On a clean determination of $\phi_2$

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Abstract

We point out that time dependent study of four decay modes $B^0(t) \to \overline{D}K_S$, $B^0(t) \to DK_S$, $\overline{B}^0(t) \to \overline{D}K_S$, and $\overline{B}^0(t) \to DK_S$ allows us to measure $\sin(\phi_1 - \phi_2)$ - contrary to previously obtained result that it measures $\sin(2\phi_1 + \phi_3)$. The time dependent study of $B^0_s(t) \to D_{s}^{\pm}K^{\mp}$, and the study of $B^- \to K^-(D, \overline{D}) \to K^- f$ decay where $D \to f$ is a doubly Cabibbo suppressed decay yield information on $\phi_3$, as previously expected.

1. Introduction

Large CP violation in $B^0 \to \psi K_S$ has been observed by Babar and Belle. Clean measurement of $\phi_1$ has been made. This is the first step toward a serious test of the standard model of CP violation. The statistical and systematic errors are large but this will improve in due time. New CDF measurements are around the corner. LHCb and BTeV is also coming up. Exciting program for CP violation study is on its way. What is next? We would like to test the unitarity relation $\phi_1 + \phi_2 + \phi_3 = \pi$. How do we measure $\phi_2$? For many years, CP asymmetry in $B^0 \to \pi\pi$ was thought to be the way to measure $\phi_2$. The existence of penguin amplitudes which prohibit a theoretically clean measurement was overcome by the isospin analysis. But, recent indication that $\text{Br}(B^0 \to \pi^0\pi^0) \sim 2 \times 10^{-7}$, and the background associated with $\pi^0$ detection makes it very difficult to extract $\phi_2$ from the isospin analysis.

A Dalitz plot analysis of $B \to \rho\pi \to 3\pi$ has been proposed. While we expect $B \to 3\pi$ to be dominated by $B \to \rho\pi$ which is a pure CP eigenstate, there is some opposite CP admixture under the $\rho$ band in the Dalitz plot. Further study of the Dalitz plot is necessary to see if this
technique can be used to extract \( \phi_2 \) without theoretical ambiguity.

2. Determination of \( \phi_2 \) from \( B^0, \overline{B}^0 \to DK_S \) Decay

Here we ask if there is a practical way to determine \( \phi_2 \) in a theoretically clean way. We point out that \( \sin(\phi_1 - \phi_2) \) can be determined without theoretical ambiguity by studying two time dependent asymmetries:

\[
a(t) = \frac{\Gamma(B^0(t) \to DK_S) - \Gamma(B^0(t) \to DK_S)}{\Gamma(B^0(t) \to DK_S) + \Gamma(B^0(t) \to DK_S)} \tag{1}
\]

Here, in addition to \( B^0 \to DK_S \), we can also use \( B^0 \to DK_0 \to DK_S \), and \( B^0 \to D^0 K_S \).

In the literature, it is stated that this asymmetry measures \( \sin(2\phi_1 + \phi_3) \) which of cause is identical to \( -\sin(\phi_1 - \phi_2) \) if we assume unitarity, \( \phi_1 + \phi_2 + \phi_3 = \pi \). Another often made statement is that \( \arg(V^*_{ub}V_{ud}V_{tb}V^*_{td}) = \phi_1 \). The correct statement is that it is related to \( \phi_2 \) as given in Eq. (1). One should remember not to use equalities which depend on a particular phase convention, for example, \( \arg(V^*_{ub}V_{ud}) = \phi_3 \) or \( \arg(V^*_{tb}V_{td}) = \phi_1 \). In the first round of theoretical analysis, we were mainly interested in identifying all possible CP asymmetries, and we did not exercise sufficient care in deriving the expression for CP asymmetry independent of the unitarity. The unitarity constraint, which we want to test, has crept into derivations.

To avoid using unitarity relation which we want to test, it is useful to express all physical observables in a rephasing invariant way, and write the asymmetries in terms of \( \phi_1, \phi_2, \phi_3 \) using the definitions:

\[
\phi_1 = \pi - \arg\left(\frac{-V^*_{tb}V_{td}}{-V^*_{cb}V_{cd}}\right), \tag{2}
\]

\[
\phi_2 = \arg\left(\frac{V^*_{tb}V_{td}}{-V^*_{ub}V_{ud}}\right), \tag{3}
\]

\[
\phi_3 = \arg\left(\frac{V^*_{ub}V_{ud}}{-V^*_{cb}V_{cd}}\right). \tag{4}
\]

![Figure 1: The unitarity triangle in the complex plane.](image)

The CP asymmetry for \( \overline{B}^0 \to \psi K_S \) is theoretically clean at the level where we neglect \( O(\sin^2 \theta_c) \) terms. Here \( \theta_c \) is the Cabibbo angle. This comes from the fact that, for \( B^0 \to \psi K_S \) decay, the tree graph, which is proportional to \( V_{cb}V^*_{cs} \), and the penguin graph, which is proportional to \( V^*_{tb}V^*_{ts} \), have the same phase if we neglect terms proportional to \( V_{ub}V^*_{us} \). The latter neglected term is down by \( O(\sin^2 \theta_c) \) compared to the former.
The $2 \times 2$ submatrices of the KM matrix are known to be approximately unitary:

\[
\begin{align*}
V_{ud}^* V_{us} & \cong -V_{cd}^* V_{cs} \\
V_{ud} V_{cd}^* & \cong -V_{us} V_{cs}^* \\
V_{cs} V_{cb}^* & \cong -V_{ts} V_{tb}^*.
\end{align*}
\] (5)

These relations have been verified experimentally to within 6%, 1%, 3%, respectively, relative to terms that are kept. So, since we are making statements that are accurate to order $\sin^2 \theta_c$, we are free to use these relations to test the unitarity relation:

\[
V_{td} V_{us}^* + V_{ts} V_{us}^* + V_{tb} V_{ub}^* = 0
\] (6)

Now, let us discuss $B^0(t) \to \overline{D}K_S$, $B^0(t) \to DK_S$, $\overline{B}^0(t) \to \overline{D}K_S$, and $\overline{B}^0(t) \to DK_S$ decays. Defining $|K_S\rangle$ state as

\[
|K_S\rangle = p_K |K^0\rangle + q_K |\overline{K}\rangle
\] (7)

and using CP symmetry of strong interaction, we obtain

\[
\begin{align*}
A(\overline{B} \to D K_S) &= e^{i\delta_-} V_{cb} V_{us}^* A_- q_K^* \\
A(\overline{B} \to D K_S) &= e^{i\delta_-} V_{ub} V_{cs}^* A_+ q_K^* \\
A(B \to D K_S) &= e^{i\delta_-} V_{cb} V_{us}^* A_- p_K^* \\
A(B \to \overline{D} K_S) &= e^{i\delta_-} V_{ub} V_{cs}^* A_+ p_K^*,
\end{align*}
\] (8)

where $\delta_\pm$ are strong phases, $A_\pm$ are the magnitudes of the matrix elements.

Note that $|A(\overline{B} \to D K_S)| = |A(B \to D K_S)|$ obviously does not hold as an identity. Setting $\Delta \Gamma_B = 0$, for simplicity, we find

\[
\begin{align*}
\Gamma(B(t) \to D K_S) &\propto |A(D K_S)|^2 \left[1 + |\overline{A}(D K_S)|^2 \right] \\
&\quad + (1 - |\overline{A}(D K_S)|^2) \cos \Delta M_{Bt} - 2 \text{Im} \left( \frac{q_B}{p_B} \overline{A}(D K_S) \right) \sin \Delta M_{Bt} \\
\Gamma(\overline{B}(t) \to \overline{D} K_S) &\propto |\overline{A}(\overline{D} K_S)|^2 \left[1 + |A(\overline{D} K_S)|^2 \right] \\
&\quad + (1 - |A(\overline{D} K_S)|^2) \cos \Delta M_{Bt} - 2 \text{Im} \left( \frac{p_B}{q_B} \overline{A}(\overline{D} K_S) \right) \sin \Delta M_{Bt} \\
\Gamma(B(t) \to \overline{D} K_S) &\propto |A(\overline{D} K_S)|^2 \left[1 + |\overline{A}(\overline{D} K_S)|^2 \right] \\
&\quad + (1 - |\overline{A}(\overline{D} K_S)|^2) \cos \Delta M_{Bt} - 2 \text{Im} \left( \frac{q_B}{p_B} A(\overline{D} K_S) \right) \sin \Delta M_{Bt} \\
\Gamma(\overline{B}(t) \to D K_S) &\propto |\overline{A}(D K_S)|^2 \left[1 + |A(D K_S)|^2 \right] \\
&\quad + (1 - |A(D K_S)|^2) \cos \Delta M_{Bt} - 2 \text{Im} \left( \frac{p_B}{q_B} A(D K_S) \right) \sin \Delta M_{Bt}
\end{align*}
\] (9)

Using

\[
\frac{q_B p_K^*}{p_B q_K} = \frac{V_{td} V_{tb}^* V_{cs} V_{cd}^*}{V_{td} V_{tb}^* V_{cs} V_{cd}}
\] (10)
we obtain:

\[
\frac{q_B}{p_B} \mathcal{P}(\overline{D}K_S) = e^{i(\delta_- - \delta_+)} \frac{V_{cb} V_{us}^* V_{td} V_{tb}^* V_{cs} V_{cd}^* A_-}{V_{ub} V_{cs}^* V_{td} V_{tb}^* V_{cs} V_{cd} A_+}
\]

\[
\frac{q_B}{p_B} \mathcal{P}(D K_S) = e^{-i(\delta_- - \delta_+)} \frac{V_{ub} V_{cs}^* V_{td} V_{tb}^* V_{cs} V_{cd}^* A_+}{V_{ub} V_{cs}^* V_{td} V_{tb}^* V_{cs} V_{cd} A_-}
\]

(11)

Note that this is rephasing invariant. Using Eq. (11), we obtain:

\[
\frac{V_{cb} V_{us}^* V_{td} V_{tb}^* V_{cs} V_{cd}^*}{V_{ub} V_{cs}^* V_{td} V_{tb}^* V_{cs} V_{cd}} = -\frac{V_{cb} V_{cd}^* V_{td} V_{tb}^*}{V_{ub} V_{ud} V_{td} V_{tb}}
\]

(12)

and using Eq. (11):

\[
\frac{q_B}{p_B} \mathcal{P}(\overline{D}K_S) = -e^{i(\delta_- - \delta_+)} \frac{A_-}{A_+} \mathcal{R} e^{i(\phi_2 - \phi_1)}
\]

\[
\frac{q_B}{p_B} \mathcal{P}(D K_S) = -e^{-i(\delta_- - \delta_+)} \frac{A_+}{A_-} \mathcal{R} e^{i(\phi_2 - \phi_1)}
\]

(13)

where

\[
\mathcal{R} = \left| \frac{V_{ub} V_{us}^*}{V_{cb} V_{cd}} \right|
\]

(14)

Measuring the coefficients of \(\cos \Delta M_{Bt}\), we can determine \(\frac{1}{A_-} \mathcal{R}\). The coefficient of \(\sin \Delta M_{Bt}\) gives \(\sin(\phi_1 - \phi_2)\).

3. Determination of \(\phi_3\) with \(B^- \to K^- D\) decays

It is useful to reanalyze the other well known methods to determine \(\phi_3\). Measurements of \(Br(B^- \to K^- (D^0, \overline{D}^0) \to K^- f)\), the doubly Cabibbo suppressed decay \(Br(D^0 \to f)\), \(Br(D^- \to K^- D^0)\), and \(Br(B^- \to K^- \overline{D}^0)\) allow us to determine \(\phi_3\). Consider

\[
B^- \to K^- (D^0, \overline{D}^0) \to K^- f
\]

(15)

Write

\[
A = A(B^- \to D^0 K^-) A(D^0 \to f)
\]

\[
\overline{A} = A(B^- \to \overline{D}^0 K^-) A(\overline{D}^0 \to f)
\]

(16)

where these amplitudes represent strong interaction part and the KM factors are taken out.

\[
A(B^- \to K^- (D^0, \overline{D}^0) \to f K^-) = [A |e^{i\delta} V_{cb} V_{us}^* V_{us} V_{cs}^* + |\overline{A}|e^{i\overline{\delta}} V_{ub} V_{cs}^* V_{ud} V_{cs}|]
\]

(17)

Here \(\delta\) and \(\overline{\delta}\) are strong interaction phases for decays involving \(D\) and \(\overline{D}\) intermediate particles, respectively. It should be noted that by choosing the doubly Cabibbo suppressed decay of \(D\) meson, and that \(B^- \to K^- \overline{D}\) is color suppressed, the two terms on the right hand side have approximately equal magnitude.
$\phi = \pi - \arg \left[ \frac{e^{i\delta} V_{cb} V_{us}^* V_{cd} V_{us}}{e^{i\delta} V_{ub} V_{cs}^* V_{ud} V_{cs}} \right] = -\phi_3 + \bar{\delta} - \delta$  \hspace{1cm} (20)

The same analysis for $B^+$ decay leads to the determination of $\phi_3 + \bar{\delta} - \delta$ so both $\phi_3$ and the relative strong interaction phase $\delta - \bar{\delta}$ can be determined.

4. Determination of $\phi_3$ with $B_s^0 \to D_s^\pm K^\mp$ decays

Both $B_s \to D_s^\pm K^\mp$ and $\bar{B}_s^0 \to D_s^\pm K^\mp$ exist and there will be CP asymmetry which is expected to lead to information on $\phi_3$. We start be denoting relevant amplitudes:

\begin{align*}
A(\bar{B}_s \to D_s^- K^+) &= e^{i\delta} V_{ub} V_{cs} A_-
A(\bar{B}_s \to D_s^+ K^-) &= e^{i\delta} V_{cb} V_{us}^* A_+
A(B_s \to D_s^- K^+) &= e^{i\delta} V_{cb} V_{us} A_+
A(B_s \to D_s^+ K^-) &= e^{i\delta} V_{ub} V_{cs}^* A_-,
\end{align*}

where $\delta_\pm$ are strong phases, $A_\pm$ are the magnitudes of the matrix elements.

\begin{align*}
\frac{q_{B_s}}{p_{B_s}} A(\bar{B}_s \to D_s^+ K^-) &= e^{i\delta} V_{tb} V_{ts}^* V_{us} V_{cb} A_+ \\
&= \frac{e^{i\delta} V_{tb} V_{ts}^* V_{us} V_{cb} A_+}{e^{i\delta} V_{ub} V_{cs} V_{cd}^* A_-} \hspace{1cm} (22)
\end{align*}
\[ \frac{V_{tb}^* V_{ts}}{V_{tb} V_{ts}^*} \frac{V_{ub}^* V_{cs}}{V_{ub} V_{cs}^*} \cong \frac{V_{tb}^* V_{cs}}{V_{tb} V_{cs}^*} \frac{V_{ub}^* V_{us}}{V_{ub} V_{us}^*} \]
\[ = \frac{V_{tb}^* V_{cs}}{V_{tb} V_{cs}^*} \frac{V_{us}^* V_{ub}}{V_{us} V_{ub}^*} \]
\[ = \frac{V_{tb}^* V_{cs}}{V_{tb} V_{cs}^*} \frac{V_{us}^* V_{ub}}{V_{us} V_{ub}^*} \]
\[ \sim = \frac{V_{tb}^* V_{cs}}{V_{tb} V_{cs}^*} \frac{V_{us}^* V_{ub}}{V_{us} V_{ub}^*} \]
\[ = \frac{1}{R} e^{-i\phi_3} \]

(23)

\[
\frac{q_{B_s} A(\overline{B}_s \to D^+_s K^-)}{p_{B_s} A(B_s \to D^+_s K^-)} = \frac{1}{R} A_+ e^{i(\delta_+ - \delta_-) - i\phi_3}.
\]

(24)

Using Eq. (5), the right hand side of Eq. (25) can be written as Using the time dependence similar to Eq. (9), we can obtain both the magnitude and the phase of \( q_{B_s} A(\overline{B}_s \to D^+_s K^-) \). Similarly we can obtain the magnitude and the phase of

\[
\frac{q_{B_s} A(\overline{B}_s \to D^-_s K^+)}{p_{B_s} A(B_s \to D^-_s K^+)} = \frac{e^{i\delta_-} V_{tb}^* V_{ts} V_{ub} V_{cs} A_-}{e^{i\delta_+} V_{tb} V_{ts}^* V_{ub}^* V_{cs} A_+} \]
\[ = R A_+ e^{i(\delta_- - \delta_+ - i\phi_3)}. \]

(25)

Thus \( \phi_3 \) can be determined.

5. Summary

The time has come to perform a serious test of the KM ansatz for CP violation. To see if the unitarity triangle is actually a triangle is the next step. For this purpose we should be careful in deriving the expression for asymmetry in terms of three angles of the unitarity triangle. A useful rule is to always make sure that the asymmetry is independent of phase conventions. We have reanalyzed processes known to yeild information on \( \phi_3 \). Asymmetries for \( B^0 \), \( \overline{B}^0 \to DK_S \) decays are shown to yield information on \( \phi_2 - \phi_1 \), and not on \( 2\phi_1 + \phi_3 \) as previously stated. The other modes analyzed yield information on \( \phi_3 \) as expected.

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