Complex links between codimension-2 bifurcations in an electronic oscillator based on hysteresis

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Abstract. The possible dynamics of a model of a hysteresis-based circuit oscillator are analyzed when varying its characteristic’s parameters. By combining brute-force simulations with continuation methods it is shown that the complexity of the scenario obtained through brute-force simulation is largely justified by the presence of a few codimension-2 bifurcation points, which organize the whole bifurcation scenario.

1. Introduction
In most of the electronic hysteresis-based oscillators already proposed in the literature [1–5], the hysteresis is provided through Schmitt-trigger-like nonlinear circuit elements, which are not hysteretic in a strict sense [6–8].

Similarly to another oscillator already considered in the literature [1], the hysteresis oscillator analyzed in this paper is made up of a linear circuit fed back through a hysteretic nonlinearity. In contrast to [1], the hysteretic cell considered here provides a real hysteretic relationship between two of the circuit state variables [9]; indeed, owing to the local memory property [7] of the hysteretic cell, the trajectories are allowed to enter within the hysteresis cycle, thus engendering dynamics richer than those exhibited by most of the known circuit oscillators.

The circuit considered here has been extensively studied in the last decade. The studies have been organized into two threads, according to the model adopted for the nonlinear resistive elements contained in the hysteretic cell. In the first stage, such elements were considered as ideal switches and modeled by means of piecewise-linear characteristics. In this case, the bifurcation analysis of the circuit was carried out through one-dimensional return maps [10, 11]. At the next stage, the nonlinear resistors were modeled by more realistic smooth functions. In this case, the bifurcation analysis was carried out initially through brute-force simulations [12, 13]. However, this approach is strongly dependent on the initial conditions used for the simulations and reveals neither unstable invariant sets nor coexisting invariant sets, even if stable. Recently, to overcome the problems related to brute-force simulations, the dependence of the oscillator dynamics on bifurcation parameters has been addressed through harmonic balance methods [14] and continuation methods combined with normal form theory [15, 16] [17,18].
Up to now, the variety of dynamical behaviors observable for the smooth model has not yet been justified. In particular, it has not yet been discovered whether the richness of the observable dynamical behaviors is due to a few characteristic nonlinear phenomena organizing the overall scenario in parameter space. In this paper, we report the bifurcation analysis of some of the periodic solutions of the smooth oscillator model. By combining results from brute-force simulations and continuation methods, it is shown that the bifurcation curves are organized in parameter space by a few peculiar codimension-2 bifurcation points.

2. The system
The hysteresis oscillator considered here is composed of two sub-circuits, which are shown in Fig. 1. The linear part of the circuit, shown in Fig. 1(a), is the same as in [1], whereas the hysteretic cell, shown in Fig. 1(b), is the simplest version of the hysteresis circuit model proposed in [9].

The two linear voltage-controlled current sources $I_a$ and $I_b$ and the linear voltage-controlled voltage source $V_c$ are driven as follows:

$$
I_a = G_a(v_1 + v_2); \quad I_b = -G_b v_3; \quad V_c = v_1;
$$

where the transconductances $G_a$ and $G_b$ are usually taken as bifurcation parameters. The nominal values of the other circuit parameters are: $R = 30 \ \text{k\Omega}$, $R_{in} = 100 \ \Omega$, and $C = 1 \ \text{nF}$.

The only nonlinear elements of the circuit are the nonlinear resistors $R_{n1}$ and $R_{n2}$. Each of these is made up of two diodes, in series with a constant ideal voltage source, connected in

![Figure 1. Electrical scheme of the hysteresis oscillator. (a) Linear part. (b) Hysteretic cell.](image)

![Figure 2. Circuit nonlinearity. (a) Nonlinear resistor $R_{nk}$ ($k = 1, 2$); (b) its equivalent circuit; its (c) piecewise-linear and (d) smooth characteristics.](image)
antiparallel, as shown in Figs. 2(a) and 2(b). These nonlinearities will be described in greater
detail in the next subsection.

2.1. The hysteretic cell

The input to the hysteretic cell is the voltage $v_1$, copied by the voltage-controlled voltage source
$V_c$, and the output is the voltage $v_3$.

To understand more easily the behavior of the hysteretic cell, it is convenient to start by
modeling the nonlinear resistors through the piecewise-linear characteristic shown in Fig. 2(c),
where the threshold voltage $V_{Tk}$ can be thought of as the sum of the constant voltage $v_{dk}$ and
the ideal diode knee-voltage (see Fig. 2(b)). In this case, for any given working condition,

![Figure 3](image)

Figure 3. Hysteretic input–output relationships for the subcircuit of Fig. 1(b), by assuming
$v_1 = V_1 \sin(2\pi f_1 t) + V_2 \sin(2\pi f_2 t)$ (with $V_1 = 3$, $V_2 = 3$ V, $f_1 = 100$ Hz, and $f_2 = 1$ kHz).
(a,b): Driving and response signal’s time series in the case of the PWL and the smooth model,
respectively. (c,d): Input ($v_1$) output ($v_3$) relationships corresponding to (a,b), respectively.
It is evident from Eqs. (2) that: (i) the equilibria of the system lie on the plane $x_2 = 5/2$. Consequently, at any given time, the hysteretic cell can be modeled by an equivalent first order linear circuit with natural frequency (eigenvalue) $\lambda = -\frac{1}{2RC}$. If the maximum frequency $\lambda_{MAX}$ in the spectrum of the input is much lower than $|\lambda|$, we can assume that the state of the cell instantaneously follows the input variations. This provides an approximately rate independent hysteretic input–output relationship. The rate-independence condition, i.e., $\lambda_{MAX} \ll |\lambda|$, can be expressed in terms of the circuit parameters as $R_{in} \ll \frac{1}{\lambda_{MAX}}$. An example of the input-output relationship exhibited by the hysteretic cell (for $V_{T1} = V_{T2} = 1$ V) is shown in Fig. 3(c), where the plot of $v_3$ (output) versus $v_1$ (input) is shown. The corresponding time series of driving and response signals are shown in Fig. 3(a).

More correctly, the diodes can be modeled through their smooth driving-point constitutive equation $i_d = I_S (\exp(v_D/V_0) - 1)$, where $v_D$ is the voltage across the diode, $I_S = 1$ fA, and $V_0 = 25.9$ mV at room temperature. In this case, the piecewise-linear characteristics of Fig. 2(c) are replaced by the more realistic smooth functions shown in Fig. 2(d) and, correspondingly, the shape of the hysteresis cycle changes as shown in Fig. 3(d), where the constant voltage sources of Fig. 2(b) are fixed at $v_{d1} = 0$ V and $v_{d2} = 0.3$ V.

In the following, we shall assume $v_{d1} = 0$ V and $v_{d2} = 0.2$ V.

### 2.2. The normalized system

By modeling each diode by its smooth constitutive equation, and by normalizing the time variable and the voltages across the capacitors, the state equations of the circuit in Fig. 1 can be written as:

$$
\begin{align*}
\dot{x}_1 &= -(x_1 + x_2) \\
\dot{x}_2 &= (2 + p_1)(x_1 + x_2) - x_2 - x_3 \\
\dot{x}_3 &= p_3 (\Psi - p_4 \sinh (x_3)) \\
x_1 - x_3 &= \text{asinh} \left( \frac{\Psi}{p_5} \right) + \Psi
\end{align*}
$$

where $\tau = \frac{t}{RC}$, $x_i = \frac{v_i}{V_0}$ ($i = 1, \ldots, 3$), and $\dot{x} = \frac{dx}{dt}$.

The bifurcation parameters become $p_1 = RG_a - 3$ and $p_2 = RG_b$, whereas $p_3 = R/R_{in} = 300$, $p_5 = 2R_{in}I_S/V_0 = 77.22E - 12$, and $p_4 = p_5 \exp(-\frac{V_0}{V_0}) = 2.97E - 24$ are held fixed. The only nonlinear equation of the ODE system is Eq. (2c), where $\Psi$ is implicitly defined by Eq. (2d).

### 2.3. System properties

It is evident from Eqs. (2) that: (i) the equilibria of the system lie on the plane $x_1 = -x_2$; (ii) their positions depend on $p_2$ only; and (iii) the origin $E_0 = (0, 0, 0)$ is a trivial equilibrium for all parameter values.

On the other hand, the stabilities of the equilibria depend on both parameters $p_1$ and $p_2$, as follows from the Jacobian matrix of the system, that can be written only partially in explicit

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1 Actually, a more accurate model of each diode would also include dynamical features induced by parasitic elements. However, their effects are negligible when reasonable oscillations frequencies are considered. Hence, for the sake of simplicity, we shall neglect all the parasitic capacitors in our model.
form:

\[
A = \begin{bmatrix}
1 & -1 & 0 \\
2 + p_1 & 1 + p_1 & -p_2 \\
\frac{p_3 \sqrt{p_5^2 + \Psi^2}}{1 + \sqrt{p_5^2 + \Psi^2}} & 0 & -p_3 \left( \frac{p_3 \sqrt{p_5^2 + \Psi^2}}{1 + \sqrt{p_5^2 + \Psi^2}} + p_4 \cosh(x_3) \right)
\end{bmatrix}
\]  

(3)

System (2) is \(Z_2\)-equivariant [18], namely it is invariant under the transformation \((x_1, x_2, x_3) \rightarrow (-x_1, -x_2, -x_3)\) or, in other words, there is a mirror symmetry with respect to the origin of the state space. As a consequence, pitchfork (or symmetry breaking) bifurcations break the \(Z_2\)-symmetry of fixed (i.e., invariant under the symmetry transformation) invariant sets and give rise to a symmetric pair of asymmetric invariant sets that are mirror image copies of each other. Henceforth, we shall denote as F-sets and S-sets the fixed and the symmetric invariant sets, respectively. Whilst \(E_0\) is of F-type, in some regions of the parameter space there are two other non-trivial S-type equilibria, denoted as \(E_+\) and \(E_-\). In the following, we refer to \(E_{\pm}\) to denote both of them. The same holds, \textit{mutatis mutandis}, for other S-sets.

3. Observed families of periodic solutions

If we properly choose the values of the bifurcation parameters, we can observe different kinds of periodic behaviors. Such behaviors can be grouped naturally into families indexed according to three integer numbers \((n^+, n^0, n^-)\), representing the numbers of revolutions (turns) of the

![Figure 4](image-url)

\textbf{Figure 4.} Families of periodic and chaotic solutions in the state space \((x_1, x_2, x_3)\); the scale is the same for all plots. Limit cycles: (a) – small (visiting half state space) S-type \((4, 4, 0)\) \pm solutions (shown in black and gray, respectively); (b) – large (visiting the whole state space) S-type \((3, 3, 3)\) (only the + solution is shown); (c) – F-type \((3, 0, 3)\); (d) – F-type \((4, 2, 4)\). (e-h) – chaotic solutions belonging to the families of periodic solutions shown in (a-d), respectively.
periodic solution (limit cycle) in the upper\(^2\), central, and lower regions of the state-space, respectively. Examples of these periodic solutions, both F- and S-type, are shown in the first row of Fig. 4, where \(x_1 \in [-180, 180]\), \(x_2 \in [-210, 210]\), and \(x_3 \in [-70, 70]\). Furthermore, as shown in Fig. 4(e-h), for each kind of periodic solution there are corresponding (shaped like) chaotic behaviors.

Experimentally, the families of periodic solutions are organized in the parameter space as shown in Fig. 5, where the colouring is organized controlling the Red Green and Blue (RGB) components proportionally to the indices \(n^-\), \(n^0\), and \(n^+\), respectively. In the figure, the transition from bluish to white colours marks a symmetry breaking of F-type attractors; the system asymptotically converges, in the light-blue region, to the one or the other of a pair of S-type attractors, similar to those shown in Fig. 4(e), whereas in the white region it converges to an F-type attractor, similar to that shown in Fig. 4(h).

The diagram turns out to be split in two parts by the central “sail”-shaped region (white, grey, dark-blue and black), where up to four stable invariants coexist [15,16]. At the left of this “sail”, there are many grey and blue “hooks”, that mark the presence of periodic windows in a dominant chaotic regime, whereas at the right of the “sail”, one can see many other “U”-shaped bi-colour periodic windows, that will be described in detail in Sec. 4.5.

It is neither clear nor evident how the families of solutions are linked to each other and how the chaos is organized by such periodic families.

In order to understand the links between families, we will carry out a bifurcation analysis based on the numerical continuation of limit cycles using the package AUTO2000 [17], starting from cycles with low values of \(n^+\), \(n^0\), and \(n^-\). According to this strategy, we will also follow, moving in the parameter space, unstable cycles that may exist also in the pale pink region or

\(^2\) Upper, lower, and central are meant with respect to the \(x_3\) variable.

**Figure 5.** Brute-force simulations bifurcation scenario (\(\ln(p_1)\) vs. \(\ln(p_2)\)). The colouring is obtained controlling the Red Green and Blue (RGB) components proportionally to the \(n^-\), \(n^0\), and \(n^+\) indices, respectively. The pale pink colour corresponds to unbounded dynamics.
even outside the portion of parameter plane shown in Fig. 5.

4. Bifurcation analysis: Linking the families
The previously studied bifurcations [15] do not justify the presence of the families of limit cycles, how they are linked, and why they lead to chaotic attractors.

We start from the limit cycle \( C_0 \) emerging from one of the previously studied bifurcations (the Hopf \( H_0 \) of the origin \( E_0 \)) and we see how a few codimension-2 bifurcation points involving \( C_0 \) justify all the families and the transitions between them. In the sketches provided in the next subsections, the dots denote unstable equilibria, with \( E_0 \) blue and \( E_{\pm} \) green, the solid (dashed) closed lines denote stable (unstable) limit cycles, with \( C_0 \) always blue.

4.1. Bogdanov-Takens: Existence of a global bifurcation
As sketched in Fig. 6, at very high values of \( p_1 \), at the intersection between \( H_0 \) and the pitchfork of the origin \( P_E \) [15], \( E_0 \) undergoes a Bogdanov-Takens (double zero) bifurcation (point \( BT \) in Fig. 6). Hence, because of the \( \mathbb{Z}_2 \) symmetry of the system, from the intersection point \( BT \) a global bifurcation curve \( Ht \) originates, on which the cycle \( C_0 \) degenerates to a double heteroclinic connection between \( E_+ \) and \( E_- \).

Figure 7 shows the taming approach of the limit cycle \( C_0 \) to its heteroclinic bifurcation. The cycle is continued with respect to \( p_2 \), for \( ln(p_1) = 5 \). While approaching the heteroclinic bifurcation, \( C_0 \) tends to (i) grow, (ii) increase its period, and (iii) settle down on the plane \( x_1 = -x_2 \), containing the equilibria. These properties can be easily inferred from Fig. 7.

However, due to the saddle (real) nature of \( E_{\pm} \), \( BT \) does not locally justify the existence of so many families of periodic solutions.

![Figure 6. System unfolding around the codimension-2 bifurcation \( BT \).](image-url)
4.2. Fold-Pitchfork: Organization of the symmetry breaking

The heteroclinic bifurcation, being global, is a good candidate for organizing more complex behaviors. Indeed, by continuing $Ht$ far-away from $BT$, we discovered a degenerate point $FP$, marking a codimension-2 Fold-Pitchfork bifurcation for the limit cycle $C_0$. At this point, the structurally unstable double heteroclinic connection undergoes a concurrent symmetry breaking. From this point, four further bifurcation curves originate:

1. A pitchfork of limit cycles (bifurcation curve $P_C$), along which the S-type limit cycles $C_{\pm}$ (in green in the sketches) emerge from the F-type limit cycle $C_0$;
2. A fold of limit cycles (bifurcation curve $F_0$), along which the two F-type limit cycles $C_0$ (in blue) and, say, $C_h$ (in magenta) collide and disappear;
3–4 A pair of coinciding homoclinic bifurcations (bifurcation curve $Hm$), where the two S-type limit cycles appearing at $P_C$ collide with the equilibria $E_{\pm}$, thus breaking and disappearing.

The aforesaid bifurcation lines are arranged around the point $FP$ as shown in Fig. 8, where sketches of the bifurcating invariant sets are shown in the corresponding regions.
Figure 8. Arrangement of the bifurcation diagram in the neighborhoods of the Fold-Pitchfork, Belyakov, and Shil’nikov-Hopf codimension-2 bifurcations.

Less evident is the existence of a period doubling bifurcation curve (PD), originating in $FP$, involving the S-type limit cycles. This suggests the non-orientability of the flow around these cycles in the neighborhood of $FP$ and, consequently, a more complex scenario [20], which, however, we did not investigate further. Indeed, in this complex scenario, the saddle (real) nature of the equilibria $E_{\pm}$ does not justify the existence of chaotic solutions.

To give an idea of the scenario linking $Ht$ and $F_0$, Fig. 9 shows the continuation of the limit cycles $C_0$ and $C_h$ with respect to $p_2$, for $ln(p_1) = 0.35$. 
Figure 9. Numerical continuation of limit cycles $C_0$ and $C_h$ with respect to $p_2$, for $\ln(p_1) = 0.35$. (a) Period vs. $p_2$. (b) Two limit cycles ($C_0$ and $C_h$) coexisting at $\ln(p_2) = 0.311$, together with equilibria $E_0$ (in black) and $E_{\pm}$ (in grey), in the state plane $(x_1, x_2)$.

4.3. Belyakov: Organization of chaotic solutions
Proceeding further, $H_t$ undergoes a Belyakov degeneracy, where the equilibria $E_{\pm}$ bearing the double heteroclinic connection change from saddle (real) to saddle-focus. Theory predicts several families, of infinite cardinality, of bifurcation curves originating in this point [19, 20]. According to the well-known Shil’nikov theorem [18], these curves are predicted to accumulate exponentially on $H_t$, and the outer ones delimit a region (light gray region in Fig. 8) where wild (chaotic) trajectories can be observed [21]. The hypothetical chaotic trajectories organized by this Belyakov point would mimic the shape of the F-type limit cycles as shown in Figs. 4(b) and 4(f). Similarly, the homoclinic curve $H_m$ originating in $FP$ undergoes the same degeneracy and, consequently, S-type wild trajectories can be observed in the parameter space (dark gray region in Fig. 8). As an example, Fig. 10 shows the continuation of one of the family of infinite limit cycles, linked to $C_0$ and $C_h$, with respect to $p_2$, for $\ln(p_1) = -0.8$.

Whilst the Belyakov points justify the existence of chaotic solutions associated with both F- and S-type limit cycles, they do not explain the existence of different families of limit cycles characterized by different numbers $n^{+,-}$ of turns (revolutions).

4.4. Shil’nikov-Hopf: Organization of the $n^{+,-}$-turns
Both $H_t$ and $H_m$ bifurcation curves end on the Hopf bifurcation curve ($H_{\pm}$) of the equilibria $E_{\pm}$ (Fig. 8). The end points $SH_{1,2}$ are codimension-2 bifurcations known as Shil’nikov-Hopf
4.5. A blue-sky catastrophe: Formation of the $n^0$-turns

The continuation of F-type limit cycles with respect to $p_2$, at values of $p_1$ below the level of the two $SH$ points, usually indicates a scenario such as the one shown in Figs. 11 and 12. This characteristic scenario, with both period and length increasing towards infinity, suggests the existence of the so-called blue-sky catastrophe, where two limit cycles collide transversally, with the trajectory of one cycle winding around the other one. In our case, the trajectories tend
Figure 11. Numerical continuation of the family of infinite limit cycles originating from $C_0$ and $C_h$ with respect to $p_2$, for $ln(p_1) = -1.2$. (a) Period vs. $p_2$. (b) Three of the limit cycles coexisting at $ln(p_2) = 0.48$, together with $E_0$ (in black) and $E_±$ (in grey), in the plane $(x_1, x_2)$.

Figure 12. Blue-sky scenario for a F-type limit cycle ($ln(p_1) = -1.2$): (a) – Period vs. $ln(p_2)$; (b) – Length vs. $ln(p_2)$. 
to roll up around the limit cycle $C_0$, thus justifying the formation of the central ($n^0$) turns. Unfortunately, blue-sky bifurcations cannot be numerically continued, at least by resorting to standard numerical packages. Extensive simulations suggest the existence of an infinity of them, one for each family of cycles with different $n^{+/-}$ turns. These bifurcation loci appear to be U-shaped with the upper part converging towards the degeneracy $FP$, and they coincide with the transitions from the bluish to the greyish regions in the “boomerang”-like shaped regions on the right side of the diagram of Fig. 5. Figure 13 shows a detail of one of the “boomerang”-like regions, where the blue-sky bifurcation curve appears (at least in part) as a green line.

**Figure 13.** Detail of a one of the “boomerang”-like regions of the bifurcation diagram.

**Figure 14.** Stable periodic solutions observable for parameter pairs $(p_1, p_2)$ corresponding to the points a to h in Fig. 13. The plot scales are the same for all figures.
Figure 14 shows the asymptotic (stable) solutions observable for parameter values corresponding to the points from a to h in Fig. 13.

By moving the parameter values along the yellow line of Fig. 13, from a to h, we observe a typical (symmetric) blue-sky scenario: an F-type periodic solution, shown in Fig. 14a, smoothly deforms winding around another small F-type limit cycle close to the origin (see Fig. 14b-e); correspondingly, its period and length grew asymptotically approaching the bifurcation boundary, as shown in Fig. 12; at the bifurcation point boundary (blue to gray transition in Fig. 13), the F-type periodic solution breaks up into a pair of S-type periodic solutions, as displayed in Fig. 14f; initially, these S-type solutions make many turns around the small F-type limit cycle; then, by further moving the parameter values along the yellow line, they unroll as shown in Fig. 14g-h, and their periods and lengths decrease correspondingly.

5. Overall bifurcation scenario

Figure 15 reports the bifurcation curves obtained by continuation superimposed to the brute-force simulations bifurcation diagram.

The colouring and labeling code of the curves is as in the sketches of Figs. 6 and 8, here summarized for the reader’s convenience:

| Colour | Type of curve | Example |
|--------|---------------|---------|
| blue   | pitchfork of equilibria | $P_E (E_0 \rightarrow E_0, E_\pm)$ |
| blue   | pitchfork of cycles | $P_C (C_0 \rightarrow C_0, C_\pm)$ |
| green  | Hopf           | $H_\pm (E_\pm \rightarrow E_\pm, C_\pm)$ |
| red    | fold of limit cycles | $F_0 (C_0, C_h)$ |
| cyan   | period doubling | $PD (C_\pm)$ |
| black  | heteroclinic (for F-cycles) | $H_t$ |
| grey   | homoclinic (for S-cycles) | $H_m$ |

Figure 15. Overall bifurcation scenario ($ln(p_1)$ vs. $ln(p_2)$).
As mentioned at the outset, for the limit cycles considered (with low values of $n^+, n^0$, and $n^-$), the continued bifurcation curves, and corresponding codimension-2 points organizing the scenario, lie, at least partially, in the region where no bounded dynamics can be observed. Nonetheless, they outline the prototype of the bifurcation diagram organizing the existence of one family of periodic solutions. The knowledge of this prototype diagram, combined with the fractal (self-similar) structure of the brute-force simulation bifurcation diagram highlights that the very same bifurcation scenario exists for all the families with different numbers $n^+, n^0$, and $n^-$ of turns. In this respect, it is interesting to notice that only the solutions with rather high numbers of loops appear to be stable.

6. Conclusions
Simulations of the hysteresis-based oscillator model show the existence of several families of periodic and chaotic solutions characterized by different numbers of turns in the upper, central, and lower part of the state-space. The detailed bifurcation analysis reported here shows how their existence is justified, as well as organized in the parameter space, by a few codimension-2 bifurcation points.

This analysis is prelude to the definition of criteria for the oscillator design.

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