MESONS IN THE QUENCHED APPROXIMATION AT LOW TEMPERATURE

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Abstract

We use an effective low energy field theory to describe the coupling between glueballs and mesons as an analog of the quenched approximation in lattice QCD. This allows us to study the temperature dependence of mesonic screening masses at temperatures below the deconfinement phase transition.

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In the last decade there has been much interest and work on a possible chiral/deconfinement phase transition in QCD at some temperature on the order of 200 MeV \cite{1,2}. In the vicinity of this temperature neither perturbation theory with quarks and gluons nor effective field theories with hadrons can give a reliable description of the thermodynamics. Lattice QCD is the best quantitative tool available. However, Monte Carlo computations with dynamical quarks are extremely time consuming. This has led to studies in the quenched approximation, which basically means that operator averages are computed in an ensemble of gluonic fields. Then Monte Carlo computations can proceed much more rapidly and with much better statistics. Not only is the quenched approximation easy to implement in the lattice simulations, it is also known to be a rather good approximation at zero temperature \textit{a posteriori}. The masses of the low-lying hadrons as computed with no dynamical quarks are very close to the experimental values \cite{2}. This should be no surprise, because many hadronic parameters are successfully calculated by means of QCD sum rules \cite{3}, which remain true within the quenched approximation. The contribution of nonvalence quarks is in the higher-order multi-loop diagrams of the perturbative sector and is naturally small. Nonvalence quarks also are unimportant in many successful phenomenological theories like the nonrelativistic quark model \cite{4} and the MIT bag model \cite{5}. Motivated by the success of the quenched approximation at zero temperature, one is naturally led to wonder about what kind of matter this approximation corresponds to at finite temperature.

Consider, for example, the correlator of two vector currents $J_\nu = \bar{\psi} \gamma_\nu \psi$ \cite{6}. After integrating out the quark fields in full QCD one has

$$
< T \left[ J_\mu(x) J_\nu(0) \right] > \propto \frac{\int [dA] e^{S(A)} \text{Tr} \left\{ \gamma_\mu D_A^{-1} \gamma_\nu D_A^{-1} \right\} \det(D_A)}{\int [dA] e^{S(A)} \det(D_A)}, \quad (1)
$$

where $T$ stands for chronological ordering, $S(A)$ is the action of glufermion propagator in the external gauge field $A$. The quenched approximation ignores the dependence of the quark determinant on the gauge field and so the determinants in
the numerator and denominator cancel in this average. This means that the only quarks which enter the average are introduced via the current operators themselves; these may be called valence quarks. There are no additional internal quark loops.

As another example, one may compute the contribution of valence quarks to the free energy of the system by

$$F_q = T \int [dA] e^{S(A)} \frac{\text{Tr} \ln (D_A^{-1})}{\int [dA] e^{S(A)}}.$$  \hspace{1cm} (2)

The free energy $F_q$ incorporates that part of the usual loop expansion of the full QCD free energy which has only one quark loop. The complete thermodynamic potential of QCD in the quenched approximation is the sum of $F_q$ and the free energy of pure gluodynamics.

How can one describe the correlation function (1) phenomenologically at low temperatures? In full QCD low temperatures are dominated by pions since they are by far the lightest hadrons. In the quenched approximation there are no dynamical pions, and so the thermodynamics will be dominated by the lowest mass glueball. Chiral symmetry no longer plays an important role \cite{7}. The pure glue theory has scale invariance which is broken by the renormalization prescription. Nevertheless, there is a memory of the classical scale invariance which leads to an infinite set of Ward identities. These identities can be saturated by one field of scalar gluonium, which can be thought of as a fluctuation of the gluon condensate, in the same sense that the pion is the low energy fluctuation of the quark condensate \cite{8, 9}. The effective La

$$\mathcal{L}_\chi = \frac{1}{2} (\partial_\mu \chi)^2 - U(\chi),$$

$$U(\chi) = B \left[ 4 \left( \frac{\chi}{\chi_0} \right)^4 \ln \left( \frac{\chi}{\chi_0} \right) - \left( \frac{\chi}{\chi_0} \right)^4 + 1 \right].$$  \hspace{1cm} (3)

This Lagrangian reproduces the scale anomaly \cite{10}. The potential is minimized when $\chi = \chi_0$, and $U(\chi_0) - U(0) = -B$, where the vacuum energy $B$ is related to the gluon condensate by $<\frac{\beta}{2g} F^a_{\mu\nu} F^a_{\mu\nu}> = -4B$. Anticipating that the gluonic
condensate will change with temperature $T$, we write $\chi = \bar{\chi} + \phi$, where $\bar{\chi}$ is the $T$-dependent condensate and $\phi$ is the fluctuation. The $\phi^2$ term in the Lagrangian gives the glueball mass as

$$m_G^2(\bar{\chi}) = \frac{16B}{\chi_0^2} \left( \frac{\bar{\chi}}{\chi_0} \right)^2 \left[ 3 \ln \left( \frac{\bar{\chi}}{\chi_0} \right) + 1 \right].$$

(4)

In the vacuum $\bar{\chi} = \chi_0$ and $m_G^2(\chi_0) = 16B/\chi_0^2$.

Now consider the coupling of a spin-1 meson $V$ to the glueball in the low energy effective theory. Since all the scale breaking is realized by the glueball Lagrangian (3), the coupling must be

$$L_V = -\frac{1}{4} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} \lambda^2 \chi^2 V_{\mu} V^{\mu},$$

(5)

where $\lambda$ is a dimensionless coupling constant. Writing the glueball field as a condensate plus a fluctuation, the mass is $m_V(\bar{\chi}) = \lambda \bar{\chi}$. Each spin-1 boson has its own coupling constant $\lambda$ which would correspond to the mass as measured by quenched lattice QCD at zero temperature. The correlator of two vector fields is

$$< T [V_\mu(x) V_\nu(0)] > = \frac{\int [d\chi \det^{-1/2} (D_{\mu\nu}) \ D_{\mu\nu}^{-1}(\chi, x)]}{\int [d\chi] e^{S(\chi)} \ det^{-1/2} (D_{\mu\nu})},$$

(6)

where the propagator $D_{\mu\nu}^{-1}(\chi, x)$ in the external field $\chi$ is the inverse of the Lagrangian differential operator in (5). Quenched QCD corresponds to ignoring the $\chi$-dependence of the determinant. The expansion of $D_{\mu\nu}^{-1}(\chi, x)$ in a power series in $\chi$ generates the usual Feynman expansion of the propagator in the Dirac representation with no internal loops of the $V$-field.

The free energy that describes hadronic matter in the quenched approximation allows for a representation similar to (2). It contains thermal loops of hadrons with hadronic propagators in the external fields of glueballs.

At low temperature it suffices to compute the meson self-energy to one-loop order; more loops either give contributions which are higher order in the particle densities, which are unimportant, or they renormalize zero temperature coupling
constants and masses. There are cubic and quartic interactions between fluctuations in the glueball field $\phi$ and the meson field $V$. The one-loop self-energy diagrams are shown in fig. 1. The tadpole involves nonvalence quarks and so is not included in the quenched approximation. In Euclidean space the self-energy is

$$\Pi_\mu^\nu (k) = \frac{1}{2} \sum_{p_4} \frac{1}{(2\pi)^3} \frac{1}{p^2 + m_G^2} \frac{\delta^{\mu\nu} m_V^2(\bar{x}) + p^\mu p^\nu}{p^2 + m_V^2(\bar{x})}.$$  

The finite temperature contribution to the static, infrared limit of the time-time component, relevant for screening

$$\Pi_4^4 (k_0 = 0, k \to 0) = \lambda^2 \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{4p^2}{m_V^2(\bar{x}) - m_T^2(\bar{x})} \left( \frac{n_V}{\omega_V} - \frac{n_G}{\omega_G} \right) - 3 \frac{n_G}{\omega_G} \right],$$  

where the $n$ are Bose-Einstein distributions, $\omega_V = \sqrt{p^2 + m_V^2(\bar{x})}$, and similarly for the glueball.

The screening mass is

$$m_{V,\text{screen}} = m_V^2(\chi_0) \left( \bar{x}/\chi_0 \right)^2 + \Pi_4^4 (k_0 = 0, k \to 0).$$  

Note that there are two contributions to the screening mass: the mean field and the one-loop scattering. The shift in the mean field at low temperature can be computed to first order in the density of thermal glueballs to be

$$m_V^2(\chi_0) \left( \frac{\bar{x}}{\chi_0} \right)^2 = m_V^2(\chi_0) - 5\lambda^2 \int \frac{d^3 p}{(2\pi)^3} \frac{n_G}{\omega_G}.$$

Thermal fluctuations act to reduce the magnitude of the condensate as one would expect. To lowest order it suffices to use the zero temperature masses on the right side of eqs. (8) and (10).

To see how important finite temperature effects are we use $B = (200 \text{ MeV})^4$ and $m_G = 1 \text{ GeV}$. The coupling $\lambda$ is chosen so as to give the appropriate zero temperature meson mass. Numerical results are shown in fig. 2 for the $\rho$ meson. The shift
in the mean field contribution at finite temperature is comparable in magnitude to the one-loop scattering contribution. The change in the mean field with temperature acts to decrease the $\rho$ mass, whereas the scattering term can increase or decrease it depending on the relative magnitude of $m_G$ and $m_\rho$ in the vacuum. Both contributions are very small because the Boltzmann factor $\exp(-m/T)$ provides a big suppression. We have plotted the mass up to $T = 200$ MeV, the calculation cannot be trusted to quite such high temperatures. Calculation of the equation of state of pure glue SU(3) on the lattice results in a first order phase transition at a critical temperature of $225 \pm 10$ MeV [2]. As this temperature is approached, higher mass glueballs become increasingly important. The density of glueballs becomes large and our low temperature/low density approximation breaks down. Probably there is an exponentially increasing glueball mass spectrum, and the thermodynamics of such a system of interacting glueballs becomes difficult to treat [11].

The same calculation can be done for spin-0 mesons. The one-loop self-energy diagrams are the same as for spin-1 mesons. Eq. (8) gets replaced by

$$
\Pi_S(k_0 = 0, k \to 0) = \lambda^2 \int \frac{d^3 p}{(2\pi)^3} \left[ \frac{n_{\rho G}}{\omega_G} - \frac{4m_S^2}{m_G^2 - m_S^2} \left( \frac{n_S}{\omega_S} - \frac{n_G}{\omega_G} \right) \right].
$$

The shift in the screening mass for any spin-0 meson whose vacuum mass lies in the mid-GeV range and above would be as small as for the $\rho$ meson.

In conclusion, we have discussed how the quenched approximation to QCD at low temperatures can be modelled by a low energy effective Lagrangian. At temperatures well below $T_c$ the lightest scalar glueball, which describes the breaking of scale invariance in QCD, plays the dominant role. In the quenched approximation it is the gluonic condensate which gives mass to the usual hadrons at zero temperature. The glueball part of the Lagrangian has two parameters which are determined at zero temperature. Each meson has its own coupling $\lambda$ to the glueball field, which is fixed by computing the meson mass at zero temperature. Then the low temperature dependence of all hadronic screening masses are predicted from this
low energy effective theory. It is very interesting that in the quenched approximation chiral symmetry, and hence dynamical pions, play essentially no role. On the contrary, previous calculations of mesonic screening masses at finite temperature using QCD sum rules \[12, 13\] or effective low energy Lagrangians \[7, 14\] have shown the dominance of pion dynamics in the full theory. The quenched approximation allows us to focus on the nature and dynamics of broken scale invariance as opposed to chiral symmetry. Unfortunately, finite temperature effects on mesonic screening masses are rather small at low temperatures in the quenched approximation. This is a consequence of the substantial difference between the spontaneous breaking of scale invariance, which leads to a very heavy scalar gluonium with mass of order 1 GeV, and the spontaneous breaking of chiral symmetry, which leads to very light pions. Although we have focussed on mesonic screening masses, there are many other quantities that can be studied at low temperatures, such as the equation of state and the screening masses of baryons. We eagerly look forward to high statistics measurements of quenched QCD on the lattice at finite temperature \[15\].

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**Figure Captions**

Fig. 1. One-loop contributions to the meson self-energy. The tadpole diagram is not included in the quenched approximation because it involves nonvalence quarks.

Fig. 2. Change in the $\rho$ meson screening mass with temperature in the quenched approximation. The dashed line is the mean field contribution. The solid line includes both the mean field and the one-loop scattering contributions.
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