Simulation of Bose-Einstein effect
using space-time aspects of Lund string fragmentation model

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Abstract

The experimentally observed enhancement of number of close boson pairs in $e^+e^-$ collisions is reproduced by local weighting according to the quantum mechanical prescriptions for production of identical bosons. The space-time picture of the process, inherently present in the Lund fragmentation model, is explicitly used.

The model is used to check systematic errors in the W mass measurements due to the Bose-Einstein effect.

The possibility of direct implementation of the Bose-Einstein effect into string fragmentation is discussed.

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1 Introduction

Recently, the Bose-Einstein (BE) effect in particle production in $e^+e^-$ annihilations received particular attention as LEP doubled its collision energy, allowing for direct production of WW pairs.

The influence of the BE effect on the measured W mass at LEP2 was first investigated in Ref. [1]. The standard JETSET implementation of BE effect (routine LUBOEI, Ref. [2]), used in this study, reshuffles momenta of generated particles to increase the fraction of close boson pairs according to a phenomenological parameterization. The method has some technical shortcomings (as local violation of energy/momentum conservation laws) but the basic problem is that it actually doesn’t make any connection between the quantum mechanical (QM) origin of the effect and its observable consequences, and therefore it has relatively low predictive power; as a result, only a very vague estimation of systematic error could be drawn out [3]. Recently, other studies [4], [5] used the phenomenological formula for global event weighting to extract the systematic uncertainty on the W mass measurement; this uncertainty was found below 20 [4] or 30 [5] MeV, but the procedure backfired by predicting a change in $R_b$ and $R_c$ of 10-20 % in $Z^0$ decay [4], which is not observed.

The problem with all studies mentioned above is that they are based uniquely on the single external appearance of the BE effect – the enhancement of production of close pairs of identical bosons – while this is probably only the most visible consequence of more fundamental physical processes taking part in the hadronization.

A fairly better way towards understanding the BE interference consists in implementing it into the simulation starting from "first principles", i.e. starting from QM formulae, and only then to check the consistency of the predictions with experimental data.

While the possibility to include QM interference effects into string fragmentation models (Artru-Mennessier, LUND) was pointed out a long time ago [6], [7], only quite recently a Monte-Carlo (MC) implementation of these ideas appeared [8].

The method presented in this paper – while having some common features with the LUND approach – simplifies the full QM treatment by resigning on higher order correlations; also, the global event weighting is replaced by “local” implementation of BE correlations.

The behaviour of simulated data is discussed, and they are compared to the experimental data. An alternative tool for study of particle correlations – factorial moments – is used to compare the standard JETSET simulation (LUBOEI) with the new one, presented in this paper.

The influence of the BE effect on the measurement of the W mass is investigated, and the systematic uncertainty due to this effect is estimated.

The last section of this paper deals with possible strategies and simplifications for future simulations of the BE effect.
2 Correlation function

The Bose-Einstein interference (or Hanbury-Brown-Twiss effect in astronomy) is experimentally seen as an enhanced probability of observing two (and more) identical bosons with a similar momentum. In the language of QM, this enhanced probability arises from the symmetrization of the amplitude with respect to the exchange of identical bosons.

If we describe the one-particle wave function by a planar wave

$$\Phi_i \sim \exp \left\{ -\frac{i}{\hbar} p_i (x - x_i) \right\}$$

where \( p \) is the 4-momentum and \( x_i \) the production vertex of the particle, then the symmetrization of an \( N \) particle wave function \( \Phi_1 \Phi_2 ... \Phi_N \) in the case of \( N \) identical bosons gives the amplitude:

$$\Psi(N) = \frac{1}{\sqrt{N!}} \sum_{i_i} \exp \left\{ -\frac{i}{\hbar} [p_{i1}(x - x_1) + p_{i2}(x - x_2) + ... + p_{iN}(x - x_N)] \right\}$$

(1)

and the probability:

$$P(N) = |\Psi(N)|^2 =$$

$$= \frac{1}{N!} \sum_{i_i, j_j} \exp \left\{ -\frac{i}{\hbar} [(p_{i1} - p_{j1})x_1 + (p_{i2} - p_{j2})x_2 + ... + (p_{iN} - p_{jN})x_N] \right\}$$

$$= 1 + \frac{1}{N!} \sum_{i \neq j, k \neq l} \exp \left\{ -\frac{i}{\hbar} [(p_i - p_j)x_k + (p_j - p_i)x_l] \right\}$$

$$+ \frac{1}{N!} \sum_{i \neq j, \neq m, \neq n, \neq l} \exp \left\{ -\frac{i}{\hbar} [(p_i - p_j)x_k + (p_j - p_m)x_l + (p_m - p_i)x_n] \right\}$$

(2)

The number of terms in the symmetrised formula increases as \( (N!)^2 \) (which already indicates the complexity of evaluating higher orders with many identical bosons).

Throughout this paper, only 2- and 3- particle interference terms will be used, rewritten in the convenient form:

- 2-particle correlations \( \sim \sum_{i < j, k \neq l} \cos [(p_i - p_j) \cdot (x_k - x_l)/\hbar] \)

- 3-particle correlations \( \sim 2 \sum_{i < j \neq m, k \neq l \neq n} \cos [(p_i - p_j)(x_k - x_n) + (p_j - p_m)(x_l - x_n)]/\hbar] \)

In general, all interference terms can be expressed in terms of \( dp \cdot dx \), the invariant product of the difference in momentum and in space-time distance of the production vertices. How this variable can be evaluated in the string fragmentation model is discussed in the next section.
3 Space-time picture of string fragmentation

The Lund string fragmentation model [3], in the form of its MC implementation JETSET [4] is commonly used in simulations of hadronic final states at high energies due to its ability to reproduce the experimental data quite well. The interesting property of this model with respect to the study of the BE effect lies in the possibility to reconstruct the space-time picture of string breaking. The initial position of string fragment – the production point of final hadron – can then be derived.

\[ a \]

\[ b \]

Figure 1:

Schematically, the situation is shown in Fig.1a); the original string, spanned between two endpoint partons, carries in its rest frame a longitudinal energy density \( |\vec{\kappa}| \simeq 1 \text{ GeV/fm} \) \((\vec{\kappa} \text{ is called "string tension"})\). The string breaks by creating a new quark-antiquark pair (“tunneling” mechanism); the new quarks are supposed to be produced with a zero longitudinal initial momentum (longitudinal with respect to the string direction) and a non-zero transverse momentum \((\pm \vec{p}_t)\); due to the string tension, they separate and move in opposite directions, acquiring a momentum \( p_{long} = \pm |\vec{\kappa}| dt \) (Fig: 1b).

\[ \text{Figure 2:} \]

Two neighbour string breakings give birth to a hadron; its energy and momentum can be expressed in terms of space-time coordinates of the string breaking (see Fig.2).

\[ E_{had} = \kappa dl = \kappa |x_i - x_{i+1}| \]

\[ \vec{p}_{had} = \vec{p}_i + \vec{p}_{i+1} + \vec{\kappa}(t_i - t_{i+1}) \]  \( (3) \)

Alternatively, the coordinates of string breaking can be expressed as a function of the momenta of final hadrons. Each breaking divide the total number of final hadrons into two parts –
left [L] and right [R]– according to the part of string they came from. Presuming that the string starts to expand from point [0,0] in its rest frame, then the coordinates of the i-th breaking are:

\[ x_i = \frac{\left( \sum_{L_i} E_{had} - \sum_{R_i} E_{had} \right) / \kappa}{t_i = \frac{\left( p_0 - \sum_{L_i} p_{had-long} \right) / \kappa}{\left(4\right)} \]

where \( p_0 \) stands for the initial momentum of the endpoint partons. Therefore, the calculation of coordinates of string breaking is straightforward for a simple \( q\bar{q} \) string.

However, things become considerably more complicated in the case of gluon radiation because of the complicated string movement around gluon corners (kinks). The algorithm finding the position of the string at the moment of its breaking is actually the most complicated part of the whole simulation of the BE effect. It follows closely the fragmentation process in JETSET and evaluates the space-time coordinates in parallel with the generation of hadron’ momenta.

Once the points where a string broke are found, the production vertices of hadrons can be calculated. A kind of convention needs to be adopted here, because the hadron is not a point-like object and because the two endpoint string breakings are causally disconnected. Therefore, by the production vertex of the hadron we will understand the barycentre of the string piece forming the hadron in the frame where the two endpoint breakings occur simultaneously. For a simple \( q\bar{q} \) string in its rest frame, the coordinates of the production vertex of the hadron will be:

\[ \vec{x}_{had} = 0.5(\vec{x}_i + \vec{x}_{i+1}); \quad t_{had} = 0.5(t_i + t_{i+1}) \]

(5)

Since one is usually only interested in the momentum spectrum of the produced hadrons, the space-time history of the fragmentation is not evaluated in JETSET. Therefore, this information had to be traced back and added into the standard event record.

Knowing the space-time distribution of the hadrons, we are now able to evaluate the \( dp \cdot dx \) terms in the correlation function of section 2. The problem is that for the moment, our correlation function (Eq.2) does not take into account the dynamics of the process of hadronization. We can however use the QM framework of the Lund fragmentation model developed in Ref.\[6\],\[7\],\[8\]. On the basis of the argumentation provided in these studies, not only the probability of string breaking can be related to the area \( A \) spanned by the string (the space-time integral over string movement, Fig.3) but also the phase of the amplitude, so that the amplitude of the string fragmentation process can be written as

\[ M = \exp(i\kappa - b/2)A \]

(6)

where \( b \) is a parameter tuned to the experimental data.
Figure 3: Fig. (a) shows the space-time diagram of string fragmentation. The shaded area A is the area spanned by the string. Fig. (b) shows the string area difference $\Delta A$ corresponding to the exchange of hadrons I and II.

The symmetrization of this amplitude with respect to the exchange of $N$ identical bosons yields

$$M \rightarrow M_{\text{sym}} = \frac{1}{\sqrt{N!}} \sum_{i=1}^{N!} \exp(i\kappa - b/2)A_i$$

and the amplitude squared can be written as

$$|M_{\text{sym}}|^2 = \frac{1}{N!} \sum_{i,j=1}^{N!} \exp[i\kappa(A_i - A_j)] \exp[-b(A_i + A_j)]$$

$$= \frac{1}{N!} \left( \sum_{j=1}^{N!} \exp(-bA_j) + \sum_{i,j,A_i \geq A_j} 2 \cos[\kappa(A_i - A_j)] \exp[-b(A_i - A_j)] \exp(-bA_j) \right)$$

$$= \frac{1}{N!} \sum_{j=1}^{N!} \exp(-bA_j) \{ 1 + \sum_{i,A_i \geq A_j} 2 \cos(\kappa \Delta A_{ij}) \exp(-b\Delta A_{ij}) \}$$

The interference appears in the formula as an additional weight depending only on the string area difference. This difference is shown in Fig. (b) for the exchange of two hadrons (I,II). It can be shown (see Appendix A) that this area difference (times $\kappa$) is equal to the $dp \cdot dx$ term:

$$\kappa \Delta A = dp \cdot dx$$

The comparison of Eq.2 to Eq.8 shows that the simple correlation function is now damped by an exponential term (see Fig. 3). The effect is concentrated in a small region around the origin of the $dp \cdot dx$ distribution; this is where the close pairs (or multiplets) are expected to be located.

5
Figure 4: The shape of the interference term in Eq.8.

4 Simulation strategy

Formula 8 provides a recipe for how to include all interference effects into the simulation via global event weights. However, the evaluation of all interference terms for all possible boson exchanges remains quite complicated; this is the way the Bose-Einstein effect is handled in [8].

As already mentioned above, we have chosen a simplified way to implement the BE interference. This works only with 2-particle, eventually 3-particle, exchanges. Without higher order interference terms, formula 8 can hardly be used as such since one cannot achieve a proper normalization nor handle safely negative weights. On the other hand, we know that the effect is very localised in the configuration space and that the observed enhancement in the production of close boson pairs is due to the peak in the $dp \cdot dx$ distribution; therefore, the generated events must contain pairs of bosons for which the products $dp \cdot dx$ lie in the interval within the shaded area of Fig. 4.

The simulation program was built from the beginning on this qualitative feature of the BE interference, and several simplifications were therefore introduced in order to have the possibility to study various aspects of the production of close boson pairs. On the level of the correlation function – built from 2- and 3- particle interference terms only – we omit the secondary peaks and minima of interference terms, setting their minimal value to 0. This allows us to force the
production of close boson pairs, because all configurations outside the central peak are rejected. We keep this simplified form of the interference term throughout this paper because it provides results which are in a good agreement with experimental data. The form of the interference term can be easily changed in the simulation program, and the dependence of the result on its modifications can be studied.

| Particle type | Production rate (LEP [10]) |
|---------------|-----------------------------|
| \( \pi^\pm \)  | 17.1 \( \pm 0.4 \)       |
| \( \pi^0 \)    | 9.9 \( \pm 0.8 \)         |
| \( K^\pm \)    | 2.42 \( \pm 0.13 \)      |
| \( K^0 \)      | 2.12 \( \pm 0.06 \)      |
| \( \eta \)     | 0.73 \( \pm 0.07 \)      |
| \( \rho^0(770) \) | 1.4 \( \pm 0.1 \)     |
| \( K^{\ast\pm}(892) \) | 0.78 \( \pm 0.08 \) |
| \( K^{\ast\ast}(892) \) | 0.77 \( \pm 0.09 \)    |

Table 1: Production rates of light mesons in hadronic \( Z^0 \) events as measured at LEP (Table 1, taken from [10]). We see that most of the BE effect can be expected from correlations between pions, eventually kaons (the production rates for other bosonic species are rather low).

| Origin of \( \pi^+ \) in \( Z^0 \) decay | Fraction [%] (JETSET 7.4) |
|------------------------------------------|-------------------------|
| direct \textit{(string fragmentation)}   | 16                      |
| decay of short-lived resonances \( \Gamma > 6.7 \text{ MeV} \) \( \rho, \omega, K^\ast, \Delta, \ldots \) | 62                      |
| decay of long-lived resonances \( \Gamma < 6.7 \text{ MeV} \) | 22                      |

Table 2: The origin of charged pions in hadronic \( Z^0 \) decay. The table shows how many of charged pions come directly from string fragmentation and from decay of resonances (the division between short and long-lived resonances is arbitrary, here it corresponds to a life-time of about 30 fm/c).

Among all bosons produced in the event, mainly direct hadrons (products of string fragmentation) and decay products of shortly living resonances are susceptible to be influenced by BE correlations. We have included BE interference for the following bosons: \( \pi, K, \rho \) and \( \omega \). Every prompt boson of one of these types goes through a local reweighting procedure at the moment of its generation, e.g. at the moment of string fragmentation or at the moment of the decay of the mother resonance. The string fragmentation cycle itself is not disturbed; all direct hadrons coming from a single string are reweighted together, which means that the fragmentation of each string is repeated until the correlation function – the product of sums of interference terms for all identical bosons – passes weighting criterium.

The decay of a short-lived resonance is affected by local weighting if – among its decay products – there are identical bosons or bosons of the same type as those already generated. (We call the weighting “local” to stress the fact that – contrary to the global weighting – we
split the total correlation function (the global weight) into a set of separate “local” weights.

The energy and momentum of the mother resonance is preserved, as well as the decay channel it started to decay into, while its life-time is allowed to vary. The weighting is used to find, in the available phase space and according to the correlation function, the configuration where daughter bosons are close to bosons already existing. We would like to point out the fact that there is no double counting of interference terms, and that the order of generation is actually irrelevant, since the individual terms in the total correlation function are Lorentz invariant.

An option is included in the MC program which allows the decay products of a resonance to be treated as if they were direct hadrons. Especially in the case of $\rho$ mesons, the resonances decay so quickly that their decay should be actually treated as part of string fragmentation. In practice however this option is of little use: the more direct bosons we have, the less effective the weighting is, and in addition the multiplicity of direct hadrons runs out of control.

The whole procedure is rather intuitive – the probability of having close bosons is enhanced step by step until the complete final state is generated, while most of the standard JETSET features are preserved. The method is obviously more effective than the global weighting \cite{8}, however the overall normalization scale being lost, we don’t know a priori how many close pairs and triplets are needed to reproduce the experimental data. As we will see in the next section, the method of “forced” generation of close bosonic pairs, a priori expected to give a somewhat exaggerated BE correlations, seems to agree rather well with experimental observations.

5 Results of simulation and comparison with experimental measurements

Fig. 5 shows the two-particle correlation function for like-sign pairs of particles from $Z^0$ decay ($E_{\text{CMS}} = 91.22$ GeV), obtained with our simulated data. The variable $Q = \sqrt{-(q_1 - q_2)^2}$ is the momentum transfer between two particles with momenta $q_1, q_2$. Only particles with momentum above 0.2 GeV/$c$ were taken into account, and – similarly to the experiment – the decay products of $K^0$ and $\Lambda$ were removed. For comparison, fits to the DELPHI data with an exponential and with a gaussian parameterization are plotted as well \cite{11}.

The simulation reproduces the enhancement of the two-particle correlation function rather well. There is a small discrepancy: a small linear rise of the correlation function with $Q$ is observed in the data, but not in the simulation. This effect is most probably due to a residual difference between the reference sample for the data (which is a sample of tracks mixed from different events) and for simulation (represented by the JETSET simulation without the BE correlation).

The simulated two particle correlation functions for neutral pions and for charged kaons are
Figure 5: Simulated two particle correlation function for like-sign pairs compared to fits of the DELPHI data (simulated sample of $10^5$ events).

Gaussian fit to DELPHI data (dotted line): $0.91 (1. + 0.05 \, Q)(1. + 0.27 \, \exp \{-2.16 \, Q\}^2)$

Exponential fit to DELPHI data (dashed line): $0.83 (1. + 0.11 \, Q)(1. + 0.61 \, \exp \{-2.82 \, Q\})$

Both were fitted with exponential parameterizations.

The enhancement in the production of close pairs of direct bosons is strong (see Fig. 8), but most of the effect observed in final hadronic states is due to the correlation between pions from resonance decays. This leads us to another observable feature of BE interference, namely the possible distortion of the resonance spectrum (observed at LEP for the $\rho^0$ [12]).

The mass spectrum of direct resonances is in principle allowed to vary in our approach. Even so, we don’t observe any significant change in the $\rho^0$ spectrum itself, contrary to [8]. What we do see, however, is a non-negligible modification of the “background” $\pi^+\pi^-$ spectrum, clearly influenced by BE correlations between identical bosons, and which would lead to a lower fitted value of the $\rho^0$ mass if not taken into account (Fig. 9). The two-particle correlation functions for direct and for all $\pi^+\pi^-$ pairs in the final state are shown in Fig. 10.

Although we have strongly influenced the distribution of identical bosons in the configuration space (Fig. 11), the changes in event shape variables are not very dramatic. Part of them are
directly related to the change of the charged multiplicity: when weighting the products of the string fragmentation we don’t fix the multiplicity of direct bosons and therefore we partly loose the control over the multiplicity of the final state. The total charged multiplicity decreases by 5% if correlations are included for all bosons mentioned above; it increases by 2% if only charged bosons are taken into account (because identical neutral bosons can be produced at closer space-time distance, and are therefore more easily correlated than equally charged bosons, see Fig. 12).

Fig. 13 shows the behaviour of the scaled momentum distribution of charged final particles, both in the case where only charged bosons are correlated and in the case when neutral bosons are correlated as well. The distribution is enhanced at both ends of the spectrum – a feature supported by the data [10]. It would probably be worthwhile to retune the JETSET parameters in order to see how much of this effect remains when the total charged multiplicity is adjusted.
Figure 8: Simulated 2-particle correlation function for pairs of direct bosons.

Figure 9: Mass distribution of prompt $\pi^+\pi^-$ pairs.

Figure 10: Simulated 2-particle correlation function for $\pi^+\pi^-$ pairs.
Figure 11: The $dp\cdot dx$ distribution of prompt equally charged pions modified by BE correlations.

Figure 12: Distribution of the squared space-time distance between pairs of direct pions (JETSET without BE correlations).

Figure 13: Scaled momentum distribution of charged particles and its logarithm, modified by BE correlations between charged and between charged+neutral prompt bosons.
An alternative tool to ordinary correlation functions in studies of particle correlation is represented by factorial moments. Originally this notion was introduced by Bialas and Peschanski in 1986 [13] in connection with intermittency. Roughly speaking, the underlying question was whether the fluctuations of local density of some quantity (like rapidity, azimuthal angle, transverse momentum) are of purely statistical nature, or have some non-trivial origin. The factorial moments have been shown to provide the suitable method for addressing these problems.

An $i$-th factorial moment can be defined as

$$F_i = \frac{1}{N_{\text{events}}} \sum_{\text{events}} \sum_{k=1}^{n_{\text{bins}}} \{n_k(n_k - 1) \cdots (n_k - i + 1)\} / n_{\text{bins}}^{i} \langle n \rangle / n_{\text{bins}}^{i}$$

where $\langle n \rangle$ is the average number of particles in the full phase space region accepted, $n_{\text{bins}}$ denotes the number of bins in this region, which is given by $(2^b)^d$, $b = 0, 1, 2...$ ($d$ is the dimension of the phase space region considered) and $n_k$ is the multiplicity in $k$-th bin. In what follows we consider factorial moments in two and three phase-space dimensions, in the conventional variables $(y, \varphi)$ and $(y, \varphi, \tilde{p}_t)$ ($y$ denotes rapidity, $\varphi$ is the azimuthal angle and $\tilde{p}_t$ is connected to the transverse momentum – it is defined as in [14]

$$\tilde{p}_t = \frac{\int_{0}^{p_{t_{\text{max}}}} P(p_t)dp_t}{\int_{0}^{p_{t_{\text{max}}}} P(p_t)dp_t}$$

where $P(p_t)$ is the probability distribution of $p_t$ in the interval $(0, p_{t_{\text{max}}})$, $p_{t_{\text{max}}}$ being some suitably chosen upper limit. The purpose of treating $p_t$ this way (and in principle any other quantity of highly non-uniform density distribution - $p_t$ itself is steeply falling) is to make the overall distribution of a quantity studied more uniform, which is a necessary condition for this type of analysis [14].

The method originally proposed in [13] consists in measuring the dependence of factorial moments defined in Eq.10 as a function of “resolution” in phase space, i.e. of $b$ in our notation. The statement is that while purely statistical fluctuations in density lead to constant behaviour of $F_i$ with respect to $b$, the presence of non-trivial correlations is signalized by its rise. The content of original concept of “intermittency” was even stronger – that the rise in double-log scale should be linear, i.e. $\log(F_i(b)) \propto \phi_i b$, where the “intermittency index” $\phi_i$ had been claimed to be connected with the fractal character of hadron or underlying parton shower and various dynamical models of fragmentation fulfilling these conditions had been formulated (see [17] and references therein).

At present the outlook somewhat changed and it is generally accepted that there is no proper “intermittency” in the above sense and BE correlations are the only cause of the short range correlation (e.g. [15], [16]). Simultaneously, the quantities studied shifted from factorial
moments to other ones, mostly two- or more-particle correlation functions in various phase-space variables. This however does not change the basic fact that the presence of any correlations should be observable in terms of factorial moments as well – it should lead to their rising as phase space variables bins decrease (though, with the original concept of intermittency all but abandoned, there is no deeper interpretation of its slope and even no deeper reasons why it should be linear in the double-log scale at all). There are even some advantages in comparison with “ordinary” correlation functions, like reasonably straightforward (from the technical point of view) construction of two- or three-dimensional (in phase-space variables) quantities and no need to construct the “uncorrelated” ensemble for normalization.

We will study, with the help of factorial moments method, the two-particle correlations of particles generated with the BE correlations switched on and off, respectively, in the JETSET generator. We define the like-charge second factorial moments as

\[
F_{2}^{\text{like}} = \frac{1}{N_{\text{events}}} \sum_{\text{events}} \left( \frac{\sum_{k=1}^{n_{\text{bins}}} \left\{ n_{k}^{+}(n_{k}^{+} - 1) \right\} / n_{\text{bins}}}{\left\langle (n^{+}) / n_{\text{bins}} \right\rangle^2} + \frac{\sum_{k=1}^{n_{\text{bins}}} \left\{ n_{k}^{-}(n_{k}^{-} - 1) \right\} / n_{\text{bins}}}{\left\langle (n^{-}) / n_{\text{bins}} \right\rangle^2} \right)
\]  

(12)
where $n_k^+$ and $n_k^-$ denotes the number of positive and negative particles, respectively, in the $k-th$ bin, while $\langle n^+ \rangle$, $\langle n^- \rangle$ are the average numbers of positive and negative particles in the full phase space region. In calculating factorial moments, we restrict ourselves to the rapidity interval $-3.2 \leq y \leq 3.2$ and we take $p_t^{\text{max}}$ (see Eq. (11)) to be 2 GeV/$c$. The phase space variables are expressed with respect to the thrust axis of each event.

The results are shown in Fig. 14. We calculated, with the help of the JETSET generator, the factorial moments for like-charge particles (Eq. (13)) produced in the decay of $Z^0$. On Fig. 14a we can see the behaviour of the two-dimensional moments $F_2^{\text{like}}(y, \phi; b)$ as a function of $b$ (see Eq. (14)), Fig. 14b shows the same for three-dimensional moments $F_2^{\text{like}}(y, \phi, \tilde{p}_t; b)$. The three sets of points on each plot have been calculated from data generated without any BE correlations, with the BE correlations included according to the original JETSET option (subroutine LUBOEI), and with the BE correlations implemented as described in the present paper, respectively. We can see that the treatment of the BE correlations based on the space-time picture of the production process leads to the right effect: the factorial moments rise. It should not be surprising that the effect is more pronounced for the three-dimensional moments than for the two-dimensional ones, as projection of the correlation effect onto the lower dimension subspace can “dilute” the effect and lead to the flattening of the behaviour of the moments [17].

In the simulation with LUBOEI, the gaussian parameterization was used with parameters $PARJ(92) = 0.35$, $PARJ(93) = 0.42$ GeV, which is in agreement with data [11]. There is practically no difference between the two methods of the treatment of the BE correlations in the two-dimensional case. However, the three-dimensional factorial moments calculated with the original JETSET BE recipe (LUBOEI) seem to behave similarly to those calculated with no BE correlations, e.g. they reach a plateau. A further rise of the three-dimensional factorial moments in data generated with LUBOEI could be achieved by setting its parameters to higher values, but this would imply much stronger enhancement of the 2-particle correlation function than observed in the experimental data.

7 Does the BE effect influence the measurement of the W mass at LEP2?

The study of hadronic WW events certainly adds a new dimension to the problematics of the BE interference. Now we have to deal with – at least – two strings. In fact, we had a multiple string configuration in $Z^0$ decays as well – as the result of a gluon splitting – but since our weighting algorithm is based on the calculation of the absolute coordinates of a hadron position, it can – technically – handle such a configuration without difficulties, and we actually didn’t ask how should the BE interference look like for bosons from different strings. Nevertheless, in the
study of the systematic error on the W mass, this question requires a detailed discussion.

Figure 15: The simulated two-particle correlation function for pairs of direct equally charged pions coming from the same W and those of a mixed origin. WW hadronic events generated at 172 GeV.

Figure 16: The simulated two-particle correlation function for pairs of all equally charged pions coming from the same W and those of a mixed origin. WW hadronic events generated at 172 GeV.

To avoid confusion, we start with the discussion of the relationship between the BE effect and colour reconnection (often they are put together and called interconnection effects). Colour reconnection is the term used for the interaction of strings which changes the string configuration ('reconnects' them), and therefore implies momentum/energy transfer between the original strings. On the other hand, while deriving the correlation function for the BE effect, we didn’t account for any explicit interaction term between different strings. In fact, we derived it only for a single string. While Eq.2 can be – at least formally – applied to bosons coming from different strings, this formula doesn’t contain the exponential suppression and, when actually used, does not produce any observable effect in the simulated data. Therefore we consider the BE interference as preserving the total string momentum and every direct string-string interaction with momentum transfer will be considered as belonging to colour reconnection. The interplay of the BE effect and colour reconnection can be investigated with the help of existing phenomenological models for simulation of colour reconnection (those based on JETSET fragmentation can be combined with simulation of BE interference without difficulties).

In agreement with the classification introduced above, the mass of the string is preserved

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3The influence of colour reconnection on W mass measurement was investigated in [18, 19, 3].
during hadronization. Which are then the remaining possibilities to see the W mass spectrum modified? One of them is purely experimental and concerns only fully hadronic WW events: since we are not able to separate completely the two hadronic systems (one belonging to the $W^+$, the other to the $W^-$), there is always a fraction of misassigned particles resulting in a smearing of the measured W mass spectrum. The Monte-Carlo simulation can be used to correct for this effect. The Bose-Einstein effect, however, with its tendency to produce boson pairs with similar momenta, can change the fraction of misassigned particles; if this effect would be missing in the simulation, we would obtain a wrong estimate of the correction to apply to the observed mass.

Another possibility to get a distorted spectrum is more fundamental, if we admit that the primary process itself (the production of WW pairs) may be influenced by the interference terms added to the hadronization part. It seems however unlikely to be so; after all, the whole simulation of the hadronization makes use of the so called factorization theorem: the amplitude of string fragmentation (Eq. 3) doesn’t appear in the total event weight nor is the hard process or parton configuration rejected because of fragmentation. Still, we don’t see really strong arguments why the hard process should not be influenced, and therefore we made a check of what happens with the W spectrum if we use our weights for direct bosons as the global event weights for the sample of semileptonic WW events. A sample of 500,000 events was generated with PYTHIA/JETSET including our BE simulation. The reweighted spectrum of the hadronic W mass was compared to the generated one (both were fitted with a Breit-Wigner distribution times a phase space factor). The result is shown in Table 3 (method I).

| Method (\(E_{CMS}=172\) GeV) | Shift of fitted W mass [MeV] |
|---------------------------------|-------------------------------|
| I: weight for direct bosons      | -10 ± 12                      |
| used as global event weight      |                               |
| (in semileptonic WW events)      |                               |
| II: BE interference included only within a string (unweighted sample,hadronic WW events) | +11 ± 11 |
| III: BE interference among strings as well (unweighted sample,hadronic WW events) | +12 ± 11 |

Table 3: The shift of the fitted W mass due to the BE effect in various scenarios (see text).

Since this is the method which is closest to the use of global weights in [3], we have also checked the effect of this weighting on the values of \(R_b, R_c\) in \(Z^0\) decays. We observed a (statistically insignificant) difference of the order of a few per-cent \((+5 ± 2\%\) for \(R_c\), \(+1 ± 2\%\) for \(R_b\)).
We don’t feel that doing the same exercise with fully hadronic events is useful – the interference across different strings is really ill defined for such a study. It can be nevertheless used to study the experimental problem of wrongly assigned particles, because it mimics rather well the situation when (for some reason) independent strings produce “mixed” pairs of bosons of similar momenta (as if Eq.8 would be valid for all bosons in the event). In fact, two studies were made: one with BE correlations allowed only inside a single string, the other with correlations of bosons coming from different strings included as well (methods II and III in Table 3). In each event, the mean W mass was reconstructed (only clear four jet events were used, i.e. with a minimal energy per jet of 20 GeV and an angular separation between jets larger than 0.5 rad), then the mass distribution was fitted and compared to the reference sample (standard PYTHIA/JETSET without BE correlations). The results are also shown in Table 3.

For illustration, Fig. 15, 16 show the two-particle correlation function for pairs of pions from decays of the same W and for those of ‘mixed’ origin. We remind once more that while the calculation of weights for mixed pairs is technically straightforward in our approach (which is based on the evaluation of hadron’s production vertex), their use is not warranted by the QM arguments as for bosons coming from a single string. (Indeed, the very first results of measurements of BE correlations in WW events at LEP [20] suggest that the interference between strings/W’s is strongly suppressed.)

The results of our studies do not signal any special danger for the W mass measurement; we don’t see how the BE effect can shift the W mass by 50 or even 100 MeV as suggested in [3], even when we take the interference between the different strings to be as strong as the interference inside a single string. The uncertainty quoted in Table 3 is based on the statistical error of the fit of the W mass distribution and could be decreased just by increasing the simulated sample. However, taking into account other related uncertainties (the study is done at the generator level, the reconstruction method we use does not necessarily correspond to the one actually used in the experiment, the shape of reconstructed W mass distribution is not a simple Breit-Wigner distribution convoluted with phase-space factor and so on), we think the quoted error is a realistic one.

To make the picture more complete, we would like to investigate a little bit more the space-time picture of hadronization. The very general argument why there should be some interference between the two W’s says that because the W’s decay close to each other, the strings overlap and are very likely to have some sort of interaction. Let us take the example of an ordinary hadronic WW event at 172 GeV: the W’s decayed at a distance of 0.05 fm, their decays were followed by parton showering and there are two or more strings around evolving towards fragmentation (the mean life-time of a string is about 1.5 fm/c). The two hadronic systems are separating (the mean velocity for W’s is around 0.4 c), the decay planes of both W’s being different. It is therefore not so evident that strings have to be in contact. In fact, the colour reconnection
study shows that in nearly 40% of all events, the overlap of strings is negligible.

Since we believe that the origins of the BE effect lie somewhere in the fragmentation, we are interested how often strings do overlap while fragmenting. Fig. 17 shows the square of the space-time interval between production vertices of equally charged direct pions for mixed pairs (one pion coming from the $W^+$, the other from the $W^-$), while Fig. 18 shows the distance in space coordinates only. The production vertices are causally disconnected and the mean distance between them exceeds the typical transverse size of a string (about 1 fm): there is no evidence of a sizeable overlapping of strings.

Figure 17: The space-time distance squared of production vertices of direct equally charged pions for pairs of mixed origin. WW hadronic events at 172 GeV.

Figure 18: The space distance of production vertices of direct equally charged pions for pairs of mixed origin. WW hadronic events at 172 GeV.
8 BE effect as flavour correlation in string fragmentation

The simulation model we have presented certainly does capture some important features of the BE effect and may be of some use for practical purposes; it is not limited by the topology of events (e.g. multiple jets), has a relatively short execution time and can be developed further. Still, it is not the best solution for the problem of BE simulation, and this for several reasons discussed hereafter.

First, a rather important amount of computing time is spent on generating and rejecting events which do not contain any bosons with similar momenta. The weighting procedure makes us to wait for the accidental generation of events which we know – more or less – how they will look like. Second, we are to some extent loosing control over some important parameters, like the multiplicity of the final state. We can in principle react by retuning the parameters of the model, but we risk to be confronted with this kind of problem again and again; in short, the approach is inconsistent with the philosophy of the fragmentation model.

We would like to devote this section to a discussion about a potential new approach to BE simulation – the direct implementation into the fragmentation scheme. Not that we have the complete solution on hand, but we are convinced that such a simulation is feasible. It would require some changes in the fragmentation model but would pay off in the long term.

To show what we have in mind, let’s take once more the case of a simple $q\bar{q}$ string in its rest frame. It will fragment into a set of hadrons, among them two identical bosons $a, b$ (let’s say charged pions, for definiteness), as in Fig. 19.

![Diagram of hadron fragmentation](image)

Figure 19:

We will calculate the $dp \cdot dx$ term for the pair $(a,b)$ using Eq.8 and 13:

\[
(p_a - p_b) \cdot (x_a - x_b) = (E_a - E_b)(t_a - t_b) - (\vec{p}_a - \vec{p}_b)(\vec{x}_a - \vec{x}_b) \\
= 0.5(x_2 - x_1 - x_4 + x_3)(t_1 + t_2 - t_3 - t_4) \\
-0.5(t_2 - t_1 - t_4 + t_3)(x_1 + x_2 - x_3 - x_4) \\
= \ldots \\
= (x_4 - x_2)(t_3 - t_1) - (x_3 - x_1)(t_4 - t_2) \\
= E_d p_u l - E_a p_d l
\]  

(13)
and we have a relationship between the Bose-Einstein effect and the flavour correlation in string
fragmentation; the correlation depends on the energy-momentum of string pieces for which
flavour is compensated (i.e. the end-point quarks are of the same flavour).

Now we can switch to the light-cone metrics in which the Lund fragmentation model is
formulated: the Lorentz invariant variable $z^+(z^-)$ determines the fraction of energy-momentum
of the end-point (massless) quark(antiquark) which the hadron takes away:

$$E_h = (z^+ + z^-)E_{q0}$$
$$p_h = (z^+ - z^-)p_{q0}$$

In the Lund fragmentation model, the hadron is defined in 3 steps:

1. the flavour of the next string breaking is chosen, as well as the hadron mass $m$;
2. the transverse momentum of the new quark/antiquark pair is generated (according to a
gaussian distribution), defining the total transverse momentum $p_t$ of the hadron;
3. $z^+$ (or $z^-$) is generated according to the Lund symmetric fragmentation function; the
momentum of the hadron is thus fully determined (the remaining $z^-(z^+)$ is calculated
from the relation

$$z^+z^-M_0^2 = m_t^2 = m^2 + p_t^2$$

where $M_0 = 2E_0$ stands for the mass of the string).

With the help of Eqs.4,14 we can translate Eq.13 into invariant variables $z$ (we use index I
for the intermediate state):

$$dp \cdot dx = \cdots = 0.5[z_I^+(z_b^- - z_a^-) - z_I^- (z_b^+ - z_a^+)] + z_I^+ z_b^- - z_I^- z_b^+]M_0^2$$

and it becomes evident that we can involve the interference by an appropriate choice of the $z$
variables (imposing a restriction on the $dp \cdot dx$ term, see again Eqs.8,14).

Let’s take a concrete example: during the fragmentation process, imagine the pion $a$ and
the arbitrary hadron system $I$ are already generated and the pion $b$ (identical with $a$) is just
about to be generated. For the two pions to be correlated, we would require their $dp \cdot dx$
term to behave according to the interference term in Eq. 8. This represents an additional
condition on the choice of $z_b$, and there is a possibility of correlations in transverse momentum
as well. We have checked that our 'weighting’ model, when applied at simple $q\bar{q}$ string, does
not predict any strong correlation in the transverse momentum (rather a small decrease of the
mean transverse momentum is observed), and therefore we just keep the random generation of
transverse momentum of standard JETSET – for simplicity. Having the transverse momentum
of pion $b$ defined, Eq. 16 becomes
\[ dp \cdot dx = 0.5 \left[ z_I^+ \left( \frac{m_{tb}^2}{z_b^+} - \frac{m_{ta}^2}{z_a^+} \right) - \frac{m_{tb}^2}{z_I^+} \left( z_b^+ - z_I^+ \right) + z_a^+ \frac{m_{tb}^2}{z_b^+} - \frac{m_{ta}^2}{z_a^+} z_b^+ \right] \]  

(17)

Obviously, to keep \( dp \cdot dx \) at the Planck scale, \( z_b \) should be close to \( z_a \). As a test, we have made a rather simple toy model for coherent fragmentation of simple \( u\bar{u}(d\bar{d}) \) strings. We have allowed only charged pions to be produced in the fragmentation, and we have included correlations by a simple rule (keeping notation of Fig. 19, where I – intermediate state – is now represented by a pion of the opposite charge):

- **Case A**: \( z_b^+ = z_a^+ \) if resulting \( dp \cdot dx \leq \hbar \)
  
  \[ ... dp \cdot dx = 0.5 \ast (1 + \frac{z_I^+}{z_a^+})(m_{tb}^2 - m_{ta}^2) \]  

  in this scenario

- **Case B**: \( z_b^+ = \frac{m_{tb}}{m_{ta}} z_a^+ \) if resulting \( dp \cdot dx \leq \hbar \)
  
  \[ ... dp \cdot dx = 0.5 \ast (m_{tb}/m_{ta} - 1)(z_I^+ z_a^- - z_I^- z_a^+)(M_0^2) \]  

  in this scenario

(Actually, we have made the string breaking in flavour \( d \) follow the pattern of string breaking in flavour \( u \) and vice versa.)

\[ Q (GeV/c) \]

As expected, we have obtained a nice enhancement of the two-particle correlation function in both cases (Fig. 20). The correlation is slightly stronger in case B since there the difference
in transverse momentum is partially compensated. A small drop in the mean multiplicity is observed: -4% in case A, -2% in case B (and consequently somewhat harder spectrum of final particles), which we think is due to the outside-in method of the fragmentation (when the string is fragmented from both its ends). Actually, the inside-out cascade would suit better the implementation of the BE interference but the discussion of possible solutions goes beyond the scope of this paper.

The direct implementation of BE interference has many advantages: we expect the algorithm to go smoothly over gluon corners, and we stress that there is no need to evaluate the absolute coordinates of string breaking and hadron position, which means extreme simplification with respect to our current simulation. We also think that a consistent way to treat the short-lived resonances can be developed. From practical point of view, the high efficiency of simulation, combined with the solid theoretical basis, would considerably simplify the study of the BE effect in $e^+e^-$ annihilations.

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Appendix A

The string area spanned by a simple $q\bar{q}$ string, parallel to the axis $x$ in its rest frame, can be expressed in the terms of the coordinates of string breakings.

In Fig.21, string breakings are represented by the full circles with coordinates $[t_i, x_i]$, while the empty circles mark the points where the quark-antiquark pairs forming the hadrons meet. If we consider the quarks to be massless, the coordinates of their 'meeting' point are

$$[t_M, x_M]_i = 0.5[x_i - x_{i-1} + t_i + t_{i-1}, x_i + x_{i-1} + t_i - t_{i-1}]$$  (18)

We introduce variables $a_i, b_i$ (i=1..N) in the following way

$$a_i = \sqrt{2}(t_{Mi} - t_{i-1}) = \frac{1}{\sqrt{2}}(x_i - x_{i-1} + t_i + t_{i-1})$$  (19)

$$b_i = \sqrt{2}(t_{Mi} - t_i) = \frac{1}{\sqrt{2}}(x_i - x_{i-1} - t_i + t_{i-1})$$  (20)
(\(a_i, b_i\) are closely related to \(z_i^+, z_i^-\) of Eq.\[4\]: \(a_i = z_i^+E_{q0}/\kappa\); \(b_i = z_i^-E_{q0}/\kappa\), \(E_{q0}\) is the initial energy of endpoint quarks.)

The string area may be then written as

\[
A = \sum_{i=1}^{N} a_i b_i + \sum_{i=1}^{N-1} a_i (b_0 - \sum_{j=i+1}^{i} b_j) \tag{21}
\]

where \(b_0 = \sqrt{2t_0} = \sqrt{2x_0} = \sqrt{2E_{q0}/\kappa}\).

The string area difference corresponding to an exchange of two hadrons \((k, l, k < l)\) can be calculated from the previous equation, and with the help of Eqs.\[3\] we obtain Eq.\[8\]( \([\tau_i, \chi_i]\) are coordinates of the production vertex of the hadron \(i\):)

\[
\Delta A = A - A(k \leftrightarrow l) = \cdots = (a_k - a_l) \sum_{i=k+1}^{l} b_i - (b_k - b_l) \sum_{i=k+1}^{l} a_i
\]

\[
= 0.5(x_k - x_{k-1} + t_k - t_{k-1} - x_l + x_{l-1} - t_l + t_{l-1})(x_l - x_k - t_l + t_k)
\]

\[
- 0.5(x_k - x_{k-1} - t_k + t_{k-1} - x_l + x_{l-1} + t_l - t_{l-1})(x_l - x_k + t_l - t_k)
\]

\[
= 0.5(x_l - x_k - t_l + t_k)(E_k + p_k - E_l - p_l)/\kappa
\]

\[
- 0.5(x_l - x_k + t_l - t_k)(E_k - p_k - E_l + p_l)/\kappa
\]

\[
= [(E_k - E_l)(t_k - t_l) - (p_k - p_l)(x_k - x_l)]/\kappa
\]

\[
= 0.5\{(E_k - E_l)[(t_k + t_{k-1}) + (t_k - t_{k-1}) - (t_l + t_{l-1}) + (t_l - t_{l-1})]
\]

\[
-(p_k - p_l)[(x_k + x_{k-1}) + (x_k - x_{k-1}) - (x_l + x_{l-1}) + (x_l - x_{l-1})]/\kappa
\]

\[
= \{(E_k - E_l)(\tau_k - \tau_l + (p_k - p_l)/2\kappa) - (p_k - p_l)(\chi_k - \chi_l + (E_k - E_l)/2\kappa)/\kappa
\]

\[
= \{(E_k - E_l)(\tau_k - \tau_l) - (p_k - p_l)(\chi_k - \chi_l)\}/\kappa \tag{22}
\]
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