Radiatively induced electron and electron-neutrino masses

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Abstract

We consider, in the context of a 331 model with a single neutral right-handed singlet, the generation of lepton masses. At zeroth order two neutrinos and one charged lepton are massless, while the other leptons, two neutrinos and two charged leptons, are massive. However the charged ones are still mass degenerate. The massless fields get a mass through radiative corrections which also break the degeneracy in the charged leptons.

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It is well known that in renormalizable theories, some masses or mass differences vanish at tree level if there are some symmetries in the theory which forbid them. This is maintained in higher order in perturbation theory. Or, if the respective higher order corrections are infinite the introduction of the counter term necessary to remove the infinity leaves these masses or mass differences as free parameters. Notwithstanding, in theories with spontaneously broken symmetry if a mass and mass counter term are forbidden by gauge structure, then higher order corrections are finite and calculable [1]. In this spirit, many mechanisms for finding fermion masses as radiative corrections have been considered in the literature [2]. Here we will show how a mechanism of this kind can be implemented in the context of the recently proposed electroweak model based on the $SU(3)_L \otimes U(1)_N$ gauge symmetry [3,4].

In this model, leptons are treated democratically with the three generations transforming as $(3,0)$ but with one quark generation (it does not matter which one) transforming differently from the other two. This condition arises because the model in order to be anomaly free must contain the same number of triplets as antitriplets. Hence, the number of generations is related to the number of quark colors.

In the minimal model the neutrinos remain massless since there is a global symmetry which prevents them to get a mass. This symmetry implies the conservation of the quantum number $\mathcal{F} = L + B$, where $L$ is the total lepton number $L = L_e + L_\mu + L_\tau$ and $B$ is the baryon number [5]. Here we will see that if we allow this symmetry to be explicitly broken and also adding a single right-handed neutrino singlet, one of the charged leptons and two neutrinos get mass through radiative corrections.

Let us introduce the following Higgs scalars, $\eta = (\eta_0, \eta_1^-, \eta_2^+)^T$, $\rho = (\rho^+, \rho^0, \rho^{++})^T$ and $\chi = (\chi^-, \chi^-, \chi^0)^T$ which transform, under $SU(3)_L \otimes U(1)_N$ as $(3,0), (3,1)$ and $(3,-1)$, respectively.

The leptonic triplets are $\psi_{aL} = (\nu''_a, l''_a, l''_c)^T \sim (3,0)$, where the double primed fields denote weak eigenstates, $l''_a = e''_a, \mu''_a, \tau''_a$ and $\nu''_a = \nu''_e, \nu''_\mu, \nu''_\tau$.

The lepton mass term transforms as $3 \otimes 3 = 3^*_A \oplus 6_S$. Thus, we can introduce a triplet, like $\eta$, or a symmetric antisextet $S = (6^*_S, 0)$. In the former case one of the charged leptons
remains massless, and the other two are mass degenerate. For this reason it was chosen in Refs. [4,6] the latter one in order to obtain arbitrary mass for charged leptons.

Here we will not introduce the sextet $S$ but only the triplets $\eta, \rho$ and $\chi$; the respective VEV will be denoted by $v_\eta, v_\rho$ and $v_\chi$.

The more general $SU(3) \otimes U(1)$ gauge invariant renormalizable Higgs potential for the three triplets is

$$V(\eta, \rho, \chi) = \mu_1^2 \eta^\dagger \eta + \mu_2^2 \rho^\dagger \rho + \mu_3^2 \chi^\dagger \chi + \lambda_1 (\eta^\dagger \eta)^2$$

$$+ \lambda_2 (\rho^\dagger \rho)^2 + \lambda_3 (\chi^\dagger \chi)^2 + (\eta^\dagger \eta) \left[ \lambda_4 (\rho^\dagger \rho)ight.$$ 

$$+ \lambda_5 (\chi^\dagger \chi) \right] + \lambda_6 (\rho^\dagger \rho) (\chi^\dagger \chi) + \lambda_7 (\rho^\dagger \eta) (\eta^\dagger \rho)$$

$$\left. + \lambda_8 (\chi^\dagger \eta) (\eta^\dagger \chi) + \lambda_9 (\rho^\dagger \chi) (\chi^\dagger \rho) \right.$$ 

$$+ [\lambda_{10} (\eta^\dagger \chi) (\eta^\dagger \rho) + \lambda'_{ijk} \eta_i \rho_j \chi_k + H.c.]. \quad (1)$$

As we said before let us define the lepto-baryon number $F = L + B$, which is additively conserved. As usually $B(l, \nu_l) = 0$ for any lepton $l, \nu_l$, and $L(q) = 0$ for any quark $q$, $F(l) = F(\nu_l) = +1$. In order to make $F$ a conserved quantum number in the Yukawa sector, we also assign to the scalar fields the following values $-F(\chi^-) = -F(\eta_2^-) = F(\rho_+^+) = -F(\chi^{++}) = +2$, and with all the other scalar fields carrying $F = 0$.

Notice that the $F$-conservation forbids the quartic term $\lambda_{10} (\eta^\dagger \chi) (\eta^\dagger \rho)$ in Eq. (1). Hence, assuming that the $\lambda_{10}$ term does exist we are violating explicitly the $F$ symmetry. We must stress that if we had introduced the scalar sextet and allow $F$ to be broken, there will be additional terms involving the sextet and the triplets as well. In this case it is not possible to maintain neutrinos with calculable masses unless a fine tune is imposed.

The $\lambda_{10}$ term has interactions like $\rho^0 \chi^0 \eta_1^- \eta_2^+, \eta^0 \rho^0 \eta_1^- \chi^+, \eta_1^- \rho^-, \eta_2^+, \chi^-$, etc. In fact, the mass matrix in the singly charged scalars sector (in the $\eta_1^-, \rho^-, \eta_2^+, \chi^-$ basis) is
where $e = \lambda' / v_\chi$, $a = v_\eta / v_\chi$, $b = v_\rho / v_\chi$. Hence we have a mixing among all singly charged scalars. The spectrum from Eq. (2) will be given elsewhere but here we must stress that it has two Goldstone bosons. Notice that if there is no $\lambda_10$ term the mixing occurs between $\eta^-_1, \rho^-$ and between $\eta^-_2, \chi^-$. 

Since right-handed neutrinos transforming as singlets under the gauge group do not contribute to the anomaly, their number is not constrained by the requirement of obtaining an anomaly free theory. Hence, we can introduce, as in the standard electroweak model, an arbitrary number of such fields. An interesting possibility is to introduce just a single neutral singlet [8].

The Yukawa interaction in the leptonic sector plus a Majorana mass term for the right-handed neutrino, is

$$L_{\eta''} = -\frac{i}{2} \sum_{a,b=e,\mu,\tau} \delta_{a\mu} F_{ab}(\psi_{La}^c)^c \psi^c_{Lb} \eta_k + \sum_a \bar{\eta}_a \psi^c_{aL} \nu^\prime_{LR} \eta - \frac{1}{2} M(\nu^\prime_R) \nu^\prime_R + H.c.$$ (3)

All the arbitrary constants in Eq. (3) may be taken real and positive. The Yukawa couplings $F_{ab}$ must be antisymmetric due to Fermi statistics. Explicitly we have

$$L_{\eta''} = -i (v_\eta + \eta^0) \bar{\nu}_{LR} F_{ab} l^\mu_{Lb} l^\mu_{La} + \frac{i}{2} \bar{\nu}_{LR} F_{ab} \nu^\prime_{Lb} \eta_1 + \frac{i}{2} \bar{(\nu^\prime_R)} F_{ab} l^b_{LR} \eta_2 + H.c.,$$ (4)

where

$$F_{ab} = \begin{pmatrix}
0 & -f_{e\mu} & -f_{e\tau} \\
-f_{e\mu} & 0 & -f_{e\tau} \\
f_{e\tau} & f_{e\tau} & 0
\end{pmatrix}. \quad (5)$$

The mass spectrum of the charge leptons is $0, m, -m$. We can always define $e''$ as the state with zero mass. That is, we can choose a basis in which $f_{e\mu} = f_{e\tau} = 0$. In this case we have $m = v_\eta f_{e\mu}^2$. In this basis the matrix in (3) can be diagonalized by an unitary matrix
\[
U = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\
0 & -\frac{i}{\sqrt{2}} & \frac{i}{\sqrt{2}}
\end{pmatrix}.
\]

Notice that one of the mass is negative. To get positive mass eigenvalues we let \( m \) be positive and redefine the respective field with a \( \gamma^5 \) factor, i.e., \( \tau' \to \gamma^5 \tau' \). Because of this \( \gamma^5 \) the \( CP \) of \( \tau' \) is \(-1\).

The neutrinos mass term is

\[
\mathcal{L}_\nu = -\sum_{a=e,\mu,\tau} h_a \bar{\nu}_{La} \nu'_R - \frac{1}{2} M(\bar{\nu}_R^c \nu'_R + H.c.,
\]

where \( h_a = \hat{v}_a \hat{h}_a \), and \( \hat{h}_a \) are arbitrary dimensionless parameters. We can write the mass term as

\[
-\frac{1}{2} \bar{N}' M' N'^c \quad \text{with} \quad N'' = (\nu''_e, \nu''_\mu, \nu''_\tau, \nu''_R)^T \quad \text{and}
\]

\[
M' = \begin{pmatrix}
0 & 0 & 0 & h_e \\
0 & 0 & 0 & h_\mu \\
0 & 0 & 0 & h_\tau \\
h_e & h_\mu & h_\tau & M
\end{pmatrix}.
\]

We can diagonalize the neutrino mass matrix by making \( N' = P N'' \) with \( P \) an orthogonal matrix,

\[
P = \begin{pmatrix}
h_\mu/(A^2 - h_\tau^2)^{\frac{1}{2}} & h_e/(A^2 - h_e^2)^{\frac{1}{2}} & 0 & 0 \\
h_e h_\mu h_\tau/(A^2 - h_e^2)^{\frac{1}{2}} & h_\tau h_\mu^2/(A^2 - h_e^2)(A^2 - h_\tau^2)^{\frac{1}{2}} & -h_\mu/(A^2 - h_e^2)^{\frac{1}{2}} & 0 \\
(h_e m'_{\nu_P} - A^2)/D_1 & h_\mu M/D_1 & h_\tau M/D_1 & h_e M/D_1 \\
(h_e (h_e m'_{\nu_P} - A^2))/D_2 & h_\mu (h_e m'_{\nu_P} - A^2)/D_2 & h_\tau (h_e m'_{\nu_P} - A^2)/D_2 & -A M (h_e m'_{\nu_P} - A^2)/D_2
\end{pmatrix}
\]

where

\[
D_1^2 = (A^2 - h_\tau^2)^2 + M^2 A^2 + (A^2 - h_e^2 - h_e m'_{\nu_P})(h_e^2 - h_e m'_{\nu_P})
\]

\[
D_2^2 = A (A^2 - h_\tau^2)(A^2 - h_e^2 + M^2 - 2h_e m'_{\nu_P}) + h_e^2 (m_{\nu_P}^2 + 2h_e^2).
\]
\[ N' = (\nu'_1, \nu'_2, \nu'_{PL}, \nu'_{FL})^T, \quad A^2 = h^2_e + h^2_\mu + h^2_\tau \] and \( \Phi \) is a diagonal phase matrix \( \Phi = diag(1, 1, i, 1) \). At this stage, the neutrino mass spectrum consists of two massless fields \( \nu'_{1,2} \) and two Majorana massive \( \nu'_{P,F} \) neutrinos \[ 8 \]

\[
m'_{\nu_p} = \frac{1}{2}[(4A^2 + M^2)^{\frac{3}{2}} - M], \quad m'_{\nu_F} = \frac{1}{2}[(4A^2 + M^2)^{\frac{3}{2}} + M],
\]

(10)

Note that \( m'_{\nu_F} \) is arbitrary and in fact could be heavier than the lepton-\( \tau \). Constraints on the masses \( m'_{\nu_P} \) and \( m'_{\nu_F} \) coming from the measured \( Z^0 \) invisible width were considered in Ref. \[ 10 \].

In the primed basis for the charged leptons, Yukawa couplings with the scalars \( \eta^-_1 \) and \( \eta^+_2 \) can be written as

\[
\frac{i}{2} \overline{R}_a (UF)_{ab} \nu'_L b \eta^-_1 + \frac{i}{2} \overline{R}_a (FU)^\dagger_{ab} l'_L \eta^+_2 + H.c.
\]

(11)

As there are four neutrinos but only three charged leptons, it is possible to extend the charged lepton column with a zero on the fourth row in such a way that in Eq. (11) all matrices are \( 4 \times 4 \). In Eq. (11) the neutrinos are still linear combinations of the mass eigenstates, \( \nu''_L = (-\Phi P^T)_{al} N'_L, \ l = 1, 2, 3, 4 \). Let us denote \( \Gamma = UF \). Notice that in Eq. (11) the \( F \) matrix is in the basis in which \( f_{e\mu} = f_{e\tau} = 0 \) and for this reason the electron does not interact with any neutrino.

Due to the mixing of \( \eta^-_1 \) and \( \eta^+_2 \) and to the non-diagonal Yukawa couplings \( \Gamma \), diagrams like the one showed in Fig. 1 exist and they are finite. Notice that the primed fields \( (\mu', \tau') \) couple with the two massive neutrinos \( \nu'_p, \nu'_F \), then the neutrino masses insertions are \( m'_{\nu_p}, m'_{\nu_F} \). Then, the diagrams in Fig. 1 induce a contribution to the mass matrix of the \( \mu', \tau' \). In fact contributions from Fig. 1 have the following form

\[
\delta_{ab} \sim \lambda_{10} m'_{\alpha_l} \beta_l \Gamma_{al} \Gamma_{vb} \left( \frac{\nu_{vb} \nu_\eta}{m^2_{\eta}} \right) \ln \left( \frac{m^2_{\eta}}{m^2_{\eta_2}} \right), \quad a, b = \mu, \tau
\]

(12)

where \( m' \) means \( m'_{\nu_p} \) or \( m'_{\nu_F} \), \( m^2_{\eta_1}, m^2_{\eta_2} \) are typical masses in the scalar sector and \( m^2_{\eta_2} \) is the greatest of them. The \( \Gamma' \)’s are the couplings appearing in Eq. (11) and \( \alpha_l, \beta_l \) denote the \( \nu'' \)’s projections in the \( \nu'_{P,F} \) components i.e., can be read off from \( \nu''_{aL} = -(\Phi P^T)_{al} N_l, \ l = 3, 4 \).
As an example the \( \nu'_F \) contribution in Fig. 1 induces the following mass matrix in the \( \mu', \tau' \) basis

\[
m' \alpha_l \beta_l \Gamma_{\alpha l} \Gamma_{\nu b} \sim \frac{1}{D_2^2} \left( \begin{array}{cc} mD_2^2 + m'_{\nu F} (h_\epsilon m'_{\nu F} - A^2)^2 (h_\tau - h_\mu)^2 & -im'_{\nu F} (h_\epsilon m'_{\nu F} - A^2)^2 (h_\tau - h_\mu)^2 \\ im'_{\nu F} (h_\epsilon m'_{\nu F} - A^2)^2 (h_\tau - h_\mu)^2 & mD_2^2 + m'_{\nu F} (h_\epsilon m'_{\nu F} - A^2)^2 (h_\tau + h_\mu)^2 \end{array} \right).
\]

(13)

Notice that the diagonal terms have the contributions of the tree level mass \( m \). We see from Fig.1 that an arbitrary mass matrix arises for the charged leptons \( \mu' \) and \( \tau' \) at the 1-loop level breaking their mass degeneracy. Since \( \nu'_F \) can be heavier than the tau lepton \( \nu_2 \), the mass difference \( m_\tau - m_\mu \) may be fitted with reasonable values for the parameters present in the model. Notwithstanding, the electron is still massless.

Since at the tree level \( \nu'_1 \) and \( \nu'_2 \) are massless we may define linear combinations, say \( \tilde{\nu}'_1 \) and \( \tilde{\nu}'_2 \) in such a way that \( \tilde{\nu}'_2 \) does not couple to one of the charged leptons, say the electron. However, if one makes this choice in the vector current, \( \tilde{\nu}'_2 \) will still couple to the electron through the Yukawa interactions in Eq. (11). This is due to the presence of the \( F \) matrix. On the contrary if we choose that \( \tilde{\nu}'_2 \) does not couple to the electron in Eq. (11) it will couple to it through the vector current.

At 1-loop level processes like those showed in Fig.2 are also possible. Here the mass insertions are \( m \) i.e., the charged lepton mass at tree level. It implies also a symmetric matrix in addition to \( M'' \) in Eq. (8). Hence, the massless neutrinos (at tree level) \( \nu'_2 \) acquire masses from 1-loop radiative corrections. In Fig. 2 we show the case of \( \nu'_2 \rightarrow \nu'_F \) mixing. However, \( \nu'_1 \) is still massless.

After these loop corrections have been taken into account we have the basis \( (e'', \mu, \tau) \) and \( (\nu'_1, \nu_2, \nu_F, \nu'_F) \) with the respective masses \( (0, m_\mu, m_\tau) \), and \( (0, m_2, m_{\nu_\mu}, m_{\nu_F}) \).

In the model there are two singly charged vector bosons \( W^\pm \) and \( V^\pm \). Their interactions with the leptons are \( \bar{\nu}_{\alpha L} V_\gamma^\mu l_L W^+_{\mu} \) and \( \bar{l}_L V_\gamma^\mu \nu_L V^+_{\mu} \) respectively. At this stage since three neutrinos are already massive we have a general mixing among them i.e., \( V = P^T \Phi P^T U^U^T \) is a \( 4 \times 4 \) matrix, \( P' \) is the arbitrary unitary \( 3 \times 3 \) matrix which diagonalize the mass
matrix of the three massive neutrinos after the contributions of Fig. 2 have been taken into account. On the other hand $U'$ has the same structure than $U$ in Eq. (3) but now with arbitrary elements. The matrices $U, U'$ and $P'$ are appropriately extended by adding zeros and with $U_{44} = U'_{44} = P'_{11} = 1$, in order to write them as $4 \times 4$ matrices and the charged lepton column written as $(e'', \mu, \tau, 0)$. Hence, it is straightforward to convince ourselves that $V_l$ has the appropriate form to induce diagrams like that showed in Fig. 3. These diagrams which exchange one $W^-$ and one $V^+$ as in Ref. [11] are possible and they are responsible for the mass of the electron as is shown in Fig. 3. In this figure, we show only the contribution involving $\nu_F$ and the tau lepton. This is a higher order contribution, however it is proportional to $m_\tau m^2_{\nu_F}$ and for this reason the electron mass can be of the appropriate size.

Let us back to the $\nu'_1$ neutrino. This particle is up to now massless, however notice that after the electron has got a mass there is a contribution similar to the Fig. 2 but now the electron in the internal line. This kind of processes mix all the four neutrinos and $\nu'_1$ acquire a mass.

We have shown that if one does not introduce the sextet $S$, it is possible to give the right mass to all leptons if at least a single right-handed neutrino and a $\mathcal{F}$-violating term in the scalar potential are added to the minimal model. It is possible, of course, the case with three right-handed neutrinos, but we will not treat this case here.

It is interesting that in a supersymmetric version of the model it is also possible to give mass to the charged leptons without introducing the sextet of scalars [12].

Hence, in this 331 model, the smallness of the masses of the electron, the lightest neutrinos and the mass difference between muon and tau arise naturally.

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FIGURES

FIG. 1. One-loop contributions to the charged lepton ($\mu', \tau'$) mass matrix. With this contribution the mass matrix appers in Eq.(13). There is a similar diagram with $\nu'_P$ on the internal line.

FIG. 2. One-loop diagram inducing a mass for the $\nu'_2$ neutrino.

FIG. 3. Diagram inducing a mass for the electron.
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