Suppression of Visibility in a Two-Electron Mach-Zehnder Interferometer

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We investigate the suppression of the visibility of Aharonov-Bohm oscillations in a two-electron Mach-Zehnder interferometer that leaves the single-electron current unchanged. In the case when the sources emit either spin-polarized or entangled electrons, partial distinguishability of electrons (coming from two different sources) suppresses the visibility. Two-particle entanglement may produce behavior similar to "dephasing" of two-particle interferometry.

Aharonov-Bohm (AB) interferometry is a centerpiece in studies of nanoscopic electronic systems. Of special interest are Mach-Zehnder interferometers (MZIs) \cite{1}: The absence of backscattering gives rise to a large interference signal (large visibility), and allows measurements in strong magnetic fields.

One conceptual step beyond single-particle interferometry is "two-particle interferometry", tailored after the Hanbury Brown – Twiss experiment. In particular, one can utilize a two-electron MZI which features two current sources and two detectors (Fig. 1). Such a device, suitable for the measurement of current cross-correlations in the detectors, has been originally proposed by Samuelson, Sukhorukov, and Böttiker (SSB). In Ref. \cite{2} it was shown that while the current (and the noise) at each particular detector is insensitive to the AB flux through the interferometer, the current cross-correlations between the two detectors do show AB oscillations. The latter are a direct consequence of particle indistinguishability: Measuring current signals at points c and d (cf. Fig. 1a) may be due to an electron from a (b) absorbed at c (d), or at d (c). The product of the amplitudes of these two processes is flux sensitive. Had the two particles been distinguishable, it would have been possible to conclude, for example, that the electron detected at c (d) originated from a (b); only one of the above amplitudes survives, hence flux insensitivity.

Trying to design an actual measurement of a two-electron interferometry, it is important to understand which manipulations can render the electrons distinguishable, in practice: suppress the AB signal \cite{2}. Below we focus on two interference-suppressing scenarios: (i) opposite spin polarizations of the two sources, and (ii) entanglement of electrons in one of the sources. We calculate how the visibility in these two scenarios is reduced. In particular, our analysis reveals that entangled electrons as non-entangled electrons act as (partially) distinguishable particles.

We first derive general expressions for the current and noise of a generic multi-terminal system, in which electrons in the same or different leads can be entangled. Let us consider a multi-terminal system, with the leads emitting (possibly) entangled electrons. A two-electron state in such a system has the form

\[ \hat{F}^\dagger_{\alpha\beta}(E, E') = \sum_{\sigma, \sigma'} g_{\alpha\sigma}^{\alpha\beta} \hat{a}^{\dagger}_{\alpha\sigma}(E) \hat{a}^{\dagger}_{\beta\sigma'}(E') , \]  

where the indices \( \alpha \) and \( \beta \) label the leads, \( \sigma \) and \( \sigma' \) label the spin projections, and the coefficient \( g \) is essentially the density matrix for the entangled electron state. If the electrons are not entangled (full product state), all coefficients \( g \) equal one. For simplicity, we assume that each lead supports a single channel: The energies \( E \) and...
The two-particle field operator depending on the coordinates, times, and spin projections of both particles, is
\[
\hat{\psi}_{\sigma_1,\sigma_2}(x_1t_1; x_2t_2) = \frac{1}{2\pi\hbar} \sum_{\beta\gamma} \int dE_1 dE_2 \\
\times \phi_\beta(x_1, E_1)\phi_\gamma(x_2, E_2) \exp(-iE_1t_1/\hbar - iE_2t_2/\hbar) \\
\times \gamma_{\sigma_2\sigma_1, \beta\sigma_1\alpha\gamma_{\sigma_2},}, (2)
\]
where for simplicity we take the electron velocities in all leads equal to \(v\), and \(\phi_\nu(x)\) is the scattering state originating from the lead \(\nu\). The asymptotic form for this function for \(x \to \infty\) is
\[
\phi_\nu(x, E) = \delta_{\alpha\nu}e^{-i(kE)x} + s_{\alpha\nu}(E)e^{i(kE)x},
\]
where, as common in the scattering approach, the coordinate \(x\) is counted from the center of the structure along each lead, and \(\hat{s}\) is the scattering matrix (below it will gain some more indices). The scattering states are orthogonal, in discrete notations \(\int dx\phi_\nu^*(x, E)\phi_\nu(x, E') = L\delta_{\nu\nu}\delta_{E E'}\), with \(L\) being the size of the system (it drops out of the final expressions) [4].

Now we produce the one-particle current operator in the lead \(\alpha\). It consists of the current operators produced by the “first” (\(\hat{I}_1\)) and the “second” (\(\hat{I}_2\)) particle. We write
\[
\hat{I}_{a1}(x_1, t_1) = -\frac{ie\hbar}{2m} \int dx_2 \sum_{\sigma_1\sigma_2} \langle \hat{\psi}^\dagger_{\sigma_1\sigma_2}(x_1, t_1; x_2, t_2) \\
\times \frac{\partial}{\partial x_1} \hat{\psi}_{\sigma_1\sigma_2}(x_1, t_1; x_2, t_2) \rangle_2 + h.c., \quad (W R O N G)
\]
where \(\langle \ldots \rangle_2\) means quantum-mechanical averaging over the state of the second particle. Using [3]
\[
\left\langle \hat{a}^\dagger_{\lambda\sigma}(E)\hat{a}_{\nu\sigma'}(E') \right\rangle = \delta_{\lambda\nu}\delta_{\sigma\sigma'}\delta(E - E')f(E - \mu_\lambda),
\]
where \(f\) is the equilibrium distribution function, and \(\mu_\lambda\) is the corresponding chemical potential, we obtain for the single-particle current operator
\[
\hat{I}_{a1}(x, t) = -\frac{e}{2\pi\hbar} \int dx_1 dE_1 e^{i(E_1 - E_2)t/\hbar} \\
\times \sum_{\beta\gamma\delta\sigma_1\sigma_2} \sum_{\lambda\nu\sigma_2} A^a_{\beta\delta\gamma\lambda\sigma_2}\gamma_{\sigma_2\sigma_1, \beta\sigma_1\alpha\gamma_{\sigma_2},} \\
\times \hat{a}^\dagger_{\beta\sigma_2}(E_1)\hat{a}_{\delta\sigma_1}(E_2)N_{\gamma\sigma_2}, \quad (S T I L L \ W R O N G)
\]
where we have introduced the following combination of the scattering matrices [3],
\[
A^a_{\beta\delta\gamma\lambda\sigma_2}(E_1, E_2) \equiv \delta_{\alpha\beta}\delta_{\sigma\sigma_2}\delta_{\alpha\gamma}(E_1 - k(E_2))x, \quad (3)
\]
\[
-s^a_{\alpha\beta}(E_1)s_{\alpha\beta}(E_2)e^{-i(k(E_1) - k(E_2))x}, \quad (4)
\]
and the quantity \(N_{\gamma\sigma_2} \equiv \int dE f_{\gamma\sigma_2}(E) dE\), which is proportional to the “number of particles” with the spin \(\gamma\) emitted by the lead \(\gamma\).

The expression for \(\hat{I}_{a1}\) is obviously wrong: It yields the average current proportional to the number of pairs of particles, not to the difference of the number of particles emitted by each lead. This is because our expression for the current operator is an overcounting: Each pair of the particles is counted twice. Thus, we need to add a normalization constant in the current operator. This normalization constant is the number of pairs which a particle incident from the lead \(\gamma\) with the spin \(\sigma_2\) forms, divided by the number of such particles. Note that this constant must depend on \(\gamma\) and \(\sigma_2\). Let us now calculate it,
\[
\# \text{ of pairs} = \int dx_1 dx_2 dE_1 dE_2 dE_3 dE_4 \frac{1}{(2\pi\hbar)^2} \\
\times \phi_\beta^*(x_1, t_1, E_1)\phi_\gamma^*(x_2, t_2, E_2)\phi_\beta(x_1, t_1, E_4) \\
\times |g_{\beta\gamma\sigma_2\sigma_1}|^2 \langle a^\dagger_{\beta\sigma_1}(E_1)\gamma_{\sigma_2\sigma_1}(E_2)\gamma_{\sigma_2\gamma}(E_3)\hat{a}_{\beta\sigma_1}(E_4) \rangle \\
= \left(\frac{L}{2\pi\hbar}\right)^2 \sum_{\beta\gamma\sigma_1} |g_{\beta\gamma\sigma_2\sigma_1}|^2 N_{\beta\sigma_1}N_{\gamma\sigma_2},
\]
and \# of particles = \((L/2\pi\hbar)n_{\gamma\sigma_2}\). Thus, including the normalization constant, we find for the current operator of the first particle,
\[
\hat{I}_{a1}(x, t) = -\frac{e}{2\pi\hbar} \int dx_1 dE_1 e^{i(E_1 - E_2)t/\hbar} \\
\times \sum_{\beta\gamma\delta\sigma_1\sigma_2} \sum_{\lambda\nu\sigma_2} \gamma_{\sigma_2\sigma_1, \beta\sigma_1\alpha\gamma_{\sigma_2},} \\
\times A^a_{\beta\delta\gamma\lambda\sigma_2}\gamma_{\sigma_2\sigma_1, \beta\sigma_1\alpha\gamma_{\sigma_2},} \\
\times \hat{a}^\dagger_{\beta\sigma_2}(E_1)\hat{a}_{\delta\sigma_1}(E_2)N_{\gamma\sigma_2}, \quad (5)
\]
and an identical expression for the current operator of the second particle.

Next we use Eq. (5) to derive the average current,
\[
\langle I_\alpha \rangle = \langle I_{a1} \rangle + \langle I_{a2} \rangle = -\frac{e}{2\pi\hbar} \sum_{\gamma\sigma_2} \\
\sum_{\beta\gamma\delta\sigma_1\sigma_2} A^a_{\beta\delta\gamma\lambda\sigma_2}\gamma_{\sigma_2\sigma_1, \beta\sigma_1\alpha\gamma_{\sigma_2},} \\
\times \hat{a}^\dagger_{\beta\sigma_2}(E_1)\hat{a}_{\delta\sigma_1}(E_2)N_{\gamma\sigma_2}, \quad (6)
\]
where we assumed that the scattering matrices are energy independent, and we added to them two additional indices, as is explained below.

Eq. (5) also yields current noise. To produce the expression for the zero-frequency noise, we take into account that the currents of the first and second particle are uncorrelated, and averaging the product of four creation and annihilation operators [3],
\[
\langle \hat{a}^\dagger_{\beta\sigma_2}(E_1)\hat{a}_{\beta\sigma_1}(E_1^\prime)\hat{a}^\dagger_{\delta\sigma_2}(E_2)\hat{a}_{\delta\sigma_1}(E_2^\prime) \rangle
\]
we obtain

\[
S_{\alpha\alpha'} = \frac{e^2}{2\pi\hbar} \left( \sum_{\beta, \gamma, \delta} N_{\gamma\delta} \right) \times \sum_{\beta, \gamma, \delta} N_{\gamma\delta} \left| g_{\beta\gamma} \right|^2 \operatorname{Re} \left[ g_{\beta\gamma}^* g_{\beta'\gamma'} \right] \times A_{\beta\beta'}(\gamma, \sigma_2) A_{\beta'\beta'}(\gamma', \sigma_2') \int dE f_{\beta\sigma_1}(E) \left[ 1 - f_{\beta'\sigma_1}(E) \right] + (\alpha \leftrightarrow \alpha') ,
\]

Eqs. (6) and (7) generalize standard expressions for the multi-terminal current and noise to the case of entangled electrons. It is easy to check that if electrons are not entangled and are in the product state, the coefficients \( g \) are equal one for all values of the arguments, and these requirements of current conservation and gauge invariance can be still generated according to Eq. (6). Usually, linear in \( \beta \) between currents at 5 and 8. We are interested in the average current through lead 5, and the cross-correlation the scattering matrix to obey

\[
\langle I_5 \rangle = -\frac{e^2 V}{2\pi\hbar} \left( s_{52} \right)^2 \left( s_{53} \right)^2 = -\frac{e^2 V}{2\pi\hbar} \left( T_A T_C + R_A R_D \right) ,
\]

where we have used the “unitarity” conditions [8].

Now we apply these expressions to the different states emitted by the reservoirs. The idea is to investigate under which conditions electrons emitted from sources 2 and 3 are “painted”, at least partially, “red” and “blue” respectively, hence suppression of the flux sensitive (coherent) term of the correlation.

(i) Full product state, \( g_{\beta'\gamma'} = 1 \). This is the case considered by SSB. Choosing \( s_{52} = t_A t_C e^{i\phi_1} \), \( s_{53} = r_A r_D e^{i\phi_4} \), \( s_{82} = r_B r_C e^{i\phi_3} \), and \( s_{83} = t_B t_D e^{i\phi_2} \), where \( t \) and \( r \) are the transmission and reflection amplitudes of the corresponding QPC’s (Fig. 1), we obtain

\[
\langle I_5 \rangle = -\frac{e^2 V}{2\pi\hbar} \left( s_{52} \right)^2 \left( s_{53} \right)^2 = -\frac{e^2 V}{2\pi\hbar} \left( T_A T_C + R_A R_D \right) ,
\]

where \( T \) and \( R \) are corresponding transmission and reflection probabilities, and

\[
S_{98} = -\frac{2e^2 V}{\pi\hbar} \left( s_{52} \right)^2 \left( s_{53} \right)^2 = -\frac{2e^2 V}{\pi\hbar} \left( T_A T_C + R_A R_D \right) ,
\]

Next, we specialize to the eight-terminal setup with edge states suggested by SSB [2]. We will always bias sources 2 and 3 with the same voltage \( V \), and measure the average current through lead 5, and the cross-correlation between currents at 5 and 8. We are interested in the linear in \( V \) effects. Assuming that \( eV \) is much smaller that the Fermi energy in the leads, we can in the leading order take all quantities \( N_{\beta\sigma} \) to be the same (voltage independent). One has then

\[
\langle I_5 \rangle = -\frac{e^2 V}{2\pi\hbar} \sum_{\gamma, \delta} \sum_{\beta, \gamma, \delta} \left| g_{\beta\gamma} \right|^2 \sum_{\alpha, \sigma_1} \left| s_{5\beta}(\gamma, \sigma_2) \right|^2 (9)
\]

and becomes unity (ideal oscillations) provided the setup is symmetric: all transmission and reflection probabilities equal 1/2. This is the result of SSB.

(ii) One source is spin polarized. Imagine that states are still not entangled, but source 2 is spin-polarized. It only emits and absorbs spin-up electrons. The coefficients \( g \) are all equal to one, with the exception of \( g_{\beta\gamma}^{\alpha} = g_{\beta\gamma}^{\sigma} = 0 \). We need now to choose the scattering matrices in such a way so that they obey the unitarity condition [8]. This is easily done intuitively. For spin-up electrons, the constraints are the same as for the full product state, and thus the scattering for spin-up electrons is described by the matrix \( s \). The scattering matrix
for spin-down electrons, which we denote \( \hat{s} \), is constrained by the condition

\[
\sum_{\nu \neq 2} \hat{s}_{\mu \nu}^{*} \hat{s}_{\mu' \nu} = \delta_{\mu \mu'}, \hat{s}_{\mu 2} = 0.
\]

We obtain the average current,

\[
\langle I_s \rangle = -\frac{e^2 V}{2\pi \hbar} \left( |s_{52}|^2 + |s_{53}|^2 + |\hat{s}_{53}|^2 \right), \tag{14}
\]

and current noise,

\[
S_{5s} = -\frac{e^2 |V|}{\pi \hbar} \left( |s_{52}^* s_{82} + s_{53}^* s_{83}|^2 + |\hat{s}_{53}^* \hat{s}_{83}|^2 \right). \tag{15}
\]

The first term in the brackets, similarly to Eq. [12], contains both phase-insensitive and AB terms. The second term is insensitive to the AB phase. Thus, the total visibility in the noise is reduced by the presence of this second term. The visibility depends on the choice of the scattering matrices \( \hat{s} \), in particular, if the setup is symmetric, and \( \hat{s}_{53} = 0 \) (unrealistic case), the oscillations are still ideal \((v = 1)\). This is because in this case a spin-down electron, originating from 3, has to go to 8 with certainty, and thus does not change the current noise. On the other hand, the natural choice would be \( |s_{52}| = |s_{23}| = |\hat{s}_{52}| \). In this case, which we call optimal, the visibility equals \( v = 2/3 \).

In the same way, we can treat a setup where both sources, 2 and 3, are polarized. If they are polarized in the same direction (spin-up), the visibility is the same as for full product states (both current and noise are reduced by the factor of 2 since now only spin-up states contribute). Provided one source is spin-up polarized and the other one is spin-down, the visibility vanishes: There are no AB oscillations in this case, since one can say with certainty from what source each electron has originated.

(iii) **Entangled electrons from one source.** Pairs in triplet \( s_z = 0 \) state are emitted from the lead 2,

\[
|2 \uparrow\rangle|2 \downarrow\rangle + |2 \downarrow\rangle|2 \uparrow\rangle,
\]

all other leads are in the full product state. This means that all the coefficients \( g \) are equal to one except for \( g_{21}^{22} = g_{11}^{22} = 0 \). Now we are obliged to choose scattering matrices which depend on the second electron in the pair — the “spouse”, otherwise the unitarity conditions cannot be fulfilled. We choose

\[
s_{\alpha \nu, \sigma}(\gamma, \sigma_2) = \begin{cases} 
s_{\alpha \nu}, & \gamma \neq 2 \text{ or } \gamma = 2, \sigma \neq \sigma_2 \\
s_{\alpha \nu}, & \gamma = 2, \sigma = \sigma_2
\end{cases}.
\]

Note that this choice (affecting the visibility) is arbitrary and definitely not unique. Our matrices now, in addition to what we have already seen, obey

\[
\sum_{\nu \neq 2} \hat{s}_{\mu \nu}^{*} \hat{s}_{\mu' \nu} = \sum_{\nu \neq 2} \hat{s}_{\mu \nu}^{*} \hat{s}_{\mu' \nu} = \delta_{\mu \mu'},
\]

We can again calculate the average current,

\[
\langle I_s \rangle = -\frac{e^2 V}{2\pi \hbar} \left( \left( \frac{7}{8} + \frac{1}{15} \right) |s_{52}|^2 + |s_{53}|^2 + \frac{1}{15} |\hat{s}_{53}|^2 \right), \tag{16}
\]

and the current noise,

\[
S_{5s} = -\frac{2e^2 |V|}{\pi \hbar} \left\{ \left[ \frac{14^2}{16^2} + 2 \times \frac{14}{15} \times \frac{16}{15} + \frac{1}{15^2} \right] \times |s_{52}^* s_{82} + s_{53}^* s_{83}|^2 + \left( \frac{14}{15} \times \frac{16}{15} + \frac{1}{15^2} \right) \times \left( |s_{53}^* s_{83}|^2 + |\hat{s}_{53}^* \hat{s}_{83}|^2 \right) \right\}. \tag{17}
\]

Again, phase-dependent contributions are only found in the first term in the braces. All other terms are phase insensitive and thus reduce the visibility. In particular, with the same optimal choice of scattering matrices, \( |s_{52}| = |s_{53}| = |\hat{s}_{52}| \), the visibility is reduced to 0.93.

(iv) Pairs in singlet state are emitted from the lead 2,

\[
|2 \uparrow\rangle|2 \downarrow\rangle - |2 \downarrow\rangle|2 \uparrow\rangle,
\]

all other leads are in the full product state. This means that all the coefficients \( g \) are equal one except for \( g_{12}^{22} = g_{11}^{22} = 0 \). We choose the scattering matrices in the following way (this choice is again arbitrary),

\[
s_{\alpha \nu, \sigma}(\gamma, \sigma_2) = \begin{cases} s_{\alpha \nu}, & \gamma \neq 2 \text{ or } \gamma = 2, (\sigma_2) = (\uparrow \uparrow) \\
\hat{s}_{\alpha \nu}, & \gamma = 2, (\sigma_2) = (\downarrow \downarrow)
\end{cases} \tag{18}
\]

The matrix \( \hat{s} \) must then obey the following constraints,

\[
\sum_{\nu} \hat{s}_{\mu \nu}^{*} \hat{s}_{\mu' \nu} = \sum_{\nu} \hat{s}_{\mu \nu}^{*} \hat{s}_{\mu' \nu} = \sum_{\nu \neq 2} \hat{s}_{\mu \nu}^{*} \hat{s}_{\mu' \nu} = \delta_{\mu \mu'}.
\]

With this choice of scattering matrices, we obtain the same current and current noise (and consequently the same visibility) as for the triplet \( s_z = 0 \) state. We should recall however that this conclusion depends on the choice of scattering matrices \( \langle 13 \rangle \): for instance, on the fact, that \( s_{\alpha \nu, \sigma}(2, \sigma) \) is the same for single and triplet entangled states.

| Scenario       | Optimal visibility |
|----------------|--------------------|
| 2, 3: product states | 1                  |
| 2: product, not polarized; 3: polarized ↑ | 2/3                |
| 2, 3: polarized ↑ | 1                  |
| 2: polarized ↑, 3: polarized ↓ | 0                  |
| 2: entangled, singlet; 3: product         | 0.93               |
| 2: entangled, triplet; 3: product          | 0.93               |

**TABLE I:** Reduction of the visibility for different scenarios and otherwise optimal conditions.
In summary, we have defined and analyzed various scenarios for which single-particle amplitudes maintain their coherence (as would be manifest in a MZ interferometry measurement), yet (partial) distinguishability of electrons emitted from different sources suppresses the flux sensitivity of the two-particle cross-correlation function (Table 1). Our analysis demonstrates how two-particle entanglement can give rise to a behavior akin to a “dephasing” of a two-particle interferometry.

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[1] Y. Ji, Y. Chung, D. Sprinzak, M. Heiblum, D. Mahalu, and H. Shtrikman, Nature 422, 415 (2003).

[2] P. Samuelsson, E. V. Sukhorukov, and M. Büttiker, Phys. Rev. Lett. 92, 026805 (2004); see also V. S.-W. Chung, P. Samuelsson, and M. Büttiker, Phys. Rev. B 72, 125320 (2005).

[3] Evidently, one can suppress the interference by introducing dephasers at the interferometer’s arms. But this will also suppress single-particle interference. We do not discuss this option here. Still, dephasing might be responsible for the suppression of visibility in the experiment [1], see F. Marquardt and C. Bruder, Phys. Rev. Lett. 92, 056805 (2004); I. Neder and F. Marquardt, cond-mat/0611372; J. T. Chalker, Y. Gefen, and M. Y. Veillette, cond-mat/0703162.

[4] It will be more natural and convenient to use continuous notations for the energy, \( \int dx \phi^*_\mu(x,E) \phi^\nu(x,E') = 2\pi \hbar v \delta_{\mu\nu} \delta(E - E') \). In particular, one can also write the “square” of the delta-function in continuous notations, \( \delta^2(E - E') = (L/2\pi v) \delta(E - E') \).

[5] Note that we made an assumption that entangled electron pairs are characterized by the Fermi distribution function. Whereas such an assumption is plausible, the actual distribution function may depend on the mechanism of entanglement, cf. G. Burkard, D. Loss, and E. V. Sukhorukov, Phys. Rev. B 61, 16303 (2000). We thank M. Büttiker for useful comments to this point.

[6] M. Büttiker, Phys. Rev. B 46, 12485 (1992); see also Ya. M. Blanter and M. Büttiker, Phys. Rep. 336, 1 (2000).