Towards Ghost-Free Gravity and Standard Model

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Abstract

This paper presents a new higher derivative gravity which in spontaneous breaking electroweak symmetry state does not have ghost in gravity sector. We show that Newton constant of the gravity and dark energy density they depend on the fundamental TeV scale and the coupling constant at the quadratic curvature term. We consider the supersymmetric extension of this model.

Keywords: Gravity; dark energy; supersymmetry.

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It is well known that the Einstein action of general relativity leads to the non-renormalizable quantum theory\(^1\). The Einstein theory should be a good approximation at classical level and has a sensible Newtonian limit.

The theory of the higher derivative gravitation, whose action contains terms quadratic in the curvature in addition to the Einstein term, is a renormalizable field theory\(^2-8\), but it is not free of defects. As such, these theories contain both second and fourth order derivative to gravitational components. It gives rise to unphysical poles in spin two sector of the tree-level propagator which breaks the unitarity. A possible way to overcome this problem is to consider nonlocal gravity\(^9-10\) and the idea proposed in\(^11\) is to modify the ultraviolet behavior of the graviton propagator in Lorenz non-invariant way.

On the other hand, induced gravity program\(^12-15\) with fourth-derivative gravitational theories do not contain dimensional coupling constants and the unphysical ghost. However, in such theories Newton’s constant is not calculable and is a free parameter\(^16\), and does not have the Newtonian limit.

In the previous work\(^17\) it was shown that the electroweak symmetry breaking can be used for the construction of the quantum gravity free of defects.

This paper is devoted to the investigation of an example of the quantum gravity with higher curvature which is ghost-free.
We show that the gravitational strength, other observed fundamental interactions, and vacuum energy density are the consequence of one fundamental dimensional scale $M_{EW} \sim 2 \times 10^{3} \text{GeV}$, which depends on vacuum expectation value of the Higgs fields $\langle \varphi^{0} \rangle \approx 250 \text{GeV}$ of the Standard Model: $v = M_{EW} = 8 \langle \varphi^{0} \rangle \approx 2 \times 10^{3} \text{GeV}$ and the dimensionless coupling constant at quadratic curvature term.

In this paper, we adopt the units $c = \hbar = 1$.

Let us consider a new model coupling of the gravity to the Standard Model with the uniquely formed action as follows

$$
S = \int d^{4}x \sqrt{-g} \left[ -\frac{\epsilon}{2} (v^{2} - 8^{2} \Phi^{+} \Phi) R + 8\beta G_{\mu\rho} R^{\mu\rho} - \right. \\
- \frac{1}{2} (D_{\mu} \Phi)^{+} (D^{\mu} \Phi) - \frac{f}{8} (8^{2} \Phi^{+} \Phi - v^{2})^{2} \right] + S_{SM},
$$

where $S_{SM}$ is a part of the action Standard Model for gauge fields and the fermion fields.

The fundamental scalar doublet is $\Phi^{T} = (\phi^{+}, \phi^{0})$ of Higgs fields, and $G_{\mu\rho} = R_{\mu\rho} - \frac{1}{2} g_{\mu\rho} R$ is the Einstein tensor, where $R_{\mu\rho}$ is the Ricci tensor and $R$ is the scalar curvature. In the action (1) $\epsilon$, $\beta$, and $f$ are dimensionless coupling constants. The action (1) is perturbatively renormalizable, but has the ghost to spin two sector and tachyon to Higgs sector in unbroken the electroweak symmetry. It is a known fact that no new ultraviolet divergences occur in theory with spontaneous symmetry breaking, over and above those in an unbroken theory. Hence, spontaneous breaking of the symmetry does not affect renormalization.

We suggest that Einstein’s action is be modified to read $-\frac{1}{2} \epsilon v^{2} R \sqrt{-g}$ in the action (1), where we have rule $\epsilon v^{2} = M_{p}^{2}$ and $M_{p} = (8\pi G)^{-\frac{1}{2}} \approx 2.4 \times 10^{18} \text{GeV}$ is the reduced Planck mass. So the Newton’s constant $G$ is not the fundamental constant and show that $\epsilon = \beta \frac{v^{2}}{G}$, where $\beta$ is the coupling constant of the higher curvature gravity.

The higher curvature term in the action (1) has little effect at low energies by compared to the Einstein term. At the lowest energy, only $-\frac{1}{2} \epsilon v^{2} R \sqrt{-g}$ is important to the current experimental tests of Newton’s law that does not contradict with coupling constant $\beta$ which has the value of $\beta \approx 2 \times 10^{60}$. The current experimental constraints from sub-millimeter tests to corrections of the higher curvature term to the Newtonian potential, give for constant $\beta$ bounding $\beta < 10^{62}$.

The field equations for metric $g_{\mu\sigma}$ and the Higgs fields $\Phi$ following from the action (1) have solutions $g_{\mu\sigma}^{(0)} = \eta_{\mu\sigma}$ is the Minkowski metric as the metrical ground state and nontrivial Higgs fields ground state is $8^{2} (\Phi^{+} \Phi)_{0} = v^{2} \approx (2 \times 10^{3} \text{GeV})^{2}$. This is ground state with energy zero. The Higgs mechanism requires that the unbroken state has $\langle \Phi \rangle = 0$, and the vacuum broken state has $\langle \Phi^{T} \rangle = (0, \frac{v}{\sqrt{2}})$.
The standard way in perturbative theory is to write the metric as \( g_{\mu\sigma} = \eta_{\mu\sigma} + \tilde{h}_{\mu\sigma} \). In the unitarity gauge Higgs fields takes the following form, avoiding Goldstone bosons

\[
\Phi = \left( \begin{array}{c} 0 \\ \frac{v}{\sqrt{2}} + \varphi \end{array} \right),
\]

where the real scalar field \( \varphi(x) \) describes the excited Higgs field connected with the Higgs particle.

We have that \( W^{\pm}_\mu \) and \( Z_\mu \) gauge bosons pick up masses from the spontaneous breaking of the electroweak symmetry: \( M^2_{W} = \frac{g^2 v^2}{4} \), \( M^2_{Z} = \frac{M^2_{W}}{\cos^2 \theta_W} \).

From the relation \( g^2 \frac{v^2}{8 M^2_{W}} = \frac{4}{\sqrt{2}} \), \( G_F \approx 1.17 \times 10^{-5} \text{GeV}^{-2} \) is the Fermi constant from muon decay, we obtain \( (\frac{v}{\sqrt{2}})^2 \approx (250 \text{GeV})^2 \).

Thus, after the spontaneous symmetry breaking we have the electroweak scale \( v \approx 2 \times 10^{-3} \text{GeV} \) fixed by the Fermi weak coupling constant \( G_F \).

The part of the action (1) quadratic in the fields \( \tilde{h}_{\mu\rho} \) and \( \varphi \) in the state of the electroweak symmetry breaking can be written as

\[
S = \int d^4x [8 \epsilon v \varphi R^{(1)}(\tilde{h}) + 8 \beta G^{(1)}_{\mu\rho}(\tilde{h}) R^{(1)\mu\rho}(\tilde{h}) - \frac{1}{2} \partial_{\mu} \varphi \partial^\mu \varphi - \frac{8^2 f v^2}{2} \varphi^2]
\]

where the Ricci tensor \( R^{(1)}_{\mu\rho}(\tilde{h}) \) and the scalar curvature \( R^{(1)}(\tilde{h}) \) can be written in a linearized form

\[
R^{(1)}_{\mu\rho}(\tilde{h}) = \frac{1}{2} (\Box \tilde{h}_{\mu\rho} - \partial_\mu \partial_\rho \tilde{h}^\nu_{\nu} - \partial_\rho \partial_\nu \tilde{h}^\sigma_{\sigma} + \partial_\mu \partial_\rho \tilde{h}),
\]

\[
R^{(1)}(\tilde{h}) = (\Box \tilde{h} - \partial_\mu \partial_\nu \tilde{h}_{\mu\nu})
\]

where \( \Box = \partial_\mu \partial^\mu \) denotes the flat space-time d’Alamberian.

The expression (3) for the fields \( \tilde{h}_{\mu\rho} \) and \( \varphi \) has the unwanted mixed term

\[
8 \epsilon v \varphi R^{(1)}(\tilde{h}).
\]

We can get rid of this term making the following redefined field

\[
\tilde{h}_{\mu\rho} = h_{\mu\rho} + \frac{\eta_{\mu\rho}}{\epsilon} \Box^{-1} \varphi
\]

where \( \Box^{-1} \) is the Green’s function of the usual d’Alamberian action on the Higgs field.

We find that the terms \( R^{(1)}(\tilde{h}) \) and \( G^{(1)}_{\mu\rho}(\tilde{h}) R^{(1)\mu\rho}(\tilde{h}) \) take the forms

\[
R^{(1)}(\tilde{h}) = R^{(1)}(h) + \frac{3v}{\epsilon} \varphi
\]
and

\[ G^{(1)}_{\mu\rho}(\tilde{h})R^{(1)\mu\rho}(\tilde{h}) = G^{(1)}_{\mu\rho}(h)R^{(1)\mu\rho}(h) - \frac{v}{\epsilon^2}R^{(1)}(h) - \frac{3v^2}{2\epsilon^2}\phi^2 \]  

we will not keep total derivative term in eq.(8).

Putting expressions (7) and (8) in the action (3) we get the following condition rid of the mixed term

\[ \epsilon^2 = \beta \]  

for gravitational constants \( \epsilon \) and \( \beta \).

Therefore, the Planck scale \( M_p \) is not the fundamental scale and depends on the coupling constant \( \beta \approx 2 \times 10^{60} \) by quadratic curvature term and the electroweak scale \( v \approx 2 \times 10^3 \) GeV which is the fundamental scale

\[ M_p = (\beta)^{\frac{1}{4}}v \approx 1.2 \times 10^{15} \cdot 2 \times 10^3 \text{GeV} \approx 2.4 \times 10^{18} \text{GeV}. \]  

As a result, expression (3) has the following form

\[ S = \int d^4x [8\beta G^{(1)}_{\mu\rho}(h)R^{(1)\mu\rho}(h) - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}(8f - 3 \cdot 8)v^2\phi^2] \]  

where \( (8f - 3 \cdot 8)v^2 = m^2\phi \) is the square mass of the Higgs particle at \( (8f - 3) \geq 0 \). The redefinition (6) does not lead to appearance ghost in the sector Higgs particle, but leads to the shift in square mass of the Higgs particle. The Higgs potential in the Standard Model is unstable against quantum corrections.

A well known problem in physics is the existence of a huge gap between the Standard Model scale and the Planck scale of gravity. The hierarchy problem, the stability of the Standard Model scale against the Planck scale, is considered to be one of the most important issues in the particle physics.

It has led to much of the original motivation for certain beyond the Standard Model: low energy SUSY, little Higgs, gauge singlet scalars, technicolor and so on.

In papers \(^{25,26}\), it is shown that theories with a warped extra dimensions and large extra spatial dimensions solve hierarchy problem without supersymmetry or technicolor.

Our model automatically lowers the Planck scale cutoff to \( M_{EW} = 2 \times 10^3 \text{GeV} \) ultraviolet (UV) cutoff, so it accounts for the quantum stability of the Standard Model. It is a new solution of the hierarchy problem.

Of course, the differential operator which appears in the gravity part of action (11) is not invertible. It is necessary to add a gauge-fixing term in this case

\[ S_{GF} = -\frac{1}{2\alpha} \int (\partial^\sigma h_{\sigma\mu}\partial^\mu h_{\rho\lambda}\Box h_{\rho\lambda})d^4x. \]  

\[ (8f - 3 \cdot 8)v^2 = m^2\phi \]
Going over to momentum space and using the projectors for the spin-two
\( P^{(2)}_{\mu\rho\lambda\sigma} \), spin-one \( P^{(1)}_{\mu\rho\lambda\sigma} \), the two spin-zero \( P^{(0-s)}_{\mu\rho\lambda\sigma} \), and \( P^{(0-w)}_{\mu\rho\lambda\sigma} \) we find for
actions (11) and (12)

\[
\bar{S} = S + S_{GF} = \frac{1}{2} \int h^{\mu\rho}( -k ) \{ k^4 [ 4\beta P^{(2)} - \frac{1}{2\alpha} P^{(1)} ] - 8\beta P^{(0-s)} + \frac{1}{\alpha} P^{(0-w)} \} h^{\lambda\sigma}(k) d^4k.
\]

Then the propagator for the fields \( h_{\lambda\rho} \) in the momentum space is

\[
D_{\mu\rho\lambda\sigma} = \frac{2}{4\beta k^4} P^{(2)}_{\mu\rho\lambda\sigma} + \frac{2\alpha}{k^4} P^{(1)}_{\mu\rho\lambda\sigma} - \frac{1}{8\beta k^4} P^{(0-s)}_{\mu\rho\lambda\sigma} + \frac{\alpha}{k^4} P^{(0-w)}_{\mu\rho\lambda\sigma}.
\]

The component projectors by \( P^{(1)} \) and \( P^{(0-w)} \) can be gauged away at \( \alpha \to 0 \).

Ignoring the terms proportional \( \alpha \) in (14), we have the following form for the propagator

\[
D_{\mu\rho\lambda\sigma} = \frac{1}{4\beta k^4} \left( P^{(2)}_{\mu\rho\lambda\sigma} - \frac{1}{2} P^{(0-s)}_{\mu\rho\lambda\sigma} \right).
\]

Thus, \( P^{(0-s)} \) residue is negative it is a ghost. There is the kind ghost related to the \( P^{(0-s)} \) projector which has precisely the coefficient. It was actually necessary for the correct cancellation of the unphysical longitudinal part of the \( P^{(2)} \) projector. Thus we conclude that the propagator describes the physical graviton state. If we have in action (1) the terms \( aW - \frac{k}{\beta} R^2 \) (see ref.17), instead of the term \( 8\beta G_{\mu\rho}R^{\mu\rho} \), then the propagator has the ghost to the \( P^{(0-s)} \) projector and there is not cancellation of the unphysical longitudinal part of the \( P^{(2)} \) projector.

As the tree-level propagator (15) do not have the ghost that in a consequence of the local Poincare symmetry loop corrections may still do not destroy the ghost absence.

Let us note that redefinition (6) brings the contribution \( \frac{v}{\beta^2 k^2} \) to some vertex of the Feynman diagrams. It leads us to the necessity of introduction of an infrared (IR) cutoff at the Feynman integrals.

We assume that the (IR) cutoff is \( k_{IR} = L^{-1} = \frac{v}{\beta^2} \) and provides one identity the scale \( L = \frac{\beta^{3/4}}{v} \) with the horizon size of the present universe \( L \sim \frac{1}{H} \), where \( H = 1.3 \times 10^{-42} \text{GeV} \) is the Hubble parameter.

According to the holographic principle \( 28,29 \) the vacuum energy density is 
\( \rho_{\text{vac}} \sim 3d^2 M_p^2 L^{-2} \), where \( d \lesssim 1 \) is a numerical parameter. The largest size \( L \) compatible with this is the infrared (IR) cutoff of the effective quantum field theory.

We have the following form of the vacuum energy density
\[
\rho_{vac} \sim 3d^2 M_p^2 L^{-2} = 3d^2 \frac{v^4}{\beta} \sim 10^{-47} \text{GeV}^4,
\]

(16)

at \( \beta \simeq 2 \times 10^{66} \). Thus in the framework the new version \( R^2 \)-gravity with one scale, which is the electroweak scale \( v \simeq 2 \times 10^3 \text{GeV} \), can be also find of the solution smallness problem of the cosmological constant. The Bekenstein-Hawking entropy for the present universe is \( S_{BH} \sim \pi M_p^2 L^2 = \pi \beta^2 \sim 10^{121} \).

Vacuum energy density (16) plays the role of the dark energy, which counts about 75 percent of the total energy density. As a result, the universe expansion is accelerating 30.

It has been known from many astrophysical measurements that the universe contains about 20 percent of the total energy matter density, non baryonic dark matter (DM) which is not included in the Standard Model (SM).

It has been studied 31,32 that a few multiplet (gauge singlets, doublet) that can be added to the SM without introducing supersymmetry (SUSY) as a potential dark matter candidate.

In this note, we make the SUSY extension of the action (1) which includes the SUSY extension of the SM. The SUSY extension of the SM to the Next - to Minimal Supersymmetric Standard Model (NMSSM) can have the DM good scenarios. In order to obtain the corresponding N=1 SUSY for the action (1), we followed the superfield approach ref.33,34.

The supersymmetry generalization of the action (1) in chiral superspace has the form

\[
S = \int d^4x \left[ d^2\theta 2E\{-3e(v^2 - Y(\Phi))R - \frac{1}{16}(\overline{D}^2 - 8R)\Phi^+ e^V \Phi - 8\beta(\overline{D}^2 - 8R)(G^a G^a - \frac{1}{4} R^+ R) + W(\Phi) + \text{gauge and quark, lepton superfields}\} + h.c. \right].
\]

The chiral superspace density or the vierbein multiplet (in Wess-Zumino gauge) reads 2\( E(x,\theta) = e(x)[1 + i\theta \sigma^a \psi_a - \theta^2(M + \psi a \sigma^a \psi b)] \), where \( e(x) = \sqrt{-\det g_{\mu\nu}} \) and \( g_{\mu\nu} = e^a_{\mu} e_{\nu a} \) is the space-time metric, \( \psi^a_\mu(x) = e^a_{\mu} \psi^a_\mu \) is a gravitino, and \( M(x) \) is the complex auxiliary field.

The gauge invariant function \( Y(\Phi) \) is a quadratic function of the chiral superfields \( \Phi(x, \theta) \) which are the Higgs and the gauge scalars supermultiplets.

The interactions of the \( SU(2)_L \) and \( U(1)_Y \) gauge field supermultiplets \( V(x, \theta, \overline{\theta}) \) with the Higgs supermultiplets are represented by the factor \( e^V \) with \( V \) in the appropriate representation.

The chiral complex scalar superfield \( R(x, \theta) \) is the curvature supermultiplet containing the scalar curvature \( R(x) \) at its \( \theta^2 \) term.

The real vector superfield \( G^a_m(x, \theta, \overline{\theta}) \) has the traceless part of the Ricci tensor, \( R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R \) in its \( \theta \sigma^a \overline{\theta} \) component.
The superpotential $W(\Phi)$ in case NMSSM depends exclusively on the Higgs chiral superfields $H_1(x, \theta)$, $H_2(x, \theta)$, and complex scalar superfield $S(x, \theta)$ can be form

$$W(\Phi) = \xi(H_1 H_2 - v^2)S + \frac{\lambda S^3}{3}. \quad (18)$$

However to (17) we have to add the Yukawa couplings of the quarks and leptons superfields.

The spontaneous supersymmetry breaking of the higher derivative supergravity theory in the case $\phi(R(x, \theta))$ was consider in article 35.

The spontaneous supersymmetry breaking for our model (17) needs further study. On the other hand, the vacuum energy density (16) can be associated with SUSY breaking.

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