Modeling Simplification and Dynamic Behavior of N-Shaped Locally-Active Memristor Based Oscillator

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ABSTRACT This paper designs a simple mathematical model for the voltage-controlled locally-active memristor (LAM), which undoubtedly simplifies the theoretical analysis. Owing to the N-shaped outline of the DC V-I curve, this LAM is called as N-shaped LAM. The small-signal analysis method is exploited to obtain the equivalent circuit of the N-shaped LAM when it is biased into the locally-active region. The calculation results show that the LAM can be equivalent to a conductance with a parasitic capacitor, which makes for a deep insight into the mechanisms at the origin of LAM based oscillator. The theoretical analysis reveals that the N-shaped LAM along with an inductor and a battery can generate oscillation, which is confirmed by the numerical simulation. In addition, observe from simulation results that as the inductance increases, the oscillation behavior of the system deviates from the sinusoidal signal and tends to a relaxation oscillation. Finally, the physical circuit realization of the N-shaped LAM based oscillator including the memristor emulator is proposed. The good agreement between the experimental results and simulation results further demonstrates the correctness and feasibility of the theoretical design and analysis.

INDEX TERMS Memristors, nonlinear circuit, oscillators.

I. INTRODUCTION

Local activity is the origin of the complexity, and complex behavior may only arise in the locally-active system [1]-[2]. Local activity renders the system capable to amplify extremely small fluctuations in energy, and thus it can be used to design the oscillating system.

Memristor, the fourth circuit element, has aroused considerable interest from the researchers and industry owing to its nano-scale and nonlinearity [3]-[10]. To date, some effort has been devoted to design the oscillators with the passive memristor [11]-[13]. Compared with passive memristor based oscillator, the locally-active memristor (LAM) is functioned as both nonlinear element and active element, and thus the circuit structure is simpler. LAM is defined as any memristor that exhibits a negative memductance or memristance for at least some values of the memristor voltage or current [14].

By replacing Chua’s diodes with memristors, several nonlinear oscillators are derived in [15], where the memristor is an active device and can be equivalent to a passive memristor connected with a negative resistance (or a negative conductance). More active memristors with different mathematical models are applied in the oscillators and more complex dynamics are observed [16]-[18]. A current-controlled active memristor along with a capacitor and an inductor is used to construct a simplest chaotic oscillating system [16]. Similarly, a voltage-controlled active memristor connected with a capacitor and an inductor forms a chaotic system [17]. In [18], a locally-active memristor model was presented for exploring the nonvolatile and switching mechanism of the memristor and the influence of local activity on the complexity of nonlinear circuits. With respect to the above active memristors [16]-[18], their memristances or memductances are negative with respect to the origin, and the voltage-current characteristic has a branch which crosses the origin into the second and the fourth quadrants. The active memristor can
provide energy via internal power supply. As for the real memristive device, it must be a passive element. However, it can be locally-active, namely, negative differential resistance (NDR), over the specific operation region where the small-signal memristance or memductance is negative. It can amplify small signals at the expense of an external power supply.

Based on the piecewise linear function, Chua proposed a passive but locally-active memristor. Due to the fact that its DC V-I plot resembles the shoelace, it is called as the Chua shoelace memristor [19]. When the Chua shoelace memristor is connected with an inductor and a battery, the oscillating behavior may generate at the particular initial conditions and DC bias [20]-[21]. Another second-order LAM with a battery is used to design an oscillator, which does not require extra energy-storage element [22].

In addition to the above memristors, it has been shown that particular types of memristor, such as niobium oxide (NbOx) and vanadium dioxide (VO2) devices, are passive but locally-active memristors [23]-[26]. Different from the non-volatile memristor, the majority of nanoscale LAMs show volatile resistive switching behavior. Pickett et al firstly reported that the NbOx devices exhibited current-controlled negative differential resistance [23]. Afterwards, HP laboratory developed a highly accurate dynamic model for NbOx LAM from the thermal feedback mechanism [27]. Based on the Chua’s unfolding theorem, another mathematical model for NbOx LAM is presented in [28] by using the parameter optimization method. However, the model expressions in [27] and [28] including many parameters are rather complicated, which will bring a limit to the theoretical analysis. In 2017, HP laboratory further reported a nano-scale NbO2 Mott memristor which exhibited both current-controlled and temperature-controlled NDRs [29]. Based on this LAM, a relaxation oscillator and even a chaotic system are obtained. In 2018, Wei Yi et.al addressed that VO2 LAM possessed most of known biological neuronal dynamics and could be used to construct an electrical equivalent of biological neurons [26]. In addition, NbOx and VO2 LAMs are used to realize relaxation oscillators that can be connected electronically to form arrays of coupled oscillators [30]-[31]. To sum up, the LAM has a number of promising applications, including self-sustained and chaotic oscillator, as well as biological neuronal dynamics. Hence, we intend to design a simple LAM mathematical model, which is conductive to the research of LAM applications. In addition, a detailed analysis of dynamic behavior of the locally-active memristor based oscillator will provide a preliminary deep understanding of its behavior.

In this paper, a simple mathematical model of voltage-controlled LAM is proposed and its characteristics are discussed in Section II. In section III, small-signal analysis method is used to explain the dynamics of the N-shaped LAM. The N-shaped LAM based oscillator is designed and numerical simulation results are given in Section IV. The hardware circuit implementations of the N-shaped LAM and the oscillator are presented as well as the experimental results are given in Section V. A short conclusion is given in Section VI.

II. LAM MODEL AND ITS THEORETICAL ANALYSIS

A. MATHEMATICAL MODEL

The memristor can be classified into voltage-controlled memristor and current-controlled memristor. By using Chua’s unfolding theorem [32], a generic voltage-controlled memristor can be represented by

\[
\begin{align*}
\dot{x} &= f(x, v) = \sum_{k=-r_1}^{r_2} a_k x^k + \sum_{k=-m_1}^{m_2} b_k v^k \\
& \quad + \sum_{k=-p_1}^{p_2} \sum_{l=-q_1}^{q_2} c_{kl} x^k v^l 
\end{align*}
\]

where \( v \) and \( i \) denote the voltage and current of the device; \( x \) is the state variable; \( G_M(x) \) is the memductance function; \( f \) is the function of the derivative of state variable; \( a_k, b_k, \) and \( c_{kl} \) are unfolding parameters.

We aim at designing a LAM model with the simple expression, which undoubtedly simplifies the theoretical analysis and is useful for exploring the oscillation mechanism. As the function \( f(x, v) \) is related to both \( x \) and \( v \), a simple formula of state equation is:

\[
\frac{dx}{dt} = f(x, v) = \frac{1}{\tau} (v - x)
\]

where \( \tau \) is a parameter determining the frequency operation range of the memristor. The DC V-I curve of the memristor is derived by equating the state equation (2) to zero, and solving for the equilibrium point for each DC input voltage \( v = V \), namely,

\[
X = V
\]

By substituting (3) into the state dependent Ohm’s Law in (1), it can be obtained as follows:

\[
I = G_M(X)V = \left( \sum_{k=-r_1}^{r_2} d_k V^k \right) V = \sum_{k=-r_1}^{r_2} d_k V^{k+1}
\]

where \(-r_1 + 1\) and \(r_2 + 1\) represent the lowest order power and the highest order power of the DC voltage \( V \), respectively. Based on (4), the differential memductance at the DC operating point can be obtained as

\[
\frac{dI}{dV} = \sum_{k=-r_1}^{r_2} (k + 1)d_k V^k
\]

Considering it is a passive but locally-active memristor, the negative differential resistance only exists in a finite interval in the DC V-I plot, which conforms to the real characteristic of the electronic devices. That is to say that the differential memductance \( dI/dV \) at the DC operating point is negative over a finite interval. Obviously, if \( r_1 = 0 \) and \( r_2 = 1 \) cannot
According to the aforementioned theoretical analysis, a simple LAM mathematical model is obtained

\[
\begin{align*}
\frac{dI}{dV} &= 3d_2V^2 + 2d_1V + d_0 \\
&= 3d_2(V - V_1)(V - V_2) < 0(V_1 < V < V_2)
\end{align*}
\]  

where \(d_1 = -3d_2(V_1 + V_2)/2\) and \(d_0 = 3d_2V_1V_2\). For real physical memristor device, it is a passive two-terminal element. Therefore, the memductance should always be positive and the values of parameters \(d_2, d_1,\) and \(d_0\) should satisfy

\[
G_M(X) = d_2X^2 + d_1X + d_0 > 0
\]

According to the aforementioned theoretical analysis, the simple LAM mathematical model is obtained

\[
\begin{aligned}
i &= (d_2x^2 + d_1x + d_0) v \\
\frac{dx}{dt} &= \frac{1}{\tau} (v - x)
\end{aligned}
\]

Herein the parameter configurations are chosen as follows: \(\tau = 1 \times 10^{-5}\), \(V_1 = 3.5V\), \(V_2 = 6.5V\), \(d_2 = 1/3 \times 10^{-4}\), \(d_1 = -5 \times 10^{-4}\), and \(d_0 = 22.75 \times 10^{-4}\).

**B. DC V-I CHARACTERISTICS**

When the input DC voltage \(V\) is connected to the LAM, the corresponding DC response current \(I\) is measured. By repeating the test with different voltages \(V\) varying from 0V to 10V with the step of 0.1V, a set of points can be obtained and its DC V-I curve is shown in Fig. 1.

The DC V-I curve shown in Fig. 1 is similar to the \(v-i\) curve of the N-shaped nonlinear resistor, like tunnel diode [33]. Hence, the proposed voltage-controlled LAM can be considered as an N-shaped LAM and may exist in physical world. With respect to N-shaped nonlinear resistor, its resistance is controlled by the voltage signal and independent of the history of state. Similarly, the N-shaped LAM is also controlled by the voltage signal. In contrast, its memductance is dependent on the history of state. This is because that each memristor is defined by a state-dependent Ohm’s law and a state differential equation.

Observe from Fig. 1 that there exists a NDR region over the range of \(3.5V < V < 6.5V\), which is consistent with the theoretical design. Under the DC bias, the N-shaped LAM could bring out the oscillation behavior. Note that, for \(I_2 < I < I_1\), a given current corresponds to three different values of \(V\). The DC V-I curve is different from the \(v-i\) pinched hysteresis loop (PHL) of the memristor. The DC V-I curve depicts the behavior of the memristor at the DC steady state regime, while PHL shows the transient performance under the AC excitation signal.

**C. PINCHED Hysteresis LOOP**

Chua addressed that a memristor is best defined as any two-terminal device that exhibits a pinched hysteresis loop in the \(v-i\) plane when driven by any periodic voltage or current signal with zero mean [34].

As for the proposed N-shaped LAM, a sinusoidal voltage signal \(v(t) = \text{Asin}(2\pi ft)\) with an amplitude of 6V and the frequency \(f = 10\text{kHz}\) is applied. Fig. 2(a) depicts the transient waveforms of the voltage \(v\), current \(i\), state variable \(x\), and memductance \(G_M\) respectively. The blue color curve in Fig. 2(b) shows the corresponding \(v-i\) characteristic curve.
It can be seen that the N-shaped LAM exhibits a PHL confined to the first and third quadrants in the \( v-i \) plane under a sinusoidal signal with zero mean. To illustrate the frequency dependence of the PHL, the momentary \( v-i \) characteristic curves measured at different input frequencies, i.e. \( f = 10\, \text{kHz}, \, 50\, \text{kHz}, \, \text{and} \, 1\, \text{MHz} \) are presented in Fig. 2(b), which manifests the dependence of the hysteresis lobe area on the frequency of the input signal. Observe that the enclosed area of the PHL decreases monotonically as the frequency, and finally shrinks to a single-value function. In contrast to ideal memristor, the LAM in general has a PHL that is asymmetric with respect to the origin.

Observe from Fig. 2(a) that the state variable \( x \) is not identical to \( v \) except for a few of specific points, namely \( dx/dt = 1/\tau \times (v-x) \) is not equal to zero. Hence, most of the points \( (v, i) \) on the PHL are unstable and they are not operating points. It follows that we cannot obtain the small-signal memductance and locally-active characteristic does not appear on the pinched hysteresis loop.

D. VOLATILITY

Based on Chua’s non-volatility theorem, all ideal memristors are non-volatile and are capable to memory the state of memristance or memductance when power is off. As for non-ideal memristor, POP (acronym for power-off plot) can be used to estimate whether it is non-volatile [14]. POP is the locus of the rate of change \( (dx/dt) \) in the memristor state variable as a function of the state variable \( x \) with the input current or input voltage set equal zero. If there are two or more intersections with a negative slope between the \( dx/dt \) and the \( x \)-axis of the memristor when the memristor is powered off, the memristor can be asymptotically stabilized to two or more equilibrium states. It follows that it has the characteristics of a non-volatile memory.

For the proposed N-shaped LAM, when input voltage \( v = 0 \), its POP is depicted in Fig. 3 and the state equation in (8) becomes

\[
\frac{dx}{dt} \bigg|_{v=0} = -\frac{1}{\tau} x
\]

When \( dx/dt > 0 \), the state variable \( x(t) \) moves to the right along the POP, and vice versa. Observe that the N-shaped LAM only has one equilibrium point \((0, 0)\). That is to say no matter what the value of \( x \) is, the locus will tend to the origin after the power is off. Hence, similarly to the nanoscale LAMs, the presented N-shaped LAM is volatile.

III. SMALL-SIGNAL ANALYSIS

Small-signal analysis method, a quasi-linear method, is used to predict the response of the LAM to a small-signal input applied at one DC operating point \((Q(V, I))\), which lies in the NDR region. This method is on basis of the Taylor series and Laplace transform. When the signal is sufficiently small, high order nonlinear terms can be ignored. Taking the first three terms of the Taylor series expansion of \( i \) about the point \( V \) and state \( X \), we obtain

\[
i = g(x, v) = g(X + \Delta x, V + \Delta v)
\]

\[
= g(X, V) + \frac{\partial g}{\partial x}(X, V) \Delta x + \frac{\partial g}{\partial v}(X, V) \Delta v
\]

\[
= I + a_{11}(Q) \Delta x + a_{12}(Q) \Delta v
\]

(10)

Based on (8), it can be easily obtained that \( I = V \) and

\[
I = G(X) = d_2 V^3 + d_1 V^2 + d_0 V
\]

\[
a_{11}(Q) = 2d_2 V^2 + d_1 V
\]

\[
a_{12}(Q) = d_2 V^2 + d_1 V + d_0
\]

(11)

Similarly, the state equation can be rewritten as

\[
\frac{dx}{dt} = f(x, v) = f(X + \Delta x, V + \Delta v)
\]

\[
= f(X, V) + \frac{\partial f}{\partial x}(X, V) \Delta x + \frac{\partial f}{\partial v}(X, V) \Delta v
\]

\[
= 0 + b_{11}(Q) \Delta x + b_{12}(Q) \Delta v
\]

(12)

Then, we obtain

\[
b_{11}(Q) = -\frac{1}{\tau}, \quad b_{12}(Q) = \frac{1}{\tau}
\]

(13)

Based on (10) and (12), small signal current \( \Delta i \) and rate of change in the state variable \( \Delta \hat{x} \) can be obtained as

\[
\Delta i = a_{11}(Q) \Delta x + a_{12}(Q) \Delta v
\]

\[
\Delta \hat{x} = b_{11}(Q) \Delta x + b_{12}(Q) \Delta v
\]

(14)

By applying Laplace transform on both sides of (14), we can get

\[
\hat{i}(s) = a_{11}(Q) \hat{x}(s) + a_{12}(Q) \hat{v}(s)
\]

\[
\hat{x}(s) = b_{11}(Q) \hat{x}(s) + b_{12}(Q) \hat{v}(s)
\]

(15)

Hence, the small-signal admittance function of the N-shaped LAM about the operating point \( Q \) is given by

\[
Y(s, Q) = \frac{\hat{i}(s)}{\hat{v}(s)} = \frac{a_{11}(Q) b_{12}(Q)}{s - b_{11}(Q)} + a_{12}(Q)
\]

\[
= a_{12}(Q) s + a_{11}(Q) b_{12}(Q) - a_{12}(Q) b_{11}(Q)
\]

\[
= \frac{b_2(Q) s + b_1(Q)}{d_2(Q) s + a_1(Q)}
\]
To quantitatively analyze the oscillation condition of the N-shaped LAM, the admittance function $Y(\omega, Q)$ is always positive when the frequency increases from zero, the conductance varies from the negative value to the positive one, whereas the capacitance decreases gradually.

Based on (18), the frequency responses of the $\text{Re}Y(\omega, Q_c)$ and $\text{Im}Y(\omega, Q_c)$ at $V = 5V$ for the N-shaped LAM over the range $0 \leq \omega \leq 8 \times 10^5 \text{ rad/s}$ are shown in Fig. 5. It can be seen from Fig. 5 that the negative real part of $Y(\omega, Q_c)$ dominates at low frequency. As the frequency increases, $\text{Im}Y(\omega, Q_c)$ first increases and then decreases.

According to Fig. 4 and (20), the small-signal equivalent circuit of the N-shaped LAM at the NDR region, i.e.,

\[ Y(\omega, Q) = G'(\omega, Q) + i\omega C'(\omega, Q) \]  

\[ \text{Re}Y(\omega, Q) = \frac{\text{Re} G(\omega, Q) b_1(\omega) + \omega^2 a_2(\omega) b_2(Q)}{a_1^2(\omega) + \omega^2 a_2^2(\omega)} \]  

\[ \text{Im}Y(\omega, Q) = \frac{\omega (\text{Re} G(\omega, Q) b_1(\omega) - a_2(\omega) b_1(\omega))}{a_1^2(\omega) + \omega^2 a_2^2(\omega)} \]  

Observe from Fig. 4 that $\text{Im}Y(\omega, Q)$ is always positive over $3.5V < V < 6.5V$ when $\omega \geq 0$. Hence, the admittance function $Y(\omega, Q)$ can be regarded as a conductance connected with a capacitance in parallel at the NDR region, i.e.,

\[ G'(\omega, Q) = \frac{a_1(\omega) b_1(\omega) + \omega^2 a_2(\omega) b_2(\omega)}{a_1^2(\omega) + \omega^2 a_2^2(\omega)} \]  

\[ C'(\omega, Q) = \frac{a_1(\omega) b_2(\omega) - a_2(\omega) b_1(\omega)}{a_1^2(\omega) + \omega^2 a_2^2(\omega)} \]
By biasing the locally-active memristor (LAM) into the locally-active region, it can generate periodic oscillation when connected with an energy-storage element. Based on the above analysis, an extra inductor needs to be added to the circuit for making up a LC oscillating system, as shown in Fig. 7. The extra L will not affect the operating point of the LAM. This is because that as for the ideal inductor, it can be considered as short circuit at DC analysis.

In order to make the memristor circuit oscillate near $V = 5\text{V}$, the value of $G'(\omega, Q_c)$ at the specific frequency should be zero. This is because that there is not energy generation and dissipation in the circuit when $G'(\omega, Q_c) = 0$. Hence, the oscillation angular frequency $\omega_0$ and oscillation frequency $f_0$ can be calculated as:

$$\omega_0 = \sqrt{-\frac{a_1(Q_c)b_1(Q_c)}{a_2(Q_c)b_2(Q_c)}} \approx 6.08 \times 10^4 \text{rad/s} \quad (21a)$$

$$f_0 = \frac{1}{2\pi} \sqrt{-\frac{a_1(Q_c)b_1(Q_c)}{a_2(Q_c)b_2(Q_c)}} \approx 9.68\text{kHz} \quad (21b)$$

According to the oscillation frequency, it can be deduced that the value of the equivalent capacitance is

$$C'(\omega_0, Q_c) = \frac{a_1(Q_c)b_2(Q_c) - a_2(Q_c)b_1(Q_c)}{a_1^2(Q_c) + a_2^2(Q_c)} \approx 6.08\text{nF} \quad (22)$$

The value of the extra inductance can be calculated as:

$$L_0 = \frac{1}{\omega_0^2C'} \approx 44.4\text{mH} \quad (23)$$

In order to further evaluate the small-signal response in the NDR region, the simulation results with an input voltage biased in the center of the NDR region ($V = 5\text{V}$) and driven with a sinusoidal signal with the amplitude of 0.5V, i.e., $v = 0.5\sin(2\pi ft)+5$, are shown in Fig. 8, where the frequency $f = 100\text{Hz}$, $f = 1\text{kHz}$, $f = 10\text{kHz}$, $f = 100\text{kHz}$ and $f = 1\text{MHz}$, respectively. Furthermore, the black line is the DC V-I plot and the insert figure at the right is the zooming of the region within the dotted box.

When the frequency is very low, such as 100Hz, $x(t)$ follows the input signal $v(t)$ and the $v-i$ curve nearly matches with the DC V-I plot, which indicates that the N-shaped LAM behaves like an N-shaped nonlinear resistor at low frequency. As the frequency of the sinusoidal signal increases, a phase shift develops between the input voltage and current signal, such as $f = 1\text{kHz}$, which is attributed to the increase of the $\text{Im}Y(i\omega, Q)$. When the frequency increases to a value close to the oscillation frequency $f_0$, i.e., $f = 10\text{kHz}$, the $v-i$ curve behaves like a capacitor without internal resistance. As the sinusoidal frequency reaches 100kHz, the positive real part $\text{Re}Y(\omega, Q)$ and $\text{Im}Y(i\omega, Q)$ coexist, resulting in a phase shift in contrast to that of 1kHz. Finally, when the frequency is equal to 1MHz, this response shrinks to the positive differential conductance.

The edge of chaos theory is used to identify whether there is complexity [1] in the system. The region of edge of chaos can be obtained according to two conditions [20]. For the condition (i), the real parts of all poles for $Y(s, Q)$ are negative. After rearranging (16), we can obtain

$$Y(s, Q) = \frac{a_{11}(Q)b_{12}(Q) + a_{12}(Q)(s - b_{11}(Q))}{s - b_{11}(Q)} = \frac{K(s - z)}{s - p} \quad (24)$$

where $p = b_{11}(Q)$ is the pole and $z = -\omega(a_{11}(Q)b_{12}(Q) - a_{12}(Q)b_{11}(Q))/a_{12}(Q)$ is the zero. It can be seen that $p$ is equal to $-1/\tau$, which satisfies the condition (i). In order to ensure that there is an edge of chaos region, the admittance $Y(s, Q)$ also must satisfy the condition (ii) which is $\text{Re}Y(i\omega, Q) < 0$ for some $\omega \in (-\infty, \infty)$ at the same operating point ($X(V), I(V)$). By setting different DC voltage, i.e., $V = 3.0\text{V}, 3.5\text{V}, 4.0\text{V}, 5.0\text{V}, 6.0\text{V}$, and 6.5V, respectively, we obtain the $\text{Re}Y(i\omega, Q)$ varying with the frequency $\omega (-8 \times 10^3 \text{rad/s} < \omega < 8 \times 10^3 \text{rad/s})$, as depicted in Fig. 9. Observe that $\text{Re}Y(i\omega, Q) < 0$ over 3.5V $< V < 6.5\text{V}$ for the specific frequency range. Based on the above theoretical analysis, we can say that the N-shaped LAM is on the edge of chaos over the interval 3.5V $< V < 6.5\text{V}$, which is identical with the locally-active region.

**IV. N-SHAPED LAM-BASED OSCILLATOR**

According to the aforementioned theoretical analysis, it can be seen that the N-shaped LAM connected with an inductor $L$ in series under the DC bias can construct an oscillator, as depicted in Fig. 10.
The state equations of the oscillating system can be described as:

\[
\begin{align*}
\frac{dx}{dt} &= \frac{1}{\tau} \left( d_2 x^2 + d_1 x + d_0 - x \right) \\
\frac{di}{dt} &= \frac{1}{L} \left( V_D - \frac{i}{d_2 x^2 + d_1 x + d_0} \right)
\end{align*}
\]  

(25)

By setting the differential equations to zero, i.e., \(dx/dt = 0\) and \(di/dt = 0\), we can obtain the equilibrium point of the system

\[
\begin{align*}
X_S &= V_D \\
I_S &= \left( d_2 X_S^2 + d_1 X_S + d_0 \right) V_D
\end{align*}
\]

(26)

At the case of \(V_D = 5V\), it can be calculated that \(X_S = 5\) and \(I_S = 3.04mA\). Hence, it can be predicted that the waveforms of the state variable and current will move around these two equilibrium states.

In order to further analysis the dynamic behavior of the N-shape LAM based oscillator, the numerical simulation results are discussed in this part. Noted that the center of the locally-active region \(Q_c\), namely, \(V_D = 5V\), is taken as a case to conduct simulation analysis.

**A. CASE I: \(L=45mH\)**

As the inductance \(L\) is 45mH which is close to and a little greater than \(L_0\), and the initial conditions are set as \(x(0) = 0\) and \(i(0) = 0A\), the time-domain waveforms and phase portraits of the oscillating circuit are shown in Fig. 11, where the arrowheads show the motion directions of the trajectories. The time-domain waveforms of the memristor voltage \(v\), state variable \(x\), and current \(i\) are shown in Fig. 11 (a), where the insert figure at the right is the stable transient waveforms. The trajectory in the \(x - i\) plane is depicted in Fig. 11 (b), where the insert figure at the right is the zooming of the stable oscillation region. Fig. 11 (c) shows the memristor voltage varies with the current, and the extra DC \(V-I\) plot is used to analysis the operation region of the LAM.
Observe from Fig. 11 (a) that the system can oscillate continuously, where the time-domain waveforms resemble the sinusoidal signal. The amplitudes of \( v, x, \) and \( i \) firstly increase to its maximum values and then decrease to the stable states. This is because that the equivalent NDR of the N-shaped LAM is the greatest when \( f \) is the smallest. The energy provided by the equivalent NDR enables the amplitudes of the \( v, x, \) and \( i \) increase rapidly firstly. However, a great voltage or current amplitude makes the LAM departs from the NDR region and the LAM turns into energy dissipation region. Hence, the amplitudes decrease gradually and tend to achieve stable values.

It can be seen from Fig. 11 (b) and 11 (c) that the trajectories start from the initial states and then tend to limit cycles, which center on the equilibrium point. Noted that the same limit cycles are obtained when the initial states are \( x(0) = 5 \) and \( i(0) = 3.16mA \), as shown in Fig. 11 (d) and Fig. 11 (e). Hence, the behavior of the system is independent on the initial conditions. This is because that there is only one DC equilibrium point for system equation, and then the system will tend to the same stable behavior under any initial state.

Observe from Fig. 11 (c) and 11 (e) that the stable limit cycle in the \( v-i \) plane is similar to a circle and located in the NDR region, where \( v \) can be considered as the voltage of the parasitic capacitance, and \( i \) is the current flowing through the inductor. This round trajectory has much common with that of the classical LC sine oscillator, where the electrical energy of capacitance and magnetic energy of inductance are exchanged back and forth.

**B. CASE II: \( L=40mH \)**

When the inductance \( (L = 40mH) \) is smaller than the oscillation inductance \( L_0 \), and the initial states are configured as \( x(0) = 3 \) and \( i(0) = 1A \), the simulation results are shown in Fig. 12. The time-domain waveforms of the memristor voltage \( v \), state variable \( x \), and current \( i \) are shown in Fig. 12 (a). The phase portrait in the \( x-i \) plane is depicted in Fig. 12 (b).

Fig. 12 shows that the transient waveforms asymptotically converge to the equilibrium point \( v = 5V, x = 5, \) and \( i = 3.04mA \). In addition, the phase portrait in the \( x-i \) plane gradually converges to the unique stable point \( S (5, 3.04mA) \). It follows that the periodical oscillation does not occur at this case. This may be due to the fact that a smaller inductance brings about a higher oscillation frequency. Meanwhile, higher frequency will make the N-shaped LAM exhibit a positive real part in the \( Y(i_0, Q) \), as shown in Fig. 5. Under this condition, the LAM will lose its NDR characteristic and the transient waveforms are converging to DC equilibrium point.

**C. CASE III: \( L=90mH \)**

When the inductance \( (L = 90mH) \) is bigger than the oscillation inductance \( L_0 \), and the initial states are configured as \( x(0) = 0 \) and \( i(0) = 0A \), the simulation results are shown in Fig. 13. The time-domain waveforms of memristor voltage \( v \), state variable \( x \), and current \( i \) are shown in Fig. 13 (a). The phase portrait in the \( x-i \) plane is depicted in Fig. 13 (b). Fig. 13 (c) shows the memristor voltage varies with the current.

It can be seen from Fig. 13 that the time-domain waveforms of the \( v, x, \) and \( i \) are the distortion of the sinusoidal signals. Hence, higher order harmonic components must exist in these waveforms. It follows that the small-signal analysis method is no longer suitable for the system. However, it still can be applied in the qualitative analysis. A greater inductance brings about a lower oscillation frequency, and then the equivalent NDR arises from the lower frequency. The energy powered by the equivalent NDR makes the amplitudes of voltage and current enter into the positive differential resistance (PDR) region. This nonlinearity results in the generation of higher-order harmonic components.

The phase portraits still exhibit shrink behavior and then tend to limit cycles. Noted that the stable limit cycle in the \( v-i \) plane has deviated from the normal round and its border has entered into the PDR region.

**D. CASE IV: \( L=1H \)**

As the inductance \( (L = 1H) \) is much greater than the oscillation inductance \( L_0 \), and the initial states are configured as \( x(0) = 0 \) and \( i(0) = 0A \), the simulation results are shown in Fig. 14. The time-domain waveforms of the memristor voltage \( v \), state variable \( x \), and current \( i \) are shown in Fig. 14 (a). The phase portrait in the \( x-i \) plane is depicted.
in Fig. 14 (b). Fig. 14 (c) shows the memristor voltage varies with the current.

Observe from Fig. 14(a) that there exists jump behavior in the time-domain waveform of the \( v \), which indicates that there are many higher-order harmonic components. Meanwhile, the phase portrait in the \( v \) – \( i \) plane has a serious deviation from the circle. Fig. 14(c) shows that the locus of oscillation revolves around the NDR region, but it does not pass through the NDR region. It jumps from one PDR region to another one. In this case, the oscillation frequency is very low as compared with the frequency \( f_0 \). It follows that the effect of the parasitic capacitance at the operating point is very small, and the circuit can be equivalent to an N-shaped nonlinear resistor connected with an inductor under a DC bias. Based on the above analysis, the oscillation behavior belongs to the first-order relaxation oscillation as \( L \) is much greater than \( L_0 \).

More detailed simulation results are shown in Table 1, which contains the frequency \( f \), period \( T \), the peak-to-peak value of the memristor voltage \( v_{pp} \), the peak-to-peak value of the memristor current \( i_{pp} \), and the oscillation type for different inductances.

| \( L \)    | \( f \)  | \( T \)  | \( v_{pp} \) | \( i_{pp} \) | Oscillation type |
|-----------|----------|----------|--------------|--------------|-----------------|
| 40mH      | -        | -        | 0            | 0            | Convergence     |
| 45mH      | 9.61 kHz | 1.04 ms  | 0.76V        | 0.28 mA      | Sinusoid        |
| 90mH      | 6.67 kHz | 0.15 ms  | 4.69V        | 1.21 mA      | Non-sinusoid    |
| 1H        | 1.48kHz  | 0.67ms   | 6.30V        | 0.74mA       | First-order     |

TABLE 1. Simulation results of the oscillator.
The inductance is smaller than the oscillation inductance of the system is determined by the value of the inductor. When the inductance is smaller than the oscillation inductance \( L_0 \), the transient waveforms will converge to the equilibrium point. As the inductance is close to the oscillation inductance \( L_0 \), the sinusoidal oscillation is observed and the oscillation frequency is nearly equal to the calculated frequency \( f_0 \). When the inductance continues to increase, the oscillation behavior deviates from the normal sinusoidal oscillation and finally tends to the first-order relaxation oscillation.

Based on the above analysis, the simulation results are consistent with the theoretical analysis. When the inductance \( L \) increases from the oscillation inductance \( L_0 \), the system can generate the periodical oscillation behavior from the sinusoidal oscillation to the first-order relaxation oscillation. The oscillation frequency decreases with the inductance, and the amplitude of \( v \) increases at the same time. In addition, the current increases and then decreases.

**V. CIRCUIT IMPLEMENTATION**

In order to further verify the local-activity of the proposed N-shaped memristor and its application in the oscillator, a physical circuit implementation of the system is designed, as shown in Fig. 15. In addition, the photograph of experimental devices is shown in Fig. 16.

It can be seen from Fig. 15 that the proposed LAM is emulated by the analog circuit composed with off-the-shelf electronic devices, of which U1 is the operational amplifier TL084 (four-channel), U2 and U3 are the multipliers AD633, and U4 is the current conveyor AD844. The detailed descriptions of each part are given as follows.

**A. EMULATOR CIRCUIT OF THE N-SHAPED LAM**

1) **PART ①**

The second generation current conveyor AD844 in Part ① is used to transform the voltage \( v_0 \) into the current \( i \). Based on the Ohm’s law and the datasheet of AD844, it can be obtained as

\[
v_w = -R_{in}i
\]  

2) **PART ②**

Part ② involves one voltage follower, with the aim of preventing the input signal from being affected by the subsequent circuits. Hence, the output of U1A is equal to the memristor voltage, i.e.,

\[
v_1 = v
\]

3) **PART ③**

This part is used to implement the state differential equation in (8) and it consists of an inverting amplifier and an integrating circuit. Assuming that \( R_1 = R_2 \) and \( R_1 = R_s \), the output of U1C can be written as:

\[
v_x = \frac{1}{R_1C_I} \int (v - v_x) \, dt
\]

Based on (8) and (29), it can be deduced that

\[
\tau = R_1C_I \quad \quad (30a)
\]

\[
v_x = x \quad \quad (30b)
\]

4) **PART ④**

The calculation of the memductance is carried out in this part, which is composed of a multiplier U2 and an operational amplifier circuit. For the operational amplifier circuit U1D, the summing and subtraction operation can be done at the same time. The superposition theorem is used here to derive the output of U1D. When only one input signal is applied to the circuit, the \( v_G \) can be expressed as:

\[
v_{G1} = -\frac{R_P}{R_{d2}} 0.1v_x^2 \quad \text{only } 0.1v_x^2 \quad (31a)
\]

\[
v_{G2} = -\frac{R_P}{R_{d0}} V_s \quad \text{only } V_s \quad (31b)
\]

\[
v_{G3} = -\frac{R_0}{R_{d1} + R_0} \left( \frac{R_{d2}/R_{d0} + R_p}{R_{d2}/R_{d0}} \right) v_x \quad \text{only } v_x \quad (31c)
\]

Hence, the output of U1D can be represented as:

\[
v_G = v_{G1} + v_{G2} + v_{G3} = -\left( -\frac{R_P}{R_{d2}} 0.1v_x^2 - \frac{R_0}{R_{d1} + R_0} \left( \frac{R_{d2}/R_{d0} + R_p}{R_{d2}/R_{d0}} \right) v_x + \frac{R_P}{R_{d0}} V_s \right)
\]

where \( R_{d2}/R_{d0} = R_{d2}R_{d0}/(R_{d2} + R_{d0}) \).
Based on the voltage signals $v_G$ and $v_1$, the output of U3 can be obtained

$$v_w = \frac{R_w + R_2}{10R_w} v_{GV1}$$  \hspace{1cm} (33)

According to (27), (32) and (33), it can be obtained

$$i = \frac{R_w + R_2}{10R_w R_{in}} \left( \frac{R_p}{d_2} \cdot 0.1 v_x^2 - \frac{R_0}{d_1 + d_0} \right) v_x + \frac{R_0}{d_0} V_s$$ \hspace{1cm} (34)

By combining (8) with (34), the parameters $d_2$, $d_1$, and $d_0$ can be written as follows:

$$d_2 = \frac{R_w + R_2}{10R_w R_{in}} \cdot \frac{R_p}{10 d_2}$$

$$d_1 = \frac{R_w + R_2}{10R_w R_{in}} \cdot \frac{R_0}{d_1 + d_0} \left( \frac{R_{d2}/R_{d0} + R_p}{R_{d2}/R_{d0}} \right)$$

$$d_0 = \frac{R_w + R_2}{10R_w R_{in}} \cdot \frac{R_p}{d_0} V_s \hspace{1cm} (35)$$

Based on the presented LAM mathematical model, the parameter configurations of the LAM emulator circuit are shown in Table 2.

| Element | Value | Element | Value |
|---------|-------|---------|-------|
| $V_{CC}$ | +15V | $V_s$ | 15V |
| $V_{EE}$ | -15V | $R_p$ | 20kΩ |
| $R_s$ | 1kΩ | $R_0$ | 20kΩ |
| $R_1, R_2$ | 10kΩ | $R_{d0}$ | 26.4kΩ |
| $R_1$ | 10kΩ | $R_{d1}$ | 7.4kΩ |
| $C$ | 1nF | $R_{d2}$ | 12kΩ |
| $R_s$ | 10kΩ | $R_{in}$ | 10kΩ |

5) PART

Based on the voltage signals $v_G$ and $v_1$, the output of U3 can be obtained

$$v_w = \frac{R_w + R_2}{10R_w} v_{GV1} \hspace{1cm} (33)$$

According to (27), (32) and (33), it can be obtained

$$i = \frac{R_w + R_2}{10R_w R_{in}} \left( \frac{R_p}{d_2} \cdot 0.1 v_x^2 - \frac{R_0}{d_1 + d_0} \right) v_x + \frac{R_0}{d_0} V_s \hspace{1cm} (34)$$

By combining (8) with (34), the parameters $d_2$, $d_1$, and $d_0$ can be written as follows:

$$d_2 = \frac{R_w + R_2}{10R_w R_{in}} \cdot \frac{R_p}{10 d_2}$$

$$d_1 = \frac{R_w + R_2}{10R_w R_{in}} \cdot \frac{R_0}{d_1 + d_0} \left( \frac{R_{d2}/R_{d0} + R_p}{R_{d2}/R_{d0}} \right)$$

$$d_0 = \frac{R_w + R_2}{10R_w R_{in}} \cdot \frac{R_p}{d_0} V_s \hspace{1cm} (35)$$

Based on the presented LAM mathematical model, the parameter configurations of the LAM emulator circuit are shown in Table 2.

Based on (30a) and (35), as well as the parameter configurations shown in Table 2, it can be calculated that $\tau = 1 \times 10^{-3}$, $d_2 = 0.33 \times 10^{-4}$, $d_1 = -5 \times 10^{-4}$, and $d_0 = 22.75 \times 10^{-4}$, which are same with the theoretical design.

Before moving onto experimental results, it is noted that the current of memristor cannot be measured directly in the circuit and must be calculated based on available data. Since $v_w$ reflects multiplied $-i$ by $R_{in}$, the memristor current can be read as $i = -v_w/R_{in}$. Therefore, the voltage $-v_w$ is used to observe the current $i$. The signal $-v_w$ is obtained by inverting the captured signal $v_w$ in the digital oscilloscope. Additionally, the value of the state variable $x$ is represented by the voltage $v_x$.

**B. PINCHED HYSTERESIS LOOP**

When the applied signal $v(t)=2.5\sin(2\pi \times 1000t)$, the experimental voltage-current characteristics is shown in Fig. 17. Observe that the proposed emulator circuit of the LAM exhibits a PHL confined to the first and the third quadrants in the $v$-$i$ plane under a sinusoidal signal with zero mean.

**C. QUASI-STABLE VOLTAGE-CURRENT PLOT**

In order to evaluate the DC $V$-$I$ characteristics of the proposed emulator circuit for the LAM, the quasi-stable $v$-$i$ plot is measured experimentally. When a triangle voltage signal with the amplitude of 10V and a low frequency of 10Hz is applied in the emulator circuit of the N-shaped LAM, the transient waveforms of $v$ and $-v_w$ are captured, as shown in Fig. 18(a). Meantime, its voltage-current characteristics plot is depicted in Fig. 18(b).

Observe that the locus in Fig. 18(b) is same with the DC $V$-$I$ curve of the N-shaped LAM, which verifies the correctness and practicability of the proposed emulator circuit. The experimentally measured NDR region is $3.47V < V < 6.53V$ and the center of NDR is $(5.0V, 3.05mA)$, which are close to the theoretical calculation. The inevitable errors may be caused...
by the measurement and non-ideals of electronic devices. Noted that this locus is not a DC $V-I$ plot, but a quasi-stable $V-I$ plot. Since the frequency of excitation signal is sufficiently low, a quasi-stable $V-I$ plot is same with the DC $V-I$ plot.

D. OSCILLATING SYSTEM

Based on the presented N-shaped LAM emulator circuit, the circuit implementation of the oscillating system shown in Fig. 10 can be easily obtained. In order to bias the N-shaped LAM in the center of the NDR region, the input DC voltage is set a little bigger than that in the simulation, namely $V_D = 5.1V$. The reason is that the parasitic resistance of the inductor needs to be taken into account in the real circuit implementation. Due to the existence of the parasitic resistance of the inductor, internal resistance of the voltage supply, and wire resistances, the small-signal equivalent conductance of the LAM should be smaller than zero for guarantee the equivalent conductance of the whole circuit is equal to zero. According to Fig. 6(b), it can be seen that the frequency will become smaller compared with $\omega_0$ as $G' < 0$. Hence, we can predict that the actual oscillation frequency is smaller than the theoretical value. In addition, an extra small capacitance 1nF connected with the LAM emulator circuit in parallel is used to avoid to generate the self-sustained oscillation originating from operational amplifier cascade.

When the inductance ($L = 40mH$) is smaller than the oscillation inductance $L_0$, the experimental results are shown in Fig. 19. It can be seen that the transient waveforms asymptotically converge to the equilibrium states $v = 5V$, $v_x = 5V$, and $-v_w = 3.12V$, which are consistent with the simulation results.

As the inductance $L$ is 42.9mH, the experimental waveforms and phase portraits of the oscillating circuit are shown in Fig. 20. The time-domain waveforms of the voltage $v$, $v_x$, and $-v_w$ are shown in Fig. 20 (a). The phase portrait of $v_x$ vs. $-v_w$ is depicted in Fig. 20 (b). Fig. 20 (c) shows the memristor voltage $v$ varies with the voltage $-v_w$, which represents the phase portrait in the $V-I$ plane.

Observe from Fig. 20 (a) that the system can oscillate continuously, and the time-domain waveforms resemble the sinusoidal signal. However, the inductance value is smaller than the theoretical value of $L_0$. This is because that the change of $f_0$ and an extra small capacitance may lead to a minor change in the real value of $L_0$. The stable limit cycle in the $V-I$ plane is similar to a circle, which matches well with the simulation results.

When the inductance $L$ is 82.5mH which is bigger than the oscillation inductance $L_0$, the experimental time-domain waveforms and phase portraits of the oscillating circuit are shown in Fig. 21. The time-domain waveforms of the voltage $v$, $v_x$, and $-v_w$ are shown in Fig. 21 (a). The phase portrait of $v_x$ vs. $-v_w$ is depicted in Fig. 21 (b). Fig. 21 (c) shows the memristor voltage $v$ varies with the voltage $-v_w$. 

![FIGURE 19. Experimental results of the oscillating system at $V_D = 5.1V$ and $L = 40mH$.](image1)

![FIGURE 20. Experimental results of the oscillating system at $V_D = 5.1V$ and $L = 42.9mH$: (a) transient waveforms of $v$, $v_x$, and $-v_w$, (b) phase trajectory of $v_x$ vs. $-v_w$, (c) $v$ changes as a function of $-v_w$.](image2)
FIGURE 21. Experimental results of the oscillating system at $V_D = 5.1\, \text{V}$ and $L = 82.5\, \text{mH}$; (a) transient waveforms of $v$, $v_x$, and $-v_w$, (b) phase trajectory of $v_x$ vs. $-v_w$, (c) $v$ changes as a function of $-v_w$.

FIGURE 22. Experimental results of the oscillating system at $V_D = 5.1\, \text{V}$ and $L = 1\, \text{H}$; (a) transient waveforms of $v$, $v_x$, and $-v_w$, (b) phase trajectory of $v_x$ vs. $-v_w$, (c) $v$ changes as a function of $-v_w$.

It can be seen from Fig. 21 that the time-domain waveforms of $v$, $v_x$, and $-v_w$ are the distortion of the sinusoidal signals and the stable limit cycle in the $V-I$ plane has deviated from the circle.

As the inductance ($L = 1\, \text{H}$) is much bigger than the oscillation inductance $L_0$, the experimental time-domain waveforms and phase portraits of the oscillating circuit are shown in Fig. 22. The time-domain waveforms of the voltage $v$, $v_x$, and $-v_w$ are shown in Fig. 22 (a). The phase portrait of $v_x$ vs. $-v_w$ is depicted in Fig. 22 (b). Fig. 22 (c) shows the memristor voltage $v$ varies with the voltage $-v_w$.

Observe from Fig. 22 that there exists jump behavior in the time-domain waveform of $v$, which indicates that there are many higher-order harmonic components. At this case, the oscillation behavior approximately belongs to the relaxation oscillation.

More detailed experimental results are shown in Table 3, which contains the frequency $f$, period $T$, the peak-to-peak value of the memristor voltage $v_{p-p}$, the peak-to-peak value of the memristor voltage $i_{p-p}$, and the oscillation type for different inductances. The good agreement between the experimental results and simulation results further demonstrates...
the correctness and feasibility of the theoretical design and analysis.

VI. CONCLUSION

Based on Chua’s unfolding theorem, a simple voltage-controlled locally-active memristor (LAM) mathematical model is proposed, which makes for a deep insight into the mechanisms at the origin of locally-active memristor based oscillator system. Due to there is only one stable point when the power supply is off, the presented N-shaped LAM is volatile. Additionally, \( v-I \) characteristics and DC \( V-I \) curve of the LAM indicate that it exhibits the pinched hysteresis loop which is dependent on the frequency in the \( v-I \) plane and possesses a NDR region, i.e., locally-active region, over the specific voltage range \( 3.5V < V < 6.5V \) in its DC \( V-I \) curve.

The small-signal equivalent circuit shows that the N-shaped LAM is equivalent to a conductance with a parasitic resistor and the second order system fades to the first order turns into a multiresonant oscillation. When the inductance is equal zero at the specific frequency \( f_0 \), the capacitance along with an inductor can form the LC resonance. Hence, the N-shaped LAM based oscillator includes an N-shaped LAM, an inductor, and a DC bias.

Simulation and experimental results of the N-shaped LAM based oscillator show that as the value of inductance is equal to \( L_0 \), the system generates the sinusoidal oscillation waveforms. As the inductance increases, oscillation behavior of the system deviates from the sinusoidal signal gradually and turns into a multiresonant oscillation. When the inductance is much bigger than \( L_0 \), the oscillation frequency is rather low and the N-shaped LAM behaves as an N-shaped nonlinear resistor and the second order system fades to the first order one. This work can offer a theoretical foundation for the research in the nanoscale LAMs.

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