There is the scale problem in the Randall-Sundrum models. Regarding this, I review the works done with B. S. Kyae and H. M. Lee. In a supersymmetric generalization in the RSI model, we discuss that the $\mu$ parameter can be obtained by an intermediate scale brane. In addition, we also discuss the cosmological constant problem with a self-tuning solution in the RSII model.

The standard model has been extremely successful phenomenologically. However, it has 20 theoretically unexplained parameters, among which the Higgs scalar mass is called the gauge hierarchy problem: Why is the electroweak scale $10^{-16}$ times the Planck mass? To solve this hierarchy problem, technicolor and supersymmetry have been extensively studied in the last twenty years.

Recently, there appeared another try toward understanding this gauge hierarchy problem, through large extra dimensions or through warp factor geometry in the so-called RSI model. There are two branes located at $y = 0$ (B1 brane) and $\pi$ (B2 brane) where $y$ is the fifth coordinate. The B1 brane tension $\Lambda_1 = 6k_1 M^3 > 0$, B2 brane tension $\Lambda_2 = 6k_2 M^3 < 0$ and the bulk cosmological constant $\Lambda_b = -6k_2 M^3 < 0$ are fine-tuned $k_1 = k = -k_2$ to have a flat geometry, where $M$ is the fundamental mass parameter in 5D. With this kind of two fine-tunings the flat-space solution is possible even if $R^2$ terms are added if their form is of the Gauss-Bonnet type.

The RSI model imposes the symmetry $S_1/Z_2$ for compactification, with the flat space metric ansatz, $ds^2 = e^{-2\sigma(y)} dx_\mu dx^\mu + r^2 dy^2$, which allows the solution $\sigma = kr_c|y|$. Integrating over $y$, we obtain an effective 4D Planck mass, $M_P = (M^3/k)(1 - e^{-kr_c})$ which is again of order $M$. This fundamental mass parameter governs the mass scale at $y = 0$. On the other hand, at $B2 (y \neq 0)$ the rescaling of the fields so that the standard kinetic energy terms result gives the mass parameter of order $m = M_{\text{input}} e^{-kr_c/2}$ which can be interpreted as a TeV scale if $r_c$ is a few tens of $M^{-1}$. Thus, interpreting B2 as the brane for housing the visible sector fields, we obtain a long anticipated exponentially suppressed electroweak mass scale compared to the Planck mass. This led to a stimulus since it can be another solution for the gauge hierarchy problem. Note that here the key point is that a warp factor is introduced, i.e. a caved space in $y$ direction even if 4D is flat.

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However, this interesting gauge hierarchy solution has a different kind of scale problem. At B2 the ultimate mass scale is TeV. Thus, the gauge coupling unification at GUT scale is not possible, which was shown to be so remarkable in \( SO(10) \) by Raby. Unification may be achieved above TeV scale by host of KK modes (which seems to be a fitting rather than prediction) but then the proton decay operator has the relevant mass parameter of order TeV which makes proton lifetime tens of orders shorter than the present bound. In addition, the TeV scale does not introduce the needed very light axion for the strong CP solution. The mm scale gravity can introduce a very light axion, but it needs a several internal dimensions not achievable in 5D RSI model. Also, the transition from inflationary phase to the Big Bang phase needs the visible sector brane tension positive which is not the case in RSI model. Thus, it seems that RSI model, designed for the gauge hierarchy solution, introduces another kind of the mass scale problem.

In this talk, I present two possible solutions for the mass scale in the Randall-Sundrum models. One is the \( \mu \) parameter which we try to understand in the RSI model, and the other is the cosmological constant which we try to understand in the RSII model.

The \( \mu \) in RSI model

Because of the scale problems we encounter in the RSI model, we do not introduce a TeV scale brane. Then the original beautiful motivation for the gauge hierarchy is lost. So, the gauge hierarchy is understood by the conventional supergravity theory. Then, there exists another scale problem: the \( \mu \) problem. In this case, we put visible sector fields at the Planck brane(B1). The second brane(B2) is interpreted as an intermediate scale brane. There is no serious cosmological problem since the brane tension at B1 is positive. A logarithmic unification of gauge couplings is possible because the visible brane is the Planck scale brane. Since the KK modes are massive at the intermediate scale, the low energy physics is very similar to the MSSM. Strong CP solution is possible by introducing a very light axion.

We introduce the chiral fields at B1, bulk and B2 as shown in Table 1, where \( A \) is the global axial charge, corresponding to the Peccei-Quinn charge. We have not specified the graviton multiplet in the bulk. The MSSM gauge multiplets live at B1.

If N=1 supersymmetry is present at B1, we can write a superpotential at B1 as

\[
W \sim \frac{\Sigma^2}{M_P}H_1H_2,
\]

which is a schematic formula. The exact meaning will be given after what are the zero mass chiral fields at low energy. If bulk and brane fields respect the N=1 supersymmetry, then at B2 we anticipate a superpotential

\[
W' \sim Z(\Sigma S - M_P^2),
\]
Table 1. The B1, bulk and B2 fields and their global charges

| Brane or bulk | B1 | Bulk | B2 |
|---------------|----|------|----|
| Fields        | $H_1$ | $H_2$ | MSSM | $\Sigma(i = 1, 2)$ | $S$ | $Z$ |
| $A$           | $-1$ | $-1$ | $+\frac{1}{2}$ | $+1$ | $-1$ | $0$ |

These fields located at $y \neq 0$ brane are required to couple supersymmetrically to $\Sigma^i$.

which is again a schematic formula. But because B2 is the intermediate scale brane, the VEV of $\Sigma$ would be an intermediate scale, and in view of Eq. (1) we expect a TeV scale $\mu$.

The 5D bulk action is

$$S_{bulk} = -\Sigma_i \int d^5 x \sqrt{-G} [g^{MN} \partial_M \Phi^i \partial_N \Phi^i + (\bar{\Psi} \gamma^M \nabla_M \Psi)^2 + M_\Psi \bar{\Psi} \Psi_R + M_\Psi \bar{\Psi} \Psi_L]$$  \hspace{1cm} (3)

where $\gamma^M = e^M_a \gamma^a$, $\nabla_M = \partial_M + \Gamma_M$, $\Gamma_\mu = \frac{1}{2} \gamma_5 \gamma_\mu \frac{d}{dy}$, $\Gamma_5 = 0$, which satisfies the N=2 supersymmetry \cite{13}. In the $AdS_5$ the fields in the same hypermultiplet must have different masses with one undetermined parameter $t$,

$$M_{\Phi_1}^2 = (t^2 + t - \frac{15}{4}) \sigma^2 + (\frac{3}{2} - t) \sigma''$$  \hspace{1cm} (4)

$$M_{\Phi_2}^2 = (t^2 - t - \frac{15}{4}) \sigma^2 + (\frac{3}{2} + t) \sigma''$$

$$M_\Psi = t \sigma'$$

where it is obvious that the fermion mass is odd under the $Z_2$ parity and the boson masses are even under the $Z_2$ parity.

The 5D field equation for the hypermultiplet is

$$\left[ e^{2\sigma} \eta^{\mu\nu} \partial_\mu \partial_\nu + e^{s\sigma} \frac{\partial}{\partial y} e^{-s\sigma} \frac{\partial}{\partial y} - M_\Phi^2 \right] \Phi(x^\mu, y) = 0$$  \hspace{1cm} (5)

where $s = 4$ for a scalar $\phi$ and $s = 1$ for the fermion $\psi_{L,R}$. Thus, $M_{\Phi_1}^2 = a k^2 + b \sigma''$ and $M_{\Phi_2}^2 = k^2 (t \pm 1) = t \sigma''$ for $\Psi_{L,R}$. The KK mode decomposition

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{2b_0 y_c}} \sum_{n=0}^{\infty} \Phi^{(n)}(x^\mu) f_n(y)$$  \hspace{1cm} (6)

gives a solution for $f_n(y)$ and hence the KK masses. For $n \geq 1$, we obtain massive KK modes. The odd fields do have massless modes due to the orbifold condition. It is easy to check that massless condition $M^2 = e^{s\sigma} \frac{\partial}{\partial y} e^{-s\sigma} \frac{\partial}{\partial y}$ gives the following pair of the massless modes from $Z_2$ even field,

$$\Phi^{1,(0)}(x, y) = \frac{e^{(\frac{3}{2} - t)\sigma(y)}}{\sqrt{2b_0 y_c N}} \phi(x) \Sigma; \quad \Psi^{(0)}_L(x, y) = \frac{e^{(\frac{3}{2} - t)\sigma(y)}}{\sqrt{2b_0 y_c N}} \psi_{2}(x)$$  \hspace{1cm} (7)
where $N^2 = (e^{c_6(1-2t)} - 1)/\sigma_c(1 - 2t)$ with $\sigma_c = kb_0 y_c$. Note that $N(t = 1/2) = 1$. One can explicitly show that the above $n = 0$ KK modes are massless for any $t$. Let us call them $\phi_\Sigma, \psi_\Sigma$, respectively. These massless bulk fields couple to the B2 brane fields $S$ and $Z$. Here comes the fixing of $t$. Only for $t = 1/2$ the couplings at B2 are maintained to be supersymmetric.

Thus, we just show the result for $t = 1/2$. The B2 fields with a proper normalization are called tilde fields. Thus, the massless bulk $\Sigma$ field becomes $\tilde{\Sigma}$ at B2

$$\tilde{\Sigma} = \{ e^{\sigma(y)} \tilde{\phi}_\Sigma(x), e^{\frac{3}{2}\sigma(y)} \tilde{\psi}_\Sigma(x) \} \rightarrow \tilde{\Sigma}(x) = \{ \tilde{\phi}_\Sigma(x), \tilde{\psi}_\Sigma(x) \}. \quad (8)$$

At B2, $\phi_i \sim e^{\sigma_c} \phi_i$ and $\psi_i \sim e^{3\sigma_c/2} \psi_i$ couple to brane fields $S = \{ \phi_S, \psi_S \}$ and $Z = \{ \phi_Z, \psi_Z \}$. For example, the following couplings result,

$$S_{int}^{B2} = \int d^4x \sqrt{-g_4} \frac{1}{2} \left[ \{ (e^{\sigma_c} \tilde{\phi}_\Sigma) \psi_S \psi_Z + \phi_S \psi_Z (e^{3\sigma_c/2} \tilde{\psi}_\Sigma) + \phi_Z (e^{3\sigma_c/2} \tilde{\phi}_\Sigma) \psi_S - h.c. \} - |\phi_S \phi_Z|^2 - |\phi_Z (e^{\sigma_c} \tilde{\phi}_\Sigma)|^2 - |(e^{\sigma_c} \tilde{\phi}_\Sigma)\phi_S|^2 + M_2^2 ( (e^{\sigma_c} \tilde{\phi}_\Sigma) \phi_S + h.c. ) - M_4 \right]

+ h.c. \right]

where $M_I = M_P e^{-\sigma_c} \sim 10^{11-13}$ GeV for $\sigma_c \simeq 11.5 - 16$. This interaction is obtained from a superpotential $W = \bar{Z}(\Sigma \tilde{S} - M_f^2)$. This kind of supersymmetric interaction is possible for $t = 1/2$. $W$ guarantees the VEV of $\Sigma$ and $S$ is of order $M_I$ when we consider the soft terms generated by supergravity. Therefore, the Peccei-Quinn symmetry is broken at the intermediate scale and there results a very light axion. At B1, the $\mu$ term is generated at the order

$$\mu = \frac{\tilde{\Sigma}^2}{M_P} \sim m_{3/2}. \quad (10)$$

We have an $N = 1$ supersymmetric theory, with the $\mu$ term generated at B2 via the PQ symmetry breaking at the intermediate scale $M_I$. Since the SUSY breaking scale in SUGRA is similar to the PQ symmetry breaking scale, it is desirable to break supersymmetry at B2 since the natural scale at B2 is $M_I$. It can be achieved by the gaugino condensation at B2, presumably $E_8$ gaugino condensation. With the fundamental scale $M$, the condensation scale is very close to $M$ since the $\beta$ function of $E_8$ is large and negative. Thus, at B2 the condensation scale of $E_8$ is of order $M_I$, and hence breaking supersymmetry at order $M_I$. Thus, the gravitino mass $m_{3/2}$ is of order electroweak scale. One can imagine that the soft mass generation of a sfermion at B1 proceeds through its coupling to gravitinos at B1, the gravitino propagation in the bulk and the gravitino mass generation at B2. However, it is more reliable to integrate an effective action with respect to $y$ to obtain the soft mass. In this way, one obtains the soft masses of the MSSM fields at B1 at order TeV.
A common supersymmetry breaking scale and the axion scale is an intriguing hypothesis, proposed several times with different contexts. Here, we achieved it by introducing an intermediate scale brane B2.

**Self-tuning solution of the cosmological constant problem**

The cosmological constant problem can be addressed from early 1920’s. But it became the modern particle physics problem since the spontaneous symmetry breaking in particle physics was accepted as the mass generation mechanism. The VEV of potential or $-\mathcal{L}$ is interpreted as the cosmological constant, $\langle -\mathcal{L} \rangle = V_0 \equiv \Lambda_{\text{eff}}$. The flat space solution is possible with $V_0 = 0$, while de Sitter space solution and anti de Sitter space solution result with $V_0 > 0$ and $V_0 < 0$, respectively. By measuring the curvature of the universe, it is known that $|V_0| \leq (0.003 \text{ eV})^4$ which needs an extreme fine tuning of parameters in the Lagrangian which are supposed to be described by parameters at the fundamental mass scale, i.e. at the Planck mass scale $M_P$. “Why is $V_0$ so accurately fine tuned?” is the cosmological constant problem. This problem is very severe in spontaneous symmetry breaking models which introduce vacuum energies in the process of seeking the true minimum of the potential.

Toward the solution of the problem, self-tuning idea has been introduced by Witten and Hawking. Their definition is “the existence of the flat space solution without any fine-tuning of parameters for a finite range of parameter space in the Lagrangian.” It was different from the current usage of self tuning, in which they need only a flat space solution excluding the possibility of de Sitter and anti de Sitter space solutions. Here, we adopt the earlier definition since in this case one does not rule out the possibility of inflation. Furthermore, one has to resolve a small cosmological constant reported recently for which quintessence ideas were proposed.

In the RSI model a flat space solution is possible with two fine tunings, $k_1 = k = -k_2$, where $k^2 = -\Lambda_b/6, k_1 = \Lambda_1/6$ and $k_2 = \Lambda_2/6$ in terms of the bulk cosmological constant $\Lambda_b$, tension $\Lambda_1$ at B1, and tension $\Lambda_2$ at B2. In the second Randall-Sundrum model(RSII), there is only one brane located at $y = 0$, which is called B1. Here, the flat space is obtained by a fine-tuning $k_1 = k$. Note, however, that these models start with nonvanishing $\Lambda$’s but allow flat space solutions, which gives a hope for obtaining a model for vanishing cosmological constant.

In the recent attempts, the study is limited to a classical action only. The bulk potential is coupled to a function $f(\phi)$ of a scalar field $\phi$, satisfying the condition $(d/d\phi)f(\phi) = f(\phi)$, which can be thought of a fine-tuning or not, depending on one’s judgement. In this case, it has been known that one fine-tuning is needed, as explained below. In general, the self-tuning solutions need to satisfy: (i) There should exists an undetermined integration
constant so that it self-tunes the cosmological constant, and (ii) there should not appear a naked singularity within the allowed region of space-time. The new attempt will require in addition that there should not exist de Sitter and anti de Sitter space solutions. Even though the recent attempt obtains an undetermined integration constant, it has a naked singularity. Resolving the naked singularity by putting a brane there requires a fine tuning. It is easy to understand this fine-tuning. As soon as there is a need to insert a brane, there appears the brane tension as a free parameter. This parameter must be fine-tuned to give a flat space since the integration with respect to $y$ pick up this brane tension as an additional vacuum energy. If total effective vacuum energy were zero at one value of the brane tension, the total vacuum energy would not be zero for another value of the brane tension. Therefore, the need to introduce another brane necessarily returns to a fine-tuning case.

Here we report a recent work which allows self-tuning of the cosmological constant. We work in the RSII type set up, i.e. one 3-brane is located at $y = 0$ in 5D. Matter fields reside at the brane. In the bulk we introduce the graviton and a three index antisymmetric tensor field $A_{MNP}$ whose field strength is $H_{MNPQ}$. The standard $H^2$ term does not give a self-tuning solution. Thus, we are led to consider the $1/H^2$ term, or more generally $1/(H^2)^n$ $(n \geq 1)$ terms. The ansatz for the flat space is $ds^2 = \beta(y)\eta_{\mu\nu}dx^\mu dx^\nu + dy^2$. For the case of $1/H^2$ term, the field equations are satisfied with the flat space solution

$$\beta(|y|) = \left(\frac{k}{a}\right)^{1/4} \left(\frac{1}{\cosh(4k|y| + c)}\right)^{1/4}$$

where the undetermined integration constant is

$$c = \tanh^{-1} \frac{\Lambda_1}{\sqrt{-6\Lambda_b}}$$

where $\Lambda_1$ is the brane tension of the brane located at $y = 0$ and $\Lambda_b$ is the bulk cosmological constant. Its shape is shown in Fig. 1. It is easy to see that for a finite range of the brane tension an integration constant can be determined to give the above flat space solution. Thus, we realize Witten and Hawking’s self-tuning idea. Note that there also exist the anti de Sitter space and de Sitter space solutions, which are shown in Figs. 2 and 3. But Hawking’s most probable universe chooses the vanishing cosmological constant, even if the electroweak or QCD phase transitions add a constant to an initial $\Lambda_1$.

We have seen that the Randall-Sundrum models offer possibilities for solving some mass hierarchy problems, in particular the $\mu$ problem in a RSI type model and the cosmological constant problem in a RSII type model.
Figure 1. The flat space solution.

Figure 2. The anti de Sitter space solution.

Figure 3. The de Sitter space solution.

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