A Mathematical Model about Neo-Classical Economy

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Abstract. We live in the era of market economy. Market and government intervention is tapering gradually. Classical planned economy has no longer adapted to the society instead of the private market in the neo-classical economic model. This paper uses mathematics as a powerful tool, with specific examples of mathematical deduction, at the same time with graphic example, we have study the neo-classical economic model, solved the neatly and a non-equilibrium market evolve toward equilibrium.

1. Introduction

We live in a time of widespread belief in an economic model [1-3] which emphasizes deregulated markets with the reduction and avoidance of government intervention in socio-economic problems [4]. This belief gained ground explosively after the collapse of the competing extreme ideology [5], communism. After many decades of rigorous attempts at central planning, communism has been thoroughly discredited in our age.

The winning side now advances globalization via rapid privatization and deregulation of markets. The dominant theoretical economic underpinning for this ideology is provided by neo-classical equilibrium theory, also called optimizing behavior, and it is taught in standard economics texts of colleges. Therefore it is necessary to know what the assumptions of mathematical model are and to understand how its predictions compare empirically with real, unmassaged data. It is even more strange that the standard equilibrium model completely excludes the profit motive as well in describing markets: the accumulation of capital is not allowed within the confines of that model, and, because of the severe nature of the assumptions required to guarantee equilibrium, cannot be included perturbatively either. This will all be discussed below.

Economists distinguish between classical and neo-classical economic ideas. Classical theory had been discussed in [6], and after neo-classical theory began with.

2. Dissecting Neo-Classical Economic Theory

In economic theory we speak of “agents.” In neo-classical theory agents consist of consumers and producers. Let \( x = (x_1, \ldots, x_n) \), where \( x_k \) denotes the quantity of asset \( k \) held or desired by a consumer. \( x_1 \) may be the number of VW Golfs, \( x_2 \) the number of Phillips TV sets, \( x_3 \) the number of ice cream cones, etc. These are demanded by a consumer at prices given by \( p = (p_1, \ldots, p_n) \). Neo-classical theory describes the behavior of a so-called “rational agent.” By “rational agent” the neo-classical mean the following.
Each consumer is assumed to perform “optimizing behavior.” By this is meant that the consumer’s implicit mental calculations are assumed equivalent to maximizing a utility function $U(x)$ that is supposed to describe his or her ordering of preferences for these assets, limited only by his or her budget constraint $M$, where

$$M = \sum_{k=1}^{n} p_k x_k = px$$  \hspace{1cm} (1)

### 3. Examples

Here, for example, $M$ equals five TV sets, each demanded at price 230 Euros, plus three VW Golfs, each wanted at 17 000 Euros, and other items. In other words, $M$ is the sum of the number of each item wanted by the consumer times the price he or she is willing to pay for it. That is, complex calculations and educated guesses that might require extensive information gathering, processing and interpretation capability by an agent are vastly oversimplified in this theory and are replaced instead by maximizing a simple utility function in the standard theory. A functional form of the utility $U(x)$ cannot be deduced empirically, but $U$ is assumed to be a concave function of $x$ in order to model the expectation of “decreasing returns” for examples and models of increasing returns and feedback effects in markets. By decreasing returns we mean that we are willing to pay less for $n$ Ford Mondeos than we are for $n-1$, less for $n-2$ than for $n-1$, and so on. An example of such a utility is $U(x) = \ln(x)$ (see Figure 1). But what about producers?

Optimizing behavior on the part of a producer means that the producer maximizes profits subject to his or her budget constraint. We intentionally leave out savings because there is no demand for liquidity (money as cash) in this theory. The only role played here by money is as a bookkeeping device. This is explained below.
4. Utility Function

Each consumer is supposed to maximize is or her own utility function while each producer is assumed to maximize his or her profit. As consumers we therefore maximize utility \( U(x) \) subject to the budget constraint (1)

\[
dU - \frac{P}{\lambda} dx = 0
\]

where \( \frac{1}{\lambda} \) is a Lagrange multiplier. We can just as well take \( \frac{P}{\lambda} \) as price \( p \) since \( \lambda \) changes only the price scale. This yields the following result for a consumer’s demand curve, describing algebraically what the consumer is willing to pay for more and more of the same item,

\[
p = \nabla U(x) = f(x)
\]

with slope \( p \) of the bidder’s price decreasing toward zero as \( x \) goes to infinity, as with \( U(x) = \ln x \) and \( p = \frac{1}{x} \), for example (see Figure 2). Equation (3) is a key prediction of neo-classical economic theory because it turns out to be falsifiable.

Some agents buy while others sell, so we must invent a corresponding supply schedule. Let \( p = g(x) \) denote the asking price of assets \( x \) supplied. Common sense suggests that the asking price should increase as the quantity \( x \) supplied increases (because increasing price will induce suppliers to increase production), so that neo-classical supply curves slope upward. The missing piece, so far, is that market clearing is assumed: everyone who wants to trade finds someone on the opposite side and matches up with him or her. The market clearing price is the equilibrium price, the price where total demand equals total supply. There is no dissatisfaction in such a world, dissatisfaction being quantified as excess demand, which vanishes.

\[
\text{Figure 3. Neo-Classical Predictions for Demand and Supply Curves } \quad p = f(x) \text{, and } p = g(x)
\]

Respectively. The Intersection Determines the Idea of Neo-Classical Equilibrium, but Such Equilibria are Typically Ruled Out by the Dynamics.

But even an idealized market will not start from an equilibrium point, because arbitrary initial bid and ask prices will not coincide. How, in principle, can an idealized market of utility maximizers clear itself dynamically? That is, how can a non-equilibrium market evolve toward equilibrium? To perform “optimizing behavior” the agents must know each other’s demand and supply schedules (or else submit them to a central planning authority) and then agree to adjust their prices to produce clearing. In this hypothetical picture everyone who wants to trade does so successfully, and this defines the equilibrium price (market clearing price), the point where the supply and demand curves \( p = g(x) \) and \( p = f(x) \) intersect (Figure 3).

There are several severe problems with this picture, and here is one: paper \cite{2} has pointed out that
supply and demand schedules for the infinite future must be presented and read by every agent (or a central market maker). Each agent must know at the initial time precisely what he or she wants for the rest of his or her life, and must allocate his or her budget accordingly. Otherwise, dissatisfaction leading to new further trades (non-equilibrium) could occur later. In neo-classical theory, no trades are made at any non-equilibrium price. Agents must exchange information, adjust their prices until equilibrium is reached, and then goods are exchanged.

The vanishing of excess demand, the condition for equilibrium, can be formulated as follows: let \( x_D = D(p) \) denote the quantity demanded, the demand function. Formally, this should be the inverse of \( p = f(x) \) if the inverse \( f \) of \( D \) exists. Also, let \( x_S = S(p) \) (the inverse of \( p = g(x) \), if this inverse exists) denote the quantity supplied. In equilibrium we would have vanishing excess demand

\[
x_D - x_S = D(p) - S(p) = 0
\]

(4)

The equilibrium price, if one or more exists, solves this set of \( n \) simultaneous nonlinear equations. The excess demand is simply

\[
\varepsilon(p) = D(p) - S(p)
\]

(5)

and fails to vanish away from equilibrium. Market efficiency \( e \) can be defined as

\[
e(p) = \min \left( \frac{S}{D}, \frac{D}{S} \right)
\]

(6)

so that \( e = 1 \) in equilibrium. Note that, more generally, efficiency \( e \) must depend on both bid and ask prices if the spread between them is large.

5. Conclusions

We see already, among other things, that although the model is used to advise governments, businesses, and international lending agencies on financial matters, the neo-classical model relies on presumptions of stability and equilibrium in a way that completely excludes the possibility of discussing money (capital) and financial markets!

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