TURBULENT CONVECTION AND PULSATIONAL STABILITY OF VARIABLE STARS. I. OSCILLATIONS OF LONG-PERIOD VARIABLES

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ABSTRACT

We have performed a linear pulsational stability survey of six series of long-period variable models with $M = 1.0 \, M_\odot$, $L = 3000–8000 \, L_\odot$, and $(X, Z) = (0.700, 0.020), (0.735, 0.005)$. The dynamic and thermodynamic couplings between convection and oscillations are treated by using a statistical theory of nonlocal and time-dependent convection. The results show that the fundamental and all the low overtones are always pulsationally unstable for the low-temperature models when the coupling between convection and oscillations is ignored. When the coupling is considered, there is indeed a “Mira” pulsational instability region outside of the Cepheid instability strip on the H-R diagram. The coolest models near the Hayashi track are pulsationally stable. Toward high temperature, the fundamental mode becomes unstable first and then the first overtone. Some of the second to fourth overtones may become unstable for the hotter models. All the modes higher than the fourth ($n > 4$) are pulsationally stable. The position and the width of such an instability region on the H-R diagram critically depends on the mass, luminosity, and metal abundance of the star. The overall properties of the dependence are the following: (1) For the same mass and luminosity, the instability region becomes slightly wider and moves to lower effective temperatures as the metal abundance increases. (2) For a given chemical abundance, the instability region becomes wider and moves to the lower effective temperature as its luminosity increases or its mass decreases. For the luminous red variables seated outside the instability strip the dynamic coupling between convection and oscillations balances or may even overtake the thermodynamic coupling. Turbulent viscosity can no longer be ignored for the pulsational instability of the low-temperature red variables. The effect of turbulent viscosity becomes more and more important for higher modes, and may finally become the main damping mechanism of the pulsation.

Subject headings: convection — stars: oscillations — stars: variables: long-period variables

1. INTRODUCTION

Observationally, there exists a group of low-temperature luminous pulsating red variables to the right of the Cepheid instability strip on the H-R diagram. They are the most heterogeneous red giants and supergiants belonging to both Population I and Population II. In the General Catalogue of Variable Stars, these luminous red variables are categorized into three different types according to their variability: Mira variables, semiregular variables, and slow irregular variables, among which the first type (Miras) have been well studied. It is currently believed that the Mira variables are stars on the asymptotic giant branch stage of evolution. Detailed summaries of the observational properties and theoretical work on Miras have been given by Whitelock (1990) and Wood (1990a, 1990b).

The nonlinear pulsation of Mira variables has been considered by Keeley (1970), Rose & Smith (1972), Wood (1974), Tuchman, Sack, & Barkat (1978, 1979), Hill & Willson (1979), Bowen (1988), and Perl & Tuchman (1990). The linear analysis is still useful for the stability survey. The linear pulsations of Mira variables were thoroughly studied by many authors (Kamijo 1962; Langer 1971; Fox & Wood 1982; Ostlie & Cox 1986; Balmforth, Gough, & Merryfield 1990; Cox & Ostlie 1991; Gong, Li, & Huang 1995), among which Ostlie & Cox (1986) and Gong et al. (1995) completely ignored the coupling between convection and oscillations. The coupling theory used by Kamijo (1962), Langer (1971), Fox & Wood (1982), and Cox & Ostlie (1991) is oversimplified. Balmforth et al. (1990) had used Gough’s local time-dependent mixing-length theory of convection in dealing with the coupling.

The red pulsating variables possess very extended convective envelopes. The convective energy transport in the H and He ionization regions well exceeds 99% of the total value. Convection overpowers the $\kappa$-mechanism of radiation and becomes the principal excitation (damping) mechanism of pulsation. The local time-dependent theory of convection has been used to interpret the red edge of the Cepheid instability strip (Baker & Gough 1979; Xiong 1980). When the surface temperature decreases, the dynamical coupling (through turbulent pressure and turbulent viscosity) between convection and oscillations becomes more and more important (Xiong 1977; Gough 1977; Stellingwerf 1984). For red stars outside the Cepheid instability strip, the dynamical coupling between convection and oscillations becomes as powerful as the thermodynamic coupling. Precisely speaking, the local time-dependent theory of convection can no longer (at least in a self-consistent way) be used to treat the dynamical coupling. For this reason, we have developed a nonlocal time-dependent statistical theory of convection (Xiong 1989). In this paper we assume that the convection is quasi-isotropic. Such an assumption excludes turbulent viscosity, which is anisotropic. We have developed a more precise version of the nonlocal time-dependent statistical theory of convection, in which the dynamic equations of the third-order correlation functions are derived, and the anisotropy

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of turbulent convection is also considered (Xiong, Cheng, & Deng 1997a). We have derived an equation of turbulent viscosity that is very similar to the Stokes equation of viscous fluid.

Compared with previous works, the present version stands on a solid base of hydrodynamics for describing the dynamic behavior of convective motion. Therefore, it would provide a better approach to both the dynamic and the thermodynamic couplings between convection and oscillations. Using this new theory, in this paper we have performed a linear pulsational stability analysis for the luminous red stars outside the Cepheid instability strip. The working equations of linear nonadiabatic oscillations are given in § 2. Section 3 describes the numerical results and theoretical arguments. A summary and concise conclusions of the present work are given at the end.

2. WORKING EQUATIONS

We have given a complete presentation of the nonlocal time-dependent theory of convection and all its mathematical formalism, i.e., the radiation-hydrodynamic equations of stellar oscillations (Xiong, Cheng, & Deng 1997a). Among all the 14 equations, three deal with fluid motion (the conservation laws of mass, momentum, and energy), two describe the radiation field (radiative transfer and energy conservation of the radiation field), and the other nine are given for the second and third correlation functions of turbulent velocity and temperature for turbulent convective motion. Within the framework of the Eddington approximation, the departure from radiative equilibrium has been accurately taken into account. The gas and radiation are treated separately, and they become coupled through the absorption and emission processes of matter. In the meantime, careful consideration has been given to the anisotropy of turbulence and the inertia of turbulent convection. The turbulent viscosity, which is nearly identical in form to the Stokes formula for viscous fluid, is included automatically in the dynamic equation of the second-order correlation of turbulent velocity. When we adopt a simplified gradient-type diffusion approximation for the third-order correlations, this reduces the number of radiation-hydrodynamic equations to 11. We have computed the linear nonadiabatic radial oscillations of p-modes for a solar model using these simplified radiation-hydrodynamic equations. The results show that the effects of departure from radiative equilibrium are negligible except for only very high order p-mode ($n \geq 25$) (Cheng & Xiong 1997). In this work we will ignore the departure from radiative equilibrium. In the context of the present study this is safe, because the giants and supergiants almost always pulsate at low-order modes. Assuming that the gas and the radiation field are always in equilibrium, we can treat them together. In this way, the number of equations is reduced to 10, and the pulsational equations are greatly simplified. Under such circumstances, the linear nonadiabatic equations can be written as

$$\frac{dy_3}{d \ln r} + 3y_3 + \delta y_1 - ax_2 = 0, \quad (1)$$

$$\frac{dy_2}{d \ln r} - \frac{d \ln T}{d \ln r}[\chi_p y_1 + (\chi_T - 4)y_2 - 4y_3 + y_4] = 0, \quad (2)$$

$$\frac{d}{d \ln r} \left\{ 3y_6 + \frac{L_e}{L} [(\delta + C_p, r)y_1 + (1 - \alpha + C_p, r)y_2 + 2y_3 + y_9] + \frac{L_e}{L} y_4 \right\} + \imath \omega \frac{4\pi r^3 \rho C_p T}{L} \left[ \frac{3}{C_p T} y_5 - \left( \frac{\delta x^2}{C_p T} \right) y_1 + \left( 1 + \frac{\delta x^2}{C_p T} \right) y_2 \right] = 0, \quad (3)$$

$$\frac{\sqrt{3} \pi c_r \rho c_p^2 P x^3}{G M_r} \frac{dy_3}{d \ln r} - \frac{L_e}{L} [(1 + \delta)y_1 - ax_2 + 6y_3 + 3y_4 - y_6] = 0, \quad (4)$$

$$\frac{\sqrt{3} \pi c_r \rho c_p^2 P x Z}{G M_r} \frac{dy_2}{d \ln r} - L_s [(1 + (2 + \delta + C_p, r))y_1 + 2(C_p - \alpha)y_2 + 6y_3 + y_5 + y_7 - y_8] = 0, \quad (5)$$

$$\frac{\sqrt{3} \pi c_r \rho c_p^2 P x Z}{G M_r} \frac{dy_3}{d \ln r} - L_s [(1 + \delta + C_p, r)y_1 + (C_p - \alpha)y_2 + 6y_3 + y_5 + y_9 + y_10] = 0, \quad (6)$$

$$\frac{1}{L} \frac{d(L_1 y_e)}{d \ln r} + \frac{4\pi r^3 \rho c_p^2 P}{3L} \left\{ \left[ \delta - 1 - \frac{\delta}{2} i \omega \xi \right] \frac{1.56 \rho x^3}{c_1 P} - ax \left( \frac{\delta}{4} + \frac{1}{2} i \omega \xi \right) \right\} y_1 + \alpha \left[ \frac{1}{2} i \omega \xi - 1 \right] \frac{1.56 \rho x^3}{c_1 P}$$

$$+ V \left[ \frac{1}{4} i \omega \xi - \alpha \right] y_2 + \left[ ax \left( 2 + \frac{r^3 \omega^2}{G M_r} - \frac{3}{4} i \omega \xi \right) - 2 \frac{1.56 \rho x^3}{c_1 P} \right] y_3 + 3 \left( \frac{1}{2} i \omega \xi - 1 \right) \frac{1.56 \rho x^3}{c_1 P} y_4 - ax x_9 = 0, \quad (7)$$
are the relative complex amplitudes of pulsation; $x$, $Z$, and $V$ are the auto- and cross-correlation functions of turbulent velocity and temperature fluctuation, respectively;

$$x^2 = \sum_{i=1}^{3} \bar{w}_i \bar{w}_j^*/3 , \quad Z = \bar{T}^2/T^2 , \quad V = \bar{w}_i \bar{T}/T ;$$

and $\tau^e = (3/16) \tau_c$, $\tau_c$ is the inertial timescale for turbulent convection, which is

$$\tau_c = \frac{c_1 Pr^2}{3 \eta_c GM_x \rho x} ,$$

while $L_1$, $L_3$, and $L_5$ are the convective variables relevant to the third correlations; $c_1$ and $c_2$ are respectively the two convective parameters relevant to the dissipation and diffusion of turbulence. In our statistical theory of turbulent convection (Xiong 1980, 1989), $l_e = c_1 H_P$ ($H_P$ is the local pressure scale height) is the average size of energy-containing eddies; $\epsilon = 2 \eta_c x^3/l_e$ ($\eta_c$ is the Heisenberg eddy-coupling constant) is the turbulent dissipation (Hinze 1975); and $l = c_2 H_P$ is the turbulent diffusion length scale. The e-folding length of convective overshooting is about $1.4(c_1 c_2)^{1/2} H_P$ (Xiong 1985); $\alpha$ and $\delta$ are the coefficients of expansion and compression for the gas. Variables with a subscript $P$ $(or \ T)$ denote the logarithmic partial derivative with respect to $P$ $(or \ T)$, such as

$$\chi_T = \left( \frac{\partial \ln \chi}{\partial \ln T} \right)_P .$$

More detailed definitions concerning the above expressions have been given in our previous work (Cheng & Xiong 1997). One thing that should be made clear is that the gas and radiation are not treated separately $(P = P_g + P_r)$ in the present work, differing from our previous manner. Therefore, the contribution by radiation to all the material properties and to all the thermodynamic quantities, such as $\delta$, $\alpha$, $C_p$, and $\nabla_{ad}$, has been included.
The boundary conditions at the bottom layer are

\[ y_3 = 0 , \]  
\[ y_4 = v_{ad} y_1 , \]  
\[ y_6 = \sqrt{\frac{12\sqrt{3} \eta_e}{1 + 2\sqrt{3} \eta_e}} (2 + i \omega_c) y_5 , \]  
\[ y_8 = \sqrt{\frac{12\sqrt{3} \eta_e}{1 + 2\sqrt{3} \eta_e}} \left( \frac{2}{\sqrt{3} \eta_e} + i \omega_c \right) y_7 , \]  
\[ y_{10} = \sqrt{\frac{12\sqrt{3} \eta_e}{1 + 2\sqrt{3} \eta_e}} \left( 2 + \frac{1}{\sqrt{3} \eta_e} + i \omega_c \right) y_9 , \]

and the boundary conditions at the surface are

\[ y_3 = 1 , \]  
\[ y_1 + \left( 4 + \frac{r^3 \omega_c^2}{GM} \right) y_3 = 0 , \]  
\[ 4y_2 + 2y_3 - y_4 = 0 , \]  
\[ y_6 = \sqrt{\frac{12\sqrt{3} \eta_e}{1 + 2\sqrt{3} \eta_e}} (2 + i \omega_c) y_5 , \]  
\[ y_8 = \sqrt{\frac{12\sqrt{3} \eta_e}{1 + 2\sqrt{3} \eta_e}} \left( \frac{2}{\sqrt{3} \eta_e} + i \omega_c \right) y_7 , \]  
\[ y_{10} = \sqrt{\frac{12\sqrt{3} \eta_e}{1 + 2\sqrt{3} \eta_e}} \left( 2 + \frac{1}{\sqrt{3} \eta_e} + i \omega_c \right) y_9 , \]

As in our previous paper (Cheng & Xiong 1997), the lower boundary is set in the convective overshooting zone, and the upper one is taken to be optically thin enough in the stellar atmosphere. Explicitly, we have adopted an optical depth of \( \tau = 0.01 \) for the upper boundary in the present work.

The convective boundary conditions (eqs. [15]–[17] and [21]–[23]) are given by the asymptotic analysis of the convective quantities within the overshooting zone. For their details we refer to another work (Xiong, Cheng, & Deng 1998), where the spatial oscillations of convective variables are amply discussed.

3. NUMERICAL RESULTS

We have calculated the linear nonadiabatic pulsation for six series of models of luminous red stars. Their masses, luminosities, chemical compositions, and pulsational instability regions are listed in Table 1. The convective parameters are \( c_1 = c_2 = 0.75; \) this leads to about the same energy transport efficiency as the original Vitense theory (Vitense 1958) when the mixing length is taken to be 1.5 times the local pressure scale height, i.e., \( l = 1.5H_p \), and the e-folding length of convective overshooting is about \( 1.4(c_1 c_2)^{1/2}H_p \approx 1.05H_p \). The reddest model in the series is very close to the corresponding Hayashi track, while for the hottest model the convective flux is much smaller than the radiation flux in the second ionization region of helium. For these hotter stars, the \( \kappa \)-mechanism functioning in the second ionization region of helium has already operated, and it has become the main excitation mechanism. Therefore, our models actually cover all the possible temperatures of the luminous red stars outside the Cepheid instability strip, for the present setting of mass, luminosity, and chemical composition.

In the present work we use a simplified MHD equation of state (Hummer & Mihalas 1988; Mihalas, Dappen, & Hummer

| TABLE 1 |
| Pulsational Instability Strip (PIS) |

| MODEL NUMBER | M/M_⊙ | L/L_⊙ | X | Z | F-mode (K) | First Overtone (K) |
|-------------|-------|-------|---|---|------------|------------------|
| 1           | 1.0   | 8000  | 0.70 | 0.02 | ~2300      | 2820–3190        |
| 2           | 1.0   | 5000  | 0.70 | 0.02 | 2660       | 3040–3490        |
| 3           | 1.0   | 3000  | 0.70 | 0.02 | 3260       | 3420–3720        |
| 4           | 1.0   | 8000  | 0.735 | 0.005 | 2740       | 3040–3560        |
| 5           | 1.0   | 5000  | 0.735 | 0.005 | 3260       | 3490–3860        |
| 6           | 1.0   | 3000  | 0.735 | 0.005 | 3790       | 3790–4010        |
1988; Dappen et al. 1988). The neutral helium has been considered as a hydrogen-like atom for convenience of calculation of its energy levels. A significant departure appears only in the ground and the lower excited states. An analytic approach to the OPAL tabular opacities (Rogers & Iglesias 1992) and the low-temperature tabular opacities (Alexander 1975) is used for the calculation of opacity.

3.1. “Mira” Instability Strip

Table 2 gives the linear amplitude growth rates of model series 2 of luminous red stars, \( \eta = -2\pi\omega_j/\omega_n \), where \( \omega = \omega_n + i\omega_t \) is the complex angular frequency of linear nonadiabatic oscillations. Column (1) gives the serial number of the model, column (2) the effective temperature \( T_e \), and columns (3)–(7) the amplitude growth rates for the fundamental through the fourth overtone (taking into account the coupling between convection and oscillations). We have actually calculated the first 12 modes \((n = 0–11)\) for nonadiabatic oscillations. However, all the modes with \( n > 4 \) are pulsationally stable for luminous red stars. For the sake of clarity, we give in Table 2 only the first five modes. With the aim of examining the effect of convection on the pulsational instability for red stars, we have also computed the nonadiabatic pulsation ignoring the coupling between convection and oscillations (however, convection has been included in the static models). If convection does not vary during the oscillation of a star, the convective variables \( y_s = y_a = y_2 = y_b = y_g = y_10 = 0 \), equations (5)–(10), and the convective boundary conditions equations (15)–(17) and (21)–(23) can be omitted. If the terms related to convection are omitted, equations (2) and (10) are reduced to

\[
\frac{d}{d\ln r} \left( \frac{P_{y_1}}{L} \right) - 4 \left( \frac{GM_p}{r} + \rho r^2 \omega^2 \right) y_3 = 0 ,
\]

\[
\frac{d}{d\ln r} \left( \frac{L_{y_4}}{L} \right) - i\omega t \frac{4\pi\rho r C_p T}{L} \left( V_{ad} y_1 - y_2 \right) = 0 .
\]

The number of pulsational equations reduces down to 4. This is the decoupling between convection and oscillations. Equations (1), (3), (24), and (25) and the boundary conditions equations (13), (14), and (18)–(20) were used to calculate the nonadiabatic pulsation of stars ignoring the coupling between convection and oscillations. The corresponding growth rates of the first five modes \((n = 0–4)\) are given as columns (8)–(12) in Table 2. Depicted in Figure 1 are the variations of the amplitude growth rates of the fundamental mode and the first overtone against the stellar effective temperatures for the six model series.

It is clear from Table 2 and Figure 1 that, when the coupling between convection and oscillations are excluded, the fundamental mode, the first overtone, and most of the low-order overtones are pulsationally unstable for all the models of luminous red stars. But this is not the case. Obviously, this is due to ignorance of the coupling between convection and oscillations. More generally speaking, no good interpretation can be made for the red edge of the Cepheid instability strip if one does not take such coupling into consideration (Iben 1971; Xiong 1980).

When the coupling between convection and oscillations is considered, most of the low-order modes and all the higher order modes \((n > 4)\) become pulsationally stable. However, there does exist a pulsationally unstable region for the luminous red stars outside the Cepheid instability strip. The coolest models near the Hayashi track are pulsationally stable. Toward high temperature the fundamental mode first becomes unstable, and then the first overtone. Some one of the second to fourth overtones may become unstable for the hotter models. All the modes higher than the fourth \((n > 4)\) are pulsationally stable. For simplicity, and to make it distinct from the other pulsationally unstable region, we will call it “the Mira instability strip" for the moment, although such a name may not be very appropriate. This instability strip includes not only the regular Mira variables but also the semiregular and irregular variables.

Our calculations show that the pulsational instability of a luminous red star depends critically on stellar metal abundance, luminosity, and mass. The overall properties are the following:

1. For the same mass and luminosity, the instability strip becomes slightly wider and its location moves to lower effective temperature in the H-R diagram as the metal abundance increases.
2. For the same abundance, the instability strip becomes slightly wider and its location moves to lower effective temperature in the H-R diagram as the luminosity increases or the mass decreases.

The above characteristics of luminous red variables are closely connected with the structure of the convective zones of these stars. Convection is the dominant factor that controls the pulsational instability for red stars. \( H_2O \) and \( TiO \) contribute most of the absorption in the low-temperature region, therefore the low-temperature opacity strongly depends on the metal abundance. Moreover, the structure of the stellar convective zone is determined by the nature of the outermost superadiabatic layer, hence the extension of the convective zone depends critically on the metal abundance. That explains why the Mira instability strip and the Hayashi track move to lower temperature as metal abundance increases. At the same time, the structure of the convective zone depends also on the mass and luminosity. The above reasons make it easy to understand the properties of the Mira instability strip.

3.2. Excitation and Damping of Oscillations

For the luminous red stars outside the instability strip, the convective energy transport exceeds 99% of the total luminosity in the ionization regions of hydrogen and helium. Figures 2 and 3 give the integrated work \( W_r \) versus depth for the luminous stars pulsating at the fundamental mode. In Figure 2, convection has been included for the static model, while the coupling between convection and oscillations is ignored in calculations of nonadiabatic oscillations. The only excitation and damping mechanism results from radiation. It can be found from Figure 2 that for the two redder models the excitation comes mainly from the outermost gradient region of radiation flux, where \( \chi_T \) approaches its maximum. This excitation mechanism functioning in the gradient region of radiation flux will be detailed in our second paper (Xiong et al. 1997b). The radiative
| Model Number | $T_e$ | Rate with Dynamic Coupling of Convection | Rate without Convection Coupling |
|--------------|-------|----------------------------------------|---------------------------------|
|              |       | 0      | 1    | 2    | 3 | 4    | 0 | 1  | 2 | 3 | 4  |
| 1           | 2400.0| 0.116E+01 | -0.408E+00 | -0.956E-01 | -0.906E-01 | -0.105E-01 | 0.194E+01 | 0.430E+00 | 0.541E-01 | 0.726E-01 | 0.847E-01 |
| 2           | 2475.0| 0.676E+00 | -0.479E+00 | -0.117E+00 | -0.709E-02 | -0.278E+00 | 0.157E+01 | 0.427E+00 | 0.459E-01 | 0.976E-01 | 0.593E-01 |
| 3           | 2550.0| 0.659E+00 | -0.229E+00 | -0.143E+00 | -0.453E-01 | -0.856E-01 | 0.137E+01 | 0.441E+00 | 0.252E-01 | 0.142E+00 | 0.141E-01 |
| 4           | 2625.0| 0.755E+00 | 0.299E-01  | -0.102E+00 | -0.106E-01 | -0.103E+00 | 0.125E+01 | 0.463E+00 | 0.285E-02 | 0.184E+00 | -0.246E-01 |
| 5           | 2700.0| 0.784E+00 | 0.369E-01  | -0.913E-01 | -0.169E-01 | -0.108E+00 | 0.116E-01 | 0.485E-00 | -0.118E-01 | 0.207E+00 | -0.374E-01 |
| 6           | 2775.0| 0.883E+00 | 0.353E-01  | -0.904E-01 | -0.249E-01 | -0.100E+00 | 0.113E-01 | 0.507E+00 | -0.207E-01 | 0.217E+00 | -0.270E-01 |
| 7           | 2850.0| 0.939E+00 | 0.114E+00  | -0.903E-01 | -0.350E-01 | -0.733E-01 | 0.111E+01 | 0.532E+00 | -0.163E-01 | 0.200E+00 | 0.264E-01 |
| 8           | 2925.0| 0.106E+01 | 0.258E+00  | -0.101E+00 | -0.318E-01 | -0.579E-01 | 0.109E+01 | 0.555E+00 | -0.275E-02 | 0.184E+00 | 0.676E-01 |
| 9           | 3000.0| 0.127E+01 | 0.426E+00  | -0.920E-01 | -0.161E-01 | -0.420E-01 | 0.109E+01 | 0.575E+00 | -0.333E-01 | 0.141E+00 | 0.123E+00 |
| 10          | 3075.0| 0.115E+01 | 0.415E+00  | -0.179E+00 | -0.746E-01 | -0.217E-02 | 0.110E+01 | 0.596E+00 | -0.724E-01 | 0.104E+00 | 0.149E+00 |
| 11          | 3150.0| -0.350E+01| 0.760E+00  | -0.996E-01 | -0.198E+00 | -0.107E+01 | 0.113E+01 | 0.616E+00 | -0.146E+00 | 0.350E-01 | 0.152E+00 |
| 12          | 3225.0| -0.185E+01| 0.615E+00  | -0.139E+00 | -0.219E+00 | -0.819E-01 | 0.117E+01 | 0.636E+00 | -0.175E+00 | 0.123E-01 | 0.143E+00 |
| 13          | 3300.0| -0.374E+01| 0.137E+00  | -0.806E-01 | -0.660E-01 | -0.821E-01 | 0.122E+01 | 0.647E+00 | -0.224E+00 | -0.330E-01 | 0.109E+00 |
| 14          | 3375.0| -0.309E+00| 0.110E+01  | 0.572E-01  | 0.907E-02  | 0.260E-01  | 0.127E+01 | 0.653E+00 | -0.202E+00 | -0.315E-01 | 0.955E-01 |
| 15          | 3450.0| -0.247E+00| 0.903E+00  | -0.190E+00 | 0.293E-01  | -0.180E+00 | 0.138E+01 | 0.649E+00 | -0.168E+00 | -0.350E-01 | 0.727E-01 |
| 16          | 3525.0| -0.609E-01| 0.191E+00  | -0.181E+01 | 0.163E+00  | 0.406E-01  | 0.166E+01 | 0.644E+00 | -0.125E+00 | -0.507E-01 | 0.391E-01 |
| 17          | 3600.0| -0.171E+00| 0.135E+00  | -0.394E+00 | 0.420E-01  | -0.266E-01 | 0.193E+01 | 0.647E+00 | -0.739E-01 | -0.884E-01 | -0.933E-02 |
| 18          | 3675.0| -0.310E+00| 0.458E-01  | 0.638E+00  | 0.162E+00  | -0.519E-02 | 0.210E+01 | 0.660E+00 | -0.257E+02 | -0.143E+00 | -0.813E-01 |
| 19          | 3750.0| -0.206E+00| 0.323E-01  | -0.427E-01 | 0.426E-01  | -0.427E-01 | 0.221E+01 | 0.654E+00 | -0.754E-01 | -0.242E+00 | -0.198E+00 |
| 20          | 3825.0| -0.196E+00| 0.510E-01  | -0.306E+00 | 0.582E-01  | -0.164E+00 | 0.227E+01 | 0.636E+00 | -0.165E+00 | -0.357E+00 | -0.389E+00 |

**TABLE 2**

Amplitude Growth Rates for a Model Series of Luminous Red Stars
excitation arising in the second ionization region of helium becomes negligible, because the radiative flux is far smaller than the convective flux here. The radiative excitation occurring in the second ionization region of helium becomes nonnegligible for the hotter model. For a clearer display, the variation of $\chi_T$ and $\log (L/L_\odot)$ in the stellar interior are given in Figure 2 as well. Because of the convective energy transport, the radiative damping in the deep interior of a star is greatly weakened. Hence, the star is always pulsationally unstable when the coupling between convection and oscillations is ignored.
Figure 2.—Variations of the integrated work $W_p$ (solid line), $\Gamma_s$ (dotted line), $\chi_r$ (dashed line), and $\log (L_r/L)$ with respect to depth ($\log T$) for three models of model series 2 ($M = 1 M_\odot$, $L = 5000 L_\odot$, $X = 0.700$, $Z = 0.020$). The coupling between convection and oscillations is ignored in the pulsation calculation, while convection is included for the corresponding static models. The three horizontal lines are the locations of the ionization regions of H and He.

Figure 3 plots the quantities for the same luminous red star model, but with the coupling between convection and oscillations being considered. $W_p$, $W_{pr}$, and $W_{vis}$ represent, respectively, the contributions of the gas (and radiation) pressure, turbulent pressure, and turbulent viscosity. Their exact definitions are referred to our previous work (Cheng & Xiong 1997). $W_{all} = W_p + W_{pr} + W_{vis}$ is the total integrated work, the value of which at the surface of a star should be the amplitude growth rate $\eta = -2\pi \nu / \omega$, given by the nonadiabatic pulsation calculation. Our results show that they are very closely matched indeed.

By comparing Figures 2 and 3, one can see that their characteristics are totally different. It can be found from Figure 3 that with consideration of the coupling between convection and oscillations the excitation (and damping) region of pulsation, where $W_{all}$ changes obviously, extends deeper into the stellar interior in comparison with Figure 2. This is surely a result of the coupling between convection and oscillations.

Convection leads to energy and momentum exchange inside a star, and hence affects the pulsational instability for variable stars. We refer to the effects of convective energy transport on variable stars as thermodynamic coupling, while the effect of momentum exchange on variable stars is called dynamic coupling.

The gas (and radiation) pressure term $W_p$ in the total integrated work contains the effect of energy transport not only by radiation but also by convection (i.e., the contribution by thermodynamic coupling between convection and oscillations). It is hard to separate the effects of convective energy transport unambiguously from radiative transport, because the two factors are closely coupled in the process of stellar oscillations. Following the conservation law of energy (momentum), any change in convective flux (turbulent pressure) must lead to the corresponding change in the radiation flux (gas pressure) in order to balance the previous factor, and vice versa. By the same reason, it is not so easy to separate the contribution of gas (and radiation) pressure $W_p$ unambiguously from that of the turbulent pressure $W_{pr}$.

The combination of $W_{pr}$ and $W_{vis}$ is the dynamic coupling between convection and oscillations. The effect of turbulent viscosity is to convert the kinetic energy of ordered pulsation into random turbulent kinetic energy, and this process happens in the low-wavenumber region of turbulent energy spectra. Through the nonlinear effects of fluid dynamics, the energy gained
in the pulsational motion is cascaded into higher and higher wavenumber regions, and is eventually converted into thermal energy at the smallest turbulent eddies. Therefore, the turbulent viscosity always works as a damping factor to pulsation, i.e., \( W_{\text{vis}} \) is always negative. Although \( W_{\text{vis}} \) is far less than \( W_p \) and \( W_{\text{pr}} \) in absolute values for the fundamental mode of luminous red stars, we cannot ignore its effect on the stability of pulsation. The contribution of turbulent viscosity increases very quickly toward higher modes for luminous red variables because viscous dissipation is proportional to the square of the velocity gradient, which grows as the oscillation mode increases. The turbulent viscosity becomes the main damping mechanism of the high overtones. This is the reason why the luminous red stars become pulsationally stable at high modes. Figure 4 demonstrates the integrated work diagram for the first, third, and fifth modes of a \( T_e = 2550 \) K model. Figure 3 (upper left-hand panel) and Figure 4 clearly show the quick increase of \( W_{\text{vis}} \) toward higher modes.

The coupling between convection and oscillations depends critically on the ratio of the timescale of convective motion \( \tau_c \) to that of pulsation \( \tau_p = 2\pi/\omega_p \), i.e., \( \omega_p \tau_c \ll 1 \), the convection is in nearly quasi-steady variation. When \( \omega_p \tau_c \) increases, convection lags behind the pulsational variation. The result is that not only does its variation amplitude decrease, but also a phase lag exists. The interaction between convection and oscillations depends on both the amplitude of the convective variations and the phase of such variations. As we will show later on (Xiong et al. 1997b), the distribution in the interior of luminous red giants is very different from that of RR Lyrae and horizontal-branch red stars. This difference results from the difference of the mass-luminosity ratios of these two types of stars. The long-period variables have very low mass-luminosity ratios. Therefore, their internal structure has very high central concentration. This may explain the difference in pulsational characteristics of these two kinds of stars. In Figures 3 and 4 we have also given the curves for \( \omega_p \tau_c \) versus depth (log \( T \)). This shows that, for such luminous red stars, \( \omega_p \tau_c \leq 1 \) holds for most of the interior of the envelope model except the extremely outermost layer.

4. CONCLUSIONS AND DISCUSSION

By using a nonlocal and time-dependent theory of convection, we have very carefully treated the coupling between convection and oscillations. The linear stability analysis for luminous red stars gives us the following results:
1. The coupling between convection and oscillations is the dominant factor for the pulsational instability. When this coupling is not considered, all the low-temperature red stars are pulsationally unstable, while, when it is taken into account, a Mira instability strip will show up outside the Cepheid instability strip.

2. Except for some slightly hotter models which may have pulsationally unstable second- to fourth-overtone modes, luminous red variable stars pulsate in the fundamental mode or the first overtone. All the modes higher than 4 are pulsationally stable.

3. For the low-temperature red stars, the dynamic coupling between convection and oscillations is of the same order of magnitude as the thermodynamic coupling, and may even overtake the latter.

4. The effect of turbulent viscosity grows very quickly toward high overtones for the luminous red stars. For high overtones, it becomes the main damping mechanism.

The troublesome spatial oscillations of the thermodynamic quantities are triggered when the local time-dependent convection theory is used in calculating the stellar oscillations. (Keeler 1977; Baker & Gough 1979; Gonczi & Osaki 1980). In the present calculations of the nonlocal convection time-dependent theory the spatial oscillations still exist but are effectively controlled. For the model of luminous red variable stars, the spatial oscillations happen at the extreme outer layer of the atmosphere, where $\omega \tau > 1$. In the stellar interiors, where $\omega \tau < 1$, the troublesome spatial oscillations do not appear. It is difficult to trace the spatial oscillations in Figures 3 and 4, since the convective energy transport is far less than the radiative energy transport, and the turbulent pressure is far less than the gas pressure in the atmosphere of a luminous red giant. It seems true that the spatial oscillations do not affect the pulsation instability of luminous red stars to a considerable level.

As for the long-disputed pulsational mode for $\omega$ Ceti, some authors think that $\omega$ Cet is pulsating in the first overtone (Kamijo 1962; Keeler 1970; Wood 1974, 1981; Tuchman et al. 1978; Tuchman 1991), while some others believe that it oscillates in the fundamental mode (Hill & Wilson 1979; Wilson 1982; Bowen 1988). The theoretical pulsation period $P$ and pulsation constant $Q$ for a luminous red star model, with $L = 5000 L_\odot$, $X = 0.700$, $Z = 0.020$, are given in Table 3. Figure 5 gives the values of $P$ and $Q$ of the fundamental and first modes versus the effective temperature for six model series. Our theoretical value of the pulsation constant $Q$ is very close to that of Ostlie & Cox (1986), but our $Q_1$ for the first overtone is
Fig. 5.—Variations of the pulsation periods $P$ and the pulsation constants $Q$ for the fundamental mode and the first overtone against the effective temperature. The coupling between convection and oscillations is included.

systematically smaller than that of Wood (1982), especially at the long-period end. Following our numerical results, there is no model having a first overtone $Q_1$ that exceeds 0.045 days. Normally, $Q_1 \leq 0.041$. Assuming for $\alpha$ Cet $M \approx M_\odot$, $T_\text{e} = 2900$ K (Wood 1990a, 1990b) and that it pulsates at the first overtone, then we have $L \approx 12,500 L_\odot$ or $M_{\text{bol}} \approx -5.5$. This would make it 0.7 mag more luminous than the value of $M_{\text{bol}} = -4.8$ given by the $P$-$L$ relation for Miras in the Large Magellanic Cloud (Glass et al. 1987; Hughes & Wood 1990). This is unlikely to be real. Wood (1990b) proposed another picture for $\alpha$ Cet. If Mira's $P$-$L$ relation depends on the metal abundance, then the discrepancy between the observed and the theoretical $Q$-values could be removed. Our results of the linear pulsational stability survey support Wood's guess. The instability strip
moves indeed toward the low temperature as the metal abundance increases, as shown by Table 2 and Figure 1. Our calculations show that if the metal abundances for LMC Mira variables are only one-half of that of o Cet, the red edge of the Mira instability strip in the Galaxy will be lowered by about 350 K compared with that in the LMC. Therefore, the magnitude of the nonadiabatic calculation will be 330 days. Following our linear stability analysis, it appears more likely that o Cet derived from the LMC P-L relation will appear 0.6 mag more luminous than it actually is. Let us suppose that the nonadiabatic calculation will be 330 days. Following our linear stability analysis, it appears more likely that o Cet pulsates in the fundamental mode than in the first overtone.

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TABLE 3
PULSATION PERIOD AND PULSATION CONSTANT FOR A LUMINOUS RED STAR MODEL.