Quark-level analogue of nuclear fusion with doubly-heavy baryons

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ABSTRACT

The recent discovery by LHCb of the first doubly-charmed baryon $\Xi^{++}_{cc} = ccu$ at $3621.40 \pm 0.78$ MeV implies a large binding energy $\sim 130$ MeV between the two $c$ quarks. This strong binding enables a quark-rearrangement exothermic reaction $\Lambda_c\Lambda_c \rightarrow \Xi^{++}_{cc} n$ with $Q = 12$ MeV, which is a quark-level analogue of deuterium-tritium nuclear fusion reaction $DT \rightarrow ^4\text{He} n$. Due to much larger binding energy between two $b$ quarks $\sim 280$ MeV, the analogous reaction with $b$ quarks, $\Lambda_b\Lambda_b \rightarrow \Xi_{bb} N$ is expected to have a dramatically larger $Q$-value, $138 \pm 12$ MeV.

Very recently LHCb has observed the doubly-charmed baryon $\Xi^{++}_{cc} = ccu$ with a mass of $3621.40 \pm 0.78$ MeV \cite{LHCb}. This value is consistent with several predictions, including our value of $3627 \pm 12$ MeV \cite{Karliner:2014ida,Karliner:2016jic}. The essential ingredient in Ref. \cite{Karliner:2014ida} is the large binding energy of the two heavy quarks in a baryon, $B(cc) = 129$ MeV and $B(bb) = 281$ MeV.

To a very good approximation this binding energy is $1/2$ of the quark-antiquark binding energy in corresponding quarkonia. This $1/2$ rule is exact in the one-gluon-exchange limit and has now been validated by the LHCb measurement of the $\Xi_{cc}$ mass. Its successful extension beyond weak coupling implies that the heavy quark potential factorizes into a color-dependent and a space-dependent part, with the space-dependent part being the same for quark-quark and for quark-antiquark. The relative factor $1/2$ is then automatic, just as in the weak-coupling limit, resulting from the color algebra.

The large binding energy between heavy quarks has some striking implications, such as the existence of a stable $b\bar{b}u\bar{d}$ tetraquark with $J^P = 1^+$ \cite{Lattice}, 215 MeV below the $B^-\bar{B}^{*0}$ threshold and 170 MeV below threshold for decay to $B^-\bar{B}^{0}\gamma$.

In the present work we point out another striking consequence of the very strong binding between heavy quarks: the quark-level analogue of nuclear fusion. To start, consider the quark-rearrangement reaction

$$\Lambda_c\Lambda_c \rightarrow \Xi^{++}_{cc} n, \quad cud cud \rightarrow ccu ddu \quad (1)$$
Table I

\[ Q \text{ value in the reaction } \Lambda_Q \Lambda_{Q'} \rightarrow \Xi_{QQ'} N, \quad Q, Q' = s, c, b. \]

| Observable (MeV) | \( Q, Q' = s \) | \( Q, Q' = c \) | \( Q, Q' = b \) | \( Q = b, Q' = c \) |
|------------------|----------------|----------------|----------------|----------------|
| \( M(\Lambda_Q) \) | 1115.7         | 2286.5         | 5619.6         | 5619.6, 2286.5 |
| \( M(\Xi_{QQ'}) \) | 1314.9         | 3621.4 ± 0.78  | 10162 ± 12b    | 6917 ± 13c     |
| \( Q \)-value    | −23.1          | +12.0 ± 0.78   | +138 ± 12      | +50 ± 13       |

\(^a\)To optimize the \( Q \)-value we take here \( \Xi_0^{ssu}, N=n \), because \( M[\Xi^-(ssd)] \) is 7 MeV larger.

\(^b\)\( \Xi_{bb} \) mass prediction from Ref. [2].

\(^c\)Average of the two values in Table XI of Ref. [2].

All the masses are known and the \( Q \)-value is 12 MeV, as shown in Table I.

Clearly, the exothermic reaction \( (1) \) is the quark-level analogue of the well-known exothermic nuclear fusion reactions involving the lightest nuclei with two or three nucleons [5],

\[
\begin{align*}
DT & \rightarrow ^4\text{He}n & Q = 17.59 \text{ MeV}, \\
DD & \rightarrow ^3\text{He}n & Q = 3.27 \text{ MeV}, \\
DD & \rightarrow ^3\text{He}p & Q = 4.04 \text{ MeV}, \\
TT & \rightarrow ^4\text{He}2n & Q = 11.33 \text{ MeV}, \\
D^3\text{He} & \rightarrow ^4\text{He}p & Q = 18.35 \text{ MeV}, \\
^3\text{He}^3\text{He} & \rightarrow ^4\text{He}2p & Q = 12.86 \text{ MeV}.
\end{align*}
\]

(2)

It is interesting that the reaction \( (1) \) involves two hadrons with three quarks each, rather than two nuclei with two or three nucleons each, and that its \( Q \)-value is of similar order of magnitude to the reactions \( (2) \).

Table I lists the \( Q \)-values of the four analogous reactions \( \Lambda_Q \Lambda_{Q'} \rightarrow \Xi_{QQ'} N, \quad Q, Q' = s, c, b \). The trend is clear: the \( Q \)-values increase monotonically with increasing quark mass. The reaction

\[ \Lambda\Lambda \rightarrow \Xi N \]

is endothermic with \( Q = -23 \text{ MeV} \). Reaction \( (1) \) is exothermic with \( Q = +12 \text{ MeV} \), while the reaction

\[ \Lambda_b\Lambda_b \rightarrow \Xi_{bb}N \]

is expected to be strongly exothermic with \( Q = +138 \pm 12 \text{ MeV} \). Finally, the reaction

\[ \Lambda_b\Lambda_c \rightarrow \Xi_{bc}N \]

is expected to have \( Q = +50 \pm 13 \text{ MeV} \), intermediate between \( cc \) and \( bb \). The two latter estimates rely on the predictions of the \( \Xi_{bb} \) and \( \Xi_{bc} \) masses in Ref. [2].

As already mentioned, the dominant effect determining the \( Q \)-value is the binding between two heavy quarks. Since these quarks interact through an effective two-body potential their binding is determined by their reduced mass, \( \mu_{red} = m_Q m_{Q'}/(m_Q + m_{Q'}) \).

In Figure 1 we plot the \( Q \)-value vs. \( \mu_{red}(QQ') \). The effective quark masses are taken as

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Figure 1: The $Q$-value in the quark-level fusion reactions $\Lambda_Q\Lambda_{Q'} \rightarrow \Xi_{Q''} N$, $Q, Q' = s, c, b$, plotted against the reduced masses of the doubly-heavy diquarks $\mu_{\text{red}}(QQ')$. The dot-dashed line denotes a linear fit $Q = -44.95 + 0.0726 \mu_{\text{red}}$.

In Ref. 2: $m_s = 538$ MeV, $m_c = 1710.5$ MeV, $m_b = 5043.5$ MeV. The straight line fit $Q = -44.95 + 0.0726 \mu_{\text{red}}$ denoted by dot-dashed line describes the data rather well, showing that to a good approximation the $Q$-value indeed depends linearly on the reduced mass.

In addition to the reactions (1), (4), and (5) which involve fusion of two heavy baryons to a doubly-heavy baryon and a nucleon, reactions involving fusion of two heavy mesons into a stable doubly-heavy tetraquark $T(bb\bar{u}\bar{d})$ are also possible,

$$B^- \bar{B}^{0*} \rightarrow T(bb\bar{u}\bar{d}), \quad Q = 215 \text{ MeV}, \quad (6)$$

and

$$B^- \bar{B}^0 \rightarrow T(bb\bar{u}\bar{d}) \gamma, \quad Q = 170 \text{ MeV}. \quad (7)$$

Reaction (6) is analogous to fusion of proton and neutron into deuteron, with $Q = 2.2$ MeV. Reaction (7) has a lower $Q$-value than (6) and it requires EM interaction on top of QCD, in order to conserve angular momentum and parity, since $T(bb\bar{u}\bar{d})$ has $J^P = 1^+$.4

It is worth noting that Table I anticipates a strong violation of the would be heavy-quark analogue of the Gell-Mann–Okubo mass formula

$$\frac{N + \Xi}{2} = \frac{3\Lambda + \Sigma}{4} \quad (8)$$
where l.h.s. = 1128.6 MeV and r.h.s. = 1135.1 MeV, i.e., Eq. (8) is accurate to \( \sim 0.6\% \). Indeed, for charm l.h.s. = 2280 MeV and r.h.s. = 2328 MeV, while for bottom l.h.s. = 5551 \( \pm 6 \) MeV and r.h.s. = 5668 MeV.

One might think that this is not surprising, given that the Gell-Mann–Okubo mass formula was derived assuming a small breaking of flavor SU(3) and there is no corresponding flavor symmetry when the s-quark is replaced by c or b. But there is more to it than this.

In the modern view Eq. (8) results from equal numbers of light and s quarks on both sides of the equation, together with the corresponding spin-dependent color-hyperfine interaction terms. Had this been the whole story, the mass formula (8) should have approximately worked also for c and b. The reason it fails is that the large binding energy between the two heavy quarks is not included in the derivation.

It should be stressed that from the point of view of the strong interactions, all the \( \Lambda_Q \) and \( \Xi_{QQ} \) baryons are stable particles. They eventually do decay, via weak interactions, but their lifetimes are at least ten orders of magnitude longer than the typical timescale \( 10^{-23} \) s of the fusion reactions (1), (4), and (5) which proceed purely through strong interactions. An analogous observation applies to reaction (6).

**Analogies with doubly strange hypernuclei**

Implications of the strong binding of two heavy quarks in \( \Xi_{QQ} \) go beyond the fusion-like reactions \( \Lambda_Q \Lambda_Q \to \Xi_{QQ} N, \ Q = b, c \). There might also be interesting ramifications for \( cc \) and \( bb \) analogues of hypernuclei.

Ref. [8] examined theoretically the production of doubly strange hypernuclei, \( \Xi^{16}\text{C} \) and \( \Lambda^{16}\text{C} \) in double-charge exchange \( ^{16}\text{O}(K^-, K^+) \) reactions, where \( \Xi^{16}\text{C} \) denotes a \( Z = 6 \) hypernucleus with \( \Xi^- \) in place one of the original nucleons.

We conjecture that in principle an analogous reaction with b instead of s-quarks might be possible, i.e., \( ^{16}\text{O}(B^-, B^+)\Xi^{16}_{bb} \), or

\[
B^- \ ^{16}\text{O} \to B^+ \ ^{16}_{bb}\text{C}.
\]

The difference between the binding energy of \( ^{16}\text{O} = 7.98 \) MeV and of ordinary \( ^{16}\text{C} = 6.92 \) MeV [9] is only about 1 MeV. On the other hand, the binding energy of \( bb \) in \( \Xi_{bb} \) is \( \sim 280 \) MeV, so the reaction (9) is expected to have a rather large \( Q \)-value.

Experimentally this reaction is extremely challenging, because the \( B^- \) lifetime is only \( 1.6 \times 10^{-12} \) s, four orders of magnitude shorter than \( \tau(K^-) = 1.2 \times 10^{-8} \) s. To put it in perspective, the distance \( d_B \) covered by \( B^- \) is \( d_B = \gamma(B) \cdot \tau(B^-) \cdot c = \gamma(B) \cdot 480 \mu \), where \( \gamma(B^-) \) is the Lorentz factor. So a 10 GeV \( B^- \) will travel \( \sim 1 \) mm before decaying.

An analogous reaction with a c instead of a b-quark might perhaps also be possible, i.e.,

\[
D^{+} N A \to D^- \ ^{16}_{cc\,A'} \Xi^{16}_{cc} A'.
\]

Such a reaction could take place in a collision in which at least one of the two projectiles is a heavy ion. The initial \( D^{+} \) would be produced as part of an open-charm process, and would then undergo double-charm-exchange in the target nucleus.

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