Measurement of the $H Z \gamma$ Coupling at the Future Linear $e^+e^-$ Collider

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Abstract

We examine the prospects for measuring the $H Z \gamma$ coupling of a Standard Model-like Higgs boson with a mass between 120 and 160 GeV at the future TESLA linear $e^+e^-$ collider, assuming an integrated luminosity of 1 ab$^{-1}$ and a center-of-mass energy of 500 GeV. We consider the Higgs boson produced in association with $\nu_e\bar{\nu}_e$ via the $WW$ fusion reaction $e^+e^- \rightarrow \nu_e\bar{\nu}_e H$, followed by the rare decay into a $Z$ boson and a photon, $H \rightarrow Z\gamma$. Accounting for all main background contributions, different selection procedures are discussed. Uncertainties on the $H \rightarrow Z\gamma$ branching fraction of approximately 48% (27%, 44%) can be achieved in unpolarised $e^+e^-$ collisions for $M_H = 120$ (140, 160) GeV. With appropriate initial state polarisations $\Delta BF(H \rightarrow Z\gamma)/BF(H \rightarrow Z\gamma)$, or the precisions on the $H \rightarrow Z\gamma$ partial width, can be improved to 29% (17%, 27%) and provide valuable information on the $H Z \gamma$ coupling. We regard our results as a convincing confirmation of the great potential of a linear collider to access and reliably measure important parameters of the Higgs boson despite initially overwhelming background with final state signature similar to the signal events.
1 Introduction

Following the discovery of the Higgs boson, one of the main tasks of a future linear $e^+e^-$ collider will be precise model-independent measurements of its fundamental couplings to fermions and bosons and its total width [1]. In particular, the determination of the couplings of the Higgs boson to the other fundamental particles will be a crucial test of the nature of the Higgs particle. In this respect future linear colliders will play a major role. Different colliding options with different beam polarisations combined with adjustable center-of-mass energies in a wide range and the clean environment in these machines will allow for rather precise determinations of these couplings.

A lot of detailed studies of the Standard Model (SM) Higgs couplings to fermions and $W$ and $Z$ bosons can be found in the literature [2]. These studies demonstrate the ability of a linear collider to access these couplings with precision of a few percent. Also the trilinear Higgs self-coupling in the double Higgs production processes $e^+e^- \rightarrow ZHH$ and $e^+e^- \rightarrow \nu\bar{\nu}HH$ [3] are within the possibilities of experimental verification, although with substantially lower precision.

Another set of important Higgs boson couplings is represented by the effective vertices $Hgg$, $H\gamma\gamma$ and $HZ\gamma$. These couplings do not occur at the tree-level but are induced by loop diagrams [4]. Since the Higgs interaction is proportional to particle masses, loop contributions of massive fermions do not decouple, and these vertices could therefore serve to count the number of particles which couple to the Higgs boson. The $Hgg$ vertex can be accessed through the $H \rightarrow gg$ decay in $e^+e^-$ collisions [5] or in the fusion reaction $gg \rightarrow H$ at the LHC [6]. The $H\gamma\gamma$ coupling can be determined either in $e^+e^-$ and LHC-$pp$ interactions when the Higgs decays into two photons [7],[8] or directly by means of the Compton back-scattering $\gamma\gamma$ fusion process $\gamma\gamma \rightarrow H \rightarrow X$, with probably the best precision [9].

In this study we explore the potential of a linear $e^+e^-$ collider to measure the $HZ\gamma$ coupling through the rare $H \rightarrow Z\gamma$ decay, for masses of the Higgs particle in the range 120 to 160 GeV. Precise electroweak data provide the existence of a light Higgs boson with a mass below 193 GeV with 95% confidence level [10], with a preference for $M_H$ close to 120 GeV. There are also hints of a signal in the direct search in $e^+e^- \rightarrow HZ$ at LEP2, with a lower mass limit of $M_H \geq 114.1$ GeV at 95% CL [11].

The reaction which will be used to explore the branching fraction $BF(H \rightarrow Z\gamma)$ is

$$e^+e^- \rightarrow \nu\bar{\nu}Z\gamma,$$

assuming a Higgs boson mass $M_H = 120$, 140 and 160 GeV, $\sqrt{s} = 500$ GeV and an integrated luminosity of $L = 1$ ab$^{-1}$.

The statistical precision for $BF(H \rightarrow Z\gamma)$ is mainly determined by $\sqrt{S+B}/S$, where $S$ and $B$ are respectively the number of signal and background events within a small interval of the $Z\gamma$ invariant mass, centered around $M_H$. Hence, evaluation of all relevant signal and background processes and optimization of selection procedures are mandatory, taking into account acceptances and resolutions of a linear collider detector.

Our analysis is, to our knowledge, the first on the study of $BF(H \rightarrow Z\gamma)$ at any (future) collider. It includes the complete irreducible background and all main reducible background contributions expected. Since the SM branching fraction $H \rightarrow Z\gamma$ is very small and there is in particular large irreducible background, the most important hadronic $Z$ decays, $Z \rightarrow q\bar{q}$, are accounted for in this analysis. Although leptonic ($e$, $\mu$) $Z$ decays provide a clean final state signature and best $Z$ boson recognition, their rates are however too small to include them at this stage of the analysis or to allow for an independent approach. $Z \rightarrow \nu\bar{\nu}$ decays are not considered since they prevent $Z$ boson reconstruction in reaction (1).

The paper is organized as follows. In Section 2 we discuss simulation of the Higgs signal and background events and their detector response. In Section 3 we present the results for unpolarised
BF($H \rightarrow Z\gamma$) measurements, using different selection procedures. In Section 4 improvements to the $H \rightarrow Z\gamma$ branching fraction measurements are discussed when e.g. beam polarisation is accounted for in signal and background events. Also, expectations on BF($H \rightarrow Z\gamma$) from the Higgs-strahlung process and possible systematic errors and the effect of overlap with $\gamma\gamma \rightarrow \text{hadrons}$ are reviewed. Section 5 summarizes the conclusions.

2 Event generation

In $e^+e^-$ collisions the Standard Model Higgs boson is predominantly produced by two different processes, the Higgs-strahlung process

$$e^+e^- \rightarrow ZH$$

and the weak boson (WW and ZZ) fusion reactions

$$e^+e^- \rightarrow \nu_e\bar{\nu}_eH$$

$$e^+e^- \rightarrow e^+e^-H$$

The ZZ fusion process (4) is strongly suppressed with respect to the WW fusion process (3) by about a factor of 10, rather independent of $\sqrt{s}$. Therefore, the production channel (4) is ignored in this study. The SM tree-level diagrams contributing to the signal reactions (2) and (3), including the $Z \rightarrow \nu_e\bar{\nu}_e$ and $H \rightarrow Z\gamma$ decays, are shown in Fig. 1.

In the production $\times$ decay approximation the processes (2) and (3) are factorizable parts of the Higgs signal diagrams for the 2-to-4 body reaction (1) with electron neutrinos in the final state. In other words, the amplitude squared of diagram 1 in Fig. 1 integrated over the phase space gives $\sigma(e^+e^- \rightarrow \nu_e\bar{\nu}_eH) \cdot BF(H \rightarrow Z\gamma)$ and the amplitudes squared of diagrams 2 and 3 give $\sigma(e^+e^- \rightarrow ZH) \cdot BF(Z \rightarrow \nu_e\bar{\nu}_e) \cdot BF(H \rightarrow Z\gamma)$. Thus, to be most general in our analysis, events of reaction (1) were generated for the complete set of tree-level diagrams (see Fig. 2 for the contributing background diagrams) by means of the program package CompHEP [12], including initial state bremsstrahlung and beamstrahlung for the TESLA linear collider option [13].

The present version of CompHEP performs analytic calculations of the matrix element squared, generates an optimized Fortran code and generates a flow of events. In addition, it provides for the user an appropriate kinematical scheme for the integration over the four-body phase space. The basic input parameters are taken from the report of the Particle Data Group [14] or are as listed here: $m_b = 4.3$ GeV, $\alpha_{EW} = 1/128$, $M_Z = 91.19$ GeV, $sin^2\theta_W = 0.23$ and $\Gamma_Z = 2.50$ GeV. The CompHEP-PYTHIA interface package [15] was used to simulate the $\nu_e\bar{\nu}_eZ\gamma \rightarrow \nu_e\bar{\nu}_e qq\gamma$ signature. In this way, Higgs boson production and the complete irreducible background as well as possible interferences are taken into account.
Figure 2: Background diagrams for the reaction $e^+e^- \rightarrow \nu_{\tau} \bar{\nu}_{\tau} Z\gamma$. 
For unstable particles, Breit-Wigner formulae have been used for the s-channel propagators. The Higgs boson width and the $H \to Z\gamma$ branching fraction were imported from the program package HDECAY [16]. $\text{BF}(H \to Z\gamma)$ depends on the Higgs mass and is largest near $M_H = 144$ GeV. Some values of this branching fraction, the total Higgs width and the $HZ\gamma$ effective coupling constant relevant for our study are summarized in Table 1.

| $M_H$, GeV | $\text{BF}(H \to Z\gamma)$ | $\Gamma_{\text{tot}}$, GeV | $\lambda_{HZ\gamma}$ |
|----------|----------------|----------------|----------------|
| 120      | 1.1·10^{-3}    | 3.6·10^{-3}    | 5.4·10^{-8}    |
| 140      | 2.5·10^{-3}    | 8.1·10^{-3}    | 6.2·10^{-8}    |
| 160      | 1.2·10^{-3}    | 8.1·10^{-2}    | 8.8·10^{-8}    |

Table 1: Branching fractions, total widths and effective coupling constants of the SM Higgs boson for $M_H = 120-160$ GeV.

As discussed in ref. [18], a favored signal to background situation is expected for the Higgs $WW$ fusion reaction (3) at e.g. $\sqrt{s} = 500$ GeV, and we will consider only this case in the following.

Due to the small cross-section expected for the signal reaction $e^+e^- \to \nu_e\bar{\nu}_eH \to \nu_e\bar{\nu}_eZ\gamma$, diagram 1 in Fig. 1, we only rely on events with the most important $Z \to q\bar{q}$ decays. Therefore, events of process (1) are characterized by two hadronic jets originating from the $Z$ boson, together with an energetic photon and large missing energy due to the two final state neutrinos. The invariant mass of the $Z$ and the photon should equal $M_H$.

The irreducible background expected from reaction (1) (the diagrams in Fig. 2) was accounted for at the same level as the signal events. Contributions from $Z$ decays into $\mu$- and $\tau$-neutrinos which would occur from diagrams 7, 9, 23, 24, 26 and 29 were effectively removed by an appropriate missing mass cut (section 3). Possible contributions from diagrams 5, 6, 21 and 22 with $n_1 = N_1 = \nu_\mu$ or $\nu_\tau$ were also calculated and found to be negligible due to large off-shell $n_1 \to ZN_1$ decay with the $Z$ boson close to its nominal on-shell mass value. The surviving $\nu_\mu$ and $\nu_\tau$ background rates were found to be smaller than 1% of the total irreducible background. An important part of the background was found to arise from the $W$-exchange diagrams, but significant contributions were also found to be due to the single bremsstrahlung production process $e^+e^- \to ZZ\gamma \to \nu_e\bar{\nu}_eZ\gamma$.

As the cross-section for the irreducible background is more than two orders of magnitude larger than the signal cross-section, we first applied the following principal cuts at the generation level, to both the signal and background events:

- the photon energy should exceed 10 GeV and
- the polar angle of the photon should lie in the range 5 to 175 degrees.

After these criteria, practically all ($\sim 96\%$) Higgs events survive, while the overwhelming irreducible background was substantially reduced. The cuts also largely avoid any infrared and collinear singularities in the calculation of the background amplitudes, as might be deduced from diagrams in Fig. 2.

Possible reducible backgrounds to $e^+e^- \to \nu_e\bar{\nu}_eH$ events which might mimic the signal such as the large event rate reactions $e^+e^- \to WW(\gamma), eeW(\gamma), e^+e^-(\gamma^*/Z)(\gamma), t\bar{t}(\gamma), WWZ(\gamma)$ and $ZZZ(\gamma)$, with beamstrahlung, initial state radiation, final state radiation and radiation from the $W$ boson itself, were generated by either PYTHIA [17] or CompHEP [12]. The $e^+e^- \to eW(\gamma), e^+e^-(\gamma^*/Z)(\gamma)$ events were obtained by $e - \gamma - e$ splitting and subsequent $\gamma e \to \nu W$ respectively $\gamma e \to e(\gamma^*/Z)$ interactions by PYTHIA, with proper cross section normalizations. Only those events were used for further analyses if at least one final state photon exists with principal cut properties. It has been
found that after detector response simulation and enforcing the same selection procedures as for the signal events (see below), $t\bar{t} (\gamma)$, $WW(\gamma)$ and $ZZZ(\gamma)$ events were effectively discarded, also if the two $W$ bosons from the top quarks and the prompt produced $W$'s decay leptonically leading together with $Z \rightarrow q\bar{q}$ to a topology similar to the signal topology. Events from the $e^+e^- \rightarrow WW(\gamma)$, $e\nu W(\gamma)$ and $e^+e^-(\gamma^*/Z)$ processes could however not be removed to a negligible level. Their contributions will be discussed below for each selection procedure applied.

A further potential background is expected from the process $e^+e^- \rightarrow q\bar{q}\gamma$, where initial state radiation and beamstrahlung reduce the center-of-mass energy available close to the Higgs mass values. Only events with center-of-mass energy below 200 GeV, a $q\bar{q}$ system consistent with the $Z$ boson and a photon with large transverse energy were accepted and enforced to the selection procedures. We found that radiative return events add some non-negligible background to the final reconstructed $Z\gamma$ mass, with some uncertainties due to the ISR model used.

Also possible contributions from Higgs-strahlung process (2) with the dominant $H \rightarrow b\bar{b}$ and $WW^*$ decays were accounted for and found to contribute with at most four events thanks to our dedicated selection procedures.

All surviving reducible background events were included in the final $Z\gamma$ invariant mass distributions and taken into account in precision estimations for the $H \rightarrow Z\gamma$ branching fraction.

3 BF($H \rightarrow Z\gamma$) measurements at 500 GeV

Based on the results of ref.[18] that the branching fraction BF($H \rightarrow Z\gamma$) should be preferably best measured in the $WW$ fusion process (3) at a high-luminosity linear $e^+e^-$ collider, we examine the prospects of measuring this quantity at $\sqrt{s} = 500$ GeV, for Higgs boson masses of 120, 140 and 160 GeV and an integrated luminosity of 1 ab$^{-1}$. Since the $WW$ fusion cross-section rises logarithmically with $\sqrt{s}$, large energies are mandatory and the very small SM $H \rightarrow Z\gamma$ branching ratio requires large accumulated luminosity.

The detector response for all generated signal and background events was simulated with the parametrized detector simulation program SIMDET-v4 [19] using parameters as presented in the Technical Design Report [20].

Throughout this paper, it was demanded that each reconstructed event involves more than three charged particles in the final state and the total visible energy is less than 240 GeV with the transverse component relative to the beam direction below 210 GeV. Important for further analyses is the requirement that the missing mass (caused by the two undetected neutrinos) lies between 180 and 400 GeV. This cut ensures clean elimination of the Higgs-strahlung process, $e^+e^- \rightarrow ZH \rightarrow \nu\bar{\nu}Z\gamma$, with a missing mass close to the $Z$ boson mass.

Large background event samples and tiny signal event rates need to pursue different strategies for extracting signal events. In order to account for the distinct properties of the final state $\nu\bar{\nu}q\bar{q}\gamma$, we start with a conventional method using consecutive cuts on kinematical variables, while more sophisticated selection procedures are followed to hopefully achieve better signal-to-background event ratios and hence smaller uncertainties on BF($H \rightarrow Z\gamma$) or the $HZ\gamma$ coupling.

3.1 Event selection using consecutive cuts

In a first attempt, a conventional method using simple consecutive cuts was applied to isolate Higgs signal events. We start to account for the distinct properties of the photon from the Higgs decay, denoted as $\gamma_H$, by demanding an energy greater than 20 GeV with a transverse component of not less than 15 GeV. In events where more than one photon candidate exists the photon with largest
transverse momentum was selected as the Higgs decay candidate. Thus, most of the bremsstrahlung photons with relatively small energy at low polar angles are eliminated. Furthermore, the $\gamma_H$ candidates should have no particle in a cone of half-angle of 10° around its direction, i.e. they are demanded to be isolated. Once a Higgs decay photon candidate had been found, it was removed from the list of all final state particles, for which in turn, the jet clustering algorithm of PYTHIA (subroutine PUCLUS) [17] was enforced to isolate two jets. Each jet was required to pass the following cuts:

- the number of particles is greater than 3;
- the jet energy exceeds 8 GeV;
- the angle between any two jets is larger than 20°;
- the jet polar angle is within $|\cos \Theta_{jet}| < 0.95$.

Thus, only well-measured and clearly separated jets were accepted. The compatibility of the dijet system with the $Z$ boson hadronic decay was quantified by demanding that its invariant mass is within 84 to 105 GeV. The lower limit of 84 GeV was chosen to reject part of the otherwise large $WW(\gamma)$ and $e\nu W(\gamma)$ background contributions. This requirement indicates the importance of sufficient dijet invariant mass resolution of the detector in order to differentiate $Z \rightarrow q\bar{q}$ from $W \rightarrow q\bar{q}'$ decays. Then this dijet system was combined with $\gamma_H$ to establish the Higgs particle in the $Z\gamma$ invariant mass. In order to enforce an improved mass resolution of the $Z\gamma$ system, a 1-constraint fit requiring $M_{jj} = M_Z$ was performed.

These selection criteria were not varied with the Higgs masses considered. It was found that changes of the cut parameters used within reasonable limits had small or negligible effects on the signal-to-background ratios. Most of the remaining reducible background was due to the $e^+e^- \rightarrow WW(\gamma) \rightarrow lvq\bar{q}'(\gamma)$ and $e\nu W(\gamma) \rightarrow e\nu q\bar{q}'(\gamma)$ channels as well as radiative return events with an event topology similar to the signal event topology. We found that this background amounts to approximately 38% of the irreducible background for Higgs masses of 140 and 160 GeV, while for $M_H = 120$ GeV it was close to 16%. For the 120 GeV Higgs case, few Higgs-strahlung $e^+e^- \rightarrow ZH$ events were found to survive. Also some $e^+e^- (\gamma^*/Z)(\gamma)$ events were retained above $M_{Z\gamma} = 130$ GeV.

By following the strategy outlined above and assuming $\mathcal{L} = 1$ ab$^{-1}$ of integrated luminosity, we obtain the $Z\gamma$ invariant mass distribution for the surviving signal and background events as shown in Fig. 3. Higgs signals are evident on non-negligible background, with best significance for $M_H = 140$ GeV. Clearly, the $H \rightarrow Z\gamma$ decay mode is not favoured for Higgs boson searches but it allows to estimate the branching fraction for Higgs masses close to 120 or 160 GeV.

Selection efficiencies for the signal and the irreducible background, the number of signal ($S$) and background ($B$) events in the $Z\gamma$ mass range between 117 and 123 (137-143, 157-163) GeV, the significances $S/\sqrt{B}$ and the statistical precisions $\sqrt{S+B}/S$ obtained on $\sigma(e^+e^- \rightarrow \nu_e\bar{\nu}_eH) \cdot BF(H \rightarrow Z\gamma) \cdot BF(Z \rightarrow q\bar{q})$ are presented in Table 2. The rather small signal selection efficiencies obtained are mainly due to the difficulty to select $\gamma_H$, the photon from the Higgs decay, out of all photons in the final state. Stringent requirements on its energy and transverse momentum, together with further cuts on event and jet properties, were found to be necessary in order to obtain acceptable signal-to-background event rates.

Accounting for all $Z$ decays and the surviving reducible background the relative precisions on the $H \rightarrow Z\gamma$ branching fraction are then deduced after convolution with the uncertainty of the inclusive $\nu_e\bar{\nu}_eH$ cross-section of few percent [20]: $\Delta BF(H \rightarrow Z\gamma)/BF(H \rightarrow Z\gamma) = 65\%$ (28%, 57%) for $M_H = 120$ (140, 160) GeV, assuming 1 ab$^{-1}$ integrated luminosity at $\sqrt{s} = 500$ GeV.
Figure 3: $M_{Z\gamma}$ invariant mass distribution for surviving signal and background events at $\sqrt{s} = 500$ GeV, assuming 1 ab$^{-1}$ integrated luminosity.

|                  | $M_H = 120$ GeV | 140 GeV | 160 GeV |
|------------------|------------------|---------|---------|
| Selection efficiency (%) | 23.9 (0.67)      | 27.7 (0.52) | 24.9 (0.45) |
| Number of events/1 ab$^{-1}$ | 18 (149)         | 45 (116) | 19 (99)   |
| Significance $S/\sqrt{B}$ | 1.47             | 4.18    | 1.91     |
| Precision $\sqrt{S + B}/S$ | 0.72             | 0.28    | 0.57     |

Table 2: Selection efficiencies (see text) for both signal and background $\nu_\ell \bar{\nu}_\ell q \bar{q} \gamma$ signature, together with significances and precisions of $\sigma(e^+e^- \rightarrow \nu_\ell \bar{\nu}_\ell H) \cdot BF(H \rightarrow Z\gamma) \cdot BF(Z \rightarrow q\bar{q})$ for $M_H = 120$, 140 and 160 GeV, at the $e^+e^-$ linear collider with $\sqrt{s} = 500$ GeV. A total integrated luminosity of 1 ab$^{-1}$ is assumed.
3.2 Event selection using a jet finder

The events generated by CompHEP or PYTHIA including the principal cuts and reconstructed by SIMDET were passed through a jet cluster algorithm. The concern is that the jet finder is able to isolate the Higgs decay photon, $\gamma_H$, from all other final state particles due to its distinct properties, namely the high transverse momentum and its relative isolation from all other particles. The remaining particles in the final state were then clustered in two hadronic jets, with an invariant mass compatible with $Z \rightarrow q\bar{q}$ decays, i.e. $M_{jj}$ was required to lie in the range 84 to 105 GeV $^1$. We applied two cluster algorithms in order to obtain some confidence on the jet finders and to control systematic uncertainties in extracting signal and background events. Subroutine PUCLUS of the PYTHIA package [17] was enforced to isolate three or more jets, with one jet to be consistent with a photon. The DURHAM algorithm [21] with $y_{cut} = 0.004$ was applied to isolate events with at least three jets, with one jet required to pass the photon selection criteria. In general, the results from both algorithms were found to be very similar except for a somewhat stronger reducible background rejection of the PYTHIA algorithm. Therefore the numbers presented in the following are from the PUCLUS jet finder. For the photon-jet candidate, $\gamma_H$, it was demanded that it involves only one dominant neutral electromagnetic shower compatible with originating from a single photon and was not associated with any charged particle. This jet should also not contain any neutral hadronic activity and if it was accompanied by one or two further photons, their total energy should not exceed 10% of the jet energy. Finally we required that $\gamma_H$ has an energy transverse to the beams, $E_T$, greater than 15 GeV, which is exploited to increase the signal-to-background ratio. A variation of $E_T$ between 8 and 20 GeV verified that the value chosen optimizes in some way $S/B$ and ensures a statistically significant signal event rate. If one of these requirements failed the photon-jet candidate was discarded and, if no other candidate was found, the event was rejected from the analysis.

The resulting dijet invariant mass, $M_{jj}$, is clearly dominated by the $Z$ boson at 91 GeV, with a total width at half-maximum of approximately 4.5 GeV. This gives us confidence that the jet clustering algorithms used isolates adequately the hadronic jets and the $\gamma_H$ candidate might result from the Higgs decay. To preserve good mass resolution for the final $Z\gamma$ system the two hadronic jets were fitted to the constraint $M_{jj} = M_Z$.

Remaining reducible background, mainly due to $e\nu W(\gamma)$, $e^+e^-(\gamma^*/Z)(\gamma)$ and radiative return events, was found to grow slowly with $M_{Z\gamma}$ having a broad maximum around 140 GeV. It contributes at most 42% of the irreducible background in the Higgs mass region considered. Fig. 4 shows the reconstructed $Z\gamma$ mass for signal and summed background events after application of the jet finder algorithm PUCLUS and the cuts mentioned above. Higgs signals are visible at the assumed masses, with most abundant Higgs production at 140 GeV. Once the Higgs mass is known from searches in dominant decay channels, reliable $BF(H \rightarrow Z\gamma)$ estimates are possible even for the worst case of $M_H = 120$ GeV.

Selection efficiencies for the $\nu_e \bar{\nu}_e Z\gamma \rightarrow \nu_e \bar{\nu}_e q\bar{q}\gamma$ signature, the number of signal ($S$) and irreducible background ($B$) events in the mass range $M_{Z\gamma}$ between 117 and 123 (137-143, 157-163) GeV, the significances $S/\sqrt{B}$ and the statistical precisions $\sqrt{S + B}/S$ obtained on $\sigma(e^+e^- \rightarrow \nu_e \bar{\nu}_e H) \cdot BF(H \rightarrow Z\gamma) \cdot BF(Z \rightarrow q\bar{q})$ are presented in Table 3.

Thus, application of the PYTHIA jet finder for Higgs event selection and accounting for all $Z$ decays as well as the surviving reducible background yields for the relative precisions on the $H \rightarrow Z\gamma$ branching fraction $\Delta BF(H \rightarrow Z\gamma)/BF(H \rightarrow Z\gamma) = 70\%$ (31\%, 57\%) for $M_H = 120$ (140, 160) GeV, after convolution with the uncertainty of the $\nu_e \bar{\nu}_e H$ production cross-section [20] and assuming 1 ab$^{-1}$ integrated luminosity at $\sqrt{s} = 500$ GeV.

$^1$For the few events with three isolated hadronic jets it was demanded that their invariant mass lies within the same limits.
Figure 4: $M_{Z\gamma}$ invariant mass distributions for surviving signal and background events at $\sqrt{s} = 500$ GeV, assuming 1 ab$^{-1}$ integrated luminosity.

| $M_H$ (GeV) | Signal | (Background) |
|-------------|--------|--------------|
| 120         | $13.0 \ (0.27)$ | $13.9 \ (0.26)$ |
| 140         | $13.9 \ (0.25)$ | $14 (55)$ |
| 160         | $14 (55)$ | $14 (55)$ |

Table 3: Jet finder selection efficiencies for both signal and background $\nu_e\bar{\nu}_e q\bar{q}\gamma$ signature, together with significances and precisions of $\sigma(e^+e^- \rightarrow \nu_e\bar{\nu}_e H) \cdot BF(H \rightarrow Z\gamma) \cdot BF(Z \rightarrow q\bar{q})$ for $M_H = 120, 140$ and 160 GeV, at the $e^+e^-$ linear collider with $\sqrt{s} = 500$ GeV. A total integrated luminosity of 1 ab$^{-1}$ is assumed.

### 3.3 Event selection by means of 'Higgs-likeness'

The results obtained by exploiting consecutive cut or jet finder techniques lead to small signal samples accompanied by large background. Demanding further cuts beyond the ones discussed so far does not
improve $S/B$ significantly, but would instead reduce signal event rates to a level precluding reasonable $H \rightarrow Z\gamma$ branching fraction measurements. This is mainly because the irreducible background has similar final state signature as the $H \rightarrow Z\gamma$ events and exceed this sample by typically two or more orders of magnitude before any selection procedure.

Such a situation calls for a more sophisticated selection approach where also slight differences between signal and background events are taken into account. In this respect, a 'likelihood factor' was constructed giving a measure of the probability that an event is part of the signal. For any particular event, kinematical variables of the final state photon, the $Z$ boson respectively the two jets and the missing neutrino system were combined into a global discriminant variable $P_H$. This quantity was constructed from a variety of normalised variables based on large statistics samples of simulated signal and background events. The variables used account for possible kinematic differences between the Higgs events (diagram 1 in Fig. 1) with the isotropic $H \rightarrow Z\gamma$ decay and the background (diagrams of Fig. 2). In particular, the transverse momentum of the photon, its cms scattering angle, the cosine of the polar angle of the $Z$ boson, the photon polar and azimuthal decay angles in the Higgs rest frame, the cms polar and azimuthal angles between the photon and the $Z$, the cms photon energy, the collinearity angle between the electron beam and the photon, the coplanarity angles of the beam, the photon and the $Z$ boson as well as the beam, the Higgs and the photon in the Higgs rest frame, the transverse masses of the photon and the missing system as well as the Higgs and the missing system were considered. In events where more than one photon candidate exists (about 48 % of the cases) the photon with largest energy was selected as the Higgs decay candidate. For each event which passes the principal cuts, the event and jet quality cuts (see sect. 3.1) and the fit constraint $M_{jj} = M_Z$, signal and background probabilities were then calculated, and by multiplication of all signal probabilities the sensitivity for an event to be a Higgs candidate was maximised. The quantity so obtained is constraint to lie in the region $[0;1]$. Background events are preferably distributed at low $P_H$ values while for Higgs signal events $P_H$ is close to unity. Since several variables included in the analysis vary with the Higgs mass, signal probabilities were individually determined for $M_H = 120$, 140 and 160 GeV. Therefore, the 'Higgs-likeness' exists for each Higgs mass considered. Fig. 5 shows, as an example, $P_H$ for $M_H = 120$ GeV signal and the sum of signal and background events. Similar distributions were obtained for $M_H = 140$ and 160 GeV. Finally, only events were retained if the energy of the Higgs photon candidate, $\gamma_H$, was greater than 20 GeV with the transverse component $E_T > 15$ GeV. Reducible background was found to arise from $WW(\gamma), e\nu W(\gamma)$ and radiative return events, with rates of at most 32% of the irreducible background at the 160 GeV Higgs mass.

Fig. 6 shows the $Z\gamma$ invariant mass spectra for the luminosity adjusted $\nu_e\bar{\nu}_e Z\gamma$ signal and background events surviving the cut $P_H > 0.98$, for $M_H = 120, 140$ and 160 GeV. In all cases, convincing Higgs signals are evident on non-negligible backgrounds. Compared to the selection procedures discussed in the previous sections best Higgs signal significancies were obtained. In particular, the excess of events at 140 GeV is very encouraging. But also the somewhat degraded event rates at $M_H = 120$ and 160 GeV allow for reliable $H \rightarrow Z\gamma$ branching fraction estimates. Variation of the discriminant variable $P_H$ between 0.82 and 0.98 does not improve $S/B$, but would rather lower the signal-to-background ratio.

Selection efficiencies for the $\nu_e\bar{\nu}_e Z\gamma \rightarrow \nu_e\bar{\nu}_e q\bar{q}\gamma$ signature, the number of signal ($S$) and irreducible background ($B$) events in the mass range $M_{Z\gamma}$ between 117 and 123 (137-143, 157-163) GeV, the significances $S/\sqrt{B}$ and the statistical precisions $\sqrt{S + B}/S$ obtained on $\sigma(e^+e^- \rightarrow \nu_e\bar{\nu}_e H) \cdot BF(H \rightarrow Z\gamma) \cdot BF(Z \rightarrow q\bar{q})$ are presented in Table 4.
Figure 5: Distribution of the discriminant variable \( P_H \) for \( e^+e^- \rightarrow \nu_e \bar{\nu}_e H \rightarrow \nu_e \bar{\nu}_e Z\gamma \) signal events (shaded histogram) and the sum of signal and background contributions.

|                  | Signal \( M_H = 120 \text{ GeV} \) | Signal \( M_H = 140 \text{ GeV} \) | Signal \( M_H = 160 \text{ GeV} \) |
|------------------|----------------------------------|----------------------------------|----------------------------------|
| Selection efficiency (%) | 17.4 (0.23)                     | 13.9 (0.20)                     | 15.9 (0.16)                     |
| Number of events/1 ab\(^{-1}\) | 16 (51)                         | 29 (44)                         | 16 (41)                         |
| Significance \( S/\sqrt{B} \) | 2.24                            | 4.37                            | 2.50                            |
| Precision \( \sqrt{S+B}/S \) | 0.51                            | 0.29                            | 0.47                            |

Table 4: "Higgs-likeness" selection efficiencies for both signal and background \( \nu_e \bar{\nu}_e q\bar{q} \gamma \) signature, together with significances and precisions of \( \sigma(e^+e^- \rightarrow \nu_e \bar{\nu}_e H) \cdot BF(H \rightarrow Z\gamma) \cdot BF(Z \rightarrow q\bar{q}) \) for \( M_H = 120, 140 \) and \( 160 \text{ GeV} \), at the \( e^+e^- \) linear collider with \( \sqrt{s} = 500 \text{ GeV} \). A total integrated luminosity of 1 ab\(^{-1}\) is assumed.

Accounting for all \( Z \) decays and the surviving reducible background the relative precisions on the \( H \rightarrow Z\gamma \) branching fraction are then deduced after convolution with the uncertainty of the inclusive \( \nu_e \bar{\nu}_e H \) production rates [20]: \( \Delta BF(H \rightarrow Z\gamma)/BF(H \rightarrow Z\gamma) = 48\% \ (27\%, \ 44\%) \) for \( M_H = 120 \ (140, \ 160) \text{ GeV} \), assuming 1 ab\(^{-1}\) integrated luminosity at \( \sqrt{s} = 500 \text{ GeV} \).
Figure 6: $M_{Z\gamma}$ invariant mass distributions for surviving signal and background events at $\sqrt{s} = 500$ GeV, for $M_H = 120$, 140 and 160 GeV and 1 ab$^{-1}$ integrated luminosity.
3.4 Discussion of the results

Relatively independent on the selection technique exploited we would register typically 15 to 30 $H \to Z\gamma$ events in reaction (3) and three or more times background events in the window $M_H \pm 3$ GeV at $\sqrt{s} = 500$ GeV for an integrated luminosity of 1 ab$^{-1}$. A comparison of the selection procedures applied favors the ‘Higgs-likeness’ method, in particular for the 120 GeV Higgs mass case. The results attainable for the relative uncertainties of the $H \to Z\gamma$ branching fraction are 48% (27%, 44%) for $M_H = 120$ (140, 160) GeV. We regard these numbers as rather encouraging, especially considering the initial value of the S/B rates. Although a complete optimization for neither selection technique has not been achieved, we are confident that room for improved $H \to Z\gamma$ branching fraction measurements is limited, mainly due to the presence of overwhelming irreducible background with final state signature similar to the signal events. Improvements may rely on the size of the Higgs window not yet adjusted to obtain optimized numbers of signal to background events for a narrow Gaussian resonance whose observed width is dominated by instrumental effects. Also Higgs-strahlung events which exist in the data sample at 500 GeV would provide an independent $H \to Z\gamma$ branching fraction, based on however different selection procedures. Although a less accurate measurement is anticipated (due to larger $e^+e^- \to ZZ\gamma$ irreducible background and more complicated analyses when both Z bosons decay hadronically), but in combination with the $WW$ fusion measurement an improved $\Delta B(H \to Z\gamma)/B(H \to Z\gamma)$ of about 10% can be expected. Furthermore, inclusion of leptonic Z boson decays and ZZ fusion events of reaction (4), so far neglected, would slightly improve the results. However, the increase of the signal event sample of about 10 to 20% would be partially compensated by more background which scales in approximately the same way.

An alternative approach relies on studying the Higgs-strahlung process $e^+e^- \to HZ \to ZZ\gamma$ at optimized lower energies, e.g. at $\sqrt{s} = 300$ GeV. Here, approximately 95% of all Z decays are useful and for $\mathcal{L} = 1$ ab$^{-1}$ about 200 $HZ \to ZZ\gamma$ events are expected. Assuming similar selection efficiencies as found for the $WW$ fusion process, the final $H \to Z\gamma$ sample would consist of some 50 events, of which approximately 50% are 4-jet events with a prompt photon. A topology of four jets with an accompanying photon is not only produced by $H \to Z\gamma \to q\bar{q}\gamma$ decays, but also via fragmentation of quarks and gluons of continuum $ZZ$ production. Refering also to the huge irreducible $e^+e^- \to ZZ\gamma$ background and the large cross-section process $e^+e^- \to WW(\gamma)$ with initial and final state photon radiation, detailed analyses are needed and preliminary results indicate less precision on $\text{BF}(H \to Z\gamma)$ [18, 22].

Linear $e^+e^-$ colliders offer the possibility for longitudinal polarised electron and positron beams, with varying polarisation degrees in right-handed or left-handed modes. Higgs boson production rates in both processes (2) and (3) depends strongly on the polarisation degree and the helicity of the incoming particles. For any given process, ratios of the cross-section for given electron and positron beam polarisations divided by the cross-section for unpolarised beams, denoted as $R$, are presented in Table 2 of [7] for different beam polarisations. The Higgs event rates in processes (2) and (3) are enhanced most for left-handed $e^-$ colliding with right-handed $e^+$ with as large a degree of polarisation as possible. For the feasible though ambitious case of collisions between an $e^-$ beam with polarisation $P_- = -0.8$ and an $e^+$ beam with polarisation $P_+ = +0.6$, $R = 1.77$ and 2.88 for the Higgs-strahlung and $WW$ fusion processes, respectively. However, the dominant irreducible background in both processes scales in approximately the same way with beam polarisations as the signal processes [7], the precision of the $H \to Z\gamma$ branching fraction improves by only a factor $\sqrt{R}$. Under such circumstances, the relative uncertainties on $\text{BF}(H \to Z\gamma)$ is lowered to 28% (16%, 26%) for $M_H = 120$ (140, 160) GeV, if the ”Higgs-likeness” selection technique is exploited. However, it should be noted that other physics processes will demand different beam polarisations and the assumption of using the full luminosity with the desired beam polarisation for this particular measurement gives some lower bound to the attainable precision.
Since the signal-to-background ratio is expected to be less than unity, it should be emphasized that large continuum $Z\gamma$ production and copious $\pi^0$ background events must be rejected by excellent geometrical resolution and stringent isolation criteria combined with excellent electromagnetic and hadronic energy resolution and hermiticity. A worse resolution would flatten the marginal signal events over large $Z\gamma$ background, thus degrading the visibility of the signal. Systematic uncertainties due to detector effects such as photon detection efficiency, energy scales and resolutions are believed to be small and can be estimated from comparison of data with well understood processes, such as $e^+e^- \to \gamma\gamma$, Compton scattering, Bhabha, $ZZ$ and $WW$ events. Systematic uncertainties on the integrated luminosity are expected to be below 0.5%, and statistical uncertainties due to final simulation sample sizes should be kept below few percent. Simulations of the Standard Model background channels are expected to yield most of the systematic uncertainties. The use of different event generators would keep this uncertainty under control and agreement between them within few percent is expected. Taking all these effects together and accounting for a precise measurement of the inclusive Higgs cross section of about 4% or less [20], it appears that the error on $BF(H \to Z\gamma)$ will be dominated by the statistical uncertainty.

The effect of overlap of $\gamma\gamma \to hadrons$ to $\nu_\ell \bar{\nu}_\ell H$ events has also been studied. The $\gamma\gamma$ events due to photons radiated in the electro-magnetic interactions of the colliding beams have been generated by the GUINEA PIG program [23] with a rate modelled in [24]. PYTHIA [17] has been used to generate the hadrons. The latest version of SIMDET [19] overlays the $\gamma\gamma$ events to $e^+e^- \to \nu_\ell \bar{\nu}_\ell H$ events, and all final state particles are then reconstructed. Without special care to isolate the particles from $\gamma\gamma$ interactions, we found that the Higgs events were recognized without notable loss or distortions after passing any of the selection procedure.

4 Conclusions

We have examined the prospects at a future linear $e^+e^-$ collider of measuring the branching fraction of a Standard Model-like Higgs boson into the $Z$ boson and a photon, $BF(H \to Z\gamma)$. Higgs boson masses of 120, 140 and 160 GeV and in integrated luminosity of $L = 1 \text{ ab}^{-1}$ at $\sqrt{s} = 500$ GeV were assumed. In order to determine the precision on $BF(H \to Z\gamma)$ which can be attained, all expected background processes were included in the analysis, and acceptances and resolutions of a linear collider detector were taken into account. In particular, by simulating the 2-to-4 particle reactions $e^+e^- \to \nu_\ell \bar{\nu}_\ell Z\gamma$, in which the signal reaction $e^+e^- \to \nu_\ell \bar{\nu}_\ell H$ is embedded, the complete irreducible background has been taken into account. Only $Z \to q\bar{q}$ decays were included so far.

Since reactions like $e^+e^- \to \nu_\ell \bar{\nu}_\ell Z\gamma, WW(\gamma), e\nu W(\gamma), e^+e^- (\gamma^*/Z)$ and radiative return $q\bar{q}\gamma$ events also constitute potentially serious background sources for the $e^+e^- \to \nu_\ell \bar{\nu}_\ell H$ signal, different selection techniques (consecutive cuts, jet finders, 'Higgs-likeness') were applied and have been shown to result in tolerable background levels and Higgs detection in the rare $H \to Z\gamma$ decay. As the favored selection procedure the 'Higgs-likeness' technique has been found, with 15 to 30 identified signal events, comparable to the other methods, but with lowest total background.

For unpolarized beams, the expected relative precision for the $H \to Z\gamma$ branching fraction was found to be 48% (27%, 44%) for $M_H = 120 (140, 160)$ GeV, after accounting for all $Z$ decays and convolution with the uncertainty on the inclusive $WW$ fusion Higgs boson cross-section.

For $e^-$ beam polarisation of -0.8 and $e^+$ beam polarisation of +0.6, the $WW$ fusion cross-section $\sigma(e^+e^- \to \nu_\ell \bar{\nu}_\ell H)$ is significantly enhanced, so improving substantially the precision on $BF(H \to Z\gamma)$ to 28% (17%, 26%), even taking into account the fact that the dominant irreducible background scales in the same way. With these uncertainties it should be possible to deduce a relative precision for the $H \to Z\gamma$ partial width of $\frac{\Delta\Gamma(H \to Z\gamma)}{\Gamma(H \to Z\gamma)} \simeq 29% (17%, 27%)$, if an uncertainty of 5% for the total Higgs width [20] is included. This in turn allows to expect a relative precision for the $HZ\gamma$ coupling of 15% (9%, 14%) for $M_H = 120 (140, 160)$ GeV.
Overlap of $\gamma \gamma \rightarrow \text{hadrons}$ to $\nu_e \bar{\nu}_e H$ events due to photons radiated in the electro-magnetic interactions of the colliding beams would not alter the uncertainties accessible.

The results presented also suggest that the $WW$ fusion reaction $e^+ e^- \rightarrow \nu_e \bar{\nu}_e H$ at 500 GeV would be superior in $H \rightarrow Z\gamma$ branching fraction measurements to the Higgs-strahlung process $e^+ e^- \rightarrow HZ$ at lower energies, e.g. at $\sqrt{s} = 300$ GeV [18, 22], in particular if polarised beams are taken into account. However, detailed analyses are needed for the latter process to establish present indications.

For Higgs masses significantly above 160 GeV, it will be difficult to determine the $HZ\gamma$ coupling with valuable precision since the $H \rightarrow Z\gamma$ branching fraction is too small to be accurately measured.

In summary, our results confirms the unique ability of a linear $e^+ e^-$ collider to access and reliably measure fundamental Higgs boson parameters in the presence of initially overwhelming background with final state signature similar to the signal events.

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