Updates of $W_R$ Effects on $CP$ Angle Determination in $B$ Decays

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The recently observed $CP$ violation in $B$ decay, and $B$-$\bar{B}$ mixing data put constraints on the mass of $W_R$ and the parameters of the right-handed current mixing matrix $V^R$ in the $SU(2)_L \times SU(2)_R \times U(1)$ gauge model. It is shown that the allowed region of parameters is severely restricted for light $W_R$ with mass of about 1 TeV or less. There exist sets of parameters that can account for the large $CP$ violation measured by Belle, $\sin 2\phi_1|_{\text{exp}} \simeq 1$, for $M_{W_R} = 1 – 10$ TeV.

The B factories at KEK and SLAC have established the existence of $CP$ violation in $B$ meson systems by measuring the time-dependent $CP$ asymmetry of neutral $B$ meson decays into $(c\bar{c})$ meson+neutral $K$($^*$) meson.

\[
A(t) = \frac{\Gamma[B^0(t) \rightarrow f_{CP}] - \Gamma[B^0(t) \rightarrow \bar{f}_{CP}]}{\Gamma[B^0(t) \rightarrow f_{CP}] + \Gamma[B^0(t) \rightarrow \bar{f}_{CP}]} = -\xi_f \sin 2\phi_1 \sin(\Delta M_B t), \tag{1}
\]

where $\xi_f$ is the $CP$ eigenvalue of the final state. The following values have been obtained: $^2,3$)

\[
\sin 2\phi_1 = \begin{cases} 0.59 \pm 0.14 \pm 0.05, & \text{(BABAR)} \\ 0.99 \pm 0.14 \pm 0.06, & \text{(Belle)} \end{cases} \tag{2}
\]

We wish to check if the above values are consistent with the 3-generation standard model with the Kobayashi-Maskawa mechanism of $CP$ violation. $^4$) With the notation of the unitarity triangle given in Fig. 1 where $V_{cb}^*V_{cd}$ is a negative real value, the geometrically defined $\sin 2\phi_1$ is given as

\[
\sin 2\phi_1 = \sin \left(2\pi - 2 \arg \left[-\frac{V_{ub}^*V_{ud}}{V_{cb}^*V_{cd}}\right]\right) = \sin \left(2 \arg \left[-1 + \left|\frac{V_{ub}^*V_{ud}}{V_{cb}^*V_{cd}}\right| e^{-i\phi_3}\right]\right). \tag{3}
\]

If no new physics beyond the 3-generation standard model enters into the measured processes of $CP$ violation, the observed $\sin 2\phi_1$ should agree with the above geometrically defined one. Assuming that any new physics does not affect the values of $|V_{ud}|$, $|V_{cd}|$ and $|V_{ub}/V_{cb}|$, which are obtained through tree-level semi-leptonic processes, the

Fig. 1. Unitarity triangle.

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prediction of $\sin 2\phi_1$ in terms of $\phi_3$ using Eq. (3) and $|(V_{ub}^*V_{ud})/(V_{cb}^*V_{cd})|$ is given as Fig. 2. The measured result $\sin 2\phi_1 > 0.4$ is consistent with $\phi_3 = 15^\circ - 145^\circ$. The neutral $B$ meson mass difference $\Delta M_B$ in the standard model is also estimated as a function of $\phi_3$. Once $|V_{ub}/V_{cb}|$ is given, $V_{tb}V_{td}^*$ can be expressed in terms of $\phi_3$ and $|V_{ub}/V_{cb}|$ by using unitarity. If we take the error of $|V_{ub}/V_{cb}|$ into account, we find that the standard model is consistent with $\Delta M_B$ for $\phi_3 = 20^\circ - 70^\circ$ as shown in Fig. 3. The measured values of $\sin 2\phi_1$ are consistent with the 3-generation standard model, considering the experimental errors, though the central value given by Belle cannot be realized in the standard model. If the large value found by Belle is confirmed in future experiments, we can conclude that some new physics beyond the standard model is necessary.

In this work we investigate the $SU(2)_L \times SU(2)_R \times U(1)$ model ($L-R$ model)$^5$ as a possible candidate of new physics that can give a larger $CP$ asymmetry in the $\sin 2\phi_1$ determination than in the standard model. We investigate constraints on the model and explore the possibility of obtaining a large value of $CP$ asymmetry, simultaneously satisfying the constraints required by $\Delta M_B$ and $K-K$ system. Some groups, including one of the present authors, have investigated the $L-R$ model and showed that the gauge boson coupled to the right-handed charged current ($W_R$) can significantly affect the values of $CP$ violation angles in $B$ decays.$^6$-$^9$ The essence of this effect is as follows. Though $W_R$ is much heavier than the ordinary $W$ boson, some elements of the right-handed current quark mixing matrix $V^R$ are not necessarily suppressed in comparison with the CKM mixing matrix elements.$^4$-$^{10}$ Then, $W_R$ can contribute significantly to some processes like $B-\bar{B}$ mixing, where the ordinary $W$ boson contribution is significantly CKM suppressed.

There exists a sizable contribution to $K-K$ mixing in the $L-R$ model from the box diagram with one $W$ and one $W_R$ exchange,$^6$-$^{11}$-$^{13}$ which allows only the following forms of $V^R$ to avoid the constraint arising from $CP$ violation in $K-K$

\begin{align*}
\sin 2\phi_1
\end{align*}

Fig. 2. $\sin 2\phi_1$ in the 3-generation standard model. Upper, middle and lower curves correspond to $|V_{ub}/V_{cb}| = 0.11$, 0.09 and 0.07, respectively.

\begin{align*}
\Delta M_B^{SM} / \Delta M_B^{exp}
\end{align*}

Fig. 3. $\Delta M_B^{SM} / \Delta M_B^{exp}$ in the 3-generation standard model. Upper, middle and lower curves correspond to $|V_{ub}/V_{cb}| = 0.07$, 0.09 and 0.11, respectively.
mixing for $W_R$ with a mass of $O(1)$ TeV or less:\(^8\)

$$V^R_I = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\omega} \end{pmatrix}, \quad V^R_{II} = \begin{pmatrix} 0 & 1 & 0 \\ \cos \theta_R & 0 & -e^{i\omega} \sin \theta_R \\ \sin \theta_R & 0 & e^{i\omega} \cos \theta_R \end{pmatrix}. \quad (4)$$

(If we allow fine tunings among the parameters of the CKM matrix and of $V^R$, there are other possibilities, which we do not consider here.) The former, $V^R_I$, does not give a significant contribution to either $B$-$\bar{B}$ mixing or $b$ decay, and therefore we concentrate on the latter type of $V^R$ here.

The contribution to $B$-$\bar{B}$ mixing can be written as

$$M^B_{12} = M^{{SM}}_{12} + M_{LR} + M_{RR}, \quad (5)$$

where $M^{{SM}}_{12}$ is the standard model contribution, $M_{LR}$ is from the box diagram with one $W$ and one $W_R$ exchange, and $M_{RR}$ is from two $W_R$ exchange. $M_{RR}$ is obtained simply by exchanging $L$ and $R$ in the standard model contribution. $M_{LR}$ is calculated from the following effective Hamiltonian: \(^7\), \(^15\), \(^16\)

$$\mathcal{H}_{LR}^{\text{eff}} = \sum_{i,j=\mu}^{\tau} \frac{2G_F^2 M_W^2}{\pi^2} \beta_y V_{id}^L V_{ib}^R V_{jd}^R V_{jb}^L J(x_i, x_j, \beta) \frac{d_R b_L d_L b_R^*}{d_R b_L d_L b_R} + (\text{h.c.}). \quad (6)$$

Here, $\beta = (M^2_W/M^2_{W_R})$, $\beta_y = (g_R/g_L)^2 \beta$ and $x_i = m^2_i/M^2_W$. The loop function is defined as

$$J(x, y, \beta) \equiv \sqrt{xy} \left[ \frac{(\eta^{(1)} + \eta^{(2)} xy \beta^2)}{4} J_1(x, y, \beta) - \frac{1}{4} (\eta^{(3)} + \eta^{(4)} \beta) J_2(x, y, \beta) \right], \quad (7)$$

with

$$J_1(x, y, \beta) = \frac{x \ln x}{(1-x)(1-x\beta)(x-y)} + (x \leftrightarrow y) - \frac{\beta \ln \beta}{(1-\beta)(1-x\beta)(1-y\beta)};$$

$$J_2(x, y, \beta) = \frac{x^2 \ln x}{(1-x)(1-x\beta)(x-y)} + (x \leftrightarrow y) - \frac{\ln \beta}{(1-\beta)(1-x\beta)(1-y\beta)};$$

where $\eta^{(1)} - \eta^{(4)}$ are QCD corrections. We use here $\eta^{(1)} = 1.1$, $\eta^{(2)} = 0.26$, $\eta^{(3)} = 1.1$ and $\eta^{(4)} = 1.0$ as the values of the QCD corrections.\(^{14}\)

Now we evaluate $M_{12}^B$, varying $\theta_R$ and $\omega$ in $V^R$ with the following inputs: $M(W_R) = 1 - 10$ TeV; $\phi_3$ in $V_{KM}$ is $45^\circ$, $90^\circ$, $135^\circ$; $|V_{ub}/V_{cb}| = 0.09$; $f_B \sqrt{B_B} = 230$ MeV. We take $g_L = g_R$ for simplicity. Then, we draw regions allowed by the experimental values of $\Delta M_B$, allowing $\pm30\%$ ambiguity from errors on $f_B \sqrt{B_B}$ and $|V_{ub}/V_{cb}|$, and estimate the $CP$ asymmetry in $B \rightarrow (c\bar{c}) + K^{(*)}$ corresponding to $\sin 2\phi_1$, which we call $\text{Asy}(\Psi K)$. First, we set $M_{W_R} = 1$ TeV and $\phi_3 = 135^\circ$. The allowed region and the predicted $\text{Asy}(\Psi K)$ are shown in Fig. 4. It can be seen that only small portions of the parameter space in $\theta_R$ and $\omega$ are allowed by $\Delta M_B$. We fix $\theta_R = 100^\circ$ and estimate $\Delta M_B$ and $\text{Asy}(\Psi K)$. The result is shown in Fig. 5. With $\theta_R = 100^\circ$, the $CP$ phase $\omega$ in $V^R$ is restricted to the range $30^\circ - 90^\circ$. If we further
impose $\text{Asy}(\Psi K) > 0.4$ (from a recent measurement), $\omega$ should be less than $60^\circ$. It is interesting that the large $CP$ asymmetry found by Belle, $\text{Asy}(\Psi K) \sim 1$, is possible for $\omega = 30^\circ - 45^\circ$. Similar figures for $\phi_3 = 90^\circ$ and $45^\circ$ are given in Figs. 6 and 7. The allowed region of $\theta_R$ and $\omega$ is severely restricted. This is because $W_R$ gives a significant contribution to $B^{-}\bar{B}$ mixing even for $M_{W_R} \sim 1$ TeV, as pointed out in Ref. 8). The standard model contribution $M_{12}^{\text{SM}}$ is CKM suppressed by a factor of $\lambda^6$ ($\lambda \equiv |V_{us}| = 0.22$), while $M_{LR}$ is suppressed by a factor of $\lambda^3$. Though another suppression of $(M_W/M_{W_R})^2$ is incorporated in $M_{LR}$, the enhancement in loop function and $\lambda^{-3}$ factor cause $M_{LR}$ and $M_{12}^{\text{SM}}$ to be of the same order of magnitude.

We have carried out the same calculations for $M_{W_R} = 2, 3, 5$ and 10 TeV. The results are displayed in Figs. 8 – 16 and 18. The area of the allowed region becomes maximal at $M_{W_R} = 3$ TeV for $\phi_3 = 135^\circ$. The standard model contribution $M_{12}^{\text{SM}}$ alone cannot give a value of $\Delta M_B$ consistent with the experimental data for $\phi_3 = 135^\circ$. With a suitable magnitude of the $W_R$ contribution, we can obtain a value of $\Delta M_B$ consistent with the experimental value. A too heavy $W_R$ cannot give a

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**Fig. 4.** Region allowed by $\Delta M_B$ (left) and $\text{Asy}(\Psi K)$ (right) for $M_{W_R} = 1$ TeV and $\phi_3 = 135^\circ$. The black regions are consistent with the experimental value of $\Delta M_B$ in the left figure.

**Fig. 5.** $\Delta M_B|_{\text{theory}}/\Delta M_B|_{\text{exp}}$ and $\text{Asy}(\Psi K)$ for $M_{W_R} = 1$ TeV, $\phi_3 = 135^\circ$ and $\theta_R = 100^\circ$. 

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Fig. 6. Same as Fig. 4 for $\phi_3 = 90^\circ$.

Fig. 7. Same as Fig. 4 for $\phi_3 = 45^\circ$.

Fig. 8. Same as Fig. 4 for $M_{WR} = 2$ TeV and $\phi_3 = 135^\circ$.

sufficient contribution to compensate for $M_{12}^{SM}$. A similar situation occurs for $\phi_3 = 90^\circ$ at larger $M_{WR}$. No allowed region remains at $M_{WR} = 10$ TeV for $\phi_3 = 90^\circ$ and $135^\circ$.

The allowed region spreads as $M_{WR}$ becomes larger for $\phi_3 = 45^\circ$, since the standard model contribution $M_{12}^{SM}$ alone gives values of $\Delta M_B$ and $\text{Asy}(\Psi K)$ consistent
with experimental data for $\phi_3 = 45^\circ$. It is interesting that allowed regions which give large CP asymmetry remains even for heavy $W_R$. For example, the figure for $M_W = 5$ TeV, $\phi_3 = 45^\circ$ and $\theta_R = 30^\circ$ is shown in Fig. 17, and the figure for $M_W = 10$ TeV, $\phi_3 = 45^\circ$ is shown in Fig. 18.

Let us comment on other CP angles, $\phi_2$ and $\phi_3$. $\phi_2$ is measured in the CP asymmetry of $B \to \pi\pi$ decay. CP violation occurs through the interference among $B-\bar{B}$ mixing, tree decay and penguin decay of $b \to u\bar{d}d$. $W_R$ can contribute significantly to $B-\bar{B}$ mixing, as in the case of $\phi_1$. There also exists a contribution to $b \to d$ penguin decay. The ratio to the standard model penguin, up to a log loop function,
is given as

$$\frac{g_L^2}{M_W^2} V_{tb}^* V_{td} : \frac{g_R^2}{M_{WR}^2} V_{tb}^R V_{td}^R = 1 : \beta_g e^{-i\omega} \sin 2\theta_R$$

The magnitude of $|\beta_g/(2V_{tb}^*V_{td})|$ is about 0.3 for $M_{WR} = 1$ TeV. The $W_R$ penguin is less than 10% of the standard model one, taking the allowed region of $\theta_R$ into account. Therefore we can neglect the $b \to d \ W_R$ penguin. Then, the effect on $\phi_2$ is the same as that of $\phi_1$. If $\phi_3$ is measured by using $B^\pm \to DK$ decays, $CP$ violation occurs through the interference between tree decays, $\bar{b} \to \bar{c}u\bar{s}$ and $\bar{b} \to \bar{u}c\bar{s}$, with a
common final state. \( W_R \) does not contribute to \( \bar{b} \to \bar{u}c\bar{s} \), as \( V_{ub}^R = 0 \), but it can affect \( \bar{b} \to \bar{c}u\bar{s} \) decay:

\[
\frac{g_L^2}{M_L^2} V_{cb}^* V_{us} : \frac{g_R^2}{M_R^2} V_{cb}^* V_{us}^R = 1 : \beta_g \left( -e^{-i\omega} \sin \theta_R \right) V_{cb}^* V_{us}.
\]

(9)

The deviation of measured \( \phi_3, \Delta \phi_3 \), from the standard model value for \( M_{W_R} = 1 \) TeV and \( \phi_3 = 135^\circ \) in the CKM matrix for \( \theta_R = 100^\circ \) is given in Fig. 19. The deviation can reach \(-45^\circ\) for \( \omega = 40^\circ \). As \( W_R \) becomes heavier, the deviation becomes smaller in proportion to \( 1/M_{W_R}^2 \). This deviation cannot be observed in the measurements of \( \phi_3 \) in \( B \to K\pi \), since \( V_{us}^R V_{ub}^* = 0 \). Hence we can expect disagreement between
the two kinds of measurements of $\phi_3$.

In conclusion, we have investigated $W_R$ effects on $B\overline{B}$ mixing and $CP$ asymmetry in $B$ decays and found that the $W_R$ effect is sizable even for $M_{W_R} = 1 - 10$ TeV. The experimental values of $\Delta M_B$ and $CP$ asymmetry in $B \to (c\bar{c}) + K^{(*)}$, $\text{Asy}(\Psi K)$, severely constrain the parameters of the right-handed quark mixing matrix $V^R$. With allowed parameter values, the $CP$ asymmetry $\text{Asy}(\Psi K)$ can be as large as 1, which is the central value of Belle. If future experiments confirm this large value of $\text{Asy}(\Psi K)$, fine measurements of $\phi_2$ and $\phi_3$ in various modes are necessary to distinguish the physics of this kind of model and other new physics.

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