Coherent emission of erbium dopants in a high-Q resonator

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The stability and outstanding coherence of dopants and other atom-like defects in tailored host crystals make them a leading platform for the implementation of distributed quantum information processing and sensing in quantum networks. Albeit the required efficient light-matter coupling can be achieved via the integration into nanoscale resonators, in this approach the proximity of interfaces is detrimental to the coherence of even the least-sensitive emitters. Here, we establish an alternative: By integrating a 19 µm thin erbium-doped crystal into a cryogenic Fabry-Perot resonator with a quality factor of 9 · 10⁶, we can demonstrate 59(6)-fold enhancement of the emission rate, corresponding to a two-level Purcell factor of 530(50), while preserving lifetime-limited optical coherence up to 0.54(1) ms. With its emission at the minimal-loss wavelength of optical fibers and its outcoupling efficiency of 46(8) %, our system enables coherent and efficient nodes for long-distance quantum networks.

The implementation of large-scale quantum networks requires emitters with long ground-state coherence and coherent, spectrally indistinguishable optical transitions [1]. In addition, scaling beyond the demonstrated two-node prototype networks requires highly efficient nodes [2]. In solid state realizations, achieving these properties simultaneously has proven difficult. Still, recent experiments suggest that this challenge can be overcome by embedding the emitters into an optical resonator [3, 4]. This reduces the emission on unwanted transitions and enables a large photon collection probability [5], thus ensuring efficient quantum network nodes. In addition, a large Purcell enhancement factor \( P \) leads to spectral broadening that can overcome detrimental effects of spectral instability.

Achieving these benefits requires resonators with large quality factor \( Q \) and small mode volume \( V \), as \( P \propto Q/V \). The latter has been achieved by confining light in nanophotonic structures [3–8]. Unfortunately, photon-mediated entanglement generation is impeded in nanostructured materials, which typically exhibit increased inhomogeneous broadening and thus spectral mismatch of embedded emitters. In addition, the proximity of interfaces causes a fluctuating charge and spin environment, which leads to reduced coherence and diffusion of the optical transition frequencies [3–8].

Therefore, good optical properties in nanophotonic structures have only been achieved with emitters that are insensitive to electric fields, such as silicon-vacancy centers in diamond [6] or rare-earth dopants in crystals with inversion symmetry [4]. Nanofabrication of suited resonators from these materials requires sophisticated techniques with limited yield and significant loss.

Here, we follow an alternative approach. Instead of using a nanostructured material, we embed a 19 µm thin crystal slab in a cryogenic Fabry-Perot resonator. To still achieve a high Purcell enhancement, instead of minimizing \( V \) we maximize \( Q \) by using atomically flat membranes and low-loss dielectric coatings. We thus achieve a Q-factor of 9 · 10⁶, about three orders of magnitude larger than state-of-the-art experiments with emitters in nanophotonic structures [3–8].

While our approach has been pioneered with cold atoms [9], cryogenic Fabry-Perot resonators with lower quality factor have recently been implemented with quantum dots [10], thin diamond membranes [11, 12], and rare-earth doped nanoparticles [13, 14]. Still, preserving the optical coherence of narrowband emitters while strongly enhancing their emission via the Purcell effect has not been demonstrated.

To this end, our experiments use erbium-dopants in yttrium-orthosilicate (YSO), whose transition wavelength around 1536.4 nm ensures minimal loss in optical fibers, as required for quantum networks. Furthermore, the optical coherence of this transition is the best ever measured in a solid, up to 4 ms [15].

A sketch of our experimental system is shown in Fig. 1. We use a plano-concave Fabry-Perot resonator with a tunable length around \( L = 50 \mu m \) and a mirror radius of curvature \( R = 155 \mu m \), giving a \( w_0 = 5.7 \mu m \) waist of the fundamental Gaussian mode. In the cavity, an atomically-flat, 19(1) µm thin YSO membrane serves as host for erbium dopants with an estimated concentration < 0.3 ppm (see methods and supplementary material).

The mirror transmissions of 7(1) and 24(1) ppm for the concave and crystal-covered side are comparable to the absorption and scattering losses, 21(8) ppm, leading to a finesse of 1.2(2) · 10⁵, a linewidth of 22(2) MHz, and
FIG. 1. Experimental setup. a. Fabry-Perot resonator (not to scale). A stable cavity mode (red) with ~46(8)% outcoupling efficiency towards one side is obtained by embedding a 19(1)µm thin membrane of erbium-doped YSO (orange) between two dielectric mirrors (blue layers of alternating refractive index). b. Optical setup. The cavity is probed by a faint laser field, polarized along the D1 axis of YSO, and on resonance with the transition of erbium dopants around 1536.4 nm. It is switched by an acousto-optical modulator (AOM) and detected by a superconducting single-photon detector (SNSPD). A second laser field at 1564 nm is resonant with another longitudinal resonator mode and can be used for resonator stabilization. c. Resonator stability over time. Off-resonant light is fully reflected from the cavity (black). Without active stabilization (left panel), vibrations lead to a fluctuating signal when tuned on resonance (blue). With thermo-optical feedback (right), the cavity frequency can be stabilized to the side (orange) or bottom (red) of the reflection.

an outcoupling efficiency of ~46(8)% to a single-mode fiber.

Keeping the cavity resonant requires to control the mirror separation to less than a picometer. Similar to other experiments with lower finesse [12,14], this turned out challenging in our closed-cycle cryostat. We therefore combine a vibration isolation platform with fine tuning via a piezo tube. Still, we observe fluctuations of the cavity resonance frequency with a root-mean-square of 8(2) MHz, comparable to the cavity linewidth, as shown in Fig. 1.

To eliminate the residual vibrations via photothermal feedback, we irradiate a laser field at a wavelength of 1564 nm, far detuned from the erbium transition but close to resonance with another longitudinal cavity mode. When the resonator fluctuates, a higher or lower fraction of the laser light is absorbed, which changes the crystal temperature and thus shifts the resonance. Choosing suited parameters, we can thus stabilize the resonator frequency [17].

Before characterizing the effects of this temperature change, we investigate the optical properties of the erbium ensemble by measuring fluorescence after resonant pulsed excitation. At temperatures around 2 K, we apply a magnetic field of 0.8 T along the D2 axis of the crystal to ensure that the population is trapped in the probed lower spin state, avoiding detrimental effects of spectral hole-burning [18] and superhyperfine couplings [19].

We first investigate the inhomogeneous broadening by sweeping the laser and cavity frequency. In contrast to nanofabricated resonators on the same host [7,8], the observed Lorentzian spectrum with a FWHM of 414(7) MHz (see supplement) is not broadened compared to bulk reference crystals. This testifies low strain in our crystalline membrane and indicates that our dopant concentration is low enough to avoid effects of collective strong coupling.

As a next step, we measure the fluorescence at the center of the inhomogeneous distribution. As the dopants are randomly distributed across the Gaussian standing-wave mode of the resonator, the observed signal is an average over many Purcell-enhanced decays with different lifetime. As shown in Fig. 2, our data is well fit by a biexponential decay. A longer lifetime is observed when the cavity is vibrating and thus spends less time on resonance with the dopants.

As detailed in the supplementary information, we can calculate the expected fluorescence using the independently determined cavity properties [20]. Without free parameters, the obtained curves are in excellent agreement with the data. Thus, our model allows us to extract the Purcell enhancement for a two-level system at the maximum of the cavity field, $P_{TL} = 530(50)$. Including the 11% branching ratio of the transition [16], this gives a 59(6)-fold lifetime reduction.

The lifetime is unchanged when the excitation pulse power and thus the number of excited dopants is increased over several orders of magnitude. We can thus preclude that cavity-enhanced superradiance contributes to the observed lifetime reduction.

We now turn to the optical coherence of the dopants which we study via photon-echo measurements [15,19]. Fig. 3a shows the measured echo area as a function of the time elapsed since the first excitation pulse, both with (red) and without (blue) photothermal feedback.

An exponential fit leads to an optical coherence of 0.14(1) ms and 0.54(1) ms, respectively. Compared to recent measurements with bulk crystals [19], we observe a tenfold improvement that is explained by the reduction of dopant interactions [18,21] at our lower concentration.

Remarkably, in both measurements the decay is well fit by a single exponential, which indicates the complete absence of spectral diffusion broadening of the homoge-
neous line for our Purcell-enhanced dopants [22]. This is further confirmed by a three-pulse echo measurement shown in Fig. 3. The observed coherence is limited by the lifetime, $T_2 = 2 \cdot T_1$, for dopants with the maximum Purcell enhancement, achieving a key requirement for remote entanglement generation [1].

We attribute the faster decay in the presence of photothermal feedback to the increased temperature. To study this effect, we first confirm that the crystal is thermalized with an adjacent thermometer by comparing the population of the two ground-state spin levels.

We then increase the crystal temperature using a resistive heater. While the lifetime remains unchanged, the dephasing rate rises with temperature, as seen in Fig. 4. Thus, we conclude that increasing the crystal temperature by $\sim 2 \text{ K}$, as required to fully stabilize the resonator frequency at the given mechanical fluctuations, comes at the price of significantly reducing the optical coherence.

In summary, we have shown that integration into high-finesse Fabry-Perot resonators allows for a strong increase of the coupling of dopants to light while preserving their outstanding optical coherence properties.

Our system thus enables efficient, cavity-enhanced quantum memories [23], in which ultra-low dopant concentration overcomes the limitations imposed by spin interactions [21]. In addition, the possibly good mode overlap with superconducting resonators facilitates efficient microwave-to-optical conversion [24] [25]. Furthermore, spectral holeburning at higher dopant concentration [26] might be used for laser stabilization [27] in the telecom C-band for fiber-based sensing or optical atomic clocks.

While the demonstrated possibility to largely decouple the vibrations of a closed-cycle cryocooler may find applications in optomechanical systems [28], it will also give a boost to quantum networking experiments with host crystals that are not amenable to nanofabrication [1]. Using fiber-coupled [10, 12–14] or chip-based [29] Fabry-Perot resonators, we still expect that scalable fabrication of quantum network nodes will be feasible.

In our system, the Purcell enhancement can be further increased. First, aligning the polarization along the D2 axis of YSO [16] increases the dipole moment twofold compared to our measurements. Second, a tenfold en-
hancement is expected for mirrors with smaller radius of
curvature [9] [10] [12] [14] [29]. Finally, the achieved surface
quality allows for increasing the finesse by another order
of magnitude with better mirror coatings [9] [10] [29].

Even without these improvements, by operating at the
tail of the inhomogeneous broadening [4] we plan to re-
solve and control single erbium dopants with good opti-
ical coherence. Combined with frequency-multiplexed
readout of individual spins [8] and the long ground state
coherence of the 167Er nuclear spin [30], our approach
may thus facilitate quantum network nodes operating at
4He temperature and at telecommunication wavelength.

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SUPPLEMENTARY INFORMATION

Experimental setup

The sample is an undoped YSO crystal grown by Scientific Materials. Trace impurities lead to an erbium concentration of < 0.3 ppm, estimated from the saturated fluorescence level and the known resonator mode volume. To tune the cavity length, the mirrors are attached to a custom-made piezo tube (PI ceramics) via titanium springs. Details about the sample and resonator fabrication are described in the supplementary information.

The cavity is mounted in a closed-cycle cryostat designed for low-vibrations and variable temperature down to 1.6 K (AttoDry 2100). To achieve sufficient passive stability of the mirror distance, we hang the resonator from a home-built vibration isolation platform based on passive air-dampers with active position control (Thorlabs PWA090) on top of the cryostat. Using a flexible edge-welded bellow filled with helium, full thermalization of the sample with its surrounding tube is achieved down to temperatures of 1.72 K. This is confirmed by resonantly exciting the fluorescence of the dopants in both spin ground states under a magnetic field of 0.8 T generated by a superconducting solenoid. Assuming a Boltzmann distribution, the ratio of the measured fluorescence is in good agreement with the reading of a resistive thermometer in close proximity to the crystal.

While we reached sub-pm stability in a previous measurement, the performance of the vibration isolation system has been degraded after replacing the variable-temperature insert of the cryostat following fatal damage caused by a short contact. Therefore, all data in this study is acquired under worse mechanical stability. A possible explanation is that the resonator is touching the vacuum tube side wall, which reduces the efficiency of the vibration isolation. This can be fixed in a future redesign with increased mechanical tolerances.

For thermo-optical feedback and fluorescence excitation, we use tunable diode laser laser systems (Toptica CTL 1550 and DLpro as well as OEwaves Gen3). The lasers are stabilized to a frequency comb (Menlo Systems) and switched via acousto-optical modulators (Gooch and Housego). The light is detected with an avalanche photodiode (Photon Spot) after suited spectral filters (Semrock). The experiment is controlled and the data is recorded by a real-time experimental control system (National Instruments Compact RIO).

Crystal and resonator fabrication

The cavity consists of two highly-reflective mirrors. Their fabrication starts from superpolished substrates with 7 mm diameter (Research Electro Optics Inc.). One of the substrates is machined with a CO$_2$ laser at a wavelength of 9.3 µm to fabricate concave depressions using a setup described in [31]. We create a regular 4 by 4 array of nearly Gaussian depressions, with a depth around 20 µm and effective radii of curvature between 150 and 250 µm.

Both mirrors are coated (Laseroptik GmbH) with a dielectric stack of alternating SiO$_2$ and Ta$_2$O$_5$ layers. The coating of the curved mirror is terminated by Ta$_2$O$_5$ and has a design transmission of 10 ppm, while the flat mirror is terminated by SiO$_2$ with a design transmission of 100 ppm.

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The mirrors are held at a stable distance by titanium springs that are attached to a piezo tube (PI ceramics). At room temperature, this allows for scanning the mirror distance over more than 10 free spectral ranges of the resonator, which reduces to less than one upon cryogenic operation.

Between the mirrors, a 19(1) µm thin membrane of YSO is used as a host for erbium dopants. While this host is well known to allow for excellent coherence properties, fabrication of membranes turned out to be challenging. We first tried reactive-ion etching with oxygen, argon and fluoride gases in different compositions. All used recipes led to a slow etch rate and increased surface roughness or surface amorphization, both of which prevent integration into a high-finesse resonator. We therefore developed a chemo-mechanical polishing procedure together with the crystal laboratory at TU Munich and Optec Munich. We now reproducibly achieve a surface roughness below 0.3 nm for membranes down to 10 µm thickness.

The procedure starts by polishing one side of the crystal to the desired surface quality. Then, this side is glued to a flat glass substrate using a thin film of low-viscosity glue. After curing, the second side of the crystal is polished until the desired thickness is achieved.

To transfer the crystal to a cavity mirror without introducing excess strain, we start by gently cleaning the crystal with isopropanol and acetone. We then cover a glass “carrier” with teflon tape and add a drop of acetone before putting it into a clean vessel. Next, we flip the glass substrate with the crystal onto the acetone drop on the carrier, such that it sticks via surface tension. We fill the vessel with acetone and wait until the glue has dissolved and all acetone has evaporated.

When the glass substrate is lifted, the crystal is left on the dry PTFE coated carrier. Still, both sides of the crystal show residues of the glue. To remove them, we add a little drop of acetone to the edge such that it creeps below the crystal, making it stick to the teflon because of surface tension. We can then gently and repeatedly clean the top surface with acetone- and isopropanol-soaked lens paper, removing residues of the glue from the top.

After getting a clean surface in a white-light microscope image, we wait until the acetone below the membrane has evaporated. We then press the mirror to the crystal, where it sticks via van-der-Waals forces. After flipping, we again gently clean the crystal surface with isopropanol-soaked lens paper until all residues of the glue are removed. This procedure works reproducibly for 5 by 5 mm² crystalline membranes.

Fig. 6a shows the center part of the crystal surface after transfer, where the height profile has been measured by a white-light interferometer. Except for a few dust particles with heights of several nm, a very flat surface is achieved.

Fig. 6b shows two line cuts at the center along the x and y direction. Over the ~10 µm mode diameter of our cavity, a root-mean-square surface roughness below 0.2 nm, likely limited by the noise of the instrument, is achieved. We thus expect that the scattering of the crystal does not limit the finesse of a surrounding resonator even for values exceeding 10⁶.

To measure the strain in the membrane at cryogenic temperature, we compare the inhomogeneous broadening of the membrane in the resonator to that of several reference crystals grown by the same company with the lowest available dopant concentration of 10 ppm. We find a shift of ~1.5 GHz, but no additional broadening, as shown in Fig. 5. This indicates that in spite of the mismatch in thermal expansion coefficients of SiO₂ and YSO, no significant strain gradient is generated in the membrane upon cooldown.

**Cavity parameters**

To model the Purcell enhancement in the assembled Fabry-Perot resonator, we start by analyzing the electromagnetic field using the methods described in [20]. To this end, we irradiate a laser (Toptica CTL 1550) and measure its transmission with a photodiode or infrared camera (NIT WiDy SWIR 320) to determine the transversal mode. The birefringence of the host crystal clearly separates the resonance frequency for orthogonal polarization. For each polarization, we observe a set of Hermite-Gaussian modes, where the higher-order modes are slightly split because of a small ellipticity of the concave cavity mirror [31].

We scan the laser from 1520 to 1630 nm and determine the frequency of all resonant modes with a calibrated wavemeter (Bristol 771 series OSA). Thus, the free-spectral range and higher-order mode splitting is de-
determined. Including a phase correction term [33] in the modeling of Ref. [20] allows us to calculate the crystal thickness $t_c = 18.2(1) \text{µm}$, air gap $t_a = 29.8(1) \text{µm}$, and radius of curvature $R = 155(3) \text{µm}$ of the concave mirror. To this end, we implement a least-square fitting procedure that combines a grid search with a local optimization algorithm.

We then continue to determine the losses of the resonator. To this end, we first measure its linewidth. We modulate the laser with an electro-optical modulator, which creates two sidebands symmetrical around the cavity resonance. We then apply a linear frequency ramp to the laser and use the sidebands to gauge the frequency axis. The linewidth is then obtained by Lorentzian fits of the cavity transmission. To reduce inaccuracy that may arise from mechanical fluctuations, the fit results of many runs are averaged.

In our hybrid cavity, the linewidth depends on the resonance frequency [20], as can be seen in Fig. 7a. For the Erbium transition frequency, we obtain a linewidth of 22(2) MHz FWHM, which gives a field decay rate $\kappa = 2\pi \cdot 11(1) \text{MHz}$. In addition to the total loss, we are also interested in the photon outcoupling rates through the mirror on the crystal-side of the cavity and the mirror on the air-side. These can be determined by measuring the transmission and reflection of the system at room temperature from both sides, which eliminates the unknown coupling efficiency of the impinging laser field. Upon cooldown, the cavity length is reduced by 5 µm. We then find $\kappa_{\text{crystal}} = 2\pi \cdot 5(1) \text{MHz}$ and $\kappa_{\text{air}} = 2\pi \cdot 1.5(3) \text{MHz}$, which gives a total outcoupling efficiency of $(\kappa_{\text{crystal}} + \kappa_{\text{air}})/\kappa = 60\%$.

In combination with the polarization decay rate $\gamma = 1/(2T_0)$, calculated from the known radiative lifetime measured in bulk crystals [22], $T_0 = 11.4 \text{ms}$, we can give the complete set of parameters that describe the dynamics of maximally coupled dopants in our cavity:

$$g_{\text{max}} = 2\pi \cdot 67(7) \text{kHz}$$
$$\kappa = 2\pi \cdot 11(1) \text{MHz}$$
$$\gamma = 2\pi \cdot 7 \text{Hz}$$

Here, the maximum coupling strength $g_{\text{max}}$ has been calculated from the cavity geometry with mode volume $V = 750(10) \text{µm}^3$ and the oscillator strength of the used transition $1 \cdot 10^{-7}$ [34]. The resulting maximum Purcell factor for an Erbium dopant, $P_{\text{Er,max}} = g_{\text{max}}^2/(\kappa \gamma) = 58(6)$, can also be calculated from the maximum Purcell factor for a two-level system, $P_{\text{TL}} = 3\lambda^3 Q/(4\pi^2 n^3 V) = 530(50)$, and a branching ratio of 11% [10].

**Numerical modeling of the fluorescence measurement**

In the preceding section, we have determined all relevant cavity QED parameters of our system, which gives a two-level Purcell enhancement of $P_{\text{TL}} = 530(50)$ for an emitter at the maximum of the mode. In this section, we explain how we calculate the expected fluorescence decay from the electromagnetic field profile $E(\rho, z) = E_{\text{max}} \cdot u(\rho, z)$ of the cavity mode within the crystal. We expect a standing wave pattern along the resonator axis $z$ and a Gaussian profile of the fundamental mode along the radial direction $\rho$:

$$u(\rho, z) = \sin(2\pi n z/\lambda) \cdot \exp(-\rho^2/w_0^2)$$  \hspace{1cm} (1)

Here, $n$ is the refractive index of the crystal, $\lambda$ the vacuum wavelength, and $w_0$ the mode waist. Assuming a homogeneous spatial distribution of dopants in the crystal, we can thus calculate a probability density function $p(P_{\text{Er}})$ depending on the Purcell factor of individual Erbium dopants, $P_{\text{Er}} = P_{\text{Er,max}} \cdot u^2(\rho, z)$.  

FIG. 6. Crystal properties. a. Surface height profile of the YSO membrane after transfer to the mirror. At its center, the membrane is fully bonded to the mirror and shows excellent flatness. Apart from a few dust particles that might be removed by further cleaning (red dots), the surface height variation is well below 1 nm. b. As can be seen from line cuts at the center, the surface roughness is below 0.2 nm over the $\approx 10 \text{µm}$ mode diameter of our resonator.
We find:
\[ p(P_{Er}) = \frac{\arctan(\sqrt{\frac{P_{Er,max}}{P_{Er} - 1}})}{4\pi P_{Er}}. \] (2)

The resulting probability density is shown in Fig. 7b (blue dotted line). Now, the effect of a finite Purcell enhancement on the contribution to the fluorescence signal is twofold: First, the probability to collect a photon emitted by a dopant into the cavity quickly drops with lower \( P_{Er} \), as \( p_{coll} = P_{Er}/(P_{Er} + 1) \) (black dotted curve). Second, also the probability \( p_{ex} \) to excite a given dopant within a time \( \tau \) by an intracavity field with \( n_{cav} \) photons depends on the coupling strength \( g = \sqrt{P_{Er} \gamma_{cav}} \) and thus the Purcell factor. We have: \( p_{ex} = \sin^2(\sqrt{n_{cav} \gamma_{cav} \tau}/2) \).

Including these effects, we obtain the overall distribution of Purcell factors that are accessible in our experiments (blue solid line). In this way, we find that the average Purcell enhancement in our experiment is less than half of the maximal value (red dashed line).

To predict the fluorescence decay curves, we numerically sum the single-exponential curves for a random distribution of dopants within the resonator mode, considering the effects mentioned above:

\[ I(t) = \int_1^{P_{Er,max}} dP_{Er} \ p(P_{Er}) \ p_{coll} \ p_{ex} \ e^{-(P_{Er}+1) t/\tau_0} \] (3)

The choice of a lower integration bound, \( P_{Er} \geq 1 \), is justified as only dopants with considerable Purcell enhancement contribute to our experimental signal, as detailed above. The obtained curve is shown in Fig. 2 of the main paper.

To accurately model the fluorescence curves in presence of cavity vibrations, we further include a reduction of the maximum Purcell factor according to the rms cavity detuning \( \Delta \nu_{rms} \):

\[ P_{Er,max} \sim \frac{1}{1 + (\frac{\Delta \nu_{rms}}{\Delta \nu_{FWMH}/2})^2} \] (4)

Leaving this detuning as a fit parameter gives the inset of Fig. 2 of the main manuscript that shows the obtained cavity resonance stability as a function of lock laser power.

**Power dependence**

To study the effect of a varying intracavity photon number and determine the number of dopants coupled to the resonator, we vary the power in the excitation pulse. Fig. 5 shows that the decay constants (blue) of a biexponential fit performed for each power setting are constant. However, the number of excited dopants, calculated from the number of fluorescence photons and the independently determined outcoupling and detection efficiencies, increases with power (red). The nonlinear rise is explained by saturation effects, from which we can estimate the number of excited dopants before entering the power broadening regime to be about 50. At this power level, the spectral width of the excitation will be comparable to the average single-dopant Rabi frequency.
**FIG. 8.** Fluorescence power dependence. In fluorescence measurements with 500 µs long excitation pulses, we scan the resonant laser power. From biexponential fits we extract the fast (filled blue squares) and slow (open blue squares) decay rates, while we calculate the number of excited dopants from the average number of photon detection events (red points). The red dashed line is a fit to a model for a saturation curve with power broadening.

Ω_{avg}, which we calculate from the Purcell factor distribution $p(P_{Er})$ and the cavity photon number $n_{cav} \approx 50$: Ω_{avg} = $\langle \sqrt{n_{cav}P_{Er}\kappa\gamma} \rangle = 2\pi \cdot 180$ kHz. Because of the intensity profile of the cavity mode, about half of the dopants in the crystal remain undetected due to their low Purcell factor. Therefore, the peak spectral density of a single magnetic class of dopants is about 0.56 kHz. By integration over the inhomogeneous linewidth, $\Gamma_{inh} = 414(7)$ MHz, correcting for the Boltzmann distribution between the ground states at finite temperature, and extrapolating the value to all four distinguishable dopant positions (two sites with two magnetic classes each), we arrive at an estimate of $10^7$ dopants in our cavity mode, corresponding to a density of $5.2 \cdot 10^{15}$ cm$^{-3}$. Comparing this with the density of yttrium sites, $1.8 \cdot 10^{22}$ cm$^{-3}$, we find a dopant concentration of 0.28 ppm, in agreement with other measurements of undoped YSO from the same supplier [7].