A tale of two cascades: Higgsing and Seiberg-duality cascades from type IIB string theory

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ABSTRACT: We construct explicitly new solutions of type IIB supergravity with brane sources, the duals of which are $\mathcal{N} = 1$ supersymmetric field theories exhibiting two very interesting phenomena. The far UV dynamics is controlled by a cascade of Seiberg dualities analogous to the Klebanov-Strassler backgrounds. At intermediate scales a cascade of Higgsing appears, in the sense that the gauge group undergoes a sequence of spontaneous symmetry breaking steps which reduces its rank. Deep in the IR, the theory confines, and the gravity background has a non-singular end of space. We explain in detail how to generate such solutions, discuss some of the Physics associated with them and briefly comment on the possible applications.

KEYWORDS: Gauge-gravity correspondence, AdS-CFT Correspondence, Supersymmetry and Duality, Duality in Gauge Field Theories

ArXiv ePrint: 1112.3350

c © SISSA 2012 doi:10.1007/JHEP02(2012)145
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1 Introduction

The rigorous way to define an interacting field theory follows the Wilsonian perspective [1]. One starts from a conformal theory, with a given basis of operators, and assumes that this provides a description of the dynamics of a fixed point which approximates the behavior at very short distances of the field theory of interest (in the case of asymptotically free field theories, the conformal theory is actually free). One then adds deformations by turning on the couplings of relevant operators of the conformal theory (or vacuum expectation values for operators), which trigger a renormalization group flow to low energies (longer distances). The flow starts in the far ultraviolet from the original CFT (which is a fixed point of the RG flow) and in the deep infrared can either end into another non-trivial fixed point, or (in the case of confinement) into a trivial theory. The RG flow itself, and the equations governing it, encode all the most important dynamical properties of the field theory, and studying it in detail provides a very clear strategy to characterize the field theory of interest. However, these flows are typically difficult to study with standard field-theoretic tools, in particular because at some stage of the RG flow (if not all along the flow) the field theory may be strongly coupled.

The AdS/CFT correspondence [2–4] conjectures the existence of a dual description to a conformal field theory in terms of a string-theory background with anti-de Sitter geometry, and its validity is supported by countless cross-checks. It hence naturally offers a new opportunity to study non-trivial RG flows by making use of what today are called gauge-string dualities [5], thereby providing a very powerful analytical tool to deal with the study of RG flows in strongly coupled theories. Indeed, many examples of flows like the one described above have been constructed. The basic idea behind such constructions is that one finds solutions to the string equations of motion which have a non-trivial dependence on a radial direction \( \rho \) in the geometry, and interprets \( \rho \) as the renormalization scale in the RG flow of some dual effective theory. Provided the space asymptotes to an anti-de Sitter geometry at large \( \rho \) (in the UV), this procedure can be interpreted as providing the dual description of the RG flow of a fundamental field theory.

A conceptually clean realization of these ideas is given by the flow from \( N = 4 \) Super-Yang-Mills to a theory with minimal SUSY called \( N = 1^* \) Yang-Mills. In the dual string description, the asymptotic space is \( AdS_5 \times S^5 \) (with a flux for a RR five-form). This is deformed by the presence of non-trivial NSNS \( H_3 \) and RR \( F_3 \) fields [6], and as a result the \( \rho \)-dependence of the background is very non-trivial, and reproduces the features expected from the RG flow of the dual field theory (see also [7] for the dual description of the flow resulting from a very special deformation of \( N = 4 \)). This statement can be tested, because the preservation of some amount of supersymmetry yields a degree of analytic control over the coupled non-linear differential equations describing the flow; and hence certain calculations can be performed on both sides of the duality, and compared.

Another non-trivial and successful application of these ideas has been developed by Klebanov and collaborators, and relies instead on the \( AdS_5 \times T^{1,1} \) geometry, where \( T^{1,1} \) is the base of the conifold. The dual conformal theory is in this case a minimally supersymmetric conformal field theory, described by a two-node quiver with gauge groups of...
equal ranks and with a given number of bi-fundamental matter fields \[8\]. A deformation is introduced in the string background that corresponds to an imbalance in the ranks of the gauge groups on the field-theory side of the correspondence. This yields an RG flow \[9\] that leads to confining field theories, in either mesonic or baryonic branches of the moduli space of the dual theory \[10, 11\], depending on the relations between the imbalance and the original rank. The downside of this construction, with respect to the previous case, is that the deformation does not switch itself off. Once a non-trivial RG flow appears, it does not reach a conformal point at high energies. Instead, the flow follows closely a line of fixed points described by quiver field theories whose rank grows without bound. Yet, by calculating correlators of some key-operators \[12, 13\], one can see that the resulting system behaves in a way which resembles very closely a four-dimensional field theory.

This particular set-up — that we will refer to as the Klebanov-Strassler (KS) system — gave numerous insights into the strongly coupled dynamics of four-dimensional field theories. These insights could find applications in Particle Physics and Cosmology, aside from giving non-trivial examples of calculable strongly-interacting field theories.

The next stage in this descendent scale of ‘conceptually clean’ set-ups is represented by the so-called wrapped-brane models. Examples of these are the ones presented in \[14, 15\], and various others. The field theory here is obtained by a twisted KK compactification of a higher-dimensional field theory on a (curved) manifold. A weak-coupling analysis of the dual theory shows that it contains an infinite tower of equidistant modes (see for example \[16, 17\]), revealing the existence of extra dimensions. Supersymmetry and R-symmetry are (partially) broken and the RG flow of the four-dimensional field theory never reaches a fixed point at high energies. These models can be thought of as dual to effective theories which require a UV completion — often non field-theoretic, after which a CFT is typically reached. See \[18–23\], for a detailed presentation of each of the set-ups described above. A very interesting fact is that the deep-IR behavior of backgrounds in this class resembles very closely the backgrounds of models of the previous classes, while the major differences arise only at large \(\rho\). Also, these models are comparatively simpler to construct and to study, which means that they are very suitable for applications.

Another development that is relevant to this paper goes in the direction of constructing the duals of field theories with matter content in the fundamental representation of the gauge groups. Adding \(N_f\) fundamental fields in a field theory with gauge group SU\((N_c)\) is by now a well-studied problem in the context of gauge/string duality. Reviews of different dynamical regimes, depending on the ratio \(N_f/N_c\), their string-theoretical description and applications can be found in \[24–26\]. It is in general very difficult, within this context, to construct backgrounds with the nice UV-asymptotic properties of the dual to a fundamental field theory.

In this paper we will construct a set of solutions that fall within the class of Klebanov-Strassler-like backgrounds, in the sense that the string background is very closely related to the KS one far in the UV and deep in the IR (i.e. the backgrounds have the same good features as the KS ones at the extrema of the RG flow). The novelty is given by the fact that the presence of \(N_f\) sources, being \(N_f \sim N_c\), modifies the ranks of the gauge groups, and drastically changes the RG flow at intermediate scales.
We will also see that such backgrounds are non-trivially related, at least at the technical level, to the wrapped-brane models, in the sense that they can be constructed by starting from one such model, modified by the addition of $N_f$ sources, and by applying an algebraic procedure that amounts to a partial UV completion. Notice however that the actual backgrounds of interest in this paper can also be constructed directly, without ever referring to the wrapped systems, and that in this sense one might think of the reference to wrapped-brane systems as purely technical. But it is suggestive, at least from the point of view of effective field theories, that finding these solutions is technically much easier by starting from the wrapped-brane systems.

1.1 General idea of this paper

Using a solution generating technique presented in [27] and further developed in [28]–[30–33] (sometimes referred to as ‘rotation’) we construct a solution that generalizes that of KS [10] and the so-called baryonic branch of [34] by the addition of source D5/D3-branes in the bulk that translate into a new numerology for the field-theory quiver. The field theory, as we will see, makes transitions between the numerology characterizing mesonic and baryonic branches and shows interesting phenomena. From the view point of the string background, it is completely smooth and solves the equations of motion of type IIB string theory coupled to the action of (smeared) sources. We use techniques developed in previous works to give a ‘profile’ to the sources avoiding any singular behavior.

We analyze the field theory, whose dynamics includes two cascades: one is the familiar cascade of Seiberg dualities that is present in [10], while the other is associated with each crossing of a source D3-brane in the radial direction (corresponding to a Higgs mechanism) as follows from the ideas of [35]. We present various matchings between field-theory expectations and calculations with the string background. We also speculate about a possible interesting application to the phenomena of *Tumbling* in theories of Extended Technicolor.

In practical terms, what we are doing is the following. We start from the wrapped-D5 system, for which it is comparatively simple to find background solutions. We add flavor branes, using a profile for the sources such that the deep IR and far UV are virtually unaffected, and solve the resulting background equations. We then apply the rotation procedure, in such a way as to cure the ills of the wrapped system in the far UV, without affecting the deep IR. The result is a very novel dynamical model, which yet preserves the good features of older constructions.

Because we believe the results we obtain are very interesting, in spite of the fact that at the technical level what we did reduces to putting together a set of well-known ideas, the work is presented in full detail and a fair amount of background material is also given, which makes the paper self-contained, if long. In a companion shorter paper [36], we explore a particular realization of the ideas described above, in which the two cascades occur at different scales. The organization of the manuscript is the following: in section 2, we present in detail the system of wrapped branes that is at the heart of our treatment. We also review the application of the solution generating technique of [27, 29]. The techniques that allow to give a profile to our sources are described in section 3. Some new solutions are displayed in that same section while some other new solutions to the type IIB equations...
with sources are discussed in section 4. Field-theoretical quantities are analyzed in section 5 and some applications are discussed in section 6. We close the paper with conclusions in section 7. Generous appendices complement the presentation.

2 Presentation of the SUSY system

In this section, we briefly summarize some well-established aspects of two particular four-dimensional supersymmetric field theories and their dual backgrounds. The two theories are the Klebanov-Strassler [10, 34] QFT (referred below as ‘theory I’) and the theory obtained when wrapping $N_c$ D5-branes on the two-cycle of the resolved conifold [15–17] (which we will refer to as ‘theory II’). We assume in the following that the reader is familiar with aspects of these two systems, and what follows is just a succint account of their properties; some reviews are [18–23]. The theory I is a quiver with gauge group $SU(n+N_c) \times SU(n)$ and bi-fundamental matter multiplets $A_i, B_\alpha$ with $i, \alpha = 1, 2$. The global symmetries are

$$SU(2)_L \times SU(2)_R \times U(1)_B \times U(1)_R.$$ (2.1)

There is also a superpotential of the form $W = \frac{1}{\mu} \epsilon_{ij} \epsilon_{\alpha \beta} \text{tr}[A_i B_\alpha A_j B_\beta]$. The field theory is taken to be close to a strongly coupled fixed point at high energies. The dual description is given by the Klebanov-Strassler background [10] and its generalizations [34]. The field theory II has one gauge group $SU(N_c)$ and the global symmetries are those of eq. (2.1) — except for the baryonic symmetry that is not present in this system. These two theories, apparently so different, can be connected as discussed in [27] and [28] via Higgsing. Indeed, giving a particular (classical) baryonic VEV to the fields $(A_i, B_\alpha)$ and expanding around it, the field content and degeneracies of [16, 17] are reproduced — see [27]. This weakly coupled field theory connection has its counterpart in the type IIB solutions dual to each of the field theories. Indeed, it is possible to connect the dual backgrounds of field theories I and II, using U-duality [27]. This connection was further studied in [28–33]. In order to lay-out some background formalism and make manifest the connection between theories I and II at strong coupling, we start from the type IIB configuration describing the strong dynamics of the ‘field theory II’ (a twisted compactification of D5-branes to four dimensions). It consists of a metric, dilaton $\Phi$ and RR-three form $F_3$. A quite generic background of this kind can be compactly written using the SU(2) left-invariant one-forms

$$\tilde{\omega}_1 = \cos \psi \, d\tilde{\theta} + \sin \psi \sin \tilde{\theta} \, d\tilde{\varphi}, \quad \tilde{\omega}_2 = -\sin \psi \, d\tilde{\theta} + \cos \psi \sin \tilde{\theta} \, d\tilde{\varphi}, \quad \tilde{\omega}_3 = \psi + \cos \tilde{\theta} \, d\tilde{\varphi},$$ (2.2)

and the vielbeins

$$E^i = e^\Phi dx^i, \quad E^\rho = e^{\Phi + k} d\rho, \quad E^\theta = e^{\Phi + h} d\theta, \quad E^\varphi = e^{\Phi + h} \sin \theta \, d\varphi,$$

$$E^1 = \frac{1}{2} e^{\Phi + g} (\tilde{\omega}_1 + a \, d\theta), \quad E^2 = \frac{1}{2} e^{\Phi + g} (\tilde{\omega}_2 - a \, \sin \theta \, d\varphi), \quad E^3 = \frac{1}{2} e^{\Phi + k} (\tilde{\omega}_3 + \cos \theta \, d\varphi).$$ (2.3)

\footnote{The R-symmetry is anomalous, breaking $U(1)_R \to Z_{2N_c}$.}
In terms of these, the background and the RR three-form, in Einstein frame, read

\[ ds_E^2 = \sum_{i=1}^{10} (E^i)^2, \]
\[ F_3 = e^{-\frac{4}{3}\Phi}(f_1 E^{123} + f_2 E^{\theta \varphi} + f_3(E^{\theta 23} + E^{\varphi 13}) + f_4(E^{\rho 16} + E^{\rho 23})), \]

where we defined

\[ E^{ijk...l} = E^i \wedge E^j \wedge E^k \wedge ... \wedge E^l, \]
\[ f_1 = -2N_c e^{-k-2g}, \quad f_2 = \frac{N_c}{2} e^{-k-2h}(a^2 - 2ab + 1), \]
\[ f_3 = N_c e^{-k-h-g}(a - b), \quad f_4 = \frac{N_c}{2} e^{-k-h-g}b'. \]

The system has a radial coordinate \( \rho \), on which \( a, b, \Phi, g, h, k \) depend, and we have set \( \alpha' g_s = 1 \). The full background is then determined by solving the equations of motion for the functions \( a, b, \Phi, g, h, k \). A system of BPS equations is derived using this ansatz (see appendix of reference [37]). These non-linear and coupled first-order equations can be arranged in a convenient form, by rewriting the functions of the background in terms of a new basis of functions that decouples the equations (as explained in [38]–[39]). We quote this change of basis in our appendix A. Using these new variables, one can solve for all of them except for a function \( P(\rho) \), that is determined by a second-order equation, see [38] and our appendix A for details. The second-order equation reads

\[ P'' + P\left(\frac{P' + Q'}{P - Q} + \frac{P' - Q'}{P + Q} - 4 \coth(2\rho)\right) = 0, \]
\[ Q(\rho) = N_c (2\rho \coth(2\rho) - 1). \]  

We will refer to eq. (2.6) as the master equation: this is the only equation that needs solving in order to generate the large classes of solutions of type IIB dual to ‘field theory II’ in different vacua, deformations of the Lagrangian, VEV’s, etc.

### 2.1 Aspects of the SUSY solutions

Let us start by describing some known solutions of the master equation (2.6). As a first example, the simple solution \( P = 2N_c \rho \) gives the background of [15, 40]. This solution and those with the same large-\( \rho \) asymptotics will not be the main focus of this paper. Other interesting solutions can be found semi-analytically (with asymptotic series expansions and numerical integration). Let us discuss the large and small-\( \rho \) expansions of the function \( P(\rho) \) that will be of interest in this work. For large values of the radial coordinate (describing the UV of the field theory II), the solution has an expansion of the form,

\[ P = e^{4\rho/3} \left[ c_+ + \frac{e^{-8\rho/3} N_c^2}{c_+} \left( 4\rho^2 - 4\rho + \frac{13}{4} \right) + e^{-4\rho} \left( c_+ - \frac{8c_+}{3} \rho \right) + \right. \]
\[ + \frac{N_c^4 e^{-16\rho/3}}{c_+^3} \left( \frac{18567}{512} + \frac{2781}{32} \rho + \frac{27}{4} \rho^2 + 36\rho^3 \right) + \mathcal{O}(e^{-20\rho/3}) \right]. \]
Notice that this expansion involves two integration constants, \( c_+ > 0 \) and \( c_- \). The background functions at large \( \rho \) are written for reference in appendix A. Regarding the IR expansion, we look for solutions with \( P \to 0 \) as \( \rho \to 0 \), in which case we find
\[
P = h_1 \rho + \frac{4h_1}{15} \left( 1 - \frac{4N_c^2}{h_1^2} \right) \rho^3 + \frac{16h_1}{525} \left( 1 - \frac{4N_c^2}{3h_1^2} - \frac{32N_c^4}{3h_1^4} \right) \rho^5 + \mathcal{O}(\rho^7),
\]
where \( h_1 > 2N_c \) is again an arbitrary constant; there is another integration constant, \( P(0) \), taken to zero here, to avoid singularities. In order to match the UV expansion, it amounts to taking as well \( c_- = 0 \) in (2.7). This gives background functions that are quoted in appendix A. Of course, there is a smooth numerical interpolation between both expansions. As one can imagine, there is then only one independent parameter; given a value for one of the constants \( \{c_+, h_1\} \), the requirement that the solution matches both expansions is sufficient to determine the value of the other. The small-\( \rho \) behavior of the expansion above is that of the exact solution \( P = 2N_c \rho \) (but notice that in eq. (2.8) the constant \( h_1 > 2N_c \)). In contrast, the large-\( \rho \) expansion \( P \sim e^{4\rho/3} \) greatly differs from the linear behavior. Let us focus our attention on the dilaton field in backgrounds like those in eq. (2.4). For small and large values of the radial coordinate, we have
\[
e^{4\Phi|_{\rho \to 0}} = e^{4\Phi(0)} \left[ 1 + \frac{64N_c^2 \rho^2}{9h_1^2} + \frac{128N_c^2 \left( -15h_1^4 + 124N_c^2 \right) \rho^4}{405h_1^4} + \mathcal{O}(\rho^6) \right],
\]
\[
e^{4\Phi|_{\rho \to \infty}} = e^{4\Phi(\infty)} \left[ 1 + \frac{3N_c^2 e^{-8\rho/3}}{4c_+^2} (1 - 8\rho) + \mathcal{O}(e^{-16\rho/3}) \right],
\]
and there is a relation between \( \Phi(0) \) and \( \Phi(\infty) \). Notice that the dilaton (that is related to the warp factor) asymptotes to a constant at large values of the radial coordinate.\(^2\) One can see [28] that this type of solutions corresponds to the addition of an irrelevant operator in the Lagrangian of the theory of the type II field theory, needing a UV completion.

### 2.1.1 A nice way of writing solutions

We summarize here a curious way of finding an exact and analytic solution to the master equation (2.6), in terms of an infinite series of non-explicit integrals. The usefulness of this formal solution will become clear in the following sections. We rewrite the master eq. (2.6) as
\[
\partial_\rho \left( \frac{(P^2 - Q^2)}{\sinh^2(2\rho)} \right) P' + \frac{4}{\sinh^2(2\rho)} P'QQ' = 0.
\]

We integrate this expression twice, taking already into account the asymptotics described above, to get
\[
P^3 - 3PQ^2 + 6 \int_0^\rho d\tilde{\rho} PQQ' - 12 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \int_\rho^\infty d\tilde{\rho} \frac{P'QQ'}{\sinh^2(2\tilde{\rho})} = 4\lambda^2 e^4 \left( \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \right).
\]

\(^2\)Some readers may find this reminiscent of what happens when ‘keeping the 1’ in the D3-brane warp factor.
where we have defined for convenience the constant $\lambda = 2^{2/3} c_+ e^{-\frac{1}{3}}$, where $\epsilon$ will be later identified with the deformation parameter of the deformed conifold.\footnote{The deformation parameter $\epsilon$ of the conifold is such that $\epsilon^{4/3}$ has dimension of length squared, the same as $c_+$ (restoring units of $g_s \alpha'$). Thus $\lambda$ is a dimensionless parameter.} We propose a solution in an inverse series expansion in the dimensionless constant $\lambda$:

$$
P = \lambda P_1 + P_0 + \frac{P_{-1}}{\lambda} + \frac{P_{-2}}{\lambda^2} + \frac{P_{-3}}{\lambda^3} + \ldots .
$$  \hspace{1cm} (2.12)

Plugging this expansion in (2.11) and matching the different powers of $\lambda$, we get

$$
P_1 = \left(4 e^4 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho})\right)^{1/3},
$$

$$
P_0 = P_{-2} = \ldots = P_{-2k} = 0,
$$

$$
P_{-1} = -\frac{1}{P_1^2} \left(- P_1 Q^2 + 2 \int_0^\rho d\tilde{\rho} P_1 Q Q' - 4 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \int_0^\infty d\hat{\rho} P_1 Q Q' \frac{1}{\sinh^2(2\hat{\rho})}\right).
$$  \hspace{1cm} (2.13)

A recurrence relation for $P_{-(2k+1)}$ can be found in eq. (B.7) of [38]. This series converges rapidly to the numerical solutions of eq. (2.6) and is a very good approximation for large values of the radial coordinate.

An interesting limit to take is $\lambda \to \infty$. We will explore this in the following sections.

### 2.2 Connecting to Theory I

As explained in [28], the solution in eqs. (2.7)–(2.8), supplemented with a suitable numerical interpolation, describes the strong dynamics of the dual field theory II in the presence of a dimension-eight operator inserted in the Lagrangian (which ultimately couples the field theory to gravity). This calls for a completion in the context of field theory which is achieved with the U-duality of [27] (a solution generating technique sometimes referred to as ‘rotation’). After the U-duality described in [27] is applied, we define the new vielbein (which we use in the following),

$$
e^{x_1} = e^{\frac{\Phi}{4} \tilde{h}^{\frac{1}{4}} d\omega_1}, \quad e^\rho = e^{\frac{\Phi}{4} + \frac{k}{4} \tilde{h}^{\frac{1}{4}} d\rho}, \quad e^\theta = e^{\frac{\Phi}{4} + \frac{1}{4} \tilde{h}^{\frac{1}{4}} d\theta}, \quad e^\varphi = e^{\frac{\Phi}{4} + \frac{1}{4} \tilde{h}^{\frac{1}{4}} \sin \theta d\varphi},
$$

$$
e^1 = \frac{1}{2} e^{\frac{\Phi}{4} + \frac{1}{4} \tilde{h}^{\frac{1}{4}}} (\tilde{\omega}_1 + a d\theta), \quad e^2 = \frac{1}{2} e^{\frac{\Phi}{4} + \frac{1}{4} \tilde{h}^{\frac{1}{4}}} (\tilde{\omega}_2 - a \sin \theta d\varphi),
$$

$$
e^3 = \frac{1}{2} e^{\frac{\Phi}{4} + \frac{k}{4} \tilde{h}^{\frac{1}{4}}} (\tilde{\omega}_3 + \cos \theta d\varphi).
$$  \hspace{1cm} (2.14)
The dilaton and RR three-form are invariant under this operation. The newly generated metric, RR and NSNS fields are

\[ ds^2_E = \sum_{i=1}^{10} (e^i)^2 , \]

\[ F_3 = e^{-\frac{2}{3} \Phi} \left[ f_1 e^{123} + f_2 e^{023} + f_3 e^{123} + f_4 (e^{\rho \theta} + e^{\varphi 2} \varphi) \right] , \]

\[ B_2 = \kappa \frac{e^{\frac{2}{3} \Phi}}{\hat{h}^{1/2}} \left[ e^{\rho 3} - \cos \alpha (e^{\theta \varphi} + e^{12}) - \sin \alpha (e^{\theta 2} + e^{\varphi 1}) \right] , \]

\[ H_3 = -\kappa \frac{e^{\frac{2}{3} \Phi}}{\hat{h}^{3/4}} \left[ - f_1 e^{\theta \varphi \rho} - f_2 e^{\rho 12} - f_3 (e^{\theta 2 \rho} + e^{\varphi 1} \rho) + f_4 (e^{1 \theta 3} + e^{\varphi 2} 3) \right] , \]

\[ C_4 = -\kappa \frac{e^{2 \Phi}}{\hat{h}} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 , \]

\[ F_5 = \kappa e^{-\frac{2}{3} \Phi} - k \frac{\hat{h}}{\rho} \partial_\rho \left( \frac{e^{\frac{2}{3} \Phi}}{\hat{h}} \right) \left[ e^{\theta 123} - e^{012} x^1 x^2 x^3 \right] . \]

We have defined

\[ \cos \alpha = \frac{\cosh(2 \rho) - a}{\sinh(2 \rho)} , \quad \sin \alpha = -\frac{2 e^{h - \eta}}{\sinh(2 \rho)} , \quad \hat{h} = 1 - \kappa^2 e^{2 \Phi} , \]

where \( \kappa \) is a constant that we will choose to be \( \kappa = e^{-\Phi(\infty)} \), requiring the dilaton to be bounded at large distances.\(^4\) The rationale for this choice is to obtain a dual QFT that is UV-complete, see [28] for explanations. The background in eq. (2.15) has the same form as the one describing the baryonic branch of the KS theory [34]. Indeed, one can check that the BPS equations written in [34] are exactly equivalent to our eq. (2.6). Another way of understanding the connection was explained in [29]: what relates the backgrounds in eqs. (2.4) and (2.15) is a rescaling of the almost Kähler and complex structure forms describing the six-dimensional internal space.

A closer inspection strongly suggests that the formal solution written in section 2.1.1 is the solution of [34]. Let us study the limit \( \lambda \rightarrow \infty \) of the special solution presented in section 2.1.1. In that case the solution reduces to

\[ P \sim \lambda P_1 = \left( 4e^4 \int \sinh^2(2 \rho) \right)^{1/3} = \lambda 2^{-\frac{3}{2}} e^{\frac{3}{2}} (\sinh(4 \rho) - 4 \rho)^{1/3} \]

(2.17)

For the different background functions the limit results in

\[ e^{2h} = \frac{\lambda \tanh(2 \rho)}{4} P_1 + \mathcal{O}(\lambda^0) , \quad \frac{e^{2g}}{4} = \lambda \frac{\coth(2 \rho) P_1}{4} + \mathcal{O}(\lambda^0) , \]

\[ \frac{e^{2k}}{4} = \frac{\lambda P_1}{8} + \mathcal{O}(\lambda^{-1}) , \quad a = \frac{1}{\cosh(2 \rho)} + \mathcal{O}(\lambda^{-1}) , \]

(2.18)

\[ b = \frac{2 \rho}{\sinh(2 \rho)} , \quad e^{4\Phi - 4\Phi_0} = \frac{2 \sinh^2(2 \rho)}{\lambda^3 P_1^2 P_1^0} + \mathcal{O}(\lambda^{-5}) , \]

\(^4\)This is the restriction on the type of solutions considered that rules out the exact solution \( P = 2N_c \rho \).
The difficulty of the one described above: \( \kappa \). Finally, rescaling the Minkowski t operator in the dual QFT, + implies that 3\( \kappa \) = 2\( e^4 \lambda^3 e^{-4\Phi_0} \), and the logic for this choice is to get a warp factor that vanishes at large \( \rho \). Indeed, plugging this value in eq. (2.18), and using the explicit expression of \( P_1 \) given in eq. (2.17), we see that the two first terms in the expansion for \( \hat{h} \) cancel out and

\[
\hat{h} = 1 - \frac{\kappa^2 e^{2\Phi_0}}{\lambda^{3/2}} \sqrt{\frac{2\sinh^2(2\rho)}{P_1^2 P_1'}} + \frac{\kappa^2 e^{2\Phi_0} \sinh(2\rho)(2P_1 P_1' P_{-1} + P_1^2 P_{-1}' - P_{-1}'(Q^2))}{\lambda^{3/2} \sqrt{2P_1^2 P_1'^2}} + \mathcal{O}(\lambda^{-11/2}).
\]

This in turn implies that we are switching off a dimension eight operator in the dual QFT, obtaining a proper UV-complete field theory. A detailed discussion of this choice, showing that the decoupled operator has dimension eight, is in [28]. Regarding \( \hat{h}_2 \) we explicitly have,

\[
\hat{h}_2 = \frac{2^{5/3}}{3\phi^3 \lambda^2} x^2 \int_0^\infty \frac{dx}{\rho} \frac{\sinh(4x)(-4x)(2x \coth(2x) - 1)}{\sinh^2(2x)(\sinh(4x) - 4x)^{2/3}},
\]

which is the Klebanov-Strassler [10] warp factor \( \hat{h}_2 = \hat{h}_{KS} \). Finally, rescaling the Minkowski coordinates \( x^i \rightarrow x^i \lambda^{-1/2} \) the metric is independent of the parameter \( \lambda \), and using that

\[
\frac{P_1'}{P_1} = \frac{8 \sinh^2(2\rho)}{3(\sinh(4\rho) - 4\rho)},
\]

the internal space metric is the deformed conifold. This is precisely the Klebanov-Strassler solution [10] that is obtained as the limit \( \lambda \rightarrow \infty \) of the formal solution in section 2.1.1 (the latter is the full baryonic branch solution [34]).

2.3 Adding sources

We consider now the addition of sources in these backgrounds and the effect of the rotation on them. The logic followed is the one described in [25, 37]. We add a bunch of \( N_f \) D5-branes in backgrounds of the form given in eq. (2.4). These D5-branes are added as explained in [37]. They are in principle localized in the compact space described by the angles [\( \theta, \hat{\varphi}, \theta, \hat{\varphi} \)]. One can run the machinery described in [29] and [41], to write a BPS system of nonlinear coupled partial differential equations (with derivatives on the radial coordinate and the angles where the sources are localized). The difficulty of the problem prompted the authors of [37, 42], to smear the sources.\(^5\) In this smeared, SU(2)_L \times SU(2)_R invariant situation one can run a very similar program to the one described above: generalize the change of variables discussed in appendix A and write a new master equation for the function \( P(\rho) \),

\[
P''' + (P' + N_f) \left[ \frac{P' + Q' + 2N_f}{P - Q} + \frac{P' - Q' + 2N_f}{P + Q} - 4 \coth(2\rho) \right] = 0,
\]

\[
Q(\rho) = \frac{2 NC - N_f}{2} (2\rho \coth(2\rho) - 1).
\]

\(^5\)Equivalently, either consider the Fourier zero mode of the solution in the compact internal space, or consider a situation where the global symmetries of the field theory without flavors/sources — eq. (2.1) — are respected after the addition of sources. See [25] for discussions on different aspects of the smearing.
See the paper [38] for technical details. In this case, solutions are known either semi-analytically (as series expansions plus a smooth numerical interpolation) and also generalizing the formalism of section 2.1.1. A classification of the solutions according to the asymptotic behavior of $P(\rho)$, together with expansions of all other background functions in the presence of sources was given in [38]. We will not quote the results here, but refer the reader to [38] and [29] for details. A feature of all these solutions is the presence of singularities for small $\rho$. In other words, one finds a divergent Ricci scalar as $R \sim \frac{N_f}{\rho^2}$. A physical reason of the existence of the singularity (namely a very large density of ‘crossing’ source-branes at the point $\rho = 0$) was given in [25], among other papers. More in detail, the solutions in the presence of the D5 sources are formally like those in eq. (2.4), with the difference that the functions $f_1, f_2, f_3, f_4$ change to reflect the presence of sources ($dF_3$ is nonzero). Indeed, we now have,

$$
    f_1 = -2N_c e^{-k-2g}, \quad f_2 = \frac{N_c}{2} e^{-k-2h} \left( a^2 - 2ab + 1 - \frac{N_f}{N_c} \right), \\
    f_3 = N_c e^{-k-h-g} (a - b), \quad f_4 = \frac{N_c}{2} e^{-k-h-g} y'.
$$

(2.23)

The other functions in the background also reflect the presence of the $N_f$ sources. After the rotation procedure is applied we will have a background that formally is the one in eq. (2.15), but with the functions $\Phi, h, g, k, a, b$ that now have contributions coming from the sources. There are two interesting points to discuss. First, as explained in detail in [29], the presence of the NSNS $B_2$ field will induce D3-brane charge on the D5 sources after the rotation. The second point is that due to the presence of D5 sources the function $P(\rho)$ gets a contribution at large values of the radial coordinate, see [38] and compare with eq. (2.7),

$$
    P(\rho) = e^{4\rho/3} \left( c_+ + \frac{9N_f}{8} e^{-4\rho/3} + \mathcal{O}(e^{-8\rho/3}) \right).
$$

(2.24)

Carefully following the details — see [29] — one finds that this new term impacts the large-radius asymptotics of the warp factor for the rotated background as

$$
    \hat{h} = N_f e^{-4\rho/3} + \frac{(2N_c - N_f)^2}{2c_+} \rho e^{-8\rho/3} + \mathcal{O}(e^{-8\rho/3}).
$$

(2.25)

Then, asymptotically, the cascading behavior of the KS system, represented above by the $e^{-8\rho/3}$ term present also in $\hat{h}_2$ of eq. (2.20), is overcome by the presence of the D3 and D5 sources. This deviates the system from the ‘near $AdS_5$’ UV.

### 2.4 About the dual field theory

We comment briefly on the field theory dual to the (singular) geometry after the rotation with sources. This was proposed in [29] to be the quiver

$$
    \text{SU}(n + N_c + n_f) \times \text{SU} \left( n + n_f + \frac{N_f}{2} \right),
$$

(2.26)

($n_f$ is the number of D3-branes induced by the sources, more precisons to be given in the following sections) with superpotential $W \sim ABAB$ for the fields $A_i, B_\alpha$ transforming as
bi-fundamentals of the gauge groups, in the same fashion as indicated around eq. (2.1).

It was also proposed that the field theory was on a mesonic branch and that as discussed in [11] the baryonic symmetry $U(1)_B$ was unbroken. Computations of the $c$-function, beta functions and anomalies supported this proposal. The presence of the small-$\rho$ singular behavior of the background (inherited from the singularity in the unrotated background), implies that we should not trust the IR dynamics as read from the geometry. At best, the solution should be thought of as a generalization of the Klebanov-Tseytlin geometry [9] in the presence of extra matter/sources. This point was made concrete by calculating Wilson loops that displayed a non-physical behavior for large separations [43].

It was clearly stated in [29] that a resolution of the singularity was very desirable as it would allow to trust the low energy dynamics of the field theory read from the type IIB background with sources. Here, we also point to the need of having a better understanding of the high-energy (and strongly coupled) dynamics. As pointed out above, this dynamics seems to be driven by higher-dimensional operators reflected in the term $N_f e^{-4\rho/3}$ in eq. (2.25). We will comment and resolve both issues in the following sections.

3 Solving the problems: sources with a profile

Let us briefly remind the reader about the recent results of [44]. The paper is written in the context of the backgrounds of eq. (2.4) for $(N_c, N_f)$ D5 color and flavor branes. The authors found that a way of getting rid of the small-radius singularity was to avoid the ‘very high density’ superposition of sources (as described above and also pointed out in [25, 45–47]). For this, the authors of [44] developed a formal way of distributing the sources such that not all the $N_f$ branes reach the point $\rho = 0$. They constructed a distribution of sources (flavors) with a particular profile that vanishes near the origin and stabilizes to a constant at large radius as indicated in the figure 1.

The distribution $S(\rho)$ is obtained rigorously solving the kappa symmetry equations for the branes and distributing them following a well-defined procedure [44, 48].
distribution can be interpreted in two ways: the first, as the addition of radius-energy-dependent masses for the flavors in the dual field theory — the profile of masses is somewhat related to $N_f S(\rho)$. The second way of interpreting the distribution of sources, uses that the field theory II couples to flavors according to the superpotential (see [37, 39])

$$W \sim \hat{Q}^\dagger \hat{\Phi}_k Q,$$

(3.1)

where ($\hat{Q}$) $Q$ are the (anti) quark superfields introduced by the sources and $\hat{\Phi}_k$ is a generic massive chiral multiplet in the field theory II. The profile is understood as an energy-dependent VEV for the field $\hat{\Phi}_k$, that effectively generates an energy-dependent mass term for the flavor multiplets. While the outcome in both cases is an energy-dependent 'distribution of mass', the second interpretation implements it without breaking the R-symmetry, hence making it our choice. Let us describe how this distribution of sources affects the string background.

Indeed, one writes a background like that in eq. (2.4), where the functions $f_1, \ldots, f_4$ now reflect the presence of smeared sources with a profile $S(\rho)$:

$$f_1 = -2N_c e^{-k-2g}, \quad f_2 = \frac{N_c}{2} e^{-k-2h} \left( a^2 - 2ab + 1 - \frac{N_f}{N_c} S(\rho) \right),$$

$$f_3 = N_c e^{-k-h-g}(a-b), \quad f_4 = \frac{N_c}{2} e^{-k-h-g} \left( b' + \frac{N_f S'(\rho)}{2N_c \cosh(2\rho)} \right).$$

(3.2)

Following [44], one can write the BPS equations for the system with the sources added with a profile. Here again, the formalism of [38] can be generalized, in this case for a distribution $\tilde{N}_f(\rho) = N_f S(\rho)$. Indeed, all functions can be immediately obtained once we know the functions $P, Q$. Following the treatment in [44] we learn that

$$Q = \coth(2\rho) \left[ \int_{0}^{\rho} dx \frac{2N_c - N_f S(x)}{\coth^2(2x)} \right],$$

(3.3)

where an integration constant has been set to zero to avoid IR singularities. Moreover, as shown in [44], one can find a master equation,

$$P'' + N_f S' + (P' + N_f S) \left( \frac{P' - Q' + 2N_f S}{P + Q} + \frac{P' + Q' + 2N_f S}{P - Q} - 4 \coth(2\rho) \right) = 0,$$

(3.4)

which reduces to eq. (2.22) when $S(\rho) = 1$ and to eq. (2.6) for $N_f = S(\rho) = 0$.

Once the function $S(\rho)$ is known (from a microscopic description of the smearing of sources), one can get $Q(\rho)$ from (3.3) and solve the second-order master equation (3.4) for $P(\rho)$. The rest of the functions have expressions that generalize those in our appendix A, as we read from [44]:

$$e^{2h} = \frac{P^2 - Q^2}{4 P \coth(2\rho) - Q}, \quad e^{2g} = P \coth(2\rho) - Q,$$

$$e^{2k} = \frac{P' + N_f S(\rho)}{2}, \quad a = \frac{P}{P \cosh(2\rho) - Q \sinh(2\rho)}.$$

(3.5)

\[\text{Compare this to eq. (2.23) to notice that the effect of the profile for the sources is not just the change } N_f \rightarrow N_f S(\rho). \text{ See [44] for details.}\]
After the rotation, D3-brane charge is induced on the D5s. As argued in section 3.1, this charge grows exponentially with the radial distance. An asymptotically constant profile $S(\rho)$ will generate an infinite amount D3-brane charge.

The dilaton is,

$$e^{4\Phi-4\Phi_0} = \frac{2 \sinh(2\rho)^2}{(P^2 - Q^2)(P' + N_f S)},$$

and the function $b(\rho)$ has the form

$$b(\rho) = \frac{2\rho}{\sinh(2\rho)} - \frac{N_f}{2N_c} \left[ \frac{S(\rho)}{\cosh(2\rho)} + \frac{2}{\sinh(2\rho)} \int_0^\rho dx \tanh(2x)^2 S(x) \right].$$

The problem just boils down to finding $P(\rho)$, given $S(\rho)$. We follow the treatment in [44] and [48] — where quite generic distributions $S(\rho)$ were constructed — to get particular expressions for the function $S(\rho)$ that make the expression of $Q(\rho)$ analytic and the resolution of the master equation simple. Performing the rotation is immediate, and we will quote below the expression for the full rotated background. As we discussed, the rotation induced D3-brane charges on the D5 sources. This is represented in figure 2.

### 3.1 The two kinds of solutions

Given a distribution function for the D5 sources $S(\rho)$, we can actually construct two kinds of solutions: those that are exact and analytic and those that are semi-analytic (in the sense of knowing either their asymptotic power series expansions or an exact expression in terms of an infinite number of integrals that cannot be evaluated exactly).

- **Exact solutions**: they are obtained in the double-limit $\lambda \to \infty$ and $N_f \to 0$, keeping $c_+ N_f = \nu$ finite. The solutions are very similar to the KS background in the sense that functions are written up to an integral, the dilaton is constant and $F_3 = *_6 H_3$. In these solutions there are D3 sources, but no D5 sources.

- **Semi-analytic solutions**: the parameters $\lambda, N_f$ in the expansion are both fixed and nonzero. These solutions are written in terms of an infinite sum of integrals. The treatment follows what is written in appendix B. For all practical purposes, this is equivalent to writing asymptotic series for large and small values of $\rho$ and a smooth numerical interpolation. In these solutions there are D3 and D5 sources.
We will give details on the semi-analytic solutions in section 4. In the case of exact solutions, the configuration in string frame reads (details of the derivation are given in appendix B)

\[ ds^2 = \hat{h}^{-1/2} dx_{1,3}^2 + \hat{h}^{1/2} ds_6^2, \]

\[ ds_6^2 = P_1 \left( \frac{P'}{2F_1}(d\rho^2 + \frac{1}{4}((\tilde{h} + \cos \theta d\varphi)^2) + \frac{\tanh(2\rho)}{4}(d\theta^2 + \sin^2 \theta d\varphi^2) + \frac{\coth(2\rho)}{4}((\tilde{h} + d\theta)^2 + (\tilde{h} - a \sin \theta d\varphi)^2) \right), \]

\[ B_2 = -\frac{N_c}{4} \frac{2\rho \coth(2\rho) - 1}{\sinh(2\rho)} \left( \frac{\cosh(2\rho)}{\sin(\theta d\theta \wedge d\varphi - \tilde{h}_1 \wedge \tilde{h}_2) - \sin \theta d\varphi \wedge \tilde{h}_1 - d\theta \wedge \tilde{h}_2} \right), \]

\[ H_3 = dB_2, \quad F_3 = *_6 H_3, \]

\[ F_5 = -(1 + 8) \partial_{\rho} \hat{h}_{-1}^{*} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\rho, \]

\[ \hat{h} = \hat{h}_2 + 4 \frac{e^{8/3} \nu}{\cosh(2\rho)} \int_{\rho}^{\infty} dx \frac{S(x)}{(-4x + \sinh(4x))^{1/3}}, \]

\[ a = \frac{1}{\cosh(2\rho)}. \]

where \( P_1 \) is written in eq. (2.13), and \( \hat{h}_2 = \hat{h}_{KS} \) is the Klebanov-Strassler [10] warp factor, as written in eq. (2.20). The dilaton is a free constant and we take \( e^\Phi = e^{\Phi(\infty)} = 1 \). Let us stress that when one takes \( S = 0 \) in eq.(3.8), one obtains precisely the Klebanov-Strassler background.

Serendipitously, we write the warp factor as

\[ \hat{h} = \hat{h}_{KS} + 4 \frac{e^{8/3} \nu}{\cosh(2\rho)} \int_{\rho}^{\infty} dx \frac{S(x)}{(-4x + \sinh(4x))^{1/3}}. \]

The function

\[ G(\rho) = \int_{\rho}^{\infty} dx \frac{1}{(-4x + \sinh(4x))^{2/3}}, \]

indicates that the solution above is just the solution to the Laplace equation (for the function \( \hat{h} \)) in the presence of fluxes \( F_3, H_3, F_3 \) and D3 sources (see eq. (101) in [10]). So, we have the KS geometry being deformed by a distribution of source D3-branes

\[ n_f \sim \nu S(\rho) (\sinh(4\rho) - 4\rho)^{1/3}, \]

that are supersymmetric when placed on the deformed conifold. Hence, this exact and analytical solution could have been written without going over all this effort, just assuming the strange distribution of D3 sources in eq. (3.11). On the other hand, the solutions with \( \lambda, N_f \) finite –presented in the next section — are new. Since the asymptotics of both kinds of solutions are quite similar, we can draw generic lessons just by studying the exact analytic solutions in eq. (3.8).

### 3.1.1 Generic features of the solutions

Let us analyze a couple of interesting points. According to what we wrote in eq. (3.11), after the rotation, the induced source D3-branes are distributed in such a way that they
pile up exponentially towards large values of the radial coordinate as \( n_f \sim e^{\rho/3} \). This is precisely what produces the solutions’ departure from the four dimensional behavior of the cascade. This large pile-up of D3-branes dominates the UV dynamics and is equivalent to the insertion of a dimension-six operator into the Lagrangian, as we can see by expanding the warp factor \( \hat{h} \) in eq. (3.8) giving as a result the one in eq. (2.25). We will analyze later how to get rid of this undesirable behavior.

The second interesting point (as anticipated above) is that for a suitably chosen distribution \( S(\rho) \), we can make the number of sources decrease towards \( \rho = 0 \). In this way, one avoids the presence of the small-\( \rho \) singularity. We can follow the papers [44]–[48] to choose an adequate \( S(\rho) \). The backgrounds constructed present no pathology at \( \rho \sim 0 \). Hence the low-energy strong dynamics of the field theory I can be calculated using the backgrounds above and is fully trustable. We still need to do something about the UV behavior, that takes us away from the cascade behavior. Let us make some comments on the field theory dual to our backgrounds that will illuminate the way to resolving our problem in the large radius region.

### 3.2 Field theory comments

In [11] it is explained that one may think the moduli space for the quiver

\[
SU(N_c + p) \times SU(p),
\]

as \((p - lN_c)\) D3-branes free to move on the deformed conifold with \(((l + 1)N_c)\)D5 together with \( lN_c \) anti-D5-branes forming a bound state at the threshold. The ways of distributing the \((p - lN_c)\) D3-branes give rise to a symmetric product of deformed conifolds as moduli space. An important point made by [11] is that if we start from a quiver \( SU((k + 1)N_c + \tilde{p}) \times SU(kN_c + \tilde{p}) \) — in the mesonic branch of the KS field theory — and Higgs down the group repeatedly, we will arrive at the quiver \( SU((k + 1)N_c) \times SU(kN_c) \times U(1)^{\tilde{p}} \). This quiver has the numerology to fit the baryonic branch of the KS field theory. The difference is that the \( U(1)_{baryonic} \) is gauged as it mixes with the \( U(1)^{\tilde{p}} \).

The authors of [11] study carefully the quantum dynamics of the quiver in the various ranges of \( p \), taking into consideration the presence of the tree-level and the quantum-induced superpotential. They show that the deformation of the conifold is always proportional to the scale at which the theory with larger gauge group goes strongly coupled. Let us now apply this information to our case with sources.

Let us consider for the following subsections the case in which the function \( S(\rho) \) vanishes for \( \rho \leq \rho_* \) and stabilizes to \( S(\rho) \to 1 \) for large values of the the radial coordinate. Also, let us focus our attention on backgrounds like the one in eq. (3.8), though the lessons will be valid also for the semi-analytic solutions that we present in section 4.

#### 3.2.1 The theory at high energies

We study the UV of the field-theory dual to the background in eq. (3.8). We interpret this solution combining the results of the papers [11] and [29]. We propose that the dual quiver is of the form

\[
SU(n + n_f + N_c) \times SU(n + n_f), \quad n = kN_c.
\]

---

(continued on next page)
The field theory is in the mesonic branch for $\rho > \rho_*$, where the function $S(\rho) \sim 1$. There are two competing processes, as explained in [29], inspired in early ideas presented in [35]. The usual cascade, represented by the $h_{KS}$ in the warp factor of eq. (3.9) and a Higgsing process represented by the term proportional to $\nu$ in eq. (3.9). In the new radial coordinate $r \sim e^{2\nu/3}$,

$$h|_{\rho \rightarrow \infty} \sim \frac{\nu r^2 + 3N_c^2 \log r}{r^4},$$

we see clearly the superposition of the cascade and the Higgsing. The presence of an exponentially increasing number of source D3s effectively behaves as the insertion of an irrelevant operator of dimension six, deforming the UV dynamics out of the near-conformal KS dynamics. This indicates the need for a UV completion.

### 3.2.2 The theory at low energies

Flowing towards the IR, and close to $\rho \sim \rho_*$ the Higgsing and cascading lower down the ranks of the groups, so eventually we will reach the numerology of the baryonic branch

$$\begin{align*}
\text{SU}(n_f + (k + 1)N_c) \times \text{SU}(n_f + kN_c) & \rightarrow \\
\text{SU}(n_f + (k + 1)N_c - 1) \times \text{SU}(n_f + kN_c - 1) \times \text{U}(1) & \rightarrow \\
& \rightarrow \ldots \text{SU}((k + 1)N_c) \times \text{SU}(kN_c) \times \text{U}(1)^{n_f} .
\end{align*}$$

(3.15)

The main difference with respect to the usual baryonic branch as discussed in detail in [11] is the fact that the baryon symmetry is in this case *gauged*. This happens because the baryonic symmetry mixes with a diagonal combination of the $\text{U}(1)^l$'s that appear when Higgsing. This symmetry being gauged impacts the low-energy phenomenology. For example, we can check in the case of the background in eq. (3.8), that the massless excitation described in [49] is not a solution to the equations of motion. This implies that the theory is on a mesonic branch.

For the rest, with a suitable choice of $S(\rho)$ vanishing conveniently fast at $\rho = 0$, the phenomenology of the low-energy field theory is very similar to that of the Klebanov-Strassler theory, as both backgrounds are quite similar. The computation of various IR quantities will give qualitatively similar results to those computed with the backgrounds of [10] and [34]. There will be numerical differences depending on the profile $S(\rho)$. We will provide examples of this in the following sections.

### 3.2.3 How to improve the UV behavior: a phenomenological approach

We would like to stop the growing number of D3 sources responsible for the pile-up of Higgsing processes and ultimately for the deviation from the nearly conformal behavior as illustrated in the discussion around eq. (3.14). A way to do so would be to choose a profile for the sources that somewhat recovers the cascade behavior. In the case of the background in eq. (3.8) this is an easy task because, as we are distributing the D3 sources on the deformed conifold, the solution preserves supersymmetry for any $S(\rho)$. This suggests that we should choose — at least at a phenomenological level — a distribution function
behaving as $S(\rho) \sim e^{-4\rho/3}$ for large values of $\rho$. A profile that does the job of fixing all the IR and UV problems is inspired in the work of [48] and [44],

$$S(\rho) = \tanh(2\rho)^4 e^{-4\rho/3}. \tag{3.16}$$

In figure 3 we have plotted the profile function (3.16). By using this distribution function, one can compute the new warp factor using eq. (3.9). We will then have a smooth IR geometry and a cascade behavior in the far UV.

In principle, one could of course try different profiles that make the distribution of D3-brane sources either constant or vanishing for large values of $\rho$. However, if we want this distribution of sources to have a positive mass density everywhere (and vanishing for $\rho \to \infty$), it seems that the only possibility is to have profiles decreasing exactly as $e^{-4\rho/3}$. See appendix C for details.

Another way to motivate why the profile $S(\rho)$ should decrease towards the UV is to consider the Klebanov-Strassler [10] geometry with $N_c$ D5-branes and add to it $lN_c$ extra D3-branes. The field theory is then in the $l$-th mesonic branch [11] of the theory I with groups $SU((k+l)N_c) \times SU((k+l+1)N_c)$. The relation between the deformation parameters $\epsilon_l$ and $\epsilon_0$ of the conifold is given in [11, 50],

$$\epsilon_l = \epsilon_0^2 e^{N_c^{2l}} = \epsilon_0^2 e^{2\pi i (N_c l)/(N_c^2)}. \tag{3.17}$$

So, we see that

$$N_c^2 \log \epsilon_l \sim lN_c \log \epsilon_0. \tag{3.18}$$

This implies (for similar deformations $\epsilon_l \sim \epsilon_0$), that $lN_c \sim N_c^2$. That is, the number of source D3-branes that we called $n_f$ should be of the same order as the number of flux D3-branes that we called $n$ in our quiver of eq. (3.13). Obviously, the situations we discussed above, where the source function $S(\rho) \rightarrow 1$ at large values of the radial coordinate, generate

---

\(^7\)Notice that one is taking $\rho_* = 0$ in the example profile (3.16). It would be trivial to introduce the scale $\rho_*$ by writing something like $S = \Theta(\rho - \rho_*) \tanh(2\rho - 2\rho_*)^4 e^{-4(\rho - \rho_*)/3}$.

\(^8\)This argument was suggested by Anatoly Dymarsky.
as explained in eq. (3.11) an exponentially large number of D3 sources $n_f$. Without a damping factor like the one proposed in eq. (3.16) or similar there is some feature of the field theory that is not captured by the solution. So, we will choose functions describing the profile of D5 sources (this will directly affect also the profile of D3 sources) of the form given in eq. (3.16). These profiles decrease fast enough in the asymptotic regions, so that the IR singularity generated by a large density of sources and the UV appearance of an irrelevant operator are both avoided.

### 3.2.4 Further comments on profiles and Goldstones

To avoid confusion with the different uses and meanings of the function $S(\rho)$, some words are in order. The function $S(\rho)$ originated in the supergravity background dual to the ‘field theory II’ with D5-brane sources [44]. Its meaning there was that of a profile for the D5-brane charge present in the background. Coming back to figure 1, we are placing several D5-branes on the geometry, each of them reaching a minimal radial distance. These minimal distances are distributed on a shell. An electrostatic analogue of this situation is given by the electric field created by a hollow cavity with a thick charged shell. In this case $S(\rho)$ would count the effective radial charge. Inside the cavity $S(\rho)=0$ as there is no electric field. As we cross the shell, $S(\rho)$ increases, and stabilizes to the total charge away from the shell. The electric field outside the shell, will be that of a point charge. Had the shell null thickness, the electric field would display a jump. This is what the $S(\rho)$ in figure 1 represents. Away from the shell, $S(\rho \to \infty) \sim 1$ in coincidence with the $S(\rho)$ generated by ‘massless’ sources.

This interpretation of $S(\rho)$ as a profile for the D5-brane charge is however lost after the rotation. We gave an interpretation of $S(\rho)$ for the exact solution of eq. (3.8). As explained after eq. (3.10), in this new solution $S(\rho)$ accounts for how the D3-branes are distributed in the geometry. So, for this exact-analytic solution, $S(\rho)$ represents a distribution of D3-branes. More precisely, the distribution is given by $S(\rho) (\sinh(4\rho) - 4\rho)^{1/3}$.

A curious fact is that the ‘physically sensible’ $S(\rho)$ for the unrotated background (which asymptotes as $S \sim 1$), is pathological for the rotated configuration as the number of D3-branes grows exponentially with $\rho$, inducing an irrelevant deformation. Conversely, a physically sensible $S(\rho)$ in the rotated background (vanishing at infinity as $e^{-4/3\rho}$) does not have — at present — a microscopic interpretation in terms of the usual flavor D5-branes in the unrotated solution. A clarification of this point remains for future work.

**About Goldstone bosons:** Let us briefly discuss the situation with Goldstone bosons. Detailed calculations needed to sort out the issue in full are left for future work.\(^9\)

First, let us state the question. Consider a background like the one in eq. (3.8). We can choose any $S(\rho)$ with a similar behavior to that in eq. (3.16), without affecting the argument. In practical terms, we are making a transition from a far UV with similar dynamics to KS and the baryonic branch, to an intermediate region, with the numerology of the mesonic branch, to a deep IR which has the gauge-group ranks of the baryonic branch of [10]. If one then calculates the solution for the massless mode in [49], the proposed ansatz

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\(^9\)Many thanks to Anatoly Dymarsky for crucial comments reflected in these following paragraphs.
of [49] does not verify the linearized equations of motion. Indeed, despite the fact that $F_3$ and $B_2$ in eq. (3.8) are the same as in the KS background, the five-form flux $F_5$ is modified by the presence of sources (as seen from the fact that $\hat{h} \neq \hat{h}_{KS}$). Then, in particular, the equation of motion for $H_3$ would be different from the standard KS one, including a term involving $dF_5$ which is non-zero because of the sources.

In line with this, as we emphasized above, transitions between mesonic and baryonic branches should produce gauged baryonic symmetries [11], and as such the Goldstone of the baryonic U(1) should not be part of the physical spectrum (it should be Higgsed away into the resulting massive gauge boson).

Nevertheless, one may speculate that a modification of the ansatz in [49] may reveal the existence of a massless mode. If we were to find a Goldstone boson on the gravity side, how could we understand it in terms of the field-theory dual? In the combination of Higgsing and cascading, there is a complicated pattern of global and gauge symmetries, which are non-trivially mixed with one another, and which undergo partial symmetry breaking. Vacuum-alignment arguments suggest that, if possible (i.e. in the absence of non-trivial obstructions within the Lie groups and cosets describing the symmetry of the system), the vacuum of the theory will align itself with the directions in the internal space of the global symmetry. In doing so, the vacuum will prefer to spontaneously break part of the symmetry which is not gauged. As we wrote, a detailed calculation would be needed in order to establish whether or not there is a normalizable massless mode in the spectrum, and we leave this for a future study.

We move now to the study of the semi-analytic solutions described in the beginning of section 3.1.

4 Details on the new solutions

In this section we give details for the semi-analytic solutions. We will concentrate on the configuration after the rotation. We consider the case in which the parameters $N_f, \lambda$ are free. While the solutions could be written in terms of an infinite series of integrals (that cannot be performed exactly), we find it more useful to give series expansions close to $\rho = 0$ and $\rho \to \infty$ and a numerical interpolation between both regimes. After solving the master equation (3.4), we will use the expressions in eqs. (3.5)–(3.7) to quote the asymptotic expansions for the functions appearing in the background. As anticipated in eq. (3.16) we will consider (this is just a possible example) the function $S(\rho) = \tanh^4(2\rho)e^{-4\rho/3}$.

We should clearly state that we are not obtaining the profile above in a rigorous fashion. Since the distribution of D3-branes on the deformed conifold can be chosen (they are SUSY), the solutions with $\lambda \to \infty, N_f \to 0$ of eq. (3.8) are correct. The ones we present in this section, lack a rigorous proof of existence (we are choosing a profile of D5-branes that we did not prove to be derived from a kappa-symmetric embedding). The healthy behavior of the backgrounds we found (together with the positivity of $T_{00}$ for the sources) suggests that the brane embedding can be rigorously derived. Let us start by studying the large-$\rho$ expansion of the solutions to the master equation (3.4) given the source term in eq. (3.16).
4.1 Large-\(\rho\) expansions

First of all we define

\[
S_\infty = \int_0^\infty d\rho \tanh^2(2\rho)S(\rho),
\]

so that we can write for the asymptotic behavior of the function \(Q(\rho)\):

\[
Q = N_c(2\rho - 1) - N_f S_\infty + \frac{3}{4} N_f e^{-4\rho/3} + (4 N_c \rho - 2 N_f S_\infty) e^{-4\rho} + O(e^{-16\rho/3}).
\]

The function \(P(\rho)\) is given at large \(\rho\) by

\[
P(\rho) = e^{4\rho/3} \left( c_+ + N_c^2 e^{-8\rho/3} \left( 4\rho^2 - 4\rho \left( 1 + \frac{N_f}{N_c} S_\infty \right) + \frac{13}{4} \frac{3c_+ N_f}{N_c^2} + 2 \frac{N_f}{N_c} S_\infty + \frac{N_f^2}{N_c^2} S_\infty^2 \right) + \frac{e^{-4\rho}}{3c_+} \left( 2 N_c N_f \rho^2 - 2\rho (4c_+^2 - N_c N_f + N_f^2 S_\infty) \right) + O(e^{-16\rho/3}) \right),
\]

Using eqs. (3.5)–(3.7), we get for the background functions

\[
\hat{h} = 3 e^{-8\rho/3} \left( N_c^2 (8\rho - 1) + 2c_+ N_f - 4 N_c N_f S_\infty \right) + \frac{N_f}{2c_+} e^{-4\rho} (2 N_c (2\rho - 1) - N_f S_\infty) + O(e^{-14\rho/3}),
\]

\[
e^{2g} = \frac{c_+}{4} e^{4\rho/3} + \frac{N_c (2\rho - 1) - N_f S_\infty}{4} + \frac{e^{-4\rho/3}}{16c_+} \left( 16 N_c^2 \rho^2 - 16 N_c N_f + N_f S_\infty \right) \rho + \frac{13 N_c^2 + 6c_+ N_f + 8 N_c N_f S_\infty + 4 N_f^2 S_\infty^2}{16c_+} + O(e^{-8\rho/3}),
\]

\[
e^{2k} = \frac{c_+}{4} e^{4\rho/3} - \frac{N_c (2\rho - 1) - N_f S_\infty}{4} + \frac{e^{-4\rho/3}}{16c_+} \left( 16 N_c^2 \rho^2 - 16 N_c + N_f S_\infty \right) \rho + \frac{13 N_c^2 + 8 N_c N_f S_\infty + 4 N_f^2 S_\infty^2}{16c_+} + O(e^{-8\rho/3}),
\]

\[
a = 2 e^{-2\rho} + 2 e^{-10\rho/3} \frac{N_c (2\rho - 1) - N_f S_\infty}{c_+} + \frac{e^{-14\rho/3}}{24c_+} \left( 4 N_c \rho - 5 N_f - 2 N_f S_\infty \right)^2 + \frac{e^{-8\rho/3}}{36c_+} \left( 4c_+^2 + N_f S_\infty \right) (8\rho - 3) - N_c N_f (8\rho^2 + 2\rho - 3) + O(e^{-4\rho}),
\]

\[
N_c b = -2 e^{-2\rho} (4 N_c \rho - 2 N_f S_\infty) + \frac{N_f}{2} e^{-10\rho/3} + \frac{e^{-14\rho/3}}{2c_+} \left( 4 N_c \rho - 2 N_f S_\infty \right) + O(e^{-6\rho}),
\]

\[
e^{4\Phi} = e^{4\Phi(\infty)} \left( 1 + \frac{3 e^{-8\rho/3}}{4c_+^2} \left[ N_c^2 (8\rho - 1) + 2c_+ N_f - 4 N_c N_f S_\infty \right] + \frac{N_f}{c_+} e^{-4\rho} (2 N_c (2\rho - 1) - N_f S_\infty) \right) + O(e^{-14\rho/3}),
\]

where \(e^{4\Phi(\infty)} = \frac{3 e^{8\rho_0}}{8c_+^2}\). Notice that one effect of the sources — with this given profile \(S(\rho)\) — is to add to the D3-brane charge in the warp factor. The sources do not interfere with
the cascade, that is now driven by the bulk D3-branes only, associated with the number \( n \) in our quiver. Let us discuss the IR expansion next.

### 4.2 Small \( \rho \) expansions

With the same \( S(\rho) \) as in (3.16), we can see that close to \( \rho = 0 \), the effect of the sources vanish as

\[
S(\rho) = 16\rho^4 - \frac{64}{3}\rho^5 - \frac{640}{9}\rho^6 + \mathcal{O}(\rho^7). \tag{4.5}
\]

It is clear that the deep IR asymptotics will suffer little modification from that of the case with \( N_f = S(\rho) = 0 \). In order to keep track of the IR modifications, we just need to look at the terms with a factor \( N_f \) and compare the expansions with those in eq. (4.6) and eqs. (A.7). The functions \( Q(\rho), P(\rho) \) read

\[
Q(\rho) = \frac{4N_c}{3}\rho^2 - \frac{16N_c}{45}\rho^4 + \left( \frac{128N_c}{945} - \frac{32N_f}{7} \right)\rho^6 + \mathcal{O}(\rho^8),
\]

\[
P(\rho) = h_1\rho + \frac{4}{15h_1^2} - \frac{4N_c^2}{15}h_1^2\rho^3 + \frac{16}{1575h_1^3}(3h_1^4 - 4h_1^2N_c^2 - 32N_c^4 - 450h_1^3N_f)\rho^5 + \mathcal{O}(\rho^7),
\]

while the functions in the background have expressions given by\(^{10}\)

\[
\hat{h} = 1 - \frac{e^{2\Phi(0) - 2\Phi(\infty)}}{2\sqrt{2}} - \frac{8\sqrt{2}N_c^2}{9h_1^2}e^{2\Phi(0) - 2\Phi(\infty)}\rho^2 - \frac{2\sqrt{2}}{135}e^{2\Phi(0) - 2\Phi(\infty)}\left( - \frac{40N_c^2}{h_1^2} + 224N_c^4 h_1^2 + 135N_f h_1 \right)\rho^4 + \mathcal{O}(\rho^5),
\]

\[
e^{2\rho} = \frac{h_1\rho}{2} + \frac{4}{45}\left( -6h_1 + 15N_c - 16N_c^2 h_1 \right)\rho^4 + \frac{16}{4725h_1^7}(267h_1^4 + 1814h_1^2N_c^2 - 700h_1N_c^3 - 328N_c^4 - 15h_1^3(77N_c + 45N_f))\rho^6 + \mathcal{O}(\rho^7),
\]

\[
e^{-2g} = \frac{1}{8} + \frac{1}{15}\left( 3h_1 - 5N_c - 2N_c^2 h_1 \right)\rho^2 + \frac{2}{1575h_1^7}(3h_1^4 + 10h_1^2N_c^2 - 45N_f) - 144h_1^2N_c^2 - 32N_c^4\rho^4 + \mathcal{O}(\rho^5),
\]

\[
e^{2k} = \frac{1}{8} + \frac{h_1^2 - 4N_c^2}{10h_1}\rho^2 + \frac{(6h_1^4 - 8h_1^2N_c^2 - 64N_c^4 - 270h_1^3N_f)\rho^4}{315h_1^3} + \mathcal{O}(\rho^5),
\]

\[
e^{4\Phi} = e^{4\Phi(0)}\left( 1 + \frac{6N_c^2\rho^2}{9h_1^2} + \frac{16( - 120N_c^2h_1^2 + 992N_c^4 + 405h_1^3N_f)\rho^4}{405h_1^3} + \mathcal{O}(\rho^5) \right),
\]

\[
a = 1 + \left( -2 + \frac{8N_c}{3h_1}\right)\rho^2 + \frac{2(75h_1^3 - 232h_1^2N_c + 160h_1N_c^2 + 64N_c^3)\rho^4}{45h_1^3} + \mathcal{O}(\rho^6),
\]

\[
b = 1 - \frac{2}{3}\rho^2 + \left( \frac{14}{45} - \frac{8N_f}{N_c} \right)\rho^4 + \mathcal{O}(\rho^5),
\]

where \( e^{4\Phi(0)} = \frac{8c e^{4\Phi_0}}{h_1^3} \). Using these expressions, one can find a smooth numerical interpolation between the two asymptotics and compute observables with them. In figure 4, we plot

\(^{10}\) Notice that for the function \( a(\rho) \) above, the first correction of the sources appears at order \( \rho^6 \).
Figure 4. For the case $N_c = N_f = 2$ and $h_1 = 5$, we show the functions defining the semi-analytic rotated background. The choice of $S(\rho)$ is the one specified in the text (eq. (3.16)).

the functions $P(\rho)$ and $Q(\rho)$ for $S(\rho) = \tanh^4(2\rho)e^{-4\rho/3}$. Using $P(\rho), Q(\rho)$ and $S(\rho)$ all other functions in the background can be obtained. Studying these functions, we do not observe any pathology or non-physical behavior. Hence, we consider the choice of $S(\rho)$ in this section to be good.

5 Further analysis of field-theory quantities

In this section, we continue the study of the background found in section 4. We compute some quantities in the proposed dual field theory. The purpose of these calculations is to investigate if the profile chosen for $S(\rho)$ produces any undesirable feature that might suggest non-physicality of our arbitrary choice. We start with the central charge in the dual QFT.

5.1 Central Charge

We calculate the central charge $c$, following the prescription developed in [51–53]. Start with a metric (in string frame) of the form

$$ds^2_{\text{st}} = \alpha dx_{1,d}^2 + \alpha \beta d\rho^2 + g_{ij} dy^i dy^j ,$$

and following [53] we define $V_{\text{int}}$ as the volume of the internal space, and $H$ as

$$H = e^{-4\Phi}V_{\text{int}}^2 \alpha^d .$$


Figure 5. Plots of the function $c(\rho)$ for $N_c = N_f = 2$ and $h_1 = 5$. We considered the backgrounds of section 4.

We have that the central charge (for $d = 3$) is

$$c = \frac{\beta^{3/2} H^{7/2}}{(H')^3}. \quad (5.3)$$

In particular, for a background like in eq. (2.15), we have

$$\alpha = e^{\Phi} \hat{h}^{-1/2}, \quad \beta = e^{2h} \hat{h}, \quad V_{\text{int}}^2 = e^{4h+4g+5\Phi+2k} \hat{h}^{5/2}, \quad H = e^{4\Phi+4g+4h+2k} \hat{h},$$

$$c = \frac{\hat{h}^2 e^{2\Phi+2h+2g+4k}}{8(\partial_\rho \log(\sqrt{\hat{h} e^{2\Phi+2h+2g+k}}))^3}. \quad (5.4)$$

We consider the semi-analytic backgrounds of section 4 and plot the central charge of eq. (5.4) in figure 5. We observe a monotonic behavior. This quantity does not show any sign of unphysical behavior for the chosen profile.\footnote{One could have arbitrarily chosen a different profile, for example $S \sim \tanh^{4}(2\rho)e^{-(\rho-\rho_0)^2}$. In this case we would have observed a non-monotonic $c$-function.}

For backgrounds like those in eq. (2.4) — before the rotation — we should take $\kappa = 0$ and $\hat{h} = 1$ in the formulas above.

5.2 Quantities defining the gauge groups

In specifying our quiver, for example around eq. (3.13), we have introduced various quantities: $n$, that is the number of bulk (flux) D3-branes; $n_f$, that is the number of source D3-branes; $N_c$ and $N_f$, the numbers of bulk and source D5-branes. Here, we give a geometric definition for these quantities, forgetting about numerical factors.

$$n + n_f = \int_{\Sigma_5} F_5 = \int_{\Sigma_5} B_2 \wedge F_3,$$

$$n = \left( \int_{\Sigma_5} F_5 \right)_{N_f = 0}, \quad n_f = \int_{\Sigma_5} F_5 - \left( \int_{\Sigma_5} F_5 \right)_{N_f = 0}. \quad (5.5)$$
where $\Sigma_5$ is the five-dimensional manifold of coordinates $(\theta, \varphi, \tilde{\theta}, \tilde{\varphi}, \psi)$. Using the expressions in eq. (2.15), we can calculate $F_5|_{\Sigma_5}$ to be

$$F_5|_{\Sigma_5} = B_2 \wedge F_3 = -\frac{\kappa}{8} e^{2\Phi} \left[ \frac{\cosh(2\rho) - a}{\sinh(2\rho)} \left( -2N_c e^{2h} + \frac{N_c}{2} e^{2g} (a^2 - 2ab + 1) - \frac{N_f}{2} e^{2g} S \right) - \frac{4N_c}{\sinh(2\rho)} e^{2h}(a - b) \right] \sin \theta \sin \tilde{\theta} d\theta \wedge d\varphi \wedge d\tilde{\theta} \wedge d\tilde{\varphi} \wedge d\psi,$$  

(5.6)

that translates into

$$n + n_f = 8\pi^2 \kappa e^{2\Phi} \left[ \frac{\cosh(2\rho) - a}{\sinh(2\rho)} \left( 2N_c e^{2h} - \frac{N_c}{2} e^{2g} (a^2 - 2ab + 1) + \frac{N_f}{2} e^{2g} S \right) + \frac{4N_c}{\sinh(2\rho)} e^{2h}(a - b) \right],$$

(5.7)

where $n = (n + n_f)|_{N_f = 0}$, $n_f = (n + n_f) - (n + n_f)|_{N_f = 0}$.  

While it is good to have the exact expressions above, for many purposes the large and small $\rho$ expansions are illuminating. For low energies in the dual QFT, we have

$$n \propto N_c^2 \rho^3 + \left( \frac{32N_c^4}{9h_1^2} - \frac{4N_c^3}{3} \right) \rho^5 + O(\rho^7),$$

$$n_f \propto N_f h_1 \left[ \rho^5 - \frac{4}{3} \rho^6 + O(\rho^7) \right],$$

(5.8)

which makes it clear that deep in the IR the Higgsing (represented by $n_f$) has stopped. The geometry is very similar to that of [34] and so is the dual QFT. To describe the regime of high energies in the field theory, we change to a radial coordinate $r = e^{2\rho/3}$ as defined around eq. (3.14). Then, for large $r$ one has

$$n \propto N_c^2 \frac{3 \log r - 1}{3} - N_c^4 \frac{36 \log^2 r - 15 \log r + 1}{8c_c^2 r^4} + O(r^{-6}),$$

$$n_f \propto N_f \left[ c_+ - 2N_c S_\infty \frac{c_-}{6} - 6N_c \log r - 5N_c - 2N_f S_\infty \right] 12r^2 + O(r^{-4}).$$

(5.9)

We see that far in the UV the cascade is at work, with a number of ‘bulk’ D3-branes $n = kN_c$ — as usual $k \sim N_c \log(r)$. The Higgsing has stopped and the source D3-branes $n_f$ just accounts for a constant charge. This situation is also reflected in the warp factor $\hat{h}$, see the discussion around eq. (4.4).

6 Towards applications

This section is devoted to provide the reader with a reason, based on field-theory arguments, why this type of solutions is interesting and might be relevant for phenomenology and model-building in high-energy Physics. In terms of the field theory dual, the backgrounds constructed and discussed in this paper have a set of very peculiar features, which can be summarized as follows.
First of all, notice that (besides the confinement scale related to the end of space in the geometry) there are three dynamical scales in the theory. One is the scale $\bar{\rho}$ controlled by the parameter $c_+$ in eq. (2.7), i.e. by the insertion of a dimension-eight operator in the single-site theory (which is also related to a dimension-two VEV, as explained in [28] and references therein). In the absence of sources, below such scale the theory is best described as a generalization of the one-site field theory dual to [15], while above this scale the rotation procedure yields a background that is dual to a two-site quiver realizing the (baryonic branch of the) Klebanov-Strassler duality cascade. There are then two other scales $\rho_*$ and $\rho_S$, $\rho_* < \rho_S$, such that the function $S$ has support in the range between them. There is no obvious relation between $\bar{\rho}$ and these other two scales, but for reasons of simplicity we will assume in this discussion that $\bar{\rho} \simeq \rho_S$. Different cases can be discussed along the same lines, in a case-by-case way that does not add to the main physical points we want to make.

We will focus only on the rotated solutions.

- In the far UV, for $\rho > \rho_S \sim \bar{\rho}$, the theory resembles the (baryonic branch of) the KS cascade: the theory is flowing close to a line of fixed points, each of which is a $N = 1$ Klebanov-Witten fixed point, a two-site quiver with gauge group $SU(n) \times SU(n)$, and with $n$ increasing towards the UV (i.e. with $\rho$). The flow never approaches an actual fixed point at finite (and large) $n$ because of a small imbalance between the ranks of the two gauge groups, but rather the flow goes up a cascade of Seiberg dualities [54] which continues indefinitely towards $\rho \to +\infty$. See [55].

- There is an intermediate range $\rho_* < \rho < \rho_S \sim \bar{\rho}$ over which the function $S$ is non-trivial. At the scale $\bar{\rho}$ and in the absence of sources, the duality cascade would stop, due to the Higgsing induced by the dimension-two condensate appearing and precipitating the theory towards the last stages of the duality cascade itself [28, 29]. On the other hand, because in this range the function $S$ is non-trivial, another cascade starts, which has a completely different interpretation: it is a cascade of Higgsings of the gauge theory. Source D3s are crossed when flowing to the IR, hence realizing the ideas proposed in [35].

- Below the scale controlled by the value of $\rho_*$, the Higgsing cascade stops, and with it most of the dynamical features related to $N_f$ (up to subtleties which have been discussed earlier), because $S$ vanishes. The theory now looks like a generalization of the single-quiver theory in [15], with the same type of dynamics.

- At very low scales (near the end of space) of the geometry, the theory shows the appearance of a non-trivial gaugino condensate, and confines in the usual sense of producing an area law for the Wilson loop.

Notice that one might as well take $\bar{\rho}$ to be very small, near the end of space: in this case the duality cascade would continue all the way to very small energies, in particular extending over the second and third of the four ranges described above. In this case, there would be a regime in which both Higgsing cascade and duality cascade coexist.
The physics taking place in the field theory at the second of the four stages (for $\rho_* < \rho < \rho_S \sim \bar{\rho}$) is very peculiar, and even more peculiar and new is the fact that such a behavior appears only over a finite range of energies. The question is: what kind of theoretical models exhibit features that are qualitatively similar to the one discussed here, and what kind of possible physical systems can they describe? In other words: are there conceivable applications of such results?

Interestingly, there is a positive answer to this question, which relates to a long-standing, very difficult and open phenomenological problem. A scenario that has some common elements with our present one is that of Extended Technicolor Models (ETC) with Tumbling dynamics [56, 57]. This is a very plausible dynamical explanation for the origin of flavor physics (SM-fermion masses, mixing angles, CP violation, FCNC interactions, . . . ), in the context of strongly coupled extensions of the Standard Model [58–63] (see also [64–69] for a sample of reviews on the subject). We will provide here a simple summary of what this means, besides referring the reader to the literature on the subject. We also want to clarify what are the actual similarities and the substantial differences, and hence the possible intrinsic limitations into attempting to use the approach of this paper in order to study tumbling ETC.

ETC addresses the following, well-known, fundamental problem, arising in the context of (strongly coupled) Dynamical ElectroWeak-Symmetry Breaking (DEWSB). DEWSB or technicolor (TC) proposes to replace the Higgs sector of the Standard Model with a new gauge theory with group $G_{TC}$ and new (techni-fermion) fields transforming non-trivially under the action of both $G_{TC}$ and the Standard Model gauge group. The strong dynamics associated with $G_{TC}$ yields the formation (at the confinement scale $\Lambda_{TC}$) of non-trivial condensates made of techni-fermions, which results in the spontaneous breaking of the SM gauge group at the ElectroWeak scale. The SM gauge interactions themselves communicate the breaking to the $W$ and $Z$ gauge bosons, which become massive. However, in the process of removing the Higgs field, one also loses the Yukawa couplings, and hence one needs a mechanism that couples the SM fermions to the techni-quarks, in order for the quarks and leptons to acquire a mass below the ElectroWeak scale.

ETC provides such a dynamical mechanism. The generic ETC model works in the following way (an example of such a construction is discussed in detail in [70–75]. For important elements used in these papers, see also [76–78]). Start from some gauge theory with gauge group $G_{ETC} \times G_{SM}$ (where $G_{SM} = SU(3)_c \times SU(2)_L \times U(1)_Y$ is the familiar SM gauge group), and a given fermionic-matter content. Assume that the dynamics of $G_{ETC}$ is such that the theory is asymptotically free, but undergoes a sequence of breaking stages

$$G_{ETC} \rightarrow G_1 \rightarrow G_2 \rightarrow G_{TC},$$

at scales $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$, respectively. At scales below $\Lambda_3$ the resulting effective field theory consists of the following.

- A gauge theory with gauge group $G_{TC} \times G_{SM}$.
- Two kinds of massless fermions: techni-quarks $\psi_{TC}$ transforming non-trivially under $G_{TC} \times G_{SM}$, and singlets of $G_{TC}$ that we denote by $\psi_{SM}$, that transform non-
Figure 6. Cartoon depiction of the running of the gauge coupling $g^2$ in an example of multi-scale (tumbling) ETC model as a function of the renormalization scale $\mu$. The sequential breaking at scales $\Lambda_1 > \Lambda_2 > \Lambda_3 > \Lambda_{TC}$ means that the gauge group and field content changes at each breaking scale, and so does the dynamics. A model that is believed to exhibit such behavior is described in detail in [70–75]. For important elements used in these papers, see also [76–78].

- Higher-order operators originating from integrating out the heavy gauge bosons of the coset $G_{ETC}/G_{TC}$. In particular, some of these are four-fermion operators coupling two SM fermions with two TC fermions, with the generic form

$$\frac{1}{\Lambda_i^2} \bar{\psi}_{TC} \psi_{TC} \bar{\psi}_{SM} \psi_{SM} .$$

We present in figure 6 a cartoon of the dynamics (in terms of the effective gauge coupling) of such a scenario.

Now, the resulting gauge theory is the TC theory. Ultimately, $G_{TC}$ will confine, and produce condensates that break $G_{SM}$. However, the presence of the four-fermion interactions means that after ElectroWeak-Symmetry Breaking the quarks and leptons will become massive. Effectively, these four-fermion operators play the same role as the Yukawa couplings in the Standard Model. Notice that, because they originate at different scales, there will be in general three families of SM fermions, and some of these operators will be suppressed as $1/\Lambda_1^2$, others as $1/\Lambda_3^2$, and so on. In general there will be a very complicated, hierarchical structure in the four-fermion couplings, which will translate after dimensional transmutation into the hierarchical structure of the masses of the SM fermions, and the mixing angles in the CKM mixing matrix.

The presence of a long (at least three-stage) sequence of breaking, and of hierarchies in the dynamical scales is absolutely necessary on phenomenological grounds, because it offers...
the only plausible and self-contained dynamical explanation for the pattern of phenomenological masses and mixing angles experimentally measured. On the other hand, it makes it extremely difficult to study, because of the strongly coupled nature of the phenomena taking place in these field theories. Many open questions about such dynamics, and its low-energy implications, require some new tools for a quantitative (and often even qualitative) analysis to be performed. The gravity duals we are finding resemble this scenario: a sequence of Higgsing stages taking place at several scales, over a finite energy range. It is hence conceivable that some of the long-standing open problems might be addressed in this context. For example, one would also like to understand what kind of low-energy spectrum one may observe, with particular reference to the presence of possible pseudo-Goldstone bosons associated with the breaking of accidental global symmetries.

There is another long-standing problem (see for instance [79] and references therein). Besides the gauge symmetries, all of the TC and ETC models also possess very large (approximate) global symmetries, which are spontaneously broken by the many condensates that form. This might yield the presence of such pseudo-Goldstones, with masses that might put them well below the current exclusion region, and hence render the models phenomenologically not viable. Computing masses and couplings of such particles requires dedicated strong-coupling calculations, and new tools are needed to perform them.

We can hence conclude that this is a first concrete example of a model which shares some of the fundamental features of tumbling, and that these types of models might be used to characterize in field-theory terms what are the features associated with the tumbling itself. It must also be stressed that the very nature of these models is such that a direct comparison to the real world must be done with caution: effectively we have an infinite, continuum number of Higgsing stages, rather than a few (three) distinct and hierarchical breaking stages. In other words, comparing to figure 6, the Higgsing cascade differs from the tumbling because between the scales $\Lambda_1$ and $\Lambda_3$ (which can be associated with $\rho_S$ and $\rho_*$, respectively), one has a continuum of breaking scales, rather than just a few steps. It hence remains to be understood what type of operators will replace the four-fermion operators of low-energy TC, and what type of phenomenological implications they have. This is somewhat analogous to the fact that the gravity dual of the duality cascade actually consists of an infinite continuum of Seiberg dualities, rather than a small number of such stages.

7 Conclusions

Let us conclude this paper by recalling the main results of this work and proposing some topics for future investigation. The reader who wants to have a simple summary of the results and main original features discussed in the paper should complement this final section with the brief summary at the beginning of section 6.

The basic idea of this paper is to construct new classes of gravity backgrounds that we interpret in terms of the very non-trivial RG flow of a dual field theory with $\mathcal{N} = 1$ supersymmetry, which exhibits a number of interesting properties.
We constructed these new solutions, which generalize the KS [10] and baryonic branch [34] solutions, by the addition of sources into the type IIB configuration. We proposed suitable profiles for the sources that avoid any small radius singularity and also provide the background with a ‘logarithmically’ AdS space in the large-radius regime. In this fashion, the backgrounds are trustable and the dual field theory associated is well defined all along the flow. It is therefore sensible to think that these backgrounds are characterizing a well-defined field theory, and hence we suggested an interpretation for the microscopic properties of such a field theory. We studied different aspects of the four-dimensional dual quantum field theory that point to the consistency of the interpretation given.

A comment on the string side of the construction must be made. While the solutions with only D3 sources on the deformed conifold are fully rigorous, our profiles for D5 and D3 sources have not been obtained from a microscopic kappa-symmetric solution to the brane equations of motion. In spite of this, various tests suggest that the backgrounds presented here are physically acceptable, both from the supergravity and from the field-theory sides.

The main element of novelty that emerged from our study is that the dual field theory undergoes two cascades: a cascade of Seiberg dualities, analogous to the one in [10], and a cascade of Higgsings, analogous to what was suggested in [35]. In both cases, the rank of the gauge groups in the dual quiver field theory shrinks when going towards the IR. But this is the result of very different phenomena. The fact that the Higgsing cascade takes place over a compact, finite range of energy scales is a significant element of novelty, also considering that traditional field theory studies of this type of behavior are peculiarly difficult.

There are several open questions we leave for future studies. On the more formal side, a well defined project would be to determine from microscopic calculations the profiles (the function $S$ in the body of the paper) that reproduce the qualitative behavior we proposed here. This would complete the picture in a satisfactory way. Also, a side project would be related to a precise discussion of the existence (or not), couplings and properties of the Goldstone mode associated with the baryonic symmetry, a problem that we only briefly touched upon in this work.

This work opens the way to several possible applications. In particular, it would be interesting to make more concrete the possible applications in the context of Dynamical Electroweak-Symmetry Breaking, for example by calculating observables related to the discussion in section 6, or by adapting to these backgrounds (or suitable modifications of them) the tests performed in [80–86]. In this way, one might make some interesting progress towards the construction of a complete model of dynamical ElectroWeak Symmetry Breaking in the rigorous context of gauge/string dualities, and hence move towards the direction of obtaining a fully computable and testable model of this type.

Acknowledgments

Discussions with various colleagues helped to improve the contents and presentation of this paper. We wish to thank Lilia Anguelova, Stefano Cremonesi, Anatoly Dymarsky (very specially), Daniel Elander, Aki Hashimoto, Tim Hollowood, Prem Kumar, Dario Martelli, Ioannis Papadimitriou and Jorge Russo. The work of J.G. was funded by the DOE Grant
DE-FG02-95ER40896. The work of E. C. and A. V. R. was funded in part by MICINN under grant FPA2008-01838, by the Spanish Consolider-Ingenio 2010 Programme CPAN (CSD2007-00042) and by Xunta de Galicia (Consellería de Educación and grant INCITE09 206 121 PR) and by FEDER. E. C is supported by a Spanish FPU fellowship, and thanks the FRont Of Galician-speaking Scientists for unconditional support.

A Technical aspects of the SUSY backgrounds without sources

We write in this appendix various technical aspects of the supersymmetric backgrounds without sources. As explained in section 2, one can partially integrate the BPS system, and rewrite the functions $\Phi, h, g, k, a$ in terms of two new functions $P$ and $Q$. This was originally derived in [38], and the solution one finds is:

\[ Q = N_c (2\rho \coth(2\rho) - 1), \quad e^{2h} = \frac{1}{4} \frac{P^2 - Q^2}{P \coth(2\rho) - Q}, \quad e^{2g} = P \coth(2\rho) - Q, \]
\[ e^{2k} = \frac{P'}{2}, \quad a = \frac{P}{P \cosh(2\rho) - Q \sinh(2\rho)}, \quad e^{4\Phi - 4\Phi_0} = \frac{2 \sinh(2\rho)^2}{(P^2 - Q^2)P'}. \quad (A.1) \]

The function $b(\rho)$ can be integrated exactly to give

\[ b(\rho) = \frac{2\rho}{\sinh(2\rho)}. \quad (A.2) \]

$P$ is given as the solution of the master equation (2.6), that we recall here:

\[ P'' + P' \left( \frac{P'}{P - Q} + \frac{P' - Q'}{P + Q} - 4 \coth(2\rho) \right) = 0. \quad (A.3) \]

The solutions of the master equation are known only numerically, but their asymptotics can be written down analytically. In the UV (that is, large values of $\rho$), we had the expansion (2.7):

\[ P = e^{4\rho/3} \left[ c_+ + e^{-8\rho/3} N_c^2 \left( 4\rho^2 - 4\rho + \frac{13}{4} \right) + e^{-4\rho} \left( c_- - \frac{8c_+}{3}\rho \right) + \frac{N_c^4 e^{-16\rho/3}}{c_+^3} \left( \frac{18567}{512} + \frac{2781}{32} \rho + \frac{27}{4} \rho^2 + 36\rho^3 \right) + O(e^{-20\rho/3}) \right]. \quad (A.4) \]
which can be plugged back into eq. (A.1) to obtain the background functions at large $\rho$,

\[
\begin{align*}
\frac{e^{2h}}{4} &= \left(\frac{c_{+} e^{4\rho/3}}{4} + \frac{N_{c}}{2} (2\rho - 1) + \frac{N_{c}^{2} e^{-4\rho/3}}{16 c_{+}} (16\rho^{2} - 16\rho + 13) + O(e^{-8\rho/3})\right), \\
\frac{e^{2g}}{4} &= \left(\frac{c_{+} e^{4\rho/3}}{4} - \frac{N_{c}}{2} (2\rho - 1) + \frac{N_{c}^{2} e^{-4\rho/3}}{16 c_{+}} (16\rho^{2} - 16\rho + 13) + O(e^{-8\rho/3})\right), \\
\frac{e^{2k}}{4} &= \left(\frac{c_{+} e^{4\rho/3}}{6} - \frac{N_{c}^{2} e^{-4\rho/3}}{24 c_{+}} (4\rho - 5)^{2} + O(e^{-8\rho/3})\right), \\
\frac{e^{4\Phi}}{e^{4\Phi(\infty)}} &= 1 + \frac{3N_{c}^{2} e^{-8\rho/3}}{4c_{+}^{2}} (1 - 8\rho) + \frac{3N_{c}^{2} e^{-16\rho/3}}{512c_{+}^{2}} (2048\rho^{3} + 1152\rho^{2} + 2352\rho - 775) + O(e^{-8\rho}), \\
\frac{a}{c_{+}} &= 2e^{-2\rho} + \frac{2N_{c}}{2} (2\rho - 1) e^{-10\rho/3} + \frac{2N_{c}^{2}}{2} (2\rho - 1)^{2} e^{-14\rho/3} + O(e^{-9\rho}), \\
b &= \frac{2\rho}{\sinh(2\rho)} = 4\rho e^{-2\rho} + 4\rho e^{-6\rho} + O(e^{-8\rho}).
\end{align*}
\]

The geometry in eq. (2.4) asymptotes to the conifold after using the expansions above. In the IR (that is, close to the origin of the space that we take to be $\rho = 0$), we have to use eq. (2.8):

\[
P = h_{1}\rho + \frac{4h_{1}}{15} \left(1 - \frac{4N_{c}^{2}}{h_{1}^{2}}\right) \rho^{3} + \frac{16h_{1}}{525} \left(1 - \frac{4N_{c}^{2}}{3h_{1}^{2}} - \frac{32N_{c}^{4}}{3h_{1}^{4}}\right) \rho^{5} + O(\rho^{7}),
\]

and (A.1), to obtain

\[
\begin{align*}
\frac{e^{2h}}{2} &= \frac{h_{1}\rho^{2}}{2} + \frac{4}{45} \left(-6h_{1} + 15N_{c} - \frac{16N_{c}^{2}}{h_{1}}\right) \rho^{4} + O(\rho^{6}), \\
\frac{e^{2g}}{4} &= \frac{h_{1}}{8} + \frac{1}{15} \left(3h_{1} - 5N_{c} - \frac{2N_{c}^{2}}{h_{1}}\right) \rho^{2} + \frac{2}{15} \left(3h_{1}^{2} + 70h_{1}N_{c} - 144h_{1}^{2}N_{c}^{2} - 32N_{c}^{4}\right) \rho^{4} + O(\rho^{6}), \\
\frac{e^{2k}}{4} &= \frac{h_{1}}{8} + \frac{(h_{1}^{2} - 4N_{c}^{2})}{10h_{1}} \rho^{2} + \frac{(6h_{1}^{2} - 8h_{1}^{2}N_{c} - 64N_{c}^{4})}{315h_{1}^{4}} \rho^{4} + O(\rho^{6}), \\
e^{4(\Phi - \Phi_{0})} &= 1 + \frac{64N_{c}^{2} \rho^{2}}{9h_{1}^{2}} + \frac{128N_{c}^{2} (-15h_{1}^{2} + 124N_{c}^{2})}{405h_{1}^{4}} \rho^{4} + O(\rho^{6}), \\
a &= 1 + \frac{8N_{c}}{3h_{1}} \rho^{2} + \frac{2}{45} \left(75h_{1}^{3} - 232h_{1}^{2}N_{c} + 160h_{1}N_{c}^{2} + 64N_{c}^{3}\right) \rho^{4} + O(\rho^{6}), \\
b &= \frac{2\rho}{\sinh(2\rho)} = 1 - \frac{2}{3} \rho^{2} + \frac{14}{45} \rho^{4} + O(\rho^{6}).
\end{align*}
\]

This space is free of singularities as can be checked by computing curvature invariants.

**B Detailed derivations in the case of sources with profile**

Let us work out the derivation of the supergravity solutions in section 3, for the case where we add sources with a profile $S(r)$. Suppose that we start with the master equation (3.4),

\[
(P'' + N_{f} S') + (P' + N_{f} S) \left[\frac{P' + Q' + 2N_{f} S}{P - Q} + \frac{P' - Q' + 2N_{f} S}{P + Q} - 4 \coth(2\rho)\right] = 0,
\]

(B.1)
that we will rewrite as
\[
\partial_\rho \left( \frac{(P^2 - Q^2)}{\sinh^2(2\rho)} (P' + N_f S) \right) + \frac{4}{\sinh^2(2\rho)} (P' + N_f S)(QQ' + PN_f S) = 0. \tag{B.2}
\]

Then, we can take this last equation and integrate it twice, to get
\[
P^3 - 3PQ^2 + 3 \int_0^\rho d\tilde{\rho} \left( 2PQQ' + N_f S(P^2 - Q^2) \right) -
-12 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \int_\tilde{\rho}^{\infty} d\tilde{\rho} \frac{(P' + N_f S)}{\sinh^2(2\tilde{\rho})} (QQ' + PN_f S) = 4\lambda^3 \epsilon^4 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}). \tag{B.3}
\]

We will now propose a solution in an inverse series expansion in the constant \(\lambda\)
\[
P = \lambda P_1 + P_0 + \frac{P_{-1}}{\lambda} + \frac{P_{-2}}{\lambda^2} + \frac{P_{-3}}{\lambda^3} + \ldots. \tag{B.4}
\]

By equating factors of \(\lambda\) we get,
\[
P_1 = \left( 4\epsilon^4 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \right)^{1/3},
\]
\[
P_0 = -\frac{N_f}{P_1^2} \left( \int_0^\rho d\tilde{\rho} P_1^2 S - 4 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \int_\tilde{\rho}^{\infty} d\tilde{\rho} \frac{P_1 P_1' S}{\sinh^2(2\tilde{\rho})} \right),
\]
\[
P_{-1} = -\frac{1}{P_1^2} \left( P_1(P_0^2 - Q^2) + 2 \int_0^\rho d\tilde{\rho} P_1 (N_f P_0 S + QQ') -
-4 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \int_\tilde{\rho}^{\infty} d\tilde{\rho} \frac{N_f P_1 S(N_f S + P_0')} {\sinh^2(2\tilde{\rho})} \right),
\]
\[
P_{-2} = -\frac{1}{P_1} \left( \frac{P_0^3}{3} + 2P_0 P_{-1} - P_0 Q^2 + \int_0^\rho d\tilde{\rho} \left( N_f P_0^2 S + N_f S(2P_1 P_{-1} - Q^2) + 2P_0 QQ' \right) -
-4 \int_0^\rho d\tilde{\rho} \sinh^2(2\tilde{\rho}) \int_\tilde{\rho}^{\infty} d\tilde{\rho} \frac{N_f P_1 P_{-1} S + N_f P_1 P_{-1}' S + (N_f S + P_0')(N_f P_0 S + QQ')} {\sinh^2(2\tilde{\rho})} \right). \tag{B.5}
\]

**B.1 The limit \(\lambda \to \infty\)**

We describe in detail the limit of large \(\lambda\). This will lead to an analytic solution. As will see, we need to take at the same time \(N_f \to 0\) to make sense of the background. First of all, we write the explicit expansions for the functions in the limit
\[
\lambda \to \infty, \quad N_f \to 0, \quad N_f \lambda \to \frac{2^{2/3}}{\epsilon^{4/3}} \nu. \tag{B.6}
\]
They are
\[
e^{2h} = \lambda \frac{\tanh(2\rho)}{4} P_1(\rho) + \frac{\tanh(2\rho)}{4} (P_0 + Q \tanh(2\rho)) +
\left( P_1P_{-1} - \frac{Q^2}{\cosh^2(2\rho)} \right) + O(\lambda^{-2}) ,
\]
\[
e^{2g} = \frac{\lambda \coth(2\rho) P_1}{4} + \frac{1}{4} (P_0 \coth(2\rho) - Q) + \frac{P_{-1} \coth(2\rho)}{4\lambda} + O(\lambda^{-2}) ,
\]
\[
e^{2k} = \frac{P_1' \lambda}{8} + \frac{Q \tanh(2\rho)}{8} \frac{P_0' + P_1'}{\lambda^2} + P_{-1} + O(\lambda^{-2}) ,
\]
\[
a = \frac{1}{\cosh 2\rho} + \frac{Q \tanh(2\rho)}{\lambda^2} (Q \sinh(2\rho) - P_0 \cosh(2\rho)) + O(\lambda^{-3}) ,
\]
\[
e^{4\Phi - 4\Phi_0} = 2 \frac{\sinh^2(2\rho)}{\lambda^2 P_1^2 P_1'} (P_1(N_f S + P_0') + 2 P_0 P_1') +
\left( \frac{2 \sinh^2(2\rho)}{P_1^2 P_1'} (P_1' + N_f S) (2 P_0' P_1 + 2 P_0 P_1' + N_f P_1 S) - P_0' P_1^2 +
\right)\left( P_1^2 (3 P_0 P_1' + P_1' Q^2 - 2 P_0 P_1' P_{-1} - P_1^2 P_{-1}') \right) + O(\lambda^{-6}) ,
\]
\[
\hat{h} = 1 - \frac{\kappa e^{2\Phi_0}}{\lambda^3} \sqrt{\frac{2 \sinh^2(2\rho)}{P_1^2 P_1'}} + \frac{\hat{h}_f}{\lambda^{1/2}} + \frac{\hat{h}_c}{\lambda^{7/2}} + O(\lambda^{-9/2}) ,
\]
where we have defined
\[
\hat{h}_f = \frac{\kappa e^{2\Phi_0}}{2 P_1 P_1'} \sqrt{\frac{2 \sinh^2(2\rho)}{P_1^2 P_1'}} (2 P_0 P_1' + P_1(N_f S + P_0') ) ,
\]
\[
\hat{h}_c = \frac{\kappa e^{2\Phi_0}}{8 P_1^2 P_1'} \sqrt{\frac{2 \sinh^2(2\rho)}{P_1^2 P_1'}} \left[ -4P_1^2 (2 P_0^2 + Q^2) + 4 P_1 P_1' (2 P_{-1} P_1' - P_0 P_0') +
\right]\left( P_1^2 (4 P_1' P_{-1}' - 3 P_0^2) - 2 N_f P_1 S [3 P_1 P_0' + 2 P_0 P_1'] + 3 N_f^2 P_1^2 S^2 \right] .
\]
Notice that the expressions for $Q(\rho), b(\rho)$ do not change from those in eqs. (3.3) and (3.7) respectively. Now, we will consider the limit in eq. (B.6). We choose the value of the constant $\kappa$ as
\[
3\kappa^4 = 2 e^4 \lambda^3 e^{-4\Phi_0} .
\]
Again the logic for this choice is to get a warp factor that vanishes at large $\rho$. In order to have a well-defined metric (where powers of $\lambda$ are absent) when taking the limit $\lambda \to \infty$, we also need to take $N_f \to 0$ as in eq. (B.6). Doing this has the following effect on the functions defining the full background:
\[
e^{2h} = \lambda \frac{\tanh(2\rho)}{4} P_1(\rho) + O(\lambda^0) ,
\]
\[
e^{2g} = \frac{\lambda \coth(2\rho) P_1}{4} + O(\lambda^0) ,
\]
\[
e^{2k} = \frac{P_1' \lambda}{8} + O(\lambda^0) ,
\]
\[
a = \frac{1}{\cosh 2\rho} + O(\lambda^{-1}) ,
\]
\[
e^{4\Phi} = \frac{3 e^{4\Phi_0}}{2 e^4 \lambda^3} + O(\lambda^{-4}) ,
\]
\[
Q = N_c(2\rho \coth(2\rho) - 1) + O(\lambda^{-1}) ,
\]
\[
b = \frac{2\rho}{\sinh(2\rho)} + O(\lambda^{-1}) ,
\]
\[\]
and on the function $\hat{h}$ this has two interesting effects. On the one hand taking $N_f \to 0$ considerably simplifies the expression for $\hat{h}_c, \hat{h}_f$. On the other hand the scaling $N_f \lambda = 2^{2/3} \epsilon^{-4/3} \nu$ fixed makes the term $\lambda^{-5/2} \hat{h}_f$ scale like $\lambda^{-7/2} \hat{h}_c$ (that is as $\lambda^{-2}$). Notice also that in the limit of eq. (B.6), $\lambda^{-5/2} \hat{h}_f$ becomes

$$
\lambda^{-5/2} \hat{h}_f = \lambda^{-2} \frac{4 \epsilon^{8/3} \nu}{\lambda^2} \int_\rho^\infty dx \frac{S(x)}{(\sinh(4x) - 4x)^{1/3}}.
$$

(B.11)

Regarding the integrals defining $\hat{h}_c$, we see that in the limit of eq. (B.6), we have

$$
\frac{\hat{h}_c}{\lambda^{7/2}} = \frac{2^{5/3} N_c^2}{\lambda^2 4 \epsilon^{8/3} \nu} \int_\rho^\infty dx \frac{(\sinh(4x) - 4x)^{1/3}(2x \coth(2x) - 1)}{\sinh^2(2x)} + \mathcal{O}(\lambda^{-3}) = \frac{\hat{h}_{KS}}{\lambda^2} + \mathcal{O}(\lambda^{-3}).
$$

(B.12)

Notice that we dropped terms that are suppressed like $\frac{\epsilon^2}{\lambda^2}$ in the previous expression. Finally, rescaling the Minkowski coordinates $x_1 \to x_i \lambda^{-1/2}$ we have a metric that is independent of the parameter $\lambda$. Using that

$$
P'_1 = \frac{8 \sinh^2(2\rho)}{3(\sinh(4\rho) - 4\rho)},
$$

(B.13)

the internal space metric is the deformed conifold. In this way, we have generated a new analytic family of solutions that depend on the function $S(\rho)$. For correct choices of $S(\rho)$, like for example $S(\rho) \sim (\tanh(2\rho))^{2n}$ as in [48] or those in eq. (3.16), this family of solutions is non-singular. Interestingly, it is also true when we do not take the limit described in eq. (B.6). In that case, our series expansion for $P(\rho)$ and $\hat{h}$ are not truncated. If we take the strict $N_f = 0$ case, this solution is the baryonic branch of [34]. The parameter $\lambda$ is the one moving between different VEV’s for the baryon and anti-baryon operators (a parameter called $U$ in [11]). For nonzero values of $N_f$, an interpretation of the parameter $\lambda$ was given in [29].

### B.2 The exact and analytic solution

We describe here the solution obtained in the limit $\lambda \to \infty$. The warp factor is

$$
\hat{h} = \frac{1}{\lambda^2} \left( \frac{4 \epsilon^{8/3} \nu}{\sinh(4x) - 4x} \right)^{1/3} + \hat{h}_{KS}.
$$

(B.14)
After rescaling \( x_i \to x_i \lambda^{-1} \) and choosing \( \Phi(\infty) = 0 \), the full configuration will read

\[
ds^2 = \hat{h}^{-1/2} dx_{1,3}^2 + \hat{h}^{1/2} ds_6^2,
\]

\[
ds_6^2 = P_1 \left( \frac{P'}{2 P_1} (d \rho^2 + \frac{1}{4}(\tilde{\omega}_3 + \cos \theta d \varphi)^2) + \frac{\tanh(2 \rho)}{4} (d \theta^2 + \sin^2 \theta d \varphi^2) + 
\]

\[+ \coth(2 \rho) \left( (\tilde{\omega}_1 + a d \theta)^2 + (\tilde{\omega}_1 - a \sin \theta d \varphi)^2 \right) \right),
\]

\[
B_2 = - \frac{N_c}{4} \frac{2 \rho \coth(2 \rho) - 1}{\sinh(2 \rho)} \left[ \cosh(2 \rho) (\sin \theta d \theta \wedge d \varphi - \sin \tilde{\theta} d \tilde{\theta} \wedge d \tilde{\varphi}) - \sin \theta d \varphi \wedge \tilde{\omega}_1 - d \theta \wedge \tilde{\omega}_2 \right],
\]

\[
H_3 = dB_2, \quad F_3 = \ast_6 H_3,
\]

\[
F_5 = -(1 + *) \partial_\rho H^{-1} dt \wedge dx_1 \wedge dx_2 \wedge dx_3 \wedge d \rho,
\]

\[
H = \hat{h}_{KS} + \frac{4}{\epsilon^{2/3} \nu} \int_{\rho}^{\infty} dx \frac{S(x)}{(\sinh(4x) - 4x)^{1/3}}.
\]

(B.15)

Here, taking the limit for \( B_2 \) is non-trivial. Indeed, when expanding it in terms of inverse powers of \( \lambda \), one gets

\[
B_2 = \lambda \frac{\epsilon^2 \sinh(2 \rho)}{2 \sqrt{3} \kappa P_1 \sqrt{F_1}} d \left( P_1 (\tilde{\omega}_3 + \cos \theta d \varphi) \right) - B_{KS} + O \left( \lambda^{-1} \right).
\]

(B.16)

Using eq.(2.17), one notices that the overall factor in front of the total derivative in the previous expression is a constant. So the leading order term in \( B_2 \) can be gauged away because it is exact. So one is left with \( B_2 = -B_{KS} + O \left( \lambda^{-1} \right) \), which has a well-defined limit when \( \lambda \to \infty \). Note that the sign difference between our result and the one of Klebanov-Strassler comes from a different choice of orientation.

C Equations of motion

In this appendix we state the equations of motion of the problem studied in this paper, that is type IIB supergravity with brane sources. The interested reader will find more details about the action from which those equations are derived in [29]. Let us first start by writing the Bianchi identities, which are modified due to the presence of sources. From equations (2.15) and (3.2), one can see that the three-form flux \( F_3 \) is not closed anymore:

\[
dF_3 = \frac{N_f}{4} \sin \theta d \theta \wedge d \phi \wedge \left[ S \tilde{\omega}^1 \wedge \tilde{\omega}^2 - S' d \rho \wedge \tilde{\omega}^3 \right] + 
\]

\[+ \frac{N_f}{8} \frac{S'}{\cosh(2 \rho)} d \rho \wedge \left[ d \theta \wedge \tilde{\omega}^2 \wedge \tilde{\omega}^3 + d \phi \wedge \left( \sin \theta \tilde{\omega}^1 \wedge \tilde{\omega}^3 + \cos \theta d \theta \wedge \tilde{\omega}^2 \right) \right].
\]

(C.1)

We then define a four-form \( \Xi_4 \) as

\[
\Xi_4 = dF_3.
\]

(C.2)

That means that we get the following Bianchi identities for the other fluxes:

\[
dH_3 = 0, \quad dF_5 = H_3 \wedge F_3 + B_2 \wedge \Xi_4.
\]

(C.3)
The equations of motion for the fluxes read
\[
d (e^{-\Phi} \ast H_3) = F_3 \wedge F_5 + e^{\Phi} \frac{\sqrt{1 - \frac{h}{\hat{h}}}}{\hat{h}} \text{Vol}_4 \wedge \Xi_4, \\
d (e^{\Phi} \ast F_3) = -H_3 \wedge F_5,
\]
where \( \text{Vol}_4 = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \). We then define the following notation
\[
\omega_{(p)} \wedge \lambda_{(p)} = \frac{1}{p!} \omega^\mu_1 \cdots \mu_p \lambda_{\mu_1 \cdots \mu_p}.
\]
We also have that
\[
\int \omega_{(p)} \wedge \lambda_{(10-p)} = - \int \sqrt{-g} \lambda_{\ast} \omega.
\]
Using these, we can write the dilaton equation of motion as
\[
\frac{1}{\sqrt{-g}} \partial_\mu \left( \sqrt{-g} g^\mu_\nu \partial_\nu \Phi \right) = \frac{1}{12} e^{\Phi} F_3^2 - \frac{1}{12} e^{-\Phi} H_3^2 - \frac{1}{2} e^{\Phi/2} \Xi_{4\perp} \left( \frac{e^{3\Phi/2}}{\hat{h}} \text{Vol}_4 \wedge J \right),
\]
where \( J = \hat{h}^{1/2} e^{-\Phi/2} \left[ e^{\theta^3} - \cos \alpha(e^{\theta^\phi} + e^{12}) - \sin \alpha(e^{\theta^2} + e^{e1}) \right] \), using the conventions of equations (2.14)–(2.16). Finally, the Einstein equation is
\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{2} \partial_\nu \Phi \partial_\mu \Phi - \frac{1}{4} g_{\mu\nu} \partial_\rho \Phi \partial_\sigma \Phi + \frac{1}{24} e^{\Phi} (6 F_{\mu\tau\sigma} F_{\nu}^{\tau\sigma} - g_{\mu\nu} F_3^2) + \frac{1}{24} e^{-\Phi} (6 H_{\mu\tau\sigma} H_{\nu}^{\tau\sigma} - g_{\mu\nu} H_3^2) + \frac{1}{96} F_{\mu\tau_1 \tau_2 \tau_3 \tau_4} F_\nu^{\tau_1 \tau_2 \tau_3 \tau_4} + T^\text{sources}_{\mu\nu},
\]
where \( T^\text{sources}_{\mu\nu} \) is the energy-momentum tensor coming from the source action. It is given by:
\[
T^\text{sources}_{\mu\nu} = - \frac{1}{12} \frac{e^{2\Phi}}{\hat{h}} \left( \Xi_{\mu\tau_1 \tau_2 \tau_3} \ast (\text{Vol}_4 \wedge J)_{\nu}^{\tau_1 \tau_2 \tau_3} - 6 g_{\mu\nu} \Xi_{4\perp} \ast \text{Vol}_4 \wedge J \right) - \frac{1}{240} e^{\Phi} \frac{\sqrt{1 - \frac{h}{\hat{h}}}}{\hat{h}} \left( (B_2 \wedge \Xi_4)_{\mu_1 \cdots \mu_5} \ast (\text{Vol}_4)_{\nu}^{\tau_1 \cdots \tau_5} - 120 g_{\mu\nu} (B_2 \wedge \Xi_4)_{\ast \text{Vol}_4} \right).
\]
From (C.9) and (C.10), one can get an equation for the Ricci tensor as:
\[
R_{\mu\nu} = \frac{1}{2} \partial_\mu \Phi \partial_\nu \Phi + \frac{1}{48} e^{\Phi} (12 F_{\mu\tau\sigma} F_\nu^{\tau\sigma} - g_{\mu\nu} F_3^2) + \frac{1}{48} e^{-\Phi} (12 H_{\mu\tau\sigma} H_\nu^{\tau\sigma} - g_{\mu\nu} H_3^2) + \frac{1}{96} F_{\mu\tau_1 \tau_2 \tau_3 \tau_4} F_\nu^{\tau_1 \tau_2 \tau_3 \tau_4} - \frac{1}{24} \frac{e^{2\Phi}}{\hat{h}} \left( 2 \Xi_{\mu\tau_1 \tau_2 \tau_3} \ast (\text{Vol}_4 \wedge J)_{\nu}^{\tau_1 \tau_2 \tau_3} - 3 g_{\mu\nu} \Xi_{4\perp} \ast \text{Vol}_4 \wedge J \right) - \frac{1}{240} e^{\Phi} \frac{\sqrt{1 - \frac{h}{\hat{h}}}}{\hat{h}} \left( (B_2 \wedge \Xi_4)_{\mu_1 \cdots \mu_5} \ast (\text{Vol}_4)_{\nu}^{\tau_1 \cdots \tau_5} - 60 g_{\mu\nu} (B_2 \wedge \Xi_4)_{\ast \text{Vol}_4} \right).
\]
Figure 7. Plots of $T_{x^0 x^0}^{\text{sources}}$ (for $N_c = 2 = N_f$ and $h_1 = 5$) for different profiles. The dot-dashed orange curve corresponds to a profile $S$ decreasing like $e^{-(4-\epsilon)\rho/3}$. It grows exponentially in the UV. The red curve is for the profile in eq. (3.16), and it behaves nicely. And the dashed blue curve corresponds to a profile $S$ decreasing like $e^{-(4+\epsilon)\rho/3}$. It becomes negative for large values of $\rho$. We take $\epsilon = 1$ to appreciate well the qualitative difference, but the results hold for any $\epsilon > 0$.

To clarify the contribution of the sources to the problem, let us express in more details the various components of the energy-momentum tensor due to the sources. In flat indices, the energy-momentum tensor of the sources gives:

$$T_{x^i x^j}^{\text{sources}} = -\frac{N_f}{2 \hat{h}^{3/2}} e^{-2g-2h-2k-\Phi/2} \left( e^{2k} S + \frac{4 e^{2h} + e^{2g} (a - \cosh(2\rho))^2}{\sinh(4\rho)} S' \right) \eta_{ij},$$

$$T_{\rho\rho}^{\text{sources}} = -\frac{N_f}{2 \hat{h}^{1/2}} e^{-2g-2h-\Phi/2} S = T^{\text{sources}}_{33},$$

$$T_{\theta\theta}^{\text{sources}} = -\frac{N_f}{\hat{h}^{1/2} \sinh(4\rho)} e^{-2g-2k-\Phi/2} S' = T^{\text{sources}}_{\phi\phi},$$

$$T_{11}^{\text{sources}} = -\frac{N_f}{2 \hat{h}^{1/2}} e^{-2g-2h-2k-\Phi/2} \frac{2 e^{2h} + e^{2g} (a - \cosh(2\rho))^2}{\sinh(4\rho)} S' = T^{\text{sources}}_{22},$$

$$T_{\theta 1}^{\text{sources}} = -\frac{N_f}{2 \hat{h}^{1/2}} e^{-g-2k-\Phi/2} \frac{a - \cosh(2\rho)}{\sinh(4\rho)} S' = T^{\text{sources}}_{\phi 2}.$$

where $\eta_{ij}$ is the flat Minkowski metric. Notice that this energy-momentum tensor is the same one (up to global factors of $\hat{h}$) as the one in the unrotated background [44]: the rotation does not change it.

One expects that the $\rho\rho$-component of the energy-momentum tensor, $T_{x^0 x^0}^{\text{sources}}$, represents the mass density of sources in our ‘static’ background. As such, it should be a positive-definite quantity (even if we perform a Lorentz transformation, its sign should be preserved). If we look at its expression from eq. (C.12), we have that

$$T_{x^0 x^0}^{\text{sources}} = \frac{N_f}{2 \hat{h}^{3/2}} e^{-2g-2h-2k-\Phi/2} \left( e^{2k} S + \frac{4 e^{2h} + e^{2g} (a - \cosh(2\rho))^2}{\sinh(4\rho)} S' \right).$$

(C.13)
It is trivial that this positivity condition will hold for profiles like the one in figure 1, as both $S$ and $S'$ are positive. However, for profiles like eq.(3.16) that we are studying in this paper, this condition is not trivially satisfied anymore.

We have seen (see figure 7) that $S$ should not decrease faster than $e^{-4\rho/3}$ in order for $T_{\text{sources}}^2$ to be positive everywhere. Moreover, we also know that $S$ should decrease at least as fast as $e^{-4\rho/3}$ in order to preserve the KS UV asymptotics. This argument seems to imply that the only physical choice is actually the one we have made in eq.(3.16). Notice that this choice is the only one that naturally gives a constant density of D3 sources in the rotated background, in virtue of eq.(3.11).

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