Bremsstrahlung in weak charged current polarized lepton-nucleon deep inelastic scattering

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Abstract

The processes of lepton-nucleon scattering, including ones with both polarized beams, at high energy provide relevant information about interaction and particles structure, allowing to analyze nucleon spin structure. As energy and experimental accuracy rise, necessity to improve Born cross sections and polarized asymmetries with higher order radiative corrections becomes substantial. In this report we stress on lowest order bremsstrahlung corrections treatment using helicity amplitudes method as applied to actual nowadays charged current lepton-nucleon deep inelastic processes, that allows to simplify matrix element calculation procedure. Real photon emission contribution is calculated by means of Lorentz-invariant formalism. Kinematical peculiarities on bremsstrahlung correction are discussed.

1 Introduction

The processes of deep inelastic lepton-nucleon scattering (DIS) are of interest at present and planned experiments, today with particular emphasis on both polarized beams interaction investigation, as it provides essential data on the internal structure of the nucleon spin. Special interest to charged current interaction is connected with the absence of large electromagnetic effects contribution to these processes. Some extensive reviews on nowadays and forthcoming experimental facilities on such processes can be found for instance in refs. [1, 2]. Asymmetries withdrew from phenomenological interaction parameters on certain experiments allow to extract detailed information on nucleon’s spin, concealed in polarized structure functions $g_1$ and $g_{2(5,6)}$ or individual quark contributions to nucleon’s spin. As expected, obtained information can be used to expand and to refine nucleon nature knowledge, to compare experimental data with other related experiments on nucleon structure (e.g. neutral current or pure electromagnetic DIS ones, which have been studied at a stretch of many years, see for instance refs. [3–5]) as well as with Standard Model predictions or perhaps to search deviations from it.

Processes in question have been investigated before mainly at Born approach, for instance in our previous papers (see in refs. [6, 7]) we realized Born level phenomenological analysis in comparison with quark-parton model approach, Born asymmetry...
analysis with stress on polarized structure functions extraction scheme. In this report we restrict oneself to detailed treatment of the bremsstrahlung correction calculation, as correct treatment with observed experimental data at high energies requires allowance for various radiative effects. Here to perform calculations we use the formalism of helicity amplitudes method offered firstly in ref. [8, 9] relevant for single and multiply bremsstrahlung processes. Using of such analytical method allows to practically avoid intermediate operations with traces of Dirac matrices products and undesirable calculations of cross-elements of $S$-matrix. This method of matrix element calculation mainly consists in special representation of 4-vectors of the photon polarization through expressions with bispinors, in using of special transformation rules likewise Chisholm identities and in treatment with $\bar{u} \pm (p) u \pm (k)$ constructions as simple scalar function of $p$ and $k$ to cancellate unnecessary terms.

2 Radiative corrections

To calculate radiative corrections we employ quark-parton model, which allows to obtain reasonable quantitative predictions for nucleon and leptonic bremsstrahlung contributions. Feynman diagrams of processes in question

$l(\overline{l}) + N \rightarrow \nu(\overline{\nu}) + X,$
$\nu(\overline{\nu}) + N \rightarrow l(\overline{l}) + X,$

are presented in FIG.1 (one of the diagram in particular case vanishes, as neutrino contain no charge).

![Feynman diagrams](image)

Figure 1: Photon bremsstrahlung diagrams for charged current lepton-nucleon DIS processes.

Let’s consider firstly the case of $l_i = l$, $l_f = \nu_i$, $q_i$, $q_f$. To make use of helicity amplitudes method one should represent photon polarization vectors as following:

$\hat{\varepsilon}^\pm = \hat{\varepsilon}^\pm = N_q \left[ \hat{q}'' \hat{q}' \hat{k}(1 \mp \gamma_5) - \hat{k} q'' q' (1 \pm \gamma_5) \right],$

$N_q = \left( 4 \sqrt{-(q''q')(q''k)(q'k)} \right)^{-1},$

$\hat{\varepsilon}^\pm = 2N_q \left[ \hat{q}'' \hat{q}' \hat{k}(1 \mp \gamma_5) - \hat{k} q'' q' (1 \pm \gamma_5) \right].$
where \( q', q'', p'_i, p''_j \) correspond to incoming \( l \) or outgoing \( \nu_l \) leptons and quarks \( q_i, q_j \) momenta, \( k \) is emitted photon momentum. This representation conserves common requirements \( \varepsilon^\pm \varepsilon'^\mp = 0, \varepsilon^\pm \varepsilon'^\pm = \varepsilon^\pm \varepsilon'^\mp = 1, \varepsilon^\pm k = 0 \). Given form suitable for calculation of the leptonic bremsstrahlung term. For hadronic terms one should involve form, dependent on quark’s momenta as following

\[
\hat{e}^\pm = e^\pm \varphi N_p \left[ p''_j p'_i \hat{k}(1 \mp \gamma_5) - \hat{k} p''_j p'_i (1 \mp \gamma_5) \right] + \beta \hat{k},
\]

where

\[
N_p = \left( 4 \sqrt{-(p''_j p'_i) (p''_j k) (p'_i k)} \right)^{-1},
\]

\[
e^\pm \varphi = (\varepsilon_q^\pm \varepsilon_p^\mp) = \frac{1}{4} S p (\hat{\varepsilon}_q^\pm \hat{\varepsilon}_p^\mp) = \frac{1}{4} S p (\hat{\varepsilon}_q^\pm \hat{\varepsilon}_p^\mp) = \frac{1}{4} S p (\hat{\varepsilon}_q^\pm \hat{\varepsilon}_p^\mp) \]

Free parameter \( \beta \) can be omitted, as longitudinal component.

One can get the following expressions for matrix element using technique thoroughly described in ref. [9]

\[
M_{++--} = -8 A N_p e^{\pm \varphi} e_{f} \sqrt{2 k^0 p_{i0} p''_0} \times u(q'') \bar{u}(p'') \bar{u}(p'_i) \times u(q') D^W, \]

\[
M_{--++} = -8 A [N_q + N_p e^{-\varphi} e_i] \sqrt{2 k^0 p_{i0} p''_0} \times \bar{u}(p''_f) u(p'_i) \times u(q') |A|^2 = \frac{e^2 G_F^2 M_W^4}{2} \frac{1}{4 q' q''}. \]

Here \( D^W \) – \( W \)-boson propagator, signs \( \pm \) refer to the particle helicity and photon polarization in the following order: \( l, \nu_l, \gamma, q_i, q_j; e_i \) and \( e_f \) – initial and final quark’s charges.

The advantage of using helicity amplitudes method is that it allows to obtain squared matrix element directly without interference terms common in straightforward calculations. It is readily to show, that squared matrix element of the lepton-quark process \( l q_i \rightarrow l q_j \nu_l \) with real photon emission have the following form

\[
|M_{++--}|^2 + |M_{--++}|^2 = \frac{4 e^2 G_F^2}{q' q'' k_0 p'_{i0} p''_0} \times \]

\[
\times \frac{M_W^4}{(Q^2 + M_W^2)^2} \left\{ \frac{1}{2} + \frac{1}{2} \left[ \frac{e_i (q' k) (q'' k)}{q' k} \left( \frac{q'' k}{q' k} - \frac{q' k}{q'' k} \right) \right] + \frac{(p'_{i0} p''_0)(p''_0 p'_{0i})}{(p'_i k)(p''_j k)} \left[ \frac{p'_i}{p''_j k} - \frac{p''_j}{p'_i k} \right]^2 \right\}.
\]
Similar formulas for other cases of electroweak lepton-quark scattering can be evolved using transformation rules

\[ l\tilde{q}_i \rightarrow \nu\tilde{q}_j : \quad p'_i \rightarrow p''_j, \quad e_i \leftrightarrow e_f \]
\[ \tilde{l}q_i \rightarrow \tilde{\nu}q_f : \quad q' \leftrightarrow q'', \]
\[ \tilde{l}\tilde{q}_i \rightarrow \tilde{\nu}\tilde{q}_f : \quad q' \leftrightarrow q'', \quad p'_i \leftrightarrow p''_j, \quad e_i \leftrightarrow e_f. \]

We use here the following common notations for kinematical variables:

\[
\begin{align*}
 Q_l &= q' - q'' , \quad Q_l^2 \approx -2q'q'', \\
 Q_h &= Q_l - k = p'' - p', \quad Q_h^2 \approx -2p'^2 - p''^2 , \\
 X_i &= -2p'_i q'', \\
 S_i &= -2p'_i q', \\
 u &= -2p'^2 , \quad z_2 = -2q''k , \\
 v &= -2p'^2 k = u - Q_l^2 + Q_h^2 , \\
 z_1 &= -2q'' k = z_2 - Q_l^2 + Q_h^2 ,
\end{align*}
\]

where \( x_{h[l]} = -Q_h^2/2p'_i Q_h , \quad y_{h[l]} = -2p'_i Q_h^2 / S \) — standard hadron and lepton scaling variables, \( p' \) and \( p'' \) — nucleon and jet 4-momenta. To obtain cross sections or polarized asymmetry including radiative corrections one should switch from lepton-quark interaction to lepton-nucleon one integrating over quark momenta being carried in the nucleon, and over radiated photon momentum. If we suppose quark to possess momentum \( p'_i = x_{ih}p' \) with the probability of \( f(x_{ih}) \), the first integration over \( p'_i \) could be performed by means of the following substitutions:

\[
 f(x_{ih}) \rightarrow \frac{f(x_{ih})}{-2p'_i Q_h} = \frac{f(x_{ih})}{y_h S} , \quad S_i \rightarrow x_{ih} S \rightarrow x_{ih} S = \frac{Q_h^2}{y_h} , \\
 X_i \rightarrow x_{ih} X \rightarrow x_{ih} X = \frac{Q_h^2}{y_h} (1 - y_h) S , \\
 u \rightarrow x_{ih} u \rightarrow x_{ih} u , \quad v \rightarrow x_{ih} u - Q_l^2 + Q_h^2 ,
\]

keeping \( Q_h^2 = (Q_l - k)^2, \quad Q_l^2, \quad z_1 \) and \( z_2 \) unaltered. Here \( S = -(p' + q')^2 \).

To integrate over photon momentum one can use covariant method of integration described, for instance, in ref. [10][11], permitting to integrate directly over Lorentz-invariant kinematical variables. Covariant calculation has advantage of missing of the sophisticated Monte-Carlo techniques but presence of the analytical integration as well as it can be carried out for various kinematical experimental configurations.

Firstly, lets imply the following suitable phase space transformation, allowing to derive from its general form the expression containing introduced before invariant variables:

\[
 d\Gamma = dM_h^2 d^3 p'' d^3 q'' d^3 k d^4 \delta(Q_l - k - Q_h) = \\
 = dM_h^2 dQ_h^2 d^3 q'' d^3 k \frac{\partial}{2q_0'' 2q_0 2k_0} \delta \left( (Q_l - k)^2 + M_h^2 \right) \times \\
 \times \delta \left( Q_h^2 - (p'' - p')^2 \right) = \frac{\pi S}{2} dy_{ih} dQ_l^2 dy_h dQ_h^2 \frac{dz}{\sqrt{R_z}}.
\]

Here \( R_z \) is the Gram determinant [12] of 4-vectors \( q', p', q'', p'' \)

\[
 R_z = -\Delta_4(q', p', q'', p'') = 
\]
which can be expressed as quadratic polynomial of \( z_1 \) or \( z_2 \) variables defined before

\[ R_z = -A z^2 + 2Bz - C, \]

where the coefficients in the ultrarelativistic limit are

\[
A_{1,2} = y^2 l S^2 + 4 M^2 Q_l^2, \\
B_1 = -2M^2 Q_l^2 (Q_l^2 - Q_h^2) + (y Q^2 - y_h Q^2) S^2 + \\
(1 - y_l) S^2 Q_l^2 (y_l - y_h) - m^2 (2M^2 Q^2 + 2M^2 Q_l^2 - S^2 y_h y_l), \\
B_2 = 2M^2 Q_l^2 (Q_l^2 - Q_h^2) + (1 - y_l)(y Q^2 - y_h Q^2) S^2 + \\
S^2 Q_l^2 (y_l - y_h) - m^2 (2M^2 Q^2 + 2M^2 Q_l^2 - S^2 y_h y_l), \\
C_1 = S^2 |Q_h^2 + (-1 + y_l - y_h) Q_l^2|^2 + \\
+4 m^2 Q_l^2 (y_l - y_h) (1 - y_l), \\
C_2 = S^2 [(1 - y_l) Q_h^2 - (1 - y_h) Q_l^2|^2 + \\
+4 m^2 Q_l^2 (y_l - y_h) (1 - y_l)^{-1}. \]

In presented above expression we simplified common phase space by means of auxiliary invariant variables \( z_1 \) or \( z_2 \). Next one can employ the following integration scheme:

\[
d\sigma \sim \int_{y_h \text{ min}}^{y_h \text{ max}} dy_h \int_{Q_h \text{ min}}^{Q_h \text{ max}} dQ_h^2 \int_{z_\text{ min}}^{z_\text{ max}} dz dy_h dQ_h^2 y_h \sqrt{R_z} A,
\]

or

\[
d\sigma \sim \int_{y_l \text{ min}}^{y_l \text{ max}} dy_l \int_{Q_l \text{ min}}^{Q_l \text{ max}} dQ_l^2 \int_{z_\text{ min}}^{z_\text{ max}} dz dy_h dQ_h^2 y_h \sqrt{R_z} A,
\]

\[
A = |M|^2 f_i (x_h, Q_h^2),
\]

and so on, dependently on desired final variables. Here matrix element \(|M|^2\) expressed in terms of \( Q_l^2, y_l, z_1, \text{ and } z_2 \) have the following form:

\[
|M(S_i, X_i, Q_h, Q_l, z_{[1,2]}))|^2 \sim \\
\frac{e^2 S^2 Q^2}{2uv} + (z_2 - Q_l^2 - X_i)^2 \times \\
\frac{[Q_h^2 u + (e_1 Q_l^2 z_1 z_2 - e_i Q_l^2 u(z_1 + z_2) + e_i Q_l^2 (u - v)(S_i z_2 - X_i z_1))]}{2uv z_1 z_2}
\]

for \( l_i = l, l_f = \nu_l, q_i, q_f \) and

\[
|M(S_i, X_i, Q_h, Q_l, z_{[1,2]}))|^2 \sim
\]
For $l_i = l$, $l_f = \nu_i$, $\bar{q}_i$, $\bar{q}_f$.

Calculation of the integral over $z$ can be carried out using simple table integrals of the form

$$I = \int_{z_{\text{min}}}^{z_{\text{max}}} \frac{z^n}{A(z - z_{\text{min}})(z_{\text{max}} - z)} \, dz, \quad n = -2 \ldots 2.$$  

To evaluate remaining integrals one should firstly use some parameterizations on quark distribution functions $f_i(x_h, Q_i^2)$, for instance QCD-based fits from ref. [13], and then choose final variables in which result cross-section or asymmetry will be expressed.

In order to calculate infrared contribution, one can apply the limits $\lim_{k \to 0} (u - v) = 0$ and $\lim_{k \to 0} (z_2 - z_1) = 0$ to presented above relations. When calculating this part requires using of some regularization method as being infrared divergent part, for instance by introducing virtual photon mass or applying dimensional regularization method. Here we didn’t stress on soft photon emission contribution calculation, as virtual loop contribution and soft emission one, infrared divergent separately, compensate partly each other, remaining uncompensated part have minor influence on the asymmetries for processes in question as reduced factor.

Kinematical peculiarities and variables limits for these integrals are thoroughly described e.g. in ref. [12], here we give only kinematical relations, necessary for determination of the integration bounds. Imposed constraints on the physical region of invariants have the following form in terms of kinematical $\lambda$-functions (see [12])

$$\lambda(\lambda_S, \lambda_I, \lambda_q) \leq 0, \quad \lambda(\lambda_q, \lambda_h, \lambda_k) \leq 0,$$

$$\lambda(\lambda_S, \lambda_r, \lambda_h) \leq 0,$$

$$\lambda_k = (y_t - y_h)^2 S^2, \quad \lambda_S = S^2 - 4m^2 M^2,$$

$$\lambda_l = (1 - y_l)^2 S^2, \quad \lambda_q = y_t^2 S^2 - 4M^2 Q_t^2,$$

$$\lambda_h = y_h^2 S^2 + 4M^2 Q_h^2, \quad \lambda_r = (1 - y_h)^2 S^2 - 4M^2 z_2,$$

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2xy - 2yz - 2xz.$$  

The boundary equations for this conditions can be expressed in the form of three equations

$$M^2(Q_t^2 - m^2)^2 + Q_t^2 y_l S^2 - m^2 S^2 y_l(1 - y_l) - Q_t^2 \lambda_S = 0,$$

$$y_l^2 S^2 Q_h^2 + y_h^2 S^2 Q_t^2 - M^2(Q_t^2 - Q_h^2)^2 - y_l y_h(Q_t^2 + Q_h^2) S^2 = 0,$$

$$(2M^2 z_2^2 - m^2 M^2 + y_h S^2 + 2M^2 Q_h^2)^2 - \lambda_S \lambda_h = 0,$$

consequently.

One can obtain certain integration bounds for chosen final variables by combining these constraints with $z_{1,\text{max}}$ and $z_{2,\text{max}}$ emerging from the condition $R_{z_{1,2}} \geq 0$.

In ref. [6] [7] we numerically calculated hard real photon emission contribution with low photon energy cut parameter $\varepsilon_{\text{cut}}$, where unpolarized quark distribution
Figure 2: Born (solid line) and next to Born order (dotted line) longitudinal polarized asymmetry $A_{\parallel}(x_l, y_l)$ for $lN \rightarrow \nu_lX$ at $E = 100$ GeV and $\varepsilon_{\text{cut}} = 1$ MeV.

functions from ref. [13] with polarized one from ref. [14] were taken. In FIG.2 we cite comparison of the corrected polarized asymmetry

$$A_{\parallel} = \left( \frac{d^2\sigma^{\uparrow\uparrow}}{dxdy} - \frac{d^2\sigma^{\uparrow\downarrow}}{dxdy} \right) / \left( \frac{d^2\sigma^{\uparrow\uparrow}}{dxdy} + \frac{d^2\sigma^{\uparrow\downarrow}}{dxdy} \right),$$

differential cross section, $\uparrow \ (\downarrow)$ corresponds to lepton helicity value $-1 \ (1)$, $\uparrow \ (\downarrow)$ for nucleon spin, parallel (antiparallel) to lepton momentum, with the Born one in lepton scaling variables $x_l$ and $y_l$. As for relative contribution, it increases with $x_l$ growth, reaching values for $x_l = 0.9$ approximately larger by 20% than for $x_l = 0.01$; it roughly inversely exponentially dependent on $y_l$, not exceeding $\approx 30\%$ for $y_l > 0.3$ and $x_l < 0.9$, $\approx 20\%$ for $y_l > 0.3$ and $x_l < 0.25$ and $\approx 10\%$ for $y_l > 0.3$ and $x_l < 0.1$. Asymmetry absolute values depend on $y_l$ noticeable unlike Born ones.

As for multiply bremsstrahlung, such contributions can be obtained on the basis of single one by applying renormalization group equations, and will be treated hereafter.

In conclusion, real photon bremsstrahlung contribution calculated here affects significantly cross-sections and polarized asymmetries, so it’s necessary to take into account such contribution, in that way it can be used for certain future experimental needs with the aim of precision DIS data extraction to improve accuracy of quark distribution functions as well as to detail nucleon’s spin structure.

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