Improved methods for hypergraphs

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[arXiv:1302.3277]

Earlier work:

“Hypersymmetry”: N=2 supersymmetry — Fayet (’76)

(cf. “Bambi Meets Godzilla”)

Hypergraphs: Ivanov, Galperin, Ogievetsky, Sokatchev (’85)

Gonzalez-Rey, Roček, Wiles, Lindström, von Unge (’97-8)

Jain, Siegel (’09-12)

Background hyperfields: Buchbinder², Ivanov, Kuzenko, Ovrut,

McArthur, Petrov (’97-’02)

This paper does for N=2 supergraphs what was done for N=1 by ...
Improved methods for supergraphs

Marcus T. Grisaru, W. Siegel (Brandeis U.), M. Roček (Cambridge U.). Jun 1979. 32 pp. Published in Nucl.Phys. B159 (1979) 429
Cited by 742 records (INSPIRE)

Actual experimental data related to supersymmetry

Cited by 910 (Google scholar)
Background field formalism

|                      | \( N = 1 \) | \( N = 2 \) (6D \( N=1 \)) |
|----------------------|-------------|-----------------------------|
| quantum superfield   | scalar      | scalar                      |
| superspace           | \( x, \text{ full } \theta \) | \( x, \text{ analytic } \left( \frac{1}{2} \right) \theta \), (internal) \( y \) |
| representation       | chiral      | analytic                    |
| background superfield| spinor      | spinor                      |
| superspace           | \( x, \text{ full } \theta \) | \( x, \text{ full } \theta \), no \( y \) |
| representation       | real        | real                        |
| nonrenormalization   | obvious*    | obvious*                    |
| effective action     | \( x, \theta \) | \( x, \theta \) (no internal) |

*As for \( N=1 \),

Quantum superfield is scalar prepotential of dimension 0;
background superfield is spinor (or maybe vector) potential with dimension \( > 0 \).
### N=4 Yang-Mills

1-loop cancelations in N=4 Yang-Mills as formulated in N=1 or N=2 superspace:

|                  | N = 1 | N = 2 |
|------------------|-------|-------|
| scalar multiplets| 3     | 1     |
| Faddeev-Popov ghosts | −2    | −2    |
| Nielsen-Kallosh ghosts | −1    | 1     |
| total            | 0     | 0     |

|                  | N = 1 | N = 2 |
|------------------|-------|-------|
| vector multiplets| 1     | 1     |
| “extra” ghosts   | 0     | 1−2   |
| total            | 1     | 0*    |

*Same propagator, different vertex  ⇒  cancels only $y$-divergence $\delta(0)$.

In both cases, vector multiplets etc. contribute only @ 4-point & higher, scalar multiplets etc. also @ lower-point.
Equations

In case there’s too much time left, some actual equations:

**scalar/FP/NK propagator:** \( \frac{1}{y_{12}^3} \nabla_{1\theta}^4 \nabla_{2\theta}^4 \delta^8(\theta_{12}) \frac{1}{2k^2} \)

**vector/XR propagator:** \( \frac{\delta(y_{12})}{y_1} \nabla_{1\theta}^4 \delta^8(\theta_{12}) \frac{1}{2k^2} \)

**scalar/FP/NK vertex:** \( \int d^4\theta \, dy \, (\hat{\Box} - \Box_0) \)

**vector vertex:** \( \int d^4\theta \, dy \, y(\hat{\Box} - \Box_0) \)

**XR vertex:** \( \int d^4\theta \, d^2y \, [-1 + y_1 \delta(y_{12})](\hat{\Box} - \Box_0) \)

Above are for just 1 loop (free quantum in background).

For vertices, use \( \nabla_{\theta}^4 \) from propagator to make \( \int d^4\theta \, \nabla_{\theta}^4 = \int d^8\theta \).
Conclusions

(1) Same kind of simplifications for N=2 as for N=1 (1 loop & higher)

(2) Quantum field \( V(x, \theta, y) \), where \( A_\theta = 0 \);
background fields \( A_\theta, A_\varphi \), where \( A_y = 0 \), trivial dependence on \( y \)

(3) Classical action in analytic superspace \( d^4x d^4\theta d_4y \), nonlocal in \( y \);
effective action in “full” superspace \( d^4x d^4\theta d^4\varphi \), no \( y \)

(4) N=3 supergraphs (for N=4 Yang-Mills): in progress

(5) Supergravity

(6) 1st-quantization?
That's a good question!

Quantum vertices (background appears only through $\nabla^4_\vartheta$):

scalar: $- \int d^4 \theta \bar{\Upsilon} (e^V - 1) \Upsilon$

vector: $\int d^4 \theta d^4 \vartheta d^n y \frac{(-1)^n (e^{V_1} - 1) \cdots (e^{V_n} - 1)}{y_1 y_2 y_3 \cdots y_{n1}}$

FP: $- \int d^4 \theta (yb + \bar{b}) \mathcal{L}_{V/2} \left[ \coth(\mathcal{L}_{V/2}) \left( c - \frac{\bar{c}}{y} \right) + \left( c + \frac{\bar{c}}{y} \right) \right]

Nonlocality in $y$ gets no background covariantization, since $A_y = 0$. 
