Cross-channel constraints on resonant antikaon-nucleon scattering

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Chiral perturbation theory and its unitarized versions have played an important role in our understanding of the low-energy strong interaction. Yet, so far, such studies typically deal exclusively with perturbative or nonperturbative channels. In this letter, we report on the first global study of meson-baryon scattering up to one-loop order. It is shown that covariant baryon chiral perturbation theory, including its unitarization for the negative strangeness sector, can describe meson-baryon scattering data remarkably well. This provides a highly non-trivial check on the validity of this important low-energy effective field theory of QCD. We show that the $K\bar{N}$ related quantities can be better described in comparison with those of lower-order studies, and with reduced uncertainties due to the stringent constraints from the $\pi N$ and $K\bar{N}$ phase shifts. In particular, we find that the two-pole structure of $\Lambda(1405)$ persists up to one-loop order reinforcing the existence of two-pole structures in dynamically generated states.

Introduction: Resolving the patterns of low-energy hadron-hadron interactions constitutes one of the most important goals of modern theoretical physics, fostering not only a better understanding of the nonperturbative nature of QCD but also motivating novel tests of fundamental symmetries including searches for beyond standard model physics. Furthermore, the quantitative understanding of hadron-hadron interactions in the different strangeness sectors has important implications for matter on the largest scales in the universe. For example, in the strangeness zero sector, $\pi N$ scattering is related to the $\sigma_{\pi N}$ term, crucial for dark matter direct-detection efforts [1, 2]. Another example with negative strangeness relates antikaon-nucleon scattering to the properties of neutron stars, where matter compressed to multiples of nuclear matter densities may allow for the appearance of kaon condensates [3, 4]. Related to this is an ongoing controversy about the existence of deeply bound $K^-$ nuclear clusters [5–11], sensitive to the $K^N$ interaction. The interest in the strangeness sector has motivated several ongoing or future experiments including SIDDHARTA-2 [12–14], AMADEUS [15, 16], BGO-OD [17], J-PARC E15, E31, E57 and E62 [18–21], PANDA [22, 23], and the proposed secondary $K_L$ beam in Hall D of Jlab [24, 25].

Bridging the different strangeness sectors of the meson-baryon interaction in a systematic fashion has been a major challenge, which is faced in this letter. Systematic theoretical approaches to such interactions are provided by lattice QCD and chiral perturbation theory (CHPT). Following the latter methodology1 baryon chiral perturbation theory (BCHPT) is known to be able to describe both pion-nucleon [28–34] and kaon-nucleon [35, 36] scattering data rather well up to one-loop order. However, in a strict perturbative application of BCHPT, such calculations do not allow to simultaneously study the antikaon-nucleon channel due to the existence of the subthreshold $\Lambda(1405)$-resonance. In this, unitarized CHPT with kernels from both leading [37–40] and next-to-leading order (NLO) [41–46] CHPT became the predominant tool, including a prediction of a second pole [39, 47]. See Refs. [47–50] for recent reviews. Such models have also been shown to be consistent with modern experimental data, such as $K^-p \to \pi^0\pi^0\Sigma^0$ and $\gamma p \to K^+ (\pi\Sigma)$ [51–55]. Nevertheless, on a quantitative level, various NLO studies obtain different results depending on the details of the implementation [48, 56, 57], particular for the pole positions of the $\Lambda(1405)$. One hypothesis for the origin of this ambiguity is related to the large number of unknown low-energy constants (LECs) appearing in the NLO kernel. In this letter, we aim to improve on this by employing novel constraints from SU(3)-flavor symmetry and its breaking, simultaneously addressing the $\pi N$, $K\bar{N}$ and $K^N$ channels.

A simultaneous study of all the meson-baryon scattering data is difficult for a number of reasons. First, it is well-known that an adequate description of pion-nucleon scattering at energies below the first resonance ($\Delta(1232)$), i.e., $\sqrt{s} \approx 1.16 \text{ GeV}$, requires the inclusion of the next-to-next-to-leading order (NNLO) contributions [31, 35]. Second, the convergence of BCHPT in the three-flavor sector is controversial. Only in recent years it has been shown that this problem can be circumvented or alleviated in the extended-on-mass-shell (EOMS) formulation of BCHPT [58, 59]. For some recent applications, see, e.g., Ref. [60] for the case of baryon magnetic moments, Ref. [61] for the case of baryon masses,

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1 For reviews on recent progress in extracting resonant hadron-hadron dynamics from lattice QCD see, e.g., Refs. [26, 27].
and Ref. [62] for a review.

Of particular importance for the present work is the unifying NNLO BCHPT calculation of \( \pi N \) and \( \bar{K}N \) scattering performed in Ref. [35]. Unifying this with a nonperturbative formulation of the \( \bar{K}N \)-amplitude is the main challenge attacked in this work, which allows one to reliably determine the free parameters (more than 30 LECs) making advantage of SU(3) flavor symmetry and abundant scattering data.

Notably, including \( \bar{K}N \) data allows one to disentangle all LECs, instead of determining only certain combinations. In addition, for the \( \bar{K}N \) channel, there is less freedom in the NNLO fits with \( \bar{K}N \) and \( \pi N \) constraints than in the NLO fits without them, which is reflected in the reduced uncertainty. Indeed, apart from providing the first NNLO study of the EOMS formulation [63], it allows for the inclusion of the chiral expansion.

Indeed, apart from providing the first NNLO study of the EOMS formulation [63, 66], the \( \Lambda(1405) \) resonance appears around the \( \bar{K}N \) threshold. Addressing this issue is guided by restoring two-body unitarity – the fundamental principle of S-matrix theory – for which we follow the method proposed in Ref. [39].

\[
T_{\text{BCHPT}}^{l\pm}(s) = T^{(1),l\pm}(s) + T^{(2),l\pm}(s) + T^{(3),l\pm}(s),
\]

(1)

up to the NNLO with respect to small meson momenta and masses, where the superscript \( l\pm \) denotes that the amplitudes are projected to the partial waves with orbital angular momentum \( l \) and total angular momentum \( J = l \pm \frac{1}{2} \). The total energy-squared is denoted by \( s \). The terms \( T^{(i)} \) can be found in Ref. [35]. As shown in the latter reference such expressions indeed allow to address the perturbative regime of the meson-baryon scattering below the lowest resonances quite successfully. Specifically, this is the case for the \( K\bar{N} \) and \( \pi N \) channels close to respective thresholds. The situation is entirely different in the \( S = -1 \) channel, where the \( \Lambda(1405) \)-resonance appears around the \( \bar{K}N \) threshold. Addressing this issue is guided by restoring two-body unitarity – the fundamental principle of S-matrix theory – for which we follow the method proposed in Ref. [39].

\[
T_{\text{K\bar{N}}}^{l\pm}(s) = \left[ 1 + N^{l\pm}(s) \cdot G(s) \right]^{-1} \cdot N^{l\pm}(s),
\]

(2)

Here, all elements are promoted to matrices in channel space, \( \{ \pi^0, \pi^\pm, \eta, \eta' \} \), while \( G(s) \) denotes a loop function, given explicitly in the Supplemental Material.

Characteristic to all unitarization procedures is the introduction of certain model-dependence [48] being also related to the appearance of the new unknown parameters \{ \( \alpha_{\pi\Lambda}, \alpha_{\eta\Sigma}, \alpha_{K\bar{N}}, \alpha_{\eta'\pi}, \alpha_{\eta'\Sigma}, \alpha_{K\bar{K}} \) \}, corresponding to each combination of multiplets of the aforementioned channel space. However, the crucial feature of the employed program lies in the fact that both amplitudes (1,2) coincide exactly when expanded to the third chiral order. Ultimately, this allows one to reduce uncertainties of extracted amplitudes and poles compared to the NLO calculations of individual strangeness sectors, by combining the analysis of perturbative \( \pi N \) and \( \bar{K}N \) and nonperturbative sector \( \bar{K}N \) corresponding to Eqs. (1) and (2). The larger number of fit parameters at NNLO compared to NLO is outweighed by the larger data base of the global analysis.

Results and discussions: The formalism proposed in this letter is derived from the \( \mathcal{O}(p^3) \) Chiral Lagrangian [73–75]. The counterterms appearing in it contain 27 LECs, 14 of \( \mathcal{O}(p^2) \) \( \{ b_0, b_7, b_{30}, b_{33,\pm}, b_{31,\pm} \} \) and 13 of \( \mathcal{O}(p^3) \) \{ \( d_{1,\pm}, \ldots, d_{14,\pm} \} \). Taking into account the six subtraction constants \( \{ \alpha_{\pi\Lambda}, \alpha_{\pi\Sigma}, \alpha_{K\bar{N}}, \alpha_{\eta\Lambda}, \alpha_{\eta\Sigma}, \alpha_{K\bar{K}} \} \) yields a total of 33 free parameters. Note that SU(3) flavor symmetry is broken both at the Lagrangian level and through the use of physical hadron masses and channel specific subtraction constants.

The challenging task of determining them is approached in a multi-step fitting strategy. In that, we note that \( \pi N \) scattering data can be described well [35] using \( \mathcal{O}(p^3) \) amplitudes with eight combinations of the 27 LECs, see the Supplemental Material. Additionally, for each of the two \( K\bar{N} \) isospin channels, eight combinations of LECs contribute up to this order which indeed decouple among all the three reaction channels \{ \( \pi N, K\bar{N}_{I=0}, K\bar{N}_{I=1} \) \}. Therefore, we leave the LECs related to the \( \pi N \) scattering data and baryon masses fixed at the values determined in Ref. [35], denoting them with "**" in the corresponding tables in the Supplemental Material. For the remaining 23 free parameters, we first search for reasonable values for the 16 effective LECs which describe the \( K\bar{N} \) elastic scattering data. Then, taking these numbers as initial values, we perform a global fit for all the 23 parameters to the experimental data in the \{ \( K\bar{N}, \bar{K}N \) \} sector. Here, the latter consists of cross sections \{ \( K^-\pi \rightarrow X \{ X = K^-p, K^0_n, \pi^-\pi^+, \pi^0\pi^0, \pi^-\Sigma^- \} \} [63–68] as well as threshold ratios \{ \gamma \}, \( \gamma' \), \( R_e \) \} [76, 77] and \( K^-\pi \) scattering length [78] extracted from the SIDDHARTA data [79].

Note that we only fit to the experimental data that are directly related to the two-body scattering amplitude \( T_{M_1B_1\rightarrow M_2B_2} \), but do not consider the \( \pi\Sigma \) spectra in the \( \gamma \) \( \Lambda \). We also perform an alternative fit where the constraints from baryon masses are excluded, referred to as "NNLO*" afterwards. See the Supplemental Material for details.

| \( \chi^2/d.o.f. \) | \( K\bar{N} \) | \( \pi N \) | \( K\bar{N}_{I=0} \) | \( K\bar{N}_{I=1} \) |
|-----------------|---------|---------|----------------|----------------|
| Data            | 173     | 78      | 60             | 60             |
the three strangeness sectors simultaneously. For the which should be compared with the equivalent value of about good description of the meson-baryon scattering data for all

The best fit $\chi^2$/d.o.f. are listed in Table I. The corresponding LECs are shown in the Supplemental Material. They show that BCHPT and its unitarized version can provide a good description of the meson-baryon scattering data for all the three strangeness sectors simultaneously. For the $\bar{K}N$ channel, with all the constraints from the $\bar{K}N$ and $\pi N$ channels, we obtain a $\chi^2$/d.o.f. = 1.56 weighting different observables by the respective number of data points [41, 43, 44, 88], which should be compared with the equivalent value of about 2 from the NLO study [43]. The $\chi^2$/d.o.f. for the $K N$ channels decrease considerably (from 3.93(2.24) to 0.46(1.46) for $KN_{f=0}(KN_{f=1})$) compared to those obtained in Ref. [35] since we take into account the $O(p^3)$ tree level contributions which were omitted there.

In Fig. 1, we show the cross sections from the global NLO $^3$ and NNLO fits for the $\bar{K} N$ coupled channels as well as $\pi N$

$^3$ The NLO study is presented only for the sake of comparison. The description of the $K N$ channel is acceptable but that of the $\pi N$ channel is much worse. See the Supplemental Material for details.
and $KN$ phase shifts. The error bands are produced by the Bayesian model for a degree of belief of 68% [89–91] (see the Supplemental Material for details). The comparison with the best NLO fits of Guo [43] reveals that the $KN$ cross sections can be described rather well already at NLO, but qualitatively better results are obtained at NNLO, in particular, those of $\{\pi^+\Sigma^+, \pi^0\Lambda, \eta\Lambda\}$ final states. It is important to note that compared to the NLO fits, only NNLO fits allow also for a simultaneous description of the $\pi N$ and $KN$ phase shifts [35].

In Fig. (1h), we also show the $\pi^-\Sigma^+$ mass spectrum in the vicinity of $\Lambda(1405)$. As explained above, these data are not fitted. They are calculated following the approach of Refs. [39, 43] but including the contributions from $\pi\Lambda$ and $\eta\Lambda$. The $\eta\Sigma$ and $K\Xi$ channels are neglected because they are too far away from the energy region of our interest. While we are facing the well-known problem that the left-hand cuts overlap with the unitary cuts below $KN$ threshold (see Supplemental Material for details), the data is indeed described well.

In Table II we compare the scattering length and three ratios with the experimental data. Clearly the agreement is very good. We show as well the results of Fit II of the NLO study of Ref. [43], which agree with ours within uncertainties.

The double pole structure of $\Lambda(1405)$ is the most interesting nonperturbative phenomenon in this coupled-channel problem. Studies on this special resonance date back to 1960s [92] where it was suggested as a $KN$ bound state (see also review [48]). It was then found that $\Lambda(1405)$ is actually a superposition of two poles [39, 93–95]. Recent discussions on this issue can be found in Refs. [42, 43, 53, 96–98]. Note that a recent lattice QCD study also supports the $KN$ bound state interpretation of $\Lambda(1405)$ [99], see also Refs. [100, 101]. In order to obtain the pole position, one needs to extend the amplitudes to the second Riemann sheet. This can be achieved by analytically extrapolating the loop function $G(s)$ to the second Riemann sheet following the standard prescription, see, e.g., Refs. [27, 43, 56]. The poles discussed in the following are all situated on the respective sheet that is closest to the physical axis. The couplings of the poles to various channels $i, j$ are obtained from the residues of the poles on the complex plane as $T^{ij}(s) = \lim_{s \rightarrow s_R} g_{ij}(s) / (s - s_R)$. With the LECs determined above, we can predict the positions of the two poles and the corresponding couplings to various channels, which are shown in Table II. In the $J = 0$ sector, the lower pole is located at $(1392, -102)$ MeV while the higher one at $(1425, -13)$ MeV. We also find a state located at $(1676, -25)$ MeV corresponding to the $\Lambda(1670)$-resonance. A selected compilation of the two-pole positions is shown in Fig. 2 including the two-pole position from the NNLO and NNLO* fits corresponding to results with or without baryon mass constraints. It is clear that though the positions of the lower pole from different studies are quite scattered, those of the higher pole are determined much more precisely. We note that compared to the NLO results, the uncertainties in the NNLO results are smaller, due to the stringent constraints from the $\pi N$ and $KN$ scattering data. It is interesting to point out that the $\Lambda(1405)$ pole positions are similar to those of Fit II of Ref. [43].

**Conclusion and outlook:** We have performed for the first time a global study of meson-baryon scattering in all three strangeness sectors $S = 0, +1, -1$. The crucial step for this was the derivation of the formalism based on covariant baryon chiral perturbation theory including next-to-next-to-leading order contributions while employing a consistent unitarization procedure for the nonperturbative $S = -1$ sector. Besides theoretical relevance, this formalism allows one to put tighter constraints on extracted amplitudes and resonances, by connecting data from the different reactions $\{\pi N, KN, \overline{K}N\}$, ensured by the $SU(3)_f$ symmetry and its breaking. Indeed, this is only possible at NNLO due to the known poor convergence of the chiral expansion in the $S = 0$ sector.

Focusing on the $\overline{K}N$ sector, we confirmed the two-pole structure of $\Lambda(1405)$ in this novel approach, simultaneously ensuring for the first time an agreement with the perturbative channels. For the corresponding pole positions, we found results consistent with most NLO studies but with reduced uncertainties due to the stringent constraints from the $\pi N$ and $KN$ scattering data. It should be stressed that for dynamically generated states, the existence of two-pole structures seems to be a common phenomenon [47]. Some recent examples that have attracted considerable attention include the $K_1(1270)$ [105] and $D_0^*(2300)$ [106]. The two-pole structure of $\Lambda(1405)$ can be understood by following trajectories on which symmetries of the hadron-hadron interactions are restored [47, 95, 107, 108]. As a result, the emergence of a two-pole structure can be viewed as a strong evidence supporting the molecular nature of the state under investigation.
| $a_{K-\rho}$ [fm] | $\gamma$ | $R_C$ | $R_K$ |
|------------------|----------|--------|--------|
| NNLO             | $(-0.71 \pm 0.07) + i(0.84 \pm 0.07)$ | $2.35 \pm 0.19$ | $0.684 \pm 0.033$ | $0.198 \pm 0.019$ |
| NLO [43]         | $-0.61^{+0.07}_{-0.08} + i(0.89^{+0.09}_{-0.08})$ | $2.36^{+0.17}_{-0.22}$ | $0.661^{+0.12}_{-0.11}$ | $0.188^{+0.028}_{-0.029}$ |
| EXP              | $(-0.64 \pm 0.10) + i(0.81 \pm 0.15)$ | $2.36 \pm 0.12$ | $0.664 \pm 0.033$ | $0.189 \pm 0.015$ |
| Pole positions [MeV] | $|g_{\pi N}|$ [GeV] | $|g_{\rho A}|$ [GeV] | $|g_{K N}|$ [GeV] | $|g_{K Z}|$ [GeV] |
| $\Lambda(1380)$  | $1392 \pm 8 - i(102 \pm 15)$ | $6.40 \pm 0.10$ | $3.01 \pm 0.15$ | $2.31 \pm 0.10$ | $0.45 \pm 0.01$ |
| $\Lambda(1405)$  | $1425 \pm 1 - i(13 \pm 4)$ | $2.15 \pm 0.07$ | $5.45 \pm 0.24$ | $4.99 \pm 0.08$ | $0.58 \pm 0.02$ |

**TABLE II.** Threshold parameters, pole positions and couplings of the two $I = 0$ states obtained in the present work in comparison with experimental data and the results of Ref. [43].

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[1] R. J. Hill and M. P. Solon, Phys. Rev. D 91, 043505 (2015), arXiv:1409.8290 [hep-ph].

[2] A. Bottino, F. Donato, N. Formenti, and S. Scopel, Astropart. Phys. 13, 215 (2000), arXiv:hep-ph/9909228.

[3] A. Gal, E. V. Hungerford, and D. J. Millener, Rev. Mod. Phys. 88, 035004 (2016), arXiv:1605.00557 [nucl-th].

[4] V. Koch, Phys. Lett. B 337, 7 (1994), arXiv:nucl-th/9406030.

[5] C. J. Batty, E. Friedman, and A. Gal, Phys. Rept. 287, 385 (1997).

[6] Y. Akaishi and T. Yamazaki, Phys. Rev. C 65, 044005 (2002).

[7] V. K. Magas, E. Oset, A. Ramos, and H. Toki, Phys. Rev. C 74, 025206 (2006), arXiv:nucl-th/0601013.

[8] E. Oset and H. Toki, Phys. Rev. C 74, 015207 (2006), arXiv:nucl-th/0509048.

[9] T. Yamazaki et al. (DISTO), Hyperfine Interact. 193, 181 (2009), arXiv:0810.5182 [nucl-ex].

[10] M. Maggiora et al. (DISTO), Nucl. Phys. A 835, 43 (2010), arXiv:0912.5116 [hep-ex].

[11] T. Yamazaki et al., Phys. Rev. Lett. 104, 132502 (2010), arXiv:1002.3526 [nucl-ex].

[12] M. Milucci et al., Acta Phys. Polon. Suppl. 14, 49 (2021).

[13] C. Cuceanu et al., Acta Phys. Polon. B 51, 251 (2020).

[14] J. Zmeskals et al., JPS Conf. Proc. 26, 023012 (2019).

[15] R. Del Grande et al. (AMADEUS), Few Body Syst. 62, 7 (2021).

[16] K. Piscicchia et al., Phys. Lett. B 782, 339 (2018).

[17] G. Schelchlin et al. (BGGOOD), Phys. Lett. B 833, 137375 (2022), arXiv:2108.12235 [nucl-ex].

[18] T. Hashimoto et al. (J-PARC E57, E62), JPS Conf. Proc. 26, 023013 (2019).

[19] J. Zmeskal et al., Acta Phys. Polon. B 46, 101 (2015), arXiv:1501.05548 [nucl-ex].

[20] Y. Sada et al. (J-PARC E15), PTEP 2016, 051D01 (2016), arXiv:1601.06876 [nucl-ex].

[21] T. Yamaga et al. (J-PARC E15), Phys. Rev. C 102, 044002 (2020), arXiv:2006.13433 [nucl-ex].

[22] M. F. M. Lutz et al. (PANDA), (2009), arXiv:0903.3905 [hep-ex].

[23] G. Barucca et al. (PANDA), Eur. Phys. J. A 57, 184 (2021), arXiv:2101.11877 [hep-ex].

[24] M. Amaryan et al. (KLF), (2020), arXiv:2008.08215 [nucl-ex].

[25] S. Dobbs (KLF), Rev. Mex. Fis. Suppl. 3, 0308032 (2022), arXiv:2207.10779 [nucl-ex].

[26] R. A. Briceno, J. J. Dudek, and R. D. Young, Rev. Mod. Phys. 90, 025001 (2018), arXiv:1706.06223 [hep-lat].

[27] M. Mai, U.-G. Meißner, and C. Urbach, (2022), arXiv:2206.01477 [hep-ph].

[28] N. Fettes, U.-G. Meißner, and S. Steininger, Nucl. Phys. A 640, 199 (1998), arXiv:hep-ph/9803266.

[29] N. Fettes and U.-G. Meißner, Nucl. Phys. A 676, 311 (2000), arXiv:hep-ph/0002162.

[30] T. Becher and H. Leutwyler, JHEP 06, 017 (2001), arXiv:hep-ph/0103263.

[31] M. Mai, P. C. Bruns, B. Kubis, and U.-G. Meißner, Phys. Rev. D 80, 094006 (2009), arXiv:0905.2810 [hep-ph].

[32] J. M. Alarcon, J. Martin Camalich, and J. A. Oller, Phys. Rev. D 85, 051503 (2012), arXiv:1110.3797 [hep-ph].

[33] Y.-H. Chen, D.-L. Yao, and H. Q. Zheng, Phys. Rev. D 87, 054019 (2013), arXiv:1212.1893 [hep-ph].

[34] B.-L. Huang, Phys. Rev. D 102, 116001 (2020), arXiv:2007.01173 [nucl-th].

[35] J.-X. Lu, L.-S. Geng, X.-L. Ren, and M.-L. Du, Phys. Rev. D 99, 054024 (2019), arXiv:1812.03799 [nucl-th].

[36] B.-L. Huang and J. Ou-Yang, Phys. Rev. D 101, 056021 (2020).
R. D. Young, Phys. Rev. Lett. 114, 132002 (2015), arXiv:1411.3402 [hep-lat].

[100] R. Molina and M. Döring, Phys. Rev. D 94, 056010 (2016), [Addendum: Phys.Rev.D 94, 079901 (2016)], arXiv:1512.05831 [hep-lat].

[101] A. Martinez Torres, M. Bayar, D. Jido, and E. Oset, Phys. Rev. C 86, 055201 (2012), arXiv:1202.4297 [hep-lat].

[102] A. Cieply and J. Smejkal, Nucl. Phys. A 881, 115 (2012), arXiv:1112.0917 [nucl-th].

[103] A. Martinez Torres, M. Bayar, D. Jido, and E. Oset, Phys. Rev. C 86, 055201 (2012), arXiv:1202.4297 [hep-lat].

[104] A. Cieply and J. Smejkal, Nucl. Phys. A 881, 115 (2012), arXiv:1112.0917 [nucl-th].

[105] N. V. Shevchenko, Phys. Rev. C 85, 034001 (2012), arXiv:1103.4974 [nucl-th].

[106] J. Haidenbauer, G. Krein, U.-G. Meißner, and L. Tólos, Eur. Phys. J. A 47, 18 (2011), arXiv:1008.3794 [nucl-th].
SUPPLEMENTAL MATERIAL

In this supplemental material, we present some details regarding the theoretical formulation, the Bayesian truncation uncertainties, the global next-to-leading order fit, the description of the πN and KN scattering phase shifts, as well as an alternative fit without baryon mass constraints.

Technical details

In this subsection, we spell out certain quantities though well-known but necessary for clarity and self-consistence. The loop function $G(s)$ can be obtained via the once-subtracted dispersion relation,

$$16\pi^2 G(s)_i = a_i(\mu) + \log \frac{m_i^2}{\mu^2} - x_+ \log \frac{x_+ - 1}{x_+} - x_- \log \frac{x_- - 1}{x_-}$$

$$x_{\pm} = \frac{s + M_i^2 - m_{\pm}^2}{2s} \pm \frac{\sqrt{4s(M_i^2 - i0^+)} + (s + M_i^2 - m_{\mp}^2)}{2s}$$

with $a_i$ the subtraction constants corresponding to the ten coupled channels in particle basis with strangeness $S = -1$, i.e., $\eta\Sigma^0, \pi^-\Sigma^+, \pi^0\Sigma^0, \pi^+\Sigma^-, \pi^0\Lambda, K^-p, K^0n, K^0\Xi^0, K^+\Xi^-$. The cross sections for $M_iB_i \to M_jB_j$ read

$$\sigma_{ij} = \frac{1}{16\pi s} \frac{|\vec{p}_i|}{|\vec{p}_j|} |T_{M_iB_i \to M_jB_j}|^2,$$

where $\vec{p}_i, \vec{p}_j$ are the momenta of incoming and outgoing particles in the center of mass frame. The three threshold ratios are directly related to the scattering cross sections at threshold as

$$\gamma = \frac{\sigma(K^-p \to \pi^+\Sigma^-)}{\sigma(K^-p \to \pi^-\Sigma^+)}, \quad R_c = \frac{\sigma(K^-p \to \text{charged particles})}{\sigma(K^-p \to \text{all})}, \quad R_n = \frac{\sigma(K^-p \to \pi^0\Lambda)}{\sigma(K^-p \to \text{all neutral states})}.$$  

The $K^-p$ scattering length is defined as

$$a_{K^-p} = \frac{1}{8\pi \sqrt{8}} T_{K^-p \to K^-p}(s)|_{\sqrt{s} = m_{K^-} + m_n}.$$  

In dealing with the $\pi\Sigma$ spectrum in Fig. 1(h), we are faced with the well-known problem in such a coupled-channel formalism that the unitary cuts overlap with the unphysical subthreshold cuts from the crossed diagrams, which include both the LO crossed Born terms and NNLO crossed loop diagrams. Coupled channels with higher thresholds, i.e., $\eta\Sigma, \eta\Lambda$, and $K\Xi$ introduce these additional cuts well above the $\pi\Sigma$ thresholds and even not far from the $KN$ thresholds. These unphysical cuts lead to logarithmic divergences and violate the unitarity in the $S$-wave amplitudes. However, these unphysical cuts are only artifacts of on-shell amplitudes and will not be present in a full theoretical calculation. In the present work, we follow the strategy of Ref. [109] and eliminate the unphysical cuts by matching the contribution of the crossed diagrams to a constant real value below a certain invariant energy $\sqrt{s_0}$. As it was pointed out in Ref. [109], the $s_0$ is somehow arbitrary as long as it is not too close to the singularities due to the numerically small contributions from crossed diagrams. Note that the cross sections, scattering lengths and the three ratios are not affected at all because these unphysical cuts are located well below the $K^-p$ threshold.

Bayesian truncation uncertainties and the NLO fit

In order to estimate the truncation uncertainties of our NNLO results, we perform a global NLO fit following the same fitting strategy adopted in the NNLO study. The results are shown in Fig. 1 of the main text and Table III.

It is clear that up to NLO, the total cross sections of the $K^-p$-induced reactions can be described quite well, similar to the results of Ref. [43], except for the $K^-p \to \eta\Lambda$ cross section which keeps increasing in the higher momentum region. However, for the elastic scattering phase shifts, significant discrepancies appear in all the partial waves except for $S_{11}$. In addition, the constraints from the $\pi N$ and $K N$ phase shifts deteriorate the description of threshold parameters in the $S = -1$ sector, as shown in Table III. Indeed, the present NLO fit (including these phase shifts) cannot fit the $K^-p$ scattering length and threshold branching ratios as well as the NLO fit of Ref. [43] that does not include these phase shifts. On the other hand, we still find two $I = 0$ poles as shown in Table III. Compared to the pole positions obtained with the NNLO potentials and the Fit II results of Ref. [43], the lower $\Lambda(1380)$ is a bit narrower and the higher $\Lambda(1405)$ is a bit broader.
for the fits obtained without the constraints from baryon masses referred to as "NNLO*. Indeed, shows the combinations of LECs corresponding to unit of GeV - substraction constants in the \( \Sigma \) though in the higher energy region the threshold in the SU(2) covariant NLO ChPT \[112\]. The description is even worse if one treats this issue non-relativistically. This is also one of the motivations for a global study up to the one-loop order in the present work. As a consequence, the truncation uncertainties will be unphysically large if we consider the LO contribution. Therefore, we choose to set the reference scale \( X_{\text{ref}} \) in the Bayesian truncation estimation as

\[
X_{\text{ref}} = \text{Max} \left[ \frac{|X_{\text{NLO}}|}{Q^2}, \frac{|X_{\text{NNLO}} - X_{\text{NLO}}|}{Q^2} \right],
\]

where we take \( Q = \frac{m_{\text{ave}}}{\Lambda_b} \) with \( m_{\text{ave}} = 0.370 \) GeV the average mass of pseudoscalar mesons and \( \Lambda_b = 1.16 \) GeV the average mass of octet baryons. We note that for the breakdown scale one can also take the chiral symmetry breaking scale \( \Lambda_{\text{ChPT}} = 4\pi f_{\pi} \approx 1.2 \) GeV. The appearance of the \( \Delta(1232)/2^+ \) in the \( P_{33} \) partial wave of the \( \pi N \) amplitude signals that the breakdown scale for this particular channel is \( m_\Delta - m_N \), beyond which either non-perturbative treatment or explicit inclusion of \( \Delta(1232) \) is required. As a result, we have limited our fits to the \( \pi N \) phase-shifts in the threshold region, like many comparable studies \[32, 33, 36, 112\]. For other details of the Bayesian model, we refer to Refs. \[89–91\].

\section*{Elastic \( \pi N \) and \( KN \) scattering}

Here, we briefly explain the \( \pi N \) and \( KN \) elastic scattering phase shifts shown in Fig. 1 of the main text with the LECs given in Table IV. Since we have fixed the LECs related to the \( \pi N \) phase shifts, they are actually the same as the results obtained in Ref. \[35\], complemented with the truncation uncertainties estimated above. We note that the \( \pi N \) phase shifts cannot be well described at NLO, particularly those of the \( P_{31} \), \( P_{33} \) and \( P_{35} \) partial waves. All the phase shifts are described well at NNLO though in the higher energy region the \( S_{31} \) phase shifts of \( \pi N \) and those of \( P_{13} \) of \( KN \) are a bit worse due to the constraints from baryon masses. See Fig. 3 for the fits obtained without the constraints from baryon masses referred to as "NNLO*". Indeed, the description of the two mentioned partial waves improves considerably at higher energies.

Table V shows the combinations of LECs corresponding to \( \pi N \), \( KN_{I=0} \), \( KN_{I=1} \) elastic scattering from Ref. \[35\] for reference. We stress again that through the inclusion of the \( KN \) sector, all LECs are fully disentangled and one can determine their individual values.
\[\text{double-pole structure of } \Lambda(1405) \text{ and phase shifts for } \pi N\]

\[\text{LECs from the baryon masses. For the selection of the } N, \text{ as well as the threshold parameters and pole positions in Table VI. Relevant meson-baryon LECs up to NNLO \((b_0, b_D, b_F, b_{1,2,3})\text{ in units of } \text{GeV}^{-4}\) and the six subtraction constants in the } KN \text{ channel.}\]

**Alternative fit strategies**

As pointed out in Refs. [35, 61], although the physical baryon masses can be reproduced accurately up to \(O(p^3)\), the lattice QCD baryon masses cannot be described satisfactorily at this order. The LECs actually contribute at different orders to the baryon masses than to the scattering observables. In Ref. [35], we found that with the constraints from baryon masses on \(b_0, b_D\) and \(b_F\), the description of \(\pi N\) phase shifts, particularly those of the \(S_{11}\) partial wave, is a bit worse. Given the fact that up to \(N^3\)LO, both the physical baryon masses and those of lattice QCD simulations, as well as the sigma terms can be described quite well, it is also interesting to see how the description of meson-baryon scattering changes if one neglects the constraints on the LECs from the baryon masses.

We show in Table VI the \(\chi^2/d.o.f.\) and LECs of the NNLO fit without the constraints from the baryon masses referred to as “NNLO*”. Clearly, the descriptions of \(\pi N\) and \(KN\) phase shifts improve significantly. We notice that the LECs change considerably and in particular the LECs \(b_0, b_D, b_F\) are somehow unnaturally large. We show the total cross sections for \(KN\) and phase shifts for \(\pi N\) and \(KN\) and in Fig. 3, as well as the threshold parameters and pole positions in Table VI. Only slight improvements are observed for the observables obtained with the nonperturbative \(KN\) amplitudes. We find again the double-pole structure of \(\Lambda(1405)\) with slightly lower masses and narrower widths.
FIG. 3. Same as Fig. 1 of the main text but without constraints from baryon masses (Fit “NNLO*”).

Two $I = 1$ poles

It is interesting to note that in our NNLO fit there exist two $I = 1$ states around the $\bar{K}N$ threshold located at $(1435, -39)$ MeV and $(1440, -135)$ MeV on the $(- + + + +)$ sheet, the order of which corresponds to $\pi\Lambda, \pi\Sigma, \bar{K}N, \eta\Lambda, \eta\Sigma, K\Xi$ respectively. Both states are well above the $\bar{K}^-p$ threshold and appear as cusps on the real axis. In the Fit “NNLO*” in which the constraints from baryon masses are omitted, the two $I = 1$ states are located at $(1364, -110)$ MeV and $(1432, -18)$ MeV also on the $(- + + + +)$ sheet. In this case, the narrower state still shows up as a cusp but the broader one becomes a broad enhancement on the $I = 1$ amplitude on the real axis. We note that the existence of a $\Sigma^*(\frac{1}{2}^-)$ state has been predicted in a number of UChPT studies [39, 113] and also in the pentaquark model [114]. In Refs. [52, 95], instead, a cusp located at the $\bar{K}N$ threshold was found, while in Ref. [43], two states were found, both of which are relatively narrow. In addition, a $\Sigma^*(\frac{1}{2}^-)$ state located at 1580 MeV was found in Ref. [95]. There is indeed some evidence for the existence of a $\Sigma^*(\frac{1}{2}^-)$ state in the $K^-p \rightarrow \Lambda\pi^+\pi^-$ [115, 116] and $\gamma n \rightarrow K^+\Sigma^*(1385)$ reactions [117]. Possible signals of this state have been studied in various other reactions [118–123]. Clearly, the exact positions and the nature of these states can not yet be settled, calling for further experimental and theoretical studies.