Muon pair production via photon-induced scattering at the CLIC in models with extra dimensions

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Abstract

The photon-induced dimuon production $e^+e^- \rightarrow e^+\gamma\gamma e^- \rightarrow e^+\mu^+\mu^-e^-$ at the CLIC is studied in the framework of three models with extra dimensions. The electron beam energies 750 GeV and 1500 GeV are considered. The total cross sections are calculated depending on the minimal transverse momenta of the final muons. The sensitivity bounds on the parameters of the models are obtained as functions of the CLIC integrated luminosity.
I. INTRODUCTION

The Standard Model (SM) has been validated by existing experiments including the LHC data and has passed many tests very successfully at the electroweak energy scale. However, many issues remain open in SM. One of the most fundamental of these problems is the hierarchy problem. This open question involves the large energy gap between the electroweak scale and the gravity scale. One of the fundamental approaches to the solution of the hierarchy problem is the theories that suggest the existence of extra dimensions (EDs). Recently, many articles have been published on these theories, which attracted great attention.

Scientists expect the LHC to elucidate many unanswered physics problems. Nevertheless, this type of collider enables precision measurements due to the nature of the proton-proton collisions. Whereas, interactions of the electrons and positrons with high-luminosity can provide higher precision than the proton-proton interaction with too much background. Compact Linear Collider (CLIC) is one of the most qualified $e^+e^-$ colliders. CLIC includes normal conducting accelerating cavities and two-beam acceleration [1]. It is used in a novel two-beam acceleration technique. In this way, accelerating gradients could be obtained as 100 MV/m. To work CLIC at maximum efficiency, three energy stages are planned [2]. First one is at $\sqrt{s} = 380$ GeV and can reach the integrated luminosity $L = 1000$ fb$^{-1}$. This era covers Higgs boson, top and gauge sectors. It is possible to search for such SM particles with the high precision [3]. Second operation is at $\sqrt{s} = 1500$ GeV. This stage is the highest center-of-mass energy available with a single CLIC drive beam complex. In the second stage, CLIC can give clues to beyond the SM physics. Moreover, detailed Higgs properties such as the Higgs self-coupling and the top-Yukawa coupling and rare Higgs decay channels could be studied [4]. In this stage, maximum integrated luminosity value is 2500 fb$^{-1}$. The last stage is that CLIC has reached its maximum center-of-mass energy value $\sqrt{s} = 3000$ GeV and integrated luminosity value $L = 5000$ fb$^{-1}$. It is possible to do the most precise examinations of the SM. Moreover, it is enable to discovery beyond the SM heavy particles of mass greater than 1500 GeV [3]. The CLIC potential for new physics is presented in [5].

At the CLIC, as with all linear accelerators, $e\gamma$ and $\gamma\gamma$ interactions are possible. Such interactions can be formed in two ways: Compton backscattering [6-8] and photon-induced reactions [9-11]. In photon-induced reactions, $\gamma\gamma$ and $e\gamma$ interactions can occur sponta-
neously, unlike the Compton backscattering process. Therefore, photon-induced reactions are much more useful than the Compton backscattering procedure search for new physics beyond the SM. This type of interactions can be studied by the Weizsäcker-Williams approximation (WWA). There are great advantages of using the WWA. Numerical calculations can be easily performed using simple formulas. In addition, this method is useful in experimental searches. Because it allows us to determine events number for the process $\gamma\gamma \rightarrow X$ approximately with use of the $e^-e^+ \rightarrow e^-Xe^+$ process [12]. Moreover, photon induced reactions have very clean backgrounds since these reactions do not involve interference with weak and strong interactions. There are many phenomenological and experimental studies in the literature on photon-induced process [13–20].

In WWA, the photons have very small virtuality. Therefore, scattered angels of the emitting photons from the electrons path along the actual beam trajectory should be very small. In this approximation, the photon spectrum in incoming electron with the energy $E$ is given by the formula [9] (see also [21])

$$\frac{dN}{dx} = \frac{\alpha_{em}}{\pi} \left[ \frac{1 - x + x^2/2}{2} \ln \frac{Q_{\text{max}}^2}{Q_{\text{min}}^2} - \frac{m_e^2 x}{2} \left( 1 - \frac{Q_{\text{min}}^2}{Q_{\text{max}}^2} \right) \right], \quad (1)$$

where $x = E_\gamma/E$ is the energy fraction of the photon, $m_e$ is the electron mass, $\alpha_{em}$ is the fine structure constant, and

$$Q_{\text{min}}^2 = \frac{m_e^2 x^2}{1 - x}, \quad Q_{\text{max}}^2 = 2 \text{ GeV}^2. \quad (2)$$

In the photon-induced collisions, the luminosity spectrum $dL^{\gamma\gamma}/dW$ can be found with using WWA as follows

$$\frac{dL^{\gamma\gamma}}{dW} = \int_{y_{\text{min}}}^{y_{\text{max}}} dy \frac{W^2}{2y} f_1(\frac{W^2}{4y}, Q_1^2) f_2(y, Q_2^2), \quad (3)$$

where $f_i = dN/dx_i$, $i = 1, 2$,

$$y_{\text{min}} = \frac{W^2}{4E(E - m_e)}, \quad y_{\text{max}} = 1 - \frac{m_e}{E}. \quad (4)$$

The cross section for the process $e^+e^- \rightarrow e^+\gamma\gamma e^- \rightarrow e^+\mu^+\mu^- e^-$ is obtained by integrating subprocess cross section $d\hat{\sigma}_{\gamma\gamma\rightarrow\mu^+\mu^-}(W)$ over the photon luminosity spectrum

$$d\sigma = \int_{W_{\text{min}}}^{W_{\text{max}}} dW \frac{dL^{\gamma\gamma}}{dW} d\hat{\sigma}_{\gamma\gamma\rightarrow\mu^+\mu^-}(W), \quad (5)$$

\[^{1}\text{When a small-angle cut is applied to the outgoing electron, a modification of this formula is needed [22].}\]
where
\[ W_{\text{min}} = 2p_{\perp \text{min}}, \quad W_{\text{max}} = 2(E - m_e), \]
(6)
and \( p_{\perp \text{min}} \) is the minimal transverse momentum of the final muons.

In the present paper, we examine the potential of the photon-induced process \( e^+e^- \rightarrow e^+\gamma e^- \rightarrow e^+\mu^+\mu^-e^- \) at the CLIC in the framework of three models with EDs. Both flat and warped metrics of the space-time are considered.

II. SCENARIO WITH EXTRA DIMENSIONS AND FLAT METRIC

One of promising possibilities to go beyond the SM is to consider a theory in a space-time with extra spatial EDs. Such an approach is motivated by the (super)string theory \cite{24}. One of the main goals of the theories with EDs is to explain the hierarchy relation between the electromagnetic and Planck scales. In the model proposed by Arkani-Hamed, Dimopolous, Dvali and Antoniadis \cite{25}-\cite{27}, called ADD, this relation looks like
\[ \bar{M}_{\text{Pl}}^2 = V_d \bar{M}_D^{d+2}, \]
(7)
where \( d \) is the number of EDs, \( V_d = (2\pi R_c)^d \) is the volume of compact EDs with the radius \( R_c \), \( \bar{M}_{\text{Pl}} = M_{\text{Pl}}/\sqrt{8\pi} \) is the reduced Planck mass, and \( \bar{M}_D \) is the reduced fundamental gravity scale in \( D = 4 + d \) dimensions, \( \bar{M}_D = M_D/(2\pi)^{d/(d+2)} \). It is assumed that the fundamental gravity scale \( M_D \) is in the TeV region. The huge gap between the Planck and TeV scales can be justified by the large value of \( V_d \) (so-called “large EDs”).

The masses of the Kaluza-Klein (KK) gravitons in the ADD model are given by the formula \cite{25}-\cite{27}
\[ m_n = \frac{n}{R_c}, \quad n = \sqrt{n_1^2 + n_2^2 + \cdots n_d^2}, \]
(8)
where \( n_i = 0,1,\ldots (i = 1,2,\ldots d) \). We see that in the scenario with large EDs the mass splitting \( \Delta m_{KK} = 1/R_c \) is very small. Thus, the mass spectrum of the gravitons can be regarded as continuous.
III. SCENARIO WITH ONE WARPED EXTRA DIMENSION

The Randall-Sundrum (RS) scenario with one ED and warped metric is based on the following background metric \[ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2, \] (9)
where $\eta_{\mu\nu}$ is the Minkowski tensor with the signature $(+, - , - , -)$, and $y$ is a compactified extra coordinate. The periodicity condition $y = y + 2\pi r_c$ is imposed, and the points $(x_\mu, y)$ and $(x_\mu, -y)$ are identified. Thus, we obtain a model of gravity in a slice of the AdS$_5$ space-time compactified to the orbifold $S^1/Z_2$ with the size $\pi r_c$. Since this orbifold has two fixed points, $y = 0$ and $y = \pi r_c$, two branes can be put at these points. They are called Planck and TeV brane, respectively. All SM fields are assumed to live on the TeV brane.

The classical action of the RS scenario looks like \[S = \int d^4x \int_{-\pi r_c}^{\pi r_c} dy \sqrt{G} (2\bar{M}_5^3 \mathcal{R} - \Lambda) + \int d^4x \sqrt{|g^{(1)}|} (\mathcal{L}_1 - \Lambda_1) + \int d^4x \sqrt{|g^{(2)}|} (\mathcal{L}_2 - \Lambda_2). \] (10)
Here $G_{MN}(x, y)$ is the 5-dimensional metric, $M, N = 0, 1, 2, 3, 4$. The quantities
\[g^{(1)}_{\mu\nu}(x) = G_{\mu\nu}(x, y = 0), \quad g^{(2)}_{\mu\nu}(x) = G_{\mu\nu}(x, y = \pi r_c) \] (11)
are induced metrics on the branes, $\mu = 0, 1, 2, 3$. $\Lambda$ is a five-dimensional cosmological constant, while $\Lambda_1$ and $\Lambda_2$ are tensions on the branes. $\mathcal{L}_1, \mathcal{L}_2$ are brane Lagrangians, and $G = \det(G_{MN})$, $g^{(i)} = \det(g^{(i)}_{\mu\nu})$. From the RS action (10) one gets 5-dimensional Einstein-Hilbert’s equations
\[6\sigma''(y) = -\frac{\Lambda}{4\bar{M}_5^3} , \] (12)
\[3\sigma'''(y) = \frac{1}{4\bar{M}_5^3} [\Lambda_1 \delta(y) + \Lambda_2 \delta(\pi r_c - y)]. \] (13)
In what follows, the reduced 5-dimensional gravity scales will be used, $\bar{M}_5 = M_5/(2\pi)^{1/3}$. Let us underline that equations (12), (13) contain only derivatives of the function $\sigma(y)$ and that equation (13) is symmetric with respect to the branes.

As it was shown in details in [29] (see also [30]), a general solution of equations (12), (13) is given by
\[\sigma(y) = \frac{\kappa}{2}(|y| - |y - \pi r_c|) + \frac{\kappa \pi r_c}{2} - C , \] (14)
where the parameter $\kappa$ with a dimension of mass defines a five-dimensional scalar curvature $\mathcal{R}^{(5)}$, and $C$ is $y$-independent quantity.\(^2\) In addition, the following fine tuning
\begin{align}
\Lambda &= -24\tilde{M}_5^3\kappa^2, \\
\Lambda_1 &= -\Lambda_2 = 12\tilde{M}_5^3\kappa
\end{align}
must be realized \[^{[29]}\]. From now on, it will be assumed that $\kappa > 0$, $\pi\kappa r_c \gg 1$. Then the hierarchy relation is of the form
\begin{equation}
\tilde{M}_{\text{Pl}}^2 = \frac{\tilde{M}_5^3}{\kappa} e^{2C} \left( 1 - e^{-2\pi\kappa r_c} \right) \bigg|_{\pi\kappa r_c \gg 1} = \frac{\tilde{M}_5^3}{\kappa} e^{2C}.
\end{equation}
The interactions of the gravitons $h^{(n)}_{\mu\nu}$ with the SM fields on the physical (TeV) brane are given by the effective Lagrangian
\begin{equation}
\mathcal{L}_{\text{int}} = -\frac{1}{\tilde{M}_{\text{Pl}}} h^{(0)}_{\mu\nu}(x) T_{\alpha\beta}(x) \eta^{\mu\alpha} \eta^{\nu\beta} - \frac{1}{\Lambda_\pi} \sum_{n=1}^{\infty} h^{(n)}_{\mu\nu}(x) T_{\alpha\beta}(x) \eta^{\mu\alpha} \eta^{\nu\beta},
\end{equation}
were $T^{\mu\nu}(x)$ is the energy-momentum tensor of the SM fields, and the coupling constant of the massive modes is
\begin{equation}
\Lambda_\pi \simeq \tilde{M}_{\text{Pl}} e^{-\pi\kappa r_c} = \left( \frac{\tilde{M}_5^3}{\kappa} \right)^{1/2} e^{C - \pi\kappa r_c}.
\end{equation}
The graviton masses $m_n$ are defined from the boundary conditions imposed on wave functions of the KK excitations. They result in the equation (see, for instance, \[^{[30]}\])
\begin{equation}
J_1(a_{1n})Y_1(a_{2n}) - Y_1(a_{1n})J_1(a_{2n}) = 0,
\end{equation}
where $J_1(x)$ and $Y_1(x)$ are the Bessel functions of the first and second kind, respectively, and the following notations are introduced
\begin{equation}
a_{1n} = \frac{m_n}{\kappa} e^{-C}, \quad a_{2n} = \frac{m_n}{\kappa} e^{\pi\kappa r_c - C}.
\end{equation}

By taking different values of $C$ in eq. \[^{(14)}\], we come to quite different physical models within the framework with the warped metric. In particular, for $C = 0$, we come to the original RS1 model \[^{[28]}\] with the hierarchy relation
\begin{equation}
\tilde{M}_{\text{Pl}}^2 = \frac{\tilde{M}_5^3}{\kappa} \left( 1 - e^{-2\pi\kappa r_c} \right) \bigg|_{\pi\kappa r_c \gg 1} = \frac{\tilde{M}_5^3}{\kappa}.
\end{equation}
\(^{2}\) The term $\kappa\pi r_c/2$ is introduced in \[^{(14)}\] for convenience only.
In order (22) to be satisfied, one has to put $\bar{M}_5 \sim \kappa \sim \bar{M}_{\text{Pl}}$ (28). The graviton masses, as one can see from (20)-(21), are given by the formula

$$m_n = x_n \kappa e^{-\kappa r_c} \ , \ \ n = 1, 2, \ldots ,$$

where $x_n$ are zeros of $J_1(x)$. The coupling constant (19) will be of the order of one TeV, if we put $\kappa r_c \simeq 11.3$. It is in agreement with our assumption $\pi \kappa r_c \gg 1$. Then the lightest graviton resonance has a mass of order one–few TeV.

Taking $C = \pi \kappa r_c$, we come to the RS-like model with a small curvature (RSSC model). For the first time, it was studied in [31], see also [29]-[30], [32]-[33]. In such a model, the hierarchy relations takes the form

$$\bar{M}_{\text{Pl}}^2 = \bar{M}_{5}^3 \left( e^{2\pi \kappa r_c} - 1 \right ) \bigg|_{\pi \kappa r_c \gg 1} = \frac{\bar{M}_{5}^3}{\kappa} e^{2\pi \kappa r_c} .$$

(24)

It is thanks to the exponential factor in (24) that the mass hierarchy can be satisfied even for moderate values of $\bar{M}_5$ and $\kappa$. For instance, this relation holds, if one puts $\bar{M}_5 \sim 1$ TeV, $\kappa \sim 1$ GeV, and $\kappa r_c = 10.2$. On the contrary, the RS1 hierarchy relation (22) does not admit the parameters $\kappa$, $\bar{M}_5$ to lie in these region. The mass spectrum of the KK gravitons in the RSSC model, as it follows from (20)-(21), is defined as

$$m_n = x_n \kappa , \ \ n = 1, 2, \ldots .$$

(25)

Note that in the limit $\kappa \rightarrow 0$, the hierarchy relation for the flat metric with one ED (7) is reproduced from (24)

$$\bar{M}_{\text{Pl}}^2 = \bar{M}_5^3 V_1 ,$$

(26)

where $V_1 = 2\pi r_c$ is the volume of ED. At the same time, $\Lambda_{\pi} \rightarrow \bar{M}_{\text{Pl}}$, and $m_n \rightarrow n/r_c$ (30).

Thus, from the point of view of a 4-dimensional observer, the models with $C = 0$ and $C = \kappa \pi r_c$ are quite different physical models. The experimental signature of the RS1 model is a production of heavy resonances, while the signature of the RSSC model is a deviation of cross sections from SM predictions.

IV. PHOTON-INDUCED DIMUON PRODUCTION

Let us consider the subprocess $\gamma \gamma \rightarrow \mu^+ \mu^-$ of the photon-induced dimuon production in $e^+e^-$ collision. It’s matrix element squared is the sum of electromagnetic, KK graviton and
interference terms

\[ |M|^2 = |M_{em}|^2 + |M_{KK}|^2 + |M_{int}|^2 , \quad (27) \]

where

\[ |M_{em}|^2 = -2 e^4 \left[ \frac{\hat{s} + \hat{t}}{\hat{t}} + \frac{\hat{t}}{\hat{s} + \hat{t}} \right] , \quad (28) \]
\[ |M_{KK}|^2 = \frac{1}{4} |S(\hat{s})|^2 \left[ -\frac{\hat{t}}{\hat{s}} (\hat{s}^3 + 2\hat{t}^3 + 3\hat{s}\hat{t}^2 + 4\hat{t}^2\hat{s}) \right] , \quad (29) \]
\[ |M_{int}|^2 = -\frac{1}{4} e^2 \text{Re} S(\hat{s}) \left[ \hat{s}^2 + 2\hat{t}^2 + 2\hat{s}\hat{t} \right] . \quad (30) \]

The quantity \( S(s) \) contains summation over \( s \)-channel massive KK excitations which can be calculated without specifying process, \( \hat{s}, \hat{t} \) are Mandelstam variables of the subprocess \( \gamma\gamma \rightarrow \mu^+\mu^- \), and \( e^2 = 4\pi\alpha_{em} \).

In the ADD model this sum is given by

\[
S_{ADD}(\hat{s}) = \frac{2}{M_{Pl}^4} \sum_{n_1,\ldots,n_d=1}^{\infty} \frac{1}{\hat{s} - m_n^2 + i\varepsilon} , \quad (31)
\]

where the masses \( m_n \) are defined by Eq. (8). Since the sum is infinite for \( d \geq 2 \), an ultraviolet procedure is needed. In the Han-Lykken-Zhang (HLZ) convention, the sum of virtual KK exchanges is replaced by the integral in variable \( m_n \) with the ultraviolet cutoff \( M_S \), that results in

\[ S_{HLZ}(s) = \frac{8\pi s^{d/2-1}}{M_D^{d+2}} \left[ 2i I(x) + \pi \right] , \quad (32) \]

where \( x = M_S/\sqrt{s} \), and

\[
I(x) = \begin{cases} 
- \sum_{k=1}^{d/2-1} \frac{x^{2k}}{2k} - \ln(x^2 - 1) , & d = \text{even} \\
- \sum_{k=1}^{(d-1)/2} \frac{x^{2k-1}}{2k - 1} + \frac{1}{2} \ln \frac{x + 1}{x - 1} , & d = \text{odd} .
\end{cases} \quad (33)
\]

In what follows, we put \( M_S = M_D \).

In the Hewett convention, sum (31) is replaced by

\[ S_H = \frac{\lambda}{M_H^4} , \quad (34) \]

\[ ^3 \text{We use the definition of the graviton field of} \quad (35): \quad g_{AB} = \eta_{AB} + \sqrt{2} M_D^{-1-d/2} h_{AB}. \]
where $M_H$ is the unknown mass scale, presumably of order $M_D$. The exact relationship between scales $M_H$ and $M_D$ is not calculable without knowledge of the full theory. The parameter $\lambda = \pm 1$ is taken in analogy with the standard parametrization for contact interactions.

Note that in the Giudice-Rattazzi-Wells convention \[37\]
\[
S_{GRW} = \frac{16\pi i}{(d-2)\Lambda_T^4}, \quad d > 2,
\]
where $\Lambda_T$ is a cutoff scale. We will not use this approximation for $S_{ADD}(s)$ in our numerical analysis.

In the RS scenario the contribution of $s$-channel gravitons is given by the sum
\[
S_{RS}(\hat{s}) = \frac{2}{\Lambda^2} \sum_{n=1}^{\infty} \frac{1}{\hat{s} - m_n^2 + i m_n \Gamma_n}.
\]
Here $\Gamma_n$ denotes the total width of the graviton with the KK number $n$ and mass $m_n$ \[33\]
\[
\Gamma_n = \frac{\rho m_n^3}{\Lambda^2},
\]
where $\rho = 0.09$.

In the RS1 model, taking into account that the KK resonances are very heavy, we put
\[
S_{RS1}(s) = \frac{2}{\Lambda^2} \sum_{n=1}^{4} \frac{1}{\hat{s} - m_n^2 + i m_n \Gamma_n}.
\]
The contribution from other resonances to the sum (36) is negligible.

In the RSSC model, graviton sum (36) can be calculated analytically \[33\]
\[
S_{RSSC}(s) = -\frac{1}{4M_5^3 \sqrt{s}} \frac{\sin(2A) + i \sinh(2\varepsilon)}{\cos^2 A + \sinh^2 \varepsilon},
\]
where
\[
A = \frac{\sqrt{s}}{\kappa}, \quad \varepsilon = 0.045 \left(\frac{\sqrt{s}}{M_5}\right)^3.
\]
As was already mentioned above, in the RSSC model the KK graviton exchanges should lead to the deviations of the cross sections from the SM predictions.

V. NUMERICAL ANALYSIS AND RESULTS

The main goal of this section is to calculate the deviations of the cross sections from the SM predictions in a number of models with EDs and to estimate the CLIC 95% C.L. search
limit for the photon-induced process $e^+e^-\rightarrow e^+\gamma\gamma e^-\rightarrow e^+\mu^+\mu^-e^-$. The expected collision energy $\sqrt{s}$ of the CLIC is 380 GeV (1st stage), 1500 GeV (2nd stage) or 3000 GeV (3rd stage), with the integrated luminosities for unpolarized beams to be equal to 1000 fb$^{-1}$, 2500 fb$^{-1}$, and 5000 fb$^{-1}$, respectively, as mentioned above. Our numerical results have shown that for the same values of the parameters of the models, the deviations from the SM are much smaller for $\sqrt{s} = 380$ GeV. That is why, we will present our result for $\sqrt{s} = 1500$ GeV and $\sqrt{s} = 3000$ GeV only.

A. ADD model

In the ADD model with the Han-Lykken-Zhang (HLZ) convention the invariant part of the subprocess $\gamma\gamma \rightarrow \mu^+\mu^-$ is given by equations (32), (33). The parameters are the number of EDs $d$ and the cutoff scale $M_S$. The latter is believed to be of order of the $(4 + d)$-dimensional gravity scale $M_D$. The results of our calculations of the total cross sections are shown in Figs. 1-4 as functions of the minimal transverse momenta of the final muons $p_{t,\text{min}}$. As one can see from these figures, the deviations from the SM take place only for $p_{t,\text{min}} \gtrsim 200$ GeV for both energies. The curves correspond to different values of the number of EDs $d$ and cutoff scale $M_S$. As one can see, for some values of $M_S$, the cross sections have nontrivial dependence on $d$. Depending on $M_S$, they can rise at mediate $p_{t,\text{min}}$, while decrease at large $p_{t,\text{min}}$, as $d$ grows. The rise of the cross sections with $d$ is due to the large contribution of the interference term.

We have calculated the statistical significance $SS$ using formula

$$SS = \sqrt{2[(S + B) \ln(1 + S/B) - S]},$$

where $S(B)$ is a number of the signal (background) events. Note that $SS \simeq S/\sqrt{B}$ for $S \ll B$. We have assumed that the uncertainty of the background is negligible. The 95% C.L. search limits for two CLIC energies are presented in Figs. 5, 6 as functions of the number of EDs and CLIC integrated luminosity for $\sqrt{s} = 1500$ GeV and $\sqrt{s} = 3000$ GeV. The bounds on $M_S$ as large as 3629(3593) GeV for $d = 2(6)$ can be achieved for the CLIC energy $\sqrt{s} = 3000$ GeV and integrated luminosity $L = 5000$ fb$^{-1}$.

The cross sections in the Hewett convention of the ADD model depend on the ultraviolet cutoff $M_H$ and sign of the parameter $\lambda$ in (54). They are shown in Figs. 7, 8 as
FIG. 1: The total cross section for the process $e^+e^- \rightarrow e^+\gamma\gamma e^- \rightarrow e^+\mu^+\mu^-e^-$ in the ADD model with the HLZ convention as a function of the muon transverse momenta cutoff $p_{t,\text{min}}$ for the CLIC invariant energy $\sqrt{s} = 1500$ GeV and scale cutoff $M_S = 2$ TeV for different values of the number of EDs. The dashed line denotes the SM contribution.

functions of $p_{t,\text{min}}$, both for positive and negative sign of the parameter $\lambda$ in (34). We see that the deviations of cross sections from the SM predictions are very small even for large values of $p_{t,\text{min}}$.

The 95% C.L. bounds on the cutoff scale $M_H$ as functions of the CLIC integrated luminosity are given in Figs. 9, 10. They demonstrate us that the case $\lambda = -1$ is clearly preferable to the case $\lambda = 1$. The bound $M_H = 2204$ GeV can be achieved for $\sqrt{s} = 3000$ GeV, $L = 5000$ fb$^{-1}$. Note that in the Hewett scheme there is no dependence on the number of EDs. The bounds obtained should be compared with the LHC bounds on the parameters in the ADD model (see, for instance, [40]).

B. RS model

We have also calculated the cross sections in the Randall-Sundrum model [28] using formula (38). The corresponding curves are shown in Figs. 11, 12 as functions of the mass.
$m_1$ of the lightest graviton for three values (0.01, 0.05, 0.1) of the ratio

$$\beta = \frac{\kappa}{M_{Pl}}.$$  \hspace{1cm} (42)

The oscillations of the curves in these figures correspond to the resonance character of the invariant part of the $s$-channel amplitude in the RS model \cite{38}. Let us indicate on the strong $\beta$-dependence of the cross section for $m_1 \lesssim 1.5(2.0)$ TeV at $\sqrt{s} = 1500(3000)$ GeV. For $\beta = 0.01$ the deviations from the SM are negligible for all $m_1$.

The estimation for the 95\% C.L. parameter exclusion region is shown in Figs. \ref{fig:13}, \ref{fig:14} for three expected values of the CLIC integrated luminosity. For both energies, the value 500 GeV was taken as the minimal transverse momenta of the final muons. The best lower bound which can be achieved is equal to $m_1 = 2629$ GeV for $\beta = 0.1$. The present experimental bounds on $m_1$ are stronger \cite{41}. Thus, we don’t expect that, instead of the very high integrated luminosity of the CLIC 3rd stage, the existing experimental bounds on $m_1$ could be improved in the photon-induced dimuon production.

C. RSSC model

Finally, we have estimated the total cross sections for the RS-like model with the small curvature (RSSC model) using formulas (39), (40). As one can see in Figs. \ref{fig:15} \ref{fig:16} the
FIG. 3: The total cross section for the process $e^+e^- \rightarrow e^+\gamma\gamma e^- \rightarrow e^+\mu^+\mu^-e^-$ in the ADD model with the HLZ convention as a function of $p_{t,\text{min}}$ for $\sqrt{s} = 3000$ GeV and scale cutoff $M_S = 3.5$ TeV for different values of the number of EDs. The dashed line denotes the SM contribution.

The total cross sections weakly depend on the curvature parameter $\kappa$ for all values of the 5-dimensional Planck scale $\bar{M}_5$. It is a well-known feature of the RSSC model, provided the condition $\kappa \ll \bar{M}_5$ is satisfied [29]-[33].

The CLIC search bounds for the scale $\bar{M}_5$ in the RSSC model are shown in Figs. [17] [18]. The value $\bar{M}_5 = 2534$ TeV can be achieved for $\sqrt{s} = 3000$ GeV, $L = 5000$ fb$^{-1}$. Let us stress that the parameter $\bar{M}_5$ has quite different magnitudes in the RS and RSSC models. In the RS model the bounds on the parameter set $(\beta, m_1)$ are searched for. On the contrary, in the RSSC model one can directly obtain bounds on the 5-dimensional Planck scale $\bar{M}_5$, while a dependence on the curvature parameter $\kappa$ is rather weak. Note that the LHC bound on $D$-dimensional scale $M_D$ (see, for instance, [40]) cannot be applied to our lower limits on the scale $\bar{M}_5$, since the RSSC model cannot be regarded as a small distortion of the ADD model with one ED [33].
FIG. 4: The same as in Fig. but for $M_S = 4.5$ TeV.

FIG. 5: The 95% C.L. CLIC search bound in the ADD model with the HLZ convention for $\sqrt{s} = 1500$ GeV, $p_t = 500$ GeV as a function of the integrated luminosity $L$.

VI. CONCLUSIONS

In the present paper we have studied the photon-induced dimuon production $e^+e^- \rightarrow e^+\gamma\gamma e^- \rightarrow e^+\mu^+\mu^-e^-$ at the CLIC in a number of models with EDs. Among these mod-
FIG. 6: The same as in Fig. 5 but for $\sqrt{s} = 3000$ GeV.

els are: (i) ADD model with the Han-Lykken-Zhang convention \[35\]; (ii) the ADD model with the Hewett convention \[36\], (iii) the original Randall-Sundrum model \[28\], (iv) the Randall-Sundrum-like model with the small curvature \[31\]. The total cross sections have been calculated for energies $\sqrt{s} = 1500$ GeV and $\sqrt{s} = 3000$ GeV. It enabled us to obtain 95% C.L. search limits on the parameters of the models as functions of the CLIC integrated luminosity $L$.

The best bounds have been derived for the $e^+e^-$ collision energy energy $\sqrt{s} = 3000$ GeV and integrated luminosity $L = 5000$ fb$^{-1}$. For the ADD model with the HLZ convention we have obtained $M_S \geq 3629$ GeV (Fig. 6), while for the ADD model with the Hewett convention we have got that $M_H \geq 2204$ GeV (Fig. 10). The best limits for the parameters $(\beta, m_1)$ of the RS model are presented in Fig. 18. The bounds on the fundamental gravity scale $\bar{M}_5$ in the RSSC model is of considerable interest, since so far there are no experimental limits on the parameters of this RS-like model. Note that the LHC discovery limits on $\bar{M}_5$ for the photon-induced process $pp \rightarrow p\gamma\gamma p \rightarrow p\mu^+\mu^- p$ have been calculated in our recent paper \[42\]. The LHC bounds obtained there are noticeably lower than our CLIC bounds.

Let us underline, the great advantage of the CLIC collider is that it has very clean backgrounds. Moreover, the CLIC detectors don’t need additional equipment for Weizsäcker-Williams photon-induced collisions analyzed in the present paper. That is why, we think
FIG. 7: The total cross section for the process $e^+e^- \rightarrow e^+\gamma e^- \rightarrow e^+\mu^+\mu^-e^-$ in the ADD model with the Hewett convention as a function of $p_{t,\text{min}}$ for $\sqrt{s} = 1500$ GeV for different values of the number of EDs.

that studying such reactions at the CLIC could be one of most important physical tasks.

It would be interesting to compare our results on the total cross section and CLIC search limits with the corresponding predictions for the processes $e^+\gamma \rightarrow e^+\gamma$ and $e^+e^- \rightarrow e^+\gamma e^- \rightarrow e^+\mu^+\mu^-e^-$, where $\gamma$ is the Compton backscattering photon [6]. It will be a subject of our separate publication.

Acknowledgments

This work is partially supported by the Scientific Research Project Fund of Sivas Cumhuriyet University under project number “F-596”.

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FIG. 8: The same as in Fig. 7 but for $\sqrt{s} = 3000$ GeV and $M_S = 3.5$ TeV.

FIG. 9: The 95% C.L. CLIC search bound in the ADD model with the Hewett convention for $\sqrt{s} = 1500$ GeV, $p_t = 500$ GeV as a function of the integrated luminosity $L$.

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FIG. 10: The same as in Fig. [9] but for $\sqrt{s} = 3000$ GeV.

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FIG. 13: The 95% C.L. exclusion region for the parameters $m_1$ and $\beta$ in the RS model for $\sqrt{s} = 1500$ GeV, $p_t = 500$ GeV and three values of the CLIC integrated luminosity $L$.

FIG. 14: The same as in Fig. 13 but for $\sqrt{s} = 3000$ GeV and different values of $L$. 
FIG. 15: The total cross sections for the process $e^+e^- \rightarrow e^+\mu^+\mu^-e^-$ in the RSSC model as a function of $p_{t,\text{min}}$ for $\sqrt{s} = 1500$ GeV and different values of $M_5$ and $\kappa$.

FIG. 16: The same as in Fig. 15 but for $\sqrt{s} = 3000$ GeV.
FIG. 17: The 95% C.L. CLIC search bound in the RSSC model for $\sqrt{s} = 1500$ GeV, $p_t = 500$ GeV as a function of the integrated luminosity $L$.

FIG. 18: The same as in Fig. 17 but for $\sqrt{s} = 3000$. 