Comment on "OPE and quark-hadron duality for two-point functions of tetraquark currents in $1/\mathcal{N}_c$ expansion"

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Abstract

Without excluding the contributions of factorizable Feynman diagrams in the color space to the QCD sum rules by hand, we cannot obtain the conclusion that the factorizable parts of the operator product expansion series cannot have any relationship to the possible tetraquark bound states. The tetraquark couplings $f_T$ are of the order $\mathcal{O}(N_c)$ rather than of the order $\mathcal{O}(N_c^0)$ in the large $N_c$ limit, the conclusion "a possible exotic tetraquark state may appear only in $N_c$-subleading contributions to the QCD Green functions" is a paradox.

A hadron has many Fock states, a tetraquark state, which has four valence quarks, maybe have color-singlet-color-singlet (11) type, color-antitriplet-color-triplet (33) type or color-sextet-color-antisextet (66) type Fock states. If a hidden-charm tetraquark state has the 11-type Fock states, we can construct the 11-type four-quark currents, which should have the same quantum numbers, to interpolate this hidden-charm tetraquark state, because the quantum field theory does not forbid such current-tetraquark couplings. The argument applies to other Fock states.

Now we construct the four-quark currents to interpolate the hidden-charm tetraquark states,

$$
\begin{align*}
J_{11}(x, \epsilon) &= \bar{c}(x + \epsilon)\Gamma q(x + \epsilon) \bar{q}'(x)\Gamma' c(x), \\
J_{33}(x, \epsilon) &= \epsilon^{ijk}\epsilon^{imn} q^T_i(x) C T c_k(x) \bar{q}_m(x + \epsilon) \Gamma'C\bar{c}_n^T(x + \epsilon), \\
J_{6\bar{6}}(x, \epsilon) &= q^T_j(x) C T c_k(x) \bar{q}'_j(x + \epsilon) \Gamma'C\bar{c}_k^T(x + \epsilon) + q^T_j(x) C T c_k(x) \bar{q}'_k(x + \epsilon) \Gamma'C\bar{c}_j^T(x + \epsilon),
\end{align*}
$$

(1)

where the $i, j, k, m, n$ are color indexes, the $\Gamma$ and $\Gamma'$ are Dirac $\gamma$-matrices, the $\epsilon$ is the spatial separation between the two clusters in the color space.

The tetraquark states are spatial extended objects, if the mean spatial sizes $\langle r \rangle \geq \epsilon$, we can choose the currents $J_{11}(x, \epsilon)$, $J_{33}(x, \epsilon)$ and $J_{6\bar{6}}(x, \epsilon)$ to interpolate the 11-type, 33-type and 66-type tetraquark states, respectively, although the diquark states in color-sextet is not favored by repulsive interaction originates from the one-gluon exchange.

In fact, it is difficult to take into account the nonlocal effects in the QCD sum rules due to the appearance of the $\epsilon$, as we have to deal with a bound state problem, so we usually take the local limit $\epsilon \to 0$. In the local limit $\epsilon \to 0$, the currents $J_{11}(x, 0)$ couple potentially to the 11-type tetraquark states rather than two-meson pairs, because in such a small spatial separation, the $\bar{c}q$ and $\bar{q}'c$ mesons lose themselves and merge into tetraquark states. Direct calculations based on the QCD sum rules support such arguments [1,2].

If the mean spatial sizes $\langle r \rangle < \epsilon$, the currents $J_{11}(x, \epsilon)$ couple potentially to the two-meson pairs, because in such large spatial separations, the $\bar{c}q$ and $\bar{q}'c$ mesons retain themselves.

Generally speaking, the 33-type and 66-type tetraquark states can have larger spatial extensions, as the confinement forbids the appearance of the free diquark states [1].

In the local limit, the currents $J_{33}(x, 0)$ and $J_{6\bar{6}}(x, 0)$ can be transformed into the currents $J_{11}(x, 0)$ freely through Fierz rearrangements in the color and Dirac-spinor spaces [4], we can study the current $J_{11}(x, 0)$ as an example. The $J_{11}(0, 0)$, $J_{33}(0, 0)$ and $J_{6\bar{6}}(0, 0)$ couple potentially to the tetraquark states ($T$),

$$
\langle 0|J_{11/33/6\bar{6}}(0, 0)|T(p)\rangle = f_T,
$$

(2)

where the $f_T$ are the pole residues or decay constants or tetraquark couplings.

Now we write down the two-point correlation functions $\Pi(p)$ in the QCD sum rules,

$$
\Pi(p) = i \int d^4 x e^{ipx} \langle 0|T\left\{J_{11}(x, 0)J_{11}^\dagger(0, 0)\right\}|0\rangle,
$$

(3)

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The lowest order Feynman diagrams shown in Fig. 1 can be divided into two color-neutral clusters, each cluster corresponds to a trace both in the color space and in the Dirac spinor space, the lowest order Feynman diagrams are proportional to $N_c^2$ in the large $N_c$ limit. In the momentum space, they are nonfactorizable diagrams, the basic integrals are of the form,

$$\int d^4q d^4k d^4l \frac{1}{(p - k - q + l)^2 - m_c^2 + i\epsilon} \frac{1}{q^2 - m_q'^2 + i\epsilon} \frac{1}{k^2 - m_q^2 + i\epsilon} \frac{1}{l^2 - m_q^2 + i\epsilon}.$$  \hspace{1cm} (4)

Such integrals certainly have an imaginary part, and we can obtain imaginary parts through dispersion relation after carrying out the integrals over $q$, $k$, $l$, and they make contributions to the QCD sum rules. From the Fig. 1 we can obtain the relation,

$$f_T \propto N_c,$$  \hspace{1cm} (5)

rather than the relation $f_T \propto N_c^0$ \cite{3} in the large $N_c$ limit. In Ref. \cite{4}, Lucha, Melikhov and Sazdjian discard the factorizable Feynman diagrams in the color space via putting the confined quark and gluons on the mass-shell and applying Landau equation to study them by hand. In Ref. \cite{2}, we present detailed discussions to show that the Landau equation is of no use to study the Feynman diagrams in the QCD sum rules for the tetraquark states, the tetraquark states begin to receive contributions at the order $\mathcal{O}(\alpha_s^2/\alpha_s^1)$ rather than at the order $\mathcal{O}(\alpha_s^2)$ claimed in Ref. \cite{4}. In Ref. \cite{3}, Lucha, Melikhov and Sazdjian re-express their viewpoint as the tetraquark states begin to receive contributions at the order $\mathcal{O}(\alpha_s^2 N_c^2)$ in the large $N_c$ limit and obtain the conclusion ”a possible exotic tetraquark state may appear only in $N_c$-subleading contributions to the QCD Green functions”, which is a paradox.

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