Methodology for estimating the magnetic Prandtl number and application to solar surface small-scale dynamo simulations

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What are the intrinsic numerical viscosity and resistivity?
How does the action of solar dynamos depend on $Pr_m$?

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Introduction

Momentum equation
\[ \frac{\partial (\rho v)}{\partial t} + \nabla \cdot (\rho vv) = -\nabla p + \rho g + F_{EM} + \nu \rho \nabla^2 v \]

Induction equation
\[ \frac{\partial B}{\partial t} - \nabla \times (v \times B) = \eta \nabla^2 B \]

- viscosity
- resistivity
Introduction

Momentum equation
\[ \frac{\partial (\rho \mathbf{v})}{\partial t} + \nabla \cdot (\rho \mathbf{v} \mathbf{v}) = -\nabla p + \rho \mathbf{g} + \mathbf{F}_{EM} + \nu \rho \nabla^2 \mathbf{v} \]

Induction equation
\[ \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = \eta \nabla^2 \mathbf{B} \]

Reynolds number
\[ Re = \frac{uL}{\nu} \]

Magnetic Reynolds number
\[ Re_m = \frac{uL}{\eta} \]

Magnetic Prandtl number
\[ Pr_m = \frac{Re_m}{Re} = \frac{\nu}{\eta} \]

For the Sun: \[ Re \gg 1 \quad Re_m \gg 1 \quad Pr_m \ll 1 \]
Is small-scale dynamo action on solar surface important?

\[ Pr_m \ll 1 \Rightarrow \text{Cannot predict } a \text{ priori which one wins between amplification and dissipation of magnetic fields} \]
Is small-scale dynamo action on solar surface important?

\[ Pr_m \ll 1 \Rightarrow \text{Cannot predict } a \text{ priori which one wins between amplification and dissipation of magnetic fields} \]

\[ \Rightarrow \text{We need numerical simulations!} \]
\[ \text{e.g., CO}^5\text{BOLD [Freytag et al., 2012]} \]
Is small-scale dynamo action on solar surface important?

\[ Pr_m \ll 1 \Rightarrow \text{Cannot predict } a \text{ priori which one wins between amplification and dissipation of magnetic fields} \]

BUT: intrinsic numerical diffusivities!

- Make it difficult to reach realistic \( Re \) and \( Re_m \)
- \( Re \) and \( Re_m \) generally unknown \( \Rightarrow \) it complicates the interpretation of results
Methodology for estimating $P_{r_m}$

Based on method of Projection of Proper elements (PoPe) [Cartier-Michaud et al., 2016]

- Step 0  \[ \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0 \]

\[ \partial_t \mathbf{B} = \sum_i w_i O_i(\mathbf{B}, \mathbf{v}) \]

\( \{w_i\} = \{1, -1\} \)

\( \{O_i\} = \{\nabla \cdot (\mathbf{Bv}), \nabla \cdot (\mathbf{vB})\} \)
Methodology for estimating $Pr_m$

Based on method of Projection of Proper elements (PoPe) [Cartier-Michaud et al., 2016]

- **Step 0**
  \[
  \frac{\partial \mathbf{B}}{\partial t} - \nabla \times (\mathbf{v} \times \mathbf{B}) = 0
  \]
  \[
  \partial_t \mathbf{B} = \sum_i w_i O_i(\mathbf{B}, \mathbf{v})
  \]

- **Step 1**
  Numerical solution from simulation code
  \[
  \delta^{sc,h} \mathbf{B}^h = \sum_i w_i O_i^{sc,h}(\mathbf{B}^h, \mathbf{v}^h)
  \]

- **Step 2**
  Introduce a post-processing scheme to compute
  \[
  r^h(\{\tilde{w}_i, \eta_{eff}\}) = \| \delta^{pp,h} \mathbf{B}^h - \sum_i \tilde{w}_i O_i^{pp,h}(\mathbf{B}^h, \mathbf{v}^h) - \eta_{eff} (\nabla^2)^{pp,h} \mathbf{B}^h \|
  \]

- **Step 3**
  Minimize $r^h(\{\tilde{w}_i, \eta_{eff}\})$ for $\tilde{w}_i, \eta_{eff}$

- **Step 4**
  Repeat with momentum equation for $\nu_{eff}$

- **Step 5**
  Compute $Pr_{m,eff} = \frac{\nu_{eff}}{\eta_{eff}}$
Results

- Dependence on height
- Intrinsic resistivity decreases with resolution
- Results independent of post-processing scheme (FD2, FD4, FD6)
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• Dependence on height
• Intrinsic resistivity decreases with resolution
• Results independent of post-processing scheme (FD2, FD4, FD6)

And several additional tests...

⇒ Robust methodology and reliable results

[Riva et al., in preparation]
Dynamo simulations

\[ Pr_m = \frac{\nu}{\eta} \approx 0.9 - 1 \]
Dynamo simulations

\[ Pr_m = \frac{\nu}{\eta} \approx 0.9 - 1 \]

\[ Re \approx 1900 - 2400 \]
Conclusions and future perspectives

Conclusions:
- Extended (i)PoPe methodology to estimating viscosity and resistivity in radiative MHD codes
- Applied methodology to CO5BOLD simulations and validated the procedure
- Demonstrated possibility of simulating self-generated magnetic fields with CO5BOLD, even at $Pr_m \approx 0.68$

Future work:
- Test with hyper-viscosity and hyper-resistivity
- Investigate impact of domain size and boundary conditions
- Investigate smaller $Pr_m$