Multi-Localization in Multi-Brane Worlds

Ian I. Kogan\(^1\), Stavros Mouslopoulos\(^2\), Antonios Papazoglou\(^3\)
and Graham G. Ross\(^4\)

*Theoretical Physics, Department of Physics, Oxford University*

1 Keble Road, Oxford, OX1 3NP, UK

Abstract

We study bulk fields in various multi-brane models with localized gravity. In the case of spin 0, \(\frac{1}{2}\) and \(\frac{3}{2}\) fields, the non-trivial background geometry itself can induce (multi-)localization of the zero modes along the extra dimension. The addition of appropriate mass terms controls the strength or/and the position of this localization and can induce (multi-)localization of spin 1 fields as well. We show that multi-brane models may also give rise to anomalously light KK modes which are also localized. The possibility of multi-localization in the context of supersymmetric brane world models in \(AdS_5\) spacetime is also discussed.
1 Introduction

Brane universe models [1–4] in more than four dimensions have been extensively studied over the last three years because they provide novel ideas for resolutions of long standing problems of particle physics such as the Planck hierarchy [3, 4]. Moreover, mechanisms to localize gravity on a brane [8, 9] have led to the realization that if extra dimensions exist, they need not be compact [10]. In the simplest formulation of these models no bulk matter states are assumed to exist and thus only gravity propagates in the extra dimensions. Nevertheless “bulk” (i.e., transverse to 3-brane space dimensions) physics turns out to be very interesting giving alternative explanations to other puzzles of particle physics. For example, by assuming the existence of a Standard Model (SM) neutral spin $\frac{1}{2}$ fermion in the bulk one can explain the smallness of the neutrino masses without invoking the seesaw mechanism [11–20]. However, it is not necessary to confine the SM fields to the brane. Assuming that the SM fields can propagate in the bulk interesting new possibilities arise. For example one can attempt to explain the pattern of the SM fermion mass hierarchy by localizing the SM fermions at different places in the bulk [21–24]. These considerations give the motivation for considering the phenomenology associated with spin 0, $\frac{1}{2}$ and 1 fields propagating in extra dimension(s). Since our discussions will be limited to models with localized gravity and in particular to Randall-Sundrum (RS) type constructions, we will be interested in the phenomenology of fields that live in a slice of $AdS_5$ spacetime. If one also wants to explore the supersymmetric version of the above models, it is also necessary to study the phenomenology of spin $\frac{3}{2}$ field on the same background geometry.

It has been shown that in the context of RS type models the graviton is localized on positive tension branes and suppressed on the negative ones [3]. In Ref. [25] it was also shown that the $AdS_5$ background geometry of these models can localize the zero mode of a massless scalar field on positive tension branes (see also [26]). Moreover, the same background localizes the spin $\frac{1}{2}$ fermions on negative tension branes [13, 20, 27–34]. The same localization behaviour holds for spin $\frac{3}{2}$ fermions [27, 31, 33]. However, the $AdS_5$ background geometry cannot localize massless Abelian gauge fields [36, 38] if one tries to get them from a higher dimensional vector field. One can, however, get localized massless Abelian fields from higher dimensional antisymmetric forms as it was done in [39].

The addition of mass terms modifies the localization properties. For example, it was shown in Ref. [2, 13, 20, 32, 40] that the addition of an appropriate mass term in the action
of a spin $\frac{1}{2}$ field can result in the localization of the zero mode that resembles that of the graviton (the magnitude of the mass term in this case controls the extent of localization). In the present paper we show that the same can occur in the case of spin 0, $\frac{3}{2}$ fields with appropriate mass terms. Furthermore we show that by adding a mass term of a particular form in the action of a massless Abelian gauge field we can achieve the desired localization of the massless zero mode which can be made to resemble that of the graviton. The mass term in this case must necessarily consist of a five dimensional bulk mass part and a boundary part.

From the above it is clear that particles of all spins, with appropriate bulk mass terms, can exhibit zero mode localization on positive tension branes, just as for the graviton. In the context of multi-brane models with localized gravity the above implies a further interesting possibility: the phenomenon of multi-localization in models that contain at least two positive tension branes. Multi-localization, as we will see, is closely related to the appearance of light, localized strongly coupled Kaluza-Klein (KK) states (their coupling to matter can be even larger than the coupling of the zero mode). Thus the mass spectrum of multi-localized fields is distinct from the mass spectrum of singly localized fields, resulting in the possibility of new phenomenological signals. Anomalously light states may also arise in theories without multi-localization (i.e. even in configurations with one positive brane) in models with twisted boundary conditions.

The appearance of light KK states can be of particular phenomenological interest. For example in the case of the graviton, multi-localization and thus the appearance of light KK graviton excitations, gives rise to the exciting possibility of Bi-gravity [41-44] (Multi-gravity [44-47]) where part (or even all) of gravitational interactions can come from massive spin 2 particle(s) (KK state(s)) [41, 52]. In this case the large mass gap between the anomalously light KK state(s) and the rest of the tower is critical in order to avoid modifications of Newton’s law at intermediate distances.

Anomalously light spin $\frac{1}{2}$ KK states can also arise when a bulk fermion is multi-localized [20]. The non-trivial structure of the KK spectrum in this case, with the characteristic mass gap between the light state(s) and the rest of the tower, can be used for example to construct models with a small number of active or sterile neutrinos involved in the oscillation (the rest will decouple since they will be heavy).

The organization of the paper is as follows: In the next Section we review the general framework and discuss the general idea of multi-localization in the context of models with localized gravity. In Section 3 we study the multi-localization properties of a bulk scalar
field. In Section 4 we review the situation of a bulk fermion field. In Section 5 we study in detail the possibilities of localization and multi-localization of an Abelian gauge field. In Section 6 present the possibility of multi-localization in the case of a gravitino and finally in Section 7 we review the same phenomena for the case of graviton [11, 12]. In section 8 we discuss how multi-localization is realized in the context of supersymmetric versions of the previous models. The overall implications and conclusions are presented in section 9.

2 General Framework - The idea of Multi-Localization

The original formulation of multi-localization of gravity was obtained in five dimensions for the case that there is more than one positive tension brane. If the warp factor has a “bounce”, in the sense that it has a minimum (or minima) between the positive tension branes, the massless graviton appears as a bound state of the attractive potentials associated with the positive tension branes with its wave function peaked around them. Moreover in this case there are graviton excitations corresponding to additional bound states with wave functions also peaked around the positive tension branes. They are anomalously light compared to the usual Kaluza Klein tower of graviton excitations. The reason for this is that the magnitude of their wavefunction closely approximates that of the massless mode, differing significantly only near the position of the bounce where the wave function is exponentially small. The mass they obtain comes from this region and as a result is exponentially suppressed relative to the usual KK excitations.

The first models of this type involved negative tension branes sandwiched between the positive tension branes [11]. That this is necessary in the case of flat branes with vanishing cosmological constant in four dimensions is easy to see because, for a single flat brane, the minimum of the warp factor is at infinity and thus any construction with another positive brane at a finite distance will have a discontinuity in the derivative of the warp factor at the point of matching of the solutions and thus at that point a negative tension brane will emerge.

Subsequently it was pointed out that a free negative tension brane(s) violate the weaker energy condition and lead to a ghost radion field(s). A way out of this problem was suggested by some of us through the use of AdS$_4$ branes [13] (see [52] for a different possibility involving an external four-form field). In this case the minimum of a single brane is at finite distance and thus one can match the solution for two positive branes without introducing a negative tension brane. However a drawback of this approach is that the four dimensional cosmological constant is negative in conflict with the current indications for a positive
Recently we have shown how it is possible to obtain the “bounce” and the related multilocalisation for zero cosmological constant in four dimensions without the need for negative tension branes \[14\]. We have shown this is possible even in cylindrically symmetric models if one allows for non-homogeneous brane tensions and/or bulk cosmological constant. In this case the structure of the effective four dimensional theory is very similar to the five dimensional case for the modes which do not depend on the new angular co-ordinate. In particular one finds a massless graviton and anomalously light massive modes with wavefunctions peaked around the positive tension branes.

In this paper we are interested in whether spin 0, \(\frac{1}{2}\), 1, and \(\frac{3}{2}\) fields can similarly show the phenomena of multi-localisation we found for the graviton in suitable curved backgrounds. We will show that the curved background can also induce localisation for the case of spin 0, \(\frac{1}{2}\) and \(\frac{3}{2}\) fields but not for a massless vector field. However, even in flat spacetime in higher dimensions, it is also possible to induce localisation by introducing mass of a very specific form for these fields. We will show that this effect can localise all fields with spin \(\leq \frac{3}{2}\), including the graviton.

For simplicity we will work mainly with the five dimensional compactification. In the context of the discussion of the multilocalisation of fields of spin \(\leq \frac{3}{2}\), what is important is the nature of the curved background as determined by the warp factor. Thus the general features are applicable to all models of multilocalisation with the same warp factor profile. However the interpretation of the mass terms needed to achieve multilocalisation differs. In the case of models with negative tension branes the masses correspond to a combination of a constant bulk mass together with a brane term corresponding to the coupling to boundary sources. In the case of models without negative tension branes the mass must have a non-trivial profile in the bulk. In certain cases this profile may be guaranteed by supersymmetry.

In the five dimensional models considered here, the fifth dimension \(y\) is compactified on an orbifold, \(S^1/Z_2\) of radius \(R\), with \(-L \leq y \leq L\). The five dimensional spacetime is a slice of \(AdS_5\) which is described by

\[
d s^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2
\]  

where the warp factor \(\sigma(y)\) depends on the details of the model considered.

\[5\]We will assume that the background metric is not modified by the presence of the bulk fields, that is, we will neglect the back-reaction on the metric from their presence.
Since we are interested in the phenomenology of fields propagating in the above slice of $AdS_5$ our goal is to determine the mass spectrum and their coupling to matter. Starting from a five dimensional Lagrangian, in order to give a four dimensional interpretation to the five dimensional fields, one has to implement the dimensional reduction. This procedure includes the representation of the five dimensional fields $\Phi(x, y)$ in terms of the KK tower of states:

$$\Phi(x, y) = \sum_{n=0}^{\infty} \Phi^{(n)}(x) f^{(n)}(y)$$  \hspace{1cm} (2)

where $f^{(n)}(y)$ is a complete orthonormal basis spanning the compact dimension. The idea behind this KK decomposition is to find an equivalent 4D description of the five dimensional physics associated with the field of interest, through an infinite number of KK states with mass spectrum and couplings that encode all the information about the five dimensions. The function $f^{(n)}(y)$ describes the localization of the wavefunction of the $n$-th KK mode in the extra dimension. It can be shown that $f^{(n)}(y)$ obeys a second order differential equation which, after a convenient change of variables and/or a redefinition of the wavefunction, reduces to an ordinary Schrödinger equation:

$$\left\{ -\frac{1}{2} \partial^2_z + V(z) \right\} \hat{f}^{(n)}(z) = \frac{m_n^2}{2} \hat{f}^{(n)}(z)$$  \hspace{1cm} (3)

The mass spectrum and the wavefunctions (and thus the couplings) are determined by solving the above differential equation. Obviously all the information about the five dimensional physics is contained in the form of the potential $V(z)$. For example in the case of the graviton the positive tension branes correspond to attractive $\delta$-function potential wells whereas negative tension branes to $\delta$-function barriers. The form of the potential between the branes is determined by the $AdS_5$ background.

### 2.1 Multi-Localization and light KK states

Multi-localization emerges when one considers configuration of branes such that the corresponding potential $V(z)$ has at least two ($\delta$-function) potential wells, each of which can support a bound state (see Fig.(1) for the "$++-$" case). If we consider the above potential

---

6. The form of the redefinitions depend on the spin of the field.
7. In the infinitely thin brane limit that we consider, the wells associated with positive branes are $\delta$-functions.
The scenario of multi-localization is realized in configurations where the corresponding form of potential has potential wells that can support bound states. Such a potential is the one that corresponds to the $''+--''$ model. Positive branes are $\delta$-function wells and negative are $\delta$-function barriers.

Wells separated by an infinite distance, then the zero modes are degenerate \emph{i.e.} and massless. However, if the distance between them is finite, due to quantum mechanical tunneling the degeneracy is removed and an exponentially small mass splitting appears between the states. The rest of levels, which are not bound states, exhibit the usual KK spectrum with mass difference exponentially larger than the one of the “bound states” (see Fig.(2)). The above becomes clearer if one examines the form of the wavefunctions. In the finite distance configuration the wavefunction of the zero mode is the symmetric combination ($\hat{f}_0 = \frac{\hat{f}_1 + \hat{f}_2}{\sqrt{2}}$) of the wavefunctions of the zero modes of the two wells whereas the wavefunction of the first KK state is the antisymmetric combination ($\hat{f}_0 = \frac{\hat{f}_1 - \hat{f}_2}{\sqrt{2}}$). Such an example is shown in Fig.(3) where are shown the wavefunctions of the graviton in the context of $''+--''$ model with two positive tension branes at the fixed point boundaries and a negative tension brane at the mid-point. From it we see that the absolute value of these wavefunctions are nearly equal throughout the extra dimension, with exception of the central region where the antisymmetric wavefunction passes through zero, while the symmetric wavefunction has suppressed but non-zero value. The fact that the wavefunctions are exponentially small in this central region results in the exponentially small mass difference between these states.

The phenomenon of multi-localization is of particular interest since, starting from a problem with only one mass scale (the inverse radius of compactification), we are able to create a second scale exponentially smaller. Obviously the generation of this hierarchy is
due to the tunneling effects in our “quantum mechanical” problem.

2.2 Locality - light KK states and separability

Summarizing, if in a single brane configuration (with infinite extra dimension) the zero mode of a field is localized on the brane, then in a multi-brane world scenario it will be multi-localized and as a consequence light KK states will appear in its spectrum. The latter is assured from the following locality argument: In the infinite separation limit of the multi-brane configuration physics on each brane should depend on local quantities and not on physics at infinity. The latter assures the smoothness of the limit of infinite separation of branes in the sense that at the end of the process the configuration will consists of identical and independent single brane configurations. As a result the above locality argument also ensures the appearance of light states which at the above limit will become the zero modes of the one brane configurations.

3 Multi-Localization of spin 0 field

3.1 " + − + " Model

Let us now explore whether multi-localization of spin 0 fields can be realized in the context of the RS type of models. We start our discussion from the simplest case of a real scalar
Figure 3: The wavefunctions of the three first modes in the "$ + - + $" model. The zero mode (solid line), first (dashed line) and second (dotted line) KK states. Note that the absolute value of the wavefunctions of the zero mode and the first KK state almost coincide except for the central region of the configuration where they are both suppressed.

Figure 4: The effect of stretching the configuration by moving the two wells further apart. Note that the wavefunction of the zero mode and the first KK state remain localized but the remaining of the modes, not being bound states, will stretch along the extra dimension.

Figure 5: The case when the distance between the branes become infinite. The zero mode (which still exists if the compactification volume is finite) and the first KK state become degenerate. The wavefunction of the second KK state spreads along the extra dimension.
field propagating in a five dimensional curved background described by the metric of eq.(1) where the function $\sigma(y)$ is the one that corresponds to the "$+ - + -$" configuration. The action for a massive bulk scalar field in this case is:

$$S = \frac{1}{2} \int d^4x \int dy \sqrt{G} \left( G^{AB} \partial_A \Phi \partial_B \Phi + m_\Phi^2 \Phi^2 \right)$$

where $G = \det(G_{AB}) = e^{-8\sigma(y)}$. Under the $Z_2$ symmetry $m_\Phi^2$ should be even. We take $m_\Phi^2$ of the form:

$$m_\Phi^2 = C + \Sigma_i D_i \delta(y - y_i)$$

where $D_i = \pm 1$ for a positive or negative brane respectively. The first term corresponds to a constant five dimensional bulk mass and the second to the coupling of the scalar field to boundary sources. The mass can be rewritten in the form

$$m_\Phi^2 = \alpha (\sigma'(y))^2 + \beta \sigma''(y)$$

Taking into account the form of the vacuum, the above action can be written as

$$S = \frac{1}{2} \int d^4x \int dy \left( e^{-2\sigma(y)} \eta^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \Phi \partial_5 (e^{-4\sigma(y)} \partial_5 \Phi) + m_\Phi^2 e^{-4\sigma(y)} \Phi^2 \right)$$

In order to give a four dimensional interpretation to this action we go through the dimensional reduction procedure. Thus we decompose the five dimensional field into KK modes

$$\Phi(x, y) = \sum_n \phi_n(x) f_n(y)$$

Using this decomposition, the above action can be brought in the form

$$S = \frac{1}{2} \sum_n \int d^4x \{ \eta^{\mu\nu} \partial_\mu \phi_n(x) \partial_\nu \phi_n(x) + m_n^2 \phi_n^2(x) \}$$

provided the KK wavefunctions obey the following second order differential equation

$$- \frac{d}{dy} \left( e^{-4\sigma(y)} \frac{df_n(y)}{dy} \right) + m_n^2 e^{-4\sigma(y)} f_n(y) = m_n^2 e^{-2\sigma(y)} f_n(y)$$

with the following orthogonality relations (taking into account the $Z_2$ symmetry):

$$\int_{-L}^{L} dy e^{-2\sigma(y)} f_m^*(y) f_n(y) = \delta_{mn}$$

Here we have assumed that the magnitude of the boundary mass term contribution is the same for all branes. This is needed in order to have a zero mode.
where we assume that the length of the orbifold is $2L$.

The linear second order differential equation can always be brought to a Schrödinger form by a redefinition of the wavefunction and by a convenient coordinate transformation from $y$ to $z$ coordinates related by: $\frac{dz}{dy} = e^{\sigma(y)}$. The coordinate transformation is chosen to eliminate the terms involving first derivatives. Thus we end up with the differential equation of the form:

$$\left\{-\frac{1}{2} \frac{d^2}{dz^2} + V(z)\right\} \hat{f}_n(z) = \frac{m_n^2}{2} \hat{f}_n(z)$$

where the potential is given by

$$V(z) = \frac{\frac{15}{4} (\sigma'(y))^2 + m_\phi^2}{2[g(z)]^2} - \frac{\frac{3}{4}}{2[g(z)]^2} \sigma''(y)$$

where $g(z) \equiv e^{\sigma(y)}$ and we have made a redefinition of the wavefunction:

$$\hat{f}_n(z) = e^{-\frac{3}{4}\sigma(y)} f_n(y)$$

Note that for $m_\phi = 0$ the above Schrödinger equation is identical to that of the graviton. This implies that the mass spectrum of a massless scalar field of even parity is identical to the graviton’s and thus supports an ultralight KK state(s). Addition of a bulk mass term ($\beta = 0$), results in the disappearance of the zero mode from the spectrum (the ultralight state also is lost). Nevertheless, by considering a mass term of the more general form (with $\alpha \neq 0$ and $\beta \neq 0$), which has the characteristic that it changes both terms of the potential of eq.(15), we can not only recover the zero mode but in addition have ultralight KK state(s). In this case the corresponding potential will be

$$V(z) = \frac{\left(\frac{15}{4} + \alpha\right)(\sigma'(y))^2}{2[g(z)]^2} - \frac{\left(\frac{3}{4} - \beta\right)}{2[g(z)]^2} \sigma''(y)$$

This is of the general form given in Appendix A. A massless mode exists if $\alpha = \beta^2 - 4\beta$ in which case the wavefunction is $\hat{f}(z) \propto e^{(\beta-3/2)\sigma(y)}$. From equation (11) we see that $f(y)e^{-\sigma(y)} \propto e^{(\beta-1)\sigma(y)}$ is the appropriately normalised wavefunction in the interval $[-L, L]$. This is localised on the positive tension brane for $\beta > 1$ and on the negative tension brane for $\beta < 1$. When the condition for the zero mode is satisfied we find the mass of the first ultralight KK state to be given by

$$m_1 \approx \sqrt{4\nu^2 - 1} \, kw \, e^{-(\nu + \frac{1}{4})x}$$
where $\nu = \frac{3}{2} - \beta$ and for the rest of the KK tower

$$m_{n+1} \approx \xi_n \ k w \ e^{-x} \quad n = 1, 2, 3, \ldots \quad (17)$$

where $\xi_{2i+1}$ is the $(i+1)$-th root of $J_{\nu - \frac{3}{2}}(x)$ $(i = 0, 1, 2, \ldots)$ and $\xi_{2i}$ is the $i$-th root of $J_{\nu + \frac{1}{2}}(x)$ $(i = 1, 2, 3, \ldots)$. The above approximations become better away from the $\nu = \frac{1}{2}, x = 0$ and for higher KK levels, $n$. The first mass is singled out from the rest of the KK tower as it has an extra exponential suppression that depends on the mass of the bulk fermion. In contrast, the rest of the KK tower has only a very small dependence on the mass of the bulk fermion through the root of the Bessel function $\xi_n = \xi_n(\nu)$ which turns out to be just a linear dependence on $\nu$.

### 3.2 $''++'''$ model

We now consider a model which exhibits Bi-gravity but does not require negative tension branes \[43\]. It is built using two positive tension branes and leads to $AdS$ in four dimensions. A discussion of this model appears in Appendix B.

The discussion of the localisation of spin 0 fields in the $''++'''$ case follows similar lines to that of the $''+-+''$ case. In the absence of any mass term for the scalar the potential has the form given in eq.(80) with $\nu = \frac{3}{2}$. Thus again the spectrum of the scalar KK tower is identical to that of the graviton. However the structure changes on the addition of a five dimensional mass term for the scalar. If one adds a constant bulk mass term there is no longer a zero mode and the ultralight state is also lost. In this case, however, it is not possible to recover the zero mode and light states by adding a boundary term corresponding to coupling to boundary sources. The reason is that a boundary term is no longer equivalent to a term proportional to $\sigma''$ (c.f. Appendix B). As a result, up to the constant bulk term we have added, the bulk potential still has the form of eq.(80) with $\nu = \frac{3}{2}$ and, due to the constant bulk mass term, there is no zero mode. We see that the multi-localisation by a constant bulk plus brane mass term was special to the case with negative tension branes. If one is to achieve the same in the case without negative tension branes it is necessary to add a mass term of the form given in eq.(6) which cannot now be interpreted as a five dimensional bulk mass term plus a coupling of the scalar field to boundary sources. Note that if one does choose a scalar mass term of the form given in eq.(6) the remainder of the discussion applies to the $''++'''$ case too and one can generate the multilocalised scalar field configurations discussed above.
Of course the question is whether such a scalar mass term can be justified. As we will discuss in Section 9 supergravity can generate such mass terms in some, but not all, cases. For the remainder it seems unlikely as $\sigma'$ and $\sigma''$ are related to the metric and the underlying geometry of the compactification and it is difficult to see why a mass term of the form given in eq.(6) should arise. One possible explanation may follow if one can realise the ideas of reference [32]. In this case the geometry of compactification is driven by scalar field vacua with kink profiles along the extra dimension. Perhaps the coupling to these scalar fields will induce a mass term of the form given in eq.(6).

4 Multi-Localization of spin $\frac{1}{2}$ field

As has been shown in Ref. [20] multi-localization can appear also to spin $\frac{1}{2}$ fields with appropriate mass terms. Here for completeness we briefly review this case. The $AdS_5$ background geometry localizes the chiral zero mode on negative tension branes. However the addition of a mass term [2, 40] can alter the localization properties of the fermion so that it is localized on positive tension branes. The starting point again will be the action for a spin $\frac{1}{2}$ particle in the curved five dimensional background of eq.(2):

$$S = \int d^4x \int dy \sqrt{G} \left\{ E^A_a \gamma^a \left( \partial_A \Psi^L - \partial_A \Psi^R \right) - m(y) \bar{\Psi} \Psi \right\}$$

(18)

where $G = det(G_{AB}) = e^{-8\sigma(y)}$. Given the convention of of eq.(2) we adopt the “mostly hermitian” representation of the Dirac matrices. The four dimensional representation of the Dirac matrices is chosen to be $\gamma^a = (\gamma^\mu, \gamma^5)$ with $(\gamma^0)^2 = -1$, $(\gamma^1)^2 = 1$, $(\gamma^5)^2 = 1$. We define $\Gamma^M = E^M_a \gamma^a$ and thus we have $\{\gamma^a, \gamma^b\} = 2\eta^{ab}$ and $\{\Gamma^a, \Gamma^b\} = 2g^{ab}(y)$, where $\eta^{ab} = diag(-1, 1, 1, 1, 1)$. The vielbein is given by

$$E^A_a = diag(e^{\sigma(y)}, e^{\sigma(y)}, e^{\sigma(y)}, e^{\sigma(y)}, 1)$$

(19)

As in Ref. [20], we choose the mass term to have a (multi-) kink profile $m(y) = \frac{\sigma(y)}{k}$. It is convenient to write the action in terms of the fields: $\Psi_L$ and $\Psi_R$ where $\Psi_{R,L} = \frac{1}{2}(1 \pm \gamma_5)\Psi$ and $\Psi = \Psi_R + \Psi_L$. The action becomes:

$$S = \int d^4x \int dy \left\{ e^{-3\sigma} \left( \bar{\Psi}_L \gamma^\mu \partial_\mu \Psi_L + \bar{\Psi}_R \gamma^\mu \partial_\mu \Psi_R \right) - e^{-4\sigma} m \left( \frac{\sigma(y)}{k} \right) \left( \bar{\Psi}_L \Psi_R + \bar{\Psi}_R \Psi_L \right) - \frac{1}{2} \left[ \bar{\Psi}_L (e^{-4\sigma} \partial_y + \partial_\mu e^{-4\sigma}) \Psi_R - \bar{\Psi}_R (e^{-4\sigma} \partial_y + \partial_\mu e^{-4\sigma}) \Psi_L \right] \right\}$$

(20)
writing $\Psi_R$ and $\Psi_L$ in the form:

$$\Psi_{R,L}(x, y) = \sum_n \psi_{R,L}^n(x) e^{2\sigma(y)} f_{n}^{R,L}(y)$$  \hspace{1cm} (21)

the action can be brought in the form

$$S = \sum_n \int d^4 x \{ \bar{\psi}_n(x)i\gamma^\mu \partial_\mu \psi_n(x) - m_n \bar{\psi}_n(x)\psi_n(x) \}$$  \hspace{1cm} (22)

provided the wavefunctions obey the following equations

$$\left(-\partial_y + m \frac{\sigma'(y)}{k}\right) f_n^L(y) = m_n e^{\sigma(y)} f_n^R(y)$$

$$\left(\partial_y + m \frac{\sigma'(y)}{k}\right) f_n^R(y) = m_n e^{\sigma(y)} f_n^L(y)$$  \hspace{1cm} (23)

and the orthogonality relations (taking account of the $Z_2$ symmetry):

$$\int_{-L}^{L} dy e^{\sigma(y)} f_m^L(y) f_n^L(y) = \int_{-L}^{L} dy e^{\sigma(y)} f_m^R(y) f_n^R(y) = \delta_{mn}$$  \hspace{1cm} (24)

where we assume that the length of the orbifold is $2L$.

We solve the above system of coupled differential equations by substituting $f_m^L(y)$ from the second in the first equation. Thus we end up with a second order differential equation, which can always be brought to a Schrödinger form by a convenient coordinate transformation from $y$ to $z$ coordinates related through

$$\frac{dz}{dy} = e^{\sigma(y)}.$$  

This gives the differential equation of the form:

$$\left\{ -\frac{1}{2} \frac{\partial^2}{\partial z^2} + V_R(z) \right\} \hat{f}_n^R(z) = \frac{m_n^2}{2} \hat{f}_n^R(z)$$  \hspace{1cm} (25)

with potential

$$V_R(z) = \frac{\nu(\nu + 1)(\sigma'(y))^2}{2[g(z)]^2} - \frac{\nu}{2[g(z)]^2} \sigma''(y)$$  \hspace{1cm} (26)

Here $\hat{f}_n^R(z) = f_n^R(y)$ and we have defined $\nu \equiv \frac{m}{k}$ and $g(z) \equiv e^{\sigma(y)}$. The left handed wavefunctions are given by

$$f_n^L(y) = \frac{e^{-\sigma(y)}}{m_n} \left( \partial_y + m \frac{\sigma'(y)}{k}\right) f_n^R(y)$$  \hspace{1cm} (27)

\footnote{Note that it can be shown that the left-handed component obeys a similar Schrödinger equation with $V_L(z) = \frac{\nu(\nu - 1)k^2}{2[g(z)]^2} + \frac{\nu}{2[g(z)]^2} \sigma''(y)$ which is given by $V_R$ with $\nu \rightarrow -\nu$.}
For \( \nu = \frac{3}{2} \) that the form of eq.(26) is exactly the same as that satisfied by the graviton. The solution has the form \( f_n^R(z) \propto e^{-\nu \sigma(y)} \). From equation (24) we see that \( f^R(y)e^{\sigma(y)/2} \propto e^{(1/2 - \nu)\sigma(y)} \) is the appropriately normalised wavefunction.

There are three regions of localization: For \( \nu < \frac{1}{2} \) the zero mode is localized on negative tension branes, for \( \nu = \frac{1}{2} \) it is not localized and for \( \nu > \frac{1}{2} \) it is localized on positive tension branes. The study of the spectrum of the above differential equation provides the spectrum (for \( \nu > \frac{1}{2} \)): For the first KK state we find (for the symmetric configuration)

\[
m_1 = \sqrt{4\nu^2 - 1} \left[ kw e^{-(\nu + \frac{1}{2})x} \right]
\]

and for the rest of the tower

\[
m_{n+1} = \xi_n k w e^{-x} \quad n = 1, 2, 3, \ldots
\]

where \( \xi_{2i+1} \) is the \((i + 1)\)-th root of \( J_{\nu-rac{1}{2}}(x) \) \((i = 0, 1, 2, \ldots)\) and \( \xi_{2i} \) is the \(i\)-th root of \( J_{\nu+rac{1}{2}}(x) \) \((i = 1, 2, 3, \ldots)\). The above approximations become better away from the \( \nu = \frac{1}{2} \), \( x = 0 \) and for higher KK levels \( n \). The first mass is manifestly singled out from the rest of the KK tower as it has an extra exponential suppression that depends on the mass of the bulk fermion. By contrast the rest of the KK tower has only a very small dependence on the mass of the bulk fermion thought the root of the Bessel function \( \xi_n = \xi_n(\nu) \) which turns out to be just a linear dependence in \( \nu \). The special nature of the first KK state appears not only in the characteristics of the mass spectrum but also in its coupling behaviour. As it was shown in Ref. [20] the coupling to matter of the right-handed component of the first KK state is approximately constant (independent of the separation of the positive branes).

It is instructive to examine the localization behaviour of the modes as the separation between the two \( \delta \)-function potential wells increases. In the following we assume that \( \nu > \frac{1}{2} \) so that multi-localization is realized. In the case of infinite separation we know that each potential well supports a single chiral massless zero mode and that the rest of the massive modes come with Dirac mass terms. This raises an interesting question: How from the original configuration with finite size which has only one chiral mode do we end up with a configuration that has two chiral modes? The answer to this question is found by examining the localization properties of the first special KK mode. From Figs (3-5) we see that the right-handed component of the first KK state (dashed line) is localized on the positive tension brane whereas the left-handed component (dotted line) is localized in the central region of the configuration. As we increase the distance separating the two potential
wells the right-handed component remains localized on the positive tension branes whereas
the left-handed state starts to spread along the extra dimension. We see that the second
chiral mode that appears in the infinite separation limit is the right-handed component of
the first KK state. This is possible since in that limit the left-handed component decouples
(since it spreads along the infinite extra dimension). In this limit, the chiral zero mode that
each potential well supports can be considered as the combination of the zero mode of the
initial configuration and the massless limit of the first KK state.

Our discussion of fermion localisation applies also to models of the “++” type without
negative tension branes. The major difference is that the fermion mass term are no-longer
constant in the bulk. As in the case of the scalars it remains to be seen whether such mass
terms actually arise in models in which the geometry is determined by non-trivial scalar
field configurations.

5 Localization and Multi-Localization of spin 1 field

We now turn to the study of an Abelian gauge field. In the context of string theory it is
natural to have gauge fields living in their world-volume of D-branes (these gauge fields
emerge from open strings ending on the D-branes). However, in the case of a domain wall
it turns out that it is difficult to localize gauge bosons in a satisfactory way. The problem
has been addressed by several authors Ref. [32, 53–56]. In this section we argue that the
localization of gauge boson fields is indeed technically possible for particular forms of its
five dimensional mass term (a similar mass term has been recently considered by [57]). Our
starting point is the Lagrangian for an Abelian gauge boson in five dimensions:

\[ S = \int d^4x \int dy \sqrt{G} \left[ -\frac{1}{4} G^{MK} G^{NL} F_{MN} F_{KL} - \frac{1}{2} \alpha (\sigma' (y))^2 A_M A^M - \frac{1}{2} \beta \sigma'' (y) A_\mu A^\mu \right] \]  (30)

where \( F_{MN} = \partial_M A_N - \partial_N A_M \). Again we have assumed a mass term allowed by the
symmetries of the action of the form:

\[ m^2 = \alpha (\sigma' (y))^2 + \beta \sigma'' (y) \]  (31)

Of course it is important to be able to generate the above mass term in a gauge invariant
way. This can be readily done through the inclusion in the Lagrangian of the term:

\[ \left( \alpha (\sigma' (y))^2 + \beta \sigma'' (y) \right) \left( (D^M \phi)^* (D_M \phi) - V(\phi) \right) \]  (32)
Here we have added a five dimensional charged Higgs field, $\phi$. If the potential $V(\phi)$ triggers a vacuum expectation value for $\phi$, it will spontaneously break gauge invariance both in the bulk and on the brane and generate a vector mass term of the required form. The resulting action (in the gauge $A_5 = 0$) is

$$S = \int d^4x \int dy \sqrt{\hat{\tilde{G}}} \left[ -\frac{1}{4} \hat{\tilde{G}}^{\mu\nu} \hat{G}^{\lambda\kappa} F_{\mu\nu} F_{\kappa\lambda} - \frac{1}{2} e^{-2\sigma(y)}(\partial_5 A_\mu)(\partial_5 A_\lambda)\hat{G}^{\mu\lambda} - \frac{1}{2} m^2 A_\mu A^\mu \right]$$  \hspace{1cm} (33)

where:

$$m^2 = \alpha (\sigma'(y))^2 + \beta \sigma''(y)$$  \hspace{1cm} (34)

Performing the KK decomposition

$$A^\mu(x,y) = \sum_n A_\mu^n(x) f_n(y)$$  \hspace{1cm} (35)

this can be brought in the familiar action form for massive spin 1 particles propagating in flat space-time

$$S = \sum_n \int d^4x \left[ -\frac{1}{4} \eta^{\mu\nu} \eta_{\rho\sigma} F_{n\mu\nu} F_{n\rho\sigma} - \frac{1}{2} m_n^2 A_\mu^n A^\mu_n \right]$$  \hspace{1cm} (36)

provided that $f_n(y)$ satisfies the following second order differential equation

$$- \frac{d}{dy} \left( e^{-2\sigma(y)} \frac{df_n(y)}{dy} \right) + m^2 e^{-2\sigma(y)} f_n(y) = m_n^2 f_n(y)$$  \hspace{1cm} (37)

with the following orthogonality relations (taking in account the $Z_2$ symmetry):

$$\int_{-L}^L dy f^*_m(y) f_n(y) = \delta_{mn}$$  \hspace{1cm} (38)

where we assume that the length of the orbifold is $2L$. As before this can be brought to a Schrödinger form by a redefinition of the wavefunction and by a convenient coordinate transformation from $y$ to $z$ coordinates related through: $\frac{dz}{dy} = e^{\sigma(y)}$. Thus we end up with the differential equation of the form:

$$\left\{ -\frac{1}{2} \partial_z^2 + V(z) \right\} \hat{f}_n(z) = \frac{m_n^2}{2} \hat{f}_n(z)$$  \hspace{1cm} (39)

where

$$V(z) = \frac{3}{4} (\sigma'(y))^2 + \frac{m^2}{2[g(z)]^2} - \frac{1}{2} \frac{1}{2[g(z)]^2} \sigma''(y)$$  \hspace{1cm} (40)
where
\[ \hat{f}_n(z) = e^{-\frac{1}{2}\sigma(y)} f_n(y) \] (41)
and we have defined for convenience \( g(z) \equiv e^{\sigma(y)} \). Let us now examine the localization properties of the gauge boson modes. For \( m = 0 \) there exists a zero mode with wavefunction:
\[ \hat{f}_n(z) = C e^{-\frac{1}{2}\sigma(y)} = \frac{C}{\sqrt{g(z)}} \] (42)
where \( C \) is a normalization constant. From eq.(38) it is clear that in this case the appropriately normalized wavefunction \( f(y) \) is constant along the extra dimension and thus that the gauge boson is delocalized. For \( m \neq 0 \) with \( \alpha \neq 0 \) and \( \beta = 0 \) the zero mode becomes massive. We can recover the zero mode by allowing for the possibility of \( \beta \neq 0 \). In this case the potential can be written as
\[ V(z) = \frac{\alpha + \frac{3}{4}}{2[g(z)]^2} (\sigma'(y))^2 - \left( \frac{1}{2} - \beta \right) \frac{\sigma''(y)}{2[g(z)]^2} \] (43)
This is of the general form given in Appendix A. A massless mode exists if \( \alpha = \beta^2 - 2\beta \) in which case the wavefunction \( \hat{f}(z) \propto e^{(\beta - 1/2)\sigma(y)} \). From equation (41) we see that \( f(y)e^{-\sigma(y)} \propto e^{\beta\sigma(y)} \) is the appropriately normalised wavefunction in the interval \([-L, L]\). This is localised on the positive tension brane for \( \beta > 0 \) and on the negative tension brane for \( \beta < 0 \). When the condition for the zero mode is satisfied we find the mass of the first ultralight KK state to be given by (for the symmetric configuration):
\[ m_1 = \sqrt{4\nu^2 - 1} \ k w \ e^{-(\nu + \frac{1}{2})x} \] (44)
where \( \nu = \frac{1}{2} - \beta \) and for the rest of the tower
\[ m_{n+1} = \xi_n \ k w \ e^{-x} \quad n = 1, 2, 3, \ldots \] (45)
where \( \xi_{2i+1} \) is the \((i+1)\)-th root of \( J_{\nu - \frac{1}{2}}(x) \) \((i = 0, 1, 2, \ldots)\) and \( \xi_{2i} \) is the \(i\)-th root of \( J_{\nu + \frac{1}{2}}(x) \) \((i = 1, 2, 3, \ldots)\). Again, the first KK is singled out from the rest of the KK tower as it has an extra exponential suppression that depends on the mass parameter \( \nu \). In contrast the rest of the KK tower has only a very small dependence on the \( \nu \) parameter thought the root of the Bessel function \( \xi_n = \xi_n(\nu) \) which turns out to be just a linear dependence in \( \nu \).
Once again our discussion applies unchanged to the case without negative tension branes. Once again the difference is that the mass no longer corresponds to a combination of brane and constant bulk terms. Perhaps the origin of such terms will be better motivated in the case that the geometry is driven by a non-trivial vacuum configuration of a scalar field with a profile in the bulk such that coupling of the gauge field to it generates the required mass term. At present we have no indication that this should be the case.

6 Multi-Localization of spin $\frac{3}{2}$ field

In this section we consider the (multi-) localization of a spin $\frac{3}{2}$ particle. The starting point will be the Lagrangian for a $\frac{3}{2}$ particle propagating in curved background is:

$$S = -\int d^4x \int dy \sqrt{G} \bar{\Psi}_M \Gamma^{MNP} \left( D_N + \frac{m}{2} \Gamma_N \right) \Psi_P$$

(46)

where the covariant derivative is

$$D_M \Psi_N = \partial_M \Psi_N - \Gamma^P_{MN} \Psi_P + \frac{1}{2} \omega^A_M \gamma_{AB}$$

(47)

with $\gamma_{AB} = \frac{1}{4} [\gamma_A, \gamma_B]$ and $\Gamma^{MNP} = \Gamma^{[M} \Gamma^{N]} \Gamma^{P]}$. The connection is given by

$$\omega^A_M = \frac{1}{2} g^{PN} e^{[A} \partial_M e^{B]} e_{N]} + \frac{1}{4} g^{PN} g^{\Sigma e} e^{[A} e^{B]} e_{\Sigma] e_{\Gamma} e_{\Delta} \eta_{\Gamma \Delta}$$

(48)

where $\Gamma^M = e^M_n \gamma^n$ with $e^M_n = diag(e^{\sigma(y)}, e^{\sigma(y)}, e^{\sigma(y)}, e^{\sigma(y)}, 1)$. As in the case of the Abelian gauge boson we will assume that we generate the mass term for this field in a gauge invariant way. Exploiting the gauge invariance we can fix the gauge setting $\Psi_5 = 0$, something that simplifies considerably the calculations. In this case, the above action becomes

$$S = -\int d^4x \int dy \sqrt{G} \bar{\Psi}_\mu \Gamma^{\mu \nu \rho} \left( D_\nu + \frac{m}{2} \Gamma_\nu \right) \Psi_\rho - \sqrt{G} \bar{\Psi}_\mu \Gamma^{\mu \nu \rho} \left( D_5 + \frac{m}{2} \Gamma_5 \right) \Psi_\rho$$

(49)

We can simplify the above further taking in account the following identities:

$$\Gamma^{\mu \rho} = e^{3\sigma(y)} \gamma^{\mu \rho}$$

$$\Gamma^\mu_5 = e^{2\sigma(y)} \gamma_5^{\mu \rho}$$

$$\gamma^{\mu \nu} \gamma_\nu = -2 \gamma^{\mu \rho}$$

$$\gamma^{\mu 5} = -\gamma^5 \gamma^{\mu \rho}$$

$$\gamma^{\mu \rho} \gamma_\mu = -2 \gamma^\rho$$

(50)
Using the above we find the following simple forms for the covariant derivatives

\[ D_\nu = \partial_\nu - \frac{1}{2} \sigma'(y)e^{-\sigma(y)}\gamma_\nu\gamma^5 \]

\[ D_5 = \partial_5 \]

Using the previous relations we write the action in the form

\[ S = -\int d^4 x \int dy e^{-\sigma(y)}\bar{\Psi}_\mu \gamma^{\mu\nu\rho} \left( \partial_\nu - \frac{1}{2} \sigma'(y)e^{-\sigma(y)}\gamma_\nu\gamma^5 + \frac{m}{2} e^{-\sigma(y)}\gamma_\nu \right) \Psi_\rho \\
- e^{-2\sigma(y)}\bar{\Psi}_\mu \gamma^{\mu5} \left( \partial_5 + \frac{m}{2} \gamma_5 \right) \Psi_\rho \]

The above can be brought in the form

\[ S = -\int d^4 x \int dy e^{-\sigma(y)}\bar{\Psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \Psi_\rho + e^{-2\sigma(y)}\bar{\Psi}_\mu \gamma^{\mu5} \left[ \frac{3m}{2} + \gamma^5 \left( \partial_5 - \sigma'(y) \right) \right] \Psi_\rho \]

At this stage it turns out, like in the spin $\frac{1}{2}$ case, that it is convenient to write $\Psi_\mu$ in terms of $\Psi_\mu^R$ and $\Psi_\mu^L$ ($\Psi_\mu = \Psi_\mu^R + \Psi_\mu^L$) which have different KK decomposition:

\[ \Psi_\mu^{R,L}(x,y) = \sum_n \psi_\mu^{R,L}(x) e^{\sigma(y)} f_\mu^{R,L}(y) \]

Substituting the above decompositions in the action we get

\[ S = -\int d^4 x \int dy e^{\sigma(y)}\bar{\Psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \Psi_\rho + \bar{\Psi}_\mu \gamma^{\mu5} \left[ \frac{3m}{2} + \gamma^5 \partial_5 \right] \Psi_\rho \\
+ \bar{\Psi}_\mu \gamma^{\mu\rho} \left[ \frac{3m}{2} + \gamma^5 \partial_5 \right] \Psi_\rho \]

this can be brought to the familiar action form for massive spin $\frac{3}{2}$ particle in flat background

\[ S = \int d^4 x \left\{ -\bar{\Psi}_\mu \gamma^{\mu\rho} \partial_\nu \Psi_\rho + m_n \bar{\Psi}_\mu \gamma^{\mu5} \Psi_\rho \right\} \]

provided that $f_\mu^R$ and $\Psi_\mu^L$ satisfy that following coupled differential equations

\[ \left( -\partial_y + \frac{3m \sigma'(y)}{2} \right) f_\mu^L(y) = m_n e^{\sigma(y)} f_\mu^R(y) \]

\[ \left( \partial_y + \frac{3m \sigma'(y)}{2} \right) f_\mu^R(y) = m_n e^{\sigma(y)} f_\mu^L(y) \]
supplied with the orthogonality relations (taking account of the $Z_2$ symmetry):

\[ \int_{-L}^{L} dy \sigma(y) f^*_m(y) f^*_n(y) = \int_{-L}^{L} dy \sigma(y) f^*_m(y) f^*_n(y) = \delta_{mn} \]  

(58)

Note that the form of the above system of differential equations is identical to the one of spin $\frac{1}{2}$ particle provided we substitute $m \rightarrow \frac{3m}{2}$. Accordingly the corresponding Schrödinger equation and thus the mass spectrum in this case is going to be the same as the spin $\frac{1}{2}$ case up to the previous rescaling of the mass parameter.

7 Multi-Localization of the graviton field

In this section, for completeness, we review the multi-localization scenario for the graviton. The gravitational field has the characteristic that it creates itself the background geometry in which it and the rest of the fields propagate. Thus, one has first to find the appropriate vacuum solution and then consider perturbations around this solution. In order to exhibit how the multi-localization appears in this case, we will again work with the " + --" configuration. The starting point is the Lagrangian

\[ S = \int d^4x \int_{-L}^{L} dy \sqrt{-G} \{ -\Lambda + 2M^3 R \} - \sum_i \int_{y=L_i} dy V_i \sqrt{-G} \]  

(59)

The Einstein equations that arise from this action are:

\[ R_{MN} - \frac{1}{2} G_{MN} R = - \frac{1}{4M^3} \left( \Lambda G_{MN} + \sum_i V_i \sqrt{-G} \delta_{ij} \delta_M \delta_N \delta(y - L_i) \right) \]  

(60)

using the metric ansatz of eq.(2) we find that the above equations imply that the function $\sigma(y)$ satisfies:

\[ (\sigma')^2 = k^2 \]  

(61)

\[ \sigma'' = \sum_i \frac{V_i}{12M^3} \delta(y - L_i) \]  

(62)

where $k = \sqrt{-\frac{\Lambda}{24M^3}}$ is a measure of the curvature of the bulk. The exact form of $\sigma(y)$ depends on the brane configuration that we consider. For example, in the case of " + --" model we have

\[ \sigma(y) = k \{ L_1 - ||y| - L_1|\} \]  

(63)
where $L_1$ is the position of the intermediate brane. with the requirement that the brane tensions are tuned to $V_0 = -\Lambda/k > 0$, $V_1 = \Lambda/k < 0$, $V_2 = -\Lambda/k > 0$. In order to examine the localization properties of the graviton, the next step is to consider fluctuations around the vacuum of eq.(1). Thus, we expand the field $h_{\mu\nu}(x,y)$ in graviton and KK states plane waves:

$$h_{\mu\nu}(x,y) = \sum_{n=0}^{\infty} h_{\mu\nu}^{(n)}(x) f_n(y)$$  \hspace{1cm} (64)$$

where $(\partial_\kappa \partial^\kappa - m_n^2) h_{\mu\nu}^{(n)} = 0$ and fix the gauge as $\partial^{\alpha}\alpha_{\mu\nu} = h_{\alpha\nu}^{(n)} = 0$. The function $f_n(y)$ will obey a second order differential equation which after a change of variables ($\frac{dz}{dy} = e^{\sigma(y)}$) reduces to an ordinary Schrödinger equation:

$$\left\{-\frac{1}{2} \partial_z^2 + V(z)\right\} \hat{f}_n(z) = \frac{m_n^2}{2} \hat{f}_n(z)$$  \hspace{1cm} (65)$$

with potential

$$V(z) = \frac{15}{4}(\sigma'(y))^2 - \frac{3}{2}\frac{\sigma''(y)}{|g(z)|^2}$$  \hspace{1cm} (66)$$

where

$$\hat{f}_n(z) \equiv f_n(y)e^{\sigma/2}$$  \hspace{1cm} (67)$$

and the function $g(z)$ as $g(z) \equiv k \{z_1 - |z - z_1|\} + 1$, where $z_1 = z(L_1)$. The study of the mass spectrum of reveals the following structure for the mass spectrum (for the symmetric configuration):

$$m_1 = 2\sqrt{2}ke^{-2x}$$  \hspace{1cm} (68)$$

$$m_{n+1} = \xi_n ke^{-x} \quad n = 1, 2, 3, \ldots$$  \hspace{1cm} (69)$$

where $\xi_{2i+1}$ is the $(i + 1)$-th root of $J_1(x)$ $(i = 0, 1, 2, \ldots)$ and $\xi_{2i}$ is the $i$-th root of $J_2(x)$ $(i = 1, 2, 3, \ldots)$. Again the first KK state is singled out from the rest of the KK tower as its mass has an extra exponential suppression.

Let us see now how the separability argument works in the case of the graviton. Starting with the familiar configuration "$+++\cdots$", the mass spectrum consists of the massless graviton, the ultra-light first KK state and the rest of the KK tower which are massive spin two

\footnote{Note that we have ignored the presence of dilaton/radion fields associated with the size of the extra dimension or the positions of the branes. For more details see Ref. \cite{58, 52}.}
particles. In the limit of infinite separation the first special KK mode becomes the second massless mode, according to our previous general discussions. However, at first sight the counting of degrees of freedom doesn’t work: we start with a massive spin 2 state (first KK state) which has five degrees of freedom and we end up with a massless mode which has two. It has been shown that in the case of flat spacetime the extra polarizations of the massive gravitons do not decouple giving rise to the celebrated van Dam-Veltman-Zakharov discontinuity in the propagator of a massive spin-2 field in the massless limit. However our separability argument is still valid: Up to this point we have ignored the presence of a massless scalar mode, the radion, which is related to the motion of the freely moving negative tension brane. It turns out that this scalar field is a ghost field, that is, it enters the Lagrangian with the wrong kinetic term sign. It can be shown that the effect of the presence of this field is to exactly cancel the contribution of the extra polarizations of the graviton making the limit of infinite brane separation smooth. Note that apart from the radion there is another scalar field in the spectrum, the dilaton, which parameterizes the overall size of the extra dimension which also decouples in the above limit.

As we have mentioned the problems associated with the presence of the ghost radion can be avoided by allowing for $AdS_4$ spacetime on the 3-branes. In this case there is no need for the negative tension brane (thus there is no radion field) and moreover the presence of curvature on the branes makes the massless limit of the massive graviton propagator smooth, meaning that in the $AdS_4$ curved background the extra polarizations of the massive graviton decouple in the massless limit, in agreement with our separability argument.

8 Multi-Localization and supersymmetry

It is interesting to investigate the multi-localization in the supersymmetric versions of the previous models. The inclusion of supersymmetry is interesting in the sense that it restricts the possible mass terms by relating the mass parameters of fermion and boson fields. It is well known that $AdS$ spacetime is compatible with supersymmetry. In contrast to the case of flat spacetime, supersymmetry in $AdS$ requires that fields belonging in the same multiplet have different masses. In the previous discussions on the localization of the fields, the mass term parameters which control the localization of the bulk states, are

\footnote{However, in Ref. was shown that in the presence of a source with a characteristic mass scale, there is no discontinuity for distances smaller than a critical one. This argument is also supported by the results of Refs. where it was shown that the limit is smooth in $dS_4$ or $AdS_4$ background.}
generally unconstrained. Let us now examine in more detail the cases of supergravity, vector supermultiplets and the hypermultiplet.

**Supergravity supermultiplet** The on-shell supergravity multiplet consists of the vielbein $e^a_M$, the graviphoton $B_M$ and the two symplectic-Majorana gravitinos $\Psi^i_M$ ($i = 1, 2$). The index $i$ labels the fundamental representation of the SU(2) automorphism group of the $N = 1$ supersymmetry algebra in five dimensions. The supergravity Lagrangian in $AdS_5$ has the form [29] (in $AdS_5$ background we can set $B_M = 0$):

$$S_5 = -\frac{1}{2} \int d^4x \int dy \sqrt{-g} \left[ M_5^3 \left\{ R + i \Psi^i_M \gamma^{MNR} D_N \Psi^j_R - i \frac{3}{2} \sigma'(y) \Psi^i_M \sigma^{MN}(\sigma_3)^{ij} \Psi^j_N \right\} \right. + 2\Lambda - \frac{\Lambda}{k^2} \sigma''(y) \right]$$  \hspace{1cm} (70)

where $\gamma^{MNR} = \sum_{\text{perm}} \frac{(-1)^p}{3!} \gamma^M \gamma^N \gamma^R$ and $\sigma^{MN} = \frac{1}{2} [\gamma^M, \gamma^N]$. From the above expression, that is invariant under the supersymmetry transformations [29], we see that the symplectic-Majorana gravitino mass term $m = \frac{3}{2} \sigma'(y)$ is such that its mass spectrum is identical to the mass spectrum of the graviton. This becomes clear by comparing eq.(26) for $\nu = \frac{3}{2}$ and eq.(66). The latter implies that, in the presence of supersymmetry, multi-localization of the graviton field implies multi-localization of the gravitinos and thus the mass spectrum of these fields will contain ultralight KK state(s).

**Vector supermultiplet** The on-shell field content of the vector supermultiplet $V = (V_M, \lambda^i, \Sigma)$ consists from the gauge field $V_M$, a symplectic-Majorana spinor $\lambda^i$, and the real scalar field $\Sigma$ in the adjoint representation.

$$S_5 = -\frac{1}{2} \int d^4x \int dy \sqrt{-g} \left[ \frac{1}{2g_5^2} F_{MN}^2 + (\partial_M \Sigma)^2 + i \bar{\lambda}^i \gamma^M D_M \lambda^i + m_\Sigma^2 \Sigma^2 + i m_\lambda \bar{\lambda}^i (\sigma_3)^{ij} \lambda^j \right]$$  \hspace{1cm} (71)

The above Lagrangian is invariant under the supersymmetry transformations if the mass terms of the various fields are of the form (for more details see Ref. [29]):

$$m_\Sigma^2 = -4(\sigma'(y))^2 + 2\sigma''(y)$$

$$m_\lambda = \frac{1}{2} \sigma'(y)$$  \hspace{1cm} (72)

Assuming that $V_M$ and $\lambda^i$ are even while $\Sigma$ and $\lambda^j$ odd then the mass spectrum of all the fields is identical. This can be easily seen if we note that for the spinors we have $\nu = \frac{1}{2}$,
for the scalar $\alpha = -4, \beta = 2$ and for the gauge boson $\alpha = 0, \beta = 0$. The even fields, for the above values of the mass parameters, they obey the eq.(83) of Appendix A with $\nu = \frac{1}{2}$ whereas the odd fields obey eq.(84) for the same value of the $\nu$ parameter. The mass spectrum of the two potentials is identical, apart from the zero modes, since they are SUSY-partner quantum mechanical potentials. The even fields have zero modes that are not localized (which is expected since the massless gauge field is not localized) in contrast to the odd fields that have no zero modes (they are projected out due to the boundary conditions).

**Hypermultiplet** The hypermultiplet $H = (H^i, \Psi)$ consists of two complex scalar fields $H^i$ ($i = 1, 2$) and a Dirac fermion $\Psi$. In this case the action setup is:

$$S_5 = - \int d^4x \int dy \sqrt{-g} \left[ |\partial_M H^i|^2 + i\bar{\Psi} \gamma^M D_M \Psi + m^2_{H^i} |H^i|^2 + im_{\lambda} \bar{\Psi} \Psi \right]$$  \hspace{1cm} (73)

Invariance under the supersymmetric transformations (see Ref. [29]) demand that the mass term of the scalar and fermion fields has the form:

$$m^2_{H^1,2} = (c^2 \pm c - \frac{15}{4})(\sigma'(y))^2 + (\frac{3}{2} \mp c)\sigma''(y)$$

$$m_{\lambda} = c\sigma'(y)$$  \hspace{1cm} (74)

from the above we identify that $\alpha = c^2 \pm c - \frac{15}{4}$ and $\beta = \frac{3}{2} \mp c$ for the scalar fields. Note that $\alpha = \beta^2 - 4\beta$ which implies the existence of zero mode for the symmetric scalar fields. Moreover, for the scalar fields we find that $\nu \equiv \frac{3}{2} - \beta = \pm c$ which implies that the wavefunctions (in z-coordinates) and the mass spectrum are identical to the Dirac fermion’s. Note that we are assuming that $H^1$ and $\Psi_L$ are even, while $H^2$ and $\Psi_R$ are odd. As expected if supersymmetry is realized, multi-localization of scalar fields implies multi-localization of Dirac fermions and the opposite.

In the five dimensional $AdS$ background, the mass terms compatible with supersymmetry are not the ones that correspond to degenerate supermultiplet partners, but are such that all members of the supermultiplets have the same wavefunction behaviour and the same mass spectrum. However, in the four dimensional effective field theory description, the states lie in degenerate SUSY multiplets, as is expected since the 4D theory is flat.
9 Discussion and conclusions

In this paper we studied the localization behaviour and the mass spectrum of bulk fields in various multi-brane models with localized gravity. We showed that the addition of appropriate mass terms controls the strength or/and the location of localization of the fields and moreover can induce localization to Abelian spin 1 fields. The localization of all the above fields can resemble that of the graviton, at least in a region of the parameter space. This means that fields of all spins (≤2) can be localized on positive tension branes. The latter implies that in the context of multi-brane models emerges the possibility of multi-localization for all the previous fields with appropriate mass terms. We have shown, giving explicit examples, that when multi-localization is realized the above fields apart from the massless zero mode support ultra-light localized KK mode(s).

In the simplest constructions with two positive branes, that we considered here, there is only one special KK state. However by adding more positive tension branes one can achieve more special light states. In the extreme example of a infinite sequence of positive branes instead of discrete spectrum of KK states we have continuum bands. In the previous case the special character of the zeroth band appears as the fact that it is well separated from the next.

Summarizing, in this paper we pointed out some new characteristics of multi-brane scenarios in the case that multi-localization is realized. The new phenomenology reveals itself through special light and localized KK states. The idea of multi-localization and its relation to new interesting phenomenology is of course general and it should not be necessarily related to RS type models\(^{12}\), although it finds a natural application in the context of these models.

Acknowledgments: We would like to thank Tony Gherghetta and Luigi Pilo for useful discussions. S.M.’s work is supported by the Greek State Scholarship Foundation (IKY) No. 811781027. A.P.’s work is supported by the Greek State Scholarship Foundation (IKY) No. 8017711802. This work is supported in part by PPARC rolling grant PPA/G/O/1998/00567, the EC TMR grant HRRN-CT-2000-00148 and HPRN-CT-2000-00152.

\(^{12}\)In the case of gravity though, such a construction (or similar) with curved background is essential.
Appendix

A  Wavefunction Solutions

In this section we briefly discuss the form of the solutions of the Schröedinger equation of the "" + − +"" configuration - a configuration that exhibits multi-localization. As we have seen in the previous sections the general form of the potential (for field of any spin) of the corresponding quantum mechanical problem is of the type:

\[ V(z) = \frac{\kappa}{2|g(z)|^2}(\sigma'(y))^2 - \frac{\lambda}{2|g(z)|^2}\sigma''(y) \]  

where \( \kappa, \lambda \) are constant parameters. For the case of "" + − +"" model we have

\[
\begin{align*}
(\sigma'(y))^2 &= k^2 \\
\sigma''(y) &= 2kg(z) \left[ \delta(z) + \delta(z - z_2) - \delta(z - z_1) \right]
\end{align*}
\]

(76)

\( z_1 \) and \( z_2 \) are the position of the second (negative) and the third (positive) brane respectively in the new coordinates (\( z_1 = z(L_1) \) and \( z_2 = z(L_2) \)). The convenient choice of variables, which is universal for fields of all spins, is:

\[
z \equiv \begin{cases} 
2^{kL_1 - e^{2kL_1 - ky} - 1} & y \in [L_1, L_2] \\
e^{ky - 1} & y \in [0, L_1]
\end{cases}
\]

(77)

(the new variable is chosen to satisfy \( \frac{dz}{dy} = e^{\sigma(y)} \) and the function \( g(z) \) is defined for convenience as \( g(z) \equiv e^{\sigma(y)} \equiv k \{ z_1 - ||z| - z_1| \} + 1 \). Note that in principle \( \kappa \) and \( \lambda \) are not related since the first gets contributions from the five dimensional bulk mass whereas the second from the boundary mass term.

13In the following expression we have assumed that boundary mass term contribution is universal (its absolute value) for all the branes, i.e. \( \lambda \) is the weight of all \( \delta \)-functions. One might consider a more general case, where each \( \delta \)-function to have its own weight. However, generally the requirement of the existence of zero mode implies that their absolute values are equal.

14The particular values of the parameters \( \kappa, \lambda \) depend on the spin of the particle. Here we are interested in the general forms of the solutions. We also assume that we have already performed the appropriate redefinition of the wavefunction \( (f(y) \rightarrow f(z)) \), which also depends on the spin of the particle.
**Even fields**  Let us consider first the case that the field under consideration is even under the reflections $y \to -y$. In this case, it is easy to show that the zero mode exists only in the case that $\kappa = \nu(\nu + 1)$ and $\lambda = \nu$ or $\lambda = -(\nu + 1)$ (this is derived by imposing the boundary conditions coming from the $\delta$-function potentials on the massless solution, see below). The zero mode wavefunctions in this case have the form:

In the case that $\lambda = \nu$,

$$\hat{f}_0(z) = \frac{A}{[g(z)]^\nu}$$

and in the case that $\lambda = -(\nu + 1)$,

$$\hat{f}_0(z) = A' [g(z)]^{\nu+1}$$

Where $A,A'$ are normalization constants. Note that the first is localized on positive tension branes whereas the second on negative tension branes. However, only the first choice gives the possibility of multi-localization on positive tension branes. Thus the existence of zero mode and light KK state requires $\kappa = \nu(\nu + 1)$ and $\lambda = \nu$. Since we are interested in configurations that give rise to light KK states, the potential of interest is:

$$V(z) = \frac{\nu(\nu + 1)k^2}{2[g(z)]^2} - \frac{\nu}{2g(z)}2k[\delta(z) + \delta(z - z_2) - \delta(z - z_1)]$$

(80)

For the KK modes ($m_n \neq 0$) the solution is given in terms of Bessel functions. For $y$ lying in the regions $A \equiv [0, L_1]$ and $B \equiv [L_1, L_2]$, we have:

$$\hat{f}_n \left\{ \begin{array}{l} A \\ B \end{array} \right\} = \sqrt{\frac{g(z)}{k}} \left\{ \begin{array}{l} A_1 \\ B_1 \end{array} \right\} J_{\nu+\nu} \left( \frac{m_n k}{g(z)} \right) + \left\{ \begin{array}{l} A_2 \\ B_2 \end{array} \right\} J_{-\nu-\nu} \left( \frac{m_n k}{g(z)} \right)$$

(81)

The boundary conditions that the wavefunctions must obey are:

$$\hat{f}_n '(0^+) + \frac{k\nu}{g(0)}\hat{f}_n(0) = 0$$

$$\hat{f}_n(z_1^+) - \hat{f}_n(z_1^-) = 0$$

$$\hat{f}_n '(z_1^+) - \hat{f}_n '(z_1^-) - \frac{2k\nu}{g(z_1)}\hat{f}_n(z_1) = 0$$

$$\hat{f}_n '(z_2^-) - \frac{k\nu}{g(z_2)}\hat{f}_n(z_2) = 0$$

(82)
The above boundary conditions give a $4 \times 4$ linear homogeneous system for $A_1, B_1, A_2$ and $B_2$, which, in order to have a nontrivial solution should have vanishing determinant. This imposes a quantization condition from which we are able to extract the mass spectrum of the bulk field.

**Odd fields**  In the case that the field is odd under the reflections $y \to -y$, there is no zero mode solution since it is not possible to make it’s wavefunction to vanish at both boundaries. In the absence of zero mode, the previous restrictions between $\lambda$ and $\nu$ do not apply - and thus they are in principle independent. However one can ask if in this case, despite the absence of zero mode, a light state can exist. Indeed we can easily find that this can be realized for special choice of parameters: It can be shown that the potential

$$V_1(z) = \frac{\nu(\nu - 1)}{2|g(z)|^2}(\sigma'(y))^2 + \frac{\nu}{2|g(z)|^2}\sigma''(y)$$  \hspace{1cm} (83)$$

considering odd parity for the fields, gives the same spectrum (apart from the zero mode) with the familiar potential for fields of even parity:

$$V_2(z) = \frac{\nu(\nu + 1)}{2|g(z)|^2}(\sigma'(y))^2 - \frac{\nu}{2|g(z)|^2}\sigma''(y)$$  \hspace{1cm} (84)$$

which according to the previous discussions supports an ultra-light special KK state. This is because the previous potentials are SUSY partners and as expected have the same spectrum apart from the zero mode.

**B  Life without negative tension branes**

It has been shown that the properties of the $''+ - +''$ model (the bounce form of the “warp” factor), which contains a moving negative tension brane can be mimicked by the $''+ + +''$ model, where the negative brane is absent provided that we allow for $AdS_4$ on the branes. Since the corresponding potential has two $\delta$-function wells that support bound states the multi-localization scenario appears also here. The previous results related to the localization properties of the various fields are valid also in this case. However, the presence of $AdS_4$ geometry on the branes, modifies the form of the potential of the corresponding Schrödinger equation and thus the details of the form of wavefunctions of the KK states. In this section we briefly discuss these modifications.
As previously mentioned, the spacetime on the 3-branes must be $AdS_4$ (in contrast to the $''++-''$ models where the spacetime is flat). Thus in this case the background geometry is described by:

$$ds^2 = \frac{e^{-2\sigma(y)}}{(1 - \frac{H^2x^2}{4})^2} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2$$  \hspace{1cm} (85)$$

By following exactly the same steps as in the case of flat branes, again the whole problem is reduced to the solution of a second order differential equation for the profile of the KK states. The differential equation is such that after the dimensional reduction the five dimensional physics is described by a infinite tower of KK states that propagate in the $AdS_4$ background of the 3-brane. It is always possible to make the coordinate transformation from $y$ coordinates to $z$ coordinates related through: $\frac{dz}{dy} = A^{-1}(y)$, where $A(y) = e^{-\sigma(y)}$, and a redefinition of the wavefunction and bring the differential equation in the familiar Schrödinger-like form:

$$\left\{ -\frac{1}{2} \frac{\partial^2}{\partial z^2} + V(z) \right\} \hat{f}_n(z) = \frac{m_n^2}{2} \hat{f}_n(z)$$  \hspace{1cm} (86)$$

where $\hat{f}_n(z)$ is the appropriate redefinition of the wavefunction.

For the $''++-''$ model the form of the potential for fields of different spin is different. However, in the case that it admits a massless mode and an anomalously light mode it has the generic form given in eq.(80) that applied to the $''++-''$ case. However the warp factor has a different form from the case with negative tension branes being given by

$$g(z) \equiv e^{\sigma(y)} = \frac{1}{\cosh(k(|z| - z_0))}$$  \hspace{1cm} (87)$$

Note that in this case $\sigma'(y)$ is not constant in the bulk and $\sigma''(y)$ is not confined to the branes. The massless modes, corresponding to the Schrodinger equation with this potential, are given by eqs.(75) and (76) as in the $''++-''$ case. Note however that the constraint on the relative magnitude of the two terms in the potential, eq.(72), is now required when solving for the propagation in the bulk whereas in the case of a negative tension brane it came when solving for the boundary conditions.

The zero mode wavefunction is given by:

$$\hat{f}_0(z) = \frac{C}{[\cos(k(z_0 - |z|))]^\nu}$$  \hspace{1cm} (88)$$

\hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{1cm} \hspace{
where $C$ is the normalization factor. By considering cases with $m_n \neq 0$, we find the
wavefunctions for the KK tower:

\[
\hat{f}_n(z) = \cos^{\nu+1}(\tilde{k}|z| - z_0)) \left[ C_1 F(\tilde{a}_n, \tilde{b}_n, \frac{1}{2}; \sin^2(\tilde{k}|z| - z_0)) + C_2 |\sin(\tilde{k}|z| - z_0)| F(\tilde{a}_n + \frac{1}{2}, \tilde{b}_n + \frac{1}{2}, \frac{3}{2}; \sin^2(\tilde{k}|z| - z_0)) \right]
\]

(89)

where

\[
\tilde{a}_n = \nu + 1 + \frac{1}{2} \sqrt{\left( \frac{m_n}{\tilde{k}} \right)^2 + \nu^2}
\]

\[
\tilde{b}_n = \nu + 1 - \frac{1}{2} \sqrt{\left( \frac{m_n}{\tilde{k}} \right)^2 + \nu^2}
\]

(90)

The boundary conditions are given by:

\[
\hat{f}_n'(0^-) + k\nu \tanh(ky_0) \hat{f}_n(0) = 0
\]

\[
\hat{f}_n'(z_L^-) - k\nu \frac{\sinh(k(L - y_0))}{\cosh(ky_0)} \hat{f}_n(z_L) = 0
\]

(91)

the above conditions determine the mass spectrum of the KK states. By studying the mass
spectrum of the KK states it turns out that it has a special first mode similar to the one
of the "$ + - +"$ model as expected.

References

[1] K. Akama, “An Early Proposal Of 'Brane World,'” Lect. Notes Phys. 176 (1982) 267 [hep-th/0001113].

[2] V. A. Rubakov and M. E. Shaposhnikov, “Do We Live Inside A Domain Wall?,” Phys. Lett. B 125, 136 (1983).

[3] M. Visser, “An Exotic Class Of Kaluza-Klein Models,” Phys. Lett. B 159, 22 (1985) [hep-th/9910093].

[4] E. J. Squires, “Dimensional Reduction Caused By A Cosmological Constant,” Phys. Lett. B 167, 286 (1986).

[5] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “The hierarchy problem and new dimensions at a
millimeter,” Phys. Lett. B 429 (1998) 263 [hep-ph/9803315].

[6] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “New dimensions at a millimeter to
a Fermi and superstrings at a TeV,” Phys. Lett. B 436 (1998) 257 [hep-ph/9804393].
[7] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, “Phenomenology, astrophysics and cosmology of theories with sub-millimeter dimensions and TeV scale quantum gravity,” Phys. Rev. D 59 (1999) 086004 [hep-ph/9807344].
[8] M. Gogberashvili, “Hierarchy problem in the shell-universe model.” [hep-ph/9812296].
[9] L. Randall and R. Sundrum, “A large mass hierarchy from a small extra dimension,” Phys. Rev. Lett. 83 (1999) 3370 [hep-ph/9905221].
[10] L. Randall and R. Sundrum, “An alternative to compactification,” Phys. Rev. Lett. 83 (1999) 4690 [hep-th/9906064].
[11] K. R. Dienes, E. Dudas and T. Gherghetta, “Light neutrinos without heavy mass scales: A higher-dimensional seesaw mechanism,” Nucl. Phys. B 557 (1999) 25 [hep-ph/9811428].
[12] N. Arkani-Hamed, S. Dimopoulos, G. Dvali and J. March-Russell, “Neutrino masses from large extra dimensions,” [hep-ph/9811448].
[13] G. Dvali and A. Y. Smirnov, “Probing large extra dimensions with neutrinos,” Nucl. Phys. B 563 (1999) 63 [hep-ph/9904211].
[14] P. N. Mohapatra, S. Nandi and A. Perez-Lorenzana, “Neutrino masses and oscillations in models with large extra dimensions,” Phys. Lett. B 466 (1999) 115 [hep-ph/9907520].
[15] Y. Grossman and M. Neubert, “Neutrino masses and mixings in non-factorizable geometry,” Phys. Lett. B 474 (2000) 361 [hep-ph/9912408].
[16] R. Barbieri, P. Creminelli and A. Strumia, “Neutrino oscillations from large extra dimensions,” Nucl. Phys. B 585 (2000) 28 [hep-ph/0002190].
[17] A. Lukas, P. Ramond, A. Romanino and G. G. Ross, “Solar neutrino oscillation from large extra dimensions,” Phys. Lett. B 495 (2000) 136 [hep-ph/0008049].
[18] N. Cosme, J. M. Frere, Y. Gouverneur, F. S. Ling, D. Monderen and V. Van Elewyk, “Neutrino suppression and extra dimensions: A minimal model,” [hep-ph/0010192].
[19] A. Lukas, P. Ramond, A. Romanino and G. G. Ross, “Neutrino masses and mixing in brane-world theories,” [hep-ph/0011295].
[20] S. Mouslopoulos, “Bulk fermions in multi-brane worlds,” JHEP 0105 (2001) 038 [hep-th/0103184].
[21] N. Arkani-Hamed and M. Schmaltz, “Hierarchies without symmetries from extra dimensions,” Phys. Rev. D 61 (2000) 033005 [hep-ph/9903417].
[22] E. A. Mirabelli and M. Schmaltz, “Yukawa hierarchies from split fermions in extra dimensions,” Phys. Rev. D 61 (2000) 113011 [hep-ph/9912267].
[23] G. Dvali and M. Shifman, “Families as neighbors in extra dimension,” Phys. Lett. B 475, 295 (2000) [hep-ph/0001072].
[24] F. del Aguila and J. Santiago, “Universality limits on bulk fermions,” Phys. Lett. B 493 (2000) 175 [hep-ph/0008143].
[25] W. D. Goldberger and M. B. Wise, “Bulk fields in the Randall-Sundrum compactification scenario,” Phys. Rev. D 60 (1999) 107505 [hep-ph/9907218].

[26] M. Mintchev and L. Pilo, “Localization of quantum fields on branes,” Nucl. Phys. B 592 (2001) 219 [hep-th/0007002].

[27] B. Bajc and G. Gabadadze, “Localization of matter and cosmological constant on a brane in anti de Sitter space,” Phys. Lett. B 474 (2000) 282 [hep-th/9912232].

[28] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, “Bulk standard model in the Randall-Sundrum background,” Phys. Rev. D 62 (2000) 084025 [hep-ph/9912498].

[29] T. Gherghetta and A. Pomarol, “Bulk fields and supersymmetry in a slice of AdS,” Nucl. Phys. B 586 (2000) 141 [hep-ph/0003124].

[30] S. Chang, J. Hisano, H. Nakano, N. Okada and M. Yamaguchi, “Bulk standard model in the Randall-Sundrum background,” Phys. Rev. D 62 (2000) 084025 [hep-ph/9912498].

[31] I. Oda, “Localization of various bulk fields on a brane,” hep-th/0009074.

[32] A. Kehagias and K. Tamvakis, “Localized gravitons, gauge bosons and chiral fermions in smooth spaces generated by a bounce,” hep-th/0010112.

[33] I. Oda, “Localization of bulk fields on AdS4 brane in AdS5,” hep-th/0012013.

[34] T. Gherghetta and A. Pomarol, Nucl. Phys. B 602, 3 (2001) [hep-ph/0012378].

[35] I. Oda, “Localization of gravitino on a brane,” hep-th/0008134.

[36] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, “Bulk gauge fields in the Randall-Sundrum model,” Phys. Lett. B 473 (2000) 43 [hep-ph/9911262].

[37] A. Pomarol, “Gauge bosons in a five-dimensional theory with localized gravity,” Phys. Lett. B 486 (2000) 153 [hep-ph/9911294].

[38] H. Davoudiasl, J. L. Hewett and T. G. Rizzo, “Experimental probes of localized gravity: On and off the wall,” Phys. Rev. D 63 (2001) 075004 [hep-ph/0006041].

[39] M. J. Duff, J. T. Liu and W. A. Sabra, “Localization of supergravity on the brane,” Nucl. Phys. B 605, 234 (2001) [hep-th/0009212].

[40] R. Jackiw and C. Rebbi, “Solitons With Fermion Number 1/2,” Phys. Rev. D 13 (1976) 3398.

[41] I. I. Kogan, S. Mouslopoulos, A. Papazoglou, G. G. Ross and J. Santiago, “A three three-brane universe: New phenomenology for the new millennium?,” Nucl. Phys. B 584 (2000) 313 [hep-ph/9912552].

[42] S. Mouslopoulos and A. Papazoglou, “$\sigma^+ - + +$ brane model phenomenology,” JHEP0011 (2000) 018 [hep-ph/0003207].

[43] I. I. Kogan, S. Mouslopoulos and A. Papazoglou, “A new bigravity model with exclusively positive branes,” Phys. Lett. B 501 (2001) 140 [hep-th/0011141].

[44] I. I. Kogan, S. Mouslopoulos, A. Papazoglou and G. G. Ross, “Multigravity in six dimensions: Generating bounces with flat positive tension branes,” hep-th/0107086.
[45] R. Gregory, V. A. Rubakov and S. M. Sibiryakov, “Opening up extra dimensions at ultra-large scales,” Phys. Rev. Lett. 84 (2000) 5928 [hep-th/0002072].

[46] I. I. Kogan and G. G. Ross, “Brane universe and multigravity: Modification of gravity at large and small distances,” Phys. Lett. B 485 (2000) 255 [hep-th/0003074].

[47] I. I. Kogan, S. Mouslopoulos, A. Papazoglou and G. G. Ross, “Multi-brane worlds and modification of gravity at large scales,” Nucl. Phys. B 595 (2001) 225 [hep-th/0006030].

[48] A. Karch and L. Randall, “Locally localized gravity,” Int. J. Mod. Phys. A 16 (2001) 780 [hep-th/0011156].

[49] A. Miemiec, “A power law for the lowest eigenvalue in localized massive gravity,” hep-th/0011160.

[50] M. D. Schwartz, “The emergence of localized gravity,” Phys. Lett. B 502 (2001) 223 [hep-th/0011177].

[51] A. Karch and L. Randall, “Localized Gravity in String Theory,” hep-th/0105108.

[52] A. Gorsky and K. Selivanov, Phys. Lett. B 485, 271 (2000) hep-th/0005066.

[53] G. Dvali and M. Shifman, “Domain walls in strongly coupled theories,” Phys. Lett. B 396 (1997) 64 [Erratum-ibid. B 407 (1997) 452] hep-th/9612128.

[54] G. Dvali, G. Gabadadze and M. Shifman, “(Quasi)localized gauge field on a brane: Dissipating cosmic radiation to extra dimensions?,” Phys. Lett. B 497 (2001) 271 hep-th/0010071.

[55] M. Shaposhnikov and P. Tinyakov, “Extra dimensions as an alternative to Higgs mechanism?,” hep-th/0102161.

[56] M. Tachibana, “Comment on Kaluza-Klein Spectrum of Gauge Fields in the Bigravity Model,” hep-th/0105180.

[57] K. Ghoroku and A. Nakamura, “Massive vector trapping as a gauge boson on a brane,” hep-th/0106147.

[58] C. Charmousis, R. Gregory and V. A. Rubakov, “Wave function of the radion in a brane world,” Phys. Rev. D 62, 067505 (2000) hep-th/9912160.

[59] L. Pilo, R. Rattazzi and A. Zaffaroni, “The fate of the radion in models with metastable graviton,” JHEP0007 (2000) 056 hep-th/0004028.

[60] A. Papazoglou, “Dilaton tadpoles and mass in warped models,” Phys. Lett. B 505, 231 (2001) hep-th/0102015.

[61] I. I. Kogan, S. Mouslopoulos, A. Papazoglou and L. Pilo, “Radion in Multibrane World,” hep-th/0105257.

[62] H. van Dam and M. Veltman, Nucl. Phys. B 22, 397 (1970).

[63] V.I. Zakharov, JETP Lett. 12 (1970) 312.

[64] A. I. Vainshtein, “To The Problem Of Nonvanishing Gravitation Mass,” Phys. Lett. B 39 (1972) 393.
[65] C. Deffayet, G. Dvali, G. Gabadadze and A. Vainshtein, “Nonperturbative continuity in graviton mass versus perturbative discontinuity,” hep-th/0106001.

[66] A. Higuchi, “Forbidden Mass Range For Spin-2 Field Theory In De Sitter Space-Time,” Nucl. Phys. B 282 (1987) 397.

[67] I. I. Kogan, S. Mouslopoulos and A. Papazoglou, “The m → 0 limit for massive graviton in dS4 and AdS4: How to circumvent the van Dam-Veltman-Zakharov discontinuity,” Phys. Lett. B 503 (2001) 173 [hep-th/0011138].

[68] M. Porrati, “No van Dam-Veltman-Zakharov discontinuity in AdS space,” Phys. Lett. B 498 (2001) 92 [hep-th/0011152].

[69] P. K. Townsend, “Cosmological Constant In Supergravity,” Phys. Rev. D 15 (1977) 2802.

[70] S. Deser and B. Zumino, “Broken Supersymmetry And Supergravity,” Phys. Rev. Lett. 38 (1977) 1433.