Super M-brane actions in \( \text{adS}_4 \times S^7 \) and \( \text{adS}_7 \times S^4 \)

Piet Claus

Instituut voor Theoretische Fysica, Katholieke Universiteit Leuven, Celestijnenlaan 200D, B-3001 Leuven, Belgium
piet.claus@fys.kuleuven.ac.be

Abstract

The world-volume action of the M2 brane and the M5 brane in an \( \text{adS}_4 \times S^7 \) and an \( \text{adS}_7 \times S^4 \) background is derived to all orders in anticommuting superspace coordinates \( \theta \). Contrary to recent constructions of super p-brane actions relying on supercoset methods, we only use 11 dimensional supergravity torsion and curvature constraints. Complete agreement between the two methods is found. The possible simplification of the action by choosing a suitable \( \kappa \)-gauge is discussed.
1. Introduction

These days a lot of research is concentrated on the connection between string-theory and M-theory on $adS_{p+2} \times S^{D-p-2}$ and extended superconformal theories in $p + 1$ dimensions [1, 2, 3]. $adS_5 \times S^5$ describes a maximally supersymmetric vacuum (besides flat space) of type IIB-supergravity, which is the near-horizon geometry of the D3-brane solution. For 11 dimensional supergravity we have $adS_4 \times S^7$ and $adS_7 \times S^4$ maximally supersymmetric vacua, which are the near-horizon limits of the M2 and M5 solutions resp.

The actions of super $p$-brane probes in these near-horizon backgrounds are described by a (modified) ‘Dirac’-type term (for $D$-$p$-branes, this is the Born-Infeld-type term) and a Wess-Zumino term. These actions are invariant under local diffeomorphisms of the world-volume and $\kappa$-symmetry and the rigid isometries of the background.

One way to realize the superconformal field theory in $p + 1$ dimensions is to gauge-fix these probe actions in their own near-horizon background [4], upon which the isometry group of the background is realized as a rigid superconformal symmetry on the world volume. The bosonic part of this project was carried out in [4].

Recently the complete super $p$-brane actions in $adS_{p+2} \times S^{D-p-2}$ background geometries have been constructed for the superstring [3, 4] and the D3 brane [10] in the $adS_5 \times S^5$ background solution to 10 dimensional type IIB supergravity and for the membrane [11, 12] in the $adS_4 \times S^7$ solution to 11 dimensional supergravity. One purpose of this paper is to construct the missing M5-brane action in $adS_7 \times S^4$, which is an important step in the further understanding of the relation between the M5 brane in $adS_7 \times S^4$ background and the nonlinear six dimensional superconformal $(0,2)$ tensor multiplet theory on its world volume [13, 1].

We will consider the near-horizon geometries of the two fundamental branes of M-theory. Firstly the $adS_4 \times S^7$ geometry of the M2-brane solution, which is given by the metric and the 3-form

$$
\begin{align*}
    ds^2 &= \left(\frac{r}{R}\right)^4 dx^m \eta_{mn} dx^n + \left(\frac{R}{r}\right)^2 dr^2 + R^2 d\Omega^2, \\
    A_3 &= dx^2 dx^1 dx^0 \left(\frac{R}{R}\right)^6.
\end{align*}
$$

Secondly we consider the $adS_7 \times S^4$ near-horizon geometry of the M5-brane solution to 11 dimensional supergravity, given by the metric and the 3-form

$$
\begin{align*}
    ds^2 &= \left(\frac{r}{R}\right) dx^m \eta_{mn} dx^n + \left(\frac{R}{r}\right)^2 dr^2 + R^2 d\Omega^2, \\
    A_3 &= -d\xi^3 d\xi^2 d\xi^1 (3R^3 \sin^3 \xi_1 \sin^2 \xi_2 \sin \xi_3 \xi_4).
\end{align*}
$$

The solution for $A_3$ can be cast in terms of its magnetic dual $A_6$ through the Bianchi identity $dA_6 = *dA_3 + \frac{1}{2} A_3 \wedge dA_3$. For this particular solution the second term vanishes

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1The 11 dimensional coordinates $X^M$ are split up into horospherical $adS$ coordinates $x^{\hat{m}} = \{x^m, r\}$ and $\xi^{\hat{m}}$, which are the coordinates on the sphere $S$. 

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and \(dA_6 = *dA_3\). We can choose \(A_6\) to be

\[
A_6 = dx^5 \ldots dx^0 \left( \frac{r}{R} \right)^3. \tag{1.3}
\]

The covariant field strengths are

\[
\begin{align*}
M2 & \quad F^4_{\bar{m}1 \ldots \bar{m}4} = \frac{6}{R} \epsilon_{\bar{m}1 \ldots \bar{m}4}, & F^7_{\bar{m}1' \ldots \bar{m}4'} = \frac{6}{R} \epsilon_{\bar{m}1' \ldots \bar{m}4'}, \\
M5 & \quad F^7_{\bar{m}1 \ldots \bar{m}7} = \frac{3}{R} \epsilon_{\bar{m}1 \ldots \bar{m}7}, & F^4_{\bar{m}1' \ldots \bar{m}4'} = \frac{3}{R} \epsilon_{\bar{m}1' \ldots \bar{m}4'}. \tag{1.4}
\end{align*}
\]

These forms define the lowest order \(\theta\) components of the super 4-form and 7-form in 11 dimensional superspace.

The rest of the paper is organized as follows. In section 2 we derive the vielbeine, connection 1-form and the 4- and 7-forms to all orders in \(\theta\) for a restricted class of 11 dimensional backgrounds. This class is characterized by the vanishing of the gravitino and covariantly constant forms. Contrary to previous work with cosets, we only rely on 11 dimensional supergravity torsion and curvature constraints to derive these results. We find complete agreement with results obtained from the supercoset approach \([12]\) and provide therefore a nice alternative description in supergravity superspace. Section 3 gives the actions for the M2 and M5 brane in these backgrounds and a detailed proof for the \(\kappa\)-invariance of the M5-brane action is given in section 4. In section 5 we discuss the killing-spinor gauge \([14]\) for \(\kappa\)-symmetry and the possible simplification of the action. We conclude in section 6 with a short discussion of the results.

2. A CLASS OF 11 DIMENSIONAL BACKGROUNDS

In this section we will derive the vielbeine \((E)\), spinconnection \((\Omega)\) and forms \((\mathcal{F}_n)\), collectively denoted as geometric superfields, for a certain class of 11 dimensional backgrounds. We will restrict the background to the case that the gravitino vanishes and the basic 11-dimensional superfield \(\mathcal{W}_{\bar{M}1 \ldots \bar{M}4}\) is covariantly constant, which restricts us essentially to the flat background (where \(\mathcal{W}\) is vanishing) the \(adS_4 \times S^7\) and \(adS_7 \times S^4\) background (where \(\mathcal{W}\) is given by (1.4)). It was shown that these are exact vacua of 11 dimensional supergravity \([10]\). The derivation of the geometric superfields in \(adS_4 \times S^7\) and \(adS_7 \times S^4\) has also been considered in \([12]\), by using coset representations. However, here we will only rely on supergravity torsion and curvature constraints.

The superspace is parametrized by the coordinates \(\bar{M}^A \equiv \{X^M, \theta^A\}\),

\[
Z^A = \{X^M, \theta^A\}, \tag{2.1}
\]
where $X^M (M = 0, \ldots, 10)$ are the bosonic coordinates of the 11 dimensional space and $\theta^A (A = 1, \ldots, 32)$ are 32 anticommuting coordinates. The vielbeine and connection satisfy ‘on shell’ torsion and curvature constraints [15]

\[
T^\bar{M} \equiv dE^\bar{M} - E^\bar{N} \Omega_{N}^{\bar{M}} = -\bar{E} \Gamma^{\bar{M}} E,
\]

\[
T^\bar{A} \equiv dE^{\bar{A}} - \frac{1}{4} (\Omega^{MN} \Gamma_{MN})^{\bar{A}} = E^\bar{M} (T_M^{\bar{N}_1 \ldots \bar{N}_4} F_{\bar{N}_4 \ldots \bar{N}_1} E)^{\bar{A}}, \tag{2.2}
\]

\[
R^{\bar{M} \bar{N}} \equiv d\Omega^{\bar{M} \bar{N}} - \Omega^{\bar{M} \bar{P}} \wedge \Omega^{\bar{P} \bar{N}} = \frac{1}{2} E^{\bar{Q}} E^{\bar{P}} R^{\bar{M} \bar{N}} \Gamma_{\bar{P} \bar{Q}} + \frac{1}{2} E S^{\bar{M} \bar{N} \bar{P}_1 \ldots \bar{P}_4} F_{\bar{P}_4 \ldots \bar{P}_1} E, \tag{2.3}
\]

where

\[
T_M^{\bar{N}_1 \ldots \bar{N}_4} = \frac{1}{288} \left( \Gamma_M^{\bar{N}_1 \ldots \bar{N}_4} - 8 \delta_M^{[N_1} \Gamma_{N_2 \ldots N_4]} \right),
\]

\[
S_{MN}^{\bar{P}_1 \ldots \bar{P}_4} = \frac{1}{72} \left( \Gamma_{MN}^{\bar{P}_1 \ldots \bar{P}_4} + 24 \delta_M^{[P_1} \delta_N^{P_2} \Gamma_{P_3 P_4]} \right) \tag{2.4}
\]

and $R^{\bar{M} \bar{N}}$ and $F$ are the bosonic curvature and 4-form of the background.

The super 4-form and 7-form are given by

\[
F_4 = \frac{1}{4!} E^{\bar{M}_1 \ldots \bar{M}_4} F_{\bar{M}_1 \ldots \bar{M}_4}^{\bar{A}_1 \ldots \bar{A}_4} + \frac{1}{2} E^{\bar{M}_1} E^{\bar{M}_2} E^{\bar{M}_3} E^{\bar{M}_4} E = dA_3
\]

\[
F_7 = \frac{1}{7!} E^{\bar{M}_1 \ldots \bar{M}_7} F_{\bar{M}_1 \ldots \bar{M}_7}^{\bar{A}_1 \ldots \bar{A}_7} + \frac{1}{5!} E^{\bar{M}_1} \ldots E^{\bar{M}_5} E^{\bar{M}_6} E^{\bar{M}_7} E = dA_6 - \frac{1}{2} A_3 \wedge A_4 \tag{2.5}
\]

The consistency conditions (Bianchi-identities) for the definitions of $A_3$ and $A_6$

\[
0 = dF_4,
\]

\[
0 = 2dF_7 + F_4 \wedge F_4, \tag{2.6}
\]

can be checked explicitly using equations (2.2) and the 11 dimensional Fierz identities [18]. These conditions also determine the relative factors in and between $F_4$ and $F_7$.

Using coset-space techniques (see e.g. [19]), various backgrounds for super $p$-brane actions have been constructed to all orders in $\theta$, especially the $\text{adS}_4 \times S^7$ and $\text{adS}_7 \times S^4$ near-horizon backgrounds in [12]. It was established that a very powerful trick to derive the vielbeine and spinconnection 1-form was to rescale the $\theta$’s, with a parameter that is put to unity in the end (see e.g. [3, 6]). However, this technique is not a privilege of coset representations and the same results can be obtained using supergravity constraints only.

Consider the transformation

\[
X^M \rightarrow X^\prime M, \quad \theta^A \rightarrow t\theta^A. \tag{2.7}
\]
Taking the derivative with respect to $t$ of $E$ and $\Omega$ leads to the coupled first-order equations in $t$

\[
\begin{align*}
\frac{d}{dt} E^\hat{A} & = d(\theta^A E^\hat{A}) - \theta^A E^\hat{A} \bar{\Omega} + E^\hat{A} \theta^A \Omega A : \Sigma \bar{\Lambda} - \theta^A E^\hat{A} \Sigma \bar{\Omega} \bar{\Lambda}, \\
\frac{d}{dt} \Omega^{\hat{M}\hat{N}} & = d(\theta^A \Omega^{\hat{M}\hat{N}}) - \theta^A E^\hat{A} \bar{\Omega} \bar{R} \bar{N} + \Omega^{\hat{M}\bar{P}} \theta^A \Omega A : \bar{P} \bar{N} - \theta^A \Omega A : \bar{P} \bar{N} \Sigma \bar{\Lambda} \Sigma \bar{\Omega} \bar{\Lambda}.
\end{align*}
\]  

(2.8)

To solve these equations we make the assumption

\[
\theta^A E^\hat{A} = \theta^A \Omega^{\hat{M}\hat{N}} = 0
\]  

(2.9)

and define

\[
\Theta^\hat{f} = \theta^A E^\hat{A}.
\]  

(2.10)

Thus (2.8) becomes

\[
\begin{align*}
\frac{d}{dt} E & = (d + \frac{1}{4} \Omega^{\hat{M}\hat{N}} \Gamma_{MN} + E^{\hat{M}} T^{\hat{N}_1...\hat{N}_4} \bar{F}_{\bar{N}_4...\bar{N}_1}) \Theta^\hat{f}, \\
\frac{d}{dt} E^{\hat{M}} & = 2 \Theta^\hat{f} \Gamma^{\hat{M}} E, \\
\frac{d}{dt} \Omega^{\hat{M}\hat{N}} & = -\Theta^\hat{f} S^{\hat{M}\hat{N} \bar{P}_1...\bar{P}_4} F_{\bar{P}_4...\bar{P}_1} E.
\end{align*}
\]  

(2.11)

These equations agree completely with the ones derived using coset-space techniques \[12\] and therefore the two approaches are completely equivalent to all orders in $\theta$.

Following \[12\], (2.11) can be solved straightforwardly, by taking multiple derivatives w.r.t. $t$ and considering the initial conditions \[12\]

\[
\begin{align*}
E|_{t=0} & = 0, \\
E^{\hat{M}}|_{t=0} & = e^M = dX^M e_M^{\hat{M}}(X), \\
\Omega^{\hat{M}\hat{N}}|_{t=0} & = \omega^{\hat{M}\hat{N}} = dX^M \omega_M^{\hat{M}\hat{N}}(X),
\end{align*}
\]  

(2.12)

where $e_M^{\hat{M}}$ and $\omega_M^{\hat{M}\hat{N}}$ are the vielbein and spinconnection components of the bosonic background.

The explicit solution reads \[12\]

\[
\begin{align*}
E & = \sum_{n=0}^{16} \frac{1}{(2n+1)!} \mathcal{M}^n D \theta^f, \\
E^{\hat{M}} & = dX^M e_M^{\hat{M}} + 2 \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta}^f \bar{M} \mathcal{M}^n D \theta^f, \\
\Omega^{\hat{M}\hat{N}} & = dX^M \omega_M^{\hat{M}\hat{N}} - \sum_{n=0}^{15} \frac{1}{(2n+2)!} \bar{\theta}^f S^{\hat{M}\hat{N} \bar{P}_1...\bar{P}_4} F_{\bar{P}_4...\bar{P}_1} \mathcal{M}^n D \theta^f.
\end{align*}
\]  

(2.13)

\[4\] On $E^\hat{A}$ we will drop the index most of the time.
where

\[(\mathcal{M})_{A}^{\bar{B}} = 2(T_{\bar{M}}^{\bar{N}_1...\bar{N}_4} F_{\bar{N}_4...\bar{N}_1} \theta_f)_{A}(\bar{\theta}_f \Gamma^M)^{\bar{B}} \]

\[-\frac{1}{4}(\Gamma_{MN} \theta_f)_{\bar{A}}(\bar{\theta}_f S^{MNP_1...P_4} F_{P_4...P_4})^{\bar{B}}, \tag{2.14}\]

\[\theta_f^{\bar{A}} = \theta_f^{A} E^{\bar{A}}_{\bar{B}}|_{t=0} \equiv \theta_f^{A} e_{\bar{A}}^{\bar{A}}(X) \tag{2.15}\]

and

\[D\theta_f = (d + \frac{1}{4} \omega \cdot \Gamma + e^{\bar{M}T_{\bar{M}} \bar{P}_1...\bar{P}_4} F_{\bar{P}_4...\bar{P}_4}) \theta_f. \tag{2.16}\]

It is straightforward to verify that (2.9) is fulfilled, using symmetry properties of 11 dimensional $\Gamma$ matrices, and therefore the derivation is consistent.

As a side remark we note that part of (2.16) is the killing spinor equation

\[0 = \delta \psi_M = (\partial_M + \frac{1}{4} \omega_M \cdot \Gamma + e_M^{\bar{M}T_{\bar{M}} \bar{P}_1...\bar{P}_4} F_{\bar{P}_4...\bar{P}_4}) \epsilon. \tag{2.17}\]

The solution to these equations can be written as

\[\epsilon_{\text{kill}}^{\bar{A}}(X) = e_0^{A} K_{\bar{A}}^{\bar{A}}(X), \tag{2.18}\]

with $e_0^{A}$ a constant 11 dimensional spinor. Therefore we can simplify the expressions for the vielbeine and connection 1-form by taking [6]

\[e_{\bar{A}}^{\bar{A}} = K_{\bar{A}}^{\bar{A}} \tag{2.19}\]

and it follows that

\[D\theta_f^{A} \rightarrow (d\theta_f^{A})K_{\bar{A}}^{\bar{A}}. \tag{2.20}\]

This gauge was for obvious reasons called the killing-spinor gauge and should be thought of as an alternative for the Wess-Zumino gauge $e_{\bar{A}}^{\bar{A}} = \delta_{\bar{A}}^{\bar{A}}$. In fact, for flat space it reduces to the Wess-Zumino gauge, because the flat-space killing spinors are constant spinors. In this killing-spinor gauge we have that

\[E^{M}_{\bar{M}}(Z) = e^{M}_{\bar{M}}(X), \quad E^{\bar{A}}_{\bar{A}}(Z) = 0, \tag{2.21}\]

which means that there are no higher order $\theta$ corrections to the bosonic solution for the vielbein $e^{M}_{\bar{M}}(X)$ and the gravitino $\Psi_{M}(X)$.

We can also obtain expressions for $A_3$ and $A_6$ to all orders in $\theta$, by considering $t$-derivatives of the forms. Using (2.11) and Fierz-identities it is non-trivial to derive that

\[\frac{d}{dt} \mathcal{F}_4 = d(E^{M_1} E^{M_2} \bar{E} \Gamma_{M_2 M_1} \Theta_f), \tag{2.22}\]

which we can easily integrate to find

\[A_3 = \frac{1}{3!} e^{M_1} e^{M_2} e^{M_3} A_{M_1 M_2 M_3} + \int_0^1 dt (E^{M_1} E^{M_2} \bar{E} \Gamma_{M_2 M_1} \Theta_f). \tag{2.23}\]

In the same way, one finds that

\[\mathcal{F}_7 = \frac{2}{5!} \left( E^{M_1} \ldots E^{M_5} \bar{E} \Gamma_{M_5 \ldots M_1} \Theta_f \right) - E^{M_1} E^{M_2} \bar{E} \Gamma_{M_2 M_1} \Theta_f \wedge \mathcal{F}_4, \tag{2.24}\]
which can be integrated to
\[ A_6 = \frac{1}{6!} e^{M_1 \ldots M_6} A_{\tilde{M}_a \ldots \tilde{M}_1} + \int_0^1 dt \left( \frac{2}{5!} E^{M_1 \ldots M_5} \tilde{E} \Gamma_{M_5 \ldots M_1} \Theta_f + \frac{1}{2} A_3 \wedge E^{M_1} E^{M_2} \tilde{E} \Gamma_{M_2 M_1} \Theta_f \right), \] (2.25)
where under the \( t \)-integrals we mean the rescaled vielbeine and 3-form.

In flat space the matrix \( \mathcal{M} \) vanishes because the field strengths vanish and therefore in the Wess-Zumino gauge the vielbeine are given by
\[ E = d\theta, \quad E^M = dX^M + \bar{\theta} \Gamma^M d\theta. \] (2.26)

It is reassuring to see that using these results in the equations for \( A_3 \) and \( A_6 \), the flat forms (see e.g. [13]) are obtained [12].

3. The M-brane actions

Super \( p \)-brane actions consist in general of two parts
\[ I = I_{\text{kin}} + I_{\text{WZ}}. \] (3.1)
\( I_{\text{kin}} \) contains the induced metric
\[ g_{\mu\nu} = E^M_{\mu} \eta_{MN} E^N_{\nu}; \quad g \equiv -\det g_{\mu\nu}, \] (3.2)
and is an integral over the world volume (with coordinates \( \sigma^\mu \)). The superspace coordinates \( Z^\Lambda = Z^\Lambda(x) \) are taken to be fields on the world volume and by \( E^M_{\mu} \) we mean the pull back to the world volume of \( E^M \). The Wess-Zumino component is an integral of a closed \( p + 2 \)-form over a \( p + 2 \) dimensional manifold that has the world volume as its boundary.

To be more specific for M2 we have (the two signs correspond to branes and anti-branes)
\[ I_{\text{kin}} = -\int_{M_3} d^3 \sigma \sqrt{g}, \quad I_{\text{WZ}} = \mp \int_{\mathcal{M}_4} \mathcal{F}_4, \] (3.3)
where \( \mathcal{F}_4 \) is the closed background super 4-form.

This action is invariant under local diffeomorphisms and \( \kappa \)-symmetry and by construction under the rigid isometries of the background, especially \( OSp(8|4) \) for \( \text{adS}_4 \times S^7 \). After gauge-fixing the local symmetries of this action we will end up with a superconformal field theory in 3 dimensions as \( OSp(8|4) \) is the superconformal group in 3 dimensions.

The M5 world-volume action is somewhat more involved. We will use the covariant formulation of [20], following [13], where the derivation of the M5-brane action in flat 11 dimensional superspace was given in detail. The covariance is achieved by introducing an extra real scalar degree of freedom, which is pure gauge for some additional gauge transformation. Besides the superspace coordinates, we also introduce
an antisymmetric tensor $B_{\mu\nu}(\sigma)$ and a real scalar $a(\sigma)$, which live only on the world volume. The two components of the action are

$$I_{kin} = \int_{\mathcal{M}_6} d^6\sigma \left( -\sqrt{-\det(g_{\mu\nu} + i\mathcal{H}^*_\mu\nu)} \mp \frac{\sqrt{g}}{4} \mathcal{H}^{\mu\nu} \mathcal{H}_{\mu\nu} \right)$$

$$\equiv -\int_{\mathcal{M}_6} d^6\sigma \left( \mathcal{G} \pm \frac{\sqrt{g}}{4} \mathcal{H}^{\mu\nu} \mathcal{H}_{\mu\nu} \right)$$  \hspace{1cm} (3.4)$$

and

$$I_{WZ} = \mp \int_{\mathcal{M}_7} I_7.$$  \hspace{1cm} (3.5)$$

We defined the following fields\footnote{We took some different definitions w.r.t.\ [13].}

$$H = dB, \quad \mathcal{H} = H + A_3,$$

$$u_\mu = \partial_\mu a, \quad u^2 = u_\mu g^{\mu\nu} u_\nu, \quad v_\mu = \frac{u_\mu}{\sqrt{u^2}},$$

$$\mathcal{H}_{\mu\nu} = v^\rho \mathcal{H}_{\mu\nu\rho}, \quad \mathcal{H}^*_{\mu\nu} = v^\rho \mathcal{H}^*_{\mu\nu\rho},$$

$$\mathcal{H}^*_{\mu\nu\rho} = \frac{\sqrt{g}}{6} \epsilon_{\mu\nu\rho\sigma\tau\phi} \mathcal{H}^{\sigma\tau\phi},$$

$$I_7 = \mathcal{F}_7 + \frac{1}{2} \mathcal{H} \wedge \mathcal{F}_4.$$  \hspace{1cm} (3.6)$$

In all these expressions it is understood that the forms have been pulled back to the world volume and indices $\mu, \nu$ are raised and lowered with the induced metric $g_{\mu\nu}$. By construction $I_7$ is a total derivative

$$I_7 = d \left( A_6 + \frac{1}{2} B \wedge \mathcal{F}_4 \right) = d \left( A_6 + \frac{1}{2} H \wedge A_3 \right)$$  \hspace{1cm} (3.7)$$

and its pull-back can be integrated over a 7 dimensional space $\mathcal{M}_7$ with the world volume as its boundary.

The gauge invariances of the M5-brane action are diffeomorphisms of the world volume, $\kappa$-symmetry (discussed in more detail in the next section) and the tensor gauge symmetry and PST-gauge symmetries\footnote{The normalization of the Wess-Zumino term can be fixed by requiring invariance under III\ [13].} I, II and III, which only act on $B_{\mu\nu}(x)$ and $a(x)$

(I) \hspace{.5cm} $\delta_I B_{\mu\nu} = 2 \partial_{[\mu} \Lambda_{\nu]}$, \hspace{.5cm} $\delta_I a = 0$; 

(II) \hspace{.5cm} $\delta_{II} B_{\mu\nu} = \frac{1}{\sqrt{u^2}} \mathcal{G} \left( \mathcal{H}_{\mu\nu} \pm 2 \frac{\delta \mathcal{G}}{\delta \mathcal{H}^*_{\mu\nu}} \right)$, \hspace{.5cm} $\delta_{II} a = \mathcal{G}$; 

(III) \hspace{.5cm} $\delta_{III} B_{\mu\nu} = \psi_{[\mu} v_{\nu]}$, \hspace{.5cm} $\delta_{III} a = 0$.  \hspace{1cm} (3.8)$$

By construction, the action is invariant under the rigid superisometries of the 11 dimensional background, which is $OSp(6,2|4)$ for $adS_7 \times S^4$ and super-Poincaré transformations in $(1,9)$ dimensions for flat space. Because $OSp(6,2|4)$ is the superconformal group in 6 dimensions, after gauge fixing local symmetries, the complete
non-linear interacting world-volume action will be superconformally invariant. For the flat case it was established in [13] that only the linearized world-volume theory is superconformally invariant.

4. $\kappa$ symmetry of the M5-brane action

In this section we will prove that the M5-brane action constructed in the previous section is $\kappa$-symmetric. The $\kappa$-symmetry has been proven in flat space both in the non-covariant form [21] and in covariant form [20] (see also [13] for a detailed proof). Unlike for $D$-p branes [22] and membranes [17], no complete detailed proof for the $\kappa$-invariance of the M5 brane in a generic 11 dimensional supergravity background, has been given. In this section we will provide such a proof for a background with vanishing gravitino and covariantly constant forms, based on exactly the same strategy as in the flat background (see [13], appendix C).

Given the variations $\delta Z^A$ of the world-volume fields $Z^A = \{X^M, \theta^A\}$, we define

$$\delta \hat{E}^\Lambda = \delta Z^A E^\Lambda_A, \quad \delta \hat{\Omega}^\Sigma = \delta Z^A \Omega_{\Lambda \Lambda}^\Lambda, \quad (4.1)$$

which are 0-forms. A universal feature of $\kappa$-symmetry is

$$\delta \kappa E^M = 0, \quad (4.2)$$

which allows one to express $\delta \kappa X^M$ in terms of $\delta \kappa \theta^A$. It follows that

$$\delta E^\Lambda = d(\delta \hat{E}^\Lambda) - \delta \hat{E}^\Lambda E^\Sigma \hat{c}_{\Sigma}^\Lambda + E^\Sigma \delta \hat{\Omega}^\Sigma - \delta \hat{E}^\Sigma \Omega^\Sigma_{\Lambda}^\Lambda, \quad (4.3)$$

which using (2.2) and (4.2) reduces to

$$\delta E^M = -2 \hat{E}^M \delta \hat{E} + \text{Lorentz},$$

$$\delta E^A = d(\delta \hat{E}^A) + E^M (T^M_{\hat{N}_1 \hat{N}_2 \hat{N}_3 \hat{N}_4} \hat{F}_{\hat{N}_4 \hat{N}_3 \hat{N}_2 \hat{N}_1} \delta \hat{E})^\Lambda + \text{Lorentz} \quad (4.4)$$

Also we define the pull-backs of the $\Gamma$-matrices

$$\gamma_\mu = \Gamma_M E^M_\mu; \quad \gamma_{\mu \nu} = \gamma_{[\mu} \gamma_{\nu]}, \ldots. \quad (4.5)$$

To demonstrate the $\kappa$-invariance of the action we begin by considering the variation of the different terms in the action. Since all building blocks of the action are manifestly Lorentz invariant we don’t have to care about the last two terms in (4.3). $^7$

$$\delta \kappa g_{\mu \nu} = 4 \delta \kappa E \gamma_{(\mu} E_{\nu)}; \quad \delta \kappa \sqrt{g} = 2 \sqrt{g} \delta \kappa E \gamma^\mu E_\mu. \quad (4.6)$$

For the 4 and 7 forms, $F_4$ and $F_7$, we find using (1.3) and (2.2) again, that

$$\delta \kappa F_4 = d(E^{M_1} E^{M_2} \hat{E} \Gamma_{M_2 M_1} \delta \kappa E). \quad (4.7)$$

$^7$From a computational point of view this derivation is essentially the same as in section 2.

$^8$In order not to confuse here with the variation of the 1-forms $\delta(E^A)$, we use hats when we mean the 0-form.

$^9$In the following expressions we drop the hats again and all $\delta E^\Lambda$ are defined as in (4.1).
As \( d \) and \( \delta_\kappa \) commute, this means
\[
\delta_\kappa A_3 = E^\bar{M}_1 E^\bar{M}_2 \bar{E} \Gamma_{\bar{M}_2 M_1} \delta_\kappa E + d A_2
\]
(4.8)
and therefore we can ‘define’ the \( \kappa \)-transformation of \( B \) to be
\[
\delta_\kappa B = -A_2, \tag{4.9}
\]
where the pull-back to the world volume of \( A_2 \) is understood. The explicit form of \( A_2 \) will of course depend on the choice of \( A_3 \), but an explicit derivation of \( A_2 \) is beyond the scope of this paper. It follows that
\[
\delta_\kappa H = E^\bar{M}_1 E^\bar{M}_2 \bar{E} \Gamma_{\bar{M}_2 M_1} \delta_\kappa E. \tag{4.10}
\]
Again using (2.2) and (4.3) one can derive that
\[
\delta_\kappa F_7 = 2 \frac{5!}{\sqrt{g}} d \left( E^{\bar{M}_1} \ldots E^{\bar{M}_5} \bar{E} \Gamma_{\bar{M}_5 \ldots M_1} \delta_\kappa E \right) - E^{\bar{M}_1} E^{\bar{M}_2} \bar{E} \Gamma_{\bar{M}_2 M_1} \delta_\kappa E \wedge F_4 \tag{4.11}
\]
and therefore
\[
\delta_\kappa I_7 = d \left( \frac{2}{5!} E^{\bar{M}_1} \ldots E^{\bar{M}_5} \bar{E} \Gamma_{\bar{M}_5 \ldots M_1} \delta_\kappa E + \frac{1}{2} H \wedge E^{\bar{M}_1} E^{\bar{M}_2} \bar{E} \Gamma_{\bar{M}_2 M_1} \delta_\kappa E \right). \tag{4.12}
\]
For the PST scalar \( a \) we make the ansatz that
\[
\delta_\kappa a = 0.
\]
Having established the \( \kappa \)-symmetry of the basic components in the action, the rest of the proof is exactly the same as in flat space (appendix C of [13]). We will repeat some of the details here to make this proof self-contained.

\[
\delta_\kappa \sqrt{u^2} = -\frac{1}{2} \sqrt{u^2} v^\mu v^\nu \delta_\kappa g_{\mu\nu}, \quad \delta_\kappa v_\mu = \frac{1}{2} v^\nu v^\rho \delta_\kappa g_{\nu\rho},
\]
\[
\delta_\kappa \bar{H}^{\mu\nu} = \frac{1}{\sqrt{g}} e^{\mu
u\rho\sigma\tau\phi} v_\mu \bar{E}_\sigma \gamma_{\tau\phi} \delta_\kappa E - \bar{H}^{\mu\nu} \left( 2 \bar{E}_\rho \gamma^\rho \delta_\kappa E + \frac{1}{2} v^\rho v^\sigma \delta_\kappa g_{\rho\sigma} \right),
\]
\[
\delta_\kappa \bar{H}_{\mu\nu} = (\delta_\kappa \bar{H}_{\mu\nu}) v^\rho - \bar{H}_{\mu\nu} \frac{\delta_\kappa \sqrt{u^2}}{\sqrt{u^2}} + \bar{H}_{\mu\rho\nu} v_\sigma \delta_\kappa g^{\rho\sigma}. \tag{4.13}
\]
Now we define
\[
\delta_\kappa \left( \mp \int d^6 x \frac{\sqrt{g}}{4} \bar{H}^{\mu\nu} \bar{H}_{\mu\nu} + I_{WZ} \right) = \mp \frac{\sqrt{g}}{2} \bar{E}_\mu T^\mu \delta_\kappa E, \quad \delta_\kappa \bar{G} = -\frac{g}{2 \bar{g}} \bar{E}_\mu U^\mu \delta_\kappa E \tag{4.14}
\]
and
\[
\tilde{\gamma} = \frac{1}{6! \sqrt{g}} e^{\mu_1 \ldots \mu_6} \gamma_{\mu_1 \ldots \mu_6} \Rightarrow \tilde{\gamma}^2 = 1. \tag{4.15}
\]
The tensors \( T^\mu \) and \( U^\mu \) are exactly the same as in the flat case and their explicit expressions can be found in equations (C.6) and (C.9) of [13]. It was established that the \( T^\mu \) and \( U^\mu \) are related by
\[
U^\mu = T^\mu \rho \quad \text{where} \quad \rho = \tilde{\gamma} + \frac{1}{2} \bar{H}^{\mu\nu} v_\rho \gamma_{\mu\nu} - \frac{1}{16 \sqrt{g}} e^{\mu\nu\rho\sigma\tau\phi} \bar{H}_{\mu\nu} \bar{H}_{\rho\sigma} \gamma_{\tau\phi} \tag{4.16}
\]
and

\[ \rho^2 = \frac{G^2}{g}. \]  

(4.17)

Defining

\[ \Gamma^{(M5)} = \frac{\sqrt{g}}{G} \rho, \]  

(4.18)

the action is \( \kappa \)-invariant if

\[ \delta \kappa E = (1 \pm \Gamma^{(M5)}) \kappa(\sigma), \quad \delta \kappa E^M = 0, \]  

(4.19)

with \( \kappa \) a generic 11 dimensional \( \sigma \)-dependent spinor. We repeat the expression for \( \Gamma^{(M5)} \)

\[ \Gamma^{(M5)} = \frac{1}{G} \left( \sqrt{g} \gamma + \frac{2}{3} H^*_{\mu\nu} \gamma^{\mu\nu\rho} - \frac{1}{16} \varepsilon^{\mu\nu\rho\sigma\tau\phi} H^*_{\mu\nu} H^*_{\rho\sigma} \gamma_{\tau\phi} \right). \]  

(4.20)

It satisfies \( \Gamma^2 = 1 \) and \( \text{tr} \Gamma = 0 \). Therefore we have established the \( \kappa \)-invariance of the M5-brane action in a 11 dimensional background with vanishing gravitino and covariantly constant forms.

For completeness and future reference we also give the \( \kappa \)-transformation in the M2 case [17]. This action is invariant under

\[ \delta \kappa E^A = (1 \pm \Gamma^{(M2)}) \kappa(\sigma), \quad \delta \kappa E^M = 0, \]  

(4.21)

where

\[ \Gamma^{(M2)} = \frac{\varepsilon^{\mu\nu\rho}}{3! \sqrt{g}} \gamma_{\mu\nu\rho}. \]  

(4.22)

5. **Simplifying the action by \( \kappa \)-gaugefixing**

In the previous section we derived the action to all orders in \( \theta \). For the near-horizon backgrounds, the vielbeine contain terms up to order 32 in \( \theta \) and the explicit expression for the action is therefore very complicated. As discussed in [14], we expect that a suitable gauge-fixing of the \( \kappa \)-symmetry related to the killing spinors will simplify the action dramatically. It is the purpose of this section to show by explicit computation that this is indeed the case.

First of all we need an expression for the matrix \( \mathcal{K} \) in (2.18). This can be obtained by explicitly solving the killing-spinor equations. For \( adS_4 \times S^7 \) (M2 source), using the coordinates defined in (1.1), the killing-spinor equations take the form

\[
\begin{align*}
0 &= \delta \psi_m = \partial_m \epsilon + \left( \frac{r}{R} \right)^2 \frac{1}{R} \Gamma_m \Gamma_r (1 - \Gamma^{012}) \epsilon, \\
0 &= \delta \psi_r = \partial_r \epsilon - \frac{1}{r} \Gamma^{012} \epsilon, \\
0 &= \delta \psi_{m'} = \bar{\nabla}_{m'} \epsilon - \frac{1}{2} \Gamma_r \epsilon_{m'} \Gamma_m \Gamma^{012} \epsilon,
\end{align*}
\]  

(5.1)
where $\tilde{\nabla}_{m'}$ is the covariant derivative and $\tilde{e}_{m'}\bar{m}'$ the vierbein on the unit 7-sphere. The first equation suggests to introduce projections of a generic 11 dimensional Majorana spinor $\lambda$

$$\lambda_{\pm} = \mathcal{P}_{\pm}^{(M2)} \lambda = \frac{1}{2} (1 \pm \Gamma^{012}) \lambda.$$

(5.2)

Acting with this projector we get 6 killing equations

\[ 0 = \partial_m \epsilon_- , \]
\[ 0 = \partial_m \epsilon_+ + 2 \left( \frac{r}{R} \right)^2 \frac{1}{R} \Gamma_{mr} \epsilon_- , \]
\[ 0 = \partial_r \epsilon_+ \mp \frac{1}{r} \epsilon_+ , \]
\[ 0 = \tilde{\nabla}_{m'} \epsilon_+ \pm \frac{1}{2} \Gamma_{r} \tilde{e}_{m'} \bar{m}' \Gamma_{m'} \epsilon_+ . \]

(5.3)

It is straightforward to solve these equations and the explicit solution reads

$$\epsilon_- = \left( \frac{R}{r} \right) \frac{1}{4} f^{-}_{(M2)} (\xi) \epsilon^0_- ,$$

$$\epsilon_+ = \left( \frac{r}{R} \right) \frac{1}{4} f^{+}_{(M2)} (\xi) \left[ \epsilon^0_+ - 2 \frac{x^m}{R} \Gamma_{mr} \epsilon^0_- \right] ,$$

(5.4)

where

$$f^{\pm}_{(M2)} (\xi) = \left( \cos \frac{\xi_1}{2} \mp \Gamma_r \xi_1 \sin \frac{\xi_1}{2} \right) \prod_{k=2}^{7} \left[ \cos \frac{\xi_k}{2} + \Gamma_{\xi_k-1 \xi_k} \sin \frac{\xi_k}{2} \right] ,$$

(5.5)

and $\epsilon^0_{\pm}$ are the two projections of a constant majorana spinor, which make 32 killing spinors, and provides us with the matrix $\mathcal{K}$ defined in (2.18).

In the same way one establishes for $adS_7 \times S^4$ (M5 source) that the killing spinors can be written as

$$\epsilon_- = \left( \frac{R}{r} \right)^{1/4} f^{-}_{(M5)} (\xi) \epsilon^0_- ,$$

$$\epsilon_+ = \left( \frac{r}{R} \right)^{1/4} f^{+}_{(M5)} (\xi) \left[ \epsilon^0_+ - \frac{1}{2} \frac{x^m}{R} \Gamma_{mr} \epsilon^0_- \right] ,$$

(5.6)

where

$$f^{\pm}_{(M5)} (\xi) = \left( \cos \frac{\xi_1}{2} \mp \Gamma_r \xi_1 \sin \frac{\xi_1}{2} \right) \prod_{k=2}^{4} \left[ \cos \frac{\xi_k}{2} + \Gamma_{\xi_k-1 \xi_k} \sin \frac{\xi_k}{2} \right] .$$

(5.7)

Now defines the projections for M5.

\[ f^{012} \] has the properties $[\Gamma^{012}, \Gamma^m] = 0$, $\{\Gamma^{012}, \Gamma^r\} = 0$ and $\{\Gamma^{012}, \Gamma^m'\} = 0$. Also \( (\Gamma^{012})^T = C^{-1} \Gamma^{012} C \), with $C$ the 11 dimensional charge-conjugation matrix. This means that $\bar{\lambda}_{\pm} = \pm \bar{\lambda}_{\pm} \Gamma^{012}$ with $\bar{\lambda} = \lambda^C$.

\[ f^{012345} \] satisfies $\{\Gamma^{012345}, \Gamma^m\} = 0$, $\{\Gamma^{012345}, \Gamma^r\} = 0$ and $\{\Gamma^{012345}, \Gamma^m'\} = 0$. Also \( (\Gamma^{012345})^T = -C^{-1} \Gamma^{012345} C \). This means that $\bar{\epsilon}_{\pm} = \mp \bar{\epsilon}_{\pm} \Gamma^{012345}$.

\[ f^{012345} \]
Next we will consider the following gauges for $\kappa$-symmetry

\begin{align}
(i) \quad & \mathcal{P}_- \theta_f = \theta_{f-} = 0, \\
(ii) \quad & \mathcal{P}_+ \theta_f = \theta_{f+} = 0,
\end{align}

where the appropriate projector (5.2) or (5.8) has to be taken for the near-horizon geometry one considers.

Firstly take case (i), which is the situation considered in [14]. For the $adS_4 \times S^7$-supergeometry in the killing-spinor gauge (2.19) we have

\begin{equation}
(D\theta_f)|_{\theta_f-=0} = \left( \frac{r}{R} \right)^{1/2w} f^+ d\theta^+_+ ,
\end{equation}

introducing $w = \frac{1}{2}$ and $w = 2$ for M2 and M5 resp. [4], and it follows that $M D\theta_f$ becomes zero in this gauge. Indeed we compute

\begin{equation}
(M D\theta_f)|_{\theta_f+=0} = -\frac{1}{2} \Gamma_{mn} \Gamma^{012} \theta_{f+} + \bar{\theta}_{f+} \Gamma^{m}(Kd\theta_f)+F_{012f} + \frac{1}{2} \Gamma_{mn} \theta_{f+} \bar{\theta}_{f+} \Gamma^{012}(Kd\theta_f)+F_{012f} ,
\end{equation}

which clearly vanishes. Also for the $adS_7 \times S^4$-supergeometry explicit computation shows that this is the case in this gauge. Therefore the vielbeine and forms simplify dramatically. Defining $\lambda = \theta_+$, we find in both cases that

\begin{align}
E^\hat{m} &= (dx^m + \bar{\lambda}^{m} d\lambda) e_{\hat{m}}^m , \\
E^\bar{p} &= d\bar{e}_{\bar{p}}^r , \\
E^{\hat{m}'} &= d\xi^{m'} e_{\hat{m}'}^{m'} , \\
E &= (-g_{00})^{\frac{1}{2}} f^+ (\xi) d\lambda ,
\end{align}

where $e_{\hat{m}}^m$ and $e_{\hat{m}'}^{m'}$ are the vielbeine of the $adS$-space and the sphere resp. $g_{00}$ is the 00-component of the metric. The expressions for the vielbeine are as simple as for the flat background (2.26). The forms can then be found by plugging in these vielbeine in (2.23)-(2.25) and the action formulas follow.

Secondly we can consider case (ii). We now apply the killing-spinor gauge (2.19) combined with $\theta_{f+} = 0$. It follows that

\begin{equation}
(D\theta_f)|_{\theta_f+=0} = \left( \frac{R}{r} \right)^{1/2w} f^-(\xi) d\theta^- + dx^m \left( \frac{r}{R} \right)^{1/w} \frac{1}{wR} \Gamma_{mr} \theta_{f-} .
\end{equation}

For both the M2 and M5 near-horizon geometries it follows that

\begin{equation}
(M D\theta_f)|_{\theta_f+=0} = M \left( \frac{r}{R} \right)^{1/w} \frac{1}{wR} \Gamma_{mr} \theta_{f-} , \quad (M^2 D\theta_f)|_{\theta_f+=0} = 0 ,
\end{equation}

by explicit computation. In this gauge the vielbeine contain terms up to order $(\theta^-)^4$. Whether these gauges are admissible, i.e. if they are compatible with the reparametrization gauge and the classical solution to the brane-wave equations one wishes to take,
has to be considered for each case at hand and goes beyond the scope of this paper.

6. Discussion

In this paper the super M-brane actions were derived to all orders in anticommuting variables $\theta$ for specific backgrounds. The main new result is the complete derivation of the M5-brane action in its near-horizon background. It was shown that the actions can be determined completely relying only on supergravity torsion and curvature constraints, in contrast to other constructions based on coset techniques. This has the advantage that one can consider different 11 dimensional backgrounds at once [12]. Also we found complete agreement with the coset construction in [12], where agreement with gauge completion results to lowest order in $\theta$ was found. However, the covariant torsion and curvature constraints can already be derived from the first order in $\theta$ gauge completion [13] and therefore as this paper only relies on these constraints, there is full agreement to all orders in $\theta$. The two approaches are therefore completely equivalent. In this paper, we restricted ourselves to backgrounds with vanishing gravitino and covariantly constant field strengths, but a study of more general backgrounds along the same lines would be interesting.

Following [14] we considered the gauge fixing of probe M-brane actions in $adS_4 \times S^7$ and $adS_7 \times S^4$. Two $\kappa$-gauges were proposed related to the killing spinors of the background and the simplification of the geometric superfields was discussed for both cases.

By construction the gauge-fixed actions will be invariant under superconformal transformations because $OSp(4|8)$ transformations, which are the isometries of the $adS_4 \times S^7$ M2 near-horizon background, and $OSp(6,2|4)$ transformations, which are the isometries of the $adS_7 \times S^4$ M5 near-horizon background, become upon gauge fixing non-linearly realized superconformal transformations on the remaining worldvolume fields. For the M2 we have a superconformal scalar multiplet in 3 dimensions and for M5 a superconformal $(0,2)$ tensor multiplet in 6 dimensions. The precise form of the transformation rules, which have been written down for the (bosonic) conformally invariant actions in [4], still have to be derived.

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