Interactive Frequency Dynamics Between Grid-Forming Inverters and Synchronous Generators in Power Electronics-Dominated Power Systems

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Abstract—Power systems are large, complex networks with system-level dynamics stemming from the individual constituent subsystem dynamics and the interactions among them. Here, we compare and analyze the interactions of multi-loop droop grid-forming inverters and synchronous generators, with an emphasis on frequency. Singular perturbation theory is applied to illuminate an order reduction in the frequency response of the grid-forming inverter, and the inverted pre-converter power–frequency relationship is discussed. Device level electromagnetic transient domain simulations corroborate these findings and confirm that a properly designed DC-side system has a negligible dynamical impacts. Simulations of the 9- and 39-bus test systems confirm the order reduction and the decoupling of the nadir and rate of change of frequency. Matrix pencil analysis of the system of oscillatory modes confirms an increase in primary mode damping with grid-forming inverters; an increase in mode frequency and decrease in damping is observed for high inverter penetrations in the 39-bus system. This same mode is found in a two-generator grid forming inverter/synchronous generator system with a low synchronous generator rating (< 20%). This indicates that the kernel is not device quantity or network complexity, but the interaction of a small rotational mass with the rapid frequency change of the grid-forming inverter.

Index Terms—grid–forming inverters, frequency response, matrix pencil method, oscillatory modes, synchronous generators,

I. INTRODUCTION

Electric power systems are a class of complex dynamic systems with dynamics primarily driven from the individual subsystems as well as the coupling, interrelated, and interactive dynamics of these subsystems. In an electric power system, the dynamically dominant subsystems are the generators, where a stable frequency response at the device-level (which is one of the coupling variables) is a primitive requirement for a stable interconnected operation at the system-level. Therefore, in order to fully understand the underlying phenomena and frequency dynamics at the system-level warrants an in-depth understanding of the frequency dynamics of the generators at the device-level first and their interactions and interrelations.

Frequency dynamics in AC power systems are traditionally dominated by Newtonian physics, where the rotor speed of a synchronous generator (SG), the conventional, typically fossil-fuel based primary source of power, follows a second or higher order trajectory upon mechanical–electrical power imbalances; the coupling of rotor speed and frequency in an SG yields an oscillatory frequency response. The integration of sustainable, variable renewable energy resources into power systems is primarily accomplished with power electronic inverters (i.e., inverter based resources (IBRs)), the majority of which have hitherto employed grid-following (GFL) control strategies that rely on other devices to establish the voltage profile that is explicitly tracked for operation [1]. As renewable shares continue to grow at an accelerating pace due to declining costs [2], and climate change driven sustainability objectives, small to medium sized power systems with significant shares penetrations of IBRs have become commonplace in many systems across the globe [3]. As the share of GFL devices increases, overall system inertia decreases due to the supplanting of SGs [7]; it is well documented that instability can occur for high/complete penetrations of GFL inverters (i.e., low–to zero inertia systems) [8]–[10], although the primary driver is ostensibly a lack of voltage forming devices and not inertia itself. As a result, attention has shifted to grid-forming (GFM) control, where instead of regulating to power set points as a current source, the IBR establishes a voltage and frequency at the point of interconnection to the grid. This work investigates the foundational power transfer dynamics of GFMs and SGs and the associated impacts on the frequency characteristics of the bulk electric power system.

A primary challenge concerning the operation of these low-inertia power systems is the maintenance of system stability, in particular the frequency response when SGs are displaced by IBRs; recent work has pointed towards the potential of GFM inverters to mitigate these stability challenges [11]–[18]. The authors of [11]–[14] have extensively studied the small-signal stability of power systems with integrated GFMs by developing high-fidelity differential-algebraic models; often, non-zero, minimum SG quantities are declared necessary to preserve system stability. Conversely, the feasibility of operating bulk power systems with 100% GFM-based generation has been demonstrated computationally in the electro-magnetic transient [15], [19], and positive sequence [16], domains. Tayyebi et al. [15] investigated the dynamic interactions between GFMs and SGs and have identified that the integration of GFMs
generally improves the system frequency response. Other studies have begun to identify the damping-like contribution of droop controlled GFM inverters to frequency dynamics [15]–[17], [20]. This paper investigates the power conversion dynamics of SGs and multi-loop droop controlled GFM devices (i.e., both are energy converters in principle and henceforth simply referred to as converters) and the associated frequency dynamics in bulk electric power systems along a shifting generation composition; namely, from a 100% SG system with standard inertia (3 – 8 s), to a 100% GFM, inertia-free system.

The primary contributions of this paper is threefold; first, it provides a high-level understanding of the power through-put processes for the SG and GFM developed from high order dynamic models; namely a reduction to first order for the GFM as compared to a second/higher order response from the SG. Second, it demonstrates that this order reduction results in a decoupling of the traditional rate of change of frequency and nadir metrics. Third, it exhibits that under normal operation, a properly designed DC-side system with a battery energy storage system does not impart considerably impacting dynamics on the GFM operation.

To this end, we characterized the response order difference between these two devices by first establishing high-order analytical models and then reduced them via singular perturbation theory to distinguish a response order difference in the dynamics pertinent to the frequency response nature. Additionally, the inverted frequency–power relation of the GFM as compared to the SG is teased out from these dynamical models; simply, it can be said that with respect to frequency, the GFM is a proactive device, whereas the SG is a reactive device. Extensive electromagnetic transient (EMT) simulations are executed that lead to a host of previously undisclosed observations; a parametric sweep in dominance between a GFM and SG in a two generator system reveals a high frequency oscillatory mode in the SG at very low rating ratios (< 20%). Simulations of full order SG and GFM devices are performed on the 9- and 39-bus test systems; the results show broad stability and corroborate the decoupling of the traditional nadir and rate of change of frequency metrics as a function of inertia, in the presence of GFM devices. Matrix pencil analyses of the electromagnetic time domain simulation system oscillatory modes show a general increase in damping of the primary mode with the addition of GFMs. In the 39-bus system, large fluctuations in the primary mode frequency occur, while the damping actually decreases with very few SGs on the system, which corroborates the two generator system findings. This is the first known presentation of a decrease in damping with dominant levels of GFM devices. All dynamical and network models, including the parallel core 39-bus system, have been made available to the public open-source.

The remainder of the paper is organized as follows. In Section II the converter terminology is extended to both the SG and GFM, followed by the high order dynamical descriptions for the GFM and discussion on the EMT model of the SG. In Section III the high-level power through-put model is developed with singular perturbation to mathematically conclude an order reduction. In Section IV-A single bus simulations are presented to confirm the order reduction, and a simple two generator system reveals the high frequency oscillatory response of the SG at low rating ratios. The DC-side dynamics are shown to have no operational impact when properly designed. In Section IV-B the computational results of extensive EMT simulations on the 9- and 39-bus test systems are presented along with frequency metrics and oscillatory mode analysis. A discussion on the impacts to frequency is presented in Section V followed by the conclusion in Section VI.

II. MATHEMATICAL DESCRIPTION OF CONVERTERS

This section presents the mathematical models that describe the power converter devices and capture the constituent dynamics. These dynamical models are systems of ordinary differential and algebraic equations for each device and are used for all analyses and simulations presented and discussed in this paper. Before in-depth discussions on the device governing processes, dynamics, and control systems, the converter terminology is presented and defined. Each device, considered here to be a converter, converts an initial power \( p_m \) referred to as pre-converter power, such as electrochemical energy from a battery for the GFM, or mechanical energy primed by steam, water, or combustion gases for the SG, into AC electrical energy \( p_e \); from hereon we refer to this as the through-put power process. Fig. I relates \( p_m \) and \( p_e \) for each device, where the second subscript (either \( G \) or \( I \)) indicates the applicable device (i.e., \( G \to SG \), and \( I \to GFM \)). The upper path applies to the SG, while the lower path is applicable to the GFM. A summary of the assumptions about what components comprise the converter is provided in the following:

- **Grid-Forming Inverter Converter:**
  - Power supplied as \( p_{m}\left(\text{DC source}\right) \), in the form of electrical or electrochemical energy.
  - Associated components include the switches and filtering components with all necessary control systems to generate the AC output.

- **Synchronous Generator Converter:**
  - Power supplied as \( p_{m,G} \) is the mechanical energy as applied to the shaft of the machine, primed by the steam, hydro, or combusted gases impinged on a turbine.
  - Associated components include the turbine/governor, rotating machine, and exciter, with necessary control systems.
  - Any electrical devices beyond the stator coils, such as the transformer, are considered part of the network.

A. Grid Forming Inverter

A variety of GFM control strategies have been presented in the literature, such as the droop [22], multi-loop droop [17], virtual synchronous machine [23], and virtual oscillator control [24]. They all achieve a similar objective, namely, the construction of a voltage and frequency at the point of interconnection with dynamics associated via a power export relationship. For this work, the device of focus is the multi-loop droop control GFM (henceforth, simply GFM), with
The instantaneous real \( (p_{c,t}) \) and reactive \( (q_{c,t}) \) powers in the \( dq \) frame are calculated as \( p_{c,t} = \Re(v_g^{dq}i_g^{dq}) \) and \( q_{c,t} = \Im(v_g^{dq}i_g^{dq}) \), where \( v_g^{dq} \) and \( i_g^{dq} \) are the \( dq \) frame grid voltages and coupling filter currents, respectively, represented in the complex domain (i.e., \( a+ji \)). The instantaneous powers \( p_{c,t} \) and \( q_{c,t} \) are passed through a low-pass filter (LPF) with cutoff frequency \( \omega_{L,f} \), as shown in (2) and (3):

\[
\dot{p}_{m,t} = \omega_{L,f}(p_{c,t} - p_{m,t}) \tag{2}
\]

\[
\dot{q}_{m,t} = \omega_{L,f}(q_{c,t} - q_{m,t}) \tag{3}
\]

The currents across the filter resistor \( (R_f) \) and inductor \( (L_f) \), \( i_f^{dq} \), are regulated with proportional integral (PI) controllers (depicted by the Current Controller block in Fig. 2):

\[
\begin{align*}
\dot{i}_f^{dq} &= i_f^{dq} - i_{f,*}^{dq} \\
\dot{i}_{f,*}^{dq} &= k_V^\xi i_f^{dq} + k_P^\xi i_f^{dq} - j\omega_L L_f i_f^{dq} + G_C v_0^{dq}
\end{align*}
\tag{4}
\]

\[
\begin{align*}
\dot{\xi}^{dq} &= v_0^{dq} - v_f^{dq} \\
\dot{v}_f^{dq} &= k_V^\xi \dot{\xi}^{dq} + k_P^\xi \dot{\xi}^{dq} - j\omega_L C_f v_f^{dq} + G_V v_f^{dq}
\end{align*}
\tag{5}
\]

where \( i_{f,*}^{dq} \) and \( \xi^{dq} \) are the integral error states (not mapped in the imaginary plane), \( k_V^\xi \) and \( k_P^\xi \) are the integral and proportional gains, respectively, \( \omega_L \) is the radian frequency from the droop relation and \( G_C \) is the voltage feed forward gain. The controller gains are tuned to cancel the inductor pole. The response time of the inner current loop is on the order of 1 millisecond. The capacitor \( (C_f) \) voltage is also regulated with a PI controller, as depicted by (6) and (7) (depicted by the Voltage Controller block in Fig. 2):

\[
\begin{align*}
\dot{\xi}^{dq} &= v_0^{dq} - v_f^{dq} \\
\dot{v}_f^{dq} &= k_V^\xi \dot{\xi}^{dq} + k_P^\xi \dot{\xi}^{dq} - j\omega_L C_f v_f^{dq} + G_V v_f^{dq}
\end{align*}
\tag{6}
\]

\[
\begin{align*}
\dot{\xi}^{dq} &= v_0^{dq} - v_f^{dq} \\
\dot{\xi}^{dq} &= v_0^{dq} - v_f^{dq}
\end{align*}
\tag{7}
\]

where \( \omega_L \) is the nominal frequency (such as 60 Hz in the United States), \( M_p \) is the frequency droop gain, \( p^\text{set} \) and \( q^\text{set} \) are the pre-disturbance power set points, \( p_{m,t} \) and \( q_{m,t} \) are as defined in (2) and (3), \( v_0^{q,*} \) is the set point for the voltage controller (capacitor voltage), \( v_f^{q,*} \) is the pre-disturbance steady state voltage, and \( M_q \) is the voltage droop gain.

In this work, a zeroth order model is used to represent the battery as shown in Fig. 3. This assumption rests on the analysis in [26], where it was found that higher order battery models, with series RL/RC branches, had a negligible impact on the battery response; a simple voltage source and resistor is therefore adopted. The governing equation of the battery output voltage is therefore Ohm’s law relation (10):

\[
v_{\text{BESS}} = v_{OC} - i_{\text{BESS}} R_{\text{BESS}} \tag{10}
\]

\( ^2 \) denotes the complex conjugate

\( ^3 \) dot notation indicates time derivative; \( \dot{x} = \frac{dx}{dt} \)
where \( V_{BESS} \) is the output voltage, \( R_{BESS} \) is the battery resistance, \( i_{BESS} \) is the delivered current, and \( V_{DC} \) is the battery characteristic open circuit voltage. The output voltage from the battery is passed to a boost converter to establish a DC voltage across the DC capacitor, which forms the voltage source for the inverter. Figure 3 depicts the boost converter and battery control block diagram. The dynamical equations describing the cascading PI controllers is the following:

\[
\begin{align*}
\dot{x} &= v_{DC}^* - v_{DC} \\
i_{ref} &= k_i \chi \left( v_{DC} - v_{DC}^* \right) + k_P i
\\\dot{\chi} &= i_{ref} - i_{in}
\\u &= k_i \chi \left( v_{DC} - v_{DC}^* \right) + k_P i
\end{align*}
\]

(11) (12) (13) (14)

where \( \chi \), \( k_i \chi \), and \( k_P \) are the integrator state, integral gain, and proportional gain of the voltage controller, respectively. \( i_{ref} \) is limited to \( \|i_{ref}\| \leq i_{rate} \). \( \chi \), \( k_i \chi \), and \( k_P \) are the integrator state, integral gain, and proportional gain of the current controller, respectively. \( d \) is output from the current controller that is limited to a range of values between [0.05, 0.90]. The inverter operation is impeded by the DC side dynamics when the 2-norm of the commanded voltages \( \|v_{DC}^*\|_2 \) exceed the DC side voltage \( v_{DC} \). When this occurs, the inverter is saturated. This is modelled by scaling the commanded voltages based on the minimum function between the 2-norm of \( v_{DC}^* \) and \( v_{DC} \), as in [15] [26], [27]:

\[
\begin{align*}
v_{i,sat}^{dq} &= \left( \min\{\|v_{DC}^*\|_2, v_{DC}\} \right) v_{i,s}^{dq} \\
\|v_{i,sat}^{dq}\|_2 &= \sqrt{(v_{i,s}^{dq})^2 + (v_{i,s}^{dq})^2}
\end{align*}
\]

where \( v_{i,sat} \) is the implemented voltage command, and \( v_{i,s}^{dq} \) and \( v_{i,s}^{dq} \) are the \( d \) and \( q \) axis components of the voltage commands, respectively. Note that \( v_{i,sat}^{dq} = v_{i,s}^{dq} \) when \( v_{DC} \) exceeds \( \|v_{DC}^*\|_2 \).

The coupling between the DC side control in Fig. 3 and the AC side implementation in Fig. 2 is \( v_{i,s}^{dq} \) which is applied to the voltage source \( v_{i} \) in Fig. 2. All of the parameters used for these devices are provided in Table VI found in Appendix VII.

B. Synchronous Generator

The SG is a well understood and documented device; see [28], [29] for in-depth discussions. In modeling of the SG, three distinct components must be considered: (1) the machine, (2) the exciter, and (3) the governor. Here, the converter is understood to be the set of all three elements combined. Most of the specific mathematics are not reproduced here, but are briefly discussed.

The fundamental machine model is a set of differential current elements in the \( dq \) frame expressed as a function of flux linkages and applied voltages [30]. The supplied parameters consist of the standard steady state, transient, and sub transient reactances/time constants, as well as windage resistance, are all provided in Appendix VII. The machines in this study are established with two \( q \) axis damper windings, consistent with the round-rotor implementation. The rotational mechanics of the machine are dictated by the swing equation, i.e., Newton’s second law in rotational form, as shown in (17) and (18):

\[
\begin{align*}
\delta_G &= \omega_G - \omega_s \\
\omega_G &= \frac{1}{M} \left( p_{m,G} - p_{e,G} - D \delta_G \right)
\end{align*}
\]

(17) (18)

where \( \delta_G \) is the SG phase angle, \( \omega_G \) is the SG rotor speed, \( \omega_s \) is the synchronous speed (equivalent to \( \omega_0 \) for a two pole machine), \( M = \frac{2H}{\omega_s} \), with \( H \) the inertia of the machine, \( p_{m,G} \) is the pre-converter mechanical power supplied by the turbine primed by gas, hydro, or steam, \( p_{e,G} \) is the electrical power delivered to the grid, and \( D \) is the damper winding component.

The exciter used is the IEEE Alternator Supplied Rectifier Excitation System (AC7B) model, which contains two cascaded PI controllers (a regulator and inner) with two feedback loops, a saturation function attached to the final feedback loop, and a final low pass filter. No derivative gains or derivative feedback are modelled. In addition to the saturation function, there are floor and ceiling limiters for each of the dynamical blocks. The governor model used here is a simple representation of a gas turbine (GAST) that acts on speed (frequency) errors. The GAST model consists of a three cascaded low pass filters and limiters associated with output limits. The overall functionality of this model can be approximated by the dominant filter, which can be expressed mathematically by (19):

\[
\dot{p}_{m,G} = R_D^{-1} (\omega_0 - \omega(G)) - (p_{m,G} - p_{m,G,o})/\tau_G
\]

(19)

where \( R_D \) is the droop setting of the device (including a factor of \( 2\pi \)), \( p_{m,G,o} \) is the pre-converter power set point, and \( \tau_G \) is the governor response time. The value of \( \tau_G \) can vary substantially depending on type and model, but is generally not less than 0.5 s [29].

III. DEVICE FREQUENCY DYNAMICS & SINGULAR PERTURBATION ANALYSIS

The through-put power processes of the GFM and SG are fundamentally different; these disparate processes heavily influence the frequency response characteristics of the respective devices. In this section, we turn to simplified models of

This GFM model is available to the public at https://github.com/NREL/PvPSCAD

[28], [29]
the GFM and SG to develop a high-level understanding of these differences and the anticipated impacts on power system frequency dynamics. The notation highlighted in Fig. 1 is of primary interest in this section, with the primary goal to observe the relationships between \( p_{m,G} \), \( \omega_G \), and \( p_{e,G} \) for the SG, and \( p_{m,1}, \omega_1, \) and \( p_{e,1} \) for the GFM. Each type of converter stores a quantity of energy, either as kinetic energy for the SG, \( E_{int,G} = \frac{1}{2} I_f \omega_{mech}^2 \), with \( I \) and \( \omega \) being the moment of inertia and shaft rotation rotational speed, respectively, or within the GFM primarily as electrical capacitive storage, \( E_{int,1} = \frac{1}{2} C_{DC} V_{DC}^2 \), with \( C_{DC} \) and \( V_{DC} \) being the DC link capacitance and capacitor voltage, respectively. Generally, \( E_{int,1} \gg E_{int,G} \) [29], [31]. Note that in a lossless system, with losses considered negligible here, conservation of energy requires that \( \dot{E} \neq 0 \) if \( p_m \neq p_e \).

The distinction between GFM pre-converter power (\( p_{m,1} \)) and electrical power (\( p_{e,1} \)) is made to create the comparison analogy; physically, these values are differentiated only by a LPF in this particular GFM control design. Here, it is assumed that \( p_{m,1} \) is readily available within the LPF rise time based on standard energy storage/inverter response times [32], [33]. The LPF dynamical relation between \( p_{m,1} \) and \( p_{e,1} \) provided in [2] can be reformulated as:

\[
\dot{p}_{m,1} = \frac{2\pi (p_{e,1} - p_{m,1})}{\tau_I} \quad (20)
\]

where \( \tau_I \) is the filter time constant; i.e., \( \omega_{I,fil} = \frac{2\pi}{\tau_I} \). The work in [12] found a limit of \( \omega_{I,fil} \approx 75 \text{ rad/s} \) in a 60 Hz system, and therefore \( \tau_I \approx 0.08 \text{ s} \). By the control design in [19], \( p_{m,1} \) has a first order relation to \( p_{e,1} \); \( p_{m,1} \) evolves based only on deviations in \( p_{e,1} \). Following, \( \delta_I \) evolves according to [8]. Therefore, GFM frequency is a function of the pre-converter power; the device meets the increase in power demand (\( p_{m,1} \)), and then adjusts the frequency according to the droop relation, again assuming sufficient head room, as discussed in the previous section. Thus, a GFM does not require local frequency deviations in order for \( p_{m,1} \) to evolve. In this control scheme, the frequency is changed to accomplish power sharing in a manner similar to the natural frequency fluctuations, and the slower governor control objectives, of the SG; but, the underlying through-put power process yields an inverted relationship between frequency and power differentials as compared to the SG, to which we turn next.

With the SG through-put process, we substitute [18] within the derivative of [19] to produce:

\[
\dot{p}_{m,G} = -\frac{R_D}{\tau_G} \omega_G - \dot{p}_{m,G} \quad (21)
\]

\[
\dot{p}_{m,G} = -\frac{(RM)_I^{-1} (p_{m,G} - p_{e,G}) - \dot{p}_{m,G}}{\tau_G} \quad (22)
\]

which exhibits that \( p_{m,G} \) has a second order relation to \( p_{e,G} \), contrasting with the first order dynamics of the GFM. Furthermore, \( p_{m,G} \) is a function of \( \omega_G \), a relation that is the inverse of the GFM pre-converter–frequency relationship. That is, a governor of the SG modulates \( p_{m,G} \) as a function of \( \omega_G \), as opposed to the GFM where \( \omega_I \) is changed only after the \( p_{m,1} \) is matched to the deficit. Simply, it can be said that with respect to frequency, whereas the SG is a reactive device, the GFM is a proactive device. We turn now to singular perturbation to reduce the dynamics of these converters at timescales of interest and discern the respective relationships between \( p_m \) and \( \omega \).

### A. Singular Perturbation Analysis

A common practice in the mathematical formulation and analysis of power systems is the utility of singular perturbation theory, wherein dynamical equations are re-cast as algebraic expressions because the associated dynamics are too fast (or too slow) to be of interest [34]. Within the context of power system dynamics analysis, this is perhaps most common in the algebraic treatment of the transmission system. Here, this mathematical technique is applied to the frequency dynamic analysis of the GFM and SG. Note that both [20] and [19], the power conversion dynamical equations for the GFM and SG, respectively, are first order differential equations, with time constants \( \tau_I \) and \( \tau_G \), respectively. There is, in general, an order of magnitude of separation between these values; i.e., \( \tau_I \ll \tau_G \). Therefore, where the time scale of interest is the settling of frequency dynamics associated with the SG, using the foundations of singular perturbation analysis, the follow relation can be established \( \dot{p}_{m,1} \approx 0 \); thus, according to [20], \( p_{m,1} \approx p_{e,1} \).

Due to the relatively slower response of the SG governor, immediately following a disturbance the difference between \( p_{m,G,o} \) and \( p_{m,G} \) is negligible; i.e., \( p_{m,G,o} \approx p_{m,G} \). Applying these approximations, the frequency dynamics relevant before substantial SG governor action of the GFM [8] and SG ([17] and [18]) can be reformulated. The approximated GFM frequency dynamics are given in [23]:

\[
\dot{\delta_I} = \frac{m_p(p_{m,1,o} - p_{e,1})}{\tau_I} \propto -p_{e,1} \quad (23)
\]

where \( \propto \) is the proportional operator. The SG dynamics are given in [24] and [25]:

\[
\dot{\delta_G} = \omega_G - \omega_o \quad (24)
\]

\[
\dot{\omega_G} = \frac{1}{M} (p_{m,G,o} - p_{e,G}) \propto -p_{e,G} \quad (25)
\]

From (23), it can be concluded that \( \omega_I \) is proportional to \( p_{e,1} \); therefore, considering the first order relation of \( p_{e,1} \) and \( p_{m,1} \) in (20), GFM frequency has a first order relation with pre-converter power. From (24) and (25), with respect to \( p_{e,G} \) the frequency dynamics of the SG follow a first order response. With the first order relation between \( p_{e,G} \) and \( p_{e,1} \) in [19], it is concluded that the frequency dynamics of the SG have a second order relation with respect to pre-converter power.

### IV. Numerical Analysis Results

This section presents the results of numerical analysis at the device-level and network-level using electromagnetic transient (EMT) simulations. The mathematical models presented in Section II are the foundation for the device implementation in EMT simulation software packages, such as Power System
Computer Aided Design (PSCAD) [30], the software used in this work; the devices as implemented are available open source at [35].

A. Device-Level Dynamics

First, the individual device responses are presented followed by the direct interaction of the two devices is analyzed on a three bus system to demonstrate the interrelated dynamics. The device-level analysis is concluded with an investigation into the impacts of the DC-side dynamical system.

1) Device Step Response: To verify the analytical findings and conclusions of Section II-A, the frequency response of the SG and GFM following a load perturbation is simulated. For this analysis, each device is operated in standalone fashion, dispatched at 50% with a constant power load connected directly to the terminals. All computer simulations are performed within the PSCAD environment with the full order dynamical models as outlined in Section I implemented. The rate of change of frequency (ROCOF) is defined as 

\[ \dot{f} = \frac{f(t + T_R) - f(t)}{T_R} \]

where \( f \) is the frequency, and \( T_R \) is the size, in seconds, of the sliding averaging window. A \( T_R = 100 \, ms \) window is used, in accordance with [36]. The nadir is defined as \( \min\{f(t)\} \), where \( t > t_{\text{perturbation}} \); i.e., the lowest frequency post disturbance.

The frequency and power step response of each device for a 10% load step are presented in Fig. 4. The SG frequency trace in Fig. 4a shows the second order negative step response following the load step with overshoot and subsequent damped oscillations that settle to the droop determined steady state. The GFM frequency traces follows a first order response; steady state is achieved with no overshoot and a far smaller response time as compared to the SG. This result is not just the result of a faster response by the GFM, but a dynamical system order reduction when compared to the SG. A summary of the nadir, settling frequency, and ROCOF are presented in Table I. Note the inverse proportionality between inertia and ROCOF and the correlation between a lower nadir and reduced inertia. Conversely, note the larger ROCOF of the GFM device but a resultant higher nadir.

![Frequency response](a) (b) Power response.

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**Fig. 4:** Frequency and power responses of isolated GFM and SG devices following a 10% load step.

The \( p_e \) and \( p_m \) responses of each device are presented in Fig. 4b. Note that the \( p_e \) response is identical for each device; because the voltage dynamics are near ideal, the electrical power is primarily determined by network changes and not the device dynamics. The first order relation of \( p_{m,1} \) and \( p_{e, G} \) is evident in the GFM traces, and occurs at a much faster rate than the SG response, corroborating the singular perturbation application. The \( p_{m,G} \) follows a second order response complete with overshoot, oscillations, initial acceleration, and inflection change.

2) Two-generator System: A simple two generator test system was constructed with only a single GFM (bus 1), a single SG (bus 3), and a single load (bus 2), connected by equal impedance lines (\( X_T = 0.084 + j0.126 \)). Figure 5 depicts this network. A set of 11 simulations were performed, wherein the aggregate rating of the devices was maintained at 30 MVA to sweep the mechanical inertia (\( H \)) space from 4.0 – 0.0 s. \( H \) is defined based on the relative ratings of the devices as \( H = \sum_{i=1}^{n} \frac{H_i}{S_{B,i}} \), where \( H_i \) is the inertia rating (in s) of device \( i \), \( S_{B,i} \) is the MVA rating of device \( i \), and \( n \) is the number of devices. Table II provides these device ratings and respective inertia. These rating ratios will match the aggregate inertia ratings for the 39-bus system simulation scenarios in Section IV-B2. The devices are dispatched at 0.5 per unit relative to the device rating, and the perturbation is a 0.1 per unit load step relative to the 30 MVA total of the system.

**TABLE I: Single Device Step Response Results**

| Device | ROCOF (Hz/s) | Nadir (Hz) | Settling Frequency (Hz) |
|--------|-------------|------------|-------------------------|
| SG (\( H = 4s \)) | 0.48 | 59.77 | 59.85 |
| SG (\( H = 3s \)) | 0.63 | 59.73 | 59.85 |
| SG (\( H = 2s \)) | 0.95 | 59.67 | 59.85 |
| SG (\( H = 1s \)) | 1.50 | 59.85 | 59.85 |
| GFM | 1.90 | 59.72 | 59.85 |

![Grid Forming Inverter](a) Synchronous Condenser

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**Fig. 5:** Three bus test system used to establish similar device rating ratios compared to the 39-bus system.

**TABLE II: Two Generator System Results**

| SG Rating (MVA) | GFM Rating (MVA) | Inertia (s) | SG ROCOF (Hz/s) | SG Nadir (Hz) |
|-----------------|------------------|-------------|-----------------|---------------|
| 30              | n/a              | 4.0         | 0.426           | 59.687        |
| 27              | 3                | 3.6         | 0.489           | 59.701        |
| 24              | 6                | 3.2         | 0.528           | 59.725        |
| 21              | 9                | 2.8         | 0.582           | 59.740        |
| 18              | 12               | 2.4         | 0.645           | 59.754        |
| 15              | 15               | 2.0         | 0.721           | 59.765        |
| 12              | 18               | 1.6         | 0.813           | 59.775        |
| 9               | 21               | 1.2         | 0.934           | 59.785        |
| 6               | 24               | 0.8         | 1.102           | 59.791        |
| 3               | 27               | 0.4         | 1.361           | 59.767        |
| n/a             | 30               | 0.0         | 1.390 (GFM)     | 59.842 (GFM)  |

The frequency results presented in Table II are for the SG alone (except for the final row), to highlight the impact of
the GFM on the SG operation. There is a clear shift with a growing GFM ratio to an increase in ROCOF, accompanied by a rising nadir. Figure 6 shows the SG frequency for a few of these cases, where it is evident that substantial damping is afforded to the SG response, until the SG represents only a small portion of the system machine rating (i.e., $< 20\%$). When the SG rating is 3 MVA, and the GFM is rated at 27 MVA, a high frequency (2.9 Hz) oscillatory mode is present that damps out within a few cycles. This oscillatory mode is present only due rapid variations in $p_{e,G}$ as $p_{m,G}$ does not change within the time frame of this oscillatory response. This mode contrasts heavily with the 0.23 Hz oscillatory mode dominating the SG only simulation (SG: 30 MVA). The primary takeaway is that for low levels of SGs as compared to GFM, with respect to total rating, high frequency oscillatory modes are excited in the SGs. The anticipated cause for these modes is the result of rapid power flow differentials generated on the network by the rapid GFM frequency change. This will incur a substantial $p_{e,G} - p_{m,G}$ differential on the SG, that when matched with the low inertia, would result in these higher frequency oscillatory modes. Further investigation into the root cause of these oscillatory modes is prime topic for future research.

![Fig. 6: Frequency response of the synchronous generator (SG) from the two generator test system with a GFM, SG, and single load. Different responses reflect different rating ratios of the two devices, where the aggregate rating is a constant at 30 MVA.](image)

3) Impact of DC-Side Dynamics: To determine an appropriate capacity of the DC-link capacitor ($C_{DC}$), and a suitable DC-link voltage across the capacitor ($V_{DC}$), a computational sensitivity analysis was performed. The capacitor sizes investigated represent either 0.1, or 1 cycle’s worth of energy contained at the rated inverter power and voltage. That is, the capacitor rating is determined according to (26):

$$ C_{DC} = \frac{2S_{base,\alpha}}{V_{DC}} $$

where $S_{base}$ is the device rating and $\alpha$ represents the time duration based on the cycles of rated power. At a minimum, the $V_{DC}$ must span the positive and negative peaks of the inverter output. Namely, $v_{DC} > V_{base, l-1} \sqrt{8/3}$. Factoring in the anticipated operation of the inverter above rated voltage, in this case it is assumed 1.05 per unit, the target $V_{DC}$ can be represented as in (27):

$$ v_{DC, set} = 1.05 \sqrt{\frac{8}{3}} V_{base, l-1} \Delta V $$

where $\Delta V$ is a parameter that represents the setpoint overlap above the requisite voltage. The parameter sweep is an executed 0.5 per unit load step, with an initial dispatch at 0.05 per unit, on an isolated GFM inverter for six different value combinations of $C_{DC}$ and $V_{DC}$. The per unit power differential between the no DC dynamics ($p_{noDC}$) and the resultant power with the specific parameters ($p_{DC}$) is integrated (discretely) across the length of the simulation, $E_{diff} = \sum (p_{noDC} - p_{DC}) \Delta t$, to quantify the impacts of the DC side dynamics. These results are shown in Table III where it is evident that for both voltage buffers of $\Delta V = 5\%$ and $\Delta V = 10\%$, the differential is very small; it is effectively zero when the capacitor is rated for one electrical cycle and $\Delta V = 10\%$. In simulations where $E_{diff} \neq 0$, the difference persists for less than 200 milliseconds. Subsequently, the DC side values of $C_{DC} = 1$ cycle and $\Delta V = 10\%$ are selected to support the immediate availability of power assumed in the perturbation analysis of Section III-A. Henceforth, all DC side dynamics are deactivated in the following simulations to improve simulation time.

| $\Delta V$ (%) | Cycles of Capacitive Energy |
|---------------|-----------------------------|
| 0             | 0.198 0.159                |
| 5             | 0.031 0.003                |
| 10            | 0.0001 0.0                 |

**B. Network-Level Dynamics**

After the device level analysis of the GFM and SG, we now turn to the network level numerical analysis results from simulations with the the IEEE 9- and 39- bus test systems in the EMT domain are presented in this section. In these simulations, a weighted frequency is calculated according to (28):

$$ f(t) = \sum_{i=1}^{M} \left( \frac{MVA_i}{MVA} \right) f_i(t) $$

where $f_i(t)$ is the frequency of device $i$ at time $t$, $MVA_i$ is the device $i$ rating, and $n$ is the number of devices. This weighted frequency is used to determine the ROCOF and nadir values. Additionally, the matrix pencil method [37]–[39] is used to calculate the frequency and decay of the modes by fitting complex exponentials to the time-series data, $\sum_{i=1}^{M} A_i e^{(-\alpha_i + j\omega_i) t} + n(t)$ where $y(t)$ is the time-series data, $M$ is the number of modes, $A_i$ is the complex amplitude, $\alpha_i$ is the damping factor, $\omega_i$ is the angular frequency, all of mode $i$, and $n(t)$ is the noise. The damping of the system is then $\zeta_i = \frac{\alpha_i}{\sqrt{\lambda_i^2 + \omega_i^2}}$ where $k$ corresponds with the dominant mode.

1) Test Case 1: IEEE 9-Bus System: Simulations on the IEEE 9 bus test system, which consists of three generation sources, were performed in the PSCAD environment. The network and all associated static elements, where all parameters are consistent with the test system implementation, are
available open source at [35], [40]. Buses 4-9 are 230 kV, buses 1-3 (the generation buses) are 16.5 kV, 18.0 kV, and 13.8 kV, respectively. All SG or GFM devices are rated at 200 MVA, with other pertinent parameters as established in Section I[11]. The load is modelled as constant power with no frequency or voltage dependence. A 10% load step (31.5 MW, 11.5 MVar) occurs at bus 6. Prior to the perturbation, the system is brought to steady state by initiating all devices as ideal sources, and then systematically releasing the associated dynamics in a manner conducive to maintaining steady state stability; see [41] for additional PSCAD start-up methodology. Different interconnection scenarios of SG and GFMs into the system are created by systematically supplanting an SG with a GFM, as shown in Table IV.

TABLE IV: 9 Bus Configuration and Results

| Scenario | Device at Bus | Inertia | ROCOF (Hz/s) | Nadir (Hz) | Damping (ζ) |
|----------|---------------|---------|--------------|------------|-------------|
| A        | SG            | 4.0     | 0.50         | 59.72      | 0.363       |
| B        | GFM          | 2.6     | 0.73         | 59.76      | 0.528       |
| C        | GFM          | 1.3     | 1.12         | 59.79      | 0.842       |
| D        | GFM          | 0.0     | 1.61         | 59.83      | 0.919       |

The results in Fig. 7 show the results for four simulations (scenarios A, B, C, and D - explained in Table IV) where the SGs are systematically replaced by the GFMs, resulting in an incremental decrease in the aggregate mechanical inertia. These reduced values are captured in Table IV along with the resultant nadir and ROCOF statistics of the average frequency for each scenario. The final column presents the damping ratio of the dominant mode of the average frequency. It is evident that as the mechanical inertia is decreased, the nadir is raised while the ROCOF increases, which is indicative of the dominant first order response of the additional GFMs as the inverse would be expected in a second order system. The system frequency oscillation period with all SGs (Scenario A) matches the single machine step response; i.e., 0.4 Hz oscillations. By inspection of Fig. 8 the dominant oscillatory mode increases in frequency with more GFMs. The damping ratio from Table IV shows a clear increase as the converters are changed to GFMs; the presence of a 200 MVA GFM yields a larger damping contribution to the system as compared to an equally rated SG.

Individual device frequencies for each Scenario are presented in Fig. 8 where it is evident that for Scenario A, all three SGs have similar frequency trajectories; the three devices maintain broad synchronization following the perturbation. Herein lies the motivation for center of inertia and average frequency metrics, which rely on the assumption of similar (i.e., second order) frequency trajectories of all devices [17], [42].

The Scenario B system frequency is second order with a damping value around 50%; the dominant mode damping is increased according to Table IV. From Fig. 8 it is obvious that the three devices are not broadly synchronized immediately following the disturbance. The Scenario C frequency exhibits overshoot, but a quasi-first order recovery; i.e. the concavity in the green trace from t = 1.5–2.5 s is the inverse of the expected second order recovery. The GFM device frequencies change too rapidly for all of the devices to remain synchronized during the first 0.5 s following the perturbation. In Scenario D, with all GFM devices, the system frequency follows a typical first order response where overshoot and similar frequency oscillations are absent. The general stability of the nine bus system with all GFM devices modelled in the EMT domain provides a strong testament to the stable operation of an all inverter based system. The bedrock assumption of average system frequency metrics is that all devices have similar frequency trajectories [17], [29]; these results show this assumption is not necessarily valid for the frequency response immediately following a perturbation with GFM device recovery contributions.

The $p_e$ and $p_m$ response for each scenario are presented in Fig. 9. The electrical powers show inter-area oscillations ($f = 1.6$ Hz) between SG 1 and SG 2 in Scenario A. Note the time separation between these $p_e,G$ Oscillations and $p_m,G$. Scenario B shows the very rapid changes in $p_m,1- p_e,1$ (these are indistinguishable at this temporal resolution) of GFM 1, which exacerbates a larger peak $p_e,G$ of SG 2 and 3, while the subsequent oscillations are more damped. The peak $p_e,G$ output of SG 3 in Scenario C is further increased, although there are no oscillations following this overshoot. The conclusion is that the rapid frequency changes of the GFMs, evident in Fig. 8 cause the network conditions to...
change rapidly, while the SG frequency follows the slower second order response and results in a larger $P_{e,G}$ extraction. Therefore, the fast frequency change of the GFM forces the SG into larger oscillations because of its slow response and exacerbates the dynamic excursions. This large $P_{e,G}$ excursion corroborates the hypothesis of an increasing initial extraction of electrical energy when the SG rating ratio is low with high shares of GFM, based on the observations in the two generator system. Scenario D power outputs show a reduction in power oscillations with minimal overshoot and a relatively rapid arrival to settling outputs.

The results from the 9 bus system suggest that the presence of GFM inverters reduces the average system frequency nadir, while increasing ROCOF. In a second order system, these two directional changes would not be correlated; the cause is due to the first order response of the GFM devices. Additionally, dominant oscillatory mode analysis indicates a monotonic increase in damping with more GFMs. Given the diverging frequency trajectories of the GFM and SG immediately following a disturbance, traditional average frequency methods may not be an appropriate metric; these metrics are founded on the basis of similar trajectories of devices, which are temporarily violated in these simulations.

2) Test Case II: IEEE 39-Bus System: The IEEE 39 bus test system [43] is presented as a larger case study, with the entire system as simulated in PSCAD and supporting Python code available open-source at [35]. The network has been partitioned into 6 subsystems using the Bergeron parallelization components; the network elements are unchanged. All buses operate at 230 kV, with a 230/18 kV generator step-up unit installed for all generation elements at 18 kV. All generation elements are rated at 1000 MVA; note that these ratings are a departure from the standard model. Dispatch and voltage set points are unchanged from the test system configuration. All ten initial SG devices are systematically replaced by GFMs, with a scenario defining each iteration; i.e. scenarios 0–10 in Table [V] The 10% load step (600 MW/140 Mvar) occurs at bus 15.

| Scenario | GFM at Buses | Inertia (s) | ROCOF (Hz/s) | Nadir (Hz) | Damping ($\zeta$) | Dom. Mode Freq. (Hz) |
|----------|--------------|------------|--------------|------------|------------------|---------------------|
| 0        | n/a          | 4.0        | 0.567        | 59.690     | 0.361            | 0.324               |
| 1        | 30           | 3.6        | 0.587        | 59.712     | 0.394            | 0.332               |
| 2        | 30–31        | 3.2        | 0.669        | 59.717     | 0.440            | 0.345               |
| 3        | 30–32        | 2.8        | 0.808        | 59.724     | 0.489            | 0.358               |
| 4        | 30–33        | 2.4        | 0.930        | 59.730     | 0.551            | 0.369               |
| 5        | 30–34        | 2.0        | 1.071        | 59.738     | 0.622            | 0.383               |
| 6        | 30–35        | 1.6        | 1.225        | 59.748     | 0.722            | 0.386               |
| 7        | 30–36        | 1.2        | 1.396        | 59.748     | 0.958            | 0.215               |
| 8        | 30–37        | 0.8        | 1.525        | 59.756     | 0.382            | 1.320               |
| 9        | 30–38        | 0.4        | 1.648        | 59.772     | 0.224            | 1.414               |
| 10       | All GFM      | 0.0        | 1.852        | 59.808     | 0.893            | 1.99                |

Figure [10] shows the average frequency for each of the 11 scenarios simulated on the 39 bus system; there are broad similarities to the 9 bus results, with a substantial reduction in overshoot following the transition to a GFM dominated system. The inverted period within the initial deceleration occurs at higher average frequency values with a larger quantity of GFM devices. The frequency statistics presented in Table [V] show that while ROCOF increases with larger quantities of GFMs and a resultant decrease in system mechanical inertia, the nadir is simultaneously raised; the average frequency trajectories expose a transition to a first order response.

Fig. 10: Average frequency response of 39 bus system for varied quantities of GFMs and SGs and a 10% load step at bus 15.
substantial decrease in damping can be seen with only a few SGs remaining on the system, though at a greatly increased dominant mode frequency. From Fig. 12 Scenario 9, a larger number of distinct oscillations are seen in the remaining SG, indicating that this device is oscillating at a faster frequency with the GFM devices. The presence of a large number of first order devices is exacerbating the second order response of the remaining SGs. The simulations in Section IV-A2 emulated the ratios of GFM to SG devices realized in the 39-bus system. The same high frequency mode was uncovered in the SG for low shares, however, this mode was realized only by varying the rating of the two devices present on the system. This indicates that neither the network, nor the quantity of the devices, plays a role in these high frequency modes present during times of low SG rating ratios.

The $f$-$p_{m,l}$ portraits for a selection of 39 bus system simulations are presented in Fig. 12 where the subtitles correspond with the Table VII entry. With no GFMs, the SGs follow the trajectory of an initial frequency deviation prior to pre-converter changes. Prior to convergence on the steady state values, the trajectories exhibit oscillations in the form of converging spirals. With half GFMs in scenario 5, the algebraic relation between frequency and pre-converter power is evident, while the SG trajectories are shortened with less overshoot. With only a single SG online in scenario 9, the SG exhibits no pre-converter power overshoot; however, substantial frequency oscillations are present, of a higher frequency as confirmed by pre-converter power overshoot; however, substantial frequency oscillations are present, of a higher frequency as confirmed by pre-converter changes frequency according to a control response, not as frequency triggered governor action to match $p_m$ after $p_m$ changes due to varying network conditions, $e,I$ not changing frequency of the GFMs, the GFMs are also the source of damping in the system as no substantial damping can be seen with only a few SGs remaining on the system, though at a greatly increased dominant mode frequency. From Fig. 12 Scenario 9, a larger number of distinct oscillations are seen in the remaining SG, indicating that this device is oscillating at a faster frequency with the GFM devices. The presence of a large number of first order devices is exacerbating the second order response of the remaining SGs. The simulations in Section IV-A2 emulated the ratios of GFM to SG devices realized in the 39-bus system. The same high frequency mode was uncovered in the SG for low shares, however, this mode was realized only by varying the rating of the two devices present on the system. This indicates that neither the network, nor the quantity of the devices, plays a role in these high frequency modes present during times of low SG rating ratios.

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V. DISCUSSION

For devices that adjust pre-converter power as a function of frequency changes such as SGs, a larger ROCOF generally yields a deeper nadir for the same magnitude power imbalance [36]. Consider the approximating equation $\Delta f_{prior} = \alpha_{ROCOF} \times t_{response}$ where $\Delta f_{prior}$ is the frequency deviation prior to substantive $p_{m,G}$ changes, $\alpha_{ROCOF}$ is the ROCOF for a generic system, and $t_{response}$ is the pre-converter power response time (as used in [20] and [19]). Evidently, a relatively larger ROCOF for the same $t_{response}$ (such as $\tau_G$, in [19]), yields a larger $\Delta f_{prior}$ prior to substantial $p_m$ changes. This is the inertial response period when rotational kinetic energy ($E_{int,G}$) is extracted from the SGs; in SG dominated systems, less inertia yields larger $\alpha_{ROCOF}$ values, increasing susceptibility to lower nadirs that can trigger frequency load shedding [44].

Due to the control design, the GFM relation is $\Delta f_{prior} = 0$. After $p_{e,l}$ changes due to varying network conditions, $p_{m,l}$ is matched prior to any change in frequency. The GFM changes frequency according to a control response, not as the result of a necessary chain of rotational kinematics and frequency triggered governor action to match $p_{e,G}$ and $p_{m,G}$. Therefore, the pre-converter–frequency relationship can be of a lower order with GFMs. Resulting from the lower order relationship embedded in droop control is a reduction in frequency dynamics in the presence of GFMs as presented in the simulation results of Sections IV-B1 and IV-B2 where it is evident that the customary second order frequency response of SG dominated systems diminishes with GFM devices. Here, it is noted that there are potential broader issues due to larger ROCOFs such as relay tripping, device disconnection, and machine shaft strain; but the analysis of these considerations is beyond the scope of this work.

An odd phenomenon was uncovered in the two generator system, where for very low levels of SGs as a rating ratio (i.e., when the rating of the SG was < 20% of the total system online generation), a previously unreported high frequency oscillatory mode was observed. While this mode was uncovered in the two generator system, it was also observed in the 39-bus system with ten generators, where similar rating ratios could be realized not by changing the device rating, but instead by binary dispatch changes where a generator is either a GFM or SG. That the mode exists in both of these systems indicates that it is not a function of generator quantity nor network interactions; it is likely the result of the rapid change frequency of GFMs, which places a substantial degree of inertia remaining on the system. Interestingly, while it is caused by the rapid change frequency in the presence of the GFMs, the GFMs are also the source of damping in the system as no substantial $p_{m,G}$ changes are observed prior the oscillatory mode dampening out. This observation is generally contradictory to the claim of a trend of...
increased damping with GFM devices [17], [20], in that there is evidently an inversion in this damping when GFMs are the dominant resource. This oscillatory mode was not observed in the 9-bus system because the binary variation of generators did not yield this low rating ratio (< 20%) where the phenomenon is present.

VI. CONCLUSION

This work investigated the power conversion system disparity of the synchronous generator and grid-forming inverter and the power system frequency impacts associated with these differences. The conversion dynamics were assessed with a simplified model from the full order dynamics of each device, yielding a lower order relation between pre-converter power and frequency with the grid-forming inverter as compared to the synchronous generator; further highlighted was the proactive nature of the grid-forming inverter with respect to frequency, contrasting with the reactive conversion process of the synchronous generator. Device level electromagnetic simulations corroborate this order reduction, as well as a previously unreported high frequency oscillatory mode for very low levels of SGs as compared to GFMs on a simple three bus system. Additionally determined was a negligible impact of DC-side dynamics during GFM operation within rating limits with appropriate capacitor sizing and DC voltage setpoints. As implemented in the electromagnetic transient domain with the 9- and 39-bus test systems, it was shown that the traditional frequency metric relations of larger rate of change and frequency with the grid-forming inverter as compared to the synchronous generator; further highlighted was the proactive nature of the grid-forming inverter with respect to frequency, contrasting with the reactive conversion process of the synchronous generator. Device level electromagnetic simulations corroborate this order reduction, as well as a previously unreported high frequency oscillatory mode for very low levels of SGs as compared to GFMs on a simple three bus system. Additionally determined was a negligible impact of DC-side dynamics during GFM operation within rating limits with appropriate capacitor sizing and DC voltage setpoints.

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TABLE VI: GFM parameters; all in per unit.

| Parameter | Value | Par... Value | Par... Value |
|-----------|-------|--------------|--------------|
| $L_f$     | 0.15  | $R_f$        | 0.005        |
| $R_{cap}$ | 0.005 | $L_{sc}$     | 0.15         |
| $k_E^L$   | 1.19  | $k_E^C$      | 0.73         |
| $k_V^L$   | 1.16  | $k_V^C$      | 0.52         |
| $k_{V,DC}$| 979   | $k_{V,DC}$  | 73.0         |
| $L_{DC}$  | 18.3  | $\Delta V$  | 10%          |
| $C_{DC}$  | 1 cycle | 1 cycle  |  |  |

TABLE VII: SG parameters; per unit unless noted.

| Parameter | Value | Par... Value | Par... Value |
|-----------|-------|--------------|--------------|
| $T_G$     | 0.5 s | $R_{Wind,dc}$| 0.01         |
| $X_d$     | 0.92  | $X_d$        | 0.169        |
| $T_{do}$  | 0.135 | $T_{do}$     | 0.032        |
| $X_q$     | 0.228 | $T_{qo}$     | 0.85         |
| $T_{qo}$  | 0.05 s| $K_{P,R}$    | 12.77        |
| $K_{P,A}$ | 20    | $K_{I,A}$    | 1            |
| $K_{I,R}$ | 20    | $T_E$        | 1.945 s      |

APPENDIX