Observation of Bose–Einstein condensation in a strong synthetic magnetic field

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Extensions of Berry’s phase and the quantum Hall effect have led to the discovery of new states of matter with topological properties. Traditionally, this has been achieved using magnetic fields or spin-orbit interactions, which couple only to charged particles. For neutral ultracold atoms, synthetic magnetic fields have been created that are strong enough to realize the Harper–Hofstadter model. We report the first observation of Bose–Einstein condensation in this system and study the Harper–Hofstadter Hamiltonian with one-half flux quantum per lattice unit cell. The diffraction pattern of the superfluid state directly shows the momentum distribution of the wavefunction, which is gauge-dependent. It reveals both the reduced symmetry of the vector potential and the twofold degeneracy of the ground state. We explore an adiabatic many-body state preparation protocol via the Mott insulating phase and observe the superfluid ground state in a three-dimensional lattice with strong interactions.

Topological states of matter are an active new frontier in physics. Topological properties at the single-particle level are well understood; however, there are many open questions when strong interactions and correlations are introduced as in the $v = 5/2$ state of the fractional quantum Hall effect and in Majorana fermions. For neutral ultracold atoms, new methods have been developed to create synthetic gauge fields. Forces analogous to the Lorentz force on electrons are engineered through the Coriolis force in rotating systems, by phase imprinting via photon recoil, or lattice shaking. Much of the research with ultracold atoms has focused on the paradigmatic Harper–Hofstadter (HH) Hamiltonian, which describes particles in a crystal lattice subject to a strong homogeneous magnetic field. For magnetic fluxes of the order of one flux quantum per lattice unit cell, the radius of the smallest possible cyclotron orbit and the lattice constant are comparable, and their competition gives rise to the celebrated fractal spectrum of Hofstadter’s butterfly, whose sub-bands have non-zero Chern numbers and Dirac points.

So far, it has not been possible to observe the ground state of the HH Hamiltonian, which for bosonic atoms is a superfluid Bose–Einstein condensate. It is unknown whether this is due to heating associated with technical noise, non-adiabatic state preparation, or inelastic collisions. These issues are complicated, as all schemes for realizing the HH Hamiltonian use some form of temporal lattice modulation and are therefore described by a time-dependent Floquet formalism. The HH model arises after time averaging the Floquet Hamiltonian, but it is an open question to what extent finite interactions and micromotion lead to transitions between Floquet modes, and therefore heating. Bose–Einstein condensation has been achieved in staggered flux configurations and in small ladder systems, further highlighting its noted absence in the uniform field configuration.

In this article, we report the first observation of Bose–Einstein condensation in Hofstadter’s butterfly. The presence of a superfluid state in the HH lattice allows us to analyse the symmetry of the periodic wavefunction by self-diffraction of coherent matter waves during ballistic expansion—analogous to a von Laue X-ray diffraction pattern, which reveals the symmetry of a lattice. Using this method, we show that the unit cell of the magnetic lattice is larger than that of the underlying cubic lattice. This reflects that the vector potential necessarily has a lower symmetry than a uniform magnetic field, because a translationally invariant field can be realized only by a vector potential that breaks this translational invariance. In our experiment, we directly observe a non-gauge-invariant wavefunction and ground state degeneracy. Finally, we explore a many-body adiabatic state preparation protocol where the many-body gap of the Mott insulator is used to switch superfluid order parameters without creating excess entropy. We realize the HH Hamiltonian superfluid ground state in a three-dimensional lattice with strong interactions.

The HH model with large synthetic magnetic fields is realized as an effective Hamiltonian engineered by laser-assisted tunnelling processes in a tilted lattice potential. As in refs 13,14, tunnelling in the $x$-direction of a two-dimensional optical lattice is suppressed by an energy offset, then subsequently restored with a resonant Raman process with a momentum transfer, $\delta k = k_x x + k_y y$. Unless otherwise stated, along the $z$-direction we have loosely confined tubes of condensate, so interactions are weak. In the time-averaged picture, and omitting the harmonic confinement in $z$, this gives rise to the Hamiltonian:

$$H = \sum_{m,n} \left( -K e^{-i\phi_{m,n}} \hat{a}^\dagger_{m+1,n} \hat{a}_{m,n} - J \hat{a}^\dagger_{m,n+1} \hat{a}_{m,n} \right) + \text{H.c.}$$

where $\phi_{m,n} = mk_x a + nk_y a = \pi(m + n)$ reflects the specific gauge implemented (Fig. 1a). $K$ and $J$ are tunnelling amplitudes in the tilted and untilted directions, respectively, and $\hat{a}^\dagger_{m,n}$ and $\hat{a}_{m,n}$ are bosonic creation and annihilation operators on lattice site $(m,n)$. An atom that travels around a single lattice unit cell picks up a phase of $\pi = k_x a$, with $a$ being the lattice spacing, so the Hamiltonian is equivalent to that of a charged particle in a magnetic field with one-half of a magnetic flux quantum per unit cell. In the tight binding limit, a uniform 1/2 flux is identical to a staggered 1/2 flux, with both realizing the fully frustrated Bose–Hubbard model. The results and understanding
Figure 1 | Observation of Bose–Einstein condensation in the Harper–Hofstadter model. a, Spatial structure of the cubic lattice with the synthetic vector potential—(dashed) x-bonds feature a spatially dependent tunnelling phase, whereas tunnelling along (solid) y-links is the normal tunnelling. The synthetic magnetic field generates a lattice unit cell that is twice as large as the bare cubic lattice (green diamond). b, The band structure of the lowest band shows a twofold degeneracy of the ground state. The magnetic Brillouin zone (green diamond) has half the area of the original Brillouin zone. Owing to the twofold degeneracy, the primitive cell of the band structure is even smaller (doubly reduced Brillouin zone, brown square). These lattice symmetries are both revealed in time-of-flight pictures (shown in c–j) showing the momentum distribution of the wavefunction. c–f, Schematics of the momentum peaks of a superfluid. The dominant momentum peak (filled circle) is equal to the quasimomentum of the ground state. Owing to the spatial periodicity of the wavefunction, additional momentum peaks (open circles) appear, separated by reciprocal lattice vectors (green arrows) or vectors connecting degenerate states in the band structure (brown arrows). g, h, Time-of-flight images. The superfluid ground state of the normal cubic lattice is shown in g compared with different repetitions of the same sequence for the superfluid ground state of the HH lattice h. In h, only one minimum of the band structure is filled, directly demonstrating the symmetry in our chosen gauge. The number of momentum components in i,j is doubled again owing to population of both degenerate ground states. The micromotion of the Floquet Hamiltonian is illustrated in e,f,i,j as a periodically shifted pattern in the x direction, analogous to a Bloch oscillation. All diffraction images have a field of view of 631 μm × 631 μm and were taken at a lattice depth of 11E, and 2.7 kHz Raman coupling, with at least 30 ms hold in the HH lattice.

derived for staggered flux apply equally to our system with uniform flux28,29,31.

After preparing the atoms in the HH lattice (see Methods), we observe the momentum distribution of the wavefunction by suddenly turning off all laser beams to allow 20 ms time of flight, and measure the resulting density distribution with absorption imaging (Fig. 1a–j). We first note that the images show sharp peaks, which are the hallmark of a superfluid Bose–Einstein condensate in a periodic potential. This demonstrates that we have successfully prepared a low-entropy state in the bulk HH Hamiltonian for the first time. In addition, the diffraction images directly show a reduced symmetry of the superfluid, despite the translational symmetry of both the lattice and the homogeneous synthetic magnetic field.

Fundamentally, the vector potential is responsible for the broken translation symmetry of the lattice, and the time-of-flight patterns can be understood by examining these symmetries. In real space, there are three relevant unit cells: the unit cell of the cubic lattice, the unit cell of the Hamiltonian determined by the experimental gauge, which we call the gauge unit cell, and the magnetic unit cell, which is gauge independent and is the smallest unit cell with an integer number of flux quanta32. For a magnetic flux $\alpha = p/q$, the magnetic unit cell is q times larger than the original unit cell and contains q indistinguishable sites30. In highly symmetric gauges—such as our experimental gauge and the Landau gauge—the gauge unit cell can serve as the magnetic unit cell and it is not necessary to distinguish between the two. In reciprocal space, these three unit cells correspond to the Brillouin zone of the underlying lattice, the gauge Brillouin zone, and the magnetic Brillouin zone.

Owing to the indistinguishability of the sites in the magnetic unit cell, every quasimomentum state within the magnetic Brillouin zone is q-fold degenerate with other quasimomentum states connected by the translation symmetry operators of the original lattice modified by a phase $2\pi/q$. This modified translation symmetry connects all degenerate ground states in the magnetic Brillouin zone and corresponds to another relevant scale $2\pi/(qa)$ useful for understanding the diffraction pattern of a superfluid in the HH Hamiltonian. This length scale defines the doubly reduced Brillouin zone, which is the primitive cell of the band structure30.

Experimentally, we observe a gauge-dependent diffraction pattern directly when the condensate occupies only one minimum of the band structure (Fig. 1a). The pattern's symmetry reflects that of the gauge potential (Fig. 1d), directly demonstrating the symmetry of the reciprocal lattice vectors $a(\hat{x} \pm \hat{y})$ of the gauge Brillouin zone. A common belief is that all observables are gauge-independent. However, for synthetic magnetic fields, gauge-dependent observations can be made in time-of-flight images of ultracold atoms when the momentum distribution of the wavefunction is observed. The sudden switch-off of all laser beams preserves the wavefunction, and canonical momentum, which is gauge-dependent, becomes mechanical momentum, which is readily observed28,29,31. Alternatively, one could define a gauge-dependent, synthetic electric field given by $E = −\partial \phi$, which changes the mechanical momentum by $A$ (Fig. 1f) during the switch-off33. This is in contrast to a sudden switch-off of a real magnetic field which creates a real electric field, $E = −\partial \phi + \mathbf{A}$, that accelerates the charged particles and leads to a momentum distribution which is gauge independent. The observed momentum distributions for both synthetic and real magnetic fields are the same only for special gauges where the electrostatic potential $\phi$ does not contribute to the electric field.

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The diffraction pattern shown in Fig. 1i reveals the symmetry of the band structure (Fig. 1b). In the experimental image, both degenerate states are equally populated and the peaks can be regarded as the sum of the diffraction pattern in Fig. 1h with an additional copy separated by the gauge-invariant scale $2\pi/(qa) = \pi/a$ in either $x$ or $y$ directions. The diffraction pattern reflects this twofold multiplicity of momentum peaks in both the $x$- and $y$-directions and features a fourfold increase of visible peaks $^{31}$. To the best of our knowledge, this work is the first direct observation of the different symmetries characterizing a simple lattice with a homogeneous magnetic field: Fig. 1g–i shows the symmetry of the lattice, the gauge potential and the band structure. Realization of a Bose condensed ground state can allow direct observation of both gauge-dependent and gauge-independent symmetry breaking.

A final feature of the observed diffraction pattern is the shot-to-shot movement of the peaks along the $x$-axis which can be caused by micromotion coming from acceleration due to the tilt and Raman drive. In the limit of a low drive amplitude, the micromotion is understood as a Bloch oscillation in the tilted lattice. More generally, it is the effect of a time-dependent transformation between the rotating frame in which the HH Hamiltonian is realized and the lab frame in which it is observed $^{31}$. Also, relative movement of the harmonic trapping potential with respect to the sites within the gauge unit cell can produce shifts in the peak positions $^{35}$. Figure 1e,f,i,j shows diffraction patterns for peak positions corresponding to micromotion at zero phase and at $\pi/2$ phase—equivalent to zero and $\pi$ phases in a Bloch oscillation due to the doubling of the unit cell. In the experiment, the observed diffraction patterns are random owing to wavelength-scale drifts of the relative positions of the stationary and Raman lattices and the harmonic confinement beams.

**Figure 2 | Population imbalance of the two ground states of the HH Hamiltonian with 1/2 flux.** a. Band mapping sequence adiabatically connecting quasimomentum to free space momentum. The Raman beams were ramped down from the initial strength of $1.4E_r$ to zero in 0.88 ms, followed by a linear ramp down of the lattice beams from $11E_r$ to zero in 0.43 ms. b. The histogram shows the relative population imbalance of the two degenerate minima. Equal population in the two diffraction peaks is suggestive of domain formation due to spontaneous symmetry breaking, but can also be driven by lattice noise and technical fluctuations. The data consists of 30 shots taken after a hold time of 29.4 ms in the HH lattice. The inset shows a raw image for the band mapping of the 1/2 flux superfluid with two degenerate ground states compared with a topologically trivial superfluid with one ground state (see text). The Brillouin zones of the cubic lattice (grey) and the gauge (green) are overlaid for clarity.

Qualitatively, almost all images show a roughly equal population of both degenerate ground states. Occupation of only one state (as in Fig. 1h) is observed in less than 1% of the shots. For a quantitative determination of the populations, we developed a band mapping technique that first adiabatically connects the HH ground states to cubic lattice quasimomentum by adiabatically lowering the modulation strength, and then maps to free momentum states with a rapid adiabatic lowering of the lattice depth (see Fig. 2a and Supplementary Information). Band mapping reduces the complicated diffraction patterns of Fig. 1 to two peaks, one for each degenerate ground state.

In the first mapping, quasimomentum in the HH ground state is conserved because the system remains locally gapped as the modulation is reduced to zero. The second mapping from the cubic lattice to free space modes is complicated by the presence of a large gradient in the system which can excite population to higher bands of the cubic lattice as the gap closes at the Brillouin zone edge. The second mapping is performed in a rapid adiabatic procedure where the ramp time is slow compared to the initial band gap, but is rapid in the regime where the gap is smaller than the Bloch oscillation frequency. The resulting momentum distributions for both a trivial superfluid and the HH superfluid are shown in Fig. 2b.

For the HH lattice, the histogram in Fig. 2b shows that the minima are most frequently populated with equal proportion, demonstrating the degeneracy of the minima and the robustness of the loading procedure to technical fluctuations. In different gauges, such as the Landau gauge, it is predicted that the weakly interacting ground state is a superposition state of the two single-particle quasimomentum eigenstates to avoid density modulation $^{29,30}$. Interestingly, in the gauge we implement, the quasimomentum eigenstates already minimize the interaction energy owing to uniform occupation of the two sites in the gauge unit cell. As a consequence, we interpret the observed equal populations to be a result of domain formation due to non-adiabatic state preparation. Domain formation in a different Floquet system has been studied recently, and it was shown that control over domain formation can be achieved with proper adiabatic state preparation $^{36}$.

**Figure 3 | Decay of Bose–Einstein condensates in modulated lattices.** The figure compares the decay of the 1/2 flux HH superfluid (red circles) against the decay of the AM superfluid (blue squares). Note that the lower visibility of the HH superfluid is due to the peak doubling, which at the same condensate fraction leads to lower visibility. Exponential fits to the decay of the visibility of the diffraction patterns give lifetimes of 142 ± 15 ms and 71 ± 8 ms, respectively. Data were taken with an $11E_r$ cubic lattice with either 2.7 kHz Raman coupling or 20% amplitude modulation, and start after a 10 ms hold time after switching on the final Hamiltonian using the non-adiabatic procedure (see Methods). Uncertainty is given by the statistical error of the mean of five repetitions of the experiment, added in quadrature to uncertainty in the peak visibility fitting.
The above results confirm many aspects of the weakly interacting superfluid ground state of the HH Hamiltonian. However, there are many open questions about topological physics in the presence of strong interactions, often involving small energy scales. To study this regime, long coherence lifetimes are necessary.

To evaluate our coherence lifetime and disentangle different sources of decoherence and heating, we extensively used a topologically trivial Floquet superfluid as a benchmark (see Methods). This system consists of a cubic lattice with a tilted potential where tunnelling in the tilted axis is restored by amplitude modulation (AM; refs 37,38). Its micromotion, which is similar to that of the HH system, and its trivial band structure allow clearer identification of technical sources of heating.

Figure 3 shows the lifetime of the visibility of the diffraction pattern for both AM and HH superfluids. The fitting routine and definition of diffraction visibility are detailed in the Supplementary Information. Initially, the HH superfluid had a vastly shorter lifetime than the AM superfluid; however, subsequent improvements in the stability of magnetic fields, beam pointing, Raman phase, and gradient alignment to the lattice direction improved the HH lifetime until both systems have a comparable lifetime in the ground state. The lifetimes for the number of trapped atoms are much longer than the coherence lifetime in either case, with no discernible loss of atom number up to 500 ms. This leads to the conclusion that two-body dipolar interactions and three-body recombination, which would lead to particle loss from the trap, are not limiting the lifetime of the ground state. The same applies to excitations to higher-lying bands which, owing to the tilt, are strongly coupled to the continuum via Landau–Zener tunnelling. Therefore the decay in Fig. 3 is dominated by transitions within the lowest band, which can be caused by technical noise or by elastic collisions that transfer energy from the micromotion into heat. Although our heating seems to be largely technical, it is useful to briefly discuss these collisions as an ultimate limit on the lifetime.

For the tilted lattice, one decay path is via overlap of neighbouring Wannier–Stark states with offset energy $\Delta$, which can be transferred to excitations of the lowest bands, or to the free particle motion along the tubes orthogonal to the two-dimensional (2D) lattice. Such processes are described by Fermi’s golden rule for transitions between Floquet states$^{39,40}$. Using this framework, we derive the scattering rate for the AM superfluid to be between $I \sim 0.30–0.68$ s$^{-1}$, where the uncertainty comes from the uncertainty in our density measurement due to redistribution during lattice ramp-up. This estimate is smaller than the observed decay rate by a factor of $\sim$10–20. See Supplementary Information for a derivation and experimental parameters.

In principle, an adiabatic procedure to prepare the ground state of the HH Hamiltonian should enable better control and higher fidelity in the final state. However, we empirically found that a sudden turn-on of the tilt and the Raman beams gave the most consistent high-contrast images, and was therefore useful in evaluating technical improvements. A slower turn-on could introduce more technical heating, and the entropy created in the sudden turn-on could be absorbed by other degrees of freedom. Such an entropy reservoir is provided in the 2D lattice schemes by the third dimension—in our case tubes of condensates with $\sim$500 atoms. For extensions to 3D lattices, we implement an adiabatic state preparation procedure.

Adiabatic methods usually require a pathway where the ground state is protected by a gap. For single-particle states, this may involve matching the size of the unit cell between a topologically nontrivial lattice and a trivial lattice$^{20,23}$. Instead, we explore the use of the many-body gap of the bosonic Mott insulator state. As the Bose–Hubbard model and the HH model with interactions have the same Mott insulating ground state for large $U$ (assuming the high-frequency limit $U \ll \Delta$), one can connect a trivial superfluid in the cubic lattice to the ground state of the HH model via two quantum phase transitions. The first transition ‘freezes’ out the original superfluid by entering the Mott insulator, where the phases of the tunnelling matrix elements can be changed to a non-zero flux configuration without adding entropy$^{46}$. The second transition ‘unfreezes’ the Mott state into the HH superfluid. Technically, this approach cannot be fully adiabatic owing to the exact degeneracy of the ground states in the HH spectrum; a Kibble–Zurek model implies that this will lead to the spontaneous formation of domains$^{46}$. However, this will not create entropy beyond the randomization of domains.

We implement this scheme to load the ground state of the HH Hamiltonian. So far, we have found the adiabatic turn-on to be less robust against technical noise, leading to a higher shot-to-shot variability; however, the best images have identical visibility and
show a marginally better lifetime than that of the non-adiabatic state preparation method (Fig. 4).

We can now use the lattice in the third dimension, required for the adiabatic loading protocol, to add stronger interactions to the physics of gauge fields. To do this, we keep the lattice depths and tunnelling strengths along the x- and y-directions constant, but vary the depth of the z-lattice, thereby adjusting the Hubbard interaction parameter U from nearly zero to ~450 Hz. With an estimated filling factor of ~5, we can access interaction energies in excess of $h \times 2k$Hz, which is close to the superfluid-to-Mott insulator transition in high magnetic fields$^3$.

We realize HH superfluids with lattice depths from 0 to 20E, and observe at least weak superfluid diffusion peaks (see Supplementary Information). Coherence lifetimes for z-lattice depths up to 11E, are presented in Fig. 5. The average visibility of the diffraction pattern is reduced at high z-lattices, but the lifetimes are reduced only by a factor of ~2–4 from the lifetime of the weakly interacting superfluid without a z-lattice. However, we observe much larger shot-to-shot variation: some shots will have clearly visible diffraction peaks, whereas other attempts show no coherence at all. We attribute the variation to technical fluctuations, which seem to have an enhanced effect at higher 3D lattices.

In conclusion, we have observed Bose–Einstein condensation in a strong synthetic magnetic field. We have directly observed how the gauge potential breaks the symmetry of the original lattice.

Methods

Methods and any associated references are available in the online version of the paper.

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Author contributions

All authors contributed to experimental work, data analysis and manuscript preparation.

Additional information

Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to C.J.K.

Competing financial interests

The authors declare no competing financial interests.
Methods
The experiment begins with a nearly pure BEC confined in a crossed dipole trap in the $|1, -1\rangle$ hyperfine state with $\sim 1 \times 10^7$ $^{87}$Rb atoms. We turn on a magnetic field gradient to levitate the atoms against gravity and simultaneously weaken the dipole traps by lowering the power to their final values. From here, the non-adiabatic and adiabatic sequences differ.

In the non-adiabatic sequence, the condensate is adiabatically loaded into a two-dimensional optical lattice composed of one vertical and one horizontal standing wave, with lattice constants 532 nm and a depth of 11 recoil energies ($E_r$) in both the $x$- and $y$-directions, in the presence of a very weak Raman lattice ($\sim 0.1 E_r$) with relative frequency detunings equal to the Bloch oscillation frequency, 3.420 kHz. The two Raman beams are derived from the same 1,064.2 nm laser source as the lattice beams, and are offset by $\sim 80$ MHz by means of an acousto-optical modulator (AOM). Raman beams propagate along the $x$- and $y$-directions of the lattice, respectively, and are made to beat together on a beam cube located as close to the atoms as our optical set-up allows. The beat note is measured with a photodiode and the Raman phase is detected by a lock-in amplifier at 3.420 kHz. The error signal is used to feedback to the phase of the radiofrequency (RF) drive of the AOM to stabilize the relative phase of the Raman beams.

Once a phase lock is achieved, we turn on a large tilt in the regime $J < \Delta < E_{\text{gap}}$ by sweeping the frequency of an RF field in 0.29 ms to transfer all the atoms to the $|2, -2\rangle$ hyperfine state, which reverses and doubles the magnetic moment. Here $J$ is the bare tunnelling energy in the lowest band, $\Delta = 3.420$ kHz is the energy offset between adjacent sites, and $E_{\text{gap}}$ is the energy gap between the lowest band and the first excited band. On completion of the RF sweep, the initial system represents an array of 1D tubes resonantly coupled in the $y$-axis and decoupled in the tilted $x$-axis.

For topologically trivial superfluids, resonant tunnelling is re-established with amplitude modulation of the vertical ($x$-axis) lattice with modulation frequency $\Delta$. For the $1/2$ flux superfluid resonant tunnelling is instead re-established by linearly ramping up both Raman beams in 0.58 ms to $2\Omega / \Delta = 1.6$, where $\Omega$ is the two-photon Rabi frequency. After a variable hold time, all laser beams are switched off to allow 20 ms time of flight, followed by absorption imaging. Our scheme for generating synthetic magnetic fields is general and adaptable to many different fluxes—we have implemented the $1/2$ flux configuration because of readily available laser sources and compatibility with the optical access in our vacuum chamber.

In the adiabatic sequence, the condensate enters the Mott insulating phase after lattices in all three directions are adiabatically ramped up to $20 E_r$. One of the Raman beams is then ramped to its final value in 15 ms, after which the hyperfine state is flipped with a 0.29 ms RF sweep, as explained previously. The lattices are then ramped down to their final values while the second Raman beam is ramped up in 35.09 ms, thereby adiabatically connecting the Mott insulator to the $1/2$ flux superfluid. Again, time-of-flight absorption imaging is performed after a variable hold time.

The Mott transition for the HH model with $1/2$ flux at a filling factor of 5 occurs at $t / U \sim 0.016$ (ref. 41). For an isotropic HH lattice, the transition happens around 15–16$E_r$, with the precise value depending on multiple lattice parameters, such as anisotropy, Raman drive strength and tilt strength. For lower-density parts of the cloud, the critical $t / U \sim 0.059$ is realized at lattice depths closer to $12E_r$. The lower critical $t / U$ value for the Mott transition in the HH model means our experiments in three-dimensional lattices happen close to the strongly interacting regime and the insulating phase.