Notes on the Quantum Corrections of Swampland and Trans-Planckian Censorship Conjecture

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We discuss the swampland and trans-Planckian censorship conjectures (TCC) from the entropy bound with quantum corrections, namely quantum version of Bousso’s bound and energy conditions. We include the typical contributions from the entanglement entropy in de Sitter spacetime. The TCC is not much corrected, whereas the bounds on swampland conjecture from energy conditions can be relaxed due to quantum corrections.

I. INTRODUCTION

The discussion around the allowed low-energy effective theories has been an interesting path to study the quantum gravity in ultraviolet (UV). Especially in the standard cosmology, early universe has a period of vacuum dominated time called inflation with quasi de Sitter (dS) geometry [1, 2]. Since the inflation, as well as the dark energy dominated late universe geometry both imply that at least an approximate de Sitter vacuum exists in nature, it is important to construct such a vacuum/quasi vacuum in possible candidates of quantum gravity. The de Sitter swampland conjecture has been proposed [3], and originally stated that any vacuum has to satisfy $|\nabla V| \geq c V/M_P$ for constant $c \sim O(1)$ to have a stringy construction, to forbid meta stable de Sitter vacuum. This conjecture, if being correct, basically states that string theory as the UV theory can not give rise to the standard ΛCDM universe. Nevertheless, it is still worth exploring more than what kinds of low energy theories can arise from string theory and what might be the low energy theory for a consistent UV quantum gravity to resolve this sharpened contradiction.

More recently, a connection from the bounds on inflationary fluctuation to the swampland conjecture has been proposed, namely trans-Planckian censorship conjecture (TCC) [4]. It comes from the statement that quantum fluctuations should remain quantum and never exit the Hubble horizon and freeze during inflation. This leads to the bound on the inflationary scale:

$$H < M_P e^{-N},$$

where $M_P$ is the reduced Planck mass and $N$ is the $e$-folding number. This further leads to very small tensor-to-scalar ratio $r < 10^{-30}$ [5], which is below current and near future observation reaches. If considering more general set-up, a higher upper bound on the inflationary Hubble expansion rate $H_{inf} < M_P T_0/T_{rh}$ was proposed in [6], where $T_{rh}$ is the reheating temperature and $T_0$ is the photon temperature today. In the lowest reheating temperature required for big bang nucleosynthesis, the bound on tensor-to-scalar ratio can be relaxed to be $r \leq 10^{-8}$. A series of discussions can be found in [7]. Moreover, the swampland and TCC can also be derived from the entropy bound [8, 9].

In this note we discuss how the quantum corrections from entanglement contribution can relax both conjectures, and hope to shed light on future discussions incorporating the quantum effects in gravity. It is known that [10] in four dimensions de Sitter space background with fluctuations the leading UV-divergent term of entanglement entropy gives rise to the well known area contributions from the surface particle entangled. The UV-finite piece of entropy contains the long range correlations that exit the Hubble horizon and freeze. For the logarithmic corrections, the UV-divergent part is proportional to $\log \left( \frac{H}{M_P} \right) \sim -O(10)$ and the finite logarithmic piece can be sub-leading and we ignore it here. This added quantum effects can correct both Bousso’s entropy bounds and null energy condition [11–13], then further modify swampland and TCC.

The paper is organized as below. In section II, we derive the TCC from the entropy bound with quantum corrections. In section III, we consider the quantum null energy condition with corrections which leads to the modified swampland conjecture. We conclude and discuss the result in section IV.

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II. QUANTUM CORRECTIONS TO THE ENTROPY BOUND

On the entropy in a region of radius \( R \), we have the quantum version of the entropy bound as \([11, 12, 14]\),

\[
S(R) \leq S_{\text{tot}} = M_\text{P}^2 R^2 + S_{\text{out}}(R),
\]

where \( M_\text{P}^2 R^2 \) is the usual area law of the Bekenstein entropy. \( S_{\text{out}}(R) \) is the contribution from the entanglement entropy across the surface. According to \([10]\), one quantum correction to the entanglement entropy is proportional to \( R^2 H^2 + \cdots \). It comes from the modes on the expanding FRW background in 4-dimensions, going out of the horizon and freeze. This part of UV cut-off independent entanglement contribution contains the long range information, and not easily calculable directly.

There are higher order corrections of logarithmic contribution, both local and long-ranged as well. We can take the following local terms and consider its corrections to the entropy bound and energy conditions.

\[
S_{\text{out}}(R) \sim (\alpha' + \beta' R^2 H^2) \log \left( \frac{H}{M_\text{P}} \right) + \cdots.
\]

During the inflationary era, \( \log \left( \frac{H}{M_\text{P}} \right) \) can be considered as a negative constant of order \( O(10) \). We can then in general consider \( S_{\text{out}} \) as the entanglement entropy \(~(\alpha + \beta R^2 H^2)\), in \( d = 4 \) dimensional de Sitter background.

A. From entropy bound to the TCC

Considering \( n_s \) relativistic particles at temperature \( T_s \), their energy and entropy are given by \([8]\)

\[
E_s = a_s n_s R^3 T_s^4, \quad S(R) = \frac{4}{3} E_s, \quad a_s = \frac{4\pi^3}{45}.
\]

The radius \( R \) need to be larger than the Schwarzschild radius of a black hole with the same energy

\[
R \geq R_s = \frac{E_s}{M_\text{P}^2} \quad \Rightarrow \quad R \lesssim R_m = \frac{M_\text{P}^2}{T_s^2 \sqrt{a_s n_s}},
\]

with \( R_m \) being the typical maximum radius \([15]\). Therefore, the maximum entropy in the sphere is

\[
S(R_m) = \frac{4}{3} n_s R^3 T_s^3 \leq M_\text{P}^2 R^2 + (\alpha + 4\pi^2 \beta R^2 T^2_s),
\]

where \( T_H \equiv \frac{H}{2\pi} \leq T_s \) has been used and here is for the case \( \beta > 0 \).

If setting \( \alpha = 0 \) and considering \( \beta \ll n_s \), we end up with

\[
T_s \leq \frac{M_\text{P}}{\sqrt{n_s}} \frac{3\sqrt{a_s}}{4} \left( 1 + \frac{9a_s \pi^2 \beta}{4 \ n_s} \right), \quad \text{or} \quad T_s \geq \frac{M_\text{P} \sqrt{n_s}}{\pi^2 \beta} \frac{1}{3\sqrt{a_s}}.
\]

The higher orders has been omitted. We take the first solution in (7). From its relation to the effective cut-off and field \( \Delta \phi \) transferred, the number of species satisfy, \( n_s > e^{2a_s N} \) with \( a_s \) being a positive number of order one \([8, 9]\). And again considering \( T_H = \frac{H}{2\pi} \leq T_s \), we can recover the (refined) TCC bounds \([9]\) from the first inequality in (7),

\[
H < M_\text{P} e^{-n_s \frac{3\pi \sqrt{a_s}}{2} \left( 1 + \frac{9a_s \pi^2 \beta}{4 \ n_s} \right)}.
\]

We can see here for this condition that the quantum correction to the TCC bound is quite small. The other condition \( T_s \geq \frac{M_\text{P} \sqrt{n_s}}{\pi^2 \beta} \frac{1}{3\sqrt{a_s}} \) may only work for some special cases. It may also provide an exception from the bound, although it is not that clear at the moment.
III. QUANTUM CORRECTIONS TO ENERGY CONDITIONS

We consider the 4-dimensional effective theories which are compactified from the $D$-dimensional spacetime. It has been shown in [3] that the strong energy condition leads to

$$\frac{|\nabla V|}{V} \geq \frac{\lambda_{\text{SEC}}}{\sqrt{\text{MP}}}, \quad \lambda_{\text{SEC}} = \sqrt{\frac{2(D - 2)}{D - 4}}, \quad (9)$$

and null energy condition leads to

$$\frac{|\nabla V|}{V} \geq \frac{\lambda_{\text{NEC}}}{\sqrt{\text{MP}}}, \quad \lambda_{\text{NEC}} = \sqrt{\frac{2(D - 4)}{D - 2}}. \quad (10)$$

Both of these bounds are greater than the bound from TCC [4] for the asymptotic region of the moduli space

$$\left(\frac{|\nabla V|}{V}\right)_{\infty} \geq \frac{\lambda_{\text{TCC}}}{\sqrt{\text{MP}}}, \quad \lambda_{\text{TCC}} = \sqrt{\frac{2}{3}}. \quad (11)$$

In this section, we consider the quantum corrections of null energy condition from [11]

$$T_{MN}k^M k^N \geq \frac{\hbar}{2\pi} S''_{\text{out}}, \quad (12)$$

where $S_{\text{out}}$ can be taken as the von Neumann entropy on the enclosed surface and we work in the unit of $\hbar = 1$. The double-prime is the derivative respected to the deformation of the surface, with $k^M$ being the null vector orthogonal to the surface [16]. This condition can also be generalized to the strong energy condition.

A. Strong Energy Condition

As a warm up, we follow the derivation of the strong energy condition in [3]. In principle the strong energy condition is also corrected by $S''_{\text{out}}$. Consider the $D$-dimensional metric

$$ds^2 = \Omega(y, t)^2(-dt^2 + a(t)dx_i^2) + \tilde{g}_{mn}dy^m dy^n, \quad (13)$$

where $x_i = x_1, x_2, x_3$ and $y^m$ being the $D - 4$ dimensional internal indices. Taking the time like normal vector as $k^N = (1, 0, ...)$, then we have

$$\left(T_{tt} + \frac{1}{D - 2} T^N_N\right) = R_{tt}^{(D)} = -\frac{\ddot{a}}{a} - 3\frac{\ddot{\Omega}}{\Omega} + \left[-\frac{1}{2} \tilde{g}^{mn}\ddot{g}_{mn} + \frac{1}{3\Omega^2} \nabla^m \nabla_m (\Omega^2)\right] \geq S''_{\text{out}}. \quad (14)$$

Then we integrate over the compact manifold and chose the $\kappa_4 = \kappa_D$. For the 4-dimensional FRW equation without kinetic energy, $\frac{\ddot{a}}{a} = \frac{\dot{a}^2}{\sigma^2} = H^2$, we end up with

$$\left(3 + \frac{\beta}{\pi}\right) H^2 \leq -\int d\Omega^{D-4} \Omega^2 \sqrt{g} \frac{\ddot{\Omega}}{\Omega}, \quad (15)$$

where $\sqrt{g} \equiv \det \tilde{g}_{mn}$. Notice here the contribution from $S''_{\text{out}}$ is taking to be 4d entanglement entropy within radius R in the large non-compact spacetime.

Solve the Einstein equations as in [3], we now can derive something similar to the weak gravity conjecture:

$$\frac{|\nabla V|}{V} \geq \sqrt{\frac{2(D - 2)}{D - 4} \left(1 + \frac{\beta}{3\pi}\right)} = \lambda_{\text{QSEC}}. \quad (16)$$

When taking $\beta \to 0$, we recover the bound $\lambda_{\text{SEC}}$ from strong energy condition in (9).
B. Quantum Null Energy Condition

For the null energy condition, consider the $D$-dimensional metric in [3],

$$ds^2 = \Omega(y, t)^2(-dt^2 + a(t)dx_i^2) + \Omega(y, t)^{-\gamma} \tilde{g}_{mn} dy^m dy^n, \quad \gamma \equiv \frac{8}{D - 6}.\quad (17)$$

Consider the null vector $k^t k^t = \Omega^{-2}$ and $k^m k^n = \frac{1}{D-4} \Omega^\gamma \tilde{g}^{mn}$, we have similar derivation and arrive at,

$$T_{MN} \langle k^M k^N \rangle = R_{MN} \langle k^M k^N \rangle = R_{tt} + \frac{1}{D-4} \Omega^\gamma R_{mn}^{(D-4)} \tilde{g}^{mn} \geq S_{\text{out}}'.\quad (18)$$

Integrating over the compact manifold sphere and choosing $\kappa_4 = \kappa_D$, and $\ddot{a}/a = H^2$, we end up with

$$\left(3 + \frac{C\beta}{\pi}\right) H^2 \leq - \int dy^{D-4} \Omega^{2\gamma} \sqrt{\frac{D-2}{D-4}} \Omega.\quad (19)$$

with constant $C$ from integration of $1/\Omega^2$ over the compact manifold. Solve the Einstein equation for the $\Omega$, we now can derive something similar to the weak gravity conjecture,

$$\frac{\nabla V}{V} \geq \sqrt{\frac{2(D-4)}{D-2}} \left(1 + \frac{C\beta}{3\pi}\right) = \lambda_{\text{QNEC}}.\quad (20)$$

When $\beta \to 0$, we recover the bound from null energy condition in (10).

It is interesting to see that both the bounds $\lambda_{\text{QSEC}}$ and $\lambda_{\text{QNEC}}$ can be relaxed due to the quantum corrections, which might be comparable with $\lambda_{TCC}$ from (11) in the asymptotic region.

IV. DISCUSSIONS

In summary, we study the swampland and trans-Planckian censorship conjectures from the entropy bound with quantum corrections. Especially, we consider the typical contributions from the entanglement entropy in de Sitter spacetime and discuss the quantum corrections to both conjectures. It is interesting to consider the entanglement entropy contributions, in the process of understanding the swampland and trans-Planckian conjectures.

It is one step from classical condition to quantum interpretation, although the exact meaning supporting these bounds still needs further developments and we just naively incorporate the leading contributions here. It would also be interesting to discuss these effects in the framework of the compactification constructions, if possible[17]. For a few more discussions on the swampland conditions relevant to the entropy bound and energy conditions, see e.g. [18].

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[1] A. H. Guth, “The Inflationary Universe: A Possible Solution to the Horizon and Flatness Problems,” Phys. Rev. D 23, 347 (1981) [Adv. Ser. Astrophys. Cosmol. 3, 139 (1987)].
[2] D. H. Lyth and A. Riotto, “Particle physics models of inflation and the cosmological density perturbation,” Phys. Rept. 314, 1 (1999) [hep-ph/9807278].
[3] G. Obied, H. Ooguri, L. Spodyneiko and C. Vafa, “de Sitter Space and the Swampland,” arXiv:1806.08362 [hep-th].
[4] A. Bedroya and C. Vafa, “Trans-Planckian Censorship and the Swampland,” arXiv:1909.11063 [hep-th].
[5] A. Bedroya, R. Brandenberger, M. Loverde and C. Vafa, “Trans-Planckian Censorship and Inflationary Cosmology,” arXiv:1909.11106 [hep-th].
[6] S. Mizuno, S. Mukohyama, S. Pi and Y. L. Zhang, “Universal Upper Bound on the Inflationary Energy Scale from the Trans-Planckian Censorship Conjecture,” arXiv:1910.02979 [astro-ph.CO].
