Testing maximal electron and muon neutrino oscillations with sub-GeV SuperKamiokande atmospheric neutrino data

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Abstract

Motivated by the Exact Parity Model and other theories, the hypothesis that each of the known neutrinos oscillates maximally with a sterile partner has been put forward as an explanation of the atmospheric and solar neutrino anomalies. We provide detailed predictions for muon and electron flux ratios induced in the Kamiokande and SuperKamiokande detectors by sub-GeV atmospheric neutrinos. Several different, carefully chosen cuts on momentum and zenith angle are proposed, emphasizing the role of up-down flux asymmetries.
The solar [1] and atmospheric [2, 3] neutrino observations and the LSND [4] experiment provide strong evidence that neutrinos have nonzero masses and oscillate. Many specific neutrino mass scenarios have been proposed as explanations for some or all of these results. In this paper we will focus on the hypothesis that each of the known neutrinos oscillates maximally with an exactly or effectively sterile partner. Though non-minimal, this scenario is theoretically well motivated by the Exact Parity Model [5] and other theories [6].

Denote the three sterile neutrinos by $\nu'_{e}$, $\nu'_{\mu}$, and $\nu'_{\tau}$. In the Exact Parity Model, all known particles are paired via an exact parity symmetry with mirror partners in order to achieve a parity symmetric dynamics. Since parity eigenstates must also be energy eigenstates, the mass eigenstate neutrinos are maximal mixtures of $\nu_{\alpha}$ and $\nu'_{\alpha}$ (where $\alpha = e, \mu, \tau$) in the limit of small intergenerational mixing. The models of Ref.[6] provide approximate maximal mixing via an approximate discrete symmetry of the neutrino mass sector. We will assume small intergenerational mixing, analogous to the quark sector, for the purposes of this paper.

Maximal $\nu_{e} - \nu'_{e}$ and $\nu_{\mu} - \nu'_{\mu}$ oscillations have been put forward as explanations of the solar and atmospheric neutrino anomalies respectively [3, 6]. This scenario is also compatible with the LSND experiment. Maximal $\nu_{e} - \nu'_{e}$ oscillations can solve the solar neutrino problem for $\delta m^{2}_{ee'}$ in the large range[7, 8],

$$3 \times 10^{-10} \lesssim |\delta m^{2}_{ee'}|/eV^2 \lesssim 0.9 \times 10^{-3},$$

where the upper bound is the most recent experimental limit [9]. Maximal $\nu_{\mu} - \nu'_{\mu}$ oscillations can explain the atmospheric neutrino anomaly [10] provided that [2, 11]

$$10^{-3} \lesssim |\delta m^{2}_{\mu\mu'}|/eV^2 \lesssim 10^{-1}.$$  

Note however that the atmospheric neutrino experiments are sensitive to both $\nu_{\mu} - \nu'_{\mu}$ and $\nu_{e} - \nu'_{e}$ oscillations in principle. The implications of this for atmospheric neutrino experiments were discussed qualitatively in Ref.[12], where rather crude analytic estimates were provided of some relevant measurable quantities. The purpose of this paper is to provide a detailed quantitative study of the implications of maximal $\nu_{\mu} - \nu'_{\mu}$ and $\nu_{e} - \nu'_{e}$ oscillations for the Kamiokande and SuperKamiokande experiments.

We will restrict our analysis to sub-GeV neutrinos. The main reason for this is as follows. Consider the zenith-angle averaged “ratio of ratios” $\langle R \rangle$, where

$$R \equiv \frac{(N_{\mu}/N_{e})_{\text{obs}}}{(N_{\mu}/N_{e})_{\text{MC}}}.$$  

The quantities $N_{e,\mu}$ are the numbers of electron- and muon-like events. The numerator denotes numbers actually observed, while the denominator the numbers expected on the basis of a Monte-Carlo simulation without oscillations. Since $\langle R \rangle$ is measured to be significantly
less than 1, we conclude that the $\nu_e$ oscillation length must be much larger than the $\nu_\mu$ oscillation length. Given this, $\nu_e$ oscillations are expected to impact more significantly on the sub-GeV data than the multi-GeV data, because the neutrino oscillation length decreases with energy. We will not comment further on the multi-GeV neutrinos, except to note that the Kamiokande and especially the preliminary SuperKamiokande data are consistent with maximal $\nu_\mu - \nu'_\mu$ oscillations and are inconsistent with the minimal standard model.

In the water-Cerenkov Kamiokande and SuperKamiokande experiments, neutrinos are detected via the charged leptons $\ell_\alpha$ ($\ell_\alpha = e$ or $\mu$) produced primarily from quasi-elastic neutrino scattering off nucleons in the water molecules: $\nu_\alpha N \rightarrow \ell_\alpha N'$ ($\alpha = e, \mu$). The total number $N(\ell_\alpha)$ of charged leptons of type $\ell_\alpha$ produced through $\nu_\alpha N \rightarrow \ell_\alpha N'$ is given by

$$N(\ell_\alpha) = n_T \int_0^\infty dE \int_{q_{\text{min}}}^{q_{\text{max}}} dq \int_{-1}^{+1} d\cos \theta \int_{-1}^{+1} d\cos \psi$$

$$\times \frac{d^2 F_\alpha(E,\theta)}{dE \ d\cos \theta} \cdot \frac{d^2 \sigma_\alpha(E,q,\cos \psi)}{dq \ d\cos \psi} \cdot P(\nu_\alpha \rightarrow \nu_\alpha'; E, \theta).$$

(4)

Here $d^2 F_\alpha/dE d\cos \theta$ is the differential flux of atmospheric neutrinos of type $\nu_\alpha$ of energy $E$ at zenith angle $\theta$. The term $n_T$ is the effective number of target nucleons. The function $d^2 \sigma_\alpha/dq d\cos \psi$ is the differential cross section for quasi-elastic scattering, $\nu_\alpha N \rightarrow \ell_\alpha N'$, where $\psi$ is the scattering angle relative to the velocity vector of the incident $\nu_\alpha$ (the azimuthal angle having been integrated over), and $q$ is the energy of the charged lepton $\ell_\alpha$. The function $P(\nu_\alpha \rightarrow \nu_\alpha'; E, \theta)$ is the survival probability for a $\nu_\alpha$ with energy $E$ after traveling a distance $L = \sqrt{(R + h)^2 - R^2 \sin^2 \theta - R \cos \theta}$, where $R$ is the radius of the Earth and $h \sim 15$ km is the altitude at which atmospheric neutrinos are produced. It can be obtained by solving the Schrödinger equation for neutrino evolution including matter effects (for a review see, for example, Ref. [14]). In our analysis we have a pair of two-flavour oscillation subsystems, $\nu_\alpha - \nu'_\alpha$ and $\nu_\mu - \nu'_\mu$, where the vacuum mixing angles are maximal ($\sin^2 2\theta_6^{\nu \nu'} = 1$ and $\sin^2 2\theta_6^{\mu \mu'} = 1$). There are thus only two free parameters: $\delta m_{ee}^2$ and $\delta m_{\mu \mu}^2$. Note that Eq. (3) must be modified in order to obtain zenith angle and momentum binned charged lepton events.

For the case of sub-GeV neutrinos, almost all the information necessary for the right hand side of Eq. (3) is available in the published references. We have used the differential cross section $d^2 \sigma_\alpha/dq d\cos \psi$ of Ref. [15]. The differential flux of atmospheric neutrinos $d^2 F_\alpha/dE d\cos \theta$ without geomagnetic effects is given in [16], but we have used the differential flux which includes geomagnetic effects [17].

For reference and for completeness, we first briefly discuss the case where only maximal $\nu_\mu - \nu'_\mu$ oscillations occur. Figure 1 shows the variation of $\langle R \rangle$ for sub-GeV neutrinos with $\delta m_{\mu \mu}^2$. The preliminary SuperKamiokande central value plus the $1\sigma$ confidence level band is also plotted. The datum implies $10^{-3} \lesssim \delta m_{\mu \mu}^2/eV^2 \lesssim 10^{-2}$ at the $1\sigma$ level, with a much larger range allowed at the $2\sigma$ level (but also constrained by the zenith angle dependence of
the multi-GeV sample). Note that while a major thrust of this paper is to examine possible manifestations of $\nu_e - \nu'_e$ oscillations, it is quite possible that $\delta m^2_{ee}$ is too small to lead to any observable effects for atmospheric neutrinos. In that case, $\nu_\mu - \nu'_\mu$ oscillations will be the only effect occurring. This case is almost indistinguishable from the oft-considered maximal $\nu_\mu - \nu_e$ oscillation solution to the atmospheric neutrino anomaly[11]. Note that future long baseline $\nu_\tau$-appearance experiments should help distinguish the two cases.

We now turn on maximal $\nu_e - \nu'_e$ oscillations. As discussed qualitatively in Ref.[12], there are several interesting effects that can occur when both $\nu_e - \nu'_e$ and $\nu_\mu - \nu'_\mu$ oscillations are present. One simple effect of $\nu_e - \nu'_e$ oscillations, for a given $\delta m^2_{\mu\mu'}$, is to increase $\langle R \rangle$ relative to what it would be in the absence of these oscillations. We illustrate this in Fig. 2, which gives $\langle R \rangle$ as a function of $\delta m^2_{ee}$ for various values of $\delta m^2_{\mu\mu'}$. We have used the usual Kamiokande momentum cuts $0.1 < p_e/GeV < 1.33$ and $0.2 < p_\mu/GeV < 1.5$. The current experimental measurements of $\langle R \rangle$ by the Kamiokande and SuperKamiokande collaborations are [13],

$$\langle R \rangle_{Kam} = 0.60^{+0.06}_{-0.05} \pm 0.05$$
$$\langle R \rangle_{SKam} = 0.635 \pm 0.033 \pm 0.053 \text{ (preliminary).}$$

In the absence of $\nu_e - \nu'_e$ oscillations (or equivalently, when $|\delta m^2_{ee}| \lesssim 2 \times 10^{-5} \text{ eV}^2$) the above measurements suggest $|\delta m^2_{\mu\mu'}| \sim 5 \times 10^{-3} \text{ eV}^2$. However, as Fig. 2 shows, much larger values of $\delta m^2_{\mu\mu'}$ can fit these data equally well when $\nu_e - \nu'_e$ oscillations also occur. For example, an $\langle R \rangle$ value of about 0.6 is obtained for $\delta m^2_{\mu\mu'} \sim 6 \times 10^{-2} \text{ eV}^2$ when $\delta m^2_{ee} \sim 10^{-4} \text{ eV}^2$. Figure 2 also shows that all of the range of Eq.(1) is consistent with the (Super)Kamiokande data.

The observation that $\nu_e - \nu'_e$ oscillations increase $\langle R \rangle$ is interesting for future long baseline neutrino oscillation experiments. If $\nu_e - \nu'_e$ oscillations are numerically important for atmospheric neutrinos, then the long baseline determination of $\delta m^2_{\mu\mu'}$ should not be consistent with that implied by atmospheric neutrinos under the assumption that $\nu_\mu - \nu'_\mu$ oscillations only are occurring. A discrepancy would signal that an additional effect is at work for atmospheric neutrinos, with $\nu_e - \nu'_e$ oscillations being a candidate.

One way to experimentally uncover the presence of both $\nu_\mu - \nu'_\mu$ and $\nu_e - \nu'_e$ oscillations is through the energy dependence of $\langle R \rangle$. It turns out that a more sensitive way is through the zenith-angle dependence of fluxes [12], as we now explain.

Consider the quantities $Y^0_e$ and $Y^0_\mu$ where

$$Y^0_e \equiv \left( \frac{N^-_e / N^+_e}{N^-_e / N^+_e} \right)_{osc}, \quad Y^0_\mu \equiv \left( \frac{N^-_\mu / N^+_\mu}{N^-_\mu / N^+_\mu} \right)_{osc}. $$

(6)

Here $N^-_e$ ($N^+_e$) is the number of electrons produced in the detector with zenith angle $\cos \Theta < 0$ ($\cos \Theta > 0$). $N^\pm_\mu$ are the analogous quantities for muons. (The zenith angle $\Theta$ for the charged leptons should not be confused with the zenith angle $\theta$ defined earlier for neutrinos.) The numerators are the predictions for $N^-_\alpha / N^+_\alpha$ ($\alpha = e, \mu$) in the model
while the denominators are the same quantities in the absence of oscillations. (Note that $N^+_{\alpha}/N^-_{\alpha}|_{no-osc}$ are close to 1 for symmetry reasons, but not exactly equal to it.) The numerators and denominators, being ratios of fluxes, should be approximately free of the systematic errors arising from uncertain absolute fluxes ($\pm 30\%$), uncertain $\mu$ to $e$ flux ratios ($\pm$ a few percent) and the uncertain cross-section. The $Y^0_{\alpha}$ are a measure of up-down flux asymmetries. It is important to observe that $Y^0_e$ depends only on $\delta m^2_{ee'}$, and $Y_\mu$ depends only on $\delta m^2_{\mu\mu'}$.

We have plotted $Y^0_e$ as a function of $\delta m^2_{ee'}$ in Fig. 3 [18]. We have used the usual Kamiokande cut $0.1 < p_e/GeV < 1.33$ as well as the alternative $0.5 < p_e/GeV < 1.33$ cut. In the former case we see that $Y^0_e$ decreases from 1 to about 0.82 as $\delta m^2_{ee'}$ is increased from $10^{-5} \ eV^2$ towards the experimental limit. The effect is much more pronounced when the alternative cut on momentum is made, with $Y^0_e$ decreasing to about 0.67. The reason for this is the greater correlation between the zenith angle distribution of the electrons and the zenith angle distribution of the incident $\nu_e$ flux when the average energy of the sample is increased. A price has to be paid in statistics since the neutrino flux is larger at smaller energies, but a gain is made in the size of the signal relative to the underlying systematic error (which is at the few percent level for flux ratios). This is important because the statistical error will be reduced as the data sample grows, whereas the systematic error will remain the same (barring great improvements in the theoretical prediction of neutrino fluxes).

Figures 4 and 5 are similar to Fig. 3, except we have redefined $N^+_{e}$ to be the number of electrons with $\cos \Theta > 0.2$ and 0.6, respectively. Similarly $N^-_{e}$ is redefined to be the number of electrons with $\cos \Theta < -0.2$ and -0.6, respectively. We use the notation $Y^{0.2}_{e}$ and $Y^{0.6}_{e}$ for the quantities corresponding to Eq. 6. Note that these cuts on the zenith angle also increase the magnitude of the effect, again with a concomitant decrease in statistics but a gain in signal relative to systematic error.

Figures 6-8 are the corresponding figures for the muon case, $Y_\mu$. Note that in this case the usual Kamiokande momentum cut is $0.2 < p_\mu/GeV < 1.5$ and we have considered the alternative cut of $0.5 < p_\mu/GeV < 1.5$ also. The peak at high $\delta m^2_{\mu\mu'}$ is due to an oscillation node in the downward-going neutrinos. For the muon case there is already interesting evidence for a zenith angle dependence of the flux of multi-GeV neutrinos. In this context, we would like to emphasize the important role that the ratios $Y_\mu$ and $Y_e$, with the various momentum and zenith angle cuts, could play in determining whether the atmospheric neutrino problem is due to $\nu_\mu$ oscillations alone or to both $\nu_\mu$ and $\nu_e$ oscillations. Current data clearly show that $\nu_\mu$ oscillations are the dominant effect. However, the presence of non-trivial $\nu_e$ oscillations cannot yet be ruled out and will be further tested as SuperKamiokande continues to acquire data.

Note the insensitivity of $Y_\mu$ to $\delta m^2_{\mu\mu'}$ for the quite large range $3 \times 10^{-4} \sim \delta m^2_{\mu\mu'}/eV^2 \sim 10^{-2}$. The corresponding values of about 0.75 – 0.80 and 0.65 – 0.70 for $Y^0_\mu$ are, in a sense, the most “likely” ones. This insensitivity may provide a useful test of maximal $\nu_\mu - \nu'_\mu$
oscillations and may also be useful in discriminating against other possible explanations of the atmospheric neutrino anomaly.

We will now quantify the decrease in statistical error expected with time. After about 1 year of running, the preliminary SuperKamiokande data show statistical errors for $Y_{e,\mu}$ of about 8% and 12% for the standard momentum cut and alternative momentum cut lower limit of 0.5 GeV, respectively. These should decrease to about 3–4% and 5–7%, respectively, after 5 years of running. These are small enough to see or rule out many of the effects plotted in Figs. 3-8.

In conclusion, we have provided a quantitative analysis of the implications of the hypothesis that both the $\nu_\mu$ and $\nu_e$ neutrinos are maximally mixed with sterile partners for the (Super)Kamiokande atmospheric neutrino experiments. Maximal mixing, motivated by the Exact Parity Model \cite{5} and other theories \cite{6}, implies that the oscillations can be described by just two parameters, $\delta m^2_{ee'}$ and $\delta m^2_{\mu\mu'}$. As Figs. 2-5 show, if $|\delta m^2_{ee'}| \lesssim 2 \times 10^{-5} \text{ eV}^2$, then the $\nu_e - \nu'_e$ oscillation length is too long to affect the experiments. As is well known, the experimental results can then be explained by maximal $\nu_\mu - \nu'_\mu$ oscillations with $\delta m^2_{\mu\mu'}$ in a range around $5 \times 10^{-3} \text{ eV}^2$. Zenith angle asymmetries for the detected sub-GeV muons produced are then expected (see Figs. 6-8). (Significant zenith angle dependence for multi-GeV muons has, of course, already been observed.) For $|\delta m^2_{ee'}| \gtrsim 2 \times 10^{-5} \text{ eV}^2$, the effects of $\nu_e - \nu'_e$ oscillations become significant. They have two main effects. First, they can increase $\langle R \rangle$ (see Fig. 2) relative to what it would be in their absence. Second, they can lead to zenith angle asymmetries for the electrons produced in the detector (see Figs. 3-5). While the current data are consistent with the minimal case of maximal $\nu_\mu - \nu'_\mu$ oscillations only, the possibility that the electron neutrino also oscillates maximally with $|\delta m^2_{ee'}| \gtrsim 2 \times 10^{-5} \text{ eV}^2$ is an interesting possibility. Furthermore, this possibility will be tested stringently in the near future as more data is collected, especially at SuperKamiokande.

**Note added**

While this paper was in the final stages of preparation, a paper by J. W. Flanagan, J. G. Learned and S. Pakvasa appeared on the hep-ph archive (9709438), which also discusses the importance of up-down zenith angle asymmetries of the charged leptons induced by atmospheric neutrinos.

**Acknowledgements**

R.F. would like to acknowledge the hospitality of the Tokyo Metropolitan University, where this work was initiated. This work was partially supported by the Australian Research Council. O.Y. was supported in part by a Grant-in-Aid for Scientific Research of the Ministry of Education, Science and Culture, #09045036.
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Figure Captions

Figure 1. The zenith angled averaged “ratio of ratios”, denoted $\langle R \rangle$, plotted as a function of $\delta m_{\mu\mu'}^2$ for the case where $\nu_e - \nu'_e$ oscillations are unimportant ($\delta m_{ee'}^2 \sim 2 \times 10^{-5} \text{ eV}^2$). The usual SuperKamiokande momentum cuts have been assumed. The band denotes the 1σ allowed region from preliminary SuperKamiokande data.

Figure 2. $\langle R \rangle$ plotted as a function of $\delta m_{ee'}^2$, for several values of $\delta m_{\mu\mu'}^2$ with the usual momentum cut used by SuperKamiokande to define its sub-GeV sample. The solid lines, going from top to bottom, correspond to $\delta m_{\mu\mu'}^2/eV^2 = 10^{-3}, 3 \times 10^{-3}, 6 \times 10^{-3}, 10^{-2}, 6 \times 10^{-2}$ and $3 \times 10^{-2}$, respectively. The band denotes the 1σ allowed region from preliminary SuperKamiokande data.

Figure 3. The quantity $Y_{e0}^0$, defined in the text, as a function of $\delta m_{ee'}^2$. The solid line uses the usual cut $0.1 < p_e/\text{GeV} < 1.33$ on the electron momentum $p_e$, while the dashed line uses the alternative cut $0.5 < p_e/\text{GeV} < 1.33$.

Figure 4. Similar to Fig. 3 except we have plotted $Y_{e0}^{0.2}$ defined in the text.

Figure 5. Similar to Fig. 3 except we have plotted $Y_{e0}^{0.6}$ defined in the text.

Figure 6. The quantity $Y_{\mu0}^0$, defined in the text, as a function of $\delta m_{\mu\mu'}^2$. The solid line uses the usual cut $0.2 < p_\mu/\text{GeV} < 1.5$ on the muon momentum $p_\mu$, while the dashed line uses the alternative cut $0.5 < p_\mu/\text{GeV} < 1.5$.

Figure 7. Similar to Fig. 6 except we have plotted $Y_{\mu0}^{0.2}$ defined in the text.

Figure 8. Similar to Fig. 6 except we have plotted $Y_{\mu0}^{0.6}$ defined in the text.
Figure 1

\[ \langle R \rangle \]

\[ \delta m^2_{\mu\mu'} \quad (\text{eV}^2) \]
Figure 2

\[ \langle R \rangle \sim \delta m^2_{ee'} (\text{eV}^2) \]
Figure 3

$Y_e^0$ vs. $\delta m^2_{ee'}$ (eV$^2$)
Figure 4

\[ Y_e^{0.2} \]

\[ \delta m^2_{ee'} \text{ (eV}^2\text{)} \]
Figure 5

$Y_e^{0.6}$ vs. $\delta m^2_{ee'}$ (eV$^2$)
Figure 6

$Y_{\mu}^0$ vs. $\delta m_{\mu\mu'}^2 \text{(eV}^2\text{)}$
Figure 7

$Y \mu^{0.2}$ vs $\delta m^2_{\mu\mu}$ (eV$^2$)
Figure 8

$Y_{\mu_0^{0.6}}$ vs. $\delta m^2_{\mu\mu'} (\text{eV}^2)$