Recent Progress in Some Active Topics on Complex Networks

J Gu\textsuperscript{1,2}, Y Zhu\textsuperscript{3,2}, L Guo\textsuperscript{4}, J Jiang\textsuperscript{5}, L Chi\textsuperscript{2}, W Li\textsuperscript{2,*}, Q A Wang\textsuperscript{3}, and X Cai\textsuperscript{2}

\textsuperscript{1} School of Science, Jiangnan University, Wuxi 214122, China
\textsuperscript{2} College of Physical Science and Technology, Huazhong Normal University, Wuhan 430079, China
\textsuperscript{3} Laboratoire de Physique Statistique et Systemes Complexes, ISMANS, 44 ave. Bartholdi, Le Mans 72000, France
\textsuperscript{4} School of Mathematics and Physics, China University of Geosciences (Wuhan), Wuhan 430074, China
\textsuperscript{5} Research Center of Nonlinear Science and College of Mathematics and Computer Science of Wuhan Textile University, Wuhan 430200, China
\textsuperscript{*} Corresponding author
E-mail: liw@mail.ccnu.edu.cn

Abstract. Complex networks have been extensively studied across many fields, especially in interdisciplinary areas. It has since long been recognized that topological structures and dynamics are important aspects for capturing the essence of complex networks. The recent years have also witnessed the emergence of several new elements which play important roles in network study. By combining the results of different research orientations in our group, we provide here a review of the recent advances in regards to spectral graph theory, opinion dynamics, interdependent networks, graph energy theory and temporal networks. We hope this will be helpful for the newcomers of those fields to discover new intriguing topics.

1. Introduction

Complex network describes the interaction among the elements of complex systems, which consider particular nature of beings as vertices and perform specified function through interactions among vertices, such as scientific collaboration network\cite{1, 2}, language network \cite{3}, spreading network \cite{4, 5}, earthquake network \cite{6, 7}, stock network \cite{8, 9}, evolutionary network \cite{10, 11} and so on. The study of networks, network theory, is an useful tool for analyzing complex system, which is an emerging area of science \cite{12}, and has many applications, especially in the interdisciplinary areas \cite{13, 14, 15}. Even though there exist myriads of real networks in the world, scientists are more willing to believe most of networks are marked by several features and governed by countable laws. In recent years, many progresses have drawn continuously increasing attentions and brought us different understandings and new insights \cite{16, 17, 18, 19, 20, 21, 22, 23}.

One of the amazing points in the research of complex networks is that the objects in networks are not separated, but interact with each other through links. This interaction can be investigated from the viewpoint of the basic structural feature of complex networks. For example, spectral graph theory studies the properties of graphs through the spectra of their...
representing matrices [24, 25]. Graph energy is associated with the graph structure in the sense of building cost [26, 27]. Entropy is a general measure of probabilistic uncertainty and proves to be related to the mathematical structure of a complex network [28, 29, 30, 31, 32].

Besides the exploration on the structure of complex networks, the study of dynamics behaviors focuses on the interaction among individuals directly. Individual changes its state through the influence of its neighbors, and also exerts its influence to its neighbours to cause the possible change of their states. For example, opinion dynamics is one of the collective dynamical phenomena in the society [33]. The convergence (or divergence) of opinions among participants of a system is realized through the interactions among participants. Moreover, the interaction phenomenon sometimes does not only happen among individuals, but among independent networks. For example, the failure of nodes in interdependent networks generally not only leads to the failure of their neighbors, but leads to the failure of their dependent nodes in other networks, which may in turn cause further damages to the first network, resulting in cascading failures and catastrophic consequences in the entire system [34].

Additionally, interaction itself is also an interesting object. Take a simple example, in real-world complex systems, many relationships are (relatively) permanent, such as the existing collaboration relationship between two authors in scientific collaboration networks. However, there also exist some relationships which do not persist over long time, but only exist during short periods of time. This temporal effects of complex systems can be taken into account by the temporal networks [35] (also called time-varying graphs [36], or evolving graphs [37]).

2. Spectral graph theory

For a long time, graph theory has played a vital role in analyzing and understanding networks structures. All the topological information of one network can be found in its connectivity matrices. As a related object to matrices, eigenvalue is naturally used to explore the structural properties of a graph. The initial question is that how much information of a graph is contained in its eigenvalues sequence. This problem is described by spectral graph theory, which is the study of properties of a graph in relationship to eigenvalues of its associated matrices.

In the early days, matrix theory were used to analyze the adjacency matrix of graph. There is a large amount of literatures on algebraic aspects of spectral graph theory, such as Biggs [38], Cvetković [39] and Godsil et al. [40]. In a way, spectral graph theory can be considered as the well-developed theory of matrices, of which the purpose is to be related to graph theory and applications with its own characteristics.

In the recent years, many developments in spectral graph theory have the geometric favors on graph, as shown by Chung [24] and Jost [25]. An important development of spectral graph theory is the interaction with Riemannian geometry. For example, the Cheeger constant from Riemannian geometry has a discrete analogue, which can provide estimation for the first non-zero eigenvalues of the Laplacian matrix [41].

In network theory, besides modeling of complex networks, considerable attention has been given to the problems of capturing topological properties. In particular, it was proved that the important information on the topological properties of a network can be extracted from its spectra [24, 39]. For example, in 1955, Wigner introduced Wigner semicircle law, for certain special classes of random matrices [42, 43]. According to this, the distribution of eigenvalues of a large real symmetric random matrix follows a semicircle distribution. Specifically, the eigenvalues of adjacent matrix and Laplacian matrix of Erdős-Rényi random networks follow the semicircle distributions [44]. The eigenvalues distributions of Watts-Strogatz small-world networks and some of power-law networks are quite far form semicircle [44, 45]. Besides, Banerjee summed up Laplacian spectra for different types of networks and introduced a tentative classification scheme for empirical networks based on their Laplacian spectra [46]. He also revealed more understanding about Laplacian spectra and some particular dynamical processes on networks.
The search on distinguishing the unique characteristics for a given system and uncovering the universal features on large set of systems attracts great interests of scientists [47, 48, 49]. Three generic models support rough classification among networks, but can not provide further classification, especially for networks within the same class. Motif supplies a deep insight into the networks functional abilities associated with evolutionary process, of which analysis implies a similarity measurement based on a comparison of subgraphs frequencies on networks [50]. One problem with this method returns back to the same one shown in graph case, if motifs are too large, then the isomorphism problem appears again. Furthermore, for one important class of networks in our world, biological systems, there are lots of evidences showing the evolutionary process on them can result in their structure changes [51, 52]. The comparison of structures of biological networks may bring us new insights about evolutionary mechanism in biological systems. Without analyzing sequence data of gene or genome, this comparison, focusing on the whole network structure, may support us one way in understanding the functions and evolutions of biological system more conjointly and more intuitively. There is no doubt that graph spectra theory becomes one important and useful tool for analyze networks and brings crucial insights on hiding geometry information of networks.

3. Graph energy
The energy of a graph is an important parameter related to the building cost of the graph. Generally speaking, low energy means low cost, and then, which can be connected to the graph with better structure. Briefly, the energy of a graph is defined as the sum of the absolute values of the eigenvalues of its adjacency matrix. This definition, introduced by Gutman in the 1970s [26], is coming from chemistry where it was used to approximate the total π-electron energy of molecules [53]. Initially, graph energy did not draw many attentions of either mathematicians or physicists. Until the coming of the 21st century, extensive research about the graph energy started.

Researchers are striving to find the upper and lower bounds of energy for some special graphs. Firstly, for a graph on N vertices, the energy is less than or equal to \( \frac{N}{2}(1 + \sqrt{N}) \), where the upper bound is related to a strongly regular graph with some special parameters [63], which are equivalent to a certain type of Hadamard matrices. Haemers surveyed constructions of the corresponding Hadamard matrices and the related strongly regular graphs [64]. While the energy of a graph with given vertices N and edges m will be less than or equal to \( \sqrt{2mN} \) according to the Cauchy-Schwarz inequality with equality holding if and only if the graph is either empty or 1-regular [65]. And for a regular graph with given vertices and average degree, the energy will also have a largest value [66], which is deduced from the energy of a very general graph on N vertices with \( m \geq N/2 \) edges, whose upper bound for the energy is \( \frac{2mN}{N} + \sqrt{(N-1)[2m - (\frac{2m}{N})^2]} \).
where equality holds for some special networks, see Ref. [63]. The results we have mentioned are limited to a general graph. However, most lower or upper bounds are just corresponding to some special graphs, such as the bipartite graph, whose eigenvalues are symmetrical about zero.

The upper bounds for graph energy we have introduced are for the graph with given vertices. However, if a graph is just given the number of edges $m$ but with unknown vertices, the graph energy will also be restricted to a region $[2\sqrt{m}, 2m]$ [67]. The lowest value will be taken if a graph consists of a complete bipartite graph and some arbitrarily isolated vertices. Except for the energy, the largest eigenvalue of adjacency matrix for a general graph will be larger than or equal to the second moment of degree sequence [68]. And this equality will be held only by regular or semiregular bipartite graph. In 2003, Koolen and Moulton strictly proved that there is a upper bound for the energy of a general bipartite graph according to the number of vertices [69]. So, for all bipartite graphs with the same vertices, there will be a graph with special topological structure corresponding to the maximal energy.

We know that the lower or upper bounds for the graph energy are just connected to some special graphs. That means that the minimal or maximal energy graphs are very limited. However, this limitation is inapplicable for those graphs without the minimal or maximal energy. In 2004, Balakrishnan established the existence of equienergetic non-cospectral graphs [66]. He defined that two graphs on the same vertices are called equienergetic if they have the same energy. When two graphs own the same spectra, then they will be called cospectral. Take as a simple example, two simple graphs with two isolated edges and with a quadrangle that are equienergetic non-cospectral with the same energy 4. While the spectra of them are $\{-1, -1, +1, +1\}$, and $\{-2, 0, 0, 2\}$ respectively. There is no doubt that two cospectral graphs are also equienergetic.

In recent years, many researches have been studying the graph energy related to the regular graphs [70, 71]. This may be driven from the isotropic of regular graphs that can be easily analyzed from the theoretical view. Gutman firstly pointed out that the energy of $k$-regular graph $G$ on any $N$-vertices is greater than or equal to $N$ [72]. Therefore, a regular graph with degree $k > 0$ will be never hypo-energetic. If $G$ is also triangle-and quadrangle-free, the minimal value for the energy will depends on $k$. In Ref. [72], Gutman also showed a remarkable narrow interval for the energy of fullerene or nanotube which can be represented as a 3-regular graph without triangles and quadrangles [73].

4. Opinion dynamics with social diversity

Opinion dynamics is one of the collective dynamical phenomena in society. Convergence (or divergence) of opinions among participants of a debate is a very important social process [74], which is similar to the phase transition from disorder to order of Ising model in statistical physics. Here, we review some works about the effect of social diversity of agents in the opinion dynamics briefly.

In real society, the diversity of agents can be described from different aspects, including the social status, the psychological attitude and mental path. The agents are diverse in their wealth and social status and have diverse influence on others. For simplicity, Guan et al. considered the two types of agents $A$ and $B$ with different influence activity in the majority rule model and find that the role of the heterogeneous influence in the order-disorder transition [75]. For example, social leaders have stronger influence compared to normal populations and have a better chance to be followed [76]. And in social networks, the social power of agent may also be quantified as a proportion of its connection degree [76, 77, 78, 79, 80], since the social leader has many followers, namely vertices to which he/she is linked generally. Kandiah and Shepelyansky introduced the PageRank method to weight the nodes’ social power and proposed the PageRank opinion formation (PROF) model [81].
The diversity of agents can also be described according to the psychological attitude and mental path. In real life, people are always rational and make decisions through team collaboration or group debate. Then they update their opinions following the rule of peer pressure in the majority-rule model. However, some people are inflexible and contrarians, which play an important role in the opinion formation [82]. The inflexible reflects the inertia effect of human during making decision. In contrast to the floater agent who updates its opinion according to the rule of opinion model, inflexible agents keep their opinions always unchanged. Galam and Jacobs [82] studied the role of inflexible minorities in the democratic opinion model following the local majority firstly. Biswas and Sen introduced the inflexible in a model of binary opinions in which the updating of agent’s opinion according to the state of their neighboring domains with the probability $\rho$ [83]. Masuda et al. introduce the parameter $\epsilon$ quantifying the strength of the intrinsic preference or partisanship in the voter model [84]. When $\epsilon = 1$, each voter becomes a zealot that never changes opinion (i.e., inflexible) after aligning with its innate preference, when $\epsilon = 0$ reduces to the classic voter model. Here, the zealot effect of voters that never change opinion (i.e., the inflexible effect) has been studied in Refs. [85, 86]. Moreover, the opinion leaders also are considered as the special inflexible [87].

The other psychological attitude is the contrarians, who are the agents that deliberately decide to oppose the prevailing choice of others. Mobilia and Redner introduced a model of opinion formation according to the majority versus minority with the probability $p$ [88]. Borghesi and Galam introduced the contrarian effect in the Galam model with a constant density of contrarian $a$ for both opinions to study the chaotic, staggered and polarized dynamics [89]. Ding et al. introduced the application of game theory to model the opinion dynamics [90]. Furthermore, the contrarian effect has been introduced in the Sznajd model [91, 92] and q-voter model [93] through a stochastic parameter $p$. However, those previous works describe the contrarian effect as a constant stochastic parameter, which is too simple to describe the heterogeneous property of agents in social networks. Probably, the contrarian effect can also be determined by the present status of agents, such as the change of its local environment. Grauwin and Jensen propose a natural, thermal noise which allows for a small probability of interaction between agents when the opinion difference $\Delta > \epsilon$ in Deffuant model with the form of $p_{\text{conv}} = [1 + \exp((\Delta/\epsilon - 1)/T)]^{-1}$, where $T$ resembles a temperature and characterize the steepness of the convergence and called interaction noise that shows the contrarian effect indirectly [94].

5. The robustness study of interdependent networks
The robustness of interdependent networks has attracted a great deal of attention and understanding how robustness is affected by the interdependence is one of the main challenges faced when designing resilient infrastructures. For example, the robustness of critical infrastructure is one of the most important topics all over the world: specially, different kinds of infrastructure have become more and more interactive under modern technology, like communication and power grid systems, water and food supply systems [95].

In 2010, the seminal model of interdependent networks has defined a one-to-one correspondence between nodes of network $A$ and nodes of networks $B$ [34]. Suppose that two networks have the same number of nodes $N$. Each node $A_i (i = 1, 2, \ldots, N)$ in network $A$ depends on a functioning node $B_i$ in network $B$, and if node $A_i$ stops functioning owing to attack or failure, node $B_i$ stops functioning, and vice versa. Based on the generating function formalism and percolation theory, a first-order discontinuous phase transition was found in this model, which is totally different from the second-order continuous phase transition found in isolated networks. In Ref. [96], it was shown that, when the strength of coupling between networks is reduced, the percolation transition becomes second-order transition at a critical coupling strength, which enhanced the robustness of the system. In addition, the vulnerability of the
system could be increased by the clustering and assortativity within the network components \[97, 98\]. And a more realistic case with both strength of coupling and connectivity links between the coupled networks was studied in Ref. \[99\].

However, the assumption that one node in network \(A\) depends on only one node in network \(B\) is not valid sometimes. In 2011, Ref. \[100\] investigated a theoretical framework to study the robustness of two interdependent networks with multiple support dependent relations.

Real interdependent networks are usually not randomly coupled: for example, well-connected ports tend to couple to well-connected airports. So Ref. \[101\] proposed two inter-similarity measures between the interdependent networks and found that the more inter-similar the entire network is, the more robust the system is \[102\]. The case in which all pairs of interdependent nodes in both networks have the same degree was studied in Ref. \[103\]. In the real world, a network is not always attacked randomly. Ref. \[104\] investigated the robustness of fully and partially interdependent networks under targeted attack, respectively.

As most of real systems are not randomly but spatially embedded, it is reasonable to consider the factor of space limitation. In interdependent lattice networks, Ref. \[105\] found that there is a change from first to second order phase transition at the critical length of dependency links and Ref. \[106\] concluded that there is no critical dependency and any small fraction of interdependent nodes leads to an abrupt collapse. Moreover, transport process has been explored in coupled spatial networks in Ref. \[107\].

There are also some other considerations beyond above ones. A network of networks (NON) is taken into account in Refs. \[108, 109\] with more realistic consideration that there are more than two interdependent networks in many real systems. Antagonistic interaction \[110, 111\], autonomous nodes \[112\] and node-weighted \[113\] are considered in study of interdependent networks, respectively, which are leading to a better understanding of the effect of dependence between networks on the dynamics of interdependent networks.

6. Temporal networks

Temporal networks, compared to static networks, emphasize on the times *when* and *how long* contact events (edges) are present. The addition of time dimension provides a new sight into the framework of complex network theory. In temporal networks, structural properties and spreading dynamics are constrained by the time ordering of edges. Consequently, the concepts and methods for temporal networks need to be extended or redefined based on the top of static graphs.

In the research of temporal networks, aggregated static networks play a crucial role in understanding the temporal effect of network structure due to the lack of methods to uncover full contact patterns. We can aggregate temporal networks to a list of snapshots of static graphs if the topological characteristics are more relevant than the temporal properties, since it is usually easier to analyze static networks. A temporal network is described as \(G(V, E, t, \delta t)\), in which the contact event happens at time \(t\) and \(\delta t\) is its duration. An edge is formed in the aggregated network if there is at least one contact happening in time window \([t, t + \Delta t]\). It is noticed that the aggregation time interval \(\Delta t\) has critical consequences on the structural properties emerging from temporal networks. Many existing tools of static graphs have been adopted to analyze temporal networks. For instance, the error and attack vulnerability of temporal networks \[114\], optimal way for constructing static snapshots in temporal networks \[115\], and so on.

The adjacency matrix in a temporal network is defined as \(a(i, j, t) = 1\) if there exists an edge between node \(i\) and node \(j\) at time \(t\), and \(a(i, j, t) = 0\) otherwise. Given the adjacency matrix as a function of time, the path of temporal networks has two distinct definitions \[36\]. One corresponds to topological distance, which is analogous to shortest path length in static networks. The other is referred to temporal distance, which means the path of minimum duration to reach each other. Keep these time-respecting paths in mind, we can redefine temporal degree, betweenness,
centrality, closeness, component, motif, to name a few. However, not all the structural concepts have their counterparts in static graphs. Holme introduced the concept of reachability as a time-ordered chain of contact leading from one node to the other [116]. Lentz et al. proposed accessibility to measure temporal networks [117]. See Ref [35] for a review of more details.

In order to find the fundamental role of time ordering and duration, the other direction is to gain insights into the effects of different time correlations. Null temporal models are served as a reference, in which the original time sequences are randomized. It allows us to distinguish between different contributions to the time correlations coming from randomizing contact times, nodes, edges, or combinations of the three.

Besides of this, scientists have also interest in the following question: how will the temporal structures affect dynamical processes on temporal networks, and vice versa? For example, concerning contact events exhibit heterogeneous inter-event time distributions, bursty characters [118] have a strong influence on dynamical processes on temporal networks.

The other effort along this line is to control or to avoid the spread. Lee et al. have introduced the concepts of Recent and Weight to investigate the immune strategy [119]. Recent and Weight are specified as the most recent contact and the most often contact, respectively. The research towards spreading dynamics of temporal networks has grown in various aspects ranging from cascades [120] and random walks [121] to synchronization [122], and so on.

Despite the promoting results in temporal networks, this field is still in its infancy and there is not yet a general framework for describing and analyzing it. For example, what is the proper (or characteristic) aggregated time window to reflect network structure over time? Dynamical approaches remain rare in describing spreading processes. Standard models are still lacking for the study of temporal networks. By extending theory and analyzing data to account for temporal networks, we can approach a better understanding of time-stamped complex system.

Acknowledgments
This work was partially supported by the National Natural Science Foundation of China under grant No. 10975057, the Programme of Introducing Talents of Discipline to Universities under Grant No. B08033. J. Jiang wants to thank the support of the National Natural Science Foundation of China under grant No. 11405118. L. Chi wants to thank the support of the Fundamental Research Funds for the Central Universities.

References
[1] Newman M 2001 Scientific collaboration networks. I. Network construction and fundamental results Phys. Rev. E 64 016131.
[2] Hui Z, Cai X, Grenache J and Wang Q A 2011 Structure and collaboration relationship analysis in a scientific collaboration network Chin. Sci. Bull. 56 3702-06.
[3] Deng W, Wang D, Li W and Wang Q A 2011 English and Chinese language frequency time series analysis Chin. Sci. Bull. 56 3717-22.
[4] Pastor-Satorras R and Vespignani A 2001 Epidemic spreading in scale-free networks Phys. Rev. Lett. 86 3200.
[5] Gu J, Gao Z and Li W 2011 Modeling of epidemic spreading with white Gaussian noise Chin. Sci. Bull. 56 3683-88.
[6] Abe S and Suzuki N 2004 Scale-free network of earthquakes Europhys. Lett. 65 581.
[7] Abe S and Suzuki N 2012 Universal law for waiting internal time in seismicity and its implication to earthquake network Europhys. Lett. 97 49002.
[8] Eom C, Oh G, Jung W, Hawoong J and Seunghwan K 2009 Topological properties of stock networks based on minimal spanning tree and random matrix theory in financial time series Physica A 388 900-06.
[9] Menezes R and Dionísio A 2011 Globalization and long-run co-movements in the stock market for the G7: An application of VECM under structural breaks Chin. Sci. Bull. 56 3707-16.
[10] Seufert A and Schweitzer F 2007 Aggregate dynamics in an evolutionary network model Int. J. Mod. Phys. C 18 1659-74.
[11] Yang H and Wang B 2011 Universal role of migration in the evolution of cooperation Chin. Sci. Bull. 56 3693-96.
[12] Dorogovtsev N and Mendes F 2013 Evolution of networks: From biological nets to the Internet and WWW (Oxford: Oxford University Press).
[13] Newman M 2010 Networks: an introduction (Oxford: Oxford University Press).
[14] Cohen R and Havlin S 2010 Complex networks: structure, robustness and function (Cambridge: Cambridge University Press).
[15] Davidson E and Levin M 2005 Gene regulatory networks P Natl. Acad. Sci. USA 102 4935.
[16] Manshour P and Montakahb A 2014 Contagion spreading on complex networks with local deterministic dynamics Communications In Nonlinear Sc. 19 2414-22.
[17] Tayurski D and Lysogorskyi Y 2011 Quantum fluids in nanoporous media—Effects of the confinement and fractal geometry Chin. Sci. Bull. 56 3617-22.
[18] Dörfler F and Bullo F 2014 Synchronization in complex networks of phase oscillators: A survey. Automatica 50 1539-64.
[19] Liu Z 2011 Signal response amplification of scale-free networks Chin. Sci. Bull. 56 3623-29.
[20] Straeten E and Beck C 2011 Skewed superstatistical distributions from a Langevin and Fokker-Planck approach Chin. Sci. Bull. 56 3633-38.
[21] Li W, Luo Y, Wang Y and Cai A 2011 A mean-field Bak-Sneppen model with varying interaction strength Chin. Sci. Bull. 56 3639-42.
[22] Robledo A 2011 Laws of Zipf and Benford, intermittency, and critical fluctuations Chin. Sci. Bull. 56 3643-48.
[23] Liu Z, Guo L and Du J 2011 Nonextensivity and the q-distribution of a relativistic gas under an external electromagnetic field Chin. Sci. Bull. 56 3689-92.
[24] Chung F 1997 Spectral graph theory (American Mathematical Soc.)
[25] Jost J and Joy M 2001 Spectral properties and synchronization in coupled map lattices Phys. Rev. E 65 016201.
[26] Gutman I 1978 The energy of a graph Ber. Math. Statist. Sekt. Forsch. Graz. 103 22.
[27] Wang R, Djeanteg S and Kaabouchi A 2011 Investigation of an energy nonadditivity for nonextensive systems Chin. Sci. Bull. 56 3661-65.
[28] Tsallis C and Plastino A 1997 Power-law sensitivity to initial conditions-new entropic representation Chaos, Solitions and Fractals 8 885-91.
[29] Jiang J, Wang R, Pezeril M and Wang Q A 2011 Application of varentropy as a measure of probabilistic uncertainty for complex networks Chin. Sci. Bull. 56 3677-82.
[30] Bi Q and Liu J. Exploring non-equilibrium statistical ensembles Chin. Sci. Bull. 56 3654-60.
[31] Ou C and Chen J 2011 Generalized entropies under different probability normalization conditions Chin. Sci. Bull. 56 3649-53.
[32] Zheng L and Li W 2011 Thermoequilibrium statistics for a finite system with energy nonextensivity Chin. Sci. Bull. 56 3666-70.
[33] Chi L 2011 Binary opinion dynamics with noise on random networks Chin. Sci. Bull. 56 3630-32.
[34] Buldyrev V, Parshani R, Paul G, Stanley E and Havlin S 2010 Catastrophic cascade of failures in interdependent networks Nature 464 1025-28.
[35] Holme P and Saramaki J 2012 Temporal networks Physics Reports 519 97-125.
[36] Casteigts A, Flocchini P, Quattrociocchi W and Santoro N 2012 Time-varying graphs and dynamic networks Int. J. of Parallel Emergent and Distributed Systems 27 346-59.
[37] Ferreira A 2004 Building a reference combinatorial model for MANETs IEEE Network 18 24-29.
[38] Biggs N 1993 Algebraic graph theory (Cambridge: Cambridge university press).
[39] Cvetkovic M, Doob M and Sachs H 1980 Spectra of graphs: Theory and application (New York: Academic press)
[40] Godsil D and Royle G 2001 Algebraic graph theory (New York: Springer).
[41] Cheeger J 1970 A lower bound for the smallest eigenvalue of the Laplacian Problems in analysis 625 195-99.
[42] Wigner E 1993 The Collected Works of Eugene Paul Wigner: Characteristic vectors of bordered matrices with infinite dimensions I (Berlin Heidelberg: Springer) 524-40.
[43] Wigner E 1993 The Collected Works of Eugene Paul Wigner: Characteristic vectors of bordered matrices with infinite dimensions II (Berlin Heidelberg: Springer) 541-45.
[44] Chung F, Lu L and Vu V 2003 Spectra of random graphs with given expected degrees P Natl. Acad. Sci. USA 100 6313-18.
[45] Farkas I, Derényi I, Barabási A and Vicsek T 2001 Spectra of real-world graphs: Beyond the semicircle law Phys. Rev. E 64 026704.
[46] Banerjee A and Jost J 2008 Spectral plot properties: Towards a qualitative classification of networks Networks and Heterogeneous Media 3 395-11.
[47] Erdős P and Rényi A 1959 On random graphs I Publ. Math. Debrecen 6 290-97.
[48] Watts D and Strogatz S 1998 Collective dynamics of small-world networks Nature 393 440-42.
[49] Barabási A and Albert R 1999 Emergence of scaling in random networks Science 286 509-12.
[50] Milo R, Shen-Orr S, Itzkovitz S, Kashtan N, Chklovskii D and Alon U 2002 Network motifs: simple building blocks of complex networks Science 298 824-27.
[51] Ispolatov I, Krapivsky L and Yuryev A 2005 Duplication-divergence model of protein interaction network Phys. Rev. E 71 061911.
[52] Kim J, Kim I, Han S, Bowie J and Kim S 2012 Network rewiring is an important mechanism of gene essentiality change Scientific reports 2 900.
[53] Gutman I 1977 Acyclic systems with extremal Hückel \( \pi \)-electron energy Theor. Chim. Acta. 45 79-87.
[54] Hall G 1955 The bond orders of alternant hydrocarbon molecules Proc. Roy. Soc. 229 251-59.
[55] Coulson C 1940 On the calculation of the energy in unsaturated hydrocarbon molecules. Proc. Cambridge Phil. Soc. 36 201-03.
[56] Ruedenberg K 1961 Quantum mechanics of mobile electrons in conjugated bond systems. III. Topological matrix as generatrix of bond orders J. Chem. Phys. 54 1884-91.
[57] McClelland B 1971 Properties of the latent roots of a matrix: The estimation of \( \pi \)-electron energies. J. Chem. Phys. 54 640-43.
[58] Gutman I 1999 Hyperenergetic molecular graphs J. Serb. Chem. Soc. 64 199-05.
[59] Gutman I and Radenković S 2007 Hypoenergetic molecular graphs Indian J. Chem. 46 1733-36.
[60] So W, Robbiano M, Nair A and Gutman I 2010 Applications of a theorem by Ky Fan in the theory of graph energy Lin. Algebra Appl. 432 2153-69.
[61] Li X and Ma H 2009 All hypoenergetic graphs with maximum degree at most 3 Lin. Algebra Appl. 431 2127-33.
[62] Hou Y and Gutman I 2001 Hyperenergetic line graphs. MATCH Commun Math. Comput. Chem. 43 29-39.
[63] Koolen J 2001 Maximal Energy Graphs Adv. Appl. Math. 26 47-52.
[64] Haemers W 2008 Strongly regular graphs with maximal energy Lin. Algebra Appl. 429 2719-23.
[65] McCelland B 1971 Properties of the latent roots of a matrix: The estimation of \( \pi \)-electron energies J. Chem. Phys. 54 640-43.
[66] Balakrishnan R 2004 The energy of a graph Lin. Algebra Appl. 387 287-95.
[67] Li X, Shi Y and Gutman I 2012 Graph Energy (New York: Springer).
[68] Cvetković Cvetkovic D, Doob M and Sachs H 1980 Spectra of Graphs: Theory and Application (New York: Academic).
[69] Koolen J H and Moulton V 2003 Maximal Energy Bipartite Graphs. Graphs and Combinatorics 19 131-35.
[70] Alinaghipour F and Ahmadi B 2008 On the energy of complement of regular line graphs MATCH Commun. Math. Comput. Chem. 60 427-34.
[71] Li X, Li Y and Shi Y 2010 Note on the energy of regular graphs Lin. Algebra Appl. 432 1144-46.
[72] Gutman I, Firoozabadi S, delaPeña J and Rada J 2007 On the energy of regular graphs MATCH Commun. Math. Comput. Chem. 57 435-42.
[73] Ecklund P 1996 Science of Fullerenes and Carbon Nanotubes (London: Academic Press).
[74] Gandica Y, Castillo-Mussot M, Vázquez G and Rojas S 2010 Continuous opinion model in small-world directed networks Physica A 389 5864-70.
[75] Guan J, Wu Z and Wang Y 2007 Effects of inhomogeneous influence of individuals on an order-disorder transition in opinion dynamics Phys. Rev. E 76 042102.
[76] Jailili M 2013 Social power and opinion formation in complex networks Physica A 392 959-66.
[77] Guo L, Gu J and Luo Z 2013 How much information is needed to be the majority during the binary-state opinion formation? Physica A 392 4373-79.
[78] Guo L and Cai X 2009 Continuous opinion dynamics in complex networks Commun. Comput. Phys 5 1045-53.
[79] Yang H, Wu Z, Zhou C, Zhou T and Wang B 2009 Effects of social diversity on the emergence of global consensus in opinion dynamics Phys. Rev. E 80 046108.
[80] Yang H and Wang B 2010 Effects of social diversity on the evolutionary game and opinion dynamics Physics Procedia 3 1859-65.
[81] Kandiah V and Shepelyansky D 2012 PageRank model of opinion formation in social networks Physica A 391 5779-5793.
[82] Galam S and Jacobs F 2007 The role of inflexible minorities in the breaking of democratic opinion dynamics Physica A 381 366-76.
[83] Biswas S and Jacobs F 2007 Model of binary opinion dynamics: coarsening and effect of disorder Phys. Rev. E 80 027101.
[84] Masuda N, Gilbert N and Redner S 2010 Heterogeneous voter models Phys. Rev. E 82 010103.
[85] Mobilia M, Petersen A and Redner S 2007 On the role of zealotry in the voter model J Stat. Mech-Theory E. 8 P08029.
[86] Mobilia M 2003 Does a single zealot affect an infinite group of voters? Phys. Rev. Lett. 91 028701.
[87] Ellero A, Fasano G and Sorato A 2013 Stochastic model of agent interaction with opinion leaders Phys. Rev. E 87 042806.
[88] Mobilia M and Redner S 2003 Majority versus minority dynamics: phase transition in an interacting two-state spin system Phys. Rev. E 68 046106.
[89] Borghesi C and Galam S 2006 Chaotic, staggered, and polarized dynamics in opinion forming: the contrarian effect Phys. Rev. E 73 066118.
[90] Ding F, Liu Y, Shen B and Si X 2010 An evolutionary game theory of binary opinion formation Physica A 389 1745–52.
[91] Schneider J 2004 The influence of contrarians and opportunists on the stability of a democracy in the Sznajd model Int. J. Mod. Phys. C 15 659-74.
[92] Lama M, Loopej J and Wio H 2005 Spontaneous emergence of contrarian-like behaviour in an opinion spreading model Europhys. Lett. 72 851.
[93] Nyczka P, Sznajd-Weron K and Cislo J 2012 Phase transitions in the q-voter model with two types of stochastic driving entities Phys. Rev. E 85 066113.
[94] Grauwin D and Jensen P 2012 Opinion group formation and dynamics: Structures that last from nonlasting entities Phys. Rev. E 85 066113.
[95] Rosato V, Issacharoff L, Tirritoce F, Meloni S, De Porcellinis S and Setola R 2008 Modelling interdependent infrastructures using interacting dynamical models Int. J. Critical Infrastructures 4 63-79.
[96] Parshani R, Buldyrev S and Havlin S 2010 Interdependent networks: Reducing the coupling strength leads to a change from first to second order percolation transition Phys. Rev. Lett. 105 048701.
[97] Huang X, Shao S, Wang H, Buldyrev S and Stanley H 2013 The robustness of interdependent clustered networks Europhys. Lett. 101 18003.
[98] Zhou D, Stanley H, D Agostino G and Scala A 2012 Assortativity decreases the robustness of interdependent networks Phys. Rev. E 86 066103.
[99] Hu Y, Ksherim B, Cohen R and Havlin S 2011 Percolation in interdependent and interconnected networks: Abrupt change from second to first order transition Phys. Rev. E 84 066116.
[100] Shao J, Buldyrev S, Havlin S and Stanley H 2011 Cascade of failures in coupled network systems with multiple support-dependence relations Phys. Rev. E 83 036116.
[101] Parshani R, Rozenblat C, Ietri D, Ducruet C and Havlin S 2010 Inter-similarity between coupled networks Europhys. Lett. 92 68002.
[102] Gu C, Zou S, Xu X, Ou Y, Jiang Y and He D 2011 Onset of cooperation between layered networks Phys. Rev. E 84 026101.
[103] Buldyrev S, Shere N and Cwilich G 2011 Interdependent networks with identical degrees of mutually dependent nodes Phys. Rev. E 83 016112.
[104] Dong G, Gao J, Tian L, Du R and He Y 2012 Percolation of partially interdependent networks under targeted attack Phys. Rev. E 85 016112.
[105] Li W, Bashan A, Buldyrev S, Stanley H and Havlin S 2012 Cascading failures in interdependent lattice networks: The critical role of the length of dependency links Phys. Rev. Lett. 108 228702.
[106] Bashan A, Berezin Y, Buldyrev S and Havlin S 2013 The extreme vulnerability of interdependent spatially embedded networks Nature Physics 9 667–72.
[107] Morris R and Barthelemy M 2012 Transport on coupled spatial networks Phys. Rev. Lett. 109 128703.
[108] Dong G, Gao J, Du R, Tian L, Stanley H and Havlin S 2013 Robustness of network of networks under targeted attack. Phys. Rev. E 87 052804.
[109] Cellai D, Lópej E, Zhou J, Gleeson J and Bianconi G 2013 Percolation in multiplex networks with overlap Phys. Rev. E 88 052811.
[110] Zhao K and Bianconi G 2013 Percolation on interacting, antagonistic networks. J Stat. Mech-Theroy. E. 5 P05005.
[111] Zhao K and Bianconi G 2013 Percolation on interdependent networks with a fraction of antagonistic interactions J Stat. Phys. 152 1069-83.
[112] Schneider C, Yazdani N, Araújo N, Havlin S and Herrmann H 2013 Towards designing robust coupled networks Scientific reports 3 169.
[113] Wiedermann M, Donges J, Heitzig J and Kurths J 2013 Node-weighted interacting network measures improve the representation of real-world complex systems Europhys. Lett. 102 28007.
[114] Trajanovski S, Scellato S and Leontiadis I 2012 Error and attack vulnerability of temporal networks Phys. Rev. E 85 066105.
[115] Holme P 2013 Epidemiologically optimal static networks from temporal network data PLOS Comput. Biol. 9 e1003142.
[116] Holme P 2005 Network reachability of real-world contact sequences Phys. Rev. E 71 046119.
[117] Lentz H, Selhorst T and Sokolov I 2013 Unfolding accessibility provides a macroscopic approach to temporal
networks Phys. Rev. Lett. 110 118701.

[118] Barabási A 2005 The origin of bursts and heavy tails in human dynamics Nature 435 207-211.

[119] Lee S, Rocha L, Liljeros F and Holme P 2012 Exploiting temporal network structures of human interaction to effectively immunize populations PloS one 7 e36439.

[120] Karimi F and Holme P 2013 Threshold model of cascades in empirical temporal networks Physica A 392 3476-83.

[121] Starnini M, Baronchelli A, Barrat A and Pastor-Satorras R 2012 Random walks on temporal networks Phys. Rev. E 85 025101.

[122] Fujiwara N, Kurths J and Díaz-Guilera A 2011 Synchronization in networks of mobile oscillators Phys. Rev. E 83 025101.