Prescribed adaptive control of unknown hysteresis in smart material actuated systems

Zhi Li*, Chun-Yi Su, Xinkai Chen and Sining Liu

*aMechanical and Industrial Engineering, Concordia University, Montreal, Canada; bDepartment of Electronic and Information Systems, Shibaura Institute of Technology, Minuma-ku, Saitama-shi, Saitama, Japan

(Received 26 November 2013; accepted 5 February 2014)

Design of controllers that ensure the closed-loop stability of the system actuated by the smart material which shows hysteresis nonlinearity is a challenging issue in the literature. In this paper, we use the Bouc–Wen model to describe the hysteresis nonlinearity and attempt to fuse the Bouc–Wen model with the available adaptive control techniques without constructing the inverse of the Bouc–Wen model. To realize such a fusion, it is necessary to utilize its solution properties. However, the general solution of the Bouc–Wen model is still not available. Therefore, an approximate solution is applied. By means of the approximate solution, a prescribed adaptive control approach is developed to achieve global stability of the closed-loop system and guarantee the transient and steady-state performance of the tracking error. Simulation results demonstrate the effectiveness of the proposed approach.

Keywords: smart material; hysteresis; Bouc–Wen model; prescribed adaptive control

1. Introduction

In line with the development of micro/nano manipulation and micro-/nano-positioning system, demands for high-precision and high-resolution actuators have been ever increasing over the past two decades. To meet these demands, smart material-based actuators, i.e. piezoelectric, magnetostrictive, and shape memory alloy (SMA), have emerged in lieu of traditional actuators, i.e. electric motors. The so-called smart materials are class of materials that are capable of changing their physical properties, such as shape, color, or size, in response to an externally stimulus (Hao & Li, 2010). In comparison with electric motors, the smart materials-based actuators have the advantages of lightness, low noise levels, low power requirements, and high reliability. Therefore, they have wide potential applications in the fields of nanotechnology, aerospace, biotechnology, mechanical engineering, civil engineering, etc.

However, the performance of smart material-based actuators, in terms of tracking precision, or stability, is severely limited by a hysteresis effect which is a nonlinear phenomenon revealing a looped and branched nonlinear relation between the input excitation and the output displacement. For most cases, the hysteresis is regarded as an undesired and detrimental effect, which will deteriorate the actuator performance and cause inaccuracy or oscillations (Su, Wang, Chen, & Subhash, 2005). For treatment of the hysteresis, many hysteresis models have been proposed in the literature for...
representing hysteresis behaviors, such as Preisach model (Rakotondrabe, Janaideh, Bienaimé, & Xu, 2013), Prandtl–Ishlinskii (PI) model (Al Janaideh & Krejci, 2013), Bouc–Wen model (Rakotondrabe, 2011), and Duhem model (Chen, Tan, Zhou, Dong, & Zhang, 2011). Among them, the Bouc–Wen model has received an increasing interest due to its capability to capture a range of shapes of hysteretic cycles which match the behavior of a wide class of hysteretic systems (Ismail, Ikhouane, & Rodellar, 2009). In particular, Bouc–Wen model has been utilized to represent the hysteresis effect in piezoelectric actuators (Lin & Yang, 2006), magnetorheological dampers (Spencer, Dyke, Sain, & Carlson, 1997), forced vibration of mechanical systems (Ismail et al., 2009) and soil-base-isolated-structure systems (Constantinou & Kneifati, 1988).

In (Su, Stepanenko, Svoboda, & Leung, 2000; Su et al., 2005; Zhou, Wen, & Zhang, 2004), robust adaptive techniques have been successfully fused with unknown backlash-like model, Prandtl–Ishlinskii (PI) model to accommodate the hysteresis nonlinearity. These techniques are achieved either because of the solution availability of the model (backlash-like model) or the explicit expression with the input signal in the model (PI model). However, for the Bouc–Wen model, its solution properties are still not clear, and the input of the model is implicitly contained in the model, which causes difficulty in controller design. To attach this problem, an approximate solution (Kozlov, 2011) of the Bouc–Wen model is adopted. Featuring with this approximate solution, the input of the Bouc–Wen model is able to be explicitly expressed, and thus the Bouc–Wen model can be fused with available control techniques such as robust adaptive control approaches to guarantee the basic stability requirement. In this paper, as a demonstration, a prescribed adaptive control approach (Bechlioulis & Rovithakis, 2008, 2009) is adopted to ensure the transient and steady-state performance of the tracking error. The effectiveness of the control scheme is validated by the simulation results.

2. Problem statement

In the literature, a common strategy for remedying hysteresis nonlinearity is to utilize an open-loop control technique, i.e. construct an inverse of the hysteresis model (Al Janaideh & Krejci, 2013; Krejci & Kuhnen, 2001; Kuhnen, 2003; Tan & Iyer, 2009) to cancel the hysteresis effect. When the hysteresis is represented by differential equations such as Bouc–Wen model, the inverse is either impossible or extremely difficult to obtain. In this case, the inverse multiplicative structure technique developed in Rakotondrabe (2011), Zhou, Wen, and Li (2012) can be used to compensate the complicated function in the Bouc–Wen model. However, this technique requires to know the exact parameters of the Bouc–Wen model, which does not appear directly applicable to the system in conjunction with unknown Bouc–Wen model. Therefore, design of controllers for nonlinear system preceded by unknown Bouc–Wen hysteresis nonlinearity is a challenging task.

In this paper, we consider a controlled system consisting of a plant preceded by an actuator with unknown hysteresis nonlinearity, that is, the hysteresis is presented as an input to the nonlinear plant. The hysteresis is represented as:

\[ u(t) = \Pi[v](t) \]  

(1)

where \( v(t) \) is the input and \( u(t) \) is the output. \( \Pi[v] \) denotes the Bouc–Wen hysteresis operator and its parameters are unknown. The nonlinear dynamic system preceded by the above hysteresis nonlinearity is described in the canonical form as,
\[ x^{(n)}(t) + \sum_{i=1}^{k} a_i Y_i(x(t), \dot{x}(t), \ldots, x^{(n-1)}(t)) = b u(t) \]  

(2)

where \( Y_i \) are known continuous, linear or nonlinear functions. Parameters \( a_i \) and control gain \( b \) are unknown constants. It should be noted that more general classes of nonlinear systems can be transformed into this structure (Isidori, 1985; Su et al., 2005).

The control objective is to design a control law for \( v(t) \) in (1), such that: P1: the plant state \( x(t) \) follows a specified desired trajectory, \( x_d(t) \), i.e. \( x(t) \rightarrow x_d(t) \) as \( t \rightarrow \infty \) and all signals in the closed-loop are bounded; P2: Both transient and steady-state performance of tracking error \( e_1(t) = x(t) - x_d(t) \) should be within the prescribed area.

Throughout the paper the following assumptions are made:

\textit{Assumption 1:} The sign of uncertain parameter \( b \) is known. It is selected as \( b > 0 \) in this paper.

\textit{Assumption 2:} The desired trajectory \( x_d = [x_{d1}, x_{d2}, \ldots, x_{dn}]^T \) is continuous. Furthermore, with \( \Omega_d \) being a compact set.

\textit{Remark:} The assumptions are very common and generally satisfied.

3. The Bouc–Wen model and its approximate analytical solution

3.1. The Bouc–Wen model

The Bouc–Wen model featured by its less identified parameters has the capability to capture a wide range of hysteresis loops in different area (Ismail et al., 2009). The expression of Bouc–Wen model is written as

\[ u(t) = \Pi [\dot{v}](t) = \zeta(t) \]  

(3)

where \( u(t) \) denotes the output of the Bouc–Wen model, \( \zeta(t) \) is defined as

\[ \zeta = A \dot{v} - B |\dot{v}|^m \zeta^{m-1} - \gamma |\dot{v}| \zeta^m \]  

(4)

Figure 1. The input–output relation of Bouc–Wen Model.
where \( A, m, \beta, \) and \( \gamma \) are the parameters that govern the scale and general shape of the hysteresis circles. \( v \) denotes the input, which contains in (4) implicitly. Figure 1 shows the input–output relation of the Bouc–Wen model with parameters \( A = 1, m = 1, 2, 3, 4, \beta = .00231, \) and \( \gamma = .00031. \)

**Remark:** The solutions of (4) are unknown. Thus we cannot derive the explicit solution of Bouc–Wen model as well as the boundedness of Bouc–Wen model. Although the boundedness of term \( \zeta \) is reported in Ismail et al. (2009), the boundedness depends on the relationship of parameters \( \beta \) and \( \gamma. \) In this paper, we deal with unknown hysteresis, which means the information of parameters of Bouc–Wen model is unavailable. Without knowing the solution properties of Bouc–Wen model, we cannot conduct the adaptive control design. Therefore, the approximate analytical solution of Bouc–Wen model is utilized.

### 3.2. The approximate analytical solution of the Bouc–Wen model

The approximate analytical solution, which was obtained by utilizing Fourier transform, was originally developed in Kozlov (2011). In this section, a brief overview of the approximate analytical solution is outlined so that the paper can be self-contained.

Equation (4) can be written as

\[
\frac{d \zeta}{dv} = A - |v|^m \Psi_\zeta(v, \dot{v}, \zeta)
\]

where \( \Psi_\zeta(v, \dot{v}, h) \) denotes

\[
\Psi_\zeta(v, \dot{v}, \zeta) = \beta \text{sign}(\dot{v}) \text{sign}(\zeta(t)) + \gamma
\]

For simplicity, function \( \Psi_\zeta(v, \dot{v}, h) = \Psi_h = \text{constant}. \) Perform a direct Fourier transform on (5) yielding

\[
\zeta(\omega) = i^{-1} \omega^{-1} \left( \Psi_h \mathcal{F}\{ |z|^m, \omega \} - 2\pi A \delta(\omega) \right)
\]

where \( \delta \) is Dirac delta-function. The first-order approximation is expressed as

\[
\zeta^{(1)}(\omega) = -2\pi A i^{-1} \omega^{-1} \delta(\omega)
\]

Conducting the inverse Fourier transform on (8)

\[
\zeta^{(1)}(v) = Av
\]

For the second-order approximation, we substitute (9) into (7) and conduct the inverse Fourier transform yielding,

\[
\zeta^{(2)}(v) = Av + F^{-1} \left\{ \frac{\Psi_h}{i \omega} \mathcal{F}\{ |Av|^m, \omega \}, v \right\}
\]

\[
= Av - \frac{|Av|^m}{m+1} |v|^{m+1} \text{sign}(v)
\]

Then, the \( k \)th approximation can be defined as

\[
\zeta^{(k)}(v) = Av + F^{-1} \left\{ \frac{\Psi_h}{i \omega} \mathcal{F}\{ |z^{(k-1)}|^m, \omega \}, v \right\}
\]

for \( k \geq 1 \) and \( z(0) = 0. \)

Following the above approach, the third-order and fourth-order approximation can be written as:
\[
\xi_p^{(3)}(v) = Av - \sum_{p=0}^{\infty} \xi_p |v|^{m(p+1)} \text{sgn}(v)
\]
(12)

where \( \xi_p = \frac{r(p-m)|A|^{m(p+1)}q^{p+1}}{p!(-m)^{p+m+1} A^{m+1}} \).

\[
\xi_p^{(4)}(v) = Av - \sum_{q=0}^{\infty} \frac{\Gamma(p-m)|A|^{m(q+1)}q^{q+1}}{q!(-m)^{q+1} A^{q+1}} \times \left( \frac{|v|^{m(q+1)} + \sum_{p=1}^{nq-1} \eta_p(q) \frac{|v|^{m(p+q+1)+1}}{m(p+q+1)+1}}{m(q+1)+1} \right) \text{sgn}(v)
\]
(13)

where

\[
\eta_1(q) = a_0^{-1} a_1 q
\]
(14)

\[
\eta_p(q) = a_0^{-1} \left( a_p q + p^{-1} \sum_{k=1}^{p-1} [q(p-k) - k]a_{p-k}\eta_k(q) \right)
\]
(15)

Figure 2 is the comparison between the third-order and fourth-order approximation of the Bouc–Wen model with input signal \( v(t) = 10 \sin(2\pi t) \). As is shown in the figure, since the third-order approximation has already had a good agreement with the Bouc–Wen model, we select the third-order approximation for the controller design in terms of the simplicity and feasibility. Based on (12), the Bouc–Wen model in (3) can be approximated as

\[
u(t) = Av - \xi_p^T v_p + \Delta_1
\]
(16)

where \( \xi_p = [\xi_0, \xi_1, \ldots, \xi_p]^T, v_p = [|v|^{m+1} \text{sgn}(v), |v|^{2m+1} \text{sgn}(v), \ldots, |v|^{m(p+1)} \text{sgn}(v)]^T \), \( \Delta_1 \) indicates the approximation error, and \( \lim_{k \to \infty} \Delta_1 = 0 \) where \( k \) denotes the \( k \)th-order approximation.
approximation. This approximation needs to be further tailored for the purpose of implementation of the designed controller, i.e.

\[ u(t) = Av - \xi_p^T v_{p-} + \Delta \]  

where \( v_{p-} = [|v_-|^{m+1} \text{sgn}(v_-), |v_-|^{2m+1} \text{sgn}(v_-), \ldots, |v_-|^{(p+1)m+1} \text{sgn}(v_-)]^T \), therein \( v_- \) is defined as \( v(t-) = v(t) - \epsilon \) with \( v(t) = \lim_{\epsilon \to 0} v(t-) \).

For simplicity, we rewrite (17) as

\[ u(t) = Av - \xi^T v_- \]  

where \( \xi = [\xi_p, \Delta] \), \( v_- = [v_{p-}, 1] \).

It should be noted that the input signal \( v(t) \) has been explicitly expressed in the third-order approximation (18) of the Bouc–Wen model in (3), which makes it possible to design the controller with available control techniques when the hysteresis is represented by the Bouc–Wen model. In the following development, we will use prescribed adaptive control approach as an illustration to show the controller design procedure with the use of the third-order approximation (18). Certainly, other approaches can also be used to present different controllers.

4. Prescribed adaptive control

Instead of constructing the inverse hysteresis model, a prescribed adaptive controller is developed in this section. Different from the standard procedure of backstepping control presented in the literature Zhou et al. (2004), Su et al. (2005), the transient and steady-state performance of tracking error are incorporated in the design procedure of prescribed adaptive control. This control approach is originally developed in Bechlioulis and Rovithakis (2009) which is the first time that provides a systematic procedure to accurately compute the required bounds, thus making tracking error converge to a predefined arbitrarily small residual set, with convergence rate no less than a pre-specified value, exhibiting a maximum overshoot less than a sufficiently small preassigned constant (Bechlioulis & Rovithakis, 2008, 2009). In order to achieve the control objective, the prescribed function and error transformation need to be introduced first.

4.1. Prescribed performance function and error transformation

The performance function is introduced in Bechlioulis and Rovithakis (2009) for the purpose of depicting a convergent zone in which the trajectory of tracking error which starts from a point in the zone remains for all future time, see Figure 3. The performance function is a decreasing smooth function, which is defined as \( \rho : R^+ \to R^+ \) with \( \lim_{t \to \infty} \rho(t) = \rho_\infty > 0 \)

It is noted that the control objective P2 can be guaranteed by satisfying

\[ M\rho(t) < e(t) < \bar{M}\rho(t) \]  

for all \( t \geq 0 \), where \( M < 0, \bar{M} > 0 \) are selected parameters. \( \bar{M}_\rho(0) \) and \( M_\rho(0) \) represent the upper bound of the maximum overshoot and the lower bound of the undershoot. Thus, the performance function as well as the parameters \( \bar{M}, M \) prescribe the convergent zone for the transient and steady state performance of the tracking error.

In order to meet the requirements P1 and P2 together with condition (19), an error transformation is developed (Bechlioulis & Rovithakis, 2009) by transforming the
original nonlinear system (2) into an equivalent unconstraint one. Define $S(\cdot)$ a smooth and strictly increasing function and $z_1$ a transformed error as

$$e(t) = \rho(t)S(z_1)$$

$S(\cdot)$ holds the following conditions:

1. $M < S(z_1) < \tilde{M}$
2. $\lim_{z_1 \to +\infty} S(z_1) = \tilde{M}, \quad \lim_{t \to -\infty} S(z_1) = M$

Since $S(\cdot)$ is strictly increasing as well as $\rho(t) > 0$, the inverse transformation can be written as:

$$z_1 = S^{-1}\left(\frac{e(t)}{\rho(t)}\right)$$

Assume $z_1(t)$ remains bounded $z_1 \in L_\infty, \quad \forall t \geq 0$, then $M < S(z_1) < \tilde{M}$ holds, and hence, the condition (19) can be guaranteed. A candidate function $S(\cdot)$ is selected as

$$S(z_1) = \frac{Me^{z_1} + Me^{-z_1}}{e^{z_1} + e^{-z_1}}$$

Conduct inverse transformation on (22), yielding

$$z_1 = S^{-1}\left(\frac{e(t)}{\rho(t)}\right) = \frac{1}{2} \ln \frac{e(t)/\rho(t) - M}{M - e(t)/\rho(t)}$$

Then the derivative of $z_1$ with respect to time can be written as

$$\dot{z}_1 = \frac{\partial S^{-1}}{\partial \frac{e(t)}{\rho(t)}} \left(\frac{e(t)}{\rho(t)}\right) = \frac{1}{2} \left[ \frac{1}{e(t)/\rho(t) - M} - \frac{1}{e(t)/\rho(t) - M} \right] \left(\dot{\rho} \frac{e_t}{\rho^2} - \dot{e}_t \right)$$

$$= r(\dot{x}_1 - \dot{x}_d - e\dot{\rho}/\rho)$$
where \( r = \frac{1}{2\rho(t)} \left[ e(t)/\rho(t) - M - e(t)/\rho(t) - M \right] \). It is noted that both \( e(t) \) and \( \rho(t) \) in (24) are available and they can be involved in controller design.

4.2. Prescribed adaptive controller design

The system (2) can be re-written as

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_3 \\
&\vdots \\
\dot{x}_{n-1} &= x_n \\
\dot{x}_n &= a^T Y + b u(t)
\end{align*}
\]

where \( a = [-a_1, -a_2, \ldots, -a_k]^T \) and \( Y = [Y_1, Y_2, \ldots, Y_k]^T \). The parameters \( a, b \) are unknown. \( v(t) \) denotes the system input. \( u(t) \) is the third-order approximation of the Bouc–Wen model as

\[
u(t) = Av - \xi^T v_-
\]

where \( \xi = [\xi_0, \xi_1, \ldots, \xi_p, \Delta]^T, v_- = [|v_-|^{m+1} sgn(v_-), |v_-|^{2m+1} sgn(v_-), \ldots, |v_-|^{m(p+1)+1} sgn(v_-), 1]^T \) parameters \( A \) and \( \xi \) are unknown.

Considering the time derivative of transformed error (24) and nonlinear system (25), the transformed nonlinear system dynamics are given by:

\[
\begin{align*}
\dot{z}_1 &= r(x_2 - \dot{x}_d - e\dot{\rho}/\rho) \\
\dot{z}_2 &= x_3 \\
&\vdots \\
\dot{z}_n &= a^T Y + b u(t) \\
&= a^T Y + bAv - b\xi^T v_-
\end{align*}
\]

The controller design can then be achieved by using the recursive backstepping technique and is summarized in Table 1, where \( b_A = bA, B(v_-) = \xi_b^T v_-, \xi_b^T = b\xi^T \).

**Theorem:** For the transformed nonlinear system (2) preceded by approximate Bouc–Wen model in (18), the prescribed adaptive controller presented by (T.6)–(T.14) guarantees that

1. All signals in the closed-loop system remain bounded;
2. The tracking control with prescribed performance condition (19) is preserved.

**Proof:**

\[
b_A v = b_A \dot{\phi} v_1 = v_1 - b_A \dot{\phi} v_1
\]

According to (T.6)–(T.8) and (28), we obtain

\[
\dot{z}_n = -c_n z_n - z_{n-1} + a^T Y + sgn(z_n) B(v_-) - B(v_-) - b_A \dot{\phi} v_1
\]

Let

\[
\dot{\phi} = \phi - \dot{\phi}
\]

\[
\tilde{a} = a - \dot{a}
\]
### Table 1. Adaptive backstepping control.

**Change of coordinates**

\[
z_1 = \frac{1}{2} \ln \left( \frac{x_1(t) - x_d(t) - \rho(t) - M}{M - (x_1(t) - x_d(t)) \rho(t)} \right) \quad (T.1)
\]

\[
z_i = x_i - x_d^{(i-1)} - \bar{x}(i-1), \quad i = 2, 3, \ldots, n \quad (T.2)
\]

\[
z_1 = -c_1 z_1 / r + e_1 \hat{\rho} / \rho \quad (T.3)
\]

\[
z_2 = -c_2 z_2 + \bar{x}_1 - rz_1 \quad (T.4)
\]

\[
z_i = -c_i z_i + \bar{x}_{i-1} - z_{i-1}, \quad i = 3, \ldots, n - 1 \quad (T.5)
\]

**Adaptive control laws**

\[
v(t) = \hat{v}_1(t) \quad (T.6)
\]

When \( n = 1 \),

\[
v_1(t) = -c_1 z_1 + \bar{x}_1 - \hat{a}^T Y + \text{sgn}(z_1) \hat{B}(v-) + e_1 \hat{\rho} / \rho \quad (T.7)
\]

When \( n \geq 2 \),

\[
v_1(t) = -c_n z_n - z_{n-1} + \bar{x}_{n-1} + x_d^{(n)} - \hat{a}^T Y + \text{sgn}(z_n) \hat{B}(v-) \quad (T.8)
\]

where \( \hat{\phi} \) and \( \hat{a} \) are estimated parameters of \( \phi = 1 / b_A \), \( a \), \( \hat{B}(v-) = \hat{\xi}_b^T v_- \) is estimated function of \( \hat{B}(v-) \).

**Parameter update laws**

When \( n = 1 \),

\[
\dot{\hat{\phi}} = -\eta_{\hat{\phi}} v_1 z_1 \quad (T.9)
\]

\[
\dot{\hat{a}} = r \Gamma_a Y z_1 \quad (T.10)
\]

\[
\dot{\hat{\xi}}_b = -r \Gamma_{\xi b} v_- |z_1| \quad (T.11)
\]

When \( n \geq 2 \),

\[
\dot{\hat{\phi}} = -\eta_{\hat{\phi}} z_n \quad (T.12)
\]

\[
\dot{\hat{a}} = \Gamma_a Y z_n \quad (T.13)
\]

\[
\dot{\hat{\xi}}_b = -\Gamma_{\xi b} v_- |z_n| \quad (T.14)
\]

where \( c_1, c_2, \ldots, c_n \) are positive design parameters, \( \eta_{\hat{\phi}} \) is positive design parameter, \( \Gamma_a \) and \( \Gamma_{\xi b} \) are positive-definite matrices.

\[
\hat{\xi}_b = \tilde{\xi}_b - \hat{\xi}_b
\]

To establish global boundedness, we define the following Lyapunov function candidate:

\[
V(t) = \sum_{i=1}^{n} \frac{1}{2} c_i z_i^2 + \frac{1}{2} \hat{a}^T \Gamma_a^{-1} \hat{a} + \frac{b_A}{2 \eta} \hat{\phi}^2 + \frac{1}{2} \hat{\xi}_b^T \Gamma_{\xi b}^{-1} \hat{\xi}_b
\]

(33)

The derivative of \( V(t) \) with regard to the time is

\[
\dot{V}(t) = -\sum_{i=1}^{n} c_i z_i^2 + \hat{a}^T Y z_n + \hat{B}(v-) |z_n| - B(v-) z_n
\]

\[
- b_A \bar{\phi} v_1 z_n + \hat{a}^T \Gamma_a^{-1} \hat{a} + \frac{b_A}{\eta} \bar{\phi} \hat{\phi} + \hat{\xi}_b^T \Gamma_{\xi b}^{-1} \hat{\xi}_b
\]

\[
\leq -\sum_{i=1}^{n} c_i z_i^2 + \hat{a}^T (Y z_n + \Gamma_a^{-1} \hat{a}) - \hat{\xi}_b^T (v_- |z_n|)
\]

\[
- \Gamma_{\xi b} \hat{\xi}_b - b_A \bar{\phi} (v_1 z_n - \frac{1}{\eta} \bar{\phi})
\]

\[
= -\sum_{i=1}^{n} c_i z_i^2
\]

Equations (33) and (34) show that \( V(t) \) is nonincreasing. Therefore, \( z_i (i = 1, \ldots, n) \), \( \hat{\phi} \), \( \hat{a} \) and \( \hat{\xi}_b \) are bounded. By utilizing the Lasalle–Yoshizawa theorem in Krstić, Kanellakopoulos,
& Kokotović (1995) to (35), it further follows that \( z_i \to 0 \) (\( i = 1, \ldots, n \)) as \( t \to \infty \), which concludes the tracking error is bounded within the prescribed zone.

5. Simulation results

In this section, the developed prescribed adaptive control approach is demonstrated on a piezo-positioning mechanic system (Zhou et al., 2012) subject to a Bouc–Wen model as in (5). The system is described as

\[
M\ddot{y}(t) + D\dot{y}(t) + Fy(t) = u(t) \tag{35}
\]

\[
u(t) = \Pi[v](t) \tag{36}
\]

where \( y(t) \), \( \dot{y}(t) \), \( \ddot{y}(t) \) denote endpoint displacement, velocity, and acceleration of the piezo-positioning stage, respectively, \( v(t) \) denotes the applied voltage to the piezo-positioning stage, \( M \), \( D \), and \( F \) are the mass, damping, and stiffness coefficients, which are unknown. The actual plant parameters are given as \( M = 1 \) kg, \( D = .15 \) Ns/m, and \( F = 1 \) M/m, and parameters of Bouc–Wen model are chosen as \( A = 1 \), \( \beta = .00231 \), \( \gamma = .00031 \), and \( n = 2.4 \). The control objective is to force the output of the piezo-positioning system to follow the desired signal \( x_d - \sin(2.3t) \) and ensure the transient and steady-state performance of the tracking error within the prescribed function area.

In the simulation, the prescribed performance function is selected as

\[
\rho = (1 - .05)e^{-3t} + .05 \tag{37}
\]

\( \bar{M} = 3 \), \( \bar{M} = -3 \). In the control law and adaptive law, the parameters are selected as \( c_1 = 1 \), \( c_2 = 1 \), \( \Gamma_a = \text{diag}[.11, .99] \), \( n_\phi = 1 \), \( \Gamma_{\phi b} = \text{diag}[10^{-3}, 10^{-6}, 10^{-7}, 10^{-8}, 10] \). The initial parameters are given by \( \hat{a}(0) = [-.4, -.1] \), \( \hat{\phi}(0) = .1 \), \( \hat{\Gamma}_{\phi b} = \text{diag} [.04, -.04, .01, -.01, .01] \). In addition, in the simulation the function \( \text{sgn}(z_n) \) is replaced by \( \text{sat}(z_n) \) to avoid the chattering effect.

![Figure 4. Comparison of state profile of \( x_1 \). The dotted black line indicates the desired trajectory. The solid red line indicates the response of the system output \( x_1 \) with considering Bouc–Wen hysteresis in controller design. The dashed blue line indicates the response of the system output \( x_1 \) without considering Bouc–Wen hysteresis.](image-url)
The simulation verification was conducted by MATLAB/SIMULINK. We use the simulink blocks to build the controller and the controlled system. The desired data, i.e. the controller output, the state of the controlled system and the tracking error, were obtained using Simulink block (named as To workspace). Then, we use the MATLAB

![Figure 5: Tracking error. The solid red line indicates the tracking error with considering Bouc–Wen hysteresis in controller design. The dashed blue line indicates tracking error without considering Bouc–Wen hysteresis. The dotted black lines indicates the prescribed error bounds.](image)

![Figure 6: Controller output. The solid red line indicates the controller output with considering Bouc–Wen hysteresis in controller design. The dashed blue line indicates the controller output without considering Bouc–Wen hysteresis.](image)
function plot to obtain Figures 4–6. Figure 4 shows comparisons of the system output $x_1$ with and without considering the effects of the Bouc–Wen hysteresis by setting $A = 1$, $\xi_p = 0$ in (18). The solid red line indicates the response of the system output with considering Bouc–Wen hysteresis in controller design, which shows that a fairly satisfactory output tracking performance is attained and the tracking error converges to the prescribed zone as shown in Figure 5 (solid red line). In addition, the controller signal, solid red line in Figure 6, is quite smooth and no oscillations occur. While for the performance without considering the Bouc–Wen hysteresis, the tracking error exceeds the prescribed zone (dashed blue line in Figure 5) and control signal (dashed blue line in Figure 6) generates higher control magnitudes and the output always exhibits oscillations, especially at the 13th second, the oscillation (dashed blue line) without considering the Bouc–Wen hysteresis is approximately six times higher than that (solid red line) considering the hysteresis effect. From the comparative results, the developed controller considering Bouc–Wen model shows smoother controller signal and superior steady-state performance than that without considering Bouc–Wen model.

Remark: The proposed control scheme is not limited to the piezo-positioning systems. It can be extended to all smart material-based actuated systems as long as the hysteresis is represented by Bouc–Wen model, such as magnetostrictive-actuated systems and SMA-actuated systems.

6. Conclusion

The Bouc–wen model is widely adopted for description of hysteresis nonlinearities, especially in the smart material-based actuators due to its general nature. However, when hysteresis nonlinearities are described by this model, the major challenge is the corresponding controller design since the general solution for the Bouc–Wen model is not available yet. Therefore, the design of stable controller is a longstanding challenge. The aim of the paper is to provide a controller design approach when the Bouc–Wen model is adopted. Focusing on a class of continuous-time nonlinear dynamic systems preceded by the Bouc–Wen model, a prescribed adaptive control scheme is developed in the paper by fusing an approximate analytical solution with the prescribed adaptive approach. In the developed control scheme, a prescribed performance function and an output error transformation are utilized to guarantee the global stability of the closed-loop system and the transient and steady-state performance of tracking error. The simulation results show that the developed control approach generates superior steady-state performance. We should mention that the proposed design approach can be thought of as an important step towards the development of general control framework for the systems with the Bouc–Wen hysteresis model.

References

Al Janaideh, M., & Krejci, P. (2013). Inverse rate-dependent Prandtl–Ishlinskii model for feedforward compensation of hysteresis in a piezomicropositioning actuator. IEEE/ASME Transactions on Mechatronics, 18, 1498–1507.

Bechlioulis, C. P., & Rovithakis, G. A. (2008). Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance. IEEE Transactions on Automatic Control, 53, 2090–2099.

Bechlioulis, C. P., & Rovithakis, G. A. (2009). Adaptive control with guaranteed transient and steady state tracking error bounds for strict feedback systems. Automatica, 45, 532–538.
Chen, H., Tan, Y., Zhou, X., Dong, R., & Zhang, Y. (2011). Identification of dynamic hysteresis based on Duhem model. *International Conference on Intelligent Computation Technology and Automation (ICICTA)*, pp. 810–814. Shenzhen.

Constantinou, M. C., & Kneifati, M. C. (1988). Dynamics of soil-base-isolated-structure systems. *Journal of Structural Engineering, 114*, 211–221.

Hao, L., & Li, Z. (2010). Modeling and adaptive inverse control of hysteresis and creep in ionic polymer–metal composite actuators. *Smart Materials and Structures, 19*, 025014.

Isidori, A. (1985). *Nonlinear control systems: An introduction*. Berlin-Heidelberg: Springer-Verlag.

Ismail, M., Ikhouane, F., & Rodellar, J. (2009). The hysteresis Bouc–Wen model, a survey. *Archives of Computational Methods in Engineering, 16*, 161–188.

Kozlov, D. V. (2011). Approximate analytical solution of the Bouc–Wen hysteresis model by the Fourier transform. *International Siberian Conference on Control and Communications (SIBCON)*, pp. 76–80. Siberia.

Krstić, M., Kanellakopoulos, I., & Kokotović, P. V. (1995). *Nonlinear and adaptive control design*. New York, NY: Wiley.

Kuhnen, K. (2003). Modeling, identification and compensation of complex hysteretic nonlinearities: A modified Prandtl–Ishlinskii approach. *European Journal of Control, 9*, 407–418.

Krejci, P., & Kuhnen, K. (2001). Inverse control of systems with hysteresis and creep. *IEE Proceedings – Control Theory and Applications, 148*, 185–192.

Lin, C.-J., & Yang, S.-R. (2006). Precise positioning of piezo-actuated stages using hysteresis-observer based control. *Mechatronics, 16*, 417–426.

Rakotondrabe, M. (2011). Bouc–Wen modeling and inverse multiplicative structure to compensate hysteresis nonlinearity in piezoelectric actuators. *IEEE Transactions on Automation Science and Engineering, 8*, 428–431.

Rakotondrabe, M., Janaideh, M., Bienaimé, A., & Xu, Q. (2013). *Smart materials-based actuators at the micro/nano-scale*. New York, NY: Springer.

Spencer, B. F., Jr, Dyke, S. J., Sain, M. K., & Carlson, J. D. (1997). Phenomenological model for magnetorheological dampers. *Journal of Engineering Mechanics, 123*, 230–238.

Su, C.-Y., Stepanenko, Y., Svoboda, J., & Leung, T. P. (2000). Robust adaptive control of a class of nonlinear systems with unknown backlash-like hysteresis. *IEEE Transactions on Automatic Control, 45*, 2427–2432.

Su, C.-Y., Wang, Q., Chen, X., & Subhash, R. (2005). Adaptive variable structure control of a class of nonlinear systems with unknown Prandtl–Ishlinskii hysteresis. *IEEE Transactions on Automatic Control, 50*, 2069–2074.

Tan, X. B., & Iyer, R. V. (2009). Modeling and control of hysteresis. *IEEE Control Systems Magazine, 29*, 26–28.

Zhou, J., Wen, C., & Li, T. (2012). Adaptive output feedback control of uncertain nonlinear systems with hysteresis nonlinearity. *IEEE Transactions on Automatic Control, 57*, 2627–2633.

Zhou, J., Wen, C., & Zhang, Y. (2004). Adaptive backstepping control of a class of uncertain nonlinear systems with unknown backlash-like hysteresis. *IEEE Transactions on Automatic Control, 49*, 1751–1757.