Fluctuating hydrodynamics for driven granular gases

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Abstract. We study a granular gas heated by a stochastic thermostat in the dilute limit. Starting from the kinetic equations governing the evolution of the correlation functions, a Boltzmann-Langevin equation is constructed. The spectrum of the corresponding linearized Boltzmann-Fokker-Planck operator is analyzed, and the equation for the fluctuating transverse velocity is derived in the hydrodynamic limit. The noise term (Langevin force) is thus known microscopically and contains two terms: one coming from the thermostat and the other from the fluctuating pressure tensor. At variance with the free cooling situation, the noise is found to be white and its amplitude is evaluated.

1 Introduction

Typically, a granular system is defined as an ensemble of macroscopic particles which collide inelastically, i.e., part of the kinetic energy of the grains is dissipated in a collision. This simple ingredient gives rise to a very rich phenomenology which is of interest not only from practical or industrial perspective, but also because of the resulting new theoretical challenges [1–4]. One of the most widely employed idealized model for granular fluids is a system of smooth hard spheres (or disks in two dimensions) whose collisions are characterized by a constant coefficient of normal restitution [5,6]. For this model, and considering that the particles move freely between collisions, kinetic equations have been derived: starting from the dynamics of the particles, it is possible to derive the corresponding Liouville equation, and the Boltzmann equation results in the low density limit [7,8]. This kinetic equation has been extensively used to address many fundamental questions such as the derivation of the hydrodynamic equations, with explicit expressions for the transport coefficients, which have been derived by the Chapman-Enskog method [9,10] and also via the linearized Boltzmann equation [11]. Due to the inelasticity of the collisions, the total energy of an isolated granular system decays monotonically in time. In the fast-flow regime, it has been shown numerically that, for a wide class of initial conditions, the system reaches the so-called Homogeneous Cooling State (HCS), in which all the time dependence of the one-particle distribution function goes through the granular temperature, which is defined as the second velocity moment of the distribution [12,13]. This state has been extensively studied in the literature and very recently the fluctuations of the transverse velocity have been analyzed [14,15]. It has been found that the transverse velocity fulfills a Langevin equation but, in contrast to the elastic case, the noise is not white and the second moment of the fluctuations is not only controlled by the viscosity but also depends on a new coefficient. Similar results are found for the other hydrodynamic equations [16]. The study of fluctuations in the HCS is important for the development of a general theory of fluctuations in granular systems because it defines the reference state from which macroscopic hydrodynamic equations can be derived [9]. In this sense, the HCS plays, for inelastic gases, a role similar to the equilibrium state for molecular gases.
On the other hand, there are situations in which the grains cannot be considered to move freely between collisions. If, for example, the grains are immersed in a medium which acts as a thermostat, the system may reach a stationary state in which the energy injected by the thermostat is compensated by the energy dissipated in collisions. Note that, if the grains are Brownian particles, the interstitial medium injects energy into the granular system, but also acts as an energy sink due to frictional forces. One of the simplest mechanisms that can be considered to thermalize the system is a white noise force acting on each grain, which results in the so-called stochastic thermostat [17–28]. One important point is that the distribution function differs from that of the HCS [17], and that it is this distribution which plays the role of the “reference state”. The non-equilibrium steady state that the system reaches in the long time limit exhibits long-range correlations which are in agreement with the predictions of fluctuating hydrodynamics [20]. The latter description was introduced phenomenologically, and is expected to be valid in the vicinity of the elastic limit only. The objective of this work is to derive these equations from a more fundamental point of view and without the restriction of small inelasticity. More precisely, we adapt the formalism worked out in [15] for the free cooling, to the present driven case. Starting from a Boltzmann-Langevin description, we derive a fluctuating equation for the transverse velocity identifying the noise of this equation. Under certain hypothesis to be clarified in the text, we obtain that the correlation function of the noise is well approximated by the one introduced in Ref. [20], where the internal noise contribution (excluding the “external” noise term directly stemming from the thermostat) fulfilled a fluctuation-dissipation relation as for conservative fluids [29].

The remainder of the paper is organized as follows. In Section 2 previous results for a system heated by a stochastic thermostat are presented, such as the equations for one-particle distribution function and the two-particle correlation function. In Section 3 the Boltzmann-Langevin equation for this system is derived and the properties of the noise are inferred. The particular case of the transverse velocity field is analyzed in Section 4 and finally, the conclusions are presented in Section 5.

2 Stochastic thermostat: Preliminary results

The system considered is a dilute gas of $N$ smooth inelastic hard particles of mass $m$ and diameter $\sigma$. The position and velocity of the $i$th particle at time $t$ will be denoted by $R_i(t)$ and $V_i(t)$, respectively. The effect of a collision between two particles $i$ and $j$ is to instantaneously modify their velocities according to the collision rule

$$V'_i = V_i - \frac{1 + \alpha}{2}(\hat{\sigma} \cdot V_{ij})\hat{\sigma}, \quad V'_j = V_j + \frac{1 + \alpha}{2}(\hat{\sigma} \cdot V_{ij})\hat{\sigma},$$

where $V_{ij} \equiv V_i - V_j$ is the relative velocity, $\hat{\sigma}$ is the unit vector pointing from the center of particle $j$ to the center of particle $i$ at contact, and $\alpha$ is the coefficient of normal restitution. It is defined in the interval $0 < \alpha \leq 1$ and it will be considered here as a constant, independent of the relative velocity. Between collisions, the system is heated uniformly by adding a random velocity to the velocity of each particle at equal times. The driving is implemented in such a way that the time between random kicks is small compared to the mean free time. Then, between collisions, the velocities of the particles undergo a large number of kicks due to the thermostat.

In addition, we will assume that the “jump moments” of the velocities of the particles verify

$$B_{i,j,\beta\gamma} \equiv \lim_{\Delta t \to 0} \frac{\langle \Delta V_{i,\beta} \Delta V_{j,\gamma}\rangle_{\text{noise}}}{\Delta t} = \xi_0^2 \delta_{ij} \delta_{\beta\gamma} + \frac{\xi_0^2}{N}(\delta_{ij} - 1)\delta_{\beta\gamma},$$

where we have introduced $\Delta V_{i,\beta} \equiv V_{i,\beta}(t+\Delta t) - V_{i,\beta}(t)$, $\langle \cdots \rangle_{\text{noise}}$, which denotes average over different realizations of the noise. The non-diagonal terms (corresponding to $i \neq j$ and $\beta = \gamma$) are necessary in order to conserve the total momentum [32].