Analysis and research on solution method of metal layer stress in fiber metal laminates

Xiaochen Zhang¹, Weiying Meng¹,*, Tian Zhang¹, Xiao Huang² and Shuai Hou³

¹ School of mechanical engineering, Shenyang Jianzhu University, Mailing address: No. 25 Hunnan Middle Road, Hunnan District, Shenyang, 110168, People’s Republic of China
² Aero Engine Corporation of China, Mailing address: No. 5 Lantianchang South Road, Haidian District, Beijing, 100097, People’s Republic of China
³ Dongpang Colliery of Jizhong Energy Resources CO., Ltd Mailing address: Team 3, Mechanized Mining Face, Dongpang Mine, Neiquiu County, Hebei, 054000, People’s Republic of China

* Author to whom any correspondence should be addressed.
E-mail: xczhang@sjzu.edu.cn, mengweiying025@163.com, zhangtian@sjzu.edu.cn, huangx@aecc.cn and h.s1116@163.com

Keywords: fiber metal laminate, stress prediction, sub-laminate stiffness method, energy method, classical laminate theory

Abstract

As a new type composite material, fiber metal laminates (FMLs) have been widely applied in the aerospace field. To further develop and apply this new composite material, a stress prediction method for the metal layer in the laminates was examined. Based on the laminate theory, the stress characteristic of each layer material was analyzed under an external load. By studying the classical laminate theory, it was found that the calculation method of the global stiffness matrix fails to satisfy the actual situation of the laminates. The concept of the equivalent stiffness matrix was therefore introduced to modify the solution of stiffness matrix and accurately predict the metal layer stress. The equivalent stiffness matrix was then obtained using the sub-laminate stiffness theory and the energy method. The sub-laminate stiffness method was improved to apply to the orthotropic FMLs, and the energy method was expanded to apply to FMLs from a three-dimensional perspective. Taking 2/1 and 3/2 laminates of FMLs as examples, the metal layer stress was determined using the optical strain method and was predicted by employing the classical laminate theory along with the two correction methods. The effectiveness and advancement of the modified model were confirmed by comparative analysis.

Nomenclature

FMLs Fiber metal laminates
RVE Representative volume element
\( \sigma_x \) Stress in the direction of the X axis
\( \sigma_y \) Stress in the direction of the Y axis
\( \sigma_z \) Stress in the direction of the Z axis
\( \tau_{yz} \) Shear stress on the YZ plane
\( \tau_{zx} \) Shear stress on the ZX plane
\( \tau_{xy} \) Shear stress on the XY plane
\( [\bar{E}]_k \) Stiffness matrix of each component material
Q Stiffness matrix of laminates in the classical laminate theory
S Flexibility matrix of laminates in the classical laminate theory
C Equivalent stiffness matrix of laminates calculated by the Sub-laminate stiffness method
\( V_k \) Volume fraction of the k layer material
\[ d_{ij}^{(k)} \] Element in the stiffness matrix of \( k \) layer material
\[ \bar{\sigma} \] Average stress
\[ \bar{\varepsilon} \] Average strain
\[ D \] Equivalent stiffness matrix of laminates calculated by energy method
\[ E \] Strain energy
\[ V \] Volume of equivalent body

1. Introduction

Fiber metal laminates (FMLs) are composed of alloy layers and embedded fiber layers. More specifically, they are composed of alternate layers of uni-directional, bi-directional, or multi-directional fibers and alloys [1]. This combination of metal and fiber layers can improve their respective performance defects, compared with conventional plates (i.e., those composed of metal or fiber layers alone). Due to their light weight and high damage tolerance, FMLs are currently used in aerospace structures instead of high strength aluminum alloys [2], with examples including the fuselage, the leading edge, and other parts of the Airbus A380 [3–5].

The metal layers of FMLs is accompanied by the fibers that resist crack growth [6], which will result in fiber bridging as the crack propagates in the metal layer, i.e., the fibers restrict crack propagation. However, in fatigue crack initiation and the initial stages of crack growth, crack-bridging mechanisms do not exist [7, 8], i.e., the fibers do not restrict crack propagation. Indeed, Alderliesten’s [9] research suggested that the critical crack lengths of metal layers, in which the fibers begin to take effect, differ for different laminates (usually between a few millimeters and tens of millimeters), with the critical crack length depending on the laminate structure and the bonding between layers. It has also been reported that the stress of the metal layer plays a major role prior to the bridging mechanism taking effect, with the metal layer stress and the bridging stress working together upon initiation of the bridging mechanism [10–13]. Thus, to predict the fatigue life, it is necessary to study the actual stress levels of metal laminates.

To estimate the crack initiation life and predict the fatigue life of FMLs, the stress of the FML metal layer must be predicted. This is commonly achieved using the classical laminate theory and derived methods to predict and assume the stresses present in composite laminates. For example, the in-plane stress of composite material laminates was analyzed by Wen [14] using the classical laminate theory. Considering the effect of tension and shear damage coupling generated by a large loading, the stress of plain woven composite materials under off-axis loading was also predicted based on the classical laminar theory by Zhen [15]. Although for fiber metal laminates the different components lead to different forms of stress distribution and deformation, their analysis remains possible using the classical laminate theory. Subsequently, Spronk [16] and Homan [1] modified the classical laminate theory by introducing the effect of the thermal expansion coefficient, and predicted the stress of metal layers in FMLs. Using these methods, calculation of the global stiffness matrix in the classical laminate theory and its derived methods involves the simple summation of the stiffness matrix of each component material based on volume fractions, and does not consider the actual laying situation or the interactions between layers. This results in a certain difference between the calculated global stiffness matrix and the actual situation, which produces a significant error for stress prediction in the metal layer.

Without considering the residual stress produced during the curing process, we herein report a modification of the classical laminate theory to predict the stress of the metal layers present in laminates. More specifically, the concept of the equivalent stiffness matrix is introduced to modify the solution of the global stiffness matrix in the classical laminate theory, thereby giving the equivalent stiffness matrix based on the sub-laminate stiffness theory and the energy method. In addition, the sub-layer stiffness method and the energy method are improved and expanded, respectively, to obtain models suitable for general laminates and special laminates. To verify the effectiveness of the two correction methods, the metal layer stresses of the two materials are tested and the accuracies of the prediction results are compared and analyzed.

2. The classical laminate theory

The classical laminate theory is summarized throughout the literature [17], but in brief, we note that the classical laminate theory model consists of three main parts: (1) The constitutive relationship of each component material; (2) the global stiffness matrix of the laminates; and (3) calculation of the stress of each layer material. The derivation of the classical laminate theory model is summarized below for clarity.
2.1. The constitutive relationship of each component material

FMLs are composed of mutually parallel metal and non-metal layers formed through alternate superposition. The constitutive relationship of each component material is as follows [18]:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yx} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix} =
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\
c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66}
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yx} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix}
\]

(1)

where

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{yx} \\
\tau_{yz} \\
\tau_{zx}
\end{bmatrix}^{T} - \text{Stress vector;}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yx} \\
\gamma_{yz} \\
\gamma_{zx}
\end{bmatrix}^{T} - \text{Strain vector;}
\begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\
c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\
c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\
c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\
c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\
c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66}
\end{bmatrix} - \text{Stiffness matrices;}
\]

In solving practical problems or designing composite materials, the global coordinate system often does not coincide with the local coordinate system. In such a case, the coordinate transformation formula of the stiffness matrix is required to convert the stiffness matrix of the local coordinate system into the stiffness matrix of the global coordinate system [19]. The coordinate transformation formula of the stiffness matrix is as follows:

\[
\tilde{\varepsilon} = T\varepsilon T'
\]

(2)

where

\[
T =
\begin{bmatrix}
l_{1}^{2} & m_{1}^{2} & n_{1}^{2} & 2m_{1}n_{1} & 2n_{1}l_{1} & 2l_{1}m_{1} \\
l_{2}^{2} & m_{2}^{2} & n_{2}^{2} & 2m_{2}n_{2} & 2n_{2}l_{2} & 2l_{2}m_{2} \\
l_{3}^{2} & m_{3}^{2} & n_{3}^{2} & 2m_{3}n_{3} & 2n_{3}l_{3} & 2l_{3}m_{3} \\
l_{1}l_{2} & m_{1}m_{2} & n_{1}n_{2} & m_{1}m_{2} + m_{2}m_{1} & n_{1}n_{2} + n_{2}n_{1} & l_{1}m_{2} + l_{2}m_{1} \\
l_{1}l_{3} & m_{1}m_{3} & n_{1}n_{3} & m_{1}m_{3} + m_{3}m_{1} & n_{1}n_{3} + n_{3}n_{1} & l_{1}m_{3} + l_{3}m_{1} \\
l_{2}l_{3} & m_{2}m_{3} & n_{2}n_{3} & m_{2}m_{3} + m_{3}m_{2} & n_{2}n_{3} + n_{3}n_{2} & l_{2}m_{3} + l_{3}m_{2}
\end{bmatrix}
\]

The direction cosines of each coordinate axis in the global coordinate system (oxyz) and the local coordinate system (o123) are shown in table 1.

The stiffness matrix of each layer in the laminates under the local coordinate system is then transformed into the stiffness matrix under the global coordinate system using the conversion formula, the general form of which is:

\[
[\tilde{\varepsilon}]_k =
\begin{bmatrix}
d_{11} & d_{12} & d_{13} & d_{14} & d_{15} & d_{16} \\
d_{21} & d_{22} & d_{23} & d_{24} & d_{25} & d_{26} \\
d_{31} & d_{32} & d_{33} & d_{34} & d_{35} & d_{36} \\
d_{41} & d_{42} & d_{43} & d_{44} & d_{45} & d_{46} \\
d_{51} & d_{52} & d_{53} & d_{54} & d_{55} & d_{56} \\
d_{61} & d_{62} & d_{63} & d_{64} & d_{65} & d_{66}
\end{bmatrix}
\]

(3)

where \(k\) means the \(k\)th layer material.

Taking the laying condition of each layer in the laminate into account, the stiffness matrix can thus be expressed as:
\[
[\bar{\epsilon}]_k =
\begin{bmatrix}
d_{11} & d_{12} & d_{13} & 0 & 0 & d_{16} \\
d_{12} & d_{22} & d_{23} & 0 & 0 & d_{26} \\
d_{13} & d_{23} & d_{33} & 0 & 0 & d_{36} \\
0 & 0 & 0 & d_{44} & d_{45} & 0 \\
0 & 0 & 0 & d_{45} & d_{55} & 0 \\
d_{16} & d_{26} & d_{36} & 0 & 0 & d_{66}
\end{bmatrix}
\]

(4)

2.2. Calculation of the stiffness matrix of the laminates

The stiffness matrix \( Q \) and the flexibility matrix \( S \) of the laminates can be expressed as:

\[
Q = \sum_{k=1}^{n} [\bar{\epsilon}]_k t_k n_{\text{lam}}
\]

(5)

\[
S = Q^{-1} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\
S_{21} & S_{22} & S_{23} & 0 & 0 & S_{26} \\
S_{31} & S_{32} & S_{33} & 0 & 0 & S_{36} \\
0 & 0 & 0 & S_{44} & S_{45} & 0 \\
0 & 0 & 0 & S_{45} & S_{55} & 0 \\
S_{61} & S_{62} & S_{63} & 0 & 0 & S_{66}
\end{bmatrix}
\]

(6)

where \( n \) is the number of plies in composite laminates, \( t_k \) is the thickness of the \( k_{th} \) layer material, \( n_{\text{lam}} \) is the total thickness of the laminate.

2.3. Calculation of the stress of each layer material

The strain in the middle plane of the laminate under an external load is:

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix} =
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & 0 & 0 & S_{16} \\
S_{21} & S_{22} & S_{23} & 0 & 0 & S_{26} \\
S_{31} & S_{32} & S_{33} & 0 & 0 & S_{36} \\
0 & 0 & 0 & S_{44} & S_{45} & 0 \\
0 & 0 & 0 & S_{45} & S_{55} & 0 \\
S_{61} & S_{62} & S_{63} & 0 & 0 & S_{66}
\end{bmatrix}
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix}
\]

(7)

The strain tensor under the global coordinate system is transformed to that of the local coordinate system, and the general form of the strain in the main direction of each layer is:

\[
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{13} \\
\gamma_{12}
\end{bmatrix} =
\begin{bmatrix}
l_1^2 & m_1^2 & m_1n_1 & n_1 & l_1 & m_1 \\
l_2^2 & m_2^2 & m_2n_2 & n_2 & l_2 & m_2 \\
l_3^2 & m_3^2 & m_3n_3 & n_3 & l_3 & m_3 \\
2l_1l_3 & 2m_1m_3 & 2n_1n_3 & m_1n_3 + m_3n_1 & n_1l_3 + n_3l_1 & l_1m_3 + l_3m_1 \\
2l_1l_2 & 2m_1m_2 & 2n_1n_2 & m_1n_2 + m_2n_1 & n_1l_2 + n_2l_1 & l_1m_2 + l_2m_1 \\
2l_2l_3 & 2m_2m_3 & 2n_2n_3 & m_2n_3 + m_3n_2 & n_2l_3 + n_3l_2 & l_2m_3 + l_3m_2
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xy} \\
\gamma_{xz} \\
\gamma_{yz}
\end{bmatrix}
\]

(8)

The stress of each layer in the main direction is:

\[
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\sigma_3 \\
\tau_{23} \\
\tau_{31} \\
\tau_{12}
\end{bmatrix} =
\begin{bmatrix}
d_{11} & d_{12} & d_{13} & 0 & 0 & d_{16} \\
d_{12} & d_{22} & d_{23} & 0 & 0 & d_{26} \\
d_{13} & d_{23} & d_{33} & 0 & 0 & d_{36} \\
0 & 0 & 0 & d_{44} & d_{45} & 0 \\
0 & 0 & 0 & d_{45} & d_{55} & 0 \\
d_{16} & d_{26} & d_{36} & 0 & 0 & d_{66}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\gamma_{23} \\
\gamma_{13} \\
\gamma_{12}
\end{bmatrix}
\]

(9)

3. Modification of the classical laminate theory

In the process of obtaining the classical laminate theory, calculation of the global stiffness matrix involves the addition of each layer stiffness matrix based on the volume fraction of the components, not taking into account
The concept of equivalent stiffness matrix is derived from performance equivalent theory. The diversity and multi-scale of the internal structures of composite materials (i.e., composite material laminates composed of multi-layer unidirectional plates, where the laying angle and laying thickness of each layer in the laminate differs) lead to each layer possessing a different material property in the global coordinate system, which renders the overall material property of the composite laminate both complex and diverse [20]. The experimental method can test the effective performance of a composite, which will obtain some empirical formula. In other words, the empirical formula is generally proposed by experimental studies on the mechanical properties of such materials, which is under the influence of various processing conditions, preparation methods and so on. As such, the results from empirical formula are often presented with certain limitations. Meanwhile, it costs a lot of manpower, financial resources and energy to directly carry out the experimental research on material properties. In order to accurately characterize the properties of composite materials at low cost, the mechanical properties of composite laminates are equivalent to those of homogeneous anisotropic materials by establishing a mechanical model. Then, the mechanical properties of the composite materials are analyzed and predicted using theoretical method. Indeed, performance equivalence is of particular importance for analysis of the structural properties and optimization of the structural design for composite materials.

In the existing prediction methods for equivalent composite properties, traditional algorithms tend to be based on simplified physical models or the assumption of uniform stress and strain fields, and so a large computation error exists for inhomogeneous media [21]. In addition, the homogenization algorithm is complicated and time-consuming, which limits its application and development [22, 23], while the sub-layer stiffness method [24] and the energy method [25] are two advanced algorithms with higher prediction accuracies. In the following subsections, these two algorithms are introduced, with the sub-laminate stiffness method being improved, and the energy method being expanded.

### 3.1 Calculation of the equivalent stiffness matrix using the sub-laminate stiffness method

The sub-laminate stiffness method proposed by Sun [24] is based on the hypothesis of interface continuity, and was directly derived from the constitutive relationship between components. The elastic tensor was considered as a unified whole in this method, and the idea of statistical averaging was developed further to deduce the analytical expressions of the elastic properties. The global equivalent stiffness matrix is as follows [24]:

\[
C = \begin{bmatrix}
  C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\
  C_{21} & C_{22} & C_{23} & 0 & 0 & C_{26} \\
  C_{31} & C_{32} & C_{33} & 0 & 0 & C_{36} \\
  0 & 0 & 0 & C_{44} & C_{45} & 0 \\
  0 & 0 & 0 & C_{54} & C_{55} & 0 \\
  C_{16} & C_{26} & C_{36} & 0 & 0 & C_{66}
\end{bmatrix}
\]  

(10)

This effective stiffness matrix was derived based on the theory that an anisotropic composite has a symmetry plane. However, laminates often have more than a single symmetry plane, for example, three symmetry planes. Thus, for orthotropic composites with three orthogonal symmetry planes, the following relationship was obtained based on the constant law of strain energy density:

\[
C_{16} = C_{26} = C_{36} = C_{45} = 0
\]  

(11)

According to the above characteristics, the equivalent stiffness matrix of an orthotropic material can be described as:

\[
C = \begin{bmatrix}
  C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\
  C_{21} & C_{22} & C_{23} & 0 & 0 & 0 \\
  C_{31} & C_{32} & C_{33} & 0 & 0 & 0 \\
  0 & 0 & 0 & C_{44} & 0 & 0 \\
  0 & 0 & 0 & 0 & C_{55} & 0 \\
  0 & 0 & 0 & 0 & 0 & C_{66}
\end{bmatrix}
\]  

(12)

The stiffness coefficient of the equivalent stiffness matrix can then be obtained from the sub-laminate stiffness formula [24]:

\[
C_{ij} = \sum_{k=1}^{n} V_k d_{11}^{(k)} + \sum_{k=2}^{n} \frac{(d_{13}^{(k)} - \lambda_{13}) V_k (d_{13}^{(k)} - d_{13}^{(k)})}{d_{13}^{(k)}}
\]
where $V_k$ is the volume fraction of the $k_{th}$ layer material.

### 3.2. The equivalent stiffness matrix calculated using the energy method

The energy method is commonly employed to study the representative volume element (RVE) of composite materials. The basic principle of this method is that based on the definition of equivalent elastic properties and the energy conservation theory between the RVE and the homogeneous equivalent body, the relationship between the elastic performance parameters of the homogeneous equivalent body and the deformation energy of the RVE can be deduced by giving the corresponding boundary conditions, thereby producing the energy expressions for the equivalent performance parameters of the homogeneous equivalent bodies [25]. The prerequisite for deducing the equivalent elasticity modulus for the energy method involves satisfaction of the following conditions: the stress tensor of the homogeneous equivalent body must be equivalent to the average stress of the RVE, $\bar{\sigma} = \frac{1}{V} \int_{\Omega} \sigma d\Omega$, and the strain tensor of the homogeneous equivalent body must be equivalent to the average strain of the RVE, $\bar{\varepsilon} = \frac{1}{V} \int_{\Omega} \varepsilon d\Omega$, where $\Omega$ denotes the RVE volume [25]. For a homogeneous equivalent, the stress and strain obey the following equivalent relationship:

$$\sigma = D\varepsilon$$

where $D$ is the global equivalent stiffness matrix of the RVE. Since the $x$-$y$ plane is a plane of symmetry for every constituent lamina, it is also a plane of symmetry for the effective solid. As a result of this symmetry, the effective elastic constant matrix assumes the following form:

$$D = \begin{bmatrix}
D_{1111} & D_{1122} & D_{1133} & 0 & 0 & D_{1166} \\
D_{1122} & D_{2222} & D_{2233} & 0 & 0 & D_{2266} \\
D_{1133} & D_{2233} & D_{3333} & 0 & 0 & D_{3366} \\
0 & 0 & 0 & D_{1212} & D_{1223} & 0 \\
0 & 0 & 0 & D_{1223} & D_{2323} & 0 \\
D_{1166} & D_{2266} & D_{3366} & 0 & 0 & D_{1313}
\end{bmatrix}$$

As the strain energies $E$ of the RVE and the homogeneous equivalent body are equal, the following relationship exists:
Based on equations (13)–(15), the relationship between the equivalent stiffness coefficients and the strain energy of the three-dimensional RVE can be identified under thirteen different boundary conditions. For example, in the first boundary condition, the average strain of the RVE is \(\varepsilon^{(1)} = (1, 0, 0, 0, 0, 0)^T\), and the corresponding average stress obtained based on equations (13) and (14) is \(\sigma^{(1)} = (D_{1111}, D_{1122}, D_{1133}, 0, 0, D_{1166})^T\). Equation (15) can then be employed to give \(D_{1111} = 2E^{(1)}\), where \(E^{(1)}\) are the strain energies corresponding to the thirteen different boundary conditions, which are as detailed below:

\[
\begin{align*}
\varepsilon^{(1)} &= (1, 0, 0, 0, 0, 0)^T, \\
\varepsilon^{(2)} &= (0, 1, 0, 0, 0, 0)^T, \\
\varepsilon^{(3)} &= (0, 0, 0, 0, 0, 0)^T, \\
\varepsilon^{(4)} &= (0, 0, 0, 0, 0, 1)^T, \\
\varepsilon^{(5)} &= (0, 0, 0, 0, 0, 0)^T, \\
\varepsilon^{(6)} &= (0, 0, 0, 0, 0, 0)^T, \\
\varepsilon^{(7)} &= (1, 0, 0, 0, 0, 0)^T, \\
\varepsilon^{(8)} &= (0, 1, 1, 0, 0, 0)^T, \\
\varepsilon^{(9)} &= (0, 0, 0, 0, 0, 0)^T, \\
\varepsilon^{(10)} &= (0, 0, 0, 0, 0, 0)^T, \\
\varepsilon^{(11)} &= (0, 0, 0, 0, 0, 0)^T, \\
\varepsilon^{(12)} &= (0, 0, 1, 0, 0, 1)^T, \\
\varepsilon^{(13)} &= (0, 0, 0, 1, 0, 0)^T.
\end{align*}
\]  

The relationship between the equivalent stiffness coefficients and the strain energy of the RVE is therefore:

\[
\begin{align*}
D_{1111} &= 2E^{(1)}, \\
D_{1122} &= 2E^{(2)}, \\
D_{1133} &= 2E^{(3)}, \\
D_{1122} &= 2E^{(4)}, \\
D_{1222} &= 2E^{(5)}, \\
D_{1313} &= 2E^{(6)}, \\
D_{1122} &= E^{(7)} - E^{(1)} - E^{(2)}, \\
D_{2222} &= E^{(8)} - E^{(2)} - E^{(3)}, \\
D_{1133} &= E^{(9)} - E^{(1)} - E^{(3)}, \\
D_{1166} &= E^{(10)} - E^{(1)} - E^{(6)}, \\
D_{2266} &= E^{(11)} - E^{(2)} - E^{(6)}, \\
D_{3366} &= E^{(12)} - E^{(3)} - E^{(6)}, \\
D_{1122} &= E^{(13)} - E^{(4)} - E^{(5)}.
\end{align*}
\]  

For the composite materials with three symmetric planes, their equivalent stiffness matrices are presented as:

\[
D = \begin{bmatrix}
D_{1111} & D_{1122} & D_{1133} & 0 & 0 & 0 \\
D_{1122} & D_{2222} & D_{2233} & 0 & 0 & 0 \\
D_{1133} & D_{2233} & D_{3333} & 0 & 0 & 0 \\
0 & 0 & 0 & D_{1122} & 0 & 0 \\
0 & 0 & 0 & 0 & D_{2233} & 0 \\
0 & 0 & 0 & 0 & 0 & D_{1133}
\end{bmatrix}
\]  

### 3.3. Model modification

The modification of the classical laminate theory carried out herein involved modification of the global stiffness matrix. More specifically, the global stiffness matrix in the classical laminate theory was replaced by the equivalent stiffness matrix derived from different performance equivalent algorithms to modify the classical laminate theory. The modified classic laminate theory based on the sub-laminate stiffness method and the energy method is therefore as follows. Firstly, the stiffness matrix of each stiffness matrix in the global coordinate system is obtained using equations (1), (2), and (4). Secondly, the equivalent stiffness matrix is obtained using the sub-laminate stiffness method and the energy method, as detailed in equations (10) or (12) and equations (14) or (18), respectively. Thirdly, the strain in the middle plane of the laminate is obtained by bringing the inverse matrix of the equivalent stiffness matrix to equation (7). For composite materials bearing a symmetric plane, the equation derived is as shown in equation (19), while for composite materials with three symmetric planes, the equation is derived as shown in equation (20). Finally, the metal layer stress of the laminate can be inferred by placing equations (19) or (20) into equations (8) and (9).

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{xz} \\
\gamma_{xy} \\
\gamma_{yz}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & A_{13} & 0 & 0 & A_{16} \\
A_{12} & A_{22} & A_{23} & 0 & 0 & A_{26} \\
A_{13} & A_{23} & A_{33} & 0 & 0 & A_{36} \\
0 & 0 & 0 & A_{44} & A_{45} & 0 \\
0 & 0 & 0 & A_{54} & A_{55} & 0 \\
A_{61} & A_{62} & A_{63} & 0 & 0 & A_{66}
\end{bmatrix} \begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xz} \\
\tau_{xy} \\
\tau_{yz}
\end{bmatrix}
\]  

(19)
4. Examples

To verify the validity of the prediction models, FMLs of the 2/1 and 3/2 structures were selected to carry out loading experiments. A plate with center holes was used as the specimen, the form and structure of which are shown in figures 1 and 2, respectively. For this purpose, the 2024-T3 aluminum was used as the metal layer of the FMLs, and the S4/SY-14 material was applied as the fiber layer in FMLs, in which the type of fiber is S4 glass fiber. The properties of the material components present in the laminate are outlined in table 2. In addition, the stress concentration factors of the specimen for 2/1 and 3/2 laminates was obtained by test, which are 2.7 and 2.6, respectively. The uniaxial tensile loading for the laminate materials was taken as an example. Taking into account the linear elastic properties of the materials, the boundary conditions were applied to the laminates with a remote stress of 50, 60, 70, 80, 90, or 100 MPa. The optical strain gauge was then used to measure the stress of each metal layer, and the classical laminate theory and modified method were adopted to predict the

![Figure 1. Structure diagrams for the 2/1 and 3/2 laminates.](image1)

![Figure 2. Schematic representation of the specimen of laminate material used for fatigue testing.](image2)

| Material     | Elastic modulus/GPa | Poisson’s ratio | Tensile strength/MPa | Yield strength at 0.2% offset/MPa | Thickness/mm |
|--------------|---------------------|-----------------|----------------------|----------------------------------|--------------|
| Metal layer  | 72                  | 0.33            | 455                  | 355                              | 0.25         |
| Fiber layer  | 50                  | 0.32            | 2000                 | —                                | 0.3          |

\[
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\varepsilon_z \\
\gamma_{yz} \\
\gamma_{xz} \\
\gamma_{xy}
\end{bmatrix} = \begin{bmatrix}
A_{11} & A_{12} & A_{13} & 0 & 0 & 0 \\
A_{21} & A_{22} & A_{23} & 0 & 0 & 0 \\
A_{31} & A_{32} & A_{33} & 0 & 0 & 0 \\
0 & 0 & 0 & A_{44} & 0 & 0 \\
0 & 0 & 0 & 0 & A_{55} & 0 \\
0 & 0 & 0 & 0 & 0 & A_{66}
\end{bmatrix}\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\sigma_z \\
\tau_{xy} \\
\tau_{xz} \\
\tau_{yz}
\end{bmatrix}
\] (20)

Table 2. Material properties and sizes of the laminate components.
corresponding metal layer stress values. As the loading direction of the specimen was equal to the length direction, the stress and strain in the length direction were deemed as the stress and strain of the metal layer.

4.1. Measurement of the metal layer stress using the optical strain gauge

A digital optical strain gauge was used to measure the strains of the 2/1 and 3/2 laminates under different levels of remote stress. The maximum strain location (hole area) was taken as the study object, and the test process is shown in figures 3–5. The strain data obtained from this test were converted into stress tensor values according to Hooke’s law, and the result was fit linearly as shown in figures 6 and 7.

4.2. Prediction of the metal layer stress using the mathematical model

The classical laminate theory and the modified laminate theory model were adopted to predict the metal layer stress values for the 2/1 and 3/2 laminates under different levels of remote stress. The test data in this study is obtained by testing the hole-containing sample with the digital optical strain gauge, the sampling stress is the
stress at the edge of the hole, and there is a stress concentration phenomenon. Therefore, it is necessary to consider the effect of stress concentration when analytical model is used to predict the stress at edge of the hole. Considering the effect of stress concentration, the results are presented in figures 6 and 7.

As shown in the figures, the results obtained for the modified laminate theory were closer to the experimental results than those obtained for the classical laminate theory. In addition, comparative analysis shows that the maximum errors between the classical laminate theory and test results for the 2/1 and 3/2 laminates were 10.15 and 13.06%, respectively, while those between the modified model based on the sub-laminate stiffness method and test results were 7.26 and 7.58%, respectively. This indicates increases in accuracy of 2.89 and 5.48%, respectively. Similarly, the maximum errors between the modified model based on the energy method and the test results for the 2/1 and 3/2 laminates were 7.36 and 6.96%, respectively, which represent increases in accuracy of 2.79 and 6.10%, respectively. Through the above comparisons and analyses, the validity and advancement of the modified model were therefore confirmed and verified.
5. Conclusion

We herein investigated a stress prediction method for the metal layers in fiber reinforced metal laminates. Without considering the residual stress produced during the curing process, the classical laminate theory was modified by introducing the concept of the equivalent stiffness matrix to replace the algorithm of the global stiffness matrix. The equivalent stiffness matrix was thereby obtained using the sub-laminate stiffness theory and the energy method. In addition, considering the special structure of the three symmetric planes for orthotropic composites, the sub-laminate stiffness method was improved to allow its facile adaptation. Similarly, considering the effect of thickness dimension for composite laminates, the energy method was expanded accordingly. Taking 2/1 laminates and 3/2 laminates of a fiber reinforced 2024-T3 aluminum alloy as examples, the metal layer stress was determined using the optical strain method and was predicted using the classical laminate theory and the two developed correction methods. The validity and advancement of the modified model were verified by comparison and analysis of the results. The research makes a significant contribution to the literature because the development of a stress prediction method for the metal layer in fiber metal laminates applied in the aerospace field is necessary for the further development and application of this new composite material.

Acknowledgments

This research was subsidized by the National Natural Science Foundation of China [No. 52005352]; the Doctoral research Start-up Foundation of Liaoning Province (No. 2019-BS-198); the Key Laboratory of Vibration and Control of Aero-Propulsion System, Ministry of Education, Northeastern University [No. VCAME202007]; and the ‘Seedling Cultivation’ Project for Young Scientific and Technological Talents of Liaoning Education Department [No. lnqn201908].

Data availability

The figures and tables data used to support the findings of this study are included within the article, and the article permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

Conflicts of interest

The authors declare that they have no conflicts of interest.
ORCID iDs

Weiying Meng © https://orcid.org/0000-0003-0202-7613

References

[1] Homan J J 2006 Fatigue initiation in fiber metal laminates Int. J. Fatigue 28 366–74
[2] Guo Y J and Wu X R 1999 A phenomenological model for predicting crack growth in fiber-reinforced metal laminates under constant-amplitude loading Composites Science & Technology 59 1825–31
[3] Chang P Y, Yeh P C and Yang J M 2008 Fatigue crack initiation in hybrid boron/glass/aluminum fiber metal laminates Materials Science and Engineering A 496 273–80
[4] Frizzell R M, Mccarthy C T and An M A M 2008 experimental investigation into the progression of damage in pin-loaded fiber metal laminates Composites Part B: Engineering 39 907–25
[5] Fu Y M, Zhong J and Chen Y 2014 Thermal postbuckling analysis of fiber–metal laminated plates including interfacial damage Composites Part B 64 65–84
[6] Marissen R 1988 Fatigue crack growth in ARALL: a hybrid aluminum-aramid composite material crack growth mechanisms and quantitative predictions of the crack growth rates Report LR-574 Delft University of Technology, Faculty of Aerospace Engineering
[7] Wilson G S, Alderliesten R C and Benedictus R 2013 A generalized solution to the crack bridging problem of fiber metal laminates Eng. Fract. Mech. 105 65–85
[8] Khan S U, Alderliesten R C and Benedictus R 2011 Delamination in fiber metal laminates (GLARE) during fatigue crack growth under variable amplitude loading Int. J. Fatigue 33 1292–303
[9] Alderliesten R C 1999 Development of an empirical fatigue crack growth prediction model for the Fibre Metal Lamine Glare Master Thesis Delft University of Technology, Faculty of Aerospace Engineering
[10] Guo Y J 1997 Fatigue damage and life prediction of fiber reinforced metal laminates Doctor Thesis Beijing Institute of Aeronautical Materials Chinese
[11] Kieboom O 2000 Fatigue crack initiation and early crack growth in Glare at different temperatures Master Thesis Delft University of Technology, Faculty of Aerospace Engineering
[12] Sen I, Alderliesten R C and Benedictus R 2013 Design optimisation procedure for fibre metal laminates based on fatigue crack initiation Compos. Struct. 116 275–84
[13] Huang Y, Liu Z, Huang X, Zhang J Z and Yue G Q 2015 Delamination and fatigue crack growth behavior in Fiber Metal Laminates (Glare) under single overloads Int. J. Fatigue 78 53–60
[14] Wei B C and Gu Z L 1989 Probabilistic prediction of laminates with or without delamination Acta Materiale Composite Sinica 2 48–54 Chinese
[15] Zhen W Q, Wang B and Li P 2014 Study of off-axis tensile properties of plain-woven C/SiC composites J. Mech. Strength 36 856–61 Chinese
[16] Spronk S W F, Sen I and Alderliesten R C 2015 Predicting fatigue crack initiation in fiber metal laminates based on metal fatigue test data Int. J. Fatigue 70 428–39
[17] Lekhnitskii S G 1968 Anisotropic Plates (New York: Gordon and Breach, Science Publishers) pp 112–5
[18] Zhang Z Y 1989 Mechanical Basis of Composite (Beijing: Aviation Industry Press) pp 101–4 Chinese
[19] Zao M Y and Tao M Z 2007 Structural Mechanics and Structural Design of Composite (Xi’an: Northwestern Poly technical University Press) pp 42–5
[20] Meng W Y 2013 Research and application of performance equivalence algorithm of composite for large scale wind turbine blade Master Thesis Inner Mongolia University of Technology Chinese
[21] Liu J 2008 Applications of the intelligence methods in the study of polymer/inorganic nanocomposites Doctor Thesis Jilin University Chinese
[22] Chung P W, Tamna K K and Namburu R R 2001 A symptotic expansion homogenization for heterogeneous media: computational issues and applications Composites Part A: Applied Science and Manufacturing 32 1291–301
[23] Feng M L and Wu C C 2001 A study of three-dimensional four-step braided piezo-ceramic composites by the homogenization method Compos. Sci. Technol. 61 1889–98
[24] Sun C T and Li S J 1988 Three dimensional effective elastic constants for thick laminates Composites Material 22 629–30
[25] Zhang W H et al 2007 Topology optimal design of material microstructures using strain energy-based method Chin. J. Aeronaut. 20 321–8