HOMOTOPY PERTURBATION METHOD FOR PERISTALTIC TRANSPORT OF MHD NEWTONIAN FLUID IN AN INCLINED TAPERED ASYMMETRIC CHANNEL WITH THE IMPACT OF POROUS MEDIUM AND CONVECTIVE THERMAL AND CONCENTRATION

Hayat A. Ali¹, Mohammed R. Salman²

¹Department of Applied Science, University of Technology, Baghdad, Iraq
²Directorate General of Education in the Holy Governorate of Karbala, Karbala, Iraq

¹hayattadel17@yahoo.com, ²mawb1967@gmail.com

Corresponding Author: Mohammed R. Salman
Email: mawb1967@gmail.com

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Abstract

The peristaltic transport of MHD Newtonian fluid under the effect of Porous Medium in an inclined tapered asymmetric channel is analyzed mathematically. Convective Thermal and concentration is discussed. The governing equations, i.e. (continuity, motion, energy, and concentration) are simplified by using a long wavelength and small Reynolds number approximations into a system of non-linear differential equations which solved approximately with the help of Homotopy perturbation method for velocity, streamlines, temperature, and concentration. The impact of important, relevant parameters on the flow is discussed graphically. We noticed that the velocity curve and trapping phenomenon reduced by increasing the Hartman number the magnetic field parameter because of the existence of Lorentz force and increasing in ascending value of permeability parameter. Further, A reduction behavior of temperature and concentration profile is depicted with the higher value of the Biot number of heat and mass transfer.

Keywords: Newtonian fluid, Homotopy perturbation, Porous medium, Convective thermal.

I. Introduction

A process in which a progressive wave of sinusoidal along the walls of the tube / channel causing in the movement of its contents is known by Peristalsis. This mechanism can be seen in the movements of the small intestine, urine flow from the kidney to the bladder through the ureter, gastrointestinal, and in many other living

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body glandular ducts. Furthermore, it gained great importance in industrial and engineering purposes like peristaltic pumps for fluid transport without internal moving parts such that mining slurries, wastewater slurries, sodium bromide and lime slurry pumping, finger and roller pumps, etc. Many researchers have investigated the peristaltic flow of Newtonian and non-Newtonian fluid in various flow geometries to see Refs. [VII, XIII, XV - XV].

Convective heat transfer is used by human and animal bodies to lose the heat generated by metabolic processes in the environment. The industrial applications include the thermal insulation, cooling of nuclear reactors, oil extraction and thermal energy storage. However, the constant thermal conductivity of the fluid condenses the mathematical difficulty of the temperature equation and the analytical solution can be achieved easily. Asghar et al. [XX] Gives a numerical study of heat transfer analysis for the peristaltic transport of viscoplastic fluid taken into consideration mixed convective boundary conditions. Also, Abbasi et al. [I] Describes the influence of an inclined magnetic field on the mixed convective peristaltic transport of Eyring-Prandtl and Sutterby fluid in an inclined channel. Sadia et al. [XVIII] modeled the peristaltic transport of third-grade nanofluid under the effect of thermal radiation and mixed convective. Ahmad et al. [III] gives a theoretical study of the mixed convection characteristics on the flow of Sutterby fluid in the squeezed channel. Paneza1 et al. [IV] Investigated the mixed convective impact on the magnetohydrodynamic heat transfer flow of Williamson fluid over a porous wedge. More studies in this aspect can be seen in refs. [II, V, VI, XII, XIX].

Discussed Muhammad et al.[VIII-XI, XIV] the effects in MHD Peristalsis of Pseudoplastic nanoFluid with Porous Medium in Tapered Channel, and also discussed Effects of MHD on Peristalsis Transport and Heat Transfer with Variables Viscosity in Porous Medium, also an Influence lesson of heat and mass transfer on inclined (MHD) peristaltic of pseudoplastic nanofluid through the porous medium with couple stress in an inclined asymmetric channel, also studied Analysis of Heat and Mass Transfer in a Tapered Asymmetric Channel During Peristaltic Transport of (Pseudoplastic Nanofluid) with Variable Viscosity Under the Effect of (MHD).

II. Mathematical Formulation

We deal with the peristaltic motion of MHD an incompressible Newtonian viscous fluid through asymmetric inclined tapered channels with a porous medium taken inclined at an angle $\alpha$ to the horizontal axis and with a width ($d_1 + d_2$). The fluid reconsidered an electrically conducting under the normal applied magnetic force $B_0$. A small magnetic Reynolds number leads to neglect of the induced magnetic field. The generated sinusoidal waves along the wall of a channel that moves at the speed (c) take the following geometrical form

$$\bar{Y}_1 = \bar{H}_1(\bar{X}, \bar{t}) = d_1 + \dot{m}\bar{X} + b_1\cos \left[ \frac{2\pi}{\lambda} (\bar{X} - c\bar{t}) \right]$$

$$\bar{Y}_2 = \bar{H}_2(\bar{X}, \bar{t}) = -d_2 - \dot{m}\bar{X} - b_2\cos \left[ \frac{2\pi}{\lambda} (\bar{X} - c\bar{t}) + \phi \right]$$

$\bar{Y}_1, \bar{Y}_2$ are the upper and lower wall respectively, $b_1, b_2$ are the amplitudes of the lower and upper walls waves, $\dot{m} \ll 1$ the non-uniform parameter. $\phi \in [0, \pi]$ the phase

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difference. \( \lambda \) is the length of the wave. Consider that when \( \phi = 0 \) approaches to symmetric channel with out of phase waves, and \( \phi = \pi \) describes in phase waves. As well as \( d_1, d_2, a, b, \phi \) satisfy condition such that the walls always parallel.

\[
b_1^2 + b_2^2 + b_1 b_2 \cos \phi \leq (d_1 + d_2)^2
\]

(3)

The fixed frame governing equation is described as bellows

\[
\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0
\]

(4)

\[
\rho \left( \frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{\partial \bar{y}} \right) = - \frac{\partial \bar{p}}{\partial \bar{x}} + \mu \left( \frac{\partial^2 \bar{u}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \right) - \frac{\mu}{\kappa_0} \bar{U} - \bar{B} \frac{\partial^2 \bar{U}}{\partial \bar{y}^2}
\]

(5)

\[
\rho \left( \frac{\partial \bar{v}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{v}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{v}}{\partial \bar{y}} \right) = - \frac{\partial \bar{p}}{\partial \bar{y}} + \mu \left( \frac{\partial^2 \bar{v}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{v}}{\partial \bar{y}^2} \right) - \frac{\mu}{\kappa_0} \bar{V}
\]

(6)

\[
\rho \bar{C}_p \left( \frac{\partial \bar{T}}{\partial \bar{t}} + \bar{U} \frac{\partial \bar{T}}{\partial \bar{x}} + \bar{V} \frac{\partial \bar{T}}{\partial \bar{y}} \right) = \frac{\partial^2 \bar{C}_p}{\partial \bar{x}^2} + \frac{\partial^2 \bar{C}_p}{\partial \bar{y}^2} + \frac{\partial \bar{K}_T}{\partial \bar{t}} \left( \frac{\partial^2 \bar{T}}{\partial \bar{x}^2} + \frac{\partial^2 \bar{T}}{\partial \bar{y}^2} \right)
\]

(7)

Identifying a wave frame \((\bar{x}, \bar{y})\) and the relationship to the fixed frame \((\bar{X}, \bar{Y}, \bar{t})\) by the expression

\[
\bar{x} = \bar{X} - c \bar{t}, \bar{y} = \bar{Y}, \bar{u} = \bar{U} - c, \bar{v} = \bar{V}, \bar{p}(\bar{x}) = \bar{P}(\bar{X}, \bar{t})
\]

(9)

And make use of the following dimensionless magnitudes

\[
\begin{align*}
x &= \frac{\bar{x}}{\lambda}, & y &= \frac{\bar{y}}{\delta}, & u &= \frac{\bar{u}}{c}, & v &= \frac{\bar{v}}{c}, & h_1 &= \frac{\bar{h}_1}{\bar{h}_1}, & h_2 &= \frac{\bar{h}_2}{\bar{h}_1}, & d &= \frac{\bar{d}}{\bar{d}_1}, & \delta &= \frac{\bar{d}_1}{\lambda}, \\
a &= \frac{b_1}{\bar{d}_1}, & b &= \frac{b_2}{\bar{d}_1}, & m &= \frac{\bar{m}}{\bar{d}_1}, & \sigma &= \frac{\bar{p}}{\bar{d}_1}, & \rho &= \frac{\bar{\rho}}{\bar{d}_1}, & \mu &= \frac{\bar{\mu}}{\bar{d}_1}, & Re &= \frac{\nu d_1}{\mu}, & \theta &= \frac{\nu d_1}{\lambda}, & \phi &= \frac{\eta d_1}{\lambda}, & Pr &= \frac{\nu d_1}{\mu}, & \beta &= \frac{\nu d_1}{\lambda}, & \gamma &= \frac{\nu d_1}{\mu}, & \chi &= \frac{\nu d_1}{\lambda}.
\end{align*}
\]

(10)

In which \( Br, M, Re, \delta, \kappa, \sigma, Pr, u, c, m, D_f, S_r, \theta, S_C \) are the Brinkman number, Hartman number, number of Reynolds, the dimensionless number of waves, the permeability parameter, the dimensionless concentration, the Prandtl number, the kinematic viscosity, the Eckert number, the non-uniform parameter, Dufour number,

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Soret number, the temperature and the Schmidt number. And define the relationship of stream function \( \psi(x, y) \) and the two velocity components \( u, v \) as:

\[
\begin{align*}
u &= \frac{\partial \psi}{\partial y}, \quad v = -\delta \frac{\partial \psi}{\partial x} \tag{11}
\end{align*}
\]

Adopting approximation of the low Re and long wavelength, the continuity equation is automatically satisfied and the rest flow equations (5)- (8) will be reduced into the dimensionless system

\[
\begin{align*}
-\frac{\partial p}{\partial x} + \frac{\partial^3 \psi}{\partial y^3} - (M^2 + \frac{1}{\kappa}) \left( \frac{\partial \psi}{\partial y} + 1 \right) &= 0 \tag{12} \\
\frac{\partial p}{\partial y} &= 0 \tag{13} \\
\frac{\partial^2 \theta}{\partial y^2} + Br \left( \frac{\partial^2 \psi}{\partial y^2} \right)^2 + D_P \frac{\partial^2 \sigma}{\partial y^2} + Br \left( M^2 + \frac{1}{\kappa} \right) \left( \frac{\partial \psi}{\partial y} + 1 \right)^2 &= 0 \tag{14} \\
\frac{1}{Sc} \frac{\partial^2 \sigma}{\partial y^2} + Sr \frac{\partial^2 \theta}{\partial y^2} &= 0 \tag{15}
\end{align*}
\]

Given the non-dimensional mean flow rates in fixed and wave frames respectively \( F, Q \) and their relation as

\[
Q = F + d + 1 \tag{16}
\]

In which

\[
F = \int_{h_2}^{h_1} \frac{\partial \psi}{\partial y} \, dy \tag{17}
\]

The dimensionless boundary conditions are:

- convective temperature condition

\[
- K \frac{\partial \theta}{\partial y} = E(T - T_w) \tag{18}
\]

- the convective concentration condition

\[
- D \frac{\partial C}{\partial y} = k_m (C - C_w) \tag{19}
\]

Where \( E \), signed to heat transfer coefficient at the walls in which \( E_1 \) at the upper wall and \( E_2 \) for the lower wall, \( T_w \) is the temperature of the walls, \( k_m \) is the mass transfer coefficient in which \( (k_{m1}, k_{m2}) \) at the upper and lower wall respectively and \( C_w \) is the concentration at the walls.0

Also the other non-dimensional boundary conditions associated with this problem are derived as

\[
\begin{align*}
\psi &= \frac{F}{2}, \quad \psi_y = -1, \quad \theta_y + B_{11} \theta = 0, \quad \sigma_y + M_{12} \sigma = 0 \quad \text{at} \quad y = h_1 \\
\psi &= -\frac{F}{2}, \quad \psi_y = -1, \quad \theta_y - B_{12} (\theta - 1) = 0, \quad \sigma_y + M_{12} (\sigma - 1) = 0 \quad \text{at} \quad y = h_2 \tag{20}
\end{align*}
\]

Notice that

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\[ h_1(x, t) = 1 + m(x + t) + a\cos(2\pi x), \]
\[ h_2(x, t) = -d - m(x + t) - b\cos(2\pi x + \phi), \]
\[ B_{i1} = \frac{E_{i1} d_1}{K}, B_{i2} = \frac{E_{i2} d_1}{K}, M_{i1} = \frac{k_{m1} d_1}{D}, M_{i2} = \frac{k_{m2} d_1}{D} \]

(21)

Where \( B_{i1,2} \) is heat transfer Biot numbers, and \( M_{i1,2} \) is mass transfer Biot numbers.

The dimensionless expression of pressure rise per wavelength is given as

\[ \Delta p_\lambda = \int_0^{2\pi} \frac{\partial p}{\partial x} dx \]  

(22)

III. Perturbation Technique for Problem Solution

Equations (12), (14), and (15) are non-linear systems, hence they are difficult to be solved. Employing a perturbation technique for a small magnitude of Brinkman number \( Br \) to solve these equations we get

\[ \psi = \psi_0 + Br\psi_1 + \cdots \]  

(23)

\[ \theta = \theta_0 + Br\theta_1 + \cdots \]  

(24)

\[ F = F_0 + BrF_1 + \cdots \]  

(25)

\[ \sigma = \sigma_0 + Br\sigma_1 + \cdots \]  

(26)

Substitute the above equations into Eqs. (12), (14), and (15), the following system obtained:

III.i. System of \( Br^0 \)

The governing equations of zero order are derived as below

\[ \frac{\partial^4 \psi_0}{\partial y^4} - \left( M^2 + \frac{1}{\kappa} \right) \frac{\partial^2 \psi_0}{\partial y^2} = 0 \]  

(27)

\[ \frac{\partial^2 \theta_0}{\partial y^2} = 0 \]  

(28)

\[ \frac{1}{Sc} \frac{\partial^2 \sigma_0}{\partial y^2} + Sr \frac{\partial^2 \theta_0}{\partial y^2} = 0 \]  

(29)

Coupled with the boundary conditions

\[ \psi_0 = \frac{r_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = -1, \quad \frac{\partial \psi_0}{\partial y} + B_{i1} \theta_0 = 0, \]

\[ \frac{\partial \sigma_0}{\partial y} + M_{i1} \sigma_0 = 0 \quad \text{at} \quad y = h_1 \quad \text{and} \]

\[ \psi_0 = -\frac{r_0}{2}, \quad \frac{\partial \psi_0}{\partial y} = -1, \quad \frac{\partial \sigma_0}{\partial y} + B_{i2} (\theta_0 - 1) = 0, \]

\[ \frac{\partial \sigma_0}{\partial y} + M_{i2}(\sigma_0 - 1) = 0 \quad \text{at} \quad y = h_2 \]  

(30)

Solving Eqs.(27)-(29), with the given boundary conditions Eq.(30), the final form for \( \psi_0, \theta_0 \) are

\[ \frac{h_1(x, t) = 1 + m(x + t) + a\cos(2\pi x), \]
\[ h_2(x, t) = -d - m(x + t) - b\cos(2\pi x + \phi), \]
\[ B_{i1} = \frac{E_{i1} d_1}{K}, B_{i2} = \frac{E_{i2} d_1}{K}, M_{i1} = \frac{k_{m1} d_1}{D}, M_{i2} = \frac{k_{m2} d_1}{D} \]

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Hayat velocity is seen at notice profile within ascending magnitude of Hartman number transfer rate, and trapping phenomenon.

IV. boundary conditions and some mathematical calculations.

\[ \psi_0 = c_3 + yc_4 + \frac{e^{-yN_1}}{N_1^2} \]  
\[ \theta_0 = r_1 + yr_2 \]  
\[ \sigma_0 = -ScSr \theta_0 \]  

Where \( N_1 = \left( M^2 + \frac{1}{\kappa} \right)^{1/2} \)

III.ii. System of \( Br \)

The first order governing equations with the suitable boundary conditions is

\[ \frac{\partial^4 \psi_1}{\partial y^4} - \left( M^2 + \frac{1}{\kappa} \right) \frac{\partial^2 \psi_1}{\partial y^2} = 0 \]
\[ \frac{\partial^2 \theta_1}{\partial y^2} - \frac{\partial^2}{\partial y^2} + D_f Pr \frac{\partial^2 \sigma_1}{\partial y^2} + \left( M^2 + \frac{1}{\kappa} \right) \left( \frac{\partial \psi_0}{\partial y} + 1 \right)^2 = 0 \]
\[ \frac{1}{Sc} \frac{\partial^2 \sigma_1}{\partial y^2} + Sr \frac{\partial^2 \theta_1}{\partial y^2} = 0 \]

\[ \psi_1 = \frac{r_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0, \quad \frac{\partial \sigma_1}{\partial y} + B_{i1} \theta_1 = 0, \quad \frac{\partial \psi_1}{\partial y} + M_{i1} \sigma_1 = 0 \quad \text{at} \: y = h_1 \} \]
\[ \psi_1 = \frac{r_1}{2}, \quad \frac{\partial \psi_1}{\partial y} = 0, \quad \frac{\partial \sigma_1}{\partial y} + B_{i2} \theta_1 = 0, \quad \frac{\partial \psi_1}{\partial y} + M_{i2} \sigma_1 = 0 \quad \text{at} \: y = h_2 \} \]

Solving the above differential equation and with the aid of the given boundary conditions the approximate solution will take the form

\[ \psi_1 = c_7 + yc_8 + \frac{e^{-yN_1}}{N_1^2} \]  
\[ \theta_1 = r_3 + yr_4 + \frac{1}{1 + D_f PrScSr} \left( \frac{(c_4 + Br c_2)^2 e^{-2yN_1}}{2N_1^2} + \frac{(c_4 + Br c_2)^2 e^{-2yN_1}}{2N_1^2} \right) \]
\[ + \frac{1}{2} \frac{1 + c_4 + Br c_2)^2 e^{-2yN_1}}{N_1^2} \]
\[ \sigma_1 = -ScSr \theta_1 \]  

In which

\( c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8, r_1, r_2, r_3, r_4 \) will be computed by applying the associated boundary conditions and some mathematical calculations.

IV. Graphical Result and Discussions

In this part of the work, we examine the impact of various flow parameters on velocity profile, pressure gradient, temperature distribution, pressure rise, heat transfer rate, and trapping phenomenon. Fig.1(a) shows mixed behavior for velocity profile within ascending magnitude of Hartman number \( M \), furthermore, we can notice the mixed relation toward the upper and lower walls whereas a reduction in velocity is seen at the center of the channel. The effect of a porous parameter \( \kappa \) on the Hayat A. Ali et al
velocity profile has reversed trends with Hartman number $M$ in impact, i.e., the velocity magnitude speeding up toward the central part of the channel while a hampering of fluid motion is noticed toward the walls see Fig. 1(b). From Fig. 2(a) we elucidate that the velocity is a decreasing function of the phase difference parameter $\phi$. Fig. 2(b) highlight the increment of axial velocity value for a larger value of the upper wall amplitude parameter $a$. Fig. 3(a) recorded the effect of Hartman number $M$ on the temperature distribution versus the axial coordinate ($y$), the temperature shows a super fast behavior when the Hartman number $M$ becomes larger. Opposite to the last result, Fig. 3(b) shown an enhancement of permeability parameter $k$ decreasing the temperature profile $\theta(y)$. It is depicted that the impact of Biot number $Bi_1$ on the upper wall is significant and it decreasing effect on $\theta(y)$, we noted that the temperature curve minimized along the whole channel as the Biot- number $Bi_1$ enhances see Fig. 4(a). However, Fig. 4(b) sketched an opposite proportion of Prantle number caused an increment in the temperature of the fluid.

The heat transfers coefficient at the upper wall $Z(x) = \frac{\partial h_1}{\partial x} \frac{\partial \theta}{\partial y}|_{y=h_1}$ is exhibited an oscillatory behavior along the peristaltic waves across the channel walls. It is found from Figs. 5(a) and 6(a),(b) that the absolute value of $Z(x)$ shows a mixed response upon larger value of Brinkman number $Br$, , Soret number $Sr$ and the Schmidt number $Sc$. We can detect creasing in $Z(x)$ at the lower wall for ($0 \leq x \leq 0.45$) of the channel while its rate decreases toward the upper wall for ($0.5 \leq x \leq 1$). However, Fig. 5(b) show a reverse result on $Z(x)$ due to rise in permeability parameter $k$.

The relationship between dimensionless flow rate $Q$ and numerically determined pressure rise per wavelength $\Delta P_1$ is analyzed graphically for $M$, $k$, $Br$ and $b$ through Figs. 7-8. The figures show a linear function with $Q$ as well as three distinct regions identified, a peristalsis pumping for ($\Delta P_1 > 0, Q > 0$), augmented pumping or co-pumping for ($\Delta P_1 < 0, Q > 0$) and free pumping ($\Delta P_1 = 0$). From Fig. 7(a) we elucidate the impact of a Hartman number $M$ on peristalsis pumping for $\Delta P_1 > 0$ is an increasing function while its variation reverse the augmented pumping ($\Delta P_1 < 0, Q > 0$) for region ($0 \leq Q \leq 1$) is depicted. However, the peristalsis region reduces for higher values of permeability parameter $k$ whereas the impact is opposite for the case of co-pumping [see Fig. 7(b)]. The $\Delta P_1$ variation depending on variation of Brinkman number $Br$ drown in Fig. 8(a), the figure shows that $\Delta P_1$ is remain unchanged, i.e. it is independent on $Br$. Fig. 8(b) demonstrate that the peristalsis pumping increases while co-pumping rate enhances for ($0 \leq Q \leq 1$) when the lower wall amplitude ratio $b$ trend to increase.

The Figs. 9-10 display the variation of the concentration profile for different values $M_{i1}$ mass transfer Biot- number, Dufour number $D_f$, Schmidt number $Sc$, and permeability parameter $k$. Reduction concentration profile $\sigma(y)$ was attributed due to rise in $M_{i1}$ value [see Fig. 9(a)]. The influence of $D_f$ And $Sc$ on $\sigma(y)$ (are illustrated Figs. 9(b)- (0) a). Higher values of $D_f$ And $Sc$ resulted in reduce of $\sigma(y)$, whereas Fig. 10(b) shows the opposite of the aforementioned, where the concentration profile $\sigma(y)$ increases with an enhances of permeability parameter $k$.

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A phenomenon of the closed bolus that splitting the streamlines as it moves along the channel walls is called trapping. Figs. 11-14 exhibit the variation of streamlines pattern for different values of $M$, $k$, $∅$, $Br$ and $m$. Fig.11 indicate that the trapping bolus decline in size when Hartman number $M$ trends to increase. Variation in permeability parameter $k$ on streamlines is interpreted via Fig.12 which depict an increasing in bolus magnitude. Fig.13 illustrates the impact of phase difference $∅$ parameter on the trapped bolus. We noted that for $∅ = 0$ symmetric channels the size of the trapped bolus is uniform and expanded along the channel wall, but for $∅ = π$ we observed that the trapped bolus shrink in size and then they merge along the upper wall. From Fig.14 we do not manifest any significant effect for the Brinkman number on the size/number of bolus or the number of enclosing streamlines.

**Fig.1:** Velocity profile for different values of (a) Hartman number $M$ (b) permeability parameter $k$ and for fixed values of the parameters $\{∅ = π/6, a = 0.5, b = 0.3, d = 1, Br = 0.02, m = 0.1, Q = 0.8, x = 2, t = 2\}$.

**Fig.2:** Velocity profile for different values of (a) phase difference parameter $∅$ (b) upper wall parameter $a$ and for fixed values of the parameters $\{M = 1, k = 0.3, b = 0.3, d = 1, Br = 0.02, m = 0.1, Q = 0.8, x = 2, t = 2\}$.
**Fig. 3:** Temperature profile $\theta(y)$ for different values of (a) Hartman number $M$ (b) permeability parameter $k$ and for fixed values of the parameters $\{\phi = \pi/2, a = 0.4, b = 0.2, d = 1, Br = 0.1, m = 0.05, Q = 0.5, Pr = 1, Sc = 0.1, Sr = 0.4, B_{i1} = 1.3, B_{i2} = 0.3, D_r = 0.1, x = 2, t = 3\}$.

**Fig. 4:** Temperature profile $\theta(y)$ for different values of (a) Biot number $B_{i1}$ (b) Prantl number $Pr$ and for fixed values of the parameters $\{\phi = \pi/2, a = 0.4, b = 0.2, d = 1, Br = 0.1, m = 0.05, Q = 0.5, Sc = 0.1, Sr = 0.4, k = 0.1, M = 0.3, B_{i2} = 0.3, Br = 0.1, x = 2, t = 2\}$. 

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**Fig. 5:** Heat transfer coefficient of variation of (a) Brinkman number $Br$ (b) permeability parameter $k$ and for fixed values of the parameters $\{\phi = \pi/2, a = 0.3, b = 0.4, d = 0.8, B_{11} = 1.3, m = 0.1, Q = 0.8, Df = 1, Pr = 1, Sc = 1, Sr = 0.4, M = 0.1, B_{12} = 0.5, y = 0.5, t = 1\}$.

**Fig. 6:** Heat transfer coefficient of variation of (a) short number $Sr$ (b) Schmidt number $Sc$ and for fixed values of the parameters $\{\phi = \pi/2, a = 0.3, b = 0.4, d = 0.8, B_{11} = 1.3, m = 0.2, Q = 0.8, Df = 3, Br = 0.1, Pr = 1, M = 0.6, B_{12} = 0.5, k = 0.3, y = 0.5, t = 1\}$.
Fig. 7: pressure rise for variation of (a) Hartman number $M$ (b) permeability parameter $k$ and for fixed values of the parameters $\{\phi = \pi, a = 0.2, b = 0.9, d = 0.5, m = 0.1, Br = 0.1, x = 1, y = 1, t = 2\}$.

Fig. 8: pressure rise for variation of (a) Brinkman number $Br$ (b) upper wall amplitude parameter $b$ and for fixed values of the parameters $\{\phi = \pi, a = 0.2, d = 0.5, m = 0.1, k = 0.1, M = 0.1, x = 1, y = 1, t = 2\}$.

Fig. 9: Concentration profile for variation of (a) Boit number $M_{i1}$ (b) Dufour number $Df$ and for fixed values of the parameters $\{\phi = \pi/2, a = 0.3, b = 0.5, d = 0.2, m = 0.1, k = 0.3, M = 0.1, Pr = 1, Sc = 0.2, Sr = 0.3, M_{i2} = 2, Br = 0.1, x = 2, t = 2\}$. 

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Fig.10: Concentration profile for variation of (a) Schmidt number $\text{Sc}$ (b) permeability parameter $k$ and for fixed values of the parameters $\{\phi = \pi/2, a = 0.3, b = 0.5, d = 0.2, m = 0.1, Sr = 0.3, M = 0.1, Pr = 1, Df = 0.2, Br = 0.1, M_{12} = 2, M_{11} = 0.3, x = 2, t = 2\}$.

Fig.11: Streamlines for multiple magnitude of Hartman number $M = \{0.5, 0.9\}$ with fixed value parameters $\{\phi = \pi/4, a = 0.3, b = 0.4, d = 0.5, Br = 0.02, m = 0.15, Q = 0.3, k = 0.3, t = 2\}$.

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Fig. 12: Streamlines for multiple magnitude of permeability parameter $k = \{.3, 0.5\}$ with fixed value parameters $\{\phi = \pi/4, a = 0.3, b = 0.4, d = 0.5, Br = 0.02, m = 0.15, Q = 0.3, M = 1, t = 2\}$.

Fig. 13: Streamlines for the multiple magnitude of the phase difference parameter $\phi = \{0, \pi\}$ with fixed values parameters $\{k = 0.3, a = 0.3, b = 0.4, d = 0.5, Br = 0.02, m = 0.15, Q = 0.3, M = 1, t = 2\}$. 

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V. Conclusions

In this paper a mathematical analysis of the peristaltic flow of Newtonian viscous fluid with the presence of a magnetic field and porous medium as moving through the inclined tapered asymmetric channel. Convective thermal and concentration is addressed as boundary conditions. The governing equations are simplified into a set of non-linear ordinary differential equations by considering the assumption of long wavelength and low-Reynolds number, yet by using the homotopy perturbation method this system has been solved. Significant main observations of the present analysis are summarized as follows:

i. Velocity profile reveals mixed behavior, i.e. (the situation in the middle of the channel reverses with the result in the walls) against Hartman number $H$ and permeability parameter $k$, furthermore they show opposite behavior on $u(y)$

ii. The temperature distribution $\theta(y)$ and the concentration $\sigma(y)$ profile show a parabolic behavior.

iii. A mixed response is seen on $\theta(y)$ and $\sigma(y)$ profile for higher value of both heat and mass transfer Biot numbers $B_{11}$ and $M_{11}$ respectively.

iv. Absolute value of heat transfer rate has an oscillatory nature, i.e. it strengthens in regions and decays in other regions along the channel wall with increment of $Br, Sr$, and $Sc$ values.

v. The pressure rise per wavelength $\Delta p_{2}$ and trapped bolus are independent on $Br$.

**Fig. 14:** Streamlines for the multiple magnitude of Brinkman number $Br = \{0.2, 0.3\}$ with fixed values parameters $\{\phi = \pi/4, \alpha = 0.3, b = 0.4, d = 0.5, m = 0.15, k = 0.3, Q = 0.3, M = 1, t = 2\}$. 
vi. The peristaltic flow pumping increases with $H$ while it reduces as $\kappa$ enhances, the impact is totally opposite for augmented pumping.

vii. The streamlines straightly depends phase difference parameter. We noted that for ($\phi = 0$ symmetric channel) the trapped bolus form is uniform and but for ($\phi = \pi$) we observed that the trapped bolus diminishes in magnitude/number and they merge along the upper wall.

viii. The influences of $H$ and $\kappa$ on the velocity profile and the formation of a circulating bolus are opposite.

Conflict of Interest:
There is no conflict of interest regarding this article.

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