Mathematical modeling of economic processes on the basis of the reflection theory

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Abstract. Doubtlessly, the process of modeling and planning economic processes is a complex procedure. The development of each specific economic system is built on plans, which are drawn up from forecasts. In this paper, the authors show the relationship between methods of exponential smoothing and autoregression. Further, the authors present the method of nonparametric selection of parameters in autoregressive models. Here, the algorithms used in the method give a comparative analysis of the results of the modeling process and retrospective forecasting is discussed. At the end of the paper, the authors give the basic recommendations on the usage of the presented method and its advantages.

1. Introduction

At the foundation of all modeling and forecasting methods lies the reflection theory, or, more precisely, the statement according to which reflection is a specific interaction of two systems. The result of the interaction is the reproduction of one system within the other. In scientific research, the property of reflection takes on the form of interaction between reality and human consciousness [1].

The main requirement for any model is its adequacy towards the studied object; otherwise the purpose of modeling will be lost. The adequacy of the model is typically understood as the degree of its compliance with the original system. During the past three decades, the vast number of models were developed using computer technology [2–4].

When planning economic processes, we use models of random stationary processes. The autoregression and exponential smoothing models are the most widely used because of their simplicity.

In the method of performing one parameter exponential smoothing, the forecast is made using the following formula [5–7]:

\[ y_{t+1} = \alpha x_t + (1-\alpha) y_t, \]

where \( y_t \) is the previously predicted value, \( \alpha \) is the smoothing parameter (0 < \( \alpha \) < 1); the value (1–\( \alpha \)) is known as the discounting coefficient; \( x_t \) is a random process.

According to the following formula, if \( y_1 \) is known, it is possible to build a forecasting model for the period \( t = 1, \ldots, T \) :

\[ y_{t+1} = \sum_{i=0}^{T-1} \alpha (1-\alpha)^i x_{t-i} + (1-\alpha)^T y_1. \]

(1)
It is possible to demonstrate that \( \sum_{i=0}^{\infty} \alpha(1 - \alpha)^i \) in (1) is a numerical series, the sum of which is equal to 1. This series may be considered normalized.

Since the \( t \to \infty \) value \( (1 - \alpha)^{t_i} \) \( y_i \to 0 \), then a relatively large \( t \) value \( (1 - \alpha)^{t} \) \( y \) in (1) can be neglected and the formula takes on the form of autoregression of the order \( t \) with the coefficients of the normalized numerical series:

\[
y_{t+1} = \sum_{i=0}^{t} \alpha(1 - \alpha)^{t_i} x_{t-i}.
\]

The formula (2) is an autoregression similar to the decomposition of a complete prehistory with coefficients selected by the modified Koyck method [8].

2. Experimental study

Consider the following generalization of the given situation. The authors shall select a normalized numerical range such as: \( \sum_{i=0}^{\infty} b_i = \sum_{i=0}^{\infty} \alpha(1 - \alpha)^i = 1 \).

In this case, the predicted value \( y_{t+1} \) is calculated as follows:

\[
y_{t+1}^{(m,p)} = \sum_{i=0}^{p-1} b_i^{(m,p)} x_{t-i},
\]

where \( m \) is the number of the normalized numerical series from a base of series, having the aforementioned properties; \( p \) is the order of the autoregression model; the upper index \( (m;p) \) – indicates the number of the series and the order of the model.

The use of normalized numerical series satisfies the stationarity condition of the process; however, for modeling non-stationary processes, it is not necessary to use the normalization condition for numerical series.

If \( p = T \), then the formula (3) is a prediction model for the complete prehistory (Wold’s decomposition [10]), with the condition that the predicted value \( y_{t+1}^{(m,p)} \) will be the sum of the prehistory of the process with the weight coefficients being elements of the numerical series (similar to Almon’s [9] and Koyck’s [8] methods).

Therefore, the error in the model will be estimated using the formula:

\[
\Delta^{(m,p)} = \frac{1}{t-1} \sum_{i=p+1}^{t} \left( \frac{x_i - y_i^{(m,p)}}{x_i} \right)^2 \times 100\%
\]

If \( x_i = 0 \) for some \( i \), then we must use a different system of coordinate \( x_i' = x_i + x_0 \), in which everything is \( x_i' \neq 0 \), where \( x_0 = \text{const} \).

To determine the optimal order of the model \( p \), as well as the form \( \sum_{i=0}^{\infty} b_i \), we shall use the function:

\[
\Theta = \left\{ \left( b_0, \ldots, b_{p-1}, p \right) : \min_{1 \leq m \leq m_0} \{ \min_{p \leq p_0} \left\{ \Delta^{(m,p)} \right\} \} \right\},
\]

where \( \rho_0 \) is the boundary of the model order.

The function (5) allows not only selecting the optimal coefficients, but also determining the order of the autoregression.

An additional parameter for assessing the performance of the model will be a retrospective forecast for the last \( k \) values of the random process. The error of the retrospective forecast is calculated using the formula:
\[ \Delta_{nvo} = \frac{1}{k_i} \sum_{i=1}^{k_i} \left( \frac{x_i - y_i^{(m,p)}}{x_i} \right) \times 100\% , \]  

(6)

where \( k_i \) is the order of the retrospective forecast.

Figure 1 shows the scheme of the algorithm.

Before proceeding to the described method of selecting parameters, it is initially necessary to assess a number of values. Further, the data is brought to a stationary form to enable further modeling and forecasting.

For the purpose of this research the authors used economic indicators for Russia for various time periods.

3. Results
In Table 1, it is possible to observe the comparative characteristics of the results obtained using the formulas (5) and (6).
According to the obtained data, the applied method is comparable in many cases with the existing ones. However, it should be noted that on short data series its numerical characteristics are significantly better. In addition, when the process is decomposed into components, the integrity of the studied phenomenon is lost; in some cases this is due to the presence of trend correlation and cyclicity. The decomposition is also a short-term dynamic process, the reduction of which, in most cases, is unacceptable. Therefore, when analyzing random dynamic processes it is necessary to use those data processing methods, which can reduce information loss, processing time, and consequently, decision-making time.

4. Conclusion

The use of numerical series as weight coefficients in autoregressive models is expedient and produces the best results in the analysis and prediction of short-term and unsteady processes.

In practice, for the decision-maker, the ease and availability of methods and the speed of obtaining possible options for future process changes are the decisive factors in selecting a predictive model. The proposed method that has been discussed in this paper meets all these requirements. This method of selecting parameters can improve the efficiency of using autoregressive models in predicting various dynamic processes. Since the method does not involve isolating the trend component (the allocation of which, sometimes, leads to the impossibility of further forecasting) from the phenomenon under consideration, its application can be expanded to other disciplines.

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