Developing a didactical design: the distance between a point and a line in three dimensional shape

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Abstract. This research was implemented to students from the 10th grade in one of high schools in Bandung city. The initial problem that became a background of this research was the existence of the learning obstacles which causes students less optimal in learning the concept of the distance in three dimensional shape so that learning becomes meaningless. The focus on this research was to develop a didactical design of the distance from a point and a line that constructed by considering the learning trajectory and the learning obstacles that have been found. This research used the qualitative method in the form of a didactical design research through three phases of the analysis that are prospective analysis, methodological analysis, and retrospective analysis, with the triangulation technique for collecting data. Based on the results and discussion of research, the didactical design that has been made is capable to overcome some learning obstacles. So, this design can be one of the recommendations on learning the concept of the distance in the geometry.

1. Introduction

Geometri is a fundamental concept in mathematics, especially in school mathematics [1,2]. In Geometri, point is undefined element. Furthermore, we know that a line is the shortest distance that connected two point [3]. There are the basic concepts that must be known by students if they want to learn advanced geometri. Learning basic concepts is very important because it relates to the cognitive system of students that must be connected to each other if they do not want to experience the learning obstacles [4,5].

However, several problems arise related to learning the concept of distance in high school mathematics [2]. Based on the results of interviews with three government school teachers in Bandung city as well as learning observations by authors, it was found that students experienced difficulties in determining projection a points and a lines and projections a points and a field, the relationship between distance and prerequisite geometrical concepts, students had difficulty determining the distance between two lines and two fields which intersects, and the appearance of space. One of the things that causes learning obstacle is teacher domination in class learning, so that students are not facilitated to build their cognitive structures independently on new knowledge [4]. This situation causes learning to be meaningless because learning begins with the definition, examples, procedures, then exercises [6].

A preliminary study done by authors was carried out to several students to ensure the existence of learning obstacle in learning the concept the distance between a point and a line, Figure 1 shows one of the questions used to find the learning obstacle. Figure 1 shows that both students can only construct the pyramid in question, but it seems that they have difficulty illustrating the distance of point A to the TC line. Based on student x answers in Figure 1 (left), we can conclude that she forgot what the distance
between a point and a line is, while in students' answers in Figure 1 (right) we can see that she tried to construct a line segment but she forgot that the line segment must be the shortest line, so the answer was incorrect.

*If there is a pyramid T.ABCD with AB 12 cm, BC 9 cm, and TA=TB=TC=TD is 24 cm. Determine the distance from point A to TC!*

![Diagram of a pyramid T.ABCD](image)

**Figure 1.** Examples of the learning obstacle found.

These difficulties are a result of the epistimological obstacle that occurs to students because they are weak in prerequisite concept which is the requirement to learn the concept of the distance between a point and a line in three dimensional shape [4]. Therefore, this study will focus on developing a didactic design about the concept of the distance between a point and a line in three dimensional shape to overcome the learning obstacle found. This study starts from develop a learning trajectory and lesson design about the concept of the distance between a point and a line in three dimensional shape, then continued with an analysis of the design implementation process that has been made, and finally the analysis of revisions needed [4-7].

The analysis in this study done with the prespective of the didactical situation theory about action, formulation, validation processes and the didactical contract about didactical and adidactical situation in learning [4]. The other theories used in the analysis are van Hiele theories about five levels of thinking and five stages of learning geometry [8], Ausubel theory about meaningfull learning [6], Piaget theory about the process of assimilation and accommodation in getting new knowledge [5], Vygosky theory about zone of proximal development between actual development and potential development of students [9].

2. Method

This qualitative study – a part of a larger qualitative study about developing didactical design about the distance in three dimensional shape – aims to develop the didactical design to overcome the learning obstacle that appears in learning the distance between a point and a line in three dimensional shape. This study used the Didactical Design Research through three phases of analysis i.e, prospective analysis, metapedadidactic analysis, and retrospective analysis with triangulation technique (interview, observation, and documentation) for collecting data [10-12].

Prospective analysis done with two steps with the result are learning trajectory and didactical design depend on learning obstacle found. First step of prospective analysis was interviewed three teachers about the learning obstacle that occurs during learning. Second step of prospective analysis was study literature about the material of distance in three dimensional shape and the learning theories. Metapedadidactic analysis was the analysis during the implementation of didactical design that has been made. Retrospective analysis was the analysis the result of the implementation of didactical design which results the revised of didactical design if needed.

The subjects of this study were 36 students (15 – 16 year old) of the 10th grade students from one of the public school in Bandung city, Indonesia. This school that selected is the school with accreditation A with most it's students coming from the area around the school.
3. Result and Discussion

3.1. Prospective Analysis
Prospective analysis began with the formulation of the learning trajectory based on the learning obstacle found, then proceed with developing a didactic design about the concept of the distance between a point and a line in three dimensional shapes based on the perspective of didactical situation theory and the other relevant learning theories. Learning trajectory is a series of learning activities, in the form of the plot to achieve goals [7]. The first step to formulate the learning trajectory was repersonalize the concept of the distance between a point and a line in three dimensional shapes which is done by reading various textbooks for high school and other books that contain a discussion of the concept. Learning trajectory that was formulated based on the repersonalization is presented in Figure 2 below.

Figure 2. Learning trajectory the distance between a point and a line

This design contained the learning trajectory about the concept of the distance between a point and a line in three dimensional shape in 100 minutes of learning. The learning process in this didactical design had used worksheets distributed for each student. Learning organized in this didactical design had adapted the process of didactical situation and adidactical situation as well as the stages of action, formulation, and validation [4]. Researcher had also adapted constructive learning theory that emphasizes the meaningful learning processes that will occur if students play an active role to form their knowledge independently through previous experience or discussion with other students [6,9].

Next, let's discuss about the worksheets that had been used in the development of didactical design. The first question (Figure 3) on the worksheets is a word problem that can facilitate students to find that the distance of the penalty point to the goal line is represented by the length of the path that forms an angle of 90° with the goal line. Visualization and analysis thinking stage was required in this phase [8].

In a futsal match, Beni gets a chance to kick from the penalty point. Make a simple sketch that showing Beni’s position (at the penalty point) and the goal line! Make at least 5 sketch the paths of Beni’s kicks (horizontal) if Beni kicks a goal! (use a ruler and protractor) Determine also the shortest path!

Figure 3. Question number 1 in worksheet

The next problem on the worksheet is about proving informally that the path they had chosen in the first question was the shortest path, which is the distance of the penalty point to the goal line. This problem required students to go to the informal deduction or even deduction thinking stage [8]. In this section, students experienced an adidactical situation because the teacher does not provide excessive intervention to students activities [4,6]. After students had a discussion in a group, some students come to the front of the class to write down their answers (other students respond). Then, the phase of didactical situation occurs when the researcher confirmed the students’ answers [4]. After that, students defined the distance between a point and a line and wrote it in the conclusion column based on the learning process they had passed.

After students wrote the conclusion, the researcher gave three exercise problems that had been done by students for forty minutes. Students did it individually while allowed to discuss in their groups. In this phase, there was a process of action, formulation, and validation occurs in the learning process [4,9].
This problem was given with the aim to be able to facilitate students to formulate steps to determine the distance between a point and a line in three dimensional shape. Finally, this didactical design ends with an evaluation. At the evaluation stage, students was given one problem with a work time of fifteen minutes. Evaluation questions are similar to exercise problem number three. It was intended that this problem can illustrate students' abilities during learning the concept of distance between a points and a line.

3.2. Metapedadidactic Analysis

In this section, the implementation process of didactical design about the concept of the distance between a point and a line in the three dimensional shapes had been analyzed. Learning began with giving worksheets to students. The first question on the worksheet (Figure 3) is about the kick from the penalty point to the goal. Apparently, this initial case provided difficulties for some female students who did not know what the penalty point was. This happened because the student has never played or watched football. So, they have no information about the location of the penalty points and the goal. To overcome this didactical obstacle, the researcher asked one of the students to draw a sketch of the penalty point and the goal in front of the class [4].

After that, the students continued the discussion in their group. Researcher walked around the classroom to see students' answers. Researcher saw that all students could make the five paths from penalty points to the goal line without difficulty, but many students always want to get the confirmation from the researcher (as a teacher) related to the correctness of their answer. This phenomena shows a low level of self efficacy [13].

The finding is all students sketched a path of the kick perpendicular to the goal line as one of the five requested paths. They had been able to conclude that the shortest path is a path that is perpendicular to the goal line. These conclusion was obtained from the results of students' intuition towards the visualization of the kicks that they have made, according to the analysis thinking stage [8].

The first type of students' informal proving, students proved by manually measuring the path length on the sketch that has been made. The second type, students used the properties of right triangles and the Pythagorean theorem to considering each path length on sketch. Both responses are accordance with the predictions of researcher [4,7]. This means that most students have successfully reached informal deduction thinking stage [8].

The next finding is when students was asked to fill in the conclusion column related to the definition of the distance between the penalty points to the goal line, it turns out that students had been able to define the meaning of the distance between a points and a lines according to their experience in solving cases and prior evidence. Students had carried out the process of action, formulations, and validation in learning regarding the concept of the shortest path connecting the point to the line [4]. The following are some conclusions written by students. The conclusions that students wrote mostly about distance is the shortest path, in the case the shortest path is the line that is perpendicular to the other line. There are also other conclusions which illustrate that students had not been able to generalize the conclusions of a learning, e.g. from the response of student with absent number 33. He only concluded that the gradient of line determines the length of path, he did not defined distance in general. This means that the process assimilation of the knowledge schemes of this student has not been optimally [5]. The last was evaluation process. The problem that used in evaluation is similar to the problem in exercise. Most students did the exercise correctly, but it seems that students had not fully understood how to choose the right concept to solve the problem. It was proven that in this evaluation there were several students who gave responses to the use of Pythagorean theorems as in Figure 4. The first response (Figure 4) arised because students were less able to choose the right concept. This response arise because of the effect of the time gap between giving the material and evaluation (students did not study as usual for 26 days because of the school exam and midterm). Whereas, the second response (Figure 4) proved that scaffolding from researcher more or less had helped students to carry out the process of accommodation the knowledge [6]. Table 1 contains data from the evaluation of the implementation of didactical designs.
that had been made. Based on Table 1 it was known that most have understood the visualization of the distance between points and lines and can solve routine problems related to the concept.

Table 1. Evaluation result of implementation

| Answers | The number of students | Percentage |
|---------|------------------------|------------|
| True    | 22                     | 63%        |
| False   | 13                     | 37%        |
| Total   | 35                     | 100%       |

3.3. Retrospective Analysis

In this section we will explain the revisions needed for the didactical design that had been implemented. The results of this didactical design implementation constrained by the time gap between giving the material and evaluation (students did not study as usual for 26 days because of the school exam and midterm), so that influenced to learning outcomes. The improvements that can be made to make this design better are doing aprespsi to previous learning, about meaning and step to calculate distance between two points [6]. The use of power point in the learning process is also felt necessary to display visualization of the initial context (penalty points and goal) and visualization of the distance between a point and a line for various example cases (without calculation) so students avoid didactical
obstacle at the beginning of the learning process [4,8]. Also required additional example cases without calculations discussed by each group of students, so that the process of connecting the concept of distance with other concepts such as Pythagorean theorem and the concept of triangle area is more optimal before being given practice questions [4,5].

The conclusion column that was placed before the practice question is not appropriate because students had not formulated the steps to determine the distance between a point and a line. So that there should be two conclusion columns on the worksheet. The first conclusion column is filled with the meaning of the distance between a point and a line (before students solve the given problem) and the second conclusion column is filled with steps to determine the distance between a point and a line (after students solve the given problem in exercise).

4. Conclusion

The didactical design of the concept of the distance between a point and a line in Three-dimensional shape has been successfully made based on the learning obstacles that have been found. The didactical design was made after having personalized and formulate the learning trajectory about the concept of the distance between a point and a line in Three-dimensional shape. This design was implemented to the subject for 100 minutes of learning. Based on the results of the design implementation, it can be concluded that the didactical designs that have been made were considered effective enough to overcome some of the learning obstacles experienced by students. In the implementation of the didactical design, students can understand the meaning of the distance between a point and a line based on the informal deduction thinking stage, but students are unable to understand the connection between the concepts of geometry which should be used in learning this concept. However, based on the implemented, several improvements that needed to make learning become more effective. These changes are not principle, only related to classroom management and the question on the worksheets.

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