Influence of the incident angle of strain wave on the sensing sensitivity of fiber Bragg grating

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Abstract: In recent years, fiber Bragg grating (FBG) strain sensors have been widely applied to measure strain and relative physical quantities. However, the direction of strain must be along the axial or transverse direction of FBG for many FBG sensors, significantly limiting application. Here, the sensing characteristics of FBG for different incident strain waves were determined, and the influences of the incident angle of static strain, low frequency strain, and high frequency strain on FBG sensitivity were characterized. The construction of an FBG strain sensor that can measure strain with any incident angle is proposed, and additional applications of sensing characteristics are discussed.

Keywords: FBG, strain, incident angle, sensitivity

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1 Introduction
Fiber Bragg grating (FBG) has undergone rapid development in recent years. FBG sensors offer advantages over conventional electrical strain gauges because they are lightweight and can easily be integrated into structures. These sensors also have immunity to electromagnetic interference (EMI) and are ideal for multiplexing [1, 2, 3].

Different kinds of measured physical quantities (such as force, vibration, pressure, and others) can be converted to strain in practical measurements. When FBG senses the strain, the center wavelength $\lambda_B$ of FBG is changed. The relative variance of $\lambda_B$ is decided by the axial strain $\varepsilon$

$$\Delta \lambda_B / \lambda_B = (1 - P_c) \varepsilon$$

where $P_c$ is the elasto-optical coefficient of FBG. For pure silica optical fiber, $P_c \approx 0.22$.

Guo et al. designed a 2D vibration sensor using four FBGs pasted on a stainless steel tube along the circumference using an interval of 90° [4]. Antunes et al. also proposed a biaxial FBG accelerometer which incorporated two FBGs [5]. When the mass vibrated along the main axial direction (x or y), one FBG elongated and the other compressed. The constraint along the z-direction in [4] and [5] was replaced with an optical fiber embedded with a pair of FBGs on the both sides of the mass block to detect the z-direction vibration [6]. Subsequently, additional three-dimen-
sional FBG accelerometers have been proposed [7, 8, 9]. These sensors all exploit the axial property of FBG.

Most studies have been limited to the axial properties of FBG. However, Li et al. presented a 2D FBG vibration sensor based on both the axial and transverse properties of a suspended optical fiber [10]. Fucai Li studied the sensing characteristic of FBG for high frequency vibration (ultrasonic) and proposed that the FBG sensor was severely direction-dependent for ultrasonic signal [11]. Meng proposed the phase shifted FBG signal intensity and the ultrasonic excitation angle presented a cosine curve with two peaks in the middle under the effect of ultrasonic wave from a random angle [12]. Culshaw designed a FBG rosette configured from three FBGs based on the relationship between incident angle and FBG sensitivity and used this design to determine the direction of ultrasonic wave [13].

In most cases, the directions of strain are not known. If the incident strain angle is not in the axial direction of FBG, then the result as calculated by Eq. (1) will be wrong. In some experiments, the sensitivity of FBG for strain may be very low due to an incorrect pasted direction of FBG. Thus, it is necessary to study the sensing characteristics of FBG for strains with different incident angle.

The aim of this study was to determine the influence of different incident angle strain on FBG sensing. The sensing characteristics of FBG for static strain and dynamic strain (including low frequency and high frequency strain) were analyzed and the applications of the findings are discussed. This paper is organized into the following sections: theoretical analysis of FBG characteristics for strain, simulated and experimental analysis, applications, and conclusion.

2 Characteristics analysis of FBG for strain

In the following, the sensing characteristics of FBG under homogeneous axial and transverse strain will be separately analyzed.

2.1 Analysis of FBG under axial strain

Supposing that FBG is only affected by homogeneous axial strain, as shown in Fig. 1, \( \sigma_z = \frac{F}{S} \), \( \sigma_x = \sigma_y = 0 \), and the shear stresses are nonexistent.

According to the principles of material mechanics, the strains can be expressed by Eq. (2):

\[
\begin{bmatrix}
  e_x \\
  e_y \\
  e_z 
\end{bmatrix}
= \begin{bmatrix}
  -\nu \frac{E}{S\lambda} \\
  -\nu \frac{E}{S\lambda} \\
  \frac{E}{S\lambda}
\end{bmatrix}
\]

where, \( \nu \), \( E \) and \( S \) are the Poisson ratio, elasticity modulus and sectional area of optical fiber, respectively.

The variance of \( \lambda_B \) is determined by the variance of effective refractive index \( \Delta n_{eff} \) and the variance of the FBG grating period \( \Delta \lambda \):

\[
\frac{\Delta \lambda_B}{\lambda_B} = \frac{2n_{eff} \cdot \Delta \lambda + 2n_{eff} \cdot A}{2n_{eff} \cdot A}
\]

\[
\frac{\Delta \lambda_B}{\lambda_B} = \frac{2n_{eff} \cdot \Delta \lambda + 2n_{eff} \cdot A}{2n_{eff} \cdot A}
\]
\[
\begin{align*}
\nu_{\text{eff}} & = \frac{1}{C_1 n_{\text{eff}}^2} \left( \frac{1}{C_0 n_{\text{eff}}^2} \right)^{1/2} p_{12} \frac{V}{L} \left( p_{11} + p_{12} \right) \varepsilon_z \\
\Delta \lambda & = \varepsilon_z \\
\Delta \lambda_b & = \left( 1 - \frac{n_{\text{eff}}^2}{2} \right) \left( p_{12} - v(p_{11} + p_{12}) \right) \varepsilon_z 
\end{align*}
\]

Where, \( p_{11} \) and \( p_{12} \) are the axial and transverse coefficients of photo-elastic tensor respectively.

Let \( K_e = 1 - \frac{n_{\text{eff}}^2}{2} \left( p_{12} - v(p_{11} + p_{12}) \right) \), so

\[
\frac{\Delta \lambda_b}{\lambda_B} = K_e \varepsilon_z 
\]

It can be found from Eq. (6) that the relative variance of \( \lambda_B \) is directly proportional to the axial strain \( \varepsilon_z \). If FBG is elongated axially, then \( \lambda_B \) will increase, but \( \lambda_B \) will decrease if FBG is compressed.

### 2.2 Analysis of FBG under transverse strain

When FBG is affected by transverse strain, as shown in Fig. 2, the stress of the point \((0, 0)\) because \( a \ll D \) [14].

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z
\end{bmatrix} = \begin{bmatrix}
\frac{2V}{\pi LD} & -\frac{6V}{\pi LD} \\
-\frac{6V}{\pi LD} & \frac{2V}{\pi LD}
\end{bmatrix}
\]

\[
\varepsilon_x = \frac{1}{2} \left[ \sigma_x - v(\sigma_y + \sigma_z) \right] \\
\varepsilon_y = \frac{1}{2} \left[ \sigma_y - v(\sigma_x + \sigma_z) \right] \\
\varepsilon_z = \frac{1}{2} \left[ \sigma_z - v(\sigma_x + \sigma_y) \right]
\]

\[
\begin{align*}
(\Delta n_{\text{eff}})_x &= -\frac{n_{\text{eff}}^2}{2} \left( (p_{11} - 2vp_{12})\sigma_x + [(1 - v)p_{12} - vp_{11}] (\sigma_y + \sigma_z) \right) \\
(\Delta n_{\text{eff}})_y &= -\frac{n_{\text{eff}}^2}{2} \left( (p_{11} - 2vp_{12})\sigma_y + [(1 - v)p_{12} - vp_{11}] (\sigma_x + \sigma_z) \right) \\
(\Delta n_{\text{eff}})_z &= -\frac{n_{\text{eff}}^2}{2} \left( (p_{11} - 2vp_{12})\sigma_z + [(1 - v)p_{12} - vp_{11}] (\sigma_x + \sigma_y) \right)
\end{align*}
\]
so the reflectance spectrum of FBG under F will be split into two harmonic peaks, one along the x-polarization direction and the other in the y-polarization direction.

\[
\begin{align*}
\Delta \lambda_{Bx} &= \left( \frac{\Delta n_{eff}}{n_{eff}} \right) x + \varepsilon z \lambda_B \\
\Delta \lambda_{By} &= \left( \frac{\Delta n_{eff}}{n_{eff}} \right) y + \varepsilon z \lambda_B 
\end{align*}
\]

Supposing that \( \sigma_z = 0 \), then the \( \Delta \lambda_{Bx}/\lambda_B \) and \( \Delta \lambda_{By}/\lambda_B \) of single mode silica fiber can be calculated by using Eq. (9) and (10):

\[
\begin{align*}
\Delta \lambda_{Bx}/\lambda_B &\approx 6.56 \times 10^{-6} \cdot F \\
\Delta \lambda_{By}/\lambda_B &\approx 1.56 \times 10^{-6} \cdot F
\end{align*}
\]

The sensitivity of FBG under axial strain \( K_c = 0.784 \), so the sensitivity \( K_F \) is equal to:

\[
K_F = \frac{K_c}{SE} = \frac{0.784}{3.14 \times (62.5 \times 10^{-6})^2 \times 74.52 \times 10^9} \approx 8.58 \times 10^{-4}
\]

The sensitivity of FBG under axial strain is much higher than that under transverse strain. So in the following analysis, the axial strain is mainly discussed.

2.3 Sensitivity analysis of FBG under different direction forces

The strains of different sections of a triangle cantilever beam are equal. Therefore, it is easy to paste FBGs on this kind of beam in such a way to insure their strains are the same. Let the length and width directions of the beam be the x and y directions, respectively, as shown in Fig. 3. A FBG is pasted on the upper surface of the beam and the angle between the FBG axial direction and the x axis is \( \theta \) (the meaning of \( \theta \) is the same in the following simulated and experimental analysis). When the free end of cantilever beam suffers a force in the up to down direction, the upper surface of the beam is stretched in the x direction and stressed in the y direction.

According to the poisson effect,

\[
e_y = -\mu e_x
\]

Therefore, the total axial strain of FBG is:

\[
\varepsilon = e_x \cos \theta + e_y \sin \theta = e_x (\cos \theta - \mu \sin \theta)
\]

If the cantilever beam is made of carbon steel, \( \mu \approx 0.3 \). Thus: (1) the sensitivity is greatest when \( \theta = 0^\circ \), (2) the sensitivity of \( \theta = 90^\circ \) is \(-0.3\) times as much as the sensitivity of \( \theta = 0^\circ \), and (3) \( \varepsilon \approx 0 \) when \( \theta = 73^\circ \).
3 Simulated and experimental analysis

3.1 Finite element simulation

The simulated model is shown in Fig. 4. The cantilever beam size is: length \( L = 60 \) mm, maximum width \( b = 24 \) mm, thickness \( h = 1 \) mm; mass: diameter = 8 mm, height = 4 mm; glue: length = 12 mm, width = 0.4 mm, thickness = 0.2 mm; the effective length of the fiber is 12 mm, diameter = 0.125 mm, and all of the fiber is embedded in the glue. The parameters of the model are listed in Table I.

Let \( \theta \) be the variable parameter “theta”, and utilizing the function “Parametric Sweep” in the study, the set range of theta is \((0, 10, 90)\). Apply “Edge Load” on the upper surface of mass, and perform stationary studies for different loads. Fig. 5 shows the relationship between the strain of FBG and the load under different theta values. The slopes of the lines shown in Fig. 5 were calculated and marked in Fig. 6 to draw the \( \theta \)-sensitivity curve.

![Fig. 4. Simulated cantilever beam model](image)

### Table I. Parameters of the cantilever beam and FBG model

|                  | Steel | Fiber | Glue | Mass  |
|------------------|-------|-------|------|-------|
| **Density (kg/m\(^3\))** | 7850  | 2457  | 1100 | 8960  |
| **Poisson ratio**         | 0.3   | 0.17  | 0.35 | 0.35  |
| **E (GPa)**              | 205   | 74.52 | 3.3  | 110   |

3.2 Experimental analysis

Experiments were performed to determine the sensing characteristics of FBG under the effects of static and dynamic strain including low frequency and high frequency vibration.

3.2.1 Static strain experiment

In the static strain experiment, as shown in Fig. 7, ten FBGs were pasted on the upper surface of a triangle cantilever beam, and the \( \theta \) values of each FBG were 0°, 10°, 20°...90°, respectively. In order to reduce the attenuation caused by the weld of fiber, 5 FBGs were cascaded in a string. Two channels of FBG interrogator with a demodulation frequency that can reach 4 kHz were utilized to demodulate the two separate FBGs strings.

The center wavelength data acquisition continued for 10 seconds in every measurement. The relative variances of \( \lambda_B \) are marked in Fig. 8, and the normalized sensitivity of average values are shown in Fig. 9. Overall, the experimental and simulated \( \theta \)-sensitivity curves were consistent.
The fitting equation Eq. (13) for static force can be obtained from the data shown in Fig. 9:

\[ f(\theta) = A[a_0 + a_1\cos(\omega\theta) + a_2\sin(\omega\theta)] \]  

(13)
where, $a_0 = 0.3494$, $a_1 = 0.6544$, $a_2 = -0.06305$, and $\omega = 0.03321$. $A$ is the maximum amplitude.

In practical application, the maximum amplitude $A$ is unknown.

### 3.2.2 Low frequency dynamic strain experiment

The experiments of dynamic low frequency force were performed similarly to the static force experiments, as illustrated in Fig. 10. A DG1022 signal generator was used to control and drive an exciter to produce dynamic strain inputs at different frequencies. The amplitude of the stimulation signals was set as 5 V, and the frequency ranged from 10 Hz to 100 Hz. The monitored voltage of the power amplifier was 6 V, and the sampling frequency was set as 4 kHz.

Fig. 11 shows the responses of the FBGs under 20 Hz vibrations. The responses of $\theta = 0^\circ$ and $\theta = 40^\circ$ FBGs were in phase but the phases of responses of $\theta = 80^\circ$ and $\theta = 40^\circ$ FBGs were opposite. This result indicates that the variances of $\lambda_B$ are
3.2.3 High frequency dynamic strain experiment

High frequency dynamic strain was generated by a piezoelectric patch. The excitation signal was a 3-cycle sine wave modulated by a Hanning window, the amplitude was amplified to 80 V by a voltage amplifier, and the frequency was 299 kHz. The FBG interrogator used in the previous experiments cannot demodulate the high frequency signal, so a different tunable laser demodulation system was adopted, as shown in Fig. 13. A FBG with 5 mm grating was chosen to sense the high frequency vibration based on previous research [15]. To avoid the effect of performance difference of FBG, only one FBG was used, and 9 piezoelectric patches were pasted on different positions of an aluminum thin plate (300 mm × 300 mm × 1 mm). The distances between each piezoelectric patch and the FBG were the same, and the range of $\theta$ was from $0^\circ$ to $90^\circ$.

The high frequency vibration is called ultrasonic and the interaction of longitudinal wave and transverse wave in the edge of the thin plate structure caused Lamb wave. Lamb wave has the frequency dispersion phenomenon and includes symmetric and anti-symmetric mode waves in which particles vibrate as Fig. 14 [16]. The frequency dispersion curves of Lamb wave in the aluminum plate are drawn in Fig. 15. Because the product of the frequency and plate thickness $fd$ was 0.299 MHz·mm, there were only S0 and A0 mode waves in the experiments. Three independent measurements were made, and responses of FBG with different $\theta$ values are shown in Fig. 16.

The peak-to-peak value $V_{p-p}$ of the S0 and A0 mode waves are shown in Fig. 17(a) and (b). It is observed that the sensing characteristic of the FBG sensor
for high frequency strain was different from that for static and low frequency strain and the sensing of FBG for S0 and A0 waves were also very different. The sensitivity difference between S0 and A0 waves is due to their different particle vibration forms. For the upper surface, the displacement caused by S0 wave is mainly in-plane, and that caused by A0 wave includes both in-plane and out-plane displacement. The curve of the $\theta$-Vp-p ratio of S0/A0 calculated based on the data in Fig. 17(a) and (b) is shown in Fig. 17(c). According to the curve, the incident angle of the Lamb wave can be determined by the response of the two FBGs.

### 4 Application

According to the above results, a strain sensor that consists of two FBGs is designed, as shown in Fig. 18. Two FBGs are pasted on the substrate and the angle between these FBGs is set as 45°. This design ensures that at least one FBG has high sensitivity. The amplitude ratio of the response signal of the two FBGs can be obtained under the effect of arbitrary angle strain. The incident angle of the
strain can be calculated by the fitting straight line equation. For static, low frequency, and high frequency dynamic strain, the fitting line equations are different. Using static strain as an example, the amplitude of FBG1 and FBG2 responses $A_1$ and $A_2$ can be expressed as:

\[ A_1 = f(\theta) = A[a_0 + a_1 \cos(\omega \theta) + a_2 \sin(\omega \theta)] \] \hspace{1cm} (14)

\[ A_2 = f(\theta + 45^\circ) = A[a_0 + a_1 \cos(\omega (\theta + 45^\circ)) + a_2 \sin(\omega (\theta + 45^\circ))] \] \hspace{1cm} (15)

$A_1$ and $A_2$ can be obtained experimentally, so $\theta$ can be calculated by Eq. (14) and Eq. (15) and the incident angle of strain on FBG can be determined.

For high frequency dynamic strain, the $\theta$-sensitivity curve can be used to determine the direction of the reflected wave caused by the presence of a defect and indicate the position of the defect.

5 Conclusion

This study examined the sensing characteristics of FBG for static strain, low frequency strain, and high frequency strain with different incident angles by theoretical, simulated, and experimental analysis. The conclusions can be drawn as follows:

(1) $\theta$-sensitivity curves of FBG for static strain, low frequency strain, and high frequency strain were obtained. The sensing features for static strain and low frequency are similar but the sensing features for high frequency are quite different.
(2) The influences of incident angle on the sensitivity of FBGs were described by the fitting lines of $\theta$-sensitivity curves.
(3) A strain FBG sensor that can measure any incident angle strain was proposed.

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