Limit load estimation for finite plates with semi-elliptical defects under tensile loading

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Abstract. The ASME code allows the designer to circumvent many of the problems arising in design by elastic analysis by performing plastic or limit analysis of the component. Unlike elastic analysis, these analyses take account of the redistribution of stress upon yield. In limit analysis, the calculation of the limit load for finite plate with different semi-elliptical defects is an important stage for mechanical safety evaluation. The elastic compensation technique proposed by Mackenzie can be used to provide the limit load of a structure with a finite element method and aims at producing a suitable stress filed for the lower bound theory. In the present study, it is assumed that the finite plate exists three kinds of semi-elliptical cracks with low aspect ratio \(a/c\) (\(a/c=0.1, 0.5\) and \(1.0\)), respectively. The improved elastic compensation technique realized by finite element method is applied in the limit load estimation. At last, the results show that the improved elastic compensation technique can be used to obtain the limit value for the finite plate with different semi-elliptical defects under tensile loading. Moreover, the improved elastic compensation technique can be easily implemented in any finite element method and enables the limit load for any complex structure to be obtained swiftly.

1. Introduction

Maintaining structural integrity is always an important effort in the nuclear power industry. In order to evaluate the failure of engineering structure containing defects, it is necessary to calculate the maximum loading capacity. Therefore, the relationship between a plastic deformation and loading is very important in the failure analysis of structure [1].

Standard design rules are based on three principal studies: elastic analysis, limit analysis and elastic-plasticity analysis. In the elastic analysis, the designer should partition the calculated stresses into peak, primary and secondary stress categories in order to apply appropriate stress limits. The limit analysis is very interesting because the results obtained with it are less pessimistic than elastic results. Limit analysis becomes more and more attractive for fracture mechanics which need reference stress in the calculation of the J integral by simplified method such as the R6 method [2, 3].

In the analysis of plastic collapse loading in the NB-3213.25 of ASME Boiler and Pressure Vessel Code, the method can be used to calculate the corresponding load for a structure with given load combination. The limit analysis is conducted at the maximum loading for a selected structure made of perfectly plastic material. For the accurate assessment of plastic collapse, the hardening modulus, the strain, and the final deformation should be considered [4]. Even for an ideal non-strain hardening material, it is difficult to get the accurate limit loading.

One of the important problems in the structure is the existence of defects. These defects are caused by manufacturing or service and may lead to the failure of the entire system. The limit analysis would be used in evaluating fracture parameters [4]. The lower bound theory and the upper bound theory are taken in determination of the limit loading. The lower bound theory is used if the stress distribution under the equilibrium applied loading can be found and below the yield stress point or at the yield stress point, then the structure will not collapse or be at the collapse point only. The limit loading value captured by the lower bound theory represents the carrying capacity of the structural material. It gives the lower or safe value of the limit loading with the proof plastic strain. The limit load prediction by the
lower-bound theory is less than or equal to the exact limit load, and such predictions are appropriate for the safe design of structural or components. The upper bound theory is taken if there is any compatible plastic deformation mode and the external force acts at a velocity equal to or exceeding the velocity of an internal dissipation, the structural will collapse. The upper bound theory indicates the fact that the structure will not stand if there is an allowed failure path. It shows an upper or unsafe value for the limit loading to the plastic deformation. So, the minimum value of upper limit loading is related to the initial plastic deformation [5].

Mackenzie [6] combined the elastic modulus adjustment procedures suggested by Marriott and obtained lower and upper-bound limit loads for each iteration by invoking the bounding theorems in limit analysis. This method, termed as “elastic compensation technique” makes use of successive linear elastic finite element analysis in an attempt to obtained convergence. The elastic compensation technique [6-11] is viewed as a simple and effective calculation method, but it creates a large error in the estimation of the limit loading for complex structures. Therefore, the improved elastic compensation technique [12] for the analysis of complex structural limit analysis is an effective method. It can obtain a good evaluation of the limit loading with plastic deformation for structure and retains the advantages of simplicity, efficiency and ease for applications.

In this work, the improved elastic compensation technique is used to determine the limit loading up to point of proof plastic deformation at yield stress for the finite plate with three kinds of semi-elliptical defects considering tensile loading. The limit loading value is obtained and analyzed. It must be emphasized that the improved elastic compensation technique would be used to analyze complex structure.

2. Model and material
In order to analyze and calculate the loading-carrying capacity for finite plates with different semi-elliptical defects under tensile loading, the improved elastic compensation technique is executed by connecting the general finite element program.

Figure 1 shows the geometric dimensions of the finite plate, where \( a \) is the semi-elliptical defect depth, \( c \) is the half length of the major axis of the semi-elliptical defect. Figure 2 depicts typical finite element calculation model and mesh for the finite plate with a special semi-elliptical defect.

A full three-dimensional (3D) model is generated considering the symmetry boundary condition. The 8-node element (type solid185 in ANSYS element library) is used in this work. The numbers of elements and nodes for a typical finite element mesh range from 4972 elements to 9950 elements and 5886 nodes to 11847 nodes. In addition, the mesh of numerical analysis model is refined to ensure convergence of calculation.

![Figure 1. Geo. (finite plate with a semi-elliptical defect).](image)
The material is assumed to the elastic-plastic behavior with the yield stress value ($\sigma_y=200$ MPa) assumed is only used to evaluate and analyze limit loading solutions.

3. Improved elastic compensation technique

The calculation the lower limit loading at the improved elastic compensation technique is as follows:

Step 1: Applying selected loading and carrying out a round of elastic finite element analysis. Then, according to the calculation, the elastic modulus value of each element is adjusted and determined. The corresponding adjusted method is as follows,

$$E_{i+1}^e = E_i^e \frac{\sigma_{n}}{\sigma_i^e}, \quad i = 1, 2, 3, \ldots$$

(1)

where subscript $i$ is the serial number of iterative step, $e$ is the finite element number=1,2,3,…,$n$, $E$ is the elastic modulus, $\sigma$ is the maximum stress in element e. The nominal stress $\sigma_n$ is arbitrary value. In the elastic compensation technique, $\sigma_n$ is calculated as,

$$\sigma_n = \frac{\min(e) + \max(e)}{2}$$

(2)

The improved elastic compensation technique can give better results for the overall plastic failure of simple and uniform loading-bearing structures. However, for the local plastic damage of complex structures, large calculation errors are often generated. In the improved elastic compensation technique, the nominal stress that considering local stress concentration factor is as follows,

$$\sigma_n = \max(e) - \lambda \left[ \max(e) - \min(e) \right]$$

(3)

where $\lambda \in (0,1]$ issued to control and adjust the number of element numbers whose elastic modulus value is changed.

Step 2: Assuming that the maximum stress is proportional to the applied external loading, the limit loading that satisfies the lower limit theorem is,

$$P_{Li} = P_n \times \frac{\sigma_y}{\max(e)}$$

(4)
where $\sigma_y$ is the yield stress.

Step 3: After the iterations (step 1 to step 2), the optimal structural lower limit loading is,

$$ p_L = \max_i (p_{Li}) $$

Repeat steps 1 to step 3 until the expected calculation elements of component reaches the yield stress condition.

Figure 3 gives the flow chart of improved elastic compensation technique. For the improved elastic compensation technique, the elastic modules adjustment equation is proposed and is applied in the limit analysis.

**Figure 3.** Flow chart of improved elastic compensation technique.

### 4. Results and analysis

The improved elastic compensation technique is adopted in determining the limit loading. Table 1 shows the results.

| $a/c$ | Defect depth $a$ / mm | Defect length $c$ / mm | Remaining areas / mm$^2$ | Limit loading / kN |
|-------|------------------------|------------------------|--------------------------|-------------------|
| 0.1   | 1.0                    | 10.0                   | 492.1                    | 301.07            |
| 0.5   | 1.0                    | 2.0                    | 498.4                    | 358.42            |
| 1.0   | 1.0                    | 1.0                    | 499.2                    | 371.71            |

Figure 4 gives variation of the limit loading versus calculation steps. It shows that a good stability of the calculation process is found to the improved elastic compensation technique. Corresponding to the calculate steps, in the limit analysis, the elastic modulus of element is reduced for these elements with the stress value exceeding yield. So, the estimation value of limit loading increases gradually. When the elastic modulus of all elements has been adjusted, the value of limit loading remains the highest point. The value of the highest point of a curve corresponds to the maximum lower boundary limit loading.
Figure 4. Limit loading for plate with semi-elliptical defect under tensile loading.

Figure 5 gives the limit loading value based on table 1. It shows that the loading value is proportional to the remaining area. In table 1, in this work, the remaining areas is gradually increasing for the defect depth $a$ is the constant value and only the ratio $a/c$ is changed. Thus, the value of limit loading is increase with the remaining areas in finite plate.

Figure 5. Limit loading versus remaining areas.
5. Conclusions
Considering finite plate with three kinds of semi-elliptical defects under tensile loading, limit loading value is analyzed based on the improved elastic compensation technique. The improved elastic compensation technique proposes the elastic modules adjustment equation and can obtain more accurate value of limit loading. For the same material, the limit loading result is obtained and the remaining areas can be presented versus determining values of the loading.

In addition, the improved elastic compensation method would be used to obtain the limit loading value for engineering case under complex structure. In future, the relationship between limit loading and remaining areas or bearing area will be studied.

References
[1] Li J 2015 Plastic limit loads for pipe bends with circumferential through-wall crack under torsion moment. Int. J. Mech. Sci. 100 283-97.
[2] Milne I, Ainsworth RA, Dowling AR and Stewart AT 1988 Background to and validation of CEGB report R/H/R6-Revision 3, Int. J. Pres. Ves. Pip. 32(1-4) 105-196.
[3] Plancq D and Berton MN 1998 Limit analysis based on elastic compensation method of branch pipe tee connection under internal pressure and out-of-plane moment loading, Int. J. Pres. Ves. Pip. 75 819-25.
[4] Miller AG 1988 Review of limit loads structures containing defects. Int. J. Pres. Ves. Pip. 32 197-327.
[5] Drucker DC 1970 Plastic deformation in brittle and ductile fracture. Eng. Fract. Mech. 1(4) 577-602.
[6] Mackenzie D and Boyle JT 1993 A method of estimating limit loads by iterative elastic analysis I- simple examples Int. J. Pres. Ves. Pip. 53 77-95.
[7] Mackenzie D, Boyle JT and Hamilton R 2005 The elastic compensation method for limit and shakedown analysis: A review J. Strain Anal. Eng. 35(3) 171-87.
[8] Mackenzie D, Shi J and Boyle JT 1994 Finite element modeling for limit analysis by elastic compensation method Comput. Struct. 51(4) 403-10.
[9] Mackenzie D, Boyle JT and Nadarajah C, et al. 1993 Simple bounds on limit loads by elastic finite element analysis J. Press. Vess-T ASME 115(1) 27-31.
[10] Hamilton R, Mackenzie D and Shi J et al. 1996 Simplified lower bound limit analysis of pressurized cylinder/cylinder intersections using generalized yield criteria, Int. J. Pres. Ves. Pip. 67(2) 219-26.
[11] Ponter ARS and Carter KF 1997 Limit state solutions based on linear elastic solutions with spatially varying elastic modulus Comput Method Appl M 140(3-4) 237-58.
[12] Chen LJ, Liu YH, Yang P and Xu BY 2007 Modified elastic compensation method for limit analysis of complex strutures J. Mech. Eng. 43(5) 187-93.