Backreaction in Semiclassical Cosmology: the Einstein-Langevin Equation

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Abstract

Using the influence functional formalism we show how to derive a generalized Einstein equation in the form of a Langevin equation for the description of the backreaction of quantum fields and their fluctuations on the dynamics of curved spacetimes. We show how a functional expansion on the influence functional gives the cumulants of the stochastic source, and how these cumulants enter in the equations of motion as noise sources. We derive an expression for the influence functional in terms of the Bogolubov coefficients governing the creation and annihilation operators of the Fock spaces at different times, thus relating it to the difference in particle creation in different histories. We then apply this to the case of a free quantum scalar field in a spatially flat Friedmann-Robertson-Walker universe and derive the Einstein-Langevin equations for the scale factor for these semiclassical cosmologies. This approach based on statistical field theory extends the conventional theory of semiclassical gravity based on a semiclassical Einstein equation with a source given by the average value of the energy momentum tensor, thus making it possible to probe into the statistical properties of quantum fields like noise, fluctuations, entropy, decoherence and dissipation. Recognition of the stochastic nature of semiclassical gravity is an essential step towards the investigation of the behavior of fluctuations, instability and phase transition processes associated with the crossover to quantum gravity.

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1 Introduction

Backreaction of quantum processes like particle creation in cosmological spacetimes [1] has been considered by many researchers in the past for the purpose of understanding how quantum effects affect the structure and dynamics of the early universe near the Planck time [2, 3]. Because of the general nature and complexity of the problem, backreaction studies have also been used as a testing ground for the development and application of different formalisms in quantum field theory in curved spacetime [4], e.g., regularization schemes to obtain finite energy-momentum tensor, perturbation methods, effective action formalism, etc. The most recent stage of development for the discussion of cosmological backreaction problems was the use of Schwinger-Keldysh (or closed-time-path, CTP) functional formalism [5], which, being formulated in the in-in boundary condition, gives rise to a real and causal equation of motion (the semiclassical Einstein equation), where the expectation value of the energy-momentum tensor of a quantum field acts as a source which drives the classical effective geometry. In this equation one can identify a nonlocal kernel in the dissipative term whose integrated dissipative power has been shown to be equal to the energy density of the total number of particles created, thus establishing the dissipative nature of the backreaction process [6, 7].

In pursuing a deeper understanding of the statistical mechanics meaning of dissipation, one of us [8] cast this backreaction problem into the conceptual framework of quantum open systems [9]. He made the observation that a Langevin-type equation is what should be expected, and predicted that for quantum fields a colored noise source should appear in the driving term. He also conjectured that the particle creation backreaction problem can be understood succinctly as the manifestation of a general fluctuation-dissipation relation for quantum fields in dynamical spacetimes, a non-equilibrium generalization of such relations depicting particle creation in black holes [10, 11] and de Sitter universe [12]. The missing piece in this search is the noise term associated with quantum fields.

To look into this aspect of the backreaction problem in semiclassical gravity, as well as exploring the quantum origin of noise and fluctuations in inflationary cosmology [13], and understanding the decoherence problem in quantum to classical transition [14], Hu, Paz and Zhang [15, 16] looked into the relation of colored noise and nonlocal dissipation in a quantum Brownian motion model with the influence functional of Feynman and Vernon [17, 18, 19]. In this formalism the effects of noise and dissipation can be extracted from the noise and dissipation kernels in the real and imaginary parts of the influence functional, their interrelation residing in the fluctuation-dissipation relation obtained as a simple functional relation. If one views the quantum field as the environment and spacetime as the system in the quantum open system paradigm, then the statistical mechanical meaning of the backreaction problem in semiclassical cosmology can be understood more clearly [8]. In particular, one can identify noise with the coarse-grained quantum fields [20, 21], derive the semiclassical Einstein equation as a Langevin equation [22], and understand the backreaction process as the manifestation of a fluctuation-dissipation relation [23]. Continuing their investigation of the backreaction problem via the CTP formalism, Calzetta and Hu [22] also found that the
results obtained from the influence functional formalism is the same as that obtained earlier (but displayed only partially) from the Schwinger-Keldysh method. This paradigm has also been applied to problems in quantum cosmology [24]. (For an account of the search and discovery of these ideas, see [25, 26].)

The specific goal of this paper is to derive the semiclassical Einstein equation in the form of a Langevin equation. Our primary task is the derivation of noise from the quantum field source, and we do this by carrying out a cumulant expansion of the influence functional. This goal is shared by two other companion papers addressing different aspects of this problem: In Ref. [22], using the closed-time-path method [6, 7], Calzetta and Hu identified the source of decoherence and particle creation to the noise kernel and showed their relation through the Bogolubov coefficients. They also showed the relation of quantum noise with classical fluctuations, and derived the semiclassical Einstein equation with a noise term. In Ref. [23] Hu and Sinha started with the density matrix of the universe in quantum cosmology in the manner of [24] and demonstrated the existence of a fluctuation-dissipation relation for the particle creation and backreaction problem in a Bianchi Type-I universe. These two papers together with this one present a quantum open system approach to the backreaction problem in semiclassical gravity and cosmology. Together they can serve as a platform for exploring the transition to quantum cosmology. It can also address the dissipative nature of effective theories [8, 27], and (to the extent that Einstein’s general relativity can be viewed as an effective theory) possible dissipative effects in the low-energy limit of any theory of quantum gravity. For a general discussion of these ideas, see [28].

This paper is organized as follows: In Sec. 2 we give a brief review of the influence functional formalism, mainly to establish notations. Readers familiar with it can skip to the next section. In Sec 3 we show how a functional expansion on the influence functional gives the cumulants of the stochastic source, and how these cumulants enter into the equations of motion as noise sources. In Sec. 4, following [21], we derive the form of the Hamiltonian for a scalar field in terms of its normal modes and consider a class of actions where the field modes are coupled parametrically to the scale factor of the universe. We derive an expression for the influence functional in terms of the Bogolubov coefficients governing the creation and annihilation operators of the Fock spaces at different times which signify particle creation. In Sec 5, we present two standard cases of cosmological particle creation and derive the Einstein-Langevin equations describing its backreaction on the background spacetime.

## 2 Influence Functional Theory

Consider the quantum system described by the action

\[ S[a, q] = S[a] + S_e[q] + S_{\text{int}}[a, q]. \]  

(2.1)

We will consider \( a \) to be our system variable and \( q \) to be our environmental variables. Typically the environment has infinite degrees of freedom which is denoted here by a bold type.
We will briefly review here the Feynman-Vernon influence functional method for deriving the evolution operator. The method provides an easy way to obtain a functional representation for the evolution operator $\mathcal{J}_r$ for the reduced density matrix $\hat{\rho}_r$. Let us start first with the evolution operator $\mathcal{J}$ for the full density matrix $\hat{\rho}$ defined by

$$\hat{\rho}(t) = \mathcal{J}(t, t_i)\hat{\rho}(t_i). \quad (2.2)$$

As $\hat{\rho}$ evolves unitarily under the action of (2.1), the evolution operator $\mathcal{J}$ has a simple path integral representation. In the position basis, the matrix elements of the evolution operator are given by

$$\mathcal{J}(a_f, q_f, a'_f, q'_f, t \mid a_i, q_i, a'_i, q'_i, t_i) = \mathcal{K}(a_f, q_f, t \mid a_i, q_i, t_i)\mathcal{K}^*(a'_f, q'_f, t \mid a'_i, q'_i, t_i)$$

$$= \int_{a_i}^{a_f} Da \int_{q_i}^{q_f} Dq \exp \left( \frac{i}{\hbar} S[a, q] \right) \int_{a'_i}^{a'_f} Da' \int_{q'_i}^{q'_f} Dq' \exp \left( -\frac{i}{\hbar} S[a', q'] \right) \quad (2.3)$$

where the operator $\mathcal{K}$ is the evolution operator for the wave functions. In a path-integral representation, the functional integrals are over all histories compatible with the boundary conditions. We have used the subscripts $i, f$ to denote the initial and final variables at $t_i, t_f$.

The reduced density matrix is defined as

$$\rho_r(a, a') = \int_{-\infty}^{+\infty} dq \int_{-\infty}^{+\infty} dq' \rho(a, q; a', q') \delta(q - q') \quad (2.4)$$

and is propagated in time by the evolution operator $\mathcal{J}_r$

$$\rho_r(a, a', t) = \int_{-\infty}^{+\infty} da_i \int_{-\infty}^{+\infty} da'_i \mathcal{J}_r(a, a', t \mid a_i, a'_i, t_i) \rho_r(a_i, a'_i, t_i). \quad (2.5)$$

By using the functional representation of the full density matrix evolution operator given in (2.3), we can also represent $\mathcal{J}_r$ in path integral form. In general, the expression is very complicated since the evolution operator $\mathcal{J}_r$ depends on the initial state. If we assume that at a given time $t = t_i$ the system and the environment are uncorrelated

$$\hat{\rho}(t = t_i) = \hat{\rho}_s(t_i) \times \hat{\rho}_e(t_i), \quad (2.6)$$

then the evolution operator for the reduced density matrix does not depend on the initial state of the system and can be written [17] as

$$\mathcal{J}_r(a_f, a'_f, t \mid a_i, a'_i, t_i) = \int_{a_i}^{a_f} Da \int_{a'_i}^{a'_f} Da' \exp \left( \frac{i}{\hbar} \{ S[a] - S[a'] \} \right) \mathcal{F}[a, a'] \quad (2.7)$$
The factor $F[a, a']$, called the ‘influence functional’, is defined as

$$F[a, a'] = \int_{-\infty}^{q_f} d\mathbf{q} \int_{-\infty}^{d\mathbf{q}} d\mathbf{q}' \int_{-\infty}^{q_{f'}} D\mathbf{q} \int_{q_i}^{q_{f'}} D\mathbf{q}' \times \exp \left\{ \frac{i}{\hbar} \left[ S_{e}[\mathbf{q}] + S_{\text{int}}[a, \mathbf{q}] - S_{\text{int}}[a', \mathbf{q}'] \right] \right\} \rho_{e}(\mathbf{q}_i, \mathbf{q}'_i, t_i)$$

(2.8)

where $S_{IF}[a, a']$ is the influence action. The effective action for the open quantum system is defined as

$$S_{\text{eff}}[a, a'] = S[a] - S[a'] + S_{IF}[a, a'].$$

It is not difficult to show that (2.8) has the representation independent form [21]

$$F[a, a'] = \text{Tr} \left( \hat{U}[a_{t,t_i}] \hat{\rho}_e(t_i) \hat{U}^\dagger[a'_{t,t_i}] \right)$$

(2.9)

where $\hat{U}$ is the quantum propagator for the action $S_{e}[\mathbf{q}] + S_{\text{int}}[a(s), \mathbf{q}]$ where $a(s)$ is treated as a time dependent classical forcing term. We have found this form to be very convenient for deriving the influence functional.

It is obvious from its definition that if the interaction term is zero, the influence functional is equal to unity and the influence action is zero. In general, the influence functional is a highly non-local object. Not only does it depend on the time history, but –and this is the more important property– it also irreducibly mixes the two sets of histories in the path integral of (2.7). Note that the histories $a$ and $a'$ could be interpreted as moving forward and backward in time respectively. Viewed in this way, one can see the similarity of the influence functional [17] and the generating functional in the closed-time-path (CTP or Schwinger-Keldysh) integral formalism [5]. The Feynman rules derived in the CTP method are very useful for computing the IF.

In those cases where the initial decoupling condition (2.6) is satisfied, the influence functional depends only on the initial state of the environment. The influence functional method can be extended to more general conditions, such as thermal equilibrium between the system and the environment [29], or correlated initial states [18, 19].

3 Stochastic Forces from the Influence Functional

In this paper we will be interested in models in which the action (2.1) has a Hamiltonian of the general form

$$H(a, \mathbf{q}) = H(a) + H_e(\mathbf{q}) + \sum_n \lambda \sigma(a, \dot{a}) \epsilon(q_n, \dot{q}_n)$$

(3.1)

where $\lambda$ is a coupling constant and $\sigma$ and $\epsilon$ are arbitrary functions of the system and environment variables. The simplification made in (3.1) is that system environment interaction is separable. This ensures that the effect of the environment on the system can be described by a single stochastic source.
Let us introduce the sum and difference system variables as
\[
\Sigma = \frac{1}{2} (\sigma(a, \dot{a}) + \sigma(a', \dot{a}')), \quad \Delta = \sigma(a, \dot{a}) - \sigma(a', \dot{a}'),
\] (3.2)
and define the real quantities
\[
C_n(t_1, ..., t_n; \Sigma_{t_1,t_i}, ..., \Sigma_{t_n,t_i}) = \left( \frac{i}{\hbar} \right)^n \frac{\delta^n \mathcal{W}[\Sigma(s), \Delta(s)]}{\delta \Delta(t_1) ... \delta \Delta(t_n)} \bigg|_{\Delta=0},
\] (3.3)
where \( \mathcal{W} = \ln \mathcal{F} \). The notation of \( C_1(t_1; \Sigma_{t_1,t_i}) \) means \( C_1 \) is a function of \( t_1 \) and also a functional of \( \Sigma \) between endpoints \( t_1 \) and \( t_i \). Writing the influence action as a functional Taylor series and generalizing the notation to \( n \) variables we find that formally
\[
\mathcal{W}[\Sigma(s), \Delta(s)] = \frac{i}{\hbar} \int_{t_i}^{t_f} dt_1 \Delta(t_1) C_1(t_1; \Sigma_{t_1,t_i})
- \frac{1}{2\hbar^2} \int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_f} dt_2 \Delta(t_1) \Delta(t_2) C_2(t_1, t_2; \Sigma_{t_1,t_i}, \Sigma_{t_2,t_i})
+ ... + \frac{1}{n!} \left( \frac{i}{\hbar} \right)^n \int_{t_i}^{t_f} dt_1 ... dt_n \Delta(t_1) ... \Delta(t_n) C_n(t_1, ..., t_n; \Sigma_{t_1,t_i}, ..., \Sigma_{t_n,t_i})
+ ...
\] (3.4)
This form of the influence functional will turn out to be useful for its physical interpretation. From (2.9) and the propagator \( \hat{U} \) given by
\[
\hat{U}[a_{t_1, t_i}] = \prod_n \mathcal{T} \exp \left[ -\frac{i}{\hbar} \int_{t_i}^{t_f} ds \left( \hat{H}_e(q, s) + \lambda \sigma(a, \dot{a}) \xi(s) \right) \right],
\] (3.5)
it is observed that \( C_n \) is of order \( \lambda^n \) in the coupling constant. We can interpret the \( C_n \) in (3.4) as cumulants of a stochastic force. Consider the action
\[
S[a(s)] = \int_{t_i}^{t_f} ds \left( L(a, \dot{a}, s) + \sigma(a, \dot{a}) \xi(s) \right)
\] (3.6)
where \( \xi(s) \) is some forcing term. We want to consider the case where \( \xi(s) \) is a stochastic force with a normalised probability density functional \( \mathcal{P}[\sigma(a, \dot{a}); \xi(s)] \). The probability density functional is taken to be conditional on the system history \( \sigma(a, \dot{a}) \). The action (3.6) generates the influence functional
\[
\mathcal{F}[\Sigma, \Delta] = \left< \exp \left[ \frac{i}{\hbar} \int_{t_i}^{t_f} \xi(s) \Delta(s) ds \right] \right>\xi
\equiv \int D\xi \mathcal{P}[\xi, \Sigma] \exp \left[ \frac{i}{\hbar} \int_{t_i}^{t_f} \xi(s) \Delta(s) ds \right]
\] (3.7)
where \( \Sigma \) and \( \Delta \) are defined in (3.2). The essential point is that the influence functional (3.7) has the exact same form as the characteristic function of the stochastic process \( \xi \).
Therefore given any influence functional $\mathcal{F}[\Sigma, \Delta]$, we can interpret the $C_n$, given by (3.3), as the cumulants of an effective stochastic force $\xi(s)$ coupled to the system in a way described by the action (3.6). The probability density functional $\mathcal{P}[\xi, \Sigma]$ of $\xi(s)$ can be obtained from a given influence functional by inverting the functional Fourier transform (3.7).

If we ignore all cumulants after the second order (the cumulants are of order $\lambda^n$) we are making a Gaussian approximation to the noise. Although $\lambda$ is usually assumed to be small for the series (3.4) to converge, the formal expansion in orders of $\lambda$ is acceptable even if $\lambda = 1$, as long as the deviations from Gaussian are small. With the Gaussian approximation we can write the influence functional as

$$\mathcal{F}[a, a'] = \int D\xi \mathcal{P}[\xi, \Sigma] \exp\left[\frac{i}{\hbar} S_{IF}[a, a', \xi]\right]$$

(3.8)

where

$$\mathcal{P}[\xi, \Sigma] = P_0 \exp\left(-\int_{t_i}^{t_f} dt_1 \int_{t_i}^{t_f} dt_2 \xi(t_1)C_2^{-1}(t_1, t_2; \Sigma_{t_1, t_i}, \Sigma_{t_2, t_i})\xi(t_2)\right)$$

(3.9)

is the normalised functional distribution of $\xi(s)$ and

$$S_{IF}[a, a', \xi] = \int_{t_i}^{t_f} dt_1 \Delta(t_1)\left(C_1(t_1, \Sigma_{t_1, t_i}) + \xi(t_1)\right).$$

(3.10)

We can use this effective action to obtain our semiclassical equation of motion which is given by

$$\frac{\delta(S_{eff}[a, a', \xi])}{\delta \Delta_a(t)}\bigg|_{\Delta_a=0} = 0$$

(3.11)

where $\Delta_a = a - a'$. We find it becomes

$$\frac{\partial L}{\partial a} - \frac{d}{dt} \frac{\partial L}{\partial \dot{a}} + \left(\frac{\partial \sigma}{\partial a} - \frac{d}{dt} \frac{\partial \sigma}{\partial \dot{a}}\right)\left(C_1(t, \sigma_{t, t_i}) + \xi(t)\right) - \frac{\partial \sigma}{\partial \dot{a}}\left(\dot{C}_1(t, \sigma_{t, t_i}) + \dot{\xi}(t)\right) = 0$$

(3.12)

where $L(a, \dot{a})$ is the system Lagrangian and $\xi(t)$ is a zero-mean Gaussian stochastic force with a correlator given by

$$\langle\xi(t)\xi(t')\rangle = C_2(t, t'; \sigma_{t, t_i}, \sigma_{t', t_i}).$$

(3.13)

Clearly both the noise and driving term are still dependent on the system history in a complex way in general. However we can further simplify things by expanding around a background $a = a_b$. In this case we approximate the first cumulant by

$$C_1(t; \sigma_{t, t_i}) = C_1(t; \sigma_{t, t_i})|_{\sigma = \sigma_b} + \int_{t_i}^{t} dt' \tilde{\sigma}(t')\mu(t, t') + ...$$

(3.14)
\[ \dot{C}_1(t; \sigma_{t,t_i}) = \dot{C}_1(t; \sigma_{t,t_i}) |_{\sigma = \sigma_b} + \int_{t_i}^t dt' \tilde{\sigma}(t') \dot{\mu}(t, t') + \ldots \]  

(3.15)

where \( \tilde{\sigma} = \sigma - \sigma_b \) and

\[ \mu(t, t') = \frac{\delta C_1(t; \sigma_{t,t_i})}{\delta \tilde{\sigma}(t')} |_{\sigma = \sigma_b} \]  

(3.16)

\[ \dot{\mu}(t, t') = \frac{\delta \dot{C}_1(t; \sigma_{t,t_i})}{\delta \tilde{\sigma}(t')} |_{\sigma = \sigma_b} \]  

(3.17)

where we have assumed in (3.15) that \( \mu(t, t') \) is antisymmetric as will be the case for our examples. The noise \( \xi(t) \) now has the correlator

\[ \langle \xi(t) \xi(t') \rangle = C_2(t, t'; \sigma_{t,t_i}, \sigma_{t',t_i}) |_{\sigma = \sigma_b}. \]  

(3.18)

These approximations greatly simplify (3.12). Our task is then to solve for the fluctuations \( \tilde{a} \equiv a - a_b \) subject to the initial condition \( \tilde{a}(t_i) = \dot{\tilde{a}}(t_i) = 0. \)

### 4 Influence Functional for Cosmological Backreaction

In this section, following the methods of [21], we will derive the form of the influence functional in terms of the Bogolubov coefficients in the transformation between the creation and annihilation operators of field amplitudes at different times. First we show how the dynamics of a general real scalar field in an expanding FRW universe can be described by a sum over quadratic time dependent Hamiltonians. Then we discuss the Bogolubov coefficients in terms of the squeeze parameters [31]. It also applies to the case of gravity wave perturbations whose two polarizations obey wave equations of the same form as a massless, minimally coupled scalar field (see [32] for details).

The action for a free scalar field in an arbitrary space-time can be written as the sum of gravitation action \( S_g \) and matter action \( S_m \) of the form

\[ S_g = l_p^2 \int d^4 x \sqrt{-g} (R - 2\Lambda) - 2l_p^2 \int d^3 x \sqrt{-h} K \]  

(4.1)

\[ S_m = \frac{l_p^2}{2} \int d^4 x \sqrt{-g} \left( g^{\mu\nu} \nabla_\mu \Phi \nabla_\nu \Phi - (m^2 + \xi_d R) \Phi^2 \right) + \xi_d l_p^2 \int d^3 x \sqrt{-h} K \Phi^2. \]  

(4.2)

where \( l_p^2 = 1/(16\pi G) \) and \( \xi_d = (n - 2)/4(n - 1) \) which in four dimensions \( d = 4 \) is equal to 0 for minimal coupling and 1/6 for conformal coupling. Adding a surface term in the action proportional to \( K \), the trace of the extrinsic curvature, is necessary for a consistent variational theory [33] in the treatment of the backreaction problem.

In the spatially- flat Friedmann-Robertson-Walker (FRW) universe with metric

\[ ds^2 = a^2(\eta) \left( d\eta^2 - \sum_i dx_i^2 \right) \]  

(4.3)

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\[ R = 6\dot{a}/a^3, K = 3\dot{a}/a^2 \] (where a dot denotes a derivative with respect to conformal time \( \eta = \int dt/a \)) we have
\[ S_g = -V l_p^2 \int d\eta \left( 6\dot{a}^2 + 2\Lambda a^4 \right) \quad (4.4) \]
\[ S_m = \frac{l_p^2}{2} \int d^4x \left[ (\dot{\chi})^2 - \sum_i (\chi_i)^2 - 2(1 - 6\xi)\frac{\dot{a}}{a}\chi\dot{\chi} - \left( m^2 a^2 + (6\xi - 1)\frac{\dot{a}^2}{a^2} \right) \chi^2 \right]. \quad (4.5) \]

Here \( \chi = a\Phi \) is the rescaled field variable and \( V \) is the volume of the universe. From now on we will absorb \( l_p \) by rescaling \( \chi \) and \( a \).

We can expand the scalar field in a box of co-moving volume \( V \) (fixed coordinate volume)
\[ \chi(x) = \sqrt{\frac{2}{V}} \sum_k \left[ q_k^+ \cos \vec{k} \cdot \vec{x} + q_k^- \sin \vec{k} \cdot \vec{x} \right] \quad (4.6) \]
which leads to the Lagrangian
\[ L(\eta) = \frac{1}{2} \sum_{\sigma} \sum_k \left[ (\dot{q}_k^\sigma)^2 - 2(1 - 6\xi_d)\frac{\dot{a}}{a} q_k^\sigma \dot{q}_k^\sigma - \left( k^2 + m^2 a^2 + (6\xi_d - 1)\frac{\dot{a}^2}{a^2} \right) q_k^\sigma q_k^\sigma \right] \quad (4.7) \]
where \( k = |\vec{k}| \) and \( S_m(\eta) = \int L(\eta)d\eta \). The canonical momentum is
\[ p_k^\sigma = \frac{\partial L(\eta)}{\partial \dot{q}_k^\sigma} = \dot{q}_k^\sigma - (1 - 6\xi_d)\frac{\dot{a}}{a} q_k^\sigma. \quad (4.8) \]

Defining the canonical Hamiltonian the usual way we find
\[ H(\eta) = \frac{1}{2} \sum_{\sigma} \sum_k \left[ p_{sk_k}^\sigma p_{sk_k}^\sigma + \left( 1 - 6\xi_d \right) \frac{\dot{a}}{a} \left( p_{sk_k}^\sigma q_k^\sigma + q_k^\sigma p_{sk_k}^\sigma \right) + \left( k^2 + m^2 a^2 + 6\xi_d(6\xi_d - 1)\frac{\dot{a}^2}{a^2} \right) q_k^\sigma q_k^\sigma \right] \quad (4.9) \]
where the sum is over positive \( k \) only since we have an expansion over standing rather than travelling waves. The system is quantized by promoting \((p_k^\sigma, q_k^\sigma)\) to operators obeying the usual harmonic oscillator commutation relations. In this way the dynamics of the field is reduced to the dynamics of time-dependent harmonic oscillators. (See [21] for details.)

In the case of a free quantized scalar field coupled to a spatially-flat FRW universe with scale factor \( a(s) \) the action thus belongs to the general form
\[ S[a, q] = \int_{t_i}^t ds \left[ L(a, \dot{a}, s) + \sum_k \left\{ \frac{1}{2} m_k(a, \dot{a})(\dot{q}_k^2 + b_k(a, \dot{a}) q_k \dot{q}_k - \omega_k^2(a, \dot{a}) q_k^2) \right\} \right]. \quad (4.10) \]

By tracing out the scalar field we can obtain an influence functional and from which derive an equation of motion for the scale factor in the semiclassical regime. Here since we work explicitly in the semiclassical regime, the environment is quantum and gravity enters classically through the scale factor \( a \).
We want to calculate the influence functional for this model. From (2.9) we see that it is formally given by
\[ F[a, a'] = \prod_k \text{Tr} \left( \hat{U}_k[a_{t,t_i}] \hat{\rho}_b(t_i) \hat{U}_k^\dagger[a'_{t,t_i}] \right) \] (4.11)
where \( \hat{U}_k \) is the quantum propagator for the bath mode in (4.10) with \( a(s) \) treated as an arbitrary classical time dependent function. We have derived this propagator before \([30]\) and will only quote the results here. The result for a particular mode is (we drop the mode label)
\[ \hat{U}[a_{t,t_i}] = \hat{S}(r, \phi) \hat{R}(\theta) \] (4.12)
where
\[ \hat{R}(\theta) = e^{-i\theta \hat{B}}, \quad \hat{S}(r, \phi) = \exp[r(\hat{A} e^{-2i\phi} - \hat{A}^\dagger e^{2i\phi})] \] (4.13)
and
\[ \hat{A} = \frac{\hat{a}^2}{2}, \quad \hat{A}^\dagger = \frac{\hat{a}^2}{2}, \quad \hat{B} = \hat{a}^\dagger \hat{a} + 1/2. \] (4.14)
\( \hat{S} \) and \( \hat{R} \) are called squeeze and rotation operators respectively. The parameters \( r, \phi, \theta \) are determined from the equations
\[ \dot{\alpha} = -ig^* \beta - i\alpha h \] (4.15)
\[ \dot{\beta} = ih \beta + ig \alpha \] (4.16)
where
\[ \alpha = e^{-i\theta} \cosh r, \quad \beta = -e^{-i(2\phi+\theta)} \sinh r \] (4.17)
and
\[ g(t) = \frac{1}{2} \left( \frac{m(t)\omega^2(t)}{c} + \frac{m(t)b^2(t)}{4c} - \frac{c}{m(t)} + ib(t) \right) \] (4.18)
\[ h(t) = \frac{1}{2} \left( \frac{c}{m(t)} + \frac{m(t)\omega^2(t)}{c} + \frac{m(t)b^2(t)}{4c} \right). \] (4.19)
c is an arbitrary positive real constant that is usually chosen so that \( g = 0 \) at \( t_i \). We must have \( \alpha = 1 \) and \( \beta = 0 \) at \( t_i \) so that the initial condition for the propagator is satisfied. The time dependence on \( g \) and \( h \) comes directly from \( a \) in (4.10).

Applying (4.12) to (4.11) we find that the influence functional for a mode in an initial vacuum state is given by
\[ \mathcal{F}_k[a, a'] = \sum_n \langle n | \hat{S}(r, \phi) \hat{R}(\theta) |0 \rangle \langle 0 | \hat{R}^\dagger(\theta') \hat{S}^\dagger(r', \phi') |n \rangle \] (4.20)
where \( |n \rangle \) are the usual number states. Using
\[ \hat{R}(\theta) |0 \rangle = e^{-i\theta/2} |0 \rangle \] (4.21)
we find
\[ \mathcal{F}_k[a, a'] = \sum_n \left[ \langle n | \hat{S}(r, \phi) |0 \rangle \langle 0 | \hat{S}^\dagger(r', \phi') |n \rangle \right] e^{-i(\theta-\theta')/2}. \] (4.22)
With
\[ S(r, \phi) \langle 0 \rangle = (\cosh r)^{-1/2} \sum_{n=0}^{\infty} \left[ -e^{2i\phi} \tanh r \frac{\sqrt{(2n)!}}{2^n n!} |2n\rangle \right] \] (4.23)
and making use of
\[ \sum_n \left[ \frac{(2n)!}{(n!)^2} x^n \right] = \frac{1}{\sqrt{1 - 4x}} \] (4.24)
we can show that
\[ F[a, a'] = \prod_k \frac{1}{\sqrt{\alpha_k[a'] \alpha_k^*[a] - \beta_k[a'] \beta_k^*[a]}}. \] (4.25)
This shows yet another way of deriving the influence functional in terms of the Bogolubov coefficients, in addition to the derivations given in [22].

5 Einstein-Langevin Equation

From the Hamiltonian (4.9) we see that the system-environment interaction is separable for two cases: the massive conformally coupled field (for which \( \sigma = a^2 \) in (3.1)) and the massless minimally coupled field (\( \sigma = \dot{a}/a \)) which also describes gravitons. For these two cases the results from Sec. 3 apply: (3.12) is the appropriate equation describing backreaction of the quantum scalar field on the metric. To derive the Einstein-Langevin equation we need to compute the first two cumulants given by (3.3) using the influence functional (4.25).

The solution of (4.15) and (4.16) can be written as
\[ U[a_{t,t_i}] = \mathcal{T} \exp \left( -i \int_{t_i}^{t} ds \ u(s) \right) \] (5.1)
where
\[ u(s) = \begin{pmatrix} h(s) & g^*(s) \\ -g(s) & -h(s) \end{pmatrix} \] (5.2)
and
\[ U[a_{t,t_i}] = \begin{pmatrix} \alpha[a_{t,t_i}] & \beta^*[a_{t,t_i}] \\ \beta[a_{t,t_i}] & \alpha^*[a_{t,t_i}] \end{pmatrix}. \] (5.3)
The key to calculating the functional derivative of (5.1) is recognizing that we can always write \( U[a_{t,t_i}] = U[a_{t,r}] U[a_{r,t_i}] \). We therefore find
\[ \frac{\delta U[a_{t,t_i}]}{\delta \Delta(\tau)} = \frac{\delta U[a_{t,r}]}{\delta \Delta(\tau)} U[a_{r,t_i}] + U[a_{t,r}] \frac{\delta U[a_{r,t_i}]}{\delta \Delta(\tau)}. \] (5.4)
Making use of the formal expression for the time ordered representation of (5.1) it is easy to see that
\[ \frac{\delta U[a_{t,r}]}{\delta \Delta(\tau)} = -i U[a_{t,r}] \int_{\tau}^{t} ds \ \frac{\delta u(s)}{\delta \Delta(\tau)} \] (5.5)
\[
\frac{\delta U[a_{\tau,t\tau}]}{\delta \Delta(\tau)} = -i \left( \int_{t_i}^{\tau} ds \frac{\delta u(s)}{\delta \Delta(\tau)} \right) U[a_{\tau,t\tau}].
\] (5.6)

Substituting (5.5) and (5.6) into (5.4) we find that
\[
\frac{\delta U[a_{\tau,t\tau}]}{\delta \Delta(\tau)} = -i U[a_{\tau,t\tau}] \left( \int_{t_i}^{t} ds \frac{\delta u(s)}{\delta \Delta(\tau)} \right) U[a_{\tau,t\tau}].
\] (5.7)

5.1 Massive conformally coupled field

For the massive conformally coupled case we have \( \sigma = a^2 \) and
\[
g = \frac{1}{2} \left[ \frac{(k^2 + m^2 a^2) t_p^2}{c} - \frac{c}{t_p^2} \right], \quad h = \frac{1}{2} \left[ \frac{(k^2 + m^2 a^2) t_p^2}{c} + \frac{c}{t_p^2} \right]
\] (5.8)
in (4.18-19). From (3.3) and (4.25) (we have reinstated the Planck length) we find the first cumulant of the stochastic force is
\[
C_1(\eta; a^2_{\eta,\eta}) = -\frac{l_p^2 m^2}{2} \sum_{\sigma} \sum_{k} \langle \tilde{q}_k^2 \rangle = -\frac{l_p^2 m^2 h}{4} \sum_{\sigma} \sum_{k} \frac{1}{c} (\alpha_{\eta} + \beta_{\eta})(\alpha_{\eta} + \beta_{\eta})
\] (5.9)

where \( \tilde{q}_k^2 = \hat{U}^\dagger[a_{\eta,\eta}] \tilde{q}^2 \hat{U}[a_{\eta,\eta}] \rangle \) and the average is with respect to the vacuum. The propagator \( \hat{U} \) is given by (4.12) with the Bogolubov coefficients determined via (4.15-16) with \( g, h \) given by (5.8). We will use this notation below as well. Similarly for the second cumulant we find
\[
C_2(\eta, \eta' ; a^2_{\eta,\eta}, a^2_{\eta',\eta'}) = -\frac{l_p^4 m^4}{8} \sum_{\sigma} \sum_{k} \left[ \langle \tilde{q}_k^2 \tilde{q}_k^2 \rangle - \langle \tilde{q}_k^2 \rangle^2 \right]
\]
\[
= -\frac{l_p^4 h^2 m^4}{16} \sum_{\sigma} \sum_{k} \frac{1}{c^2} \left[ (\beta_{\eta} + \alpha_{\eta})^2 (\alpha_{\eta}^* + \beta_{\eta}^*)^2 + (\beta_{\eta'}^* + \alpha_{\eta'})^2 (\alpha_{\eta'} + \beta_{\eta'})^2 \right].
\] (5.10)

Applying (3.16) to (5.9) we find the dissipation kernel to be
\[
\mu(\eta, \eta') = \frac{i l_p^4 m^4}{4h} \sum_{\sigma} \sum_{k} \left[ \langle \tilde{q}_k^2 \tilde{q}_k^2 \rangle - \langle \tilde{q}_k^2 \rangle^2 \right] \bigg|_{a^2 = a^2_{\eta}}
\]
\[
= \frac{i l_p^4 h m^4}{8} \sum_{\sigma} \sum_{k} \frac{1}{c^2} \left[ (\beta_{\eta} + \alpha_{\eta})^2 (\alpha_{\eta}^* + \beta_{\eta}^*)^2 - (\beta_{\eta}^* + \alpha_{\eta}^*)^2 (\alpha_{\eta'} + \beta_{\eta'})^2 \right] \bigg|_{a^2 = a^2_{\eta}}
\] (5.11)

Again we see the close relation between the noise and dissipation kernels.

Using (4.4) and \( \sigma = a^2 \) we find that the equation of motion (3.12) with the background approximation becomes
\[
\ddot{a} - \frac{2}{3} \Lambda a^3 + \frac{a(\eta)}{6V t_p^2} \left[ C_1(\eta; a^2_{\eta,\eta}) \bigg|_{a^2 = a^2_{\eta}} + \int_{t_i}^{\eta} ds' \dot{a}^2(\eta') \mu(\eta, \eta') \right] = -\frac{a(\eta)}{6V t_p^2} \xi(\eta)
\] (5.12)

where \( \xi \) is a zero mean gaussian stochastic force with the correlator (5.10) evaluated on the background \( a_b \).
5.2 Massless minimally coupled case

For the massless minimally coupled case, $\sigma = \dot{a}/a$,

\[ g = -i \frac{\dot{a}}{a} + \frac{1}{2} \left( \frac{l_p^2 k^2}{c} - \frac{c}{l_p^2} \right), \quad h = \frac{1}{2} \left( \frac{l_p^2 k^2}{c} + \frac{c}{l_p^2} \right), \quad (5.13) \]

we get

\[ C_1(\eta; (\dot{a}/a)_{\eta, \eta}) = -\frac{i}{2} \sum_{\sigma} \sum_{k} \langle (pq + qp)_{\eta} (pq + qp)_{\eta} \rangle = -\frac{i\hbar}{2} \sum_{\sigma} \sum_{k} [\alpha_{\eta}^* \beta_{\eta} - \alpha_{\eta} \beta_{\eta}^*] \quad (5.14) \]

where $p$ is the canonical momentum from the Lagrangian (4.7) with $m = \xi = 0$. For this case $\frac{\partial \sigma}{\partial a} - \frac{d}{d\eta} \frac{\partial \sigma}{\partial a} = 0$ so we see from (3.12) we must find $\dot{C}_1$. Taking the derivative of (5.14) and using (4.15-16) and (5.13) (with $c = l_p^2 k$) we find

\[ \dot{C}_1(\eta; (\dot{a}/a)_{\eta, \eta}) = \hbar \sum_{\sigma} \sum_{k} k[\alpha_{\eta}^* \beta_{\eta} + \alpha_{\eta} \beta_{\eta}^*]. \quad (5.15) \]

For the second cumulant we find

\[ C_2(\eta, \eta'; (\dot{a}/a)_{\eta, \eta}, (\dot{a}/a)_{\eta', \eta'}) = \frac{1}{8} \sum_{\sigma} \sum_{k} \left[ \langle (pq + qp)_{\eta} (pq + qp)_{\eta'} \rangle + \langle (pq + qp)_{\eta'} (pq + qp)_{\eta} \rangle \right] - 2 \langle (pq + qp)_{\eta} \rangle \langle (pq + qp)_{\eta'} \rangle \]

\[ = \frac{\hbar^2}{4} \sum_{\sigma} \sum_{k} \left[ (\alpha_{\eta}^2 - \beta_{\eta}^2) (\alpha_{\eta'}^2 - \beta_{\eta'}^2) + (\alpha_{\eta}^* \beta_{\eta'}^* - \beta_{\eta}^* \alpha_{\eta'}^*) \right]. \quad (5.16) \]

From (3.17) and (5.15) the dissipation kernel is given by

\[ \dot{\mu}(\eta, \eta') = -\hbar \sum_{\sigma} \sum_{k} k \left[ (\beta_{\eta}^2 + \alpha_{\eta}^2)(\alpha_{\eta'}^* \beta_{\eta'}^* - \beta_{\eta'}^* \beta_{\eta}^*) + (\beta_{\eta}^2 + \alpha_{\eta}^* \beta_{\eta'}^*)(\alpha_{\eta'}^2 - \beta_{\eta'}^2) \right] \bigg|_{\dot{a}/a = (\dot{a}/a)_{b}}. \quad (5.17) \]

The equation of motion (3.12) with the background approximation becomes

\[ \ddot{a} - \frac{2}{3} \Lambda a^3 - \frac{1}{12V l_p^2 a(\eta)} \left[ \dot{C}_1(\eta, (\dot{a}/a)_{\eta, \eta}) \bigg|_{\dot{a}/a = (\dot{a}/a)_{b}} + \int_{\eta_1}^{\eta} d\eta' \frac{\dot{a}(\eta')}{a(\eta')} \dot{\mu}(\eta, \eta') \right] = \frac{\dot{\xi}(\eta)}{12V l_p^2 a(\eta)}. \quad (5.18) \]

We need to know the stochastic properties of $\dot{\xi}(\eta)$ given that $\dot{\xi}(\eta)$ is a zero mean gaussian stochastic force with the correlator (5.16) evaluated on a background. We can deduce this by integrating by parts the noise term in the effective action (3.4). We find that (relaxing the notation for $C_2$)

\[ \int_{\eta_1}^{\eta_2} d\eta_1 d\eta_2 \Delta(\eta_1) \Delta(\eta_2) C_2(\eta_1, \eta_2) = \text{surface term} \]
\[ + \int_{\eta_1}^{\eta_f} d\eta_1 \Gamma(\eta_1) \left[ \frac{dC_2}{d\eta_1}(\eta_1, \eta_i) \Gamma(\eta_i) - \frac{dC_2}{d\eta_1}(\eta_1, \eta_f) \Gamma(\eta_f) \right] \]
\[ + \int_{\eta_1}^{\eta_f} d\eta_1 \int_{\eta_1}^{\eta_f} d\eta_2 \Gamma(\eta_1) \Gamma(\eta_2) \frac{d^2C_2(\eta_1, \eta_2)}{d\eta_1 d\eta_2} \]
\[ (5.19) \]

where \( \Gamma(\eta) = \int d\eta \Delta(\eta) = \ln a - \ln a' \). The surface term will not contribute to the equation of motion but the last term of (5.19) shows clearly that \( \dot{\xi}(t) \) corresponds to a zero mean gaussian stochastic force with the correlator

\[ C_{2\xi}(\eta, \eta') = \frac{d^2C_2(\eta_1, \eta_2)}{d\eta_1 d\eta_2}. \]
\[ (5.20) \]

The meaning of the middle term of (5.19) is more difficult to interpret. It vanishes only when the noise is stationary since we then have \( C_2(\eta, \eta') = C_2(\eta - \eta') \). We will not discuss this term further since it will vanish in the example we consider next. Clearly though, its meaning will need to be considered for a study about nonstationary backgrounds.

### 5.3 Backreaction of graviton fluctuations about flat space

A simple case to study is a massless minimally-coupled field around a flat background \( (\tilde{a} = a) \). In this case \( \alpha(\eta) = e^{-ik\eta} \) and \( \beta(\eta) = 0 \). We see that in this case the first cumulant \( (5.14) \) vanishes. This should be compared to the massive field where the first cumulant is divergent around a flat background. The noise kernel \( (5.16) \) becomes

\[ C_2(\eta - \eta') = \frac{\hbar^2 V}{32 \pi^2} \int_0^\infty dk \ k^2 \cos[k(\eta - \eta')] = -\frac{\hbar^2 V}{32 \pi} \delta''(\eta - \eta') \]
\[ (5.21) \]

where a prime on a function denotes a derivative taken with respect to its argument. From (5.20) we have

\[ C_{2\xi}(\eta - \eta') = \frac{\hbar^2 V}{32 \pi^2} \int_0^\infty dk \ k^4 \cos[k(\eta - \eta')] = \frac{\hbar^2 V}{32 \pi} \delta'''(\eta - \eta'). \]
\[ (5.22) \]

The dissipation kernel \( (5.17) \) becomes

\[ \dot{\mu}(\eta - \eta') = -\frac{\hbar V}{16 \pi^2} \int_0^\infty dk \ k^3 \cos[k(\eta - \eta')]. \]
\[ (5.23) \]

The Einstein-Langevin equation \( (5.18) \) becomes

\[ \ddot{a} - \frac{2}{3} \Lambda a^3 - \frac{1}{12Vl_p^2 a(\eta)} \int_\eta^\eta d\eta' \frac{\dot{a}(\eta')}{a(\eta')} \dot{\mu}(\eta - \eta') = \frac{1}{12Vl_p^2 a(\eta)} \dot{\xi}(\eta). \]
\[ (5.24) \]

where \( \dot{\xi} \) is a zero-mean Gaussian force with the correlator \( (5.22) \). The solution of the Einstein-Langevin equations discussed here are beyond the scope of the present paper. We plan to consider these solutions in the future in the context of a general study into the dynamics of second order Langevin equations with non-local dissipation and colored noise [34].
6 Summary

Together with two related work [22, 23], this paper seeks to establish a new framework for the study of semiclassical gravity theory based on the Einstein-Langevin equation. In [22] the noise and fluctuation terms are identified from the closed-time-path formalism and the Einstein-Langevin equation derived for perturbances off the Robertson-Walker spacetime. In [23] the influence functional method is used to derive an equation of motion for the anisotropy matrix of the Bianchi Type-I universe. Dissipation of anisotropy from particle creation in a quantum scalar field is seen to be driven by an additional stochastic source (noise) term related to the fluctuations of particle creation and shown to be a manifestation of a fluctuation-dissipation relation. In this paper, we have derived the following results:

• By carrying out a functional Taylor series expansion on the influence functional we show how the successive orders measure the higher cumulants of noise in its most general (colored and multiplicative) forms, the lowest order truncation yielding a Gaussian noise. The second cumulant gives the autocorrelation function for the stochastic force (noise), which drives the Einstein-Langevin equation.

• Using a general form for the Hamiltonian of a quantum field whose normal modes are coupled to a curved spacetime parametrically, we showed a new way to derive the influence functional in terms of the Bogolubov coefficients between the second-quantized operators of Fock spaces at two different times. This relation connects our new influence functional / effective action method with the traditional canonical quantization approach and thus incorporates the established body of knowledge in quantum field theory in curved spacetimes.

• With the previous two results we were able to express the noise and dissipation kernels in terms of the Bogolubov coefficients. This connection offers a more transparent interpretation of the physical meaning of the many statistical mechanical processes such as decoherence and dissipation in terms of particle creation and related quantum effects.

• We have also derived the form of the Einstein-Langevin equations for some well-studied cases of scalar fields in Robertson-Walker and de Sitter spacetimes. They form the starting points of the next stage of work, which is the solution of these equations for the analysis of fluctuations, instability and phase transition. We hope to report on these problems in future communications.

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