Research Article

New Results on the Radial Solutions to a Class of Nonlinear \(k\)-Hessian System

Guotao Wang and Zhuobin Zhang

School of Mathematics and Computer Science, Shanxi Normal University, Taiyuan, Shanxi 030031, China

Correspondence should be addressed to Guotao Wang; wgt2512@163.com

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This paper investigates the positive radial solutions of a nonlinear \(k\)-Hessian system.

\[
\begin{cases}
\Lambda (S_k^{1/k} (\lambda (D^2 z_1))) S_k^{1/k} (\lambda (D^2 z_1)) = b(|x|) \psi(z_1, z_2), & x \in \mathbb{R}^N \\
\Lambda (S_k^{1/k} (\lambda (D^2 z_2))) S_k^{1/k} (\lambda (D^2 z_2)) = h(|x|) \psi(z_1, z_2), & x \in \mathbb{R}^N,
\end{cases}
\]

where \(\Lambda\) is a nonlinear operator and \(b, h, \psi, \varphi\) are continuous functions. With the help of Keller–Osserman type conditions and monotone iterative technique, the positive radial solutions of the above problem are given in cases of finite, infinite, and semi-infinite. Our results complement the work in by Wang, Yang, Zhang, and Baleanu (Radial solutions of a nonlinear \(k\)-Hessian system involving a nonlinear operator, Commun. Nonlinear Sci. Numer. Simul. 91(2020), 105396).

1. Introduction

It is well known that Laplace equation has a wide range of applications in mathematics and physics, for instance [1], it can be used to describe the relationship between the curvature of liquid surface and the surface pressure of liquid [2] as well as in solving electrostatic field problems [3, 4]. In a mathematical sense, the existence of solutions of Laplace equation has attracted increasing attention, and numerous excellent results have been obtained.

In 1957, under the condition

\[
\int_1^\infty \frac{1}{\sqrt{\int_0^t \psi(s)ds}} dt < \infty, \quad \forall \alpha > 0.
\]  

(1)

Keller [5] studied the existence of solutions for the nonlinear equation \(\Delta z = \varphi(z)\), and Osserman [6] studied the existence of solutions for the nonlinear differential inequality \(\Delta z \geq \varphi(z)\).

In 2011, under the Keller–Osserman condition

\[
\int_1^\infty \frac{1}{\sqrt{\int_0^t \varphi(s)ds}} dt < \infty,
\]

(2)

Peterson and Wood [7] presented the existence of entire positive blow up radial solutions of semilinear elliptic system

\[
\begin{cases}
\Delta z_1 = b(|x|) \psi(z_2), & x \in \mathbb{R}^N,
\\
\Delta z_2 = h(|x|) \psi(z_1), & x \in \mathbb{R}^N.
\end{cases}
\]  

(3)

In 2019, under the Keller–Osserman type conditions

\[
\int_1^\infty \frac{1}{\sqrt{\int_0^t \psi(s, \varphi(s))ds}} dt < \infty,
\]

(4)

\[
\int_\beta^\infty \frac{1}{\sqrt{\int_0^t \psi(s, \varphi(s, s))ds}} dt < \infty, \quad \forall \alpha, \beta > 0.
\]
works of many authors in [7, 8, 15–17, 37, 38].

Addition, our results complement the work in [1] and extend the system (8) under the cases of finite, infinite, and semifinite. For the existence results on positive radial solutions of the $k$-Hessian system (8) under the Keller–Osserman type condition, we establish some new results in [1].

For example, for a family of operators including many well-known operators, the existence and uniqueness of solutions with strong isolated singularity [13], and the existence of solutions for the boundary blow up problem in one dimensional case [14].

we define the $k$-Hessian operator $(k = 1, 2, \ldots, N)$ as follows:

\[
S_k(\lambda(D^2z)) = \sum_{1 \leq i \leq j \leq N} \lambda_i \cdots \lambda_j,
\]

where $D^2z = (\partial^2z(x)/\partial x_i \partial x_j)_{N \times N}$, $(i, j = 1, 2, \ldots, N)$ is the Hessian matrix of $z \in C^2(\mathbb{R}^N)$ and the $\lambda(D^2z) = (\lambda_1, \ldots, \lambda_N) \in \mathbb{R}^N$ is the vector, which consists of eigenvalues of $D^2z$. It is easy to observe that $S_k(\lambda(D^2z))$ is a family of operators including many well-known operators. For example, for $k = 1$, $S_1(\lambda(D^2z))$ is Laplace operator, which is studied widely in [15–19], for $k = N$, $S_N(\lambda(D^2z))$ is the Monge–Ampère operator, which is studied extensively in [20–26].

The $k$-Hessian equations play an important role in differential geometry [27, 28]. They can describe Weingarten curvature or reflector shape [29] and some phenomena of non-equilibrium phase transitions and statistical physics [30, 31]. In 2019, Zhang and Feng [32] considered the existence and asymptotic behavior of $k$-convex solution to the boundary blow up problem for the following $k$-Hessian equation:

\[
\begin{align*}
\lambda & \in \mathcal{S}_N(\lambda(D^2z)) = b(x)\varphi(z), & \text{in } \Omega, \\
u & = +\infty, & \text{on } \partial \Omega,
\end{align*}
\]

where $b$ and $\varphi$ are smooth positive functions and $\Omega$ is a smooth, bounded, strictly, convex domain of $\mathbb{R}^N$ with $N \geq 2$.

In 2020, by means of monotone iterative technique, Wang, Yang, Zhang, and Baleanu [1] established the existence of the entire positive bounded radial solutions and entire positive blow up radial solutions for the following $k$-Hessian system:

\[
\begin{align*}
\lambda & \in \mathcal{S}_N(\lambda(D^2z)) = b(x)\varphi(z), & x \in \mathbb{R}^N, \\
\lambda & \in \mathcal{S}_N(\lambda(D^2z)) = h(x)\varphi(z), & x \in \mathbb{R}^N,
\end{align*}
\]

where $b, h \in C([0, \infty), t[0, \infty))$, $\Lambda$ is a nonlinear operator in the set

\[
\Theta = \{ \lambda \in C^2((0, \infty), [0, \infty))\mid (\Lambda (l \sigma) \leq l^p \lambda (x) \text{ for all } l \in (0, 1), s \in [0, \infty) \text{ and } p \in (0, \infty) \}.
\]

\section{2. Preliminaries}

For the convenience of subsequent proofs, we list a definition, notations, assumptions, and related lemmas.

We first recall the classification of solutions.

\begin{definition}
(see [8]). A solution $(z_1, z_2) \in C^2[0, \infty) \times C^2[0, \infty)$ of system (8) is called an entire bounded solution if condition (12) holds; it is called an entire blow up solution if condition (13) holds; it is called a semifinite entire blow up solution if condition (14) or (15) holds.

Finite case: both components $z_1, z_2$ are bounded, namely

\[
\begin{align*}
\lim_{|x| \to \infty} z_1(|x|) & < \infty, \\
\lim_{|x| \to \infty} z_2(|x|) & < \infty.
\end{align*}
\]

Infinite case: both components $z_1, z_2$ are blow up, namely

\[
\begin{align*}
\lim_{|x| \to \infty} z_1(|x|) & = \infty, \\
\lim_{|x| \to \infty} z_2(|x|) & = \infty.
\end{align*}
\]

A continuation of previous work, in this paper, by employing monotone iterative method, we establish some new existence results on positive radial solutions of the $k$-Hessian system (8) under the cases of finite, infinite, and semifinite. For details of the monotone iterative method, see [1, 7, 8, 33–36]. In addition, our results complement the work in [1] and extend works of many authors in [7, 8, 15–17, 37, 38].

\section{Main results}
Semifinite case: one of the components is bounded, whereas the other is blow up, namely
\[
\lim_{|x| \to \infty} z_1(|x|) < \infty, \quad \lim_{|x| \to \infty} z_2(|x|) = \infty.
\] (14)

or

Next, we introduce the notations as follows: \(r = |x|\) and \(\alpha, \beta, c_1, c_2 \in (0, \infty)\) are suitably chosen,

\[
G_1(r) = \int_0^r \left( \frac{k}{t^{N-k}} \int_0^{s^{N-1}} \left[ R^{-1}(b(s)) \right]^k ds \right)^{1/k} dt,
\]

\[
G_2(r) = \int_0^r \left( \frac{k}{t^{N-k}} \int_0^{s^{N-1}} \left[ R^{-1}(h(s)) \right]^k ds \right)^{1/k} dt,
\]

\[
H(r) = \int_{s \in \beta} \frac{1}{(\varphi + \psi)(t,t) + 1} dt,
\]

\[
H(\infty) = \lim_{r \to \infty} H(r),
\]

\[
G(r) = G_1(r) + G_2(r),
\]

\[
G_i(\infty) = \lim_{r \to \infty} G_i(r), \quad i = 1, 2,
\]

\[
\omega_1(r) = \varphi \left( 1, \frac{\beta}{(\psi(\alpha, \alpha))^{1/(p+1)}} + \left( c_2 \psi \left( 1, 1 + \frac{1}{\alpha} H^{-1} (G(r)) \right) + \frac{1}{\psi(\alpha, \alpha)} \right)^{1/(p+1)} G_2(r) \right),
\]

\[
\omega_2(r) = \psi \left( \frac{\alpha}{(\varphi(\beta, \beta))^{1/(p+1)}} + \left( c_1 \varphi \left( 1 + \frac{1}{\beta} H^{-1} (G(r)), 1 \right) + \frac{1}{\varphi(\beta, \beta)} \right)^{1/(p+1)} G_1(r), 1 \right),
\]

\[
P_1(r) = \int_0^r \left( \frac{k}{t^{N-k}} \int_0^{s^{N-1}} \left[ R^{-1}(b(t) \varphi \left( \alpha, \beta + \frac{1}{\psi(\alpha, \beta)} + 1 \right)^{1/(p+1)} G_2(t) \right) \right]^{1/k} dt \right)^{1/k} dv,
\]

\[
Q_1(r) = \int_0^r \left( \frac{k}{t^{N-k}} \int_0^{s^{N-1}} \left[ R^{-1}(h(t) \psi \left( \alpha + \frac{1}{\varphi(\alpha, \beta)} + 1 \right)^{1/(p+1)} G_1(t), \beta \right) \right]^{1/k} dt \right)^{1/k} dv,
\]

\[
\phi_1(s) = \max_{0 \leq s \leq r} b(s),
\]

\[
\phi_2(s) = \max_{0 \leq s \leq r} h(s),
\]

\[
P_2(r) = \int_0^r \left( c_1 \omega_1(s) + 1 \right) \left( \frac{k}{(p+1)(k+1)} \right) \frac{1}{\left( \psi(\alpha, \alpha) \right)^{1/(p+1)}} \left[ R^{-1}(\phi_1(s)) \right]^{1/(p+1)} ds,
\]

\[
Q_2(r) = \int_0^r \left( c_2 \omega_2(s) + 1 \right) \left( \frac{k}{(p+1)(k+1)} \right) \frac{1}{\left( \varphi(\beta, \beta) \right)^{1/(p+1)}} \left[ R^{-1}(\phi_2(s)) \right]^{1/(p+1)} ds,
\]

\[
P_i(\infty) = \lim_{r \to \infty} P_i(r),
\]

\[
Q_i(\infty) = \lim_{r \to \infty} Q_i(r), \quad \text{for} \quad i = 1, 2,
\]
We see that

\[ H'(r) = \frac{1}{(\varphi + \psi)(r,r) + 1} > 0, \quad \forall r > 0, \tag{23} \]

\[ F_1'(r) = \frac{1}{\sqrt{\int_0^r (\varphi(t, (\varphi(t,t)))^{1/(p+1)} + 1)^{k/(p+1)} \, dt}} > 0, \quad \forall r > 0, \tag{24} \]

\[ F_2'(r) = \frac{1}{\sqrt{\int_0^r (\psi(t, (\varphi(t,t)))^{1/(p+1)} + 1)^{k/(p+1)} \, dt}} > 0, \quad \forall r > 0, \tag{25} \]

which mean that \( H(r) \) has the inverse function \( H^{-1}(r) \) on \([0, H(\infty))\), \( F_1(r) \) has the inverse function \( F_1^{-1}(r) \) on \([0, F_1(\infty))\), and \( F_2(r) \) has the inverse function \( F_2^{-1}(r) \) on \([0, F_2(\infty))\).

We assume that \( \varphi \) and \( \psi \) satisfy the following assumptions:

(N1) \( \varphi, \psi \in C([0,\infty) \times [0,\infty), [0,\infty)) \) are increasing for each variable and \( \varphi(x, y) > 0, \psi(x, y) > 0 \) for all \( x, y > 0 \);

(N2) For fixed parameters \( \alpha, \beta \in (0,\infty) \), there exist \( c_1, c_2 \in (0,\infty) \) such that

\[ \varphi(t_1 s_1, t_2 s_2) \leq c_1 \varphi(t_1, t_2) \varphi(s_1, s_2), \tag{26} \]

\[ \psi(t_1 s_3, t_2 s_4) \leq c_2 \psi(t_1, t_2) \psi(s_3, s_4), \tag{27} \]

\[ \varphi(a, \beta) \geq \frac{\sqrt{5} - 1}{2}, \tag{28} \]

\[ \psi(a, \beta) \geq \frac{\sqrt{5} - 1}{2}. \]

where \( t_1 \geq \min\{\alpha, \beta\}, \ s_1 \geq 1, \ t_2 \geq \min\{\beta, (\psi(a, \alpha))^{1/(p+1)}\}, \ s_2 \geq \min\{1, \beta (\psi(a, \alpha))^{-(1/(p+1))}\}, \ t_3 \geq \min\{\alpha, (\varphi(\beta, \beta))^{1/(p+1)}\}, \)

Lemma 1 (see [39]). If \( \Lambda \in \Theta \), letting \( R(s) = s\Lambda(s) \), then

1: \( R(s) \) has a nonnegative increasing inverse mapping \( R^{-1}(s) \);

2: when \( 0 < l < 1 \), one has

\[ R^{-1}(Is) \geq l^{1/(p+1)} R^{-1}(s). \tag{29} \]

3: when \( l \geq 1 \), one has

\[ R^{-1}(Is) \leq l^{1/(p+1)} R^{-1}(s). \tag{30} \]

Lemma 2 (see [40]). We assume that \( y(r) \in C^2[0, R] \) is radially symmetric and \( y'(0) = 0 \), then the function \( z(|x|) = y(r) \) with \( r = |x| < R \) belongs to \( C^2(B_R) \), and

\[ \lambda(D^2 z) = \begin{cases} \left( y''(r), \frac{y'(r)}{r}, \ldots, \frac{y'(r)}{r} \right), & r \in (0, R), \\ (y''(0), y''(0), \ldots, y''(0)), & r = 0, \end{cases} \tag{31} \]
where $B_R = \{x \in \mathbb{R}^N : |x| < R \}$ and $C_N^k = N!/(k!(N-k)!)$.

Lemma 3 (see [1]). $(z_1(x), z_2(x))$ is a radial solution of the $k$-Hessian system (4) if and only if $(y_1(r), y_2(r))$ is a solution of the following ordinary differential system:

$$
\left\{ \begin{array}{l}
\frac{r^{N-k}}{k} (y_1(r)) \frac{y_1(r)}{r} = \frac{r^{N-1}}{C_{N-1}^{N-k}} \left[ b(r)\varphi(y_1(r), y_2(r)) \right]^k, \quad r \geq 0, \\
\frac{r^{N-k}}{k} (y_2(r)) \frac{y_2(r)}{r} = \frac{r^{N-1}}{C_{N-1}^{N-k}} \left[ h(r)\psi(y_1(r), y_2(r)) \right]^k, \quad r \geq 0.
\end{array} \right.
$$

(33)

3. Entire Positive Bounded Radial Solution

In this section, we investigate the entire positive bounded radial solution of system (8), and the main results are as follows.

Theorem 1. We assume that (N1),(N2) hold, then system (8) has an entire positive bounded radial solution $(z_1, z_2) \in C^2[0, \infty) \times C^2[0, \infty)$.

Proof. Obviously, the solutions of system (33) are equivalent to the solutions of the following system:

$$
\left\{ \begin{array}{l}
y_1(r) = y_1(0) + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{N-k}} \left[ b(s)\varphi(y_1(s), y_2(s)) \right]^k \right)^{1/k} ds \right) \frac{dr}{dr}, \\
y_2(r) = y_2(0) + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{N-k}} \left[ h(s)\psi(y_1(s), y_2(s)) \right]^k \right)^{1/k} ds \right) \frac{dr}{dr}.
\end{array} \right.
$$

(34)

We define the sequences $\{y_1^{(m)}(r)\}_{m \geq 0}$ and $\{y_2^{(m)}(r)\}_{m \geq 0}$ on $[0, \infty)$ by

$$
\left\{ \begin{array}{l}
y_1^{(0)}(r) = y_1(0) = \alpha, \quad y_2^{(0)}(0) = y_2(0) = \beta, \quad r \geq 0, \\
y_1^{(m)}(r) = \alpha + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{N-k}} \left[ b(s)\varphi(y_1^{(m-1)}(s), y_2^{(m-1)}(s)) \right]^k \right)^{1/k} ds \right) \frac{dr}{dr}, \quad r \geq 0, \\
y_2^{(m)}(r) = \beta + \int_0^r \left( \frac{k}{t^{N-k}} \int_0^t \frac{s^{N-1}}{C_{N-1}^{N-k}} \left[ h(s)\psi(y_1^{(m-1)}(s), y_2^{(m-1)}(s)) \right]^k \right)^{1/k} ds \right) \frac{dr}{dr}, \quad r \geq 0.
\end{array} \right.
$$

(35)

Using the same arguments as in [1], we get the sequences $\{y_1^{(m)}(r)\}_{m \geq 0}$ and $\{y_2^{(m)}(r)\}_{m \geq 0}$ that are increasing for $\forall m \geq 0$ and

$$
\left( y_1^{(m)}(r) + y_2^{(m)}(r) \right) r \left( \varphi + \psi \right) \left( y_1^{(m)}(r) + y_2^{(m)}(r) \right) \left( y_1^{(m)}(r) + y_2^{(m)}(r) \right) + 1 \leq G_1(r) + G_2(r).
$$

(36)
Next, integrating the above inequality from 0 to \( r \), we get
\[
\int_{\alpha}^{r} \frac{\frac{\alpha}{\psi + \beta} \rho(t)}{\rho(t) + 1} \frac{dt}{\rho(t)} \leq G(r).
\]  
(37)
Consequently,
\[
H(\eta_1^{(m)}(r) + \eta_2^{(m)}(r)) \leq G(r).
\]  
(38)
It follows from the above inequality and the fact that \( H \) is a bijection with the inverse function \( H^{-1} \) strictly increasing on \([0, H(\infty)) \) that
\[
\eta_1^{(m)}(r) + \eta_2^{(m)}(r) \leq H^{-1}(G(r)).
\]  
(39)
Since \( \psi, \psi, \left\{ \eta_1^{(m)}(r) \right\}_{m \in \mathbb{N}^+} \) and \( \left\{ \eta_2^{(m)}(r) \right\}_{m \in \mathbb{N}^+} \) are increasing, by means of Lemma 1, (N2), (35), (39), we get
\[
\eta_1^{(m)}(r) \leq \int_{0}^{r} \left( \int_{0}^{t} \frac{k}{N-1} \left[ R^{-1} \left( b(s) \phi \left( \eta_1^{(m)}(s), \eta_2^{(m)}(s) \right) \right) \right] \frac{ds}{\rho(t)} \right)^{1/k} dt \leq \alpha + \left[ \frac{\phi \left( \eta_2^{(m)}(r), \eta_2^{(m)}(r) \right) + 1}{\alpha} \right]^{1/(p+1)} G_1(r)
\]
\[
= \left( \frac{\phi \left( \eta_2^{(m)}(r), \eta_2^{(m)}(r) \right)}{\alpha} \right)^{1/(p+1)} \left[ \frac{\psi \left( \eta_1^{(m)}(r), \eta_1^{(m)}(r) \right)}{\psi \left( \eta_2^{(m)}(r), \eta_2^{(m)}(r) \right)} \right]^{1/(p+1)} + \left( \frac{c_1 \psi \left( \frac{\eta_2^{(m)}(r)}{\eta_2^{(m)}(r)} \right) + \frac{1}{\psi \left( \eta_2^{(m)}(r), \eta_2^{(m)}(r) \right)}}{\psi \left( \eta_2^{(m)}(r), \eta_2^{(m)}(r) \right)} \right)^{1/(p+1)} G_1(r)
\]
(40)
\[
\eta_2^{(m)}(r) \leq \beta + \int_{0}^{r} \left( \int_{0}^{t} \frac{k}{N-1} \left[ R^{-1} \left( h(s) \psi \left( \eta_1^{(m)}(s), \eta_2^{(m)}(s) \right) \right) \right] \frac{ds}{\rho(t)} \right)^{1/k} dt \leq \beta + \left[ \frac{\psi \left( \eta_1^{(m)}(r), \eta_1^{(m)}(r) \right) + 1}{\psi \left( \eta_1^{(m)}(r), \eta_1^{(m)}(r) \right)} \right]^{1/(p+1)} G_2(r)
\]
\[
= \left( \frac{\psi \left( \eta_1^{(m)}(r), \eta_1^{(m)}(r) \right)}{\beta} \right)^{1/(p+1)} \left[ \frac{c_2 \psi \left( \eta_1^{(m)}(r), \eta_1^{(m)}(r) \right) + \frac{1}{\psi \left( \eta_1^{(m)}(r), \eta_1^{(m)}(r) \right)}}{\psi \left( \eta_1^{(m)}(r), \eta_1^{(m)}(r) \right)} \right]^{1/(p+1)} G_2(r)
\]
(41)
Similar to the above, by Lemma 3, (N2), (40), (41), we obtain
\[
\left\{ \frac{r^{N-k}}{k} \left[ (y_1^{(m)}(r))^\prime \right] \right\}^{\prime} \leq \frac{r^{N-k}}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( b(r)c_1 \psi \left( (y_1^{(m)}(r), y_2^{(m)}(r)) \right) \right) \right]^{\prime} \leq \frac{r^{N-k}}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( b(r)c_1 \psi \left( (y_1^{(m)}(r), y_2^{(m)}(r)) \right) \right) \right]^{\prime} \left( \frac{\alpha}{(\varphi(\beta, \beta))^{1/(p+1)}} \right) G_1(r) \left( y_2^{(m)}(r) \right) \right) \right] \right\} \leq \frac{r^{N-k}}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( h(r)c_2 \psi \left( (y_2^{(m)}(r), y_2^{(m)}(r)) \right) \right) \right]^{\prime} \left( y_2^{(m)}(r) \right) \omega_2(r) \right) \right]^{\prime}.
\]

Next, we have

\[
\frac{r^{N-k}}{k} \left[ \left( y_1^{(m)}(r) \right) \right]^{\prime} \leq \frac{r^{N-k}}{k} \left[ \left( y_1^{(m)}(r) \right) \right]^{\prime} \leq \frac{r^{N-k}}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( b(r)c_1 \psi \left( (y_1^{(m)}(r), y_2^{(m)}(r)) \right) \right) \right]^{\prime} \left( \frac{\alpha}{(\varphi(\beta, \beta))^{1/(p+1)}} \right) G_1(r) \left( y_2^{(m)}(r) \right) \right) \right] \right\} \leq \frac{r^{N-k}}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( h(r)c_2 \psi \left( (y_2^{(m)}(r), y_2^{(m)}(r)) \right) \right) \right]^{\prime} \left( y_2^{(m)}(r) \right) \omega_2(r) \right) \right]^{\prime}.
\]

Multiplying the last inequality in (44) by \((y_1^{(m)}(r))^\prime\) and the last inequality in (45) by \((y_2^{(m)}(r))^\prime\), we arrive at

\[
\left\{ \left[ \left( y_1^{(m)}(r) \right) \right]^{\prime} \right\}^{\prime} \left( y_1^{(m)}(r) \right)^{\prime} \leq \frac{r^{N-k}}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( b(r)c_1 \psi \left( (y_1^{(m)}(r), y_2^{(m)}(r)) \right) \right) \right]^{\prime} \left( \frac{\alpha}{(\varphi(\beta, \beta))^{1/(p+1)}} \right) G_1(r) \left( y_2^{(m)}(r) \right) \right) \right\} \left( y_1^{(m)}(r) \right)^{\prime},
\]
\[
\left(\left[y_1^{(m)}(r)\right]\right)^{k+1} \leq \frac{k+1}{k} \int_0^r \frac{k^{k-1}}{C_{N-1}} \mathfrak{R}^{-1}\left(h(r)c_1\psi\left(\left(y_1^{(m)}(r),y_2^{(m)}(r)\right)\right)^{1/(p+1)}\right)\omega_1(s)\] 
\left(y_1^{(m)}(s)\right)^k ds
\]

By (N1) and Lemma 1, integrating (46) and (47) from 0 to r, we get

\[
\left(\left[y_1^{(m)}(r)\right]\right)^{k+1} \leq \frac{k+1}{C_{N-1}} \int_0^r s^{k-1}\mathfrak{R}^{-1}\left(c_1\phi_1(r)\omega_1(r)\right)\left[\mathfrak{R}^{-1}\left(\phi_1(r)\right)^k\int_0^s \left[y_1^{(m)}(t)\right]^{k/(p+1)} dt + 1\right]^{k/(p+1)} ds
\]

\[
\left(\left[y_2^{(m)}(r)\right]\right)^{k+1} \leq \frac{k+1}{k} \int_0^r \frac{k^{k-1}}{C_{N-1}} \mathfrak{R}^{-1}\left(h(r)c_2\psi\left(\left(y_1^{(m)}(r),y_2^{(m)}(r)\right)\right)^{1/(p+1)}\right)\omega_2(s)\] 
\left(y_2^{(m)}(s)\right)^k ds
\]

By (48) and (49), we get
\( (y_1^{(m)}(r))' \leq (c_1 \omega_1(r) + 1)^{k((p+1)(k+1))} \frac{\sqrt{k + 1 - 1}}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( \phi_1(r) \right) \right]^k \times \left[ \int_a^{y_1^{(m)}(r)} \left[ \mathcal{W} (\mathcal{W}(\psi(s,s),s) + 1) \right]^{k/(p+1)} ds \right]^{1/(k+1)}, \)  

(50)

From the above two inequalities, we easily deduce that

\( (y_2^{(m)}(r))' \leq (c_2 \omega_2(r) + 1)^{k((p+1)(k+1))} \frac{1}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( \phi_2(r) \right) \right]^k \)

\[ \times \left[ \int_{y_2^{(m)}(r)}^{y_2^{(m)}(r)} \left[ \mathcal{W} (\mathcal{W}(\psi(s,s),s) + 1) \right]^{k/(p+1)} ds \right]^{1/(k+1)} \]  

(51)

Integrating (52) and (53) from 0 to \( r \), we arrive at

\[
\int_a^{y_1^{(m)}(r)} \frac{1}{\sqrt{[\int_0^\mu (\psi(t,\psi(t,t)))^{1/(p+1)} + 1]^{k/(p+1)}}} d\mu \\
\leq \int_0^r (c_1 \omega_1(s) + 1)^{k((p+1)(k+1))} \frac{1}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( \phi_1(s) \right) \right]^k ds,
\]

(54)

\[
\int_{y_2^{(m)}(r)} \frac{1}{\sqrt{[\int_0^\mu (\psi(t,\psi(t,t)))^{1/(p+1)} + 1]^{k/(p+1)}}} d\mu \\
\leq \int_0^r (c_2 \omega_2(s) + 1)^{k((p+1)(k+1))} \frac{1}{C_{N-1}} \left[ \mathcal{R}^{-1} \left( \phi_2(s) \right) \right]^k ds.
\]

(55)

Now, the above two inequalities can be written as

\[ F_1(y_1^{(m)}(r)) \leq P_2(r), \quad \forall r \geq 0, \]

(56)

and

\[ F_2(y_2^{(m)}(r)) \leq Q_2(r), \quad \forall r \geq 0. \]

(57)

Finally, by the fact that \( F_1^{-1} \) and \( F_2^{-1} \) are strictly increasing on \([0,F_1(\infty))\) and \([0,F_2(\infty))\) separately, we get

\[ y_1^{(m)}(r) \leq F_1^{-1}(P_2(r)), \quad \forall r \geq 0, \]

(58)

\[ y_2^{(m)}(r) \leq F_2^{-1}(Q_2(r)), \quad \forall r \geq 0. \]

(59)

Then, we prove that the sequences \( \{y_1^{(m)}(r)\}_{m \geq 0} \) and \( \{y_2^{(m)}(r)\}_{m \geq 0} \) are bounded on \([0,c_0]\) for arbitrary \( c_0 > 0 \). Indeed, since

\[ y_1^{(m)}(r) \geq 0, \]

\[ y_2^{(m)}(r) \geq 0, \]

\[ \forall r \geq 0, \]

(60)

It follows that

\[ y_1^{(m)}(r) \leq y_1^{(m)}(c_0) \leq C_1, \]

\[ y_2^{(m)}(r) \leq y_2^{(m)}(c_0) \leq C_2, \]

\[ \forall r \geq 0, \]

(61)

where \( C_1 = F_1^{-1}(P_2(c_0)) \) and \( C_2 = F_2^{-1}(Q_2(c_0)) \) are positive constants. Moreover, from (50) and (51), we can deduce that \( \{y_1^{(m)}(r)\}_{m \geq 0} \) and \( \{y_2^{(m)}(r)\}_{m \geq 0} \) are bounded on \([0,c_0]\) for arbitrary \( c_0 > 0 \). Thus, the sequences \( \{y_1^{(m)}(r)\}_{m \geq 0} \) and \( \{y_2^{(m)}(r)\}_{m \geq 0} \) are bounded and equicontinuous on \([0,c_0]\) for arbitrary \( c_0 > 0 \). By Arzela–Ascoli theorem, there exist
subsequences of $\{y_1^{(m)}(r)\}_{m \geq 0}$ and $\{y_2^{(m)}(r)\}_{m \geq 0}$ converging uniformly to $y_1(r)$ and $y_2(r)$ on $[0, c_0]$. Since $\{y_1^{(m)}(r)\}_{m \geq 0}$ and $\{y_2^{(m)}(r)\}_{m \geq 0}$ are increasing on $[0, \infty)$, then $\{y_1^{(m)}(r)\}_{m \geq 0}$ and $\{y_2^{(m)}(r)\}_{m \geq 0}$ converge uniformly to $y_1(r)$ and $y_2(r)$ on $[0, \infty)$. By the arbitrariness of $c_0$, we deduce that $(y_1, y_2)$ is an entire positive radial solution of system (32). Thus, by Lemma 3, we get that $(z_1, z_2)$ is an entire positive radial solution of system (8). Repeating the proof in Theorem 1, we get $y_1, y_2 \in C^2[0, \infty) \times C^2[0, \infty)]$. Thus, by Lemma 3, we deduce that system (8) has an entire positive radial solution $(z_1, z_2) \in C^2[0, \infty) \times C^2[0, \infty)]$. This completes the proof. □

**Theorem 2.** We assume (N1), (N2), (S1), and (S2) hold, then system (8) has an entire positive bounded radial solution $(z_1, z_2)$ such that

$$
\begin{align*}
\frac{\alpha + P_1(r)}{\beta + Q_1(r)} & \leq z_1(r) \leq F_1^{-1}(P_2(r)), \\
\frac{\beta + Q_1(r)}{\beta + Q_1(r)} & \leq z_2(r) \leq F_2^{-1}(Q_2(r)).
\end{align*}
$$

**Proof.** On the basis of (N1), (N2), by a proof similar to one of Theorem 1, it is easy to prove that (8) has an entire positive radial solution $(z_1, z_2) \in C^2[0, \infty) \times C^2[0, \infty)]$. Moreover, it follows from (56), (57), and (S1) that

$$
F_1\left(y_1^{(m)}(r)\right) \leq P_2(\infty) < F_1(\infty) < \infty, \quad \forall r \geq 0.
$$

Letting $m \to \infty$, we can prove the conclusions.

Then, it follows from (N1), (N2), (S2), and Lemma 1 that

$$
y_1(r) = \alpha + \int_0^r \left(\frac{k}{\nu^{N-k}} \int_0^t \frac{1}{\nu^{N-k}} \left[\mathcal{R}^{-1}(b(s)\psi(y_1(s), y_2(s)))\right]^{k} \text{d}s\right)^{1/k} \text{d}t
$$

$$
\geq \alpha + \int_0^r \frac{k}{\nu^{N-k}} \int_0^t \frac{1}{\nu^{N-k}} \left[\mathcal{R}^{-1}(b(t)\psi(\alpha, \beta))\right]^{k} \text{d}t
$$

$$
\geq \alpha + \int_0^r \frac{k}{\nu^{N-k}} \int_0^t \frac{1}{\nu^{N-k}} \left[\mathcal{R}^{-1}(b(t)\psi(\alpha, \beta + \frac{1}{\nu^{N-k}}))\right]^{k} \text{d}t
$$

$$
= \alpha + P_1(r).
$$

As in the preceding lines, we can prove

$$
y_2(r) \geq \beta + Q_1(r).
$$

This completes the proof. □

**4. Entire Positive Blow up Radial Solution**

In this section, we investigate entire positive blow up radial solution of system (8), and the main result is as follows:

$$
F_2\left(y_2^{(m)}(r)\right) \leq Q_2(\infty) < F_2(\infty) < \infty, \quad \forall r \geq 0.
$$

Since $F_1^{-1}$ and $F_2^{-1}$ are strictly increasing on $[0, F_1(\infty))$ and $[0, F_2(\infty))$ separately, we get

$$
y_1^{(m)}(r) \leq F_1^{-1}(P_2(\infty)) < \infty, \quad \forall r \geq 0.
$$

$$
y_2^{(m)}(r) \leq F_2^{-1}(Q_2(\infty)) < \infty, \quad \forall r \geq 0.
$$

Letting $m \to \infty$ in the above two inequalities, we obtain

$$
y_1(r) \leq F_1^{-1}(P_2(\infty)) < \infty, \quad \forall r \geq 0.
$$

$$
y_2(r) \leq F_2^{-1}(Q_2(\infty)) < \infty, \quad \forall r \geq 0.
$$

Then, it follows from (N1), (N2), (S2), and Lemma 1 that

$$
F_1\left(y_1^{(m)}(r)\right) \leq P_2(\infty) < F_1(\infty) < \infty, \quad \forall r \geq 0.
$$

**Theorem 3.** We assume that (N1), (N2), (S3), and (S4) hold, then the nonlinear $k$-Hessian system (8) has an entire positive blow up radial solution $(z_1, z_2) \in C^2[0, \infty) \times C^2[0, \infty)]$.

**Proof.** It follows from the conditions (N1), (N2) that a similar proof of Theorem 1 ensures that system (8) has an entire positive radial solution $(z_1, z_2) \in C^2[0, \infty) \times C^2[0, \infty)]$. Moreover, it follows from (56) and (57) that
inequalities, we obtain

\[ F_1 \left( y_1^{(m)}(r) \right) \leq P_2(\infty), \quad \forall r \geq 0, \]  
\[ F_2 \left( y_2^{(m)}(r) \right) \leq Q_2(\infty), \quad \forall r \geq 0. \]  

Since \( F_1^{-1} \) and \( F_2^{-1} \) are strictly increasing on \([0, F_1(\infty))\) and \([0, F_2(\infty))\) separately, we get

\[ y_1^{(m)}(r) \leq F_1^{-1}(P_2(\infty)), \quad \forall r \geq 0, \]  
\[ y_2^{(m)}(r) \leq F_2^{-1}(Q_2(\infty)), \quad \forall r \geq 0. \]

When (S3) holds, we see that \( F_1^{-1}(\infty) = F_2^{-1}(\infty) = \infty \). By the condition (S3), letting \( m \to \infty \) in the above two inequalities, we obtain

\[ y_1(r) \leq F_1^{-1}(P_2(\infty)), \quad \forall r \geq 0, \]  
\[ y_2(r) \leq F_2^{-1}(Q_2(\infty)), \quad \forall r \geq 0. \]

By the condition (S3), letting \( r \to \infty \) in the above two inequalities, we obtain

\[ \lim_{r \to \infty} y_1(r) = \infty, \quad \forall r \geq 0, \]  
\[ \lim_{r \to \infty} y_2(r) = \infty, \quad \forall r \geq 0. \]

Then, it follows from (S4), (72), and (73) that

\[ \lim_{r \to \infty} y_1(r) \geq \alpha + \lim_{r \to \infty} P_1(r) > P_1(\infty) = \infty, \]  
\[ \lim_{r \to \infty} y_2(r) \geq \beta + \lim_{r \to \infty} Q_1(r) > Q_1(\infty) = \infty. \]

Consequently,

\[ \lim_{r \to \infty} y_1(r) = \infty, \]  
\[ \lim_{r \to \infty} y_2(r) = \infty. \]

Therefore, system (33) has an entire positive blow up radial solution \((y_1, y_2) \in C^2([0, \infty) \times C^2(0, \infty))\). By Lemma 3, system (8) has an entire positive blow up radial solution \((z_1, z_2) \in C^2([0, \infty) \times C^2(0, \infty))\). The proof is completed.

5. Semifinite Entire Positive Blow up Radial Solution

In this section, we investigate semifinite entire positive blow up radial solution of system (8), and the main results are as follows:

**Theorem 4.** We assume that (N1), (N2), (S5), and (S6) hold, then the nonlinear \( k \)-Hessian system (8) has one semifinite entire positive blow up radial solution \((z_1, z_2) \in C^2([0, \infty) \times C^2(0, \infty))\).

**Proof.** In view of (N1), (N2), the same arguments as in Theorem 1, we can know that system (8) has an entire positive radial solution \((z_1, z_2) \in C^2([0, \infty) \times C^2(0, \infty))\). By (S3), (79), and (81), we get

\[ \lim_{r \to \infty} y_1(r) \leq F_1^{-1}(P_2(\infty)) = \infty, \]  
\[ \lim_{r \to \infty} y_1(r) \geq \alpha + \lim_{r \to \infty} P_1(r) > P_1(\infty) = \infty, \]  
which imply that

\[ \lim_{r \to \infty} y_1(r) = \infty. \]

Moreover, by (S6), (69), and (73), we obtain

\[ \lim_{r \to \infty} y_2(r) \leq F_2^{-1}(Q_2(\infty)) < \infty, \]  
\[ \lim_{r \to \infty} y_2(r) \geq \beta + \lim_{r \to \infty} Q_1(r) > Q_1(\infty), \quad Q_1(\infty) < \infty, \]  
which imply that

\[ \lim_{r \to \infty} y_2(r) < \infty. \]

Therefore, system (33) has a positive semifinite entire blow up radial solution \((y_1, y_2) \in C^2([0, \infty) \times C^2(0, \infty))\). By Lemma 3, system (8) has a positive semifinite entire blow up radial solution \((z_1, z_2) \in C^2([0, \infty) \times C^2(0, \infty))\). The proof is finished.

**Theorem 5.** We assume that (N1), (N2), (S7), and (S8) hold, then the nonlinear \( k \)-Hessian system (8) has a positive semifinite entire blow up radial solution \((z_1, z_2) \in C^2([0, \infty) \times C^2(0, \infty))\).

**Proof.** Same as Theorem 4, we can know that system (8) has an entire positive radial solution \((z_1, z_2) \in C^2([0, \infty) \times C^2(0, \infty))\). By (S7), (81), and (83), we get

\[ \lim_{r \to \infty} y_2(r) \leq F_2^{-1}(Q_2(\infty)) = \infty, \]  
\[ \lim_{r \to \infty} y_2(r) \geq \beta + \lim_{r \to \infty} Q_1(r) > Q_1(\infty) = \infty, \]  
which imply that

\[ \lim_{r \to \infty} y_2(r) = \infty. \]

Moreover, by (S8), (68), and (72), we obtain

\[ \lim_{r \to \infty} y_1(r) \leq F_1^{-1}(P_2(\infty)) < \infty, \]  
\[ \lim_{r \to \infty} y_1(r) \geq \alpha + \lim_{r \to \infty} P_1(r) > P_1(\infty), \quad P_1(\infty) < \infty, \]  
which imply that

\[ \lim_{r \to \infty} y_1(r) = \infty. \]
which imply that
\[ \lim_{r \to \infty} y_1(r) < \infty. \] (95)

Therefore, system (33) has a positive semifinite entire blow up radial solution \((y_1, y_2) \in C^2[0, \infty) \times C^2[0, \infty). By Lemma 3, system (8) has a positive semifinite entire blow up
\[
\Lambda(S_4^{\frac{1}{4}}(\lambda(D^2 z_2)))S_4^{\frac{1}{4}}(\lambda(D^2 z_2)) = \frac{3}{4} \frac{2 - 2|x|}{4|\epsilon|^{4/3}} e^{1/2}, \quad x \in \mathbb{R}^6.
\]

Letting \( \Lambda(s) = s^3, \ p = 3, \) then \( \Lambda \in \Theta. \) Here, \( b(s) = (2 - s)/s^4 e^s, \ h(s) = (2 - 2s)/s^4 e^s, \) \( \varphi(z_1, z_2) = (1/4)z_1^{1/3} z_2^{2/3}, \) \( \varphi(z_1, z_2) = (3/4)z_1^{1/3} z_2^{2/3}, \) then \( \varphi \) and \( \psi \) are increasing for each variable which satisfies (N1). Obviously, when \( \alpha = \beta = 4, \) we have \( t_1 \geq 4, \ s_1 \geq 1, \ t_2 \geq \sqrt{3}, \ s_2 \geq 1, \ t_3 \geq 1, \)
\[
\varphi(t_1 s_1, t_2 s_2) = \frac{1}{4} t_1^{1/4} s_1^{1/4} t_2^{1/4} s_2^{1/4} \leq \frac{1}{4} t_1^{1/4} t_2^{1/4} s_1^{1/4} s_2^{1/4}
\]
and
\[
\psi(t_3 s_3, t_1 s_1) = \frac{3}{4} t_1^{1/4} t_2^{1/2} s_1^{1/2} s_2^{1/2} \leq \frac{3}{4} t_1^{1/4} t_2^{1/2} s_1^{1/4} s_2^{1/4} \leq \frac{3}{4} t_1^{1/4} t_2^{1/4} s_1^{1/4} s_2^{1/4}.
\]

6. Example

Example 1. we consider
\[
\varphi(t_1 s_1, t_2 s_2) = \frac{1}{4} t_1^{1/4} t_2^{1/4} s_1^{1/4} s_2^{1/4} \leq \frac{1}{4} t_1^{1/4} t_2^{1/4} s_1^{1/4} s_2^{1/4}, \quad \forall c_1 \geq 1,
\]
\[
\psi(t_3 s_3, t_1 s_1) = \frac{3}{4} t_1^{1/4} t_2^{1/2} s_1^{1/2} s_2^{1/2} \leq \frac{3}{4} t_1^{1/4} t_2^{1/2} s_1^{1/4} s_2^{1/4} \leq \frac{3}{4} t_1^{1/4} t_2^{1/4} s_1^{1/4} s_2^{1/4}.
\]

Example 2. we consider
\[
\varphi(t_1 s_1, t_2 s_2) = \frac{1}{4} t_1^{1/4} t_2^{1/4} s_1^{1/4} s_2^{1/4} \leq \frac{1}{4} t_1^{1/4} t_2^{1/4} s_1^{1/4} s_2^{1/4}, \quad \forall c_1 \geq 1,
\]
\[
\psi(t_3 s_3, t_1 s_1) = \frac{3}{4} t_1^{1/4} t_2^{1/2} s_1^{1/2} s_2^{1/2} \leq \frac{3}{4} t_1^{1/4} t_2^{1/2} s_1^{1/4} s_2^{1/4} \leq \frac{3}{4} t_1^{1/4} t_2^{1/4} s_1^{1/4} s_2^{1/4}.
\]
\[ \varphi(\alpha, \beta) \geq \frac{\sqrt{5} - 1}{2}, \quad (103) \]

which mean that \( (N2) \) is satisfied. After a simple calculation, one has

\[ P_2(\infty) = \int_0^\infty (c_1 \omega_1(s) + 1)^{1/5} \sqrt{\frac{1}{2} 5s^3 \phi_1(s)} ds > \int_0^\infty \frac{1}{\sqrt{5}} \frac{d}{ds} \]

\[ = \sqrt{\frac{1}{2}} \int_0^\infty s^{8/5} ds = \infty, \quad (104) \]

\[ Q_2(\infty) = \int_0^\infty (c_2 \omega_2(s) + 1)^{1/5} \sqrt{\frac{1}{2} 5s^3 \phi_2(s)} ds > \int_0^\infty \frac{1}{\sqrt{5}} \frac{d}{ds} \]

\[ = \sqrt{\frac{1}{2}} \int_0^\infty s^{7/5} ds = \infty, \quad (105) \]

\[ F_1(\infty) = \int_0^\infty \frac{1}{\sqrt{\int_0^\infty (t^4 + t^3 + 1) dt}} \frac{d}{dt} = \int_0^\infty \frac{1}{\sqrt{17/5 t^5 + 1/4 t^4 + \mu}} \frac{d}{d\mu} > \int_0^\infty \frac{1}{\sqrt{(\mu + 1)^5}} d\mu \]

\[ = \int_0^\infty \frac{d\mu}{\mu + 1} = \infty, \quad (106) \]

\[ F_2(\infty) = \int_0^\infty \frac{1}{\sqrt{\int_0^\infty (t^{15/4} + t^2 + 1) dt}} \frac{d}{dt} = \int_0^\infty \frac{1}{\sqrt{4/19 t^{19/4} + 1/3 t^3 + \mu}} \frac{d}{d\mu} > \int_0^\infty \frac{1}{\sqrt{(\mu + 1)^5}} d\mu \]

\[ = \int_0^\infty \frac{d\mu}{\mu + 1} = \infty, \quad (107) \]

which mean that \( (S3) \) is satisfied. Then, we have

\[ G_1(r) = \int_0^r \left( \frac{4}{t^2} \int_0^t s^5 \frac{d}{ds} \right)^{1/4} dt = \left( \frac{\sqrt{5}}{3} \right)^{13/4} \int_0^r t^{9/4} dt = \left( \frac{\sqrt{5}}{3} \right)^{13/4} \]

\[ G_2(r) = \int_0^r \left( \frac{4}{t^2} \int_0^t s^5 \frac{d}{ds} \right)^{1/4} dt = \left( \frac{\sqrt{5}}{3} \right)^{13/4} \int_0^r t^{9/2} dt = \left( \frac{\sqrt{5}}{3} \right)^{13/4}, \quad (108) \]
which mean that (S4) is satisfied. By Theorem 3, the 4-Hessian system (101) has an entire positive blow up radial solution \((z_1, z_2) \in C^2[0, \infty) \times C^2[0, \infty)\).

7. Conclusion

The k-Hessian equations, a deep extension of the Laplace equation, have played an important role in mathematics and other applied sciences. In this paper, we studied the radial solutions of a class of k-Hessian system involving a nonlinear operator. The Keller–Osserman type conditions are also crucial throughout the proof process. By employing the monotone iterative method, we establish some new existence results on positive radial solutions of the k-Hessian system (8) in cases of finite, infinite, and semifinite. Our results complement the work in [1] and extend the existing results [7, 8, 15–17, 38], which is a meaningful contribution to the topic of nonlinear elliptic system.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

All authors equally contributed this manuscript and approved the final version.

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