Study of the phase transition of a medium using reverberated signals

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Abstract. Non-destructive testing (NDT) methods nowadays are essential in materials and structures control due to their capability to not affect the integrity of material. These methods are particularly efficient for the monitoring of resin or composites formation. In particular, ultrasonic techniques can be used to evaluate mechanical properties of materials by measuring acoustic waves characteristics. In general classical characterization techniques, based on linear measurements, give local information's of medium. Reverberation methods, classically used in room acoustics, allow global characterization. This approach is not so common in NDT or structure monitoring. The reverberation signals which contain global information about propagation media properties, are highly dependent on the boundary conditions. Sabine\'s formula gives the relationship between reverberation time RT and wall attenuation coefficient can be adapted in many acoustic applications. Previous studies established a relation between RT and solid interfaces absorption. In the case of a sol-gel material, the reverberation time measured in a metallic block can be linked to the material properties such as density or viscosity. The RT will vary during the phase transition of our material from liquid to solid state. Hence, our motivation is to derive the equation of the acoustic intensity for solid interface absorption. Furthermore, to estimate the reverberation time from the determined acoustic intensity (simulation part) and to compare the results with the experimental part. For this reason, experiments were performed to validate the simulation part. Measurements are preformed in an aluminium mold using five piezoelectric (PZT) patches, one being used as emitter and the others as receivers has been studied and it\'s evolution is shown to lead to a good estimation of the phase transition time.

1. Introduction

Testing and quality check-up during the life of products is a crucial challenge for several industries. Non destructive evaluation (NDE) can be considered a powerful tool to reach this goal. NDE is defined as a classification of physical and chemical tests that permit the detection and/or measurement of the significant properties of a material or a structure without diminishing its utility [1].

Ultrasonic techniques (UT) is an acoustic evaluation technique, which is expanding rapidly into many areas of manufacturing and in-service detection [2]. Its ability to detect and distinguish the surface and subsurface of the tested material [3], [4], [5], [6], [7], [8], has opened up impressive
opportunities. Since most of these methods have several drawbacks to detect defects like only obtaining local information along the direct propagation path, as needed to find complementary techniques to obtain global information about media properties.

We consider a wave created by a sound source that propagates through a room. If the wave arrives directly at the receiver, then we called this direct path. It will be reflected many times before they reaching the receiver, the arrangement of these reflections called reverberation. Classical technique depends only on the direct path or on the first received packet [9], [10], which depends only on the first reflections without taking into account any others reflections. These techniques provide only local useful information along the direct path about media properties. Although, the reverberated signal contains useful information about the entire room properties [8]. Due to this advantage, our work focuses on them in order to recover information about media characterization. Thus, the extraction of useful informations is the subject of techniques such as time reversal [11], coda interferometry [12], and correlation fields [13]. In our case, the aluminum block is equivalent to acoustic room where reverberation occurs and the tested material which can be air, water, oil, sol-gel is equivalent to the walls where the attenuation occur.

Reverberation time (RT) is defined as the time required for the energy of a sound to decay by a specific decay level (60dB in room acoustic) below its initial level after switching off the sound source. RT is crucial estimate acoustic parameters like the absorption coefficient [14]. RT should be directly linked to phase transition of the material under test since its mechanical properties will slightly change during the phase transition from liquid to solid state. RT is estimated through a linear curve fitting applied to the logarithm of Schroeder’s integrals average [15].

In the other side, if an acoustic wave strikes on an interface between two media with an oblique incidence, and if one of these media is solid, a complicated phenomenon called mode conversion occurs. It results a conversion from longitudinal wave to shear wave, and vice versa, for the reflected and transmitted waves. The relations between the reflected, transmitted, and the incident wave, defined in terms of amplitude and intensity, depend on the characteristic acoustic impedances of the media, the angles of incidence, and the types of the incident wave (longitudinal or shear wave). The acoustical intensity of the acoustic pulse in an acoustic room has been described. In this paper, we aim to derive the equation of the acoustic intensity for solid interface by considering the shear and the longitudinal waves. Furthermore, the reverberation time from the determined acoustic intensity (simulation part) is estimated and then is compared with the experimental part.

This paper is organized as follows: in section 2, some information about reverberation characteristics and reverberation time are described in details. In section 3, we introduce the sound intensity in room acoustic. We will then derive the theoretical expressions of the sound intensity for solid interface (Solid-fluid). in section 4 a simulation part for a theoretical expression to estimate the RT using simulation. An experiment setup is represented in section 5, to estimate the reverberation time experimentally. The theoretical approach will then be validated by comparison between numerical and experimental results.

2. Reverberation Characteristics

Many studies on extraction of useful information from reflections of sound inside acoustic room have been done. The reflections of sound happening inside the acoustic room continues after the sound source switched off. For example, when a sound wave propagates inside a room, several reflections occur but the propagating wave is attenuated by the air (damping factor) and when it hits the wall, some of the energy is absorbed by the walls. The set of these reflections is called reverberation. The reverberation process can be interpreted as the combination of some reflections arriving at different time at the receiver.

A reverberant medium is created when a sound wave is reflected causing multi-reflections
that are received. The received wave could be decomposed into direct path, early reflections, and late reflections as shown in Figure 1.

Reverberation time (RT) is defined as the time required for the energy of a sound to decay by $60\,\text{dB}$ below its initial level inside acoustic room after switching off the sound source. For this reason, RT is generally denoted by $RT_{-60\,\text{dB}}$. This definition is widely accepted and used in scientific researches. It is possible to estimate the RT value using Sabine’s Formula which gives a relationship between reverberation time and wall attenuation coefficient.

$$RT_{-60\,\text{dB}} = 0.16 \frac{V_r}{S \alpha}$$

where the constant 0.16 depends on the celerity of the wave in room acoustic and for an attenuation gain of $60\,\text{dB}$, $V_r$ is the room volume and $S$ is the total area of the walls and $\alpha$ is the wall absorption coefficient...

Schroeder used the impulse response of the room to estimate the experimental RT. Schroeder demonstrated the equivalence between the sound level decay of an impulse response and the average of a large number of decays of an interrupted stationary noise when his backward integration method is used to extract the decay curve from the impulse response[15].

$$y(t) = \int_{t}^{t_u} h^2(\tau) \, d\tau$$

Where $y(t)$ is a schroeder integral and $t_u$ is the upper limit of integration, and $h(t)$ is the impulse response of the studied medium. Note that the squared room impulse response gives an energy decay.

3. Theoretical background
The acoustic intensity of the acoustical pulse $I(t)$ over the time in a finite acoustic room is given by [16]:

$$I(t) = I_0 (1 - \alpha) I_m^L e^{-T_L c L t}$$

Figure 1: Received signal on oscilloscope
where $I_0$ is the initial intensity and $\alpha$ is the mean absorption coefficient of the wall and $t_{mL}$ is the average period of time between two successive reflections, since that in acoustic room depends only in one mode of propagation which is the longitudinal one, and $\Gamma_L$ is the attenuation coefficient for the longitudinal wave in neper per meter and $c_L$ is the celerity of the longitudinal wave in fluid.

In case of solid homogeneous and isotropic medium, there are at least two modes of propagation which are longitudinal and transverse modes for any type of excitation. Moreover, a mode conversion occurs when an acoustic waves arrives at the interface between two media having different acoustic impedances[17].

Consider an aluminium block of volume $V$ exposed to a punctual acoustic excitation of finite duration, we can derive the acoustic intensity over the time by taking into account mode conversion, let us consider, we have a longitudinal incidence as shown in figure 2.

For oblique incident (Longitudinal and shear wave incidence) between solid-fluid (Aluminium-Air) interface the reflection and transmission coefficients is calculated as shown the following figure 3.

In our notation, (RSS) the first element is either $R$ or $T$ (Reflected or transmitted waves) the second element is for the incident wave, $S$ or $L$ (Shear or Longitudinal) and the third element is for (Shear or Longitudinal)[18]. Our aim is the calculation of the acoustic intensity of each mode over the time by taking into account mode conversion.

From figure 3a for longitudinal incidence, we take the mean value of $RLL$ from 0 to 89 degree, since the wave will strikes at different angle. While for Shear incidence the mean value of RSS is taken from 0 to 28 degree, until the first critical angle occurs, corresponding to the propagative mode of the shear wave.

For longitudinal incidence as shown in figure 2, a part of the energy is converted to shear wave and a part is transmitted into the fluid medium , the acoustic intensity for longitudinal wave after a reflection from longitudinal incidence $E_{LI}$ may be written as:

\[
I_{LL}(t) = E_{LI} (RLL) e^{-\Gamma_L c_L t}
\]
where $\Gamma_L$ is the acoustic attenuation coefficient for longitudinal wave and $c_L$ is the wave celerity for longitudinal wave.

By taking into account the acoustic intensity for the converted energy, which means the calculation of the acoustic intensity for the shear wave that is converted from longitudinal incident. After, applying the law of energy conservation, we can write:

$$I_{converted} + I_{LL} = I_{LI} \tag{5}$$

Note that, $I_{converted}$ is the sum of the transmitted energy and the converted energy, in our case energy converted from longitudinal to shear wave, as shown in figure 2. Thus, it can be written as:

$$I_{converted} = I_T + I_{LS} \tag{6}$$

and

$$I_{LS} = \frac{RLS}{RLS + T} I_{converted} = \frac{RLS}{1 - RLL} I_{converted} \tag{7}$$

by substituting equation (4) in equation (5) we realize that:

$$I_{converted} + E_{LI}(RLL)e^{-\Gamma_L c_L t} = E_{LI}e^{-\Gamma_L c_L t} \tag{8}$$
Thus dividing equation (8) by $E_{LI}$ has no effect, then derivating with respect to time since energy of the wave (Longitudinal or Shear wave) will change at each reflection at the interface between two media, we suppose we have one reflection at the interface, so get that:

$$\frac{\partial}{\partial t} \left( \frac{I_{\text{converted}}}{E_{LI}} \right) = \frac{\partial}{\partial t} (e^{-\Gamma_{LCLt}} - (RLL)e^{-\Gamma_{LCLt}}), \tag{9}$$

By substituting equation (6) in equation (9) and after integrating equation (9), we can see the variation of the converted energy with time. The acoustic intensity for the converted energy from the longitudinal to shear can be written:

$$I_{LS}(t) = E_{LI}(t) \frac{RLS}{1 - RLL} e^{-\Gamma_{LCLt}} \left[ 1 - (RLL)\frac{I}{m} \right] $$

For transverse incident, the acoustic intensity for transverse wave with respect to transverse incidence will change at each reflection at the interface between two media, can be written as:

$$I_{SS}(t) = E_{SI}(t)(RSS)\frac{I}{m} e^{-\Gamma_{SCST}} \tag{10}$$

Then, the acoustic intensity for the longitudinal wave converted from the transverse to longitudinal wave can be written as:

$$I_{SL}(t) = E_{SI}(t) \frac{RSL}{1 - RSS} \left[ 1 - (RSS)\frac{I}{m} \right] e^{-\Gamma_{SCST}} \tag{11}$$

Next, the derived equations just for one strike at the interface between two media are listed as follows:

$$\begin{cases} 
I_{LL}(t) = E_{LI}(t) (RLL)\frac{I}{m} e^{-\Gamma_{LCLt}} \\
I_{LS}(t) = E_{LI}(t) \frac{RLS}{1 - RLL} \left[ 1 - (RLL)\frac{I}{m} \right] e^{-\Gamma_{LCLt}} \\
I_{SS}(t) = E_{SI}(t) (RSS)\frac{I}{m} e^{-\Gamma_{TSSST}} \\
I_{SL}(t) = E_{SI}(t) \frac{RSL}{1 - RSS} \left[ 1 - (RSS)\frac{I}{m} \right] e^{-\Gamma_{SCST}} 
\end{cases} \tag{13}$$

Since two mode exist at the interface between two media, it is preferable to discretize the wave taking into account every single strike happening at the interface. If we assume that the energy is initially one in the longitudinal mode, we can write that:

$$\begin{cases} 
E_{LI}(0) = 1 \\
E_{SI}(0) = 0 
\end{cases} \tag{14}$$

For longitudinal incident, the energy in the longitudinal mode after the first strike at the interface is smaller than one. And the energy of shear mode is greater than zero. For each strike at the interface we have a new value for $E_{LI}(n\Delta t)$ and $E_{SI}(n\Delta t)$ . After repeating this for a specific number of iteration as shown in figure 4:

$$E_{LI}((n+1)\Delta t) = E_{LI}(n\Delta t)(RLL)\frac{\Delta t}{m} e^{-\Gamma_{LCLt}} + E_{SI}(n\Delta t) \frac{RSL}{1 - RSS} (1 - (RSS)\frac{\Delta t}{m}) e^{-\Gamma_{SCST}} \tag{15}$$

and

$$E_{SI}((n + 1)\Delta t) = E_{SI}(n\Delta t)(RSS)\frac{\Delta t}{m} e^{-\Gamma_{SCST}} + E_{LI}(n\Delta t) \frac{RSL}{1 - RLL} (1 - (RLL)\frac{\Delta t}{m}) e^{-\Gamma_{LCLt}} \tag{16}$$

where $\Delta t$ is a time step used to transform form continuous to discrete function to create a simulation, that calculate the new values at each reflection that happens at the interface between two media.
4. Simulation Part

The value of the time step, should be slightly smaller than the median travel time of longitudinal and shear waves $t_{mL} = 3.18 \times 10^{-5}$ s and $t_{mS} = 6.45 \times 10^{-5}$ s, was taken at $(10^{-5}$ s).

The value of the attenuation for longitudinal and shear waves [19], [20] is $\Gamma_L=0.01969$ dB/m $\Gamma_S=0.0657$ dB/m respectively.

The dimension of our aluminium block is, $20 \times 20 \times 5$ cm$^3$, giving surface of 0.12 m$^2$ since the air is surrounded all the faces of our aluminium block and the volume of 0.002 m$^3$ and also we take the mean value of RLL=0.5048, RSS=0.7266, RSL=0.2734 and RLS=0.4951 for aluminium-air interface using the values described in figure 3.

The velocity was taken at $c_L=6280$ m/s for longitudinal wave and $c_S=3100$ m/s for shear wave.

Figure 5 shows the results for longitudinal incidence and shear incidence using Matlab software after repeating this for specific duration. The total time in the experimental results is 0.014 s for that reason the number of iteration is $n=1400$, thus the total time ($n\Delta t$) in our simulation results will be identical to the experimental time.

As described in figure 5, some energy of the incident wave is converted into another mode (Shear wave), where longitudinal into a shear wave and vice versa. On the other hand, in solid isotropic and homogeneous medium, the acoustic waves can be either transverse or longitudinal for any type of excitation and, thus, we can calculate the total acoustic intensity for shear and longitudinal wave incidence:

$$I_{Total}(t) = I_L(t) + I_S(t)$$

Figure 6 shows the simulation is preformed only on aluminum-air interface, then the reverberation time has been estimated at -10 dB from the normalized total acoustic intensity which gives that the RT is equal to 11.3 ms.

5. Experimental Setup

The Experiment setup is necessary to validate the simulation part. For our experience, considering an aluminium block as shown in figure 7 with dimensions $20 \times 20 \times 5$ cm$^3$, giving a volume $V$ of 0.002 m$^3$ and a total surface $S$ of 0.12 m$^2$. In our case, aluminium block is equivalent to an acoustic room where the reverberation occurs and the material deposed on the aluminium is equivalent to the wall where the attenuation occurs. The aluminium has been connected with a set of five piezoelectric (PZT) patches randomly distributed on the bottom of
Figure 5: acoustical intensity (a) for longitudinal incidence (b) for shear incidence for an Aluminium/air interface.

our aluminium. One of piezoelectric patches, used as emitter powered with an electrical signal generated from arbitrary function generator (Agilent 33220a), and four patches as receivers, connected to oscilloscope channels (LECROY HDO4104).

This electrical signal is set to a 10v and corresponds to one 110 kHz sine cycle. Reverberated signals with bulk acoustic waves are created due to this frequency since the wavelength is smaller than the dimension (depth) of our aluminium block. An accurate estimation of reverberation time is obtained by calculating the average of the wave energy over many receivers in our case over four receivers. An averaging 1024 sweep is done in order to increase the signal to noise ratio (SNR). Then the reverberation signals are recorded in our digital oscilloscope, and they are transferred to our computer for signal processing in order to estimate the reverberation time at -10 dB instead of -60 dB. According to the volume and celerity in the aluminium block, it is expected that the reverberation time should decay faster than in acoustic room.

Indeed, to estimate an accurate RT is through a log-linear curve fitting process applied to Schroeders integrals average. Therefore, linear regression is used to approximate the curve with
Figure 6: RT of aluminum air interface (simulation results)

Figure 7: Aluminium block and Margarine. Considering aluminum mold equivalent to room, where reverberation occurs, and Margarine equivalent to the walls, where discrete attenuation occurs.

a straight line, by applying log linear curve fitting over the received signals. The measurement is performed in three distinct cases. In the first case, the measurement is performed only on aluminium block, without margarine. In the second case, the margarine, nearly to liquid state is deposed at the top of the aluminium block. In the third case, the margarine viscosity has changed and is near to solid state after one hour and a half.

Figure 7 (a) correspond to the response of the air surrounded volume although plot (b) correspond to the response after deposed the margarine on the top of our aluminium, while plot (c) correspond to the response after the margarine phase transition from liquid to solid.
Figure 8: Schroeder’s integrals (a), (b) and (c), the first figure corresponds to Aluminium-Air, Whereas the second figure corresponds to Aluminium-margarine (liquid), the third figure corresponds to Aluminium-margarine (solid) interfaces.
6. Results and discussion
In the first case (aluminium block surrounded by air), the attenuation is very low compared to the other cases and this leads to an important reverberation time \((RT=11.86\text{ms})\) because the acoustic impedance’s differ greatly between aluminium and air while from simulation \((RT=11.34\text{ms})\), leads a virtually complete reflection. In the second case (the margarine is in liquid state), less reverberation occur in the aluminium mold \((RT=4.7\text{ms})\) since part of the energy is absorbed by the margarine. In the third case (the margarine is in the solid state), this presents a higher absorption due to better impedance matching \((RT=3.3\text{ms})\).

During the transition of margarine from liquid to solid state, the data are recorded at first 14 ms from every 30 seconds. Then, Schroeder, is applied to every 14 ms signal to determine the reverberation time over all recorded data from the receivers.

Figure 9 shown the variation of the reverberation time in function of time during the phase transition of margarine. A smooth variation of RT from 4.7 ms to 3.3 ms have been observed. It show a relation between RT and phase transition of margarine. In another way, we can link the RT to the phase transition and can be used to monitor margarine transition. It shows, that the reverberation time depends on the viscosity of the material.

![Figure 9: Reverberation time in function of time. (Aluminum-margarine)](image)

7. Conclusion
This paper, present first a short introduction about reverberation characteristics and gives a good estimation of reverberation time from Schröder’s curves, then a theoretical background has been discussed about the sound intensity in room acoustic. The acoustic intensity equation for the two mode of propagation (longitudinal and shear) is described. Using Matlab software, results showed when a longitudinal incident wave strikes an interface, the acoustic intensity for longitudinal wave decrease but the shear one increase before decreasing. This increasing is justified by the energy converted from longitudinal to shear one. An experimental setup has been designed to verify theoretical result. An accurate estimation of reverberation time is obtained during the phase transition of margarine from liquid to solid. We can deduce that the viscosity have an important role in influencing the reverberation time. For example, when the margarine was liquid the reverberation time was 4.7 ms higher compared when the margarine was solid 3.3 ms.
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