Analysis of radial segregation of granular mixtures in a rotating drum

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Abstract. This paper considers the segregation of a granular mixture in a rotating drum. Extending a recent kinematic model for grain transport on sandpile surfaces to the case of rotating drums, an analysis is presented for radial segregation in the rolling regime, where a thin layer is avalanching down while the rest of the material follows rigid body rotation. We argue that segregation is driven not just by differences in the angle of repose of the species, as has been assumed in earlier investigations, but also by differences in the size and surface properties of the grains. The cases of grains differing only in size (slightly or widely) and only in surface properties are considered, and the predictions are in qualitative agreement with observations. The model yields results inconsistent with the assumptions for more general cases, and we speculate on how this may be corrected.

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1 Introduction

The production of many goods, ranging from pharmaceuticals and foods to polymers and semiconductors, depends on reliable uniform mixing of granular materials. Although there have been several recent advances, particulate mixing is poorly understood, and one cannot \textit{a priori} predict the effectiveness of any mixing process. Indeed, mixing operations often result in segregation or de-mixing, and even the parameters that control mixing and segregation are not fully understood.

Rotating drums, or kilns, are employed in industry to carry out a range of operations; some examples are the calcination of limestone, reduction of oxide ore, clinkering of cementitious materials, waste incineration and calcination of petroleum coke. Owing to its industrial importance the rotary kiln has been the subject of numerous investigations. Significant im-
Fig. 1. Streamlines in the active and passive layers in the rolling regime of bed motion in a rotating drum. Grains cascade rapidly down the active layer, which is usually very thin in comparison with the passive layer.

Improvement in kiln performance may be achieved by better understanding grain transport during operation.

In a rotating drum several regimes of bed motion have been identified, namely, slipping, slumping, rolling, cascading, cataracting and centrifuging [2]. The most desirable bed motion for many industrial operations is the rolling mode, as it helps in promoting good mixing of the particles along with rapid renewal of the exposed material. The rolling regime is characterized by two distinct regions: the “active” and “passive” layers [3]. Grains in the passive layer execute rigid body rotation along with the drum and the streamlines are circular (see figure 1). The active layer is usually very thin in comparison with the extent of the passive layer. Grains roll down rapidly in the active layer and streamlines have been observed to be straight [3].

While rotating drums are used to mix particles of varying sizes and shapes to obtain a homogeneous mixture necessary for certain industrial processes, there is considerable evidence of segregation when the charge is a granular mixture of different properties. A mixture of grains differing either in size and/or roughness [1]-[4], or density [5] when rotated in a drum is seen to undergo radial segregation, characterized by the formation of a core of smaller or rougher grains. This radially segregated core is seen to evolve into alternate bands of larger (or smoother) and smaller (or rougher) grains along the length of the drum [6,7]. Axial segregation is found to occur over a considerably larger time scale than radial segregation, which is usually complete within a few drum rotations. In a rotary kiln, radial segregation could lead to poor contact between the gas flowing above and the particles in the core, which would result in poor heat transfer and/or lower rate of reaction. Axial segregation, would lead to products of fluctuating qualities.

Analysis of granular segregation has been undertaken in some recent studies via discrete element simulations [8], and using coarse-grained continuum models [8]-[10]. A recent approach, which is the one followed in this work, is based on the theoretical formalism of Bouchaud, Cates, Ravi Prakash and Edwards [11]. Henceforth referred to as BCRE in this work, this study models grain transport on a sandpile surface, and has been employed in [12] for studying segregation during the filling of a silo with a mixture of two species. The “minimal” model of [12] describes the case of grains with equal sizes but with small differences in surface properties. The same formalism was used in [13] to demonstrate the possibility of complete segregation in a mixture of large smooth grains and small rough grains, and spontaneous stratification, i.e. alternating layers of the two species, for a mixture of large rough grains and small
smooth grains. The minimal model of [12] was generalised by [14] to accommodate grains differing slightly in surface properties as well as size.

In this work, we extend the formalism of [14] to address the problem of segregation in rotating drums of a granular mixture whose constituents differ in size and surface properties. The problem of segregation in rotating drums has been addressed in [15] for some special cases; however, our approach differs from theirs in some fundamental ways, which we will point out in this paper.

## 2 The model

Following BCRE, we assume the existence of a sharp interface between the active and passive layers. Grains in the active layer are referred to as rolling grains and those in the passive layer as immobile. Collisions between rolling and immobile grains occur at the interface, which lead to exchange of grains between the two regions. We are interested in studying the case of a bimodal grains in the passive layer at the interface $\phi_\alpha(x,t)$ and $\phi_\beta(x,t)$. The surface of the active layer is inclined at an angle $\theta_r$, the dynamic angle of repose, with the horizontal.

The conservation equations for the two species in the active layer are

$$\partial_t R_\alpha(x,t) = -\partial_x (v(x,t)R_\alpha(x,t)) + \Gamma_\alpha$$

$$\partial_t R_\beta(x,t) = -\partial_x (v(x,t)R_\beta(x,t)) + \Gamma_\beta$$

where $v(x,t)$ and $D$ are the mean velocity and the diffusivity of the grains, respectively. The active layer is assumed to be very thin, and variations across it are not resolved in this model. All the interesting physics in this model is buried in $\Gamma_\alpha$ and $\Gamma_\beta$, which are the rate of conversion of immobile grains of each type into rolling grains by collisions at the interface, and we will specify their functional form shortly.

In the passive layer, grain transport is only by advection due to the rotation of the drum. The equation for conservation of $\alpha$ type grains, integrated across the passive layer, is

$$\rho_\alpha \partial_t \int_0^h \phi^\alpha_\alpha(x,y,t) dy = \phi_\alpha \rho_\alpha v_\theta \cdot n - \Gamma_\alpha,$$

and similarly for $\beta$. Here, $\phi^\alpha_\alpha(x,y,t)$ is the number fraction of $\alpha$ type grains inside the passive layer, $v_\theta$ is the velocity field due to solid body rotation, $\rho_\alpha$ is the number density of grains in the passive layer (taken to be constant), and $n$ is the unit normal to the interface, pointing from the passive layer to the active. (Due to the circular streamlines in the passive layer $\phi^\beta_\alpha$ can be related to the number fraction at the interface $\phi_\alpha$.)
conservation for all grains then assumes the form

\[ \rho_n \partial_t h = \rho_n v_\theta \cdot n - \sum_{i=\alpha,\beta} \Gamma_i \]  

\hspace{1cm} \text{(4)}

\[ 2.1 \text{ Model for } \Gamma_i \]

As in BCRE, we write an expression for \( \Gamma_\alpha \) in terms of the rolling grain densities \( R_i \) and the number fraction of immobile grains \( \phi_i \) at the interface. Boutreux and de Gennes (1996) consider the four possible outcomes of binary collisions between rolling and immobile grains at the interface; amplification, capture, exchange and recoil. Amplification of species \( \alpha \) refers to the event of an \( \alpha \) type immobile grain being dislodged into the active layer by a rolling grain. This could happen either by auto-amplification, when the colliding grain is also of type \( \alpha \), or by cross-amplification, when it is of type \( \beta \). The auto-amplification (cross-amplification) rate of \( \alpha \) type species will increase with \( R_\alpha (R_\beta), \phi_\alpha \) and the collision frequency \( f \). Capture of species \( \alpha \) refers to the event of an \( \alpha \) type rolling grain being rendered immobile and absorbed by the passive layer. Arguing similarly for the rate of capture (and noting that exchange and recoil leave the net transfer of grains unchanged), we write the simplest form for \( \Gamma_\alpha \) as

\[ \Gamma_\alpha = \gamma_{\alpha \alpha} \frac{v}{d_p} R_\alpha \phi_\alpha g^a(\psi) + \gamma_{\alpha \beta} \frac{v}{d_p} R_\beta \phi_\alpha g^a(\psi) - \gamma_{\alpha} \frac{v}{d_p} R_\alpha g^c(\psi) \]  

\hspace{1cm} \text{(5)}

wherein we have taken the collision frequency, \( f \), to vary as \( \frac{v}{d_p} \), \( d_p \) being the average grain diameter. The coefficients \( \gamma_{\alpha \alpha}, \gamma_{\alpha \beta} \) and \( \gamma_{\alpha} \) are constants of order unity and \( g^a(\psi) \) and \( g^c(\psi) \) functions of the slope of the interface.

We note that unlike in previous studies investigating granular segregation [12,14,15], we do not set the velocity \( v \) to a constant, but allow it to vary along the length of the active layer by solving the momentum balance in conjunction with the species balances [16].

To complete the formulation, the functional forms of \( g^a(\psi) \) and \( g^c(\psi) \) must be specified. Recognizing that the rates of amplification and capture depend not just on the local slope of the interface but on the difference between the local slope and the angle of repose, we set \( \psi = \theta(x) - \theta_r(x) \), where \( \theta(x) \) is local slope of the interface and \( \theta_r(x) \) is the angle of repose for the mixture. Next, \( \Gamma^a_\alpha \) (\( \Gamma^c_\alpha \)) should rise (diminish) with \( \psi \), but we do not expect it to vanish altogether at \( \psi = 0 \), since we expect a fraction of the grains striking a surface at the angle of repose to dislodge other grains (get captured). However, for large negative (positive) values of \( \psi \), the rate of amplification (capture) should become negligibly small. In accordance with these ar-
arguments, we assume the following continuous forms:

\[ g^a(\psi) = \frac{A \exp(B\psi)}{1 + \exp(B\psi)} \]  
\[ g^c(\psi) = \frac{A}{1 + \exp(B\psi)} \]

where \( A \) and \( B \) are constants of \( O(d_p L) \) and \( O(L d_p) \), respectively.

The notable feature of the above forms is that \( g^a + g^c \) is constant - while we are unable to ascribe a physical meaning to this constraint now, it is essential if the solutions are to be symmetric about \( x = 0 \). The issue of symmetry is commented on in greater detail in section 3.4.

We note here that Makse [15] considered special cases of the above slope functions for very small and very large differences of angles of repose. He also assumed that differences in size and surface properties between the species result only in differences in their angles of repose, which is the ultimate cause of segregation, and that the coefficients for auto-amplification and cross-amplification (\( \gamma_{\alpha\alpha} \) and \( \gamma_{\alpha\beta} \)) are equal. We believe that ascribing segregation purely to a difference in the angle of repose is unrealistic; it surely does not address segregation of grains of different sizes, but made of the same material. In this case, segregation arises because the probability of a larger grain dislodging a smaller one is greater than that of the larger one dislodging another larger one, as we shall demonstrate below.

We propose that segregation due to size and surface properties are driven primarily by amplification and capture respectively, as elaborated in section 3.1. This has been done by allowing (unlike [15]) the coefficients \( \gamma_{\alpha\alpha}, \gamma_{\alpha\beta}, \gamma_{\alpha} \) etc. to depend on the size, roughness and type of interacting grains. In addition, the effect of differences of angle of repose caused by differences on size or surface properties has been incorporated through the slope functions (eq. 6). Our results, shown in sections 3.1 – 3.3 illustrate quite clearly that the qualitative nature of size and roughness segregation is different, which is in agreement with previous experimental observations [2] – [5]. We allow the the angle of repose to depend on the grain concentration in our model, but assume it to be constant in this paper for the sake of simplicity.

### 2.2 Momentum Balance

Previous studies that used the same theoretical framework made the unrealistic assumption that the velocity of grains in the active is constant – clearly, the velocity must start with zero at the top end of the active layer, reach a maximum at an intermediate point, and again vanish at the lower end of the active layer.

We allow the velocity to vary along the length of the layer, and write a momentum balance for the rolling grains, averaged over the thickness of the active layer:

\[
\partial_t (Rv) + \partial_x (Rv^2) = v \sum_{\epsilon=\alpha,\beta} \Gamma_{\epsilon} + R g \sin \theta_r - \sigma_f - \sigma_c.
\]

The first term in the right hand side accounts for momentum influx due to conversion of immobile grains into rolling grains, the second is the momentum generated due to the gravitational body force, and the third and fourth terms are the shear stresses at the interface due to frictional grain interactions at the interface and grain collisions [16], respectively. We assume a Coulombic form for the frictional stress,

\[
\sigma_f = \mu R g \cos \theta_r,
\]

where \( \mu \) is the coefficient of friction of the material. The collisional shear stress, \( \sigma_c \), is generated by collisions of the ex-
change and recoil type, discussed earlier, that do not lead to inter-conversion of grains but do result in loss of momentum. Therefore, we choose a constitutive form of $\sigma_c$ that is similar to the momentum flux due to inter-conversion,

$$\sigma_c = \lambda R \frac{v^2}{d_p},$$

where $\lambda$, like $A$, is a constant of proportionality of $O(\frac{d_p}{L})$.

### 2.3 Scaling and Leading Order Analysis

To clearly illustrate the relative magnitude of the different terms in our model equations, we define the following dimensionless variables,

$$x^* = \frac{x}{L}, \quad v^* = \frac{v}{\omega L}, \quad \delta^* = \frac{\delta}{d_p}, \quad R^*_\alpha = \frac{R_\alpha}{\rho_\alpha L}, \quad h^* = \frac{h}{L}$$

and the following dimensionless parameters,

$$\epsilon = \frac{d_p}{L}, \quad r^* = \frac{r}{L}, \quad D^* = \frac{D}{\omega L d_p}, \quad g^* = \frac{g}{\omega^2 L}, \quad \lambda^* = \frac{L \lambda}{d_p}$$

where $L$ is the half-length of the free-surface, $\omega$ is the rotational velocity of the drum, $d_p$ is the average grain size, $r$ is the radius of the drum and $\rho_\alpha$ is the number of grains per unit area of the passive layer.

The active layer is expected to be only a few grains deep, and its thickness $\delta$ is therefore scaled with $d_p$. The diffusion coefficient, $D$, being a material property scales as the product of the velocity and the grain size. Using the above normalized variables, and dropping the asterisks for the sake of simplicity, (11) transforms to the following dimensionless form at steady state:

$$\frac{d(v R)}{dx} = \sum_{i=\alpha,\beta} \Gamma_i,$$  

$$\frac{d(v R_\alpha)}{dx} = \Gamma_\alpha,$$  

$$-x = \sum_{i=\alpha,\beta} \Gamma_i,$$  

$$-\phi_\alpha x = \Gamma_\alpha,$$  

with

$$R_\alpha + R_\beta \equiv R,$$  

$$\phi_\alpha + \phi_\beta \equiv 1.$$
An additional condition is required for solving for $\phi_\alpha$, and this is the loading condition specifying the total amount of material loaded into the drum,

$$\int_{-L}^{L} \int_{y_1}^{y_2} \phi_\alpha^p dy dx = V \Phi_\alpha,$$

where $V$ and $\Phi_\alpha$ are the total volume and number fraction of $\alpha$ type grains in the entire charge, respectively. Since $\phi_\alpha$ is constant along each circular streamline in the passive layer, the above condition reduces at leading order to the following dimensionless form:

$$\int_{-1}^{1} \phi_\alpha(x) x dx = \Phi_\alpha.$$

### 3 Model Predictions

Substituting (14) in (12), and applying the boundary condition $vR = 0$ at $x = -1$, we get

$$vR = \frac{(1 - x^2)}{2}.$$

Substituting (20) in (16), and solving for the velocity subject to the boundary condition $v = 0$ at $x = -1$, we get

$$v = \sqrt{\frac{C}{\lambda} \left(1 - \exp[-\lambda(1 + x)]\right)},$$

where

$$C = g \left[\sin \theta_r - \mu \cos \theta_r\right].$$

The above velocity profile does not vanish at the lower end of the active layer, $y = 1$; this boundary condition may be enforced by putting in a bulk viscosity for the flowing medium. The correction to the velocity would be of higher order in $\epsilon$ everywhere, except in a thin boundary layer near $y = 1$. Also notable is that while the velocity is not symmetric about $x = 0$, the flux $vR$ is.

Substituting (20) and (15) in (13) and simplifying yields

$$\frac{d u_\alpha}{d x} = \frac{(u_\alpha - \phi_\alpha)x}{R v},$$

where $u_\alpha \equiv R_\alpha/R$ is the number fraction of $\alpha$ grains in the active layer. Now (14), and (15) can be used to express $\phi_\alpha(x)$ in terms of $u_\alpha$ and obtain the solution of (22). The constant of integration for this solution must be evaluated by enforcing the loading condition given by (19).

We consider various cases where the grains of the two species differ either in size or in surface properties. The mean number fraction $\Phi_\alpha$ has been set to 0.5 in all the cases.

#### 3.1 Grains differing only in surface properties

It is reasonable to suppose that amplification is largely guided by differences in sizes while capture is driven by differences in surface properties. Then for this case, we may set

$$\gamma_\alpha^a = \gamma_\alpha^a = \gamma^a_\beta = \gamma^a_\beta = \gamma^a.$$

The differences in surface properties would, of course, lead to the capture coefficients being different, and we set

$$\gamma_\beta^c = \gamma^c, \gamma_\alpha^c = s \gamma^c.$$

A value of $s > 1$ implies that $\alpha$ type grains are rougher than $\beta$, and hence more easily captured by the passive layer. With these simplifications, it follows that the solution of (13)–(15) is:

$$\phi_\alpha = \frac{s u_\alpha}{1 + (s - 1) u_\alpha} \quad \text{and} \quad \frac{u_\alpha}{(1 - u_\alpha)} = C_1 (1 - x^2)^{s-1}\left(23\right)$$

where $C_1$, the constant of integration is obtained by numerical integration of (19). Figures 3 and 4 present the variation of the
number fraction of $\alpha$ type grains in the active and the passive layers, respectively, along the length of the interface for two cases, one where the $\alpha$ type grains are slightly rougher than the $\beta$ type grains ($s = 1.5$), and another where the $\alpha$ type grains are much rougher ($s = 10$). The model predicts depletion of the rougher grains at the two ends of the drum and accumulation near the center of the drum. The degree of segregation, of course, depends upon the difference between the surface properties of the two species, quantified in this model by $s$.

### 3.2 Grains differing slightly in size

Following our arguments in §3.1, we will assume that the difference in size leads to a difference in the rate of amplification of the two species but not that of capture. We investigate the case where grains have identical surface properties but differ slightly in size. For the case of grains differing neither in size nor surface properties (i.e. identical species), the solution is, of course, that of constant number fraction in the active and passive layers,

$$u_\alpha(x) = \phi_\alpha(x) = \Phi_\alpha,$$  \hspace{1cm} (24)

where $\Phi_\alpha$ is the mean number fraction of $\alpha$ type grains during filling of the drum. For a mixture comprising species that differ very slightly in size, we allow the amplification constants for the two species to differ by a small amount $\vartheta$. The probability of a large rolling grain dislodging a large immobile grain is equal to that of a small rolling grain dislodging a small immobile grain. Hence,

$$\gamma_{\alpha\alpha} = \gamma_{\beta\beta} = \gamma^\alpha,$$  \hspace{1cm} (25)

However, if the $\beta$ type grains are larger, the probability of a $\beta$ type grain dislodging an $\alpha$ type grain is greater than that of it dislodging one of its kind, and the probability that an $\alpha$ type grain dislodges a $\beta$ type grain is less than that of it dislodging.
one of its kind. Hence, for small differences in sizes, we set

\[ \gamma^a_{\alpha \beta} = \gamma^a (1 + \vartheta), \]  
\[ \gamma^a_{\beta \alpha} = \gamma^a (1 - \vartheta). \]  

We now seek the solution as a perturbation to the uniform solution (24) to leading order in \( \vartheta \),

\[ u_\alpha(x) = \Phi_\alpha + \vartheta u'_\alpha(x), \quad \phi_\alpha(x) = \Phi_\alpha + \vartheta \phi'_\alpha(x) \]  

and solve for \( u'_\alpha(x) \) and \( \phi'_\alpha(x) \). Substituting (25) – (28) in (14) – (15), we can express \( \phi'_\alpha(x) \) in terms of \( u'_\alpha(x) \) and \( x \) as

\[ \phi'_\alpha(x) = u'_\alpha(x) - \gamma^c \Phi_\alpha \Phi_\beta (\frac{\gamma^c R v - x}{\gamma^c R v + \gamma^a x}) \frac{x}{R v}. \]  

Using (28), we can write (22) as:

\[ \frac{d u'_\alpha}{dx} = (u'_\alpha - \phi'_\alpha) x \frac{1}{R v}. \]  

We now consider special cases for which simple analytical solutions of (30) are possible. The first is the case of the grains of both species being very smooth, \( i.e. \gamma^c \gg 1 \). The solution is

\[ u_\alpha(x) = \Phi_\alpha + \vartheta \frac{\gamma^c}{\gamma^a} \Phi_\alpha \Phi_\beta \ln(1 - x^2), \]  
\[ \phi_\alpha(x) = \Phi_\alpha + \vartheta \frac{\gamma^c}{\gamma^a} \Phi_\alpha \Phi_\beta [1 + \ln(1 - x^2)]. \]  

Figures 5 and 6 present these results for \( \vartheta = 0.1 \), where it is clear that the central core is richer in smaller grains, in agreement with experimental observations.

The second case is when both species are very rough, \( i.e. \gamma^c \ll 1 \). The solution is

\[ u_\alpha(x) = \Phi_\alpha - \vartheta \Phi_\alpha \Phi_\beta \ln(1 - x^2), \]  
\[ \phi_\alpha(x) = \Phi_\alpha - \vartheta \Phi_\alpha \Phi_\beta [1 + \ln(1 - x^2)]. \]  

Figures 7 and 8 present the profiles of \( u_\alpha \) and \( \phi_\alpha \) for \( \vartheta = 0.1 \), where it is apparent that, unlike in the case of smooth grains,
that size segregation is then due to percolation or “kinematic sieving”, as the smaller rolling grains tend to fall through the gaps between the large grains, forming a sub-layer above the interface. The larger rolling grains are therefore not in contact with the bulk, and are not captured to the extent they would have been if in contact. To account for this phenomenon, we follow \cite{14} and modify $\Gamma_\alpha$ by a factor $\exp(p u_\alpha)$, which mimics the screening of the large grains from the interface by the sub-layer of the small grains. The dimensionless parameter $p$, which is proportional to the size ratio of $\beta$ to $\alpha$ type grains, measures the degree of percolation.

For a large difference in sizes, segregation due to percolation effect would far exceed the segregation effects due to differences in amplification and capture coefficients, as suggested by \cite{14}. Hence we assume,

\begin{equation}
\gamma_{a\alpha} = \gamma_{a\beta} = \gamma_{\beta\alpha} = \gamma_{\beta\beta} = \gamma_a. \tag{35}
\end{equation}

and

\begin{equation}
\gamma_c^a = \gamma_c^\beta = \gamma_c. \tag{36}
\end{equation}

Using (35) and (36), we solve (14) - (15) to obtain:

\begin{equation}
\phi_\alpha(x) = \frac{u_\alpha(x)}{u_\alpha(x) + (1 - u_\alpha(x)) \exp[p u_\alpha(x)]}, \tag{37}
\end{equation}

and solve (22) numerically. The results shown in figures 9 and 10 for $p = 2$ indicate that the larger grains are heavily depleted in the central region of the drum and accumulate at the ends both in the active and passive layers. This is in qualitative agreement with experimental observations \cite{7}.

\section{3.3 Grains differing widely in size}

For large differences in sizes between the two species, experimental observations in sandpiles reveal either complete segregation or stratification; it has been reported \cite{17} that this occurs when the size ratio exceeds 1.5. Boutreux \textit{et al.} \cite{14} proposed...
Fig. 9. Profiles of the number fraction of $\alpha$ (smaller) and $\beta$ (larger) type grains in the active layer for the case of grains differing widely in size. See text for parameter values.

Fig. 10. Profiles of the number fraction of $\alpha$ (smaller) and $\beta$ (larger) type grains in the passive layer at the interface for the case as in Fig. 9.

3.4 Other cases: Model inconsistency

The solutions described in the previous section are symmetric about the mid-plane $x = 0$. However this is not a generic result, rather only for the special cases we have considered, and also because the slope functions have been chosen to satisfy $g^a + g^c = \text{constant}$. If the grains differ in size and shape, or if the slope functions do not satisfy the above constraint, the solutions for $R_i$ and $\phi_i$ are not symmetric. This result is inconsistent with our assumption that the passive layer executes solid body rotation: circular streamlines in the passive layer necessitates symmetry of $R_i$ and $\alpha_i$

It thus appears that the assumption of solid body rotation in the passive layer must be relaxed for consistency, and solutions in general need not be symmetric about the mid-plane. This may be accomplished by postulating an interface of finite thickness within which streamlines are not circular, which is the approach we are following currently and hope to report the results soon. The recent study of Shinbrot et al. [18] reports the existence of an interface between the active and the passive layers, wherein deformation occurs by periodic stick-slip motion. In this light, further experimentation is necessary to ascertain whether the concentration of species in the active and passive layers are symmetric.

4 Conclusions

Granular mixtures, consisting of grains that differ in size and/or surface properties, have been observed to undergo segregation when they are either poured in a heap or rotated in a drum. Several analytical models have been proposed to predict segregation in case of a two dimensional sand-pile. The theoretical formalism proposed by Bouchaud et al. [11] and later extended by [12,13,14], successfully addresses the problem of segregation in poured heaps. The present work attempts to extend this framework to the rotating drum problem.

We argue that the qualitative nature of segregation of two mixtures, one of which differ in surface properties and the other
in size, may be quite different even if their angles of repose are equal. Our model recognizes collisional interactions between grains at the interface as the cause for segregation. Such interactions are significantly influenced by the size of grains and their surface properties, apart from the angle of repose. This provides a canonical framework under which segregation of grains differing in size and shape could be studied in various geometries. The model gives satisfactory predictions for grains differing either in surface properties or size. These results are in good qualitative agreement with experimental observations. For the case of segregation due to small differences in size of grains when both the species are very rough, the prediction of our model is counter-intuitive - such a system has not yet been studied experimentally, and this work provides a motivation for it.

Our predictions of symmetric profiles of the species concentrations in the active and passive layers are only for particular cases of grain properties. This is not always the case, and the solution is in general not symmetric; this is an inconsistency in the model as it is contrary to the assumption of circular streamlines in the passive layer. We believe that the inconsistency arises from the assumption of a sharp interface between the active and passive layers, and are currently working to incorporate a finite sized interface in which streamlines deviate from that of rigid body motion. This will result in asymmetric profiles of the species concentration. While recent experiments suggest asymmetry in the active layer thickness, more experimental studies are necessary to resolve this issue.

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