Pressure drag and friction drag for truncated pyramids in a turbulent open channel flow

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Abstract
The purpose of this study is to establish a method that more efficiently reduces drags for a three-dimensional object whose surfaces are shaped in some kind of wavy form. We use a truncated pyramid that is a relatively simple three-dimensional object. Experiments were carried out for truncated pyramids covered with flat surfaces or wavy surfaces in a turbulent open channel flow. The total drag was obtained by measuring the force acting on the pyramids. The local wall shear stress was estimated from the local mean velocity profile in the direction normal to the surfaces. The difference between the pressure on the uphill surface and that on the downhill surface of the pyramid was also measured with a pressure difference gauge. It was found that the total drag and the pressure difference for the truncated pyramid covered with the wavy surfaces were at their highest 7.9% and 13.7% lower respectively than the equivalent figures for the truncated pyramid covered with flat surfaces. The reductions of the pressure difference and total drag were caused by the recirculation flow intermittently occurring near the top face and the downhill face of the truncated pyramids.

Keywords: Drag reduction, Wavy surface, Truncated pyramid, Measurements, Recirculation flow

1. Introduction

The reduction of drag acting on a moving solid-body in fluid when its velocity is different from that of the fluid around the solid-body has been focused on for many years. There are various methods used for reducing the pressure drag caused by the body shape and the friction drag on the body’s surface. Some specific methods of the friction drag reduction, such as riblet forms and ship bottom paints, are inspired by large aquatic life. The reduction of drag acting on a dolphin has also received a lot of attention. Several drag reduction mechanisms found in dolphins, such as viscous damping by hydrophilic skin, skin folds and skin separation, have been discussed (Choi et al., 1997; Endo and Himeno, 2002; Fish, 2006).

Our research group has focused on the skin folds which appear on dolphins’ chests and abdominal areas only when the dolphins swim at a high speed: for example, Yoshitake et al. (2008) carried out the total drag measurement for a wavy plate which mimicked the skin on a dolphin. They found that the total drag of the wavy plate increased and the friction drag for this plate decreased when compared with a flat plate. In addition, they found that a circulating flow appeared intermittently at the valley of the wavy surface if the value of the ratio \( a/\lambda \) of the amplitude to wavelength in the wavy form was \( a/\lambda = 0.035 \). Ozaki et al. (2009) and Trieu et al. (2013) carried out the total drag measurement for an angled-wavy plate which was angled towards the main flow direction. In addition, Kuroda et al. (2009) researched the relationship between the difference of the angle of a wavy surface to a flat plate and the total drag. However, wavy
surfaces were formed on flat plates in these studies. Therefore, studies where wavy surfaces are formed for a three-dimensional object have not yet been conducted.

Studies on open channel flow around relatively simple three-dimensional objects have also been conducted: Agelinchaab et al. (2006) located the hemispherical ribs on the bottom wall of an open channel in the streamwise direction. Abbaspour et al. (2014) located the trapezoidal ribs on the bottom of an open channel. They carried out the velocity measurement of flow around ribs using various heights and intervals of the ribs. However, the purposes of these studies were to discuss the influence on the flow around a three-dimensional object, not to discuss the reduction of the drag acting on this object. Mizutani et al. (1999) carried out the measurement of wave forces on a circular truncated cone, a truncated pyramid or a rectangular parallel piped installed on a submerged breakwater. In this case also, the purpose of this study was not to reduce the drag acting on these objects. On the other hand, Aider et al. (2010) aimed at reducing the drag or lift force of a bluff body. However, they used air, rather than water, to apply their technology to moving solid-bodies such as cars. The present authors also previously measured the total drag of swimming goggles whose surfaces were shaped with wavy forms (Shintani et al., 2012). In this study, a part of the surfaces were carved to obtain wavy surfaces with the amplitude \( a = 0.1 \) mm. The results showed a reduction of total drag when compared with that in the case of goggles without wavy forms, although the samples of both types of goggles had the same projected area to the main flow direction. However, so far no one has elucidated the mechanism for a wavy form which affects the total drag.

The purpose of our current study is to establish a method that more efficiently reduces drags for a three-dimensional object whose surfaces are shaped in some kind of wavy form. In this study, we carry out experiments on drags for a truncated pyramid that is a relatively simple three-dimensional object. We measure the total drag, the pressure difference between the front and rear surfaces and changes in flow in the vicinity of surfaces with wavy forms.

2. Experimental set-up

2.1 Apparatus

Figure 1 shows the apparatus. This is the same of those in our previous studies (Trieu et al., 2013). Water flows from the upper tank to the chamber through a pipe and a valve for adjusting the flow rate. After the water in the chamber passes a flow-straightening part and a contraction-flow part, it flows into the open channel whose size is 2000 mm in length, 270 mm in width and 150 mm in height. The water in the open channel then flows into the lower tank and is returned to the upper tank using a submersible pump (Tsurumi manufacturing, 80PU23.7).

A tripping wire and an emery paper were placed at the channel inlet in order to advance the development of a turbulent boundary layer flow. The bottom wall of the open channel from the inlet to the downstream end of the test section was covered with a natural rubber sheet, except for the area where the test sample was positioned. The reason for using the rubber sheet was to reduce the drag and the disturbance of flow caused by the stainless-steel plate on which the test samples were placed.

The \( x \)-, \( y \)- and \( z \)-axes were arranged respectively in the streamwise, upward and spanwise directions. The origin of the coordinate system was centered on the channel inlet bottom. The test section was placed at 1150 mm from the inlet in the streamwise direction. The mean velocity \( u_e \) outside the turbulent boundary layer was 1.12 m/s in the case of the flat bottom of an open channel. \( u_e \) was determined from a typical swimming speed of the breast stroke. The uncertainty of the velocity was 0.04 m/s for \( u_e = 1.12 \) m/s. The Reynolds number based on \( u_e \) and the streamwise distance from the inlet of the channel was 1.2×10^6. The Reynolds number based on \( u_e \) and the water depth was 4.8×10^4. We set the water depth to 54 mm and the water temperature to 17±2 \(^\circ\)C. The thickness \( \delta_{99.5} \) of turbulent boundary layer at the test section in the case without the truncated pyramid was 8.9 mm. The Reynolds number based on the mean velocity \( u_e \) and the thickness \( \delta_{99.5} \) was 0.9×10^6. We obtained the friction velocity \( u_f \) from the mean velocity profile (See Appendix A) with the Clauser plot method in the case of flat plate, and the value of \( u_f \) was 53.8 mm/s.

The turbulence intensities at the test section in the case without the truncated pyramid were in the ranges of 0.76 < \( u'_{rms}/u_e < 2.41 \) in the streamwise direction and 0.56 < \( v'_{rms}/u_e < 1.02 \) in the upward direction (See Appendix A). The values of the turbulence intensities which were in the non-dimensional form with the mean velocity \( u_e \) were in the range of 0.00 < \( u'_{rms}/u_e < 0.12 \) and 0.03 < \( v'_{rms}/u_e < 0.05 \), respectively. Thus, the turbulence intensity was low.

The Kolmogorov length scale \( l_k \) and Kolmogorov time scale \( t_k \) were estimated by the following equations:
\[ l_k = \frac{1}{A^2} \frac{v}{u_f}, \quad t_k = \frac{1}{A^2} \frac{v}{u_f^2} \]  

where \( A \) is the dissipation rate of the turbulent kinetic energy; \( v \) is the kinematic viscosity; \( u_f \) is the friction velocity. We adopted the value of \( A \) in the DNS results obtained by Iwamoto et al. (2002), and the value of \( A \) was 0.329 near the wall. \( l_k \) and \( t_k \) were 0.027 mm and 0.68 ms respectively.

### 2.2 Test samples

Figure 2 (a) shows a truncated pyramid. Figures 2 (b) - (d) show truncated pyramids whose surfaces were shaped in wavy forms. The height of the truncated pyramid was determined from the height of a swimming goggle. The wavy forms on the surface are shown in Fig. 3. These wavy forms were shaped by changing the value of the amplitude \( a \) and the wavelength \( \lambda \). The values of \( a \) and \( \lambda \) were in the non-dimensional form with the friction velocity \( u_f \) and the kinematic viscosity. The values of \( a^* \), \( \lambda^* \) and the ratio \( a/\lambda \) of the amplitude to wavelength for each wavy surface are described in Table 1. The top of the wavy form was located on the edge of each surface of the pyramid. Therefore, all of these test samples have the same projected area to the mainstream direction of the flow. Parts which are shown with a dotted line in Fig. 2 were hollowed out and stainless-steel sinkers were fitted into these parts in order to increase the gross weight of the test samples.

Figure 4 shows a truncated pyramid used for the pressure difference measurement. The pressure measuring holes of 0.7 mm in diameter were allocated at the uphill and downhill surfaces of the truncated pyramid. The pressure taps which were attached to the connecting tubes were located at the side surface of the truncated pyramid. The gently curved cavities of 0.7 mm in diameter were placed at the inside of the truncated pyramid for joining the pressure measuring holes to the pressure taps on the side surface. The cavities are indicated with red dotted curves in Fig. 4 (a). Figure 4 (b) shows the positions of holes for the pressure measurement. Each hole was located at the center of three small regions on the uphill and downhill parts of the flat surfaces (model 1), and each pressure measuring hole was placed in the position of 2.5 mm, 7.5 mm and 12.5 mm from the open channel bottom to the upward direction. The pressure difference of the uphill and downhill surfaces for the truncated pyramid was measured at the regions between (1) and (9), (2) and (8), and (3) and (7) for each pressure hole. Three samples were manufactured by combinations of pressure measuring holes for each model. The pressure taps on the side surface were located at the same position in each model. The wavy form of the surface was similar to that shown in Table 1. Each hole was located at the positions which were the same distance from the channel bottom in the case of the wavy surfaces. The method by which the sample is fixed at the open channel will be mentioned in Section 3.1.
Fig. 2 Test samples (Unit : mm).

(a) Flat surface (model 1)
(b) Wavy surface (model 2)
(c) Wavy surface (model 3 or model 5)
(d) Wavy surface (model 4 or model 6)

Fig. 3 Wavy form.

Fig. 4 Test sample for pressure difference measurement.

Table 1 Amplitude and wavelength of wavy surface and weight.

| Sample    | \(a\) (mm) | \(a^*\) | \(\lambda\) (mm) | \(\lambda^*\) | \(a/\lambda\) | Weight (g) |
|-----------|------------|---------|-------------------|---------------|--------------|------------|
| model 1   | ---        | ---     | ---               | ---           | ---          | 44.0       |
| model 2   | 0.10       | 5.4     | 2.86              | 153           | 0.035        | 43.1       |
| model 3   | 0.06       | 3.2     | 1.72              | 92.2          | 0.035        | 43.4       |
| model 4   | 0.30       | 16      | 8.58              | 460           | 0.035        | 41.2       |
| model 5   | 0.10       | 5.4     | 1.72              | 92.2          | 0.058        | 43.0       |
| model 6   | 0.10       | 5.4     | 8.58              | 460           | 0.012        | 43.0       |
3. Experimental procedures

3.1 Total drag

The measurement system for the total drag acting on the test sample is indicated in Fig. 5. This measurement system is the same as that in our previous studies (Trieu et al., 2013). We carried out an experiment, in which a cylinder was supported above the plate, to verify the measurement system. The measurement system was accurate from this experiment result (See Appendix B). Each test sample was fixed on a T-shaped stainless-steel plate with a double sided tape. The length of the stainless-steel plate was 300 mm in the streamwise direction. The test samples were placed in the position of 150 mm from the forefront edge part of the stainless-steel plate. The stainless-steel plate was not fixed to the bottom wall of the open channel so that the test samples could slide freely to the mainstream direction. Many particles of 0.25 mm in diameter were arranged between the lower surface of the stainless-steel plate and the surface of the channel in order to reduce the friction force between the stainless-steel plate and the bottom of the channel. Each test sample was supported by two vertical cantilevers, which were made of phosphor-bronze strips. One end of each cantilever was clamped above the free surface. The other end of each cantilever made contact with one of the edges of a U-shaped part. This part was fixed to the edge of each wing of the stainless-steel plate. The edge of the stainless-steel plate on the downstream side was fixed with a stop. When the stop was slid slowly, the truncated pyramid received drag from the flow and moved to the downstream. Then the cantilevers were deflected slightly by the movement of the truncated pyramid, due to the total drag acting on the sample. We measured the strain at a location on each cantilever by using a strain gauge (Kyowa electronic instruments, KFWS-type). The gauges were attached to the cantilevers and connected to bridge circuits (Kyowa electronic instruments, PCD-300A). The outputs from the circuits were recorded on a PC. We calculated the total drag acting on the test sample from the calibration lines. In order to reduce the strain errors caused by the flow which collides with the cantilevers, the cantilevers were positioned inside the indented parts of the sidewalls. The indented parts were covered with a clear resinous sheet. The error of the measurement for the total drag was approximately ±0.054 (N).

We carried out a calibration experiment to obtain the total drag acting on each truncated pyramid by using the relationship between the load by weight, \( W \), and the strain, \( \varepsilon \). The calibration measurement system is shown in Fig. 6. One end of a cantilever which was made of phosphor-bronze strips was fixed. Several different weights were hung on the other end of the cantilever. The cantilever was deflected slightly by the weight. The calibration lines in Fig. 7 show the proportional relationship between the load by weight and the strain.

![Fig. 5 Measurement system of total drag.](image_url)

![Fig. 6 Schematic of calibration system.](image_url)

![Fig. 7 Calibration of strain gauge.](image_url)
3.2 Velocity field

The instantaneous velocity field was measured from images of flow visualized by adding tracer particles. Tracer particles (Orgasol, specific gravity: 1.03) whose diameters were in the range of 0.048-0.052 mm were used. The upper limit of the frequency $f_c$, beyond which these particles cannot respond at all to fluid sinusoidal fluctuations, was estimated to be 1540 Hz from the equations obtained by Hjelmfelt Jr. and Mockros (1966) and from the density ratio $\rho_p/\rho_w$ and the particle diameter. On the other hand, the highest frequency calculated from the Kolmogorov time scale was $1/t_K = 1482$ Hz. Therefore, these particles can respond to any fluctuation of flow. Thus, the tracer particles adopted in the present study are reasonable.

The measurement system is shown in Fig. 8. Light from an Nd:YVO$_4$ laser (Jenoptik, $\lambda = 532$ nm) was used as the light source. The laser beam was expanded with a plano-convex cylindrical lens and a plano-concave lens in order to obtain a laser light sheet. The laser light sheet passed through a slit of 5 mm in width and provided illumination from above the open channel. Scattered light from the particles was captured with a CMOS camera (Photron, FASTCAM-1024PCI 100K). The captured images were directly recorded in a PC.

We adopted the following three-step processing of images, which is the same as that adopted in our previous studies (Kitagawa et al., 2007; Trieu et al., 2013).

1. The particle-mask correlation method, developed by Etoh et al. (1999), was used to remove any weak scattered light from particles in the images.

2. The PTV (Particle Tracking Velocimetry) technique, based on the velocity gradient tensor method proposed by Ishikawa et al. (2000), was applied to the preprocessed images for obtaining velocity vectors. In this method, the matrix including the velocity gradient tensor was calculated for pairs of neighboring particles in a specific region around a single particle.

3. Then, the sum of the square of errors in the matrix was evaluated. This procedure was repeated for all the candidate particles until the sum reached its minimum value. This method has the advantage of accurately reproducing strongly-deformed velocity fields.

The flow visualization with path-lines of tracer particles was carried out by changing the exposure time. Each image-capturing condition is described in Table 2.

The directions of the mean velocity of flow along the uphill face, top face and downhill face are shown in Fig. 9. The normal velocity and the tangential velocity of the surfaces were calculated by the following equation. $\theta$ is the angle between the bottom of the open channel and the uphill surfaces.

$$u_{uphill} = u\cos\theta + v\sin\theta, \quad v_{uphill} = v\cos\theta - u\sin\theta$$

$$u_{downhill} = u\cos\theta - v\sin\theta, \quad v_{downhill} = u\sin\theta + v\cos\theta$$

![Fig. 8 Velocity measurement system.](image_url)

![Fig. 9 Velocity vectors of front and rear surfaces.](image_url)
3.3 Pressure difference

The pressure difference measurement system is shown in Fig. 10. The stainless-steel plate with the truncated pyramid was fixed to the bottom wall of the open channel. The pressure measuring holes of 0.7 mm in diameter were placed at the uphill and downhill surfaces of the truncated pyramid. Polyurethane tubes of 1.8 mm in outer diameter were used as connecting tubes. The pressure taps on the side surface were connected to the pressure difference gauge (Nagano keiki, GC-50) using the polyurethane tubes. First, the truncated pyramid was placed properly and the pressure difference gauge was proofread at the zero point. Secondly, water flowed into the open channel and the drains of the pressure difference gauge were opened. The drains were kept open for ten minutes to remove air. Next, the drains were closed and the pressure difference measurement was started. The measurement was carried out at a water temperature of 20±2 °C. The error for the pressure difference was approximately ±0.01 (KPa).

![Fig. 10 Measurement system of pressure.](image)

### Table 2 Image-capturing conditions.

|                      | PTV     | Path-lines |
|----------------------|---------|------------|
| Pixel numbers        | 1024×336| 1024×512   |
| Frame rate (fps)     | 3000    | 125        |
| Shutter speed (s)    | 1/9000  | 1/125      |
| Laser power (a)      | 25.0    | 8.5        |
| Number of frames     | 9752×5  | 3000       |
| Pixel resolution     | (uphill)| 0.051      |
|                      | (top)   | 0.028      |
|                      | (downhill) | 0.056     |
|                      |         | 0.034      |

4. Results and discussion

4.1 Total drag

Table 3 shows the mean values and standard deviation of the total drag coefficient. The value of the total drag coefficient $C_T$ was calculated by the following equation:

$$C_T = \frac{D_T}{\frac{1}{2} \rho u_e^2 S}$$  (4)

where $D_T$ is the measured value of the total drag; $\rho$ is the density; $u_e$ is the mean velocity; $S$ is the sum of the
stainless-steel surface area and the surface area except for the side faces of the truncated pyramid. The RMS values of the total drag in Table 3 were obtained from the temporal variation of the total drag. The values of the total drag for the truncated pyramids covered with wavy surfaces (model 2 - model 6) were lower than that of model 1. The difference of the total drag was higher than the RMS value for each sample. Thus, we found that this difference was not an error of the measurement but an effect due to the wavy surfaces.

The value of the total drag for model 2 was the lowest value. The value of the total drag for this model was 7.9% lower than that of model 1. The flow around the truncated pyramid was changed by the wavy surfaces. Therefore, the values of the total drag were reduced. We considered that the values of the total drag were effectively reduced by the influence of the amplitude and the wavelength of the wavy forms.

We will compare the results for each model in detail below. Firstly, we compared the results of model 4 with those of model 6. The wavelength $\lambda^+$ of these models was the same. The value of the total drag for model 4 was higher than that of model 6. Therefore, we found that the increase in the total drag for model 4 was caused by the large amplitude.

Secondly, we compared the results for models 2, 3 and 4 whose values of $a/\lambda$ were identical. The value of the total drag for model 4 was the highest in these three models. The values of the two other models did not have many differences. We obtained an approximately 8% reduction of the total drag if the value of the amplitude was lower than 5.4. Therefore, we found that the value of the total drag was effectively reduced when the amplitude was a certain level, but this effect was reduced if the amplitude was higher than this level.

Thirdly, we compared the results for model 2, model 5 and model 6, whose wavy surfaces had the identical amplitude of $a^+ = 5.4$. The value of the total drag for model 2 was lower than that for model 5, whose wavelength was lower than that for model 2. The total drag for model 2 was also lower than that for model 6, whose wavelength was higher than that for model 2. Therefore, we found that the value of the total drag was reduced most when the wavelength was 153 and thus the ratio $a/\lambda$ was equal to 0.035. It can be concluded that the value of the total drag was effectively reduced when $a^+ = 5.4$ and $\lambda^+ = 153$ in the ranges of $3.2 < a^+ < 16$ and $92.2 < \lambda^+ < 460$.

### Table 3 Mean and RMS values of total drag coefficient.

| Sample | $a^+$ | $\lambda^+$ | $a/\lambda$ | Total drag coefficient $C_T$ | RMS       |
|--------|-------|-------------|-------------|------------------------------|-----------|
| model 1 | ---   | ---         | ---         | 0.0240                       | 0.00026   |
| model 2 | 5.4   | 153         | 0.035       | 0.0221                       | 0.00023   |
| model 3 | 3.2   | 92.2        | 0.035       | 0.0222                       | 0.00021   |
| model 4 | 16    | 460         | 0.035       | 0.0234                       | 0.00024   |
| model 5 | 5.4   | 92.2        | 0.058       | 0.0224                       | 0.00109   |
| model 6 | 5.4   | 460         | 0.012       | 0.0230                       | 0.00080   |

#### 4.2 Velocity field

Small-scale recirculation flow appeared intermittently near the uphill face and the top face, while large-scale recirculation flow appeared near the downhill face for all the samples. These recirculation flows appeared clockwise but were not synchronized. The duration and frequency of the recirculation flow will be discussed in Section 4.4.

We obtained the wall shear stress and thus the friction drag from the mean velocity profiles along the faces using Eqs. (2) and (3). We divided each surface of the pyramid (except for the side surfaces) in nine areas in the case of flat surface. We divided the envelope surface of the hilltops of wavy surfaces of the pyramid in nine areas in the case with wavy surfaces. The flat surface and the envelope surface of the hilltops of wavy surfaces were the same position. Figure 11 shows the areas where the velocity profiles were calculated along the axes normal to the pyramid surfaces. The area size was decided by considering the appearance of the recirculation flow mentioned above.

Figure 12 shows the mean velocity profiles of each area. The horizontal axis shows the distance from each surface and the vertical axis shows the value for velocity of the parallel and perpendicular direction for each surface, respectively. The velocities and the distance were in the non-dimensional form with the friction velocity without the pyramid and the kinematic viscosity.

Firstly, the values of $u^{+}_{uphill}$ in the area shown in Fig. 12 (a) were negative near the uphill face for all the samples.
This negative velocity resulted in the summation of negative velocity caused by the recirculation flow and positive velocity in the case without the recirculation flow in this area. The values of \( u^{+}_{\text{uphill}} \) in the area shown in Fig. 12 (b) and (c) were positive for all the samples. This result showed that the flow along the inclined plane of the uphill face was accelerated in this area.

Secondly, the values of \( u^{+}_{\text{top face}} \) in the area shown in Fig. 12 (e) and (f) were also negative near the top face for all the samples. This negative velocity also resulted in the summation of negative velocity caused by the recirculation flow and positive velocity in the case without the recirculation flow in this area. The values of \( u^{+}_{\text{top face}} \) for model 1 and model 2 in the area shown in Fig. 12 (d) - (f) increased in the area close to the top surface when compared with those of the other models. In addition, the values of \( u^{+}_{\text{top face}} \) for model 1 and model 2 in the areas shown in Fig. 12 (e) and (f) changed to positive in the area close to the top surface. We considered that the separation of the main flow on the top surface was thin and the area in which recirculation flow appeared was narrow.

Thirdly, the values of \( u^{+}_{\text{downhill}} \) and \( v^{+}_{\text{downhill}} \) in the area shown in Fig. 12 (g) - (i) were also negative near the downhill face for all the samples. This negative velocity also resulted in the summation of negative velocity caused by the recirculation flow and positive velocity in the case without the recirculation flow in this area. The values of \( v^{+}_{\text{downhill}} \) for model 1, model 2 and model 3 in the area shown in Fig. 12 (g) were positive near \( y^{+}_{\text{downhill}} = 250 \). We considered that the flow which approached the downhill face was decreased by the recirculation flow of the downhill face.
We calculated the wall shear stress from the velocity gradient adjacent to the surface in each area using the following equation:

\[
\begin{align*}
\tau_{w \text{ uphill}} &= \mu \cdot \frac{du_{\text{uphill}}}{dy_{\text{uphill}}} \\
\tau_{w \text{ top face}} &= \mu \cdot \frac{du_{\text{top face}}}{dy_{\text{top face}}} \\
\tau_{w \text{ downhill}} &= \mu \cdot \frac{du_{\text{downhill}}}{dy_{\text{downhill}}}
\end{align*}
\]  

(5)

We calculated the velocity gradient from the mean velocity at the first position of 4.64 wall units from the flat surface or the envelope surfaces. The wall units were defined with \( u_\tau \) and the kinematic viscosity. The wall units of amplitude of the wavy surfaces were comparable to the resolution except for the model 4. Thus, the measured velocity well represents the velocity adjacent to the wavy surfaces in almost all areas of the wavy surfaces except for the valley region of the models 2, 3, 5 and 6. Even for model 4, the measured velocity on one-third of the wavy surface can represent the actual velocity.

The friction drag was calculated by multiplying each wall shear stress by the area of each region, and was discussed using the component of the friction drag in the streamwise direction. The value of the friction drag coefficient \( C_F \) was calculated by the following equation:

\[
C_F = \frac{\tau_{\text{plate}} S_{\text{plate}} + \tau_{\text{pyramid}} S_{\text{pyramid}}}{\frac{1}{2} \rho u_e^2 S}
\]

(6)

where \( \tau_{\text{plate}} \) is the mean wall-shear stress of the plate surface; \( S_{\text{plate}} \) is the stainless-steel surface area except for the area which the truncated pyramid is arranged; \( \tau_{\text{pyramid}} \) is the wall-shear stress at each region of the truncated pyramid; \( S_{\text{pyramid}} \) is the surface area of the truncated pyramid except for the side faces. Note that the friction drag due to the
stainless-steel plate which fixed the pyramid was included in the total drag. Thus, we estimated the friction drag of the stainless-steel plate from the shear stress of the plate surface, which was obtained from the mean velocity profiles in the upstream and downstream areas of the pyramid. The friction drag coefficient of the stainless-steel plate was 0.00111. Table 4 shows the mean values of the friction drag coefficient for each sample. The values of the friction drag for the truncated pyramids covered with the wavy surfaces (models 2 - 6) were lower than that for the pyramid without wavy surfaces (model 1). Here, we will discuss the difference of the results for models 2 - 6. Firstly, we compared model 2, model 3 and model 4, whose surfaces were shaped in a wavy form where $a/\lambda = 0.035$. The value of the friction drag for model 3 was lowest. The value of the friction drag for model 4, whose wavelength was longest, was higher than those of model 2 and model 3. Therefore, we found that the friction drag was reduced when the value of $a/\lambda$ is low in relation to $\lambda^*$ in the case of $a/\lambda = 0.035$.

Secondly, we compared the results for models 2, 5 and 6, whose amplitudes were identical ($a^* = 5.4$). In this case, we obtained similar results to models 2, 3 and 4, whose values of $a/\lambda$ were identical. Therefore, we found that the value of the friction drag was effectively reduced when the wavelength was decreased because the opposite flow near the surface caused the reduction of the friction drag.

We considered that the recirculation flow has influenced these results. The flow at the downhill face changed as a result of the recirculation flow near the downhill face. When recirculation flow appeared at the rear of a truncated pyramid, the flow toward the downhill face was decreased by the recirculation flow. The flow along the downhill face became weak and the value of the friction drag decreased.

We cannot explain the dependence of the total drag in the models shown in Table 3 only by the results of the friction drag. We will therefore discuss the pressure drag predominant to the total drag in Section 4.3.

### Table 4 Mean values of friction drag coefficient.

| Sample  | $a^*$ | $\lambda^*$ | $a/\lambda$ | Friction drag coefficient $C_f$ |
|---------|------|-------------|-------------|-------------------------------|
| model 1 | ---  | ---         | ---         | 0.00109                       |
| model 2 | 5.4  | 153         | 0.035       | 0.00105                       |
| model 3 | 3.2  | 92.2        | 0.035       | 0.00104                       |
| model 4 | 16   | 460         | 0.035       | 0.00105                       |
| model 5 | 5.4  | 92.2        | 0.058       | 0.00104                       |
| model 6 | 5.4  | 460         | 0.012       | 0.00105                       |

### 4.3 Pressure difference

Table 5 Pressure difference values (Unit:KPa).

| Sample  | $a^*$ | $\lambda^*$ | $a^*/\lambda^*$ | (a) | (b) | (c) |
|---------|------|-------------|----------------|-----|-----|-----|
| Holes   |      |             |                | (1) - (9) | (2) - (8) | (3) - (7) |
| model 1 | ---  | ---         | ---            | 0.28 | 0.34 | 0.16 |
| model 2 | 5.4  | 153         | 0.035          | 0.25 | 0.29 | 0.14 |
| model 3 | 3.2  | 92.2        | 0.035          | 0.27 | 0.29 | 0.18 |
| model 4 | 16   | 460         | 0.035          | 0.29 | 0.29 | 0.27 |
| model 5 | 5.4  | 92.2        | 0.058          | 0.27 | 0.29 | 0.18 |
| model 6 | 5.4  | 460         | 0.012          | 0.28 | 0.31 | 0.21 |

Table 5 (a) shows the mean values in the differences of pressures measured at hole (1) and those at the hole (9) in Fig. 4, except for the first 5 seconds of the measurement. Similarly, Table 5 (b) shows the mean values in the differences of pressures measured at hole (2) and those at hole (8) in Fig. 4. Table 5 (c) shows the mean values in the differences of pressures measured at hole (3) and those at hole (7) in Fig. 4.

In Table 5 (a), recirculation flow in the clockwise direction appeared intermittently in areas (1) and (9) for all the
samples. The size and duration time of the recirculation flow depended on the samples. We will discuss this dependency in the next section. We compared the results for models 2, 3 and 5, whose wavelengths were shorter than 153. The values of the pressure difference for these three models were lower than those for model 1, model 4 and model 6. Therefore, we found that the values of the pressure difference were effectively reduced for the truncated pyramids covered with wavy surfaces whose wavelengths were shorter than 153.

In Table 5 (b), the value of the pressure difference for model 1 was highest. The areas of the recirculation flow for the truncated pyramids covered with wavy surfaces (model 2 - model 6) were wider than the area in the case of model 1. The flow toward area (2) was decreased by the circulation flow near the uphill face. Therefore, we found that the value of the pressure difference for model 1 was increased by the flow toward the uphill face.

In Table 5 (c), we focused on areas (3) and (7). Except for model 2, the values of the pressure difference for the truncated pyramids covered with wavy surfaces (model 2 - model 6) were higher than the value in the case of model 1. In particular, the increase in the pressure difference in case of model 4 was remarkable. The values of the pressure difference for the truncated pyramids whose wavelengths were shorter than 153 were lower than those of the truncated pyramids whose wavelength was 460.

The RMS values of the pressure difference \( P_{\text{di, rms}} \) were obtained from the temporal variation of the pressure difference of five runs. The RMS values were in the non-dimensional form with the mean pressure difference \( \bar{P} \). The non-dimensional values for model 5 were in the range of \( 0.05 < \frac{P_{\text{di, rms}}}{\bar{P}} < 0.08 \) at the regions between (1) and (9), \( 0.09 < \frac{P_{\text{di, rms}}}{\bar{P}} < 0.12 \) at the regions between (2) and (8), and \( 0.14 < \frac{P_{\text{di, rms}}}{\bar{P}} < 0.16 \) at the regions between (3) and (7). Thus, the temporal variation of the pressure difference was small.

We carried out calculations of mean values of the pressure drag coefficient using the area-weight summation of the pressure difference shown in Tables 5 (a), (b) and (c) for each model. For this calculation, we obtained relationship between the pressure differences and the vertical positions of the pressure holes in the Section 2.2 by using the least-square second-order polynomial approximation. We evaluated the pressure difference at thirty imaginary points in the vertical direction by using this relationship. The value of the pressure drag coefficient \( C_P \) was calculated by the following equation:

\[
C_P = \frac{1}{2 \rho u_e^2} \sum_{i=0}^{30} P_{di} \Delta S_i
\]

(7)

where \( P_{di} \) is the value of pressure difference at the \( i \)-th imaginary point and \( \Delta S_i \) is the narrow area including the \( i \)-th imaginary point in the uphill and downhill surfaces. Table 6 shows the mean values of the pressure drag coefficient for each sample. The result in Table 6 shows that the dependency of the pressure drag coefficient on the models was similar to that of the total drag shown in Table 3. It is found by comparing the values in Tables 3, 4 and 6 that the pressure drag was predominant to the total drag. Note that the ratio of the sum of the friction drag coefficient \( C_F \) in Table 4 and the pressure drag coefficient \( C_P \) in Table 6 to the total drag coefficient \( C_T \) in Table 3, \( (C_F + C_P)/C_T \), was in the range of 0.82 - 0.99. This is possibly due to the error of the approximation of distribution of the pressure difference.

Table 6 Mean values of pressure drag coefficient.

| Sample  | \( a^* \) | \( \lambda^* \) | \( a/\lambda \) | Pressure drag coefficient \( C_P \) |
|---------|-----------|----------------|-------------|----------------------------------|
| model 1 | ---       | ---            | ---         | 0.0195                           |
| model 2 | 5.4       | 153            | 0.035       | 0.0169                           |
| model 3 | 3.2       | 92.2           | 0.035       | 0.0192                           |
| model 4 | 16        | 460            | 0.035       | 0.0220                           |
| model 5 | 5.4       | 92.2           | 0.058       | 0.0192                           |
| model 6 | 5.4       | 460            | 0.012       | 0.0202                           |
4.4 Influence of the recirculation flow

Figures 13 (a) and (b) show typical images of path-lines with and without recirculation flow near the uphill face, respectively. It is found from Fig. 13 (a) that recirculation flow occurred between the bottom wall and the main flow impinging with the uphill face. The main flow of the front area approached the uphill face. The main flow, impinging on the inclined uphill face, was divided into upward secondary flow and downward secondary flow along the uphill face. The downward flow along the uphill face maintained the recirculation flow. On the other hand, it is found from Fig. 13 (b) that all the fluid flowed along the inclined uphill face at a distance from the surface.

Figures 14 (a) and (b) show typical images of path-lines with and without recirculation flow near the downhill face, respectively. It is found from Fig. 14 (a) that a large recirculation flow occurred in the downstream area of the downhill face. The direction of the flow toward the downhill face changed to the upward direction in the clockwise recirculation flow. In addition, the direction of the flow along the downhill face was changed to an upward direction by this recirculation flow. On the other hand, in Fig. 14 (b), the recirculation flow disappeared and the main flow along the downhill face approached the bottom wall in the area.

We considered that the pressure at the downhill face of the truncated pyramid was decreased by the appearance of the recirculation flow. However, the pressure of the uphill face was increased by the main flow which collided with the uphill face. The pressure difference between the uphill face and the downhill face was increased when the recirculation flow appeared at the downhill face. We examined the duration and the frequency of the recirculation flow which was generated at the uphill, top and downhill faces for each sample in order to discuss the relationship between the appearance of recirculation flow and the drags. For this purpose, we observed 3000 images with path-line. Table 7 shows the results for the duration and the frequency. In Table 7 (a), the average rate of existence time of the recirculation flow ($\eta=T_{x_{f_i}}$) became higher in the order of models 3 < 6 < 2 < 5 < 1 < 4. The value of $\eta$ became high when the amplitude became high, and the value of $\eta$ also became high when $a/\lambda$ became high if the value of the amplitude was the same.

In Table 7 (b), the value of $\eta$ became higher in the order of models 2 < 3 < 5 < 1 < 6 < 4. This order corresponded with that of the value of $\eta$ at the uphill face except for model 2 and model 6. We considered that the upward flow along the uphill face was affected by the surfaces in regions (2) and (3) because the flow which collided with the uphill face...
flowed upward along the inclined uphill face. In Fig. 12 (b) and (c), the values of $u^{+}_{\text{uphill}}$ for model 2 were the lowest and the values of $u^{+}_{\text{uphill}}$ for model 6 were the highest in these two regions. We considered that the flow in regions (2) and (3) affected the average rate of existence time of the recirculation flow at the top face for model 2 and model 6.

In Table 7 (c), the value of $\eta$ became higher in the order of models $2 < 3 < 1 < 5 < 6 < 4$. This order corresponded with that of the value of $\eta$ at the top face except for model 1. We think that the recirculation flow of the top face usually triggered the generation of the recirculation flow of the downhill face. In addition, this order for the value of $\eta$ corresponded with that of the mean values of the pressure drag coefficient shown in Table 6 and that of the mean values of the total drag coefficient except for model 1. Thus, we found that the existence of the recirculation flow at the downhill face was a controlling factor of pressure drag and total drag.

The value of the total drag was decreased by decreasing the value of the pressure drag. The value of the pressure drag was decreased when the average rate of existence time of the recirculation flow at the downhill face was low, because the value of the pressure drag depended on the average rate of existence time of the recirculation flow at the downhill face. The values of $u^{+}_{\text{uphill}}$ for model 2 were the lowest in the regions (2) and (3), the area in which a recirculation flow appeared at the top face was narrow and the flow which approached the downhill face was decreased by the recirculation flow of the downhill face. We consider that these factors are the reasons for the reduction of the value of the total drag for model 2. Thus, we consider that the value of the total drag for model 2 was decreased because the average rate of existence time of the recirculation flow at the downhill face was decreased.

We now discuss the mechanism of drag reduction by wavy surfaces. It is found from Tables 2, 3 and 5 that the pressure drag and the total drag in the cases of wavy surfaces particularly with short wavelength were lower than those in the case of flat surface, respectively. In addition, it is found from Table 7 that the average rate of existence time of the recirculation flow in the cases of wavy surfaces particularly with short wavelength was lower than that in the case of flat surface. Thus, the lower rate of appearance of the recirculation flow caused the drag reduction.

It was observed that the main flow over the top face usually induced small-scale recirculation flow above the top face. The recirculation flow grew not only above the top face but also in the wake flow region of the pyramid (See Fig. 15). Some of the detached recirculation flow was merged into recirculation flow in rear of the downhill face. In this procedure, the wavy surface with short wavelength on the top face can enhance the perturbation the recirculation flow. Then, the recirculation flow in rear of downhill face became more turbulent and moved more widely than that in the case of flat surface. Thus, the pressure on the downhill face of the wavy surface was higher than that of the flat surface. In addition, the average rate of existence time decreased.

Table 7 Duration and frequency of recirculation flow.

| model | $a$ | $\lambda$ | $a/\lambda$ | $T_T$ (s) | $f_T$ (Hz) | $\eta_{TV\times f_T}$ | $T_T$ (s) | $f_T$ (Hz) | $\eta_{TV\times f_T}$ | $T_T$ (s) | $f_T$ (Hz) | $\eta_{TV\times f_T}$ |
|-------|----|-------|------------|---------|---------|----------------------|---------|---------|----------------------|---------|---------|----------------------|
| 1     | --- | ---   | ---        | 0.14    | 5.23    | 0.73                 | 0.25    | 3.05    | 0.76                 | 0.29    | 2.51    | 0.73                 |
| 2     | 5.4 | 153   | 0.035      | 0.16    | 4.28    | 0.68                 | 0.25    | 2.96    | 0.74                 | 0.25    | 2.45    | 0.61                 |
| 3     | 3.2 | 92.2  | 0.035      | 0.15    | 4.48    | 0.67                 | 0.29    | 2.57    | 0.75                 | 0.38    | 1.86    | 0.71                 |
| 4     | 16  | 460   | 0.035      | 0.23    | 3.41    | 0.78                 | 0.28    | 2.83    | 0.79                 | 0.41    | 1.88    | 0.77                 |
| 5     | 5.4 | 92.2  | 0.058      | 0.15    | 4.57    | 0.69                 | 0.30    | 2.53    | 0.76                 | 0.29    | 2.54    | 0.74                 |
| 6     | 5.4 | 460   | 0.012      | 0.18    | 3.79    | 0.68                 | 0.35    | 2.23    | 0.78                 | 0.37    | 2.05    | 0.76                 |

Fig. 15 Mechanism of an appearance of recirculation flow (downhill face).
5. Conclusions

Measurements were conducted for the velocity field and the pressure difference of turbulent boundary-layer flow for truncated pyramids covered with flat surfaces and wavy surfaces and for the total drag acting on these samples. The main conclusions obtained are as follows:

1) The values of the total drag for the truncated pyramids covered with wavy surfaces were lower than those of the truncated pyramid covered with flat surfaces. In addition, the value of the total drag for the sample shaped with the values $a^+ = 5.4$, $\lambda^+ = 153$ and $a/\lambda = 0.035$ was 7.9% lower than that of the sample covered with flat surfaces.

2) The value of the total drag was effectively reduced when the amplitude was a certain level and the value of the ratio of amplitude to wavelength for the wavy surfaces was $a/\lambda = 0.035$.

3) The value of the total drag was reduced by the friction drag which was caused by the opposite flow when the wavelength was decreased, but this reduction showed only a small value.

4) The values of the pressure difference for the truncated pyramids covered with a wavy surface whose wavelengths were smaller than 153 were low. In addition, the value of the pressure difference for the sample shaped with the values $a^+ = 5.4$, $\lambda^+ = 153$ and $a/\lambda = 0.035$ was 13.7% lower than that of the sample covered with flat surfaces.

5) The value of the pressure difference between the uphill face and the downhill face was effectively reduced by inhibiting the generation of the recirculation flow at the downhill face.

6) The recirculation flow near the top face usually triggered the generation of the recirculation flow near the downhill face.

7) The existence of the recirculation flow near the downhill face was a controlling factor of the pressure drag and total drag.

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Appendix

A. Mean velocity profiles and turbulence intensities

Figures A (a) and A (b) show mean velocity profiles and turbulence intensities at the test section in the case without the truncated pyramid.

B. Verification of the measurement system of total drag

The measurement was carried out using the measurement system which was the same as that was described in Section 3.1. The test sample was a cylinder whose diameter $d_c$ was 10 mm and length was 150 mm. The cylinder was fixed on a stainless-steel plate with L-shaped supports which were attached on end surfaces of a cylinder. The cylinder was arranged so that its axis was in the spanwise direction and was 20 mm in height from the surface of the stainless-steel plate in order to remove an effect of the mean velocity gradient. The Reynolds number based on $u_e$ and $d_c$ was $1.0 \times 10^3$. The experimental result of $C_T$ was 1.23. There was an error of 8.9% between this experimental value and the value of the following empirical formula in the textbook written by Munson et al. (2006).

$$C_D = \frac{5.93}{\sqrt{Re}} + 1.17$$

Munson, B.R. and Young, D.R., Fundamental of Fluid Mechanics, 5th Ed., John Wiley & Sons. Inc (2006), pp.518-537.

C. Measurement of duration and frequency of a recirculation flow

Figure C shows a typical result for the occurrence of recirculation flow, which was obtained from the images in the case of model 5. It is obtained from this figure that the frequency is $f_v = 8/2.25 = 3.6$ Hz, and the average duration $T_v$ is 0.20 s. Thus, the average rate of existence time is $\eta = 0.693$. The actual rate of existence was $1.55/2.25 = 0.689$. Thus, the equation of $\eta = T_v \times f_v$ is reasonable.

$C_T$ Typical example of the detection of a recirculation flow (model 5).