Spin Coulomb Drag

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Abstract

We introduce a distinctive feature of spin-polarized transport, the Spin Coulomb Drag: there is an *intrinsic* source of friction for spin currents due to the Coulomb interaction between spin “up” and spin “down” electrons. We calculate the associated “spin transresistivity” in a generalized random phase approximation and discuss its dependence on temperature, frequency, and electron density. We show that, in an appropriate range of parameters, such resistivity is measurable and propose an experiment to measure it.
Interest in spin-polarized transport has been growing dramatically in the last few years, spurred by the hope of realizing practical spin-electronic devices in a not too distant future [1]. In particular it has been shown that spin coherence can be maintained over large distances \( s > \sim 100 \mu m \) and for long times \( T_2 \sim 10^{-9} - 10^{-8} s \) both in metals [2] and in semiconductors [3].

In this paper we introduce a distinctive feature of spin-polarized transport: in a conductor, due to the Coulomb interaction, there is an intrinsic mechanism for friction between electrons of different spin, the “Spin Coulomb Drag” (SCD). For simplicity we shall restrict our discussion to the case in which the spin state of each electron can be classified as “up” or “down” relative to the \( z \) axis. In the absence of impurities the total momentum \( P = \sum_i p_i \), where \( p_i \) is the momentum of the \( i \)-th electron, is a conserved quantity. On the contrary, the “up” and the “down” components of the total momentum \( P_\uparrow = \sum_i p_i \uparrow \) and \( P_\downarrow = \sum_i p_i \downarrow \), where \( \hat{\sigma}_z \) is the the Pauli matrix for the \( z \) component of the \( i \)-th electron’s spin, are not separately conserved even in the absence of impurities: Coulomb scattering can transfer momentum between spin up and spin down electrons thereby effectively introducing a “friction” for relative motion of the two spin components. If, for example, one of the two spin components is set into motion relative to the other, it will tend to drag the latter in the same direction. Or, if a finite spin current is set up through the application of an external field, then the Coulomb interaction will tend to equalize the net momenta of the two spin components, causing the difference \( \langle P_\uparrow \rangle - \langle P_\downarrow \rangle \) to decay to zero when the external field is turned off.

The most dramatic manifestation of the SCD is the appearance of a finite trans-resistivity defined as the ratio of the gradient of the spin-down electro-chemical potential to the spin-up current density when the spin-down current is zero. This is completely analogous to the trans-resistivity measured in Coulomb drag experiments with electrons in two separate layers [4–6], but in this case what makes the two electron populations distinguishable is not a physical separation but the different spin. In SCD the non conservation of the spin, caused mainly by the spin-orbit interaction, represents a “leakage” mechanism analogous to the interlayer tunneling in the usual Coulomb drag.

First of all let us describe the SCD from a phenomenological point of view. Let \( E_\uparrow(t) \) and \( E_\downarrow(t) \) be uniform effective electric fields that couple to spin \( \uparrow \) and spin \( \downarrow \) electrons respectively. These fields are sums of the ordinary electrostatic field plus the gradient of the local chemical potential, which can be spin-dependent. We restrict ourselves to the linear response regime, assume weak electron-electron and electron-impurity scattering, and ignore spin-flipping processes altogether. If \( \mathbf{v}_\sigma \) is the velocity of the center of mass of electrons of spin \( \sigma \), and \( N_\sigma \) the number of such electrons, the phenomenological equation of motion has the form

\[
mN_\sigma \dot{\mathbf{v}}_\sigma = -eN_\sigma \mathbf{E}_\sigma + \mathbf{F}_{\sigma\bar{\sigma}} - \frac{m}{\tau_D} N_\sigma \mathbf{v}_\sigma
\]  

where \( \tau_D \) is the Drude scattering time and \( \mathbf{F}_{\sigma\bar{\sigma}} \) is the Coulomb force exerted by spin \( \bar{\sigma}(= -\sigma) \) electrons on spin \( \sigma \) electrons. By Newton’s third law \( \mathbf{F}_{\sigma\bar{\sigma}} = -\mathbf{F}_{\bar{\sigma}\sigma} \) and by Galilean invariance this force can only depend on the relative velocity of the two components. Hence, for weak Coulomb coupling we write

\[
\mathbf{F}_{\sigma\bar{\sigma}} = -\gamma mN_\sigma \frac{n_{\bar{\sigma}}}{n} (\mathbf{v}_\sigma - \mathbf{v}_\bar{\sigma}),
\]  

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where \( n_{\sigma} \) is the number density of electrons of spin \( \sigma \) and \( n = n_{\uparrow} + n_{\downarrow} \) is the total density. Eq. (2) defines the spin drag coefficient \( \gamma \). Fourier transforming Eq. (1) with respect to time, and making use of the relationship \( j_{\sigma}(\omega) = -en_{\sigma}v_{\sigma}(\omega) \) between current density and velocity, we obtain

\[
i\omega j_{\sigma}(\omega) = -\frac{n_{\sigma}e^2}{m}E_{\sigma}(\omega) + \left( \frac{n_{\sigma}}{n} \gamma + \frac{1}{\tau_D} \right) j_{\sigma}(\omega)
- \frac{n_{\sigma}}{n} \gamma j_{\sigma}(\omega).
\]

(3)

The resistivity matrix \( \rho_{\sigma,\sigma'} \) is defined as the coefficient of proportionality between the electric field and the current: \( E_{\sigma} = \sum_{\sigma'} \rho_{\sigma,\sigma'} j_{\sigma'} \). A quick comparison between this definition and Eq. (3) shows that \( \gamma \) is directly proportional to the spin trans-resistivity

\[
\gamma = -\frac{ne^2}{m} \rho_{\uparrow\downarrow}.
\]

(4)

Let us now proceed to the microscopic calculation of the spin trans-resistivity. We start from the Kubo formula [7] for the uniform conductivity matrix

\[
\sigma_{\sigma,\sigma'}(\omega) = -\frac{1}{i\omega m} \left[ n_{\sigma} \delta_{\sigma,\sigma'} + \frac{\langle \langle P_{\sigma}; P_{\sigma'} \rangle \rangle_\omega}{m} \right],
\]

(5)

where \( \langle \langle A; B \rangle \rangle_\omega \) represents, as usual [4], the retarded response function for the expectation value of \( A \) under the action of a field that couples linearly to \( B \). The resistivity matrix is the inverse of the conductivity matrix. In the limit that both the electron-impurity and the electron-electron scattering are weak the \( P_{\sigma} \)’s are almost constants of the motion and therefore \( \langle \langle P_{\sigma}; P_{\sigma'} \rangle \rangle_\omega \to 0 \). This means that the second term in the square bracket of Eq. (5) can be treated as a small correction to the first [8]. Inverting Eq. (5) to first order in \( \langle \langle P_{\sigma}; P_{\sigma'} \rangle \rangle_\omega \) and selecting the \( \uparrow\downarrow \) matrix element we obtain

\[
\rho_{\uparrow\downarrow}(\omega) = -\frac{i\omega}{e^2} \frac{\langle \langle P_{\uparrow}; P_{\downarrow} \rangle \rangle_\omega}{n_{\uparrow}n_{\downarrow}}.
\]

(6)

It is convenient to recast this equation in a form that emphasizes the importance of the non conservation of \( P_{\uparrow} \) and \( P_{\downarrow} \). To this end we make use twice of the general equation of motion

\[
\langle \langle A; B \rangle \rangle_\omega = \frac{1}{\omega} \langle \langle [A, B] \rangle \rangle + i \langle \langle \dot{A}; B \rangle \rangle_\omega,
\]

(7)

where \( \dot{A} \equiv i[A, H] \) is the time derivative of the operator \( A \), and \( \langle .. \rangle \) denotes the thermal average. Thus, Eq. (6) can be rewritten as

\[
\rho_{\uparrow\downarrow}(\omega) = -\frac{i}{e^2 n_{\uparrow} n_{\downarrow}} \frac{\langle \langle \dot{P}_{\uparrow}; P_{\downarrow} \rangle \rangle_\omega + i \langle \langle [\dot{P}_{\uparrow}, P_{\downarrow}] \rangle \rangle_\omega}{\omega}.
\]

(8)

The commutator term controls the high frequency behavior of \( \rho_{\uparrow\downarrow}(\omega) \) and can be expressed in terms of ground-state properties [9]. This term however gives a purely imaginary contribution to the trans-resistivity. Our present interest is in the real part of the trans-resistivity, which is controlled by the imaginary part of the force-force response function.
The force operator is given by

\[
\dot{P}_\sigma = - \frac{i}{V} \sum_q \mathbf{q} v_q \rho_{q\sigma} \rho_{-q\sigma} - \frac{i}{V} \sum_q \mathbf{q} \bar{v}_q^{e-i} \rho_q \rho_{-q\sigma},
\]

where \(v_q = 4\pi e^2/q^2\) is the Fourier transform of the Coulomb interaction, \(v_q^{e-i}\) is the Fourier transform of the electron-impurity interaction, \(\rho_{q\sigma}\) is the electronic spin density fluctuation operator, \(\rho_q^i\) is the Fourier transform of the impurity density (a number), and \(V\) is the volume of the system.

In calculating the force-force response function special attention must be paid to the contributions of the electron-impurity interaction. In the theory of the ordinary Coulomb drag [10] such contributions are zero on the average because the electrons in the two layers interact with two \(different\) sets of impurities, which are uncorrelated to each other. In the present case, however, electrons of opposite spin interact with the \(same\) set of impurities, so that electron-impurity terms, generated from the substitution of Eq. (9) into Eq. (8), do not vanish upon disorder averaging. Happily, it turns out that these terms cancel out exactly at low frequency \((\omega << E_F)\) and to leading order in the electron-electron and electron-impurity interactions [11]. Thus, the real part of the spin trans-resistivity takes the form

\[
Re\rho_{\uparrow\downarrow}(\omega) = -\frac{1}{n^\uparrow n^\downarrow e^2 V^2} \sum_{qq'} \frac{\mathbf{q} \cdot \mathbf{q'}}{3} v_q v_{q'},
\]

where we have made use of the isotropy of the electron gas.

We have calculated the four point response function \(\chi_4(q, q', \omega) \equiv \langle \langle \rho_{-q\uparrow} \rho_{q\downarrow}; \rho_{q'\uparrow} \rho_{-q'\downarrow} \rangle \rangle_\omega\) at finite temperature in a generalized Random Phase Approximation (RPA). The selected diagrams are shown in Fig. 1. Because of its infinite range, the Coulomb interaction must be treated to infinite order, even when weak. The sum of the RPA diagrams has been evaluated by standard methods [12] with the following result:

\[
Re\rho_{\uparrow\downarrow}(\omega, T) = -\frac{1}{n^\uparrow n^\downarrow e^2 V^2} \sum_q \frac{q^2}{3} v_q^2 \frac{(e^{-\beta\omega} - 1)}{\omega} \int_{-\infty}^{\infty} \frac{d\omega'}{\pi} \left[ \chi''_{\uparrow\downarrow}(q, \omega') \chi''_{\downarrow\uparrow}(q, \omega - \omega') - \chi''_{\uparrow\downarrow}(q, \omega') \chi''_{\downarrow\uparrow}(q, \omega - \omega') \right] (e^{-\beta\omega'} - 1)(e^{-\beta(\omega-\omega')} - 1).
\]

Here \(\beta = 1/k_B T\), with \(k_B\) the Boltzmann constant, \(\chi''_{\sigma\sigma'}(q, \omega)\) is the imaginary part of the RPA spin-resolved density-density response function, which is related to the noninteracting response function \(\chi_{0\sigma}(q, \omega)\) as follows

\[
[\chi^{-1}(q, \omega)]_{\sigma\sigma'} = [\chi_{0\sigma}]^{-1}(q, \omega) \delta_{\sigma\sigma'} - v_q.
\]

It is possible to show by simple but tedious algebraic calculations that this expression for the spin trans-resistivity \(\rho_{\uparrow\downarrow}(\omega, T)\) reduces, in the case of finite temperature and \(\omega = 0\), to the well known result of memory function and diagrammatic theories for the Coulomb drag [10], [13]. Furthermore, for \(T = 0\) and \(\omega \neq 0\), the RPA is equivalent to the decoupling approximation for the four-point response function used in [14] to calculate the dynamical
exchange-correlation kernel. Thus our calculation demonstrates that those two approximations, quite different at a first sight, are simply RPAs performed in different limits.

Let us focus on the low-temperature and low-frequency regime \( k_B T \ll E_F \) and \( \omega \ll E_F \), with \( E_F \) the Fermi energy. In this regime the imaginary part of the density-density response functions \( \chi''_{\sigma\sigma}(q,\omega) \) is a linear function of \( \omega \). In the limit of vanishing impurity concentration \( \chi_{\sigma\sigma}(q,\omega) \) is simply the Lindhard function, whose imaginary part, at low frequency, is given by \( \chi''_{\sigma\sigma}(q,\omega \to 0) = -(m^2/4\pi)(\omega/q) \) and whose real part can be approximated by its value at \( \omega = 0 \). Making use of this limiting form, the calculation of \( \rho_{\uparrow\downarrow} \) can be carried in an essentially analytical fashion. The result is

\[
\text{Re} \rho_{\uparrow\downarrow}(\omega, T) = \frac{\hbar a 4\pi^2(k_B T)^2 + \omega^2}{6(R_y)^2} \frac{1}{24\pi^3 \bar{n}_s \bar{n}_\uparrow} \int_{-\infty}^{\infty} \frac{d\bar{q}}{q^2 |\epsilon(q/a, 0)|^2}
\]

where \( a \equiv \hbar^2/me^2 \) is the effective Bohr radius, \( R_y = e^2/2a \) is the effective Rydberg, \( \bar{q} \equiv qa \), \( \bar{n}_\sigma \equiv n_\sigma a^3 \) and \( \epsilon(q, \omega) = 1 - v_q \chi_{0\uparrow}(q, \omega) - v_q \chi_{0\downarrow}(q, \omega) \) is the RPA dielectric function. Eq. (14) shows that, in the absence of impurities, \( \rho_{\uparrow\downarrow}(\omega, T) \) is proportional to \( \omega^2 \) for \( k_B T \ll \omega \) and to \( T^2 \) for \( \omega \ll k_B T \).

Modifications in the form of \( \chi_{\sigma\sigma}(q, \omega) \) due to the presence of impurities can be taken into account through Mermin’s approximation scheme [13]. These modifications amount to replacing \( \omega/q v_F \) by \( \omega/D q^2 \) (\( D = v_F^2 \tau/3 \) being the diffusion constant) for \( \omega < 1/\tau \) and \( q < 1/v_F \tau \) where \( v_F \) is the Fermi velocity and \( \tau \) is the electron-impurity mean scattering time. The \( \omega \) and \( T \) dependencies of Eq. (14) are not affected.

Writing explicitly in Eq. (14) the dependence over \( r_{s\sigma} \) (where \( r_{s\sigma} = (4\pi n_\sigma a^3/3)^{-1/3} \) is the usual electron gas parameter) one can also see that \( \rho_{\uparrow\downarrow}(\omega, T) \sim r_{s\uparrow}^3 r_{s\downarrow}^3 \) so that \( \rho_{\uparrow\downarrow} \) will strongly increase with decreasing electron density. In Fig. 2 we plot \( \rho_{\uparrow\downarrow}(\omega = 0, T) \) for \( n_\uparrow = n_\downarrow \), at metallic densities (roughly \( 2 < r_s < 6 \)) in the temperature range \( 10K < T < 70K \). It can be seen that for temperatures of the order of \( 40 - 60K \) (at which for example experiments on spin relaxation time using spin polarized currents have been performed [3]), the spin trans-resistivity is appreciable \( (\rho_{\uparrow\downarrow}(\omega = 0, T) \gtrsim 0.01 \mu\Omega cm) \).

In the remaining part of this paper, we describe an experiment aimed at detecting the effect of the Spin Coulomb Drag and measuring the spin trans-resistivity. The setup is shown in Fig.3: a paramagnetic metal film of thickness \( L \) is sandwiched between two ferromagnets polarized in the same direction. A battery is connected to the ferromagnets inducing a spin-polarized current from the first ferromagnet (“injector”) through the paramagnet and toward the second ferromagnet (“receiver”). The injector and receiver are chosen to be semi-metals, i.e., they have only electron states of spin \( \uparrow \) at the Fermi level (see Fig.3). It follows that the injected current \( j_\uparrow \) is carried only by spin \( \uparrow \) electrons. If we choose \( L \ll \delta_s \), where \( \delta_s \) the spin relaxation length, we can safely neglect spin-flipping processes and the polarized current entering the paramagnet will not relax before reaching the receiver. Spin relaxation lengths are relatively large in some materials (\( \delta_s \approx 100 \mu m \) in Al [3]), so the condition \( L \ll \delta_s \) is not particularly restrictive. Due to the SCD, the injected \( J_\uparrow \) will drag spin \( \downarrow \) electrons toward the junction with the receiver. But, since there is no conduction band available in the receiver for spin \( \downarrow \) electrons the circuit will behave as an open circuit.
for spin $\downarrow$ electrons, i.e., $j_\downarrow = 0$. The vanishing of $j_\downarrow$ is an indication that the Coulomb drag force is exactly balanced by the gradient of the electro-chemical potential for spin down

$$-eE_\downarrow + m\gamma \frac{j_\uparrow}{n_\uparrow} = 0.$$  \hspace{1cm} (15)

where $E_\downarrow = \nabla\mu_\downarrow/e + E$ is the sum of the electrostatic field $E$ and the gradient of the chemical potential $\mu_\downarrow$. What Eq. (15) tells us is that due to the SCD there will be a measurable electro-chemical potential difference $eE_\downarrow l = em\gamma j_\downarrow l/n_\uparrow$ for spin $\downarrow$ electrons between two points within the metal separated by a distance $l$ along the direction of the current.

To measure this potential difference a second circuit including a voltmeter of very large resistance is connected to the regions of the paramagnet close to the junctions (See Fig. 3). Our purpose is to measure $E_\downarrow$, so this second circuit must be driven by the spin $\downarrow$ electro-chemical potential only. In order to accomplish this, we propose to use as contacts two semi-metallic ferromagnetic electrodes (“detectors”), similar to the injector and the receiver, but polarized in the opposite direction. In this way, for the same reasons explained before, the detection circuit will be “open” as far as spin $\uparrow$ electrons are concerned, and the current flowing in the voltmeter will be exclusively driven by the electro-chemical potential difference of spin $\downarrow$ electrons. The spin trans-resistivity will then be given by $\rho_{\uparrow\downarrow} = (\Delta V_D/I_\uparrow)(A/l)$, where $\Delta V_D$ is the voltage measured by the meter, $A$ is the cross-section of the paramagnetic metal, $l$ is the distance between the detectors, and $I_\uparrow$ the current flowing between injector and receiver. As shown by our calculations, we expect a resistivity of the order of $10^{-2}\mu\Omega\text{cm}$ that is proportional to $T^2$ for $k_B T >> \omega$.

In summary we have pointed out a novel effect in spin-polarized transport, the Spin Coulomb Drag, and have proposed an experiment to observe it. We hope that this paper will stimulate experimental work aimed to the detection of this effect.

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[11] The reader can verify this noting that in the limit of weak Coulomb interaction the spin up and spin down subsystems are decoupled so that $\langle \rho_{q} \rho_{\sigma} - \rho_{-q} \rho_{\sigma} \rangle \sim \delta_{q,q'} \langle \rho_{q} \rho_{\sigma} - \rho_{-q} \rho_{\sigma} \rangle$ and that $\langle \rho_{q} \rho_{\sigma} \rangle = \chi_{\sigma}(q,0)v_{q}^{\sigma} e^{-i\omega} \rho_{-q}$. where $\chi_{\sigma}(q,0)$ is the static density-density response function for spin $\sigma$. Inclusion of higher order terms would take us beyond the level of accuracy of the approximation leading to Eq. (6).

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FIGURES

FIG. 1. The two series of “bubble” diagrams for the four-point response function $\chi_{4\rho}$ in the RPA. The vertices represent spin-density fluctuations $\rho_{q\sigma}$ as labelled.

FIG. 2. Density and temperature dependence of $\rho_{\uparrow\downarrow}(0,T)$ in a paramagnetic metal. Top and bottom lines correspond to $T = 70K$ and $T = 10K$ respectively. Temperature is incremented in steps of $10K$ starting from the bottom.

FIG. 3. (a) Experimental setup to detect the SCD effect: the voltage $\Delta V$ is applied between two parallel semi-metallic ferromagnets (injector (inj.) and receiver (rec.)) that sandwich a paramagnet (P). The voltage $\Delta V_D$ is detected using two ferromagnetic electrodes (d) similar to the injector and the receiver, but polarized in the opposite direction. (b) Schematic bandstructure of injector, receiver, d and P.
Fig. 1
Fig. 2
Fig. 3