Abstract: We discuss the use of the variable $\sqrt{\hat{s}}_{\text{min}}$, which has been proposed in order to measure the hard scale of a multi parton final state event using inclusive quantities only, on a SUSY data sample for a 14 TeV LHC. In its original version, where this variable was proposed on calorimeter level, the direct correlation to the hard scattering scale does not survive when effects from soft physics are taken into account. We here show that when using reconstructed objects instead of calorimeter energy and momenta as input, we manage to actually recover this correlation for the parameter point considered here. We furthermore discuss the effect of including $W + \text{jets}$ and $t\bar{t} + \text{jets}$ background in our analysis and the use of $\sqrt{\hat{s}}_{\text{min}}$ for the suppression of SM induced background in new physics searches.

Keywords: Supersymmetry Phenomenology

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1 Introduction and motivation

Since the startup and the following successful data taking of the LHC, the LHC experiments have already published a large number of result for exclusion limits for BSM physics, where for many BSM scenarios the actual limits within specific parameter regions have been strongly pushed to higher scales [1–3]. However, most of these analyses have been performed within specific models, and more generic variables, which provide information about the generic scale of new physics without additional assumptions about the decay topologies or decay chains, have not been fully exploited. Furthermore, many variables which are currently proposed for mass or scale determination for new physics processes only make use of the transverse momentum of the event, thereby neglecting the information which can be obtained by additionally including the longitudinal information.\footnote{Excellent reviews about different mass determination variables and their use, including advantages and disadvantages, have recently been published in [4, 5].} Examples for variables which make use of transverse momentum only are eg \( M_{T^2} \) [6] or \( M_{CT} \) [7]. On the other hand, some more traditional variables as eg invariant masses of composite objects as used in edges studies [8–11] implicitly use all visible information, including the longitudinal momentum of the visible decay products. In [12] a new fully inclusive calorimeter-level variable, \( \sqrt{s_{\text{min}}} \),...
was proposed which promises to give information about the hard scale of the underlying new physics processes without further assumptions or specification of the decay products, and additionally makes use of the longitudinal information of the inclusive event. In that paper, the authors propose a conjecture which relates the peak position of $\sqrt{s_{\text{min}}}$ to the actual rise of the hard new physics production cross section. However, this variable was subsequently shown to have a strong dependence on the soft physics in terms of ISR and underlying event. This is mainly caused by the fact that in this case, the energy of the additional soft particles equally enters in the calorimeter-level definition of $\sqrt{s_{\text{min}}}$, which then boosts the variable and its peak position to higher values with respect to the parton-level quantity. In [12], the authors tried to circumvent this problem by introducing a pseudorapidity cut in order to suppress the unwanted effects originating from soft physics from entering $\sqrt{s_{\text{min}}}$. However, the introduction of the cut destroyed the correlation between peak position and hard cross section threshold which holds at parton level; this has explicitly been shown analytically for effects arising from initial state radiation (ISR) [13, 14]. In this case, the peak position is basically determined by the value of the pseudorapidity cut. Subsequently, $\sqrt{s_{\text{min}}}$ was promoted to $\sqrt{s_{\text{min}}}(\text{reco})$ [15], using reconstructed objects at analysis level; for this variable, the correlation between its peak and the threshold of the hard production cross section was recovered, such that a determination of the hard scale of the BSM process was again made possible using experimentally accessible detector level objects.

Apart from providing information about the hard scale of the underlying parton level BSM process, new variables can equally be used as cut parameters for SM background suppression, and several of these variables have already made their way into the current BSM searches at the LHC experiments. It is therefore equally important to determine the use of $\sqrt{s_{\text{min}}}$ for SM background suppression. Although this constitutes a slightly weaker use of the variable per se, it is still an important issue to investigate, especially as it has been proposed on a fully inclusive level and can therefore be applied without any further assumptions on the model or the specific decay chains and topology.

In this report, we therefore investigate the properties of $\sqrt{s_{\text{min}}}$ at analysis level using reconstructed objects. For this, we use a full sample for the mSugra point SPS1a$^2$ [17], which contains all strong production as well as all decay chains; for this parameter point, our sample therefore corresponds to the full data set which would be obtained from strongly interacting initial cascade particles in a realization of SPS1a.$^3$ We include soft physics in terms of initial and final state radiation (ISR/FSR), as well as a fast detector simulation. We test the correlation between the threshold of the hard process and the peak of $\sqrt{s_{\text{min}}}$ on parton level for the inclusive sample as well as exclusive dominant final states. In addition, $^2$We are aware that this parameter point has recently been excluded by ATLAS measurements in the quark/gluon plus missing transverse energy channel [16]. However, we here want to show that the parton level $\sqrt{s_{\text{min}}}$ can actually be recovered with sufficient accuracy from analysis level objects. Spectra which evade current exclusion limits typically exhibit higher initial cascade particle masses, and our arguments are generically not affected by the actual position of the particle production threshold. This is only important in the studies of background suppression presented in section 5; here, a higher peak value for the BSM induced variable should actually even enhance the SM background suppression which can be obtained using $\sqrt{s_{\text{min}}}$.

$^3$Gaugino-gaugino initial cascade states, which have not been considered here, would contribute an additional 5% to the total production cross section.
we show that, for our sample, this relation can be regained on reconstruction level using quite simple analysis object definitions, and that major discrepancies between analysis and parton level quantities can be traced back to uncertainties in the reconstruction of tau jets. We equally comment on the power of the analysis level variable for SM background suppression, and compare to similar variables. All analyses are done for a LHC-like proton proton collider with a center of mass energy of 14 TeV and an integrated luminosity $\int \mathcal{L} = 1 \text{ fb}^{-1}$.

The report is organized as follows: in section 2, we briefly review the variable definition of $\sqrt{s_{\text{min}}}$ and define other kinematic quantities which were used in our study. In section 3, we describe the data set we use in this study. Section 4 contains the comparison of parton and analysis level $\sqrt{s_{\text{min}}}$, and section 5 describes the inclusion of SM background and the use of $\sqrt{s_{\text{min}}}$ for SM background suppression, as well as a brief comparison with other (transverse) variables. We conclude in section 6.

2 Variable definition

In this section, we will briefly review the original variable definition as well as its RECO level version; the interested reader is referred to [12, 15] for a more detailed discussion.

In general, $\sqrt{s_{\text{min}}}$ is defined on an event by event basis as the minimal value for the partonic center-of-mass energy $\sqrt{s}$ which is in agreement with the events’ momentum configuration. It can be derived through a minimization process [12] as

$$\sqrt{s_{\text{min}}} (M_{\text{inv}}) = \sqrt{E_T^2 + M_{\text{vis}}^2} + \sqrt{E_T^2 + M_{\text{inv}}^2}$$

with

$$M_{\text{vis}}^2 = E^2 - P^2$$

being the effective visible mass and the invisible total mass

$$M_{\text{inv}} = \sum_{\text{invisible}} m$$

the sum over all masses of invisible particles. For completeness, we give the minimization procedure leading to eq. (2.1) in appendix B. Note that $M_{\text{vis}}$ and $M_{\text{inv}}$ are not defined equivalently, and especially that $M_{\text{inv}}$ is not the Lorentz-invariant mass of the total invisible system, but rather the sum over all invisible particles rest masses. In the definition of $\sqrt{s_{\text{min}}}$, $M_{\text{inv}}$ is therefore an external input parameter, as it is the only quantity which cannot be measured directly from experiment. Therefore, all results which are derived in the following sections have an implicit dependence on the value of $M_{\text{inv}}$. Throughout our study, we have usually set this to its “true” BSM value $M_{\text{inv}} = 2m_{\chi^0} (= 193.4 \text{ GeV})$.

The translation to experimentally accessible quantities is then straightforward and gives

$$\sqrt{s_{\text{min}}} (M_{\text{inv}}) = \sqrt{E^2 - P^2_Z} + \sqrt{E_T^2 + M_{\text{inv}}^2}.$$  

For consistency, we here adopted the notation introduced in [12] for $M_{\text{inv}}$ and hope that the potentially misleading nomenclature does not cause confusion in the remainder of our discussion.
Note that the use of transverse energy and momentum strongly depends on the definition of the specific quantity; we define

\[
\mathbf{P}_T = -\mathbf{P}_T, \quad E_T = |\mathbf{P}_T| \tag{2.4}
\]

where \(E, P\) are the total energy and four momentum of all visible objects

\[
P^\mu = \sum_{\text{vis}} p_i^\mu, \quad E = P^0, \quad \mathbf{P}_T = \left( \begin{array}{c} P_X \\ P_Y \end{array} \right) \tag{2.5}
\]

and the \(z\)-direction defines the beam-line. In the original proposal, all visible quantities are taken from calorimeters, and soft background is suppressed by a cut in the pseudorapidity \(\eta\). Subsequently, for the correct value of \(M_{\text{inv, true}}\), a conjecture

\[
\sqrt{s_{\text{min}}}^{\text{peak}} \sim \sqrt{s_{\text{th}}} \tag{2.6}
\]

is empirically derived, which links the peak position of \(\sqrt{s_{\text{min}}}^{\text{peak}}\) to the actual threshold \(\sqrt{s_{\text{th}}}\) of the hard matrix element process. However, in [13] it was subsequently pointed out that, using different values for the \(\eta\) cut, the peak position for the calorimeter-based variable \(\sqrt{s_{\text{min}}}^{\text{peak}}\) could actually be arbitrarily shifted around; the same effect has been observed in [18], which applies the original calorimeter-based definition of \(\sqrt{s_{\text{min}}}^{\text{peak}}\) on our data sample. In answer to this criticism, new reconstruction and subsystem level variables were proposed in [15]. We here use \(\sqrt{s_{\text{min}}}^{\text{peak}}\) on an inclusive level using reconstructed objects, which basically corresponds to the RECO variable definition given in [15]. In our work, the suppression of effects from the parton shower has been achieved by quite simple object-level definitions given in table 2. We will show that we obtain the parton-level \(\sqrt{s_{\text{min}}}^{\text{peak}}\) quite accurately in our sample, and that we indeed observe a similar peak of the RECO-level \(\sqrt{s_{\text{min}}}^{\text{peak}}\) close to the production threshold for the correct input value of \(M_{\text{inv}}\). In addition, we investigate the actual sources of discrepancies between the parton and reconstruction level \(\sqrt{s_{\text{min}}}^{\text{peak}}\) in more detail. In our work, we equally present the first study of \(\sqrt{s_{\text{min}}}^{\text{peak}}\) as a variable for SM background suppression in BSM searches.

In this study, we use the term “leptons” for all three SM lepton generations; in cases when we are concerned with tau leptons alone we will mention this explicitly. Equally, the tau jets at analysis/reconstruction level are defined by the tau jet reconstruction algorithm in Delphes [19] and differ from the parton level tau lepton by the four-momenta of the invisible tau decay products, specifically the associated third generation neutrino. In the following, we use the term ”tau” for the parton level and ”tau jet” for the reconstruction level quantity, which for an ideal reconstruction of the visible tau decay products four-momenta only differ by the four-momentum of the invisible decay products.

### 3 Data sample and event generation

In this report, we have made use of the BSM data samples which have been generated in the course of the 2009 BSM Les Houches mass determination study; first results using these data for studies of various mass determination methods were presented in [18].
Table 1. SPS1a production cross sections in pb for $pp \rightarrow X_1X_2$ using Madgraph $2 \rightarrow 2$ parton level production cross sections, convoluted with PDFs, for a hadronic center-of-mass energy of 14 TeV. CTEQ6L1 PDFs [29] were used.

| $X_1X_2 \rightarrow 2$ | 6.56  \\
| $\tilde{q}\tilde{g}$ | 19.96  \\
| $\tilde{g}\tilde{g}$ | 4.53   |

We use a SUSY spectrum for the point SPS1a, where the spectrum was generated with the spectrum generator SOFTSUSY [20]. Parton level events have been generated using Madgraph [21, 22] for the generation of the heavy initial cascade particles (i.e. the squark-squark, squark-gluino, and gluino-gluino initial states). The heavy pair-produced particles have then been fully decayed according to the respective branching ratios into all possible decay products using Bridge [23] within the Madgraph framework; we therefore consider a complete sample for this parameter point, which contains all possible final states. The SM background has been generated using Alpgen [24]. For parton shower and hadronization, we used Pythia [25], where the parton shower evolution follows the Pythia 6.4 default, ie is $Q^2$ ordered with additional modifications to guarantee color coherence, as well as matrix element corrections where these are available (Pythia switches are given in appendix C). The detector simulation has been performed with Delphes [19] in its default mode. For specific input parameters and setups, we refer to the specifications which can be found in the data base for our samples [26]. Data analysis as well as fitting has been done within the ROOT [27, 28] framework. All our results have been obtained with a data sample for a center-of-mass energy $\sqrt{S_{\text{hadr}}} = 14$ TeV and an integrated luminosity $\int L = 1 \text{ fb}^{-1}$. Table 1 lists the production cross sections for the hard $2 \rightarrow 2$ process; these numbers were obtained using the Madgraph parton level $2 \rightarrow 2$ production cross sections, with the electroweak scale spectrum obtained from SOFTSUSY, convoluted with PDFs to account for the parton to hadron transition for the incoming states. For this study, we restrict ourselves to the leading order predictions for the hard process in both signal and background simulation.\(^5\)

### 4 Parton and analysis level $\sqrt{s_{\text{min}}}$

As already discussed in section 2, the variable $\sqrt{s_{\text{min}}}$ has undergone several developments since its original proposal. Initially defined as a calorimeter-based variable, it was shown to be quite sensitive to effects of soft physics for the respective processes. Especially the original merit of this variable, namely the correlation of the peak position and the threshold of the heavy pair-produced particles at the beginning of the decay chain, is

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\(^5\)A fully differential study including NLO contributions to account for cut effects would require $2 \rightarrow n$ event generators for both BSM signal and SM background, which additionally include the matching of parton shower and NLO contribution; although fast progress has been made in this field for SM processes, no fully differential higher order BSM generator is currently publicly available.
strongly influenced by the soft physics of the event. The original suggestion of the authors was to introduce a cut on the pseudorapidity; however, the authors in [13, 14] have shown analytically that the position of the peak position in this case is completely cut-value dependent; similar results have been observed in [18].

In this study, we show that, if $\sqrt{s}_{\text{min}}$ is defined at analysis object level rather than on calorimeter level, the parton level variable $\sqrt{s}_{\text{min}}$ can be reconstructed quite well using simple object definitions. This recovery of the parton level peak position using a reco-level variable for both inclusive and exclusive final states, however for different parameter points, have equally been presented in [15]. For our sample, we equally observe that the conjectured correlation between the rise of the hard scattering event cross section and the peak position of $\sqrt{s}_{\text{min}}$ holds; however, we want to emphasize that this is on the level of a conjecture which has not been systematically studied or proven on an analytic level, although some preliminary studies indicate a kinematic origin which emerges after the convolution with PDFs\(^\text{6}\) [30]. Therefore, even on parton level, it is currently unclear whether this conjecture necessarily holds for all BSM parameter points and scenarios. Equally, this conjecture only holds for a correct input value of $M_{\text{inv}}$.

In the following, we will compare quantities derived on the parton level with the same quantities which have been derived from analysis level objects. For the identification of the former, we consider the hard process, i.e. our data sample after the complete decay to SM particles and the LSP, but before the parton shower, hadronization, and detector simulation. All particles are considered as visible apart from neutrinos and the LSP. The invisible total four-momentum is then the sum of the latter particles' four-vectors

$$P_{\text{parton inv}} = \sum_{\nu, s, \tilde{\chi}} p_i,$$

and the same holds for the missing transverse momentum. At analysis level, we require all physical objects to fulfill the object definition requirements given in table 2 on detector level; these object definitions basically follow the Delphes predefinitions, where we introduced slightly more stringent requirements for lepton isolation and jet criteria and equally set a lower limit of 100 GeV on the total missing energy.\(^\text{7}\) Visible and invisible quantities are then defined according to eqs. (2.4) and (2.5) in section 2.

We first study the variable $\sqrt{s}_{\text{min}}$ for a complete inclusive sample, i.e. we sum over all final states of the hard process. Our main results are shown in figure 1, where we compare the true $\sqrt{s}$, parton level $\sqrt{s}_{\text{min}}$, reconstruction level $\sqrt{s}_{\text{min}}$ as well as the original calorimeter based variable with and without a cut in pseudo rapidity $\eta$, as originally suggested in [12]. We see that the parton level $\sqrt{s}_{\text{min}}$ peaks quite close to the actual heavy

\(^{6}\text{This result has been obtained with a unit matrix element as well as unit PDFs; in this case, the peak position of } \sqrt{s}_{\text{min}} \text{ arises from the lower PDF integration boundary following from the kinematic lower limit which guarantees that } \sqrt{s}_{\text{part}} \geq \sqrt{s}_{\text{threshold}}. \text{ More realistic scenarios with non-uniform PDFs and matrix elements are currently under investigation.}\)

\(^{7}\text{These cuts closely follow cuts used in the SUSY analysis studies in [10, 32]. Due to the relatively high } p_T \text{ jet cuts, together with a high } E_T^{\text{miss}} \text{ cut and lepton isolation criteria, we expect minimum bias events to be sufficiently suppressed ([33], as well as section 6.1 in [34]).}\)
Table 2. Physical object definitions in terms of the single objects pseudorapidity $\eta$, absolute value of transverse momentum $p_T$, and (jet) cone radius $R$ for analysis level objects on detector level. We basically adapt the Delphes predefinitions, with slightly more stringent requirements for isolated leptons and jet definitions. We equally set a lower limit $E_T^{\text{miss}} \geq 100 \text{ GeV}$ for events with missing transverse energy.

particle production threshold as suggested in [12]; equally, we observe that the same variable from reconstructed objects again peaks close to the threshold, but is shifted to slightly lower values with respect to the parton level quantity. We will comment on this in more detail below. In contrast, the pure calorimeter based variable exhibits a peak position at quite high values and can therefore not be used for a scale measurement of the new physics process. Restricting the contributions to calorimeter energy deposits with a minimal pseudorapidity improves this behaviour and brings the peak closer to lower values; however, this approach suffers from the drawbacks pointed out in [13, 14].

For a more accurate determination of the peak position and a viable assessment of the error in its position, we fit the $\sqrt{s}_{\text{min}}$ distribution with a Gaussian around its peak, where we use the largest fit region which is still in agreement with $\chi^2/\text{d.o.f} \sim O(1)$. Specifically, we use $600 \text{ GeV} \leq \sqrt{s}_{\text{min}} \leq 1400 \text{ GeV}$ and $400 \text{ GeV} \leq \sqrt{s}_{\text{min}} \leq 1400 \text{ GeV}$ to determine the parton level and analysis level peak positions respectively. We then obtain

\[
\begin{align*}
\text{parton level } \sqrt{s}_{\text{min}}^{\text{peak}} & : (1152 \pm 4) \text{ GeV} \\
\text{analysis level } \sqrt{s}_{\text{min}}^{\text{peak}} & : (1083 \pm 4) \text{ GeV}
\end{align*}
\]

We see that the reconstruction level variable for the overall sample peaks close to the "true" maximum of the parton level variable, the difference being $O(100 \text{ GeV})$. In order to pin down the major sources of this shift, we have performed detailed studies for specific final state signatures; we will discuss this in more detail in section 4.1. To summarise the result of this section, we observe that, in our sample, larger shifts in the peak positions stem from processes with one or more leptons in the final state. One source of this is the imperfect reconstruction of tau jets from parton to analysis level objects. We can test this
Table 3. Peak positions for separate heavy initial cascade particles for parton level ($\sqrt{s_{\text{peak;parton}}_{\text{min}}}$) and analysis level ($\sqrt{s_{\text{peak;ana}}_{\text{min}}}$) quantities as well as analysis level quantity for idealized tau jets ($\sqrt{s_{\text{peak;ana,}\tau = \tau_{p}}_{\text{min}}}$) in GeV, where the value in the respective first line arises from a Gaussian fit around the peak, while the second corresponds to the more simplified definition of the peak position by maximal number of bin entries. In addition, we give the average threshold value $\sqrt{s_{\text{th}}} = (m_1 + m_2)$ for each sample, where $m_{1,2}$ are the masses of the heavy initial cascade particles. We see that the peak position from both peak position definitions are close to the actual thresholds; in addition, the effect of imperfect tau reconstruction account for an approximate shift $O(100 \text{ GeV})$ for all initial state pairings.

by taking an "idealistic" approach, where we use the parton level four-vector values for taus in the analysis level objects; this simple "gedankenexperiment" trick, where we assume a perfect reconstruction of tau jets at analysis level, significantly reduces this difference, cf. figure 2. While such a requirement is in fact not possible in reality, it however shows that our (quite loose) lepton definitions and resulting poor tau reconstruction are a major source of this shift, and more dedicated algorithms might further reduce this discrepancy. Fitting the "new" analysis level distribution within the range $600 \text{ GeV} \leq \sqrt{s_{\text{min}}} \leq 1400 \text{ GeV}$, we obtain

$$\text{analysis level } \sqrt{s_{\text{min}}}^{\text{peak}}, \tau = \tau_{\text{parton}} : \ (1163 \pm 4) \text{ GeV}$$

and we see that the discrepancy with the parton level value of $(1152 \pm 4) \text{ GeV}$ reduces to the permill level, cf. figure 2. The average heavy particle threshold in our sample is

$$\text{true (average) } \overline{(m_1 + m_2)} : \ 1146 \text{ GeV}$$

which again agrees with the parton level value of $\sqrt{s_{\text{min}}^{\text{peak}}}$ on permill level within the error bars. In addition, the object definitions in table 2 equally allow for an adequate recovery of the parton level distribution shape, and, more specifically, we are able to suppress distribution tails for higher $\sqrt{s_{\text{min}}}^{\text{peak}}$ values appearing in the reco-level definition of this variable in [15]. A breakdown in terms of pairs of initially produced particles prior to the cascade decays is given in table 3.

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8In this work, we only want to demonstrate that the approximate peak position of the parton level variable can actually be obtained from analysis level objects; for more dedicated analyses, the peak position could also be determined by other means, eg. a fit to a more variable-specific function.
Figure 1. Sum of $\tilde{q}\tilde{q}$, $\tilde{g}\tilde{g}$, and $\tilde{g}\tilde{g}$ initial states, where $\tilde{g}\tilde{g}$ initial states dominate. All final states which fulfill object definitions from table 2 are included. True $\sqrt{s}$ (red; solid), parton level $\sqrt{s_{\text{min}}}$ (blue; dashed), analysis level $\sqrt{s_{\text{min}}}$ (green; dotted), $\sqrt{s_{\text{min}}}$ using calorimeters (pink; dash-dotted) and same with an $|\eta| < 1.4$ cut (black; dash-dot-dot-dotted). $\sqrt{s_{\text{hadr}}} = 14$ TeV, $\int L = 1$ fb$^{-1}$; corresponds to 31050 events. Shift between parton level and analysis level peak is about 70 GeV. The calorimeter based distribution without a pseudorapidity cut exhibits a peak at much larger $\sqrt{s_{\text{min}}}$ values.

Figure 2. As figure 1, where the hard matrix element tau four-vectors were used for the analysis level $\sqrt{s_{\text{min}}}$. With the differences due to tau identification at the analysis level removed, parton and analysis level peak positions agree within error bars. Explicit numbers are given in section 4.

For illustration purposes and completeness, we also investigate the parton level $\sqrt{s_{\text{min}}}$ dependence on the input value for $M_{\text{inv}}$; similar results have already been presented in [12] and [18] for the analysis level quantity. For R-parity conserving SUSY scenarios, as con-
Figure 3. Parton level $\sqrt{s}_{\text{part}}$ (red; solid), and parton level $\sqrt{s}_{\text{part min}}$ dependence on the input value $M_{\text{inv}}$ for varying values. Shown are results for $M_{\text{inv}} = 0$ GeV (dark green; long dashed), 200 GeV (black; solid), 400 GeV (pink; dotted), 600 GeV (green; dash-dotted), 800 GeV (dark blue; dash-dot-dot-dotted), 1000 GeV (light blue; short dashed). The corresponding peak positions for increasing $M_{\text{inv}}$ input values as given above, using a maximal bin definition, are obtained as $\sqrt{s}_{\text{peak; part min}} = (1035, 1170, 1305, 1440, 1620, 1845)$ GeV. As before, the analysis level peak positions using the parton level tau leptons coincide with the parton level peaks (not shown here).

Considered in this study, this corresponds to the guess of the LSP mass, as in this case $M_{\text{inv}} = 2 m_{\text{LSP}}$ when neutrino masses are neglected. Figure 3 shows the shift of the parton level $\sqrt{s}_{\text{min}}$ distribution for different input values $M_{\text{inv}}$. We see that a variation of the $M_{\text{inv}}$ mass leads to a shift in the peak of a similar magnitude. Therefore, we again emphasize that the results presented in this study concerning the correlation of the peak of the $\sqrt{s}_{\text{min}}$ distribution and the hard scale of the underlying production process have indeed an implicit dependence on the correctness of the guessed input value for $M_{\text{inv}}$, as already discussed in the original proposal of this variable [12], and therefore generically only allow for a measurement of the hard scale as a function of this variable.\(^9\) As before, the parton level distributions and peak positions could be reproduced using analysis level objects in the idealized version, ie replacing the analysis level tau-jets with parton-level tau leptons in $\sqrt{s}_{\text{min}}$.

4.1 Signal based searches

In this section, we consider the variable $\sqrt{s}_{\text{min}}$ for several exclusive final states. The (parton level) dominant decay modes of our sample are given in table 4. Most of the dominant final states can be tracked down to a couple of competing processes, and can be broken down to the following parton-level decay chains

\(^9\)Several other widely used variables, as eg the original definition of $M_{T_2}$ [6], equally exhibit a dependence on an input value for the LSP mass.
Table 4. Number of events for dominant parton level decay modes, characterized by specific visible final states, on parton level ($N_{\text{hard}}$) and at analysis level ($N_{\text{ana}}$). At analysis level, the jet number requirement for event selection is changed from an exact equality to a minimal number of jets. If not stated otherwise, the main source provides all events with a specific signature on the parton level. Examples for dominant decay chains leading to the specific parton-level final states are given in section 4.1.

- $\tilde{q}\tilde{g}$, 3 jet channel
  $$\tilde{q}R\tilde{g} \to qR\tilde{q}R\tilde{q}R\tilde{\chi}_1^0 \to qR\tilde{q}R\tilde{q}R\tilde{\chi}_1^0\tilde{\chi}_1^0$$ (90%)

- $\tilde{q}\tilde{g}$, 3 jet 2 lepton channel
  $$\tilde{q}_L\tilde{g} \to q_L\tau^+\tau^-\tilde{\chi}_1^0\tilde{q}_R\tilde{\chi}_1^0$$ (27%)
  $$\tilde{q}_R\tilde{g} \to q_R\tilde{\chi}_1^0\tilde{b}\tilde{\tau}^+\tau^-\tilde{\chi}_1^0$$ (22%)
  $$\tilde{q}_L\tilde{g} \to q_L'\tau^+\chi_1\tilde{q}_L'\tilde{\chi}_1^0$$ (17%)
  $$\tilde{q}_L\tilde{g} \to q_L'\tau\nu\tau\tilde{\chi}_1^0$$ (17%)

- $\tilde{q}\tilde{g}$, 3 jet 1 lepton channel
  $$\tilde{q}R\tilde{g} \to q_R\tilde{\chi}_1^0\tilde{b}\tilde{\tau}\nu\tilde{\chi}_1^0$$ (45%, BR $\sim 0.09$)
  $$\tilde{q}_L\tilde{g} \to q_L\tilde{\chi}_1^0q_L\tilde{\tau}\nu\tilde{\chi}_1^0$$ (30%, BR $\sim 0.06$)
  $$\tilde{q}_L\tilde{g} \to q_L\tau\nu\tilde{\chi}_1\tilde{q}_R\tilde{\chi}_1^0$$ (25%, BR $\sim 0.05$)

- $\tilde{q}\tilde{q}$, 2 jet 2 lepton channel
  $$\tilde{q}_L\tilde{q}_L' \to q_L'\tau\nu\tilde{\chi}_1\tilde{q}_L''\tau\nu\tilde{\chi}_1^0$$ (36%)
  $$\tilde{q}_R\tilde{q}_L' \to q_R\tilde{\chi}_1^0\tilde{q}_L\tau^+\tau^-\tilde{\chi}_1^0$$ (64%)

- $\tilde{q}\tilde{q}$, 2 jet 1 lepton channel
  $$\tilde{q}R\tilde{q}_L' \to q_R\tilde{\chi}_1^0q_L'\nu\tau\tilde{\chi}_1^0$$ (100%)

At the analysis/reconstruction level, we require a minimal jet multiplicity, which leads to much larger event numbers especially for signatures with a smaller number of leptons. We equally do not apply any dedicated additional channel-based cuts.
Figures 4 and 5 show the true $\sqrt{s}$, parton level $\sqrt{s}_{\text{min}}$, reconstruction level $\sqrt{s}_{\text{min}}$ as well as the original calorimeter based variable with and without a cut in the magnitude of the pseudo rapidity $|\eta| < 1.4$ for several explicit final states. We observe a similar behavior as in the overall sample, cf. figure 1: the parton level variable peaks around the actual production threshold, while there is a shift to lower peak values for the analysis level quantity. In order to understand the origin of this shift, we investigate this for a final state which initially exhibits a large difference between these quantities. We consider the 2 jet 1 tau-lepton channel, where originally the $\sqrt{s}_{\text{min}}$ peak positions differ by about 170 GeV. From eq. (2.3), we see that the definition of $\sqrt{s}_{\text{min}}$ depends on the following independently measured quantities:

$$E_{\text{vis}}, P_{Z}, |P_{T}| = P_{T} = E_{T} = \not{E}_{T}.$$ 

In order to investigate the origin of the shift between the parton level and analysis level peak positions, we therefore consider each of these variables separately and plot the difference between the respective parton level and analysis level quantity; the results are shown in figures 6 and 7. We observe that, while the differences between parton and analysis level $P_{Z}, P_{T}$ basically peak around zero, there is an average discrepancy $\sim 50 - 100$ GeV between the parton and analysis level total visible energy. However, this discrepancy is accounted for by the fact that when changing from parton to hadron level, we replace the (visible) parton level tau by the (visible) tau-jet and the (invisible) tau-neutrino:

$$\tau_{\text{part}} \rightarrow \tau_{\text{jet}} + \nu_{\tau}.$$ 

In this transition, we equally shift the four momenta of the tau neutrinos from the visible to the invisible contribution of the definition of $\sqrt{s}_{\text{min}}$ (cf. eq. (2.3)):

$$P_{\text{vis}} = \ldots + p_{\tau_{\text{part}}} + \ldots, \quad M_{\text{part}} \equiv M_{\text{inv}}$$
$$P_{\text{ana}} = \ldots + p_{\tau_{\text{jet}}} + \ldots, \quad M_{\text{ana}} = M_{\text{inv}} + m_{\nu_{\tau}}.$$ 

We consider the neutrinos to be massless; therefore, we can leave the sum of all invisible particles’ masses $M_{\text{inv}}$ unchanged. As the original variable definition of $\sqrt{s}_{\text{min}}$ and the subsequent correlation in eq. (2.6) only depend on the heavy initial cascade particles, but not on the actual number of visible and invisible decay products, the observed change in the visible energy due to the escaping neutrinos at analysis level should then be compensated by associated changes in $P_{Z}, P_{T}$ on an event by event basis, leading to a similar peak behavior of the $\sqrt{s}_{\text{min}}$ distribution at parton and analysis level. Here, in order to assess the overall impact of this shift and a possible poor reconstruction of the tau decay products,\textsuperscript{10} we perform a gedankenexperiment and change into a more ideal world where we idealistically reverse the analysis level tau-jet reconstruction and take the parton level tau four-vectors for the analysis level variable. In this case, the shift in the peak position of $\sqrt{s}_{\text{min}}$ reduces to roughly 130 GeV. An alternative though less sophisticated way to determine the peak

\textsuperscript{10}Note that, in case the shift cannot be explained by poor reconstruction of the decay products alone, this equally opens the window to a possible topology-dependence of $\sqrt{s}_{\text{min}}$; we thank K. Sakurai for pointing this out.
Figure 4. As figure 1 $\ell = 1 \text{ fb}^{-1}$: Left figure for exactly 0 leptons in the final state, 3 hardest jets, (14249 events), and shift between parton level and analysis level $\sqrt{s_{\text{min}}}$ is about 100 GeV. Right figure for exactly 2 leptons in the final state, 2 hardest jets (2745 events). Here, the shift between parton level and analysis level $\sqrt{s_{\text{min}}}$ is about 200 GeV (reduces to 100 GeV if parton level tau vectors are used).

Figure 5. As figure 1 $\ell = 1 \text{ fb}^{-1}$, exactly 1 tau lepton in the final state, 2 hardest jets (3260 events) Shift between parton level and analysis level peak is 170 GeV(left) and reduces to 130 GeV(right) when parton level tau vectors are used for analysis level objects.

position is to consider the bin which contains a maximal number of entries; using this definition of the distribution peak position, the original shift between parton level and analysis level $\sqrt{s_{\text{min}}}$ reduces from 200 GeV to 90 GeV if the parton level tau vectors are used at analysis level. A similar study for the 2 tau lepton 2 jet channel, which originally equally exhibits a quite large shift between the peak positions, shows that the effect of tau misidentification is $\mathcal{O}(100\text{GeV})$ for both peak position definitions, reducing to $\sim 100$ GeV in both cases when tau misidentification is removed. A similar effect can be observed for other specific final state signatures, cf. table 5.

Although we still obtain a quite large shift for specific final state signatures, we have seen that, when using parton level taus for the analysis level observable and therefore suppressing possible effects from poor tau reconstruction, the inclusive sample peaks at the same value for both parton and analysis level $\sqrt{s_{\text{min}}}$ distributions, cf. figure 2. We therefore conclude that, with correctly identified analysis level objects, the peak position of
Figure 6. Difference between parton level and analysis object level total transverse momentum (left) and tau jet energy (right) for the 1 tau 2 jet channel. While the shift between the two values for the transverse momentum peaks around around zero, the shift between the parton level tau and analysis level tau jet energy is quite large, due to the escaping neutrino in the tau jet reconstruction. The difference in the $P_T$ distribution (not shown here) exhibits a similar behaviour as the $P_T$ distribution.

Figure 7. Difference between parton level and analysis object level total visible energy for the 1 tau 2 jet channel. Left side shows the real difference, while on the right hand side analysis level tau jets were replaced by parton level taus. While we originally observe a large shift between the two values, originating from the escaping tau neutrinos on reconstruction object level and with a peak on the order of $O(200 \text{ GeV})$, the distribution of the difference peaks around zero when parton level tau four-vectors are used for the calculation of the analysis level observable.

the parton level $\sqrt{s_{\text{min}}}$ can indeed be reconstructed from generator level measurements; however, we want to emphasize that the correlation between the peak position and the actual heavy particle production threshold only exists in the form of a conjecture which lacks a rigorous proof. In case the conjecture proves to hold in all cases, the analysis level $\sqrt{s_{\text{min}}}$ variable indeed gives a quite easy grasp on the threshold of the new physics

\[\text{We want to point out that the reconstruction level objects in table 2, through their definition by } p_T \text{ and } \eta \text{ cuts, still depend on these two parameters; therefore, a recovery of the hard scale from reconstruction level objects will always be obstructed by an implicit dependence on the cut values in the analysis object definitions. However, in contrast to the calorimeter-based variable and the cut in pseudorapidity originally proposed in [12], we here use object level definitions which are more optimized to the reconstruct the hard scattering event. We thank B. Webber for bringing this point to our attention.}\]
pair-produced particles. Although this analysis was done in a specific scenario, where only certain initial heavy particle spin states are allowed, we saw that our conclusions hold for all possible spin combinations we considered. As our study relies on purely kinematic variables, we are therefore confident that these also hold for other spin combinations both for the heavy initial pair-produced particles as well as the particles in the decay chains, i.e. especially for other (also non-SUSY) BSM scenarios.

4.2 Comment on additional soft physics effects

The data set used in this study contains soft physics in the form of initial and final state radiation as described in section 3, but no simulation of underlying event or pileup. However, the criticism which was expressed by the authors of [13] exactly concerns the dependence of soft physics in terms of ISR, which has been addressed in this work. Additionally, soft physics can enter in the form of minimum bias events, underlying event and pileup. We believe that the cuts in table 2 are sufficiently hard enough to suppress minimum bias events ([33], as well as section 6.1 in [34])). Underlying event as well as pileup effects can still distort the overall result for the peak position; however, we believe that these issues should be pursued in an experimental study, in combination with a collaboration internal full detector simulation. We can give a first estimate of the effect of underlying event fake $P_T$ contributions by adding $\Delta P^\text{fake}_T = 10$ GeV, which corresponds to a conservative upper limit of the average $P_T$ from the underlying event [34, 35], in the definition of $E_T$ in eq. (2.3); in this case, the best fit value from $400 \text{ GeV} \leq \sqrt{s}_\text{min} \leq 1400 \text{ GeV}$ is again given by

$$\sqrt{s}_\text{min} (\Delta P^\text{fake}_T = 10 \text{ GeV}) : (1082 \pm 4) \text{ GeV}$$

which completely agrees with the value without the addition of $\Delta P^\text{fake}_T$. Changing the fake

| final state  | $\sqrt{s}^\text{peak;parton}_{\text{min}}$ [GeV] | $\sqrt{s}^\text{peak;ana}_{\text{min}}$ [GeV] | $\sqrt{s}^\text{peak;ana,}\tau = \tau_p_{\text{min}}$ [GeV] |
|-------------|---------------------------------|---------------------------------|---------------------------------|
| 0$\ell$3j   | 1190 ± 5                         | 1072 ± 6                        | 1072 ± 6                        |
| 2$\ell$3j   | 1271 ± 8                         | 1128 ± 8                        | 1257 ± 8                        |
| 1$\tau$3j   | 1204 ± 7                         | 1123 ± 8                        | 1210 ± 8                        |
| 2$\ell$2j   | 1231 ± 7                         | 1001 ± 7                        | 1105 ± 6                        |
| 1$\tau$2j   | 1157 ± 7                         | 990 ± 7                         | 1031 ± 8                        |

Table 5. Comparison of peak positions from Gaussian fits for parton level $\sqrt{s}^\text{peak;parton}_{\text{min}}$ and analysis level $\sqrt{s}^\text{peak;ana}_{\text{min}}$ for specific final states, specified by the number of visible final state leptons ($\ell$), jets ($j$), and $\tau$-leptons ($\tau$), corresponding to dominant decay chains in the complete SPS1a sample. Values for the peak position of the analysis level quantity with perfect tau jet reconstruction ($\sqrt{s}^\text{peak;ana,}\tau = \tau_p_{\text{min}}$) are also given. For most final states, the effect of the peak shift due to imperfect tau jet reconstruction is $O(100 \text{ GeV})$. 


Table 6. Additional filters on magnitude of the total missing transverse momentum $|\slashed{P}_T|$ applied for SM background generation, depending on the number of final state leptons $n_{\text{leptons}}$. Leptons are required to obey the cut criterium $p_T > 5\,\text{GeV}$ for the magnitude of the transverse momentum and $|\eta| < 3.2$ for the magnitude of pseudorapidity.

| $n_{\text{leptons}}$ | $|\slashed{P}_T,\text{min}|$ |
|---------------------|------------------|
| < 2                 | 80 GeV           |
| 2                   | 40 GeV           |
| > 2                 | 0 GeV            |

Table 7. Cross sections in pb for SM background with Alpgen; filters in table 6 were applied in the generation stage.

| $n_{\text{jets}}$ | $W + n\text{ jets}$ | $t\bar{t} + n\text{ jets}$ |
|-------------------|---------------------|-----------------------------|
| 0                 | 75.2                |                             |
| 1                 | 48.5                |                             |
| 2                 | 188                 | 20.0                        |
| 3                 | 53.6                | 6.3                         |
| 4                 | 12.6                | 1.4                         |

additional transverse momentum to $\Delta P_T^{\text{fake}} = 100\,\text{GeV}$ leads to the result\(^{12}\)

\[ \sqrt{s}^{\text{peak}}_{\text{min}} \left( \Delta P_T^{\text{fake}} = 100\,\text{GeV} \right) : (1093 \pm 4)\,\text{GeV}. \]

In fact, as $M_{\text{inv}}$ and $\hat{E}_T$ appear in the same form in the definition of $\sqrt{s}^{\text{min}}$, the generic effects of additional fake $P_T$s can be estimated from figure 3. From underlying events, $\Delta P_T^{\text{fake}} \lesssim 100\,\text{GeV}$, and we therefore estimate the uncertainty related to underlying event fake transverse momentum to be generically much smaller than the tau reconstruction effects discussed above. A more realistic investigation of these experimentally dominated effects, which should include a full detector simulation, is beyond the scope of this work.\(^{13}\)

5 SM background

In this section, we investigate $\sqrt{s}^{\text{min}}$ when SM background is included, as well as its use for the reduction of SM background in new physics searches. As an example, we consider $W + \text{jets}$ and $t\bar{t} + \text{jets}$ background. Due to the large cross sections, we applied an additional $\slashed{P}_T$ filter in the generation of the SM data sample, cf. table 6. The cross sections after these additional cuts are given in table 7. As before, we investigate the peak position for $\sqrt{s}^{\text{min}}$

\(^{12}\)Simulations using more recent tunes for a 14 TeV LHC, as eg the Pythia C4 tune [36], point to additional $\Delta P_T^{\text{fake}} \sim 30 - 40\,\text{GeV}$ for similar/ less stringent object definition values [37]; however, following the above considerations the induced error then is certainly $\sim \text{GeV}$, which again corresponds to a relatively small uncertainty in the determination of $\sqrt{s}^{\text{peak}}_{\text{min}}$.

\(^{13}\)In fact the experimental collaborations are already applying algorithms to subtract $E_T$ due to underlying event; cf eg [38]. We thank S. Wahrmund for bringing this to our attention.
Figure 8. Analysis level $\sqrt{s_{\text{min}}}$ after a cut $\sqrt{s_{\text{min}}} > 700$ GeV. Dominant six SM backgrounds after cut ($W + 2 j, W + 3 j, W + 4 j, t \bar{t}, t \bar{t} + 1 j, t \bar{t} + 2 j$) are included. Left: SM+BSM (green, dotted; 136834 events) and SM only (black, solid; 108017 events). In the sum and without further suppression cuts, the peak structure disappears. Right: Difference between (SM+BSM) and (SM). Assuming the SM background is well-known, the peak structure of the BSM signal is recovered. The difference is much larger than the statistical error.

Figure 9. Difference between (BSM+SM) and (SM) for $M_{\text{inv}} = 0$ GeV (left; 29815 events) as well as $M_{\text{inv}} = 1000$ GeV (right; 27802 events). Assuming the SM background is well-known, the peak structure of the BSM signal is recovered. The difference is much larger than the statistical error.

at parton and analysis level when using the true BSM value $M_{\text{inv}} = 2 m_{\chi_1^0}$. From figure 8 we see that in the total number of events after the cuts the peak structure disappears when no further SM cuts are applied. However, assuming an accurate enough (data or Monte Carlo driven) background subtraction, the peak structure is clearly visible again and much larger than the statistical error, cf. figure 8. A similar behavior is observed when we vary the input variable $M_{\text{inv}}$: figure 9 shows the behavior for $M_{\text{inv}} = 0$ GeV, 1000 GeV respectively after SM background subtraction; we see we obtain a clear BSM signal. We therefore conclude that, at least for the parameter point studied here, with SM background being well-known, $\sqrt{s_{\text{min}}}$ can be used as a BSM discovery variable and that for a true input value of $M_{\text{inv}}$, the scale of the new physics can be derived from the peak of both parton level and (properly defined) analysis level quantities.

We furthermore assess the use of $\sqrt{s_{\text{min}}}$ as a cut variable for SM background suppression. For this, we investigate the position of the peak for the different SM background
### Table 8

Cross sections $\sigma$ for SM background processes and $\sqrt{\hat{s}}_{\text{min}}$ maximal bin positions for parton level ($\sqrt{\hat{s}}_{\text{maxbin;parton}}$) and analysis level ($\sqrt{\hat{s}}_{\text{maxbin;analysis}}$) with standard analysis object definitions only; $\mathcal{M}_{\text{inv}} = 2 m_{\chi_1^0}$. Last two columns give cross sections $\sigma$ after $\sqrt{\hat{s}}_{\text{min}}$ cuts respectively. After a minimal analysis level cut on $\sqrt{\hat{s}}_{\text{min}}$, the $W$ and $t\bar{t}$ backgrounds are reduced by factors $3-6$, while we maintain roughly $90\%$ of the BSM signal.

| process | $\sigma$ [pb] | $\sqrt{\hat{s}}_{\text{min;parton}}$ [GeV] | $\sqrt{\hat{s}}_{\text{min;analysis}}$ [GeV] | $\sigma, \sqrt{\hat{s}}_{\text{min}} > 700$ GeV [pb] | $\sigma, \sqrt{\hat{s}}_{\text{min}} > 800$ GeV [pb] |
|---------|--------------|-------------------|-----------------|-------------------|-------------------|
| $W + 2j$ | 188 | 450 | 405 | 29.03 | 19.07 |
| $W + 3j$ | 53.6 | 630 | 585 | 23.47 | 17.51 |
| $W + 4j$ | 12.6 | 900 | 765 | 9.03 | 7.63 |
| $\sum W + \text{jets}$ | 254.2 | | | 61.53 | 44.21 |
| $t\bar{t}$ | 75.2 | 540 | 450 | 11.50 | 3.35 |
| $t\bar{t} + 1j$ | 48.5 | 675 | 585 | 20.71 | 14.41 |
| $t\bar{t} + 2j$ | 20.0 | 900 | 720 | 14.28 | 11.68 |
| $t\bar{t} + 3j$ | 6.3 | 1215 | 900 | 5.31 | 4.85 |
| $t\bar{t} + 4j$ | 1.4 | 1530 | 1215 | 1.29 | 1.24 |
| $\sum t + \text{jets}$ | 151.4 | | | 53.09 | 35.53 |
| $g\bar{g}$ | 6.56 | 1080 | 1035 | 5.59 | 5.03 |
| $g\bar{g}$ | 4.53 | 1260 | 1170 | 4.38 | 4.20 |
| $g\bar{g}$ | 19.96 | 1170 | 1035 | 18.85 | 17.82 |
| BSM | 31.05 | | | 28.82 | 27.05 |

channels considered here, where we again use $\mathcal{M}_{\text{inv}} = 2 m_{\chi_1^0}$. The respective values are given in table 8. For a first estimate of these positions, we do not need to perform a more sophisticated fit, and we therefore follow the simplified approach by defining the peak positions according to the bin which has the maximal number of entries. We see that the most dominant SM background channels have distribution peaks around 500 – 700 GeV, while the BSM signals peak at higher values. We therefore apply two different cuts of $\sqrt{\hat{s}}_{\text{min}} \geq 700$ GeV and $\sqrt{\hat{s}}_{\text{min}} \geq 800$ GeV on all samples; the cross sections after these cuts are summarized in table 9. We see that, for both cut values, while we only cut out around 10% of the BSM signal, the dominant SM channels are suppressed by factor 3-6. We therefore conclude that $\sqrt{\hat{s}}_{\text{min}}$ can easily be used as a variable for SM background suppression, even for wrong guesses for the total invisible mass $\mathcal{M}_{\text{inv}}$. In the previous sections, we discussed how in our sample the peak of the $\sqrt{\hat{s}}_{\text{min}}$ variable is correlated with the real threshold for the hard production cross sections only if the correct value input for $\mathcal{M}_{\text{inv}}$ is used, cf. eq. (2.6). For the background suppression, however, a correct guess or estimate of this value from other sources is not necessary, and we equally obtain a good SM background suppression with wrong input values for $\mathcal{M}_{\text{inv}}$.\(^{14}\)

\(^{14}\)We want to remind the reader that the same value of $\mathcal{M}_{\text{inv}}$ for the calculation of $\sqrt{\hat{s}}_{\text{min}}$ needs to be used in both SM and BSM samples; the correlation with the threshold however only holds for the sample with the equivalent correct $\mathcal{M}_{\text{inv}}$. We thank K. Matchev for reemphasizing this point.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
 & no cut & $M_{\text{inv}} = 2 m_{\chi_1^0}$ & 0 GeV & 400 GeV & $M_{\text{vis, min}}$ & $M_{\text{eff, min}}$ \\
 & & & $10^3$ GeV & $[\text{GeV}]$ & $\text{400}$ & $\text{500}$ \\
\hline
$W+$ jets & 254.2 & 62.53 & 81.13 & 48.73 & 46.73 & 70.29 & 42.41 & 75.89 & 43.3 \ 
$t\bar{t}$+ jets & 151.4 & 52.99 & 64.67 & 51.77 & 50.14 & 64.63 & 46.3 & 63.98 & 40.12 \ 
BSM & 31.05 & 28.82 & 29.82 & 28.42 & 27.80 & 27.07 & 24.43 & 29.99 & 28.72 \\
\hline
\end{tabular}
\caption{Total cross sections $\sigma \text{[pb]}$ for BSM as well as $W+$ jets and $t\bar{t}$+ jets backgrounds, without and with several cuts on different inclusive quantities. First column: no cut; second to fifth column: values for $\sqrt{s_{\text{cut}}} = M_{\text{inv}} + 500$ GeV, with varying $M_{\text{inv}}$ values; last four columns: cuts on $M_{\text{vis}}$ (eq. (5.1)) and $M_{\text{eff}}$ (eq. (5.4)) respectively, where $M_{\text{inv}} = 2 m_{\chi_1^0}$. While the maximal suppression factor for the BSM signal is around 1.11, the SM backgrounds are suppressed by factors $2-5$. Equal results, however, can easily be obtained by a cut on $M_{\text{vis}}$ or $M_{\text{eff}}$.}
\end{table}

5.1 Comparison with other (transverse) variables

The strength of $\sqrt{s_{\text{min}}}$ vs other (transverse) variables lies in the conjecture, given by eq. (2.6), about the direct correlation between its peak position (given the correct mass $M_{\text{inv}}$) and the rise of the parton level production cross section. However, this correlation has so far not risen beyond the status of a conjecture. Therefore briefly discuss two other variables which might serve a similar purpose in background subtraction, namely

$$M_{\text{vis}} = \sqrt{E_{\text{vis}}^2 - \vec{P}_{\text{vis}}^2} \quad (5.1)$$

and

$$E_T = |\vec{P}_T|, \quad (5.2)$$

where everything is defined at analysis object level. In [12], two more variables, namely $E_T$ and $H_T = E_T + \vec{E}_T$, are studied; however, as we define

$$\vec{P}_T = -\vec{P}_T \quad (5.3)$$

these two variables are only variations of $E_T$ and therefore not discussed here. Figure 10 shows the subtracted (BSM+SM) $-$ (SM) distributions of $M_{\text{vis}}$ and $E_T$ respectively; especially the former looks quite promising. Indeed, a cut $M_{\text{inv}} > 500$ GeV reduces the SM background by a factor 6 ($W+$ jets) and 3 ($t\bar{t}$+jets), cf. table 9. We equally compare to the frequently used variable $M_{\text{eff}}$ [8, 39]

$$M_{\text{eff}} = \sum_{\text{vis}} |p_T| + \vec{E}_T, \quad (5.4)$$

which exhibits a similar power for background suppression as $M_{\text{vis}}$, cf. table 9. We therefore conclude that, for background suppression, both $M_{\text{vis}}, M_{\text{eff}}$ as well as $\sqrt{s_{\text{min}}}$ work in a similar way; the advantage of the latter variable is the (conjectured) $M_{\text{inv}}$-dependent correlation between the its peak position and the hard (=parton level) process center-of-mass.

\footnote{Note that the definition of $H_T$ differs in [15], where it basically is set to the variable $M_{\text{eff}}$ [8, 39].}
energy, if this can be proven to hold in all cases. Indeed, a similar conjecture of a linear
correlation between the SUSY scale and the peak position of $M_{\text{eff}}$ [8, 39] has recently been
shown not to hold in all cases [40]; however, an equivalent systematic study of $\sqrt{s_{\text{min}}}$ is
still lacking.

6 Conclusion and outlook

We investigated the variable $\sqrt{s_{\text{min}}}$ for a complete BSM sample which includes all strong
production as well as all possible decay chains. In our analysis, we include both soft as
well as detector effects by including a complete parton shower as well as a generic detector
simulation and reconstruction-level objects, which have been defined such that the parton-
level variable can be recovered quite accurately. We investigate the variable $\sqrt{s_{\text{min}}}$ for
a fully inclusive sample which sums over all possible final states of the hard scattering
process, as well as for dominant exclusive final states. We see that, on parton level and
for a correct input value of $M_{\text{inv}}$, the $\sqrt{s_{\text{min}}}$ variable peaks closely to the heavy particle
production cross section which is in agreement with the conjecture made by the authors
of [12]. In a comparison between parton level and analysis level quantities, we see that in
our sample the largest shift between these arises from the transition from tau-leptons which
were used for the parton level quantity to the tau-jets and associated invisible neutrinos,
which were used at analysis level. In order to assess this effect, we used the true tau-
lepton four-vectors in the analysis level quantities. The effect is usually of the order of 100
GeV, and in the totally inclusive sample the shift completely disappears for the idealized
case of perfectly reconstructed tau jets. We therefore conclude that for the parameter
point considered here, even at analysis level, the parton level $\sqrt{s_{\text{min}}}$ peak position can be
sufficiently reconstructed.\footnote{We point out that, for the scenario considered in this study, even when using analysis tau jets the maximal shift between parton level and analysis level peak positions was $\sim 200$ GeV, which effectively leads in an error $\sim 100$ GeV in the estimation of the initial heavy particles masses. The magnitude of this effect can of course differ depending on the BSM model and as well as specific model scenario point.} In case the correlation between the threshold of the parton
level cross section and the peak of the $\sqrt{s_{\text{min}}}$ distribution could be proven rigorously, this
would indeed provide a quite elegant and straightforward way to assess the scale of the new physics signal as a function of the total invisible mass of the process. Furthermore, we present the first study which investigates the use of \( \sqrt{s_{\text{min}}} \) in order to suppress SM background for BSM searches. For this, we considered \( W^+ \text{jets} \) as well as \( t\bar{t} + \text{jets} \) background. We saw that these backgrounds could be sufficiently reduced by cuts on \( \sqrt{s_{\text{min}}} \), leading to suppression factors around \( 2 - 6 \), while we retained \( 90\% \) of the BSM signal. This feature was independent of the input value of the total invisible mass \( M_{\text{inv}} \). However, we could achieve similar results by a cut on the total visible mass \( M_{\text{vis}} \), which is a simpler variable which additionally does not require the input of \( M_{\text{inv}} \). A further comparison with \( M_{\text{eff}} \) as a cut variable lead to similar results. We therefore conclude that, unless the conjecture about mass particle threshold and peak position of \( \sqrt{s_{\text{min}}} \) can be rigorously proven, the latter does not exhibit significant advantages over other (transverse) variables. However, if the correlation between the threshold and \( \sqrt{s_{\text{min}}} \) could be rigorously proven, it would indeed provide a simple and elegant hold on the scale of new physics processes. A further investigation of this relation is in the line of future work.

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### A SPS1a spectrum

| \( l_L \) | \( 568.4 \) | \( l_R \) | \( 545.2 \) | \( \tilde{u}_L \) | \( 561.1 \) | \( \tilde{u}_R \) | \( 549.3 \) | \( \tilde{b}_1 \) | \( 513.1 \) | \( \tilde{b}_2 \) | \( 543.7 \) | \( \tilde{t}_1 \) | \( 399.7 \) | \( \tilde{t}_2 \) | \( 585.8 \) |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| \( \tilde{l}_L \) | 202.9 | \( \tilde{l}_R \) | 144.1 | \( \tilde{\tau}_1 \) | 134.5 | \( \tilde{\tau}_2 \) | 206.9 | \( \tilde{\nu}_L \) | 185.3 | \( \tilde{\nu}_\tau \) | 184.7 | \( \tilde{g} \) | 607.7 |
| \( \chi_1 \) | 181.7 | \( \chi_2 \) | 380.0 | \( \chi_3^0 \) | 96.7 | \( \chi_4^0 \) | 181.1 | \( \chi_1^0 \) | 363.8 | \( \chi_2^0 \) | 381.7 |

**Table 10.** Relevant masses for SPS1a in GeV. \( u = (u, c) \), \( d = (d, s) \), \( l = (e, \mu) \).

### B Minimization of \( \sqrt{s} \)

As stated in section 2, \( \sqrt{s_{\text{min}}} \) denotes the *minimal* center of mass energy \( \sqrt{s} \) of an event with a measured visible four-vector \( P_{\text{vis}}^\mu = (E, P_T, P_Z) \) which is still in agreement with total energy momentum conservation as well as onshellness of all outgoing particles. We
equally assume the event to be at rest in the transverse plane such that eq. (2.4) holds. The generic expression for \( \hat{s} \) in this case is given by

\[
\hat{s}(\vec{p}_T = -\vec{P}_T) = \left( E + \sum_j E_j \right)^2 - \left( P_z + \sum_j p_{jz} \right)^2
\]

(B.1)

where the index \( j \) goes over the invisible particles in the event with the respective energies

\[
E_j^2 = m_j^2 + p_{jT}^2 + p_{jz}^2.
\]

Here and in the following, we omit the vector notation in \( p_T \) for simplification, but all transverse quantities should be read as \( p_{jT} = (p_{jx}, p_{jy}) \), \( \vec{P}_T = (\vec{P}_X, \vec{P}_Y) \), etc.

We use a Lagrange multiplier \( \lambda \) to take the additional constraint for the vector sum of the transverse momenta

\[
\sum_j p_{jT} = \vec{P}_T \tag{B.2}
\]

into account. We therefore aim at minimizing

\[
\mathcal{L} = \hat{s} - \lambda \left( \sum_j p_{jT} - \vec{P}_T \right),
\]

ie we try to find the values of the invisible particles’ three-momenta \( \vec{p}_i \), such that

\[
\frac{\partial \mathcal{L}}{\partial \vec{p}_i} = 0.
\]

If we consider the case of \( n_{\text{inv}} \) invisible particles in the event, we obtain \( 3 \times n_{\text{inv}} \) equations

\[
\frac{\partial \mathcal{L}}{\partial p_{iT}} = 2 \left( E + \sum_j E_j \right) \frac{p_{iT}}{E_i} - \lambda = 0,
\]

\[
\frac{\partial \mathcal{L}}{\partial p_{iz}} = 2 \left( E + \sum_j E_j \right) \frac{p_{iz}}{E_i} - 2 \left( P_z + \sum_j p_{jz} \right) = 0. \tag{B.3}
\]

Together with the constraint in eq. (B.2), we now have in total \( 3 n_{\text{inv}} + 1 \) constraints for \( 3 n_{\text{inv}} + 1 \) unknowns \((p_{ix}, p_{iy}, p_{iz}; \lambda)\). From eqs. (B.3), we immediately see that

\[
\frac{E_i}{E_j} = \frac{p_{iT}}{p_{jT}} = \frac{p_{iz}}{p_{jz}} = c_{ij},
\]

with \( c_{ij} = \frac{m_i}{m_j} \equiv \text{const.} \) Combining this with eq. (B.2), we obtain

\[
\frac{m_i}{p_{iT}} = \frac{\sum_j m_j}{\vec{P}_T} = \frac{M_{\text{inv}}}{\vec{P}_T}, \tag{B.4}
\]

where we defined \( M_{\text{inv}} = \sum_j m_j \) to be the sum over the masses of all invisible particles, cf. eq. (2.2). We therefore have

\[
p_{iT} = \frac{\vec{P}_T}{M_{\text{inv}}} m_i.
\]
We can now rewrite the second equation in (B.3) and obtain

\[
\left( E + \frac{E_i}{p_i T} p_T \right) P_{iz} - \left( P_z + \frac{p_{iz}}{p_T} p_T \right) = 0.
\]

Solving this for \( p_{iz} \) leads to

\[
p_{iz} = \frac{P_z m_i}{\sqrt{E^2 - p_T^2}} \sqrt{1 + \frac{p_T^2}{M_{inv}^2}}.
\]

We here have reproduced the solutions for \( p_{i,z}, p_{i,T} \) given in [12] which minimize \( \hat{s} \). Inserting these into eq. (B.1) then leads to \( \sqrt{s_{\text{min}}} \) given by eq. (2.1), which denotes the minimal hard scattering center of mass energy which is allowed by energy momentum conservation for a specific visible total four vector \((E, P_T, P_Z)\) obtained from measurement. The only unknown quantity is \( M_{inv} \) defined according to eq. (2.2), which has to be treated as an external input parameter for \( \sqrt{s_{\text{min}}} \).

We want to comment that the transverse mass variable \( M_T \) [41–44] has a functional form similar to \( \sqrt{s_{\text{min}}} \) as given in eq. (2.1). For a system with visible and invisible total four-vectors \( P^\mu_{\text{vis}}, P^\mu_{\text{inv}} \), this variable is defined as

\[
M_T^2 = (E_{T,\text{vis}} + E_{T,\text{inv}})^2 - (P_T + P_T')^2.
\]

with the transverse energies

\[
E_{T,\text{vis}}^2 = M_{\text{vis}}^2 + P_T^2, \quad E_{T,\text{inv}}^2 = (M_{\text{inv}}')^2 + P_T'^2.
\]

Here, \( M_{\text{vis}}, M_{\text{inv}}' \) denote the Lorentz-invariant masses of the total visible and invisible system respectively,

\[
M_{\text{vis}}^2 = P_{\text{vis}}^2, \quad (M_{\text{inv}}')^2 = P_{\text{inv}}^2,
\]

which vary on an event by event basis. Assuming eq. (2.4) to hold, it follows that

\[
M_T = \sqrt{E_T^2 + M_{\text{vis}}^2 + (M_{\text{inv}}')^2}.
\]

We now see that the functional forms of \( M_T(M_{\text{inv}}') \) and \( \sqrt{s_{\text{min}}}(M_{\text{inv}}) \) as given in eq. (2.1) are identical, and differences between the variables only stem from the difference between \( M_{\text{inv}}' \) and \( M_{\text{inv}} \). Indeed, for a correct guess of \( M_{\text{inv}}, \sqrt{s_{\text{min}}} \) and \( M_T \) coincide if for the invisible particles in the event

\[
\sum_{i \neq j} m_i m_j = \sum_{i \neq j} p_i \cdot p_j.
\]

Componentwise, this equation is only fulfilled if we either have a complete set of massless particles which are all collinear with each other such that \( \cos \theta_{ij} = 1 \) for all \((i, j)\) pairs or for a complete set of massive particles which are all produced at rest. In general, however,

\[
M_{\text{inv}}' > M_{\text{inv}}^{\text{(true)}},
\]

and therefore

\[
M_T > \sqrt{s_{\text{min}}}(M_{\text{inv}}^{\text{(true)}})
\]

on an event by event basis.
C Pythia 6.4 ISR/ FSR default setup

All switch descriptions here are taken from [25]. We equally refer the reader to section 10 of this reference for a more detailed discussion of the parton shower model and its implementation in Pythia.

MSTP(32): (D = 8) $Q^2$ definition in hard scattering for $2 \to 2$ processes. For resonance production $Q^2$ is always chosen to be $\hat{s} = m_R^2$, where $m_R$ is the mass of the resonance. The newer options 6–10 are specifically intended for processes with incoming virtual photons. These are ordered from a ‘minimal’ dependence on the virtualities to a ‘maximal’ one, based on reasonable kinematics considerations. The old default value MSTP(32) = 2 forms the starting point, with no dependence at all, and the new default is some intermediate choice. Notation is that $P^2_1$ and $P^2_2$ are the virtualities of the two incoming particles, $p_\perp$ the transverse momentum of the scattering process, and $m_3$ and $m_4$ the masses of the two outgoing partons. For a direct photon, $P^2$ is the photon virtuality and $x = 1$. For a resolved photon, $P^2$ still refers to the photon, rather than the unknown virtuality of the reacting parton in the photon, and $x$ is the momentum fraction taken by this parton.

- 2: $Q^2 = (m_{\perp 3}^2 + m_{\perp 4}^2)/2 = p_\perp^2 + (m_3^2 + m_4^2)/2$.
- 8: $Q^2 = p_\perp^2 + (P^2_1 + P^2_2 + m_3^2 + m_4^2)/2$. ensure that the $Q^2$ scale is always bigger than $P^2$.

MSTP(62): (D = 3) level of coherence imposed on the space-like parton-shower evolution.

- 3: $Q^2/p_\perp^2$ values and opening angles of emitted (on-mass-shell or time-like) partons are both strictly ordered, increasing towards the hard interaction.

MSTP(63): (D = 2) structure of associated time-like showers, i.e. showers initiated by emission off the incoming space-like partons in PYSSPA.

- 2: a shower may evolve, with maximum allowed time-like virtuality set by phase space or by PARP(71) times the $Q^2$ value of the space-like parton created in the same vertex, whichever is the stronger constraint.

MSTP(64): (D = 2) choice of $\alpha_s$ and $Q^2$ scale in space-like parton showers in PYSSPA.

- 2: first-order running $\alpha_s$ with argument PARP(64) $k_\perp^2 = \text{PARP}(64)(1 - z)Q^2$.

MSTP(65): (D = 1) treatment of soft-gluon emission in space-like parton-shower evolution in PYSSPA.

- 1: soft-gluon emission is resummed and included together with the hard radiation as an effective $z$ shift.

MSTP(66): (D = 5) choice of lower cut-off for initial-state QCD radiation in VMD or anomalous photoproduction events, and matching to primordial $k_\perp$.

- 1: for anomalous photons, the lower $Q^2$ cut-off is the larger of PARP(62)$^2$ and VINT(283) or VINT(284), where the latter is the virtuality scale for the $\gamma \to q\bar{q}$ vertex on the appropriate side of the event. The VINT values are selected logarithmically even between PARP(15)$^2$ and the $Q^2$ scale of the parton distributions of the hard process.
a stronger damping at large $k_\perp$, like $dk_\perp^2/(k_\perp^2 + Q^2/4)^2$ with $k_0 < k_\perp < p_{\perp \text{min}}(W^2)$. Apart from this, it works like $= 1$.

$= 5$: a $k_\perp$ generated as in $= 4$ is added vectorially with a standard Gaussian $k_\perp$ generated like for VMD states. Ensures that GVMD has typical $k_\perp$'s above those of VMD, in spite of the large primordial $k_\perp$'s implied by hadronic physics. (Probably attributable to a lack of soft QCD radiation in parton showers.)

MSTP(67): (D = 2) possibility to introduce colour coherence effects in the first branching of the backwards evolution of an initial-state shower in PYSSPA; mainly of relevance for QCD parton-parton scattering processes.

$= 2$: restrict the polar angle of a branching to be smaller than the scattering angle of the relevant colour flow.

Note 1: azimuthal anisotropies have not yet been included.

Note 2: for subsequent branchings, MSTP(62) = 3 is used to restrict the (polar) angular range of branchings.

MSTP(68): (D = 3) choice of maximum virtuality scale and matrix-element matching scheme for initial-state radiation. To this end, the basic scattering processes are classified as belonging to one or several of the following categories (hard-coded for each process):

$= 0$: maximum shower virtuality is the same as the $Q^2$ choice for the parton distributions, see MSTP(32). (Except that the multiplicative extra factor PARP(34) is absent and instead PARP(67) can be used for this purpose.) No matrix-element correction.

$= 3$: as $= 0$, but ME corrections are applied where available.

MSTP(69): (D = 0) possibility to change $Q^2$ scale for parton distributions from the MSTP(32) choice, especially for $e^+e^-$. $= 0$: use MSTP(32) scale.

MSTP(72): (D = 1) maximum scale for radiation off FSR dipoles stretched between ISR partons in the new $p_\perp$-ordered evolution in PYPTIS.

$= 1$: the $p_{\perp \text{max}}$ scale of FSR is set as the $p_\perp$ production scale of the respective radiating parton. Dipoles stretched to remnants do not radiate.

The additional switches/variables appearing above are given by

PARP(15): (D = 0.5 GeV) lower cut-off $p_0$ used to define minimum transverse momentum in branchings $\gamma \rightarrow q\bar{q}$ in the anomalous event class of $\gamma p$ interactions, i.e. sets the dividing line between the VMD and GVMD event classes.

PARP(62): (D = 1. GeV) effective cut-off $Q$ or $k_\perp$ value (see MSTP(64)), below which space-like parton showers are not evolved. Primarily intended for QCD showers in incoming hadrons, but also applied to $q \rightarrow q\gamma$ branchings.

PARP(64): (D = 1.) in space-like parton-shower evolution the squared transverse momentum evolution scale $k_\perp^2$ is multiplied by PARP(64) for use as a scale in $\alpha_s$ and parton distributions when MSTP(64) = 2.
PARP(67): (D = 4.) the $Q^2$ scale of the hard scattering (see MSTP(32)) is multiplied by PARP(67) to define the maximum parton virtuality allowed in $Q^2$-ordered space-like showers. This does not apply to $s$-channel resonances, where the maximum virtuality is set by $m^2$. It does apply to all user-defined processes, however.

PARP(71): (D = 4.) the $Q^2$ scale of the hard scattering (see MSTP(32)) is multiplied by PARP(71) to define the maximum parton virtuality allowed in time-like showers. This does not apply to $s$-channel resonances, where the maximum virtuality is set by $m^2$. Like for PARP(67) this number is uncertain.

VINT(283), VINT(284): virtuality scale at which a GVMD/anomalous photon on the beam or target side of the event is being resolved. More precisely, it gives the $k_\perp^2$ of the $\gamma \rightarrow q\bar{q}$ vertex. For elastic and diffractive scatterings, $m^2/4$ is stored, where $m$ is the mass of the state being diffracted. For clarity, we point out that elastic and diffractive events are characterized by the mass of the diffractive states but without any primordial $k_\perp$, while jet production involves a primordial $k_\perp$ but no mass selection. Both are thus not used at the same time, but for GVMD/anomalous photons, the standard (though approximate) identification $k_\perp^2 = m^2/4$ ensures agreement between the two applications.

VDM/ GVDM are acronyms for vector meson dominated/ generalized vector meson dominated events in photo production respectively (cf. section 7.7.2 of [25]).

References

[1] G. Tonelli, Highlights and searches at CMS, presentation at International Europhysics Conference on High Energy Physics, Grenoble France (2011), http://eps-hep2011.eu/.

[2] D. Charlton, Highlights and searches at ATLAS, presentation at International Europhysics Conference on High Energy Physics, Grenoble France (2011), http://eps-hep2011.eu/.

[3] H. Bachacou, BSM Results from LHC, presented at Lepton-Photon 2011, Mumbai India (2011), http://www.tifr.res.in/~lp11.

[4] A.J. Barr and C.G. Lester, A Review of the Mass Measurement Techniques proposed for the Large Hadron Collider, J. Phys. G 37 (2010) 123001 [arXiv:1004.2732] [INSPIRE].

[5] A.J. Barr et al., Guide to transverse projections and mass-constraining variables, Phys. Rev. D 84 (2011) 095031 [arXiv:1105.2977] [INSPIRE].

[6] C. Lester and D. Summers, Measuring masses of semi invisibly decaying particles pair produced at hadron colliders, Phys. Lett. B 463 (1999) 99 [hep-ph/9906349] [INSPIRE].

[7] D.R. Tovey, On measuring the masses of pair-produced semi invisibly decaying particles at hadron colliders, JHEP 04 (2008) 034 [arXiv:0802.2879] [INSPIRE].

[8] I. Hinchliffe, F. Paige, M. Shapiro, J. Soderqvist and W. Yao, Precision SUSY measurements at CERN LHC, Phys. Rev. D 55 (1997) 5520 [hep-ph/9610544] [INSPIRE].

[9] H. Bachacou, I. Hinchliffe and F.E. Paige, Measurements of masses in SUGRA models at CERN LHC, Phys. Rev. D 62 (2000) 015009 [hep-ph/9907518] [INSPIRE].

[10] ATLAS collaboration, ATLAS detector and physics performance: technical design report. Vol. 2, ATLAS-TDR-015, CERN, Geneva Switzerland (1999).
[11] B. Allanach, C. Lester, M.A. Parker and B. Webber, Measuring sparticle masses in nonuniversal string inspired models at the LHC, JHEP 09 (2000) 004 [hep-ph/0007009] [SPIRE].

[12] P. Konar, K. Kong and K.T. Matchev, $\sqrt{s}_{\text{min}}$: A Global inclusive variable for determining the mass scale of new physics in events with missing energy at hadron colliders, JHEP 03 (2009) 085 [arXiv:0812.1042] [SPIRE].

[13] A. Papaefstathiou and B. Webber, Effects of QCD radiation on inclusive variables for determining the scale of new physics at hadron colliders, JHEP 06 (2009) 069 [arXiv:0903.2013] [SPIRE].

[14] A. Papaefstathiou and B. Webber, Effects of invisible particle emission on global inclusive variables at hadron colliders, JHEP 07 (2010) 018 [arXiv:1004.4762] [SPIRE].

[15] P. Konar, K. Kong, K.T. Matchev and M. Park, RECO level $\sqrt{s}_{\text{min}}$ and subsystem $\sqrt{s}_{\text{min}}$: Improved global inclusive variables for measuring the new physics mass scale in $E_T$ events at hadron colliders, JHEP 06 (2011) 041 [arXiv:1006.0653] [SPIRE].

[16] ATLAS collaboration, G. Aad et al., Search for squarks and gluinos using final states with jets and missing transverse momentum with the ATLAS detector in $\sqrt{s} = 7$ TeV proton-proton collisions, Phys. Lett. B 701 (2011) 186 [arXiv:1102.5290] [SPIRE].

[17] B.C. Allanach et al., The Snowmass points and slopes: Benchmarks for SUSY searches, Eur. Phys. J. C 25 (2002) 113 [hep-ph/0202233] [SPIRE].

[18] New Physics Working Group collaboration, G. Brooijmans et al., New Physics at the LHC. A Les Houches Report: Physics at TeV colliders 2009 — New Physics Working Group, arXiv:1005.1229 [SPIRE].

[19] S. Ovyn, X. Rouby and V. Lemaitre, Delphes, a framework for fast simulation of a generic collider experiment, arXiv:0903.2225 [SPIRE].

[20] B. Allanach, SOFTSUSY: a program for calculating supersymmetric spectra, Comput. Phys. Commun. 143 (2002) 305 [hep-ph/0104145] [SPIRE].

[21] T. Stelzer and W. Long, Automatic generation of tree level helicity amplitudes, Comput. Phys. Commun. 81 (1994) 357 [hep-ph/9401258] [SPIRE].

[22] F. Maltoni and T. Stelzer, MadEvent: Automatic event generation with MadGraph, JHEP 02 (2003) 027 [hep-ph/0208156] [SPIRE].

[23] P. Meade and M. Reece, BRIDGE: Branching ratio inquiry / decay generated events, hep-ph/0703031 [SPIRE].

[24] M.L. Mangano, M. Moretti, F. Piccinini, R. Pittau and A.D. Polosa, ALPGEN, a generator for hard multiparton processes in hadronic collisions, JHEP 07 (2003) 001 [hep-ph/0206293] [SPIRE].

[25] T. Sjöstrand, S. Mrenna and P.Z. Slands, PYTHIA 6.4 Physics and Manual, JHEP 05 (2006) 026 [hep-ph/0603175] [SPIRE].

[26] http://www.lpthe.jussieu.fr/LesHouches09Wiki/index.php/Mass_methods.

[27] I. Antcheva et al., ROOT: A C++ framework for petabyte data storage, statistical analysis and visualization, Comput. Phys. Commun. 180 (2009) 2499 [SPIRE].

[28] http://root.cern.ch.
[29] D. Stump et al., Inclusive jet production, parton distributions and the search for new physics, *JHEP* **10** (2003) 046 [hep-ph/0303013] [inSPIRE].

[30] T. Robens, work in progress.

[31] CDF collaboration, F. Abe et al., The Topology of three jet events in $\bar{p}p$ collisions at $\sqrt{s} = 1.8$ TeV, *Phys. Rev.* **D 45** (1992) 1448 [inSPIRE].

[32] ATLAS collaboration, G. Aad et al., Expected Performance of the ATLAS Experiment — Detector, Trigger and Physics, arXiv:0901.0512 [inSPIRE].

[33] P. Bechtle, R. Brunelière, M. Kobel, J.-R. Lessard and S. Wahr mund, private communication.

[34] A. Moraes, C. Buttar and I. Dawson, Prediction for minimum bias and the underlying event at LHC energies, *Eur. Phys. J.* **C 50** (2007) 435 [inSPIRE].

[35] ATLAS collaboration, A. Tricoli, Underlying event studies at ATLAS, ATL-PHYS-PROC-2009-048, CERN, Geneva Switzerland (2009).

[36] R. Corke and T. Sjöstrand, Interleaved Parton Showers and Tuning Prospects, *JHEP* **03** (2011) 032 [arXiv:1011.1759] [inSPIRE].

[37] D. Kar, private communication.

[38] ATLAS collaboration, Jet energy scale and its systematic uncertainty for jets produced in proton-proton collisions at $\sqrt{s} = 7$ TeV and measured with the ATLAS detector, ATLAS-CONF-2010-056, CERN, Geneva Switzerland (2010).

[39] D. Tovey, Measuring the SUSY mass scale at the LHC, *Phys. Lett. B* **498** (2001) 1 [hep-ph/0006276] [inSPIRE].

[40] J.A. Conley, J.S. Gainer, J.L. Hewett, M.P. Le and T.G. Rizzo, Supersymmetry Without Prejudice at the LHC, *Eur. Phys. J.* **C 71** (2011) 1697 [arXiv:1009.2539] [inSPIRE].

[41] W.L. van Neerven, J.A.M. Vermaseren and K.J.F. Gaemers, Lepton-jet events as a signature for $W$ production in $p\bar{p}$ collisions, NIKHEF preprint NIKHEF-H/82-20 (1982) [inSPIRE].

[42] UA1 collaboration, G. Arnison et al., Further Evidence for Charged Intermediate Vector Bosons at the SPS Collider, *Phys. Lett. B* **129** (1983) 273 [inSPIRE].

[43] UA1 collaboration, G. Arnison et al., Recent Results on Intermediate Vector Boson Properties at the CERN Super Proton Synchrotron Collider, *Phys. Lett. B* **166** (1986) 484 [inSPIRE].

[44] V.D. Burger, T. Han and R. Phillips, Improved transverse mass variable for detecting Higgs boson decays into $Z$ pairs, *Phys. Rev. D* **36** (1987) 295 [inSPIRE].