Calculated Inter-band B(E2)’s in the p-f Shell

Levering Wolfe and Larry Zamick (Corresponding author)
Department of Physics and Astronomy,
Rutgers University, Piscataway, New Jersey 08854

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1 Abstract

Whereas previous works for B(E2)’s in the even-even Ti isotopes focused on yrast transitions we here also consider inter-band transitions to a second group, i.e. states like $0^+_2$, $1^+_1$, $2^+_2$, $3^+_1$, $4^+_2$, $5^+_1$, etc. We focus on variations from one even-even Ti isotope to the next. We make a qualitative comparison with similar transitions in a heavier deformed nucleus.

2 Introduction

In the past one of us (LZ) was involved in simple single j shell calculations in the $f_{7/2}$ shell e.g. Bayman et al.[1], McCullen et al. [2], see also Sherr et al. [3]. Despite the simplicity of the model one could find meaningful results for magnetic moments, beta decay transitions [1,2] and especially spectroscopic factors in single nucleon transfer reactions such as $(p,d)$[3]. However in the single j shell model, with $j=f_{7/2}$, one could not make meaningful calculations of B(E2) transitions. The rates from the $J=0^+$ ground state of an even-even nucleus to the first $2^+$ state were much too weak. It was clear that the collectivity built up in these states could not be handled in a single j-shell model. At the other extremes we have the rotational model of Bohr and Mottelson [4]. It is clear from previous work [5] that although there is indeed considerable collectivity, one has not yet reached the stage where the rotational model is applicable.

Previous studies of even-even Ti isotopes showed reasonably strong B(E2)’s in the yrast band: $J=0_1$ to $2_1$, $2_1$ to $4_1$, etc. [5]. In this work we study transitions from states in the yrast band to a second group of states: $1_1$, $2_2$, $3_1$, $4_2$, $5_1$, i.e. second excited states of even J and lowest states of odd J. For orientation purposes we show the calculated spectrum of $^{46}$Ti in Fig 1. The shell model code NUSHELLX@MSU [6] was used to perform these calculations. Further details are given in the work of Honma et al. [7]. We show results for the GX1A[7] and FPD6 Interactions[8]. For both interactions the effective charges
of the proton and neutron are 1.5 and 0.5 respectively. Most of the B(E2)'s will be shown with J_i less than J_f. To turn things around one can use the relation

\[(2J_i + 1)B(E2, J_i \rightarrow J_f) = (2J_f + 1)B(E2, J_f \rightarrow J_i)\]  

We make comparisons with the rotational model as described by Bohr and Mottelson[4]. They give simple formulas for B(E2)'s and static quadrupole moments (they use I instead of J for angular momentum).

\[B(E2, I_i K \rightarrow I_f K) = \left( \frac{5}{16\pi} \right) Q_0^2 (I_i K^20|I_f K)^2\]  

\[Q(I, K) = \frac{3K^2 - I(I + 1)}{(I + 1)(2I + 3)} Q_0\]  

Where Q is the static quadrupole moment Q_0 is the intrinsic quadrupole moment. Our first group should be compared with the K=0 band of Bohr and Mottelson although we recognize that the shell model and rotational model are not exactly the same. Our second group differs from a K=2 band inasmuch as we include states 0_2 and odd J's - 1_1, 3_1 etc.

Our main concern will be transitions from the yrast band to the second group for which there are no such clear cut formulas.

The nuclei considered are ^{44}\text{Ti}, ^{46}\text{Ti}, ^{48}\text{Ti}, ^{50}\text{Ti} and ^{48}\text{Cr}. Whereas Robinson et al. [5] considered the even J yrast band we here show results which also include a second group as described in the introduction.

## Comments on the two interactions

The values of B(E2, 0_1 \rightarrow 2_1) are consistently larger with the FPD6 interaction than with GX1A. For example in ^{48}\text{Cr} the respective values are 1570 and 1254 e^2fm^4. A contributing factor for this can be found by looking at the single particle energies relative to f_{7/2} in Table 1. For example the p_{3/2}-f_{7/2} splitting with FPD6 is 1.8942 MeV, which is significantly smaller than the corresponding value for GX1A of 2.9447 MeV. There will therefore be more configuration mixing with FPD6 and this leads to an enhancement of the B(E2) strength.

| Table 1: Single Particle energies | FPD6       | GX1A       |
|----------------------------------|------------|------------|
| 0f_{7/2}                         | 0 (-8.3876)| 0 (-8.6240)|
| 1p_{3/2}                         | 1.8942     | 2.9447     |
| 0f_{5/2}                         | 6.4910     | 7.2411     |
| 1p_{1/2}                         | 3.9093     | 4.4870     |
4 The Tables

In Table 2 we compare $B(E2)$'s from $J_1$ to $(J + 2)_1$ (intra-band), $J_1$ to $(J + 2)_2$ (inter-band) and $J_2$ to $(J + 2)_2$ (intra-band). Comparisons are made between the FPD6 and GXIA interactions.

Table 2: Selected $B(E2)$'s from $J$ to $J + 2$ with the FPD6 interaction (left) and GXIA interaction (right) in $e^2$fm$^4$

|       | $0_1$ to $2_1$ | $0_1$ to $2_2$ | $0_2$ to $2_2$ |       | $0_1$ to $2_1$ | $0_1$ to $2_2$ | $0_2$ to $2_2$ |
|-------|---------------|---------------|---------------|-------|---------------|---------------|---------------|
| $^{44}$Ti | 699.00        | 0.11          | 212.00        |       | 526.00        | 28.30         | 330.00        |
| $^{46}$Ti | 780.50        | 46.03         | 50.80         |       | 624.00        | 4.03          | 161.00        |
| $^{48}$Ti | 638.00        | 108.40        | 13.50         |       | 521.00        | 99.10         | 7.28          |
| $^{50}$Ti | 569.00        | 0.68          | 0.61          |       | 502.00        | 0.07          | 77.90         |
| $^{52}$Cr | 1570.00       | 52.10         |               |       | 1254.00       | 3.08          | 294.00        |

|       | $2_1$ to $4_1$ | $2_1$ to $4_2$ | $2_2$ to $4_2$ |       | $2_1$ to $4_1$ | $2_1$ to $4_2$ | $2_2$ to $4_2$ |
|-------|---------------|---------------|---------------|-------|---------------|---------------|---------------|
| $^{44}$Ti | 343.00        | 1.29          | 118.00        |       | 246.00        | 0.01          | 100.00        |
| $^{46}$Ti | 399.00        | 5.73          | 14.90         |       | 286.00        | 11.70         | 50.60         |
| $^{48}$Ti | 349.00        | 0.36          | 134.00        |       | 269.00        | 5.62          | 83.60         |
| $^{50}$Ti | 212.00        | 0.85          | 18.80         |       | 176.00        | 1.10          | 9.87          |
| $^{52}$Cr | 789.00        | 11.70         | 366.00        |       | 609.00        | 12.60         | 89.20         |

|       | $4_1$ to $6_1$ | $4_1$ to $6_2$ | $4_2$ to $6_2$ |       | $4_1$ to $6_1$ | $4_1$ to $6_2$ | $4_2$ to $6_2$ |
|-------|---------------|---------------|---------------|-------|---------------|---------------|---------------|
| $^{44}$Ti | 232.00        | 9.54          | 87.80         |       | 155.00        | 33.80         | 83.50         |
| $^{46}$Ti | 314.00        | 6.97          | 0.39          |       | 228.00        | 4.38          | 1.54          |
| $^{48}$Ti | 73.03         | 147.00        | 44.12         |       | 87.80         | 95.50         | 57.10         |
| $^{50}$Ti | 88.50         | 15.50         | 107.40        |       | 67.90         | 10.50         | 102.00        |
| $^{52}$Cr | 657.00        | 15.08         | 487.00        |       | 487.00        | 8.91          | 68.90         |

|       | $6_1$ to $8_1$ | $6_1$ to $8_2$ | $6_2$ to $8_2$ |       | $6_1$ to $8_1$ | $6_1$ to $8_2$ | $6_2$ to $8_2$ |
|-------|---------------|---------------|---------------|-------|---------------|---------------|---------------|
| $^{44}$Ti | 146.00        | 1.90          | 47.60         |       | 94.70         | 2.97          | 39.40         |
| $^{46}$Ti | 250.10        | 0.01          | 119.00        |       | 190.00        | 0.12          | 107.00        |
| $^{48}$Ti | 91.50         | 9.06          | 74.50         |       | 102.00        | 5.70          | 73.90         |
| $^{50}$Ti | 0.00          | 31.40         | 5.98          |       | 0.43          | 14.40         | 2.62          |
| $^{52}$Cr | 561.00        | 19.30         | 468.00        |       | 403.00        | 3.82          | 121.00        |
5 Discussion of the B(E2) Tables

We start with a crude overview of the results. For the yrast transitions if we go in the opposite direction, namely from \( J \) to \((J-2)\), then in the rotational and vibrational models the B(E2)’s increase with \( J \) but in our shell model calculations they decrease with \( J \) after \( J=4 \) [4].

On the whole the \( J_1 \) to \((J+2)_1 \) (yrast) transitions are the largest and the \( J_2 \) to \((J+2)_2 \) are often large as well but not as much. To partially understand this we use the rotational formula for B(E2)’s as given in the introduction.

Consider for example a transition from \( J=2 \) to \( J=4 \). If we assume Band 1 has \( K=0 \) and band 2 has \( K=2 \), and both have the same intrinsic quadrupole moment \( Q_0 \), then the ratio

\[
\frac{B(E2; 2\rightarrow 4_2)}{B(E2; 2\rightarrow 4_1)} = \begin{vmatrix} 2 & 2 & 0 & 0 \\ 4 & 2 \end{vmatrix}_2 = \begin{vmatrix} 2 & 0 & 2 & 0 \\ 4 & 0 \end{vmatrix}_2 = 5 \frac{12}{12} = 0.41667.
\]
In detail from Table 2, the ratios are smaller than that except for $^{44}\text{Ti}$ and $^{48}\text{Cr}$.

Although some of the $J_2$ to $(J + 2)_2$ transitions are reasonably large this is not always the case. For example consider the $2_2$ to $4_2$ transitions using the FPD6 interaction. The values for $^{44}\text{Ti}$, $^{46}\text{Ti}$, $^{48}\text{Ti}$ and $^{50}\text{Ti}$, and $^{48}\text{Cr}$ are respectively, 118.0, 14.90, 134.0, 18.8, and 360.0 e$^2$ fm$^4$. Some are large and some are small. One main point of this study is that the inter-band transitions are quite small with some glaring exceptions e.g. $0_1$ to $2_2$ in $^{48}\text{Ti}$ – 108.40 with FPD6 and 99.1 e$^2$ fm$^4$ with GX1A. It is difficult to see a simple trend with neutron number for these inter-band transitions. Using again the example of $2_1$ to $4_2$ the respective values for FPD6 are 1.29, 5.73, 0.36, 0.85 and 11.70 e$^2$ fm$^4$. In the Ti isotopes we go from low to high to low to high, so it is difficult to find a clear-cut trend. The results may be useful however to prevent excessive hand waving.

Also by using 2 interactions we get a feel about how far we can go in making quantitative remarks about the inter-band transitions. There is unfortunately much variation in the results. For example, again for $2_1$ to $4_2$, the values for FPD6 (GX1A) are respectively: 1.90 (0.01), 5.73 (11.70), 0.36 (5.62), 0.85 (1.10), 11.70 (12.60). We can however make the qualitative remarks that the $J_1$ to $(J + 2)_2$ B(E2)'s are much smaller than the yrast B(E2)'s and in the majority of cases also smaller than the $J_2$ to $(J + 2)_2$ B(E2)'s.

6 Comparisons with Collective Models

The shell model calculations can be compared with predictions of the rotational model. The rotational expression is as follows:

$$\frac{B(E2, J \to (J + 2))}{B(E2, 0 \to 2)} = 3 \frac{(J + 2)(J + 1)^2}{(2J + 1)(2J + 2)(2J + 3)}$$

The values from J=0 to 16 are respectively

| J  | $\frac{B(E2, J \to (J + 2))}{B(E2, 0 \to 2)}$ |
|----|---------------------------------------------|
| 0  | 1.000                                       |
| 2  | 0.514                                       |
| 4  | 0.455                                       |
| 6  | 0.431                                       |
| 8  | 0.418                                       |
| 10 | 0.410                                       |
| 12 | 0.404                                       |
| 14 | 0.400                                       |
| 16 | 0.397                                       |

Table 3: Ratio of E2 transitions from J to (J+2) over E2 transition from 0 to 2
For the transition from $J=14$ to 16 the value is 0.400. On the other hand the value of this ratio for say $^{48}\text{Cr}$ with FPD6 is $^{78.6}_{1570}=0.050$. There is a tremendous difference. This discrepancy is also present in the vibrational model, even more so. $B(E2,J+2 \rightarrow J) = \frac{J+2}{J}$. This is equivalent to saying that $B(E2,J+2 \rightarrow J)$ is proportional to the number of phonons that are present in the initial state. We see that although comparisons with collective models are suggestive there are important differences, and shell model calculations of increasing complexity are needed to get quantitative results in this part of the periodic table.

7 Comparisons with other work

In Table 4 we make a comparison of the behavior in the Ti isotopes with what occurs in more deformed nuclei. It is convenient to choose the work of Clément et al. [9] on $^{98}\text{Sr}$ because they show several measured $B(E2)$'s between states in the yrast band and those in the next band. The comparison is somewhat hybrid because we are listing experimental results for Sr and theoretical results for Ti. The $B(E2)$'s in Weisskopf units (WU) are 19.4 in $^{46}\text{Ti}$ and 95.5 in $^{98}\text{Sr}$. This shows that the latter nucleus is indeed more strongly deformed than any of the Ti isotopes. In their Table 4 Clément et al. [9] show reduced matrix elements. In our Table 3 we show rather the ratio of a given $B(E2)$ to the intra-band $0_{1} \rightarrow 2_{2}$ $B(E2)$ with the GX1A interaction. The ratio of this transition to $2_{1} \rightarrow 2_{2}$ in $^{98}\text{Sr}$ is quite small whereas for $^{44}\text{Ti}$ and $^{46}\text{Ti}$ the values are 0.2909 and 0.1694 respectively. A Ratio close to 0.2 is also found for $0_{1} \rightarrow 2_{2}$ in $^{48}\text{Ti}$.

| $J_i \rightarrow J_f$ | $^{98}\text{Sr}$ | $^{44}\text{Ti}$ | $^{46}\text{Ti}$ | $^{48}\text{Ti}$ | $^{50}\text{Ti}$ | $^{48}\text{Cr}$ |
|----------------------|----------------|----------------|----------------|----------------|----------------|----------------|
| $0_{1} \rightarrow 2_{2}$ | 0.00799 | 0.05380 | 0.006458 | 0.1902 | 0.000142 | 0.00246 |
| $2_{1} \rightarrow 0_{2}$ | 0.02556 | 0.0113 | 0.0208 | 0.0195 | 0.00219 | 0.0221 |
| $2_{1} \rightarrow 2_{2}$ | 0.00767 | 0.2909 | 0.1694 | 0.0845 | 0.00703 | 0.0451 |
| $4_{1} \rightarrow 2_{2}$ | 0.004603 | 0.02567 | 0.01651 | 0.00263 | 0.00365 | 0.00606 |
| $2_{1} \rightarrow 3_{1}$ | 0.0123 | 0.009295 | 0.0595 | 0.000448 | 0.0151 |
| $4_{1} \rightarrow 3_{1}$ | 0.04297 | 0.006490 | 0.04626 | 0.0307 | 0.00177 |

8 Even J to Odd J Transitions

In Table 5 we show some selected even J to odd J $B(E2)$'s. We show in Table 5 some selected ones in $^{48}\text{Cr}$. While most of them are small, there are some surprisingly large ones from $4_{2}$ to $5_{1}$, $6_{2}$ to $5_{1}$, and $6_{2}$ to $7_{1}$.

| $J_i \rightarrow J_f$ | $^{48}\text{Cr}$ |
|----------------------|----------------|
| $4_{2} \rightarrow 5_{1}$ | 0.2909 |
| $6_{2} \rightarrow 5_{1}$ | 0.1694 |
| $6_{2} \rightarrow 7_{1}$ | 0.0451 |

Table 5: $B(E2)$’s from even J to odd J in $^{48}\text{Cr}$ in $e^2\text{fm}^4$
In Tables 6 and 7 we show results for static quadrupole moments of $2^1$ and $2^2$ states. Note that for $^{46}$Ti, $^{48}$Ti and $^{48}$Cr the quadrupole moments of the $2^1$ states are negative and those of the $2^2$ states are positive. In $^{44}$Ti with FPD6 we get the same behavior as in other nuclei, $Q_1$ negative and $Q_2$ positive, but this is not the case with GX1A. We get negative values for both $Q$’s with GX1A. This may be explained in part by the fact that with only 4 valence nucleons it is difficult to establish collective motion and, as mentioned before, the single particle energy splittings are larger with GX1A than with FPD6. This favors single particle behavior over collective behavior for GX1A.

In the rotational model (see introduction) the value of $Q(2^1)$ is equal to $\frac{2}{7} Q_0$ whilst the value for $Q(2^2)$ is $+\frac{2}{7} Q_0$. Indeed, the quadrupole moment of $J=2^+$ for a $K=2$ band are equal and opposite of those of a $K=0$ band. Note that for the $K=0$ band the laboratory quadruple moment is of opposite sign to the intrinsic quadrupole moment. One can imagine a pencil which clearly has a positive quadrupole moment along it’s axis. (prolate). But if one rotates the pencil about an axis perpendicular to the line of the pencil and about its center of mass one traces out a pancake which will have a negative quadrupole moment.

### Table 6: FPD6 quadrupole moments in e fm$^2$

| $J_i$ | $J_f$ | $^{44}$Ti | $^{46}$Ti | $^{48}$Ti | $^{50}$Ti | $^{48}$Cr |
|-------|-------|----------|----------|----------|----------|--------|
| $2^1$ | $1^1$ | -21.60   | -23.60   | -18.90   | 3.64     | -35.50 |
| $2^2$ |       | 14.40    | 3.71     | 2.34     | 12.90    | 36.98  |

### Table 7: GX1A quadrupole moments e fm$^2$

| $J_i$ | $J_f$ | $^{44}$Ti | $^{46}$Ti | $^{48}$Ti | $^{50}$Ti | $^{48}$Cr |
|-------|-------|----------|----------|----------|----------|--------|
| $2^1$ | $1^1$ | -6.01    | -13.6    | -14.5    | 6.53     | -30.8  |
| $2^2$ |       | -0.89    | 7.1      | 5.02     | 13.3     | 21.9   |
10 B(E2)’s from the J=0+ ground state to several 2⁺ states

In Tables 8 to 17 we present B(E2)’s, Energies and B(E2)*Energy for the J=0+ ground state to 15 J=2⁺ excited states. In all cases the largest transition is to the 2₁ state. After that the 2 interactions sometimes differ in which state has the next strongest strength. For example in ⁴⁸Cr the first 3 B(E2)’s with FPD6 are 1569, 52.10 and 15.50 e² fm⁴ whereas with GX1A they are 1254, 3.10 and 75.60 e² fm⁴. With FPD6 the second 2⁺ state has the second most strength but with GX1A it is the third. Things are steadier if we look at the summed strength and summed energy weighted strength.

Table 8: B(E2) and Energy Weighted B(E2) of transitions from 0₁ in ⁴⁴Ti with FPD6 interaction in e²fm⁴ MeV

| 0₁       | B(E2) | Energy | B(E2) * Energy |
|----------|-------|--------|---------------|
| 2₁       | 698.90| 1.30   | 908.43        |
| 2₂       | 0.11  | 4.34   | 0.47          |
| 2₃       | 10.28 | 6.11   | 62.78         |
| 2₄       | 4.24  | 7.07   | 29.92         |
| 2₅       | 6.36  | 7.99   | 50.83         |
| 2₆       | 6.22  | 8.11   | 50.48         |
| 2₇       | 1.51  | 8.34   | 12.60         |
| 2₈       | 11.20 | 9.29   | 104.09        |
| 2₉       | 0.14  | 9.75   | 1.37          |
| 2₁₀      | 0.01  | 9.92   | 0.11          |
| 2₁₁      | 0.02  | 10.02  | 0.23          |
| 2₁₂      | 0.13  | 10.17  | 1.29          |
| 2₁₃      | 0.07  | 10.44  | 0.74          |
| 2₁₄      | 0.00  | 10.48  | 0.00          |
| 2₁₅      | 0.34  | 10.69  | 3.59          |
| SUM(15)  | 739.53| x      | 1226.93       |
| SUM(50)  | 747.23| x      | 1318.23       |

Table 9: B(E2) and Energy Weighted B(E2) of transitions from 0₁ in ⁴⁴Ti with GX1A interaction in e²fm⁴ MeV

| 0₁       | B(E2) | Energy | B(E2) * Energy |
|----------|-------|--------|---------------|
| 2₁       | 526.20| 1.29   | 677.43        |
| 2₂       | 28.34 | 3.17   | 89.78         |
| 2₃       | 14.82 | 5.30   | 78.51         |
| 2₄       | 3.13  | 6.49   | 20.34         |
| 2₅       | 0.88  | 6.77   | 5.92          |
| 2₆       | 11.74 | 7.24   | 84.99         |
| 2₇       | 0.06  | 7.89   | 0.51          |
| 2₈       | 0.01  | 8.55   | 0.09          |
| 2₉       | 0.57  | 8.87   | 5.01          |
| 2₁₀      | 14.22 | 8.95   | 127.23        |
| 2₁₁      | 0.11  | 9.35   | 1.00          |
| 2₁₂      | 4.40  | 9.46   | 41.63         |
| 2₁₃      | 0.77  | 9.47   | 7.30          |
| 2₁₄      | 4.04  | 9.66   | 39.01         |
| 2₁₅      | 0.01  | 9.74   | 0.07          |
| SUM(15)  | 609.29| x      | 1178.82       |
| SUM(50)  | 617.38| x      | 1271.91       |
Table 10: B(E2) and Energy
Weighted B(E2) of transitions from
\(0_1\) in \(^{46}\text{Ti}\) with FPD6 interaction in
e\(^2\text{fm}^4\) MeV

| \(\ell\) | B(E2) | Energy | B(E2) \* Energy |
|---|---|---|---|
| 2_1 | 780.50 | 0.98 | 762.24 |
| 2_2 | 46.03 | 3.23 | 148.59 |
| 2_3 | 1.51 | 3.89 | 5.87 |
| 2_4 | 7.56 | 4.34 | 32.83 |
| 2_5 | 0.38 | 4.91 | 1.86 |
| 2_6 | 0.05 | 5.53 | 0.27 |
| 2_7 | 3.75 | 5.92 | 22.21 |
| 2_8 | 7.24 | 6.22 | 45.00 |
| 2_9 | 6.95 | 6.34 | 44.09 |
| 2_10 | 10.34 | 6.62 | 68.44 |
| 2_11 | 2.37 | 6.85 | 16.26 |
| 2_12 | 0.22 | 7.00 | 1.52 |
| 2_13 | 0.12 | 7.48 | 0.87 |
| 2_14 | 0.27 | 7.69 | 2.07 |
| 2_15 | 0.03 | 7.93 | 0.25 |
| SUM(15) | 867.31 | x | 1152.35 |
| SUM(50) | 875.9495 | x | 1234.837 |

Table 11: B(E2) and Energy
Weighted B(E2) of transitions from
\(0_1\) in \(^{46}\text{Ti}\) with GX1A interaction in
e\(^2\text{fm}^4\) MeV

| \(\ell\) | B(E2) | Energy | B(E2) \* Energy |
|---|---|---|---|
| 2_1 | 624.40 | 1.01 | 627.81 |
| 2_2 | 4.03 | 2.59 | 10.42 |
| 2_3 | 43.56 | 3.39 | 147.48 |
| 2_4 | 8.22 | 4.28 | 35.17 |
| 2_5 | 0.04 | 5.01 | 0.19 |
| 2_6 | 21.26 | 5.44 | 115.73 |
| 2_7 | 0.29 | 5.54 | 1.58 |
| 2_8 | 0.36 | 5.82 | 2.09 |
| 2_9 | 1.74 | 6.10 | 10.63 |
| 2_10 | 5.44 | 6.38 | 34.70 |
| 2_11 | 6.44 | 6.53 | 42.05 |
| 2_12 | 0.70 | 6.65 | 4.66 |
| 2_13 | 1.03 | 6.90 | 7.13 |
| 2_14 | 0.59 | 7.03 | 4.17 |
| 2_15 | 0.11 | 7.13 | 0.76 |
| SUM(15) | 718.20 | x | 1044.57 |
| SUM(50) | 733.6302 | x | 1176.939 |
Table 12: B(E2) and Energy
Weighted B(E2) of transitions from 0$^+_1$ in $^{48}$Ti with FPD6 interaction in $e^2 fm^4$ MeV

|   | B(E2) | Energy | B(E2) * Energy |
|---|-------|--------|----------------|
| 2_1 | 638.30 | 1.18 | 751.15 |
| 2_2 | 108.40 | 2.47 | 267.27 |
| 2_3 | 10.86 | 3.72 | 40.39 |
| 2_4 | 3.91 | 4.16 | 16.25 |
| 2_5 | 2.85 | 4.87 | 13.90 |
| 2_6 | 20.12 | 5.21 | 104.87 |
| 2_7 | 4.33 | 5.65 | 24.48 |
| 2_8 | 0.14 | 5.88 | 0.82 |
| 2_9 | 1.80 | 6.04 | 10.87 |
| 2_10 | 1.27 | 6.13 | 7.78 |
| 2_11 | 1.06 | 6.32 | 6.71 |
| 2_12 | 0.01 | 6.55 | 0.10 |
| 2_13 | 0.03 | 7.00 | 0.21 |
| 2_14 | 0.67 | 7.03 | 4.68 |
| 2_15 | 1.59 | 7.21 | 11.44 |
| SUM(15) | 795.34 | x | 1260.92 |
| SUM(50) | 811.7956 | x | 1392.518 |
| SUM(15) | 520.80 | 1.01 | 525.90 |
| SUM(50) | 699.94 | x | 1091.40 |

Table 13: B(E2) and Energy
Weighted B(E2) of transitions from 0$^+_1$ in $^{48}$Ti with GX1A interaction in $e^2 fm^4$ MeV

|   | B(E2) | Energy | B(E2) * Energy |
|---|-------|--------|----------------|
| 2_1 | 520.80 | 1.01 | 525.90 |
| 2_2 | 99.13 | 2.18 | 216.11 |
| 2_3 | 26.51 | 3.32 | 87.94 |
| 2_4 | 0.13 | 4.03 | 0.52 |
| 2_5 | 19.56 | 4.52 | 88.42 |
| 2_6 | 24.18 | 4.71 | 113.87 |
| 2_7 | 0.70 | 5.21 | 3.64 |
| 2_8 | 0.77 | 5.67 | 4.37 |
| 2_9 | 3.38 | 5.78 | 19.51 |
| 2_10 | 0.35 | 5.93 | 2.10 |
| 2_11 | 0.59 | 6.14 | 3.62 |
| 2_12 | 0.69 | 6.23 | 4.32 |
| 2_13 | 0.22 | 6.59 | 1.43 |
| 2_14 | 1.73 | 6.69 | 11.55 |
| 2_15 | 1.21 | 6.70 | 8.11 |
| SUM(15) | 699.94 | x | 1091.40 |
| SUM(50) | 721.5041 | x | 1259.665 |
Table 14: B(E2) and Energy
Weighted B(E2) of transitions from 0\(^+_1\) in \(^{50}\)Ti with FPD6 interaction in \(e^2 fm^4\) MeV

|    | B(E2) | Energy | B(E2) * Energy |
|----|-------|--------|----------------|
| 2_1| 568.80| 1.83   | 1039.65        |
| 2_2| 0.68  | 4.19   | 2.84           |
| 2_3| 57.08 | 4.56   | 260.18         |
| 2_4| 9.13  | 5.08   | 46.40          |
| 2_5| 1.32  | 5.92   | 7.79           |
| 2_6| 0.35  | 6.23   | 2.16           |
| 2_7| 2.90  | 6.46   | 18.74          |
| 2_8| 0.00  | 6.63   | 0.02           |
| 2_9| 0.63  | 6.78   | 4.29           |
| 2_{10}| 2.63 | 6.97   | 18.30          |
| 2_{11}| 0.01 | 6.97   | 0.06           |
| 2_{12}| 3.05 | 7.16   | 21.86          |
| 2_{13}| 3.03 | 7.43   | 22.53          |
| 2_{14}| 4.66 | 7.55   | 35.16          |
| 2_{15}| 0.05 | 7.67   | 0.35           |
| SUM(15)| 654.30 | x | 1480.32 |
| SUM(50)| 662.7534 | x | 1556.158 |

Table 15: B(E2) and Energy
Weighted B(E2) of transitions from 0\(^+_1\) in \(^{50}\)Ti with GX1A interaction in \(e^2 fm^4\) MeV

|    | B(E2) | Energy | B(E2) * Energy |
|----|-------|--------|----------------|
| 2_1| 502.30| 1.62   | 815.74         |
| 2_2| 0.08  | 3.90   | 0.30           |
| 2_3| 56.92 | 4.22   | 240.07         |
| 2_4| 2.53  | 5.03   | 12.71          |
| 2_5| 6.38  | 5.31   | 33.86          |
| 2_6| 1.23  | 5.88   | 7.25           |
| 2_7| 1.78  | 6.04   | 10.71          |
| 2_8| 1.57  | 6.53   | 10.27          |
| 2_9| 1.60  | 6.60   | 10.55          |
| 2_{10}| 6.41 | 6.74   | 43.26          |
| 2_{11}| 0.09 | 6.80   | 0.63           |
| 2_{12}| 2.98 | 6.93   | 20.67          |
| 2_{13}| 3.35 | 7.07   | 23.72          |
| 2_{14}| 0.64 | 7.20   | 4.60           |
| 2_{15}| 0.01 | 7.37   | 0.09           |
| SUM(15)| 587.88 | x | 1234.43 |
| SUM(50)| 604.1513 | x | 1373.675 |
We now comment on the tables in this section-B(E2)'s from the J=0+ ground state to many 2+ excited states. Note that with both interactions the overwhelming fraction of the strength is to the first 2+ state. For example in 46Ti with FPD6 the B(E2) to the first 2+ state is 780.5 e^2fm^4 while the sum over 50 states is 875.9 e^2fm^4. Thus, 91% of the strength goes to the lowest state. This is typical of what happens for all the other nuclei. Another striking feature is that the strength to the lowest 2+ state and SUM(50) is consistently larger for FPD6 than for GX1A. A quantitative comparison is made in Table 18 with % deviations ranging from 9.25% to 19.2% As mentioned before the single particle splittings are larger with GX1A than with FPD6 and this leads to reduced collectivity with GX1A. We note that although there is always an uncertainty of what effective charges should be used in calculating B(E2)'s. One can bring the results of B(E2) to the lowest 2+ state SUM(50) with the 2 interactions closer together by making the effective charges for GX1A a bit larger than for FPD6.

The cumulative SUM(n) for 46Ti and 48Cr as well as the energy weighted sums are shown in Figs 2 and 3. They are smooth and do not show too much structure. However, if we look at the B(E2)'s to the next few 2+ states there are discrepancies that cannot be eliminated by adjusting the effective charges. For example compare the B(E2)'s to the second and third 2+ states. In 46Ti with FPD6 they are 46.03 and 1.51 respectively while with GX1A the values

Table 16: B(E2) and Energy
Weighted B(E2) of transitions from 0_1 in 48Cr with FPD6 interaction in e^2fm^4 MeV

| 2 | B(E2) | Energy | B(E2) * Energy |
|---|-------|--------|----------------|
| 2_1 | 1569.00 | 0.79 | 1237.78 |
| 2_2 | 52.11 | 3.66 | 190.97 |
| 2_3 | 14.96 | 4.57 | 68.32 |
| 2_4 | 17.00 | 5.55 | 94.42 |
| 2_5 | 12.43 | 6.35 | 78.96 |
| 2_6 | 1.21 | 6.67 | 8.08 |
| 2_7 | 20.35 | 6.96 | 141.69 |
| 2_8 | 0.03 | 7.40 | 0.21 |
| 2_9 | 1.95 | 7.47 | 14.60 |
| 2_10 | 0.00 | 7.54 | 0.00 |
| 2_11 | 12.20 | 7.69 | 93.87 |
| 2_12 | 0.04 | 7.90 | 0.30 |
| 2_13 | 5.47 | 8.06 | 44.12 |
| 2_14 | 2.35 | 8.13 | 19.14 |
| 2_15 | 0.99 | 8.33 | 8.26 |
| SUM(15) | 1710.10 | x | 2000.70 |
| SUM(50) | 1737.61 | x | 2266.909 |

Table 17: B(E2) and Energy
Weighted B(E2) of transitions from 0_1 in 48Cr with GX1A interaction in e^2fm^4 MeV

| 2 | B(E2) | Energy | B(E2) * Energy |
|---|-------|--------|----------------|
| 2_1 | 1254.00 | 0.79 | 989.16 |
| 2_2 | 3.09 | 3.39 | 10.50 |
| 2_3 | 75.59 | 4.10 | 309.81 |
| 2_4 | 34.57 | 4.62 | 159.86 |
| 2_5 | 12.04 | 5.50 | 66.17 |
| 2_6 | 9.39 | 5.69 | 53.42 |
| 2_7 | 1.44 | 5.98 | 8.63 |
| 2_8 | 0.21 | 6.41 | 1.37 |
| 2_9 | 1.35 | 6.77 | 9.13 |
| 2_10 | 0.02 | 6.78 | 0.15 |
| 2_11 | 2.05 | 6.90 | 14.12 |
| 2_12 | 9.81 | 6.98 | 68.45 |
| 2_13 | 6.73 | 7.14 | 48.03 |
| 2_14 | 0.07 | 7.21 | 0.49 |
| 2_15 | 0.29 | 7.41 | 2.18 |
| SUM(15) | 1410.66 | x | 1741.47 |
| SUM(50) | 1446.022 | x | 2041.868 |
are 4.03 and 43.56. Thus FPD6 predicts a strong transition to the second $2^+$ state and a weak one to the 3rd $2^+$ state, but GX1A gives the opposite. A similar story in $^{48}$Cr where the FPD6 numbers are 52.11 and 14.96 while the GXiA numbers are 3.09 and 75.41. On the other hand there is qualitative agreement with the 2 interactions with regards to $^{48}$Ti and $^{50}$Ti. In the former the second $2^+$ state is the most strongly excited (108.4, 99.13), while in $^{50}$Ti it is the 3rd one (57.08, 56.93). It should be noted that the rotational model does not give much guidance for interband B(E2) transitions. If there were a simple formula for a transition from the J=0+ yrast band to the $2^+$ state of the gamma band, Bohr and Mottelson [4] would surely have shown it. Our shell model results suggest that more experimental work should be done in this regard, just as has been done by Clement et al. [9] as shown in a previous section.

In Table 18 we show the percent deviation in the summed strength and the energy weighted strength between the 2 interactions.

|       | SUM(50) | EWS(50) |
|-------|---------|---------|
| $^{44}$Ti   | 19.03%  | 3.58%   |
| $^{46}$Ti   | 17.68%  | 4.80%   |
| $^{48}$Ti   | 11.78%  | 10.02%  |
| $^{50}$Ti   | 9.25%   | 12.46%  |
| $^{48}$Cr   | 18.32%  | 10.45%  |

Table 18: Percent deviation $\frac{(FPD6-GX1A)}{(FPD6+GX1A)/2}$
Figure 1: $^{46}$Ti Energy Levels using GX1A interaction

$^{46}$Ti

1st band

2nd band

9.3332, 14$^{(1)}$ ———— 17.0274, 17$^{(1)}$

7.4722, 12$^{(1)}$ ———— 12.68, 15$^{(1)}$

5.5567, 10$^{(1)}$ ———— 12.2288, 14$^{(2)}$

4.281, 8$^{(1)}$ ———— 9.577, 13$^{(1)}$

3.899, 6$^{(2)}$ ———— 9.2199, 12$^{(2)}$

3.7697, 5$^{(1)}$ ———— 7.049, 11$^{(1)}$

2.8329, 6$^{(1)}$ ———— 6.6321, 10$^{(2)}$

1.8359, 4$^{(1)}$ ———— 6.3299, 9$^{(1)}$

1.0053, 2$^{(1)}$ ———— 5.3429, 8$^{(2)}$

0.0, 0$^{(1)}$ ———— 5.2527$^{(1)}$

3.995, 1$^{(1)}$ ———— 4.2487, 0$^{(2)}$

3.759, 3$^{(1)}$ ———— 2.9, 4$^{(2)}$

2.586, 2$^{(2)}$
In Figures 2 and 3 we show the cumulative summed strength for $^{46}$Ti and $^{48}$Cr up to 50 states. The curves show a rise at low excitation energies but they quickly flatten out which indicates that there is not much strength left. We must modify this statement by noting that at much higher excitation energies there is a new "Giant quadrupole strength" which our model space cannot describe. This involves excitations through two major shells. This is discussed in many places including Bohr and Mottelson [4]. Our model spaces have only one major shell.

11 B(E2)s from the lowest $2^+$ state to several $2^+$ states

In Tables 19 and 20 we show B(E2)'s from the $2_1^+$ state to various other $2^+$ states. We see there is considerable fragmentation. The $2_1^+$ is not really a transition but can be related to the quadrupole moment. When we include $2_1^+$ (second last column) we get a behavior inline with what we had for other
result, with FPD6 yielding larger results than GX1A. When we remove \( 2\) (last column) for the most part we get the opposite-GX1A bigger than FPD6. The exception is the semi-magic nucleus \(^{50}\)Ti. The behavior is much simpler when \( 2\) is included in the sum.

Table 19: B(E2)s from the \( \frac{1}{2}^+ \) state to various \( 2^+ \) states with FPD6 interaction in e\(^2\)fm\(^4\)

|       | \( \frac{1}{2} \) | \( 2 \) | \( 3 \) | \( 4 \) | \( 5 \) | SUM(50) | SUM(50) - \( \frac{1}{2} \) |
|-------|------------------|--------|--------|--------|--------|---------|-------------------|
| \(^{44}\)Ti | 162.8            | 62.0   | 0.2    | 0.8    | 0.2    | 233.4   | 70.6              |
| \(^{46}\)Ti | 193.7            | 12.9   | 29.4   | 20.2   | 0.0    | 264.7   | 71.0              |
| \(^{48}\)Ti | 124.0            | 20.2   | 62.3   | 11.2   | 0.0    | 231.4   | 107.4             |
| \(^{50}\)Ti | 4.6              | 48.7   | 112.1  | 16.2   | 1.5    | 191.5   | 186.9             |
| \(^{48}\)Cr | 438.9            | 15.9   | 30.3   | 14.2   | 0.6    | 514.2   | 75.3              |

Table 20: B(E2)s from the \( \frac{1}{2}^+ \) state to various \( 2^+ \) states with GX1A interaction in e\(^2\)fm\(^4\)

|       | \( \frac{1}{2} \) | \( 2 \) | \( 3 \) | \( 4 \) | \( 5 \) | SUM(50) | SUM(50) - \( \frac{1}{2} \) |
|-------|------------------|--------|--------|--------|--------|---------|-------------------|
| \(^{44}\)Ti | 12.6             | 152.9  | 0.1    | 0.1    | 2.4    | 178.5   | 166.0             |
| \(^{46}\)Ti | 64.6             | 105.7  | 19.2   | 8.5    | 5.2    | 213.2   | 148.6             |
| \(^{48}\)Ti | 72.8             | 43.7   | 54.2   | 0.6    | 0.7    | 195.0   | 122.2             |
| \(^{50}\)Ti | 14.8             | 35.3   | 104.0  | 1.2    | 4.4    | 172.2   | 157.4             |
| \(^{48}\)Cr | 330.2            | 56.5   | 8.6    | 19.8   | 0.5    | 428.9   | 98.7              |

12 Additional remarks

There have been recent re-measurements of B(E2)'s by K. Arnswald et al. [10] and they are somewhat different from those used for comparison in ref [4]. The new (old) B(E2)'s for \(^{44}\)Ti and \(^{48}\)Cr from \( 2^+ \) to \( 0^+ \) are respectively 205 (136) and 279 (274) e\(^2\) fm\(^4\). There is a recent compilation of B(E2)'s from the lowest \( 2^+ \) state to ground by Pritychenko et al. [11]. This article also includes shell model calculations as support.

Early on, Gerace and Green [12] showed that admixtures of highly deformed (intruder) states are important in the lower half of the p-f shell and can lead to enhanced B(E2)'s. Hertz-Kintish et al. [13] noted that the measured ratio \( \frac{B(E2, \frac{1}{2}^+ \rightarrow 0^+)}{B(E2, \frac{1}{2}^+ \rightarrow 0^+)} \) in \(^{48}\)Cr was smaller than the predictions of the shell, rotational and vibrational models.

There has been recent work on vibrational spectra of even-even nuclei, especially \(^{92}\)Pd [14,15,16]. Robinson et al. [17,18] made a comparison of \(^{92}\)Pd and \(^{48}\)Cr.

In working with the SU(3) Model of Elliott [19], Kingan and Zamick [20] noted that there are no non-zero B(E2)'s from the ground S=0 (80) band to the S=1
(61) first excited band. This is because the E2 operator has no spin dependence and therefore cannot connect S=0 to S=1. This is an extreme model which gives insight into why the inter-band transitions are small. Indeed in $^{20}$Ne they found only 2 non zero B(E2)’s from the J=0$^+$ T=0 ground state. The first was an intra-band transition to the lowest 2$^+$ state with a value of 427 e$^2$fm$^4$, while the second was to a T=1 (S=0) state with a value of 12.56 e$^2$fm$^4$. The smallness of the latter can be explained by the fact that T=0 to T=0 B(E2)’s are proportional to $(e_p + e_n)^2$ whereas T=0 to T=1 are proportional to $(e_p - e_n)^2$, where the e’s are effective charges.

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