Up-Down Asymmetry:
A Diagnostic for Neutrino Oscillations

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Abstract

We propose a simple test of underground neutrino data to discriminate amongst neutrino oscillation models. It uses the asymmetry between downward-going events and upward-going events, for electron and muon events separately. Because of the symmetry of typical underground detectors, an asymmetry can be compared with calculations with little need for the intermediary of a simulation program. Furthermore, we show that the various oscillation scenarios give rise to dramatically differing trajectories of asymmetry versus energy for muons and electrons. This permits a clean distinction to be drawn between models.

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I. INTRODUCTION

The atmospheric neutrino anomaly has been with us now for more than ten years. It was discovered serendipitously in the largest detectors built for nucleon decay searches, IMB [1] and Kamioka [2], and confirmed recently in Soudan [3]. It appeared as a deficit in the ratio of muon-like to electron-like neutrino interactions within the fiducial volumes of these massive Cherenkov detectors. The atmospheric neutrinos in the range of 1 GeV of energy originate high in the atmosphere from the decay of pions and muons. The ratio of the $\nu_\mu$ flux to $\nu_e$ flux is thus closely constrained by well known decay kinematics to be in the ratio of 2:1 (actually, 2.2:1). The experiments have found that the apparent ratio of observed flux flavor ratio to expected flux flavor ratio is closer to $R = 0.60 \pm 0.05$. The result is usually presented in terms of this ratio of ratios because the absolute neutrino fluxes are not very well known ($\sim 20\%$, or perhaps even worse), whereas the $\phi(\nu_e)/\phi(\nu_\mu)$ is known to several percent. The simplest explanation for the anomaly seems to be the one originally suggested, namely neutrino oscillations [4].

Until the present time the size of the underground detectors has limited the number of events to a few hundred, and the energy of those events to less than about 1.5 GeV. Previous data have not shown conclusive zenith angle dependence as expected from neutrino oscillations. At energies of a few GeV however, we must begin to observe angular effects, if neutrino oscillations are the cause of the anomaly. Indeed, the first public presentations of data from the new massive Super-Kamiokande experiment do seem to indicate some angular variation [5].

Our purpose in this note is to set the stage to interpret future data in terms of discriminating amongst the many scenarios which have been constructed to explain one or more of the three outstanding “problems” in neutrinos: the solar neutrino problem [6], the LSND effect [7], and the presently considered atmospheric neutrino anomaly.

We define an asymmetry in direction as simply

$$A = \frac{D - U}{D + U}$$

(1)
where \( D \) are the number of downward-going events and \( U \) are upward-going events, for each of muon neutrino events and electron neutrino events. We assume the detector to be up/down symmetric, and the data set to be free of significant (presumably downward-going) background and crossover between muon and electron type of charged-current events.

We ignore the effect of \( \nu_\tau \) in our calculations, as well as the contamination of \( \nu_e \) events by neutral current interactions. The \( \nu_\tau \) charged-current cross section is sufficiently small at the energies discussed herein that it makes a negligible contribution (to both muon and electron events). Thus for present considerations, oscillations between \( \nu_\mu \) and \( \nu_\tau \) are indistinguishable from oscillations between \( \nu_\mu \) and a new sterile neutrino species. Neutral currents are a small fraction of events, and in any case should show no asymmetry (except in the sterile neutrino case).

**II. CALCULATIONS**

We now calculate this asymmetry for a wide variety of oscillation scenarios. We consider the following cases:

- (A) two-flavor mixing (a) \( \nu_\mu - \nu_\tau \) and (b) \( \nu_\mu - \nu_e \);
- (B) three-flavor mixing with a variety of choices for mass-mixing parameters;
- (C) sterile maximal mixing; and
- (D) massless neutrino mixing.

It will be shown that these various scenarios lead to very different predictions in the sign, magnitude or the energy dependence of the asymmetry and hence are (relatively) easy to distinguish from one another once there is sufficient data available.

**A. Two-Flavor Mixing**

- **a. \( \nu_\mu - \nu_\tau \) Mixing.** This is the simplest case possible \[^4\]. The \( \nu_e \) flux is unaffected and the \( \nu_\mu \) flux modified as
\[ N_\mu = N_\mu^0(P_{\mu\mu}) \]
where
\[ P_{\mu\mu} = 1 - \sin^2 2\theta \sin^2 \frac{\delta m^2 L}{4E}. \] (2)

Hence \( A_e \) is essentially zero and independent of energy. At low enough energies \( P_{\mu\mu} \approx 1 - \frac{1}{2} \sin^2 2\theta \) and is independent of \( L \) and hence \( A_\mu \to 0 \) at low energies. At high energies; \( L/E \) is negligible for down \( \nu'_\mu \)'s and hence
\[ N^d_\mu = N_\mu^0. \] (3)

For upward-going \( \nu'_\mu \)'s \( P_{\mu\mu} \approx (1 - \frac{1}{2} \sin^2 2\theta) \) and \( N^u_\mu \approx (1 - \frac{1}{2} \sin^2 2\theta) N_\mu^0 \) and hence:
\[ A_\mu = \frac{1/2 \sin^2 2\theta}{1 + 1/2 \sin^2 2\theta} \] (4)

\( A_\mu \) has a maximum of 1/3 when \( \sin^2 2\theta = 1 \). Note that at high enough energies \( A_\mu \) will come back asymptotically to zero.

b. \( \nu_\mu - \nu_e \) Mixing In this case \[ 4 \]
\[ N_\mu = N_\mu^0(P + r(1 - P)) \]
\[ N_e = N_e^0(P + (1/r)(1 - P)) \] (5)

where \( P = P_{\mu\mu} = P_{ee} \), as in Equation (2), and \( r = N^0_e/N^0_\mu \). Again at low energies \( A_e = A_\mu \approx 0 \). At high energies
\[ N^d_e = N^0_e, \quad N^u_e = N^0_e (P + r(1 - P)) \]
\[ N^d_\mu = N^0_\mu, \quad N^u_\mu = N^0_\mu (P + (1/r)(1 - P)) \]
\[ A_\mu = \frac{(1 - P) - r(1 - P)}{(1 + P) + r(1 - P)} \] (6)
\[ A_e = \frac{(1 - P) - 1/r(1 - P)}{(1 + P) + 1/r(1 - P)} \]

For \( P = 1/2 \) we get the limiting values:
\[ A_\mu = \frac{1 - r}{3 + r} \] (7)

and

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Recall that $r \sim 0.45$ at low energies, decreases to 0.3 at $E_\nu \sim 5$ GeV and eventually becomes negligible. Note that $A_\mu$ and $A_e$ always have opposite signs in this case.

### B. Three-Flavor Mixing

There are two ways to account for all three neutrino anomalies (solar, atmospheric and LSND) with just three flavors. In one, due to Cardall and Fuller [8], a single $\delta m^2$ is expected to account for both the atmospheric low energy anomaly as well as the LSND observations. In particular, $0.3 \, eV^2 \sim \delta m^2_{31} \sim \delta m^2_{32} >> \delta m^2_{12} \sim 10^{-5} \, eV^2$, with large $\nu_\mu - \nu_\tau$ mixing. In this case the resulting probabilities are very similar to the two-flavor $\nu_\mu - \nu_e$ mixing, but with a large $\delta m^2$ of $0.3 \, eV^2$. As a result of the large $\delta m^2$ very little zenith angle dependence or asymmetry (neither $A_e$ nor $A_\mu$) is expected.

The other three-flavor solution, due to Acker and Pakvasa [9], accounts for both solar and atmospheric anomalies with a single $\delta m^2$ and with large mixing between $\nu_e$ and $\nu_\mu$, the mass pattern being $2 \, eV^2 \sim \delta m^2_{31} \sim \delta m^2_{32} >> \delta m^2_{12} \sim 5 \times 10^{-3} \, eV^2$. The probabilities in this case are essentially identical to the two-flavor $\nu_\mu - \nu_e$ case with large mixing. In both of these scenarios, it is possible to have nearly degenerate neutrinos with cosmologically significant total mass.

There are also other scenarios with three-neutrino mixing with a wide range of mixing patterns [10]. In general, we expect them to yield asymmetries which will interpolate between the two limiting cases of $\nu_\mu - \nu_\tau$ and $\nu_\mu - \nu_e$ mixing.

We consider as a unique and interesting example, the maximal mixing proposal of Harrison, Perkins and Scott [11]. The assumption is that $\delta m^2_{32} \sim \delta m^2_{31} >> \delta m^2_{12} \sim 10^{-11} eV^2$ and $\delta m^2_{31}$ is in the range of $10^{-2} - 10^{-3} \, eV^2$. With the assumed maximal mixing, the probabilities for $L/E$ in the appropriate atmospheric range are given by

$$P_{\mu\mu} = P_{ee} = 1 - \frac{8}{9} \sin^2 \frac{\delta m^2_{31} L}{4E}$$
\[ P_{\mu e} = P_{e\mu} (= P_{e\tau} = P_{\mu\tau}) = 4/9 \sin^2 \frac{\delta m_{31}^2 L}{4E} \quad (9) \]

The expected asymmetries can be easily written down

\[ A_\mu = \frac{(1 - P_{\mu\mu}) - rP_{\mu e}}{(1 - P_{\mu\mu}) + rP_{\mu e}}, \quad A_e = \frac{(1 - P_{ee}) - 1/rP_{e\mu}}{(1 + P_{ee}) - 1/rP_{e\mu}} \quad (10) \]

Hence both \( A_\mu \) and \( A_e \) are small at low energies and at high energies \( P_{d\mu\mu} = P_{dee} = 1 \) and \( P_{d\mu e} = 0 \), whereas \( P_{u\mu\mu} = P_{uee} = 5/9 \) and \( P_{u\mu e} = 2/9 \). And so

\[ A_\mu = \frac{4/9 - r(2/9)}{14/9 + r(2/9)} = \frac{2 - r}{7 + r} \]

and

\[ A_e = \frac{4/9 - 1/r(2/9)}{14/9 + 1/r(2/9)} = \frac{1 - 2r}{7r + 1} \quad (11) \]

C. Sterile Maximal Mixing

In the scheme of Foot and Volkas [12], \( \nu_\mu \) mixing maximally with a new sterile \( \nu_{\mu'} \) with a \( \delta m_{\mu\mu'}^2 \sim 5 \times 10^{-3} \text{ eV}^2 \) accounts for the low-energy atmospheric neutrino anomaly. Hence, the muon asymmetry is identical to the one in the case of \( \nu_\mu - \nu_\tau \) oscillations, as discussed above. In addition, \( \nu_e \) mixes maximally with a sterile \( \nu_{e'} \), and when \( \delta m_{ee'}^2 \) is in the range of \( 10^{-3}\text{eV}^2 \) electron-neutrinos will also oscillate and get depleted. The resulting electron asymmetry is strikingly different from the case of \( \nu_\mu - \nu_e \) oscillations in having the opposite sign, which makes it unique and easy to distinguish. \( A_e \) and \( A_\mu \) will differ only in slightly different energy (or L/E) dependence but be otherwise similar and always have the same sign.

D. Massless Neutrinos

If neutrinos are massless, there can still be mixing and oscillations. Two possibilities have been considered in the literature. One is the case where different flavors couple differently to gravity [13] and the other is a breakdown of Lorentz invariance where each particle may
have its own maximum speed \[14\]. The oscillation phenomenology is identical for both cases. The survival probability in the two-flavor limit is given by

\[
P_{\mu\mu} = P_{ee} = 1 - \sin^2 2\theta \sin^2 \left(\frac{1}{2} \delta v \ E L\right)
\]

(12)

where \(\theta\) is the flavor mixing angle, and \(\delta v\) is the small parameter characteristic of violation of equivalence principle or Lorentz invariance. Note the strikingly different dependence on \(L\) and \(E\): \(L \times E\) instead of \(L/E\). Remarkably, an allowed choice of parameters is able to account for both solar and atmospheric neutrinos \[15\]: \(\sin^2 2\theta \approx 0.8\) to 1 and \(\frac{\delta v}{2} \sim 10^{-2} - 10^{-3} (km - GeV)^{-1}\). The expressions are the same as in the \(\nu_\mu - \nu_e\) case except that \(\sin^2 \frac{\delta m^2}{4E}\) is replaced by \(\sin^2 (\delta v/2LE)\). As a result, the roles of low and high energy are reversed. The asymmetries \(A_\mu\) and \(A_e\) become rather small at high energies; at low energies they are given by Eq.(12).

III. NUMERICAL RESULTS

We have performed numerical calculations of the models discussed above. They are explicitly:

1. Simple two-flavor oscillations between \(\nu_\mu\) and \(\nu_\tau\) \[4\]. The example is for \(\delta m^2 = 0.005\ eV^2\), \(\sin^2 2\theta = 1\).

2. Two-flavor oscillations between \(\nu_\mu\) and \(\nu_e\) with the same parameters as above. (The Acker-Pakvasa \[9\] scheme leads to the same result).

3. Three-flavor mixing \(a' la\ Cardall-Fuller\ \[8\].

4. Three-flavor maximal mixing scheme of Harrison-Perkins-Scott \[11\].

5. Sterile maximal mixing of Foot-Volkas \[12\].

6. Massless neutrino mixing, where we take \(\delta v/2 \sim 10^{-3} (km - GeV)^{-1}\) \[15\].
In Figure 1 we show results for $A_\mu$ and $A_e$ as functions of energy from more detailed calculations. We calculated energy spectra between 0.2 and 5.0 GeV for a detector with an exposure of 22 kiloton-years (approximately one year of Super-Kamiokande data). We use the Bartol flux model, and a simple quark model for the charged-current cross section, and assume a perfect detector \[16\]. Detailed calculations for a particular instrument will of course vary, but the asymmetry will change little, the general behavior illustrated being insensitive to the details.

We show the trajectories of $A_\mu$ versus $A_e$ in Figure 2 for the six models. Note that there are small asymmetries at low energies, due to the inhomogeneity of the earth’s magnetic field, as incorporated into the atmospheric flux model we employ \[16\]. It is clear that with good statistics all scenarios can be clearly distinguished by both energy dependence and relative signs of $A_\mu$ and $A_e$. In particular it is noteworthy that the first model, currently seemingly favored in preliminary reports from Super-Kamiokande \[5\], stands out distinctly from all others. It is straightforward to plot the expected asymmetries in other scenarios or different choices of parameters.

**IV. CONCLUSIONS**

In the foregoing we have presented a case for employing the up-to-down asymmetry of neutrino interactions in underground detectors as a discriminator for some of the many neutrino oscillation schemes which have been discussed as solutions to various combinations of the current three neutrino puzzles (solar, atmospheric and LSND). The asymmetry has the virtue that it can be calculated directly from data (using only particle identification, energy and direction), without aid of simulation programs. It is self-normalizing and independent of flux model calculations, and tests electron and muon data separately.
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FIG. 1. The muon (solid lines) and electron (dashed lines) asymmetries versus energy (GeV), for the 6 oscillations models considered herein (see text).
FIG. 2. The trajectories of the muon asymmetry and electron asymmetry for the 6 oscillation models considered herein. The arrowheads point in the direction of increasing charged lepton energy, which ranges from 0.2 to 5.0 GeV in these calculations.