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**Abstract**

Beginning with a review of the arguments leading to the so-called $c=1$ barrier in the continuum formulation of noncritical string theory, the pathology is then exhibited in a discretized version of the theory, formulated through dynamical triangulation of two dimensional random surfaces. The effect of embedding the string in a superspace with fermionic coordinates is next studied in some detail. Using techniques borrowed from the theory of random matrices, indirect arguments are presented to establish that such an embedding may stabilize the two dimensional world sheet against degeneration into a branched polymer-like structure, thereby leading to a well-defined continuum string theory in a spacetime of dimension larger than 2.

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1 The $c_m = 1$ Barrier

1.1 Continuum Formulation

Bosonic string theory in twenty six dimensional spacetime is characterized by two important properties: (i) quantum fluctuations of the string world sheet metric (Liouville mode) decouple, leaving behind a free theory of matter and ghost fields in two dimensions; (ii) the theory has exact two dimensional conformal symmetry. The first property is no longer true for embeddings of the string in spacetimes of dimensionality $D \neq 26$, thus requiring a proper quantum formulation of the Liouville mode. It is however possible to deal with this within the 2d conformal field theory framework provided the 2d cosmological constant is treated perturbatively. In this formulation, the Liouville mode behaves like a free scalar field with a background charge $Q$, such that its contribution to the central charge of the Virasoro algebra is given by $c_L = 1 + 3Q^2$. Requiring now that the total central charge, consisting of contributions from the matter, ghost and the Liouville sectors, vanish fixes $Q$ to be

$$Q = \left(\frac{25 - c_m}{3}\right)^{\frac{1}{2}} \tag{1}$$

The fixed area partition function scales as

$$Z(\lambda A) = \lambda^{\frac{Q}{2}} Z(A), \tag{2}$$

where, $\gamma \equiv -\frac{1}{2}Q + \left(\frac{1-c_m}{12}\right)^{\frac{1}{2}}$. Clearly, $Q/\gamma$ is complex for $1 \leq c_M < 25$, so that the string susceptibility has complex critical exponents for $c_M$ in this interval. This is the so-called $c_M = 1$ barrier [1, 2]. If we interpret the Liouville mode as an extra dimension of spacetime [3], then the interval $2 < D < 26$ is the forbidden region for the bosonic string. What exactly happens to string dynamics for these dimensions of the embedding space is not clearly understood; the malady has been variously attributed to tachyons in the string spectrum [4], a strongly coupled phase of two dimensional gravity or to the disintegration of the string world sheet to branched polymers.

A tachyon-free string theory is of course one with target space supersymmetry which allows naturally for spacetime fermions. The continuum Green Schwarz superstring [5], which is classically consistent in spacetime dimensions 3, 4, 6 and 10 [6] is an important candidate because, in addition to the usual bosonic string coordinates $X^\mu(\sigma, \tau)$, $\mu = 1, \ldots, D - 1$ it also has spinorial coordinates $\theta^\alpha(\sigma, \tau)$, $\alpha = 1, \ldots, 2^{(D-1)/2}$, which are spinors in target space but (anticommuting) scalars on the world sheet much like the $b - c$ ghost system of the bosonic string. Now recall that the latter always contribute negatively to the total conformal anomaly (central charge). If the fermionic coordinates indeed have a similar dynamics, then quite conceivably $c_\theta < 0$, so that, with $c_X = D - 1$ and $c_M = c_X + c_\theta$, one has $D > c_M + 1$. This implies that the $c_M = 1$
barrier no longer restricts the allowed dimensionality of spacetime. While it is known that for critical $D = 10$ Green Schwarz superstrings this is indeed true, the situation for $D \neq 10$ is far from clear. The major problem has to do with the fermionic gauge symmetry known as $\kappa$ symmetry\(^7\) and its Lorentz covariant fixing\(^8\). This, however, is not relevant for the discretized version of the model formulated as a dynamically triangulated world sheet embedded in superspace\(^5\). We next turn to this version of the theory.

1.2 Discretized Formulation

The discretized bosonic string \(^9\)\(^10\)\(^11\) is given, for a random world sheet of spherical topology, by

$$Z_B(\beta, \Lambda) = \sum_T \frac{e^{-\Lambda|T|}}{\rho(T)} Z_T^B(\beta)$$

where,

$$Z_T^B(\beta) \equiv \prod_{\mu=1}^{D-1} \int dX^\mu \prod_{<ij>} dP_{ij} e^{-\beta S_B(X,P)} ,$$

and the action, expressed in first order form, is given as a sum over links by

$$S_B(X, P) \equiv \sum_{<ij>} \left[ \frac{1}{2}(P_{ij}^\mu)^2 + iP_{ij}(X^i_\mu - X^j_\mu) \right] .$$

The variables $P_{ij}$ are basically link variables on the triangular lattice, which we define to be antisymmetric under interchange of the indices $i$ and $j$. Integrating over these variables yields the more familiar version of the discretized Polyakov string whose action is

$$S_{Polyakov} \sim \sum_{<ij>} (X_i^\mu - X_j^\mu)^2 .$$

If we scale

$$X_i^\mu \rightarrow \beta^{-1/2} X_i^\mu ; \ P_{ij}^\mu \rightarrow \beta^{-\frac{1}{2}} P_{ij}^\mu ,$$

then

$$Z_T^B(\beta) = \beta^{-\frac{1}{2}(D-1)(|T|+V-3)} Z_T^B(1)$$

which implies that $Z_B(\beta, \Lambda)$ can be thought of as a power series in $\beta^{-1}$. For a fixed genus of the world sheet, the total number of triangles into which the world sheet has actually been triangulated, $|T|$, increases, with the number of vertices $V \rightarrow \infty$, as $[c(g)]^V$, where $c(g)$ is a number of order 1. Therefore there is a large $\beta = \beta_0^B$ such that

$$Z_B(\beta) < \infty \text{ for } \beta > \beta_0^B ,$$

and diverges as $(\beta-\beta_0)^\alpha$ at criticality. Correspondingly, the string susceptibility $\chi_B(\beta)$ diverges at criticality with exponent $\Gamma$. 

The pathology inherent in the existence of a forbidden range of embedding space dimension manifests in the discretized version in the critical behaviour of the lattice string tension. For the existence of a proper continuum limit, the lattice string tension should scale to zero at criticality. But both for hypercubic and triangular latticizations of the random world sheet, there exists a non-zero absolute lower bound on the lattice string tension, proportional to the critical temperature (inverse string coupling) \[13\] for \( D > 2 \). Although it is very likely that random surfaces of minimal area (spikes) dominate the partition function, leading to a degeneration of the world sheet into branched polymers, the only evidence comes from numerical simulations which show this behaviour for \( D > 11 \) \[12\]. Another aspect of this malady manifests in the Hausdorff dimension

\[ d_H \equiv \lim_{|T| \to \infty} \frac{< X^2 >_{|T|}}{\ln |T|} . \]

For \( D > 2 \), numerical studies indicate that

\[ < X^2 >_{|T|} \sim |T|^{\frac{1}{2}} \]

so that \( d_H \to \infty \). While reflection positivity is a good working hypothesis ensuring absence of tachyons from the spectrum, the above sickness might well originate from tachyons in the continuum. Thus, spacetime supersymmetry is a likely cure. Additional motivation comes from the theory of random walks: for supersymmetric walks the Hausdorff dimension is 1 compared to 2 for bosonic walks.

## 2 Discretized superstring in \( D = 3 \)

### 2.1 Scaling properties

The partition function for a discretized superstring in a superspace with 3 bosonic Euclidean directions is given by \[14, 5\],

\[ Z^T_S(\beta) = \int \prod_{\mu=1}^{3} \prod_{i=1}^{|V(T)|} dX^\mu_i \prod_{ij} dP^\mu_{ij} \prod_{i,\alpha} d\theta^\alpha_i e^{-\beta S_S} , \quad (10) \]

where, the action, in a first order form, is

\[ S_S \equiv \sum_{\mu,ij} \left\{ \frac{1}{2} (P^\mu_{ij})^2 + iP^\mu_{ij}[X^\mu_i - X^\mu_j + \frac{1}{2} i\theta_{ij} \sigma^\mu \theta_{ij}] \right\} . \quad (11) \]

Upon integration over the momentum variables \( P^\mu_{ij} \), one obtains the conventional discretized version of the Green Schwarz action sans the Wess-Zumino type term needed for the action to be \( \kappa \) invariant. We recall that the latter is not realized on the lattice\[5\]; further, in \( D = 3 \) the WZ term is not relevant even in the continuum because one does not need it for \( \kappa \) invariance. One
further remark is that, unlike the second order form in which the fermionic coordinates appear through quartic couplings, in the first order form in (11) they appear quadratically just like the $X_s$; thus they can be integrated out to yield an effective action as a functional of the link variables alone.

Performing a scaling of the variables in (10)

$$X^\mu_i \rightarrow \beta^{-\frac{1}{2}} X^\mu_i, \quad P_{ij}^\mu \rightarrow \beta^{-\frac{1}{2}} P_{ij}^\mu, \quad \theta^\alpha_i \rightarrow \beta^{-\frac{1}{4}} \theta^\alpha_i$$

(12)

we get

$$Z^T_S(\beta) = \beta^{-\frac{3}{2}} [ |T| + V - 3 ] . \beta^{-V} Z^T_S(1) . \quad (13)$$

As a power series in $\beta^{-1}$, $Z_S(\beta)$ has a radius of convergence given by $(\beta_0^S)^{-1}$. A crucial question is, if we assume that the absolute lower bound on the lattice string tension derived by Ambjorn and Durhuus in ref. [13] in terms of the critical temperature is still valid, then is $\beta_0^S < \beta_0^B$? If so, the minimal value of the critical lattice string tension will be smaller for the superstring than for the bosonic string signifying that supersymmetry is a step in the right direction. Evidence for the latter is already available in the work of Ambjorn and Varsted who present numerical results on the average ratio of the radius to circumference of the random surface [5]. These authors show that this ratio is closer to 1 for the superstring than for the bosonic string, although the results are inconclusive. In the sequel we present an analytical approach to this problem. We show that $\beta_0^S < \beta_0^B$, under some assumptions. Work is in progress to ascertain whether the critical string tension is indeed zero.

2.2 Representation as a Matrix Model: bosonic string

Even though the first order action (11) is quadratic in the $\theta$ and $X$ variables, integrating over them yields a rather complicated action which, in the literature has only been dealt with numerically. In this subsection we use the theory of random matrices towards an analytical approach to this problem. Before turning to the case of the superstring, we take a closer look at the bosonic string to illustrate our approach. Recall that for a bosonic string in $D - 1 = 3$ spatial dimensions, the partition function is given, after integration over $X$ as

$$Z_B^T(1) = \prod_{\mu=1}^3 \int \prod_{<ij>} dP_{ij}^\mu e^{-\sum_{<ij>} (P_{ij}^\mu)^2} \prod_i \delta(\sum_j P_{ij}^\mu) . \quad (14)$$

On the other hand, if we do the $P$ integration followed by integration over the $X$ variables, we get

$$Z_B^T(1) \sim det^{-3/2}D_{ij} , \quad (15)$$

where $D_{ij}$ is the adjacency matrix,

$$D_{ij} \equiv \begin{cases} -1 & \text{if } i \text{ and } j \text{ are nearest neighbours} \\ 0 & \text{otherwise} \end{cases}$$
With this definition,

\[ Z_T^B = \int \prod_{\mu;\langle ij\rangle} dP_{ij}^\mu e^{\sum_{\mu;\langle ij\rangle} (P_{ij}^\mu)^2} \cdot \left\{ \prod_{\mu;i} \delta\left( \sum_j P_{ij}^\mu \right) \prod_{\mu;j} \delta\left( (1 - D_{ij}) P_{ij}^\mu \right) \right\}, \tag{16} \]

where, the terms above in curly brackets are constraints that enforce that the integration variables are in reality link variables.

We now make that assumption that, on the above constraint surface, link fluctuations in different directions are independent, and as such, the antisymmetric matrices \( P^\mu \) commute for different \( \mu \). This assumption now allows us to cast eqn \((16)\) in the form of a matrix model with ‘non-singlet’ delta function constraints for three antisymmetric matrices. If \( O_{ij} \) is the orthogonal transformation that reduces each \( P^\mu \) to its Jordan canonical form with eigenvalues \( p_i^\mu \), and \( \Delta(p) \) is the Van der Monde determinant, we have

\[ Z_T^{T,\text{matr}}(1) = \prod_{\mu} \int \prod_i dp_i^\mu e^{\sum_i (p_i^\mu)^2} \Delta^2(p) \int \mathcal{D}O \{\text{constraints}\} \left[ O \right] \tag{17} \]

If we ignore the non-singlet constraints,

\[ Z_T^B(1) = \left[ V! \det \mathcal{H} \right]^3, \tag{18} \]

where \( \mathcal{H} \) is the Hadamard matrix:

\[ \mathcal{H}_{ij} = \left\{ \begin{array}{ll} \frac{(i+j-1)!!}{2^\frac{(i+j)}{2}} & \text{if } i + j \text{ is even} \\ 0 & \text{if } i + j \text{ odd, } i, j = 0, 1, \ldots V - 1. \end{array} \right. \]

The rationale for considering the case when the non-singlet constraints are absent is simply that these constraints are the same for the bosonic and the supersymmetric cases, and as such, the ratio of the partition functions in their absence is expected to be a good approximation to the ratio when these constraints are included.

### 2.3 Matrix Model Representation for the Discretized Superstring

Integrating over the \( X \) and \( \theta \) variables we get

\[ Z_T^S(1) = \int \prod_{\mu;\langle ij\rangle} dP_{ij}^\mu e^{-\sum_{\mu;\langle ij\rangle} (P_{ij}^\mu)^2} \text{Det} \left( \sum_\mu (\sigma^\mu P^\mu) \right) \prod_{\mu;i} \delta\left( \sum_j P_{ij}^\mu \right), \tag{19} \]
where $\sigma^\mu$ are the Pauli matrices. Using elementary matrix theory this can be rewritten

$$\tilde{Z}^T_m = \int \prod_{\mu; <ij>} dP^\mu_{ij} e^{-\sum_{\mu; <ij>} (P^\mu_{ij})^2 - \ln(\sum_{\mu} (P^\mu)^2)} \cdot \text{constraints},$$

where, constraints refer to the delta function constraints in the rhs of eq. (17). Once again, we have a (constrained) matrix model of three antisymmetric matrices with a potential which is a sum of Gaussian and logarithmic parts, the latter being due to the fermionic coordinates. In terms of eigenvalues $p^\mu_i$, this is reexpressed as

$$Z^T_S(1) = \int \prod_{\mu} dp^\mu_i e^{-\sum_{\mu} (p^\mu_i)^2} \prod_{i} \left(\sum_{\mu} (p^\mu_i)^2\right)$$

$$\cdot \prod \Delta^2(p^\mu) \int \text{constraints} \ , (21)$$

Let us now define a real symmetric matrix of order $V \times V$ in the following manner:

$$A_d \equiv \text{diag}(a_1, a_2, \ldots, a_V) \ , \ a_i \text{ real} \ ,$$

$$A \equiv \mathcal{O}A_d\mathcal{O}^{-1} \ ; (23)$$

Thus, the matrix $A$ shares its angular parts with those of the random matrices $P^\mu$, while its eigenvalues $\{a_i\}$ are classical, since they are not integrated over. $A$ is therefore a rather special type of external matrix field. Consider now the partition function

$$Z(a_1, \ldots, a_V) \equiv \prod_{\mu} \int dp^\mu_i \Delta^2(p) \cdot C \ ;$$

here $C \equiv \int \text{constraints}$, with the constraints being once again the delta function constraints. Observe that $C$ is quite independent of the eigenvalues $\{a_i\}$, so that one obtains the following results [15],

$$Z^T_B(1) = Z(a_1, \ldots, a_V)|_{a_1=a_2=\ldots=a_V=1}$$

$$Z^T_S(1) = \prod \frac{\partial}{\partial a_i} Z|_{a_1=a_2=\ldots=a_V=1} . \ (26)$$

In fact, if we were to expand $Z(\{a_i\})$ around the point $a_i = 1$ for all $i$, the bosonic and supersymmetric partition functions become coefficients of the first and $V$th terms in this expansion. Thus, the theory described by the partition function $Z$ is interesting on its own right, especially if any of the other coefficients in the above expansion could be identified as partition functions of some new string theories.
Once again, if the non-singlet constraints are ignored,
\[
\mathcal{Z}^{\text{matr}}(a_i) = \prod_\mu \int \mathcal{D}P^\mu e^{-\text{Tr}[A \sum (P^\mu)^2]}
\]
\[
\equiv \prod_\mu \mathcal{Z}_\mu ,
\]
(27)

where \(\mathcal{Z}_\mu\) describes a random (antisymmetric) matrix model with a ‘diagonal’ external field \(A\). Recall that for \(A\) being the identity, the bosonic partition function was exactly calculable a la Bessis \cite{16} in terms of the determinant of the Hadamard matrix. For arbitrary real \(a_i\) not necessarily all equal to 1, one can define a generalized Hadamard matrix \(\tilde{H}\) whose elements are defined as
\[
\tilde{H}_{ij} = a_{i+1}^{-(i+j+1)/2}H_{ij} .
\]
(28)

In terms of this matrix, the partition function in (27) can be expressed as
\[
\mathcal{Z}_\mu = \det \tilde{H} \text{ for every } \mu .
\]
(29)

The proof of the above consists of a straightforward generalization of the proof of Bessis for the case when all the external eigenvalues \(a_i\) are unity, and is not included here \cite{17}. It lends itself to generalization quite freely to the cases of non-Gaussian measures.

Specializing to the case of the discretized superstring imbedded in a superspace of 3 bosonic dimensions, we have
\[
\mathcal{Z}^{\text{matr}}(a_1, \ldots, a_V) = \det \tilde{H} .
\]
(30)

Now, from eqns \(\text{24}, \text{27}, \text{28}\) and \(\text{29}\), one can derive an upper bound on the ratio of the supersymmetric to the bosonic partition function in the absence of the delta function constraints :
\[
\frac{\mathcal{Z}^{\text{matr}}_S}{\mathcal{Z}^{\text{matr}}_B} < \left( \frac{1}{2^V} \frac{\det \left[ (i+j+1)! \right]}{\det \left[ (i+j)! \right]} \right) \frac{1}{2^{(i+j)}} .
\]
(31)

For the limit \(V \to \infty\), the ratio on the rhs is certainly less than 1. If we now assume that
\[
\frac{\mathcal{Z}^T_S(1)}{\mathcal{Z}^{\text{matr}}_S} \sim \frac{\mathcal{Z}^T_B(1)}{\mathcal{Z}^{\text{matr}}_B} ,
\]
(32)

because, as already mentioned, the non-singlet constraints in both cases are identical, we can infer that
\[
\mathcal{Z}^T_S(1) < \mathcal{Z}^T_B(1) ,
\]
(33)

which implies that \(\beta^S_0 < \beta^B_0\). Thus if we believe that analogous to the bosonic case the critical lattice string tension for the superstring is bounded from above
by a quantity proportional to the inverse critical temperature, then it follows
that this minimal value is lower in the supersymmetric case as compared to the
bosonic case.

Two crucial questions remain at this point: (1) is this minimal value of the
string tension actually zero, as one would like it to be? (2) Is the geometrical
proof of Ambjorn and Durhuus \[13\] for the bosonic string generalizable to the
case of the superstring, as we have assumed above. We do not have any answer
to the first question at this point. What one requires is a better technique to
deal with matrix models with delta function constraints which are functions of
the ‘angular’ variables left over after diagonalization. As for the second issue,
we argue below that there are reasons to suspect that the proof in the bosonic
case may not generalize.

2.4 Is $T_{\text{lat}}^S \geq 2/\beta$?

We first briefly review the proof given in ref. \[13\] for the absolute lower bound
of the lattice string tension in case of the bosonic string. Consider a closed
loop $\gamma_{L,n}$ of length $L$ on the random world sheet, which has $4n$ vertices with
coordinates $X_i$, $i = 1, \ldots n$. The loop correlation function for the bosonic
string is defined as

$$ G_\beta(\gamma_{L,n}) \equiv \sum_{T \in T(n)} \rho(T) \int \prod_{i \in T} dX_i e^{-\beta \sum_{<ij>} (X_i - X_j)^2}. \quad (34) $$

The lattice string tension is defined in terms of this loop Green’s function as

$$ T \equiv -\lim_{L \to \infty} \frac{1}{L^2} \ln G_\beta(\gamma_{L,L}) \quad . \quad (35) $$

For fixed $L$, total number of vertices $|T|$ on the entire world sheet, and for a
fixed loop $\gamma$, one can decompose the discretized superstring action into a term
which may be thought of as the minimum (or saddle point) or classical part
of the action, subject to a fixed boundary which maps into the fixed loop $\gamma$,
and a term which is a function only of the fluctuating variables, and as such,
is independent of $\gamma$:

$$ S[|T|, \gamma] = S_{\text{min}}(|T|, \gamma) + S'(T) \quad , \quad (36) $$

with

$$ S_{\text{min}} = \sum_{<ij>} (X_{0i} - X_{0j})^2 \quad . \quad (37) $$

Here $X_0$ indicate ‘saddle-point’ or classical solutions of the discretized equations
of motion. Note that, this classical part corresponds to a fixed triangulation
of the world sheet. Consequently, it is a sum of squared Euclidean distances
between points which are vertices of triangles. It follows from the relation
between the area and squared length of each side of an equilateral triangle that

$$ S_{\text{min}} \geq 2L^2 \quad . \quad (38) $$
This implies that
\[ G_\beta(\gamma_{L,L}) \leq e^{-2\beta L^2} G'_\beta, \] (38)
where \( G'_\beta \) is independent of \( \gamma_{L,L} \). In fact, Ambjorn and Durhuus have shown that \( G'_\beta \leq e^{cL} \), \( c \geq 0 \); thus
\[ G_\beta \leq e^{2\beta L^2}, \] (39)
so that
\[ T_{\text{lat}}^B(\beta) \geq 2\beta. \] (40)
As \( \beta \to \beta_0 \), therefore, the lattice string tension has an absolute lower bound given by twice the inverse critical temperature.

The question now is whether a similar geometrical result exists for embeddings of the random surface in a superspace of three Euclidean bosonic dimensions. Observe that the geometric result sketched above depends crucially on the interpretability of the classical (minimal) action in terms of a sum of squared Euclidean lengths. For the superstring, the action in second order form is given by the square of a ‘current’ which is manifestly supersymmetric under spacetime supersymmetry transformations:
\[ S_S = \sum_{<ij>} \left( X_i - X_j + i\theta_i\sigma\theta_j \right)^2. \] (41)
Now the infinitesimal squared super-invariant interval on flat superspace is given as
\[ ds^2 = \left\{ dx + i\theta\sigma d\theta \right\}^2. \]
It is not at all clear whether the finite form of this ‘superlength’ has anything to do with the superstring action. This is more so because, unlike ordinary Minkowski space, flat superspace has torsion. One might of course inquire as to whether, the effective action obtained upon integrating over the fermionic coordinates in a first order form, can indeed be given a geometric interpretation akin to the bosonic situation. This issue is currently investigation [17]. Basically one needs to reformulate the proof of Ambjorn and Durhuus for the first order form of the bosonic string action and then attempt a supersymmetric generalization.

3 Summary and Conclusions

We have presented evidence (albeit indirect) that the discretized superstring in a superspace with \( D = 3 \) has better chances of metamorphosing into a well defined continuum string theory than its bosonic counterpart. A more accurate calculation of the superstring partition function is necessary, especially one in which the non-singlet constraints are handled adequately, to show explicitly that the lattice string tension does indeed scale to zero at criticality.
The generalization of Bessis’ approach to the case when a special type of external matrix field is present whose eigenvalues are classical and angular parts are random, can be extended to higher polynomial measures as well. Thus, if we have a matrix model whose potential is

$$V(M, A) = AM^2 + gA^2M^4,$$

with $M$ being an $N \times N$ hermitian matrix which is random, and $A$ is a hermitian matrix defined as in eqn. (23), i.e., its angular part shares the randomness of $M$ while its eigenvalues remain classical, then the partition function can be shown to be given by

$$Z(A) = N! \det \tilde{H},$$

where,

$$\tilde{H}_{ij} \equiv a_{\frac{i}{i+1}}^{(1+j+1)} H_{ij},$$

and

$$H_{ij} \equiv \int d m e^{-(m^2 + gm^4)m(i+j)} \text{ for } i + j \text{ even}.$$ 

Finally, apart from the simplicity of $D = 3$ for the choice of the embedding space for the discretized superstring, there is the expectation that, if a continuum limit exists then the Liouville mode will play the role of an extra dimension of spacetime, and may lead to a realistic situation, of course, one without the full $D = 4$ Poincaré supersymmetry. Since it is quite likely that this continuum theory (if it exists) will not have tachyons in its spectrum, ‘space supersymmetry’ may turn out to be enough to make amplitudes finite. If so, the problem of the cosmological constant gets decoupled from the hitherto unsolved problem of a nonperturbative mechanism for spacetime supersymmetry breaking. This could have far-reaching implications for future research in string theory.

4 References

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