Crumpling an ordinary thin sheet transforms it into a structure with unusual mechanical behaviors, such as enhanced rigidity, emission of crackling noise, slow relaxations, and memory retention. A central challenge in explaining these behaviors lies in understanding the contribution of the complex geometry of the sheet. Here we combine cyclic driving protocols and three-dimensional (3D) imaging to correlate the global mechanical response and the underlying geometric transformations in unfolded crumpled sheets. We find that their response to cyclic strain is intermittent, hysteretic, and encodes a memory of the largest applied compression. Using 3D imaging we show that these behaviors emerge due to an interplay between localized and interacting geometric instabilities in the sheet. A simple model confirms that these minimal ingredients are sufficient to explain the observed behaviors. Finally, we show that after training, multiple memories can be encoded, a phenomenon known as return point memory. Our study lays the foundation for understanding the complex mechanics of crumpled sheets and presents an experimental and theoretical framework for the study of memory formation in systems of interacting instabilities.

Significance

When a thin sheet is crumpled, it undergoes irreversible plastic deformation, resulting in a permanent network of folds and creases. This process also changes its mechanical behaviors; in particular, it allows the thin sheet to encode memories of past driving, similarly to various amorphous solids. Here we provide a link between the intricate structure of crumpled sheets and their emergent mechanics. We find that the geometry gives rise to a disordered array of bistable effective degrees of freedom, acting as “mechanical bits.” These interact and frustrate each other, suggesting that crumpled sheets have a complex energy landscape, reminiscent of a mechanical spin glass.
Results and Discussion

Global Response: Memory and Intermittency. A thin Mylar sheet is crumpled several times and then opened to an approximately flat configuration. In contrast to its floppy, uncrumpled ancestor, the crumpled-then-flattened sheet is stiffer and can easily carry its own weight if held at one end. When forcefully bent, the sheet yields but via a series of discrete jumps accompanied by audible popping sounds (4, 5). If the force is removed, the crumpled sheet tends to retain its bent shape and to resist reversing the deformation (5, 7).

To measure this response we load the sheets into two custom mechanical testers, shown in Fig. 1A. The sheets are held at two opposite edges, where one edge is fixed and the displacement of the other is controlled by a linear motorized stage. The force exerted back by the sheet is measured using a load cell connected to the fixed end (Materials and Methods). The rather flattened configuration of the sheets prevents friction from playing a role, as opposed to 3D configurations (9). We note that in all experiments below, the measured stresses are much smaller than those applied during crumpling; therefore, we expect negligible generation of new plastic deformations (16).

We begin by considering the mechanical response of a sheet that is strained periodically. We vary the stage displacement \( \Delta \) at a slow constant rate between \( \Delta_{\text{min}} \) and \( \Delta_{\text{max}} \). A typical example of the resulting force-displacement curve is presented in Fig. 1B. Two prominent features are observed. First, the mechanical response is hysteretic—the force depends on the direction of change in the displacement. Second, the force curves are not smooth but rather decorated with a multitude of sudden force jumps. After a small number of cycles the hysteresis curves converge to an approximate limit cycle, in which force jumps occur at nearly the same displacement values. Over many cycles we observe slow creep: a sluggish decrease in the measured force and a slow drift of the displacement values of the force jumps (Fig. 1B, Inset). We attribute these to material aging, presumably due to plastic flow in the creases (10). Here we regard these creep processes as secondary effects and focus on the prominent, near-repeating features.

Next, we study the effect of changing the maximal displacement, after the system has reached a limit cycle in the interval \([\Delta_{\text{min}}, \Delta_{\text{max}}]\). We find two distinct behaviors, depending on whether the new upper limit \( \Delta_{\text{max}}^{\text{new}} \) is larger or smaller than the previous, \( \Delta_{\text{max}}^{\text{old}} \). For any \( \Delta_{\text{max}}^{\text{new}} < \Delta_{\text{max}}^{\text{old}} \) the system immediately falls into a new limit cycle, in which the force–displacement curve is fully enclosed by (and partially overlaps with) the previous curve, as shown, for example, in Fig. 1C. This transition is reversible: upon returning to the initial \( \Delta_{\text{max}}^{\text{old}} \), the previous limit cycle is immediately recovered, identified by the same pattern of force jumps (Fig. 1C, Top Left Inset).

In contrast, for any \( \Delta_{\text{max}}^{\text{new}} > \Delta_{\text{max}}^{\text{old}} \), the force–displacement curve changes irreversibly, as shown, for example, in Fig. 1D. After a short transient, the system converges to a new limit cycle, which generally has small overlap with the previous curve. Moreover, returning to the interval \([\Delta_{\text{min}}, \Delta_{\text{max}}] \) does not recover the original limit cycle. The new limit cycle is tilted with respect to the previous one, and exhibits a different pattern of force jumps (Fig. 1D, Top Left Inset). This indicates that the response is history dependent and encodes a memory of the largest applied strain.

Return Point Memory. Having shown that when completing a cycle in the reversible regime, the system recovers its initial state, we investigate whether more complex displacement cycles obey the same rule. The phenomenon where the system returns to its initial state irrespective of the displacement trajectory, given it is confined to a previously reached limit cycle, is known as RPM (23, 28, 34). Systems exhibiting RPM can be used to encode and retrieve multiple memories repeatedly (23, 35).

Fig. 1. Global response. (A) Illustration of experimental setup: two custom mechanical testers with sheets held at both ends in cylindrical (Left) and flat (Right) geometries. (B) The force as a function of the displacement under oscillatory drive (every 3rd cycle is shown). The curves converge to approximate limit cycles with repeating features. (C) Reversible behavior occurs when lowering and then restoring the initial displacement amplitude. (D) Irreversible behavior occurs when the displacement surpasses previous maximal value, indicating a memory of largest displacement. (Bottom Right Insets) The driving protocols. (Top Left Insets) The force derivative \( dF/d\Delta \) during compression. The sharp spikes identify the instabilities. Note that the pattern of spikes changes in the irreversible case.
One of the hallmarks of RPM is a hierarchy of nested hysteresis loops. Each time the displacement returns to a previously visited value, it recovers its previous state, closing a hysteresis loop. As a result, a displacement sequence that backtracks and closes smaller inner loops results in a set of hysteresis loops that are confined within each other. Fig. 2, Bottom Right Inset, shows the displacement sequence that forms three loops (indicated in yellow, orange, and purple). The resulting force–displacement curve yields three nested hysteresis loops in agreement with the occurrence of RPM. Finally, repeating a cycle in the parent interval (shown in blue) yields a hysteresis loop that overlaps the initial cycle (yellow) with a matching pattern of force jumps. This amounts to strong evidence that RPM is present.

**Mesoscopic Excitations.** We now turn to investigate the mechanism underlying the mechanical and memory response reported above. Since the force jumps are a central feature of the mechanical response, we start by investigating their geometric origin. We characterize a single force jump by varying the motor displacement periodically in a small interval around one such event. The resulting force–displacement curve shows that each event is reversible. An example of one such measurement is shown in Fig. 3A. However, the threshold for the transition in the increasing strain direction is larger than the threshold required to reverse the transition. Thus, a single event defines a two-state hysteresis loop, a hystereron. For each hystereron we denote the flipping thresholds \( \Delta_{i}^{+} > \Delta_{i}^{-} \). Collecting the magnitudes of many force jumps in the increasing displacement phase of the cycle, we find their distribution is broad, spanning several decades in magnitude (Fig. 3B).

The planar geometry of our crumpled sheets enables direct imaging of the geometric transformations occurring at an instability. Using a 3D scanner, we measure the topography of the sheet before and after a single force jump. Subtracting the two measurements reveals that each force jump is a result of a localized event, in which a small region of the sheet snaps suddenly, in a direction perpendicular to the plane of the sheet. An example is shown in Fig. 3C. The bistable geometric features are localized and for the most part consist of tips of d-cones, vertices formed at the crossing points of ridges, or small flat facets that undergo buckling (5, 36). We note that the instabilities are accompanied by audible acoustic emissions, and the resulting crackling noise was shown to exhibit a universal power law distribution of intensities (4, 5, 37).

To relate this mesoscopic picture to the macroscopic memory of largest strain, we cycle the displacement between \( [\Delta_{0}^{\text{min}}, \Delta_{i}^{\text{max}}] \) and compare the topography of the sheet at the end of the cycle to the topography obtained after varying the upper limit \( \Delta_{i}^{\text{max}} \). For the reversible regime of Fig. 1C, we find no discernible difference between the topographies (Fig. 3D). Namely, by the end of the protocol, all hysterons return to their original state. This holds for any \( \Delta_{i}^{\text{max}} < \Delta_{0}^{\text{max}} \). In contrast, for the irreversible transition of Fig. 1D, the topography changes significantly as well (Fig. 3D). In particular, several hysterons have changed their state. This is generally true for any \( \Delta_{i}^{\text{max}} > \Delta_{0}^{\text{max}} \). We therefore deduce that the memory of largest strain is encoded in the collective configuration of the hysterons.

**Fig. 2.** A set of nested hysteresis loops indicating RPM. (Top Left Inset) The force derivative \( dF \) during compression. (Bottom Right Inset) The displacement sequence.

**Fig. 3.** Mesoscopic excitations. (A) Experimental measurement of a single hysteretic event demonstrates bistability. (B) Distribution of the magnitude of the force jumps. (C) Difference in the whole sheet’s topographies following a force jump, measured through 3D scanning. Note the changes are localized to a small region in the sheet. (D) Negligible change in the sheet’s topography following the sequence of cycles shown in Fig. 1C, leading to reversible behavior. This indicates that by the end of the protocol, all hysterons returned to their initial state. (E) Significant changes in sheet’s topography following the sequence of cycles shown in Fig. 1D, leading to irreversible behavior. This indicates that by the end of the protocol, some of the hysterons changed their state irreversibly.
We note that this mechanism may also play a role in the previously observed memory of the largest applied load (1). However, in those experiments, the formation of new creases could also contribute to the reported memory. Presumably, new creases were formed due to the confined geometry and the large loads (15).

**Interactions between Hysterons.** To complete the mesoscopic description, we ask whether interactions between the bistable elements are present and affect the sheet’s mechanical response. Interactions between hysterons are expected to be mediated through the sheet’s elasticity; flipping a hysteron presumably modifies the local strain field, affecting the flipping thresholds of its neighbors (7, 38). Thus, interactions may be inferred from shifts in the activation thresholds which are configuration dependent (27, 29, 31, 32). In experiments, we find evidence for such interactions both by probing flipping thresholds locally and by carefully examining the global response of the sheet.

To probe interactions locally, one must measure how the flipping threshold of a hyster on depends on the state of a neighboring one. However, the experimental setup of Fig. 1A measures in-plane forces and does not allow isolating the response of a single hyster on. To this end, we attach a probe to a load cell and use it to push locally on a hyster on in a direction perpendicular to the sheet (out of plane), until it flips. We control the displacement of the probe δ while measuring the normal force. The measured force first increases, until an instability occurs—the hyster on flips, and the force drops to zero. This instability identifies the local flipping threshold, denoted δ⁺

We identify two neighboring hysterons in the sheet that are flipped down and denote their state [00]. We then measure the flipping threshold for the first hyster on δ⁺ and its dependence on the state of the second hyster on. We compare the transitions [00] → [01] and [10] → [11], as shown in Fig. 4 A and B. Doing so for several pairs of hysterons, we find that their flipping thresholds are state dependent. We find both antiferromagnetic and ferromagnetic interactions: the flipping threshold of a hyster on may increase or decrease depending on the state of its neighbor. Examples for both types of interactions are shown in Fig. 4 C and D. We also observe cases in which locally flipping a hyster on results in the spontaneous flip of another one in a small avalanche. Overall, this amounts to evidence for significant interactions between hysterons in the sheet.

Interactions between hysterons are also apparent in the global response of the sheet to strain applied at the boundaries, as reported above. This is most clear in Fig. 1D, when comparing the two limit cycles before and after surpassing the maximal displacement (yellow and blue curves, respectively). Even though the displacement range is identical for the two cycles, the pattern of instabilities changes considerably. This is identified from the spikes in the derivative of the force, shown in Fig. 1

Overall, this amounts to evidence for significant interactions between hysterons in the sheet.

**A Coupled Hysterons Model.** Our experiments have shown that the effective degrees of freedom of crumpled sheets are localized, coupled, bistable elements. To formulate a model we make further simplifying assumptions. We assume that the planar sheet can be approximated by a 2D system embedded with bistable elements. Although the majority of the motion in experiments occurs in the lateral dimension, we assume it yields an in-plane strain. Last, we incorporate interactions by assuming that the strain results in forces between the bistable elements.

A simple realization of such a model is a 2D disordered bonded network, where each bond is a bistable spring (44, 45) (Fig. 5A). Namely, each bond is governed by a potential with two minima, given by

\[ U_i = \frac{c_i}{4} (\delta r_i)^4 - \frac{a_i}{2} (\delta r_i)^2. \]  

Here \( \delta r_i \) denotes the distance from the local maximum. Each bond is defined by the two parameters \( a_i \) and \( c_i \), which set the locations of the two minima, \( \delta r_{\text{min}} \) which is \( \pm \sqrt{a_i/c_i} \). The overall length of the bond is taken to be significantly larger than \( \delta r_{\text{min}} \). Additional details are presented in Materials and Methods and SI Appendix. In the simulations, the network is strained
isotropically and quasi-statically; at each step the energy is minimized to reach force balance.

We measure the stress–strain relation and find smooth elastic intervals separated by sharp stress drops. These instabilities originate from bonds transitioning between their two minima, as illustrated in Movie S1. Under periodic drive the system may converge to a precise limit cycle, depending on the parameters of the model. Even when it does not converge, often consecutive cycles yield very similar stress–strain curves and appear as approximate limit cycles.

Following the same protocols as in experiments, we find that the model recovers all essential experimental findings: memory of the largest strain, a tilt in the stress–strain curves, and change to the activation pattern when increasing the strain amplitude and, after training, the nested loops which characterize RPM (Fig. 5 B and C). Further characterization is provided in SI Appendix.

Interestingly, we observe that bonds generally deviate from their minima due to a mismatch between their rest lengths. Namely, their geometrical constraints cannot all be satisfied simultaneously, and the network is geometrically frustrated (46). This is demonstrated in Fig. 5A and in Movie S1, where residual stresses within the network are visible, even when the global stress vanishes. Such frustration also develops in networks which were initiated at an unfrustrated state. We expect such frustration to emerge in crumpled sheets as well, due to their disordered geometry and the antiferromagnetic interactions between hysterons.

Conclusions and Outlook

We have studied the mechanical response of unfolded crumpled sheets to cyclic strain. We found that the force–displacement curves are intermittent and hysteretic and encode a memory of the largest strain. All these effects can be traced back to the collective response of multiple bistable snap-through instabilities spread across the sheet. These mesoscopic, localized, and interacting degrees of freedom form the basis for understanding the mechanics of crumpled sheets. A model of a coupled hysterons network reproduces the observed behaviors and sheds light on the role of interactions. Altogether, our work offers an experimental and theoretical framework for the study of memory formation in systems composed of interacting instabilities.

Remarkably, our system exhibits RPM with high precision, allowing us to encode multiple memories through a series of nested hysteretic loops. The occurrence of RPM is surprising given the complexity and strength of the interactions. Thus, a sheet that is crumpled mindlessly and with little effort is transformed into a programmable material, without the careful design required in engineered mechanical metamaterials (47–51).

Further research is required to fully map the distribution of hysterons across the crumpled sheet, the patterns and range of their interactions, and how these are related to the geometry. Particular attention should be given to the formulation of the interactions between hysterons, which may not be of a pairwise nature and may affect the activation thresholds $\Delta^\pm$ differently (29).

The description of crumpled sheets as a frustrated network of coupled bistable degrees of freedom suggests that they can be viewed as a mechanical spin glass with a complex energy landscape. Similar suggestions have been made in the context of origami (38, 52, 53). This raises the question of whether the glasslike behaviors of crumpled sheets, such as logarithmic aging and Kovacs-like memory retention (1, 6), could also be understood using this framework. Furthermore, as geometric frustration generically gives rise to an extensive multistability, this description may relate to the shape memory of crumpled sheets (7). We hope our work promotes further understanding of these phenomena, both in crumpled sheets and in other systems that can be effectively described as a network of strongly coupled metastable elements.

Materials and Methods

Experimental. The experiments are performed on Mylar sheets, 25 and 90 microns thick and $\sim$20 cm by 20 cm across. These were crumpled manually several times and then loaded into the mechanical testers. The difference between the two setups is in the geometry of the testers’ edge constraints. The first has straight constraints, keeping the sheet approximately flat and allowing direct imaging of its topography. However, in this setup the response is limited as the sheet tends to bulge along the axis of strain, leaving the transverse dimension flat. The second cylindrical configuration avoids these localized deformations as the transverse curvature frustrates the bending along the strain axis. In all the results displayed above, topographic 3D imaging is done with the flat setup using opaque 90-mm-thick Mylar, and a HP David 5 3D scanner. Force–displacement curves are measured using the cylindrical setup and transparent 25-mm-thick Mylar. We note that the flat setup exhibits the same characteristic features in its response, including the memory of the largest strain, the signature of interactions, and RPM.

Numerical Model. The picture suggested by the experiments is that a crumpled sheet can be modeled as a disordered collection of coupled bistable...
mechanical degrees of freedom. To test if these ingredients are sufficient to explain the observed behaviors, we study a simple model. We assume that a planar sheet can be approximated by a 2D elastic system embedded with bistable elements. A simple realization of this is a disorderedbonded network where each bond is bistable, implemented through a potential with two minima. The length of the bond is denoted by \( r = r_0 + \delta r \), where \( \delta r \) is smaller than \( r_0 \), which signifies the average between the two bistable states. For simplicity the potential is chosen to be symmetric:

\[
V_U(r) = \frac{C}{2} (\delta r)^2 - \frac{a_1}{2} (\delta r)^2.
\]

Here \( a_1 > 0 \) and the two minima are at \( \delta r = \pm \sqrt{a_1/C} \). For simplicity, the constant \( C \) is chosen to be the same for all bonds. We consider two cases: 1) \( a_1 = 0 \) is the same for all bonds and 2) \( a_1 \) is uniformly distributed between \([0, a_2]\). To avoid boundary effects we employ periodic boundary conditions.

We simulate quasi-static dynamics, by straining the system between 0 and \( \epsilon_{\text{max}} \). The strain is discretized into small steps, below \( 10^{-4} \). Each step of the dynamics consists of a small change to the strain and then an energy minimization step to reach force balance, using the Fast Inertial Relaxation Engine (FIRE) algorithm (54). The qualitative behavior appears to be independent of the particular deformation, and throughout the paper we strain both axes equally.

In Fig. 4A the number of nodes is \( N = 512 \). In Fig. 4B and C the number of nodes is \( N = 2,048 \), and they are uniformly distributed between \([0, 0.04]\). \( C = 1 \), and the average \( \langle \theta \rangle > 1 \).

**Disordered Network Preparation.** We prepare our 2D network from amorphous packings of spheres at zero temperature (55). This choice is made for convenience, and we believe it has little impact on the results. A pair of spheres interact when the interparticle distance, \( r \), is below the sum of the radii \( R_i + R_j \), via the potential

\[
V_{ij}(r) = \begin{cases} 
V_0 \left(1 - \frac{r}{r_{ij}} \right)^2 & r \leq R_i + R_j, \\
0 & r > R_i + R_j.
\end{cases}
\]

Force balance configurations are reached by minimizing the energy using the FIRE algorithm (54). The properties of packings depend on the distance from the jamming transition, which can be tuned through the pressure exerted on the box. In the limit of zero applied pressure the system has critical-like behavior, characterized by anomalous elasticity and diverging length scales. To avoid these atypical behaviors we focus on the regime that is far from the jamming transition. The distance from the isostatic transition is often characterized by the excess coordination number \( \Delta Z = 2 - Z_e \). Here \( Z_e = 2N_i / N \), where \( N_i \) is the number of bonds, \( N \) is the number of nodes, and \( Z_i \approx 2d(5d-58) \). All our simulations are at \( \Delta Z \approx 1.5 \), which is far from the isostatic point \( \Delta Z = 0 \).

The packings are converted into a bonded network, by identifying the nodes with the centers of spheres and connecting pairs of overlapping particles with a bond. The length of the bond \( r_0 \) is set by the distance between the particle pair. Initially, we set \( \delta r = 0 \); however, this is the local unstable maximum, and the system relaxes into a stable basin of attraction.

**Data Availability.** All study data are included in the article and/or supporting information.

**ACKNOWLEDGMENTS.** We are grateful to Martin van Hecke, Ido Regev, Naomi Oppenheimer, Yohai Bar Sinai, Roy Beck, and Yair Shokef for enlightening discussions. We thank Liz Nizan for 3D graphic design. This work was supported by the Israel Science Foundation grant 2096/18 (Y.L.) and grant 2385/20 (D.H.).
49. J. L. Silverberg et al., Applied origami. Using origami design principles to fold reprogrammable mechanical metamaterials. Science 345, 647–650 (2014).
50. J. P. Udani, A. F. Arieta, Programmable mechanical metastructures from locally bistable domes. Extreme Mech. Lett. 42, 101081 (2021).
51. S. Waitukaitis, R. Menaut, B. G. Chen, M. van Hecke, Origami multistability: From single vertices to metasheets. Phys. Rev. Lett. 114, 055503 (2015).
52. M. Stern, M. B. Pinson, A. Murugan, The complexity of folding self-folding origami. Phys. Rev. X 7, 041070 (2017).
53. B. G. Chen, C. D. Santangelo, Branches of triangulated origami near the unfolded state. Phys. Rev. X 8, 011034 (2018).
54. E. Bitzek, P. Koskinen, F. Gähler, M. Moseler, P. Gumbsch, Structural relaxation made simple. Phys. Rev. Lett. 97, 170201 (2006).
55. C. S. O’Hern, L. E. Silbert, A. J. Liu, S. R. Nagel, Jamming at zero temperature and zero applied stress: The epitome of disorder. Phys. Rev. E Stat. Nonlin. Soft Matter Phys. 68, 011306 (2003).
56. J. C. Phillips, Topology of covalent non-crystalline solids I: Short-range order in chalcogenide alloys. J. Non-Cryst. Solids 34, 153-181 (1979).
57. S. Alexander, Amorphous solids: Their structure, lattice dynamics and elasticity. Phys. Rep. 296, 65-236 (1998).
58. C. F. Moukarzel, Isostatic phase transition and instability in stiff granular materials. Phys. Rev. Lett. 81, 1634 (1998).