Magnetized SASI: its mechanism and possible connection to some QPOs in XRBs

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ABSTRACT

The presence of a surface at the inner boundary, such as in a neutron star or a white dwarf, allows the existence of a standing shock in steady spherical accretion. The standing shock can become unstable in 2D or 3D; this is called the standing accretion shock instability (SASI). Two mechanisms – advective-acoustic and purely acoustic – have been proposed to explain SASI. Using axisymmetric hydrodynamic and magnetohydrodynamic simulations, we find that the advective-acoustic mechanism better matches the observed oscillation time-scales in our simulations. The global shock oscillations present in the accretion flow can explain many observed high frequency (\(\gtrsim 100\) Hz) quasi-periodic oscillations (QPOs) in X-ray binaries. The presence of a moderately strong magnetic field adds more features to the shock oscillation pattern, giving rise to low frequency modulation in the computed light curve. This low frequency modulation can be responsible for \(\sim 100\) Hz QPOs (known as hHz QPOs). We propose that the appearance of hHz QPO determines the separation of twin peak QPOs of higher frequencies.

Key words: accretion, accretion discs – hydrodynamics – instabilities – magnetic fields – MHD – shock waves.

1 INTRODUCTION

Spherically symmetric steady-state accretion of adiabatic gas on to a point mass that can accrete the supersonically infalling gas (e.g. a black hole) is characterized by the classical transonic solution given by Bondi (Bondi 1952). On the other hand, if the central accretor has a surface that accretes very slowly, a standing shock may form within the sonic radius (McCrea 1956). For a detailed discussion, see section 5.1 of Dhang, Sharma & Mukhopadhyay (2016, hereafter Paper I).

The standing shock is stable in 1D under radial perturbations, but is unstable in 2D. The shock structure oscillates with \(l = 1\) and higher order modes (axisymmetric sloshing modes). In the context of supernovae, Herant et al. (1994) advocated convective instability as the possible mechanism behind the oscillations of the stalled shock front. But, Foglizzo, Scheck & Janka (2006) showed that in presence of advection in the post-shock region, negative entropy gradient is no longer a sufficient condition for convective instability; advection acts as a stabilizing factor. The shock instability exists even in the absence of an entropy gradient (Blondin, Mezzacappa & DeMarino 2003; Dhang et al. 2016). Blondin et al. (2003) named this instability as standing accretion shock instability (SASI) and identified advective-acoustic feedback (Foglizzo 2002) as its possible mechanism. Later, Blondin & Mezzacappa (2006) attributed SASI to a purely acoustic cycle, and thus triggering the debate on the physical origin of SASI. Some other studies reached divergent conclusions. While studies of Ohnishi, Kotake & Yamada (2006) and Scheck et al. (2008) identified advective-acoustic cycle as the possible mechanism, Laming (2007) in his analytical studies claimed that both advective-acoustic and purely acoustic cycles can be possible depending on the ratio of the shock radius to the inner radius.

In 3D, in addition to these axisymmetric modes, SASI also shows a non-axisymmetric spiral mode \((m = 1)\) (Blondin & Mezzacappa 2007). Fernández (2010) interpreted spiral modes as the combination of two sloshing modes, whereas Blondin & Shaw (2007) showed that sloshing modes can be constructed by combining two equal and opposite non-axisymmetric spiral modes. According to Kazeroni, Guilet & Foglizzo (2016), spiral modes dominate the dynamics of SASI only if the ratio of the initial shock radius to the neutron star radius exceeds a critical value. Otherwise, dynamics is dominated by the sloshing mode. The actual mechanism behind the shock instability is still not fully understood.

SASI has been studied extensively in the context of stellar collapse simulations over the years including different aspects of physics (e.g. neutrino transport, cooling, rotation, magnetic fields). There are state of the art realistic simulations (Burrows et al. 2006; Bruevich et al. 2006; Marek & Janka 2009) in which neutrino transport, self-gravity of stellar gas, and nuclear equation of

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state are considered. Also, there are simplified planar toy models of SASI without any extra physics (Foglizzo 2009; Sato, Foglizzo & Fromang 2009). Models of SASI considering the angular momentum of the infalling gas are markedly different from models without angular momentum (Blondin & Mezzacappa 2007). Spiral modes become more prominent relative to sloshing modes in presence of rotation both in linear (Yamasaki & Foglizzo 2008) and in non-linear regime (Iwakami et al. 2009). Endeve et al. (2010) and Endeve et al. (2012) explored the effects of a weak magnetic field in the absence and presence of rotation both in axisymmetric and non-axisymmetric simulations. While axisymmetric models give magnetic field amplification of the order of 2, non-axisymmetric models provide an amplification of the order of 4. They also observe that magnetic field beyond a certain strength stabilizes SASI.

As discussed earlier, different studies reached divergent conclusions by inspecting the linear properties of eigen modes, including the fundamental mode and its harmonics. In this paper, we study the physics of SASI in the non-linear regime using numerical simulations and try to shed some light on its mechanism by two different approaches: (i) by changing the ratio of the shock radius to the inner radius in hydrodynamic (HD) simulations; and (ii) by changing the magnetic field strength in magnetohydrodynamic (MHD) simulations. If SASI is an outcome of a meridional acoustic cycle, the weak magnetic field in the downstream region close to the shock should not affect the oscillation time-scales. On the other hand, a somewhat stronger magnetic field close to the centre can affect the radial advective-acoustic cycle (Guilet & Foglizzo 2010).

In Paper I, using our HD axisymmetric simulations, we proposed that SASI in accretion flows may give rise to some of the quasi-periodic oscillations (QPOs) observed in the light curves of X-ray binaries (XRBs). Most of the proposed QPO mechanisms are based on the physics of test particle motion (e.g. Strohmayer et al. 1996; Miller, Lamb & Psaltis 1998; Stella & Vietri 1999; Kluzniak & Abramowicz 2002; Kluzniak et al. 2004; Mukhopadhyay 2009), which is not affected by pressure and magnetic fields. However, for a particular model, the QPO frequencies obtained considering bulk motion significantly differ from the ones corresponding to free particles (Blaes et al. 2007). Along with our model, there are few models (e.g. Kato & Fukue 1980; Kato 1990; Ipser & Lindblom 1991; Ryu et al. 1995; Yang & Kafatos 1995; Molteni, Sponholz & Chakrabarti 1996; Chakrabarti & Manickam 2000; Wagoner, Silbergleit & Ortega-Rodriguez 2001; Mukhopadhyay et al. 2003) where bulk motion of the flow is considered to explain the origin of QPOs. In reality, accreting matter around a compact object has angular momentum and is magnetized. As a first step, here we incorporate magnetic field and explore the origin of QPOs appearing in the light curve due to SASI in a magnetized accreting medium. This will help to understand the sole effect of magnetic field on SASI and QPOs. Our particular emphasis is QPO frequencies ≥ 100 Hz in XRBs, the origin of which is still not understood. We show that the presence of magnetic fields, hence magnetized SASI, appears to uncover some of the important characteristics of QPOs. In other words, the inclusion of magnetic fields introduces important physics in the SASI model to predict certain QPOs, which is absent in an unmagnetized case.

The paper is organized as follows. In Section 2, we briefly discuss the two different mechanisms proposed to explain SASI. In Section 3, we describe the physical set-up and the solution method. In Section 4, we qualitatively discuss the effects of a split-monopolar magnetic field on steady Bondi accretion. In Section 5, we describe the results obtained from our numerical simulations. In Section 6, we discuss the possible mechanism behind SASI and its astrophysical implications (in particular, QPOs), and summarize in Section 7.

2 WHAT IS SASI AND WHY?

Two different mechanisms, namely advective-acoustic and acoustic mechanisms, have been proposed to explain SASI. Most recent studies (e.g. Foglizzo et al. 2007; Foglizzo 2009) favour the former. For a comparative and detailed discussion of the two mechanisms, see Guilet & Foglizzo (2012).

2.1 Advective-acoustic cycle

Advective-acoustic cycle was first proposed by Foglizzo & Tagger (2000) in the context of Bondi–Hoyle–Lyttleton accretion. Two different waves – an outward propagating acoustic wave and an inward propagating entropy-vorticity wave – contribute to this mechanism and complete a single cycle (Foglizzo 2002, Foglizzo et al. 2007). Due to the compression of gas in the post-shock region (specially near the surface of neutron star), an acoustic wave is produced. The acoustic wave (propagation direction need not be purely radial) reaching the shock surface distorts it. The distortion of the shock surface, in turn, creates entropy-vorticity wave that advects down to the central neutron star and decelerates near the surface. Deceleration creates a positive acoustic feedback that completes the cycle. Over many cycles, the instability attains an exponential growth. With appropriate boundary conditions (like ours in this paper), the system reaches a quasi-steady state with stable non-linear oscillations.

2.2 Acoustic cycle

Acoustic cycle is thought to be driven by a trapped acoustic wave in the post-shock cavity. Blondin & Mezzacappa (2006) proposed that any density inhomogeneity produces sound waves near the shock surface. Due to refraction, these sound waves propagate around the circumference of the shock until they meet on the other side. There their excess pressure produces a shock deformation that sends another pair of sound waves back again. The growth of the mode depends on how pressure perturbation in the post shock region interacts with the shock front.

A comparison of sonic and advection time-scales should help us to distinguish these two mechanisms.

3 METHOD

To study SASI, we set up an initial value problem, in which a central accretor (e.g. a neutron star) is embedded in a stationary, spherically symmetric uniform medium. We solve the MHD equations to study the problem.

3.1 Equations solved

We use the PLUTO code (Mignone et al. 2007) to solve the Newtonian MHD equations in spherical coordinates (r, θ, φ). The equations are

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) = 0, \quad (1)$$

$$\frac{\partial}{\partial t} (\rho \mathbf{v}) + \nabla \cdot (\rho \mathbf{v} \mathbf{v} - \mathbf{BB}) = -\rho \nabla \Phi - \nabla P^*, \quad (2)$$

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\[ \frac{\partial E}{\partial t} + \nabla \cdot ((E + P^*) v - B(B) v) = -\rho \nabla \Phi \cdot v, \]  
(3)

\[ \frac{\partial B}{\partial t} + \nabla \cdot (\varepsilon B - B v) = 0, \]  
(4)

where \(\rho\) is the gas density, \(v\) is the velocity, \(B\) is the magnetic field (a factor of \(1/\sqrt{4\pi}\) is absorbed in the definition of \(B\)), \(P^* = P + B^2/2\) is the total pressure (\(P\) is gas pressure), and \(E\) is the total energy density related to the internal energy density \(e\) as \(E = e + \rho v^2/2 + B^2/2\). The adiabatic index \(\gamma\) relating pressure and internal energy density \((P = \gamma e - 1)e\) is chosen to be 1.4. Gravitational potential due to the central accretor is given by the Newtonian potential due to a point mass at the origin, \(\Phi = -GM/r\).

PLUTO uses a Godunov-type scheme, which solves the equations in conservative form. We use the HLLD solver with second-order slope limited reconstruction. For time-integration, second-order Runge–Kutta (RK2) is used with a CFL number of 0.4. Divergence free constraint on magnetic field is enforced by solving a modified system of conservation laws, in which the induction equation is coupled to a generalized Lagrange multiplier (Dedner et al. 2002; Mignone & Tzeferacos 2010). In this scheme, magnetic fields retain a cell centred representation.

We solve equations (1)–(4) in dimensionless form, we express our results in both code units (e.g. time-scales are expressed in units of \(r_g/c\), \(r_g = GM/c^2\)) and in CGS units. In the latter case, we use the central compact object mass to be 1 M_{\odot}. It is straightforward to convert from one system of units to another.

3.2 Grid and boundary conditions

Our spherical computational domain \((r, \theta, \phi)\) extends from an inner boundary \(r_g = 6r_g\) to an outer boundary \(r_{out} = 10^7r_g\) in the radial direction and from 0 to \(\pi\) in the meridional \((\theta)\) direction. Here, \(r_g = GM/c^2\) is the gravitational radius, where \(G\) is gravitational constant and \(M\) is mass of the central accretor. We use two logarithmic grids along radial direction, one from \(r_{in}\) to \(50r_{in}\) with 512 grid points and another from \(50r_{in}\) to \(r_{out}\) with 256 grid points. In the meridional direction, we use a uniform grid with 256 grid points.

We fix the values of velocity components at the inner boundary; radial component \(v_r\) is set to \(v_{in}\), whereas meridional component \(v_\theta = 0\) (we obtain similar results even if \(v_\theta\) is copied in the inner radial ghost zones). The fiducial value of \(v_{in}\) is 0.05c, but we change it to control the equilibrium shock radius. Density, pressure, and magnetic field components in the ghost zones are copied from the last computational zone near the inner boundary. At the outer boundary, the values of pressure, density, and velocity field components are set to their initial values. The values of magnetic field components in the outer ghost zones are copied from the last computational zone. Axisymmetric boundary conditions (scalars and tangential components of vector fields are copied and normal components of vector fields are reflected) are used at both the \(\theta\) boundaries (\(\theta = 0, \pi\)).

3.3 Initial conditions

We carry out 2D, axisymmetric MHD simulations in spherical \((r, \theta, \phi)\) co-ordinates in an initially static \((v_r = v_\theta = 0)\) uniform ambient medium of density \(\rho_{\text{ini}}\). Initial pressure of the medium is also uniform and is given by \(P_{\text{ini}} = \rho_{\text{ini}}c_{\text{ini}}^2\). We choose the value of \(c_{\text{ini}}^2\) to be 0.002\(c_\odot^2\) to mimic the typical proton temperature \((\approx 10^{11}\text{K})\) of the sub-Keplerian hot flow in XRBs (Narayan & Yi 1995; Rajesh & Mukhopadhyay 2010). Moreover, this choice of temperature gives rise to a sonic radius \(r_s \approx 71.43r_g\) and the Bondi radius \(r_B \approx 714r_g\), which are well inside the computational domain. We initialize a split monopolar magnetic field given by

\[ B_r = \frac{C}{r^2} \text{sign} (\cos \theta). \]

The advantage of using this magnetic field configuration is that the flow structure is expected to change only close to the central accretor (i.e. at small \(r\) where the field is strong), whereas at larger radii the solution remains unaffected (see Section 4). The strength of magnetic field is determined by the value of the constant \(C\).

4 BONDI ACCRETION WITH SPLIT-MONOPOLAR FIELD

Before discussing the simulation results, we want to investigate the effects of magnetic field configuration given in equation (5) on the standard Bondi accretion. Taking the spherically symmetric form of equations (1) and (2) and using a polytropic equation of state, \(P = K \rho^\gamma\) (\(K\) is a constant related to entropy), and rearranging, we get the following set of equations

\[ \frac{\partial \rho}{\partial t} = \frac{\partial}{\partial r} \left( \frac{\rho v}{\gamma - 1} \right), \]
\[ \frac{\partial v}{\partial t} = \frac{\partial}{\partial r} \left( \frac{v^2}{\gamma - 1} \right) - \frac{\partial}{\partial r} \left( \frac{\rho v^2}{\gamma - 1} \right), \]
\[ \frac{\partial E}{\partial t} + \frac{\partial}{\partial r} \left( \rho v c_s^2 \right) = 0, \]
\[ \frac{\partial B}{\partial t} + \frac{\partial}{\partial r} \left( \varepsilon B v \right) = 0, \]

where \(n = 1/(\gamma - 1)\) and \(c_s(r)\) is the adiabatic sound speed given by \(c_s = \sqrt{\gamma P/\rho}\).

Equation (6) has a critical point (sonic point) where \(c_s = v\). The location of the sonic point \(r_s\) can be obtained if we set the numerator of equation (6) to zero to avoid divergence, namely

\[ r_s = \frac{GM}{2c_{\text{sh}}^2}, \]

where \(c_{\text{sh}}\) is sound speed at the critical point. Note that the expression for \(r_s\) is identical to the HD Bondi solution. So the presence of a split-monopolar magnetic field does not affect the steady spherically symmetric accretion solution. Physically, the current is concentrated in the equator where the field vanishes and therefore \(J \times B\) force vanishes everywhere. But if spherical symmetry is broken, as it happens due to SASI, magnetic fields will have an effect especially at smaller radii where the field strength is large.

5 RESULTS

In this section, we present results from our simulations with and without magnetic fields. We begin with results in the HD limit.

5.1 HD

To study SASI in the HD regime, we choose \(C\) in equation (5) to be very small such that the terms involving magnetic field in equations (2) and (3) vanish. We run simulations to study unmagnetized SASI with five different radial velocities imposed at the inner radial ghost zones \((v_{in};\text{see Table 1})\). We change \(v_{in}\) to control the mean shock radius \(r_{sh}\), a larger value of \(v_{in}\) gives rise to smaller \(r_{sh}\) (for details see section 5.1 of Paper I). This way we can study SASI for different values of \(r_{sh}/r_{in}\).

5.1.1 Flow evolution

Fig. 1 shows the density snapshots at different times for our fiducial run of unmagnetized SASI. The details of flow evolution in an
unmagnetized medium are described in Paper I; here, we only give a brief description. We can divide the time evolution into three phases: the early non-equilibrium phase, the intermediate transition phase, and the final quasi-stationary oscillating non-linear phase. At $t = 0$, the ambient medium is uniform and static. As the central gravitating object starts accreting, matter attains supersonic velocity. Both density and pressure build up near the accretor. Unlike classical Bondi accretion, here the supersonic matter falling under gravity feels an obstruction at the inner boundary as the radial velocity there is fixed.

The accretion shock can be easily seen at $t = 2685.49 \, r_g/c$. With time, thermal pressure builds up behind the shock due to the conversion of kinetic energy to thermal energy and shock surface starts expanding. The initial expansion is purely radial, but with time the radial expansion is accompanied by non-spherical global oscillations with $l = 1$ and higher order modes. This can be seen in the snapshots at $t = 3919.36 \, r_g/c$, $3991.94 \, r_g/c$, and $4173.39 \, r_g/c$. As the shock becomes aspherical, it becomes oblique, resulting in the generation of meridional component of velocity ($v_\theta$) in the post-shock region (see the change in direction of velocity arrows in the post-shock region for the snapshots at and after $t = 3919.36 \, r_g/c$), as the mass flux ($\rho v_z$) and the tangential component of velocity ($v_\phi$) have to be conserved across the shock. Due to the build up of thermal pressure, the shock overcomes the inward gravitational pull and the post-shock cavity expands out (see snapshots at $t = 5733.88 \, r_g/c$, $5879.04 \, r_g/c$, $6024.20 \, r_g/c$). With the advection of mass and thermal energy across the inner boundary, after a few adjustments the systems attains a quasi-stationary state, in which the inward gravitational pull is balanced by outward thermal pressure. In this state, the post-shock cavity incessantly oscillates about the equatorial plane (the last five panels in Fig. 1 show one full oscillation period).

The equilibrium standing shock is linearly unstable to aspherical SASI modes but non-linearly the systems settle into stable, long-lived, large-amplitude oscillations. The effective potential for such oscillations can be thought of as a local maximum within a stable potential well experienced at large amplitudes.

### 5.1.2 Mode analysis

Shock surface can be easily identified just by looking at the density jumps in different snapshots of Fig. 1. We see that in the non-linear quasi-stationary state, shock surface can be considered as a sphere with sub-structure on top of it. To quantify the sub-structures, we perform mode analysis using the method of spherical harmonics decomposition (SHD).

Any spherical function $f(\theta, \phi)$ can be expanded as a linear combination of spherical harmonics as

$$f(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} a_{lm} Y_{lm}(\theta, \phi),$$

where the spherical harmonics $Y_{lm}(\theta, \phi)$ are given by

$$Y_{lm}(\theta, \phi) = \sqrt{\frac{(2l+1)(l-|m|)!}{4\pi(l+|m|)!}} P_m(\cos \theta) e^{im\phi},$$

where $P_m(\cos \theta)$ are the associated Legendre polynomials. For an axisymmetric system, $m = 0$ and $Y_{00}$ reduces to Legendre polynomial $P_0(\cos \theta)$ with a normalization factor. Then, the deformed (from spherical shape) shock surface $R_s(\theta)$ can be decomposed as

$$R_s(\theta) = \sum_{l=0}^{\infty} a_l P_l(\cos \theta).$$
Figure 1. Density snapshots of the unmagnetized fiducial run at different times. Arrows represent the direction of velocity. Top left-hand panel shows the initial uniform density distribution. Due to accretion of matter from the surrounding medium, both density and pressure increase, which is shown in the second panel. Unlike the moderately magnetized case (cf. Fig. 7), the shock appears at a later time. The snapshot at $t = 2685.49$ (where $t$ is in units of $r_g/c$) shows the first development of the shock surface. The next four panels (from $t = 3266.13$ to $t = 4173.39$) show radial expansion of the shock surface as well as the initial buildup of the vertical oscillation modes. Then, the post-shock cavity goes through a vigorously oscillating phase (snapshots at $t = 5733.88$, $5879.04$, and $6024.20$). Finally, the system enters a quasi-stationary non-linear phase in which the post-shock cavity oscillates with a definite time period. The last five panels show a full period of coherent oscillations of global modes.

where the coefficients can be calculated as

$$a_l = \frac{2l + 1}{2} \int_0^{\pi} R_l(\theta) P_l(cos \theta) \sin \theta d\theta.$$  \hspace{1cm} (12)

Fig. 2 shows the time evolution of mode amplitudes $a_l$ for the fiducial unmagnetized SASI run. $R_l(\theta)$ is computed using the pressure jump across the shock. Initially, the vanishing mode amplitudes reflect the absence of a shock. The first emergence of shock is reflected in the non-vanishing value of $a_0$, while other mode amplitudes are still zero, as the shock is spherical. As the shock starts oscillating vertically about the equatorial plane, it becomes aspherical in nature and $l = 1$ and 2 modes become prominent. In the fully non-linear regime, we see that apart from $l = 0$ mode, $l = 1$, 2, and 3 are the most prominent modes present. The higher order modes (specially $l \leq 8$) are also present but with a smaller amplitude.

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5.1.3 Methods to measure SASI time period

It is clear from Fig. 1 that there are global non-linear oscillation modes associated with the post-shock cavity. We want to determine the time period of oscillations. We use two different methods to find the precise oscillation period:

(i) Following Ohnishi et al. (2006), we fit the mode amplitude (in quasi-steady state) associated with $l = 1$ with a sin $e$ curve given by $\psi_1 = A_1 \sin(\omega_l t + \phi_1)$. Time period is obtained from the value of $\omega_l$ as $T_{a_l} = 2\pi/\omega_l$. Top panel of Fig. 3 shows the temporal variation of $a_1$ for our fiducial unmagnetized SASI run ($v_{in} = 0.05c$, $C = 1$). After the initial growing phase, $a_1$ attains a quasi-steady state and oscillates about a mean value close to 0. In the bottom panel of Fig. 3, the simultaneous plots of $a_1$ and the fitting function $\psi_1$ are shown. The original data and the fitting function match well and the measured time period of SASI is $T_{a1} = 580.24$ $rg/c$.

(ii) The second method to obtain the time period of oscillations is based on calculating the temporal variation of a local quantity at a single point in space. We choose $v_g$ to be the local quantity because $v_g$ changes sign as the post-shock cavity goes from the upper hemisphere to the lower hemisphere. We compute $v_g(t)$ in the equatorial plane ($\theta = \pi/2$) at $r = 8rg$ as a function of time, which is shown in the top panel of Fig. 4 for the fiducial HD run. Note that $v_g(t)$ is not a purely sinusoidal function (see the inset in top panel of Fig. 4). To find the time period associated with it, we take the fast Fourier transform (FFT) and identify the most prominent peak in the power spectrum (defined here as simply the absolute value of the Fourier transform). To find the centroid frequency ($f_{v_g}$), we fit the prominent peak with a Lorentzian given by

$$
P(f) = \frac{A_k \Gamma}{(f - f_{v_g})^2 + \Gamma^2/4},
$$

\begin{equation}
(13)
\end{equation}

where $A_k$ is the normalization and $\Gamma$ is the full width at half-maximum. Inverse of $f_{v_g}$ gives the oscillation period ($T_{v_g}$) measured by this method. Bottom panel of Fig. 4 shows the power spectrum of $v_g(t)$; the fitting function $P(f)$ is plotted on top of it to compare the actual and fitted values of the power spectrum. The time period of the oscillations obtained by this method is $T_{v_g} = 581.62$ $rg/c$. We note that both methods (i) and (ii) give almost identical results.

5.1.4 Timescales from linear theory

We measure the time period of oscillations in the quasi-steady, non-linear phase with large amplitude. Linear theories of SASI predict important time-scales related to the propagation of various perturbations. While not strictly valid, the various signal propagation time-scales are expected to provide an appropriate scaling even...
for the non-linear oscillations. The following arguments based on simple signal propagation time-scales stem from the fact that the disturbances have to reflect and travel back to the origin of waves to interfere and create a standing wave. In some mechanisms, mode conversion (e.g. from acoustic to vorticity/entropy modes and vice versa) is invoked at the boundaries.

First, we define an advective-acoustic time-scale (Foglizzo et al. 2007) as the sum of the radial advection time from shock surface to the inner boundary and the acoustic time to return back to the shock surface in radial approximation

\[ t_{aac} = \int_{r_{in}}^{r_{sh,0}} \frac{dr}{|v_{r}(r)|} + \int_{r_{sh,0}}^{r_{sh,1}} \frac{dr}{|\bar{v}_{r}(r) - |v_{r}(r)||}, \]

(14)

where \( \bar{v}_{r}(r) \) is the \( \theta \)-averaged (throughout the paper we use an overline to represent angle average) radial velocity within the shock and the integrals are performed within the shock, in the sense that (the following also applies to the other radial time-scales that follow; cf. equation 15)

\[ \bar{v}_{r}(r) = \frac{\int_{0}^{\pi} H (v_{r}(r, \theta) \sin \theta) \, d\theta}{\int_{0}^{\pi} H \sin \theta \, d\theta}, \]

where, \( H = H(r_{sh}(\theta) - r) \) is the Heaviside step function whose value is zero for negative argument and one for positive argument. Here, we want to emphasize that there are two shock surfaces at certain times (e.g. see snapshot at \( t = 23,915.35 \) \( r_{g}/c \) in Fig. 1), and for calculating the advection time (or any time associated with signals propagating inward), we compute the time taken by the fluid element to reach inner boundary \( r_{in} \) from the maximum outer shock radius \( r_{sh,0} \). But for calculating the acoustic time (or any time associated with outward-propagating signals), we compute the time taken by the outward-propagating sound wave to reach the maximum inner shock radius \( r_{sh,1} \) from the inner boundary \( r_{in} \), as acoustic signals cannot propagate outside the shock at \( r_{sh,1} \).

The time-scales vary with time because of the finite amplitude of the shock oscillations, but average time-scales should be indicative of the fundamental mode.

Secondly, we compute the radial acoustic time-scale, sum of the times taken by the sound waves to reach the shock surface from the inner boundary and back,

\[ t_{racs} = \int_{r_{in}}^{r_{sh,0}} \frac{dr}{|\bar{v}_{r}(r) + |v_{r}(r)||} + \int_{r_{sh,0}}^{r_{sh,1}} \frac{dr}{|\bar{v}_{r}(r) - |v_{r}(r)||}, \]

(15)

Thirdly, we compute the meridional acoustic time-scale (Blondin & Mezzacappa 2006), considering the propagation of sound wave along the circumference of the shock,

\[ t_{mcs} = \int_{0}^{\pi} \frac{r_{sh}(\theta) \, d\theta}{c(\bar{v}_{r}(\theta))}. \]

(16)

Fig. 5 shows the plots of the observed time period \( T_{\Delta 1} \) (since \( T_{\Delta 1} \) and \( T_{\Delta 2} \) are almost identical, we plot only \( T_{\Delta 1} \)) with the above time-scales obtained from linear theory for the fiducial unmagnetized SASI simulation. In the quasi-steady state, the theoretical time-scales oscillate in time with a large amplitude (80–300 \( r_{g}/c \)). While both the advective-acoustic time \( t_{aac} \) and meridional acoustic time \( t_{mcs} \) contain \( T_{\Delta 1} \) within their range of variations, radial acoustic time \( t_{racs} \) is shorter.

As the variations in time-scales are large, we take the time average between \( t = 50,000 \) \( r_{g}/c \) and 82,000 \( r_{g}/c \). The time averaged values of advective-acoustic scales (\( t_{aac} = 567.41 \) \( r_{g}/c \) ) and meridional acoustic time-scales (\( t_{mcs} = 567.81 \) \( r_{g}/c \)) are close to the observed time period \( T_{\Delta 1} = 580.24 \) \( r_{g}/c \). It is a coincidence that \( t_{aac} \) and \( t_{mcs} \) are so close. Note that according to Blondin & Mezzacappa (2006), the time period of SASI oscillations is expected to be \( 2t_{mcs} \) (so that the two waves originated at one point near the shock surface can interfere on the other side and return back to the origin), whereas we find a close match of \( t_{mcs} \) to the measured SASI time period. On the contrary, the time averaged value of radial acoustic time is \( t_{racs} = 368.03 \) \( r_{g}/c \), much less than the SASI time period. So there appears to be a degeneracy between the two time-scales \( t_{aac} \) and \( t_{mcs} \) derived from two different physical mechanisms, namely advective-acoustic and purely acoustical cycles.

To break this degeneracy (between \( t_{aac} \) and \( t_{mcs} \)) because of our choice of parameters, we change the shock location by tuning the value of \( r_{in} \) and measure the oscillation period as well as the relevant time-scales. Top panel of Fig. 6 shows the time averaged values of velocity oscillation time period \( (T_{v_{\theta}}, \text{described in (ii) in Section 5.1.3}) \), the advective-acoustic time \( (t_{aac}) \), and the meridional acoustic time \( (t_{mcs}) \) as a function of \( r_{in} \). The bottom panel of Fig. 6 shows the absolute value of the difference between different relevant time-scales (\( |\Delta_{v_{\theta}, \text{aac}}| = |T_{v_{\theta}} - t_{aac}|; |\Delta_{v_{\theta}, \text{mcs}}| = |T_{v_{\theta}} - t_{mcs}|; |\Delta_{\text{aac}, \text{mcs}}| = |t_{aac} - t_{mcs}| \)) – as a function of \( r_{in} \). For smaller \( r_{in} \), the advective-acoustic time \( t_{aac} \) matches the SASI time-period measured by \( T_{v_{\theta}} \). For larger \( r_{in} \), the radial advective-acoustic time-scale is shorter perhaps because of non-radial propagation of sound waves. Also note the closeness between \( t_{aac} \) and \( t_{mcs} \) for \( r_{in} \gtrsim 3.9 \), which makes it harder to choose between the two cycles in this regime.

5.2 MHD

In this section, we present results from our simulations of initially split-monopolar magnetic fields with varying field strengths.

5.2.1 Evolution of the flow

In this section, we discuss the time evolution of the magnetized flow with the radial inward velocity at the inner boundary set to \( v_{in} = 0.05c \), as in the fiducial HD run. We focus on two different cases with moderate and high field strengths.
Magnetized SASI

Figure 6. Top panel: Variation of different average time-scales (namely, advective-acoustic cycle, \( t_{\text{aac}} \); radial acoustic cycle, \( t_{\text{rca}} \); meridional acoustic cycle, \( t_{\text{mc}} \); and SASI time period \( T_{\text{SASI}} \)) in units of \( r_g/c \) with change in the ratio of the mean shock radius and the inner radius, \( r_{\text{sh}}/r_g \) for unmagnetized SASI. Bottom panel: Change in the absolute value of difference (\( \Delta \)) between two time-scales with \( r_{\text{sh}}/r_g \): \( \Delta_{\text{sh,aac}} = |T_{\text{sh}} - t_{\text{aac}}| \); \( \Delta_{\text{sh,mcs}} = |T_{\text{sh}} - t_{\text{mcs}}| \); \( \Delta_{\text{aac,mcs}} = |t_{\text{aac}} - t_{\text{mcs}}| \). Note that of the various time-scales, the advective-acoustic time-scale most closely matches the observed SASI time period.

Case I: in which the magnetic field strength is moderate and SASI (identified by coherent shock oscillations) exists; this is the fiducial MHD run with \( C = 5 \times 10^3 \) (see equation 5) marked in Table 1. Case II: in which a strong magnetic field prevents a shock from existing at late times, with \( C = 8 \times 10^3 \).

Fig. 7 shows the density snapshots for Case I. Streamlines show magnetic field lines. At \( t = 0 \), the ambient density is uniform and the magnetic pressure is comparable to the thermal pressure close to the accretor. Like the unmagnetized simulations, the magnetized runs with moderate field strengths go through three phases: an early phase in which a shock develops (top panels in Fig. 7), the intermediate transition period (middle panels in Fig. 7), and a final quasi-stationary phase (bottom panels in Fig. 7). The flow undergoes a very early transient phase (see snapshot at \( t = 115.77 r_g/c \)), during which thermal pressure builds up due to the conversion of gravitational (via kinetic energy) to thermal energy, and the shock surface starts expanding radially outwards (see snapshots at \( t = 385.90 r_g/c \) and \( t = 2006.67 r_g/c \)). Finally, the shock executes coherent oscillations.

Even with a slightly higher magnetic field strength (1.6 times \( C \)), the temporal evolution of Case II is qualitatively different because the shock is absent at late times. Fig. 8 shows the density snapshots for Case II, overplotted with arrows showing the velocity unit vectors. Like in Case I, after undergoing a transient phase (see snapshot at \( t = 126 r_g/c \)), a spherical shock is formed that expands in time (see snapshots at \( t = 588 r_g/c \) and \( 1134 r_g/c \)). However, the shock does not stall but keeps on expanding and becoming weaker, as gravitational pull is unable to balance the outward (thermal + magnetic) pressure. When shock reaches the sonic point \( (r_s \sim 71 r_g/c) \), the flow becomes subsonic and the shock disappears. Eventually, a hydrostatic atmosphere is formed. Snapshots at \( t = 1848 r_g/c \) and at \( 2478 r_g/c \) show the outward propagation of shock, while snapshots at \( t = 7182 r_g/c \) and at \( 10920 r_g/c \) show the flow structure when shock disappears.

Fig. 9 shows the snapshots of plasma \( \beta \) (the ratio of thermal and magnetic pressures) in quasi-steady state for Case I. Plasma \( \beta \) close to the shock surface is \( \lesssim 10^3 \), and therefore magnetic fields are not expected to noticeably change the shock oscillation period if the underlying mechanism for SASI involves meridional propagation of fast MHD waves (generalization of sound waves in the MHD regime), characterized by the meridional acoustic time-scale \( t_{\text{mcs}} \) (equation 16). Later, we shall see that the SASI oscillation frequency in presence of magnetic field changes noticeably (cf. black solid line with square symbols in Fig. 15), ruling out the meridional acoustic mechanism for SASI.

To quantify the magnetic field strength within the shock, we define a volume averaged quantity

\[
\beta_V = \frac{\int_V P dV}{\int_V (B^2/2) dV} = \frac{(\gamma - 1)E_{\text{th},V}}{E_{B_r,V} + E_{B_\theta,V}},
\]

(17)

where the volume \( V \) over which the integral is done extends from inner boundary to \( r = 30r_g \); this radius is well inside the sonic radius \( r_s (=7r_g \) for our parameters; equation 8), and the shock radius \( r_{\text{sh}} \) is always within it. Similarly, \( E_{\text{th},V} \) is the volume averaged thermal energy, and \( E_{B_r,V} \) and \( E_{B_\theta,V} \) are the volume averaged magnetic energies of the radial and meridional components of the magnetic field.

The top panel of Fig. 10 shows the evolution of volume averaged magnetic and thermal energies within \( 30r_g \) (top panel) with time for Cases I and II; bottom panel shows the evolution of \( \beta_V \) (equation 17). In the top panel of Fig. 10 for Case I, during purely radial expansion of post-shock cavity, thermal energy (yellow line) increases rapidly with time due to shock heating, but magnetic energy remains roughly constant because radial flows cannot amplify a radial field. As a result, \( \beta_V \) increases during this phase of evolution. Later, the radial expansion of the shock is accompanied by global oscillations with \( l = 1 \) and higher order modes (see snapshots at \( t = 2662.69 r_g/c \), \( 2739.87 r_g/c \), and \( 2817.05 r_g/c \) in Fig. 7). The turbulent (in transition phase) and oscillatory meridional velocities associated with non-spherical modes amplify magnetic fields at late times. Simultaneous increase of thermal and magnetic pressure causes further expansion of the post-shock cavity (in Fig. 7). see snapshots at \( t = 6097.18 r_g/c \), \( 6251.54 r_g/c \), and \( 6405.90 r_g/c \). Though, both thermal and magnetic energies increase simultaneously, the build up of magnetic energy is more erratic. Eventually, Case I attains a quasi-stationary state, in which both thermal and magnetic energies start oscillating about a mean value.

For Case II, magnetic field amplification happens earlier compared to Case I due to presence of aspherical shock from the very beginning of the flow evolution (see snapshots at \( t = 126 r_g/c \), \( 588 r_g/c \), and \( 1134 r_g/c \) in Fig. 8). Once shock disappears, magnetic energies (both \( E_{B_r,V} \) and \( E_{B_\theta,V} \)) saturate. This early amplification of magnetic field results in a low \( \beta_V \) (close to \( \sim 5 \)) during the flow evolution which, in turn, choking the flow. The temporal evolution of the flow in Case II is equivalent to a hydro set-up with reflective inner radial boundary condition, or more precisely, if \( v_{\text{in}} \) is smaller than the lower limit of velocity (at the inner boundary) for which a stationary shock solution is possible (see Fig. 15 and section 5.1 in Paper I).

5.2.2 Mode analysis

Fig. 11 shows the evolution of mode amplitudes \( a_l \) (measured by decomposing the shock radius into spherical harmonics; see Section 5.2.2) with time for the Case I MHD run. As in HD evolution, \( l = 0 \) is always the dominant mode. But unlike HD, during the very early evolution of the flow (\( t < 400 r_g/c \)), all the even order modes (\( l = 2, 4, 6 \) etc.) are more dominant compared to the odd

\[ \beta = \frac{B^2}{\rho c^2} \]

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Figure 7. Case I: Evolution of the flow with time for the fiducial MHD run. Colour represents the density and streamlines show the magnetic field lines. First panel shows the initial uniform density distribution. A transient phase is shown in second panel. The next two panels (from $t = 385.90$ to $2006.67$) describe growing phase of post-shock cavity. A transition period from an growing phase to an quasi-steady phase of SASI is shown in next six panels (from $t = 2662.69$ to $6405.90$). The last five panels shows a full period of coherent oscillations of global modes. $t$ is in units of $r_g/c$.

modes ($l = 1, 3, 5$ etc.). This can be attributed to the anisotropic nature of the initial transient phase of evolution, which is symmetric about $\theta = \pi/2$ (see snapshot at $t = 115.77 r_g/c$ in Fig. 7). When the post-shock cavity attains an almost spherical shape and starts expanding radially, contribution from even order modes with $l \geq 2$ becomes negligible. As the shock starts oscillating vertically about the equatorial plane, $l = 1$ mode starts to dominate the higher order modes. In the fully non-linear quasi-steady regime, apart from the $l = 0$ mode, $l = 1, 2$ and 3 are the most prominent modes.

Comparing Fig. 11 and Fig. 2, it is very difficult to quantitatively study the differences in modal contribution of the HD and MHD runs. Fig. 12 shows the temporal evolution of $a_0$ (see equation 12), a measure of spherical radius of the aspherical shock, for different magnetic field strengths. At the very early stage of evolution, a
large value of $a_0$ reflects the transient phase at $t = 115 r_g/c$ (see Fig. 7). After the transient phase, a spherical shock emerges, and the value of $a_0$ drops. After showing large fluctuations in $a_0$ in the transition phase, the shock attains a quasi-stationary state with $a_0$ oscillating about a mean value. The inset at top right-hand side shows the zoomed-in view of $a_0(t)$ in steady state. As expected, the average shock radius increases with an increase in the magnetic field strength. Even in the quasi-steady state, $a_0$ does not show sinusoidal variation at a single frequency.

Fig. 13 shows the variation of time-averaged mode amplitude for $l = 0, 1, 2,$ and $3$ with the initial magnetic field strength. For $a_0$ and $a_3$, we take time average in the quasi-steady state. For $l = 1$ and $3$, as $a_1$ and $a_3$ oscillate about a vanishing mean value, we fit the quasi-steady $a_1(t)$ and $a_3(t)$ with a sinusoidal curve, and plot the variation of the amplitudes $A_1$ and $A_3$ with the magnetic field strength. As expected, Fig. 13 shows that a stronger magnetic field suppresses the higher order modes due to higher magnetic tension.

5.2.3 Timescales from linear theory

Any disturbance in HD is carried by either sound waves propagating at $\pm c_s$ relative to the inflow or by entropy/vorticity waves travelling at the local flow velocity. In MHD, the sound wave generalizes to the fast mode and the entropy mode still consists of perturbations in total (thermal+magnetic) pressure balance. However, there are two new modes in MHD: the shear Alfvén wave and the slow magnetosonic waves. Therefore, the advective part of the advective-acoustic cycle is expected to split into five different cycles: an entropy wave,
Figure 9. Snapshots of plasma $\beta (\equiv 2P/B^2)$ at late times for Case I; $t$ is in units of $r_g/c$. The colour scale has been cut off at the minimum and maximum value shown in the colour bar. The inner and equatorial regions have a higher field strength. Close to the mid-plane, one can clearly see an oscillating current sheet.

Figure 10. Top panel: Temporal evolution of average (over the volume $V$ enclosed within $r = 30r_g$) thermal energy $E_{th,V}$ and magnetic energies associated with radial ($E_{B,r,V}$) and meridional ($E_{B,\theta,V}$) components of magnetic field for Case I and Case II. Bottom panel: Temporal evolution of the $\beta_{V}$, the ratio of volume averaged gas pressure ($P$) to magnetic pressure ($B^2/2$) for the same runs. While time averaged $\beta_{V}$ in quasi-stationary state is $\sim 300$ and shock persists, in Case II due to presence of stronger initial magnetic field, the flow gets choked and the final $\beta_{V} \sim 5$ and shock disappears at late times.

Figure 11. Results of mode analysis for the fiducial MHD run. Colour represents the absolute value of mode amplitude. $l = 0$ is the most dominant mode at all time. During the very early transient phase, only even modes ($l = 0, 2, 4, 6$ etc) dominate. After the transient phase, during pure radial expansion of shock, only $l = 0$ mode dominates. As soon as shock-structure starts oscillating in the vertical direction, apart from $l = 0$ mode, $l = 1, 2$ and 3 modes dominate over the higher order modes.

Figure 12. Temporal evolution of the amplitude $a_0$ (mean shock radius) of $l = 0$ mode for different initial strength of magnetic fields and $v = 0.05c$. Zoomed-in view of the initial phase is shown in inset figure on bottom left-hand side. The inset figure on the top right-hand side shows the zoomed-in view of the quasi-steady phase. Shock radius increases with time and eventually settles into an oscillating value. The mean shock radius is larger for a stronger field (see inset in the top right-hand side).

two Alfvén waves, and two slow magnetosonic waves (Guilet & Foglizzo 2010).

To interpret the SASI oscillation time-scales in presence of magnetic fields, we compute two more time-scales in addition to the three time-scales introduced in Section 5.1.4. For computing time-scales in the MHD set-up, we assume that for radial propagation
In Section 4, we showed that a radial magnetic field does not modify the magnetic field on the SASI. Now, we discuss the key results of our study.

Fig. 14 shows the temporal variation of all the relevant time-scales for the fiducial MHD run (Case I). While \( T_{\text{tot}} = 617.01 \) \( \tau_g/c \) (SASI time-scale measured by the period of \( l = 1 \) perturbation in the shock location) still lies in the range of variations of \( t_{\text{acc}}, t_{\text{acc}A+} \), and \( t_{\text{accA}} \) (equations 14, 19, and 18) related to the advective-acoustic cycle, it is longer than the meridional and radial sonic time-scales, \( t_{\text{mcs}} \) and \( t_{\text{rcs}} \) (equations 16 and 15).

Fig. 15 shows the variation of SASI time-scale (measured by both methods, \( T_{\text{tot}} \) and \( T_{\text{acc}} \); see Section 5.1.3) and the time-averaged time-scales (obtained from various signal propagation time-scales) as a function of the initial magnetic field strength (quantified by \( C \); see equation 5). While the maximum relative change (compared to HD) in SASI time period is \( \approx 15.84 \) per cent, that in \( t_{\text{mcs}} \) is only \( \approx 1.75 \) per cent. In presence of a weak magnetic field (\( \beta \gg 1 \)), SASI time period is not expected to be affected if the mechanism is purely acoustic. Even the variation in the HD advective-acoustic time-scale is small. However, the \( t_{\text{accA}} \) time-scale in which the inward-propagating signal travels at \( |v_A| - v_A \) (i.e. Alfvén wave travels outwards relative to the flow) matches the variation of the observed SASI time-scale fairly well. In principle, the inward propagating signal should consist of five waves (Guilet & Foglizzo 2010), but a cycle consisting of outward propagating fast waves and inward-propagating Alfvén disturbances (travelling inward at \( |v_A| - v_A \)) seems to quantitatively describe the shock oscillations observed in our simulations.

6 DISCUSSIONS AND CONCLUSIONS

In this paper, we describe the effects of an initial split-monopolar magnetic field on the SASI. Now, we discuss the key results of our work and draw conclusions.

6.1 Flow structure

In Section 4, we showed that a radial magnetic field does not modify the Bondi accretion solution. In this section, we discuss how the...
flow structure changes in the saturated state for \( v_{in} = 0.05c \) and different magnetic field strengths. Beyond a certain magnetic field strength (for \( v_{in} = 0.05c \); the critical value is \( C = 7.94 \times 10^5 \)), SASI does not occur. We choose four different strength of magnetic field: \( C = 1 \), unmagnetized; \( C = 5 \times 10^7 \), moderately magnetized; \( C = 7.94 \times 10^5 \), the strongest magnetic field for which SASI occurs and \( C = 8 \times 10^5 \), the strength of magnetic field at which SASI cannot occur.

Fig. 16 shows the average flow profiles for four different initial magnetic field strengths. We take the time and angle average of the quantity \( q \) as

\[
\langle q \rangle = \frac{1}{T} \int_{0}^{T} dt \int_{0}^{\pi} \frac{q(r, \theta, t) \sin \theta d\theta}{\sin \theta d\theta},
\]

where \( T \) is the averaging period. We represent time average by \( 'a n d \) \( ' \). Top panel shows the average density \( \langle \rho \rangle \) as a function of \( r \) for different value of \( C \).}

\[ M = \frac{\langle \rho \rangle}{\langle v \rangle}, \]

The critical magnetic field strength depends on the ratio \( v_{rs} \), the ratio of Alfvén, and radial advection speeds.

6.2 SASI mechanism

Unlike most of the previous simulations, our set-up leads to a quasi-steady state in which the non-linear oscillations essentially last forever. We compare the SASI time period with time-scales obtained from two different mechanisms (namely advective-acoustic and purely acoustic) in HD and MHD.

In HD regime, we change the ratio of mean shock radius \( (r_{sh}) \) to inner radius \( (r_{in}) \) keeping \( r_0 \) fixed, and study the variation of different time-scales with this ratio. For small values of \( r_{sh}/r_{in} \), the match between the advective-acoustic time-scale \( t_{aac} \) and the SASI time period \( (T_{sh} \text{ or } T_{rs}) \) is excellent. With an increase in \( r_{sh}/r_{in} \), the deviation of the time-scale becomes larger (see Fig. 6). Purely acoustic mechanism gives rise to two different time-scales: the radial acoustic time \( t_{rA} \) (considering purely radial propagation) and the meridional acoustic time \( t_{mc} \) (considering meridional propagation). In all cases, \( t_{rA} \) is always much less than the SASI time period, so we can discard purely radial acoustic mechanism as the possible cause for SASI. The match between the \( t_{mc} \) and SASI time period becomes best around \( r_{sh}/r_{in} \sim 3.9 \). But it is to be noted that according to Blondin & Mezzacappa (2006), SASI time period is expected to be equal to the round trip time of two sound waves advancing along the shock surface i.e. \( 2t_{rA} \). Instead, we observe the closeness between \( t_{mc} \) and SASI time period.

In MHD regime, we study the variation of SASI time-scales with the change in initial magnetic field strength. In presence of a weak magnetic field, the advective-acoustic cycle is expected to split into five different cycles that constitute the actual cycle (see Section 5.2.3). We compute three time-scales to quantify five cycles, \( t_{aac} \) – outgoing fast magnetosonic wave + ingoing entropy wave, \( t_{aacA} \) – outgoing fast magnetosonic wave + ingoing (with respect to local inflow) Alfvén/slow wave, \( t_{aacA-} \) – outgoing fast magnetosonic wave + outgoing (with respect to local inflow) Alfvén/slow wave. While the maximum relative change (to HD) in time-scales obtained from the advective-acoustic mechanism is \( \approx 18 \% \) (for \( t_{aacA} \)), in the meridional acoustic mechanism it is only \( \approx 1.75 \% \) (for \( t_{mc} \)), compared to the change in SASI time period of \( \approx 15.84 \% \). In purely acoustic mechanism, weak magnetic fields do not affect the SASI time period, but in advective-acoustic mechanism weak magnetic fields affect the SASI time period. The effects depend on the ratio \( v_{sh}/v_{r} \), the ratio of Alfvén, and radial advection speeds.

Both in HD and MHD regimes, advective-acoustic mechanism is favoured as the possible mechanism for SASI. Further, if SASI is a purely acoustic mechanism, the dispersion relation in the local limit is given by \( \omega = k_{c} \), which means that the frequency of different modes should be proportional to the wavenumber. To find the frequency associated with different modes, we take the temporal FFT of shock deformation modes \( (a_{l}(t)) \) and best fit the lowest frequency peak with a Lorentzian (equation 13); centroid frequency gives the frequency of the corresponding mode. Fig. 17 shows the representative examples of FFT of \( a_{l} \) for \( l = 0, 1, 15, \text{ and } 16 \) for our fiducial hydro run. Bar plot in Fig. 18 shows the variation of mode frequency with mode number. All the even modes \( (l = 0, 2, 4 \text{ etc.}) \) have frequency \( \approx 700 \text{Hz} \), and odd modes \( (l = 1, 3, 5 \text{ etc.}) \) have frequency \( \approx 350 \text{Hz} \), which is against the expectation \( f \propto l \) for acoustic signals.

Guilet & Foglizzo (2010), in their toy model, showed that the effects of magnetic field on the advective-acoustic cycle depend on the ratio of Alfvén speed to advection speed \( (v_{sh}/v_{r}) \) instead of the ratio of gas pressure to magnetic pressure \( (\beta) \). Even a weak
encounter a significant change from the time-scales in HD case for in Fig. 19. SASI time period (pressure in the post-shock volume (β steady state, the ratio of average gas pressure to average magnetic

take

r decreased, and

v volume (∼sh)
v,

≈ vsh

t, 

AAC, l = 0, 1, 15, 16 for the fiducial hydro run. The lowest frequency peaks are fitted individually by a Lorentzian using the least squares fit method. Centroid of the Lorentzian gives the frequency associated with the corresponding mode.

Figure 17. FFT of the mode amplitudes \( a_l \) for \( l = 0, 1, 15, 16 \) for the fiducial HD run. The lowest frequency peaks are fitted individually by a Lorentzian using the least squares fit method. Centroid of the Lorentzian gives the frequency associated with the corresponding mode.

Figure 18. Lowest frequency associated with different modes \((l)\) of oscillation for the fiducial HD run.

magnetic field is able to significantly affect the advective-acoustic cycle provided \( v_A/v_l \) is of the order of

\[
\frac{v_A}{v_l} \approx \frac{r_{sh}}{2h\sqrt{l(l+1)}},
\]

where \( r_{sh} \) is the shock radius, \( h \) is the distance over which flow is decelerated, and \( l \) is mode number (see their equation 18). If we take \( r_{sh} \approx h \), we get the ratio 0.353 and 0.204 for \( l = 1 \) and 2 modes, respectively. Fig. 19 shows the change in the ratio of Alfvén speed to advection speed considering the average over the whole post-shock volume \( (v_A/v_l) \) and over the volume within the half shock radius \( (v_{A,h}/v_{l,h}) \) with the change in initial magnetic field strength \( (C) \). To quantify the magnetization of the accreting medium in the quasi-steady state, the ratio of average gas pressure to average magnetic pressure in the post-shock volume \( (\beta) \) is plotted as a function of \( C \) in Fig. 19. SASI time period \( (T_{w}, T_{w}) \) and time-scales corresponding to the advective-acoustic mechanism \( (T_{AAC}, T_{AAC,A}, T_{AAC,A}) \) encounter a significant change from the time-scales in HD case for \( C = 1.67 \times 10^7 \) (as seen in Fig. 15), for which \( v_A/v_l = 0.053 \) and \( v_{A,h}/v_{l,h} = 0.11 \) and \( \beta \approx 3200 \). So it appears that in our set-up, SASI is affected at a smaller value of \( v_A/v_l \) compared to the estimates of

\[
\frac{v_A}{v_l} \approx \frac{r_{sh}}{2h\sqrt{l(l+1)}},
\]

if \( v_A < v_l \).

Equation (21) tells that for the same strength of magnetic field, the higher order modes are more affected compared to lower order modes, which is clear from Fig. 13. We also see that the SASI period is not a monotonically increasing functions of magnetic field strength; there are irregularities that are expected in the framework of advective-acoustic mechanism due to interference of different cycles (see fig. 6 of Guilet & Foglizzo 2010).

So, we conclude that the physical mechanism behind SASI (at least in the parameter regime that we explored) is more likely to be the advective-acoustic mechanism instead of a purely acoustic mechanism (either meridional or radial).

6.3 QPOs and SASI

SASI in an accretion flow gives rise to an intrinsic time variability in the flow, which may explain some of the QPOs observed in XRBs. In this section, we try to connect different time-scales associated with magnetized SASI with different high frequency (HF ≳ 100 Hz) QPOs observed in XRBs (both in black hole and neutron star binaries).

Kilohertz (kHz) QPOs are the fastest variability components in neutron star XRBs (van der Klis 2004), seen in most Z and atoll sources. Sometimes kHz QPOs appear in pairs; the peak with the higher frequency is called the upper kHz QPO at frequency \( f_{u} \) and

\[
\frac{v_A}{v_l} \approx \frac{r_{sh}}{2h\sqrt{l(l+1)}},
\]

\[
\frac{v_{A,h}}{v_{l,h}} = 0.11 \text{ and } \beta \approx 3200.
\]

\[
\frac{v_A}{v_l} \approx \frac{r_{sh}}{2h\sqrt{l(l+1)}},
\]

\[
\frac{v_{A,h}}{v_{l,h}} = 0.11 \text{ and } \beta \approx 3200.
\]

\[
\frac{v_A}{v_l} \approx \frac{r_{sh}}{2h\sqrt{l(l+1)}},
\]

\[
\frac{v_{A,h}}{v_{l,h}} = 0.11 \text{ and } \beta \approx 3200.
\]

\[
\frac{v_A}{v_l} \approx \frac{r_{sh}}{2h\sqrt{l(l+1)}},
\]

\[
\frac{v_{A,h}}{v_{l,h}} = 0.11 \text{ and } \beta \approx 3200.
\]

\[
\frac{v_A}{v_l} \approx \frac{r_{sh}}{2h\sqrt{l(l+1)}},
\]

\[
\frac{v_{A,h}}{v_{l,h}} = 0.11 \text{ and } \beta \approx 3200.
\]
the other is called lower kHz QPO with frequency $f_l$. Many models associate orbital motion in the disc with one of the kHz QPOs (Strohmayer et al. 1996; Miller et al. 1998; Mukhopadhyay et al. 2003). While Mukhopadhyay et al. 2003 attributes global shock oscillations as the origin of upper kHz QPOs for the first time, some other models argue that both QPOs arise via non-linear resonance between fundamental frequencies, e.g. between radial and vertical epicyclic oscillation frequencies along with the spin frequency of neutron star (Kluźniak et al. 2004; Petri 2005; Blaes et al. 2007; Mukhopadhyay 2009). Parametric resonance models are particularly attractive if $f_h - f_l$ is linked with the spin frequency of the neutron star, when $f_h - f_l \sim v_i/2$ (if $v_i \gtrsim 400$ Hz; e.g. KS 1731-260, 4U 1636-53) or $f_h - f_l \sim v_i$ (if $v_i \lesssim 400$ Hz; e.g. 4U 1728-34, 4U 1702-429) (Strohmayer et al. 1996; van der Klis et al. 1996; Ford et al. 2000; Wijnands et al. 2003). However, later on this interpretation was questioned (Méndez & Belloni 2007).

For black hole sources, on the other hand, the observed twin HF QPOs are often argued to be seen in 2 : 3 ratio [e.g GRO J1655-40 (300, 450 Hz; Remillard et al. 1999; Strohmayer 2001), XTE J1550-564 (184, 276 Hz; Homan et al. 2001), and GRS 1915+105 (113, 168 Hz; Remillard 2004)], which again was explained based on non-linear resonance by the groups mentioned above. Some recent observations question the 2 : 3 appearance of HF QPOs in black hole XRBs [e.g IGR J17091-3624 (66, 164 Hz; Altamirano & Belloni 2012)].

Another $\gtrsim 100$ Hz variability phenomenon is the hecetoertz (hHz) QPO (Ford & van der Klis 1998) with a frequency $f_{\text{hHz}}$, in the range 100–270 Hz (e.g see Altamirano et al. 2008) in atoll sources in most states. Fragile, Mathews & Wilson (2001) proposed that accreting material passing through the transition region formed due to Bardeen–Peterson effect may generate hHz frequencies. Kato (2007) proposed that a warp in accretion disc gives rise to the hecetoertz QPOs in atoll sources. The black hole sources also exhibit QPO frequency of order hHz or slightly less, e.g. GRS 1915+105, XTE J1550–564, simultaneously with HF ones (e.g. Remillard et al. 2002). Earlier non-linear resonance models can be modified to explain it (Mukhopadhyay, Bhattacharya & Sreekumar 2012).

Fig. 20 shows the power spectrum of the light curve $L(t)$ obtained in the quasi-steady state for different initial magnetic field strengths (quantified by $C$; see equation 5) and $v_{\text{in}} = 0.05c$. Luminosity $L(t)$ is assumed to be due to free–free emission (a similar time variability is expected for other mechanisms such as synchrotron and inverse-Compton) from the volume $V$, and computed as

$$L = \int_V \frac{1}{4} \times 10^{-27} \left( \frac{\rho}{m_p} \right)^2 T^2 \Delta V \text{ erg s}^{-1}. \quad (22)$$

where $V$ is the spherical volume of radius $r = 30r_g$, dominated by post-shock region. The post-shock temperature in simulations is very high ($T \sim 10^{11} K$). The electrons in hot accretion flows are at lower temperature compared to that of the ions and other emission process may be important (e.g. Sharma et al. 2007; Rajesh & Mukhopadhyay 2010; Yuan & Narayan 2014). Therefore, light curves from simulations (which assume a single temperature fluid) should be only taken as trends.

First panel of Fig. 20 shows the power spectrum for the unmagnetized ($C = 1$) SASI run and the power spectrum has the most prominent peak at $f_l = 700.24$ Hz along with its harmonics (the peak frequency is obtained by fitting with a Lorentzian). This is the frequency associated with the $l = 0$ mode and double the frequency of $l = 1$ mode. There is some low frequency noise present in the power spectrum. With the increase in magnetic field strength, the prominent peak frequency shifts to lower value and the low frequency noise becomes less prominent (for $C = 10^5$) and vanishes for $C = 1.25 \times 10^7$. As the magnetic field strength is increased more, some extra peaks arise at low and intermediate frequencies along with the main peak (e.g. $C = 1.67 \times 10^7$ and $2 \times 10^8$). The lowest frequency is associated with the modulation frequency on top of a regular frequency of mode amplitude $a_0(t)$ (e.g. see the variation of $a_0(t)$ for $C = 7.94 \times 10^7$ in Fig. 12). With the increase in magnetic field strength, low frequency features appear and disappear non-monotonically.

In this analysis, the origin of QPOs (whether kHz, HF or hHz) is different from past proposals. Fig. 21 shows the power spectrum for $C = 2 \times 10^5$ and $v_{\text{in}} = 0.05c$. The most prominent peak appears at 681.78 Hz, which can be related to the upper kHz QPOs at $f_h$. The lowest frequency peak is at 135.96 Hz, which can be identified as the hHz QPO at $f_{\text{hHz}}$. In between these two peaks, there are three more peaks. One of them is the harmonic of the hHz QPO ($f_{\text{hHz}} = 273.44$ Hz $\approx 2f_{\text{hHz}}$). Other two peaks are the beat frequencies, $f_h = 554.40$ Hz $\approx f_h - f_{\text{hHz}}$ and $f_s = 408.21$ Hz $\approx f_h - f_{\text{hHz}}$, one of which can be related to the lower kHz QPO. Whereas, $f_s$ is equal to the frequency associated with $l = 0$ mode, $f_{\text{hHz}}$ is the frequency of modulation in the mode amplitude $a_0$ due to magnetic field, as seen in inset at upper right-hand side of Fig. 12. With magnetic field strength ($C$), the upper kHz QPO frequency $f_h$ tracks $2/T_{\text{a1}}$, the frequency of $l = 0$ mode (which is double the $l = 1$ mode frequency). For all field strengths, the hHz QPO frequency remains constrained in the range (110–135) Hz.

If the shock location is changed by tuning the value of $v_{\text{in}}$, SASI time period changes (as shown in Fig. 6), so does $f_h$, as $f_h \approx 2/T_{\text{a1}}$. On the other hand, the hHz QPO arises due to magnetic effects. To see the variation in $f_{\text{hHz}}$, with the change in shock location, we decrease and increase the shock radius by changing $v_{\text{in}}$ to 0.06c and 0.048c, respectively. For $v_{\text{in}} = 0.048c$, power spectrum of the light curve for $C = 2 \times 10^7$ is shown in Fig. 22; three peaks are present in the power spectrum. Upper kHz QPO frequency $f_h = 599.90$ Hz is shifted to a lower value compared to the fiducial case ($v_{\text{in}} = 0.05c$), so does the hHz QPO frequency ($f_{\text{hHz}} = 105.22$ Hz). The frequency of lower kHz QPO is $f_l = 495.82$ Hz $\approx f_h - f_{\text{hHz}}$. Fig. 23 shows the power spectrum of light curve for $v_{\text{in}} = 0.06c$ and $C = 5 \times 10^9$, with a smaller shock radius. As expected, the upper kHz QPO frequency ($f_h = 1081.10$ Hz) related to SASI time period increases. Also, the hHz QPO frequency ($f_{\text{hHz}} = 225$ Hz) moves to a higher value. The lower kHz QPO ($f_l = 869.54$ Hz) structure becomes fainter (this might be immersed in noise in real observations).

Our idealized simulations suggest shock oscillations as the origin of QPOs, in particular kHz/HF/hHz ones. We identify the $l = 0$ SASI mode frequency (which is double the frequency of $l = 1$ mode) as the frequency of upper kHz QPO. It is the appearance of the hHz QPO that determines the separation of twin QPO peaks. We do not observe the hHz QPOs in our simulations without magnetic fields, indicating that they originate only in the presence of a magnetic field. Hence, one does not necessarily need to introduce the spin of the compact objects to explain QPOs.

### 6.4 Caveats of the model

The present model is very simplistic. In reality, accretion flows have complicated magnetic field geometry, angular momentum, cooling (depending on the spectral state of the XRBs), which might change the above results. A brief discussion of the above-mentioned factors is given below.
Figure 20. Power spectrum of the light curves assuming free–free emission for different magnetic field strength ($C$) and $v_\infty = 0.05c$. With the increment in magnetic field strength, low frequency features appear and disappear non-monotonically. The most prominent peak whose frequency is the frequency of $l=0$ mode and half the frequency of $l=1$ mode can be related to upper kHz QPO. For some strengths of magnetic field, alongside the main peak, there are some comparatively low frequency peaks that can be related to some other types of HF QPOs such as lower kHz QPO and hHz QPO.

We initialize the simplest magnetic field configuration, a split-monopole. Because of the absence of magnetic force in the pre-shock flow, the equilibrium is not affected by the magnetic field (see Section 4), but the mode frequencies are. The mode frequencies are expected to behave differently for different field geometries (Guilet & Foglizzo 2010).

Accreting matter in XRBs is expected to have angular momentum. QPOs are observed in the hard state, in which the inner flow is expected to be hot, quasi-spherical, and sub-Keplerian (e.g see Chakrabarti 1989). To approximate that we study the spherical, adiabatic, non-rotating accretion flow on to a compact object. However, even small angular momentum can affect the global shock oscillations (Blondin & Mezzacappa 2006; Yamasaki & Foglizzo 2008; Kazeroni, Guilet & Foglizzo 2017). Shock instabilities in rotating accretion flows were invoked to explain time variability (mostly low frequency phenomena) in accreting systems (Molteni et al. 1996; Nagakura & Yamada 2009).

We also assume axisymmetry, breakdown of which may significantly alter the oscillation frequencies. While in the non-rotating flow, non-axisymmetric modes of SASI redistribute angular
The prominent peaks appear at $f_u = 681.78 \text{ Hz}$, $f_{hHz} = 135.96 \text{ Hz}$, and at its harmonics, $f_{hHz}^2 = 273.44 \text{ Hz}$, and at the beat frequencies, $f_1 = 545.40 \text{ Hz} \approx f_u - f_{hHz}$, $f_2 = 408.21 \text{ Hz} \approx f_u - f_{hHz}^2$. The peaks are fitted individually by Lorentzians using least squares fit method.

Figure 22. Power spectrum of light curve for $v_{in} = 0.048c$ and $C = 2 \times 10^7$. The prominent peaks appear at $f_u = 599.90 \text{ Hz}$, $f_{hHz} = 105.22 \text{ Hz}$, and at the beat frequency of the former two, $f_1 = 405.82 \text{ Hz} \approx f_u - f_{hHz}$. The peaks are fitted individually by Lorentzians using least squares fit method.

Figure 23. Power spectrum of light curve for $v_{in} = 0.06c$ and $C = 5 \times 10^7$. The most prominent peaks appear at $f_u = 1081.10 \text{ Hz}$, and at $f_b = 225.04 \text{ Hz}$, which can be considered as upper kHz QPO and hHz QPO, respectively. A fainter peak is seen at $f_b = 869.54 \text{ Hz}$, which can be considered as lower kHz QPO. The peaks are fitted individually by Lorentzians using least squares fit method.

We expect shock oscillations in inner, hot, transonic accretion flows (rotating or non-rotating). But for comparing with the observations, one needs to study them in more realistic 3D simulations with rotation, magnetic fields.

7 SUMMARY

In this work, we study SASI in unmagnetized and magnetized spherical accretion flow around a central gravitating accretor, in particular the ones with a hard surface. The key findings of the work are listed below.

(i) A standing shock does not occur above a critical strength of magnetic field as the sum of outward magnetic and thermal pressure becomes high enough to overcome the inward gravitational attraction and the shock moves into the subsonic region and vanishes.

(ii) A comparison of various signal propagation time-scales and the observed shock oscillation frequency agrees with the advective acoustic mechanism, and not a purely acoustic one (at least for our parameters).

(iii) The global shock oscillations in the accretion flow give rise to a prominent peak in the power spectrum of the light curve, which can be related to the upper kHz QPOs. In presence of magnetic field, there are a few low frequency peaks that can be related to lower kHz and hHz QPOs.

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