Simulation of 2D Waves in Circular Membrane Using Excel Spreadsheet with Visual Basic for Teaching Activity

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Abstract. We propose a teaching instrument demonstrating the circular membrane waves using the excel interactive spreadsheets with the Visual Basic for Application (VBA) programming. It is based on the analytic solution of circular membrane waves involving Bessel function. The vibration modes and frequencies are determined by using Bessel approximation and initial conditions. The 3D perspective based on the spreadsheets functions and facilities has been explored to show the 3D moving objects in transitional or rotational processes. This instrument is very useful both in teaching activity and learning process of wave physics. Visualizing the vibration of waves in the circular membrane which is showing a very clear manner of $m$ and $n$ vibration modes of the wave in a certain frequency has been compared and matched to the experimental result using resonance method. The peak of deflection varies in time if the initial condition was working and have the same pattern with matlab simulation in zero initial velocity.

Key words: Visual Basic for Application, Vibration Modes, MS Excel

1. Introduction

Teaching of physics sometimes difficult to be delivered without demonstration or visualization tools. Simulations can help teacher in creating or performing the real conditions in front of the class. Simulation can be expected to be an effective tool in helping students to explore their higher levels of learning capabilities such as analysis, synthesis and evaluation. Many authors plead for the use of spreadsheets demonstrating their multiple advantages, describing their facile use in physics lessons with examples of data processing according to tables and their graphic representation [1]. Moreover, an emphasis is placed upon their calculus capacity through the functions available for users, and the rapid feedback when modifying data [2]. Studies have shown that students have a better grasp of the definition of certain measures by dealing with basic calculi, as for instance the notion of velocity, while the instantaneous graphic response of the finite product represents an advantage in acquiring the concepts of physics [3]. In a series of recent papers, excel spreadsheets have helped to describe the simulation of motion in the case of a projectile motion in a gravitational field [4].

Successful in learning of physics is determined by the better understanding to the concept, because physics is generally a collection of concepts explaining the natural phenomena experienced by the students in their everyday life. The facts from natural phenomena are investigated and tested through repeated experiments, and then based on them, the theory is formulated.

In this report, we propose the simulation of vibration modes and its frequency on the circular membrane waves using the Excel interactive spreadsheets with the Visual Basic for Application (VBA) programming. The wave deflection on circular membran was analysed theoretically by using the separated variable method. Here, the Fourier series involving the Bessel function was selected as an initial condition. We have explored a series of facilities available on Excel spreadsheets, such as...
the perspective view of the 3D object in the further way as well as the 3D graphical scattering of in translational and rotational processes. The simulation results were compared to the experimental results from Radiansyah et al., using the audio frequency generator as the wave source in resonance condition with vibration modes of mica membrane (diameter of 23.4 cm) poured by sand beach [5].

2. Methods

a. Two Dimensional Wave In Circular Membrane

The equation motion of 2D wave in circular membrane is described as

$$\begin{align*}
\frac{\partial^2 Z}{\partial t^2} &= c^2 \frac{\partial^2 Z}{\partial r^2} + \frac{1}{r} \frac{\partial Z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 Z}{\partial \theta^2},
\end{align*}$$

(1)

with $0 \leq r \leq a$ and $t \geq 0$ [6]. Here, $a$ is the radius of membrane. By performing the separation variable method, the deflection of wave in polar coordinate $Z(r, \theta, t)$ showed as

$$Z(r, \theta, t) = J_m(k_{mn}r)(A_{mn} \cos m\theta + B_{mn} \sin m\theta)\cos \omega_{mn} t + J_m(k_{mn}r)\left(A^*_{mn} \cos m\theta + B^*_{mn} \sin m\theta\right)\sin \omega_{mn} t$$

If the boundary condition at $r = a$ is $Z(a, \theta, t) = 0$, the wave number can be deduced from zeros of approximation of Bessel function [7].

$$J_m(k_a) = \frac{2}{\pi k_a} \cos (k_m a (2m + 1) \frac{\pi}{4}) = 0, \quad k_m = (4n + 2m - 1) \frac{\pi}{4a} ; k_{mn} = \frac{\omega_{mn}}{c}$$

For the case of wave moving stationary with the tip banded in circular membrane [8], the initial deflection and the initial velocity are respectively given by

$$Z(r, \theta, 0) = f(r, \theta) \quad \text{and} \quad \frac{\partial}{\partial t}[Z(r, \theta, 0)] = g(r, \theta)$$

(2)

Then, by performing the separation variable method and taking into account boundary condition, the deflection can be represented as

$$Z_{mn}(r, \theta, t) = \sum_{n=1}^{\infty} A_{0n} J_0 \left(\frac{\omega_{mn}}{a} r\right) \cos \omega_{mn} t + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} J_m \left(\frac{\omega_{mn}}{a} r\right)$$

$$\left(A_{mn} \cos m\theta + B_{mn} \sin m\theta\right) \cos \omega_{mn} t + \sum_{n=1}^{\infty} A^*_{0n} J_0 \left(\frac{\omega_{mn}}{a} r\right) \sin \omega_{mn} t + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} J_m \left(\frac{\omega_{mn}}{a} r\right)$$

$$\left(A^*_{mn} \cos m\theta + B^*_{mn} \sin m\theta\right) \sin \omega_{mn} t$$

(3)

The initial deflection function of 2D waves in circular membrane, $f(r, \theta)$, is calculated using the initial conditions, $Z(r, \theta, 0) = f(r, \theta)$ and $\frac{\partial}{\partial t}[Z(r, \theta, 0)] = 0$. 


\[ f(r, \theta) = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} J_m \left( \frac{u_{mn}}{a} r \right) \left( A_{mn} \cos m\theta + B_{mn} \sin m\theta \right) \]

\[ = \sum_{m=1}^{\infty} A_{0n} J_0 \left( \frac{u_{0n}}{a} r \right) + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} J_m \left( \frac{u_{mn}}{a} r \right) \cos m\theta \]

\[ + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} B_{mn} J_m \left( \frac{u_{mn}}{a} r \right) \sin m\theta \]  

(4)

**b. Shaped 2D Wave in Circular Membrane Using Spreadsheets Excel**

On the process of making the wave shaped in circular membrane using spreadsheet excel, first we need to draw the circular membrane by combining two parameters, the radius \((0 \leq r \leq a)\) and angle \((0 \leq \theta \leq 360)\). Fig. 1 shows spreadsheet display of (a) circles drawing at interval of 0.05 m and (b) the circular membrane including angle interval of 18°. Here, the radius is chosen for 1 m. Other parameters, then, can be calculated by using several assumptions displayed in spreadsheet excel shown in Fig. 2.

**Figure 1.** (a) The combination of circles with interval radius \(0 \leq r \leq a, a = 1m\), (b) The circular membrane in spreadsheet excel display.

**Figure 2.** The proponent parametric of wave in circular membrane.

Furthermore, in order to show the coefficient of \(A_{0n}, A_{mn},\) and \(B_{mn}\) in the spreadsheet excel cell, we must consider the principle of Trapezoidal rule for performing the numerical integration

\[ b \int_{a}^{f(x)dx} \approx \frac{\Delta x}{2} \left\{ f(x_0) + 2f(x_1) + 2f(x_2) + ... + 2f(x_{n-1}) + f(x_n) \right\} \]  

(5)
This process are displayed in spreadsheet cells shown in Fig. 3.

**Figure 3.** Calculation process of $A_{0n}$, $A_{mn}$, and $B_{mn}$ coefficients shown in spreadsheet excel display.

c. Application of Moving Shapes, Perspective and $(XY)$ Scatter Chart

Due to the limitation of MS Excel in providing the 3D scattering of object, here the 2D or $XY$ Scatter Chart function can be modified for moving shape and 3D perspective by applying the translation and rotation processes. The translation process is from $(x,y,z)$ to $(x_{tr} = x + \Delta x, y_{tr} = y + \Delta y, z_{tr} = z + \Delta z)$. In cases rotation in $z$-axis, the vector components velocity becomes $\vec{v}_x = i v_x \cos \theta + j v_x \sin \theta$, $\vec{v}_y = j v_y \cos \theta + k v_y \sin \theta$, and $\vec{v}_z = k v_z$ whereas rotation in $y$-axis is $X_{rot,y} = v_y \sin \theta + v_x \cos \theta$, $Y_{rot,y} = v_y$, $Z_{rot,y} = v_x \cos \theta + v_y \sin \theta$. The calculation process of moving shape in spreadsheet is shown in FIG. 4. The 3D object is look like 2D object illustrated using perspective application. An example of 2D circular wave plotted by using the $XY$ scatter chart where $m = 0$ and $n = 1$ is shown Fig. 5.

**Figure 4.** The frame of moving shaped in spreadsheet.

**Figure 5.** 2D wave in circular membrane $m=0$ and $n=1$. 


\[ d. \quad \textbf{Application of VBA Macro} \]

VBA macro is a tool in MS. Excel used to move data with action button for \( m, n, dt, \text{Eye to screen (ES)}, \Delta x, \Delta y, \Delta z, \text{rot}_z, \text{rot}_x, \text{dan rot}_y \). The purpose of using this, is to move and to visualize those figures according to the real situation where they can be modified rapidly. The script program of 2D wave in circular membrane showed in following Table 1.

**Table 1. VBA macro script of 2D circular membrane simulation.**

| Parameter | Action Button | Action Script |
|-----------|---------------|---------------|
| \( M \)   | \( m_o \)     | Private Sub \( m_o \)_Change() |
|           |               | \( \text{Range}("m") = m_o.Value \) |
| \( N \)   | \( n_o \)     | Private Sub \( n_o \)_Change() |
|           |               | \( \text{Range}("n") = n_o.Value \) |
| \( \Delta t \) | \( \Delta t_o \) | Private Sub \( \Delta t_o \)_Change() |
|           |               | \( \text{Range}("\Delta t") = \Delta t_o.Value / 10 \) |
| rotation\(_z\) | \( z\_rot \) | Private Sub \( z\_rot \)_Change() |
|           |               | \quad \text{If } z\_rot > 71 \text{ Then } z\_rot = 0 |
|           |               | \quad \text{If } z\_rot < 0 \text{ Then } z\_rot = 71 |
|           |               | \quad \text{Range}("rot\_z") = 5 * z\_rot.Value |
| rotation\(_y\) | \( y\_rot \) | Private Sub \( y\_rot \)_Change() |
|           |               | \quad \text{If } y\_rot > 71 \text{ Then } y\_rot = 0 |
|           |               | \quad \text{If } y\_rot < 0 \text{ Then } y\_rot = 71 |
|           |               | \quad \text{Range}("rot\_y") = 5 * y\_rot.Value |

**3. Results and Discussions**

The vibration mode is characterized by a modal frequency and shape. In general all modes have different frequencies and mode shapes. The low mode number have lower frequency. The node is the line on the structure that does not move (zero displacement) while the rest of the structure is vibrating. In a 2D system such as the circular membrane, each mode is entirely independent of all other modes. Some of the simulations of 2D wave in circular membrane using spreadsheet excel with several values of \( m \) (mode) and \( n \) (node) are represented in Table 2. The frequency of each mode is shown as a certain value multiplied to \( f_{01} \) (frequency of (0,1) vibration mode).

**Table 2. Simulation results of circular membrane wave**

| \( m \) \( n \) | 0   | 1   | 2   | 3   |
|-----------------|-----|-----|-----|-----|
| 1               |     |     |     |     |
|                 | \( f_{01} \) | 1.667 \( f_{01} \) | 2.333 \( f_{01} \) | 3.000 \( f_{01} \) |
The frequency of vibration 2D wave in circular membrane in this research was calculated by using the Bessel approximation of the wave number. On the other side, the wave number is calculated from the roots of Bessel function [7]. Some conditions are given for verification of this simulation. In the case of m = 0 and n = 1 or mode (0,1), the circular shape with no deflection occurs at the radius of r_0 = 0.117 m and frequency of 205 Hz. This result is similar to the experimental result observed from membrane diameter of 23.4 cm using sand beach under vibration at a specific frequency [5]. Using the same condition with experiment such as the radius and density of membrane, the simulation result can be verified and differentiation between them can be analyzed. In the simulation, the stress of 1000 N/m is introduced to the membrane. The membrane density is 0.2444 kg/m². For mode (0,2), the calculated frequency is 478 Hz where the circular shape formed at the radius of r_0 = 0.117 m and r_1 = 0.0501 m. The experimental result gave the circular shape at radius of r_0 = 0.117 m and r_1 = 0.0525 m (Fig. 6). The location of vibration mode is found a little bit different from resonance model in experiment but all of them have the same vibration pattern. The second way verify this simulation is to compare them with simulation results using MATLAB at the same conditions with zero initial velocity [9] (see in Fig. 7). A little bit difference result between them occur from different method in calculation of the wave number. In Matlab, the wave number were calculated from the root value of Bessel function.

| Mode (0,1) | Mode (0,2) | Mode (0,3) | Mode (2,2) | Mode (4,1) |
|------------|------------|------------|------------|------------|
| Frequency result 205 Hz r_0 = 0.117 m | Frequency result 478 Hz r_0 = 0.117 m r_1 = 0.0501 m | Frequency result 752 Hz r_0 = 0.117 m r_1 = 0.0744 m r_2 = 0.0319 m | Frequency result 752 Hz r_0 = 0.117 m r_1 = 0.0744 m | Frequency result 752 Hz r_0 = 0.117 m |

Figure 6. Vibration mode simulated by MS. Excel including frequency and circular node position.
4. Conclusion

Simulation of 2D wave in circular membrane has been created using the spreadsheets and macro VBA in Ms Excel with presence of initial deflection and initial zero velocity. The vibration modes have the same pattern compared to the resonance method in experiment with the membrane diameter of 23.4 cm. The peak of deflection in this simulation have also the same pattern compared to MATLAB simulation.

References

[1] Webb L 1993 J. Physics Education 28 2 p 77-82
[2] Cooke B A 1997 J. Physics Education 32 2 p 80-87
[3] Fowler M 2004 J. Science & Education 13 (7-8) p 791-796.
[4] Benacka J 2010 J. Math. Ed. Sci. Technol. 41 3 p 373-378
[5] Radiansyah H et al 2006 The Analysis Of Circular Membrane Vibration Modes (UNY: skripsi Jurusan Pendidikan Fisika)
[6] Ross S L 1984 Differential Equations (New York: John Wiley and Sons Inc)
[7] Boas L M 1983 Mathematical Methods in the-Physical Sciences (USA : John Wiley and Sons inc)
[8] Chaudhuri R N 2010 Waves and Oscillations (New Delhi : New Age International)
[9] Asmar N H 2005 Partial Differential Equations with Fourier Series and Boundary Value Problems (USA: Pearson Prentice Hall)