The reconstruction of inflationary potential with non-minimal derivative coupling model without high friction limit

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Abstract

We derive the reconstruction formulae for the inflation model with the non-minimal derivative coupling term. If reconstructing the potential from the tensor-to-scalar ratio, we could obtain the potential without using the high friction limit. As an example, we reconstruct the potential from the parametrization \( r = 8\alpha/(N + \beta)^\gamma \), which is a general form of the \( \alpha \)-attractor. The reconstructed potential has the same asymptotic behavior as the T- and E-model if we choose \( \gamma = 2 \) and \( \alpha \ll 1 \).

We also discuss the constraints from the reheating phase preceding the radiation domination by assuming the parameter \( w_{re} \) of state equation during reheating is a constant. The scale of big-bang nucleosynthesis could put a up limit on \( n_s \) if \( w_{re} = 2/3 \) and a low limit on \( n_s \) if \( w_{re} = 1/6 \).
I. INTRODUCTION

In the standard big-bang cosmology, inflation successfully has solved various problems, such as the flatness, horizon and monopole problems. Besides, it’s quantum fluctuation can produce the seed of the formation of large-scale structure [1–4]. A scalar field with a flat potential is usually chosen to investigate inflation. The most economical and fundamental candidate for the inflaton is therefore the Standard Model Higgs boson. However, the Higgs boson is disfavored by the observational data [3, 5] when minimally coupled to gravity due to its large tensor-to-scalar ratio. If the kinetic term of the scalar field is non-minimally coupled to Einstein tensor, the tensor-to-scalar ratio $r$ could be reduced to being consistent with the observational data, and the effective Higgs self-coupling $\lambda$ could be the order of 1 [6, 7]. This inflation model with non-minimal derivative coupling belongs to the subclass of the Horndeski theory [8], which is a general scalar-tensor theory, with field equations that are at most of the second order derivatives of both the metric $g_{\mu\nu}$ and scalar field $\phi$ in four dimensions [9]. Therefore, the non-minimal derivative coupling inflation model could save the Higgs model without introducing a new degree of freedom. For more about the non-minimal derivative coupling inflation model, refer to [10–14].

The most important observables of inflation are the spectral tilt $n_s$ and the tensor-to-scalar ratio $r$. To be compared with the observational data easily, they are usually expressed in terms of the $e$-folding number $N_*$ before the end of inflation at the horizon exit of the pivotal scale. For example, the Starobinsky model[1] gives $n_s = 1 - 2/N_*$ and $r = 12/N_*^2$, which is consistent with the Planck 2018 results $n_s = 0.9649 \pm 0.0042$ and $r_{0.002} < 0.064$ with $N_* = 50 - 60$. A usual method to research the inflation is to obtain the observables from a given physical potential. In this paper, we choose the inverse way, starting from the observational data and parameterizing the observables with $N_*$ to reconstruct the potential [15, 16]. By this reconstruction, the model parameters can be constrained easily and the reconstructed potential would be always consistent with the observational data [16, 39].

After the inflation, it is followed by the reheating phase which may provide further constraints on inflationary models [40]. Assuming that the effective parameter $w_{re}$ of state equation during reheating is a constant and the entropy is a conserved quantity, we can relate the $e$-folding number and the energy scale during reheating to that during inflation [40–45]. From these relations, the constraints on the energy scale during reheating would
transfer to the constraints on the inflation model.

In this paper, we use the reconstruction method to reconstruct the inflationary potentials and discuss additional constraints from the reheating. The paper is organized as follows. In section II, we review the inflation model with non-minimal derivative coupling and the reconstruction method. In section III, we reconstruct the potential from the parametrization of tensor-to-scalar ratio. We discuss the constraints from the reheating in section IV, and the paper is concluded in section V.

II. THE RELATIONS

In this section we develop the formulae for the reconstruction of the inflationary potential with the kinetic term non-minimally coupled to Einstein tensor. We start from the action

\[ S = \frac{1}{2} \int d^4x \sqrt{-g} \left[ R - g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{M^2} G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - 2V(\phi) \right], \]  

(1)

where we choose the unit \( c = M_{pl}^2 = 1/(8\pi G) = 1 \) and \( M \) is the coupling constant with the dimension of mass. The Friedmann equation is

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{3} \left[ \frac{\dot{\phi}^2}{2} (1 + 9F) + V(\phi) \right], \]  

(2)

where \( F = H^2/M^2 \) is the friction parameter. The equation of motion for the scalar field \( \phi \) is

\[ \frac{d}{dt} \left[ a^3 \dot{\phi}(1 + 3F) \right] = -a^3 \frac{dV}{d\phi}. \]  

(3)

If the scalar field slowly rolls down the potential, the slow-roll conditions is then

\[ \frac{1}{2} (1 + 9F) \dot{\phi}^2 \ll V(\phi), \]

\[ |\dddot{\phi}| \ll |3H \dot{\phi}|, \]  

| \frac{2\dot{H}}{M^2 + 3H^2} | \ll 1. \]  

(4)

Under these slow-roll conditions, the background Eqs. (2) and (3) become

\[ H^2 \approx \frac{V(\phi)}{3}, \]  

(5)

\[ 3H \dot{\phi}(1 + 3F) \approx -V_\phi, \]  

(6)
where \( V_\phi = dV/d\phi \). With Eq. (5), the friction parameter becomes

\[
F \approx \frac{V(\phi)}{3M^2}.
\]

(7)

To quantify those slow-roll conditions (4), we introduce the slow-roll parameters

\[
\epsilon_V = \frac{1}{2} \left( \frac{V_\phi}{V} \right)^2 \frac{1 + 9F}{(1 + 3F)^2},
\]

\[
\eta_V = \frac{1}{1 + 3F} \frac{V_{\phi\phi}}{V}.
\]

(8)

(9)

Using Eqs. (5), (6) and (8), we obtain

\[
\frac{3\dot{\phi}^2(1 + 9F)}{2V(\phi)} \approx \epsilon_V.
\]

(10)

The derivative of \( \epsilon_V \) with respect to \( t \) is [10]

\[
\dot{\epsilon}_V = 2H \epsilon_V \left[ \frac{2 + 21F + 81F^2}{(1 + 9F)^2} \epsilon_V - \eta_V - \frac{4 + 72F + 603F^2 + 2538F^3 + 5103F^4}{3(1 + 3F)(1 + 9F)^3} \epsilon_V^2 
+ \frac{2(2 + 48F + 441F^2 + 1944F^3 + 3645F^4)}{3(1 + 3F)(1 + 9F)^3} \epsilon_V \eta_V - \frac{1}{3} \eta_V^2 \right].
\]

(11)

By using the relation \( dN = -Hdt \) to the first slow-roll parameters, Eq. (11) becomes

\[
\frac{d \ln \epsilon_V}{dN} = 2 \left[ \eta_V - \frac{2 + 21F + 81F^2}{(1 + 9F)^2} \epsilon_V \right],
\]

(12)

where \( N \) is the e-folding number before the end of inflation at the horizon exit. To the first order of slow-roll parameters, the power spectrum of the scalar perturbation is [10]

\[
P_\zeta \approx \frac{1 + 9F}{1 + 3F} \times \frac{H^2}{8\pi^2 \epsilon_V}.
\]

(13)

The power spectrum of the tensor perturbation is [10]

\[
P_T \approx \frac{2H^2}{\pi^2}.
\]

(14)

The scalar tilt \( n_s \) and the tensor-to-scalar ratio \( r \) are [10] [46]

\[
n_s - 1 = 2\eta_V - \frac{6(1 + 4F)}{1 + 9F} \epsilon_V,
\]

(15)

\[
r = \frac{16(1 + 3F)}{1 + 9F} \epsilon_V.
\]

(16)
From Eqs. (12) and (15), we obtain the relation between $n_s$ and $\epsilon_V$,

$$n_s - 1 = \frac{d \ln \epsilon_V}{dN} = \frac{2 + 36F + 54F^2}{(1 + 9F)^2} \epsilon_V. \quad (17)$$

From Eqs. (2) and (10), we obtain the relation between $\phi$ and $N$,

$$d\phi = \pm \sqrt{\frac{2\epsilon_V}{1 + 9F}} dN, \quad (18)$$

where the sign $\pm$ is the same as the sign of $dV/d\phi$. Without loss of generality, in this paper, we only research the ‘$+$’ case. Combining Eqs. (8) and (18), we get the relation between the potential and the slow-roll parameter,

$$\epsilon_V = \frac{1 + 9F}{2 + 6F} (\ln V)_{,N}. \quad (19)$$

By using Eqs. (7) and (16), the Eqs. (13), (17) and (19) become

$$P_\zeta = \frac{2H^2}{\pi^2 r}, \quad (20)$$

$$n_s - 1 = \frac{d \ln r}{dN} - \frac{r}{8}, \quad (21)$$

$$r = 8(\ln V)_{,N}. \quad (22)$$

These relations escape from the friction parameter $F$, so it is possible to reconstruct the potential from the tensor-to-scalar ratio without using the high friction limit. In the following sections, we will discuss this issue.

**III. THE RECONSTRUCTION**

In this section, we will reconstruct the potential from the tensor to scalar ratio. The observational data favor small $r$, and the $\alpha$-attractor gives $r = 12\alpha/N^2$ which is very small when $\alpha \ll 1$. In this section, we consider the general parametrization of the $\alpha$-attractor

$$r = \frac{8\alpha}{(N + \beta)^\gamma}, \quad (23)$$

where $\gamma > 1$, and $\beta$ accounts for the contribution from the scalar field $\phi_e$ at the end of the inflation. From the relation (21), we obtain the spectral tilt

$$n_s - 1 = -\frac{\gamma}{N + \beta} - \frac{\alpha}{(N + \beta)^\gamma}. \quad (24)$$
With the help of relation (22), we obtain the potential

\[ V = V_0 \exp \left[ -\frac{\alpha}{(\gamma - 1)(N + \beta)^{\gamma - 1}} \right], \quad (25) \]

where

\[ V_0 = \frac{3}{2} \pi^2 A_s r \exp \left[ \frac{\alpha}{\gamma - 1} \left( \frac{r}{8\alpha} \right)^{\frac{\gamma - 1}{\gamma}} \right]. \quad (26) \]

Substituting Eq. (25) into Eq. (19), we obtain the slow-roll parameter

\[ \epsilon_V = 1 + \frac{3F_0 \exp \left[ \alpha(1 - \gamma)^{-1}(N + \beta)^{1 - \gamma} \right]}{2 + 2F_0 \exp \left[ \alpha(1 - \gamma)^{-1}(N + \beta)^{1 - \gamma} \right] \times \frac{\alpha}{(N + \beta)^\gamma}}, \quad (27) \]

where the amplitude of the friction parameter \( F_0 = \frac{V_0}{M^2} \). From the condition of the end of inflation, \( \epsilon_V(0) = 1 \), we obtain the relation among \( \alpha \), \( \beta \) and \( \gamma \)

\[ \frac{1 + 3F_0 \exp \left[ \alpha(1 - \gamma)^{-1}(N + \beta)^{1 - \gamma} \right]}{2 + 2F_0 \exp \left[ \alpha(1 - \gamma)^{-1}(N + \beta)^{1 - \gamma} \right] \times \frac{\alpha}{\beta^\gamma}} = 1. \quad (28) \]

In the GR limit \( F_0 \ll 1 \), \( \alpha = 2\beta^\gamma \); in the high friction limit \( F_0 \gg 1 \), \( \alpha = 2\beta^\gamma/3 \). From Eq. (23), the tensor-to-scalar ratio \( r \) in the high friction limit is therefore smaller than that in the GR limit when \( \beta \) and \( \gamma \) is unchanged. Substituting Eq. (27) into Eq. (18), we get the relation between \( \phi \) and \( N \),

\[ d\phi = \sqrt{r \left( 8 + 8F_0 \exp \left[ \alpha(1 - \gamma)^{-1}(N + \beta)^{1 - \gamma} \right] \right)^{-1}} \ dN. \quad (29) \]

Combining it with Eq. (23), the relation becomes

\[ d\phi = \sqrt{r \left( 8 + 8F_0 \exp \left[ \alpha(1 - \gamma)^{-1}(8\alpha)^{(1 - \gamma)/\gamma} \times r^{(\gamma - 1)/\gamma} \right] \right)^{-1}} \ dN. \quad (30) \]

To the first order of tensor-to-scalar ratio \( r \), it becomes

\[ d\phi = \sqrt{\frac{r}{8 + 8F_0}} \ dN, \quad (31) \]

and the solution is

\[ \phi - \phi_0 = \begin{cases} \frac{2}{2 - \gamma} \sqrt{\frac{\alpha}{1 + F_0}} (N + \beta)^{\frac{2 - \gamma}{2}}, & \gamma \neq 2, \\ \sqrt{\frac{\alpha}{1 + F_0}} \ln(N + \beta), & \gamma = 2, \end{cases} \quad (32) \]
where $\phi_0$ is the integration constant. Substituting Eq. (32) into Eq. (25), we get the reconstructed potential

$$V(\phi) = \begin{cases} 
V_0 \exp \left[ -\lambda_2 \left( \sqrt{1 + F_0 \phi_0 - \sqrt{1 + F_0 \phi}} \right)^{\frac{2\gamma - 2}{\gamma}} \right], & \gamma \neq 2, \\
V_0 \exp \left[ -\alpha e^{-\sqrt{1+F_0(\phi-\phi_0)/\sqrt{\alpha}}} \right], & \gamma = 2,
\end{cases}$$

(33)

where

$$\lambda_2 = \frac{\alpha}{\gamma - 1} \left( \frac{\gamma - 2}{2\sqrt{\alpha}} \right)^{\frac{2\gamma - 2}{\gamma - 2}}.$$  

(34)

Therefore, we reconstruct the potential from the parameterization (23) without using the high friction limit. Furthermore the potential (33) shows that the effect of the no-minimally derivative coupling term is the rescaling of the inflaton field by a factor $\sqrt{1+F_0}$. For the $\alpha$-attractors parametrization $\gamma = 2$ in the GR limit $F_0 \ll 1$, the potential reduces to

$$V(\phi) = V_0 \exp \left[ -\alpha e^{-(\phi-\phi_0)/\sqrt{\alpha}} \right].$$

(35)

When $\alpha \ll 1$, this potential reduces to

$$V(\phi) = V_0 \left[ 1 - \alpha e^{-(\phi-\phi_0)/\sqrt{\alpha}} \right],$$

(36)

which is asymptotic behavior of the T model and E model.

Comparing the results (23) and (24) with the Planck 2018 observations [5], taking $N_* = 60$ and $F_0 \gg 1$, we get the constraints on the parameters $\beta$ and $\gamma$ shown in Fig. 2. Taking $\gamma = 1.99$, $\beta = 0.47$ and $N_* = 60$, we get $n_s = 0.968$ and $r = 0.00076$. With these parameters, the plot of the potential is shown in Fig. 1.

**IV. REHEATING**

When the inflaton rolls down to the minimum of the potential, inflation ends and the inflaton field begins to oscillate around the minimum and to reheat the cold universe. Although the physics of the reheating is uncertain, the reheating process may provide additional constraint on inflationary models. In this section, we will research the constraint from the reheating phase on the reconstructed model under the high friction limit $F \gg 1$.

The relation between the pivotal scale $k_* = 0.002 \text{Mpc}^{-1}$ and the current Hubble horizon is

$$\frac{k_*}{a_0 H_0} = \frac{a_* H_*}{a_0 H_0} = \frac{a_*}{a_0} \frac{a_{re}}{H_*} \frac{H_*}{H_0} = e^{-N_*-N_{re}} \frac{a_{re}}{a_0} \frac{H_*}{H_0},$$

(37)
FIG. 1. The reconstructed potentials are normalized with $V_0$ from Eq. (26), and the inflaton field is normalized with $1/\sqrt{F_0}$. We choose the value of $\phi_0$ that could make $\phi_e = 0$.

FIG. 2. The marginalized 68% and 95% confidence level contours for $n_s$ and $r_{0.002}$ from Planck data [5] and the theoretical predictions for the parametrization [23] in the high friction limit. The left panel shows the $n_s$-r contours and the right panel shows the constraints on $\beta$ and $\gamma$ for $N_s = 60$. The red and blue regions correspond to 68% and 95% confidence level.

where $a_{re}$ denotes the value of scale factor at the end of reheating, $N_{re}$ denotes the number of e-folds during reheating, and we assume that radiation domination begins immediately after the reheating, and reheating begins immediately after inflation. Assuming the parameter $w_{re}$ of state equation is a constant during reheating, we obtain

$$N_{re} = \frac{1}{3(1 + w_{re})} \ln \frac{\rho_e}{\rho_{re}},$$

where the relation between $\rho_{re}$ and the temperature $T_{re}$ is

$$\rho_{re} = \frac{\pi^2}{30} g_{re} T_{re}^4,$$
and $g_{re}$ is the effective number of relativistic species at reheating. From the entropy conservation, the relation between temperature $T_{re}$ and the current cosmic microwave background temperature $T_0 = 2.725K$ is

$$a_{re}^3 g_{s,re} T_{re}^3 = a_0^3 \left( 2T_0^3 + 6 \times \frac{7}{8} T_{\nu 0}^3 \right), \quad (40)$$

where $g_{s,re}$ is the effective number of relativistic species for entropy and the current neutrino temperature $T_{\nu 0} = (4/11)^{1/3} T_0$. Combining the above results, we get [40][41]

$$N_{re} = \frac{4}{1 - 3w_{re}} \left[ -N_* - \ln \frac{\rho_e^{1/4}}{H_*} + \frac{1}{3} \ln \frac{43}{11g_{s,re}} + \frac{1}{4} \ln \frac{\pi^2 g_{re}}{30} - \ln \frac{k_*}{a_0 T_0} \right], \quad (41)$$

$$T_{re} = \exp \left[ -\frac{3N_{re} (1 + w_{re})}{4} \right] \left[ \frac{30\rho_e}{\pi^2 g_{re}} \right]^{1/4}. \quad (42)$$

These equations show that $N_{re}$ and $T_{re}$ depend on $g_{re}$ and $g_{s,re}$ logarithmically, so it is safe to take $g_{re} = g_{s,re} = 106.75$. At the end of inflation, we have $\epsilon_V \approx 1$; from Eq. [10], we obtain the relation $\dot{\phi}^2 = 2V_e/(27F)$, so we have $\rho_e = 4V_e/3$. By using the observational value[5]

$$A_s = 3H_*^2/(8\pi^2 \epsilon_V) = 2.2 \times 10^{-9}, \quad (43)$$

Eqs. (41) and (42) become

$$N_{re} = \frac{4}{1 - 3w_{re}} \left( 56.46 - N_* - \frac{1}{4} \ln V_e + \frac{1}{2} \ln \epsilon_V \right), \quad (44)$$

$$T_{re} = \exp \left[ -\frac{3N_{re} (1 + w_{re})}{4} \right] \left[ \frac{4V_e}{10.675\pi^2} \right]^{1/4}. \quad (45)$$

By using Eqs. (24) and (27), under the high friction limit, we obtain the constraint from the reheating process on the model parameters,

$$N_{re} = \frac{4}{1 - 3w_{re}} \left[ 60.45 + \frac{\alpha}{4(\gamma - 1)\beta^{\gamma - 1}} + \frac{1}{4} \ln \alpha - N_* - \gamma \frac{\alpha}{4} \ln(N_* + \beta) - \frac{\alpha}{4(\gamma - 1)(N_* + \beta)^{\gamma - 1}} \right], \quad (46)$$

$$T_{re} = 0.01 \left( \frac{\alpha^{1/4}}{(N_* + \beta)^{\gamma/4}} \right) \exp \left[ \frac{\alpha}{4(\gamma - 1)(N_* + \beta)^{\gamma - 1}} - \frac{\alpha}{4(\gamma - 1)^{\beta^{\gamma - 1}}} \right] \left[ -\frac{3N_{re} (1 + w_{re})}{4} \right], \quad (47)$$

where $\alpha = 2\beta^{\gamma}/3$. For different values of $\beta, \gamma, N_*$ and $w_{re}$, we calculate $n_s, N_{re}$ and $T_{re}$ from Eqs. (24), (46) and (47), and the results are shown in figure 3. Different model parameters
FIG. 3. $N_{re}$ (upper panels) versus $n_s$ as determined from Eqs. (24) and (46), and $T_{re}$ (lower panels) versus $n_s$ as determined from Eqs. (24) and (47). The values of $\beta$ and $\gamma$ are indicated in each panel. The gray band corresponds to the 1$\sigma$ Planck constraint $n_s = 0.9649 \pm 0.0042$ [5], and the 1$\sigma$ constraint on $N_*$ is also given. In each panel, the black, red, blue and green lines denote $w_{re} = -1/3, 0, 1/6$ and 2/3, respectively; the arrow indicates $N_*$ increases along the line. The horizontal gray solid and dashed lines in lower panels correspond to the electroweak scale $T_{EW} \sim 100$ GeV and the big bang nucleosynthesis scale $T_{BBN} \sim 10$ MeV, respectively.

$\beta$ and $\gamma$ and the value of $w_{re}$ give different constraints on $N_{re}$ and $T_{re}$, but the parameter $\beta$ has little impact on the reheating phase. For the parameters $\beta$ and $\gamma$ that make $n_s$ consistent with the observation, reheating with $-1/3 \leq w_{re} \leq 2/3$ are all consistent with the observations. As $n_s$ becomes larger, the allowed reheating epoch becomes longer for $w_{re} = -1/3, 0$ and 1/6 while the allowed reheating epoch becomes shorter for $w_{re} = 2/3$. 

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V. CONCLUSION

The non-minimal derivative coupling term in the inflation model could reduce the tensor to scalar ratio to save the large tensor-to-scalar ratio model. We derive the reconstructed formulae of the inflation model with non-minimal derivative coupling. To reconstruct the potential without using the high friction limit, we consider the parameterization of the tensor to scalar ratio $r = 8\alpha/(N + \beta)^\gamma$ inspired from the $\alpha$-attractor. For $\gamma = 2$, which is the $\alpha$ attractor, we get the same potential as obtained in Ref. [31], in the GR limit $F \ll 1$. When $\alpha \ll 1$, this potential has the same asymptotic behavior as that of $\alpha$-attractor. For $\gamma \neq 2$, the potential is the exponential form. The observational constraints on the parameters are $1.2 < \gamma < 2.7$ and $\beta < 10$. The reconstruction also show that the observational data favor the $\alpha$ attractor case with $\gamma \approx 2$.

The constraints on the spectral tilt $n_s$ could provide additional constraints on the reheating phase. The different model parameters provide different constraints on $N_{re}$, $T_{re}$ and $w_{re}$. If the model parameters are chosen to make $n_s$ consistent with the observations, then reheating with $-1/3 \leq w_{re} \leq 2/3$ are all consistent with the observations. The energy scale of the reheating could also provide additional constraints on the inflation. If $\gamma = 2$, $\beta = 1$ and $w_{re} = 2/3$, the big bang nucleosynthesis scale requires $n_s < 0.967$; if $\gamma = 2$, $\beta = 1$ and $w_{re} = 1/6$, the big bang nucleosynthesis scale requires $n_s > 0.962$.

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