Abstract—Random phase screen method was used to simulate the light intensity and phase distribution of the synthesized double vortex beams transmitted in atmospheric turbulence given the light intensity characteristics and phase characteristics of double Laguerre–Gaussian beams (DLGB) generated by coaxial superposition. Results show that the scintillation coefficient decreases initially, and then increases with the topological charge difference of the double vortex beam, and the topological charge difference is related to the number of split spots. When the number of split spots is small, the intensity distribution is concentrated, exhibiting a good penetrating effect on atmospheric turbulence.

Index Terms—Double vortex beams, atmospheric channel, atmospheric turbulence, bit error rate.

I. INTRODUCTION

Atmospheric turbulence is a random movement of air, in which irregular changes in temperature, humidity, and pressure lead to uneven refractive index distribution of the atmosphere in general, and many constantly moving air vortices in the flowing atmosphere pose challenges to laser transmission in the atmosphere [1], [2], [3], [4], [5], [6]. The vortex beam carrying orbital angular momentum has infinite quantum eigenstates of orbital angular momentum by each photon in the beam due to its infinite possibility of topological charge value, which can greatly improve the channel capacity and information transfer efficiency of the current free-space optical communication system [7], [8], [9], [10], [11]. In addition, vortex beams have been widely used in optical communication, remote sensing, and super-resolution imaging due to their specific spiral phase structure and dark empty ring light intensity distribution [12], [13], [14], [15]. By the way, many new modulation method of vortex beams has been carried out by researchers [16], [17], [18]. With the deepening of the research on vortex beams, researchers find that the double vortex beams formed by the co-axial superposition of two vortex beams produce more novel characteristics and have great research value. The study of the interaction mechanism between double vortex beam and atmospheric turbulence may provide a theoretical basis for solving the problem of laser communication in atmospheric channels.

The theory of atmospheric turbulence is well developed. Soviet mathematician Kolmogorov introduced structural functions to study the statistical properties of atmospheric turbulence, and the “Law of 2/3 power” was proposed [19], [20]. On this basis, Tatarski introduced the power spectral density function of atmospheric turbulent dissipation region [21] given that the origin of the two power spectral densities is not integral. Von Karman proposed the modified Kolmogorov spectrum [22] and established the current universal model of atmospheric turbulence spectrum. In the research of double vortex beams, He et al., [23] selected the new composite vortex beam formed by superposition of two LG beams with zero radial index and different topological charges as the research object. They also analyzed the light intensity distribution and transmission characteristics of the composite vortex beam. Ke et al., [24] considered a composite vortex beam formed by collinear superposition of two LG beams with the same high-order radial index and negative orbital angular momentum (OAM) states to analyze the influence of beam waist parameters and transmission distance on its light intensity distribution from theoretical and experimental aspects. The effects of atmospheric turbulence on double vortex beam propagation have not been studied.

Studies have shown that dual-vortex beams can carry more angular momentum modes in the source field and are difficult to diverge during transmission [25], [26], [27]. In this study, the coaxial superposition of two zero-order radial LG vortex beams with different topological charges is used to study the intensity and phase characteristics of the composite vortex beams, the random phase screen method is used to study their propagation characteristics in atmospheric turbulence, and the scintillation coefficients after atmospheric turbulence are calculated. In the second section, the theoretical derivation of the co-axial superimposed double Laguerre–Gaussian vortex beam and random phase screen of atmospheric turbulence is introduced. In the third section, the physical model of the co-axial superimposed Laguerre–Gaussian vortex beam propagation in atmospheric turbulence is presented. In the fourth section, the intensity and phase distribution of the superposition vortex beam under the
background of atmospheric turbulence are calculated, and the scintillation index and bit error rate (BER) under different modulation modes are analyzed. Finally, the conclusion is discussed.

II. Propagation Theory of Double Laguerre–Gaussian Beams (DLGB) in Atmospheric Turbulence

A. Coaxial Superposition of Double Laguerre–Gaussian Vortex Beams

LGB is a high-order Gaussian beam, which is described by the combination of Laguerre polynomial and Gaussian distribution function when transmitted along the Z direction, as follows:

\[
L_G(r, \varphi, z, m) = \frac{A}{w_0(z)} \left( \frac{\sqrt{2}}{w(z)} \right)^m L_n^{|m|} \left( \frac{2r^2}{w^2(z)} \right) \exp \left( -\frac{r^2}{w^2(z)} \right) \exp(i m \varphi) \exp(i \Phi) \tag{1}
\]

where \( A \) is a constant, \( w_0(z) \) is the beam waist radius of the beam source plane, \( \varphi \) is the azimuth angle of the beam, \( m \) is the topological charge number as an arc mode indicator, and \( n \) is the radial mode indicator. \( w(z) \) is the radius of the beam at \( z \), expressed as follows: \( w(z) = w_0\sqrt{1 + (z/z_R)^2} \), and \( z_R \) is the confocal parameter, also known as Rayleigh radius, and \( \Phi = (n + 2m + 1) \arctan(z/z_R) - k(z + r^2/R) \), \( R = z + z_R^2/z \).

For radial mode \( N = 0 \), the co-axial superposition of the two LGB with different topological charges \( m \) generates compound LGB, which is expressed as follows:

\[
L_G(r, \varphi, z, m_1, m_2) = L_G(r, \varphi, z, m_1) + L_G(r, \varphi, z, m_2) \exp(i \delta) \tag{2}
\]

where \( \delta \) is the phase difference of one of the vortex beams, and \( k \) is an arbitrary integer. \( m_1 \) and \( m_2 \) are the topological charges of the two superimposed vortex beams, and \( v = |m_1 - m_2| \), the difference in the topological charges of the two beams, is used as the mode of the DLGBs. When the value of \( v \) is zero, the DLGBs degenerated to LGBs.

B. Random Phase Screen Transmission Theory of Atmospheric Turbulence

The power spectral density of refractive index \( \Phi_n(\kappa) \) is a physical quantity used to describe the fluctuations of atmospheric refractive index. Here, Von Karman’s spectral model is used to describe the following:

\[
\Phi_n(\kappa) = 0.033C_n^2 \exp \left[ \frac{-\kappa^2(L_0/5.92)^2}{\kappa^2 + (2\pi/L_0)^2} \right]^{11/6} \tag{3}
\]

where \( L_0 \) is the outer scale of turbulence, \( L_0 \) is the inner scale, \( C_n^2 \) is the structure constant of atmospheric refractive index, \( \kappa \) is the space wave number, and the value range is as follows: \( 2\pi/L_0 \leq \kappa \leq 2\pi/L_0 \). The relation between the power spectrum density of phase and that of refractive index is obtained as follows: \( \Phi(\kappa) = 2\pi^2 k^2 \Delta z \Phi_n(\kappa) \), where, \( \Delta z \) is the propagation distance of laser in the atmosphere turbulence. In this case, the atmospheric coherence length can be expressed as

\[
\Delta x = (0.423k^2 \Delta z C_n^2)^{-3/5}
\]

and the random phase screen of power spectrum inversion method is as follows:

\[
\varphi = \text{Re} \left\{ \mathcal{F}^{-1} \left[ h(k_x, k_y) \sqrt{\Phi_n(\kappa)} \right] \right\} \tag{4}
\]

where \( h(k_x, k_y) \) is the generation of complex Gaussian random matrix; it is the standard normal distribution function in the frequency domain. \( \Phi_n(\kappa) \) is the power spectral density matrix. Therefore, combined with Fresnel diffraction integral, the light field transmitted through an atmospheric turbulent phase screen can be expressed as follows:

\[
E(r, \varphi, z) = -\frac{ik \exp(ikz + i\varphi)}{2\pi z} \int \int dp d\Phi E(r, \varphi, z) \times \exp \left\{ \frac{ik}{2z} (\rho^2 + r^2 - 2\rho r \cos(\varphi - \phi)) \right\} \tag{5}
\]

C. Scintillation Coefficient, Beam Jitter, and Broadening

During laser transmission, the distribution of light intensity changes due to the influence of atmospheric turbulence. To describe the fluctuation degree quantitatively and eliminate the chance of random generation of phase screen as much as possible, the Scintillation Index (SI) of light intensity could be calculated by taking the mean value of multiple simulations, as follows:

\[
SI(r, \varphi, z) = \frac{I(r, \varphi, z)^2}{I(r, \varphi, z)} - 1 \tag{6}
\]

In (6), \( I \) represents the light intensity after turbulence. Probability density of light intensity accepted by free-space optical communication system in atmospheric turbulence is obtained as follows:

\[
p(I) = \frac{1}{I \sqrt{2\pi SI}} \exp \left\{ -\frac{(\ln(I/I_0) + SI/2)^2}{2SI} \right\} \tag{7}
\]

where, \( I_0 \) is the received light intensity without turbulence; it is the received light intensity under turbulence. The scintillation index is calculated in (5).

For OOK intensity modulation, the average BER can be written as follows:

\[
P_e = \int_0^\infty p(I)Q(\sqrt{SNR}I) dI \tag{8}
\]

where SNR is the average signal-to-noise ratio, which can be expressed as in the free-space optical communication system, \( R \) represents the response degree of photodetector, and \( \delta \) is the optical modulation index. The properties of Q function can be expressed as follows:

\[
Q(x) = \frac{1}{\pi} \int_0^{2\pi} \exp \left( -\frac{x^2}{2\sin^2\theta} \right) d\theta, \ x > 0
\]

Taking \( x \) as the exponential term of turbulence probability density function and combining with the Gaussian–Hermite orthogonal integration, the average BER can be simplified as...
follows:

$$P_{e_{OOK}} = \frac{1}{\sqrt{\pi}} \sum_{i} w_i Q \left\{ \sqrt{SNR} \exp \left( \sqrt{2SI} x_i - SI/2 \right) \right\}$$

(10)

Similarly, the average BER of BPSK and DPSK can be solved as shown in Eqs. (11) and (12), as follows:

$$P_{e_{BPSK}} = \frac{1}{\sqrt{\pi}} \sum_{i} w_i Q \left\{ \sqrt{SNR_0} \cdot \exp \left( SNR_1 \left( \sqrt{2SI} x_i - SI/2 \right) \right) \right\}$$

(11)

$$P_{e_{DPSK}} = \frac{1}{2\sqrt{\pi}} \sum_{i} w_i \exp \left[ -SNR \exp \left( 2\sqrt{2SI} x_i - SI \right) \right]$$

(12)

Where, $w_i$ and $x_i$ are the weight factors and zeros of m-order Hermite polynomials, respectively.

III. PHYSICAL COMPUTING MODEL

Fig. 1 represents the proposed physical calculation model. During the calculation, the transmission distance is 1000 m, a randomly generated atmospheric turbulence phase screen is placed every 200 m, and the light field passing through the first phase screen is synthesized as the source field passing through the next phase screen. The transmission result of atmospheric turbulence passing through 1000 m is obtained five times in one iteration.

IV. NUMERICAL SIMULATION RESULTS

A. Light Intensity and Phase Distribution Under Atmospheric Turbulence

The calculation parameters are shown in Table I, and the results are shown in Fig. 2.

Fig. 2 shows the light intensity and phase distribution of modes $\nu = 0(m_1 = 1, m_2 = 1), \nu = 1(m_1 = 1, m_2 = 2), \nu = 3(m_1 = 1, m_2 = 4),$ and $\nu = 5(m_1 = 1, m_2 = 6)$ under different turbulence intensity. The light field distribution characteristics of the source field indicate that the superimposed double-vortex beam produces spot splitting, and the number of split spots is the same as the difference in topological charge. Moreover, the phase presents an internal and external double-ring structure. The inner and outer double rings reflect the helical phase information of the superimposed two beams without interfering with each other. After atmospheric turbulence, the intensity distribution and phase of the light field are blurred and become more intense with the increase in turbulence intensity. This condition is due to the turbulence intensity that reflects the fluctuation of atmospheric refractive index, and the larger fluctuation indicates more intense beam refraction and energy attenuation.

| Parameters | Value  | Parameters | Value  |
|------------|--------|------------|--------|
| Wavelength $\lambda$ | 1550nm  | Screen Width $D$ | 0.6m  |
| Outer Scale $L_a$ | 100m   | Screen spacing $\Delta x$ | 200m |
| Inner Scale $L_i$ | 0.01m  | Grid Width | 200  |
| Waist radius $w_i$ | 0.03m  | Statistics Times | 100   |
| Total distance $L$ | 1000m  | Radial mode $p$ | 0     |

![Image of intensity and phase distribution of two-vortex beams in atmospheric turbulence.](image)
B. Analysis of Scintillation Coefficient and BER of Superimposed Vortex Beam

Simulation for 100 times was carried out for each group of superimposed vortex beams to calculate their scintillation coefficients by averaging to reduce the randomness of phase screen generation. Beam parameters and phase screen parameters were consistent with the results in Table I, and other calculation parameters are shown in Table II.

As shown in Fig. 3(a), when the topological charge difference of DLGBs is small, the scintillation index after turbulence is lower than that of non-superposition traditional LGB \((v = 0)\), and the scintillation index decreases with the increase in topological charge difference, when the topological charge difference \(v\) is greater than 10, the scintillation index increases with the increase in topological charge difference. At the same time, the stronger turbulence intensity indicates lower scintillation index of the DLGB. Lower topological charge difference indicates more advantages than the traditional LGB. The reason is that when the mode difference is small, the number of spot splitting is also small, the energy density of the unit spot focusing is large, the penetration performance of turbulence is good, and the scintillation index is low. When the number of spot splitting increases, the beam energy is dispersed. As the area of the light field affected by turbulence increases, the scintillation index increases, and finally exceeds the traditional LGB. When the turbulence is stronger, the vortex beam with lower topological charge difference has more evident energy focusing characteristics. As shown in Fig. 3(b), the atmospheric turbulence intensity is positively correlated with light intensity scintillation. Compared with the traditional LGB, the DLGB with lower topological charge difference can maintain relatively stable and lower scintillation index in the environment with increasing turbulence.

The BER of BPSK modulated signal of LGB compared with DLGB is considered to further analyze the influence of different DLGBs on the modulation mode, and the calculation parameters are shown in Table III.

| Fig. 4(a) | \(v\) | \(C_s^2\) |
|-----------|-------|----------|
| (a)       | 0, 10, 30 | \(1 \times 10^{-14}\) |
| (b)       | 10      | \(1 \times 10^{-15}, 1 \times 10^{-14}, 1 \times 10^{-13}\) |

Fig. 4. Scintillation coefficient of double vortex beams in atmospheric turbulence.

Fig. 5. BER comparison of DLGBs under different modulation modes in atmospheric turbulence.

Fig. 4(a) shows that the BER decreases with the increase in receiving intensity. Meanwhile, the BER of the beam with small topological charge difference \((v = 10)\) is smaller than that of the dual-vortex beam with large topological charge difference \((v = 30)\), consistent with the change law of scintillation coefficient, and the reason is in accordance with (16). Fig. 4(b) reflects the general rule that the stronger turbulence intensity indicates greater BER, consistent with the trend of Fig. 3.

In order to analyze the communication performance of the superposition vortex beam in different debugging modes, BPSK, DPSK, and OOK modulation modes are selected for BER comparison under the condition that the topological charge difference is 10 and the turbulence intensity is \(1 \times 10^{-14}\), and the results are shown in Fig. 5.

Under the same calculation conditions, Fig. 5 presents the BER of three modulation modes, BPSK, DPSK, and OOK. These results show that OOK has the best communication quality among the three modulation modes because OOK does not need the local carrier to participate in demodulation, thereby reducing the possibility of error generation, but it has the disadvantage of requiring adaptive threshold. DPSK is the highest under the same
conditions because the modulation of DPSK requires absolute phase detection, leading to error diffusion between symbols.

V. CONCLUSION

Random phase screen method is applied in this article, which studies the DLGB through the atmospheric turbulence intensity and phase distribution. The scintillation index was evaluated on this basis. The topological charge difference $\nu$ is related to the number of split spots. The scintillation index of DLGBs decreases initially and then increases as the value of $\nu$ increases continuously because low $\nu$-value ($\nu < 10$) means concentrated power of light, which can help penetrate the turbulence. Then, the three modulation error rates of BPSK, DPSK, and OOK are calculated on the basis of scintillation index. The results show that OOK modulation has superior BER performance, whereas DPSK is unsuitable for transmission in atmospheric turbulence due to the symbol error diffusion. These results provide theoretical basis for the study of atmospheric optical communication.

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