Entanglement and correlation functions following a local quench: a conformal field theory approach

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Abstract. We show that the dynamics resulting from preparing a one-dimensional quantum system in the ground state of two decoupled parts, then joined together and left to evolve unitarily with a translational invariant Hamiltonian (a local quench), can be described by means of quantum field theory. In the case when the corresponding theory is conformal, we study the evolution of the entanglement entropy for different bipartitions of the line. We also consider the behavior of one- and two-point correlation functions. All our findings may be explained in terms of a picture, that we believe to be valid more generally, whereby quasiparticles emitted from the joining point at the initial time propagate semiclassically through the system.

Keywords: correlation functions, conformal field theory (theory), entanglement in extended quantum systems (theory)

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The understanding of the degree of entanglement in extended quantum systems has prompted an enormous effort in a field at the border of condensed matter physics, quantum information theory, and quantum field theory. It would take too long to mention all the topics and problems addressed so far: we refer the reader to a recent thorough review and extensive bibliography [1].

Several measures have been introduced to quantify the entanglement in an extended system [1]. Among them we consider here only the entanglement entropy, which is defined as follows. Suppose a whole system is in a pure quantum state $|\Psi\rangle$, with density matrix $\rho = |\Psi\rangle\langle\Psi|$, and an observer A measures only a subset $A$ of a complete set of commuting observables, while another observer B may measure the remainder. A’s reduced density matrix is $\rho_A = \text{Tr}_B \rho$. The entanglement entropy is just the von Neumann entropy

$$S_A = -\text{Tr}_A \rho_A \log \rho_A$$

associated with this reduced density matrix. For an unentangled product state, $S_A = 0$. Conversely, $S_A$ should be a maximum for a maximally entangled state. One of the most striking features of the entanglement entropy is its universal behavior displayed at and close to a quantum phase transition. In fact, close to a quantum critical point, where the
correlation length $\xi$ is much larger than the lattice spacing $a$, the long-distance behavior of the correlations in the ground state of a quantum spin chain are effectively described by a 1+1-dimensional quantum field theory. At the critical point, where $\xi$ diverges, the field theory, if Lorentz invariant, is also a conformal field theory (CFT). In the latter case, it has been shown that, if $A$ is an interval of length $\ell$ in an infinite chain, $S_A \simeq (c/3) \log \ell$ [2–4], where $c$ is the central charge of the corresponding CFT. When the correlation length $\xi$ is large but finite (more precisely $\xi \ll \ell$), it has been shown that, increasing $\ell$, $S_A$ saturates [3] to $S_A \simeq (c/3) \log \xi$ [4,5].

Recently the interest in the properties of entanglement has been extended to understanding its dynamical behavior. The natural question is how the entanglement propagates through the system when it is prepared in a state that is not an eigenstate and then is left to evolve in the absence of any dissipation. The most common situation studied so far concerns a sudden quench of some coupling of the model Hamiltonian [6–10]. The main motivation for the large number of studies of this dynamics is that it may actually be realized and measured in ultra-cold atomic systems [11]. For a complete list of references on the subject we refer to our earlier paper [12].

Here we consider a different situation, known in the literature as local quench [13]. We will concentrate on the case when a physical one-dimensional system (e.g. a spin chain) is physically cut into two parts that are in their own ground state. We then join the two halves at a given time $t = 0$ and study the subsequent evolution according to a translational invariant Hamiltonian. Here, the entanglement entropy is the most natural quantity to be studied because the two halves are clearly unentangled for $t < 0$ whereas the initial energy differs from that of the ground state only by a finite amount. This differs from the case of a global quench, when the energy of the initial state above the ground state is always extensive, and correspondingly the entanglement can be extensively larger. We will only consider gapless models that are described asymptotically by a boundary CFT, since in this case we can use powerful analytic results [14,15].

The paper is organized as follows. In section 2 we briefly recall the CFT approach to the entanglement entropy. We outline the setup for the local quench which is then applied to different situations in the following section 3. Then we consider the time evolution of correlation functions in section 4. In section 5 we describe how this approach can be generalized to the case with two joining points and we derive a heuristic solution. Finally, we conclude the paper with a discussion of open problems and possible generalizations in section 6.

2. The CFT approach to entanglement entropy and a local quench

2.1. Entanglement entropy and CFT

The entanglement entropy $S_A$ defined by equation (1) can be studied in a general quantum field theory through the replica trick [4]

$$S_A = -\left. \frac{\partial}{\partial n} \text{Tr} \rho^A_n \right|_{n=1}. \quad (2)$$

This is particular useful in a CFT because, when $A$ consists of disjoint intervals with $N$ boundary points with $B$, $\text{Tr} \rho^A_n$ transforms under a general conformal transformation as
the $N$-point function of a primary field $\Phi_n$ with conformal dimension \[4\]

\[x_n = \frac{c}{12} \left( n - \frac{1}{n} \right), \quad (3)\]

where $c$ is the central charge of the underlying CFT.

In particular this implies that the entanglement entropy of a slit of length $\ell$ in an infinite system is given by

\[\text{Tr} \rho^n_A = c_n \left( \frac{\ell}{a} \right)^{-2x_n} \Rightarrow S_A = \frac{c}{3} \log \frac{\ell}{a} + c'_1, \quad (4)\]

where $a$ is an ultraviolet cutoff (for example, in a spin chain it is the lattice spacing). The constants $c_n$ are also non-universal and so is the derivative for $n = 1$, $c'_1$, entering in $S_A$. However, in any particular model, $a$ and $c'_1$ can be fixed unambiguously by specific requirements (see e.g. \[5\]).

Another important result we will use in the following is the entanglement of the slit $A = [0, \ell]$ in a semi-infinite line with specific boundary conditions at $r = 0$, that is given by \[4\]

\[\text{Tr} \rho^n_\Lambda = \tilde{c}_n \left( \frac{2\ell}{a} \right)^{-x_n} \Rightarrow S_A = \frac{c}{6} \log \frac{2\ell}{a} + \tilde{c}'_1. \quad (5)\]

In analogy to the $c_n$, the constants $\tilde{c}_n$ are not universal and depend on the boundary condition at $r = 0$. However, comparing the finite temperature result of \[4\] with the standard thermodynamic entropy for systems with boundaries \[16\] we have \[4, 17, 18\]

\[\tilde{c}'_1 - c'_1 / 2 = \log g, \quad (6)\]

where $\log g$ is the boundary entropy first introduced by Affleck and Ludwig \[16\]. We recall that $g$ is universal and depends only on the boundary CFT.

### 2.2. CFT for global quenches

CFT is also able to predict the time-dependence of the entanglement entropy after a global quench \[6\]. We briefly summarize these results here, because some of them will be useful in understanding the local case. The density matrix has the path integral representation \[6\]

\[\langle \psi''(r') | \rho(t) | \psi'(r') \rangle = Z_1^{-1} \langle \psi''(r') | e^{-itH-\epsilon H} | \psi_0(r) \rangle \langle \psi_0(r) | e^{+itH-\epsilon H} | \psi'(r') \rangle, \quad (7)\]

where we included damping factors $e^{-\epsilon H}$ in such a way as to make the path integral absolutely convergent. Each of the factors may be represented by an analytically continued path integral in imaginary time: the first one over fields $\psi(r, \tau)$ which take the boundary values $\psi_0(r)$ on $\tau = -\epsilon - it$ and $\psi''(r)$ on $\tau = 0$, and the second with $\psi(r, \tau)$ taking the values $\psi'(r)$ on $\tau = 0$ and $\psi_0(r)$ on $\tau = \epsilon - it$. When $H$ is critical and the field theory is a CFT, under the renormalization group any translationally invariant boundary condition is supposed to flow into a boundary fixed point, satisfying conformal boundary conditions. Thus we may assume that the state $| \psi_0 \rangle$ corresponds to such boundary conditions on sufficiently long length scales. Thus, for real $\tau$, the strip geometry described above may be obtained from the upper half-plane (where the entanglement entropy is known from \[4\])

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by the conformal mapping \( w = (2\epsilon/\pi) \log z \). In this way one easily get the result for imaginary time. The real time evolution is obtained by continuing \( \tau = \epsilon + it \). The final result is \([6]\)

\[
S_A(t) \simeq \begin{cases} 
\frac{\pi ct}{6\epsilon} & t < \ell/2, \\
\frac{\pi c \ell}{12\epsilon} & t > \ell/2.
\end{cases}
\] (8)

From this we note that \( \epsilon \) enters in the calculation in an essential way. A careful analysis \([19,12]\) shows that in fact it corresponds to the correlation length in the initial state and so it is not only a useful tool, but a physically important parameter.

The fact that \( S_A(t) \) increases linearly until it saturates at \( t^* = \ell/2 \) has a simple interpretation in terms of quasiparticle excitations emitted from the initial state at \( t = 0 \) and freely propagating with velocity \( v = 1 \). This phenomenon is very general and holds even for non-critical systems, and it has been confirmed by exactly integrable dynamics and numerics (see e.g. \([6,7,13]\)) However, since in lattice models there are particles moving more slowly than \( v \), after \( t^* \) the entropy does not saturate abruptly, but is a slowly increasing function of the time. The same picture is valid for the correlation functions. Firstly incoherent quasiparticles arriving a given point from well-separated sources cause relaxation of (most) local observables towards their ground state expectation values. Secondly, entangled quasiparticles (emitted from an initially correlated region) arriving at the same time \( t \) induce correlations between local observables. In the case where they travel at a unique speed \( v \), therefore, there is a sharp ‘horizon’ effect: the connected correlations do not change significantly from their initial values until time \( t \sim |r|/2v \). After this they rapidly saturate to time-independent values. For large separations, these decay exponentially differently from the power law dependence in the ground state. Also for correlation functions, this picture has been shown to be valid in several models (see e.g. \([19,12]\), \([20]\)–\([23]\)).

It is clear that a very similar interpretation should apply also for local quenches, but with the fundamental difference that now the quasiparticle excitations are emitted only from the point where the quench happened and not from everywhere.

2.3. CFT setup for local quenches

Suppose we physically cut a spin chain at the boundaries between two subsystems \( A \) and \( B \), and prepare a state where the individual pieces are in their respective ground states. In this state the two subsystems are completely unentangled, and its energy differs from that of the ground state by only a finite amount. Let us join up the pieces at time \( -t \) and watch the system evolve up to \( t = 0 \).

The procedure for the global quenches does not apply because the initial state is not translationally invariant and will not flow under the renormalization group toward a conformally invariant boundary state. One may try to handle the problem introducing proper boundary condition changing operators (see e.g. \([24]\)). However we prefer to take a different approach. In fact, we can simply represent the corresponding density matrix in terms of path integral on a modified world-sheet. The physical cut corresponds to having a slit parallel to the (imaginary) time axis, starting from \(-\infty\) up to \( \tau_1 = -\epsilon - it \) (the time when the two pieces have been joined), and analogously the other term of the density

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Figure 1. Space–time region for the density matrix (left) mapped to the half-plane (right) by means of equation (9). \( z_1 = i\tau \) and \( z_2 = i\tau + \ell \) during the computation and in the end \( \tau \to it \).

matrix, like in equation (7), gives a slit from \( \tau_2 = \epsilon - it \) to \( +\infty \). Again we introduced the regularization factor \( \epsilon \), that we will interpret \textit{a posteriori}.

For computational simplicity we will consider the translated geometry with two cuts starting at \( \pm i\epsilon \) and an operator inserted at imaginary time \( \tau \). This should be considered real during the course of all the computation, and only at the end can be analytically continued to \( it \). This plane with the two slits is pictorially represented on the left of figure 1, where \( i\tau \) corresponds to \( z_1 \). As shown in the same figure, the \( z \)-plane is mapped into the half-plane \( \text{Re} w > 0 \) by means of the conformal mapping

\[
 w = \frac{z}{\epsilon} + \sqrt{\left(\frac{z}{\epsilon}\right)^2 + 1} \quad \text{with inverse } z = \epsilon \frac{w^2 - 1}{2w}. \tag{9}
\]

On the two slits in the \( z \)-plane (and so on the imaginary axis in the \( w \)-plane) conformal boundary conditions compatible with the initial state must be imposed. For example, in the most natural situation when the boundary ‘spins’ are left free, we require free boundary conditions. In contrast, when the boundary spins are supposed to stay in a particular state we require fixed boundary conditions.

Note that this setting is valid in any dimension when the system is prepared in two spatially divided halves. In this case one can try to tackle the problem with the methods of boundary critical phenomena [25], but this will be extremely difficult, if not impossible, since conformal invariance is far less powerful.

3. Entanglement entropy

In this section we consider the time evolution of the entanglement entropy after the local quench. We consider two half-chains joined together at the point \( r_D = 0 \) at a given time. The various subcases we consider correspond to different spatial partitions of the system among which we calculate the entanglement. We consider the four different situations depicted in figure 2.

We start with the more natural division, considering the entanglement entropy between the two parts in which the system was divided before the quench (case I). As case II we consider the entanglement of the region \( r > \ell \) with \( r < \ell \). This will allow us to identify simple horizon effects. To highlight the interaction of the two previous effects we then consider the entanglement of the part \( B = [0, \ell] \) with the rest (case III). Finally,
we consider as case IV the most general slit $B = [\ell_1, \ell_2]$. Clearly cases I and III are particular choices of cases II and IV respectively, and can be derived as simple limits. However, we prefer to present the results in such order because the physical effects and their interpretation will emerge in a more natural way.

### 3.1. Case I: entanglement of the two halves

This is the case when $B$ is the positive real axis and $A$ is the negative real axis. $\text{Tr} \rho^n_A$ transforms like a one-point function that in the $w$-plane is $[2 \text{Re } w_1]^{-x_n}$. Thus in the $z$-plane at the point $z_1 = (0, i\tau)$ we have

$$\langle \Phi_n(z_1) \rangle = \tilde{c}_n \left( \frac{d w}{d z} \bigg|_{z_1} \right)\frac{a}{[2 \text{Re } w_1]} x_n$$

that using

$$\epsilon w_1 = i\tau + \sqrt{\epsilon^2 - \tau^2} \quad, \quad \left| \frac{d w}{d z} \right|_{z_1} = \frac{\epsilon}{\sqrt{\epsilon^2 - \tau^2}}$$

becomes

$$\langle \Phi_n \rangle = \tilde{c}_n \left( \frac{a \epsilon / 2}{\epsilon^2 - \tau^2} \right)^{x_n}.$$  \hspace{1cm} (12)

Continuing this result to real time $\tau \rightarrow it$ we obtain

$$\langle \Phi_n(t) \rangle = \tilde{c}_n \left( \frac{a \epsilon / 2}{\epsilon^2 + t^2} \right)^{x_n}.$$ \hspace{1cm} (13)

Finally, using the replica trick to find the entanglement entropy, we have

$$S_A = -\frac{\partial}{\partial n} \text{Tr} \rho^n_A \bigg|_{n=1} = \frac{c}{6} \log \frac{t^2 + \epsilon^2}{a \epsilon / 2} + \tilde{c}_1'. $$ \hspace{1cm} (14)

There are two main pieces of information that we can extract from this result, coming from the long and the short time behavior. For $t \gg \epsilon$ we have

$$S_A(t \gg \epsilon) = \frac{c}{3} \log \frac{t}{\epsilon} + k_0,$$ \hspace{1cm} (15)

i.e. the leading long time behavior is only determined by the central charge of the theory in analogy with the ground state value for a slit. This could result in a quite powerful tool to extract the central charge in time-dependent numerical simulations. This behavior resembles the slow increasing of $S_A$ observed numerically in [26] (unfortunately these data were never fitted, to understand the quantitative predictive power of our formula). The constant $k_0$ is given by $k_0 = \tilde{c}_1' + (c/6) \log(2a/\epsilon)$.

Figure 2. The four different bipartitions of the line we consider here.
The behavior for short time allows us instead to fix the regulator $\epsilon$ in terms of the non-universal constant $\tilde{c}'_1$. In fact we have

$$S_A(t = 0) = \frac{c}{6} \log \frac{2\epsilon}{a} + \tilde{c}'_1 = 0 \Rightarrow \epsilon = \frac{a}{2} e^{-6\tilde{c}'_1/c}.$$ (16)

Note that, even if non-universal, for a given lattice model the constant $\tilde{c}'_1$ can be fixed only by ground state quantities. Consequently equation (14) has no free dynamical parameter.

The parameter $\epsilon$ depends in a specific manner on the boundary contribution to the entanglement $\tilde{c}'_1$. This is completely different from what happens for a global quench where the equivalent regulator is connected to the correlation length in the initial state [19,12]. Furthermore, the order of magnitude of $\epsilon$ is fixed by the lattice spacing $a$ (i.e., the UV cutoff) and so considering the limit of times and lengths larger than $\epsilon$ is equivalent to the standard condition to apply the field theory for distances larger than $a$.

### 3.2. Case II: decentered defect

Let us now consider the entanglement of the region $r > \ell$ with the rest of the system. In this case $\text{Tr} \rho^n = \rho^n_A$ is equivalent to the one-point function in the plane $z$ at the point $z_2 = \ell + i\tau$ as in figure 1. Under the conformal mapping (1) this point goes into

$$\epsilon w_2 = \ell + i\tau + \sqrt{\epsilon^2 + (\ell + i\tau)^2} \equiv \ell + i\tau + \rho e^{i\theta}$$ (17)

with

$$\rho^2 = \sqrt{(\epsilon^2 + \ell^2 - \tau^2)^2 + 4\ell^2\tau^2}, \quad \theta = \frac{1}{2} \arctan \frac{2\ell\tau}{\sqrt{\epsilon^2 + \ell^2 - \tau^2}}$$ (18)

and

$$\left| \frac{dw}{dz} \right|_{z_2} = \frac{\sqrt{(\ell + \rho \cos \theta)^2 + (\tau + \rho \sin \theta)^2}}{\epsilon \rho}. \quad \text{(19)}$$

Thus $\langle \Phi_n \rangle$ is given by equation (12) with $w_1 \rightarrow w_2$, resulting in

$$\langle \Phi_n \rangle = \tilde{c}_n \left( \alpha \frac{\sqrt{(\ell + \rho \cos \theta)^2 + (\tau + \rho \sin \theta)^2}}{2\rho(\ell + \rho \cos \theta)} \right)^{x_n}. \quad \text{(20)}$$

The real time evolution for $t, \ell \gg \epsilon$ is obtained by analytically transforming $\tau \rightarrow it$. The calculation can seem very cumbersome, but it is greatly simplified by the fact that in this limit we have $\rho^2 \rightarrow |\ell^2 - t^2|$ and $\rho \cos \theta \rightarrow \max[\ell, t], \rho \sin \theta \rightarrow \text{im} \min[\ell, t]$. Care must also be taken when the zeroth order in $\epsilon$ is vanishing. Finally, we have

$$\sqrt{(\ell + \rho \cos \theta)^2 + (\tau + \rho \sin \theta)^2} \rightarrow \begin{cases} \frac{1}{2|\ell|} & t < \ell, \\ \frac{\epsilon}{2(t^2 - \ell^2)} & t > \ell, \end{cases}$$ (21)

that leads to

$$S_A = \begin{cases} \frac{c}{6} \log \frac{2\ell}{a} + \tilde{c}'_1 & t < \ell, \\ \frac{c}{6} \log \frac{t^2 - \ell^2}{a^2} + k_0 & t > \ell, \end{cases} \quad \text{(22)}$$

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with $k_0$ the same as in equation (15). The interpretation of this result is quite direct. Indeed at $t = 0$ the joining procedure produces a quasiparticle excitation at $r = 0$ that propagates freely with the corresponding speed of sound $v_s$ that in the CFT normalization is $v_s = 1$. This excitation takes a time $t = \ell$ to arrive at the border between $A$ and $B$ and only at that time will start modifying their entanglement. The following evolution for $t \gg \ell$ is the same as in equation (15).

Also, the constant value for $t < \ell$ deserves a comment: it is exactly the value known from CFT for the slit in the half-line equation (5). This is a non-trivial consistency check. Note that a finite $\epsilon$ smooths the crossover between the two regimes and makes the entanglement entropy a continuous function of the time.

3.3. Case III: the slit with the defect at the border

Let us consider again the same physical situation as before, but we now calculate the entanglement entropy of $A = [0, \ell]$ and $B$ the remainder. For $t < 0$ the real negative axis is decoupled from the rest and does not contribute to the entanglement entropy, that is just the one of a slit in half-chain, i.e., the initial entropy is given by equation (5).

The entanglement entropy is obtained from the replica trick considering the scaling of a two-point function between the endpoints of the slit. In the $z$-plane these two points are $z_1 = i\tau$ and $z_2 = \ell + i\tau$ (we adopt the same notation as before to make direct the use of previous formulas). As usual, the plane with cuts is mapped into the half-plane Re $w > 0$ by the transformation (9). On the $w$-plane we have [4]

$$\langle \Phi_n(w_1)\Phi_{-n}(w_2) \rangle = \tilde{c}_n^2 \left( \frac{a^2|w_1 + \bar{w}_2||w_2 + \bar{w}_1|}{|w_1 - w_2||\bar{w}_2 - \bar{w}_1||w_1 + \bar{w}_1||w_2 + \bar{w}_2|} \right)^{x_n},$$

so the mapping to the original plane $z$ is

$$\langle \Phi_n(z_1)\Phi_{-n}(z_2) \rangle = \tilde{c}_n^2 \left( \left| \frac{\text{d}w}{\text{d}z} \right|_{z_1} \left| \frac{\text{d}w}{\text{d}z} \right|_{z_2} \frac{a^2|w_1 + \bar{w}_2||w_2 + \bar{w}_1|}{|w_1 - w_2||\bar{w}_2 - \bar{w}_1||w_1 + \bar{w}_1||w_2 + \bar{w}_2|} \right)^{x_n},$$

where the various terms are functions of $z_i$ through equations (11), (17)–(19), and

$$\epsilon^2|w_1 - w_2|^2 = (\ell + \rho \cos \theta - \sqrt{\epsilon^2 - \tau^2})^2 + \rho^2 \sin^2 \theta,$$

$$\epsilon^2|w_1 + \bar{w}_2|^2 = (\ell + \rho \cos \theta + \sqrt{\epsilon^2 - \tau^2})^2 + \rho^2 \sin^2 \theta.$$

Putting everything together, we get

$$\langle \Phi_n(z_1 = i\tau)\Phi_{-n}(z_2 = i\tau + \ell) \rangle = \tilde{c}_n^2 \left( \frac{a^2 \epsilon (\ell + \rho \cos \theta + \sqrt{\epsilon^2 - \tau^2})^2 + \rho^2 \sin^2 \theta}{\epsilon^2 - \tau^2 (\ell + \rho \cos \theta - \sqrt{\epsilon^2 - \tau^2})^2 + \rho^2 \sin^2 \theta} \times \frac{\sqrt{(\ell + \rho \cos \theta)^2 + (\tau + \rho \sin \theta)^2}}{4\rho(\ell + \rho \cos \theta)} \right)^{x_n}.$$

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Again this simplifies considering the limit \( t, \ell \gg \epsilon \) as explained after equation (20). We finally obtain

\[
\text{Tr} \rho^n_A = \begin{cases} 
\tilde{c}_n^2 \left( \frac{a^2 \ell + t}{\ell^2 - t} \right)^x, & t < \ell, \\
\tilde{c}_n^2 \left( \frac{a^2}{\ell^2} \right)^x, & t > \ell.
\end{cases}
\]  

(28)

Note that for \( t > \ell \) the calculation is slightly more cumbersome because we should take into account the first order in \( \epsilon \) of some expressions.

Finally, using the replica trick we get for the entanglement entropy

\[
S_A(t > \ell) = \frac{c}{3} \ln \frac{t}{a} + \frac{c}{6} \ln \frac{\ell}{\epsilon} + \frac{c}{6} \ln \frac{4 \ell - t}{\ell + t} + 2\tilde{c}_1', \quad t < \ell,
\]

\[
S_A(t > \ell) = \frac{c}{3} \ln \frac{\ell}{a} + 2\tilde{c}_1', \quad t > \ell.
\]  

(29)

The crossover time \( t^* = \ell \) is again in agreement with the quasiparticle interpretation.

There are several interesting features of this result. For very short time \( t \ll \ell \) it reduces to the \( \ell = \infty \) case, equation (15), as it should. The leading term for \( t > \ell \) is just the ground state value for a slit in an infinite line. However, the subleading term is not the same; in fact we have

\[
S_A(t > \ell) - S_A(\text{eq}) = 2\tilde{c}_1' - c_1' = \log g,
\]  

(30)

where we also use the value of \( \epsilon \) in equation (16). This is a signal that for long time the system still remembers something of the initial configuration as a boundary term that is unable to ‘dissipate’. Since the extra energy never dissipates under unitary evolution, there is no reason for the constant terms to be the same.

Another interesting feature is the behavior for \( t < \ell \). This is very similar to the form proposed in [13] to fit the numerical data, i.e.,

\[
S_A = \frac{c_0}{3} \log \ell + \frac{c_1}{3} \log(t/\ell) + \frac{c_2}{3} \log(1 - t/\ell) + k'.
\]  

(31)

Only the term in \( t + \ell \) was missing in [13]. However, this behaves smoothly for \( 0 < t < \ell \) and its effect can be well approximated by a constant factor in \( k' \). It will be interesting to check whether and how the use of this term in \( t + \ell \) changes the quality of the fit. Furthermore, the fit of [13] is also a strong confirmation of our result; indeed they found \( c_0 \simeq 1 + c_2, \quad c_1 \simeq 1, \) and \( c_2 \simeq 1/2 \) (plot 7 in [13] with \( t' = 0 \)) that are exactly our predictions for \( c = 1 \) (the known central charge of the XX model).

Furthermore, equation (29) display a large plateau for \( 0.2 < t/\ell < 0.8 \) as shown in figure 3 (the black line in both the plots) and already noticed in [13]. This plateau can be studied considering the value at the maximum \( t/\ell = (\sqrt{5} - 1)/2 \) where we have

\[
S_A(\text{plateau}) = \frac{c}{2} \log \frac{\ell}{a} + k_1 = \frac{3}{2} S_A(\text{eq}) + k_2,
\]  

(32)

with the constants \( k_p \) simply related to \( a, \epsilon, \) and \( \tilde{c}_1' \).

Finally, we can study the result for \( t = 0 \) when \( \epsilon \ll \ell \):

\[
S_A(t = 0) = \frac{c}{6} \log \frac{4\epsilon\ell}{a^2} + 2\tilde{c}_1' = \frac{c}{6} \log \frac{2\ell}{a} + c_1',
\]  

(33)
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where in the last equality we used the value of $\epsilon$ given by equation (16). This result is exactly the initial value for a slit of length $\ell$ in the half-line. Again this is an highly non-trivial consistency check of all the method. A final curiosity is that

$$2S_A(t = 0) - S_A(t > \ell) = \frac{c}{3} \log 2,$$

is independent of the details of the theory.

### 3.4. Case IV: the general slit

Let us now consider the most general case of a slit $A = [\ell_1, \ell_2]$. In the $z$-plane we have to calculate the two-point function between $z_1 = \ell_1 + i\tau$ and $z_2 = \ell_2 + i\tau$. Under the mapping (1) these correspond to $w_p = \ell_p + i\tau + \rho_p e^{i\theta_p}$ with $\rho_p$ and $\theta_p$ given by equation (18) with $\ell \rightarrow \ell_p$ and $p = 1, 2$. In imaginary time, after long but straightforward algebra, we have

$$\left( \frac{\langle \Phi_n(z_1 = i\tau + \ell_1)\Phi_{-n}(z_2 = i\tau + \ell_2) \rangle}{\mathcal{C}_n^2} \right)^{1/x_n}$$

$$= \sqrt{(\ell_1 + \rho_1 \cos \theta_1)^2 + (\tau + \rho_1 \sin \theta_1)^2} \sqrt{(\ell_2 + \rho_2 \cos \theta_2)^2 + (\tau + \rho_2 \sin \theta_2)^2}$$

$$\frac{4\rho_1\rho_2(\ell_1 + \rho_1 \cos \theta_1)(\ell_2 + \rho_2 \cos \theta_2)}{((\ell_1 - \ell_2 + \rho_1 \cos \theta_1 + \rho_2 \cos \theta_2)^2 + (\rho_1 \sin \theta_1 - \rho_2 \sin \theta_2)^2)^2}.$$ (35)

The analytic continuation can be simplified for $t, \ell_1, \ell_2 \gg \epsilon$. For simplicity in the notation, but without loss of generality, we indicate as $\ell_1$ the larger in absolute value of the two $\ell_p$ and we assume $\ell_1 > 0$. $\ell_2$ can be either positive or negative. For $t < |\ell_2|$ we have a different result if the two points are on the same side or on different sides of the
defect, namely:

\[
S_A(t < |\ell_2|) = \begin{cases} 
\frac{c}{6} \log \frac{4\ell_1 |\ell_2|}{a^2} + 2c_1' & \ell_2 < 0, \\
\frac{c}{6} \log \frac{(\ell_1 - \ell_2)^2 4\ell_1 \ell_2}{(\ell_1 + \ell_2)^2 a^2} + 2c_1' & \ell_2 > 0.
\end{cases}
\]

(36)

The first is just the sum of the entanglement entropies of two slits of length \(\ell_1\) and \(|\ell_2|\) in the half-line starting at the origin. In fact, in the initial state the two parts are unentangled and the total entropy is the sum of the two. Also the second result is just the ground state value for the slit \([\ell_2, \ell_1]\) in the half-line [4].

The entanglement between A and B becomes sensitive to the joining of the system when the quasiparticles emitted from the origin arrive at \(\ell_2\). The behavior for \(|\ell_2| < t < \ell_1\) does not depend on the relative sign of the \(\ell_p\):

\[
S_A(|\ell_2| < t < \ell_1) = \frac{c}{6} \log \frac{(\ell_1 - \ell_2)(\ell_1 - t) 4\ell_1 (t^2 - \ell_2^2)}{(\ell_1 + \ell_2)(\ell_1 + t) \epsilon a^2} + 2c_1'.
\]

(37)

Finally, for \(t > \ell_1\) we have almost the ground state value:

\[
S_A(t > \ell_1) = \frac{c}{3} \ln \frac{\ell_1 - \ell_2}{a} + 2c_1',
\]

(38)

that is exactly the same as for case III.

The resulting \(S_A\) for different values of \(\ell_p\) is plotted in figure 3. For \(|\ell_2| < t < \ell_1\) there is again a large plateau whose actual extension and value depend on both \(\ell_p\).

The analytical value of the plateau is quite complicated and not really illuminating (it contains cubic roots, as can be realized solving the equation for the maximum). The most important features are that it decreases when the defect penetrates deeply in the slit (left part of figure 3) and it stays almost constant when the slit moves far from the defect (right part of the figure).

Finally, one can easily read off from our formula the result for a defect exactly at the center of the slit: the entanglement entropy is independent of the time. This is different from what is found numerically in a lattice model [13] and will be discussed (among the other things) in the next subsection.

3.5. Comparison with numerical works

As far as we are aware, the entanglement entropy after a local quench has been discussed in only two papers [13, 26]. There are a few qualitative differences between these results and the asymptotic ones of the CFT that are easily understood in terms of the general scenario we draw for quasiparticles emitted from the origin. These differences are attributable to slow quasiparticles, that exist as consequence of a non-linear dispersion relation \(v_k = \partial_k E_k\), emitted from the joining point at \(t = 0\).

The first difference is that the lattice numerics present, on top of a smooth curve, very fast oscillations. These oscillations have been discussed in the context of global quenches [19, 12], and are due to the modes at the zone boundary \(|ka| = \pi\) that have \(v_k = 0\). However, they are only corrections to the asymptotic result for \(t, \ell \gg a\), since their amplitude remains constant while the CFT result diverges. Furthermore, for any specific model they can be easily predicted in a quantitative way.
In [13] different configurations of slits were considered for electron hopping on a chain (XX model in spin language). The sharp horizon effect is present even in the numerics, since there cannot be (by definition) quasiparticles faster than the CFT ones. We already discussed that the numerical results for a defect at the boundary of the slit are in quantitative agreement with the CFT in the region \( t < \ell \). However, for \( t > \ell \) the numerics show an asymptotic value that appears to be exactly that of the ground state. According to our analysis this is possible only when the boundary entropy \( \log g \) vanishes.

The most relevant qualitative difference is that for \( t \) greater than the maximum length the asymptotic result is not constant but it is slowly decaying (like \( \log(t)/t \)) smoothing the sharp transition from the plateau. This, for example, changes completely the behavior in the case of central defect. We expect that this phenomenon can be ascribed to the slow quasiparticles that, after the quickest ones have arrived entangling the two parts, then disentangle them for very long times. This is the most plausible explanation, but we do not have an argument to put it on a more quantitative level.

4. Correlation functions

In this section we derive the time-dependence of correlation functions after the local quench. We find that the one-point function whose functional form is completely fixed by conformal invariance. The two-point functions instead depend on the particular (boundary) CFT. We will discuss all the details in the simplest case of a Gaussian theory and then show how from general CFT arguments we can obtain part of the asymptotic behavior. Some of these results may have been previously derived in the context of quantum impurity problems from a different point of view (see e.g. [27]).

4.1. One-point function

The one-point function of a primary field in the half-plane \( \Re w > 0 \) is

\[
\langle \Phi(w) \rangle = \frac{A^\Phi_b}{[2 \Re w]^{x_{\Phi}}},
\]

where \( x_{\Phi} \) is the scaling dimension of the field and \( A^\Phi_b \) is a non-universal amplitude that can be fixed in terms of the normalization of the two-point function of the same operator. We also fix the lattice spacing \( a \) to 1. \( A^\Phi_b \) is known for the simplest universality classes as the Gaussian theory and the Ising model [28].

With the mapping (9) we can get the one-point correlation, that obviously assumes the same form as equation (20). At the point \( r \), after continuing to real time we have

\[
\langle \Phi(r, t) \rangle = \begin{cases} 
A^\Phi_b(2r)^{-x_{\Phi}} & t < r, \\
A^\Phi_b \left( \frac{\epsilon}{2(t^2 - r^2)} \right)^{x_{\Phi}} & t > r.
\end{cases}
\]

Thus for short times the correlation takes its initial value, until the effect of the joining arrives at time \( t = r \) when it decays for \( t \gg r \) like \( t^{-2x_{\Phi}} \) (note that this exponent is twice the boundary one).
4.2. A two-point function: the Gaussian model

For the Gaussian model the two-point function of a primary field \( \Phi \) in the half-plane is given by equation (23) with \((\ell_2, \ell_1) \to (r_2, r_1)\) and \(c_n \to (A_0^\Phi)^2 = 1\) [14]. As a consequence we can obtain its scaling as a by-product of the result for the entanglement entropy for a general slit in section 3.4.

For simplicity in the notation we assume here and in the following section \( r_1 \) to be positive and to be larger than the absolute value of \( r_2 \) that can be either positive or negative. Thus we have that for \( t < |r_2| \) the two-point function keeps its initial value that has a different form depending on the relative signs of \( r_2 \) given by \(|4r_1 r_2|^{-x_\Phi}\) (different sides of the defect) and \(|4r_1 r_2/(r_1 + r_2)^2|^{-x_\Phi}\) (same side).

For \( t > r_1 \), it reaches the ground state value \(|r_1 - r_2|^{-2x_\Phi}\). An interesting and non-trivial behavior is displayed for \(|r_2| < t < r_1\)

\[
\langle \Phi(r_1, t)\Phi(r_2, t) \rangle = \left[ \frac{(r_1 + r_2)(r_2 + t)}{(r_1 - r_2)(r_1 - t)} \right] \frac{\epsilon}{4r_1(t^2 - r_2^2)} \right)^{x_\Phi}. \tag{41}
\]

4.3. The general two-point function

There are some features of the previous result that are expected to be valid in general, not only for a Gaussian theory, as for example the horizon effect and the final equilibrated value. The natural expectation is that these results are obtainable with CFT, and in fact we will show that this is the case.

The two-point function in the half-plane can be always written as [14]

\[
\langle \Phi(w_1)\Phi(w_2) \rangle = \left( \frac{|w_1 + \bar{w}_2||w_2 + \bar{w}_1|}{|w_1 - w_2||\bar{w}_2 - \bar{w}_1||w_1 + \bar{w}_1||w_2 + \bar{w}_2|} \right)^{x_\Phi} F(\eta), \tag{42}
\]

where \( \eta \) is the four-point ratio

\[
\eta = \frac{|w_1 + \bar{w}_1||w_2 + \bar{w}_2|}{|w_1 + \bar{w}_2||w_2 + \bar{w}_1|}, \tag{43}
\]

and the function \( F(\eta) \) depends explicitly on the considered (boundary) model. It is known for the simplest models as e.g. the Gaussian \( F(\eta) = 1 \) and the Ising universality class (see below).

Thus the behavior of the two-point function depends mainly on the value on the ratio \( \eta \) that we now study. Using the analytic structure of the previous section we have for \( t < |r_2| \) a different behavior if the two points are on the same or on different sides of the defect:

\[
\eta(t < |r_2|) = \begin{cases} 
\frac{4r_1 r_2}{(r_1 + r_2)^2} & r_2 > 0, \\
\frac{\epsilon^2 r_1 |r_2|}{(r_1^2 - t^2)(r_2^2 - t^2)} & r_2 < 0.
\end{cases} \tag{44}
\]

For intermediate times we have

\[
\eta(|r_2| < t < r_1) = \frac{2r_1 (r_2 + t)}{(r_1 + r_2)(r_1 + t)}, \tag{45}
\]

while for larger times \( \eta \) is 1 constantly.

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From the previous subsection we already know how the first part of equation (42) transforms under the conformal mapping (9). Thus we need only to map \( F(\eta) \) that, in general, is an unknown function. However, we know its behavior in two special circumstances. Indeed when \( \eta \sim 1 \) the two points are deep in the bulk, meaning \( F(1) = 1 \). Instead, for \( \eta \ll 1 \), from the short-distance expansion, we have

\[
F(\eta) \simeq (A_b^\Phi)^2 \eta^{x_b},
\]

where \( x_b \) is the boundary scaling dimension of the leading boundary operator to which \( \Phi \) couples and \( A_b^\Phi \) is the bulk-boundary operator product expansion coefficient that equals the one introduced in equation (39) (see e.g. [28]).

From this we easily understand the behavior for \( t > r_1 \). Since \( \eta \to 1 \) so \( F(1) = 1 \), the two-point function is just the one in the ground state,

\[
\langle \Phi(r_1 < t) \Phi(|r_2| < t) \rangle = \frac{1}{|r_1 - r_2|^{2x_\Phi}}.
\]

Although this result could have been expected, it is important to have recovered it only from CFT arguments.

The behavior for \( |r_2| < t < r_1 \) is instead more complicated: combining the result of the previous section with the value of \( \eta \) we have

\[
\langle \Phi(r_1, t) \Phi(r_2, t) \rangle = \left[ \frac{(r_1 + r_2)(r_2 + t)}{(r_1 - r_2)(r_1 - t)} \right]^{x_\Phi} F \left( \frac{2r_1(r_2 + t)}{(r_1 + r_2)(r_1 + t)} \right) .
\]

For example, when \( \Phi \) is the order parameter in the Ising universality class we have [14]

\[
F(\eta) = \frac{\sqrt{1 + \eta^{1/2}} \pm \sqrt{1 - \eta^{1/2}}}{\sqrt{2}},
\]

and \( x_\Phi = 1/8 \). The sign \( \pm \) depends on the boundary conditions. \( + \) corresponds to fixed boundary conditions and \( - \) to free ones. This completely fixes the behavior of the two-point function in the intermediate regime. It would be very interesting to check this prediction.

The time evolution for \( t < |r_2| \) depends on the sign of \( r_2 \). For \( r_2 > 0 \) we have

\[
\langle \Phi(r_1, t) \Phi(r_2, t) \rangle = \left[ \frac{1}{4r_1 r_2 (r_1 - r_2)^2} \right]^{x_\Phi} F \left( \frac{4r_1 r_2}{(r_1 + r_2)^2} \right) ,
\]

i.e., it keeps its initial value until \( t = r_2 \), that is nothing but correct boundary value at zero time.

More complicated is the behavior for \( r_2 < 0 \); in fact, we get

\[
\langle \Phi(r_1, t) \Phi(r_2, t) \rangle \simeq \frac{1}{|4r_1 r_2|^{x_\Phi}} F \left( \frac{\epsilon}{(r_1^2 - t^2)(r_2^2 - t^2)} \right) \left( \frac{\epsilon^2 r_1}{r_2} \right)^{x_b},
\]

where in the last approximation we used that \( \epsilon \ll t, r_1, r_2 \) and the behavior of \( F(\eta) \) for \( \eta \ll 1 \). In particular we note that this is time independent only when \( x_b = 0 \), which corresponds to the case of a non-zero one-point function. Similar anomalous time-behavior was found when \( x_b > 0 \) in [6].
5. Decoupled finite interval

A natural question arising is how the results we just derived change when we introduce more than one defect in the line. It is straightforward to have a path integral for the density matrix: we only need to have pairs of slits for $-\infty$ to $-i\epsilon$ and from $i\epsilon$ to $+i\infty$ everywhere there is a defect. However, it becomes prohibitively difficult to treat this case analytically. In order to begin to understand the case when a finite interval is initially decoupled, we consider the case when it lies at the end of a half-line.

So, let us consider a semi-infinite chain in which the A subsystem is the finite segment $(-\ell, 0)$ and the B is the complement $(0, \infty)$ and with the initial defect at $r_D = 0$. The space–time geometry describing this situation is like the one just considered, with a wall at $-\ell + iy$ ($y$ real) that represents the boundary condition. This is depicted in the left panel of figure 4.

In these circumstances the inverse conformal mapping between the $z$-plane and the half-plane can be worked out using the Schwarz–Christoffel formula. After long algebra one obtains

$$z(w) = i\left(\frac{\ell}{\pi} \log(iw) + b \frac{-iw - 1}{-iw + 1}\right), \quad (52)$$

with the parameter $b$ related to $\ell$ and $\epsilon$ in a non-algebraic way. (A slit in the full line is closely related to this transformation; the last piece is replaced by $(w^2 - 1)/(w^2 + 1)$.) Unfortunately the mapping (52) is not analytically invertible and its exact use is limited to numerical calculations that do not help us, since we need to perform an analytical continuation. We will develop in the remaining part of the section an approximate solution for $\ell \gg \epsilon$, that however is not always justified: this limit is allowed only after the analytical continuation to real time.

The limit $\ell \gg \epsilon$ before the analytical continuation simplifies the calculation because under the conformal transformation (9) the boundary at $x = -\ell$ is approximately a disk tangent to the imaginary axis at the origin and with radius $R = \epsilon/4\ell \ll 1$, as depicted in the central part of figure 4. This is simply checked applying the inverse transformation (9)
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to the disk $\zeta = R(1 + e^{i\theta})$ with $\theta \in [0, 2\pi]$. In fact we have

$$z = \frac{\zeta^2 - 1}{2\zeta} = \frac{R^2(1 + e^{i\theta})^2(1 + e^{-i\theta}) - (1 + e^{-i\theta})}{4R(1 + \cos \theta)} = -\frac{\epsilon}{4R} - i\frac{\epsilon}{4R} \tan \theta/2 + O(R). \tag{53}$$

The half-plane minus the disk $R(1 + e^{i\theta})$ is mapped in the half-plane $Re w > 0$ by the transformation

$$w = -i \exp(2\pi i R/\zeta). \tag{54}$$

Let us note that, when $\epsilon \ll \ell$, the parameter $b$ is given by $b = \pi\epsilon^2/(8\ell) + O(\epsilon^4/\ell^2)$.

Combining the two transformations we can map the space–time region on the left of figure 4 into the half-plane (only in the limit $\ell \gg \epsilon$) by means of

$$w = -i \exp \left[ \frac{\pi i \epsilon}{2\ell} \left( \sqrt{z^2/\epsilon^2 + 1} - z/\epsilon \right) \right], \tag{55}$$

with inverse

$$z(w) = \frac{\ell}{\pi} \log(iw) + \frac{\epsilon^2}{\ell^2} \frac{1}{4 \log(iw)}. \tag{56}$$

Note that the residue of the pole at $w = -i$ is the same as the exact one in equation (52). Furthermore, calculating the difference between the approximate and the exact solution, one easily checks that it is of the order of $\epsilon^4/\ell^2$ everywhere in the complex plane.

We then have that $w(i\tau)$ can be written as

$$w(i\tau) = -i e^{\pi/2\ell} \left[ \cos(\sqrt{1 - \tau^2/\epsilon^2} \pi \epsilon/2\ell) + i \sin(\sqrt{1 - \tau^2/\epsilon^2} \pi \epsilon/2\ell) \right]. \tag{57}$$

Thus

$$\langle \Phi_n(i\tau) \rangle = c_n \left[ \frac{\pi \epsilon}{4\ell} \frac{1}{\sqrt{\epsilon^2 - \tau^2}} \sin(\pi \sqrt{\epsilon^2 - \tau^2}/2\ell) \right]^{x_n}. \tag{58}$$

Note that for $\ell \to \infty$ it reduces to the previous result, as it should. Furthermore, it is clear that it is well defined only for $\ell \gg \epsilon$.

Continuing to real time $\tau \to it$, and for $t \gg \epsilon$, we have

$$\langle \Phi_n(i\tau) \rangle = c_n \left[ \frac{\pi \epsilon}{4\ell t} \frac{1}{\sin(\pi t/2\ell)} \right]^{x_n}. \tag{59}$$

Clearly this cannot make sense when the argument of the power law becomes negative (i.e., for $t > 2\ell$), signaling that there is something wrong in the derivation. However, using the replica trick, for the entanglement entropy we obtain

$$S_A = \frac{c}{6} \log \left( \frac{4\ell}{\pi \epsilon} t \sin(\pi t/2\ell) \right) + c'_n. \tag{60}$$

One is tempted to assume that this result can be correct only for $t < \ell$ and that for larger time it saturates as suggested by the quasiparticle interpretation. Only an exact calculation via the exact mapping (52) can resolve these doubts. However, exact solutions of integrable models or numerical density matrix renormalization group could already be able to eventually exclude equation (60) and to shed some light on the problem. We mainly mention this topic here to encourage further studies in this direction.

Finally, let us point out that transformations similar to equation (52) appear when studying finite-size effects.

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6. Discussion

We have presented a detailed study of the unitary evolution that results after a local quench in a quantum one-dimensional system. All our findings have been obtained by means of CFT and so they are limited to the evolution of a gapless quantum model with a linear dispersion relation. We have presented calculations for the entanglement entropies for different bipartitions of the system and for one- and two-point correlation functions in a general CFT. In particular we studied the case when two half-systems are joined at time $t = 0$.

All our results are interpretable in terms of a scenario that we believe to be valid in general, not only for gapless systems (in analogy to the case of global quenches [6,19,12]). In fact, the initial state is expected to generate quasiparticle excitations at $r_D = 0$ that then propagate freely through the system and carry all the information about entanglement and correlations. In the case of a CFT all these excitations travel at the same speed $v_s$ (= 1 by normalization). However, in general, as a consequence of a non-linear dispersion relation, a full spectrum of velocities is expected.

Although the physical cut may seem a rather specific situation, we believe that our findings are quite general. For example, a general defect (e.g. a weakened bond) is asymptotically equivalent to the results we obtained here as long as the defect is a relevant perturbation. This excludes the XX chain that is the only model studied so far [13], where the defect is believed to be marginal [29], but should apply to XXZ spin-chains [30]. Furthermore, we expect a qualitatively similar behavior when a system has been artificially prepared in a configuration that is only locally different from the ground state. For example there are already several studies where the local horizon effect is evident (see for example those considered in [31]). A full quantitative analysis of most of these settings should be possible properly adapting our CFT treatment (e.g. with the theory of boundary condition changing operators [24]). On the other hand, also the case of an initial state with one or more kinks displays a time-evolution with an horizon effect similar to the one considered here (see e.g. [32] where also finite temperature results are obtained in a rigorous manner). It would be interesting to understand whether these results can be recovered analytically continuing some CFT results. Work in this direction is in progress.

We described here several different physical situations, but many problems are still left for future investigations. A first goal would be to check our predictions in exactly solvable models. In fact, as far as we are aware only the paper by Eisler and Peschel [13] discussed these topics in the context of the XX model, and their results are fully compatible with ours.

Another simple generalization of our results is to study gapped exactly solvable models, like the Ising model in a transverse magnetic field, or others admitting a free-field representation. In these case it is also interesting to understand whether some predictions can be made on the basis of the generalized Gibbs ensemble [33].

A more difficult question to study analytically concerns the role played by quenched disorder. This is expected to change at a qualitative level the quasiparticle scenario as a consequence of Anderson localization [34]. In this case the time-dependent density matrix renormalization group [35] should be quite effective, since for clean systems we know that the entanglement entropy never increases dramatically as it does in the case of global quenches.
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