The effective action of $N = 1$ Calabi-Yau orientifolds

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ABSTRACT

We determine the $N = 1$ low energy effective action for compactifications of type IIB string theory on compact Calabi-Yau orientifolds in the presence of background fluxes from a Kaluza-Klein reduction. The analysis is performed for Calabi-Yau threefolds which admit an isometric and holomorphic involution. We explicitly compute the Kähler potential, the superpotential and the gauge kinetic functions and check the consistency with $N = 1$ supergravity. We find a new class of no-scale Kähler potentials and show that their structure can be best understood in terms of a dual formulation where some of the chiral multiplets are replaced by linear multiplets. For $O_3$- and $O_7$-planes the scalar potential is expressed in terms of a superpotential while for $O_5$- and $O_9$-planes also a $D$-term and a massive linear multiplet can be present. The relation with the associated F-theory compactifications is briefly discussed.
1 Introduction

In recent years it became clear that string theory is not only a theory of fundamental strings but also contains higher-dimensional extended objects such as D-branes and orientifold planes [1, 2, 3]. D-branes have massless excitations associated with the attached open strings while orientifold planes carry no physical degrees of freedom. Their relevance arises from the fact that they can have negative tension and are often necessary ingredients to ensure the consistency of a compactifications. In order to cancel gravitational and electro-magnetic tadpoles on a compact manifold in the presence of D-branes and/or background fluxes objects with negative tension have to be included [4].

Phenomenologically the most interesting case are compactifications which lead to a (spontaneously broken) \( N = 1 \) supersymmetry in four space-time dimensions (\( D = 4 \)). Such theories naturally arise as Calabi-Yau compactifications of heterotic or type I string theories. In type II string compactifications on Calabi-Yau threefolds \( Y \) one obtains instead an \( N = 2 \) theory in \( D = 4 \). However, this \( N = 2 \) can be further broken to \( N = 1 \) by introducing appropriate BPS D-branes and/or orientifold planes. Turning on additional background fluxes in such compactifications generically breaks (spontaneously) the left over \( N = 1 \) supersymmetry [4]–[34].

The purpose of this paper is to determine the \( N = 1, D = 4 \) low energy effective action for Calabi-Yau orientifold compactifications in the presence of background fluxes. To do so we use a standard Kaluza-Klein reduction which is valid in the large volume limit. For special classes of type IIB Calabi-Yau orientifolds this effective action has already been determined in refs. [4, 19, 21, 20, 25, 26]. Here we generalize the class of orientifolds but continue to focus on type IIB string theory leaving the discussion of type IIA and mirror symmetry to a separate publication. For some of the cases we discuss consistency requires the presence of background D-branes. However, in our analysis we freeze all of the associated massless fluctuations and solely concentrate on the couplings of the orientifold bulk. Including D-branes fluctuations has recently been discussed in \( N = 1 \) theories in refs. [22, 28, 29, 32].

More precisely, we start from type IIB string theory and compactify on Calabi-Yau threefolds \( Y \). In addition we mod out by orientation reversal of the string world-sheet \( \Omega_p \) together with an ‘internal’ symmetry \( \sigma \) which acts solely on \( Y \) but leaves the \( D = 4 \) Minkowskian space-time untouched. Consistency requires \( \sigma \) to be an isometric and holomorphic involution of \( Y \) [35, 36]. Hence in our analysis we focus on the class of Calabi-Yau threefolds which admit such an involution but within this class we leave the threefolds arbitrary. One can show that for such threefolds \( \sigma \) leaves the Kähler form \( J \) invariant but can act non-trivially on the holomorphic three-form \( \Omega \). Depending on the transformation properties of \( \Omega \) two different symmetry operations \( \mathcal{O} \) are possible [37, 38, 35, 36]. One can have either

\[
\mathcal{O}_{(1)} = (-1)^{F_L} \Omega_p \sigma^* , \quad \sigma^* \Omega = -\Omega , \quad (1.1)
\]

or

\[
\mathcal{O}_{(2)} = \Omega_p \sigma^* , \quad \sigma^* \Omega = \Omega . \quad (1.2)
\]

\(^1\) The present paper should be regarded as a companion paper to ref. [32].
$\Omega_p$ is the world-sheet parity, $F_L$ is the space-time fermion number in the left-moving sector and $\sigma^*$ denotes the action of $\sigma$ on forms (the pull-back of $\sigma$). Modding out by $O^{(1)}$ leads to the possibility of having $O3$- and $O7$-planes while modding out by $O^{(2)}$ allows $O5$- and $O9$-planes.

This paper is organized as follows. In order to set the stage we briefly recall the compactification of type IIB on Calabi-Yau threefolds in section 2 following refs. [39, 8, 10, 18]. In section 3 we analyze orientifold theories which arise when the string theory is modded out by the symmetry (1.1). We first determine the low energy spectrum and show how it assembles into $N = 1$ supermultiplets (section 3.1). In section 3.2 we compute the effective action in the presence of background fluxes from a Kaluza-Klein reduction and show that it obeys the constraints of $N = 1$ supergravity by explicitly determining the Kähler potential, the superpotential and the gauge-kinetic functions. We find that the potential is positive semi-definite and can be expressed entirely in terms of a superpotential [7, 10, 4, 19]. The gauge kinetic functions turn out to depend holomorphically on the complex structure deformations but they are independent of all other moduli. The Kähler metric is block diagonal with one factor descending from the standard metric of the complex structure deformations [10]. The second factor has a ‘no-scale’ form [11] and is a non-trivial mixing of the other moduli including the dilaton and the Kähler deformations. Aspects of this Kähler potential have also been discussed recently in ref. [43] from a slightly different perspective. In section 3.3 we rewrite the effective action in the linear multiplet formalism following [42] and show that some of its properties can be understood more conceptually in this dual formulation. As a byproduct we discover a new class of no-scale Kähler potentials.

In section 4 we repeated the same analysis for the projection (1.2). In 4.1 we determine the massless spectrum while in 4.2 we compute the effective action. As before the consistency with $N = 1$ supergravity can be established by explicitly determining the Kähler potential, the superpotential and the gauge kinetic function. However, due to the different projection (1.2) the effective action turns out to have a different structure. Analogously to the situation in $N = 2$ compactifications we find that depending on the choice of background fluxes a universal two-form can become massive [18]. The potential is again positive semi-definite but this time only the RR-fluxes contribute to the superpotential while the NS-fluxes instead lead to a $D$-term and a mass-term for the scalar partner of the massive two-form. The gauge kinetic function again depends holomorphically on the complex structure deformations. As before the Kähler potential and the couplings of the massive two-form can be best understood in the linear multiplet formalism.

Section 5 contains our conclusions and some of the details of the computations are relegated to four appendices. In appendix A we summarize our conventions. In appendix B we present the detailed computation of the scalar potentials while appendix C provides the details of the computation of the Kähler metric.

2 Type IIB compactified on Calabi-Yau threefolds

In order to set the stage for the following sections let us briefly recall the compactification of type IIB supergravity on Calabi-Yau manifolds following refs. [39, 8, 10, 18].
sections 3 and 4 we then repeat the analysis including the orientifold projections \([1.1]\) and \([1.2]\), respectively.

The massless bosonic spectrum of type IIB in \(D = 10\) consists of the dilaton \(\hat{\phi}\), the metric \(\hat{g}\) and a two-form \(\hat{B}_2\) in the NS-NS sector and the axion \(\hat{l}\), a two-form \(\hat{C}_2\) and a four-form \(\hat{C}_4\) in the R-R sector.\(^2\) Using form notation (our conventions are summarized in appendix A) the type IIB low energy effective action in the \(D = 10\) Einstein frame is given by \([2]\)

\[
S_{IIB}^{(10)} = -\int \left( \frac{1}{2} \hat{R} \ast 1 + \frac{1}{4} \hat{d}\hat{\phi} \wedge * \hat{d}\hat{\phi} + \frac{1}{4} e^{-\hat{\phi}} \hat{H}_3 \wedge * \hat{H}_3 \right) - \frac{1}{4} \int \left( e^{2\hat{\phi}} \hat{d}l \wedge * \hat{dl} + e^{\hat{\phi}} \hat{F}_3 \wedge * \hat{F}_3 + \frac{1}{2} \hat{F}_5 \wedge * \hat{F}_5 \right) - \frac{1}{4} \int \hat{C}_4 \wedge \hat{H}_3 \wedge \hat{F}_3 ,
\]

where \(*\) denotes the Hodge-* operator and the field strengths are defined as

\[
\hat{H}_3 = d\hat{B}_2 , \quad \hat{F}_3 = d\hat{C}_2 - \hat{l} d\hat{B}_2 , \quad \hat{F}_5 = d\hat{C}_4 - \frac{1}{2} d\hat{B}_2 \wedge \hat{C}_2 + \frac{1}{2} \hat{\bar{B}}_2 \wedge d\hat{C}_2 .
\]

The self-duality condition \(\hat{F}_5 = * \hat{F}_5\) is imposed at the level of the equations of motion.

In standard Calabi-Yau compactifications one chooses the 10-dimensional background metric to be block diagonal or in other words the line element to take the form

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu + g_{ij} dy^i dy^j ,
\]

where \(g_{\mu\nu}\), \(\mu, \nu = 0, \ldots, 3\) is a Minkowski metric and \(g_{ij}, i, j = 1, \ldots, 3\) is the metric on the Calabi-Yau manifold \(Y\). Deformations of this metric which respect the Calabi-Yau condition correspond to scalar fields in \(D = 4\). The deformations of the Kähler form \(J = ig_{ij} dy^i \wedge dy^j\) give rise to \(h^{(1,1)}\) real scalar fields \(v^A(x)\) and one expands

\[
J = v^A(x) \omega_A , \quad A = 1, \ldots, h^{(1,1)},
\]

where \(\omega_A\) are harmonic \((1,1)\)-forms on \(Y\) which form a basis of the cohomology group \(H^{(1,1)}(Y)\). Deformations of the complex structure are parameterized by complex scalar fields \(z^K(x)\) and are in one-to-one correspondence with harmonic \((1,2)\)-forms

\[
\delta g_{ij} = \frac{i}{||\Omega||^2} z^K(\bar{\chi}_K)_{ij} \Omega^{j\bar{j}} , \quad K = 1, \ldots, h^{(1,2)} ,
\]

where \(\Omega\) is the holomorphic \((3,0)\)-form, \(\bar{\chi}_K\) denotes a basis of \(H^{(1,2)}\) and we abbreviate \(||\Omega||^2 \equiv \frac{1}{3!} \Omega_{ijk} \bar{\Omega}^{ijk} .

Using the Ansatz \([2.4]\) the type IIB gauge potentials appearing in the Lagrangian \(2.1\) are similarly expanded in terms of harmonic forms on \(Y\) according to

\[
\hat{B}_2 = B_2(x) + b^A(x) \omega_A , \quad \hat{C}_2 = C_2(x) + c^A(x) \omega_A , \quad A = 1, \ldots, h^{(1,1)} ,
\]

\[
\hat{C}_4 = D_2^A(x) \wedge \omega_A + V^K(x) \wedge \alpha_K - U_K(x) \wedge \beta^K + \rho_A(x) \bar{\omega}^A , \quad \hat{K} = 0, \ldots, h^{(1,2)} .
\]

\(^2\)The hats ‘*’ denote ten-dimensional fields.
In Table 2.1 we summarize the non-trivial cohomology groups on $Y$ and denote their basis elements which appear in the expansion of the ten-dimensional fields. As we already indicated the $\omega_A$ are harmonic $(1, 1)$-forms while the $\tilde{\omega}^A$ are harmonic $(2, 2)$-forms which form a basis of $H^{(2,2)}(Y)$ dual to the $(1, 1)$-forms $\omega_A$. $(\alpha_K, \beta^L)$ are harmonic three-forms and form a real, symplectic basis on $H^{(3)}(Y)$ in that they satisfy

$$\int \alpha_K \wedge \beta^L = \delta^K_L, \quad \int \alpha_K \wedge \alpha_L = 0 = \int \beta^K \wedge \beta^L.$$  \hspace{1cm} (2.7)

| cohomology group | dimension | basis |
|------------------|-----------|-------|
| $H^{(1,1)}$      | $h^{(1,1)}$ | $\omega_A$ |
| $H^{(2,2)}$      | $h^{(1,1)}$ | $\tilde{\omega}^A$ |
| $H^{(3)}$        | $2h^{(2,1)} + 2$ | $(\alpha_K, \beta^L)$ |
| $H^{(2,1)}$      | $h^{(2,1)}$ | $\chi_K$ |

Table 2.1: Cohomology groups on $Y$ and their basis elements.

The four-dimensional fields appearing in the expansion are the scalars $b^A(x)$, $c^A(x)$ and $\rho_A(x)$, the one-forms $V^K(x)$ and $U_K(x)$ and the two-forms $B_2(x), C_2(x)$ and $D_A^2(x)$. The self-duality condition of $\tilde{F}_5$ eliminates half of the degrees of freedom in $\tilde{C}_4$ and in this section we choose to eliminate $D_A^2$ and $U_K$ in favor of $\rho_A$ and $V^K$. Finally, the two type IIB scalars $\hat{\phi}, \hat{l}$ also appear as scalars in $D = 4$ and therefore we drop the hats henceforth and denote them by $\phi, l$.

Altogether the massless $D = 4$ spectrum consists of the gravity multiplet with bosonic components $(g_{\mu\nu}, V^0)$, $h^{(2,1)}$ vector multiplets with bosonic components $(V^K, z^K)$, $h^{(1,1)}$ hypermultiplets with bosonic components $(v^A, b^A, c^A, \rho_A)$ and one double-tensor multiplet with bosonic components $(B_2, C_2, \phi, l)$ which can be dualized to an additional (universal) hypermultiplet. The four-dimensional spectrum is summarized in Table 2.2.

| multiplet              | rank | components          |
|------------------------|------|---------------------|
| gravity multiplet      | 1    | $(g_{\mu\nu}, V^0)$ |
| vector multiplets      | $h^{(2,1)}$ | $(V^K, z^K)$ |
| hypermultiplets        | $h^{(1,1)}$ | $(v^A, b^A, c^A, \rho_A)$ |
| double-tensor multiplet| 1    | $(B_2, C_2, \phi, l)$ |

Table 2.2: $N = 2$ multiplets for Type IIB supergravity compactified on a Calabi-Yau manifold.

The $N = 2$ low energy effective action is computed by inserting into the action and integrating over the Calabi-Yau manifold. For the details we refer the
reader to the literature [39, 8, 16, 18] and only recall the results here. One finds

\[ S_{IIB}^{(I)} = \int -\frac{1}{2} R * 1 + \frac{1}{4} \text{Re} \mathcal{M}_{KL} F^K \wedge F^L + \frac{1}{4} \text{Im} \mathcal{M}_{KL} F^K \wedge * F^L \]

\[-G_{KL} dz^K \wedge * dz^L - G_{AB} dv^A \wedge * dv^B - \frac{1}{4} d \ln K \wedge * d \ln K - \frac{1}{4} d \phi \wedge * d \phi \]

\[-\frac{1}{4} e^{2 \phi} \, dl \wedge * dl - e^{-\phi} G_{AB} db^A \wedge * db^B - e^\phi G_{AB} \left( dc^A - l db^A \right) \wedge * \left( dc^B - l db^B \right) \]

\[-\frac{9 G^{AD}}{4 K^2} \left( d \rho_A - \frac{1}{2} \mathcal{K}_{ABC} \left( e^B db^C - b^B dc^C \right) \right) \wedge * \left( d \rho_D - \frac{1}{2} \mathcal{K}_{DEF} \left( e^E db^F - b^E dc^F \right) \right) \]

\[-\frac{K^2}{144} e^{-\phi} db_2 \wedge * db_2 - \frac{K^2}{144} e^\phi \left( d C_2 - l db_2 \right) \wedge * \left( d C_2 - l db_2 \right) \] (2.8)

\[ + \frac{1}{2} \left( db^A \wedge C_2 + c^A db_2 \right) \wedge \left( d \rho_A - \mathcal{K}_{ABCC} db^C \right) + \frac{1}{4} \mathcal{K}_{ABC} b c^B db_2 \wedge db^C, \]

where \( F^K = d V^K \). The gauge kinetic matrix \( \mathcal{M}_{KL} \) is related to the metric on \( H^3(Y) \) and defined by [45]

\[ \int \alpha^\kappa \wedge * \alpha^\lambda = \left( \text{Im} \mathcal{M} + (\text{Re} \mathcal{M})(\text{Im} \mathcal{M})^{-1}(\text{Re} \mathcal{M}) \right)^\kappa, \]

\[ \int \beta^\kappa \wedge * \beta^\lambda = \left( \text{Im} \mathcal{M} \right)^{-1} \beta^\kappa \]

\[ \int \alpha^\kappa \wedge * \beta^\lambda = \left( \text{Re} \mathcal{M} \right)^{-1} \beta^\lambda \] (2.9)

\[ \mathcal{M}_{KL} = \mathcal{F}^\kappa = \int Y \wedge \beta^\kappa, \quad \mathcal{F}^\kappa = \int Y \wedge \alpha^\kappa, \quad \mathcal{F}_{KL} = \frac{\partial \mathcal{F}^\kappa}{\partial X^L}. \] (2.10)

where \( X^\kappa \) and \( \mathcal{F}^\kappa \) are the periods of the holomorphic three-form \( \Omega(z) \) and \( \mathcal{F}_{KL} \) is the period matrix defined as

\[ \mathcal{M}_{KL} = \mathcal{F}^\kappa \wedge 2i \frac{\left( \text{Im} \mathcal{F} \right)_{KN} X^M \left( \text{Im} \mathcal{F} \right)_{LN} X^N}{X^N \left( \text{Im} \mathcal{F} \right)_{NM} X^M}, \]

As a consequence \( \Omega \) enjoys the expansion \( \Omega(z) = X^\kappa(z) \alpha^\kappa - \mathcal{F}^\kappa(z) \beta^\kappa \) where both \( X^\kappa(z) \) and \( \mathcal{F}^\kappa(z) \) depend holomorphically on the complex structure deformations \( z^K \) and \( \mathcal{F}^\kappa \) is the derivative of a holomorphic prepotential \( \mathcal{F} \), i.e. \( \mathcal{F}^\kappa = \frac{\partial \mathcal{F}}{\partial X^K} \). Finally, there is a set of coordinates – called special coordinates – where one chooses \( X^\kappa = (1, z^K) \).

The metric \( G_{KL}(z, \bar{z}) \) which appears in (2.8) is the metric on the space of complex structure deformations given by [40]

\[ G_{KL} = \frac{\partial}{\partial z^K} \frac{\partial}{\partial \bar{z}^L} K_{cs}, \quad K_{cs} = - \ln \left[ -i \int_Y \Omega \wedge \bar{\Omega} \right] = - \ln \left[ X^K \mathcal{F}^\kappa - X^\kappa \mathcal{F}^\kappa \right]. \] (2.12)
It is a special Kähler metric in that it is entirely determined by the holomorphic prepotential $F(z)$ \cite{16,17}.

The metric $G_{AB}$ in (2.8) is the metric on the space of Kähler deformations defined as \cite{48,40}

$$G_{AB} = \frac{3}{2K} \int_Y \omega_A \wedge \omega_B = -\frac{3}{2} \left( \frac{K_{AB}}{K} - \frac{3}{2} \frac{K_A K_B}{K^2} \right),$$  \hspace{1cm} (2.13)

where we abbreviated

$$K_{ABC} = \int_Y \omega_A \wedge \omega_B \wedge \omega_C, \quad K_{AB} = \int_Y \omega_A \wedge \omega_B \wedge J = K_{ABC} v^C, \quad K_A = \int_Y \omega_A \wedge J \wedge J = K_{ABC} v^B v^C, \quad K = \int_Y J \wedge J \wedge J = K_{ABC} v^A v^B v^C.$$  \hspace{1cm} (2.14)

Recall that $J$ is the Kähler form and we repeatedly used (2.4) in (2.14). Note that with this convention the volume of the Calabi-Yau manifold is given by $\text{Vol}(Y) = \frac{1}{6} K$.

The $D = 4$ two-forms $B_2, C_2$ can be dualized to scalar fields such that the action (2.8) is entirely expressed in terms of vector- and hypermultiplets. One finds \cite{19, 16, 17}

$$S_{IIB}^{(4)} = \int -\frac{1}{2} R \ast 1 + \frac{1}{4} \text{Re} \mathcal{M}_{KL} F^K \wedge F^L + \frac{1}{4} \text{Im} \mathcal{M}_{KL} F^K \wedge \ast F^L,$$

$$-G_{KL} dz^K \wedge \ast d\bar{z}^L - h_{\hat{A}\hat{B}} dq^\hat{A} \wedge \ast dq^\hat{B},$$  \hspace{1cm} (2.15)

where $q^\hat{A}$ collectively denotes all $h^{(1,1)} + 1$ hypermultiplets and $h_{\hat{A}\hat{B}}$ is a quaternionic metric which can be found in \cite{50}. In this basis the scalar manifold $\mathcal{M}$ of the $N = 2$ theory is the product of a quaternionic manifold $\mathcal{M}^Q$ spanned by the scalars $q^\hat{A}$ in the hypermultiplets and a special Kähler manifold $\mathcal{M}^{SK}$ spanned by the scalars $z^K$ in the vector multiplets

$$\mathcal{M} = \mathcal{M}^{SK} \times \mathcal{M}^Q.$$  \hspace{1cm} (2.16)

This ends our brief summary of type IIB compactified on Calabi-Yau threefolds and its $N = 2$ low energy effective action. Let us now turn to the main task of this paper and impose the orientifold projections (1.1), (1.2) and derive the resulting $N = 1$ low energy effective action.

## 3 Calabi-Yau Orientifolds with $O3/O7$ planes

In this section we focus on the first type of orientifold projection $O(1) = (-1)^F L \Omega_p \sigma^*$ with $\sigma^* \Omega = -\Omega$ which we already gave in (1.1). This projection has been discussed in refs. \cite{37, 38, 39, 40} and here we closely follow their analysis. $\Omega_p$ is the world sheet parity transformation under which the type IIB fields $\hat{\phi}, \hat{g}$ and $\hat{C}_4$ are even while $\hat{B}_2, \hat{l}, \hat{C}_4$ are odd. $F_L$ is the ‘space-time fermion number’ in the left moving sector and therefore $(-1)^F_L$ leaves the NS-NS fields $\hat{\phi}, \hat{g}, \hat{B}_2$ invariant and changes the sign of the RR fields $l, \hat{C}_2, \hat{C}_4$. $\sigma$ is an ‘internal’ symmetry which acts on the compact Calabi-Yau manifold but leaves the $D = 4$ Minkowskian space-time invariant. In addition, $\sigma$ is required to
be an involution, i.e. to satisfy $\sigma^2 = 1$, and to act holomorphically on the Calabi-Yau coordinates \[35, 36\]. As a consequence the possible orientifold-planes in type IIB are necessarily even-dimensional.

The induced action of $\sigma$ on forms is denoted by the pull-back $\sigma^*$. Since $\sigma$ is a holomorphic isometry it leaves both the metric and the complex structure invariant. As a consequence also the Kähler form $J$ is invariant. However, the action of $\sigma^*$ on the holomorphic three-form $\Omega$ is not fixed and one can have $\sigma^* \Omega = \pm \Omega$. In this section we analyze the projection \[1.1\] where $\sigma^* \Omega = -\Omega$.

Since the four-dimensional Minkowski space is left invariant by $\sigma$ the orientifold planes are necessarily space-time filling. Together with the fact that they have to be even-dimensional (including the time direction) this selects $O3$-, $O5$-, $O7$- or $O9$-planes as the only possibilities. The actual dimensionality of the orientifold plane is then determined by the dimensionality of the fix point set of $\sigma$ in $Y$. In order to determine this dimensionality we need the induced action of $\sigma$ on the tangent space at any point of the orientifold plane. Since one can always choose $\Omega \propto dy^1 \wedge dy^2 \wedge dy^3$ we see that for $\sigma^* \Omega = -\Omega$ the internal part of the orientifold plane is either a point or a surface of complex dimension two. Together with the space-time filling part we thus can have $O3$- and/or $O7$-planes. The same argument can be repeated for $\sigma^* \Omega = \Omega$ which then leads to the possibility of $O5$- and/or $O9$-planes.

In this section we impose the projection \[1.1\] on the type IIB theory and derive the massless spectrum (section \[3.1\]) and its low energy $N = 1, D = 4$ effective supergravity action (section \[3.2\]). This generalizes similar derivations already performed in refs. \[1, 10\] in that we also allow for the presence of $O7$-planes. We restrict our analysis to the bosonic fields of the compactification keeping in mind that the couplings of the fermionic partners are fixed by supersymmetry. The compactification we perform is closely related to the compactification of type IIB string theory on Calabi-Yau threefolds reviewed in the previous section. The orientifold projection \[1.1\] truncates the massless spectrum from $N = 2$ to $N = 1$ multiplets and also leads to a modification of the couplings which render the low energy effective theory compatible with $N = 1$ supergravity. Such truncation procedures from $N = 2$ to $N = 1$ supergravity has been carried out from a purely supergravity point of view in refs. \[51\].

### 3.1 The massless spectrum of $N = 1$ Calabi-Yau orientifolds

Before computing the effective action let us first determine the massless spectrum when the orientifold projection is taken into account and see how the fields assemble in $N = 1$ supermultiplets \[36\]. In the four-dimensional compactified theory only states invariant under the projection are kept. From the previous discussion one immediately infers that the scalars $\hat{\phi}, \hat{l}$, the metric $\hat{g}$ and the four-form $\hat{C}_4$ are even under $(-1)^{F_L} \Omega_\phi$ while both two forms $\hat{B}_2, \hat{C}_2$ are odd. Using \[1.1\] this implies that the invariant states have to obey

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\[3\]Calabi-Yau manifolds have only discrete isometries. For example in the case of the quintic, $\sigma$ could act by permuting the coordinates such that the defining equation is left invariant. A classification of all possible involutions of the quintic can be found in ref. \[35\].

\[4\]Whenever $\sigma^* = id$ the theory has $O9$-planes and coincides with type I if one introduces D9-branes to cancel tadpoles.
\[
\begin{align*}
\sigma^* \hat{\phi} &= \hat{\phi}, & \sigma^* \hat{l} &= \hat{l}, \\
\sigma^* \hat{g} &= \hat{g}, & \sigma^* \hat{C}_2 &= -\hat{C}_2, \\
\sigma^* B_2 &= -B_2.
\end{align*}
\] (3.1)

In addition, \( \sigma^* \) is not arbitrary but required to satisfy
\[
\sigma^* \Omega = -\Omega.
\] (3.2)

Since \( \sigma \) is a holomorphic involution the cohomology groups \( H^{(p,q)} \) (and thus the harmonic \((p,q)\)-forms) split into two eigenspaces under the action of \( \sigma^* \)
\[
H^{(p,q)} = H_+^{(p,q)} \oplus H_-^{(p,q)}.
\] (3.3)

\( H_+^{(p,q)} \) has dimension \( h_+^{(p,q)} \) and denotes the even eigenspace of \( \sigma^* \) while \( H_-^{(p,q)} \) has dimension \( h_-^{(p,q)} \) and denotes the odd eigenspace of \( \sigma^* \). The Hodge \( * \)-operator commutes with \( \sigma^* \) since \( \sigma \) preserves the orientation and the metric of the Calabi-Yau manifold and thus the Hodge numbers obey \( h_+^{(1,1)} = h_-^{(1,1)} \). Holomorphicity of \( \sigma \) further implies \( h_+^{(2,1)} = h_-^{(1,2)} \) while (3.2) leads to \( h_+^{(3,0)} = h_-^{(0,3)} = 0 \) and \( h_-^{(0,3)} = h_+^{(0,3)} = 1 \). Furthermore, the volume-form which is proportional to \( \Omega \wedge \tilde{\Omega} \) is invariant under \( \sigma^* \) and thus one has \( h_+^{(0,0)} = h_+^{(3,3)} = 1 \) while \( h_-^{(0,0)} = h_-^{(3,3)} = 0 \). We summarize the non-trivial cohomology groups including their basis elements in Table 3.1.

| cohomology group | dimension | basis |
|------------------|-----------|-------|
| \( H_+^{(1,1)} \) | \( h_+^{(1,1)} \) | \( \omega_\alpha \) |
| \( H_-^{(1,1)} \) | \( h_-^{(1,1)} \) | \( \bar{\omega}^\alpha \) |
| \( H_+^{(2,2)} \) | \( h_+^{(1,1)} \) | \( \bar{\omega}^\alpha \) |
| \( H_-^{(2,2)} \) | \( h_-^{(1,1)} \) | \( \bar{\omega}^\alpha \) |
| \( H_+^{(2,1)} \) | \( h_+^{(2,1)} \) | \( \chi_\kappa \) |
| \( H_-^{(2,1)} \) | \( h_-^{(2,1)} \) | \( \chi_\kappa \) |
| \( H_+^{(3)} \) | \( 2h_+^{(2,1)} \) | \( \langle \alpha_\kappa, \beta_\lambda \rangle \) |
| \( H_-^{(3)} \) | \( 2h_-^{(2,1)} + 2 \) | \( \langle \alpha_\kappa, \beta_\lambda \rangle \) |

Table 3.1: Cohomology groups and their basis elements.

The four-dimensional invariant spectrum is found by using the Kaluza-Klein expansion given in eqs. (2.4), (2.5) and (2.6) keeping only the fields which in addition obey (3.1). We see immediately that both \( D = 4 \) scalar fields arising from \( \hat{\phi} \) and \( \hat{l} \) remain in the spectrum and as before we denote them by \( \phi \) and \( l \). Since \( \sigma^* \) leaves the Kähler form \( J \) invariant only the \( h_+^{(1,1)} \) even Kähler deformations \( v^\alpha \) remain in the spectrum and we expand
\[
J = v^\alpha(x) \omega_\alpha, \quad \alpha = 1, \ldots, h_+^{(1,1)},
\] (3.4)
where \( \omega_\alpha \) denotes a basis of \( H_+^{(1,1)} \). Similarly, from eq. (2.5) we infer that the invariance of the metric together with (3.2) implies that the complex structure deformations kept in the spectrum correspond to elements in \( H_+^{(1,2)} \) and (2.5) is replaced by
\[
\delta g_{ij} = \frac{i}{||\Omega||^2} \tilde{z}^k(\tilde{\chi}_k)_{ij} \Omega_{ij} \chi_k, \quad k = 1, \ldots, h_+^{(1,2)},
\] (3.5)
where $\chi_k$ denotes a basis of $H^{(1,2)}$.  

From eqs. (3.1) we learn that in the expansion of $\hat{B}_2$ and $\hat{C}_2$ only odd elements survive while for $\hat{C}_4$ only even elements are kept. Therefore the expansion (2.6) is replaced by

$$\hat{B}_2 = b^a(x) \omega_a, \quad \hat{C}_2 = c^a(x) \omega_a, \quad a = 1, \ldots, h_{-1}^{(1,1)},$$

$$\hat{C}_4 = D^a_2(x) \wedge \omega_a + V^\kappa(x) \wedge \alpha_\kappa + U_\kappa(x) \wedge \beta^\kappa + \rho_\alpha(x) \tilde{\omega}^\alpha, \quad \kappa = 1, \ldots, h_+^{(1,2)},$$

where $\omega_a$ is a basis of $H^{(1,1)}$, $\tilde{\omega}^\alpha$ is a basis of $H^{(2,2)}_+$ which is dual to $\omega_a$, and $(\alpha_\kappa, \beta^\kappa)$ is a real, symplectic basis of $H^{(3)}_+ = H_+^{(1,2)} \oplus H_+^{(2,1)}$ (c.f. table 3.1). As for Calabi-Yau compactifications imposing the self-duality on $\hat{F}_5$ eliminates half of the degrees of freedom in the expansion of $\hat{C}_4$. For the one-forms $V^\kappa, U_\kappa$ this corresponds to the choice of electric versus magnetic gauge potentials. On the other hand choosing the two forms $D^a_2$ or the scalars $\rho_\alpha$ determines the structure of the $N = 1$ multiplets to be either a linear or a chiral multiplet and below we discuss both cases.

Altogether the resulting $N = 1$ spectrum assembles into a gravitational multiplet, $h_+^{(2,1)}$ vector multiplets and $(h_-^{(2,1)} + h^{(1,1)} + 1)$ chiral multiplets and is summarized in table 3.1 [36]. As we already mentioned we can replace $h_-^{(1,1)}$ of the chiral multiplets by linear multiplets.

| gravity multiplet | 1 | $g_{\mu\nu}$ |
|-------------------|---|--------------|
| vector multiplets | $h_+^{(2,1)}$ | $V^\kappa$ |
| chiral multiplets | $h_-^{(2,1)}$ | $z^k$ |
|                  | $h_-^{(1,1)}$ | $(\phi, l)$ |
|                  | $h_+^{(1,1)}$ | $(b^a, c^a)$ |
| chiral/linear multiplets | $h_+^{(1,1)}$ | $(v^\alpha, \rho_\alpha)$ |

Table 3.2: $N = 1$ spectrum of $O3/O7$-orientifold compactification.

Compared to the $N = 2$ spectrum of the Calabi-Yau compactification given in table 2.2 we see that the graviphoton ‘left’ the gravitational multiplet while the $h_-^{(2,1)} N = 2$ vector multiplets decomposed into $h_-^{(2,1)} N = 1$ vector multiplets plus $h_-^{(2,1)}$ chiral multiplets. Furthermore, the $h^{(1,1)} + 1$ hypermultiplets lost half of their physical degrees of freedom and are reduced into $h^{(1,1)} + 1$ chiral multiplets. This is consistent with the theorem of [52, 51] where it was shown that any Kähler submanifold of a quaternionic manifold can have at most half of its (real) dimension.

Note that the two $D = 4$ two-forms $B_2$ and $C_2$ present in the $N = 2$ compactification (see (2.6)) have been projected out and in the expansion of $\hat{B}_2$ and $\hat{C}_2$ only the scalar fields $c^a, b^a$ appear. The non-vanishing of $c^a, b^a$ and $V^\kappa$ is closely related to the appearance of $O7$-planes. To understand this in more detail we recall, that $O3$-planes appear when

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5In ref. [36] it is further shown that the $h_-^{(1,2)}$ deformations form a smooth submanifold of the Calabi-Yau moduli space.
the fix point set of \( \sigma \) is zero-dimensional in \( Y \) or in other words all tangent vectors at this point are odd under the action of \( \sigma \). This in turn implies that locally two-forms are even under \( \sigma^* \), while three-forms are odd. However, this is incompatible with the expansions given in (3.6) for non-vanishing \( b^a, c^a \) and \( V^\kappa \). For a setup also including \( O7 \)-planes we locally get the correct transformation behavior, so that harmonic forms in \( H^{(1,1)}_- \) and \( H^{(2,1)}_+ \) can be supported.

3.2 The effective action in terms of chiral multiplets

Before we derive the effective action let us recall that in type IIB string theory it is possible to allow background three-form fluxes \( H_3 \) and \( F_3 \) on the Calabi-Yau manifold [8, 10, 11, 4]. The Bianchi identity together with the equation of motion imply that \( H_3 \) and \( F_3 \) have to be harmonic three-forms. In orientifold compactifications they are further constrained by the orientifold projection. From (3.1) we see that for the projection given in (1.1) they both have to be odd under \( \sigma^* \) and hence are parameterized by elements of \( H^{(3)}_-(Y) \). It is convenient to combine the two three-forms into a complex \( G_3 \) according to

\[
G_3 = F_3 - \tau H_3 , \quad \tau = l + ie^{-\phi} .
\]  

(3.7)

\( G_3 \) can be explicitly expanded into a symplectic basis of \( H^{(3)}_- \) as

\[
G_3 = m^k \alpha_k - e_k \beta^k , \quad \hat{k} = 0, \ldots, h^{(1,2)} , \quad (3.8)
\]

with \( 2(h^{(1,2)} + 1) \) complex flux parameters

\[
m^k = m^k_H - \tau m^k_F , \quad e_k = e^H_k - \tau e^F_k . \quad (3.9)
\]

However, most of the time we do not need this explicit expansion and express our results in terms of \( G_3 \).

The presence of background fluxes and localized sources requires a deviation from the standard Calabi-Yau compactifications in that a non-trivial warp factor \( e^{-2A} \) has to be included into the Ansatz for the metric (2.3) [4, 53]

\[
ds^2 = e^{2A(y)} g_{\mu\nu}(x) dx^\mu dx^\nu + e^{-2A(y)} g_{ij}(y) dy^i dy^j .
\]  

(3.10)

However, in this paper we perform our analysis in the unwarped Calabi-Yau manifold since in the large radius limit the warp factor approaches one and the metrics of the two manifolds coincide [4, 54]. This in turn also implies that the metrics on the moduli space of deformations agree and as a consequence the kinetic terms in the low energy effective actions are the same. The difference appears in the potential when some of the Calabi-Yau zero modes are rendered massive.

The four-dimensional effective action is computed by redoing the Kaluza-Klein reduction of the ten-dimensional type IIB action given in (2.1) for the truncated orientifold spectrum including the background fluxes (3.7). Inserting (3.6) into (2.2) we arrive at

\[
\hat{H}_3 = db^a \wedge \omega_a + H_3 , \quad \hat{F}_3 = dc^a \wedge \omega_a - l db^a \wedge \omega_a + F_3 - l H_3 ,
\]

\[
\hat{F}_5 = dD_2^a \wedge \omega_a + dV^\kappa \wedge \alpha_\kappa - dU_\kappa \wedge \beta^\kappa + d\rho_\alpha \omega^\alpha - \frac{1}{2} (c^a db^b - b^a dc^b) \wedge \omega_a \wedge \omega_b ,
\]

This uses the fact that the exterior derivative on \( Y \) commutes with \( \sigma^* \).
where we allowed for the presence of background fluxes $H_3$ and $F_3$. The next step is to insert (3.11) into (2.1) and perform the integration over $Y$. In order to do so we first need to reconsider the structure of the metrics (2.12), (2.13) and the intersection numbers (2.14) for the orientifold.

Let us start with the complex structure deformations. Due to the split of the cohomology $H^{(3)} = H^{(3)}_+ \oplus H^{(3)}_-$ the real symplectic basis $(\alpha, \beta)$ of $H^{(3)}$ also splits into $(\alpha_1, \beta_1)$ of $H^{(3)}_+$ and $(\alpha_2, \beta_2)$ of $H^{(3)}_-$. Eqs. (2.7) continue to hold which implies that both bases are symplectic and obey

$$\int \alpha_\kappa \wedge \beta_\lambda = \delta_\kappa^\lambda, \quad \int \alpha_\kappa \wedge \beta_\lambda = \delta_\lambda^\kappa, \quad (3.12)$$

with all other intersections vanishing. Furthermore, we saw in the previous section that out of $h^{(2,1)}$ complex structure deformation $z^k\bar{z}^l$ only $h^{(2,1)}_\perp$ (denoted by $z^k$) survived. The three-form $\Omega$ being an element of $H^{(3)}_\perp$ can thus be expanded according to

$$\Omega(z) = X^k \alpha_k - F_{k\bar{l}}^{(3)} \beta^l, \quad \hat{k} = 0, \ldots, h^{(1,2)}_\perp, \quad (3.13)$$

while the ‘other’ periods $(X^\kappa, F_\kappa)$ vanish

$$X^\kappa = \int_Y \Omega \wedge \beta^\kappa = 0, \quad F_\kappa \big|_{z^n = 0} = \int_Y \Omega \wedge \alpha_\kappa = 0, \quad \kappa = 1, \ldots, h^{(1,2)}_+ \quad (3.14)$$

As a consequence the metric on the space of complex structure deformations reduces to

$$G_{kl} = \frac{\partial}{\partial z^k} \frac{\partial}{\partial z^l} K, \quad K = -\ln \left[ -i \int_Y \Omega \wedge \bar{\Omega} \right] = -\ln \left[ \int_X X^k F_k - X^{\bar{k}} \bar{F}_{\bar{k}} \right], \quad (3.15)$$

replacing (2.12).

Let us now turn to the Kähler deformations. Corresponding to the decomposition $H^{(1,1)} = H^{(1,1)}_+ \oplus H^{(1,1)}_-$ also the harmonic $(1,1)$-forms $\omega_A$ split into $\omega_A = (\omega_\alpha, \omega_\beta)$ such that $\omega_\alpha$ is a basis of $H^{(1,1)}_+$ and $\omega_\beta$ is a basis of $H^{(1,1)}_-$. This in turn results in a decomposition of the intersection numbers $K_{ABC}$ given in (2.13). Under the orientifold projection only $K_{\alpha\beta\gamma}$ and $K_{abc}$ can be non-zero while $K_{ab\gamma} = K_{abc} = 0$ has to hold. Since the Kähler-form $J$ is invariant we also conclude from (2.14) that $K_{ab} = 0 = K_a$. To summarize, keeping only the invariant intersection numbers results in

$$K_{\alpha\beta\gamma} = K_{abc} = K_{ab} = K_a = 0, \quad (3.16)$$

while all the other intersection numbers can be non-vanishing. Inserting (3.16) into (2.13) we derive

$$G_{\alpha\beta} = -\frac{3}{2} \left( \frac{K_{\alpha\beta}}{K} - \frac{3}{2} \frac{K_{\alpha}}{K^2} \right), \quad G_{ab} = -\frac{3}{2} \frac{K_{ab}}{K}, \quad G_{ab} = G_{a\beta} = 0, \quad (3.17)$$

where

$$K_{\alpha\beta} = K_{\alpha\beta\gamma} v^\gamma, \quad K_{ab} = K_{ab\gamma} v^\gamma, \quad K_{a} = K_{a\beta\gamma} v^\beta v^\gamma, \quad K = K_{ab\gamma} v^a v^b v^\gamma. \quad (3.18)$$

Note that $H_3$ and $F_3$ do not effect $\tilde{F}_3$ since the only possible terms would be of the form $H_3 \wedge C_2$ or $B_2 \wedge F_3$ but both $C_2$ and $B_2$ are projected out by the orientifold projection.

From a supergravity point of view this has been also observed in refs. [51].
We see that the metric $G_{AB}$ given in (2.13) is block-diagonal with respect to the decomposition $H^{(1,1)} = H_+^{(1,1)} \oplus H_-^{(1,1)}$. For later use let us also record the inverse metrics

$$G^{\alpha \beta} = -\frac{2}{3} \kappa k^{\alpha \beta} + 2 v^\alpha v^\beta, \quad G^{ab} = -\frac{2}{3} \kappa k^{ab}, \quad (3.19)$$

where $K^{\alpha \beta}$ and $K^{ab}$ are the inverse of $K_{\alpha \beta}$ and $K_{ab}$, respectively.

To calculate the four-dimensional action we insert (3.4), (3.5) and (3.11) into (2.1), integrate over $Y$ using (3.5)–(3.19). Furthermore we impose the self-duality condition $e_5 = *F_5$ by adding the following total derivative to the action (16)

$$\delta S_{O3/07}^{(4)} = \frac{1}{4} dV^\kappa \wedge dU_\kappa + \frac{1}{4} dD_2^a \wedge d\rho_\alpha. \quad (3.20)$$

Then the equation of motions for $D^a_2$ and $U_\kappa$ (or equivalently for $\rho_\alpha, V^\kappa$) coincide with the self-duality condition and we can consistently eliminate $D^a_2$ and $U_\kappa$ (or $\rho_\alpha, V^\kappa$) by inserting their equations of motions into the action (16). Keeping $V^\kappa$ corresponds to the choice of expressing the action in terms of an electric instead of a magnetic gauge potential $U_\kappa$. Choosing to eliminate $D^a_2$ or $\rho_\alpha$ corresponds to the choice of expressing the action in terms of linear or chiral multiplets. The standard $N = 1$ supergravity formulation uses chiral multiplets and thus from this point of view it is more convenient to eliminate $D^a_2$ in favor of $\rho_\alpha$ and express everything in terms of chiral multiplets. However, the resulting geometry of the $N = 1$ moduli space can be understood more conceptually by using linear multiplets. For this reason we supplement our analysis with a discussion of the low energy effective action in terms of linear multiplets in section 3.3.

Eliminating $D^a_2$ and $U_\kappa$ by its equations of motion and performing a Weyl rescaling of the four-dimensional metric $g_{\mu \nu} \rightarrow \frac{\kappa}{6} g_{\mu \nu}$ to obtain the canonically normalized Einstein-Hilbert term we arrive at

$$S_{O3/07}^{(4)} = \int_{M_{3,1}} -\frac{1}{2} \mathcal{R} * \mathbf{1} - G_{k l} dz^k \wedge * d\bar{z}^l - G_{\alpha \beta} dv^\alpha \wedge * dv^\beta - \frac{1}{4} d\ln K \wedge * d\ln K$$

$$- \frac{1}{4} d\phi \wedge * d\phi - \frac{1}{4} e^{2\phi} dl \wedge * dl - e^{-\phi} G_{ab} db^a \wedge * db^b$$

$$- e^{\phi} G_{ab} (dc^a - ld^b) \wedge * (db^b - ld) \quad (3.21)$$

$$- \frac{9G^{\alpha \beta}}{4K^2} \left( d\rho_\beta - \frac{1}{2} K_{\alpha \beta} (a^c db^c - b^a dc^c) \right) \wedge * \left( d\rho_\beta - \frac{1}{2} K_{\beta \alpha c} (c^d db^d - d^a de^a) \right)$$

$$+ \frac{1}{4} \Im \mathcal{M}_{\kappa \lambda} F^\kappa \wedge * F^\lambda + \frac{1}{4} \Re \mathcal{M}_{\kappa \lambda} F^\kappa \wedge F^\lambda - V * \mathbf{1},$$

where $F^\kappa = dV^\kappa$ and $\mathcal{M}_{\kappa \lambda}$ is the $N = 2$ gauge kinetic matrix given in (2.11) evaluated at $z^\kappa = \bar{z}^\kappa = 0$. The potential $V$ is manifestly positive semi-definite and found to be

$$V = \frac{18i}{K^2} \int \frac{e^{\phi}}{\Omega \wedge \bar{\Omega}} \left( \int \frac{\Omega \wedge G_3}{\Omega \wedge \bar{G}_3} \int \frac{\bar{\Omega} \wedge G_3}{\bar{\Omega} \wedge G_3} + G^{kl} \int \frac{\chi_k \wedge G_3}{\chi_k \wedge G_3} \int \bar{\chi}_l \wedge G_3 \right), \quad (3.22)$$

\textsuperscript{9}For vanishing fluxes this action can also be obtained by directly inserting the truncated spectrum into (2.8) and the fluxes only add the potential $V$. 

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where $\chi_k$ is a basis of $H^{(2,1)}$ as defined in (3.5) and the background flux $G_3$ is defined in (3.7). For completeness we include the details of the computation of $V$ in appendix B.1

Strictly speaking the additional term $L^{(4)}_{\text{top}} \sim \int_Y H_3 \wedge F_3$ arises in the Kaluza-Klein reduction. However, consistency of the compactifications requires its cancellation against Wess-Zumino like couplings of the orientifold planes to the R-R flux [4].

Our next task is to transform the action (3.21) into the standard $N = 1$ supergravity form where it is expressed in terms of a Kähler potential $K$, a holomorphic superpotential $W$ and the holomorphic gauge-kinetic coupling functions $f$ as follows [55, 56]

$$S^{(4)} = - \int \frac{1}{2} R \ast 1 + K_{IJ} D_I W D_J \bar{W} - 3 |W|^2 + \frac{1}{2} (\text{Re } f)^{-1} \kappa^{\lambda} D_\alpha D_{\lambda},$$

where

$$V = e^K (K^{IJ} D_I W D_J \bar{W} - 3 |W|^2) + \frac{1}{2} (\text{Re } f)^{-1} \kappa^{\lambda} D_\alpha D_{\lambda}.$$  

Here the $M^I$ collectively denote all complex scalars in the theory and $K_{IJ}$ is a Kähler metric satisfying $K_{IJ} = \partial_I \partial_J K(M, \bar{M})$. The scalar potential is expressed in terms of the Kähler-covariant derivative $D_I W = \partial_I W + (\partial_J K) W$.

We first need to find a complex structure on the space of scalar fields such that the metric computed in (3.21) is manifestly Kähler. As we saw in (3.15) the complex structure deformations $z^k$ are already good Kähler coordinates with $G_{kl}$ being the appropriate Kähler metric. For the remaining fields the definition of the Kähler coordinates is not so obvious. Guided by refs. [14, 19] we define

$$\tau = l + i e^{-\phi}, \quad G^a = c^a - \tau b^a, \quad T_\alpha = \frac{3i}{2} \rho_\alpha + \frac{3}{4} \mathcal{K}_\alpha(v) - \frac{3}{2} \zeta_\alpha(\tau, \bar{\tau}, G, \bar{G}),$$

where

$$\mathcal{K}_\alpha = \kappa_{\alpha \beta \gamma} v^\beta v^\gamma, \quad \zeta_\alpha = -\frac{i}{2(\tau - \bar{\tau})} \kappa_{abc} G^b(G - \bar{G})^c.$$ 

In appendix C.1 we check explicitly that in terms of these coordinates the metric of (3.21) is Kähler with the Kähler potential

$$K = K_{cs}(z, \bar{z}) + K_k(\tau, T, G), \quad K_{cs} = -\ln \left[ -i \int \Omega(z) \wedge \bar{\Omega}(z) \right],$$

$$K_k = -\ln \left[ -i(\tau - \bar{\tau}) \right] - 2 \ln \left[ \kappa(\tau, T, G) \right].$$

$K \equiv \kappa_{\alpha \beta \gamma} v^\alpha v^\beta v^\gamma \equiv 6 \text{Vol}(Y)$ should be understood as a function of the Kähler coordinates $(\tau, T, G)$ which enter by solving (3.25) for $v^\alpha$ in terms of $(\tau, T, G)$. Unfortunately this solution cannot be given explicitly and therefore $K$ is known only implicitly via $v^\alpha(\tau, T, G)$.\(^{11}\) In the next section we show that the definition of the Kähler coordinates

\(^{10}\)The definition of $\zeta_\alpha$ is unique up to a constant which does not enter into the metric. The possibility of a non-zero constant is important for the formulation in terms of linear multiplets in section 3.3.

\(^{11}\)This is in complete analogy to the situation encountered in compactifications of M-theory on Calabi-Yau fourfolds studied in [14]. This is no coincidence and can be understood from the fact that this theory can be lifted to F-theory on Calabi-Yau fourfolds which in a specific limit is related to orientifold compactifications of type IIB [37].
and the Kähler potential (3.27) can be understood somewhat more conceptually in a dual formalism using linear multiplets \( L^a \) instead of the chiral multiplets \( T_\alpha \). From a slightly different perspective these ‘dual’ variables have recently also been discussed in ref. [43].

Let us return to the Kähler potential (3.27). The first two terms are the standard Kähler potentials for the complex structure deformations and the dilaton, respectively. \( \mathcal{K} \) also depends on \( \tau \) and therefore the metric mixes \( \tau \) with \( T_\alpha \) and \( G^a \). It is block diagonal in the complex structure deformations which do not mix with the other scalars. Thus, the moduli space has the form

\[
\mathcal{M} = \mathcal{M}_{cs}^{(1,2)} \times \mathcal{M}_k^{h(1,3)}+1 ,
\]

where each factor is a Kähler manifold and \( \mathcal{M}_{cs}^{(1,2)} \) even is a special Kähler manifold in that \( K_{cs} \) satisfies (3.15).

Although not immediately obvious from its definition \( K_k \) obeys a no-scale type condition in that it satisfies

\[
\frac{\partial K_k}{\partial M^I} (K^{-1})^{IJ} \frac{\partial K_k}{\partial M^J} = 4 ,
\]

where \( M^I = (\tau, G^a, T_\alpha, z^k) \). For \( G^a = 0 \) this has already been observed in [41, 19, 21, 43] while for \( G^a \neq 0 \) we explicitly check (3.29) in appendix C.1. From (3.24) we see that (3.29) implies \( V \geq 0 \) which we also show in the linear multiplet formalism in the next section 3.3. For \( \tau = \text{const} \) the right hand side of (3.29) is found to be equal to 3 as it is the case in the standard no-scale Kähler potentials of [11].

Let us relate (3.27) to the known Kähler potentials in the literature. First of all, for \( G^a = 0 \) and thus \( T_\alpha = \frac{3i}{2} \rho_\alpha + \frac{3}{4} K_\alpha \) the Kähler potential (3.27) reduce to the one given in [19]. Secondly, for one overall Kähler modulus \( v \) parameterizing the volume (i.e. for \( h_+^{(1,1)} = 1, T_\alpha \equiv T \), but keeping all \( h_-^{(1,1)} \) moduli, eq. (3.29) can be solved for \( v \) and one finds

\[
-2 \ln \mathcal{K} = -3 \ln \left[ \frac{2}{3} \left( T + \bar{T} - \frac{3i}{4(\tau - \bar{\tau})} K_{\alpha \beta}(G - \bar{G})^\alpha (G - \bar{G})^\beta \right) \right] .
\]

If in addition we set \( G^a = 0 \) and defines \( -\frac{3i}{2} \rho \equiv T = \frac{3i}{2} \rho + \frac{3}{4} K^{2/3} \) (3.30) reduce to \( K = -3\ln(-i(\rho - \bar{\rho})) \) which coincides with the Kähler potential determined in [11].

Before we turn to the discussion of the gauge kinetic functions and the superpotential let us note that \( K \) is invariant under the \( SL(2, \mathbb{R}) \) transformations inherited from the ten-dimensional type IIB theory. In the orientifold theory this symmetry acts on \( \tau \) by fractional linear transformations \( \tau \to \frac{a\tau + b}{c\tau + d} \) exactly as in \( D = 10 \) and transforms \( (b^a, c^d) \) as a doublet. Under the \( SL(2, \mathbb{R}) \) only the second term of \( K \) given in (3.27) transforms but this transformation is just a Kähler transformation. The two other terms are invariant as can be seen from (3.29) and the fact that \( v^\alpha \) and \( z^k \) are invariant.15

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12We like to thank Geert Smet and Joris Van Den Bergh for drawing our attention to an error in an earlier version of this paper.
13Recall that this corresponds to the situation where only O3-planes are present.
14Since \( K_{\alpha \beta \gamma \delta} v^\alpha v^\beta v^\gamma v^\delta = K \) we find for the case of only one Kähler modulus \( v = K^{1/3} \).
15Contrary to the Calabi-Yau compactifications discussed in section 2 the Kähler moduli \( v^\alpha \) are not redefined with powers of the dilaton [39] and as a consequence they stay \( SL(2, \mathbb{R}) \) invariant.
Our next task is to determine the gauge-kinetic coupling functions \( f_{\kappa\lambda} \) and show that they are holomorphic in the moduli. By comparing the actions \((3.21)\) and \((3.23)\) we find

\[
 f_{\kappa\lambda} = -\frac{i}{2} \mathcal{M}_{\kappa\lambda} \bigg|_{z^k = \bar{z}^k = 0} , \tag{3.31}
\]

where \( \mathcal{M}_{\kappa\lambda} \) is the \( N = 2 \) gauge kinetic matrix given in \((2.10)\) evaluated at \( z^k = \bar{z}^k = 0 \). Its holomorphicity in the complex structure deformations \( z^k \) is not immediately obvious but can be shown by using \((2.9)\) and \((2.10)\). More precisely, from \((2.9)\) together with the decomposition of \( H^{(3)} \) expressed by \((3.3)\) and \((3.12)\) we infer that \( \mathcal{M}_{\hat{K}L} \) is block diagonal or in other words \( \mathcal{M}_{\kappa\hat{l}} = 0 \).

Multiplying \( \mathcal{M}_{\kappa\hat{l}} \) with \( \mathcal{X}_{\hat{l}} \) and using \( \mathcal{X}_{\lambda} = 0 \) together with \((2.10)\) we further conclude

\[
 F_{\kappa\hat{l}} \bigg|_{z^{\hat{k}} = 0 = \bar{z}^{\hat{k}}} = 0 , \tag{3.32}
\]

Finally inserting \((3.13)\) and \((3.32)\) into \((2.10)\) we arrive at

\[
 f_{\kappa\lambda}(z^k) = -\frac{1}{2} i \mathcal{F}_{\kappa\lambda} \bigg|_{z^k = \bar{z}^k = 0} , \tag{3.33}
\]

which is manifestly holomorphic since \( \mathcal{F}_{\kappa\lambda}(z^k) \) are holomorphic functions of the complex structure deformations \( z^k \).

Finally, we show that the potential \((3.22)\) can be derived from a superpotential \( W \) via the expression given in \((3.24)\) with vanishing \( D \)-term \( D_{\kappa} = 0 \). For orientifolds with \( c^a = b^a = 0 \) (corresponding to only \( O3 \)-planes present) \( W \) was shown to be \[7, 10, 11, 19, 21\]

\[
 W(\tau, z^k) = \int_Y \Omega \wedge G_3 . \tag{3.34}
\]

This continues to be the correct superpotential also if \( c^a \) and \( b^a \) are in the spectrum. Indeed, using \((3.25)\), \((3.27)\), \((3.36)\) and \((C.6)\) one computes the Kähler-covariant derivatives to be

\[
 D_{\tau} W = \frac{i}{2} e^\phi \int \Omega \wedge \bar{G}_3 + i G_{ab} b^a b^b W , \quad D_{T_{\alpha}} W = K_{T_{\alpha}} W = -2 v^a_{\alpha} \mathcal{K} W ,
\]

\[
 D_{G^{ab}} W = K_{G^{ab}} W = 2 i G_{ab} b^b W , \quad D_{z^k} W = i \int \chi_k \wedge G_3 , \tag{3.35}
\]

where we used that \( \chi_k \) has the additional property \[40\]

\[
 D_{z^k} \Omega = i \chi_k . \tag{3.36}
\]

Due to the fact that \( K_k \) satisfies the no-scale condition \((3.29)\) the potential is unchanged if one includes \( b^a \) and \( c^a \). Inserting \((3.35)\) and \((3.29)\) into \((3.24)\) one shows that \((3.22)\) is reproduced.

It is interesting to note that both \( W(\tau, z^k) \) and \( f_{\kappa\lambda}(z^k) \) depend on the complex structure deformations \( z^k \) but not on the Kähler deformations. This can be directly understood from the Peccei-Quinn symmetries of the theories or equivalently from the fact that the Kähler deformations can be viewed as members of linear multiplets. Let us turn to this aspect now.
3.3 The effective action in terms of linear multiplets

In section 3.1 (table 3.1) we already observed that for the $h^{(1,1)}_{\perp}$ chiral multiplet $(v^\alpha, \rho_\alpha)$ including the Kähler deformations $v^\alpha$ there exist an dual description in terms of linear multiplets. In this section we rewrite the effective action using the linear multiplet formalism of ref. [42]. In this way we will be able to understand the definition of the Kähler coordinates given in (3.25) as a superfield duality transformation and furthermore discover the no-scale property (3.29) of $K_k$ somewhat more conceptually. In an analog three-dimensional situation this has also been observed in [24].

Let us first briefly review $N = 1$ supergravity coupled to $h^{(1,1)}_{\perp}$ linear multiplets $L^\alpha, \alpha = 1, \ldots, h^{(1,1)}_{\perp}$ and $r$ chiral multiplets $N^A, A = 1, \ldots, r$ following [12]. Linear multiplets are defined by the constraint

$$ (D^2 - 8\bar{R})L^\alpha = 0 = (\bar{D}^2 - 8R)L^\alpha, $$

where $D$ is the superspace covariant derivative and $R$ is the chiral superfield containing the curvature scalar. As bosonic components $L$ contains a real scalar field which we also denote by $L$ and the field strength of a two-form $D_2$. The superspace Lagrangian (omitting the gauge interactions) is given by

$$ \mathcal{L} = -3 \int E F(N^A, \bar{N}^A, L^\alpha) + \frac{1}{2} \int \frac{E}{R} e^{K/2} W(N) + \frac{1}{2} \int \frac{E}{R} e^{K/2} \bar{W}(\bar{N}), $$

where $E$ is the super-vielbein and $W$ the superpotential. The function $F$ depends implicitly on the Kähler potential $K(N^A, \bar{N}^A, L^\alpha)$ through the differential constraint

$$ 1 - \frac{1}{3} L^\alpha K_{L^\alpha} = F - L^\alpha F_{L^\alpha}, $$

which ensures the correct normalization of the Einstein-Hilbert term. The subsrcripts on $K$ and $F$ denotes differentiation, i.e. $K_{L^\alpha} = \frac{\partial K}{\partial L^\alpha}, F_{L^\alpha} = \frac{\partial F}{\partial L^\alpha},$ etc.

Here we are not interested in the most general couplings but our aim is to rewrite the action (3.21) in the linear multiplet formalism. As we are going to show this is achieved by the Kähler potential

$$ K = K_0(N^A, \bar{N}^A) + \alpha \ln(K_{\alpha\beta\gamma} L^\alpha L^\beta L^\gamma), $$

where we leave $K_0(N^A, \bar{N}^A)$ and the normalization constant $\alpha$ arbitrary for the moment. Inserting (3.40) into (3.39) determines $F$ to be

$$ F = 1 - \alpha + L^\alpha \zeta^R_\alpha(N^A, \bar{N}^A), $$

where the real functions $\zeta^R_\alpha(N^A, \bar{N}^A)$ are not further determined by (3.39). In that sense the $\zeta^R_\alpha$ are additional input functions which determine the Lagrangian.

The (bosonic) component Lagrangian derived from (3.38) is found to be

$$ \mathcal{L} = -\frac{1}{2} R \ast 1 - \tilde{K}_{AB} dN^A \wedge \ast d\bar{N}^B + \frac{1}{4} K_{L^\alpha L^\beta} dL^\alpha \wedge \ast dL^\beta - V $$

$$ + \frac{1}{4} K_{L^\alpha L^\beta} dD_2^\alpha \wedge \ast dD_2^\beta + \frac{3i}{2} dD_2^\alpha \wedge (\zeta^R_{\alpha A} dN^A - \zeta^R_{\alpha A} d\bar{N}^A), $$

\[16\]Strictly speaking $K(N^A, \bar{N}^A, L^\alpha)$ is not a Kähler potential but as we will see it determines the kinetic terms in the action.

\[17\]This is a straightforward generalization of the Lagrangian for one linear multiplet given in [42]. The potential for this case has also been given in [14].
\[ \tilde{K}_{AB} \equiv K_{AB} - 3L^\alpha \zeta^R_{\alpha,AB}, \quad V = e^K \left( \tilde{K}^{AB} D_A W D_B \bar{W} - (3 - L^\alpha K_{L^\alpha})|W|^2 \right). \quad (3.43) \]

We see that the effective action is determined in terms of \( K(N, \bar{N}, L) \) and \( \zeta^R_{\alpha}(N, \bar{N}) \). \( K \) determines the kinetic terms of the fields \( N^A \) and \( L^\alpha \) while the \( \zeta^R_{\alpha}(N, \bar{N}) \) determine the couplings of the two-forms \( D^\alpha_2 \) to the chiral fields \( N^I \). Note that only derivatives of \( \zeta^R_{\alpha} \) appear leaving a constant piece in \( \zeta^R_{\alpha} \) undetermined.

From here we can proceed in two ways. We can dualize the two-forms \( D^\alpha_2 \) in components and show the equivalence with the action (3.21). This is done at the end of this section. However, performing the duality in superspace yields directly the proper Kähler coordinates \( T_\alpha \) and thus gives a more conceptual understanding of the definition (3.25).

The duality transformation in superfields is performed in detail in [42] and here we only repeat the essential steps. One first considers the linear multiplets \( L^\alpha \) to be unconstrained real superfields and modifies the action (3.38) to read\(^\text{18}\)

\[ S = -3 \int E \left( F(N^A, \bar{N}^A, L^\alpha) + \frac{2}{3} L^\alpha (T_\alpha + \bar{T}_\alpha) \right) + \ldots, \quad (3.44) \]

where the \( T_\alpha \) are chiral superfields and in order to be consistent with our previous conventions we have included a factor \( \frac{2}{3} \) in the second term. Variation with respect to \( T_\alpha \) results in the constraint that \( L^\alpha \) are linear multiplets and one arrives back at the action (3.38). Variation with respect to the (unconstrained) \( L^\alpha \) yields the equations\(^\text{19}\)

\[ \frac{2}{3}(T_\alpha + \bar{T}_\alpha) + F_{L^\alpha} - \frac{1}{3} K_{L^\alpha} \left( F + \frac{2}{3} L^\beta (T_\beta + \bar{T}_\beta) \right) = 0, \quad (3.45) \]

where we have used \( \delta_L E = -\frac{1}{3} E K_{L^\alpha} \delta L^\alpha \). This equation determines \( L^\alpha \) in terms of the chiral superfields \( N^A, T_\alpha \) and is the looked for duality relation. However, depending on the specific form of \( F \) and \( K \) one might not be able to solve (3.45) explicitly for \( L^\alpha \) but instead only obtain an implicit relation \( L^\alpha(N, \bar{N}, T + \bar{T}) \). Nevertheless one should insert \( L^\alpha(N, \bar{N}, T + \bar{T}) \) back into (3.44) which then expresses the Lagrangian (implicitly) in terms of \( T_\alpha \) and therefore defines a Lagrangian in the chiral superfield formalism. The unusual feature being that the explicit functional dependence is not known.

A correctly normalized Einstein-Hilbert term is ensured by additionally imposing

\[ F(N, \bar{N}, L) + \frac{2}{3} L^\alpha (T_\alpha + \bar{T}_\alpha) = 1. \quad (3.46) \]

Contracting (3.45) with \( L^\alpha \) and using equation (3.46) one obtains (3.39). Thus \( F \) has to have the same functional dependence as before and therefore eqs. (3.40) and (3.41) are unmodified but one should insert \( L(N, \bar{N}, T + \bar{T}) \) implicitly determined by (3.45). In other words eqs. (3.40) should be red as

\[ K = K_0(N, \bar{N}) + \alpha \ln \left[ \mathcal{K}_{\alpha \beta \gamma} L^\alpha(N, T) L^\beta(N, T) L^\gamma(N, T) \right]. \quad (3.47) \]

\(^\text{18}\)We omit the superpotential terms here since they only depend on \( N \) and play no role in the dualization.

\(^\text{19}\)Notice that there is a misprint in the equivalent equation given in [42]. We thank R. Grimm for discussions on this point.
Inserting (3.46) into (3.45) and using (3.47) we arrive at
\[ T_\alpha + \bar{T}_\alpha + \frac{3}{2} \zeta_R^\alpha = \frac{1}{2} K_{L^\alpha} . \] (3.48)

Comparing (3.47) and (3.48) with (3.25) and (3.26) we are led to identify
\[ \alpha = 1 , \quad L^\alpha = \frac{\nu^\alpha}{K} , \quad \zeta_R^\alpha = \zeta_\alpha + \bar{\zeta}_\alpha , \quad \zeta_\alpha = -\frac{i}{2(\tau - \bar{\tau})} K_{abc} G^b (G - \bar{G})^c . \] (3.49)

As promised we just showed that the somewhat ad hoc definition of the Kähler coordinates in (3.25) is nothing but the duality relation (3.48) obtained from the superfield dualization of the linear multiplets \( L^\alpha \) to chiral multiplets \( T^\alpha \).

The case \( \alpha = 1 \) is a somewhat special situation in that the function \( F \) does not have a constant piece but only the term linear in \( L^\alpha \). This in turn requires that the \( \zeta_\alpha \) cannot be chosen zero but that they have at least a constant piece so that \( F \) does not vanish. This constant is otherwise irrelevant since it drops out of all physical quantities.\(^{21}\) (In a slightly different context the case \( \alpha = 1 \) has also been discussed in ref. \[57\].)

The discussion so far is valid for arbitrary \( K_0 \). By comparing the kinetic terms of the chiral fields in (3.42) one determines
\[ K_0 = K_{cs}(z, \bar{z}) - \ln \left[ -i(\tau - \bar{\tau}) \right] . \] (3.50)

Inserting (3.49) and (3.50) into (3.47) we finally arrive at
\[ K = K_{cs}(z, \bar{z}) - \ln \left[ -i(\tau - \bar{\tau}) \right] - 2 \ln \left[ K_{\alpha\beta\gamma} \nu^\alpha(T, \tau, G) \nu^\beta(T, \tau, G) \nu^\gamma(T, \tau, G) \right] , \] (3.51)
where \( \nu^\alpha(T, \tau, G) \) is determined by (3.48). This \( K \) indeed agrees with the \( K \) previously determined in (3.27).

Let us summarize the story so far. \( N = 1 \) supergravity coupled to chiral and linear multiplets is determined (in the formalism of ref. \[42\] and apart from \( W \) and \( f \) which we can neglect for this discussion) by the independent functions \( K \) and \( \zeta_\alpha \). For the orientifold compactification under consideration the chiral multiplets are \((z^k, \tau, G^a)\), the linear multiplets are \( L^\alpha \) and we determined
\[ K(z, \tau, L) = K_{cs}(z, \bar{z}) - \ln \left[ -i(\tau - \bar{\tau}) \right] + \ln \left[ K_{\alpha\beta\gamma} L^\alpha L^\beta L^\gamma \right] , \] (3.52)
\[ F(L, \tau, G) = L^\alpha (\zeta_\alpha + \bar{\zeta}_\alpha) , \quad \zeta_\alpha = -\frac{i}{2(\tau - \bar{\tau})} K_{abc} G^b (G - \bar{G})^c + \text{const.} . \]

In the dual formulation where the linear multiplets \( L^\alpha \) are dualized to chiral multiplets \( T^\alpha \) the Lagrangian is entirely determined by the Kähler potential given in (3.51) with the ‘unusual’ feature that it is not given explicitly in terms of the chiral multiplets but only implicitly via the constraint (3.48). In this sense the orientifold compactifications (and similarly the compactifications of F-theory on elliptic Calabi-Yau fourfolds considered in [14]) lead to a more general class of Kähler potentials then usually considered in supergravity. In fact the same feature holds for arbitrary \( K_0 \) and arbitrary \( \zeta_\alpha \).

\(^{20}\)Strictly speaking (3.48) only determines the real part of \( \zeta_\alpha \). However, the imaginary part can be redefined into the imaginary part of \( T^\alpha \). We return to this point at the end of this section.

\(^{21}\)We thank R. Grimm for discussions about this point.
Furthermore, these ‘generalized’ Kähler potentials are all of ‘no-scale type’ in that they lead to a positive semi-definite potential $V$. For $\alpha = 1$ (and arbitrary $K_0$ and $\zeta_\alpha$) the Kähler potential (3.40) obeys
\[
L^\alpha K_{L^\alpha} = 3 ,
\]
and hence the the second term in the potential (3.43) vanishes leaving a positive semi-definite potential with a supersymmetric Minkowskian ground state. Since in the chiral formulation $K$ cannot even be given explicitly one can consider such $K$’s as a ‘generalized’ class of no-scale Kähler potentials.\footnote{It would be interesting to show that all positive semi-definite potentials can be derived from such Kähler potentials involving linear multiplets and in this way offering a classification of non-scale Kähler potentials.}

The analogous property has also been observed in refs.\cite{14,19,43}. Finally note with what ease the no-scale property follows in the linear formulation compared to the somewhat involved computation in the chiral formulation performed in appendix C.1.

To complete our analysis we also perform the duality transformation in components, i.e. we dualize the two-forms $D_2^\alpha$ to scalars $\rho_\alpha$. This can be done by the standard procedure \cite{42} where one replaces $dD_2^\alpha$ in (3.42) by an unconstrained three-form $D_3^\alpha$ and in addition adds the term
\[
\delta L = - 3 D_3^\alpha \wedge d\rho_\alpha ,
\]
to the Lagrangian (3.42). The $\rho_\alpha$ act as Lagrange multipliers in that their equations of motions imply $D_3^\alpha = dD_2^\alpha$. On the other hand the equations of motion for $D_3^\alpha$ are now algebraic and can be used to eliminate $D_3^\alpha$ in favor of $\rho_\alpha$. They are given by
\[
D_3^\alpha = 6 K^{L^\alpha L^\beta} (d\rho_\beta - i(\bar{\zeta}_{\beta,A}dN^A - \zeta_{\alpha,A}\bar{d}N^A)) ,
\]
where in order to make contact with (C.8) we have further defined $\rho_\alpha = \rho_\alpha + \frac{i}{2}(\zeta_\alpha - \bar{\zeta}_\alpha)$. Inserting (3.55) into $L + \delta L$ one finally obtains
\[
\mathcal{L} = - \frac{1}{2} R \ast 1 - (K_{AB} - 3 L^\alpha \zeta^{R}_{\alpha,AB}) dN^A \wedge \ast d\bar{N}^B + \frac{1}{4} K^{L^\alpha L^\beta} dL^\alpha \wedge \ast dL^\beta
\]
\[
+ 9 K^{L^\alpha L^\beta} (d\rho_\beta - i(\bar{\zeta}_{\alpha,A}dN^A - \zeta_{\alpha,A}\bar{d}N^A)) \wedge \ast (d\rho_\beta - i(\bar{\zeta}_{\beta,A}dN^A - \zeta_{\beta,A}\bar{d}N^A)) .
\]
For $\alpha = 1$ and $L^\alpha = \frac{v_\alpha}{\kappa}$ the metric in (3.56) coincides with the metric (C.8) obtained in the chiral description.

Let us close this section by stressing once more that the analysis just performed can easily be generalized to other situations in particular including background D-branes. For example, D3-branes couple to $\hat{C}_4$ and therefore also to $D_2^\alpha$. As shown in \cite{32} the low energy dynamics and couplings of the D3-brane scalars is fully encoded in $\zeta_\alpha$ and $K(z, \tau, L)$ remains unchanged. However, in the dual formulation with chiral multiplets the Kähler potential (3.47) changes due a modified (3.48). By differentiating (3.48) with respect to the chiral fields $T_\alpha, N^A$ one obtains
\[
\frac{\partial L^\alpha(T,N)}{\partial T_\beta} = 2 K^{L^\alpha L^\beta} , \quad \frac{\partial L^\alpha(T,N)}{\partial N^A} = 3 K^{L^\alpha L^\beta} \zeta^{R}_{\alpha,A} .
\]
which determines the Kähler metric even so $L^\alpha(T,N)$ is not given explicitly.
4 Calabi-Yau orientifolds with $O5/O9$ planes

In this section we repeat the analysis of section 3 for orientifold compactifications which satisfy the second possible projection (1.2) which reads $O_{(2)} = \Omega_p \sigma^*$ with $\sigma^* \Omega = \Omega$. As we already discussed earlier, in this case the possible orientifold planes present are $O5$- or $O9$-planes. As in this previous section let us start by imposing (1.2) on the ten-dimensional massless spectrum and in this way determine the massless $N = 1$ supermultiplets in $D = 4$.

4.1 The massless spectrum

In order to determine the invariant spectrum we first recall the transformation of the ten-dimensional massless fields under the world-sheet parity transformation $\Omega$. As we discussed in section 3 the dilaton $\hat{\phi}$, the metric $\hat{g}$ and the two form $\hat{C}_2$ are even while $\hat{l}$, $\hat{B}_2$ and $\hat{C}_4$ are odd. Using (1.2) this implies that the invariant states have to obey

\[
\begin{align*}
\sigma^* \hat{\phi} &= \hat{\phi}, & \sigma^* \hat{l} &= -\hat{l}, \\
\sigma^* \hat{g} &= \hat{g}, & \sigma^* \hat{C}_2 &= \hat{C}_2, \\
\sigma^* \hat{B}_2 &= -\hat{B}_2, & \sigma^* \hat{C}_4 &= -\hat{C}_4.
\end{align*}
\]

In addition, $\sigma^*$ now is required to satisfy

\[
\sigma^* \Omega = \Omega. \tag{4.2}
\]

As before $\sigma$ has to be a holomorphic isometry of $Y$. Therefore the Kähler form is invariant and eq. (3.4) remains unchanged implying that $h_{(1,1)}^{(1,1)}$ Kähler deformations $v^\alpha(x)$ survive the projection. However, due to (1.2) instead of (3.2) the deformations of the complex structure change in that now $h_{(1,2)}^{(1,2)}$ complex structure deformation $z^\kappa$ survive the projection or in other words (3.5) is replaced by

\[
\delta g_{ij} = \frac{i}{||\Omega||^2} \tilde{z}^\kappa(\tilde{\chi}_\kappa)_{ij} \Omega^\kappa_j, \quad \kappa = 1, \ldots, h_{+}^{(1,2)}, \tag{4.3}
\]

where $\tilde{\chi}_\kappa$ denotes a basis of $H_{+}^{(1,2)}$.

From eqs. (4.1) we learn that in the expansion of $\hat{B}_2$ the odd elements survive while for $\hat{C}_2$ and $\hat{C}_4$ the even elements are kept. Thus the expansion (3.6) is replaced by

\[
\begin{align*}
\hat{B}_2 &= b^a(x) \omega_a, & a &= 1, \ldots, h_{(-1,1)}, \\
\hat{C}_2 &= C_2(x) + c^\alpha(x) \omega_\alpha, & \alpha &= 1, \ldots, h_{+}^{(1,1)}, \\
\hat{C}_4 &= D^a_2(x) \wedge \omega_a + V^k(x) \wedge \alpha_k - U_k(x) \wedge \beta^k + \rho_a(x) \tilde{\omega}^a, & k &= 1, \ldots, h_{(-1,2)}.
\end{align*}
\]

As before the self-duality constraint $\ast \hat{F}_5 = \hat{F}_5$ eliminates half of the degrees of freedom in $\hat{C}_4$. Again we have the choice to eliminate $D^a_2(x)$ or $\rho_a$ and express the action in terms of chiral or linear multiplets. Similarly, the axion $\hat{l}$ is now projected out and replaced by the $D = 4$ antisymmetric tensor $C_2(x)$. As a consequence the $N = 1$ spectrum
contains a ‘universal’ linear multiplet which in the massless case can be dualized to a chiral multiplet.

Altogether the resulting $N = 1$ spectrum assembles into a gravitational multiplet, $h_{(2,1)}^{-}$ vector multiplets and either $(h_{(2,1)}^{+} + h_{(1,1)}^{+})$ chiral multiplets or $(h_{(2,1)}^{+} + h_{(1,1)}^{-})$ chiral multiplets and $(h_{(1,1)}^{-} + 1)$ linear multiplets. We summarize the spectrum in table 4.1.

| gravity multiplet | 1 | $g_{\mu\nu}$ |
|-------------------|---|-------------|
| vector multiplets  | $h_{(2,1)}^{-}$ | $V^{k}$ |
| chiral multiplets  | $h_{(2,1)}^{+}$ | $z^{\kappa}$ |
|                   | $h_{(1,1)}^{+}$ | $(v^{\alpha}, c^{\alpha})$ |
| chiral/linear multiplets | $h_{(1,1)}^{-}$ | $(b^{a}, \rho_{a})$ |
|                   | 1 | $(\phi, C_{2})$ |

Table 4.1: $N = 1$ spectrum of $O5/O9$-orientifold compactification.

Compared to the spectrum of the first projection given in table 3.1 we see that the vectors and complex structure deformations have switched their role with respect to the decomposition in $H^{(3)}$. Furthermore, different real fields combine into the complex scalars of the chiral/linear multiplets or in other words the complex structure on the moduli space has changed. Now $(v, c)$ and $(b, \rho)$ combine into chiral multiplets whereas before $(v, \rho)$ and $(b, c)$ formed the chiral multiplets.  

4.2 The effective action

To evaluate the four-dimensional effective action we proceed as in the $O3/O7$ case, by first evaluating the field strengths (2.2) including the possibility of background three-form fluxes $H_{3}$ and $F_{3}$. Since $\hat{B}_{2}$ and hence $H_{3}$ is odd it is again parameterized by $H_{3}^{(3)}^{-}$ while $\hat{C}_{2}$ and $F_{3}$ are even and therefore parameterized by $H_{3}^{(3)}^{+}$. Due to (4.2) there is a slight change in $H^{(3)}_{3}$ compared to the previous case in that now we have $H_{3}^{(3,0)} = h_{(0,3)}^{(0,3)} = 1$ and $h_{3}^{(0,3)} = h_{(0,3)}$ = 0 or in other words we have the split $H^{(3)}_{3} = H_{3}^{(3,0)} \oplus H_{3}^{(1,2)} \oplus H_{3}^{(2,1)} \oplus H_{3}^{(3,0)}$ and $H^{(3)}_{3} = H_{3}^{(1,2)} \oplus H_{3}^{(2,1)}$. As a consequence the explicit expansions of the background fluxes $H_{3}$ and $F_{3}$ are given by

$$H_{3} = m_{k}^{H} \alpha_{k} - e_{k}^{H} \beta^{k}, \quad k = 1, \ldots, h_{(2,1)}^{-},$$

$$F_{3} = m_{k}^{F} \alpha_{\hat{k}} - e_{\hat{k}}^{F} \beta^{\hat{k}}, \quad \hat{k} = 0, \ldots, h_{(2,1)}^{+},$$

where the $(m_{k}^{H}, e_{k}^{H})$ are $2h_{(2,1)}^{-}$ constant flux parameters determining $H_{3}$ and $(m_{k}^{F}, e_{\hat{k}}^{F})$ are $2h_{(2,1)}^{+} + 2$ constant flux parameters corresponding to $F_{3}$. Inserting (4.4) and (4.5)

---

Note that again the complex structure which combines $(v, b)$ and which is natural from the $N = 2$ Calabi-Yau point of view does not appear.
where we defined

\[
\tilde{F}_k = dV_k - m^k H C_2, \quad \tilde{G}_k = dU_k - e^H k C_2.
\]

As before the self-duality condition on \( \tilde{F}_5 \) is imposed by adding to the action the total derivative [16]

\[
\delta S^{(4)}_{O_5/O_9} = \frac{1}{4} dV_k \wedge dU_k + \frac{1}{4} dD_2^a \wedge d\rho_a.
\]

Again we have the choice which fields to eliminate. In order to make contact with the standard \( N = 1 \) supergravity we first eliminate \( D_2^a \) and \( U_k \) by inserting their equations of motion into the action. After Weyl rescaling the four-dimensional metric with a factor \( \lambda/6 \) the \( N = 1 \) effective action reads

\[
S^{(4)}_{O_5/O_9} = \int -\frac{1}{2} R \ast 1 - G_{\kappa \lambda} d\zeta^\kappa \wedge \ast d\zeta^\lambda - G_{\alpha \beta} dV^\alpha \wedge \ast dV^\beta - \frac{1}{4} d\ln K \wedge \ast d\ln K
\]

\[
- \frac{1}{4} d\phi \wedge \ast d\phi - e^{\phi} G_{\alpha \beta} dc^\alpha \wedge \ast dc^\beta - e^{-\phi} G_{ab} db^a \wedge \ast db^b
\]

\[
- \frac{K^2}{144} e^{\phi} dC_2 \wedge \ast dC_2 - \frac{1}{4} dC_2 \wedge (\rho_a db^a - b^a d\rho_a)
\]

\[
- \frac{9}{4K^2} G^{ab}(d\rho_a - K_{\kappa \alpha} c^\alpha db^b) \wedge \ast (d\rho_b - K_{\beta \beta} c^\beta db^b) - V \ast 1
\]

\[
+ \frac{1}{4} \text{Re} \mathcal{M}_{kl} \tilde{F}^k \wedge \tilde{F}^l + \frac{1}{4} \text{Im} \mathcal{M}_{kl} \tilde{F}^k \wedge \ast \tilde{F}^l + \frac{1}{4} \epsilon_k (dV^k + \tilde{F}^k) \wedge C_2
\]

where

\[
V = \frac{18i e^{\phi}}{K^2} \int \frac{1}{\Omega \wedge \Omega} \left( \int \Omega \wedge F_3 \int \Omega \wedge F_3 + G_{\kappa \lambda} \int \chi_\kappa \wedge F_3 \int \chi_\lambda \wedge F_3 \right)
\]

\[
- \frac{9}{K^2} e^{-\phi} \left[ m_H^k (\text{Im} \mathcal{M})_{kl} m_H^l + (e^H_k - (m_H \text{Re} \mathcal{M})_k) (\text{Im} \mathcal{M})^{-1kl} (e^H_l - (m_H \text{Re} \mathcal{M})_l) \right].
\]

The derivation of this potential is performed in appendix B.2 where we also show that the first term in the potential can be written in terms of \( (e^F_k, m_F^k) \) exactly as the second term.\(^{24}\)

The action (4.9) has the standard one-form gauge invariance \( V^k \rightarrow V^k + d\lambda^k_0 \) but due to the modification in (4.7) also a modified (Stückelberg) two-form gauge invariance given by

\[
C_2 \rightarrow C_2 + d\Lambda_1, \quad V^k \rightarrow V^k + m_H^k \Lambda_1.
\]

\(^{24}\)Note that in this class of orientifolds the topological term \( \int_{\Sigma} H_3 \wedge F_3 \) vanishes since there is no intersection between \( H_3^{(3)} \) and \( H_3^{(1)} \). Thus strictly speaking background D-branes have to be included in order to satisfy the tadpole cancellation condition.
Thus for \( m^k_H \neq 0 \) one vector can be set to zero by an appropriate gauge transformation. This is directly related to the fact that (4.9) includes mass terms proportional to \( m^k_H \) for \( C_2 \) arising from (4.7). In this case gauge invariance requires the presence of Goldstone degrees of freedom which can be ‘eaten’ by \( C_2 \).\(^{25}\) Finally note that the last term in (4.9) also includes a standard \( D = 4 \) Green-Schwarz term \( F^k \wedge C_2 \).

### 4.2.1 Vanishing magnetic fluxes \( m^k_H = 0 \)

The next step is to show that \( S_{O5/O9}^{(4)} \) is consistent with the constraints of \( N = 1 \) supergravity. However, due to the possibility of \( C_2 \) mass terms this is not completely straightforward. A massive \( C_2 \) is no longer dual to a scalar but rather to a vector. We find it more convenient to keep the massive tensor in the spectrum and discuss the \( N = 1 \) constraints in terms of a massive linear multiplet. However, in this case a ‘standard’ \( N = 1 \) action is not available in the literature and thus we first discuss the situation \( m^k_H = 0 \) where \( \tilde{F}^l = F^l \) holds. In this case the \( C_2 \) remains massless and can be dualized to a scalar field \( h \) which together with the dilaton \( \phi \) combines to form a chiral multiplet \((\phi, h)\). Using the standard dualization procedure (see, for example, [18]) one obtains

\[
S_{O5/O9}^{(4)} = \int \left( -\frac{1}{2} R * 1 - G_{\kappa \lambda} \, d\zeta^\kappa \wedge *d\zeta^\lambda - G_{\alpha \beta} \, dv^\alpha \wedge *dv^\beta \right. \\
\left. - \frac{1}{4} d\ln k \wedge *d\ln k - \frac{1}{4} d\phi \wedge *d\phi \right. \\
\left. - e^{H} G_{\alpha \beta} \, dc^\alpha \wedge *dc^\beta - e^{-H} G_{ab} \, db^a \wedge *db^b \right. \\
\left. - \frac{9}{k^2} e^{-\phi}(Dh + \frac{1}{2}(d\rho_a b^a - \rho_a db^a)) \wedge *(Dh + \frac{1}{2}(d\rho_a b^a - \rho_a db^a)) \right. \\
\left. - \frac{9}{4k^2} G^{ab}(d\rho_a - K_{a\alpha \beta} c^\alpha db^\beta) \wedge *(d\rho_b - K_{bd\beta} c^\beta db^d) \right. \\
\left. + \frac{1}{4} \text{Im} \mathcal{M}_{kl} \, dV^k \wedge *dV^l + \frac{1}{4} \text{Re} \mathcal{M}_{kl} \, dV^k \wedge dV^l - V \wedge 1 \right.
\]

where \( V \) is given by (4.10) evaluated at \( m^k_H = 0 \) and the covariant derivative of \( h \) is defined as

\[
Dh = dh - \epsilon_k H V^k.
\]

\( h \) couples non-trivially to the gauge fields as a direct consequence of the Green-Schwarz coupling \( F^k \wedge C_2 \) in (4.9). In the dualized action the scalar \( h \) then is charged under the corresponding \( U(1) \) gauge transformation. More precisely, the gauge invariance reads

\[
h \rightarrow h + \epsilon_k H A^k_0, \quad V^k \rightarrow V^k + dA^k_0,
\]

which leaves the covariant derivative (4.13) invariant. Note that the gauge charges are set by the electric fluxes.

In the action (4.12) we immediately see that the complex structure deformations \( z^\kappa \) are again already good Kähler coordinates. For the remaining fields we find the appropriate

\[^{25}\text{Exactly the same situation occurs in Calabi-Yau compactifications of type IIB with background fluxes where both } B_2 \text{ and } C_2 \text{ can become massive [18].}\]
Kähler coordinates to be
\[ t^\alpha = \hat{v}^\alpha + ic^\alpha , \]
\[ A_a = -\frac{2}{3} \hat{G}_{ab} b^b + i \left( \mathcal{K}_{abcd} c^d b^c + \rho_a \right) , \tag{4.15} \]
\[ S = e^{\frac{i}{2} \phi} \mathcal{K} + ih - \frac{1}{4} (\text{Re} \Theta^{-1})^{ab} A_a (A + \bar{A})_b , \]
\[ = e^{\frac{i}{2} \phi} \mathcal{K} + ih + \frac{1}{3} \hat{G}_{ab} b^a b^b - \frac{i}{2} \left( \mathcal{K}_{abcd} c^d b^c + \rho_a b^a \right) , \]
where we abbreviated
\[ \Theta_{ab}(t) = -\frac{2}{3} \hat{G}_{ab} + i \mathcal{K}_{aba} c^a = \mathcal{K}_{aba} t^a , \quad \hat{v}^\alpha = e^{\frac{1}{4} \phi} \mathcal{K}^{-\frac{1}{2}} v^\alpha . \tag{4.16} \]

Note that the quantities with the hat (i.e. \( \hat{\mathcal{K}}, \hat{\mathcal{G}}_{ab} \)) in (4.15) are calculated using the redefined Kähler moduli \( \hat{v}^\alpha \). Furthermore, \( \Theta_{ab} \) depends holomorphically on the coordinates \( t^\alpha \) and the covariant derivative of \( h \) given in (4.13) translates into the covariant \( DS = dS - ie_k V^k \). In the variables given in (4.15) the Kähler potential reads
\[ K = -\ln \left[ -i \int \Omega \wedge \bar{\Omega} \right] - \ln \left[ \frac{1}{48} \mathcal{K}^{\alpha \beta \gamma} (t + \bar{t})^\alpha (t + \bar{t})^\beta (t + \bar{t})^\gamma \right] \]
\[ = -\ln \left[ S + \bar{S} + \frac{1}{4} (A + \bar{A})_a (\text{Re} \Theta^{-1})^{ab} (A + \bar{A})_b \right] . \tag{4.17} \]

The corresponding Kähler metric is computed in appendix C.2 and the check that it indeed reproduces (4.12) is straightforward, since (4.17) is closely related to the quaternionic ‘Kähler potential’ given in [50] and we can make use of their results.\(^{27}\) The same reference already observed that for a holomorphic matrix \( \Theta \) the quaternionic geometry is also Kähler. This situation was also found in compactifications of the heterotic string to \( D = 3 \) on a circle [14].

Finally, let us note that inserting (4.15) back into \( K \) results in
\[ K = -\ln \left[ -i \int \Omega \wedge \bar{\Omega} \right] - \ln \left[ 2e^{-\phi} - 2\ln \left( \mathcal{K} \right) \right] , \tag{4.18} \]
which is exactly the same \( K \) as (3.27) when expressed in terms of the variables \( \phi \) and \( v^\alpha \). This can be understood from that fact that these NS-sector variables are the same in both types of orientifolds and the difference in the two cases only arises when one expresses \( K \) in terms of proper Kähler coordinates.

Let us now turn to the gauge couplings and the potential. Comparing (4.9) with (3.23) we determine
\[ f_{kl}(z^\alpha) = -\frac{i}{2} \bar{\mathcal{M}}_{kl} = -\frac{i}{2} \mathcal{F}_{kl} \big|_{z^k = 0 = \bar{z}^k} , \tag{4.19} \]
\[ ^{26} \text{In this case it is necessary to redefine the Kähler moduli and the dilaton in analogy to Calabi-Yau compactifications of type IIB [39].} \]
\[ ^{27} \text{Note however, that the complex structure changed non-trivially. In [50] the standard } t \sim v + ib \text{ formed complex coordinates.} \]
where the second equation holds in complete analogy to eqs. (3.32), (3.33). As a consequence the \( f_{kl} \) are again manifestly holomorphic functions of the complex structure deformations \( z^\kappa \).

From eq. (4.14) we see that the axion is charged and as a consequence we expect a non-vanishing \( D \)-term in the potential. Recall the general formula for the \( D \)-term

\[
K_{IJ}\bar{X}^J_k = i\partial_I D_k ,
\]

(4.20)

where \( X^I \) is the Killing vector of the \( U(1) \) transformations defined as \( \delta M^I = \Lambda^k_0 X^J_k \partial_J M^I \). Inserting (4.17), (C.13) and (4.15) we obtain

\[
D_k = -e^{H_k} \frac{\partial K}{\partial S} = 3 e^{H_k} e^{-\frac{1}{2}\phi} K^{-1} .
\]

(4.21)

Using also (4.19) we arrive at the \( D \)-term contribution to the potential

\[
\frac{1}{2} (\text{Re} f)^{-1} C_{kl} D_k D_l = -9 e^{-\phi} K^{-2} e^{H} (\text{Im} \mathcal{M})^{-1} C_{kl} e^{H} ,
\]

(4.22)

which indeed reproduces the last term in (4.10) for \( m^k_H = 0 \).

The first term in (4.10) arises from the superpotential

\[
W = \int_{Y} \Omega \wedge F_3 .
\]

(4.23)

Using (4.17) the Kähler covariant derivatives of \( W \) are calculated to be

\[
D_\kappa W = i \int \chi^\kappa \wedge F_3 , \quad D_S W = K_S W = -3 e^{-\frac{1}{2}\phi} K^{-1} W ,
\]

\[
D_{t^a} W = K_{t^a} W = \frac{3}{2} \left( K_\alpha + e^{-\frac{1}{2}\phi} K^{-1} K_{a\alpha b^a b^b} \right) W ,
\]

\[
D_{A_a} W = K_{A_a} W = -3 e^{-\frac{1}{2}\phi} K^{-1} b^a W ,
\]

(4.24)

where we used \( D_\kappa \Omega = i\chi^\kappa \). Inserting (C.14) and (1221) into (5.21) one obtains the first term in (4.10).

It is interesting that for this class of orientifolds the RR-flux \( F_3 \) results in a contribution to the superpotential while the NS-flux \( H_3 \) contributes instead to a \( D \)-term.

### 4.2.2 Non-vanishing magnetic fluxes \( m^k_H \neq 0 \)

Let us now turn to the case where both electric and magnetic fluxes are non-zero and the two-form \( C_2 \) is massive. In this case \( C_2 \) is dual to a massive vector or equivalently the massive linear multiplet is dual to massive vector multiplet. Here we do not discuss this duality but instead show how the couplings of a massive linear multiplet is consistent with the action (4.9).\(^{28}\)

In section 3.3 we already reviewed some properties of the linear multiplet. The kinetic terms in the effective action are determined by a generalized Kähler potential (3.40) and

\(^{28}\)More details will appear in [58].
the couplings $\zeta$ defined in (3.41) which determine the couplings of the two-form to the chiral multiplets. These functions are not affected by any mass terms and can be read off directly from (4.9). Comparing (4.9) with (3.42) and using (4.15) and (4.16) we determine

$$K = K_0 + \ln L, \quad \zeta^R = \frac{1}{12}(A + \bar{A})_a(\text{Re}\Theta^{-1})^{ab}(A + \bar{A})_b$$

(4.25)

with

$$K_0 = -\ln \left[ -i \int \Omega \wedge \bar{\Omega} \right] - \ln \left[ \frac{1}{48} K_{\alpha\beta\gamma} (t + \bar{t})^\alpha(t + \bar{t})^\beta(t + \bar{t})^\gamma \right],$$

$$L = 3e^{-\frac{1}{2}\phi}K^{-1}, \quad D_2 = \frac{1}{2}C_2.$$  

(4.26)

Performing the duality transformation from the linear multiplet $L$ to a chiral multiplet $S$ as outlined in section 3.3 we obtain

$$\frac{1}{L} = S + \bar{S} + 3\zeta^R.$$  

(4.27)

Inserted back into (4.25) we determine the dual Kähler potential to be

$$K = K_0 - \ln \left[ S + \bar{S} + 3\zeta^R(A, \bar{A}, t, \bar{t}) \right],$$

(4.28)

which indeed coincides with (4.17). Thus we have shown that the kinetic terms can consistently be described either in the chiral- or the linear multiplet formalism and that (4.15) are the appropriate coordinates.

Let us now briefly discuss the situation of a massive linear multiplet coupled to $N = 1$ vector- and chiral multiplets. For simplicity we discuss the situation in flat space and do not couple the massive linear multiplet to supergravity. However, we expect our results to generalize to the supergravity case. More details can be found in [56, 58].

As we already said, a linear multiplet $L$ contains a real scalar (also denote by $L$) and the field strength of a two-form $C_2$ as bosonic components. However, it does not contain the two-form itself which instead is a member of the chiral ‘prepotential’ $\Phi$ defined as

$$L = D\Phi + \bar{D}\bar{\Phi}, \quad D\Phi = 0.$$  

(4.29)

This definition solves the constraint (3.37) (in flat space). The kinetic term for $L$ (or rather for $\Phi$) is given in (3.38) and a mass-term can be added via the chiral integral

$$\mathcal{L}_m = \frac{1}{4} \int d^2 \theta \left[ f_{kl}(N)(W^k - 2im^k_H\Phi)(W^l - 2im^l_H\Phi) + 2e^H_k(W^k - im^k_H\Phi)\Phi \right] + \text{h.c.},$$

(4.30)

where $W^k = -\frac{1}{4}D^2DV^k$ are the chiral field strengths supermultiplets of the vector multiplets $V^k$ and $f_{kl}(N)$ are the gauge kinetic function which can depend holomorphically on the chiral multiplets $N$. $(m^k_H, e^H_k)$ are constant parameters which will turn out to correspond to the flux parameters defined in (4.5). The Lagrangian (4.30) is invariant

29 We suppress the spinorial indices and use the convention $D\Phi \equiv D^\alpha\Phi_\alpha, \bar{D}\Phi \equiv \bar{D}^\dot{\alpha}\Phi_{\dot{\alpha}}$. 

26
under the standard one-form gauge invariance $V^k \rightarrow V^k + \Lambda_0^k + \bar{\Lambda}_0^k$ ($\Lambda_0^k$ are chiral super-fields) which leaves both $W^k$ and $\Phi$ invariant. In addition (4.30) has a two-form gauge invariance corresponding to (4.11) given by

$$\Phi \rightarrow \Phi + \frac{i}{8} \bar{D}^2 D \Lambda_1, \quad V^k \rightarrow V^k + m^k_H \Lambda_1,$$

where $\Lambda_1$ now is a real superfield. From (4.31) we see that one entire vector multiplet can be gauged away and thus plays the role of the Goldstone degrees of freedom which are ‘eaten’ by the massive linear multiplet.

In components one finds the bosonic action

$$\mathcal{L}_m = -\frac{1}{2} \text{Re} f_{kl} \tilde{F}^k \wedge * \tilde{F}^l - \frac{1}{2} \text{Im} f_{kl} \tilde{F}^k \wedge \tilde{F}^l + \frac{1}{4} e_k (dV^k + \tilde{F}^k) \wedge C_2 - V \ast 1,$$

where $\tilde{F}^l$ is defined exactly as in (4.7) $\tilde{F}^l = dV^l - m^k_H C_2$ and the potential $V$ receives two distinct contributions

$$V = \frac{1}{2} (\text{Re} f)^{-1kl} D_k D_l + 2 m^k_H \text{Re} f_{kl} m^l_H L^2, \quad D_k = (e^H_k + 2 \text{Im} f_{kl} m^l_H) L.$$

The first term arises from eliminating the $D$-terms in the $U(1)$ field strength $W^k$ while the second term is a ‘direct’ mass term for the scalar $L$.\(^{30}\) Inserting the $D$-term yields a second contribution to the mass term and one obtains altogether

$$V = \frac{1}{2} [(e^H_k + 2 \text{Im} f_{kp} m^p) (\text{Re} f)^{-1kl} (e^H_l + 2 \text{Im} f_{lr} m^r) + 4 m^k_H \text{Re} f_{kl} m^l_H] L^2. \quad (4.34)$$

Using (4.26) and (4.19) this precisely agrees with the second term in the potential (4.10).

As before the first term in (4.10) can be derived from the superpotential (4.23). Inserting (4.25) into (3.43) using (4.26) indeed yields the first term of (4.10).

5 Conclusions

In this paper we determined the low energy effective action for Calabi-Yau orientifolds in the presence of background fluxes from a Kaluza-Klein reduction. In our analysis we did not specify a particular Calabi-Yau manifold but merely demanded that it admits a holomorphic and isometric involution $\sigma$. Depending on the explicit action of $\sigma$ on the holomorphic three-form $\Omega$, we analyzed two distinct cases: (1) orientifolds with $O3/O7$-planes and (2) orientifolds with $O5/O9$-planes. For each case we calculated the Kähler potential, the superpotential and the gauge kinetic functions and showed the consistency with $N = 1$ supergravity.

In the first case the background fluxes induce a non-trivial scalar potential which is determined in terms of a superpotential previously given in $[7, 10, 4, 19]$. We also included the scalar fields $\{b^a, c^a\}$ arising from the two type IIB two-forms $B_2$ and $C_2$ and which are related to the presence of $O7$-planes into the analysis. We showed that in this case

\(^{30}\)Note that this second term is a contribution to the potential which is neither a $D$- nor an $F$-term but instead a ‘direct’ mass term whose presence is enforced by the massive two-form.
the potential is unmodified which can be traced to the no-scale property of the Kähler potential. This in turn (as well as the choice of the proper Kähler coordinates) can be easily understood in terms of the ‘dual’ formulation where the chiral multiplets containing the Kähler deformations of the Calabi-Yau orientifold are replaced by linear multiplets. This formulation of the effective action is particularly suitable for also including the couplings of background $D$-branes to the bulk moduli fields as given in [21, 59, 32]. As a byproduct we determined an entire new class of no-scale Kähler potentials which in the chiral formulation can only be given implicitly as the solution of some constraint equation. In the linear multiplet formalism on the other hand they are defined straightforwardly and their no-scale property is easily displayed.

For orientifolds with $O5/O9$ planes the influence of background fluxes is more involved. This is due to the fact that the space-time two-form $C_2$ arising in the expansion of the RR field $\hat{C}_2$ remains in the spectrum. It combines with the dilaton into a linear multiplet, which only if it is massless can be dualized to a chiral multiplet. However, generic NS three-form background fluxes render this form massive. We therefore first restricted our attention to the case were the mass term vanishes which occurs if the magnetic fluxes arising from the NS three-form $H_3$ are set to zero. In the resulting chiral description the axion dual to $C_2$ is gauged with the gauge charges set by the electric fluxes. The scalar potential now consists of two distinct contributions. The term which depends on the RR fluxes arising from $F_3$ is obtained from a (truncated) superpotential of the previous case whereas the second contribution depends on the electric fluxes of $H_3$ and arises from $D$-terms which are present due to the gauged isometry. Finally, we also analyzed non-vanishing magnetic fluxes in the NS sector which can be described by an $N = 1$ theory including a massive linear multiplet coupled to vector and chiral multiplets. In this case the scalar potential as additional contributions again arising from $D$-terms but furthermore a direct mass term for the scalar in the linear multiplet which is neither a $D$- nor an $F$-term.

Orientifolds with $O3/O7$-planes can also be obtained as a limit of F-theory compactified on elliptic Calabi-Yau fourfolds [60, 37]. At the level of the effective action this can indeed be seen by compactifying the effective action given in section 3.2 on a circle to $D = 3$ and comparing it with the action of M-theory compactified on Calabi-Yau fourfolds computed in [14]. Notice that the Kähler potential in three dimensions is the sum of the $D = 4$ Kähler potential plus an additional term which arises when the three-dimensional vectors are dualized to scalars. Out of the vectors $V^k$ present in the orientifold one obtains complex scalars $w^k$ in $D = 3$, such that [14]

$$K_{(3)} = K_{(4)}(\tau, T, G, z) + K_{(vec)}(w, z) . \quad (5.1)$$

Turning to the M-theory compactification, we recall that the massless spectrum of the fourfold $Y_4$ consists of $h^{(3,1)}$ complex structure deformations $Z$, $h^{(1,1)}$ complexified Kähler deformations $(M, P)$ and $h^{(2,1)}$ ‘non-geometrical’ scalars $N$. Together they span a Kähler manifold determined by a non-trivial Kähler potential computed in [14]. In the orientifold limit this metric splits into two parts and can be compared to the Kähler potential (5.1) of the compactified orientifold. One finds agreement if one identifies

$$Z \sim \tau, z^k , \quad (M, P) \sim T_\alpha , \quad N \sim G^a, w^k . \quad (5.2)$$

The inverse radius of the compactification circle can be identified with the volume of the torus fiber. In lifting the M-theory compactification to F-theory one shrinks the
volume of the torus fiber \cite{60}, which on the other hand corresponds to going back to the four-dimensional orientifold. Further details about this F-theory lift will be presented elsewhere.

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Appendix

A Conventions

In this appendix we summarize our conventions.

- The coordinates of the four-dimensional Minkowski space-time are denoted by $x^\mu, \mu = 0, \ldots, 3$. The corresponding metric is chosen to have signature $(-, +, +, +)$. The coordinates of the compact Calabi-Yau manifold $Y$ are $y^i, \bar{y}^{\bar{i}}, i, \bar{i} = 1, 2, 3$.

- $p$-forms are expanded into a real basis according to

$$A_p = \frac{1}{p!} A_{\mu_1 \ldots \mu_p} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p}. \quad (A.1)$$

- $(p, q)$-forms are expanded into a complex basis as

$$A_{p,q} = \frac{1}{p!q!} A_{i_1 \ldots i_p \bar{\bar{i}}_1 \ldots \bar{\bar{i}}_q} dy^{i_1} \wedge \ldots \wedge dy^{i_p} \wedge d\bar{y}^{\bar{i}_1} \wedge \ldots \wedge d\bar{y}^{\bar{i}_q}. \quad (A.2)$$

- The exterior derivative is defined as

$$dA_p = \frac{1}{p!} \partial_\mu A_{\mu \mu_1 \ldots \mu_p} dx^{\mu_1} \wedge \ldots \wedge dx^{\mu_p}. \quad (A.3)$$

- The field strength of a $p$-form $F_{p+1} = dA_p$ is given by

$$F_{\mu_1 \ldots \mu_{p+1}} = (p + 1) \partial_{[\mu_1} A_{\mu_2 \ldots \mu_{p+1}]} \quad (A.4)$$
The inner product for real forms is defined by using the Hodge-* operator. In components we have
\[ \int F_p \wedge * F_p = \frac{1}{p!} \int d^d x \sqrt{-g} F_{\mu_1 \ldots \mu_p} F^{\mu_1 \ldots \mu_p}. \] (A.5)
\[ *1 = d^d x \sqrt{-g} \] is the d-dimensional measure.

The Hodge-* satisfies \[ **F_p = (-1)^{p(d-p)+\kappa} F_p, \] where \( \kappa = 1 \) for Lorentzian signature and \( \kappa = 0 \) for Euclidean signature.

On a Calabi-Yau manifold the non-trivial cohomology groups \( H^{(p,q)} \) are
\( H^{(0,0)}, H^{(1,1)}, H^{(3,0)}, H^{(2,1)}, H^{(1,2)}, H^{(0,3)}, H^{(2,2)}, H^{(3,3)} \).

Their dimensions \( h^{(p,q)} \) obey
\[ h^{(0,0)} = h^{(3,0)} = h^{(0,3)} = 1, \quad h^{(1,1)} = h^{(2,2)}, \quad h^{(2,1)} = h^{(1,2)}. \] (A.6)

For the indices labeling the basis elements of these cohomology groups we use the conventions
\[ \omega_A \in H^{(1,1)}, \quad \bar{\omega}^A \in H^{(2,2)}, \quad A, B = 1, \ldots, h^{(1,1)}, \]
\[ \chi_K \in H^{(2,1)}, \quad \bar{\chi}_K \in H^{(1,2)}, \quad K, L = 1, \ldots, h^{(2,1)}, \]
\[ (\alpha_{\hat{K}}, \beta^{\hat{L}}) \in H^{(3)}, \quad \hat{K}, \hat{L} = 0, \ldots, h^{(2,1)}. \] (A.7)

For orientifolds the cohomology groups split according to \( H^{(p,q)} = H^{(p,q)}_+ \oplus H^{(p,q)}_- \).

The basis elements are denoted as follows
\[ \omega_a \in H^{(1,1)}_+, \quad \bar{\omega}^a \in H^{(2,2)}_+, \quad \alpha, \beta = 1, \ldots, h^{(1,1)}_+, \]
\[ \omega_a \in H^{(1,1)}_-, \quad \bar{\omega}^a \in H^{(2,2)}_-, \quad a, b = 1, \ldots, h^{(1,1)}_-, \] (A.8)
\[ \chi_\kappa \in H^{(2,1)}_+, \quad \kappa, \lambda = 1, \ldots, h^{(2,1)}_+, \]
\[ \chi_k \in H^{(2,1)}_-, \quad k, l = 1, \ldots, h^{(2,1)}_- \].

For the cohomology \( H^{(3)} \) there is a difference depending on the transformations property of \( \Omega \). For \( \sigma^* \Omega = -\Omega \) (O3/O7 case) one has \( \Omega \in H^{(3)}_- \) and thus \( 2h^{(2,1)}_+ = h^{(3)}_+ \) and \( 2h^{(2,1)}_- + 2 = h^{(3)}_- \) holds. As in the Calabi-Yau case we put a ‘hat’ on the index if it starts from 0 and we have
\[ (\alpha_\kappa, \beta^\lambda) \in H^{(3)}_+, \quad \kappa, \lambda = 1, \ldots, h^{(2,1)}_+, \]
\[ (\alpha_{\hat{k}}, \beta^{\hat{l}}) \in H^{(3)}_-, \quad \hat{k}, \hat{l} = 0, \ldots, h^{(2,1)}_-. \] (A.9)

For \( \sigma^* \Omega = \Omega \) (O5/O9 case) one has \( \Omega \in H^{(3)}_+ \) and thus \( 2h^{(2,1)}_+ + 2 = h^{(3)}_+ \) and \( 2h^{(2,1)}_- = h^{(3)}_- \) holds. In this case we have
\[ (\alpha_\kappa, \beta^\lambda) \in H^{(3)}_+, \quad \kappa, \lambda = 0, \ldots, h^{(2,1)}_+, \]
\[ (\alpha_{\hat{k}}, \beta^{\hat{l}}) \in H^{(3)}_-, \quad \hat{k}, \hat{l} = 1, \ldots, h^{(2,1)}_- \]. (A.10)
B The Potentials

B.1 The scalar potential for the O3/O7 orientifolds

In this appendix we give some details about the derivation of (3.22). This already exists in the literature [10, 4, 19, 21] and we include it here in order to make the paper more self-contained.

\[ V \text{ arises from the terms of (2.1) given by} \]
\[ S^{(10)} = -\frac{1}{4} \int \left( e^{-\hat{\phi}} \hat{H}_3 \wedge \hat{H}_3 + e^{\hat{\phi}} \hat{F}_3 \wedge \hat{F}_3 \right) + \ldots . \]  
\[ \text{(B.1)} \]

Inserting only the background fluxes \( H_3 \) and \( F_3 \) which are defined in (3.7) and (3.11) and which are harmonic three-forms on the Calabi-Yau manifold \( Y \) with no four-dimensional space-time dependence we arrive at

\[ L = -\frac{1}{4} e^\phi \int_Y G_3^+ \wedge *_6 \overline{G}_3^+ + \ldots , \]
\[ \text{(B.2)} \]

where \( *_6 \) is the Hodge \( * \)-operator on \( Y \). One defines the imaginary self- and anti-self-dual parts of \( G_3 \) by

\[ *_6 G_3^\pm = \mp i G_3^\pm , \quad \text{i.e.} \quad G_3^\pm = \frac{1}{2} (G_3 \pm i *_6 G_3). \]  
\[ \text{(B.3)} \]

Inserting (B.3) into (B.2) we arrive at

\[ L = -\frac{1}{2} e^\phi \int_Y G_3^+ \wedge *_6 \overline{G}_3^+ + \frac{i}{4} e^\phi \int_Y G_3 \wedge \overline{G}_3 + \ldots . \]  
\[ \text{(B.4)} \]

The second term in (B.4) is a topological term contributing to the flux tadpoles while the first term corresponds to the four-dimensional scalar potential \( V \). Including the Weyl rescaling of the four-dimensional metric \( g_{\mu \nu} \rightarrow \kappa^2 g_{\mu \nu} \) results in the overall factor \( 36/\kappa^2 \) and we obtain

\[ V = \frac{18}{\kappa^2} e^\phi \int_Y G_3^+ \wedge *_6 \overline{G}_3^+ . \]  
\[ \text{(B.5)} \]

In eq. (3.7) we observed that \( G_3 \) can be expanded in a basis of \( H_3^{(-)} \). The decomposition (B.3) says that \( G_3^+ \) can be expanded along \( H_3^{(3,0)} \oplus H_3^{(1,2)} \) and a choice of basis is \( (\Omega, \bar{\chi}_k) \) where the \( \chi_k \) are defined in (3.36). Thus we have

\[ G_3^+ = -\frac{1}{\int \Omega \wedge \Omega} \left( \Omega \int \bar{\Omega} \wedge G_3 + G_{kl} \bar{\chi}_k \int \chi_l \wedge G_3 \right) , \]  
\[ \text{(B.6)} \]

where \( G^{kl} \) is the inverse of the metric defined in (3.15) and we have furthermore used \( \int \bar{\Omega} \wedge G_3^+ = \int \bar{\Omega} \wedge G_3 \) and \( \int \chi_k \wedge G_3^+ = \int \chi_k \wedge G_3 \). Inserting (B.6) into (B.5) we finally arrive at the expression already given in (3.22)

\[ V = \frac{18i e^\phi}{\kappa^2 \int \Omega \wedge \Omega} \left( \int \Omega \wedge \bar{G}_3 \int \bar{\Omega} \wedge G_3 + G^{kl} \int \chi_k \wedge G_3 \int \bar{\chi}_l \wedge \bar{G}_3 \right) . \]  
\[ \text{(B.7)} \]
An alternative form of this potential can be obtained by inserting the explicit expansion of $G_3$ given in (4.8) into (B.2) and expressing the potential in terms of the $2(h^{1,2} + 1)$ complex flux parameters given in (3.9). Using (2.6) and (3.12) one finds (10)

$$V = -\frac{9e^\phi}{K^2} \left[ m^k (\text{Im } M)_{ki} \tilde{m}^i + (e^H_k - (m\text{Re } M)_k) (\text{Im } M)^{-1kl}(e^H_l - (m\text{Re } M)_l) \right]. \quad (B.8)$$

### B.2 The scalar potential for $O5/O9$

Here we supply some details computing the potential (4.10). The difference to the previous case is that the two background fluxes $H_3$ and $F_3$ take values in different cohomologies with no non-trivial intersections. $F_3$ is expanded in a basis of $H^{(3)}$ while $H_3$ is expanded in a basis of $H^{(3)}$ given in (4.5). Since $\Omega$ is an element of $H^{(3)}$ the expansion of $F_3$ is completely analogous to the previous case while $H_3$ is only expanded along $H^{(2,1)}$. As a consequence the derivation of the potential changes slightly. First we decompose $G_3 = F_3 - ie^{-\phi}H_3$ into self-dual $G_3^+$ and anti self-dual $G_3^-$ components. Analogous to (3.6) $G_3^+$ enjoys an expansion

$$G_3^+ = F_3^+ - ie^{-\phi}H_3^+$$

$$= -\frac{1}{\int \Omega \wedge \bar{\Omega}} \left( \int \bar{\Omega} \wedge F_3 + G^{\lambda \kappa} \bar{\chi}_\kappa \int \chi_\lambda \wedge F_3 \right)$$

$$-\frac{i}{2} e^{-\phi} \left( m^H_k \alpha_k - e^H_k \beta_k + i(m^F_k \alpha_k - e^F_k \beta_k) \right)$$

$$= -\frac{1}{2} \left( m^F_k \alpha_k - e^F_k \beta_k + i(m^H_k \alpha_k - e^H_k \beta_k) \right)$$

$$-\frac{i}{2} e^{-\phi} \left( m^H_k \alpha_k - e^H_k \beta_k + i(m^F_k \alpha_k - e^F_k \beta_k) \right),$$

where we have used (B.6) and the self-dual combinations of (4.5). Inserted into (B.5) and using the fact that $H_3$ and $F_3$ have no non-trivial intersections we arrive at

$$V = \frac{18i e^\phi}{K^2 \int \Omega \wedge \bar{\Omega}} \left( \int \Omega \wedge F_3 \int \bar{\Omega} \wedge F_3 + G^{\kappa \lambda} \int \chi_\kappa \wedge F_3 \int \bar{\chi}_\lambda \wedge F_3 \right)$$

$$-\frac{9e^\phi}{K^2} \left[ m^H_k (\text{Im } M)_{kl} m^l_H + (e^H_k - (m\text{Re } M)_k) (\text{Im } M)^{-1\lambda l}(e^H_\lambda - (m\text{Re } M)_\lambda) \right]$$

$$= -\frac{9e^\phi}{K^2} \left[ m^F_k (\text{Im } M)_{k\lambda} m^\lambda_F + (e^F_k - (m\text{Re } M)_k) (\text{Im } M)^{-1\lambda \bar{l}}(e^F_\lambda - (m\text{Re } M)_\lambda) \right]$$

$$-\frac{9e^\phi}{K^2} \left[ m^H_k (\text{Im } M)_{k\lambda} m^\lambda_H + (e^H_k - (m\text{Re } M)_k) (\text{Im } M)^{-1\lambda l}(e^H_\lambda - (m\text{Re } M)_\lambda) \right].$$
C The Kähler metrics

C.1 The Kähler metric and its inverse for $O3/O7$

In this appendix we supply the details for the calculation of the Kähler metric obtained from the Kähler potential $K(\tau, T, G, z)$ given in (3.27). It turns out to be instructive to do this computation for a more general Kähler potential than the one given in (3.27) and only in the end restrict ourselves to this case.

Let $M^I = (T_\alpha, N^A)$ be the coordinates of a Kähler manifold with Kähler potential

$$K = K_0(N, \bar{N}) + K_K(T + \bar{T}, N, \bar{N}),$$  \hspace{1cm} (C.1)

where $K_K$ has the special form

$$K_K = -2\ln(K) = -2\ln(K_{\alpha\beta\gamma}v^\alpha v^\beta v^\gamma).$$  \hspace{1cm} (C.2)

The $v^\alpha(T + \bar{T}, N, \bar{N})$ are functions of the Kähler coordinates defined implicitly by the equation

$$T_\alpha = \frac{3i}{2} \rho_\alpha + \frac{3}{4} K_{\alpha\beta\gamma}v^\beta v^\gamma - \frac{3}{2} \zeta_\alpha(N, \bar{N}),$$  \hspace{1cm} (C.3)

where the $\zeta_\alpha$ are arbitrary complex functions depending on the coordinates $N^A, \bar{N}^A$ but not on $T_\alpha$. We see that for

$$K_0 = K cs - \ln\left[-i(\tau - \bar{\tau})\right], \hspace{1cm} \zeta_\alpha = -i\frac{2}{3(\tau - \bar{\tau})} K_{abc} G^b(G - \bar{G})^c$$  \hspace{1cm} (C.4)

the $K$ of (C.1) coincides with the $K$ given in (3.27).

The Kähler metric is computed by differentiating the real part of (C.3) with respect to all Kähler coordinates $T_\alpha, N^A$. This yields

$$\frac{\partial v^\alpha}{\partial T_\beta} = \frac{1}{3} K_{\alpha\beta}, \hspace{1cm} \frac{\partial v^\alpha}{\partial N^A} = \frac{1}{2} K_{\alpha\beta} \zeta_R^{\beta, A},$$  \hspace{1cm} (C.5)

where $\zeta_R^{\alpha} \equiv \zeta_\alpha + \bar{\zeta}_\alpha$ and $K_{\alpha\beta}$ is the inverse of $K_{\alpha\beta} = K_{\alpha\beta\gamma} v^\beta v^\gamma$. Using (C.5) one computes the first derivatives of (C.1) to be

$$K_{T_\alpha} = -2 \frac{v^\alpha}{K}, \hspace{1cm} K_{N^A} = K_{0,A} - 3 \frac{v^\alpha}{K} \zeta_R^{\alpha, A}. $$  \hspace{1cm} (C.6)

Similarly, the Kähler metric is found to be

$$K_{T_\alpha T_\beta} = \frac{G_{\alpha\beta}}{K^2}, \hspace{1cm} K_{T_\alpha N^A} = \frac{3}{2} \frac{G_{\alpha\beta}}{K^2} \zeta_R^{\beta, A},$$  \hspace{1cm} (C.7)

where $G_{\alpha\beta}$ is given in (3.14) and we abbreviated $Q_{AB} = -3 \frac{v^\alpha}{K} \zeta_R^{\alpha, AB}$. Finally, using (C.6) and (C.3) one can express the Kähler metric in the (non-Kähler) coordinates $(v^\alpha, \rho_\alpha, N^A)$

$$K_{M^I M^J} dM^I \wedge *dM^J =$$

$$G_{\alpha\beta} dv^\alpha \wedge *dv^\beta + \frac{1}{4} d\ln K \wedge *d\ln K + (K_{0,AB} - 3 \frac{v^\alpha}{K} \zeta_R^{\alpha, AB}) dN^A \wedge *dN^B$$

$$+ \frac{9}{4} \frac{G_{\alpha\beta}}{K^2} (d\rho_\alpha - i(\bar{\zeta}_\alpha dN^A - \zeta_{\alpha, A} d\bar{N}^A)) \wedge *\left(d\rho_\beta - i(\bar{\zeta}_\beta dN^B - \zeta_{\beta, B} d\bar{N}^B)\right).$$
Let us now discuss two special cases. First we assume that the Kähler manifold is a direct product with block-diagonal metric and $K_0$ ($K_K$) is the Kähler potential of the first (second) factor. An example for this situation is the case $\tau = \text{const}$. In this case (for arbitrary $\zeta_\alpha$) the Kähler metric of the Kähler potential $K_K$ can be inverted as

$$K_K^{T_\alpha T_\beta} = K^2 G_{\alpha\beta} + \frac{9}{4} Q^{AB} \zeta_{\alpha A} \zeta_{\beta B} , \quad K_K^{T_\alpha \bar{N}^A} = -\frac{3}{2} Q^{AB} \zeta_{\alpha A} , \quad K_K^{N^A \bar{N}^B} = Q^{AB} , \quad (C.9)$$

where $Q^{AB}$ is the inverse of $Q_{AB}$. Using (C.6) and (C.9) one finds

$$\frac{\partial K_K}{\partial M^I} (K^{-1})^I_J \frac{\partial K_K}{\partial M^J} = 3 . \quad (C.10)$$

This implies that $K_K$ obeys the standard no-scale condition \[\text{(III)}\].

The second case corresponds to a Kähler potential given in (3.27). One inserts (C.4) and $N^A = (\tau, G^a, z^k)$ into (C.7) to obtain

$$K_{T_\alpha T_\beta} = \frac{1}{K^2} G^{\alpha\beta} ,$$
$$K_{T_\alpha \bar{G}^a} = -\frac{3i}{2K^2} G^{\alpha\beta} K_{\beta ab} b^b ,$$
$$K_{T_\alpha \bar{G}^a} = -\frac{3i}{4K^2} G^{\alpha\beta} K_{\beta ab} b^b ,$$
$$K_{G^a \bar{G}^b} = e^{\phi} G_{ab} + \frac{9}{4K^2} G^{\alpha\beta} K_{\alphaac} b^c K_{\beta bd} b^d ,$$
$$K_{G^a \bar{G}^b} = e^{\phi} G_{ab} b^b + \frac{9}{8K^2} G^{\alpha\beta} K_{\alphaac} b^c K_{\beta bd} b^d ,$$
$$K_{G^a = \frac{1}{4} e^{2\phi} + e^{\phi} G_{ab} b^b + \frac{9}{16K^2} G^{\alpha\beta} K_{\alphaac} b^c K_{\beta bd} b^d ,}$$
$$K_{\zeta z^l} = G_{kl} ,$$

and the inverse metric

$$K^{T_\alpha T_\beta} = K^2 G_{\alpha\beta} + \frac{9}{4} e^{-\phi} G^{ab} K_{aabc} b^c K_{\beta bd} b^d + \frac{9}{4} e^{-2\phi} K_{aabc} b^c K_{\beta bd} b^d ,$$
$$K^{T_\alpha \bar{G}^a} = -\frac{3i}{2} e^{-\phi} G^{ab} K_{aabc} b^c - 3i e^{-2\phi} K_{aabc} b^c b^a ,$$
$$K^{T_\alpha \bar{G}^a} = 3i e^{-\phi} K_{aabc} b^c ,$$
$$K^{G^a \bar{G}^b} = -4 e^{-2\phi} b^a ,$$
$$K^{G^a \bar{G}^b} = -4 e^{-2\phi} b^a ,$$
$$K^{G^a = 4 e^{-2\phi} ,}$$

$$K^{G^a \bar{G}^b} = 4 e^{-2\phi} ,$$
$$K_{\zeta z^l} = G_{kl} . \quad (C.12)$$

Using (C.12) and (C.6) one verifies (3.29). In section 3.3 we trace this property to the ‘dual’ formulation where instead of the chiral superfields $T_\alpha$ one uses the dual linear multiplets $L^\alpha$.

**C.2 The Kähler metric and its inverse for O5/O9**

For completeness let us also give explicitly the Kähler metric of the Kähler potential (4.17) expressed in the (non-Kähler) variables $(v^\alpha, e^\alpha, \rho_\alpha, b^a, \phi, h, z^k)$. By straightforward
differentiation one finds for the metric
\[ K_{t\alpha}^{\bar{t}\beta} = e^{\phi} G_{\alpha\beta} + \frac{9}{4} \kappa^2 G^{ab} \kappa_{ac} b^c \kappa_{bd} b^d + \frac{9}{4} \kappa^2 e^{-\phi} \kappa_{aab} b^a b^b \kappa_{bc} b^c b^d, \]
\[ K_{Aa}^{\bar{t}a} = -\frac{9}{4} \kappa^2 e^{\phi} \kappa_{cdd} b^c b^d a^b, \]
\[ K_{S\bar{t}a} = -\frac{9}{4} \kappa^2 G^{ab} + \frac{9}{4} \kappa^2 e^{\phi} b^a b^b, \]
\[ K_{Aa}^{\bar{t}A} = -\frac{9}{4} \kappa^2 e^{-\phi} b^a b^a, \]
\[ K_{SS} = \frac{9}{4} \kappa^2 e^{-\phi}, \]
\[ K_{z\kappa}^{\bar{z}\lambda} = G_{\kappa\lambda}. \]

Inverting this metric yields
\[ K^{t\alpha}_{\bar{t}\beta} = e^{-\phi} G^{\alpha\beta}, \]
\[ K^{t\alpha}_{\bar{A}a} = e^{-\phi} G^{\alpha\beta} \kappa_{ab} b^b, \]
\[ K^{t\alpha}_{S} = -\frac{1}{2} e^{\phi} G^{\alpha\beta} \kappa_{ab} b^a b^b, \]
\[ K^{t\alpha}_{\bar{A}b} = \frac{4}{9} \kappa^2 G_{ab} + e^{-\phi} G^{\alpha\beta} \kappa_{ac} b^c \kappa_{bd} b^d, \]
\[ K^{t\alpha}_{S} = -\frac{4}{9} \kappa^2 G_{ab} b^b - \frac{1}{2} e^{-\phi} G^{\alpha\beta} \kappa_{ac} b^c \kappa_{bd} b^d b^d, \]
\[ K^{S}_{S\bar{S}} = \frac{1}{4} e^{\phi} \kappa^2 + \frac{4}{9} \kappa^2 G_{ab} b^a b^b + \frac{1}{4} e^{-\phi} G^{\alpha\beta} \kappa_{ac} b^c b^d \kappa_{bd} b^d b^d, \]
\[ K^{z\kappa}_{\bar{z}\lambda} = G_{\kappa\lambda}. \]

Notice that the $O5/O9$ metric for $(S, t^a, A_a)$ in non-Kähler coordinates is up to factors $\kappa$ and $e^\phi$ the inverse of the $O3/O7$ metric for $(\tau, T, G^a)$. This can be understood by noticing that $T_a$ and $t^a$ are 'dual' coordinates [43].

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