Frequency domain control based on quantitative feedback theory for vibration suppression in structures equipped with magnetorheological dampers

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Abstract
This paper addresses the problem of designing quantitative feedback theory (QFT) based controllers for the vibration reduction in a structure equipped with an MR damper. In this way, the controller is designed in the frequency domain and the natural frequencies of the structure can be directly accounted for in the process. Though the QFT methodology was originally conceived of for linear time invariant systems, it can be extended to nonlinear systems. A new methodology is proposed for characterizing the nonlinear hysteretic behavior of the MR damper through the uncertainty template in the Nichols chart. The resulting controller performance is evaluated in a real-time hybrid testing experiment.

1. Introduction

The protection of civil engineering structures has always been a major concern especially when these structures are built in places prone to hazardous weather conditions (e.g. hurricanes, tsunamis), in zones of intense seismic activity, or when the structure is subjected to heavy loadings (e.g. heavy traffic on a bridge). If a structure is not well protected against these phenomena, they can suffer severe damage, and as a consequence, cause injuries or deaths as could be seen during the earthquakes in Mexico City (1985), Kobe (1995), northwestern Turkey (1999), those that struck southern Asia in 2004 following the tsunamis, or more recently in China (2008).

In order to make structures more resistant against these phenomena, passive and active dampers were initially proposed. Passive dampers alleviate the energy dissipation of the main structure by absorbing part of the input energy, without the need of external power sources. However, once installed, they are not adaptable to different loading conditions (Yoshioka et al 2002). Active dampers, on the other hand, can respond to variations of the loading conditions and structural dynamics but require large power sources and additional hardware such as sensors and DSP’s to operate. Active dampers can also inject energy to the structure and may destabilize it in a bounded-input bounded-output sense (Spencer and Sain 1997).

Semiactive devices provide an effective solution to overcome the disadvantages of passive and active dampers (Dyke et al 1998). They have been shown to perform significantly better than passive devices and also active devices. Despite requiring additional hardware to operate, semiactive devices need less power than active devices because they cannot inject energy to the system. Thus, semiactive devices can be powered with batteries (Spencer and Song 1999). The main characteristics of semiactive devices are the rapid adaptability of their dynamic properties in real time but without injecting any energy into the system. Among the diverse semiactive devices, MR fluid dampers are the most...
attractive and useful ones. MR dampers can generate a high yield strength, have a low cost of production, require low power, and have a fast response and small size. However, they are characterized by a complex nonlinear dynamics (typically hysteresis), which makes mathematical treatment challenging, especially in the modeling and identification of the hysteretic dynamics and the development of control laws for its implementation through MR dampers for vibration mitigation purposes.

A number of control techniques have been developed for vibration control of structures equipped with MR dampers. Clipped optimal control (Dyke et al. 1996) was one of the first controllers developed for this class of systems. An optimal controller is designed to estimate the force that mitigates the vibrations in the structure and the control signal takes only two values according to an algorithm, in which the MR damper dynamics are ignored. Control techniques based on Lyapunov’s stability theory have been proposed, and successfully tested, in structures such as buildings, bridges and car suspension systems (Jansen and Dyke 2000, Yang 2001, Wang and Gordaninejad 2002, Park and Jeon 2002, Luo et al. 2003, Nagarajaiah et al. 2006). The general control objective is achieved through the choice of control inputs that make the Lyapunov function derivative as negative as possible and consequently obtain the maximum energy dissipation. Other control methods have also been proposed, such as bang–bang control (McClamroch et al. 1994, McClamroch and Gavin 1995, Jansen and Dyke 2000); sliding mode control (Luo et al. 2003, Villamizar et al. 2003, Moon et al. 2003); backstepping control (Villamizar et al. 2005, Luo et al. 2007, Zapateiro et al. 2008); and intelligent control, such as fuzzy logic control (Kim and Roschke 2006) and neuro fuzzy control (Schurter and Roschke 2001).

Most of these semiactive structural control strategies are based on the idea of attenuating vibrations or maintaining the structural time response within certain acceptable ranges when external forces such as earthquakes or strong winds act on the structures. The effectiveness of control strategies depends on the knowledge of structural dynamics (for example, the precision of estimated or measured variables of displacement, acceleration, etc.). The controller design is usually done in the time domain by considering that the system model and its associated parameters are known or uncertain, but with known upper and lower bounds.

Since the behavior of controlled structures depends not only on the magnitude of the external excitation but also on its frequency modes, modal frequency control is of great interest for achieving structural safety and human comfort. As one of the linear structural control strategies, the quantitative feedback theory (QFT) control technique was first introduced by Luo et al. (2004) for vibration reduction in linear structures and was extended to structures equipped with MR dampers by Villamizar et al. (2004). Numerical simulations and experiments on small-scale specimens showed the feasibility of applying QFT control in larger systems. In these works, however, an algorithm similar to the clipped optimal control was followed, i.e. the nonlinear dynamics of the MR damper were ignored. A further step was taken by Zapateiro et al. (2008) in proposing the inclusion of the hysteretic dynamics of MR dampers in the QFT control design and its feasibility was proved by numerical simulations. In this paper, a QFT controller that includes the MR damper dynamics is proposed as an extension of the previous works. The design is experimentally evaluated in a real-time hybrid testing basis to analyze the performance of vibration control in large-scale structures.

This paper is organized as follows. Section 2 explains the basic concepts of quantitative feedback control. Section 3 describes the experimental environment and the structure model that will be used for control design. Section 4 presents the details of the QFT controller formulation for the experimental structure. Section 5 presents the experimental results and the controller performance analysis. Finally, the conclusions are outlined in section 6.

2. Quantitative feedback theory

QFT is a frequency control methodology based on the notion that feedback is only necessary when there is uncertainty and nonmeasurable disturbances actuating on the plant. The basic developments with QFT are focused on the control design problem for uncertain linear time invariant (LTI) systems, as shown in figure 1. In this figure, \( R(s) \) represents the command input set, \( P \) the plant set and \( T \) the transfer function set. For each \( R(s) \in R \), \( P(s) \in P \), the closed loop output will be \( Y(s) = T(s)R(s) \) for some \( T(s) \in T \). For a large class of such problems, QFT is executable, i.e., a pair of controllers \( F(s) \) and \( G(s) \) can be found to guarantee that \( Y(s) = T(s)R(s) \). Suppose that the plant \( P(s) \) is uncertain but known member of the set \( P \). The designer is free to choose the prefilter \( F(s) \) and the compensator \( G(s) \) in order to ensure that the system transfer function \( T(s) = F(s)P(s)G(s)/(1 + P(s)G(s)) \) satisfies the assigned specifications.

The uncertain plant model \( P(s) \) and its frequency and time domain specifications are represented in the Nichols chart through the use of the Horowitz–Sidi bounds. These bounds

![Figure 1. Schematic of the QFT control system.](image-url)
determine the regions where the nominal open loop transfer function \( L_0(s) = G(s)P_0(s) \) may lie so that all the design specifications can be achieved.

The QFT methodology design can be summarized as follows (Houpis et al 2005):

1. **Plant model, template generation and nominal plant selection.** The plant is represented in the Laplace domain; each uncertain parameter is assigned a range of variation and the frequencies of interest are chosen within the expected operation range. At each frequency of interest and for each possible value of the uncertain parameters, the plant model \( P(j\omega) \) becomes a complex number that can be represented in the Nichols chart (dB, Φ). This set of complex numbers is called the template.

2. **Design specifications.** The inputs to the system of figure 1 are \( R(s) \) (the reference), \( W(s) \), \( D_1(s) \) and \( D_2(s) \) (the disturbances) and \( N(s) \) (the noise). \( Y(s) \) is the variable to be controlled, \( E(s) \) is the error and \( U(s) \) is the control signal. The following transfer functions can be obtained:

\[
Y = \frac{1}{1 + PHG}D_2 + \frac{P}{1 + PHG}D_1 + \frac{PG}{1 + PHG}(W + FR) - \frac{PHG}{1 + PHG}N \quad (1)
\]

\[
U = \frac{G}{1 + PHG}(W + FR) - \frac{GH}{1 + PHG}(N + D_2 + D_1) \quad (2)
\]

\[
E = \frac{1}{1 + PHG}D_2 + \frac{PH}{1 + PHG}D_1 + \frac{PHG}{1 + PHG}W + \frac{1}{1 + PHG}FR - \frac{H}{1 + PHG}N. \quad (3)
\]

By limiting the transfer function magnitudes of equations (1)–(3), it is possible to set the stability and robustness specifications such as disturbance rejection, tracking and noise rejection.

3. **Bound generation.** Once the nominal plant has been chosen, the next step is to transform the closed loop specifications of uncertainty plants in a set of restriction curves or bounds, known as Horowitz–Sidi bounds, for each frequency of interest on the Nichols chart. This information synthesis allows the design of the controller using only the nominal plant. For each frequency and for each design specification there is one bound. When all these bounds are calculated, then the most restrictive one per frequency is kept.

4. **Loop shaping.** When the most restrictive bounds are found, the controller is synthesized by adding a gain, poles and zeros such that the loop function \( L_0(j\omega) \) lies in the Nichols chart in the regions where the design specifications can be achieved. The optimal controller is the one that has the minimum gain, and lies on the bounds at each frequency of interest. In this case, it is possible to affirm that the controller accomplishes all the design specifications.

5. **Prefilter.** When tracking specifications are required, the prefilter \( F(s) \) must be designed. The prefilter synthesis is similar to that of the controller.

6. **Design validation.** This step involves the performance evaluation of the controller and its adjustment until all the design specifications are satisfied within acceptable limits.

Most of the design process can be done with the help of software packages such as the QFT toolbox for Matlab. The loop shaping process is left to the ability and experience of the designer.

### 3. Experimental setup

This section describes the experimental environment where the controller will be tested. Experiments are executed in a real-time hybrid testing configuration available at the Smart Structures Technology Laboratory, University of Illinois Urbana-Champaign (USA), as shown in figure 2. It consists of a computer that simulates the structure to be controlled and generates the commanding signals (displacements and control signals); a small-scale MR damper that is driven by a hydraulic actuator which in turn is controlled by a servo-hydraulic controller; and DSP, A/D and D/A hardware for signal processing. Sensors available include a linear variable displacement transformer (LVDT) for displacement measurements and a load cell to measure the MR damper force. In figure 1, \( x_{cmd} \) is the commanded displacement, \( f_{mr} \) is the MR damper force measured by the load cell, \( x_{meas} \) is the displacement measured by the LVDT and \( i \) is the control current sent to the hydraulic actuator. A fully detailed description of this real-time hybrid testing implementation can be found in Carrion and Spencer (2007).

#### 3.1. Structure model

The schematic of the 3-story building to be controlled is shown in figure 3. The building can be modeled with the second order motion equation:

\[
M_s \ddot{x} + C_s \dot{x} + K_s x = G_s f - M_s L_s \ddot{x}_g, \quad (4)
\]

where the matrices and vectors \( M_s, C_s, K_s, G_s \) and \( L_s \) are given by:

\[
M_s = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} = \begin{bmatrix} 20253 & 0 & 0 \\ 0 & 20253 & 0 \\ 0 & 0 & 20253 \end{bmatrix} \text{ kg} \quad (5)
\]

\[
C_s = \begin{bmatrix} c_{11} & c_{12} & 0 \\ c_{21} & c_{22} & c_{23} \\ 0 & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 7243.2 & -2070 & 0 \\ -2070 & 4138.2 & -2070 \\ 0 & -2070 & 2070 \end{bmatrix} \text{ N s m}^{-1} \quad (6)
\]

\[
K_s = \begin{bmatrix} k_{11} & k_{12} & 0 \\ k_{21} & k_{22} & k_{23} \\ 0 & k_{32} & k_{33} \end{bmatrix} = \begin{bmatrix} 9932 & -5661 & 0 \\ -5661 & 11338 & -5661 \\ 0 & -5661 & 5661 \end{bmatrix} \text{ N m}^{-1} \quad (7)
\]

\[
G_s = [-1, 0, 0]^T, \quad L_s = [1, 1, 1]^T \quad (8)
\]
\( \mathbf{x} \) is the vector of relative displacements, i.e., with respect to the ground; \( f \) is the MR damper force and \( \ddot{x}_g \) is the incoming earthquake acceleration. \( \ddot{x}_{ai} \) is the absolute acceleration of the \( i \)th floor. The absolute displacement is measured with respect to an inertial frame, so the relationship between relative and absolute coordinates is: 
\[
\mathbf{x} = \mathbf{x}_a - \mathbf{x}_g.
\]

The natural frequencies and the damping ratios of the structure corresponding to the first, second and third mode are 1.09 Hz (0.31%), 3.17 Hz (0.62%) and 4.74 Hz (0.63%), respectively.

### 3.2. MR damper

The MR damper used in the experiments is the RD-1005 manufactured by the Lord Corporation (www.lord.com), shown in figure 4. The damper is 216 mm long in its extended position, is 38.1 mm in diameter and has a stroke of 25.4 mm. It contains 50 ml of MR fluid and can generate forces up to approximately 3000 N. The magnetic field is generated by the current from a PWM amplifier (the RD-1002 Wonder Box, from Lord Corp.).

The dynamics of the damper can be modeled by the Bouc–Wen model (Spencer et al 1997), as shown in equations (9) and (10):

\[
f_{mr} = (c_{0a} + c_{0b}u)\dot{x}_{1r} + (k_{0a} + k_{0b}u)x_{1r} + (\alpha_a + \alpha_bu)z \tag{9}
\]

\[
\dot{z} = -\gamma|\dot{x}_{1r}|z|z|^{n-1} - \beta|\dot{x}_{1r}|z^n + A\dot{x}_{1r}, \tag{10}
\]

where \( \dot{x}_{1r} \) is the damper piston velocity, \( z \) is an evolutionary variable that describes the hysteretic behavior of the damper; \( A, \gamma, \beta \) and \( n \) are parameters that are chosen so to adjust the hysteretic dynamics of the damper. \( u \) is the voltage output of the first order filter introduced to account for the dynamics involved in the MR fluid when reaching the rheological equilibrium:

\[
\dot{u} = -\eta(u - v), \tag{11}
\]

where \( \eta \) is a parameter that is empirically identified. The parameters of the MR damper specimen are: \( \alpha_a = 33.27 \text{ N m}^{-1}, \alpha_b = 182.65 \text{ N m}^{-1} \text{ V}^{-1}, c_{0a} = 754.41 \text{ N s m}^{-1}, c_{0b} = 712.73 \text{ N s m}^{-1} \text{ V}^{-1}, k_{0a} = 1137.57 \text{ N m}^{-1}, k_{0b} = 1443.50 \text{ N m}^{-1} \text{ V}^{-1}, x_0 = 0 \text{ m}, \gamma = 4209.8 \text{ m}^{-2}, \beta = 4205.2 \text{ m}^{-2}, A = 10246, n = 2, \)
Figure 5. Time behavior of the MR damper characteristics: sinusoidal displacement and switching voltage.

Figure 6. Time behavior of the MR damper characteristics: random displacement and voltage.

Figure 7. Numerical model and physical system interaction.

\[ \eta = 57 \, \text{s}^{-1} \] The following scaling factors are used to integrate the physical small-scale MR damper to the numerical large-scale structure: the first floor relative displacement is reduced by a factor \( S_L = 7.25 \) to obtain the damper piston displacement and the MR damper force is increased by a factor \( S_F = 60 \) to obtain the input force to the structure.

Figures 5 and 6 show a comparison between the experimental dynamics of the MR damper and those predicted by the Bouc–Wen model. In the first case, the damper is subject to a sinusoidal displacement at 5 Hz and 0.254 cm amplitude. The voltage periodically switches from 0 to 5 V. In the second case, the damper is subject to random displacement and random voltage excitation.

3.3. Hydraulic actuator dynamics

The MR damper is driven by a hydraulic actuator which receives a command signal from the computer where the simulation runs to impose a displacement to it. A block diagram that shows the interaction between the numerical model and the dynamic system is illustrated in figure 7.

The entire physical system can be modeled by a transfer function \( G_{xy}(s) \) whose input \( u_c \) is the commanded displacement and whose output \( x \) is the piston displacement. Modeling the dynamic systems is useful for simulating the real-time hybrid testing experiments. The transfer function \( G_{xy}(s) \) varies according to the MR damper input voltage. Two cases are identified corresponding to the damper operating
at $V_0 = 0$ V ($G_{xu,V_0}(s)$) and $V_{\text{max}} = 5$ V ($G_{xu,V_{\text{max}}}(s)$) respectively. These transfer functions are given by:

$$
G_{xu,V_0}(s) = \frac{1}{(0.0062s + 1)(2.639 \times 10^{-5}s^2 + 0.0059s + 1)}
$$

(12)

and

$$
G_{xu,V_{\text{max}}}(s) = \frac{1}{(0.0094s + 1)(2.618 \times 10^{-5}s^2 + 0.0058s + 1)}
$$

(13)

In order to identify each transfer function, the response of the hydraulic actuator to a displacement input was measured in the cases where the MR damper voltage was set at 0 and 5 V. Transfer functions of different orders were obtained. Finally, those transfer functions of reduced order that accurately represented the dynamics of the system were chosen. An algorithm was designed by Carron and Spencer (2007) to provide a smooth transition from $G_{xu,V_0}(s)$ to $G_{xu,V_{\text{max}}}(s)$ and vice versa when the damper voltage varies during the experiments. A block diagram of this algorithm is shown in figure 8. The Laplace transform of the model is described by:

$$
X(s) = X_a(s) + X_b(s)W(s)
$$

(14)

$$
X_a(s) = G_a(s)U_a(s) = G_{xu,V_0}(s)U_a(s)
$$

(15)

$$
X_b(s) = G_b(s)U_c(s) = (G_{xu,V_{\text{max}}}(s) - G_{xu,V_0}(s))U_c(s)
$$

(16)

$$
W(s) = G_i(s)V(s),
$$

(17)

where $G_i(s)$ is used to model the dynamics of the actuator associated with the change in the voltage of the MR damper, providing a smooth transition between $G_a(s)$ and $G_b(s)$, and is given by:

$$
G_i(s) = \frac{0.2}{0.0048s + 1}.
$$

(18)

Figure 8. Block scheme of the actuator dynamics with bumpless transfer.

Due to the inherent dynamics of the physical system (e.g., time delays), a pre-compensator $G_{\Pi}(s)$ is added to the system for compensation purposes. In this way, the commanded displacement ($u_c$, input to the physical system) is calculated based on the desired displacement ($d$, output from the simulations) and the inverse dynamics of the physical system. As a result, $x \approx d$. A schematic of the compensated system is shown in figure 9.

Once again, two compensators are designed: one for the MR damper operating at $V_0 = 0$ V ($G_{I\Pi,V_0}(s)$) and the other for the damper operating at $V_{\text{max}} = 5$ V ($G_{I\Pi,V_{\text{max}}}(s)$). The transfer functions are given by:

$$
G_{I\Pi,V_0}(s) = \frac{(0.0062s + 1)(2.639 \times 10^{-5}s^2 + 0.0059s + 1)}{(4.129 \times 10^{-4}s + 1)(1.173 \times 10^{-7}s^2 + 3.909 \times 10^{-4}s + 1)}
$$

(19)

$$
G_{I\Pi,V_{\text{max}}}(s) = \frac{(0.0094s + 1)(2.618 \times 10^{-5}s^2 + 0.0058s + 1)}{(6.289 \times 10^{-4}s + 1)(1.164 \times 10^{-7}s^2 + 3.857 \times 10^{-4}s + 1)}
$$

(20)

A similar approach to that of figure 8 is followed to provide a smooth transition between both compensators. The block diagram is shown in figure 10 and the model is described by:

$$
U_{I\Pi}(s) = U_a(s) + U_b(s)W(s)
$$

(21)

$$
U_a(s) = G_{I\Pi,U_a}(s)D(s) = G_{I\Pi,V_0}(s)D(s)
$$

(22)

$$
U_b(s) = G_{I\Pi,U_b}(s)D(s) = (G_{I\Pi,V_{\text{max}}}(s) - G_{I\Pi,V_0}(s))D(s)
$$

(23)

$$
W(s) = G_i(s)V(s)
$$

(24)

Figure 9. Diagram of the complete system with dynamics compensation.
Figure 11. Comparison between the desired, commanded and measured piston displacement.

Table 1. Performance indices.

| Index | Description                                      |
|-------|--------------------------------------------------|
| $J_1$ | $\max_i \left( \frac{\|\dot{x}_1(t)\|}{\|\dot{x}_1(t)\|} \right)$ Normalized peak floor acceleration |
| $J_2$ | $\max_i \left( \frac{\|\ddot{x}_1(t)\|}{\|\ddot{x}_1(t)\|} \right)$ Normed peak acceleration |
| $J_3$ | $\max(\|x_1(t)\|)$ 1st floor peak relative displacement under control |
| $J_4$ | $\max(\|\ddot{x}_1(t)\|)$ 1st floor peak absolute acceleration under control |
| $J_5$ | $\max(\|\dddot{x}_1(t)\|)$ Maximum control force |
| $J_6$ | $(\frac{1}{\tau} \int_0^\tau [\ddot{x}_m(t)]^2 dt)^{1/2}$ RMS control power |

where $G_i(s)$ is used to provide a smooth transition between both compensators:

$$G_i(s) = \frac{0.2}{0.0048s + 1}.$$  (25)

Figure 11 shows a comparison between the desired, the commanded and the measured piston displacement during the execution of an experiment. The lower curve is a close-up of the upper one.

4. QFT controller formulation

To begin the design of the QFT controller, the structure model of equation (4) is divided into two subsystems accounting for
Figure 15. QFT controller closed loop analysis. Upper: robust performance. Lower: disturbance rejection.

Figure 16. Graphical representation of the clipped optimal algorithm.

Figure 17. Example of a hysteresis loop.

Figure 18. Description of the MR damper as an uncertain plant.

Figure 19. Templates of the system of equation (44).

Figure 20. QFT controller initial loop: $G_2(s) = 1$.  

by the following set of equations:

$$S_m: M_s \ddot{x}_m + C_s \dot{x}_m + K_s x_m = B_s x_m + F_g$$  

(26)

$$S_i: m_i \ddot{x}_i + c_{11} \dot{x}_i + k_{11} x_i = -f_{mr} - f_c + F_g$$  

(27)

where $S_m$ stands for the main structure subsystem (the two upper floors) and $S_i$ is the first floor subsystem. The $a$ sub-
and \( f_g \) is the force due to the seismic motion:

\[
f_c = c_{12} \ddot{x}_2 + k_{12} x_2,
\]

(29)

\[
f_g = (c_{11} + c_{12}) \dot{x}_c + (k_{11} + k_{12}) x_c.
\]

(30)

The following propositions about the intrinsic stability of the structure will be used in formulating some control laws (Luo et al 2000):

**Proposition 1.** The unforced main structure subsystem \( S_m \) is globally exponentially stable for any bounded initial conditions.

**Proposition 2.** If the coordinates \( (x, \dot{x}) \) of the base and the coupling term \( B_2 x_{1a} \) are uniformly bounded, then the main structure subsystem is stable and the coordinates \( (\ddot{x}, \dot{x}) \) of the main structure are uniformly bounded for all \( t \geq 0 \) and any bounded initial conditions.

In this way, the controller is designed for the first floor subsystem assuming that it will stabilize the overall system. Now consider the system of equation (27). This equation written in relative coordinates becomes:

\[
m_1 \ddot{x}_1 + c_{11} \dot{x}_1 + c_{12} \ddot{x}_2 + k_{11} x_1 + k_{12} x_2 = -f_m - m_1 \ddot{x}_g.
\]

(31)

Taking the Laplace transform of equation (31) yields:

\[
m_1 s^2 X_1(s) + c_{11} s X_1(s) - c_{12} s X_2(s) + k_{11} X_1(s) + k_{12} X_2(s) - k_{12} X_2(s) = -F_m(s) - m_1 \ddot{X}_g(s).
\]

(32)

Rearranging terms from equation (32) yields:

\[
X_1(s) = \frac{1}{m_1 s^2 + c_{11} + k_{11}}[-F_m(s) + c_{12} s X_2(s) + k_{12} X_2(s) - m_1 \ddot{X}_g(s)].
\]

(33)

Therefore, the plant \( P(s) \) in figure 1 is given by:

\[
P_1 = \frac{1}{m_1 s^2 + c_{11} + k_{11}}
\]

(34)

while the perturbation, i.e. the seismic motion, is the term \( m_1 \ddot{X}_g(s) \). The control input \( U_1(s) \) is given by:

\[
U_1(s) = -F_m(s) + c_{12} s X_2(s) + k_{12} X_2(s)
\]

(35)

from where the MR damper force can be estimated. The controller design specifications are: \( c_{11} = 7243.2 \pm 5\% \text{ N s m}^{-1}, k_{11} = 9932 \pm 5\% \text{ N m}^{-1} \); the frequencies of interest are the natural frequencies of the system, i.e., 1.09, 3.17 and 4.74 Hz. The controller performance should achieve the following bounds: robust performance \( W_{1a} = 2 \) and disturbance rejection \( W_{23} = 3 \times 10^{-2} \).

Figures 12–14 show different stages of the QFT controller design: the templates, the initial and final loops. Figure 15 shows the analysis of the closed loop response for the robust performance and disturbance rejection problems for the range of frequencies studied. The final controller, with input \( X_1(s) \) (the displacement, measured in meters) and output \( U_1(s) \) (force, measured in Newtons) is given by:

\[
G_1(s) = \frac{27541034(0.002s + 1)(0.068s + 1)(1.960s + 1)}{(0.151s + 1)(0.188s + 1)^2}
\]

(36)

The control law of equation (36) cannot be implemented directly because the force to the MR damper cannot be commanded. Instead, a voltage signal must be sent to the damper to approximately generate the desired force. Two approaches are now considered to determine the voltage to the MR damper that can produce the damping force required to mitigate the vibrations.
Figure 23. Records of the El Centro, Loma Prieta and Northridge earthquakes.

Figure 24. Clipped QFT: structure response under the Loma Prieta earthquake.

The first approach is based on the clipped optimal control algorithm by Dyke et al (1996). The algorithm is graphically depicted in figure 16. The dynamics of the MR damper are ignored and the control signal (i.e., the voltage) takes only two values, 0 and 5 V, according to the following algorithm:

\[ v = V_{\text{max}} H \{ (f_{\text{meas}} - f_{\text{meas}}) f_{\text{meas}} \} \]  

where \( H\{\cdot\} \) is the Heaviside function, \( f_{\text{meas}} \) is the force generated by the QFT controller and \( f_{\text{meas}} \) is the actual damping force actuating on the system.

The second approach consists of replacing the term \( F_{\text{mr}}(s) \) with a Laplace representation of the damper dynamics. However, as has been stated throughout the paper, the MR damper is a nonlinear device. To solve this problem, it is proposed to represent the damper as a linear plant with uncertain parameters in such a way that this new representation approximates the nonlinear dynamics. Consider again the Bouc–Wen model of the MR damper of equation (9). It can be decomposed into two parts: one linear and the other nonlinear. Thus:

\[ f_{\text{lin}} = (c_{0a} + c_{0b}u_{0})\dot{x} + (k_{0a} + k_{0b}u_{0})x = a_{1}\dot{x} + a_{2}x \]  

\[ f_{\text{nonlin}} = (\alpha a + \alpha bu)z_{0} = u_{d}z_{0} \]  

\[ f_{\text{meas}} = f_{\text{lin}} + f_{\text{nonlin}} \]  

\[ u_{d} = (\alpha a + \alpha bu) \]  

From equation (38), it is observed that the parameters \( a_{1} \) and \( a_{2} \) vary only with the input voltage. The third parameter, \( z_{0} \) in equation (39), is a bounded parameter (see figure 17). At high velocities, \( z \) is approximately constant and thus, \( z_{0} \) could take either the maximum or the minimum value depending on
the signs of velocity. In this way, equations (38) and (39) can be seen as a plant with 3 uncertain parameters namely, \(a_1\), \(a_2\) and \(z_0\) that describe the dynamics of the damper while \(u\) is the input to this system. In this way, the damper dynamics appear to follow the Bingham model (Zapateiro et al 2007). Figure 18 illustrates this approach with a sinusoidal displacement excitation at 3 levels of voltage.

The representation of the MR damper as an uncertain linear plant can now be incorporated into equation (32). The Laplace transform of equations (38)–(41) yields:

\[
F_{mr}(s) = a_1 S_F S_L s X_{1r}(s) + a_2 S_F S_L X_{1r}(s) + S_F z_0 U_D(s). \tag{42}
\]

Substitution of equation (42) into equation (32) yields:

\[
X_{1r}(s) = \frac{-S_F z_0 U_D(s)}{m_1 s^2 + (c_1 + a_1 S_F S_L) s + (k_{11} + \frac{a_2 S_F S_L}{S_L})} + \frac{c_{12} s X_{2r}(s) + k_{12} X_{2r}(s) - m_1 \ddot{X}_g(s)}{m_1 s^2 + (c_1 + \frac{a_2 S_F S_L}{S_L}) s + (k_{11} + \frac{a_2 S_F S_L}{S_L})}. \tag{43}
\]

The plant \(P_2(s) (P(s) in figure 1) is now given by:

\[
P_2(s) = \frac{S_F z_0}{m_1 s^2 + (c_1 + a_1 S_F S_L) s + (k_{11} + a_2 S_F S_L)}. \tag{44}
\]
and the voltage can be estimated by manipulating the following equation:

\[ U_i(s) = -U_D(s) + \frac{1}{\nu_{c_0}}(c_{12}X_2(s) + k_{12}X_2(s)). \] (45)

The uncertain parameters and QFT controller specifications are: \( a_1 = [754.41, 4318.06] \) N s m\(^{-1}\), \( a_2 = [1137.57, 685.05] \) N m\(^{-1}\) and \( z_0 = [-1.11, 1.11] \) m. The frequencies of interest are the natural frequencies of the system, i.e., 1.09, 3.17 and 4.74 Hz. The controller performance should accomplish the following bounds: robust performance \( W_{s1} = 2 \) and disturbance rejection \( W_{s3} = 3 \times 10^{-2} \).

Figures 19–21 depict different stages of the controller design: templates and initial approach (\( G_2(s) = 0 \)) and final loop. Figure 22 shows the analysis of the closed loop response for the robust performance and disturbance rejection problems for the range of frequencies studied. The final controller \( G_2(s) \) with a displacement input measured in meters and output \( U_i(s) \) (measured in Newton-meter), is given by:

\[ G_2(s) = \frac{298(0.0165s^2 + 0.073s + 1)(7.3 \times 10^{-4}s^2 + 0.051s + 1)(4.7 \times 10^{-3}s^2 + 2.15 \times 10^{-3}s + 1)}{(0.017s + 1)(0.033s + 1)(0.015s^2 + 0.095s + 1)} \times (4.65 \times 10^{-3}s^2 + 0.060s + 1)^{-1}. \] (46)

5. Experimental results

The QFT controllers are now tested in the real-time hybrid testing setup described previously. The numerical model, i.e. the 3-story building and the controller, are implemented in Matlab/Simulink. The ordinary differential equation solver used is the 4th order Runge–Kutta method with a time step \( T_s = 5 \times 10^{-4} \) s. The structure is subject to three different earthquake records, namely, El Centro, Loma Prieta and Northridge as shown in figure 23; the scale amplitude used is 0.4. Table 1 shows the performance indices used to evaluate the controller performance. Indices \( J_1 \) and \( J_2 \) compare the first floor acceleration in the controlled case and the maximum acceleration reached by any floor in the uncontrolled case. Indices \( J_3 \) and \( J_4 \) are the first floor peak displacement and acceleration respectively in the controlled case. Indices \( J_5 \) and \( J_6 \) measure the control effort.

The performance indices for the different seismic excitations are shown in table 2. Figures 24–27 show the structure response and the MR damper performance when subject to the Loma Prieta seismic excitation. Figures 25 and 27 show the performance of the MR damper (the actual damper, i.e. not scaled), and particularly, a comparison of the dynamics predicted by the Bouc–Wen model and that obtained experimentally.

Performance indices \( J_1 - J_4 \) show that both controllers have a similar performance and in most cases, the QFT controllers
based on the MR damper dynamics is better than the other. However, performance indices $J_3$ and $J_6$ show that for this controller to perform better in reducing the structure response, it makes use of a greater control effort. According to the performance indices and the structure response, the control objectives were satisfactorily accomplished, i.e. a reduction in displacement and acceleration response was achieved with both QFT controllers.

6. Conclusions

In this paper, a new semiactive controller based on the quantitative feedback theory method has been proposed to reduce the vibrations in a 3-story building equipped with an MR damper. Two variations were presented: one based on a modification of the clipped optimal algorithm and another based on the nonlinear dynamics of the MR damper. The controllers were experimentally tested in a real-time hybrid testing setup. Both controllers successfully achieved the proposed goal of reducing the structure response when subject to a seismic motion.

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