Effect of dynamic screening on the electron capture process in nonideal plasma

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Abstract. In this paper, the process of electron capture in nonideal plasma has been investigated. For this goal the Bohr–Lindhard method has been applied to obtain the electron capture cross section. All calculations have been done in the framework of the trajectories of incident electron near hydrogen ion (proton) obtained by the numerical simulation of the equations of motion. The effective interaction potential, which takes into account the static or dynamic screening effects at large distances and quantum diffraction effects at short distances, was used. The results of numerical calculations of the electron capture radius, differential and total cross section for different values of the density parameter and velocity are presented. It is shown that dynamic charge screening increases the capture cross sections in comparison with static screening.

1. Introduction

To solve the actual problems of thermonuclear controlled fusion (TCF) with inertial confinement [1], as well as to study the processes taking place in astrophysical objects (white dwarfs, the Sun, the bowels of giant planets, etc.), reliable data on the physical characteristics of a nonideal semiclassical plasma arising on Earth and in the Cosmos in many processes associated with the heating and compression of matter are needed. A dense nonideal plasma is observed, for example, when the target substance is compressed by a high–power laser radiation in nuclear fusion, in nuclear explosions, at supersonic motion of bodies in dense layers of planetary atmospheres, at impact of high–intensity energy fluxes on the surface of various materials. Femtosecond laser pulses with high intensities are available now to produce hot dense plasmas in the laboratory [2]. The development in the fields of laser produced plasmas [3] and heavy ion beam plasma [4, 5] interactions led to a growing interest in the kinetics of ionization [6] and recombination [6, 7] and also in other processes in the nonideal semiclassical plasmas [8–13].

Recombination can take place at the collision of an electron with an ion if the latter capture the electron. The process of electron capture by an atom has been investigated in many studies [14, 15]. A neutral hydrogen atom can be converted to a negative hydrogen ion because of the polarization capture of the electron. In [14] the electron capture cross section was theoretically considered, and a method for finding the capture radius based on perturbation theory was proposed. To find the capture radius, capture time, and probability of electron capture, the Bohr–Lindhard method [15,16] was applied. The influence of the electron–exchange and plasma was discussed in the Ref. [17]. The influence of the nonthermal screening on the formation of
the negative hydrogen ion in partially ionized generalized Lorentzian plasmas degeneracy was investigated in [14]. In Ref. [15] for calculation of the electron capture cross section the applying of the calculated electron trajectories near target particle instead of the linear trajectories, which are used in the perturbation theory, has been made. In the present work we have used this approach to calculate the cross section of the electron capture by the hydrogen ion (proton) on the basis of the numerical simulation of the equations of electron motion near proton. The effective potential of electron–ion interactions, taking into account the effects of static screening and diffraction [12,18] has been used for this goal.

2. Effective interaction potential
The effective potential of electron–ion interactions, taking into account the effects of static screening [12,18–20], can be written as

$$\Phi_{ei}(r) = -\frac{Z\pi e^2}{\mu_{ei}}(e^{-Br_{ei}} - e^{-Dr_{ei}}),$$

where $A_{ei}^2 = (1 + C_{ei})/(2\lambda_{ei}^2)$, $B_{ei}^2 = (1 - C_{ei})/(2\lambda_{ei}^2)$, $C_{ei} = 1 - 4\lambda_{ei}^2/r_0^2$, $r_0 = r_D$ (static screening) or $r_0 = r_D(1 + u^2/\nu_{Th}^2)^{1/2}$ (dynamic screening) [8, 10, 12, 16, 21, 22], $u$ is the relative velocity of the colliding particles, $\nu_{Th} = (kB_T/m_e)^{1/2}$ is the thermal velocity, and $r_D = (kB_T/(8\pi e^2 n_e))^{1/2}$ is the Debye length. At small velocities of the colliding particles $r_0$ tends to the static Debye length, and at high velocities potential (1) take into account the effect of dynamic screening. Also, $n_e$ is the numerical density of electrons; $T$ is the plasma temperature; $k_B$ is the Boltzmann constant, and $Z$ is the charge number of the ion. $\lambda = \hbar/\sqrt{2\pi\mu k_B T}$ is the de Broglie thermal wavelength, $\mu_{ei} = m_e m_i/(m_e + m_i)$ is the reduced mass of the ion and the electron. In this work the following dimensionless parameters were used: $\Gamma = e^2/(ak_B T)$ is the coupling parameter, the average distance between particles is $a = (3/(4\pi n))^1/3$, $n$ is the density of the charged particles. The density (Brückner) parameter $r_A = a/a_B$ ($a_B = \hbar^2/(m_e e^2)$ is the Bohr radius, $\hbar$ is the reduced Plank constant).

3. Theory and methods
The Bohr–Lindhard method has been applied to obtain the electron capture radius, capture time and electron capture probability. In this method [23], it has been stated that the electron capture can happen when the distance between moving electron and ion is smaller than the electron capture radius $r_{cap}$. This electron capture radius is determined by equating the kinetic energy of moving electron and the interaction energy $\Phi_{ei}(r)$ between the electron and the hydrogen ion.

$$\Phi_{ei}(r_{cap}) = \frac{1}{2} m_e v_e^2,$$

where $v_e$ is the velocity and $m_e v_e^2/2$ is the kinetic energy of moving electron. According to the Refs. [14–16] the capture time can be determined as the time of the electron travelling within circle with radius $r_{cap}$. It can be found from the ratio of the traversed path of the electron to the velocity of the electron, within perturbation theory it can be written as:

$$t_{cap} = \begin{cases} \frac{2\sqrt{r_{cap} - b^2}}{v_e} & \text{for } b < r_{cap}, \\ 0 & \text{for } b \geq r_{cap}. \end{cases}$$

In this case, the electron is captured when the impact parameter $b$ is less than the capture radius, $b < r_{cap}$.
The electron capture probability is defined by the ratio of the collision time to the electron orbital time:

\[ P_{\text{cap}}(b, v_e) = \frac{1}{\tau} \int_{-\tau}^{\tau} dt. \]  

(4)

Using the electron capture probability, the electron differential and total capture cross sections can be calculated on the basis of the following expression:

\[ \frac{d\sigma_{\text{cap}}}{db} = 2\pi b P_{\text{cap}}(b, v_e), \]  

(5)

\[ \sigma_{\text{cap}} = 2\pi \int b P_{\text{cap}}(b, v_e) db, \]  

(6)

where \( \tau = a_n/v_n \) is the electron orbital time, \( a_n = n^2a_B/Z \) is the \( n \)-th Bohr radius of the hydrogenic ion, \( Z \) is atomic number of the ion, \( v_n = Z\alpha c/n \) is the electron velocity of the \( n \)-th Bohr orbit. In this work the pair collision of impacting electron with hydrogen ion was considered. The influence of other plasma particles is taken into account by the effective potential \( (1) \). The equation of motion of the electron in the field of the motionless hydrogen ion was numerically solved. Obtained trajectories were used for evaluation of capture time and capture cross section.

4. Results

The trajectories of an electron near a hydrogen ion on the basis of the effective potential with static screening and on the basis of the effective potential with dynamic screening, calculated on the basis of the numerical solution of the equations of motion of electron with initial velocity

![Figure 1](image-url)

**Figure 1.** The trajectories of an electron near the hydrogen ion on the basis of the effective potential with static screening length \( r_0 = r_D \) (right hand side, red lines in electronic version) and on the basis of the effective potential with dynamic screening length \( r_0 = r_D (1 + \delta^2/\nu_{Th}^2)^{1/2} \) (left hand side, blue lines in electronic version) obtained for the different impact parameters \( (\beta = b/a_B) \) and initial velocity \( \delta = v/\nu_{Th} = 0.5 \) by calculating the equations of motion, \( \Gamma = 1, r_s = 12. \)
\[ \delta = \frac{\upsilon}{\upsilon_{Th}} = 0.5 \text{ and impact parameter } (\beta = \frac{b}{a_B}), \text{ are shown in figure 1. In this figure, the scattering center (hydrogen ion particle) is located at } x = 0, y = 0. \]

Figure 2 shows the dependence of the electron capture radius on the velocity of the moving electron. It can be seen that with an increase in velocity the capture radius decreases. When the density parameter decreases, the electron capture radius decreases too. Electron capture radius calculated within the effective potential with dynamic screening are larger than electron capture radius derived from the static model, since the dynamic screening of the field is weaker than the static one. It is also seen that with increase in \( r_s \) (fall in the density) electron capture radius grow, since decrease in the screening effect.

![Figure 2](image)

**Figure 2.** The electron capture radius as function of the velocity of the moving electron for different values of the density parameter, \( \Gamma = 1 \). 1 – on the basis of the effective potential with static screening (1) (red lines in electronic version); 2 – on the basis of the effective potential with dynamic screening (blue lines in electronic version).

Figure 3 represents the differential cross section of electron capture as a function of the impact parameter for various values of the velocity of the moving electron. This electron capture cross section was obtained by equation (6). In this figure, one can see that the electron capture cross section decreases with growing of the velocity of the moving electron, i.e., very fast electrons do not have enough capture time to interact with the hydrogen ion.

Figure 4 shows a comparison of the total cross sections for the electron capture by the hydrogen ion. As can be seen from this figure, the total cross section obtained on the basis of the effective potential with dynamic screening lies higher than the total cross section obtained on the basis of the effective potential with static screening \( r_0 = r_D \).

5. Conclusion

Based on the effective interaction potentials, which take into account the static or dynamic screening effect at large distances and the diffraction effect at short distances, the electron capture process of the nonideal plasma has been investigated. The results showed that the electron capture radius decreases with an increase in the velocity of the electrons. Differential and total cross sections were obtained on the basis of the equations of motion. Analysis of the results showed that the electron capture radiiases, as well as the cross section scattering obtained...
Figure 3. Differential capture cross sections obtained on the basis of the simulation for different values of the velocity of the moving electron, $\Gamma = 1, r_s = 12$. 1 – on the basis of the effective potential with static screening $r_0 = r_D$ (red lines in electronic version); 2 – on the basis of the effective potential with dynamic screening $r_0 = r_D(1 + v^2/v_{Th}^2)^{1/2}$ (blue lines in electronic version).

Figure 4. Total cross sections obtained on the basis of simulation for different density parameters, $\Gamma = 1$. 1 – On the basis of the effective potential with static screening $r_0 = r_D$ (red lines in electronic version); 2 – On the basis of the effective potential with dynamic screening $r_0 = r_D(1 + v^2/v_{Th}^2)^{1/2}$ (blue lines in electronic version).

with taking into account the dynamic screening of the field are larger than the data obtained with consideration of static charge screening.
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