Improving Causal Effect Estimation of Weighted Regression Based Estimator using Neural Networks

Plabon Shaha*  
University of Dhaka  
Dhaka, Bangladesh

Ismat Rahman  
University of Dhaka  
Dhaka, Bangladesh

Talha Islam Zadid†  
University of Dhaka  
Dhaka, Bangladesh

Md. Mosaddek Khan  
University of Dhaka  
Dhaka, Bangladesh

ABSTRACT

Estimating causal effects from observational data informs us about which factors are important in an autonomous system, and enables us to take better decisions. This is important because it has applications in selecting a treatment in medical systems or making better strategies in industries or making better policies for our government or even the society. Unavailability of complete data, coupled with high cardinality of data, makes this estimation task computationally intractable. Recently, a regression-based weighted estimator has been introduced that is capable of producing solution using bounded samples of a given problem. However, as the data dimension increases, the solution produced by the regression-based method degrades. Against this background, we introduce a neural network based estimator that improves the solution quality in case of non-linear and finitude of samples. Finally, our empirical evaluation illustrates a significant improvement of solution quality, up to around 55%, compared to the state-of-the-art estimators.

KEYWORDS

Causal Estimation, Weight-based Generalised Estimator, Neural Networks

1 INTRODUCTION

Causal inference is a data-driven discipline that focuses on study design, experimentation, and estimation methodologies to find cause-and-effect linkages based on how actions (or treatments) affect a system’s outcome of interest. Unlike other state-of-the-art statistical models, a causal model combines data with a context. In other words, it focuses on data generation procedures, making it inherently interpretable and robust in dynamic conditions. Specifically, causal identification techniques help us to identify the causal effects between actions and outcomes, which are both referred to as causal variables of a causal model. Due to small number of samples and high cardinality, identifying causal effects from observational and experimental data is often infeasible. In effect, estimation of causal effects between variables still remains an important problem and is studied intensively in many disciplines, such as Medical Science [9, 12, 13, 19], Genetics [2, 4, 18], Business Studies [3, 23], Epidemiology [7, 17], Economics [11, 22] and Social Science [5].

The problem of causal identification comes under the form of do-calculus. In causal literature, the so-called do operator is used to denote interventions or actions that modifies parts of the causal model. The do-calculus is a set of inference directives that helps the transformation of these interventions into more interpretable probabilistic sentences, and as such, enables an user to derive or confirm causal claims about interventions [14]. Results inferred from do-calculus is well understood on the whole but its application is still questionable [10]. This is because do-calculus assumes that the distributions being used are error-free, but in practice, we do not have sufficient samples to confirm that. In case of limited samples, a popular criterion, namely back-door criterion, is employed to estimate causal effects. In more detail, back-door (BD) adjustment formula estimates causal effects when a set of covariates (observed variables) satisfies the BD criterion relative to an ordered pair [15].

The Back-door adjustment formula depends on the cardinality or dimension of the covariate set, which imposes a significant computational overhead when the covariate set is high dimensional. Over the years, a number of approaches have been introduced to deal with the problem of estimating effects from finite samples. In Inverse probability weighting (IPW) [8], a two stage sampling scheme was proposed that estimates the sampling variance from limited samples without the need of any additional assumptions about the sample. [21] developed a propensity score based method that provides score on the treatment variable of interest given a set of observed covariates. This method showed that only the adjustment of the scalar propensity score is enough to remove bias and also depicted great success not only in univariate but also in multivariate scenario. Stabilized weighting (SW) [6] uses the combination of marginal structural models and inverse probability weighting to estimate causal effects in the presence of time dependent confounders (i.e. unobserved common causes). Doubly robust estimators [1] provide a way to estimate causal effects in the scenario of limited observational data. Bayesian additive regression trees focuses more on the flexibility of modelling data and it can handle large number of covariates, continuous treatment variables and missing data. [20] introduced g-formula that can be used to estimate causal effect of time varying treatment in the presence of time varying confounders.

In practical BD-admissible settings, the aforementioned techniques have been proved to be quite effective in calculating causal effects. However, many scenarios exist in reality where causal effects are identifiable, but not BD-admissible (often termed as non-BD settings). For example, in some cases, confounders affect both treatment and outcome, necessitating the introduction of a new set of variables to compute effect. Besides, there are situations when we are unable to control the treatment variable and must

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*Both authors contributed equally to the paper
establish a new set of variables to control it. To address these issues, [10] recently developed a Composition of weighting operator (CWO) based causal estimation method. The building block of this method is a weight-based operator that can be estimated using statistical techniques such as the BD (Back-door) estimand. CWO is capable of estimating the causal effects of non-back-door scenarios solely by the weighting operator or its composition.

The CWO-based model utilizes regression techniques (e.g. weighted least square, logistic regression, linear regression) to estimate the causal effects between variables. The data becomes increasingly non-linear as the dimension of the covariates rises, and regression methods fail to completely interpret these non-linear relationships. To address this drawback, we propose a neural network based estimator, that we call NN-CWO, where the operator of CWO is modeled using an artificial neural network.

Specifically, a back propagation method is deployed to learn the parameters of the operator that helped the NN-CWO model to learn the causal relations from a very small sample. As neural networks can effectively learn the complex relationships, NN-CWO model produces good results in case of non-linear data. Our empirical findings show that it outperforms the state-of-the-art approach in both low and large data dimension scenarios.

The remainder of this paper is structured as follows. We describe the problem of estimating causal effects in case of non-BD scenarios in more detail in the section that follows. Then, in Section 3, we discuss the complete process of NN-CWO with worked examples. Afterwards, in Section 4, we present the empirical results of our method compared to the current state-of-the-art, and Section 5 concludes.

2 BACKGROUND AND PROBLEM DEFINITION

In this section, we first define the Structural Causal Models (SCM) followed by some necessary definitions. We then discuss how CWO’s weighting operator deals with non-back-door scenarios.

Structural Causal Models (SCM) are sufficient to express the underlying causal relationships between causal variables and is denoted by a tuple \( M = < V, U, F, P(u) > \). Here, \( U \) denotes the set of unobserved (exogenous) variables, \( V \) denotes endogenous variables, \( F \) is a set of functions such that each \( f_i \in F \) determines each variable \( V_i \in V \) from \( U \cup P(a_i) \), where \( P(a_i) \subseteq V \setminus \{V_i\} \) and \( P(u) \) denotes the probability distribution defined over the domain of exogenous variables \( U \). An SCM \( M \) can be interlinked with a causal diagram \( G \) in which every node resembles a variable and each link signifies a causal connection from \( P(a_i) \cup U \) to \( V_i \). The endogenous variable set can be further decomposed into three disjoint sets \( V = \{X, Z, Y\} \), where \( X \) is the set of treatment variables, \( Z \) is the set of observed variables (covariates) and finally \( Y \) is the set of outcome variables. Interventions or actions on a causal model can be thought of as an external force that modifies the function \( f_i \) between \( X \) and its parents by setting it to an external value(s) \( x \). This operation is denoted by \( \text{do}(X=x) \) and this induces a sub-model \( M_{X=x} \) and a probability distribution \( P_X(x) \) that is defined over the set of observed variables \( V \) (see [15] for more detail). The causal effect of \( X \) on \( Y \) (\( P_X(Y) \)) is identifiable if \( P(Y \mid \text{do}(X=x)) \) can be uniquely computed from the probability distribution over \( V \). The identifiability conditions assume that the probability distributions are flawless but in real-life scenarios we often do not have enough samples to support that. Here, we aim to estimate causal effects of control variables \( X \) on outcome \( Y \) in non-back-door scenarios in the context of limited samples.

Note that, we represent each variable with capital letters (e.g. \( A, B, C \)) and their corresponding registered values by small letters (e.g. \( a, b, c \)). A set of variables is denoted by bold capital letters (e.g. \( A, B, C \)). In an ordered set of variables, \( A = \{A_1, A_2, A_3, \ldots, A_n\} \), \( A^{(i)} = \{A_1, A_2, \ldots, A_i\} \) and \( A^{\geq i} = \{A_i, \ldots, A_n\} \). In the graphs, we used the notations Pa(A), Ch(A), An(A), De(A) to represent the parents, children, Ancestors and Descendants of \( A \), respectively. Here \( G_{C \rightarrow Z} \) denotes the graph \( G \) where all the incoming edges into \( C \) and all the outgoing edge from \( C \) are removed. Finally, bi-directional dashed lines depicts the variables being confounded by confounders.

Definition 2.1 (Weighted Distribution, \( P^W(v) \)). Given a set of variables \( V \), their probability distribution \( P(V) \) and a weight function \( W(v) \), where \( W(v) > 0 \). The weighted probability is given by,

\[
P^W(v) = \frac{P(v)W(v)}{E_W}; \text{ where } E_W = E[ W(v) ]
\]

Weighting-based estimators such as IPW [8], SW [6] have been used to estimate BD-admissible estimands which used the aforementioned formulation.

Definition 2.2 (Back-door adjustment). If a set of variables \( Z \) satisfies the back-door criterion relative to an ordered set of variables \( (X, Y) \), then we say \( P_X(Y) \) is identifiable by the BD-adjustment formula which is,

\[
E[P_X(Y)] = \sum_y (Y=y) \sum_z P(Y=y \mid X=x, Z=z)P(Z=z)
\]

2.1 Weighting Operator and the Formulation of Non Back-door Scenarios

The CWO approach has extended the BD scenario using weighting-based operator \( \beta \) and this weighting operator is also used as the building block of our proposed method.

Definition 2.3 (Weighting Operator, \( \beta \)). Given a set of treatment variables \( X = x \), a weight function \( W \) and a function of variables \( h(Y) \), the weighting operator is defined as,

\[
E[P_X(h(Y)) = E[P^W(h(Y)|x)] = \beta(h(Y)|x; W); \text{ where } W = \frac{x}{P(x|z)}
\]

The CWO method performs the complicated task of formulating expected value of effect for a cause with the help of a weighting function to give weight to the samples. This helps to estimate the causal effect from small samples. After using the weighting operator to weight the samples, the resulting weighted distribution can be treated as the post intervention distribution, which simplifies the operation of intervention [15].

Despite the aforementioned advantages, CWO utilizes regression methods, which axiomatically fail to interpret the underlying causal relationships as the data gets increasingly non-linear. In the wake of this, we are going to extend CWO using neural networks with the intent to improve solution quality for BD and non-BD settings with
low and high data dimensions. Before that, we are going to explore how the weighting operator $\beta$ of CWO can be used to formulate all the non-BD scenarios.

2.1.1 Multi-outcome Sequential Back-door Criterion. Estimating effects of sequential plans $X = \{X_1, X_2, X_3, ..., X_n\}$ on outcome $Y$ in the presence of confounders is an age-old question and has been solved effectively by the development of the Sequential Back-door (SBD) adjustment formula [16]. SBD assumes the outcome $Y$ to be a singleton. CWO extends the SBD criterion to fit in with the scenarios where the outcome $Y$ is a set of variables, and named it as Multi-outcome Sequential Back-Door criterion (mSBD) [10]. Here, if a set of covariates $Z$ satisfies mSBD criterion relative to $(X,Y)$, then the causal effect is identifiable and given by,

$$P_x(y) = \sum_{z} \prod_{k=0}^{n} P(x_k|x^{(k)}, z^{(k)}, y^{(k-1)}) \times \prod_{j=1}^{n} P(z_j|x^{(j-1)}, z^{(j-1)}, y^{(j-1)})$$

(2)

This can be formulated using CWO’s weighting operator as follows,

$$E[P_x[h(Y)]] = \beta[h(Y)|x; W], \text{ where}$$

$$W = W_{mSBD}(x, y, z) = \frac{P(x)}{\prod_{k=1}^{n} P(x_k|x^{(k-1)}, y^{(k-1)}, z^{(k)})}$$

(3)

2.1.2 Surrogate Criterion. The Surrogate Criterion is another setting where causal effect can be efficiently estimated using the weighting operator of CWO. In Figure 2, we see the causal model of such a criterion. Here, $Z$ is a set of confounders for the causal effect of $X$ on $Y$. Here, if $X$ is hard to ascertain, it can be substituted by the variable $W$. The causal effect can be estimated using weighting operators using the weight function $W_{mSBD}$ from Equation 3.

In this setting, if a set of covariates $\{W,Z\}$ satisfies Surrogate criterion relative to $(X,Y)$, then the causal effect is identifiable and given by,

$$E[P_x[h(Y)]] = \beta[h(Y)|x \cup W; W], \text{ where}$$

$$W = W_{mSBD}(W, x \cup y, z)$$

(4)

2.1.3 Front-door Criterion. There are non-BD scenarios where both the treatment $X$ and the outcome $Y$ are confounded by covariates $U_{x,y}$. Front-door criterion effectively solves the problem by introducing covariate set $Z = \{Z_1, Z_2, ..., Z_n\}$. Front-door can be expressed as a composition of multiple weighting operators to effectively estimate causal effects. If $Z$ is front-door admissible relative to ordered pair $(X,Y)$, then $P_x(y)$ is identifiable and is formulated here.

The equation for estimating causal effect of $X$ on $Y$ can be split into two weighting operators of two back-door scenarios. Here, $\beta_1$ estimates the causal effect of $X$ on covariate set $Z$ with no back-door admissible node as follows,

$$\beta_1 = \beta[h(z)|x; W_1] = \sum_z h(z)P^{W_1}(z|x); \text{ where } W_1 = \frac{p(x)}{p(x|\emptyset)} = 1$$

(5)

On the other hand, $\beta_2$ estimates the effect of $Z$ on $Y$ where the node $X$ is back-door admissible as follows,

$$\beta_2 = \beta[y|z; W_2] = \sum_y yP^{W_2}(y|z); \text{ where } W_2 = \frac{p(z)}{p(z|x)}$$

(6)
Algorithm 1: NN-CWO Algorithm

| Function | NN_CWO(X, y, X_pred, w; hp): |
|-----------|-------------------------------|
| 1         | d ← number of columns of X; |
| 2         | if d ≠ 1 then |
| 3         | Beta ← Sequential Neural Network; |
| 4         | + Dense("linear", hp[input_units], d); |
| 5         | + Dropout(hp[dropout_rate]); |
| 6         | for i ← 1 to hp[n_layers] do |
| 7         | + Dense("relu", hp[units_i]); |
| 8         | + Dropout(hp[dropout_rate_i]); |
| 9         | end |
| 10        | + Dense("linear", 1); |
| 11        | Beta.compile("adam", hp[learning_rate], "mean squared error"); |
| 12        | Beta.fit(X, y, w); |
| 13        | end |
| 14        | else |
| 15        | Beta ← LinearRegression() ; |
| 16        | Beta.fit(X, y, w); |
| 17        | end |
| 18        | y_pred ← Beta.predict(X_pred); |
| 19        | return y_pred |

A composition of β1 and β2 operators is done to estimate the direct effect of X on Y as follows,

\[ E(P_x(y)) = (\beta_1 \beta_2)(x) = \beta_2(z|x; w_1) \]

3 THE NN-CWO ALGORITHM

NN-CWO is a Neural Network based algorithm that effectively solves the challenges faced in causal effect estimation of non-BD conditions. The algorithm estimates causal effects in both linear and non-linear relations. In so doing, we introduce a neural network-based operator. We use a composition of this operator to effectively estimate causal effects in the non-BD conditions: Front-door Criterion, Surrogate Criteron and mSBD.

Algorithm 1 shows the NN-CWO method. The inputs of this algorithm are the training data where X is the cause variable and y is the effect variable, X_test is the data on which predictions for the effect variable are made, w is a column of sample weights and hp is a set of hyper-parameters. The algorithm uses a sequential neural network model to estimate causal effects of the respective layers are taken from hp. Back propagation is done while fitting the data where the samples are weighted using weights predefined for the respective causal model using the procedure getWeight(). Here, Dropouts and callback functions are used to tackle overfitting. In more detail, the algorithm first finds the dimension d of the cause variable and specifies which model to use. When d is not 1, the neural network-based model is used where the steps in lines 4-13 are executed. Here, a sequential model named Beta is created. Then a dense input layer is added to Beta with the input dimension set to d. The ReLu activation function is used and the unit is taken from hp (line 8). Then, a dropout layer is added to Beta with the dropout rate taken from hp. Similarly, other layers with similar properties are added to Beta iteratively in lines 7-9 where the number of iterations is also taken from hp. In each iteration, one dense layer is added with the ReLu activation function along with one dropout layer, where the unit and the dropout rate of the respective layers are taken from hp. Then, in line 11, a dense output layer is added to Beta with unit 1 and a Linear activation function. Beta is compiled in line 12 with the Adam optimizer having a learning rate obtained from hp and with mean squared error as the loss function. Finally, the model Beta is fit with X and y and the weight as the sample weight. This is where the model feeds forward the input data towards the output layers and also performs Back Propagation to tune the parameters. On the other hand, if d is equal to 1 (checked in Line 3), the algorithm executes from line 16, where Beta is defined as a linear regression model. X and y with w as the sample weight is fit into the model. Finally, after the model Beta is trained for the corresponding d, y_pred is set as the predicted values of y for the given set X_pred from the model. y_pred is then returned. A composition of NN_Beta is done consecutively depending on the causal scenario.

3.1 Front-door

In Algorithm 2, the causal effect for the Front-door criterion of the causal model, as seen in Figure 3, is estimated. The causal effect of X on Y is estimated using two Back-door considerations. Firstly, the dimension d of the variable Z is determined. The data is processed to figure out X, Y and Z. Here, a composition of two back-doors is made. For the first back-door, the causal effect of Z on Y with X as the back-door admissible node is modeled, where weight1 is generated using a getWeight() function using Equation 6. In line 7, NN_CWO is called with parameters (Z, X, Z, weight1, hp)

\[ 1 \text{Although other optimizers can be used, we choose Adam optimizer as it gives good results at the expense of small computational time, and also for its ease of configuration.} \]
Algorithm 3: Function that estimates the ATE in Surrogate Criterion

```python
def surrogate_NNCWO(data, hp):
    d ← dimension of columns of Z of data;
    X ← data[x];
    Y ← data[y];
    X_train ← data[x, w];
    weight ← getWeight();
    X_test ← [[0,1],[1,0]];
    y_pred ← NN_CWO(X_train, Y, X_test, weight, hp);
    μ0 ← y_pred[0];
    μ1 ← y_pred[1];
    return μ0, μ1
```

Algorithm 4: Function that estimates the ATE in mSBD Criterion

```python
def mSBD_NNCWO(data, hp):
    X ← data[x1, x2];
    Y ← data[y2];
    weight ← getWeight();
    X_test ← [[0,0], [0,1], [1,0], [1,1]];
    y_pred ← NN_CWO(X, Y, X_test, weight, hp);
    μ0 ← y_pred[0];
    μ1 ← y_pred[1];
    μ10 ← y_pred[2];
    μ11 ← y_pred[3];
    return μ0, μ1, μ10, μ11
```

Finally, the predicted values are stored in \( y_{pred} \). This gives the estimates of \( Y \) for a given \( X \) which is \( \mu_0 = y_{pred}[0] \) when \( x = 0 \) and \( \mu_1 = y_{pred}[1] \) when \( x = 1 \). These predictions are returned.

### 3.2 Surrogate Criterion

Algorithm 3 estimates the causal effect for the Surrogate criterion described in the causal model shown in Figure 2. The causal effect of \( X \) on \( Y \) can be visualized as the causal effect of \( [X, W] \) on \( Y \) with \( Z \) being a Back-door admissible node. Here, the \( NN_CWO \) is called once. Though composition is not required but the weighted Neural Network learns the complex relations of surrogate effectively. To begin with, the data is processed to figure out \( X, W, Z_1 \) to \( Z_d \) and \( Y \). In line 5, \( X \) and \( W \) is put in a single variable \( X_{train} \). Then, the model is trained for the causal effect of \( X_{train} \) on \( Y \) with \( Z \) as the Back-door admissible node. \( weight \) is generated with the help of \( getWeight() \) function using equation \( W = \frac{P(W)}{P(W|Z)} \). An array \( X_{test} \) with only the possible combinations of \( X \) and \( W \) is used. Here \( X_{test} \) is a 2-D array \([0,1],[1,0] \). In line 8, we call \( NN_CWO \) with parameters \( (X_{train}, Y, X_{test}, weight, hp) \) (Equation 4). Finally, the predicted values are stored in \( y_{pred} \). This gives the estimates of \( Y \) for a given \( x \) which is \( \mu_0 = y_{pred}[0] \) when \( x = 0 \) and \( \mu_1 = y_{pred}[1] \) when \( x = 1 \). These predictions are returned.

### 3.3 mSBD

Algorithm 4 estimates the causal effect for the Multi-Sequential outcome back-door criterion as described in the causal model Figure 1. Here, \( Z \) is mSBD admissible relative to \( (X, Y) \). So, the causal effect of \([X_1, X_2] \) on \( Y_2 \) is estimated using Equation 3. Firstly, the data is processed keeping columns \( X_1 \) and \( X_2 \) in variable \( X \) and column \( Y_2 \) is set as variable \( Y \). In line 4, the sample weights, \( weight \) are generated. Here, \( X_{test} \) is a 2-D array \([0,0],[0,1],[1,0],[1,1] \) which are all possible combinations of \( X_1 \) and \( X_2 \) respectively. In line 6, \( NN_CWO \) is called with parameters \( (X_{train}, Y, X_{test}, weight, hp) \).

In our experiments, as recommended in the CWO paper, we use Median Average Absolute Error or MAAE to report our comparative results. For each of the three non-BD situations, we perform the tests with different dimensions of covariates and a wide range of sample sizes. We estimate \( \mu(x) = E(P_x) \) from a sample of size \( N=10^7 \) by performing \( do(X=x) \) and generating a model after performing intervention on \( X \). We use this as the ground truth, as it is done in the CWO paper. In a similar way, we estimate \( E(P_{xy}) \) as \( \mu_{cwo}(x) \). We then use this as the baseline to compare the performance of the NN-CWO algorithm. We then estimate the expected value \( \mu_{in}(x) \) using NN-CWO approach. Moreover, we estimate the weights using assumed parametric models from the data. For multi-dimensional cases, we used chain rule of probability. Logistic regression is used for binary cases. We compute Average absolute error AAE as \( |\mu(x) - \mu_{cwo}(x)| \) and \( |\mu(x) - \mu_{in}(x)| \) averaged over \( x \). So, if \( X=x_1, x_2, x_3, ..., x_n \), the AAE for method \( R \) is calculated as

\[
\frac{\sum_{i=1}^{n} |\mu(x_i) - \mu_{R}(x_i)|}{n}
\]

For each sample size, we generate 100 AAEs and determine the median AAE or MAAE. MAAE is used for comparison because the
results might vary over 100 AAEs and picking the median solves this issue. A plot of MAAE vs sample size \( N \) makes the MAAE plot which is the basis for our performance comparison. The lower the MAAE curve, the better the performance of the algorithm.

4.2 Results

We make estimations for the different scenarios over a range of dimensions. For efficient estimation, we perform all the experiments over a range of samples \([500, 1000, 1500, ..., 10000]\). Here, dashed lines in a graph represents the results for CWO algorithm and solid lines represents the results for NN-CWO algorithm.

**Front-door Criterion**: To test our NN-CWO with the CWO in case of front-door, as done in the CWO paper, we set \( X \) to be binary, \( Y \) is continuous in \([0,1]\) and \( Z = \{ Z_1, Z_2, ..., Z_D \} \). The graphs in Figure 4a and Figure 4b depict the results for low and high dimension of the data respectively. It can be observed that, both NN-CWO and CWO starts with a slightly higher MAAE. However, MAAE tends to drop with the increase of sample sizes. This is due to the fact that more samples help both of these models in better understanding the causal relationships between the variables. We can see from Figure 4b that MAAE for higher dimensions tend to converge less and the convergence is much smaller for CWO algorithm.

**Surrogate Criterion**: We estimate \( \mathbb{E}(P_{x'y}) \) from the SCM in Figure 2 using Algorithm 3. Following the SCM in the CWO paper, \( X, Z \) and \( W \) are binary variables, \( Y \) is continuous between 0 and 1, and \( D \) is the dimension of \( Z \). We generate two MAAE plots in Figure 5a and Figure 5b similar to that of Front-door criterion. Here, as it is observed in the Front-Door criterion experiment, in case of lower sample sizes, the MAAE curve for NN-CWO starts lower than that of CWO. As the sample size grows, both curves gradually decline, with the gradual dissension being higher for NN-CWO than for CWO. We observe that NN-CWO consistently outperforms CWO in both higher and lower dimensions, as well as for different sample sizes.

**mSBD Criterion**: We estimate \( \mathbb{E}(P_{x'y}) \) for mSBD Criterion from the SCM in Figure 1 using Algorithm 4. Here, the variables \( X_1, X_2, Y_1 \) and D-dimensions of \( Z_1 \) and \( Z_2 \) are all binary whereas \( Y_2 \) is continuous. Here, similar to the previous two criteria, we follow the CWO paper. We generate the MAAE plot for mSBD criterion in Figure 6. The curves for both CWO and NN-CWO converges become increasingly nonlinear, and the CWO approach underperforms in this situation, but our algorithm performs better. Finally, we can conclude that NN-CWO is more robust in all dimensions and all sample sizes, it outperforms CWO in all the cases.
We see that NN-CWO gives better estimates than CWO in the linear causal relations, NN-CWO learns non-linear causal relations significantly well. Our extensive empirical evaluations illustrate that our algorithm outperforms the state-of-the-art algorithm. NN-CWO learns causal relations from limited samples which is a major problem in the field of Causal Inference. In future, we want to explore ways to handle model mis-specifications, and conduct experiments to see how our algorithm performs in mSBD composition criterion. Finally, we would like to extend the work to handle time series-based data using Recurrent Neural Networks.

5 CONCLUSIONS AND FUTURE WORKS

This paper introduces an algorithm, NN-CWO, for the causal estimation of non Back-door scenarios with the incorporation of a Neural Network-based Composition of Weighting Operators. Along with the linear causal relations, NN-CWO learns non-linear causal relations significantly well. Our extensive empirical evaluations illustrate that our algorithm outperforms the state-of-the-art algorithm. NN-CWO learns causal relations from limited samples which is a major problem in the field of Causal Inference. In future, we want to explore ways to handle model mis-specifications, and conduct experiments to see how our algorithm performs in mSBD composition criterion. Finally, we would like to extend the work to handle time series-based data using Recurrent Neural Networks.

REFERENCES

[1] Heejung Bang and James M Robins. 2005. Doubly robust estimation in missing data and causal inference models. Biometrics 61, 4 (2005), 962–973.
[2] Stephen Burgess and Simon G Thompson. 2015. Multivariable Mendelian randomization: the use of pleiotropic genetic variants to estimate causal effects. American journal of epidemiology 181, 4 (2015), 251–260.
[3] M Keith Chen, Yilin Zhou, Malena la Fuente, Ryne Rohla, and Elisa F Long. 2020. Causal estimation of stay-at-home orders on SARS-CoV-2 transmission. arXiv preprint arXiv:2005.05469 (2020).
[4] Alexis Diamond and Jasjeet S Sekhon. 2013. Genetic matching for estimating causal effects: A general multivariate matching method for achieving balance in observational studies. Review of Economics and Statistics 95, 3 (2013), 932–945.
[5] Ana V Dier Roux. 2004. Estimating neighborhood health effects: the challenges of causal inference in a complex world. (2004).
[6] Miguel Ángel Hernán, Babette Brumback, and James M Robins. 2000. Marginal structural models to estimate the causal effect of zidovudine on the survival of HIV-positive men. Epidemiology (2000), 561–570.
[7] Miguel A Hernán and James M Robins. 2006. Estimating causal effects from epidemiological data. Journal of Epidemiology & Community Health 60, 7 (2006), 578–586.
[8] Daniel G Horvitz and Donovan J Thompson. 1952. A generalization of sampling without replacement from a finite universe. Journal of the American statistical Association 47, 260 (1952), 663–685.
[9] Kosuke Imai, Gary King, and Elizabeth A Stuart. 2014. 8. Misunderstandings Between Experimentalists and Observationalists About Causal Inference. In Field Experiments and Their Critics. Yale University Press, 196–227.
[10] Yonghan Jung, Jin Tian, and Elias Bareinboim. 2020. Estimating causal effects using weighting-based estimators. In Proceedings of the AAAI Conference on Artificial Intelligence. Vol. 34. 10186–10193.
[11] Michael Lechner. 2001. Identification and estimation of causal effects of multiple treatments under the conditional independence assumption. In Econometric evaluation of labour market policies. Springer, 43–58.
[12] María Manuela Moreno-Fernández and Helena Matute. 2020. Biased sampling and causal estimation of health-related information: Laboratory-based experimental research. Journal of medical Internet research 22, 7 (2020), e17502.
[13] Hiroshi Nakajima, Hiroshi Tasaki, Naoki Truchiyi, Takehiro Hamaguchi, and Toshikazu Shiga. 2011. Visceral fat estimation method by bioelectrical impedance analysis and causal analysis. In Independent Component Analyses, Wavelets, Neural Networks, Biosystems, and Nanoengineering IX, Vol. 8058. International Society for Optics and Photonics. 80580Z.
[14] Judea Pearl. 1995. Causal diagrams for empirical research. Biometrika 82, 4 (1995), 669–688.
[15] Judea Pearl. 2009. Causality. Cambridge university press.
[16] Judea Pearl and James M Robins. 1995. Probabilistic evaluation of sequential plans from causal models with hidden variables. In UAI. Vol. 95. Citeseer, 444–453.
[17] Maya L Petersen, Sandra E Sinisi, and Mark J van der Laan. 2006. Estimation of direct causal effects. Epidemiology (2006), 276–284.
[18] Aashad Puli, Adler J Perotte, and Rajesh Ranganath. 2020. Causal Estimation with Functional Confounders. Advances in neural information processing systems 33 (2020), 5115.
[19] Yoshiko Raita, Carlos A Camargo Jr, Liming Liang, and Kohei Hasegawa. 2021. Big data, data science, and causal inference: A primer for clinicians. Frontiers in Medicine 8 (2021).
[20] James Robins. 1986. A new approach to causal inference in mortality studies with a sustained exposure period—application to control of the healthy worker survivor effect. Mathematical modelling 7, 9–12 (1986), 1393–1512.
[21] Paul R Rosenbaum and Donald B Rubin. 1983. The central role of the propensity score in observational studies for causal effects. Biometrika 70, 1 (1983), 41–55.
[22] James H Stock and Mark W Watson. 2018. Identification and estimation of dynamic causal effects in macroeconomics using external instruments. The Economic Journal 128, 610 (2018), 917–948.
[23] Zuzana Szkorupová. 2014. A causal relationship between foreign direct investment, economic growth and export for Slovakia. Procedia economics and finance 15 (2014), 123–128.