Numerical Investigation of the Scale Effects of Rock Bridges

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Abstract
Due to the challenge of measuring rock bridges in the field and the negligence of progressive damage and changes in stresses within a rock mass when defining rock bridges, it is questionable to evaluate mechanical properties of rock bridges using only geometric parameters. A demonstration is the scale effects of rock bridges, because the same geometric parameter may refer to different sizes and numbers of rock bridges, leading to erroneous equivalent rock mass responses. In this context, in-plane rock bridges in rock slope engineering were equivalent to rock bridges subjected to direct shear by conducting numerical simulations employing the Universal Distinct Element Code (UDEC) and described by constant geometric parameters, i.e., joint persistence, while the sizes and numbers of rock bridges were variant. In this way, the scale effects of rock bridges were investigated from the perspective of load–displacement curves, stress and displacement fields, crack propagations and AE characterizations. The results revealed that the mechanical properties of rock bridges deteriorated with decreasing scales. More specifically, the shear resistance and the area and value of stress concentration decreased with decreasing scale. Furthermore, an uneven distribution of displacement fields in an arc manner moving and degrading away from the load was observed, indicating the sequential failure of multiple rock bridges. It was also found that the propagation of tensile wing cracks was insensitive to scale, while the asperity of macro shear fracture mainly formed by secondary cracks decreased with decreasing scale. In addition, increasing the dispersion of rock bridges would overlap the failure precursors identified by intense AE activities. Based on the abovementioned results, the scale effects of rock bridges were characterized using existing rock bridge potential (RBP) index and degree of persistence (DoP) index. Interestingly, a scale threshold to possibly identify a rock bridge was found.

Highlights
• The mechanical properties of rock bridges deteriorated with decreasing scales.
• The scale effects of rock bridges were appropriately characterized using existing indices.
• A possible scale threshold to identify a rock bridge was found.

Keywords Rock bridge · Scale effect · Joint persistence · Numerical simulation · Acoustic emission

1 Introduction
Researchers have long recognized that the discontinuity within natural rock masses appears to have an important role in rock engineering (Terzaghi 1962; Lajtai 1969; Einstein et al. 1983; Stead and Eberhardt 2013). As a typical discontinuity, rock bridges have always been considered to provide key resistance to reduce the potential for damage in rock masses to develop (Diederichs and Kaiser 1999; Cai et al. 2004; Hencher and Richards 2015). To evaluate the resistant capacity of rock bridges, joint persistence, K, was introduced as a measure of rock mass discontinuity and was...
defined as the ratio of the joint length (1D) or area (2D) of the joint surface:

\[ K = \frac{\sum J_i}{\sum J_i + \sum R_i} \]  \hspace{1cm} (1)

where \( J_i \) and \( R_i \) are the ith length or area of the joint and rock bridge, respectively (Fig. 1). Accordingly, Jennings’ criterion (1970) was proposed to compute the combined strength of joints and rock bridges:

\[ \tau = K(c_j + \sigma \tan \phi_j) + (1 - K)(c_r + \sigma \tan \phi_r) \]  \hspace{1cm} (2)

where \( \tau \) and \( \sigma \) are the shear stress and normal stress, \( c_j, c_r \) are the equivalent cohesion of joints and rock bridges, and \( \phi_j, \phi_r \) are the equivalent friction angle of joints and rock bridges, respectively.

Benefitting from the simplicity of statistical linear weighing of the discontinuity, Jennings’ criterion has been widely used in quantification of the resistant role of rock bridges (Alzo’ubi 2012; Jiang et al. 2015; Zare et al. 2021). Nevertheless, Prudencio and Jan (2007) noticed that this criterion assumed simultaneous failure of rock bridges and joints and thus disregarded the impact of joints on the stress field. Furthermore, Elmo et al. (2018) pointed out that this criterion was based on the definition of rock bridges as the distance between existing discontinuities, which might be flawed because of the negligence of progressive rock mass damage and changes in stresses within a rock mass. Moreover, the difficulty in defining rock bridges is observation and measurement of rock bridges within a rock mass in the field. In this context, rock bridges should be reflections of brittle failure mechanisms instead of simple distinct geological structures (Elmo et al. 2021b). In other words, it is questionable to evaluate mechanical properties of rock bridges using only geometric parameters. Typically, if the \( K \) value was invariant, while the sizes and numbers of rock bridges were variant, the strength and failure process of the rock mass would be consistent according to Jennings’ criterion, leading to erroneous evaluation of the discontinuity, as shown in Fig. 1. This misleading understanding presented herein results from the scale effects of rock bridges.

However, it appears to be difficult to investigate the scale effects of rock bridges in the field due to the invisibility and the difficulty of measurement of rock bridges within a natural rock mass (Shang et al. 2017). Moreover, rigorous measurement of rock bridges is available only after failure has occurred, leading to the difficulty in comparing mechanical behaviours of rock bridges on different scales before failure. Accordingly, researchers always perform a back analysis to study the mechanical behaviours of rock bridges on a laboratory scale by setting pre-existing intact rock bridges (Der-Showitz and Einstein 1988), with a classical method being the direct shear test with a constant normal stress applied to the discontinuous plane because of the consistency of boundary conditions between direct shear tests and engineering practices (e.g., rock slope stability and surface excavation stability) (Lajtai 1969; Muralha et al. 2014). In this context, although a plethora of researchers have investigated the effects of normal stress (Wong et al. 2001; Cundall et al. 2016), joint length (Zhang et al. 2005; Ghazvinian et al. 2007; Asadizadeh et al. 2018), joint orientation (Gehle and Kutter 2003; Zhong et al. 2020) and joint overlap (Kemeny 2004; Sarfarazi et al. 2014) on rock bridges in direct shear, the investigation of the scale effects of rock bridges in a laboratory is still difficult because of the high cost and high failure rate of handcraft specimens with multiple rock bridges (Shang et al. 2018). Thus, it is necessary to introduce numerical simulations into this research. In this paper, in-plane rock bridges in rock slope engineering (Fig. 2) were equivalent to rock bridges subjected to simulated direct shear and described by constant geometric parameters, \( K \) values, while the sizes and numbers of rock bridges were variant. In this way, rock bridges on different scales were set and their mechanical responses could be investigated.

The main objective of this paper is to study the scale effects of rock bridges in direct shear by numerical simulation. Methodology, calibration and simulation are presented first, followed by the numerical results and interpretations.

As indicated below, \( K_2 = K_1 = K_3 \). According to Jennings’ criterion, their mechanical properties are consistent, which is questionable due to the negligence of scale effects of rock bridges.
of the scale effects of rock bridges from the perspective of load–displacement curves, stress and displacement fields, crack propagations and AE characterizations. Based on the above investigations, the scale effects were characterized using existing indices, providing new insights into the definition of a rock bridge.

2 Methods

2.1 Discrete Element Modelling

The discrete element modelling (DEM) introduced by Cundall and Strack (1979) has been widely used in numerical simulations of rock behaviours because of its ability to explicitly represent fractures and bond failure of rocks. One of the mainstream commercial software programs based on the DEM is the Universal Distinct Element Code (UDEC) developed by the ITASCA Consulting Group (USA) (Itasca 2014), in which a rock material is modelled as an assembly of blocks bonded by contacts, as shown in Fig. 3a. Blocks usually respond according to Newton’s second law, and contacts among blocks are prescribed by a force–displacement law, as shown in Fig. 3b. In this way, the modelling properties of blocks and contacts determine the macroscale mechanical behaviours of numerical models. Normally, the bulk modulus ($K_{\text{block}}$) and shear modulus ($G_{\text{block}}$) of blocks in addition to normal stiffness ($k_n$) and shear stiffness ($k_s$) of contacts determine the deformation behaviours, whereas cohesion ($c_{\text{cont}}$), friction angle ($\phi_{\text{cont}}$) and tensile strength ($\sigma_{\text{cont}}^{\text{t}}$) of contacts determine the strength. More specifically, in the direction perpendicular to a contact surface, the stress–displacement relation is assumed to be linear:

$$\Delta \sigma_n = -k_n \Delta u_n$$

where $\Delta \sigma_n$ is the effective normal stress increment and $\Delta u_n$ is the normal displacement increment. If $\sigma_{\text{cont}}^{\text{t}}$ is exceeded by $\sigma_n$, the latter equals zero. In the direction tangential to a contact surface, the stress–displacement relation is always assumed to be controlled by Coulomb friction law:

$$|\tau_s| \leq c_{\text{cont}} + \sigma_n \tan \phi_{\text{cont}} = \tau_{\text{max}}$$

where $\tau_s$ is the shear stress, then:

$$\Delta \tau_s = -k_s \Delta u_s^e$$

however, if $|\tau_s| \geq \tau_{\text{max}}$, then:

$$\tau_s = \text{sign}(\Delta u_s^e) \tau_{\text{max}}$$

where $\Delta u_s^e$ is the elastic component of the incremental shear displacement and $\Delta u_s$ is the total incremental shear displacement.

In the initial version of the UDEC, simulated cracks could only propagate in a predetermined direction. To solve this problem, a polygon block pattern in UDEC was proposed by Lorig and Cundall (1989) and later evolved into Voronoi, allowing more reasonable crack propagation in a numerical model. This Voronoi block pattern has been widely adopted in the simulations of rock behaviours (Christianson et al. 2006; Lan et al. 2010; Stavrou et al. 2019). Furthermore,
Granitic rock specimens collected from a quarry in Qingshdao City, Shandong Province, China, were used as physical prototypes (Yang and Kulatilake 2019). These granites were machined as cylindrical (diameter 50 mm × height 100 mm) and rectangular (length 200 mm × width 100 mm × height 100 mm) specimens to conduct uniaxial compression tests and direct shear tests, respectively. The cylindrical specimens were used to estimate macroscale mechanical properties, e.g., unconfined compressive strength (UCS). The three rectangular specimens were further processed into specimens with nonpersistent open joints using a high-pressure water jet cutting machine, and the lengths of the rock bridges were set as 50 mm ($K = 0.75$), 100 mm ($K = 0.50$) and 150 mm ($K = 0.25$), as shown in Fig. 4. Then, the lower parts of the specimens were fixed, and the upper parts were subjected to a constant normal load and an invariant left velocity boundary. More details related to the experiments are available in the reports by Yang and Kulatilake (2019).

UDEC was adopted to simulate the above experiments. The numerical model used to calibrate the uniaxial compression tests had a width of 50 mm and a height of 100 mm and was generated by UDEC Trigon logic to 4409 blocks bonded by 25,522 contacts with an average block edge length of 1.50 mm, which was proven to be low enough to have less influence on the simulated failure patterns. Large stiffness of contacts was set to eliminate boundary effect and to avoid penetration between blocks (Huang et al. 2021). An upper velocity boundary of 0.01 m/s was applied considering the amount of calculation (Gao and Stead 2014).

To calibrate the direct shear tests of rock bridges, another numerical model with a width of 200 mm and a height of 100 mm was generated using the modified UDEC Trigon logic. Inspired by Tatone and Grasselli (2015) and Kemeny et al. (2016), the block size linearly increased as a function of the distance from the shear interface to underline the region of rock bridges. Furthermore, this numerical model was cut into three different models with $K$ values of 0.75, 0.50 and 0.25, corresponding to 8876 blocks bonded by 57,163 contacts, 9030 blocks bonded by 58,512 contacts and 9204 blocks bonded by 59,906 contacts, respectively. According to the physical prototypes, the lower parts of the numerical models were fixed, and a constant normal stress boundary of 4 MPa was applied until the numerical models were stable (the maximum unbalanced force was less than $10^{-3}$ N). Then, this normal stress boundary was kept invariant, and a left velocity boundary of 0.01 m/s was added to the upper parts.

In terms of calibration, multiparameter sensitivity analysis was introduced to set the default initial modelling parameters and their possible ranges. Next, based on the physical prototype constraints, the modelling parameters and their ranges were further determined. Finally, these parameters were calibrated according to previous macro-modelling parameter relationships until the numerical results were generally in line with the physical prototype results. In this study, the modelling parameters shown in Table 1 have been calibrated for a range of macroparameters of both uniaxial compression tests and direct shear tests with these presented in Table 2, indicating that the simulation in UDEC agreed well with the prototype results.

### 2.3 Numerical Models and Monitorings

Scenarios of different scales of rock bridges were implemented by extending the above calibrated numerical models in direct shear to models with different sizes and numbers of rock bridges, $n$, while maintaining constant $K$ values, as shown in Fig. 5. The $n$ value was set as integers from 1 to 6, meaning that the intact rock bridge was dispersed into equivalent lengths. To maintain comparability, the dispersed part of the multiple rock bridges should maintain the same total length of joints and rock bridges, and the boundary conditions were the same as those in the calibration process. In this way, the dispersion of rock bridges became the only independent variable and was used to represent the scale, that is, negatively correlated with scale.

In the simulation process, the monitorings of shear load (reaction force on the left velocity boundary), shear displacement (horizontal displacement on the left velocity boundary), stress field and displacement field were implemented by writing various FISH functions. In addition, simulated acoustic emission (AE) by monitoring and analysing element velocity in DEM was introduced to characterize the failure processes of rock bridges (Bu et al. 2022).
3 Results

3.1 Peak Shear Resistance

Figure 6 shows the variation in shear load versus shear displacement of models with three constant $K$ values and six different scales of rock bridges. The 18 acquired curves showed similar variation trends; that is, with increasing displacement, the shear load increased in an approximately linear manner at first, and then, the increasing rate decreased gradually before reaching the peak shear resistance. After that, the curves were in the post-peak stages, and the load dropped significantly. While differences among the curves of the models with three $K$ values were conspicuous, as expected, the peak shear resistance decreased with increasing $K$ value. These differences are not discussed further in this paper, and more attention is given to the differences among scenarios with various scales of rock bridges with constant $K$ values.

As shown in Fig. 7, the peak shear resistance decreased with decreasing scale in an approximately linear manner, and the absolute decreasing slope was directly proportional to the joint persistence. Precisely, the absolute slopes of $0.35$, $0.73$ and $1.03$ corresponded to $K$ values of $0.25$, $0.50$ and $0.75$, respectively. In addition, as shown in Fig. 8, the peak shear displacement also decreased with decreasing scale. The above results were consistent with the experimental studies of scale effects on shear behaviours of rock joints (Bandis et al. 1981).

3.2 Shear Stress Field

Progressive failure processes of rock bridges have been widely claimed to be due to the stress concentrations at the tips of pre-existing cracks (Yang et al. 2009; Zhang and Wong 2012; Fan et al. 2018); thus, it is necessary to investigate the stress field to interpret the scale effects on progressive failure processes. Numerical simulations with a $K$ value of $0.50$ were taken as an example. When the numerical models were subjected to a constant normal compression without shear load and were stable, as shown in Fig. 9, the scale effects of rock bridges on the normal stress ($\sigma_n$) field were appreciable. Although significant stress concentrations at the tips of both end pre-existing joints were similar in number and shape, the stress concentrations at the inside rock bridges among the six numerical models showed differences in that the concentrated region and value decreased with decreasing scale, indicating that the end rock bridges bore the key normal compression, while the inside ones shared it almost equally.

Upon shearing, the orientation of the principal stress ($\sigma_1$) undergoes a transition from normal to lateral. Figure 10 shows the shear stress ($\tau_{xy}$) field states of the pre-peak stages (displacement of $0.20$ mm), approximate peak stages (displacement of $0.80$ mm) and post-peak stages (displacement of $1.33$ mm). During the pre-peak stages, $\tau_{xy}$ distribution of rock bridges was not uniform, and significant stress concentration appeared at the upper tips of the left joints and the lower tips of the right joints, which was highly consistent with the analytical results by Segall and Pollard (1980) and the experimental results by Allersma (2005). In terms of multiple rock bridges, these above phenomena remained visible, while the concentrated region and value decreased with decreasing scale.

After the pre-peak stages, the load–displacement curves of the numerical models reached the peak stress points. As shown in Fig. 10, a significant stress concentration appeared at the tips of the rock bridges further from the shear load, while the concentrated region decreased with decreasing scale. In contrast, rock bridges closer to the shear load bore less shear resistance compared with those at pre-peak stages, because most of these rock bridges had been broken, as concluded from Sect. 3.3.

When the numerical models lay in post-peak stages, the bearing capacity dropped significantly, manifested by smaller stress concentration values and areas. The concentrated positions were also at the tips of the rock bridges further from the shear load, and the concentrated values and areas also decreased with decreasing scale.

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**Table 1** Calibrated modelling properties used in UDEC to present the granite specimens

| Properties | Values |
|------------|--------|
| Young’s modulus of blocks, $E_{\text{block}}$ (GPa) | 27.20 |
| Poisson’s ratio of blocks, $\mu_{\text{block}}$ | 0.12 |
| Normal stiffness of contacts, $k_n$ (GPa/m) | 140,600 |
| Shear stiffness of contacts, $k_s$ (GPa/m) | 56,240 |
| Contact cohesion, $c_{\text{cont}}$ (MPa) | 24.80 |
| Contact friction angle, $\phi_{\text{cont}}$ (°) | 23.00 |
| Contact tensile strength, $\sigma_{t\text{cont}}$ (MPa) | 4.00 |

**Table 2** Calibrated results of macroproperties

| Properties | Experiment | Modelling |
|------------|------------|-----------|
| Young’s modulus, $E$ (GPa) | 21.48 | 21.53 |
| Poisson’s ratio, $\mu$ | 0.16 | 0.16 |
| UCS (MPa) | 87.80 | 87.75 |
| Cohesion strength, $c$ (MPa) | 23.40 | 21.39 |
| Internal friction angle, $\phi$ (°) | 33.50 | 32.08 |
| Shear strength ($K = 0.75$) (MPa) | 10.74 | 10.63 |
| Shear strength ($K = 0.50$) (MPa) | 14.16 | 13.82 |
| Shear strength ($K = 0.25$) (MPa) | 15.53 | 16.16 |
3.3 Displacement Field

Compared with stress field, displacement field is also important, because it can intuitively reflect the transfer of inside strain, reasonably interpret macro failure modes (Sarfarazi et al. 2014; Zhang and Wong 2014) and better explain crack initiation and propagation (Cao et al. 2017). As shown in Fig. 11, the displacement magnitude fields vary with the states of the numerical models. This uneven distribution and evolution of the displacement fields were due to the presence of pre-existing rock bridges according to Paronuzzi et al. (2016).

At the pre-peak stages, an intuitive result was the arc distribution of displacement, similar to the research by Cundall et al. (2016). This arc formed at the position of rock bridges and had transmitted away from the load. The shapes and
areas of the arc among models with different scales of rock bridges were similar. Furthermore, this arc was asymmetrical and approximately continuous; the former was due to the progressive failure of rock bridges, and the latter was greatly affected by the pre-existing joints and formed cracks. The formed cracks could be clearly observed, because the displacement field would vary abruptly in magnitude within a short distance in the vicinity of a crack (Zhang and Wong 2014). As shown in Fig. 11, clear tensile cracks that initiated from the tips of the left joints were observed, and their propagation was constrained by Griffith theory; that is, tensile wing cracks initiated in the direction normal to the axis of pre-existing joints first, followed by the propagation along a curvilinear path that aligned with the direction of the principal stress, which had been proven by Sarfarazi and Haeri (2016) to be a typical crack pattern in rock bridges. In addition to the tensile wing cracks, another typical mode was the secondary cracks described as shear zones formed by the accumulation of tensile cracks that initiated in a direction coplanar or quasi-coplanar to the pre-existing joints (Ghazvinian et al. 2012). The propagation of secondary cracks at this stage had some differences in that the length decreased with decreasing scale.

When the load–displacement curves of the numerical models approximately reached the peak stress points, the position, shape and area of the arc and the crack propagation changed appreciably. In terms of the arc, it moved further away from the load, and the displacement field on the left side of the arc varied significantly, indicating the failure of the left rock bridges. Another change was that the area of the arc decreased, while the reduction increased with decreasing scale. In terms of crack propagation, tensile wing cracks at the tips of the left joints were not clear, because the range of displacement contours increased several times; in other words, displacement in the shear zone was much larger than the displacement at the tensile wing cracks. Thus, secondary cracks were visually the main pattern at this stage. Furthermore, the left-formed fracture had a different morphology.
in which the asperity basically decreased with decreasing scale, since the smaller intact rock bridges would be more easily broken in shear.

The displacement fields at the post-peak stages are also shown in Fig. 11. The arc at the previous two stages was invisible, and the displacement fields strongly separated along the rock bridges, indicating that the cracks had a large-scale coalesce. Formed rough macrofractures provided residual shear resistance, and the roughness basically decreased with decreasing scale.

### 3.4 AE Characterization

AE characterization is becoming a popular interest in rock bridge studies (Chen et al. 2015; Jiang et al. 2020). Normally, AE signals accompanied by fracturing events are always operated in the form of waveform analysis to acquire AE parameters, e.g., AE events and AE energy, to quantify the failure process of rocks. The average $\tau_{xy}$ of each rock bridge was also recorded by summing the shear stress of each grid and averaging the results. As shown in Fig. 12a, two sharp increases in both AE events and AE energy at displacements of 0.62 mm and 0.90 mm could be observed. Corresponding points on the load–displacement curve could be determined as the volume-expansion point and the peak stress point according to Moradian et al. (2016). Both points play a crucial role in the evolution of damage in heterogeneous rocks and can be used to predict macroscopic rupture (Xue et al. 2014a, b; Chen et al. 2022).

In addition to the intact rock bridge, multiple rock bridges had similar characteristics in curves of $\tau_{xy}$. Although $\tau_{xy}$ might not be an extremely accurate index to present the damage of each rock bridge, it could reflect the failure sequence and the transmission of stored strain energy to a certain degree. As shown in Fig. 12b–f, all of the $\tau_{xy}$ curves were the first to exhibit significant drops, and the last plunge always occurred at the $\tau_{xy}$ curves of the final (rightmost) rock bridge, agreeing with the deduction using stress and displacement fields. These phenomena illustrated that the rock bridge closer to the load end was prior to a fracture, releasing partial strain energy. Another part of the strain energy was transmitted and stored in the unbroken rock bridges further from the load end. With more stored energy, the last rock bridges provided key resistance, meaning that the breakage of the final rock bridges would result in the macrofailure of the whole model.

The intensive AE activities always indicated fracturing events on a relatively large scale. Several sudden increases in AE events and cumulative AE energy can be observed in Fig. 12, of which the last increase was the strongest, corresponding to the peak stress point and significant drops in $\tau_{xy}$ curves of the final rock bridges. It was also valuable to analyse the intensive AE activities at pre-peak stages as the precursors to macroscopic rupture. As shown in Fig. 12, the precursor of an intact rock bridge was easier to identify than the precursors of multiple rock bridges. With decreasing scale, the shear resistant zone was more crushed, leading to a superposition of the precursors.

### 4 Discussion

#### 4.1 Definition of a Rock Bridge: Insights from Scale Effects

Elmo et al. (2018) proposed several important but unanswered questions in the rock bridge domain, of which the first is “What is a rock bridge and what parameters govern whether a given intact rock portion of the rock mass can or should be defined as a rock bridge?” The above investigations of the scale effects of rock bridges provide us with new insights to answer this question, since the shear resistance of a rock bridge would be reduced with a smaller scale of individual rock bridges until the original intact rock bridge was deprived of the resistance and could not be identified as a rock bridge. In this way, there should be a scale threshold between rock bridges and non-rock bridges. According to Sect. 3.1,
a negative linear relationship between scale and peak shear resistance was found. To explore this possible scale threshold, the scale of rock bridges was further narrowed, and the corresponding dispersion was extended to 20, as shown in Fig. 13. The absolute slope experienced a perceptible drop when $n$ fell in the range of [9,10], after which the absolute slope perceptibly decreased. In this range, the $K$ value of a unit (defined as a combination of an individual rock bridge and an adjacent joint) was in the vicinity of 0.20. Interestingly, according to the report by Diederichs and Kaiser (1999), this was a threshold that a crack was long enough to begin to act significantly in the mechanical behaviours of rocks. Thus, a

| $n$ | Displacement = 0.20 mm | Displacement = 0.80 mm | Displacement = 1.33 mm |
|-----|------------------------|------------------------|------------------------|
| 1   | ![Image](image1.png)    | ![Image](image2.png)    | ![Image](image3.png)    |
| 2   | ![Image](image4.png)    | ![Image](image5.png)    | ![Image](image6.png)    |
| 3   | ![Image](image7.png)    | ![Image](image8.png)    | ![Image](image9.png)    |
| 4   | ![Image](image10.png)   | ![Image](image11.png)   | ![Image](image12.png)   |
| 5   | ![Image](image13.png)   | ![Image](image14.png)   | ![Image](image15.png)   |
| 6   | ![Image](image16.png)   | ![Image](image17.png)   | ![Image](image18.png)   |

**Fig. 10** Shear stress ($\tau_{xy}$) field distribution of numerical models with different scales of rock bridges while keeping a constant $K$ value of 0.50 at displacements of 0.20 mm, 0.80 mm and 1.33 mm.
A value of 0.20 of a unit might be a possible scale threshold to identify a rock bridge. In addition, as shown in Fig. 13, apparent fluctuations occurred when \( n \) was more than 12 in number, since with smaller scales of rock bridges, mechanical responses became more dependent on grain size (Schultz 1996); thus, more dispersion would be questionable.

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**Fig. 11** Displacement magnitude field distribution of numerical models with different scales of rock bridges while keeping a constant \( K \) value of 0.50 at displacements of 0.20 mm, 0.80 mm and 1.33 mm.
4.2 Characterization of the Scale Effects of Rock Bridges

As noted by this research and previous reports (Shang et al. 2018; Tang et al. 2021), the $K$ value neglects the scale effects of rock bridges. In this context, we investigated the scale effects by assuming that an intact rock bridge was dispersed as multiple rock bridges on smaller scales while maintaining constant $K$ values, which was in an ideal state. Thus, it seemed to be unreasonable to propose a new method according to this assumption to characterize the scale effects of rock bridges. With regard to rock engineering designs, discontinuities have been widely considered in rock mass classification systems. The excellent rock mass classification systems are RMR (Bieniawski 1973), Q (Barton et al. 1974) and GSI (Hoek and Kaiser 1995). These classical and inductive approaches are always based on the experience and subjectivity of engineers, leading to potential inevitable errors in rock mass characterization (Elmo et al. 2021a). According to the report by Kim et al. (2007), rock bridges were involved in the RMR system, but their weighted strength was underestimated; the Q system gave insufficient consideration to rock bridges in practice; and the GSI descriptively referred to rock bridge as block interlocking.

**Fig. 12** Average shear stress ($\tau_{xy}$), load, AE events and normalized cumulative AE energy versus displacement of numerical models with different scales of rock bridges while keeping a constant $K$ value of 0.50, where $\eta$ is the left-to-right serial number of rock bridges.
Several researchers have also made positive contributions to referring to rock bridges in rock mass classification systems. Laubscher (1990) proposed a measurement using the average fracture frequency rating (FFR) in the MRMR classification system. The rock bridge percentage $P_{rb}$ was determined by

$$P_{rb} = \frac{\text{FFR} \times a}{40}$$

(7)

where $a$ is the constant and can be determined using experience (Dempers et al. 2011). This measurement was intended to neglect the fundamental role of fracture persistence (Elmo et al. 2018). Furthermore, Elmo et al. (2021a) proposed a network connectivity index (NCI) based on a combination of fracture intensity, fracture density and fracture intersection density parameters, expressed by

$$\text{NCI} = \frac{P_{21}}{P_{20}} I_{20}$$

(8)

where $P_{21}$, $P_{20}$ and $I_{20}$ are the areal fracture intensity, areal fracture density and areal fracture intersection density, respectively. Elmo et al. (2021b) introduced the concept of NCI$_{rb}$ by considering the impact of stress-driven fractures, defined by

$$\text{NCI}_d = \text{NCI} + \text{NCI}_{rb}$$

(9)

where NCI$_{rb}$ was the calculated NCI considering only the induced fractures. The ratio between NCI$_{rb}$ and NCI$_d$ was defined as the rock bridge potential (RBP):

$$\text{RBP} = \frac{\text{NCI}_{rb}}{\text{NCI}_d} = \frac{\text{NCI}_{rb}}{\text{NCI} + \text{NCI}_{rb}}$$

(10)

In this way, the higher RBP meant the higher contribution of rock bridges to the overall rock mass behaviours. In this research, each model with a designated scale of rock bridge points to a designated RBP pre-failure, because NCI$_{rb}$ pre-failure is the same, while NCI pre-failure is different. Thus, RBP pre-failure was adopted as an index to characterize the scale effects of rock bridges, as shown in Fig. 14.

In addition to the $K$ value, some other calculations of discontinuity were proposed. For example, Park (2005) proposed a probabilistic analysis by treating joint persistence as a random variable, implemented by comparing the individual joint length with the maximum sliding dimension in each interval on the premise that the discontinuity on the failure plane was fully persistent. Wasantha et al. (2014) also proposed an index named the degree of persistence (DoP) by considering the impact of discontinuous joint tips on stress distribution characteristics:

$$\text{DoP} = \frac{\sum J_i}{n_d} \times \left( \sum J_i + \sum R_i \right)$$

(11)

where $J_i$ and $R_i$ are the $i$th length of the joint and rock bridge, respectively, and $n_d$ is the number of discontinuous joint tips. The DoP index was also used to characterize the scale effects of rock bridges and was compared with RBP pre-failure, as shown in Fig. 14. Although the span was different, both indices showed consistent feasibility in evaluating the scale effects.

### 4.3 New Insights into the Understanding of Rock Bridges

Our research revealed erroneous equivalent rock mass responses in detail when using the concept of joint persistence. With invariant $K$ values, the mechanical properties of rock bridges deteriorated with decreasing scale, manifested...
by lower shear resistance, lower stress concentration area and value, higher displacement arc field reduction and lower asperity of macro shear fracture. It is thus necessary to consider failure mechanisms in addition to geometric parameters when defining rock bridges. A negative piecewise linear relationship between the scale and shear resistance was found, on which the cutoff point was in the vicinity of the K value of 0.20 for a unit; at this point, the joint began to act significantly as well. Thus, this value could be used to identify a rock bridge. In addition, both RBP and DoP were appropriate indices when characterizing the scale effects of rock bridges.

4.4 Limitations of the Present Study

In this study, numerical simulations were implemented at the 2D level, bringing the limitations of high consistency with the 3D physical prototypes, although the investigations at the 2D level might theoretically appear compatible at the 3D level and could save plenty of time to calculate and analyze. Furthermore, this research focused on filling the gap of scale effects of rock bridges, but other factors, such as minerals, joint length and orientation, normal stress and loading rate, were not included. In addition, our understanding of the scale effects of rock bridges was still at the laboratory scale and based on a traditional back analysis method due to the invisibility of rock bridges, which might be improved by introducing some techniques to measure rock bridges in the field.

5 Conclusions

To understand the erroneous equivalent rock mass responses resulting from neglecting the scale effects of rock bridges when using only geometric parameters, i.e., joint persistence, numerical simulations were performed on direct shear tests with different scales of rock bridges using UDEC while maintaining constant joint persistence. From the perspective of load–displacement curves, stress and displacement fields, crack propagations and AE characterizations, the following key conclusions can be drawn.

1. It is necessary to consider failure mechanisms in addition to geometric parameters when defining rock bridges.
2. Shear resistance decreased with decreasing scale, and the reduction increased with increasing joint persistence.
3. Rock bridges at the end bore the initial key resistance, and the stress concentration area and value decreased with decreasing scale.
4. The uneven distribution of the displacement field of rock bridges was in an arc manner moving and degrading away from the load, illustrating the subsequent failure of multiple rock bridges. The relative degradation area of this arc increased with decreasing scale.
5. Propagation of wing cracks was not sensitive to the scale, while the asperity of macro shear fractures mainly formed by secondary cracks basically decreased with decreasing scale.
6. Dispersion of rock bridges might lead to the overlap of the precursors identified by intense AE events and abrupt AE energy.
7. A unit with joint persistence of 0.20 might be a scale threshold to identify a rock bridge.
8. Both RBP and DoP were appropriate indices when characterizing the scale effects of rock bridges.

Acknowledgements This work was financially supported by the National Natural Science Foundation of China under Grant Nos. 41977249, 42090052 and U1704243, and the National Key Research and Development Program of China under Grant No. 2019YFC1509701.

Author Contributions Conceptualization: FB and LX; Methodology: FB; Formal analysis and investigation: FB and LX; Software: FB; Writing—original draft preparation: FB; Validation: LX; Writing—review and editing: LX; Supervision: MZ, CX and YC.

Data Availability The data sets generated during and/or analysed during the current study are available from the corresponding author on a reasonable request.

Declarations

Conflict of Interest The authors declare no competing interests.

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