Spin Susceptibility of a $J = 3/2$ Superconductor

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We discuss the spin susceptibility of superconductors in which a Cooper pair consists of two electrons having the angular momentum $J = 3/2$ due to strong spin-orbit interactions. The susceptibility is calculated analytically for several pseudospin quintet states in a cubic superconductor within the linear response to a Zeeman field. The susceptibility for $A_{1g}$ symmetry states is isotropic in real space. For $E_g$ and $T_{2g}$ symmetry cases, the results depend sensitively on choices of order parameter. The susceptibility is isotropic for a $T_{2g}$ symmetry state, whereas it becomes anisotropic for an $E_g$ symmetry state. We also find in a $T_{2g}$ state that the susceptibility tensor has off-diagonal elements.

I. INTRODUCTION

Spin-orbit interaction is a source of exotic electronic states realized in topological semimetals [1, 2], topological insulators [3, 4], and topological superconductors [5–8]. In the presence of strong spin-orbit interactions, spin $S = 1/2$ and orbital angular momentum $L = 1$ of an electron are inseparable degrees of freedom. Electronic properties of such materials are characterized by an electron with pseudospin $J = L + S = 3/2$. Recent studies have suggested a possibility of superconductivity due to Cooper pairing between two electrons with $J = 3/2$ [9, 10, 11]. The large angular momentum of an electron enriches the symmetry of the order parameter such as pseudospin-quintet even-parity and pseudospin-septet odd-parity [12, 13] in addition to conventional spin-singlet even-parity and spin-triplet odd-parity. Such high angular-momentum pairing states would feature superconducting phenomena of $J = 3/2$ superconductors [14–19]. In particular, a large angular momentum of a Cooper pair would qualitatively change the magnetic response of a superconductor to an external magnetic field.

The spin susceptibility reflects well the internal spin structures of a Cooper pair. It is well known in spin-singlet superconductors that the spin susceptibility decreases monotonically with the decrease of temperature below $T_c$ and vanishes at zero temperature [20]. This phenomenon occurs independently of the direction of a Zeeman field $H$ because a Cooper pair has no spin. In spin-triplet superconductors, on the other hand, the susceptibility can be anisotropic depending on the relative alignment between a Zeeman field and a $d$ vector in the order parameter. For $d \perp H$, the spin susceptibility is constant independent of temperature. Thus, the unchanged Knight shift across $T_c$ in experiments could be a strong evidence of spin-triplet superconductivity. For $d \parallel H$, the susceptibility decreases with decreasing temperature below $T_c$. Such anisotropy is more remarkable when the number of components in a $d$ vector is smaller. For $J = 3/2$ superconductors, however, our knowledge of the spin susceptibility is very limited to a theoretical paper that reported vanishing the spin susceptibility at zero temperature for a singlet-quintet mixed state in a centrosymmetric superconductor [21].

In this paper, we study theoretically the response of pseudospin-quintet even-parity superconductors to an external Zeeman field. The angular momentum of a Cooper pair in such superconductors is $J = 2$. Since the pairing symmetries of the pseudospin-quintet states are not well understood, we decided to calculate the spin susceptibility for the plausible pair potentials at a cubic superconductor preserving time-reversal symmetry. The spin susceptibility is analytically calculated based on the linear response formula [22]. The pair potential of pseudospin-quintet states is described by a five-component vector that couples to five $4 \times 4$ matrices in pseudospin space. Such complicated internal structures of the pair potential enrich the magnetic response of $J = 3/2$ superconductors. When the symmetry of the pair potential is high in pseudospin space ($A_{1g}$ state), the magnetic response is isotropic in real space and the spin susceptibility decreases monotonically with the decrease of temperature. The results are similar to those of $^3$He B-phase. When the pair potential is independent of wavenumber ($T_{2g}$ and $E_g$ states), the spin susceptibility shows unique features to pseudospin-quintet states. The magnetic response becomes anisotropic in real space in an $E_g$ state and the susceptibility tensor has finite off-diagonal elements in a $T_{2g}$ state.

This paper is organized as follows. In Sec. II, we described the electronic structure and the pair potential of a $J = 3/2$ superconductor in terms of five $4 \times 4$ matrices. The characteristic behaviors of the spin-susceptibility are discussed in Sec. III. The conclusion is given in Sec. IV. Algebras of $4 \times 4$ matrices and a number of mathematical relationships used in the paper are summarized in Appendices. Throughout this paper, we use the system of units $\hbar = k_B = c = 1$, where $k_B$ is the Boltzmann constant and $c$ is the speed of light.

II. J=3/2 SUPERCONDUCTOR

We begin our analysis with the normal state Hamiltonian adopted in Ref. [13]. The electronic states have
four degrees of freedom consisting of two orbitals of equal
parity and spin 1/2. In the presence of strong spin-orbit inter-
actions, the effective Hamiltonian for a $J = 3/2$ elec-
tron is given by

$$
\mathcal{H}_N = \sum_{k} \Psi_k^\dagger \mathcal{H}_N(k) \Psi_k,
$$

(1)

$$
\Psi_k = [c_{k,3/2}, c_{k,1/2}, c_{k,-1/2}, c_{k,-3/2}]^T,
$$

(2)

where $T$ means the transpose of a matrix and $c_{k,j_z}$ is the
annihilation operator of an electron at $k$ with the $z$-
component of angular momentum being $j_z$. The normal
state Hamiltonian is represented by

$$
H_N(k) = \alpha k^2 + \beta (k \cdot J)^2 - \mu = \xi_k h 4 \times 4 + \epsilon_k \cdot \hat{\gamma}
$$

(3)

with $\xi_k = \epsilon_{k,0} - \mu$ and

$$
c_{k,0} = \left(\alpha + \frac{5}{4} \widetilde{\gamma}^2\right) k^2, \quad c_{k,j} = \beta k^2 e_j c_j(k),
$$

(4)

$$
c_1(k) = \sqrt{15} \hat{k}_x \hat{k}_y, \quad c_2(k) = \sqrt{15} \hat{k}_x \hat{k}_z,
$$

(5)

$$
c_3(k) = \sqrt{15} \hat{k}_x \hat{k}_z, \quad c_4(k) = \frac{\sqrt{15}}{2} (\hat{k}_x^2 - \hat{k}_y^2),
$$

(6)

$$
c_5(k) = \frac{\sqrt{5}}{2} (2 \hat{k}_z^2 - \hat{k}_x^2 - \hat{k}_y^2),
$$

(7)

where $\hat{k}_j = k_j / |k|$ for $j = x, y$ and $z$ represents the di-
rection of wavenumber on the Fermi surface. The con-
stants $\alpha > 0$ and $\beta$ determine the normal state property.
The spin-orbit interactions increase with the increase of $\beta > 0$. The coefficients $c_j$ are normalized as

$$
\langle c_i(k) c_j(k) \rangle_k = \int \frac{dk}{4\pi} c_i(k) c_j(k) = \delta_{i,j},
$$

(8)

where $\left\langle \cdots \right\rangle_k$ means the integral over the solid angle on
the Fermi surface. The normalized five-component vector
$\vec{\epsilon} = (\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4, \epsilon_5)/|\vec{\epsilon}|$ determines the dependence of the normal state dispersions on pseudospins. The spinors
for the angular momentum of $J = 3/2$ are described by,

$$
J_x = \frac{1}{2} \begin{bmatrix}
0 & \sqrt{3} & 0 & 0 \\
\sqrt{3} & 0 & 2 & 0 \\
0 & 2 & 0 & \sqrt{3} \\
0 & 0 & \sqrt{3} & 0
\end{bmatrix},
$$

(9)

$$
J_y = \frac{1}{2} \begin{bmatrix}
0 & -i\sqrt{3} & 0 & 0 \\
i\sqrt{3} & 0 & -2i & 0 \\
0 & 2i & 0 & -i\sqrt{3} \\
0 & 0 & i\sqrt{3} & 0
\end{bmatrix},
$$

(10)

$$
J_z = \frac{1}{2} \begin{bmatrix}
3 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 \\
0 & 0 & 0 & -3
\end{bmatrix}.
$$

(11)

The $4 \times 4$ matrices in pseudospin space are defined as

$$
\gamma^1 = \frac{1}{\sqrt{3}} (J_x J_y + J_y J_z),
\gamma^2 = \frac{1}{\sqrt{3}} (J_y J_z + J_z J_x),
$$

(12)

$$
\gamma^3 = \frac{1}{\sqrt{3}} (J_z J_x + J_x J_y),
\gamma^4 = \frac{1}{\sqrt{3}} (J_x^2 - J_y^2),
$$

(13)

and $4 \times 4$ is the identity matrix. They satisfy the follow-
ing relations

$$
\gamma^\nu \gamma^\lambda + \gamma^\lambda \gamma^\nu = 2 \times 4 \times 4 \delta_{\nu,\lambda},
$$

(15)

$$
\gamma^1 \gamma^2 \gamma^3 \gamma^4 \gamma^5 = -4 \times 4,\n$$

(16)

$$
\{\gamma^\nu\}^* = (\gamma^\nu)^T = U_T \gamma^\nu U_T^{-1}, \quad U_T = \gamma^1 \gamma^2.
$$

(17)

where $U_T$ is the unitary part of the time-reversal operation
$T = U_T K$ with $K$ meaning complex conjugation. The superconducting pair potential is represented as

$$
\Delta(k) = \tilde{\eta}_k \cdot \gamma U_T,
$$

(18)

where a five-component vector $\tilde{\eta}_k$ represents an even-
parity pseudospin-quintet state. Throughout this paper, we assume that all components of $\tilde{\eta}_k$ are real values. As a result of the Fermi-Dirac statistics of electrons, the pair potential is antisymmetric under the permutation of two pseudospins, (i.e., $\Delta_T(k) = -\Delta(k)$). The Bogoliubov-de Gennes Hamiltonian reads,

$$
H_{\text{BdG}}(k) = \left[ \begin{array}{cc}
H_N(k) & \Delta(k) \\
-\Delta^T(k) & -H_N(k)
\end{array} \right],
$$

(19)

where $X(k, i\omega) \equiv X^*(-k, i\omega)$ represents the particle-
hole conjugation of $X(k, i\omega)$.

The interaction with a uniform Zeeman field $H$ is described by

$$
H_Z = -\mu_B \vec{J} \cdot \vec{H},
$$

(20)

$$
\vec{J} = g_1 J_x + g_3 J_z,
$$

(21)

for $j = x, y,$ and $z$, where $\mu_B$ is the Bohr’s magneton, and $g_1$ and $g_3$ are the coupling constants. The matrix
structures of $J_j$ and $J_z^3$ are displayed in Appendix A

The angular momenta in the Zeeman Hamiltonian are then given by

$$
\vec{J}_x = \frac{-i}{2} \left(\sqrt{3} p_1 \gamma^2 \gamma^5 + p_2 \gamma^1 \gamma^3 + p_1 \gamma^2 \gamma^4 \right),
$$

(22)

$$
\vec{J}_y = \frac{i}{2} \left(\sqrt{3} p_1 \gamma^3 \gamma^5 + p_2 \gamma^1 \gamma^2 - p_1 \gamma^3 \gamma^4 \right),
$$

(23)

$$
\vec{J}_z = \frac{i}{2} \left(p_2 \gamma^2 \gamma^3 + 2 p_1 \gamma^1 \gamma^4 \right),
$$

(24)

$$
p_1 = g_1 + \frac{7}{4} g_3, \quad p_2 = g_1 + \frac{13}{4} g_3.
$$

(25)

In the linear response theory, the spin susceptibility is
calculated by using the formula \[ \chi_{\mu\nu} = \chi_N \delta_{\mu\nu} - \left( \frac{\mu B}{2} \right)^2 \frac{1}{T} \sum_{\omega_n} \int \frac{dk}{(2\pi)^3} \times \text{Tr} \left[ G(k, i\omega_n) \tilde{J}_\mu G(k, i\omega_n) \tilde{J}_\nu \right. \\
+ F(k, i\omega_n) \tilde{J}_\mu F(k, i\omega_n) \tilde{J}_\nu^* \left. - G_N(k, i\omega_n) \tilde{J}_\mu G_N(k, i\omega_n) \tilde{J}_\nu \right]. \] (26)

The summation over the Matsubara frequency and that over the wavevector are regularized by introducing the Green’s functions in the normal state \( G_N \) and the spin susceptibility \( \chi_N \) in the normal state [24].

The Green’s function for a superconducting state can be obtained by solving the Gor’kov equation

\[
\left[ i\omega_n - H_{\text{BDG}}(k) \right] \begin{bmatrix} G(k, i\omega_n) & F(k, i\omega_n) \\ -F(k, i\omega_n) & -G(k, i\omega_n) \end{bmatrix} = 1_{8 \times 8}. \] (27)

The anomalous Green’s function results in

\[
F^{-1}(k, i\omega_n) = \frac{U_T}{\eta_k} \left[ (\omega_n^2 + \xi_k^2 + \eta_k^2)\eta_k \cdot \gamma \\
+ i\omega_n[\eta_k \cdot \gamma, \tilde{e}_k \cdot \gamma] - \\
+ 2\xi_k \eta_k \cdot \tilde{e}_k + (\tilde{e}_k \cdot \gamma)(\eta_k \cdot \gamma)(\tilde{e}_k \cdot \gamma) \right]. \] (28)

Generally speaking, it is not easy to calculate analytically the inversion of \( 4 \times 4 \) matrices.

To proceed analytic calculation, we consider a cubic symmetric superconductor [14], in which even-parity pair potentials are classified into \( A_{1g} \), \( E_g \), and \( T_{2g} \) states according to irreducible representations (irreps) of cubic symmetry. With focusing on pseudospin-quintet Cooper pairs, their pairing states are explicitly represented as

\[
A_{1g} : \tilde{\eta}_k \cdot \gamma = \Delta \sum_{j=1}^5 \eta_j c_j(\hat{k}) \gamma_j, \] (29)

\[
E_g : \tilde{\eta}_k \cdot \gamma = \Delta (l_4 \gamma^4 + l_5 \gamma^5), \] (30)

\[
T_{2g} : \tilde{\eta}_k \cdot \gamma = \Delta (l_1 \gamma^1 + l_2 \gamma^2 + l_3 \gamma^3), \] (31)

where the unit vector \( \hat{h} \) determines the pseudospin structure of \( A_{1g} \) state and \( l_i \in C \), \( i = 1 - 5 \). \( A_{1g} \) states involve momentum-dependent coefficients, which comes from the fact that \( \gamma^4 \) and \( \gamma^5 \) \( \gamma^1, \gamma^2, \) and \( \gamma^3 \) themselves belong to \( E_g \) \( T_{2g} \) irreps of cubic symmetry.

Generally speaking, the coefficients \( l_i \) for \( i = 1 - 5 \) are determined from the steady states of the free energy [23]. For \( E_g \) state, three distinct steady states exist: \( (l_1, l_5) = (1, 0), (0, 1), \) and \( (1, 1)/\sqrt{2} \). The first two states preserve time-reversal symmetry, while the last state breaks time-reversal symmetry. For \( T_{2g} \) state, there are four distinct steady states: \( (l_1, l_2, l_3) \in (1, 1, 1)/\sqrt{3}, (1, 0, 0), (1, e^{i2\pi/3}, e^{i4\pi/3})/\sqrt{3}, (1, i, 0)/\sqrt{2} \). The last two states break time-reversal symmetry. Time-reversal symmetry-breaking superconducting states have a problem specific to them: the formation of Bogoliubov-Fermi surfaces [13, 14, 19]. Since the relations between the pseudospin structures of a Cooper pair and the magnetic response of a superconductor is the main issue in this paper, we focus on time-reversal symmetry respecting superconducting states. In addition to the steady states, we also consider another pseudospin states described by \( (l_1, l_5) = (1, 1)/\sqrt{2}, (l_1, l_2, l_3) = (1, 1, 0)/\sqrt{2}, \) and \( (l_1, l_2, l_3, l_5) = (1, 1, 1, 1)/\sqrt{5} \). The last one is the admixture of \( E_g \) and \( T_{2g} \) states. These states complement possible combinations of \( \gamma_i \) \( i = 1 - 5 \). The comparison between the calculated results for such states and those for the steady states helps us to understand the relations between the pseudospin structures of a Cooper pair and the spin susceptibility. Note that \( (l_1, l_2, l_3) = (1, 1, 0)/\sqrt{2} \) represents a possible order parameter in tetragonal symmetric superconductors with the high symmetry axis being the \( y \) direction [23].

III. SPIN SUSCEPTIBILITY

As shown in the second term in Eq. (28), the anomalous Green’s function contains the pairing correlation belonging to odd-frequency symmetry class. The stable superconducting states can be described by

\[
[\eta_k \cdot \gamma, \tilde{e}_k \cdot \gamma] = 0, \] (32)

which means the absence of odd-frequency pairs. Odd-frequency pairs increase the free-energy of a uniform su-
perconducting state [26] because they indicate the paramagnetic response to a magnetic field [27,29]. A phenomenological argument on the paramagnetic response of odd-frequency Cooper pairs is given in Appendix A of Ref. [30]. Eq. (32) gives a guide that relates the stable pair potential \( \bar{\eta} \) to the electronic structures \( \bar{c} \). To understand this situation, we briefly summarize the case of spin-triplet superconductors in the presence of a strong Rashba spin-orbit interaction \( \lambda \cdot \sigma \) with
\[
\lambda = \lambda_{\alpha}(\bar{b}_y e_x - \bar{b}_x e_y),
\]
where \( \lambda_{\alpha} \) represents the amplitude of spin-orbit interaction, \( \sigma_j \) and \( e_j \) for \( j = x, y \) and \( z \) are the Pauli matrix and the unit vector in spin space, respectively. The stable order parameter \( d \cdot \sigma \cdot \lambda \) is determined as
\[
|d \cdot \sigma \cdot \lambda \cdot \sigma_y| = 0 \quad \text{or} \quad d \parallel \lambda,
\]
so that odd-frequency pairs are absent and the transition temperature is optimal [26,31]. Namely, the order parameter of a spin helical state is stable in this case. The pair potentials other than the helical state would be realized when the Rashba spin-orbit interaction is sufficiently weak. The choice in Eq. (34) and that in Eq. (32) are equivalent to each other. We choose the normal state dispersion \( \bar{e}_k \) so that Eq. (32) is satisfied for the pair potentials in Eqs. (29)-(31). The Green’s function in the superconducting state can be expressed simply and analytically under Eq. (32). The results of the Green’s function in such a case are shown in Appendix B.

The spin susceptibility results in
\[
\chi_{\mu\nu} = \chi_{N} \delta_{\mu\nu} \left[ 1 - \frac{1}{2} \left( 1 + \frac{P_-}{3P_+} \right) Q \right],
\]
with \( \Omega = \sqrt{n_0^2 + n_1^2} \). The tensor is defined by
\[
L_{\mu\nu}(\bar{\eta}) \equiv \text{Tr} \left[ \bar{\eta} \cdot \bar{\gamma} \mu \bar{\eta} \cdot \bar{\gamma} \nu \right],
\]
and its elements are calculated to be
\[
\begin{align*}
L_{xx} &= P_+ (n_1^2 - n_2^2 + n_3^2) + R_+ n_1^2 - R_- n_2^2 - 4\sqrt{3} P_1 n_4, \\
L_{yy} &= P_+ (n_1^2 + n_2^2 - n_3^2) + R_+ n_1^2 - R_- n_2^2 + 4\sqrt{3} P_1 n_4, \\
L_{zz} &= P_+ (n_1^2 + n_2^2 + n_3^2) + R_+ n_1^2 + R_- n_2^2 + P_1 n_{4}, \\
L_{xy} &= 2 \left[ P_+ n_2 n_3 + \sqrt{3} n_1 n_2 n_3 \right], \\
L_{xz} &= 2 \left[ P_+ n_1 n_3 + \sqrt{3} n_1 n_2 n_3 \right], \\
L_{yz} &= 2 \left[ P_+ n_1 n_2 + \sqrt{3} n_1 n_2 n_3 \right], \\
P_\pm &= \mp 2P_1^2 \pm P_2^2, \quad R_\pm = 2P_1^2 \pm 2P_2.
\end{align*}
\]
The results in Eq. (37) of [42] describe the characteristic features of the susceptibility of \( J = 3/2 \) superconductors.

### A. \( A_{1g} \) state

In the presence of attractive interactions in an \( A_{1g} \) channel, the pair potential is represented by Eqs. (18) and (29). We always set \( \bar{\bar{c}} = \bar{b} \) so that Eq. (32) is satisfied. Different from the \( s \)-wave spin-singlet pairing that also belongs to the \( A_{1g} \) irrep, an \( A_{1g} \) state in the pseudo-spin quintet pairing has the pseudospin degrees of freedom, which allows us to choose a variety of pseudospin structures. We first choose a pseudospin structure of a \( T_{2g} \) irrep characterized by \( \bar{h} = \bar{h}_{T_{2g}} = (1,1,1,0,0)/\sqrt{5} \). The spin susceptibility is calculated as
\[
\chi_{\mu\nu} = \chi_{N} \delta_{\mu\nu} \left[ 1 - \frac{1}{2} \left( 1 + \frac{p_+^2}{3p_-^2} \right) Q \right],
\]
In BCS theory, \( Q \) represents the fraction of Cooper pairs to quasiparticles on the Fermi surface. Indeed \( Q = 0 \) at \( T = T_c \), increases monotonically with the decrease of \( T \), and becomes unity at \( T = 0 \). The susceptibility tensor is diagonal and isotropic in real space. The susceptibility for a pseudospin structure of an \( E_g \) irrep characterized by \( \bar{h} = \bar{h}_{E_g} = (0,0,0,1,1)/\sqrt{2} \) results in,
\[
\chi_{\mu\nu} = \chi_{N} \delta_{\mu\nu} \left[ 1 - \frac{1}{2} \left( 1 + \frac{p_+^2}{3p_-^2} \right) Q \right].
\]
The off-diagonal elements in the susceptibility tensor are always zero in these cases, \( \chi_{\mu\nu} = 0 \) for \( \mu \neq \nu \).

In Fig. 1(a), we plot the susceptibility at \( g_3 = 0 \) as a function of temperature. The dependence of the pair potential on temperature is calculated by solving the gap equation in the weak coupling limit. As shown in Appendix C the gap equation is common for all the superconducting states considered in the present paper and is identical to that in BCS theory. The results show that the susceptibility decreases with the decrease of temperature below \( T_c \). The results are independent of such choices of the pseudospin structures as \( T_{2g} \), \( E_g \), and \( T_{2g} + E_g \) at \( g_3 = 0 \). Although a Cooper pair has the angular momentum of \( J = 2 \) in the quintet states, the susceptibility is isotropic for all the pseudospin structures. The characteristic behaviors are independent of the strength of spin-orbit interaction \( \beta/\alpha \). The isotropic feature of the spin susceptibility is considered as a result of high symmetry of the pair potential. The pseudospin structures in the pair potential are characterized by \( \gamma^j \) which can be described by the linear combination of \( J_\mu J_\nu \), as shown in Eqs. (12), (14). For a \( T_{2g} \) irrep, \( \gamma^j \) for \( j = 1-3 \) characterize both the normal state dispersion and the pair potential. The pair potential with \( \bar{h}_{T_{2g}} \) is symmetric under...
the cyclic permutation among $J_x$, $J_y$ and $J_z$. For an $E_g$ irrep, the pair potential is symmetric under interchanging $J_x \leftrightarrow J_y$. The structure of $h_{E_g}$ results in the isotropic susceptibility including the $z$ direction. Thus, the direction of a Zeeman field $H$ in real space does not point a specific direction in pseudospin space. For comparison, we briefly mention the spin susceptibility in superfluid $^3$He B-phase described by

$$d = \Delta(\hat{k}_x e_x + \hat{k}_y e_y + \hat{k}_z e_z).$$

The pair potential is symmetric under the cyclic permutation among $x, y$ and $z$. As a result, the susceptibility plotted with a broken line in Fig. (1b) is isotropic in real space.

At the end of the subsection, we briefly discuss the effects of $J^3_\mu$ term on the spin susceptibility by choosing $g_3 = g_1$. The results are shown in Fig. (1c). The isotropic nature of the susceptibility remains unchanged even for $g_3 = g_1$. The amplitude of the susceptibility depends on the pseudospin structure of the pair potential; the amplitude for a $T_{2g}$ irrep becomes slightly larger than that for an $E_g$ irrep.

### B. $T_{2g}$ state

When attractive interactions between two electrons work in a $T_{2g}$ channel, the order parameters in Eqs. (30) and (31) are isotropic in momentum space. To satisfy Eq. (32), we switch off $\hat{c} = 0$ and consider a simple pseudospin quintet superconductor in the following subsections. In other words, the pair potentials independent of momenta are stable when $|\hat{c}|$ is sufficiently smaller than the Fermi energy $\mu$. Even if we choose $\hat{c} = 0$, superconductors show the rich magnetic response depending on the pseudospin structures of the pair potential. The effects of the pseudospin-dependent dispersions $\hat{c} \neq 0$ on the magnetic response will be discussed later.

We first discuss a $T_{2g}$ state with $(l_1, l_2, l_3) = (1, 1, 1)/\sqrt{3}$ in Fig. (2a). The pair potential includes off-diagonal terms $J_\mu J_\nu$ with $\mu \neq \nu$ through $\gamma^j$ for $j = 1 - 3$ as shown in Eqs. (40)-(42). The first term of $L_{\mu\nu}(\eta)$ in Eqs. (40)-(42) is the direct results of such pseudospin structure. Since the pair potential is independent of wavenumber, these off-diagonal terms remain nonzero values even after averaging over directions in momentum space on the Fermi surface. Thus, the appearance of the off-diagonal elements in the susceptibility tensor is a characteristic feature of a $T_{2g}$ state.

Secondly, we display the susceptibility for a $T_{2g}$ state with $(l_1, l_2, l_3) = (1, 1, 0)/\sqrt{2}$ in Fig. (2b), where we delete $\gamma^3$ component from Eq. (31). As a result, $J_y$ is no longer equivalent to $J_x$ and $J_z$, which explains the anisotropy of the diagonal elements in Fig. (2b). As shown in Eqs. (40)-(42), the off-diagonal elements are finite only for the multi-component pair potentials. At the present case, only the $\chi_{zz}$ element remains finite because of $l_3 = 0$.

Finally, we display the susceptibility for a $T_{2g}$ state with $(l_1, l_2, l_3) = (1, 0, 0)/\sqrt{2}$ in Fig. (2c), which has only $\gamma^1$ component. The diagonal elements are anisotropic as they are in Fig. (2b). All the off-diagonal elements vanish because the pair potential has only one pseudospin component. Thus, the off-diagonal elements emerge if $\gamma^1$ and $\gamma^j (i, j = 1, 2, 3; i \neq j)$ coexist in the pair potential. In particular, the nonzero off-diagonal ele-

\[ \chi_{xy} = \chi_{yz} = \chi_{zx} = -\chi N \frac{1}{3} Q. \]
ments are determined from the direction of high symmetry axis in tetragonal symmetric superconductors, e.g., \( \chi_{xy} = \chi_{yz} = 0 \) and \( \chi_{zx} \neq 0 \) for the pair potential with \((l_1, l_2, l_3) = (1, 1, 0)/\sqrt{2}\) since the high symmetry axis is the y direction.

\[E_g\] state

The susceptibility for an \(E_g\) state with \((l_4, l_5) = (1, 1)/\sqrt{2}\) is calculated as

\[
\chi_{xx} = \chi_N \left[ 1 - \frac{1}{2} \left( 1 + \frac{p_2^2 - \sqrt{3}p_1^2}{P_+} \right) Q \right],
\]

\[\text{FIG. 3. The spin susceptibility is plotted as a function of temperature for an } E_g \text{ state in (a).}\]

that the anisotropic response like Eq. (54) and the absence of off-diagonal elements are the common feature of the single component order parameter.

D. Admixture of \(T_{2g}\) and \(E_g\) states

The results for a the admixture of \(T_{2g}\) and \(E_g\) states, i.e., \((l_1, l_2, l_3, l_4, l_5) = (1, 1, 1, 1)/\sqrt{5}\), are calculated as

\[
\chi_{xx} = \chi_N \left[ 1 - \frac{1}{2} \left( 1 + \frac{P_+ - 2\sqrt{3}p_1 p_2}{5P_+} \right) Q \right],
\]

\[
\chi_{yy} = \chi_N \left[ 1 - \frac{1}{2} \left( 1 + \frac{P_+ + 4\sqrt{3}p_1^2}{5P_+} \right) Q \right],
\]

\[
\chi_{zz} = \chi_N \left[ 1 - \frac{1}{2} \left( 1 + \frac{1}{\gamma} \right) Q \right].
\]

The off-diagonal elements are calculated in the similar way,

\[
\chi_{xy} = -\chi_N Q \frac{P_+ - 2\sqrt{3}p_1 p_2}{5P_+},
\]

\[
\chi_{yz} = -\chi_N Q \frac{P_+ + \sqrt{3}p_1 p_2(1 - \sqrt{3})}{5P_+},
\]

\[
\chi_{zx} = -\chi_N Q \frac{P_+ + \sqrt{3}p_1 p_2(1 + \sqrt{3})}{5P_+}.
\]

The calculated results for \(g_3 = 0\) are plotted in Fig. 3(d). Not only the diagonal elements but also the off-diagonal elements are anisotropic. All the elements in Eqs. (57)-(62) are finite and different from one another. The degree
We have studied theoretically the spin susceptibility of pseudospin-quintet pairing states in a $J = 3/2$ superconductor that preserves cubic lattice symmetry and time-reversal symmetry. Within the linear response to a Zeeman field, we calculate the spin susceptibility by using the Green’s function that is obtained by solving the Gor’kov equation analytically. The pair potentials are chosen so that a superconducting state is stable under the pseudospin structures in the normal state. The calculated results indicate that the magnetic response of pseudospin-quintet states depends sensitively on the pseudospin structures of the pair potential. The susceptibility tensor in $A_{1g}$ states is isotropic in real space as that in the B-phase of superfluid $^3$He. For $E_g$ states, the susceptibility tensor becomes anisotropic in real space. We found in a $T_{2g}$ state that the susceptibility tensor has the off-diagonal elements.

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Appendix A: Normal state

The spin susceptibility in the normal state is given by

$$\chi_N = -\left(\frac{\mu_B}{2}\right)^2 T \sum_{\omega_n} \int \frac{d\mathbf{k}}{(2\pi)^d} \text{Tr} \left[ G_N(\mathbf{k}, i\omega_n) J_\mu G_N(\mathbf{k}, i\omega_n) J_\nu \right],$$

(A1)

$$G_N = \frac{i\omega_n - \xi + \bar{c}_k \cdot \gamma}{(i\omega_n - \xi)^2 - \bar{c}_k^2} = \alpha_N + \beta_N \bar{c}_k \cdot \gamma, \quad \alpha_N = 1 \left[ \frac{1}{z_{N+}} + \frac{1}{z_{N-}} \right], \quad \beta_N = \frac{1}{2|\bar{c}_k^2|} \left[ \frac{1}{z_{N+}} - \frac{1}{z_{N-}} \right],$$

(A2)

$$z_{N\pm} = i\omega_n - \xi_{\pm}, \quad \xi_{\pm} = \xi \pm |\bar{c}_k|.$$  

(A3)

The the Green’s function is the solution of

$$[i\omega_n - H_N] G_N(\mathbf{k}, \omega_n) = 1, \quad H_N(\mathbf{k}) = \xi_k + \bar{c}_k \cdot \gamma.$$  

(A4)

The trace of the Green’s function is calculated as

$$\text{Tr} \left[ G_N(\mathbf{k}, i\omega_n) \tilde{J}_\mu G_N(\mathbf{k}, i\omega_n) \tilde{J}_\nu \right] = \delta_{\mu,\nu} P_+ \alpha_N^2 + \alpha_N \beta_N M_{\mu,\nu}(\bar{c}_k) + \beta_N^2 L_{\mu,\nu} \bar{c}_k,$$  

(A5)

where we use $\text{Tr}(\tilde{J}_\mu \tilde{J}_\nu) = P_+ \delta_{\mu,\nu}$ and define the tensor

$$M_{\mu,\nu}(\bar{c}) \equiv \text{Tr} \left[ \bar{c} \cdot \gamma (\tilde{J}_\mu \tilde{J}_\nu + \tilde{J}_\nu \tilde{J}_\mu) \right].$$  

(A6)
The angular momenta \( J_\nu \) are expressed in terms of \( \gamma^\nu \)

\[
J_x = \frac{i}{2} \left[ \sqrt{3} \gamma^3 \gamma^5 + \gamma^1 \gamma^3 + \gamma^2 \gamma^4 \right], \quad J_y = \frac{i}{2} \left[ \sqrt{3} \gamma^3 \gamma^5 + \gamma^1 \gamma^2 - \gamma^3 \gamma^4 \right], \quad J_z = \frac{i}{2} \left[ \gamma^2 \gamma^3 + 2 \gamma^1 \gamma^4 \right]. \tag{A7}
\]

They obey the relation \( U_T (J^\nu)^* U_T^{-1} = -J^\nu \), which simply means that the angular momenta are antisymmetric under the time-reversal operation. The expression of \( J^3_y \)

\[
J_x^3 = -\frac{i}{8} \left( 7 \sqrt{3} \gamma^2 \gamma^5 + 13 \gamma^1 \gamma^3 + 7 \gamma^2 \gamma^4 \right), \quad J_y^3 = \frac{i}{8} \left( 7 \sqrt{3} \gamma^3 \gamma^5 + 13 \gamma^1 \gamma^2 - 7 \gamma^3 \gamma^4 \right), \tag{A8}
\]

suggested that \( J_\nu \) and \( J^3_\nu \) share the common matrix structures. The elements of the tensor are calculated as

\[
M_{xx}(\vec{\epsilon}) = 4p_1 p_2, (\sqrt{3} \epsilon_4 - \epsilon_5), \quad M_{yy}(\vec{\epsilon}) = -4p_1 p_2, (\sqrt{3} \epsilon_4 + \epsilon_5), \quad M_{zz}(\vec{\epsilon}) = 8p_1 p_2, \epsilon_5, \tag{A10}
\]

\[
M_{xy}(\vec{\epsilon}) = M_{yx}(\vec{\epsilon}) = 4\sqrt{3} \epsilon_1^2, \quad M_{xz}(\vec{\epsilon}) = M_{zx}(\vec{\epsilon}) = 4\sqrt{3} \epsilon_3^2, \quad M_{yz}(\vec{\epsilon}) = M_{zy}(\vec{\epsilon}) = 4\sqrt{3} \epsilon_2^2, \tag{A11}
\]

where \( \epsilon_j = \beta \vec{k} \cdot e_j \gamma_j (\hat{\epsilon}) \) as defined in Eq. \( \ref{relation} \) and we have used the relations

\[
\text{Tr}[\gamma^\nu] = 0, \quad \text{Tr}[\gamma^\nu \gamma^\nu] = 4 \delta_{i,j}, \quad \text{Tr}[\gamma^i \gamma^j \gamma^k] = 0, \quad \text{Tr}[\gamma^i \gamma^j \gamma^k \gamma^l] = 4 \left[ \delta_{i,j} \delta_{k,l} - \delta_{i,k} \delta_{j,l} + \delta_{i,l} \delta_{j,k} \right]. \tag{A12}
\]

The summation over the wavenumber is replaced by the integration as

\[
\int \frac{d\mathbf{k}}{(2\pi)^d} F(\mathbf{k}) \to N_0 \int_{-\infty}^{\infty} d\xi \langle \langle F(\xi, \mathbf{k}) \rangle \rangle_{\mathbf{k}} = \int \frac{d\mathbf{k}}{4\pi} F(\xi, \mathbf{k}), \tag{A13}
\]

where \( \hat{\mathbf{k}} \) is the unit vector on the Fermi surface. By using the relations

\[
\langle M_{\mu,\nu}(\vec{\epsilon}_k) \rangle_{\mathbf{k}} = 0, \quad \langle L_{\mu,\nu}(\vec{\epsilon}_k) \rangle_{\mathbf{k}} = \langle L_{\mu,\mu}(\vec{\epsilon}_k) \rangle_{\mathbf{k}} \delta_{\mu,\nu}, \tag{A14}
\]

the susceptibility in the normal state becomes

\[
\chi_N = -\left( \frac{\mu_B}{2} \right)^2 T \sum_{\omega_n} \frac{d\mathbf{k}}{(2\pi)^d} \left[ P_+ \alpha_N^2 + \beta_N^2 \langle L_{\mu,\mu} \rangle_{\mathbf{k}} \right] \delta_{\mu,\nu}. \tag{A15}
\]

The summation over the Matsubara frequency is carried out as

\[
T \sum_{\omega_n} \alpha_N^2 = \frac{1}{4} T \sum_{\omega_n} \left[ \frac{1}{z_{N+}} + \frac{1}{\epsilon_k} \left( \frac{1}{z_{N+}} - \frac{1}{z_{N-}} \right) + \frac{1}{z_{N-}} \right], \tag{A16}
\]

\[
= \frac{1}{4} \left[ -\frac{1}{4T} \cosh^{-2} \frac{\xi}{2T} - \frac{1}{\epsilon_k} \left( \tanh \frac{\xi}{2T} - \tanh \frac{\xi}{2T} \right) - \frac{1}{4T} \cosh^{-2} \frac{\xi}{2T} \right]. \tag{A17}
\]

The integration over \( \xi \) after the summation over the frequency can be calculated exactly as

\[
\int_{-\infty}^{\infty} d\xi T \sum_{\omega_n} \alpha_N^2 = -1, \quad \int_{-\infty}^{\infty} d\xi T \sum_{\omega_n} \beta_N^2 = 0. \tag{A18}
\]

The resulting spin susceptibility

\[
\chi_N = \left( \frac{\mu_B}{2} \right)^2 P_+ N_0, \tag{A19}
\]

is diagonal and isotropic independent of the direction of a Zeeman field.

The normal Green’s function can be described alternatively as

\[
G_N = \frac{-1}{Z_N(\omega_n)} [A_N + B_N \epsilon_k \cdot \gamma], \quad Z_N = \epsilon^4 + 2 \epsilon^2 (\omega_n^2 - \epsilon_k^2) + (\omega_n^2 + \epsilon_k^2)^2 \tag{A20}
\]

\[
A_N = (\omega_n^2 + \epsilon^2 + \epsilon_k^2) \omega_n + (\omega_n^2 + \epsilon^2 - \epsilon_k^2) \xi, \quad B_N = -\left\{ (i \omega_n - \xi)^2 - \epsilon_k^2 \right\}. \tag{A21}
\]
When we carry out the summation over the wavenumber first as
\[ N_0 \int_{-\infty}^{\infty} d\xi \langle \text{Tr} \left[ G_N(\mathbf{k}, i\omega_n) \hat{J}_\mu \, G_N(\mathbf{k}, i\omega_n) \hat{J}_\nu \right] \rangle_k = N_0 \int_{-\infty}^{\infty} d\xi \frac{1}{Z_N^2} \left[ \delta_{\mu,\nu} P + A_N^2 + B_N^2 \langle L_{\mu,\nu} \rangle_k \right], \]  
we find \( \chi_N = 0 \) because of
\[ \int_{-\infty}^{\infty} d\xi \frac{A_N^2}{Z_N^2} = \int_{-\infty}^{\infty} d\xi \frac{B_N^2}{Z_N^2} = 0. \]  
The discrepancy is derived from the fact that the integration over the wavenumber and the summation over the frequency do not converge.\[24\] On the way to Eq. (A23), we have used the following relations
\[ I_0 = \int_{-\infty}^{\infty} d\xi \frac{1}{Z_N^2} = \frac{\pi}{2|\omega_n|\omega_n^2(\omega_n^2 + \varepsilon^2)^2}, \quad J_n = \int_{-\infty}^{\infty} d\xi \frac{\xi^2}{Z_N^2}, \quad J_0 = \int_{-\infty}^{\infty} \frac{I_0}{8\omega_n^2} (\omega_n^2 + \varepsilon^2), \quad J_4 = \int_{-\infty}^{\infty} \frac{I_0}{8\omega_n^2} (\omega_n^2 + \varepsilon^2)(5\omega_n^2 + \varepsilon^2). \]
We approximately replace \( \tilde{\varepsilon}_k^2 > 0 \) by \( \varepsilon^2 = (\alpha/\beta + 5/4)^{-2} \mu^2 \).

Appendix B: Superconducting state

The Green’s function under Eq. (A22) is calculated to be
\[ G(\mathbf{k}, \omega_n) = \frac{1}{Z_N} \left[ A_g + B_g \tilde{\varepsilon}_k \cdot \tilde{\gamma} \right], \]
with \( \Omega = \sqrt{\omega_n^2 + \tilde{\eta}_k^2} \). When we carry out the summation over the wavenumber, we find
\[ N_0 \int_{-\infty}^{\infty} d\xi \left( \text{Tr} \left[ G_S(\mathbf{k}, i\omega_n) \hat{J}_\mu \, G_S(\mathbf{k}, i\omega_n) \hat{J}_\nu \right] \right)_k, \]
\[ = N_0 \int_{-\infty}^{\infty} d\xi \left\langle \left[ \frac{A_N^2}{Z_N^2(\Omega^2)} + \frac{\tilde{\eta}_k^2(\Omega^2 + \tilde{\varepsilon}_k^2 + \varepsilon^2)}{Z_S^2} \right] \hat{J}_\mu \hat{J}_\nu + \left[ \frac{B_N^2}{Z_N^2(\Omega^2)} + \frac{4\tilde{\varepsilon}_k^2 \varepsilon^2}{Z_S^2} \right] L_{\mu,\nu}(\tilde{\eta}_k) \right\rangle_k, \]
\[ = \left\langle \frac{\pi N_0}{4\Omega^2(\Omega^2 + \varepsilon^2)} \left[ P_+ \tilde{\eta}_k^2 (2\Omega^2 + \varepsilon^2) \delta_{\mu,\nu} + \varepsilon^2 L_{\mu,\nu}(\tilde{\eta}_k) \right] \right\rangle_k, \]
The average \( \langle L_{\mu,\nu} \rangle_k \) describes the anisotropy and the off-diagonal response of the spin susceptibility.

Appendix C: Gap equation

The attractive interactions between two electrons are necessary for Cooper pairing. Some bosonic excitation usually mediates the attractive interactions. In this Appendix, we assume the attractive interaction phenomenologically and
derive the gap equation for superconducting states discussed in this paper. The pair potential of the superconducting states is defined by

$$\Delta_{\alpha,\beta}(\hat{k}) = \frac{1}{V_{\text{vol}}} \sum_{\lambda,\tau} \sum_{\nu} V_{\alpha,\beta;\lambda,\tau}(\hat{k} - \hat{k}') \langle c_{\nu}^{\dagger}(\lambda,\tau) c_{\nu}(\lambda,\tau) \rangle = -\frac{1}{V_{\text{vol}}} \sum_{\lambda,\tau} T \sum_{\omega_n} \sum_{\nu} V_{\alpha,\beta;\lambda,\tau}(\hat{k} - \hat{k}') F_{\lambda,\tau}(\hat{k}',\omega_n),$$

(C1)

where $\alpha, \beta, \lambda, \tau$ are the indices of pseudospin of an electron. The attractive interaction $V_{\alpha,\beta;\lambda,\tau}$ works on two electrons with $\lambda$ and $\tau$ and generates the pair potential between two electrons with $\alpha$ and $\beta$. The attractive interaction can be decomposed as

$$V_{\alpha,\beta;\lambda,\tau}(\hat{k} - \hat{k}') = \sum_{\nu=1-5} g_{\nu}(\hat{k} - \hat{k}') \left( \gamma_{\nu} U_T \right)_{\alpha,\beta} \left( \gamma_{\nu} U_T \right)_{\lambda,\tau}^*.$$

(C2)

For $A_{1g}$ states in Sec. III A, we choose

$$g_{\nu}(\hat{k} - \hat{k}') = \begin{cases} g c_{\nu}(\hat{k}) c_{\nu}(\hat{k}') & \nu = 1 - 3 \\ 0 & \nu = 4, 5 \end{cases} \quad T_{2g} \text{ irreps}$$

(C3)

$$g_{\nu}(\hat{k} - \hat{k}') = \begin{cases} 0 & \nu = 1 - 3 \\ g c_{\nu}(\hat{k}) c_{\nu}(\hat{k}') & \nu = 4, 5 \end{cases} \quad E_g \text{ irreps}$$

(C4)

$$g_{\nu}(\hat{k} - \hat{k}') = g c_{\nu}(\hat{k}) c_{\nu}(\hat{k}') \quad \nu = 1 - 5 \quad T_{2g} + E_g \text{ irreps}.$$  

(C5)

By substituting the anomalous Green’s function in Eq. (B3) into Eq. (C1), we obtain the gap equation

$$\Delta = T \sum_{\omega_n} g N_0 \int d\xi \frac{B_1}{Z_S} \Delta,$$

(C6)

where we have used Eq. (3). After integrating over $\xi$, we obtain

$$1 = g N_0 T \sum_{\omega_n} \frac{1}{\sqrt{\omega_n^2 + \Delta^2}}.$$  

(C7)

The results coincide with the gap equation in BCS theory. For $T_{2g}$, $E_g$, and an admixture state of them in Sec. III B and C, we replace $c_{\nu}(\hat{k})$ by 1 for all $\nu$ in Eqs. (C3)-(C5). The gap equation for such states is identical to Eq. (C7).

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