Radiation reaction and quantum damped harmonic oscillator

F. Kheirandish\(^{1}\) *and M. Amooshahi\(^{1}\) †

\(^{1}\) Department of Physics, University of Isfahan, Hezar Jarib Ave., Isfahan, Iran.

December 2, 2017

Abstract

By taking a Klein-Gordon field as the environment of an harmonic oscillator and using a new method for dealing with quantum dissipative systems (minimal coupling method), the quantum dynamics and radiation reaction for a quantum damped harmonic oscillator investigated. Applying perturbation method, some transition probabilities indicating the way energy flows between oscillator, reservoir and quantum vacuum, obtained.

Keywords: Radiation reaction, Dissipative systems, Minimal coupling

PACS numbers: 03.65.Ca, 03.65.Sq

1 Introduction

There are some treatments for investigating quantum mechanics of dissipative systems, one can consider the interaction between two systems via an irreversible energy flow [1,2], or use a phenomenological treatment for a
time dependent Hamiltonian which describes damped oscillations, here we can refer the interested reader to Caldirola-Kanai Hamiltonian for a damped harmonic oscillator [3]. There are significant difficulties about the quantum mechanical solutions of the Caldirola-Kanai Hamiltonian, for example quantizing with that Hamiltonian violates the uncertainty relations or canonical commutation rules, also the uncertainty relations vanish as time tends to infinity, [4,5,6,7,8].

When a quantum particle of mass $m$ moving in a one-dimensional potential $V(q)$ coupled to a heat bath, the macroscopic equation describing the time development of the particle motion is the quantum Langevin equation

$$m\ddot{q} + \int_{-\infty}^{t} dt' \mu(t-t')\dot{q}(t') + \frac{dV}{dq} = F(t),$$

(1)

where $\mu(t)$ is a memory function and $F(t)$ is a noise operator associated with absorption of energy of the particle by heat-bath. This is the Heisenberg equation of motion of the particle. For a charged particle with charge $e$, by taking the heat-bath as a quantum electromagnetic field, Ford et. al [10], obtained the Langevin equation (1) using the Hamiltonian

$$H = \left(\frac{p-eA_x}{2m}\right)^2 + V(q) + \sum \hbar c a_{\vec{k},s}^{\dagger} a_{\vec{k},s},$$

(2)

where $a_{\vec{k},s}, a_{\vec{k},s}^{\dagger}$ are annihilation and creation operators of photon field and $A_x$ is the $x$-component of vector potential given by

$$A_x = \sum_{\vec{k},s} \left(\frac{2\pi \hbar c}{kV}\right)^{\frac{1}{2}} \left[f_{\vec{k}} a_{\vec{k},s}^{\dagger} \hat{e}_{\vec{k},s}^{\dagger} \hat{x} + f_{\vec{k}} a_{\vec{k},s} \hat{e}_{\vec{k},s} \hat{x}\right],$$

(3)

where $f_{\vec{k}}$ is the form factor, $\hat{e}_{\vec{k},s}$ are polarization vectors and $V$ is the volume. In a paper [11] we generalized the Hamiltonian (2) for a quantum particle which is not necessarily a charged particle and its environment modeled by a Klein-Gordon type field, the Hamiltonian for a harmonic oscillator is written as

$$H = \frac{(p-R)^2}{2m} + \frac{1}{2}m\omega^2 q^2 + H_B,$$

(4)

where $m$, $\omega$ are mass and frequency of the oscillator and $q$ and $p$ are position and canonical conjugate momentum operators of the oscillator, respectively. $H_B$ is the Hamiltonian of the reservoir

$$H_B(t) = \int_{-\infty}^{+\infty} d^3k \omega_k b_{\vec{k}}^{\dagger}(t) b_{\vec{k}}(t), \quad \omega_{\vec{k}} = |\vec{k}|.$$

(5)
Annihilation and creation operators $b_{\vec{k}}$, $b_{\vec{k}}^\dagger$, in any instant of time, satisfy the following commutation relations

$$[b_{\vec{k}}(t), b_{\vec{k}}^\dagger(t')] = \delta(\vec{k} - \vec{k}').$$

(6)

Operator $R$ have the basic role in interaction between oscillator and reservoir and is defined as

$$R(t) = \int_{-\infty}^{+\infty} d^3k [f(\omega_{\vec{k}})b_{\vec{k}}(t) + f^*(\omega_{\vec{k}})b_{\vec{k}}^\dagger(t)].$$

(7)

It can be shown easily that combination of Heisenberg equation for $q(t)$ and $b_{\vec{k}}$ lead to

$$\ddot{q} + \omega^2 q + \int_0^t dt' \dot{q}(t')\gamma(t-t') = \xi(t),$$

$$\gamma(t) = \frac{8\pi}{m}\int_0^\infty d\omega_{\vec{k}} |f(\omega_{\vec{k}})|^2 \omega^3_k \cos \omega_{\vec{k}} t,$$

$$\xi(t) = \frac{i}{m} \int_{-\infty}^{+\infty} d^3k \omega_{\vec{k}} (f(\omega_{\vec{k}}) b_{\vec{k}}(0)e^{-i\omega_{\vec{k}} t} - f^*(\omega_{\vec{k}}) b_{\vec{k}}^\dagger(0)e^{i\omega_{\vec{k}} t}),$$

(8)

Where we have taken $\hbar = 1$. It is clear that the expectation value of $\xi(t)$ in any eigenstate of $H_B$, is zero. For the following special choice of coupling function

$$f(\omega_{\vec{k}}) = \sqrt{\frac{\beta}{4\pi^2 \omega^3_{\vec{k}}}},$$

(9)

equation (8) takes the form

$$\ddot{q} + \omega^2 q + \frac{\beta}{m} \dot{q} = \tilde{\xi}(t),$$

$$\tilde{\xi}(t) = i \sqrt{\frac{\beta}{4\pi^2 m^2}} \int_{-\infty}^{+\infty} \frac{d^3k}{\omega^3_{\vec{k}}} (b_{\vec{k}}(0)e^{-i\omega_{\vec{k}} t} - b_{\vec{k}}^\dagger(0)e^{i\omega_{\vec{k}} t}).$$

(10)

In QED, a charged particle in quantum vacuum interacts with the vacuum field and its own field, known as the radiation reaction. In classical electrodynamics, there is only the radiation reaction field that acts on a charged particle in the vacuum. The vacuum and radiation reaction fields have a fluctuation-dissipation connection and both are required for the consistency of QED. For example the stability of the ground state, atomic transitions and
lamb shift can only be explained by taking into account both fields. If self reaction was alone the atomic ground state would not be stable [12]. When a quantum mechanical system interacts with the quantum vacuum field, the coupled Heisenberg equations for both system and field give us the radiation reaction field, for example it can be shown that the radiation reaction for a charged harmonic oscillator is \( \frac{2e^2}{3c^3} \) [12]. In this paper we investigate the dynamics of a quantum damped harmonic oscillator interacting with the quantum vacuum and a reservoir by a minimal coupling method appropriate for dissipative quantum systems.

## 2 Quantum dynamics

When a damped Harmonic oscillator with charge \( e \), interacts with the quantum vacuum, the total Hamiltonian can be written like this

\[
H = \frac{(p - R - eA_x)^2}{2m} + \frac{1}{2}m\omega^2q^2 + H_B + H_F, \tag{11}
\]

where \( H_B \) and \( R \), are defined by relations (5) and (7). \( H_F \) is the vacuum Hamiltonian defined by

\[
H_F = \int d^3k \sum_{\lambda=1}^{2} \omega_k a_{k\lambda}^\dagger a_{k\lambda}, \tag{12}
\]

and \( A_x \) is the \( x \)-component of the vector potential or the vacuum field which in the dipole approximation [15] is

\[
A_x = \int_{-\infty}^{+\infty} \frac{d^3k}{\sqrt{2(2\pi)^3\omega_k}} \sum_{\lambda=1}^{2} \varepsilon_x(\vec{k}, \lambda)[a_{k\lambda}(t) + a_{k\lambda}^\dagger(t)], \tag{13}
\]

\( a_{k\lambda} \) and \( a_{k\lambda}^\dagger \) satisfy the usual commutation relations

\[
[a_{k\lambda}, a_{k'\lambda'}^\dagger] = \delta_{\lambda\lambda'}\delta(\vec{k} - \vec{k}'). \tag{14}
\]

Applying Heisenberg equation for operators \( q \) and \( p \), we find

\[
\dot{q} = \frac{p - eA_x - R}{2m}, \quad \dot{p} = -m\omega^2q, \tag{15}
\]
and their combination leads to
\[ \ddot{q} + \omega^2 q = -\frac{\dot{R}}{m} - e\frac{\dot{A}_r}{m}, \]  
(16)
also the Heisenberg equation for \( b_{\vec{k}} \) and \( a_{k\lambda} \) is as follows
\[ \dot{b}_{\vec{k}} = -i\omega_{\vec{k}} b_{\vec{k}} + i\dot{q} f^*(\omega_{\vec{k}}), \]
\[ \dot{a}_{k\lambda} = -i\omega_{\vec{k}} a_{k\lambda} + ie \dot{q} \frac{\varepsilon_{x}(\vec{k}, \lambda)}{\sqrt{2(2\pi)^3 \omega_{\vec{k}}}}, \]  
(17)
with the following formal solution
\[ b_{\vec{k}}(t) = b_{\vec{k}}(0)e^{-i\omega_{\vec{k}}t} + i\int_0^t dt' e^{-i\omega_{\vec{k}}(t-t')} \dot{q}(t'), \]
\[ a_{k\lambda}(t) = e^{-i\omega_{\vec{k}}t} a_{k\lambda}(0) + ie \int_0^t dt' e^{-i\omega_{\vec{k}}(t-t')} \dot{q}(t'), \]  
(18)
substituting \( \dot{b}_{\vec{k}} \) and \( \dot{a}_{k\lambda} \) into the right hand side of (16), we obtain
\[ \ddot{q} + \omega^2 q + \int_0^t dt' \gamma(t-t') \dot{q}(t') = \xi(t) - \frac{e}{m} E_0(t) - \frac{e}{m} E_{RR}(t), \]  
(19)
where \( \gamma(t) \) and \( \xi(t) \) are defined by (8) and \( E_0(t) \) is the vacuum field defined by
\[ E_0(t) = i \int_{-\infty}^{+\infty} d^3k \sqrt{\frac{\omega_{\vec{k}}}{2(2\pi)^3}} \sum_{\lambda=1}^2 \varepsilon_{x}(\vec{k}, \lambda)[a_{k\lambda}(0)^\dagger e^{i\omega_{\vec{k}}t} - a_{k\lambda}(0)e^{-i\omega_{\vec{k}}t}], \]  
(20)
\( E_{RR} \) is the radiation reaction electrical field [15]
\[ E_{RR}(t) = \frac{e}{3\pi^2} \int_0^t dt' \dot{q}(t') \int_{-\infty}^{+\infty} \omega^2 \cos \omega(t-t') d\omega = -\frac{e}{6\pi} \frac{\partial^3 q}{\partial t^3}, \]  
(21)
now substituting the special choice \( \xi(t) \), we find
\[ \ddot{q} + \omega^2 q + \frac{\beta}{m} \dot{q} + \tau \frac{\partial^3 q}{\partial t^3} = E_0(t) + \tilde{\xi}(t), \]  
(22)
where \( \tau = \frac{e^2}{6\pi m} \) and \( \tilde{\xi}(t) \) is defined by (10).
3 Transition probabilities

Let us write the Hamiltonian \( \mathbf{11} \) as

\[
H = H_0 + H',
\]

\[
H_0 = \frac{p^2}{2m} + \frac{1}{2}m\omega^2q^2 + H_F + H_B,
\]

\[
H' = -\frac{R}{m}p - \frac{e}{m}A_xp + \frac{e}{m}A_xR + \frac{R^2}{2m} + \frac{e^2}{2m}A^2_x.
\]

Because \( \frac{R^2}{2m} \) is of the second order of damping and \( \frac{e^2}{2m}A_x \) is small in comparison with \( -\frac{e}{m}A_xp \), then for sufficiently weak damping, we can approximate \( H' \) by

\[
H' = -\frac{R}{m}p - \frac{e}{m}A_xp + \frac{e}{m}A_xR,
\]

and in interaction picture, we can write

\[
H'_I(t) = e^{iH_0t}H'(0)e^{-iH_0t} = \frac{e}{m} \int_{-\infty}^{+\infty} d^3k \int_{-\infty}^{+\infty} d^3k' \sum_{\lambda'=1}^{2} \varepsilon_x(k', \lambda')
\]

\[
\times [f(\omega_k)b_k(0)a^\dagger_{k'\lambda'}(0)e^{i(\omega_{k'}-\omega_k)t} + f^*(\omega_k)a_{k'\lambda'}(0)b^\dagger_k(0)e^{i(\omega_k-\omega_{k'})t}]
\]

\[
+i\sqrt{\frac{\omega}{2m}} \int_{-\infty}^{+\infty} d^3k [f^*(\omega_k)ab^\dagger_{k}(0)e^{i(\omega_k-\omega) t} - f(\omega_k)a^\dagger_{k}(0)e^{i(\omega-\omega_k)t}]
\]

\[
+ie\sqrt{\frac{\omega}{2m}} \int_{-\infty}^{+\infty} \sum_{\lambda=1}^{2} [aa^\dagger_{k\lambda}(0)e^{i(\omega_k-\omega) t} - a^\dagger_{k\lambda}(0)e^{i(\omega-\omega_k)t}] \varepsilon_x(k, \lambda),
\]

where we have ignored the terms containing \( aa_{k\lambda}, ab^\dagger_{k}, b^\dagger_{k}a_{k\lambda} \) and their adjoints according to rotating-wave approximation\([13]\), because in the first order perturbation these terms at the same time destroys an excited state of harmonic oscillator, quantum vacuum and reservoir or create at the same time an excited state of harmonic oscillator, quantum vacuum and reservoir and so violate the conservation of energy. The density operator in interaction picture \( \rho_I \) can be obtained from \([13]\)

\[
\rho_I(t) = U_I(t, t_0)\rho_I(t_0)U_I^\dagger(t, t_0),
\]
where \( U_I \), up to the first order perturbation is

\[
U_I(t, 0) = 1 - i \int_0^t dt_1 H'_I(t_1) = 1 - \\
-\sqrt{\frac{\omega}{2m}} \int_{-\infty}^{+\infty} d^3k [f(\omega \vec{k}) a^\dagger \vec{b}_k(0) e^{i(\omega - \omega \vec{k})t} - f^*(\omega \vec{k}) a_k^\dagger(0) e^{-i(\omega - \omega \vec{k})t}] \sin \left( \frac{\omega - \omega \vec{k}}{2} \right) t
\]

\[
-\sqrt{\frac{\omega}{2m}} \int_{-\infty}^{+\infty} \frac{d^3k}{\sqrt{2(2\pi)^3}\omega_k} \sum_{\lambda=1}^2 \varepsilon_x(\vec{k}, \lambda) [a^\dagger a_k \lambda(0) e^{i(\omega - \omega \vec{k})t} - a_k^\dagger(0) e^{-i(\omega - \omega \vec{k})t}] \sin \left( \frac{\omega - \omega \vec{k}}{2} \right) t
\]

\[
-\sqrt{\frac{e}{2m}} \int_{-\infty}^{+\infty} d^3k \int_{-\infty}^{+\infty} \frac{d^3k'}{\sqrt{2(2\pi)^3}\omega_{k'}} \sum_{\lambda' \lambda=1}^2 \varepsilon_x(k', \lambda') \times
\]

\[
\times [f(\omega \vec{k}) b_{k'}^\dagger(0) a_{k' \lambda'}(0) e^{i(\omega - \omega \vec{k})t} + f^*(\omega \vec{k}) b_{k}^\dagger(0) a_{k \lambda'}(0) e^{-i(\omega - \omega \vec{k})t}] \sin \left( \frac{\omega - \omega \vec{k}}{2} \right) t.
\]

(27)

Now let us calculate the transition probability \( |n\rangle_\omega \rightarrow |n-1\rangle_\omega \), which indicates the way energy flows between the subsystems in some different cases. The interested reader is referred to N. D. Hari Dass et. al [14], for some related decaying rates obtained trough path integral techniques.

1. If \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes |0\rangle_F \langle 0| \otimes |0\rangle_B \langle 0| \), where \( |0\rangle_B \) and \( |0\rangle_F \), are the vacuum state of reservoir and quantum vacuum respectively and \( |n\rangle_\omega \) is an excited state of the harmonic oscillator, then substituting \( U_I(t, 0) \) from (27) in (26) and tracing out the degrees of freedom of both reservoir and quantum vacuum, we obtain the following transition probability for the transition \( |n\rangle_\omega \rightarrow |n-1\rangle_\omega \) in long time approximation

\[
\Gamma_{n\rightarrow n-1} = Tr_s[|n-1\rangle_\omega \langle n-1| \rho_{sI}(t)]
\]

\[
= \frac{4\pi^2 \omega^3 nt}{m} |f(\omega)|^2 + \frac{n\omega^2 e^2 t}{6\pi m},
\]

(28)

where \( Tr_s \) denotes tracing over the harmonic oscillator and \( \rho_{sI} \) is defined by tracing out the quantum vacuum and reservoir \( \rho_{sI} = Tr_{F,B}[\rho_I(t)] \). For special choice (9) we find

\[
\Gamma_{n\rightarrow n-1} = \frac{n\beta t}{m} + \frac{n\omega^2 e^2 t}{6\pi m},
\]

(29)
and there is no transition from $|n\rangle_\omega$ to $|n-1\rangle_\omega$ in this case.

2. $\rho_I(0) = |n\rangle_\omega \omega\langle n| \otimes |0\rangle_F \otimes |\vec{p}_1, \ldots \vec{p}_j\rangle_B$ $B \langle \vec{p}_1, \ldots \vec{p}_j| |\vec{p}_1, \ldots \vec{p}_j\rangle_B$ denotes a state of reservoir that contains $j$ quanta with corresponding momenta $\vec{p}_1, \ldots \vec{p}_j$, from

$$Tr_B[b^\dagger_k|\vec{p}_1, \ldots \vec{p}_j\rangle_B \langle \vec{p}_1, \ldots \vec{p}_j| b_{k'}] = \delta(k - k'),$$

$$Tr_B[b^\dagger_k|\vec{p}_1, \ldots \vec{p}_j\rangle_B \langle \vec{p}_1, \ldots \vec{p}_j| b_{k'}] = \sum_{l=1}^{j} \delta(k - p_l)\delta(k' - p_l),$$

and waiting for a long time, we obtain

$$\Gamma_{n\rightarrow n-1} = \frac{n\beta t}{m} + \frac{n\omega^2 e^2 t}{6\pi m},$$

$$\Gamma_{n\rightarrow n+1} = \frac{\beta(n+1)t}{4\pi m\omega^2} \sum_{l=1}^{j} \delta(\omega p_l - \omega),$$

(31)

for special choice (9).

3. $\rho_I(0) = |n\rangle_\omega \omega\langle n| \otimes |0\rangle_F \otimes \rho_T^T$ where $\rho_T^T = e^{-\frac{H_B}{kT}}$ $T_R B(e^{-\frac{H_B}{kT}})$ is the Maxwell Boltzmann distribution of reservoir, then using relations

$$Tr_B[b^\dagger_k\rho_T^T b_{k'}] = Tr_B[b^\dagger_k\rho_T^T b_{k'}] = 0,$$

$$Tr_b[b^\dagger_k\rho_T^T b_{k'}] = \frac{\delta(k - k')}{e^{\frac{kT}{\omega}} - 1},$$

$$Tr_B[b^\dagger_k\rho_T^T b_{k'}] = \frac{\delta(k - k')e^{\frac{kT}{\omega}}}{e^{\frac{kT}{\omega}} - 1},$$

(32)

one can obtain the transition probabilities

$$\Gamma_{n\rightarrow n-1} = \frac{n\beta t e^{\frac{kT}{\omega}}}{m(e^{\frac{kT}{\omega}} - 1)} + \frac{n\omega^2 e^2 t}{6\pi m},$$

$$\Gamma_{n\rightarrow n+1} = \frac{(n+1)\beta t}{m} \frac{1}{e^{\frac{kT}{\omega}} - 1},$$

(33)

4. $\rho_I(0) = |n\rangle_\omega \omega\langle n| \otimes |\vec{p}_1, \lambda_1, \ldots \vec{p}_j, \lambda_j\rangle_F \otimes |0\rangle_F \otimes |\vec{p}_1, \lambda_1, \ldots \vec{p}_j, \lambda_j\rangle_B$ $B \langle \vec{p}_1, \lambda_1, \ldots \vec{p}_j, \lambda_j| \otimes |0\rangle_B \otimes |0\rangle_B$ where $|\vec{p}_1, \lambda_1, \ldots \vec{p}_j, \lambda_j\rangle_B$ denotes a state of quantum vacuum that contains $j$ photon
with corresponding momenta $\vec{p}_1, ... \vec{p}_j$, and polarizations $\lambda_1, ..., \lambda_j$ respectively then by making use of

$$Tr_F[a_k^{\dagger}\lambda^i_1|\vec{p}_1, \lambda_1 \ldots \vec{p}_j, \lambda_j,F a_k^{\dagger}\lambda^i_j]|\vec{p}_1, \lambda_1 \ldots \vec{p}_j, \lambda_j,F] = \delta(\vec{k} - \vec{k}')\delta_{\lambda^i_1 \lambda^i_j},$$

$$Tr_F[a_k\lambda^i_1|\vec{p}_1, \lambda_1 \ldots \vec{p}_j, \lambda_j,F a_k^{\dagger}\lambda^i_j]|\vec{p}_1, \lambda_1 \ldots \vec{p}_j, \lambda_j,F] = \sum_{l=1}^j \delta(\vec{k} - \vec{p}_l)\delta(\vec{k}' - \vec{p}_l)\delta_{\lambda_l \lambda^i_1 \lambda^i_j} \delta_{\lambda^i_1 \lambda^i_l}.$$  

(34)

we find

$$\Gamma_{n\rightarrow n-1} = \frac{n\beta t}{m} + \frac{n\omega^2 e^2 t}{6\pi m},$$

$$\Gamma_{n\rightarrow n+1} = \frac{(n+1)e^2 \omega t}{4\pi m \omega^2} \sum_{l=1}^j \delta(\omega p_l - \omega) + \frac{(n+1)e^2 \omega t}{16\pi^2 m} \sum_{l=1}^j \frac{\varepsilon_x(\vec{p}_l, \lambda_l)^2}{\omega p_l} \delta(\omega p_l - \omega).$$

(35)

5. $\rho_I(0) = |n\rangle_\omega \langle n| \otimes |\vec{p}_1, \lambda_1, ... \vec{p}_j, \lambda_j\rangle_F F |\vec{p}_1, \lambda_1, ... \vec{p}_j, \lambda_j\rangle \otimes |\vec{q}_1, ... , \vec{q}_j\rangle_B B |\vec{q}_1, ... , \vec{q}_j\rangle$ using (30) and (34) we have

$$\Gamma_{n\rightarrow n-1} = \frac{n\beta t}{m} + \frac{n\omega^2 e^2 t}{6\pi m},$$

$$\Gamma_{n\rightarrow n+1} = \frac{\beta(n+1)t}{4\pi m \omega^2} \sum_{l=1}^j \delta(\omega p_l - \omega) + \frac{(n+1)e^2 \omega t}{16\pi^2 m} \sum_{l=1}^j \frac{\varepsilon_x(\vec{p}_l, \lambda_l)^2}{\omega p_l} \delta(\omega p_l - \omega).$$

(36)

6. $\rho_I(0) = |n\rangle_\omega \langle n| \otimes |\vec{p}_1, \lambda_1, ... \vec{p}_j, \lambda_j\rangle_F F |\vec{p}_1, \lambda_1, ... \vec{p}_j, \lambda_j\rangle \otimes \rho_B^T$ using (32), gives

$$\Gamma_{n\rightarrow n-1} = \frac{n\beta t}{m(e^{\frac{\pi t}{\tau}} - 1)} + \frac{n\omega^2 e^2 t}{6\pi m},$$

$$\Gamma_{n\rightarrow n+1} = \frac{(n+1)\beta t}{m} \frac{1}{e^{\frac{\pi t}{\tau}} - 1} + \frac{(n+1)e^2 \omega t}{16\pi^2 m} \sum_{l=1}^j \frac{\varepsilon_x(\vec{p}_l, \lambda_l)^2}{\omega p_l} \delta(\omega p_l - \omega).$$

(37)
7. \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes \rho_F^T \otimes |0\rangle_B \langle 0| \) where \( \rho_F^T \) is the Maxwell-Boltzmann distribution of quantum vacuum, using

\[
\begin{align*}
    Tr_F[a_{k\lambda} \rho_F a_{k'\lambda'}] &= Tr_F[a_{k\lambda} \rho_F a_{k'\lambda'}] = 0, \\
    Tr_F[a_{k\lambda} \rho_F a_{k'\lambda'}^\dagger] &= \frac{\delta(\vec{k} - \vec{k}')\delta_{\lambda\lambda'}}{e^{\frac{\omega}{kT}} - 1}, \\
    Tr_F[a_{k\lambda} \rho_F a_{k'\lambda'}] &= \frac{\delta(\vec{k} - \vec{k}')e^{\frac{\omega}{kT}}}{e^{\frac{\omega}{kT}} - 1},
\end{align*}
\]

leads to

\[
\begin{align*}
    \Gamma_{n \to n-1} &= \frac{n\beta t}{m} + \frac{n\omega^2 e^2 t}{6\pi m} \frac{e^{\frac{\omega}{kT}}}{e^{\frac{\omega}{kT}} - 1}, \\
    \Gamma_{n \to n+1} &= \frac{(n+1)e^2 \omega^2 t}{6\pi m} \frac{1}{e^{\frac{\omega}{kT}} - 1}. \tag{38}
\end{align*}
\]

8. \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes \rho_F^T \otimes |\vec{q}_1, ..., \vec{q}_j\rangle_B \langle \vec{q}_1, ..., \vec{q}_j| \) using (30), we find

\[
\begin{align*}
    \Gamma_{n \to n-1} &= \frac{n\beta t}{m} + \frac{n\omega^2 e^2 t}{6\pi m} \frac{e^{\frac{\omega}{kT}}}{e^{\frac{\omega}{kT}} - 1}, \\
    \Gamma_{n \to n+1} &= \frac{(n+1)e^2 \omega^2 t}{6\pi m} \frac{1}{e^{\frac{\omega}{kT}} - 1} + \frac{\beta(n+1)t}{4\pi m} \sum_{l=1}^{j} \delta(\omega_{\vec{p}_l} - \omega). \tag{39}
\end{align*}
\]

9. Finally for \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes \rho_F^T \otimes \rho_B^T \), and using (32) and (38), we find

\[
\begin{align*}
    \Gamma_{n \to n-1} &= \left[ \frac{n\beta t}{m} + \frac{n\omega^2 e^2 t}{6\pi m} \right] \frac{e^{\frac{\omega}{kT}}}{e^{\frac{\omega}{kT}} - 1}, \\
    \Gamma_{n \to n+1} &= \frac{\left[(n+1)e^2 \omega^2 t\right]}{6\pi m} \frac{1}{e^{\frac{\omega}{kT}} - 1} + \frac{(n+1)\beta t}{m} \frac{e^{\frac{\omega}{kT}}}{e^{\frac{\omega}{kT}} - 1}. \tag{40}
\end{align*}
\]

So in all cases the rate of energy transmitted from oscillator to the environment is a constant.

3.1 Transition probability between quantum vacuum and reservoir

Energy flows between reservoir and quantum vacuum with a non-zero probability while the state of oscillator remains unchanged. This is due to the first
integral in [25]. For example if \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes |\vec{p}, r\rangle_F \langle \vec{p}, r| \otimes |0\rangle_B \langle 0| \), where \( |\vec{p}, r\rangle_F \) denotes a state that contains a photon with momentum \( \vec{p} \) and polarization \( r \), then probability of absorbing \( |\vec{p}, r\rangle_F \) by reservoir in the long time approximation is

\[
\frac{e^2 \omega_{\vec{p}}}{2\pi m^2} |f(\omega_{\vec{p}})|^2 (\varepsilon_x(\vec{p}, r))^2.
\] (42)

For the initial state \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes |\vec{p}, r\rangle_F \langle \vec{p}, r| \otimes \rho^T_F \), the probability that a photon with momentum \( \vec{p} \) and polarization \( r \) be created, in the long time approximation, is

\[
\frac{e^2 \omega_{\vec{p}}}{2\pi m^2} |f(\omega_{\vec{p}})|^2 (\varepsilon_x(\vec{p}, r))^2 \frac{1}{e^{\frac{\omega_{\vec{p}}}{kT}} - 1},
\] (43)

where we have used [32]. Also if the initial state is \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes |\vec{p}, r\rangle_F \langle \vec{p}, r| \otimes \rho^T_F \), then the probability that the mentioned photon be absorbed by the reservoir, is

\[
\frac{e^2 \omega_{\vec{p}}}{2\pi m^2} |f(\omega_{\vec{p}})|^2 (\varepsilon_x(\vec{p}, r))^2 \frac{e^{\frac{\omega_{\vec{p}}}{kT}}}{e^{\frac{\omega_{\vec{p}}}{kT}} - 1}.
\] (44)

If the initial state is \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes |\vec{p}, r\rangle_F \langle \vec{p}, r| \otimes \rho^T_F \), then using [32], it can be shown that the probability of absorbing a photon with momentum \( \vec{p} \) and polarization \( r \), by the quantum vacuum, in the long time approximation, is

\[
\frac{e^2 t}{8m^2 \pi^2 \omega_{\vec{p}}} (\varepsilon_x(\vec{p}, r))^2 \sum_{l=1}^{j} |f(\omega_{\vec{p}_l})|^2 \delta(\omega_{\vec{p}_l} - \omega_{\vec{p}}),
\] (45)

and for \( \rho_I(0) = |n\rangle_\omega \langle n| \otimes |\vec{p}, r\rangle_F \langle \vec{p}, r| \otimes |\vec{p}_1, ..., \vec{p}_j\rangle_B \langle \vec{p}_1, ..., \vec{p}_j| \), the probability that this photon be absorbed by the reservoir, is

\[
\frac{e^2 t \omega_{\vec{p}}}{2m^2 \pi^2 (2\pi)^2} (\varepsilon_x(\vec{p}, r))^2 |f(\omega_{\vec{p}})|^2.
\] (46)

### 4 Concluding remarks

By taking a Klein-Gordon field as the environment of an harmonic oscillator and using minimal coupling method, we could obtain a Heisenberg equation
that contained a dissipative term proportional to velocity and also a radiation reaction electromagnetic field. In this way we observed that energy can be exchanged between harmonic oscillator, quantum vacuum and reservoir and the rate of dissipation of energy of the harmonic oscillator was a constant.

References

[1] H. Haken, Rev. Mod. Phys. 47 (1975) 67.

[2] G. Nicolis, I. Prigogine, Self-Organization in Non-Equilibrium system, Wiley, new York, (1977).

[3] P. Caldirola, Nuovo Cimento 18 (1941) 393.

[4] I. R. Svinin, Teor. Mat. Fiz. 27 (1972) 2037.

[5] W. E. Brittin, Phys. Rev. 77 (1950) 396.

[6] P. Havas, Bull. Am. Phys. Soc. 1 (1956) 337.

[7] G. Valentini, Rend. Ist. Lomb. So: A 95 (1961) 255

[8] M. Razavy, Can. J. Phys. 50 (1972) 2037

[9] G. W. Ford, J. T. Lewis, R. F. OConnell, Phys. Rev. A 37 (1988) 4419

[10] G. W. Ford, J. T. Lewis, R. F. OConnell, Phys. Rev. Lett. 55 (1985) 2273

[11] F. Kheirandish, M. Amooshahi, quant-ph/0502076 (2005)

[12] P. W. Milonni, The quantum vacuum, Academic press, New York, (1994)

[13] M. O. Scully, M. S. Zubairy, Quantum optics, Cambridge, (1997).

[14] N. D. Hari Dass, S. Kalyana and B. Sathiapalan, Int. J. Mod. Phys. A, Vol. 18, No.17, (2003) 2947