Origin of resonances in chiral dynamics

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Dynamical state and CDD pole

Chiral unitary approach

Natural renormalization scheme

Effective interaction: origin of resonance

Application: Λ(1405) and N(1535)

T. Hyodo, D. Jido, A. Hosaka, arXiv:0803.2550 [nucl-th]
Resonances in two-body scattering

- Knowledge of interaction (potential)
- Experimental data (phase shift, cross section)

**Dynamical state**: molecule, quasi-bound, ...

- \( e.g. \) Deuteron in NN, positronium in \( e^+e^- \), (\( \sigma \) in \( \pi \pi \)), ...

**CDD pole**: elementary, independent, ...

\( L. \) Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)

- \( e.g. \) \( J/\Psi \) in \( e^+e^- \), (\( \rho \) in \( \pi \pi \)), ...
Dynamical state and CDD pole

Dynamical state and CDD pole (notes)

Model space and dynamical/CDD

Notion of dynamical/CDD depends on the scattering particles under consideration. It is not an inherent property of the resonance state.

e.g.) J/Ψ : CDD in e^+e^−, dynamical in cū

Quark structure (for baryon resonances)

dynamical ~ CDD ~

For hadron resonances, dynamical/CDD is not directly related to quark structure.

Mixing of dynamical and CDD

When both exist in one system, relative weight is important.
Chiral unitary approach

**S = -1, \bar{K}N s-wave scattering : \Lambda(1405) in I=0**

- Interaction \( \leftarrow \) chiral symmetry
- Amplitude \( \leftarrow \) unitarity (coupled channel)

\[
T = \frac{1}{V^{-1} - G}
\]

**Chiral (WT interaction)**

By construction, generated resonances are all dynamical? 

**Not always...**

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N. Kaiser, P. B. Siegel, W. Weise, Nucl. Phys. A594, 325 (1995)
E. Oset, A. Ramos, Nucl. Phys. A635, 99 (1998)
J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)
M.F.M. Lutz, E. E. Kolomeitsev, Nucl. Phys. A700, 193 (2002), .... many others
Scattering theory: N/D method

Single-channel scattering, masses: $M_T$ and $m$

G.F. Chew, S. Mandelstam, Phys. Rev. 119, 467 (1960)

\[ s = W^2 \]

unphysical cut \[ s^- = (M_T - m)^2 \] unitarity cut \[ s^+ = (M_T + m)^2 \]

Divide $T$ into N(umerator) and D(inominator)

unitarity cut $\rightarrow$ D, unphysical cut $\rightarrow$ N

\[ T(s) = N(s)/D(s) \]

phase space (optical theorem)

\[
\begin{align*}
\text{Im}D(s) &= \text{Im}[T^{-1}(s)]N(s) = \rho(s)N(s)/2 \quad \text{for } s > s^+ \\
\text{Im}N(s) &= \text{Im}[T(s)]D(s) \quad \text{for } s < s^- 
\end{align*}
\]

Dispersion relation for N and D

$\rightarrow$ set of integral equations, input: \[ \text{Im}[T(s)] \quad \text{for } s < s^- \]
Neglect unphysical cut (crossed diagrams), set $N=1$

\[ T^{-1}(\sqrt{s}) = \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)} \]

**subtraction constant**, not determined

**pole (and zero) of the amplitude**

L. Castillejo, R.H. Dalitz, F.J. Dyson, Phys. Rev. 101, 453 (1956)

\[ s^- = (M_T - m)^2 \]

\[ s^+ = (M_T + m)^2 \]

**CDD pole(s), $R_i$, $W_i$ : not known in advance**

\[ T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - \sqrt{s_i}} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)} \]

**CDD pole contribution --> independent particle**

G.F. Chew, S.C. Frautschi, Phys. Rev. 124, 264 (1961)
Identify loop function $G$, the rest contribution $\to V^{-1}$

\[
T^{-1}(\sqrt{s}) = \sum_i \frac{R_i}{\sqrt{s} - \sqrt{s}_i} + \tilde{a}(s_0) + \frac{s - s_0}{2\pi} \int_{s^+}^{\infty} ds' \frac{\rho(s')}{(s' - s)(s' - s_0)}
\]

\[
= -i \int \frac{d^4q}{(2\pi)^4} \frac{2M_T}{(P - q)^2 - M^2_T + i\epsilon} \frac{1}{q^2 - m^2 + i\epsilon} \bigg|_{\text{dim.reg.}}
\]

\[
= - \frac{2M_T}{(4\pi)^2} \left\{ a + \frac{m^2 - M^2_T + s}{2s} \ln \frac{m^2}{M^2_T} + \frac{\bar{q}}{\sqrt{s}} \ln \frac{\phi_{++}(s) \phi_{+-}(s)}{\phi_{-+}(s) \phi_{--}(s)} \right\}
\]

\[
= -G(\sqrt{s}; a)
\]

\[
T(\sqrt{s}) = [V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)]^{-1}
\]

$V$? chiral expansion of $T$, (conceptual) matching with ChPT

J. A. Oller, U. G. Meissner, Phys. Lett. B500, 263 (2001)

\[
T^{(1)} = V^{(1)}, \quad T^{(2)} = V^{(2)}, \quad T^{(3)} = V^{(3)} - V^{(1)}GV^{(1)}, \ldots
\]
**Summary of chiral unitary approach**

**Scattering amplitude\( T \)**

\[
T(\sqrt{s}) = \frac{1}{V^{-1}(\sqrt{s}) - G(\sqrt{s}; a)}
\]

- \( V(\sqrt{s}) \) : interaction (ChPT at given order)
- \( G(\sqrt{s}; a) \) : loop function
- \( a \) : subtraction constant (cutoff parameter)

|                      | ChPT                  | ChU                  |
|----------------------|-----------------------|----------------------|
| **Unitarity**        | perturbative          | exact                |
| **Dynamical resonance** | \( \times \)       | \( \bigcirc \)      |
| **Crossing symmetry** | exact                | (perturbative)       |
| **Chiral counting**  | \( \bigcirc \)       | \( \times \)        |

**Nonrenormalizable --> cutoff theory**

**CDD pole contribution --> V (interaction)**
Chiral unitary approach

(Known) CDD pole in chiral unitary approach

Explicit resonance field in $V$ (interaction)

$$\begin{align*}
\text{Diagram 1} & + \quad \text{Diagram 2} + \ldots
\end{align*}$$

U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)
D. Jido, E. Oset, A. Ramos, Phys. Rev. C66, 055203 (2002)

Contracted resonance propagator in $V$

$$\begin{align*}
\text{Diagram 3} & \quad M \to \infty \quad \text{Diagram 4}
\end{align*}$$

G. Ecker, J. Gasser, A. Pich, E. de Rafael, Nucl. Phys. B321, 311 (1989)
V. Bernard, N. Kaiser, U.G. Meissner, Nucl. Phys. A615, 483 (1997)

$$\begin{align*}
\text{Diagram 5} & + \quad \text{Diagram 6} + \ldots
\end{align*}$$

J.A. Oller, E. Oset, J.R. Pelaez, Phys. Rev. D59, 074001 (1999)

Is that all? subtraction constant?
Phenomenological (standard) scheme

--> V is given, “a” is determined by data

\[ T = \frac{1}{(V^{(1)})^{-1} - G(a)} \]  

leading order

\[ T = \frac{1}{(V^{(1)} + V^{(2)})^{-1} - G(a')} \]  

next to leading order

↑pole question mark

“a” represents the effect which is not included in V. The CDD pole contribution in G?

Natural renormalization scheme

--> fix “a” first, then determine V

exclude CDD pole contribution from G, based on theoretical argument.
Loop function below threshold

Below threshold, $G$ is real and NEGATIVE
(\sim assume no states below threshold)

$$G(\sqrt{s}) = \leq 0 \quad \text{(for } \sqrt{s} \leq M_T + m)$$

It is automatically satisfied in 3d cutoff. However, ...

$$G(\sqrt{s}; a) = \frac{2M_T}{(4\pi)^2} \left\{ a + \frac{m^2 - M_T^2 + s}{2s} \ln \frac{m^2}{M_T^2} + \frac{\bar{q}}{\sqrt{s}} \ln \frac{\phi_{++}(s)}{\phi_{--}(s)} \right\}$$

Large (positive) “a” can make $G$ positive.
Avoid this for s-channel region (above $M_T$),

$$a \leq a_{\text{max}}(M_T, m)$$

or equivalently (G: decreasing),

$$G(\sqrt{s} = M_T) \leq 0$$
(Explicit) matching with ChPT

V is given by ChPT. At a “low energy”, T should be matched with V:

\[ G(\sqrt{s} = \mu_m) = 0, \quad \Leftrightarrow \quad T(\mu_m) = V(\mu_m) \]

subtraction constant: real

\[ \Rightarrow \quad M_T \leq \mu_m \leq M_T + m \]

consistent with “low energy” requirement

\[ \sqrt{s} = M_T + m \quad \Rightarrow \quad p = 0, \quad \sqrt{s} = M_T \quad \Rightarrow \quad \omega \sim 0 \]
Natural renormalization scheme

Natural renormalization condition

- Loop function should be negative below threshold
- \( T \) matches with \( V \) at low energy scale

“\( a \)” is uniquely determined such that

\[
G(\sqrt{s} = M_T) = 0, \quad \Leftrightarrow \quad T(M_T) = V(M_T)
\]

matching with low energy interaction

K. Igi, K. Hikasa, Phys. Rev. D59, 034005 (1999)
U.G. Meissner, J.A. Oller, Nucl. Phys. A673, 311 (2000)

crossing symmetry (matching with u-channel amplitude)

M.F.M. Lutz, E. Kolomeitsev, Nucl. Phys. A700, 193 (2002)

We regard this condition as the exclusion of the CDD pole contribution from \( G \)
Two renormalization schemes

**Phenomenological scheme**

\( V \) is given by ChPT (for instance, leading order term), fit cutoff in \( G \) to data

**Natural renormalization scheme**

determine \( G \) to exclude CDD pole contribution, \( V \) is to be determined

Same physics (scattering amplitude \( T \))

\[
T = \frac{1}{V_{\text{ChPT}}^{-1} - G(a_{\text{pheno}})} = \frac{1}{(V_{\text{natural}})^{-1} - G(a_{\text{natural}})}
\]

↑Effective interaction
Origin of the resonance
Pole in the effective interaction

Leading order V : Weinberg-Tomozawa term

\[ V_{WT} = -\frac{C}{2f^2}(\sqrt{s} - M_T) \]

\[ T^{-1} = V_{WT}^{-1} - G(a_{\text{pheno}}) = (V_{\text{natural}})^{-1} - G(a_{\text{natural}}) \]

\[ \uparrow \text{ChPT} \quad \uparrow \text{data fit} \quad \uparrow \text{given} \]

Effective interaction in natural scheme

\[ V_{\text{natural}} = -\frac{C}{2f^2}(\sqrt{s} - M_T) + \frac{C}{2f^2} \frac{(\sqrt{s} - M_T)^2}{\sqrt{s} - M_{\text{eff}}} \]

\[ M_{\text{eff}} = M_T - \frac{16\pi^2 f^2}{CM_T \Delta a}, \quad \Delta a = a_{\text{pheno}} - a_{\text{natural}} \]

Physically meaningful pole : \( C > 0, \ \Delta a < 0 \)

There is always a pole for \( a_{\text{pheno}} \neq a_{\text{natural}} \)

--> energy scale of the effective pole is relevant.
S=-1 and S=0 meson-baryon scatterings

Models for the Meson-baryon scattering:

E. Oset, A. Ramos, C. Bennhold, Phys. Lett. B527, 99 (2002),
T. Inoue, E. Oset, M.J. Vicente Vacas, Phys. Rev. C. 65, 035204 (2002)
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Phys. Rev. C. 68, 018201 (2003)
T. Hyodo, S.I. Nam, D. Jido, A. Hosaka, Prog. Thor. Phys. 112, 73 (2004)

\[ T^{-1} = V_{WT}^{-1} - G(\alpha_{\text{pheno}}) = (V_{\text{natural}})^{-1} - G(\alpha_{\text{natural}}) \]

Pole of the full amplitude
physical state

Pole of the effective interaction (M_{eff})

pure CDD pole contribution
(can be complex for coupled-channel case)

Pole of the V_{WT} + natural
pure dynamical contribution
Comparison of pole positions

Pole of the full amplitude
physical state

\[
\begin{align*}
\Lambda_1^* &= 1429 - 14i \text{ MeV}, \\
\Lambda_2^* &= 1397 - 73i \text{ MeV} \\
N^* &= 1493 - 31i \text{ MeV}
\end{align*}
\]

Pole of the effective interaction (\(\text{M}_{\text{eff}}\))
pure CDD pole contribution

\[
\begin{align*}
\Lambda_{\text{eff}}^* &\sim 7.9 \text{ GeV} \quad \text{irrelevant!} \\
N_{\text{eff}}^* &= 1693 \pm 37i \text{ MeV} \quad \text{relevant?}
\end{align*}
\]

Pole of the \(V_{\text{WT}} + \text{natural}\)
pure dynamical contribution

\[
\begin{align*}
\Lambda_1^* &= 1417 - 19i \text{ MeV}, \\
\Lambda_2^* &= 1402 - 72i \text{ MeV} \\
N^* &= 1582 - 61i \text{ MeV}
\end{align*}
\]

Application: \(\Lambda(1405)\) and \(N(1535)\)
Example: \( \Lambda(1405) \) and \( N(1535) \)

Difference of interactions \( \Delta V \equiv V_{\text{natural}} - V_{\text{WT}} \)

Application: \( \Lambda(1405) \) and \( N(1535) \)

Difference of interactions:

- \( \Delta V_{11} \)
- \( \Delta V_{22} \)
- \( \Delta V_{33} \)
- \( \Delta V_{44} \)

Meff \( \sim 8 \) GeV

Important CDD pole contribution to \( N(1535) \)

Meff \( \sim 1.7 \) GeV
Application: $\Lambda(1405)$ and N(1535)

**N(1535) coupling strengths**

Residues of the pole --> coupling strengths

\[
T_{ij}(\sqrt{s}) \sim \frac{g_i g_j}{\sqrt{s} - M_R + i\Gamma_R/2}
\]

| pole in          | property | $\pi N$ | $\eta N$ | $K \Lambda$ | $K \Sigma$ |
|------------------|----------|---------|----------|-------------|------------|
| full T           | physical | 0.949   | 1.64     | 1.45        | 2.96       |
| $V_{\text{natural}}$ | CDD      | 4.67    | 2.15     | 5.71        | 7.44       |
| $WT+\text{natural}$ | Dynamical | 0.353   | 2.11     | 1.71        | 2.93       |

Coupling properties of the physical pole is **similar with those of dynamical pole.**

**Dynamical component is also important?**
We study the origin (dynamical/CDD) of the resonances in the chiral unitary approach

Natural renormalization scheme
Exclude CDD pole contribution from the loop function, consistent with N/D.

Comparison with phenomenology
--> Pole in the effective interaction
We extract the CDD pole contribution hidden in the subtraction constant into effective interaction V.
Summary: application

\( \Lambda(1405) \) : predominantly dynamical consistent with \( N_c \) scaling

T. Hyodo, D. Jido, R. Loca, Phys. Rev. D77, 056010 (2008)
R. Loca, T. Hyodo, D. Jido, arXiv:0804.1210 [hep-ph]

\( \to \Lambda(1405) \) is non-qqq dominant

\( N(1535) \) : mixture of both components
Energy of the pole in the effective interaction \( \to \) CDD pole nature
Analysis of the coupling strengths \( \to \) dynamical nature

T. Hyodo, D. Jido, A. Hosaka, arXiv:0803.2550 [nucl-th]