Antiproton–nucleus quasi-bound states within the 2009 version of the Paris $\bar{N}N$ potential

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Abstract. We studied the $\bar{p}$ interactions with the nuclear medium within the 2009 version of the Paris $\bar{N}N$ potential model. We constructed the $\bar{p}$–nucleus optical potential using the Paris $S$- and $P$-wave $\bar{p}N$ scattering amplitudes and treated their strong energy and density dependence self-consistently. We considered a phenomenological $P$-wave term as well. We calculated $\bar{p}$ binding energies and widths of the $\bar{p}$ bound in various nuclei. The $P$-wave potential has very small effect on the calculated $\bar{p}$ binding energies, however, it reduces the corresponding widths noticeably. Moreover, the $S$-wave potential based on the Paris amplitudes supplemented by a phenomenological $P$-wave term yields the $\bar{p}$ binding energies and widths in very good agreement with those obtained within the RMF model consistent with $\bar{p}$-atom data.

1 Introduction

The antiproton–nucleus interaction below threshold have been so far studied within phenomenological RMF approaches [1, 2]. The G-parity motivated $\bar{p}$ coupling constants were used to construct the $\bar{p}$–nucleus potential. The absorption of $\bar{p}$ was accounted for in terms of a purely phenomenological optical potential. The $\bar{p}$ optical potential was confronted with $\bar{p}$ atom data. It was found that the $\bar{p}$ coupling constant have to be properly scaled in order to be consistent with the data. Consequently, the $\bar{p}$ potential was applied in the calculations of $\bar{p}$ quasi-bound states in various nuclei [2].

However, it is desirable to study the $\bar{p}$ interactions with the nuclear medium within other theoretical approaches, such as microscopic models of $\bar{N}N$ interaction based on meson-exchange models [3–5] or chiral $\bar{N}N$ interaction models [6, 7]. Comparison between these $\bar{N}N$ interaction models could bring valuable information about in-medium $\bar{p}$ interactions in the direct confrontation with the data from $\bar{p}$ atoms and $\bar{p}$ scattering off nuclei, as well as predictions for $\bar{p}$-nuclear quasi-bound states.

Recently, the 2009 version of the Paris $\bar{N}N$ potential [3] was confronted by Friedman et al. with the $\bar{p}$-atom data and antinucleon interactions with nuclei up to 400 MeV/c, including elastic scattering and annihilation cross sections [8]. The analysis revealed the necessity to include the $P$-wave interaction in order to describe the $\bar{p}$ atom data. The Paris $S$-wave potential supplemented by a phenomenological $P$-wave term was found to fit the data on low-density, near-threshold $\bar{p}$-nucleus interaction. This fact stimulated us to apply it in the present calculations of $\bar{p}$-nuclear quasi-bound states and explore the effect of the $P$-wave interaction on $\bar{p}$ binding energies and widths of $\bar{p}$-nuclear states.

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In Section 2, we briefly introduce the model applied in our calculations. Section 3 presents few representative results together with the discussion of the main findings of our study.

## 2 Methodology

The binding energies $B_p$ and widths $\Gamma_p$ of $\bar p$ quasi-bound states in a nucleus are obtained by solving self-consistently the Dirac equation with the optical potential

$$[-i\alpha \cdot \nabla + \beta p + V_{\text{opt}}(r)]\psi_p = \epsilon_p \psi_p,$$

where $m_p$ is the mass of the antiproton and $\epsilon_p = -B_p - i\Gamma_p/2$ ($B_p > 0$). The $S$-wave $\bar p$–nucleus optical potential $V_{\text{opt}}$ enters the Dirac equation as the time component of a 4-vector and is constructed in a ‘t$p\bar p$’ form as follows:

$$2E_p V_{\text{opt}}(r) = -4\pi \left( F_0 \frac{1}{2} \rho_p(r) + F_1 \left( \frac{1}{2} \rho_p(r) + \rho_n(r) \right) \right).$$

Here, $E_p = m_p - B_p$, $F_0$ and $F_1$ are isospin 0 and 1 in-medium amplitudes, and $\rho_p(r)$ [$\rho_n(r)$] is the proton (neutron) density distribution calculated within the RMF NL-SH model [9]. The in-medium amplitudes $F_0$ and $F_1$ entering Eq. (2) account for Pauli correlations in the nuclear medium. They are constructed from the free-space $\bar pN$ amplitudes using the multiple scattering approach of Was et al. [10] (WRW)

$$F_1 = \frac{\sqrt{x}}{m_N} f^S_{\bar p n}(\sqrt{s}) \cdot, \quad F_0 = \frac{\sqrt{x}}{m_N} \left[ 2f^S_{\bar p n}(\sqrt{s}) - f^S_{\bar p p}(\sqrt{s}) \right] \cdot,$$

Here, $f^S_{\bar p n}$ ($f^S_{\bar p p}$) denotes the free-space c.m. $\bar p n$ ($\bar p p$) $S$-wave scattering amplitude derived from the Paris $\bar NN$ potential as a function of Mandelstam variable $\sqrt{s}$, $m_N$ represents the mass of the nucleon and $\rho(r) = \rho_p(r) + \rho_n(r)$. The factor $\sqrt{s}/m_N$ transforms the amplitudes from the two-body frame to the $\bar p$–nucleus frame. The Pauli correlation factor $\xi_k$ is defined as follows

$$\xi_k = \frac{9\pi}{k_F^2} \left[ 4 \int_0^\infty \frac{dr}{r} \exp(ikr)f_j(kr) \right],$$

where $j_1(kr)$ is the spherical Bessel function, $k_F$ is the Fermi momentum and $k = \sqrt{(\epsilon_p + m_p)^2 - m_p^2}$ is the antiproton momentum. The integral in Eq.(4) can be solved analytically. The resulting expression is of the form

$$\xi_k = \frac{9\pi}{k_F^2} \left[ 1 - \frac{q^2}{6} + \frac{q^2}{4} \left( 2 + \frac{q^2}{6} \right) \ln \left( 1 + \frac{4}{3} q \left( \frac{\pi}{2} - \arctan \left( \frac{q}{2} \right) \right) \right) \right].$$

where $q = -ik/k_F$.

The analysis of $\bar p$ atom data [8] revealed that it is necessary to supplement the Paris $S$-wave potential by the $P$-wave interaction to make the real $\bar p$ potential attractive in the relevant low-density region of a nucleus. To incorporate the $P$-wave interaction in our model we supplement the r.h.s. of the $S$-wave optical potential in Eq. (2) [$2E_p V^S_{\text{opt}} = q(r)$] by a gradient term [8]:

$$2E_p V_{\text{opt}}(r) = q(r) + 3\nabla \cdot \alpha(r) \nabla.$$
The in-medium amplitudes supplement the r.h.s. of the wave potential by the $\rho$ and is constructed in a 'two-body frame' to the $\bar{B}_2$ Methodology. The analysis of Ref. [8] also revealed that it is necessary to supplement the Paris optical potential constructed from the Paris $\bar{p}n$ quasi-bound states in a nucleus are obtained by using the multiple amplitudes using the multiple $\alpha$.

The factor $2l + 1 = 3$ in the $P$-wave part is introduced to match the normalization of the Paris $\bar{N}N$ scattering amplitudes and

$$\alpha(r) = 4\pi \frac{m_N}{\sqrt{3}} \left( f_{\bar{p}p}^P(\sqrt{s})\rho_p(r) + f_{\bar{p}n}^P(\sqrt{s})\rho_n(r) \right).$$

(7)

Here, $f_{\bar{p}p}^P(\sqrt{s})$ and $f_{\bar{p}n}^P(\sqrt{s})$ represent the Paris $P$-wave $\bar{p}p$ and $\bar{p}n$ free-space c.m. scattering amplitudes, respectively. We do not consider any medium modifications of the $P$-wave amplitudes since we assume that the $P$-wave potential should contribute mainly near the surface of the nucleus due to its gradient form.

The analysis of Ref. [8] also revealed that the optical potential constructed from the Paris $S$- and $P$-wave amplitudes fails to reproduce the $\bar{p}$ atom data and that it is mainly due to the $P$-wave amplitude — its real and imaginary parts had to be scaled by different factors to get reasonable fit. On the contrary, the optical potential based on the Paris $S$-wave potential supplemented by a purely phenomenological $P$-wave term with $f_{\bar{p}N}^P = 2.9 + i1.8$ fm$^3$ fits the data well. In our calculations, we adopt both $P$-wave amplitudes, Paris as well as phenomenological, in order to study their effect on the binding energies and widths of $\bar{p}$-nuclear states.

The Paris amplitudes used in our calculations are shown in Fig. 1. There are $\bar{p}p$ (top) and $\bar{p}n$ (bottom) medium modified $S$-wave amplitudes (3) at saturation density $\rho_0 = 0.17$ fm$^{-3}$.

**Figure 1.** Energy dependence of real (left) and imaginary (right) parts of the Paris 09 $\bar{p}p$ (top) and $\bar{p}n$ (bottom) two-body c.m. scattering amplitudes used in the present calculations: in-medium (Pauli blocked) $S$-wave amplitudes at $\rho_0 = 0.17$ fm$^{-3}$ and free-space $P$-wave amplitudes.
and free-space $P$-wave scattering amplitudes plotted as a function of the energy shift $\delta \sqrt{s} = E - E_{\text{th}}$ with $E_{\text{th}} = m_\bar{p} + m_N$. The $S$-wave amplitudes vary considerably with energy below threshold. The real in-medium $\bar{p}p$ amplitude is attractive in the entire energy region below threshold. The real part of the in-medium $\bar{p}n$ amplitude is attractive for $\delta \sqrt{s} \leq -70$ MeV with a small repulsive dip near threshold. The imaginary parts of the $S$-wave amplitudes are comparable or even larger than the corresponding real parts. The energy dependence of the free-space $P$-wave amplitudes is less pronounced than in the $S$-wave case. Moreover, the $P$-wave amplitudes are considerably smaller than the in-medium $S$-wave amplitudes in the region relevant to $\bar{p}$-nuclear states calculations.

Strong energy dependence of the $\bar{p}N$ amplitudes presented in Fig. 1 requires a proper self-consistent scheme for evaluating the $\bar{p}$ optical potential. The energy argument $\sqrt{s}$ of the amplitudes is expressed in the $\bar{p}$–nucleus frame where the contributions from antiproton and nucleon kinetic energies are not negligible [11]

$$\sqrt{s} = E_{\text{th}} \left(1 - \frac{2(B_\bar{p} + B_{N\text{av}})}{E_{\text{th}}} + \frac{(B_\bar{p} + B_{N\text{av}})^2}{E_{\text{th}}^2} - \frac{T_\bar{p}}{E_{\text{th}}} - \frac{T_{N\text{av}}}{E_{\text{th}}}\right)^{1/2}. \quad (8)$$

Here, $B_{N\text{av}} = 8.5$ MeV and $T_{N\text{av}}$ are the average binding and kinetic energy per nucleon, respectively, and $T_\bar{p}$ represents the $\bar{p}$ kinetic energy. The kinetic energies are evaluated as corresponding expectation values of the kinetic energy operator $\hat{T} = -\frac{\hbar^2}{2m} \Delta$. Since the $B_\bar{p}$ appears as an argument in the $\sqrt{s}$, which in turn serves as an argument for $V_{\text{opt}}$, $\sqrt{s}$ has to be determined self-consistently. Namely, its value obtained by solving Eq. (8) should agree with the value of $\sqrt{s}$ which serves as input in Eq. (3) and thus Eq. (1), as well.

### 3 Results

We performed self-consistent calculations of $\bar{p}$-nuclear quasi-bound states in selected nuclei within the model presented in the previous section. We explored the energy and density dependence of the $S$-wave $\bar{p}$–nucleus potential as well as the role of the $\bar{p}N$ $P$-wave interaction, and compared the predictions for $\bar{p}$ binding energies and widths with the phenomenological RMF approach [2].

The $\bar{p}N$ amplitudes are strongly energy and density dependent, as was shown in Fig. 1. Consequently, the depth and shape of the $\bar{p}$–nucleus potential depend greatly on the energies and densities pertinent to the processes under consideration. It is demonstrated in Fig. 2 where we present the $\bar{p}$ potential in $^{40}\text{Ca}$ calculated for different energies and densities: i) using the Paris free-space $S$-wave amplitudes at threshold (denoted by ‘th free’), ii) using in-medium Paris $S$-wave amplitudes at threshold (denoted by ‘th medium’), iii) using in-medium Paris $S$-wave amplitudes at energies relevant to $\bar{p}$ atoms (constructed following Ref. [8]), and iv) using in-medium Paris $S$-wave amplitudes at energies relevant to $\bar{p}$ nuclei. The $\bar{p}$ potential constructed using the free-space amplitudes has a repulsive real part and fairly absorptive imaginary part. When the medium modifications of the amplitudes are taken into account, the $\bar{p}$ potential becomes attractive and more absorptive. At the energies relevant to $\bar{p}$ atoms, the $\bar{p}$ potential is more attractive and weakly absorptive. Finally, at the energies relevant to $\bar{p}$ nuclei, the $\bar{p}$ potential is strongly attractive, however, also strongly absorptive. The figure clearly shows that proper self-consistent evaluation of the energy $\sqrt{s}$ is essential.

Next, we performed static and dynamical calculations of $\bar{p}$ binding energies and widths using the Paris $\bar{N}N$ potential. In the static calculations, the core nucleus is not affected by the presence of extra $\bar{B}$. In the dynamical calculations, the polarization of the nuclear core due to $\bar{B}$, i.e., changes in the nucleon binding energies and densities, is taken into account. The response of the nuclear core to the extra antiproton is not instant — it could possibly
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due to \( \bar{p} \). We present the \( \bar{p} \) and densities pertinent to the processes under consideration. It is demonstrated in Fig. 2 where self-consistent scheme for evaluating the \( \bar{p} \) threshold. The real part of the in-medium \( \bar{p} \) free-space \( P \) with a small repulsive dip near threshold. The imaginary parts of the \( pp \) wave amplitudes are considerably smaller than the in-medium \( \bar{p} \) potential constructed using the free-space amplitudes has a repulsive real part and fairly strong energy dependence of the \( \bar{p} \) potential is strongly attractive, however, also strongly absorptive. Finally, at the energies corresponding expectation values of the kinetic energy operator \( \hat{\text{E}}_{\text{kin}} \). Here, \( \hat{\text{E}}_{\text{kin}} = \frac{\hbar^2 A^{2/3}}{2m} \). Since the \( \hat{\text{E}}_{\text{kin}} \) represents the \( \bar{p} \) potential in \( 40\text{Ca} \) calculated for different energies and densities: i) using the \( \bar{p} \) atom and \( \bar{p} \) nucleus, calculated for \( ^{40}\text{Ca}+\bar{p} \) with in-medium Paris \( S \)-wave amplitudes and static RMF densities. The \( \bar{p} \) potential calculated using free-space amplitudes at threshold is shown for comparison (‘th free’).

**Figure 2.** The potential felt by \( \bar{p} \) at threshold (‘th medium’), in the \( \bar{p} \) atom and \( \bar{p} \) nucleus, calculated for \( ^{40}\text{Ca}+\bar{p} \) with in-medium Paris \( S \)-wave amplitudes and static RMF densities. The \( \bar{p} \) potential calculated using free-space amplitudes at threshold is shown for comparison (‘th free’).

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\begin{align*}
\text{Figure 3.} & \quad 1s \; \bar{p} \; \text{binding energies (left panel) and widths (right panel) in various nuclei, calculated} \\
& \quad \text{statically (triangles) and dynamically (circles) using } S \text{-wave Paris potential (red) and including phenomenological } P \text{-wave potential (black). The } \bar{p} \text{ binding energies and widths calculated dynamically using the Paris } S + P \text{-wave potential (blue circles) are shown for comparison.}
\end{align*}
\]

last longer than the lifetime of \( \bar{p} \) inside a nucleus [12, 13]. As a result, the antiproton could annihilate before the nuclear core is fully polarized. Our static and dynamical calculations of \( \bar{p} \) binding energies and widths may be thus considered as two limiting scenarios.

In Fig. 3, we present 1s \( \bar{p} \) binding energies (left) and widths (right) as a function of mass number \( A \), calculated statically (triangles) and dynamically (circles) with the Paris \( S \)-wave and Paris \( S \)-wave + phen. \( P \)-wave potentials. We present the \( \bar{p} \) binding energies and widths calculated dynamically using the Paris \( S + P \)-wave potential for comparison as well.
In dynamical and static calculations alike, the $P$-wave interaction does not affect much the $\bar{p}$ binding energies — they are comparable with the binding energies evaluated using only the $S$-wave potential. On the other hand, the $\bar{p}$ widths are reduced significantly when the phenomenological $P$-wave term is included in the $\bar{p}$ optical potential. The effect is even more pronounced for the Paris $P$-wave interaction.

The $\bar{p}$ widths calculated dynamically are noticeably larger than the widths calculated statically. It is caused by the increase of the central nuclear density, which exceeds the decrease of the $\bar{p}N$ amplitudes due to the larger energy shift with respect to threshold ($\delta \sqrt{s} \sim -255$ MeV in the dynamical case vs. $\delta \sqrt{s} \sim -200$ MeV in the static case). On the other hand, the $\bar{p}$ binding energies increase only moderately and get closer to each other when the dynamical effects are taken into account. The $\bar{p}$ widths exhibit much large dispersion then the $\bar{p}$ binding energies for the different potentials.

We explored the $\bar{p}$ excited states in selected nuclei as well and compared the results with those obtained within the RMF approach [2]. Fig. 4 shows $\bar{p}$ spectra in $^{40}$Ca calculated using the Paris $S$-wave + phen. $P$-wave potential and phenomenological RMF approach. The Paris $S$-wave + phen. $P$-wave potential yields the $1p$ and $1d$ binding energies slightly larger and thus the $s$-$p$ and $s$-$d$ level spacing smaller than the RMF approach. It is an effect of a broader $\bar{p}$ potential well generated by the Paris $S$-wave + phen. $P$-wave potential. Nevertheless, both approaches yield comparable $\bar{p}$ widths as well as energies and the overall agreement is surprisingly good.

It is to be noted that there is no spin-orbit splitting of the $p$ and $d$ levels presented in Fig. 4 since the $V_{opt}$ is a central potential constructed from angular momentum-averaged scattering amplitudes. In the RMF approach, the $\bar{p}$ binding energies in $1p$ and $1d$ spin doublets are nearly degenerate, the difference in $\bar{p}$ energies (as well as $\bar{p}$ widths) is up to $\sim 1$ MeV. This is in agreement with spin symmetry in antinucleon spectra within the RMF approach [14, 15]. In the left panel of Fig. 4 we show the spin-averaged $1p$ and $1d$ $\bar{p}$ binding energies and widths for better comparison with the results obtained with the central Paris potential.

In conclusion, we performed self-consistent calculations of $\bar{p}$-nuclear quasi-bound states using a microscopic potential, namely the Paris $\bar{NN}$ potential, for the first time. We explored

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure.png}
\caption{1s, 1p and 1d binding energies (lines) and widths (boxes) of $\bar{p}$ in $^{40}$Ca calculated dynamically within the phenomenological RMF $\bar{p}$ optical potential and Paris $S$-wave + phen. $P$-wave potential.}
\end{figure}
the effect of the $P$-wave interaction on $\bar{p}$ binding energies and widths. We found that the $P$-wave interaction almost does not affect the binding energies of $\bar{p}$-nuclear states. This is in sharp contrast to the case of $\bar{p}$ atoms where it was found necessary to include the $P$-wave interaction in order to increase attraction of the $\bar{p}$ optical potential [8]. Moreover, we found good agreement between the results obtained using the phenomenological RMF potential and the Paris $S$-wave + phenomenological $P$-wave potential which are the two potentials consistent with antiprotonic atom data and $\bar{p}$ scattering off nuclei at low energies.

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