Optimizing Empty Container Repositioning and Fleet Deployment via Configurable Semi-POMDPs

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Abstract—With the continuous growth of the global economy and markets, resource imbalance has risen to be one of the central issues in real logistic scenarios. In marine transportation, this trade imbalance leads to Empty Container Repositioning (ECR) problems. Once the freight has been delivered from an exporting country to an importing one, the laden will turn into empty containers that need to be repositioned to satisfy new goods requests in exporting countries. In such problems, the performance that any cooperative repositioning policy can achieve strictly depends on the routes that vessels will follow (i.e., fleet deployment). Historically, Operation Research (OR) approaches were proposed to jointly optimize the repositioning policy along with the fleet of vessels. However, the stochasticity of future supply and demand of containers, together with black-box and non-linear constraints that are present within the environment, make these approaches unsuitable for these scenarios.

In this paper, we introduce a novel framework, Configurable Semi-POMDPs, to model this type of problems. Furthermore, we provide a two-stage learning algorithm, “Configure & Conquer” (CC), that first configures the environment by finding an approximation of the optimal fleet deployment strategy, and then “conquers” it by learning an ECR policy in this tuned environmental setting. We validate our approach in large and real-world instances of the problem. Our experiments highlight that CC avoids the pitfalls of OR methods and that it is successful at optimizing both the ECR policy and the fleet of vessels, leading to superior performance in worldwide trade environments.

Index Terms—Configurable environments, empty container repositioning, fleet deployment, reinforcement learning.

I. INTRODUCTION

Nowadays marine transportation is crucial for the world’s economy: 80-90% of the global trade is carried by sea [1], and most of the world’s marine cargo is transported in containers [2], [3]. In 2019, global containerized trades accounted for more than 150 million TEUs (i.e., Twenty feet Equivalent unit, the standard unit measure for cargo transportation), with a yearly average growth over the last 20 years of over 5%. When using containers to convey goods, problems arise due to the joint combination of the inner nature of container flow with the imbalance of global trade between different regions. For instance, once the freight has been delivered from an exporting country to an importing one, the laden will turn into empty containers that need to be repositioned to satisfy new goods requests in exporting countries. This problem takes the name of Empty Container Repositioning (ECR) [4]. In ECR, when a vessel arrives at a port to discharge laden, the port has two options: discharge a certain amount of empty containers that are present on the vessel, or load the vessel with empty containers from its stock. The goal is to find cooperative repositioning policies that minimize the shortage of demand for empty containers in a given horizon. ECR can be a very costly activity in complex logistic networks and, even if it does not directly generate income, it can account for about 20% of the total costs for shipping companies [4]. Thus, building efficient ECR strategies is a crucial point for real-world logistic scenarios.

As highlighted in previous studies [5], the demand for empty containers that can be satisfied in a given horizon by any ECR policy strictly depends on the routes that the vessels of the given shipping company will follow. From an intuitive point of view, this is clear if we consider a scenario with two routes that have no ports in common. Suppose that most of the demand for empty containers concerns ports that are present on the first route. If most of the vessels that the shipping company owns follow the second route, any ECR policy will have poor performance since the cooperation ability of the network to exchange containers is limited by the poor assignments of vessels to routes. In the marine transportation literature, the problem of assigning vessels to routes to maximize some given target function takes the name of Fleet Deployment (FD) [5]. More specifically, in FD, the set of routes is predetermined by the shipping company, and the task is to select the starting port on a route for each vessel that the shipping company owns.

In this paper, we study the adoption of FD techniques to improve ECR policies. Historically, Operation Research (OR) approaches were proposed to jointly optimize the repositioning policy along with the fleet of vessels [6]. However, as noted in recent studies [3], even in the easier setting in which the assignments of vessels to routes are predetermined and given to the learners, the stochasticity within the environment together with black-boxed and non-linear constraints that are present in the real world, makes OR methods unsuitable for such complex scenarios. To overcome these limitations, researchers have recently investigated, with promising results, the adoption of Multi-agent Reinforcement Learning (MARL) techniques [3], [7]. They model the problem as a Semi-Partially Observable Markov Decision Process (Semi-POMDP) and propose ad-hoc neural architectures to learn cooperative ECR policies.¹

¹Notice that the “Semi” component arises from the fact that the ECR problem is intrinsically event-driven: a repositioning action needs to be taken only when a vessel arrives at a given port.
ECR+FD problem. For such purpose, we start by noticing that, from the agents’ perspective, the assignment of the fleet of vessels to routes can be seen as features of the environment that can be optimized to reach higher performances. In this sense, for single-agent problems, Configurable Markov Decision Processes (Conf-MDPs) [8], [9], [10] have recently been introduced to extend the Markov Decision Process (MDP) [11] framework to account for environmental configurations. In Conf-MDPs, an agent and a configurator are responsible for finding the optimal policy-configuration pair. This is clearly related to our application scenario: our agents are in charge of deciding which container repositioning policy to play, whereas the configurator is entitled to select the fleet of vessels. While the early works [8], [9], [10] focused on the case in which the agent and the configurator share the same objective, in [12], the setting has been extended to the case in which the configurator and the agent have different (and, possibly, adversarial) goals. Although these approaches have strong theoretical guarantees, how to successfully scale them to more complex domains remains an open question. Indeed, in a multi-agent cooperative setting, the dimension of the problem explodes with the number of agents. Moreover, the intrinsic non-stationarity present in multi-agent systems significantly complicates the learning process [13]. We also note that all the previous methods assume the state to be fully observable; in ECR, however, the agents operate under partial observability, which introduces an additional challenge.

The contributions of our work are summarized as follows:

- We introduce the Configurable Semi-POMDPs (Conf-SemiPOMDPs), a novel framework whose goal is extending Conf-MDPs to the more complex multi-agent, partially-observable dynamics of the ECR+FD problem (Section IV). In particular, we focus on the cases in which the configuration of the environment (i.e., assignment of vessels to routes) is decided by a central entity (i.e., configurator); that, in our case, is the shipping company. To the best of our knowledge, this is the first time in which the ECR+FD problem is fully modeled through a learning formalism, as well as the first time in which Conf-MDPs are extended in multi-agent settings.

- To solve Conf-Semi-POMDP, we propose a general two-step solution algorithm called “Configure & Conquer” (CC) (Section V). The goal of CC is to build solutions that successfully scale the joint optimization process (i.e., policy and configurations) to large multi-agent systems. CC first optimizes the configurator to output an approximation of the optimal configuration (i.e., fleet deployment) and then “conquers” it by learning a policy in this tuned environmental setting (i.e., the ECR cooperative policy). The main intuition that CC exploits in its configure step is that to compare two distinct configurations one can even leverage suboptimal policies. Furthermore, as we shall show, the configurator adopts an ad-hoc neural architecture that exploits similarities between configurations.

- We validate our approach in large and real-world instances of the ECR+FD problem (Section VI). Our experiments show that CC is successful at optimizing both the ECR policy and the fleet of vessels, leading to superior performance w.r.t. competitive baselines in world trade environments. We stress the significance of the problem: finding efficient ECR policies together with the fleet configurations can generate significant value for shipping companies.

II. RELATED WORKS

Conf-MDPs have been introduced in [8] for finite spaces and extended in [10] for more complex continuous environments. In these seminal works, the agent is fully responsible for the configuration activity of the environment, which, in turn, results in an auxiliary task to optimize performance. As highlighted in [8], this leads to a clear distinction between Conf-MDPs and multi-task learning [14]. Indeed, in Conf-MDPs, the agent is not interested in learning and gathering experience samples in sub-optimal configurations; its interest is solely toward the optimal policy in the optimal environmental configuration. The configuration activity within the environment, as shown in more recent works [9], [12], can also be carried out by an external entity (i.e., configurator) whose goals can even be adversary w.r.t. the ones of the agent [12]. None of the previous methods, however, have been designed to handle the more complex multi-agent multi-cooperative setting, in which numerous additional challenges are present (e.g., partial observability, highly dimensional states, intrinsic non-stationarity). Environment design literature [15], [16] is also related to configurable environments. However, substantial differences are present since these approaches assume that the configurator (interested party) has (partial) access to the agent’s best response to a given environment. In addition, the agent’s policies, given an environment, are fixed, and, consequently, the optimization process is not joint.

The literature on artificial intelligence techniques applied to maritime transportation problems is broad, and several different peculiar aspects have been studied (e.g., [5], [17], [18], [19]). In our work, we focus on the joint ECR+FD setting. ECR problems [4] were historically solved using OR methods [2]. However, the environment stochasticity, together with the non-linear and black-boxed constraints that are present in the problem, have led researchers to explore MARL solutions [3], [7], [20]. In these works, the authors propose different formulations of the problem and investigate deep-reinforcement learning algorithms and architectures to find efficient cooperative strategies for fixed FD strategies. However, since the performance that a method can achieve in terms of satisfied demand in a given horizon strictly depends on the routes that the vessels will follow, network design [21], [22], [23] and fleet deployment [24], [25] techniques have been proposed to jointly optimize the policy along with the fleet of sailing boats [5]. The difference between network design and fleet deployment is that, in network design methods, routes are generated together with assignments of vessels to routes; in FD, instead, the set of routes is fixed and pre-determined. The joint optimization problem has been investigated from an OR point of view in [6], [24], [25], and [26]. These works provide different mixed-integer linear programming formulations based on several sets of assumptions and modeling choices; we refer the reader to [5] and [26] for complete surveys on the topic. However, we remark that in this more challenging setting, the aforementioned pitfalls of OR methods are even amplified by the complexity of the problem. Furthermore, for large networks of vessels and ports and/or for long planning horizons, solving the joint problem with a mathematical programming formulation leads to high computational requirements. For these reasons, hybrid approaches that adopt heuristics and Genetic Algorithms (GA) [27], have also been considered [21], [28], [29]. Among this line of work, the one that is most closely related to ours is [21]. In [21] the authors study the network design setting and propose a method to generate paths for each of the vessels. Each network configuration is evaluated using the value of the objective function of the mixed-integer formulation of the ECR problem in that specific configuration. Then, they use GAs to optimize for the configuration with the best objective function. This, however, inherits some of the pitfalls of the OR approach. As our experiments will show, the plan can diverge from reality, leading to suboptimal solutions.

To conclude, we remark that comparing ourselves with these existing approaches, we cast the ECR+FD problem under a, to the best of our knowledge, novel perspective, that grounds itself at the
core of a recent strand of modern reinforcement learning literature (i.e., Conf-MDPs). Furthermore, we propose a general, sample-based solution framework that can successfully scale to high-dimensional and real-world inspired experiments. As far as we know, CC represents the first learning-based method to solve the ECR+FD problem.

III. PRELIMINARIES

A. Configurable MDPs

A Configurable Markov Decision Process (Conf-MDP) \cite{8} is defined as a tuple \((\mathcal{S}, \mathcal{A}, r, \gamma, \mu, \mathcal{P}, \mathcal{O})\), where \(\mathcal{S}\) is the set of states, \(\mathcal{A}\) is the set of actions, \(r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}\) is the reward function specifying the reward \(r(s, a)\) for taking action \(a\) in state \(s\), \(\gamma \in (0, 1)\) is the discount factor, \(\mu \in \Delta(\mathcal{S})\) is the distribution of the initial state, \(\mathcal{P}\) and \(\mathcal{O}\) are the model and policy spaces respectively. In particular, every \(p \in \mathcal{P}\) is a transition function \(p : \mathcal{S} \times \mathcal{A} \rightarrow \Delta(\mathcal{S})\) that specifies a probability distribution \(p(\cdot|s, a)\) over next state upon taking action \(a\) in state \(s\), and every \(\pi \in \mathcal{O}\) is a policy \(\pi : \mathcal{S} \rightarrow \Delta(\mathcal{A})\) specifying a probability distribution \(\pi(\cdot|s)\) over actions for every states \(s\). In Conf-MDPs, the goal is to find the optimal model-policy pair \((p^*, \pi^*) \in \mathcal{P} \times \mathcal{O}\) that maximizes the expected return: 

\[
J^p(\pi) := \mathbb{E} \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \right] \quad \text{s.t. } \pi, p, s_0 \sim \mu, 
\]

where the expectation is taken w.r.t. the randomness of \(\pi\), \(p\), and \(\mu\). This joint optimization is solved by two cooperating entities: the agent, responsible for improving the policy \(\pi\), and the configurator, whose goal is to learn a configuration \(p\).

B. Semi-POMDPs

A Semi-Partially Observable Markov Decision Process (Semi-POMDP) \cite{7} is defined as a tuple \((\mathcal{D}, \mathcal{S}, \mathcal{A}, p, r, \mathcal{O}, o, \gamma, \mu)\), where \(\mathcal{S}\), \(\gamma\), and \(\mu\) have the same meaning as before. \(\mathcal{D}\) is the set of agents, \(\mathcal{A} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_d\) is the set of joint actions the agents can perform, \(r : \mathcal{S} \times \mathcal{A} \rightarrow \mathbb{R}\) is the reward function extended to joint actions, \(\mathcal{O}\) is the set of joint observations that are perceived by the agents, \(o : \mathcal{O} \rightarrow \Delta(\mathcal{O})\) is the observation function that, for every joint state \(s\) and joint action \(a\) provides a probability distribution \(o(\cdot|s, a)\) over joint observations. The transition function \(p : \mathcal{S} \times \mathcal{A} \rightarrow \Delta(\mathcal{S} \times \mathcal{O})\) provides a probability distribution \(p(\cdot|s, a)\) over next state and the time interval \(k\) associated to the current state \(s\) to the next state \(s'\). A joint policy \(\pi\) maps its observations \(\tau = (o_0, a_0, t_0, o_1, a_1, t_1, \ldots)\) to a distribution over joint actions \(\pi(\cdot|\tau)\). The goal consists in finding an optimal policy \(\pi^*\) that maximizes the expected return: 

\[
J^{p(\cdot)}(\pi) := \mathbb{E} \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \right] \quad \text{s.t. } p, s_0 \sim \mu, \quad t_0 = 0 \quad \text{and} \quad t_t = t_{t-1} + k, \quad \text{for } t \geq 1. 
\]

C. Empty Container Repositioning

As highlighted in \cite{3}, the ECR problem can be modeled as a graph \(\mathcal{G} = (H, V, E)\), where \(H, V, E\) are the set of harbor, vessels and routes respectively. More specifically, each harbor \(h \in H\) has a stock of empty containers of maximum capacity \(C_h\). We denote with \(C_h^t\) the number of containers available at day \(t\). Each route \(e \in E\) is a directed cycle of consecutive harbors in \(H\), namely \(e_k := (h_{1k}, \ldots, h_{\kappa k})\), where \(h_{1k} = h_{\kappa k}\). Routes can intersect with each other. Each vessel \(v \in V\) is associated with a maximum container capacity \(C_v\) and a route \(e \in E\). We denote with \(C_v^t\) the amount of empty space at day \(t\) on vessel \(v\). Finally, we denote with \(u_{hv}\) the speed function of vessel \(v\). Given source and destination harbors \(h_i, h_j\), \(u_{hv}\) provides a probability distribution over the number of days required by \(v\) to travel \(h_i\) to \(h_j\). Order of goods between ports are described by a stochastic function \(q\). Given two ports \(h_i, h_j \in H\) and a day \(t\), \(q\) provides a probability distribution on the number of goods that are requested to be shipped from \(h_i\) to \(h_j\) at day \(t\).

More specifically, \(h_i\) can satisfy this demand using stock of empty containers at the previous day (i.e., \(C_h^{t-1}\)). Whenever this amount is not enough, a shortage of containers will happen. We denote with \(L_h^t\) the total shortage on harbor \(h\) at day \(t\). When vessels arrive at harbors, the following happens:

- laden containers for that destination will be discharged to the port; after some days these containers will turn empty containers and will accumulate in the port’s stock;
- empty containers can be loaded/discharged on/from the vessel.

These are the actions \(a_t\) that the ECR policy is responsible for optimizing.

As mentioned in \cite{3}, we remark that the behavior with which containers (both full and empty) are loaded/discharged to/from vessels is complex to be modeled. Indeed, this mechanism is subject to black-boxed and non-linear country regulations.

The goal is to find a policy that minimizes the total shortage, namely \(\sum_{h \in H} L_h^t\). A full mathematical model is available in Appendix of \cite{3}. As one can verify, the ECR problem can be formalized as a Semi-POMDP, where each agent is mapped to a port that takes repositioning actions (i.e., \(a_t\)) whenever a vessel arrives. The (unobserved) state of the network contains information such as the position of each vessel and its future path, the remaining capacity, the laden, the laden’s destination, the number of empty containers available in each port, the daily demand for goods, and so on. The observation that an agent receives when requested to take an action is composed of its own information (e.g. port capacity, historical information on the number of orders), and that of related ports and vessels (e.g., the vessel that triggered the action, ports in the same route of the vessel). Vessels’ arrivals at ports are governed by the stochastic duration function \(u_v\) that induces a distribution on the next time interval \(k\) in which repositioning actions are requested. Between different time intervals, the state will evolve both according to the taken action \(a_t\) (which impacts the empty containers available at ports and vessels) and the order function \(q\) that generates daily demand.

To conclude, we remark that the function \(u_v\) has crucial consequences on the transition between states \(p\) and that changing the assignment of vessels to routes (i.e., Fleet Deployment) induces a completely different \(u_v\). This, in turn, leads to a different transition model \(p\), and, potentially, to a different optimal policy \(\pi^*\), that might improve the overall performance of the system, i.e., the minimal total shortage of goods. For this reason, in the next section, we extend the formalism of Semi-POMDP to configurable environments, so as to express the joint optimization problem of \(p\) and \(\pi\).

IV. THE CONFIGURABLE SEMI-POMDP FRAMEWORK

As we have seen, the Conf-MDP framework \cite{8} models scenarios in which a configurator and a single agent cooperate to improve overall performance. In this section, we generalize the formulation to account for the peculiarities of the ECR+FD problem, i.e., the presence of multiple agents, the partial observability, and the semi-Markov property.

Definition 1: A Configurable Semi-POMDP (Conf-Semi-POMDP) is a tuple \((\mathcal{D}, \mathcal{S}, \mathcal{A}, r, \mathcal{O}, o, \gamma, \mu, \mathcal{P}, \mathcal{O})\), where \((\mathcal{D}, \mathcal{S}, \mathcal{A}, r, \mathcal{O}, o, \gamma, \mu)\) is a Semi-POMDP without transition function, and \(\mathcal{P}\) and \(\mathcal{O}\) are the model and policy spaces.

More precisely, \(\mathcal{P} = \prod_{l=1}^{d} \prod_{j=1}^{d} \prod_{d} \prod_{j=1}^{d}\) is the set of history-dependent policies that the agents have access to (i.e., the set of ECR repositioning policies). Thus, we can look at the novel Conf-Semi-POMDP framework as either (i) an extension of the Conf-MDP setting to semi-Markov, multi-agent, partially-observable environments or (ii) an extension of the Semi-POMDP to configurable environments in which we have no transition model \(p\), that can indeed be altered as

\footnote{We denote with \(\Delta(X)\) the set of probability distributions over a set \(X\).}

\footnote{Further details can be found in e.g., \cite{3} and \cite{7}.}
an effect of the environment configuration activity. We focus on the case where $\mathcal{P}$ is a parametric space of transition probability functions. This assumption, which is usual in Configurable MDPs [8], nicely fits the ECR+FD domain, in which each $p \in \mathcal{P}$ encodes assignments of vessels to routes. More specifically, each configuration $p \in \mathcal{P}$ corresponds to $\{(v_i, e_i, h_i)\}_{i=1}^{V}$, where each element $(v, e, h)$ encodes the fact that vessel $v$ follows route $e$ and starting from port $h$. We also enforce the constraint that $h$ must belong to $e$. Notice that different assignments of vessels to routes induce a different arrival function $u_e$, that, as discussed in the previous section, has a significant impact on the transition model between states.

The performance of a model-policy pair $(p, \pi) \in \mathcal{P} \times \Pi$ is defined via the expected return, as for Semi-POMDPs:

$$J^p,\pi := \mathbb{E}\left[\sum_{t=0}^{+\infty} \gamma^t r(s_t, a_t)|s_0 \sim \mu, \pi, p\right],$$

where $t_0 = 0$ and $t_{i+1} = t_i + k_i$ for $i \geq 1$. Thus, the goal, as for Conf-MDPs, consists of finding the optimal model-policy pair $(p^*, \pi^*) \in \mathcal{P} \times \Pi$ such that $J^{p^*,\pi}$ is maximized. We denote with $\pi^*_p$ an optimal policy for a generic configuration $p \in \mathcal{P}$. For ease of notation, whenever it is clear from the context, we drop the specification of the environment in which a policy is run. For instance, $J^{p^*_p}$ measures the performance of policy $(p, \pi^*_p)$. In our work, we consider the case in which a central entity (i.e., the shipping company) is responsible for optimizing/taking decisions on the adopted configuration $p$.

V. CONFIGURE & CONQUER

We now introduce Configure & Conquer, our method to solve Conf-Semi-POMDPs. To appreciate its generality, we first present CC to solve a generic Conf-Semi-POMDP, and then discuss how it works in the ECR+FD domain.

Imagine having an oracle that, given a model $p \in \mathcal{P}$, provides the performance index $J^{p^*_p}$ of the optimal policy $\pi^*_p$ for that specific environment $p$. In this case, the original joint optimization problem described in Section IV reduces to:

$$p^* = \arg\max_{p \in \mathcal{P}} J^{p^*_p}.$$  \hspace{1cm} (2)

In practice, however, we do not have access to such an oracle. Nevertheless, given a configuration $p$, it is possible to train an algorithm $\mathcal{A}_c$ of our choice to learn an approximation of the optimal policy $\pi^*_p$ and, consequently, $J^{p^*_p}$. In particular, there might exist sample efficient (yet suboptimal) algorithms that can be used to obtain such approximations. In that case, we can leverage these methods to optimize the empirical version of the objective function $\tilde{J}^{p^*_p}$, where the expectation in $J^{p^*_p}$ is estimated with trajectories collected within model $p$ using $\tilde{\pi}^*_p$. We can notice that, with this new formulation, the contribution of the configurator to the optimal solution is completely decoupled from the problem of finding the optimal agents’ policy. Indeed, when the approximation of $p^*$ is found (i.e., configure step), CC optimizes the agents’ policy in the tuned environment with a more complex and expensive algorithm $\mathcal{A}_a$ (i.e., conquer step), which aims at obtaining better approximations of $\pi^*_p$. For this reason, CC is a two-stage optimization algorithm. The general pseudo-code is reported in Algorithm 1. With respect to the ECR+FD setting, Algorithm 1 evaluates a given assignment $p$ of vessels to routes exploiting a cheaper algorithm $\mathcal{A}_c$ to train a cooperative ECR policy in $p$. Once this is done, $\mathcal{A}_a$ is used to train the final ECR policy that will be deployed in the approximation of optimal fleet $\tilde{p}_a$. We now discuss how to choose $\mathcal{A}_a$ and $\mathcal{A}_c$.

A. Choosing $\mathcal{A}_c$ and $\mathcal{A}_a$

The choice of algorithms for computing approximations of the optimal policy in some configuration $p$ depends on the specific problem at hand. For what concerns $\mathcal{A}_c$, there is a trade-off between computational/sample efficiency and performance. Indeed, since $\mathcal{A}_c$ is used to train the configurator, the ideal method should be fast to compute and provide good approximations of $\pi^*_p$. The main issue is that the faster the method, the more configurations we can evaluate in a reasonable amount of time so to find $\tilde{p}^*$. However, this usually comes at the cost of precision, which might impact the optimization landscape of $\tilde{J}^{\tilde{p}^*_p}$. For these reasons, depending on the problem, one might use heuristics, mathematical programming, experts, reinforcement learning agents trained on limited data and so on. On the other hand, the choice of $\mathcal{A}_a$ is more critical for the final performance of the system since it is responsible for computing agents’ policy that will be actually deployed in $\tilde{p}^*$. In this sense, the optimal choice for $\mathcal{A}_a$ is the state-of-the-art for the considered industrial setting.

In our experiments, we analyze the performance of CC varying $\mathcal{A}_c$ and $\mathcal{A}_a$ among traditional methods usually employed in ECR domains. More specifically, we consider algorithms that ranges from simple heuristics, to the more complex OR [3] and MARL approaches [3, 7]. As last, we conclude by remarking that our contribution is orthogonal to the chosen algorithms. Indeed, we propose a general learning framework that can be adopted to solve Conf-Semi-POMDPs.

B. Configurator Optimization

We have seen how CC decouples the joint optimization process into two subsequent stages: configure and conquer. More specifically, once the configuration step is over, the conquer step consists in simply applying any algorithm of choice in the tuned environmental setting. For this reason, the crucial step of CC consists in finding an approximation of $p^*$. We now provide an in-depth description of how one can use Reinforcement Learning (RL) to solve this issue.

Suppose w.l.o.g. that $D$ is the dimension of the parametric space $\mathcal{P}$ (i.e., the number of parameters required to define every $p \in \mathcal{P}$). At this point, it is possible to construct an MDP $\mathcal{M}_{\text{conf}}$ whose sequence of actions will define a configuration $p \in \mathcal{P}$. More specifically, at each timestep the agent chooses a dimension-value pair $(d, z)$ that will assign the value $z$ for dimension $d$. At time $t$, the state contains information about the previously selected pairs $\{(d_i, z_i)\}_{i=1}^{t-1}$ and dummy values for the missing configuration parameters. After $D$ timestamps, an entire configuration $p$ is produced and the agent will receive a reward according to $\tilde{J}^{p^*_p}$, where $\tilde{p}^*_p$ is the

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**Algorithm 1** Configure & Conquer (CC) Framework.

**Require:** Algorithms $\mathcal{A}_c$, $\mathcal{A}_a$, model and policy spaces $\mathcal{P}$, $\Pi$

1: Solve $\tilde{p}^*_p \in \arg\max_{p \in \mathcal{P}} J^{\tilde{p}^*_p}$ using $\mathcal{A}_c$ to estimate $\tilde{\pi}^*_p$

2: Solve $\tilde{\pi}^*_p \in \arg\max_{p \in \Pi} J^{\tilde{p}^*_p,\pi}$ using $\mathcal{A}_a$

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**Algorithm 2** Reinforcement Learning Configurator

**Require:** MDP $\mathcal{M}\text{conf}(\mathcal{A}_c)$; Algorithm $\mathcal{A}_c$, Batch size $B$

1: Initialize configurator policy $\nu_c$

2: while not done do
3: for $b = 0, \ldots, B$ do
4: for $t = 1, \ldots, D$ do
5: Sample $(d_t, z_t) \sim \nu_c(\cdot | \{(d_i, z_i)\}_{i=1}^{t-1})$
6: end for
7: Compute $\tilde{p}_c^*$ where $p = \{(d_i, z_i)\}_{i=1}^{D}$ using $\mathcal{A}_c$
8: Evaluate $J^{\tilde{p}_c^*,\pi}$ to obtain the final reward
9: end for
10: Optimize $\nu_c$ via e.g., PPO [31]
11: end while

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approximation of the optimal policy of algorithm $\Delta E$ in model $p$. For all previous timestamps, the reward is fixed at 0 for any action. Given this formulation, it is easy to show that the expected discounted reward of the fixed initial state (i.e., no value-dimension assignment done) is proportional to $\mathbb{E}_{p \sim v_c} \left[ \mathcal{J}^p \right]$, where $v_c$ is the single-agent configurator policy. To highlight the dependency between $\mathcal{M}_{\text{conf}}$ and $\Delta E$, we write $\mathcal{M}_{\text{conf}}(\Delta E)$. At this point, one can use any RL algorithm to train the configurator to solve this MDP. Algorithm 2 reports the pseudo-code for the RL configurator. In our experiments, we model $v_c$ with neural networks, and rely on PPO [31] for its optimization. Notice, however, that this choice is not bonded to the CC framework. Indeed, due to its versatility, CC can easily be adopted in conjunction with other deep-reinforcement learning algorithms (e.g., [32], [33], [34], [35]). In our work, we chose to adopt PPO thanks to (i) its well-known stability in the training process and (ii) the fact that it is not highly sensitive to the adopted hyper-parameters compared to other deep RL approaches. Ablations that target the comparison of PPO against other deep-RL algorithms are deferred to the appendix. Overall, we found PPO to perform better.\(^5\)

At this point, we recall that in the ECR+FD setting, the configurator, at each step, needs to assign a vessel to a route and an initial port; i.e., it selects a triplet $(v, e, h)$. It follows that, with the most naive implementation of $v_c$, the action space would be given by the caribbean product of $H \times V \times E$, which, in practice, can be quite large, thus making the learning process hard, unstable, and inefficient.\(^6\) Moreover, we expect a good configurator to be able to exploit the inner structure within the parametric space $\mathcal{P}$, such as similarities between configurations. Imagine two distinct configurations $p_1, p_2 \in \mathcal{P}$ in which the only difference is that a given vessel $\nu \in V$ is assigned to the same route $\tau \in E$ but two different initial ports $h_1$ and $h_2$ in $\mathcal{E}$. Clearly, we can expect the rewards $\mathcal{J}^p$ that the configurator will obtain for the two configurations to be very similar. Therefore, to make training efficient and effective, we rely on the following more complex architecture for $v_c$. First of all, we significantly reduce the action space size using autoregressive policies [36]. More specifically, each action (i.e., triplet $(v, e, h)$), is split into three components (i.e., $v$, $e$ and $h$) that will be sequentially picked one before the next. We first pick the route $e$, then given the route, we pick the port $h$, and given the route and the port we pick the vessel $v$. By doing so the size of the action space is reduced from $|H| \times |V| \times |E|$ to $|H| + |V| + |E|$. Moreover, to exploit similarity between configurations, we enlarge the state of the agent at timestep $t$ with features of the current uncompleted configuration (e.g., number of vessels assigned to each route, total capacity of the vessels assigned to each route). These features are processed by a neural network $f$ to create an embedding of the current state of the agent. This embedding is used to select the first sub-action (i.e., the route), and it is concatenated to previous selected sub-actions to compute the next sub-actions. Moreover, to further exploit structure the parametric space $\mathcal{P}$, any unfeasible action (e.g. a port that does not exist in a route) is masked.

VI. EXPERIMENTS

A. Experimental Setup

Similar to previous studies [3], [7], our experiments aim at testing our approach in scenarios that mimic dimensions and behaviors of international transportation companies. To this end, we rely on a patched version of the MARO simulator [30]. More specifically,

\(^5\)We notice that this might be a consequence of the easier hyper-parameter tuning process. Nevertheless, benchmarking different RL algorithms on the considered domains is outside the scope of the present work.

\(^6\)In our experiments, we consider real-world instances of 46 ships, 22 ports and 13 routes. This means that the action space would have dimension 13156.

We test our method on two different topologies $W W T_1$ and $W W T_2$. $W W T_1$ is composed of 46 vessels, 22 ports, and 13 routes (as in [7]); in $W W T_2$, the number of ports and vessels is the same, but the number of routes is 6. Moreover, in $W W T_1$, we consider an optimization horizon of 400 days, while in $W W T_2$, 200 days are considered. For both problems, as in previous works [7], order distributions have complex trigonometric shapes with multiple periods.

Given this real-world inspired setup, our experiments aim at answering the following questions: (i) Can CC find better configurations in which the agents operate? (ii) How does the performance of $\Delta E$ and $\Delta E$ impact the final results? (iii) How does CC compare to existing algorithms? To this end, we conduct an extensive empirical study of CC in $W W T_1$ and $W W T_2$, picking as $\Delta E$ and $\Delta E$ the following algorithms:

- **Random policy (Rand).** A random repositioning action is taken every-time a vessel arrives at a certain port.
- **Heuristic policy (Heur).** This is a stochastic intuitive heuristic that we propose to solve ECR problems. If a port is an exporting one (i.e., it exports much more goods w.r.t. the ones that it imports), then, whenever a vessel arrives, we randomly discharge at least parts of the empty containers that it carries. If a port is an importing one (i.e., it imports much more goods than the ones it exports), instead, we randomly load empty containers on the vessel.
- **Operation Research (OR).** Noisy estimates of future orders and vessel arrivals are used at the beginning of the interaction with the environment to compute a plan by solving the mathematical formulation of the problem (see Appendix in [3]).
- **Operation Research methods with iterative plan (OR(I)).** Noisy estimates of future orders and vessel arrivals are used to solve the mathematical formulation of the ECR problem. The plan is computed for a long horizon but executed only for a short window. Once the window expires, a new plan is recomputed using the current state of the environment, so to prevent the plan to diverge from reality [2], [3].
- **MARL system.** The application of MARL techniques to ECR problems has been studied in several works [3], [7], [20]. In our experiments, we use a variant of [7].

We select a subset of combinations of $\Delta E$ and $\Delta E$ that highlight our contributions and that successfully answer to the previous questions. We remark that the purpose of this comparison is not selecting the best combination of $\Delta E$ and $\Delta E$ but, instead, lies in performing an ablation over the algorithms that define the general CC framework. The notation that will be used for CC is $CC-\Delta E-\Delta E$.

Moreover, we compare CC with the following baselines:

- **LS-NET [21].** LS-NET [21] was originally proposed to tackle the joint problem of network design and ECR, however, the extension to the ECR+FD is direct. We define elements of a GA population so that each element describes a configuration $p \in \mathcal{P}$ (i.e., assignments of vessels to routes and initial ports), and we evaluate each of the elements in the population using as fitness function the value of the objective function of the OR formulation of the problem. Once the method has reached convergence, OR(I) is used to evaluate the performance of the approximation of the optimal configuration.
- **Genetic Algorithm - Joint (GA joint).** GAs are used to jointly optimize the configuration and the agents policy. The agents’ policy is represented as a matrix in which cell $(i, j)$ specifies the repositioning $j$-th action of the $i$-th vessel. Configurations are represented as assignments of vessels to routes. An element in the GA population is, thus, a concatenation of a policy with a configuration.
these are heuristic methods that can be used to solve complex optimization problems. More specifically, configurations are optimized following both TS and SA. The value of the objective function that is optimized is the percentage of satisfied demand when using OR(I) as ECR policy.

- **Random Configuration and OR (RandomConf-OR(I)).** Configurations are generated at random; OR(I) computes the policy on these sampled configurations.

More specifically, comparing CC against LS-NET, GA-JOINT, TS, and SA aims to evaluate whether CC is successful at building competitive configurations in real-world inspired domains. On the other hand, as we shall see, the performance RandomConf-OR(I) provides insight into the relevance of optimizing the fleet of vessels to increase the demand satisfaction of ECR policies in the considered scenarios.

### B. Results

Table I reports mean and 95% confidence intervals (10 runs) of the percentage of satisfied demand of different algorithms on WTT1 and WTT2. We highlight in bold the highest performance reached in each domain. In the following, we aim at answering our set of questions; further details on the experiments are provided in Appendix.

First of all, as long as we choose good algorithms for $\alpha_a$ (i.e., OR, OR(I), MARL), we can notice that CC is able to find configurations in which the agents operate that are better than random. This is confirmed by the fact that a good method on random configurations (i.e., RandomConf-OR(I)) reaches lower performance w.r.t. cases in which configurations have been tuned using CC. More generally, we remark that the sub-optimal performance level of RandomConf-OR(I) clearly shows the importance of optimizing the fleet of vessels to satisfy a larger amount of demand in the considered domains.

Furthermore, inspecting thoroughly Table I, we can also see that the choice of $\alpha_a$ highly impact the final performance of our two-stage optimization algorithm. This is expected; indeed, even though we find highly performing configurations, if our agents ignore how to behave (see CC-OR(I)-Rand; 74.24% and 38.72%), the performance will be highly sub-optimal (even worse than using a good method on random configurations such as RandConfig-OR(I) does). Furthermore, we notice that the more sophisticated $\alpha_a$ is (e.g., MARL and OR(I)) the better the performance we can obtain. As for choice of $\alpha_c$, instead, we highlight that, even when the reward for the configurator is computed using random policies, one can still significantly improve the performance. For instance, we notice that in WTT2, even if the random policy is highly suboptimal (see CC-Rand-Rand; 42.70%), CC is still able to find configurations that lead to significantly good performance (see CC-Rand-OR(I); 87.21%). Indeed, we remark that, in this case, CC-Rand-OR(I) performs better then competitive baselines such as LS-NET and GA joint. We notice that this behavior is of particular interest. Indeed, the configurator training process, when using sub-optimal algorithms for $\alpha_c$ is significantly cheaper (we refer the reader to Table II in the Appendix for computational details).

Finally, we investigate how CC compares against existing algorithms that can be used to solve the complex and joint ECR+FD optimization problem. To this end, Table I shows the comparison between CC and several algorithms (i.e., GA joint, LS-NET, TS, and SA). As we can see, all these algorithms underperform w.r.t. all the non-ablation versions of CC. In this sense, CC is successful at obtaining significant performance improvements compared to competitive methods in real-world inspired domains. As a consequence, we can appreciate the ability of CC to build high-performance level fleet deployment configurations that can be exploited by ECR state-of-the-art algorithms (e.g., OR(I)).

### VII. Conclusion

In this work, we have studied the ECR+FD setting under the novel perspective of Configurable Semi-POMDPs. We have modeled the fleet deployment problem as configurations of the environment that can be tuned to increase the overall performance of the multi-agent system. In particular, we focused on the case in which decisions on the fleet of vessels are taken by a central entity (i.e., the shipping company) that cooperates with the agents to minimize the total shortage of containers. We proposed a novel two-stage optimization algorithm (CC) that, as our experiments show, successfully solves the joint ECR+FD optimization problem in large and real-world inspired problem instances, outperforming competitive baselines.

We remark on the generality of the proposed approach. Indeed, a broad number of multi-agent environments can be configured to maximize the performance of the system. In this sense, CC represents a viable option that can successfully scale to large and complex domains. Our work represents a further step in the literature of Configurable MDPs, that, so far, have managed to solve problems of much smaller dimensions only. We also notice that CC can be directly applied as-is in single-agent problems as well.

To conclude, we highlight several exciting lines for future research. First, we plan to extend the Conf-Semi-POMDPs framework and the CC algorithm to the more complex ECR+network design setting. In this case, the set of routes is not pre-determined, but paths for each of the vessels are computed by the configurator. Although this adds a significant level of complexity to the training process, it expands the range of potential configurations that can be uncovered, potentially leading to the discovery of higher-quality solutions. Secondly, our experiments have shown that when sub-optimal algorithms are used as $\alpha_c$, the configurator is still able to improve the quality of the fleet configuration substantially. Nevertheless, as expected, the best performance is obtained when the most computationally expensive algorithms are used. In this sense, future research should investigate dynamic schedules of $\alpha_c$ (e.g., via curriculum learning techniques

| Algorithm | WTT1 | WTT2 |
|-----------|------|------|
| CC-Heur-Heur | 86.18 ± 0.10 | 78.68 ± 0.21 |
| CC-Heur-OR(I) | 90.98 ± 0.23 | 90.50 ± 0.25 |
| CC-Rand-Rand | 77.70 ± 0.17 | 42.70 ± 0.08 |
| CC-Rand-OR(I) | 87.89 ± 0.09 | 87.21 ± 0.26 |
| CC-OR | 88.33 ± 0.16 | 86.14 ± 0.13 |
| CC-OR-MARL | 88.80 ± 0.27 | 94.80 ± 0.46 |
| CC-OR-Joint | 91.04 ± 0.09 | 92.97 ± 0.44 |
| CC-OR(I)-OR(I) | 92.31 ± 0.26 | 94.62 ± 0.36 |
| CC-OR(D)-MARL | 80.32 ± 0.23 | 95.32 ± 0.53 |
| CC-OR(I)-Rand | 74.24 ± 1.20 | 38.72 ± 3.88 |
| GA joint | 86.35 ± 0.13 | 83.86 ± 0.58 |
| LS-NET | 88.26 ± 0.38 | 84.78 ± 1.15 |
| RandomConf-OR(I) | 77.38 ± 0.13 | 68.82 ± 0.43 |
| Tabu Search | 83.45 ± 1.16 | 80.25 ± 1.28 |
| Simulated Annealing | 85.35 ± 0.81 | 83.29 ± 1.45 |
Concerning Figures 1 and 2, they show the training curves of MARL methods when applied as \( \alpha_{KL} \) in \( WTT_1 \) and \( WTT_2 \). Results are mean and 95% confidence intervals over 10 runs. The focus of this figure is on the optimization plateau.

Figure 3, instead, shows the configurator training curves varying \( \alpha_{KL} \) within Heur, Random, OR(I), and OR. This has the purpose of showing the speed at which the approximation of \( \tilde{p} \) is carried out. Indeed, we notice that the configurator step aims at solving the following optimization problem: \( \tilde{p}^* \in \arg \max_{p \in P} J(\tilde{p}) \). To carry out this step, the performance of policies computed with \( \alpha_{KL} \) is used (namely, \( \tilde{p}^*_n \)). The curves exactly report how the performance of these policies evolve through the training of the configurator. When they reach convergence, the configuration step also reaches convergence. In this sense, these curves show the convergence speed of the configuration step. As a final remark, we notice that \( \alpha_{KL} \) is not referenced in this figure; indeed, the configurator step is independent w.r.t. the conquer step.

Table II aims at comparing the computational requirements of the different algorithms of Table I. More specifically, we report the number of iterations together with the iteration computing time when executing a single run in parallel over a cluster of 40 Intel(R) Xeon(R) CPU E7-8880 v4 @ 2.20GHz CPUs and 94 GB of RAM. More specifically, for CC, we report the aforementioned values for the configuration step varying \( \alpha_{KL} \), which is the most demanding one in all methods. In all cases, except when using MARL as \( \alpha_{KL} \), the computational requirements of the conquer step are negligible w.r.t. the configuration one. For these reasons, we report an additional row for the computational requirements of training the MARL system.

Finally, Table III shows the comparison varying the deep RL algorithm used in the configuration step. More specifically, we report the comparison when OR(I) is used both as \( \alpha_{KL} \) and \( \alpha_{CC} \). We report the performance of PPO, TRPO [33], A2C [40], and DQN [32]. For the sake of fairness, all algorithms are trained on the same amount of environment steps. Overall, we found PPO to reach higher performance levels. A2C and TRPO both reach competitive performance levels compared to the other baselines of Table I, while DQN instead shows sub-optimal performance and instabilities during the training process. Although we conjecture that this might be due to sub-optimal hyper-parameters choice, we remark that DQN usually suffers training instabilities and high sensitivity to the hyper-parameters.

### Appendix

We report here further details on the experiments. More specifically, Figure 1, 2 and 3 aims at reporting the training speed of the different methods.

Concerning Figures 1 and 2, they show the training curves of MARL methods when applied as \( \alpha_{KL} \). Performance metrics are reported both for the case in which OR and OR(I) are used as \( \alpha_{KL} \). More precisely, we can observe that CC-OR(I)-MARL and CC-OR-MARL exhibit very similar performance, as evident from Table I. Their behaviors almost coincide, although CC-OR(I)-MARL shows a slightly superior performance compared to CC-OR-MARL, in particular in \( WTT_1 \). In \( WTT_2 \), the performances are not statistically different at the end of the training process. We conjecture that the reason lies in the complexity of the MARL system together with the fact that the approximately optimal configuration \( \tilde{p}^* \) that is obtained from the configurator when using from OR(I) and OR as \( \alpha_{KL} \) are similar. Although when using other methods as \( \alpha_{KL} \) (e.g., OR(I)) the variations in the configurations still lead to significant performance difference, this is not always the case for the MARL system (e.g., WTT2). We attribute this phenomenon to the complexity of the MARL system, which struggles to capture the differences in the final configurations, possibly due to issues like local maxima.
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