The effect of negative viscosity of a fluid flowing over a right circular cylinder of infinite length

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Abstract: Negative viscosity seems to be an impossible parameter for any thermodynamic system. But for some special boundary conditions the viscosity of a fluid has apparently become negative, like for secondary flow of a fluid or in a plasma flow interacting with a dominant magnetic field. This work focuses on the effect of apparent negative viscosity for a fluid flow over a cylinder. Five different viscosities are applied, which consist of zero viscosity and two negative and positive viscosities. The results show a vast difference in the vortex formation, pattern and their sustainability. General incompressible Navier Stokes equation has been employed for the analysis. The stability of the Navier Stokes equation with negative viscosity has been studied using CFL criterion. The vortex formation and the subsequent analysis of their kinetic energies has been performed using the spatially averaged and time averaged vorticity magnitude and the magnitude of Enstrophy. The sustainability of the vortices with respect to the overall flow kinetic energy has been studied by using the Vorticity Sustainability Number (VSN), which has been also defined in the same work. This parameter provides a parameterization of the sustainability of the vorticities in a flow.

Keywords: Negative Viscosity; Enstrophy; External Flow; Vorticity Sustainability Number (VSN).

Introduction: Every physical system in the universe undergoes some kind of losses while in operation. When a fluid flow is considered as a physical system, it undergoes some losses in its flow fields or parameters. Among the different parameters of a fluid flow, viscosity is an important parameter which is a dominant cause of frictional losses. This parameter affects the losses related to the pressure-velocity fields. The viscosity of a fluid basically resists its flow in between two adjacent layers. This resistance generates due to the action of cohesive force in a liquid and momentum transfer for gases [1, 2]. In practice the viscosity varies with temperature. As the temperature increases, the energy of each molecules increases, which eventually decreases the cohesive force in between the two molecules of liquid. This effect reduces the viscosity of that liquid with increasing temperature. But, for gaseous fluid, the
increasing temperature increases the mobility of the molecules and as a result the momentum transfer increases. This increases the viscosity of gases with increasing temperature [3]. The viscosity of a fluid is represented in two different forms, kinematic viscosity and dynamic viscosity. Here in this study, the viscosity is represented in the kinematic form. The magnitude of viscosity of any practical fluid is always positive. But the effective viscosity can be sometimes negative under some special boundary conditions. In an isotropic medium when a steady external force gets applied, the effective viscosity of the fluid may become negative based on the direction [4, 5]. The effective viscosity may become negative for plasma flow under special boundary conditions when the dominant magnitude of the Lorentz force opposes the viscous force, which is inferior compared to the former force [6, 7].

This work addresses a similar kind of boundary condition, where the plasma flow has been considered. The magnitude of the Lorentz force is larger than that of the viscous force and their directions are opposite to each other. The flow domain has been considered as a flow over a right circular cylinder of infinite length, as it’s a benchmark problem. The Fig.1 shows the problem pictorially for easy understanding purpose.

![Fig.1. Problem description](image)

Flow over a cylinder shows an important phenomenon of Von Karman vortex street [8]. The effects of the vortex street are very much common for the positive effective viscosity. But the effects and types of vortex street in an effective negative viscosity environment still not studied much. Sivashinsky, G. et al. [5] claims, when the effective viscosity becomes negative in a large-scale flow, small scale eddies generate initially. This gradually form large scale eddies. Therefore, in this work also, this similar result can be expected. But an important point may be taken into account about the sustainability of the vortices that formed as the flow interacts with the cylinder. The vorticity formation is an important phenomenon in the case of flow instability, combustion instability and mixing etc. When vortices form, it develops more flow instability
due to which fluid power losses occur [9]. Also, vortex motion develops combustion instability [10]. Therefore, more vorticity formation indicates more flow instability and more combustion instability. However, the vortex motion helps mixing [11, 12]. So, more vorticity inside the flow can be beneficial for mixing, but develops flow instability and combustion instability. Hence, it’s important to study the formation and sustainability of the vorticity in a flow domain, which has been studied in this work. The following discussion is going to explain the computational modeling of the CFD domain.

Computational modeling: The problem has been studied using the simplest incompressible Navier Stokes equation. The inlet velocity has been taken as 0.1307 m/s. The fluid temperature is 300 K. The negative viscosity concept has been derived as following way.

\[
\rho \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u}
\]  
(Eq.1)

Eq.1 is the Navier Stokes equation for an incompressible flow with no influence on any external force [13]. But when the Lorentz appears, the Eq.1 converts to MHD equation [14].

\[
\rho \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u} + (\vec{j} \times \vec{B})
\]  
(Eq.2)

The Eq.2 expresses the simplest form of MHD equation for an incompressible plasma flow with \(\vec{j}\) as the current density and \(\vec{B}\) as the magnetic field [14]. The Eq.2 can be rewritten as,

\[
\rho \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u} + |\vec{j}| |\vec{B}| \sin \theta
\]  
(Eq.3)

When \(180^\circ \leq \theta \leq 270^\circ \Rightarrow 0 \leq \sin \theta \leq -1\). Therefore, the Eq.3 will be transformed into the following form.

\[
\rho \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u}, \text{ when } \theta = 180^\circ
\]  
(Eq.4a)

\[
\rho \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u} - |\vec{j}| |\vec{B}|, \text{ when } \theta = 270^\circ
\]  
(Eq.4b)

Therefore, for \(180^\circ < \theta < 270^\circ\), the Eq.3 can be as follows,

\[
\rho \frac{D\vec{u}}{Dt} = -\nabla p + \mu \nabla^2 \vec{u} - |\vec{j}| |\vec{B}| \sin \theta
\]  
(Eq.5)

Now if, the magnitude of Lorentz force is greater than the viscous force, then mathematically the function \(NG\) is,

\[NG = \mu \nabla^2 \vec{u} - |\vec{j}| |\vec{B}| \sin \theta| < 0 \Rightarrow NG = \zeta \nabla^2 \vec{u} < 0\]
Therefore, the Eq.5 can be rewritten as,

\[ \rho \frac{D\vec{u}}{Dt} = -\nabla p + \zeta \nabla^2 \vec{u} \]  

(Eq.6)

The Eq.6 shows an effective negative viscosity of the fluid and this equation has been used in this work. The Fig.2 shows the meshed CFD domain. To perform a high-fidelity simulation with a reasonable computational expense, the minimum grid size along the x axis has been taken as 1.33 mm and that of for the y axis has been taken as 0.99 mm. There are 60744 number of nodes are created.

![Fig.2. Meshed CFD domain](image)

The purpose of this study is to closely monitor the formation and sustainability of vortices formed under varying viscosity condition from negative to positive including inviscid flow. The vortex formation across the cylinder wall is the most important phenomena, which is responsible to form the vortex street. To study this the meshed region across the cylinder region has been refined to the order of 2.

**Vorticity sustainability number:** Vorticity is an important flow parameter which determines how fast a rotating flow rotates. Mathematically the vorticity can be defined as follows [15],

\[ \vec{\omega} = \nabla \times \vec{u} \]  

(Eq.7)
The kinetic energy of the vortex motion is called the Enstrophy [16]. Mathematically the Enstrophy can be defined as follows [17],

\[ \varepsilon = \iint |\omega|^2 dS \]  

(Eq.8)

Now the ratio of Enstrophy to the kinetic energy of the overall flow can be expressed as follows,

\[ f_{KEv} = \frac{2\varepsilon}{|u|^2} \]  

(Eq.9)

The term \( f_{KEv} \) represents the how much the kinetic energy of the vortex is dominant than the kinetic energy of the overall flow. When \( f_{KEv} = 0 \), means there no vortex formation occurred. This condition occurs when there is no obstruction available inside the flow. However, when \( f_{KEv} > 1 \), means the kinetic energy of the vortex is dominant than the kinetic energy of the overall flow. This condition generally occurs when there’s an obstruction in the flow is present, which resist the fluid flow. So, the flow becomes unstable. Therefore, the flow instability increases when the \( f_{KEv} \) increases by potentially obstructing the overall flow.

The Fig.3 represents a rotational fluid element in an overall flow. When a rotational motion develops inside a fluid flow the vorticity comes into the discussion. However, following the basic physics, whenever a rotational motion develops a centrifugal force generates. This force always tries to obstruct the circular motion. Here also, when vortex motion develops in a fluid flow, the centrifugal force tries to minimize the vorticity magnitude. Therefore, the high vorticity magnitude is more prone to less sustainable rotational motion due to high centrifugal force. This is why, studying the Enstrophy and kinetic energy of the overall flow is important.

To minimize the fluctuations in the Enstrophy and kinetic energy of the overall flow with time, the parameter \( f_{KEv} \) has been square rooted. Here, to estimate the sustainability of a vortex in the flow domain, it’s necessary to determine the magnitude of the parameter \( \sqrt{f_{KEv}} \) w.r.t. the angular displacement of the fluid element in a single timestep. Therefore mathematically,

\[ VSN = \frac{\sqrt{f_{KEv}}}{|d\theta|} = \frac{\sqrt{f_{KEv}}}{|\omega|dt} \]  

(Eq.10)

Hence, this is called the Vorticity Sustainability Number (VSN). The VSN has been designed to determine the effectiveness of kinetic energy responsible to develop a vortex flow w.r.t. the overall flow per unit angular displacement of the fluid element.
Results: The positive viscosity resists a flow, whereas, an effective negative viscosity should accelerate the flow. Therefore, the effect of negative viscosity can be studied by closely monitoring some important flow parameters. External flow over a cylinder shows an important feature of vortex street. Here, the study has been performed by following the Eq.6. Therefore, the values of apparent viscosities are as follows,

\[
\begin{align*}
\zeta &= -1.51E - 05 \text{ m}^2/\text{s}^2 \\
\zeta &= -1E - 06 \text{ m}^2/\text{s}^2 \\
\zeta &= 0 \text{ m}^2/\text{s}^2 \\
\zeta &= 1E - 06 \text{ m}^2/\text{s}^2 \\
\zeta &= 1.51E - 05 \text{ m}^2/\text{s}^2
\end{align*}
\]

The inlet velocity has been considered as 0.1307 m/s and the total simulation time has been taken as 5 s. The timestep is adaptive and the initial timestep is 0.001 s. The inlet pressure is 1 bar with a temperature of 300 K. While applying the Eq.6, it’s important to study the stability of this equation. The Fig.4 shows the comparison of the CFL number for different viscosities. Studying the Fig.4(a), it’s clear that when the viscosities are negative, more disturbances generate over time. This is the reason why the CFL number oscillates with larger amplitude for the \( \zeta = -1.51E - 05 \text{ m}^2/\text{s}^2 \) and \( \zeta = -1E - 06 \text{ m}^2/\text{s}^2 \) than that of for the other values of \( \zeta \).

The increased magnitude of the negative \( \zeta \), the disturbances in the flow domain increases. The Fig.4(b) confirms the findings from the Fig.4(a). The time averaged CFL number also the highest for the \( \zeta = -1.51E - 05 \text{ m}^2/\text{s}^2 \) and the least for \( \zeta = 1.51E - 05 \text{ m}^2/\text{s}^2 \). The time averaged CFL number decreases almost following \( \zeta^{-2.16} \). Hence, the disturbances inside the flow domain might have decreased with increasing the value of \( \zeta \) from negative to positive.
Variation of CFL numbers with time for different viscosities

Fig. 4. Comparison of CFL numbers for different viscosities

In this work the flow instability has been studied using the vorticity magnitude, Enstrophy and the sustainability of the vortices formed has been studied using the VSN.

**Variation of the vorticity magnitude:** The spatial distribution of the vorticity magnitude plotted at $t = 5$ s for different viscosities in the Fig. 5. The range of the vorticity magnitude lies in between 0 to 100 rad/s. Closely analyzing the subfigures, it’s clear that the vorticity magnitude decreases as the viscosity increases from $\zeta = -1.51E - 05 \text{ m}^2/\text{s}^2$ to $\zeta = 1.51E - 05 \text{ m}^2/\text{s}^2$. This happened as a result of increasing the resistances in between adjacent fluid layers. The Fig. 5(a and b) shows the existence of high vorticity magnitude inside the flow domain due to the attraction between the adjacent fluid layers than the Fig. 5(c, d and e). The effective negative viscosity helps to develop more vortices with increasing angular velocities, which is just opposite of the positive viscosity. Hence, disturbances develop inside the flow domain and the disturbances decrease with increasing viscosity from $\zeta = -1.51E - 05 \text{ m}^2/\text{s}^2$ to $\zeta = 1.51E - 05 \text{ m}^2/\text{s}^2$. 

(a) Viscosity = - 1.51E-05 
(b) Viscosity = - 1E-06
The time averaged vorticity magnitude for different viscosities has been shown in the Fig.6. The standard case has been taken as the inviscid flow where $\zeta = 0 \text{ m}^2/\text{s}^2$. Following the same trend of the Fig.5, the time averaged vorticity magnitude is the highest of the $\zeta = -1.51E - 05 \text{ m}^2/\text{s}^2$ and least for the $\zeta = 1.51E - 05 \text{ m}^2/\text{s}^2$. The time averaged vorticity magnitudes for the $\zeta = -1.51E - 05 \text{ m}^2/\text{s}^2$ and $\zeta = -1E - 06 \text{ m}^2/\text{s}^2$ are 94 times and 10 times larger than that of for the $\zeta = 0 \text{ m}^2/\text{s}^2$, which is due to the effect the negative viscosity. However, the time averaged vorticity magnitude for the $\zeta = 1E - 06 \text{ m}^2/\text{s}^2$ and $\zeta = 1.51E - 05 \text{ m}^2/\text{s}^2$ are 36% and 56% lower than that of for the $\zeta = 0 \text{ m}^2/\text{s}^2$ due to positive viscosity. The time averaged vorticity magnitude decreases almost following $\zeta^{-3.556}$. Therefore, the vorticity magnitude decreases with increasing viscosity.
Fig. 6. Comparison of time averaged vorticity magnitude for different viscosities

**Variation of Enstrophy:** The Enstrophy provides a quantitative estimate of the kinetic energy of the vortices in a flow. High Enstrophy indicates the higher vorticity magnitude and more irregularity or disturbances in the flow domain. The Fig. 7 shows the comparison of logarithmic Enstrophy for different viscosities. From the Fig. 7(a) shows the Enstrophy sharply increases when \( \zeta = -1.51 E - 05 \ m^2/s^2 \) and \( \zeta = -1E - 06 \ m^2/s^2 \) over time. This happened as a result of negative viscosity which introduced more vorticities inside the flow domain by increasing the angular velocity of the vortices. However, the Enstrophy almost remain constant when the \( \zeta = 0 \ m^2/s^2 \) and decreases when \( \zeta = 1E - 06 \ m^2/s^2 \) and \( \zeta = 1.51E - 05 \ m^2/s^2 \) over time, which is obvious due to zero and positive viscosities respectively.
On the other side, analyzing the Fig.7(b), it’s clear that the time averaged Enstrophy decreases sharply with the increase of viscosity from $\zeta = -1.51E - 05 \, m^2/s^2$ to $\zeta = 1.51E - 05 \, m^2/s^2$. The time averaged Enstrophy of the case with $\zeta = 0 \, m^2/s^2$ is 7537 $m^2$ rad$^2$/s$^2$. The time averaged Enstrophy for the $\zeta = -1.51E - 05 \, m^2/s^2$ and $\zeta = -1E - 06 \, m^2/s^2$ are almost 15000 times and 84 times higher than that of for the $\zeta = 0 \, m^2/s^2$ due the effect of negative viscosity. However, the time averaged Enstrophy for the $\zeta = 1E - 06 \, m^2/s^2$ and $\zeta = 1.51E - 05 \, m^2/s^2$ are 91% and 99% lower than that of for the $\zeta = 0 \, m^2/s^2$ due to the effect of positive viscosity. The decrement follows with $\zeta^{-8.947}$ and this closely follows the trend shown in the Fig.7(a).

**Variation of fKEv and VSN:** Vorticities are important phenomena in a fluid flow, which can be analyzed by vorticity magnitude, Enstrophy etc. But it’s important to study their sustainability. High vorticity magnitude can increase the centrifugal force which acts to dissipate the rotational motion of the fluid element. Therefore, there’s no guarantee that the vortex motion will be sustained when the vorticity magnitude as well as the Enstrophy are high and hence, it’s important to discuss the sustainability of the vortex motion in this work. The Fig.8 shows the comparison of fKEv for different viscosities. This parameter will provide the effectiveness of kinetic energy of the vortices to the kinetic energy of the overall flow. Fig.8(a) shows, the fKEv for $\zeta = -1.51E - 05 \, m^2/s^2$ is around 350 rad$^2$ with considerable fluctuations due to more disturbances in the flow field (Fig.5a). The fKEv of the $\zeta = 0 \, m^2/s^2$ shows a decreasing trend and but low fluctuations than the $\zeta = -1.51E - 05 \, m^2/s^2$ and $\zeta = -1E - 06 \, m^2/s^2$ due to no external influence on the vortices formed. The fKEv for the $\zeta = 1E - 06 \, m^2/s^2$ is lower than that of the $\zeta = 0 \, m^2/s^2$ but higher than that of for the $\zeta = 1.51E - 05 \, m^2/s^2$. The reason behind this is the same reason of the effect of viscosity. However, the fKEv for $\zeta = 1E - 06 \, m^2/s^2$ and $\zeta = 1.51E - 05 \, m^2/s^2$ shows the least fluctuations due to the least disturbance in the flow field (Fig.5d and e).
An almost similar trend of fKEv can be noticed in the Fig.8(b) where the time averaged fKEv has been compared for different viscosities. The fKEv for the $\zeta = 0 \ m^2/s^2$ is almost 212 rad$^2$. The highest time averaged fKEv can be observed for $\zeta = -1.51E - 05 \ m^2/s^2$ which is 53% higher than that of for the $\zeta = 0 \ m^2/s^2$. This is due to assistance to develop rotational motion. However, the time averaged fKEv for $\zeta = -1E - 06 \ m^2/s^2$ decreased by 15% than the $\zeta = 0 \ m^2/s^2$ due to the sudden decrease of the fKEv after 1.5 s (Fig.8a). The time averaged fKEv decreases for $\zeta = 1E - 06 \ m^2/s^2$ and $\zeta = 1.51E - 05 \ m^2/s^2$ as the viscosity increases due to increase in the resistance to maintain a high vorticity magnitude and hence the Enstrophy decreases. The time averaged fKEv for $\zeta = 1E - 06 \ m^2/s^2$ and $\zeta = 1.51E - 05 \ m^2/s^2$ are 74% and 94% lower than that of for the $\zeta = 0 \ m^2/s^2$. Therefore, when the viscosity increases, the vortex dissipation increases and hence, the Enstrophy decreases.

Hereafter, it’s important to quantify the sustainability of the vortices formed. The magnitude of the fKEv and the vorticity will indicate how much dominant the rotational flow is, but these won’t provide any estimation about the sustainability of the vortex motion. The Fig.9 shows the variation and comparison of VSN with time for different viscosities. The Fig.9(a) shows an almost exponential decay of the VSN for all the viscosities except for $\zeta = -1E - 06 \ m^2/s^2$ with time. The overall VSN for the $\zeta = 0 \ m^2/s^2$ maintains the highest value over time among all the other values of $\zeta$ followed by that of for $\zeta = 1E - 06 \ m^2/s^2$, $\zeta = 1.51E - 05 \ m^2/s^2$ due to the absence of viscous resistance and gradual increase of viscous resistance respectively. However, the least VSN can be seen over time for the $\zeta = -1.51E - 05 \ m^2/s^2$ due to high disturbances in the flow, which increases the dissipation of the rotational motion of the fluid elements (Fig.5a).
(a) Variation of VSN with time for different viscosities

(b) Variation of time averaged VSN with viscosity

Fig. 9. Comparison of VSN for different viscosities

The Fig. 9(b) shows the comparison of the time averaged VSN with varying viscosities. The highest VSN of 76114 can be observed when the $\zeta = 0 \, m^2/s^2$ due to no viscous force to resist the rotational motion of the fluid element to maintain a sustainable vortex motion. But when the viscosity increases to $\zeta = 1E-06 \, m^2/s^2$ and $\zeta = 1.51E-05 \, m^2/s^2$, the VSN decreases by 51% and 73%, respectively, due to the increase in the viscous force. However, when the magnitude of the viscosity increases in the negative side, the VSN also decreases. The VSN for $\zeta = -1.51E-05 \, m^2/s^2$ and $\zeta = -1E-06 \, m^2/s^2$ are 92% and 51% lower than that of for the $\zeta = 0 \, m^2/s^2$ due to the effect of high flow disturbances in the flow (Fig. 5a and b). Hence, the sustainability of vorticity decreases by increasing the magnitude of viscosity irrespective of positive and negative types.

**Conclusion:** The discussion mainly discussed the effects and comparison of the negative, zero and positive viscosity in a fluid flow. The following points can be summarized.

- Applying negative value of effective viscosity, the stability of the incompressible Navier Stokes equation decreases, due to which the CFL number increases. Also, by increasing the effective viscosity from positive to negative the time averaged CFL number decreases following a power law profile.
- The vorticity magnitude, Enstrophy and fKEv decreases following a power law profile with increasing viscosity due to increase in the viscous resistance.
- Negative viscosity develops more flow instability in a flow by creating more disturbances inside the flow domain than the positive viscosity.
- The vorticity become lesser sustainable with an increase in the viscosity magnitude irrespective of positive or negative type.
Acknowledgement: The author declares an immense support from the SIMFLOW Technologies for providing SIMFLOW CFD Software Free License for this research.

Disclaimer: This research was started while the author’s affiliation was Vel Tech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology, Chennai - 600 062, India.

Nomenclature:

| Symbols | Names of parameter                  | Symbols | Names of parameter                  |
|---------|-------------------------------------|---------|-------------------------------------|
| VSN     | Vorticity Sustainability Number     | ρ       | Density                             |
| 𝜀       | Velocity of flow                    | p       | Pressure                            |
| μ       | Actual viscosity of fluid           | 𝐟       | Current density                     |
| 𝐵̅      | Magnetic field                      | θ       | Angle                               |
| ζ       | Effective viscosity                 | 𝜌̇      | (\frac{∂}{∂x} \hat{i} + \frac{∂}{∂y} \hat{j} + \frac{∂}{∂z} \hat{k}) |
| 𝜔̃      | Vorticity magnitude or Angular      | 𝜀       | Enstrophy                           |
|         | Velocity                             |         |                                     |
| S       | Surface vector                       | fKEv    | Ratio of Enstrophy to the kinetic   |
|         |                                     |         | energy of the overall flow          |
| 𝜃̃      | Angular displacement                | t       | Time                                |

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