Calculation procedures for oil free scroll compressors based on mathematical modelling of working process

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Abstract
Basic propositions of calculation procedures for oil free scroll compressors characteristics are presented. It is shown that mathematical modelling of working process in a scroll compressor makes it possible to take into account such factors influencing the working process as heat and mass exchange, mechanical interaction in working chambers, leakage through slots, etc. The basic mathematical model may be supplemented by taking into account external heat exchange, elastic deformation of scrolls, inlet and outlet losses, etc. To evaluate the influence of procedure on scroll compressor characteristics calculations accuracy different calculations were carried out. Internal adiabatic efficiency was chosen as a comparative parameter which evaluates the perfection of internal thermodynamic and gas-dynamic compressor processes. Calculated characteristics are compared with experimental values obtained for the compressor pilot sample.

1. Introduction
Oil free scroll vacuum pumps and compressors hold much favour on the world market, which can be explained by their undeniable advantages such as low energy consumption, high reliability, counter balance and space saving. Scroll pumps and compressors are very science intensive type of production. The upgrading of scroll machines is carried out on the basis of physically valid and approbated mathematical models.

In this paper different approaches to modelling of working process of oil free scroll machines are considered.

The main characteristics of positive-displacement machine working as the compressor are volumetric efficiency ($\eta_v$), adiabatic efficiency ($\eta_{ad}$), and total adiabatic efficiency ($\eta_{ad,t}$).

2. Calculation procedure of working process taking into account heat and mass exchange in working chambers
Mathematical modelling of working process in a scroll machine makes its possible to take into account different factors influencing working process on each stage (heat and mass exchange, mechanical interaction in working chambers, leakage through slots, etc. [1-4]). The basic mathematical model may be supplemented by taking into account external heat exchange, elastic deformation of scrolls, inlet and outlet losses, etc., similar to [5-7], for example.

Working process in a scroll machine is performed with variable mass of working fluid. Mathematical model of a process is based on differential equations of working fluid parameters change which are widely used in compressors [1,2] and vacuum pumps [3,4] modelling. Differential equations describing the gas state in scroll machine chambers may be written in different forms. In this paper we’ll use the form presented in [8-14]

\begin{equation}
\frac{dP_i}{V_i} = \frac{1}{V_i} \left[ m_i RdT_i + RT_i dm_i - P_i dV_i \right] \quad (1)
\end{equation}

\begin{equation}
\frac{dT_i}{T_i} = \frac{dT_{vi}}{T_{vi}} + \frac{dT_{wrti}}{T_{wrti}} + \frac{dT_{ma}}{T_{ma}} + \frac{dT_{li}}{T_{li}} \quad (2)
\end{equation}

The first of these expressions is obtained from the equation of state in differential form and takes into account the influence of mass change $dm$, temperature change $dT$ and volume change $dV$ on
The change of gas temperature $dT_i$ in working chamber over an infinitely small period of time; $dT_i$ is the temperature change due to adiabatic volume change; $dT_{inlet}$ is the temperature change in the chamber due to entering of gas into this chamber with some other temperature; $dT_{inlet}$ is the temperature change due to change of gas mass in the chamber; $dT_T$ is the temperature change due to heat exchange of gas with the working chamber walls ($dT_T$ may have both positive and negative sign depending on flow direction). The parameters of the chamber under study are given subscript “i”; $V$ is the current volume value; $R$ is the individual gas constant.

The following main assumptions are taken for the mathematical model:

- continuity and homogeneity of gas medium;
- compressible gas obeys ideal gas laws;
- when gas enters the working chamber from adjacent chambers parameters of state of the working gas change simultaneously over the whole volume of the working chamber;
- change of gas potential and kinetic energy is negligible;
- gas leakage through scrolls face seals is negligible;
- parameters of state in suction and discharge chambers are constant;
- gas-dynamic pressure loss at the suction is not taken into account;
- gas-dynamic friction loss between the gas and the chamber walls is not taken into account;
- working gas flow through design clearances is assumed to be isothermic;
- working gas flow through discharge port is assumed to be adiabatic;
- heat exchange between the gas and working chambers walls is convective and may be described by Newton-Richman law with a correction coefficient depending on torsion angle of the fixed scroll involute on all heat exchanging surfaces of the working chamber.

In figure 1 a design diagram of a scroll machine taking into account heat and mass exchange in working chambers is presented where $Q_i$, $Q_{i+1}$, $Q_{i-1}$ is the heat supplied to (or removed from) the walls of working chambers $i$, $i+1$, $i-1$.

The total differential of mass change in a working chamber is given as

$$dm_i = dm_{i+1} - dm_{i-1} - dm_{Di},$$

where $dm_{i+1}$ is the gas mass increase in the working chamber connected with gas flow from the foregoing working chamber with high pressure; $dm_{i-1}$ is the gas mass decrease in the working chamber connected with gas flow into the working chamber with low pressure; $dm_{Di}$ is the gas mass increase or decrease in the working chamber connected with gas flow into the discharge port or from the discharge port.

![Figure 1. Design diagram of a scroll machine taking into account heat and mass exchange in working chambers.](image-url)
Taking into account mass exchange between chambers of a scroll machine is necessary for calculation. The mass of gas flowing through the clearances (slots) of a working chamber formed by working elements of a scroll compressor or the discharge port is given as

\[ dm_i = \phi_i F_i W_i \rho_i d \tau, \tag{4} \]

where \( \phi_i \) is the gas flow rate coefficient through the slot or the discharge port; \( F_i \) is the slot or the discharge port cross-section area; \( W_i \) is the gas adiabatic exhaust velocity; \( \rho_i \) is the gas density in the chamber from which gas flows; \( d \tau \) is the period of time over which exhaust occurs.

The mass of flowing gas is given \([8]\) as

\[ m = K_f / F \left[ \frac{1}{RT_2} (P_2^2 - P_1^2) \right]^{1/2}, \tag{5} \]

where \( K_f = \frac{1}{\sqrt{\ln(P_2/P_1)^2 + \xi + \varnothing}} \)

is the dimensionless experimental flow rate coefficient;

\( P_1, P_2 \) is the gas pressure before and after the slot; \( \xi \) is the coefficient taking into account local resistance at the inlet and the outlet of the slot; \( \varnothing \) is the friction coefficient of gas flow in the slot; \( \Sigma \) is the slot form coefficient; \( F = \delta_p h \) is the slot cross-section area; \( h \) is the scroll height; \( \delta_p \) is the profile clearance (the clearance between the scroll profile surfaces).

\( K_f \) takes into account flow rate decrease due to losses when gas flows through the slot.

When a form and dimensions of the slot, physical properties and parameters of gas before and after the slot are known then determining of flow rate through the slot reduces to determining of the flow rate coefficient \( K_f \).

The differential equations system of the working process is solved by the method of successive approximations. As a result, gas finite parameters in the chamber depending on shaft rotation angle are obtained including effective indicator and temperature charts of a compressor or a pump. In calculation the profile clearance \( \delta_p \) is specified in initial data as the constant value not depending on rotation angle of the orbiting scroll.

Finally, the main characteristics of the positive-displacement machines such as delivery rate, volumetric efficiency \( (\eta_v) \), adiabatic efficiency \( (\eta_{ad}) \) and total adiabatic efficiency \( (\eta_{ad.t}) \) are determined.

This calculation procedure is used for qualitative evaluation of main characteristics of scroll machines and for study and optimization of profile working elements geometry of such machines.

3. Calculation procedure of working process taking into account heat and mass exchange and elastic deformation of scrolls

Mathematical models which take into account change of operation clearances due to thermal and force deformations of scroll machine elements and heat exchange with external medium and heat exchange of gas with working chambers walls \([6, 7, 13, 14]\) are used for quantitative evaluation of main characteristics of scroll machines.

The following additional assumptions are taken:

- heat from face seals friction is not supplied to gas;
- heat exchange on external surfaces of the orbiting and the fixed scrolls is convective;
- heat-transfer coefficient for the section of the heat exchange surface under study is constant;
- heat conductivity processes in scrolls are considered in steady-state conditions;
- scrolls material is uniform, isotropic, and elastic;
- heat conductivity of scrolls is constant and doesn’t depend on temperature.

The design diagram of a scroll machine taking into account heat and mass exchange and elastic deformation of scrolls is presented in figure 2 where \( T_{Cs}, T_{cover}, T_{sh} \) are the temperatures of the casing, the cover and the shaft of the machine, respectively; \( T_a \) and \( T_w \) are the temperatures of cooling air and water; \( \alpha_p, \alpha_{cool1}, \alpha_{cool2}, \alpha_{air} \) are the heat exchange coefficients between the working fluid and working chamber walls, between cooling medium and the orbiting scroll surface, between cooling medium and
modulus of elasticity, $E_{OSC}$ and $E_{FSc}$ are the Poisson coefficients, $G_{OSC}$ and $G_{FSc}$ are the shear modulus, $ψ_{OSC}$ and $ψ_{FSc}$ are the coefficients of material linear expansion, $λ_{OSC}$ and $λ_{FSc}$ are the heat conductivity of the orbiting and the fixed scroll material, respectively.

For the fixed and the orbiting scrolls three-dimensional wedge cells mesh is built (figure 3).

Figure 2. Design diagram of an air scroll machine taking into account heat and mass exchange and elastic deformation of scrolls (dQ are heat fluxes through structural elements).

Figure 3. General view of three-dimensional mesh on the fixed scroll.

Scroll machines characteristics are determined by working fluid backward leakage through slots between scroll elements, and the leakage depends on clearances values.

There are two types of working clearances in a scroll machine: between scrolls profile surfaces (profile clearances) and between the top of one scroll and face disc of the other scroll (face clearances).

To minimize clearances is the most important line in efficiency increase of scroll machines working process. Moreover, for such machines to be failure proof it is necessary to set minimal
Face clearance in a scroll machine is

\[ \delta_f = \delta_{fm} + \Delta \delta_T^C + \Delta \delta_T^S - \Delta \delta_B^c - \Delta \delta_B^S + \Delta \delta_B^{ball} + \Delta \delta_F^S + \Delta \delta_F^B - \Delta \delta_F^{ball}, \tag{7} \]

where \( \delta_{fm} \) is the mounting face clearance value; \( \Delta \delta_T^C, \Delta \delta_T^S, \Delta \delta_B^c, \Delta \delta_B^S, \Delta \delta_B^{ball}, \Delta \delta_F^S, \Delta \delta_F^B, \Delta \delta_F^{ball} \) is the change of the face clearance due to axial thermal deformations of the casing, of one of the scrolls, of the adjoining base in the point with the maximal axial shift, and balls of the anti-rotation mechanism; \( \Delta \delta_T^F, \Delta \delta_F^S, \Delta \delta_F^B \) is the change of the face clearance due to axial force deformations of one of the scrolls and adjoining base in the point with the minimal axial shift; \( \Delta \delta_F^{ball} \) is the change of the face clearance due to elastic deformation in the point of contact of the most loaded ball with the rolling path.

In commercial machines, grooves are made at the top of both scrolls where seals are placed which nearly lean against the face disc providing the “tightness” increase of the face clearance. Face channel represents a plane rectangular slot the length of which is equal to the seal width along gas flow.

Delivery rate of the scroll compressor [9] is more sensitive to the change of face clearances than that of profile clearances (up to ten times).

Technological errors of manufacturing of scroll machine parts forming working chambers are not taken into consideration in equations (6) and (7) because they are manufactured with the help of high precision equipment.

Scrolls and casing thermal deformations due to high thermal load have the main effect on the change of working chambers clearances of oil free scroll compressors with pressure ratio \( \pi \) up to ten. Force deformations for the mentioned pressure ratio range are negligible.
The steady-state temperature distribution in a scroll is described by three-dimensional heat conductivity equation (Laplace’s law)

$$\Delta T_i(x) = 0, \quad x \in \Omega_i, \quad i = 1, 2.$$  

where $\Omega_i$ is the three-dimensional space occupied by a scroll; $x = (x_i, y_i, z_i)$ is the point of the three-dimensional space; $\Delta T_i(x) = \frac{\partial^2 T_i(x)}{\partial x_i^2} + \frac{\partial^2 T_i(x)}{\partial y_i^2} + \frac{\partial^2 T_i(x)}{\partial z_i^2}$ is the three-dimensional Laplace operator. $i = 1$ corresponds to the fixed scroll, $i = 2$ corresponds to the orbiting scroll, $T$ is the scroll temperature.

The boundary $B$ of the area $\Omega_i$ is divided into two parts $B = B_1 + B_2$, where $B_1$ is the boundary part where the temperature is specified. The boundary condition is set for $B_1$

$$T(x) = T_d(x), \quad x \in B_1,$$

where $T_d$ is the specified function.

The boundary condition describing heat exchange with external medium with the specified temperature and the opposite scroll is set for $B_2$

$$k_i \frac{\partial T_i(x)}{\partial n} + \left[ \gamma(x) [T_i(x) - \gamma_1(x)T_1(x)] - \gamma_2(x)T_2(x) \right] = q(x), \quad x \in B_2.$$

where $k_i$ is the heat conductivity of the scroll material; $i = 1, 2$; $\nu$ is the vector of the external normal to the scroll surface; $\gamma(x) = \alpha(x) + \beta(x)$; $\alpha$ is the heat exchange coefficient with external medium (air, cooling water, shaft, casing); $\beta = \lambda / \nu$ is the heat exchange coefficient with the opposite scroll; $\lambda$ is the air heat conductivity coefficient; $\nu$ is the distance between the corresponding sections of the orbiting and the fixed scrolls; $\gamma_1(x) = \alpha(x)/[\alpha(x) + \beta(x)]$; $T_0$ is the external medium temperature; $q$ is the heat flux (due to bearings heat release and seal lip friction); $T_{s \gamma}$ is the opposite scroll temperature.

The equation (8) with boundary conditions (9) and (10) was approximated by linear algebraic equations system according to finite elements method. Beforehand for the orbiting and the fixed scrolls three-dimensional mesh with triangular bases was built (figure 3). The mesh was adapted to the surfaces of the scroll rib, the base (discharge port is taken into consideration for the fixed scroll) and the boss. Isoparametric elements with nodes placed in the wedge vertices were used for approximation.

Elasticity equations for linear and angular deformations taking into account thermal deformations are

$$\epsilon_x = \frac{1}{E} \left[ \sigma_x - \mu \left( \sigma_y + \sigma_z \right) \right] + a \Delta T,$$

$$\epsilon_y = \frac{1}{E} \left[ \sigma_y - \mu \left( \sigma_x + \sigma_z \right) \right] + a \Delta T,$$

$$\epsilon_z = \frac{1}{E} \left[ \sigma_z - \mu \left( \sigma_x + \sigma_y \right) \right] + a \Delta T,$$

$$\gamma_{xy} = \frac{1}{G} \tau_{xy}, \quad \gamma_{yz} = \frac{1}{G} \tau_{yz}, \quad \gamma_{zx} = \frac{1}{G} \tau_{zx}$$

where $\epsilon_x$, $\epsilon_y$, $\epsilon_z$ are linear deformations in the directions $x$, $y$, $z$; $\gamma_{xy}$, $\gamma_{yz}$, $\gamma_{zx}$ are angular deformations; $\tau_{xy}$, $\tau_{yz}$, $\tau_{zx}$ are tangent stresses; $\sigma_x$, $\sigma_y$, $\sigma_z$ are normal stresses in the directions $x$, $y$, $z$; $a$ is the coefficient of material linear expansion; $\Delta T$ is the temperature change on heating; $E$ is the modulus of elasticity; $\mu$ is Poisson coefficient; $G$ is the shear modulus. There are two subscripts in angular deformations and tangent stresses symbols: the first denotes the direction of the normal to the plane, the second denotes the direction of tangent stress vector. Equations (11) present the general form of the elasticity law (Hooke’s law). The problem is solved in linear approach.

At present method of finite elements which is realized in software ANSYS [15] is widely used for calculation of stresses and deformations in design elements of intricate shape.
The obtained components of deformation vectors with the help of ANSYS are displayed in the form of isolines and fields. Temperature fields and pressure diagrams of the orbiting and the fixed scrolls are obtained according to the calculation programme of thermal fields and in the form of a data file are sent to the initial data file of the deformation calculation programme.

Calculation results are presented in the form of radial thermal and force shifts distribution at the scrolls wrap middle height section (to determine \( \Delta \delta_{FSc}(\varphi) \) and \( \Delta \delta_{OSc}(\varphi) \)) and in the form of axial thermal and force shifts distribution at the section over scrolls wrap face surface (to determine \( \Delta \delta_{Sc} \) and \( \Delta \delta_{B} \)).

The calculation sequence of the compressor working process according to this method is the following. Equations (1)-(5) are numerically solved. Coincidence of indicator and temperature diagrams of two consecutive approximations with predetermined accuracy is obtained by successive approximations. Then coincidence of working chamber walls temperature curves is obtained by successive approximations. Temperature of working chambers walls and scrolls temperature fields are obtained according to equation (8) with boundary conditions (9), (10). Temperature fields are used in equations (11) for calculation of scrolls thermal and force shifts, and gas pressure diagrams in working chambers are calculated according to (1). To solve equations (11) the following boundary conditions are set: shifts following three spatial directions are lacking in the scroll and casing mating area.

Thermal and force shift values are used for profile and face clearances according to equations (6) and (7). These clearances are used for mass transfer calculation according to (5).

This method of calculation makes it possible to obtain information about influence of any scroll machine parameter on its characteristics and to predict the main machine characteristics with sufficiently high accuracy on the design development stage.

4. Effect of calculation procedure on exactness of characteristics evaluation

To evaluate the influence of procedure on oil free air scroll compressor characteristics calculation accuracy different calculation procedures were used: without taking into account heat exchange in the working chamber; taking into account heat exchange with one wall of the working chamber; taking into account external heat exchange, heat exchange with all walls of the working chamber and the change of the working clearances caused by elastic deformation of scroll compressor parts.

The compressor worked with water cooling (\( \pi \) is the pressure ratio; \( n \) is the rotation speed; \( V_w \) is the flow rate of water cooling the fixed scroll from the rear side). In this machine cast iron scrolls were used.

Internal adiabatic efficiency \( \eta_{ad,int} \), was chosen as a comparative parameter. Some calculation characteristics are presented in figure 4 where solid lines correspond to calculated data and the dashed line corresponds to experimental data [12] \((n=3000 \text{ rpm}, V_w = 4 \text{ l/min})\).

The characteristics analysis makes it possible to make the following conclusions:

- heat exchange between gas and one wall of the working chamber (profile rib surface of the fixed scroll) results in inconsiderable decrease in \( \eta_{ad,int} \) by 1.0 – 3.2% (characteristic 2) in comparison with characteristic 1 obtained without regard for heat exchange despite the fact that the fixed scroll is water-cooled. This can be explained by heating of the sucked gas caused by convective heat exchange with the hotter fixed scroll rib and therefore input energy increase. Experimental temperatures were used in this procedure. \( \eta_{ad,int} \) may be significantly increased if more effective cooling of the fixed scroll is used but it results in design complication and considerable increase of the scroll compressor mass and bulk characteristics. For example, decrease of scroll rib temperature by 30 K results in \( \eta_{ad,int} \) increase by 6 – 9% (characteristic 3) in comparison with characteristic 1. The maximal increase corresponds to the optimal value of \( \pi \).
Figure 4. Effect of calculation procedure on exactness of internal adiabatic efficiency evaluation

1 - not taking into account heat exchange; 2, 3 - taking into account heat exchange with one wall of working chamber; 4 - taking into account external heat exchange and heat exchange in working chamber, 5 - taking into account external heat exchange, heat exchange in working chamber and elastic deformation of a scroll compressor elements.

- taking into account heat exchange with all walls of the working chamber (temperature of which was obtained by calculation according to the presented procedure) results in $\eta_{ad, int}$ decrease by $6.5-9.5\%$ (characteristic 4) in comparison with characteristic 1. It can be explained by the sucked gas heating due to additional convective heat exchange with the orbiting scroll surfaces with temperatures higher than the fixed scroll temperatures. It should be noted that characteristics 1 – 4 were obtained with the constant clearance $\delta_p$ (which doesn’t depend on $\phi$ and $\pi$) which is equal to the average calculation clearance value over the angle $\phi$ at $\pi=7$ of characteristic 5 ($\delta_p=0.035$ mm);

- taking into account heat exchange with all walls of the working chamber and the working clearances $\delta_p$ change over the shaft rotation angle $\phi$ results in $\eta_{ad, int}$ decrease by $8.5-19.5\%$ (characteristic 5) in comparison with characteristic 1, the larger decrease corresponding to larger $\pi$. It can be explained by the increase and redistribution of working clearances with increase of $\pi$ caused by elastic deformation of the compressor parts. The clearances increase due to repeated compression caused by leakage in intermediate chambers results in input energy increase.

According to the figure 4 the most close to the experimental characteristic is characteristic 5 (the difference is no more than $3-5\%$ over all experimental range).

It should be noted that an open type refrigeration scroll compressor (scrolls didn’t have developed cooling surface) reconstructed as an oil free air scroll compressor is used as an object of the research. The feature of this compressor construction is the presence of the chamber at the discharge side which was intended for separation of oil and refrigerant. This chamber after the compressor reconstruction is used for water cooling of the fixed scroll, and the orbiting scroll is not cooled purposely. On some working regimes in the experiment the temperature in the centre of the orbiting scroll reached 140°C and at the periphery (at the inlet port) it reached 80°C. For the cooled fixed scroll these values were 70°C and 40°C respectively. Due to imperfection of the compressor cooling system the heating resulted in considerable decrease of efficiency in general.

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6. References
[1] Jand D and Jeond S 2006 Int. J. Refrig. 29 744-53
[2] Uchikava N, Terrada H and Arata T 1987 Hitachi Rev. 36 155
[4] Chen Y, Halm N, Groll E and Braun J 2000 *Int. Compressor Engineering Conf.* (West Lafayette: Purdue University) pp 715-24

[5] Kauder K and Unvert T 2003 *Compressornaya tehnika i pnevmatika* (*Compressors and pneumatics*) 1 6-10 (in Russian)

[6] Lee G and Kim G 2001 *Int. Conf. on Compressors and their Systems* (London: City University) pp 123-32

[7] Peng B, Zhang H, Zhang L and Liu Z 2008 *Int. Compressor Engineering Conf.* (West Lafayette: Purdue University) pp 14-7

[8] Ibragimov E 2009 *Povishenie effektivnosti spiralnogo compressor suhogo szhatiya* (*Oil free scroll compressor efficiency increase*) (thesis) (Kazan: Kazan State Technological University) (in Russian)

[9] Paranin Yu, Yakupov R and Burmistrov A 2013 *Bulletin of Kazan Technological University* 16 (19) 267-71 (in Russian)

[10] Salikeev S, Burmistrov A and Raykov 2013 *Compressornaya tehnika i pnevmatika* (*Compressors and pneumatics*) 4 37-42 (in Russian)

[11] Burmistrov A, Raykov A and Salikeev S 2012 *Bulletin of Kazan Technological University* 15 (8) 257-8 (in Russian)

[12] Paranin Y 2011 *Sovershenstvovanie metoda rascheta rabochego processa spiralnogo compressor suhogo szhatiya s ispolzovaniem rezultatov experimentalnyh issledovanii* (*Oil free scroll compressor working process calculation method development using experimental results*) (Kazan: Kazan State Technological University) (in Russian)

[13] Paranin Yu and Khisameev I 2011 *Compressornaya tehnika i pnevmatika* (*Compressors and pneumatics*) 5 16-23 (in Russian)

[14] Paranin Yu, Yakupov R and Burmistrov A 2014 *Bulletin of Kazan Technological University* 17(1) 248-52 (in Russian)

[15] Structural Analysis Solutions 2014 *ANSYS Inc.*

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