Entanglement generation and perfect state transfer in ferromagnetic qubit chains

Giulia Gualdi, Irene Marzoli and Paolo Tombesi
Dipartimento di Fisica, Università degli Studi di Camerino, I-62032 Camerino, Italy
E-mail: giulia.gualdi@unicam.it

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Abstract. We propose to use ferromagnetic systems for entanglement generation and distribution together with perfect state transfer between distant parties in a qubit chain. The scheme relies on an effective two-qubit dynamics, realized by leaving two empty sites in a uniformly filled chain. This allows long-range interacting qubit chains to serve as quantum channels for both tasks with optimal performances. Remarkably, the entanglement between sender and receiver sites is independent of both the transmission distance and the system size. This property opens new perspectives for short- and mid-range quantum communication with qubit chains.

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1 Author to whom any correspondence should be addressed.
1. Introduction

Quantum tasks such as quantum teleportation require the generation of entanglement as well as the transfer of a quantum state between distant parties [1]. Several theoretical works have shown that antiferromagnetic systems are suited for generating and distributing entanglement between distant parties [2]–[4]. This capacity is related to the specific ground state properties of the different antiferromagnetic systems. So far, all proposals have concentrated on nearest-neighbour (NN) interacting spin systems, due to the difficulty in finding the ground state for an arbitrary long-range interacting system. Antiferromagnetic systems allow to generate entangled states, but the amount of entanglement decreases with the distance between the members of the entangled pair.

On the other hand ferromagnetic systems may serve as quantum channels for short- and mid-range quantum communication [5]. These systems are traditionally regarded as being capable of transmitting a quantum state [5]–[8], but as not being able to generate any entanglement between two distant parties [4]. In general, entanglement generation and perfect state transfer have not been put into direct relation for ferromagnetic systems. Rather, different classes of physical systems have been proposed for accomplishing the two goals separately. One exception is represented by the ferromagnetic branched chain analysed in [9]. Here, one can generate entanglement between two receivers by means of a state transfer protocol. This implies that entanglement generation is not distinguishable from state transfer, so that the two tasks are not separately addressed. Moreover, the procedure is not scalable, since it relies on specifically engineered qubit couplings [6, 7].

In this paper, we prove that starting from an \( N \)-qubit system, whenever it is possible to restrict the dynamics to an effective two-qubit subspace, one obtains a quantum channel suitable for both optimal state transfer and entanglement generation and distribution. Remarkably enough our procedure applies to ferromagnetic systems as well. According to the time at which the final measurement takes place one can select, on the same quantum channel, one of the two tasks—state transfer or entanglement generation. Moreover, the generated entanglement is independent of both system size and transmission distance.

Resorting neither to dynamical control [10] nor to ancillary systems [11, 12], perfect state transfer between two distant parties of a ferromagnetic system is attained only if its spectrum is mirror-periodic [6, 7], [13]–[15]. This means that the time evolution maps an arbitrary quantum state into its mirror symmetric with respect to the centre of the system. For example, when the information is initially encoded at one end of an open chain, after the so-called transfer time we will retrieve the message at the opposite end. There are two possible strategies to achieve this goal. The first one is to design an \( N \)-qubit mirror-periodic system by carefully adjusting the coupling strengths between the members of an \( N \)-qubit array [6, 7, 9], [13]–[15]. The second one is to map an \( N \)-qubit system into an effective two-qubit system, which is by definition mirror-periodic. This task is accomplished by slightly detaching sender and receiver from the channel [2], [17]–[19]. In this paper, we show that a system which exhibits perfect state transfer by means of this second approach, is also capable of generating maximally entangled states between distant parties. In contrast, \( N \)-qubit mirror-periodic systems are prevented from reaching this goal.

This paper is organized as follows: in section 2, we derive analytically the conditions for generating maximally entangled states between the two ends of a generic \( N \)-qubit chain. We then discuss, in section 3, the relationship between entanglement generation and perfect
state transfer, and suggest a scalable and straightforward-to-implement protocol to achieve both tasks. As an example, in section 4, we apply our strategy to a long-range interacting spin chain, described by the Heisenberg XYZ isotropic model. In section 5, we provide numerical results supporting our theoretical insight. Finally, we draw our conclusions in section 6.

2. Conditions for the generation of maximally entangled states

Consider an \( N \)-spin system, whose Hamiltonian \( \mathcal{H} \) preserves the total magnetization \( M = \sum_{i=1}^{N} \sigma_i^z \), where \( \sigma_i^z \) is the \( z \)-component of the Pauli spin operator acting on the spin at the \( i \)th site. We consider the case of transmitting a single excitation from sender to receiver (figure 1(a)). As demonstrated in [17], perfect state transfer for a generic system, without a mirror-periodic spectrum, implies that the \( N \times N \)-dimensional Hilbert space \( \mathcal{H} \) of the system must be expressed as the sum of two disjoint subspaces, \( \mathcal{H}^{N \times 2 \times N} \) and \( \mathcal{H}^{(N-2) \times (N-2)} \), corresponding, respectively, to the sender–receiver subspace and to the rest of the chain, i.e. the channel. Ideally, perfect state transfer of an excitation from the sender site \( s \) to the receiver site \( r \) only involves a pair of eigenvectors

\[
|\lambda\pm\rangle = \frac{1}{\sqrt{2}}(|s\rangle \pm |r\rangle),
\]

where \(|s\rangle\) denotes the basis state with all the qubits in the \(|0\rangle\) state except the one at the \( i \)th site which is in \(|1\rangle\). In a truly two-qubit space, restricted to sender and receiver only, these eigenvectors can be written as

\[
|\lambda\pm\rangle = \frac{1}{\sqrt{2}}(|10\rangle \pm |01\rangle) \equiv |\Psi\pm\rangle.
\]

We note that the ideal system eigenvectors correspond to the two Bell states, \(|\Psi\pm\rangle\), with the same magnetization for both members of the entangled state. Hence, perfect state transfer is achieved owing to the interplay between the two maximally entangled states \(|\Psi^+\rangle\) and \(|\Psi^-\rangle\). This observation suggests that the same system should be able to generate entanglement as well.

To this end, we now compute the concurrence [20] as a measure of the entanglement between a generic sender–receiver pair of an \( N \)-qubit system. The system, initially prepared
in the separable state
\[ |\Psi(0)\rangle = \cos \frac{\theta}{2} |0\rangle + e^{-i\phi} \sin \frac{\theta}{2} |s\rangle, \]
(3)
evolves according to the unitary time-evolution operator
\[ U(t) = \exp(-iHt) \quad \text{with} \quad \hbar = 1 \]
(4)
into
\[ |\Psi(t)\rangle = U(t)|\Psi(0)\rangle. \]
(5)
Following [20], the concurrence is defined as
\[ \mathcal{C} = \text{max}\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \]
where the \( \lambda_i \)'s are, in decreasing order, the square roots of the eigenvalues of the non-Hermitian matrix \( \rho_{s,r}(t) \). The reduced density matrix \( \rho_{s,r}(t) \) is obtained after tracing over all the qubits in the channel besides sender and receiver, whereas \( \tilde{\rho}_{s,r}(t) = (\sigma^y \otimes \sigma^y) \rho_{s,r}(t)(\sigma^y \otimes \sigma^y) \) and \( \sigma^y \) is the Pauli spin-flip operator. After some algebraic manipulations, we reach the following expression for the concurrence between the two sites \( s \) and \( r \), i.e. between the two states \( |s\rangle \) and \( |r\rangle \):
\[ \mathcal{C}(t) = 2 \sin^2 \frac{\theta}{2} \left| f_{s,s}(t) \right| \left| f_{s,r}(t) \right|, \]
(6)
where
\[ f_{s,n}(t) = \langle n | U(t) | s \rangle \]
(7)
is the probability amplitude of propagating the information from the sender site \( s \) to the generic site \( n \). We note that the concurrence equation (6) depends on the initial probability of having one excitation in the chain, from which the system can generate the entangled state. Indeed, in our case, this probability is given by \( \sin^2(\theta/2) \). Therefore, we set \( \theta = \pi \). Thus the concurrence \( \mathcal{C}(t) \) becomes directly comparable to the transfer fidelity \( F(t) = |f_{s,r}(t)|^2 \), when the latter is not averaged over the Bloch sphere\(^2\).

We now examine the conditions under which the concurrence \( \mathcal{C}(t) \) reaches the unitary value as a function of the transition amplitudes. For each \( t \), this search is subject to the probability conservation constraint \( |f_{s,s}(t)|^2 + |f_{s,r}(t)|^2 + |f_{s,r}(t)|^2 = 1 \), where \( \Gamma^2(t) = \sum_{i \neq s,r} |f_{s,i}(t)|^2 \) represents the dispersion of information in the channel, i.e. on all the sites besides \( s \) and \( r \). This condition leads to the individuation of a curve of local maxima given by \( |f_{s,s}(t)|^2 = |f_{s,r}(t)|^2 \). Hence, at time \( t \) we find a local maximum of \( \mathcal{C}(t) \) if
\[ |f_{s,s}(t)|^2 = |f_{s,r}(t)|^2 = \frac{1 - \Gamma^2(t)}{2}, \]
(8)
which becomes the absolute maximum iff
\[ \Gamma^2(t) = 0 \quad \text{and} \quad |f_{s,s}(t)|^2 = |f_{s,r}(t)|^2 = \frac{1}{2}. \]
(9)
Equations (8) and (9) provide the conditions for generating maximally entangled states. In the next section, we show that these conditions are satisfied by systems that attain perfect state transfer by means of an effective two-qubit dynamics.

\(^2\) Usually the fidelity is averaged over all the possible input states to give \( F(t) = |f_{s,r}(t)|^2/6 + |f_{s,r}(t)|/3 + 1/2. \)
3. Entanglement generation and perfect state transfer

It is convenient to express the propagator in equation (7) in terms of the system eigenvalues \( \{E_j\} \) and eigenvectors \( \{|\lambda_j\rangle\} \)

\[
f_{s,n}(t) = \sum_{j=1}^{N} \langle n | \lambda_j \rangle \langle \lambda_j | s \rangle \exp(-iE_j t)
\]  

(10)

in order to recast equation (8) as

\[
\left| \sum_{j=1}^{N} |\sigma_j|^2 \exp(-iE_j t) \right| = \left| \sum_{j=1}^{N} \sigma_j \rho_j^* \exp(-iE_j t) \right|.
\]  

(11)

The relevant quantities are

\[
\sigma_j = \langle \lambda_j | s \rangle, \quad \rho_j = \langle \lambda_j | r \rangle, \quad |\gamma_j|^2 = \sum_{i \neq (s,r)} \langle \lambda_j | i \rangle^2,
\]  

(12)

which represent the projection of the \( j \)th eigenvector \( |\lambda_j\rangle \) on, respectively, the sender state, the receiver state and the remaining system states. For each \( j \), the normalization condition \( |\sigma_j|^2 + |\rho_j|^2 + |\gamma_j|^2 = 1 \) is fulfilled. If one looks for a solution of equation (8) independent of the size \( N \) of the system, i.e. scalable, and of related specific properties of the spectrum, then equation (11) provides a set of conditions for each \( |\sigma_j|, |\rho_j| \) namely

\[
|\sigma_j|^2 = |\rho_j|^2 = \frac{1 - |\gamma_j|^2}{2} \quad \forall j.
\]  

(13)

which represent exactly the local maximum conditions for perfect state transfer \cite{17} in the case of a generic spectrum.

Next, in order to satisfy also equation (9), we consider

\[
\Gamma^2(t) = \sum_{j=1}^{N} |\sigma_j|^2 |\gamma_j|^2 + 2 \sum_{j < j'} |\sigma_j| |\sigma_{j'}| |\gamma_j\gamma_{j'}| \cos \left[ (E_j - E_{j'}) t + \xi_{j,j'} \right],
\]  

(14)

where \( |\gamma_j\gamma_{j'}| = \sum_{i \neq s,r} |\langle \lambda_j | i \rangle||\langle \lambda_{j'} | i \rangle| \) and \( \xi_{j,j'} \) is a phase factor. Obviously

\[
\Gamma^2(t) \leq \sum_{j=1}^{N} |\sigma_j|^2 |\gamma_j|^2 + 2 \sum_{j < j'} |\sigma_j| |\sigma_{j'}| |\gamma_j\gamma_{j'}|,
\]  

(15)

regardless of the specific spectrum of the system. Moreover, due to the normalization \( |\gamma_j\gamma_{j'}| \leq |\gamma_j|^2 \). Hence, we can further maximize equation (15), by writing

\[
\Gamma^2(t) \leq \sum_{j=1}^{N} |\sigma_j|^2 |\gamma_j|^2 + 2 \sum_{j < j'} |\sigma_j| |\sigma_{j'}| |\gamma_j|^2 \leq N \sum_{j=1}^{N} |\sigma_j|^2 |\gamma_j|^2 \equiv N \Gamma_M.
\]  

(16)

The necessary and sufficient condition in order to have \( \Gamma^2(t) = 0 \) is that \( |\gamma_j|^2 = 0 \) for each \( j \) for which \( |\sigma_j|^2 \neq 0 \). Now, we prove that there can be only two eigenvectors for which \( |\sigma_j|^2 \neq 0 \).
and $|\gamma_j|^2 = 0$. Given the normalization constraint $\sum_j |\gamma_j|^2 = N - 2$, in order to minimize $\Gamma_M$ we need to find its lowest extreme, with respect to $|\gamma_j|^2$. When we use the local maximum conditions, equation (13), we obtain

$$\Gamma_M = \sum_{j=1}^{N} \frac{1 - |\gamma_j|^2}{2} |\gamma_j|^2. \quad (17)$$

The extremal point is then reached for $|\gamma_j|^2 = (N - 2)/N$ for which $\Gamma_M = (N - 2)/N$. The absolute minimum, $\Gamma_M = 0$, is attained for $N = 2$, i.e. for a truly two-qubit system. We note that for $N = 2$ also $|\gamma_j|^2 = 0$. Hence, to generate a maximally entangled state, when $N > 2$, only two eigenvectors must have finite projections on sender and receiver states and zero projections on the other system states. This means that the $N$-qubit Hilbert space of the system can be decomposed as $\mathcal{H}^{N\times N} = \mathcal{H}^{2\times 2}_{s,r} \otimes \mathcal{H}^{N-2\times N-2}_{\text{channel}}$, which is precisely the perfect state transfer condition for a generic system [17].

### 4. The isotropic Heisenberg model

We now study the properties and time behaviour of the concurrence $C(t)$ for a finite linear chain of interacting spins according to the isotropic XYZ Heisenberg model

$$H = \frac{1}{2} \sum_{i,j=1}^{N} J_{i,j} (S_i \cdot S_j - 3S_i^z S_j^z), \quad (18)$$

where $S_i$ and $S_j$ are the total spin operators at sites $i$ and $j$ and $S_i^z$ and $S_j^z$ are the respective $z$ components. According to the specific form of the coupling strength $J_{i,j}$ between spins $i$ and $j$, the Hamiltonian equation (18) encompasses different cases ranging from NN to long-range interaction. For instance, when $J_{i,j} = J_s \delta_{i+1,j}$ the coupling is nonzero for nearest-neighbouring spins only. In the present work, we consider, instead, a long-range interaction characterized by

$$J_{i,j} = \frac{C}{(a |i - j|)^\nu}, \quad (19)$$

where $\nu > 0$, $a$ is the fixed inter-spin distance and $C$ is a model-dependent constant. In particular, the case $\nu = 3$ corresponds to the dipolar coupling [16].

Sender and receiver are located at the ends of a chain of $N$ equally spaced sites. In the truly two-qubit sender–receiver subspace the propagator equation (10) of the excitation from sender to receiver can be calculated analytically [17]

$$|f_{s,r}(t)|^2 = \sin^2 \left( \frac{\Delta}{2} t \right), \quad (20)$$

where $\Delta = (E_+ - E_-) = 2J_{s,r}$ is the difference between the two system energy eigenvalues $E_+$ and $E_-$, corresponding to the eigenvectors $|\lambda_\pm \rangle$ of equation (2). We see that $\Delta$ is proportional to the coupling strength $J_{s,r} = C/[a(N - 1)]^\nu$ between sender and receiver, which in turns depends on the distance ($\propto (N - 1)$) between the two ends of the chain. Perfect state transfer takes place at the time $T = \pi/\Delta$ when $|f_{s,r}(T)|^2 = 1$ (figure 1(c)). The transmission distance
affects only the transfer time through the explicit expression of $J_{s,r}$. This is also true for a $N$-qubit system whenever the dynamics is confined to the sender–receiver subspace, regardless of the actual system size. We note that the maximum concurrence, equation (9), is reached at the time $t = T/2$, when $|f_{s,s}(t)|^2 = |f_{s,r}(t)|^2 = 1/2$ (figure 1(b)). We emphasize that entanglement generation between sender and receiver shares the same properties as the information transfer. Both concurrence and fidelity peak to the same maximum value regardless of the distance between the two parties and of the number of qubits in the channel.

The correspondence between perfect state transfer and entanglement generation applies only to non mirror-periodic systems. Instead, for mirror-periodic systems [6, 7] perfect state transfer represents the mark of minimum concurrence between $s$ and $r$. In these cases, the system evolves in a $N$-qubit space (see, for instance, the fictitious-particle representation in [6, 7]), such that $|\gamma_j|^2 \neq 0$ always, for each system eigenvector. This implies that $\Gamma^2(t) \neq 0$, for each value of $t$ except for $T$, the time at which perfect state transfer takes place. At this time, however, $|f_{s,s}(T)|^2 = 0$ and $|f_{s,r}(T)|^2 = 1$, thus the conditions expressed in equation (9) are never simultaneously fulfilled. This fact is reasonable from a physical point of view. In fact, in mirror-periodic systems, perfect state transfer is reached at the time $T$ when all the $N$ eigenfrequencies of the system recombine coherently and the message revives at the sender-mirroring point in the chain. This implies that all degrees of freedom of the system are strongly coupled. The formation of an entangled state, instead, takes place only when two degrees of freedom are strongly correlated and fully detached from the others. Indeed $|\gamma_j|^2$, which represents the mixing between the sender–receiver subspace and the rest of the chain, quantifies how closely a system resembles a two-qubit one.

5. Numerical results

To provide numerical evidence of our analytical results, in figure 2, we plot the concurrence and the transfer fidelity for the interacting XYZ Heisenberg ferromagnetic qubit chain, equation (18). We compare the performances of mirror-periodic cases, (a) and (b), to those of systems with a generic spectrum, (c) and (d). In figure 2(a), the mirror-periodic Hamiltonian for $N = 10$ sites is characterized by a NN interaction with a non-uniform $J$ coupling, chosen according to the procedure outlined in [6, 7]. Plot (b) pertains to the example presented in [15] of a mirror-periodic Hamiltonian with dipole-like interaction and non-uniform coupling strength. We note that in both cases the fidelity achieves the unitary value. The concurrence, instead, is practically zero in case (a), hence indistinguishable from the horizontal axis, and very close to zero in case (b). Plot (c) depicts the behaviour of a uniform chain with dipolar interaction. We note that both fidelity and concurrence approximately achieve the same maximum value, though at different times. In this case the dynamics is approximately dominated by the two lowest-energy eigenvectors. Therefore $|\gamma_j|^2$ is small, but not negligible. The truly two-qubit dynamics is attained by the so-called double-hole (DH) model [17], obtained from the same system by removing sender and receiver nearest-neighbouring spins (figure 2(d)). This situation is represented by the Hamiltonian, equation (18), with the coupling strength $J_{i,j}$ set equal to zero whenever $i, j = 2, N − 1$. In the DH case $|\gamma_j|^2$ is zero for any practical purpose. In other words, no information is left in the channel, but it periodically oscillates between sender and receiver. Therefore, the DH chain not only attains perfect state transfer but also generates maximally entangled states. Also relevant for experimental implementations, it is the regular time-behaviour of both concurrence and fidelity, which greatly relaxes the time resolution.
Figure 2. Concurrence (solid line) and fidelity (dashed line) as a function of time for mirror-periodic, (a) and (b), and generic, (c) and (d), spin chains. (a) NN interaction with \( N = 10 \) sites. (b) Dipolar interaction with \( N = 6 \) sites engineered according to [15]; (c) \( N = 10 \) sites and uniform filling; (d) \( N = 10 \) sites in the DH setup. Time is measured in units of \( a^3/C \) (see equation (19)). In case (a) the parameter \( \lambda \) of [6] is set equal to \( 2C/a^3 \).

requested for the measurement. This increased regularity also reflects the enhanced robustness of the DH system with respect to errors due to thermal excitation. Indeed, as a consequence of the reduced mixing between the two-qubit subspace and the rest of the chain, the energy gap between the two lowest eigenvalues and the rest of the spectrum is greatly increased. This decreases, according to the Boltzmann distribution, the transition probability to higher energy states. Therefore, the DH chain is more robust against thermal noise with respect to the complete chain. Moreover, reducing the portion of the chain effectively involved in the communication process makes the system less exposed to decohering processes, taking place in the channel between sender and receiver.

In figure 3, we plot the concurrence as a function of the distance between sender and receiver for a chain of equally spaced qubit sites. In the case of the dipolar complete chain (uniform filling) the concurrence strongly decreases with increasing number of spins between sender and receiver. This behaviour can be understood in terms of the spectral properties of the system. The longer the chain, the smaller the difference in energy between the two lowest eigenvectors and all the others. Therefore the mixing between the sender–receiver subspace and the channel becomes less and less negligible (\(|\gamma_j|^2 \neq 0\)). To restore the ideal two-qubit dynamics it is sufficient to leave empty the sender and receiver nearest-neighbouring sites. Indeed, from figure 3 we see that the concurrence of the DH system is practically insensitive to the distance between sender and receiver and to the number of spins in between.

The DH scheme can be regarded as the implementation of a \( N \)-to-2 qubit mapping, which allows recovery of the Rabi oscillations between the two states of the system. In general the dynamics of the system can always be traced back to Rabi oscillations, but in the wrong basis. Indeed, in the case of the complete chain, Rabi oscillations involve the two halves of the chain. Hence, in order to recover a two-state dynamics we should adopt two blocks of the chain as sender and receiver states. However, since we have access only to two spins, our aim is to

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Figure 3. The maximum concurrence between sender and receiver as a function of the number $N$ of sites, for a complete chain (red circles) and for the DH chain (blue diamonds).

confine Rabi oscillations to the portion of the chain where these two spins are located. This goal is accomplished by removing the states which have non-negligible overlap with the eigenvectors carrying the information.

6. Conclusions

In this paper, we have proved that a ferromagnetic qubit system can be used as a quantum channel not only for optimal state transfer but also for generating maximally entangled states over an arbitrary distance. Starting from a long-range interacting spin chain, with a generic spectrum, we are able to confine the dynamics of the system to an effective two-qubit subspace. Once the dynamics is governed by these two qubits, one can either retrieve the information with unitary fidelity or prepare the system in one of the two Bell states $|\Psi^\pm\rangle$. Our approach is fully scalable, since it only requires to leave empty the nearest-neighbouring sites of sender and receiver. Our results include in a more exhaustive theoretical frame the schemes proposed in [18, 19], whereas they extend to ferromagnetic systems the concepts about long-distance entanglement generation outlined in [3].

Given the minimal spatial and temporal control, requested by our approach, over the interacting spin chain, we expect that the system performances are relatively robust even in the presence of thermal noise, fluctuations in the relevant parameters, and other sources of decoherence. We can envisage a straightforward implementation of the proposed scheme with strings of trapped particles [21]–[28]. Indeed, in a linear ion trap [21] the DH scheme could be implemented shifting out of resonance, with respect to the driving lasers which allow to establish the effective spin–spin interaction, the two ions trapped, respectively, at the sites 2 and $N - 1$. Instead, when using an array of microtraps, each one holding a single particle, the DH scheme requires only to switch-off the coupling between the sender and receiver nearest-neighbouring traps and the rest of the system. Alternatively, the experimenter may decide to leave empty these two traps, having the possibility to individually control the trap filling.
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