Crack detection in a shaft using finite difference technique

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Abstract. Fatigue cracks arise in a rotor shaft due to cyclic loading. A growing crack in a machine element may lead to a calamitous failure of the machinery. Identification of crack at an early stage supports in safe and economic running of machinery. The appearance of crack in a shaft reduces local stiffness and causes a slope discontinuity in the shaft elastic line during vibration. Detection of slope discontinuity reveals the presence of crack in a shaft. However, in the presence of measurement noise, identification of slope discontinuity is difficult. In the present work, finite element model of a cracked simply supported shaft is considered for the simulation work. A transverse surface crack is considered for the cracked model. The shaft is assumed to be non-rotating and an external sinusoidal forcing is used to excite the shaft. To imitate actual experimentation, Gaussian noise is added in the simulated cracked shaft response. A dominant peak is observed at the crack position, which is obtained by using the central finite difference approximation on the simulated response of simply supported shaft. The method is used for the localization of the cracks present in the shaft. It is based on detecting the slope discontinuity in the elastic line of the shaft. The finite difference method based evaluation is also implemented for varying crack depth ratios at different signal to noise ratio (SNR).

1. Introduction
Condition monitoring of rotating machines is important for continuously running plant such as power generation industries. Some rotor faults, such as cracks in the shaft need to be detected at early stage. The crack appears in rotating machinery due to the periodic stresses imposed in the shaft. Presence of crack in a shaft affects flexibility and entire dynamic vibrational response of the shaft. Vibration measurements from rotating machines have important information regarding the condition of the machineries. A crack in the shaft has mainly two parameters its depth and position. The crack position can’t be seen directly from the shaft response. The investigation of the dynamic behaviour of cracked rotor can lead to proper diagnosis of machinery and prevent the machinery from sudden failure [1-2]. A thorough state-of-the art reviews are presented by [3-4]. In the earlier literature [5-6], the damage identification has been mainly focused on modal analysis of vibration signals. These analyses mainly use frequency and mode shape information.
The appearance of crack in shaft reduces local stiffness which causes slope discontinuity in the shaft elastic line. Resulting slope discontinuity is localized and hence its identification reveals the crack existence in the shaft. A multi crack detection and localization algorithm (MCDLA) is developed [7-8] for the detection of slope discontinuity due to cracks in a shaft. The reduction of stiffness causes abrupt change in the first derivative of deflection i.e. slope and second derivative of deflection i.e. curvature. The observation of these modal derivatives helps to identify the crack in the structure [9]. A Vibration-based Structural Health Monitoring (SHM) method was developed for cantilever beam with the artificial delamination to analyse mode shapes for different crack depth and locations. The analysis was performed on ANSYS software. Curvature mode shapes were calculated by using a central finite difference approximation to identify the crack position [10]. In the present paper, the finite difference method is used to detect the crack position in the rotor shaft.

The purpose of the present paper is to present the potentiality of the proposed method using simulated signals. The present paper is structured as follows: Section (2) introduces the finite difference method and the term signal to noise ratio (SNR). Section (3) presents the mathematical model of the rotor shaft with a transverse open crack which is based on finite element method. The simulated response of the cracked rotor shaft with measurement noise is evaluated in section (4). In this section, we also discussed the finite difference method based crack detection results at different SNR values. Finally, the paper is concluded in Section (5).

2. Theory

2.1 The proposed finite difference method
The finite difference method plays an imperative role for the numerical solution of differential equations, mainly in boundary value problems. In the present paper we are using second order central difference approximation, as defined below

\[ \frac{d^2 y}{dz^2} \approx \frac{y(i+\Delta z) - 2y(i) + y(i-\Delta z)}{(\Delta z)^2} \]  

(1)

where, \( y(i) \) is the point of interest, \( y(i+\Delta z) \) is the point of interest plus step size, \( y(i-\Delta z) \) is the point of interest minus step size, \( \Delta z \) is the step size.

2.2. Signal to noise ratio (SNR)
Finite difference based evaluation is required to be consistent in the presence of measurement noise for efficient analysis. In order to specify the effect of white Gaussian noise in the signal, the SNR is defined in dB as \( \text{SNR} = 20 \log_{10}(Y_s/Y_n) \) dB. Here \( Y_s \) and \( Y_n \) is the standard deviation of signal and white noise respectively.

3. Mathematical model of a cracked rotor shaft
The Timoshenko beam theory is used in the modelling of the rotor shaft. A discretized model of the cracked simply supported shaft is developed by using finite element method. The finite element model of the rotor shaft with a transverse open crack is shown in figure 1. Each node of shaft element has two rotational and two translational degrees of freedom.
Figure 1. Finite element model of the rotor shaft

These degrees of freedom describes the position and rotation at the \( i^{th} \) node of a shaft element, which is collected in the generalized displacement vector \( X_i \).

\[
X_i^T = [u_{xi} \quad u_{yi} \quad \theta_{xi} \quad \theta_{yi}]
\]

A cracked shaft element and its generalized displacements at the nodes \( i \) and \( i+1 \) are shown in the figure 2.

Figure 2. Cracked shaft element with generalized displacement at nodes \( i \) and \( i+1 \)

Here \( u_{xi} \) and \( u_{yi} \) are translational degree of freedom along x and y direction and \( \theta_{xi}, \theta_{yi} \) are rotational degree of freedom along x and y direction respectively.

The governing equation of motion for non-cracked rotor shaft system is

\[
[M]\ddot{\{X\}} + [C]\{\dot{X}\} + [K]\{X\} = \{F\}
\]

(2)

In case of cracked rotor-shaft system, a crack is assumed to be placed within finite element of the model. The presence of crack increases the local flexibility, which affect stiffness. The total flexibility matrix of the cracked shaft element is given by,

\[
[S] = [S_0] + [S_c]
\]

(3)

where \([S_0]\) is the flexibility in an uncracked element and \([S_c]\) is the additional flexibility in the shaft element due to the crack. The term \([S_c]\) is expressed as [2],

\[
[S_c] = \begin{bmatrix}
S_{22} & 0 & 0 & 0 \\
0 & S_{33} & 0 & 0 \\
0 & 0 & S_{44} & S_{45} \\
0 & 0 & S_{54} & S_{55}
\end{bmatrix}
\]

where \([S_{22}], [S_{33}], [S_{44}], [S_{55}] \) are the direct and \([S_{45}], [S_{54}] \) are the cross-coupled flexibility coefficients. Now, the stiffness of the cracked shaft element is obtained as [2]

\[
[K_c]^e = [T]\ [S]^{-1}\ [T]^T
\]

(4)

where \([T]\) is a transformation matrix.

The assembled mass matrix of the non-cracked system is considered to be same as that of the cracked system. Now, the governing equation of motion for cracked rotor shaft system can be written as:

\[
[M]\ddot{\{\dot{X}\}} + [C_c]\{\dot{X}\} + [K_c]\{X\} = \{F\}
\]

(5)
Here, $[M]$ is assembled mass matrix. The term $[K]$ and $[K_c]$ are assembled stiffness matrices of a rotor shaft in the absence and presence of crack respectively. The effect of the proportional damping is also included in the rotor-shaft system, so the assembled damping matrices of a rotor shaft in the absence and presence of crack are $[C] = a_0[M] + a_1[K]$ and $[C_c] = a_0[M] + a_1[K_c]$ respectively, where $a_0$ and $a_1$ are the Rayleigh damping factors. The term $\{X\}$ is the nodal displacement vector of the rotor shaft and $\{F\}$ represents the excitation force vector. Dimension and material properties for the rotor shaft system is given in table 1.

### Table 1. Details of the rotor shaft parameters

| Parameters                             | Physical dimensions |
|----------------------------------------|---------------------|
| Shaft length, $L$ (m)                  | 1                   |
| Shaft diameter, $D$ (m)                | 0.01                |
| Number of elements on the shaft        | 500                 |
| Cracked element position              | 300                 |
| Young’s modulus, $E$ (N/m$^2$)         | 2.06x10$^{11}$      |
| Poisson’s ratio, $\nu$                | 0.3                 |
| Density, $\rho$ (Kg/m$^3$)            | 7800                |

4. Results and discussions

4.1. Response of a cracked shaft

A simply supported rotor shaft with a transverse open crack is considered for the simulation. The shaft is supposed to be non-rotating and an external forcing is applied in vertical direction to excite the shaft. The cracked shaft is excited at 110 radian/sec of forcing frequency. This excitation frequency is near the first natural frequency of shaft. The vertical response in the space domain is continuous by nature. In the real scenario, the shaft response is contaminated by measurement noise. Hence to imitate real experimentation, white Gaussian noise is added in the simulated shaft response. The response of the cracked rotor shaft is shown in figure 3. The noise level (SNR) is kept at 90 dB. This shaft response is further used as an input signal for the finite difference method based analysis.

![](image.png)

**Figure 3.** Deflection response of the shaft with crack

4.2. Detection of crack position at different SNR level

The appearance of crack in a rotor shaft causes slope discontinuity in the simply supported shaft response. The resulting slope discontinuity is very difficult to visualize. Derivative of the shaft elastic line amplifies any discontinuity in the shaft elastic line due to presence of crack. In the present work, the finite difference method (equation 1) is used for the localization of the cracks present in the shaft. The presence of local discontinuity is revealed as a spike at the crack location. Figure 4 shows the results obtained by applying the finite difference method, which is represented by formula in equation
1, on the cracked shaft response at different SNR values. The crack depth ratio \((\alpha/D)\) is taken 0.33 in this case, where \(\alpha\) is the crack depth. Here, the dominant peak shows the position of the crack. The crack is detectable up to 85 dB of SNR.

![Figure 4. Crack identification for relative crack depth ratio \((\alpha/D)=0.33\)](image)

When the crack depth ratio \((\alpha/D)\) is increased to 0.50 then the crack is detectable at lower SNR. In figure 5, the dominant peak corresponding to the crack location is observed at different SNR values.
Figure 5. Crack identification for relative crack depth ratio ($\alpha/D=0.50$)

Hence the present method is employed successfully to detect the crack position in the rotor shaft. This method can also be employed to detect the cracks in the beams and plates. The present algorithm needs high resolution measurement of shaft deflection. High resolution measurement of shaft deflection can be obtained by using scanning laser vibrometer based measurement and photographic methods.

5. Conclusion
A finite difference based approach is proposed to detect slope discontinuity in the shaft elastic line. Finite element modelling of a rotor shaft system has been performed for the study of vibration features of the rotor shaft with a single transverse open crack. The vibration response of the rotor shaft is analysed in the spatial domain. White Gaussian noise is added to the simulated response to imitate actual experimentation. Slope discontinuity is detected as a sharp peak in the second derivative of the
shaft deflection. The finite difference method based evaluation successfully identified the crack position for varying crack depth ratios ($\alpha/D$) at different signal to noise ratios (SNR) conditions.

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References
[1] Sekhar A S, Balaji P P 1999 Journal of Sound and Vibration 208 457-474
[2] Sekhar A S, Prabhu B S 1994 Journal of Sound and Vibration 173 415–421
[3] Dimarogonas A D 1996 Engineering Fracture Mechanics 55 831-857
[4] Wauer J 1990 Applied Mechanics 43
[5] Schultz A B, Warwick D N 1971 Journal of Composite Materials 5 394-404
[6] Cawley P, Adams R D 1979 Journal of Strain Analysis 14 49-57
[7] Singh S K, Tiwari R 2010 Mechanism and Machine Theory 45 1813-1827
[8] Singh S K, Tiwari R 2012 Fatigue & Fracture of Engineering Materials & Structures 36 85-91
[9] Chance J C, Tomlinson G R, Worden K 1994 In the Proceedings of the 12 International Modal Analysis Conference 778-785
[10] Raju G, Ramesh L 2016 International Journal for Innovative Research in Science & Technology 3 282-289