Methods for detecting fatigue cracks in gears

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Abstract. A crack in the tooth root is the least desirable damage caused to gear units and it often causes failure of gear unit operation. For fault analyses presented in this article, gear units with real damages or faults produced on the basis of numerical simulations of real operating conditions are used; tests were carried out in a laboratory test plant. Damages in gear units can be identified by monitoring vibrations. The influences of a crack in a single-stage gear unit on produced vibrations are presented. Significant changes in tooth stiffness are caused by a fatigue crack in the tooth root whereas, in relation to other faults, changes of other dynamic parameters are more expressed. Non-stationary signals are analysed, using the family of Time Frequency Analysis tools, which include Wavelets and Joint Time Frequency Analyses.

1. Introduction
The aim of maintenance is to keep a technical system (e.g. a gear unit) in the most suitable working condition, whereas its purpose is to discover, to diagnose, to foresee, to prevent and to eliminate damages. On the other hand, the purpose of modern maintenance is not only to eliminate failures but also to identify the stage before a sudden failure of system operation. The objectives of diagnostics are to define the current condition of the system and the location, shape and reason of damage formation.

A gear unit is a complex dynamic model. Its elements enable transmission of rotating movement. The movement is usually periodical, faults and damages represent a disturbing quantity or impulse. Disturbances are denoted by local and time changes in vibration signals. Consequently, time-frequency changes can be expected [1,2]. This idea is based on kinematics and operating features.

Fourier, adaptive and wavelet transforms, and Gabor expansion represent various time-frequency algorithms [3]. The basic idea of all linear transforms is to perform, in advance, the comparison with the elementary function [4]. Different signal presentations can be obtained by means of different elementary functions.

2. Introduction
Many authors had been developing algorithms without interference parts that reduce usability of individual transforms. Qian [4] significantly enhanced the adaptive transform of a signal, which represented a difference to Cohen’s class.

The adaptive transform of a signal $x(t)$ is expressed in the following way:

$$x(t) = \sum_p B_p \cdot h_p(t)$$  \hspace{1cm} (1)
where the following equations were used to analyse coefficients

\[ B_p = \langle x, h_p \rangle \]  

(2)

In this way, a similarity between the measured signal \( x(t) \) and elementary functions \( h_p(t) \) of transform is expressed.

It is possible to determine the time-dependent adaptive spectrum as follows

\[ P_{ADT}(t, \omega) = \sum_p |B_p|^2 \cdot P_{WV} h_p(t, \omega) \]  

(3)

As a rule, the choice of elementary functions has no impact upon the adaptive transform since it allows arbitrary elementary functions. Elementary functions, applied for adaptive representation of a signal with equation (1), are usually very general. In practice, however, this is not always so. To stress the time dependence of a signal, the elementary functions should be localised in regard to time and frequency. Also, it is desirable that they can use the presented algorithm in a relatively simple manner. A Gauss type signal has very favourable characteristics, and it is considered a basic choice when it comes to an adaptive representation.

3. Wavelet Analysis

This is how the continuous wavelet transform of function \( x(t) \in L^2(\mathbb{R}) \) at the time and scale is expressed [5]:

\[ W_x(t, s) = \langle x, \psi_{u,s} \rangle = \int_{-\infty}^{+\infty} x(t) \cdot \frac{1}{\sqrt{s}} \cdot \psi_s^*(t) \cdot dt = x(t) \otimes \overline{\psi}_s(t) \]  

(4)

When it comes to the continuous wavelet transform, the observed function \( x(t) \) is multiplied by a group of shifted and scaled wavelet functions. Time and frequency dissemination of the continuous wavelet transform changes simultaneously. Locally limited functions (wavelets) are used to analyse the observed function \( x(t) \); therefore, the continuous wavelet transform is very sensitive to local non-stationarities.

Gabor wavelet function, which is acquired using a frequency modulation of the Gauss window function, represents an approximately analytical wavelet function [5]:

\[ \psi_{Gabor}(t, \sigma, \eta) = \frac{1}{4\sqrt{\sigma^2 \cdot \pi}} \cdot e^{-\frac{t^2}{2\sigma^2}} \cdot e^{j\eta \cdot t} \]  

(5)

4. Practical Example

The measurements were carried out in the test plant (presented in Fig. 1) of the Computer Aided Design Laboratory of the Faculty of Mechanical Engineering, University of Maribor. A one-stage helical gear unit is at the spot where vibrations were measured [6].

A single stage gear unit was used. A helical gear unit with straight teeth was integrated into the gear unit. A carburised spur gear pair of module 4 mm was in each gear unit, the pinion had 19 and the wheel 34 teeth. The presented results are relating to a nominal pinion torque of 20 Nm and nominal pinion speed of 1200 rpm (20 Hz), which is, in industry, a load condition very much characteristic of this type of gear units. Tests were performed under constant loads. Accelometers, fixed on the housings, were used to measure vibrations.
A gear unit with a fatigue crack in the tooth root of a pinion was measured; the operating conditions were such as they are normally associated with this type of a gear unit. A standard gear pair, teeth quality 6, that was used had a crack in the tooth root of a pinion.

The characteristics of elementary functions are restricted. Consequently, adaptive spectrogram has a fine adaptive time-frequency resolution. Time-frequency resolution of the transform is adapted to signal characteristics. Gauss function (impulse) and linear chirp with Gauss window can be used as an elementary function.

The signal was 1 s long; on average, it had 14000 measuring points. It is possible to note some pulsation sources but, in relation to adaptive spectrogram, they are not very expressed (Fig. 2). This indicates a higher level of energy accumulation in the origins. Monitoring the increase or decrease (complete disappearance) in appropriate frequency components with rotational frequency of 20 Hz is of particular interest. This is typical of the 3rd harmonic of mesh frequency. 1530 Hz is expressed only in association to the presence of a crack. This phenomenon can be noted in the adaptive spectrogram (Fig. 3). In relation to a single engagement of a gear pair with a crack in one rotation of a shaft, the pulsation (the area marked with a continuous line) is expressed. Similarly, between the 6th and the 9th harmonics (the area marked with a dashed line), sources indicating pulsating portions of individual components, with the frequency of 20 Hz, can be observed.
A scalogram of analytical wavelet transform with Gabor wavelet function represents square values of amplitudes of wavelet coefficients. On the grounds of the connection between the scale and frequency, the representation is performed in a time-frequency domain. This is favourable when it
comes to technical diagnostics as it is much simpler to determine adequate characteristics in time-frequency domain (frequency scalogram) than in time-scale domain (scalogram). The transform matches the Parseval characteristic of energy preservation on the basis of normalization, meaning that the energy of wavelet transform is equal to the energy of the original signal in time domain.

Wavelet analysis is primarily appropriate in relation to non-stationary phenomena with local changes. Therefore, the analysis was performed to determine the condition associated with the presence of a crack in a tooth root. More specifically, the purpose of the analysis was to establish the location of the crack.

The analytical continuous wavelet transform (parameters: $\eta = 8$ and $\sigma = 2$) was used for the analysis. Nyquist frequency and the frequency of sampling the measured time signal were used to obtain the highest frequency in the signal (7000 Hz). After that, based on the known connection between scale and frequency, the scale type for constant frequency distribution was determined. The representation of the frequency scalogram is given in the form of wavelet coefficients or their square values.

In relation to the faultless gear, the figures in the frequency scalogram indicate no particularities in expressed components that would denote local changes. This is so in relation to a square representation (Fig. 4) of wavelet coefficients. In relation to the analysis of the signal produced by a gear with a crack, a local change in wavelet coefficients, in time at the value of 12 ms, is very clearly indicated in frequency scalograms (Fig. 5). In relation to the tooth with the crack in its root, a local change, i.e. the presence of transients, can be observed. If the wavelet length is 50 ms, which represents one rotation of the pinion, and if there are 19 teeth along the circumference, the increased amplitude is located at 12 ms. It belongs to the fourth tooth in the direction of rotation from the reference positional point of the gear unit.

![Figure 4: Frequency scalogram of square wavelet coefficient of the reference gear unit](image_url)
5. Conclusions
The purpose of vibration analysis is to detect a fault in industrial gear units. On the basis of the methods presented, the safety of operation and, as a result, the reliability of monitoring operational capabilities can be improved. The reliability of monitoring life cycle of a gear unit can be increased by means of suitable spectrogram samples and a clear presentation of the pulsation of individual frequency components. By using wavelet transform, changes can be identified very quickly and the presence of a damage can be determined at the level of an individual tooth.

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