Radiative decays with $a_0(980)$ and $f_0(980)$ from ChPT at order $p^4$

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Abstract. A consistent description of the $\pi^0\pi^0$ invariant mass distribution in the $\phi(1020) \to f_0(980)\gamma \to \pi^0\pi^0\gamma$ decay and the $\pi^0\eta$ in $\phi(1020) \to a_0(980)\gamma \to \pi^0\eta\gamma$ is suggested. A search for the consequences of the flavor $SU(3)$ symmetry for the scalar mesons can be based on such an analysis. In order to accurately treat the pseudoscalar meson dynamics, which is very important for the scalar meson decays, we employ Resonance Chiral Theory.

Keywords: Flavor symmetries, Chiral symmetries, Chiral Lagrangians, Hadronic decays, Properties of mesons, Scalar mesons

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INTRODUCTION

This paper focuses on the isotriplet $a_0(980)$ and isosinglet $f_0(980)$ scalar mesons ($S$). These particles, probably together with the controversial $\sigma$ meson, represent the lightest members of the scalar meson spectrum. To test this assumption it is important to search for consequences of the flavor $SU(3)$ symmetry for these particles in nature. Among different observables, the invariant mass distributions of $\pi\pi$ and $\pi\eta$ pairs in the $\phi(1020) \to \pi\pi\gamma$ (or $\pi\eta\gamma$) decays are of considerable interest, because the $a_0(980)$ and $f_0(980)$ are important intermediate states in these decays and thereby show up in the spectra. The $e^+e^-$ experiments in Novosibirsk [1, 2, 3, 4, 5] and Frascati [6, 7, 8, 9, 10] allow one to study the $\phi$ decays. From the point of view of data extraction, the neutral final states are preferable since in the decays $\phi(1020) \to \pi^0\pi^0\gamma$, $\pi^0\eta\gamma$ the photon may be radiated only from the final state. Therefore the above processes are rather suitable for study of chiral dynamics.

The analysis of the data (see Refs. [2, 11] and others) shows that

- the decays are mediated by the $S$ resonances, $f_0(980)$ and $a_0(980)$, in the isoscalar and isovector channels, respectively;
- the kaon loop (KL) coupling of $S$ to the vector meson $\phi$ is very important.

Fig.1 shows schematically the processes in question. The KL coupling scheme allows one to reproduce a cusp behavior of the amplitude dependence on the invariant energy [12]. Many authors relate the dominance of the KL mechanism to a large $K\bar{K}$ com-
ponent in the \(a_0(980)\) and \(f_0(980)\) mesons and to the proximity of the \(K\bar{K}\) threshold to the scalar meson mass. We argue that the KL mechanism is a feature of the chiral dynamics and reflects the important role of pseudoscalar mesons in low- and intermediate-energy interactions. We also emphasize that the kaon loops in the \(\phi \to S\gamma\) transitions naturally arise in the leading order in Resonance Chiral Theory \((R\chi T)\), irrespectively of the threshold or mass position, and the structure of scalars. In addition, \(R\chi T\) accounts for an important feature – momentum dependence of the vertex functions.

We have recently obtained [13] a complete set of \(O(p^4)\) contributions to various radiative decays with the scalar mesons from the \(O(p^4)\) \(R\chi T\) Lagrangian. The parameters were fixed from the average values of the decay widths. In the following we discuss the \(R\chi T\)-based study of the \(\pi^0\pi^0\) and \(\pi^0\eta\) pair mass distributions in the \(\phi(1020)\to S\gamma\) decays and the assumptions used. We fix the parameters and compare predictions with the data and other models.

We conclude that the invariant-mass distribution is a more reliable source of information than the average width from PDG, because the width values may depend on models used in the data analysis.

**FORMALISM**

The \(R\chi T\) Lagrangian terms [14]

\[
L_{\text{scalar}} = c_d \langle S^{\text{oct}} u_\mu u^\mu \rangle + c_m \langle S^{\text{oct}} \chi_+ \rangle + \bar{c}_d S^{\text{sing}} \langle u_\mu u^\mu \rangle + \bar{c}_m S^{\text{sing}} \langle \chi_+ \rangle ,
\]

\[
L_{\text{vector}} = \frac{F_V}{2\sqrt{2}} \langle V_{\mu
u} f_+^{\mu\nu} \rangle + \frac{i G_V}{\sqrt{2}} \langle V_{\mu
u} u_\mu u^\nu \rangle
\]

describe interaction of the scalar singlet \(S^{\text{sing}}\), octet \(S^{\text{oct}}\), pseudoscalar mesons parameterized by \(u_\mu\), vector mesons \(V_{\mu\nu}\) and the external electro-magnetic field at order \(O(p^4)\) (for notation we refer to the original paper [14]). The couplings for the scalar singlet \(\bar{c}_d\) and \(\bar{c}_m\) are in general independent of the octet couplings \(c_d\) and \(c_m\). The values of \(F_V\) and \(G_V\) are accurately determined from different observables. The flavor \(SU(3)\) symmetry is explicitly broken by the mass term contained in \(\chi_+\) and the relevant couplings have the subscript \(m\): \(c_m\) and \(\bar{c}_m\). We mention below the important aspects of the model [13].

- In order to estimate the \(a_0 \to \pi\eta\) decay width, one has to construct the physical \(\eta\) state from the pseudoscalar singlet \(\eta_1\) and the eighth component \(\eta_8\) of the octet. We use the two-parameter mixing scheme: \(\eta = \eta_8 \cos \theta_8 - \eta_1 \sin \theta_1\) with \(\theta_8 = -9.2^\circ\) and \(\theta_1 = -21.2^\circ\) [15].
FIGURE 2. The invariant mass distributions in \( \phi \rightarrow \pi^+ \pi^- \gamma \) (left) and \( \phi \rightarrow \pi^+ \pi^- \eta \gamma \) (right). The solid lines correspond to the choice of parameters in [13]. The dashed lines are calculated with “classical” values for the scalar meson couplings [14] \( (c_m = 42 \text{ MeV}, c_d = 32 \text{ MeV}, \theta = -35.26^\circ) \). The dotted lines are calculated in the present work. Data: [2] (left) and [5] (right).

- For the isoscalar state \( f_0(980) \) we use mixing with the angle \( \theta \): \( f_0 = S^{\text{sing}} \cos \theta - S^{\text{oct}}_8 \sin \theta \), and for the isovector state \( a_0(980) \) we take \( a_0 = S^{\text{oct}}_3 \). This scheme is rather flexible and allows us to study the mixing within the lightest nonet.
- Although the usual large \( N_c \) behavior of a resonance may not hold for some physical scalar mesons (see discussion in Refs. [16, 17]), the scalars \( S^{\text{sing}} \) and \( S^{\text{oct}}_8 \) in the present model obey this behavior, as these resonances are introduced similarly to the vector and axial-vector resonances. Thus we employ the relations \( \tilde{c}_m = c_m / \sqrt{3} \) and \( \tilde{c}_d = c_d / \sqrt{3} \) between the singlet and octet couplings.
- The effect of the resonance finite width in the invariant mass distributions is important [18]. We choose the propagator of the scalar meson with the mass \( m_S \) in the form

\[
D_S(p^2) = [p^2 - m_S^2 + im_S \Gamma_{S, \text{tot}}(p^2)]^{-1},
\]

where the total width \( \Gamma_{S, \text{tot}}(p^2) \) depends on the meson momentum squared \( p^2 \). It may also be important to use a more advanced form of the propagator including both real and imaginary parts of the self-energy, as suggested in Ref. [19].

Following approach of Ref. [13] one arrives at the matrix elements for the \( \phi \) radiative decays.

DISCUSSION

Let us summarize advantages of the present approach.

- There are no contact \( V \gamma S \) and \( \gamma \gamma S \) couplings at the lowest (tree-level) order. Therefore the leading contribution includes one-loop diagrams with intermediate pseu-
TABLE 1. The width estimates and comparison with other models

| observable        | estim.   | [13]* | [22]† | model “a” Ref. [23] | model “b” Ref. [23] | [24]** | PDG [21] |
|-------------------|----------|-------|-------|---------------------|---------------------|--------|----------|
| $\Gamma_{\phi\to\eta_0}$ | $1.7 \times 10^{-4}$ | 4.92 ± 0.07 | 0.46 ± 0.09 | 6.1 ± 0.6 |
| $\Gamma_{\phi\to\eta_0}$ | $3.16 \times 10^{-4}$ | 4.92 ± 0.07 | 0.46 ± 0.09 | 6.1 ± 0.6 |
| $\Gamma_{\phi\to\eta_0}$ | $4.99$ | 2.6 | 0.26 ± 0.06 | 6.1 ± 0.6 |
| $\Gamma_{a_0\to\pi\eta}$, MeV | 21.11 | 14.2 | — | — | — | — |
| $\Gamma_{f_0\to\pi\pi}$, MeV | 54.56 | 41.8 | — | — | — | 34.2 ± 22.7 |
| $\Gamma_{a_0\to\gamma\gamma}$, keV | 0.16 | 0.24 | 0.28 ± 0.09 | 0.28 ± 0.09 | 0.30 ± 0.10 |
| $\Gamma_{f_0\to\gamma\gamma}$, keV | 0.13 | 0.24 | 0.39 ± 0.13 | 0.39 ± 0.13 | 0.31 ± 0.08 |
| $\Gamma_{a_0\to\gamma\gamma}$, keV | 5 | 9.1 | 3.4 | 3.0 ± 1.0 | 11 ± 4 | — |
| $\Gamma_{f_0\to\gamma\gamma}$, keV | 4.1 | 9.6 | 3.4 | 4.2 ± 1.0 | 19 ± 5 | 3.3 ± 2.0 | — |
| $\Gamma_{a_0\to\gamma\gamma}$, keV | 4.8 | 8.7 | 3.4 | 641 ± 87 | 641 ± 87 | 31 ± 13 | — |
| $\Gamma_{f_0\to\gamma\gamma}$, keV | 8.4 | 15.0 | 3.4 | 4.3 ± 1.3 | 126 ± 20 | 88 ± 17 | — |

* an asterisk marks the input values 
† the kaon loop model estimates are selected from Ref. [22] 
** the model of Ref. [24] accounts also for vector mesons in the loops 
‡ both pion and kaon loops are included in the current model

doscalar mesons. This holds for any scalar meson mass irrespectively of their internal structure.

• The sum of the loop contributions is convergent, gauge invariant and universal: one can use either the analytical expression of [20], or calculate it numerically.

One should also stress an influence of the pseudoscalar meson dynamics on the above decays, as interaction of the scalar mesons is intimately connected with pseudoscalar loops.

The predictive possibilities of the model are illustrated in Fig. 2. The observables shown in the figure are the invariant mass distributions for the neutral pseudoscalar meson pairs in the $\phi$ radiative decays. We do not make a numerical fit to the data and just illustrate flexibility of the model. The masses of the scalar mesons in the analysis are taken $m_{f_0} = 980.0$ MeV and $m_{a_0} = 984.7$ MeV. The parameters are chosen to reproduce the form of the distribution in the vicinity of the peak (see the dotted line). Such a choice, $c_m = -38.32$ MeV, $c_d = -9.47$ MeV and $\theta = -11.62^\circ$, also preserves the reasonable agreement with the decay widths, see Table 1. The table also shows our previous results [13], which were based on analysis of the average width values given by PDG [21]. A comparison with predictions of other models [22, 23, 24] is presented in Table 1 as well. We also update the predictions for the $S\to\gamma\nu$ because of the growing interest to these decays [13, 22, 23, 24]. One can see that the current approach describes the hadronic decays more accurately than the two-photon decays.

We should notice that the other resonance contributions, which interfere with the scalar meson mechanism, are quite complicated, even in case of the neutral particles in
the final state (e.g., there are $\phi \to \rho^0\pi^0 \to \pi^0\pi^0\gamma$, $\phi \to \omega\eta \to \eta\pi^0\gamma$, $\phi \to \rho^0\pi^0 \to \eta\pi^0\gamma$ and other contributions). In this connection we note that a problem of the distribution data analysis is discussed in Ref. [25]. It is a challenge to calculate consistently the interfering contributions in framework of $R\chi T$ and build a unified analysis of the available data.

The model used here for the mass distributions in the $\phi$ decays can be applied to a detailed study of other radiative decays involving the light scalar mesons. Among the most interesting ones we should mention the $S \to \gamma V$ decays currently studied experimentally in Jülich [26, 27]. An option to apply the model for the above decays and $\phi(1020) \to \pi^+\pi^-\gamma$ data is to include it in a Monte Carlo generator for the Dalitz plot or mass distribution analysis.

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