Dilaton-driven brane inflation in type IIB string theory

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Abstract

We consider the cosmological evolution of the three-brane in the background of type IIB string theory. For two different backgrounds which give nontrivial dilaton profile we have derived the Friedman-like equations. These give the cosmological evolution which is similar to the one by matter density on the universe brane. The effective density blows up as we move towards the singularity showing the initial singularity problem. The analysis shows that when there is axion field in the ambient space the recollapsing of the universe occurs faster compared with the case without axion field.

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I. INTRODUCTION

The idea that our universe might be a domain wall embedded in a higher dimensional space \([1]\) has attracted much interest recently. It is possible that the fundamental scale of gravity can be lowered to the electroweak scale by introducing large extra dimensions \([2]\). Randall and Sundrum \([3]\) proposed scenarios that our observed universe is embedded in a five-dimensional bulk, in which the background metric is curved along the extra dimension due to the negative bulk cosmological constant. These models have been studied extensively because they might provide the solution to the gauge hierarchy problem and cosmological constant problem \([4]\). The cosmology can be different from the conventional four dimensional one.

One can naturally attempt to justify the scenario within a well defined framework for higher dimensional quantum theory of gravity such as string theory. Many attempts have been made to apply this idea to string theory in the context with D-branes \([5]\), where the standard model gauge bosons as well as charged matter arise as fluctuations of the D-branes. An early example of this is the Horava-Witten picture for the nonperturbative heterotic \(E_8 \times E_8\) string \([6]\). The spacetime includes a compact dimension with an orbifold structure. Matter is confined to the hypersurface which forms the boundaries of the spacetime. Within the string theory context, it is natural that our observable four-dimensional world is a three-brane embedded in ten dimensional string. In such theories, one of the important issue is the cosmological evolution of our universe. Many cosmological models associated to brane universe have been suggested. The models can be classified into two categories. The first is that the domain walls (branes) are static solution of the underlying theory and the cosmological evolution of our universe is due the time evolution of energy density on the domain wall (brane) \([7]\). The second is that the cosmological evolution of our universe is due to the motion of our brane-world in the background of gravitational field of the bulk \([8,9]\). We will focus on the second approach in this paper.

It is shown that the motion of the brane in ambient space induces cosmological expansion (or contraction) on our universe simulating various kinds of matter or a cosmological constant. In other words the cosmological expansion is not due to energy density on our universe but somewhere else. This is the idea by mirage cosmology \([9]\). Friedman-like equations were derived for various bulk background field solutions. In \([10]\), the motion of a three-brane, in a background of type 0 theory was examined.

In this paper, employing the formalism of \([11]\), we will study how the presence of matter field on the background geometry affects the cosmological evolution of the brane universe. More specifically we will consider cosmological evolution of type IIB theory with two different background geometries, one without axion and the other with axion. We will compare the two and see the difference.

The organization of the paper is as follows. In Sec. II we will briefly review the formalism of mirage cosmology in ref. \([11]\) and set up some preliminaries for our calculation. In Sec. III we consider the type IIB theory and its background solution with and without axion field. In Sec. IV, using the background solutions of Sec. III, we find the cosmological evolution of the three-brane under the background. Finally section V is devoted to conclusions and discussion.
II. FORMALISM

In this section, we consider a probe brane moving in a generic static spherically symmetric background. We ignore its back reaction to the ambient space. As the brane moves in a geodesic, the induced world-volume metric becomes a function of time. The cosmological evolution is possible from the brane resident point of view. We will focus on a D3-brane case. For this purpose we parametrize the metric of a D3-brane as

$$ds_{10}^2 = g_{00}(r)dt^2 + g(r)(d\vec{x})^2 + g_{rr}(r)dr^2 + g_S(r)d\Omega_5^2,$$

and there are dilaton field $\phi$ as well as RR (Ramond-Ramond) background $C(r) = C_{0\ldots3}(r)$. The probe brane will in general move in this background and its dynamics is governed by the Dirac-Born-Infeld (DBI) action. In maximally supersymmetric case, ignoring the fermions, it is given by

$$S = T_3 \int d^4\xi e^{-\phi} \sqrt{-\det(\hat{G}_{\alpha\beta} + (2\pi\alpha') F_{\alpha\beta} - B_{\alpha\beta})}$$

$$+ T_3 \int d^4\xi \hat{C}_4 + \text{anomaly terms},$$

where the induced metric on the brane is

$$\hat{G}_{\alpha\beta} = G_{\mu\nu} \frac{\partial x^\mu}{\partial \xi^\alpha} \frac{\partial x^\nu}{\partial \xi^\beta}$$

with similar expressions for other fields. Generally the motion of a probe D3-brane have a nonzero angular momentum in the transverse directions. We can write the relevant part of the Lagrangian, in the static gauge $x^\alpha = \xi^\alpha$ ($\alpha = 0, 1, 2, 3$), as

$$L = \sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j} - C(r),$$

where $h_{ij}\varphi^i\varphi^j$ is the line element on the unit five-sphere ($i, j = 5, \cdots, 9$),

$$A(r) = g^3(r)|g_{00}(r)|e^{-2\phi}, \quad B(r) = g^3(r)g_{rr}(r)e^{-2\phi}, \quad D(r) = g^3(r)g_S(r)e^{-2\phi},$$

and $C(r)$ is the RR background. The momenta of the system are given by

$$p_r = -\frac{B(r)\dot{r}}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}},$$

$$p_i = -\frac{D(r)h_{ij}\dot{\varphi}^j}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}}.$$  

Calculating the Hamiltonian and demanding the conservation of energy, we have

$$H = C(r) - \frac{A(r)}{\sqrt{A(r) - B(r)\dot{r}^2 - D(r)h_{ij}\dot{\varphi}^i\dot{\varphi}^j}} = -E,$$

where $E$ is the total energy of the brane. Also from the conservation of the total angular momentum $h^{ij}p_ip_j = \ell^2$, we have
\[ h_{ij} \dot{\varphi}^i \dot{\varphi}^j = \frac{\ell^2 (A(r) - B(r) \dot{r}^2)}{D(r)(D(r) + \ell^2)}. \] (8)

Substituting equation (8) into (7) and solving with respect to \( \dot{r}^2 \), we have the equation for the radial variable as

\[ \dot{r}^2 = \frac{A}{B} \left\{ 1 - \frac{A}{(C + E)^2} \frac{D + \ell^2}{D} \right\}. \] (9)

Plugging equation (9) back into (8), we have the equation for the angular variable

\[ h_{ij} \dot{\varphi}^i \dot{\varphi}^j = \frac{A^2 \ell^2}{D^2(C + E)^2}. \] (10)

The induced four-dimensional metric on the three-brane universe is

\[ ds_{4d}^2 = (g_{00} + g_{rr} \dot{r}^2 + g_s h_{ij} \dot{\varphi}^i \dot{\varphi}^j) dt^2 + g(dx)^2. \] (11)

Using equation (9) and (10), this reduces to

\[ ds_{4d}^2 = -\frac{g_{00} g_{rr} g_s}{(C + E)^2} e^{-2\phi} dt^2 + g(dx)^2 = -d\eta^2 + g(r(\eta))(dx)^2, \] (12)

where we defined, for the standard form of a flat expanding universe, the cosmic time \( \eta \) as

\[ d\eta = \frac{|g_{00}| g^{3/2} e^{-\phi}}{|C + E|} d\ell. \] (13)

If we define the scale factor as \( a^2 = g \), we can calculate, from the analogue of the four-dimensional Friedman equation, the Hubble constant \( H = \dot{a}/a \)

\[ \left( \frac{\dot{a}}{a} \right)^2 = \frac{(C + E)^2 g_s e^{2\phi} - |g_{00}| (g_s g^3 + \ell^2 e^{2\phi})}{4|g_{00}| g_{rr} g_s g^3} \left( \frac{g'}{g} \right)^2, \] (14)

where the dot denotes the derivative with respect to cosmic time and the prime denotes the derivative with respect to \( r \). The right hand side of (14) can be interpreted as the effective matter density on the probe brane

\[ \frac{8\pi}{3} \rho_{\text{eff}} = \frac{(C + E)^2 g_s e^{2\phi} - |g_{00}| (g_s g^3 + \ell^2 e^{2\phi})}{4|g_{00}| g_{rr} g_s g^3} \left( \frac{g'}{g} \right)^2. \] (15)

We have also

\[ \frac{\ddot{a}}{a} = \left( 1 + \frac{g}{g'} \frac{\partial}{\partial r} \right) \frac{(C + E)^2 g_s e^{2\phi} - |g_{00}| (g_s g^3 + \ell^2 e^{2\phi})}{4|g_{00}| g_{rr} g_s g^3} \left( \frac{g'}{g} \right)^2 \]
\[ = \left( 1 + \frac{1}{2} \frac{\partial}{\partial a} \right) \frac{8\pi}{3} \rho_{\text{eff}}. \] (16)

Equating the above to \(-(4\pi/3)(\rho_{\text{eff}} + 3p_{\text{eff}})\), we can find the effective pressure.
\[ p_{\text{eff}} = -\rho_{\text{eff}} - \frac{1}{3} \frac{\partial}{\partial a} \rho_{\text{eff}}. \] (17)

The apparent scalar curvature of the four-dimensional universe is

\[ R_{4d} = 6 \left\{ \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 \right\} = 8\pi (4 + a\dot{a}) \rho_{\text{eff}}. \] (18)

We have given the formalism for simple D3-brane case. The geodesic motion of Dp-brane in the background of the Dp'-brane with \( p' > p \) can be generalized easily. In the case \( p = p' \), there exists the additional Wess-Zumino term \( T_p f \hat{C}_{p+1} \) in the DBI action which modifies the equation of the probe brane as well as the induced metric. This modification turn out to be the shift \( E \rightarrow E + C \) where \( C = C_{0 \ldots p} \).

**III. THE TYPE IIB BACKGROUND SOLUTION**

Here we will consider the background geometry of type IIB theory with five-form flux through an \( S^5 \). We will also assume, for the metric, \((3+1)\)-dimensional Poincaré invariance \( \text{ISO}(1,3) \) since we need the theory defined on the Minkowski space-time. In addition we will preserve the \( \text{SO}(6) \) symmetry of the \( \text{AdS}_5 \times S^5 \). As a result, the \( \text{ISO}(1,3) \times \text{SO}(6) \) invariant ten-dimensional metric with \( N \) units of five-form flux through an \( S^5 \), in the Einstein frame, can be written as

\[ ds_{10}^2 = \hat{g}_{MN} dx^M dx^N = e^{-\frac{2\chi}{\pi} + 2\sigma} (-dt^2 + d\vec{x}^2 + dr^2) + L^2 e^{2\chi} d\Omega_5^2, \] (19)

\[ F_5 = \frac{N\sqrt{\pi}}{2\text{Vol}S^5} (\text{vol}_{S^5} + *\text{vol}_{S^5}), \] (20)

where \( \chi, \sigma \), and also the dilaton \( \phi \) and the axion \( \eta \) are allowed to depend only on the radial coordinate \( r \).

The equations of motion in type IIB supergravity, truncated to the fields of our interests, are given by \([11]\)

\[ \hat{\nabla}^2 \phi = -e^{2\phi} (\partial \eta)^2, \]

\[ \hat{\nabla}^2 \eta = -2(\partial M \phi)(\partial^M \eta), \] (21)

\[ \hat{R}_{MN} = \frac{1}{2} \partial_M \phi \partial_N \phi - \frac{1}{2} e^{2\phi} \partial_M \eta \partial_N \eta + \frac{\kappa^2}{6} F_{MKLPQ} F_N^{KLPQ}, \]

where hat means that the operators are expressed in ten-dimensional terms and \( M, N, \cdots = 0, \cdots, 9 \). The equation of motion for the five-form field is

\[ \hat{\nabla}_M F^{MKLPQ} = 0, \] (22)

which is satisfied with the self-duality condition \([20]\). The Einstein equation in \( S^5 \) direction is

\[ \hat{R}_{ij} = \left( \frac{\kappa N}{2\pi^{5/2}} \right)^2 g_{ij}, \quad i, j = 5, \cdots, 9 \] (23)
which is automatically satisfied if
\[ L^4 = \frac{\kappa N}{2\pi^{5/2}}. \]  

The remaining equations can be expressed in purely five-dimensional terms
\begin{align*}
\nabla^2 \phi &= -e^{2\phi}(\partial_\mu \eta)^2, \\
\nabla^2 \eta &= -2(\partial_\mu \phi)(\partial^\mu \eta), \\
\nabla^2 \chi &= \frac{4}{L^2} \left( e^{-\frac{40}{3} \chi} - e^{-\frac{40}{3} \chi} \right), \\
R_{\mu\nu} &= \frac{1}{2} \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} e^{2\phi} \partial_\mu \eta \partial_\nu \eta + \frac{40}{3} \partial_\mu \chi \partial_\nu \chi - \frac{g_{\mu\nu}}{L^2} \left( \frac{20}{3} e^{-\frac{16}{3} \chi} - \frac{8}{3} e^{-\frac{40}{3} \chi} \right),
\end{align*}

where the new metric
\[ ds_5^2 = g_{\mu\nu} dx^\mu dx^\nu = e^{2\sigma}(-dt^2 + d\vec{x}^2 + dr^2) \]

should be used to compute \( R_{\mu\nu} \) and to contract indices. The equations in (25) can be derived from the five-dimensional action \([12]\)
\begin{equation}
S = \frac{1}{2\kappa_5^2} \int d^5 x \sqrt{-g} \left\{ R - \frac{1}{2}(\nabla \phi)^2 + \frac{1}{2} e^{2\phi}(\nabla \eta)^2 - \frac{40}{3} (\nabla \chi)^2 + \frac{1}{L^2} \left( 20 e^{-\frac{16}{3} \chi} - 8 e^{-\frac{40}{3} \chi} \right) \right\},
\end{equation}

where the gravitational couplings \( \kappa_5 \) and \( \kappa \) in five and ten dimensions are related by
\[ \frac{1}{2\kappa_5^2} = \frac{\pi^3 L^5}{2\kappa^2} = \frac{N^2}{8\pi^2 L^3}. \]

Note the minus sign in front of the axion kinetic term, which is the result of the Hodge-duality rotation of the type-IIB nine-form \([13]\).

### A. Solution without axion

In general one can reduce the equations of motion (25) to a set of coupled non-linear second order ordinary differential equations in \( \phi, \eta, \chi, \) and \( \sigma \). These equations are too complicated to solve in general, but there is an obvious simplification with \( \chi = \eta = 0 \) \([14]\). Then the equations in (25) become much simpler
\begin{align*}
e^{-5\sigma} \partial_r e^{3\sigma} \partial_r \phi &= 0, \\
\partial^2 \sigma + 3(\partial_r \sigma)^2 &= \frac{4}{L^2} e^{2\sigma}, \\
4\partial_r^2 \sigma &= -\frac{1}{2} (\partial_r \phi)^2 + \frac{4}{L^2} e^{2\sigma}.
\end{align*}

Equations (30) and (31) are obtained from \((tt)\) and \((rr)\) components of the Einstein equation. Because \( \chi = 0 \) there is no distinction between the five-dimensional Einstein metric and ten-dimensional metric restricted to the five-dimensional noncompact subspace. The equation (29) can be integrated to give
\[
\phi(r) = \phi_\infty + \frac{B}{L} \int_0^r d\tilde{r} e^{-3\sigma(\tilde{r})},
\]

where \(\phi_\infty\) is the value of the dilaton at the boundary of the asymptotically \(AdS_5\) geometry and \(B\) is an integration constant. Substituting (32) into (30) and (31), defining new variable \(u \equiv r/L\), we obtain

\[
(\partial_u \sigma)^2 = e^{2\sigma} + \frac{B^2}{24} e^{-6\sigma},
\]

where \((\partial_u \sigma)^2 = \frac{e^{2\sigma} - 3\sigma}{8}\). The second equation follows from differentiating the first, so we see that (32), (33) and (34) are consistent system of equations despite being overdetermined.

One can understand (33) as a mechanical analog of a classical particle with unit mass moving in the potential

\[
V(\sigma) = -\frac{1}{2} e^{2\sigma} - \frac{B^2}{48} e^{-6\sigma},
\]

with zero energy. If \(B = 0\), the solution is pure \(AdS_5\) with constant dilaton. To have a solution with nonconstant dilaton, we take \(B > 0\). However, the \(B \neq 0\) geometry is geodesically incomplete and singular at some point \(u = u_0\). To find \(u_0\) explicitly, we integrate equation (33)

\[
u = \int_{\sigma}^{\infty} \frac{d\tilde{\sigma}}{\sqrt{e^{2\tilde{\sigma}} + \frac{B^2}{24} e^{-6\tilde{\sigma}}}} = \frac{3^{1/8} \Gamma(3/8) \Gamma(1/8)}{8^{7/8} \sqrt{\pi} B^{1/4}} - \sqrt{\frac{8}{3} \frac{e^{3\sigma}}{B}} F\left(\frac{3}{8}, \frac{1}{2}, \frac{11}{8}; \frac{24 e^{8\sigma}}{B^2}\right),
\]

where \(F(\alpha, \beta; \gamma; z)\) is the usual hypergeometric function. The second term vanishes as \(\sigma \to -\infty\), so we find

\[
u_0 = \frac{3^{1/8} \Gamma(3/8) \Gamma(1/8)}{8^{7/8} \sqrt{\pi} B^{1/4}}.
\]

Also we find the dilaton in terms of \(\sigma\) by solving the equation (32)

\[
\phi = \phi_\infty + \frac{3}{2} \coth^{-1} \sqrt{1 + (24/B^2) e^{8\sigma}}
\]

\[
\phi = \phi_\infty + \frac{3}{2} \ln \frac{\sqrt{1 + (24/B^2) e^{8\sigma}} + 1}{\sqrt{1 + (24/B^2) e^{8\sigma}} - 1}.
\]

We can write the ten-dimensional Einstein metric explicitly if we use \(\sigma\) as the radial variable

\[
ds_{10}^2 = e^{2\sigma} (-dt^2 + dx^2) + \frac{L^2 d\sigma^2}{1 + (B^2/24) e^{-8\sigma}} + L^2 d\Omega_5^2.
\]

We can cancel the factors of \(B^2/24\) from (38) and (39) by replacing by \(\sigma \to \sigma + (1/8) \ln(B^2/24)\), \(t \to (B^2/24)^{-1/8} t\) and \(x_i \to (B^2/24)^{-1/8} x_i\) for \(i = 1, 2, 3\). If we also rescale \(r \to (B^2/24)^{-1/8} r\), then the net result is the same as if we had set \(B^2 = 24\). Choice of the radial coordinate in \(AdS_5\) corresponds to choice of one out of a given class of conformally equivalent boundary metrics. Thus the freedom to change \(B\) in the solution corresponds to the asymptotic scale invariance of the boundary theory.
B. Solution with axion

In the previous subsection we described the simplest case in which one can have a solution with nontrivial dilaton. Here we will consider the case with only $\chi = 0$ to find the solution with nontrivial axion field $[15]$. The equations in (25) can be written

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu\nu} \partial_{\nu} \phi \right) = -e^{2\phi} \partial_{\nu} \eta \partial_{\mu} \eta g^{\mu\nu}, \tag{40}
\]

\[
\frac{1}{\sqrt{-g}} \partial_{\mu} \left( \sqrt{-g} g^{\mu\nu} e^{2\phi} \partial_{\nu} \eta \right) = 0, \tag{41}
\]

\[
R_{\mu\nu} = -\frac{4}{L^2} g_{\mu\nu} + \frac{1}{2} \partial_{\mu} \phi \partial_{\nu} \phi - \frac{1}{2} e^{2\phi} \partial_{\mu} \eta \partial_{\nu} \eta. \tag{42}
\]

Integral of the axion in equation (41) is

\[
\eta' = \eta_0 e^{-3\sigma} e^{-2\phi}, \tag{43}
\]

where the prime denotes derivative with respect to $u = r/L$ and $\eta_0$ is an integration constant. Inserting this expression for $\eta$ into equation (40) we obtain the differential equation for dilaton

\[
e^{3\sigma} (e^{3\sigma} \phi')' = -\eta_0^2 e^{-2\phi}, \tag{44}
\]

and integrating once we have

\[
e^{6\sigma} \phi'^2 = \eta_0^2 e^{-2\phi} + \tilde{B}^2, \tag{45}
\]

where $\tilde{B}$ is another arbitrary constant. Equations (13) and (15) are sufficient to proceed and solve for the function $\sigma(u)$ that appears in the metric (12). The Einstein equation (12) becomes

\[
\partial_{u}^2 \sigma = e^{2\sigma} - \frac{1}{8} (\partial_{u} \phi)^2 + \frac{1}{8} (\partial_{u} \eta)^2 \tag{46}
\]

\[
\partial_{u}^2 \sigma + 3 (\partial_{u} \sigma)^2 = 4 e^{2\sigma}. \tag{47}
\]

Inserting (13) and (15) into (16) and (17), we obtain

\[
\partial_{u}^2 \sigma = e^{2\sigma} - \tilde{B}^2 \eta_0^2 e^{-6\sigma}, \tag{48}
\]

\[
(\partial_{u} \sigma)^2 = e^{2\sigma} - \frac{\tilde{B}^2}{24} e^{-6\sigma}. \tag{49}
\]

The above equations are exactly the same as those without axion (see (33) and (34)). So the expressions for $u$ and $u_0$ in (33) and (37) are still valid in the presence of axion field. Also we find the dilaton in terms of $\sigma$ by solving the equation (13)

\[
e^{\sigma} = \frac{\tilde{B}}{\eta_0} \sinh \left\{ \ln \left( \frac{\sqrt{1 + (24/\tilde{B}^2)} e^{8\sigma} + 1}{\sqrt{1 + (24/\tilde{B}^2)} e^{8\sigma} - 1} \right)^{(1/2)} \sqrt{3/2} \right\}
\]

\[
= \frac{\tilde{B}}{2\eta_0} \left\{ \left( \frac{\sqrt{1 + (24/\tilde{B}^2)} e^{8\sigma} + 1}{\sqrt{1 + (24/\tilde{B}^2)} e^{8\sigma} - 1} \right)^{(1/2)} \sqrt{3/2} - \left( \frac{\sqrt{1 + (24/\tilde{B}^2)} e^{8\sigma} + 1}{\sqrt{1 + (24/\tilde{B}^2)} e^{8\sigma} - 1} \right)^{-(1/2)} \sqrt{3/2} \right\}. \tag{50}
\]
Finally from equation (53), we can find the solution for axion

\[
\eta = \frac{\eta_0}{B} \left\{ \frac{\sqrt{1 + (24/B^2)e^{8\sigma}} + 1}{\sqrt{1 + (24/B^2)e^{8\sigma}} - 1} \right\}^{3/2} \right\}.
\]

IV. BRANE COSMOLOGY

In this section we will consider the cosmology probe D3-brane when it is moving along a geodesic in the background type IIB solutions of the previous section.

A. Without axion field

The metric of D3-brane (1) using the background solution (39) is

\[
|g_{00}(r)| = e^{2\sigma}, \quad g(r) = e^{2\sigma}, \quad g_{rr}(r) = \frac{L^2 d\sigma^2}{1 + (B^2/24)e^{-8\sigma}}, \quad g_s(r) = L^2.
\]

To apply the formalism of Sec. II we also need to express RR field in terms of \(\sigma\). From the ansatz for the RR field

\[
C_{0123} = C(r), \quad F_{0123r} = \frac{dC(r)}{dr},
\]

equation (22) becomes

\[
\frac{dC(r)}{dr} = 2Qg^2 g_s^{-5/2} \sqrt{g_{rr}}
\]

where \(Q\) is a constant. Using the solution of the metric in (52), the RR field can be integrated with appropriate normalization,

\[
C = \sqrt{1 + (24/B^2)e^{8\sigma}}.
\]

Now we can calculate the effective density on the brane using equations (38), (52) and (55)

\[
\frac{8\pi}{3} \rho_{\text{eff}} = \frac{1 + (24/B^2)e^{-8\sigma}}{L^2} \left\{ E + \sqrt{1 + (24/B^2)e^{8\sigma}} \right\}^2 e^{-8\sigma} \left( \frac{\sqrt{1 + (24/B^2)e^{8\sigma}} + 1}{\sqrt{1 + (24/B^2)e^{8\sigma}} - 1} \right)^{3/2}
\]

\[
- \left\{ 1 + \frac{\ell^2}{L^2} e^{-6\sigma} \left( \frac{\sqrt{1 + (24/B^2)e^{8\sigma}} + 1}{\sqrt{1 + (24/B^2)e^{8\sigma}} - 1} \right) ^{3/2} \right\}.
\]

If we rescale with \(B^2 = 24\) and using \(a = e^\sigma\), we get

\[\]
\[ \frac{8\pi}{3} \rho_{\text{eff}} = \frac{1 + a^{-8}}{L^2} \left[ \left( \sqrt{1 + a^{-8} + \frac{E}{a^4}} \right)^2 \left( \frac{\sqrt{1 + a^8} + 1}{\sqrt{1 + a^8} - 1} \right)^{3/2} - \left\{ 1 + \frac{\ell^2}{L^2} a^{-6} \left( \frac{\sqrt{1 + a^8} + 1}{\sqrt{1 + a^8} - 1} \right)^{3/2} \right\} \right] , \] (57)

where the range of \( a \) is \( 0 < a < \infty \), while the range of \( \sigma \) is \( -\infty < \sigma < \infty \). When the universe brane is moving towards the singularity (\( \sigma \to -\infty, \ a \to 0 \)) the universe is contracting while it is moving outward (\( \sigma \to \infty, \ a \to \infty \)) it is expanding. We also can calculate the scalar curvature of the four-dimensional universe from (18).

Far from the black brane, one can see that \( \rho_{\text{eff}} \sim a^{-4} \). The cosmological expansion due to the brane motion is indistinguishable from the one by radiation on the brane. This is the idea of the mirage cosmology. If we use the effective density (57) it blows up \( \rho_{\text{eff}} \sim a^{-8(2+\sqrt{3}/2)} \) as \( a \to 0 \). Also if we move \( a \to 0 \), the ten-dimensional metric becomes

\[ ds_{10}^2 = a^2(-dt^2 + (d\vec{x})^2) + \frac{L^2 da^2}{a^2} + L^2 d\Omega_5, \] (58)

which is an \( \text{AdS}_5 \times S^5 \) space. Thus the brane develops an initial singularity as it reaches \( a = 0 \) where the description of our formalism breaks down.

B. With axion field

In this case, the only difference on the effective density comes from the form of the dilaton. Still with \( \tilde{B}^2 = 24 \), \( e^\sigma = a \), we have

\[ \frac{8\pi}{3} \rho_{\text{eff}} = \frac{1 + a^{-8}}{L^2} \times \left[ \left( \frac{12}{\eta_0} \right)^2 \left( \sqrt{1 + a^{-8} + \frac{E}{a^4}} \right)^2 \left( \frac{\sqrt{1 + a^8} + 1}{\sqrt{1 + a^8} - 1} \right)^{3/2} + \left( \frac{\sqrt{1 + a^8} + 1}{\sqrt{1 + a^8} - 1} \right)^{-\sqrt{3}/2} - 2 \right] - 1 - \frac{\ell^2}{L^2} \left( \frac{12}{\eta_0} \right)^2 a^{-6} \left\{ \left( \frac{\sqrt{1 + a^8} + 1}{\sqrt{1 + a^8} - 1} \right)^{3/2} + \left( \frac{\sqrt{1 + a^8} + 1}{\sqrt{1 + a^8} - 1} \right)^{-\sqrt{3}/2} - 2 \right\} . \] (59)

Near the brane the effective density blows up \( \rho_{\text{eff}} \sim a^{-8(2+\sqrt{3}/2)} \) as \( a \to 0 \) which has the same functional dependence as in the case without axion. However, far from the brane (\( a \to \infty \)), it gives \( \rho_{\text{eff}} \sim -1/L^2 \). This negative cosmological constant means that the expansion of the universe stops at somewhere and eventually recollapses. Comparing equation (59) with (57), we see that the effective density becomes negative faster if there is axion field. The presence of the axion field do not play any important role in the early stage of the evolution but its coupling to other field gives different evolution at late stage.

V. DISCUSSION

We considered the motion of a brane universe moving in a background bulk space of type IIB string theory. For two different backgrounds which give nontrivial dilaton profile, one
without axion field and the other with axion, we have derived the Friedman-like equations. These give the cosmological evolution which is similar to the one by matter density on the universe brane. As the brane moves towards the singularity (smaller values of radial coordinate) it contracts and while if it moves away from the black brane it expands. So an observer on the three-brane will see that the universe is expanding. The presence of axion field in the background changes the dilaton profile but it does not change the induced metric. Since dilaton, as well as the induced metric, plays an important role in the effective density, the cosmological evolutions are different for two different backgrounds. For both cases, the effective density blows up as we move toward the singularity showing the initial singularity problem and becomes negative due to the angular momenta $\ell^2$ on the brane meaning the recollapse of the universe. The functional dependence on the radial coordinate shows that when there is axion field in the ambient space the recollapsing of the universe occurs faster compared with the case without axion field. It seems that this phenomenon is true if we do the same calculation with field other than axion.

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