POLARIZATION PHENOMENA IN SMALL-ANGLE PHOTOPRODUCTION OF $e^+e^-$ PAIRS AND THE GDH SUM RULE

A.I. L’JOV $^a$
Lebedev Physical Institute, Russian Academy of Sciences, Moscow, 117924, Russia

S. SCOPETTA
Department of Physics, University of Perugia, via A. Pascoli, I-06100 Perugia, Italy
and Institut für Kernphysik, Universität Mainz, D-55099, Germany

D. DRECHSEL, S. SCHERER
Institut für Kernphysik, Universität Mainz, D-55099, Germany

We discuss a possibility to measure the spin-dependent part of the forward Compton scattering amplitude through interference effects of the Bethe-Heitler and virtual Compton scattering mechanisms in photoproduction of $e^+e^-$ pairs at small angles.

1 Introduction

In studies of spin-dependent structure functions of nucleons and nuclei with real and virtual photons, the verification of the Gerasimov–Drell–Hearn sum rule is of special interest. The GDH sum rule is based in essence only on the assumption of spin-independence of high-energy forward Compton scattering and thus provides a very clean test of the spin dynamics.

The forward Compton scattering amplitude on a spin-1/2 target is described by two even functions $f_{1,2}$ of the photon energy $\omega$,

$$f = (e' \cdot e) f_1(\omega) + i\omega \sigma \cdot (e' \times e) f_2(\omega), \quad (1)$$

which, at $\omega = 0$, are constrained by the low-energy theorem:

$$f_1(0) = -\frac{\alpha Z^2}{M}, \quad f_2(0) = -\frac{\alpha \kappa^2}{2M^2}. \quad (2)$$

Here $M, eZ, \kappa$ are the mass, electric charge, and anomalous magnetic moment of the target, and $\alpha = e^2/4\pi \simeq 1/137$. Within the framework of the Regge pole model, these functions behave like

$$f_1(\omega) \propto \omega^{\alpha_R(0)}, \quad f_2(\omega) \propto \omega^{\alpha_R(0)-1} \quad \text{for} \quad \omega \rightarrow \infty. \quad (3)$$

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where $\alpha_R(0)$ is the intercept of the leading $t$-channel Regge exchange contributing to the amplitudes. With the usual assumption of $\alpha_R(0) \lesssim 1$, both $f_{1,2}$ satisfy once-subtracted dispersion relations. The optical theorem allows to find the imaginary parts of $f_{1,2}$ in terms of the total photoabsorption cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$, where the subscript refers to total helicities $1/2$ and $3/2$, respectively:

$$\text{Im} \, f_1 = \frac{\omega}{8\pi}(\sigma_{1/2} + \sigma_{3/2}) = \frac{\omega}{4\pi}\sigma_{\text{tot}}, \quad \text{Im} \, f_2 = \frac{1}{8\pi}(\sigma_{1/2} - \sigma_{3/2}) \frac{df}{d\omega} = \frac{1}{4\pi}\Delta \sigma. \quad (4)$$

Therefore, one can write the dispersion relation for $f_2$ in the form

$$f_2(\omega) = -\frac{\alpha K^2}{2M^2} + \frac{\omega^2}{2\pi^2} \int_{\omega_{\text{thr}}}^{\infty} \frac{d\sigma(\omega')}{\omega'^2 - \omega^2 - i0^+} \frac{d\omega'}{\omega'} \quad (5)$$

The GDH sum rule,

$$\int_{\omega_{\text{thr}}}^{\infty} \frac{\Delta \sigma(\omega')}{\omega'} d\omega' = -\frac{\pi^2 \alpha K^2}{M^2}, \quad (6)$$

arises from (5) under the stronger assumption that $f_2(\omega) \to 0$ for $\omega \to \infty$, or alternatively $\alpha_R(0) < 1$. Numerical investigations of the integral on the l.h.s. of (6) in the case of the nucleon reveal a different behavior for the two isospin combinations of the proton and neutron amplitudes,

$$f_{1/2}^{I=0} = \frac{1}{2} (f_{1/2}^p + f_{1/2}^n), \quad f_{1/2}^{I=1} = \frac{1}{2} (f_{1/2}^p - f_{1/2}^n), \quad (7)$$

which correspond to isoscalar and isovector exchanges in the $t$-channel. The $I = 0$ GDH sum rule seems to work very successfully, thus supporting the conjecture that the leading isoscalar Regge exchange (Pomeron) is decoupled from the spin-dependent transitions. It is very surprising, however, that the $I = 1$ GDH sum rule seems to be violated, despite the fact that all known isovector exchanges (such as the $a_2$-meson) have an intercept $\alpha_R(0) \lesssim 0.5$ and hence cannot spoil the assumption of $f_2^{I=1} \to 0$ for $\omega \to \infty$.

This discrepancy stimulated a lot of efforts to explore the amplitude $f_2$ experimentally. A straightforward way to study $\text{Im} \, f_2$ is provided by measuring the photoproduction cross sections $\sigma_{1/2}$ and $\sigma_{3/2}$ over a wide energy region, using $4\pi$ detectors for the final hadrons. In the present paper we discuss a possibility to study both $\text{Re} \, f_2$ and $\text{Im} \, f_2$ by measuring photoproduction of small-angle $e^+e^-$ pairs which shows a specific azimuthal asymmetry due to the interference of Bethe–Heitler and virtual Compton contributions. Such a method was already used to determine $\text{Re} \, f_1$ at the energy of 2.2 GeV. Using polarized particles, one can determine $f_2$ as well.
2 Bethe–Heitler and virtual Compton contributions

Considering the reaction
\[ \gamma N \rightarrow e^+ e^- N' \]  
(8)
in the lab frame, we denote the 4-momentum and helicity of \( e^+ \) by \( \varepsilon_1, p_1, \frac{1}{2} h_1 \); we use \( \varepsilon_2, p_2, \frac{1}{2} h_2 \) for those of \( e^- \) and \( \omega, k, h_\gamma \) for the photon. Also, we denote the nucleon spin projections to the beam direction by \( \frac{1}{2} h_N, \frac{1}{2} h'_N \).

Aside from the overall azimuth of the final particles, the kinematics of \( e^+ e^- \) photoproduction is specified by four variables. We choose two of them to be the invariant mass of the \( e^+ e^- \) pair and the invariant momentum transfer:
\[ W^2 = k'^2, \quad k' \equiv p_1 + p_2, \quad Q^2 = -q^2, \quad q \equiv p'_N - p_N. \]  
(9)

Furthermore, we introduce the fractions of the energy carried by \( e^+ \) and \( e^- \),
\[ x_1 = \frac{\varepsilon_1}{\omega'}, \quad x_2 = \frac{\varepsilon_2}{\omega'}, \quad \omega' = \varepsilon_1 + \varepsilon_2, \]  
(10)
the total momentum of the pair, \( k' = p_1 + p_2 \), which determine the angles \( \theta_{1,2} \) between \( p_{1,2} \) and \( k' \) through
\[ 2|p_i| |k'| \cos \theta_i = 2\varepsilon_i \omega' - W^2, \quad i = 1, 2, \]  
(11)
and denote the azimuthal angles of \( e^+ \) and \( e^- \), with respect to the direction of \( k' \), by \( \phi_1 \) and \( \phi_2 = \pi + \phi_1 \), see Fig. 1. In terms of these variables, the recoil nucleon has the kinetic energy and momenta
\[ q_0 = \frac{Q^2}{2M}, \quad q_z = q_0 + \frac{W^2 + Q^2}{2\omega}, \quad q_{\perp}^2 = Q^2 + q_0^2 - q_z^2 \geq 0. \]  
(12)
Hence, the energy and momentum carried by the \( e^+ e^- \) pair is \( \omega' = \omega - q_0 \) and \( k' = k - q \), respectively.

The differential cross section of (8) reads:
\[ d\sigma = \frac{1}{(4\pi)^3} \frac{\omega'}{|k'|} \frac{W dW Q dQ}{\omega^2} dx_1 \frac{d\phi_1}{2\pi} |T|^2, \]  
(13)
where the appropriate sum and average over spins is implied.

To the lowest order in the electromagnetic coupling \( e \), the amplitude \( T = T_{BH} + T_{VCS} \) consists of the Bethe–Heitler and virtual Compton scattering contributions shown in Fig. 2. Typically, we consider the kinematical region of
\[ \omega \sim 1 \text{ GeV}, \quad W \sim 10 \text{ MeV}, \quad Q \sim 100 \text{ MeV}, \quad x_1 \sim x_2 \sim 0.5, \]  
(14)
which corresponds to small angles $p_{1\perp}/\epsilon_1 \ll 1$ and $p_{2\perp}/\epsilon_2 \ll 1$ of $e^+$ and $e^-$, respectively.

Neglecting the electron mass $m_e$ and keeping leading terms in the expansion of the matrix elements in powers of the transverse momenta $p_{1,2\perp}$, we arrive at the following expressions for the amplitudes diagonal in the spin variables:

$$T_{\text{BH}} \approx -\frac{2e^3Z}{Q^2}\omega \sqrt{x_1x_2}(e \cdot D)(x_1 - x_2 + h_1 h_\gamma)\delta_{h_1,-h_2}\delta_{h_N,h_N'}$$

and

$$T_{\text{VCS}} \approx -\frac{4\pi e}{W^2}\sqrt{x_1x_2}(e \cdot d)(x_1 - x_2 + h_1 h_\gamma)(f_1 - h_\gamma h_N\omega f_2)\delta_{h_1,-h_2}\delta_{h_N,h_N'}.$$  

Here the photon polarization vector $e$ and the vectors $D, d$ read

$$e = \frac{1}{\sqrt{2}}(-h_\gamma e_x + ie_y), \quad D = \frac{p_{1\perp}}{p_{1\perp}^2} + \frac{p_{2\perp}}{p_{2\perp}^2}, \quad d = \frac{p_{1\perp}}{x_1} - \frac{p_{2\perp}}{x_2}. $$
The absolute value of \( d \) is related to the invariant mass of the \( e^+e^- \) pair, 
\[ W^2 \approx x_1x_2d^2. \] 
Another relation, \(|D| \approx (x_1x_2Q)^{-1}\), is valid provided \( W \ll Q \).

We will use these formulae below to discuss the possibility of measuring \( f_2 \).

First, a few comments are in order.

(i) Exact calculations of the amplitudes, accounting for the finite mass \( m_e \) and keeping higher terms in \( p_{1,2 \perp} \), agree to within 1–2\% with the approximations (15) and (16) for all cross sections and asymmetries considered.

(ii) Equation (15) only accounts for the contribution of the Coulomb field to pair production. Magnetic effects due to the spin of the target and recoil corrections are of higher order in \( p_{1,2 \perp} \) and were found to be numerically small and thus irrelevant for our conclusions.

(iii) To obtain Eq. (16), we neglected that the Compton scattering amplitude (1) varies slowly between \( 0^\circ \) and a typical angle of \( q_\perp/\omega \approx 0.1 \approx 6^\circ \). Based on dispersion calculations of \( \gamma p \) scattering at energies up to 1 GeV, we estimate this deviation to be less than 1–2\%. In general, the momentum transfer \( Q^2 \sim 0.01 \text{ GeV}^2 \) is too small in comparison with the typical hadronic scale \( m_\rho^2 \approx 0.6 \text{ GeV}^2 \) to visibly change the amplitude \( T_{VCS} \) found in the forward approximation (1).

(iv) To derive Eq. (16), we also neglected the contribution of longitudinal photons. Considering a resonance model of \( \gamma p \) scattering at energies \( \sim 1 \text{ GeV} \), we have estimated the longitudinal contribution to \( T_{VCS} \) to be of the order of \( q_\perp W/\omega^2 \sim 10^{-3} \) with respect to the dominating transverse contribution (16).

3 Azimuthal asymmetries and determination of \( f_2 \)

Generally, the VCS amplitude (16) is small in comparison with the BH amplitude (15) and can be observed only through the VCS–BH interference. Note that the \( e^+e^- \) pairs produced by a single virtual photon, like in the VCS process, or in the background reaction
\[ \gamma p \rightarrow \pi^0 X \rightarrow e^+e^-\gamma X, \] 
(18)
have negative intrinsic C-parity. Therefore, all the appropriate amplitudes are even under the interchange
\[ \varepsilon_1, p_1, h_1 \leftrightarrow \varepsilon_2, p_2, h_2, \] 
(19)
while the BH amplitude is odd under (19), and its interference with the VCS contribution leading to the same final state results in a \( 1 \leftrightarrow 2 \) asymmetry.

The amplitude (16) gives the Bethe–Heitler cross section
\[ \sigma_{12}^{\text{BH}} \overset{def}{=} 2\pi W Q \frac{d^4\sigma_{12}^{\text{BH}}}{dWdQ dx_1d\phi_1} \approx \frac{8\alpha^3 Z^2 W^2}{Q^2} x_1x_2 (x_1^2 + x_2^2)D^2, \] 
(20)
which is symmetric under \( 1 \leftrightarrow 2 \) and depends on the azimuth \( \phi_1 \) only weakly. In the kinematics of (14) it equals 2.4 nb.
The excess of $e^+$ over $e^-$ at some angles is due to the interference term:

$$\sigma_{12} - \sigma_{21} \approx \frac{16\alpha^2 Z}{\omega} x_1 x_2 (x_1^2 + x_2^2)|D| A(\omega, \phi_1),$$

where the function

$$A(\omega, \phi_1) = (\text{Re} f_1(\omega) - h_N \omega \text{Re} f_2(\omega)) \cos \phi_1 + (-h_N \text{Im} f_1(\omega) + h_N \omega \text{Im} f_2(\omega)) \sin \phi_1$$

(22)
carries information on the real and imaginary parts of $f_{1,2}$. This function can be measured through the $e^+-e^-$ asymmetry.

$$\Sigma = \frac{\sigma_{12} - \sigma_{21}}{\sigma_{12} + \sigma_{21}} \approx \frac{2 \text{Re} [T_{VCS} T_{BH}^*]}{|T_{BH}|^2} \approx \frac{Q^3 \sqrt{x_1 x_2}}{\alpha Z \omega W} A(\omega, \phi_1),$$

(23)

provided the background contribution such as (18) is subtracted. The available knowledge on $f_1$ can be used for checking the whole procedure.

It is interesting to note that even with unpolarized photons one can measure the GDH cross section $\Delta \sigma \propto \text{Im} f_2$. The appropriate asymmetry

$$\frac{1}{2} \Sigma(h_N) - \Sigma(-h_N) \approx h_N \sin \phi_1 \frac{Q^3 \sqrt{x_1 x_2}}{4\pi \alpha Z W} \Delta \sigma$$

(24)
is proportional to the target polarization ($h_N$) and equals 14% for $\Delta \sigma = 100 \mu b$ in the kinematics of (14), independently of the photon energy $\omega$. With circularly-polarized photons and a polarized target one can measure $\text{Re} f_2$ as well and use the dispersion relation (5) to learn about $\Delta \sigma$ at asymptotically high energies. Preliminary estimates of count rates and backgrounds support a feasibility of such measurements both for nucleons and nuclei.

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