TOTAL GALAXY MAGNITUDES AND EFFECTIVE RADII FROM PETROSIAN MAGNITUDES AND RADII

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ABSTRACT

Petrosian magnitudes were designed to help with the difficult task of determining a galaxy’s total light. Although these magnitudes (taken here as the flux within $2R_P$, with the inverted Petrosian index $1/\eta(R_P) = 0.2$) can represent most of an object’s flux, they do of course miss the light outside of the Petrosian aperture ($2R_P$). The size of this flux deficit varies monotonically with the shape of a galaxy’s light–profile, i.e., its concentration. In the case of a de Vaucouleurs $R^{1/4}$ profile, the deficit is 0.20 mag; for an $R^{1/8}$ profile this figure rises to 0.50 mag. Here we provide a simple method for recovering total (Sérsic) magnitudes from Petrosian magnitudes using only the galaxy concentration ($R_{90}/R_{50}$ or $R_{80}/R_{20}$) within the Petrosian aperture. The corrections hold to the extent that Sérsic’s model provides a good description of a galaxy’s luminosity profile. We show how the concentration can also be used to convert Petrosian radii into effective half–light radii, enabling a robust measure of the mean effective surface brightness. Our technique is applied to the SDSS DR2 Petrosian parameters, yielding good agreement with the total magnitudes, effective radii, and mean effective surface brightnesses obtained from the NYU–VAGC Sérsic $R^{1/n}$ fits by Blanton et al. (2005). Although the corrective procedure described here is specifically applicable to the SDSS DR2 and DR3, it is generally applicable to all imaging data where any Petrosian index and concentration can be constructed.

Subject headings: galaxies: fundamental parameters — galaxies: structure — methods: analytical — methods: data analysis

1. INTRODUCTION

Galaxies are known to possess well–defined correlations between their various structural and kinematic parameters (e.g., Faber & Jackson 1976; Tully & Fisher 1977; Djorgovski & Davis 1987; Dressler et al. 1987; Caon, Capaccioli, & D’Onofrio 1993; Graham, Trujillo, & Caon 2001; De Rijcke et al. 2005; Matković & Guzmán 2005). These empirical “scaling–laws” provide key observational constraints needed to test current theoretical models of galaxy formation and evolution. Obviously even the most basic of these, the luminosity–size relation (e.g., Dutton et al. 2005; McIntosh et al. 2005), relies on our ability to accurately measure robust photometric parameters.

In this vein, the need for a more unified approach to galaxy photometry has recently been highlighted by Cross et al. (2004). They noted that much of the discrepancy in galaxy magnitudes (and sizes) between various groups is because of the varying methodology applied. For example, some Authors use Kron magnitudes, others Petrosian magnitudes, some extrapolate fitted models to large radii, while others use somewhat limited aperture photometry. This can impact significantly on global measures of the galaxy population, such as the luminosity function (e.g., Norberg et al. 2002; Blanton et al. 2003a; Driver et al. 2005), the color–magnitude relation (e.g., Scodeggio 2001; Chang 2005, and references therein), and the luminosity density (e.g., Yasuda et al. 2001; Cross et al. 2001). It also, for example, impacts on studies of the supermassive black hole mass function derived using the galaxy luminosity–black hole mass relation (e.g., McLure & Dunlop 2004, Shankar et al. 2004, and references therein). Perhaps less obvious however, are the consequences for the calculation of size and surface brightness distributions (e.g., Kormendy 1977; Cross et al. 2001; Shen et al. 2003; Driver et al. 2005). These are typically derived from the half–light radius which in turn depends critically on an accurate assessment of the total flux. If the magnitude is underestimated, the size and surface brightness distributions will

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be affected.

One of the great strengths of the Petrosian (1976) index, the average intensity within some projected radius divided by the intensity at that radius, and similarly the radii themselves corresponding to some fixed Petrosian index, is that they do not depend on a galaxy’s distance. That is, because surface brightness dimming does not change the shape of a galaxy’s light–profile, it does not affect the Petrosian index, nor does it affect the observed galaxy concentration. Furthermore, due to the Petrosian index’s ability to define aperture sizes which contain the bulk of an object’s light, and due to the large influx of small, faint images of high–redshift galaxies that are now available, the Petrosian index has experienced a resurgence (e.g., Wirth, Koo, & Kron 1994; Bershady, Lowenthal, & Koo 1998; Dalcanton 1998; Takamiya 1999; Bershady et al. 2000; Blanton et al. 2001; Lubin & Sandage 2001; Conselice, Gallagher, & Wyse 2002; Yagi et al. 2002). Strauss et al. (2002; their section 3.2) do however stress the fact that a different fraction of galaxy light is missed depending on whether a galaxy has an $R^{1/4}$ light–profile or an exponential light–profile, and they emphasize the subsequent need to account for this in analyses of the Sloan Digital Sky Survey (SDSS; York et al. 2000) galaxy data.

In an effort to account for the flux missed by Petrosian apertures, this paper outlines a corrective procedure to convert Petrosian magnitudes into total (Sérsic) galaxy apertures, this paper outlines a corrective procedure to convert Petrosian magnitudes into total (Sérsic) galaxy apertures, this paper outlines a corrective procedure to convert Petrosian magnitudes into total (Sérsic) galaxy apertures, this paper outlines a corrective procedure to convert Petrosian magnitudes into total (Sérsic) galaxy apertures, this paper outlines a corrective procedure to convert Petrosian magnitudes into total (Sérsic) galaxy apertures, this paper outlines a corrective procedure to convert Petrosian magnitudes into total (Sérsic) galaxy apertures, this paper outlines a corrective procedure to convert Petrosian magnitudes into total (Sérsic) galaxy apertures.

In the absence of measurement errors, all $R^{1/4}$ light–profiles have exactly the same concentration index $R_{90}/R_{50}$ — the ratio of radii containing 90% and 50% of the Petrosian flux (e.g., Blanton et al. 2001; Strauss et al. 2002; Goto et al. 2003). The same is true for an exponential $R^{1/4}$ light–profile, although the specific value of the concentration will be different in this case. It follows that the observed range of galaxy concentrations, if not due to errors, reflects a range of light–profile shapes; that is, galaxies do not simply have exponential or $R^{1/4}$ light–profiles.

Recognizing this in the SDSS data, Blanton et al. (2003b) adopted Sérsic’s (1963, 1968) $R^{1/n}$ model3 to represent the range of galaxy light–profile shapes and provide estimates of their total luminosities, sizes, and surface brightnesses. Indeed, in the case of (dwarf and ordinary) elliptical galaxies, such an approach is crucial if one is to properly understand the various relationships between such terms (e.g., Graham & Guzmán 2003, their Section 4). However, for low signal–to–noise data or where the spatial resolution is lacking, it can become difficult to obtain reliable Sérsic fits. One therefore needs an alternative strategy to obtain the total galaxy flux and associated half–light terms.

Using only the observed concentration, $R_{90}/R_{50}$, within the Petrosian aperture, we provide an easy prescription to recover the total (Sérsic) flux from the Petrosian flux while maintaining the distance–independent qualities of the Petrosian system. We also explain how one can recover the effective radii and associated surface brightness terms. The corrective formula presented here are not only valid for pure $R^{1/4}$ or exponential profiles, but applicable to galaxies having intermediary light–profile shapes and a range of more extreme stellar distributions.

2. PETROSIAN RADI AND MAGNITUDES

The Petrosian (1976) index was initially introduced with the goal of measuring galaxy evolution. It gained additional popularity from its potential to determine the cosmological parameters (e.g., Djorgovski & Spinrad 1981; Sandage & Perelmuter 1990). Indeed, under the assumption of structural homology, it provided a means to obtain “standard rods” that could be used to constrain cosmogonic models. Nowadays it is often used as a tool for defining aperture sizes from which to measure galaxy magnitudes.

The Petrosian index, $\eta(R)$, is a function of a galaxy’s projected radius $R$, and can be written as

$$\eta(R) = \frac{2}{\pi} \frac{1}{R^{4}} \frac{I(R') R'dR'}{I(R)} = \frac{L(< R)}{\pi R^{2} I(R)} = \frac{(I)_R}{I(R)},$$

(1)

where $I(R)$ is an object’s (projected) intensity at some radius $R$, and $(I)_R$ is the average intensity within that radius. Following, for example, Blanton et al. (2003b), who modeled 183,487 SDSS galaxies, we adopt Sérsic’s (1963) $R^{1/n}$ model to represent the possible range of light–profiles $I(R)$. This can be written as

$$I(R) = I_e \exp \left\{ -b_n \left[ \left( \frac{R}{R_e} \right) ^{1/n} - 1 \right] \right\},$$

(2)

where $I_e$ is the intensity of the light–profile at the effective radius $R_e$, and $n$ defines the ‘shape’ of the profile. The term $b_n$ is simply a function of $n$ and chosen4 to ensure the radius $R_e$ encloses half of the profile’s total luminosity. Using the substitution $x = b_n(R/R_e)^{1/n}$, and thus $R = x^n R_e/(b_n)^n$ and $dR = R_e n/(b_n)^n x^{n-1} dx$, the Petrosian index reduces to

$$\eta(x, n) = \frac{2n \gamma(2n, x)}{e^{-x} x^{2n}},$$

(3)

where $\gamma(2n, x)$ is the incomplete gamma function defined as

$$\gamma(2n, x) = \int_0^x e^{-t} t^{2n-1} dt.$$  

(4)

The inverted Petrosian index, $1/\eta(R)$, is more commonly used in the literature (e.g., Bershady, Jangren, & Conselice 2000; Blanton et al. 2001) and is shown in Figure 1. It has a value of 1 at $R = 0$ and falls to zero at large radii. Although the Petrosian index can be used to determine a galaxy’s ‘Petrosian radius’ $R_P$ — the radius where the index equals some fixed value — it is important to determine the relation between this radius and the effective radius $R_e$: as shown inset to Fig. 1, $R_P/R_e$ varies with Sérsic index $n$ for a fixed $1/n$. Constant multiples of $R_P$ have been used as a means to define an appropriate aperture size for purposes of deriving galaxy magnitudes.

3 A useful compilation of various Sérsic expressions can be found in Graham & Driver (2005).

4 The value $b_n$ is such that $\Gamma(2n) = 2\gamma(2n, b_n)$, with $\gamma$ and $\Gamma$ the incomplete and complete gamma functions respectively (Ciotti 1991).
For example, some Authors have chosen to use $2R_P$ with $1/\eta(R_P) = 0.2$ (e.g., Bershady et al. 2000; Blanton et al. 2001) and others $3R_P$ with $1/\eta(R_P) = 0.5$ (e.g., Conselice et al. 2002, 2003). The SDSS consortium adopted the former criteria. From Figure 1, one can see that different light–profile shapes reach constant values of $1/\eta$ at different fractions of their effective half–light radii $R_e$. (The absolute value of $R/R_e$ and the value of $n$ determine $1/\eta$, see equation 3.)

This is shown more clearly in the inset Figure where one can see, as a function of profile shape $n$, the number of effective radii that $1/\eta = 0.2$ and 0.5 correspond to. If $n=4$, then $1/\eta(R_P)$ = 0.2 occurs at $R_P = 1.82R_e$. However, when using $1/\eta(R_P) = 0.5$, if $n=4$ then $R_P$ equals only $0.16R_e$. Although this particular radius is multiplied by a factor of three before determining the Petrosian magnitude, this still amounts to an aperture size less than $0.5R_e$. This particular Petrosian magnitude therefore greatly underestimates an object’s total magnitude.

For a Sérsic profile, the Petrosian magnitude, $m_P$, is given by the expression

$$m(<NR_P) = m_e - 5\log R_e - 2.5\log \left[ \frac{\gamma(2n)}{(b_n)^{2n}} \right] (2n, x_P),$$

where $x_P = b_n(NR_P/R_e)^{1/n}$. $N$ is the multiplicative factor (usually 2 or 3), and $m_e = -2.5\log I_e$. The total magnitude is obtained by replacing $\gamma(2n, x_P)$ with the (complete) gamma function $\Gamma(2n)$. The extent to which the Petrosian magnitude underestimates the total magnitude is shown in Figure 2 under two conditions. The first is when the Sérsic profiles extend to infinity (panel a). Although galaxies are recognized not to have sharp edges, the second condition assumes that the Sérsic profiles truncate at $5R_e$ (panel b). There is no physical justification for a truncation at $5R_e$; this is simply chosen in order to explore the magnitude deficit if the light–profiles don’t extend to infinity. While elliptical galaxy light–profiles continue into the background sky–noise (e.g., Caon et al. 1993), there is evidence that some disk galaxies may truncate at ~4 scale–lengths, or change their exponential slope at these radii (van der Kruit 2001; Pohlen et al. 2004; Erwin, Beckman, & Pohlen 2005), but see Narayan & Jog (2003) and also Bland-Hawthorn et al. (2005) who present a light–profile for the late-type disk galaxy NGC 300 which extends to 10 scale–lengths. Figure 2 thus provides a boundary of sorts to the extent that Petrosian magnitudes may underestimate a galaxy’s total magnitude as a function of the underlying profile shape. To avoid possible confusion, we note that $R_e$ shall always refer to the value associated with the $R^{1/n}$ model extended to infinity. Thus, when the profile is assumed to truncate at $5R_e$, the value of $R_e$ does not change.

One can see from Figure 2a that the use of $1/\eta(R_P)=0.5$ results in magnitude differences of 0.5 mag when $n \sim 2.5$; 1.25 mag when $n = 4$; 2 mag when $n \sim 5.5$ and considerably worse for galaxies with yet higher values of $n$. Obviously such an approach to determine galaxy magnitudes should be used with caution. When dealing with dwarf galaxies, because of their faint central surface brightnesses, the sky flux can often dominate at the radius where $1/\eta(R_P) = 0.2$. The use of $1/\eta(R_P) = 0.5$ is thus more practical, and for galaxies with $n \lesssim 2$ the bulk of their flux is still recovered. Overall, however, the use of $1/\eta(R_P) = 0.2$ does much better at recovering a galaxy’s true magnitude. When $n=4$, the magnitude difference $^7$ is only 0.20 mag, rising to 0.64 mag when $n = 10$. Things are better for profiles with smaller values of $n$ and if one considers the profiles truncate at $5R_e$ (Figure 2b).

3. RECOVERING EFFECTIVE RADIi AND TOTAL (SÉRSIC) MAGNITUDES

In this section we will only consider the case where $1/\eta(R_P) = 0.2$, using $2R_P$ to define the Petrosian magnitude. The merits of this precise definition, which was used by the SDSS team, are exalted in Strauss et al. (2002, their section 3.2).

3.1. Concentration

If one had some way of knowing the underlying light–profile shape ‘$n$’, then from Figure 2 one could correct the Petrosian magnitudes for the missing flux beyond $2R_P$. Given that the Petrosian index was developed in part to avoid fitting a model to an object’s light–profile, one may prefer not to fit the $R^{1/n}$ model, or indeed one may not have the resolution to do so. Conveniently, central light ‘concentration’ is monotonically related to the shape of a light–profile (Trujillo, Graham, & Caon 2001a; Graham et al. 2001). One can therefore use concentration as a proxy for the value of $n$.

The SDSS consortium have been using a ratio of two radii $(R_{50}/R_{90})$ as a measure of an object’s concentration, both of which are available from the SDSS public data releases. These radii enclose 90% and 50% of the Petrosian flux and their ratio is shown in Figure 3 and Table 1 as a function of the underlying light–profile shape $n$. The use of such radii avoids the inner seeing–affected part of a light–profile and also avoids the outer noisier part of a profile while still providing a useful range of concentrations for the various galaxies. For an $n = 1$ and $n = 4$ profile$^8$, $R_{90}/R_{50}$ equals 2.29 and 3.42 respectively. The inverse ratio is sometimes used, giving 0.44 and 0.29 (Blanton et al. 2001, their section 4.5). Not surprisingly, the $R_{90}/R_{50}$ concentration index correlates with galaxy type and also color (e.g., Strateva et al. 2001; Kauffmann et al. 2003).

3.2. Magnitudes

Although Section 3.3 of Bershady et al. (2000) reports that $1/\eta(R_P) = 0.5$ roughly corresponds to $R_P = 1R_e$, this is only true for Sérsic profiles with $n \lesssim 2$ (see Figure 1). Exact values for $R_P$, in terms of $R_e$, when $1/\eta(R_P) = 0.5$ are given in Bershady et al. (1998) for $n = 0.5, 1, 2, 3$.

$^5$ The use of $1/\eta(R_P) = 0.2$ has also been adopted by some Authors because it results in a minimal variation of $R_P/R_e$ with $n$. For $n \lesssim 4$, this value is around 2, and so to approximate the total light, one could measure the light within $R_P/2$ and multiply by 2.

$^6$ Bershady et al. (2000) reported a difference of 0.13 mag, but this is appropriate for an $n = 3$ profile rather than an $n = 4$ profile. The value of 0.13 mag was based on photometry of IRAF artdata simulations, and therefore is likely to be a round-off or discretization error in the IRAF rendering.

$^7$ If an $n = 1$ and $n = 4$ profile are integrated to infinity, rather than only $2R_P$, then $R_{50} = 2.32R_e$ and $5.55R_e$ respectively.
Combining Figures 2 and 3, Figure 4a shows the required magnitude correction in order to account for the missing flux beyond the Petrosian aperture 2RP, when 1/η(RP) = 0.2, as a function of concentration within the Petrosian aperture (i.e., curve (iii) from Figure 3). When n = 4, R90/R50 = 3.42, and Δmag = 0.20 and 0.07 mag to recover the total flux and that within 5Re, respectively (see Table 1).

To an accuracy of ∼ 0.01 mag, over the Sersic interval 0.1 < n < 10, the missing flux (Figure 4a) can be approximated by the expression

\[ \Delta m \approx P_1 \exp \left( \frac{R_{90}/R_{50}}{P_2} \right) \equiv m_P - m_x, \]  

(6)

where \( P_1 \) and \( P_2 \) equal 5.1×10^{-4} and 1.451 in order to recover the total flux (Figure 5a), and 5.9×10^{-5} and 1.597 in order to recover the flux within 5Re (Figure 5b). Here, \( m_P \) is the Petrosian magnitude and \( m_x \) is the corrected magnitude which, to an accuracy of ∼ 0.01 mag, is equivalent to the Sersic magnitude \( m_S \).

### 3.3. Radii

The effective radii containing half of the total (Sersic) flux can be computed in a number of ways.

Figure 4b is the combination of Figure 3 and Figure 1’s inset figure, and allows one to determine the effective radius \( R_e \), from the Petrosian radius and concentration \( R_{50}/R_{90} \) within the Petrosian aperture. Alternatively, from the total galaxy magnitude (approximated by \( m_x \)) one may empirically determine the aperture containing half a galaxy’s light and therefore obtain the half-light radius this way.

Alternatively still, for 0.1 < n < 10, the \( R_{50}/R_e \) curve seen in Figure 4c can be approximated in terms of Petrosian concentration by the expression

\[ R_e \approx \frac{R_{50}}{1 - P_3 (R_{90}/R_{50})^{P_4}} \equiv R_e, \]  

(7)

where \( P_3 \) and \( P_4 \) equal 8.0 × 10^{-6} and 8.47, respectively. This approximation is shown in Figure 5c.

### 3.4. Surface brightnesses

It turns out one can also easily transform \( \mu_{50} \), the surface brightness at \( R_{50} \), and \( \langle \mu \rangle_{50} \), the mean surface brightness within \( R_{50} \), into the effective surface brightness, \( \mu_e \), and the mean effective surface brightness \( \langle \mu \rangle_e \). From Graham & Driver (2005, their equation 6 and their section 2.2), one has that

\[ \mu_{50} - \mu_e = \frac{2.5b_n}{\ln(10)} \left[ \left( \frac{R_{50}}{R_e} \right)^{1/n} - 1 \right], \]  

(8)

and

\[ \langle \mu \rangle_{50} - \langle \mu \rangle_e = 2.5 \log \left[ \frac{R_{50}}{R_e} \right] \frac{x_2}{x_1}, \]  

(9)

where \( x_{50} = b_n (R_{50}/R_e)^{1/n} \) and \( x_{e} = b_n \). One can see that for a given value of n, the ratio \( R_{50}/R_e \) (Figure 4c) is sufficient to solve equations 8 and 9. Again using the relation between \( n \) and \( R_{90}/R_{50} \) (Figure 5, curve (iii)), Figures 4d and 4e show the surface brightness differences as a function of the concentration \( R_{90}/R_{50} \). In practice, one can directly measure the surface brightness at \( R_e \) and/or the mean surface brightness within \( R_e \), the latter of which can also be computed from the expression \( I_{rot} = 2\pi R_e^2 \langle I \rangle_e \). Approximations to equations 8 and 9 are therefore not given.

Because the above values of \( R_e, \mu_e \), and \( \langle \mu \rangle_e \) are those pertaining to a non–truncated Sersic profile, equations 7–9 are independent of any possible profile truncation beyond the Petrosian aperture.

### 4. Application

#### 4.1. Demonstration with SDSS data

The SDSS consortium adopted a slightly modified form of the Petrosian index. Rather than dividing the average intensity within \( R_P \) by the intensity at \( R_P \) (Equation 1), they divided the average intensity within \( R_P \) by the average intensity within 0.8–1.25 \( R_P \). This was done so as to reduce the sensitivity of the index to noise and (possible) real small–scale fluctuations in the light–profile (Strauss et al. 2002). As a result, their definition of the Petrosian index is given by the expression

\[ \eta(x, n) = \frac{\gamma(2n, x_{1.25 R}) - \gamma(2n, x_{0.8 R})}{[(1.25)^2 - (0.8)^2] \gamma(2n, x)}, \]  

(10)

where \( x_{1.25 R} = b_n (1.25 R/R_e)^{1/n} \) and \( x_{0.8 R} = b_n (0.8 R/R_e)^{1/n} \).

In general, where the SDSS Petrosian index (Equation 10) equals 1/0.2, the associated Petrosian radii are slightly smaller than the radii obtained previously with Equation 3 equal to 1/0.2 (see Table 1). The required magnitude corrections are therefore slightly greater when using the SDSS definition. The overall appearance of Figures 1–4 do not change, but the exact numbers are different by a few percent and therefore provided in the middle section of Table 1. The approximations given by equations 6 and 7 also require a slight modification. Using the SDSS definition of the Petrosian index (equation 10), to recover the total magnitude one now has \( P_1 = 4.2 \times 10^{-4} \) and \( P_2 = 1.514 \) (Figure 5d), and to recover the flux within 5Re \( P_1 = 8.0 \times 10^{-5} \) and \( P_2 = 1.619 \) (Figure 5e). The values for \( P_3 \) and \( P_4 \) are 6.0 × 10^{-6} and 8.92, respectively (see Figure 5f).

We have tested the applicability and need for these corrections using real data from the SDSS consortium. Specifically, we have taken the Petrosian magnitudes, 50–percent Petrosian radii, and surface brightnesses \((m_P, R_{50}, \langle \mu \rangle_{50})\) from the SDSS Second Data Release3 (Abazajian et al. 2004) and the Sersic–derived total magnitudes, effective radii, and surface brightnesses \((m_S, R_e, \langle \mu \rangle_e)\) from the New York University Value–Added Galaxy Catalog (NYU–VAGC, Blanton et al. 2005)10. From this sample we then removed those galaxies whose Petrosian–derived \( R_{50} \) values are biased high by seeing; we crudely did this by excluding galaxies with \( R_e < 5'' \) (90% of SDSS DR2 seeing was between 1.0–1.6’’). In deriving Sersic parameters, Blanton et al. (2005) prevented the fits from obtaining

9 SDSS–DR2: http://www.sdss.org/dr2/

10 NYU–VAGC: http://sdss.physics.suny.edu/vagc/
Sérsic indices greater than 5.9 (and less than 0.2). To avoid any objects piled up at this boundary, and which therefore may not have accurate Sérsic quantities, we only use galaxies with \( n < 5.8 \) (and greater than 0.21). This left us with a sample of 16128 galaxies, more than enough to test our method\(^{11}\).

Figure 6 plots the difference between the Petrosian and Sérsic values as a function of the (SDSS-tabulated) concentration indices \( R_{90}/R_{50} \) within the Petrosian apertures. A similar figure using Sérsic \( n \) rather than concentration is given in Blanton et al. (2003a, their figure 14). The solid curves show the expected differences that we have derived. The dashed curves show the selection boundary imposed upon the data due to the restriction used by Blanton et al. (2005). Confining \( n \) to values smaller than 5.8 can prevent an accurate recovery of the total Sérsic flux in large galaxies, and (artificially) prevents Sérsic magnitudes getting more than 0.37 mag brighter than the SDSS-defined Petrosian magnitudes (see Table 1, middle section). Of course, measurement errors in either the Petrosian or Sérsic magnitudes, at least to a concentration of \( R_e = 0.087 \) can result in differences outside of this selection boundary.

In Figure 6a, the Sérsic magnitudes are clearly brighter than the Petrosian magnitudes for concentrations greater than about 2.7 (\( n \sim 1.7 \)). In Figure 6b it is obvious that our corrected Petrosian magnitudes agree with the Sérsic magnitudes, at least to a concentration of \( \sim 3.3 \) (\( n \sim 4 \)) at which point the selection boundary from the Sérsic fits start to dominate the figure. In the concentration bin \( 2.75 \leq R_{90}/R_{50} < 3.0 \), the mean difference between the Petrosian and Sérsic magnitudes, \( \langle m_P - m_S \rangle \), equals 0.087\pm0.005 mag: close to the expected offset of 0.06 mag at \( R_{90}/R_{50} = 2.87 \) (the mean concentration inside this bin). In this same bin, \( \langle m_P - m_S \rangle = 0.027 \pm 0.005 \) mag. In the next bin, \( 3.0 < R_{90}/R_{50} < 3.25 \), \( \langle m_P - m_S \rangle = 0.144 \pm 0.004 \) mag, with the expected value at \( R_{90}/R_{50} = 3.12 \) (the mean concentration inside this bin) equal to 0.13 mag. Similarly, \( \langle m_S - m_S \rangle = 0.028 \pm 0.004 \) mag in this bin. The mean value of \( m_P - m_S \) in the bin \( 3.25 \leq R_{90}/R_{50} < 3.5 \) is 3.34, for which the expected value of \( \langle m_P - m_S \rangle \) is 0.22 mag. However, the measured value is only 0.148\pm0.006 mag. The reason for this is partly due to the selection boundary, but primarily due to the underestimation of the Sérsic magnitude by Blanton et al. (2005). Figure 9 from Blanton et al. reveals that at \( n = 4 \), corresponding to \( R_{90}/R_{50} = 3.35 \), their Sérsic fluxes are underestimated by \( \sim 7\% \). This therefore accounts for the value of 0.148\pm0.006 instead of \( \sim 0.22 \), and for the negative value of \( \langle m_S - m_S \rangle = -0.063 \pm 0.006 \) mag in this bin.

For a system with \( n = 5 \), Blanton et al. (2005) typically obtained \( n = 4.0 - 4.6 \) and recovered only \( \sim 90\% \) of the actual flux from their simulated test galaxies. The situation is systematically worse for galaxies with higher values of \( n \) and accounts for much of the apparent over-correction in Figure 6b at high-concentrations. Due to this problem, the simple corrective procedure presented here to allow for the missing flux outside of the Petrosian aperture actually provides a means to acquire (total) Sérsic magnitudes which are more accurate than those obtained from the direct application of Sérsic models to the data. For a typical elliptical galaxy, our method improves the accuracy on the galaxy magnitude from a couple of tenths of a mag to a couple of hundredths of a mag (assuming that the Sérsic profile within the Petrosian aperture continues outside of it).

We do note that \( >90\% \) of the SDSS Main Galaxy Sample (\( m_r < 17.77 \)) have \( R_{90}/R_{50} < 3.5 \) (Blanton et al. 2003b; Nakamura et al. 2003). Thus, cases where corrections are most severe are rare. This, however, is not to say that the high-concentration objects are uninteresting. Due to the \( M_{B_{vir}} \)-concentration relation (Graham et al. 2001), they are the galaxies expected to have the most massive supermassive black holes.

In passing we also note that Figures 6a and 6b reveal that the SDSS Petrosian magnitudes are too faint, in the sense that they underestimate the total galaxy flux, at the high-luminosity end. Correcting for this increases the galaxy number counts at the bright-end of the (Petrosian-derived) luminosity function (see Blanton et al. 2003a).

Figure 6c shows the difference between the Petrosian radii \( R_{90} \) and the Sérsic effective radii. There is a clear offset at the high-concentration end of the diagram, albeit constrained within the selection boundary which prevented Sérsic indices greater than 5.8 and thus, from Table 1, \( R_{90}/R_e < 0.53 \). At the concentration \( R_{90}/R_{50} = 3.4, n \sim 4.5 \) and from Figure 9 in Blanton et al. (2005) one can see that their Sérsic effective radii are only \( \sim 80\% \) of the true effective radii. Allowing for this, the average ratio \( R_e/R_e \) should be \( \sim 1.25 \) at \( R_{90}/R_{50} = 3.4 \), exactly as observed in Figure 6d. That is to say, the apparent mis-match at high-concentrations is understood: it arises from the systematics which Blanton et al. (2005) identified in their fitted Sérsic quantities.

Figure 6e reveals that the early-type galaxies, defined to be those with concentrations greater than 2.86 (e.g., Shimasaku 2001; Nakamura et al. 2003), have Petrosian surface brightnesses which are clearly brighter than the Sérsic-derived surface brightness. Figure 6f shows that the procedure does a good job at correcting for the missing flux beyond the Petrosian apertures, at least until a concentration of around 3.3 where the selection boundary and the underestimation of the Sérsic flux once more come into play.

**4.2. Application in CAS space**

Another popular setup is that used in the CAS system (Conselice 2003), which measures the concentration (C), asymmetry (A), and stellar/star-forming clumpiness (S) of a galaxy’s light distribution. Within the CAS code, Petrosian apertures have sizes of \( 1.5R_p \), with \( 1/\eta(R_p) = 0.2 \). Concentration is also defined slightly differently from that used by the SDSS consortia: it is the ratio of radii containing \( 80\% \) and \( 20\% \) (rather than \( 90\% \) and \( 50\% \)) of the flux within the Petrosian aperture (Bershady et al. 2000).

Theoretically, it doesn’t matter what choice of radii one uses to define concentration. For example, radii containing \( 100\% \) and \( 50\% \) of the flux within the Petrosian aperture would work. In practice, the use of a large upper-

\(^{11}\) For those who need or wish to use the full galaxy sample, the seeing-affected radii \( R_{90} \) will first need to be corrected (see Trujillo et al. 2001b and 2001c for a prescription to do this). The Sérsic radii \( R_e \) were obtained by Blanton et al. (2005) from the application of seeing-convolved Sérsic profiles and therefore need no further seeing-correction.
percentage and a small lower–percentage leads to a greater range of concentrations and thus a clearer distinction between different profile types. However, not using too large an upper–percentage allows one to work with a less noisy part of the light–profile. Not using too small a lower–percentage means that one is less vulnerable to the effects of seeing. An analysis of the optimal concentration index (e.g., Graham et al. 2001) is, however, beyond the intended scope of this paper and will be addressed elsewhere. We note that such a search need not be limited to differing percentages, but could include radii based on different values of the Petrovian index. Here we simply provide the corrections using the popular SDSS and CAS definitions for the concentration index and Petrovian index/aperture.

The various size and flux transformation and corrective terms for the specific Petrovian setup (1.5R_P, with 1/η(R_P) = 0.2) and concentration (R_{80}/R_{20}) used by the CAS code of Conselice et al. (2003) are given in the lower section of Table 1. To approximate the total Sérsic magnitude, over a range in n from 0.1 to 10, this missing flux ∆m can be approximated by

\[ \Delta m \approx -0.096 + 0.021(R_{80}/R_{20}) + 0.0044(R_{80}/R_{20})^2 \]

\[ \approx m_P - m_x. \]  

To obtain the flux within 5R_e, the following provides a good approximation

\[ \Delta m \approx -0.012 - 0.0013(R_{80}/R_{20}) + 0.00346(R_{80}/R_{20})^2. \]  

The effective Sérsic radius \( R_e \) is given by

\[ R_{50}/R_e \approx 0.903 + 0.102(R_{80}/R_{20}) - 0.0276(R_{80}/R_{20})^2 \]

\[ +0.00119(R_{80}/R_{20})^3. \]  

4.3. General applicability

Caon et al. (1993) have shown Sérsic’s \( R^{1/n} \) model fits early–type galaxies remarkably well (see their figure 2) down to faint surface brightness levels (\( \mu_B \approx 27 \) mag arcsec\(^{-2} \)). Spiral galaxies, however, usually have two clearly distinct components, namely a bulge and a disk. It is therefore pertinent to inquire how the above corrections may apply when a galaxy is clearly better represented by two components, as is the case for the intermediate–type galaxies.

When dealing with luminous galaxies, the rough divide between early–type and late–type galaxies has been taken to occur at \( R_{80}/R_{50} = 2.86 \) (n \( \approx 2 \)) (Shimasaku 2001; Nakamura et al. 2003; Shen et al. 2003, using the SDSS definition of Petrovian index and aperture).\(^{12} \) It should be kept in mind that this concentration based definition of galaxy type is only applicable to luminous systems. Dwarf elliptical galaxies have exponential–like profiles and thus low concentrations (e.g., Graham et al. 2001; their figure 8), but are obviously not late–type galaxies. The magnitude correction for galaxies having concentrations of this size or smaller is less than 0.06 mag (Table 1). In late–type galaxies, the exponential disk, rather than the bulge, dominates the flux in the outer parts. Whether or not these disks continue for many scale–lengths or truncate at a few scale–lengths, if one applies the corrections presented here, one will not over–correct the galaxy flux by more than 0.06 mag for the 2–component late–type galaxies, nor overestimate the half–light radius by more than 7% (Table 1).

We caution that intermediate objects, such as lenticular galaxies, with half their light coming from their disk and half from their bulge may not be well approximated with a single Sérsic profile. In such cases the concentration prescription given here should be used with care as it is intended for systems which can be approximated with a single Sérsic function. In general, however, Section 4.1 and in particular Figure 6 reveal that the method developed here seems to work rather well.

A different issue pertains to how one actually measures the Petrovian index. For a disk system, the Petrovian index is independent of disk inclination if one uses appropriate elliptical apertures reflecting the inclination of the disk. If one instead measures the index from the slope of the major–axis light–profile, such that \( 2/\eta = d[ln L(R)]/d[ln R] \) (Gunn & Oke 1975), then again the index is not dependent on the inclination of the disk. However, the use of circular apertures applied to highly–inclined disk galaxies will result in erroneously high concentrations compared to what these values would be if such galaxies were viewed with a face–on orientation. Although we have not quantified this effect, comparisons between eye–ball morphology and concentration index yields reasonable agreement for the late-type galaxies (e.g., Shimasaku et al. 2001), suggesting that in practice this effect is not a significant problem.

Lastly, we note that if image distortion due to seeing is not corrected for and is such that the PSF has altered the Petrovian 50% radius \( R_{50} \) and thus \( R_{90}/R_{50} \) (or \( R_{80}/R_{20} \)) to the extent that the measured value no longer reflects the intrinsic galaxy concentration, then obviously one should not be including such objects in plots or analyses that use this quantity. The concentration–based corrections outlined in this study cannot be applied in such circumstances, at least until the radii are corrected for seeing (e.g., Trujillo et al. 2001b,c). We also direct readers to Blanton et al. (2001) who present a study of how the SDSS Petrovian magnitudes are affected by seeing, and Wirth et al. (1994) who considered the affects of both poor image resolution and low signal–to–noise on image concentration.

5. SUMMARY

In commenting on the adopted Petrovian system of the SDSS consortium, Strauss et al. (2002) wrote: “Any scientific analysis that uses the [SDSS] redshift survey must consider how the fact that a different fraction of light is included for elliptical and spiral galaxies affects the result”.

\(^{12} \) Other Authors have used \( n = 2.5 \) (e.g., Blanton et al. 2003b) or \( R_{80}/R_{20} = 2.6 \) (e.g., Strateva et al. 2001; Kauffmann et al. 2003) to mark the rough divide between luminous early– and late–type galaxies.
This was both a recognition that a) their particular Petrosian system likely captures a different fraction of galaxy light compared to other surveys and that b) any practical definition of Petrosian parameters will inevitably miss some fraction of a galaxy’s light, and a different fraction in different galaxies. For example, a galaxy with an exponential light–profile will have 99% of its total flux encompassed by the SDSS Petrosian magnitude, but if a galaxy has an $R^{1/4}$ profile, then the flux beyond the SDSS Petrosian aperture accounts for an additional 0.22 mag, i.e., roughly 1/5 th of that galaxy’s total light. In this paper we outline a simple method for accurately recovering the missed flux.

Differences between galaxy light–profile shapes result in a range of galaxy concentrations. We have shown how these can be used to correct for the flux outside of the Petrosian apertures and thus provide a galaxy’s total (Sérsic) magnitude. The corrections presented here hold to the extent that Sérsic’s $R^{1/n}$ model provides a good description of the various galaxy light–profiles. For early–type galaxies this is known to be the case (e.g., Caon et al. 1993), while for late–type galaxies the correction is <0.06 mag and one can argue whether or not to apply it.

For typical elliptical galaxies, Petrosian magnitudes can fall shy of the total magnitude by a couple of tenths of a magnitude. Application of our method to the SDSS Petrosian magnitudes yielded total (Sérsic) magnitudes accurate to a couple of hundredths of a magnitude. Moreover, it actually provided a more accurate measurement of this quantity than obtained from direct Sérsic fits to the data (Blanton et al. 2005).

We have shown how the galaxy concentration can also be used to determine the effective half–light radius $R_e$ from the Petrosian radius $R_p$, and thereby additionally leading to a determination of the associated surface brightness terms. We also provide magnitude corrections under the assumption that galaxy light–profiles truncate at 5 effective radii.

In regard to the SDSS data, Blanton et al. (2005) showed how their Sérsic fits to objects with increasingly higher values of $n$ resulted in increasingly underestimated effective radii. For example, their code obtained radii that were ~80% of the true effective radii for simulated objects with $n = 4.5$ ($R_{n0}/R_{200} = 3.4$). Due to this, the concentration–corrected effective radii derived from the Petrosian 50% radii are in fact preferable to those obtained from the direct Sérsic fits. The same is obviously true for the associated surface brightness terms.

In passing we warn that the use of $3R_p$, with $1/\eta(R_p) = 0.5$, misses more than half of an $R^{1/4}$ profile’s flux and should not be used in the analysis of bright elliptical galaxies. The use of $2R_p$, with $1/\eta(R_p) = 0.2$, as justified by the SDSS consortium, is suitably appropriate. The optimal choice of concentration remains an outstanding issue.

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Fig. 1.— Inverted Petrosian index, $1/\eta$ (equation 3), as a function of normalized radius, $R/R_e$, for light–profiles having a range of Sérisc shapes $n=0.5, 1, 2, 3,..., 10$ (equation 2). Inset Figure: Number of effective radii that the Petrosian radius $R_P$ — the radius $R$ at which the inverted Petrosian index equals some value — corresponds to when the $1/\eta = 0.2$ and 0.5 (i.e., the dot–dashed lines in the main figure).
Fig. 2.— Panel a) shows the difference, as a function of light–profile shape ‘$n$’, between the Petrosian magnitude (equation 5) inside (i) twice the radius where $1/\eta=0.2$ and (ii) thrice the radius where $1/\eta=0.5$ and the total magnitude obtained by integrating the $R^{1/n}$ profile to infinity. Panel b) is similar to panel a) but assumes the $R^{1/n}$ light–profiles are truncated at $5R_e$.

Fig. 3.— Concentration index, defined as the ratio of radii $R_{90}/R_{50}$, is shown as a function of the light–profile shape $n$. (i) $R_{90}$ and $R_{50}$ are simply the radii containing 90% and 50% of the total flux (obtained by integrating the Sérsic profile to infinity, note that $R_{50} = R_e$ in this case). (ii) Similar to (i) except that the total flux is now considered to be only that within $5R_e$ (i.e., the Sérsic profile is truncated at $5R_e$). (iii) $R_{90}$ and $R_{50}$ are now the radii containing 90% and 50% of the flux within $NR_P$, where we have chosen $N = 2$ and $R_P$ is the Petrosian radius such that $1/\eta(R_P) = 0.2$. 
Fig. 4.— a) Magnitude differences shown in Figure 2 for the $1/\eta(R_p) = 0.2$ curves versus the associated Petrosian concentration index $R_{90}/R_{50}$ within the Petrosian aperture $2R_p$ (as shown by curve (iii) in Figure 3). The solid curve gives the correction to obtain the total (Sérsic) magnitude, while the dash-dot curve gives the correction if the underlying Sérsic profile is truncated at $5R_e$. b) The ratio of Petrosian radii $R_p$ (where $1/\eta(R_p) = 0.2$) to effective radii $R_e$. c) Radius containing 50% of the Petrosian flux, $R_{50}$, divided by the effective radius $R_e$. d) Difference between the surface brightness at $R_{50}$ and $R_e$. e) Difference between the mean surface brightness within $R_{50}$ and the mean surface brightness within $R_e$. All panels are relative to $R_{90}/R_{50}$ within $2R_p$ and with $1/\eta(R_p) = 0.2$ using the “standard” definition for the Petrosian index (equation 3).
Fig. 5.— Panel a and b) Difference between the exact magnitude corrections seen in Figure 4a and the approximate magnitude corrections given by equation 6. Panel c) Relative error in the radius $R_x$, the approximation used to represent the effective radius $R_e$ (equation 7). Panels d–f) Similar to panels a–c) but based upon the SDSS definition of the Petrosian index (equation 10).
Fig. 6.—Panel a) The points show the difference between the SDSS Petrosian magnitude \( m_P \) and the NYU–VAGC Sérsic magnitudes \( m_S \) as a function of concentration for 16128 galaxies. The solid curve is the expected difference based on the correction we have formulated to obtain the flux beyond the Petrosian aperture. The dashed line reflects the selection boundary imposed on the data by the restriction that \( n \leq 5.9 \) in the Sérsic fitting process used by Blanton et al. (2003b). We made a cut at \( n = 5.8 \) to avoid any galaxies which may have piled up at the upper boundary to \( n \). Panel b) Our corrected Petrosian magnitudes \( m_x \) versus the Sérsic magnitudes. The selection boundary in panel a) has been propagated into this panel. Panel c) Petrosian 50–percent radii \( R_{50} \) divided by the Sérsic effective radii \( R_e \) vs. concentration. From Table 1, one can see that the restriction \( n \) be less than 5.8 (artificially) prevents \( R_{50}/R_e \) from getting smaller than 0.53. Panel d) Our corrected values for \( R_{50} \), denoted here by \( R_x \), are shown divided by the Sérsic radii \( R_e \). Again we have propagated the selection boundary. Panels e) and f) are similar to the other panels but show the mean surface brightness.
| (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|
|     | $R_p$ | $R_{50}$ | $R_{90}$ | $R_{50}/R_{90}$ | $R_{50}/R_{20}$ | $R_{20}/R_{80}$ | $R_{80}/R_{20}$ | $R_{20}/R_{50}$ |
| 0.1 | 1.57 | 1.00 | 1.40 | 0.71 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.2 | 1.70 | 1.00 | 1.51 | 0.66 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.3 | 1.80 | 1.00 | 1.62 | 0.62 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.5 | 1.96 | 1.00 | 1.82 | 0.55 | 0.00 | 0.00 | 0.00 | 0.00 |
| 0.7 | 2.06 | 1.00 | 2.02 | 0.49 | 0.00 | 0.00 | 0.00 | 0.00 |
| 1.0 | 2.16 | 0.99 | 2.29 | 0.44 | 0.01 | 0.01 | 0.01 | 0.00 |
| 2.0 | 2.20 | 0.93 | 2.90 | 0.34 | 0.13 | 0.09 | 0.06 | 0.02 |
| 3.0 | 2.04 | 0.84 | 3.23 | 0.31 | 0.36 | 0.26 | 0.13 | 0.04 |
| 4.0 | 1.82 | 0.73 | 3.42 | 0.29 | 0.63 | 0.48 | 0.20 | 0.07 |
| 5.0 | 1.59 | 0.62 | 3.56 | 0.28 | 0.94 | 0.74 | 0.28 | 0.11 |
| 6.0 | 1.36 | 0.53 | 3.65 | 0.27 | 1.27 | 1.03 | 0.35 | 0.15 |
| 7.0 | 1.15 | 0.44 | 3.72 | 0.27 | 1.62 | 1.33 | 0.43 | 0.20 |
| 8.0 | 0.97 | 0.37 | 3.78 | 0.26 | 1.98 | 1.65 | 0.50 | 0.25 |
| 9.0 | 0.81 | 0.31 | 3.83 | 0.26 | 2.35 | 1.98 | 0.57 | 0.30 |
| 10.0 | 0.67 | 0.25 | 3.87 | 0.26 | 2.73 | 2.33 | 0.64 | 0.36 |

**Note.** — Col.(1): Sérsic index $n$. Col.(2): Petrosian radius $R_p$ such that $1/\eta(R_p) = 0.2$. Col.(3): Radius containing 50% of the Petrosian flux. Col.(4): Concentration index defined by the ratio of radii that contain 90% & 50% (and 80% & 20%) of the Petrosian flux. Col.(5): The inverse of column (4). Col.(6): Difference between the surface brightness at $R_e$ and the value at $R_{50}$. Col.(7): Difference between the mean surface brightness inside the radii $R_e$ and $R_{50}$. Col.(8): Difference between the Petrosian magnitude and the total magnitude (obtained by integrating the Sérsic profile to infinity). Col.(9): Difference between the Petrosian magnitude and the Sérsic magnitude if the profile is truncated at $5R_e$. 

Table 1: 

**Petrosian Radii, Concentration, and Magnitude Corrections**