Protocol for universal gates in optimally biased superconducting qubits

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We present a new experimental protocol for performing universal gates in a register of superconducting qubits coupled by fixed on-chip linear reactances. The qubits have fixed, detuned Larmor frequencies and can remain, during the entire gate operation, biased at their optimal working point where decoherence due to fluctuations in control parameters is suppressed to first order. Two-qubit gates are performed by simultaneously irradiating two qubits at their respective Larmor frequencies with appropriate amplitude and phase, while one-qubit gates are performed by the usual single-qubit irradiation pulses.

Single quantum bits displaying coherence in the time domain have now been implemented in various superconducting integrated electrical circuits[1]. Microwave spectroscopy[2], coherent temporal oscillations[3], and a conditional gate operation[4] have been reported in experiments on pairs of capacitively coupled qubits. In all these implementations, decoherence is by far the largest obstacle to be overcome for applications to quantum information processing. Yet, as Vion et al. have demonstrated, appropriate symmetries in circuit architecture and bias conditions can be exploited for suppressing to first order decoherence due to fluctuations in control parameters.

The schemes for performing two-qubit gates proposed so far require dynamical tuning of either the qubit transition frequencies[2] or of the subcircuit controlling the qubit-qubit interaction[5]. The former typically requires dc pulses that move the qubits away from their optimal bias points for coherence, while the latter requires additional control lines and non-linear elements that inevitably introduce additional couplings to uncontrolled degrees of freedom in the environment. In this Letter, we present a novel scheme which minimizes decoherence by maintaining both qubits at their optimal bias points, and by employing only noise-free fixed linear coupling reactances. Furthermore, this scheme takes advantage of the spread in circuit parameters occurring naturally in fabrication, instead of suffering from it.

Our strategy consists of constructing a circuit with fixed, detuned Larmor frequencies and fixed coupling strengths—a sort of “artificial molecule” —and realizing gates with protocols inspired by those of nuclear magnetic resonance (NMR) quantum computation[6]. The essential difference between our “molecules” and those used in NMR resides in the form of the qubit-qubit couplings and the way they are exploited. In NMR, the secular terms in the coupling Hamiltonian (i.e. those that commute with the Zeeman Hamiltonian and thus act to first order) dominate the spin-spin interaction. Two-qubit gates are realized as the spins precess freely, while refocusing pulses are applied in order to “do nothing”. In our scheme, the coupling is purely non-secular, and has no effect to first order. So unlike in NMR, we must construct pulses to enhance the second-order effect of the coupling. We refer to this strategy with the (NMR-style!) nickname: FLICFORQ, for Fixed LInear Couplings between Fixed Off-Resonant Qubits.

The superconducting register we have in mind may consist of charge qubits (controlled via charges on gate capacitors) interacting through on-chip capacitors or of flux qubits (controlled via fluxes through superconducting loops) interacting through mutual inductances. We focus for the moment on two-qubit registers (Fig. 1), the simplest that allow the realization of a universal set of quantum gates, leaving the extension to larger systems to the discussion. The optimal bias conditions for the circuits shown are \( N_1^z = N_2^z = 1/2 \) for charge qubits, where \( N^z = C^0 U/2 e \) is the dimensionless gate charge, or \( N_1^φ = N_2^φ = 1/2 \) for flux qubits, where \( N^φ = Φ_{ext}/Φ_o \) is the flux frustration. Under these conditions, the systems become immune, to first order, to variations in the control parameters, such as \( 1/f \) charge noise in the Josephson junctions or substrate or noise due to the motion of trapped flux[7].

![FIG. 1: Superconducting two-qubit circuits for performing universal quantum gates at optimal bias point. (a) Charge qubits (Saclay style) coupled by a capacitor. (b) Flux qubits (Delft style) coupled by a mutual inductance.](image)

At optimal bias, these two-qubit systems are described by the reduced Hamiltonian

\[
\mathcal{H}/\hbar = \frac{1}{2} \left[ \omega_1^x \sigma_1^x + 2 \omega_1(t) \cos(\omega_1^t t) \sin(\omega_1^t t) \right] \sigma_1^z
+ \omega_2^x \sigma_2^x + 2 \omega_2(t) \sin(\omega_2^t t) \sin(\omega_2^t t) \sigma_2^z
+ \omega_{xx} \sigma_1^x \sigma_2^x,
\]

where \( \omega_1^x/2π \) (\( ω_2^x/2π \)) is the Larmor frequency of qubit
\[ \frac{\omega f}{2\pi} = \frac{\omega y}{2\pi} \] is the frequency of the signal applied to the “write” port of qubit 1; \( \omega f \) and \( \omega y \) are the amplitudes of the in-phase and quadrature components of the applied signals, respectively, and, when divided by 2\( \pi \), are directly interpretable as Rabi frequencies; and \( \omega x / 2\pi = (t_{\text{swap}})^{-1} \) is the “swap” frequency. The Larmor frequencies are detuned from one another, as occurs naturally during fabrication, by \( \delta = \omega 1 - \omega 2 \), and remain fixed throughout. The swap frequency is fixed at the time of circuit fabrication and should satisfy \( \pi / t_{\text{rf}} \) and we perform all possible gates by playing with only one-qubit unitaries, \( (1 + \sigma x 1 \sigma x 2) / \sqrt{2} \) (Fig. 2, outer levels). If we choose \( \delta = 0 \), \( \sigma y 1 \sigma y 2 \) rotation taking place during the first half of the pulse will be fully undone during the second half. This technique resembles the refocusing schemes used in NMR, though here we are modifying the pulse shape rather than performing additional \( \pi \) rotations.

Implementing the refocusing flip leads to the pure \( \pi / 2 \) rotation \( (X_1 X_2)^{1/2} = (1 + i \sigma x 1 \sigma x 2) / \sqrt{2} \). This rotation, when augmented by one-qubit \( \pi / 2 \) rotations, is known to generate the two-qubit Clifford group \( C_2 \). So along with all one-qubit unitaries, \( (X_1 X_2)^{1/2} \) therefore constitutes a universal set of rotations.

We can thus turn to the construction of a protocol to perform \( U_{\text{CNOT}} \), the rotation corresponding to the standard rotations:

\[
D = \frac{(1 - i \sigma x 1 \sigma y 2)(1 + i \sigma x 1 \sigma y 2)}{2} = (Z_1 Z_2)^{-1/2}(X_1 X_2)^{1/2},
\]

where we have used the rotation operator notation in which \( X_1 = \sigma x 1 \), \( X_2 = \sigma x 2 \), etc. This rotation maps the computational basis states to the Bell states with a relative phase, e.g., \( D: |00\rangle \rightarrow (|00\rangle + |11\rangle) / \sqrt{2} \), and can therefore be used to generate and study maximally entangled states. However, it is easy to verify that \( D^2 = -1 \), indicating that \( D \) is a \( \pi \) rotation, and therefore is not universal \( \sigma y 1 \sigma x 2 \).

The mechanism allowing the very weak interqubit coupling \( \omega x \) to produce maximally entangled two-qubit states is easily understood in the dressed atom picture of quantum optics. When the RF fields and qubits are uncoupled, each qubit + field system has an infinite discrete ladder of doubly-degenerate energy levels, labelled by the qubit state \( |1\rangle \) or \( |0\rangle \) and the photon number \( |N\rangle \), and separated by \( \omega y \) (Fig. 2, outer levels). Taking the qubit-field coupling into account lifts the degeneracy, causing the two states in each manifold to be split symmetrically by the field strength \( \omega y \) (Fig. 2, inner levels). The two dressed qubits may then absorb and emit energy at frequencies \( \omega 1 \pm \omega y \) and \( \omega 2 \pm \omega y \), respectively. The result of the irradiation is thus to split the single-mode qubit line into two sidebands at these frequencies.

Choosing the RF amplitudes \( \omega 1,2 = \delta / 2 \) causes the upper sideband of qubit 1 to overlap the lower sideband of qubit 2, allowing the qubits to exchange photons of energy \( h(\omega 1 - \omega 2) = \hbar(\omega 1 + \omega 2) \) through the coupling reactance.

The simplest protocol for generating entangled states in this system is to simultaneously irradiate each qubit with a signal of amplitude \( \delta / 2 \) \( \omega 1,2 = \delta / 2 \) and \( \omega 1,2 = 0 \), and initialize the state to \( \rho_{in} = |00\rangle \langle 00| \); the qubits will become maximally entangled after a pulse time \( 4\pi / \omega x = 2t_{\text{swap}} \). If the RF is then switched off, the system will remain in an entangled state until it is measured in a local basis or it undergoes decoherence or relaxation. Note that this scheme allows us to produce entanglement on demand without dc excursions from the optimal bias point of either qubit.

The rotation realized when the system is irradiated in this manner, which we call “D”, is not a pure \( \sigma x 1 \sigma y 2 \) rotation, but is rather a product of two commuting \( \pi / 2 \) rotations:

\[
D = \frac{(1 - i \sigma x 1 \sigma y 2)(1 + i \sigma x 1 \sigma y 2)}{2} = (Z_1 Z_2)^{-1/2}(X_1 X_2)^{1/2},
\]

where we have used the rotation operator notation in which \( X_1 = \sigma x 1 \), \( X_2 = \sigma x 2 \), etc. This rotation maps the computational basis states to the Bell states with a relative phase, e.g., \( D: |00\rangle \rightarrow (|00\rangle + |11\rangle) / \sqrt{2} \), and can therefore be used to generate and study maximally entangled states. However, it is easy to verify that \( D^2 = -1 \), indicating that \( D \) is a \( \pi \) rotation, and therefore is not universal \( \sigma y 1 \sigma x 2 \).

We propose to circumvent this problem by nulling out the \( \sigma y 1 \sigma x 2 \) factor of D. This is done by flipping the sign of the RF signal amplitude on one of the two qubits midway through the pulse. With this “refocusing flip” the unwanted \( \sigma y 1 \sigma x 2 \) rotation taking place during the first half of the pulse will be fully undone during the second half. This technique resembles the refocusing schemes used in NMR, though here we are modifying the pulse shape rather than performing additional \( \pi \) rotations.

Implementing the refocusing flip leads to the pure \( \pi / 2 \) rotation \( (X_1 X_2)^{1/2} = (1 + i \sigma y 1 \sigma x 2) / \sqrt{2} \). This rotation, when augmented by one-qubit \( \pi / 2 \) rotations, is known to generate the two-qubit Clifford group \( C_2 \). So along with all one-qubit unitaries, \( (X_1 X_2)^{1/2} \) therefore constitutes a universal set of rotations.

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![FIG. 2: Energy levels of qubit + RF photons systems with (inner levels) and without (outer levels) qubit-photon coupling. Outer: systems have an infinite ladder of doubly-degenerate levels corresponding to products of a photon number state (green, orange) and a qubit state (red, blue). Inner: Photons–qubit coupling lifts degeneracy in each manifold by Rabi frequency \( \omega y \). Transitions between adjacent manifolds (wavy arrows) correspond to absorption/emission of a photon from dressed qubit system. Transition of qubit 1 at \( \omega 1,2 = \omega y \) and qubit 2 at \( \omega 1 \pm \omega y \) coincide when \( \omega 1 - \omega 2 = \delta / 2 \), putting qudits on speaking terms.](image-url)
FIG. 3: Pulse sequence for $U^{\text{CNOT}}$ using FLICFORQ. (a) Amplitude of in-phase ($\omega^x$) and quadrature ($\omega^y$) components of resonant RF signal on first (top) and second (bottom) qubits. Protocol constructs $U^{\text{CNOT}}$ from sequence of five “primitive” $\pi/2$ rotations. One-qubit $\sigma^x$ and $\sigma^y$ pulses have amplitude $\delta/8$ and duration $t^\text{sync}_0 = 4\pi/\delta$; two-qubit pulses have amplitude $\delta/2$. The $\sigma^z$ rotation occurs last, where it can be ignored if followed by measurement in computational basis. Gate is completed at time $t^\text{CNOT}$. (b) Description of sequence in Heisenberg picture[18]. First and last columns (red, green) are connected by $U^{\text{CNOT}}$. Standard two-qubit logical gate CNOT. We first decompose $U^{\text{CNOT}}$ into a sequence of rotations that draws only on $(X_1X_2)^{1/2}$ and one-qubit $\pi/2$ rotations. We use the sequence (time running left to right),

$$X_2^{1/2}Y_1^{1/2}(X_1X_2)^{1/2}Y_1^{-1/2}Z_1^{1/2},$$

which, as required, performs the $U^{\text{CNOT}}$ mapping in the Heisenberg picture: \{\sigma_1^x, \sigma_1^y, \sigma_2^x, \sigma_2^y\} $\rightarrow$ \{\sigma_1^x, \sigma_1^y, \sigma_2^x, \sigma_2^y\}[14]. Though similar in spirit to decompositions of $U^{\text{CNOT}}$ given elsewhere for other systems, e.g. [12], expression (2) is presented in the general language of Pauli rotation operators, making it applicable to any physical implementation. It can, with simple algebra, be adapted to systems where the core two-qubit gate is other than $(X_1X_2)^{1/2}[11]$. The full $U^{\text{CNOT}}$ pulse sequence is constructed by concatenating the pulses generating each of the constituent rotations in expression (2) (see figure 3).

We must briefly comment on how the difference between the Larmor frequencies is dealt with. In the absence of irradiation the natural evolution of the system consists of continuous rotations of each qubit about the $\sigma^x$ axis, resulting in a time-dependent phase between the qubits that vanishes every $t^\text{sync}_0 = 2\pi/\delta$. This phase is unimportant when considering one-qubit gates, as compensatory $\sigma^z$ rotations may be realized through simple waiting periods[12]. However, it must be taken into account when doing two-qubit rotations by initiating two-qubit pulses only at $t^\text{sync}_m = mt^\text{sync}_0$ for integer $m$, i.e. when the qubits are in synchrony. This condition can be met by using one-qubit pulse amplitudes such that the one-qubit $\sigma^x$ and $\sigma^y$ rotations last $t^\text{sync}_m$. For the above chosen parameters, an amplitude of $\delta/8$ for $\pi/2$ pulses is convenient (finer rotations may be generated by weaker pulses with the same duration), corresponding to $t^\text{sync}_m = 4\pi/\delta$. The associated timing grid is shown with dashed vertical lines in figure 3. This scheme is generalizable to multiquubit registers (see below).

We have simulated the pulse sequence of figure 3 by numerically solving a set of fifteen coupled differential equations describing each component of the two-qubit density operator[18]. The simulation technique is exact in the sense that it does not rely on any approximations or perturbative expansions of the time-dependent Hamiltonian. Figure 4 shows the results of a simulation of two representative evolutions. The simultaneous vanishing of each component of the two reduced density operators indicates the generation of a fully entangled two-qubit state[12].

FIG. 4: Evolution of sample input states during $U^{\text{CNOT}}$ sequence. Components of the reduced density operators $\rho_1 = Tr_2\rho$ (row 1) and $\rho_2 = Tr_1\rho$ (row 2). $\rho_1$ and $\rho_2$ are plotted in reference frames rotating at $\omega^x_1$ and $\omega^x_2$, resp. Dashed vertical line denotes $t^\text{CNOT}$. Error visible at $t^\text{CNOT}$ is due to Bloch-Siegert shift and effect of coupling during one-qubit rotations[13].

What level of gate fidelity can we expect from this scheme? We first discuss the error in one-qubit gates. Choosing the one-qubit pulse time to be $t^\text{sync}_m$ means that the coupling term $\omega^x$ will perform a parasitic rotation in this time by an angle $\arccos(\omega^x_0 t^\text{sync}_m)$. This rotation can be nullified altogether using dynamic decoupling schemes, as is done in NMR[13]. In the present
system, this would be done by performing appropriately timed $\pi$ rotations about $\sigma^y$, which anticommutes with the coupling term $\sigma^z \cdot \sigma^z$. However, for the range of practical circuit parameters $\delta \gtrsim 0.1 \omega_0$ and $\omega_{xx} \lesssim 0.1 \omega_0$, the one-qubit gate error rate resulting from this parasitic rotation is already below $10^{-5}$, or two orders of magnitude better than the fidelity of presently available read-out schemes, and the correction is an easily dispensable luxury. Also, though our simulations have used only square pulses, realistic pulse shapes should not cause a significant further loss of fidelity in the one-qubit operations, since, as is commonplace in NMR, pulse shapes requiring far less bandwidth could be used.

The two-qubit gate error rate will likely be dominated by errors resulting from imperfect RF pulses. The strength of the qubit-qubit interaction depends strongly on the amplitude of the simultaneous RF signals, so entangling gates will be sensitive to jitter or ringing in the pulse amplitudes. This problem could be minimized by using slowly-rising pulses which require less bandwidth rather than trying to approximate square pulses. The pulses could be constructed so that intended one-qubit rotations are implemented during the rise time.

Nonetheless, there is still some error present in our simulations of two-qubit gates, even though we have used ideal pulses. We have verified that this can be attributed to the counter-rotating term in the rotating wave framework, as the qubits are irradiated with strong fields for many Larmor periods during a two-qubit gate. This error can be reduced by choosing a stronger coupling $\omega_{xx}$, thereby reducing the time required to generate entanglement, or by reducing the detuning $\delta$, which reduces the required field strength. Since these changes would increase the one-qubit gate error rate due to the fixed coupling, an NMR-style decoupling scheme will likely be needed once we require a gate error rate $\lesssim 10^{-5}$.

We believe a main advantage of the gate scheme presented in this paper is that it can be directly generalized to multiqubit registers with minimal extra hardware. A fixed linear coupling reactance between all pairs of qubits could easily be achieved by coupling each qubit to a common superconducting island, loop or cavity. Selective one-qubit gates would be realized by applying an RF signal at just the target qubit transition frequency, while the protocol generating $D$ or the universal two-qubit rotation $(X_1 X_2)^{1/2}$ could be realized on any pair by simultaneously applying RF signals at the resonant frequencies of the two targeted qubits. Since each qubit in the register would be detuned from all the others, all these write pulses could be multiplexed on a single RF control line, a decisive advantage in seeking to limit stray couplings to the environment or crosstalk between qubits. Applying $D$ to several qubits in a pairwise fashion would allow the direct production of multiqubit entangled states of the form $|\text{GHZ}_n\rangle = (|0\rangle^{\otimes n} + e^{i \phi} |1\rangle^{\otimes n})/\sqrt{2}$, which, for $n > 2$ can display maximal violations of Bell-type inequalities.

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