Order parameter of A-like phase of $^3$He in aerogel.

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Phenomenological criterion of a choice of the order parameter for superfluid phases of $^3$He in aerogel in a near vicinity of the transition temperature is formulated. Except for the BW-phase this criterion is met by the order parameter of axiplanar phase for a special choice of its parameters. It is proposed as a limiting at $T \to T_c$ form of the order parameter for the observed A-like phase.

I.

Two superfluid phases of $^3$He in aerogel are usually referred as A-like and B-like. This reflects both their relation to the phases in the pure (free of impurities) $^3$He and the existing uncertainty in their identification. The pulse-NMR experiments [1] indicate strongly that in the B-like phase the order parameter after averaging over small scale fluctuations has the same BW-form as that in the B-phase of a pure $^3$He. The measured magnetic susceptibility of A-like phase is the same as in the normal phase [2]. This is an evidence that the order parameter in the A-like phase has an equal spin pairing (ESP) form i.e. there are no Cooper pairs with zero spin projection on a direction of magnetic field ($z$-axis) Formally it means that the order parameter – matrix $A_{\mu j}$ can be represented as

$$A_{\mu j} = \hat{x}_\mu a_j + \hat{y}_\mu b_j,$$

where $\hat{x}_\mu$ and $\hat{y}_\mu$ are the orts of the corresponding axes in spin space, $a_j$ and $b_j$ are complex vectors in momentum space. A-phase of pure $^3$He (axial) is a particular case of (1). Its order parameter has a form:

$$A_{\mu j} = \Delta \frac{1}{\sqrt{2}} \hat{d}_\mu (\hat{m}_j + i\hat{n}_j),$$

i.e. it contains only one spin vector $\hat{d}_\mu$. The "orbital" vectors $\hat{m}_j$ and $\hat{n}_j$ are real and orthonormal. Volovik has shown [3], that aerogel destroys the long range ordering described by the order parameter of a form, given by eq. (2). Direction of the vector $\hat{l} = \hat{m} \times \hat{n}$ experiences a random walk and an average of $A_{\mu j}$ turns to zero. There remains possibility of transition in a superfluid glass state [4] with the order parameter formed by average of four creation and (or) annihilation operators. It will be shown in what follows that there exist a possibility to preserve long range ordering for a p-wave Cooper paired phase in aerogel with the order parameter of ESP type. Definite form of the order parameter is suggested for the A-like phase of $^3$He in a near vicinity of $T_c$.

II.

In a vicinity of $T_c$ interaction of aerogel with the superfluid $^3$He can be described phenomenologically [5] by adding to the Ginzburg and Landau functional an energy density of a form:

$$f_\eta = g_\eta \eta_{jl}(r) A_{\mu j} A^{\mu l},$$

where $\eta_{jl}(r)$ is a random real symmetric tensor, $g_\eta$ – an interaction constant. This term takes into account fluctuations in position and configurations of strands. The functional takes the form:

$$F_{GL} = \int d^3r \{ \alpha (T - T_c) A_{\mu j} A^{\mu j} + f_\eta + f_\nabla + f_4 \},$$

where $f_\nabla$ and $f_4$ are correspondingly the gradient energy and the fourth order terms. The isotropic part of the added tensor $\eta_{jl}(r)\delta_{jl}$ can be absorbed in the $T_c$, then $\eta_{jl} = 0$. Aerogel is assumed to be isotropic on the average i.e. $< \eta_{jl}(r) > = 0$, but locally it introduces deviations from spherical symmetry. Tensor $\eta_{jl}(r)$ describes the splitting of $T_c$ due to these deviations. In a pure $^3$He $T_c$ is the same for all spherical harmonics with $l = 1$. In aerogel in volumes with a size $> \xi_0$ one can speak of a “local transition temperature” which is different for different projection of angular momentum. The aerogel $^3$He-interaction $f_\eta$ – is of the second order on $A_{\mu j}$. If $< A_{\mu j} > \neq 0$, in a close vicinity of $T_c$
this term gives a principal order contribution to the functional (4). In that region the proper choice of combinations of spherical harmonics forming $A_{\mu j}$ is determined by the perturbation $f_\eta$, as in a degenerate case of perturbation theory. Of particular interest are combinations of spin and angular momentum projections, which are not splitted by the field $\eta_j(r)$, i.e. which meet the condition

$$\eta_j A_{\mu j} A_{\mu l}^* = 0.$$  

(5)

In that case $f_\eta$ is not a dominating contribution and the other terms, driving the phase transition become essential. If $A_{\mu j}$ is an extremum of the functional (4) and simultaneously satisfies eq. (5), then $\frac{\partial F_{G\eta}}{\partial \eta_l} = 0$ and a change of the interaction with aerogel does not change the energy of $^3$He. This is the case for the BW-phase: $A_{\mu j} = \Delta e^{i\phi} R_{\mu j}$, where $R_{\mu j}$- orthogonal matrix. It can be checked by direct substitution in eq. (2):

$$\eta_j R_{\mu j} R_{\mu l} = \eta_j \delta_{jl} = \eta_{jj} = 0.$$  

(6)

For the axial, phase going through the same argument one arrives at a finite contribution to the energy functional: $f_\eta \sim -\eta_{j n} l_{j l}$ which results in the loss of orientational long range order. Let us find out whether condition (5) can be satisfied by an ESP-type order parameter. For that we write explicitly real and imaginary parts of the vectors $\mathbf{a}$ and $\mathbf{b}$ in the definition (1): $\mathbf{a} = \mathbf{m} + i \mathbf{n}$, $\mathbf{b} = 1 + i \mathbf{p}$, where $\mathbf{m}, \mathbf{n}, \mathbf{l}$ – are real vectors, and substitute eq. (1) into eq. (5).

The imaginary part of the resulting expression turns to zero automatically since $\eta_j$ is symmetric. The real part is zero if the following equation is satisfied:

$$m_j m_l + n_j n_l + l_j l_l + p_{j l} p_l = \delta_{jl} \cdot \text{const.}$$  

(7)

The constant in the r.h.s. can be set to unity by normalization. Eq. (7) has following solutions: one of the four vectors, for example $\mathbf{p} = 0$, Three remaining vectors $\mathbf{m}, \mathbf{n}, \mathbf{l}$ form an orthonormal set. As a result the order parameter has the form:

$$A_{\mu j} = \Delta \frac{1}{\sqrt{3}} [\delta_{\mu j} (\tilde{m}_j + i \tilde{n}_j) + \tilde{e}_{\mu l} \tilde{l}_j].$$  

(8)

One can check directly that matrix (8) satisfies eq. (5). It need not be an extremum of the functional of free energy for pure $^3$He [6], nevertheless the order parameter which is found, is a particular case of axiplanar phase [7,8], its order parameter is proportional to

$$\mathbf{d} + i \mathbf{e} [\tilde{m} v_x + i (\tilde{n} v_y + \tilde{l} v_z)] + (\mathbf{d} - i \mathbf{e} [\tilde{m} v_x + i (\tilde{n} v_y - \tilde{l} v_z)].$$  

(9)

This expression contains three real parameters $v_x, v_y, v_z$, bound by a condition: $v_x^2 + v_y^2 + v_z^2 = 1$. The isotropic ESP-phase (8) is obtained at $v_x^2 = v_y^2 = v_z^2 = 1/3$, and the axial phase (2) - at $v_x^2 = v_y^2 = 1/2, v_z = 0$. Both limiting cases belong to a one-parametric family $v_x = v_y \equiv u, v_z \equiv w, 2u^2 + w^2 = 1$ with the order parameter proportional to

$$u \mathbf{d} (\tilde{m} + i \tilde{n}) - w \tilde{l}$$  

(10)

When $w \neq 0$ this order parameter corresponds to a non unitary phase with a symmetry which is different from that of the axial phase. In particular, the symmetry with respect to combination of a gauge transformation with the rotation of $\tilde{m}$ and $\tilde{n}$ around $\tilde{l}$ is absent.

III.

According to the above argument in aerogel a phase transition characterized by 3 by 3 matrix order parameter $A_{\mu j}$ at $T = T_c$ can take place either in the BW-phase or in the symmetric ESP-phase (8). In a magnetic field the phase (8) is favored by its greater magnetic susceptibility. When temperature is lowered, the coefficients $u$ and $w$ can deviate from the value $u = w = 1/\sqrt{3}$. To estimate a distance from $T_c$, for substantial deviations to occure one has to extrapolate fluctuational corrections to the average value of $A_{\mu j}$ from a region far from $T_c$, where the corrections are small into a region closer to $T_c$, where the corrections become of the order of 1. In conventional superconductors [9] this happens at $(T_c - T)/T_c \sim (\lambda_{corr}/T_c)^2$. Here $\lambda_{corr}$ is a correlation length for the field $\eta_j (r)$, $l_{tr}$ – a transport mean free path for Fermi excitations, $\xi_0$ – the superfluid coherence length in the liquid $^3$He. Assuming $\lambda_{corr} \sim 500 \text{Å}$, $l_{tr} \sim 2000 \text{Å}$, $\xi_0 \sim 200 \text{Å}$, one arrives at $(T_c - T)/T_c \sim 1/30$. This estimation is not very accurate because poorly known
λ_{corr} enters it in the sixth power. More reliable estimation one can extract from the data on the smearing of the specific heat jump [10]. According to this data \((T_c-T)/T_c \sim 1/25\). At a smaller deviation from \(T_c\) the order parameter must be close to the symmetric ESP-phase. This region can be even larger if the phase in question is close to a stable minimum of the functional (4) for \(f_4=0\) and for the existing \(f_2\). The symmetric ESP-phase does not experience the disorienting effect of aerogel. When \(u \neq w\) the disorienting term is finite \((w^2 - u^2)\eta_{jl}/l_{jl}\), but if \(|w^2 - u^2| \ll 1\) the corresponding correlation length [3] is much greater then the dipole length. In that case directions of \(l\) and \(m\) are fixed by \(d\) and \(e\). To find a limit of stability of the symmetric ESP-phase one needs a detailed quantitative analysis, which would involve poorly known parameters.

Possible experiments which would make distinction between the axial and axiplanar phases were discussed in a literature and performed in a pure \(^3\)He [8]. All of them can, in principle, be applied to helium in aerogel. More direct are measurements of the orbital properties, like anisotropy of the superfluid density. In the phase (8) the superfluid density must be isotropic. Transverse NMR frequency shift for an equilibrium configuration in the axiplanar phase must be positive in a contrast to the observed negative shift in the A-like phase [2]. This contradiction does not exclude an identification of that phase which was proposed here. One knows that in non-equilibrium configurations one can have negative shift. It happens in the A-phase when \(d\) deviates from \(l\) because of a presence of longitudinal oscillations [11] or because of an orienting effect of the walls [12].

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