Studying problems on choosing stable orbits of nanosatellites to provide passive and periodic relative trajectories

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Abstract. The paper considers the passive perturbed inspection movement of one nanosatellite (NS) relative to another. The inspection movement is considered in local vertical local horizontal associated with the inspected NS moving in a near-circular low Earth orbit. NS2 has an inspection movement around NS1. Under the influence of J2 perturbation the real movement diverges from the inspection one and the relative trajectory of satellites becomes open. The paper shows that by choosing the initial conditions of both NS motion, the perturbation of the inspection motion can be reduced. As strict J2 invariant trajectory conditions exists [1] to provide quasi-periodic relative orbit [2], the perturbed inspection movement will slightly differ from the reference trajectory. Thus, NS can fulfill the target task without orbital correction for a certain time interval. Paper provides analytical relations for the conditions of quasi-periodic relative orbit and a methodology for selecting the initial conditions for the NS motion.

Introduction
Inspection motion is a type of relative motion, which provides one spacecraft (or group) motion around another (central) spacecraft for visual sensing and receiving data from it. Nowadays, the inspection movement is an urgent topic because the development of space technology involves conducting sophisticated operations in orbit including rendezvous, docking (undocking), spacecraft formation flight.

Inspection movement can be divided into passive and periodic relative trajectories and active ones. In passive and periodic relative trajectories (PPRT) the inspector satellite’s orbit is selected to provide the inspection movement, which is maintained for a certain period without correction. In case of active and periodic relative trajectories, the orbit correction is performed to maintain a certain relative distance. An example of active relative trajectories is the mission repairing the Hubble telescope using the Endeavor shuttle as part of the STS-61 mission [3]. The other example is the Orbital Express program. Two spacecraft ASTRO and NEXTSat were launched in 2007 in frame of this program. The main task was to transfer fuel between the spacecraft and replace the layout of the batteries in automatic mode [4]. The technology of active inspection movement is used in missions to study asteroids, to study the surface with scientific instruments and to select a place for soil sampling [5]. The use of nanosatellites (NS) reduces the cost of developing and testing of the technology for inspection movement. In mission [6], one NS flew around another with a one-degree relative orientation deviation and a one meter distance error.

During long inspection missions, orbit corrections are required to maintain the spacecraft periodic relative trajectories. It is possible to reduce the number of correcting pulses using PPRT. The
possibility of implementing PPRT is shown in [7]. In [8], orbits were found that made it possible to maintain satisfactory quality of the tetradial formation of satellites for 42 orbital revolutions (about 170 days) with passive movement in highly elliptical orbits. In [9], the PPRT is implemented using certain inclinations of the spacecraft orbits with small eccentricities. In [10], it was shown that PPRT could be created by selecting the argument of pericenter and the mean anomaly when both SC are in the same orbital plane with an eccentricity less than 0.3. In Samara National Research University, the relative motion in general and PPRT problems are also developing [11-14].

The paper studies the possibility of implementing PPRT by selecting the orbits and motion conditions, which provide the inspection movement maintained for a certain time interval.

1. **Formulation of the problem**

   It is considering the possibility of providing the technical sustainability of a nanosatellite inspector (NS2) perturbed motion in PPRT. It is provided by choosing of the initial conditions for the movement of NS2. In the problem of passive inspection movement, the term technical sustainability means perturbed movement in PPRT with the deviation from the selected reference by a value not exceeding the specified within a fixed time interval. The term technical sustainability is used for systems that do not have a stable state [15]. To describe the relative motion, two coordinate systems will be used: the Earth-centered inertial (ECI) coordinate system (XYZ) and the local vertical local horizontal (LVLH) rotating coordinate system (xyz), centered at the inspected nanosatellite (NS1). The fundamental plane of LVLH is the orbital plane; the unit vector x is directed from the spacecraft radially outward, z is normal to the fundamental plane, positive in the direction of the (instantaneous) angular momentum vector, and y completes the setup.

2. **Choosing of the inspection movement nominal trajectory of the (unperturbed movement)**

   With the assumptions that the distance between two nanosatellites much less than distance to the Earth’s center, the relative motion will be described by the well-known Hill’s equations [9]:

   \[
   \ddot{x} - 2n\dot{y} - 3n^2x = 0 \\
   \ddot{y} + 2n\dot{x} = 0 \\
   \ddot{z} + n^2z = 0
   \]  

   Here \(n\) is an orbital angular speed of NS1 around the Earth.

   Equations (1) have an analytical solution:

   \[
   \vec{R} = M, \vec{R}_0
   \]  

   Fig. 1. Coordinate systems: a) ECI and LVLH; b) inspection ellipse
where \( \vec{R} = (x, y, z, v_x, v_y, v_z)^T \) – current state vector; \( \vec{R}_0 = (x_0, y_0, z_0, v_{x0}, v_{y0}, v_{z0})^T \) – initial state vector; \( M_t \) – matrix of coefficients.

The relative motion described by (2) is taken as the unperturbed motion. From (2), the conditions for motion along an ellipse in the LVLH can be found:

\[
\vec{R}_0 = \left( x_0, y_0, 0, v_{x0} = \frac{V_0}{2}, v_{y0} = -2x_0w, 0 \right)^T
\]

When conditions (3) are satisfied, NS2 will move along the ellipse with the major semi-axis \( a \) twice greater than the major semi-axis \( b \), and the NS1 will be in the geometric center of the ellipse. Such a movement of NS2 in this paper is called inspection, and an ellipse is called an inspection ellipse (IE). The period of revolution in IE is equal to orbital one. IE is an ideal trajectory of NS2. Using PPRT this trajectory can be almost gained during the movement in a normal gravity field at a given time interval.

3. Perturbed inspection movement of NS2

Taking into account \( J_2 \) perturbation, the perturbed motion of NS2 will represent a cycloid, whose geometric shape depends on the initial conditions (IC). It is possible to choose such IC of the NS1 and NS2 motion, to provide insignificant deformation of the PPRT from \( J_2 \) perturbation. The criterion for this deformation is the shift of the trajectory reference points of the NS2 relative motion. These are the intersection points of the PPRT with the y-axis in the LVLH. Technical sustainability is a condition to provide the shift of the reference points in a given range.

To estimate the shift of the reference points, it was simulated the motion of both nanosatellites taking into account \( J_2 \) perturbation in ECI coordinate system. It is necessary to convert the initial conditions of both nanosatellites motion from LVLH into ECI coordinate system, then simulate their motion and calculate the relative distance vector. After that, convert results to the LVLH centered in NS1 and gain a PPRT.

Conversion of NS1 IC from orbital elements into ECI coordinate system:

\[
\begin{bmatrix}
    r_1 \\
    v_1
\end{bmatrix} = M^{-1} \begin{bmatrix}
    r_i & 0 & 0
\end{bmatrix}^T
\]

Conversion of NS2 IC from LVLH into ECI coordinate system:

\[
\begin{bmatrix}
    r_2 \\
    v_2
\end{bmatrix} = \begin{bmatrix}
    r_i \\
    v_i
\end{bmatrix} + M^{-1} \mathbf{\hat{\rho}}
\]

\[
\begin{bmatrix}
    v_2 \\
    M^{-1} \mathbf{\hat{\rho}} + \mathbf{\hat{w}} \times (M^{-1} \mathbf{\hat{\rho}})
\end{bmatrix}
\]

where \( \mathbf{\hat{\rho}} \) – velocity of NS2 in LVLH, \( M \) – directional cosines matrix for conversion from ECI coordinate system into LVLH.

\[
M = \begin{bmatrix}
    \cos \Omega \cos u - \sin \Omega \sin u \cos \sin \Omega + \sinu \cos \cos \sin \Omega & \sinu \sinu \\
    -\sin \Omega \cos \Omega - \cos \Omega \sin \Omega \cos \sin \Omega - \sin \Omega \sinu + \cos \Omega \cos \sinu \\
    \sinu \sin \Omega & -\sin \Omega \cos \Omega \cosu
\end{bmatrix}
\]

Relative distance vector in ECI coordinate system:

\[
\Delta \mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2
\]

Conversion of relative distance vector from ECI coordinate system into LVLH:

\[
\mathbf{\hat{\rho}} = M \Delta \mathbf{r}
\]
4. Analysis of the perturbed PPRT of NS2

A parametric analysis of the perturbed inspection movement was made with a wide range of NS2 IC and orbital elements of the NS1. Simulation was made using the fourth-order Runge-Kutta method. IC of the NS1: $i_0=45^\circ, 60^\circ, 75^\circ, 95^\circ$; $\Omega_0=0^\circ$; $H_0=400$ km; $e_0=0$, $u_0=0^\circ\ldots360^\circ$ in increments of $5^\circ$. IC of the NS2 were considered for two points A (0; 1000) and B (500; 0) in LVLH. Figure 3 shows cases of significant deformation of the perturbed PPRT. Figure 4 shows that there is such an initial value of the argument of latitude $u_0$ for NS1, which provides slight deformation of the perturbed NS2 PPRT (technical sustainability). The simulation time interval is 2 days. The value of the reference point shift is indicated by $\Delta$ (Fig. 4).

Under the influence of $J_2$ perturbation, the perturbed trajectory moves in the orbital plane along the y axis of LVLH in the positive or negative direction and it does not move along the x axis. Change in the initial orbit height in the range from 500 to 1000 km and different values of the ascending node longitude do not affect the perturbed motion. A significant influence is exerted by $i_0$, $u_0$ and the initial coordinates of NS2. From figure 5 it is seen that with NS2 IC at the point B (500; 0), the value of $\Delta$ is less than with NS2 IC at point A (0; 1000). This is because at the initial time moment both NS are on the same direction of the position vector and the gravitational acceleration difference acts less. Simulation showed that if NS2 is in the orbit plane at the initial time, only planar motion occurs.

It can be seen from Figure 5 that for certain argument of latitude initial values, the $\Delta$ value is close to zero. This is because the orbit parameters of nanosatellites have values that tend to fulfill strict $J_2$ invariant trajectory conditions [1]:

$$\delta i = 0, \delta u = 0, \delta v = 0$$  \hspace{1cm} (4)
The fulfillment of condition (4) is provided by the equality of the orbital energies and the orbital angular momentum of the first and second nanosatellites [18]:

\[ \delta H = H_1 - H_2 = 0 \]  \hspace{1cm} (5)
\[ \delta E = E_1 - E_2 = 0 \]  \hspace{1cm} (6)

Conditions (5) and (6) are not satisfied when NS2 has PPRT (\( \delta E \neq 0 \) and \( \delta H \neq 0 \)).

Conditions (5) and (6) can be written through the parameters of the first and second nanosatellites orbits:

\[ \delta H = \cos i_o \sqrt{\mu a_o} \delta \eta - \eta_0 \sin i_o \sqrt{\mu a_o} \delta i + \frac{1}{2} \eta_0 \cos i_o \sqrt{\mu a_o} \delta a \]  \hspace{1cm} (7)
\[ \delta E = \left( \frac{\mu}{2a_0^2} + \frac{3J_2 \mu R_e^2}{4a_0^2 \eta_0^2} (1 - 3\cos^2 i_0) \right) \delta a + \frac{3J_2 \mu R_e^2}{4a_0^2 \eta_0^2} \left( \sin(2\eta_0) \eta_0 \delta i (1 - 3\cos^2 i_0) \right) \delta \eta \]  \hspace{1cm} (8)

where: \( a_o, a_{ns2} \) – major semi-axes of the first and second nanosatellites orbits in the ECI coordinate system; \( \mu = 398602 \) km\(^3\)/s\(^2\) – gravitational parameter of the Earth; \( R_e = 6378116 \) km – equatorial radius of the Earth; \( J_2 = -1082.6274 \cdot 10^{-6} \) – \( J_2 \) perturbation; \( \eta_0 = \sqrt{1 - e_o^2} \); \( \delta \eta = \eta_0 - \eta_{ns2} \); \( \delta i = i_o - i_{ns2} \); \( \delta a = a_o - a_{ns2} \).

Since NS2 has a perturbed PPRT close to IE with NS1 in its center, the conditions (5) and (6) will not be satisfied, but \( \delta E \) and \( \delta H \) are small for a small value of \( \Delta \). The parameter \( \Delta \) is used to evaluate the performance of technical sustainability:

\[ \Delta = \frac{\Delta \left( i_o, u_H, e_o, \overline{R_o}, t' \right)}{\Delta_{max}} \leq 1 \text{ when } t' \in [0, T_k] \]

Condition (5) in LVLH can be presented as:

\[ \nu_{x_1}^2 + \nu_{y_1}^2 + \nu_{z_1}^2 - \nu_{x_2}^2 - 2K \left( \frac{1}{r_{z_2}^3} - 1 \right) - 2K \left( \frac{1}{3r_{z_2}^3} - \frac{r_{x_2}^2}{r_{z_2}^5} + \frac{\sin^2 u \cos^2 u}{r_{z_2}^3} \right) = \delta E \]  \hspace{1cm} (9)

where
\[
\begin{align*}
v_{x_1} &= V_x + \dot{r} - y_2 w_z; \\
v_{y_2} &= V_y + (r_1 + x_2) w_z - z_2 w_x; \\
v_{z_2} &= V_z + y_2 w_x; \\
r_{z_2} &= (r_1 + x) \sin i \sin u + y \sin i \cos u + z \cos i; \\
w_s &= -\frac{K_{J_2} \sin 2i \sin u}{hr_1^3}; \\
K_{J_2} &= -\frac{3J_2 \mu R^2}{2}; \\
h &= \hat{r}_1 \times \hat{r}_1.
\end{align*}
\]

Condition (5) in LVLH can be presented as:

\[
v_{x_1} \left( z \cos u \sin i - y \cos i \right) + v_{y_2} \left( (r_1 + x) \cos i - z \sin u \sin i \right) + \\
v_{z_2} \left( y \sin u - (r_1 + x) \cos u \right) \sin i - h \cos i = \delta H
\]

Substituting (3) into (10), we obtain the quadratic equation with respect to the variable \(y\). Its solution will have two roots:

\[
y_{i,2} = \frac{1}{2w_s \sin u \cdot \sin i + w_c \cos i} \left( rw_y \cos u \sin i + w_x \cos u \sin i \right) + V_c \cos (i) \pm \\
\pm \left( r^2 w_c^2 \cos^2 u \sin^2 i + 2rw_y^2 \cos^2 u \sin^2 i + w_y^2 x^2 \cos^2 u \sin^2 i - 4r^2 w_y \cos i \sin u \sin i + \\
+ 4w_y w_x x^2 \cos i \sin u \sin i - 2r^2 w_y^2 \cos^2 i + 2w_y x^2 \cos^2 i + 2V_y \cos \cos u \sin i + \\
+ 2V_y w_x \cos \cos u \sin i + 4hw_x \cos i \sin u \sin i + V_c \cos \cos i + 2hw_x \cos^2 i + \\
+ 4w \delta H \sin u \sin i + 2w \delta H \cos i \right)^{1/2}.
\]

Substituting in (11) different variants of the NS1 orbital elements and \(\delta H\), we obtain two sets of initial conditions for the motion of NS2.

\[
L_1 = \{x_0, y_{01}\}; \\
L_2 = \{x_0, y_{02}\}.
\]

Since (7) and (8) provide strict J2 invariant trajectory conditions, it is necessary to verify that condition (8) is satisfied.

\[
L'_1 = \{x_0, y_{01}'\}; \\
L'_2 = \{x_0, y_{02}'\}.
\]

PPRT corresponding to the IC of NS2, selected from sets (13) and (14) will deform slightly. To determine the IE with IC of NS2, we use the formula to connect the IC and the minor semi-axis of the IE [17]:

\[
b = \sqrt{\left(3y \left(x_0\right) + \frac{2}{w} V_{yo} \left(x_0\right)\right)^2 + \left(\frac{3y \left(x_0\right)}{w} \right)^2}.
\]

5. Method for selecting the initial conditions of NS2 to provide small deformation of the PPRT

The initial data for the method are the parameters of the NS1 reference orbit and the dimensions of the IE, depending on the inspection mission.

1. Calculation of the \(\delta E\) and \(\delta H\) values using (7) and (8).
2. The set of NS2 initial conditions obtaining, to provide small deformation of the PPRT using formulas (9), (11).
3. The choice from the obtained sets of initial conditions by the formula (15), which will lead to perturbed PPRT close to a given IE.
4. Verification of the technical sustainability condition fulfillment for the perturbed PPRT with the selected IC of NS2 for the NS1 reference orbit.
There is a restriction of the method. Choosing the initial argument of latitude NS1 $u_0 = 0^\circ, 90^\circ, 180^\circ, 270^\circ$ the set of initial conditions will be complex conjugate.

6. An example of using the method
We consider the NS1 reference orbit with the parameters: $i_0 = 60^\circ$, $u_0 = 140^\circ$, $H_0 = 400$ km, $\Omega_0 = 0^\circ$, $e_0 = 0$. The condition is the fulfillment of technical sustainability is $\Delta = 0.05$. The parameters of the inspection ellipse are: $a = 500$ m, $b = 250$ m. Using (9), (11) the set of initial conditions NS2 was obtained to provide small deformation of the perturbed PPRT. In Figure 6, this set is displayed in two colors. The blue color indicates the values that fulfill the conditions (7) and (8), and the red color indicates the values that does not fulfill the condition (8). In order to select from the obtained set a pair of IC corresponding to a given IE, expression (15) was used: $x_0 = 250$ m, $y_0 = 26$ m, $V_{x0} = 0.15$ m, $V_{y0} = -0.57$ m/s (Fig. 7).

Verification of the technical sustainability performance was made by the perturbed PPRT simulation, using the obtained initial conditions for NS2. In Figure 8, IE (reference trajectory) is shown in red and perturbed PPRT in blue. For a given orbit of NS1 and selected IC of NS2 technical sustainability condition will be fulfilled at a time interval of 40 days.

Conclusion
The parametric analysis of the perturbed PPRT in the wide range of IC NS2 and orbital elements of the NS1 has been done. The analysis made it possible to prove the existence of conditions which provides a slight deformation of the NS2 perturbed PPRT. It became possible because the nanosatellites orbital parameters have values that tend to fulfill strict J2 invariant trajectory conditions. Based on these conditions, a method for choosing the initial conditions of NS2 has been developed. The method provides the technical sustainability for the PPRT at a certain time interval. The method can be used to choose the IC not only for the reference orbit of NS1, but also to choose the parameters of the reference orbit for a fixed IE. The approach used for developing this method, can be also used for developing the solution methods of formation flight problems.

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