Loop Quantum Gravity Modification of the Compton Effect

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Abstract

Modified dispersion relations (MDRs) as a manifestation of Lorentz invariance violation, have been appeared in alternative approaches to quantum gravity problem. Loop quantum gravity is one of these approaches which evidently requires modification of dispersion relations. These MDRs will affect the usual formulation of the Compton effect. The purpose of this paper is to incorporate the effects of loop quantum gravity MDRs on the formulation of Compton scattering. Using limitations imposed on MDRs parameters from Ultra High Energy Cosmic Rays (UHECR), we estimate the quantum gravity-induced wavelength shift of scattered photons in a typical Compton process. Possible experimental detection of this wavelength shift will provide strong support for underlying quantum gravity proposal.

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1 Introduction

Historically, Compton effect is one of the most important evidence of particle nature of electromagnetic radiation. Compton scattering or equivalently Compton effect, is the reduction of energy (or increase of wavelength) of an X-ray or gamma ray photon, when it interacts with matter. The amount the wavelength increases by is called the Compton shift. Although nuclear Compton scattering exists, what is meant by Compton scattering usually is the interaction involving only the electrons of an atom. This effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon. Thomson scattering, the classical theory of charged particles scattered by an electromagnetic wave, cannot explain any shift in wavelength. Light must behave as if it consists of particles in order to explain the Compton scattering. Compton’s experiment convinced physicists that light can behave as a stream of particles whose energy are proportional to the frequency [1]. When a high energy photon collides an electron, part of its initial energy will transfer to electron and cause it to recoil. The other part of initial photon energy leads to creation of a new photon and this photon moves in a direction which satisfy the total momentum conservation. Compton scattering occurs in all materials and predominantly with photons of medium energy, i.e. about 0.5 to 3.5 MeV.

Recently it has been revealed that Lorentz symmetry is not an exact symmetry of the nature. Possible violation of Lorentz invariance has been studied from several view points [2,3,4]. From a loop quantum gravity point of view, a Lorentz invariance violation can be formulated in the modification of standard dispersion relation. Since formulation of ordinary Compton effect is based on the standard dispersion relation, possible modification of this dispersion relation may affect the calculations and their interpretations. Here we are going to incorporate these quantum gravity effects in the formulation of the Compton effect. Although numerical values of these modifications are very small, possible detection of these small effects will support underlying quantum gravity proposal. We use UHECR data to estimate typical wavelength shift due to these quantum gravity effects. In this manner we constraint threshold momentum to a lower value relative to existing prescriptions.

The paper is organized as follows: section 2 is devoted to a brief review of standard Compton effect and its formulation focusing on the central role played by dispersion relation. Section 3 gives an overview of modified dispersion relations. Our calculations are presented in section 4. The paper follows by a numerical estimation of the wavelength
shift and related discussion.

2 Compton Effect

Suppose that a photon with known wavelength collides with a thin metallic surface. In classical theory of light scattering, photon will be reflected by oscillating electron in such a way that its angular distribution varies as \(1 + \cos^2 \theta\). Compton had been noticed that scattered photons consist of two different wavelength: a part with the same wavelength as original photons. These photons have been scattered by the whole of the atom. Another part of scattered photons have shifted wavelength relative to incident photons and their wavelength depend on the angular parameter. If we consider an elastic collision between photon and electron, total energy and momentum should be conserved in this process.

Suppose that incident photon has energy \(h\nu\) and momentum \(\vec{p}\) where \(p = \frac{h\nu}{c}\). From standard dispersion relation between energy and momentum we have

\[
E = [(m_0c^2)^2 + (pc)^2]^{\frac{1}{2}}, \tag{1}
\]

where \(m_0\) is the rest mass of the particle. The speed of the particle is given by

\[
v = \frac{\partial E}{\partial p} = \frac{pc^2}{E} = \frac{pc^2}{\sqrt{m_0^2c^4 + p^2c^2}} \tag{2}
\]

where leads to \(E = pc\) for photons. Now consider a photon with initial momentum \(\vec{p}\) which collides with an electron at rest. After collision, we have an electron with momentum \(\vec{P}\) and a new photon with momentum \(\vec{p}'\). From momentum conservation one can write

\[
\vec{p} = \vec{p}' + \vec{P} \tag{3}
\]

which leads to

\[
(\vec{P})^2 = (\vec{p} - \vec{p}')^2 = (\vec{p})^2 + (\vec{p}')^2 - 2\vec{p} \cdot \vec{p}' \tag{4}
\]

From conservation of energy, one can write

\[
h\nu - h\nu' = E - E_0 = (P^2c^2 + E_0^2)^{\frac{1}{2}} - E_0, \tag{5}
\]

where \(E_0\) is electron rest energy. Therefore, we obtain

\[
m^2c^4 + P^2c^2 = (h\nu - h\nu' + mc^2)^2 = (h\nu - h\nu')^2 + 2mc^2(h\nu - h\nu') + m^2c^4 \tag{6}
\]
Using equation (4) we can write

\[ P^2 = \left( \frac{h\nu}{c} \right)^2 + \left( \frac{h\nu'}{c} \right)^2 - 2\left( \frac{h\nu}{c} \right)\left( \frac{h\nu'}{c} \right) \cos\theta \] (7)

which means

\[ P^2 c^2 = (h\nu - h\nu')^2 + 2(h\nu)(h\nu')(1 - \cos\theta) \] (8)

where \( \theta \) is the angle of photon scattering. A simple calculation leads to the following expression

\[ h\nu\nu'(1 - \cos\theta) = mc^2(\nu - \nu') \] (9)

which can be rewritten as follows

\[ \lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \] (10)

Figure 1: Standard Compton Effect.

where \( \frac{h}{mc} \) is called the Compton wavelength. Note that scattered electron and wavelength-shifted photon appear simultaneously. Figure 1 shows the variation of \( \Delta\lambda \) versus scattering angle \( \theta \).
3 Modified Dispersion Relations (MDRs)

Lorentz invariance violation at quantum gravity level can be addressed by the modification of the standard dispersion relations. Recently these modified dispersion relations have been used to describe anomalies in astrophysical phenomena such as the GZK cutoff anomaly[5,6] as well as a large number of problems in the spirit of quantum mechanics(see for instance [5] and references therein). Modified dispersion relations have support on several alternative approaches to quantum gravity problem[7,8]. Here we consider those MDRs formulations that contain Planck length (\(l_p\)) explicitly. In this framework, there is a new length scale, \(L \gg l_p\), which is called ”weave” scale. For distance \(d \ll L\) the quantum loop structure of spacetime is manifest, while for distances \(d \gg L\) the continuous flat geometry is regained[7]. In this context, dispersion relation \(E = E(p)\) for particle with energy \(E\) and momentum \(p\) is given by (\(\hbar = c = 1\)) [8,9]

\[
E^2 = A^2 p^2 + m^2
\]  

where \(E\), \(p\) and \(m\) are the respective energy, momentum and mass of the particle and \(A\) is a Lorentz invariance violation parameter which can be interpreted as the maximum velocity of the particle\(^1\). For Fermions( Majorana Fermions), these modified dispersion relations can be written as follows[7,8]

\[
E^2_{\pm} = (Ap \pm \frac{B}{2L})^2 + m^2(\alpha \pm \beta p)^2
\]  

where

\[
A = 1 + \kappa_1 \frac{l_p}{L} + \kappa_2 \left(\frac{l_p}{L}\right)^2 + \frac{\kappa_3}{2} l_p^2 p^2
\]  

\[
B = \kappa_5 \frac{l_p}{L} + \kappa_6 \left(\frac{l_p}{L}\right)^2 + \frac{\kappa_7}{2} l_p^2 p^2
\]  

\[
\alpha = (1 + \kappa_8 \frac{l_p}{L}) \quad and \quad \beta = \frac{\kappa_9}{2} l_p
\]

\(^1\)It can be shown that these Lorentz invariance violation can significantly modify the kinematical conditions for a reaction to take place[8].
Here $\kappa_i$ are unknown adimensional parameters of order one and the $\pm$ signs stand for the helicity of propagation. For simplicity we write

$$E_{\pm}^2 = A^2 p^2 + \eta p^4 \pm 2\Gamma p + m^2$$

(16)

where now $A = 1 + \frac{\kappa_4 l_p}{L}$ and $\kappa_1, \kappa_3$ and $\kappa_5$ are of the order of unity with $\eta = \kappa_3 l_p^2$ and $\Gamma = \kappa_5 \frac{l_p}{L}$ which depends on the helicity.

The following MDR has been suggested also

$$E^2 = p^2 + m^2 + \frac{|p|^{2+n}}{M^n}$$

(17)

where $M^n$ is the characteristic scale of Lorentz violation[9,10]. In which follows, we use these relations to incorporate quantum gravitational effects in the formulation of the Compton effect. The possible values of parameters in equation (16) have been discussed in references[7,8] using UHECR data. We use these values to estimate the order of wavelength shift due to loop quantum gravity effect.

4 Generalized Compton Effect

In this section we use modified dispersion relations as given by (16) and (17) to incorporate quantum gravitational effects in the formulation of the Compton scattering. In a typical Compton scattering, the conservation of linear momentum leads to

$$\vec{p} = \vec{p}' + \vec{P}$$

(18)

where $\vec{p}$ is linear momentum of the incident photon, $\vec{p}'$ is momentum of the wavelength-shifted secondary photon and $\vec{P}$ is electron linear momentum. In this situation, we can write

$$\vec{P}^2 = (\vec{p} - \vec{p}')^2 = \vec{p}^2 + \vec{p}'^2 - 2\vec{p}.\vec{p}'$$

(19)

In the first step, we assume that photon dispersion relation has no loop quantum gravity modification. This assumption will be justified later. For conservation of energy, one should consider modified dispersion relation for electron. If we use relation (16), we find

$$\nu - \nu' = E - E_0 = (A^2 P^2 + \eta P^4 \pm 2\Gamma P + m^2)^{\frac{1}{2}} - m$$

(20)

where we have set $\hbar = c = 1$. $m = E_0$ is rest energy and $E$ is the final energy of the electron. If we rearrange this relation we find

$$(\nu - \nu' + m)^2 = A^2 P^2 + \eta P^4 \pm 2\Gamma P + m^2$$

(21)
where leads to the following expression

\[ P^2 = \frac{1}{A^2} \left[ (\nu - \nu')^2 + 2(\nu - \nu')m - \eta P^4 \mp \Gamma P \right] \]  

(22)

We can write \( A = 1 + \epsilon \) where \( \epsilon = \frac{\kappa L}{\ell} \) is a small quantity, then using the formula \((1 + \epsilon)^{-2} \simeq 1 - 2\epsilon\) we find

\[ P^2 = (\nu - \nu')^2 + 2(\nu - \nu')m - \eta P^4 \mp \Gamma P - 2\epsilon(\nu - \nu')^2 - 4\epsilon(\nu - \nu')m + 2\epsilon\eta P^4 \pm 2\epsilon\Gamma P \]  

(23)

From equation (19) we can write

\[ P^2 = (\nu - \nu')^2 + 2\nu\nu'(1 - \cos \theta) \]  

(24)

Finally, combining equations (23) and (24) we find

\[ 2\nu\nu'(1 - \cos \theta) = (1 - 2\epsilon)[2(\nu - \nu')m - \eta P^4 \mp \Gamma P] - 2\epsilon(\nu - \nu')^2 \]  

(25)

The frequency shift of the scattered photon can be obtained using this relation. If we look at equation (25) and demand that both photon frequencies be positive, then we learn that \( \eta \) should be negative and one of the polarizations is ruled out. This is in agreement with the result of ref. [7]. Equation (25) contains some parameters which should be constraint using phenomenological evidences of these quantum gravity effects.

We can proceed also using relation (17) for modified dispersion relation. A simple calculation leads to the following result

\[ P^2 = (\nu - \nu')^2 + 2m(\nu - \nu') - \frac{|P|^{2+n}}{M^n} \]  

(26)

which can be transformed to the relation

\[ 2\nu\nu'(1 - \cos \theta) = 2m(\nu - \nu') - \frac{|P|^{2+n}}{M^n} \]  

(27)

The quantum gravity-corrected Compton effect now takes the following form

\[ \lambda' - \lambda = \frac{1}{m} (1 - \cos \theta) + \frac{\lambda\nu'|P|^{2+n}}{2M^n m} \]  

(28)

where \( M^n \) is the characteristic scale of Lorentz invariance violation. The matter which should be stressed is the fact that wavelength shift due to loop quantum gravity effect is wavelength dependent itself. In another words, the value of this shift depends on the wave length of incident photon. This is a novel feature. In ordinary Compton effect such
a wavelength dependence has not been observed. Relation (28) can be written as follows

\[ \lambda' = \frac{\lambda + \frac{1}{m}(1 - \cos \theta)}{1 - \frac{\lambda|P|^2 + n}{2M^*m}}. \] (29)

Note that equation (29) should be supplemented by an arbitrary factor \( F \) of order 1, then equation (26) will say that \( F < 0 \) and the contradiction of having negative wavelength is avoided.

In MKS system of units, this relation takes the following form

\[ \lambda' = \frac{\lambda + \frac{\hbar}{mc}(1 - \cos \theta)}{1 - \frac{\lambda|P|^2 + n}{2\hbar(Mc^2)^2mc^3}}. \] (30)

Since for reasonable values of \( |P| \), the quantity \( \frac{\lambda|P|^2 + n}{2\hbar(Mc^2)^2mc^3} \) is a positive small quantity, loop quantum gravity induces an increase of wavelength shift. Figure 2 shows the variation of \( \lambda' \) versus \( \theta \) based on equation (30).

Now we consider the more general case where photon dispersion relation is modified by quantum gravity effect also. For photons, the modified dispersion relation can be written as [11]

\[ E_{\pm} = p[A_\gamma - \theta_3(l_p)^2 \pm \theta_3l_pp] \] (31)

where

\[ A_\gamma = 1 + \kappa_\gamma (\frac{l_p}{L})^{2 + 2\Upsilon} \] (32)

In this relation, \( E_{\pm} \) and \( p \) are the respective energy and momentum of the photon, while \( \kappa_\gamma \) and \( \theta_3 \) are adimensionl parameters of order one, and \( \Upsilon \) is a free parameter that still needs interpretation\(^2\). A similar contribution was also suggested by Ellis et al [13],[14] (in this case, without helicity dependance). They have found

\[ E^2 = p^2 \left[ 1 - 2M_D^{-1}p \right], \] (33)

where \( M_D \) is a mass scale coming from D-brane recoil effects for the propagation of photons in vacuum. When Gamma Ray Burst (GRB) data are analyzed to restrict \( M_D \) [14], the following condition arises

\[ M_D \geq 10^{24} \text{eV} \] (34)
For photon’s dispersion relation which we have considered, (32) can be interpreted as the bound $\theta \leq 10^4$. To consider photon (Boson) dispersion relation in our analysis, we start with equation (20) and we obtain

$$p[A_\gamma - \theta_3 (l_p p)^2 \pm \theta_8 l_p p] - p[A_\gamma - \theta_3 (l_p p')^2 \pm \theta_8 l_p p'] = E - E_0 = (A^2 P^2 + \eta P^4 \pm 2\Gamma P + m^2)^{\frac{1}{2}} - m$$

This equation has several parameters which should be constraint from experimental or observational data. Another form of this relation can be obtained using equation (31)

$$p \left[1 - 2M_D^{-1} p\right]^\frac{1}{2} - p' \left[1 - 2M_D^{-1} p'\right]^\frac{1}{2} = E - E_0 = (A^2 P^2 + \eta P^4 \pm 2\Gamma P + m^2)^{\frac{1}{2}} - m.$$  

These two relations are more general than (25) and (29). One can use these relations for an estimation of induced wavelength shift. However, in which follows, for simplicity we consider only relation (30) to estimate a typical wavelength shift.

5 Threshold Analysis

In this paper we have focused on an interaction which contains photons and electrons. In this way, we are able to obtain rather strong constraints on the allowed parameter space from threshold analysis[18]. Based on these analysis the photon decay rate goes like $E$ above threshold, so any gamma ray which propagates over macroscopic distances must have energy below the threshold. This threshold now is supposed to be $|p_{th}| = 10^{13}$eV [17,18]. If we accept this threshold, we can estimate the value of the wavelength shift due to loop quantum gravity effect in a typical Compton effect. For simplicity we consider relation (30). In this relation, $M$ causes the appearance of threshold effects at momenta $|p| \geq |p_{th}|$, where $|p_{th}|^{2+n} = m^2 M^n$. It is these effects which allows to explore quantum gravity at energies much lower than $M$ [15]. Carmona and Cortes have argued that for $M = M_P$ and a typical hadronic process, one gets $|p_{th}| = 10^{15}$eV in the case $n = 1$ and $|p_{th}| = 10^{18}$eV in the case $n = 2$. In both cases, one has modifications to relativistic kinematics at energies below the GZK cutoff, so that the observed violations of this cutoff in the cosmic ray spectrum[15] could be a footprint of a Lorentz invariance violation at high energies[8]. First we try to find a wavelength shift using Carmona-Cortes threshold. Suppose that a photon with wavelength $0.71\lambda = 0.71 \times 10^{-10} m$ (which is the wavelength of photon in original Compton experiment) is scattered by electron via Compton process at angle $\theta = \frac{\pi}{2}$. Using the relation (30) we find an unacceptable wavelength shift: an
extremely small negative shift which is unacceptable due to its negative sign. This may reflect the fact that Carmona-Cortes threshold is not acceptable on physical ground. Using relation (30) we can obtain a reasonable wavelength shift of the scattered photon. Our calculations show a shifted wavelength of $\lambda' = 7.376636 \times 10^{-11} m$ which leads to $\Delta \lambda_{LQG} = 0.033636 \times 10^{-11} m$. Note that we have used $|P| \leq 10^{12} eV$ to have a reasonable wavelength shift. Looking back to relation (30) we see that the only arbitrary quantity is $|P|$. In our situation, only when $|P| \leq 10^{12} eV$ we find a reasonable wavelength shift. This threshold is one order of magnitude smaller than threshold presented by Jacobson et al. So, we may conclude that the reasonable threshold should be $|P| \leq 10^{12} eV$. Note that $0.343 \times 10^{-11} m$ is the ordinary wavelength shift via Compton process. In summary, we conclude that to have a reasonable wavelength shift due to loop quantum gravity effect, threshold effects should take place at $|p_{th}| = 10^{12} eV$. For comparison, note that Carmona and Cortes[10] have considered $|p_{th}| = 10^{15} eV$ for $n = 1$ while Jacobson et al have considered $|p_{th}| = 10^{13} eV$[17]. Our analysis shows that Jacobson et al framework gives more reliable result of threshold effect. Finally, we should emphasize that Lowering of this threshold may affect several of arguments in this research area.

Figure 2: Variation of $\lambda'$ in modified Compton effect for X-rays with wavelength 0.71Å. Due to smallness of the modification, it has been multiplied with an arbitrary factor.
6 summary

The results of our analysis of Compton effect within the framework of modified dispersion relations can be summarized as follows:

- The wavelength shift due to loop quantum gravity effect is wavelength dependent itself. In another words, the value of the wavelength shift depends on the wavelength of incident photon.

- Through a threshold analysis we have found a threshold for the momentum of electron as $|P| \leq 10^{12} eV$. Our analysis shows that Jacobson et al framework\cite{17,18} gives more reliable result of threshold effect.

- The analysis presented here provides a direct test of loop quantum gravity and Lorentz invariance violation. Any wavelength shift of scattered photon after subtraction of ordinary Compton shift will show the violation of Lorentz invariance and provides a direct test of loop quantum gravity.

References

[1] S. Gasiorowicz, *Quantum physics*, John Weily, 3rd Edition, 2003

[2] - G. Amelino-camelia, *Int. J. Mod. Phys. D* 11 (2002) 35
  - G. Amelino-Camelia et al, *Phys. Rev. D* 70 (2004) 107501
  - G. Amelino-Camelia et al, *Class. Quant. Grav.* 23 (2006) 2585-2606

[3] J. Maguijo and L. Smolin, *Phys. Rev. Lett.* 88 (2002) 190403

[4] K. Nozari and A. S. Sefiedgar, *Phys. Lett. B* 635 (2006) 156

[5] G. Amelino-Camelia, C. Lammerzahl, A. Macias and H. Muller, (2005), arXiv:gr-qc/0501053

[6] A. A. Andrianov, R. Soldati and L. Sorbo, *Phys. Rev. D* 59 (1999) 025002

[7] J. Alfaro and G. Palma, *Phys. Rev. D* 65 (2002) 103516, arXiv:hep-th/0111176

[8] - J. Alfaro and G. A. Palma, *Phys. Rev. D* 67 (2003) 083003, arXiv:hep-th/0208193
  - J. Alfaro and G. A. Palma, arXiv:hep-th/0501116
[9] K. Nozari and S. D. Sadatian, "On the Phenomenology of the Lorentz Invariance Violation", preprint, (2006)

[10] J. M. Carmona and J. L. Cortes, *Phys. Rev. D* **65** (2002) 025006, Xiv:hep-th/0012028

[11] J. Alfaro, H. A. Morales-Tecotl and L.F. Urrutia, hep-th/0108061 , (2001)

[12] J. Alfaro, H.A. Morales-Técotl and L.F. Urrutia, *Phys. Rev. Lett.* **84** (2000) 2318

[13] J. Ellis, N. E. Mavromatos and D.V. Nanopoulos, *Gen. Rel. Grav.* **32**(2000) 127

[14] J. Ellis, N. E. Mavromatos and D.V. Nanopoulos, gr-qc/9909085 , (1999)

[15] R. Aloisio, P. Blasi, P.L. Ghia and A. Grillo, *Phys. Rev. D* **62** (2000) 053010

[16] A. A. Watson, *Phys. Rep.* **333-334** (2000) 310

[17] T. Jacobson et al, *Annals Phys.* **321** (2006) 150-196

T. Jacobson et al, *Phys. Rev. D* **66** (2002) 081302

[18] T. Jacobson, S. Liberati, and D. Mattingly, hep-ph/0209264