Numerical and experimental study of an annular pulse tube used in the pulse tube cooler

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Abstract: Multi-stage pulse tube coolers normally use a U-type configuration. For compactness, it is attractive to build a completely co-axial multi-stage pulse tube cooler. In this way, an annular shape pulse tube is inevitable. Although there are a few reports about previous annular pulse tubes, a detailed study and comparison with a circular pulse tube is lacking. In this paper, a numeric model based on CFD software is carried out to compare the annular pulse tube and circular pulse tube used in a single stage in-line type pulse tube cooler with about 10 W of cooling power at 77 K. The length and cross sectional area of the two pulse tubes are kept the same. Simulation results show that the enthalpy flow in the annular pulse tube is lower by 1.6 W (about 11% of the enthalpy flow) compared to that in circular pulse tube. Flow and temperature distribution characteristics are also analyzed in detail. Experiments are then conducted for comparison with an in-line type pulse tube cooler. With the same acoustic power input, the pulse tube cooler with a circular pulse tube obtains 7.88 W of cooling power at 77 K, while using an annular pulse tube leads to a cooling power of 7.01 W, a decrease of 0.9 W (11.4%) on the cooling performance. The study sets the basis for building a completely co-axial two-stage pulse tube cooler.

Keywords: Co-axial; pulse tube; annular; circular; expansion efficiency

1 Introduction

Pulse tube coolers (PTC) can be divided into three types according to their geometrical configuration [1]: in-line type, U type and co-axial type. In-line types normally have the best performance [2], but the cold end heat exchanger is in the middle and not convenient for applications. The U-type configuration eases the situation but occupies a larger space and has extra losses at the connecting tube [3-4]. The co-axial configuration is the most compact
system [5] and popular for applications.

In a conventional single stage co-axial PTC, the regenerator has an annular cross section surrounding the pulse tube. Manufacturing annular screen meshes is relatively complicated compared to circular screen meshes. In 2004, T. Haruyama et al developed a co-axial PTC with an annular pulse tube for a liquid xenon calorimeter [6]. The cylindrical regenerator was placed inside of the pulse tube in order to save space and simplify fabrication. It provides a cooling power of 70 W at 165 K by using a 2.2 kW compressor. However, the authors did not provide further detailed analysis about this configuration.

For a two stage PTC, the conventional configuration is U-type for both stages. There are almost a few reports about a completely co-axial configuration as shown in Fig 1. This configuration has the 1\textsuperscript{st} stage pulse tube constructed as an annular-shaped space surrounding the high temperature section of the 2\textsuperscript{nd} stage pulse tube. The overall dimensions are reduced significantly compared with a U-type two stage PTC. In 2006, T. Koettig et al [7] first demonstrated a G-M type two stage co-axial PTC. The second stage cold end reached a no-load temperature of 6.6 K with 6 kW of input power. Their paper does not compare the cooling performance with the in-line and U-shaped configuration. In 2012, another Stirling type co-axial two stage PTC was developed by I. Charles et al [8]. The system reaches a no-load temperature of 28 K at the 2\textsuperscript{nd} stage and 108 K at the 1\textsuperscript{st} stage, when the input acoustic power is maintained at 120 W. Compared with the no-load temperature of 30 K and 89 K with U-type configuration, the performance of 2\textsuperscript{nd} stage was slightly better than the U-type, but the 1\textsuperscript{st} stage was worse to some extent. The authors attributed this to the annular pulse tube configuration of the first stage but gave no further analysis.

In this paper, CFD software is employed to analyze the influence of the annular pulse tube based on an in-line pulse tube cooler system. A two dimensional axisymmetric model of the pulse tube with two heat exchangers is set up. Detailed analyses regarding flow and temperature characteristics have been made. Subsequently experiments are carried out for comparison.

![Figure 1. Schematic of a completely co-axial two stage pulse tube cooler.](image)

1. Ambient heat exchanger
2. 1\textsuperscript{st} stage regenerator
3. 1\textsuperscript{st} stage pulse tube
4. 2\textsuperscript{nd} stage pulse tube
5. 1\textsuperscript{st} stage cold end heat exchanger
6. 2\textsuperscript{nd} stage regenerator
7. 2\textsuperscript{nd} stage cold end heat exchanger

2 Physical model
Schematics for the two dimensional axisymmetric simulation models are shown in Fig 2, which only includes partial components of a pulse tube cooler system: the cold end heat...
exchanger, pulse tube and heat exchanger. Table 1 shows the details of the components. The cross-sectional area and length of the annular pulse tube are maintained the same as those of the circular pulse tube.

The model has a mass flow inlet boundary condition:

$$m_a = m_s \sin(\omega t + \theta)$$

(1)

and a pressure outlet boundary condition:

$$p_{\text{out}} = p_a + p_s \sin(\omega t)$$

(2)

where $p_c$ represents the mean pressure. $m_a$ and $p_s$ are the mass flow rate amplitude and pressure amplitude, respectively, and $\theta$ is the phase angle between mass flow rate and pressure wave.

Figure 2. Schematics of the simulation models of the pulse tubes; (a) Case A (circular pulse tube); (b) Case B (annular pulse tube).

| Components               | Diameter (mm) | Length (mm) | Wall thickness (mm) | Porosity | Materials                           |
|--------------------------|---------------|-------------|---------------------|----------|-------------------------------------|
| cold end heat exchanger  | 10            | 9.5         | 0.25                | 0.245    | Copper                              |
| pulse tube               |               |             |                     |          |                                     |
| Circular                 | 10            | 54          | 0.25                | 1        | Stainless steel tube                |
| Annular                  |               |             |                     |          | Material of the rod 2) in center of inner dia. 8 | |
|                          |               |             |                     |          | PEEK 1)Stainless steel tube 2)Material of the rod |
| ambient heat exchanger   | 10            | 10          | 0.25                | 0.69     | Copper mesh                         |
wall is set as an adiabatic boundary.

Helium is treated as ideal gas since its temperature remains higher than 40 K. The operating frequency is 100 Hz and the mean pressure is 3.5 MPa. Grid independence and time step independence are performed before the results are finalized. Quadrilateral cells are adopted in all the regions including gas and solid area. The total number of computational cells in the solid area is 980 for the circular pulse tube and 4165 for the annular pulse tube. For the gas flow area, a boundary layer mesh is used near the wall. The total grids number in gas flow area is 5145 for the circular pulse tube and 5635 for the annular pulse tube. The number of computational time steps in one cycle is 250, and the simulation may be considered to have reached steady state when the changes of all the important parameters, like pressure, mass flow and especially temperature, are less than 1% every ten periods.

A linear temperature distribution is used to set the initial axial temperature profile of the gas inside the pulse tube and the wall. The simulation reveals that the pressure and velocity easily reach a steady state; however, the temperature and related enthalpy and entropy flow are much slower to reach a steady state due to the large density and specific heat capacity of the pulse tube wall. To overcome this difficulty, a linear extrapolation methodology is used. Below we first introduce the definitions of some important concepts.

The definition of PV power, enthalpy flow, entropy flow and total power \[10\] is as follows:

\[
\langle PV \rangle = \frac{1}{\tau} \int \int \rho \bar{u} \bar{v} dtdA
\] (3)

\[
\langle H \rangle = \frac{1}{\tau} \int \int \rho \bar{u} \bar{v} C \bar{T} dtdt
\] (4)

\[
\langle S \rangle = \frac{1}{\tau} \int \int \rho \bar{u} \bar{s} dtdt
\] (5)

\[
\langle H_i \rangle = \frac{1}{\tau} \int \int \rho \bar{u} \bar{s} dtdA - \left( \frac{dT_{in}}{dx} + A_{co} k co \frac{dT_{in}}{dx} \right)
\] (6)

in which \( \tau \) is the cycle time, \( \rho, \bar{u}, \bar{v}, \bar{k}, \bar{s} \) are dynamic pressure (without the mean pressure), velocity, mass flow rate, specific enthalpy and specific entropy, respectively. In the post-processing, FFT analysis is used to obtain the components at fundamental frequency, then integration is performed to get these quantities.

The pulse tube with the pulse tube wall and the rod in the center are considered as the control volume (the dotted line box in Fig 2). The total power flows into and flows out the control volume is defined as \( H_{in} \) and \( H_{out} \), respectively. The external surface of the control volume is adiabatic. Thus, the unbalanced quantity of the total power is \( \Delta H = H_{in} - H_{out} \).

In fact, the mean temperature in the pulse tube (including the gas temperature and the solid temperature) deviates from the initial linear distribution after a certain number of calculation cycles, which should lead to \( \Delta H_i = 0 \). To reduce the calculation time, we record the mean temperature data and unbalanced quantity of the total power of the system as \( T_i, \Delta H_i \) after \( n \) cycles, and \( T_{in}, \Delta H_{in} \) after \( n+m \) cycles and use them to predict temperatures at the final state according to the total energy balance. With these values, we modify the mean temperature data from \( T_{in} \) to \( T \) after \( n+m \) cycles in the pulse tube as Eq.(7) through a patch option inherent in Fluent and then continue the calculation until all the parameters reach a steady state. The method has proved to be very effective.
\[ T = \frac{T_{\text{in}} - T_{\text{out}}}{\Delta H_{\text{in}} - \Delta H_{\text{out}}} (0 - \Delta H_{\text{in}}) + T_{\text{in}} \]  

(7)

3 Simulation results and analyses

3.1 Flow and temperature field

The flow characteristics are initially investigated. The pressure distributions of the two types of pulse tubes are almost the same since the hydraulic diameter of each pulse tube is much larger than the viscous penetration depth. The impedances at the cold end of the pulse tube are listed in Table 2. The impedance amplitude and phase angle of the annular pulse tube are close to that of the circular pulse tube. This further validates that the influence of the tube shape on the tube impedance is small when the cross-sectional flow area of the tube is kept the same.

|                | Case A     | Case B     |
|----------------|------------|------------|
| Impedance amplitude (Pa.s/m²) | 1.612E9    | 1.612E9    |
| Impedance angle (Deg)           | 41.50      | 41.44      |

Table 2. Comparison of the impedance at the cold end of the pulse tube

Figure 3. Velocity distributions at X=36 mm (middle of the pulse tube) at four moments.

(a) Circular pulse tube  
(b) Annular pulse tube

Figure 4. Temperature distributions at X=36 mm (middle of the pulse tube) at four moments.

(a) Circular pulse tube  
(b) Annular pulse tube

To further study the effect on the flow and temperature field, Fig 3 and Fig 4 present the
radial distributions of axial velocity and the radial transient temperature distributions of the gas in the pulse tube at x=36mm (middle of the pulse tube) at four moments during a cycle based on case A (circular pulse tube) and case B (annular pulse tube), respectively. The maximum velocity occurs near the pulse tube wall, which is well-known as the skin effect [11]. Compared with the circular pulse tube, the influencing area occupies a larger fraction of the total flow area in the annular pulse tube, which may lead to an apparent change of performance. The temperature distribution has a similar skin effect, and the larger temperature inhomogeneity may contribute to more thermal losses in the annular pulse tube.

Area-weighted mean temperature distributions of the gas in the pulse tube are shown in Fig 5. The mean temperature distribution curves are similar, and the temperature in the annular pulse tube is slightly higher than in the circular pulse tube. Both of them deviate from the linear distribution (the dotted line in Fig 5).

![Figure 5. Area-weighted mean temperature distribution of the gas in the pulse tube.](image)

3.2 Energy flow distribution

To evaluate the performance of the pulse tube, the expansion efficiency of the pulse tube is defined as the ratio of the enthalpy flow and the PV power at the cold end of the pulse tube [12].

The negative entropy flow shown in Fig 6 represents the total loss of the pulse tube. Entropy generation [10] can be expressed as Eq.(8). Entropy generation mainly comes from viscous dissipation and imperfect heat transfer. Both the temperature and velocity inhomogeneities will increase the entropy generation and affect the performance.

\[
\sum \dot{S}_{gen} = \frac{1}{T} \int \rho \dot{h} dv dt = \frac{1}{T} \int \left( \frac{\kappa \dot{V}^2}{T^2} + \frac{1}{T} (\sigma^T \dot{v})^2 \right) dv dt
\]

Figure 6. The distribution of energy flow in the pulse tube.

where \( \sigma^T \) is the nine-component viscous stress tensor.

Fig 6 also shows the distribution of the enthalpy flow and acoustic power along X axis. The curves of the acoustic power distributions almost overlap each other. However, the average enthalpy flow in the annular pulse tube is lower by about 1.6 W (11%) compared to that in the circular pulse tube. The results indicate that the expansion efficiency of the pulse tube reduces from 88% to 78% when the pulse tube configuration changes from the circular to the annular shape.

3.3 Influence of the cold end heat exchanger porosity

The reason for this study is that, the porosity of the cold end heat exchanger used in the
experiments is not the same for the two different pulse tube shapes. Fig 7, shown in the following experimental section, displays the two different configurations of the cold end heat exchanger with the slot width kept the same. Although the different configurations may influence the cooling performance, here, the simulations focus on the influence of the porosity itself. In correspondence to the experimental system, the porosity of the cold end heat exchanger is set at 41% for the circular pulse tube and 24.5% for the annular pulse tube, respectively. Simulation results indicate that the expansion efficiency of the pulse tube is 88.7% and 88%, respectively, which means that this variation has a minor effect on the system performance.

4 Experimental results

Based on the simulation results, various experiments are carried out. Fig 7 shows the schematic of the system. It consists of a moving-magnet type linear compressor and an in-line type pulse tube cooler. The main geometric parameters of the cooler are listed in Table 3.

The influence of the pulse tube configuration on the system performance is studied here. The components of the pulse tube cooler are the same except for the cold end heat exchanger and the pulse tube, which are shown in Fig 7. For the annular pulse tube, a rod made of PEEK (polyetheretherketone) is inserted inside a thin stainless pipe. The gap between the outer pipe and the PEEK rod serves as annular pulse tube. The slit width of the two cold end heat exchangers is kept the same. However, the porosity of the cold end heat exchanger is 24.5% for the annular pulse tube, and 41% for the circular pulse tube, respectively.

| Components                        | Diameter/mm | Length/mm |
|-----------------------------------|-------------|-----------|
| Main ambient exchanger            | 20          | 30        |
| Regenerator                       | 20          | 35        |
| Cold end heat exchanger           | Refer to Table 1 |
| Pulse tube                        | Refer to Table 1 |
| Secondary ambient heat exchanger  | Refer to Table 1 |
| Inertance tube                    | 3           | 1500      |
| Reservoir                         |             | 300 cm³   |

Figure 7. Schematic of the experimental setup.
The cross sectional area and the length of the two pulse tubes are the same. The inner and external diameter of the flow channel of the annular pulse tube is 8 mm and 12.8 mm, respectively. The dimensions are listed as case A and case B shown in Table 1.

The working gas is helium. The mean pressure is maintained at 3.5 MPa. The ambient heat exchanger is cooled by water maintained at 300 K. The system operates at 100 Hz. A platinum resistance thermometer with an accuracy of ±0.1 K is attached to the cold end heat exchanger to measure the temperature. Cooling power is measured through a heating wire that is mounted on the cold end heat exchanger powered by a DC voltage source.

No-load temperature curves are shown in Fig. 8. The no-load temperature increases by about 5.5 K when the pulse tube changes from circular shape to annular shape. Fig 9 presents the dependence of cooling power and relative Carnot efficiency on input acoustic power. Better cooling performance is achieved by employing the circular pulse tube. With an acoustic power of 105 W, the cooling powers at 77 K of the circular and annular pulse tubes are 7.88 W and 7.01 W, respectively. Assuming the losses coming from regenerator are the same, the experimental results show that the difference in enthalpy flow at the cold end of the pulse tube is about 0.9 W. In the simulation, the enthalpy flows at the cold end of the circular and annular pulse tube are 14.38 W and 12.78 W, respectively, with a difference of 1.6 W.

![Figure 8. No-load temperature vs. Input acoustic power.](image1)

![Figure 9. Cooling power & relative Carnot efficiency at 77 K vs. input acoustic power.](image2)

5 Conclusion
An annular pulse tube is the inevitable choice when one wants to build a compact, completely co-axial multi-stage pulse tube cooler. A two dimensional simulation investigation has been conducted using Fluent software to compare the performance of two different types of in-line pulse tube coolers working at liquid nitrogen temperature, an annular shape pulse tube and a circular pulse tube. The inhomogeneities of the velocity and temperature profiles are stronger in the annular pulse tube than in the circular pulse tube. The difference of acoustic power in the two pulse tubes is negligible and the difference of enthalpy flow is 1.6 W (about 11.4% of the enthalpy flow in circular pulse tube). The expansion efficiency of the annular pulse tube (78%) is lower than that of the circular pulse tube (88%).

Experiments have also been carried out to verify the differences in performance. The cooling power with the annular pulse tube is lower by 11% compared to the circular pulse tube. Both the simulations and experiments show that the performance deterioration of the annular pulse tube in this ~10 W@77 K class pulse tube cooler is acceptable. The analyses and comparisons in this study are helpful for a better understanding of the losses in the pulse tube.
tube and set the basis for building a completely co-axial two-stage pulse tube cooler system.

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