Answering queries using pairings

Alberto Trombetta\textsuperscript{1}, Giuseppe Persiano\textsuperscript{2}, and Stefano Braghin\textsuperscript{3}

\textsuperscript{1} University of Insubria, DiSTA, Varese, Italy
\texttt{alberto.trombetta@uninsubria.it}

\textsuperscript{2} University of Salerno, DIA, Salerno, Italy
\texttt{giuper@dia.unisa.it}

\textsuperscript{3} IBM Research – Ireland, Smarter Cities Technology Centre, Dublin, Ireland
\texttt{stefanob@ie.ibm.com}

Abstract. Outsourcing data in the cloud has become nowadays very common. Since – generally speaking – cloud data storage and management providers cannot be fully trusted, mechanisms providing the confidentiality of the stored data are necessary. A possible solution is to encrypt all the data, but – of course – this poses serious problems about the effective usefulness of the stored data. In this work, we propose to apply a well-known attribute-based cryptographic scheme to cope with the problem of querying encrypted data. We have implemented the proposed scheme with a real-world, off-the-shelf RDBMS and we provide several experimental results showing the feasibility of our approach.

1 Introduction

The widespread and ever-growing deployment of cloud computing allows users to access a wide array of services, such as online data storage. The advantages in adopting such solutions is mitigated by the fact that outsourced data is not under the direct control of the legitimate owner, who has uploaded it to the storage service and one has to fully trust the service provider. As a typical solution, the data owner encrypts with a classical symmetric scheme its data before uploading it to the storage provider. This is not optimal in the case the storage manager – or some other party other than the data owner – has to perform some kind of computation over the encrypted data. For example, it is impossible to search for a given content over encrypted data, or, more specifically, in the case of encrypted structured data (such as relational data), it is not possible to perform expressive, SQL-like queries. Several solutions have been proposed that solve the problem of searching over encrypted data. However, most of such solutions are not well-tailored for performing searches over structured data, such as relational tables. We refer to Section 2 for a discussion.

Our motivating scenario. Consider a data owner $U$ that collects data coming from an information source (say, a sensor network) and stores them in a cloud-based database, managed by a (possibly untrusted) third party. We assume that the outsourced database is formed by a single table $T$. In the following we describe the security requirements related to the collection and querying of such
data. The information collected in the table $T$ is sensitive and should not be unconditionally disclosed. Henceforth, in this scenario the solutions typically adopted for ensuring the confidentiality (and then enforcing proper access policies to the data) do not apply. In fact, the DBMS is typically demanded to enforce the access control to the data stored in it, but – since in our case the DBMS could be untrusted – this solution is not viable. The first few rows of an instance of the table $T$ are as follows:

| ServiceId | TypeId | Availability | Certificate | Position  | Description       | Timestamp            |
|-----------|--------|--------------|-------------|-----------|--------------------|----------------------|
| 10        | 3      | yes          | cert1       | District2 | infrared camera   | 2014-09-07 12:45:34  |
| 23        | 3      | yes          | cert2       | District3 | temperature        | 2012-12-03 11:52:34  |
| 41        | 1      | yes          | cert3       | District3 | camera             | 2012-06-12 11:52:45  |
| 12        | 2      | no           | cert4       | District1 | camera             | 2012-04-07 14:33:28  |

Furthermore, the data should be accessed by the legitimate users that can pose standard, select-from-where SQL queries like:

$Q1$

```sql
select * from T where TypeId = 3 and Position = "District1";
```

$Q2$

```sql
select ServiceId, TypeId from T where Position = "District1" and Availability = "yes";
```

Fig. 1. Two conjunctive SFW queries

As an additional layer of security, the queries themselves are considered sensitive by the data owner $U$. Points (i), (ii), (iii) are conflicting and to find a practical and efficient solution satisfying all of them is a challenging task.

In this work, we propose an application-level database encryption technique, based on an attribute-based cryptographic scheme, for processing obfuscated selection-projection queries over an encrypted version of the table $T$, such that: (i) data owner $U$ stores an encrypted version of the table $T$ on a non-private (possibly untrusted) RDBMS; (ii) Each user $V$ entitled for running a query $Q$ over $T$ receives a corresponding decryption key $K_q$ encoding the query. That is, only users satisfying a given access policy may access data satisfying a corresponding query, and only such data. (iii) The data owner issues the decryption keys in an initial setup phase; (iv) The processing of the query is done by a trusted, separate query processor. Depending on architectural choices, such query processor may be under the control of end users, or managed by a trusted third party, detached from the rdbms server; (v) Data stored in the encrypted table may be modified by the owner $U$ without the need of re-encrypting the entire table, nor recomputing existing decryption keys. Note that in the proposed approach, access control is performed by the data owner by safely distributing to each user entitled to perform a query a corresponding decryption key. The encryption procedure is completely oblivious of the access control policy and this is a good feature since data can be encrypted even before potential readers with still to be defined access rights enroll into the system. Specifically, the addition of new readers does not force the data owner to modify the encryption of the current data. Given that $U$ wants the encrypted data to be searched by
queries like query $Q_1$ and $Q_2$ stated above, $U$ performs the following operations: it encrypts the tuples of table $T$ using an attribute-based encryption scheme that takes as additional input a vector $X$ containing (an encoding of) the values contained in the tuples. That is, there is a different vector $x$ for every different row; this has to be repeated as many times as there are attributes to be projected. In order to do this, we add a special attribute whose domain ranges over the indices encoding the positions of attributes; Afterwards, $U$ computes the corresponding secret restricted decryption keys, that depend on the values to be searched, encoded in a vector $Y$; since the attribute-based cryptographic scheme obfuscates the search values contained in the vector $Y$, it is referred to as hidden vector encryption (or HVE) scheme. The decryption keys are then distributed by $U$ (or some other trusted authority) to the users satisfying the corresponding access policy. This is the last time in which users interact. After the decryption keys’ distribution, the data owner can go offline. Note that in the scenario just described the query are entirely defined by the data owner $V$, that fixes – among other things – the values to be searched in the table $T$. However, in real-world database-backed applications, it is the case that the end user $V$ may specify the values to be searched filling empty placeholders in an a-priori defined query. We show that a very simple modification of the proposed scheme allows the data owner $U$ to issue a “parametric” token $K$ corresponding to a query with unspecified search values, that are to be provided by the user receiving such token. Note that the encryption depends on the data contained in the tuples and decryption keys only depend on the query. Note also that it is possible to adjoin or delete tuples from the database without the need to re-encrypt its entire content, but only the modified content has to be re-enciphered. We remark also that a single encryption of the database is sufficient for answering different obfuscated queries. The experimental results show that – quite expectedly – the space overhead for storing the encrypted data grows linearly with the number of columns of the cleartext database. This linear overhead holds for the execution time of an encrypted query, as well. The paper is structured as follows: in Section 2 are discussed the most relevant related works; in Section 3, a simplified version of the attribute-based encryption scheme of Okamoto and Takashima suitable for our context is presented; in Section 4 we show how to use the above mentioned scheme for executing obfuscated SQL queries over an encrypted database; in Section 5 the main functional components of an architecture implementing our approach are presented; in Section 6 the results of the experiments with our implementation are shown; finally, in Section 7 the conclusions are drawn, along with a description of future extensions we are already working on.

2 Related works

The ever-growing adoption of cloud-based data storage and management service has prompted several research efforts aiming at assuring an high security level of the stored data using cryptographic mechanisms that do not assume the service to be trusted. The problem of how to provide an adequate security level to the
stored data by deploying cryptographic techniques has been addressed by the database research community as well as the cryptographic research community, each of them focusing on different aspects that are relevant for the respective community. See, for example, the scenarios described in [1].

There are several works in the database literature that deal with encrypted databases. Seminal works like [2] and [3] have underlined the need to devise novel mechanisms that complement the usual access control approaches in order to achieve a greater level of security, by storing an encrypted version of the data, as well as the need to devise efficient indexing techniques that allow reasonable query performance over encrypted data. The advent of fast networks and cheap online storage has made viable the management of encrypted data at application-level. One of the first works to present the paradigm database-as-a-service is [4]. In [5] a database architecture based on a trusted hardware cryptographic module is presented. In [6], the authors propose a full-fledged system for performing SQL queries over an encrypted database. Compared to such works, our approach is far less complex – albeit with admittedly less search functionalities – while offering a strong degree of security. All the major commercial RDBMS releases provide functionalities to encrypt the data they store, see for example [7]. All commercial solutions are based on database-level encryption, thus limiting the functionalities over encrypted data, that have to be deciphered at server side in order to processing queries over them. Apart from encryption techniques to deal with large quantities of data that are to be managed and searched over, issues like key management and indexes over encrypted data have been addressed in [8], [9] are presented two short surveys of the major challenges that are relevant to the design of encrypted database as well as useful reference architectures. In the cryptographic (and more broadly, security) research community, the problem of how to query an an encrypted database has been viewed as a (very relevant) example of the broader problem of computing over encrypted data. As it is well known, the first general, fully homomorphic encryption scheme has been defined in [10]. However, it is too impractical to be applied to real-world scenarios and a very active research area in cryptography is to find Therefore, more specialized techniques have been proposed in order to solve the (less general) problem of searching over encrypted data. One of the first works addressing the problem is [11], in which the authors define a public key scheme that – given a search keyword – allows for the creation of a corresponding decryption key that tests whether such keyword is contained in the ciphertext. A subsequent work has introduced the notion of hidden vector encryption, in which it is possible to pose conjunctive and range queries over encrypted data in such a way that the query is itself obfuscated [12]. Among the many works based on homomorphic encryption, we mention [13], that uses a “limited” (yet more efficient) version of homomorphic encryption to query an encrypted repository. We point out though that homomorphic encryption is at the moment very inefficient and thus the approaches based on it cannot be considered practical. A similar approach to ours is described in [14], where the authors propose an attribute-based encryption scheme for keyword searching over unstructured textual data, along with
a mechanism for proving the truthfulness of the search results. An extensive amount of work has been done concerning how to access data without disclosing sensitive information using anonymous credentials see for example [15] and [16]. Finally, a related area is private information retrieval in which the goal is to preserve the privacy of queries. That is retrieve information from a database (typically represented as an unstructured sequence of bits) without letting the database know anything about the query [17]. While it is a very interesting approach from a theoretical point of view, its practical applicability is rather scarce and it only allows very simple queries (like retrieving a cell in a table) and does not enforce any control on what is accessed.

The approach presented in this work takes inspiration from attribute-based encryption schemes as defined, for example, in [18] or [12]. However, since such schemes are optimized for a binary alphabet and thus not very efficient over larger alphabets, such as those found in practice. The technical core of our contribution is a novel encryption scheme for HVE [19] that is based on the dual pairing vector space abstraction of [20] (which supports very large alphabets) and allows for an efficient amortized way of encrypting the rows of a table in such a way that selection-projection queries on encrypted data can be easily performed. A self-contained presentation of the cryptographic tool we develop is found in Section 3.

3 The cryptographic scheme constructions

In this section we describe our construction of HVE. We start by describing the dual pairing vector space framework (the DPVS framework) of [20].

3.1 Dual Pairing Vector Space

In the DPVS framework, we have an additive group \((\mathbb{G}, 0)\) and a multiplicative group \((\mathbb{G}_T, 1_T)\) of the same prime size \(q\) and a bilinear map \(e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T\). That is, for \(a, b, c \in \mathbb{G}\), we have

\[
e(a + b, c) = e(a, c) \cdot e(b, c) \quad \text{and} \quad e(a, b + c) = e(a, b) \cdot e(a, c)
\]

and

\[
e(0, a) = 1_T \quad \text{and} \quad e(a, 0) = 1_T.
\]

The above imply that for all \(s, t \in \mathbb{F}_q\) we have

\[
e(s \cdot a, t \cdot b) = e(a, b)^{st}.
\]

The bilinear map is extended to vectors over \(\mathbb{G}\) as follows. For two vectors \(X = (x_1, \ldots, x_n)\) and \(Y = (y_1, \ldots, y_n)\) over \(\mathbb{G}\), define

\[
e(X, Y) = \prod_{i=1}^{n} e(x_i, y_i).
\]
Note the abuse of notation by which we use $e$ to denote the bilinear map defined over $G$ and over the vector space over $G$. Also, we observe that the extended bilinear map $e$ is still bilinear in the sense that

$$e(X + Y, Z) = e(X, Z) \cdot e(Y, Z) \quad \text{and} \quad e(X, Z + Y) = e(X, Z) \cdot e(X, Y)$$

for all vectors $X, Y, Z$.

For a fixed $g \in G$, we define the canonical base $A_1, \ldots, A_n$ with respect to $g$ where, for $i = 1, \ldots, n$,

$$A_i = (0, \ldots, 0, g, 0, \ldots, 0).$$

Notice that

$$e(A_i, A_i) = e(g, g) \quad \text{and} \quad i \neq j \Rightarrow e(A_i, A_j) = 1_T.$$  

Let $B$ and $B^*$ be two $n \times n$ matrices with columns $B = (B_1, \ldots, B_n)$ and $B^* = (B_1^*, \ldots, B_n^*)$ and let $\psi \in \mathbb{F}_q$. We say that $(B, B^*)$ is a pair of $\psi$-orthogonal matrices if, for all $1 \leq i < j \leq n$,

$$e(B_i, B_i^*) = g_T^\psi \quad \text{and} \quad e(B_i, B_j^*) = 1_T,$$

where we set $g_T = e(g, g)$. A pair of $\psi$-orthogonal matrices can be constructed as follows. Let $X = (x_{i,j}), X^* = (x^*_{i,j}) \in \mathbb{F}_q^{n \times n}$ be matrices such that

$$X^T \cdot X^* = \psi \cdot I.$$  

for $i = 1, \ldots, n$, vectors $B_i$ and $B_i^*$ are defined as follows

$$B_i = \sum_{j=1}^{n} x_{i,j} A_j \quad \text{and} \quad B_i^* = \sum_{j=1}^{n} x^*_{i,j} A_j.$$  

For a vector $(x_1, \ldots, x_n) \in F_q$ and a matrix $B = (B_1, \ldots, B_n)$, we define the vector

$$(x_1, \ldots, x_n)_B = \sum_{i=1}^{n} x_i B_i.$$  

3.2 HVE in the DPVS framework

In this section we first define the concept of an HVE encryption scheme and then give an implementation using the DPVS framework.

The HVE function over $\mathbb{F}_q^\ell$ is defined as follows: for $X = (x_1, \ldots, x_\ell) \in \mathbb{F}_q^\ell$ and $Y = (y_1, \ldots, y_\ell) \in (\mathbb{F}_q \cup \{\star\})^\ell$, we define

$$\text{HVE}(X, Y) = 1 \quad \text{iff} \quad \forall i \ y_i \neq \star \Rightarrow x_i = y_i.$$
Definition 1. An HVE encryption scheme is a quadruple of algorithms HVE = (Setup, Encrypt, KeyGen, Decrypt) with the following syntax:

1. The Setup algorithm takes integers \( q \) and \( \ell \) and returns a master public key \( \text{MPK} \) and a master secret key \( \text{MSK} \).
2. The Encrypt algorithm takes a plaintext \( M \), an attribute vector \( X = (x_1, \ldots, x_\ell) \in \mathbb{F}_q^\ell \) and a master public key \( \text{MPK} \) and returns a ciphertext \( \text{Ct} \).
3. The KeyGen algorithm takes an attribute vector \( Y = (y_1, \ldots, y_\ell) \in (\mathbb{F}_q \cup \{\ast\})^\ell \) and master key \( \text{MSK} \) returns a key \( K \) for \( Y \).
4. The Decrypt algorithm takes a ciphertext \( \text{Ct} \) for plaintext \( m \) and an attribute vector \( X \) computed using master public key \( \text{MPK} \) and a key \( K \) for attribute vector \( Y \) computed using master secret key \( \text{MSK} \) and returns the value \( m \) iff \( \text{HVE}(X,Y) = 1 \).

We are now ready to describe our implementation \( \text{HVE} \) of an HVE encryption scheme. We assume that a DPSV framework \( (\mathbb{G}, \mathbb{G}_T, e, q) \) is given.

1. Setup(\( \ell \)). Randomly choose \( \psi \in \mathbb{F}_q \) and \( g \in \mathbb{G} \) and set \( g_T = e(g,g)^\psi \). For \( i = 0, \ldots, \ell \) generate a pair \( (B^i, C^i) \) of \( \psi \)-orthogonal 3 \( \times \) 3 matrices. Return \( \text{MPK} = (B_0, \ldots, B_\ell, g_T) \) and \( \text{MSK} = (C_0, \ldots, C_\ell) \).
2. Encrypt(\( m, X, \text{MPK} \)). We assume \( m \in \mathbb{G}_T \) and \( X = (x_1, \ldots, x_\ell) \in \mathbb{F}_q^\ell \). Randomly choose \( z, w_0, \ldots, w_\ell \in \mathbb{F}_q \), set \( c = g_T^z \cdot m \) and \( c_0 = (w_0, z, 0)_{B^0} \).

and, for \( t = 1, \ldots, \ell \), set \( c_t = (w_t, w_t \cdot x_t, w_0)_{B^t} \).

Return \( \text{Ct} = (c, c_0, c_1, \ldots, c_\ell) \).
3. KeyGen(\( Y, \text{MSK} \)). Assume \( Y = (y_1, \ldots, y_\ell) \in (\mathbb{F}_q \cup \{\ast\})^\ell \) and let \( S \) be the set of \( 1 \leq t \leq \ell \) such that \( y_t \neq \ast \).

Pick random \( \eta \in \mathbb{F}_q \) and, for \( t \in S \), pick random \( d_t, s_t \in \mathbb{F}_q \) and set \( s_0 = -\sum_{t \in S} s_t \). Set \( k_0 = (s_0, 1, \eta)_{C^0} \) and \( k_t = (d_t \cdot y_t, -d_t, s_t)_{C^t} \).

Return \( K = (k_0, (k_t)_{t \in S}) \).
4. Decrypt(\( K, \text{Ct} \)). Write \( K \) as \( K = (k_0, (k_t)_{t \in S}) \) and \( \text{Ct} \) as \( (c, c_0, \ldots, c_\ell) \). Return \( e(k_0, c_0) \cdot \prod_{t \in S} e(k_t, c_t) \).

Let us now show that our scheme is correct.

Suppose that \( \text{HVE}(X,Y) = 1 \). Then by the \( \psi \)-orthogonality of \( B^0 \) and \( C^0 \) we have \( e(k_0, c_0) = g_T^{s_0 \cdot w_0 + z} \).
Moreover, the $\psi$-orthogonality of $B^i$ and $c^i$ gives
\[ e(k_t, c_t) = g_T^{d_i \cdot w_i \cdot (x_t - y_t) + w_0 \cdot s_t}. \]

Therefore if $x_t = y_t$ for $t \in S$ we have
\[ \prod_{t \in S} e(k_t, c_t) = g_T^{w_0 \cdot \sum_{t \in S} s_t} = g_T^{-s_0 \cdot w_0} \]
which implies that the Decrypt algorithm returns $m$. On the other hand if $x_t \neq y_t$ for some $t \in S$ the Decrypt algorithm returns a random value in $\mathbb{G}_T$.

For security we observe that a ciphertext for plaintext $m$ with attribute vector $X$ does not reveal any information on $m$ and on attribute vector $X$. On the other hand, no security guarantee is made for a key $K$.

### 3.3 An amortized scheme

In our construction of secure database queries we will often have to encrypt $n$ messages $m^1, \ldots, m^n \in \mathbb{G}_T$ with closely related attributes. More specifically, message $m^j$ is encrypted with attributes $(x_1, \ldots, x_t, x_{t+1}^j) \in \mathbb{F}_q^{t+1}$; that is, the attributes of two messages coincide except for the $(t + 1)$-st. If we use the HVE implementation of the previous section, the sum of the sizes of the ciphertexts of all messages is $\Theta(\ell \cdot n)$. In this section we describe a scheme that reduces the size to $\Theta(\ell + n)$.

1. **Setup**. Same as Setup.
2. **Encr**($m^1, \ldots, m^n \in \mathbb{G}_T, x_1, \ldots, x_t, x_{t+1}^1, \ldots, x_{t+1}^n, \text{MPK}$).
   Randomly choose $z, w_0, \ldots, w_\ell \in \mathbb{F}_q$ and set
   \[ c_0 = (w_0, z, 0)_B^0 \quad \text{and} \quad c_t = (w_t, t \cdot x_t, w_0)_B^t \quad \text{for} \quad t = 1, \ldots, \ell. \]

   The encryption of the $j$-th message $m^j$ is computed as follows: Pick random $z^j, w^j, w_0^j \in \mathbb{F}_q$ and set
   \[ c^j = g_T^{z^j + z'} \cdot m^j \]
   and
   \[ c_0^j = (w_0^j, z^j, 0)_B^{0.\ell} \quad \text{and} \quad c_{t+1}^j = (w_t^j, w^j \cdot x_{t+1}^j, w_0^j)_B^{t+1}. \]

   The cumulative ciphertext consists of
   \[ \text{Ct} = (c_0, c_1, \ldots, c_t, (c_1^1, c_0^1, c_{t+1}^1), \ldots, (c_n^a, c_0^a, c_{t+1}^a)). \]

   The ciphertext corresponding to $m^j$ is
   \[ \text{Ct}^j = (c_0, c_1, \ldots, c_t, (c^j_1, c_0^j, c_{t+1}^j)). \]
3. KeyGen\textsuperscript{sm}(Y, MSK).

Write $Y$ as $Y = (y_1, \ldots, y_t, y_{t+1})$ and let $S$ be the set of $1 \leq t \leq \ell$ such that $y_t \neq \star$. We assume that $y_{t+1} \neq \star$ and we stress that $\ell + 1 \notin S$.

Randomly choose $\eta \in F_q$ and, for $t \in S$, randomly choose $d_t, s_t \in F_q$ and set $s_0 = -\sum_{t \in S} s_t$. Set

$$k_0 = (s_0, 1, \eta)_{C^0} \quad \text{and} \quad k_t = (d_t \cdot y_t, -d_t, s_t)_{C^t}.$$ 

Randomly choose $s_0^*, \eta^*, d_{\ell+1} \in F_q$ and output

$$k_0^* = (s_0^*, 1, \eta^*)_{C^0} \quad \text{and} \quad k_{\ell+1}^* = (d_{\ell+1} \cdot y_{\ell+1}, -d_{\ell+1}, -s_0^*)_{C^{\ell+1}}.$$ 

Return key $K$

$$K = (k_0, (k_t)_{t \in S}, k_0^*, k_{\ell+1}).$$

4. Decrypt\textsuperscript{sm}(K, Ct). Write $K = (k_0, (k_t)_{t \in S}, k_0^*, k_{\ell+1})$ and $Ct = (c_0, c_1, \ldots, c_\ell, (c_j, c_0^j, c_{\ell+1}^j)_{n=1}^n)$. Compute $m^j$ as

$$e(c_0^j, k_0^*) \cdot e(c_{\ell+1}^j, k_{\ell+1}^*) \cdot e(c_0, k_0) \cdot \prod_{t \in S} e(c_t, k_t).$$

For correctness, suppose $\text{HVE}(X, Y) = 1$. By the $\psi$-orthogonality we have

$$e(c_0, k_0) = g_T^{s_0 \cdot w_0 + z}.$$ 

and, for $t \in S$

$$e(c_t, k_t) = g_T^{s_t \cdot w_0}.$$ 

Therefore

$$\prod_{t \in S} e(c_t, k_t) = g_T^{\sum_{t \in S} s_t \cdot w_0} = g_T^{-s_0 \cdot w_0}.$$ 

and thus

$$e(c_0, k_0) \cdot \prod_{t \in S} e(c_t, k_t) = g_T^z.$$ 

Moreover we have

$$e(c_0^j, k_0^*) = g_T^{s_0^* \cdot w_0^j + z^j} \quad \text{and} \quad e(c_{\ell+1}^j, k_{\ell+1}) = g_T^{-s_0^* \cdot w_0^j}$$

and thus

$$e(c_0^j, k_0^*) \cdot e(c_{\ell+1}^j, k_{\ell+1}) = g_T^{z^j}.$$ 

4 Private queries

In this section we describe how a table is encrypted using the HVE scheme of Section 3. Let us start by fixing our notation.

We assume that data owner $U$ holds table $T$, composed of $l$ columns $A_1, \ldots, A_l$ and of $u$ rows $R^1, \ldots, R^u$ and we write the $i$-th row $R^i$ as $R^i = (v^1_i, \ldots, v^l_i)$. 


Encrypting a table. We assume that the data owner $U$ has selected a DPVS framework with groups of size $q$ and an authenticated private-key block cipher $ABC = (E,D)$ with key length $k < q$.

1. Generating the system parameters.
   $U$ runs algorithm $\text{Setup}^m(q, l)$ and obtains a pair of master public and secret key $(\text{MPK}, \text{MSK})$.

2. Encrypting row $R = \langle v_1, \ldots, v_l \rangle$.
   $U$ picks $l$ random keys $k_1, \ldots, k_l$ for $ABC$ and use them to encrypt $R$. Specifically, compute $\tilde{R} = \langle \tilde{v}_1, \ldots, \tilde{v}_l \rangle$, where $\tilde{v}_i = E(k_i, v_i)$ for $i = 1, \ldots, l$.
   Then $U$ encrypts keys $k_i$ using the HVE scheme. Specifically, for $i = 1, \ldots, l$,
   $$ \hat{k} = \text{Encrypt}^m(k_1, \ldots, k_l, v_1, \ldots, v_l, 1, \ldots, l, \text{MPK}). $$
   That is, key $k_i$ is encrypted with attribute vector $(v_1, \ldots, v_l, i)$.
   The encryption of row $R$ consists of the pair $(\tilde{R}, \hat{k})$ and the encrypted table is simply the sequence $\tilde{T} = (\langle \tilde{R}^1, \hat{k}^1 \rangle, \ldots, (\tilde{R}^u, \hat{k}^u))$ of the encrypted rows.

Generating a key for a selection-projection query. A typical selection-projection query (like $Q_2$ in Figure 1 "select Serviceld, TypeId from $T$ where Position = 'District1' and Availability = 'yes' ") is described by specifying the search values, along with “don’t care” entries $\star$, and the columns that are to be projected in the corresponding attribute vectors (one for every column to be projected).

More specifically, consider a query that specifies value $a_i$ for $i = 1, \ldots, l$ (it is possible that $a_i = \star$ for some values $i$ corresponding to don’t care entries in the query) and column $c$ to be projected. We assume without loss of generality that only one columns is to be projected; in general one computes a different key for each column to be projected. For example, a selection-only query, like query $Q_1$:"select * from $T$ where Typeld = 3 and Position = 'District1' ", is a special case of selection-projection query in which all the columns of the table are to be projected. The key for such a query is obtained by running the procedure described below with $c = 1, \ldots, l$.

The data owner $U$ releases key $K$ for the query $Q1$ by running $\text{KeyGen}^m(Y, \text{MSK})$ for attribute vector $Y = (a_1, \ldots, a_l, c)$ and using master secret key $\text{MSK}$ computed as part of the generation of the system parameter generation. The key $K$ is then sent to an user $V$ entitled to execute the query.

We point out that in the description above we have implicitly assumed that values $a_i$ are elements of the field $\mathbb{F}_q$ or $\star$. This does not hold in general as in most applications values $a_i$ are strings over an alphabet; thus we derive values $a_i$ by applying an hash function that for each string returns an element of $\mathbb{F}_q$.

Executing a selection-projection query. Upon receiving key $K$, the user $V$ applies $K$ to the encrypted table as follows. For $i = 1, \ldots, u$, row $i$ $(\tilde{R}^i, \hat{k}^i)$ of the encrypted table is used to compute $\hat{k}^i_c$ by selecting the $c$-th output of the
Decrypt^am algorithm on input $K$ and $\hat{k}^c_i$. Key $\hat{k}^c_i$ is then used to run the decryption algorithm $D$ for ABC to decrypt $\hat{v}^c_i$. If the decryption algorithm succeeds then it returns $v^c_i$. Otherwise the row is not selected.

Indeed, if the key attribute vector of $K$ matches the attribute vector used to obtain $\hat{k}^c_i$ then this means that row $i$ is to be selected and thus $\hat{k}^c_i = k^c_i$ and the decryption algorithm Decrypt^am does not fail. If instead the key attribute vector of the key $K$ does not match the attribute vector of the row (that is, the row is not to be selected) then $\hat{k}^c_i$ is a random element of $\mathbb{G}_T$ and thus with very high probability the decryption algorithm fails.

4.1 Parametric queries

In the previous sections, we have assumed that the data owner, in constructing the key $K$, completely determines the query that can be run over the encrypted database. That is, the values that are to be searched (and their corresponding columns) and the columns that are to be returned. However, in real-world, database-backed applications what usually happens is that the user asking for the query execution has the ability to fix by itself the values to be searched. Specifically, the data owner might want to be able to generate a key that allows to search a specified column (or set of columns) for values to be specified later. We remark that the way tokens are computed by the KeyGen procedure easily allows for a two-step computation of a token encoding a given query: namely, the first step is performed by the data owner which computes an intermediate, “parametric” decryption key; the second step then consists in specifying the parameters into the parametric decryption key thus obtaining a complete decryption key. More precisely, we observe that the computation of decryption key components $k_t$, for $1 \leq t \leq l$ (the ones that depend on the values to be searched (see Section 3) can be performed by the following modified version of the KeyGen^am:

KeyGen^am_par: The decryption key is computed by a two-step process: (i) for every decryption key $K$, for $t \in S$, pick random $s_t, d_t$ from $\mathbb{F}_q$ and set $\overline{k}_t = (d_t, -d_t, s_t) C_t$ ($k_0$ is defined as in KeyGen^am); (ii) multiply element-wise the vector $\overline{k}_t$ (which is composed by three elements) with the vector $(y_t, 1, 1)$, obtaining the component $k_t$ of the decryption key $K$.

We note that Steps (i) and (ii) may be performed by different entities. In fact, with respect to Query Q2, following Step 1 user $U$ computes the parametric decryption keys $\overline{K}_1, \overline{K}_2$ that correspond to the parametric query “select columns ServiceId and TypeId of all the rows of the table $T$ such that columns Availability and Position contain values ‘value1’ and ‘value2’ respectively”. A (possibly different) user $V$ then after having received $\overline{K}_1, \overline{K}_2$ from $U$, computes the decryption keys $K_1, K_2$ by specifying values $y_1, y_2$ corresponding to search values ‘yes’, ‘District3’. In other words the data owner $U$ can delegate restricted search capabilities to user $V$.

For the security we observe that the parametric key does not allow to generate keys other than the ones that can be obtained by specifying the values of the parameters.
5 Architectural Overview

The high-level overview of the architecture (as shown in Figure 2) assumes that there are users $V, V', \ldots$, a data owner $U$, and a server split in two components: a query processor $P$ hosted on a trusted, separate application server, and a (possibly untrusted) DBMS server $S$. The encryption and decryption operations are thus delegated outside the DBMS server (compare with, for example, with the application-level architecture in the taxonomy presented in [9]). For sake of simplicity, we assume that the data owner $U$ generates the public parameters, as well as the key material deployed in the execution of the cryptographic schemes described in the previous sections.

Also, we assume that the proxy query processor $P$ (whose task is to decrypt the (parts of the) rows having matching values with the encryption attribute vector) communicates with the other components via secure and authenticated channels. Confidentiality of both data and queries hold as long as the proxy $P$ and the server $S$ do not collude. We now illustrate the workflow occurring among the data owner, the trusted proxy query processor, the untrusted server and users who wish to pose queries to the encrypted database. We consider the non-amortized case, being the amortized one rather similar in the sequence of actions to be performed.

![Fig. 2. The high-level architecture](image)

The data owner $U$ runs the Setup procedure (see Section 3), producing the secret master key $C^0, \ldots, C^l$ and the public key $g_T = e(g, g)^{\psi}, B^0, \ldots, B^l$, $u$ randomly generates the secret ABC keys $k_1, \ldots, k_u$, as well. $U$ then encrypts the table in two steps: (i) rows are encrypted with symmetric scheme ABC and

---

4 As a more realistic setting, we may assume that the tokens are generated by users having proper credentials or by other trusted third parties that check such credentials.
(ii) the corresponding secret keys are encrypted with the public keys using the HVE-based Encrypt procedure. The encrypted database $\text{Enc}(T)$ (composed of the ABC-encrypted rows and the HVE-encrypted ABC keys and an additional column that stores a row counter) is uploaded in the untrusted DBMS server. Afterwards, given a query $Q$, $U$ proceeds in computing the corresponding token $tkQ$, using the secret master key and the the vector containing the values specified in the query $Q$. Finally, the decryption key $K_Q$ is sent to the query processor. At this point, the data owner $u$ may go offline. Upon receiving a request from user $V$ for executing query $Q$, The proxy query processor $P$ performs the following steps (see Figure 3):

(i) retrieve with a table scan from the encrypted database (stored in the servers $S$) the columns containing the row counter and the HVE-encrypted ABC keys (which we denote $I$ and $KHVE$, respectively) from the encrypted database;

(ii) execute the procedure $\text{Decrypt}$ with $K_Q$ on each HVE-encrypted ABC key in $KHVE$; for every successfully decrypted AES key, put the corresponding value of the row counter into $I_{\text{ok}}$; put the deciphered ABC keys in $KHVE_{\text{ok}}$;

(iii) execute the following SQL query over the encrypted database:

$$Q_{\text{ok}} = \text{select * from } E(T) \text{ where } I \in I_{\text{ok}}$$

(iv) retrieve the answer $\text{Ans}(Q_{\text{ok}})$;

(v) decipher the ABC-encrypted rows in $\text{Ans}(Q_{\text{ok}})$ with the keys in $KHVE_{\text{ok}}$;

(vi) send the deciphered rows to the issuer of query $Q$.
The case of parametric queries. Note that the workflow presented in the previous section requires that the query \( Q \) to be fully specified by the data owner. In particular, \( U \) knows the value(s) to be searched. In reality, what happens – as already explained in Section 4.1 – is that user \( V \) specifies the search values in a query \( Q \), provided by the data owner \( U \). The architecture just presented allows as well for the execution of a query following the steps specified in Section 4.1: the data owner \( U \) sends the parametric token \( \bar{tk}_Q \) to the proxy \( P \). When user \( V \) wants to execute the query \( Q \), it sends (along with a proper request) the vector \( Y \) (containing the values to be searched) to \( P \), that uses it to form the decryption key \( K_Q \).

6 Experimental evaluation

We now describe the experimental results of the application of the schema presented in Section 3 and 4. We implemented the schemes presented in Section 3 in Python, using the Charm library\(^5\) using the relational database SQLite\(^6\). Charm is a Python-based library that offers support for the rapid prototyping and experimentation of cryptographic schemes. In particular, we rely on the support offered by Charm for dealing with pairings, which is in turn based on the well-known C library PBC\(^7\). The implementation has been tested on a machine with a 2-core 2GHz Intel Core i7 processor with 8 GB 1600 MHz DDR3 RAM, running OS X 10.9.1.

Asymptotic complexity. Tables 1 and 2 show the complexities of the procedures presented respectively in Section 3.2 and Section 3.3. The operations we take into account in measuring time complexity are: pairings (denoted as \( P \)), exponentiations in \( G_T \) (denoted as \( EG_T \)), row products (denoted \( RW \), see Point 6 in Section 5). As for space complexity, we denote with \(|g|\) and \(|g_T|\) the size of elements in \( G \) and \( G_T \). Given a SWF query to be privately executed over an encrypted version of table \( T \), we remind that \( c \), \( t \) and \( l \) respectively denote the number of projected columns in the select clause, the number of columns in the table \( T \), specified in the from clause and the number of search predicates in the where clause.

| Function    | number of operations | output size |
|-------------|----------------------|-------------|
| Setup       | \( 1 \cdot P \)       | \( |g_T| + 18l \cdot |g| \) |
| Enc         | \( EG_T + (l + 1) \cdot RW \) | \( |g_T| + 10l \cdot |g| \) |
| KeyGen      | \( (t + 1) \cdot RW \) | \( 3(t + 1) \cdot |g| \) |
| Dec         | \( 3(t + 1) \cdot P \) |             |

Table 1. Asymptotic complexity of non-amortized scheme

\(^5\) http://www.charm-crypto.com/Main.html
\(^6\) http://www.sqlite.org
\(^7\) http://crypto.stanford.edu/pbc/
Table 2. Asymptotic complexity of amortized scheme

|  | number of operations | output size |
|---|----------------------|-------------|
| Setup | $1 \cdot P$ | $|g_T| + 18l \cdot |g|$ |
| Enc | $(l \cdot E_G + (l^2 + l) \cdot RW)$ | $|g_T| + 13l \cdot |g|$ |
| KeyGen | $e(t + 1) \cdot RW$ | $3(t + 3) \cdot |g|$ |
| Dec | $3e(t + 1) \cdot P$ | |

Experiments with real data. We have executed several tests on datasets of growing sizes, deploying different curve parameters. More precisely, we have generated three synthetic relational tables of size 234, 554, 463, 999 and 934, 347 bytes respectively; we have encrypted them at row level with AES and subsequently we have encrypted the corresponding 256-bit AES keys with HVE-IP using as curves parameters MNT159 ($G_1$, $G_2$, $G_T$ elements’ bitsizes are respectively 159, 477, 954), SS512 ($G_1$, $G_2$, $G_T$ elements’ bitsizes are respectively 512, 512, 1024) and MNT224 (with $G_1$, $G_2$, $G_T$ elements’ bitsizes of 224, 672, 1344) MNT159 parameters have a security level equivalent to 954-bit DLOG, while SS512 and MNT have security levels equivalent respectively to 1024 and 1344-bit DLOG. In Figure 4 are shown the execution times of encrypted queries corresponding, respectively, to the following SQL queries with search conditions of increasing length:

(i) select * from $T$ where ServiceId = 42;
(ii) select * from $T$ where ServiceId = 42 and TypId = 3;
(iii) select * from $T$ where ServiceId = 42 and TypId = 3 and Availability = ‘no’;

As it is expected, for a given row in the cleartext table, the encryption execution time linearly depends on the number of table columns and the major time cost is due to the computation of pairings. The queries have been performed on the largest database. We remind that an encrypted query execution is composed of the following steps (see Section 5): (i) retrieve from the encrypted database the rows index and the the hve-ip-encrypted aes keys, (ii) for every retrieved encrypted key, run the decryption procedure with the attribute vector encoding the query and, for each key that has a successful match, store the corresponding row index in the index result set $I_{ok}$, (iii) retrieve from the encrypted database the aes- encrypted rows whose row indexes are contained in $I_{ok}$, (iv) finally, decipher the rows with the corresponding aes keys that have been successfully decrypted in Step (iii). In Figure 5 the space overhead of the encrypted database is shown, resulting from the AES encryption of the data stored in the plaintext database and the subsequent encryption of the corresponding AES keys using HVE-IP. Again as expected, the expansion factor linearly depends on the number of columns employed by the HVE-IP encryption procedure. With respect to the datasets used in the experiments, the expansion factor of the hve-ip encryption is roughly 5.5. Such factor is directly proportional on the number of columns in the cleartext database schema – as already pointed out in Table 2

\footnote{In this case, the pairing is asymmetric}
– and inversely proportional to the size of the cleartext database. This is due, of course, to the fact that values in the cleartext database (which can be of arbitrary size) are mapped to $\mathbb{F}_q$ elements (which are of fixed size). As a rule of thumb for decreasing the size of the encrypted database, one should limit the columns on which the encryption depends only on the ones that are actually involved in the search predicates.

![Fig. 4. Queries execution time](image)

![Fig. 5. Encrypted database space overhead](image)

### 7 Conclusions

In this work we have presented an attribute-based encryption scheme based on the DPVS framework and we have used it to define a system for privately
query with standard SQL conjunctive queries an encrypted, off-the-shelf, non-private relational database without the need to decipher the stored data during the execution of the intermediate query processing steps. We have implemented our scheme using a standard relational RDBMS, and we provide experimental results that show the feasibility of our approach. Regarding the extensions of the present work, we are currently finishing a full-fledged implementation in C in a client-server architecture, that will provide us more realistic experimental results, also, we are currently working on privately executing (i) join queries and (ii) aggregate queries on encrypted tables. Also, we are investigating ways for deploying indexes in order to avoid full table scans; also we plan to add mechanisms for verifying whether the query processor has faithfully executed the query.

References

1. Samarati, P., di Vimercati, S.D.C.: Data protection in outsourcing scenarios: issues and directions. In: Proceedings of the 5th ACM Symposium on Information, Computer and Communications Security, (ASIACCS), Beijing, China. (2010) 1–14
2. Davida, G.I., Wells, D.L., Kam, J.B.: A database encryption system with subkeys. ACM Trans. Database Syst. 6(2) (1981) 312–328
3. Bayer, R., Metzger, J.K.: On the encipherment of search trees and random access files. ACM Trans. Database Syst. 1(1) (1976) 37–52
4. Hacigümüş, H., Iyer, B.R., Li, C., Mehrotra, S.: Executing sql over encrypted data in the database-service-provider model. In: Proceedings of the ACM SIGMOD Conference on Management of Data. (2002) 216–227
5. Bajaj, S., Sion, R.: Trusteddb: A trusted hardware based outsourced database engine. PVLDB 4(12) (2011) 1359–1362
6. Popa, R.A., Redfield, C.M.S., Zeldovich, N., Balakrishnan, H.: Cryptdb: protecting confidentiality with encrypted query processing. In: Proceedings of the 23rd ACM Symposium on Operating Systems Principles (SOSP). (2011) 85–100
7. Corp., O.: Oracle advances security transparent data encryption best practices. white paper (2012)
8. Shmueli, E., Vaisenberg, R., Elovici, Y., Glezer, C.: Database encryption: an overview of contemporary challenges and design considerations. SIGMOD Record 38(3) (2009) 29–34
9. Bouganim, L., Guo, Y.: Database encryption. In: Encyclopedia of Cryptography and Security (2nd Ed.). (2011) 307–312
10. Gentry, C.: Fully homomorphic encryption using ideal lattices. In: Proceedings of the 41st Annual ACM Symposium on Theory of Computing, STOC 2009, Bethesda, MD, USA. (2009) 169–178
11. Boneh, D., Crescenzo, G.D., Ostrovsky, R., Persiano, G.: Public key encryption with keyword search. In: Proceedings of the Inteerational Conference on the Theory and Applications of Cryptographic Techniques (EUROCRYPT). (2004) 506–522
12. Boneh, D., Waters, B.: Conjunctive, subset, and range queries on encrypted data. In: Proceedings of the 4th Theory of Cryptography Conference (TCC). (2007) 535–554
13. Boneh, D., Gentry, C., Halevi, S., Wang, F., Wu, D.J.: Private database queries using somewhat homomorphic encryption. In: Applied Cryptography and Network Security - 11th International Conference, ACNS 2013, Banff, AB, Canada, Proceedings. (2013) 102–118
14. Zheng, Q., Xu, S., Ateniese, G.: Vabks: Verifiable attribute-based keyword search over outsourced encrypted data. IACR Cryptology ePrint Archive 2013 (2013)
15. Bangerter, E., Camenisch, J., Lysyanskaya, A.: A cryptographic framework for the controlled release of certified data. In: Proceedings of the 12th International Workshop on Security Protocols. (2004) 20–42
16. Camenisch, J., Dubovitskaya, M., Lehmann, A., Neven, G., Paquin, C., Preiss, F.S.: Concepts and languages for privacy-preserving attribute-based authentication. In: Policies and Research in Identity Management - Third IFIP WG 11.6 Working Conference, IDMAN 2013. Proceedings. (2013) 34–52
17. Gasarch, W.I.: A survey on private information retrieval (column: Computational complexity). Bulletin of the EATCS 82 (2004) 72–107
18. Iovino, V., Persiano, G.: Hidden-vector encryption with groups of prime order. In: Proceedings of the 2nd International Conference on Pairing-based Cryptography (Pairing). (2008) 75–88
19. Katz, J., Sahai, A., Waters, B.: Predicate encryption supporting disjunctions, polynomial equations, and inner products. J. Cryptology 26(2) (2013) 191–224
20. Okamoto, T., Takashima, K.: Adaptively attribute-hiding (hierarchical) inner product encryption. In: Proceedings of the International Conference on the Theory and Applications of Cryptographic Techniques (EUROCRYPT). (2012) 591–608