Zero modes for the quantum Liouville model

L. D. Faddeev

St. Petersburg Dept. of Steklov Mathematical Institute
and
St. Petersburg State University

Abstract
The problem of definition of zero modes for quantum Liouville model is discussed and corresponding Hilbert space representation is constructed.

1 Introduction and statement of the problem

Quantum Liouville model is a basic example of conformal field theory. It represents a quantization of the dynamical system for real-valued field $\phi(x,t)$, defined on the cylinder $(x,t) \in S^1 \times \mathbb{R}$ by hamiltonian

$$H = \frac{1}{2\gamma} \int_0^{2\pi} (\pi^2 + \phi_x^2 + e^{2\phi}) dx$$

and canonical Poisson bracket

$$\{\phi(x),\phi(y)\} = 0, \quad \{\pi(x),\pi(y)\} = 0, \quad \{\pi(x),\phi(y)\} = \gamma \delta(x-y)$$

in terms of the initial data $\phi(x) = \phi(x,0)$, $\pi(x) = \phi_t(x,0)$. The corresponding equation of motion looks as follows

$$\phi_{tt} - \phi_{xx} + e^{2\phi} = 0.$$  \hspace{1cm} (1)

The quantization developed by several authors (see survey [4] and the literature cited there) is based on the classical Liouville formula

$$e^{2\phi} = -4 \frac{f'(x-t)g'(x+t)}{(f(x-t) - g(x+t))^2},$$  \hspace{1cm} (2)

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and parametrization of functions $f(x)$ and $g(x)$ via free quantum fields and zero modes like this

$$\ln f(x) = Q - \frac{Px}{\pi} + \chi(x),$$

where $Q$ and $P$ are canonical variables, whereas the periodic field $\chi(x)$ is expressed by oscillators. The field $g(x)$ is parametrized analogously with the same $P$ and $Q$. There arises a picture of the corresponding Hilbert space

$$\mathcal{H} = L_2(\mathbb{R}) \otimes \mathcal{H}_L \otimes \mathcal{H}_R,$$

where $\mathcal{H}_L$ and $\mathcal{H}_R$ represents oscillators and $L_2(\mathbb{R})$ is a space for zero modes $(P,Q)$. However on some stage an invariance to reflection $P \to -P$ appears, the origin of which is usually quite vague. In this paper, which follows the previous publication [4], I shall present arguments for this invariance and describe its realization in Hilbert space.

## 2 Classical theory

The equation (1) is a condition of zero curvature

$$[\hat{L}_1, \hat{L}_0] = 0$$

for the connection

$$\hat{L}_1 = \frac{d}{dx} - L_1, \quad \hat{L}_0 = \frac{d}{dt} - L_0,$$

where $2 \times 2$ matrices $L_1$ and $L_0$ are parametrized by the field $\phi(x,t)$

$$L_1 = \frac{1}{2} \begin{pmatrix} \phi_t & e^\phi \\ e^{-\phi} & -\phi_t \end{pmatrix}, \quad L_0 = \frac{1}{2} \begin{pmatrix} \phi_x & -e^\phi \\ e^{-\phi} & -\phi_x \end{pmatrix}.$$  

Let

$$T(x,t) = \begin{pmatrix} A(x,t) & B(x,t) \\ C(x,t) & D(x,t) \end{pmatrix}$$

be the corresponding holonomy, i.e. a solution of the compatible equations

$$T_x = L_1 T, \quad T_t = L_0 T$$

with initial condition

$$T(0,0) = I.$$  

\[2\]
It is easy to see, that the projective components
\[ f(x, t) = \frac{A(x, t)}{B(x, t)}, \quad g(x, t) = \frac{C(x, t)}{D(x, t)} \]
satisfy the equations
\[ f_x = f_t, \quad g_x = -g_t \]
and thus
\[ f(x, t) = f(x - t), \quad g(x, t) = g(x + t). \]
It follows from (4), that the following initial condition
\[ f(0) = \infty, \quad g(0) = 0 \]
and bounds
\[ f'(0) < 0, \quad g'(0) > 0 \]
are valid. It follows, that \( f(x) \) and \( g(x) \) are positive. They realize the Liouville formula (2).

Let \( M \) be a corresponding monodromy for a circle \( t = 0 \)

\[ M = T(2\pi, 0) = \begin{pmatrix} a & b \\ c & d \end{pmatrix}. \]

It is a hyperbolic unimodular matrix with positive matrix elements. It is clear, that \( f \) and \( g \) satisfy the condition of quasiperiodicity
\[ f(x + 2\pi) = \frac{af(x) + c}{bf(x) + d}, \quad g(x + 2\pi) = \frac{ag(x) + c}{bg(x) + d}. \]

These conditions simplify if one diagonalizes the monodromy taking
\[ MN = ND, \]
where
\[ D = \begin{pmatrix} \lambda & 0 \\ 0 & 1/\lambda \end{pmatrix}, \quad N = \begin{pmatrix} 1 & 1/\xi \\ \eta & 1 \end{pmatrix} \]
The parameter \( \lambda \) defines the eigenvalues of monodromy and \( \xi, \eta \) are its fixed points, i.e. solutions of the quasiperiodic equation
\[ b\xi^2 - (a - d)\xi - c = 0, \quad a + d = \lambda + \frac{1}{\lambda}. \quad (5) \]
Leon Takhtajan and me in 1984 calculated the Poisson brackets for matrix elements $a, b, c, d$ of the monodromy $M$ (published in 1986 in [5])

\[
\begin{align*}
\{a, b\} &= \frac{1}{2} \gamma ab, & \{a, c\} &= \frac{1}{2} \gamma ac, \\
\{d, b\} &= -\frac{1}{2} \gamma bd, & \{d, c\} &= -\frac{1}{2} \gamma cd, \\
\{b, c\} &= 0, & \{a, d\} &= \gamma bc.
\end{align*}
\]

It is instructive to note that these relations became a base of quantum groups [6], [7].

It follows from these relations, that $\lambda, \xi$ and $\eta$ can be parametrized in the form

\[
\lambda = e^{-P}, \quad \xi = \alpha e^Q, \quad \eta = -\alpha e^{-Q}
\]

with brackets

\[
\{P, Q\} = \frac{\gamma}{2}, \quad \{P, \alpha\} = 0, \quad \{Q, \alpha\} = 0.
\]

In term of these variables the monodromy $T$ assumes the form

\[
M = \frac{1}{\text{ch} Q} \begin{pmatrix}
\text{ch}(Q - P) & \alpha^{-1} \text{sh} P \\
\alpha \text{sh} P & \text{ch}(Q + P)
\end{pmatrix}.
\] (6)

Now for the Möbius transformed $f$

\[
\hat{f} = N(f) = \frac{f + \eta}{f/\xi + 1}
\]

we have

\[
\hat{f}(0) = \xi = \alpha e^Q, \quad \hat{f}(2\pi) = e^{-2P} \hat{f}(0),
\]

so that

\[
\ln \hat{f}(x)/\alpha = Q - \frac{Px}{\pi} + \chi(x),
\]

where $\chi(x)$ is periodic. We get the formula (3). However the elements of monodromy $M$ are positive only under condition

\[
P > 0.
\]

Thus the phase space of the zero modes $P, Q$ is a halfplane, which raises the problem of correct quantization. We shall do it by initial choice of alternative canonical variables and appropriate canonical transformation.
Introduce change of variables
\[
u = \frac{\operatorname{ch} Q}{\operatorname{ch}(Q - P)} = \frac{1}{a}, \quad v = \frac{\operatorname{sh}^2 P}{\operatorname{ch} Q \operatorname{ch}(Q - P)} = \frac{bc}{a},
\]
which maps the upper halfplane \((P, Q)\) into the positive quadrant
\[u > 0, \quad v > 0.
\]
The brackets of \(u\) and \(v\) are as follows
\[
\{u, v\} = -\gamma uv,
\]
so that their logarithms are canonical variables in \(\mathbb{R}^2\) and their quantization is trivial.

The monodromy in new variables assumes the form
\[
M = \left( \begin{array}{cc}
u^{-1} & \alpha \sqrt{v/u} \\
\alpha^{-1} \sqrt{v/u} & u + v \end{array} \right)
\]
and now we can construct its quantum realization.

### 3 Quantization

Quantum analogue of the variables \(u\) and \(v\) is a Weyl pair \(u\) and \(v\) with relation
\[
uv = q^2 vu, \quad q = e^{i\gamma/2}.
\]
The variable \(\alpha\) remains a central element. To be closer to the notation of the theory of automorphic functions we put
\[
\gamma = 2\pi \tau
\]
and parametrize \(\tau\) via imaginary halfperiods \(\omega, \omega'\)
\[
\tau = \frac{\omega'}{\omega}, \quad \omega \omega' = -\frac{1}{4},
\]
lying in the upper halfplane.

The operators \(u\) and \(v\) can be realized in \(L_2(\mathbb{R})\) in the form
\[
u f(x) = e^{-i\pi x/\omega} f(x), \quad v f(x) = f(x + 2\omega'),
\]
as essentially selfadjoint operators with the domain $D$, consisting of analytic functions like
\[ e^{-\alpha x^2}e^{\beta x}P(x), \]
with $\alpha > 0$, $\beta \in \mathbb{C}$ and $P(x)$ being a polynomial with complex coefficients.

Quantum analogue of the monodromy is given by matrix
\[ M = \begin{pmatrix} u^{-1} & \alpha q^{-1/4}u^{-1/2}v^{1/2} \\ \alpha^{-1} q^{-1/4}u^{-1/2}v^{1/2} & u + v \end{pmatrix} \]
after the natural ordering of the factors.

Our first problem is to diagonalize $\text{tr} M$, namely the operator
\[ L = u + u^{-1} + v. \]

This operator is very well known nowadays and it acquired a lot of interpretations. For example in the quantum Teichmuller theory [8], [9] it is called operator of minimal geodesic length, in the theory of the decomposition of the tensor product of two irreducible representations of modular double $SL_q(2, \mathbb{R})$ into irreducibles it appears as a main part of corresponding Casimir [10]. Spectral theory for $L$ was investigated by Kashaev [11].

Operator $L$ has continuous spectrum of multiplicity 1 on the halfline $2 < \lambda < \infty$. The corresponding generalized eigenfunctions can be expressed in terms of function
\[ \gamma(x) = \exp \left( -\frac{1}{4} \int_{-\infty}^{\infty} \frac{e^{itz}}{\sin \omega t \sin \omega \tau} dt \right), \]
where the singularity at $t = 0$ is passed from above. This function acquired recently quite a popularity. I call it “quantum modular dilogarithm” [12]. Detailed description of its properties is given in [13]. Those of them, used in this paper, are collected in Appendix.

The basic property — the functional equation
\[ \frac{\gamma(x + \omega')}{\gamma(x - \omega')} = 1 + e^{-i\pi x/\omega} \]
(7)
can be used to show, that the generalized function
\[ \psi(x, s) = \gamma(x - s - \omega'' + i0)\gamma(x + s - \omega'' + i0)e^{-i\pi(x-\omega'')^2} \]
for real \( s \) is an eigenfunction of operator \( L \) with eigenvalue
\[
e^{i\pi s/\omega} + e^{-i\pi s/\omega} = 2 \cos \pi s/\omega,
\]
where
\[
\omega'' = \omega + \omega'.
\]
Function \( \gamma(z - \omega'') \) has a pole at \( z = 0 \) and \(+i0\) is added in the expression for \( \psi(x, s) \) to define how it is understood.

Kashaev [11] proved orthonormality of functions \( \psi(x, s) \) in the form
\[
\int \overline{\psi(x, s)} \psi(x, s') dx = \frac{1}{\rho(s)} \left( \delta(s - s') + \delta(s + s') \right)
\]
\[
\int_0^\infty \psi(x, s) \overline{\psi(y, s)} \rho(s) ds = \delta(x - y),
\]
where the measure \( \rho(s) \) is given by
\[
\rho(s) = -4 \sin \frac{\pi s}{\omega} \sin \frac{\pi s}{\omega'}.
\tag{8}
\]
Thus the natural statement is as follows: the diagonal representation for operator \( L \) is Hilbert space \( L_2(\mathbb{R}^+, \rho) \).

Let us calculate the expression for the nondiagonal element of monodromy in this representation. Denote by \( U \) an integral operator with kernel \( \psi(x, s) \), which connects spaces \( L_2(\mathbb{R}^+, \rho) \) and \( L_2(\mathbb{R}) \). Let \( r \) and \( Z \) be a Weyl pair acting on functions \( F(s) \) for \( s \in \mathbb{R} \)
\[
rF(s) = F(s + \omega'), \quad Zf = e^{i\pi s/\omega}F(s).
\]
Later we shall find contributions of \( r \) and \( Z \) having sense in \( L_2(\mathbb{R}^+, \rho) \).

From the functional equation (7) we get the expressions for the kernels of integral operators \( u^{1/2}Ur \) and \( u^{1/2}Ur^{-1} \)
\[
\psi(x + \omega', s - \omega') = (1 - \frac{u}{Z}) \frac{1}{i} q^{-1/2} u^{-1/2} \psi(x, s)
\]
\[
\psi(x + \omega', s + \omega') = (1 - uZ) \frac{1}{i} q^{-1/2} u^{-1/2} \psi(x, s),
\]
and from this we get
\[
q^{1/4} u^{-1/2} v^{1/2} U = U \frac{1}{i} (Z - Z^{-1})(r - r^{-1})^{-1}.
\]
Operator in the RHS makes sense in the subspace of even functions of $s$ and is selfadjoint in $L_2(\mathbb{R}_+, \rho)$.

Thus we get a new representation for the matrix elements of monodromy

$$U^{-1}(a + d)U = Z + Z^{-1}, \quad U^{-1}bU = \frac{\alpha}{t}(Z - Z^{-1})\frac{1}{r - r^{-1}}.$$ 

It is instructive to compare these quantum formulas with the classical ones in (6)

$$a + d = 2 \text{ch} P, \quad b = \alpha \frac{\text{sh} P}{\text{ch} Q}.$$ 

It is natural to identify $Z$ with $e^P$, and $r^{-1}$ with $e^Q$, however classical $\text{ch} Q$ is changed into $i \text{sh} Q$; this corresponds to shift $Q$ by $i\pi/2$.

I was not able to solve the quantum analogue of equation (5) to get the representations of $u$ and $v$.

4 Reflection coefficient

It is natural to want to get rid of the measure $\rho(s)$ and find the realization of operators $P$ and $b$ in usual $L_2(\mathbb{R})$. It is clear, that such a realization can be done only in a subspace. We shall show, that the corresponding projector looks like

$$\Pi = \frac{1}{2}(I + KS),$$

where $K$ is a simple reflection

$$KF(s) = F(-s),$$

and $S$ is an operator of multiplication by a function $S(s)$ such that

$$\overline{S(s)} = S(-s) = S^{-1}(s).$$

We call this factor the reflection coefficient.

The explicit calculation of $S(s)$ is based on factorization

$$1/\rho(s) = M(s)M(-s),$$

where

$$M(s) = \text{const} e^{-2i\pi(s-\omega'')}\gamma(2s - \omega''),$$

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with constant which guarantees the relation

\[ M(s) = M(-s), \]

with the help of relations (11), (13) from the Appendix. The explicite value of this constant is inessential for us in what follows.

From the functional relation (7) it follows

\[ M(s + \omega') = M(s) \frac{1}{i}(Z - Z^{-1}) \]
\[ M(s - \omega') = M(s)i(q^{-1}Z - qZ^{-1})^{-1}. \]

Let us define an operator

\[ V = U \frac{1}{M(s)}. \]

The image of \( V \) consists of functions \( F(s) \) satisfying condition

\[ F(-s) = S(s)F(s), \]

where

\[ S(s) = \frac{M(s)}{M(-s)}, \]

and thus belonging to subspace

\[ \Pi L_2(\mathbb{R}), \]

which we shall take as a natural Hilbert space for the zero mode \( P \).

Operator \( b^{-1} = q^{1/4}u^{1/2}v^{-1/2} \) is transformed by \( V \) as follows

\[ b^{-1}V = U(r - r^{-1})(Z - Z^{-1})^{-1} \frac{1}{M(s)} = \]
\[ = V(r^{-1} + \frac{1}{Z - Z^{-1}}r\frac{1}{Z - Z^{-1}}). \]

The formula in the RHS was proposed in [4] using the quasiclassical considerations. Here we get its quantum derivation.

To conclude: we have defined a natural quantum Hilbert space for the zero modes \( P \) and \( Q \).

In papers [14], [15] the alternative realization was proposed with the reflection coefficient expressed in terms of classical \( \Gamma \)-function. The factorization of the measure \( \rho(s) \) [5] makes sense also for this reflection coefficient. It will be interesting to make a more close contact between the two approaches.
5 Evolution

In [4] it was shown, that under discrete shift of time
\[ t \to t + \pi \]
the zero modes \( P \) and \( Q \) transforms elementary
\[ P \to P, \quad Q \to P + Q. \]
This is connected with the fact that this evolution is product of those for left and right motion and oscillator degrees of freedom do not change.

For the variables \( u \) and \( v \) this evolution is given by
\[ u \to u + v, \quad v \to u^{-1}v(u + v)^{-1}. \]
We shall assume that this is true also in quantum case with already prescribed order of factors. We look for the corresponding evolution operator
\[ K^{-1}uK = u + v, \quad K^{-1}vK = u^{-1}v(u + v)^{-1}. \]
It is easier to find the operator \( K^{-1} \)
\[ (u + v)K^{-1} = K^{-1}u, \quad Ku^{-1}vK^{-1} = K^{-1}vu, \]
where we simplified the second equation with proper order of factors.

It follows from the function equation (7) that \( K^{-1} \) is an integrable operator with kernel
\[ K^{-1}(x, y) = e^{2\pi ixy} \gamma(x - y - \omega''). \]
Let us show, that \( \psi(x, s) \) is its eigenfunction. Consider
\[ K^{-1}\psi = \int e^{2\pi ixy} \gamma(x - y - \omega'')\gamma(y + s - \omega'')\gamma(y - s - \omega'')e^{-i\pi(y-\omega'')^2}dy = I(x, s). \]
The integral in the RHS can be explicitly calculated with the use of a particular variant of so called \( \beta \)-integral (see e.g. [13]).

Begin by rearranging the expression for \( \psi(x, s) \) using the reflection property
\[ \psi(x, s) = c(s)\frac{\gamma(x + s - \omega'')}{\gamma(s - x + \omega'')}e^{2\pi isx}, \]
where
\[ c(s) = \beta e^{-is^2}. \]

Then use the integral identity (15)
\[ \frac{\gamma(x - y - \omega'')}{\gamma(s - y + \omega'')} = \frac{c}{\gamma(s - x + \omega'')} \int \frac{\gamma(s - x + \omega'' + z)}{\gamma(z + \omega'')} e^{2\pi iz(x-y)} dz. \]

for two factors in the integral \( I(x,s) \) and formula (17) for the integral of the last factor
\[ \int \gamma(y + s - \omega'')e^{2\pi i(x-z-s)} dy = c \frac{1}{\gamma(z - x + s + \omega'')}. \]

Factors \( \gamma(s - x + \omega'' - z) \) cancel and as result
\[ I(x, s) = c^2 c(s) \frac{e^{2\pi is^2}}{\gamma(s - x + \omega'')} \int \frac{e^{2\pi iz(x+s)}}{\gamma(z + \omega'')} dz = c^2 e^{2\pi is^2} \psi(x, s), \]

with the help of (16).

Thus Kashaev function \( \psi(x, s) \) is an eigenfunction of operator \( K^{-1} \) with the eigenvalue \( c^2 e^{2\pi is^2} \). The constant factor \( c^2 \) can be omitted, so that the evolution operator \( K \) in our representation is simply the multiplication by \( e^{-2\pi is^2} \) so that
\[ K^{-1} Z K = Z, \quad K^{-1} r K = e^{2\pi is^2} e^{-2\pi i(s+\omega')^2} r = q^{1/2} e^{i\pi s/\omega} r = q^{1/2} Z r. \]

Exactly this evolution was obtained in [4].

6 Conclusion

It is shown, that the natural inclusion of the zero modes of the Liouville model into the monodromy matrix of the Lax operator \( L_1 \) gives the description of the Hilbert space and the evolution operator for the quasimomentum \( P \).

7 Appendix

The properties of the modular quantum dilogarithm are given in detail in the paper [13] as well as in many other places. In this paper we used the following properties besides the fundamental equation (7):
The reflection condition
\[ \gamma(z)\gamma(-z) = \beta e^{i\pi z^2}, \quad \beta = e^{\frac{2\pi}{\tau} (\gamma + \frac{1}{\tau})}. \tag{11} \]

The description of the first pole
\[ \gamma(z + \omega'') = \frac{c}{z}, \quad c = -\frac{1}{2\pi i \beta} e^{-i\pi/4} \tag{12} \]

“Reality”
\[ \overline{\gamma(z)} = \frac{1}{\gamma(\bar{z})} \tag{13} \]

Asymptotics
\[ \gamma(z) \to 1 \tag{14} \]

in the sector \(-\pi/4 < \text{arg} \ z < \pi/4\).

Integral identity
\[ \frac{\gamma(t + a)}{\gamma(t + b)} = \frac{c}{\gamma(b - a - \omega'')} \int \frac{\gamma(b - a - \omega'' + z)}{\gamma(z + \omega'')} e^{2\pi i z(t + a + \omega'')} dz, \tag{15} \]

its limit for \(b \to \infty\)
\[ \gamma(t + a) = c \int \frac{e^{2\pi i z(t + a + \omega'')}}{\gamma(z + \omega'')} dz \tag{16} \]

and its inversion
\[ \int \gamma(t + a)e^{-2\pi i s} dt = c \frac{e^{2\pi i s(a + \omega'')}}{\gamma(s + \omega'')}. \tag{17} \]

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