Observation of Emergent $\mathbb{Z}_2$ Gauge Invariance in a Superconducting Circuit

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Lattice gauge theory (LGT) is one of the most fundamental subjects in modern quantum many-body physics, and has recently attracted many research interests in quantum simulations. Here we experimentally investigate the emergent $\mathbb{Z}_2$ gauge invariance in a 1D superconducting circuit with 10 transmon qubits. By precisely adjusting the staggered longitude and transverse fields to each qubit, we construct an effective Hamiltonian containing a LGT and gauge-broken terms. The corresponding matter sector can exhibit localization, and there also exist a 3-qubit operator, of which the expectation value can retain nonzero for long time in a low-energy regime. The above localization can be regarded as confinement of the matter field, and the 3-body operator is the $\mathbb{Z}_2$ gauge generator. Thus, these experimental results demonstrate that, despite the absent of gauge structure in the effective Hamiltonian, $\mathbb{Z}_2$ gauge invariance can still emerge in the low-energy regime. Our work paves the way for both theoretically and experimentally studying the rich physics in quantum many-body system with an emergent gauge invariance.

I. INTRODUCTION

Lattice gauge theories (LGTs) [1–4] play a significant role in a wide range of modern physics: from standard model to condensed matter physics. In high energy physics, gauge invariance is one of the most fundamental principle of quantum field theory. For example, LGTs can help us understand the confinement of quarks [1]. In condensed matter systems, LGTs are closely related to the quantum spin liquid [5–7] and deconfined quantum critical points [8]. Recently, as the rapid development of quantum simulations [9, 10], realizing LGTs in synthetic quantum many-body systems becomes possible and have drawn many interests from both theoretical and experimental physicists [32–38]. On the one hand, many unresolved problems associated with LGTs are potentially solvable via large scale quantum simulations. On the other hand, quantum simulations provide a new viewpoint to study LGTs, i.e., non-equilibrium dynamics [11, 12]. The corresponding experimental studies have been centred on ultracold atoms [4, 40, 41].

Due to the good scalability, long decoherence time, and high-precision full control, the superconducting circuit [15, 16] becomes one of the most competitive candidates for achieving the universal quantum computation [31], and it is also a much suitable platform for performing quantum simulations [1, 17–29]. Thus, it is natural to ask whether we can benefit advantages of superconducting circuits to simulate and further study LGTs.

In fact, it is generally challenging to construct an exact gauge invariant Hamiltonian in artificial quantum many-body system [4, 34]. Thus, another natural question is whether the gauge invariance can be approximately realized or emergent in the low-energy physics, and is also detectable using state-of-the-art quantum simulators. In Ref. [5], some of us theoretically verify that the gauge invariance can emerge at the ground state, although the gauge invariance is absent in the Hamiltonian. Furthermore, this emergent gauge invariance can be probed by the quench dynamics, which is expected to be observable in experiments.

In this article, we experimentally demonstrate the emergent $\mathbb{Z}_2$ gauge invariance in a 1D superconducting processor with 10 qubits. Following the scheme in Ref. [5], we apply the staggered longitude and transverse fields to the system to construct an effective Hamiltonian, which is the mixture of a $\mathbb{Z}_2$ LGT and some gauge-broken terms. To detect the emergent $\mathbb{Z}_2$ gauge invariance, we firstly study the charge spreading of the matter degrees. The experimental results show that it can exhibit localization under proper parameters, which is a strong dynamical signature of the confinement induced by emergent gauge invariance. Then, benefiting the joint readout of three qubits under arbitrary basis, we can study the time evolution of 3-qubit operators. We find that there is a 3-qubit operator, whose expectation value can retain nonzero for long time. We verify that this operator can be regarded as the $\mathbb{Z}_2$ gauge generator. Therefore, this ex-

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experimental result is a direct evidence of the emergent $\mathbb{Z}_2$ gauge invariance. Furthermore, from relations between charge spreading and initial state, we find that this gauge structure mainly emerges in the low-energy regime.

II. MODEL AND SET-UP

This experiment is performed in a superconducting circuit with 10 transmon qubits ($Q_1$–$Q_{10}$) arranged into a chain, see Fig. 1(a). Due to the large and stagger anharmonicity [43], the system can be described by an isotropic 1D XY model with tunable transverse and longitudinal fields [4, 40, 41]. In addition, the next-nearest neighbor (NNN) coupling cannot be neglected in the following experiment. Therefore, the Hamiltonian can be written as

$$\hat{H} = \sum_j \left( g_j \hat{\sigma}_j^x \hat{\sigma}_{j+1}^x + \lambda_j \hat{\sigma}_j^z \hat{\sigma}_{j+2}^z + \text{H.c.} \right)$$

$$+ \sum_{j=1}^{\ell} \left( \frac{V_j}{2} \hat{\sigma}_j^x + h_{x,j} \hat{\sigma}_j^x \right),$$

(1)

where $\hat{\sigma}_j^x = (\hat{\sigma}_j^x \pm i \hat{\sigma}_j^y)/2$, $\hat{\sigma}_j^{x,z}$ are Pauli matrices, $g_j/2\pi = 12$ MHz and $\lambda_j/2\pi = 0.7 – 1.1$ MHz are the nearest-neighbor (NN) and NNN coupling strengths [43], respectively, $V_j$ is the longitude field tuned by Z pulses, and $h_{x,j}$ is the transverse field controlled by XY drivings.

Here, we let the longitude field at each odd qubit is $V_{2\ell-1}/2\pi = -80$ MHz and the detuning between NN qubits is much larger than the coupling strength, i.e., $V_{2\ell} - V_{2\ell-1} \gg g_j$. In addition, the transverse field is only applied on the even qubits with equal strength $h_z$. Thus, according to Ref. [5], we can obtain an effective
Hamiltonian $\hat{H}_{\text{eff}}$, which reads

$$
\hat{H}_{\text{eff}} = \hat{H}_1 + \hat{H}_2 + \hat{H}_3,
$$

$$
\hat{H}_1 = \sum_{\ell=1}^{4} \tilde{g}_{\ell}(\tilde{\sigma}_{\ell+\frac{1}{2}}^x \tilde{\sigma}_{\ell+\frac{1}{2}}^z + \text{H.c.}) + \sum_{\ell=1}^{5} \tilde{h}_{\ell} \tilde{\sigma}_{\ell+\frac{1}{2}}^x,
$$

$$
\hat{H}_2 = - \sum_{\ell=1}^{4} \tilde{g}_{\ell}(\tilde{\sigma}_{\ell+\frac{1}{2}}^x \tilde{\sigma}_{\ell+\frac{1}{2}}^z + \text{H.c.}),
$$

$$
\hat{H}_3 = \sum_{\ell=1}^{5}(\lambda_{2\ell-1} \tilde{\sigma}_{\ell+\frac{1}{2}}^x + \lambda_{2\ell} \tilde{\sigma}_{\ell+\frac{1}{2}}^z + \text{H.c.}) + \sum_{\ell=1}^{5} \tilde{h}_{\ell} \tilde{\sigma}_{\ell+\frac{1}{2}}^x,
$$

where $\tilde{\sigma}_{\ell} = \tilde{\sigma}_{2\ell-1}$ and $\tilde{\sigma}_{\ell+\frac{1}{2}} = \tilde{\sigma}_{2\ell}$ are also Pauli matrices labeling the odd and even qubits, respectively, and $\tilde{g}_{n/\ell,t} \approx g_0^2/\Delta \approx -1.8$ MHz is the effective three-body coupling strength, see Fig. 1(b).

Here, $\hat{H}_1$ is nothing but a $\mathbb{Z}_2$ lattice gauge field (τ-site) coupled to a matter field (s-site), where the corresponding $\mathbb{Z}_2$ gauge generator reads $G^0_{\ell} = \tilde{\tau}_{\ell-\frac{1}{2}} \tilde{\sigma}_{\ell+\frac{1}{2}}^x$, i.e., $[G^0_{\ell}, \hat{H}_1] = 0$ [3]. However, we can find that the additional terms $\hat{H}_2$ and $\hat{H}_3$ both violate this gauge invariance, so the whole Hamiltonian is not a rigorous LGT. However, by numerical simulation via density matrix renormalization group (DMRG) method [6, 7] (The details are shown in SM [43]), we find that a new $\mathbb{Z}_2$ gauge invariance can still emerge in the ground state for large transverse field $h_x$ and proper longitudinal field $h_z$. The corresponding new gauge generator is

$$
\hat{G}_{\ell} = \tilde{\tau}_{\ell-\frac{1}{2}} \tilde{\sigma}_{\ell+\frac{1}{2}}^x,
$$

where $\tilde{\tau}^z = \sin \beta \tilde{\tau}^z + \cos \beta \tilde{\tau}^x$ and $\beta = \arctan(h_z/h_x)$.

Next, we will experimentally investigate the quench dynamics of this system to probe the emergent gauge invariance. Here, we consider the system containing only one s-spin, i.e., $\sum_{\ell=1}^{5} \tilde{\sigma}_{\ell} = 1$. The initial state is chosen as $|\psi_0\rangle = |s\rangle \otimes |\tau\rangle$, where $|s\rangle$ and $|\tau\rangle$ label the states of s- and τ-sectors, respectively. We let $|s\rangle = |00100\rangle$ and $|\tau\rangle = |\Phi_0\Phi_0\Phi_0\Phi_0\Phi_0\rangle$, where $|\Phi_0\rangle = \cos \frac{\theta}{2}|1\rangle + \sin \frac{\theta}{2}|0\rangle$.

Here, according to the effective Hamiltonian $\hat{H}_{\text{eff}}$, we know that different $\theta$ means different energy during the quench dynamics. The experimental procedure can be summarized as follows: Firstly, all qubits are at their idle frequencies, and we use single-qubit rotational gates to prepare the initial state $|\psi_0\rangle$. Then, we bias the frequency of each qubit to the corresponding working point with Z-pulses, i.e., let the local potential of each qubit be $V_{j}$, Meanwhile, we apply an ac driving with amplitude $h_z$ to each even qubit through XY line to realize the transverse field. Finally, after the system evolving with time $t$, we bias back all qubits to their idle points and readout the corresponding observable. The pulse sequence of each qubit is shown in Fig. 1(c).

### III. EXPERIMENTAL RESULTS

In this section, we report the experimental results to demonstrate the existence of emergent $\mathbb{Z}_2$ gauge invariance. We first study the charge transport of s-sector to probe whether the system locates in confined phase. Then we explore the dynamics of a 3-qubit operator to find the corresponding $\mathbb{Z}_2$ gauge generator.

#### A. Confinement dynamics

In 1D $\mathbb{Z}_2$ LGTs, the system is generally in a confined phase, where the matter field can exhibit a localization [3, 4]. Thus, we can study the dynamics of s-spins to demonstrate whether it can localize, which is a signature for identifying the existence of emergent gauge invariance. Here, we measure the density distributions of the photon at odd qubits, i.e., the spin density distribution of the s-sector, defined as

$$
P_f(t) := \langle \psi(t) | \sigma_j^+ \sigma_j^- | \psi(t) \rangle,
$$

where $f$
where $|\psi(t)\rangle = \exp(-i\hat{H}t)|\psi_0\rangle$ is the wavefunction of the system at time $t$.

When $h_2/2\pi = 2$ MHz, $V_{2d}/2\pi = 0$ MHz, and $\theta = -\pi/3$, we can find that the $s$-spin is delocalized and spread to the whole system very quickly, see Fig. 2(a). However, the situation is different when $h_2/2\pi = 6$ MHz, $V_{2d}/2\pi = 15$ MHz. Figs. 2(b–c) show that $s$-spin can indeed localize when $\theta = -\pi/3$ or $-\pi/2$, which is an evidence of existing confinement. When we continue to change the initial state, e.g., $\theta = \pi$, the localization of $s$-spin disappears, see Fig. 2(d). These experimental results indicate that the confinement of $s$-spin can indeed emerge for the proper transverse and longitudinal fields, and it also depends on the energy of the system.

Next, we further explore the relation between confinement and the system energy, which can be characterized by the localization strength of $s$-spins. Here, we define the extended imbalance of $s$-spins [29]

$$\mathcal{I} := \sum_{j=\text{odd}} \eta_j P_j,$$

where $\eta_j = 1/N_1 (-1/N_0)$ if the initial state of $Q_j$ is $|1\rangle$ ($|0\rangle$), and $N_1$ ($N_0$) is the number of $|1\rangle$ ($|0\rangle$) for the initial state of $s$-sector. In Fig. 3(a), we show the dynamics of extended imbalance for different $\theta$ under the condition of $h_2/2\pi = 6$ MHz and $V_{2d}/2\pi = 15$ MHz. It shows that $\mathcal{I}$ can stabilize at different values indicating the different localization strength. Now we use the steady value of extended imbalance $\mathcal{I}_\infty$ to quantify the localization strength, which can be calculated as the average of $\mathcal{I}$ during the last 0.8 $\mu$s. Here, the larger $\mathcal{I}_\infty$ means the stronger localization strength. In Fig. 3(b), the relation between $\mathcal{I}_\infty$ and $\theta$ is presented. We can find that, when $\theta = \theta_m \approx -0.35$, the corresponding $\mathcal{I}_\infty$ is the largest indicating the strongest localization strength in this case. In the following discussion, we will verify that this is because that emergent gauge invariance can only exist in a low-energy regime, and the initial state is close to the ground state when $\theta = \theta_m$.

### B. Gauge invariance

Now we start to directly study the gauge invariance of the system. Here, when deriving the effective Hamiltonian $\hat{H}_{\text{eff}}$, the correction of longitudinal field $h_2$ from high-order terms can hardly be directly confirmed accurately. Thus, $\beta$ or $\mathbb{Z}_2$ gauge generator $\hat{G}_\ell$ is in fact unknown. However, we can define an ansatz of $\mathbb{Z}_2$ gauge generator as

$$\hat{G}_\ell(\alpha) := \hat{T}_{\ell - \frac{1}{2}}(\alpha) \hat{\phi}_x^\ell \hat{\phi}_{x}^{-\ell} (\alpha),$$

where $\hat{T}_{\ell - \frac{1}{2}}(\alpha) = \cos(\alpha) \hat{\phi}_{-\ell - \frac{1}{2}}^* + \sin(\alpha) \hat{\phi}_{-\ell + \frac{1}{2}}$, so $\hat{T}_{\ell + \frac{1}{2}}(\beta) = \hat{T}_{-\ell - \frac{1}{2}}^*$ and $\hat{G}_\ell(\beta) = \hat{G}_\ell$ is the emergent $\mathbb{Z}_2$ gauge generator defined in Eq. (3).

If the specific eigenstate has emergent $\mathbb{Z}_2$ gauge invariance and the initial state has a large overlap with this state, then the expectation value of $\hat{G}_\ell(\alpha)$ during the quench dynamics will be nearly time-independent. Furthermore, the steady expectation value of $\hat{G}_\ell(\alpha)$ during the dynamics approaches the minimum/maximum when $\alpha = \beta$ [43]. Therefore, we can determine $\beta$ and thus fix $\hat{G}_\ell$ in the experiment based on these considerations. To measure the expectation value of $\hat{G}_\ell(\alpha)$, we need the joint readout of adjacent three qubits under the specific basis, which is accessible in superconducting circuits. We should measure the expectation values of $\hat{\phi}_{-\ell - \frac{1}{2}}^* \hat{\phi}_{-\ell + \frac{1}{2}}$, $\hat{\phi}_{-\ell}^* \hat{\phi}_{-\ell + \frac{1}{2}}$, $\hat{\phi}_{-\ell - \frac{1}{2}}^* \hat{\phi}_{\ell}^\ell$, $\hat{\phi}_{-\ell + \frac{1}{2}}^* \hat{\phi}_{\ell}^\ell$, and $\hat{\phi}_{-\ell - 1}^* \hat{\phi}_{\ell + 1}^\ell$, respectively. Then, we combine these four values linearly according to Eq. (6).

In Fig. 4(a), we show the relation between $(\bar{G}_3)_{\infty}$ and $\alpha$ when $h/2\pi = 6$ MHz, $V_{2d}/2\pi = 15$ MHz, and $\theta = -\pi/3$. Here, $(\bar{G}_3)_{\infty}$ is the average of $\langle \hat{G}_3 \rangle$ during the
an evidence that some specific eigenstates of \( \hat{H}_{\text{eff}} \) for effective Hamiltonian (The oscillation is from the high-order term and absent ance. 

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the effective Hamiltonian can emerge a maximum when \( \alpha \approx 1 \). We find that the gauge invariance indeed mainly emerges in the low-energy regime. 

In summary, we have experimentally investigated the emergent \( \mathbb{Z}_2 \) gauge invariance in a 10-qubit superconducting processor. Our experimental results demonstrate that \( \mathbb{Z}_2 \) gauge invariance can indeed emerge in low-energy regime, even though the \( \mathbb{Z}_2 \) gauge structure is absent in the effective Hamiltonian. Moreover, this emergent gauge invariance can lead exotic dynamical behaviors, for instance confinement-induced localization, which has been observed in this experiments.

Our results can scale up to the larger quantum system and enable the further study of emergent LGTs in superconducting circuits. For instance, the dynamics of string breaking [11], the thermalization of the effective Hamiltonian \( H_{\text{eff}} \) and whether existing disorder-free many-body localization [47, 48] are interesting issues. In addition, how to realize a truly gauge-invariant Hamiltonian on superconducting circuits is another relevant question.

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**FIG. 4.** Gauge invariance. (a) The relation between \( \langle \hat{G}_3 \rangle_\infty \) and \( \alpha \) with the parameters being identical to the cases in Figs. 2(b). Here, \( \langle \hat{G}_3 \rangle_\infty \), calculated as the average of \( Z \) from 0.2 to 1 \( \mu \)s with the standard deviation being the error bar, represents the steady expectation value of 3-qubit operator \( \hat{G}_3(\alpha) \), defined in Eq. (6). The curve is almost a sine-shape curve with the valley at \( \alpha = -1.19 \). (b) Time evolution of 3-qubit operator \( \langle \hat{G}_3(\alpha = -1.19) \rangle \).
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Supplemental Material

Observation of Emergent $\mathbb{Z}_2$ Gauge Invariance in a Superconducting Circuit

In this Supplemental Material, we mainly present more details about experiments, including information of the device, corrections of pulses, and other extended experimental and numerical data.

EXPERIMENTAL DEVICE

A. Parameters of the chip

The chip of this experiment is a chain-like superconducting circuit consisting of 10 Xmon qubits (from left to right of the chain is $Q_1$ to $Q_{10}$, see Fig. 1(a) in the main text), which are fabricated on a 10 mm×10 mm×0.43 mm sapphire substrate with two steps of aluminum deposition [S1].

The detailed parameters of the device are listed in Tab. S1. The frequency of readout resonator $f_r$ increases by about 20 MHz from $Q_1$ to $Q_{10}$ distributed from 6.51 GHz to 6.69 GHz. Each Xmon qubit can reach its maximum frequency $f_m$ at the sweetpoint, at which the qubit is insensitive to flux noise and exhibits long dephasing time. In this experiment, all qubits are initialized to the ground state ($|0\rangle$) at their idle frequencies $f_i$, which are in the range from 4.934 GHz to 5.692 GHz. The single-qubit gate used in the experiment is also performed at their idle frequencies. Additionally, in this experiment, the working points of odd and even qubits are about 5.14 GHz and 5.22 GHz, respectively. Due to the different capacitors between odd and even qubits, anharmonicity $\eta$ of the qubits is staggered. Here, the energy relaxation time (decoherence time) $T_1$ and the dephasing time $T_2^*$ are both measured at the idle point. We optimize the quadrature correction term with DRAG coefficient $\alpha$ to minimize leakage to higher levels [S2]. Finally, we use the randomized benchmark (RB) method to characterize the error of $X/2$ and $Y/2$ gate. The nearest neighbor (NN) coupling strength is about 12 MHz, and the next nearest neighbor (NNN) coupling strength between the odd qubits and between the even qubits are different due to the different effective capacitance of even and odd qubits.

B. Experimental setup

In Fig. S1, we present the diagram of the experimental setup. This system is consisted of some main function boards and some auxiliary boards. The main function boards, displayed at the left top, includes control board, bias board, and readout board. The control board is composed of 6 DACs controlled by FPGA, and is used to realize the full XY control and partial Z control. DACs together with the nearby microwave source output microwaves for the XY control of each qubits. The dc-bias wires compose the bias board, which is used to complete Z control together with the Z wire in control board and bias tee. The readout board provides the measurement function of qubits. The DAC in the readout board and the nearby microwave source output a ten-tone microwave pulse targeting all readout resonators. The readout signal is amplified sequentially by the Josephson parametric amplifier (JPA) (JPA is unused in this experiment), high electron mobility transistor (HEMT), and room temperature amplifiers before demodulated by the ADC. All control lines go through various stages of attenuation and filter to prevent unwanted noises from disturbing the operation of the device.

CALIBRATION

C. Readout calibration

The qubit readout pulse consists of a 1.6 $\mu$s microwave pulse, which contains information of all readout resonance. After demodulation by FPGA, we can obtain IQ data, see Fig. S2. Due to the unwanted noise, readout has errors. Here, we can use the calibration matrix to calibrate this error, which reads

$$
\begin{pmatrix}
P^g_g \\ P^e_g \\ P^g_e \\ P^e_e
\end{pmatrix} =
\begin{pmatrix}
F_{gg} & 1 - F_{eg} & P^g_g \\ 1 - F_{gy} & F_{ey} & P^e_g \\ P^g_e \\ P^e_e
\end{pmatrix},
$$

(S1)
TABLE S1. Basic device parameters. \( f_r \) is the readout resonator frequency, \( f_m \) is the qubit maximum frequency, and \( f_i \) is the qubit idle frequency. \( \eta \) is the qubit anharmonicity. \( T_1 \) and \( T_2^* \) are the energy relaxation time and dephasing time of the qubit at idle point. \( F_{gj} \) and \( F_{ej} \) are the readout fidelities for the ground and first-excited states, respectively. The errors of \( \pm X/2 \) and \( \pm Y/2 \) gates are also presented. In addition, \( g_{j,j+1} \) and \( g_{j,j+2} \) are the coupling strengths of nearest-neighbor (NN) and next-nearest-neighbor (NNN) qubits, respectively.

where \( F_{gj} \) and \( F_{ej} \) are readout fidelities of state \( |0\rangle \) and \( |1\rangle \) of \( Q_j \), respectively, see Tab. S1, and \( P_{c}^g (P_{c}^e) \) and \( P_{g} (P_{e}) \) are calibrated and original probabilities (calculated directly from IQ data) of \( |0\rangle \) (\( |1\rangle \)), respectively.

### D. Crosstalk correction of Z pulse

The crosstalk of Z lines will decrease the accuracy of the experiment, and thus should be corrected. Firstly, we need determine the Z crosstalk matrix \( M_z \), which can be calculated by measuring the offset and the compensation offset of the qubit through different Z lines. The measured Z crosstalk matrix in this system is shown in Tab. S2. Here, if we applied the Z pulse to each qubit with strength \( Z_{\text{app}} \), then the qubits can actually feel the strength \( Z_{\text{act}} = M_z \cdot Z_{\text{app}} \). Thus, we can get the compensation value through the above formula to calibrate the error caused from Z crosstalk.

### E. Distortion calibration of Z pulse

In the experiment, we need adjust the qubit level by applying a square wave pulse to the Z line. However, due to the presence of parasitic inductance and capacitance, an ideal square pulse is usually distorted when reaching to the chip showing overshoot or undershoot near the rising edge and tailed falling edge. In Fig. S3, we show the results for distortion calibration of Z pulse. We can find that, after third order calibration, Z pulse can nearly become a perfect square wave.
LOW-ENERGY PHYSICS OF THE EFFECTIVE HAMILTONIAN

In this section, we use matrix product state (MPS) based methods to study the ground state and quench dynamics properties of the effective Hamiltonian, i.e., Eq. (2) in the main text. Without loss generality, we consider a homogeneous system, namely, we fix $\tilde{g}_{s/l, t} = g = 1.8$, $\lambda_{2\ell -1} = \lambda_s = 1.1$, and $\lambda_{2\ell} = \lambda_r = 0.7$. Thus, the Hamiltonian reads

$$\hat{H}_{\text{eff}} = \hat{H}_1 + \hat{H}_2 + \hat{H}_3,$$

$$\hat{H}_1 = -\sum_{\ell=1} g(s_{\ell+\frac{1}{2}} + s_{\ell+1} + \text{H.c.}) + \sum_{\ell=1} h_x \hat{r}_{\ell+\frac{1}{2}}^x,$$

$$\hat{H}_2 = \sum_{\ell=1} g(\hat{r}_{\ell-\frac{1}{2}}^y - s_{\ell+\frac{1}{2}} + \text{H.c.}),$$

$$\hat{H}_3 = \sum_{\ell=1} (\lambda_s \hat{s}_{\ell+\frac{1}{2}}^x \hat{s}_{\ell+1} + \lambda_r \hat{r}_{\ell+\frac{1}{2}}^y \hat{r}_{\ell+1}^y + \text{H.c.}) + \sum_{\ell=1} h_z \hat{r}_{\ell+\frac{1}{2}}^z.$$  

Thus, there are only two driving parameters, i.e., $h_x$ and $h_z$.

Here, we can find that $\hat{H}_1$ is a typical $\mathbb{Z}_2$ LGT coupled with a matter field [S3, S4], where $\tau$ and $s$ are gauge and matter fields, respectively, and the transverse field $h_x$ is the corresponding $\mathbb{Z}_2$ electric field. The $\mathbb{Z}_2$ gauge transformation can be defined as $\hat{G}_\ell^0 = \hat{r}_{\ell+\frac{1}{2}}^x \hat{r}_{\ell+1}^y \hat{r}_{\ell+\frac{1}{2}}^y$, satisfying $[\hat{G}_\ell^0, \hat{H}_1] = 0$. However, $\hat{H}_2$ and $\hat{H}_3$ both lack this $\mathbb{Z}_2$ invariance, i.e., $[\hat{G}_\ell^0, \hat{H}_2] \neq 0$ and $[\hat{G}_\ell^0, \hat{H}_3] \neq 0$. Therefore, the whole effective Hamiltonian $\hat{H}_{\text{eff}}$ is not $\mathbb{Z}_2$ gauge invariant.

To analyze the emergent gauge invariance of $\hat{H}_{\text{eff}}$, we map the $\tau$ spin (matter field) to another frame, i.e., performing the following replacement

$$\hat{r}_{\ell+\frac{1}{2}}^x = \cos \beta \hat{r}_{\ell+\frac{1}{2}}^x + \sin \beta \hat{r}_{\ell+\frac{1}{2}}^y,$$

$$\hat{r}_{\ell+\frac{1}{2}}^y = \frac{\sin \beta \hat{r}_{\ell+\frac{1}{2}}^x}{\cos \beta},$$

$$\hat{r}_{\ell+\frac{1}{2}}^z = \cos \beta \hat{r}_{\ell+\frac{1}{2}}^z - \sin \beta \hat{r}_{\ell+\frac{1}{2}}^x,$$

where $\beta = \frac{\pi}{4}$.
where \( \beta = \arctan(h_x/h_z) \). Thus, the effective Hamiltonian \( \hat{H}_{\text{eff}} \) can be transformed as

\[
\hat{H}_{\text{eff}} = \hat{H}_1 + \hat{H}_2 + \hat{H}_3, \tag{S4}
\]

\[
\hat{H}_1 = -\sum_{\ell=1} g \cos \beta (\hat{s}_t^x \hat{\tau}_t^x \hat{s}_{\ell+1} + \text{H.c.}) + \sum_{\ell=1} \hat{h} \hat{s}_t^x, \tag{S5}
\]

\[
\hat{H}_2 = \sum_{\ell=1} \hat{s}_t^x (\lambda_s - g \sin \beta \hat{\tau}_t^x \hat{s}_{\ell+1} + \text{H.c.}), \tag{S6}
\]

\[
\hat{H}_3 = \frac{1}{2} \sum_{\ell=1} \left[ \hat{\tau}_t^x (\lambda_r - \hat{s}_t^z) \hat{\tau}_{\ell+\frac{3}{2}}^x + \cos^2 \beta \hat{\tau}_t^x (\lambda_r - \hat{s}_t^z) \hat{\tau}_{\ell+\frac{3}{2}}^x + \sin \beta \cos \beta \hat{\tau}_t^x (\lambda_r - \hat{s}_t^z) \hat{\tau}_{\ell+\frac{3}{2}}^x \right], \tag{S7}
\]

where \( \hat{h} = \sqrt{h_x^2 + h_z^2} \). Similarly, we can find that \( \hat{H}_1 \) is also a \( \mathbb{Z}_2 \) LGT with gauge generator

\[
\hat{G}_\ell = \hat{\tau}_t^x \hat{s}_t^z \hat{\tau}_{\ell+\frac{1}{2}}^x. \tag{S8}
\]

However, \( \hat{H}_2 \) and \( \hat{H}_3 \) violate such \( \mathbb{Z}_2 \) gauge invariance.

Now we discuss the ground state of \( \hat{H}_{\text{eff}} \). When \( h \ll g \), \( \hat{\tau}_t^x \) sector is almost polarized in the \( \hat{\tau}_t^x \) channel under the ground state. Thus, \( \langle \hat{\tau}_t^x \rangle \neq 0 \), where \( \langle \cdot \rangle \) represents taking expectation value towards the ground state. Then, using mean-field approximation,

\[
\hat{H}_2 \approx \sum_{\ell=1} \hat{s}_t^z (\lambda_s - \sin \beta \langle \hat{\tau}_t^x \rangle) \hat{s}_{\ell+1} + \text{H.c.}. \tag{S9}
\]

Under the specific fields \( h_x \) and \( h_z \), we can have \( \lambda_s = \sin \beta \langle \hat{\tau}_t^x \rangle = 0 \), i.e., \( \hat{H}_2 = 0 \). Therefore, following the argument in Ref. [S5], we know that \( \mathbb{Z}_2 \) gauge invariance can indeed possibly emerge in the ground state, which suppresses the spreading of \( s \)-spin in the low-energy regime.

To verify the emergent \( \mathbb{Z}_2 \) gauge invariance of \( \hat{H}_{\text{eff}} \) in the ground state, numerically, we define the \( \mathbb{Z}_2 \) charge

\[
\tilde{W}_{s_+}(i,j) = \prod_{i \leq k \leq j} \hat{s}_k^z, \tag{S10}
\]

and the flux

\[
\tilde{C}_{\tau_+}(i,j) := \hat{\tau}_t^x \hat{s}_t^x \hat{\tau}_{\ell+\frac{1}{2}}^x. \tag{S11}
\]

We know that \( \tilde{W}_{s_+}(i,j) = \pm \tilde{C}_{\tau_+}(i,j) \) must be satisfied rigorously for \( \mathbb{Z}_2 \) gauge invariant systems (e.g., \( \hat{H}_1 \)), which is Gauss law of 1D \( \mathbb{Z}_2 \) LGT. Now, we first use density matrix renormalized group (DMRG) method to calculate \( \mathbb{Z}_2 \) charge and flux at the ground state. Here, open boundary condition is used for the numerical simulation. Furthermore, we set the \( s \)-spins to be half-filling, i.e., \( \sum_{j=1}^L \hat{s}_j^z = 0 \). From Figs. S4(a-c), one can find, in our system,

\[
\langle \tilde{C}_{\tau_+}(i,j) \rangle = (-1)^f(i,j) \langle \tilde{W}_{s_+}(i,j) \rangle, \tag{S12}
\]

when \( h_x = 6 \) and \( h_z = -4.45 \), i.e., \( \beta \approx -0.638 \). Here, \( f(i,j) = \frac{\langle \hat{\tau}_t^x \rangle + 1}{2} \) is an integer. Therefore, the effective Hamiltonian \( \hat{H}_{\text{eff}} \) can indeed emerge \( \mathbb{Z}_2 \) gauge invariance at the ground state under proper transverse and longitude field, although the \( \mathbb{Z}_2 \) gauge invariance is absent in \( \hat{H}_{\text{eff}} \). In Figs. S4(d), we present the expectation values of \( \mathbb{Z}_2 \) flux and charge for all eigenstates of \( \hat{H}_{\text{eff}} \) at half-filling regime and periodic boundary condition. We can find that \( \mathbb{Z}_2 \) gauge invariance is almost absent in high excited state.

Now we use time evolving block decimation (TEBD) method [S8] to study the quench dynamics of \( \hat{H}_{\text{eff}} \). Here, we choose second-order Suzuki-Trotter decomposition. We also enlarge the maximum bond dimension and decrease time of single step till the final results converge. We set the total \( s \)-charge to \( \sum_{j=1}^L \hat{s}_j^z \hat{s}_j^- = 1 \). The initial state is chosen as \( |\psi_0\rangle = |s\rangle \otimes |\tau\rangle \), where \( |s\rangle \) and \( |\tau\rangle \) label the states of \( s \) and \( \tau \) sectors, respectively. We let \( |s\rangle = |... \uparrow \downarrow \uparrow \uparrow \uparrow \ldots \rangle \) and
\(|\tau\rangle = |...\Phi_0\Phi_0\Phi_0\Phi_0\Phi_0\rangle\), \(|\Phi_0\rangle = \cos \frac{\theta}{2} |\uparrow\rangle + \sin \frac{\theta}{2} |\downarrow\rangle\). Thus, this initial state is closely related to the experiment in main text. In addition, we know that, when \(\theta = -\pi/2 - \beta\), the system is much close to the ground state.

Firstly, we calculate the time evolution of the extended imbalance of \(s\)-spins defined in Eq. (5) of main text. According to Fig. S5(a), we can find \(s\)-sector can exhibit a localization in a short time regime when \(h_x = 6\) and \(h_z = -4.45\), and localization strength depends on the initial state. Fig. S5(b) shows that the localization strength of \(s\)-sector approach the strongest when \(\theta \approx -\pi/2 - \beta\). Now we study the time evolution of \(Z_2\) gauge generator \(\hat{G}_\ell\). Similar to the main text, we define an ansatz of \(Z_2\) gauge generator as

\[
\hat{G}_\ell(\alpha) := \hat{T}_{\ell-\frac{1}{2}}(\alpha) \hat{s}_z^x \hat{T}_{\ell+\frac{1}{2}}(\alpha), \tag{S10}
\]

where \(\hat{T}_{\ell-\frac{1}{2}}(\alpha) = \cos(\alpha) \hat{\tau}_z^{\ell-\frac{1}{2}} + \sin(\alpha) \hat{\tau}_x^{\ell-\frac{1}{2}}\), so \(\hat{T}_{\ell+\frac{1}{2}}(\beta) = \hat{\tau}_x^{\ell+\frac{1}{2}}\) and \(\hat{G}_\ell(\beta) = \hat{G}_\ell\) is the emergent \(Z_2\) gauge generator.

According to Figs. S5(c–d), we can find that the stead value of \(\hat{G}_\ell(\alpha)\) indeed approach the minimum when \(\alpha = \beta\). These quench dynamics is completely consistent with our experimental results. Here we note that there is nearly no oscillation during the dynamics of \(\hat{G}_\ell(\beta)\) for \(\hat{H}_{\text{eff}}\) [see Fig. S5(d)], which is distinct to the corresponding experimental results [see Fig. 4(b) in main text]. Thus, we conjecture that this oscillation of the dynamics for original Hamiltonian results from the high-order term.

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FIG. S1. Diagram of the experimental setup. The left bottom is the electronic control instrument system.
FIG. S2. 10 qubit Readout. The red dots means the measurement state is in |1⟩ and the blue dots means the measurement state is in |0⟩.

FIG. S3. Z pulse calibration.
FIG. S4. Expectation values of $Z_2$ charge and the flux at ground state with $h_x = -6$ and (a) $h_z = 0$, (b) $h_z = -4.45$, (c) $h_z = -6$. The results are calculated via density matrix renormalized group (DMRG) method [S6, S7] with $L = 80$. (d) Expectation values of $Z_2$ charge and the flux for all eigenstates with $h_z = -4.45$. The result is calculated via exact diagonalization method with $L = 6$. The inset shows the $Z_2$ charge and the flux for low-energy eigenstates.
FIG. S5. Quench dynamics of $\hat{H}_{\text{eff}}$ with $h_x = -6$, $h_z = -4.45$ and $L = 21$. (a) Time evolution of the extended imbalance of $s$-spins for different initial states. (b) The relation between stead values of the extended imbalance $I_\infty$ and initial states. Here, $I_\infty$ is the average of $I(t)$ for $t \in [4, 8]$. (c) The stead values of $\hat{G}_{11}(\alpha)$, and $\langle \hat{G}_{11}(\alpha) \rangle_\infty$ is also the average of $\langle \hat{G}_{11}(\alpha) \rangle(t)$ for $t \in [4, 8]$. (d) Time evolution of $Z_2$ gauge generator $\hat{G}_\ell(\alpha = \beta = -0.638)$, and there is nearly no oscillation. Here, the results are calculated via TEBD method.