Small-Scale Robots in Fluidic Media

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One of the most promising uses of miniature robots (MRs) in the biomedical field is performing local in situ diagnosis and therapy. Researchers have proposed numerous swimming methods utilizing various actuation principles. Herein, the different propulsion methods of MRs are evaluated by analyzing their scalability. Comparing various actuators, how their performance changes with size reduction is evaluated. The swimming of natural flagellar swimmers such as spermatozoa and nematodes is analyzed. It is found that although the fluidic regime and the geometry of these organisms change considerably, there are nondimensional features that remain almost constant; most importantly, the variation of the swimming velocity is much smaller than the variation of the Reynolds number in natural swimmers. Then, several methods of propulsion and actuation principles are compared, and it is found that among the swimming methods examined, the downscaling of a piezoelectrically driven vibrating elastic beam is the most favorable. Similar to natural swimmers, the swimming velocity of a piezoelectric active swimming tail does not depend on the geometry given that its power requirements can be met. This comparative approach tool aids in the development of future actuation methods for MRs and other active microsystems.

1. Introduction

Low Reynolds number swimmers, typically microsized, exhibit different gaits and body shapes compared with regular size swimmers[1–5] because the reciprocal motion that is often used by the later is ineffective in this flow regime. Microorganisms such as bacteria, spermatozoa,[6,7] eukaryotic cells with cilia organelles[8] or amoebae[9] create undulating motion to swim.

For a swimming miniature robots (MRs), such motion can be created by rotating an object with nonzero off-diagonal elements in the fluidic resistance tensor[10,11] or by deforming the object’s shape and creating undulating motion.[12–15] Other devices use body forces invoked by external force fields.[16,17] (most commonly a magnetic field with constant gradient, driving a permanent magnet) or self-propel by ejection of a microjet[18] or are driven by microorganisms.[19–21]

In this study, we consider miniature-sized robots starting from a characteristic volume of 2–3 cm³, such as the different pill cams,[22] down to microrobots sized 1–2 mm³ and lower.[23] We analyze here how scaling down effects the different propulsion methods of these robots in a fluidic medium.

We differentiate between the “swimming method” which is the mechanism that creates the motion in the fluidic medium and the “actuation principle” which is the physical principle used to drive the swimming method. Figure 1 classifies the MRs in the current study according to the propulsion method (columns) and actuation principles they use (rows). The diagonal split in the cells indicates whether the MRs are internally (upper triangle) or externally (lower triangle) driven. The asterisk in part of the cells points out MRs that are internally driven but externally steered.

Different actuation principles were suggested for swimming: piezoelectric, electro active polymer (EAP), shape memory alloy (SMA), and magnetic actuators.

A piezoelectric swimming actuator for MRs was introduced by Fukuda et al.[46] The robot exhibited a vibrating tail with a motion-enhancing mechanism and it was 55 mm long. Using two vibrating tails, the robot swam at a velocity of 44.4 mm s⁻¹. Watson et al.[47] developed a piezoelectric micromotor that rotates a helical tail in a fluidic medium. Another piezoelectric swimming actuator was developed by Kosa et al.[13] based on creation of a traveling wave in a bending piezoelectric actuator.

Ionic polymer metal composite (IPMC) also known as EAP actuators was used by Guo et al.[41] to propel a 30 mm swimming
robot with a swimming velocity of 5 mm s$^{-1}$. Guo et al. added also a floating actuator to control the buoyancy of the MR and two “walking” actuators to advance along the basin of the vessel. Nguyen et al.$^{[40]}$ used IPMC to build a swimming actuator that creates an undulating motion. The actuator’s overall dimensions were $54 \times 8 \times 0.2$ mm, and it achieved a swimming velocity of 1.5 mm s$^{-1}$. Nguyen et al.$^{[40]}$ developed a 25 mm long multiaxial swimming tail inspired by the Caenorhabditis elegans nematode’s swimming. Lately, EAP actuators for swimming were studied thoroughly by Porfiri et al.$^{[58-60]}

Kyu-Jin et al.$^{[39]}$ presented an aquatic propulsion system made of three-linked swimming actuators with two SMA actuators at the joints.

Magnetic forces are widely used for propulsion of MRs. There are two main operational principles: placing a permanent magnet in an alternating magnetic field to create rotation or vibration of a propeller, or placing a permanent magnet into a magnetic field with constant gradient.

Dreyfus et al.$^{[28]}$ were the first who made a magnetic MR. The swimmer was based on a linear chain of colloidal magnetic particles linked by a DNA chain and attached to a red cell. The MR was stabilized and actuated by an external magnetic field creating swimming by inducing an undulating motion to the filaments. The filament’s length was 24 $\mu$m, and it achieved a propulsive velocity of $3.9 \mu$m s$^{-1}$.

Honda et al.$^{[34]}$ made a 21 mm long helix with a 1 mm$^3$ SmCo magnet at its head and achieved a swimming velocity of 20 mm s$^{-1}$. Sendoh et al.$^{[35]}$ used the same principle wrapping the helix around a capsule and placing a permanent magnet at the center of the capsule. In addition to swimming, the capsule can crawl in the small intestine by a screwing motion. Rahmer et al.$^{[36]}$ used this propeller like motion to move through gel and muscle tissue.

Bell et al.$^{[33]}$ applied the same principle as Honda in a smaller scale and manufactured nanohelices out of rectangular strips (40 $\mu$m length, 3 $\mu$m diameter, 150 nm helix strip thickness) and attached them to magnetic microbeads. The propulsive velocity of the nanohelix was also $3.9 \mu$m s$^{-1}$. Many other studies developed magnetic propellers driven by an external magnetic field, e.g., Gong et al.$^{[37]}$

Yesin et al.$^{[17]}$ used external coils to drive an MR with a constant gradient magnetic field. The MR was a 1 mm long ellipsoid

| Actuation Principle | Swimming Method | Traveling Wave in an elastic beam | Undulating motion in an elastic beam | Propeller | Linear Momentum |
|---------------------|----------------|---------------------------------|------------------------------------|-----------|-----------------|
| Electro-Magnetic    | [24, 25]       | [12]                            | [26-29]                            | [10, 11, 32-37] | [16, 17, 38] |
| SMA                 | [39]           |                                 |                                    |           |                 |
| EAP                 | [40]           |                                 |                                    |           |                 |
| Piezoelectric /Acoustic | [13, 25, 43-45] | [46]                          | [47]                                |           |                 |
| Light               | [12]           |                                 |                                    |           |                 |
| Chemical            | [18, 49, 50]   |                                 |                                    |           | [19, 21, 54-57] |
| Biological          | [51]           |                                 |                                    |           |                 |

Figure 1. Classification of MRs according to their propulsion method and actuation principle. The diagonal splits between internally (upper triangle) and externally (lower triangle). $^{[10-13, 16-21, 24-37]}$ The asterisk designates MRs that are driven internally but steered by an external mean.
made of Nickel. Mathieu et al.\[38\] used the gradient coil of the magnetic resonance imaging (MRI) to propel 600 μm magnetized steel beads (carbon steel 1010/1020). This MR was developed into a five degrees of freedom (DoF) wireless manipulation system.\[14\]

Kosa et al.\[24,25\] suggested a different magnetic propulsion method using the constant magnetic field of an MRI device.

Another principle of micropropulsion is the creation of a microjet. The MR advances as a result of ejection of a gas bubbles into the liquid medium.\[18,61,62\] The bubbles can be created by a catalytic reaction\[49,63\] or ultrasound interaction with the fuel.\[48\]

Likewise, Steager et al.\[57\] propelled a 50 μm size triangular structure using S. Marcescens bacteria. Lately, Kim et al.\[56\] suggested using automated obstacle avoidance in a S. Marcescens driven MR.

Engineered living muscle can be also used to propel a swimming robot as pioneered by Herr and Dennis\[51\] who built a 7 cm long robot actuated by two frog semitendinosus muscles. More modern biohybrid muscle cell-based actuators were developed by Ricotti and Menciassi\[64\] who were able to create 2D and 3D structures using the muscle cells.\[65\] A comprehensive review on biohybrid actuators for robotics was written by Ricotti et al.\[66\]

Other studies surveyed MRs\[67,68\] and categorized them by their components and operating environment. In this study, we focus on MRs in a fluidic medium and the actuators that drive them. There are several studies reviewing MRs, capsule endoscopy, and micro/nano motors.\[5,22,66-75\] These studies do not compare quantitatively the different propulsion methods.

We use scaling theory to assess the different swimming methods. Scaling analysis of microsystems was originally suggested by Trimmer\[76\] who analyzed a microsystem’s performance by a generalized size parameter, 𝑠, and found the power of 𝑠 for different actuation methods. Other studies continued his work extending it to other components such as sensors\[79\] and power sources (PSs).\[80\] Wautellet\[81\] also used scaling analysis analyzing biology, micromachines, and nanotechnologies.

This work continues and expands Abbott et al.'s\[32\] study that investigated the scaling of the swimming of a spherical permanent magnet with radius 𝑏 in an external magnetic field with constant gradient and the propulsion created by the rotation of a helix driven by a rotating magnetic field (the head section of the helix is a permanent magnet sphere with radius 𝑏). We compare those results with various swimming methods using several actuation principles.

In this article, we present in Section 2 the scaling of natural microorganisms as a comparative model and source of inspiration for swimming MRs. Section 3 analyzes different swimming and propulsion methods used today by MRs. Section 4 discusses the results and draws conclusions.

### 2. Natural Microswimmers

The biological study of microorganisms with flagella dates back to the start of the 20th century\[82\] and continues to date because of the improvement of rapid photography and image processing.\[7\] A large number of studies characterized the motion of flagellar swimmers such as different kinds spermatozoa\[6,7,83-87\] and nematodes.\[88,89\]

Flagellar motion’s theoretical aspects have also drawn vast attention\[5,90-98\] although these studies focused on prediction of swimming velocity and energy consumption of microorganisms. For further data on the hydrodynamic aspects microorganism swimming, see Lauga and Powers.\[3\]

In contrast, the scaling of swimming microorganisms was not fully investigated. Wu showed,\[99\] based on experimental data from Brokaw and Gibbons,\[100\] that the power consumption 𝑃 in scales with the bacteria’s mass, 𝑚, and characteristic dimension, 𝑠 (assuming the density of the bacteria is constant) almost linearly, 𝑃 ∝ 𝑚0.28 ∝ 𝑠0.84.

Wu\[99\] also found that the propulsive velocity, 𝑉, is virtually independent of body mass, 𝑉 ∝ 𝑚−0.04 ∝ 𝑠−0.013.

Holwill\[101\] pointed out that in microswimmers such as bacterial flagella, eukaryotic flagella, sperm, and small worms, the ratio between the undulating motion’s amplitude 𝐴 and wavelength 𝜆 is 𝐴/𝜆 = 1/2π = 0.15915. This ratio is the exact value of optimal propulsive efficiency.\[102\]

To investigate scaling of natural flagellar swimmers, we gathered data on swimming using several biological studies. Although the length, 𝐿, and diameter, 𝐷, of in these organisms vary, we found that the nondimensional geometrical parameter’s variation is quite small in relation to the hydrodynamic conditions. We characterize the flow around the flagella by defining the Reynolds number as

\[
\text{Re} = \frac{\rho A U d}{\mu} = \frac{2\pi \rho A U d}{\mu \lambda} = \frac{2\pi}{\mu} A d f
\]

\(\rho\) and \(\mu\) are the fluid medium’s density and absolute viscosity, \(A\) is the average amplitude of the tail’s motion, \(\kappa\) and \(U\) are the wave number and advance velocity respectively of the traveling wave in the tail, \(d\) is the diameter of the tail, \(f\) is the frequency of tail beat, and \(\lambda\) is the wavelength. The characteristic velocity is based on the flagella’s cross-section diameter and its maximal velocity, \(\kappa A U\).

The Re number is defined according to the motion of the tail and not the whole organism because the beating tail generates the flow and characterizes it.\[3\]

Figure 2 presents a meta-analysis based on 11 biological studies (detailed in the caption of Figure 2) on the motion of several kinds of spermatozoa and nematodes.

We find that despite the large variation of the Re number, \(\text{Re} \in [3.7 \times 10^{-8}, 1.4 \times 10^{-1}]\) the variations of \(A/\lambda \in [6.4 \times 10^{-2}, 2.6 \times 10^{-1}]\) and \(\lambda/\ell \in [0.54, 0.66]\) are quite small. The linear trend line of these parameters in the log-log domain indicates the following relations.
The meta-analysis data were evaluated from the following sources:

- highly viscous oil collected by the authors.

The characteristic size, $\lambda$, plus $\nu$ parameters to the mass, $m$, and the higher is it, the smaller is the swimmer, the lower is its beating amplitude, $A$, and the frequency $f$. According to the meta-analysis, the smaller is the swimmer, the lower its beating frequency, $f$. By drawing a log-log plot of $f$ and $A$, we find the following relation

$$f = 128.8 A^{-1.19}$$

(4)

Substituting Equation (4) into the Re number definition Equation (1) and replacing geometrical dimensions by $s$ leads to an almost linear scaling of the Re number with the size $s$.

$$\text{Re} \propto A f \Rightarrow \text{Re} \propto s^{1.19}$$

(5)

Similar to the results of Wu,[99] we find that the swimming velocity is almost independent of the organism’s characteristic size, $V_p \propto s^{0.21}$. Nonetheless, we find that the swimming velocity increases with the size of the swimmer and does not decrease as in Wu’s results.[94]

Although, to the best of our knowledge, there is no direct measurement of the propulsive force created by a natural microscopic swimmer, the propulsive force can be estimated by assuming Stokes drag force.[108] Consequently, the propulsive force’s scaling is $F_p \propto s^{4.21}$.

The energy dissipation rate (e.g., output power of the organism) is the scalar product of the force and the velocity and its scaling is $P_p \propto s^{4.52}$.

Using the output power, $P_p$, and the power requirement, $P_{in}$, the efficiency of the swimmer $\eta = P_p / P_{in}$ is defined. The scaling of the efficiency of natural microswimmers is $\eta = s^{0.58}$.

Table 1 summarizes the swimming ability of natural microswimmers (starting from spermatozoa in glycerine and finishing with nematodes in water).

$$A/\lambda = 0.14 \cdot \text{Re}^{-0.018}$$

$$\lambda/L = 1.28 \cdot \text{Re}^{0.054}$$

The mean value and standard deviation of $A/\lambda$ is $A/\lambda = 0.171 \pm 0.05$, which matches earlier observations.[101]

The mean value and standard deviation of $\lambda/L$ is $\lambda/L = 0.83 \pm 0.277$.

The low power value of Equation (2) indicates that scaling theory is applicable for the different flagellar swimmers and we can find the relation between the swimming velocity, $V$, and output power, $P_{out}$, and the Re number. Wu[99] related these parameters to the mass, $m$; however, we prefer to use the characteristic size, $s$, as the scaling parameter. There are MRs with varying mass (Section 3.5) in which $m$ cannot be used. The characteristic size of a microscopic organism or an MR advancing in a fluidic medium, $s$, is the dominant geometric dimension that influences the flow it creates. This measure is also used in dimensionless numbers characterizing flow regimes such as Re number Equation (1).

Figure 3 shows the scaling of the swimming velocity $V$ and the velocity ratio $V/U$.

The linear trend lines of the velocities in Figure 3 indicate the following relations between the variables

$$V_p = 559 \cdot \text{Re}^{0.18} \, [\mu m \, s^{-1}]$$

$$V_p / U = 0.08 \cdot \text{Re}^{-0.11}$$

(3)

The dependencies in Equation (3) can be compared to earlier results[99] by finding the scaling of the Re number. The geometrical dimensions in the Re number definition Equation (1) are the diameter of the swimmer’s tail, $d$, the undulating motion’s amplitude $A$, and the frequency $f$. According to the meta-analysis, the smaller is the swimmer, the higher is its beating amplitude, $A$, and the higher is it’s beating frequency, $f$. By drawing a log-log plot of $f$ and $A$, we find the following relation

$$f = 128.8 A^{-1.19}$$

Substituting Equation (4) into the Re number definition Equation (1) and replacing geometrical dimensions by $s$ leads to an almost linear scaling of the Re number with the size $s$.

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An interesting feature of Natural Swimmers using Traveling Waves (NSTW) is that the propulsive velocity is almost independent of the geometrical scale, $V_p \propto s^{0.21}$. Artificial swimming methods can achieve similar scaling as detailed in the following section.

3. MRs in Fluidic Medium

We compare NSTW to artificial methods of self-propulsion in fluidic media. Firstly, we present the swimming method we developed creating traveling waves in an elastic tail by applying torque (piezoelectric) or force (magnetic) at different locations on the tail. Secondly, we analyze swimming methods based on the diffusion of the wave in an elastic tail. Thirdly, we analyze swimmers based on EAP actuators. EAP MRs require further attention because of the multiphysics domain interaction in these actuators. Fourthly, we analyze MRs that use a propeller to drive themselves. Fifthly, we analyze MRs that use gas microjets for self-propulsion, in particular bubble generating tubular catalytic motors. We conclude this section with the analysis of biohybrid MRs and MRs driven by bacteria.

Similarly, to the previous section, we compare the different MRs evaluating their performance criteria’s downscaling potential. We derive their swimming velocity, $V_p$, propulsive force, $F_p$, and propulsive output, $P_p$. Using the power consumption of the actuator, $P_{in}$, we find the scaling of the efficiency of the MR, $\eta = P_p / P_{in}$.

3.1. Traveling Waves in an Elastic Beam

The simplest undulating motion that one can formulate with a vibrating beam is a planar traveling wave.\(^{[44]}\) A traveling wave MR has two variants: piezoelectric traveling wave (PETW)\(^{[13,43]}\) and magnetic traveling wave (MTW).\(^{[25]}\). PETW is driven by piezoelectric beams (Figure 4a) and MTW is driven by electromagnetic coils (Figure 4b). In Figure 4, the tails are placed in a trilateral symmetry to enable 5 DoF motion. To drive the robots, an internal PS and a command and control unit (CCU) are added to the MR.

PETW and MTW are based on the decomposition of a sinusoidal traveling wave,

\[
w(x, t) = W \sin(\kappa(x - Ut))
\]

\[
= W(\sin \kappa x \cos \kappa Ut - \cos \kappa x \sin \kappa Ut)
\]

into the natural modes of a vibrating beam, $\phi_k(x)$ as follows

\[
w(x, t) = \sum_{k=1}^{\infty} \left( C_{sk} \cos \kappa x Ut - C_{ck} \sin \kappa x Ut \right) \phi_k(x)
\]

\[
= \sum_{k=1}^{\infty} \sum_{j=1}^{d} g^{(d)}_{kj}(t) \phi_k(x) = \sum_{k=1}^{\infty} G_k \sin(\Omega t - \Phi_k) \phi_k(x)
\]

$W$ is the amplitude of the traveling wave, $\phi_k(x)$ are the vibration modes of the beam, and $C_{sk}$ and $C_{ck}$ are the coefficients of

| Parameter               | NSTW |
|-------------------------|------|
| Wave amplitude $w$      | $s$  |
| Swimming velocity $V_p$ | $s^{0.21}$ |
| Propulsive force $F_p$  | $s^{1.21}$ |
| Output power $P_o$      | $s^{0.84}$ |
| Input power $P_{in}$    | $s^{0.58}$ |

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Table 1. Scaling performance of swimming microorganisms.
the decomposition of the functions, \( \sin \lambda x \) and \( \cos \lambda x \), respectively.

To approximate the traveling wave, one has to form the desired time functions \( b_{k}^{(t)} = G_{k} \sin(\Omega t - \Phi_{k}) \) by calculating the input signals of the actuator from the multiphysics model. In this article, we do not present the governing equations, rather we analyze the parameters that influence the scaling of the system. More detailed modeling and analysis can be found in Kosa et al.[13]

The multiphysics model we used is based on the vibration of a beam in viscous fluid. The Euler–Bernoulli beam model is divided in \( n \) elastic sub-domains (in former publications specifically to three sub-domains[13]). The boundary conditions are semiclamping by springs at \( x = 0 \) and free at \( x = L \). The posing of the problem is continued by comparison between the domains.

The actuation of a piezoelectric beam (see Figure 4a) is modeled by a torque, \( M_{Ei}(t) \), applied at the subdomain boundaries. The index \( i \in [1, n] \) represents a specific subdomain in the beam. \( M_{Ei}(t) \) is created by the electric fields, \( \mathbf{E}_{i}(t) = -V_{g}(t)/t_{g} \) in the piezoelectric layers of the beam (\( V_{g}(t) \) and \( t_{g} \) are the voltage and thickness of layer \( j \in [1, m] \) at subdomain \( i \in [1, n] \), respectively). For more details regarding piezoelectric bending beam modeling, please see previous study[109]. There is a maximal applicable electrical field, \( \mathbf{E}_{i}(t) \), on the piezoelectric material before electric breakdown. Thus, the driving voltage, \( V_{g}(t) \), has to be scaled down according to \( s \) (or by a higher power of \( s \)), to avoid depolarization, \( V_{g}(t) \propto s \). The driving torque \( M_{Ei}(t) \) scales down cubically as \( M_{Ei}(t) \propto s^{3} \), with \( s \). The hypothesis in this scaling analysis is that the material properties (Young modulus, piezoelectric, and dielectric coefficients) remain constant if the size of the actuator is scaled down.

The actuation of a magnetically driven beam (see Figure 4b) is modeled by the Lorentz force, \( F_{Li}(t) \), created at the subdomain boundaries. The force is the vector product of the constant magnetic field, \( B_{0} \), and the current in the coils, \( I_{i}(t) \).

To analyze the scaling of the propulsive force, we discuss first the scaling of the current in the coils. \( I_{i}(t) \) can be scaled according to three assumptions as detailed in previous study[76];

1. Constant current density, \( J \propto s^{0} \). Constant current density is the conservative assumption and it leads to the downsizing of the current in the coils by the cross-section area of the conductor: \( I_{i}(t) \propto s^{2} \).
2. Constant heat flow from the boundary of the conductor, \( Q \propto s^{0} \). The dominant limiting factor in this assumption is the temperature increase in the conductor due heat conduction into the insulator circumscribing the wire. This assumption suits electromagnetic coils that are bundled or a single wire that is embedded in an insulating material. In this case, the scaling of the current is \( I_{i}(t) \propto s^{1/2} \).
3. Constant temperature difference between the wire and its environment, \( \Delta T \propto s^{0} \), is the most liberal assumption because it leads to an input current that scales down linearly with the scale of the wire \( I_{i}(t) \propto s \). This assumption may be used for a standalone wire with a small insulating layer. In addition, Peirs et al.[78] showed that this assumption fits the thermal boundary conditions of natural convection from the wire.

In Table 2, we summarize the known current limitations. The current’s scaling is divided to conservative and realistic approaches of the system and the thermal boundary assumptions.

We assume from here and on that a coil embedded in a polymer (see Figure 4c) is scaled by constant heat flow \( I_{i}(t) \propto s^{1/2} \) which leads to \( J_{i} \propto s^{-1/2} \). Consequently, the Lorentz force in the coils, \( F_{Li}(t) \), scales \( F_{Li}(t) \propto s^{3/2} \).

The motion of the MR is determined by the drag force resisting the motion of the beam. This drag force is modeled by a distributed force \( q_{d}(x, t) \), see Wiggins and Goldstein.[110] \( q_{d}(x, t) \) is proportional to the lateral velocity of the beam, \( \dot{x} \). The MR is driven at the natural frequency of the beam, \( \Omega = \omega_{0} \). It is important to note that natural frequency increases linearly with the size of the MR, \( \omega_{0} \propto s^{-1} \).

Considering the actuation, hydraulic forces, the beams elasticity, and the operating frequency, one can derive an analytical solution for the motion of the beam, \( u(x, t) \). For further details, see Kosa et al.[13]

The propulsive velocity of the actuator, \( V_{p} \), is derived from the solution of the flow field created by a traveling wave in a finite cylindrical body,[110] originally solved by Taylor.[90] \( V_{p} \) is linearly dependent to the traveling wave’s advance velocity, \( U \). The propulsive force, \( P_{p} \), is equal to the drag of a cylindrical rod moving at the swimming velocity, \( V_{p} \). It increases linearly with the size of the MR (for details see ref.[110]). The output power \( P_{o} \) is the scalar product of \( F \) and \( V \).

The power requirement of PETW scales down with the area, \( P_{in}^{(PE)} \propto s^{2} \). The power requirement of MTW, \( P_{in}^{(MC)} \), is dominated by ohmic resistance; therefore, it is proportional to the square of the current in the coil \( P_{in}^{(MC)} \propto R^{(MC)} I^{2} \). The resistance \( R^{(MC)} \) decreases linearly with size, \( R^{(MC)} \propto s^{-1} \).

The efficiency of the swimming methods can be evaluated by the ratio between the power output and the power requirement, \( \eta = P_{o}/P_{in}^{(X)} \).

Table 3 summarizes the scaling of the performance criteria of the MRs using a traveling wave in an elastic beam as the propulsive actuator.

![Table 2. Scaling of the current in a conducting wire.](image)

| Geometry                        | Scaled variable | Conservative | Thermal |
|---------------------------------|-----------------|--------------|---------|
|                                 |                 | approach     | analysis |
| Single wire cooled by convection| \( I(t) \)      | \( s^{2} \)   | \( s \)      |
|                                 | \( f(t) \)      | \( s^{0} \)   | \( s^{-1} \) |
| Wire in a coil cooled by conduction| \( I(t) \)      | \( s^{2} \)   | \( s^{3/2} \) |
|                                 | \( f(t) \)      | \( s^{0} \)   | \( s^{1/2} \) |

![Table 3. Scaling of the performance in an elastic beam.](image)

| Parameter          | PETW\(^{(X)}\) | MTW\(^{(X)}\) |
|--------------------|----------------|--------------|
| Figure             | 4a             | 4b           |
| Wave amplitude \( \omega \) | \( s \)         | \( s^{3/2} \)  |
| Swimming velocity \( V_{p} \) | \( s \)         | \( s^{3/2} \)  |
| Propulsive force \( F_{p} \) | \( s \)         | \( s^{3/2} \)  |
| Output power \( P_{o} \) | \( s \)         | \( s^{3/2} \)  |
| Input power \( P_{in} \) | \( s^{2} \)     | \( s^{2} \)    |
| Efficiency \( \eta \) | \( s^{-1} \)   | \( s^{1/2} \)  |
3.2. Wave Diffusion in an Elastic Beam

Next, we analyze MRs driven by wave diffusion in an elastic beam. An elastic tail’s harmonic motion driven by a relatively low frequency (an order of 10 Hz) (such as natural spermatozoa,[7] or a magnetic microswimmer[25,26,29]) may be modeled as an Euler–Bernoulli beam without inertia. Wiggins and Goldstein[111] coined the term hyper-diffusive equation (shortened as HD from here forward) to describe this beam model. The HD equation was used to model the swimming of biological[112] and artificial[29,39] microswimmers. The HD equations boundary conditions are periodic excitation at the base of the tail and free boundary at $x = L$. The periodic driving in an MR is a torque $M_{HD}(t) = M_{HD} \cos \Omega t$ or a force $F_{HD}(t) = F_{HD} \cos \Omega t$. From here and on, we will use the superscripts ($^{MHD}$) and ($^{FHD}$), respectively, to designate the origin of the motion in the beam. Both cases are solved and analyzed analytically by Wiggins and Goldstein.[111] The actuation creates an undulating motion, $u(x,t)$, in the beam.

Several actuation principles can be used for producing $M_{HD}(t)$ and $F_{HD}(t)$. Figure 5 illustrates various MRs propelled by the diffusive wave propulsive principle. The elastic tails can be driven by an internal PS and CCU (Figure 5b,d,e,g) or by an external magnetic field (Figure 5a,c,f). Figure 5g–f describes MRs driven by $M_{HD}(t)$ and Figure 5g,h describes MRs driven by $F_{HD}(t)$.

The simplest propulsion method is shown in Figure 5a, at the head of MR is a permanent magnet (PM, its magnetization vector is designated by $M$) and an external magnetic field $B(t)$ creates a torque $M_{HD}(t)$, that drives the diffusive wave in the elastic tail. The external magnetic field, $B(t)$, also orients the MR. In Figure 5b, the MR uses the same driving principle, but the CCU and PS create the alternating magnetic fields $B_i(t)$ ∀ $i = \{1, 2, 3\}$ internally using a magnetic circuit. Differential driving of $B_i(t)$ enables the orientation of the MR.

Figure 5c,e describes an MR in which the PM from the MR of Figure 5a is replaced by a magnetic coil with alternating current $I(t)$ and the external magnetic field is constant $B_0$. Such an MR was realized in Kosa et al.,[25] as shown in Figure 5e. Similarly, to

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**Figure 5.** Illustration of MRs using wave diffusion in an elastic tail for propulsion. The MRs can be driven externally (left column) or internally (right column). As in Figure 4, internally driven actuators are placed in a trilateral symmetry. The externally driven variants are: a) a PM driven by a varying magnetic field $B(t)$ or (c) a coil with varying current $I(t)$ directed by a constant magnetic field $B_0$, this variants image is taken from (e) previous study[25], or (g) a PM driven by a varying external magnetic gradient. The internally driven variants are: (b) a driving torque created by a PM in a magnetic field $B_i(t)$ or (d) by a coil in a constant magnetic field created by a PM or (f) by an SMA springs heated by constant current or (h) a driving force created by a PM between Helmholtz coils creating a varying magnetic gradient. The internally driven MR’s body has a PS and CCU.
Figure 5b, the driving principle can be realized by an internal constant magnetic fields and coils as shown in Figure 5d. Figure 5f illustrates creation of $M_{\text{HM}}(t)$ by SMA.

$F_{\text{HD}}(t)$ (see previous study\cite{3}) is created by the gradient of the product of a magnetic field, $B(t)$, and the PM’s magnetization $M$, $V(B(t) \cdot M)$. A miniature PM is placed in a magnetic field gradient as illustrated in Figure 5g. Figure 5h presents the internally driven version of this actuation principle using Helmholtz coils creating the gradients $V\delta B(t)/V_i = \{1, 2, 3\}$. Three PMs connected to elastic tails are placed in the coils.

The swimming velocity $V_p$ and propulsive force $F_p$ of an HD swimmer are calculated by integrating the tangential and normal field, $F(t)$ and the PM’s magnetization $M$, $\Omega F(t)$ and $\Omega M$, respectively. The time and space derivatives $\partial \nu/\partial t$ and $\partial \nu/\partial x$, of the beam’s motion. The power output is the product of the force and the velocity $P_p = V_p F_p$.

Next, we discuss the utilization and scaling of the MRs described in Figure 5. Firstly, we analyze the MRs driven by a torque at the base of the elastic tail, $M_{\text{HM}}(t)$, e.g., MHD swimmers. A driving torque can be created in a PM, as in Guo et al\cite{27} (see Figure 5a) or a coil, as in Kosa et al\cite{28} (see Figure 5c,e).

The actuation principle of the MR in Figure 5a is based on the alignment of the magnetization vector $M$ and the oscillating magnetic field, $B(t)$. The resulting torque $M_{\text{PM}}^{(\text{MHD})}$ scales with the volume, $V$, of the PM and it is a product of the magnetization coefficient $M$ and the magnetic field $B(t)$. Consequently, the driving torque of the MR variant in Figure 5a has a cubic scaling with the size of the PM, $M_{\text{PM}}^{(\text{MHD})} \propto S^3$.

The magnetization vector $M$ of a current loop $I(t)$ is the product of the current in the loop and the area it circumscribes $A = W_C \times B_C$ (assuming a rectangular coil seized $W_C \times B_C$). In an electromagnetic coil, it is multiplied by the number of turns, $N$. In Figure 5c, MR, the external magnetic field is constant, $B_0$. Accordingly, the torque’s scaling is $M_{\text{PM}}^{(\text{MHD})} \propto S^{7/2}$.

An external PS (as the MRs in the left-hand side of Figure 5) has its drawbacks. Creating a large enough magnetic field necessitates a relatively large external generator which limits the MR’s applications. Next, we analyze internally driven MHD MRs.

An internal varying magnetic field $B(t)$ at the head of the MR can be created by a magnetic circuit, Figure 5b. A magnetic circuit has coil elements that drive the circuit (also known as MMF Magneto-Motive Force), $N(t)$ and resistive elements, $AR$ (A is the cross-section area of the ferro-magnetic material and $R$ is the reluctance of the circuit). Subsequently, a field $B(t)$ created by a magnetic circuit scales down as the square of its size, $B(t) \propto S^{1/2}$. The driving torque of the MR variant Figure 5b has a scaling as $M_{\text{PM}}^{(\text{MHD})} \propto S^{3.5}$, less favorably compared with the previously analyzed external $B(t)$ driven MR (Figure 5a).

In contrast, creating a constant magnetic field, $B_0$, is simpler. As shown in Figure 5d, the PM is positioned at the head of the MR generating a constant magnetic field. The scaling of the magnetic field in this case is $B_0 \propto S^2$, and consequently, the scaling of the driving torque is $M_{\text{PM}}^{(\text{MHD})} \propto S^{5.5}$. Such high downsizing power indicates that variant Figure 5d is impractical for miniaturization.

In addition to magnetic actuation principle (Figure 5a–e), the torque in the head of the MR can be created by SMA actuators, $M_{\text{SMA}}^{(\text{MHD})}$ (Figure 5f). Figure 5f illustrates an MR actuated by two antagonistic SMA springs on opposite sides of a pivot. The scaling of a SMA spring is limited by the temperature increase in the wire, $\Delta T \propto \nu^{7/8}\omega^{3/4}$\cite{78} Peirs et al\cite{78} showed that the force created by an SMA wire scales according to its cross-sectional area, $F_{\text{SMA}} \propto S^2$. Using the mechanism suggested in Figure 5f, the scaling of the torque is $M_{\text{SMA}}^{(\text{MHD})} \propto S^4$.

As mentioned earlier, a hyper-diffusive wave can be created by an attenuating force $F_{\text{HD}}(t)$ at the head of the MR as shown in Figure 5g,h. It is created by placing a PM in a magnetic gradient (see illustration in Figure 5g), $B(t) = dB(x, t)/dx$\cite{12} The interaction between the magnetic field and the magnetization creates $F_{\text{HD}}(t) \propto V(M \cdot B)$. In the MR variant of Figure 5g, magnetic gradient is created externally, and in Figure 5h variant, the magnetic gradient is created at the head of the MR. The scaling of $F_{\text{HD}}(t)$ in Figure 5g variant depends on the volume, $V$, of the PM (similarly to $M_{\text{PM}}^{(\text{MHD})}$), $F_{\text{HD}}(t) \propto S^4$.

In the internally driven variant, Figure 5h, the magnetic field generator influences the scaling of $F_{\text{PM}}^{(\text{MHD})}$ too. A simple method for generating a magnetic gradient is a Helmholtz coil. Abbott et al\cite{32} showed that the scaling of such a gradient is $B(t) \propto S^{-1/2}$. Consequently, the force in the actuator is scaled as $F_{\text{PM}}^{(\text{MHD})} \propto S^{2.5}$.

The motion of an MR driven by HD depends also on the driving frequency of the actuation principle, $\Omega$. In MRs with PM (Figure 5a,b,g,h), $\Omega$ is limited by the step-out frequency (see previous study\cite{44}). Thus, the driving frequency, $\omega_{\text{PM}} < \Omega_{\text{Step-out}}$, does not depend on the size of the PM, $\Omega_{\text{PM}} \propto 1$.

In contrast, we observed that in coil actuators (Figure 5c,d,e), the best performance is at the mechanical resonance of the elastic tail\cite{25} Although this observation contradicts the hyper-diffusion assumption, to achieve the best performance, the operating frequency, $\Omega_{\text{C}}$, should be the resonance frequency of the elastic tail, $\Omega_{\text{C}} = \omega_{\text{N}}$. We showed previously that $\omega_{\text{N}}$ increases linearly with the downscale of the MR, thus $\Omega_{\text{C}} \propto S^{-1}$.

The driving frequency of the MR in Figure 5f is limited by the heating/cooling cycle of the SMA wire. The thermal analysis of Peirs et al\cite{78} shows that the maximum operating frequency is proportional to $\Omega_{\text{SMA}} \propto S^{-2}$.

Adding multiple actuating elements to the elastic beam will not affect their scaling. Examples of such MRs with multiple elements are PMs, developed by Dreyfus et al\cite{28} or SMAs wires, developed by Kuo-Jin et al\cite{39}.

The scaling of the undulating motion $u(x, t)$ of the tail is evaluated using the HD equation\cite{11} and the results above. From the motion of the tail $u(x, t)$, using the resistive force model, the propulsive velocity's, $V_p$, and force's, $F_p$ scaling are found. Their product represents the scaling of the output power, $P_p$, of the MR.

The power requirement of MRs driven by a hyper-diffusive wave, $P_{\text{in}}$, is the power required to drive the actuator. The power consumption of the coil and SMA driven variants (Figure 5b,d,f,h) is based on the consumption of the resistive part because the induction part of the coil is only relevant in high frequencies that are not reachable in a mechanical system. $P_{\text{in}}$ is proportional to the resistance and the square of the current in the wire and it scales as follows: $P_{\text{in}} \propto S^2$. The power consumption of externally driven MRs (Figure 5a,g) is not relevant to the current analysis. The scaling of the efficiency, $\eta$, is the ratio of the scaling of the output, $P_p$, and input, $P_{\text{in}}$, powers, respectively. The scaling
of the different MR performance measures is summarized and compared in Table 4.

Table 4. Scaling of the performance in an elastic beam using wave diffusion.

| Parameter          | PM<sub从小就</sub> | PM<sub从小就</sub> | C<sub从小就</sub> | C<sub从小就</sub> | SMA<sub从小就</sub> | PM<sub从小就</sub> | PM<sub从小就</sub> |
|--------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| Figure             | 5a                  | 5b                  | 5c, e               | 5d                  | 5f                  | 5g                  | 5h                  |
| Wave amplitude w   | s                   | s<sup>3/2</sup>     | s<sup>3/2</sup>     | s<sup>3/2</sup>     | s                   | s                   | s<sup>3/2</sup>     |
| Swimming velocity V<sub>p</sub> | s                 | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s                   | s                   | s<sup>2</sup>       |
| Propulsive force F<sub>p</sub> | s                 | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       |
| Output power P<sub>o</sub> | s<sup>2</sup>   | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       |
| Input power P<sub>i</sub> | NR                | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       |
| Efficiency η       | NR                  | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | s<sup>2</sup>       | NR                  | s<sup>2</sup>       |

Table 4<sup>ab</sup>—Externally/internally driven permanent magnet actuator creating an oscillatory torque M<sub>PM</sub><sup>(MHD)</sup>(t); Table 4<sup>c</sup>—Externally/Internally driven permanent magnet actuator creating an oscillatory torque M<sub>SMA</sub><sup>(MHD)</sup>(t); Table 4<sup>d</sup>—Externally/Internally driven permanent magnet actuator creating an oscillatory force F<sub>PM</sub><sup>(MHD)</sup>(t).

3.3. EAP-Based Swimmers

EAP-based MRs, in particular the most commonly used IPMC swimmers, are elastic tails similar to HD swimmers. In contrary to HD swimmers, the elastic beam is activated by a distributed force along the beam and not by a torque or force at the base of the tail. In addition, in an IPMC, the mechanical domain is coupled with the electrical domain by an electrochemical interaction. In IPMC swimmers, the inertia of the elastic shaft is significant. An MR driven by three IPMC actuators is illustrated in Figure 6.

Modeling properly an EAP actuator is a challenging task, thus many simplified models have been suggested.<sup>113</sup> To analyze the scaling of such an actuator, one has to use a reduced order model that is deduced from physical laws and not by empirically determined parameters. We chose a control-oriented physics-based distributed model derived by Chen and Tan<sup>114</sup> as illustrated in Figure 7. The suggested model is considered as a gray model (an analogy to black-box models) because the parameters are derived experimentally. Even though other effects such as the surface roughness of the electrodes, steric effects,<sup>59</sup> and mixture in the ionic polymer<sup>115</sup> influence the model, taking them into consideration will result in a complex model that is less amenable to swimming MR design (e.g., previous study<sup>142</sup>.

In contrary to piezoelectric actuation, the coupling between the electrical and mechanical domains is not direct (as shown in Appendix A, Supporting Information). The electrical energy is converted into an ionic migration between the anode and the cathode in one direction defined by the coordinate z (z represents the normal direction of the EAP layer; see Figure 7). The charge density’s <i>ρ</i>(z, t)<sup>116</sup> dynamics depend on the concentration, diffusivity, and temperature of the ionic solution.

The boundary conditions of <i>ρ</i>(z, t) domain are the electric potentials, <i>φ</i>(z, t), at the anode and cathode of the EAP actuator, <i>φ</i>(−h, t) and <i>φ</i>(h, t). The potentials are determined by the distributed resistance elements designated by <i>ri</i> and <i>Rp</i> in Figure 7.

The EAP model’s basic hypothesis is that charge density, <i>ρ</i>(z, t), is proportional to the mechanical stress in the beam.<sup>59,115,117</sup> The mechanical stress, <i>T</i>(x, t), is summed along the cross-section resulting in a distributed moment <i>M</i><sub>EAP</sub>(x, t).

The mechanical model of the EAP beam is determined utilizing the Euler–Bernoulli assumptions. The EAP beam’s
model differs from the hyper diffusion (Section 3.2) and the traveling wave (Section 3.1) swimming methods by its characteristic. The beam’s density is close to the surrounding fluid, thus the inertia term is important. An additional mass term representing the mass of the fluid that moves with the beam is added to the inertia. Since the beam’s motion is large relative to its length, we use the damping term developed by Sader et al.\cite{118,119} and later modified by Aureli et al.\cite{120} and Facci et al.\cite{121}

The driving frequency of an ICPF beam is determined by the natural frequency of the beam $\omega_N$, consequently $\Omega_{EAP} \propto s^{-1}$ (similarly to Section 3.1). Using the multiphysics model described above and the scaling of $\Omega_{EAP}$, we find that the scaling of the torque created by an EAP actuator is almost cubic, $M_{EAP}(L, I) \propto s^{2.96}$.

The large motion of the EAP beam necessitates the use of different models describing the propulsion of the MR. The propulsive velocity $V_p$ and trust force $F_p$ are modeled by Lighthill’s\cite{122} swimming model that fits these scales.\cite{122} The scaling of these performance parameters and the propulsive power, $P_p$, are summarized in Table 5. An interesting aspect of the EAP MR is the influence of the head section on the swimming velocity. In contrary to previous swimming methods (Section 3.1 and 3.2), the scaling of the swimming velocity depends on the drag added by the head section. This influence is illustrated in Figure 8.

The EAP’s power consumption is derived from the impedance of the electrical model and the ion migration. In contrary to the input power in previous MRs, the input power of an EAP actuator increases as the swimming tail is downscaled, $P_{in} \propto s^{-1}$. This is a disadvantage for EAPs compared with previous MRs and it affects the scaling of the efficiency, as shown in Table 5.

### 3.4. Swimming with a Propeller

A propeller is a rigid body that converts rotation into a propulsive force. In contrast to the swimming methods discussed above (Section 3.1, 3.2, and 3.3), a propeller does not deform its shape to create an undulating motion. The propeller has a rotational symmetry without reflective symmetries (mathematical definition: $Z_2, Z_3, Z_4$...).

![Figure 8. Logarithm of the propulsive velocity as a function of the logarithm of the scaling of an EAP actuator with various additional drag forces due the body of the MR. The projected area of the added load at scaling $s = 1$ is $0 \text{ mm}^2$ (red x), $0.218 \text{ mm}^2$ (green circle), $2.73 \text{ mm}^2$ (blue rhombus), and $2184 \text{ mm}^2$ (orange star). The black lines are the linear approximations of the scaling.](image)

| Parameter | EAP |
|-----------|-----|
| Figure   | 6   |
| Wave amplitude $\omega$ | $\propto s^{0.78}$ |
| Swimming velocity $V_p$ | $\propto s^{-0.23}$ |
| Propulsive force $F_p$ | $\propto s^{1.69}$ |
| Output power $P_p$ | $\propto s^{1.76}$ |
| Input power $P_{in}$ | $\propto s^{1.46}$ |
| Efficiency $\eta$ | $\propto s^{-0.23}$.

The variation of the scaling is caused due to the relatively high Re number of the flow, and as a result, the increase of the influence of the inertia added by the load. This variation is not very high and consequently one can assume the propulsive velocity is independent of scaling. The scaling, $s^{0.23}$, was found when no additional load (except self-drag) was applied. A slight increase with downscaling, $s^{-0.23}$ with a 10 times larger load than the load detailed in previous study.\cite{123}

This swimming method is based on the mobility matrix of a rigid body in low Reynolds number hydrodynamics.\cite{108} In this flow regime, the drag force, $F(D)$, and torque, $M(D)$, acting on a rigid body are proportional to the linear and angular velocity, $V(D)$ and $\Omega(D)$, respectively. The symmetry of the propeller enables direct conversion of angular rotation into trust.

Figure 9 illustrates two MRs that use propeller swimming. Figure 9a presents a propeller actuated by a PM at its head driven and steered by a rotating external magnetic field $B(t)$. In Figure 9b variant, the propeller is driven by a miniature electric motor powered internally and has a trilateral symmetry. The scaling analysis of a propeller driven MR was first pointed out by Purcell\cite{124} and the magnetically driven propeller (see Figure 9a) is discussed in previous study.\cite{32} Abbott et al.\cite{32} identified this setup as the preferable propulsion method. Internally driven MRs using a propeller (illustrated in Figure 9b) are actuated by miniature DC or stepping motors\cite{30,31} (illustrated in Figure 9b). The *Escherichia coli* (*E. coli*)\cite{125} is an internally driven microorganism with a propeller and a molecular motor.

Next, we analyze the scaling of the actuators. The MR variant of Figure 9a is rotated by an external torque, $B(t)$. As shown in
Section 3.2, the scaling of the torque created PM by in a magnetic field, \( B(t) \), is \( M_{PM} \propto 1^3 \).

The MR variant of Figure 9b is driven by electric motors. The structure of a miniature electric motor is complex, and it is difficult to derive the scaling of its performance from basic principles. The torque these motors provide, \( M_{DC}^{(P)} \) or \( M_S^{(P)} \), (The suffix represents: direct current and stepping motors) is derived using the specifications of commercially available miniature motors. The motors specifications are supplemented in Appendix B, Supporting Information. Figure 10a presents the estimation of the scaling of \( M_{DC}^{(P)} \) and \( M_S^{(P)} \) with the volume of the motor. The trend lines indicate that the scaling is \( M_{DC}^{(P)} \propto 1^2.6 \) and \( M_S^{(P)} \propto 1^2.3 \).

The driving frequency of the PM driven MR (Figure 6a) is constant with size because of the step-out frequency, \( \Omega_{PM} \propto 1 \) as explained in Section 3.2. The driving frequency of the motor driven MR (Figure 6b) is derived from the trendlines derived from Figure 10b. The scaling of the driving frequencies is \( \Omega_{DC} \propto 0.068 \) and \( \Omega_S \propto 0.1 \). Since the scaling in both motor types is close to 1, it is reasonable to assume that the cut-off frequency here is the limiting factor too, \( \Omega_{DC/S} \propto 1 \).

We notice that the range of the volume of the miniature stepping motors is limited to one order, \( V_S \in [74.6, 790] \) mm\(^3\), and similar in DC motors too, \( V_{DC} \in [38, 708] \) mm\(^3\). The extrapolation in the downscaling may not be valid in smaller scales.

To maintain a constant angular velocity, the torque required by the MR scales by the volume \( M_{DC}^{(P)} \propto 1^3 \). As a result, the scaling of the driving torque will be determined by the higher scaling factor and the torque required from the motors is \( M_{DC/S}^{(P)} \propto 1^3 \).

Table 6 summarizes the scaling of propeller swimmers. The scaling of the propulsive velocity, \( V_p \), and propulsive force, \( F_p \), is derived from the driving torque, the driving frequency, and mobility matrix. The scaling of the output power \( P_p \) is derived as in the previous sections.

The power input of the motor driven by MR, \( P_{in} \), is the product of the current consumption and the operating voltage. The current input in a DC motor is proportional to the torque, \( M_{DC}^{(P)} \propto 1^3 \), (defined by the torque constant, see Appendix B, Supporting Information); therefore by assuming a constant operational voltage, we deduce that \( P_{in} \) scales as \( P_{in} \propto 1^3 \). The scaling with the volume is further substantiated by the empirical relation depicted in Figure 11. The trend line of the DC motors indicates that the input power is proportional to the volume, \( P_{in} \propto V^{0.979} = 2^9.3^7 \). The scaling of the stepping motor’s input

| Parameter | PMP\(^{a}\) | DCMP\(^{b}\) | SMP\(^{c}\) |
|-----------|------------|------------|------------|
| Figure    | 9a         | 9b         | 9b         |
| Swimming velocity \( V_p \) | \( s \) | \( s \) | \( s \) |
| Propulsive force \( F_p \) | \( s^2 \) | \( s^2 \) | \( s^2 \) |
| Output power \( P_p \) | \( s^1 \) | \( s^1 \) | \( s^1 \) |
| Input power \( P_{in} \) | \( NR^{d}\) | \( s^1 \) | \( s^{1.536} \) |
| Efficiency \( \eta \) | \( NR^{d}\) | 1 | \( s^{1.464} \) |

\(^{a}\)PMP—permanent magnet driven MR with propeller; \(^{b}\)DCMP—DC motor driven MR with propeller; \(^{c}\)SMP—stepping motor driven MR with propeller; \(^{d}\)NR—not relevant because the power supply is external.

Figure 11. Input power of DC motors (blue x) and stepping motors (purple triangle) as a function of their volume. Black lines represent the trend line of the motors scaling. Miniature motors with volumes of 40–700 mm\(^3\) are included.

Figure 10. a) The torque created of the motor at peak efficiency, and b) the maximal angular velocity, for DC motors (blue x) and the maximal torque (a) and angular velocity (b), for stepping motors (purple triangle) as a function of the volume. The black lines represent the trend line of the motors scaling. The motors represent miniature motors with volumes of 40–700 mm\(^3\).
power derived from the trend line is \( P_{\text{in}} \propto V^{0.512} = s^{1.536} \). The efficiency, \( \eta \), is provided in Table 6.

### 3.5. Propulsion by Linear Momentum

In the previous section, we discussed MRs that convert angular momentum into linear momentum and subsequently create a propelling force. Linear momentum and consequently self-propulsion can be also produced directly by an external magnetic gradient or an internal chemical reaction.

**Figure 12a** illustrates an MR moved by a trust force generated by the interaction of a PM with an externally created magnetic field gradient.[16,32] The actuating mechanism is the same as the MR’s mechanism analyzed in Section 3.2 described in Figure 5g. The scaling of the propulsive force is \( F_{\text{LM}}^\text{PS} = F_p \propto s^3 \).

A different MR creating linear momentum is illustrated in Figure 12b. The MR is propelled by a microjet and steered by an external magnetic field. Such MRs have been realized in the nano/micro scales using catalytic micromotors or ultrasonic vaporization.[18,62] A gas bubble grows in the hollow conical tubular structure and released from its larger orifice.[63] The propulsion in the MR is due to the sucking of fluid from the smaller orifice and the trusting of fluid from the larger orifice of the MR. A detailed analysis of this microjet actuator shows that the trust force depends on the gas production rate and the geometry of the MR.[126] Assuming a constant gas production rate, the propulsive force of a microjet MR scales by the square of the size, \( F_{\text{LM}}^\text{MJD} = F_p \propto s^2 \). The drag force acting on the MR is modeled by the mobility matrix.[108] The scaling of the propulsive velocity, \( V_p \), is derived from the force equilibrium. The scaling of the output power \( P_p \) is derived as in the previous sections. The performance of the MRs is summarized in Table 7.

The scaling of the input power of a microjet-driven MR depends on the chemical reaction on the surface of the MR, thus \( P_{\text{in}} \propto s^2 \). The efficiency is provided in Table 7.

### 3.6. Biological Actuation

The MR we analyzed up to now was artificially made. There are MRs that use for propulsion micro-organisms and biological tissues, e.g., biological actuation. The different micro-organisms using methods are carrying a payload by a bacterial agent[53] or pushing an MR by creating a selective incentive (chemotaxis) in a fluidic media with bacteria[53] (for further details, see the review of previous study[75]). A possible implementation for steering and driving such an MR is illustrated in **Figure 13a**.

Another actuation method in this category is using a biological muscle tissue. The tissue may be artificially grown[65] or retrieved from an organism.[51] The combination of microstructures and biological microscopic organisms, tissue, or cells is defined as a biohybrid microsystems.[30,66] A possible conversion of a biohybrid actuator into an MR is illustrated in Figure 13b.

The advantage of biologically driven MRs is its low-energy consumption and the exploitation of the microorganisms already available in nature.

The propulsive force of an MR propelled by bacteria (see Figure 13a) depends on the area of contact with the robot’s head section. Assuming the density of the bacterial agents remains constant, their number, \( N \), determines the propulsive force. If each agent delivers a force of \( F_{1B}^B \), the propulsive force downscales by the square of the size of the MR, \( N F_{1B}^B = F_p \propto s^2 \). Notice that the minimal force in biological propulsion method is \( F_{1B} \).

The drag force acting on the MR is modeled by the mobility matrix[108] as previously in Section 3.5. The propulsive velocity is derived from the force equilibrium and it scales down linearly with the size of the MR, \( V_p \propto s \). The scaling of the propulsive velocity, force, and power are summarized in Table 8.

The force created by a biohybrid actuator made of tissue is proportional to the cross-sectional area of the muscle tissue driving the MR. Let us assume that the embodiment of the MR is similar to the SMA-driven MR in Figure 5f. The linear motion

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**Table 7. Scaling of a linear momentum driven MR.**

| Parameter | MGDM\(^{(i)}\) | MJD\(^{(i)}\) |
|-----------|----------------|------------|
| Figure    | 12a            | 12b        |
| Swimming velocity \( V_p \) | \( s^2 \) | \(-\) |
| Propulsive force \( F_p \)  | \( s^3 \) | \( s^2 \) |
| Output power \( P_p \)      | \( s^3 \) | \( s^2 \) |
| Input power \( P_{\text{in}} \) | NR\(^{(i)}\) | \( s^2 \) |
| Efficiency \( \eta \)       | NR\(^{(i)}\) | \(-\) |

\(^{(i)}\)MGDM—magnetic gradient driven MR; \(^{(i)}\)MJD—micronjet driven MR; \(^{(i)}\)NR—not relevant because the power supply is external.

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**Figure 12.** a) Illustration of linear momentum driven MRs using a microjet with magnetic steering or (b) by an external magnetic gradient. The propellers in (b) have protective cages to protect adjacent tissue from harm.

**Figure 13.** Illustration of biologically driven MRs using a (a) bacteria or (b) artificially grown muscle. The bacteria drive the MR by directed chemotaxis or magnetotaxis cues created in the excitation electrodes (a). The muscles’ contractions move the elastic tails and the tails returned to the start of the stroke cycle by springs (b).
Table 8. Scaling of a biologically driven MR.

| Parameter               | BDM(6)  | MDM(6) |
|-------------------------|---------|---------|
| Figure                  | 13a     | 13b     |
| Swimming velocity $V_p$ | $s$     | $s$     |
| Propulsive force $F_p$  | $s^2$   | $s^2$   |
| Output power $P_o$      | $s^3$   | $s^3$   |
| Input power $P_in$      | $s^{2.84}$ | $s^{2.84}$ |
| Efficiency $\eta$       | $s^{-0.16}$ | $s^{-0.16}$ |

BDM—bacteria-driven MR; MDM—muscle-driven MR.

of the actuator is converted into a torque that scales as $M_\text{BH} \propto s^3$. Using the HD model from Section 3.2, we derive the scaling of the propulsive velocity, force, and power, summarized in Table 8.

In Section 2, we discussed the scaling of natural micro swimmers. The scaling of the power consumption of a microscopic organism was $P_\text{in} \propto s^{0.84}$. Because the actomyosin complex has a similar structure to the microtubules in the sperm’s tail, we assume that the muscle power consumption’s scales as $P_\text{iB} \propto s^{0.84}$. The total power consumption is multiplied by the effective or the number of bacteria that both scale by the square of the MR’s size. Consequently, the scaling of the power consumption is $P_\text{in} \propto s^{2.84}$. The efficiency’s scaling is the ratio between the propulsive power to the input power.

4. Discussion

In this study, we compare different propulsive methods for microrobots in a fluidic medium. We use scaling analysis to compare between them. In most of the swimming methods, we use analytical models to derive the scaling and we partially rely on empirical analysis. The results of the scaling analysis are summarized in Table 3–8. It seems that is possible to rate the different propulsion methods using scaling, and the favorite ones are apparently PETW and EAP.

In most of the actuation methods (see: MTW, PMM, SMA, …), the propulsive velocity, $V_p$, is proportional to the characteristic size of the MR, $V_p \propto s$. The prominent propulsion methods regarding V are PETW, $V_p \propto 1$, and the EAP-driven MR, $V_p \propto s^{0.07} \ldots 0.23$. In both methods, the swimming velocity remains constant with the downsizing of the MR (in EAP it even increases slightly with downsizing). Other propulsion methods (CMM, PMM, MGDM) are less efficacious if the scaling of V is considered as the major mean of evaluation.

In most swimming methods (see: PETW, Table 4, 6–8), the difference between the power of the scaling of the propulsive force $F_p$ and velocity $V_p$ is 0, i.e., $F_p / V_p \propto s$. This ratio is typical to propulsive methods in which the force or torque is applied directly on the MR and equaled by the viscous drag force. From the propulsive force, $F_p$, perspective, the best propulsion method is PETW that scales proportionally with the size. EAP is the second-best scaling, $F_p \propto s^{2.69}$. As a result of the $s$ ratio, most other propulsion methods scale by the square of the size of the MR, $F_p \propto s^2$ (PMM, SMA, Table 6, MJDM, Table 8).

The actuator output power, also known in natural micro swimmer’s literature as propulsive power, is the power required to move the MR at a velocity of $V_p$. Consequently, the scaling of output power is the product of the scaling of the propulsive force and the swimming velocity of the MR. Since PETW leads in both the scaling $V_p$ and $F_p$, it also has the best scaling of power output, $P_o \propto s$, followed by EAP, $P_o \propto s^{1.43 \ldots 1.76}$, and MTW, $P_o \propto s^{2.5}$. The characteristic scaling of the power is $P_o \propto s^3$ (see: PMM, SMA, Table 6, MJDM, Table 8).

For the comparison of the efficiency, $\eta$, one distinguishes between internally (see: Table 4, PMM, CMM, SMA, PMM, EAP, DCMP, SMP, MJDM, Table 8) and externally (see: PMM, CMM, PMM, MGDM, PMP) powered swimming. In externally powered MRs, the input power’s, $P_i$, scaling is not relevant because the size of external transmitter is almost unlimited, and it is not scaled down as the MR’s size is reduced. From the internally powered propulsion methods again PETW has the best scaling, $\eta \propto s^{-1}$. Piezoelectrically driven swimming MRs are more efficient as their size is reduced.

We notice that the input power of PETW scales down by the square of the size as other swimming methods (see: PMM, CMM, SMA, PMM, MJDM, Table 8) and this can be problematic knowing that most PSs scale down cubically, $P_i \propto s^3$.

Another efficient propulsion method is DC motor actuated MRs whose efficiency does not vary by downsizing, $\eta \propto 1$. From the point of view of the efficiency’s scaling, the methods that follow the first two are the biologically driven MRs ($\eta \propto s^{0.16}$), MTW ($\eta \propto s^{1/2}$), SMA, and MJDM ($\eta \propto s$). The EAP swimming efficiency’s method’s efficiency is relatively low, $\eta \propto s^{0.16 \ldots 1.76}$, because of the bad scaling of the input power it requires $P_i \propto s^{-1}$. The efficiency’s scaling criterion enables only the comparison between internal PSs, therefore it is not generic enough to be used as a design directive for MRs.

This analysis leads to the conclusion that a MR driven by traveling waves generated in a piezoelectric beam, PETW, has an advantage compared with other propulsion methods (examined here). The current study examines the different propulsion methods theoretically, and it is an indicator for the best MR propulsion. Other aspects of the MRs realization such as manufacturing limitation and a proper PS may jeopardize these results. We assumed that physical properties of the materials remain constant with scaling. Piezoceramics differ in thin films compared with bulk material; however, the variation of properties depends not only on size’s scaling but also on the ferroelectric orientation of the material and the electrode layer of the actuator. It is difficult to model the irregularity of the structural domains because of their inconsistency.

Currently the smallest actuators that we developed are multilayer piezoelectric actuators sized 20 × 1.8 × 0.267 mm, but in the literature piezoelectric microbeams sized 1000 × 100 × 5.8 μm have been realized, indicating that “there’s plenty of room at the bottom”.

The powering of PETW can be problematic in small scales. The power rating of a battery scales by its volume. However the power requirement of PETW scales by the square of $s$, $P_i \propto s^2$. The powering problem can be solved by reducing the performance of the PETW (for example changing the scaling of the input voltage, $V$, of the PETW actuators to: $V \propto s^{3/2}$,

Adv. Intell. Syst. 2019, 1, 1900035

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instead of the physical bound of $V \propto s$) or finding an alternative PS with favorable scaling. PSs such as optical power transmission to a photo-voltaic cell$^{[130]}$ (the scaling is $P_{in} \propto s^2$) or thin film batteries made of 3D structure$^{[131]}$ may fit better to the PETW’s requirements.

It is interesting to notice that PETW resembles natural flagellar swimmers (see Table 1). The similarity is in the independence of the propulsive velocity, $V_p$, in size of the microswimmer and the linear scaling of the propulsive force, $F_p$, and power, $P_p$. Because of the low input power scaling ($P_{in} \propto s^0.09$) of microscopic organisms the scaling of the efficiency is different ($\eta \propto s^{-2.25}$ in organisms opposed to $\eta \propto s^{-1}$ in PETW). Another aspect of similarity is the form of the undulating motion and the linear scaling of the amplitude of the motion. Though the operating parameters of the two swimming methods are quite different, the operating frequency (order of magnitude of 10 Hz in natural swimmer opposed to order of magnitude of 1 kHz in PETW) and the $A/\lambda$ ratio’s value of the ratio (0.15 in organisms Equation (2), and the order of $10^{-2}$ in PETW) are separated by two orders of magnitude.

5. Conclusion and Future Work

In this study we identified PETW as a highly competitive propulsion method for swimming MRs. Consequently, we are developing such actuators to estimate all the aspects of their downsizing.$^{[43,132,133]}$

We proposed a generic comparison method for the propulsion of MRs in fluidic medium. It is difficult to find a method that comprehensively describes all possible swimming methods and actuation principles. For this reason, we chose scaling analysis. If we consider other possibilities such as optimization, the large parameter space will blunt the comparison and effective conclusions are difficult to derive. Another pitfall is the definition of a target task for the MR. Here, we lose the advantages of the generalization and the ability to compare the different actuations.

The focus of this study is the actuation for positioning of MRs. We intend to expand the scaling analysis for MRs to a full microsystem analysis by scaling other critical components such as the PSs (batteries, energy harvesters, wireless power transmission, etc.), and examining the synergy of the components. In addition, the applicability of the downsizing will be tested in the future by experiments.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

Acknowledgements

The authors would like to thank Prof. Netta Cohen and Dr. Jordan Doyle for providing data on C. elegans. The author would like to thank Prof. Yossi Yovel for providing data on C. elegans and Prof. Oksana Stalnov for helping with the presentation of the meta-analysis of the natural microswimmers.

Conflict of Interest

The authors declare no conflict of interest.

Keywords

microactuation, microrobots, microswimmers, scaling-theory

Received: April 3, 2019
Revised: July 1, 2019
Published online: September 12, 2019

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