Remark on the vectorlike nature of the electromagnetism and the electric charge quantization

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Abstract

In this work we study the structure of the electromagnetic interactions and the electric charge quantization in gauge theories of electroweak interactions based on semi-simple groups. We show that in the standard model of the electroweak interactions the structure of the electromagnetic interactions is strongly correlated to the quantization pattern of the electric charges. We examine these two questions also in all possible chiral bilepton gauge models of the electroweak interactions. In all they we can explain the vectorlike nature of the electromagnetic interactions and the electric charge quantization together demanding nonvanishing fermion masses and the anomaly cancellations.

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Why nature arranges the things so that the electromagnetic interactions among fermions are vectorial and electric charge quantization (ECQ) comes with the pattern: \( Q_\nu = 0, \ Q_e = -e, \ Q_u = \frac{2}{3}e, \ Q_d = -\frac{1}{3}e \) is a question not altogether closed. The QED, the natural place to investigate these questions, is unable to explain them because of the arbitrariness of the \( U(1)_{\text{em}} \) quantum numbers. Gauge theories of electroweak interactions based on semi-simple group face also the same difficulty as in QED: the \( U(1) \) factor compounding the semi-simple groups.

In spite of this difficulty the ECQ was analyzed in gauge theories of electroweak interactions. It was found that in some models difficulties may be overcome by two types of constraints. One type comes from the requirement that some fermions be massive. In such models the fermions obtain masses through the Yukawa sector. Demanding this sector to be invariant under the gauge symmetry we get constraints among the \( U(1) \) quantum numbers. These are the nonvanishing fermion masses constraints also called classical constraints. Another type of constraints come from the requirement of theoretical consistency of the model which means the model be free from anomalies. Anomaly cancellations [1] are expressed in terms of relations among the \( U(1) \) quantum numbers. These are the quantum constraints. Using these two constraints the ECQ can be explained in some extension of the standard model(SM) [2–8]. In this work we extend such analysis to include the structure of the electromagnetic interactions. Such study will be done in the SM with massless and massive neutrinos and in all possible chiral bilepton gauge(CBGM) versions.

This paper is organized as follows. In Sec. II we analyze the VLNE and the ECQ in the SM with one and three generations. In sec. III we extend our analysis to the case of CBGM. In Sec. IV we summarize our conclusions.

I. THE ELECTRIC CHARGE QUANTIZATION AND THE VECTORLIKE NATURE OF THE ELECTROMAGNETISM IN THE FRAMEWORK OF THE STANDARD MODEL
A. The case of one generation

In the SM with one generation the quarks and leptons come in the following representations

\[ L_L = \left( \begin{array}{c} \nu \\ e \end{array} \right) \sim (1,2,Y_L), \quad e_R \sim (1,1,Y_l), \]

\[ Q_L = \left( \begin{array}{c} u \\ d \end{array} \right) \sim (3,2,Y_q), \quad u_R \sim (3,1,Y_u), \quad d_R \sim (3,1,Y_d). \] (1)

In order to break symmetry spontaneously and give masses to the gauge bosons \( W^\pm \) and \( Z^0 \) we need to introduce a Higgs doublet \( \phi \sim (1,2,Y_\phi) \) that acquires a vacuum expectation value

\[ \langle \phi \rangle_0 \sim \left( \begin{array}{c} 0 \\ v \end{array} \right). \] (2)

After the spontaneous symmetry breaking (SSB) and due to the mixing among \( W^3 \) and \( B \) we find the following charge operator \[[4]\]

\[ Q/a = T_3 + b/a Y \] (3)

Where \( a = g/e \sin \theta_W \) and \( b = g'/e \cos \theta_W \). Since we want the generator \( Q \) unbroken, \( Q \langle \phi \rangle_0 \) must be zero. With this condition we find \( a = b Y_\phi \). Using the freedom in assigning the scale of the electric charge we set \( a = 1 \) \([3][4]\). Then we have \( g \sin \theta_W = e \) and \( g' \cos \theta_W = e/Y_\phi \).

After these steps we find the following electromagnetic interactions among the fermions

\[ \mathcal{L}_{em} = -\frac{e}{4Y_\phi} [2(\gamma^\mu Y_L + Y_\phi)\overline{\nu}_L \gamma^\mu \nu_L + \\
+ \overline{e} ( ( -Y_L - Y_l + Y_\phi ) + ( -Y_L + Y_l + Y_\phi ) \gamma_5 ) \gamma^\mu e \\
+ \overline{u} ( ( -Y_q - Y_u - Y_\phi ) + ( -Y_q + Y_u - Y_\phi ) \gamma_5 ) \gamma^\mu u \\
+ \overline{d} ( ( -Y_q - Y_d + Y_\phi ) + ( -Y_q + Y_d + Y_\phi ) \gamma_5 ) \gamma^\mu d ] A_\mu. \] (4)

In order to give to fermions their masses we introduce the Yukawa interaction
\[ -\mathcal{L}^Y = g^L \bar{L}_L \phi e_R + g^u \bar{Q}_L \phi u_R + g^d \bar{Q}_L \phi d_R + \text{H.c.} \]  

(5)

By demanding U(1)$_Y$ gauge invariance we find from (5)

\[ Y_L - Y_t - Y_\phi = 0, \quad Y_q - Y_u + Y_\phi = 0, \quad Y_q - Y_d - Y_\phi = 0. \]  

(6)

Substituting (6) in (4) we find the following structure to the electromagnetic interactions

\[
\mathcal{L}_{em} = -\frac{e}{2Y_\phi} [(Y_L + Y_\phi) \bar{\nu}_L \gamma^\mu \nu_L + \bar{\nu}(\nu_L + Y_\phi) \gamma^\mu e \\
- (Y_q + Y_\phi) \bar{u} \gamma^\mu u + (-Y_q + Y_\phi) \bar{d} \gamma^\mu d] A_\mu.
\]  

(7)

Now we must impose the anomaly cancellation. The conditions in (6) leaves only two nontrivial anomalies, which are sufficient to fix the hypercharges $Y_L$ and $Y_q$

\[
[SU(2)_L]^2 U(1)_Y \implies Y_q = -\frac{1}{3} Y_L, \\
[U(1)_Y]^3 \implies Y_L = -Y_\phi \implies Y_q = \frac{1}{3} Y_\phi.
\]  

(8)

Substituting (8) in (7) we obtain the ECQ and the VLNE

\[
\mathcal{L}_{em} = e \bar{\nu} \gamma^\mu e A_\mu - \frac{2e}{3} \bar{u} \gamma^\mu u A_\mu + \frac{e}{3} \bar{d} \gamma^\mu d A_\mu.
\]  

(9)

Cancellation of anomalies and demanding the fermions are massives one obtains the ECQ and the VLNE in the SM with one generation and massless neutrinos.

Adding a right-handed Dirac neutrino, $\nu_R \sim (1,2,Y_\nu)$, to the SM we get the following electromagnetic interaction

\[
-\frac{e}{4Y_\phi} \bar{\nu}_L [(Y_L + Y_\nu + Y_\phi) + (Y_L - Y_\nu + Y_\phi) \gamma_5] \gamma^\mu \nu_L A_\mu.
\]  

(10)

Its Yukawa term is $\tilde{L}_L \tilde{\nu}_R$. Its $U(1)_Y$ gauge invariance provides the following relation: $-Y_L + Y_\nu - Y_\phi = 0$, which cancels the axial term in (10). But now only one nontrivial anomaly constraint remains: $[SU(2)_L]^2 U(1)_Y$. Then we have two arbitrary hypercharges from the gauge invariance of the Yukawa sector and one constraint from the anomaly cancellation. In this case we have no ECQ. This result is the dequantization effect [2–7]. Nevertheless we have the VLNE automatically.
\[ L_{em} = -\frac{e}{2Y_\phi} [(Y_L + Y_\phi) \bar{\nu}_e \gamma^\mu \nu + (Y_L + Y_\phi) \bar{\nu}_e \gamma^\mu e \\
+ \frac{Y_L}{3} + Y_\phi) \bar{u} \gamma^\mu u + (-\frac{Y_L}{3} + Y_\phi) \bar{d} \gamma^\mu d] A_\mu. \quad (11) \]

Babu and Mohapatra made the important observation in Ref. [5] that if we suppose that neutrino is a Majorana particle we can fix all the hypercharges restoring thus the ECQ and consequently the VLNE.

B. The case of three Generations

With three generations the representation content is

\[ L_{aL} = \begin{pmatrix} \nu_a \\ e_a \end{pmatrix}_L \sim (1, 2, Y_{L_a}), \quad e_{aR} \sim (1, 1, Y_{l_a}), \]
\[ Q_{aL} = \begin{pmatrix} u_a \\ d_a \end{pmatrix}_L \sim (3, 2, Y_{q_a}), \quad u_{aR} \sim (3, 1, Y_{u_a}), \quad d_{aR} \sim (3, 1, Y_{d_a}). \quad (12) \]

With \( a = 1, 2, 3 \) being the flavor index. Now we have the following structure for the electromagnetic interactions

\[ L_{em} = -\frac{e}{4Y_\phi} \sum_a Y_{L_a} \bar{L}_{aL} \phi e_{aR} + \frac{Y_\phi}{3} \bar{L}_{aL} \phi \bar{e}_{aR} \gamma^5 \nu_{aL} \\
+ \bar{e}_a (\bar{Y}_{L_a} - Y_{l_a} - Y_\phi) + \bar{Y}_{L_a} + Y_{l_a} + Y_\phi) \gamma_5 \nu_a \\
+ \bar{u}_a (\bar{Y}_{q_a} - Y_{u_a} - Y_\phi) + \bar{Y}_{q_a} + Y_{u_a} - Y_\phi) \gamma_5 \gamma^\mu u_a \\
+ \bar{d}_a (\bar{Y}_{q_a} - Y_{d_a} + Y_\phi) + \bar{Y}_{q_a} + Y_{d_a} + Y_\phi) \gamma_5 \gamma^\mu d_a] A_\mu. \quad (13) \]

The Yukawa sector takes the following form [4]

\[ -L^Y = \sum_{a,b}^{1,2,3} Y_{L_a} \bar{L}_{aL} \phi \bar{e}_{aR} + \frac{Y_\phi}{3} \bar{L}_{aL} \phi \bar{e}_{aR} + g_{ab}^d \bar{Q}_{aL} \phi d_{bR} + g_{ab}^u \bar{Q}_{aL} \phi u_{bR} \] + H.c. \quad (14) \]

From its \( U(1)_Y \) gauge invariance we find the following relations among the hypercharges

\[ Y_{L_a} - Y_{l_a} - Y_\phi = 0, \quad Y_{q_a} - Y_{u_a} - Y_\phi = 0, \quad Y_{q_a} - Y_{d_a} + Y_\phi = 0, \quad (15) \]

From the two last terms above we obtain
\[ Y_{qa} = Y_q, \quad Y_{ua} = Y_u, \quad Y_{da} = Y_d, \]  

which lead (15) to

\[ Y_{La} - Y_{la} - Y_{\phi} = 0, \quad Y_q - Y_u - Y_{\phi} = 0, \quad Y_q - Y_d + Y_{\phi} = 0. \]  

(17)

Substituting (17) into (13) we obtain the following electromagnetics interaction among fermions

\[
\mathcal{L}_{em} = -\frac{e}{2Y_{\phi}} \sum_a^3 \left[(Y_{La} + Y_{\phi})\bar{\nu}_{aL}\gamma^\mu \nu_{aL} A_\mu + \bar{e}_a(-Y_{La} + Y_{\phi})\gamma^\mu e_a A_\mu \right. \\
- \bar{u}_a(Y_q + Y_{\phi})\gamma^\mu u_a A_\mu + \bar{d}_a(-Y_q + Y_{\phi})\gamma^\mu d_a A_\mu.
\]

(18)

After this we have only two nontrivial anomaly constraints

\[
[SU(2)_L]^2 U(1)_Y \implies 9Y_q + \sum_a Y_{La} = 0,
\]

\[
[U(1)_Y]^3 \implies 18Y_q^3 - 9Y_u^3 - 9Y_d^3 + \sum_a (2Y_{La}^3 - Y_{la}^3) = 0.
\]

These two constraints are insufficient for fix the four arbitrary hypercharges in (18). Differently of the SM with one generation we have neither explanation to the ECQ nor to the VLNE. This is the dequantization effect [2–7]. Nevertheless we have through (18) a correlation among the quantization pattern and the structure of the electromagnetic interactions. Such correlation permits us to conclude that nature arranges the things so that the electric charge quantization comes with the quantization pattern \( Q_\nu = 0, \quad Q_e = -e, \quad Q_u = \frac{2}{3}e, \quad Q_d = -\frac{1}{3}e \) because the nature of the electromagnetic interactions is vectorial. Thus whether we wish explain the ECQ we need to take as constraints the nonvanishing fermion masses, anomaly cancellations and the VLNE.

Let us suppose a Dirac-like massive neutrinos. In this case also we have the dequantization effect [2–4]. Nevertheless the VLNE is automatic
where \( Y_{L_1} = Y_{L_2} = Y_{L_3} = Y_L \) \[1\].

If we suppose a Majorana-like massive neutrinos we must restore the ECQ and the VLNE like in the case of the SM with one generation \[2\-3\].

**II. THE ELECTRIC CHARGE QUANTIZATION AND THE VECTORLIKE NATURE OF THE ELECTROMAGNETISM IN CHIRAL BILEPTON GAUGE MODELS**

Chiral bilepton gauge models are extensions of the \( SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \) symmetry group to the \( G_{3X1} = SU(3)_C \otimes SU(X)_L \otimes U(1)_N \) one, with \( X = 3, 4 \). \( X = 3 \) gives us the simplest versions and \( X = 4 \) the largest version. Whether we do not consider exotic leptons we can have only two independent simplest versions of CBGM \[9\-10\] and one largest version \[11\]. Their key feature are bilepton gauge bosons with lepton number \( L = \pm 2 \) and exotic quarks. They give in some sense a answer to the family problem \[12\] because they require a minimal of three families to cancel anomalies. They are a multi-higgs model, nevertheless FCNC with the standard neutral gauge boson is strongly suppressed \[13\]. Some studies of their phenomenology were done in \[14\].

**A. Version A**

Here we analyze the VLNE and the ECQ in a CBGM based in the \( SU(3)_C \otimes SU(3)_L \otimes U(1)_N \) symmetry which has as electric charge operator the following linear combinations of their diagonal generators \[9\]

\[
Q = \frac{1}{2}(\lambda_3 - \sqrt{3}\lambda_8) + bN,
\]

where \( N \) is the generator operator of the \( U(1)_N \) group \( \lambda_3 \) and \( \lambda_8 \) the two diagonal Gell-mann matrices.

The minimal set of scalars necessary to give correct masses to the fermions and bosons are three triplets and one sextet. They get vacuum expectation value different from zero.
and transform by $\text{SU}(3)_C \otimes \text{SU}(3)_L \otimes \text{U}(1)_N$ in the following manner
\[
\begin{align*}
\langle \eta \rangle_0 &= \begin{pmatrix} v_\eta \\ 0 \\ 0 \end{pmatrix} \sim (1,3, N_\eta), \quad \langle \rho \rangle_0 = \begin{pmatrix} 0 \\ v_\rho \\ 0 \end{pmatrix} \sim (1,3, N_\rho) \\
\langle \chi \rangle_0 &= \begin{pmatrix} 0 \\ 0 \\ v_\chi \end{pmatrix} \sim (1,3, N_\chi), \quad \langle S \rangle_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & v_{\sigma_2} & 0 \end{pmatrix} \sim (1,3, N_S). \quad (22)
\end{align*}
\]

Requiring the electric charge operator annihilates the vacuum of the scalars we set
\[
N_\eta = 0, \quad b = \frac{1}{N_\rho}, \quad N_\chi = -N_\rho, \quad N_S = 0. \quad (23)
\]

After these steps the electric charge operator acquire the following form
\[
Q = \frac{1}{2}(\lambda_3 - \sqrt{3}\lambda_8) + \frac{N}{N_\rho}. \quad (24)
\]

The fermions come in the following representations
\[
L_{aL} = \begin{pmatrix} \nu_a \\ e_a \\ e_a^c \end{pmatrix}_L \sim (1,3, N_{L_a}),
\]
\[
Q_{1L} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (3, 3^*, N_{Q_1}),
\]
\[
u_{1R} \sim (3, 1, N_{u_1}), \quad d_{1R} \sim (3, 1, N_{d_1}), \quad J_{1R} \sim (3, 1, N_{J_1}),
\]
\[
Q_{iL} = \begin{pmatrix} d_i \\ -u_i \\ J_i \end{pmatrix}_L \sim (3, 3^*, N_{Q_i}),
\]
\[
d_{iR} \sim (3, 1, N_{d_i}), \quad u_{iR} \sim (3, 1, N_{u_i}), \quad J_{iR} \sim (3, 1, N_{J_i}), \quad (25)
\]

with $a = 1, 2, 3$ and $i = 2, 3$ being flavor index. There are no lepton singlets. The quarks $u$'s and $d$'s are the usual ones with $J$'s being the exotic quarks.
After SSB SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U($1$)$_N$ $\rightarrow$ SU(3)$_C$ $\otimes$ U($1$)$_{em}$ we find the following structure for the electromagnetic interactions

$$L_{em} = -\frac{e}{2N_\rho} \sum_a \sum_{i=1,2,3} \left[ 2 N_{La} \bar{\nu}_{aL} \gamma^\mu \nu_{aL} + 2 \bar{e}_a \left( -N_\rho + N_{La} \gamma_5 \right) \gamma^\mu e_a \right]$$

$$+ \bar{u}_1 \left( (N_{Q_1} + N_{u_1}) - (N_{Q_1} + N_{u_1}) \gamma_5 \right) \gamma^\mu u_1$$

$$+ \bar{u}_i \left( (N_{Q_i} + N_{u_i} + N_\rho) - (N_{Q_i} - N_{u_i} + N_\rho) \gamma_5 \right) \gamma^\mu u_i$$

$$+ \bar{d}_1 \left( (N_{Q_1} + N_{d_1} - N_\rho) - (N_{Q_1} - N_{d_1} - N_\rho) \gamma_5 \right) \gamma^\mu d_1$$

$$+ \bar{d}_i \left( (N_{Q_i} + N_{d_i}) - (N_{Q_i} - N_{d_i}) \gamma_5 \right) \gamma^\mu d_i$$

$$+ \bar{J}_1 \left( (N_{Q_1} + N_{J_1} + N_\rho) - (N_{Q_1} + N_{J_1} - N_\rho) \gamma_5 \right) \gamma^\mu J_1$$

$$+ \bar{J}_i \left( (N_{Q_i} + N_{J_i} - N_\rho) - (N_{Q_i} - N_{J_i} - N_\rho) \gamma_5 \right) \gamma^\mu J_i \gamma^\mu A_\mu. \quad (26)$$

From the U($1$)$_N$ invariance of the Yukawa sector

$$- L_Y = \frac{1}{2} G_{ab} \bar{L}_{aL} S^* L_{bL}$$

$$+ \lambda_{1i} \bar{Q}_{1L} J_{1R} \chi + \lambda_{ij} \bar{Q}_{iL} J_{iR} \chi^*$$

$$+ \lambda'_{1a} \bar{Q}_{1L} d_{aR} \rho + \lambda'_{ia} \bar{Q}_{iL} u_{aR} \rho^*$$

$$+ \lambda''_{1a} \bar{Q}_{1L} u_{aR} \eta + \lambda''_{ia} \bar{Q}_{iL} d_{aR} \eta^* \quad + \text{H.c.}, \quad (27)$$

we find the following relations between the $N$ quantum numbers

$$N_{u_1} = N_{u_2} = N_{u_3} = N_u,$$

$$N_{d_1} = N_{d_2} = N_{d_3} = N_d,$$

$$N_{Q_2} = N_{Q_3} = N_Q,$$

$$N_{J_2} = N_{J_3} = N_J,$$ \quad (28)

and

$$N_{L_1} = N_{L_2} = N_{L_3} = 0,$$

$$N_J = N_Q - N_\rho,$$

$$N_d = N_Q,$$
\[ N_u = N_Q + N_\rho, \]
\[ N_{J_1} = N_Q + 2N_\rho, \]
\[ N_{Q_1} = N_Q + N_\rho. \]  

(29)

Substituting these relations in (29) we obtain the following electromagnetic interactions

\[
\mathcal{L}_{em} = e\bar{e}_a \gamma^\mu e_a A_\mu + \frac{e}{N_\rho}([N_Q + N_\rho])u_\alpha \gamma^\mu u_\alpha + N_Q \bar{d}_a \gamma^\mu d_a \\
+ (N_Q + 2N_\rho)\bar{J}_1 \gamma^\mu J_1 + (N_Q - N_\rho)\bar{J}_i \gamma^\mu J_i]A_\mu.
\]

(30)

Note that the classical constraints alone provide the VLNE and the ECQ among the leptons.

After the relations in (29) only one nontrivial anomaly constraint remains

\[ [SU(3)_L]^2U(1)_N \implies 3N_{Q_1} + 3N_{Q_2} + 3N_{Q_3} + N_{L_1} + N_{L_2} + N_{L_3} = 0, \]  

(31)

which together with the relations in (29) gives \( N_Q = -N_\rho/3 \). This fixes uniquely the \( N \)'s quantum numbers as a function of \( N_\rho \), furnishing thereby the electric charge quantization of all fermions and the VLNE

\[
\mathcal{L}_{em} = e\bar{e}_a \gamma^\mu e_a A_\mu + \frac{2e}{3}u_\alpha \gamma^\mu u_\alpha A_\mu + \frac{e}{3}d_a \gamma^\mu d_a A_\mu + \\
- \frac{5e}{3}\bar{J}_1 \gamma^\mu J_1 A_\mu + \frac{4e}{3}\bar{J}_i \gamma^\mu J_i A_\mu.
\]

(32)

Let us now analyze this model with massive neutrinos. In the case of Majorana neutrino, it is sufficient to modify the sextet of scalars to

\[
\langle S \rangle_0 = \begin{pmatrix} \sigma_1 & 0 & 0 \\ 0 & 0 & \sigma_2 \\ 0 & \sigma_2 & 0 \end{pmatrix} \sim (1, 3, N_S).
\]

(33)

This does not change the above steps. So the result in (32) remains the same with or without Majorana neutrinos.
If we add right-handed neutrinos, $\nu_{aR} \sim (1,1, N_{Ra})$, their electromagnetic interaction become this

$$\mathcal{L}_{\text{em}} = -\frac{e}{N_\rho} \sum_a (N_{La} + N_{Ra})\bar{\nu}_a \gamma^\mu \nu_a A^\mu. \quad (34)$$

From their Yukawa interactions $\bar{L}_{aL} \eta \nu_{aR}$, we find: $N_{Ra} - N_{La} = 0$, which together with (29) gives $N_{Ra} = 0$, annulling their electromagnetic interactions. In short, in this simplest CBGM version we explain, with or without neutrinos are massives, the ECQ and the VLNE.

**B. Version B**

This is another possible variant of the simplest CBGM versions. Its Higgs sector is more economic than the one in the first version. It present Dirac-like massive neutrinos inevitably in tree level. Its fermion content is the following

$$L_{aL} = \begin{pmatrix}
\nu_a \\
e_a \\
\nu_a^c
\end{pmatrix}_L \sim (1,3, N_{La}), \quad e_{aR} \sim (1,1, N_{Ra}),$$

$$Q_{aL} = \begin{pmatrix}
d_\alpha \\
u_\alpha \\
\lambda_\alpha \\
j_\alpha
\end{pmatrix}_L \sim (3, \bar{3}, N_{Qa}),$$

$$u_{aR} \sim (3, 1, N_{ua}), \quad d_{aR} \sim (3, 1, N_{da}), \quad J_{Ra} \sim (3, 1, N_{Ja}),$$

$$Q_{3L} = \begin{pmatrix}
u_3 \\
d_3 \\
\lambda_3 \\
j_3
\end{pmatrix}_L \sim (3, 3, N_{Q3}),$$

$$d_{3R} \sim (3, 1, N_{d3}), \quad u_{3R} \sim (3, 1, N_{u3}), \quad J_{3R} \sim (3, 1, N_{J3}), \quad (35)$$

with $a = 1, 2, 3$ and $\alpha = 1, 2$ being the flavor index.

Its electric charge operator takes the following linear combination [10]

$$Q = \frac{1}{2} (\lambda_3 - \frac{1}{\sqrt{3}} \lambda_8) + b' N. \quad (36)$$
We need three triplets of scalars to break spontaneously the symmetry and give mass to the fermions. Their vacuum expectation value is different from zero and transforms by $SU(3)_C \otimes SU(3)_L \otimes U(1)_N$ in the following manner

$$\langle \eta \rangle_0 = \begin{pmatrix} v_{\eta} \\ 0 \\ 0 \end{pmatrix} \sim (1,3, N_{\eta}), \quad \langle \rho \rangle_0 = \begin{pmatrix} 0 \\ v_{\rho} \\ 0 \end{pmatrix} \sim (1,3, N_{\rho})$$

$$\langle \chi \rangle_0 = \begin{pmatrix} 0 \\ 0 \\ v_{\chi} \end{pmatrix} \sim (1,3, N_{\chi}).$$

(37)

With the requirement that the electric charge operator annihilates the vacuum of the scalars we set

$$N_{\rho} = -2N_{\eta}, \quad N_{\chi} = N_{\eta}, \quad b' = -\frac{1}{3N_{\eta}}.$$  

(38)

After the break of the symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_N \rightarrow SU(3)_C \otimes U(1)_{em}$ we find the following structure for their electromagnetic interactions

$$\mathcal{L}_{em} = -\frac{e}{6N_{\eta}} \sum_a \sum_{\alpha} (N_{L_a} - N_{N_{\eta}})\bar{\nu}_a \gamma^\mu v_a$$

$$\bar{e}_a ( (-N_{L_a} - N_{R_a} - 2N_{\eta}) - (-N_{L_a} + N_{R_a} - 2N_{\eta}) \gamma^\mu e_a$$

$$+ \bar{\nu}_a ( (-N_{Q_a} - N_{u_a} + 2N_{\eta}) - (-N_{Q_a} + N_{u_a} + 2N_{\eta}) \gamma^\mu \nu_a$$

$$+ \bar{u}_3 ( (-N_{Q_3} - N_{u_3} + N_{\eta}) - (-N_{Q_3} + N_{u_3} + N_{\eta}) \gamma^\mu u_3$$

$$+ \bar{d}_a ( (-N_{Q_a} - N_{d_a} - N_{\eta}) - (-N_{Q_a} + N_{d_a} - N_{\eta}) \gamma^\mu d_a$$

$$+ \bar{d}_3 ( (-N_{Q_3} - N_{d_3} - 2N_{\eta}) - (-N_{Q_3} + N_{d_3} - 2N_{\eta}) \gamma^\mu d_3$$

$$+ J_a ( (-N_{Q_a} - N_{J_a} - N_{\eta}) - (-N_{Q_a} + N_{J_a} - N_{\eta}) \gamma^\mu J_a$$

$$+ \bar{J}_3 ( (-N_{Q_3} - N_{J_3} + N_{\eta}) - (-N_{Q_3} + N_{J_3} + N_{\eta}) \gamma^\mu J_3) A_\mu.$$  

(39)

The Yukawa sector here is \[10\]

$$-\mathcal{L}_Y = G_{ab}\epsilon^{lmn} (\bar{L}_{aL})_l (L_{bL})_m (\rho_*)_n + G'_{ab} \bar{L}_{aL} e_{bR} \rho$$
\[ + \lambda_1 \bar{Q}_3 J_{3x} \chi + \lambda_{2\alpha\beta} \bar{Q}_\alpha L J_{\beta R} \chi^* \\
+ \lambda_{1a} \bar{Q}_3 d_a \rho + \lambda_{2\alpha a} \bar{Q}_\alpha L u_{aR} \rho^* \\
+ \lambda_{3a} \bar{Q}_3 u_{aR} \eta + \lambda_{4\alpha a} \bar{Q}_\alpha L d_a \eta^* + H.c., \] (40)

with \(a, b = 1, 2, 3\) and \(\alpha, \beta = 1, 2\). From \(U(1)_N\) invariance, this sector supplies us with the following relations between the \(N\) quantum numbers

\[
N_{u_1} = N_{u_2} = N_{u_3} = N_u, \\
N_{d_1} = N_{d_2} = N_{d_3} = N_d, \\
N_{Q_1} = N_{Q_2} = N_Q, \\
N_{J_1} = N_{J_2} = N_J, \\
\] (41)

and

\[
N_{L_1} = N_{L_2} = N_{L_3} = N_\eta, \\
N_{R_1} = N_{R_2} = N_{R_3} = 3N_\eta, \\
N_u = N_Q - 2N_\eta, \\
N_d = N_Q + N_\eta, \\
N_{Q_3} = N_Q - N_\eta, \\
N_{J_3} = N_Q - 2N_\eta, \\
N_J = N_Q + N_\eta. \\
\] (42)

Substituting these relations into (39) we obtain

\[
\mathcal{L}_{em} = e \bar{e}_a \gamma^\mu e_a A_\mu \\
- \frac{e}{3N_\eta} [(-N_Q + 2N_\eta) \bar{u}_a \gamma^\mu u_a - (N_Q + N_\eta) \bar{d}_a \gamma^\mu d_a \\
+ (N_Q + 2N_\eta) \bar{J}_3 \gamma^\mu J_3 - (N_Q + N_\eta) \bar{J}_\alpha \gamma^\mu J_\alpha] A_\mu. \] (43)

Note that, as in the version A, the classical constraints lead to ECQ and the VLNE in the leptonic sector while in the quark sector lead only to the VLNE.
Again, as in the previous section, only one nontrivial anomaly constraint remains

\[ [SU(3)_L]^2 U(1)_N \implies 3N_{Q_1} + 3N_{Q_2} + 3N_{Q_3} + N_{L_1} + N_{L_2} + N_{L_3} = 0, \]  

(44)

which together with the relations in \[ \text{(12)} \] gives \( N_Q = 0 \). This result fixes uniquely the \( N \)'s quantum numbers in function of \( N_\eta \) explaining the ECQ for all fermions and leading to the VLNE

\[
\mathcal{L}_{em} = e e_a \gamma^\mu e_a A_\mu + \\
- \frac{2e}{3} \bar{u}_a \gamma^\mu u_a A_\mu + \frac{e}{3} \bar{d}_a \gamma^\mu d_a A_\mu + \\
\frac{e}{3} \bar{J}_a \gamma^\mu J_a A_\mu - \frac{2e}{3} \bar{J}_3 \gamma^\mu J_3 A_\mu. 
\]  

(45)

The largest CBGM version is based on the \( SU(3)_C \otimes SU(4)_L \otimes U(1)_N \) symmetry group \[ [11] \]. In it the ECQ takes place in the same way as in the first simplest version, as was showed recently in \[ [16] \]. Thus in it the VLNE and the ECQ must be explained in the same way as in the simplest versions A. We close this section saying that in CBGM inevitably we explain the ECQ and the structure of the electromagnetic interactions.

### III. CONCLUSIONS

In this paper we have examined the correlation among ECQ and VLNE in some gauge theory of electroweak interactions in order to understand the structure of one of the four fundamental forces of the nature. Depending on the model and on their representation content ECQ and the VLNE are strongly correlated. This is the case with the SM with three generations and massless neutrinos. In this case, through the classical and quantum constraints, we do not explain neither the ECQ nor the VLNE. Nevertheless we can understand why the electric charge is quantized with the pattern required by nature through a correlation among the ECQ and the VLNE obtained in \[ (18) \]. There we can see that such required pattern occurs because the QED is vectorial. Also such correlation say that whether we wish explain the ECQ we must require as constraints the nonvanishing fermion masses,
the anomaly cancellations and the VLNE. In the case of Dirac-like massive neutrinos we lose such correlations in the sense that we have the VLNE but not the ECQ. In the case of Majorana-like massive neutrinos we restore the VLNE and the ECQ. In chiral bilepton gauge model we can explain the ECQ and the VLNE together. This takes place in all versions, with or without massive neutrinos, through the nonvanishing fermion masses and anomaly cancellations. These results make CBGM an interesting extension of the SM. Principally whether we hope that a final theory of matter and forces explains the VLNE and the ECQ together.

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