Restrictions on the lifetime of sterile neutrinos from
primordial nucleosynthesis

Oleg Ruchayskiy∗ and Artem Ivashko†‡

Abstract

We analyze the influence of sterile neutrinos with the masses in the MeV
range on the primordial abundances of Helium-4 and Deuterium. We solve
explicitly the Boltzmann equations for all particle species, taking into account
neutrino flavour oscillations and demonstrate that the abundances are sensitive
mostly to the sterile neutrino lifetime and only weakly to the way the active-
sterile mixing is distributed between flavours. The decay of these particles also
perturbs the spectra of (decoupled) neutrinos and heats photons, changing the
ratio of neutrino to photon energy density, that can be interpreted as extra
neutrino species at the recombination epoch. We derive upper bounds on the
lifetime of sterile neutrinos based on both astrophysical and cosmological mea-
urements of Helium-4 and Deuterium. We also demonstrate that the recent
results of Izotov & Thuan [1], who find 2σ higher than predicted by the stan-
dard primordial nucleosynthesis value of Helium-4 abundance, are consistent
with the presence in the plasma of sterile neutrinos with the lifetime 0.01 – 2
seconds.

1 Introduction

The characteristic feature of the physical processes in the early Universe is a pec-
culiar interplay of gravity and microscopic physics. Gravity introduces the Hubble
time parameter \( \tau_H \) that indicates the timescale on which the global properties of the
Universe (geometry, temperature, etc.) change significantly. The Hubble time is de-
termined solely by the energy density of the matter filling the space. The microscopic
matter constituents, particles, are involved in the interaction processes, that are believed to be described fundamentally by three known forces — electromagnetic, weak and strong. As long as the timescale $\tau$ of any given microscopic physical process is much smaller than $\tau_H$, the expansion can be neglected on that timescale. If time $\tau$ is enough to establish thermal equilibrium between the particles, then the equilibrium is maintained in the course of the Universe expansion, while $\tau \ll \tau_H$ holds. When this inequality ceases to hold, the state of equilibrium is lost. The main reason for that is that interparticle distances become larger, while the corresponding densities become lower, hence interactions are less likely to occur.

In this paper we are consider the formation of light nuclei in the primordial environment – Big Bang nucleosynthesis (BBN). All three fundamental interactions are important for this phenomenon, all playing different roles. Charged particles together with photons are subject to electromagnetic forces and the equilibration timescale of corresponding processes is tiny with respect to the expansion time. Therefore the particles are kept in thermal equilibrium at the common temperature $T$. Due to expansion the temperature is decreasing with time. The equilibration time of the weak interactions changes abruptly so that at $T \gtrsim \text{few MeV}$ weakly interacting neutral particles (neutrinos and neutrons) stay in equilibrium, while at lower temperatures they fall out of it (freeze out).

At high temperatures processes like $n + \nu_e \rightarrow p + e^-$ maintain chemical equilibrium, that is the neutron-to-proton conversion exhibits the same finite intensity as the opposite processes. Chemical and thermal equilibria are interconnected, so they are lost simultaneously, when neutron-to-proton ratio freezes out. Finally, the strong interactions are responsible for the production of nuclei comprising more than one nucleon. The most important fusion reaction for the formation of the first nucleus, deuteron, $n + p \rightarrow D$, releases energy of at least the binding energy of deuteron $E_D \approx 2.2 \text{ MeV}$, and proceeds effectively in dense primordial medium. At temperatures of the order of $E_D$, however, energetic photons collide with deuteron and lead to its destruction. As baryon density is much lower than the density of photons [2], there are many photons with energies much higher than $E_D$ that collide with deuterons and hence postpone the production of the significant deuteron density until the temperature when the photodissociation is not effective anymore, $T \approx 80\text{keV}$, much lower than the binding energy. The net abundance of deuterium is, however, non-zero at all times till this moment and is given by the equilibrium Boltzmann distribution. Deuterium that is created at lower temperatures, serves as a fuel for the formation of $^3\text{He}$, $^4\text{He}$ and other nuclides.

Although the times of elements’ production and the moment of the departure from the chemical $p-n$ equilibrium are well-separated, the former process is very sensitive to the latter. Firstly, the details of the freeze-out set the ratio of the neutron to proton densities, and secondly, the time elapsed between the two moments determines the fraction of neutrons that have decayed since then (recalling that neutron is an unstable particle).
The seminal ideas of the primordial synthesis of light elements were first outlined in the so-called \( \alpha \beta \gamma \) paper, [3], published in the late 1940s. Since then the theory of Big Bang nucleosynthesis has evolved and its main predictions were confirmed, making it a well-developed model from both theoretical and observational points of view. A lot of reviews of the standard BBN scenario and its implication for particle physics models exist (see e.g. [4, 5, 6]).

The predictions of the primordial nucleosynthesis can change once one replaces the Standard Model of particle physics underlying the processes considered so far by some of its “beyond the Standard Model” (BSM) extensions. Therefore the BBN plays the role of a benchmark for testing physical models.

In this paper we investigate the influence of sterile neutrinos on primordial nucleosynthesis. Sterile neutrinos are hypothetical massive super-weakly-interacting particles (see e.g. [7, 8] for reviews), as opposed to their weakly-interacting counterparts – ordinary Standard Model neutrinos \( \nu_e, \nu_\mu, \nu_\tau \), that are called “active” in this context. Sterile neutrinos carry no charges with respect to the Standard Model gauge groups (hence the name), but via their quadratic mixing to active neutrinos they effectively participate in weak reactions and at energies much below the mass of the \( W \)-boson their interaction can be described by the analog of the Fermi theory with the Fermi coupling constant \( G_F \) replaced by \( G_F \times \vartheta_\alpha \), where the \textit{active-sterile mixing angle} \( \vartheta_\alpha \ll 1 \) (see Fig. 1). Here \( \alpha \) is a flavour index, \( \alpha = e, \mu, \tau \), indicating that sterile neutrino can mix differently with neutrinos of different flavours. Massive sterile neutrinos can decay, but due to their feeble interaction strength their lifetime can be of order seconds (even for masses as large as MeV). The decay products of the sterile neutrinos are injected into the primordial environment, increasing its temperature and shifting the chemical equilibrium.

In this work we concentrate on sterile neutrinos with the masses in the MeV range, motivated by the recent observations [9, 10, 11, 12, 13] that particles with such masses can be responsible simultaneously for neutrino oscillations and generation of baryon and lepton asymmetry of the Universe and can influence the subsequent generation of dark matter [14]. The corresponding model has been dubbed \( \nu \text{MSM} \) (\textit{Neutrino Minimal Standard Model}, see [7] for review).

Several works had previously considered the influence of MeV-scale particles on primordial nucleosynthesis. Compared to the Refs. [15, 16] this paper accounts for the neutrino flavour oscillations in the plasma and employs more accurate strategy of solving Boltzmann equations, which results in the revision of the bounds of [15, 16] (see Section 4 for detailed comparison). The authors of [17] developed a new code that can perform treatment of active and sterile neutrinos with arbitrary distribution functions, non-zero lepton asymmetry, etc. However, as of time of writing this code has not been made publicly available and the Ref. [17] did not derive bounds on sterile neutrino parameters. The work [18] concentrated on the bounds that cosmic microwave background measurements could provide on decaying sterile neutrinos with the masses \( 100 - 500 \) MeV, leaving BBN analysis for the future work. A number
Figure 1: Fermi-like super-weak interactions of sterile neutrino

of other works (19, 20, 21, 22, 23) analyzed the influence of decaying MeV particles on BBN. We compare with them in the corresponding parts of the paper.

The paper is organized as follows. We explain the modifications of the standard BBN computations due to the presence of sterile neutrinos in the plasma and describe our numerical procedure in Sec. 2. The results are summarized in Sec. 3. We conclude in Sec. 4. Appendixes A–C provide the details of our numerical procedure.

2 Big Bang Nucleosynthesis with sterile neutrinos

The section below summarizes our setup for the BBN analysis with decaying particles. The notations and conventions closely follow the series of works [16, 24, 25].

We will be interested only in the tree-level Fermi interactions of sterile neutrinos with the primordial plasma. In this case the interaction is fully determined by the squares of their mixing angles. We will consider one Majorana particle with 4 degrees of freedom\(^1\) and three active-sterile mixing angles \(\vartheta^2_{e}\). Matrix elements of interactions of sterile neutrinos with the Standard Model particles are summarized in Appendix B (Tables 3–4 on page 27).

\(^1\)This number corresponds to \(g_s = 2\) of additional chiral singlets (i.e. “neutrino-like” species). Actual number of degrees of freedom is of course twice larger: \(\text{dof} = 2 \times g_s = 4\), because every chiral fermion has 2 different helicity states.
We consider in this work only sterile neutrinos with the masses in the range $1 \text{ MeV} < M_s < M_a \approx 140 \text{ MeV}$. For heavier particles, two-particle decay channels appear (e.g. $\nu_S \rightarrow \pi_0 \nu_\alpha, \pi^\pm e^\mp$) and our procedure of solving Boltzmann equations (described below) should be significantly modified. The lower bound was chosen to be around 1 MeV by the following considerations. The sterile neutrino lifetime $\tau_s$ is

$$
\tau_s^{-1} = \frac{G_F^2 M_s^5}{96\pi^3} \left[ (1 + \tilde{g}_L^2 + g_R^2)(\vartheta^2_\mu + \vartheta^2_\tau) + (1 + g_L^2 + g_R^2)\vartheta^2_\epsilon \right]
$$

approx 6.9 sec$^{-1} \left( \frac{M_s}{10 \text{ MeV}} \right)^5 \left[ 1.6 \vartheta^2_\epsilon + 1.13(\vartheta^2_\mu + \vartheta^2_\tau) \right]$

(1)

where $\theta_W$ is the Weinberg’s angle and $g_R = \sin^2 \theta_W \approx 0.23$, $g_L = \frac{1}{2} + \sin^2 \theta_W$, $\tilde{g}_L = -\frac{1}{2} + \sin^2 \theta_W$. From this expression one sees that sterile neutrinos lighter than about 2 MeV have lifetime of at least several hundred seconds even for very large mixing angles $\vartheta \sim 1$. Therefore, such particles survive till the onset of the BBN, and freeze-out at temperatures $T \sim 2 - 3$ MeV. They would be relativistic at that time, i.e. their average momentum would be of the order of temperature, $\langle p \rangle \sim T$, and their contribution to the number of relativistic neutrino species would be significant, $\Delta N_{\text{eff}} \approx 2$. In the course of the Universe expansion $\langle p \rangle$ would scale as temperature due to the gravitational redshift, and at some point would become smaller than the mass of sterile neutrino. At that moment the energy density of sterile neutrinos would start to change with expansion as $a^{-3}$ rather than $a^{-4}$ (where $a$ is a scale-factor) so that the contribution of these massive particles to the energy density would quickly become dominant, making $N_{\text{eff}} \gg 1$ (or could even overclose the Universe) before the production of light elements starts. It contradicts the current bound that puts $N_{\text{eff}} = 3.74^{+0.8}_{-0.7} \pm 0.06(\text{syst})$ at 2$\sigma$.

Additionally, in the $\nu$MSM the successful baryogenesis is possible only for the masses of sterile neutrinos above few MeV [11, 13]. Therefore we restrict the analysis to the region of masses higher than 1 MeV.

2.1 Expanding Universe and distributions of particles

We consider expansion of the homogeneous and isotropic Universe with the flat Friedmann–Robertson–Walker metric in the form $ds^2 = dt^2 - a^2 dx^2$, where $a = a(t)$ is a time-dependent scale factor, whose evolution is described by the Friedmann equation

$$
H \equiv \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G_N}{3} \rho} \ ,
$$

(2)

The expression is for Majorana particle. For Dirac particle the lifetime would be twice larger.

Here the systematic error is due to the different values of neutron lifetime between the average value from Particle Data group, [27] and the recent measurement of [28].
with the quantity on the left-hand side being the Hubble expansion rate, reciprocal to the expansion timescale \( \tau_H \) discussed above. The total energy density \( \rho \) is the sum of all the energy densities present in the medium, and \( G_N \) is the Newton’s constant. The energy density together with the total pressure density \( p \) satisfy the “energy conservation” law

\[
a \frac{d\rho}{da} + 3(p + \rho) = 0. \tag{3}
\]

At the temperatures of interest the dominant components of the plasma are photons \( \gamma \), electrons and positrons \( e^\pm \), three flavours of active neutrinos \( (\nu_e, \nu_\mu, \nu_\tau) \) and sterile neutrinos. Working with the particle kinematics in the expanding Universe it is convenient to use *conformal momentum* \( y \) instead of the usual physical momentum \( p \). The two are related through \( y = pa \). The quantitative description of the plasma population is provided by the distribution functions \( f_\alpha \), that are the numbers of particles \( \alpha \) per “unit cell” of the phase space \( d^3p \, d^3x = (2\pi)^3 \). At keV–MeV temperatures the medium is homogeneous and the distribution functions are independent of spatial coordinates of particles, and due to isotropy they do not depend on the direction of the particle momentum. That simplifies the description of their evolution and therefore

\[
\frac{df}{dt} = \left( \frac{\partial f}{\partial t} - H_p \frac{\partial f}{\partial p} \right) = \frac{\partial f(t, y)}{\partial t} \tag{4}
\]

holds. The goal is to find the distribution functions of all relevant particles and to use them to compute the energy density and pressure as a function of time and scale-factor, closing the system of Eqs. (2)–(3) via

\[
\rho = \sum_i \frac{g_i}{2\pi^2} \int f_i E_i p^2 dp \quad ; \quad p = \sum_i \frac{g_i}{6\pi^2} \int f_i E_i^4 p^4 dp \tag{5}
\]

Here the summation goes over all plasma particles, \( g_i, m_i \) is the number of degrees of freedom and mass of \( i \)-th particle respectively, \( E_i = \sqrt{p^2 + m_i^2} \).

If interaction rate of the particles is much faster than the Hubble expansion rate, their distribution functions are given by either the Bose-Einstein, or the Fermi-Dirac distributions. This is the case for photons, electrons and positrons — that are kept in equilibrium due to intensive electromagnetic interactions

\[
f_\gamma = \frac{1}{e^{E/T} - 1}, \quad f_e = \frac{1}{e^{E/T} + 1}. \tag{6}
\]

Their contribution to the energy and pressure in Eqs. (2), (3) is hence determined by the single parameter – temperature. However, to describe the contributions of the other particles one has to solve *kinetic* equations involving them (see Secs. 2.3–2.4 below).

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4Muons may appear in plasma from the decays of the sterile neutrinos with \( M_s > 106 \text{ MeV} \). See Sec. 2.4 for details.
2.2 Baryonic matter

The contribution of the baryonic matter to the evolution of the hot plasma of relativistic species is proportional to the so-called baryon-to-photon ratio $\eta_B = n_B/n_\gamma$. The measurements of relic radiation [2] yield $\eta_B = (6.19 \pm 0.15) \times 10^{-10}$. One can see that baryons are present in negligible amount, and do not influence the dynamics of the remaining medium. This allows to analyze our problem in two steps. At step I we omit baryonic species and study how the temperature of the plasma, the expansion factor and neutrino distributions evolve in time from temperatures of the order of 100 MeV, when sterile neutrinos typically start to go out of equilibrium, down to $T_{\text{Fin}} \approx 10$ keV when nuclear fusion reactions have ended. At step II we use these results to determine the outcome of the nuclear reaction network against the background of evolving electromagnetic plasma (Sec. 2.5).

2.3 Active neutrinos at MeV temperatures

Weak interactions are not able to maintain the thermal equilibrium of active neutrinos with the plasma during all the expansion period we consider. A simple comparison of the weak collision rate $G^2 T^5$ and $H(T)$ tells that neutrino maintain their equilibrium with the rest of the plasma down to temperatures $T_{\text{dec}} \sim$ few MeV. The process of neutrinos going out of equilibrium is usually referred to as neutrino decoupling. Throughout the paper we assume that no large lepton asymmetry is present so that the number of neutrinos is equal to the number of antineutrinos. At temperatures higher than $T_{\text{dec}}$ the distribution is therefore given by the Fermi-Dirac one, while at lower temperatures we have to solve the set of three Boltzmann equations

$$\frac{df_{\nu_\alpha}}{dt} = I_\alpha, \quad \alpha = e, \mu, \tau$$

(7)

The details of the interactions, such as particle collisions, are encoded in the so-called collision terms $I_\alpha$. The terms are explicitly [33]

$$I_\alpha = \frac{1}{2E_\alpha} \sum_{\text{in},\text{out}} \int S |\mathcal{M}|^2 F[f](2\pi)^4 \delta^4(p_{\text{in}} - p_{\text{out}}) \prod_{i=2}^{Q} \frac{d^3 p_i}{(2\pi)^3 2E_i}$$

(8)

The sum runs over all the possible initial states “in” involving $\nu_\alpha$ (represented by a particle set $\nu_\alpha$, 2, 3, . . . , $K$) and the final states “out” ($K+1$, . . . , $Q$). Matrix element $\mathcal{M}$ corresponds to the probability of the transition “in”–“out” to occur and the delta-function ensures the conservation of 4-momentum $p_{\text{in}} = p_{\text{out}}$. Symmetrization factor $S$ is equal to 1, except of the transitions involving identical particles either in

5The exact “freeze-out” temperature depends on the mixing angle.

6For the previous studies of the BBN outcomes with the large lepton asymmetry present see e.g. [29, 30, 31, 17, 32].
initial or in a final state. Relevant matrix elements together with the symmetrization factors are listed in Appendix B. The interaction rates are dependent on the population of the medium, and the functional $F[f]$ describes this. In case when all the incoming and outgoing particles are fermions,

$$F[f] = (1 - f_{\nu_{\alpha}}) \cdots (1 - f_{K}) f_{K+1} \cdots f_{Q} - f_{\nu_{\alpha}} \cdots f_{K}(1 - f_{K+1}) \cdots (1 - f_{Q}).$$ \hspace{1cm} (9)

When some of particles are bosons, one has to replace $(1 - f_{R})$ by $(1 + f_{R})$ for every bosonic particle $R$.

A simple estimate (see Appendix C) demonstrates that the rates of transitions between neutrinos of different flavours are much faster than weak reactions. We argue that this phenomenon can be approximately described by the following modification of the Boltzmann equations

$$\frac{df_{\nu_{\alpha}}}{dt} = \sum_{\beta} I_{\beta} P_{\beta\alpha}. \hspace{1cm} (10)$$

Summation is carried out over three active flavours and expressions for $P_{\beta\alpha}$ are listed in Appendix C (Eqs. 26).

2.4 The impact of sterile neutrinos

As already mentioned, sterile neutrinos interact much more feebly than active neutrinos do. Nevertheless, at some high temperature sterile neutrinos may enter thermal equilibrium. Whether this happens or not depends on the thermal history of the Universe before the onset of the synthesis\(^7\) Even if they were in thermal equilibrium at early times, sterile neutrinos then necessarily decouple at temperatures higher than those of active neutrino decoupling. If sterile neutrinos were light and stable (or very long-lived), they would be relativistic and propagate freely in the medium, yielding $N_{\text{eff}} \approx 3 + g_{S}$ together with active neutrinos ($g_{S}$ is the number of sterile neutrinos). However, sterile neutrinos decay into active neutrinos and other particles. The energies of the decay products may be very different from the typical energies of plasma particles. For particles that equilibrate quickly (such as electrons or photons), this “injection” results in the fast redistribution of the energy between all particles in equilibrium and effectively the process looks like a temperature increase (more precisely, it just slows down the cooling of the Universe). But for particles that either are not in equilibrium or are about to fall out of it, such as active neutrinos at few MeV, the “injection” modifies the form of their spectra. The other mass-induced effect is that sterile neutrinos may switch from the relativistic regime (when their average momentum is larger than mass), that is established at large temperatures, to the non-relativistic one, due to the gravitational redshift.

\(^7\)For example in the $\nu$MSM model at early times ($T \gg 100$ GeV) initial densities of sterile neutrinos are negligible \[^{34}\]. Then the neutrinos come into equilibrium at temperature $T_{\nu}$ (typically $T_{\nu} = 10 \div 100$ GeV) and freeze-out at temperatures $T_{\nu} \sim 0.5 - 5$ GeV \[^{12}\].
For the quantitative description of sterile neutrino dynamics we utilize the Boltzmann equation similar to (7), replacing active neutrino everywhere therein by sterile neutrino $\nu_S$

$$\frac{df_S}{dt} = I_S$$  \hspace{1cm} (11)

Reactions contributing to the right-hand side together with their probabilities are listed in Tables 3, 4 on page 27 of Appendix B. Note that we neglect the processes involving baryonic particles. However, they become important for temperatures near the QCD crossover temperature $T_{QCD} \simeq 200\text{MeV}$, when their density is not negligible anymore. More scattering channels of sterile neutrino would appear and their proper account is involved. However it seems to be reasonable to assert that the only modification the account will bring is to lower the decoupling temperature of sterile neutrinos.

Oscillation phenomenon does not affect significantly sterile neutrinos and therefore Boltzmann equation in its original form (11) is still valid, contrary to what we have found out for active neutrinos. An argument in favor of this statement is explained in Appendix C.

When sterile neutrino is heavier than muon, the former particle can appear in the decay $\nu_S \rightarrow \mu^- + e^+ + \bar{\nu}_e$. However, the branching fraction of this decay mode does not even reach a percent for masses of sterile neutrino we consider (see e.g. [26]). Therefore we can neglect influence of both muons and other particles, appearing in the decay.

As a result we have six equations (2), (3), (10), and (11) describing primordial plasma at temperatures of interest. These equations contain six unknowns – scale factor $a(t)$, temperature $T(t)$ and four neutrino distribution functions, $f_{\nu_\alpha}$ and $f_S$. The system of equations is therefore closed and we have solved it numerically at the step i.

### 2.5 Course of nuclear reactions

Outcome of the nuclear reaction chains is found numerically. For the Standard BBN model one of the earlier attempts was made with the code written by L. Kawano [35, 36]. However, the program in its original form is inappropriate for the account of the BSM physics, and we modified it for this work. Two technical remarks are in order here. First, we used the 1992 version of the program [36] as a starting point, and not the 1988 one. [35]. Therefore, the integration time steps were taken small enough, so that the integration procedure did not introduce an error, that was compensated as a shift in the resulting value of the $Y_p$\(^8\) the so-called “Kernan correction” [37]. Second, the code did not take into account the Coulomb and the nucleon finite-mass corrections to weak interaction rates, as well as radiative and finite-temperature

\(^8\)We denote by $Y_p$ the mass fraction of the $^4\text{He}$, that is a fraction of the total baryon mass stored in the form of Helium-4
We do not calculate directly these effects, but assume their net result to be in the form of the additive correction, which we took to be \( \Delta Y_p = -0.0003 \). The tests described in Appendix A.1 demonstrate an agreement of this modified “Kawano code” with the results of the other code, PArthENoPE \([42]\), that takes a proper account of these effects.

Presence of sterile neutrinos alters the standard dynamics of the temperature and the expansion rate as well as the rates of weak interactions involving neutrons and protons. These quantities are known from the step 1, so we have implemented the import of these data. Together with the change of \( \Delta Y_p \) indicated above, it has lead to the code, that became an essential tool of step II in our approach. The computations of nuclide evolution started from temperatures of several MeV, when the chemical equilibrium ceases to hold, up to temperatures \( T_{\text{Fin}} \).

### 2.6 Adopted values of abundances of the light nuclei

The observables of the BBN are concentrations, or abundances, of light nuclides dispersed in the cosmos. The most relevant abundance in our problem is that of \(^4\text{He}\), as it is sensitive to the expansion rate of the Universe at MeV temperatures and neutrino distribution functions. The presence of sterile neutrinos in plasma typically increases the concentration of \(^4\text{He}\), described by \( Y_p \). Accurate calculations carried out in the Standard Model \([42]\) predict the values

\[
\begin{align*}
Y_{p,\text{SBBN}} &= 0.2480 \quad (\tau_n = 885.7 \text{ sec}) \quad (12) \\
Y_{p,\text{SBBN}} &= 0.2465 \quad (\tau_n = 878.5 \text{ sec}) \quad (13)
\end{align*}
\]

depending on the lifetime of neutron, \( \tau_n \), see below.

There are two main methods of experimental determination of primordial Helium abundance. The first one is related to the studies of low-metallicity astrophysical environments and extrapolating them to zero metallicity case. The \( Y_p \) measurements are known to be dominated by systematic uncertainties. Therefore we adopt the \( Y_p \) values from the two most recent studies, Refs. \([1, 43]\) that have slightly different implications. For recent discussion of various systematic uncertainties in \(^4\text{He}\) determination, see \([44]\).

In Ref. \([1]\) the value \( Y_p = 0.2565 \pm 0.0010(\text{stat.}) \pm 0.0050(\text{syst.}) \) was obtained. Therefore, the 2\(\sigma\) intervals that we adopt in our studies are \([10]\)

\[
Y_p = 0.2495 - 0.2635 \quad (\text{Ref.} \ [1], \ 2\sigma \text{ interval}) \quad (14)
\]

One notices that this result is more than 2\(\sigma\) away from the Standard Model BBN predicted value of \( Y_p \), Eq. \((12)\).

\[^9\text{For the accurate account of these corrections, see e.g.} \ [43, 39, 40].\]

\[^{10}\text{We add the systematic errors linearly}\]
Using a subsample of the same data of [1], a different group had independently determined $Y_p$ [3]. From their studies we adopt $Y_p = 0.2574 \pm 0.0036$ (stat.) $\pm 0.0050$ (syst.). As a result,

$$Y_p = 0.2452 - 0.2696 \text{ (Ref. [3], 2$\sigma$ interval)}$$

(this values of $Y_p$ coincide with the Standard BBN one, [2], at about 1$\sigma$ level).

Second method of determination of Helium abundance is based on the CMB measurements. This method is believed to determine truly pristine value of $Y_p$, not prone to the systematics of astrophysical methods. However currently its uncertainties are still much larger than of the first method. The present measurements put it at

$$Y_p = 0.22 - 0.40, \quad N_{\text{eff}} = 3 \text{ (Refs. [4, 2], 2$\sigma$ interval)}$$

again consistent with the Standard Model BBN at 1.5$\sigma$. Here $N_{\text{eff}}$ is the so-called effective number of neutrino species

$$N_{\text{eff}} = \frac{120 \rho_\nu_e + \rho_\nu_\mu + \rho_\nu_\tau}{7 \pi^2} T^4,$$

proportional to the ratio of the total energy, deposited into the active neutrino species to that of photons. Notice, that the bound (16) is based on assumption that before the onset of the recombination epoch the effective number of neutrino species is close to its SM value $N_{\text{eff}} \approx 3$. As we will see later, sterile neutrinos can significantly distort $N_{\text{eff}}$. For the values of $N_{\text{eff}}$ strongly deviating from 3 the CMB bounds on $Y_p$ gets modified. For example, the analysis carried out in [4] reveals that

$$Y_p = 0.10 - 0.33, \quad N_{\text{eff}} = 6 \text{ (Ref. [4], 2$\sigma$ interval)}.$$  

The similar conclusion is reached if one employs the data of [5].

The other element produced during the BBN is the Deuterium, and recent observations determine its abundance to be

$$D/H = (2.2 - 3.5) \times 10^{-5} \text{ (Ref. [4], 3$\sigma$ interval)}.$$  

This value is sensitive both to the baryon-to-photon ratio and to $N_{\text{eff}}$. In this work we adjust the value of baryon-to-photon ratio $\eta$ at the beginning of the computation so that by $T_{\text{Fin}} \sim 10$ keV it is equal to the value given by cosmic microwave background measurements [2].

Finally, we mention another important uncertainty originating from the particle physics. There are two different measurements of neutron lifetime $\tau_n$ that are at tension with each other. Particle Data Group [27] provides $\tau_n = 885.7 \pm 0.8$ sec, while measurements performed by Serebrov et al. [28] result in $\tau_n = 878.5 \pm 0.8$ sec. We employ both results and explore the differences they lead to in what follows.

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11 We use the average value over metallicities, $\langle Y_p \rangle$ (Eq. (8.2) of [3]) and leave the systematic error from [1].

12 A study of [5], based on the independent dataset, provides the value $Y_p = 0.2477 \pm 0.0029$. Its upper bound becomes very close to that of [1] if one employs an additional systematic uncertainty at the level $\Delta Y_{\text{syst}} = 0.010$ (twice the value of systematic uncertainty of [1]).
3 Results

In this Section we present our main results: the bounds on sterile neutrino lifetime as a function of their masses and mixing patterns, as well as the bounds on the mixing angles. As discussed in the previous Section, there are several systematic uncertainties in the determination of the $^{4}\text{He}$ abundance and therefore the results will depend on the adopted values of $Y_{p}$ (together with the neutron lifetime, $\tau_{n}$). We summarize these systematic effects below.

We start with comparing the upper bounds on sterile neutrino lifetime for different values of $Y_{p}$ (see Section 2.6). The Fig. 2a shows that the bounds from the two recent works [1, 43] are quite similar (the difference is of the order of 30%). The bound, based on [45] would give a result, similar to [43] as discussed above.

For the CMB bound in Fig. 2b, we present only the result for masses $M_{s} > 40$ MeV where $N_{\text{eff}} \approx 3$. Fig. 3 indicates that for smaller masses the number of effective neutrino species increases significantly. It in turn affects the CMB helium bounds (c.f. Eqs. (16) and (17)). The accurate account of this effect goes beyond the scope of this work and we choose instead to plot stronger deuterium-based bounds (those of Fig. 3) in Fig. 2a for $M_{s} \lesssim 40$ MeV.

The lower bound on $Y_{p}$ from the recent work of [1] is above the Standard BBN value (12) at $\sim 2\sigma$ level (see however [44]). The presence of sterile neutrinos in plasma of course relaxes this tension and therefore at $2\sigma$ the adopted values of $Y_{p}$ (Eq. 14) provide both upper and lower bounds on sterile neutrino lifetime. This is
Figure 3: **Left:** Deuterium abundance, with the shaded region corresponding to the *allowed* 3σ range, based on [4]. **Right:** Effective number of neutrino species (the ratio of the effective neutrino temperature to the photon temperature at $T \sim \text{fewkeV}$) as a result of decay of sterile neutrino. The horizontal “SM” lines indicate $N_{\text{eff}}$ that corresponds to the boundary of the 3σ range [4], in the SM with the number of relativistic species deviating from $N_{\text{eff}} \approx 3$. In both panels, parameters of sterile neutrinos correspond to the upper bound on $Y_p$ from [1] (see Eq. (14)), except of the “CMB” line that corresponds to the upper bound from [46, 2] (see Eq. (16)).

shown in Fig. 2, right panel. At 3σ level the measurements of [1] are consistent with Standard BBN and the lower bound disappears.

Fig. 3 shows the changes in Deuterium abundance and in the effective number of neutrino species, caused by sterile neutrinos (with parameters corresponding to the upper bound based on [4]). For these values of parameters the abundance lies within the 3σ boundaries (19). And for the highest effective number of neutrinos reached, $N_{\text{eff}} = 6$, $D/H$ is close to the 3σ upper bound. Notice that the same relation between $N_{\text{eff}}$ and $D/H$ is observed in the model without new particles but with the effective number of neutrinos different from 3. The effective number of neutrino species does not define the Helium abundance though. Otherwise the same $Y_p$ bound [1] would predict *only one particular* value of $N_{\text{eff}}$, which is not case, as the inspection of Fig. 3 shows.

The influence of another systematic uncertainty (the lifetime of neutron, $\tau_n$) is negligible. Indeed, the relative difference between sterile neutrino lifetimes were found to be of the order of 5% for two choices of $\tau_n$ – from [28] and from [27] (taking the same $Y_p$ bound from [43]).

Next we investigate the dependence of the resulting bounds on the mixing patterns of sterile neutrinos. Naively, one would expect that sterile neutrinos mixing “only with $\nu_e$” and “only with $\nu_\mu$” should have different effect of $Y_p$. However, it is the energy “injection” rate (i.e. the overall decay rate of sterile neutrinos) that is *more important* for the dynamics of plasma before the onset of nucleosynthesis. This quantity depends on the lifetime $\tau_s$ and the mass $M_s$ of the neutrino. Mixing patterns affect mostly the concentration of particular decay products, but not the
Figure 4: Upper bound for sterile neutrino lifetime for different mixing patterns: mixing with $\nu_e$-only (red dashed line), $\nu_\mu$-only (green dashed-dotted line) and equal mixing with $\nu_e$ and $\nu_\mu$ flavours (black solid line). All bounds are derived for the lifetime of neutron $\tau_n$ adopted from [27]. The effect of different mixing patterns is at the level $\sim 10 - 50\%$ and can only be seen in the right panel because of the different y axis. In the right panel, only the masses $M_s > 40$ MeV are presented. For details, see Sec. 3 and Fig. 3.

(a) Bounds based on astrophysical measurements of $Y_p$ of [43]

(b) Bounds from CMB (notice different y-axis range).

Figure 5: Lower bound on mixing angles of sterile neutrinos for different mixing patterns: mixing with $\nu_e$-only (red dashed line), $\nu_\mu$-only (green dashed-dotted line) and equal mixing with $\nu_e$ and $\nu_\mu$ flavours (black solid line). Both types of bounds are derived by assuming lifetime of the neutron $\tau_n$ from [27]. In the right panel, only the masses $M_s > 40$ MeV are presented. For details, see Sec. 3 and Fig. 3.

(a) Bounds based on astrophysical measurements of $Y_p$ of [43]

(b) Bounds from CMB

injection rate. In addition, the neutrino oscillations (fast at the BBN epoch) make the difference between flavours less pronounced (see Appendix C). As a result, mixing patterns give essentially the same results with the difference at the level of tens of per cent (see Figs. 4, 5).
4 Discussion

In this work we considered the influence of decaying particles with the masses few MeV – 140 MeV on the primordial abundance of light elements ($D$ and $^4$He). Such particles appear in many cosmological scenarios [12, 13, 18, 19, 20, 21, 22, 23, 48, 49, 50]. Particularly, we concentrated on the properties of sterile neutrinos and derived constraints on their lifetime imposed by the present measurements of primordial Helium abundance $Y_p$. Sterile neutrinos are super-weakly interacting particles, quadratically mixed with the active flavours.

We analyzed the case of one Majorana sterile neutrino with 4 degrees of freedom (if sterile neutrinos were kept in thermal equilibrium it would be equivalent to $g_s = 2$ species of active neutrinos). Since the plasma evolution is mostly affected by the overall decay rate of sterile neutrinos, the lifetime bounds that we obtained are essentially independent of the particular mixing patterns, as Figs. 4, 5 demonstrate.

In the paper [16] a similar model was considered with one Dirac sterile neutrino. Dirac sterile neutrino has the same 4 degrees of freedom and influences primordial plasma in the same way (if it has the same spectrum, lifetime and mixing pattern). However, in [16] effect of active-neutrino oscillations was not taken into account, and some simplifying approximations like Boltzmann statistics were employed. To provide corresponding analysis we wrote code that solves more accurate Boltzmann equations describing kinetics of neutrino than what were used in [16]. We compare the results of this work with the previous bounds [15, 16] in Fig. 6. We see that our results are broadly consistent with the previous works. The differences for a given mixing pattern of sterile neutrinos can be as large as a factor of 2.5 for some masses.

The presence of sterile neutrinos in the plasma affects the effective number of neutrino degrees of freedom, $N_{\text{eff}}$. Fig. 3 right panel shows that $N_{\text{eff}}$ between 2.7 and 6 are possible for different mixing angles and masses, which could explain a larger than 3 values of $N_{\text{eff}}$, reported recently in several CMB observations (see e.g. [46, 51, 47], but also [52]).

Decaying sterile neutrinos with the masses 100 – 500 MeV and lifetimes from
seconds to minutes and their influence on $N_{\text{eff}}$ and entropy production have been recently considered in [18] (see also [53]) where it was demonstrated that they can lead to $N_{\text{eff}} \neq 3$ and can therefore be probed with the CMB measurements. The results of the present work demonstrate that in the region $100 - 140$ MeV where we overlap with the parameter space, studied in [18], the primordial nucleosynthesis restricts the lifetime of sterile neutrinos to be well below 1 sec (see Fig. 2, left panel).

Finally, it is interesting to compare the upper bound on sterile neutrino lifetime, derived in this paper with the lower bounds that come from direct experimental searches for sterile neutrinos (see [26, 54, 55]). These latter bounds are based on the assumption that sterile neutrinos with four degrees of freedom are solely responsible for the observed pattern of neutrino oscillations via the see-saw mechanism [55]. The appropriate comparison, based on [55], is presented in Fig. 7. No allowed values of sterile neutrino lifetimes for $1$ MeV $\lesssim M_s < 140$ MeV exist for either type of neutrino mass hierarchy (i.e. the upper bound is smaller than the lower bound, see the purple double-shaded region in Fig. 7). Notice, that if the astrophysical bounds on Helium [1, 43] were used for $M_s \gtrsim 40$ MeV in Fig. 7 instead of the CMB bound, the resulting lifetime bounds would become stronger (by as much as a factor of 4) in this mass range. We stress that for this conclusion it is essential that MeV sterile neutrinos are responsible for neutrino oscillations. For example, a model in which sterile neutrinos couple to $\nu_\tau$ only (and therefore do not contribute to the mixing between active neutrino flavours), is allowed even if one confronts the strongest BBN bounds (based on the astrophysical Helium measurements) with the direct accelerator bounds, see Fig. 8 for details.
Figure 8: Comparison of direct accelerator constraints and BBN bounds, based on the Helium-4 measurements of $^{11}$ in the model where sterile neutrinos mix with $\nu_\tau$ only. Unlike the case, presented in Fig. 7 there is an allowed region of parameter space for most of the masses below 140 MeV.

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A Tests of the numerical approach

The Section below summarizes the comparison of the present work with the previous ones that analyzed the influence of the MeV particles on primordial nucleosynthesis. Throughout this Section, we normalize scale factor by imposing condition $aT = 1$ at the initial moment. Conformal momentum is $y = pa$ with the same normalization of the scale factor. In the figures that contain both the solid and the dashed curves, the former correspond to the results obtained with our code, and the latter – to the
Table 1: Values of Helium abundance $Y_p$ in the Standard Model BBN (SBBN) and their dependence on the neutron lifetime, $\tau_n$.

| Code                        | $Y_p$ for $\tau_n$ from PDG | $Y_p$ for $\tau_n$ from [28] |
|-----------------------------|-------------------------------|-------------------------------|
| (Modified) Kawano code [36] | 0.2472                        | 0.2457                        |
| PArthENoPE code [42]       | 0.2480                        | 0.2465                        |
| Difference                  | -0.0008                       | -0.0008                       |

A.1 Standard Model BBN

First we considered the nucleosynthesis in Universe filled with the Standard Model particles only. We compute the actual non-equilibrium form of the active neutrino spectra during their decoupling. The results of the present work are compared with those of [25, 24, 57]. In [25, 24] neutrino oscillations were neglected, while in [57] the effect was taken into account. Fig. 9 shows the evolution of the quantity $aT$ as a function of temperature. It is identical to the Fig. 1 in Ref. [25]. Figures 10, 11 show how distorted neutrino spectra $f_{\nu_\alpha}$ are, compared to the thermal distribution $f^{eq} = (e^y + 1)^{-1}$. One can see good agreement between the results. We believe that the difference, that is present nevertheless, arises solely due to our one-step time integration method of the stiff kinetic equations, that is not as accurate as the method employed in Refs. [24, 57].

We turned off flavour oscillations and compared asymptotic values of ratio $aT$ at low temperatures together with the effective number of neutrino species, $N_{eff}$. For the former quantity, Refs. [24, 57] present values 1.3991 and 1.3990, respectively. On the other hand, we derived 1.3996. For the number of neutrino species in absence of neutrino oscillations, the same Refs. [24, 57] provide numbers 3.034 and 3.035, respectively, while we get 3.028.\(^\text{15}\)

The resulting $Y_p$ is summarized in Table 1 for different values of neutron lifetime $\tau_n$. We also provide a comparison of the modified version of the Kawano code [36] that we adopted for computing nuclear reactions with a newer code, PArthENoPE [42]. By comparing the results of PArthENoPE and the modified KAWANO code, we find the former to be larger by 0.0008 than the latter. We use the shift $\Delta Y_p = -0.0008$ as a correction in our subsequent results.

\(^\text{15}\) Ref. [57] the takes into account both the effects of neutrino oscillations and QED corrections, the latter changes the result significantly. As a result we could not compare the effect of neutrino oscillations only.
Figure 9: $T/T_\nu$ as a function of inverse temperature $T^{-1}$. The solid line is produced by the code of the present work, the dashed – the result of [25].

Figure 10: Relative distortions of neutrino spectra before the onset of BBN. **Left:** neutrino flavour oscillations are neglected, **right:** the oscillations are taken into account, with the parameter choice $\theta_{13} = 0$, $\sin^2 \theta_{23} = 0.5$, $\sin^2 \theta_{12} = 0.3$ used in [57]. In both panels, the pair of upper curves shows the distortion of the electron neutrino, the lower – of $\nu_\mu$. In each pair, the solid curve is the result of this work, and the dashed is from Fig. 2 of [57].
Figure 11: **Left:** Relative distortion of $\nu_e$ spectra $\delta f_{\nu_e}/f_{\text{eq}}$ for conformal momenta $y = 3, 5, 7$ (from bottom to up). **Right:** The same, but for muon neutrino. In each pair of curves the solid one corresponds to this work and the dashed one is from [25].

### A.2 Test of energy conservation

If all weak reactions involving electrons and positrons are turned off, neutrinos decouple from the rest of plasma. Then the energy conservation law (3) holds separately for the neutrino component and for the remaining particles. In approximation of zero mass of electron we obtain

$$\frac{d(aT)}{dt} = 0$$

similar to Eq. (22). As a corollary, product $aT$ is conserved. On the other hand, our code solves the equation (3) involving all medium components simultaneously. And it turns out that the relation (20) is not a trivial consequence of the numerical computation. Therefore the check of the conservation serves as a test of the code. We considered separately scattering and decay processes involving neutrinos and observed conservation of $aT$ with precision of order 0.2%.

### A.3 Heavy sterile Dirac neutrino

Next we have tested model with *one* sterile Dirac neutrino $\nu_S$ with mass $M_s = 33.9$ MeV, mixed with $\nu_\tau$ [15]. This neutrino was assumed to be in thermal equilibrium with plasma at $T \gtrsim 50$ MeV. To simplify the problem, the authors of [15] used the Boltzmann equilibrium statistics for active species in collision integral for a sterile neutrino.

Being in equilibrium the sterile neutrino spectrum becomes more and more non-relativistic with time due to the redshift. Therefore the ratio $\rho_s/M_s n_s$ of the energy density $\rho_s$ to the mass times number density $n_s$ should approach 1 at lower temperatures. We have recomputed the evolution of the system using our code, without the Boltzmann approximation. Fig. [12] shows the comparison of the results with those...
Figure 12: Ratio $\rho_s/n_s M_s$ as a function of scale factor for $M_s = 33.9$ MeV sterile neutrino. The upper curve is the result of Ref. [15], the lower curve is the present work.

of [15] for sterile neutrino lifetime $\tau_s = 0.3$ sec. Both results coincide till $T \approx 5$ MeV and after that moment ratio $\rho_s/M_s n_s$ of [15] stops decreasing, while the numerical result we obtained shows the expected behaviour — the ratio continues to decrease, approaching 1.

A.4 Massive $\nu_\tau$

Next we considered a model with the massive tau neutrino [19, 21]. Fig. 13 presents relative deviation of the energy densities of massless neutrinos $\delta \rho_\nu/\rho_{eq}$ produced by our code and plotted in [19]. $\rho_{eq} = \rho_{eq}\nu = 7\pi^2 T^4/120$ is the equilibrium energy density of one neutrino specie, and $\delta \rho_\nu = \rho_\nu - \rho_{eq}\nu$. In Fig. 13 distortion of electron neutrino spectrum $y^2 \delta f_\nu/eq$ is depicted. Here one observes good agreement between the results.

A.5 Late reheating model

To test the treatment of MeV decaying particles, we considered the low-reheating models with the reheating temperature of several MeV [22, 23]. In [22] heavy non-relativistic particles were considered, that dominated the energy density of the Universe once and then decayed into electrons, positrons or photons (so that decay products are quickly thermalized). The most important output is the effective number of active neutrino species $N_{eff}$ (defined in Eq. (17)). Dependence of this quantity on decay width of heavy particle is presented in Fig. 14. We have noticed some difference between the results of cited papers and those of our code. We believe
Figure 13: **Left:** Relative deviation from its equilibrium value of $\nu_e$ energy density $\delta \rho_{\nu_e}/\rho_{eq}$ in a model where tau neutrino is massive. **Right:** Spectrum distortion $y^2 \delta f_{\nu_e}/f_{eq}$ for the same model. In both panels $M_{\nu_\tau} = 0, 3, 7, 20$ MeV from bottom to top, the solid curves depict the numerical results of this work, and the dashed – the results of [19].

that this is due to the different approximations made. For example, in both works [22, 24] the scattering processes involving only neutrinos were not taken into account, approximation of Boltzmann statistics was used throughout and electron mass was neglected. We checked that the account of finite electron mass gives a gain of 5% to the $N_{\text{eff}}$ for $\tau = 0.1s$, while the account of scatterings involving only neutrinos gives rise of 1%.

### A.6 Instant thermalization of decay products

Next we considered a model with two heavy Majorana sterile neutrinos, similar to the $\nu$MSM. However, we assumed that for any mass of sterile neutrino it can decay only via channels listed in Table 4 of Appendix B. It is not a natural assumption, because usually sterile neutrinos heavier than pion decay dominantly into states containing mesons [26]. Also we approximated sterile neutrino spectrum as a non-relativistic one, while all the other particles are relativistic and in equilibrium all the time. In this case the system may be adequately described by the kinetic equation

$$\frac{d \rho_s}{dt} + 3 \frac{a}{\dot{a}} \rho_s = -\Gamma_s \rho_s \quad (21)$$

together with the Friedmann equations (2–3). The latter of these equations can be rewritten as

$$\frac{d(aT)}{dt} = \frac{30a\Gamma_s \rho_s}{43\pi^2 T^3} \quad (22)$$

$\Gamma_s$ is the decay width of sterile neutrino, $\rho_s$ is the energy density of sterile neutrinos, and we have used expression for the energy and pressure densities of relativistic species $\rho_{\text{rel}} = 3p_{\text{rel}} = 43\pi^2 T^4/120$. 

22
Figure 14: Effective number of neutrino species $N_{\text{eff}}$ depending on decay width of heavy non-relativistic particles. Comparison of the results of this work and Refs. [22, 23].

In Figs. 15 the evolution of quantities $aT$ and $\rho_s/\rho_{\text{rel}}$ is compared between the results of our code and the semi-analytic integration of Eqs. (21)–(22) for three different sets of masses and lifetimes. One can see very good agreement between these results, maximum relative deviation is 1%.

B Tree-level matrix elements

In this Appendix we summarize the matrix elements we used for computing the collision integrals in Boltzmann equation. The squares of the matrix elements for Standard Model particles only are listed in Table 2, while the squares of the matrix elements of processes with sterile neutrinos are summarized in Tables 3, 4. In these expressions, averaging over helicities of incoming particles and summation over those of outgoing products is assumed. The reactions are considered for two cases. In the first one sterile neutrino is a right-chiral Majorana neutrino that has 2 helicity degrees of freedom. That is actually the case in our problem, where we have two neutrinos of this kind. The other case corresponds to sterile neutrino of Dirac nature. Dirac fermions have both right- and left-chiral components, hence yielding 4 degrees of freedom in total. Expressions listed in Tables 3, 4 are applicable for both cases of the neutrino nature. Moreover, to complete the list of possible tree-level reactions, one has to consider charge-conjugated channels and take into account that Dirac particle is distinct from its antiparticle, while Majorana neutrino is not.

Throughout this Section we use the notations $g_R = \sin^2 \theta_W$, $g_L = 1/2 + \sin^2 \theta_W$, $\tilde{g}_L = -1/2 + \sin^2 \theta_W$, where $\theta_W$ is the Weinberg angle so that $\sin^2 \theta_W \approx 0.23$. The resulting expressions coincide with [25, 15].
C Neutrino oscillations

The active neutrinos of different flavours $\nu_e, \nu_\mu, \nu_\tau$ are related to the mass eigen-state basis $\nu_1, \nu_2, \nu_3$ via a non-diagonal Pontecorvo-Maki-Nakagava-Sakata (PMNS) matrix $V |\nu_\alpha\rangle = \sum V_{\alpha i} |\nu_i\rangle$ (see e.g. [58] for reviews):

$$V = \begin{pmatrix}
1 & 0 & 0 \\
0 & c_{23} & s_{23} \\
0 & -s_{23} & c_{23}
\end{pmatrix}
\begin{pmatrix}
c_{13} & 0 & s_{13} \\
0 & e^{i\phi} & 0 \\
-s_{13} & 0 & c_{13}
\end{pmatrix}
\begin{pmatrix}
c_{12} & s_{12} & 0 \\
-s_{12} & c_{12} & 0 \\
0 & 0 & 1
\end{pmatrix}.$$  

here $c_{ij} = \cos \theta_{ij}$ and $s_{ij} = \sin \theta_{ij}$ are functions of the active-active neutrino mixing angles $\theta_{ij}$.

Exact treatment of active neutrino oscillation in the early Universe is a difficult task (see e.g. [59, 60, 61]) Characteristic timescale of oscillation between $i$ and $j$ mass eigen-states for a neutrino with energy $E$ is [58]

$$\tau_{ij} = \frac{4\pi E}{|m_i^2 - m_j^2|} \approx 8.3 \times 10^{-6}s \frac{E}{\text{MeV}} \frac{10^{-3} \text{eV}^2}{|m_i^2 - m_j^2|}$$  \hspace{1cm} (23)

Average energy of relativistic Fermi particles in equilibrium is $\langle E \rangle = 3.15T$ [33]. Applying this relation to active neutrinos and using their measured mass differences $m_2^2 - m_1^2 \approx 7.6 \times 10^{-5} \text{eV}^2$, $|m_3^2 - m_1^2| \approx 2.5 \times 10^{-3} \text{eV}^2$, we obtain

$$\tau_{12} \approx 1.0 \times 10^{-3}\text{sec} \frac{T}{3 \text{MeV}}, \hspace{0.5cm} \tau_{13} \approx 3.1 \times 10^{-5}\text{sec} \frac{T}{3 \text{MeV}},$$  \hspace{1cm} (24)

provided that influence of the surrounding environment on neutrino propagation is neglected. One sees therefore that about the moment active neutrino decouples...
Table 2: Squared matrix elements for weak processes involving active species only. $S$ is the symmetrization factor; $\alpha, \beta = e, \mu, \tau$. In all processes we take $\alpha \neq \beta$. The results coincide with those of Ref. [25].

$T \simeq 3\text{MeV}$ typical oscillation timescales are much smaller than the Hubble expansion time given by Eq. (2)

$$\tau_H = \sqrt{\frac{15}{4\pi^3 g^*_{s} G_N T^4}} \simeq 0.16 \text{ sec} \left(\frac{3 \text{ MeV}}{T}\right)^2.$$ (25)

Here $g^*_{s} \approx 11$ (at $T \sim \text{MeV}$) [33] is the so-called number of relativistic species that enters energy-temperature relation $\rho = \frac{\pi^2 g^*_{s} T^4}{30}$. Therefore, active neutrinos oscillate many times between the subsequent reactions involving them. In quantitative terms it means that probabilities $P_{\alpha\beta}$ to transform from flavour $\alpha$ to flavour $\beta$ are oscillating functions of time. In realistic situation neutrinos do not have a definite momentum but are created in wave packets that are superpositions of states which have one. Since oscillation periods are momentum-dependent according to Eq. (23), each state in the superposition will have his own period. Therefore after sufficiently many periods initial phases characterizing superposition will change, and there is no reason
| Process $(1 + 2 \rightarrow 3 + 4)$ | $S$ | $SG_{F}^{-2} |\mathcal{M}|^2$ |
|----------------------------------|-----|----------------------------------|
| $\nu_s + \nu_\beta \rightarrow \nu_\alpha + \nu_\beta$ | 1   | $32\theta_{\alpha}^2(p_1 \cdot p_2)(p_3 \cdot p_4)$ |
| $\nu_s + \bar{\nu}_\beta \rightarrow \nu_\alpha + \bar{\nu}_\beta$ | 1   | $32\theta_{\alpha}^2(p_1 \cdot p_4)(p_2 \cdot p_3)$ |
| $\nu_s + \nu_\alpha \rightarrow \nu_\alpha + \nu_\alpha$ | 1/2 | $64\theta_{\alpha}^2(p_1 \cdot p_2)(p_3 \cdot p_4)$ |
| $\nu_s + \bar{\nu}_\alpha \rightarrow \nu_\alpha + \bar{\nu}_\alpha$ | 1   | $128\theta_{\alpha}^2(p_1 \cdot p_4)(p_2 \cdot p_3)$ |
| $\nu_s + \bar{\nu}_e \rightarrow e^+ + e^-$ | 1   | $128\theta_{\alpha}^2[g_L^2(p_1 \cdot p_4)(p_2 \cdot p_3) + g_R^2(p_1 \cdot p_3)(p_2 \cdot p_4) + g_{LR}m_s^2(p_1 \cdot p_2)]$ |
| $\nu_s + e^- \rightarrow \nu_e + e^-$ | 1   | $128\theta_{\alpha}^2[g_L^2(p_1 \cdot p_4)(p_3 \cdot p_4) + g_R^2(p_1 \cdot p_4)(p_2 \cdot p_3) + g_{LR}m_s^2(p_1 \cdot p_3)]$ |
| $\nu_s + e^+ \rightarrow \nu_e + e^+$ | 1   | $128\theta_{\alpha}^2[g_L^2(p_1 \cdot p_4)(p_2 \cdot p_3) + g_R^2(p_1 \cdot p_2)(p_3 \cdot p_4) + g_{LR}m_s^2(p_1 \cdot p_3)]$ |
| $\nu_s + \bar{\nu}_{\mu(\tau)} \rightarrow e^+ + e^-$ | 1   | $128\theta_{\mu(\tau)}^2[g_L^2(p_1 \cdot p_4)(p_2 \cdot p_3) + g_R^2(p_1 \cdot p_3)(p_2 \cdot p_4) + g_{LR}m_s^2(p_1 \cdot p_2)]$ |
| $\nu_s + e^- \rightarrow \nu_{\mu(\tau)} + e^-$ | 1   | $128\theta_{\mu(\tau)}^2[g_L^2(p_1 \cdot p_4)(p_3 \cdot p_4) + g_R^2(p_1 \cdot p_4)(p_2 \cdot p_3) + g_{LR}m_s^2(p_1 \cdot p_3)]$ |
| $\nu_s + e^+ \rightarrow \nu_{\mu(\tau)} + e^+$ | 1   | $128\theta_{\mu(\tau)}^2[g_L^2(p_1 \cdot p_4)(p_2 \cdot p_3) + g_R^2(p_1 \cdot p_2)(p_3 \cdot p_4) + g_{LR}m_s^2(p_1 \cdot p_3)]$ |

Table 3: Squared matrix elements for scatterings of sterile neutrinos $\nu_s$. Here $S$ is the symmetrization factor; $\alpha, \beta = e, \mu, \tau; \alpha \neq \beta$. $\theta_\alpha$ is the mixing angle of sterile neutrino with $\nu_\alpha$. The results are applicable for one right-chiral Majorana neutrino as well as for one Dirac neutrino, for details see text.
Table 4: Squared matrix elements for decays of sterile neutrinos \( \nu_S \). Here \( S \) is the symmetrization factor; \( \alpha, \beta = e, \mu, \tau; \ \alpha \neq \beta \). \( \vartheta_\alpha \) is the mixing angle of sterile neutrino with \( \nu_\alpha \). The results are both for Majorana and Dirac neutrinos, for details see text.

To understand what happens with a neutrino, consider example of electron-neutrino created in electron-positron annihilation. At the production time this particle has probability 1 to oscillate into \( \nu_e \) and zero for other final state. After long enough time for many oscillations to happen and before the time when a collision with other particle becomes quite probable, the decoherence comes into play. So now we may find the \( \nu_e \) with probability \( P_{ee} \), \( \nu_\mu \) with probability \( P_{e\mu} \) and \( \nu_\tau \) with \( P_{e\tau} \).

The production rate of the initial specimen per unit time is proportional to collision integral \( I_e \), according to the Boltzmann equation (7). But the actual number of produced electron neutrinos is actually reduced by factor \( P_{ee} \). And even if (consider this hypothetical situation) muon neutrino does not interact with plasma, it will be

\[
P_{ee} = 1 - \frac{1}{2} (\sin^2 2\theta_{13} + \cos^4 \theta_{13} \sin^2 2\theta_{12})
\]

\[
P_{e\mu} = P_{\mu e} = \frac{1}{2} \cos^2 \theta_{13} \sin^2 2\theta_{12}
\]

\[
P_{e\tau} = P_{\tau e} = \sin^2 \theta_{13} \cos^2 \theta_{13} \left( 2 - \frac{1}{2} \sin^2 2\theta_{12} \right)
\]

\[
P_{\mu\mu} = 1 - \frac{1}{2} \sin^2 2\theta_{12}
\]

\[
P_{\mu\tau} = P_{\tau\mu} = \frac{1}{2} \sin^2 \theta_{13} \sin^2 2\theta_{12}
\]

\[
P_{\tau\tau} = 1 - \sin^2 \theta_{13} \left( 2 \cos^2 \theta_{13} + \frac{1}{2} \sin^2 \theta_{13} \sin^2 2\theta_{12} \right)
\]
anyway produced, at rate $P_{ee} I_e$. Generalization to other neutrino flavours leads us
to conclusion that the modified Boltzmann equation
\[
\frac{df_\alpha}{dt} = \sum P_{\alpha\beta} I_\beta
\]  
(27)
describes neutrino dynamics correctly (that is not the case for the initial equation
(7)). For the actual computations we use the following experimental best-fit values:
$\sin^2 \theta_{12} = 0.31$, $\sin^2 \theta_{23} = 0.52$ from [62], and $\sin^2 2\theta_{13} = 0.09$ from the Daya Bay [63].
The latter number is close to the result $\sin^2 2\theta_{13} = 0.11$ indicated by another recent
experiment, RENO [64].

However, in dense medium oscillations proceed differently due to considerable
effects of the plasma on properties of a single particle. Still, the phenomenon can
be described by the formalism of the PMNS matrix. The difference is that mixing
parameters together with masses now depend on properties of the environment. In
case of plasma close to equilibrium with no non-trivial conserving charges present
the parameter describing it is the temperature. So the parameters of the PMNS
become temperature-dependent. In language of the effective Hamiltonian approach
the system of three neutrinos is described by the addition of medium potential $\Delta H_M$
to the Hamiltonian $H_V$ of the system in vacuum [65]
\[
H_M = H_V + \Delta H_M, \quad H_V = \frac{1}{2E} V^* \text{diag}(m_1^2, m_2^2, m_3^2) V^\dagger,
\]  
(28)
where $E$ is the neutrino energy. Diagonalization of the total propagation Hamiltonian
$H_M$ gives effective masses and mixings.

The medium potential comprises effects of neutrino interactions. Since neutrino-
take part only in charged- and neutral-current interactions, matter potential has
two terms $\Delta H_{CC}$ and $\Delta H_{NC}$, respectively. All neutrinos couple to neutral currents
identically, so $\Delta H_{NC}$ is proportional to unit matrix. Therefore this term just renor-
malizes energy, and does not affect oscillations. In contrast, the charged-current term
is non-diagonal and is present only for $\nu_e$. The reason is that due to abundance of
electrons in plasma, $\nu_e$ couples effectively to charged currents, while at temperatures
below the muon’s mass there is no significant contribution of muons and tau-leptons
to realize coupling of other neutrinos to W boson.

Expressly matter potential is [58]
\[
\Delta H_{CC} = -\frac{14\sqrt{2} G_F}{45 M_W^2} E \text{ diag}(1, 0, 0)
\]  
(29)
in the flavour neutrino basis ($\nu_e, \nu_\mu, \nu_\tau$). $M_W$ is the mass of the W-boson.

So far we have dropped sterile neutrinos from consideration. But their mixing
properties are also altered in hot plasma. Using the approach of the effective Hamilton-
ian for them, one finds that their effective mixing angles in medium $\theta_M$ differ
from that in vacuum $\theta_V$ as \[65\]

$$\frac{\theta_M - \theta_V}{\theta_V} \sim \frac{G_F T^5}{M_W^2 M_S^2} \sim 10^{-11} \times \left(\frac{T}{100 \text{ MeV}}\right)^6 \left(\frac{10 \text{ MeV}}{M_S}\right)^2$$

(30)

for small mixing angles $\theta_V$. Therefore the mixing angle is not altered significantly for sterile neutrinos and matter effects are negligible for their dynamics.

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