Analysis method of brittle failure probability of steel members

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Abstract. In recent years, researchers have been proposed various analytical methods to calculate the structural reliability. However, a series of low stress brittle fracture accidents occurred in steel structure, the brittle failure of steel members cannot be ignored. The analytical method of a steel structure with potentially brittle steel members was developed. The brittle failure of a steel members is represented by the brittle failure probability. However, the calculating method of brittle failure probability is still not clear. In order to evaluate structural reliability, the brittle failure probability should be more accurate. In this paper, a method was applied to calculate the brittle failure probability of steel members.

1 Introduction

The analysis of structural reliability is a discipline based on the theory of probability and mathematical statistics. It systematically analyzes the objective function of structure and provides a good scientific basis for structural design. Various analytical methods for calculating reliability have been developed [1-3]. For the design of steel structures, designers often only consider the ductile failure [4]. A series of low stress brittle fracture accidents have occurred in engineering; thus, the brittle failure cannot be ignored. A Heldweiller [5] proposed an analytical method which has considered the brittle failure of a structure. Then, Mai [6] proposed analytical method of a Steel Structure with potentially brittle steel members based on the relaxed linear programming bounds method (RLP) [7]. However, the brittle failure probability is replaced with the experience value in the method. In order to evaluate structural reliability, the brittle failure probability should be more accurate. A method was applied to calculate the brittle failure probability of steel members in this paper. The curve of brittle failure probability of steel members is obtained.

2 Calculating the brittle failure probability of members

After a large number of fracture accident analysis, researchers found that the occurrence of fracture accidents is related to the existence of cracks (inherent or caused by damage during manufacturing or using) in steel members [8]. Open-type (Type I) cracks are the main reason for the brittle failure of members, so the open-type (Type I) crack is the main point of this paper. There are many forms of Type I crack in a member, such as the center penetration Type I crack and the edge penetration Type I crack. The central penetration Type I crack is the main form considered in this paper.

From tensile member with crack damage (Figure 1), A Heldweiller [5] proposed that the failure form of member was determined by ductile strength $R_d$ and brittle strength $R_b$. Both of them are random. Therefore, the member has a probability $P\{R_b > R_d\}$ of responding in a ductile behavior and a probability $P\{R_b < R_d\}$ of responding in a brittle behavior. The basic assumption of this method is that the material has sufficient deformation capacity, which can be represented by the solid line in Figure 2. $c_b$ is half width of crack when $R_b = R_d$. The formulas are expressed as:

$$R_d = \sigma_y \cdot 2b \cdot t$$ (1)
$$R_b = \sigma_u \cdot (2b - 2c) \cdot t$$ (2)

where $\sigma_y$ is the yield strength; $\sigma_u$ is the ultimate strength; $2b$ and $t$ are the geometric dimensions of plate; $2c$ is the width of crack.

Figure 1. A schematic of tensile member.
In Figure 2, when the half width of crack \( c \) is changing from 0 to \( c_b \), the member with crack damage mainly occurs ductile failure, and the ductile failure probability of member is \( P \{ R_b > R_d \} \); when the half width of crack \( c \) is changing from \( c_b \) to \( b \), the member mainly occurs brittle failure, and the brittle failure probability of member is \( P \{ R_b < R_d \} \). Therefore, as shown in Figure 3, when the ductile strength \( R_d \) is smaller than the brittle strength \( R_b \), the strength of member is determined by its ductile strength \( R_d \). When the brittle strength \( R_b \) is smaller than the ductile strength \( R_d \), the strength of member is determined by its brittle strength \( R_b \). \( c_b \) is half width of crack when \( R_b = R_d \).

For this method, when the geometric dimensions of member are determined, the ductile strength \( R_d \) and brittle strength \( R_b \) have supposed to be independent. Nevertheless, there is a correlation between the ductile strength \( R_d \) and brittle strength \( R_b \) because of the crack in a member. The correlation will not be considered in this paper. Then, the distribution function of yield strength and ultimate strength should be normal or lognormal distribution [9]. According to formula (1) and (2), the \( R_b \) and \( R_d \) of member are the normal distribution. Therefore, based on the nature of the normal distribution, the brittle failure probability of a member is:

\[
P \{ R_b < R_d \} = \Phi \left( \frac{\mu_b - \mu_d}{\sqrt{\sigma_b^2 + \sigma_d^2}} \right)
\]

where \( \Phi (\cdot) \) is the distribution function of standard normal distribution; \( \mu_b \) and \( \mu_d \) are the mean values of \( R_b \) and \( R_d \), respectively; the corresponding standard deviations are \( \sigma_b \) and \( \sigma_d \).

### 3 Examples

The materials of members are used Q235 steel. The main physical and mechanical parameters of these materials are shown in Table 1. The geometric dimensions of member are shown in Table 2. According to Table 1, when the thickness is from 0 to 16mm, the yield ratio of Q235 is 0.6351.

| Yield strength (MPa) | Ultimate strength (MPa) | Elasticity modulus (MPa) |
|----------------------|-------------------------|--------------------------|
|                      | \( t = 0 - 16\text{mm} \) | \( t = 0 - 16\text{mm} \) |
| Q235                 | 235                     | 370                      | 206000                   |

| Geometric dimensions  | Thicknesses | Widths |
|-----------------------|-------------|--------|
| Values (mm)           | 4           | 500    |

According to formula (1) and (2), with the width of crack from 0 to 500mm, the ductile strength \( R_d \) and brittle strength \( R_b \) are shown in Table 3.

| Yield ratio | Yield strength (MPa) | Ultimate strength (MPa) | Width (mm) | Thickness (mm) | The width of crack (mm) | Ductile strength (kN) | Brittle strength (kN) |
|-------------|----------------------|-------------------------|------------|----------------|------------------------|-----------------------|------------------------|
| 0.6351      | 235                  | 370                     | 500        | 4              | 0                      | 470.0000              | 740.0000               |
| 0.6351      | 235                  | 370                     | 500        | 4              | 2                      | 470.0000              | 737.0400               |
| 0.6351      | 235                  | 370                     | 500        | 4              | 4                      | 470.0000              | 734.0800               |
Taking the Group 4 data in Table 3 as an example. The distribution function of ductile strength $R_d$ and brittle strength $R_b$ are as follows:

$$R_d \sim N(470.0000, 47^{2}) ; \quad R_b \sim N(731.1200, 73.112^{2}) .$$

The coefficient of variation $\text{Cov} = \sigma_{d} / \mu_{d} = \sigma_{b} / \mu_{b} = 0.1$. Generally, the values of Cov from 0.03 to 0.1 is used in engineering. Considering the worse situation, the coefficient of variation Cov = 0.1 is preferred.

According to formula (3), brittle failure probability of the member is $P \{R_b < R_d\} = 1.33 \times 10^{-3}$. With the width of crack from 0 to 500mm, Curve of brittle failure probability of members made of Q235 steel is shown in Figure 4.

![Figure 4. Curve of brittle failure probability of members made of Q235 steel.](image)

4 Conclusion

Based on the ductile strength $R_d$ and the brittle strength $R_b$ of member, this paper applied a method to calculate the brittle failure probability of steel members. For the method, the ductile strength $R_d$ and brittle strength $R_b$ have supposed to be independent. When the geometric dimensions of member are determined, the curve of brittle failure probability of steel members is obtained.

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