Fixed point variations of a frequency comb generated by a passively mode-locked fiber laser

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Abstract: We report fixed point variations of a frequency comb generated by a passively mode-locked fiber laser. We measured the pump induced changes in the repetition frequency and the carrier-envelope offset frequency in slightly different intracavity polarization states. The responses of the two frequencies varied by changing the polarization state. Each frequency had a specific polarization state in which it did not depend on the pump power. The fixed point for a pump induced change varied across ∼1 PHz range. This study shows the possibility of realizing a robust and low-noise frequency comb.

Keywords: optical frequency comb, fiber laser, mode-locked laser, fixed point

Classification: Optical systems

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1 Introduction

Mode-locked lasers generating ultra-short pulse trains have been employed in various fields including the observation of ultrafast dynamics owing to their temporal energy concentration [1, 2] and the generation of a frequency comb thanks to their equal mode spacing [3, 4]. Of the various types of mode-locked lasers, fiber-based lasers [5, 6] are of great importance because of their high robustness [7, 8] and broad servo bandwidths [9, 10]. The frequency of each comb mode of an optical frequency comb is stabilized by, for example, phase-locking the repetition frequency ($f_{\text{rep}}$) and the carrier-envelope offset frequency ($f_{\text{CEO}}$). Two different actuators are needed to stabilize the two frequencies simultaneously. Generally, the fiber length is changed by using piezo-electric transducers (PZTs) [11], electro-optic modulators [10, 12] or distinct optically pumped rare-earth-doped fibers [13, 14] for $f_{\text{rep}}$ control and the pump power is changed for $f_{\text{CEO}}$ control [15]. Employing a pair of actuators with control orthogonality for the two frequencies is preferable because it enables us to lock them without the actuator that we use for $f_{\text{rep}}$ control changing $f_{\text{CEO}}$ and vice versa. However, the actuators inside the laser cavity often change both frequencies simultaneously [11, 16]. A
“fixed point ($v_{fix}$)” can be used as an index for discussing the control orthogonality of two given actuators [17]. The fixed point is a specific frequency around which the entire set of comb modes expand and contract on the frequency domain when we operate an actuator. The comb mode frequency at the fixed point does not change when we operate the actuator. Thus, the comb modes around the fixed point have small frequency fluctuation. It is preferable to be able to tune $v_{fix}$ for a given actuator so that we can control orthogonality between two actuators and reduce phase noise of comb modes around $v_{fix}$.

In ref. [17], they theoretically estimated that $v_{fix}$ for the pump power of a mode-locked fiber laser (oscillator) based comb is around the carrier frequency but $v_{fix}$ can deviate from that frequency under a specific laser condition. $v_{fix}$ variation was experimentally confirmed by changing a pump power bias of a fiber oscillator [11]. However, large pump power change of an oscillator tends to make the mode-locking unstable. An alternative method to vary $v_{fix}$ is attractive to enhance design flexibility of a fiber oscillator based comb.

In this work, we report $v_{fix}$ variations of a frequency comb generated using a soliton fiber laser passively mode-locked by nonlinear polarization rotation (NPR) as an oscillator. We experimentally measured the pump-induced changes in $f_{rep}$ ($\Delta f_{rep}$) and $f_{CEO}$ ($\Delta f_{CEO}$) of the comb as we slightly changed the polarization state (PS) in the oscillator cavity. The center wavelength of the oscillator spectrum ($\lambda_c$) was also changed by the PS change. The change rates and signs of the frequencies varied with PS, and each frequency showed no pump-induced change at a specific PS. $v_{fix}$ for the pump power change varied $\sim$1 PHz range by the slight PS change.

### 2 Experimental setup

Fig. 1 shows a schematic diagram of the experimental setup used in this study. The oscillator had a cavity length of around 2.5 m that consisted of a 0.5-m-long erbium-doped fiber (EDF) and a 2.0-m-long standard single-mode fiber. The $f_{rep}$ of the oscillator was about 80 MHz. The EDF was pumped by a 1480 nm laser diode in a backward direction through a wavelength-division-multiplexing coupler. An isolator ensured stable single direction lasing. The oscillator output was extracted through a 30% output coupler. Passive mode-locking was obtained based on NPR by appropriately controlling the polarizer and the polarization controller. The threshold pump power for the mode-locking was approximately 90 mW. From the fiber specifications that we used for the oscillator, we estimated that the oscillator had a total cavity dispersion of $-0.035(3)$ ps². We chose the relatively large negative dispersion to easily determine $\lambda_c$. In general, the oscillator with a large negative dispersion emits a narrow and clean spectrum of a soliton-like pulse and it is easy to determine $\lambda_c$ from such the spectrum.

The output was propagated through an isolator and divided into two branches by a 10:90 coupler. 90% of the light was made more than 1 octave wider with an erbium-doped fiber amplifier and a highly nonlinear fiber [18], and then used to detect an $f_{CEO}$ signal through an $f2f$ interferometer. 10% of the light was further divided by a 1:99 coupler for the simultaneous measurement of $f_{rep}$ in a 1-%-split path and an optical spectrum in a 99-%-split path.
We changed the intracavity PS by slightly rotating the quarter-wave plate (QWP) in the oscillator. Along with the QWP rotation, the center wavelength $\lambda_c$ was swept between 1558 and 1560 nm at a pump power of 94 mW. Here, we define $\lambda_c$ as the -3 dB center wavelength of the output spectrum. The rotation angle of the QWP was less than 2 degrees for the entire wavelength sweep. The oscillator output had no continuous wave component during the PS control. In each PS, $f_{\text{rep}}$ and $f_{\text{CEO}}$ are measured as increasing the pump power.

To suppress thermal frequency drift, we placed the oscillator in a box in which the temperature was controlled within $\pm 0.1^\circ$C. In addition to the temperature control, we applied step-wise pump power changes and monitored the resulting step-wise $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$ to suppress the measurement error caused by the thermal drift. The power and pulse width of the oscillator output were in the 4.5–5.1 mW and 300–400 fs ranges, respectively. When the pump power was constant, the output had a higher power and a wider spectrum, indicating a shorter pulse width, at a longer $\lambda_c$.

Fig. 2 shows the pump-power dependence of the output spectra from the oscillator with an intracavity PS that lased at 1558 nm with a 94-mW pump.

![Fig. 1. Experimental setup for fixed point tuning. LD: laser diode, pol: polarizer, HWP: half-wave plate, QWP: quarter-wave plate, iso: optical isolator, EDF: erbium-doped fiber, OSA: optical spectrum analyzer, PD: photo detector, HNLF: highly nonlinear fiber, BPF: optical band pass filter, PPLN: periodically poled lithium niobate.](image1)

Fig. 2. Optical spectra change with different pump powers at $\lambda_c = 1558.0$ nm. (a) logarithm scale, (b) magnified spectra around $\lambda_c$ in linear scale. In (b), only three spectra are shown for simplicity and points show $\lambda_c$ of each spectrum.
Figs. 2(a) and 2(b) show whole spectra in logarithm scale and magnified normalized spectra around $\lambda_c$ in linear scale, respectively. The spectra had Kelly sidebands, which typically appear in mode-locked soliton fiber laser outputs. In this PS, an increase in the pump power shifted $\lambda_c$ to a shorter wavelength with broadening spectral widths.

3 Experimental result and discussion

Fig. 3(a) and 3(b), respectively, show pump induced $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$ with a $\lambda_c$ shift from multiple $\lambda_c$ values swept by rotating the intracavity QWP at the 94-mW pump. Here, we experimentally confirmed the sign of $f_{\text{CEO}}$ by monitoring the variation of the beat frequency between the outputs of the oscillator and a frequency-stable continuous-wave laser while changing the phase locked $f_{\text{rep}}$ and $f_{\text{CEO}}$ individually. As shown in Figs. 3(a) and 3(b), an increase in the pump power shifted $\lambda_c$ to a shorter/longer wavelength when $\lambda_c$ was shorter/longer than 1559.2 nm with a 94-mW pump. $f_{\text{rep}}$ increased/decreased with a $\lambda_c$ shift to a shorter/longer wavelength, which is consistent with the fact that the cavity dispersion was negative. $\lambda_c$ did not shift from 1559.2 nm when we changed the pump power.

To understand the pump-induced responses in $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$, we consider changes in the group ($v_g$) and phase velocities ($v_p$) of the pulse (trains) inside the laser cavity. When we change the pump power, $f_{\text{rep}}$ is affected by $v_g$ change. For example, a $v_g$ increase reduces the round-trip time of the pulse and results in an increase in $f_{\text{rep}}$. On the contrary, a decrease in $v_g$ results in a decrease in $f_{\text{rep}}$. The pump power modulation changes the EDF gain profile. This causes $\lambda_c$ shift, which is a combination of linear and nonlinear shifts [16]. We assume that the $\lambda_c$ shift, accompanied by a $v_g$ change, showed PS dependence because net loss of the cavity

![Graph showing pump induced changes of $f_{\text{rep}}$ and $f_{\text{CEO}}$ at different $\lambda_c$.](image)

Fig. 3. Pump induced changes of (a) $f_{\text{rep}}$, (b) $f_{\text{CEO}}$ at different $\lambda_c$. The measurement uncertainty of $\lambda_c$ was $\sim0.01$ nm which was limited by the OSA used in the experiment. The free-running $f_{\text{rep}}$ fluctuation was of the order of 0.1 Hz at averaging time of 1 s. Measurement uncertainty of $\Delta f_{\text{rep}}$, which is shown by error bars in (a), was dominated by the fluctuation and had little effect on the measurement result as shown in (a). The error bars in (b) shows a visual observation uncertainty of $\Delta f_{\text{CEO}}$ which was estimated from the frequency fluctuation and the linewidth of the $f_{\text{CEO}}$ signal.
had PS dependence. The balance between the linear and nonlinear shifts may be changed by the PS change. Therefore, the $\Delta f_{\text{rep}}$ response depended on $\lambda_c$, which was swept by the QWP rotation.

In contrast, $f_{\text{CEO}}$ is affected by changes in both $v_g$ and $v_p$ when the pump power is changed. $f_{\text{CEO}}$ is expressed as $(f_{\text{rep}}\Delta \phi_{\text{CEO}})/(2\pi)$ where $\Delta \phi_{\text{CEO}}$ is a carrier-envelope phase shift between neighboring pulses. When the pump power is changed, $\Delta f_{\text{CEO}}$ is affected by both the $\Delta f_{\text{rep}}$ and the $\Delta \phi_{\text{CEO}}$ change. $\Delta f_{\text{rep}}$ and the $\Delta \phi_{\text{CEO}}$ change were induced by the changes in $v_g$ and in the strength of the self-phase modulation (SPM), respectively. An increase in the pump power enhances the SPM strength, and decreases $v_p$ and $\Delta \phi_{\text{CEO}}$. An increase in the pump power can only decrease $f_{\text{CEO}}$ through SPM while it can either increase or decrease $f_{\text{CEO}}$ through the $f_{\text{rep}}$ change. As described in Fig. 3, $f_{\text{rep}}$ can either increase or decrease depending on the PS. We assume that the fact that the $f_{\text{CEO}}$ did not depend on the pump power at $\lambda_c = 1559.5$ nm is the result of balancing the effects of $v_g$ and SPM strength changes in this PS. In addition, $f_{\text{rep}}$ did not change with the pump power at $\lambda_c = 1559.2$ nm as a result of the constant $v_g$. In this case, $f_{\text{CEO}}$ decreased mainly because of the SPM strength enhancement along with the pump power increase.

Here, we describe $v_{\text{bx}}$ variations for the pump power change in terms of the above-mentioned $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$ behavior. Fig. 4(a) summarizes the relationship between the $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$ at each $\lambda_c$. The wavelengths in Fig. 4 are $\lambda_c$ at a pump power of 94 mW. Each data point shows $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$ at pump powers of 94 to 109 mW. $v_{\text{bx}}$ for a given actuator is expressed as $-f_{\text{rep}}(df_{\text{CEO}}/df_{\text{rep}}) + f_{\text{CEO}}$ [16]. In Fig. 4(a), each data series has widely different slopes ($df_{\text{CEO}}/df_{\text{rep}}$), which shows large $v_{\text{bx}}$ variation.

Fig. 4(b) summarizes the result of a similar measurement for the oscillator with a different set of intracavity PS for mode-locking. We define the PS in the measurements for Figs. 4(a) and 4(b) as PS set #1 and #2, respectively. For the PS set #2, $\lambda_c$ was swept from 1559.2 to 1560.3 nm at a pump power of 94 mW by rotating the intracavity QWP. The measurement procedure of $\Delta f_{\text{rep}}$, $\Delta f_{\text{CEO}}$ and $\lambda_c$ was same as in the above-mentioned experiment. In contrast to PS set #1, the output power and spectral bandwidth were higher and wider at a shorter $\lambda_c$ with a constant pump power, respectively. An increase in the pump power induced a $\lambda_c$...
shift to a shorter/longer wavelength at a longer/shorter $\lambda_c$. Fig. 4(b) summarizes the relationship between $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$ at each $\lambda_c$ for PS set #2. The relationship showed the opposite tendency to one for PS set #1.

Fig. 5 summarizes $v_{\text{fix}}$ at different $\lambda_c$ calculated from $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$ in the two sets of PS shown in Fig. 4. $v_{\text{fix}}$ showed large variation of $\sim 1$ PHz for only a 2 nm range of $\lambda_c$. As shown in Figs. 4(a) and 4(b), the oscillator had a specific $\lambda_c$, or PS, such that $df_{\text{CEO}}/df_{\text{rep}} = +/\infty$ in both PS sets, which corresponds to that $v_{\text{fix}} = -/\infty$.

4 Conclusion

We reported $v_{\text{fix}}$ variation for a pump power change to a mode-locked fiber laser-based frequency comb by slightly rotating the intracavity QWP. The pump-induced $\Delta f_{\text{rep}}$ and $\Delta f_{\text{CEO}}$ varied with PS and each frequency had a specific PS where it did not depend on the pump power. $v_{\text{fix}}$ varied across the $\sim 1$ PHz range within $\lambda_c$ change of 2 nm corresponding to the slight PS change. We assume that the balance between the linear and nonlinear spectral shifts depended on intracavity PS and this is a probable reason for the wide $v_{\text{fix}}$ variation range. This study enables us to obtain a new perspective for a fiber based frequency comb design. In many cases, cavity length control and pump power control are employed as two control actuators to simultaneously stabilize $f_{\text{rep}}$ and $f_{\text{CEO}}$, respectively. It is preferable that the $v_{\text{fix}}$ for a pump power change is far from zero frequency to ensure high control orthogonality since the $v_{\text{fix}}$ for a cavity length change is known to be near zero frequency. On the other hand, the comb lines near $v_{\text{fix}}$ have relatively small frequency fluctuation caused by the pump power fluctuation compared with the lines far from $v_{\text{fix}}$. By tuning $v_{\text{fix}}$, we may be able to realize the wavelength selective suppression of the frequency fluctuation without any stabilizing with a broad servo bandwidth and/or optical system utilizing nonlinear effects. This suppression may provide further sophisticated optical frequency measurement and/or spectroscopy.

Fig. 5. $v_{\text{fix}}$ for pump power change at different $\lambda_c$ with two sets of PS. Vertical lines show the diverging $\lambda_c$ for the two sets of PS. The error bars show least square errors, which are dominated by the slope nonlinearity of each data series.
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