On the propagation of waves in dissipative layered cylindrical bodies

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Abstract. In work questions of distribution of characteristic waves in the dissipative stratified cylindrical bodies interacting with deformable (viscoelastic) environments are considered. The dynamic behavior of cylindrical bodies is described by the equations of a mechanics of continua. The spectral task comes down to the solution of system of ordinary differential equations of first order with variable complex coefficients. The solution of system of differential equations is expressed by means of Bessel and Hankel’s cylindrical higher transcendental functions. The frequenciest equations are solved numerically by methods of Müller and Gauss. Change of a natural frequency and phase speed depending on a wave number is investigated. For dissipative heterogeneous mechanical systems nonmonotone dependence of imaginary parts of phase speeds on wave numbers is found.

1. Introduction
Regularities of distribution of indignations in continuous mediums are of the considerable interest to many fields of science and technology [1,2].

In the work, unlike the known ones, the propagation of natural waves is considered, dissipative inhomogeneous (the elements of the mechanical system have different rheological behavior) a cylindrical body [3,4,5,6].

The analysis of an advance of waves in an elastic medium is based on an assumption about validity of a Hooke law according to which tension in this point of the environment in instants is proportional to deformations in the same instants [7,8]. The fact that energy of a series of waves or an impulse remains to a constant is a consequence of this assumption. As a rule, the influence of these "non-guk" relationships on metals and metal alloys is not sufficient stress, but they are of great importance for rubber and polymer structural materials that find significant time effects [9]. Also different viscoelastic (rheological) behavior of the dissipatively nonuniform bodies distribution and attenuations of waves significantly influences the law [10,11,12].

2. Statement of the problem and methods of solution
The cylindrical coordinate system \(\{r, \theta, z\}\) the propagation of natural waves in an isotropic viscoelastic body \((V)\), consisting of a piecewise-homogeneous cylindrical bodies \(V_k\) \((k=1, N)\), immersed in a uniform viscoelastic medium (Figure 1). The first paragraph after a heading is not indented (Bodytext style).

Linear equations of motion of the mechanical system in vector form in the absence of body forces takes the form:
\[ \hat{\mu}_k \nabla^2 \ddot{u} + (\hat{\lambda}_k + \hat{\mu}_k) \text{grad} \text{div} \ddot{u} = \rho_k \frac{\partial^2 \ddot{u}}{\partial t^2}, \quad (k = 1, 2, 3, \ldots, N) \]  

where

\[ \hat{\lambda}_k f(t) = \lambda_{k0} \left[ f(t) - \int_{-\infty}^{t} R_{3k}(t-\tau)f(\tau)d\tau \right], \]

\[ \hat{\mu}_k f(t) = \mu_{k0} \left[ f(t) - \int_{-\infty}^{t} R_{4k}(t-\tau)f(\tau)d\tau \right], \quad (2) \]

\( f(t) \) – arbitrary function of time; \( R_{3k}(t-\tau) \) and \( R_{4k}(t-\tau) \) – relaxation kernel; \( \lambda_{k0}, \mu_{k0} \) – instant elastic moduli \((k=1, 2, \ldots, N)\); \( \ddot{u} \) – displacement vector; \( \rho_k \) – density of the medium, \( k \) – ordinal number of layers.

Between the layers is put hard conditions or sliding contact [13]

\[ r = a_k : \quad \sigma_{rk} = \sigma_{r(k+1)}; \quad \sigma_{r0k} = \sigma_{r0(k+1)}; \quad \sigma_{rzk} = \sigma_{rzk+1} ; \]  

\[ u_k = u_{k+1}; \quad v_k = v_{k+1}; \quad w_k = w_{k+1}. \quad (3a) \]

If the outer surface is free from multilayer cylinder stresses, whereas

\[ r = a_{k+1} : \quad \sigma_{r(k+1)} = 0; \quad \sigma_{r0(k+1)} = 0; \quad \sigma_{rzk+1} = 0. \]  

If the displacement vector presented in the form of potential and solenoidal parts
\[ \ddot{u} = \nabla \varphi + \nabla \times \psi \]

the wave equation have the form

\[ \Delta \varphi_k - \frac{1}{c_{jk}^2} \frac{\partial^2 \varphi_k}{\partial t^2} = 0; \]

\[ \Delta \psi_{sk} - \frac{1}{c_{sk}^2} \frac{\partial^2 \psi_{sk}}{\partial t^2} = 0; \]

\[ \Delta \psi_{sk} - \frac{\psi_{sk}}{r^2} + \frac{2}{r^2} \frac{\partial \psi_{sk}}{\partial \theta} - \frac{1}{c_{sk}^2} \frac{\partial^2 \psi_{sk}}{\partial t^2} = 0; \]

\[ \Delta \psi_{sk} - \frac{\psi_{sk}}{r^2} + \frac{2}{r^2} \frac{\partial \psi_{sk}}{\partial \theta} - \frac{1}{c_{sk}^2} \frac{\partial^2 \psi_{sk}}{\partial t^2} = 0; \]

\[ \Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}; \]

\[ u_{rk} = \frac{\partial \varphi_k}{\partial r} + \frac{2}{r} \frac{\partial \psi_{sk}}{\partial \theta} - \frac{\partial \psi_{sk}}{\partial z}; \]

\[ u_{rk} = \frac{1}{r} \frac{\partial \varphi_k}{\partial \theta} + \frac{\partial \psi_{sk}}{\partial z} - \frac{\partial \psi_{sk}}{\partial r}; \]

\[ u_{sk} = \frac{\partial \varphi_k}{\partial r} + \frac{\partial \psi_{sk}}{\partial r} - \frac{1}{r} \frac{\partial \psi_{sk}}{\partial \theta}; \]

\[ k = 1, 2, \ldots, N. \]

where \( \varphi \) – potential of longitudinal waves; \( \psi (\psi, \psi_\theta, \psi_z) \) – potential transverse waves.

In the construction of representations for motion vector components a multilayer cylinder

\[ 0 < r_1 \leq a_1; a_1 \leq r_2 \leq a_2; \ldots; a_{k-1} \leq r_k \leq a_k; a_{k-1} \leq r_k \leq \infty \quad (k = 1, 2, \ldots, N); |z_1| < \infty \]

we proceed from the equation (4) for the scalar \( \varphi (r, \theta, z, t) \) and the vector \( \psi (r, \theta, z, t) \) potentials. The geometry of the object and the natural assumption about the nature of wave motion along the axis \( OZ \) allow largely to predict the shape of the desired scalar and vector functions. They should represent waves traveling along the \( OZ \) axis. The solutions of (4) in the form:

\[ \varphi_k (r, \theta, z, t) = \sum_{n=0}^{\infty} \phi_n (\alpha_k r) \begin{bmatrix} \cos n\theta \\ -\sin n\theta \end{bmatrix} e^{i\gamma_k z} e^{-i\omega t}; \]

\[ \psi_{rk} (r, \theta, z, t) = \sum_{n=0}^{\infty} \psi_{nr} (\beta_k r) \begin{bmatrix} \sin n\theta \\ -\cos n\theta \end{bmatrix} e^{i\gamma_k z} e^{-i\omega t}; \]

\[ \psi_{rk} (r, \theta, z, t) = \sum_{n=0}^{\infty} \psi_{nr} (\beta_k r) \begin{bmatrix} \cos n\theta \\ -\sin n\theta \end{bmatrix} e^{i\gamma_k z} e^{-i\omega t}; \]

\[ \psi_{sk} (r, \theta, z, t) = \sum_{n=0}^{\infty} \psi_{nr} (\beta_k r) \begin{bmatrix} \sin n\theta \\ \cos n\theta \end{bmatrix} e^{i\gamma_k z} e^{-i\omega t}; \]

where \( n \) - integer; \( \gamma_{pk} \) - constant wave propagation; \( \omega \) - natural frequency; \( r = \frac{r_k}{a_0}, z = \frac{z_k}{a_0} \).
At infinity \( r \to \infty \) put Sommerfeld conditions for each component. Substituting (5) into (4) we obtain the following ordinary differential equation:

\[
\begin{align*}
\frac{d^2 \varphi_k}{dr^2} + \frac{1}{r} \frac{d\varphi_k}{dr} + \left( \alpha_k^2 - \frac{n^2}{r^2} \right) \varphi_k &= 0; \\
\frac{d^2 \psi_{zk}}{dr^2} + \frac{1}{r} \frac{d\psi_{zk}}{dr} + \left( \beta_k^2 - \frac{n^2}{r^2} \right) \psi_{zk} &= 0; \\
\frac{d^2 \psi_{ok}}{dr^2} + \frac{1}{r} \frac{d\psi_{ok}}{dr} + \frac{1}{r^2} \left( -n^2 \psi_{ok} + 2m \psi_{ok} - \psi_{ok} \right) \beta^2 \psi_{ok} &= 0; \\
\frac{d^2 \psi_{zk}}{dr^2} + \frac{1}{r} \frac{d\psi_{zk}}{dr} + \frac{1}{r^2} \left( -n^2 \psi_{zk} + 2m \psi_{zk} - \psi_{zk} \right) \beta^2 \psi_{zk} &= 0;
\end{align*}
\]

were \( \alpha_k^2 = \frac{\Omega_k^2}{r_k^2} - \gamma_p^2; \quad \beta_k^2 = \Omega_k^2 - \gamma_p^2; \quad \Omega_k = \frac{\omega c}{c_k}; \quad \gamma_p^2 = \frac{2(1-v_i)}{1-2v_i}. \)

The first two equations in (6) the following solutions for \( k = 1 \) of the first (solid) and the outer cylinder \((k=N)\):

\[
\begin{align*}
\varphi_k(r) &= \begin{cases} F_{nk} J_n(\alpha_k r), & k = 1; \\
F_{nk} H_n^{(1)}(\alpha_k r), & k = N; \end{cases} \tag{7a} \\
\psi_{zk}(r) &= \begin{cases} M_{nk} J_n(\beta_k r), & k = 1; \\
M_{nk} H_n^{(1)}(\beta_k r), & k = N; \end{cases} \tag{7b}
\end{align*}
\]

where \( J_n(\alpha_k r) \) – Bessel function \( n \)-th order. \( H_n^{(1)}(\beta_k r) \) – Hankel function of the first kind \( n \)-th order.

The decision of the other two equations in (6) is also expressed in terms of Bessel functions and Hankel:

\[
\psi_{ok}(r) = \begin{cases} L_{nk} J_{n-1}(\beta_k r) + L_{n+1} J_{n+1}(\beta_k r), & k = 1; \\
L_{nk} H_{n-1}^{(1)}(\beta_k r) + L_{n+1} H_{n+1}^{(1)}(\beta_k r), & k = N; \end{cases} \tag{8}
\]

Solutions (7a), (7b) and (8) of the system of differential equations (6) will contain a \( 6k-2 \) arbitrary constants. I take this opportunity to go to a large extent arbitrary values when choosing a permanent believe further \( L_{nk} = L_{nk}^* = 0 \), i.e. \( \psi_{ok} = \psi_{ok}^* \).

Moving \( k \) cylinder is expressed in terms of Bessel and Neumann functions of \( n \)-th order complex argument

\[
\begin{align*}
\xi_{zk} &= \sum_{n=0}^{\infty} r \gamma_n \left[ A_{zkn} J_n(\gamma_k r) + A_{zkn} Y_n(\gamma_k r) \right] + \frac{r}{r_n} \left[ A_{zkn} J_n(\gamma_z r) + A_{zkn} Y_n(\gamma_z r) \right] - \\
&\quad \frac{r \gamma_k}{\mu_k} \left[ A_{zkn} J_n(\gamma_k r) + A_{zkn} Y_n(\gamma_k r) \right] \cos n \phi e^{i(-\omega t + \gamma_k z)}; \\
\xi_{ok} &= \sum_{n=0}^{\infty} n \gamma_n \left[ A_{zkn} J_n(\gamma_k r) + A_{zkn} Y_n(\gamma_k r) \right] + \frac{r \gamma_k}{n} \left[ A_{zkn} J_n(\gamma_z r) + A_{zkn} Y_n(\gamma_z r) \right] - \\
&\quad \frac{r}{\mu_k} \left[ A_{zkn} J_n(\gamma_k r) + A_{zkn} Y_n(\gamma_k r) \right] \sin n \phi e^{i(-\omega t + \gamma_k z)}; \tag{9}
\end{align*}
\]
\[ u_{ik} = \sum_{n=0}^{\infty} \left[ \gamma_k J_n(\gamma_k r) + A_{2kn} Y_n(\gamma_k r) \right] + \frac{\gamma_{2k}^2}{n} \left[ A_{4kn} J_n(\gamma_{2k} r) + A_{4kn} Y_n(\gamma_{2k} r) \right] \cos \omega \varphi e^{i(\omega \varphi + \gamma_k z)}, \]

where

\[ \gamma_k^2 = \overline{\rho}_{ik}^2 - \gamma^2; \quad \overline{\rho}_{ik} = \frac{\omega}{a_{ik} \Gamma_k}; \quad \gamma_{2k}^2 = \overline{\rho}_{2k}^2 - \gamma^2; \]
\[ \overline{\rho}_{ik} = \omega / c_{ik} \Gamma_{2k}; \quad c_{ik}^2 = \frac{1}{\rho_k^2}; \quad \gamma = m \pi / l, \quad (m = 1, 2, ..., \quad k = 1, 2, 3). \]

To determine the arbitrary constants \( A_{1kn}, A_{2kn}, A_{3kn}, A_{4kn}, A_{5kn}, A_{6kn} \) used the boundary conditions (3). Then we obtain a system of homogeneous algebraic equations \( 6k + 3 \) unknown and \( 6k + 3 \) equations. A necessary and sufficient condition for the existence of solutions of this system, is the vanishing of its determinant. The order of the primary identifier \( (6k + 3 \times 6k + 3) \) and elements are expressed in terms of Bessel and Neumann functions of \( n \)-th order complex argument. This equation gives the dispersion equation for the dissipative systems.

Under the dispersion characteristics are understood according to the phase \( (C = C_r + i C_i) \) and group velocity \( V = V_r + i V_i \), the wave number (\( \gamma_p \)) while various parameters of the mechanical system. It is known that the magnitudes of \( C \) and \( V \) related to the value of the root of the dispersion equation

\[ \Delta(C, \omega, \gamma_p, \lambda) = 0, \quad (10) \]

where \( \omega \) - complex frequency; \( \gamma_p \) - wave number, \( \lambda \) - wavelength. The phase and group velocity is related to the root meaning of the dispersion equation some complex dependencies. Thus, to be able to calculate the dispersion characteristics, it is necessary to make a study of the roots of the equation (10) at the points of the complex plane, and also to develop a method of numerical determination.

The work for the solution of the transcendental equation (10) applies the method of Mueller, at each iteration of the method applied by Muller Gaussian with the release of the main element. Thus, the solution of equation (10) does not require disclosure of the determinant. As an initial approximation we choose the phase velocity of the waves corresponding elastic system. For complex roots Muller method simplifies the calculations and provides faster convergence [14,15,16,17].

3. The propagation of transverse waves in an infinitely long cylindrical shell, located in an elastic medium

The main objective of the research study of the existence of the phase velocity of propagation of the geometric and physical and mechanical parameters of the system. The basic equations of the theory of elasticity for such tasks are reduced to a plane problem \( (u_r = u_z = 0) \). The components of the displacement vector in the cylinder and its environment are represented as:

\[ u_{r01} = -\frac{\partial \psi_1}{\partial r} = \left[ A_i \frac{d}{dr} H_0^{(1)}(\overline{K}_r r) + B_i \frac{d}{dr} H_0^{(2)}(\overline{K}_r r) \right] e^{K_r z}; \]

\[ u_{r02} = -C_i \frac{d}{dr} K_0(\overline{K}_r r) e^{iK_r z}; \]

\[ \gamma_{r01} = - \left[ A_i \frac{d^2}{dr^2} H_0^{(1)}(\overline{K}_r r) + B_i \frac{d^2}{dr^2} H_0^{(2)}(\overline{K}_r r) \right] e^{-K_r z} + \frac{1}{r} \left[ A_i \frac{d}{dr} H_0^{(1)}(\overline{K}_r r) + B_i \frac{d}{dr} H_0^{(2)}(\overline{K}_r r) \right] e^{-iK_r z}. \]
To determine the arbitrary constants $A^i$, $B^i$ and $C^i$. It used the boundary conditions $r = a_1$: $\sigma_{r01} = 0$ and $r = a_2$: $u_{r1} = u_{r2}$, $\sigma_{r01} = \sigma_{r02}$ we obtain a homogeneous system of algebraic equations of the third order. Their conditions of existence of nontrivial solutions we obtain the following dispersion equation:

$$
\begin{bmatrix}
-\bar{K}_1^2 H_0^{(1)}(\bar{K}_1, a_1) & -\bar{K}_1 H_0^{(2)}(\bar{K}_1, a_1) & 0 \\
-\bar{K}_1 H_1^{(1)}(\bar{K}_1, a_2) & -\bar{K}_1 H_0^{(2)}(\bar{K}_1, a_2) & -\bar{K}_2 K_1 (\bar{K}_2, a_2) \\
-\bar{K}_1^2 \mu H_0^{(1)}(\bar{K}_1, a_2) & -\bar{K}_1^2 \mu H_0^{(2)}(\bar{K}_1, a_2) & -\bar{K}_2 \mu K_0 (\bar{K}_2, a_2)
\end{bmatrix} = 0,
$$

(11)

where $K_1^2 > K_2^2$.

The numerical results are shown in Figure 2. and Figure 3. Note that an increase in the thickness of the layer of the first and second mode phase velocity gradually decreases. From the figures it can be seen that the phase and group velocity in the area of wavelengths in the range of $\lambda$. In the of short waves phase velocity may be less than the velocity of shear waves.

![Figure 2. Changing the phase and group velocities, depending on the wavelength.](image)

$C_{\beta1} = 200 \text{ m/s}$; $C_{\beta2} = 1100 \text{ m/s}$
4. Distribution of natural waves in the three-layered cylindrical body (Figure 1)

Internal ($r=a_1$) and external ($r=a_4$) the surface of the three-layer cylinder is free from stress. Then the order of the determinant of the system of homogeneous algebraic equations ($18 \times 18$), which is the dispersion equations

$$(a_2-a_1 = \Delta h_1, a_3-a_2 = \Delta h_2, a_4-a_3 = \Delta h_3).$$

In the calculations we take the following values:

$\Delta h_1 = 0.02 \text{ m}, \Delta h_2 = 0.03 \text{ m}, \Delta h_3 = 0.02 \text{ m}, E_1 = 5.5 \times 10^8 \text{ Pa}, E_2 = 3 \times 10^8 \text{ Pa}, E_3 = 5.5 \times 10^8 \text{ Pa},

\rho_1 = 27 \text{ kg/m}^3, \rho_2 = 11 \text{ kg/m}^3, \rho_3 = 27 \text{ kg/m}^3, A = 0.048, \beta = 0.05; \alpha = 0.1.$

Consider two options for a dissipative system. In a first embodiment, dissipative system is structurally homogeneous.

The dimensionless wave number $\gamma_p^*$ it varies between 0 - 2. The calculation results are shown in Figure 4 as well. The dependence of the real and imaginary parts of the phase velocity of the dimensionless wave number $\gamma_p^*$ It was monotonous, and dependence is the same for the real and imaginary part of the phase velocity. In the second embodiment, a dissipative system is structurally inhomogeneous, i.e. Rheological properties of the core layer ($n = 2$) is equal to zero, the other parameters the same as adopted above. The calculation results are shown in Figure 4b. The dependence of the real part of the phase velocity of the dimensionless wave number of $\gamma_p^*$ It is the same as for the homogeneous system: corresponding curves coincide up to 5%. Dependence of the imaginary part of the phase velocity of the $\gamma_p^*$ nonmonotonic it turned out.

Of particular interest is the minimum value $\xi$ a fixed damping coefficient:

$$\delta_\omega = \min(-\omega_n), \delta_\xi = \min(-c_n), k = 1, 2, ..., K,$$

here $\delta c$ – coefficient determining the damping properties of the system.
For a homogeneous system coefficient $\delta_c$ entirely determined by the imaginary part of the first modulo complex phase velocity. For inhomogeneous systems as a factor $\delta_c$ can act as an imaginary part of the first and the second frequency depending on their values. "Turn the Tables" occurs when the characteristic value $\gamma^0$, while the value of the real parts of the first and second frequencies are closest.

Coefficient $\delta_\omega$ and $\delta_c$ at the specified characteristic value has a pronounced maximum.

**Figure 4a.** Changing the complex natural frequencies of the wave number (Dissipative homogeneous mechanical system).
5. Conclusions

1. In the course of solving the problem of propagation of waves in dissipative-inhomogeneous media, the monotonic dependences of the natural frequencies (or the imaginary part of the phase velocity) on the physicomechanical and geometric parameters of the system were found.

2. It is found that the phase velocity of the higher forms of tension and torsion waves exceeds the maximum possible speed $C$ of waves in an infinite medium, the group velocity never exceeds $C$. It has also been found that the group velocity of 10-15% exceeds the dispersive medium in comparison with the dispersive medium. In other words, the impulse forms remain unchanged as they propagate, as in homogeneous elastic bodies.

3. Increase in viscosity reduces the real and imaginary parts of the phase velocity (or real and imaginary parts of the natural frequencies) of up to 20% for the homogeneous dissipative mechanical systems. For inhomogeneous dissipative mechanical systems increase in viscosity reduces the real part...
of the phase velocity (or the real part of the natural frequencies) to about 18%, and their imaginary parts, changed radically.

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