CONTRIBUTION OF HIGH-MASS BLACK HOLES TO MergERS OF COMPACT BINARIES

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ABSTRACT

We consider the merging of compact binaries consisting of a high-mass black hole and a neutron star. From stellar evolutionary calculations that include mass loss, we estimate that a zero-age main sequence (ZAMS) mass of \( \gtrsim 80 \, M_\odot \) is necessary before a high-mass black hole can result from a massive O star progenitor. We first consider how Cyg X-1, with its measured orbital radius of \( \sim 17 \, R_\odot \), might evolve. Although this radius is substantially less than the initial distance of two O stars, it is still so large that the resulting compact objects will merge only if an eccentricity close to unity results from a high kick velocity of the neutron star in the final supernova explosion. We estimate the probability of the necessary eccentricity to be \( \sim 1\% \); i.e., 99% of the time the explosion of a Cyg X-1-type object will end as a binary of compact stars, which will not merge in Hubble time (unless the orbit is tightened in common envelope evolution, which we discuss later). Although we predict \( \sim 7 \) massive binaries of Cyg X-1 type, we argue that only Cyg X-1 is narrow enough to be observed, and that only Cyg X-1 has an appreciable chance of merging in Hubble time. This gives us a merging rate of \( \sim 3 \times 10^{-8} \, \text{yr}^{-1} \) in the galaxy, the order of magnitude of the merging rate found by computer-driven population syntheses, if extrapolated to our mass limit of \( 80 \, M_\odot \) ZAMS mass for high-mass black hole formation. Furthermore, in both our calculation and in those of population syntheses, almost all of the mergings involve an eccentricity close to unity in the final explosion of the O star. From this first part of our development we obtain only a negligible contribution to our final results for mergers, and it turns out to be irrelevant for our final results. In our main development, instead of relying on observed binaries, we consider the general evolution of binaries of massive stars. The critical stage is when the more massive star A has become a black hole and the less massive star B is a giant reaching out to A. We then have a common envelope, and we expect hypercritical accretion to star A. Star A will accept a small fraction of the mass of the envelope of star B, but it will plunge deep into star B while expelling the envelope of star B. We expect that star B can at least be in the mass range \( 15 \sim 35 \, M_\odot \), while the black hole A has a mass of \( 10 \, M_\odot \). About 20% of the binaries of this type are found to end up in a range of orbital radii favorable for merging; i.e., outside of the relevant Roche lobes, but close enough so that these final binaries of compact objects will merge in Hubble time. The narrow black hole O star orbits do not seem to be found in population syntheses, because in them mergers happen almost completely as a result of kick velocities. In the exception (case H of Portegies Zwart & Yungelson, which includes hypercritical accretion), common envelope evolution is more effective and we are in agreement with their results. We find that the high-mass black hole neutron star systems contribute substantially to the predicted observational frequency of gravitational waves. We discuss how our high-mass black hole formation can be reconciled with the requirements of nucleosynthesis, and we indicate that a bimodal distribution of masses of black holes in single stars can account, at least qualitatively, for the many transient sources that contain high-mass black holes.

Subject headings: binaries: close — black hole physics — gravitation — stars: neutron

1. INTRODUCTION

The supernova (SN) community once believed that stars above a certain mass, about 30–40 \( M_\odot \) zero-age main sequence (ZAMS), will collapse into a massive black hole (MBH) of mass of order 10 \( M_\odot \). The argument was that in these stars the mantle was bound with a binding energy well above 10^{51} \( \text{ergs} \) so that the SN shock was not strong enough to expel it.

Whereas this may be true for single stars, Woosley, Langer, & Weaver (1995) showed that in binaries, where the hydrogen envelope of the primary star has been transferred to the companion in Roche lobe overflow (RLOF), the evolution of the resulting "naked" He star (i.e., the star without a hydrogen envelope) led to a substantially smaller pre-SN core than that of a single star with a hydrogen envelope. A comparison of compact core masses from naked He stars and those evolved by Woosley & Weaver (1995) for single stars is shown in Figure 1, taken from Brown, Weingartner, & Wijers (1996). Detailed reasons for the great difference in the evolution of "clothed" and naked He cores are given in Woosley et al. (1995).

Stars with ZAMS masses \( \gtrsim 40 \, M_\odot \) lose their masses by strong winds, whether in binaries or not, and become Wolf-Rayet (W-R) stars. In an earlier paper, Woosley, Langer, & Weaver (1993) investigated ZAMS masses of 35, 40, 60, and 85 \( M_\odot \). In those up through 60 \( M_\odot \), the hydrogen envelope was blown off early enough for the He cores to evolve as naked cores, and compact core masses were around 1.5 \( M_\odot \).
back in the later SN explosion can be made as in Brown, Weingartner, & Wijers (1996). If the center collapses into a black hole, the outer part of the star has no support and will also collapse. We then get a massive black hole containing the entire mass of the pre-SN star, perhaps of order \( \lesssim 10 M_\odot \). (At least in binaries the companion star will have removed the hydrogen envelope by either RLOF or by common envelope evolution, depending on the mass of the companion, and wind will diminish the He core before the SN explosion.)

If the mass of the core is less than \( M_{pc} \), the electrons will be captured by protons,

\[
p + e^- \rightarrow n + \nu ,
\]

and the neutrinos will diffuse out of the core. This process takes of order 10 s, as has been shown by the duration of the neutrino signal from SN 1987A. The result is a neutron star with a small concentration of protons and electrons. The Fermi pressures of the core are chiefly from the nucleons, with small correction from the electron. On the other hand, the nucleon energy is increased by the symmetry energy; i.e., by the fact that we now have nearly pure neutrons instead of an approximately equal number of neutrons and protons. Thorsson et al. (1994) have calculated that the maximum mass of the neutron star \( M_{ns} \) is still about \( 1.8 M_\odot \); i.e., the symmetry energy compensates the loss of the Fermi energy of the leptons. Corrections for thermal pressure are small (Prakash et al. 1997).

The important fact is that the 10 s of neutrino diffusion from the core give ample time for the development of a shock, which expels most of the mass of the progenitor star.

But this is not the end of the story. The neutrons can convert into protons plus \( K^- \) mesons,

\[
n \rightarrow p + K^- .
\]

Since the density at the center of the neutron star is very high, the energy of the \( K^- \) is very low, as confirmed by Li, Lee, & Brown (1997) using experimental data. By this conversion the nucleons can again become half neutrons and half protons, thereby saving the symmetry energy needed for pure neutron matter. The \( K^- \), which are bosons, will condense, saving the kinetic energy of the electrons they replace. The reaction (eq. [2]) will be slow, since it is preceded by

\[
e^- \rightarrow K^- + \nu ,
\]

which is actually effected in the star by reactions such as \( e^- + p \rightarrow K^- + p + \nu \), with strangeness breaking. (Times are long enough for chemical equilibrium to be realized.) It becomes energetically advantageous to replace the fermion-electrons by the bosonic \( K^- \) at higher densities. Initially, the neutrino states in the neutron star are filled up to the neutrino chemical potential with trapped neutrinos, and it takes some seconds for them to leave the star. These must leave before new neutrinos can be formed from the process (eq. [3]). Thorsson et al. (1994) have calculated that the maximum mass of a star in which reaction (2) has gone to completion is

\[
M_{np} \approx 1.5 M_\odot ,
\]

where the lower suffix “np” denotes their nearly equal content of neutrons and protons, although we continue to
use the usual name of neutron star. This is the maximum mass of neutron stars, which is to be compared with the masses determined in binaries. The masses of 19 neutron stars determined in radio pulsars (Thorsett & Chakrabarty 1999) are consistent with this maximum mass.

The core mass $M_c$ formed by the collapse of SN must, therefore, be compared to the two limiting masses, $M_{pc}$ and $M_{np}$. If

$$ (1) \quad M_c > M_{pc},$$

we get a high-mass black hole of mass essentially equal to the full mass of the pre-SN star. If

$$ (2) \quad M_{pc} > M_c > M_{np},$$

we get a low-mass black hole of mass $M_c$. Only if

$$ (3) \quad M_c < M_{np}$$
do we get a neutron (more precisely, “nucleon”) star from the SN. Only in this case can we observe a pulsar. In cases (2) and (3), only initial neutrinos from electrons captured in the collapse before $M_c$ becomes greater than $M_{pc}$ can be observed, but no light would reach us.

We tentatively choose the lower limit of ZAMS mass for making MBHs to be 80 $M_\odot$. On the other hand, it is believed that ZAMS above 100 $M_\odot$ do not exist because of excessive formation of electron pairs. So we assume that the range of 80–100 $M_\odot$ is available.

### 3. RATE OF FORMATION

We are interested in massive binaries containing one star of ZAMS mass between 80 and 100 $M_\odot$. As in Bethe & Brown (1998), we start from the assumption that there is one SN per century per galaxy in a binary. Assuming also that 10 $M_\odot$ is required for a star to end up as an SN of Type II (or Ib or Ic), the formation of binaries of $M > 10 M_\odot$ is also $10^{-2} \text{ yr}^{-1} \text{ galaxy}^{-1}$. We assume a Salpeter function with index $n = 1.5$; then the fraction of such stars between 80 and 100 $M_\odot$ is

$$ f = 8^{-3/2} 10^{-3/2} = 1.26 \times 10^{-2}. $$

So the rate of formation is

$$ \alpha_1 = 1.26 \times 10^{-4} \text{ yr}^{-1} \text{ galaxy}^{-1}. $$

We require that this star $A$ be accompanied by a companion $B$ of $M > 10 M_\odot$. Assuming that the distribution of mass of the companion is $dq$, with

$$ q = M_B/M_A, $$

and having assumed $M_A = 90 M_\odot$ on average, we need $q > \frac{1}{2}$, which has a probability

$$ 1 - q = 0.9, $$

hence a formation rate

$$ \alpha_2 = (1 - q)\alpha_1 = 1.13 \times 10^{-4} \text{ yr}^{-1}. $$

In order to observe strong X-rays from the MBH, the distance $a$ between star A (the MBH) and star B (an O or B star) should not be too large, let us say less than 150 $R_\odot$. On the other hand, it must not be less than 30 $R_\odot$, because otherwise the two stars would merge already at this stage of evolution. Assuming as in Bethe & Brown (1998) a distribution $da/7a$, the probability that $a$ falls in the desired limits is

$$ p = 7^{-1} \ln(150/30) = 0.23, $$
giving for the probability of formation (per galaxy)

$$ \alpha_3 = 0.23\alpha_2 = 2.6 \times 10^{-5} \text{ yr}^{-1}. $$

Although we take the same logarithmic distribution as in Bethe & Brown (1998), we think of the lower limit as greater than $2 \times 10^7$ km so that the more massive stars we deal with here lie inside their Roche lobes, and the upper limit as less than $2 \times 10^{10}$ km, although the latter is uncertain because O star binaries probably will not be recognized for such a large separation (Garmany, Conti, & Massey 1980). High-mass transfers from the O star occur only when it nearly fills its Roche lobe. It is then bright for a time (Massevich et al. 1979)

$$ \tau = 2.7 \times 10^5 \text{ yr}, $$

so the expected number of strong X-ray–emitting binaries in the galaxy is

$$ N = \alpha_3 \tau = 7. $$

Only one such binary has been observed, Cyg X-1, at a distance of 2.5 kpc. It might be thought that X-ray binaries at larger distances have escaped detection because of absorption of X-rays in the highly ionized galactic medium. But, in fact, X-rays of energy greater than 3 keV should be seen throughout the galaxy, penetrating even the Galactic disc. On the other hand, we are probably seeing the closest binary of Cyg X-1 type, with orbital separation only about half of the initial separation in the double O star progenitor. Thus other such objects may well have substantially lower luminosities. The O star in such a binary would probably be in the disc and might not be observed at greater distances, so that the X-rays could not be associated with the binary. In any case, it is clear that in the disc there is only one bright Cyg X-1 type object, a fact we use below.

In the Magellanic clouds with less than 1/10 the mass of the galaxy, two high-mass black hole binaries, LMC X-3 with a B star companion and LMC X-1, where the donor is probably an O star, have been observed. We believe the high incidence of high-mass black holes in the LMC to be not only a consequence of the large amount of star formation there, but also of the low metallicity compared with solar. We return to a discussion of high-mass black hole formation in §8.

### 4. MERGERS

In the case of low-mass black holes, the rate of mergers is limited chiefly by the disruption of the binary due to the recoil that star B experiences when it goes SN. For MBHs this is of little concern since the orbital velocity of the binary before the SN process is already of order 600 km s$^{-1}$ (see below), comparable to the higher recoil velocities, so that the recoil is unlikely to disrupt the binary.

Instead of this, the concern for MBHs is that two stars of the binary may be too far apart so they will not merge.
Contributions of mergers of binaries

during Hubble time (assumed to be $10^{10}$ yr). Note that the progenitor binary must have a separation greater than 30 $R_\odot$ in order that the two O stars do not fill their Roche lobes. At present, the separation in Cyg X-1 is only slightly more than half this, which is permissible because now only the O star needs to be in its Roche lobe, and the black hole has negligible radius. But as we show below, even this small separation is still large enough to substantially cut down the probability of merging within Hubble time. Because of rapid mass loss in such massive stars of ZAMS 80–100 $M_\odot$, there is no well-established procedure for calculating their evolution in binaries. Starting from a separation greater than 30 $M_\odot$, mass exchange by the very massive star with the lower mass companion when the former is on the main sequence (case A mass transfer) could decrease the separation substantially, perhaps causing the stars to coalesce. However, the resulting binary would then widen (roughly back to its original separation) as the W-R, which remains after mass transfer from the very massive star, loses mass by wind. We do not have any evidence of the net change (S. Portegies Zwart 1998, private communication).

Because of the uncertainties in the evolutionary scenario, we adopt an empirical approach beginning from the measured separation $a_i$ in Cyg X-1. The requirement that the two stars merge in Hubble time then sets a lower limit on the eccentricity $e$ of the final neutron star black hole orbit:

$$a_i(1 - e^2) < a_0,$$  

where $a_i$ is the semimajor axis of the orbit after the SN event, and $e$ is the eccentricity of the orbit; $a_0$ will be calculated in § 5.

The left-hand side of equation (17) is related to the angular momentum $J$ of the post-SN orbit. In fact,

$$J = a_2 V_3(1 - e^2)^{1/2},$$  

where $V_3$ is the orbital velocity if the orbit were circular. (We do not need to consider the mass of the system.) By Newton’s laws,

$$a_2 V_3^2 = G(M_A + M_n),$$  

where $M_A$ is the mass of the MBH and $M_n$ is that of the neutron star resulting from the SN of star B. Hence

$$J^2 = G(M_A + M_n)a_2(1 - e^2).$$  

On the other hand, immediately after the SN the distance between stars A and B is still $a_1$, and the relative velocity is

$$V_2 = V_1 + Q,$$  

where $Q$ is the recoil velocity due to the SN. The angular momentum is

$$J = a_1 \times V_2,$$

$$J^2 = a_1^2 V_2^2 \sin^2 \psi,$$  

where $\psi$ is the angle between $V_2$ and $a_1$.

This angle can be calculated, but it is complicated. We now make the approximation of replacing $\sin^2 \psi$ by

$$\langle \sin^2 \psi \rangle = \frac{3}{2}.$$  

Then, equating equations (20) and (22) and using equation (17),

$$a_1^2 V_2^2 < \frac{3}{2} G(M_A + M_n)a_0.$$  

Equation (23) involves a double approximation: (1) $\psi$ is uniformly distributed over the sphere, and (2) only the average of $\sin^2 \psi$ is used instead of calculating the limit on $a_1 V_2$ separately for each possible direction of $Q$ and then averaging the results. We believe that a correct calculation would give a somewhat higher limit on $a_1 V_2$.

The product $a_1 V_2^2$ can be calculated assuming that the orbit of the binary is circular before the SN event, an assumption that is probably very nearly correct. Then, similar to equation (19),

$$a_1 V_2^2 = G(M_A + M_B),$$  

where $M_B$ is the mass of star B, the companion of the MBH, before the SN event. In Cyg X-1, it is believed that the mass of the MBH is $M_A = 10 M_\odot$, and that of the O star companion is $M_B = 17 M_\odot$ (Herrero et al. 1995). After RLOF of the O star,¹ these masses happen to be just exchanged, assuming conservative mass transfer, so that immediately before the SN

$$M_A = 17 M_\odot, \quad M_B = 10 M_\odot.$$  

However, we shall use “standard” values

$$M_A = M_B = 10 M_\odot.$$  

The SN changes star B to a neutron star, $M_n = 1.4 M_\odot$, while star A remains unchanged. We now have

$$a_1^2 V_1^2 = G(M_A + M_B)a_1.$$  

Dividing equation (24) by equation (28),

$$\frac{V_2^2}{V_1^2} < \frac{3}{2} \left( \frac{M_A + M_n}{M_A + M_B} \right) \frac{a_0}{a_1} = B.$$  

Usually, $a_i > a_0$, so $V_2 < V_1$. From equation (21), this means that the recoil velocity $Q$ of the SN must be approximately opposite to the pre-SN orbital velocity $V_1$ and of the same order of magnitude. Setting

$$Q \cdot V_1 = -Q V \mu,$$

$$Q/V_1 = \omega,$$  

thus equation (29) becomes

$$1 + \omega^2 - 2\omega \mu < B$$  

$$1 - \mu < \frac{B - (1 - \omega)^2}{2\omega}.$$  

Since B is, in general, considerably smaller than 1, we thus get the condition

$$|1 - \omega| < B^{1/2},$$  

¹ Later we shall show that common envelope evolution of the black hole in the evolving O star is more likely.
stating once more that \( Q \) must be of the same order as \( V_1 \). The fraction of the total solid angle available to \( Q \) is \((1 - \mu)/2\).

Now, according to equation (28),
\[
V_1^2 = \frac{2}{3} \times 10^{-7} \times 20 \times 2 \times 10^{33} \frac{a_1}{a_1} = \frac{2.7 \times 10^{15}}{10^{12}} \frac{a_1}{a_1},
\]
(35)
\[
V_1 = 520 \text{ km s}^{-1} \left(\frac{10^{12}}{a_1}\right)^{1/2}.
\]
(36)

In Cyg X-1, the distance between the two stars is
\[
a_1 = 17 \ R_\odot = 1.2 \times 10^{12} \text{ cm}.
\]
(37)

So \( a_1 \) is likely to be of order \( 10^{12} \). This means \( V_1 \) is of order 500 km s\(^{-1}\), and \( Q \) is likewise. Now Cordes \& Chernoff (1997) have found that the recoil velocities of pulsars are distributed bimodially with \( \sigma = 175 \) and 700 km s\(^{-1}\), respectively. The smaller velocity Gaussian is negligible for our \( Q \). For the larger \( \sigma \), we set
\[
x = Q/\sigma.
\]
(38)

The Gaussian distribution is
\[
f dx = 2\pi^{-1/2} x^2 \text{d}x e^{-x^2}.
\]
(39)

We assume \( V_1 = \sigma \) for the Gaussian with the greater \( \sigma \), and then for this Gaussian \( \omega = x \), and the probability is (cf. eq. [33])
\[
\int \frac{1}{4\pi} \left[ B - (1 - x^2) \right] f dx \approx \frac{2\sigma^1}{3\sqrt{\pi}} B^{3/2} = 0.14 B^{3/2}.
\]
(40)

For large \( B \), substitute 0.9 for \( B^{3/2} \). Considering that only 20\% of pulsar recoils are in the larger \( \sigma \) Gaussian, the fraction of binaries containing massive black holes that merge during Hubble time is
\[
\phi = 0.03 B^{3/2},
\]
(41)

with \( B \) given by equation (29). Inserting our values for \( M_A, M_B \), and \( M_n \),
\[
B = 0.85 a_0 / a_1.
\]
(42)

It remains to calculate \( a_0 \).

5. NUMERICAL MERGERS

If the orbit is circular, a merger will take place within Hubble time if the initial distance is less than \( R_{max} \), with \( R_{max} \) given by equation (7.7) of Bethe \& Brown (1998):
\[
R_{max}^4 = 1.6 \times 10^{25} \text{ km}^4 \frac{M_A M_B (M_A + M_B)}{M_\odot^3}.
\]
(43)

\( R \) is the same as the semimajor axis introduced in \( \S \) 4, so \( a_0 = R_{max} \). If the orbit is eccentric, the merger time for a given major axis is diminished by a factor (P. Eggleton 1998, private communication)
\[
Z(e) = (1 - e^2)^{3.689 - 0.243 e - 0.058 e^2}.
\]
(44)

Therefore, the condition on the initial semimajor axis, \( a_2 \), is
\[
a_2^4 Z(e) < R_{max}^4.
\]
(45)

Taking the fourth root,
\[
a_2 Z^{1/4} < R_{max}.
\]
(46)

\( Z^{1/4} \) is almost \( 1 - e^2 \), so the left-hand side of equation (46) is almost \( a_2 (1 - e^2) \), thus
\[
a_2 (1 - e^2) < R_{max} (1 - e^2)^{0.0777 + 0.0608e + 0.0195e^2}.
\]
(47)
The last factor depends only slightly on the eccentricity. So we replace it in \( e^2 \) by an average of 0.5. This makes that factor
\[
0.5^{0.130} = 0.914.
\]
(48)

Inserting in equation (43) \( M_A = 10 \ M_\odot, M_B = 1.4 \ M_\odot, \)
\[
R_{max} = 7.1 \times 10^6 \text{ km}.
\]
(49)
The upper limit of \( a_2 (1 - e^2) \) is the definition of \( a_0 \), according to equation (17). Thus the right-hand side of equation (47) equals \( a_0 \), and we have
\[
a_0 = 6.5 \times 10^6 \text{ km},
\]
(50)

and equation (29) becomes
\[
B = 5.1 \times 10^6 \text{ km} / a_1.
\]
(51)

The measured distance between the two stars in Cyg X-1 is \( 1.2 \times 10^7 \) km. If this distance does not change in RLOF, then \( a_1 = 1.2 \times 10^7 \) km,
\[
B = 0.42,
\]
(52)
\[
\phi = 0.009.
\]
(53)

This estimate can be checked from Figure 4a of Kalogera (1996). In our situation, her parameters are \( \beta \approx 0.6, \xi \approx 0.9. \)

Using the Eggleton formula (eq. [44]), we find that \( e \gtrsim 0.76 \) for a merger. The average survival probability is \( \sim 0.3 \) for these eccentricities, and our \( \xi \) is \( \sim 0.3 \), so that the final probability is \( \frac{1}{2} \times 0.24 \times 0.3 \approx 0.014 \), close to our \( \phi \).

Multiplying this by the formation rate \( x \), equation (14), we find for the rate of mergers
\[
x \phi = 2 \times 10^{-7} \text{ yr}^{-1}.
\]
(54)

So this is a rare event, as we might expect. In fact, equation (54) is probably still an overestimate because we predicted seven X-ray-emitting binaries in the galaxy (eq. [16]), and we see only Cyg X-1. In our assumed distribution of these objects out to 150 \( R_\odot \), the others would be expected to have substantially larger \( a_1 \) than Cyg X-1, in which case their merger rate would be negligible. Thus a possible guess of the rate of mergers is not much larger than 1/7 of equation (54).

We compare our result with results from population synthesis:

1. First, we note that the smallness of the rate of mergers results from the separation at the black hole O star stage being too large for merger (for spherical orbits) without a high eccentricity from kick velocities. Whereas Portegies Zwart \& Yungelson (1998) obtain a (high-mass) black hole neutron star birthrate of \( 1.8 \times 10^{-5} \) yr\(^{-1}\) in the galaxy without kick velocities, their merger rate is zero for this case. Lipunov, Postnov, \& Prokhorov (1997) find an order
of magnitude greater merger rate with inclusion of kick velocities than without.

2. Portegies Zwart & Yungelson (1998)² use 40 $M_\odot$ for their ZAMS mass for the black hole limit, and we compare with those for 40 $M_\odot$ from Lipunov et al. (1997). Increasing the mass limit to 80 $M_\odot$ and taking into account the factor $1 - q$ would decrease the results of these authors by a factor of ~5. Thus Portegies Zwart & Yungelson (1998) would have a merging rate of $2 \times 10^{-7}$ yr$^{-1}$, and Lipunov et al. (1997) would have one of ~$4 \times 10^{-8}$ yr$^{-1}$ for our mass limit of 80 $M_\odot$. Our rate obtained by taking $q$ of equation (54) is

$$R = 3 \times 10^{-8} \text{ yr}^{-1},$$

about the same as the latter. We believe that our simple considerations give some understanding of the results obtained in the latter population synthesis, and we shall return to a discussion of the work of Portegies Zwart & Yungelson (1998) later.

We can also try an “observational estimate” of the merging of Cyg X-1 type objects, although we have only one such object near its Roche lobe. Assuming, as in equation (15), Cyg X-1 to be bright for a time $\tau = 2.7 \times 10^5$ yr, the birth rate of such bright objects is

$$\tau^{-1} = 3.7 \times 10^{-6} \text{ yr s}^{-1}.$$  

Given the merging probability of the resulting black hole neutron star binary $\phi = 0.009$ from equation (53), we arrive at a merging rate of $4 \times 10^{-8}$ yr$^{-1}$ for the Galaxy, in rough agreement with equation (55). This may be an underestimate since Cyg X-1 will probably go into stable mass transfer after losing substantial mass to the black hole. LMC X-3 with roughly equal black hole and B star masses (Kuiper et al. 1997) may already be in stable mass transfer. Thus the $2.7 \times 10^5$ yr may well be an underestimate.

Although the last two sections give results for mergers that are negligible compared to our final results in §7, once hypercritical accretion is included in our common envelope evolution we believe that they are instructive in that they match rather well the results of population synthesis, especially those of Lipunov et al. (1997). In Portegies Zwart & Yungelson (1998), (generally unstable) mass transfer starts on the dynamical timescale of the donor, changing along the way to the thermal timescale, and the last part of the mass is sometimes even transferred on nuclear timescales (S. Portegies Zwart 1998, private communication). Their initial separations of the O star binaries are nearly double our separations. (We take ours before substantial main-sequence burning, which increases the stellar radius by a factor of ~2.) Thus their common envelope evolution does tighten the orbits, but clearly not enough for a substantial merger of the compact objects evolved later. The exception is case H of Portegies Zwart & Yungelson (1998), the case similar to ours with inclusion of hypercritical accretion, which gives a merger rate ~35 times that of cases B–G. We develop in the next section why hypercritical accretion is so effective.

6. COMMON ENVELOPE EVOLUTION

In the last two sections we have addressed the evolution of massive binaries in which the primary evolves into a high-mass black hole in an empirical way. We match rather well the population synthesis results of Lipunov et al. (1997). In particular, these authors find that introduction of kick velocities increases their merging rate by an order of magnitude. Portegies Zwart & Yungelson (1998) find zero mergers without kick velocities. We believe this to result from the fact that their black hole O star binaries lie too far apart for the resulting binary of compact objects to merge in Hubble time without the increase in merger distance given by an eccentricity close to unity; i.e., these authors have very little (or no) merging from circular black hole neutron star orbits, except for their case H that includes hypercritical accretion, which we discuss below.

Clearly the population syntheses of Lipunov et al. (1997) do not include efficient common envelope evolution that brings the compact object (high-mass black hole) close to the He core of the companion star.

We now develop our scenario for common envelope evolution, which hypercritical accretion makes particularly plausible, in that we have quantitative control over the crucial quantities. As the envelope of the giant expands to meet the compact object one can see from Rasio & Livio (1996) that the common envelope evolution begins rather quickly. Soon after the beginning of mass transfer the compact object creates a tidal bulge in the evolving companion, transferring angular momentum to the companion. As the giant companion loses mass, the isentropic envelope responds by expanding. The compact object plunges into the companion, the chief loss in orbital energy occurring in ~1 yr in time (Terman, Taam, & Hernquist 1995; Rasio & Livio 1996).

In the literature, common envelope evolution is thought to occur when the two stars involved differ in mass more than a factor of 2–3. This condition arises because at higher mass ratios the mass-receiving star will then not be able to accept the large amount of mass transferred to it on the short thermal timescale of the companion. This thermal timescale is (van den Heuvel 1994)

$$\tau_{\text{th}} = \frac{GM^2}{RL} \sim 3 \times 10^7 \left(\frac{M}{M_\odot}\right)^2.$$  

In fact, the factor of 2–3 in $q$ initially came from a factor of ~10 in $\tau_{\text{th}}$ from the work of Kippenhahn & Meyer-Hofmeister (1977) who considered case A mass transfer (during main-sequence evolution). In case B mass transfer, the transfer takes place as the giant traverses the Herzsprung gap, which is much more quickly than in case A, so it is not clear why $q$ could not be substantially reduced and still have common envelope evolution.

The situation with hypercritical accretion is different from either case A or case B (RLOF) mass transfer. Initially, our ~10 $M_\odot$ black hole is met by the expanding red giant or supergiant envelope. The black hole accretes some of the matter and transfers enough energy to the remaining matter to expel it. The convective envelope has constant entropy and must expand in order to replace the accreted and expelled matter. The black hole drops in gravitational potential closer to the He core of the companion in order to furnish the necessary energy to expel most of the matter. This whole process happens very rapidly. (In our case of

² We compare with their case H in which hypercritical accretion was included in the next section.
hypercritical accretion, an accretion disc will be set up around the black hole, which cannot immediately accept the matter because of the high angular momentum of the latter. We assume the viscosity to be high enough so that angular momentum, but little mass, will be advected outward. Observed masses of \( \sim 7 M_\odot \) of high-mass black holes in transient sources discussed by Brown, Lee, & Bethe (1999 substantiate this scenario.)

Hypercritical accretion sets in when the envelope density reaches \( \sim 10^{-9} \text{ cm}^{-3} \). From equations (5.3) and (5.7) of Bethe & Brown (1998), we can show that the contribution to the coefficient of dynamical friction \( c_d \) from matter inside the accretion radius \( R_{ac} \), i.e., from matter that can accrete onto the compact object, is \( \delta c_d = 2 \). Now the total coefficient of dynamical friction is \( c_d = 6-8 \) (Ruffert 1994; Ruffert & Arnett 1994). The remainder of \( c_d \) comes chiefly from the wake, at greater distances than \( R_{ac} \). Thus for

\[
c_d \gg 2 , \tag{58}
\]

we encounter highly nonconservative mass transfer, and it is plausible that common envelope evolution ensues for a wider range of \( q \) than usually thought. In fact, as we show below, out of the \( \sim 20 M_\odot \) hydrogen envelope of a 30 \( M_\odot \) companion star, only \( \sim 3 M_\odot \) is accepted by the black hole.

From the above argumentation we believe that \( c_d \gg 2 \) strongly favors common envelope evolution. (For \( c_d = 2 \), the mass transfer is nearly conservative.) The companion star should have mass less than 35–40 \( M_\odot \), because for higher masses it loses mass in an LBV phase. It should have somewhat higher mass than the black hole, say, 15 \( M_\odot \) in our case, so that mass transfer is unstable throughout the transfer of envelope mass, but the final transfer is so rapid that it will overshoot into the range of stable mass transfer. As noted, Cyg X-1 is probably included in our interval of masses favorable for common envelope evolution.

Having established the plausibility of common envelope evolution for the black hole in the expanding H envelope of an O star with ZAMS mass of 15–35 \( M_\odot \), we now carry out this common envelope evolution for a typical O star mass of 20 \( M_\odot \) following Bethe & Brown (1998). The He core is 6 \( M_\odot \).

We choose the coefficient of dynamical friction \( c_d \) to be 6. In the Bethe & Brown (1998) notation, \( M_A \) is the compact object mass, \( M_B \) the companion mass, and

\[
Y = M_B a^{-1} . \tag{59}
\]

The initial and final \( Y \) are related by

\[
\left( \frac{Y_f}{Y_i} \right)^{1+1/(c_d-1)} = \frac{2.4 M_B}{M_A} , \tag{60}
\]

depending only on the initial masses. The ratio of final to initial black hole masses is given by

\[
\frac{M_{A_f}}{M_{A_i}} = \left( \frac{Y_f}{Y_i} \right)^{1/(c_d-1)} = \left( \frac{Y_f}{Y_i} \right)^{1/5} = 1.3 , \tag{61}
\]

so the final black hole mass is 13 \( M_\odot \), for an initial 10 \( M_\odot \). We find

\[
a_i = \frac{M_{B_f}}{M_{B_f} Y_f} = 12 , \tag{62}
\]

where \( M_{B_f} \) is the companion He core mass of 6 \( M_\odot \).

7. MERGERS AFTER COMMON ENVELOPE

The most important result of the calculation at the end of § 6 is that \( a_i/a_f \) is large, of the order of 12. In equation (62), there are two factors: \( Y_f/Y_i \) is fairly large, about 4; \( M_B/M_{B_f} \) is also fairly large, the ratio of the entire mass of the star to its He core taken to be (0.3) \(-1 \) by Bethe & Brown (1998).

Thus the radius of the orbit shrinks by a large factor in the common envelope. We know the range of the final radius \( a_f \): its maximum is given by the condition that the merger should occur within Hubble time. According to equation (49), it would be about \( 7 \times 10^6 \) km if the orbit were circular. Eccentricity may raise this about 30%,

\[
a_f^\text{max} \approx 9 \times 10^{11} \text{ cm} . \tag{63}
\]

The minimum \( a_f \) is some multiple of the radius of the He star, which star B becomes after removal of its hydrogen envelope. The He star radius may be about \( 5 \times 10^{10} \), and we estimate

\[
a_f^\text{min} = (1.5-3) \times 10^{11} \text{ cm} , \tag{64}
\]

so \( a_f^\text{max}/a_f^\text{min} = 3 \sim 6 \). We assume the distribution of \( a \) to be \( da/7a \), so the probability of having \( a_f \) in the permitted range is

\[
p = \frac{\ln 3}{7} - \frac{\ln 6}{7} = 0.15 \sim 0.25 . \tag{65}
\]

The center of the useful range is at about \( 5 \times 10^{11} \) cm. Multiplying by \( a_i/a_f = 12 \), we get

\[
a_i \lesssim 10^{13} \text{ cm} . \tag{66}
\]

This is too large a distance to permit appreciable X-rays to come from the black hole. If the two compact stars are to merge by gravitational waves within Hubble time, their precursors (a black hole and an O or B star) presumably cannot be observed as emitting X-rays.

Conversely, objects like Cyg X-1 probably will not lead to an observable gravitational wave merger. When the O star expands as a giant and goes into a common envelope with the black hole, the orbit will contract to the extent that the black hole falls into the He core of the giant; there is a merger, but its gravitational waves will be too low a frequency to be observed. Thus the discussions in §§ 4 and 5 are actually irrelevant, given our scenario for common envelope evolution.

Returning to our discussion up to equation (65), we must examine the values of the initial mass of star B. It should be greater than 15 \( M_\odot \) to permit common envelope evolution, but less than 35 \( M_\odot \) so as to avoid undue mass loss by wind. With \( M_B = 90 M_\odot \), this means a range of \( q \) of 0.2. Multiplying this by \( a_i \) in equation (9) and by the mean of \( p \) in equation (65), the rate of merger of a massive black hole with another compact star is

\[
a_i = (4-6) \times 10^{-6} \text{ yr}^{-1} \text{ galaxy}^{-1} . \tag{67}
\]

This is in remarkably good agreement with the results for case H with hypercritical accretion of Portegies Zwart & Yungelson (1998) if we decrease their number by a factor of 5 to take into account our greater high-mass black hole
mass limit; namely, with this decrease they would have \[ R = 7 \times 10^{-6} \text{ yr}^{-1} \text{ galaxy}^{-1}. \]

The great uncertainty is in \( x_4 \), because we do not know very well the minimum ZAMS mass that leads to a massive black hole; thus \( x_4 \) is likely to be uncertain by a factor of 2 either way. Together with \( p \) in equation (65), the uncertainty of merger is about a factor 3, so

\[ x_4 = 1 \times 10 \times 10^{-6} \text{ yr}^{-1} \text{ galaxy}^{-1}. \] (68)

This rate is much smaller than that of mergers of small black holes with neutron stars, \( x_4 \approx 10^{-4} \), as one might expect from the difficulty of forming massive black holes. But the signal-to-noise ratio in the gravitational wave detector depends on the chirp mass as \( (M_{\text{chirp}})^{2.5} \). The chirp mass is

\[ M_{\text{chirp}} = \mu^{0.6} M^{0.4}, \] (69)

where \( \mu \) is the effective mass and \( M \) is the total system mass. We have \( M_{\text{chirp}} \sim 3.3 M_\odot \) for our high-mass black hole neutron star binaries, as compared with \( \sim 1.6 M_\odot \) for the low-mass black hole neutron star binaries. Thus effectively the high-mass black hole merging rate should be multiplied by a factor of \( \sim 6 \). The two types can be distinguished by the chirp mass.

Star B, after its SN event, will be either a neutron star or a small black hole of mass slightly over \( 1.5 M_\odot \), and it has no occasion to accrete extra mass. Thus we estimate that signals from massive black hole mergers with neutron stars should have a frequency about one-third of those from smaller black holes. Taking the central value of \( 5 \times 10^{-6} \) from equation (67) and multiplying it by 6, we obtain an effective increase of \( \sim 30\% \) over the merging rate of \( 10^{-4} \) yr\(^{-1} \) for the low-mass black hole neutron star binaries.

8. OTHER EFFECTS OF HYDROGEN CLOTHING

We have seen that the He core of stars evolves quite differently according to whether it is clothed with an H envelope or not. We believe that this difference may explain problems in some recent investigations.

8.1. Formation of Black Holes with Light Companions

Portegies Zwart, Verbunt, & Ergma (1997) have discussed the formation of black holes in low-mass X-ray binaries. Many of these transient X-ray sources have been discussed recently with black hole masses in the probable range of \( \sim 6-7 M_\odot \). Assuming a lower limit of \( \sim 40 M_\odot \) for black hole formation, Portegies Zwart et al. (1997) find a rate of formation much too low.

In their evolution, the black hole originates from the more massive component (star A) of the binary. The less massive component (star B) will spiral into star A when star A becomes a giant and touches star B. They show that star B will survive this spiral-in only if its original distance from star A is at least several hundred solar radii. Otherwise, star B will spiral into the He core of star A, will merge with star A, and thus be lost. In order for star A to have a radius of several hundred \( R_\odot \), it must have completed He core burning. This means that star A has burned He while clothed with most of its H envelope. Consequently, the upper curve in Figure 1 applies; the massive star A essentially burns as a single star. In this case stars of ZAMS mass as low as \( 20-25 M_\odot \) can go into a black hole. We suggest this as a possible solution to the Portegies Zwart et al. (1997) underproduction of high-mass black holes.

Our scenario also suggests an explanation of why the transient black holes are generally accompanied by low-mass companions. As the massive progenitor star A of the black hole evolves as a giant, RLOF will generally transfer its H envelope to a massive companion (if one exists) during hydrogen shell burning or early in the He core-burning phase. After RLOF, the He core of the primary will burn as a naked He core, with its possible fate as either a low-mass black hole or a neutron star (Brown et al. 1996), but not as a high-mass black hole. Therefore, a high-mass black hole will generally not have a massive companion, except in the relatively rare cases.

If, on the other hand, star B has small mass, it can accept only very little mass from star A in RLOF. Instead, interaction of the two stars will wait until star A becomes a supergiant and its surface reaches star B. Then in the common envelope star B spirals in as described by Portegies Zwart et al. (1997). The He core of star A, mass about one-third of the original ZAMS mass, evolves as described by Portegies Zwart et al. (1997), finally collapses as an SN, and leaves a remnant of mass approximately equal to the He core mass, \( 6-7 M_\odot \).

8.2. Cyg X-3

Cyg X-3 may be a progenitor of a binary of two low-mass black holes. We believe the envelope of the O star progenitor of the compact object (star A) in Cyg X-3 was probably lifted off in RLOF. This star A then had an SN event making it into a neutron star. When star B, the present He star, became a giant, star A would spiral in, which would have converted the neutron star into a low-mass black hole (if it was not one already). The He star B in Cyg X-3 is certainly burning as naked. Our estimate is that the ZAMS mass of the progenitor of this He star is about the maximum mass that does not go into an LBV stage (because the envelope must have been used up in common envelope evolution), say, \( 35 M_\odot \). From our Figure 1, the most likely fate of such a star with an H envelope lifted off in RLOF is a low-mass black hole, but a neutron star of mass \( \sim 1.5 M_\odot \) cannot be excluded.

8.3. Nucleosynthesis

The galactic ratio of oxygen to iron depends on the ZAMS mass above which single stars evolve into high-mass black holes (cut-off mass), and therefore do not return matter to the galaxy. Oxygen is chiefly produced in quiescent burning before the SN explosion, and the amount is roughly proportional to ZAMS mass. Fe, on the other hand, is produced explosively in the SN explosion, and the amount is roughly independent of ZAMS mass, possibly decreasing slightly as the latter increases. Tsujimoto et al. (1997) have recently used the [O/Fe] ratio and observations in metal poor stars to determine a cutoff mass of

\[ M_{\text{cutoff}} = 50 \pm 10 M_\odot. \] (70)

Tsujimoto et al. (1997) remark that the influence of the metallicity dependence of stellar wind losses may be significant, but they do not take it into account. We believe, however, that the larger wind losses of stars with solar metallicity have important effects. Over a wide range of ZAMS masses, between \( \sim 35-40 M_\odot \) and \( \sim 80 M_\odot \) accord-
ing to Woosley et al. (1993), the winds remove the H envelope sufficiently rapidly that the He cores evolve as naked. Thus the cores evolve into either low-mass black holes or neutron stars, their (gravitational) core masses following the lower line heading slightly above 1.5 \( M_\odot \) in Figure 1. Brown et al. (1996) found that the primary in 1700–37 probably evolved into a low-mass black hole (i.e., it does not pulse), whereas 1223–62 is known to contain a neutron star. In both cases, the ZAMS mass of the primary was found to be \( \sim 40 M_\odot \), so this approximately locates the ZAMS mass corresponding to \( M_\text{ap} \) of §2 in the case of solar metallicity. (As noted earlier, we estimate the ZAMS mass corresponding to \( M_\text{pc} \) to be \( \sim 80 M_\odot \).) It may well be that for metal-poor stars with much weaker winds the hydrogen envelope is not removed rapidly enough for their He cores to evolve as naked. In this case the limit (eq. [70]) for ZAMS masses giving core mass \( M_\text{pc} \) could be more appropriate.

We have not attempted to carry out a calculation of nucleosynthesis in our scenario, but we wish to point out features of our scenario that will tend to increase the \( M_\text{cutoff} \), above which element production for the galaxy ceases.

Disregarding mass loss, single stars above a certain mass \( M_{\text{min}} \) evolve into high-mass black holes. From the upper line in Figure 1, one might take the minimum He mass to be 6–8 \( M_\odot \), corresponding to a ZAMS mass of 20–25 \( M_\odot \). But as the mass of the single star increases, there is big mass loss before the star ever reaches the SN stage. So with extensive mass loss, Woosley et al. (1993) find that ZAMS mass 35 \( M_\odot \) leads, after SN, to a compact object of about 1.5 \( M_\odot \), thus a low-mass black hole. Thus high-mass black holes are only formed by single stars in a limited mass range, from 20–25 \( M_\odot \) to about 35–40 \( M_\odot \). Stars in this intermediate mass range, of width 10–20 \( M_\odot \), do not return their matter, especially their Fe, to the galaxy. This group of stars is in addition to the stars above \( M_\text{cutoff} \). To compensate for this fact, \( M_\text{cutoff} \) must be raised above the Tsujimoto et al. (1997) value. Because the abundance of stars decreases with increasing mass, the raise must be more than 10–20 \( M_\odot \). Thus we suggest that \( M_\text{cutoff} \) may be as high as the 80 \( M_\odot \), which we used for other reasons in §3.

We note from the Woosley et al. (1993) calculations that naked He cores evolve into less massive carbon/oxygen cores than clothed ones. Thus we see that inclusion of mass loss will tend to move the mass above which all nucleo-synthesis ceases even higher. Therefore, we believe that inclusion of the mentioned effects may decrease the apparent discrepancy between \( M_\text{cutoff} \) of equation (70) and our ZAMS mass of 80 \( M_\odot \) for making MBHs.

We return to the higher observed incidence of high-mass black holes (LMC X-1 and LMC X-3) in the LMC. Suppose that the limit of equation (70) applies for the lower metallicity of the LMC; i.e., stars with ZAMS masses \( \sim 40-80 M_\odot \) in the LMC would not experience as large a wind loss as those in the disc because of the lower metallicity in the LMC. This could help explain why two high-mass X-ray binaries containing high-mass black holes are observed in the LMC, and only Cyg X-1 is seen in the Galactic disc.

9. CONCLUSIONS

Our chief result is that those massive binary systems in which the primary is sufficiently massive, \( \gtrsim 80 M_\odot \), to go into a high-mass black hole can contribute importantly to observable gravitational waves upon merger of the final binary of compact objects. These mergers come chiefly from companion O stars in the range of 15–35 \( M_\odot \) ZAMS mass.

We argue that the situation created by hypercritical accretion is favorable to common envelope evolution in this range of companion masses, and that the timescale of this evolution is very fast.

We show that population syntheses do not have a common envelope with efficiency comparable with ours, possibly because they generally do not include hypercritical accretion. Certainly they do not have as effective a tightening of orbits as we have. The exception is case H of Portegies Zwart & Yungelson (1998), which does include hypercritical accretion. For the same mass limit for black holes, their results are in good agreement with ours. At first sight it may seem surprising that we match so well the Portegies Zwart & Yungelson (1998) work, provided they include hypercritical accretion, as we do. Their calculation includes various phases of mass transfer before forming the binary consisting of a neutron star and a black hole. However, results depend only on the ratio of the logarithmic interval favorable for gravitational merger with avoidance of coalescence to the total logarithmic interval over which binaries are distributed. Whereas the favorable logarithmic interval is shifted around in the various mass transfers, its magnitude is unchanged because the only scale is the radial separation of the two objects.

Our final merger rate of \( \sim (4-6) \times 10^{-6} \text{ yr}^{-1} \) increases the Bethe & Brown (1998) rate of gravitational waves by a factor of \( \sim 1.3 \), largely because of the higher chirp mass with high-mass black holes.

This important effect comes in spite of a much higher mass limit ZAMS 80 \( M_\odot \) for evolution into high-mass black holes in binaries. We justify this high-mass limit from results for the evolution of naked He stars by Woosley et al. (1993). Namely, the more massive primary in binaries has its mass lifted off early, either RLOF or in LBV stage for the stars with ZAMS masses greater than 35–40 \( M_\odot \). The resulting naked W-R or He star is deprived of the H envelope, which normally "insulates" He cores during their burning, so the convective carbon-burning stage is not skipped, as it is in single stars of ZAMS mass \( \gtrsim 20 \ M_\odot \). The great entropy loss during the long duration of this stage results in a low-mass compact object.

We show that the high-mass limit for evolution into high-mass black holes in binaries is consistent with nucleo-synthesis, because single stars with ZAMS masses in the range of 20–35 or 40 \( M_\odot \) evolve into high-mass black holes without return of matter to the galaxy.

We also indicate that we could evolve enough transient sources, binaries of a high-mass black hole, and a low-mass main-sequence star.

There is interest in the merging of binaries composed of two high-mass black holes (Brady et al. 1998). With our mass limit of 80 \( M_\odot \) for high-mass black holes, the initial separation of the two massive O star progenitors must be greater than 40 \( R_\odot \) for both of them to lie inside their Roche lobes. Case A mass transfer between two nearly equal mass stars can only widen the binary, since the orbit is tightened is excluded. The W-R stars that result after loss of the H will lose most of their mass by wind, down to \( \sim 10-15 \ M_\odot \) before going into black holes. With spherically symmetric...
mass loss, \(a_f/a_i = M_f/M_i \sim 4\). Thus the separation of the two final black holes will be several times the initial greater than \(40 \, R_\odot\), which precludes merging in Hubble time. We are in agreement with Portegies Zwart & Yungelson (1998) on this point.

We believe that in Brown & Bethe (1998) together with this paper we have given a consistent description of the evolution of most binaries of compact stars containing a black hole.

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