In a quantum ring structure of nanoscale dimension, the confined electrons exhibit a topological quantum coherence, the celebrated Aharonov–Bohm (AB) effect [1]. The characteristics of the energy spectrum (non-interacting) for a ring-shaped geometry, pierced by a magnetic flux \(\Phi\), correspond to a periodically shifted parabola with period of one flux quantum, \(\Phi_0 = \hbar/e\) [2]. All physical properties of this system, most notably, the persistent current (magnetization) [3] and optical transitions [4], have this periodicity. Experimental observations of the AB effect were reported in metal rings [5] and in semiconductor rings [6]. Persistent currents were also measured in metal [7] and semiconductor [8] rings. The role of electron–electron interactions on the AB effect was explored systematically via the exact diagonalization scheme for few interacting electrons in a quantum ring [9, 10]. Interactions were found to introduce fractional periodicity of the AB oscillations [11]. Major advances in fabrication of semiconductor nanostructures have resulted in creation of nanoscale quantum rings in e.g. GaAs and InAs systems containing only a few electrons [12, 13]. In those experiments, the AB effect manifests itself in optical transitions [11, 14], and magneto-conductance [15]. The electron energy spectrum in a ring geometry has also been measured [16]. Those experiments have confirmed the theoretical predictions about the influence of electron–electron interactions on the persistent current, that was previously predicted [10, 12, 13]. The AB effect has also been studied in Dirac materials, such as graphene [17], both theoretically [18] and experimentally [19]. One major advantage of all these nanoscale quantum rings is that here the ring size and the number of electrons in it can be externally controlled [12, 13]. There are several recent works that propose the controlling of the AB effect in semiconductor quantum rings via the external fields, such as the lateral electric field (see e.g. [20–22]). The problem of controlling the AB oscillations in the QRs is still an open problem that can play an important role in fundamental and applied researches involving quantum rings.

In all these years, for investigations of nanoscale quantum structures, such as the quantum dots (QDs) (or, the artificial atoms) [23, 24] and quantum rings (QRs), the materials of choice had been primarily the conventional semiconductors, viz. the GaAs or InAs heterojunctions, where the high-mobility
two-dimensional electron gas (2DEG) was quantum confined. In recent years, very exciting developments have taken place with the creation of high-mobility 2DEG in heterostructures involving insulating complex oxides. Unlike in traditional semiconductors, electrons in these systems are strongly correlated [25]. These should then exhibit effects ranging from strong electron correlations, magnetism, interface superconductivity, tunable metal-insulator transitions, among others, and of course, the exciting possibility of all-oxide electronic devices. Many surprising results were found in the fractional quantum Hall states [26] discovered in the MgZnO/ZnO heterojunction [27, 28]. Preparation of various nanostructures, such as nanorings, nanobelts, etc have been reported in ZnO [29]. Here we report on the AB effect in a ZnO quantum rings and compare that in a conventional semiconductor QR, namely in GaAs. Quite remarkably, we found that while in the non-interacting case the AB effect remains unaltered for both systems, the combination of strong Zeeman interaction and the strong Coulomb interaction, two signature effects of the ZnO 2DEG, make the effect disappear in ZnO QR for electron number larger than one.

We consider here a two-dimensional QR with inner radius \( R_1 \) and outer radius \( R_2 \) having cylindrical symmetry, containing few electrons, in a magnetic field applied in the growth direction. The Hamiltonian of our system then is

\[
H = \sum_{i}^{N_e} H_{SP} + \frac{1}{2} \sum_{ij} V_{ij},
\]

(1)

where \( N_e \) is the number of electrons in the QR, \( V_{ij} = e^2/\epsilon |r_i - r_j| \) is the Coulomb interaction term, with dielectric constant of the ring material \( \epsilon \), and \( H_{SP} \) is the single-particle Hamiltonian in the presence of an external perpendicular magnetic field.

\[
H_{SP} = \frac{1}{2m} \left( \mathbf{p} - \frac{e}{c} \mathbf{A} \right)^2 + V_{conf}(r) + \frac{1}{2} g \mu_B B \sigma_z,
\]

(2)

where \( \mathbf{A} = B/(2\pi \gamma x, y, 0) \) is the vector potential of the magnetic field, \( m \) is the electron effective mass. We chose the confinement potential of the QR with infinitely high borders: \( V_{conf}(r) = 0 \), if \( R_1 \leq r \leq R_2 \) and infinity outside of the QR. This choice of the confinement potential is suitable for ZnO/MgZnO heterostructures due to the large values of the conduction band offset and the electron effective mass [30]. The last term of (2) is the Zeeman interaction.

We take as basis states the eigenfunctions of \( H_{SP} \) for \( B = 0 \). The eigenfunctions of this Hamiltonian then have the form [31]

\[
\phi_{nl}(r, \theta) = \frac{C}{\sqrt{2\pi}} e^{i \theta} \left( J_0(\gamma_n r) - \frac{J_1(\gamma_n R_1)}{Y_0(\gamma_n R_1)} Y_1(\gamma_n r) \right),
\]

(3)

where \( J(r) \) and \( Y(r) \) are Bessel functions of the first and second kind respectively, \( \gamma_n = 2mE_{nl}/\hbar^2 \), where \( E_{nl} \) are the eigenstates defined by the boundary conditions of the single-electron wave functions at the inner and outer radii of the ring \( R_1 \) and \( R_2 \), the constant \( C \) is determined from the normalization integral, and \( n \) and \( l \) are the radial and angular quantum numbers respectively. In order to evaluate the energy spectrum of the many-electron system, we need to diagonalize the matrix of the Hamiltonian (1) in a basis of the Slater determinants constructed from the single-electron wave functions [4, 32]

\[
I = \langle f | \sum_{i=1}^{N} re^{i \theta} | i \rangle
\]

(4)

when the transition goes from the initial \( N \)-particle state \( | i \rangle \) to the final state \( | f \rangle \). In this work we always consider \(| i \rangle \) to be the \( N \)-particle ground state. To evaluate (4) we need to calculate the dipole matrix elements between the one electron states \(| n, l \rangle \) and \(| n', l' \rangle \).

\[
M = \int_{R_1}^{R_2} \int_0^{2\pi} \phi_n^*(r, \theta)(r e^{i \theta})^p \phi_{n'}(r, \theta) r dr d\theta.
\]

(5)

After the angular integration we arrive at the optical transition selection rule for the total angular momentum \( L \).

The numerical studies were carried out for the ZnO QR with parameters \( m = 0.24 m_0 \), \( g = 4.3 \), \( \epsilon = 8.5 \) [30]. For the purpose of comparison we have also presented similar studies for the GaAs QR with parameters \( m = 0.067 m_0 \), \( g = -0.44 \), \( \epsilon = 13.18 \) respectively [32]. We consider here the two QRs of same sizes with radii \( R_1 = 10 \text{ nm} \) and \( R_2 = 40 \text{ nm} \).

The low-lying energy levels of the ZnO QR with one and two electrons are presented in figure 1 as a function of the magnetic field \( B \). For comparison similar results are also presented for the GaAs QR in figures 1(c) and (d). In all these figures different colors correspond to different values of the total angular momentum \( L \) of the electrons. In the QR with only one electron in both systems, the ground state changes periodically with increasing magnetic field (figures 1(a) and (c)). This is the direct signature of the AB effect in a QR. For the ZnO QR the energy eigenvalues are lower due to the larger value of the electron effective mass. Additionally, the states with different spin are highly split due to the larger value of the \( g \)-factor for ZnO. However, for the non-interacting electrons the AB effect survives in both systems.

For QRs with two interacting electrons, there are several substantial differences between the energy spectra of the ZnO and GaAs QRs. For instance, in the GaAs QR we see the usual and well observed AB oscillations due to level crossings between the singlet and triplet ground states, and for each crossing of the two-electron ground state the total angular momentum \( L \) changes by unity. On the other hand, for the ZnO QR containing two electrons (figure 1(b)), due to the combined effect of the strong Zeeman splitting and the strong Coulomb interaction, the singlet-triplet crossings disappear from the ground state. Interestingly, the periodic crossings happen only in the excited states. For small values of the magnetic field the ground state is a singlet with \( L = 0 \) and the total electron spin \( S = 0 \). With an increase of the magnetic field the ground state changes to a triplet with \( L = -1 \) and \( S = -1 \). With further increase of the magnetic field all the observed crossings of the ground state correspond to
triplet-triplet transitions between the states with odd number of total angular momentum \( (L=1, 3, 5,...) \). These interesting and unexpected results will manifest themselves in optical transitions in the ZnO QR.

The dipole allowed optical transition energies are presented in figure 2 as a function of the magnetic field for the ZnO ((a) and (c)) and the GaAs ((b) and (d)) QRs containing one and two electrons respectively. Different colors in figure 2 correspond to the value of the ground state angular momentum (see figure 1) of the optical transition and the sizes of the points are proportional to the intensity of the optical transitions.

For the QRs containing only one electron for both materials we can see the familiar picture: periodic optical AB oscillations. Comparing figures 2(a) and (b) we notice that although the strong Zeeman effect changes the one-electron energy spectra of the ZnO QR, it does not change the periodicity of the optical AB oscillations. In the case of the QRs with two electrons again we see considerable differences between the two systems. In the case of the two-electron GaAs QR, we see the periodic optical AB oscillations with the period that is half the flux quantum, which is a well-known result [11, 15]. In contrast, for the two-electron ZnO QR we notice an aperiodic behavior of optical AB oscillations. The first oscillation which corresponds to the singlet-triplet transition from the state with \( L = 0 \) (the \( 1S_0 \) state) to the state with \( L = -1 \) (the \( 1P_1 \) state) has a smaller period compared to the other oscillations that correspond to transitions between the triplet states with odd angular momentum. The period of these triplet–triplet oscillations is almost equal to the period of the single electron case. This unexpected effect is caused by the different properties of the energy spectra of the two-electron ZnO QR discussed above and can be explained by the combined effect of the strong Zeeman interaction and the strong electron–electron interaction in the ZnO.

In figures 3(a) and (b) the low-lying energy levels for the ZnO and GaAs QRs containing three electrons are presented as a function of the magnetic field \( B \). For the three-electron GaAs QR we again note the periodic ground state transitions and during each transition the ground state angular momentum changes by one. In contrast to that, for the three-electron ZnO QR we notice an aperiodic behavior of optical AB oscillations. The first oscillation which corresponds to the singlet-triplet transition from the state with \( L = 0 \) (the \( 1S_0 \) state) to the state with \( L = -1 \) (the \( 1P_1 \) state) has a smaller period compared to the other oscillations that correspond to transitions between the triplet states with odd angular momentum. The period of these triplet–triplet transitions is almost equal to the period of the single electron case. This unexpected effect is caused by the different properties of the energy spectra of the two-electron ZnO QR discussed above and can be explained by the combined effect of the strong Zeeman interaction and the strong electron–electron interaction in the ZnO.

![Figure 1.](image1.png)

Figure 1. The low-lying energy levels versus the magnetic field for (a) ZnO QR with one electron, (b) the ZnO QR with two electrons, (c) the GaAs QR with one electron and (d) the GaAs QR with two electrons. Different colors correspond to different values of the total angular momentum \( L \).

![Figure 2.](image2.png)

Figure 2. Dipole allowed optical transition energies versus the magnetic field, for (a) the ZnO QR with one electron, (b) the GaAs QR with one electron, (c) the ZnO QR with two electrons and (d) the GaAs QR with two electrons. The size of the colored dots is proportional to the intensity of the calculated optical transitions.
Therefore we can state that in the range of the magnetic field considered here the Aharonov–Bohm effect disappears. The corresponding optical transition energies for the three-electron ZnO QR are shown in figure 3(c). That figure clearly illustrates the disappearance of the optical Aharonov–Bohm oscillations in a ZnO quantum ring.

For a better understanding of the role of Coulomb interaction in the remarkable results for the ZnO QR shown above, we have considered also the case of two-electron ZnO QR with a screened Coulomb interaction. We have used the Yukawa-type screened Coulomb interaction potential $V_{\text{scr}} = \lambda/e |\mathbf{r}_i - \mathbf{r}_j|$, where $\lambda$ is the screening parameter. The energy spectra and the optical transition energies are presented in figures 4(a) and (b) respectively for $\lambda = 1\text{nm}^{-1}$. The screened interaction shows that the first optical oscillation is observed for almost the same range of the magnetic field, as for the two-electron GaAs, but the periodicity is still absent. This clearly illustrates that interaction alone can not destroy the AB effect, but a strong Zeeman interaction is also important for that.

To summarize, we have studied the electronic states and optical transitions of a ZnO quantum ring containing a few interacting electrons in an applied magnetic field via the exact diagonalization scheme. These results are also compared with similar quantities for a conventional GaAs ring. We have found that the strong Zeeman interaction and the strong electron–electron Coulomb interaction, two major characteristics of the ZnO system, exert a profound influence on the electron states and as a consequence, on the optical properties of the ring. In particular, we find that the AB effect is strongly electron number dependent. Our results indicate that in the case of two interacting electrons in the ZnO ring, the AB oscillations become aperiodic. For three electrons (interacting) we have found that the AB oscillations actually disappear. These unusual properties of the ZnO QR are explained in terms of the energy level crossings that are very different from those of the conventional semiconductor QRs, such as for the GaAs. The AB effect (and thereby the persistent current) in a ZnO quantum ring can therefore be controlled by varying the electron number.

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