Models of strong wind acting on buildings and infrastructure

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Abstract. In the paper, the general concept of tornado is presented. A tornado wind distribution is analysed with use of modified Rankine, Burgers-Rott and Bjerkens vortex model. These models are very useful in the analysis of assessment of damages of buildings and infrastructure. Differences between these models are presented. The formation of the tornado and phenomena that appears during the tornado were presented. It is not easy to measure parameters of strong winds, especially tornado, in situ, due to fast movement and not predictable location, many researchers investigate the analytical models of strong wind, especially used for tornadoes. Rankine, Burgers-Rott and Bjerkens vortex models were presented in the paper.

1. Introduction
Since last years, the most attention has been focused on the phenomena and effects of strong winds. The results of such load consist in destroying building structures, their elements and infrastructure. Such objects are unsuitable for further exploitation, requiring demolition or extensive and costly repairs. Strong winds, including tornadoes, appear quite often in all continents except Antarctica. The largest number of tornadoes is found in the United States with the majority on the central and in the south-eastern states, e.g. Tornado Alley, in Europe and in Asia. They can occur throughout the year at any time of the day. Tornado is a rotating column characterized by small size, axial symmetry and short-term action. The area of action of tornadoes is usually smaller than e.g. earthquakes or hurricanes, but they appear more frequently and bring more deaths than earthquakes and hurricanes in total. Tornadoes are known as the most violent and destructive meteorological phenomena, because they cause huge damage to property and pose serious threats to life. Significant wind speed changes are accompanied by sudden changes in pressure. Tornadoes are very different from each other. The average return period is 1000 years.

2. Formation of tornado
During the formation of the tornado, the air column approaches the ground surface as a funnel of air, dust or debris. The whirl usually rotates around a circle at a speed of 30 m/s to 140 m/s. The diameter of tornadoes usually does not exceed 2 km. Some tornadoes may also contain secondary eddies also referred to as suction vortices, subvortices, multiple, and satellite vortices. There are some features, which characterize tornados, i.e. they exist as single cells or multiple cells (Figure 1). The reasons for the formation of tornadoes are the same and are related to the difference in space in temperature, air density and pressure.
Due to the short duration of action of tornados, rapid changes in the amplitude and direction of velocity, and changes in pressure, the conventional procedures for design of structures under wind pressure cannot be used.

The nature of wind tornado differs from the synoptic wind. Because direct speed measurement is very difficult, several empirical and theoretical models of tornadoes have been developed. During the last decades, there has been significant progress in the design of buildings and structures under the influence of the synoptic wind. Special requirements are set in the field of the safety of the objects particularly sensitive to strong winds, such as light engineering constructions of power transmission lines. Hence, that is the need to create design requirements for non-synoptic winds, in particular tornados.

Due to the extending scope of information and the serious effects of strong winds, the number of studies devoted to these phenomena is increasing, indicating the need to analyse models that more precisely describe non-linear effects [1-4].

First works concern the analytical models were investigated in [5-9]. Many analytical and experimental models are created that describe the tornadoes and the effects they cause. In the paper the general concept of tornado is presented. A tornado wind distribution is analysed with use of modified Rankine, Burgers-Rott and Bjerkens vortex model. Differences between these models are presented.

### 3. The aim of the research on strong winds

During the use of buildings and other structures, the strong winds act, that significantly exceeding the speed specified by code [10]. Due to the frequency of occurrence of tornadoes in the world, there are proposals for taking into account the tornado load on structures, from which the safety of ultimate and serviceable limit states. The general purpose of the researches is to describe the character of strong winds to predict the influence of the wind on buildings and infrastructure, to know how to act during the emergency. Despite the climate change, the general character of the strong wind, e.g. tornado is the same as it was in the past. That is why many researchers look for the historical data. Direct measurement of tornadoes parameters, its velocity is difficult; hence, information about the tornado velocity is often obtained from the assessment of damage of buildings, infrastructure and trees.

### 4. Models of strong wind

#### 4.1. Fluid flow

The model of incompressible density-uniform ideal fluid is used for analysis of vortex motion. Viscous liquids are described by Navier – Stokes equations [11]
\[
\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u} \cdot \nabla) \cdot \mathbf{u} = \mathbf{g} \frac{\nabla p}{\rho} + \nu \Delta \mathbf{u}
\]  

(1)

Where \( \nu \) is the coefficient of kinematic viscosity, \( \rho \) is the density of the air, \( t \) is the time, \( p \) is the static pressure and \( \mathbf{g} \) is the gravity and \( \mathbf{u} \) is the velocity. Equation of mass conservation has the following form:

\[
\frac{\partial \rho}{\partial t} + \text{div}(\rho \mathbf{u}) = 0
\]

(2)

After mathematical improvements equation of continuity:

\[
\text{div}(\rho \mathbf{u}) = 0
\]

(3)

can be performed in orthogonal coordinates as:

\[
\frac{\partial (L_2 L_3 u_1)}{\partial q_1} + \frac{\partial (L_2 L_3 u_2)}{\partial q_2} + \frac{\partial (L_1 L_2 u_3)}{\partial q_3} = 0
\]

(4)

where:

\[
L_k(q_1, q_2, q_3) = \sqrt{\left(\frac{\partial x}{\partial q_k}\right)^2 + \left(\frac{\partial y}{\partial q_k}\right)^2 + \left(\frac{\partial z}{\partial q_k}\right)^2}
\]

(5)

are the Lame coefficients and \( q_k \) are the curvilinear coordinates.

In the cylindrical coordinate system \( (r, \theta, z) \) shown in Figure 2., the Lame coefficients have the form: \( L_1 = 1, L_2 = r, L_3 = 1 \).

\[\text{Figure 2. Velocity components in cylindrical coordinates}\]

After substituting the Lame coefficients to the equations of motion (1) and equation of continuity (3), the following formula is received:

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial r} + W \frac{\partial U}{\partial z} - \frac{V^2}{r} = 0
\]

(6)
\[ \frac{\partial V}{\partial t} + U \frac{\partial V}{\partial r} + W \frac{\partial V}{\partial z} + UV = g_{\theta} - \frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \frac{\partial^2 V}{\partial r^2} + \frac{2}{r} \frac{\partial V}{\partial r} - \frac{V}{r^2} \right), \]  

(7)

\[ \frac{\partial W}{\partial t} + U \frac{\partial W}{\partial r} + W \frac{\partial W}{\partial z} + UV = g_z - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \Delta W, \]  

(8)

\[ \frac{\partial(rU)}{\partial r} + \frac{\partial V}{\partial \theta} + \frac{\partial(rW)}{\partial z} = 0. \]  

(9)

Where \( U, V, W \) are the radial, tangential and axial components of the velocity vector, and

\[ \Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2} \]  

(10)

4.2. Modified Rankine vortex model

In the modified Rankine vortex model [8] the flow field is one-dimensional, steady state and flow is inviscid. In the analysis, body forces are neglected.

The Rankine model is characterized by two areas: internal and external. In the inner region defined by \( r < r_{\text{max}} \), the tangential velocity increases linearly from the value of zero to the maximum value at the point with radius \( r_{\text{max}} \). It is assumed that the air in this area rotates like a solid. In the outer area, the tangential velocity decreases with the increase of the radius. The flow at this point is called potential flow because there are no vortices. The above assumptions reduce Navier-Stokes equations (6-9) to the equation:

\[ \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} + \frac{V}{r^2} = 0 \]  

(12)

Hence, the tangential velocity has the following form [8]:

\[ V = V_{\text{max}} \left( \frac{r}{r_{\text{max}}} \right)^\phi, \quad r \leq r_{\text{max}} \]  

(13)

\[ V = V_{\text{max}} \left( \frac{r_{\text{max}}}{r} \right)^\phi, \quad r \geq r_{\text{max}} \]  

(14)
\[ V_{\text{max}} = \frac{\Gamma_{\infty}}{2\pi r_{\text{max}}} \]  

where \( \varphi \) is decay index assumed according to Figure 3, \( \Gamma_{\infty} \) is the vortex circulation m²/s at infinity. Velocity distribution for modified Rankine model is presented in Figure 3.

### 4.3. Burgers-Rott vortex model

Burgers-Rott model [5-6] is a one-cell vortex model. The following assumption are introduced to the equations of Navier-Stokes (6-9):

\[ V = V(r), \quad W = W(z), \quad U = U(r), \quad p = p(r, z) \]  

The flow field is steady state, the viscosity is constant, body forces can be neglected. Equations (6-9) after the above arrangements have the form:

\[ \frac{U}{r} \frac{\partial U}{\partial r} - \frac{V^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left( \frac{\partial^2 U}{\partial r^2} + \frac{1}{r} \frac{\partial U}{\partial r} - \frac{U}{r^2} \right), \]  

\[ \frac{U}{r} \frac{\partial V}{\partial r} + \frac{U V}{r} = \nu \left( \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r} - \frac{V}{r^2} \right), \]  

\[ \frac{W}{z} \frac{\partial W}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \frac{\partial^2 W}{\partial z^2}, \]  

\[ \frac{\partial (rU)}{\partial r} + \frac{\partial (rW)}{\partial z} = 0. \]  

Solutions that fulfilled the equations 17-20 [5-6] are:

\[ U = -ar \]  

\[ V = \frac{\Gamma_{\infty}}{2\pi r} \left\{ 1 - \exp \left( -\frac{ar^2}{2\nu} \right) \right\} \]  

\[ W = 2az \]  

where \( a \) is the velocity gradient [1/s].

After differentiating the tangential velocity by the radius, the maximum tangential velocity can be found and the equation (16) can be written as:

\[ V = V_{\text{max}} \frac{1}{K_1} \frac{r_{\text{max}}^2}{r} \left\{ 1 - \exp \left( -K_1 \left( \frac{r}{r_{\text{max}}} \right)^2 \right) \right\} \]  

where \( K_1 \) is 1.26 and \( K_2 \) is 0.72.

### 4.4. Bjerknes vortex model

Bjerknes is the author of the model, which was created from the observation of winds in Scandinavia. Bjerknes noticed that the tornadoes' air inflow usually concentrated along the lines oriented in front of
the tornado path. The warm air in the part of the tornado is closed by the remaining air masses. The tornado is characterized by the highest intensity when the warm air mass rises towards the upper parts of the tornado and when the potential energy of the air mass is converted into the kinetic energy of the wind. Based on the Bjerknes model, in [12] the formula for tangential velocity using meteorological records obtained from some observatories along the path of the tornado that hit cities in Japan is presented:

\[
V = 2V_{\text{max}} \frac{\left( \frac{r}{r_{\text{max}}} \right)}{1 + \left( \frac{r}{r_{\text{max}}} \right)^2}
\]  

(25)

The distribution of tangential velocity of Rankine, Burgers – Rott and Bjerknes model are presented in Figure 3 for different distance from the centre of tornado. In some study, the rotational velocity is introduced with the components of radial and tangential velocity.

![Figure 3. Distribution of tangential velocity for analysed model](image1)

![Figure 4. Radial component of the velocity in Rankine model](image2)
The analysis presented in Figure 4 shows that in the range of $r > r_{\text{max}}$ the highest values of the dimensionless coefficient $V / V_{\text{max}}$ depending on the dimensionless distance from the tornado centre $r/r_{\text{max}}$ are in the Bjerknes model, while the smallest in the Rankine model for $\phi = 1.0$. The maximum and minimum values for both models differ almost twice (1.79). In the range of $r < r_{\text{max}}$, the values of the dimensionless $V / V_{\text{max}}$ coefficient in all models are similar. On the basis of the analysis of the dimensionless radial velocity coefficient (Figure 5.), for $\phi = 0.65$ the radial coefficient is higher by 1.65 times compared to $\phi = 1.0$. In both analyses concerning the Rankine model it should be stated that the smaller decay coefficient is, the higher the tangential and radial velocity coefficients are.

5. Conclusions

In the paper the description of strong winds that may act on building and infrastructure was presented with special attention to the tornado. The formation of the tornado and phenomena that appears during the tornado were presented. Because of difficulties in measurements of parameters of strong winds, especially tornado, in situ, due to fast movement and not predictable location, many researchers and administrative service investigate the analytical models of strong winds, especially used for tornadoes. These Rankine, Burgers-Rott and Bjerkess vortex models were presented in the paper.

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