An Emergence Analysis Method for System of Systems Capability Items Based on Soft Set

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Abstract. Oriented emergence new capability items for system of systems (SoS), a measurement model for SoS capability items emergence was proposed in this paper. The soft set which has obvious advantages in dealing with the uncertainty problem based on the nature of its data, as well as measuring attribute significance and knowledge dependence. The decision division between component objects set of SoS and capability items set has been defined by the adopt soft set. The importance of the emerging capability items in the overall SoS is analyzed through changes in decision division in order to obtain the measurement of SoS capability items emergence, then the measurement method for SoS capability items emergence is formed. A case study is then conducted to verify the effectiveness of method.

Introduction

The concept of “System of Systems” (SoS) has been around for a long time [1]. Scholars and organizations in many different fields have proposed definitions of SoS based on their respective backgrounds and perspectives, but none of these definitions have been universally accepted. SoS is a collection of connected systems that communicate and cooperate with each other, for example “C4I” (Command, Control, Computers, Communications and Information) and ISR (Intelligence, Surveillance and Reconnaissance) systems [2]. SoS is an integration of systems which aims to use the evolution, coordination and optimization of those systems to ultimately achieve the purpose of improving overall efficiency [3]. SoS is the integration of collaborative systems. Its components have two additional characteristics: operational autonomy and management autonomy [4,5]. As pointed out by the United States Department of Defense, “The combined links of interdependent systems provide far greater capabilities than the sum of the capabilities of individual systems” [6]. According to this definition, the systems are linked by a network of member systems distributed in different locations, thereby forming a complex organic whole.

The holistic behavior of SoS during the execution of functions to achieve its objectives is not available to or cannot be exhibited by its constituent parts (components). This is the “emergence” characteristic of SoS. Such emergence can only be observed in the SoS as a whole, and is not available to local member systems. The SoS emergence can be divided into two types. One is emergence new capability (or property) items of SoS because have new properties or new functions, and changes in original functions or attributes. Another kind of emergence is not a change in SoS capability (or performance) item and its structure, but a sudden change in the value of capability items. It is difficult to predict, restore and trace SoS emergence, but such emergence can be measured after the SoS is constructed. The measurement of SoS emergence can reversely analyze and trace the mechanisms of SoS emergence, providing directions for SoS design, integration and verification. In this paper, oriented emergence new capability items for SoS, a measurement model for SoS capability items emergence was proposed. The soft set which has obvious advantages in dealing with the uncertainty problem based on the nature of its data, as well as measuring attribute significance and knowledge...
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**Soft Set Theory [7]**

In practical terms, there is a lot of uncertainty information contained in things, which cannot be directly measured by specific values. Molodtsov proposed a method to deal with such uncertainty information: the soft set. Widely used in solving problems of uncertainty, this method is characterized by its simple operation and easily understandable nature. It is more intuitive in its representation and can provide a satisfying description of the relationships between the parts and the whole in a system.

**Definition 1.** Assume \( U \) is a finite non-empty set called a “scheme set” and \( E \) is a finite non-empty set called a “parameter set”. \( A \subseteq E \), ordered pair \((F, A)\) is called a “soft set” when (and only when) \( F \) is the mapping from \( A \) to the power set on set \( U \), namely \( F : A \rightarrow P(U) \).

According to the definition, soft set \((F, A)\) on a domain of \( U \) can be considered a parameter family of a subset of domain \( U \), and a soft set gives an approximation of domain \( U \) (soft description). For any parameter, \( e \in A \), \( F(e) \subseteq U \) can be regarded as \( e \) - approximate set in a soft set \((F, A)\).

The following example is a more in-depth explanation of the definition of a soft set.

**Example 1.** Assume that a SoS contains a capability items set \( \{h_1, h_2, h_3, h_4\} \), and for these capability items, the component sets contained in this SoS are \( E = \{e_1, e_2, e_3, e_4, e_5, e_6\} \), assuming \( F(e_1) = \{h_1, h_2\} \), \( F(e_2) = \{h_1, h_2, h_3\} \), \( F(e_3) = \{h_1, h_3\} \), \( F(e_4) = \{h_1, h_2, h_4\} \), \( F(e_5) = \{h_1\} \) and \( F(e_6) = \{h_2, h_3, h_4\} \), in which \( F \) represents the mapping of component set \( E \) to all subsets on domain \( U \), and soft set \((F, A)\) describes the advantages of these 4 capability items relative to SoS.

In order to better represent a soft set, scholars have proposed a variety of different representation methods to adapt to different scenarios. Graphic representation is proposed by Maji et al [8]. Take Example 1 the purpose of formatting and storing data is shown in Table 1.

| \( U \) | \( e_1 \) | \( e_2 \) | \( e_3 \) | \( e_4 \) | \( e_5 \) | \( e_6 \) |
|--------|----------|----------|----------|----------|----------|----------|
| \( h_1 \) | 1        | 1        | 1        | 1        | 1        | 0        |
| \( h_2 \) | 1        | 1        | 0        | 1        | 0        | 1        |
| \( h_3 \) | 0        | 0        | 1        | 1        | 0        | 1        |
| \( h_4 \) | 0        | 1        | 0        | 0        | 0        | 1        |

It is difficult to predict, restore or trace SoS emergence. During the evaluation and measurement of SoS emergence, decision-makers will encounter considerable uncertainty information. Data-oriented soft set theory can analyze and measure SoS emergence more reasonably and effectively.

**Measurement Model for SoS Capability items Emergence**

The emergence of SoS capability means the emergence of new capability items after SoS establishment which are not included among the original SoS components. The emergence of SoS capability items can be measured by analyzing the importance of the capability items to the overall SoS.
Definition of SoS Capability Items Emergence

In order to discuss the new capability items emerging of SoS, the following definitions are given:

Definition 2. For one soft set \((F,A)\) and object set \(U=\{h_1, h_2, \ldots, h_n\}\), \(h_1, h_2, \ldots, h_n\) is capability items. The value of \(f_E(\ast)\) shall be calculated according to the indiscernibility relation, and data objects in universal set \(U\) are divided and ranked according to this value to obtain \(C_E=\left\{\{h_1, h_2, \ldots, h_i\}_f \mid \{h_{i+1}, \ldots, h_j\}_f, \ldots, \{h_k, \ldots, h_n\}_f\right\}\). It is defined as a “decision division”.

For a soft set \((F,A)\) components set \(E=\{e_1, e_2, \ldots, e_m\}\) and capability sets \(U=\{h_1, h_2, \ldots, h_n\}\) and \(C_E=\left\{\{h_1, h_2, \ldots, h_i\}_f \mid \{h_{i+1}, \ldots, h_j\}_f, \ldots, \{h_k, \ldots, h_n\}_f\right\}\) belong to one decision division. If \(e_i\) is deleted from the parameter set, the decision division may be changed and expressed as:

\[
C_{E-e_i}=\left\{\{h_1, h_2, \ldots, h_i\}_f \mid \{h_{i+1}, \ldots, h_j\}_f, \ldots, \{h_k, \ldots, h_n\}_f\right\}
\]

For convenience, we use \(C_E=\{E_{f_1}, E_{f_2}, \ldots, E_{f_l}\}\) and \(C_{E-e_i}=\{E-e_{f_1}, E-e_{f_2}, \ldots, E-e_{f_l}\}\) to indicate the above results.

Definition 3. For a soft set, the related components set \(E\) is \(\{e_1, e_2, \ldots, e_m\}\), and object set \(U=\{h_1, h_2, \ldots, h_n\}\) represents various capability items. The decision division of such a soft set and the decision division after deleting a component are respectively \(C_E=\{E_{f_1}, E_{f_2}, \ldots, E_{f_l}\}\) and \(C_{E-e_i}=\{E-e_{f_1}, E-e_{f_2}, \ldots, E-e_{f_l}\}\), and the importance of \(e_i\) to the decision division is defined as:

\[
r_{e_i} = \frac{1}{U} \left\{ \alpha_{1,e_i} + \alpha_{2,e_i} + \cdots + \alpha_{s,e_i} \right\}
\]

\[
\alpha_{k,e_i} = \begin{cases} 
|E_{f_k} - E-e_{f_k}|, & \text{if the presence of } z' \text{ lead to } f_k=f_{k'}, 1 \leq z' \leq s', 1 \leq k \leq s \\
|E_{f_k}|, & \text{other}
\end{cases}
\]

Among them, \(|\cdot|\) represents the cardinality of the set.

Assume that the SoS should have a specific set of capability items \(U=\{h_1, \ldots, h_n\}\), then component importance reflects that the SoS shall have impacts of knowledge (i.e. component) \(E\) on the overall capabilities of the SoS based on capability items set \(U\). The component importance has a maximum value of 1. If the component importance approximates 1, this indicates that the component is of greater importance to the SoS.

Definition 4. The SoS capability items emergence measurement is:

\[
\eta_{e_i} = \frac{r_{e_i}}{\sum r_{e_i}}
\]

Measurement Process of SoS Capability Items Emergence

The new nature and new function lead to a new capability items after SoS establishment. The emergency is analyzed by changes in the decision division of the capability items set in soft set theory.

We have defined the component set contained in the SoS as \(E=\{e_1, e_2, \ldots, e_m\}\) an object set \(U=\{h_1, h_2, \ldots, h_n\}\) as a set of various capability items. The measurement of SoS capability items emergence is analyzed through the following steps:

Step 1: A component-capability items data sheet is established according to the evaluation results of evaluation subjects for each capability item, in which the evaluation object is the capability items of each SoS component and the evaluation result is the component capability items. If such a
capability item is not available, the value of capability item will be 0. If that capability is available, the range of capability value will be $[1, 5]$. Among them, 1 indicates that the device has little capability while 5 indicates that it has great capability. **Step 2:** First, the component-capability items data sheet on the decision division of capability items set $E$ is obtained; that is, the decision division after additional capability items due to emergence. Then, the decision division after removing one emerging capability items from set $E$ will be obtained in order; that is, the overall SoS did not have such a capability in its internal nature before the emergence. The importance of $e_i$ to the decision division is obtained according to Definition 3.

**Step 3:** The measurement of SoS capability items emergence shall be obtained according to Definition 4.

Through the above process, the measurement of SoS capability items emergence can be analyzed reasonably and effectively through changes in the data related to various SoS capability items set.

### Case Analysis

In order to illustrate the effectiveness of the proposed method, the SoS for aircraft carrier formation in attack missions against sea targets is taken as an example for analysis. The SoS consists of different weapon sets, as shown in Figure 2 below.

![Figure 1. Composition diagram of SoS for attack mission against sea targets.](image)

**Table 2. Weapon capabilities data sheet with $e_6$.**

| Capability | Weapons | $f(\bullet)$ |
|------------|---------|-------------|
|            | $e_1$   | $e_2$ | $e_3$ | $e_4$ | $e_5$ | $e_6$ | $f(\bullet)$ |
| $A_2$      | 2       | 0     | 4     | 0     | 0     | 4     | 10          |
| $B_1$      | 1       | 0     | 4     | 0     | 0     | 3     | 8           |
| $C_2$      | 2       | 3     | 0     | 5     | 4     | 0     | 14          |
| $D_0$      | 0       | 0     | 0     | 4     | 3     | 0     | 7           |
| $E_1$      | 1       | 1     | 3     | 0     | 0     | 3     | 8           |
| $F_1$      | 1       | 3     | 5     | 0     | 1     | 4     | 14          |
| $G_2$      | 2       | 3     | 2     | 0     | 2     | 3     | 14          |
| $H_0$      | 0       | 3     | 0     | 4     | 0     | 3     | 10          |

For this SoS, the set $E = \{e_1, e_2, \cdots, e_n\}$ is a set of weapons that constitutes the SoS. Table 2 is a weapon capability items data sheet established in accordance with the evaluation results given by the evaluation subjects for each capability, and the evaluation objects are capabilities of the weapons under SoS.
In table 2, A represents detection and tracking guidance capability, B represents command coordination control capability, C represents firepower striking capability, D represents protection and survival capability, E represents communication transmission capability, F represents sustained combat capability, G represents rapid response capability and H represents ultra-long-range attack capability.

In table 2, if early warning aircraft (e6) is removed from the SoS, it loses its ultra-long-range attack capability. Table 3 is the weapon capability items data sheet established according to the evaluation results of the evaluation objects for each capability after the early warning aircraft is removed from the SoS.

**Table 3. Weapon capabilities data sheet without e6.**

| Capability | Weapons | e1 | e2 | e3 | e4 | e5 | f(●) |
|------------|---------|----|----|----|----|----|------|
| A          |         | 2  | 0  | 4  | 0  | 0  | 6    |
| B          |         | 1  | 0  | 4  | 0  | 0  | 5    |
| C          |         | 2  | 3  | 0  | 5  | 4  | 14   |
| D          |         | 0  | 0  | 0  | 4  | 3  | 7    |
| E          |         | 1  | 1  | 3  | 0  | 0  | 5    |
| F          |         | 1  | 3  | 5  | 0  | 1  | 10   |
| G          |         | 2  | 3  | 2  | 0  | 2  | 9    |

It can be concluded from the above table that when the early warning aircraft is present in the SoS, not only do the early warning aircraft in the SoS have the corresponding capabilities but the attack aircraft and missiles also gain ultra-long-range attack capability. Therefore, H can be considered a capability item which emerged after the SoS was established. The decision division of table 2 is as follows:

\[ C_e = \{(C, F, G)_{14}, (A, H)_{10}, (B, E)_{8}, (D)_{7}\} \]

\[ C_{E-e_1} = \{(F)_{13}, (C, G)_{12}, (H)_{10}, (A)_{8}, (B, D, E)_{7}\} \]

\[ C_{E-e_2} = \{(C, F, G)_{14}, (A)_{10}, (B)_{8}, (D, E, H)_{7}\} \]

\[ C_{E-e_3} = \{(C, G)_{12}, (H)_{10}, (F)_{9}, (D)_{7}, (A)_{6}, (E)_{5}, (B)_{4}\} \]

\[ C_{E-e_4} = \{(F, G)_{14}, (A)_{10}, (C)_{8}, (B, E)_{6}, (H)_{6}, (D)_{5}\} \]

\[ C_{E-e_5} = \{(F)_{13}, (G)_{12}, (A, C, H)_{10}, (B, E)_{8}, (D)_{7}\} \]

\[ C_{E-e_6} = \{(C)_{14}, (F)_{10}, (G)_{7}, (D)_{5}, (A)_{6}, (B, E)_{6}, (H)_{9}\} \]

According to the above classification results, the importance \( e_6 \) can be obtained as:

\[ r_{e_6} = \frac{1}{8} \times (2 + 2 + 2 + 0) = \frac{6}{8} \].

Therefore, the measurement results of emergence after the early warning aircraft is added to the SoS are as:

\[ \eta_{e_6} = \frac{r_{e_6}}{\sum r_{e_i}} = \frac{6 / 8}{29 / 8} = \frac{6}{29} \].

In fact, during the interaction of SoS components, new properties and functions are likely to emerge. The above cases show that the importance of capability items to the overall SoS can be analyzed through changes in the data related to various SoS capability items, thereby enabling the measurement of SoS capability emergence.
Summary

There is emergence characteristic of SoS, and such emergence can only be observed in the SoS as a whole. The emergence of SoS is difficult to predict, restore and trace SoS emergence, but it can be measured after the SoS is constructed. In this paper, a measurement model for SoS capability items emergence was proposed. The soft set which has obvious advantages in dealing with the uncertainty problem based on the nature of its data, as well as measuring attribute significance and knowledge dependence. The decision division between component object set of SoS and capability items set has been defined by the adopt soft set. The importance of the emerging capability items in the overall SoS is analyzed through changes in decision division in order to obtain the measurement of SoS capability items emergence, then the measurement method for SoS capability items emergence is formed. The measurement method of SoS emergence can reversely analyze and trace the mechanisms of SoS emergence, providing directions for SoS design, integration and verification.

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