ABBSTRACT

The powertrain of battery electric vehicles can be optimized to maximize the travel distance for a given amount of stored energy in the traction battery. We present a systematic optimization approach to derive optimal powertrain designs for battery electric vehicles that is based on geometric programming. The benefits of this approach are a rapid solution time, a global optimal solution, and the ability to derive local sensitivities in the optimal design point with no extra cost. The presented approach is suitable for the evaluation of a complete drive cycle, as this is commonly done in powertrain system design, or for usage in a stochastic approach, where multiple scenarios are sampled from a given probability density function. The latter is useful, to account for the uncertainty in the driving loads and it can generate solutions that are optimal in an average sense for a high variety of vehicles and drive conditions. For the powertrain model we use a gearbox with one selectable gear ratio and an electric motor model that is based on an efficiency map representation. We can easily model a continuously variable transmission to show beneficial energy savings based on this additional degree of freedom. Furthermore, we show approaches to add further component designs, as for instance a multi-gear gearbox in the optimization model.

1 Introduction

To improve the air quality, especially in cities, and to enable a more sustainable transportation, battery electric vehicles (BEV) are a promising technology. With rapidly decreasing investment costs for the traction battery, as shown by [1], the profitability of these vehicles increase and therefore the number of new registrations. As a result the market share of electric vehicles increased over the last years, as shown by [2]. This increase of new registrations of BEV is also driven by environmental protection regulations in major markets, like for example China as mentioned in [3]. The major drawback of BEV in comparison to vehicles with an internal combustion engine, as stated by [4], is a shorter travel distance. The lower ratio of stored energy to weight within the car and the higher recharging times of BEV in comparison to fuel powered vehicles lead to requirements to improve the overall efficiency of the powertrain as much as possible.

In the following, we present a brief overview about geometric programming and convex powertrain design. Afterwards, we present the developed geometric programming model for a powertrain design and present exemplified results. Then, we show an outlook on the usage of geometric programming in powertrain design and close with a conclusion and summary.
2 Geometric Programming

The presented optimization model relies on the geometric programming approach, which was first developed by Duffin, Peterson, and Zener in 1967, cf. [5]. It is a modeling approach in which a log-transformation is used to convert a non-convex nonlinear program in a convex program. This transformation is possible, as shown in [6], if the objective and constraints either consist of monomials

\[ m(x) = \alpha_0 x_1^{\alpha_1} x_2^{\alpha_2} \cdots x_n^{\alpha_n}, \]  

or posynomials

\[ p(x) = \sum_k \alpha_0 k x_1^{\alpha_1 k} x_2^{\alpha_2 k} \cdots x_n^{\alpha_n k}. \]  

In this example the \( n \) variables are given by \( x \), while \( \alpha \) represents specific parameters for each constraint and \( k \) the total number of terms of the sum. The underlying optimization program can be transformed in a convex program by using a log-transformation if it has a monomial as an objective and monomial or posynomial constraints of the following form, [6]:

\[
\begin{align*}
\min_x & \quad m(x) \\
\text{s.t.} & \quad m(x) = 1 \\
& \quad p(x) \leq 1 \\
& \quad x > 0.
\end{align*}
\]

The resulting log-transformed problem is convex. In general, convex programs can be solved very efficiently by state-of-the-art solvers. We refer to [6] as a general introduction on geometric programming and to [7] for an overview about convex optimization. The geometric programming approach is currently used in aircraft design, as for example shown by [8, 9].

Besides the geometric programming approach shown below, further research was done to derive convex optimization models for the powertrain component sizing and control. For instance [10] present a detailed modeling approach that relies on a convex optimization program. Further approaches for convex modeling and optimization are given in [11, 12, 13]. To the best of the author’s knowledge, the powertrain design and control problem for battery electric vehicles with an explicit gear ratio design has not yet been modeled as a geometric program. Hence, we present in the following section an easily extendable modeling approach, which is also compliant with the underlying physical principles.

3 Preprocessing

The powertrain of battery electric vehicles primarily consists of the battery, a power electronics, an electric motor and a gearbox with one or more gears. For simplicity, we first present only the integration of a gearbox with one variable single transmission ratio and an example electric motor. Further components are currently neglected, but can be added later, if necessary. In an early design stage in powertrain development commonly a backwards dynamic longitudinal vehicle model is used to derive the torque and speed requirements at the wheel, [14]. In the following we rely on a given driving load representation \( \Lambda \):

\[
\Lambda = \begin{bmatrix}
\mathbf{v}^T \\
\mathbf{s}^T \\
\pi^T
\end{bmatrix}, \Lambda \in \mathbb{R}^{3 \times l}.
\]  

Here, \( \mathbf{v} \) represents the vector of velocity requirements and \( \mathbf{s} \) represents the slope requirements. This representation is equivalent to a given drive cycle representation, cf. [15], if the additional weights \( \pi_t, t \in \{1, \ldots, l\} \) are neglected. In the following we represent the set of used discrete time steps with \( \mathcal{T} \). We can also interpret the given data representation as a finite event set approximation of a stochastic program with recourse. In this case an uncertain variable is given by the representation of a drive conditions. This interpretation enables us to evaluate multiple vehicle representations and drive conditions at once by using for instance specific weights \( \pi_t \) and not a drive cycle but a set of representative scenarios that approximate the underlying probability density function of the uncertain drive conditions. This set of scenarios is usually smaller than the complete cycle. This approach is useful in many situations, as most time steps in the drive cycle do not affect the final optimal solution. The usage of a comparable stochastic approach is for instance shown by [16].

We use the following longitudinal vehicle model for modeling the torque and speed requirements in the cycle [17, p. 13ff.]:

\[
T_t = \left( \lambda \dot{m} \dot{v} + mg \left( \sin (\alpha) + \lambda_t \right) + \frac{1}{2} \rho c_w A v_t^2 \right) r \quad \forall t \in \mathcal{T}.
\]
Table 1: Used vehicle parameters for dimensioning the powertrain.

| PARAMETER | VALUE | UNIT | DESCRIPTION |
|-----------|-------|------|-------------|
| $M_0$     | 1100  | kg   | vehicle mass |
| $M_a$     | 75    | kg   | additional component mass in powertrain |
| $M_d$     | 75    | kg   | mass of the considered driver |
| $M_b$     | 550   | kg   | mass of the used battery |
| $g$       | 9.81  | m s$^{-2}$ | specific gravitational constant |
| $\alpha$  | 0     | –    | terrain slope angle |
| $\rho$    | 1.2041| kg m$^{-3}$ | air density at 20$^\circ$C and sea level |
| $c_w$     | 0.3   | –    | drag coefficient |
| $A$       | 2.2   | m$^2$ | vehicle reference area |
| $\lambda_r$ | 0.01 | –    | rolling resistance coefficient |
| $r$       | 0.3   | m    | wheel radius |
| $\lambda_i$ | 1.0  | –    | inertia factor |

Figure 1: Input based on WLTC, [18]. (a) velocity requirements. (b) acceleration requirements derived by using a central difference scheme.

We refer to Tab. 1 for the meaning and values of the used parameters. For the longitudinal vehicle model, we only use the speed $v$ and acceleration $\dot{v}$ in each time step in the given drive cycle as inputs. We derive acceleration values for each given discrete time step $t \in T$ in the cycle based on a central difference scheme. For simplicity, we assume $s_t = 0 \forall t \in T$. Nevertheless, this assumption can be easily dropped, as the modeling approach is independent of the slope values. It only differs in the preprocessing. As a reference cycle, we use the worldwide harmonized light vehicles test cycle (WLTC), [18], as shown in Fig. 1. Furthermore, we only consider positive acceleration ($\dot{v} > 0$ m s$^{-2}$) and speed ($v > 0$ m s$^{-1}$) values in the following. We also present a further model extension to integrate recuperation in section 6.

As the power demand in the cycle depends on the total mass $m$ of the vehicle, we can not estimate the power directly before optimization, if the used mass of relevant components are modeled as variables within the model. For a first approach, we use in this example a fixed value which is estimated as follows: $m = M_0 + M_d + M_a + M_b$. Here, $M_0$ is
the basic vehicle mass, $M_d$ the mass for the considered passengers, $M_a$ the additional mass of powertrain components, and $M_b$ the battery mass. Furthermore, we show a possible extension to a variable mass in section 6.

Within BEV, permanent magnet synchronous motors (PMSM) are commonly used, as they have a weight and efficiency benefit compared to other motor technologies. Therefore, the underlying electric motor model used in this contribution represents a PMSM. We use multiple monomials to represent specific power losses of the motor. For instance [19, 20] showed that PMSM partial losses, like the friction loss in bearings, or stray losses can be approximated by using monomials. As a reference for comparison, we use a scaled version of an efficiency map shown by [21] for a motor developed for the usage in electric vehicles. For simplicity we currently do not model the power electronics and battery explicitly. Instead, we only use an aggregated efficiency map and assume that it represents the motor as well as the required power electronic components. We scale the derived efficiency map within the optimization according to the motor maximum torque $T^{\text{M}}$. This scaling is useful in the early design stage and for instance also shown in [19]. It assumes that the efficiency map of a derived electric motor is comparable with the efficiency map of a reference motor of the same product series. The benefit of the shown approximation method is that it is compliant with the GP approach, as it is modeled with the help of multiple monomial and posynomial constraints. An example result of the used motor efficiency approximation method is shown in Fig. 2 for the 100 kW reference motor.

Next to the given drive cycle this optimization method allows to integrate easily further constraints, as for instance high speed requirements or starting constraints based on the gradeability on a slope. They can also be interpreted as further scenarios / time steps that must be met but have a zero probability $\pi_t$. This ensures that they are not accounted for in the objective but have to be fulfilled by the final solution.

4 Used Model

The complete model is shown in Figure 3. The used parameters, sets and variables are given in Tab. 2. The model reads as follows: As an objective, (6a), we consider an upper bound of the average approximation of the total power losses in all time steps $T^+$, which is given in constraint (6b). The transmission ratio $i$ must lay between an upper bound $I$ and a given lower bound $I_0$ (6c). Constraints (6d) and (6e) model the transmission with the variable transmission ratio $i$ and the fixed gearbox efficiency $\eta_g$. We use the speed in min$^{-1}$ of the given motor. If desired, it is also possible to use the angular speed $\omega_M = 2\pi n_M/60$ instead. As the motor can be scaled in the torque direction, we model the variable motor power with (6f) and the upper hyperbola limit of the feasible motor domain with constraint (6g). All torque values $t_M$ and speed values $n_M$ must lay in the feasible motor domain. Next to the previously modeled hyperbola, we also model the upper torque and speed limits in (6h) and (6i). As the efficiency map of the underlying PMSM should be scaled according to the maximum torque, we use a scaling parameter $k^{\text{MP}}$ and introduce it in a monomial in constraint (6j). As a reference we use the motor power $P_{\text{ref}}$. Constraints (6k) – (6n) represent specific
power losses, which lead to the already shown efficiency map model in Fig. 2 (a). We used a nonlinear least-square method to estimate the reference values and exponents. We use the scaling factor $k^{\text{MP}}$ in each loss term to get an according scaling of all losses. The constant losses in constraint (6n) for instance can represent air gap losses and cooling power requirements, cf. [19]. These losses could be estimated with further monomials and posynomials as well. Nevertheless, we omit these further details, as they don’t affect the shown optimization approach and could be added later, if desired. The shaft power in each time step is given in (6o) and the total required power that must be supplied by the battery is given in a relaxed form in (6p). We additionally model an upper bound for $\bar{P}_{\text{ref, out}}$ in (6q).

This upper bound is not active in the final result, but required for solving the GP. The complete model represents the physical properties of the given system. But especially the posynomial constraints (6b, 6p) have to be relaxed to allow the usage in the GP. Nevertheless, in the final solution they meet the equality condition to derive a system design that has the lowest power losses. Therefore, these constraint relaxations do not affect the found solution.

## 5 Results

We use the weights $\pi_t = 1/\sum_{k=1}^{n}1$ \forall $t \in T^+$ for each step based on the set $T^+$ which was derived from the drive cycle. Here, we only consider positive speed and acceleration values, which results for the WLTC in approximately 760 time steps. We additionally use two further restrictions, which must be fulfilled by the optimized powertrain. First a start torque at speed $n^M = 0.2 \text{ min}^{-1}$ that represents a gradeability on a slope of 66 %. The speed value is set arbitrarily above zero to enable a GP solution. As the selected low speed value lays below the motor design speed, it does not affect the solution. Furthermore, we require a high-speed limit of $160 \text{ km h}^{-1}$ with an acceleration of $0.3 \text{ m s}^{-2}$. These two constraints are modeled with two additional time steps, that must be fulfilled in the final motor domain, but are not used in the objective. In total, this results in approx. 6100 variables and approx. 20800 constraints for the complete program. The final optimized gearbox has a gear ratio of 7.06 for this specific case. The selected motor has a maximum power $\bar{P}_{\text{M, out}}$ of 185 kW which results in a maximum torque of $\bar{R}^{M} \approx 424 \text{ N m}$. These values are comparable with a solution derived by a genetic algorithm with a higher fidelity model that uses the original motor efficiency map next to recuperation.

Besides the specific gear ratio $i$ in the single-speed gearbox, we can also consider a continuously variable transmission (CVT). In this case the gear ratio is not modeled as a single variable but instead as a vectorized variable for all time steps. This additional degree of freedom results in a more efficient system design and can be seen as a physical

| PARAMETER    | VALUE  | UNIT | DESCRIPTION |
|--------------|--------|------|-------------|
| $n_p$        | 0.98   | –    | transmission efficiency of gearbox |
| $A$          | 0.4161 | –    | motor hyperbola constant |
| $I$          | 1.0    | –    | minimum transmission ratio |
| $R$          | 18.0   | –    | maximum transmission ratio |
| $\pi$        | $1/\sum_{k=1}^{n}$ | 1     | probability of occurrence for all $|T^+|$ time steps in the given cycle |
| $\tau^W$     | based on cycle | N m  | wheel torque |
| $N^W$        | based on cycle | min$^{-1}$ | wheel rotational speed |
| $N^M$        | 10000  | min$^{-1}$ | motor maximum rotational speed |
| $P_{\text{ref}}$ | 100    | kW   | power of reference motor |
| $P_{\text{M, ref, 1}}$ | 787.35 | W    | power reference loss 1 |
| $P_{\text{M, ref, 2}}$ | 1566.67 | W    | power reference loss 2 |
| $P_{\text{M, ref, 3}}$ | 9904.85 | W    | power reference loss 3 |
| $\tau^{\text{M, ref}}$ | 1059.34 | W    | power reference constant loss |
| $T^{\text{M, up}}$ | 8000   | kW   | upper limit of required motor power |

| VARIABLE      | DOMAIN      | UNIT | DESCRIPTION |
|---------------|-------------|------|-------------|
| i             | $[L, T]$   | –    | transmission ratio |
| $r^M$         | $[0, R^M]$ | N m  | motor torque |
| $p^M$         | $[R^+, N^M]$ | N m  | motor maximum torque |
| $n^M$         | $[0, N^M]$ | min$^{-1}$ | motor speed |
| $\bar{P}_{\text{M, avg}}$ | $R^+$ | W | average used power in cycle |
| $\bar{P}_{\text{M, in}}$ | $R^+$ | W | motor power requirement drawn from battery |
| $\bar{P}_{\text{M, out}}$ | $R^+$ | W | motor maximum design power |
| $\bar{P}^M$   | $R^+$      | W    | motor design power |
| $k^M$         | $R^+$      | –    | motor power factor |

Table 2: GP model notation

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We use the weights $\pi_t = 1/\sum_{k=1}^{n}1$ \forall $t \in T^+$ for each step based on the set $T^+$ which was derived from the drive cycle. Here, we only consider positive speed and acceleration values, which results for the WLTC in approximately 760 time steps. We additionally use two further restrictions, which must be fulfilled by the optimized powertrain. First a start torque at speed $n^M = 0.2 \text{ min}^{-1}$ that represents a gradeability on a slope of 66 %. The speed value is set arbitrarily above zero to enable a GP solution. As the selected low speed value lays below the motor design speed, it does not affect the solution. Furthermore, we require a high-speed limit of $160 \text{ km h}^{-1}$ with an acceleration of $0.3 \text{ m s}^{-2}$. These two constraints are modeled with two additional time steps, that must be fulfilled in the final motor domain, but are not used in the objective. In total, this results in approx. 6100 variables and approx. 20800 constraints for the complete program. The final optimized gearbox has a gear ratio of 7.06 for this specific case. The selected motor has a maximum power $\bar{P}_{\text{M, out}}$ of 185 kW which results in a maximum torque of $\bar{R}^{M} \approx 424 \text{ N m}$. These values are comparable with a solution derived by a genetic algorithm with a higher fidelity model that uses the original motor efficiency map next to recuperation.

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\[ \begin{align*}
\min & \quad p_{M,\text{in,avg}}^t \\
\text{s.t.} & \quad \sum_{t \in T^+} \pi_t p_{t,\text{in}}^M \leq p_{M,\text{in,avg}}^t \quad (6a) \\
& \quad I \leq i \leq T \quad (6b) \\
& \quad t_{\text{M},i} \eta_0 = T_t^W \quad \forall t \in T \quad (6c) \\
& \quad n_{t}^M = i N_t^W \quad \forall t \in T \quad (6d) \\
& \quad p_{M,\text{out}}^t = \frac{N^M 2\pi}{60} A_t^M \quad (6e) \\
& \quad n_t^M \frac{2\pi}{60} t_t^M \leq p_{M,\text{out}}^t \quad \forall t \in T \quad (6f) \\
& \quad n_t^M \leq N^M \quad \forall t \in T \quad (6g) \\
& \quad t_t^M \leq t_t^T \quad \forall t \in T \quad (6h) \\
& \quad k_{M,p} = \frac{p_{M,\text{out}}^t}{P_{\text{ref}}} \quad (6i) \\
& \quad p_{t,\text{M,L,1}}^t = k_{M,p} P_{\text{L,ref,1}} \left( \frac{t_t^M}{T_t^M} \right) \left( \frac{n_t^M}{N_t^M} \right)^{3.93} \quad \forall t \in T \quad (6j) \\
& \quad p_{t,\text{M,L,2}}^t = k_{M,p} P_{\text{L,ref,2}} \left( \frac{t_t^M}{T_t^M} \right)^2 \quad \forall t \in T \quad (6k) \\
& \quad p_{t,\text{M,L,3}}^t = k_{M,p} P_{\text{L,ref,3}} \left( \frac{t_t^M}{T_t^M} \right)^2 \quad \forall t \in T \quad (6l) \\
& \quad p_{t,\text{M,L,\text{const}}} = k_{M,p} P_{\text{L,\text{ref,\text{const}}}} \quad \forall t \in T \quad (6m) \\
& \quad p_t^M = t_t^M \frac{2\pi}{60} n_t^M \quad \forall t \in T \quad (6n) \\
& \quad p_t^M \geq p_t^t + p_{t,\text{M,L,1}}^t + p_{t,\text{M,L,2}}^t + p_{t,\text{M,L,3}}^t + p_{t,\text{M,L,\text{const}}}^t \quad \forall t \in T \quad (6o) \\
& \quad p_t^M \leq \eta_t^M \quad \forall t \in T \quad (6p) \\
\end{align*} \]

Figure 3: GP model

lower bound for multi-speed gearboxes. We used an upper bound \( \bar{T} = 18 \) besides the lower bound \( L = 1.0 \). In this domain, the optimized system design uses a motor with 73 kW and a maximum torque \( \bar{t}_t^M \approx 167 \text{ N m} \). The CVT result also reduces the objective value compared to the single-speed transmission by approx. 8%. This value depends on the given upper limit \( \bar{T} \). Nevertheless, we can conclude that a CVT can improve the overall system performance. Furthermore, the usage of multiple gears in a BEV powertrain, which can be interpreted as a discrete approximation of the CVT, can result in potential energy savings, cf. [22]. These will be lower than the benefits of the shown CVT.

We showed that the developed GP model is suitable for the early design stage of powertrain design. With the given model it is possible to compute optimized powertrain design solutions rapidly in comparison to a non-convex modeling or commonly used heuristics. We developed the model and preprocessing in Python. Therefore, we used GPkit [23] for modeling and Mosek 9.2 as a solver. The computations were done on a Debian-based machine with an Intel i5-5200U and 12 GB RAM. On this hardware, we were able to compute the optimized system designs in less than three seconds each. This emphasizes the high potential for usage in rapid development in an early design stage.

6 Model Extensions

With the help of the shown GP model we were able to transform the non-convex original problem to a convex program, without the need of further polynomial or piecewise-affine model approximations of components or the preliminary selection of a gear ratios. The underlying motor representation is based on physical properties of the modeled PMSM. The shown model 3 can be extended in multiple ways. For a more detailed mass estimation besides the shown fixed
mass, we can model the total mass \( m \), based on variable partial masses of different components. For instance we can model the motor mass \( m^M \) with a specific mass \( ρ^M \) and the maximum motor power \( p^M_{\text{out}} \), as shown in constraint (7a). This results in a new variable mass \( m \) which can be estimated with fixed reference masses for the basic vehicle \( M_0 \), for additional powertrain components \( M_a \), for the passengers \( M_p \), and for the battery \( M_b \) (7b). Furthermore, the torque at the wheel now depends on the mass, which is linear in \( m \) and can be approximated in a posynomial by using (7c).

\[
\begin{align*}
m^M &= ρ^M p^M_{\text{out}} \\
m &\geq M_0 + M_a + M_p + M_b + m^M \\
i^W_t &\geq δ_i m + γ_t 
\end{align*}
\]  

Furthermore, besides the shown CVT, we are also able to derive results for multi-gear gearboxes if the control strategy is given in advance. A discrete gear selection is useful in BEV powertrain design, as it can result in lower power requirements and a higher overall efficiency, cf. [24]. We can replace the transmission ratio variable \( i \) in the given set of time steps \( T \) with a new transmission ratio \( i_g \) for the selected gear \( g \). This results in a GP that has additional variables for the newly introduced transmission ratios of the selected gears but is equivalent besides this extension to the already introduced problem. Therefore, the solution time should be comparable. If the control strategy is not known in advance, the previously extended multi-gear GP model can be solved multiple times with different gear–time step assignments to derive an optimized control strategy. For this, commonly used heuristics, as for instance simulated annealing or tabu search could be used to derive good primal solutions with a low number of GP-evaluations. A further advantage of these methods is the possible usage in a parallel computation framework where multiple GP-runs are evaluated in parallel. A further extensions would be the definition of a Mixed-Integer Geometric Program (MIGP), cf. [6], that can then be solved with a branch-and-bound scheme.

Additionally, we can easily extend the given model to account for recuperation. For this step, we have to model the fourth quadrant of the torque-speed domain with a second efficiency map model. This second model is comparable to the given model of the first quadrant, but differs slightly in the given parametrization. We can add the recuperation domain by using the absolute values of the given torque to map it to a new positive domain. In this domain, we use the newly defined efficiency map model and integrate it in the final model with a second weighted sum in constraint (6b). To account for the relation between the driving and recuperation, we add further positive weights, which weigh both sides against each other.

The shown GP-approach is also suitable for the already mentioned stochastic representation. In this, we can use multiple scenarios and weights to represent the given uncertain driving conditions. For instance, the \( k \)-means clustering algorithm is one possibility to derive suitable scenarios, as shown by [25]. To ensure robust final solutions of the optimized system that fulfill all desired drive conditions in the underlying representative drive cycle or probability density function, we have to add further loads that lay on the convex hull of each cluster or at least at the highest required power demands. They are added as constraints but do not affect the objective.

Within this contribution, we presented a white-box reference-based model for the motor efficiency. Next to this approach further more detailed component models can be cast as a geometric program. For instance a more detailed motor mass estimation besides the density based approach would be possible. Furthermore, each component model representation which can be represented as a quantity of monomial or posynomial constraints can be used. This is often possible by adding further variables as shown in the motor model. If it is not possible, we can also fit a posynomial approximation, as for instance shown by [26].

7 Summary and Conclusion

We presented a geometric programming model for the early design stage of battery electric vehicles. We showed that the optimal design of a given gearbox and electric motor can be represented as a convex program, without specific requirements on the variables and constraints. This easily extendable basic model can therefore be used in numerous possibilities. The shown efficiency map modeling is for instance also suitable for electric aircraft design. The presented model was developed based on publicly available software and the usage of an of-the-shelf optimizer. Therefore it is easily applicable in industry and academia. System model details next to the motor efficiency and gear transmission can be added easily to the shown model, which makes it very suitable for early design proof-of-concepts and the comparison of optimal powertrain designs with different system layouts or components. In the future, we will extend this approach to account for discrete gear selection with two gears and an according control strategy. Furthermore, we extend the given model with an explicit consideration of recuperation and validate the shown method in more detail with more drive cycles and other optimization methods.
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