A unified picture of the post-merger dynamics and gravitational wave emission in neutron-star mergers

A. Bauswein$^{1}$ and N. Stergioulas$^{1}$

$^{1}$Department of Physics, Aristotle University of Thessaloniki, GR-54124 Thessaloniki, Greece

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We introduce a classification scheme of the post-merger dynamics and gravitational-wave emission in binary neutron star mergers, after identifying a new mechanism by which a secondary peak in the gravitational-wave spectrum is produced. It is caused by a spiral deformation, the pattern of which rotates slower with respect to the double-core structure in center of the remnant. This secondary peak is typically well separated in frequency from the secondary peak produced by a nonlinear interaction between a quadrupole and a quasi-radial oscillation. The new mechanism allows for an explanation of low-frequency modulations seen in a number of physical characteristics of the remnant, such as the central lapse function, the maximum density and the separation between the two cores. We find empirical relations for both types of secondary peaks between their gravitational-wave frequency and the compactness of nonrotating individual neutron stars, that exist for fixed total binary masses. Our classification scheme may form the basis for the construction of detailed gravitational-wave templates of the post-merger phase. We find that the quasi-radial oscillation frequency of the remnant decreases with the total binary mass. For a given merger event our classification scheme may allow to determine the proximity of the measured total binary mass to the threshold mass for prompt black hole formation, which can, in turn, yield an estimate of the maximum neutron-star mass.

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Introduction: Neutron star (NS) mergers are strong emitters of gravitational waves (GW) and thus among the prime targets for the upcoming GW detectors Advanced LIGO [1] and Advanced Virgo [2]. Future GW observations of such events could reveal the properties of high-density matter and NSs (see e.g. [3–4] for reviews). The merger will likely result in the formation of a differentially rotating, strongly oscillating remnant [8–34]. Specifically, detections of its post-merger GW frequency would strongly constrain the radius and the maximum mass of nonrotating NSs [24, 26, 29, 32–34, 37–39]. We confirm this by adding a quasi-radial density perturbation to the remnant at late times. The frequency of the excited quasi-radial oscillation is determined by a form of the waveform $h_{\text{eff}, \chi} = h_\chi(f) \cdot f$, where $h_\chi$ is the Fourier transform of the waveform $h_\chi$ and $f$ is frequency. Besides the dominant $f_{\text{peak}}$ frequency, there are two secondary peaks at lower frequencies ($f_2$, $f_3$) with comparable signal-to-noise ratio (both are generated in the post-merger phase).

In this letter we find that two different mechanisms produce the low-frequency secondary peaks: apart from a nonlinear combination frequency $f_{\text{peak}}$, a distinct mechanism generates a secondary GW peak by the rotating pattern of a deformation of spiral shape. This leads to a unified classification scheme for the post-merger dynamics and GW emission. We find mass-dependent empirical relations for the different peaks as a function of NS compactness and rule out the existence of a universal, mass-independent relation for the secondary peaks recently proposed in [32, 33]. The understanding of prominent GW features is a prerequisite for constructing GW templates for the post-merger phase, which could enhance the detection prospects compared to unmodelled searches [36, 39].

Simulations: We investigate mergers of equal-mass, intrinsically non-spinning NSs with a 3D relativistic smoothed particle hydrodynamics (SPH) code, which imposes the conformal flatness condition on the spatial metric and incorporates energy and angular momentum losses by a GW backreaction scheme (see [11, 16, 24, 26, 43] for details). Comparisons to other numerical setups show good agreement in determining the post-merger spectrum [24, 26, 29, 32–34]. We explore ten microphysical, fully temperature-dependent equations of state (EoSs) and consider total binary masses $M_{\text{tot}}$ between 2.4 $M_\odot$ and $3.0 M_\odot$.

Nature of secondary GW peaks: First, we focus on a reference model for the DD2 EoS with $M_{\text{tot}} = 2.7 M_\odot$. The left panel of Fig. 1 shows the effective amplitude $h_{\text{eff}, \chi} = h_\chi(f) \cdot f$, where $h_\chi$ is the Fourier transform of the waveform $h_\chi$ and $f$ is frequency. Besides the dominant $f_{\text{peak}}$ frequency, there are two secondary peaks at lower frequencies ($f_2$, $f_3$) with comparable signal-to-noise ratio (both are generated in the post-merger phase).

The secondary peak at $f_2$ is a nonlinear combination frequency between the dominant quadrupolar $f_{\text{peak}}$ oscillation and the quasi-radial oscillation of the remnant [38]. We confirm this by adding a quasi-radial density perturbation to the remnant at late times. The frequency $f_0$ of the excited quasi-radial oscillation is determined by a Fourier analysis of the density or central lapse function. It perfectly coincides with $f_{\text{peak}} - f_2$. As in [38], the extracted eigenfunction at $f_0$ confirms the quasi-radial nature.

The secondary $f_{\text{spiral}}$ peak is produced by a strong deformation initiated at the time of merging, the pattern
of which then rotates (in the inertial frame) slower than the inner remnant and lasts for a few rotational periods, while diminishing in amplitude. Figure 4 shows the density evolution in the equatorial plane, in which one can clearly identify the two antipodal bulges of the spiral pattern. In this early phase the inner remnant is still composed of two dense cores rotating around each other (this is the nonlinear generalization of an $m = 2$ quadrupole oscillation producing the dominant $f_{\text{peak}}$). Extracting the rotational motion of the antipodal bulges, their frequency equals $f_{\text{spiral}}/2$ producing gravitational waves at $f_{\text{spiral}}$ (compare the times in the right panels). Examining the GW spectrum for different time intervals shows that the presence of the $f_{\text{spiral}}$ peak agrees with the duration of the spiral deformation.

In the upper right panel of Fig. 2, the spiral deformation can be seen to initially reach deep inside the remnant. The two antipodal bulges contain several tenth of $M_\odot$, which can explains the strength of the $f_{\text{spiral}}$ peak. Moreover, the $f_{\text{spiral}}$ peak can be roughly reproduced in a toy model, where two bulges orbit as point particles around the central double-core structure for a duration of few milliseconds.

By the quadrupole formalism we compute GW spectra considering only certain parts of the remnant, which are defined by using a density cut-off. In the right panel of Fig. 1 the dominant quadrupole $f_{\text{peak}}$ frequency is generated mainly by regions of the remnant which encompass densities exceeding 50% of the instantaneous maximum density $\rho_{\text{max}}$. In contrast, most power of the $f_{\text{spiral}}$ peak originates from densities below 0.5$\rho_{\text{max}}$ (outer parts of the remnant with the two bulges).

In models where $f_{\text{spiral}}$ dominates over $f_{2-0}$ (see our classification below), the presence of the two rotating antipodal bulges explains a particular low-frequency mod-
double-core structure alone are not sufficient to interpret the secondary peaks, and the simultaneous presence of both \(f_{\text{peak}}\) and \(f_{\text{spherical}}\) cannot simply be attributed to a single instantaneous angular frequency of the system, as suggested in \(^{[32, 34]}\).

Classification of post-merger dynamics: We have applied the above analysis tools (GW spectra, determination of \(f_0\) from perturbed models, rotational frequency of the antipodal bulges and of the double cores, GW spectra with different cut-off densities) for a number of representative models, varying the binary mass and stiffness of the EoS. Based on the relative strength between \(f_{2-0}\) and \(f_{\text{spherical}}\) we identify three different types of post-merger dynamics and GW spectra for remnants which survive for more than several milliseconds.

Type I: When \(M_{\text{tot}}\) is not too far from the threshold mass for prompt quasi-radial collapse of the remnant for a given EoS, the evolution of the central lapse function (and of \(\rho_{\text{max}}\)) is dominated by a strong quasi-radial oscillation of the remnant, see lower curve in Fig. 3. The two initial NSs are more centrally condensed and they merge with higher impact velocity (Fig. 3 in \(^{[40]}\)). \(f_{2-0}\) is the strongest secondary peak, while \(f_{\text{spherical}}\) is much weaker (there can be partial overlap between \(f_{2-0}\) and \(f_{\text{spherical}}\), see Fig. 4).

Type II: For intermediate masses \(f_{2-0}\) and \(f_{\text{spherical}}\) have a comparable strength and are well separated (see Fig. 1, reference model).

Type III: When \(M_{\text{tot}}\) is significantly below the threshold mass for quasi-radial collapse, the time evolution of the central lapse function (as well as of \(\rho_{\text{max}}\) and of the radius of the remnant) is dominated by the \(f_{\text{peak}} - f_{\text{spherical}}\) modulation explained in the previous section. This modulation typically has a smaller amplitude than Type I variations (Fig. 3). The quasi-radial oscillation \(f_0\) is also present, but with much smaller amplitude than the dominant modulation. The smaller NS compactness implies a smaller impact velocity and allows for a stronger spherical deformation. Consequently, the dominant secondary peak is \(f_{\text{spherical}}\), while \(f_{2-0}\) is very weak or hidden inside the background (see Fig. 1).

For a given EoS there is a continuous transition between the different types of post-merger dynamics, depending on \(M_{\text{tot}}\). Types I and III are the limiting cases of the more generic Type II. Since the threshold for quasi-radial collapse is EoS-dependent \(^{[28]}\), the different types cover a different mass range for each EoS.

For \(2.4 \, M_{\odot} \leq M_{\text{tot}} \leq 3.0 \, M_{\odot}\) we find that \(f_{\text{spherical}}\) typically ranges between \(f_{\text{peak}} - 0.5 \, \text{kHz}\) and \(f_{\text{peak}} - 0.9 \, \text{kHz}\), while \(f_{2-0}\) ranges between \(f_{\text{peak}} - 0.9 \, \text{kHz}\) and \(f_{\text{peak}} - 1.3 \, \text{kHz}\). This will be useful for identifying either \(f_{2-0}\), or \(f_{\text{spherical}}\) (or both) in future GW observations. \(f_{\text{peak}} - f_{2-0} = f_0\) decreases with increasing \(M_{\text{tot}}\), in agreement with the fact that the quasi-radial frequency decreases near the threshold to collapse. Very near the threshold one may expect an overlap \(f_{2-0} \rightarrow f_{\text{peak}}\). Typically, \(f_{\text{peak}} - f_{\text{spherical}}\) increases with increasing \(M_{\text{tot}}\), and above the threshold to collapse a spiral pattern during the dynamical collapse could still produce a weak peak in the GW spectrum as in \(^{[50]}\).

Empirical relations for dominant and secondary peak frequencies: For our sample of EoSs Fig. 4 (left panel) shows \(f_{\text{peak}}\), \(f_{\text{spherical}}\) and \(f_{2-0}\) as a function of the compactness \(M/R\) of the nonspinning, individual NSs (at infinite separation) for \(M_{\text{tot}} = 2.7 \, M_{\odot}\) (with the compactness in units of \(c = G = 1\)). We find strong correlations that can be described by the following quadratic fits:

\[
\begin{align*}
    f_{\text{peak}}[\text{kHz}] &= 199(M/R)^2 - 28.1(M/R) + 2.33, \\
    f_{\text{spherical}}[\text{kHz}] &= 358(M/R)^2 - 82.1(M/R) + 6.16, \\
    f_{2-0}[\text{kHz}] &= 392(M/R)^2 - 88.3(M/R) + 5.95.
\end{align*}
\]

The accuracy of these empirical relations is similarly good for \(f_{\text{peak}}\) and \(f_{\text{spherical}}\) and somewhat worse for \(f_{2-0}\). Such empirical relations hold, with varying accuracy, for each total binary mass. As pointed out and explained for \(f_{\text{peak}}\) in \(^{[20]}\), even better empirical relations are obtained for 1.35-1.35 \(M_{\odot}\) binaries when the above three frequencies are examined as function of the radius of a nonrotating NS with 1.6 \(M_{\odot}\). All three empirical relations follow similar trends. Because the frequency of the \(f_{\text{spherical}}\) peak agrees with the rotational frequency of the antipodal bulges near the surface of the remnant, a scaling of \(f_{\text{spherical}}\) with the compactness is not unexpected, which explains the similarities with the behavior of \(f_{\text{peak}}\).

A measurement of \(f_{\text{peak}}\) is already sufficient to accurately constrain the radius of nonrotating NSs (and thus the EoS) \(^{[24, 27, 35]}\) if the total mass has been obtained accurately from the inspiral signal, as it is likely for distances within which \(f_{\text{peak}}\) has the required signal-to-noise ratio to be detectable with second-generation interferometers (e.g. \(^{[24, 51]}\)). A detection of the weaker secondary peaks could further optimize the constraints on the EoS.

The middle panel of Fig. 4 displays \(f_{\text{spherical}}\) as a func-
tion of the compactness $M/R$ of the nonspinning, individ-ual NSs for different EoSs and for different $M_{\text{tot}}$ in the range $2.4-3.0 ~M_\odot$ (the most likely range of total binary masses [52]). The same figure, but for $f_{2-0}$, is shown in the right panel of Fig. 4. Especially for $f_{\text{spiral}}$ we discover that there exist tight relations between the compactness and the secondary frequencies for fixed binary masses.

In [32, 33] the existence of a single, universal, mass-independent relation (dashed line in Fig. 1) between the frequency of the strongest secondary GW peak (denoted there as $f_1$) and the compactness $M/R$ was proposed (there was no distinction of two different secondary peaks, as we find here). However, this result was based on using a limited set of five EoSs of soft or moderate stiffness (with corresponding maximum masses of nonrotating NS only up to $2.2 ~M_\odot$) as well as on different chosen mass ranges for each EoS with a spread of only $0.2 ~M_\odot$ in the total binary mass.

In contrast to [32, 33], within our larger sample of ten EoSs (that includes stiff EoSs with maximum masses reaching up to $2.8 ~M_\odot$) and for a more representative to-tal binary mass range of $2.4-3.0 ~M_\odot$ (same for all EoSs), such a mass-independent, universal relation does not ex-ist. The middle and right panel of Fig. 4 show that there is a large spread both in the $f_{\text{spiral}}$ vs. $M/R$ relation and in the $f_{2-0}$ vs. $M/R$ relation. Even if one consistently chooses the strongest secondary peak among $f_{\text{spiral}}$ and $f_{2-0}$ in each case, there does not exist a mass-independent universal relation with compactness. Notice also that in the left panel of Fig. 4, the relation proposed in [32, 33] describes either $f_{\text{spiral}}$ at low compactness or $f_{2-0}$ at high compactness. For intermediate values of compactness the merger will be of Type II and both types of secondary peaks can be present with comparable amplitude, which further complicates the definition of a single $f_1$ frequency, as was assumed in [32, 33].

Instead of a universal mass-independent relation, we find that there exist useful empirical relations only for specific masses (shown as thin line segments in the middle and right panels in Fig. 4), such as the case shown in the left panel in Fig. 4.

**Outlook:** Identifying the merger type and determining the quasi-radial frequency can yield estimates of the threshold mass for black-hole collapse and of the maximum mass of nonrotating NSs [25]. Our classification scheme and the frequency dependencies can provide a basis for constructing detailed GW templates, increasing the detectability of the post-merger GW emission [30].

We will further investigate whether the frequency differences between the dominant and secondary peaks can clarify the nature of the latter or if detailed comparisons between spectra of different binary masses are needed. We will also explore unequal-mass binaries (anticipating a strong impact of the mass-ratio on $f_{\text{spiral}}$) and analyze the relevance of our classification scheme for the mass ejection and torus formation of NS mergers and for accompanying phenomena, such as r-process nucleosynthesis [53, 54], electromagnetic counterparts [55–57] and short gamma-ray bursts [54, 58].

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[1] G. M. Harry and LIGO Scientific Collaboration, Classical Quantum Gravity 27, 084006 (2010).

[2] F. Acernese, P. Amico, M. Alshourbagy, F. Antonucci, S. Aoudia, S. Avino, D. Babusci, G. Ballard, F. Barone,
