CONSTRaining SUB-Parsec Binary Supermassive Black Holes (BHs) in Normal Broad-line Quasars at z < 0.8, using multi-epoch Sloan Digital Sky Survey (SDSS) spectroscopy of the broad Hβ line. Our working model is that (1) one and only one of the two BHs in the binary is active; (2) the active BH dynamically dominates its own broad-line region (BLR) in the binary system, so that the mean velocity of the BLR reflects the mean velocity of its host BH; (3) the inactive companion BH is orbiting at a distance of a few RBLR, where RBLR ∼ 0.01–0.1 pc is the BLR size. We search for the expected line-of-sight acceleration of the broad-line velocity from binary orbital motion by cross-correlating SDSS spectra from two epochs separated by up to several years in the quasar rest frame. Out of ~700 pairs of spectra for which we have good measurements of the velocity shift between two epochs (1σ error ∼40 km s⁻¹), we detect 28 systems with significant velocity shifts in broad Hβ, among which 7 are the best candidates for the hypothesized binaries, 4 are most likely due to broad-line variability in single BHs, and the rest are ambiguous. Continued spectroscopic observations of these candidates will easily strengthen or disprove these claims. We use the distribution of the observed accelerations (mostly non-detections) to place constraints on the abundance of such binary systems among the general quasar population. Excess variance in the velocity shift is inferred for observations separated by longer than 0.4 yr (quasar rest frame). Attributing all the excess to binary motion would imply that most of the quasars in this sample must be in binaries, that the inactive BH must be on average more massive than the active one, and that the binary separation is at most a few times the size of the BLR. However, if this excess variance is partly or largely due to long-term broad-line variability, the requirement of a large population of close binaries is much weakened or even disfavored for massive companions. We perform a systematic search for sub-parsec binary supermassive black holes (BHs) in normal broad-line quasars at z < 0.8, using multi-epoch Sloan Digital Sky Survey (SDSS) spectroscopy of the broad Hβ line. Our working model is that (1) one and only one of the two BHs in the binary is active; (2) the active BH dynamically dominates its own broad-line region (BLR) in the binary system, so that the mean velocity of the BLR reflects the mean velocity of its host BH; (3) the inactive companion BH is orbiting at a distance of a few RBLR, where RBLR ∼ 0.01–0.1 pc is the BLR size. We search for the expected line-of-sight acceleration of the broad-line velocity from binary orbital motion by cross-correlating SDSS spectra from two epochs separated by up to several years in the quasar rest frame. Out of ~700 pairs of spectra for which we have good measurements of the velocity shift between two epochs (1σ error ∼40 km s⁻¹), we detect 28 systems with significant velocity shifts in broad Hβ, among which 7 are the best candidates for the hypothesized binaries, 4 are most likely due to broad-line variability in single BHs, and the rest are ambiguous. Continued spectroscopic observations of these candidates will easily strengthen or disprove these claims. We use the distribution of the observed accelerations (mostly non-detections) to place constraints on the abundance of such binary systems among the general quasar population. Excess variance in the velocity shift is inferred for observations separated by longer than 0.4 yr (quasar rest frame). Attributing all the excess to binary motion would imply that most of the quasars in this sample must be in binaries, that the inactive BH must be on average more massive than the active one, and that the binary separation is at most a few times the size of the BLR. However, if this excess variance is partly or largely due to long-term broad-line variability, the requirement of a large population of close binaries is much weakened or even disfavored for massive companions.

Theoretical studies on BBHs in the past two decades have laid down the basic framework for the dynamical evolution of BBHs within a merged galaxy (for a review, see, e.g., Merritt & Milosavljević 2005). The two BHs sink to the center of the merged host via dynamical friction and form a BBH. The BBH orbit continues to decay by scattering stars in the galactic nucleus until it depletes the loss cone. In the absence of other mechanisms to further extract energy from the binary, or to refill the loss cone, the BBH will stall at ~pc scale, establishing the “final parsec barrier” (Milosavljević & Merritt 2001). A number of mechanisms have been suggested to circumvent this barrier, though in most cases their effectiveness is quite uncertain: non-spherical stellar potentials (e.g., Yu 2002; Merritt & Poole 2004; Khan et al. 2013), interactions with gas (e.g., Armitage & Natarajan 2002; Escala et al. 2005; Mayer et al. 2007; Cuadra et al. 2009), or hierarchical triple BH systems (e.g., Blaes et al. 2002) through the Kozai–Lidov mechanism (Kozai 1962). Once the BBH reaches orbital separations below ~0.01–0.05 pc (for a ~10⁶ M⊙ BBH), gravitational radiation will dominate the successive decay of the binary orbit, and merge the two BHs within a Hubble time (e.g., Peters 1964). However, there is considerable theoretical uncertainty in the BBH evolution from
a few parsec to the gravitational regime, and the time the BBH spends during this phase is not well constrained.

The strongest indirect evidence for BBHs perhaps comes from the observed central light deficit in massive ellipticals (e.g., Ferrarese et al. 1994; Faber et al. 1997), where the scouring of the central stellar cusp is most easily explained by star interactions with the BBH (e.g., Ebisuzaki et al. 1991; Milosavljević & Merritt 2001). Other indirect evidence includes unusual radio morphologies such as helical/winged-X-shaped jet or double-lobed morphologies, which are commonly explained by BBH models (e.g., Roos et al. 1993; Romero et al. 2000; Merritt & Ekers 2002; Liu et al. 2003; Liu 2004; Liu & Chen 2007)—although the BBH interpretation is not unique for these observations.

Directly observing BBHs during different merger stages is challenging, given the stringent resolution requirement and the difficulty of identifying active galactic nuclei (AGNs). Although close quasar pairs (binary quasars) on tens of kiloparsec scales have been known for a while (e.g., Djorgovski 1991; Kochanek et al. 1999; Hennawi et al. 2006, 2010), clear examples of kpc-scale binary AGNs were substantially rarer until recently (e.g., Owen et al. 1985; Komossa et al. 2003; Ballo et al. 2004; Bianchi et al. 2008).

Considerable progress has been made in the last decade on the identification of binary AGNs on kpc scales, mostly owing to the advent of modern, large-scale sky surveys. Below a few kpc, a growing number of confirmed or candidate binary AGNs have been selected from multiwavelength surveys (e.g., Zhou et al. 2004; Comerford et al. 2009a, 2009b, 2012; Wang et al. 2009; Xu & Komossa 2009; Liu et al. 2010a, 2010b, 2011, 2012, 2013; Smith et al. 2010; Shen et al. 2011a; Rosario et al. 2011; Koss et al. 2011, 2012; McGurk et al. 2011; Fu et al. 2011, 2012; Barrows et al. 2012; Ge et al. 2012), increasing the inventory of confirmed kiloparsec-scale binary AGNs already by several fold. Below ~100 pc, there is only one clear example, 0402+379, where a pair of BHs with a projected separation of ~7 pc were detected in the radio (e.g., Rodriguez et al. 2006) with the Very Long Baseline Array; in addition, there is one claimed binary AGN with a projected separation of ~150 pc in the nearby Seyfert galaxy NGC 3393 (Fabbiano et al. 2011).

So far there is no confirmed case of a BBH with sub-parsec separation. This scale is unresolvable with current instruments except with long baseline radio interferometry. Thus the observational searches for sub-parsec BBHs almost exclusively rely on spatially unresolved signatures.

The first potential indirect signature is a periodic photometric light curve, where the periodicity may indicate the presence of a BBH. The best known candidate is OJ 287, a z ~ 0.306 blazar with its light curve showing a quasi-period of ~9 yr in rest frame, which is interpreted as a sub-parsec BBH with a mass ratio q < 0.01 in the latest models (e.g., Valtonen et al. 2008). Other examples have been claimed in the literature but these are not as robust as OJ 287 (see Merritt & Milosavljević 2005, and references therein).

A second signature commonly utilized for searching sub-parsec BBHs is the velocity offset between the broad emission line centroid and the systemic velocity in broad-line quasars (e.g., Gaskell 1983; Peterson et al. 1987; Gaskell 1996; Popović et al. 2000; Shen & Loeb 2010). Since the broad-line region (BLR) is gravitationally confined to the BH on sub-parsec scales (e.g., Peterson 1997, and see below), any velocity offset may reflect the orbital motion of the BH in a hypothetical binary. Two classes of broad-line velocity offset have attracted attention in the literature as indicators of candidate BBHs: (1) a double-peaked broad-line profile, which is interpreted to mean that both BHs are active and co-rotating along with their own distinct BLRs; (2) a single-peaked broad-line profile, with the broad-line centroid offset from the systemic velocity, in which case only one BH is active, or both BHs are active but their BLR emission cannot be separated in velocity space. The second feature is also often used to argue for recoiling BHs after the coalescence of the BBH (e.g., Loeb 2007).

It is straightforward to select such candidate sub-parsec BBHs based on these broad-line diagnostics, especially in the era of massive spectroscopic surveys. Examples are presented in either large samples (e.g., Eracleous & Halpern 1994; Strateva et al. 2003; Shen et al. 2011b; Shen & Loeb 2010; Eracleous et al. 1997; Shapovalova et al. 2001). The main difficulty here involves the poorly understood BLR geometry and kinematics even for single BHs, which may mimic a BBH. For instance, the rival model for the double-peaked or offset broad lines is a disk emitter model (e.g., Chen et al. 1989; Eracleous & Halpern 1994; Eracleous et al. 1995), which does not require a BBH to explain the peculiar broad-line profile. Thus extraordinary caution is required when interpreting these unusual broad-line profiles as BBHs (e.g., Gaskell 2010; Shen & Loeb 2010; Eracleous et al. 2012; Popović 2012). In addition, some theoretical arguments suggest that the double-peaked broad-line profile is a very ineffective indicator for BBHS, given the requirement that both BLRs are largely dynamically distinct while their relative motion must be large enough to separate the two peaks in the spectrum—such conditions are difficult to fulfill in reality (e.g., Shen & Loeb 2010).

The most effective, and the most definitive, way to improve the power of the broad-line diagnostics to identify BBHs is multi-epoch spectroscopy. Long-term spectroscopic monitoring of the broad lines can confirm or rule out the BBH hypothesis based on changes in the broad-line radial velocity (acceleration), given that the time baseline is long enough to see the expected binary motion and the spectral quality is adequate. Earlier spectral monitoring of samples of double-peaked disk emitters did not focus on testing the BBH scenario (e.g., Gezari et al. 2007; Lewis et al. 2010). Spectroscopic monitoring programs that are specifically designed to find BBHs in quasars are just starting (e.g., Eracleous et al. 2012; Decarli et al. 2013)

We are conducting systematic searches for sub-parsec BBHs in broad-line quasars using multi-epoch spectroscopy of large statistical quasar samples. In contrast to previous work (e.g., Eracleous et al. 2012; Decarli et al. 2013) which only focused on objects with offset or double-peaked broad lines, we are also targeting normal quasars with no broad-line velocity offset. There are two reasons for targeting normal quasars. First, quasars with double-peaked or significantly offset broad lines are only a minority of the entire quasar population (~10%, e.g., Shen et al. 2011b). While they are more likely to be BBHs by
selection, they do not represent the general quasar population where a BBH can be easily hidden in the spectrum (since we only observe the line-of-sight (LOS) velocity). Second, the acceleration in the radial velocity from the BBH is largest at zero velocity offset (see Section 2), therefore on average it is easier to detect acceleration for BBHs with non-offset broad lines than those with offset broad lines.

In this paper we present results for the general quasar population, i.e., without pre-selection of a subsample with double-peaked or offset broad lines, based on multi-epoch spectroscopy separated by up to several years. As shown in Figure 1, the vast majority of our quasars have broad-line velocity offset within 1000 km s$^{-1}$ of the systemic velocity (based on [O III] λ5007). Thus these quasars will not have been included in the samples of Eracleous et al. (2012) and Decarli et al. (2013). Our sample is thus substantially larger than the other two samples, allowing better statistical constraints. In the second paper of the series (hereafter Paper II), we will focus on a subset of quasars selected with offset broad lines, and present our own multi-epoch spectroscopic data and analysis in the context of the BBH scenario.

The organization of the paper is as follows. In Section 2, we provide preliminaries on the broad-line diagnostics in the BBH scenario. The data and our approach to measure radial velocity acceleration with multi-epoch spectra are described in Section 3. We pay particular attention to a proper error analysis, which is key to this kind of study. In Section 4 we present candidate detections of acceleration (which implies a possible BBH), and the statistical constraints on the general BBH population in quasars using (mostly) non-detections. In Section 5 we discuss the caveats of our approach and compare to previous studies, and we summarize our conclusions in Section 6. By default all time separations are in the rest frame of the quasar, and L refers to the quasar bolometric luminosity unless otherwise specified. We use “offset” to refer to the velocity difference between two lines in single-epoch spectra, and “shift” to refer to the changes in the line velocity between two epochs. For simplicity we assume circular BBH orbits, but the general methodology can be applied to a BBH population with an eccentricity distribution as well.

2. PRELIMINARIES

Since essentially all quasars in our sample have single-peaked broad lines, we initially assume that the broad-line emission is dominated by one active BH. We will return to the possibility that the broad-line emission comes from both BHs in Section 5.1.

Consider a postulated SMBH binary on a circular orbit, where only BH 1 is active and powering its BLR. The orbital period, LOS velocity, and acceleration (relative to the systemic frame) of the active BH are

$$P = 2\pi d^{3/2} (GM_{\text{tot}})^{-1/2} = 9.4 d_{0.01}^2 M_{\text{tot}}^{-1/2} \text{yr},$$

$$V_1 = M_2^2 M_{\text{tot}}^{-1/2} d / d_0^{1/2} \sin I \sin \beta \text{ km s}^{-1},$$

$$a_1 = G M_2 / d_0^2 \sin \phi \text{ km s}^{-1} \text{yr}^{-1},$$

where subscripts 1 and 2 refer to BH 1 and 2. $M_2 = M_1 + M_2$ is the inclination of the orbit, $d$ is the binary separation, and $\phi = \phi_0 + 2\pi t / P$ is the orbit phase. We use the conventions $M_{\text{tot}} = (M_1 + M_2) / 10^8 M_\odot$ and $d_0 = (d / 0.01 \text{ pc})$. For zero velocity offset relative to the systemic velocity, $\phi = 0$, and the acceleration is at its maximum.

The typical size of the BLR (for H$\beta$) around a single BH with mass $M_1$ is \footnote{This BLR size is approximate only, assuming a fixed slope $\alpha = 0.5$ in the BLR $R - L$ relation, and so slightly differs from directly observed relations (e.g., Kaspi et al. 2005; Bentz et al. 2009). It also changed slightly from Shen & Loeb (2010) to reflect the most recent measurements of the $R - L$ relation. In our analyses in Sections 3 and 4 we estimate the BLR size using the best-fit relation reported in Bentz et al. (2009) instead of the approximation in Equation (2).} (Shen & Loeb 2010)

$$R_{\text{BLR}} \sim R_0 (L / L_\odot)^{1/2} \sim 2.7 \times 10^{-2} \left( \frac{L}{10^{45} \text{ erg s}^{-1}} \right)^{1/2} \text{ pc},$$

$$R_{\text{BLR}} \sim 3 \times 10^{-2} \left( \frac{L}{10^{45} \text{ erg s}^{-1}} \right)^{1/2} \left( \frac{M_1}{10^8 M_\odot} \right)^{1/2} \text{ pc},$$

following the observed $R - L$ relation for the reverberation mapping AGN sample (e.g., Kaspi et al. 2005; Bentz et al. 2009). Here $R_0$ and $L_0$ are constants, and $M_1 \equiv L / L_\odot$ is the luminosity Eddington ratio with $L_\odot = 1.26 \times 10^{38} (M_1 / M_\odot) \text{ erg s}^{-1}$ the Eddington luminosity. This observed relation is tight, with only <40% scatter in the predicted BLR size (e.g., Bentz et al. 2009). The dynamical time of the BLR is

$$t_{\text{dyn}} \sim R_{\text{BLR}} / \text{FWHM} \sim 24 (R_{\text{BLR},0.1} / \text{FWHM}_{4000}) \text{ yr},$$

where FWHM$_{4000}$ is the broad-line FWHM in units of 4000 km s$^{-1}$, and $R_{\text{BLR},0.1}$ is the BLR size in units of 0.1 pc.

Now we can derive some crude criteria for the detectability of the binary motion with two-epoch spectroscopy separated by rest frame time $\Delta t$. For simplicity, we ignore the detailed shape of the line profile and consider only the width (FWHM) of the broad line. Suppose the sensitivity of our velocity shift measurement is a fraction of the FWHM of the broad line,
where $f_d = |\sin l \cos \phi|$. $f_d$ is typically inversely proportional to the S/N of the line (e.g., Lindegren 1978, and see Section 3.3). Obviously, detection of binary motion is easier if the time separation $\Delta t$ is large (so long as $\Delta t \ll P$, which is generally true), the binary separation $d$ is small, and the FWHM is small.

The radius of the BLR must be smaller than the separation of the BHs, so $R_{\text{BLR}} \leq f_d d$, where $f_r$ is less than unity. Otherwise the BLR becomes circumbinary, the dynamics of the BLR becomes more complicated, and there may be no coherent velocity drift in the broad-line emission (e.g., Shen & Loeb 2010). $f_d d$ is then the maximum size of the BLR before it is dynamically affected by the companion dormant BH in the system. We can define $f_r$ as the average radius of the Roche lobe in a circular binary system (e.g., Paczyński 1971):

$$f_r = 0.38 - 0.2 \log q, \quad 0.05 < q < 1.88$$

$$= 0.46224(1 + q)^{-1/3}, \quad q > 1.88$$  

(5)

where $q \equiv M_2/M_1$ is the binary mass ratio.

Now we consider this maximum BLR size, i.e., we set $R_{\text{BLR}} = f_d d$, and derive some crude constraints on the detectability based on the properties of the active BH 1. Since the BLR around BH 1 is dominated by the gravity of BH 1, we have

$$M_1 = f_m \text{FWHM}^2 R_{\text{BLR}}/G,$$  

(6)

where $f_m$ is the virial coefficient, determined by the geometry of the BLR. An empirical estimate is $f_m \approx 1.4$ (e.g., Onken et al. 2004). Substituting Equation (6) to Equation (4) we obtain

$$f_m f_a f_d^2 M_2 \text{FWHM} \Delta t \equiv f_d M_1 R_{\text{BLR}}/R_0 > 1,$$  

(7)

Using the observed $R-L$ relation we can recast the above equation into

$$f_m f_a f_d^2 M_2 \text{FWHM} \Delta t \equiv M_1 R_0 (L/L_0)^{1/2} > 1,$$  

(8)

from which we can define a figure of merit (FoM) of the detection:

$$\text{FoM} = \text{FWHM}/L^{1/2}.$$  

(9)

This FoM is based on two observables, line width and luminosity, and the larger it is, the more likely we will detect the velocity shift due to binary motion in a hypothetical BBH, assuming everything else is equal in Equation (8).

If we substitute FWHM in Equation (8) with Equation (6) and use the $R-L$ relation (2) we obtain

$$C f_m f_a M_2 \Delta t (L/L_{\text{Edd}})^{1/2} / L^{5/4} > 1,$$  

(10)

where all the other known quantities are absorbed in $C$. We can therefore define an alternative FoM based on the luminosity and Eddington ratio of the active BH 1:

$$\text{FoM}' = (L/L_{\text{Edd}})^{1/2} / L^{5/4}.$$  

(11)

From this we see that if all quasars have the same Eddington ratio and other conditions are the same, including the companion mass $M_2$, lower luminosity (or equivalently, lower $M_1$) quasars are more likely to have detectable velocity shifts due to binary motion (if instead the mass ratio $M_2/M_1$ were constant, FoM' would vary as $L^{-1/4}$ at fixed Eddington ratio, so the conclusion would be similar but weaker). One caveat here for both FoM definitions is that $f_d$ in conditions (8) and (10) scales inversely with S/N, while generally lower luminosity objects have lower S/N at fixed redshift in a given survey, which will reduce the detectability.

Note that in our postulated binary system there is only one active BH. This is different from the scenario studied in Shen & Loeb (2010), where both BHs are active. Therefore we do not require the additional condition that the difference in LOS velocities of the two BLRs must exceed the FWHM of the line to make a double-peaked profile (e.g., the upper limit of $d$ in Equation (6) of Shen & Loeb 2010).

### 3. DATA

We start from the DR7 compilation of the spectroscopic quasar catalog (Schneider et al. 2010) from Sloan Digital Sky Survey (SDSS). This sample contains 105,783 quasars brighter than $M_r = -22.0$ that have at least one broad emission line with FWHM larger than 1000 km s$^{-1}$ or have interesting complex absorption features. The reduced and calibrated one-dimensional (1D) spectral data used in this study are available through the SDSS Data Archive Server. The spectral resolution is $R \sim 1850-2200$, and the 1D spectra are stored in vacuum wavelength, with a pixel scale of $10^{-4}$ in log-wavelength, which corresponds to 69 km s$^{-1}$. All spectra are wavelength calibrated to the heliocentric reference, with an accuracy of better than 5 km s$^{-1}$. Throughout the paper, we refer to the signal-to-noise ratio per pixel as S/N.

Several thousand of these DR7 quasars have multiple spectra taken at different epochs by SDSS. Most of them have two epochs, and a small fraction of them have multiple epochs. The default spectrum reported in the DR7 quasar catalog is usually the one with the highest S/N, and we always take that default spectrum as Epoch 1 regardless of the relative time between the two epochs. Epoch 2 is then the other available spectra in SDSS (could be more than one). We call each Epoch 1/2 combination a pair of observations. These quasars with repeated spectroscopy in SDSS form the sample in which we search for velocity shift of the broad line. We will focus on the H$\beta$ line for this systematic study, which restricts us to $z \lesssim 0.9$, and reduces the number of available quasars to below 2000.

We choose the H$\beta$ line rather than other broad lines for the following reasons: (1) it has robust systemic redshift from the [O III] lines (which are shifted out of SDSS spectrum for $z \gtrsim 0.8$); (2) the presence of [O III] line in the SDSS spectrum provides an empirical way to subtract the narrow H$\beta$ component; (3) the rest frame time separation $\Delta t$ between two epochs is longer for the H$\beta$ sample than for higher-redshift quasars; (4) the broad Balmer lines are better studied and better understood than Mg II or C IV lines, especially in terms of reverberation mapping studies, therefore their BLR size and virial BH mass estimates are the most reliable (e.g., Shen 2013 and references therein); (5) Mg II is difficult because of the complication of the strong Fe II emission underneath Mg II; C IV is difficult because this line is more asymmetric and may arise
from a disk-wind component; in addition, for both Mg ii and C iv the narrow line subtraction will be ambiguous; and (6) finally, Hα is a line complex enclosing more than one narrow line ([N ii] λλ 6548, 6584, narrow Hα), and the broad-line component isolation is seemingly difficult.

Nevertheless, if we detect any Hβ velocity shift, we will investigate the Mg ii and Hα lines at the two epochs, if covered in the SDSS spectrum.

The basic sample statistics are summarized in Table 1. For rest frame Δt > 100 days, there are 834 pairs of observations over 674 unique quasars; most of them have a rest frame time span of less than a year. We do not remove duplicated objects, and treat multiple pairs for the same object as independent measurements in our statistical analysis in Section 4.

### 3.1. Measurements of Spectral Properties

In order to characterize the properties of quasars in our sample, we measure the continuum and emission line properties using a combination of model fitting and direct measurements on the spectrum. This process also produces the broad-line spectra (i.e., continuum and narrow line removed) on which we will perform the cross-correlation to measure velocity shifts in Section 3.2.

The basic spectral fitting approach follows our earlier work in Shen et al. (2008, 2011b). In short, a Fe ii template plus a power-law continuum is fit to several line-less windows around Hβ to form a pseudo-continuum; this pseudo-continuum model is then subtracted from the spectrum, leaving the emission lines. The narrow emission lines are modeled by Gaussians (in logarithmic wavelength) with the same width and their relative velocity offsets fixed to those based on their rest frame wavelengths. Whenever necessary we fit two Gaussians to the [O iii] lines to account for the core and the wing components often seen in [O iii]; the narrow Hβ is tied to the core [O iii] component in such cases. The broad Hβ profile is fit with a mixture of Gaussians (up to five, but three are enough in most cases). These Gaussian functions are merely used to reproduce the line profile and we assign no physical meaning to individual Gaussian components. We visually inspect all fits to make sure they are reasonable in the sense that the line is well reproduced by the model, and we require a reduced $\chi^2 < 5$ for the fits. Figure 2 shows two examples of the decomposition of different spectral components. The [O iii] λλ 5007/narrow Hβ flux ratio is tied to be the same at both epochs. This is important for cases where the narrow Hβ contributes a substantial amount to the Hβ profile, and incorrect subtraction of the narrow Hβ emission will likely impact the cross-correlation analysis in Section 3.2. Due to aperture effects and seeing/guiding variations between the two epochs, as well as the extended nature of the narrow line regions, the absolute narrow line flux could change, and so we do not fix the normalization of the narrow line flux at the two epochs. Visual inspections were performed to ensure there is no artificial line variation near the systemic velocity between the two epochs due to the narrow line removal.

Once we have decomposed the spectrum around the Hβ region, we measure a variety of broad-line properties both using the model fit and using the raw (broad-line only) spectrum:

1. Line peak, FWHM, line flux, and rest frame equivalent width (EW) of the broad line from the model fit;
2. Line centroid or first moment of the line: $\langle \lambda \rangle \equiv \sum \lambda_i f_i / \sum f_i$, where $\lambda_i$ and $f_i$ are the pixel wavelength and flux density. To compute the centroid and higher order moments we use the raw spectrum, and we only use pixels that enclose $2 \times$ FWHM of the line peak. This is to reduce the adverse effects of the noisy wings in determining the line moments.

We also compute the second moment, skewness, and kurtosis of the broad line, again using all the pixels enclosing $2 \times$ FWHM

### Table 1

Sample Statistics

|       | All   | Good | Superior |
|-------|-------|------|----------|
| N_pair| N_qso| N_pair| N_qso| N_pair| N_qso|
| All   | 1910  | 1347 | 688      | 521  | 193  | 163  |
| Δt < 30 days | 757  | 613  | 235      | 207  | 0    | 0    |
| Δt > 100 days | 834  | 674  | 316      | 268  | 193  | 163  |
| Δt > 1 yr | 251  | 223  | 104      | 94   | 98   | 90   |
| Δt > 2 yr | 117  | 106  | 53       | 49   | 53   | 49   |
| Δt > 3 yr | 69   | 65   | 27       | 26   | 27   | 26   |
| Δt > 5 yr | 6    | 6    | 4        | 4    | 4    | 4    |

**Notes.** All times are in the rest frame of the quasar. The “good” sample is defined in Section 3.2, in which we search for binary candidates. The “superior” sample is a subset of the good sample, defined in Section 4.1, which we use to place statistical constraints on the binary population.
Line centroid (or line peak) is not a robust indicator for the velocity shift we are trying to measure, because it is only one characteristic of the broad line: slight changes in the line shape may produce artificial centroid shifts or hide a real shift. In the next section, we will measure the broad-line velocity shift with a cross-correlation technique (ccf), which is more robust in measuring the broad-line velocity shift between the two epochs.

The measured broad Hβ velocity shift will be used in our analyses in Section 4.

### 3.2. χ² Cross-correlation between Two Epochs

In order to constrain the velocity shift between two epochs and hence the orbital acceleration in the hypothetical binary, we deploy a χ² ccf similar to that used in Eracleous et al. (2012). We describe this approach in detail below.

We consider the pseudo-continuum and narrow-line subtracted spectra from Section 3.1, i.e., broad-line spectra only. This is different from the approach in Eracleous et al. (2012), which used several broad-line windows that avoid the narrow Hβ line and did not subtract the underlying pseudo-continuum in the cross-correlation. We prefer to use the broad-line spectra in the cross-correlation, because possible changes in the continuum slope will potentially bias the cross-correlation result if the full spectrum is used. We also want to use a large portion of the broad-line profile to increase the statistical power in the cross-correlation, which requires removal of the central narrow Hβ component. As discussed in Section 3.1, we carefully controlled the systematics from narrow Hβ subtraction in our fitting and visually inspected all fits to ensure that the narrow line subtraction does not introduce artificial changes between the two epochs near the systemic velocity.

We shift the second epoch broad-line spectrum by an increasing number of pixels, and compute the χ² as a function of the shift:

$$\chi^2 = \sum_i \frac{(f_{1,i} - f'_{2,i})^2}{\sigma_{1,i}^2 - \sigma_{2,i}^2},$$

where $f_{1,i}$ and $\sigma_{1,i}$ are the flux density and flux density errors of the $i$th pixel in the Epoch 1 spectrum, and $f'_{2,i}$ and $\sigma_{2,i}$ are the corresponding quantities of the $i$th pixel in the shifted Epoch 2 spectrum. We only use pixels within [4800, 4940] Å to compute the $\chi^2$, which encloses most of the broad-line flux and excludes the noisy wings. During the $\chi^2$ calculation, we also scale the second-epoch spectrum by the ratio of the integrated broad-line flux within [4800, 4940] Å of the two epochs. The latter step is necessary because the broad-line flux could vary between the two epochs, which will reduce the statistical power of the cross-correlation.

One great advantage of using the SDSS data is that both spectra were reduced using the same software and binned on the same logarithmic wavelength grid, i.e., 1 pixel shift corresponds to 69 km s⁻¹ shift. The step size in the grid of shifts that we use to compute the cross-correlation is 1 pixel, i.e., we do not use sub-pixel shifts and rebinning of the second-epoch spectra as in Eracleous et al. (2012). We chose to do so because the S/N per pixel is typically several, and so by interpolating on a sub-pixel grid and rebinning we would likely introduce systematics, which would complicate the resulting error estimation of the measured velocity shift.

Once we have the $\chi^2$ values, we determine the minimum $\chi^2$ and its location as follows: we fit the 20 points enclosing the minimum $\chi^2$ point with a sixth-order B-spline function.

Figure 3 shows how the measurement error in the broad-line centroid, $\sigma_{\text{cent}}$, changes with various quantities. The most significant trends are the correlations of $\sigma_{\text{cent}}$ with S/N and FWHM: $\sigma_{\text{cent}}$ decreases as S/N increases, and increases as FWHM increases. This is expected, as the uncertainty in measuring the line centroid is proportional to line width, and is inversely proportional to S/N, i.e., $\sigma_{\text{cent}} \propto \text{FWHM}/\text{SN}$. However, the trends shown in Figure 3 deviate somewhat from the expected dependence on FWHM and S/N, which is likely caused by the following: (1) there is a slight anti-correlation between FWHM and S/N for our objects; (2) the line shape also changes with line width, which may increase the difficulty of measuring line centroid for large line widths. Finally, there is a weak dependence of measurement errors on the strength of the line, such that stronger lines on average have smaller errors in measuring their centroid. This weak trend with line strength is likely a secondary effect of S/N, i.e., given fixed continuum level and photon noise, weaker lines have smaller S/N.

**Figure 3.** Measurement error in line centroid as functions of various broad-line properties: (a) median S/N per pixel for the broad-line flux; (b) Continuum luminosity at 5100 Å; (c) rest frame equivalent width of broad Hβ; (d) FWHM of the broad Hβ. The measurement error decreases as S/N increases, and increases as line width increases, as expected. In addition, stronger lines (larger EW) have on average smaller measurement errors in line centroid, but the trend is not as strong as those with S/N and FWHM, and is likely a secondary effect of S/N.
The minimum $\chi^2$ value is then determined from the spline, which allows us to measure sub-pixel shifts, as well as reduce the impact of noisy $\chi^2$ curve. We estimate the confidence range by finding the intercepts of the B-spline at $\Delta \chi^2 = \chi^2_{\text{min}} + 6.63$, which corresponds to 99% confidence range ($\sim 2.5\sigma$). In this way we measure the velocity shift $V_{\text{corr}}$ and the associated uncertainty between the two epochs.

We also apply this cross-correlation method to the [O III] $\lambda$5007 line, using pixels within [4995, 5020] Å. This allows us to check possible issues of bad wavelength calibration associated with either epoch. In most cases the shift of the [O III] $\lambda$5007 line is constrained to be $<20$ km s$^{-1}$, which is expected as the SDSS wavelength calibration is better than 5 km s$^{-1}$. We will use the uncertainty in the [O III] shift as an additional constraint to select individual detections of broad H$\beta$ velocity shift in Section 4.2.

Figure 4 shows an example of our $\chi^2$ cross-correlation procedure, where a velocity shift is detected at the $2.5\sigma$ significance level, and the rest frame time separation is $\sim 6$ months. Figure 5 shows another example, where no significant velocity shift is detected over a rest frame timescale of $\sim 1$ year. In both examples the shape of the broad H$\beta$ remains more or less the same, as indicated by the FWHM.

Many SDSS spectra are of low quality and often the resulting $\chi^2$ curve is too noisy to measure a well-defined minimum and confidence ranges. To eliminate these, we first filter our sample using a simple S/N cut, keeping only objects where both epochs have detected broad H$\beta$ at $> \sigma$ significance (where line fluxes and errors are taken from our spectral fits in Section 3.1). We then visually inspect the B-spline fits for all objects and discard cases where the $\chi^2$ curve is noisy, such that either a meaningful spline fit is not possible or that the formal confidence range from the spline fit is questionable. For such cases the two epochs cannot constrain the velocity shift well using this cross-correlation method. These objects usually have noisy spectra or very broad lines (FWHM $\gtrsim 10,000$ km s$^{-1}$). For the remaining objects, the $\chi^2$ curve shows a well-determined minimum and the B-spline provides reasonably good measurements of its minimum location and associated confidence levels.

While our rejection based on visual inspection of the $\chi^2$ curves is not rigorous, it is necessary to reject false positives and obtain a clean sample. As long as the remaining objects probe the same parameter space as the parent sample, we are not biasing the sample selection by this visual screening (see below). We kept 688 objects in our final sample, which we call the “good” sample. We will focus on this sample in the following analysis.

Figure 6 shows the statistical quasar properties of the good sample and the parent sample. Objects at high redshift and low luminosities are preferentially removed from our good sample because they have low quality spectra. Other than that, the
Figure 6. Distributions of quasar properties for different samples. From top to bottom, bolometric luminosity, virial BH mass estimates, and Hβ BLR size vs. redshift. The bolometric luminosity and virial BH mass estimates were taken from the Shen et al. (2011b) catalog. The BLR sizes were estimated from the 5100 Å continuum luminosity using the best-fit $R - L_{5100}$ relation in Bentz et al. (2009), therefore they have explicit dependence on luminosity. The gray points are for our parent sample and the black points are the “good” subsample defined in Section 3.2 in which we search for velocity shifts and statistical constraints on the general quasar population. The green points are the “superior” sample defined in Section 4.1, which is a subset of the “good” sample. Finally the red points are individual quasars in which a significant velocity shift has been detected (see Section 4.2). Other than removing objects with low-quality spectra at high redshift and low luminosity, the good sample is statistically indistinguishable from the parent sample. In addition, the “superior” sample is statistically indistinguishable from the “good” sample, as confirmed by standard KS tests.

(A color version of this figure is available in the online journal.)

Figure 7. Top: distributions of the measurement errors in the velocity shift $V_{ccf}$ between two epochs using the $\chi^2_{ccf}$ method. Bottom: distributions of the measurement errors in the LOS acceleration measured from two epochs, i.e., $a_{ccf} = V_{ccf}/\Delta t$. The black lines are for the “good” sample (Section 3.2), while the green dashed lines are for the “superior” sample (Section 4.1).

(A color version of this figure is available in the online journal.)

good sample probes the same parameter space of the general SDSS quasar population, thus there is no obvious bias from our sample refining. The resulting error distributions of the measured velocity shift and acceleration of the good sample are shown in Figure 7.

The virial BH masses probed by our sample have a median value of $1.8 \times 10^8 M_\odot$ with a dispersion of $\sim 0.4$ dex. We fully appreciate the uncertainty in these mass estimates (for a comprehensive review of the caveats of virial BH masses, see, e.g., Shen 2013). The $\sim 0.4$ dex dispersion in these virial BH masses is comparable to the typical virial mass uncertainty, and so it is reasonable to assume that our sample only probes a limited dynamic range in BH mass. The BLR size of our quasars is typically below 0.1 pc, with a median value of $\sim 0.06$ pc. The uncertainty of these BLR size estimates based on $L_{5100}$ is $< 40\%$ (e.g., Bentz et al. 2009), much better than that for the BH mass estimates, and so we will use these BLR size estimates as true values. These physical properties will be used in our later analyses in Section 4.
3.3. Tests of the $\chi^2$ Cross-correlation Method

To use the cross-correlation results in our statistical analyses, we must make sure that our error estimation of the velocity shift is reliable. We first test this using simple Monte Carlo experiments on mock data.

We generate mock broad-line spectra with different FWHM and S/N, sampled on the same wavelength grid as the SDSS spectra. For simplicity we assume a single-Gaussian intrinsic profile for the broad line, and we use the peak S/N (per pixel) as the indicator for the spectral S/N. We then run exactly the same $\chi^2$ cross-correlation procedure on the mock data, and derive the errors of the measured velocity shift between two epochs.

Figure 8 summarizes the results. The data points show the median and standard deviations from 5000 trials in our Monte Carlo mock tests. The dashed lines are not a fit to the points, but straight lines with a power-law slope of $-1$. This test indicates that the measurement error decreases as S/N increases and FWHM decreases, as expected. The two-sided arrows at the top indicate the 68% quantile range of actual peak line S/N for the “good” sample (black) and the “superior” sample (green). Our quasars have typical broad Hβ FWHM ~ 3500 km s$^{-1}$. Thus the expected typical measurement error is $\sim$40 km s$^{-1}$ (1$\sigma$), fully consistent with the actual values shown in Figure 7.

(A color version of this figure is available in the online journal.)

![Figure 8](image_url)

**Figure 8.** Tests on the measurement errors in the velocity shift using the $\chi^2$ ccf method, with mock spectra. Shown here are the measurement errors as functions of peak line S/N (per pixel) and FWHM. The points and error bars are the median and standard deviations from 5000 trials in our Monte Carlo mock tests. The dashed lines are not a fit to the points, but straight lines with a power-law slope of $-1$. This test indicates that the measurement error decreases as S/N increases and FWHM decreases, as expected. The two-sided arrows at the top indicate the 68% quantile range of actual peak line S/N for the “good” sample (black) and the “superior” sample (green). Our quasars have typical broad Hβ FWHM ~ 3500 km s$^{-1}$. Thus the expected typical measurement error is $\sim$40 km s$^{-1}$ (1$\sigma$), fully consistent with the actual values shown in Figure 7.

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Figure 8 summarizes the results. The data points show the median and standard deviations from 5000 trials. For a given FWHM, the measurement uncertainty decreases with S/N roughly as $\sigma_{Vccf} \propto (S/N)^{-1}$. This trend breaks down at both the large FWHM/low-S/N end and the small FWHM/high-S/N end. In the former regime it is very difficult to measure the velocity shift of a noisy and broad-line profile, as indicated by the large scatter in the error estimations. The latter is likely inherent to our $\chi^2$ error estimation procedure, such as the finite bin size in the ccf and the B-spline, but is not a concern since it is beyond the regime of interest in terms of S/N and line width. On the other hand, at fixed S/N, the measurement uncertainty increases as FWHM increases. This trend with FWHM is slightly slower than $\sigma_{Vccf} \propto$ FWHM because we are using the peak S/N instead of the S/N integrated over the line profile.

Overall these trends are fully consistent with the expectations of the precision in measuring the profile center with the cross-correlation method (e.g., Lindegren 1978), suggesting that our error estimation is reasonable.

The second test is to study the distributions of measured velocity shifts for the good sample. Figure 9 shows the distributions of measured velocity shifts for pairs separated by $\Delta t < 30$ and $\Delta t > 30$ days in rest frame (black histograms). For $\Delta t > 30$ days, if we assume there is no intrinsic change in the broad line, given that this time is much shorter than the BLR dynamical timescale or possible binary period, then the expected velocity shift distribution should be consistent with zero, broadened by the error distribution. The blue dashed line in the upper panel shows the expected zero velocity shift shuffled by our measurement errors for these objects, and it is not too much different from the observed distribution. If we add an additional error component of 100 km s$^{-1}$, the resulting velocity shift distribution would be the red histogram, which is inconsistent with what we observe. Therefore we are not underestimating our errors in the velocity shift by much. To quantify this statement we scale the errors up by a constant factor, and perform
Kolmogorov–Smirnov (KS) tests between the observed distribution and a simulated distribution of zero velocity shifts shuffled by the scaled-up errors. The results are shown in Figure 10, where we plot the median and standard deviations from 5000 random realizations at each error scale. We achieve the best agreement with the observed distribution if our nominal errors were scaled up by only 30%.

We also verify that the measurement error in the velocity shift, $\sigma_{vel}$, does not vary with the time separation between two epochs, as also indicated by the similar distributions of the blue dashed lines in the two panels of Figure 9. Thus the observed velocity shift distribution for $\Delta t > 30$ days shows excess variance compared with zeros broadened by errors. However, the errors in the measured velocity shifts are still quite large for many objects, prohibiting tight constraints on the intrinsic velocity shift distribution. Therefore in Section 4.1 we use a subset of the good sample with small measurement errors to place more stringent constraints on the intrinsic distribution.

The tests performed in this section suggest that our error estimation of the measured velocity shift is reliable, and hence we will proceed to statistical constraints and individual detections in the next section.

4. RESULTS

For the 688 pairs of observations that define our good sample, we have derived meaningful constraints on the velocity shift between two epochs from the cross-correlation analysis. As shown in Figure 7, the typical uncertainty in the measured velocity shift is $\sigma_{vel} \sim 40$ km s$^{-1}$. Although our spectra are generally of lower quality, this typical uncertainty is comparable to or better than that achieved in Eracleous et al. (2012), mainly due to the fact that we are using a larger portion of the broad line in the cross-correlation than in Eracleous et al. (2012).

We have 30 detections ($\sim 2.5\sigma$, 28 unique objects) among the 688 pairs in the good sample, 2 of which are detections with a rest frame time difference of less than one month. Figure 11 shows the detection fraction with Poisson errors for objects in the good sample, as a function of time separation. There is an apparent increase of the detection fraction as the time baseline increases. The detection fraction is $\lesssim 1%$—this is consistent with our detection threshold of $\sim 2.5\sigma$. This should be considered as an upper limit on the false-positive rate as both of the detections could be real with large accelerations (see further discussion of these two objects in Section 4.2). Thus the majority of detections with longer time baselines should be genuine.

The detection rate at $\Delta t < 30$ days can be used to estimate the false-positive rate due to statistics and systematics. We only detected 2 out of $\sim 230$ pairs with such short time intervals, or $\sim 1%$—this is consistent with our detection threshold of $\sim 2.5\sigma$. This should be considered as an upper limit on the false-positive rate as both of the detections could be real with large accelerations (see further discussion of these two objects in Section 4.2.2). Thus the majority of detections with longer time baselines should be genuine.

Figure 12 shows the broad H$\beta$ line shape properties of the “good,” “superior” (defined in Section 4.1) and the detected samples, which suggest that objects with detected broad H$\beta$ velocity shifts are not a distinct population from the parent sample. In addition, significant profile changes (in terms of FWHM and skewness) are not common for the timescales probed by our sample.

Below we will first derive constraints on the general population with the full distribution of observed velocity shifts (mostly non-detections) in Section 4.1 and then discuss individual detections in Section 4.2.
4.1. Constraints on the General Population

Most of our objects do not show detectable acceleration between the two epochs. We can use these non-detections to place constraints on a hypothetical binary population. Specifically, we will compare the distribution of measured accelerations with predictions from a binary population using the measurement error distribution in our \( \chi^2 \) ccf approach. In doing so, we have to assume that all the observed accelerations are either due to binary motion, or due to errors in the measurements, i.e., we neglect the complication that some apparent “acceleration” may be due to broad-line variability. We also stick to the simplified scenario described in Section 2 that only one BH is active and dominates its BLR dynamics. We will come back to the caveats in our assumptions in Section 5. Throughout Section 4.1 we adopt a constant mass for BH 1, \( M_1 = 1.8 \times 10^8 M_\odot \), i.e., the median virial BH mass for our sample. The reason for adopting a constant value rather using the full distribution of virial masses is because the latter is dominated by the large uncertainties associated with these virial BH masses (see discussion in Section 3.2). We will, however, use the full distribution of the measured BLR sizes, since the uncertainties associated with these estimates are substantially smaller.

To get the strongest constraints possible, we consider a further refinement of the “good” sample, with stringent limits on the precision of the measured acceleration between the two epochs, i.e., \( \sigma_{\text{acc}} < 50 \text{ km s}^{-1} \text{ yr}^{-1} \). Figure 13 shows the acceleration distributions for subsets of the “good” sample with different cuts on \( \sigma_{\text{acc}} \). As we increase the threshold error, the observed distribution broadens. Therefore to better constrain the intrinsic acceleration distribution, while still keeping a sufficient number of observations, we use the stringent 50 km s\(^{-1}\) yr\(^{-1}\) cut in refining the “good” sample. There are 193 pairs that satisfy this criterion, and all pairs are separated by a rest frame time span >0.4 yr. We refer to this sample as the “superior” sample. As shown in Figure 6, the distribution of these objects in the quasar parameter space is statistically indistinguishable from the “good” sample, hence we are not biasing the sample by imposing this cut on \( \sigma_{\text{acc}} \). We also verified that the error distribution in measured accelerations does not correlate with the quasar continuum luminosity (hence the broad-line size via the \( R-L \) relation) or virial BH masses, which simplifies our simulations below using the measured BLR size distribution.

Figure 14 demonstrates how we use the observed acceleration distribution to constrain the binary population. In this example, we fix the companion BH mass to be \( M_2 = 1.8 \times 10^8 M_\odot \), the median virial BH mass of the active BHs in the “good” sample. We also consider a 100% binary fraction, i.e., all the objects in our sample are in a binary system with only one BH active. To obtain the expected acceleration distribution for this hypothetical binary population, we must also assume the distribution of binary separations. We consider an ideal case where the binary separation is a fixed scaling of the BLR size of the active BH, \( d = (1/f) R_{\text{BLR}} \) with \( f < 1 \). The intrinsic distribution of the radial acceleration can then be derived using Equation (1) assuming random orientations and orbital phases of the binary, and using the distribution of BLR sizes measured from \( L_{5100} \) using the best-fit \( R-L \) relation in Bentz et al. (2009). We then convolve the intrinsic distribution with the measurement error distribution to obtain the simulated acceleration distribution expected from a binary BH population.

The black histogram in Figure 14 is the observed acceleration distribution, and the blue dashed histogram is a simulated distribution of zero acceleration broadened by the error distribution. The simulated distribution of zero accelerations with errors is too narrow to account for the observed distribution, indicating non-negligible accelerations in the intrinsic distribution. The magenta and red histograms are the simulated distributions for a binary population with \( d = 2 R_{\text{BLR}} \) and \( d = R_{\text{BLR}} \), convolved...
with measurement errors. The case with \( d = 2 \) \( R_{\text{BLR}} \) produces an acceleration distribution consistent with the observed one, but the case with \( d = R_{\text{BLR}} \) is strongly ruled out (again, assuming that all quasars are in equal-mass binaries with this separation).

As we have discussed in Section 3.3, it is likely that our estimated measurement errors in velocity shifts are too small by \( \sim 30\% \). We therefore scale our formal errors up by 30\% and repeat the above exercise in Figure 15. Again, this test suggests there is non-negligible intrinsic acceleration, as the simulated distribution of zero accelerations is still narrower than the observed one. The 100\% binary scenario with \( d = R_{\text{BLR}} \) is even more strongly ruled out.

Now we consider the general case with different binary fractions and binary separations. We determine the KS probability that the expected acceleration distribution is drawn from the same distribution as the observed one, for a series of values of \( d/R_{\text{BLR}} \), the binary fraction, and the mass of the companion BH, \( M_2 \). As discussed earlier, we adopt a constant mass \( M_1 = 1.8 \times 10^8 M_\odot \) for BH 1. Each evaluation is performed for a mock sample of \( 10^5 \) systems.

We choose to parameterize the KS probability in terms of \( d/R_{\text{BLR}} \) instead of \( d \), because objects in our sample spread over a range of BLR sizes. So for a fixed value of \( d \), some objects will not satisfy the criterion \( d > R_{\text{BLR}} \) required in our binary scenario (see Section 2), which will then complicate the expected distribution.

If we consider the more restrictive condition that the BLR size must be smaller than the Roche radius, then we must have \( d > f_r^{-1} R_{\text{BLR}} \), where \( f_r \) is given in Equation (5).

Figure 16 shows the contours of the KS probability as functions of the binary fraction and separation, at several different values of \( M_2 \), using the “superior” sample with our fiducial error estimates in the measured acceleration. The cyan lines show the 5\% probability contour, and darker regions are more probable. The dashed vertical lines in each panel mark the minimum separation of the hypothetical BBHs in units of the BLR size (i.e., the Roche limit, Equation 5) in order for the broad-line diagnostics to work. In each panel, regions with larger binary separations and lower binary fractions are disfavored, because they contribute negligible acceleration and thus cannot explain the observation that the observed distribution of accelerations is larger than expected from the measurement errors. Thus the observed acceleration distribution (error broadened) favors islands in the two-dimensional (2D) parameter space with high KS probabilities, bounded by the minimum separation (the vertical line). Since the mass for the active BH is \( 1.8 \times 10^8 M_\odot \) in our sample, these results suggest that the companion inactive BH must be several times more massive than the active BH to account for the observed...
acceleration distribution. In addition, we require a high binary fraction for the companion mass range probed ($\sim 5-10 \times 10^8 M_\odot$) and the companion BH must be orbiting not far outside the BLR.

The above conclusions are based on the assumption that all the excess variance in acceleration we see is from binary motion. However, as argued below, apparent acceleration between two epochs can also be induced by BLR variability in single BHs. If some, or most, of the observed excess variance is caused by broad-line variability, the constraints on the BBH population will be significantly weakened.

In the extreme case where the observed acceleration distribution can be entirely explained by intrinsic broad-line variability and measurement errors, our method only provides upper limits to the inactive BH mass and binary fraction, and lower limits to the separation. To illustrate this, we take the observed distribution as is, and randomly add intrinsic acceleration due to a BBH population as we did before, and see when the simulated distribution becomes significantly different from the observed one.

Figure 17 shows the resulting KS contours for two values of companion BH mass, $q \equiv M_2/M_1 = 1$ and $q = 5$, where $M_1 = 1.8 \times 10^8 M_\odot$ is the mass for the active BH in our sample. Since all the variance has already been accounted for, we no longer require a BBH population to contribute to the excess variance in the observed accelerations. As a result, there is essentially no constraint on the allowable binary parameters when the companion BH is less massive than $M_1$ (left panel). On the other hand, for more massive companion masses, certain regions in the 2D parameter space are ruled out, as the resulting accelerations will exceed the observed variance (right panel). The constraints become stronger for more massive companion BHs. With a companion mass $M_2 = 9 \times 10^8 M_\odot$, the scenario of a $\gtrsim 90\%$ binary fraction with separations $d/R_{BLR} < 5$ can be ruled out at $\gtrsim 95\%$ confidence.

The real situation is probably in between the two extreme cases considered in Figures 16 and 17, with the excess acceleration due both to a BBH population and to broad-line variability in single BHs. Future spectroscopic surveys may provide the prior information on the (stochastic) variance due to the latter

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**Figure 16.** Parameter constraints on a hypothetical binary BH population, using the observed acceleration distribution. We have assumed a constant mass $M_1 = 1.8 \times 10^8 M_\odot$ (see discussion in text). In each panel, the companion BH mass $M_2$ is fixed, and the contours show the logarithmic KS probability between the simulated acceleration distribution with the given binary fraction and the separation/BLR size ratio, and the observed distribution. The simulated distribution at each grid point has $10^5$ mock systems, and we have convolved the intrinsic distribution of accelerations in the hypothetical binary BH population with the measurement errors. The dashed vertical lines mark the threshold on binary separation, $d > f_r^{-1} R_{BLR}$, where $f_r$ is given in Equation (5). The cyan lines indicate the 5% probability contour. Darker regions are more probable.

(A color version of this figure is available in the online journal.)
2.2. Individual Detections

We show the 30 detections (28 unique systems) in Figures 18–20, where we plot the two-epoch spectra around the broad Hβ line and the [O iii] \( \lambda 5007 \) line in velocity space. We also show the Hα or Mg II line of these objects, if it is covered in SDSS spectra and the line is detected at >6σ. We follow the same \( \chi^2 \) cross-correlation approach to measure the velocity shift between two epochs based on Mg II or Hα. But we note that the inferred velocity shift from Mg II or Hα should be interpreted with caution, for the reasons discussed in Section 3. The wavelength range for the \( \chi^2 \) cross-correlation is [2750, 2850] Å for Mg II, and [6450, 6650] Å for Hα. For Hα we carefully subtracted the narrow line components, fixing the relative flux ratios of all narrow lines at both epochs. In contrast, for Mg II, we use the full line flux (broad and narrow Mg II lines) in the \( \chi^2 \) cross-correlation, because the narrow Mg II component cannot be reliably constrained.

In Figure 21 we show the distribution of the detected quasars in the BH 1 mass and binary orbit separation plane, along with other samples defined earlier. Again, we use the virial BH mass estimates for the active BH 1 mass and we caution that there are large uncertainties in these mass estimates. We also use the measured BLR sizes for the binary separation. As discussed in Sections 2 and 4.1, the orbit separation of the hypothetical binary must be larger than the BLR size, i.e., all points in Figure 21 would move up if the true binary separations were known. The cyan dashed lines indicate constant maximum acceleration, as defined in Equation (1), where we assume a constant binary mass ratio \( q = 1 \). The magenta dotted lines indicate constant orbital decay time due to gravitational radiation in a circular binary with a mass ratio of \( q = 1 \) (Peters 1964)

\[
\tau_{\text{gr}} = \frac{5}{256} \left( \frac{c}{G} \right)^4 \frac{d^4}{M_1^2},
\]

(13)

Therefore most quasars in our sample are in the regime where the orbit decays from gravitational radiation in less than a Hubble time (if the virial mass estimate is correct and the orbital separation is not much bigger than the size of the BLR). Obviously, we expect BBHs to be rare in the lower right region of the diagram, where the orbital decay time is much less than the Hubble time.

The biggest concern with our detections is that the broad-line profile could change over multi-year timescales, which may lead to an artificial line shift with the \( \chi^2 \) cross-correlation method. While visual inspection can lend some confidence to the interpretation as a genuine shift rather than a profile change (or vice versa), the correct interpretation is still ambiguous in some cases.

Below we give an object-by-object discussion of individual detections, and comment on the robustness of the detection, as well as possible causes of the detected velocity shift. To guide our discussion we divide these objects into three categories: (1) binary candidates, (2) broad Hβ line variability, and (3) ambiguous cases. These categories are our best efforts to assign the “most likely” scenario to individual detections, and are not a rigorous classification of these systems.

A useful constraint on the expected acceleration from a BBH can be derived using the equations in Section 2:

\[
|a_1| < 44q f_{\alpha}^2 \left( \frac{M_1}{10^8 M_\odot} \right) \left( \frac{R_{\text{BLR}}}{0.1 \text{ pc}} \right)^{-2} \text{ km s}^{-1} \text{ yr}^{-1},
\]

(14)

where \( f_\alpha(q) < 1 \) is given by Equation (5). The product \( q f_\alpha^2 \) < 0.6 is a monotonic increasing function of \( q \) for 0.05 < \( q \) < 20.
Figure 18. Line spectra at both epochs for the 30 detections in our good sample. Black is for Epoch 1 and red is for Epoch 2. The dotted lines are the corresponding flux density errors. We show the broad Hβ and [OIII] λ5007 lines in velocity space relative to the systemic velocity. We also show the broad MgII or Hz line, if covered in the spectra and detected at >5σ significance. The measured velocity shift in units of pixels (1 pixel = 69 km s⁻¹) is marked in each sub-panel, which encloses the 2.5σ confidence range in our χ² cross-correlation approach. Discussions of individual detections are presented in Section 4.2 and summarized in Table 2.

(A color version of this figure is available in the online journal.)
Figure 19. Same as Figure 18, for another set of detections.
(A color version of this figure is available in the online journal.)
Figure 20. Same as Figure 18, for another set of detections. Note that for J2355+0049 there is significant absorption in the Epoch 1 spectrum near +1500 km s$^{-1}$ of H$\beta$; this is not due to our narrow-line subtraction, but likely due to incorrect data reduction. Masking out these affected pixels in our cross-correlation still yields a consistent, negative shift.

(A color version of this figure is available in the online journal.)
4.2.2. Broad Hβ Line Variability

J030639.57+000343.1. For this object we have eight different secondary epochs to cross-correlate with the default Epoch 1; all are in our good sample with good measurement precision. In Figure 22 (top) we show the broad-line spectra at different epochs, along with the measured velocity shift with respect to the reference Epoch 1. We see a narrowing of the line profile from the first three spectra taken ~10 months earlier than Epoch 1, then the line profile stays more or less constant. In the bottom panel we show the measured radial velocity curve, again using Epoch 1 as reference. The time baseline is only ~1.8 yr in the quasar rest frame. The expected binary period is $P > 86(1 + q)^{-1/2} f_{r}^{-3/2}$ yr given the constraints of the binary separation (from the BLR size) and the virial BH mass estimate, where $q \equiv M_{2}/M_{1}$ and $f_{r} < 1$ is a function of $q$ as defined in Equation (5). For $q > 0.05$ we have $(1 + q)^{-1/2} f_{r}^{-3/2} > 1.9$ hence $P > 160$ yr. Thus within such short time interval the expected BBH velocity should approximately have constant acceleration. If this is a binary system with an LOS acceleration inferred from the observations up to and including Epoch 1, we should expect to see a velocity shift within $\sim[-2.5, -0.5]$ (pixels) between the last epoch (gray spectrum/point in Figure 22) and Epoch 1. While this is still consistent with what we measure, the fact that the last epoch has the same line profile as Epoch 1 while the first two epochs have very different profile strongly suggest that the detected velocity shift is more likely due to broad-line variability.

J135829.58+010908.6. This object is an unambiguous disk emitter with double-peaked broad-line profiles. While we detected velocity shifts between the three epochs we have in the good sample, it is hard to ascribe this average velocity shift to the shape is more symmetric, in Epoch 2 than in Epoch 1. It is possible that there is only a portion of the broad-line emission moving along with BH 1 in the hypothetical binary system, and the remaining broad-line emission comes from a circumbinary region, which does not accelerate between the two epochs. It is also possible that some of the broad-line emission comes from BH 2, and BH 2 is much more massive than BH 1. The inferred acceleration is $\sim 6-26$ km s$^{-1}$ yr$^{-1}$ (2.5$\sigma$).

J155053.16+052112.1. For this object we detected the largest velocity shift in our sample, for both Hβ and Hα, over 3.5 yr. Interestingly, the velocity shift for Hβ is larger than that for Hα (significant at the $> 3\sigma$ level). At face value, this may argue against the binary hypothesis if the Hβ and Hα broad-line emission comes from the same BLR. However, if the Hβ BLR is mostly confined to the active BH, while the Hα BLR also contains a circumbinary component (which does not accelerate), a larger velocity shift in Hβ is expected. The width of the broad Hβ is substantially larger than that of the broad Hα, which is consistent with this scenario. The inferred acceleration based on Hβ is then $\sim 250-340$ km s$^{-1}$ yr$^{-1}$ (2.5$\sigma$).

J234932.77−003645.8. For this object both Hβ and Hα show consistent velocity shifts over a time period of $\sim 1.6$ yr. The shape of the broad-line profile remains more or less constant with consistent FWHM at both epochs. The narrow Hβ subtraction is not perfect for both epochs, but does not affect the detected velocity shift. The inferred radial acceleration is in the range of $11-75$ km s$^{-1}$ yr$^{-1}$ (2.5$\sigma$).

Since the last spectroscopic observations of these candidates were a few years ago, taking a third-epoch spectrum now would be very helpful to confirm (or rule out) the proposed binary hypothesis.

4.2.1. Binary Candidates

J030100.23+000429.3. For this object consistent velocity shifts between two epochs are detected for both Hβ and Mg ii over 261 days, despite the apparently noisy broad-line spectrum for Mg ii. The shape of the broad Hβ is not significantly different between the two epochs. The inferred acceleration is $\sim 220-760$ km s$^{-1}$ yr$^{-1}$ (2.5$\sigma$).

J032213.89+005513.4. The S/N for this object is very high for both epochs. Consistent velocity shifts are detected for both Hβ and Hα over $\sim 3.4$ yr. There is a slight change in the broad-line profile between the two epochs. The inferred acceleration is $\sim 6-23$ km s$^{-1}$ yr$^{-1}$ (2.5$\sigma$).

J122909.52−003530.0. For this object both Hβ and Mg ii show consistent velocity shifts over a time period of $\sim 4$ yr. The change in line shape is insignificant, although there is some hint that the red-side flux changes a little between the two epochs. In the binary scenario, the inferred radial acceleration based on Hβ is $50-110$ km s$^{-1}$ yr$^{-1}$ (2.5$\sigma$).

J141020.57+364322.7. For this object we detected a velocity shift for Hβ but not for Mg ii. However, the limits we obtained from Mg ii are fully consistent with the velocity shift detected in Hβ. Upon visual inspection (see Figure 19), both Hβ and Mg ii show similar profile changes between the two epochs, with extra flux on the red-side in Epoch 1. Therefore we classify this object as a good BBH candidate. The inferred radial acceleration based on Hβ is $50-110$ km s$^{-1}$ yr$^{-1}$ (2.5$\sigma$).

J153705.95+005522.8. For this object consistent velocity shifts between two epochs are detected for both Hβ and Hα, over $\sim 7$ yr. The broad-line profile also changed dramatically between the two epochs: the line is substantially narrower, and...
binary motion. It appears that the relative strength of the two peaks changed over this period (seen in both Hβ and Hα). Variability of this kind has been seen in long-term monitoring of disk emitters (e.g., Gezari et al. 2007; Lewis et al. 2010), which usually invokes a non-binary interpretation (but see, e.g., Bon et al. 2012 for an alternative BBH interpretation). We therefore classify this object as exhibiting broad-line variability, although the BBH scenario cannot be completely ruled out given only three epochs. Similar objects were reported in Eracleous et al. (2012).}

\[ J074700.19+285608.5 \text{ and } J165638.86+362121.0 \text{. These two objects are detections with the shortest time separation (Δt < 30 days) between the two epochs, which implies rather large accelerations. Using Equation (14) and assuming the BLR sizes are accurate, it is difficult to get such large acceleration \] unless the BH mass \( M_1 \) is \( \sim 10 \) times larger than the virial BH mass estimate (see Table 2) and the unseen BH is at least as massive for both objects. While this is not entirely ruled out, it is more likely that the velocity shifts detected for these two objects are due to short-term broad-line variability. In addition, for J0747 the velocity shift based on Hβ is \textit{inconsistent} with that based on Hα, which also favors the BLR variability scenario.

4.2.3. Ambiguous Cases

We classify the remaining detections as ambiguous cases. This category includes more than half of our detections, which reflects the general ambiguity in interpreting the observed broad-line velocity shifts. They have neither a second broad line (Mg II or Hα) showing consistent velocity shifts, nor more than two epochs with sufficient quality to rule out the hypothetical binary acceleration. Thus, they could still be binary candidates, although the binary probability is perhaps lower than those presented in Section 4.2.1. Additional high-quality spectroscopic observations are needed to further investigate these systems.

5. DISCUSSION

5.1. Caveats with the Broad-line Diagnostics

The basic assumption required to utilize the broad-line shift as an indicator for the BBH orbital motion is that the BLR must move along with its associated BH. This means this method is only applicable to BBHs with separations \( d \geq f_{r,1}^{-1} R_{\text{BLR}} \) (see Section 2 and Shen & Loeb 2010). Fortunately, the typical BLR size for our sample is \( \sim 0.06 \) pc, so this method is still probing the interesting sub-parsec regime where presumably the BBH has passed the final parsec barrier.

The primary uncertainty is that the BLR is unlikely to be in a steady state, and changes in the BLR may produce artificial broad-line velocity shifts. Velocity-resolved reverberation mapping results for a handful of objects have shown diverse kinematics of the BLR gas (e.g., Sergeev et al. 1999; Denney et al. 2009; Grier et al. 2013) involving inflows and outflows. Changes of the kinematic structure of the BLR will occur on the dynamical time \( t_{\text{dyn}} \sim 24(R_{\text{BLR,0.1}}/\text{FWHM}_{4000}) \) yr, i.e., the same timescale over which we search for the BBH signature. Broad-line shifts without line shape changes are perhaps better explained by the BBH scenario, but the broad-line shape may vary even in a BBH system. Occasionally an apparent velocity shift might be induced due to excess flux on one side of the broad line. This may indicate a BLR variability origin rather than a BBH; however, the BBH scenario cannot be completely ruled out if the BLR is composed of spatially distinct components, and the “excess flux” is indeed associated with the motion of one BH. In a similar spirit, the broad-line profile could also change in the BBH scenario, if both BHs are active and the broad line is composed of two spectrally unresolved components (e.g., Shen & Loeb 2010). Invoking multiple broad lines sometimes helps confirm the detected velocity shift, but cannot be used to confirm or rule out the BBH scenario given the possible differences in the BLRs for different lines with various ionization potentials.

It is practically impossible to distinguish between the BBH case and the BLR variability case for the observed broad-line velocity shifts with only two epochs. The best (perhaps the only) way to differentiate these two cases is continued spectral monitoring of the broad line. BLR variability will lead to stochastic velocity shifts, while the BBH scenario will lead to coherent velocity shifts, either Keplerian (over long
Table 2
Summary of the Detections

| SDSS Designation | z_{sys} | \log M_{1,\,\text{vir}} (M_\odot) | R_{BLR} (pc) | Epoch 1 Fiber | Plate | Epoch 1 MJD | Plate | Epoch 2 Fiber | Epoch 2 MJD | Δt (days) | \nu^{\text{ccf}} (km s^{-1}) | \alpha^{\text{ccf}} (km s^{-1} yr^{-1}) | Category |
|------------------|--------|-------------------------------|----------|--------------|-------|-------------|-------|--------------|-------------|------------|----------------|---------------------|----------|
| 012016.72−092028.8 | 0.495 | 8.42 | 0.083 | 0660 | 586 | 52177 | 2864 | 621 | 54467 | 1532 | 158 | 102 | 215 | 37 | 24 | 51 | 3 |
| 015836.26+010052.0 | 0.724 | 8.38 | 0.201 | 2045 | 417 | 53350 | 2851 | 417 | 54459 | 643 | -103 | -200 | -14 | -58 | -113 | -7 | 3 |
| 021225.56+010056.1 | 0.513 | 8.70 | 0.075 | 0405 | 380 | 51816 | 0703 | 334 | 52209 | 259 | 131 | 58 | 201 | 184 | 81 | 282 | 3 |
| 030100.23+000429.3 | 0.486 | 8.50 | 0.056 | 0411 | 265 | 51817 | 0709 | 477 | 52205 | 261 | 420 | 163 | 545 | 588 | 229 | 762 | 1 |
| 030639.57+000343.1 | 0.108 | 7.51 | 0.030 | 0412 | 400 | 52258 | 5412 | 397 | 51871 | 349 | 117 | 43 | 190 | 122 | 45 | 199 | 2 |
| 041201.04+000422.9 | 0.250 | 7.93 | 0.040 | 0679 | 327 | 52618 | 0719 | 330 | 52592 | 20 | 296 | 151 | 422 | 5241 | 2670 | 74672 | 2 |
| 041213.58+000425.1 | 0.382 | 8.17 | 0.060 | 0656 | 397 | 51871 | 0719 | 330 | 52592 | 20 | 296 | 151 | 422 | 5241 | 2670 | 74672 | 2 |
| 041234.50+000428.8 | 0.350 | 8.14 | 0.050 | 0639 | 330 | 52618 | 0719 | 330 | 52592 | 20 | 296 | 151 | 422 | 5241 | 2670 | 74672 | 2 |
| 041235.59+000430.8 | 0.382 | 8.17 | 0.060 | 0656 | 397 | 51871 | 0719 | 330 | 52592 | 20 | 296 | 151 | 422 | 5241 | 2670 | 74672 | 2 |
| 041236.50+000431.8 | 0.350 | 8.14 | 0.050 | 0639 | 330 | 52618 | 0719 | 330 | 52592 | 20 | 296 | 151 | 422 | 5241 | 2670 | 74672 | 2 |
| Notes. *Velocity shift and acceleration are calculated based on the broad Hβ line, and the first column is the value corresponding to the minimum χ^2 and the last two columns enclose the 2.5σ confidence range (Δχ^2 = 6.63). We classify the detections into three categories (see Section 4.2): (1) binary candidates; (2) broad-line variability; (3) ambiguous cases.
timescales comparable to the binary orbit period) or linear (over timescales much shorter than the binary orbit period). Of course, one also needs high S/N to make the measurements meaningful for this purpose. Currently there is no adequate spectroscopic monitoring data to provide priors on the stochastic velocity shifts induced by BLR variability, but future time-domain spectroscopic surveys may provide such information (see Section 5.4).

Finally, although we have focused on circular binary orbits, the methodology developed in this work can be easily applied to a binary population with a distribution of eccentricities. Such an application will be appropriate once we have a good understanding of the velocity shifts due to broad-line variability.

5.2. Uncertainty in the BLR Size Estimation

Throughout this work we have used BLR sizes estimated from the BLR $R - L$ relation found in reverberation mapping studies (e.g., Bentz et al. 2009). This is of course an approximation, as the BLR size measured in reverberation mapping represents the emissivity-weighted average radius of the BLR, and the actual BLR could extend beyond this typical size. It is nevertheless reasonable to expect that the bulk of the line emission comes from within this typical BLR size. If we have significantly underestimated the BLR sizes, the allowable binary parameter space shown in Figure 16 will diminish, because then the allowable binary orbit separations (i.e., $d > f_{\text{BLR}}^{-1}$) will be too large to account for the observed excess variance in broad-line velocity shifts; in this case the observed excess variance must be due to BLR variability.

An alternative way to estimate the BLR size is to use theoretical predictions based on photoionization calculations (e.g., Dumont & Collin-Souffrin 1990): $R_{\text{BLR}} \sim 10^4 R_G$, where $R_G \equiv 2GM/\alpha^2$ is the gravitational radius of the active BH. In our opinion, such theoretical estimates are more uncertain than the BLR sizes derived empirically from reverberation mapping. In addition, such estimates have an explicit dependence on the highly uncertain BH mass estimates (e.g., Shen 2013). Despite these concerns we have constructed models using $R_{\text{BLR}} \sim 10^4 R_G$. Following our earlier discussion in Section 4.1, we assume a constant mass of $1.8 \times 10^9 M_\odot$ for BH 1 and use the new BLR size estimates (which are then the same for all quasars in our sample) to repeat the analysis in Figure 16. The new BLR size is $10^5 R_G \sim 0.17$ pc, larger than our fiducial BLR size estimates (with a median value of $\sim 0.06$ pc) by almost a factor of three. Therefore the allowable binary parameter space as shown in Figure 16 disappears, and BLR variability is needed to explain the observed excess variance in broad-line velocity shifts.

5.3. Comparisons with Other Work

Most theoretical work on BBHs with a gaseous disk does not make specific predictions for the broad lines (e.g., Armitage & Natarajan 2002; Escala et al. 2005; Dotti et al. 2006, 2007; Mayer et al. 2007; Haiman et al. 2009; Kocsis et al. 2012; Rafikov 2012) with few exceptions (e.g., Bogdani\’evi\’c et al. 2008). In addition, their models often involve a circumbinary accretion disk, which presumably complicates the applicability of the broad-line diagnostics used in this work, if this circumbinary disk also contributes to the broad-line emission. Using smoothed particle hydrodynamics simulation coupled with photoionization calculations, Bogdani\’evi\’c et al. (2008) studied the temporal behavior of the broad Hα line from the circumbinary accretion disk. The line profile is often double-peaked, showing the characteristics of a disk, and evolves with time, similar to the results in Shen & Loeb (2010) with toy models for the BLR. However, it is not yet feasible to make detailed comparisons between the simulation predictions and observations of quasars with double-peaked broad-line profiles. Montuori et al. (2011) considered a BBH scenario where the BLR has a stratified structure for different line species with the higher-ionization lines deeper within the gravitational potential of the BH. Tidal erosion from the companion BH then reduces the flux ratio of the lower-ionization lines to higher-ionization lines. While this process may be at work for very close BBHs, it is difficult to differentiate this scenario from single BHs with abnormal broad-line flux ratios, thus difficult to confirm the BBH nature with this feature alone. Finally, recent predictions of sub-parsec BBH fractions in quasars from hierarchical cosmological merger models (e.g., Volonteri et al. 2009; Kulkarni & Loeb 2012) still have large uncertainties, and a clear pathway connecting these predictions to direct observables is currently absent. Clearly, more theoretical work is needed to probe the full potential of various broad-line diagnostics, and to provide more theoretical guidance on the expected binary frequency.

There are three recent observational studies that are closely related to the current work. Eracleous et al. (2012) and Decarli et al. (2013) both obtained second-epoch spectra for samples ($\sim 70$ and $\sim 30$ objects, respectively) of $z < 0.7$ SDSS quasars selected to have peculiar broad-line characteristics, such as large offsets ($\gtrsim 1000$ km s$^{-1}$) between the broad Hβ and narrow lines, and/or double-peaked or asymmetric line profiles. They used the two spectra (one from the SDSS) to measure broad Hβ velocity shifts between the two epochs and reported several detections. The detection fraction in their samples is roughly consistent with ours if we restrict to the same rest frame time separation (see Figure 11). But we caution that our statistics are poor (we only have seven detections for the largest $\Delta t$ bin), and there are considerable differences in the sample and the quality of the spectroscopic data, in the approaches to measuring the velocity shift and in the associated error analysis among the three studies. The most significant difference is that our sample is typical of normal quasars, while theirs were selected to be more likely to host a BBH. Therefore they could not provide constraints on the general population as we do in Section 4.1. A more appropriate comparison will be provided in Paper II, where we also target quasars selected to be more likely BBHs.

The third study, by Ju et al. (2013), also used repeated spectroscopy from the SDSS, and focused on the general quasar population, as we do here. However, there are several key differences between the two studies: (1) their sample probes quasars at $0.4 < z < 2$ (although their reported detections are mostly at $z < 0.7$) while our sample is at $z < 0.8$ and has lower quasar luminosities; (2) they used the Mgλγ line instead of Hβ. We did not choose the Mgλγ line as our primary line for the reasons discussed in Section 3; (3) they used the several “detections” to constrain BBH models, while we believe it is more informative, and more appropriate, to use the full distribution in constraining the general BBH population; and (4) they did not utilize individual measurement errors for their sample, and used a typical ensemble value for the measurement errors instead. This could be problematic, as the several “detections” in their study could be due to large individual measurement errors, and their modeling is sensitive to this detected number. All these issues may explain the different conclusions (in particular, Ju et al. find a much lower detection fraction for their sample, and used a typical ensemble value for the measurement errors instead). This could be problematic, as the several “detections” in their study could be due to large individual measurement errors, and their modeling is sensitive to this detected number. All these issues may explain the different conclusions (in particular, Ju et al. find a much lower detection
fraction) reached in the two studies, but one should also bear in mind the different assumptions that went into both analyses. We defer a more careful comparison to future work when we perform our own analysis on the Mg II line.

5.4. Future Perspectives

We have demonstrated the potential of using the broad-line diagnostics, i.e., velocity shifts between epochs, to identify and characterize the BBH population in broad-line quasars. However, to improve the power of this technique, there is both observational and theoretical need to understand the broad-line emission in single and binary BHs.

From an observational point of view, we need long-term spectroscopic monitoring of large samples of quasars to construct the power spectrum of the stochastic velocity shifts due to BLR variability in single BHs. Analogous to studying the photometric quasar continuum variability, we can construct a stochastic velocity shift “structure function,” i.e., the random fluctuation of broad-line shifts between epochs as a function of time separation. This must be done for a large sample to capture the diversity in the quasar population, and must be done over multi-year timescales and many epochs for the same objects, and with adequate S/N, to confirm the stochastic nature of the velocity shifts. A by-product of these programs would be potential discoveries of coherent velocity shifts, which would be the best candidates for BBHs. There are also general benefits from improved spectral quality, increased time baseline and more epochs for our statistical constraints. For instance, merely increasing the time baseline can shrink the measurement errors in acceleration. Parallel observational efforts are needed to better understand the BLR structure with reverberation mapping.

From a theoretical perspective, we need BBH simulations with recipes for the BLR gas to predict the temporal evolution of the broad-line emission and its correlation with the orbital motion of the BBH, as well as the effects of the (postulated) circumbinary disk on the broad lines (e.g., how much broad-line emission comes from this circumbinary disk), with empirical constraints from observations, such as the BLR size, geometry, and kinematics.

6. CONCLUSIONS

We have performed a systematic search for sub-parsec BBH candidates in optically selected, broad-line quasars with multi-epoch (mostly two-epoch) SDSS spectroscopy. The spectra are separated by up to several years in the quasar rest frame. Our sample is composed of normal quasars, which makes it different from the samples in other studies (e.g., Eracleous et al. 2012; Decarli et al. 2013). We have shown that targeting these normal quasars is not only as efficient as targeting offset or double-peaked broad-line quasars in terms of finding BBH candidates, but also provides much more robust constraints on the BBH population in quasars.

Our working hypothesis was that only one BH is active and that this BH dynamically dominates its own BLR, and that the companion BH is orbiting at a large enough distance such that the BLR is not tidally distorted or disrupted. Our sample is composed of $z < 0.8$ quasars, with typical virial BH mass estimates $1.8 \times 10^{9} M_{\odot}$ (for the active BH) and BLR sizes $\sim 0.01–0.1$ pc. We searched for the expected acceleration (velocity shift) in the broad H$\beta$ line from the binary motion between the two epochs, using a cross-correlation function. Our main findings and their implications are summarized below.

1. The broad H$\beta$ line shape (FWHM, skewness, etc.) can change over multi-year timescales, but dramatic changes in FWHM (>20%) are rare.
2. Out of 688 pairs of observations for which we have successfully measured the velocity shift between two epochs, we detected 28 systems with significant acceleration at the $\sim 2.5\sigma$ confidence level. Object-by-object inspections suggest that seven of them are the best examples of sub-parsec BBH candidates, four of them are best explained by broad-line variability in single BHs, and the rest are ambiguous.
3. The full distribution of acceleration (mostly non-detections) for a subset from the 688 pairs with better-quality measurements and $\Delta t > 0.4$ yr (rest frame) was used to constrain the BBH population among general quasars. After accounting for measurement errors, excess variance is required to explain the observed distribution. If all the excess variance in velocity shifts is due to a BBH population, then (1) a large binary fraction is required; (2) the companion BH must be orbiting within a few times the BLR radius (details depend on the companion mass; see Section 4.1); (3) the companion BH must be more massive than the active one. On the other hand, if the excess variance is largely due to stochastic BLR variability in single BHs, then the requirement for a BBH population is substantially weakened, and even disfavored in the case of a very massive companion—otherwise the predicted acceleration distribution will be broader than observed (after convolving with errors).
4. Long-term spectroscopic monitoring of the detected BBH candidates with more epochs are required to confirm or rule out the BBH hypothesis. In addition, long-term spectroscopic monitoring of normal quasars will provide vital information on the stochastic velocity shift induced by BLR variability, and thus improve statistical constraints on the BBH population in broad-line quasars with the methodology developed in this paper.

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