Electromagnetic Mean Squared Radii of $\Lambda(1405)$ in Meson-baryon Dynamics with Chiral Symmetry

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Electromagnetic mean squared radii of $\Lambda(1405)$ are evaluated in chiral unitary approach. In this approach we regard $\Lambda(1405)$ as dynamically generated resonances in the octet meson and octet baryon scattering, and also as $\bar{K}N$ bound state. Especially for the $\Lambda(1405)$ as $\bar{K}N$ bound state we obtain negative electric mean squared radius. With the small binding energy of $\Lambda(1405)$ in the chiral unitary approach, our results imply that $\Lambda(1405)$ has structure that $K^-$ is widely spread around $p$.

§1. Introduction

One of the most interesting physics in hot and/or dense QCD is restoration of chiral symmetry. In order to verify the symmetry restoration in hot and/or dense system and extract some properties of QCD vacuum from the symmetry restoration, many studies have been extensively done from both experimental and theoretical sides. Among these studies, mesic nuclei in which Nambu-Goldstone bosons are bound in nuclei are considered as a good subject of the study because Nambu-Goldstone bosons are strongly affected by QCD vacuum in finite density. Especially kaonic nuclei are interesting because kaon is expected to be bound in nuclei due to the strong attractive interaction between kaon and nucleon required by chiral symmetry.

It is suggested that binding energy of $\bar{K}N$ system in the $\Lambda(1405)$ plays an important role for the kaonic nuclei.$^{1–4}$ Further the structure of the $\bar{K}N$ system might be essential to the properties of kaonic nuclei. Related to the kaonic nuclei, we explain our study of electromagnetic mean squared radii of $\Lambda(1405)$ in chiral unitary approach, in which $\Lambda(1405)$ is described by meson-baryon dynamics based on chiral symmetry.$^{5–9}$

In chiral unitary approach, the scattering amplitude is obtained by an algebraic equation:

$$T_{ij}(\sqrt{s}) = V_{ij}(\sqrt{s}) + \sum_k V_{ik}(\sqrt{s})G_k(\sqrt{s})T_{kj}(\sqrt{s}), \quad (1.1)$$

with an $s$-wave interaction kernel $V$ given by the lowest order chiral perturbation theory, that is the Weinberg-Tomozawa interaction, and the meson-baryon loop function $G$. Both $V$ and $G$ are functions of the center-of-mass energy of the meson-baryon system, $\sqrt{s}$, in matrix forms of the meson-baryon channels of strangeness $-1$ and...
charge 0. This approach has reproduced well the scattering cross sections of $K^- p$ to various channels and the mass spectrum of the $\Lambda(1405)$ resonance below the $\bar{K}N$ threshold, giving two states for the $\Lambda(1405)$ as poles of the scattering amplitudes in the complex energy plane, $(z_1 = 1390 - 66i \text{ MeV})$ and $(z_2 = 1426 - 17i \text{ MeV})$.\(^7,9\)

§2. Form Factors and Mean Squared Radii of Excited Baryons

In this section, we discuss the formulation to evaluate the electromagnetic form factors and the mean squared radii of excited baryons described by the amplitudes obtained from Eq. (1.1). First of all, let us define the electromagnetic form factors of an excited baryon with spin $1/2$, $|H^*\rangle$, as matrix elements of the electromagnetic current $J^{\mu}_{\text{EM}}$ in the Breit frame:\(^10,11\)

$$\langle H^* \big| J^\mu_{\text{EM}} \big| H^* \rangle_{\text{Breit}} \equiv \left( G_E(Q^2), G_M(Q^2) \frac{i \sigma \times q}{2M_p} \right), \quad (2.1)$$

with the electric and magnetic form factors, $G_E(Q^2)$ and $G_M(Q^2)$, the virtual photon momentum $q^\mu$, $Q^2 = -q^2$ and the Pauli matrices $\sigma^a$ $(a = 1, 2, 3)$. The magnetic form factor $G_M(Q^2)$ is normalized as the nuclear magneton $\mu_N = e/(2M_p)$ with the proton charge $e$ and mass $M_p$. From these form factors, electromagnetic mean squared radii, $\langle r^2 \rangle_E$ and $\langle r^2 \rangle_M$, are calculated by

$$\langle r^2 \rangle_E \equiv -6 \left. \frac{dG_E}{dQ^2} \right|_{Q^2=0}, \quad \langle r^2 \rangle_M \equiv -6 \left. \frac{dG_M}{G_M(0) dQ^2} \right|_{Q^2=0}. \quad (2.2)$$

The matrix elements of $J^\mu_{\text{EM}}$ are related to the residue of the double pole of the $MB\gamma^* \rightarrow MB$ amplitude $T_{\gamma ij}^{\mu}$.\(^10,11\) In our approach, the amplitude $T_{\gamma ij}^{\mu}$ is calculated under the assumption that the photon interacts with the $\Lambda(1405)$ through the photon couplings to the constituent meson and baryon of the $\Lambda(1405)$. The calculation of the photon coupling should be performed in a gauge invariant way, since the gauge invariance guarantees to give the correct electric charge to the excited baryons, $G_E(Q^2 = 0) = Q_H$. Within the framework of the gauge invariant calculation proposed by Ref. 12), we find the relevant diagrams for our purpose, which have the double poles, as shown in Fig. 1. The photon couplings to the mesons and baryons are given by gauging the kinetic terms and the effective interaction. Calculating these three diagrams and summing up them, we obtain the $MB\gamma^* \rightarrow MB$ amplitude,

$$T_{\gamma ij}^{\mu} \equiv T_{\gamma 1 ij}^{\mu} + T_{\gamma 2 ij}^{\mu} + T_{\gamma 3 ij}^{\mu}. \quad (2.3)$$

Fig. 1. Diagrams for the form factor of the $\Lambda(1405)$. The shaded ellipses represent the meson-baryon scattering amplitudes.
The detailed calculation and the proof of gauge invariance of form factors in our scheme is given in Ref. 11).

In the present calculation, we did not introduce the form factors for the ground state mesons and baryons and treat them as point particles, because we are interested in the sizes of the excited baryon generated by meson-baryon dynamics and in estimation of pure dynamical effects. In this formulation, however, inclusion of the form factors for the mesons and baryons is straightforward; we simply multiply the meson and baryon form factors to each vertex. It is worth noting that the electric mean squared radii for neutral excited baryons do not depend on the inclusion of the meson and baryon form factor, due to the neutrality of the excited baryons.11)

§3. Results

First of all, we briefly mention the results of electromagnetic mean squared radii of two states of $\Lambda(1405)$, $z_1$ and $z_2$. As is well known, mean squared radii of resonant states, which have decay width, are in general complex. We obtain complex mean squared radii of $\Lambda(1405)$. The mean squared radii are second moments of the form factor. Taking the absolute value, we obtain the second moment for the higher $\Lambda(1405)$ state $z_2$ as $0.32\text{fm}^2$, which is larger than that of neutron $\sim -0.12\text{fm}^2$. This means that the $\Lambda(1405)$ has a softer form factor than the neutron. The detailed discussions about the complex mean squared radii are given in Ref. 11).

In order to extract the information of the electromagnetic sizes, we perform the following analysis. If the decay width of the resonance is small, the mean squared radii are close to real numbers. For an estimate of the size, we consider $\Lambda(1405)$ as a $\bar{K}N$ bound state, which is generated only by the attractive interaction of the $\bar{K}N$ channel$^{1-3)}$ and considered to be a origin of higher $\Lambda(1405)$ state $z_2$.

Using the same parameter as in the fully coupled-channel case, we obtain $\langle r^2 \rangle_E = -2.19\text{fm}^2$ and $\langle r^2 \rangle_M = 1.97\text{fm}^2$ for the $\bar{K}N$ bound state with mass 1429 MeV. The negative sign for the electric mean squared radius implies that the $K^-$ is surrounding around the proton. In addition, with the fact that the electromagnetic size of the proton is roughly 0.9 fm, our result of the electric root mean squared radius $\sqrt{\langle r^2 \rangle_E} \approx 1.48$ fm implies that $\Lambda(1405)$ has structure of widely spread $K^-$ clouds around the core of proton with larger size than that of typical ground state baryons.

In Fig. 2, we show the electric mean squared radius as a function of the mass of the $\bar{K}N$ bound state. The mass of the $\bar{K}N$ bound state is controlled by a subtraction constant, which is the only one parameter in our approach appearing in the loop integral $G$. As one can see, our result is consistent with the expectation that the deeper bound states have the smaller radii.

For deep binding energy (about more than 40 MeV), electric mean squared radius is as small as that of neutron (about less than $-0.1\text{fm}^2$). This implies that overlap between kaon and nucleon is large, therefore picture of meson-baryon bound state for $\Lambda(1405)$ may be destroyed.$^{13)}$ For small binding energy (about a few MeV), on the other hand, absolute value of electric mean squared radius is about larger than 1 fm$^2$, which exceeds the typical size of ground state baryons, and overlap between kaon and nucleon is small. In the chiral unitary approach, we have $\Lambda(1405)$ with about a
few MeV binding energy as the pole position of the higher state $z_2$. Therefore, our result shown in Fig. 2 is consistent with the picture that $\Lambda(1405)$ is generated by $KN$ meson-baryon dynamics.

§4. Summary

We have calculated electromagnetic mean squared radii of $\Lambda(1405)$ in the chiral unitary approach. Calculations are performed in two ways: fully coupled-channel approach and approach of $KN$ bound state by neglecting all the coupling of $KN$ to the other channels. We obtain large negative value ($\lesssim -1\text{ fm}^2$) for electric mean squared radius for $KN$ bound state with small binding energy (about a few MeV). Furthermore, our result is consistent with the picture that $\Lambda(1405)$ is generated by $KN$ meson-baryon dynamics.

References

1) Y. Akaishi and T. Yamazaki, Phys. Rev. C 65 (2002), 044005.
2) T. Yamazaki and Y. Akaishi, Phys. Rev. C 76 (2007), 045201.
3) T. Hyodo and W. Weise, Phys. Rev. C 77 (2008), 035204.
4) A. Dote, T. Hyodo and W. Weise, Nucl. Phys. A 804 (2008), 197.
5) N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A 594 (1995), 325.
6) E. Oset and A. Ramos, Nucl. Phys. A 635 (1998), 99.
7) J. A. Oller and U. G. Meissner, Phys. Lett. B 500 (2001), 263.
8) M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A 700 (2002), 193.
9) D. Jido, J. A. Oller, E. Oset, A. Ramos and U. G. Meissner, Nucl. Phys. A 725 (2003), 181.
10) D. Jido, A. Hosaka, J. C. Nacher, E. Oset and A. Ramos, Phys. Rev. C 66 (2002), 025203.
11) T. Sekihara, T. Hyodo and D. Jido, arXiv:0803.4068 [nucl-th].
12) B. Borasoy, P. C. Bruns, U. G. Meissner and R. Nissler, Phys. Rev. C 72 (2005), 065201.
13) Y. Kanada-En’yo and D. Jido, [arXiv:0804.3124 [nucl-th]].