Different D-brane Interactions

H. Arfaei
and
M.M. Sheikh Jabbari

Institute for Studies in Theoretical Physics and Mathematics IPM
P.O.Box 19395-5746, Tehran, Iran
Department of Physics Sharif University of Technology
P.O.Box 11365-9161
e-mail Arfaei@theory.ipm.ac.ir
e-mail jabbari@netware2.ipm.ac.ir

Abstract

We use rotation of one D-brane with respect to the other to reveal the hidden structure of D-branes in type-II theories. This is done by calculation of the interaction amplitude for two different parallel and angled branes. The analysis of strings with different boundary conditions at the ends is also given. The stable configuration for two similar branes occurs when they are anti-parallel. For branes of different dimensions stability is attained for either parallel or anti-parallel configurations and when dimensions differ by four the amplitude vanishes at the stable point. The results serve as more evidence that D-branes are stringy descriptions of non-perturbative extended solutions of SUGRA theories, as low energy approximation of superstrings.
I. Introduction

The observation that D-branes provide stringy descriptions of topological objects in string theory has been source of many recent developments [1,2,3]. They are sources of RR charges and are argued to provide short distance probes in some special cases, where strings fail [4,5]. Like solitonic solutions they interact via exchange of the basic quanta of the theory i.e. closed strings which are intrepreted as D-strings in the dual channel.

It is important to verify that the D-branes which are topological defects, able to trap the end of the strings, and solitonic black solutions of the low energy limit which are non-perturbative objects of the theory are identical objects. If this is so we have our hands on a stringy description of a non-perturbative effects without having a non-perturbative formalism.

There have been attempts to present justifications of this view [6]. The main contribution coming from work of Polchinski [1,2], where it has been shown that D-branes carry RR charges. It was shown that the gravitional interaction of two D-branes is balanced by their RR repulsion. On the other hand solitonic or black branes solutions to SUGRA [7,8] have more structure than D-branes which are merely given by hyperplanes.

In this work we consider the interaction of two different D-branes which are not parallel and find the orientation dependence of their interaction. In Nilsen-Olsen vortices (or similarly in typeII super conductivity), rotating a vortex by 180 degrees, the repulsive force of the parallel flux lines turns into attractive force of the two anti-parallel vortices. We see similar effects for the RR force between two non-parallel D-branes and find its orientation dependence [9] and furthermore the angle between D-branes can be considered as a collective dynamical varible. The stable situation is where two branes are anti-parallel, which is justified by field theoretical arguments. This observation gives further evidence for the identity of D-branes and solitonic black branes, where their interactions is described by $\int J.A$ term in which $J$ is the related P-brane current. We also find the velocity dependence of the interactions of two D-branes which confirms the intuitive expectation to see the corresponding magnetic RR force in agreement with the low velocity results of [4] and [6]. For completeness we also include the ordinary bosonic strings results in the appendix, in this case only graviton and dilaton are present.

II. Interaction of two parallel D-branes of $P$ and $P'$ dimensions in type-II theories:

Consider two D-branes of dimensions $p$ and $p'$ ($p \geq p'$). We assume that they interact via exchange of closed strings, i.e. the basic object of the theory. Following [1] to calculate the amplitude we consider the open string loop in the dual channel. In this part we assume the
branes to be parallel and non intersecting. The case of angled (non-parallel non-intersecting) branes will be considered later. The $D_p$-brane is located at $X^\mu = Y^\mu \quad \mu = p + 1, \ldots, 9$ and the $D_{p'}$-brane at $X^\mu = 0 \quad \mu = p' + 1, \ldots, 9$.

We realize three types of open D-strings [10,11]: i) both ends on $D_p$-brane, ii) both ends on $D_{p'}$-brane and iii) one end on $D_p$ and the other on $D_{p'}$-brane. The first two types represent the excitations of the individual branes but the third is responsible for their interaction. The first $p'$ components satisfy Neumann boundary conditions at both ends, the $\Delta = p - p'$ components satisfy Neumann at $\sigma = 0$ and Dirichlet at $\sigma = \pi$ and remaining $9 - p$ components Dirichlet at both ends. This leads to half integer modes for the $X^\mu \quad \mu = p' + 1, \ldots, p$ which are not present in $p$ and $p'$ case. Hence the open strings stretching between two branes satisfy the conditions:

$$\sigma = 0 \begin{cases} X^\mu = 0 \quad \mu = p + 1, \ldots, 9 \\ \partial_\sigma X^\mu = 0 \quad \mu = 0, \ldots, p \end{cases}$$ (1)

$$\sigma = \pi \begin{cases} X^\mu = Y^\mu \quad \mu = p' + 1, \ldots, 9 \\ \partial_\sigma X^\mu = 0 \quad \mu = 0, \ldots, p' \end{cases}$$ (2)

The boundary conditions for the world-sheet fermions $\psi^\mu_+$ and $\psi^\mu_-$ follows from world-sheet super symmetry transformation:

$$\delta X^\mu = \bar{\epsilon} \psi^\mu$$ (3)

where $\bar{\epsilon}$ is a 2-D world-sheet fermion. There are, as expected, two types of open strings, Ramond($R$) type and Neveu-Schwarz($NS$)type. Putting all these together mode expansion for $X^\mu$ are:

$$X^\mu = p^\mu \tau + \sum_{n \in \mathbb{Z}} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \cos n\sigma \quad \mu = 0, \ldots, p'$$

$$= \sum_{r \in \mathbb{Z} + 1/2} \frac{1}{r} \alpha_r^\mu e^{-ir\tau} \sin r\sigma \quad \mu = p' + 1, \ldots, p$$

$$= Y_{\frac{\mu}{\pi}}^{\frac{\sigma}{\pi}} + \sum_{n \in \mathbb{Z}} \frac{1}{n} \alpha_n^\mu e^{-in\tau} \sin n\sigma \quad \mu = p + 1, \ldots, 9.$$ (4)

for both $R$ and $NS$ types, and for the $\psi^\mu_{\pm}$:

$$\begin{cases} \psi^\mu_+ = \sum_{n \in \mathbb{Z}} d_n^\mu e^{-in(\tau + \sigma)} & \psi^\mu_- = \sum_{n \in \mathbb{Z}} d_n^\mu e^{-in(\tau - \sigma)} \quad \mu = 0, \ldots, p' \\
\psi^\mu_+ = \sum_{n \in \mathbb{Z}} d_n^\mu e^{-in(\tau + \sigma)} & \psi^\mu_- = -\sum_{n \in \mathbb{Z}} d_n^\mu e^{-in(\tau - \sigma)} \quad \mu = p + 1, \ldots, 9 \\
\psi^\mu_+ = \sum_{r \in \mathbb{Z} + 1/2} d_r^\mu e^{-ir(\tau + \sigma)} & \psi^\mu_- = -\sum_{r \in \mathbb{Z} + 1/2} d_r^\mu e^{-ir(\tau - \sigma)} \quad \mu = p' + 1, \ldots, p \end{cases}$$ (5)
for the R-sector and
\[
\begin{align*}
\psi^\mu_+ &= \sum_{r \in \mathbb{Z} + 1/2} b_r^\mu e^{-ir(\tau + \sigma)} \\
\psi^\mu_- &= \sum_{r \in \mathbb{Z} + 1/2} b_r^\mu e^{-ir(\tau - \sigma)} \\
\psi^\mu_+ &= \sum_{n \in \mathbb{Z}} b_n^\mu e^{-in(\tau + \sigma)} \\
\psi^\mu_- &= -\sum_{n \in \mathbb{Z}} b_n^\mu e^{-in(\tau - \sigma)}
\end{align*}
\]
(6)
for the NS-sector. Upon quantization we get the following commutation relations:
\[
[\alpha^\mu_r, \alpha^\nu_s] = \delta_{r+s}\delta^{\mu\nu}
\]
(7)
\[
\{d^\mu_r, d^\nu_s\} = \delta_{r+s}\delta^{\mu\nu}
\]
(8)
\[
\{b^\mu_n, b^\nu_m\} = \delta_{n+m}\delta^{\mu\nu}
\]
(9)
Commutations of the other components are the same as usual. The mass spectrum of the strings are given by:
\[
\alpha' M^2 = \frac{Y^2}{4\pi^2\alpha'} + N - \left(\frac{1}{2} - \frac{\Delta}{8}\right) \quad \Delta = p - p'
\]
(10)
\[
N = \sum_{n > 0}^{8 - \Delta} \alpha_{-n} \cdot \alpha_n + \sum_{r > 0}^{\Delta} \alpha_{-r} \cdot \alpha_r + \sum_{r > 0}^{8 - \Delta} rb_{-r} \cdot b_r + \sum_{n > 0}^{\Delta} nb_{-n} \cdot b_n.
\]
(11)
for the NS sector and by
\[
\alpha' M^2 = \frac{Y^2}{4\pi^2\alpha'} + N \quad \Delta = p - p'
\]
(12)
\[
N = \sum_{n > 0}^{8 - \Delta} \alpha_{-n} \cdot \alpha_n + \sum_{r > 0}^{\Delta} \alpha_{-r} \cdot \alpha_r + \sum_{n > 0}^{8 - \Delta} nd_{-n} \cdot d_n + \sum_{r > 0}^{\Delta} rd_{-r} \cdot d_r.
\]
(13)
for R sector, \(\sum_{n > 0}^{8 - \Delta}\) and \(\sum_{r > 0}^{\Delta}\) mean that the sums include \((8 - \Delta)\) and \(\Delta\) component, which are related to \(DD\) and \(NN\) components and \(DN\) components respectively.

The amplitude for the exchange of one closed string expressed in terms of the open string loop in the dual channel is:
\[
A = \int \frac{dt}{2\tau} \sum_{i,p} e^{-2\pi\alpha'(p^2 + M_i^2)}
\]
(14)
where \(i\) indicates the modes of the open string and \(p\) the momentum which has non-zero components in the first \(p' + 1\) dimensions.

Before calculating the amplitude we must ensure that in closed string channel no tachyon will emerge. The following generalized GSO projections [12] are consistently sufficient for this purpose:
\[
NS: \quad P = 1/2(1 - G); \quad G = (-1)^{\sum_{r > 0}^{8 - \Delta} b_{-r}b_r + \sum_{n > 0}^{\Delta} b_{-n}b_n}.
\]
(15)
\[ R : \quad P = \frac{1}{2(1 + \Gamma)}; \quad \Gamma = (-1)^{\sum_{n \geq 0} d_n + \sum_{r > 0} d_r d_r}. \]  

The GSO again suggests that, the \( NS \) contribution must be subtracted from the \( R \) contribution, thus we obtain:

\[ A = 2V_{p'+1} \int \frac{dt}{2t} (8\pi^2 \alpha' t)^{-(p'+1)/2} e^{-\frac{\pi^2 t}{24\pi^2 \alpha'}} (NS - R). \]  

where \( NS \) and \( R \) are given by

\[ NS = 2^{\Delta/2 - 1} q^{-1} \left( \prod \left( \frac{1 - q^{2n}}{1 - q^{2n-1}} \right)^{\Delta} \left( \prod \left( \frac{1 + q^{2n}}{1 + q^{2n-1}} \right)^{\Delta} \right)^8 \right) \]  

\[ R = 2^{3-\Delta/2} \left( \prod \left( \frac{1 - q^{2n}}{1 - q^{2n-1}} \right)^{\Delta} \left( \prod \left( \frac{1 + q^{2n}}{1 + q^{2n-1}} \right)^{\Delta} \right)^8 \right) \]  

When examined in the closed string channel we see that no \( RR \) particle is exchanged due to the fact that the term coming from \((-G)^\Delta\) is absent here. The interaction comes from the exchange of \( NSNS \) particle where in the small \( t \) limit consists of graviton and dilaton [6].

Lack of \( RR \) term agrees with our field theoretic intuition. If \( D_p \)-brane is the manifestation of solitonic black branes their \( RR \) field is described by a \( p + 1 \) from. This potential can be probed by a \( RR \) charge of the same type, i.e. only by a P-brane and not \( P' \)-branes.

If we take the small \( t \) limit to isolate the contribution of the massless closed strings, long range interaction amplitude is obtained to be:

\[ A = V_{p'+1} (4\pi^2 \alpha')^{3 - \frac{p-p'}{2}} (2 - \Delta/2) \pi G_{9-p}(Y^2), \]  

in agreement with result of [2] that \( T_p = (4\pi^2 \alpha')^{3-p} \).

Note that when \( \Delta = 4 \) the amplitude \( \text{vanishes in all orders of} \ t \), indicating the existence of SUSY [12] and stable classical BPS solutions with non-vanishing two \( \text{different} \ RR \) charges.

III. \textbf{Angled branes in type-II theories:}

In this part we consider two non-intersecting D-branes which make an angle in the plane of \((p,p+1)\). Thus the second brane is like the first one but it is rotated by the angle \( \pi \theta \) in the \((p,p+1)\) plane. The boundary conditions for the \( X^\mu \) are:

\[ \sigma = 0 \left\{ \begin{array}{ll}
X^\mu = 0 & \mu = p + 1, \ldots, 9 \\
\partial_\sigma X^\mu = 0 & \mu = 0, \ldots, p
\end{array} \right. \]
where $\pi \theta$ is the angle between two non-intersecting $0 \leq \theta \leq 1$ D-branes. The world-sheet fermion boundary conditions again are obtained using the world-sheet SUSY transformation written above and the mode expansions are the same as usual except in the $(p,p+1)$ components where the modes look like twisted string modes with indices ranging over $\mathbb{Z}$:

\[
X^p = \sum_{n_+} \frac{1}{n_+} \alpha^p_{n_+} e^{-in_+ \theta} \cos n_+ \sigma + \sum_{n_-} \frac{1}{n_-} \alpha^p_{n_-} e^{-in_- \theta} \cos n_- \sigma. \tag{23}
\]

\[
X^{(p+1)} = \sum_{n_+} \frac{1}{n_+} \alpha^p_{n_+} e^{-in_+ \theta} \sin n_+ \sigma - \sum_{n_-} \frac{1}{n_-} \alpha^p_{n_-} e^{-in_- \theta} \sin n_- \sigma.
\]

\[
\begin{align*}
\psi^p_+ &= \sum_{n_+} d^p_{n_+} e^{-in_+ \theta} (\tau+\sigma) \\
\psi^p_- &= \sum_{n_+} d^p_{n_+} e^{-in_+ \theta} (\tau-\sigma) \\
\psi^{p+1}_+ &= i \sum_{n_+} d^p_{n_+} e^{-in_+ \theta} (\tau+\sigma) \\
\psi^{p+1}_- &= -i \sum_{n_+} d^p_{n_+} e^{-in_+ \theta} (\tau-\sigma) \\
\psi^{p+1}_+ &= \sum_{n_-} d^p_{n_-} e^{-in_- \theta} (\tau+\sigma) \\
\psi^{p+1}_- &= \sum_{n_-} d^p_{n_-} e^{-in_- \theta} (\tau-\sigma)
\end{align*}
\tag{24}
\]

\[
\begin{align*}
\psi^{p+1}_+ &= -i \sum_{n_-} d^p_{n_-} e^{-in_- \theta} (\tau+\sigma) \\
\psi^{p+1}_- &= i \sum_{n_-} d^p_{n_-} e^{-in_- \theta} (\tau-\sigma)
\end{align*}
\tag{25}
\]

for the $R$ sector, where $n \pm$ stands for $n \pm \theta$. $NS$ sector can be obtained by changing $n_\pm$ to $r_\pm$, where $r$ is in $Z + 1/2$. The commutation relation of the twisted operators reads as:

\[
[\alpha_{-n_\pm}, \alpha_{-n_\pm'}] = n_\pm \delta_{n+n'} \tag{26}
\]

If we insert these commutations the corresponding mass spectrum becomes as:

\[
\alpha' M^2 = \frac{Y^2}{4\pi^2 \alpha'} + N(\theta) \tag{27}
\]

\[
N(\theta) = \sum_{n>0} \alpha_{-n} \alpha_n + \sum_{n>0} n d_{-n} d_n + \sum_{n>0} \alpha_{n+}^p \alpha_{n+}^p + \sum_{n>0} \alpha_{-n-}^p \alpha_{n-}^p
\]

\[
+ \sum_{n>0} n_+ d_{-n_+}^p d_{n_+}^p + \sum_{n>0} n_- d_{-n_-}^p d_{n_-}^p + \frac{1}{2} (\alpha_{\theta} \alpha_{-\theta} + \alpha_{-\theta} \alpha_{\theta}) + \frac{1}{2} \theta (d_{-\theta}^p d_{\theta}^p - d_{\theta}^p d_{-\theta}^p) \tag{28}
\]

for the $R$ sector, $d$ and $\alpha$ are similar to what was defined before.

In $NS$ sector the oscillator part is the same as $R$ sector but $d$ is replaced by $b$, $n$ by $r$ and but the ordering constant is $1/2$, as we see the ordering constant doesn’t depend
on $\theta$ in both $R$, $NS$ cases. The GSO projection is again needed to remove tachyons from the closed string channel. This is implemented by using:

$$G = (-1) \sum_{r>0} b_r r + \sum_{r\geq 0} b_r r + b_r^p r + \sum_{r\geq 0} b_r^- r^-.$$  \hfill (29)$$

$$\Gamma = (-1) \sum_{n\geq 0} d_n n + \sum_{n\geq 0} d_n n^p + \sum_{n\geq 0} d_n^- n^-.$$  \hfill (30)$$

So amplitude is:

$$A(\theta) = 2V_p \int \frac{dt}{2t} (8\pi^2 \alpha' t)^{-p/2} e^{-\frac{\gamma^2 t}{2\pi^2 \alpha'}} (NS - R).$$  \hfill (31)$$

where $NS, R$ are given by

$$R = 8\pi^3 \left( \frac{\Theta_2(0 \mid it)}{\Theta_1(0 \mid it)} \right)^3 \frac{i\Theta_3(it \theta \mid it)}{\Theta_1(it \theta \mid it)}$$  \hfill (32)$$

$$NS = 8\pi^3 \left[ \left( \frac{\Theta_2(0 \mid it)}{\Theta_1(0 \mid it)} \right)^3 \frac{i\Theta_3(it \theta \mid it)}{\Theta_1(it \theta \mid it)} - \left( \frac{\Theta_4(0 \mid it)}{\Theta_1(0 \mid it)} \right)^3 \frac{i\Theta_4(it \theta \mid it)}{\Theta_1(it \theta \mid it)} \right].$$  \hfill (33)$$

To analyze this result consider the massless contribution by looking the small $t$ limit:

$$A(\theta) = V_p \int \frac{dt}{t} (8\pi^2 \alpha' t)^{-p/2} e^{-\frac{\gamma^2 t}{2\pi^2 \alpha'}} (8t^3 \tan(\pi \theta / 2) \sin^2(\pi \theta / 2))$$

$$= 4V_p \tan(\pi \theta / 2) \sin^2(\pi \theta / 2) (4\pi^2 \alpha')^{3-p} G_{8-p}(Y^2).$$  \hfill (34)$$
Following comments are in order:

1. At $\theta = 0$ $A$ vanishes (reproducing the result of [1]).

2. The potential altogether is proportional to $(1 - \cos \pi \theta)^2$, which has a minimum at $\theta = 1$ and a maximum at $\theta = 0$. This shows that, two P-branes tend to rotate so that they become anti-parallel. The anti-parallel case was also considered in [9], whose result is a special case of ours. It is instructive to compare the type-II results with the bosonic one which are given in the appendix. In type-II the $RR$ interaction of the D-branes reveal their structure more which is not obvious in their simple definition as hyperplanes. In the bosonic theory the amplitude is symmetric under $\theta \leftrightarrow 1 - \theta$ since the structure of the extended object is invariant under parity.

3. $RR$ and $NSNS$ massless contributions are:

$$V_{RR} = 8V_0 \cos \pi \theta \quad V_0 = 4V_p \frac{1}{\sin \pi \theta} (4\pi^2 \alpha')^{3-p} G_{8-p}(Y^2).$$

$$V_{NSNS} = -4V_0 (1 + \cos^2 \pi \theta)$$

$$V_{tot} = V_{NSNS} + V_{RR} = -V_0 (1 - \cos \pi \theta)^2.$$  

The $\theta$ dependence of the amplitude may be considered as further evidence for the identity of D-brane and extended low energy topological objects. The black branes have non-zero $RR$ gauge fields which are proportional to $\epsilon^{\mu_0 \ldots \mu_p}$ in the $p + 1$ dimensional P-brane hyperplane. The charge density is proportional to $\epsilon$ in the orthogonal space hence having an orientation. Close examination of the $\int J.A$ term as the interaction, clarifies the $\cos \pi \theta$ dependence of the $V_{RR}$.

Although the stringy description of D-branes is very simple with no input, the structure has become clear under rotation of one of them.

4. If we look at $\theta$ as a collective coordinate the amplitude suggests that two D-branes system will oscillate around the equilibrium position with frequency proportional to the $RR$ charge density (equal to the D-brane tension).

If $\theta$ is fixed the oscillator modes of $X$ corresponding to $\alpha_\theta$ and $\alpha_{-\theta}$ shows excitations with frequency proportional to $\theta$ since:

$$[\alpha_\theta, \alpha_{-\theta}] = \theta$$

These oscillations are much more easily excited than the collective mode oscillations of $\theta$, which requires infinite energy if the space is not compactified along $(p, p+1)$ plane. The limit $\theta$ going to zero is also interesting, in this limit only relevant part is $n = 0$ term in
summations

\[ X^p = \frac{1}{2\theta} \alpha_\theta^p e^{-i\theta\tau} \cos \theta \sigma - \frac{1}{2\theta} \alpha_{-\theta}^p e^{i\theta\tau} \cos \theta \sigma + \text{oscil}. \]  
\[ \simeq \frac{1}{2\theta} (\alpha_\theta^p - \alpha_{-\theta}^p) - i(\alpha_\theta^p + \alpha_{-\theta}^p) \tau + \text{oscil}. \]  
\[ X^{(p+1)} = \frac{1}{2\theta} \alpha_\theta^p e^{-i\theta\tau} \sin \theta \sigma - \frac{1}{2\theta} \alpha_{-\theta}^p e^{i\theta\tau} \sin \theta \sigma + \text{oscil}. \]  
\[ \simeq (\alpha_\theta^p - \alpha_{-\theta}^p) \sigma + \text{oscil}. \]

and as it’s seen from commutation relations:

\[ \frac{1}{\theta} (\alpha_\theta^p - \alpha_{-\theta}^p), i(\alpha_\theta^p + \alpha_{-\theta}^p) \right] = 2i. \]  
reproducing the results of the parallel branes results by the following identifications:

\[
\begin{array}{l}
\partial_\theta \alpha^p \mid_{\theta=0} = x^p \\
\alpha^p = p^p (\text{momentum}) \\
X^{(p+1)} \simeq \theta x^p \sigma.
\end{array}
\]

In this limit the low frequency spectrum turns into extra translational mode present in the case of parallel branes.

5. If we take more complicated relative orientations of the D-branes we will find further evidence for \( \int J.A \) form of the interaction.

\textit{IV.P and \( P' \) angled D-branes in type-II theories:}

In this case we follow the method we have used in the previous cases. Since we don’t have \( RR \) interactions, we expect the amplitude to be invariant under \( \theta \rightarrow 1 - \theta \). In this case we have (31) but \( R \) and \( \text{NS} \) are replaced by:

\[ R = (2\pi)^{3-\Delta/2} \left( \frac{\Theta_2(0 \mid it)}{\Theta_1(0 \mid it)} \right)^{3-\Delta/2} \left( \frac{\Theta_3(0 \mid it)}{\Theta_1(0 \mid it)} \right)^{\Delta/2} \frac{i\Theta_2(it\theta \mid it)}{\Theta_1(it\theta \mid it)}. \]

\[ \text{NS} = (2\pi)^{3-\Delta/2} \left( \frac{\Theta_3(0 \mid it)}{\Theta_1(0 \mid it)} \right)^{3-\Delta/2} \left( \frac{\Theta_2(0 \mid it)}{\Theta_1(0 \mid it)} \right)^{\Delta/2} \frac{i\Theta_3(it\theta \mid it)}{\Theta_1(it\theta \mid it)}. \]

Then the massless contribution to amplitude reads as:

\[ A(\theta) = -2^\Delta V_p \frac{F(\theta)}{\sin(\pi \theta)} (4\pi^2 \alpha')^{3-(p+p')/2} G_{8-p'}(Y^2). \]

\[ F(\theta) = 3 - \Delta + \cos 2\pi \theta \]
This amplitude is similar to the gravitational interaction of D-brane in the bosonic case given in the appendix. When $P$ and $P'$ are different two branes can not detect their $RR$ charge. Both $\theta = 0$ and $\theta = 1$ are stable points and the frequency of small oscillations around equilibrium points is proportional to $\sqrt{T_pT_{p'}} = (4\pi^2\alpha')^{3-(p+p')/2}$.

It is worth noting the amplitude vanishes at all orders of $t$ for parallel branes when $\Delta = 4$ and for perpendicular branes when $\Delta = 2$.

V. Moving branes:

Moving D-brane result can be obtained by the following transformations from the angled D-branes:

\[ i\theta \to \epsilon \quad \tanh \pi \epsilon = v. \]  

(47)

where $v$ is the relative velocity of D-branes for both, the bosonic case or the type-II. This is exactly boosting one of the branes respect to the other one. If we do so the results are:

$P$ and $P$ case:

\[ V_{RR} = 8V_0\frac{1}{\sqrt{1-v^2}} \]

\[ V_{NSNS} = -8V_0\left(\frac{1-v^2/2}{1-v^2}\right) \]

(48)

$P$ and $P'$ case:

\[ V_{NSNS} = -2V_0\frac{4-\Delta - v^2(2-\Delta)}{1-v^2} \quad V_{RR} = 0. \]

(49)

They can be expected as "magnetic" forces induced by Lorentz transformations and again can be justified by the field theoretic interaction term $\int J.A$. The $v^2$ dependence is due to two factors one from $J$ and the other from $A$. In the limit of small velocities we recover the results of [6],[4].

As we have seen the relative orientation or velocity of the D-branes reveal more details about their structure and bringing them closer to the P-brane extended solutions of low energy SUGRA.

These correspondences make us believe that finally we are approaching a totally string theoretic language, a simple description of non-perturbative objects.

Acknowledgement:

Authors would like to thank Porf. Ardalan for fruitful discussions.
Appendix:

Here we summarize the results for the bosonic strings case in 26 dimensions:

\[
A = 2V_{p+1} \int \frac{dt}{2t} (8\pi^2 \alpha' t)^{-\left(p'+1\right)/2} e^{-\frac{y^2}{2\pi^2 \alpha'}} (q^{-2} \prod (1 - q^{2n})^{-24}) \left(q^{1/8} \prod \frac{(1 - q^{2n})}{(1 - q^{2n - 1})} \right)^\Delta
\]

small \( t \) limit:

\[
A = V_{p+1} (24 - 2\Delta) (4\pi^2 \alpha')^{11-\frac{p+p'}{2}} \frac{\pi}{210} G_{25-p}(Y^2)
\]

Angled branes:

\[
\alpha'M^2 = \frac{Y^2}{4\pi^2 \alpha'} + N(\theta) - \left(1 + \theta^2/2\right)
\]

\[
N(\theta) = \sum_{n>0} \alpha_{-n} \cdot \alpha_n + \sum_{n>0} \alpha_{n+} \cdot \alpha_{n+} + \sum_{n>0} \alpha_{-n-} \cdot \alpha_{n-} + \frac{1}{2} (\alpha_{p\theta} \alpha_{\theta p} + \alpha_{p\theta} \alpha_{-\theta})
\]

If we put these in \( A \) it reads to be

\[
A(\theta) = 2V_p \int \frac{dt}{2t} (8\pi^2 \alpha' t)^{-p/2} e^{-\frac{y^2}{2\pi^2 \alpha'}} \left(\frac{\Theta_1(0 | it)}{2\pi} \right)^{-7} \frac{iq^{-\theta^2}}{\Theta_1(i\theta t | it)}
\]

in small \( t \) limit

\[
A(\theta) = V_p (4\pi^2 \alpha')^{11-p} \frac{\pi}{210} F(\theta) \frac{1}{\sin^2 \pi \theta} G_{21-p}(Y^2)
\]

\[
F(\theta) = (22 - 4 \sin^2 \pi \theta)
\]

and small \( t \) limit of the angled \( P \) and \( P' \) branes:

\[
A(\theta) = V_p' (4\pi^2 \alpha')^{11-(p+p')/2} \frac{\pi}{210} F(\theta) \frac{1}{\sin^2 \pi \theta} G_{24-p}(Y^2)
\]

\[
F(\theta) = (22 - 2\Delta - 4 \sin^2 \pi \theta) \quad \Delta = p - p'.
\]

References

[1] J. Polchinski, S. Chaudhuri, and C.V. Johnson, "Notes on D-Branes," [hep-th/9602052]

[2] J. Polchinski, Phys. Rev. Lett. 75 (1995) 4724.

[3] E. Witten, Nucl. Phys. B460 (1996) 335, [hep-th/9510133].
[4] M.R. Douglas, D. Kabat, P. Pouliot, and S.H. Shenker, "D-branes and Short Distances in String Theory", hep-th/9608024.

[5] S.H. Shenker, "Another Length Scale in String Theory?" preprint RU-95-53, hep-th/9509132.

[6] G. Lifschytz, "Comparing D-branes to Black-branes", BRX-TH-394, hep-th/9604156.

[7] G.T. Horowitz and A. Strominger, Nucl. Phys. B360 (1991) 197.

[8] M.J. Duff and J.X. Lu, "Black and Super P-brane in Diverse Dimensions", CERN-TH.6675/93, CTP/TAMU-54/92.

[9] T. Banks and L. Susskind, "Brane-Antibrane Forces", RU-95-87, hep-th/9511194.

[10] J.M. Maldacena, "Black Holes in String Theory", PhD thesis, Princeton University June 1996 hep-th/9607235.

[11] A. Hashimoto, "Dynamics of Dirichlet-Neumann Open Strings on D-branes", PUPT-1642, hep-th/9608127.

[12] H. Arfaei and M.M. Sheikh Jabbari, Work in progress.