Heat transfer in three-dimensional intersecting shock-wave/turbulent boundary-layer interactions with wall-modeled large-eddy simulations

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The accurate prediction of aerothermal surface loading is of paramount importance for the design of high speed flight vehicles. In this work, we consider the numerical solution of hypersonic flow over a double-finned geometry, representative of the inlet of an air-breathing flight vehicle, characterized by three-dimensional intersecting shock-wave/turbulent boundary-layer interaction at Mach 8.3. High Reynolds numbers ($Re_L \approx 11.6 \times 10^6$ based on free-stream conditions) and the presence of cold walls ($T_w/T_o \approx 0.27$) leading to large near-wall temperature gradients necessitate the use of wall-modeled large-eddy simulation (WMLES) in order to make calculations computationally tractable. The comparison of the WMLES results with experimental measurements shows good agreement in the time-averaged surface heat flux and wall pressure distributions, and the WMLES predictions show reduced errors with respect to the experimental measurements than prior RANS calculations. The favorable comparisons are obtained using an LES wall model based on equilibrium boundary layer approximations despite the presence of numerous non-equilibrium conditions including three dimensionality, shock-boundary layer interactions, and flow separation. Lastly, it is also demonstrated that the use of semi-local eddy viscosity scaling (in lieu of the commonly used van Driest scaling) in the LES wall model is necessary to accurately predict the surface pressure loading and heat fluxes.

Nomenclature

\begin{align*}
x, y, z &= \text{coordinate in the inertial coordinate system, m} \\
L_r &= \text{reference length for normalization, m} \\
t &= \text{physical time, s} \\
\rho &= \text{density, kg/m}^3 \\
P &= \text{pressure, Pa} \\
P_0 &= \text{total pressure, Pa} \\
T &= \text{temperature, K}
\end{align*}

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I. Introduction

Hypersonic wall-bounded flows for realistic flight vehicles can be characterized by high Reynolds numbers and cold surface temperatures compared to the free-stream stagnation temperature. The prohibitive computational cost associated with high Reynolds numbers are well known [1], while the cold wall conditions exacerbate near-wall
resolution requirements associated with the large temperature gradients in the vicinity of peak viscous dissipation. As a result, direct numerical simulations of these flows have been largely limited to simple geometries with low Reynolds numbers, e.g. the high-speed compressible boundary layer flows [2], the hypersonic boundary-layer transitional flow for a flared cone [3], the turbulent boundary layer along a compression ramp [4], the transitional shock/boundary-layer interaction [5].

When more realistic geometries and conditions are considered, the RANS approach is commonly used in industrial settings due to reduced computational costs compared to DNS strategies. However, RANS based approaches have been demonstrated to have limited accuracy in hypersonic flow regimes; significant errors in peak aerodynamic heating ($\approx 25\%$) [6] are observed and macroscopic flow characteristics are misrepresented, in particular when laminar/turbulent transition or boundary layer separation are present [7][8]. Cold wall conditions in hypersonic flow regimes also challenge traditional RANS models (e.g., Spalart-Allmaras, $k\rightarrow\omega$) to predict near-wall turbulent fluctuations or transverse heat fluxes [9] even in zero-pressure gradient boundary layers. Algebraic RANS closures, such as the Baldwin-Lomax model [10], have been shown to offer reasonable predictions in high Mach number boundary layer flows [11]. However, the application of these models to the double-finned inlet flow presently considered show substantial errors in the surface heat fluxes and in the extent of flow separation [12].

Large-eddy simulations have been shown to offer superior accuracy in the prediction of many of these flow regimes, however, near-wall resolution requirements are prohibitive in high Reynolds number conditions. Alternatively, the WMLES approach, where flow structures that scale with the boundary layer thickness while effects of unresolved near-wall eddies (at viscous length scales, $l_v = \nu/u_\tau$) are modeled, has been shown to be computationally tractable for high Reynolds number flows and predictive of complex flow phenomena [13]. In the context of high-speed flows, these approaches have been successfully applied to the prediction of shock-induced separation [14][15], oblique shock wave interaction with lateral confinement and boundary layer separation [16][17], transitional flows [18][19][20], and aerodynamic heating [21][22]. However, most of the high speed applications of WMLES have been conducted in relatively simple geometries or without the presence of relevant cold wall conditions.

To this end, the present work considers a canonical model of a realistic inlet of an air-breathing hypersonic vehicle [23]. The configuration consists of two sharp fins mounted on a flat plate. An incident hypersonic turbulent boundary layer approaches the two vertical fins generating a crossing shock pattern resulting in high local aerothermal loading and flow separation. The objective of this investigation is to assess the suitability of wall modeled LES in this complex geometry and flow regime with emphasis on the ability to predict surface heat fluxes, mechanical loading, and separation that arises from the impinging shock structure.

The remainder of this paper is organized as follows. In Section II, the governing equations, the wall model and the numerical method are briefly reviewed. In Section III, the flow conditions and corresponding computational setup are described. Section IV analyzes the WMLES results and assesses their accuracy with respect to existing experimental measurements and prior RANS simulations. It is additionally demonstrated that the cold wall conditions in this configuration necessitate augmentation of wall model eddy viscosity to be scaled on semi-local conditions rather
than solely on wall quantities typically used in prior WMLES calculations. Concluding remarks and discussions are
provided in Section V.

II. LES methodology

A. LES governing conservation equations

The Favre-averaged compressible Navier-Stokes equations in the conservative form are

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho \tilde{u}_j}{\partial x_j} = 0,
\]

(1)

\[
\frac{\partial \rho \tilde{u}_i}{\partial t} + \frac{\partial \rho \tilde{u}_i \tilde{u}_j}{\partial x_j} + \frac{\partial P}{\partial x_i} = \frac{\partial \sigma_{ij}}{\partial x_j} - \frac{\partial \tau_{ij}^{sgs}}{\partial x_j},
\]

(2)

\[
\frac{\partial E}{\partial t} + \frac{\partial ([E + P]) \tilde{u}_j}{\partial x_j} = \frac{\partial}{\partial x_j} (k \frac{\partial \tilde{T}}{\partial x_j}) + \frac{\partial (\tilde{u}_i \sigma_{ij})}{\partial x_j} - \frac{\partial Q_j^{sgs}}{\partial x_j} - \frac{\partial (\tilde{u}_i \tau_{ij}^{sgs})}{\partial x_j},
\]

(3)

where \( \rho, P, \) and \( T \) denote the fluid density, pressure, and temperature, respectively. \( \tilde{u}_i \) denotes the velocity component in the \( x_i \) coordinate direction. \( \tilde{T} = \tilde{u} \rho + \rho \tilde{u}_k \tilde{u}_k / 2 \) denotes the total energy, \( \tilde{\sigma}_{ij} = \mu(\tilde{T})(\tilde{S}_{ij} - 1 / 3 \tilde{S}_{kk} \delta_{ij}) \) is the resolved deviatoric stress tensor, and \( \tilde{S}_{ij} = 1 / 2(\partial \tilde{u}_i / \partial x_j + \partial \tilde{u}_j / \partial x_i) \) is the resolved strain rate tensor. The subgrid stress \( \tau_{ij}^{sgs} \) and heat flux \( Q_j^{sgs} \) arising from the effect of unresolved eddies are defined as

\[
\tau_{ij}^{sgs} = \bar{p}(\tilde{u}_i \tilde{u}_j - \bar{u}_i \bar{u}_j), \quad Q_j^{sgs} = \bar{p}(e \tilde{u}_j - \bar{e} \tilde{u}_j),
\]

(4)

The subgrid stresses and heat fluxes are presently closed with a Vreman eddy viscosity model [24] supplemented
with a constant turbulent Prandtl number (\( Pr_t = 0.9 \)). The equation of state for the fluid is a calorically perfect gas,
\( P = \rho R \tilde{T} \), where \( R \) denotes the specific gas constant. The relation between the dynamic viscosity and the temperature
is characterized by the Sutherland’s law with a model constant \( S/T_r = 1.38, T_r = 80 \) K, and the Prandtl number is
0.72. The calorically perfect gas assumption is adopted based on the low free-stream temperature for the configuration
under consideration, \( T_\infty = 80 \) K. For brevity, the operator symbol \( \bar{\cdot} \) (Reynolds average) and \( \tilde{\cdot} \) (Favre average) are
omitted hereafter.

B. LES wall model based on equilibrium boundary layer approximations

As the near-wall eddies whose size are characterized by viscous length scales are not resolved in the present formulation,
their aggregate effect on the wall stress and heat flux must be modeled. (For a detailed description of the wall model
formulation, see [13] [25].) The present LES wall model assumes that the pressure gradient and convection effects can be
neglected for unresolved eddies between the wall and the local LES resolution (of grid size, \( \Delta \)), and
that these eddies reach a statistically stationary state over the duration of the simulation time step (\( \Delta t \)). With these
approximations, the simplified momentum and total energy equations can be written as

\[
\frac{d}{dy} \left[ (\mu + \mu_{t,wm}) \frac{du_u}{dy} \right] = 0,
\]

(5)

\[
\frac{d}{dy} \left[ (\mu + \mu_{t,wm}) u_\parallel \frac{du_\parallel}{dy} + c_p \left( \frac{\mu}{Pr} + \frac{\mu_{t,wm}}{Pr_{t,wm}} \right) \frac{dT}{dy} \right] = 0,
\]

(6)
where \( y \) and \( u_{||} \) denote the wall-normal coordinate and the velocity component parallel to the wall, respectively. \( c_p \) and \( Pr \) denote the specific heat capacity at constant pressure, and the molecular Prandtl number, respectively. Turbulent stresses and heat fluxes are modeled with an eddy viscosity, \( \mu_{t,wm} \), given by the following mixing length model:

\[
\mu_{t,wm} = \kappa y \sqrt{\frac{T_w}{\rho}} D,
\]

where \( \kappa = 0.41 \) is the von Kármán constant. In order to recover the linear behavior in the viscous sublayer and the buffer layer in a zero-pressure gradient limit, the damping function \( D \) is defined as

\[
D = \left[ 1 - \exp \left( \frac{y_D^+}{A^+} \right) \right]^2,
\]

where the dimensionless constant \( A^+ = 17 \) indicates the characteristic length scale of the exponential damping. \( y_D^+ = y/(\nu_w/\tau_w) \), where \( \nu_w \) and \( \tau_w \) denote the kinematic viscosity and friction velocity at the wall. However, it is shown in [26][27] that the van Driest transformation performs poorly in collapsing the compressible velocity profile onto the incompressible counterpart for wall-bounded flows with cold wall condition. Due to this, we additionally consider in this work a semi-local scaling

\[
y^{+}_{SL} = \frac{\rho(y)\sqrt{\tau_w/\rho}y}{\mu(T(y))},
\]

where the dynamic viscosity is computed based on the local conditions at a off-wall distance, \( y \), and \( y^{+}_{SL} \) is used in place of \( y^{+}_{D} \) in Eq. (8).

The boundary conditions for Eqns. (5) and (6) for the velocity and temperature are no-slip, isothermal conditions at the wall \( (u_{||}(y = 0) = 0, T(y = 0) = T_w) \) and the interior LES conditions \( (u_{||}(h_{wm}) = u_{LES}, T(h_{wm}) = T_{LES}) \) at a distance, \( h_{wm} \), from the wall. In this work, the matching location is chosen as the first off-wall cell center in the local wall-normal direction. Although Kawai and Larsson [28] propose that the matching location should be several cells away from the wall to provide better resolved solution for the wall model as the boundary condition, recent investigations have shown that the first-cell coupling is sufficient for the prediction of relevant quantities of interest such as the skin friction or wall heat fluxes (see [18]). This choice is further validated by this study through the comparisons with the available experimental data in later sections. It is worthwhile to note that while the wall model does not explicitly contain the non-equilibrium pressure gradient or convection effects, the influence of these phenomena are implicitly present in the time dependent far field boundary conditions that the interior LES provides.

C. Numerical methods

The compressible, finite-volume code charLES [29], is used to conduct the numerical simulations herein. The numerical method consists of a low dissipation, approximate entropy-preserving scheme and utilizes artificial properties to capture solution discontinuities. The LES governing equations are temporally integrated by the explicit third-order strong-stability-preserving (SSP) Runge-Kutta method [30]. The spatial and temporal schemes converge to second- and third-order with respect to the nominal mesh spacing and time step, respectively. Computational meshes based on arbitrary polyhedra are constructed from the computation of Voronoi diagrams associated with the specification of the
location of the degrees of freedom clipped against the domain boundaries [31]. Additional details of the numerical method and relevant solver validation can be found in [32], [33], [34], [29], and [35].

III. Double-finned problem definition and computational setup

The present geometry and computational setup follow those described in the experiment of a 15° double-finned configuration [23]. The geometry is composed of two sharp fins with wedge angle $\alpha = 15^\circ$ fastened to a flat plate, as shown in Fig. 1. Specifically, each fin is 20 cm high and 40.6 cm long, and the flat plate is 220 cm long and 10 cm high. The double fins are placed 165 cm downstream of the leading edge of the flat plate such that there is sufficient length for a turbulent boundary layer to develop. The free-stream flow measured 3 cm ahead of the double fins (i.e. at $x_0 = 162$ cm) has a Mach number $Ma_\infty = 8.23$ and Reynolds number $Re_\delta = 1.7 \times 10^5$ based on the local boundary layer thickness $\delta_0 = 3.25$ cm. The wall is isothermal at $T_w = 300$ K which is substantially colder than the stagnation temperature $T_\infty = 1177$ K. In the following discussion, velocity, temperature, density and length are normalized by the sound speed $c_r = 179$ m/s, the reference temperature $T_r = 80$ K, the reference density $\rho_r = 0.0186$ kg/m$^3$ and the reference length $L_r = 1$ cm.

![Figure 1. The geometry parameters and the computational coordinate system.](image)

Figure 1. The geometry parameters and the computational coordinate system. The wedge angle for the double fins is $\alpha = 15^\circ$. The arrow indicates that the fluid flows from the leading edge of the flat plate towards the double fins. The leading edges of both the flat plate and the double fins are sharp and the model is symmetric with respect to the plane $z = 0$ cm. The free-stream condition is defined 3 cm ahead of the double fins at $x_0 = 162$ cm. This figure is adapted from Fig. 1 of [23].

The computational geometry is given in Fig. 2(a). The computational domain is sufficiently large such that there are no artificial reflections from the far-field outflow boundaries. The inlet flow condition is imposed by combining a uniform flow with the turbulence fluctuation generated by the synthetic turbulence generation method. Freestream conditions upstream of the sharp leading edges are adjusted such that the Mach number (behind the leading edge
shock) matches the experimental measurements upstream of the double fin entrance. The computational domain is discretized with the $7 \times 10^7$ Voronoi mesh elements adaptively clustering near the wall, as shown in Fig. 2(b,c,d). Based on the resolution of the finest Voronoi mesh element near the wall, the turbulent boundary layer at $x_o = 162$ cm is resolved by approximately 40 cells. (The present resolution is much coarser than that previously employed for studying the confinement effects in shock wave/turbulent boundary layer interactions, see Table 1 of [16]). The resolution is coarsened further away from the wall to a maximum of $\approx 0.1\delta_0$ (see Fig. 2(d)). It is infeasible to further coarsen the deployed grid for the following two reasons. First, further coarsening the grid will lead to insufficient resolution to properly capture the boundary layer growth from the sharp leading edge. Second, the side wall separation is small, and this resolution is sufficient to capture the side wall separation bubble (see Fig. 12). Capturing the side wall separation is necessary in order to properly predict the surface heat flux distribution. Nonetheless, the present first off-wall mesh resolution is roughly 400-time coarser when compared to that required by the RANS simulations (see Table 2 of [36]).

| $(\Delta x, \Delta y, \Delta z)/\delta_0 |_{\text{min}}$ | $(\Delta x, \Delta y, \Delta z)/\delta_0 |_{\text{max}}$ |
|---------------------|---------------------|
| $2.5 \times 10^{-2}$ | $1.0 \times 10^{-1}$ |

Figure 2. Computational geometry and mesh sketch for the double-fin simulations: (a) the overview of the computational geometry; (b,c,d) the zoom-in views of the Voronoi mesh distributions (70M total mesh elements).

7 of 27
IV. Results and discussion

In this section, the numerical results from WMLES with the semi-local scaling based damping function (Eq. (9)) are analyzed and compared against the experimental measurements. The predictions of the WMLES using the van Driest scaling (Eq. (8)) will also be assessed. Unless otherwise indicated, the bracket \( < \cdot > \) denotes time-average operator, \( f' = f - < f > \) denotes the fluctuation defined based on the Reynolds average, and \( f'' = f - < \rho f > / < \rho > \) denotes the fluctuation defined based on the Favre average.

A. WMLES with semi-local scaling based damping function

1. Overall statistics

Fig. 3 shows the time- and spanwise-averaged Mach number contour on a wall-normal plane and the instantaneous streamwise velocity distribution on a wall-parallel plane at \( y/L_r = 0.3616 \) between the leading edge of the flat plate and that of the double fins. A weak shockwave is generated at the leading edge of the flat plate, and slightly decreases the Mach number downstream as shown in Fig. 3(a). The instantaneous streamwise velocity field in Fig. 3(b) reveals that the boundary layer transitions, and eventually becomes fully turbulent ahead of the double fins. The turbulent boundary layer appears sustained for approximately \( 25\delta_0 \) upstream of the double find entrance. More quantitative comparisons of the flow statistics between the experimental data and the WMLES results at \( x/L_r = 162 \), just upstream of the fins, are given in Fig. 4. While all the statistics close to the boundary layer edge are in good agreement, there are notable discrepancies inside the boundary layer. The discrepancies are in part due to the lingering effect of artificial inflow conditions and relatively short developing length of the incoming boundary layer from the sharp leading edge of the plate.

The results of a grid convergence study at \( x/L_r = 162 \) is shown in Fig. 5. The fine grid denotes the mesh with parameters given in Table 2. The resolutions of the medium and coarse grids are 50% and 70% coarser than that of the fine grid in each coordinate direction, respectively. While the density profile is not sensitive to mesh refinement, the mean streamwise velocity just upstream of the fins exhibits considerable sensitivity to the grid resolution. In particular, the boundary layer thickness predicted from the medium and coarse grids is 38% smaller than that given by the experiment, and consequently the local effective Reynolds number differs from the experimental setup as well. Hereafter, only the simulation results from the fine grid will be discussed and the rationality for this resolution choice will be addressed by comparing the WMLES results with the experimental data at downstream locations.

Fig. 6 shows the time-averaged \( y^+ \) at the first off-wall cell centers, i.e. the matching locations for the wall model. The largest \( y^+ \) appears around the leading edges of the double fins and the regions where the shock waves impinge on the surfaces. It is clear that the near-wall flow is not directly resolved in the simulations and the wall model plays a pivotal role in the predicted flow states. The \( y^+ \) values near the shock impingement location below \( y^+ \approx 20 \) suggesting that the details of the LES wall model damping function, \( D(y^+) \), may significantly impact the flow solutions.

The instantaneous surface heat flux distribution, the time-averaged surface heat flux, surface pressure, and surface shear stress distributions are shown in Fig. 7. The instantaneous surface heat flux distribution in Fig. 7(a) demonstrates
Figure 3. (a) Time- and spanwise-averaged Mach number contour on the wall-normal x-y plane and (b) the instantaneous streamwise velocity distribution on the wall-parallel x-z plane of $y/L_r = 0.3616$. In panel (b), the high-speed region is associated with the oblique shock wave originating from the leading edge of the flat plate at $x/L_r = 0$.

Figure 4. Time- and spanwise-averaged distributions of (a) Mach number, (b) density, and (c) streamwise velocity at $x/L_r = 162$. The blue and red lines denote the results from WMLES and experiment [23], respectively.

that the flow is highly unstable after the double shockwaves induced by the fin leading edges intersect around the shoulders. As shown in Fig. 7(b)(c)(d), right downstream of the shock waves intersection, the distributions of the time-averaged surface heat flux, surface pressure and surface stress attain local maxima around the centerline on the plate. Furthermore, the excessive aerodynamic heating and friction are generated around the shock impingement locations on the fin surfaces.
Figure 5. Time- and spanwise-averaged distributions of (a) density and (b) streamwise velocity at $x/L_r = 162$. The fine grid denotes the mesh with parameters given in Table 2. The resolutions of the medium and coarse grids are 50% and 70% coarser than that of the fine grid in each coordinate direction, respectively. EXP denotes the experimental data [23].

Figure 6. Distribution of the time-averaged $y^+$ at the first off-wall cell centers. For facilitating the presentation, only the data over the flat plate and one vertical fin are shown.

2. Data analyses in x-z and x-y planes

The time-averaged surface pressure and heat flux distributions along the centerline of the plate between the two fins as well as the double-shock intersection location based on the inviscid theory are given in Fig. 8. The predicted time-averaged pressure distribution from WMLES is in good agreement with the experiment including in the region downstream of the shock intersection. The static pressure first increases significantly due to the shockwave intersection and subsequently exhibits a rapid drop due to the expansions emanating from the fin shoulders, as depicted by Fig. 9(a). It is also observed that the peak surface pressure $< P_w > / P_{w,\infty} \approx 22$ is considerably lower than the prediction from the inviscid theory, $< P_w > / P_{w,\infty} \approx 45$ [37]. Further downstream at $x/L_r \approx 198$, a smaller pressure peak appears due to the second crossing of the reflected shock waves. In terms of the heat flux distribution, the agreement with the experimental data is also favorable across the entire channel between the double fins. The streamwise variation of the surface heat flux follows that of the surface pressure qualitatively. In detail, both the experiment and the
WMLES results exhibit an initial decline at $x/L_r \approx 170$, and the predicted heat flux is 20% smaller than that from the experiment in the pre-shock region of $x/L_r \approx 180$, which is the location of a secondary small flow separation (see the discussions of Fig. 11). Downstream of the shock wave intersection, the peak heat flux shows 4% discrepancy between the WMLES results and the experimental data. Similar differences are also observed further downstream at $x/L_r = 194$ in the low pressure region and heat flux valley (see also Fig. 9). Nevertheless, the present prediction of both quantities shows a much better agreement with the experiment than those from the RANS simulations [37][36]. In the RANS solutions, the heat flux plateau around $x/L_r = 180$ before the shock wave intersection is completely missed and great sensitivity is noticed with regard to the choice of length scale definition in the Baldwin-Lomax model (see Fig. 4 and Fig. 5 in [37]). Both the zero-equation BL model and the two-equation $k-\epsilon$ model overpredict the peak
pressure and the peak heat flux by about 20% (see Fig. 3 and Fig. 9 in [36]). As shown in Fig. 9, the predicted nominal shock impingement location on the side fins is around $x/L_r = 192$ and is similar to the RANS method prediction (see Fig. 3 of [37]).

![Graph](image)

**Figure 8.** Streamwise distributions of the time-averaged (a) surface pressure and (b) surface heat flux on the flat plate at $y/L_r = 0$ and $z/L_r = 0$. The green lines and the red dots denote the results from the WMLES simulation and the experiment, respectively. The location of the double-shock intersection based on the inviscid theory is also shown in the plots.

To characterize the boundary-layer flow separation, Fig. 10 shows the time-averaged surface skin friction lines on the right fin and the corresponding sketch from the experiment. It is observed that the flow separates around the region where the shock wave impinges on the fin surface. While the overall agreement is good, the predicted streamwise separation close to the fin surface starts at $x/L_r = 190$, which is delayed compared to the sketch from the experiment at $x/L_r = 187$. As summarized in [36], the two-equation $k - \epsilon$ model does not capture this streamwise separation.

The skin friction lines on the flat plate are given in Fig. 11. The WMLES result is in excellent agreement with the experimental measurement, and show similarities with the RANS solutions using the Baldwin-Lomax model [37]. There are two lines of coalescence, i.e. the principal line of separation (PLS) and the secondary line of separation (SLS). Accordingly, two lines of divergence are also well captured, i.e. the principal line of attachment (PLA) and the center line of attachment (CLA). Close to the centerline, the secondary separations are formed and characterized by a pair of streamwise counter-rotating vortical structures. Specifically, downstream the location at $x/L_r = 175$, the secondary separation continuously shrinks and disappears near $x/L_r = 197$, where the upside and downside SLS lines converge to the CLA line, i.e. the centerline of the double fins. Fig. 12 shows the zoom-in view of the time-averaged streamlines on the symmetry plane. The maximum height of the secondary separation is roughly 0.5 cm at $x/L_r \approx 178$, which is much smaller than the inlet boundary layer thickness of 3.25 cm around the leading edge of the double fins, and the predicted flow structure is in accordance with that reported in Fig. 11 of [37]. This secondary
Figure 9. Distributions of the time-averaged (a) surface pressure and (b) surface heat flux on the flat plate at \(y/L_r = 0\). The location of the double-shock intersection based on the inviscid theory is also shown in the plots. Also shown is the shock impingement location around \(x/L_r = 192\).

separation bubble size is resolved with \(O(5)\) points across its height on the present case.

The near-wall root-mean-square (r.m.s) statistics of the pressure, temperature and streamwise velocity are given in Fig. 13. The near-wall peak temperature and streamwise velocity fluctuations mainly occur around the secondary lines of separation, in particular at the location where the shockwaves intersect. On the other hand, the peak pressure fluctuations occur on the fin surfaces.

The distributions of the time-averaged wall-normal turbulent heat flux and Reynolds shear stress in the central wall-normal plane are plotted in Fig. 14. The magnitudes of both quantities grow rapidly due to the presence of flow separation at \(x/L_r = 175\), and are further amplified by the shock intersection around \(x/L_r = 185\). In addition, the primary vortex (see Fig. 17) and the centerline counter-rotating vortex also flip the sign of the wall-normal turbulent heat flux and the Reynolds stress at a certain wall-normal distance, resulting in a layered distribution. The impact of this layered distribution on the time-averaged temperature distribution will be discussed in the next section.

3. Data analyses in \(y-z\) planes

The spanwise distributions of the time-averaged surface pressure and surface heat flux at different streamwise stations are given in Fig. 15 and Fig. 16, respectively. Considering the reported 10% uncertainties in the experimental data, the spanwise profiles of both quantities are well captured by the present WMLES simulation for all the considered
streamwise stations. In terms of the time-averaged pressure profile, deviations from the experimental measurements are visible around the region with $1.2 \leq z/L_r \leq 2.7$ at station $x/L_r = 183.2$ before the shock intersection. The agreement becomes remarkably better at the further downstream two stations. On the other hand, the agreement of the heat flux distribution between the experimental data and the WMLES results is excellent for all three stations. The present results are also better than that from the RANS prediction with the zero-equation Baldwin-Lomax turbulence model [37] and that from the two-equation $k - \epsilon$ model [36]. As seen in Fig. 6 of [37] and the present Fig. 16(b), the spanwise heat flux distribution at $x/L_r = 185.8$ is substantially over-predicted by the Baldwin-Lomax turbulence model. In Fig. 10 of [36], it is concluded that both the zero-equation BL model and the two-equation $k - \epsilon$ model perform poorly in terms of predicting the transverse profiles of flat plate surface heat transfer for all three streamwise stations.

Similar to previous investigations of the crossing shock interaction [38][37][36], due to the principal and secondary flow separation analyzed in Fig. 11, the salient feature of the streamline structure is a low total pressure jet (i.e. the primary vortex) as shown in Fig. 17. For instance, on the right side, the counter-clockwise rotating structure around the junction is associated with the corner vortex and the central clockwise rotating vortical structure denotes the primary vortex, which forms the low total pressure jet [36]. The third counter-clockwise rotating centerline vortex by the secondary separation is close to the plate and only partially visible due to the coarse LES mesh.

Fig. 18, Fig. 19, and Fig 20 show the total pressure contours on the transverse $y$-$z$ planes at the pre-shock station, $x/L_r = 183.2$, the peak pressure station, $x/L_r = 187.5$, and the further downstream station, $x/L_r = 192$, respectively. The low total pressure regions are associated with the primary vortex comprising two helical counter-rotating
Figure 11. The time-averaged surface skin friction lines on the flat plate. Panel (a) denotes the sketch from the experiment and adapted from Fig. 7(b) of [23]. Panel (b) denotes the CFD result of RANS approach and adapted from Fig. 9 of [37]. In Panel (b), PLS denotes the principal line of separation, PLA denotes the principal line of attachment, CLA denotes the center line of attachment, SEP denotes the separatrix, and SLS denotes the secondary line of separation. Panel (c) denotes the present WMLES result and the colored contour levels represent the time-averaged streamwise skin friction.

vortices as shown in Fig. 17. At all the considered streamwise stations, the predicted flow structures are qualitatively similar with those from the experiments. Moreover, the predicted flow structure shape shows a better collapse to the experimental data in the post-shock intersection regions than that in the pre-shock region. As corroborated by Fig. 18, the total pressure contour at $y/L_r \leq 3$ is triangular from WMLES versus round from experiment at the pre-shock station. It is also observed that the low total pressure jet tends to gradually move upward from the streamwise station $x/L_r = 183.2$ to the downstream station $x/L_r = 192$. As shown in Fig. 5 of [36], the present WMLES predictions of the transverse total pressure profiles are also better than those from both the zero-equation Baldwin-Lomax model and the two-equation $k – \epsilon$ model in the context of RANS approach.

Fig. 21 shows the distributions of the time-averaged temperature, r.m.s temperature fluctuations, and r.m.s stream-
Figure 12. The zoom-in view of the time-averaged streamlines on the symmetry plane. Around $x/L \approx 174.5$, there is a critical stagnation point located very close to the plate.

Figure 13. Included in this figure are the r.m.s statistics of the (a) pressure, (b) temperature and (c) streamwise velocity fluctuations. These statistics are projected to the plate surface from the first off-wall cell centers.

Streamwise velocity fluctuations at $x/L = 183.2$, 187.5, and 192. Before the shock intersection, at station $x/L = 183.2$, the boundary layer is dramatically lifted up and filled with hot fluids due to the primary vortex, and the centerline vor-
Figure 14. Distributions of (a) the time-averaged wall-normal turbulent heat flux and (b) the time-averaged Reynolds stress on the central wall-normal x-y plane of $z/L_r = 0$.

Figure 15. Spanwise distributions of the time-averaged wall pressure $< P_w > / P_{w,\infty}$ at the streamwise stations: (a) $x/L_r = 183.2$, (b) 187.5 and (c) 192. The experimental data (denoted as EXP) are adapted from the Table 4 of [23].

tex induced by the shock/boundary-layer interaction on the flat plate. Owing to the strong centerline counter-rotating vortex by the secondary separation, the peak temperature appears close to the flat plate, where the peak temperature and velocity fluctuations are also present. Immediately downstream of the shock intersection, at station $x/L_r = 187.5$, the peak temperature region further moves upward while the overall flame-like structure of the temperature distribution remains a similar shape. Conversely, the peak temperature and streamwise velocity fluctuations show up on the outer edge of the flame-like structure, and the effects of the centerline secondary separation become weaker. The prominent temperature and streamwise velocity fluctuations also correlate with the unsteadiness of the off-plate primary
Figure 16. Spanwise distributions of the time-averaged wall heat flux $<Q_w > / Q_{w, \infty}$ at the streamwise stations: (a) $x/L_r = 181.5$, (b) 185.8 and (c) 190.3. The experimental data (denoted as EXP) are from the Table 4 of [23]. In panel (b), the RANS result is computed from the Baldwin-Lomax model with grid E - IB, which is elaborated in Fig. 6 of [37]. For all three panels, the RANS predictions in [36] with the zero-equation Baldwin-Lomax model and the two-equation $k-\epsilon$ (Rodi) model are also shown for comparisons. The locations of the spanwise measurements of the surface heat flux are not coincident with those of the surface pressure in Fig. 15.

Figure 17. The sectional streamlines on a transverse y-z plane at the streamwise station $x/L_r = 187.5$.

vortex after the shock intersection. Further downstream at $x/L_r = 192$, the effects of the secondary separation almost disappear and the flame-like structure becomes flatter with a significant amount of hot fluids accumulated within the thin layers attached to the side fins. This redistribution of hot fluids is probably due to the presence of the separation induced by the shock impingement on the fin surfaces, see also Fig. 10.

Fig. 22 shows the time-averaged wall-normal turbulent heat flux and the time-averaged Reynolds stress on the transverse y-z planes at three streamwise stations. The distribution of the time-averaged wall-normal turbulent heat flux generally shows a similar pattern as that of the time-averaged Reynolds stress, but with a reversed sign. Taking the distribution of the wall-normal turbulent heat flux at $x/L_r = 187.5$ for instance, along the centerline of $z/L_r = 0$, $< \rho T'v' >$ remains negative pushing the hot fluids downward until approximately $y/L_r \approx 2.2$. Conversely, $< \rho T'\nu' >$ becomes positive lifting the hot fluids up for the region with $y/L_r > 2.2$. Consequently, the temperature peak is produced around $y/L_r \approx 2.2$, which is consistent with the time-averaged temperature distribution as in
Figure 18. Distribution of the total pressure $\langle P_0 \rangle / P_{0,\infty}$ on a transverse y-z plane at $x/L_r = 183.2$. Panel (b) denotes the experimental result and is adapted from the Fig. 11(a) of [23].

Figure 19. Distribution of the total pressure $\langle P_0 \rangle / P_{0,\infty}$ on a transverse y-z plane at $x/L_r = 187.5$. Panel (b) denotes the experimental result and is adapted from the Fig. 11(b) of [23].

Figure 20. Distribution of the total pressure $\langle P_0 \rangle / P_{0,\infty}$ on a transverse y-z plane at $x/L_r = 192$. Panel (b) denotes the experimental result and is adapted from the Fig. 11(c) of [23].
Figure 21. Distributions of the time-averaged temperature (a,d,g), the r.m.s temperature fluctuations (b,e,h), and the r.m.s streamwise velocity fluctuations (c,f,i) on the transverse y-z planes at the streamwise stations with $x/L_r = 183.2$ (a,b,c), $187.5$ (d,e,f), and $192$ (g,h,i).

Fig. 21(d). The similar analyses are applicable for interpreting the time-averaged temperature distribution at other streamwise stations.

Fig. 23 and Fig. 24 show the flow yaw angle distributions on the y-z planes at $x/L_r = 183.2$ and $x/L_r = 187.5$, respectively. Before the shock waves intersect around the fin corners, the yaw angle distribution predicted from WMLES qualitatively coincides with that from the experiment as shown in Fig. 23. The discrepancy indicated in Fig. 24, however, becomes tremendous at the streamwise station $x/L_r = 187.5$, which is slightly downstream the shock wave intersection. The erroneous prediction of the yaw flow angles in the WMLES in regions where the non-equilibrium effect is significant has also been reported in [39]. As shown in Fig. 6 of [36], both the zero-equation Baldwin-Lomax model and the two-equation $k-\epsilon$ model feature the similar deficiencies in terms of predicting the yaw angle distribution in the context of RANS approach, particularly for the streamwise stations downstream the shock intersection.
Figure 22. Distributions of the time-averaged wall-normal turbulent heat flux (a,c,e), and the time-averaged Reynolds stress (b,d,f) on the transverse $y$-$z$ planes at the streamwise station $x/L_r = 183.2$ (a,b), 187.5 (c,d), and 192 (e,f). The predicted flow fields are marginally asymmetric mainly due to the staggered nature of the deployed unstructured Voronoi mesh and the coarse resolution.

B. WMLES with van Driest scaling based damping function

The sensitivity of the results to the coordinate scaling the wall model eddy viscosity is further evaluated. Wall modeled LES calculations on the “fine” grid using identical freestream boundary conditions described above were additionally performed using the van Driest damping function [28]. The comparison of the time-averaged pressure and heat flux distributions between the WMLES with the van Driest scaling and that with the semi-local scaling is provided by Fig. 25 and Fig. 26. For all the concerned quantities, the accuracy of the WMLES deteriorates substantially when the van Driest scaling is deployed for the damping function in the near-wall eddy viscosity model. In terms of the streamwise distributions, both the mean pressure and heat flux are notably over-predicted around the entrance of the double fins. Moreover, the variation trends of both quantities are also poorly captured after the shock intersection. For
Figure 23. Distribution of the time-averaged flow yaw angle (deg) on the transverse y-z plane at \( \frac{x}{L_r} = 183.2 \). Panel (b) is adapted from the Fig. 13(a) of [23].

Figure 24. The distribution of the time-averaged flow yaw angle (deg) on the transverse y-z plane at \( \frac{x}{L_r} = 187.5 \). Panel (b) is adapted from the Fig. 13(b) of [23].

the spanwise distributions, the maximum discrepancy rises up to 50% when compared to the experimental data and the WMLES result with the semi-local scaling.

Further evidenced by Fig. 27, the predicted wall pressure distribution is significantly different from the result with semi-local scaling, and the peak pressure drops by 25% when compared to Fig. 9(a). This result is consistent with the observations that semi-local scaling functions better collapse compressible velocity profiles to their incompressible counterparts compared to the van Driest transformation, particularly in the viscous sublayer with isothermal wall conditions [40][26][27].
V. Conclusions

In this study, the three-dimensional intersecting shock-wave/turbulent boundary-layer interaction flow over double-fin geometry is investigated using wall modeled large eddy simulation. Despite the complexity of the shock/boundary layer interaction and presence of small separation bubbles, the WMLES agrees favorably with experimental data for mechanical loading, surface heat fluxes, and for the prediction of the secondary separation in both the shock impingement and post-shock regimes. The wall modeled LES calculations include the semi-local scaling of the eddy
Figure 27. Distribution of the time-averaged pressure on the flat-plate surface at \( y/L_r = 0 \). The location of the double-shock intersection based on the inviscid theory is also shown in the plot. The result reported in this plot is from WMLES with van Driest scaling based damping function.

viscosity, which was necessary to predict accurately the heat fluxes and wall pressure distributions. The resultant errors in the WMLES are shown to be significantly smaller than prior RANS simulations of this configuration using either Baldwin-Lomax or \( k - \epsilon \) models. The coarseness of the WMLES calculations (relative to the boundary layer thickness or size of the separation bubble) suggest that this approach may be tractable for more complex configurations.

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