Chiral condensate in neutron matter 1
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Abstract
A recent chiral perturbation theory calculation of the in-medium quark condensate $\langle \bar{q}q \rangle$ is extended to the isospin-asymmetric case of pure neutron matter. In contrast to the behavior in isospin-symmetric nuclear matter we find only small deviations from the linear density approximation. This feature originates primarily from the reduced weight factors (e.g. 1/6 for the dominant contributions) of the $2\pi$-exchange mechanisms in pure neutron matter. Our result suggests therefore that the tendencies for chiral symmetry restoration are actually favored in systems with large neutron excess (e.g. neutron stars). We also analyze the behavior of the density-dependent quark condensate $\langle \bar{q}q \rangle(\rho_n)$ in the chiral limit $m_\pi \rightarrow 0$.

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1 Introduction and framework
The quark condensate $|\langle 0|\bar{q}q|0 \rangle|$ is an order parameter of spontaneous chiral symmetry breaking in QCD. With increasing temperature the quark condensate decreases (or “melts”). For low temperatures $T$ this effect can be systematically calculated in chiral perturbation theory. The estimate $T_c \approx 190$ MeV [1, 2] for the critical temperature, where chiral symmetry will eventually be restored, has been found. This extrapolated value is remarkably consistent with $T_c = (192 \pm 8)$ MeV [3] obtained in recent lattice QCD simulations (modulo still persisting disputes between different lattice groups [4]).

The chiral condensate $|\langle \bar{q}q \rangle|$ drops also with increasing baryon density. Presently, it is not feasible to study this phenomenon rigorously in lattice QCD due to the problems arising from the complex-valued fermion determinant at non-zero quark chemical potential. As an alternative, the density dependence of $\langle \bar{q}q \rangle(\rho)$ can be extracted by exploiting the Feynman-Hellmann theorem applied to the chiral symmetry breaking quark mass term $m_q \bar{q}q$ in the QCD Hamiltonian. The leading linear term in the nucleon density $\rho$ is then readily derived by differentiating the energy density of a nucleonic Fermi gas, $\rho M_N + O(\rho^{5/3})$, with respect to the light quark mass $m_q$. This introduces the nucleon sigma-term $\sigma_N = \langle N|m_q \bar{q}q|N \rangle = m_q \partial M_N / \partial m_q = (45 \pm 8)$ MeV [5] as the driving term for the density evolution of the chiral condensate. Following this leading linear density approximation one would estimate that chiral symmetry gets restored at $(2.5 - 3)\rho_0$, with $\rho_0 = 0.16$ fm$^{-3}$ the nuclear matter saturation density.

Corrections beyond the linear density approximation arise from the nucleon-nucleon correlations which transform the nucleonic Fermi gas into a nuclear Fermi liquid. Because of the Goldstone boson nature of the pion, with its characteristic mass relation $m_\pi^2 \sim m_q$, the pion-exchange dynamics in nuclear matter plays a particularly important role for the in-medium quark condensate. In a recent work [6] we have used in-medium chiral perturbation theory to calculate systematically the corrections to the linear density approximation. This calculation

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has treated in detail the long- and medium-range correlations arising from $1\pi$-exchange (with $m_\pi$-dependent vertex corrections), iterated $1\pi$-exchange, and irreducible $2\pi$-exchange including also virtual $\Delta(1232)$-isobar excitations, with Pauli-blocking corrections up to three-loop order. It was furthermore necessary to estimate the quark mass dependence of a NN-contact term which encodes unresolved short-distance dynamics. Employing a recent computation of the NN-potential in lattice QCD [7] (at three different pion masses) we have found that the contact term has a negligible influence on the in-medium quark condensate (in agreement with ref.[8] which follows a somewhat different approach). As a result, we have obtained in ref.[6] a strong and non-linear dependence of the “dropping” condensate on the actual value of the pion mass $m_\pi$. In the chiral limit $m_\pi = 0$, chiral symmetry would seem to be restored already at about 1.5$\rho_0$. By contrast, for the physical pion mass $m_\pi = 135$ MeV, the in-medium condensate stabilizes at about 60% of its vacuum value around that same density (see Fig. 2 in the present paper).

Having found such pronounced deviations from the linear density approximation, with a hindered tendency towards chiral symmetry restoration in isospinymmetric nuclear matter, it is a logical next step to explore the effects of the additional isospin degree of freedom present in nuclear many-body systems. Such a novel study is the purpose of the present short paper where we consider the extreme isospin-asymmetric case of pure neutron matter. Interestingly, we find that the same chiral pion-exchange dynamics which stabilizes the quark condensate in isospin-symmetric nuclear matter does not alter (in a significant way) its linear decrease in pure neutron matter (for densities $\rho_n < 0.35$ fm$^3$). At the same time the behavior in the chiral limit $m_\pi \to 0$ changes prominently once a nonvanishing isospin-asymmetry is present in the nuclear medium.

As in ref.[6], our starting point is the Feynman-Hellmann theorem. It relates the in-medium quark condensate $\langle \bar{q}q \rangle(\rho_n)$ to the quark mass derivative of the energy density of pure neutron matter. Using the Gell-Mann-Oakes-Renner relation $m_\pi^2 f_\pi^2 = -m_q \langle 0|\bar{q}q|0 \rangle$ one obtains for the ratio of the in-medium to the vacuum quark condensate:

$$\frac{\langle \bar{q}q \rangle(\rho_n)}{\langle 0|\bar{q}q|0 \rangle} = 1 - \rho_n \left\{ \frac{\sigma_N}{f_\pi^2} \left( 1 - \frac{3k_n^2}{10M_N^2} + \frac{9k_n^4}{56M_N^4} \right) + D_n(k_n) \right\}, \quad (1)$$

with $k_n$ the neutron Fermi momentum and $\rho_n = k_n^3/3\pi^2$ the neutron density. The term proportional to $\sigma_N = \langle N|m_q \bar{q}q|N \rangle = m_q \partial M_N/\partial m_q$ comes from the non-interacting Fermi gas including the (relativistically improved) kinetic energy. We mention here that $f_\pi$ denotes the pion decay constant in the chiral limit and $m_\pi^2$ stands for the leading linear term in the quark mass expansion of the squared pion mass. Interaction contributions beyond the linear density approximation are collected in the function:

$$D_n(k_n) = \frac{\partial \bar{E}_n(k_n)}{\partial m_\pi^2}, \quad (2)$$

defined as the derivative of the interaction energy per particle $\bar{E}_n(k_n)$ with respect to $m_\pi^2$. Note that small (explicit) isospin breaking effects [9] (in interactions etc.) are not included in our calculation. The asymmetry in isospin is entirely given by the filled Fermi sea of neutrons (and the empty one for protons).

2 Selected interaction contributions

In this section we present analytical results for the contributions to the function $D_n(k_n)$ as given by various classes of $1\pi$- and $2\pi$-exchange diagrams, calculated up to three-loop order in the
energy density $\rho_n E_n(k_n)$. At the level of individual diagrams pure neutron matter and isospin-symmetric nuclear matter differ only with respect to the (overall) isospin factor. In Table I several of these relative isospin factors are listed. Therein, we refer to the corresponding equation for $D(k_f)$ in ref.[6] and prescribe to substitute the Fermi momentum: $k_f \rightarrow k_n$. For completeness and clearness the remaining 2π-exchange contributions to $D_n(k_n)$ are better written out explicitly. We are following the enumeration scheme introduced in Sec. II of ref.[6].

| Eq. in ref.[6] | (4) | (5) | (6) | (7) | (8) | (9) | (12) | (22) | (23) |
|----------------|-----|-----|-----|-----|-----|-----|------|------|------|
| isospin factor | 1/3 | 1/3 | 1/6 | -1/3| 1/6 | -1/3| 1/6  | 1/6  | 1/3  |

Table I: Relative isospin factors for several 1π- and 2π-exchange contributions to $D_n(k_n)$.

The irreducible 2π-exchange with only nucleons in the intermediate state leads to the following contribution [10]:

$$
D_n(k_n)^{\text{(2π)}} = \frac{m_\pi^4}{(4\pi f_\pi)^4} \left\{ \left[ \frac{1}{8u} \left( 83g_\pi^4 + 6g_\Lambda^2 - 1 \right) + \frac{3}{4u} \left( 47g_\Lambda^4 + 2g_\Lambda^2 - 1 \right) \right] \ln^2(u + \sqrt{1 + u^2}) 
+ \left[ \frac{1}{4u^2} \left( 1 - 6g_\Lambda^2 - 83g_\pi^4 \right) - \frac{4}{3} - 2g_\Lambda^2 + \frac{86}{3} g_\Lambda^4 - \frac{u^2}{3} \left( g_\Lambda^4 + 6g_\Lambda^2 + 1 \right) \right] \sqrt{1 + u^2} 
\times \ln(u + \sqrt{1 + u^2}) + \frac{1}{8u} \left( 83g_\pi^4 + 6g_\Lambda^2 - 1 \right) + \frac{u}{24} \left( 47 + 30g_\Lambda^2 - 1285g_\Lambda^4 \right) 
+ \frac{u^3}{12} \left( 9 + 46g_\Lambda^2 - 55g_\Lambda^4 \right) + \frac{u^3}{3} \left( 1 + 6g_\Lambda^2 - 15g_\Lambda^4 \right) \ln \frac{m_\pi}{\lambda} \right\},
$$

(3)

with $u = k_n/m_\pi$ and $\lambda$ the regularization scale. The three-body Fock term related to 2π-exchange with virtual Δ-excitation reads:

$$
D_n(k_n)^{\text{(Δ,F3)}} = \frac{g_\pi^4 m_\pi^4}{4(4\pi f_\pi)^4 u^3} \int_0^u dx \left\{ G_S \left( x \frac{\partial G_S}{\partial x} + u \frac{\partial G_T}{\partial u} - 4G_S \right) + 2G_T \left( x \frac{\partial G_T}{\partial x} + u \frac{\partial G_T}{\partial u} - 4G_T \right) \right\},
$$

(4)

with the auxiliary functions $G_{S,T}(x, u)$ written in Eqs.(7,8) of ref.[11]. The dominant two-body terms scaling reciprocally with the delta-nucleon mass splitting $\Delta = 293$ MeV take the form:

$$
D_n(k_n)^{\text{(Δ2)}} = \frac{\pi g_\pi^4 m_\pi^4}{70(2\pi f_\pi)^4} \left\{ \left( 140 + 42u^2 + 15u^4 \right) \arctan u
- \frac{89 + 315u^2}{4u^3} \ln(1 + u^2) + \frac{89}{4u} - \frac{579u}{8} - \frac{1115u^3}{12} \right\}.
$$

(5)

For the remaining two-body terms with a more complicated $\Delta$-dependence we employ the spectral-function representation [11] and differentiate directly the imaginary parts of the $\pi N\Delta$-loop functions with respect to $m_\pi^2$. This gives:

$$
D_n(k_n)^{\text{(Δ2')}} = \frac{g_\Lambda^2}{(4\pi f_\pi)^4} \int_{2m_\pi}^\infty d\mu \left\{ 3\mu k_n - \frac{4k_n^3}{3\mu} - \frac{\mu^3}{2k_n} - 4\mu^2 \arctan \frac{2k_n}{\mu} + \frac{\mu^3}{8k_n^2} \left( 12k_n^2 + \mu^2 \right) \right\}
\times \ln \left( 1 + \frac{4k_n^2}{\mu} \right) \left\{ \left( \frac{2\Delta}{\mu} + \frac{g_\Lambda^2}{8\mu\Delta} \left( 40\Delta^2 + 72m_\pi^2 - 17\mu^2 \right) \right) \arctan \sqrt{\frac{\mu^2 - 4m_\pi^2}{2\Delta}}
- \frac{2g_\Lambda^2 \mu m_\pi^2}{\Delta^2 \sqrt{\mu^2 - 4m_\pi^2}} + \sqrt{\mu^2 - 4m_\pi^2} \left[ \frac{7g_\Lambda^2 \mu (m_\pi^2 - \Delta^2)}{(\mu^2 + 4\Delta^2 - 4m_\pi^2)^2} - \frac{2 + 5g_\Lambda^2}{2\mu} 
+ \frac{4g_\Lambda^2 \mu (m_\pi^2 - \Delta^2) - \mu \Delta^2}{2\Delta^2 (\mu^2 + 4\Delta^2 - 4m_\pi^2)} \right] \right\},
$$

(6)
after subtracting a term linear in the neutron density $\rho_n = k_n^3/3\pi^2$. The associated subtraction constant includes also pion-loop contributions with a nonanalytical dependence on the quark mass $m_q$. We reinstore these distinguished pieces by the term:

$$D_n(k_n)(dt) = \frac{g_A^2 k_n^3}{4\pi f_\pi^4} \left\{ (5g_A^2 - 2) \ln \frac{m_\pi}{2\Delta} + \frac{5g_A^2 (2\Delta^2 - 9m_\pi^2) - 4\Delta^2}{2\Delta \sqrt{\Delta^2 - m_\pi^2}} \ln \frac{\Delta + \sqrt{\Delta^2 - m_\pi^2}}{m_\pi} \right\}. \quad (7)$$

Finally, there is the $2\pi$-exchange two-body term generated by the $\pi\pi NN$-contact vertex proportional to the low-energy constant $c_1 = -\sigma_N/4m_\pi^2 + \mathcal{O}(m_\pi)$ (measuring explicit chiral symmetry breaking in the $\pi N$-interaction). Its contribution to the function $D_n(k_n)$ reads:

$$D_n(k_n)(c_1) = \frac{3g_A^2 c_1 m_\pi^4}{280\pi^3 f_\pi^4} \left\{ (14u^2 + 3u^4) \arctan u + \frac{27 + 49u^2}{4u^3} \ln(1 + u^2) - \frac{27}{4u} - \frac{71u}{8} - \frac{93u^3}{4} \right\}. \quad (8)$$

In comparison to Eq.(20) in ref.[6] only the numerical coefficient of the last $u^3$-term has changed. This comes from the different weighting of the Hartree and Fock contributions in neutron matter as compared to isospin-symmetric nuclear matter.

The sum of all terms, Eqs.(3-8) together with those obtained via the relative isospin factors in Table I, comprise the $m_\pi^2$-derivative of the complete set of in-medium $1\pi$- and $2\pi$-exchange processes up to three-loop order in the energy density (with inclusion of explicit $\Delta(1232)$ degrees of freedom).

### 3 Results and discussion

We are using consistently the same parameters in the chiral limit as in our previous work [6], namely: $f_\pi = 86.5$ MeV, $g_A = 1.224$, $c_1 = -0.93$ GeV$^{-1}$ and $M_N = \lambda = 882$ MeV. For the quark mass dependence of the short-distance dynamics (not controlled by the underlying chiral effective field theory) we adopt the result derived in ref.[6] via the short range part ($r \leq 0.6$ fm) of the NN-potential from lattice QCD [7]. Its effect on the in-medium condensate $\langle \bar{q}q\rangle(\rho_n)$ is then again negligibly small.

Collecting all the pieces entering into Eq.(1), the condensate ratio $\langle \bar{q}q\rangle(\rho_n)/\langle \bar{q}q\rangle(0)$ in pure neutron matter comes out as shown by the full line in Fig.1. The dashed line therein corresponds to the linear density approximation using the empirical central value of the nucleon sigma-term, $\sigma_N = 45$ MeV [5]. In contrast to the behavior in isospin-symmetric nuclear matter (redisplayed in Fig.2 for comparison) the chiral pion-exchange dynamics in pure neutron matter generates only small deviations from the linear dropping of the in-medium condensate with density $\rho_n$. This feature is straightforwardly explained by the reduced isospin weight factors of the $2\pi$-exchange mechanisms (1/6 for the dominant contributions). Fig.3 shows separately the effects of the five classes of interaction contributions for neutron densities $\rho_n \leq 0.35$ fm$^{-3}$. They are consecutively added up in the sequence: linear density approximation $\rightarrow 1\pi$-exchange $\rightarrow$ iterated $1\pi$-exchange $\rightarrow$ irreducible $2\pi$-exchange $\rightarrow$ $2\pi$-exchange with virtual $\Delta(1232)$-excitation $\rightarrow$ chiral symmetry breaking $c_1$-term. One observes that the effects from $2\pi$-exchange cancel here almost completely such that the total result (full line) lies close to the $1\pi$-exchange approximation (dashed-dotted line). This behavior is markedly different from the situation in isospin-symmetric nuclear matter (see Fig.6 in ref.[6]). Note also that all the results discussed so far refer to the physical value of the pion mass, $m_\pi = 135$ MeV.

For the sake of completeness, we show in Fig.4 the neutron matter equation of state resulting from our choice of parameters. Besides the $1\pi$- and $2\pi$-exchange contributions described in
Fig. 1: Ratio of the in-medium chiral condensate in pure neutron matter to its vacuum value at the physical pion mass, $m_\pi = 135$ MeV. The dashed line corresponds to the linear density approximation using the empirical central value $\sigma_N = 45$ MeV [5].

Fig. 2: Ratio of the in-medium chiral condensate in isospin-symmetric nuclear matter to its vacuum value. The dashed line shows the linear density approximation.
neutron matter

Fig. 3: Ratio between the in-medium chiral condensate in pure neutron matter and its vacuum value. The five classes of interaction contributions are consecutively added in the sequence: linear $\rightarrow$ $1\pi$ $\rightarrow$ iterated $\rightarrow$ $2\pi$ $\rightarrow$ $\Delta$ $\rightarrow$ $c_1$.

refs.\cite{10, 11} and those proportional to $c_1$, it includes an adjusted short-distance term of the form: $E_n(k_n)(adj) = -1.04 \text{ GeV}^{-2} k_n^3 - 18.4 \text{ GeV}^{-4} k_n^5$. For not too high neutron densities, $\rho_n \leq 0.2 \text{ fm}^{-3}$, our perturbative calculation reproduces fairly well the result of the Urbana group \cite{12} based on a sophisticated many-body calculation. At higher neutron densities we get (as in ref.\cite{11}) a stiffer neutron matter equation of state. This has to do with a repulsive $k_n^6$-term generated by the $2\pi$-exchange three-neutron interaction. Note however that this $k_n^6$-term drops out when taking the derivative with respect to $m_\pi^2$.

Finally, we turn to the behavior of the density-dependent quark condensate $\langle \bar{q}q \rangle(\rho_n)$ in the chiral limit $m_\pi \rightarrow 0$. As demonstrated in Sec. II F of ref.\cite{6} the (singular) chiral logarithms $\ln(m_\pi/\lambda)$ from irreducible $2\pi$-exchange and those from the $m_\pi$-dependent vertex corrections to the $1\pi$-exchange cancel exactly in the case of isospin-symmetric nuclear matter. This subtle balance does not work anymore for pure neutron matter. The following singular piece remains:

$$D_n(k_n)|_{m_\pi \rightarrow 0} = \frac{2k_n^3}{3(4\pi f_\pi)^4}(1 + 6g_A^2 - 11g_A^4) \ln \frac{m_\pi}{\lambda}.$$  \hspace{1cm} (9)

In order to understand the physical origin behind this infrared singularity let us consider the simplified situation with the $p$-wave pion-nucleon coupling set to zero: $g_A = 0$. In that case only the $2\pi$-exchange interaction generated by the Weinberg-Tomozawa term (at second order) survives. It gives rise to an isovector ($\approx \vec{\tau}_1 \cdot \vec{\tau}_2$) central NN-interaction with $m_\pi^2$-derivative equal to:

$$\frac{\partial W_C(q)}{\partial m_\pi^2} = \frac{1}{64\pi^2 f_\pi^4} \left\{ \frac{1}{2} - \ln \frac{m_\pi}{\lambda} - \frac{\sqrt{4m_\pi^2 + q^2}}{q} \ln \frac{q + \sqrt{4m_\pi^2 + q^2}}{2m_\pi} \right\}.$$ \hspace{1cm} (10)

The two-body Hartree term (left diagram in Fig. 5) in infinite neutron matter is proportional to its value at zero momentum transfer $q = 0$, thus exhibiting the chiral logarithm, $-1/2 -$
Fig. 4: Energy per particle $\bar{E}_n(k_n)$ of pure neutron matter as a function of the neutron density $\rho_n = k_n^3/3\pi^2$. It includes an adjusted short-distance term of the form: $\bar{E}_n(k_n)^{(adj)} = -1.04 \, \text{GeV}^{-2} k_n^3 - 18.4 \, \text{GeV}^{-4} k_n^5$. The dashed-dotted line stems from the sophisticated many-body calculation of the Urbana group [12].

$\ln(m_\pi/\lambda)$. On the other hand the limit $m_\pi \to 0$ of Eq.(10) exists and it is proportional to $1/2 - \ln(q/\lambda)$. Consequently, the two-body Fock term (right diagram in Fig.5), involving the integral: $\int_0^{2k_n} dq \, q^2(2k_n - q)^2(4k_n + q)[1/2 - \ln(q/\lambda)] = 4k_n^6[5 - 4\ln(2k_n/\lambda)]/3$, stays finite in the chiral limit. The coordinate-space "potential" associated with the logarithm $\ln(q/\lambda)$ in momentum space actually behaves as $r^{-3}$ (for $r > 0$). Its long-range character causes the interaction energy density for a homogeneous neutron-sphere of radius $R$ to grow logarithmically with the system size:

$$
\frac{3}{4\pi R^3} \int d^3 r_1 d^3 r_2 \int \frac{d^3 q}{(2\pi)^3} e^{i\vec{q} \cdot (\vec{r}_1 - \vec{r}_2)} \ln \frac{\lambda}{q} \ln R x = 6 \pi \int_0^\infty dx \frac{(x \cos x - \sin x)^2}{x^4} \ln \frac{2\lambda R}{x} = \ln(2\lambda R) + \gamma_E - \frac{7}{3}. \tag{11}
$$

In other words, the usual the thermodynamic limit $N \to \infty$, $R \to \infty$, with $N/R^3 = \text{constant},$
does not exist for such a long-ranged interaction. In this context the quantity $\langle \tau_3 \rangle / f_\pi^2$ plays the role of an isovector “polarizability” for the coupling and exchange of a pair of massless pions which causes the infrared singular behavior. Since the expectation value of $\tau_3$ does not vanish whenever the nuclear medium is asymmetric in isospin, the corresponding in-medium quark condensate will encounter an infrared singularity in the chiral limit $m_\pi \to 0$.

In passing we note that the result for the quark condensate in pure proton matter (ignoring the Coulomb interaction) is the same as that for pure neutron matter, shown in Figs. 1, 3. Our results for the in-medium chiral condensate can also be interpreted in terms of a density-dependent effective nucleon sigma-term, $\sigma_{N,\text{eff}}(\rho) = \sigma_{N,\text{eff}}^0(\rho) + \langle \tau_3 \rangle^2 \sigma_{N,\text{eff}}^{(1)}(\rho)$, with $\langle \tau_3 \rangle = (Z - N)/(Z + N)$ the relative isospin-asymmetry. Its isoscalar piece $\sigma_{N,\text{eff}}^0(\rho)$ drops from the value $\sigma_N \simeq 45$ MeV in vacuum to about 20 MeV at $\rho = 0.35$ fm$^{-3}$ (see Fig. 7 in ref.[6]). The isovector piece $\sigma_{N,\text{eff}}^{(1)}(\rho)$, which is maximally active in pure neutron or proton matter, brings it then back to almost the vacuum value in the density region $\rho = 0.35$ fm$^{-3}$ considered here. As a consequence, the nuclear scalar mean-field associated in certain models [13, 14] with the in-medium condensate $\langle \bar{q}q \rangle(\rho)$, would have an attractive isoscalar component as well as an attractive isovector component with approximately opposite density dependence.

In summary, we have used in-medium chiral perturbation theory to calculate the density-dependent quark condensate $\langle \bar{q}q \rangle(\rho_n)$ in pure neutron matter. We have found that the same $2\pi$-exchange dynamics which stabilizes the chiral condensate in isospin-symmetric nuclear matter [6] does not alter (in a significant way) its linear decrease with the neutron density $\rho_n$. This different behavior originates from the reduced weight factors of the $2\pi$-exchange mechanisms in pure neutron matter. We can conclude that the tendencies for chiral symmetry restoration are actually favored in systems with large neutron excess (e.g. neutron stars). Possible observable consequences of this feature should be further investigated.

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