Low energy signatures of nonlocal field theories

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The response of inertial particle detectors coupled to a scalar field satisfying nonlocal dynamics described by nonanalytic functions of the d’Alembertian operator $\Box$ is studied. We show that spontaneous emission processes of a low energy particle detector are very sensitive to high-energy nonlocality scales. This allows us to suggest a nuclear physics experiment ($\sim$ MeV energy scales) that outperforms the sensitivity of LHC experiments by many orders of magnitude. This may have implications for the falsifiability of theoretical proposals of quantum gravity.

INTRODUCTION

Quantum field theories with nonlocal dynamics were originally studied in the 1950s and 1960s with the goal of sidestepping the infinities of local interacting QFTs [1]. With the advent of Wilson’s understanding of renormalization and the birth of the Standard Model however, these attempts were by and large abandoned only to be revived in the last two decades, mainly because they seem to emerge ubiquitously in models of quantum gravity [2–4], and also because they provide examples of consistent, renormalizable theories of gravity [5].

Nonlocal field theories are simply defined as field theories whose equations of motion have an infinite number of derivatives. For example, the equations of motion for a nonlocal free massless scalar field can be written in the form $f(\Box)\phi(x) = 0$, where $f$ is some (nonpolynomial) function of $\Box$. These theories can be subdivided into two subclasses: a) those defined by entire analytic functions $f$ and, b) those with nonanalytic $f$s. In both cases the nonlocality of the theory can lead to a much improved ultraviolet (UV) behaviour of the propagators [6,7], which is the reason why these theories were originally studied. The qualitative behaviour of the two subclasses a) and b) is, however, radically different. The underlying reason for this is that unlike an entire analytic function, nonanalytic functions contain a branch cut, i.e. a 1-dimensional subspace of the complex plane where the function has a discontinuity. In the Green function this branch cut corresponds to a continuum of massive modes, even though the original field itself is massless. Note that this is very similar to what happens to the Green function of local interacting QFTs [8]. As we will discuss below, the presence of this continuum of massive modes modifies all $n$-point functions of the theory, thus giving rise to nontrivial modifications to many physical observables.

Much of the early literature on nonlocal field theories was devoted to understanding properties such as stability and unitarity [9–11]. Recently however, there has been considerable interest in the extraction of phenomenological consequences of this kind of nonlocality, for both analytic and nonanalytic $f$s [12–15].

Ideally, one would like to find experimentally accessible signatures of nonlocality so that its existence can be put to the test. However, if such a scale is assumed to be near the Planck scale, finding an experimental setup in which the nonlocal features of the theory can be seen becomes extremely challenging.

The fact that the low energy behaviour of particle detectors is sensitive to high-energy effects was recently pointed out by Kajuri [16], and Louko and Husain [17]. They showed that some features of low energy particle detectors can be sensitive to violations of Lorentz invariance at high energies. For example, in [17] it is shown that polymer quantization (motivated by loop quantum gravity) may induce a Lorentz violation at high energies that is perceived by low energy detectors (below current ion collider energy scales). More concretely, they found low energy Lorentz violations in the response of atoms modelled as Unruh-DeWitt detectors (which capture the features of the atom–light interactions [18,19]) for a general family of quantum fields with modified dispersion relations at high (Planckian) energies.

In contrast to [17], we will consider nonlocal theories with nonanalytic $f$s that, crucially, preserve Lorentz Invariance (LI). It should be noted that LI violations are strictly constrained by various experimental observations, making theories that preserve LI particularly appealing [20,21].

In this paper we show that the existence of a nonlocality scale in a scalar field theory has phenomenological consequences on the low energy behaviour of particle detectors. In particular, we study how the existence of a nonlocality scale influences the spontaneous emission of an atomic species: a very well understood and easy to test experimental setup. We will show that it is possible, in principle, to devise a finite-time low energy experiment with a resolution similar to that of particle collider setups.
NONLOCAL DYNAMICS

We study a real scalar field obeying a special class of nonlocal dynamics given by real, retarded, Poincaré invariant wave operators, $□ := f(□)$. The retarded nature of these operators implies that $f$ is nonanalytic. Interest in this particular kind of operators can be traced back to the original construction of R. Sorkin of a d’Alembertian operator on a 2 dimensional causal set. His results were then extended to higher dimensions in, finally culminating in a comprehensive study of all possible generalizations in all dimensions in [7]. The operators $□$ depend on a nonlocality scale $l_n$, thus consistency with the local d’Alembertian requires that $□ \rightarrow □$ in the limit $l_n \rightarrow 0$ (for further details see [7]). A spectral analysis of these operators reveals that as a function of spacetime momenta the operators depend on both $k^2$ and $\text{sgn}(k^0)$, i.e.,

$$□ e^{ik \cdot x} = B(\text{sgn}(k^0), k^2) e^{ik \cdot x}. \quad (1)$$

The function $B$ possesses a branch cut along $k^2 \leq 0$ that represents a continuum of massive modes, much like those present in interacting local quantum field theories, except for the lack of a mass gap.

Free and interacting scalar quantum field theories based on this family of dynamics have been constructed using different quantization schemes [14, 25], all of which lead to the same quantum theory, at least at the free level. In particular, the Wightman function $D^{(+)}(x, y) := \langle 0 | \phi(x) \phi(y) | 0 \rangle$ for the free theory is given by [14]

$$D^{(+)}(x - y) = \int \frac{d^4k}{(2\pi)^4} \tilde{W}(k^2) e^{ik \cdot (x - y)}, \quad (2)$$

where

$$\tilde{W}(k^2) = \frac{2\text{Im}(B) \theta(k^0)}{|B|^2}. \quad (3)$$

The Wightman function can be re-written as

$$D^{(+)}(x - y) = \int \frac{d^4k}{(2\pi)^4} 2\pi \theta(k^0) \delta(k^2) e^{ik \cdot (x - y)} + \int_0^\infty d\mu^2 \rho(\mu^2) \int \frac{d^4k}{(2\pi)^4} 2\pi \theta(k^0) \delta(k^2 + \mu^2) e^{ik \cdot (x - y)}, \quad (4)$$

where $2\pi \theta(-k^2) = \tilde{W}(k^2)$ and $\tilde{W}(\mu^2) = \delta(\mu^2) + \rho(\mu^2)$. One can see that $D^{(+)}$ is a sum of two parts, one is the standard Wightman function for a local massless scalar field, $D^{(+)}_0$, and the other is an integral over the Wightman function of a local massive field, $G^{(+)}_\mu$, weighted by the finite part of the discontinuity function, $\rho(\mu^2)$.

For every choice of $□$ there corresponds a specific $\rho$. In this paper we are interested in two different kinds of d’Alembertians whose discontinuity functions are given by

$$\rho(\mu^2) = \lim_{\epsilon \rightarrow 0^+} \frac{-2 e^{2\mu^2/2} \Im[E_2(l_n^2(-\mu^2 + i\epsilon)/2)]]}{\mu^2} \Im[E_2(-l_n^2\mu^2/2)^2]. \quad (5)$$

and

$$\rho(\mu^2) = l_n^2 e^{-\alpha l_n^2 \mu^2}. \quad (6)$$

where $\alpha$ is an order one numerical coefficient [14]. The former choice of $\rho$ can be shown to give rise to a stable interacting QFT [14], while the latter is a much simpler function which captures all the fundamental features of [5] (see [7, 26]) and allows us to check that our results are largely independent of the specific form of the discontinuity function. Note that the asymptotic limit of the discontinuity function for small masses is given by $\rho(\mu^2) = l_n^2 [13]$, while for large masses it is exponentially suppressed (see Appendix B of [7]).

COUPLING THE FIELD TO A PARTICLE DETECTOR

The interaction of our nonlocal field with a two-level Unruh-DeWitt detector is described by the interaction Hamiltonian $H = g \chi(\tau/T) m(\tau) \phi(x(\tau))$, where $g$ is a small coupling constant, $m$ is the detector’s monopole moment, and $\chi^\mu(\tau)$ are the detector’s worldline coordinates parametrized by proper time $\tau$. We have included a switching function $\chi$ that controls the time dependence of the detector’s coupling strength, and is strongly supported for a timescale $T$. This detector model captures the fundamental features of the light-matter interaction in the absence of angular momentum exchange [15, 19].

The response function of an Unruh-Dewitt detector is [27, 28]

$$F(\Omega, T) = \int_{-\infty}^\infty d\tau' e^{-i\Omega \Delta \tau} D^{(+)}(\Delta \tau) \chi \left( \frac{\tau}{T} \right) \chi \left( \frac{\tau'}{T} \right), \quad (7)$$

where $\Delta \tau = \tau - \tau'$, and $\Omega$ is the frequency difference between the two detector states and $\Delta \tau = \tau - \tau'$. Using [1] one can see that the response function (7) splits into the response function of a detector coupled to a local massless scalar field, $F_0$, plus the response function of a local massive scalar field, $F_\mu$, integrated over $\mu$ weighted by $\rho(\mu^2)$. We can therefore write (7) as

$$F(\Omega, T) = F_0(\Omega, T) + \int_0^\infty d\mu^2 \rho(\mu^2) F_\mu(\Omega, T). \quad (8)$$

Since the first term is common to both local and nonlocal theories, in what follows we will study the relative difference in the detector’s response, i.e.,

$$\Delta(l_n, \Omega, T) := \frac{F(\Omega, T) - F_0(\Omega, T)}{F_0(\Omega, T)}. \quad (9)$$
It is a well known fact that in a local QFT an inertial detector in the ground state, switched on for an infinite time, \( T \to \infty \), will not click because of Poincaré invariance. A straightforward calculation along the lines of [27] shows that this is also true in the nonlocal theories studied in this paper. This should not come as a surprise given that such theories are also Poincaré invariant (and stable) by construction. We now ask what happens when the inertial detector is switched on for a finite time, \( T \), which we implement by inserting non-trivial switching functions \( \chi(\tau/T) \) in the Unruh-Dewitt interaction. Within this context, the most interesting case is that of spontaneous emission, i.e. when the detector starts out in an excited state, since in this case there can be differences between the behaviour of the detector coupled to local and nonlocal field theories even in the limit \( T \to \infty \). Furthermore, spontaneous emission is a well-understood, experimentally accessible phenomenon [29].

We will assume that the nonlocality (length) scale is much smaller than any other length scale in the problem. In particular we assume that \( |\Omega|/l_n \ll 1 \), \( T/l_n \gg 1 \), where the first condition defines the “low energy” condition, and the second ensures that the detector is switched on for a reasonable amount of time. We first consider the behaviour of the detector’s vacuum response and the spontaneous emission for short detector timescales, where we are able to perform an analytical analysis of the dependence of the results on the shape of the switching function. Secondly we will analyze the more relevant and experimentally accessible case of spontaneous emission when the detector interacts with the field for long times compared to the detector’s Heisenberg time \( \Omega^{-1} \). In this experimentally accessible regime the detector’s response is independent of the details of the switching function. We will show how a low energy detector can resolve nonlocality scales with a precision comparable to a high-energy particle collider experiment.

| \( \Omega > 0, \Omega T \gg 1 \) | \( |\Omega|T \ll 1 \) | \( \Omega < 0, |\Omega|T \gg 1 \) |
|---|---|---|
| \( \chi(t) \) | \( e^{-[l]} \) | \( \sin(t)/t \) | \( 1/2\pi T^2 \) | \( e^{-\pi^2 \omega^2} \) |
| \( \approx l_n^2/T^2 \) | \( 0 \) | \( l_n^2/t^2 \) | \( l_n^2/T^2 \) | \( l_n^2/T^2 \) |

**TABLE 1.** Detector’s response \( \mathcal{F} - F_0 \) for various switching functions (taking a dimensionless argument \( t = \tau/T \)) and for both the exponential spectral function eq. (6) and the causal and experimentally accessible case of spontaneous emission function. We will show how a low energy detector can resolve nonlocality scales with a precision comparable to a high-energy particle collider experiment.

**SPONTANEOUS EMISSION**

Consider now the case in which \( \Omega < 0 \), corresponding to the process of spontaneous emission. We are interested here in the regime characterized by \( |\Omega|T \gg 1 \), which corresponds to assuming that the detector is turned on for times much larger than the Heisenberg time of the atomic system. In this regime, we expect the detector’s response to be largely independent of the specific form of the switching function.

Using the following dimensionless variables \( t = \tau/T, k = Tp, m = T\mu \), and defining the Fourier transform of the switching function as \( \tilde{\chi}(\omega) = \int dt e^{-i\omega t} \chi(t) \), one can show that

\[
\mathcal{F} - F_0 = \frac{1}{T^2} \int dm^2 \rho(m^2/T^2) \times \int d^4k \delta(k^2 + m^2)|\tilde{\chi}(k^0 + \Omega T)|^2.
\]

For switching functions whose Fourier transform decays asymptotically faster than polynomially (e.g. Gaussian, Lorentzian or sinc), and assuming that \( |\Omega|T \gg 1 \) and \( |\Omega|l_n \ll 1 \), we get the asymptotic result

\[
\mathcal{F} - F_0 \approx \frac{4\pi}{3} Tl_n^2 |\Omega|^3 \left( 1 + O(T^2 |\Omega|^2) \right) \int_{-\infty}^{\infty} dx |\tilde{\chi}(x)|^2.
\]

Performing a similar calculation in the local, massless case yields \( F_0 \). Finally we find that the relative response of eq. (9) goes like

\[
\Delta(l_n, \Omega, T) \approx e^{-2|\Omega|^2l_n^2}, \tag{12}
\]

where we have reintroduced the speed of light for dimensional reasons. This asymptotic expression should hold for any spectral function that is exponentially suppressed with \( l_n \), such as (5) and (6). Although, rigorously speaking, (11) was obtained for the exponential spectral function (6), the asymptotic result (12) is also confirmed by a numerical analysis with the spectral function in (3) (see Tab. 1). Finally, we note that the use of switching functions whose Fourier transform decays faster than polynomially is just a sufficient condition for (12) to hold: We can see in Table 1 that (12) also applies to all the switching modalities considered, including the exponential switching function, whose Fourier transform decays polynomially.
FIG. 1. (Color online.) Detector’s relative response, ∆ (eq. (9)) for the exponential switching function and spectral function eq. (5). From left to right we have: a) |Ω|T ≫ 1 for both positive (blue circles) and negative (black squares) Ω, i.e. vacuum noise and spontaneous emission respectively; b) |Ω|T ≪ 1 for both positive (blue circles) and negative (black squares) Ω. The two data sets overlap which is consistent with the behaviour reported in table I for |Ω|T ≪ 1; c) Logarithmic-scaled contour plot of ∆. Note from plot (a) that although the vacuum response (Ω > 0) has a larger relative difference compared to spontaneous emission (Ω < 0), measuring the latter is experimentally easier.

DISCUSSION

In all the physically reasonable regimes studied for an inertial detector coupled for a finite time to a nonlocal field, we find that the nonlocal contribution to the detector’s response is polynomial in the nonlocality scale, i.e. ∝ T^3. The behaviour of the relative response (eq. (5)) in different regimes is reported in Fig. 1. This result is independent of the specific form of the switching function χ(t). The fact that nonlocal effects are not exponentially suppressed opens up interesting phenomenological windows.

In the case |Ω|T ≪ 1, we see from Table I that the detector’s response (and, indeed, also the relative response) is independent of the detector’s gap, at leading order. As for the case of vacuum excitation with large ΩT we find that while the polynomial scaling with the nonlocality scale persists, the response is in general dependent on the details of the switching function. This is not surprising given that the a non-trivial dependence also occurs in the standard local, massless case.

In the case of spontaneous emission with |Ω|T ≫ 1, we see from eq. (11) that the nonlocal contribution to the detector’s response grows like T^3Ω^3, a fact which can be used to amplify the signature of nonlocality in an experimental setting. Note that this regime is particularly interesting because spontaneous emission for times greater than the detector’s Heisenberg time is an experimentally very well understood process [29] (indeed spontaneous emission is far easier to observe than vacuum noise).

Substituting in some realistic numbers we can estimate the expected magnitude of the nonlocal signal. Consider an experimental tolerance for the relative response of ∆ ≈ 10^{-10}. Such a tolerance implies that the experimenter has the ability to repeat the experiment of the order of billions of times and accumulate statistics in order to distinguish between the two probability distributions. Then using a frequency gap of the order of 10^{22} Hz, corresponding to γ-ray transitions, we can cast a bound on l_n ≲ 10^{-19} m. Note that this constraint is of the same order as present constraints on nonlocality coming from LHC data [12].

At first sight these numbers may seem experimentally far fetched, but recall that we are analyzing the process of spontaneous emission and we could have a large number of events. Let us analyze some realistic experimental testbeds in nuclear physics. Consider for example 22Na. This nuclear species has a half-life of T_{1/2} ≈ 500 ms and decays into electromagnetically excited, highly unstable, 20\% Ne, which then spontaneously decays to its ground state emitting ∼ 11 MeV gamma radiation [30, 31]. Suppose now that one has ∼ 20 grams of 22Na (∼ N_A ≈ 6 × 10^{23} atoms), then according to the radioactive decay laws the number of gamma emission events in time τ is given by

\[ N_\gamma(\tau) = N_A(1 - e^{-\frac{T_{1/2}}{\tau}}). \]  

(13)

But in a time of τ ∼ 10 s, N_γ ∼ N_A ∼ 10^{23} ≫ ∆^{-1}. Assuming that gamma ray detection is not 100% efficient (which it is not), and in particular assuming a very conservative 0.1% experimental detection efficiency (at least one order of magnitude more conservative than realistic estimates [32]), there are still orders of magnitude more detection events than ∆^{-1}. In other words a low energy nuclear physics experiment (∼ 10 MeV scale) would already yield a higher resolution than the LHC experiments. In theory, following the reasoning above, if we assume that we have 200 grams 22Na (i.e., we have ∼ 10 N_A of the nuclear species) a very conservative estimate for this number of emission events yields that the detectable relative response would be of order ∆ ∼ 10^{-23}, which in turn implies that the experiment could detect nonlocality scales of l_n ≲ 10^{-25} m, many orders of magnitude better than the resolution of the LHC. Furthermore, there are more than a dozen different nuclear species that provide a reliable source of spontaneous emission of gamma rays.
 Due to the similarity of Eq. [4] with the standard Källén-Lehmann representation for interacting theories [33], one may wonder whether it is possible to discern the nonlocal contribution to spontaneous emission from the similar effect that would arise through interaction with a secondary massive field. In fact, one can show that such a contribution, in the case of long time spontaneous emission, vanishes unless the massive field’s mass $2m < |\Omega|$. Therefore, for EM nuclear decay, considering $|\Omega| < 2m_e \sim 1$ MeV would suffice to guarantee that the only non-trivial contribution to $l_n$ comes from $l_n$. Doing so would worsen the bound on $l_n$ discussed above by one order of magnitude – which is still better than the LHC bound – but it would also greatly increase the number of experimentally viable nuclear species. Furthermore, contributions from local massive fields can in principle always be accounted for a priori and subtracted when defining $\Delta$ (see Eq. [9]).

**CONCLUSION**

We have studied the low energy response of particle detectors coupled to a Lorentz Invariant nonlocal QFTs characterized by a nonanalytic functions of $\Box$, a kind of nonlocality that finds its roots in models of LI discrete spacetimes [2] [7] [25]. For the cases considered (eqns. [3] and [6]), we gave both numerical and analytical evidence that the detector’s relative response depends quadratically on the nonlocality scale, and argued that this result should hold for any exponentially suppressed spectral function $\rho$.

We exploited this fact to show that experimentally feasible setups – involving detectors with energy gaps of the order of MeVs (e.g. gamma emission following the $\beta$ decay of $^{22}$Na) – can potentially probe nonlocality scales of the order of $10^{-25}$ m, six orders of magnitude better than a TeV-scale experiment at the LHC [12]. This paves the way for low energy experimental tests of high-energy theories and models of quantum gravity.

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