Vibration characteristics of an operating ball mill

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Abstract. A ball mill, which is used to finely grind materials, causes high levels of vibration and sound during grinding operations. The vibration and sound of mills provide significant information about the internal conditions and can be used to estimate the status of the ground material. We developed a simulation model for the vibration of a mill wall to better understand the relationship between the operating conditions and wall vibration characteristics. The discrete element method can be used to predict the motion of the contents, which are grinding balls and material particles, and estimate the collision force between the balls and the mill wall. A finite element method vibration analysis is performed to calculate the time history response of the vibration from the estimated collision force by DEM. Simulations and experiments are performed to evaluate the influence of changing material particle diameter on the vibration characteristics. The results of experiments and simulations show the same tendencies. A decrease in material diameter reduces the vibration velocity of the mill. The developed simulation is useful for understanding the vibration characteristics of the mill because it could obtain information, such as individual particle element velocities, that is not available from the experiments.

1. Introduction
A ball mill is a type of grinding equipment used to finely grind materials. Many balls, referred to as media, are placed into the mill with the material to be ground, and the mill is rotated. The material is finely ground by collision forces among the material, balls, and mill wall. Operation time depends on the type of material and desired fineness, but a direct measurement of material fineness is usually difficult. The grinding process and the conditions inside the ball mill must be monitored during the operation to improve its efficiency and enable automated operation. To directly measure the internal state of a mill, Martins et al. [1] proposed the use of sensors installed in each ball. However, this method limits the measuring conditions (e.g., the size of the ball and the sensor). To indirectly measure the inside of the mill, a method using vibration and the radiated sound of the mill is available. The vibration of the mill and the sound radiated from the mill wall can provide a significant amount of information about the operating conditions and internal state of the mill. The vibration and radiated sound change with operation conditions and fineness of the material. Therefore, the sound and the vibration during mill operation have been analysed in many studies [2] [3]. Moreover, the automation of ball mills based on measured vibrations has been studied experimentally [4].

To understand the interior of mills during operation, the motions of balls have been analysed by the discrete element method (DEM). The DEM predicts the motion of each ball during operation and...
provides an understanding of the internal state in terms of the velocity distribution and collision forces between balls [5]. Hosseini et al. [6] experimentally obtained the transfer functions between the force of collisions with the mill wall and the sound pressure at a reference point. Using this transfer function, the sound pressure was estimated based on collision forces calculated by the DEM simulation. Moreover, the collision force calculated by the DEM simulation can be used as the input data for a vibration analysis of the mill wall and for sound-radiation analysis. By this approach, the prediction of the sound radiation based on the mill conditions can provide an understanding of the relationship between the vibration of the mill, sound radiation, and grinding progression. Characterizing the relationship between the grinding condition and the information of vibration and sound would allow the estimation of the grinding progression based on the measured vibration and sound during operation.

In this study, we developed an experimental ball mill and simulation model to better understand the relationship between the operating conditions and vibration characteristics of the mill. We estimated the influence of the diameter of material particles on the vibration characteristics to confirm the validity of the developed simulation model. In the simulation, the motion of the balls and the material was predicted by DEM, and the vibration of the mill was predicted by a finite element method (FEM) model.

2. Experimental ball mill

2.1. Dimension and measurement setup

Table 1 shows the dimensions of the experimental ball mill, and Figure 1 shows the experimental ball mill and measurement setup. The vibration of the mill wall is measured at a point 45° from the horizontal plane using a laser displacement sensor. Although the acrylic disc shown in Figure 1(b) was mounted to observe the mill interior, a steel disc was mounted when the mill wall vibration is measured.

Table 1. Dimension of the experimental mill (mm)

| Inner diameter $D_v$ | 190 |
|----------------------|-----|
| Width                | 125 |
| Thickness            | 2.5 |
| Ball diameter $D_b$  | 19.8 |

(a) Experimental ball mill   
(b) Position of laser sensor

Figure 1. Experimental setup

2.2. Rotational speed and motion inside mill

When the rotational velocity exceeds the critical velocity $N_c$, the centrifugal force acting on a ball is equal to the gravitational force. As a result, the ball rotates while clinging to the mill wall and the grinding capability of the mill decreases. The following equation describes the critical velocity [7]:
\[ N_c = \frac{60}{2\pi} \sqrt{\frac{g}{R_v - R_b}} \]  

(1)

where \( R_v \) is the inner radius of the mill and \( R_b \) is the radius of the ball. To keep the appropriate state for the grinding operation, the rotational velocity of the mill may not exceed the critical velocity \( N_c \). Therefore, mills are usually operated at speeds lower than the critical velocity. Figure 2 shows the motions of the balls and materials at rotational velocities of 30.8 rpm, 61.5 rpm, and 92.3 rpm, which correspond to 30\%, 60\%, and 90\% of the critical velocity of 102.5 rpm. When the rotational velocity is slow, the balls circulate on the surface of an aggregated group of balls. In contrast, an increase in the rotational velocity causes some balls to separate from this group. This behaviour promotes the grinding capability by increasing the collision force among the balls, material particles, and mill wall. Furthermore, the increasing rotational velocity increases the vibration displacement, which was obtained in the previous paper \[9\]. In the experiment and simulation discussed in this paper, the rotational velocity was set to 92.3 rpm, which is 90\% of the critical velocity of 102.5 rpm.

![Figure 2. Motion of media and material at selected rotation speeds](image)

(a) \( N=30.8 \text{ rpm (30\%)} \)  
(b) \( N=61.5 \text{ rpm (60\%)} \)  
(c) \( N=92.3 \text{ rpm (90\%)} \)

3. Analytical method

3.1. Motion analysis of balls and material by DEM

The motion of the balls and material was modelled with particle elements using DEM. Based on the Voigt model shown in Figure 3, the DEM was used to calculate the contact force between two particle elements in the normal direction \( F_{n_{ij}} \) and in the tangential direction \( F_{s_{ij}} \). In the simulation, collisions between the elements are detected in each time step, and the contact forces are calculated when two elements come into contact. It is possible to calculate the behaviour of the particle elements by numerically integrating the following equations of motion:

\[ m_i \frac{d^2x_i}{dt^2} = mg + \sum F_{n_{ij}} + \sum F_{s_{ij}} \]  

(2)

\[ I_i \frac{d^2\theta_i}{dt^2} = \sum T_j \]  

(3)

where \( m \) is the mass of particle \( i \), \( \mathbf{x}_i \) is the displacement vector of particle \( i \), \( \mathbf{g} \) is the gravity vector, \( I_i \) is the moment of inertia of particle \( i \), \( \mathbf{\theta}_i \) is the angular displacement vector of particle \( i \), and \( T_j \) is the moment due to contact with particle \( j \). The suffixes \( i \) and \( j \) represent parameters for particles \( i \) and \( j \), respectively. The above equations are solved numerically by the Euler method and predict the motion of the elements in three-dimensional space.

The normal force \( F_n \) is calculated based on the relative displacement \( u_n \) by the following equations:

\[ F_n = K_n u_n^{3/2} + \eta_n \frac{du_n}{dt} \]  

(4)

\[ u_n = (r_i + r_j) - \| \mathbf{x}_i - \mathbf{x}_j \| \]  

(5)
Figure 3. Modelling the contact between two particles in the DEM

where $K_n$ is the stiffness in the normal direction, $\eta_n$ is the coefficient of viscosity in the normal direction, and $r$ is the particle radius. Based on the Hertz contact theory [10], the stiffness in the normal direction $K_n$ is determined by

$$K_n = \frac{4E_i E_j}{3(E_i(1-\nu_i^2)+E_j(1-\nu_j^2))} \sqrt{\frac{r_i}{r_i+r_j}}$$  \hspace{1cm} (6)

where $\nu$ is Poisson’s ratio and $E$ is Young’s modulus. The coefficient of viscosity in the normal direction is calculated by

$$\eta_n = 2\zeta_{ij}\sqrt{mK_n}$$  \hspace{1cm} (7)

where $E$ is the elastic modulus and $\zeta_{ij}$ is the damping ratio of the elements [11].

The stiffness in the shear direction $K_s$ and the coefficient of viscosity in the shear direction $\eta_s$ may be calculated from the coefficient of transformation $s$ as follows:

$$K_s = s K_n$$  \hspace{1cm} (8)

$$\eta_s = \eta_n \sqrt{s}$$  \hspace{1cm} (9)

$$s = \frac{G}{E} = \frac{1}{2(1+\nu)}$$  \hspace{1cm} (10)

where $G$ is the shear modulus of rigidity. A friction force $\mu F_n$, where $\mu$ is the coefficient of friction, acts between the particles. When the force due to the spring and the dashpot in the shear direction exceeds $\mu F_n$, the friction slider starts to move. Therefore, the shear force $F_s$ can be calculated by the following conditional equation when $\mu$ is given:

$$F_s = \begin{cases} \mu F_n & (\mu F_n < K_s u_s + \eta_s \frac{du_s}{dt}) \\ K_s u_s + \eta_s \frac{du_s}{dt} & (\mu F_n \geq K_s u_s + \eta_s \frac{du_s}{dt}) \end{cases}$$  \hspace{1cm} (11)

Not only the interaction forces between particle elements of the DEM but also the contact forces with the mill wall and the particle elements is calculated by the Hertz contact theory in the DEM simulation. The mill wall is assumed to be a rigid body since the deformation of the wall is considered to be small and does not affect the motion of the particle elements. Before the vibration analysis described the following section, the wall contact forces are calculated at the all time steps of a simulation. After, the calculated forces are processed as the input forces of vibration analysis by the FE model.
3.2. Vibration analysis of mill wall
In the analysis of the vibration of the mill wall, the side of the mill cylinder was modelled in the FEM software ANSYS with four-node shell elements, Shell181, whose nodes have six degrees of freedom. The side of the mill cylinder was divided into 100 elements in the circumferential direction and 25 elements along the rotational axis direction of the mill. The edges of the cylinder were given fixed boundary conditions because the actual cylinder edges are welded or bolted to flat plates.

The time history response was calculated by the modal superposition method with a frequency in the analytical target range of less than 10 kHz. The modal damping ratio was set to 0.2% for all modes. The Newmark-β method was used for time integration. The vibration response of the mill wall was then calculated using the contact forces calculated by the DEM analysis as the input forces. Furthermore, input nodes are set considering the rotation of the mill. The vibration displacement and velocity each time step also calculate from the nodes which corresponds measurement point with the laser displacement sensor.

4. Experiment and result
4.1. Balls and material
Table 2 shows the ball diameter $D_b$, material particle diameter $D_m$, and number of balls and material particles. As a fundamental principle, the material used in this study is not ground by collisions with balls and the mill wall; rather, the material grinding is represented by changing particle diameter. The balls and material fill half the volume of the mill, and the weight ratio of the balls and material is 7:3 under all conditions. The DEM simulations were conducted with the parameters shown in Table 3.

| Media ball (steel) | Particle diameter $D_b$ (mm) | Number of particles |
|-------------------|----------------------------|--------------------|
|                  | 19.8                       | 147                |

| Material (ceramic Al$_2$O$_3$) | Particle diameter $D_m$ (mm) | Number of particles |
|--------------------------------|-----------------------------|--------------------|
|                                | 19.8, 15.1, 10.3, 4         | 63, 144, 448, 7662 |

Table 3. DEM parameters

| Vessel | Young’s modulus E (GPa) | 205 |
|--------|-------------------------|-----|
|        | Poisson’s ratio $\nu$   | 0.3 |
| Media ball (steel) | Density (kg/m$^3$) | 7800 |
|        | Young’s modulus (GPa)   | 210 |
|        | Poisson’s ratio         | 0.3 |
| Material (ceramic Al$_2$O$_3$) | Density (kg/m$^3$) | 360 |
|        | Young’s modulus (GPa)   | 330 |
|        | Poisson’s ratio $\nu$   | 0.23 |
| Coefficient of friction $\mu$ | Ball - Ball | 0.3 |
|                               | Ball - Vessel            | 0.3 |
|                               | Ball - Materia           | 0.32 |
|                               | Material - Material      | 0.32 |
|                               | Material - Vessel        | 0.32 |
| Damping ratio $\zeta$ | Ball - Ball | 0.26 |
|                           | Ball - Vessel            | 0.26 |
|                           | Ball - Materia           | 0.55 |
|                           | Material - Material      | 0.55 |
|                           | Material - Vessel        | 0.55 |
4.2. Influence of material size on motion of ball and material

Figure 4 shows simulation results for the motion of the balls and material as material size $D_m$ was decreased. The colour of particles shows translational velocity of the balls and material, and the balls are identified as meshed particles. The height of the top of the particles consisted of a clump as the material size increased. When the material size was 19.8 mm or 15.1 mm, the particles rotated faster in the container and some balls separated at the top of this clump. As a result, the particles collided with other particles and the wall surface of the mill at a high speed. In contrast, when the material size was 10.3 mm or 4.0 mm, the particles rotated and fell into a clump of particles. The smaller material size decreased the translational velocity. These results demonstrate that the material size affects the vibration of the mill.

Figure 4. Motion of media and material for various particle sizes (92.3 rpm)

4.3. Frequency response of operation mill

This section discusses the vibration response when the material diameter is changed at a rotational speed of 92.3 rpm. The experimental results are shown in Figure 5, and the simulation results are shown in Figure 6. In these figures, the line colour represents material diameters. Multiple peaks are observed between 1 and 6 kHz in both results. These peak frequencies are related to the natural frequency of the mill.
Figures 5 and 6 show that a decrease in the material diameter reduces the vibration velocity, probably because the decrease in mass per particle weakened the vibration input. Although peak values of the simulation results do not quantitatively agree with those of the experimental results, the influence of changes in particle diameter can be evaluated qualitatively by the developed simulation. From the result of the dynamic analysis, not only the decrease in weight of a material particle but also the decrease in velocities of the balls and material, which is shown Figure 4, affect the vibration response. In the simulation, since the velocities of particle elements can be obtained in detail, the influence of vibration can be evaluated based on information that cannot be measured in experiments.

At high frequencies, the vibration velocity in the simulation results is smaller than that in the experiment results. This difference is caused by the characteristics of DEM analysis, in which the size of the time step affects numerical stability [12]. Since the numerical solutions become more unstable at higher frequencies, this instability may reduce the energy imparted to the mill by collisions at high frequencies. As a result, the simulation result shows that the vibration velocity is smaller than that in the experimental result, and the influence of the change in the particle diameter is magnified at high frequencies.
4.4. Kinetic energy of ball and material

The time-averaged kinematic energy of each particle element was calculated from the results of DEM simulation, and the calculated values are shown Figure 7. In this figures, the horizontal axes show the number of elements. Numbers from 1 to 147 represent balls (green lines) and numbers larger than 147 represent material particles (red lines). Table 4 reports the total energy of individual ball and material elements.

A part of the kinematic energy of the balls and the material is changed into vibration energy by collisions with the mill wall. The decrease in the material particle diameter reduces the kinematic energy. As a result, energy imparted to the mill is also reduced. Moreover, the decrease in the velocities of the balls and particles also reduces the energy imparted to the mill. Calculation of kinematic energy by DEM simulation is useful for understanding mill operations because the motion of the balls and material particles has a relation with mill vibration.

![Figure 7. Kinetic energy of media and material](image)

Table 4. Total kinetic energy of media and material

| Diameter | Ball       | Material   |
|----------|------------|------------|
| 19.8 mm  | 2.89×10⁻¹ J| 8.82×10⁻² J|
| 15.1 mm  | 1.74×10⁻¹ J| 4.26×10⁻² J|
| 10.3 mm  | 3.68×10⁻² J| 5.48×10⁻³ J|
| 4.0 mm   | 2.20×10⁻³ J| 1.00×10⁻⁴ J|
5. Conclusions
Vibration in a ball mill provides a significant amount of information about the operating conditions and the internal state of the mill. We investigated the influence of material particle diameter on vibration characteristics of a ball mill with an experimental ball mill and a simulation model. The simulation model included both DEM and FEM models. From the experiment and simulation results, we have shown that decreasing material diameter reduces the vibration velocity of the mill wall. Thus, the results of the simulation qualitatively agree with the experiment results, and the simulation model is valid for evaluating the vibration characteristics of the ball mill. Thus, it may be possible to correlate the vibration and internal state of a mill by DEM analysis.

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