Measurements of the parameters of the $\phi(1020)$ resonance through studies of the processes $e^+e^- \rightarrow K^+K^-$, $K_SK_L$ and $\pi^+\pi^-\pi^0$

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The cross sections of the processes $e^+e^- \rightarrow K^+K^-$, $K_SK_L$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ were measured in the SND experiment at the VEPP-2M collider in the energy region near the $\phi(1020)$ meson. These measurements were based on about $10^6 K^+K^-$, $0.5 \times 10^6 K_SK_L$ and $0.4 \times 10^6 \pi^+\pi^-\pi^0$ selected events. The measured cross sections have been analyzed in the framework of the vector meson dominance model and the main parameters of the $\phi$-resonance were obtained, such as its mass, width, the production cross section and branching ratios of the main decay modes. The measured value of the $\phi$ meson total width, $\Gamma_\phi = 4.21 \pm 0.04$ is lower than the present world average of $4.458 \pm 0.032$ MeV. Contributions in addition to the conventional vector meson dominance model were found in the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ reaction cross section.

I. INTRODUCTION

Cross sections of the reactions $e^+e^- \rightarrow K\overline{K}$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^0$, are determined by strong and electromagnetic interactions of the light quarks ($s,u,d$) and cannot be evaluated at present from first principles. But a rather good description of these cross sections is provided by the vector meson dominance model (VDM) with phenomenologically adjusted coupling constants ($g_{V\gamma\gamma}$, $g_{V\rho\pi}$, $g_{VP\gamma}$, $g_{VP\rho}$). In the VDM, the cross sections are determined by the amplitudes of vector meson V transitions ($V = \phi, \omega, \rho$) into the final state: $V \rightarrow \rho\pi \rightarrow 3\pi$, $V \rightarrow K\overline{K}$. In the energy range from 980 to 1070 MeV, the main contributions to the $e^+e^- \rightarrow K\overline{K}$ and $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross sections come from the $\phi$-meson. Therefore the measurement of these cross sections allows us to determine the $\phi$-meson main parameters and study its interference with other vector mesons. Such studies provide important information about light meson physics.

Earlier these processes were studied in several experiments: $e^+e^- \rightarrow 3\pi$, $e^+e^- \rightarrow K_SK_L$, and $e^+e^- \rightarrow K^+K^-$. The SND (Spherical Neutrat Detector) experiment continues these efforts at the VEPP-2M collider. Here we present the results of this investigation.

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II. EXPERIMENT

The SND has been operating at the VEPP-2M collider in the energy range from 360 to 1400 MeV since 1995 (the SND detailed description can be found in Ref.\[12]. The detector contains several subsystems. The tracking system includes two cylindrical drift chambers. The three-layer spherical electromagnetic calorimeter is based on NaI(Tl) crystals. The muon/veto system consists of plastic scintillation counters and two layers of streamer tubes. The calorimeter energy and angular resolution depend on the photon energy as $\sigma_E/E(\%) = 4.2\%/\sqrt{E(\text{GeV})}$ and $\sigma_{\phi,\theta} = 0.82^\circ/\sqrt{E(\text{GeV})} \oplus 0.63^\circ$. The tracking system angular resolution is about $0.5^\circ$ and $2^\circ$ for azimuthal and polar angle respectively. The energy loss resolution $dE/dx$ in the drift chamber is about 30% – good enough to provide charged kaon identification in the region of $\phi$-meson production.

This work is based on the data sample collected in the SND experiments during 1998. The data were collected at 32 energy points in the energy range from 984 to 1060 MeV (Table I). The total integrated luminosity accumulated in these experiments is 8.5 pb$^{-1}$. The beam energy was calculated from the magnetic field value in the bending magnets and revolution frequency of the collider. In the vicinity of the $\phi$-meson peak we also used the beam energy calibrated by the Cryogenic Magnetic Detector (CMD-2), which operated at VEPP-2M at the same time. This calibration is based on the measurements of the charged kaon momenta in the drift chamber of CMD-2. The energy calibration accuracy in each point is about 0.1 MeV, the common shift of the energy scale is estimated to be 0.04 MeV, and the energy spread of the beams is about 0.37 MeV.

For the luminosity measurements, $e^+e^-\rightarrow e^+e^-$ and $e^+e^-\rightarrow \gamma\gamma$ processes were used. The corresponding luminosity values were respectively used for normalizing the events containing charged particles and those with neutral particles only. They are different by about 10% (Tab.I) due to the dead time of the trigger, which selected events containing neutral particles only. The systematic error of the integrated luminosity determination is estimated to be 2%. Since luminosity measurements by $e^+e^-\rightarrow e^+e^-$ and $e^+e^-\rightarrow \gamma\gamma$ reveal a systematic spread of about 1.3%, this was added to the statistical error of the luminosity determination in each energy point. The statistical accuracy was better than 1%.

The experimental conditions were rather stable during data taking: the SND systems counting rate, average currents and luminosity of the collider were stable at the level of about 20% and no correlation between different energy points was observed.

III. DATA ANALYSIS

The procedure of the experimental data selection consists of several stages. During the experimental runs, the first-level trigger was used. The first-level trigger selects events of different types: events with photons only and events with charged particles. The thresholds applied on the energy deposition in the calorimeter were about 200 MeV for events containing charged particles and 300 MeV for events with neutral particles only.

During the processing of experimental data, the event reconstruction is performed in the following sequence. The first step is a search for separated clusters in the calorimeter. Track reconstruction in the drift chambers is then performed. Tracks are linked to the calorimeter clusters. Clusters with energy depositions of more than 20 MeV, and not linked to tracks in the drift chambers are considered as photons.
For charged tracks no requirements on the energy depositions in the calorimeter are imposed.

For further analysis, events with $|z| < 10$ cm and $r < 1$ cm for each charged particle were selected. Here $z$ is the coordinate of the charged particle production point along the beam axis (the longitudinal size of the interaction region is $\sigma_z \sim 2.5$ cm); $r$ is the distance between a charged particle track and the beam axis in the $r - \phi$ plane. For each process individual selection criteria were then applied.

The detection efficiency was obtained from Monte-Carlo (MC) simulation. Simulated events were reconstructed and the actual selection criteria were applied. To take into account the apparatus resolution of the first-level trigger thresholds, for simulated and experimental events the trigger conditions were applied with the threshold values $10\%$ higher than during the experimental runs.

As a result of uncertainties in the simulation of the distributions over some selection parameters, the detection efficiency was multiplied by correction coefficients, which were obtained in the following way. The experimental events were selected without any conditions on the parameter under study, using the cut parameters uncorrelated with the studied one. The same selection was applied for simulated events. Then the cut was applied to the parameter and the correction coefficient was calculated:

$$\delta = \frac{n/N}{m/M},$$

where $N$ and $M$ are the number of events in experiment and simulation selected without any cuts on the parameter under study; $n$ and $m$ is the number of events in experiment and simulation, when the cut on the parameter was applied. As a rule, the error in the coefficient $\delta$ determination is connected with the uncertainty of background subtraction. This systematic error was estimated by varying other selection criteria.

The overlap of the beam background with the events containing charged particles can result in track reconstruction failure and a decrease of detection efficiency. To take into account this effect, background events (experimental events collected when detector was triggered with an external generator) were overlapped with the simulated events. It was found that the detection efficiency decreased by about $3\%$ and therefore the correction coefficient $\delta_{\text{over}} = 0.97 \pm 0.03$ was used for the processes containing charged particles. In the analysis described further simulated events are not overlapped with beam background events.

A. Selection of $e^+e^- \to K^+K^-$ events

In the studied energy region the $e^+e^- \to K^+K^-$ events have the following features:

- The charged kaons are rather slow ($\beta\gamma \sim 0.2 - 0.4$) and therefore have large $dE/dx$ losses in the drift chamber.
- Events often contain several photons due to the kaon decays ($K^\pm \to \pi^\pm \pi^0$, $\pi^\pm \pi^0 \pi^0 \pi^0 \mu^\pm \nu_\mu$, $\pi^0 e^\pm \nu_e$) in the tracking system.
- Tracks attributed to the kaon decays in the tracking system (secondary particles) are also present.

To select the $e^+e^- \to K^+K^-$ events, detection of two charged particles was required, with polar angles in the range $20^\circ < \theta < 160^\circ$, with acollinearity angle in the azimuthal plane $\Delta \phi < 10^\circ$ and the energy losses in the drift chamber
$dE/dx > 3 \cdot (dE/dx)_{\text{min}}$, where $(dE/dx)_{\text{min}}$ is an average $(dE/dx)$ loss of a minimum ionizing particle. The events must also contain two or more photons and at least one secondary particle from kaon decay in the tracking system. The distribution of the angle $\theta$ for charged particles in the selected events has the typical shape for the $e^+e^- \rightarrow K^+K^-$ reaction (Fig. 1).

After these cuts, the background processes $e^+e^- \rightarrow \pi^+\pi^-$, $\mu^+\mu^-$ and also beam and cosmic backgrounds still contribute. The data accumulated under the $K\bar{K}$ production threshold, at the energy point $\sqrt{s} = 984$ MeV, were used to estimate the number of background events. After selection criteria were applied, only $N_{\text{bkg}}(984) = 5$ events were found at this point. The number of background events in each energy point was calculated as

$$N_{\text{bkg}}(s) = N_{\text{bkg}}(984) \cdot \frac{IL(s)}{IL(984)},$$

where $IL(s)$ is the integrated luminosity, and this number was subtracted from the number of selected events.

The detection efficiency was multiplied by correction coefficients $\delta_{dE/dx}$, $\delta_{\text{over}}$, $\delta_{NP}$, $\delta_{\text{tot}}$. Here $\delta_{dE/dx} = 0.93 \pm 0.02$ is a correction due to the inaccuracy in the simulation of the $dE/dx$ distributions. This coefficient does not depend on the energy in the explored region. The coefficient $\delta_{NP}$ is a correction to the efficiency of detection of secondary particles and at least two photons. Its value varies in this energy range by $\sim 10\%$ and in the vicinity of the $\phi$-resonance peak is $\delta_{NP} = 0.99 \pm 0.015$. The correction $\delta_{\text{tot}} = 0.91 \pm 0.05$ is due to the inaccuracy in the simulation of the average energy deposition of $e^+e^- \rightarrow K^+K^-$ events. The total systematic error of the detection efficiency determination is 6.8%. The number of selected $e^+e^- \rightarrow K^+K^-$ events (after background subtraction) and the detection efficiency are shown in Table 1. The detection efficiency rises with energy because the probability of kaon absorption in the detector passive material decreases with energy. Above 1035 MeV the efficiency goes down since the probability of kaon decay in the detector decreases.

B. Selection of $e^+e^- \rightarrow K_SK_L$ events

The $K_S$ and $K_L$ mesons decay lengths in the studied energy region are $c\tau\beta\gamma \sim 0.3$ – 1 cm and 2 – 5.6 m respectively. The $K_S$ meson decays inside the collider vacuum chamber or tracking system, and the $K_L$ reaches the calorimeter, where it can either produce signals (“photons”) due to nuclear interactions or decay in flight, or it can pass through the detector without interacting. The analysis of the $e^+e^- \rightarrow K_SK_L$ reaction is based on the detection of $K_S \rightarrow \pi^+\pi^-$ or $K_S \rightarrow \pi^0\pi^0$ decays.

In the first case (charged mode) the events which contained two charged particles and at least one “photon” were used. The polar angles of the charged particles and their acollinearity angle were bounded by the criteria: $36^\circ < \theta < 144^\circ$, $|\Delta\theta| < 3^\circ$ and $10^\circ < |\Delta\phi| < 50^\circ$. The cut $10^\circ < |\Delta\phi|$ rejects the background from $e^+e^- \rightarrow K^+K^-$ events. To suppress the same background, the following cuts were also applied: $(dE/dx) < 5 \cdot (dE/dx)_{\text{min}}$ for each charged particle and $(dE/dx) < 3 \cdot (dE/dx)_{\text{min}}$ at least for one.

The $K_S$ meson decay length is about 0.5 cm, so to suppress the background from $e^+e^- \rightarrow 3\pi$, $\eta\gamma (\eta \rightarrow \pi^+\pi^-\pi^0)$, $\omega\pi^0$, $e^+e^-\gamma\gamma$ events, the cut $0.2$ cm $< r < 1$ was used. The beam and cosmic backgrounds were suppressed by the criteria: $E_p > 0$ and $E_{p3} < 50$ MeV, where $E_p$ and $E_{p3}$ are the total energy deposition and the energy deposition in the third calorimeter layer of any charged particle. Pions in the process under study have energies of about $\sim 200 – 300$ MeV.
and actually give a very low energy deposition in the third layer. The distribution of the angle $\psi$ between charged particles in the selected events is shown in Figure 2. It has a characteristic peak at the minimum angle between pions, close to 150°.

To estimate the number of background events, the data accumulated at the energy point $\sqrt{s} = 984$ MeV, i.e. under the reaction threshold, were used. After the cuts, $N_{bkg}(984) = 30$ events were left. Their production points are uniformly distributed along the beam direction, so these events can be attributed to the beam and cosmic backgrounds. The number of background events in other energy points was calculated according to the formula (2) and then subtracted from the number of selected events.

The detection efficiency was multiplied by correction coefficients $\delta_{dE/dx}$, $\delta_{\text{over}}$ and $\delta_{r}$. Here $\delta_{dE/dx} = 0.95 \pm 0.01$ is a correction due to an inaccuracy in the simulation of the $dE/dx$ distribution. The coefficient $\delta_{r}$ is a correction due to an inaccuracy in the simulation of the $r$ distribution (Fig.3). Its value varies in this energy region by $\sim 10\%$ and in the vicinity of the $\phi$-resonance peak is $\delta_{r} = 0.91 \pm 0.015$. The total systematic error of the detection efficiency determination is $3.7\%$. The numbers of selected $e^+e^- \rightarrow K_SK_L$ events (after background subtraction) and the detection efficiencies are shown in Table III. The detection efficiency depends on the energy due to the dependence of $\Delta\phi$ and $\Delta\theta$ distributions on the $K_S$ energy and the dependence of $K_L$ nuclear interactions and decay length on its energy.

For analysis of the $e^+e^- \rightarrow K_SK_L$ decay in the neutral mode ($K_S \rightarrow \pi^0\pi^0$), we used events where at least four photons were detected and no charged particles were present. To reject beam and cosmic backgrounds, constraints were imposed on the total energy deposition ($E_{\text{tot}} > 0.35 \cdot \sqrt{s}$) and total momentum of events ($P_{\text{tot}} < 0.6 \cdot \sqrt{s}$). To suppress cosmic background even further, the events where most of the hit crystals could be fitted by a single track, were rejected.

For events satisfying selection criteria described above, the kinematic fit was performed under the following assumptions: the event contains two photon pairs originating from the $\pi^0 \rightarrow \gamma\gamma$ decay and invariant mass of these four photons is equal to the $K_S$-meson mass. The value of the likelihood function $\chi^2_{K_S}$ is calculated. In events with more than four photons, extra photons are considered as spurious ones and rejected. To do this, all possible subsets of four photons were tested and one, corresponding to the maximum likelihood, was selected. Figures 2 and 4 represent invariant masses of photon pairs and of all four photons found in the reconstruction. To suppress the beam background, polar angles of the photons, reconstructed as $\gamma$-quanta from the $K_S \rightarrow 2\pi^0 \rightarrow 4\gamma$ decay, were limited to $40^\circ < \theta < 140^\circ$. The background from the multi-photon events (pure QED events, $e^+e^- \rightarrow \pi^0\pi^0\gamma$ and $\eta\gamma$) was rejected by the following cuts: $\chi^2_{K_S} < 20$ and $\Delta p < 100$ MeV. Here $\Delta p = p_{K_S} - \sqrt{s/4 - m^2_{K_S}}$ and $p_{K_S}$ is the measured kaon momentum. The $\chi^2_{K_S}$ and $p_{K_S}$ distributions are shown in Figures 2, 3 and 4.

After these cuts were applied, only three events were found at the energy point $\sqrt{s} = 984$ MeV. The number of background events for each energy point was calculated according to Eq. (3) and then subtracted from the number of selected events. The number of background events from the $e^+e^- \rightarrow \eta\gamma$ decay was estimated using simulation and even in the vicinity of the $\phi$-resonance peak, their number is less than 0.2 % of the selected $e^+e^- \rightarrow K_SK_L$ events.

The detection efficiency was multiplied by correction coefficients: $\delta_{\phi}$ and $\delta_{\chi^2_{K_S}}$. The $\delta_{\chi^2_{K_S}} = 0.97 \pm 0.01$ is a correction due to an inaccuracy in the simulation of the $\chi^2_{K_S}$ distribution. The experimental $e^+e^- \rightarrow K_SK_L$ events can contain false photons, caused by the beam background overlaps. The energy of these photons is similar to the energy of the $\gamma$-quanta from the $K_S$ decay, and they are localized at the polar angles close to the beam direction. During reconstruction, some false
photons can be mistaken as originating from the $K_S$ decay. In other words, the reconstruction program can reject with some probability a valid photon from the $K_S$ decay and substitute it by the false one. An event reconstructed in such a way will not pass our cut on the photon polar angles (described above) because the beam related false photons have small polar angles. Therefore the beam background overlaps lead to the decrease of the detection efficiency – an effect not taken into account in the MC simulation. Hence the necessity of the correction coefficient $\delta_g = 0.90 \pm 0.03$ arises. The detection efficiency does not depend on the energy and equals 0.068 after all corrections were implemented. The total systematic error of the detection efficiency determination is 3.5%. The selected event numbers (after background subtraction) are shown in Table IV.

C. Selection of $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ events

For analysis of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ process, events containing two charged particles and two or more photons were used. Extra photons can appear because of the beam background overlap, splitting of the electromagnetic showers, and nuclear interactions of the charged pions in the calorimeter. Under these selection conditions, the background sources are $e^+e^- \rightarrow K^+K^-$, $K_SK_L(K_S \rightarrow \pi^+\pi^-\gamma)$, $\eta\gamma(\eta \rightarrow \pi^+\pi^-\pi^0, \pi^+\pi^-\gamma)$, $\omega\pi^0$, $e^+e^-\gamma\gamma$ processes and the beam background.

The polar angles of the charged particles were bounded by the criterion: $20^\circ < \theta < 160^\circ$. To suppress the beam background, the following cuts on the spatial angle between two charged particles and energy deposition of the neutral particles were applied: $\psi > 40^\circ$, $E_{\text{neu}} > 0.1\sqrt{s}$.

To suppress the background from the $e^+e^- \rightarrow K^+K^-$ process, the following cuts were imposed: $(dE/dx) < 5 \cdot (dE/dx)_{\text{min}}$ for each charged particle, $(dE/dx) < 3 \cdot (dE/dx)_{\text{min}}$ at least for one and $\Delta\phi < 10^\circ$. To reject the $e^+e^- \rightarrow e^+e^-\gamma\gamma$ events the energy deposition of the charged particles was bounded by the criterion: $E_{\text{cha}} < 0.5\sqrt{s}$.

For events left after these cuts, a kinematic fit was performed under the following constraints: the charged particles are considered to be pions, the system has zero momentum, the total energy is $\sqrt{s}$, and the photons originate from the $\pi^0 \rightarrow \gamma\gamma$ decays. The value of the likelihood function $\chi^2_{3\pi}$ is calculated. In events with more than two photons, extra photons are considered as spurious ones and rejected. To do this, all possible subsets of two photons were tested and the one, corresponding to the maximum likelihood was selected. Two-photon invariant mass and $\chi^2_{3\pi}$ distributions are shown in Figures 3 and 13. After the kinematic fit, the following additional cuts were applied: $36^\circ < \theta_\gamma < 144^\circ$, $NNP = 2$, and $\chi^2_{3\pi} < 20$ for energy points with $\sqrt{s} < 1028$ MeV, and $\chi^2_{3\pi} < 5$ for energy points above 1028 MeV. Here $\theta_\gamma$ is polar angle of any photon selected by the reconstruction program as originated from the $\pi^0$-decay. $NNP$ is the number of detected photons. A harder cut on $\chi^2_{3\pi}$ at the five last energy points is necessary to suppress $e^+e^- \rightarrow 3\pi\gamma_{\text{rad}}$ events, where $\gamma_{\text{rad}}$ is a photon emitted by initial particles.

As experimental data were accumulated in a rather narrow energy region, the detection efficiency for the events without $\gamma$-quantum radiation by initial particles does not depend on energy in the entire region. The detection efficiency dependence on the radiated photon energy, obtained from simulation, is shown in Figure 11.

The detection efficiency was multiplied by correction coefficients: $\delta_{\text{over}}, \delta_{dE/dx} = 0.95 \pm 0.01$, $\delta_{\chi^2_{3\pi}} = 0.93 \pm 0.03$ and $\delta_{\text{NNP}} = 0.87 \pm 0.005$. The last correction factor is due to inaccuracies in the simulation of extra photons. The detection efficiency (without $\gamma$-quantum radiation) equals 0.183 (under condition the $\chi^2_{3\pi} < 20$) and 0.086 (under condition the $\chi^2_{3\pi} < 5$) after all corrections were implemented. The
The obtained number accuracy of background event numbers determination is less than 15% at all energy and even at the φψ numbers of the IL taking into account the radiative corrections. Outside this region the background is strongly determined by the Kπ process. The background from the γγ process are written as follows

\[ \sigma_{K^+K^-}(s) = \frac{8\pi\alpha}{3s^{3/2}} \left| \frac{g_{\phi\gamma g_{K^+K^-}}}{D_\phi(s)} e^{i\phi_{K^+K^-}} - \frac{g_{\omega\gamma g_{K^+K^-}}}{D_\omega(s)} \right|^2 \]  

\[ \sigma_{K_SK_L}(s) = \frac{8\pi\alpha}{3s^{3/2}} \left| \frac{g_{\phi\gamma g_{K_SK_L}}}{D_\phi(s)} e^{i\phi_{K_SK_L}} - \frac{g_{\omega\gamma g_{K_SK_L}}}{D_\omega(s)} + \frac{g_{\rho\gamma g_{K_SK_L}}}{D_\rho(s)} + A_{K_SK_L} \right|^2 \]  

\[ \sigma_{3\pi}(s) = \frac{4\pi\alpha}{s^{3/2}} W(s) \left| \frac{g_{\phi\gamma g_{3\pi}}}{D_\phi(s)} e^{i\phi_{3\pi}} + \frac{g_{\omega\gamma g_{3\pi}}}{D_\omega(s)} + A_{3\pi} \right|^2 \]  

where \( i \) is a background process number, \( \sigma_{RI}(s) \) is the cross section of the process taking into account the radiative corrections, \( IL(s) \) is the integrated luminosity, \( e_i(s) \) is the detection efficiency and \( \delta_i \) is a correction to the efficiency. For \( e^+e^- \rightarrow K\bar{K} \) reactions the \( \sigma_{RI}(s) \) were taken according to the SND measurements presented in this article and for the process \( e^+e^- \rightarrow \omega\gamma \) other SND measurements were used. The \( e^+e^- \rightarrow K_SK_L \) is the main background in the vicinity of the φ-resonance peak \( \sqrt{s} = 1015.75 \pm 0.23 \) MeV and its contribution is \( \sim 90\% \) of all background. Outside this region the background is strongly determined by the \( e^+e^- \rightarrow \omega\gamma \) and \( e^+e^- \rightarrow \gamma\gamma \) events. The background from the \( e^+e^- \rightarrow K^+K^- \) events is negligible, and even at the φ-meson peak it is only \( \sim 0.5\% \) of all background events. The accuracy of background event numbers determination is less than 15% at all energy points. The obtained number \( N_{bkg}(s) \) was subtracted from the selected events. The numbers of the \( e^+e^- \rightarrow 3\pi \) and background events are shown in Table V.

### IV. THEORETICAL CROSS SECTIONS

In the VDM framework the cross sections of the \( e^+e^- \rightarrow K^+K^-, K_SK_L \) and \( \pi^+\pi^-\pi^0 \) process are written as follows[3,4]:

\[ \sigma_{K^+K^-}(s) = \frac{8\pi\alpha}{3s^{3/2}} 4K^+K^-(s) \left| \frac{g_{\phi\gamma g_{K^+K^-}}}{D_\phi(s)} e^{i\phi_{K^+K^-}} - \frac{g_{\omega\gamma g_{K^+K^-}}}{D_\omega(s)} \right|^2 \]  

\[ \sigma_{K_SK_L}(s) = \frac{8\pi\alpha}{3s^{3/2}} 4K_SK_L(s) \left| \frac{g_{\phi\gamma g_{K_SK_L}}}{D_\phi(s)} e^{i\phi_{K_SK_L}} - \frac{g_{\omega\gamma g_{K_SK_L}}}{D_\omega(s)} + \frac{g_{\rho\gamma g_{K_SK_L}}}{D_\rho(s)} + A_{K_SK_L} \right|^2 \]  

\[ \sigma_{3\pi}(s) = \frac{4\pi\alpha}{s^{3/2}} W(s) \left| \frac{g_{\phi\gamma g_{3\pi}}}{D_\phi(s)} e^{i\phi_{3\pi}} + \frac{g_{\omega\gamma g_{3\pi}}}{D_\omega(s)} + A_{3\pi} \right|^2 \]  

where

\[ D_V = m_V^2 - s - i\sqrt{s}\Gamma_V(s), \]  

\[ \Gamma_V(s) = \Gamma(V \rightarrow 3\pi, s) + \Gamma(V \rightarrow \eta\gamma, s) + \Gamma(V \rightarrow \pi^0\gamma, s) + \Gamma(V \rightarrow K^+K^-, s) + \Gamma(V \rightarrow K_SK_L, s) + \Gamma(V \rightarrow \pi^+\pi^-, s). \]  

Here \( V \) denotes the vector mesons \( \rho, \omega, \phi; q_{K^+K^-}(s), q_{K_SK_L}(s) \) are kaon momenta, \( W(s) \) is a phase space factor including \( \rho\pi \) intermediate states, \( A_{K^+K^-}, A_{K_SK_L} \)
and $A_{4\pi}$ are amplitudes corresponding to contributions additional to the conventional VDM (for example $\rho$, $\omega$ and $\phi$ primes), and $\chi_{\phi-\omega}$ and $\phi_{K\bar{K}}$ are the relative interference phases. The $\phi_{K\bar{K}}$ and $\chi_{\phi-\omega}$ phases are equal to $180^\circ$ in the framework of the naive quark model.

The partial decay width dependences on energy are written as:

$$\Gamma(V \to 3\pi, s) = \frac{|g_{V\gamma}|^2}{4\pi} W(s), \quad \Gamma(V \to PP, s) = \frac{|g_{VP}|^2}{6\pi s} q_{PP}(s),$$

$$\Gamma(V \to P\gamma, s) = \frac{1}{3} |g_{VP\gamma}|^2 q_{P\gamma}(s), \quad (8)$$

where $P$ denotes the pseudoscalar meson $\pi$ or $K$.

The coupling constants are determined through the decay branching ratios in the following way:

$$|g_{V\gamma}| = \left[ \frac{3m^2 \Gamma V B(V \to e^+e^-)}{4\pi \alpha} \right]^{1/2}, \quad |g_{VP}| = \left[ \frac{3 \Gamma V B(V \to P\gamma)}{q_{P\gamma}(m_V)} \right]^{1/2},$$

$$|g_{\omega\rho}\phi_{K+K^-}| = \frac{1}{\sqrt{2}} |\phi_{K+K^-}|, \quad |g_{\omega\rho}\phi_{K_SK_L}| = \frac{1}{\sqrt{2}} |\phi_{K_SK_L}| \quad (9)$$

Here $\Gamma V = \Gamma V (m_V)$. To restrict the growth of the partial width $\Gamma(\omega \to 3\pi, s)$ with energy, the form factor is usually included in the $g_{\omega\rho\pi}$ coupling constant: $g_{\omega\rho\pi} \to g_{\omega\rho\pi} C_{\omega\rho\pi}(s)$, where

$$C_{\omega\rho\pi}(s) = \left[ 1 + \left( \frac{R_{\omega\rho\pi}^2 m_{\omega}}{1 + (R_{\omega\rho\pi}^2 s)^2} \right) \right]^{1/2} \quad (10)$$

We did not use this form factor for the experimental data approximation in this work: the parameter $R_{\omega\rho\pi}$ was set to zero.

The Coulomb interaction of the charged kaons in the final state was taken into account by multiplying the coupling constant $g_{\phi_{K+K^-}}$ by the appropriate form factor: $g_{\phi_{K+K^-}} \to g_{\phi_{K+K^-}} \cdot \sqrt{Z(s)/Z(m_\phi)}$, where

$$Z(s) = 1 + \alpha \frac{1 + v^2}{2v}, \quad v = \left( 1 - \frac{4m_{K+}^2}{s} \right)^{1/2}$$

In the vicinity of the $\phi$-meson peak, the value $Z(s)$ is nearly constant and the influence of the $\sqrt{Z(s)/Z(m_\phi)}$ form factor on the resonance width and mass is negligible.

For the experimental data approximation, theoretical cross sections were reduced to the following expressions:

$$\sigma_{K+K^-}(s) = \frac{1}{s^{5/2}} q^4_{K+K^-}(s) \frac{\Gamma_{K+K^-} m_{\phi}^3 \sigma_{K+K^-}(s)}{D_{K+K^-}(s)} e^{i\phi_{K+K^-}}$$

$$- \frac{\sqrt{\Gamma_{K+K^-} m_{\phi}^3 m_{\omega}^2 6\pi B(\omega \to e^+e^-) B(\phi \to K+K^-)}}{D_{\omega}(s)} + A_{K+K^-} \quad (11)$$
\[ \sigma_{KSKL}(s) = \frac{1}{s^{5/2}} \frac{g^3_{KSKL}}{g^3_{KSKL}(m_\phi)} \left| \frac{\Gamma_\phi m_\phi^3 \sqrt{\sigma(\phi \to KSKL)m_\phi}}{D_\phi(s)} e^{i\phi_\pi} \right. \\
\left. \sqrt{\Gamma_\phi \Gamma_\omega m_\phi^2 m_\omega^2 6\pi B(\phi \to e^+e^-)B(\phi \to KSKL)} \right| \\
\left. + \sqrt{\Gamma_\phi \Gamma_\rho m_\phi^2 m_\omega^2 6\pi B(\rho \to e^+e^-)B(\phi \to KSKL)} \right| D_\rho(s) \right)^2 + A_{KSKL}^0 \bigg| ^2 \right) \]

\[ \sigma_{3\pi}(s) = \frac{1}{s^{3/2}} \frac{W(s)}{W(m_\phi)} \left| \frac{\Gamma_\phi m_\phi^2 \sqrt{\sigma(\phi \to 3\pi)m_\phi}}{D_\phi(s)} e^{i\chi_\omega} \right. \\
\left. \sqrt{\frac{W(m_\omega)}{W(m_\phi)} \frac{\Gamma_\omega m_\omega^2 \sqrt{\sigma(\omega \to 3\pi)m_\omega}}{D_\omega(s)} \sqrt{C_{\omega 3\pi}(s)} + A_{3\pi}^0 \bigg| ^2 \right) \]

where

\[ \sigma(V \to X) = \frac{12\pi B(V \to e^+e^-)B(V \to X)}{m_V^2}, \]

and \( A_{KSKL}^0, A_{KSKL}^0, A_{3\pi}^0 \) are complex constants, corresponding to the contributions of the higher radial excitations of the \( \rho, \omega \) and \( \phi \) mesons in the cross section [3, 8, 2].

V. FITTING OF THE EXPERIMENTAL DATA

The experimental cross section \( \sigma(s) \) can be calculated as follows:

\[ \sigma(s) = \frac{N(s)}{IL(s)\epsilon(s)(1 + \delta_{rad}(s))}, \]

where \( N(s) \) is the number of selected events, \( IL(s) \) is the integrated luminosity, \( \epsilon(s) \) is the detection efficiency, and \( \delta_{rad}(s) \) is the radiative correction calculated according to Ref. [19]. The cross sections were fitted with the theoretical functions and the values of parameters and their errors were obtained using the procedure described in Ref. [20] which takes into account the beam energy spread and the error in the beam energy determination.

For the approximation of the \( e^+e^- \to 3\pi \) cross section, to take into account the detection efficiency dependence \( \epsilon(E_\gamma) \) on the emitted \( \gamma \)-quantum energy \( E_\gamma \), in the formula (14) the factor \( \epsilon(s)(1 + \delta_{rad}(s)) \) was replaced with the following expression:

\[ \frac{\int_0^{E_{\gamma max}} \sigma(s, E_\gamma) F(s, E_\gamma) \epsilon(E_\gamma) dE_\gamma}{\sigma(s)}, \]

where \( E_{\gamma max} \) is the emitted photon energy and \( F(s, E_\gamma) \) is the electron “radiator” function.
A. Fitting of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section

The fitting of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section was performed with the $m_\phi$, $\Gamma_\phi$, $\sigma(\phi \rightarrow 3\pi)$, $A^0_{3\pi}$ and in some cases with the phase $\chi_{\phi-\omega}$ as free parameters. Approximations were performed under the following assumptions on the $\chi_{\phi-\omega}$ value:

- $\chi_{\phi-\omega} = 180^\circ$;
- $\chi_{\phi-\omega} = 180^\circ + \Delta \chi_{\phi-\omega}(s)$;
- $\chi_{\phi-\omega}$ is a free parameter.

Here the expression for $\Delta \chi_{\phi-\omega}(s)$ was taken from Ref. [2] and it is equal to $\Delta \chi_{\phi-\omega}(m_\phi) \simeq -17^\circ$, when $s = m_\phi^2$.

The results of the fit are shown in Table VII. The first three variants correspond to the case when both the real and imaginary parts of the $A^0_{3\pi}$ amplitude were free parameters. The value of the Re($A^0_{3\pi}$) differs from zero by more than three standard deviations. The value of Im($A^0_{3\pi}$) is consistent with zero. Then the imaginary part was fixed at Im($A^0_{3\pi}$) = 0 and only the Re($A^0_{3\pi}$) was taken as a free parameter (four to six variants in Table VII). The values of $m_\phi$ and $\Gamma_\phi$ do not depend on the theoretical model. The model dependence of the parameter $\sigma(\phi \rightarrow 3\pi)$ is about $\sim 10\%$ and the difference of its values for variants 4 and 5 is much greater than the errors. The energy dependence of the $e^+e^- \rightarrow 3\pi$ cross section for the variant 5 is shown in Figure 12. At the right side from the resonance peak, the cross section has a minimum due to the interference of the $\phi$-meson amplitude with the nonresonant part of the cross section. The cross section value does not actually depend on the model parameters in the energy range $\sqrt{s} = 984 - 1028$ MeV, but in the region of the interference minimum at $\sqrt{s} = 1040$ MeV, it can differ by a factor of 2 for different models (Fig. 13). This uncertainty is connected with the difference in the values of $\delta_{\text{rad}}(s)$ correction in Eq. (3), calculated for different models.

B. Fitting of the $e^+e^- \rightarrow K\overline{K}$ cross sections

The $e^+e^- \rightarrow K^+K^-$ and $K_SK_L$ cross sections were fitted with $m_\phi$, $\Gamma_\phi$, $\sigma(\phi \rightarrow K^+K^-)$, $\sigma(\phi \rightarrow K_SK_L)$ as free parameters. The parameters $A^0_{K^+K^-}$, $A^0_{K_SK_L}$, and $\phi_{K\overline{K}}$ were either fixed ($\phi_{K\overline{K}} = 180^\circ$ and $A^0_{K^+K^-}(A^0_{K_SK_L}) = 0$), or were free parameters. In particular, various fits were performed under the following assumptions:

1. $\phi_{K\overline{K}} = 180^\circ$, $A^0_{K^+K^-} = A^0_{K_SK_L} = 0$;
2. $\phi_{K\overline{K}}$ was free parameter, $A^0_{K^+K^-} = A^0_{K_SK_L} = 0$;
3. $\phi_{K\overline{K}}$ and Re($A^0_{K^+K^-}$), Re($A^0_{K_SK_L}$) were free parameters;
4. $\phi_{K\overline{K}} = 180^\circ$, Re($A^0_{K^+K^-}$), Re($A^0_{K_SK_L}$), Im($A^0_{K^+K^-}$), Im($A^0_{K_SK_L}$) were free parameters;
5. $\phi_{K\overline{K}} = 180^\circ$, Re($A^0_{K^+K^-}$) and Re($A^0_{K_SK_L}$) were free parameters.

The results of the fit are shown in Tables VII and VIII.

For the $e^+e^- \rightarrow K^+K^-$ process, the phase $\phi_{K\overline{K}}$ agrees with the theoretically predicted value $180^\circ$. The imaginary part of the $A^0_{K^+K^-}$ amplitude is consistent with zero, and its real part is different from zero by about only one standard deviation (variants 4 and 5 in Table VII). The values of $m_\phi$ and $\Gamma_\phi$ do not depend
on the applied theoretical model. The value of $\sigma(\phi \to K^+ K^-)$ varies by 2% for different variants, but these biases are less than its error. The cross section is almost independent of the applied model (Fig. 14).

The data for the $e^+ e^- \to K^0_S K^+ K^-$ reaction obtained in charged and neutral modes were fitted simultaneously. For this process the phase $\phi_K$ is different from the expected 180° by more than five standard deviations and is close to 90° (variant 2 in Table VIII). The imaginary part of the $A_0$ amplitude is consistent with zero, and its real part is different from zero by about two standard deviations (variants 5 and 4 in Table VIII). The parameters $m_\phi$ and $\Gamma_\phi$ are slightly dependent on the theoretical assumptions. The value of $\sigma(\phi \to K^0_S K^+ K^-)$ varies by 2% for different variants and is shown in Fig. 15. The cross section is almost insensitive to the choice of the model.

C. Combined fitting of the cross sections

To obtain the $\phi$-meson production cross section and branching ratios of the main decay modes, the combined fitting of the experimental data, taking into account the $e^+ e^- \to \eta \gamma$ cross section also, was carried out. As this work does not contain the analysis of the $e^+ e^- \to \eta \gamma$ reaction, it was natural to use the cross section $\sigma(\phi \to \eta \gamma)$ from the previous SND measurements presented in Ref. 25. In the Ref. 25 the correction $\delta_{\text{corr}} = 0.97 \pm 0.03$ to the detection efficiency was not applied. The value $\sigma(\phi \to \eta \gamma) = 53.2 \pm 2$ nb, presented there, was increased by 3% and the value $54.8 \pm 2.1$ nb was used in the fit. The results of the fit, for the different model assumptions, are shown in Table IX. In all fits $\text{Im}(A_0^{K^0_S K^+ K^-}) = 0$, $\text{Im}(A_0^{K^0_S K^+ K^-}) = 0$, and $\text{Im}(A_0^{3\pi}) = 0$ were set.

Using these data, the following $\phi$-meson parameters were obtained: the $\phi$-meson production cross section

$$\sigma(\phi) = \sum_X \sigma(\phi \to X), X = K^+ K^-, K^0_S K^+ K^-, \pi^+ \pi^- \pi^0, \eta \gamma$$ (16)

the branching ratios of the main decay modes

$$B(\phi \to X) = \frac{\sigma(\phi \to X)}{\sigma(\phi)},$$ (17)

and the branching ratio of the decay into an $e^+ e^-$ pair:

$$B(\phi \to e^+ e^-) = \frac{\sigma(\phi) m_\phi^2}{12\pi},$$ (18)

The final results are

$$B(\phi \to e^+ e^-) = (2.93 \pm 0.02 \pm 0.14 \pm 0.02) \times 10^{-4},$$

$$B(\phi \to K^+ K^-) = 47.6 \pm 0.3 \pm 1.6 \pm 0.3\%$$

$$B(\phi \to K^0_S K_L) = 35.1 \pm 0.2 \pm 1.2 \pm 0.3\%$$

$$B(\phi \to 3\pi) = 15.9 \pm 0.2 \pm 0.7 \pm 0.4\%$$

$$B(\phi \to \eta \gamma) = 1.33 \pm 0.03 \pm 0.05 \pm 0.01\%$$
These values correspond to the fit where \( \chi_{\phi-\omega} = 180^\circ + \Delta \chi_{\phi-\omega}(s), \phi K^0 \chi = 180^\circ, \text{Im}(A^{K^0}_{3\pi}) = 0, A^{K^0}_{K_S K_L} = 0 \) and \( A^{K^+ K^-} = 0 \) (variant 2 in Table IX). The third error is a model uncertainty. The presented values are in good agreement with the world average and with other experimental data (Table IX). Using the SND data, the following coupling constants were calculated: \( |g_{\phi\gamma}| = 6538 \pm 156 \text{ MeV}^2, |g_{\phi K^+ K^-}| = 4.391 \pm 0.078, \text{ and } |g_{\phi K_S K_L}| = 4.664 \pm 0.086. \) The uncertainty of the measured cross sections and of the related parameters \( \sigma(\phi \rightarrow K^+ K^-), \sigma(\phi \rightarrow 3\pi) \) are 7.1% and 5% respectively and include the systematic uncertainties in the detection efficiency and luminosity determinations. For the \( e^+e^- \rightarrow K_S K_L \) cross section, the systematic error is 4.2% in the charged mode and 4.0% in the neutral mode. The systematic error of the parameter \( \sigma(\phi \rightarrow K_S K_L) \) is 3.2%.

VI. DISCUSSION

The \( \phi \)-meson mass and width measured in the three studied main decay modes are consistent with each other. The measurements of the \( \phi \)-meson parameters in all channels have high accuracy. Let us mention that all these processes have different energy dependence for detection efficiency. The dependence of the mass value on the applied theoretical model is less than the measurement accuracy. As a result of the combined approximation, we find:

\[
m_\phi = 1019.42 \pm 0.02 \pm 0.04 \text{ MeV},
\]

where the systematic error of 0.04 MeV is attributed to the possible common shift of the collider energy scale. This value is consistent with the world average: 1019.417 \pm 0.014 MeV [23]. The value of the total \( \phi \)-meson width, obtained by using different models, can vary from 4.18 to 4.21 MeV and was found to be

\[
\Gamma_\phi = 4.21 \pm 0.03 \pm 0.02 \text{ MeV}
\]

(the systematic error 0.02 MeV is due to model dependence), and differs from the world average 4.458 \pm 0.032 MeV [23] by 4.8 standard deviations and from the closest measurement \( \Gamma_\phi = 4.477 \pm 0.036 \pm 0.022 \text{ MeV} [23] \) by 4.5 standard deviations.

The \( e^+e^- \rightarrow \pi^+\pi^-\pi^0 \) process, as the approximation showed, cannot be described by only \( \phi \) and \( \omega \) meson decays and it is necessary to include additional amplitude \( A^0_{3\pi} \), which can be explained as a contribution of the higher resonances. The presence of such an amplitude is also expected from the experimental data on the \( e^+e^- \rightarrow \pi^+\pi^-\pi^0 \) cross section measured by the SND detector in the energy region above the \( \phi \)-resonance, which cannot be described with the conventional VDM. The value of the amplitude \( A^0_{3\pi} = 0.046 \pm 0.003 \text{ MeV}^{1/2} \) (Table XI) is only 2 times lower than the real part of \( \omega \)-meson amplitude in the vicinity of the \( \phi \)-meson peak \( \text{Re}(A_\omega) \approx 0.09 \text{ MeV}^{1/2} \) and approximately equals its imaginary part \( \text{Im}(A_\omega) \approx 0.05 \text{ MeV}^{1/2} \). The presence of the \( A^0_{3\pi} \) amplitude prevents a test of the deviation of the \( \chi_{\phi-\omega} \) phase from 180°. Both \( \chi_{\phi-\omega} = 180^\circ \) and \( 180^\circ + \Delta \chi_{\phi-\omega}(s) \) are consistent with the experimental data. If the \( \chi_{\phi-\omega} \) phase is considered to be a free parameter with no energy dependence, then its fitted value is 178° \pm 27° for free real and imaginary parts of \( A^0_{3\pi} \) (variant 5 in Table VII), and \( 165^\circ \pm 19^\circ \), if \( \text{Im}(A^0_{3\pi}) = 0 \) is set (variant 3 in Table IX).

The parameter:

\[
\sigma(\phi \rightarrow 3\pi) = 659 \pm 6 \pm 33 \pm 20 \text{ nb}
\]

12
has a rather large model error $(\pm 20 \text{ nb})$, due to the uncertainty in the choice of the phase $\chi_{\omega - \phi}$. The quoted value of the $\sigma(\phi \rightarrow 3\pi)$ corresponds to the choice $\chi_{\omega - \phi} = 180^\circ + \Delta \chi_{\omega - \phi}(s)$ and $\text{Im}(A_{3\pi}^0) = 0$. It agrees with other measurements, for example, $\sigma(\phi \rightarrow 3\pi) = 619 \pm 39 \pm 12$ from Ref. [2] and $\sigma(\phi \rightarrow 3\pi) = 564 \pm 26 \pm 30$ from Ref. [3].

In case of the $e^+e^- \rightarrow K^+K^-$ and $K_SK_L$ processes, the $\text{Im}(A_{K^+K^-}^0)$, $\text{Im}(A_{K_SK_L}^0)$ agree with zero (variant 4 in Tables VII and VIII) and $\text{Re}(A_{K^+K^-}^0)$, $\text{Re}(A_{K_SK_L}^0)$ deviates from zero by about two standard deviations (variant 5, Table IX). The phase $\phi_{K\bar{K}}$ extracted from the combined fit, or from the fitting of the $e^+e^- \rightarrow K^+K^-$ process, is consistent with $180^\circ$ (variants 2 and 4 in Tables VII and IX), but the fitting of the $e^+e^- \rightarrow K_SK_L$ data gives the phase value close to $90^\circ$. Such a result can be attributed to the uncertainty of the nonresonant contributions to the cross section and was used to estimate the model error in the cross section determination. The results

$$\sigma(\phi \rightarrow K^+K^-) = 1968 \pm 20 \pm 140$$

$$\sigma(\phi \rightarrow K_SK_L) = 1451 \pm 10 \pm 48 \text{ nb}$$

agree with the world average [2].

Isotopic symmetry predicts the equality of the $g_{\phi K^+K^-}$ and $g_{\phi K_SK_L}$ coupling constants. The electromagnetic interactions of charged kaons in the final state lead to the relation $g_{\phi K^+K^-} / \sqrt{Z(m_\phi)} = g_{\phi K_SK_L}$. Using the Particle Data Group (PDG) data [2], one can find:

$$\frac{g_{\phi K^+K^-}}{g_{\phi K_SK_L}} \frac{1}{\sqrt{Z(m_\phi)}} = \sqrt{\frac{B(\phi \rightarrow K^+K^-)g_{K^+K^-}(m_\phi)}{B(\phi \rightarrow K_SK_L)q_{K^+K^-}(m_\phi)}} = 0.95 \pm 0.01 \quad (19)$$

This value differs considerably (five standard deviations) from the expected value of unity. It is possible that the Coulomb correction is less than expected due to finite dimensions of the final particles. A similar problem in the $\Upsilon_{4s} \rightarrow BB$ decay was discussed in Refs. [2, 4]. Quite recently the ratio of the constants $g_{\phi K^+K^-}$ and $g_{\phi K_SK_L}$ was discussed in Ref. [3], where it was suggested to study this ratio in $e^+e^-$ annihilation around the $\phi$ resonance peak. The drawback of this proposal is a very low sensitivity of such studies to the nonresonant contribution to the total cross section. The extracted coupling constant values depend on the nonresonant contribution to the total cross section from the $\rho$ and $\omega$ mesons and from higher radial excitations ($\rho', \omega', \phi', ...$). So one of the possible approaches to the solution of this problem can be a precise study of these contributions to the total cross section, by means of the measurement of the $e^+e^- \rightarrow K\bar{K}$ cross section in a wide energy range. Unfortunately the SND data have rather high systematic errors, which preclude such studies, and they are accumulated in a rather narrow energy region. From SND data (Table IX) one can obtain

$$\frac{g_{\phi K^+K^-}}{g_{\phi K_SK_L}} \frac{1}{\sqrt{Z(m_\phi)}} = 0.92 \pm 0.03. \quad (20)$$

This value agrees with the quoted world average, but is also consistent with the expected value of 1 (the difference is about 2.5 standard deviations).
VII. CONCLUSION

The main parameters of the $\phi$-resonance were obtained in experiments with the SND detector at the VEPP-2M collider. The integrated luminosity used was about $8.5 \, \text{pb}^{-1}$. The processes $e^+e^- \rightarrow K^+K^-$, $e^+e^- \rightarrow K_SK_L$, and $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ were studied. The measured cross sections were approximated within the VDM, taking into account $\rho$, $\omega$ and $\phi$-mesons. Possible contributions from higher resonances $\rho'$, $\omega'$, $\phi'$ were included as constant amplitudes. The $\phi$-meson parameters obtained by the approximation (Table X) mainly agree with PDG data and have accuracies comparable with the world averages. The $\phi$-meson total width was found to be $4.21 \pm 0.04 \, \text{MeV}$ in contradiction to the PDG value $4.458 \pm 0.032 \, \text{MeV}$. For a good approximation of the $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section the additional contribution is strongly required, and its value is about 2 times lower than the real part of the $\omega$-meson contribution. This contribution can be attributed to the higher resonances, for example to the resonance structure found by SND.26

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1. G. Parrou et al., Phys. Lett. B 364, 362 (1976).
2. A.D. Bukin et al., Yad. Fiz. 27, 976 (1978) [Sov. J. of Nucl. Phys. 27, 516 (1978)].
3. A. Cordier et al., Phys. Lett. B 364, 13 (1980).
4. S.I. Dolinsky et al., Phys. Rep. 202, 99 (1991).
5. R.R. Akhmetshin et al., Phys. Lett. B 364, 199 (1995).
6. B.R. Akhmetshin et al., Phys. Lett. B 434, 426 (1998).
7. L.M. Kurdadze et al., Preprint INP 84-07, Novosibirsk, 1984.
8. R.R. Akhmetshin et al., Phys. Lett. B 466, 385 (1999).
9. P.M. Ivanov et al., Phys. Lett. B 107, 297 (1981).
10. V.E. Balakin et al., Phys. Lett. B 34, 328 (1971).
11. A.N. Skrinsky, in Proc. of Workshop on physics and detectors for DAΦNE, Frascati, Italy, April 4-7, 1995, p.3
12. M.N. Achasov et al., Nucl. Instr. and Meth. A 449, 125 (2000).
13. R.R. Akhmetshin et al., Preprint, Budker INP 99-11, Novosibirsk, 1999.
14. M.N. Achasov et al., Phys. Lett. B 499, 122 (1999).
15. N.N. Achasov et al., Yad. Fiz. 54, 1097 (1991) [Sov. J. Nucl. Phys. 54, 664 (1991)];
   N.N. Achasov, et. al., Int. J. of Mod. Phys. A vol.7 No.14 3187 (1992).
16. E. Cremmer and M. Gourdin, Nucl.Phys. B 9, 451 (1969).
17. N.N. Achasov, A.A. Kozhevnikov, Phys. Rev. D 55, 2663 (1997); N.N. Achasov, A.A. Kozhevnikov, Yad. Fiz. 60, 1131 (1997) (in Russian);
   N.N. Achasov, A.A. Kozhevnikov, Phys. Rev. D 57, 4334 (1998).
18. N.N. Achasov, A.A. Kozhevnikov, Yad. Fiz. 60, 2212 (1997) [Phys. At. Nucl. 60, 2029 (1997)]
19. E.A. Kuraev, V.S. Fadin, Yad. Fiz. 41, 733 (1985) [Sov. J. Nucl. Phys. 41, 466 (1985)].
20. A.V. Bozhnenok, et. al., Preprint, Budker INP (in Russian) 99-103, 1999.
21. Particle Data Group, D.E. Groom, et al., Eur.Phys.J. C 15, 1 (2000).
22. D. Atwood and W.J. Marciano, Phys.Rev. D 41, 1736 (1990).
23. G.P. Lepage, Phys.Rev. D 42, 3251 (1990).
24. N.N. Achasov, A.A. Kozhevnikov, Phys. Rev. D 61 054005 (2000).
25. M.N. Achasov et al., Zh. Eksp. Teor. Fiz. 117, 22 (2000) [JETP 90, 17 (2000)].
26. M.N. Achasov et al., Phys. Lett. B 462, 365 (1999).
27 A. Bramon et al., Phys. Lett. B 486, 406 (2000)
| \( \sqrt{s} \) (MeV) | \( \sigma \) (MeV) | \( L_{cha} \) (nb\(^{-1}\)) | \( L_{neu} \) (nb\(^{-1}\)) | \( \sqrt{s} \) (MeV) | \( \sigma \) (MeV) | \( L_{cha} \) (nb\(^{-1}\)) | \( L_{neu} \) (nb\(^{-1}\)) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 984.21±0.10     | 0.36            | 173.7±2.3       | 159.7±2.4       | 984.02±0.10     | 0.36            | 200.9±2.7       | 186.7±2.7       |
| 1003.91±0.10    | 0.36            | 209.9±2.8       | 193.6±2.8       | 1003.71±0.10    | 0.36            | 181.1±2.4       | 171.4±2.6       |
| 1010.17±0.13    | 0.37            | 156.0±2.1       | 140.8±2.2       | 1010.34±0.13    | 0.37            | 158.1±2.1       | 147.6±2.2       |
| 1015.75±0.07    | 0.37            | 168.2±2.2       | 160.1±2.4       | 1015.43±0.07    | 0.39            | 200.9±2.7       | 183.7±2.7       |
| 1016.68±0.07    | 0.37            | 304.7±4.0       | 286.8±4.1       | 1016.78±0.07    | 0.38            | 333.0±4.4       | 299.1±4.2       |
| 1017.59±0.07    | 0.37            | 483.3±6.3       | 444.2±6.1       | 1017.72±0.07    | 0.37            | 528.4±6.9       | 478.7±6.6       |
| 1018.78±0.07    | 0.38            | 527.0±6.9       | 482.8±6.6       | 1018.62±0.07    | 0.38            | 536.5±7.0       | 484.1±6.6       |
| 1019.79±0.07    | 0.37            | 560.2±7.3       | 511.4±7.0       | 1019.51±0.07    | 0.37            | 523.6±6.9       | 472.7±6.5       |
| 1020.65±0.08    | 0.37            | 323.7±4.3       | 296.3±4.2       | 1020.43±0.08    | 0.37            | 355.6±4.7       | 330.7±4.6       |
| 1021.68±0.08    | 0.37            | 163.2±2.2       | 150.4±2.3       | 1021.41±0.08    | 0.37            | 184.3±2.4       | 172.1±2.6       |
| 1023.27±0.09    | 0.37            | 202.6±2.7       | 186.2±2.8       | 1022.32±0.09    | 0.39            | 178.1±2.4       | 166.3±2.5       |
| 1028.23±0.14    | 0.37            | 167.9±2.2       | 150.1±2.3       | 1027.52±0.14    | 0.38            | 218.4±2.9       | 211.5±3.1       |
| 1033.84±0.10    | 0.37            | 167.8±2.2       | 147.7±2.3       | 1033.58±0.10    | 0.36            | 182.2±2.4       | 172.3±2.6       |
| 1039.59±0.10    | 0.39            | 162.7±2.1       | 151.5±2.3       | 1039.64±0.10    | 0.37            | 185.1±2.5       | 169.2±2.5       |
| 1049.81±0.10    | 0.38            | 196.9±2.6       | 178.9±2.7       | 1049.60±0.10    | 0.38            | 187.2±2.5       | 176.3±2.6       |
| 1059.66±0.10    | 0.38            | 169.8±2.3       | 158.6±2.4       | 1059.52±0.10    | 0.50            | 216.3±2.9       | 205.6±3.0       |

**TABLE I.** Experimental main parameters: \( \sqrt{s} \) – the energy in the center of mass system, \( \sigma \) – beam energy spread, \( L_{cha} \), \( L_{neu} \) – integrated luminosities used for normalizing the events containing charged particles and with neutral particles only.

| \( \sqrt{s} \) (MeV) | \( N_{K^+K^-} \) | \( \epsilon_{K^+K^-} \) | \( \sqrt{s} \) (MeV) | \( N_{K^+K^-} \) | \( \epsilon_{K^+K^-} \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1010.17         | 306±23          | 0.055±0.012     | 1010.34         | 272±22          | 0.0599±0.012    |
| 1015.75         | 8370±108        | 0.1669±0.0045   | 1015.43         | 8524±110        | 0.1638±0.0045   |
| 1016.68         | 27723±195       | 0.1749±0.0046   | 1016.78         | 31181±207       | 0.1758±0.0046   |
| 1017.59         | 69921±321       | 0.1814±0.0047   | 1017.72         | 72263±325       | 0.1821±0.0047   |
| 1018.78         | 12459±431       | 0.1878±0.0049   | 1018.62         | 121718±421      | 0.1869±0.0048   |
| 1019.79         | 14933±472       | 0.1922±0.0049   | 1019.51         | 139295±452      | 0.1909±0.0049   |
| 1020.65         | 73994±321       | 0.1958±0.005   | 1020.43         | 88199±342       | 0.1948±0.005    |
| 1021.68         | 26342±187       | 0.2001±0.0052   | 1021.41         | 33916±211       | 0.1992±0.0051   |
| 1023.27         | 20637±167       | 0.2037±0.0053   | 1022.32         | 22111±169       | 0.2021±0.0052   |
| 1028.23         | 7009±97         | 0.2115±0.0059   | 1027.52         | 9865±112        | 0.2106±0.0059   |
| 1033.84         | 3747±72         | 0.2163±0.0064   | 1033.58         | 4437±75         | 0.2164±0.0064   |
| 1039.59         | 2349±52         | 0.1975±0.0063   | 1039.64         | 2745±58         | 0.1972±0.0063   |
| 1049.81         | 1270±40         | 0.1330±0.0005   | 1049.6          | 1298±40         | 0.1334±0.0005   |
| 1059.66         | 672±27          | 0.1228±0.0053   | 1059.52         | 961±33          | 0.1233±0.0053   |

**TABLE II.** Event numbers \( N \) and detection efficiencies \( \epsilon \) for the \( e^+e^- \rightarrow K^+K^- \) process versus energy.
### TABLE III. Event numbers $N$ and detection efficiencies $\epsilon$ for the $e^+e^- \rightarrow K_S K_L$ process with $K_S \rightarrow \pi^+\pi^-$ decay.

| $\sqrt{s}$ (MeV) | $N_{K_S K_L}$ | $\epsilon_{K_S K_L}$ | $\sqrt{s}$ (MeV) | $N_{K_S K_L}$ | $\epsilon_{K_S K_L}$ |
|------------------|---------------|------------------------|-------------------|---------------|------------------------|
| 1003.91          | 56$\pm$9      | 0.0475$\pm$0.019       | 1003.71           | 46$\pm$9      | 0.0469$\pm$0.019       |
| 1010.17          | 277$\pm$18    | 0.532$\pm$0.0069       | 1010.34           | 249$\pm$17    | 0.0534$\pm$0.0069       |
| 1015.75          | 2372$\pm$50   | 0.0614$\pm$0.0028      | 1015.43           | 2594$\pm$53   | 0.0614$\pm$0.0028       |
| 1016.68          | 7092$\pm$88   | 0.0612$\pm$0.0016      | 1016.78           | 8314$\pm$95   | 0.0612$\pm$0.0016       |
| 1017.59          | 18335$\pm$141 | 0.0634$\pm$0.0011      | 1017.72           | 19545$\pm$145 | 0.0632$\pm$0.0011       |
| 1018.78          | 30820$\pm$182 | 0.0631$\pm$0.0011      | 1018.62           | 31703$\pm$185 | 0.0631$\pm$0.0011       |
| 1019.79          | 36358$\pm$198 | 0.0636$\pm$0.0012      | 1019.51           | 35272$\pm$195 | 0.0636$\pm$0.0012       |
| 1020.65          | 17276$\pm$137 | 0.0632$\pm$0.0011      | 1020.43           | 21071$\pm$150 | 0.0633$\pm$0.0011       |
| 1021.68          | 5975$\pm$80   | 0.0642$\pm$0.0019      | 1021.41           | 7726$\pm$90   | 0.0643$\pm$0.0019       |
| 1023.27          | 4453$\pm$69   | 0.0633$\pm$0.0022      | 1022.32           | 4895$\pm$72   | 0.0637$\pm$0.0022       |
| 1028.23          | 1361$\pm$39   | 0.0627$\pm$0.0035      | 1027.52           | 2079$\pm$47   | 0.0630$\pm$0.0035       |
| 1033.84          | 679$\pm$27    | 0.0607$\pm$0.0043      | 1033.58           | 901$\pm$31    | 0.0608$\pm$0.0043       |
| 1039.59          | 471$\pm$23    | 0.0620$\pm$0.0057      | 1039.64           | 517$\pm$24    | 0.0626$\pm$0.0056       |
| 1049.81          | 311$\pm$19    | 0.0633$\pm$0.0074      | 1049.60           | 330$\pm$20    | 0.0626$\pm$0.0073       |
| 1059.66          | 234$\pm$17    | 0.0395$\pm$0.0069      | 1059.52           | 245$\pm$17    | 0.0395$\pm$0.0069       |

### TABLE IV. Event numbers $N$ for the $e^+e^- \rightarrow K_S K_L$ process with $K_S \rightarrow \pi^0\pi^0$ decay.

| $\sqrt{s}$ (MeV) | $N_{K_S K_L}$ | $\sqrt{s}$ (MeV) | $N_{K_S K_L}$ |
|------------------|---------------|-------------------|---------------|
| 1003.91          | 74$\pm$9      | 1003.71           | 57$\pm$8      |
| 1010.17          | 241$\pm$16    | 1010.34           | 250$\pm$16    |
| 1015.75          | 2340$\pm$50   | 1015.43           | 243$\pm$51    |
| 1016.68          | 7267$\pm$87   | 1016.78           | 7865$\pm$90   |
| 1017.59          | 17558$\pm$135 | 1017.72           | 20422$\pm$146 |
| 1018.78          | 29870$\pm$176 | 1018.62           | 29376$\pm$175 |
| 1019.79          | 35002$\pm$190 | 1019.51           | 32910$\pm$185 |
| 1020.65          | 16908$\pm$132 | 1020.43           | 20454$\pm$145 |
| 1021.68          | 5968$\pm$79   | 1021.41           | 7680$\pm$89   |
| 1023.27          | 4453$\pm$68   | 1022.32           | 4945$\pm$72   |
| 1028.23          | 1376$\pm$38   | 1027.52           | 2187$\pm$48   |
| 1033.84          | 767$\pm$28    | 1033.58           | 936$\pm$31    |
| 1039.59          | 521$\pm$24    | 1039.64           | 559$\pm$24    |
| 1049.81          | 372$\pm$20    | 1049.60           | 390$\pm$20    |
| 1059.66          | 234$\pm$16    | 1059.52           | 333$\pm$19    |

TABLE IV. Event numbers $N$ for the $e^+e^- \rightarrow K_S K_L$ process with $K_S \rightarrow \pi^0\pi^0$ decay.
### TABLE V. Event numbers \(N_{3\pi}\) and \(N_{bkg}\) of the \(e^+e^- \rightarrow \pi^+\pi^-\pi^0\) (after background subtraction) and background processes respectively. The accuracy of background event numbers determination is less than 15% at all energy points.

| \(\sqrt{s}\) (MeV) | \(N_{3\pi}\) | \(N_{bkg}\) | \(\sqrt{s}\) (MeV) | \(N_{3\pi}\) | \(N_{bkg}\) |
|-------------------|-------------|-------------|-------------------|-------------|-------------|
| 984.21            | 498 ± 23   | 5           | 984.02            | 552 ± 25   | 5           |
| 1003.91           | 1145 ± 36  | 8           | 1003.71           | 1026 ± 34  | 7           |
| 1010.17           | 1548 ± 42  | 11          | 1010.34           | 1592 ± 43  | 12          |
| 1015.75           | 5610 ± 79  | 52          | 1015.43           | 6094 ± 83  | 55          |
| 1016.68           | 14833 ± 129| 154         | 1016.78           | 15956 ± 133| 177         |
| 1017.59           | 31862 ± 190| 394         | 1017.72           | 36261 ± 202| 458         |
| 1018.78           | 45502 ± 228| 657         | 1018.62           | 46722 ± 230| 643         |
| 1019.79           | 46590 ± 229| 770         | 1019.51           | 47414 ± 232| 735         |
| 1020.65           | 20172 ± 151| 381         | 1020.43           | 25387 ± 169| 448         |
| 1021.68           | 6339 ± 86  | 144         | 1021.41           | 8588 ± 99  | 180         |
| 1023.27           | 4160 ± 69  | 104         | 1022.32           | 4969 ± 76  | 121         |
| 1026.23           | 444 ± 22   | 4           | 1027.52           | 715 ± 27   | 5           |
| 1033.84           | 203 ± 16   | 3           | 1033.58           | 254 ± 16   | 4           |
| 1039.59           | 118 ± 12   | 2           | 1039.64           | 133 ± 12   | 2           |
| 1049.81           | 91 ± 12    | 2           | 1049.60           | 87 ± 9     | 2           |
| 1059.66           | 54 ± 8     | 4           | 1059.52           | 66 ± 8     | 5           |

### TABLE VI. Fit results for the process \(e^+e^- \rightarrow \pi^+\pi^-\pi^0\). The column number \(N\) corresponds to the different variants of choice of the phase \(\chi_{\phi-\omega}\) and the imaginary part of the amplitude \(\text{Im}(A_{3\pi})\). Here \(\Psi_{\phi-\omega}(s) = 180^\circ + \Delta\chi_{\phi-\omega}(s)\) is the phase value predicted in Ref. 24.

| \(N\) | 1   | 2   | 3   | 4   | 5   | 6   |
|-------|-----|-----|-----|-----|-----|-----|
| \(m_\phi - 1000\) (MeV) | 19.41 ± 0.03 | 19.42 ± 0.03 | 19.41 ± 0.03 | 19.41 ± 0.03 | 19.42 ± 0.03 | 19.42 ± 0.03 |
| \(\Gamma_\phi\) (MeV) | 4.18 ± 0.06 | 4.18 ± 0.06 | 4.18 ± 0.06 | 4.18 ± 0.06 | 4.18 ± 0.06 | 4.18 ± 0.06 |
| \(\sigma(\phi \rightarrow 3\pi)\) (nb) | 668±27 | 657±33 | 668 ± 24 | 704 ± 8 | 661 ± 7 | 666±2.18 |
| \(\text{Re}(A_{3\pi}^0) \cdot 10^3\) (MeV\(^{1/2}\)) | -54±15 | -48±15 | -53 ± 6 | -47 ± 3 | -47 ± 3 | -47 ± 3 |
| \(\text{Im}(A_{3\pi}^0) \cdot 10^3\) (MeV\(^{1/2}\)) | -35±40 | -4±30 | -33 ± 60 | 0 | 0 | 0 |
| \(\chi_{\phi-\omega}\) (degree) | 180 | \(\Psi_{\phi-\omega}(s)\) | 178 ± 27 | 180 | \(\Psi_{\phi-\omega}(s)\) | 165±30 |
| \(\chi^2/N_{df}\) | 20.39/27 | 20.46/27 | 20.39/26 | 20.56/28 | 20.49/28 | 20.49/27 |
\[
\begin{array}{c|ccccc}
N & 1 & 2 & 3 & 4 & 5 \\
\hline
m_{\phi} & 1019.46 \pm 0.03 & 1019.46 \pm 0.03 & 1019.44 \pm 0.03 & 1019.44 \pm 0.03 & 1019.44 \pm 0.04 \\
(\text{MeV}) & & & & & \\
\Gamma_{\phi} & 4.28 \pm 0.06 & 4.26 \pm 0.06 & 4.22 \pm 0.08 & 4.22 \pm 0.08 & 4.22 \pm 0.08 \\
(\text{MeV}) & & & & & \\
\sigma(\phi \rightarrow K^+ K^-) & 1940 \pm 31 & 1962 \pm 38 & 1972^{+63}_{-58} & 1969 \pm 64 & 1950 \pm 32 \\
(\text{nb}) & & & & & \\
\phi_{K^+ K^-} & 180 & 157 \pm 24 & 156^{+31}_{-39} & 180 & 180 \\
(\text{degree}) & & & & & \\
\text{Re}(A_{K^+ K^-}^0) & 0 & 0 & 4.7^{+1.4}_{-1.3} & 7.1^{+2.5}_{-1.7} & 5.9 \pm 4.8 \\
(\text{MeV}^{3/2}) & & & & & \\
\text{Im}(A_{K^+ K^-}^0) & 0 & 0 & 0 & 12 \pm 33 & 0 \\
(\text{MeV}^{3/2}) & & & & & \\
\chi^2/N_{df} & 22.15/25 & 21.2/24 & 20.45/23 & 20.54/23 & 20.65/24 \\
\end{array}
\]

**TABLE VII.** Fit results for the process \(e^+ e^- \rightarrow K^+ K^-\). The column number \(N\) corresponds to the different variants of choice of the phase \(\phi_{K^+ K^-}\) and the real and imaginary parts of the amplitude \(\text{Re}(A_{K^+ K^-}^0), \text{Im}(A_{K^+ K^-}^0)\).

\[
\begin{array}{c|ccccc}
N & 1 & 2 & 3 & 4 & 5 \\
\hline
m_{\phi} & 1019.40 \pm 0.02 & 1019.38 \pm 0.02 & 1019.38 \pm 0.02 & 1019.38 \pm 0.02 & 1019.38 \pm 0.02 \\
(\text{MeV}) & & & & & \\
\Gamma_{\phi} & 4.20 \pm 0.04 & 4.15 \pm 0.04 & 4.15 \pm 0.05 & 4.16 \pm 0.04 & 4.16 \pm 0.04 \\
(\text{MeV}) & & & & & \\
\sigma(\phi \rightarrow K_SK_L) & 1454 \pm 12 & 1436 \pm 12 & 1450^{+73}_{-71} & 1440^{+70}_{-69} & 1458 \pm 12 \\
(\text{nb}) & & & & & \\
\phi_{K_SK_L} & 180 & 102 \pm 14 & 120 \pm 51 & 180 & 180 \\
(\text{degree}) & & & & & \\
\text{Re}(A_{K_SK_L}^0) & 0 & 0 & 8^{+30}_{-40} & 4^{+5}_{-7} & 5.7 \pm 2.8 \\
(\text{MeV}^{3/2}) & & & & & \\
\text{Im}(A_{K_SK_L}^0) & 0 & 0 & 0 & -3^{+50}_{-26} & 0 \\
(\text{MeV}^{3/2}) & & & & & \\
\chi^2/N_{df} & 64.11/57 & 59.95/56 & 59.94/55 & 59.95/55 & 59.96/56 \\
\end{array}
\]

**TABLE VIII.** Fit results for the process \(e^+ e^- \rightarrow K_SK_L\). The column number \(N\) corresponds to the different variants of choice of the phase \(\phi_{K_SK_L}\) and the real and imaginary parts of the amplitude \(\text{Re}(A_{K_SK_L}^0), \text{Im}(A_{K_SK_L}^0)\).
TABLE IX. Results of the combined fit for the processes $e^+e^- \rightarrow KK$ and $3\pi$. The column number $N$ corresponds to the different variants of choice of the phases $\phi_{KK}$, $\chi_{\phi-\omega}$ and the real parts of the amplitudes Re($A_{K^+K^-}^0$), Re($A_{KSKL}^0$), Re($A_{3\pi}^0$).

| $N$ | 1      | 2      | 3      | 4      | 5      |
|-----|--------|--------|--------|--------|--------|
| $m_{\phi} - 1000$ (MeV) | 19.417 ± 0.014 | 19.419 ± 0.014 | 19.419 ± 0.014 | 19.418 ± 0.014 | 19.40 ± 0.02 |
| $\Gamma_{\phi}$ (MeV) | 4.21 ± 0.04 | 4.21 ± 0.03 | 4.21 ± 0.03 | 4.21 ± 0.03 | 4.18 ± 0.03 |
| $\sigma(\phi \rightarrow KSKL)$ (nb) | 1451 ± 10 | 1451 ± 10 | 1451 ± 10 | 1441 ± 12 | 1455 ± 10 |
| $\sigma(\phi \rightarrow K^+K^-)$ (nb) | 1968 ± 20 | 1968 ± 20 | 1967 ± 20 | 1994 ± 27 | 1961 ± 21 |
| $\sigma(\phi \rightarrow 3\pi)$ (nb) | 701 ± 6 | 659 ± 6 | 664±18 | 702 ± 6 | 705 ± 6 |
| Re($A_{K^+K^-}^0$) (MeV$^{3/2}$) | 0 | 0 | 0 | 0 | 8.6 ± 3 |
| Re($A_{KSKL}^0$) (MeV$^{3/2}$) | 0 | 0 | 0 | 0 | 4.3 ± 2.5 |
| Re($A_{3\pi}^0$) · $10^3$ (MeV$^{1/2}$) | $-46 ± 3$ | $-46 ± 3$ | $-46 ± 3$ | $-46 ± 3$ | $-46 ± 3$ |
| $\chi_{\phi-\omega}$ (degree) | 180 | 180 + $\Delta \chi_{\phi-\omega}(s)$ | 165±19 | 180 | 180 |
| $\phi_{KK}$ (degree) | 180 | 180 | 180 | 147±27 | 180 |
| $\chi^2/N_{dof}$ | 110.29/114 | 110.09/114 | 110.1/113 | 108.67/113 | 103.39/111 |

TABLE X. The comparison of the main results of this work with PDG data and results of the other experiments.

|                      | SND                  | Other data                  |
|----------------------|----------------------|-----------------------------|
| $\sigma(\phi \rightarrow KSKL)$, nb | 1451 ± 49 | 1367 ± 29 |
| $\sigma(\phi \rightarrow K^+K^-)$, nb | 1968 ± 142 | 2001 ± 104 |
| $\sigma(\phi \rightarrow 3\pi)$, nb | 659 ± 35 | 654 ± 40 |
| $B(\phi \rightarrow e^+e^-) \times 10^4$ | 2.93 ± 0.14 | 2.91 ± 0.09 |
| $B(\phi \rightarrow K^+K^-)$, % | 47.6 ± 1.7 | 49.2 ± 0.75 |
| $B(\phi \rightarrow KSKL)$, % | 35.1 ± 1.3 | 33.8 ± 0.7 |
| $B(\phi \rightarrow 3\pi)$, % | 15.9 ± 0.8 | 15.5 ± 0.62 |
| $B(\phi \rightarrow \eta\gamma)$, % | 1.33 ± 0.06 | 1.297 ± 0.033 |
| $m_{\phi}$, MeV | 1019.42 ± 0.05 | 1019.417 ± 0.041 |
| $\Gamma_{\phi}$, MeV | 4.21 ± 0.04 | 4.458 ± 0.032 |
| √s (GeV) | \(\sigma_{K^+K^-}\) (charged mode) | \(\sigma_{K_S K_L}\) (neutral mode) | \(\sigma_{K^-}\) (charged mode) | \(\sigma_{K_S K_L}\) (neutral mode) | \(\sigma_{3\pi}\) |
|-----------|---------------------------------|---------------------------------|------------------------------|---------------------------------|------------------|
| 984.21    | 18.1 ± 0.9                      |                                 |                              |                                 |                  |
| 1003.91   | 7.7 ± 3.0                       | 7.7 ± 1.0                       | 36.2 ± 1.3                   |                                 |                  |
| 1010.17   | 48.1 ± 13.3                     | 45.2 ± 6.1                      | 34.0 ± 2.8                   | 68.5 ± 2.4                     |                  |
| 1015.75   | 409.0 ± 20.3                    | 315.5 ± 20.2                    | 294.5 ± 13.3                 | 243.1 ± 7.5                    |                  |
| 1016.68   | 717.1 ± 32.3                    | 525.5 ± 25.7                    | 513.1 ± 20.6                 | 358.9 ± 10.6                   |                  |
| 1017.59   | 1112.0 ± 51.8                   | 836.3 ± 37.6                    | 809.1 ± 32.5                 | 493.6 ± 14.9                   |                  |
| 1018.78   | 1794.1 ± 64.8                   | 1324.7 ± 40.4                   | 1295.0 ± 33.7                | 658.6 ± 11.6                   |                  |
| 1019.79   | 1898.1 ± 56.9                   | 1388.4 ± 36.2                   | 1372.4 ± 23.9                | 595.5 ± 14.1                   |                  |
| 1020.65   | 1471.9 ± 54.5                   | 1053.0 ± 35.6                   | 1051.9 ± 30.3                | 399.8 ± 14.5                   |                  |
| 1021.68   | 928.4 ± 37.7                    | 648.0 ± 29.0                    | 667.2 ± 22.3                 | 217.4 ± 8.5                    |                  |
| 1023.27   | 514.4 ± 20.0                    | 352.0 ± 16.7                    | 359.5 ± 11.7                 | 92.2 ± 3.4                     |                  |
| 1028.23   | 161.1 ± 6.0                     | 103.0 ± 7.0                     | 108.1 ± 4.0                  | 11.2 ± 0.65 ± 0.036           |                  |
| 1033.84   | 72.9 ± 2.9                      | 45.1 ± 4.0                      | 52.0 ± 2.1                   | 1.400 ± 0.113±0.100           |                  |
| 1039.59   | 47.4 ± 2.0                      | 28.3 ± 3.1 ± 0.1                | 31.0 ± 1.5 ± 0.4             | 0.096 ± 0.010±0.059           |                  |
| 1049.81   | 29.0 ± 1.5                      | 13.3 ± 2.1 ± 0.6                | 16.4 ± 0.9 ± 0.5             | 0.613 ± 0.081±0.040           |                  |
| 1059.66   | 18.6 ± 1.2                      | 17.0 ± 2.6 ± 2.2                | 11.0 ± 0.7 ± 0.5             | 1.304 ± 0.194±0.070           |                  |

TABLE XI. The \(e^+e^- \rightarrow K^+K^-\) and 3\(\pi\) cross sections. First error is statistical and the second is the model uncertainty. The systematic errors are 7.1% for \(e^+e^- \rightarrow K^+K^-\), 4% and 4.2% for \(e^+e^- \rightarrow K_S K_L\) at charged and neutral modes and 5% for \(e^+e^- \rightarrow 3\pi\).
FIG. 1. The $\theta$ distribution of charged particles from the $e^+e^- \rightarrow K^+K^-$ reaction.

FIG. 2. Angle $\psi$ between pion pairs from the $K_S \rightarrow \pi^+\pi^-$ decay.

FIG. 3. The $r$ distribution of pions from the $K_S \rightarrow \pi^+\pi^-$ decay.
FIG. 4. The invariant mass of photon pairs from the $K_S \rightarrow \pi^0\pi^0$ decay.

FIG. 5. The invariant mass of pions from the $K_S \rightarrow \pi^0\pi^0$ decay

FIG. 6. The $\chi^2_{K_S}$ distribution for the $K_S \rightarrow \pi^0\pi^0$ events.
FIG. 7. The kaon momentum distribution from the $K_S \to \pi^0\pi^0$ decay at the energy $\sqrt{s} = 1020$ MeV.

FIG. 8. The kaon momentum distribution from the $K_S \to \pi^0\pi^0$ decay at the energy $\sqrt{s} = 1060$ MeV. The left peak corresponds to the case when the photon was emitted by the initial particles.

FIG. 9. Two-photon invariant mass distribution in the $\phi \to \pi^+\pi^-\pi^0$ events.
FIG. 10. The $\chi^2_{3\pi}$ distribution in $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ events.

FIG. 11. The detection efficiency $\epsilon(E_\gamma)$ dependence on the radiated photon energy $E_\gamma$ for $e^+e^- \rightarrow 3\pi + \gamma$ events under the conditions: $\chi^2_{3\pi} < 20$ (dots) and $\chi^2_{3\pi} < 5$ (circles), obtained by simulation.
FIG. 12. The $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section. The dots are experimental data, the curve is the fit.

FIG. 13. The $e^+e^- \rightarrow \pi^+\pi^-\pi^0$ cross section in the interference minimum region. Solid curve and dots: fit with $\chi_{\phi-\omega} = 180^\circ$, Im($A_{3\sigma}^0$) = 0; dashed curve and squares: the fit with $\chi_{\phi-\omega} = 180^\circ + \Delta \chi_{\phi-\omega}(s)$, Im($A_{3\sigma}^0$) = 0.
FIG. 14. The $e^+e^- \rightarrow K^+K^-$ cross section. The dots are experimental data, the curve is the fit.

FIG. 15. The $e^+e^- \rightarrow K_S K_L$ cross section. The dots are experimental data, the curve is the fit.