The Pulsation Properties of Ultra-Massive DA White Dwarf Stars with ONe cores

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ABSTRACT

Context. Ultra-massive hydrogen-rich white dwarfs stars are expected to harbour oxygen/neon cores resulting from the progenitor evolution through the super asymptotic giant branch phase. As evolution proceeds during the white dwarf cooling phase, crystalization process resulting from Coulomb interactions in very dense plasmas is expected to occur, leading to the formation of a highly crystallized core. In particular, pulsating ultra-massive white dwarfs offer an unique opportunity to infer and test the occurrence of crystallization in white dwarfs interiors as well as physical processes related with dense plasmas.

Aims. This paper is aimed to assess the adiabatic pulsation properties of ultra-massive hydrogen-rich white dwarfs with oxygen/neon cores.

Methods. We study the pulsation properties of ultra-massive hydrogen-rich white dwarf stars with oxygen/neon cores. We employ a new set of ultra-massive white dwarf evolutionary sequences of models with stellar masses in the range $1.10 \leq M / M_\odot \leq 1.29$ computed taking into account the complete evolution of the progenitor stars and the white dwarf stage. During the white dwarf cooling phase, we consider element diffusion. When crystalization sets on in our models, we take into account latent heat release and also the expected changes in the core chemical composition due to phase separation according to a phase diagram suitable for oxygen and neon plasmas. We compute nonradial pulsation $g$ modes of our sequences of models at the ZZ Ceti phase by taking into account the presence of a solid core. We explore the impact of crystallization on their pulsation properties, in particular the structure of the period spectrum and the distribution of the period spacings.

Results. We find that the chemical rehomogeneization due to phase separation during crystallization leaves clear imprints in the pulsation spectrum, particularly strong features in the diagrams of period spacing versus periods. We also find that it is not possible, in principle, to discern whether a white dwarf has a nucleus made of carbon and oxygen or a nucleus of oxygen and neon by simply studying the spacing between periods.

Conclusions. The features found in the period-spacing diagrams could be used as a seismological tool to study the processes that occur during crystallization in white dwarfs. In addition, we found that the only way to discern the core composition is, probably, through detailed asteroseismic analysis using the individual periods observed in ZZ Ceti stars.

Key words. stars — pulsations — stars: interiors — stars: evolution — stars: white dwarfs

1. Introduction

White dwarf (WD) stars are the final stage of evolution for low- and intermediate-mass stars. Most of these stars are post-Asymptotic Giant Branch (AGB) remnants with carbon-oxygen cores and stellar masses $M_{\text{wd}} \sim 0.6 M_\odot$. Mass distribution also exhibits a tail of massive WDs peaking at $\sim 0.82 M_\odot$. The existence of massive WDs with $M_{\text{wd}} \gtrsim 0.8 M_\odot$ (Castanheira et al. 2010, 2013) and ultra-massive WDs with $M_{\text{wd}} \gtrsim 1.15 M_\odot$ (Hermes et al. 2013; Curd et al. 2017) is also revealed.

Theoretical studies suggest that progenitor stars initially more massive than $\sim 7 M_\odot$ are expected to evolve through the Super AGB (S-AGB) phase, igniting carbon in their interiors and eventually resulting in the formation of oxygen and neon (ONe) WDs with masses $M_{\text{wd}} \gtrsim 1.05 M_\odot$ (García-Berro et al. 1997). Specifically, oxygen is ignited in semi-degenerate conditions, leading to repeated carbon burning shell flashes that propagate inwards, turning the CO core into ONe mixture (García-Berro & Iben 1994; Siess 2006). After carbon is exhausted in the core, and as a result of mass loss, the remnant evolves towards the WD domain, and a WD is formed with a core made mostly of $^{16}$O and $^{20}$Ne with traces of $^{23}$Na and $^{24}$Mg (Siess 2007).

As is well known, many WDs exhibit multi-periodic luminosity variations caused by pulsations. In particular, variable H-rich (DA) WDs, called ZZ Ceti (or DAV), are the most numerous class of compact pulsators. These stars show variations in their luminosity in a narrow instability strip $10,500 \leq T_{\text{eff}} \leq 12,300$ K (Fontaine & Brassard 2008; Winget & Keplinger 2008; Althaus et al. 2010b) with photometric variations up to 0.3 mag. These variations are the result of spheroidal non-radial $g$-mode pulsations with low harmonic degree ($\ell \leq 2$) and periods in the range $70-1500$ s. In particular, the first attempt to study the adiabatic pulsational properties of ultra-massive ONe-core WDs was done by Córscico et al. (2004), who demonstrated that pulsational expectations such as the forward and mean period spacing of ONe WDs are markedly different from those of CO WDs.
One interesting point studying ultra-massive pulsating DA WDs lies in the fact that these stars are expected to harbour a crystallized core resulting from Coulomb interactions in very dense plasmas. Although the occurrence of crystallization in WDs was theoretically suggested several decades ago by Kirzhnits (1960); Abrikosov (1961); Salpeter (1961); van Horn (1968) (see also recent studies by Montgomery & Winget 1999; Metcalfe et al. 2003; Corsico et al. 2005; Brassard & Fontaine 2005) it was not until quite recently that the existence of crystallized WDs was inferred from the study of WD luminosity function of stellar clusters (Winget et al. 2003; Garcia-Berro et al. 2010). The fact that ultra-massive pulsating DA WDs are expected to be crystallized, turns these stars into unique objects to also infer the occurrence of crystallization in WD interiors. The first star studied in this sense was the ultra-massive ZZ Ceti star BPM 37093 (Kanaan et al. 1992; 2005) expected to have a partially crystallized core (Metcalfe et al. 2004; Brassard & Fontaine 2005).

This paper is aimed to assess the adiabatic pulsation properties of ultra-massive H-rich WDs with ONe cores on the basis of full evolutionary models that incorporate the most updated physical ingredients governing the progenitor and WD evolution. This investigation constitutes a substantial improvement over the study done in Corsico et al. (2004) in three major aspects: first, the chemical profiles for all of our WD models are consistent with predictions of the progenitor evolution with stellar masses in the range 9 \textless M_{ZAMS}/M_⊙ \textless 10.5 from the zero age main sequence (ZAMS) to the end of the the S-AGB phase (Siess 2010). Thus, not only a realistic ONe inner distribution expected for each WD mass is considered in the WD modeling, but also realistic chemical profiles and intershell masses built up during the S-AGB are taken into account. Second, we take into account for the first time the changes in the core chemical composition resulting from phase separation as WDs crystallize. To this end, we consider phase diagrams suitable for 16O and 20Ne plasmas (Medin & Cumming 2010). We will show that the changes in the chemical profiles resulting from phase separation processes leave strong signatures in the theoretical pulsational spectrum and must be taken into account in realistic computations of the pulsational properties of ultra-massive WDs. Finally, element diffusion was included for all model sequences, from the beginning of the WD cooling track. Element diffusion smooths the inner chemical profiles, thus altering the run of the Brunt-Väisälä frequency, and hence the period spectrum and mode trapping properties. The pulsational study presented in this work is based on evolving models of ultra-massive WDs recently computed in Camisassa et al. (2018, submitted). These evolutionary sequences, that incorporate all the above mentioned improvements, were computed from the very beginning of the cooling track down to very low surface luminosities.

The DA WD evolutionary models developed in this work were computed with the LPCODE evolutionary code, (see Althaus et al. 2005, 2010a; Renedo et al. 2010; Romero et al. 2005, Miller Bertolami 2016, for detailed physical description). This numerical tool has been employed to study various aspects of the evolution of low-mass stars (Wachlin et al. 2011; Althaus et al. 2013, 2015), formation of horizontal branch stars (Miller Bertolami et al. 2008), extremely low-mass WDs (Althaus et al. 2013), AGB, and post-AGB evolution (Miller Bertolami 2016) among other. More recently the code has been used to assess the impact of the uncertainties in progenitor evolution on the pulsation inferences of ZZ Ceti stars (De Geronimo et al. 2017, 2018). Next we describe the main input physics of the code relevant for the computation of the ultra-massive WD models: 1) convection is treated according to the mixing length formulation (ML2, Tassoul et al. 1990); 2) radiative and conductive opacities are those from OPAL (Iglesias & Rogers 1996) and Cassisi et al. (2007) respectively; 3) molecular radiative opacities for low-temperature regime are those from Ferguson et al. (2005); 4) the equation of state for low-density regime is taken from Magni & Mazzitelli (1979) while for high-density regime from Segretain et al. (1994), which accounts for both the solid and liquid phases; 5) element diffusion, including gravitational settling and chemical and thermal diffusion is considered; 6) energy release from crystallization –latent heat and gravitational energy associated to ONe phase separation– are included, by considering phase diagram suitable for the dense ONe interiors of ultra-massive WD (Medin & Cumming 2010). To our knowledge, these are the first pulsatational analysis of ultra-massive WD models that includes phase separation processes in ONe cores.

2.2. Treatment of crystallization and phase separation

Cool WD stars are expected to crystallize as a result of strong Coulomb interactions in their very dense interior (van Horn 1968). Crystallization sets in when the ion coupling constant $\Gamma \equiv (Z^{2/3}) e^2/\alpha a_B k_B T$ is larger than a certain value, which depends on the adopted phase diagram. Here $a_i$ is the interelectronic distance, $(Z^{2/3})$ an average (by number) over the ion charges, and $a_B$ Boltzmann’s constant. The other symbols have their usual meaning. The occurrence of crystallization leads to two additional energy sources: the release of latent heat and the release of gravitational energy associated with changes in the chemical composition profile induced by crystallization (Garcia-Berro et al. 1988; Winget et al. 2009). In our study, the inclusion of these two additional energy sources is done self-consistently and locally coupled to the full set of equations of stellar evolution. In particular, the luminosity equation is appropriately modified to account for both the local contribution of energy released from the core chemical redistribution and the latent heat. At each timestep, the crystallization temperature and the change of the chemical profile resulting from phase separation are computed using the appropriate phase diagram. In particular, the oxygen-enhanced convectively unstable liquid layers overlying the crystallizing core are assumed to be instantaneously mixed, a reasonable assumption considering the long evolutionary timescales of WDs. The chemical redistribution due to phase separation and
the associated release of energy have been considered following the procedure described in Althaus et al. (2010a) appropriately modified by Camisassa et al. (2018) for ONe plasmas. To assess the enhancement of $^{20}\text{Ne}$ in the crystallized core we used the azetropic-type phase diagram of Medin & Cumming (2010).

After computing the chemical composition of both the solid and the liquid phases, we evaluated the net energy released in the process as in Althaus et al. (2010c), see also Isern et al. (1997).

\[ \Pi = \frac{\partial \ln P}{\partial \ln \rho} \bigg|_{\rho, \chi} = \frac{\partial \ln T}{\partial \ln \rho} \bigg|_{\rho, \chi}. \]

2.3. Pulsation Code

We computed non-radial g-mode pulsations of our complete set of ultra-massive ONe-core DA WD models using the adiabatic version of the LP-PUL pulsation code described in Córsico & Althaus (2006). The pulsation code is based on the general Newton-Raphson technique that solves the full fourth order set of equations and boundary conditions governing linear, spheroidal, adiabatic, non-radial stellar pulsations following the dimensionless formulation of Dziembowski (1971). We have not considered torsional modes, since these modes are characterized by very short periods (up to 20 s; see Montgomery & Winget 1999), and are not observed in ZZ Ceti stars. To account for the effects of crystallization on the pulsation spectrum of $g$ modes, we adopted the “hard sphere” boundary conditions, which assume that the amplitude of the eigenfunctions of $g$ modes is drastically reduced below the solid/liquid interface due to the non-shear modulus of the solid, as compared with the amplitude in the fluid region (see Montgomery & Winget 1999). In our code, the inner boundary condition is not the stellar center but instead the mesh-point corresponding to the crystallization front moving toward the surface (see Córsico et al. 2004, 2005; Romero et al. 2013). Specifically, the hard-sphere boundary condition at the radial shell corresponding to the outward-moving crystallization front (r$_c$ = r(M$_c$)) reads:

\[ y_1 = 0, \quad y_2 = \text{arbitrary}, \quad \ell y_3 - y_4 = 0. \]

Here $y_1$ and $y_2$ represent the radial and horizontal displacements, respectively, and $y_3$ and $y_4$ are the Eulerian perturbation of the gravitational potential and its derivative. Note that the last condition is the same as for the normal case in which the core is in a fluid state and the boundary condition is applied at the stellar centre – see Appendix B of Montgomery & Winget (1999).

The asymptotic period spacing is computed as in Tassoul et al. (1990). For $g$ modes with high radial order $k$ (long periods), the separation of consecutive periods ($|\Delta k| = 1$) becomes nearly constant at a value given by the asymptotic theory of non-radial stellar pulsations. Specifically, the asymptotic period spacing is given by:

\[ \Delta \Pi_\ell^k = \Pi_0 \sqrt{\ell (\ell + 1)}, \]

where

\[ \Pi_0 = 2\pi \left[ \int_{r_1}^{r_2} N \frac{d r}{r} \right]^{-1}. \]

The squared Brunt-Väisälä frequency ($N$, one of the critical frequencies of non-radial stellar pulsations) is computed as

\[ N^2 = \frac{\rho^2 \chi_T \nabla_{ad} - \nabla + B}{P X_\rho}, \]

where the compressibilities are defined as Tassoul et al. (1990); Brassard et al. (1991):

\[ \chi_p = \frac{\partial \ln P}{\partial \ln \rho} \bigg|_{T, \chi} \quad \chi_T = \frac{\partial \ln T}{\partial \ln \rho} \bigg|_{\rho, \chi}. \]

The Ledoux term $B$ is computed as

\[ B = -\frac{1}{\chi_T} \sum_{i=1}^{M-1} \chi_i \frac{\partial \ln \rho_i}{\partial \ln P} \bigg|_{\rho, T, \chi_i}. \]

The computation of the Ledoux term $B$ has been generalized in order to include the effects of having multiple chemical species ($^1\text{H}$, $^4\text{He}$, $^{12}\text{C}$, $^{16}\text{O}$, $^{20}\text{Ne}$, $^{23}\text{Na}$, $^{24}\text{Mg}$) varying in abundance.

When a fraction of the WD core is crystallized, the lower limit of the integral in Eq. (3) coincides with the radius of the crystallization front ($r_1 = r_c$), which is moving outward as the star cools down, and the fraction of crystallized mass increases. Hence, the integral in Eq. (5) decreases, leading to an increase in the asymptotic period spacing (Eq. 2), and also in the periods themselves.

3. Evolutionary models

We computed the evolution and pulsation properties of four ultra-massive WD sequences with stellar masses $M_\ast =$ 1.10, 1.16, 1.22, and 1.29 $M_\odot$ resulting from the complete evolution of the progenitor stars through the S-AGB phase. The core and intershell chemical profiles of our models at the start of the WD cooling phase were obtained from Siess (2010). The cores are composed mostly of $^{16}\text{O}$ and $^{20}\text{Ne}$ and smaller amounts of $^{12}\text{C}$, $^{23}\text{Na}$ and $^{24}\text{Mg}$. Since element diffusion and gravitational settling operate all along the WD evolution, our models develop pure hydrogen envelopes. In Table 1 we show the $^1\text{H}$ and $^4\text{He}$ mass content for each sequence, as well as the $T_{\text{eff}}$ and log $g$ values corresponding to the onset of crystallization, and the fraction of crystallized mass at the boundaries of the ZZ Ceti instability strip. The He content of our WD sequences is given by the evolutionary history of progenitor star, but instead, the H content (M$_{\text{He}} \sim 10^{-6} M_\ast$) has been set by imposing that the further evolution does not lead to H thermonuclear flashes on the WD cooling track. The higher mass model sequences start to crystallize at effective temperatures well above the instability strip, thus harboring a core almost completely crystallized at the time the sequences reach the ZZ Ceti stage. As consequence of element diffusion, the chemical profiles at low $T_{\text{eff}}$ are strongly smooth as compared with those at the beginning of the WD stage (see Figs. 8, 9 of Camisassa et al. 2018). In particular, the He buffer, which is located above the ONe core, is strongly eroded, therefore developing a region rich in $^3\text{He}$, $^{12}\text{C}$, $^{16}\text{O}$ and $^{20}\text{Ne}$.

3.1. Crystallization and chemical redistribution due to phase separation

Theoretical evidence suggests that if the core of a WD is composed initially of a mixture of $^{16}\text{O}$ and $^{20}\text{Ne}$ (Medin & Cumming...
In our scheme for mixing, we first consider a crystallized, azeotropic-type phase diagram for a given layer crystallizes, we adopt the Medin & Cumming (2005). To derive the chemical profile is strongly modified, even in regions far from the crystallization front. In particular, at the end of the rehomogenized region a very pronounced step is formed in the chemical profiles, which, as we will show later, has important consequences on the pulsation properties of these stars.

Table 1. He mass content of our ONe-core ultra-massive DA WD models, together with the effective temperature and surface gravity at the onset of crystallization, and the fraction of crystallized mass at the blue and red edges of the ZZ Ceti instability strip.

| $M_\star/M_\odot$ | $M_{\text{He}}/M_\star$ | $T_{\text{eff}}$ | $\log g^e$ | $M_\star/M_\odot$ | $M_\star/M_\odot$ |
|-------------------|------------------------|----------------|----------|-------------------|-----------------|
| 1.998             | 29.6                   | 19881          | 8.83     | 0.81              | 0.92            |
| 1.159             | 15.7                   | 23291          | 8.95     | 0.90              | 0.96            |
| 1.226             | 6.38                   | 28425          | 9.12     | 0.96              | 0.98            |
| 1.292             | 1.66                   | 37309          | 9.33     | 0.994             | 0.998           |

Fig. 1. Internal chemical profiles of $^{16}$O (solid lines) and $^{20}$Ne (dashed lines) in terms of the fractional mass for the 1.29$M_\odot$ ONe-core WD sequence corresponding to various percentages of crystallization. The thick black lines correspond to the profiles just before the onset of crystallization, and the thin lines of different colors correspond to the chemical profiles resulting from chemical rehomogenization for different effective temperatures at increasing percentages of crystallized mass fraction, as indicated for some selected cases. The thick red curves correspond to a model with $T_{\text{eff}} = 4046$ K and a percentage of 99.99 % of crystallized mass.

We show in Figs. 2, 3, 4, and 5 the chemical abundances by mass of $^1$H, $^4$He, $^{12}$C, $^{16}$O, $^{20}$Ne, $^{23}$Na, and $^{24}$Mg (upper panel), through the fluid and the process stops when further mixing no longer decreases the $^{20}$Ne content of the fluid between this point and the crystallization boundary. When the crystallization front moves outward due to cooling, the aforementioned procedure is repeated.

In Fig. 1 we show the $^{16}$O and $^{20}$Ne chemical profiles (solid and dashed lines, respectively) in terms of the fractional mass for the 1.29$M_\odot$ ONe-core WD sequence corresponding to various degrees of crystallization. We show with thick black lines the profile just before the onset of crystallization, and with thin lines of different colors the chemical profiles resulting from chemical rehomogenization for different effective temperatures at increasing percentages of crystallized mass fraction, indicated for some selected cases. The thick red curves correspond to a model with $T_{\text{eff}} = 4046$ K and a percentage of 99.99 % of crystallized mass. Note that the shape of the chemical profile is strongly modified, even in regions far from the crystallization front. In particular, at the end of the rehomogenized region a very pronounced step is formed in the chemical profiles, which, as we will show later, has important consequences on the pulsation properties of these stars.

4. Pulsation Calculations

We compute adiabatic pulsation periods of $\ell = 1, 2$ g modes in a range of periods covering the period spectrum typically observed in ZZ Ceti stars (70 s $\ll \Pi 

1. Crystallization does not take place (hereinafter, NC case).
2. Crystallization happens, but there is only a release of latent heat and the core chemical profile remains unaltered (from here, LH case).
3. Crystallization does take place, involving a release of latent heat and also a chemical redistribution due to phase separation, as it is shown in Fig. 1 (hereinafter, LH+PS case).

The NC case is considered merely to assess the period spectrum of ultra-massive WDs when we neglect crystallization, that is, when the ordinary boundary conditions at the stellar centre are adopted to solve the pulsation equations. The LH case is considered to show the pulsation properties of ultra-massive WDs when account is taken of the presence of a solid core in the computation of the pulsational eigenspectra, that is, when we use the hard-sphere boundary conditions to compute the eigenfunctions of modes, but the core chemical profiles remain fixed as the WD crystallizes. Finally, we consider the LH+PS case, that constitutes the most physically plausible situation in which crystallization is considered and the core chemical profiles are being modified continuously during crystallization as a result of phase separation.

We show in Figs. 2, 3, 4, and 5 the chemical abundances by mass of $^1$H, $^4$He, $^{12}$C, $^{16}$O, $^{20}$Ne, $^{23}$Na, and $^{24}$Mg (upper panel),
the Ledoux term $B$ (middle panel), and the logarithm of the squared Brunt-Väisälä frequency, which is more notorious for the more massive models, is irrelevant for their pulsation properties because it is outside the propagation zone of the $g$ modes. A detailed inspection of the central panels of Figs. 2 and 3 in the region of the crystallized core reveals that the Ledoux term $B$ has non-zero values that increase towards the center of the star as a result of the spatial variation of the abundances of $^{16}$O and $^{20}$Ne due to the chemical redistribution by phase separation. This small contribution of $B$ to the Brunt-Väisälä frequency, which is more notorious for the more massive models, is irrelevant for their pulsation properties because the eigenfunctions of $g$ modes are excluded from the solid region.

4.1. The Impact of Crystallization on the Pulsation Spectrum

In this section we shall analyze the pulsation properties of our set of One-core ultra-massive WD models. In doing so, we will compare the impact of crystallization by comparing the results for the cases NC, LH and LH+PS described before.

We begin by examining the asymptotic period spacing of our sequences, computed according to Eqs. (3) and (4). In Fig. 4 we depict $\Delta \Pi^f_\ell$ for $\ell = 1$ modes in terms of $T_{\text{eff}}$ for the One-core WD evolutionary sequences with masses $1.10, 1.16, 1.22$ and $1.29 M_\odot$. The results correspond to the case in which crystallization has been computed taking into account latent heat release and chemical rehomogenization due to phase separation (LH+PS case). The results correspond to the case in which crystallization has been computed taking into account latent heat release and chemical rehomogenization due to phase separation (LH+PS case). The results correspond to the case in which crystallization has been computed taking into account latent heat release and chemical rehomogenization due to phase separation (LH+PS case).

Finally, the innermost peak in $B$ that is due to the small step located exactly at the edge of the crystallized core. This also appears in the Brunt-Väisälä frequency as a small peak. In contrast with what happens with the previous mentioned features of $N^2$, the peak at the boundary of the crystallized core has no consequences for the pulsation spectrum because it is outside the propagation zone of the $g$ modes. A detailed inspection of the central panels of Figs. 2 and 3 in the region of the crystallized core reveals that the Ledoux term $B$ has non-zero values that increase towards the center of the star as a result of the spatial variation of the abundances of $^{16}$O and $^{20}$Ne due to the chemical redistribution by phase separation. This small contribution of $B$ to the Brunt-Väisälä frequency, which is more notorious for the more massive models, is irrelevant for their pulsation properties because the eigenfunctions of $g$ modes are excluded from the solid region.

The abundances by mass of $^1$H, $^4$He, $^{12}$C, $^{16}$O, $^{20}$Ne, $^{23}$Na, and $^{24}$Mg as a function of the fractional mass (upper panel), the Ledoux and Lamb frequencies (lower panel), corresponding to a One-core WD model with $M_\star = 1.10 M_\odot$ and $T_{\text{eff}} \sim 11600$ K. The model has been computed taking into account latent heat release and chemical redistribution due phase separation during crystallization (LH+PS case). The gray area marks the domain of crystallization. $M_\ell / M_\star$ is the fraction of the crystallized mass of the model.

![Fig. 2.](image1.png)  
![Fig. 3.](image2.png)
of the NC, LH, and consequent increase of \( \Delta \Pi \) which induces a lower value of the integral in Eq. (3) and the is due to the appearance of surface convection in the models, with mass \( M \) effective temperature for the ONe-core WD evolutionary sequence electronic degeneracy (\( \Delta \Pi \)). For this sequence, the onset of crystallization result from \( \Delta \Pi \) at certain effective temperatures, the crystallized mass. Clearly, for this sequence, the star is almost 99% of their mass crystallized throughout the ZZ Ceti instability strip.

According to Eqs. (2) and (3), the dependence of \( \Delta \Pi \) on the Brunt-Väisälä frequency is such that the asymptotic period spacing is larger when the mass and/or effective temperature of the model is lower. This trend is clearly visible in Fig. 6. The higher values of \( \Delta \Pi \) for lower \( M_\star \) comes from the dependence \( N \propto g \) (Eq. 4), with \( g \) the local gravity \( (g \propto M_\star/R_\star^2) \). On the other hand, the higher values of \( \Delta \Pi \) for lower \( T_{\text{eff}} \) result from the dependence \( N \propto \sqrt{T} \) (Eq. 4). The abrupt change in the slope of the curves representing \( \Delta \Pi \) at certain effective temperatures is due to the appearance of surface convection in the models, which induces a lower value of the integral in Eq. 3 and the consequent increase of \( \Delta \Pi \).

In order to show the different expected behaviour of the asymptotic period spacing when we consider the NC, LH and LH+PS treatments, we display in Fig. 3 the \( \Delta \Pi \) in terms of the effective temperature for the ONe-core WD evolutionary sequence with mass \( M_\star = 1.29 M_\odot \). The thick curve corresponds to the LH+PS case, the intermediate-thickness curve depicts the LH case, and the thin curve is associated to the NC case. Again, the palette of colors corresponds to the fraction of crystallized mass \( (M_c/M_\star) \). For this sequence, the onset of crystallization occurs at \( T_{\text{eff}} \sim 37500 \) K. For higher effective temperatures, the three curves coincide. When the model cools below that effective temperature, the curves of the asymptotic period spacing corresponding to the LH and LH+PS cases begin to separate from the curve associated to the NC case. This is because, as mentioned, when crystallization begins the lower limit of the integral in Eq. 3 coincides with the radius of the crystallization boundary, which is continuously moving outward as the star cools down. But for a ONe-core WD model with \( M_\star = 1.22 M_\odot \). Then, the fraction of crystallized mass increases and the integral in Eq. 3 decreases, leading to an increase in \( \Delta \Pi \) (Eq. 2). Note that the asymptotic period spacing for the LH and LH+PS cases is always longer than for the NC case, including at the stages along the ZZ Ceti instability strip, where \( \Delta \Pi \) is \( \sim 3 \) s shorter if we do not consider crystallization. On the other hand, the value of \( \Delta \Pi \) for the LH case is slightly larger than for the LH+PS case, the larger difference (that amounts to \( \sim 1.5 \) s) being at the red edge of the ZZ Ceti instability strip. This small difference is due to the presence of the peak in \( N^2 \) located at the end of the rehomogenized region in the LH+PS case \((\log(1-M_\star/M_\odot) \sim 4)\); see Fig. 5, which is within the Brunt-Väisälä frequency for the LH case. The case of the sequence with mass \( M_\star = 1.29 M_\odot \) is representative of the results for the sequences with masses 1.10, 1.16 and 1.22 \( M_\odot \), and they are not shown for brevity.

It is illustrative to examine how the pulsation periods evolve as the WD models cool, according to the different treatments considered. In Fig. 3 we show the periods of \( \ell = 1 \) g modes as a function of the effective temperature, for the range of \( T_{\text{eff}} \) of the ZZ Ceti instability strip, corresponding to the 1.29 \( M_\odot \) ONe-core WD model sequence. In the left panel we show the results when crystallization has been neglected (NC case). In the middle panel we depict the results when the presence of a solid core has been considered in the computation of the eigenmodes, although chemical rehomogenization upon crystallization has been neglected (LH case). Finally, in the right panel we show the case in which chemical rehomogenization due to phase separation has been considered (LH+PS case). The numbers at the upper part of the central and right panels correspond to the percentage of the crystallized mass. Clearly, for this sequence, the star is almost completely crystallized at the stage of the ZZ Ceti instability strip. Generally, the periods get longer for decreasing effective temperature, although this increase is hardly noticeable in
modes of low radial order \((k \leq 4)\). In the NC case (left panel) the increase of the periods is due solely to the cooling of the star. This is because the Brunt-Väisälä frequency gets smaller with cooling due to the increase of electronic degeneracy \((\chi_T \rightarrow 0)\).

A close inspection of the figure reveals some weak patterns of avoided crossing. When crystallization is taken into account (LH case, middle panel), the size of the propagation region becomes smaller as the crystallization front moves outward, leading to an additional increase in the period values. Some avoided crossing patterns can be appreciated also in this case, particularly for periods shorter than \(\sim 400\) s. Finally, when the chemical profile at the region surrounding the crystallized core is modified by rehomogenization due to phase separation (LH+PS case, right panel), strong avoided crossing patterns are produced, as can be clearly appreciated in the figure. This is similar to the avoided crossing phenomena found by Montgomery & Winget (1999) and Córsico et al. (2005) for massive CO-core WD when the crystallized mass fraction of their models is varied in a continuous fashion. Note that the avoided crossing patterns propagate to longer periods as the effective temperature decreases.

An important consequence of avoided crossing is that if we consider a fixed \(T_{\text{eff}}\) value, the separation between consecutive periods has strong minima each certain number of modes. It is clearly emphasized in the \(\Delta \Pi - \Pi\) diagram shown in the upper right panel of Fig. 9 that show the forward period spacing \(\Delta \Pi_k = \Pi_{k+1} - \Pi_k\) in terms of the periods of \(\ell = 1\) pulsation modes for a \(1.29 M_\odot\) ONe-core WD model at \(T_{\text{eff}} \sim 11 \text{ 600 K}\) for the LH+PS case. Left and middle panels correspond to the NC and LH cases, respectively, for the same \(M_*\) and \(T_{\text{eff}}\). From the left panel of Fig. 10 it is evident the presence of strong minima of \(\Delta \Pi_k\) with values \((\sim 15\) s) well below the asymptotic period spacing \((\Delta T_{\text{eff}} \sim 22.5\) s; dashed horizontal line). This is in contrast with the cases NC and LH (left and middle panels), for which the forward period spacing exhibits maxima and minima whose amplitudes decrease as we consider higher-order modes (long periods). Indeed, the \(\Delta \Pi_k\) values approximate to \(\Delta T_{\text{eff}}\) for high values of \(k\). Results for models with other masses are similar than for the \(1.29 M_\odot\) model. In particular, we show in Fig. 10 the case of the \(1.10 M_\odot\) model at the same effective temperature \((T_{\text{eff}} \sim 11 \text{ 600 K})\). Again in this case we found strong minima with values of \(\Delta \Pi_k\) much lower than the asymptotic period spacing.

The remarkable difference in the distribution of period spacing in the case in which rehomogenization of the core chemical profiles due to phase separation is taken into account (LH+PS treatment) in comparison with the case in which it is ignored (LH case), could be used as an asteroseismological tool in real ZZ Ceti stars to shed light about the existence (or not) of phase separation in crystallized WDs.

In order to gain additional information about the nature of the pulsation modes, it is useful to know the amount of kinetic energy involved by the oscillation. In the lower panels of Figs. 9 and 10 we plot the logarithm of the pulsation kinetic energy in terms of the periods for the \(1.29 M_\odot\) and \(1.10 M_\odot\) models, respectively. The LH+PS case is particularly interesting because of the presence of the strong minima in \(\Delta \Pi_k\). A close examination of the lower left panels of the figures reveals that these minima are associated to \(g\) modes characterized by relatively high kinetic energy when compared to the remainder ones. Indeed, each minima in \(\Delta \Pi_k\) has a clear correspondence with a local maxima in
log($E_{1,0}$). We find that these modes, which are characterized by high kinetic energy values, are g modes confined to the internal region of the model bounded by the limit of the crystallized core and the step in the chemical profile at the extreme of the rehomogenization region. This step leads to the peak in the Brunt-Väisälä frequency located at $-\log(1 - M_*/M_\odot) \sim 4$ for the $1.22 M_\odot$ and $1.29 M_\odot$ models, and $-\log(1 - M_*/M_\odot) \sim 3$ for the $1.10 M_\odot$ and $1.16 M_\odot$ models (see the upper panels of Figs. 2, 3, 4, and 5). This step is absent if we do not consider chemical rehomogenization due to phase separation (LH case).

4.2. Comparison with CO-core WD Models

The mass limit that separate the WDs harboring CO cores from those having ONe cores is a matter of debate. Therefore, it is interesting to compare the pulsation properties of WDs with CO cores and ONe cores with the same stellar mass. The aim of the present analysis is to explore the possibility of using WD asteroseismology to distinguish between the two types of objects. This has been precisely the goal of the study in Córsico et al. (2004), who compared the pulsation properties of a 1.06 $M_\odot$ WD model with CO core and a model with the same mass but with a core made of O and Ne. Here, we re-examine the topic by taking into account the chemical rehomogenization due to phase separation. This piece of physics was ignored in Córsico et al. (2004) for the ONe-core WD model employed in that work.

For the present analysis, we have computed an additional evolutionary sequence of WD models with $M_\ast = 1.10 M_\odot$ and a core made of $^{12}$C and $^{16}$O, taking into account latent heat release and chemical rehomogenization due to phase separation during crystallization (LH+PS treatment). We have employed the azeotropic phase diagram for a mixture of C and O of Horowitz et al. (2010). For this sequence, we have computed the adiabatic pulsation periods of g modes in the range of periods observed in ZZ Ceti stars, as well as the asymptotic period spacing. In Fig. 11 we show the dipole ($\ell = 1$) asymptotic period spacing as a function of $T_{\text{eff}}$, for the ONe evolutionary cooling sequence (thick curve) and the CO evolutionary cooling sequence with stellar mass $M_\ast = 1.10 M_\odot$. Both sequences were computed taking into account latent heat release and chemical redistribution due to phase separation during crystallization (LH+PS case). The effective-temperature range of the ZZ Ceti instability strip is marked with the two vertical dotted lines. The crystallized mass fraction is indicated with a palette of colors. From the figure it is evident that the asymptotic period spacing for both sequences is very similar, in particular in the regime of effective temperatures of the ZZ Ceti instability strip. This is a somewhat expected result given that the asymptotic period spacing of g modes in ZZ Ceti stars depends basically on the stellar mass, the thickness of the H envelope, and the effective temperature, and these three parameters are the same for both sequences. Note, however, that by virtue of the different core chemical compositions, the degree of crystallization for a given $T_{\text{eff}}$ is different for both types of WD models. We conclude that the asymptotic period spacing is not a useful quantity to distinguish between ZZ Ceti harboring CO cores from those with cores made of ONe.

An alternative way to differentiate WDs with CO cores from those with ONe cores for a fixed stellar mass is to examine the period spacing values in terms of the periods. This has been the approach employed by Córsico et al. (2004). To this aim, we have picked up a 1.10 $M_\odot$ CO-core template WD model at an effective temperature at the middle of the ZZ Ceti instability strip ($T_{\text{eff}} \sim 11 600$ K). In Fig. 12 we show the abundances by mass of $^1$H, $^4$He, $^{12}$C, and $^{16}$O as a function of the fractional mass (upper panel), the Ledoux term B (middle panel), and the logarithm of the squared Brunt-Väisälä and Lamb frequencies (lower panel).
Fig. 9. The forward period spacing ($\Delta \Pi$, upper panels) and the logarithm of the kinetic energy ($E_{\text{kin}}$, lower panels) in terms of the periods of $\ell = 1$ pulsation modes for a 1.29$M_\odot$ ONe-core WD model at $T_{\text{eff}} \sim 11\ 600$ K. The left panels correspond to the case in which crystallization has not been taken into account (NC case). The central panels show the same quantities for the situation in which crystallization has been considered but phase separation has not (LH case). Finally, the right panels show the case in which both crystallization and phase separation have been taken into account (LH+PS case). In the three cases, the percentage of the crystallized mass is indicated. In the upper panels, horizontal red dashed lines correspond to the asymptotic period spacing.

We conclude that neither the average (asymptotic) period spacing nor the forward period spacing can be used to distinguish the chemical composition of the cores of ultra-massive ZZ Ceti stars. We foresee that the only plausible way to know if a ultra-massive ZZ Ceti star has a CO core or a ONe core is to carry out detailed asteroseismological studies involving period-to-period comparisons of real stars employing two sets of ultra-massive WD models, one of them characterized by CO-core WDs and the other one with ONe-core WDs. This analysis is beyond the scope of the present study, and will be the focus of a future work.

5. Summary and conclusions

In this work, we have assessed the adiabatic pulsation properties of ultra-massive H-rich WDs with ONe cores on the basis of full evolutionary models that incorporate the most updated physical inputs governing the progenitor and the WD evolution. This investigation constitutes a substantial improvement from that done in Córsico et al. (2004) in several aspects. On one hand, the chemical profiles of our WD models are consistent with the predictions of the progenitor evolution through the S-AGB phase for all the WD sequences with different stellar masses considered. On the other hand, during WD evolution, we have taken into account for the first time the changes in the core chemical composition resulting from phase separation upon crystallization, by using phase diagrams suitable for $^{16}$O and $^{20}$Ne plasmas. Finally, element diffusion was included for all model sequences, from the beginning of the WD cooling track. Element diffusion smooths the inner chemical profiles which strongly affects the run of the Brunt-Väisälä frequency, and thus the period spectrum and mode trapping properties.

We assessed the pulsational properties of our models by computing their dipole ($\ell = 1$) and quadrupole ($\ell = 2$) g-mode
period spectra for a wide range of effective temperatures, and in particular, for the $T_{\text{eff}}$ interval defining the ZZ Ceti instability strip. Because our models are very massive ($1.10 \leq M_*/M_0 \leq 1.29$), when they reach the ZZ Ceti instability strip, they have a very large fraction of their mass in a crystalline phase. In order to explore in depth the impact of crystallization on their pulsation properties, we considered three cases: (i) we neglect crystallization in the equilibrium models and also in the computation of the pulsation modes; (ii) we consider crystallization with the ensuing release of latent heat, and compute the pulsation modes adopting the hard-sphere boundary conditions, and (iii) we consider crystallization with latent heat release and take into account also phase separation that induces a chemical rehomogenization in the liquid part surrounding the crystallized core, and the eigen-modes are computed using the hard-sphere boundary conditions. We found strong differences in the pulsation spectra of our WD models for the different cases analyzed. In particular, the differences we found in the distribution of period spacings in the case in which the changes of the core chemical profiles due to phase separation are taken into account, in comparison with the case in which it is ignored, could be exploited as an asteroseismological tool in real ZZ Ceti stars to elucidate if phase separation occurs in the deep interior of WDs stars.

We also revisited the possibility of using asteroseismology to distinguish ultra-massive DA WDs harboring ONe cores from those having CO cores. In particular, we compared the pulsation properties (specifically, periods and period spacings) of WD models characterized by the same stellar mass (1.10$M_0$) but in one case having a core made of $^{16}$O and $^{20}$Ne and in another case a core composed by $^{12}$C and $^{16}$O. We did not find sizeable differences neither in the mean period spacing nor in the period-spacing distribution at the ZZ Ceti stage. So, in contrast to the claim of Córsico et al. (2004), we conclude that these pulsation quantities do not allow to differentiate the chemical composition of the cores of ultra-massive ZZ Ceti stars. Instead, we believe that in order to infer the core composition of ultra-massive DA WDs it will be necessary to carry out detailed asteroseismic analysis using the individual periods observed in ultra-massive ZZ Ceti stars like BPM 37093 (Kanaan et al. 1992, 2005), GD 518 (Hermes et al. 2013), and SDSS J0840+5222 (Curd et al. 2017). This will be the focus of a future work.

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during crystallization (LH + latent heat release and chemical redistribution due to phase separation (thick curve) and the CO evolutionary cooling sequence with mass \( M_\odot = 1.10 M_\odot \). Both sequences have been computed taking into account latent heat release and chemical redistribution due to phase separation during crystallization (LH+PS case). Vertical dotted lines show the \( T_{\text{eff}} \) interval of the ZZ Ceti instability strip.

\[ T_{\text{eff}}[\text{K}] = \text{asymptotic period spacing as a function of mass \( M_\odot \).} \]

\[ \Delta \Pi a \sum \log(N_i) \frac{L_i}{L_\odot} \frac{1}{1/s} \]

\[ \log(N_i^2), \log(L_i^2) \]

The abundances by mass of \(^1\text{H}, ^4\text{He}, ^12\text{C},\) and \(^16\text{O}\), as a function of the fractional mass (upper panel), the Ledoux term B (middle panel), and the logarithm of the squared Brunt-Väisälä and Lamb frequencies (lower panel), corresponding to a CO-core WD model with \( M_\odot = 1.10 M_\odot \), \( \log(M_\odot/M_\odot) = -6 \), and \( T_{\text{eff}} \sim 11 600 \text{ K} \). Latent heat release and chemical redistribution due to phase separation have been taken into account during crystallization (LH+PS case) The gray area marks the domain of crystallization. \( M_\odot/M_\odot \) is the fraction of the crystallized mass of the model.

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Fig. 11. The dipole (\( \ell = 1 \)) asymptotic period spacing as a function of the effective temperature, for the ONe evolutionary cooling sequence (thick curve) and the CO evolutionary cooling sequence with mass \( M_\odot = 1.10 M_\odot \). Both sequences have been computed taking into account latent heat release and chemical redistribution due to phase separation during crystallization (LH+PS case). Vertical dotted lines show the \( T_{\text{eff}} \) interval of the ZZ Ceti instability strip.

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