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PI robust fault detection observer for a class of uncertain switched systems using LMIs *

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Abstract: This paper addresses a method for robust fault detection and estimation by minimizing the disturbance and uncertainties to residual sensitivity. It consists in the design of proportional integral observer while minimizing the well known $H_\infty$ norm for worst case uncertainties and disturbance attenuation, and combining a transient response specification. This multi-objective problem is formulated as linear matrix inequalities (LMI) feasibility problem in which a cost function is minimized subject to LMI constraints. This approach is employed to generate a set of robust observers for uncertain switched systems.

Keywords: PI Observer, robust fault detection and estimation, uncertain switched systems, $H_\infty$, LMI, MLF.

1. INTRODUCTION

Modern systems (vehicles, aircrafts, trains...) are increasingly equipped with new mechanisms to improve passengers safety Isermann (2005); Venkatasubramanian et al. (2003); Koenig et al. (2013). These new systems have often active parts using data from sensors and actuator, for detection and diagnosis of process faults.

There is an abundance of literature on fault detection (FD) techniquesChen and Patton (1999); Varrier et al. (2014). The idea is to compute a residual signal that represents the inconsistency between the actual plant variables and the mathematical model, to extract information on possible changes caused by faults Zhang (2009). In practical applications, the residuals are corrupted by unknown inputs such as noises, disturbances, and uncertainties in the system model. Hence, the main objective of FD methods is to generate stable robust residuals that are insensitive to these noise and uncertainties, while sensitive to faultsShi and Patton (2012); Liu and Zhou (2007).

The Linear Matrix Inequalities (LMI) formulation is often used to mathematically express robust fault detection problem, for classes of uncertain systems with bounded uncertainties, or non-linear system with Lipschitz nonlinearities. The idea of finding a formulation with bounded unknowns has been widely studied. Some authors have proposed adaptive observerPaesa et al. (2010); Pourgholi and Majd (2013), adding an adaptive term to the observer. Nevertheless, there are two potential problems in their implementation: first the adaptive term might increase unboundedly and become infeasible in computationGu and Yang (2011). Second, it is an online adaption, that leads to more calculation and power consumption. Therefore, instead of using an adaptive term in the observer, the proposed method is to extend the Lyapunov function into a Multiple Lyapunov Function (MLF) resulting to a feasible LMI problem.

The specifications and objectives under consideration include $H_\infty$ performance and time domain constraints. The motivations for using this mixed performances are as follows:

- The time domain constraint that is expressed by pole region assignment is useful to tune the transient response Moore (1976); Patton and Chen (1991, 1997); Liu and Patton (1998).
- The integral term in the fault observer is convenient to ensure a zero fault estimation error in steady state regime.
- The $H_\infty$ performance is useful to ensure the residual robustness to model uncertainties, disturbances and unknown inputs.

In this paper, a proportional integral observer based filter is designed with the mixed $H_\infty$ / eigen region assignment objectives. The desired observer is computed by solving a set of LMI. A compromise between fault sensitivity, unknown input rejection, uncertainty robustness and eigen region assignment is optimized via a convex optimization algorithm.

The outline of this paper is as follows. After the introduction, problem formulation is given in Section II. In section III, preliminaries for the $H_\infty$ synthesis, the eigen region synthesis and the fault Proportional integral (PI) observer formulation. The multi-objective switched robust fault detection observer scheme is given in Section IV. The set of LMIs are then solved as an optimization problem.

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The above results are illustrated by a numerical example in Section V with filed data from a car as application to lateral vehicle control. Finally, Section VI shows the concluding remarks and the possible future work.

Notations: The notation used in this paper is standard. $X^T$ is the transposed of matrix $X$, the star symbol (*) in a symmetric matrix denotes the transposed block in the symmetric position. The notation $P > (>=) 0$ means $P$ is real symmetric positive (negative) definite matrix. $0$ and $I$ denote zeros and identity matrix of appropriate dimensions.

2. PROBLEM FORMULATION

Consider the state space representation of the linear time uncertain switched system:

\[
\begin{align*}
\dot{x}(t) &= A_\alpha(t)x(t) + B_\alpha(t)u(t) + E_{d,\alpha}(t)d + E_{f,\alpha}(t)f(t) \\
y(t) &= C_\alpha(t)x(t) + D_\alpha(t)u(t) + F_{d,\alpha}(t)d + F_{f,\alpha}(t)f(t)
\end{align*}
\]

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^p$ is the measurement output vector, $u \in \mathbb{R}^{nu}$ is the input vector, $d \in \mathbb{R}^{nd}$ is the disturbance vector, $f \in \mathbb{R}^{nf}$ is the vector of faults to be detected, $\alpha(t)$ is the switching signal, it is assumed known and measured.

Model uncertainties can be represented in different forms, in this study additive form is considered as:

\[
\Delta_{\alpha}(t) = \Delta_{\alpha}(t) + \Delta_{x,\alpha}(t)N_{x,\alpha}(t)
\]

The matrices $A_\alpha$, $B_\alpha$, $E_{d,\alpha}$, $E_{f,\alpha}$, $C_\alpha$, $D_\alpha$, $F_{d,\alpha}$ and $F_{f,\alpha}$ are the nominal LTI system matrices, they are known and in appropriate dimensions. $\Delta_{x,\alpha}$ is the state uncertainty matrix that is bounded $\|\Delta_{x,\alpha}\|_2 \leq \epsilon_{x,\alpha}$. $N_{x,\alpha}$ define the directions of these uncertainties.

In the following the subscript $t$ is omitted without confusion for typing simplifications.

\[
\begin{align*}
\dot{x} &= (A_\alpha + \Delta_{x,\alpha}N_{x,\alpha})x + B_\alpha u + E_{d,\alpha}d + E_{f,\alpha}f \\
y &= C_\alpha x + D_\alpha u + F_{d,\alpha}d + F_{f,\alpha}f
\end{align*}
\]

Assumption 1. In this study the pair $(\tilde{A}_\alpha, \tilde{C}_\alpha)$ defined in (9) is assumed observable, or without loss of generality is detectable. It is a standard assumption for all fault detection problems.

That is using Popov criterion, $\forall p$ s.t. $\mathfrak{R}(p) \geq 0$:

\[
\begin{align*}
\text{rank} \begin{bmatrix} pI - A_\alpha & -E_{f,\alpha} \\ C_\alpha & 0 \end{bmatrix} = n + \text{rank}(E_{f,\alpha})
\end{align*}
\]

Assumption 2. In this study the system is assumed stable. This assumption is explained with Theorem 1 and will be used in Theorem 2.

Introducing local variable $\chi$, the system can be put in the form:

\[
\begin{align*}
\dot{x} &= A_\alpha x + B_\alpha u + E_{d,\alpha}d + E_{f,\alpha}f + \chi \\
\dot{\chi} &= \Delta_{x,\alpha}N_{x,\alpha}x \\
y &= C_\alpha x + D_\alpha u + F_{d,\alpha}d + F_{f,\alpha}f
\end{align*}
\]

The switched robust PI observer for fault detection and estimation has the form:

\[
\begin{cases}
\dot{x} = A_\alpha \dot{x} + B_\alpha u + L_{P,\alpha}(y - \dot{y}) + E_{f,\alpha} \dot{f} \\
\dot{\chi} = L_{I,\alpha}(y - \dot{y}) \\
y = C_\alpha \dot{x} + D_\alpha u
\end{cases}
\]

where $L_{P,\alpha}$ and $L_{I,\alpha}$ are respectively the proportional and integral gain for the robust PI fault observer.

Assuming the static fault case (i.e. $\dot{f} = 0$), and define the errors $e_x = x - \hat{x}$ and $e_f = f - \dot{f}$, then the following could be written:

\[
\begin{align*}
\dot{e}_x &= A_\alpha e_x + B_\alpha u + E_{d,\alpha}d + E_{f,\alpha}f + \chi \\
&= (A_\alpha - L_{P,\alpha}C_\alpha)e_x + E_{f,\alpha}e_f + \chi \\
&= (E_{d,\alpha} - L_{P,\alpha}F_{d,\alpha})d - L_{P,\alpha}F_{f,\alpha}f \\
\dot{e}_f &= -L_{I,\alpha}(y - \dot{y}) \\
&= -L_{I,\alpha}C_\alpha e_x - L_{I,\alpha}F_{d,\alpha}d - L_{I,\alpha}F_{f,\alpha}f
\end{align*}
\]

In matrix form:

\[
\begin{align*}
\begin{bmatrix}
\dot{e}_x \\
\dot{e}_f
\end{bmatrix} &= \begin{bmatrix}
A_\alpha E_{f,\alpha}L_{I,\alpha} & -L_{P,\alpha}C_\alpha \\
E_{d,\alpha} & -L_{I,\alpha}F_{d,\alpha}
\end{bmatrix} \begin{bmatrix}
e_x \\
e_f
\end{bmatrix} \\
&+ \begin{bmatrix}
E_{d,\alpha} - L_{P,\alpha}F_{d,\alpha} \\
-L_{I,\alpha}F_{f,\alpha}
\end{bmatrix}d + \begin{bmatrix}
I \\
0
\end{bmatrix} \chi
\end{align*}
\]

(8a)

Let:

\[
\begin{align*}
\tilde{A}_\alpha &= [A_\alpha E_{f,\alpha}] \\
\tilde{B}_\alpha &= [B_\alpha 0] \\
\tilde{C}_\alpha &= [C_\alpha 0] \\
\tilde{E}_{d,\alpha} &= [E_{d,\alpha} 0] \\
\tilde{I} &= [I 0] \\
\tilde{L}_{I,\alpha} &= [L_{P,\alpha} L_{I,\alpha}]
\end{align*}
\]

(9)

Then the residual generator in (7) with $x = [e_x^T e_f^T]^T$ becomes:

\[
\begin{align*}
\dot{x} &= (\tilde{A}_\alpha - L_{I,\alpha}\tilde{C}_\alpha)x + (\tilde{E}_{d,\alpha} - L_{I,\alpha}F_{d,\alpha})d \\
&+ (\tilde{L}_{P,\alpha}F_{f,\alpha} + I)\chi \\
r_{\alpha} &= \tilde{C}_\alpha \dot{x} + \tilde{F}_{d,\alpha}d + \tilde{F}_{f,\alpha}f
\end{align*}
\]

(10)

And the PI observer in (5) with $\tilde{x} = [e_x^T e_f^T]^T$:

\[
\begin{align*}
\dot{\hat{x}} &= \tilde{A}_\alpha \hat{x} + \tilde{B}_\alpha u \\
&+ (\tilde{L}_{P,\alpha}F_{d,\alpha}d + \tilde{L}_{I,\alpha}F_{f,\alpha}f + \tilde{L}_{I,\alpha}\tilde{C}_\alpha \tilde{x}
\end{align*}
\]

(11)

Let: $A_\alpha^* = \tilde{A}_\alpha - L_{I,\alpha}\tilde{C}_\alpha$, $E_{d,\alpha}^* = \tilde{E}_{d,\alpha} - L_{I,\alpha}F_{d,\alpha}$, $E_{f,\alpha}^* = -L_{I,\alpha}F_{f,\alpha}$.

Then the sensitivity functions of disturbance to the residual is defined as:

\[
T_{rd,\alpha}(s) = \tilde{C}_\alpha(sI - A_\alpha^*)^{-1}E_{d,\alpha}^* + F_{d,\alpha}
\]

(12)

The objective of the $H_\infty$ switched fault detection PI observer is resumed by the following condition:

\[
\|T_{rd,\alpha}\|_\infty < \gamma_\alpha
\]

(13)
Then a robust PI fault detection observer can be designed where the gain filter $L_o = -P_{1, o}^{-1} U_o$

**Proof 2.** The followings are the constraints for a general robust fault detection observer design:

- If there exists $P_o > 0$, the sufficient stability condition considering the candidate Multiple Lyapunov Function (MLF):

$$V_o = \ddot{x}^T P_o \ddot{x} + \dot{x}^T P_o \dot{x}$$
$$\dot{V}_o < 0$$

- For a positive scalar $\gamma_o$, the $\mathcal{H}_\infty$ disturbance rejection condition (13) is formulated as:

$$\|r_o\|_2 < \gamma_o \|d\|_2$$

- The boundedness property of the uncertainties is:

$$\chi^T = x^T N_{x,o} \Delta x_o \Delta x_o N_{x,o} x < c^T x^T N_{x,o} N_{x,o} x \chi$$

Using $x = e_x + \ddot{x} = \dddot{\dot{x}} + \dddot{\dot{x}}$, and with the property in Lemma 1, we can write:

$$x^T N_{x,o} \Delta x_o \Delta x_o N_{x,o} x = (\dddot{\dot{x}} + \dddot{\dot{x}}) + I^T \dot{P}_o \dot{\dot{x}} + \dot{P}_o (E_{d,o}^T d + E_{f,o}^T f)\dot{x} + \dot{P}_o \dot{\dot{x}} + \chi^T \dot{\dot{x}} \dot{\dot{x}}$$

Combining the equations (17a) - (17c) yields to:

$$\dot{V}_o + \dot{r}_o^T r_o - \gamma_o^2 d^T d < 0$$

Let $V_o = V_{1, o} + V_{2, o}$; $V_{1, o} = \dddot{\dot{x}}^T P_o \dddot{\dot{x}}$ and $V_{2, o} = \dddot{\dot{x}}^T P_o \dddot{\dot{x}}$.

Then using the properties (16g) and (17), the general form of the MLF derivatives are:

$$V_{1, o} = \dddot{\dot{x}}^T P_o \dddot{\dot{x}} + \dot{P}_o (E_{d,o}^T d + E_{f,o}^T f) + \dot{P}_o \dot{\dot{x}}$$

$$\dot{V}_o + \dot{r}_o^T r_o - \gamma_o^2 d^T d < 0$$

where

$$\Omega_{d,o} = P_o \dot{A}_o + U_o \dddot{\dot{C}}_o + \dddot{\dot{A}}^T P_o + \dddot{\dot{C}}^T U_o + \dddot{\dot{C}}^T \dddot{\dot{C}}_o + 2\dot{\dot{x}}^T I N_{x,o} x + \dot{\dot{x}}^T P_o \dot{\dot{x}}$$

$$\dot{U}_{d,o} = F_{d,o} \dddot{\dot{x}} + \dddot{\dot{C}}_o \dddot{\dot{x}} + \dddot{\dot{C}}^T \dddot{\dot{C}} + 2\dot{\dot{x}}^T I N_{x,o} x + \dot{\dot{x}}^T P_o \dot{\dot{x}}$$

$$\dot{J}_{d,o} = E_{d,o} \dddot{\dot{x}} + \dddot{\dot{C}}_o \dddot{\dot{x}} + \dddot{\dot{C}}^T \dddot{\dot{C}} + 2\dot{\dot{x}}^T I N_{x,o} x + \dot{\dot{x}}^T P_o \dot{\dot{x}}$$

$$\dot{\Pi}_o = \dddot{\dot{C}}^T P_o + P_o \dddot{\dot{A}} + 2\dot{\dot{x}}^T I N_{x,o} x + \dddot{\dot{C}}^T \dddot{\dot{C}}_o + \dot{\dot{x}}^T P_o \dot{\dot{x}}$$

Then a robust PI fault detection observer can be designed where the gain filter $L_o = -P_{1, o}^{-1} U_o$
\[ \dot{V}_{2,\alpha} = \dot{x}^T P_{\alpha} \dot{x} = \dot{x}^T (\dot{A}_x \dot{x} + \dot{B}_x u + L_x \dot{C}_x \dot{x} + L_x F_{d,a} d) + \dot{x}^T P_{\alpha} (\dot{A}_x \dot{x} + \dot{B}_x u + L_x \dot{C}_x \dot{x} + L_x F_{d,a} d + L_x F_{j,a} f) \\
= \dot{x}^T (\dot{A}_x \dot{x} + \dot{B}_x \dot{A}_x) \dot{x} + \dot{x}^T P_{\alpha} \dot{A}_x \dot{x} + \dot{x}^T P_{\alpha} \dot{B}_x u + u^T \dot{B}_x^T P_{\alpha} \dot{x} + \dot{x}^T P_{\alpha} L_x \dot{C}_x \dot{x} + \dot{x}^T P_{\alpha} \dot{C}_x L_x^T P_{\alpha} \dot{x} + \dot{x}^T \dot{x} - \dot{x}^T P_{\alpha} L_x F_{d,a} d + d^T F_{d,a} L_x^T P_{\alpha} \dot{x} + \dot{x}^T P_{\alpha} L_x F_{j,a} f + \dot{x}^T F_{j,a}^T L_x^T P_{\alpha} \dot{x} + \dot{x}^T \dot{C}_x L_x^T P_{\alpha} \dot{x} + \dot{x}^T \dot{C}_x L_x^T \dot{C}_x \dot{x} - \dot{x}^T \dot{C}_x L_x^T \dot{x} \tag{21} \]

In the fault free case, the inequalities (19)-(21) yield to:

\[ \dot{V}_{a} = \dot{r}^T \dot{r} - \gamma^2 d^T d < 0 \]

\[ \dot{V}_{\alpha} = \dot{x}^T (\dot{A}_x \dot{x} + \dot{B}_x \dot{A}_x) \dot{x} + \dot{x}^T P_{\alpha} \dot{A}_x \dot{x} + \dot{x}^T P_{\alpha} \dot{B}_x u + u^T \dot{B}_x^T P_{\alpha} \dot{x} + \dot{x}^T P_{\alpha} L_x \dot{C}_x \dot{x} + \dot{x}^T P_{\alpha} \dot{C}_x L_x^T P_{\alpha} \dot{x} + \dot{x}^T \dot{x} - \dot{x}^T P_{\alpha} L_x F_{d,a} d + \dot{x}^T \dot{C}_x L_x^T \dot{x} \tag{22} \]

Solving the set of inequalities \( \Gamma_{\alpha} < 0 \) guarantees the solution for (19).

In the quadratic form:

\[ \dot{V}_{2,\alpha} = \dot{x}^T P_{\alpha} \dot{x} + \dot{x}^T P_{\alpha} \dot{x} \\
= \dot{x}^T (\dot{A}_x \dot{x} + \dot{B}_x \dot{A}_x) \dot{x} + \dot{x}^T P_{\alpha} \dot{A}_x \dot{x} + \dot{x}^T P_{\alpha} \dot{B}_x u + u^T \dot{B}_x^T P_{\alpha} \dot{x} + \dot{x}^T P_{\alpha} L_x \dot{C}_x \dot{x} + \dot{x}^T P_{\alpha} \dot{C}_x L_x^T P_{\alpha} \dot{x} + \dot{x}^T \dot{x} - \dot{x}^T P_{\alpha} L_x F_{d,a} d + \dot{x}^T \dot{C}_x L_x^T \dot{x} \tag{21} \]

\[ \begin{bmatrix} \Omega_{d,a} & \Upsilon_{d,a} & P_{\alpha} \dot{B}_a \dot{C}_a^T \dot{L}_a^T P_{\alpha} \\ \ast & j_{d,a} & 0 \\ \ast & \ast & 0 \\ \ast & \ast & \Pi_a \end{bmatrix} < 0 \tag{23} \]

where

\[ \Omega_{d,a} = P_{\alpha} A_{a}^T + A_{a}^T P_{\alpha} + 2 P_{\alpha}^2 + 2 \epsilon_2 \dot{I} N_{x,a} N_{x,a}^T + \tilde{C}_a \tilde{C}_a^T \]

\[ \Upsilon_{d,a} = P_{\alpha} C_{d,a} + \tilde{C}_a^T F_{d,a} \]

\[ j_{d,a} = F_{d,a}^T \dot{d}_a \]

\[ \Pi_a = \tilde{A}_a^T P_{\alpha} + \tilde{A}_a^T \dot{A}_a + 2 \epsilon_2 \dot{I} N_{x,a} N_{x,a}^T + \tilde{C}_a \tilde{C}_a^T \]

This inequality holds \( \forall \left[ \dot{x} \ det \ u^T \right] = 0 \) , thus:

\[ \begin{bmatrix} \Omega_{d,a} & \Upsilon_{d,a} & P_{\alpha} \dot{B}_a \dot{C}_a^T \dot{L}_a^T P_{\alpha} \\ \ast & j_{d,a} & 0 \\ \ast & \ast & 0 \\ \ast & \ast & \Pi_a \end{bmatrix} < 0 \tag{24} \]

This BMI is transformed into LMI by replacing \( u_a = -P_{\alpha} L_x \), and using Schur complement formula for \( P_{\alpha}^2 \).

It follows:

\[ \begin{bmatrix} \Omega_{d,a} & \Upsilon_{d,a} & P_{\alpha} \dot{B}_a \dot{C}_a^T \dot{L}_a^T P_{\alpha} \\ \ast & j_{d,a} & 0 \\ \ast & \ast & 0 \\ \ast & \ast & \Pi_a \end{bmatrix} < 0 \tag{25} \]

\[ \begin{bmatrix} \Omega_{d,a} & \Upsilon_{d,a} & P_{\alpha} \dot{B}_a \dot{C}_a^T \dot{L}_a^T P_{\alpha} \\ \ast & j_{d,a} & 0 \\ \ast & \ast & 0 \\ \ast & \ast & \Pi_a \end{bmatrix} < 0 \tag{25} \]

\[ \begin{bmatrix} \Omega_{d,a} & \Upsilon_{d,a} & P_{\alpha} \dot{B}_a \dot{C}_a^T \dot{L}_a^T P_{\alpha} \\ \ast & j_{d,a} & 0 \\ \ast & \ast & 0 \\ \ast & \ast & \Pi_a \end{bmatrix} < 0 \tag{25} \]

Remark 1. In this theorem, a MLF has been used. The second term \( V_2 \) has been added to ensure the feasibility of the LMI (16). Doing that adds the Assumption 2 on the stability of the system. We can consider that our interest here focuses on stable systems, and we work only on the observability and FD problems.

Theorem 3. For a given square \( n \times n \) matrix \( A_{a} \), if there exists a symmetric matrix \( P_{\alpha} > 0 \) and a positive scalar \( \xi_{\alpha} \) such that the following inequality holds:

\[ A_{a}^T P_{\alpha} + P_{\alpha} A_{a} - \xi_{\alpha} \rho_{\alpha} < 0 \tag{26} \]

Then all eigenvalues of \( A_{a} \) are on left plane of \( \xi_{\alpha} \).

Proof. (26) is a result of a classical Lyapunov function for sufficient condition of stability.

The system \( \dot{x} = (A_{a} - \xi_{\alpha} I) x \) is stable if there exist a symmetric matrix \( P_{\alpha} > 0 \) where \( \dot{V} = x^T P_{\alpha} \dot{x} < 0 \).

Thus:

\[ (A_{a} - \xi_{\alpha} I)^T P_{\alpha} + P_{\alpha} (A_{a} - \xi_{\alpha} I) < 0 \tag{27} \]

which is equivalent to (26).

Remark 2. The rise time property of the system is the time that takes to reach 90% of the steady state, it is approximated in the first order system to:

\[ t_{\text{rise,}\alpha} = \frac{2.2}{\xi_{\alpha}} \tag{28} \]

This shows the strong connection between the dominant pole region \( \xi_{\alpha} \) and the time specification.

4. PI ROBUST FAULT DETECTION OBSERVER DESIGN

The multi-objectives of the observer are: (a) robustness against uncertainties, (b) robustness and perturbation rejection, (c) sensitivity toward faults and (d) a correct time response for fault detection. In order to meet all these constraint, the developed design in this section can be adopted.

Whilst the rise time constant is predefined and is inversely proportional to \( \xi_{\alpha} \), the coefficient \( \gamma_{\alpha} \) has to be minimized. This consists in solving a set of LMIs as an optimization problem.

These LMIs are:

\[ P_{\alpha} \dot{A}_a + \tilde{A}_a^T P_{\alpha} + U_{\alpha} \dot{C}_a + \tilde{C}_a^T U_{\alpha} - 2 \xi_{\alpha} P_{\alpha} < 0 \tag{29} \]

and the same LMI in (24).

The gain filter is \( \dot{L}_a = -P_{\alpha}^{-1} U_{\alpha} \).

Using Matlab optimization tools such YALMIP or SeDuMi, the set of LMIs can then be solved minimizing \( \gamma_{\alpha} \).
The bicycle-model is widely used as representation of the vehicle. Consider the problem of the FD in the lateral control of a vehicle.

### 5.1 Bicycle switched model

Consider the problem of the FD in the lateral control of a vehicle.

The bicycle-model is widely used as representation of the system from Mammar and Koenig (2002). Though, this model is non-linear since it has \( \frac{1}{v} \) and \( \frac{1}{v^2} \) terms in it:

\[
\begin{bmatrix}
\dot{\alpha}(t) \\
\dot{\psi}(t)
\end{bmatrix} =
\begin{bmatrix}
\frac{c_r + c_f}{m v(t)} & \frac{c_f l_r - c_f l_f}{I_z(t)} \\
\frac{c_r l_r - c_f l_f}{I_z(t)} & \frac{c_l f + c_l t_f}{I_z(t)}
\end{bmatrix}
\begin{bmatrix}
\alpha(t) \\
\psi(t)
\end{bmatrix} +
\begin{bmatrix}
\frac{c_f}{m} \\
\frac{c_l f}{m}
\end{bmatrix} u_L(t) +
\begin{bmatrix}
\frac{1}{m v(t)} \\
\frac{1}{m v(t)}
\end{bmatrix} F_w(t)
\]

(32)

The measured output is the lateral acceleration \( \gamma_L \), the entry command is the steering angle \( u_L \), the sates are the side slip angle \( \alpha \) and the yaw rate \( \psi \), and we consider the wind force as a perturbation signal \( F_w \).

In this approach, the system is linearized around multiple points \( v_o \) as shown in dashed curve of figure 3. It is calculated as the integer part of the output of the division: \( \frac{v(t)}{\alpha} \) at \( v_o \) is the switching signal.

Using a Taylor expansion in the neighborhood of \( v_o \):

\[
A = \begin{bmatrix}
A_0 + \frac{1}{v_o} A_1 + \frac{1}{v^2_o} A_2 \\
A_0 + \frac{1}{v^2_o} A_1 - 2 \frac{1}{v^3_o} A_2 \left( v - v_o \right)
\end{bmatrix}
\]

(36)

5.2 Simulation scenario

Experimental data have been taken from a "Renault Scenic" car, provided by the french laboratory MIPS. In this example, we consider the scenario of the evasive manoeuvre test more commonly known as the moose test.

Figure 1 shows the measured steering angle for this scenario. The avoidance occurs between \( t = 18 \) to \( t = 24s \).

On figure 2, a comparison between the measured data and the simulated switched bicycle model for lateral acceleration validates the switched model, it can be used in simulations.
The fault considered in this application is an actuator fault, that occurs on the actuator. The uncertain switched state space representation in this case becomes:

\[
\begin{align*}
\dot{x} &= (A_\alpha + \Delta_{x,\alpha}N_{x,\alpha})x \\
&\quad + B_\alpha(u + f) + E_d d
\end{align*}
\]

(37)

Applying the set of LMIs detailed in section 4, a robust proportional integral fault observer can be designed, meeting the desired objectives.

For a fault that occurs between \( t = 21 \) and \( t = 24 \)s, we can see on figure 4 the fault estimation by the switched observer. The estimated signal rises within 1s.

In the non-faulty case, the estimated signal is very low, but non-zero. This is due to the uncertainties, but also to the parameters variation and the sensors calibration in the vehicle. These parameters can never be accurately known, the design of the observer is robust to these though.

6. CONCLUSION AND FURTHER WORK

The technique presented in this paper provides a framework for generating a class of robust fault detection observers.

Indeed, we have analyzed the problem of fault detection using proportional integral observer. We showed that the \( H_\infty \) PI observer structure formulated using LMIs makes it possible to decouple the disturbances while estimating the states and faults with satisfactory convergence properties.

Several time- and frequency-domain specifications have been expressed as LMI constraints on the observers state-space matrices. These analysis are then used for multi-objective synthesis purposes. A compromise of these objectives is proposed as a criterion to minimize. A LMIs feasibility problem is formulated. It is then solved as optimization problem by using efficient LMI solver. A particular example with field data was given to illustrate and validate this approach.

In future work, this design can be extended to a class of non-linear Lipschitz system, were the same idea of boundedness can be used.