C_0 = -e\mu \Omega r_0^2 / 3r_h^3$$

$$\varsigma = 7r_h^2 - 8r_h r + 2r^2 \ , \text{ and } \ C_1 = 2C_0 \delta \epsilon \lambda^4.$$ 

The numerical solutions of the equation (40) will determine the radii of marginally stable circular orbits for slowly rotating wormholes with magnetic dipole momenta as functions of the parameter $\delta$, the angular velocity $\Omega$, as well as of the magnetic dipole $\mu$ of the source. In the Table 3.1, we numerical solutions for the radii of the stable circular orbits of charged test particles for different values of the parameter $\delta$ and magnetic dipole moment $\mu$ of the wormhole. With the increase of the $\delta$, radii of stable circular orbits shift to observer at the infinity, while existence of magnetic dipole moment and a possible increase in it displace the orbits to the gravitational source.

\begin{table}[h]
\centering
\caption{Period of the radial oscillating of charged particles near the slowly rotating magnetized WH with respect to $\mu$ and $\delta$.}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
$\delta$ & 0.001 & 0.01 & 0.02 & 0.05 & 0.1 \\
\hline
$\mu = 4$ & 4.02782 & 4.07698 & 4.11094 & 4.21451 & 4.39281 \\
$\mu = 6$ & 4.41272 & 4.46564 & 4.52357 & 4.69716 & 4.9919 \\
$\mu = 12$ & 4.93646 & 4.99437 & 5.05869 & 5.25325 & 5.58609 \\
$\mu = 20$ & 5.22254 & 5.28301 & 5.35083 & 5.55727 & 5.91227 \\
\hline
\end{tabular}
\end{table}
Table 2  Radii of the stable circular orbits of test particles near the slowly rotating magnetized wormhole with respect to $\mu$ and $\delta$.

| $\delta$   | 0.01 | 0.02 | 0.03 | 0.04 | 0.05 |
|-----------|------|------|------|------|------|
| $\mu = 0.3$ | 7.81538 | 9.31114 | 10.3082 | 11.0739 | 11.7021 |
| $\mu = 0.7$ | 6.31054 | 7.52564 | 8.34287 | 8.97572 | 9.49905 |
| $\mu = 2$   | 4.86001 | 5.79180 | 6.41939 | 6.90621 | 7.30947 |
| $\mu = 5$   | 3.90299 | 4.64910 | 5.15093 | 5.53995 | 5.86208 |
| $\mu = 13$  | 3.18162 | 3.79127 | 4.19940 | 4.51495 | 4.77577 |
| $\mu = 21$  | 2.93889 | 3.50249 | 3.87818 | 4.16793 | 4.40702 |

4 Astrophysical Applications and Conclusions

In this paper we consider electromagnetic field of slowly rotating WH i.e. neglecting quadratic and higher order terms of angular velocity we first find exact vacuum solutions of Maxwell equations in spacetime of slowly rotating magnetized WH. In the paper (Kim 2005) it has been justified that electric charge could exist in WH and did not change the structure of the spacetime around WH seriously, which gives us the right to consider the existence of magnetic dipole momenta of the WH due to possible motion of the electric charge.

Astrophysical processes and effects around black hole and WH different models are distinguishable such as absence of event horizon on WH, passage of the radiation and particles through WH, appearance of blueshift effect in addition to the gravitational redshift effect near WH etc (see for the details (Kardashev et al. 2007, Shatskii 2007)). In Ref. (Kardashev et al. 2006) circular orbits of test particles around a WH and their periods were studied. Here we extended these results to motion of charged particles and have shown the strong dependence of particle motion from WH shape parameter $\delta$ and magnetic field strength.

Here we address two basic question: are there any differences between astrophysical processes around WH and standard models for compact object? If there are so, how can we distinguish WH from other compact objects using observational data? To answer these questions we consider magnetic dipole momentum of the WH and develop the existing WH model constructed by (Kardashev et al. 2006). For such models we perform a charged particle motion analysis around magnetized WH. Finally we can conclude that from astrophysical point of view the following differences between WH and standard compact objects can be detected observationally:

1. Oscillations of bodies in the vicinity of a WH throat (radial orbits) could give rise to a peculiar observational phenomenon. Signals from such sources detected by an external observer will display a characteristic periodicity in their spectra. All objects (stars, BHs) other than WHs absorb bodies falling onto them irrecoverably. Periodic radial oscillations are a characteristic feature of magnetized WHs as it was first shown by (Kardashev et al. 2007) for other WH models.

2. As it was shown by (Kardashev et al. 2007) the difference of the circular motion of particles around Reisner-Nordström black hole and around the charged WH model described by (Kardashev et al. 2006) is negligible at $r = 2r_h$ (or more). Thus all conclusions about circular orbit around a WH are the same as in the limiting Reisner-Nordström geometry at the corresponding distances. However, on the contrary for our WH model which has a magnetic dipole momentum one can easily obtain the difference on circular motion of charged particle around WH and compact object as stars, BH etc. As we have shown in subsection 3.1 with the increase of the $\delta$, radii of stable circular orbits shift from central object (WH) to observer at the infinity. From the numerical results given in Table 2, one can easily see that the influence of $\delta$ parameter on particle motion depends on the strength of magnetic field of WH: radii of the stable circular orbits differ almost 2 times bigger when $\mu = 21$, while this increasing almost disappears when magnetic field of WH is margin.

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