Robust parameter design based on response surface model under considering measurement errors

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Abstract: Response surface approach is effective for robust parameter design. Previous response surface methodology assumes that the independent variables are measured without errors. However, this assumption might be violated due to the low capability of measurement system. This paper is concerned with applying response surface method for robust parameter design when there are measurement errors in variables. We present an unbiased estimator when there are some measurement errors and an optimal setting which is determined to minimize the expected quadratic loss. An example is illustrated to verify the effectiveness of the proposed approach. The results show that the proposed method can achieve better operating conditions under situations with measurement errors than the conventional method.

Keywords: robust parameter design; response surface model; measurement errors; unbiased estimator; quality loss

1. Introduction

Robust parameter design (also known as parameter design) is a quality improvement technique proposed by Taguchi [1,2], which is a cost effective approach for reducing variation in products and processes. As summarized in Nair [3] and Phadke [4],
Taguchi classifies the inputs to the system into two groups: control variables \( x \) and noise variables \( z \). The former can be easily controlled and manipulated, whereas the latter are difficult, expensive or impossible to control during the normal design or production process. Let \( y \) denote the response, which is actually some quality characteristic that measures the output performance of the system. There could be many combinations or settings of \( x \) at which the system can produce the desired level of \( y \) (also called the target) on an average. Out of these, there will be some settings at which the system is insensitive to the effect of the noise variables \( z \). Variation in \( z \) during the manufacturing process adds unwanted variation into the system. The basic idea of parameter design is to select the levels of control variables in such a way to achieve robustness, or insensitivity to the noise variation \( z \). This is done by exploiting interactions between control variables and noise variables. A traditional approach in robust parameter design study is the signal-to-noise ratio approach [5], which combines mean and variance together. The signal-to-noise ratio approach can mitigate the deleterious effects of the noise variables, but it is limited since that it ignores the interactions between control variables and may be invalid sometimes. Robust design problems can be handled by a response surface approach which can include interactions between control variables [6]. Response surface model approach establishes relationships between process responses and process variables, which contains control variables, noise variables and interactions of control variables and noise variables. The control variables are set by minimizing the quality loss function, which contains deviation from the target and variance (see, for example, Ouyang et al. [7], Zhou [8], Emdadi et al. [9], and Bellucci and Jr. [10]).

To improve the product quality, the first issue is to identify process variables that potentially affect the product quality characteristics. Once the influential variables are identified, the next step is to seek appropriate settings for the control variables to optimize process output and improve product quality. No doubt the product quality data are obtained by measurement in both the design and the production stages. Zhai et al. [11] pointed that measurement errors are often inevitable, due to such various factors as human errors and limited precision of the measurement device. Gauge
measurement errors has been extensively discussed in the literatures (e.g., see Grau D [12], Rakhmawati et al. [13]). Loken and Gelman [14] pointed that measurement error adds noise to predictions, increases uncertainty in parameter estimates, and makes it more difficult to discover new phenomena. Feng et al. [15] pointed that all measurement processes have some extent of uncertainty, emphasizing that measurement result should be reported with quantitative statement of its uncertainty. The classical theory of robust design assumes that the independent variables are measured without errors. Giovagnoli and Romano [16] introduced a modification of the dual response surface modeling, which incorporates the option of stochastically simulating some of the noise variables when their probabilistic behavior is known. They applied the method to the design of a high-precision optical profilometer and insisted that their method seems suitable for designing complex measurement systems. In practice, however, measurement system usually is not precise enough. A firm needs to design the experiment and analysis the data in the design stage and maintain the level of control variables in the production stage. Measurement errors can occur in design or production stages, which may not be recognized by the design or production engineers. The effect of errors in the factor levels on the statistical properties of the parameters obtained from a two-level factorial and fractional factorial designs was first studied by Box [17]. Draper and Beggs [18] measured the robustness of experimental designs to errors in the factor levels by the sum of squared differences between the observed and the predicted values. However, they also recognized that finding an analytical proof for any optimality conditions when there are two or more factors is very difficult and recommend searching numerically for a solution. Steiner and Hamada [19] assumed that during regular production, some measurement errors may occur, but negligible measurement errors occur during the design process. Under this assumption, the response model derived from the experimental results is unaffected by the measurement errors. Donev [20] is concerned with the statistical properties of experimental designs where the factor levels cannot be set precisely and proposed that the criterion of D-optimality should be based on the inverse of the information matrix. They recognized that errors in variables can affect the mean and
the variance of responses. In the D-optimality the influence of noise variables is ignored. Moreover, measurement errors also have a significant influence on the variance of responses in the fitted model. Zhong et al. [21] pointed that the errors in measuring noise variables have a significant influence on process responses. Ardakani [22] pointed that error in noise variables causes inappropriate estimates of the response model, which consequently affects the optimal setting of the control variables. But they didn’t consider measurement errors on the control variables. Over the past years, the problem of estimating unknown parameters with measurement errors has been extensively discussed in the literatures (e.g., see Wolter and Fuller [23], Chang et al. [24], Zhang [25], and Firpo [26]). But it has not received enough attention. Design engineers usually do not consider measurement errors. Since measurement errors are not ignorable in model fitting and optimization, it is important to investigate the impacts of modeling and measurement errors on the performance of robust parameter design, as well as further optimizing the control variables to improve the robustness to measurement errors.

The purpose of this paper is first to analyze the influence of the measurement errors on the response surface modeling and optimization, and then to explore the best setting of control variables in this situation. Specifically, the next section presents the response surface model to robust parameter design. The influence of measurement errors on response surface model is analyzed in Section 3. Section 4 presents how to optimize the control variables setting under measurement errors. The optimization scheme is illustrated with an example in Section 5. Concluding remarks are given in the final section.

2. Response surface model to robust parameter design

Welch et al. [27] proposed a single response surface model which utilizes control variables and noise variables. A general statistical model can be expressed as:

\[ y = f(x, z) + \varepsilon \]  \hspace{1cm} (1)

In Equation (1), \( x \) and \( z \) represent the vectors of control and noise variables. \( \varepsilon \) is a
term representing other sources of variability not accounted for in \( f \).

The regression model in Equation (1) is often described including linear, quadratic, and interaction terms. A second-order statistical model with main effects and control-by-noise interactions is used:

\[
y = \beta_0 + \beta_1^T x + \beta_2^T z + x^T B_1 x + x^T B_2 z + \varepsilon
\]  

(2)

In Equation (2), \( x = (x_1, x_2, \ldots, x_p)^T \), \( z = (z_1, z_2, \ldots, z_q)^T \), and other vectors and matrices are of appropriate dimension, i.e., \( \beta_1 = (\beta_{11}, \beta_{12}, \ldots, \beta_{1p})^T \), \( \beta_2 = (\beta_{21}, \beta_{22}, \ldots, \beta_{2q})^T \),

\[
B_1 = \begin{pmatrix}
B_{11} & 1/2 B_{12} & \cdots & 1/2 B_{1p} \\
1/2 B_{12} & B_{22} & \cdots & 1/2 B_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
1/2 B_{1p} & \cdots & \cdots & B_{pp}
\end{pmatrix},
\]

\[
B_2 = \begin{pmatrix}
B_{21} & 1/2 B_{22} & \cdots & 1/2 B_{2p} \\
1/2 B_{22} & B_{22} & \cdots & 1/2 B_{2p} \\
\vdots & \vdots & \ddots & \vdots \\
1/2 B_{2p} & \cdots & \cdots & B_{pp}
\end{pmatrix}.
\]

It needs to be noted that the regression model should be built using significant terms. In general, the model in Equation (2) is fairly practicable in many engineering applications, which assumes the following:

a) The product system performance is time-invariant. That is, the real-life manufacturing process is stable and the model parameters (\( \beta \)'s and \( B \)'s) don’t vary with time.

b) \( z \) and \( \varepsilon \) are independent each other, with the mean of \( z \) and \( \varepsilon \) are 0 and the variance of \( z \) and \( \varepsilon \) are \( \Sigma_z \) and \( \sigma_z^2 \), respectively. The regression error \( \varepsilon \)'s are independently and identically distributed.

The model in Equation (2) can be rewritten as

\[
y = g(x, z)^T \theta + \varepsilon
\]

(3)

where we let

\[
g(x, z) = (1, x_1, x_2, \ldots, x_p, z_1, z_2, \ldots, z_q, x_{11} x_1, x_{12} x_2, x_{13} x_3, \ldots, x_{1p} x_p, x_{21} z_1, x_{22} z_2, \ldots, x_{2q} z_q)^T,
\]

and \( \theta = (\beta_0, \beta_{11}, \beta_{12}, \ldots, \beta_{1p}, \beta_{21}, \beta_{22}, \ldots, \beta_{2q}, B_{11}, B_{12}, \ldots, B_{1p}, B_{21}, B_{22}, \ldots, B_{2q})^T \). The linear regression coefficients \( \theta \) and the error variance \( \sigma_z^2 \) can be estimated using data obtained from the experiment by the least squares estimator

\[
\hat{\theta} = (X'X)^{-1} X'y
\]

(4)
and

\[
\hat{\sigma}^2 = \frac{\text{SSE}}{n - p - q - \frac{p(p + 1)}{2} - pq - 1}
\]  

(5)

where \(\mathbf{X} = (g(x_1, z_1), g(x_2, z_2), \ldots, g(x_n, z_n))^T\), \(\mathbf{y} = (y_1, y_2, \ldots, y_n)\) and \(\text{SSE}\) denotes the sum of the squared errors, that is \(\text{SSE} = (\mathbf{y} - \mathbf{X}\hat{\mathbf{\theta}})^T(\mathbf{y} - \mathbf{X}\hat{\mathbf{\theta}})\).

The model proposed by Welch et al. could be used to formulate dual response surfaces (see, for example, Vining and Myers [28], Myers et al. [29], and Myers et al. [30]). The choice of optimum setting \(\mathbf{x}^*\) could be obtained via the joint exploration of the response surfaces generated by the mean and variance of the response. The model for the location response (the mean response) is found by taking the expectation of \(f(x, z)\) as

\[
\mu = E_{x,z}[f(x, z)] = \beta_0 + \beta_1^T \mathbf{x} + \mathbf{x}^T \mathbf{B}_1 \mathbf{x}
\]  

(6)

The dispersion response (variance) of \(f(x, z)\) in model (2) is given by

\[
\sigma^2 = \text{Var}_{x,z}[f(x, z)] = (\beta_1^T + \mathbf{x}^T \mathbf{B}_2) \Sigma_z (\beta_1^T + \mathbf{x}^T \mathbf{B}_2)^T + \sigma^2_{\epsilon}
\]  

(7)

An objective function must be defined to obtain the optimal setting of the control variables. Ames et al. [31] and Murphy et al. [32] studied different types of loss functions. One of most commonly used loss function is a quadratic loss function. There are three types of quadratic loss function. Equation (8) gives the quadratic loss function for a nominal-the-best (NTB) quality characteristic \(y\) where the deviation on either side of the target value \(\tau\) is undesirable.

\[
L = K(y - \tau)^2
\]  

(8)

where \(K\) is an economic coefficient. The quadratic loss function considers off-target penalty and variance by measuring the deviation between the response and target values. Equation (9) is the quadratic loss function for a smaller-the-better (STB) quality characteristic which can be obtained by substituting \(\tau = 0\) in the Equation (8):

\[
L = Ky^2
\]  

(9)

For the larger-the-better quality characteristic \(y\) which has positive values, making \(y \to -y\)
larger is equivalent to making \( y' = 1/y \) smaller. So, a smaller-the-better (LTB) type quality loss function can be used by replacing \( y \) with its reciprocal value \( y' = 1/y \) as

\[
L = Ky'^2
\]

Equations (9) and (10) represent well-tested functions to calculate quality loss for smaller-the-better and larger-the-better quality characteristic when the quality characteristic does not take negative values and zero. For larger-the-better quality characteristic, the response model should be built based on the relationship between new quality characteristic \( y' \) and process input variables (control variables \( x \) and noise variables \( z \)).

Based on the quality loss function, expected quality loss can be computed:

\[
J(x) = E(L) = \begin{cases} 
K[(\mu_y - \tau)^2 + \sigma_y^2] & \text{(NTB)} \\
K[\mu_y^2 + \sigma_y^2] & \text{(STB)} \\
K[\mu_y^2 + \sigma_y^2] & \text{(LTB)} 
\end{cases}
\]  
(11)

The economic coefficient \( K \) is assumed to be 1 in finding optimal setting of control variables without loss of generality. Then the expected quality loss will be simple as follows:

\[
J(x) = E(L) = \begin{cases} 
(\mu_y - \tau)^2 + \sigma_y^2 & \text{(NTB)} \\
\mu_y^2 + \sigma_y^2 & \text{(STB)} \\
\mu_y^2 + \sigma_y^2 & \text{(LTB)} 
\end{cases}
\]  
(12)

The optimal setting of control variables can be derived by minimizing the expected loss in Equation (12):

\[
x^* = \min_{x \in \Theta} J(x)
\]  
(13)

where \( \Theta \) is the region of control variables. The procedures of optimization will be illustrated with an example in the section 4.

3. **Response surface model with measurement errors**

Response surface models for robust parameter design are widely implemented by
engineers to improve product or process quality. However, if the data collected are measured with errors, they do not represent the true values of the variables of the product or process being measured. Therefore, it is important to do a valid measurement analysis beforehand to ensure the data collected are accurate and precise. Repeatability and Reproducibility (R&R) are most basic concepts used in the identification of the variation of the measurement system. Gauge R&R addresses the magnitude of measurement errors of a measurement system. The variance of the errors in measurement system can easily be calculated by statistical software.

As mentioned in the first section, there often exist measurement errors in practical situations. If there are some measurement errors occurring in the product or process design stage, the data obtained from experiments will be imprecise. Assume that the Equation (3) is the actual form of the regression model. In an experiment to estimate the relationship, suppose that one observes

\[ \begin{aligned}
    \bar{y} &= y + v \\
    (\bar{x}, \bar{z}) &= (x, z) + (w_x, w_z)
\end{aligned} \quad (14) \]

In Equation (14), \( y, x \) and \( z \) denote error-free response, control, and noise variables, and \( v \) and \( w \) denote the measurement errors of the corresponding variables. It is assumed that the measurement errors follow the assumptions:

\[
\begin{bmatrix}
    v \\
    w
\end{bmatrix} \sim MVN \left( 0, \begin{bmatrix} \sigma_v^2 & 0 \\
                        0 & \Sigma_w \end{bmatrix} \right)
\]

where MVN reads ‘multivariate normal’ and \( \Sigma_w \) is a \((p+q) \times (p+q)\) covariance matrix of measurement errors vector \( w \). Note that \( v \) is assumed to be independent of each component of \( w \) and components of \( w \) are independent of each other, i.e.,

\[ \Sigma_w = \text{diag} (\sigma_{w_1}^2, \sigma_{w_2}^2, \ldots, \sigma_{w_p}^2) \].

However, one can only observe the experimental variables \( \bar{y} \) and \( g(\bar{x}, \bar{z}) \) with the relationship \( y = g(x, z)^\top \theta + \varepsilon \). The ordinary least squares (OLS) estimator \( \hat{\theta} \) based on the observed variables is

\[ \hat{\theta} = (X^\top X)^{-1}X^\top y + (X^\top X)^{-1}X^\top \varepsilon \quad (15) \]

where \( X = (g(x_1, z_1), g(x_2, z_2), \ldots, g(x_n, z_n))^\top \) and \( \tilde{X} = (g(\bar{x}_1, \bar{z}_1), g(\bar{x}_2, \bar{z}_2), \ldots, g(\bar{x}_n, \bar{z}_n))^\top \). \( \hat{\theta} \) is an
unbiased estimator for $\theta$ when there is no measurement errors. However, when measurement errors exist, which often happens in practice, the OLS estimator is biased.

Fuller gave a summary of results concerning the measurement error models. To facilitate our discussion, let $\varphi = (\tilde{y}, \tilde{g}(\tilde{x}, \tilde{z})) = (y, g(x, z)) + \eta$, 

$$\tilde{g}(\tilde{x}, \tilde{z}) = g(x, z) + f
= (1, \tilde{x}_1, \tilde{x}_2, \cdots, \tilde{x}_p, \tilde{z}_1, \tilde{z}_2, \cdots, \tilde{z}_q, \tilde{x}_p \tilde{x}_1 - \sigma^2_{w_1}, \tilde{x}_p \tilde{x}_2, \cdots, \tilde{x}_p \tilde{x}_q, \tilde{x}_1 \tilde{z}_1, \tilde{x}_1 \tilde{z}_2, \cdots, \tilde{x}_p \tilde{z}_q)^T,$$

$$f = (0, w_1, \cdots, w_q, 2x_1w_1 + w_1^2 - \sigma^2_{w_1}, x_1 w_2 + x_2 w_1 + w_1 w_2, \cdots, x_p w_1 + z_q w_p + w_1 w_q, \psi_1, \psi_2),$$

$$\eta = (\nu, f)$$

Note that we have defined $\tilde{g}(\tilde{x}, \tilde{z})$ and $\eta$ so that $E(\eta) = 0$. We also define the moment matrix

$$M = n^{-1}\sum_{i=1}^{n} \varphi_i \varphi_i^T = \begin{bmatrix} M_{yy} & M_{yx} \\ M_{xy} & M_{xx} \end{bmatrix}$$

(16)

and the covariance matrices

$$\Omega_i = E(\eta_i^T \eta_i) = \begin{bmatrix} \sigma^2 & \Omega_{i\nu} \\ \Omega_{i\nu} & \Omega_{ii} \end{bmatrix}$$

(17)

$$\Omega = n^{-1}\sum_{i=1}^{n} \Omega_i$$

(18)

Because the measurement errors are distributed as multivariate normal, the covariance matrix of $\eta_i$ is symmetric. Diagonal elements in Equation (17) are the variance of the corresponding terms, e.g., $\text{Var}(\tilde{y}) = \sigma^2_y$, $\text{Var}(\tilde{x}_i) = \sigma^2_{w_i}$, $\text{Var}(\tilde{z}_i) = \sigma^2_{z_i}$, $\text{Var}(\tilde{x}_i^2) = 2\sigma^4_{w_i} + 4x_i \sigma^2_{w_i}$, and $\text{Var}(\tilde{x}_i \tilde{z}_i) = \sigma^2_{w_i} \sigma^2_{z_i} + x_i^2 \sigma^2_{w_i} + x_j^2 \sigma^2_{z_j}$. Other elements are the covariance of different terms, e.g., $\text{cov}(\tilde{x}_i, \tilde{x}_j) = 0$, $\text{cov}(\tilde{x}_i, \tilde{z}_i) = 2x_i \sigma^2_{w_i}$, $\text{cov}(\tilde{x}_i, \tilde{x}_j) = x_j \sigma^2_{w_i}$, $\text{cov}(\tilde{z}_i, \tilde{x}_i, \tilde{x}_j) = 2x_i x_j \sigma^2_{w_i}$, $\text{cov}(\tilde{x}_i, \tilde{x}_j, \tilde{z}_k) = x_j x_k \sigma^2_{w_i}$, and $\text{cov}(\tilde{x}_i, \tilde{x}_j, \tilde{z}_k) = 0$.

$\hat{\Omega}_i$, which is an unbiased estimator of $\Omega_i$, can be obtained by replacing $x_i$ with $\tilde{x}_i$,
$x_i x_j$ with $\tilde{x}_i \tilde{x}_j$, and $x_i^2$ with $\tilde{x}_i^2 - \sigma_{\tilde{x}_i}^2$. Then the unbiased estimator of $\Omega$ can be constructed by

$$\hat{\Omega} = n^{-1} \sum_{i=1}^{n} \hat{\Omega}_i$$  \hspace{1cm} (19)

The unbiased estimator of $\theta$, which is the extended version of Wolter and Fuller$^{18}$ for multiple independent variables, is

$$\hat{\theta} = (M_{xx} - \hat{\alpha} \hat{M}_{II})^{-1} M_{xy}$$  \hspace{1cm} (20)

Where $\hat{\alpha}$ is the smallest root of the determinant equation $|M - \hat{\alpha} \hat{M}| = 0$.

4. Response surface optimization with measurement error model

When the levels of the control variables are set with errors, their values become random variables. If $x$ is the vector of intended value for the control variables, the factual value $\tilde{x}$ will be observed as

$$\tilde{x} = x + w_x$$  \hspace{1cm} (21)

In Equation (21) $w_x$ is the vector of measurement errors of the control variables. Naturally, we suppose that the measurement system is unbiased, the covariance of measurement system is $\Sigma_{w_x}$, and the measurement errors are independent of each other. i.e. $w_x \sim N(0, \Sigma_{w_x})$ and $\Sigma_{w_x} = diag(\sigma_{w_{x1}}^2, \sigma_{w_{x2}}^2, \ldots, \sigma_{w_{xp}}^2)$. Substituting $\tilde{x}$ for $x$ of Equation (21) in Equation (2) gives

$$y = \beta_0 + \beta'_1 (x + w_x) + \beta'_2 z + (x + w_x)^T B_1 (x + w_x) + (x + w_x)^T B_2 z + \varepsilon$$  \hspace{1cm} (22)

Hence, the expectation and variance of the responses can be derived

$$E_{w_x, \varepsilon}(y) = \beta_0 + \beta'_1 x + tr(B_1 \Sigma_{w_x}) + x^T B_2 x$$  \hspace{1cm} (23)
\[
Ve_{w,z}(y) = E\{\text{var}_i(y)\} + \text{var}_i[E_i(y)]
\]
\[
= E\{[(\beta^T_2 + x^T B_2)\Sigma_{i}(\beta^T_2 + x^T B_2)] + \text{var}_i[\beta_0 + \beta^T_1 x + x^T B_1 x] + \sigma^2
\]
\[
= tr(B_2^T \Sigma_{w} B_2 \Sigma_{i}^T) + (\beta^T_2 + x^T B_2)(\Sigma_{i}^2 B_2^T x + \Sigma_{i} \beta_2) + 2tr(B_1^T \Sigma_{w} B_1 \Sigma_{w}^T) + 4(x + \frac{1}{2}B_1^T \beta_1)^T B_1^T \Sigma_{w} B_1 (x + \frac{1}{2}B_1^T \beta_1) + \sigma^2
\]

\section*{5. Example}

In this section, the proposed method is illustrated using a turning operation process case study introduced in Kirby et al. [33]. A typical turning operation produces parts which have critical features requiring a specified surface roughness. The objective of the experiment is to reduce surface roughness by controlling four control variables. The control variables are spindle speed($x_1$), feed rate($x_2$), depth of cut($x_3$), and tool nose radius($x_4$). At the same time, tool inserts($z_1$) and room temperature($z_2$) are noise variables that cannot be controlled. A modified orthogonal array was created using the
orthogonal array L9(3⁴) and the selected parameters as shown in Table 1. A total of 36 experiments were run and the design and experimental data are given as Table 2. The control variables and noise variables are coded from -1 to 1. Supposing that the variance of noise variables is equal to 1/3 (\( \frac{USL - LSL}{6} \)), so that most of noise variables range from -1 to 1.

To fit empirical model, terms up to the second order are taken into account and significant terms are included in the model for further analysis. Based on the experimental data, a stepwise regression was used to fit the turning process model by using \( \alpha = 0.05 \) for both enter and remove tests in MINITAB. It is assumed that there are some measurement errors in the design stage, and all variables are independently and identically distributed with the variance of measurement errors ranging from 0 to 0.07. We can get the estimator of the parameters by the method proposed in section 3. The results are given as Table 3.

From the above results, the control variable \( x_3 \) (depth of cut) is not significant in reducing the surface roughness, so that it can be set according to economic advantages during the production. At the same time, the noise variable \( z_2 \) (room temperature) is also not significant to reduce the surface roughness. And the responses surface model can be obtained:

\[
y = \beta_0 + \beta_1^T x + \beta_2^T z + x^T B_1 x + x^T B_2 z
\]  

(27)

where \( x = [x_1, x_2, x_4]^T \) and \( z = z_1 \). When there are no measurement errors, that is the variance of measurement errors of all variables equal to 0, estimated parameters are

\[
\beta_0 = 139.72, \quad \beta_1 = [-5.71, 69.89, -48]^T, \quad \beta_2 = 4.17, \\
B_1 = [-16.2, 0, -2.45; 0, 0, -10.755; -2.45, -10.755, 16.46]
\]

and
\[ \mathbf{B}_2 = [0, -8.5, -13.75]^T. \] From the Table 4, it can be seen that even though the errors in independent variables are at a low level, they have a great impact on model fitting. As the variance of the measurement errors increases, the estimated coefficients of the parameters deviate further from those obtained under no measurement errors case.

Since the experimenters are interested in minimizing surface roughness, the optimization function is the STB type expected loss. In the surface operation process, variance of measurement errors can be obtained by measurement system analysis.

If there are no measurement errors in the design stage, meaning the variables are measured with precision in applying design of experiments, errors can only occur in the production stage. As can be seen in Equation (21), optimal setting \( x^* \) cannot be set precisely in production. The actual input value is \( \bar{x} \) due to measurement errors. Assume that the measurement system is unbiased and the variance ranges from 0 to 0.07, i.e., \( \sigma_{w_s}^2 \in [0, 0.07] \). When the variance of measurement system equals to 0, the optimal setting can be obtained by the traditional responses surface optimization approach proposed by Welch et al.\textsuperscript{21}. The optimization problem in Equation (26) will be given as Equation (28) below, which would be solved by Genetic Algorithm using MATLAB.

\[
\begin{align*}
\underset{x}{\text{Min}} & \quad J_{u,z}(x) = u_{w,s}(x) + \sigma_{w,s}^2(x) \\
\text{s.t.} & \quad x \in [-1,1] \\
\end{align*}
\] (28)

Optimization results are presented in Table 4. As can be seen in Table 4, the initial optimal setting \( (x_1, x_2, x_3) = (1.000, -1.000, 0.970) \) is calculated by the traditional response surface approach when there are no measurement errors. And the actual optimal setting changes along with the variance of measurement errors of control variables. Table 4 also gives the actual optimal setting and corresponding expected quality loss with respect to different measurement errors. Figure 1 shows the comparison of the performances of the estimators with and without consideration of
measurement errors in the production stage. $J_{ios}$ and $J_{tos}$ are expected quality losses under initial optimal setting which is obtained with no measurement errors assumption and under true optimal setting obtained considering measurement errors in the production stage. If the initial optimal setting with no measurement errors assumption is still used as the process input with measurement errors, the expected quality loss will be larger than that obtained with measurement errors consideration. However, the gaps between the two expected quality losses are small and can be ignored.

[Insert Table 4 here]

[Insert Figure 1 here]

Suppose the levels of the control variables can be measured with errors in the design stage. From the Table 3, we find that measurement errors have a great impact on model fitting when they occur in the design stage. If measurement errors occur in both the design stage and the production stage, the true optimum condition may be different from the initial optimal setting. We suppose that the distributions of measurement errors in the design stage are the same as the ones in the production stage when the variance of measurement errors ranges from 0 to 0.07. That is, the diagonal elements of covariance matrix of $MVN \left\{ 0, \begin{bmatrix} \sigma_r^2 & 0 \\ 0 & \Sigma_w \end{bmatrix} \right\}$ are same. Table 5 presents the actual optimal setting and corresponding expected quality loss under different magnitude of measurement errors. It is seen that when there are measurement errors, if the initial optimal setting obtained with no measurement errors assumption is still used, the expected quality loss will be larger than that under the optimal setting considering the measurement errors. This happens even when the variance of the measurement errors is very small. If there are measurement errors in the design and the production stage, not only the regression coefficients are poorly estimated by the usual OLS but also the prediction property is severely damaged. At the same time, the optimum operating condition of control variables is located far from the true optimum condition obtained by the unbiased estimation method of Equation (20). Figure 2 shows the comparison of the performances of the estimators.
with and without consideration of measurement errors in the design stage. $J_{los}$ is quality loss under initial optimal setting and $J_{los}$ is quality loss under true optimal setting obtained considering measurement errors. When the variance of measurement errors is larger than 0.04, considering measurement errors is very imperative. Comparing Figures 1 and 2, we can see that measurement errors in the production stage have a very negligible impact on the expected loss, while the errors influence a lot in the design stage. As seen in Figure 2 and Table 5, expected quality loss soars up when the magnitude of measurement errors is 0.07 or larger.

[Insert Table 5 here]

[Insert Figure 2 here]

6. Discussion and concluding remarks

To find optimal setting of the control variables for robust design based on response surface methodology, design engineers usually do not consider measurement errors. However, in reality, measurement errors exist, which leads to poor estimation of the regression coefficients and damage the prediction capability. This paper proposes an approach to properly estimate a response surface model, which takes into account measurement errors both in the design stage and the production stage. We present a modeling method when there are some measurement errors in the design stage. Location and dispersion performances of quality characteristics are given, and then optimal setting of control variables are derived in the production stage. With a turning operation case study, the performances of these settings are compared with the performance of the setting which is obtained with no measurement errors assumption. Analysis results shows that errors in variables are needed to be taken into account for parameter estimation and prediction to reduce the expected quality loss in the design stage.

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Table 1. Factors, codes, and level values used for the orthogonal array

| Parameter                      | Code | -1  | 0   | 1   |
|-------------------------------|------|-----|-----|-----|
| **Control variables**         |      |     |     |     |
| Spindle speed (rpm)           | $x_1$| 1500| 2250| 3000|
| Feed rate (ipr)               | $x_2$| 0.004| 0.008| 0.012|
| Depth of cut (in)             | $x_3$| 0.010| 0.020| 0.030|
| Tool radius (in)              | $x_4$| 0.008| 0.016| 0.032|
| **Noise variables**           |      |     |     |     |
| Tool number                   | $z_1$| 1   |     | 2   |
| Temperature range (°F)        | $z_2$| 65–75|     | 95–100|
| **Response variable**         |      |     |     |     |
| Surface roughness (μin)       | $Y$  |     |     |     |
Table 2. Data of the designed experiment

| Order | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $z_1$ | $z_2$ | Y   |
|-------|-------|-------|-------|-------|-------|-------|-----|
| 1     | -1    | -1    | -1    | -1    | -1    | -1    | 47  |
| 2     | -1    | 0     | 0     | 0     | -1    | -1    | 137 |
| 3     | -1    | 1     | 1     | 1     | -1    | -1    | 145 |
| 4     | 0     | -1    | 0     | 1     | -1    | -1    | 32  |
| 5     | 0     | 0     | 1     | -1    | -1    | -1    | 159 |
| 6     | 0     | 1     | -1    | 0     | -1    | -1    | 230 |
| 7     | 1     | -1    | 1     | 0     | -1    | -1    | 44  |
| 8     | 1     | 0     | -1    | 1     | -1    | -1    | 64  |
| 9     | 1     | 1     | 0     | -1    | -1    | -1    | 244 |
| 10    | -1    | -1    | -1    | -1    | -1    | -1    | 47  |
| 11    | -1    | 0     | 0     | 0     | -1    | 1     | 137 |
| 12    | -1    | 1     | 1     | 1     | -1    | 1     | 147 |
| 13    | 0     | -1    | 0     | 1     | -1    | 1     | 31  |
| 14    | 0     | 0     | 1     | -1    | -1    | 1     | 162 |
| 15    | 0     | 1     | -1    | 0     | -1    | 1     | 214 |
| 16    | 1     | -1    | 1     | 0     | -1    | 1     | 49  |
| 17    | 1     | 0     | -1    | 1     | -1    | 1     | 69  |
| 18    | 1     | 1     | 0     | -1    | -1    | 1     | 240 |
| 19    | -1    | -1    | -1    | -1    | 1     | -1    | 61  |
| 20    | -1    | 0     | 0     | 0     | 1     | -1    | 124 |
| 21    | -1    | 1     | 1     | 1     | 1     | -1    | 117 |
| 22    | 0     | -1    | 0     | 1     | 1     | -1    | 26  |
| 23    | 0     | 0     | 1     | -1    | 1     | -1    | 178 |
| 24    | 0     | 1     | -1    | 0     | 1     | -1    | 207 |
| 25    | 1     | -1    | 1     | 0     | 1     | -1    | 50  |
|   |   |   |   |   |   |   |   |   |   |   |
|---|---|---|---|---|---|---|---|---|---|---|
| 26 | 1  | 0  | -1 | 1  | 1  | -1 |   |   |   | 52 |
| 27 | 1  | 1  | 0  | -1 | 1  | -1 |   |   |   | 248|
| 28 | -1 | -1 | -1 | -1 | 1  | 1  |   |   |   | 64 |
| 29 | -1 | 0  | 0  | 0  | 1  | 1  |   |   |   | 130|
| 30 | -1 | 1  | 1  | 1  | 1  | 1  |   |   |   | 116|
| 31 | 0  | -1 | 0  | 1  | 1  | 1  |   |   |   | 36 |
| 32 | 0  | 0  | 1  | -1 | 1  | 1  |   |   |   | 171|
| 33 | 0  | 1  | -1 | 0  | 1  | 1  |   |   |   | 210|
| 34 | 1  | -1 | 1  | 0  | 1  | 1  |   |   |   | 40 |
| 35 | 1  | 0  | -1 | 1  | 1  | 1  |   |   |   | 46 |
| 36 | 1  | 1  | 0  | -1 | 1  | 1  |   |   |   | 247|
### Table 3. Parameter estimation under different magnitude of measurement errors

| Term | Variance of measurement errors |
|------|--------------------------------|
|      | 0     | 0.01  | 0.02  | 0.03  | 0.04  | 0.05  | 0.06  | 0.07  |
| constant | 139.72 | 141.88 | 144.71 | 148.68 | 155.08 | 136.37 | 173.81 | 214.76 |
| $x_1$    | -5.71  | -6.46  | -7.31  | -8.24  | -8.86  | -29.76 | -16.44 | -19.30 |
| $x_2$    | 69.89  | 71.60  | 73.51  | 75.65  | 77.85  | 91.74  | 87.04  | 92.23  |
| $x_4$    | -48.00 | -49.00 | -50.08 | -51.25 | -52.52 | -53.93 | -55.51 | -57.31 |
| $z_1$    | -4.17  | -4.34  | -4.52  | -4.72  | -4.93  | -5.17  | -5.43  | -5.73  |
| $x_1x_1$ | -16.20 | -17.63 | -19.42 | -21.87 | -26.14 | 14.36  | -25.76 | -32.63 |
| $x_4x_4$ | -16.46 | -18.65 | -21.57 | -25.63 | -31.69 | -41.69 | -61.30 | -117.18 |
| $x_1x_4$ | -4.90  | -3.19  | -1.16  | 1.25   | 3.76   | 27.90  | 17.46  | 25.25  |
| $x_2x_4$ | -21.51 | -22.81 | -24.27 | -25.84 | -26.76 | -65.26 | -40.08 | -44.75 |
| $x_2z_1$ | -8.50  | -8.99  | -9.54  | -10.19 | -10.94 | -11.84 | -12.91 | -14.24 |
| $x_4z_1$ | -13.75 | -14.54 | -15.44 | -16.48 | -17.70 | -19.15 | -20.89 | -23.03 |
Table 4. Optimal setting and expected quality loss (EQL) with different magnitudes of measurement errors in the production stage

| Variance | True optimal setting | Quality loss under true optimal setting | Quality loss under initial optimal setting |
|----------|----------------------|----------------------------------------|-------------------------------------------|
|          | A       | B       | D       |                       |                                        |
| 0        | 1.000   | -1.000  | 0.991   | 29.21                 | 29.21                                  |
| 0.01     | 1.000   | -1.000  | 0.982   | 112.68                | 112.95                                 |
| 0.02     | 1.000   | -1.000  | 0.974   | 196.13                | 197.22                                 |
| 0.03     | 1.000   | -1.000  | 0.966   | 279.57                | 282.01                                 |
| 0.04     | 0.932   | -1.000  | 1.000   | 362.16                | 367.32                                 |
| 0.05     | 0.914   | -1.000  | 1.000   | 443.97                | 453.16                                 |
| 0.06     | 0.896   | -1.000  | 1.000   | 525.23                | 539.52                                 |
| 0.07     | 0.877   | -1.000  | 1.000   | 605.94                | 626.41                                 |
Table 5. Optimal setting and expected quality loss (EQL) with different magnitudes of measurement errors in the design stage

| Variance | True optimal setting | EQL under true optimal setting | EQL under initial optimal setting |
|----------|----------------------|-------------------------------|----------------------------------|
|          | A  | B   | D    |                      |                                |                                |
| 0        | 1.00 | -1.00 | 0.991 | 29.23               | 29.23                          |
| 0.01     | 1.00 | -1.00 | 0.953 | 117.26              | 123.55                         |
| 0.02     | 1.00 | -1.00 | 0.911 | 217.02              | 247.16                         |
| 0.03     | -1.00 | -1.00 | 1.000 | 376.22              | 424.30                         |
| 0.04     | -1.00 | -1.00 | 0.931 | 569.72              | 750.06                         |
| 0.05     | 0.275 | -1.00 | 1.000 | 508.44              | 913.49                         |
| 0.06     | 1.00 | -1.00 | 0.756 | 974.17              | 1607.07                        |
| 0.07     | 1.00 | -1.00 | -0.522 | 3660.60           | 5816.29                        |