Critical region in the spatiotemporal dynamics of a turbulent thermoacoustic system and smart passive control

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We develop a passive control strategy for suppressing thermoacoustic instability (TAI) in a bluff-body stabilized premixed turbulent combustor where the dynamics is primarily driven by flame-vortex interaction. Variation in the equivalence ratio leads to a transition from combustion noise to thermoacoustic instability via intermittency in the combustor. Simultaneous acoustic pressure, 2D-PIV and CH* chemiluminescence measurements were performed to capture the pressure fluctuations, the velocity field, and the heat release rate (HRR) field during the transition. We measure the spatial distribution of turbulent velocity amplitude, time-averaged vorticity, time-averaged HRR and Rayleigh index. We implement a passive control strategy by targeting various regions inside the combustor determined based on these measures with a steady injection of secondary micro-jet of air to optimize for the location of injection. When secondary injection of optimum velocity targets the optimum region, the so-called critical region, inside the combustor, we observe a suppression of greater than 20 dB of the dominant thermoacoustic mode. We observe that the coherent structure forming from the shear layer following the dump plane gets suppressed, leading to an incoherent spatial distribution of HRR fluctuations. We find that the velocity amplitude correctly identifies the critical region for optimized passive control. In contrast, Rayleigh index identifies the region of most significant acoustic driving, but it does not always identify the region most sensitive to control. Finally, we spatially extend the analysis using Hurst exponent by calculating the scaling associated with the intensity of turbulent velocity fluctuations. We show that Hurst exponent not just identifies the critical region during the state of TAI, but also predicts the critical region during the state of intermittency, unlike the other physical measures considered in this study. Thus, we develop a smart passive control method by combining the need for finding critical regions in the combustor with the predictive capabilities of Hurst exponent.

I. INTRODUCTION

Thermoacoustic instability refers to very large amplitude periodic pressure oscillations arising as a result of positive feedback between unsteady combustion and the acoustic modes of the combustion chamber [1]. The problem is exacerbated in land-based gas turbine combustors which usually operate in fuel-lean conditions to reduce the emission of NOx and meet emission norms. Such a susceptibility of lean combustion systems to thermoacoustic oscillations is a result of the high sensitivity of lean premixed flames to harmonic perturbations arising from pressure oscillations and flow instabilities [2]. Each year, thermoacoustic instability causes billions of dollars of loss in revenue directly through repair and replacement costs of failed combustors in gas turbine industries, and indirectly due to combustor downtime and associated power outages in power plants [1]. The complex interactions between turbulence, combustion and acoustics of the combustion chamber leads to a variety of nonlinear dynamics [3] and has necessitated the use of different approaches in making the problem tractable. These include the use of flame transfer and describing function [4], distributed time-lag modeling [5], adjoint methods [6] and complex systems approach [7] in characterizing and controlling thermoacoustic oscillations.

A. Transition to thermoacoustic instability

During stable combustor operation, the sound generated from turbulent flames is due to non-steady volumetric expansion and convective entropy modes [8]. This state is referred to as combustion noise as radiated noise lacks any characteristic time scales and has a broadband signature [8]. Later studies revealed that combustion noise displays scale invariance [9] and possess signatures of multifractality [10]. During unstable combustor operation, i.e., the state of thermoacoustic instability, feedback between heat release rate (HRR) and acoustic pressure fluctuations lead to the collapse of broadband frequencies into a single, or at most a few, characteristic frequencies. By systematically varying the control parameters, it is possible to transition from stable to unstable combustor operation. In turbulent combustors, the transition is often associated with an intermediate state known as intermittency [11]. Intermittency is characterized by the presence of periodic bursts interspersed with chaotic oscillations. During this transition, there is a loss in multifractality associated with the temporal dynamics of acoustic pressure and HRR oscillations at the onset of thermoacoustic instability [7, 10, 12]. In other words, there is a transition from a state possessing multiple time scales to one possessing a single characteristic time scale.

Concomitantly, the flow field undergoes drastic changes during the transition. There are small vortices that are shed aperiodically during the state of combustion noise; however, during thermoacoustic instability, vortices are shed periodically from the backwards-facing step of the combustor [13].

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As for the spatial dynamics, during the state of combustion noise and aperiodic part of intermittency, there is a distinct lack of large-scale vortices, and the HRR field remains spatially incoherent. In contrast, during the periodic epochs of intermittency and the state of thermoacoustic instability, periodically shed vortices develop into large-scale coherent structures of air and fuel mixture, which upon impingement with combustor walls result in regions with intense HRR [13, 14]. The multifractal spectra calculated from the spatial distribution of wrinkles on the flame surface show that the span of the spectra increases during thermoacoustic instability in comparison to that during combustion noise, indicating the significant increase in the spatial scales over which HRR fluctuations occur [15].

Thus, the dynamics of a thermoacoustic system are controlled by the spatiotemporal evolution of the interaction between the three key subsystems – turbulent flow field, combustion and acoustic field of the combustor. Quantifying this spatiotemporal evolution using the right mathematical tool is crucial in forewarning an impending thermoacoustic instability. Nair et al. [11] observed that the scaling of the acoustic pressure oscillations (quantified by the Hurst exponent, $H$) observed during combustion noise and intermittency fares better than corresponding measures based on tracking the root-mean-squared (rms) or Fourier transformed amplitude of the pressure oscillations when predicting thermoacoustic instability. However, such single point acoustic measurements cannot be used to predict in advance the changes required in the combustor to prevent instability. Thus, there is a need for extending such an analysis in the spatial domain.

### B. Passive control of thermoacoustic instability

The different approaches in combating the problem of thermoacoustic instability can be broadly classified into passive and active control strategies. Passive control refers to the change in some aspect (geometry, injector arrangement, etc.) of the combustor independent of the operation of the combustor [16]. In contrast, active control refers to monitoring the state of combustor operation and undertaking measures based on the state of the combustor through the means of external perturbations to the system [16]. Passive control strategies are widely found in practical combustion systems owing to the limitations posed by active control methods which are rather difficult to integrate with full-scale combustion systems.

Active control studies with modulated injection of secondary fuel or air modulation have been shown to suppress thermoacoustic instability [17–20] and have been reviewed in [16]. Modulation of secondary fuel or air relies crucially on the response of high-speed actuators being robust. As such, the combustor dynamics show significant nonlinear dependence on the forcing frequency [21]. Ensuring fast response of actuators at a frequency where the combustor dynamics is most sensitive to forcing is a challenge and leads to significant complications in the combustor design.

Motivated by the above-mentioned limitations, Ghoniem and co-workers [22–24] considered steady injection of secondary air for achieving control in a dump combustor. Ghoniem et al. [22] found that momentum-ratio of the jet to the main flow above unity leads to a compact flame structure which is less driven by the wake vortex. Later, Altay et al. [24] compared the suppression observed during transverse and streamwise secondary air injection. For optimum transverse injection, they observed suppression due to compact flame structure obtained through anchoring of the flame upstream of the step. In contrast, for optimum streamwise injection, suppression was through prevention of unstable vortex formed at the step.

The next important aspect that we need to consider is the location of the injection of secondary air. One disadvantage of injecting air near the location of flame anchoring is the related issue of flame stability. Consequently, Ghoniem et al. [22] and Altay et al. [24] injected secondary $H_2$ injection to increase flame stability and prevent blow-out. However, an increase in $H_2$ leads to an increase in NOx levels due to the higher temperature of the reacting flow field. Thus, Ghoniem et al. [22] had to optimize the main flow after the injection of secondary air and $H_2$ in order to reduce the temperature inside the combustor and decrease the NOx levels. Later, Oztarlik et al. [25] showed that secondary $H_2$ injection in small fraction is capable in suppressing TAI.

In order to by-pass the back and forth adjustments in the main and secondary flow, prevent flame blow-out and reduce the complexity involved in maintaining expensive $H_2$ plumbing, we simply consider unmodulated, steady injection of secondary air optimized based on the secondary injection away from the location of flame anchoring. Optimal region for injection can be selected if local regions responsible for TAI can be identified. Uhm and Acharya [19] considers this region to be the region of local maxima in HRR, whereas Ghoniem and co-workers [22–24] rationalize the region of flame anchoring, which leads to flame stabilization problems. In a similar study, Tachibana et al. [26] use a distribution of Rayleigh index to optimize for the choice of the secondary fuel injector. Very recently, Krishnan et al. [27] were able to determine the optimum location of the secondary air-injection based on network-based measures calculated from correlations in the velocity field. Targeting regions with large values of network measure led to control of thermoacoustic instability.

### C. Scope

In the discussion above, we note the particular importance of quantifying the spatiotemporal behaviour of the combustor during the transition to the state of TAI. We also note the significant challenges in developing even a relatively simple passive control strategy where unmodulated secondary microjet of air is used. Thus, the key objectives of this article are:

1. To optimize for the location of secondary air-injection targeting certain critical region(s) in the combustor obtained from spatiotemporal measurables under the constraint that the flame stability is maintained. These measurables include the amplitude of velocity fluctuations, mean vorticity, Rayleigh index and averaged HRR field.
To extend the analysis using Hurst exponent (\(H\)) in the spatial domain. As mentioned earlier, \(H\) measured from single-point pressure measurements is invaluable in the prediction of impending instability. A spatial analysis using \(H\) may provide useful insights in optimizing the location of secondary air injection during the states leading up to TAI. Consequently, the advantages of optimized passive control can be complemented by the predictive capability of the Hurst exponent resulting in a smart passive control strategy. To the best of our knowledge, predicting optimum location for targeted passive control during the states leading up to thermoacoustic instability have not been achieved until now.

The manuscript is structured as follows: The experimental setup with the turbulent combustor is described in §IIA and the combustor is characterized in §IIB. The methodology of determining the spatial distribution of \(H\) from the flow field is described in §IIC. In §IIA, we discuss the spatiotemporal dynamics through measures based on the amplitude of velocity fluctuations, vorticity, averaged HRR and Rayleigh index. In §IIB, we discuss the result of passive control based on the information from the physical measures discussed in §IIA. In §IIC, we discuss the results based on the distribution of \(H\) and highlight the key advantages of using \(H\) over physical measures. Lastly, in §IV, we emphasize the key contributions of this study in conclusion.

II. METHODOLOGY

A. Experimental setup and measurements

We perform experiments in a bluff-body stabilized turbulent combustor (Fig. 1a). The turbulent combustor has a cross-section of 90 × 90 mm² and is 1100 mm long. The bluff-body comprises of a circular disk of 47 mm diameter and 10 mm thickness. It is centrally mounted on a hollow shaft of \(d_i = 16\) mm diameter. The bluff-body is located 35 mm from the dump plane of the combustor (Fig. 1b). Air first passes through a settling chamber before being guided into the combustor through an inlet of diameter \(d_i = 40\) mm.

Fuel (Liquified Petroleum Gas or LPG, 60% butane and 40% propane) is injected through holes of 1.7 mm diameter present on the hollow shaft, 110 mm upstream of the backwards-facing step. The partially premixed fuel-air mixture is ignited using a spark plug connected to an 11 kV transformer and mounted on the dump plane. Air and fuel flow rates are controlled through mass flow controllers (Alicat Scientific, MCR series) and have a measurement uncertainty of \(±0.8\%\) of reading + 0.2% of full-scale. In our experiments, the fuel flow rate was set to 0.95 g/s, and the air flow rate was varied from 9.80 g/s to 15.92 g/s such that the equivalence ratio (\(\phi\)) varied in the range of 0.95 to 0.53. The air velocity varies in the range of \(\nu_a = 8.1\) m/s to 14 m/s. The air flow Reynolds number, calculated as \(Re = \nu_a(d_i - d_s)/\nu\), varies from 12500 to 18000, where \(\nu\) is the kinematic viscosity of air. The maximum uncertainty in the indicated value of \(\phi\), \(\nu_a\), and \(Re\) are \(±1.6\%\), \(±0.8\%\) and \(±0.8\%\), respectively.

Secondary air injection ports of 5 mm diameter are present on either side of the centerline, as shown in Fig. 1b. These ports target different regions of the flow field and are used for passive control. Note that the position of the injection ports is different from the ones utilized in previous studies [19, 22, 24] where the flame anchoring point was targeted. The particular choice of the injection ports was made so as to not compromise the flame anchoring and stability during control. Secondary air is controlled through a separate mass flow controller. Secondary air is injected into the combustor during the state of thermoacoustic instability in steps of 0.16 g/s till 1.90 g/s.

Pressure measurements are performed using a PCB103B02 piezoelectric transducer (sensitivity: 217 mV/kPa and uncertainty: \(±0.15\) Pa) mounted on the combustor wall, 17 mm from the dump plane. Two-component 2-D high-speed particle image velocimetry (PIV) is performed to acquire the velocity field during the experiments. The reactive flow field is seeded using 1 \(\mu\)m diameter TiO₂ particles. Mie scattering images were acquired using a high-speed CMOS camera (Photron SA-4). The procedure for determining the velocity field from Mie scattering images is detailed in [28]. The camera is equipped with a ZEISS 100 mm camera lens at \(f/2\) aperture. Chemiluminescence images are captured using Phantom - V12.1 with a ZEISS 50 mm camera lens and outfitted with a bandpass filter centred around 435 ± 10 nm to capture the emissions from CH* radicals from a region spanning 87 × 78 mm around the bluff-body (Fig. 1a). The flow-

FIG. 1. (a) The bluff-body stabilized turbulent combustor used for the present study. (b) Schematic of the combustor cross-section indicating the PIV and CH* field of view. Also shown are the ports (P1–P5) present on either side of the combustor through which secondary air is injected targeting different regions of the flow field. All dimensions in mm. Reproduced and adapted with permission from [28].
field spanning 45 × 40 mm between the dump plane and the bluff-body is imaged (Fig. 1b). The sampling frequencies for pressure, chemiluminescence and PIV measurements are 20 kHz, 4 kHz, and 2 kHz respectively.

Pressure measurements were acquired at progressively leaner flow conditions to obtain the transition to thermoacoustic instability. We performed optical diagnostics at specific fuel and airflow rates which corresponded to states which were representative of combustion noise, intermittency, and thermoacoustic instability. We also performed diagnostics during the control experiment to evaluate the effect of secondary jet on the flow characteristics. For a more detailed discussion on the experimental setup and uncertainty measurements associated with measurement devices, please refer to [13, 27].

B. Characterizing the turbulent combustor

The transition of the turbulent combustor from the state of combustion noise to thermoacoustic instability through the state of intermittency is depicted in Fig. 2. In Fig. 2a, we show the change in \( p'_{rms} \) as a function of the nominal velocity \( u_y \) (bottom axis) of air and equivalence ratio \( \phi \) (top axis). Markers ’A’, ’B’ and ’C’ correspond to three points representative of the states of combustion noise, intermittency, and thermoacoustic instability for which the subsequent spatiotemporal analysis has been performed. Figure 2b shows the change in the dominant frequency of the acoustic pressure and spatially averaged velocity time series during the transition to thermoacoustic instability. We can observe the two separate time scales – acoustic (1/\( f_a \)) and hydrodynamic (1/\( f_h \)) – associated with the high amplitude bursts and the low amplitude weakly periodic oscillations present during the state of intermittency, respectively [29]. During thermoacoustic instability, they undergo mutual synchronization and attain a common frequency [30]. We discuss the spatiotemporal dynamics from the perspective of optimization of the location of secondary injection. A more thorough discussion on the spatiotemporal dynamics of bluff-body stabilized combustors can be found elsewhere [13, 14, 29, 31, 32].

C. Nonlinear time series analysis: The Hurst exponent

The time-series of velocity fluctuations obtained from different locations of the combustor are generally non-stationary, and hence, central moments diverge over time [11]. Consequently, it is instructive to calculate the scaling associated with the dependence of the moments on the length of the time interval over which the moments are evaluated. The scaling exponent is called the Hurst exponent. We measure the Hurst exponent using Multifractal Detrended Fluctuation Analysis (MF DFA)[33], which we briefly discuss below.

From the velocity field obtained from PIV, we calculate the resultant velocity as \( u_T(x,y,t) = \sqrt{\langle u(x,y,t)^2 \rangle + \langle v(x,y,t)^2 \rangle} \). From the time series of resultant velocity at a given location \((x,y)\), we calculate the cumulative deviate series:

\[
y_k = \sum_{t=1}^{k} [u_T(t) - \bar{u}_T],
\]

where \( \bar{u}_T \) indicates the time-averaged resultant velocity. The deviate series is divided into \( n_s \) non-overlapping segments \( \{y_i(t), i = 1, 2, \ldots, n_s\} \) of equal span \( s \). Local trends are removed by local linear fit \( \tilde{y}_i \) onto the deviate series \( y_i \). Local fluctuations are obtained by subtracting the fit from the deviate series. The minimum and maximum length of the window was chosen as \( 2/f_a \) ms to \( 4/f_a \) ms, where \( f_a = 140 \) Hz. In other words, minimum and maximum window size corresponds to two to four acoustic time periods [10, 34].

The second-order structure function scales as \( F_2(s) \sim s^H \) within the bounds set by the minimum and maximum window size. The Hurst exponent \( H \) is then determined as:

\[
H = \frac{\log F_2(s)}{\log s} \forall s \in [2/f_a, 4/f_a].
\]

We plot the second-order structure function \( F_2 \) as a function of the scale \( s \) measured from the time series of turbulent velocity fluctuations \( u_T(t) \) at a representative point in the flow field.
where we observe that the range of x-axis is not too low so as to be within the operating parameters indicated in Fig. 2. As can be seen, the scaling is evaluated over the range of $s \in [2/f_a, 4/f_a]$. We observe that the Hurst exponent $H = 0.66$ and $H = 0.67$ during combustion noise and intermittency, respectively. In contrast, during thermoacoustic instability, $H = 0.31$ is comparatively much lower. We repeat this process over the complete flow field and obtain the spatial distribution of $H$ over the entire flow field.

Measuring the scaling of the structure-function within the bounds $s \in [2/f_a, 4/f_a]$ has two advantages. First, this ensures that the range of x-axis is not too low so as to be within a periodic cycle and not too large as to become completely uncorrelated [34]. Such behaviour is also evident from Fig. 3 where we observe that $F_2$ is relatively flat for low values of $s (< 2/f_a)$, indicating the high correlation between the segments by virtue of them being in the same periodic cycle. Similarly, for large $s (> 4/f_a)$, $F_2$ shows oscillations, indicating contributions from decorrelated segments. Second, the acoustics of the combustor imposes a characteristic length scale on the turbulent flow. Consequently, one only needs to be concerned with the scaling of the structure-function of turbulent intensity in the range of time scales comparable to the acoustic time scale. The onset of thermoacoustic instability is captured by the gradual disappearance of scaling of the structure-function in the bounded range of $s$.

Finally, we note that $H$ measures the correlation and persistence in a time series. If a large (small) value is more likely to be followed by another large (small) value, the signal is said to be persistent and long-range correlated. Such signals have $H > 0.5$. If a large (small) value is more likely to be followed by a small (large) value, the signal is anti-persistent. For such signals, $H < 0.5$ and only short-range correlations exist [10]. For a purely periodic signal, $H = 0$ and $H = 0.5$ for white noise. Thus, the spatial distribution of $H$ indicates the persistence and correlation in the velocity fluctuations over the flow-field.

### III. RESULTS AND DISCUSSIONS

#### A. Spatiotemporal analysis during the transition to thermoacoustic instability

We start by analyzing the spatiotemporal dynamics during the transition from combustion noise to thermoacoustic instability. As already discussed, we acquire simultaneous data associated with velocity, CH$^+$ chemiluminescence and pressure fluctuations. In Fig. 4, we plot the Fourier transformed amplitude of velocity fluctuations measured at the acoustic frequency $|\tilde{u}(x, y, f_a)|$, the time-averaged vorticity $\langle \tilde{\omega}(x, y) \rangle$, and the time-averaged heat release rate field $\langle \tilde{q}(x, y) \rangle$ during combustion noise, intermittency and thermoacoustic instability at parametric points A, B and C indicated in Fig. 2a. We also plot the Rayleigh index, which is defined as:

$$RI(x, y) = \frac{1}{NT} \int_0^{NT} p'(t) \tilde{q}'(x, y, t) dt.$$  \hspace{1cm} (4)

Here, $N (= 686)$ denotes the total number of cycles, and $T (= 1/f_a)$ denotes the time-period of oscillations. The spatial distribution of the Rayleigh index quantifies the strength of acoustic power sources and sinks depending upon positive or negative feedback between pressure and heat release rate oscillations, respectively.

During the state of combustion noise at $\phi = 0.86$ and $v_a = 8.1$ m/s, the hydrodynamic mode at $f_a$ dominates. Thus, the Fourier transformed amplitude $|\tilde{u}(f_a)|$ at the acoustic mode $f = f_a$ is naturally very low (Fig. 4a). Notice that the scale only extends till 0.1 m/s, which is very low. The time-averaged vorticity field shown in Fig. 4b indicates that the vortices evolve only along the shear layer (SL). The transverse span of vorticity contour indicates that the size of vortices is very small. We also notice the absence of the outer recirculation zone. The time-averaged heat release rate field also shows very low values (Fig. 4c). Likewise, the Rayleigh index shows very low values throughout the combustor (Fig. 4d).

During the state of intermittency at $\phi = 0.66$ and $v_a = 11.1$ m/s, $|\tilde{u}(f_a)|$ shows higher values compared to the state of combustion noise due to intermittent periodic oscillations of turbulent velocity induced by intermittent acoustic pressure oscillations (fig. 4e). During epochs of periodic oscillations, larger vortices are shed, leading to the flow recirculating at the dump plane. This can be seen from the rather large distribution of $\tilde{\omega}_z$.
FIG. 4. Comparison of amplitude of turbulent velocity fluctuations $|\hat{u}(f_a)|$ (m/s), time-averaged vorticity $\bar{\omega}_z$ (s$^{-1}$), time-averaged heat release field $\bar{q}$ (a.u.) and time-average Rayleigh index $\bar{RI}$ (a.u.) during the states of (a-d) combustion noise, (e-h) intermittency and (i-l) thermoacoustic instability. The flow conditions are indicated in Fig. 2. The span of the ordinate and abcissa are indicated in Fig. 1b.

FIG. 5. Phase-averaged heat release rate field at the indicated phase of the acoustic cycle during the state of thermoacoustic instability.

in Fig. 4f. The maximum of $\bar{\omega}_z$ along the shear layer indicates that most of the vortices are shed along the shear layer but only recirculate intermittently. The time-averaged heat release rate field (Fig. 4g) and Rayleigh index (Fig. 4b) also show higher values when compared with that during combustion noise.

There is a significant change in the spatiotemporal behaviour during the state of thermoacoustic instability at $\phi = 0.63$ and $u_a = 12.3$ m/s (Figs. 4i-4l). Thermoacoustic instability in bluff-body stabilized combustors is associated with the phenomenon of vortex-acoustic lock-on wherein the frequency of vortex shedding matches the acoustic frequency $f_a = f_h$ (as shown in Fig. 2b) and is central to the establishment of the thermoacoustic feedback in the combustor [14, 29–31]. Accordingly, we observe a clearly defined region with a very high value of $|\hat{u}(f_a)|$ (Fig. 4i). The periodically shed vortices from the dump plane and the tip of the bluff-body, develop into large coherent structures which recirculates into the outer recirculation zone (ORZ) and can be observed from the high value of $\bar{\omega}_z$ (Fig. 4j).

The large coherent structures upon impingement with the bluff-body and the combustor wall lead to intense heat release. The time-averaged heat release rate field in Fig. 4k indicates that the maxima in the heat release rate fluctuations take place downstream of the bluff-body. In order to get further information from the heat release field, we plot the phase-averaged heat release rate field at four points – $0^\circ$, $90^\circ$, $180^\circ$ and $270^\circ$ – of the acoustic cycle in Fig. 5. Such a phase-averaged field is indicative of the evolution of the flame structure at different points of the acoustic cycle. At $90^\circ$ and $270^\circ$, i.e., at pressure maxima and pressure minima, we observe very intense structure in the CH* field. We notice that the fluctuations intensify from moderate to high values as one moves closer to maxima ($90^\circ$) and minima ($270^\circ$) of the pressure fluctuations. The effect of such an intensified and localized heat release rate can be seen on the acoustic power sources from the spatial distribution of Rayleigh index shown in Fig. 4l. The acoustic power source is concentrated in the region above the bluff-body where we observe very high, positive Rayleigh index.

As mentioned earlier, precursor based methods are great for predicting impending thermoacoustic instability. However, such methods do not provide any information required for implementing passive control measures. Passive control based on design changes requires knowledge about the relative importance of different regions in the flow field. Based on the considerations made above, different measures indicate different regions of interest. For instance, the region between the dump plane and the bluff-body appears significant based on the large amplitude of $|\hat{u}(f_a)|$ during thermoacoustic instability (Fig. 4i). Similarly, the maxima in $\bar{\omega}_z$ during thermoacoustic instability (Fig. 4j) emphasizes the importance of the outer
FIG. 6. Illustration of control of thermoacoustic instability through secondary air injection targeting different regions of the combustor. (a) Amplitude of pressure fluctuations, $p'_{\text{rms}}$, as a functions of the momentum ratio, $(v_{\text{inj}}/v_a)^2$, and mass flow ratio, $\dot{m}_{\text{inj}}/\dot{m}_a$. Representative (b) time series and (c) sound pressure level (SPL) observed during thermoacoustic instability, point A in (a), and subsequent control, point B in (a). Subfigure (a) adapted from [28] with permission.

FIG. 7. (a) Amplitude of velocity fluctuations $|\hat{u}(f_a)|$, (b) time-averaged vorticity $\omega_z$, (c) time-averaged HRR $q$ and (d) Rayleigh index $RI$ during suppression corresponding to point B in Fig. 6a.

recirculation zone. Likewise, $\bar{q}(x,y)$ points towards the region after the bluff-body (Fig. 4k), while phase-averaged heat release field (Fig. 5) and Rayleigh index (Fig. 4l) highlights the importance of the region above the bluff-body. We test the efficacy of passive control when these “critical” regions are selectively targeted with secondary air injection. Finally, as with the precursor, we would like to find out whether it is possible to predict such critical regions inside the combustor during the state of intermittency itself.

B. Passive control of thermoacoustic instability

We now attempt passive control of thermoacoustic instability using secondary air injection from various ports mounted on the combustor side-walls (see Fig. 1b). Secondary air injected through these ports target various regions identified through the physical measures considered in the previous sec-
FIG. 8. Heat release rate field during the state of suppression of thermoacoustic instability for point B indicated in Fig. 6a. (a) Intermittent acoustic pressure fluctuations during suppression and (b) an enlarged portion showing alternate cycles of periodic and aperiodic fluctuations. (c) Points i-iv correspond to $\dot{q}'(x,y)$ at four points of the periodic cycle indicated in (b). Points v-viii corresponds to $\dot{q}'(x,y)$ at the indicated points during aperiodic oscillations as indicated in (b).

not lead to any reduction in $p'_{rms}$, and in fact, lead to increase in the amplitude of limit cycle oscillations.

The above exercise shows us the relative importance of different regions of the flow field as far as passive control is concerned. The region between the dump plane and bluff-body, as identified by the amplitude of velocity fluctuations $|\tilde{u}(f_a)|$ in Fig. 4i, is “critical” to the spatiotemporal dynamics of thermoacoustic instability. This is why targeting the critical region through P1 and P2+P3 leads to effective control. Other regions such as the top of the bluff-body or region downstream of the bluff-body identified respectively from Rayleigh index (Fig. 4l) and averaged HRR (Fig. 4k) are not as important and, hence, cannot be used for optimizing the location of secondary injection. This is in direct contrast to Tachibana et al. [26] who optimized for the fuel injector based on microjet injection targeting the region of large Rayleigh index in their swirl-stabilized combustor. While Rayleigh index identifies the region of most significant driving, it does not always identify the region most sensitive to control. Thus, the local Rayleigh index or average HRR cannot always be used for determining critical region.

Next, we analyze the effect of secondary air injection on different measures. Figure 7 depicts the spatial dynamics associated during the state of suppression (point B indicated in Fig. 6a). We notice many differences from the spatiotemporal dynamics during thermoacoustic instability. The maxima of $|\tilde{u}(f_a)|$ moves from the region between the dump plane and the bluff-body (Fig. 4i) to the top of the bluff-body (Fig. 7a) albeit with comparatively lower amplitude. The time-averaged vorticity field $\bar{\omega}_z$ shows vortices concentrated along a small region along the shear layer, indicating the suppression of the large coherent structures formed during thermoacoustic instability (Fig. 4a). We further notice that the mean flame structure (Fig. 7c) is no longer as concentrated downstream of the bluff-body, as was the case during thermoacoustic instability (Figs. 4k). In fact, the HRR gets more distributed and extends far downstream. Accordingly, the Rayleigh index is very low value throughout the combustor, indicating the reduction in the strength of acoustic power sources inside the combustor due to secondary injection.

During the state of suppression, the pressure fluctuations show intermittent characteristics (Fig. 8a,b). So, the time-averaged HRR field is ill-suited to depict the spatial flame dynamics appropriately. We, thus, plot the instantaneous mean-subtracted HRR field $\dot{q}'(x,y)$ for four different phases from the epoch of periodic oscillations in Figs. 8ci-iv and from the epoch of aperiodic oscillations in Fig. 8cv-viii. Secondary injection too close to the lip of the dump plane leads to significant instability in flame anchoring [22, 24] which was solved by secondary $H_2$ injection. We note that secondary air injection does not affect the flame anchoring and stability as the flame can be observed along the shear layer after the dump plane and extends along the shear layer following the bluff-body both during the periodic and aperiodic epochs. The instantaneous HRR field during periodic oscillations show no concentrated spots in the HRR field as were present during thermoacoustic instability (Fig. 5) and is distributed throughout the field. The HRR field during aperiodic oscillations shows incoherence. Finally, the instantaneous HRR field does not show any visible correlation of large HRR fluctuations with pressure fluctuations either during periodic or aperiodic oscillations leading to very low values of the Rayleigh index (Fig. 7d).
the difference in the nature of the velocity fluctuations present there. The presence of values of $H > 0.5$ indicates that the dynamics of velocity fluctuations are persistent, i.e., a large (small) value is more likely to be followed by another large (small) value. In fact, the velocity fluctuations in regions with $0.5 < H < 1$ have fractal characteristics and possess long-range correlations typically associated with fully-developed turbulent flows [35]. The observed temporal dynamics may be due to the presence of small-scale vortices observed during the state of combustion noise [13].

Figure 9b shows the distribution of $H$ during the state of intermittency at $\phi = 0.66$ & $\nu_a = 11.1$ m/s. During this state, we observe large-amplitude periodic bursts embedded randomly amidst the low amplitude aperiodic fluctuations in the measured pressure signal. The field of $H$ is shown in Fig. 9b. We notice that the values of $H$ in the range of $[0.2, 0.7]$ are distributed across the flow field. Regions with $H < 0.5$ indicate anti-persistent behaviour in that large (small) values associated with velocity fluctuations are more likely to be followed by a small (large) fluctuation as would be the case for a periodic signal. This indicates that the velocity field exhibits periodicity temporally. The region with $H$ close to 0.2 indicates the region over which spatial coherence is maximum in the field.

During thermoacoustic instability at $\phi = 0.63$ & $\nu_a = 12.3$ m/s (Fig. 9c), we observe that the flow field of the combustor has $H$ values predominantly lesser than 0.5. We can observe that the region immediately after the dump plane has very low values of $H$ close to 0.1. Signals with $H$ near zero imply the absence of scaling of the structure-function. In other words, the fluctuations are bounded. As a consequence, the signal displays a strong anti-persistent behaviour, characteristic of periodic signals. Thus, in the region with $H < 0.3$, the temporal dynamics of velocity fluctuations are nearly periodic, and much of the spatial region display coherence.

Figure 9d shows the distribution of $H$ measured from the flow field when control has been achieved. We observe that the expansive region with $H \approx 0$ during the state of thermoacoustic instability, which was located between the backwards-facing step and the bluff-body is no longer present. In other words, the region with coherence in spatiotemporal dynamics is disrupted. The resultant flow field has a distribution of $H$ values ranging from 0.5 to 1 indicating that the periodic and coherent spatiotemporal dynamics associated with the state of thermoacoustic instability have been suppressed (Fig. 9a).

Thus, we see that the Hurst exponent correctly identifies the critical region in the flow field as was also done by Fourier transformed velocity amplitude $|\hat{u}(f_a)|$ (Fig. 4i). As noted earlier, $|\hat{u}(f_a)|$ identified the critical region only during thermoacoustic instability. In contrast, we observe that $H$ is able to capture the critical region even during the state of intermittency (Fig. 9b), which is something we do not observe from the field of $|\hat{u}(f_a)|$ during the state of intermittency (Fig. 4e). Hence, it is possible to predict the critical region from the velocity field obtained during the state of intermittency. Thus, Hurst exponent proves to be a very robust measure in analyzing the temporal and spatiotemporal dynamics of thermoacoustic systems.

C. Optimized passive control using Hurst exponent

We have established that effective passive control of thermoacoustic instability depends crucially on the region targeted using secondary air-injection. Determination of the critical region is non-trivial as different physical measures point to different regions. Of these, we observed that amplitude of velocity fluctuations $|\hat{u}(x, y, f_a)|$ correctly identifies the critical region, targeting which led to the suppression of thermoacoustic instability. However, the measure was unable to distinguish the critical region during the state of intermittency. We remedy this by implementing the spatiotemporal analysis using Hurst exponent calculated from the turbulent intensity ($u_T$) as discussed in §II C.

During the state of combustion noise at $\phi = 0.86$ & $\nu_a = 8.1$ m/s, $H$ values in the range $[0.5, 1.2]$ are distributed throughout the combustor (Fig. 9a). The different regions indicate

FIG. 9. Field of Hurst exponent ($H$) calculated from the intensity of velocity fluctuations during (a) combustion noise, (b) intermittency and (c) thermoacoustic instability. The experimental conditions for (a-c) are indicated in Fig. 2 and (d) corresponds to B in Fig. 6.
IV. CONCLUSIONS

In the present study, we develop a smart passive control strategy of combating thermoacoustic instability in a bluff-body stabilized turbulent combustor. We analyze the spatiotemporal behaviour of the thermoacoustic system during the transition from combustion noise to thermoacoustic instability through the state of intermittency using physical measures such as the amplitude of velocity fluctuations, time-averaged vorticity, averaged heat release rate (HRR) and Rayleigh index. We optimize the location of the secondary air injection based on these physical measures and find that the local region exhibiting large-amplitude velocity fluctuations, measured using Fourier transformed velocity amplitude at the acoustic frequency, is most suited for targeting secondary air. In doing so, we observe a 20 dB drop in the sound pressure level. Further, the choice of injection port is such that it does not jeopardize flame stability during control, thus requiring no secondary fuel injection for flame stabilization as was necessary for some of the past studies. Moreover, local regions identified using Rayleigh index and HRR did not lead to any significant suppression, in contrast to the inferences of past study, indicating the system-specific nature of these physical measures. Further, this makes the optimization problem of finding the right location of secondary air injection non-trivial.

It has been found that the Hurst exponent is a much more robust measure than the velocity amplitude for predicting thermoacoustic instability [11]. We extend this into the spatiotemporal domain in a bid to combine the predictive abilities of Hurst exponent with the idea of optimization of the location of secondary air injection. Using the spatial distribution of the Hurst exponent during the state of thermoacoustic instability, we are able to correctly identify the critical region recognized by the amplitude of velocity fluctuations, thus validating our approach. We then find that the spatial distribution of Hurst exponent is able to predict the critical region during the state of intermittency, something which none of the other physical measures is able to do. This constitutes the most important finding of our study. Thus, aided with the predictive capabilities of the Hurst exponent, we are able to develop a smart passive control strategy.

In closing, we note that the present methodology has potential for wide application in combustors. First, it can be used to determine critical regions most suited for aiming control strategies without infringing upon the stability of the flame. Second, the spatial distribution of Hurst exponent can be used to predict critical region if the flow-field is known during the states prior to full-blown thermoacoustic instability. This indeed is a boon as the extent of spatial and temporal scales during the state of intermittency is lesser than that observed during thermoacoustic instability [15]. Indeed, CFD simulations of intermittent states would be numerically cheaper to simulate and used for predicting critical region which develops during thermoacoustic instability. Thus, for already commissioned combustors, critical regions can be obtained from LES simulations and used to determine the right combination of secondary air-injection ports for efficient control of thermoacoustic instability. Such a control can expand the operational regime of combustors leading to reduced costs and energy saving in gas turbine combustors.

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