Cosmic flows in the nearby Universe: new peculiar velocities from SNe and cosmological constraints

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Abstract

The peculiar velocity field offers a unique way to probe dark matter density field on large scales at low redshifts. In this work, we have compiled a new sample of 465 peculiar velocities from low redshift ($z < 0.067$) Type Ia supernovae. We compare the reconstructed velocity field derived from the 2M++ galaxy redshift compilation to the supernovae, the SFI++ and the 2MTF Tully-Fisher distance catalogue. We used a forward method to jointly infer the distances and the velocities of distance indicators by comparing the observations to the reconstruction. Comparison of the reconstructed peculiar velocity fields to observations allows us to infer the cosmological parameter combination $f$/$\sigma_8$, and the bulk flow velocity arising from outside the survey volume. The residual bulk flow arising from outside the 2M++ volume is inferred to be $159^{+9}_{-11}$ km s$^{-1}$ in the direction $l = 295^\circ \pm 4^\circ$ and $b = 5^\circ \pm 3^\circ$. We obtain $f$/$\sigma_8 = 0.401 \pm 0.017$, equivalent to $\sigma_8(\Omega_m/0.3)^{0.55} = 0.775 \pm 0.033$, a $\sim 4\%$ statistical uncertainty on the value of $f$/$\sigma_8$. Our inferred value is consistent with other low redshift results in the literature.

Key words: Galaxy: kinematics and dynamics – galaxies: statistics – large-scale structure of Universe – cosmology: observations

1 Introduction

Peculiar velocities, the deviation from the regular Hubble flow of the galaxies, are sourced by inhomogeneities in the universe, making them an excellent probe of its large-scale structure. In fact, the peculiar velocity field is the only probe of very large-scale structures in the low-redshift universe.

In linear perturbation theory, the relationship between peculiar velocity, $v$, and the dark matter overdensity, $\delta$, is given as

$$v(r) = \frac{H_0 f}{4\pi} \int d^3 r' \delta(r') \frac{r' - r}{|r' - r|^3}.$$  (1)

where $\delta = \rho/\bar{\rho} - 1$, with $\rho$ being the density and $\bar{\rho}$ the mean density of the Universe. As can be seen from the above equation, the velocity field is sensitive to the dimensionless growth rate, $f = \frac{dnD}{dn} \frac{D}{a}$ and the typical size of density fluctuations. Here, $D$ is the growth function of linear perturbations, and $a$ is the scale factor. Consequently, the peculiar velocity field has been used to constrain the degenerate cosmological parameter $f$/$\sigma_8$ (Pike & Hudson 2005; Davis et al. 2011; Carrick et al. 2015; Adams & Blake 2017; Dupuy et al. 2019), where $\sigma_8$ is the root mean squared fluctuation in the matter overdensity in a sphere of radius $8 \ h^{-1} \ Mpc$. In the ΛCDM cosmological model, $f = \Omega_m^{0.55}$ (Wang & Steinhardt 1998). However, in modified theories of gravity, the growth rate could be different, i.e. $f = \Omega_m^{\gamma}$ with $\gamma \neq 0.55$. Therefore, peculiar velocities can also be used to constrain theories of gravity (Abate & Lahav 2008; Nusser & Davis 2011; Hudson & Turnbull 2012; Huterer et al. 2017).

However, analysing peculiar velocities poses several challenges. The measured redshift, $cz$, of a galaxy gets a contribution from both the recessional velocity due to Hubble flow, $Hr$, and the peculiar velocity, $v$. Therefore, to analyse peculiar velocities, one needs to determine the distances to these galaxies in order to separate these two contributions. There are several ways to measure distances directly. The most popular of these use empirical galaxy scaling relations. For example, SFI++ (Masters et al. 2006) and the 2MTF (Masters et al. 2008) catalogues use the Tully-Fisher (TF) relation (Tully & Fisher 1977), and the 6dF velocity survey (Springob et al. 2014) uses the Fundamental Plane (hereafter FP) relation (Dressler et al. 1987; Djorgovski & Davis 1987). Another distance indicator relies on Type Ia supernovae (Riess et al. 1997; Radburn-Smith et al. 2004; Turnbull et al. 2012; Huterer et al. 2017; Mathews et al. 2016). Since the distance errors from Type Ia supernovae ($\sim 5-10\% \ )$ are much lower than those obtained using galaxy scaling relationships ($\sim 20-25\% \ )$, a smaller sample of
Type Ia supernovae can give comparable results to that of a larger catalogue based on the TF or FP relations. In this work, we combine low redshift supernovae from various surveys to produce the largest peculiar velocity catalogue based on Type Ia supernovae to date.

The different approaches to analysing peculiar velocities can be separated into two categories: i) those which use only the distance indicator data for peculiar velocity analysis, and ii) those which ‘reconstruct’ the cosmic density field from a redshift survey and then compare the velocity field predictions with the observed peculiar velocity data. Some examples of the first category are the POTENT (Bertschinger & Dekel 1989) and the forward-modelled VIRBIUS (Lavaux 2016) method. Our approach falls into the second category, where we compare the reconstructed velocity field to distance observations. In particular, we use the distribution of galaxies (δg) as a tracer of the mass density field, δ. We can then use a modified version of Equation (1) to predict the peculiar velocities. In this approach, we can constrain the degenerate parameter combination \( \beta = f / b \), where \( b \) is the linear galaxy bias. The cosmological parameter combination \( f \sigma_8 \) is then related to \( \beta \) as \( f \sigma_8 = \beta \sigma_{8,0} \), where \( \sigma_{8,0} \) is the typical fluctuation in the galaxy overdensity field at a radius of 8 \( h^{-1} \) Mpc. Specifically, we compare the observed data from the peculiar velocity surveys to the reconstructed velocity field from the 2M++ redshift compilation. In doing so we use an inverse reconstruction scheme which was used in Carrick et al. (2015). More examples of reconstruction-based peculiar velocity analyses are given in Lavaux et al. (2010) and Erdogdu et al. (2006).

This paper is structured as follows: Section 2 describes the peculiar velocity catalogue we use in this work, primarily, the new compilation of type Ia supernovae. Section 3 describes the 2M++ galaxy catalogue and the reconstruction scheme used in this work. In Section 4, we elaborate on the methods used to compare the reconstructed velocity field to the observations of the peculiar velocity catalogue. The results are presented in section 5. We compare our results to other results in the literature and discuss future prospects of peculiar velocity analysis in section 6 before we summarise our results in section 7. Throughout this work, \( h = H_0 / (100 \text{ km s}^{-1} \text{ Mpc}^{-1}) \), where \( H_0 \) is the local Hubble constant.

## 2 PECULIAR VELOCITY CATALOGUE

In this section, we describe the two main peculiar velocity catalogues used in this paper. In Section 2.1, we present a new compilation of low redshift Type Ia supernovae from various different surveys. In Section 2.2, we summarise the data from SFI++ catalogue where the distance has been estimated using the Tully-Fisher relationship.

### 2.1 Second amendment (A2) supernovae compilation

Several distance indicators have been used over the past decades. Among these, distances from the FP and TF relations have found a central place in peculiar velocity analysis. In this section, we focus on distances derived from SNe-Ia light curves. Even with the recent addition of gravitational wave “standard siren” distances (see e.g. Abbott et al. 2017), SNe distances (typically ~ 5%) are still the best in terms of distance errors (see Figure 1). The peak luminosity of a Type Ia supernova is correlated with the rate of decline of its light curve, making these ‘standardsizable candles’ (Phillips 1993).

Type Ia supernovae have been previously used in many works to probe the velocity field of the nearby universe (Riess et al. 1997; Radburn-Smith et al. 2004; Turnbull et al. 2012; Huteter et al. 2017).

Turnbull et al. (2012) previously presented the ‘First Amendment’ (A1) compilation of Type Ia supernovae in the local universe. This was based on the addition of 26 SNe from the first data release (DR1) of the Carnegie Supernovae Project (CSP, Folatelli et al. 2010) to the low-z set of ‘Constitution’ supernovae (Hicken et al. 2009). In this work, we add to the First amendment catalogue additional supernovae from the third data release (DR3) of CSP (Krisiunias et al. 2017), the Lick Observatory Supernova Search (LOSS, Li et al. 2000) and the Foundation Supernova Survey data release 1 (DR1) (Foley et al. 2018; Jones et al. 2019), resulting in the ‘Second Amendment’ (A2) compilation of SN peculiart velocities. For each of these sub-catalogues, we only use the supernovae that are within the 2M++ volume and remove the duplicates from different catalogues. We also do a simple \( \chi^2 \) fit (described in section 4.1) to determine the ‘flow model’, which refers to the set of four parameters consisting of the rescaling factor, \( \beta \) and the three components of the residual bulk flow velocities. We then reject iteratively the outliers from this fit until there are no outliers. The rejection level is chosen such that in a catalogue of size \( N \), the probability of getting a chi-squared value as extreme as the rejection value is 1/(2N). For the different samples, this value is 2.6\( \sigma \) for the LOSS and CSP-DR3 samples and 2.9\( \sigma \) for the Foundation sample. The LOSS supernova sample was taken from Ganeshalingam et al. (2013). Removing the duplicates from the A1 catalogue and outliers from the \( \chi^2 \) fit, we arrive at a sample of 55 SNe. We reject a total of 4 outliers: (SN2005ls, SN2005mc, SN2006on, SN2001e) in the process. The data set for the CSP-DR3 was obtained from Burns et al. (2018). From this catalogue, we remove the duplicates in the First Amendment or the LOSS sample. We also remove supernovae outside the 2M++ volume or the outliers of the \( \chi^2 \) fit. This yields us a total of 53 supernovae after rejection of 2 outliers (SN2006os, SN2008gp). For the Foundation DR1 sample we rejected a total of 12 outliers in the fitting procedure: (SN2016ckc, SN2016gkt, ASASSN-15go, ASASSN-15mi, PS15akf, SN2016eqb, SN2016gfr, SN2017cjv, ASASSN-15la, PS15bhn, SN2016aqs, SN2016cyt).

The Foundation DR1 and the LOSS sample provides the supernovae light-curve stretch parameter, \( x_1 \), color parameter \( c \) and
the amplitude, $m_B$ after fitting the light curves using the SALT2 (Guy et al. 2007) fitter. The distance modulus for the SALT2 model is given by the Tripp formula (Tripp 1998),

$$
\mu = m_B - M + \alpha x_1 - 2c.
$$

To determine the global parameters, we use a self consistent method to jointly fit for the flow model and the global parameters to determine the distances. We fit for the global parameters $\alpha, \beta, M$ and the intrinsic scatter for this sample of supernovae in addition to the flow model using a modified forward likelihood method, which is described in section 4.2.1. We note that, conventionally, the parameter $\beta$ is denoted with $\beta_i$ in the SN literature. We avoid this notation to avoid confusion with $\beta = f/h$. For the different samples, we define an uncertainty weighted characteristic depth for a sample as

$$
d_i = \frac{\sum_{i=1}^{N} r_i / \sigma_i^2}{\sum_{i=1}^{N} 1 / \sigma_i^2},
$$

where $\sigma_i$ is the uncertainty in the distance estimates. The characteristic depth of the different sub-samples of supernovae and the combined catalogue is presented in Table 1. Note that the newly added LOSS and Foundation samples probe higher redshifts compared to the earlier A1 sample. The characteristic depth of the full A2 sample is 41 $h^{-1}$ Mpc.

A Hubble diagram for the supernovae in our compilation is shown in Figure 2. The redshift distribution of the supernovae in the A2 compilation is shown in Figure 3. The distribution of the supernovae on the sky is shown in a Mollweide projection in Figure 4. Note that the CSP sample is distributed primarily in the southern sky.

2.2 Tully-Fisher catalogues

The Tully-Fisher relation (Tully & Fisher 1977) is an empirical scaling relationship between the luminosity and the velocity width of spiral galaxies. It is commonly expressed in terms of the variable, $\eta = \log W - 2.5$, where $W$ is the velocity width of the galaxies in km s$^{-1}$. This relationship can be used to determine distances to galaxies. The distance modulus to a galaxy in terms of the apparent magnitude ($m$), and $\eta$ is given as,

$$
\mu = m - (\alpha_{TF} + b_{TF} \eta),
$$

where $\alpha_{TF}$ and $b_{TF}$ are the zero-point and the slope of the Tully-Fisher relationship. The intrinsic scatter is denoted with $\sigma_{int}$.

For the analysis of the Tully-Fisher samples, we jointly fit for the distances and the flow model using the method described in section 4.2.1. This requires fitting for three additional TF parameters, $\alpha_{TF}, b_{TF}$ and $\sigma_{int}$ in addition to the flow model. In this work, we use the data from two TF catalogues: the SFI++ catalogue and the 2MASS Tully-Fisher (2MTF) survey. We present the details of data processing for the two catalogues in the next subsections. The value of the TF parameters for the SFI++ and the 2MTF catalogues as inferred in our fitting procedures are presented in Table 2.

### Table 1. Properties of the different catalogues of the Second Amendment compilation.

| Catalogue | Number of Supernovae | $d_i$ ($h^{-1}$ Mpc) |
|-----------|----------------------|----------------------|
| A1        | 232                  | 31                   |
| CSP (DR3) | 53                   | 40                   |
| LOSS      | 55                   | 61                   |
| Foundation| 125                  | 59                   |
| A2        | 465                  | 41                   |

Note that the CSP sample is distributed primarily in the southern sky.

### Figure 2. The Hubble diagram for the supernovae in the A2 compilation.

The error bars for the magnitude include the intrinsic scatter for each sample. The black solid line is the expected distance-redshift relation in a $\Lambda$CDM cosmological model with $\Omega_m = 0.30$. The lower panel shows the residual from the given relation.

### Figure 3. The redshift distribution of the supernovae in the different catalogues in the A2 compilation. Note that the LOSS and the Foundation samples probe higher redshifts, i.e., they have a higher characteristic depth ($r_c \sim 60$ $h^{-1}$ Mpc) compared to the A1 and CSP samples. The characteristic depth is shown in Table 1.

### Table 2. Tully-Fisher parameters inferred using our fitting procedure

| Tully-Fisher | $\alpha_{TF}$ + $5 \log_{10}(h)$ | $b_{TF}$ | $\sigma_{int}$ (mag) |
|--------------|---------------------------------|---------|----------------------|
| SFI++        | $-20.915 \pm 0.006$            | $-6.42 \pm 0.07$ | $0.299 \pm 0.006$ |
| 2MTF         | $-22.586 \pm 0.013$            | $-6.56 \pm 0.13$ | $0.392 \pm 0.010$ |

2.2.1 SFI++

The SFI++ catalogue (Masters et al. 2006; Springob et al. 2007) consists of 4052 galaxies and 736 groups. After restricting to the groups and galaxies inside the region covered by 2M++, we are left with 3915 galaxies and 734 groups. For the set of galaxies, we then use the redshift distance as the distance estimate and fit for the Tully-Fisher relations. It was noted in Davis et al. (2011) that the $J$-band Tully-Fisher relation deviates from a linear relationship at the faint end. Since we are fitting using a forward method, the selection cuts should be a function of $\eta$ only for an unbiased estimate.
Figure 4. The sky distribution of the A2 supernovae in Equatorial coordinates. The above shows the Mollweide projection of the right ascension and the declination of the supernovae in the different samples. As can be seen in the figure, the CSP sample is primarily in the southern hemisphere.

Figure 5. The deviation from the inferred linear Tully-Fisher relationship in bins of \( \eta \). We calculated the mean absolute magnitude in bins of \( \eta \) of width 0.04 and calculate its deviation from the inferred linear relationship (shown on the y-axis). As can be seen, it deviates from the linear relationship in both in the faint end (low \( \eta \)) and in the bright end (high \( \eta \)).

We plot the mean relation as inferred from the data and how it deviates from the inferred linear relationship in Figure 5. As can be seen from the figure, there is a deviation from the inferred linear relationship in both the faint end (low \( \eta \)) and the bright end (high \( \eta \)). Therefore, we reject the objects with \( \eta < -0.15 \) and \( \eta > 0.2 \) from the SFI++ dataset. We then iteratively reject the points which are not within 3.5\( \sigma \) of the inferred TF relation in the magnitude. Finally, we compare the peculiar velocity predicted using bulk flow parameters inferred using the \( \chi^2 \) minimization method (described in section 4.1) to the reported peculiar velocities in the SFI++ dataset. We reject the 3.5\( \sigma \) outliers (17 objects) from this comparison.

For fitting the bulk flow parameters, we use both the galaxy and the group catalogues from the SFI++ dataset. Therefore, we remove the duplicates from the galaxy catalogue in the group catalogue. We also reject the groups for which all the corresponding galaxies in the dataset are rejected during one of the cuts described in the earlier paragraph. After these cuts, we are left with a total of 1996 field galaxies and 599 groups (containing 1167 galaxies).

The characteristic depth of the field galaxy sample was found to be 38 \( h^{-1} \) Mpc and that of the group sample is 22 \( h^{-1} \) Mpc.

2.2.2 2MTF

The 2MTF survey contains TF data for 2062 galaxies in the nearby Universe. It is restricted to distances < 100 \( h^{-1} \) Mpc. To remove duplicates from 2MTF that have distances in the SFI++ catalogue, we cross-match the galaxies by considering all galaxies which are within an angular distance of 20 arcseconds and the difference in redshift \( |\Delta z| < 150 \) km s\(^{-1}\). We find a total of 384 galaxies which are in both catalogues, and we remove these from the 2MTF catalogue. We also restrict to the 2M++ region which removes another 22 galaxies. The 2MTF provide the galaxy magnitudes in \( H \), \( J \) and \( K \) bands. For the purposes of this paper, we only use the \( K \) band magnitudes. As with the SFI++ data, we observe a deviation from the inferred linear relationship in the faint and bright end. We therefore keep only galaxies with \(-0.1 < \eta < 0.2\). We then fit the Tully-Fisher relationship by using the redshift-space distance and iteratively exclude the 3.5\( \sigma \) outliers. The final sample has a total of 1247 galaxies. The characteristic depth of the 2MTF sample is 21 \( h^{-1} \) Mpc.

3 DENSITY AND VELOCITY FIELD RECONSTRUCTION

In this section, we present details on the density and velocity reconstruction that we use for predicting the peculiar velocities. In section 3.1, we describe the 2M++ redshift compilation, which has been used in our reconstruction. In section 3.2, we present details of the reconstruction scheme used.

3.1 2M++ galaxy redshift compilation

Peculiar velocities are sourced by the density fields on large scales. Therefore to study peculiar velocities, we require our galaxy catalogue to have a large sky coverage and be as deep as possible. With this as a goal, the 2M++ compilation of galaxy redshifts was constructed in Lavaux & Hudson (2011). The 2M++ redshifts are derived from the 2MASS redshift survey (2MRS) (Erdogdu et al. 2006), 6dF galaxy redshift survey-DR3 (Jones et al. 2009) and the Sloan Digital Sky Survey (SDSS) Data Release 7 (Abazajian et al. 2009). The apparent K-band magnitude was corrected by taking into account Galactic extinction, k-corrections, evolution and surface brightness dimming. The Zone of Avoidance (ZoA) due to the Galactic Plane is masked in the process. The resulting catalogue consists of a total of 69160 galaxies. The catalogue was found to be highly complete up to a distance of 200 \( h^{-1} \) Mpc (or \( K < 12.5 \)) for the region covered by the 6dF and SDSS and up to 125 \( h^{-1} \) Mpc (or \( K < 11.5 \)) for the region that is not covered by these surveys. Hereafter, ‘2M++ volume/region’ is restricted to less than 200 \( h^{-1} \) Mpc for the region in the 2M++ catalogue which is covered by SDSS and 6dF survey and to less than 125 \( h^{-1} \) Mpc for the region covered only by 2MRS.

In Carrick et al. (2015), the ZoA was filled by “cloning” galaxies above and below the plane. We elaborate on the reconstruction process in section 3.2. For further details on the 2M++ catalogue, see Lavaux & Hudson (2011) and the references therein.

3.2 Reconstruction scheme

In Carrick et al. (2015), the density field was reconstructed with an iterative scheme modelled on Yahil et al. (1991). We use the luminosity-weighted density field from Carrick et al. (2015) in this
work. A luminosity weight was assigned to each galaxy in the 2M++ catalogue after fitting the luminosity function with a Schechter function. Galaxy bias depends on luminosity: Westover (2007) found that

$$\frac{b}{b_*} = (0.73 \pm 0.07) + (0.24 \pm 0.04) \frac{L}{L_*},$$

where $b_*$ is the bias of an $L_*$ galaxy. This luminosity-dependent bias function was used to normalize the density contrast to a uniform $b_*$ at all radii.

Finally, the mapping from the redshift data of 2M++ to comoving coordinates is done using an iterative scheme. First, the galaxies are grouped using the ‘Friends of friends’ algorithm (Huchra & Geller 1982). Then, galaxies are initially placed at the comoving distance corresponding to its redshift. Then, the luminosity-weighted density field is calculated and smoothed using a Gaussian filter at 4 $h^{-1}$ Mpc. From this density field, the peculiar velocity is calculated using linear theory for each object. The comoving coordinates in the next iteration are then corrected for using this peculiar velocity prediction. This process is repeated, slowly increasing $\beta$ from $\beta = 0$ to $\beta = 1$. For more details on this reconstruction procedure, refer to Carrick et al. (2015). The reconstructed density and the radial velocity in the supergalactic plane is shown in Figure 6.

4 COMPARING PREDICTED AND OBSERVED PECULIAR VELOCITIES

We want to compare the reconstructed velocity field to the observations of the peculiar velocity catalogues. To do this, we fit for $\beta = f/b$ and a coherent residual bulk flow velocity $V_{\text{ext}}$, which may arise from the large-scale structures outside of the 2M++ sur-

4.1 $\chi^2$ minimization

In the first approach, we compare the observed redshift to the predicted redshift of a galaxy by assuming it is at the distance reported in the peculiar velocity survey. This is therefore a Forward-Method I approach. This approach suffers from Malmquist bias (Strauss & Willick 1995). In Section 4.2, we correct for the Malmquist bias by integrating the measured inhomogeneities along the line-of-sight. It is difficult to correct for it in a simple $\chi^2$ fitting method used in this subsection. Because of this bias, the inferred value of $\beta$ is biased high, in this approach. Nevertheless, we use this method because of its interpretability and to check consistency.

The predicted redshift for a galaxy is dependent on the flow model and the reconstructed velocity. That is, $z_{\text{pred}} = z_{\text{pred}}(r, \beta, V_{\text{ext}})$. The dependence of $z_{\text{pred}}$ on these quantities is given as

$$1 + z_{\text{pred}} = \left(1 + z_{\cos}(r)\right)\left[1 + \frac{1}{c}(\beta v + V_{\text{ext}}) \cdot \hat{r}\right],$$

where $r$ is obtained by taking the distance as being equal to the reported distance in the peculiar velocity catalog and $v$ is the velocity predicted from our reconstruction. In what follows, we do not explicitly show the dependence of $z_{\text{pred}}$ on the peculiar velocity, whereas a forward distance.

Given the parameters $(\beta, V_{\text{ext}})$ and a reconstructed velocity field $v$, the discrepancy between the observations and the predictions is given by,

$$\chi^2(\beta, V_{\text{ext}}) = \sum_{i=1}^{N_{\text{red}}} \frac{(c \cos \theta - c z_{\text{pred}})^2}{2(\sigma^2 + \sigma^2_{\text{d}})},$$

where $\sigma_{\text{d}}$ is the additional uncertainty in modelling the velocity field and $\sigma_{\text{d}}$ is the error on the distance estimate converted to the units of km s$^{-1}$. The predicted redshift, $z_{\text{pred}}$ is obtained using Equation (6) by assuming that the tracer is at the radial position reported in the peculiar velocity catalogue. Unless mentioned otherwise, throughout this work, we fix $\sigma_{\text{d}} = 150$ km s$^{-1}$. This value was obtained in Carrick et al. (2015) by comparing the linear theory predictions with the observed velocities of halos in N-body simulations. However, changing this value (or fitting it as an additional parameter) does not change the results.

We minimize the $\chi^2$ given in Equation (8) with respect to $\beta$ and $V_{\text{ext}}$ to infer the best-fit flow model.

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1 Available at https://cosmicflows.iap.fr

2 There remains, a weak Malmquist-like bias due to the scatter in the flow model used to assign a distance given a redshift (Kaiser & Hudson 2015) but this is much smaller than the one due to the scatter in the distance indicator.
4.2 Forward Likelihood

As mentioned in the previous section, the Forward-Method I approach is affected by inhomogeneous Malmquist bias. Pike & Hudson (2005) introduced an approach to take care of these difficulties. We call this approach Forward likelihood. A virtue of this method is that we can include any distance indicator data in this method. Like the $\chi^2$ minimization method introduced in section 4.1, the difference in the observed and predicted redshifts are minimized in this approach. To correct the inhomogeneous Malmquist bias, we need to take the inhomogeneities along the line of sight into account. This is done by assuming the following prior on the radial distribution.

$$P(r) = \frac{r^2 \exp \left( - \frac{(r-d)^2}{2\sigma_d^2} \right)}{\int_0^\infty dr' r'^2 \exp \left( - \frac{(r'-d)^2}{2\sigma_d^2} \right) (1 + \delta_g(r'))},$$

where $d$ is the distance reported in the peculiar velocity survey and $\delta_g$ is the overdensity in the galaxy field. As a proxy for the galaxy field, the luminosity weighted density was used. For the distance estimates which have already been corrected for homogeneous Malmquist bias, we drop the $r^2$ term from the prior. Instead, to correct for possible scale errors in the reported distance, we marginalize over a nuisance parameter, $\tilde{h}$, which rescales the reported distance

$$P(r|\tilde{h}) = \frac{1}{N(\tilde{h})} \exp \left( - \frac{(r-\tilde{h}d)^2}{2\sigma_d^2} \right) (1 + \delta_g(r')),$$

where $N(\tilde{h})$ is the normalization term that depends on $\tilde{h}$. To account for the errors that arise because of the triple-valued regions and inhomogeneities along the line of sight, the likelihood is marginalized over the above radial distribution. The likelihood, $P(\zobs|v, V_{\text{ext}}, \beta)$, can therefore be written as

$$P(\zobs|v, V_{\text{ext}}, \beta) = \int_0^\infty dr P(\zobs|r, v, V_{\text{ext}}, \beta) P(r),$$

where

$$P(\zobs|r, v, V_{\text{ext}}, \beta) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp \left( - \frac{(\zobs - \zpred)^2}{2\sigma^2} \right).$$

and $P(r)$ is given by Equation (9) and $\zpred \equiv \zpred(r, v, V_{\text{ext}}, \beta)$ as given in Equation (6).

We infer the flow model, $\{\beta, V_{\text{ext}}\}$ by sampling from the following posterior distribution,

$$P(V_{\text{ext}}, \beta|\zobs) = \frac{P(\zobs|v, V_{\text{ext}}, \beta) P(V_{\text{ext}}, \beta)}{P(\zobs)}.$$

Assuming a uniform prior on $V_{\text{ext}}, \beta$ and ignoring the denominator in Equation (13) as it does not depend on the parameters of interest, the posterior turns out to have the same functional form as the likelihood. For the dataset of all galaxies, $\{z_i\}$, assuming independent probabilities, we maximize the joint posterior, which is given by

$$P(V_{\text{ext}}, \beta|\zobs) \propto \prod_i P(z_i|V_{\text{ext}}, \beta).$$

The results from the forward likelihood fit are presented in Section 5.1.2.

4.2.1 Jointly inferring distances and flow model with a modified Forward likelihood method

Measuring distances to distance indicators usually requires a calibration step for the distance indicator relationship. In this section,
Reconstructed velocity comparison with SN

The predicted velocity ($V_{\text{pred}}$) vs the observed velocity ($V_{\text{obs}}$) for objects in the peculiar velocity catalogues: the A2 supernovae, SFI++ groups, SFI++ field galaxies and 2MTF. The predicted velocity is scaled to $\beta = 1$. The fitted slope therefore gives an estimate for $\beta$, although this will be biased somewhat high due to inhomogeneous Malmquist bias (see text for details). The red solid line is the best fitted line and the shaded area is the corresponding 1σ error.

Figure 7. The predicted velocity ($V_{\text{pred}}$) vs the observed velocity ($V_{\text{obs}}$) for objects in the peculiar velocity catalogues: the A2 supernovae, SFI++ groups, SFI++ field galaxies and 2MTF. The predicted velocity is scaled to $\beta = 1$. The fitted slope therefore gives an estimate for $\beta$, although this will be biased somewhat high due to inhomogeneous Malmquist bias (see text for details). The red solid line is the best fitted line and the shaded area is the corresponding 1σ error.

we introduce a method to jointly calibrate the distance indicator relationship while fitting for the flow model. To fit the LOSS and Foundation supernovae data and the field galaxies sample of the SFI++ catalogue, we modify the forward likelihood method to jointly fit for both the flow model and the parameters of the distance indicator. For the SALT2 model, the distance is a function of the global parameters, $\alpha, \beta, M$ and $\sigma_{\text{int}}$. We jointly denote these parameters with $\Theta_{\text{SN}}$. Similarly, for the Tully-Fisher relationship, the distances depend on the TF parameters, $\Theta_{\text{TF}} = \{a_{\text{TF}}, b_{\text{TF}}, \sigma_{\text{int}}\}$. In order to jointly fit the parameters of distance indicator and the flow model, we therefore fit for these global parameters in addition to the flow model. In this approach, the Equation (9) is modified to

$$P(r|\Theta) = \frac{1}{N(\Theta)} r^2 \exp \left( -\frac{(r - d(\Theta))^2}{2\sigma_d^2(\Theta)} \right) (1 + \delta_g(r)), \hspace{1cm} (15)$$

where $d(\Theta)$ is obtained from equation (2) or (4) and $\sigma_d$ is obtained by adding in quadrature the intrinsic scatter and the measurement uncertainty. Here, $\Theta$ stands for either $\Theta_{\text{SN}}$ or $\Theta_{\text{TF}}$. $N(\Theta)$ is the normalization term that depends on $\Theta$. Using Bayes’ Theorem as in the usual approach, we can then write the joint posterior for $\Theta, V_{\text{ext}}, \beta$. We sample from this posterior to infer the parameters. The results for fitting the supernovae data in using this method are presented in section 5.

To sample from the posterior distribution of this section, we used the MCMC package emcee (Goodman & Weare 2010; Foreman-Mackey et al. 2013). The autocorrelation time for the MCMC chains is $O(10-20)^3$ This gives an effective sample size of ~ 1000.

5 RESULTS

In this section, we will present the results of the comparison between the predicted and the measured peculiar velocities. In Section 5.1, we present our peculiar velocity analysis of the different catalogues. In Section 5.2, we present the constraints on the cosmological parameters and the external bulk flow.

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3 We note that finding the autocorrelation of an ensemble sampler is not trivial as the walkers are not independent. To get our estimate, we calculated the autocorrelation for each walker and then average over them. This has been suggested in https://emcee.readthedocs.io/en/stable/tutorials/autocorr/
5.1 Peculiar velocity analysis with different catalogues

In this section, we present the results of analysis of the different catalogues we use in our peculiar velocity analysis. First, we analyse these catalogues using the $\chi^2$-minimization method presented in section 5.1.1. In section 5.1.2, we present the analysis of the same catalogues using the forward likelihood method.

5.1.1 $\chi^2$ minimization

While the $\chi^2$ minimization method is affected by the inhomogeneous Malmquist bias, it is advantageous to get interpretable results. We present the results of the $\chi^2$-minimization method in Table 3. For each sub-sample, we infer the external bulk flow velocity, $V_{\text{ext}}$, its direction in the galactic coordinates, l and b. We also infer the velocity rescaling factor for the predicted velocity from reconstruction. Note that this rescaling factor is equal to $\beta = f/b$. In Table 3, we also report the value of the $\chi^2$ over the number of degrees of freedom. In this section, for the SFI++ catalogue, we use the distance as reported in the catalogue. For the supernovae samples, we use a variant of the $\chi^2$ minimization method where we also fit for the intrinsic scatter. For the LOSS and the Foundation sample, we fit for the light curve parameters in addition to the flow model parameters.

In Figure 7, we compare the predicted peculiar velocities to the observations from the peculiar velocity surveys. In the $\chi^2$ minimization method, the difference between the two is minimized by fitting for the flow model. The observed peculiar velocities usually have a large uncertainty. Nonetheless, when taken together, the trend is clearly visible. We also show the results of the $\chi^2$ fitting method in the plot. We plot the predicted velocities from the reconstruction against the observed velocities and fit for the slope. This slope roughly corresponds to the value of $\beta$. However, the obtained value is biased high due to inhomogeneous Malmquist bias. One can also observe this by comparing the value of $\beta$ as found in Table 3 and Table 4.

5.1.2 Forward likelihood

We also analysed the peculiar velocity samples using the forward likelihood method of Section 4.2. The result of this analysis is presented in Table 4. For the analysis in this section, wherever possible, we use the modified forward likelihood method, presented in Section 4.2.1 to jointly fit for the distance indicator parameters and the flow model. For the Foundation and the LOSS SNe samples and the Tully-Fisher galaxy samples, we fit the parameters of the distance indicator relation and the flow model for the sample. In this method, the likelihood is still given by Equation (11) but Equation (9) is modified to Equation (15). We jointly fit $\beta$, $V_{\text{ext}}$, $M$, $\alpha$, $B$ and $\sigma_{\text{int}}$ for the supernovae samples and $\beta$, $V_{\text{ext}}$, $\sigma_{\text{TF}}$, $b_{\text{TF}}$, $\sigma_{\text{int}}$ for the Tully-Fisher samples using this modified version of forward likelihood. Similarly, for the CSP-DR3 sample, we also fit the intrinsic scatter. The results of these fits and a comparison of the intrinsic scatter for the LOSS, CSP and Foundation sample is presented in Table 5.

5.2 Constraining $f\sigma_8$ and the bulk flow

In this section we present the results of inferring the cosmological parameter, $f\sigma_8$, and the bulk flow using the forward likelihood method. Note that while inferring the flow model parameters with multiple catalogues, we jointly fit the distance indicator parameters of each peculiar velocity catalogue and the flow model parameters.

Figure 8. The results of forward likelihood inference with our reconstruction scheme. The numerical values are presented in Table 4. The panels show the two dimensional marginal posteriors for $\beta$, $V_{\text{ext}}$, l, b. The different samples corresponds to the results obtained from the taking the different datasets. The ‘combined’ dataset is obtained by combining the A2 supernovae, SFI++ field galaxies, SFI++ groups and 2MTF samples.

5.2.1 Constraint on $f\sigma_8$

Using the forward likelihood method of Section 4.2, we inferred the parameter $\beta = f/b$ for the reconstructed velocity field. The relation between $\beta$ and the factor $f\sigma_8$ is given as, $f\sigma_8 = \beta\sigma_8^g$, where $\sigma_8^g$ is the root mean squared fluctuation in the galaxy field. Carrick et al. (2015) found the value of $\sigma_8^g$ to be 0.99 ± 0.04. To convert our constraints on $\beta$ to the constraints on $f\sigma_8$, we use this value of $\sigma_8^g$.

It should be noted however, the value of $\sigma_8$ inferred from peculiar velocities is sensitive to the non-linear evolution of structures. To compare with values of $\sigma_8$ inferred at high redshifts, we need to correct for the non-linear evolution. This is done using the recipe of Juszkiewicz et al. (2010). This linearized value is denoted by $f\sigma_{8,\text{lin}}$. We assume $\Omega_m = 0.3$ to convert the constraint on $f\sigma_8$ into the constraint on $f\sigma_{8,\text{lin}}$. This value is then converted into the linearized value. The result for $f\sigma_{8,\text{lin}}$ as inferred from the two reconstruction schemes and the different datasets is presented in Table 4 and in Figure 8. We find consistent results from the different datasets. The value of $f\sigma_{8,\text{lin}}$ inferred by comparing the combined dataset of the A2 supernovae, 2MTF and the SFI++ to the predictions of our reconstruction is 0.401 ± 0.017.

5.2.2 Bulk Flow

We also infer the external bulk flow in the forward likelihood method. The bulk flow may be thought of as the coherent flow in the reconstructed volume. Comparison with our reconstruction yields an external bulk flow of magnitude $159_{-11}^{+12}$ km s$^{-1}$ in the direction $l = 295^\circ \pm 4^\circ$, $b = 5^\circ \pm 3^\circ$. We also compare the reconstructed bulk flow centred on the Local Group at an effective radius of 40 h$^{-1}$ Mpc. At this scale, we find a bulk flow of 246 ± 11 km s$^{-1}$ in the direction $l = 292^\circ \pm 5^\circ$, $b = 14^\circ \pm 5^\circ$. To obtain this flow,
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Table 3. Results of the $\chi^2$ minimization with the different catalogues.

| Sample          | $\beta$ | $V_{\text{ext}}$ (km/s) | $f$ (deg) | $b$ (deg) | $\chi^2$/d.o.f. |
|-----------------|---------|-------------------------|-----------|-----------|-----------------|
| A1              | 0.445 ± 0.042 | 130 ± 37               | 314 ± 29  | 26 ± 17   | 0.882           |
| CSP-DR3         | 0.588 ± 0.092 | 231 ± 63               | 14 ± 41   | −50 ± 18  | 0.825           |
| LOSS            | 0.483 ± 0.077 | 264 ± 100              | 282 ± 42  | −24 ± 16  | 0.841           |
| Foundation      | 0.389 ± 0.060 | 375 ± 64               | 251 ± 10  | 18 ± 7    | 0.958           |
| A2              | 0.439 ± 0.033 | 132 ± 30               | 285 ± 47  | 16 ± 13   | 0.758           |
| SFI++ Groups    | 0.431 ± 0.040 | 184 ± 41               | 282 ± 22  | 23 ± 14   | 0.832           |
| SFI++ Field Galaxies | 0.458 ± 0.031 | 192 ± 30               | 283 ± 11  | 4 ± 8     | 0.732           |
| 2MTF            | 0.504 ± 0.041 | 190 ± 36               | 285 ± 16  | 17 ± 11   | 0.934           |
| Combined        | 0.457 ± 0.016 | 163 ± 17               | 283 ± 8   | 15 ± 6    | 0.802           |

Table 4. Results of forward likelihood analysis for different peculiar velocity datasets. For the A2 and the combined results, we jointly fit the flow model parameters and the global parameters of the each sample.

| Sample          | $\beta$ | $f \sigma_{R,\text{lin}}$ | $V_{\text{ext}}$ (km/s) | $f$ (deg) | $b$ (deg) |
|-----------------|---------|---------------------------|-------------------------|-----------|-----------|
| A1              | 0.417 ± 0.030 | 0.393 ± 0.030            | 154 ± 24               | 309 ± 12  | 8 ± 9    |
| CSP-DR3         | 0.458 ± 0.106 | 0.427 ± 0.099            | 215 ± 55               | 16 ± 32   | −4 ± 16  |
| LOSS            | 0.479 ± 0.079 | 0.445 ± 0.073            | 152 ± 65               | 285 ± 106 | −26 ± 22 |
| Foundation      | 0.342 ± 0.064 | 0.328 ± 0.062            | 318 ± 54               | 248 ± 12  | 16 ± 8   |
| A2              | 0.405 ± 0.024 | 0.382 ± 0.026            | 130 ± 23               | 299 ± 11  | −1 ± 9   |
| SFI++ groups    | 0.407 ± 0.026 | 0.381 ± 0.029            | 170 ± 29               | 293 ± 10  | 2 ± 7    |
| SFI++ field galaxies | 0.407 ± 0.018 | 0.382 ± 0.022            | 171 ± 20               | 289 ± 7   | 17 ± 6   |
| 2MTF            | 0.481 ± 0.022 | 0.440 ± 0.025            | 168 ± 21               | 305 ± 8   | −1 ± 7   |
| All combined    | 0.430 ± 0.011 | 0.401 ± 0.017            | 159 ± 11               | 291 ± 5   | 8 ± 4    |

Table 5. Light curve parameters and intrinsic scatter inferred using the modified forward likelihood analysis for the LOSS, Foundation and the CSP samples.

| Sample          | $M + 5 \log_{10}(h)$ | $\alpha$ | $\beta$ | $\sigma_{\text{int}}$ |
|-----------------|-----------------------|----------|---------|----------------------|
| LOSS            | −18.195 ± 0.021        | 0.123 ± 0.018 | 3.52 ± 0.15 | 0.123 ± 0.017       |
| Foundation      | −18.555 ± 0.010        | 0.135 ± 0.009 | 2.88 ± 0.10 | 0.064 ± 0.010       |
| CSP-DR3         | —                     | —        | —       | 0.053 ± 0.018       |

we added the external flow to the velocity obtained by smoothing the reconstructed velocity flow at $R = 40$ h$^{-1}$ Mpc with a Gaussian filter. We compare our results for the bulk flow at 40 h$^{-1}$ Mpc with other results from the literature in section 6.1.2.

6 DISCUSSION

In this section, we compare our inferred value of $f \sigma_R$ and the bulk flow to that of other results in the literature and also discuss the prospects for the future.

6.1 Comparison with the literature

In Section 6.1.1, we compare our results to other results of $f \sigma_R$ based on a variety of cosmological probes. In section 6.1.2, we compare our results for the bulk flow to $\Lambda$CDM prediction and to other results in the literature.

6.1.1 Comparison of the value of $f \sigma_R$

In our analysis of the peculiar velocity, we find a value of $f \sigma_{R,\text{lin}} = 0.401 ± 0.017$. In this section, we compare this result to other results from the literature. This include cosmological constraints obtained from CMB anisotropies, cluster abundance, weak lensing, redshift space distortions and other peculiar velocity analysis. The results of this comparison is shown in Figure 9.

Different cosmological probes are sensitive to different combination of parameters. In particular, peculiar velocities are sensitive to $\Omega_m^{0.55} \sigma_8$. Similarly, cosmological constraints from weak lensing are usually reported in terms of the parameter, $S_C = \sigma_8 / (\Omega_m/0.3)^{0.55}$. In comparing the results here, we use $\Omega_m = 0.3$ to convert the constraints on $S_C$ to constraints on $f \sigma_R$. We compare our results to the results from DES-Y1 (Abbott et al. 2018), KiDS450 (Hildebrandt et al. 2017) and the HSC (Hamana et al. 2019). The $S_C$ value reported in these studies are as follows: 0.773 ± 0.026 (DES-Y1), 0.745 ± 0.039 (KiDS450), 0.804 ± 0.032 (HSC), 0.774 ± 0.055 (CFHT).

We also compare our results also to the results obtained from CMB anisotropies. We use the publicly available MCMC chains for Planck 2018 (Planck Collaboration 2018) and Wilkinson Microwave Anisotropy Probe (WMAP) 9 year (Hinshaw et al. 2013) results to obtain the constraints on $f \sigma_R$. For WMAP 9 year results, we obtain, $f \sigma_R = 0.407 ± 0.034$. For Planck, $f \sigma_R = 0.429 ± 0.008$. For the Planck results, we use the combination of TT,TE,EE+lowE+lensing results.

Cluster abundances are also a powerful probe of cosmology. Cluster abundance is sensitive to the cosmological parameter combination, $\sigma_8 \Omega_m^{0.55}$, where $\alpha$ is the local slope of the matter power spectrum (White et al. 1993). Depending on the specific survey, $\alpha \sim 0.2 - 0.4$. We compare the results of 4 cluster abundance
Figure 9. Comparison of different results for $\sigma_8(\Omega_m/0.3)^{0.55} = f\sigma_8/(0.3)^{0.55}$ in the literature. CMB: The CMB results from WMAP9 (Hinshaw et al. 2013) and Planck CMB (Planck Collaboration 2018) are obtained using the publicly available MCMC chains. For the Planck results, we use the combination of TT,TE,EE+lowE+lensing. Cluster Abundance: The results for cosmological constraints with cluster abundance are obtained from Planck Collaboration (2016) (Planck - SZ), Bocquet et al. (2019) (SPT-SZ), Mantz et al. (2015) (WiG - Weighing the Giants), Costanzi et al. (2018) (RedMapper). Lensing: Lensing results are quoted in terms of $S_8 = \sigma_8\sqrt{\Omega_m}/0.5$. We use $\Omega_m = 0.3$ to convert these constraints into that of $f\sigma_8$. The DES-Y1 results are taken from Abbott et al. (2018). The KiDS-450 lensing are from Hildebrandt et al. (2017). The HSC and the CFHT results are from Hamana et al. (2019) and Heymans et al. (2013) respectively. RSD: The RSD values are given in terms of $f\sigma_8$ at an effective redshift. We use linear theory to extrapolate the value of $\sigma_8$ to $z = 0$ and $f(z)$ is obtained as a function of redshift using $\Omega_m = 0.3$ and $\Omega_L = 0.7$. The 6dFGS RSD results are obtained from Beutler et al. (2012). BOSS-RSD is reported in Alam et al. (2017). Peculiar Velocity: The peculiar velocity results are quoted in terms of $f\sigma_8$ which is sensitive to non-linear structure formation. To convert these constraints into linear value constraint for $\sigma_8$, we use the prescription of Juszkiewicz et al. (2010). The 6dFGSv results are obtained from Adams & Blake (2017). The 6dFGSv + SuperCal results are from Huterer et al. (2017). The CosmicFlows results were presented in Dupuy et al. (2019). Our results are shown alongside. The horizontal line corresponds to the uncertainty weighted mean for the measurement from the different datasets, excluding the CMB and our result. The shaded grey region is the weighted uncertainty for these same studies.

We finally compare our results to the constraints on $f\sigma_8$ from other analyses of peculiar velocity in the local universe. In Adams & Blake (2017), the authors used the cross correlation between the density and velocity field of the 6dF galaxy redshift survey (6dFGRS) to obtain, $f\sigma_8 = 0.424\pm0.007$. In Huterer et al. (2017), the authors used the ‘SuperCal’ sample of supernovae in addition to the 6dFGSv catalogue to obtain the constraint, $f\sigma_8 = 0.422^{+0.048}_{-0.045}$. The CosmicFlows result (Dupuy et al. 2019) is obtained using the Cosmicflows-3 data (Tully et al. 2016). The obtained value of $f\sigma_8$ is $0.43 \pm 0.03$. Note that these three results are not independent since they all use the 6dFGSv catalogue. To maintain consistency, when plotting in Figure 9, we convert the results obtained through peculiar velocity into the linear theory results using the prescription of Juszkiewicz et al. (2010) as used in Section 5.2.1.

Our result is in good agreement with a simple error weighted average of lower redshift results: the uncertainty-weighted of the measurements from the different datasets, excluding the CMB and our result, is $0.400\pm0.006$. While our result appears to be in tension with Planck, the difference is not statistically significant (1.6$\sigma$) Moreover, the systematic uncertainties in our measurement have not been fully quantified at this time.

6.1.2 Comparison of the bulk flow

The bulk flow in the local universe has been studied in the literature by many groups (see e.g. Carrick et al. 2015; Scrimgeour et al.
Table 6. Bulk flow results - comparison with other studies. We quote our bulk flow result at 40 h⁻¹ Mpc for easy comparison with other studies

| Work            | Peculiar Velocity survey | Effective radius | | | | Reference                      |
|-----------------|--------------------------|------------------|---|---|---|--------------------------------|
| 6dFGRSv         | 6dFGRSv                  | 40 h⁻¹ Mpc       | 248 ± 58 | 318 ± 20 | 40 ± 13 | Scrimgeour et al. (2016)     |
| 2MTF            | 2MTF                     | 40 h⁻¹ Mpc       | 292 ± 28 | 296 ± 16 | 19 ± 6  | Hong et al. (2014)           |
| THF             | A1 Supernovae            | 50 h⁻¹ Mpc       | 249 ± 76 | 319 ± 18 | 7 ± 14  | Turnbull et al. (2012)       |
| WFH             | COMPOSITE                | 40 h⁻¹ Mpc       | 407 ± 81 | 287 ± 9  | 8 ± 6   | Watkins et al. (2009)        |
| This Work       | A2 Supernovae + SF1++     | 40 h⁻¹ Mpc       | 246 ± 11 | 292 ± 5  | 14 ± 5  | —                              |

Figure 10. Comparison of the bulk flow amplitude. Our result for the bulk flow amplitude is compared to other results from the literature and to the ΛCDM prediction, which is calculated using a Gaussian filter of the given scale radius. The green shaded area shows the 68% confidence region for ΛCDM predictions. Our bulk flow amplitude is calculated by adding the residual bulk flow inferred in the earlier sections to the Gaussian smoothed bulk velocity centered on the Local Group at different scales. Our result is shown with a hatched blue region. The 2MTF (Hong et al. 2014) bulk flow is denoted with a black symbols, 6dFGRS (Scrimgeour et al. 2016) with a red pentagon, THF (Turnbull et al. 2012) with a brown cross, WFH (Watkins et al. 2009) with an orange triangle.

2016; Hong et al. 2014). In this section, we compare our results to the predictions from the ΛCDM model and to other results in the literature.

One can use linear perturbation theory to calculate the expected bulk flow in a ΛCDM universe. The variance of the bulk flow on a scale, R, is given as (Gorski 1988),

\[ \sigma_B^2(R) = \frac{H_0^2 f^2}{2\pi^2} \int_0^\infty dk P(k) \tilde{W}^2(k, R), \]  

where \( P(k) \) is the matter power spectrum and \( \tilde{W} \) is the window function used to smooth the field at the scale, R. We calculate the matter power spectrum using the publicly available CAMB software (Lewis et al. 2000).

The distribution of velocity on a scale, R, with standard deviation, \( \sigma_B \) is given by the Maxwellian distribution if the density field is Gaussian. On large-scales, where linear theory holds, this is a valid assumption. Hence, the distribution of bulk flow velocity, \( V \), for ΛCDM universe is given as,

\[ P(V) \, dV = \sqrt{\frac{2}{\pi \sigma_B^2}} \frac{3}{B} \frac{\alpha}{\beta} V^{3/2} \exp \left( -\frac{V^2}{2\sigma_B^2} \right) \, dV. \]

We plot the mean and standard deviation of this distribution as a function of the scale, R, in Figure 10. These results are calculated assuming \( \Omega_m = 0.3 \). We compare our results along with other results of bulk flow in the literature. At a radius of 125 h⁻¹ Mpc, up to which 2M++ has high all-sky completeness, the mean and standard deviation of the predicted bulk flow for ΛCDM is 100 km s⁻¹ and 42 km s⁻¹ respectively. The results for the external bulk flow obtained from our reconstruction are consistent with the ΛCDM predictions. We also compare our results with other studies of bulk flow in the literature. The details of these other studies are given in Table 6. In this comparison, we quote our bulk flow results at \( R = 40 \) h⁻¹ Mpc. The direction of the bulk flow as found in this study is also similar to what has been found before in other studies.

We compare some of these in Figure 11.

6.2 Future Prospects

In the near future, we anticipate an order-of-magnitude increase in peculiar velocity data from new surveys. The “Transforming Astronomical Imaging surveys through Polychromatic Analysis of Nebulae” (TAIPAN) survey (da Cunha et al. 2017) will acquire the distances to ~ 45,000 galaxies up to z ~ 0.1 in the southern sky using the FP relation. The Widefield ASKAP L-band Legacy All-sky Blind Survey (WALLABY, Johnston et al. 2008) survey is a HI-survey which will observe 3 quarters of the sky. Using the Tully-Fisher relation, it is expected to acquire distances to ~ 40,000
galaxies (Howlett et al. 2017). In comparison, at present, the largest Tully-Fisher catalogue is the SFI++ catalogue with ∼ 4 500 galaxies. It has been forecast that using a combination of the WALLABY and the TAIPAN peculiar velocity data, the constraints on \( f_{\sigma_8} \) will reach ∼ 3% (Howlett et al. 2017; Koda et al. 2014). There will also be an increase in the number of low-redshift Type Ia supernovae usable for peculiar velocity studies. The full Foundation supernovae sample will consist of up to 800 supernovae at \( z \leq 0.1 \) (Foley et al. 2018; Jones et al. 2019). In the near future, Large Synoptic Survey Telescope (LSST) will also start taking data. It is expected to greatly increase the number of supernovae known in the local universe, although many will be at redshifts \( z > 0.2 \) and will therefore have large uncertainties (Garcia et al. 2019). Together, the use of these peculiar velocity estimates could provide us with unprecedented constraints on the growth rate in the local universe.

Given the statistical precision of peculiar velocity studies, it would be timely to more clearly understand the systematics of the density-velocity comparison. Carrick et al. (2015) used N-body simulations to show that, when dark matter haloes are used as tracers of the density field, the inverse reconstruction procedure used here should have biases of order of 1%. However, in practice, luminosity-weighting is used as a proxy for halo mass, and linear biasing is assumed, with the bias factor fit from the data. The limits of this approximation have not been fully tested.

Improvement in the methods of analysis may also tighten these constraints. Forward-modelled reconstruction is a promising framework for the analysis of the large-scale structure. In Lavaux (2016), a forward-modelled approach, virbius, was introduced to analyse the 3-dimensional velocity field by jointly inferring the distances to the peculiar velocity data. virbius was used in Graziani et al. (2019) to analyse the CosmicFlows-3 data. In Boruah et al. (in preparation), we compare the velocity field of the forward modelled reconstruction scheme, noro (Jasche & Wandelt 2013; Jasche & Lavaux 2019) to study the peculiar velocity field of the local universe. Non-linear structure formation models such as Second order Lagrangian Perturbation Theory (2LPT, Bouchet et al. 1995), Particle-Mesh (see e.g., Hockney & Eastwood 1988) and COMoving Lagrangian Acceleration (COLA, TassKY et al. 2013) can be incorporated into noro, likely providing a better approximation to the non-linear velocity field.

7 SUMMARY

In this work, we used peculiar velocity analysis to infer the cosmological parameter combination \( f_{\sigma_8} \) and the bulk flow in the local universe. We compiled a new peculiar velocity catalogue of low-z Type Ia supernovae, called the Second Amendment (A2) sample. We also used the SFI++ and the 2MTF Tully-Fisher catalogues for our analysis. We used an inverse reconstruction scheme used in Carrick et al. (2015) to compare the predicted velocities from the reconstruction to the observations in order to infer \( f_{\sigma_8} \) and the bulk flow. To make this comparison, we introduced a variant of the original forward likelihood method, in which the distances to the peculiar velocity tracers are fitted jointly with the flow model and hence, do not require prior calibration. The comparison yielded \( f_{\sigma_8, \text{lin}} = 0.401 \pm 0.017 \), with ∼ 4% statistical uncertainties on the value of \( f_{\sigma_8} \). These results are consistent with other low redshift results from the literature, as shown in section 6.1.1. We also fit for an external bulk flow which is not accounted for in our reconstruction process. We compare our constraint of the bulk flow with the ΛCDM prediction in Figure 10. With our reconstruction method, we obtain a residual bulk flow of \( 159_{-11}^{+12} \) km s\(^{-1}\) in the direction \( l = 295^\circ \pm 4^\circ, b = 5^\circ \pm 3^\circ \). At an effective radius of \( 40 h^{-1} \) Mpc, this corresponds to a bulk flow of \( 246 \pm 11 \) km s\(^{-1}\) in the direction \( l = 292^\circ \pm 5^\circ, b = 14^\circ \pm 5^\circ \) for our reconstruction scheme.

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