PCB board heat transfer model based on heat transfer mechanism analysis

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Abstract. Temperature control plays a vital role in the circuit board production process. In order to control the temperature change in the reflow oven, we establish a differential equation model of the PCB heat transfer based on the theory of heat transfer and use the boundary conditions to solve the temperature distribution in the oven. We also accurately simulate the heating and cooling process of the PCB. According to the requirements for the shape of the oven temperature curve in actual production, we respectively use the heating factor and the area enclosed by the temperature as evaluation indicators, establish a target optimization model, and obtain the best set temperature for each temperature zone and the best transmission speed of PCB board. Through analysis of different cases, we could get the conclusion that the model in this paper is reasonable and effective.

1. Introduction
In this paper, we establish a mathematical model based on CUMCM-2020 Question A. First, we set up a single-objective optimization model with the minimum heating factor [1] as the goal, taking the process limits and the temperature range of each temperature zone as the constraints. When solving the model, we use a function optimization algorithm based on genetic algorithm and nonlinear programming. First, the genetic algorithm is used to calculate the global optimal solution, which is used as the initial value of the nonlinear programming algorithm and continues to find the local optimal solution, and the high accuracy result is obtained finally.

In actual production, in order to ensure welding quality, the cooling curve also should be mirror-symmetrical to the heating curve as much as possible. Therefore, we take the size of the area enclosed by the symmetrical curve of the temperature curve on the right side of the temperature peak and the temperature curve on the left as a measure of the degree of symmetry. We take the minimum index and the minimum heating factor as the goal, the process limit and the temperature range of each temperature zone as the constraint conditions, establish a multi-objective optimization model, and finally obtain an oven temperature curve model that is more suitable for actual production.

2. Assumptions
1) Before the PBC board enters, the temperature in the reflow oven has stabilized. We need to consider the temperature change of the PCB board.
2) Ignore the heat absorbed by the conveyor belt.
3) The heating power and blowing wind speed in different temperature zones are equal and remain unchanged.
4) Ignore the heat conduction and heat radiation between the PCB board and the high-temperature gas.

3. Model building and solution

3.1 Basic principles of modeling
The research object of this paper is an all-hot-air reflow oven. There are three types of heat transfer in the reflow oven [2], which are convective heat transfer, heat conduction and thermal radiation heat transfer. Heat convection is the heat transfer caused by the macroscopic movement between the parts of the fluid at different temperatures, which can only occur in the fluid; heat conduction occurs inside or between solids, and the heat is transferred from the high temperature area to the low temperature area; thermal radiation refers to the temperature Objects higher than absolute zero will emit energy to the outside in the form of electromagnetic waves, thereby realizing heat transfer.

3.2 PBC plate heat transfer model
The single heating temperature of the reflow oven is divided into upper and lower temperature zones. The heating tube and hot air motor are installed in the temperature zone, and the hot air formed is blown onto the PCB board of the conveyor belt through the air duct. Realize the heating of the PCB board. Therefore, there is mainly heat convection between the PCB board and the single temperature zone, and it is an instantaneous heat transfer process. The upper temperature zones and lower temperature zones are very close to the PCB board on the conveyor belt, the high-temperature gas blown from the temperature zone hits the PCB board instantly, so we can assume that the high-temperature gas has no heat loss before touching the PCB board, i.e., the inside of the temperature zone is a constant temperature system. Only the temperature of the PCB board is changing.

Assuming that the total heat received by the PCB board is $\Phi$, without considering the heat conduction and heat radiation, the heat absorbed by the PCB board in the oven can be expressed by the following formula:

$$\Phi = cm_{PCB}(T_{PCB}(t) - T_{PCB}(t_0))$$ (1)

Where $c$ is the specific heat capacity of the PCB board, $m_{PCB}$ is the quality of the PCB board, $T_{PCB}(t_0)$ refers to the temperature of the PCB board at $t_0$ moment, and $T_{PCB}(t)$ refers to the temperature of the PCB board at $t$ moment.

The quality of the PCB board is:

$$m_{PCB} = \rho \times L \times S$$ (2)

Where $\rho$ is the density of the PCB board, $L$ is the thickness of the PCB board, and $S$ is the area of the PCB board.

According to the thermal convection formula [3], the heat absorbed by the PCB board can be obtained as:

$$\Phi = h_c A_{PCB} (T_{air} - T_{PCB}(t))$$ (3)

Where $h_c$ is the convective heat transfer coefficient, $A_{PCB}$ is the area of the PCB board, and $T_{air}$ is the temperature of the gas in the oven. The temperature is different in different temperature zones.

From formula (1), formula (2), and formula (3), we can get:

$$T_{PCB}(t) - T_{PCB}(t_0) = -\frac{1}{\rho c L} \int_{t_0}^{t} h_c (T_{PCB} - T_{air}) dt$$ (4)

From equation (4), we can get the PCB board temperature at $t$ moment.
3.3 Temperature distribution model of reflow oven

The reflow oven is composed of 11 temperature zones, 10 gaps, the front and back areas of the oven. The temperature in the 11 temperature zones has been given, and the temperature of the area before and after the oven and the gap is determined by the adjacent temperature zones. Therefore, finding the temperature distribution in the oven is transformed into finding the temperature division between the front and back areas of the oven and the gap.

The gas flow in the oven is mainly concentrated in the small temperature zone, and the air in the gap in the low temperature zone is in a static state. Therefore, the heat transfer method in the gap is mainly heat conduction.

![Figure 1. Schematic diagram of gap heat transfer](image)

As shown in Figure 1, assuming the gap thickness is \( \delta \), the left and right sides of the surface are maintained at a uniform and stable temperature \( t_1, t_2 \). According to the heat conduction formula [3]:

\[
\frac{d^2 T_{gap}}{dx^2} = 0
\]

s.t. \( \begin{cases} x = 0, T_{gap} = T_1 \\ x = \delta, T_{gap} = T_2 \end{cases} \)

According to formula (5), we can get:

\[
T_{gap}(x) = \frac{T_2 - T_1}{\delta} x + T_1
\]

In formula (6), \( \delta, T_1, \) and \( T_2 \) are all fixed values, so the temperature in the gap is linearly distributed.

For the area in front of the oven, we consider the temperature on the left as room temperature and the temperature on the right as the temperature gap of small temperature zone 1. The same goes for the area behind the oven.

In summary, the temperature distribution of the reflow furnace is a constant temperature distribution in a small temperature zone and a linear temperature distribution in the gap.

3.4 Optimal oven temperature curve under given conditions

3.4.1 Analyze modeling conditions. The quality of reflow solder joints plays a vital role in the realization of chip functions after the PCB board is processed, and the quality of solder joints depends on the control of the reflow curve. Solder joint connection is essentially a metallurgical bonding process [1]. The molten solder paste alloy forms a metal compound on the interface with the base metal to realize the solder joint connection.
The temperature time integral of the reflux curve above the melting point of the metal is defined as the heating factor, which is expressed by \( Q_n \):

\[
Q_n = \int_{t_1}^{t_2} (T_{pcb}(t) - T_{lim}) dt
\]  \( (7) \)

In the formula (7), \( T_{lim} \) refers to the melting point of the metal. \( t_1 \) and \( t_2 \) represent the time when PCB reaches metal melting point and the time when PCB reaches temperature peak respectively.

### 3.4.2 Model establishment

In order to ensure the high reliability of welding, it is necessary to control the size of the heating factor to make it in an optimal range. Therefore, we established a single-objective optimization model, with the minimum heating factor as the goal, and the process limit as the constraints. Build the model as follows:

\[
\min Q_n = \int_{t_1}^{t_2} (T_{pcb}(t) - T_{lim}) dt
\]  \( (8) \)

\[
\begin{aligned}
T_{PCB}(t + \Delta t) &= T_{PCB}(t) + \frac{k}{L}*(T_{PCB}(t) - T_{air}(t))*\Delta t \\
T_{lim} &= 217 \\
0 &\leq \tan \theta \leq 3 \\
-3 &\leq \tan \theta \leq 0 \\
240 &< T_{MAXpcb} < 250 \\
60 &< t_{190^\circ C} - t_{190^\circ C} < 120 \\
40 &< |t_{217^\circ C} - t_{217^\circ C}| < 90 \\
65 &< V < 100 \\
165 &< T_{air,i} < 185 \quad i = 1, 2, 3, 4, 5 \\
185 &< T_{air,i} < 205 \quad i = 6 \\
225 &< T_{air,i} < 245 \quad i = 7 \\
245 &< T_{air,i} < 265 \quad i = 8, 9 \\
T_{air,i} &= 25 \quad i = 10, 11
\end{aligned}
\]

In the formula (8), \( V \) is the speed of the conveyor belt.

### 3.4.3 Model solution and analysis

In order to obtain the optimal heating factor, we use a function optimization algorithm based on genetic algorithm and nonlinear programming. Nonlinear programming algorithm has strong local search ability, but weak global search ability. The way of genetic algorithm to search is the selection, crossover and mutation of operators. As a result, it has a strong global search capability, but a weak local search capability. Therefore, we combine the advantages of two algorithms. First, we use genetic algorithm for global search and use nonlinear programming algorithm based on the result of genetic algorithm for local search, so as to obtain the global optimal solution of the problem [4].

The algorithm flow chart is as follows:
Figure 2. Algorithm flow chart

Take $n = 10$, and get the optimal solution of heating factor as shown in the figure below:

Figure 3. The optimal solution of the heating factor

We can see that the difference between the minimum value and the maximum value is about 10%, which is acceptable. We use 495.8066 as the minimum value of the heating factor and use this as a condition to calculate the temperature and conveyor speed in different temperature zones.
Figure 4. Temperature in different zones
The temperatures of temperature zone 1-5, temperature zone 6, temperature zone 7, and temperature zone 8-9 are 171.5169°C, 193.3525°C, 229.6853°C, and 262.8098°C, respectively. The conveyor speed through the oven is 80.1523 cm/min.

3.5 Optimal oven temperature curve when the oven temperature curve is as symmetric as possible
We can refine the solder crystal lattice and increase the bonding strength by rapid cooling of the solder joints. So we need to find an optimal oven temperature that is mirror-symmetrical with the heating curve.

Figure 5. The area enclosed by a temperature curve
We take the area of the symmetrical curve on the right side of the peak temperature and the curve on the left side as a measure. The larger the area is, the more asymmetric the left and right sides are. The smaller the area is, the more symmetrical the left and right sides are. As shown in Figure 5, the blue curve in the figure indicates the temperature curve in the furnace when the temperature exceeds the melting point of the solder paste, and the blue dotted line indicates the symmetrical curve of the original curve. The red curve indicates the difference between the blue dotted line and the blue solid line, i.e., the area enclosed by the symmetrical curve of the peak temperature on the right and the temperature curve on the left.

\[ M = \int_{t_1}^{t_{\text{end}}} |T(t) - T(2t_{\text{half}} - t)| dt \]  \hspace{1cm} (9)
In the formula (9), $M$ is the measurement index, $t_1$ and $t_2$ respectively represent the two moments when the temperature reaches the melting point of the solder paste. $t_{\text{half}} = \frac{t_1 + t_2}{2}$.

On the basis of the previous model, we add a new optimization goal to form a multi-objective optimization model, and convert it into a single-objective optimization model by adding a weight $\lambda$ [5]:

$$
\min Z = \lambda \frac{M}{M_{\text{max}}} + (1 - \lambda) \frac{Q}{Q_{\text{min}}} 
$$

$$
T_{\text{PCB}}(t + \Delta t) = T_{\text{PCB}}(t) + \frac{k}{L} \times (T_{\text{PCB}}(t) - T_{\text{air}}(t)) \times \Delta t
$$

$$
Q = \int_{t_i}^{t_f} (T_{\text{pol}}(t) - T_{\text{air}}) dt
$$

$$
M = \int_{t_i}^{t_f} |T(t) - T(2t_i - t)| dt
$$

$$
T_{\text{air}} = 217
$$

$$
0 \leq \tan \theta \leq 3
-3 \leq \tan \theta \leq 0
240 < T_{\text{MAXPAS}} < 250
60 < t_{\text{previous}} - t_{\text{previous}} < 120
40 < |t_{\text{previous}} - t_{\text{previous}}| < 90
65 < V < 100
$$

In the formula (10), $V$ is the speed of the conveyor belt.

Take $\lambda = 0.5$, we get the temperature curve in the oven that meets the conditions as shown in the figure below, the index value at this time is 4.4933, and the heating factor is 593.3767.

![Figure 6. Temperature curve in the oven](image)

The temperature curve of each temperature interval under current conditions is:
Figure 7. Temperature curve of each temperature interval

The temperatures of temperature zone 1-5, temperature zone 6, temperature zone 7, and temperature zone 8-9 are respectively $172.64\,^\circ C$, $192.52\,^\circ C$, $229.21\,^\circ C$, $263.62\,^\circ C$. The speed of the conveyor belt passing the oven is $78.0181\,cm/min$.

4. Summary

In this paper, in response to CUMCM-2020 problem A, we established the heat transfer model of the PCB board and the temperature distribution model in the oven based on the analysis of the heat transfer mechanism. The model can be used for known heat convection coefficients, known welding material parameters, and at the same time Simulation of the temperature of the welding center in a hot-air reflow oven with no complicated internal changes. Using the theory of heat transfer and convection heating, combined with the actual physical process of reflow soldering equipment heating the PCB, it can well solve the problem of difficult analysis and simulation of the two boundary conditions of reflow soldering. In order to get accurate results, in this paper, we use the function optimization algorithm based on the genetic algorithm and nonlinear programming. At the same time, in practice, we can also use the model optimization control strategy to carry out the optimization control of lead-free reflow welding.

5. References

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