A non-uniform travelling-wave current source model for designing OAM antenna: theory, analysis and application

ZELIN ZHU1, (Graduate Student Member, IEEE), SHILIE ZHENG1, (Member, IEEE), XIAOWEN XIONG1, (Graduate Student Member, IEEE), YUQI CHEN1, (Graduate Student Member, IEEE), XIAONAN HUI1,2, (Member, IEEE), XIAOFENG JIN1, XIANMIN ZHANG1, (Member, IEEE), and XIANBIN YU1,2, (Member, IEEE)

1 College of Information Science and Electronic Engineering, Zhejiang University, Hangzhou 310027, Zhejiang Province, China
2 Zhejiang Lab, Hangzhou 310000, China

Corresponding author: Shilie Zheng (e-mail: zhengsl@zju.edu.cn).

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ABSTRACT Orbital angular momentum (OAM) has become a research hotspot in radar, communication, and other fields because of its spatial spiral phase distribution in azimuthal domain. However, it encounters some problems in practical application scenarios owing to its inherent defects. Recently, several particular OAM waves and novel antenna structures have been proposed to make up for its defects or optimize its characteristics. However, there lack a universal and systematic theoretical analysis for these schemes. In this paper, the non-uniform travelling-wave current source (NTCS) model for OAM generation is proposed. This model is universal and suitable to be applied to many proposed antennas. The far field pattern and OAM spectrum distribution are theoretically derived based on the uniform travelling-wave source model. Bridged by the OAM spectrum analysis, an application-oriented antenna design process is presented. A PSOAM MG antenna optimal for the MIMO system is designed as an example. This work not only unifies the generation model of different OAM waves, but also provides a new view for the OAM based antenna design.

INDEX TERMS Orbital angular momentum (OAM), non-uniform travelling-wave current source (NTCS), antenna design.

I. INTRODUCTION

Thanks to its unique spiral phase distribution [1][2], the orbital angular momentum (OAM) wave in radio frequency (RF) has been proved to have the application prospect in the fields of communication [3][4][5], radar [6][7], etc. It features a linear phase distribution $\exp(-jl\varphi)$ along the azimuthal direction, where $l$ is the OAM order reflecting the vorticity characteristics and can take any integer. Two OAM waves with different order are orthogonal to each other. When OAM order $l$ covers all integers from negative infinity to positive infinity, all OAM waves form a set of complete orthogonal bases in azimuthal domain. These above characteristics construct the cornerstone for promoting the applications of OAM waves.

However, the OAM wave has a doughnut-like radiation pattern with a dark zone in the center, and the divergence angles vary with the variation of order $l$ [8]. Hence, it is inconvenient to multiplex or superpose multi-mode OAM waves in the same elevation direction. To improve its characteristics, several particular OAM waves have been proposed. For instance, the plane spiral OAM (PSOAM) wave which propagates along the horizontal plane, and thus the divergence angles of all the PSOAM waves can be the same of nearly $90^\circ$, making the multiplexing and superposition possible [9]. OAM mode-group (MG), which is superposed by specific OAM waves, and thus improves the beam gain while maintains the vortex and orthogonality of conventional OAM [10][11]. PSOAM MG, the special form of OAM MG, which can be easily obtained as all the PSOAM waves propagate along the transverse plane [12][13]. The application potential of PSOAM MG in the MIMO communication system has been analysed and verified through experiments [12][14]. In the context, the single mode doughnut-like OAM wave is termed as the conventional OAM wave as a distinction. Fig.1
show the far field pattern of the conventional OAM wave, PSOAM, OAM MG and PSOAM MG, respectively.

With the development of the OAM-based application and the non-conventional OAM wave, how to design and analyse the OAM-related antenna is becoming more and more targeted. Till now, many kinds of antenna models have been proposed to improve the OAM wave’s characteristics to meet some specific requirements, such as the mode purity, the OAM order, the beam gain and the bandwidth, etc. Fig. 2 shows some typical OAM antennas. The travelling-wave current source model (Fig. 2(a)) has the ability to generate high-purity single-mode OAM wave [15][16], which may reduce the mode interference to improve the isolation in multiplexed system. In contrast, the uniform circular array (UCA) scheme (Fig. 2(b)) has the advantages of flexibility and reconfigurability owing to the variousness of the antenna element, however makes concessions in mode purity and system complexity [17][18]. The UCA scheme can be regarded as the discrete travelling-wave current source. In [11], a partial arc transmitting (PAT) antenna (Fig. 2(c)), which uses an arc current instead of the entire circumference compared with the travelling-wave current source model, was reported to generate the OAM MG with high equivalent OAM order of \( l = \pm 40 \). Similarly, literatures [19][20] present the array on an arc (Fig. 2(d)) to generate OAM MG (called “Q-OAM” in their papers), which can be regarded as a discrete PAT scheme. Although many antenna structures have been proposed, there lack a universal and systematic theoretical analysis. The common ground of these schemes is that the radiation sources are distributed on a circle, which provides an inspiration for the unification of theory.

In this paper, a non-uniform travelling-wave current source (NTCS) model is proposed to generalize the generation of the OAM waves. Most OAM generation schemes could be viewed as various types of the NTCS model. The far field distribution and the OAM spectrum distribution function of the NTCS are theoretically derived. Two classical OAM generation schemes: the PAT scheme and the UCA scheme are analysed by the proposed model to verify its correctness.

Since the NTCS model can give a direct OAM spectrum in the beginning of the antenna design, an application-oriented OAM antenna design process is proposed. To demonstrate the antenna design process in detail, the PSOAM MG antenna design for the application in MIMO system is discussed as an example. The proposed NTCS model provides a new way for both the OAM antenna analysis and the OAM-related antenna design.

II. NON-UNIFORM TRAVELLING-WAVE CURRENT SOURCE

A. SOURCE MODEL

It is well known that the travelling-wave current source model is capable of generating high purity single-mode OAM wave due to its consecutive source [11][15]. Fig. 3(a) shows its source model, \( \tilde{I}_0(l_0, \varphi') = I_0 e^{-j\lambda l_0 \varphi' / c} \), which can generate single mode OAM wave of \( l_0 = \frac{2m}{\lambda} \). The
subscript ‘u’ indicates that this parameter belongs to the uniform travelling-wave current source. Here, \( \vec{e}_l \) is direction of current which could be an axis of the rectangular \((\vec{e}_x, \vec{e}_y, \vec{e}_z)\) or the cylindrical coordinate system \((\vec{e}_r, \vec{e}_\theta, \vec{e}_z)\), \( \lambda_g \) is the guided wavelength of the electromagnetic field in the current source and \( a \) is the circular radius of the ring current. It is noted that the current here can be either electric or magnetic current source. The field analysis can be linked according to the principle of duality. The travelling-wave current source can be viewed as a special circle source with each point having the uniform amplitude and linear phase along the azimuth angle. Thus, a more general model can be proposed. It is the NTCS model. In this model, both the amplitude and the phase of each point along the circle source are the function of the azimuthal angle. The source function of the NTCS model can be expressed as

\[
\vec{I}_n'(\varphi') = I_n(\varphi')\vec{e}_i = I_{n0}(\varphi')e^{-j\lambda_n(\varphi')\varphi'}\vec{e}_i \tag{1}
\]

where \( I_{n0}(\varphi') \) and \( I_n(\varphi') \) are current amplitude distribution function and mode distribution function, respectively. The subscript ‘n’ indicates that this parameter belongs to the non-uniform travelling-wave current source. Based on this definition, the OAM generation schemes mentioned in Fig.2 can be unified to the NTCS models having different amplitude distribution, which are shown in Fig.4. Hence, the NTCS is a more universal model. As for the non-uniform \( I_n(\varphi') \), there has been no report till now owing to the difficulty of the practical realization.

However, if the radiation source is complex, it is not easy to obtain its analytic solution expression. Here we derive the far field expression of the NTCS based on the uniform travelling-wave current model. For the uniform travelling-wave current source generating OAM wave of mode \( l_0 \), the far field \( \vec{E}_u(r, \theta, \varphi) \) at the point \( P(r, \theta, \varphi) \) can be described as [21][15]:

\[
\vec{E}_u(r, \theta, \varphi) = -\frac{j\mu_0\omega}{4\pi} \int_0^{2\pi} I_{n0}(\varphi')e^{-j\lambda_n(\varphi')\varphi'}|r^2 - r'^2|d\varphi' \\
\approx C_0(-j)^l J_{l0}(\kappa \sin\theta) e^{-j\lambda_0e\varphi'} \vec{e}_r \\
= C_0 J_{|l|}(\kappa \sin\theta) e^{-j(\omega/c)(r^2 - r_0^2/\lambda_0^2)\varphi'} \vec{e}_r 
\tag{2}
\]

where \( C_0 \propto \frac{\mu_0 I_{l0}\omega \kappa r}{4\pi} \), \( \mu_0 \) is the permeability of free space, \( \omega \) is the angular frequency and \( k = \omega/c \) is the wave number. \( \vec{e}_r \) is the polarization direction of the electric field, which is related to the current direction \( \vec{e}_i \). In the far field, \( C_0 \) is a constant term which is independent on \( \theta \) and \( \varphi \).

According to the dual relation between the azimuthal domain and the OAM domain[22], the Fourier transform (FT) of the source \( I_u \) is

\[
M_u(l) = \mathcal{F}[I_u(l_0, \varphi')] = \delta(l - l_0) \tag{3}
\]

Eq.3 means that the uniform travelling current source with \( I_u(l_0, \varphi') \) can generate the OAM mode of \( l_0 \). When it comes to the non-uniform travelling-wave current source, its source \( \vec{I}_n(\varphi') \) can be expressed as the \( I_u(l_0, \varphi') \) with same direction \( \vec{e}_i \) multiplied by a modulation function \( D(l_0, \varphi') \):

\[
D(l_0, \varphi') = \frac{I_n(\varphi')}{I_u(l_0, \varphi')} \tag{4}
\]

The FT of the source \( I_n(\varphi') \) can be calculated as

\[
M_n(l) = \mathcal{F}[D(l_0, \varphi')I_u(l_0, \varphi')] = \mathcal{F}[D(l_0, \varphi')] \ast D(l_0, \varphi') \tag{5}
\]

where \( (\ast) \) is the convolution operator. Based on Eq.5, the non-uniform source \( I_n(\varphi') \) can be equivalent to a sum of a series of single mode source with weight function \( M_n(l) \):

\[
\vec{I}_n(\varphi') = \sum_{l=-\infty}^{\infty} M_n(l)\vec{I}_u(l, \varphi') \tag{6}
\]

Since the far field of the uniform travelling-wave source is known as Eq.2, the radiation field of the source \( \vec{I}_n(\varphi') \) can be organized into:

\[
\vec{E}_n(r, \theta, \varphi) = C_0 \sum_{l=-\infty}^{\infty} \mathcal{F}[D(l - l_0, \varphi')] \\
J_{|l|}(\kappa \sin\theta) e^{-j(\omega/c)(r^2 - r_0^2/\lambda_0^2)\varphi'} \vec{e}_r \tag{7}
\]

Obviously, Eq.7 can be used to obtain the far field expression of all the NTCS models. Compared with the schemes of vector potential and the field superposition of hertz dipole, this method is easy and directly perceived. The field is

FIGURE 4. The amplitude distributions of the classical non-uniform current sources in Fig.2: (a) the travelling-wave current source based antenna; (b) the UCA based antenna; (c) the PAT based antenna; (d) the arc array based antenna.

B. FAR FIELD AND OAM SPECTRUM ANALYSIS

The far field of the antenna is usually obtained by the vector potential or the superposition of the hertz dipole, however if the radiation source is complex, it is not easy to obtain its analytic solution expression. Here we derive the far field expression of the NTCS based on the uniform travelling-wave current model. For the uniform travelling-wave current source generating OAM wave of mode \( l_0 \), the far field \( \vec{E}_u(r, \theta, \varphi) \) at the point \( P(r, \theta, \varphi) \) can be described as [21][15]:

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\]

Obviously, Eq.7 can be used to obtain the far field expression of all the NTCS models. Compared with the schemes of vector potential and the field superposition of hertz dipole, this method is easy and directly perceived. The field is
obtained by the decomposition of the non-uniform travelling-wave current source and the field superposition of each travelling-wave current source. As travelling-wave current source can generate one single mode OAM wave, the radiation of the NTCS, which is the superposition of different travelling wave current source, is no longer single-mode, but multi-mode. Based on of the decomposition of the current source, the OAM spectrum can be easily obtained.

This model is an extension of the classical Fourier-Bessel integrals solution in the generation of OAM waves [23], which could extract the significant information: the OAM spectrum. In the RF domain, the receiving antenna is usually placed at the maximum radiation ring, hence the normalized phase of each mode. The main lobe direction of the NTCS, which is the superposition of different modes, is the main lobe direction, therefore the normalized complex weight function can be found by calculating the maximum point.

\[ A_i(l) \propto D[l-l_0, \varphi'] J_l(k \sin \theta_0) e^{j l \varphi} = |A_i(l)| e^{j \phi_i(l)} \]  

where \(|A_i(l)|\) is the OAM spectrum and \(\phi_i(l)\) is the initial phase of each mode. The main lobe direction \(\theta_0\) in the \(\theta\) direction can be found by calculating the maximum point.

\[ \theta_0 = \arg \max_\theta |E_n(\theta, \varphi)| \]  

OAM MG is the superposition of different OAM waves with pre-designed complex weight. Thus the NTCS model and the analysis method based on travelling-wave current source provides a useful way to construct the OAM MG.

III. CLASSICAL SCHEMES OF THE NTCS MODEL

In this section, two classical OAM generation schemes: the PAT scheme and the UCA scheme are analysed based on the NTCS model and the corresponding analysis method. The antennas are simulated in the commercial software CST to verify the accuracy of theoretical analysis.

A. THE PAT SCHEME

Fig.3(b) shows the model of the PAT scheme which can be regarded as a uniform travelling-wave current source covered by a sector ring mask. According to the NTCS model, its modulation function is:

\[ D_{PAT}(\varphi') = g(\varphi') = \begin{cases} 1, & 0 < \varphi' \leq \beta \\ 0, & \beta < \varphi' < 2\pi \end{cases} \]  

The FT of the PAT source are

\[ \mathcal{F}[D_{PAT}(l-l_0, \varphi')] = \text{Sinc}(\frac{\beta(l-l_0)}{2}) e^{j l \varphi} \]  

Then the normalized complex weight function and the far field of the PAT scheme can be further calculated:

\[ A_{PAT}(l) \propto |A_{PAT}(l)| e^{j \phi_{PAT}(l)} = |\text{Sinc}(\frac{\beta(l-l_0)}{2}) J_l(k \sin \theta_0) e^{j l \varphi} | e^{j l \varphi} \]  

\[ \vec{E}_{PAT}(r, \theta, \varphi) = C_0 \sum_{l=-\infty}^{\infty} \text{Sinc}(\frac{\beta(l-l_0)}{2}) J_l(k \sin \theta_0) e^{j l \varphi} \]  

\[ J_l(k \sin \theta_0) e^{-j l \theta_0} e^{-j l (\varphi - \varphi_0)} e^{-r} \]  

\[ \varphi_0 = \frac{l \pi}{\beta} + \frac{\beta}{2} \]  

where \(\theta_0\) is the main lobe direction in elevation angle, as shown in Eq.9. \(\varphi_0\) is the theoretical main lobe direction in azimuthal direction. The radiation characteristics of the PAT scheme, such as \(\varphi_0\), \(\theta_0\) and \(|A_{PAT}(l)|\) can be controlled by adjusting parameters \(a\), \(\beta\) and \(l_0\).

Next we are going to use a one-ninth aperture PAT antenna [11] as an example to verify the correctness of the NTCS analysis method. The main antenna parameters are listed in Table 1. It is a kind of leaky-wave antenna based on a partial slotted curved waveguide. And the direction of its principal equivalent current component \(\vec{e}_i\) is the \(\vec{e}_r\) axis of the cylindrical coordinate system.

| Parameter | Value |
|-----------|-------|
| \(f\) (GHz) | 60 |
| \(l_0\) | \(\pm 40\) |
| \(a\) (mm) | 80 |
| \(\beta\) (rad) | \(\frac{2\pi}{3}\) |

The maximum point of the simulated far field is obtained at point \(P_{max}(\varphi_s = 24.0^\circ, \varphi_r = 112.0^\circ)\). Fig.5(a) shows the simulated radiation pattern in elevation direction when \(\varphi = \varphi_s\) (the red line) and the theoretical one when \(\varphi = \varphi_0\) calculated by Eq.13 (the blue one). The simulated and theoretical radiation patterns agree very well. The simulated main lobe direction \(\theta_s\) in elevation direction are also well matched with theoretical one \(\theta_0 = 23.8^\circ\), the error is less than 0.2\(^\circ\). Fig.5(b) shows the simulated and theoretical results of the amplitude distribution in azimuthal direction when \(\theta = \theta_s\) and \(\theta = \theta_0\), respectively. The patterns are consistent on the whole, and the simulated \(\varphi_s\) is 111.4\(^\circ\). Compared with the theoretical value \(\varphi_0 = 110.0^\circ\), there is only a 1.4\(^\circ\) deviation.

Fig.6(a) shows the simulated beam pattern and phase distribution results in the azimuthal direction. According to Eq.14, all negative modes are superimposed with the same initial phase in the opposite direction of the main lobe direction \(\varphi_r = \varphi_0 + \pi = \frac{-l \pi}{\beta} + \frac{\beta}{2}\), while the positive modes are not. Therefore, there is a side lobe peak with an opposite phase slope in \(\varphi_r\), where the contribution of the negative modes in the MG is much greater than the positive modes.

Fig.6(b) shows the simulated results of the phase distribution and the calculated one in the azimuthal main lobe, which is highlighted in blue in Fig.6(a). The calculated one is \(arg(\vec{E}_{PAT}(\theta, \varphi))\), where \(arg(\cdot)\) is the argument function. It can be seen that the two curves agree very well. The wave has a stable phase slope and the beam directionality of OAM MG compared with the conventional OAM waves. And its equivalent order \(l_c = 40\) is same with the desired order \(l_0\).
The latter is calculated by conjugately filtering the E-field in \((\theta = \theta_s, \varphi \in (0, 2\pi))\) and then be normalized as Eq.15. It can be seen that they are well matched, and the calculated OAM spectrum can be identified as the integer sampling of the theoretical OAM spectrum function. The simulated OAM purity around \(-l_o\) is a bit larger than the theoretical one. It is because that there exists a reflected wave \(E_r \propto e^{j\varphi}\) in the waveguide, which generates an opposite source \(I_r = |\Gamma_L| e^{-j(-l_o)\varphi}\), where \(\Gamma_L\) is the reflection coefficient. It amplifies the side lobe peak in the opposite direction and then worsens the front-to-rear ratio of the far field, which could be optimized by reducing the standing wave coefficient. Moreover, the OAM spectrum of the PAT scheme is quasi-symmetric centered by the desired OAM order \(l_o\). The OAM spectrum of the PAT is the product of the Sinc function and the Bessel function. Centred by the desired OAM order \(l_o\), the Sinc function is symmetric while the Bessel function is not. Hence, the quick vanishment of the high order Bessel function results the asymmetric centered by the designed order \(l_o\). The spectrum symmetry of the OAM MG can be further improved by optimizing the antenna parameters. In part IV, the characteristics of the OAM MG is analysed based on the symmetry of the OAM spectrum.

### B. THE UCA SCHEME

The UCA scheme is a commonly used scheme to generate OAM waves. In the former literatures, the radiation field distribution is calculated based on the traditional field theory[21], and then the OAM spectrum distribution is further analysed. Using the NTCS model, its field distribution and OAM characteristics can be calculated more simply and directly, as it can be explained from the perspective of OAM spectrum. Its modulation functions meet:

\[
D_U(\varphi') = \begin{cases} 
1, & \frac{2\pi(k_U-1)}{N_U} < \varphi' \leq \frac{2\pi(k_U-1)}{N_U} + d_U \\
0, & \frac{2\pi(k_U-1)}{N_U} + d_U < \varphi' < \frac{2\pi k_U}{N_U}
\end{cases}
\]

Obviously, its current amplitude distribution function is a periodic gate function, where \(N_U\) is the quantity of the units in the UCA, \(k_U \in (1, N_U)\) is the serial number, \(d_U\) is the duty cycle of the gate function, which depends on the ratio of the size of the unit to \(\frac{2\pi}{N_U}\). The direction of its principal equivalent current component \(\vec{e}_i\) is the \(\vec{e}_z\) axis. According to the NTCS model, its modulation function is: The FT of the periodic gate function is a discrete function at \(l = m\delta l_0\), where \(\delta l_0 = N_U\) and \(m\) is arbitrary integer. It is sampled at a Sinc function. Hence, the OAM spectrum function and the far field can be calculated:

\[
|A_U(l)| \propto |H(l)\text{Sinc}(\frac{d_U\pi(l - l_0)}{N_U})J_l(k\sin\theta_0)|
\]

\[
H(l) = \begin{cases} 
1, & l = l_0 + m\delta l_0 \\
0, & \text{others}
\end{cases}
\]
\[ \vec{E}_U(r, \theta, \varphi) = C_0 \sum_{l=\delta l_0}^{\infty} \text{Sinc}\left(\frac{d_U \pi (l - l_0)}{N_U}\right) J_l(\text{kasin}\theta)e^{j\left(\frac{d_U \pi}{\pi} - \frac{d_U \pi}{\pi} - \frac{d_U \pi}{\pi}\right) e_r} \]

where \(H(l)\) is the sampling function. In particular, when the duty cycle \(d_U\) is 1, its OAM spectrum function is \(|A_U(2,1)| \approx J_0(\text{kasin}\theta_0)|\), which means that it is equivalent to a uniform travelling-wave current source scheme, though the mode purity of the UCA scheme is poor. As shown in Eq.17, it will have the OAM side-lobes dependent on the number of antenna unit. The more the antenna units, the larger the difference of the desired mode and the side-lobe. The antenna unit can be supposed to be infinite in the uniform travelling-wave current source, which show the less interference of the side-lobes and the superiority in the desired mode purity.

Taking two dipole UCA OAM antennas as an example, which are with various \(N_U = 12\) or 18 while keeping other parameters the same: \(a = 35\) mm, \(l_0 = 5\) and operating frequency \(f_c\) is 7.56 GHz. Here, \(\text{Sinc}\left(\frac{d_U \pi (l - l_0)}{N_U}\right)\) becomes a constant because of the infinitesimal \(d_U\) of the dipole units. The Bessel function \(J_l(\text{kasin}\theta)\) presents a low-pass filter characteristics around \(l (-10, 10)\) as exhibited in Fig.8, where \(k = \frac{2\pi f c}{3 \times 10^8} = 158.34\) and \(\theta_0 = \frac{\pi}{2}\) rad.

\[ J_l(\text{kasin}\theta) \quad \text{Fig. 8. The low-pass filter characteristics of the Bessel function} \]

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model can be determined. Comprehensively evaluating other requirements, such as bandwidth, polarization, etc., a kind of appropriate antenna structure can be selected to design the antenna in line with the source distribution. Next, taking the PSOAM MG antenna design for the application in MIMO system as an example, the application oriented antenna design process is discussed.

In the PSOAM MG based MIMO system, the directionality and the vorticity of OAM MG can improve the signal-to-noise ratio and reduce the channel correlation, respectively. To bring the OAM MG’s superiority into full play, it’s highly anticipated for the generated beam with a stable high equivalent order and a high gain. Moreover, if the beams at the transmitting end have the orthogonality, the complexity of demodulation will be greatly reduced. In addition, the other requirements such as the wider bandwidth, the dual-polarization and the low complexity and cost should also be considered in the antenna design.

As analysed before, the conventional single-mode OAM wave is omnidirectional in azimuthal direction, because its OAM spectrum distribution is an impulse function \(\delta(l - l_0)\) [24]. According to the dual relationship between OAM order and azimuthal angle, broadening OAM spectrum could make the beam narrowing, which can increase the directionality. The method of generating a high directional beam by superposition of multiple modes OAM waves is called OAM-based beam-forming [10]. If the OAM spectral components are continuous, an energy-convergent pencil beam can be obtained. Hence, the directionality of the OAM MG needs many consecutive OAM modes when the NTCS model is applied in antenna design.

The vorticity of the MG means that the phase of the main lobe has a linear distribution. The necessary condition for an OAM MG to have a stable vorticity is that it must possess a symmetric OAM spectrum distribution. Any OAM MG can be expressed as:

\[
E_{MG}(\varphi) = \sum_{i=1}^{N} A_i(l)e^{-jO_i(l)\varphi} \tag{20}
\]

where \(N\) is the quantity of superposed single-mode OAM waves, \(i\) is the serial number, \(O_i\) is the OAM order and \(A_i(l) = |A_i(l)|e^{j\Phi(i)}\) is the normalized complex weight function, which is the product of its OAM purity \(|A_i(l)|\) and initial phase factor \(e^{j\Phi(i)}\).

For a symmetrical OAM spectral components, its \(O_i\) and \(A_i\) have to satisfy:

\[
\begin{cases}
O_i(l) + \delta O_i(l) = O_i(N - i + 1) - \delta O_i(l) = l_c \\
A_i(l) = A_i(N - i + 1)
\end{cases}
\tag{22}

where \(l_c\) is the center mode, \(\delta O_i\) refers to the difference values between each mode and the center mode.

In this case, Eq.20 can be rewritten as

\[
E_{MG}(\varphi) = ((\sum_{i=1}^{[\frac{N}{2}]} 2A_i(l)cos(\delta O_i(l))) + A_{l_c})e^{-jl_c\varphi} \tag{23}
\]

where \(A_{l_c}\) is the complex weight of the center mode \(l_c\), which varies when \(N\) is odd or even

\[
A_{l_c} = \begin{cases}
A_i(N + 1), & \text{mod}(N, 2) = 1 \\
0, & \text{mod}(N, 2) = 0
\end{cases} \tag{24}
\]

Eq.23 shows that the OAM MG has a linear azimuthal phase distribution \(exp(-jl_c\varphi)\), when the OAM spectrum distribution is entirely symmetrical. The phase slope is equal to \(l_c\), which is the center mode and called the equivalent order of the MG. To reduce the channel correlation in the MIMO system, the high equivalent order could be obtained by setting a higher center mode. For the practical application, when the OAM spectrum shows quasi-symmetric, the OAM MG can also maintain the linear phase distribution in some degrees, which is the case of the PAT scheme discussed in Part III.

As for the orthogonality, for the conventional single-mode OAM waves, they are orthogonal when they have different orders. Usually, we use the correlation coefficient \(\eta_{MG}\) to judge the orthogonality between two different OAM MGs by their electric field distribution[25]:

\[
\eta_{MG} = \frac{\int_{0}^{2\pi} E_{MG1}(l)E_{MG2}(l)dl}{\sqrt{\int_{0}^{2\pi} E_{MG1}(l)^2 dl \int_{0}^{2\pi} E_{MG2}(l)^2 dl}} \tag{25}
\]

When \(\eta_{MG}\) is close to 0, these two OAM waves are quasi-orthogonal.

According to the orthogonality between different OAM modes and the relationship between the electric field and the OAM spectrum. The orthogonality for the OAM MG can be simplified as.
where \( A_{11}(l) \) and \( A_{12}(l) \) are the complex weight functions of two OAM MGs, respectively.

Apparently, the field orthogonality can be calculated by the OAM spectral orthogonality. To the extreme, if there is no mode overlap between the OAM spectrum of two OAM MGs, they are perfectly orthogonal to each other. Besides, if Eq.26 approaches 0, they are also orthogonal. Therefore, two quasi-orthogonal beams can be easily constructed by the OAM spectral orthogonality analysis using the NTCS model.

According to the above analysis, there are rules to design the OAM MG antennas optimal for the PSOAM MG based MIMO system based on NTCS model. The PAT scheme is such kind of the NTCS model, whose beam characteristics can meet all these requirements. Owing to its partial arc source distribution, its OAM spectrum can be broadened accordingly and is continuous and quasi-symmetric. It is feasible to obtain the OAM MG with high equivalent order \([11]\) and easy to obtain the orthogonal OAM MGs only by separating the OAM spectrum distribution in the NTCS design.

**TABLE 2. parameter list of the PAT source in Fig.11**

| Source | Equivalent order | \( \alpha (\text{mm}) \) |
|--------|-----------------|------------------------|
| Orange one in Fig.11(a) | \(-40\) | 34 |
| Blue one in Fig.11(a) | 40 | 34 |
| Orange one in Fig.11(b) | 20 | 17 |
| Blue one in Fig.11(b) | 40 | 34 |

Fig.11 shows two PAT schemes to generate two orthogonal OAM MGs using a shared aperture. Main various parameters of the PAT sources are listed in Tab.2. They are working at same operating frequency of 60 GHz and have the same arc angle \( \beta \) of \( \frac{\pi}{2} \) rad. Fig.11(a) is to design two OAM MG with opposite equivalent orders \( l_{c1} = 40 \) and \(-40\). Two sources are placed at symmetrical arcs \( \beta_1 \in (0, \beta) \) and \( \beta_2 \in (\pi, \pi + \beta) \) of the same radius, respectively. They have opposite travelling-wave direction (as the arrow direction in Fig.11) to ensure the same main lobe direction at \( \phi_0 = \frac{\beta_1}{\pi} + \frac{\beta_2}{\pi} \) and opposite equivalent orders. Fig.11(b) corresponds to two independent OAM MGs with various equivalent order of \( l_{c1} = 20 \) and \( l_{c2} = 40 \). Their sources have same arc angle but different circular radii. Fig.11(c) displays the phase distribution of the four OAM MGs generated by the PAT sources, they all have a stable linear phase distribution in their main lobe region around \( \phi_0 = 135^\circ \).

Due to the symmetry of the current sources, the OAM spectrum and field distributions generated by the two sources of the first scheme are symmetrical. Hence, the two OAM MGs have the same beam gain and beam width as shown in the dotted box in the Fig.11(a). The two OAM MGs of the second scheme could not maintain a similar pattern as the two current sources have different radius. However, from the view of the OAM spectrum, the overlap of both the two cases is small, which verifies that the two OAM MGs are quasi-orthogonal. The calculated correlation coefficient of the two source schemes are 0.0055 and 0.0069, respectively. For the first scheme, it maximizes the aperture and generates OAM MGs with the largest mode difference \( \Delta l = 2l_{c} \), thus it can minimize the channel correlation of the MIMO system. However, the second scheme has the advantage of smaller actual aperture. In \( N \times N \) MIMO system, these two schemes can be combined to design more mutually orthogonal OAM MGs.

After the source distribution of the PAT schemes are determined for the PSOAM MG antennas from the OAM spectrum analysis, various antenna models can be used for designing PAT antenna, for instance, the leaky-wave antenna based on a partial slotted curved waveguide in [11], the travelling-wave resonator antenna, the spoof surface plasmon polaritons (SSPP) antenna, etc. Hence, there must be a satisfactory antenna structure to construct the PAT source distribution according to the requirements in this application scenario such as wide-band operating frequency, high polarization isolation, low antenna cost and low design difficulty.

In summary, a NTCS model based application-oriented antenna design process has been presented in this section. Analysis of the NTCS model and the OAM spectrum bridges the application requirements and antenna design.

V. CONCLUSIONS

In this paper, the non-uniform travelling-wave current source model is proposed as a universal method of generating the OAM waves. The analytical expression of the far field pattern and the OAM spectrum distribution generated by any NTCS are derived and calculated. The PAT scheme and the UCA scheme are analysed as two examples to verify its correctness. The results shows that the NTCS model can provide an
effective analysis for different kinds of the OAM antennas. Since the NTCS model considers the OAM spectrum directly, an application-oriented antenna design process can be obtained based on it. The antenna design of the PSOAM MG which is applied in the MIMO system is detailed discussed as an instance.

All the demonstrations show that the NTCS model not only proposes a universal source model, which generalizes the creation of the OAM waves, but also gives a new aspect for the OAM antenna design, which bridges the application requirements and antenna design. This work will provide a new perspective for promoting the OAM-related applications.

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Z. Zhu et al.: A non-uniform travelling-wave current source model for designing OAM antenna: theory, analysis and application

XIAOWEN XIONG received the B.S. degree in communication engineering from Nanjing University of Posts and Telecommunications, Nanjing, China, in 2017. He is currently pursuing the Ph.D. degree with electronic science and technology, Zhejiang University, Hangzhou, China. His current research interests are in the areas of orbital angular momentum (OAM) waves, OAM based beamforming technique and wireless communication system. He was a recipient of the Best Student Paper Honorable Mention in the 2021 IEEE MTT-S International Wireless Symposium (IW2021) held in Nanjing, China. He served as a TPC member of the 2021 IEEE International Conference on Communications Workshops (ICC2021 Workshop on OAM Transmission) held in Montreal, Canada.

YUQI CHEN received the B.S. degree in University of Electronic Science and Technology of China, in 2019. He is currently pursuing the Ph.D. degree in electronic science and technology with Zhejiang University, Hangzhou, China. His research interests include orbital angular momentum antenna and its applications in wireless communication.

XIAONAN HUI received the B.S. degree from Northeastern University, China, in 2012, the M.S. degree from Zhejiang University, China, in 2015, and the Ph.D. degree from Cornell University, USA, in 2021, all in electrical engineering. In 2021, he joined the College of Information Science and Electronic Engineering, Zhejiang University as an assistant professor. His research interests include wireless sensing, RFID, IoT, and wireless communication systems.

XIAOFENG JIN received the B.S. degree in optical engineering from Huazhong University of Science and Technology, Wuhan, China, in 1990, the M.S. degree in Underwater Acousto-electronics Engineering from China Ship Building Institute in 1993, and the Ph.D. degree in optical engineering from Zhejiang University, Hangzhou, China, in 1996. In 1999, he was appointed as an Associate Professor at the Department of Information and Electronic Engineering, Zhejiang University, and a Full Professor in 2006. His current research interests include microwave photonics, photonic circuits, components and modules, and smart sensing systems.

XIANBIN YU is currently a Research Professor at the Zhejiang University, Hangzhou, China. From 2005 to 2007, he was a postdoctoral researcher in Tsinghua University, China. Since 2007, he was employed at the Technical University of Denmark (DTU), as a postdoc, assistant professor and Senior Researcher. He has co-authored 2 book chapters and 180+ peer-reviewed international journal and conference papers in the area of optical communications. His current research interests are in the areas of THz/microwave photonics and its applications, optical communications, ultrafast photonic RF signal processing and high speed photonic wireless access technologies.

XIANMIN ZHANG received the B.S. and Ph.D. degrees in physical electronics and optoelectronics from Zhejiang University, Hangzhou, China, in 1987 and 1992, respectively. He was appointed as an Associate Professor of information and electronic engineering at Zhejiang University in 1994 and a Full Professor in 1999. He was a Research Fellow with the University of Tokyo, Tokyo, Japan, and Hokkaido University, Sapporo, Japan, from November 1996 to September 1997, and October 1997 to September 1998, respectively. In 2007, he spent 2 months with the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, MA, USA, as a Visiting Research Fellow. He was the Dean of the Department of Information Science and Electronic Engineering, Zhejiang University, from September 2005 to November 2017, the Dean of the School of Microelectronics, Zhejiang University, from May 2015 to September 2018, the Vice Dean of Polytechnic Institute, Zhejiang University, from July 2016 to September 2018, the President of Zhejiang University Ningbo Institute from July 2018 to April 2020, and the Dean of Ningbo Campus, Zhejiang University, from September 2018 to November 2020. He is currently the Vice President of NingboTech University, Ningbo, China. His research interests include microwave photonics, electromagnetic wave theory and applications.