Rayleigh waves in thermo elastic medium with double porosity

Abstract

The present paper deals with the propagation of Rayleigh waves in isotropic homogeneous thermoelastic half–space with double porosity whose surface is subjected to stress–free, thermally insulated/isothermal boundary conditions. The compact secular equations for thermoelastic solid half–space with voids are deduced as special cases from the present analysis. In order to illustrate the analytical developments, the secular equations have been solved numerically. The computer simulated results for copper materials in respect of determinant of Rayleigh wave secular equation, Rayleigh wave velocity and attenuation coefficient have been presented graphically for different values of phase velocity.

Keywords: rayleigh waves, double porosity, thermoelastic, secular equation

Introduction

Porous media theories play an important role in many branches of engineering including material science, the petroleum industry, chemical engineering, biomechanics and other such fields of engineering. Biot\(^1\) proposed a general theory of three–dimensional deformation of fluid saturated porous salts. Biot\(^1\) theory is based on the assumption of compressible constituents and till recently, some of his results have been taken as standard references and basis for subsequent analysis in acoustic, geophysics and other such fields. One important generalization of Biot’s\(^1\) theory of poroelasticity that has been studied extensively started with the works by Barenblatt et al.,\(^2\) where the double porosity model was first proposed to express the fluid flow in hydrocarbon reservoirs and aquifers. The double porosity model represents a new possibility for the study of important problems concerning the civil engineering. It is well–known that, under super–saturation conditions due to water of other fluid effects, the so called neutral pressures generate unbearable stress states on the solid matrix and on the fracture faces, with severe (sometimes disastrous) instability effects like landslides, rock fall or soil fluidization (typical phenomenon connected with propagation of seismic waves). In such a context it seems possible, acting suitably on the boundary pressure state, to regulate the internal pressures in order to deactivate the noxious effects related to neutral pressures; finally, a further but connected positive effect could be lightening of the solid matrix/fluid system.

Wilson and Aifantis\(^1\) presented the theory of consolidation with the double porosity. Khaled et al.,\(^4\) employed a finite element method to consider the numerical solutions of the differential equation of the theory of consolidation with double porosity developed by Aifantis.\(^4\) Wilson et al.,\(^3\) discussed the propagation of acoustics waves in a fluid saturated porous medium. Beskos et al.,\(^4\) presented the theory of consolidation with double porosity–II and obtained the analytical solutions to two boundary value problems. Aifantis\(^5\)–\(^10\) introduced a multi–porous system and studied the mechanics of diffusion in solids. Khalili et al.,\(^11\) presented a fully coupled constitutive model for thermo–hydro–mechanical analysis in elastic media with double porosity structure. Straughan\(^12\) studied the stability and uniqueness in double porous elastic media. Svanadze\(^13\)–\(^17\) investigated some problems on elastic solids, viscoelastic solids and thermoelastic solids with double porosity. Rayleigh waves are always generated when a free surface exists in a continuous body. Rayleigh firstly introduced them as solution of the free vibration problem for an elastic half–space (on waves propagated along the plane surface of an elastic solid). Rayleigh wave play an important role in the study of earthquakes, seismology, geophysics and geodynamics. During earthquake, Rayleigh waves play more drastic role than other seismic waves because these waves are responsible for destruction of buildings, plants and loss of human lives etc. Geophysical and thermal problems consist of the study of propagation of progressive elastic and thermoelastic waves and hence the effect of voids on the surface waves propagating in the thermoelastic media has got its due importance where the situation so demands. The cooling and heating of the medium also results in the expansion and contraction of the voids along with the core material which contributes towards thermal stress and vibration developments in solids. In coating or casting applications, the voids that are not detected and removed, can result in defects that compromise the adhesion, electric properties, surface finish and durability of the product.

Rayleigh\(^18\) investigated the propagation of waves along the plane surface of an elastic solid. Lockett\(^19\) studied the effect of thermal properties on Rayleigh wave’s velocity. Propagation of Rayleigh waves along with isothermal and insulated boundaries discussed by Chadwick et al.,\(^20\) Kumar et al.,\(^21\)\(^22\) presented the problem of Rayleigh waves in an isotropic generalized thermoelastic with diffusive half–space medium. Sharma et al.,\(^21\) presented the problem of Rayleigh...
waves in rotating thermoelastic with voids. Kumar et al. discussed the problem of Rayleigh waves in isotropic micro stretch thermoelastic diffusion solid half-space. Kumar and Gupta discussed the problem of Rayleigh waves in generalized thermoelastic medium with mass diffusion. Abd–Alla et al. investigated the propagation of Rayleigh waves in different theories. Singh et al. examined the propagation of the Rayleigh wave in an initially stressed transversely isotropic dual phase lag magneto–thermoelastic half space. Kumar et al. studied the propagation of Rayleigh waves in generalized thermoelastic medium with mass diffusion. Biswas et al. investigated the Rayleigh surface wave propagation in orthotropic thermoelastic solids under three–phase lag model. Singh et al. examined the propagation of Rayleigh wave in two–temperature dual–phase–lag thermo elasticity. Biswas et al. studied the effect of phase–lags on Rayleigh wave propagation in initially stresses magneto–thermoelastic orthotropic medium. Hussien et al. investigated the effect of rotation on Rayleigh waves in a fiber–reinforced solid anisotropic magneto–thermo–viscoelastic media. In the present paper, we investigate the propagation of Rayleigh waves in homogenous isotropic elastic materials with double porosity structure. Secular equations are derived mathematically for the boundary conditions. The values of determinant of Rayleigh wave structure. Secular equations are derived mathematically for the present paper, we investigate the propagation of Rayleigh waves in reinforced solid anisotropic magneto–thermo–viscoelastic media. In

**Basic equations**

Following Iesan et al., the constitutive relations and field equations for homogenous elastic material with double porosity structure without body forces, intrinsic equilibrium body forces and without heat sources can be written as:

### Constitutive Relations

$$\tau_{ij} = \lambda \epsilon_{n} \delta_{ij} + 2 \mu \epsilon_{ij} + b \delta \sigma_{ij} - \beta \delta \sigma_{n} T,$$  

$$\sigma_{ij} = \alpha \epsilon_{ij} + b \delta \sigma_{ij},$$  

$$\tau_{ij} = b \delta \sigma_{ij} + \gamma \sigma_{ij}.$$  

### Equations of motion

$$\rho \nabla \cdot \ddot{u} + (\lambda + \mu) \nabla \cdot \dot{u} + b \nabla \sigma + d \nabla \psi - \beta \partial T = \rho \frac{\partial^2 \ddot{u}}{\partial t^2},$$  

$$u_1 = u_1(x_1, x_2, t), u_2 = 0, u_3 = u_3(x_1, x_2, t), \phi = \phi(x_1, x_2, t), \psi = \psi(x_1, x_2, t), T = T(x_1, x_2, t)$$  

We define the following non–dimensional quantities:

$$x_1 = \frac{\alpha x_1}{c_1}, x_2 = \frac{\alpha x_2}{c_1}, u_1 = \frac{\alpha u_1}{c_1}, u_3 = \frac{\alpha u_3}{c_1}, t_0 = \frac{t_0}{T_0}, \sigma' = \frac{\sigma}{\alpha T_0}, \tau' = \frac{\tau}{\beta T_0},$$  

$$\phi' = \frac{\kappa \sigma^2}{\alpha^2}, \psi' = \frac{\kappa \sigma^2}{\alpha^2},$$  

$$\sigma' = \frac{c_1}{\alpha \omega}, \tau' = \frac{c_1}{\alpha \omega},$$  

$$\phi' = \frac{c_1}{\alpha \omega}, \psi' = \frac{c_1}{\alpha \omega},$$  

Where  

$$c_1 = \sqrt{\lambda + 2 \mu} / \rho, \alpha = \frac{\rho C_p^* c_1^2}{K}$$  

### Equilibrated Stress Equations of motion

$$\alpha \dot{\phi} + b_1 \nabla \psi - b \nabla \cdot \dot{u} - \alpha_1 \phi - \alpha_2 \psi + \gamma_1 T = \kappa_1 \frac{\partial^2 \phi}{\partial t^2},$$  

$$b_1 \dot{\psi} + b_2 \nabla^2 \psi - d \nabla \cdot \ddot{u} - \alpha_3 \phi - \alpha_4 \psi + \gamma_2 T = \kappa_2 \frac{\partial^2 \psi}{\partial t^2},$$  

### Equation of Heat conduction

$$K \nabla^2 T - \beta \nabla \dot{u} - \gamma_1 T_0 \phi - \gamma_2 T_0 \psi - \rho C^* \dot{T} = 0$$  

where $\ddot{u}$ is the displacement vector; $\tau_0$ is the stress tensor; $\kappa_1$ and $\kappa_2$ are coefficients of equilibrated inertia; $\phi$ and $\psi$ are the volume fraction fields corresponding to pores and fissures respectively; $\sigma_1$ is the equilibrated stress corresponding to fissures; $\tau_1$ is the equilibrated stress corresponding to fissures; $K$ is the coefficient of thermal conductivity; $C^*$ is the specific heat at constant strain; $\beta$ is the mass density; $\beta = (3 \lambda + 2 \mu) \alpha$; $\alpha$ is the linear thermal expansion; $\lambda$ and $\mu$ are Lame’s constants and $b, d, \beta_1, \gamma_1, \beta_2, \gamma_2$ are constitutive coefficients; $\delta_{ij}$ is the Kronecker’s delta; $T$ is the temperature change measured from the absolute temperature $T_0$; $T_0 \neq 0$; a superscribed dot represents differentiation with respect to time variable $t$.

$$\nabla \cdot \ddot{u} + (\lambda + \mu) \nabla \cdot \dot{u} + b \nabla \sigma + d \nabla \psi - \beta \partial T = \rho \frac{\partial^2 \ddot{u}}{\partial t^2}$$  

$$\nabla^2 \psi = \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_1^2}$$  

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_1^2}$$  

**Formulation of the problem**

We consider homogenous isotropic thermoelastic with double porous half space. We take the origin of the coordinate system $(x_1, x_2, x_3)$ at any point plane on the horizontal surface and $x_1$ – axis in the direction of the wave propagation and $x_2$ – axis pointing vertically downward to the half-space so that all particles on line parallel to $x_2$ – axis are equally displaced. Therefore, all the field quantities will be independent of $x_2$ – coordinate.

For the two–dimensional problem, we take

$$\nabla \cdot \ddot{u} + (\lambda + \mu) \nabla \cdot \dot{u} + b \nabla \sigma + d \nabla \psi - \beta \partial T = \rho \frac{\partial^2 \ddot{u}}{\partial t^2}$$  

$$\nabla^2 \psi = \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_1^2}$$  

$$\nabla^2 \phi = \frac{\partial^2 \phi}{\partial x_1^2} + \frac{\partial^2 \psi}{\partial x_1^2}$$  

### Here $\omega$ and $c_1$ are the constants having the dimension of frequency and velocity in the medium respectively.

Using (8) in Eqs. (4)–(7) and with the aid of (9), after suppressing the primes, we obtain

$$\frac{\lambda + \mu}{\rho c_1^2} \ddot{u} = \frac{\mu}{\rho c_1^2} \nabla^2 u_1 + a_1 \frac{\partial \phi}{\partial x_1} + a_2 \frac{\partial \psi}{\partial x_1} - a_3 \frac{\partial T}{\partial x_1} = \frac{\partial^2 u_1}{\partial t^2}$$  

$$\frac{\lambda + \mu}{\rho C_p^* c_1^2} \ddot{u} = \frac{\mu}{\rho C_p^* c_1^2} \nabla^2 u_3 + a_1 \frac{\partial \phi}{\partial x_1} + a_2 \frac{\partial \psi}{\partial x_1} - a_3 \frac{\partial T}{\partial x_1} = \frac{\partial^2 u_3}{\partial t^2}$$  

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\[ a_4 \nabla^2 \phi + a_5 \nabla^2 \psi - a_8 e - a_9 \phi - a_{10} \psi - a_{11} T = \frac{\partial^2 \phi}{\partial t^2} \]

\[ a_{10} \nabla^2 \phi + a_{11} \nabla^2 \psi - a_{12} e - a_{13} \phi - a_{14} \psi - a_{15} T = \frac{\partial^2 \psi}{\partial t^2} - \frac{\partial \phi}{\partial t} - a_{16} \frac{\partial \phi}{\partial t} - a_{17} \frac{\partial \psi}{\partial t} - a_{18} \frac{\partial \psi}{\partial t} + a_{19} \nabla^2 T - \frac{\partial T}{\partial t} = 0 \]

Where

\[ a_1 = \frac{b a_1}{\rho c_i^2}, a_2 = \frac{d a_1}{\rho c_i^2}, a_3 = \frac{\beta T_0}{\rho c_i^2}, a_4 = \frac{\alpha}{\rho c_i^2}, a_5 = \frac{b a_1}{\rho c_i^2}, a_6 = \frac{b}{\rho c_i^2}, a_7 = \frac{a_1}{\rho c_i^2}, a_8 = \frac{a_3}{\rho c_i^2}, a_9 = \frac{a_5}{\rho c_i^2}, a_{10} = \frac{b}{\rho C}, a_{11} = \frac{\gamma T_0}{a_1}, a_{12} = \frac{d k_i^1}{\rho c_i^2}, a_{13} = \frac{a_3}{\rho c_i^2}, a_{14} = \frac{a_5}{\rho c_i^2}, a_{15} = \frac{\beta}{\rho C}, a_{16} = \frac{\gamma^2 T_0}{a_1}, a_{17} = \frac{\gamma^2 a_1}{\rho C^2}, a_{18} = \frac{\gamma^2 a_1}{\rho C^2}, a_{19} = \frac{K^*}{\rho c_i^2} \]

Here \( \nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} \), \( e = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} \)

The displacement components \( u_1 \) and \( u_3 \) are related by potential functions \( \phi \) and \( \psi \) as

\[ u_1 = \frac{\partial \phi}{\partial x_1} - \frac{\partial \psi}{\partial x_3}, \quad u_3 = \frac{\partial \phi}{\partial x_3} + \frac{\partial \psi}{\partial x_1} \]

Making use of (15) in equations (10)–(14), we obtain

\[ \left( \nabla^2 - \frac{\partial^2}{\partial x_3^2} \right) \phi_1 + a_3 \phi_1 + a_4 \psi_1 - a_2 T_0 = 0 \]

\[ -a_8 \nabla^2 \phi_1 + \left( a_4 \nabla^2 - a_2 - \frac{\partial^2}{\partial t^2} \right) \phi_1 + \left( a_5 \nabla^2 - a_8 \right) \psi_1 + a_9 T_0 = 0 \]

\[ -a_9 \nabla^2 \phi_1 + \left( a_5 \nabla^2 - a_8 \right) \phi_1 + \left( a_9 \nabla^2 - a_9 \right) \psi_1 + a_{10} T_0 = 0 \]

\[ -a_{10} \frac{\partial}{\partial t} \left( \nabla^2 \phi_1 \right) - a_{11} \frac{\partial \phi_1}{\partial t} - a_{12} \frac{\partial \psi_1}{\partial t} + \left( a_{13} \nabla^2 - \frac{\partial}{\partial t} \right) T_0 = 0 \]

and

\[ \left( a_{14} \nabla^2 - \frac{\partial^2}{\partial t^2} \right) \psi_1 = 0 \]

Here

\[ a_{20} = \frac{\mu}{\rho c_i^2} \]

**Solution of the problem**

We assume the solution of the form

\( (\phi_1, \phi, \psi, T, \psi_1) = (\phi_1^*, \phi^*, \psi^*, T^*, \psi_1^*) e^{i(\xi - ct)} \)

Where \( \xi \) is the wave number, \( \omega = \xi c \) is the angular frequency and \( C \) is the phase velocity of the wave.

Making use of (21) in Eqs.(16)–(20), we obtain four homogeneous equations in four unknowns and these equations have non–trivial solutions if the determinant of the coefficient \( \phi_1, \phi, \psi, T \) vanishes, which yield to the following characteristics equation:

\[ E_1 \frac{d^2}{dx^2} + E_2 \frac{d^6}{dx^6} + E_3 \frac{d^4}{dx^4} + E_4 \frac{d^2}{dx^2} + E_5 = 0 \]

Where \( E_1, E_2, E_3, E_4 \) and \( E_5 \) are given in the appendix and

\[ \left( \frac{d^2}{dx^2} - \xi_5^2 \right) = 0 \]

Where

\[ \xi_5^2 = \xi^2 + \xi_2^2 - \frac{a_{20}}{a_9} \]

Since we are interested in surface waves only, it is essential that the motion is confined to the free surface \( x_3 = 0 \) of the half-space. Therefore, to satisfy the radiation conditions, \( \phi_1, \phi, \psi, T, \psi_1 \rightarrow 0 \) as \( x_3 \rightarrow \infty \) are given by

\[ (\phi_1, \phi, \psi, T) = \sum_{i=1}^{4} (1, r_i, s_i, h_i) B_i e^{-m_{13}^i} e^{i(\xi - ct)} \]

and from (21), we get

\[ \psi_1 = B_i e^{-m_{13}^i} e^{i(\xi - ct)} \]

where \( m_i (i = 1, 2, 3, 4) \) are roots of the equation (22) and \( m_i \) is root of equation (23). \( B_i (i = 1, 2, 3, 4, 5) \) are arbitrary constants in equation(21) and (22).

Here the coupling constants are

\[ r_i = \frac{D_{90}}{D_{80}}, s_i = \frac{D_{90}}{D_{80}}, h_i = -\frac{D_{90}}{D_{80}}; \quad i = 1, 2, 3, 4 \]

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$D_{ij}, D_{2i}, D_{3i}$ are given in the Appendix.

**Boundary conditions**

The boundary conditions at the free surface $x_3 = 0$. Mathematically these can be written as

1. $T_{ij} = 0$
2. $T_{ij} = 0$
3. $\sigma_{ij} = 0$
4. $\tau_{ij} = 0$
5. $\frac{\partial T}{\partial x_3} = 0$

**Derivation of the secular equation**

Making use of (21) and (22) in the boundary conditions (26)–(30) and with the aid of (1)–(3), we obtain a system of five simultaneous homogeneous linear equations

$$\sum_{j=1}^{5} Q_{ij} B_j = 0 \quad \text{for } i=1,2,3,4,5 \quad (31)$$

where

$$Q_{ij} = \begin{cases} p_1 m_j^2 - \xi^2 & \text{for } j = 1, 2, 3, 4, 5 \end{cases}$$

$$Q_{ij} = \begin{cases} 2i \xi m_j & \text{for } j = 1, 2, 3, 4, 5 \end{cases}$$

$$Q_{ij} = \begin{cases} -m_j (p_s j + p_s j) & \text{for } j = 1, 2, 3, 4, 5 \end{cases}$$

$$Q_{ij} = \begin{cases} -m_j (p_s j + p_s j) & \text{for } j = 1, 2, 3, 4, 5 \end{cases}$$

$$Q_{ij} = \begin{cases} h_j & \text{for } j = 1, 2, 3, 4, 5 \end{cases}$$

**Particular case**

If $h_1 = \gamma = \alpha_2 = b = \gamma_2 \to 0$, Eq. (37) yields the expressions for thermoelectrical material with voids.

**Numerical results and discussion**

The material chosen for the purpose of numerical computation is copper, whose physical data is given by Sherief et al., as,
3. The magnitude of the determinant of Rayleigh wave secular equation, Rayleigh wave velocity and Attenuation coefficient increase with the increase in the value of phase velocity for higher values of wave number.

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**Conflict of interest**

The author declares there is no conflict of interest.

**References**

1. Biot MA. General theory of three–dimensional consolidation. *J Appl Phys*. 1941;12(2):155–164.
2. Barenblatt GI, Zheltov IP, Kochina IN. Basic Concept in the theory of seepage of homogeneous liquids in fissured rocks (strata). *J Appl Math Mech*. 1960;24(5):1286–1303.
3. Wilson RK, Aifantis EC. On the theory of consolidation with double porosity. *Int J Engg Sci*. 1982;20(9):1009–1035.
4. Khaled MY, Beskos DE, Aifantis EC. On the theory of consolidation with double porosity–III. *Int J Numer Anal Meth Geomech*. 1984;8:101–123.
5. Wilson RK, Aifantis EC. On the theory of consolidation with Double Porosity–II. *Int J Engg Sci*. 1986;24(11):1697–1716.
6. Beskos DE, Aifantis EC. A Double Porosity Model for Acoustic Wave propagation in fractured porous rock. *Int J Engg Sci*. 1984;22(8–10):1209–1227.
7. Wilson RK, Aifantis EC. On the theory of consolidation with Double Porosity–II. *Int J Engg Sci*. 1986;24(11):1697–1716.
8. Aifantis EC. On the theory of consolidation with Double Porosity–III. *Int J Engg Sci*. 1984;22(8–10):1209–1227.
9. Aifantis EC. The mechanics of diffusion in solids. *Acta Mechanics*. 1980;37(3–4):265–296.
10. Aifantis EC. The mechanics of diffusion in solids. TAM Report No. 440, Dept. of Theor Appl Mech, University of Illinois, Urbana, Illinois, India; 1980.
11. Khalili N, Selvadurai APS. A Fully Coupled Constitutive Model for Thermo–hydro–mechanical Analysis in Elastic Media with Double Porosity. *Geophys Res Lett*. 2003;30(24):2268–2271.
12. Straughan B. Stability and Uniqueness in Double Porosity Elasticity. *Int J Engg Sci*. 2013;65:1–8.
13. Svanadze M. Fundamental solution in the theory of consolidation with double porosity. *J Mech Behav Mater*. 2005;16:123–130.
14. Svanadze M. Dynamical Problems on the Theory of Elasticity for Solids with Double Porosity. *Proc Appl Math Mech*. 2010;10:209–310.
15. Svanadze M. Plane Waves and Boundary Value Problems in the Theory of Elasticity for solids with Double Porosity. *Acta Appl Math*. 2012;122(1):461–470.
16. Svanadze M. On the Theory of Viscoelasticity for materials with Double Porosity. *Disc and Cont Dynam Syst Ser B*. 2014;9(7):2335–2352.
17. Svanadze M. Uniqueness theorems in the theory of thermoelasticity for solids with double porosity. *Meccanica*. 2014;49(9):2099–2108.
18. Rayleigh L. On waves propagated along the plane surface of an elastic medium.
solid. *Proc London Math Soc.* 1885;17(1):4–11.

19. Lockett FJ. Effect of thermal properties of a solid on the velocity of Rayleigh waves. *J of Mech Phys Solids.* 1958;7(1):71–75.

20. Chadwick P, Windle DW. Propagation of Rayleigh waves along isothermal and insulated boundaries. *Proc R Soc Lond A.* 1964;280(1380):47–71.

21. Kumar R, Kansal T. Effect of rotation on Rayleigh waves in transversely isotropic generalized thermoelastic diffusive half-space. *Arch Mech.* 2008;60(5):421–433.

22. Kumar R, Kansal T. Propagation of Rayleigh waves in transversely isotropic generalized thermoelastic diffusion. *J Enng Phys Thermophys.* 2009;82(6):1199–1210.

23. Sharma JN, Kaur D. Rayleigh waves in rotating thermoelastic solids with voids. *Int J Appl Math Mech.* 2010;6(3):43–61.

24. Kumar R, Ahuja S, Garg SK. Rayleigh waves in isotropic micro stretch thermoelastic diffusion solid half-space. *L Amer J solid Struct.* 2014;11:299–319.

25. Kumar R, Gupta V. Rayleigh waves in generalized thermoelastic medium with mass diffusion. *Canadian J phys.* 2015;93(10):1–11.

26. Abd–Alla M, Hammad HS, Abo–Dahab SM. Rayleigh waves in a magnetoelastic half–space of orthotropic material under influence of initial stress and gravity field. *Appl Math Comp.* 2004;154(2):583–597.

27. Abd–Alla N, Abo–Dahab SM. Rayleigh waves in magneto–thermo–viscoelastic solid with thermal relaxation times. *Appl Math Comp.* 2004;149(3):861–877.

28. Abd–Alla M, Abo–Dahab SM, Hammad HA, et al. On generalized magneto–thermoelastic Rayleigh waves in a granular medium under influence of gravity field and initial stress. *J Vib Control.* 2011;17(1):115–128.

29. Abd–Alla M, Hammad HA, Abo–Dahab SM. Propagation of Rayleigh waves in generalized magneto–thermoelastic orthotropic material under initial stress and gravity field. *Appl Math Model.* 2011;35(6):2981–3000.

30. Abd–Alla M, Abo–Dahab SM, Bayones FS. Rayleigh waves in generalized magneto thermo–viscoelastic granular medium under the influence of rotation, gravity field, and initial stress. *Math Prob Eng.* 2011;1–47.

31. Abd–Alla M, Abo–Dahab SM, Al–Thamali TA. Propagation of Rayleigh waves in a rotating orthotropic material elastic half–space under initial stress and gravity. *J Mech Sci Tech.* 2012;26(9):2815–2823.

32. Abd–Alla M, SM Abo–Dahab SM, Bayones FS. Propagation of Rayleigh waves in magneto–thermo–elastic half–space of a homogeneous orthotropic material under the effect of the rotation, initial stress and gravity field. *J Vib Control.* 2013;19(9):1395–1420.

33. Abd–Alla M, Aflah Khan, Abo–Dahab SM. Rotational effect on Rayleigh, Love and Stoneley waves in fibre–reinforced anisotropic general viscoelastic media of higher and fraction orders with voids. *J of Mech Sci Tech.* 2015;29(10):4289–4297.

34. Singh S, Kumar J, Singh. Propagation of the Rayleigh wave in an initially stressed transversely isotropic dual phase lag magneto–thermoelastic half space. *J of Enng phys and Thermophysics.* 2014;87(6):1539–1547.

35. Kumar R, Gupta V. Rayleigh waves in generalized thermoelastic medium with mass diffusion. *Canad J of Phys.* 2015;93(10):1039–1049.

36. Biswas S, Mukhopadhay B, Shaw S. Rayleigh surface wave propagation in orthotropic thermoelastic solids under three–phase lag model. *J Therm Stresses.* 2017;40(4):403–419.

37. Singh S, Kumar J, Singh. Propagation of Rayleigh wave in two–temperature dual–phase–lag thermoelasticity. *Mech Mech Enng.* 2017;21(1):105–116.

38. Biswas S, Abo–Dahab SM. Effect of phase–lags on Rayleigh wave propagation in initially stressed magneto–thermoelastic orthotropic medium. *Applied Mathematical Modeling.* 2018;59:713–727.

39. Hassien NS, Bayones FS. Effect of rotation on Rayleigh waves in a fiber–reinforced solid anisotropic magneto–thermo–viscoelastic media. *Mechanics of Advanced materials and Structures.* 2018.

40. Iesan D, Quintanilla R. On a theory of thermoelastic materials with a double porosity structure. *J Therm Stresses.* 2014;37(9):1017–1036.

41. Sherief H, Saleh H. A half space problem in the theory of generalized thermoelastic diffusion. *Int J Solid and Structures.* 2005;42(15):4484–4493.

42. Khalili N. Coupling effects in double porosity media with deformable matrix. *Geophys Res Lett.* 2003;30(22):2153.