The effects of tool eccentricity on individual P and S head waves in monopole acoustic LWD

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Abstract
Velocities of P and S waves are main goals of downhole acoustic logging. In this work, we study the effects of an off-center acoustic tool on formation P and S head waves in monopole logging while drilling (LWD), which will be helpful for accurate interpretation of recorded logs. We first develop an analytic method to solve the wavefields of this asymmetric LWD model. Then using a branch-cut integration technique, we evaluate the contributions of branch points associated with P and S waves, and further investigate the effects of tool eccentricity on their characteristics of excitation, attenuation and waveforms. The analyses reveal that the variation of the excitation and attenuation of both P and S head waves with eccentricity depends on frequencies and receiver azimuths strongly. Besides, new resonance peaks appear in excitation spectra due to influences of poles of multipole modes near branch points when the monopole tool is off-center. According to semblance results of individual compressional and shear waveforms, extracted velocities are not affected by tool eccentricity in both fast and slow formations. In fast formations, spectra analyses indicate that S-wave excitation is more sensitive to tool eccentricity than P-wave. Moreover, resonance peaks in P-wave excitation spectra increase with the increasing eccentricity in all directions. In slow formations, off-center tools almost have no influence on both P and S waves at low frequencies, which suggests that the effects of tool eccentricity can be reduced by adjusting the source’s operating frequency.

Keywords: logging while drilling, tool eccentricity, analytic solutions, head waves

1. Introduction
Studies on how tool eccentricity affects recorded logs have always been of interest in acoustic logging, since it may not be possible to maintain logging tools in perfect alignment in inclined, deviated or horizontal boreholes even using centralisers (Roever et al. 1974; Willis et al. 1982; Byun & Toksöz 2006). Compared with wireline logging, the problem of tool eccentricity may be more pronounced and more complex in logging while drilling (LWD). Unlike traditional wireline logging tools, acoustic LWD tools take up most of the borehole space, resulting in a thin fluid annulus between the drill collar and wellbore (Zheng et al. 2004). Therefore, even if the tool is slightly off-center, the relative change in the thickness of the fluid annulus can be significant. Knowledge about the effects of an eccentric acoustic LWD tool on borehole wavefields will help accurate interpretation of recorded data.

Several studies have been done on an off-center LWD tool in a fluid-filled borehole by means of some numerical simulation techniques such as the finite-element method (FEM) or finite-difference method (FDM). Wang & Tang (2003)}
investigated the influences of tool eccentricity on simulated waveforms in the quadrupole LWD case and concluded that acquired data with an off-center tool could be corrected. Zheng et al. (2004) and Huang et al. (2004) evaluated wavefield dispersion features with varying tool eccentricity using FEM and FDM, respectively. Their studies indicate that tool eccentricity may lead to two remarkable changes: generating more modes and altering dispersion features. Pardo et al. (2013) compared the differences of acoustic responses between wireline logging and LWD when a monopole tool is not centered. Wang et al. (2013) studied multipole LWD wavefields in horizontal or highly deviated wells and proposed a method to invert the degree of tool eccentricity.

Wang et al. (2015) investigated high-frequency wavefields of an off-center monopole acoustic LWD tool and observed prominent collar flexural and quadrupole modes. Wang et al. (2017) evaluated the reliability of velocity measurements made by eccentric monopole acoustic LWD tools in two types of fast formation.

More attention should be paid to formation compressional and shear head waves (P and S waves) in a real measuring environment, which are much more important practically for formation evaluation. Although the previously mentioned publications have discussed much about the influences of tool eccentricity on dispersion features of borehole modes and simulated LWD full wavefields, no research has been involved in how an off-center acoustic LWD tool affects individual P and S head waves. In general, we think that head waves recorded in the borehole are theoretically contributed to by corresponding branch points and only the analytic method can cope with the problem. Haugland (2004a, 2004b) derived analytic solutions for an eccentric solid mandrel in the borehole. Nevertheless, he did not consider the effects of mandrel eccentricity on head waves individually, either. Besides, his physical model is different from the true acoustic LWD that we state, as the real LWD tool is a fluid-filled cylinder, not a solid mandrel.

Therefore, in this study, we first derive analytic solutions of borehole acoustic fields in this asymmetric cylindrically layered structure. Then by applying a branch-cut integration technique (Kurkjian & Chang 1986), we evaluate the contributions of compressional and shear branch points with varying eccentricity to investigate the effects of tool eccentricity on individual formation P and S waves.

2. Method

First, we establish the theoretical acoustic LWD model with an off-center monopole tool, which is an eccentric drill collar in a fluid-filled borehole encircled by an infinite rock, as shown in figure 1. In figure 1a, from the inside out, these four layers of media represent the internal fluid, drill collar, external fluid annulus and formation, respectively. Both the metallic collar and formation are supposed to be macroscopically homogeneous and isotropic. A symmetric ring normal-stress source is installed on the external surface of the collar directly, as shown in figure 1a. Receivers locate directly above the acoustic source and are arranged circumferentially on the external wall of the tool.

Unlike previous studies (Byun & Toksöz 2006; Xu & Hu 2019) in wireline logging in which tool boundaries are ignored, our work here considers problems of the eccentric elastic boundary (i.e. the off-center collar) in a fluid-filled borehole. For the LWD model with an off-center collar we studied here, the whole structure is no longer symmetric. Therefore, to obtain analytic solutions of wavefields, two sets of cylindrical coordinate systems \((r_1, \theta_1, z_1)\) and \((r_2, \theta_2, z_2)\) are needed for characterising this model, as depicted in figure 1. Here we define that the first and second coordinate systems are referenced to the collar and borehole, respectively. In figure 1b, \(a\), \(b\) and \(c\), respectively, represent the internal and external radii of the drill collar as well as the radius of the borehole. The origin-to-origin separation \(e\) is the tool offset value and satisfies \(0 < e = |O_1O_2| < c - b\). As the cylinder axes of the collar and borehole are parallel, we only use \(z\) to express both \(z_1\) and \(z_2\) in the following.

The displacement potentials of the wave motion in the frequency and wavenumber domains in each layer can be respectively expressed in terms of their relevant coordinate systems as

\[
\varphi (r, \theta, k, \omega) = \sum_{n=-\infty}^{\infty} [a_n I_n (\beta r) + b_n K_n (\beta r)] \cos (n\theta),
\]

(1a)

\[
\psi (r, \theta, k, \omega) = \sum_{n=-\infty}^{\infty} [c_n I_n (\xi r) + d_n K_n (\xi r)] \sin (n\theta),
\]

(1b)

\[
\chi (r, \theta, k, \omega) = \sum_{n=-\infty}^{\infty} [e_n I_n (\eta r) + f_n K_n (\eta r)] \cos (n\theta),
\]

(1c)

\[
\phi_m (r, \theta, k, \omega) = \sum_{n=-\infty}^{\infty} g_n I_n (\eta' r) \cos (n\theta),
\]

(2)

\[
\phi_{e2} (r, \theta, k, \omega) = \sum_{n=-\infty}^{\infty} [h_n I_n (\eta' r) + i_n K_n (\eta' r)]
\]

\[
\times \cos (n\theta),
\]

(3)

\[
\phi_m (r, \theta, k, \omega) = \sum_{n=-\infty}^{\infty} j_n K_n (\beta r) \cos (n\theta),
\]

(4a)
\[
\phi_m(\rho_2, \theta_2, k, \omega) = \sum_{v=-\infty}^{\infty} k_v K_v(\xi_m \rho_2) \sin(v \theta_2),
\]
\[
\chi_m(\rho_2, \theta_2, k, \omega) = \sum_{v=-\infty}^{\infty} l_v K_v(\xi_m \rho_2) \cos(v \theta_2),
\]
where \(\omega\) and \(k\) are the angular frequency and axial wavenumber, respectively; \(I\) and \(K\) are the modified Bessel functions; \(\psi\) and \(\xi\) are the compressional and shear radial wavenumbers in the collar, respectively; \(\eta\) represents the compressional radial wavenumber in both internal and external fluids; \(\beta_m\) and \(\xi_m\) represent the compressional and shear radial wavenumbers in the formation, respectively; \(a_n-i_n\) and \(j_n-l_n\) are the unknown coefficients of each layer that need to be solved by mean of boundary conditions and the subscript numbers \(n\) and \(v\) denote the circumferential periodic numbers in the first and second coordinate systems, respectively. Note that \(e^{(kz-\omega t)}\) is left out in equations (1)–(4). \(\phi\) and \(\chi\) in equation (1a–c) are scalar potentials in the collar; \(\phi_m\) and \(\phi_n\) in equations (2) and (3) are compressional potentials in the internal and external fluids, respectively. These five displacement potentials are expressed in terms of the first coordinate system. \(\phi_m\), \(\psi_m\) and \(\chi_m\) in equation (4a–c) indicate scalar potentials in the formation and they are derived with respect to the second coordinate system.

Then field components such as stresses or displacements in each layer can be derived from these potentials and some literature has given detailed relationships (Pao & Mow 1971; He et al. 2017). For the drill collar layer, the field quantities and six unknown coefficients can be related by
\[
(\mathbf{u}_r, \tau_{rr}, \tau_{r\theta}, \tau_{rz})_n = \mathbf{B} \cdot (a_n, b_n, c_n, d_n, e_n, f_n)^T,
\]
where \(\mathbf{u}_r\) is the radial displacement; \(\tau_{rr}\) is the normal stress and \(\tau_{r\theta}\) and \(\tau_{rz}\) represent two shear stresses. Considering that two shear stresses are zero at both inner and outer walls of the collar, four of the six coefficients in equation (5) can be eliminated. As a result, the radial displacement and normal stress of the order \(n\) at the two surfaces can be expressed, respectively, by
\[
\left. \begin{pmatrix} u_r \\ \tau_{rr} \\ \tau_{r\theta} \\ \tau_{rz} \end{pmatrix} \right|_{r=a} = \mathbf{B}_1 \cdot (a_n, b_n)^T 
\]
and
\[
\left. \begin{pmatrix} u_r \\ \tau_{rn} \end{pmatrix} \right|_{r=b} = \mathbf{B}_2 \cdot (a_n, b_n)^T. 
\]
Similarly, for the innermost fluid layer and the external fluid annulus, the radial displacement and the pressure at the two surfaces of the collar can be written as
\[
\left. \begin{pmatrix} u^m_r \\ p^m \end{pmatrix} \right|_{r=a} = \mathbf{A} \cdot g_n 
\]
and
\[
\left. \begin{pmatrix} u^e_r \\ p^e \end{pmatrix} \right|_{r=b} = \mathbf{C} \cdot (h_n, i_n)^T. 
\]
Combining equations (6–9) and considering the normal-stress source, the unknown coefficients in the inner and outer fluid domains can be connected by
\[
\left( \begin{array}{c} h_n \\ i_n \end{array} \right)^T = \mathbf{X} \cdot \mathbf{g}_n - \mathbf{C}^{-1} \cdot \begin{pmatrix} 0 \\ \sigma^* \end{pmatrix}_n, \quad \mathbf{X} = \mathbf{C}^{-1} \cdot \mathbf{B}_2 \cdot \mathbf{B}_1^{-1} \cdot \mathbf{A}
\]
where the symbol \(\sigma^*\) in equation (10) denotes the monopole normal-stress source and when order \(n\) is zero \(\sigma^*\) is a
positive constant, otherwise it is zero. The time and frequency domain information of the source can refer to Cui (2004).

For the outermost formation layer, taking radiation conditions at infinity into account, there are only three unknown coefficients need to be solved. At the borehole wall, also considering that two shear stresses equal to zero, the radial displacement and radial normal stress can be expressed by only one coefficient in the second coordinate system as follows:

$$\left( u^m \, T \right)_{rr} |_{r = c} = D \cdot j_v. \quad (11)$$

To set up equations at the borehole wall, a transformation needs to be performed to make acoustic fields of both sides of the interface in the same coordinate system through the translational addition theorem. The translational addition theorem for cylindrical Bessel functions can be expressed as (Watson 1944; Guz et al. 2002)

$$I_n(\eta' r_1) \left\{ \cos (n \theta) \right\} \sin (n \theta_1) \right\} = \sum_{\nu = -\infty}^{\infty} I_{n-\nu}(\eta' e) I_{\nu}(\eta' r_2) \left\{ \cos (\nu \theta_2) \right\} \sin (\nu \theta_1) \right\}, \quad (12a)$$

$$K_n(\eta' r_1) \left\{ \cos (n \theta) \right\} \sin (n \theta_1) \right\} = \sum_{\nu = -\infty}^{\infty} (-1)^{n-\nu} I_{n-\nu}(\eta' e) K_{\nu}(\eta' r_2) \left\{ \cos (\nu \theta_2) \right\} \sin (\nu \theta_1) \right\}. \quad (12b)$$

By substituting equation (12a) and (12b) into equation (3), the coordinate system of the potential in the external fluid domain is transformed from the first to second, as shown in equation (13a) and (13b):

$$\varphi_{ex} (r_2, \theta_2, k, \omega) = \sum_{\nu = -\infty}^{\infty} \left[ H_{\nu} I_{\nu} (\eta' r_2) + I_{\nu} K_{\nu} (\eta' r_2) \right] \times \cos (\nu \theta_2), \quad (13a)$$

$$H_{\nu} = \sum_{m = -\infty}^{\infty} h_m I_{m-\nu}(\eta' e), \quad I_{\nu} = \sum_{m = -\infty}^{\infty} i_m (-1)^{m-\nu} I_{m-\nu}(\eta' e). \quad (13b)$$

Accordingly, at the borehole wall, the radial displacement and pressure in the external fluid can be expressed with reference to the second coordinate system as follows

$$\left( u^m \, T \right)_{rr} |_{r = c} = C' \cdot (H_{\nu} \, I_{\nu})^T. \quad (14)$$

Combining equations (11) and (14), we can obtain

$$\left( H_{\nu} \, I_{\nu} \right)^T = N \cdot j_\nu, \quad N = C'^{-1} \cdot D. \quad (15)$$

Next, substituting equations (10) and (13b) into (15), the innermost and outermost layers can be related by

$$\sum_{m = -\infty}^{\infty} \left( X_2 (1)^{n-\nu-m} I_{m-\nu}(\eta' e) \right) \cdot g_{m} - \left( N_2 \right) \cdot j_{\nu} = \sum_{m = -\infty}^{\infty} \left( C^{-1} \sigma I_{n-\nu}(\eta' e) \right) \cdot \left( X_2 (1)^{-n-m} I_{n-\nu}(\eta' e) \right). \quad (16)$$

Through a series of simplifications and decoupling, the following system of infinite equations, which only contain one unknown coefficient $g_{m}$ can be obtained and then solved:

$$\sum_{m = -\infty}^{\infty} \left( -N_2 X_1 I_{n-\nu}(\eta' e) + N_1 X_2 (1)^{-n-m} I_{n-\nu}(\eta' e) \right) \cdot g_{m} = \sum_{m = -\infty}^{\infty} \left( -N_2 \cdot C^{-1} \sigma I_{n-\nu}(\eta' e) + N_1 \cdot C^{-1} \sigma I_{n-\nu}(\eta' e) \right). \quad (17)$$

In the practical calculation, $n$ and $\nu$ should be truncated to a finite number not infinity. By solving equation (17), $g_{m}$ can be identified and further the other remaining unknown coefficients can be derived from $g_{m}$.

In general, we think that formation P and S arrivals are mainly contributed to by compressional and shear branch points, respectively. This developed analytic method can evaluate branch points’ contributions through the vertical branch-cut integration (Kurkjian 1985; Zhang et al. 1996; He & Hu 2010). The responses of the two waves in the frequency-space domain are expressed by

$$P_b (r_0, \theta_0, z_0, \omega) = \sum_{m = -\infty}^{\infty} \int_{k_0}^{k_0 + \infty} \left[ h_m I_{m}(\eta' r_0) + i_n K_{n}(\eta' r_0) \right] \times \cos (n \theta_0) e^{ikz} dk, \quad (18)$$

where $r_0 = 0.09 \, m$, $\theta_0 = 0.45, 90, 135$ and $180^\circ$, and $z_0 = 3 \, m$ indicate the receiver positions, in terms of the first cylindrical coordinate system; ‘+’ and ‘−’ indicate the two sides of the integral path (see figure 2 in Kurkjian’s paper [1985]), respectively, and $k_0$ represents the branch point associated with P or S waves. By multiplying equation (18) by the source function in the frequency domain and then going through a Fourier transform, we can obtain synthetic waveforms of individual P and S waves. Besides, P- and S- wave attenuation characteristics can also be studied by comparing excitation spectra with different axial distance $z$. The attenuation parameter $\alpha$ is defined as follows

$$\alpha = -20 \log A_2 (f) / A_1 (f) / L_1, \quad (19)$$
Table 1. Parameters of each layer in the model

| Layer            | P velocity (m s⁻¹) | S velocity (m s⁻¹) | Density (kg m⁻³) | Radius (m) |
|------------------|--------------------|--------------------|------------------|------------|
| Internal fluid   | 1470               | 0                  | 1000             | 0.027      |
| Drill collar     | 5860               | 3130               | 7800             | 0.090      |
| External fluid   | 1470               | 0                  | 1000             | 0.117      |
| Fast formation   | 3972               | 2455               | 2320             | —          |
| Slow formation   | 2742               | 1067               | 2100             | —          |

where \( A_2 \) and \( A_1 \) represent the excitation spectra at \( z = 4 \) and \( 3 \) m, respectively; \( L \) is the distance between the two receivers; \( f \) is the frequency and the unit of the attenuation parameter \( \alpha \) is dB/ft.

Finally, by changing eccentricity values in the calculations, the effects of tool eccentricity on individual formation primary and shear head waves can be evaluated.

3. Computational results

Based on these theoretical analyses, we investigate characteristics of individual P and S head waves with varying tool eccentricity in both fast and slow formations in this section. The model parameters are given in Table 1. To describe the extent of tool eccentricity from the borehole axis quantitatively, we define a normalised eccentricity coefficient \( \varepsilon \) as following:

\[
\varepsilon = \frac{e}{(c - b)} \times 100\% \tag{20}
\]

where \( e \) represents the distance between the axes of the off-center tool and the borehole, as shown in figure 1b; \( (c - b) \) is the thickness of the external fluid annulus when the collar is in the center of the borehole. During the operation of LWD tools, a centraliser is usually used that can prevent large tool eccentricity. Therefore, in this study, we only focus on cases where the eccentricity coefficient \( \varepsilon \) is within 30%. Besides, due to the symmetry of both the source and the structure, the acoustic fields are axisymmetric with respect to the extended line formed by the connection of \( O_1 O_2 \) (Wang et al. 2015). In this work we just need to consider the cases where receiver azimuths are from \( 0^\circ \) to \( 180^\circ \).

3.1. Fast formations

We first study the effects of tool eccentricity on individual P and S waves in a fast formation. By evaluating equation (18), figure 2a presents P-wave excitation spectra with varying eccentricity at \( 0^\circ, 45^\circ, 90^\circ, 135^\circ \) and \( 180^\circ \) (from bottom to top), respectively. We can see that when the tool becomes off-center, a new resonance appears at around 2 kHz in the spectra.

Figure 2. (a) P-wave excitation spectra with varying tool eccentricity at different receiver azimuths in the fast formation. (b) Resonance peak values in P-wave excitation spectra with varying tool eccentricity at different receiver azimuths. (c) P-wave attenuation spectra with varying tool eccentricity at \( 0^\circ \) and \( 180^\circ \).
Besides, the intrinsic resonance peak at approximately 14 kHz increases with increasing eccentricity in all the five directions. By selecting peak values in the excitation spectra with varying eccentricity at different azimuths, figure 2b can be plotted. It indicates that amplitudes of resonance peaks at different azimuths show small differences within 15% eccentricity, while present relatively large differences more than 15% eccentricity. Moreover, peak values at 0° and 180° are larger than those at other azimuths. Figure 2a also illustrates that in the other frequency range except the area near the two resonance frequencies, the excitation variation is small with the eccentricity changing. Attenuation characteristics of P waves are also investigated by evaluating equation (19). The left and right clusters of curves in figure 2c indicate receiver azimuths of 0° and 180°, respectively. Figure 2c reveals that the P-wave attenuation is almost unaffected by tool eccentricity in the full frequency range, except at some specific frequencies (i.e., approximately 2 and 14 kHz).

Further, we present individual waveforms of P waves with varying eccentricity at different receiver azimuths, as shown in figure 3. Figure 3 parts a–c adopt the source center frequencies of 12, 8 and 4 kHz, respectively. In the actual calculations, we found that waveforms only contributed by branch points of P waves at 12 kHz are chaotic and meaningless due to the effects of poles near the P-wave branch point. To obtain meaningful P-wave pack at this frequency, we also take the contribution of poles near the branch point into account (Liu & Chang 1996). Figure 3 parts a and b indicate that 12 and 8 kHz waveforms of P waves are almost unaffected by tool eccentricity. But for the 4 kHz case, in figure 3c, some trailing appears when the tool is off-center except for waveforms at 90°. To explore whether the trailing affects the extracted P-wave velocity, the time-slowness semblance method (Kimball & Marzetta 1984) is used to process those waveforms. Here, we only take the case where the receiver azimuth is 180° and the eccentricity parameter $\varepsilon$ is 30% as an example. Figure 3d shows that the trailing generated from eccentricity does not influence the accurate extraction of P-wave velocity.

Next, we focus on how an LWD tool being off-center affects S waves propagating along the borehole. Figure 4a depicts S-wave excitation spectra with varying tool eccentricity at the above five azimuths. We can see that a new resonance peak appears owing to the effects of multiple modes generated from eccentricity. When poles corresponding to these newly generated modes are close to branch points, associated head waves will be affected. For the spectra at 135 and 180°, almost in the full frequency range, the excitation amplitude increases with the increase of eccentricity. The S-wave excitation in the direction of 90° is almost unaffected by
tool eccentricity. The characteristics of S-wave excitation with the azimuths smaller than 90° seem relatively complex. When the frequency is larger than about 10 kHz, the excitation changes little with eccentricity; while during 6–10 kHz, the excitation increases with the increasing eccentricity. In the range of 4–6 kHz, the excitation intensity decreases as eccentricity increases. Attenuation spectra of S waves with varying eccentricity at 0° and 180° are shown in figure 4b. We know that S-wave attenuation seems more sensitive to tool eccentricity than P waves. When the tool becomes eccentric, a steep decrease appears around 0–5 kHz and the decreased amplitude increases with the increasing eccentricity. In another frequency area, the eccentricity has relatively little effect on attenuation.

The synthetic individual waveforms of S waves are also calculated with different eccentricity, azimuths and center frequencies of the source. In figure 5a, with the center frequency 12 kHz, the waveforms change little with the eccentricity. At 8 kHz, figure 5b shows that the amplitudes of waveforms at all the five azimuths increase with the increasing eccentricity. The waveforms become disordered due to suffering from the effects of poles near S branch points when the tool is off-center. However, we can still extract S-wave velocity accurately from those waveforms even with 30% eccentricity, as shown in figure 5d. We also processed individual 4-kHz waveforms of S waves in figure 5c. The accurate S-wave velocity can also be identified and semblance plots are left out here. Note that there are rippers from the very beginning in figure 5. These artifact ripples are generated due to the existence of poles near the S-wave branch point, such as the poles associated with the leaky modes and pseudo-Rayleigh modes, which also contribute to the full waveform (Kurkjian 1985; Zheng & Hu 2017). Summing up the contribution of the shear branch point and those poles nearby, the waveform with a clear arrival can be obtained, which is the mixture of shear and pseudo-Rayleigh waves. As stated by Kurkjian (1985), separation of the shear head waves from the pseudo-Rayleigh modes without introducing artifact ripples is a tough work. Although the integration associated with the shear branch point may not result in the signal with a sharp arrival, its amplitude can still stand for the excitation characteristics of the shear head wave (e.g. Zhang et al. 1994; Zheng & Hu 2017). In this paper, we focus on revealing the trend how the tool eccentricity influences the amplitudes of P and S head waves, so we only investigate the component waveforms contributed by the branch-cut integral with varying eccentricity.

### 3.2. Slow formations

Referring to the means of analysis in the fast formation case, we now pay attention to the effects of tool eccentricity on head waves when outside the borehole is a slow formation case. Figure 6a presents the excitation spectra of P waves with varying eccentricity at different azimuths. We can see that before about 6 kHz the P-wave excitation is almost unaffected by eccentricity. At about 8 kHz, a new resonance peak appears due to tool eccentricity. The amplitude of this newly generated resonance peak increases with the increasing eccentricity at all the five azimuths. When the azimuth is less than 90°, the excitation intensity decreases with the increase of eccentricity in the range of 9–15 kHz, while it increases above 15 kHz. For the other three azimuths, when the frequency is larger than the frequency corresponding to the newly generated peak, the excitation increases with the increasing eccentricity. Figure 6b demonstrates P-wave attenuation features in the frequency domain with varying eccentricity at 0° and 180°. Through the two sets of curves, it can be seen that before about 6 kHz, the attenuation is nearly not influenced by eccentricity. Above 6 kHz, overall, the attenuation decreases at 0° and increases at 180°, with the increasing eccentricity.

Next, figure 7a–c investigates the effects of an off-center LWD tool on individual P-wave waveforms at 12, 8 and 4 kHz, respectively. From figure 7a, we can observe that the variation of these 12 kHz waveforms’ amplitudes with eccentricity is consistent with what we analyse from the
Figure 5. Individual waveforms of S waves with varying tool eccentricity at different receiver azimuths in the fast formation when the source center frequencies are (a) 12, (b) 8 and (c) 4 kHz. (d) The time-slowness semblance for 8-kHz waveforms of S waves with the tool eccentricity parameter $\varepsilon$ 30% and receiver azimuth 180°.

Figure 6. (a) P-wave excitation spectra with varying tool eccentricity at different receiver azimuths in the slow formation. (b) P-wave attenuation spectra with varying tool eccentricity at 0° and 180°.

In general, we think that there are no refracted shear waves in the borehole in a slow formation. But recent studies from Fang & Cheng (2018) found that formation S waves can be detected in a slow formation using a monopole acoustic LWD tool. Therefore, in this work, we also discuss the effects of tool eccentricity on S waves by evaluating the contribution of S-wave branch point in a slow formation. Figure 8a presents S-wave excitation spectra with varying eccentricity at different azimuths. These curves indicate that when receiver azimuth angles are less than 90°, the excitation decreases with the increasing eccentricity, while in the
direction of azimuths larger than 90°, the excitation increases. The variation of excitation amplitude with eccentricity is more evident in the high-frequency region. The excitation at 90° changes little with eccentricity. Further, figure 8b investigates how eccentricity affects S-wave attenuation in the direction of 0° and 180°. The results reveal that as eccentricity increases, the attenuation increases at 0°, while decreases at 180°.

Finally, we calculate synthetic waveforms of S waves with different eccentricity and receiver azimuths. Figure 9a–c refers to cases where the center frequencies are 12, 8 and 4 kHz, respectively. As no poles affect S-wave branch points in slow formations, recorded S head waves in the borehole are contributed to completely by associated branch points, leading to complete and desirable wave shapes in figures 9a–c. From the three figures, we can conclude that the eccentricity only affects waveforms’ amplitudes slightly and has no effects on S-wave velocities and wave arrivals.

4. Conclusions
In this paper, we develop an analytic method to investigate the effects of an off-center monopole acoustic LWD tool on individual compressional and shear head waves. By
evaluating the contribution of branch points associated with P and S waves, their characteristics of excitation, attenuation and waveforms with varying eccentricity are revealed.

Semblance results indicate that extracted velocities of the two head waves in both fast and slow formations are not affected by tool eccentricity. In the case of small eccentricity in this work, wave arrivals are almost not influenced. The variation of the excitation and attenuation of both P and S head waves with eccentricity strongly depends on frequencies and receiver azimuths. The details are as follows.

In a fast formation, a new resonance peak appears around 2 kHz in the P-wave excitation spectra when the tool is off-center. The intrinsic resonance peak at around 14 kHz increases with the increase of tool eccentricity at all the five azimuths. Except for the area near the two resonance peaks, the excitation variation is small with changing eccentricity in the other frequency range. For S-wave spectra at 135° and 180°, the excitation amplitude increases with eccentricity almost in the full frequency range. However, at 0° and 45°, the effects of tool eccentricity on the S-wave excitation are complex and strongly dependent on frequencies.

In a slow formation, before about 6 kHz the P-wave excitation is almost unaffected by eccentricity. At about 8 kHz, a new resonance peak appears due to tool eccentricity and its peak value increases with the increasing eccentricity at all the five azimuths. For S-wave excitation spectra, when the receiver azimuth angle is less than 90°, the excitation intensity decreases with the increasing eccentricity, while at azimuths larger than 90°, it increases with the increasing eccentricity. The analyses indicate that a low frequency source is a good choice in a slow formation because both P and S waves are almost unaffected by tool eccentricity at low frequencies and excitation amplitudes of the two waves are relatively large.

In both fast and slow formations, new resonance peaks are generated in the excitation spectra when the monopole tool is off-center, which results from the effects of multipole poles near branch points. Although semblance plots of individual P and S waves indicate that the extracted velocity is not affected by eccentricity even there is a trailing section in the waveforms, we have to process recorded full-wave data instead of single compressional or shear waveforms in the real field. Therefore, it is better to avoid or reduce disturbances of these newly generated resonance peaks by selecting appropriate source frequencies or directly using receiver information at 90°.

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