A Magnonic Analogue of Black/White Hole Horizon in Superfluid $^3$He-B

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We report on theoretical model and experimental results of the experiment made in a limit of absolute zero temperature ($\sim 600$ µK) studying the spin wave analogue of black/white hole horizon using spin (magnonic) superfluidity in superfluid $^3$He-B. As an experimental tool simulating the properties of the black/white horizon we used the spin-precession waves propagating on the background of the spin super-currents between two Bose-Einstein condensates of magnons in form of homogeneously precessing domains. We provide experimental evidence of the white hole formation for spin precession waves in this system, together with observation of an amplification effect. Moreover, the estimated temperature of the spontaneous Hawking radiation in this system is about four orders of magnitude lower than the system’s background temperature what makes it a promising tool to study this radiation.

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Recently, it has been shown that Hawking radiation should be related not only to physics of the astronomical black holes [1], but should be viewed as a general phenomenon of a dynamic nature of various physical systems having capability, at certain conditions, to create and form a boundary - an event horizon [2]. According to theory, a fundamental dynamical property of any event horizon analogue is a spontaneous emission of thermal Hawking radiation, the temperature of which depends on a velocity gradient at the horizon [2, 3]

$$T = \frac{\hbar}{2\pi k_B} \frac{\partial v}{\partial r} \sim 10^{-12} \frac{\partial v}{\partial r}. \quad (1)$$

Belong a set of physical systems used to model black/white hole horizon can be included: the sound waves in trans-sonic fluid flow [3, 5], the surface waves on flowing fluid [6, 7], hydraulic jumps in flowing liquids [8, 9], and light in optical fibre [10]. It turns out, however, that temperature of the spontaneous Hawking radiation is typically a several orders of magnitude lower than the background temperature of the physical systems used as an experimental tool to study this phenomenon. Perhaps, solution to this problem is to find another, suitable condensed matter systems, which can model the event horizon, but with background temperature approaching absolute zero temperature [11, 12].

In this Letter we present as a theoretical model, so experimental results of the first experiment made in a limit of absolute zero temperature ($\sim 600$µK) studying the black/white hole horizon analogue using a physical system based on the spin (magnonic) superfluidity in superfluid $^3$He-B. The concept of the experiment is quite simple [15]. The experimental chamber with superfluid $^3$He-B consists of two cylindrical cells mutually connected by a channel (see Fig. 1). The chamber is placed in steady magnetic field $B_0$ and magnetic field gradient $\nabla B$, both oriented along $z$-axis. Using cw-NMR technique, we created a Bose-Einstein condensate of magnons in a form of the homogeneously precessing domain (HPD) in both cylindrical cells [11]. The HPD is a dynamical spins structure formed in a part of the cell placed in lower magnetic field which, with an aid of the dipole-dipole interaction, coherently precess around a steady magnetic field at the angular frequency $\omega_{rf}$. Within the rest of cell, the spins are co-directional with the steady magnetic field and do not precess. These two spin domains are separated by a planar domain wall, the position of which is determined by the Larmor resonance condition $\omega_{rf} = \gamma (B_0 + \nabla B \cdot z)$, where $\gamma$ is the gyromagnetic ratio.
of the \(^3\)He nuclei.

To model black/white hole horizon in superfluid \(^3\)He-B, we used two fundamental physical properties of the HPD: the spin superfluidity and the presence of the HPD’s collective oscillation modes in a form of the spin precession waves \([17, 20]\). The spin superfluidity allows to create and manipulate the spin flow, i.e. the spin super-currents flowing between precessing domains as a consequence of the phase difference \(\Delta \alpha_{rf}\) between the phases of spin precession in individual domains \([21]\). The spin precession waves serve as a probe testing formation and presence of a black/white hole horizon inside the channel: channel has a restriction allowing to reach the different regimes of the velocity of the spin super-current flow with respect to the group velocity of the travelling spin-precession waves. This is similar to the experiment suggested by R. Schützhold and W. Unruh \([6]\) and later performed by S. Weinlurfer et al \([2]\). In the presented experiment, the magnetic field gradient \(\nabla B\) plays the role of the gravitational acceleration, the spin super-currents represent the water flow, and the spin-precession waves correspond to the gravity waves on the water’s surface.

In order to develop a mathematical model, we initially consider a simplified problem - a volume of \(^3\)He-B placed into a large steady magnetic field \(B_0\) with gradient field \(\nabla B\) applied in the \(z\)-direction and a small magnetic rf-field \(B_{rf}\), which rotates in the “horizontal” \(x - y\) plane at the angular frequency \(\omega_{rf}\), with the phase of rotation varying linearly with \(x\). Cartesian components of the resultant magnetic field are \(B_x = -B_{rf} \cos(\omega_{rf}t + \nabla \alpha_{rf} x)\), \(B_y = B_{rf} \sin(\omega_{rf}t + \nabla \alpha_{rf} x)\) and \(B_z = -B_0 + \nabla B.z\). This model problem is treated theoretically by adapting the method we presented elsewhere \([20]\).

The steady-state response is a layer of magnetization precessing with the frequency and local phase of the rf-field lying over a layer of stationary magnetization. The domain with precessing spins (magnetization) may oscillate about its steady state. The principal variable describing small oscillations is the perturbation \(a(t, x, y, z)\) to the phase of spin precession. In the long wavelength approximation and for a thin domain of precessing spins, the perturbation propagating along the domain wall is found to be governed by the equation

\[
\left( \frac{\partial}{\partial t} - \frac{\partial}{\partial x} u \right) \left( \frac{\partial}{\partial t} - u \frac{\partial}{\partial x} \right) a_{dw} - \frac{\partial}{\partial x} \left( c^2 \frac{\partial a_{dw}}{\partial x} \right) - \frac{\partial}{\partial y} \left( c^2 \frac{\partial a_{dw}}{\partial y} \right) + \frac{3}{5} \gamma^2 \nabla B B_{rf} L a_{dw} = 0. \quad (2)
\]

Here, \(a_{dw}(t, x, y) = a(t, x, y, z_{dw})\), where \(z_{dw}\) denotes \(z\)-coordinate of the domain wall position and \(L\) is the thickness of precessing domain. Two terms \(u\) and \(c\) represent the spin flow and the group wave velocities, respectively, and they can be expressed as

\[
u = \frac{(5c_L^2 - c_T^2)}{2c_{rf}} \nabla \alpha_{rf},\]

\[
c^2 = \frac{(5c_L^2 + 3c_T^2) \gamma \nabla B L}{4c_{rf}}. \quad (3)
\]

where and \(c_L\) and \(c_T\) denote the longitudinal and transverse spin wave velocities with respect to the field orientation, respectively. We shall assume that \(\nabla \alpha_{rf}\) is localized on the length of the sharpest restriction in the channel of the order \(dl = 0.5\) mm, therefore \(\nabla \alpha_{rf} \sim \Delta \alpha_{rf}/dl\).

The long spin-precession waves travelling along the surface of a thin layer of precessing and flowing spins are governed by the same equation as a scalar field in a \((2+1)\)-dimensional curved space-time. Thus, these waves experience the background as an effective space-time with the effective metric

\[
ds^2 = c^2 \left[ -c^2 dt^2 + (dx + u dt)^2 + dy^2 \right]. \quad (4)
\]

As it is implied by this equation, an “event horizon” for the long spin-precession waves is formed where and when \(u^2 = c^2\). For the sake of completeness, the exact dependence of the angular frequency \(\omega\) of any spin-precession wave on components \(k_x\) and \(k_y\) of its wave vector is

\[
(\omega + u k_x) = \frac{\gamma \nabla B}{2c_{rf}} c^2 \kappa \tanh(\kappa L), \quad (5)
\]

where \(c^2 = (5c_L^2 - c_T^2)\) and \(\kappa\) is defined as

\[
\kappa^2 = \frac{3}{2c_L^2} \left[ \frac{4}{15} \omega_{rf}^2 B_{rf} + \frac{3}{4}(5c_L^2 + 3c_T^2)(k_x^2 + k_y^2) \right]. \quad (6)
\]

It is important to note that the applied rf-field explicitly determines the “vacuum”, that is to say, it prescribes the angular frequency and the variation of the phase for the precessing magnetization (spins) representing a steady state of the system considered. But Eq. \((2)\) for small oscillations superposed on the steady state is determined primarily by the properties of that state, no matter how the steady state was created (except for the “mass” term that is determined by the external field explicitly). So, the Eq. \((2)\) is capable of describing the perturbations of the steady state in the absence of rf-field, if the precession of background magnetization has maintained its given angular frequency and phase. Although derived for a background represented by a uniform magnetization flow parallel to the flat top of the chamber, the above presented relations can be used for qualitative analysis and quantitative estimation of the situation where \(\nabla \alpha_{rf}\) and \(L\) slightly vary on spatial scales larger than \(L\).

As mentioned above, we performed the experiment in the chamber shown in Fig. \(1\). The chamber was attached to Košice’s diffusion nuclear stage \([22]\), filled with \(^3\)He at 3 bars and using the adiabatic demagnetization cooled down to temperature of \(\sim 0.5T_c\). The temperature of the \(^3\)He was measured using a powder
Pt-NMR thermometer immersed in the liquid and calibrated against the $^3$He superfluid transition temperature $T_c$. The HPDs were simultaneously and independently excited in both cells using cw-NMR method at angular frequency $\omega_{rf} = 2\pi \times 10^3$ rad/sec. To achieve this, we used two rf-generators working in phase-locked mode and with zero phase difference $\Delta\alpha_{rf}$ between excitation signals. The induced voltage signals from NMR coils were amplified by preamplifiers and measured by two rf-lock-in amplifiers, each controlled by its own generator. In order to reduce the mutual crosstalk between the rf-coils, each rf-coil was covered by a shield made of a copper foil. The longitudinal coils provided an additional alternating magnetic field used for the spin-precession waves generation [18].

Once two HPDs were generated, the position of the domain wall was adjusted into channel. This step is easy to accomplish by means of the homogeneous field $B_0$ (with aid of a small longitudinal oscillations), as the position of the domain wall follows the plane where the Larmor resonance condition is satisfied. Specifically, the precision of the domain wall adjustment is given by $\Delta B_0/\nabla B$, where $\Delta B_0 = 0.76 \mu T$ is the field step controlled by the current source and $\nabla B = 15$ mT/m giving a spatial precision of $\sim 50$ µm [23]. For comparison, the domain wall thickness $(\lambda_T = c_{L}^{2/3}/(\gamma \nabla B \omega_{rf})^{1/3})$ for the above parameters is $\sim 0.34$ mm. As the height of the channel is 0.4 mm, an estimated length of the precessing layer $L$ in the channel is $L \sim 100$ µm $\pm 50$ µm. The spin flow between HPDs can be established in both directions depending on the sign of the phase difference $\Delta\alpha_{rf}$, while the spin flow velocity $u$ depended on the magnitude of $\Delta\alpha_{rf}$. The details of the experiment are provided in [23].

The spin-precession waves in the source domain were generated by 8 sinusoidal pulses at an appropriate low frequency using the separate generator. The low frequency response from the source and detection HPD for a particular value of $\Delta\alpha_{rf}$ was extracted from rf-signal by a technique based on application of a rf-detector and a low-frequency filter. The low frequency signals were stored by a digital oscilloscope for the data analysis. The examples of the signals representing the excited spin-precession wave in the source domain and incoming wave in the detection domain are showed in Fig. 2. When the pulse is finished, there are clear free decay signals of the spin-precession waves from both domains, and these parts of the signals were analyzed by the methods of the spectral analysis as a function of the phase difference $\Delta\alpha_{rf}$, i.e. as a function of the spin flow velocity $u$.

![FIG. 2: (Color on line) Voltage signals corresponding to spin-precession waves: the source domain (red), the detection domain (blue). The rectangular signal shows the time window when 8 sinusoidal excitation pulses were applied in order to excite the spin-precession waves. Insets: the picture of the experimental cell on the bench before rf-shield installation.](image)

![FIG. 3: (Color on line) The power spectral density (PSD) of the free decay signals as function of the phase difference measured from the source domain (upper) and the detection domain (bottom). Insets show a schematic illustration of the spin waves dynamics in channel.](image)
at region of $\Delta \alpha_{rf}$ corresponding to $\sim 10^\circ$. Secondly, the weaker PSD signals are observed in the detection domain in the range of negative values of $\Delta \alpha_{rf}$ up to $10^\circ$, above which strong PSD signals were measured. Thirdly, no PSD signals within experimental resolution were measured in the detection domain for values of $\Delta \alpha_{rf} \gtrsim 25^\circ$. How can one interpret these data?

For the negative values of $\Delta \alpha_{rf}$, the spin super-currents flow from the source domain towards the detection one. Thus, the spin-precession waves excited in the source domain are dragged by the spin super-currents and they travel downstream to the detection domain, where they are detected. The amplitude of the detected waves is reduced by process of the energy dissipation inside channel due to spin diffusion, and by the spin flow modifying the frequency of spin-precession waves which is slightly different from the resonance frequency of the standing waves.

The deep signal minimum in the source domain and corresponding maximum in the detection domain for $\Delta \alpha_{rf} \sim 10^\circ - 15^\circ$ is consequence of zero spin flow between domains that leads to a resonance match [26]. Therefore, when the spin-precession waves are excited in the source domain at this condition, all energy is transferred to and absorbed by the detection domain at once. For $\Delta \alpha_{rf} > 10^\circ$ the direction of the spin flow is reversed, i.e. the spin super-currents flow towards the source domain and the emitted spin-precession waves propagate against this flow. The change in direction of the spin super-currents is also seen on the phase of the decay signal from the source domain as the gradual phase shift by $180^\circ$ [25].

As one can see from Fig. 3 there are no PSD signals detected in the detection domain for $\Delta \alpha_{rf} \gtrsim 25^\circ$. We interpret this as a formation of the white hole horizon in the channel: spin-precession waves sent from the source domain towards the detection domain are blocked by the spin flow and never reach the detection domain. This interpretation is supported by calculation using above presented model: the white hole horizon is formed in a place, where and when the condition $c^2 = u^2$ is satisfied. For given experimental parameters and assuming that $\nabla \alpha_{rf}$ is localized on the length of $dl = 0.5 \text{ mm}$, in order to satisfy the condition $c^2 = u^2$ for the phase difference $\Delta \alpha_{rf} \sim 30^\circ$, estimated length of the precessing layer $L$ is $L \sim 100 \mu m$, what reasonably corresponds to the experimental value [25].

Finally, Fig. 4 shows the cross power spectral density between free decay signals measured in source and detector domains as function of the phase difference $\Delta \alpha_{rf}$. For values of $\Delta \alpha_{rf} < 0^\circ$, i.e. when the spin flow drags excited spin-precession waves from the source domain towards to the detection domain, the decay signals from both domains are correlated. For values of $\Delta \alpha_{rf} > 0^\circ$, reduction and following reversion of the spin flow affects the dynamics of the propagation of the spin-precession waves that leads to the change in correlation between the decay signals detected in both domains. When the spin flow approaches zero value, due to resonance match between the domains, the energy is transferred in both directions that is manifested as correlation/anti-correlation peaks. However, when the white horizon is formed ($\Delta \alpha_{rf} \gtrsim 25^\circ$), the decay signals are anti-correlated. We may interpret this in way that the rise of the decay signal in the source domain is paid by the spin flow flowing from the detection domain towards to the source domain - in agreement with theoretically predicted amplification of the wave on the horizon paid by the energy of the flow [27]. This interpretation is supported by dependence presented in Fig. 3 (upper dependence), where a notable feature regarding to the absolute values is showed: when spin current flows from the detection domain to the source domain, the waves in source domain have a tendency to have a higher power spectral density amplitude than those for the spin current flowing in opposite direction. However, to confirm the physical origin of the observed phenomena additional measurement have to be done.

In conclusion, we performed the experiment in a limit of absolute zero temperature probing the black/white hole horizon analogues in superfluid $^3$He-B using the spin-precession waves propagating on the background of the spin super-currents between two mutually connected HPDs and provided the evidence of the white hole formation for spin precession waves. Moreover, the presented theoretical model and experimental results demonstrate that the spin-precession waves propagating on the background of the spin flow between two HPDs possess all physical features needed to elucidate physics associated with the presence of the event horizons, e.g. to test the spontaneous Hawking process. In fact, assuming that the spin super-currents velocity of the order of $u \sim 1 \text{ m/s}$
varies on the length of $dl \sim 10^{-4}$ m one can estimate the temperature of the Hawking radiation in this system to be of the order of 10 nK, what is a temperature only four orders of magnitude lower that the background temperature, and this makes presented system a promising tool to investigate this radiation [25].

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[1] S. W. Hawking, Nature 248, 30 (1974).
[2] S. J. Robertson, J. Phys B: At. Mol. Opt. Phys. 45, 163001 (2012).
[3] W. G. Unruh, Phys. Rev. Lett. 46, 1351 (1981).
[4] R. Schützhold and W. G. Unruh, Phys. Rev. D 78, 041504(R) (2008).
[5] M. Visser, Class. Quantum Grav. 15,1767 (1998).
[6] R. Schützhold and W. G. Unruh, Phys. Rev. D 66, 044019 (2002).
[7] S. Weinfurtner, E. W. Tedford, M. C. J. Penrice, W. G. Unruh and G. A. Lawrence, Phys. Rev. Lett. 106, 021302 (2011).
[8] G. E. Volovik, JETP Lett. 82, 624 (2005).
[9] G. Jannes, R. Piquet, P. Maissa, C. Mathis, G. Rousseaux, Phys. Rev. E 83, 056312 (2011).
[10] T. G. Philbin, Ch. Kuklewicz, S. Robertson, S. Hill, F. König and U. Leonhard, Science 319, 1367 (2008).
[11] L. J. Garay, J.R. Anglin, J.I. Cirac, P. Zoller, Phys. Rev. Lett. 85, 4643 (2000).
[12] S. Giovanazzi, Phys. Rev. Lett. 84, 061301 (2005).
[13] O. Lahav, A. Itah, A. Blumkin, C. Gordon, S. Rinott, A. Zayats, J. Steinhauser, Phys. Rev. Lett. 105, 240401 (2010).
[14] A. Roldán-Molina, A. S. Nunez, and R. A. Duine, Phys. Rev. Lett. 118, 061301 (2017).
[15] P. Skyba, Superfluid He-3 as a model system for cosmology - Experimental point of view. Lecture Notes in Physics 718 Ed. W. G. Unruh and R. Schützhold, DOI 10.1007/3-540-70859-6-5, Springer Verlag (2007).
[16] A. Feher, R. Harakály, L. Lokner, E. Gažo, M. Kupka, J. Nyéké, P. Skyba, Yu. M. Bunkov, O. D. Timofeevskaya, J. Low Temp. Phys.108, 461 (1997).
[17] L. Lokner, A. Feher, R. Harakály, M. Kupka, R. Scheibel, Yu. M. Bunkov, P. Skyba, Europhys. Lett. 40, 539 (1997).
[18] E. Gažo, M. Kupka, M. Medeová, P. Skyba, Phys. Rev. Lett. 91, 055301 (2003).
[19] Človečko, E. Gažo, M. Kupka, P. Skyba, Phys. Rev. Lett. 100, 155301 (2008).
[20] M. Kupka, P. Skyba, Phys. Rev. B 85, 184529 (2012).
[21] A. S. Borovik-Romanov, Yu. M. Bunkov, A. De Waard, V. V. Dmitriev, V. Makróczyová, Yu. M. Mukharskii, D. A. Sergackov, JETP Lett. 47, 478 (1988).
[22] P. Skyba, J. Nyéké, E. Gažo, V. Makróczyová, Yu.M. Bunkov, D. A. Sergackov, A. Feher, Cryogenics 37, 293 (1997).
[23] P. Skyba, Rev. Sci. Instrum. 62, 2666 (1991).
[24] M. Človečko, E. Gažo, M. Kupka, P. Skyba, Journal of Physics Conference Series 150, 032016 (2009).
[25] M. Človečko, E. Gažo, M. Kupka, P. Skyba, to be submitted to PRB.
[26] The $10^2$ shift from $\Delta \alpha_{rf} = 0$ for zero spin flow is caused by small discrepancy in the resonance frequencies settings of two resonance circuits used for HPD excitation.
[27] W. G. Unruh, [arXiv:1107.2669] [gr-qc].