Thermodynamics of a Fermi liquid in a magnetic field

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We extend previous calculations of the non-analytic terms in the spin susceptibility \(\chi_s(T)\) and the specific heat \(C(T)\) to systems in a magnetic field. Without a field, \(\chi_s(T)\) and \(C(T)/T\) are linear in \(T\) in 2D, while in 3D, \(\chi_s(T) \propto T^2\) and \(C(T)/T \propto T^2 \log T\). We show that in a magnetic field, the linear in \(T\) terms in 2D become scaling functions of \(\mu_B H/T\). We present explicit expressions for these functions and show that at high fields, \(\mu_B H \gg T\), \(\chi_s(T, H)\) scales as \(|H|\). We also show that in 3D, \(\chi_s(T, H)\) becomes non-analytic in a field and at high fields scales as \(H^2 \log |H|\).

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Landau Fermi liquid theory \cite{1} provides the basis for our present understanding of correlated electronic systems. The theory predicts that, in any Fermi liquid, the spin susceptibility \(\chi_s(T)\) and the specific heat coefficient \(\gamma(T) = C(T)/T\) tend to a constant at \(T \to 0\) \cite{1, 2}. Later, Landau theory has been extended to include the leading temperature dependence of \(\chi_s(T)\) and \(\gamma(T)\) which turn out to be non-analytic in dimensions \(D \leq 3\) \cite{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}. Like the zero-temperature terms, the thermal corrections come from fermions in the immediate vicinity of the Fermi surface. In 2D systems, both \(\chi_s(T)\) and \(\gamma(T)\) are linear in \(T\) \cite{3, 4, 5, 6, 7, 8, 9, 10}, while \(\gamma(T)\) is non-analytic and scales as \(T^2 \log T\) \cite{3, 4, 5, 6, 7, 8, 9, 10}.

In this communication, we extend previous works to systems in a magnetic field \(H\). We consider \(S = 1/2\) charge-less fermions (like 3He atoms) for which the magnetic field adds spin-dependent Zeeman term \(\pm \mu_B H\) to the fermionic dispersion. We show that, in the presence of a field, \(\Delta \chi_s(T, H) = \chi_s(T, H) - \chi_s(0, 0)\) and \(\Delta \gamma(T, H) = \gamma(T, H) - \gamma(0, 0)\) become scaling functions of \(\mu_B H/T\): \(\Delta \chi_s(T, H) = \chi_s(T, 0)f_s(\mu_B H/T)\), \(\Delta \gamma(T, H) = \gamma(T, 0)f_\gamma(\mu_B H/T)\). We present the expressions for these functions to second order in the interaction potential \(U\). For 2D systems, we show that at \(\mu_B H \gg T\) (but still, \(\mu_B H \ll E_F\)), \(\chi_s(T, H)\) scales as \(|H|\) and weakly depends on \(T\). In the same field range, \(\gamma(T, H)\) is still linear in \(T\), but the prefactor is different from that at \(H = 0\). For 3D systems, we show that \(\chi_s(T, H)\) becomes non-analytic at a non-zero \(H\). The non-analytic term in \(\chi_s(T, H)\) scales as \(H^2 \log[\max(\mu_B H, T)/E_F]\). The specific heat coefficient \(\gamma(H, T)\) in a field still scales as \(T^2 \log T\), but, like in 2D, the prefactor changes between \(H = 0\) and \(\mu_B H \gg T\).

The analysis of the behavior of \(\Delta \chi_s(T, H)\) and \(\Delta \gamma(T, H)\) in a magnetic field may be useful for experimental verifications of the non-analytic behavior of thermodynamic variables. It is more straightforward to analyze the dependence on the magnetic field rather than the dependence on the temperature. In particular, recent measurements of the temperature dependence of the spin susceptibility in Si inversion layers \cite{13} didn’t yield conclusive results on whether the \(T\) dependence of \(\chi_s(T)\) is indeed linear, as some temperature dependence inevitably comes from spins on the substrate. We propose to measure the field dependence of the spin susceptibility at a given \(T\) and use our scaling functions to fit the data.

The point of departure for our calculations is the Luttinger-Ward expression for the thermodynamic potential. To simplify the presentation, we assume that the interaction potential \(U(q)\) is independent on \(q\). We restore the momentum dependence of \(U(q)\) in the final formulas. To second-order in \(U\), the thermodynamic potential is given by

\[
\Phi = \Phi_0 - \frac{U^2}{2} \sum_n \int \frac{d^d q}{(2\pi)^d} \Pi^{\uparrow \uparrow}(q, \Omega_n, T)\Pi^{\downarrow \downarrow}(q, \Omega_n, T),
\]

where \(\Phi_0\) is the thermodynamic potential for free fermions, and \(\Pi^{\uparrow \uparrow}(q, \Omega_n, T)\) and \(\Pi^{\downarrow \downarrow}(q, \Omega_n, T)\) are the particle-hole bubbles composed of fermions with spin up or spin down, respectively.

Previous studies of the spin susceptibility and the specific heat in a zero magnetic field established that the non-analytic temperature behavior of \(\Delta \chi_s(T)\) and \(\Delta \gamma(T)\) originates from the non-analyticity of the polarization operator either near \(q = 0\) (Landau damping) \cite{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13} or near \(q = 2k_F\) (a dynamic Kohn anomaly) \cite{3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13}. The \(2k_F\) non-analyticity contributes to the spin susceptibility and the specific heat, while the \(q = 0\) non-analyticity only contributes to the non-analyticity in the specific heat. This can be easily understood as the non-analytic term in the zero field spin susceptibility \(\Delta \chi_s(T)\) describes a singular response to an infinitesimally small magnetic field. A magnetic field
FIG. 1: The diagram for $\Delta \Phi_{2k_F}$, and the trick to compute it. As the non-analytic part of $\Delta \Phi_{2k_F}$ comes from small $k$ and $p$, it can be re-expressed as a product of two bubbles $\Pi^{\uparrow\downarrow}(q, \omega)$ with small momentum transfer $q = \bar{k} - \bar{p}$.

splits Fermi momentum $k_F$ into $k_F^\uparrow$ and $k_F^\downarrow$. The small $q$ form of $\Pi^{\uparrow\uparrow}(q, \Omega_n, T)$ and $\Pi^{\downarrow\downarrow}(q, \Omega_n, T)$ is unaffected by this splitting, up to terms of order $(\mu_B H/E_F)^2$, hence the response to the infinitesimal field must be analytic in $T$. At the same time, singular $2k_F$ contribution to $\Phi(T)$ at zero field originates from the fact that the two polarization operators in Eq. (4) are non-analytic at the same $q = 2k_F$. In a field the singularities in spin-up and spin-down polarization operators occur at different $q = 2k_F^\uparrow$ and $q = 2k_F^\downarrow$. Accordingly, a magnetic field regularizes $2k_F$ non-analyticity in the thermodynamic potential, but for a price that the linear response to the field, i.e. the spin susceptibility $\Delta \chi(T, H = 0)$, becomes non-analytic.

Our goal is to analyze the forms of the susceptibility and the specific heat at a finite $H$, i.e., beyond the linear response theory. We consider the fields for which $\mu_B H$ is comparable to $T$, but still $\mu_B H \ll E_F$. For these fields, the non-analytic contribution to $\Phi$ from small $q$ are unaffected by the field. However the $2k_F$ contribution is field dependent and evolves at $\mu_B H \sim T$.

The calculation of $\Delta \Phi = \Phi - \Phi_0$ is somewhat tricky. In principle, all one has to do is to evaluate particle-hole bubbles for fermions with up and down spins at a finite $T$, substitute the results into Eq. (1), integrate over moment $q$ and sum over Matsubara frequencies $\Omega_n$. In practice, however, this computation is easy to perform only for small $q$ part as for $q \ll k_F$, the non-analytic part of the polarization bubble is associated with the Landau damping, which does not depend on $T$, apart from regular $(T/E_F)^2$ corrections. Accordingly, one can safely use the known analytical forms of $\Pi(q, \Omega)$ at $T = 0$.

For $q$ near $2k_F$, non-analytic terms in $\Pi^{\uparrow\uparrow}(q, \Omega_n, T)$ and $\Pi^{\downarrow\downarrow}(q, \Omega_n, T)$ contain scaling functions of $T/\omega$, which are only available in integral forms [10]. This substantially complicates direct calculation of the $2k_F$ term. There exists, however, a way to compute the $2k_F$ term, which avoids dealing with the $2k_F$ polarization bubbles at a finite $T$. This method exploits the fact that only the non-analytic parts $\Pi^{\uparrow\uparrow}(q, \Omega_n, T)$ and $\Pi^{\downarrow\downarrow}(q, \Omega_n, T)$ for $q$ near $2k_F$ contribute the non-analyticity in the thermodynamic potential. Earlier works have demonstrated that the $2k_F$ non-analyticity in $\Pi(q, \Omega_n, T)$ comes from fermions in the particle-hole bubble with momenta near $\pm \bar{q}/2$ [11,12]. This implies that, out of four fermions in the second order, two-bubble diagram for the thermodynamic potential in Fig. 1, two fermions with opposite spins have momenta near $\bar{q}/2$, while the other two fermions have momenta near $-\bar{q}/2$. Then the $2k_F$ part of $\Delta \Phi$ can be re-written as the integral over small $\bar{k}$ and small $\bar{p}$ of

$$
\Delta \Phi_{2k_F} = -\frac{U^2}{2} \sum_{\omega_n, \omega'_m, \omega''_m} \int d^2 q \int d^2 k d^2 p \ G^{\uparrow}(\bar{q}/2 + \bar{k}, \omega_m + \omega'_m) G^{\downarrow}(\bar{q}/2, \omega'_m) \times G^{\downarrow}(\bar{q}/2 + \bar{k}, \omega_m + \omega''_m) G^{\uparrow}(\bar{q}/2 + \bar{p}, \omega''_m)
$$

or, equivalently, as

$$
\Delta \Phi_{2k_F} = -\frac{U^2}{2} \sum_n \int \frac{d^4 q}{(2\pi)^4} \left[ \Pi^{\uparrow\downarrow}(\bar{q}, \Omega_n, T) \right]^2,
$$

where the integration is confined to small $\bar{q} = \bar{k} - \bar{p}$. In other words, the non-analytic $2k_F$ contribution to the thermodynamic potential can be re-expressed in terms of the particle-hole bubble for fermions with opposite spins and a small momentum transfer. The non-analytic term in $\Pi$ at small $\bar{q}$ does not depend on temperature (apart from irrelevant corrections), hence $\Pi^{\uparrow\downarrow}(\bar{q}, \Omega_n, 0)$ can be safely approximated by $\Pi^{\uparrow\downarrow}(\bar{q}, \Omega_n, 0)$, $\Pi^{\downarrow\downarrow}(\bar{q}, \Omega_n, 0)$ strongly depends on the magnetic field (contrary to $\Pi^{\uparrow\downarrow}(\bar{q}, \Omega_n, 0)$), and this gives rise to the scaling dependence on $\mu_B H/T$.

Combining the $q = 0$ and $2k_F$ contributions, we obtain for the thermodynamic potential

$$
\Delta \Phi = -\frac{U^2}{2} \sum_n \int \frac{d^4 q}{(2\pi)^4} \left[ \Pi^{\uparrow\downarrow}(\bar{q}, \Omega_n, 0) \right]^2 + \Pi^{\uparrow\downarrow}(\bar{q}, \Omega_n, 0) \Pi^{\downarrow\downarrow}(\bar{q}, \Omega_n, 0),
$$

where the integration involves only small $q$.

We next proceed separately with $2D$ and $3D$ cases. In $2D$ we have

$$
\Pi^{\uparrow\downarrow}(\bar{q}, \Omega_n, T) = \frac{m}{2\pi} \frac{|\Omega_n|}{\sqrt{\Omega_n^2 - (\delta \mu)^2 + (v_F q)^2}} + ..., \\
\Pi^{\downarrow\downarrow}(\bar{q}, \Omega_n, T) = \frac{m}{2\pi} \frac{|\Omega_n|}{\sqrt{\Omega_n^2 + (v_F q)^2}} + ...,\tag{5}
$$

where dots stand for analytic terms, expandable in powers of $\Omega_n^2$ or $q^2$, and $\delta \mu = \mu_+ - \mu_-$ is $2\mu_B H$. Substituting Eq. (5) into Eq. (4) and integrating explicitly over momentum $q$ we obtain

$$
\Delta \Phi = \left( \frac{m}{2\pi} \right)^2 \frac{U^2 T}{8 \pi v F} \sum_n \Omega_n^2 \log \frac{(\Omega_n - 2\mu_B H)^2 \Omega_n^2}{E_F^4}.
$$

(6)
Differentiating Eq. \( E \) with respect to \( H \), we obtain

\[
\Delta M = -\frac{\partial \Delta \Phi}{\partial H} = \frac{\mu_B m^4 U^2 H^3}{4\pi^3 k_F^2} T \sum_n \frac{1}{\Omega_n^2 + (2\mu_B H)^2}
\]

The sum over Matsubara frequencies can be easily evaluated and yields

\[
\Delta M = \frac{\mu_B m^4 U^2}{4\pi^3 k_F^2} T^2 \left[ \left( \frac{\mu_B H}{T} \right)^2 \coth(\frac{\mu_B H}{T}) \right] = \mu_B A T^2 x^2 \left[ 1 + 2n_B(2x) \right], \quad (7)
\]

where

\[
A = \frac{m^4 U^2}{4\pi^3 k_F^2}, \quad x = \frac{\mu_B H}{T}. \quad (8)
\]

We see from Eq. \( E \) that \( \Delta M \) increases in a field by two reasons. First, the field leads to a finite magnetization at \( T = 0 \), and second, a finite field populates the system with spin waves precessing at the energy \( \mu_B H \). Differentiating \( E \) again over \( H \), we obtain the spin susceptibility in the form

\[
\Delta \chi(T, H) = \chi(T, H) - \chi(0, 0) = \mu_B^2 A T f_\chi(x), \quad (9)
\]

where

\[
f_\chi(x) = \frac{x}{\sinh(x)} \left[ \sinh(2x) - x \right]. \quad (10)
\]

For vanishing \( H \), i.e., at \( x \to 0 \), \( f_\chi(0) = 1 \), and

\[
\Delta \chi(T, H) = \chi(T, 0) - \chi(0, 0) = \mu_B^2 A T. \quad (11)
\]

This coincides with the earlier result \( E \). In the opposite limit of large \( x \), \( f_\chi(x) \approx 2x \), and

\[
\Delta \chi(T, H) = 2\mu_B^2 A T |x| = 2\mu_B^3 A |H|. \quad (12)
\]

We see that at high fields, the spin susceptibility scales as \( |H| \), i.e., is non-analytic in \( H \).

In Fig. 2 we plot the susceptibility as a function of temperature at a given \( H \), and as a function of the magnetic field at a given \( T \). Note that at a finite \( H \), the

\[
\Delta \chi(H) = \frac{\mu_B^4 m^4 U^2 H^3}{\pi^3 k_F^2} T \sum_n \frac{1}{\Omega_n^2 + (2\mu_B H)^2}
\]

Bose term in Eq. \( E \) gives rise to a negative derivative of \( \partial \Delta \chi/\partial T \). This in turn gives rise to a shallow minimum in the temperature dependence of \( \Delta \chi(T, H) \).

The specific heat \( \Delta \gamma(T, H) = \gamma(T, H) - \gamma(0, 0) \) is obtained by differentiating Eq. \( E \) twice over \( T \). At \( H = 0 \),

\[
\Delta \gamma(T, H) = -6AT\zeta(3) \times \left( (x \cosh x - \sin x) = -ATf_\gamma(x), \quad (13) \right.
\]

where

\[
f_\gamma(x) = 3\left( Li_3(e^{-2x}) + 2x Li_2(e^{-2x}) - 2x^2 \log(1 - e^{-2x}) \right)
\]

+6\zeta(3) - 2x^3 + 4x^3 \cosh x - x^3 \frac{1}{\sinh^2 x} (\sinh 2x - x)

and \( Li \) are polylogarithmic functions. At \( x \ll 1 \), \( f_\gamma(x) \approx 6\zeta(3) - \frac{x^2}{6} \) and

\[
\Delta \gamma(\frac{\mu_B H}{T} \ll 1) \approx -AT \left( 6\zeta(3) - \frac{1}{6} \left( \frac{\mu_B H}{T} \right)^4 \right). \quad (14)
\]

In the opposite limit of \( x \gg 1 \), \( f_\gamma(x) = 3\zeta(3) + 4x^4 e^{-2x} \), and

\[
\Delta \gamma(\frac{\mu_B H}{T} \gg 1) \approx -AT \left( 3\zeta(3) + 4 \left( \frac{\mu_B H}{T} \right)^4 e^{-2\mu_B H/T} \right). \quad (15)
\]

We see that in both limits the temperature dependence of the specific heat is linear in \( T \), but the prefactor changes by a factor of 2 between small and high fields. This result could be anticipated as a high magnetic field eliminates the non-analyticity in the polarization bubble \( \Pi^{\uparrow\downarrow}(\vec{q}, \Omega_n) \), such that only the second term in Eq. \( E \) contributes to the \( T \) term in \( \Delta \gamma(T, H) \).

The extension of the above results to an arbitrary \( U(q) \) is straightforward. For the susceptibility, the prefactor in Eq. \( E \), contains \( U(2k_F) \) instead of \( U \). For the specific heat coefficient, we have, instead of \( E \),

\[
\Delta \gamma(T, H) = -3\zeta(3) m^4 \frac{U^2 (2k_F)}{2\pi^3 k_F^2} T \times \left( U(0) - \frac{1}{2} U(2k_F) \right)^2
\]

\[
+ \frac{U^2 (2k_F)}{4} \left( 1 + 2f_\gamma(x) - 3\zeta(3) \right) \left( \frac{3\zeta(3)}{3\zeta(3)} \right). \quad (16)
\]
The combinations $U(0) - 1/2U(2k_F)$ and $-1/2U(2k_F)$ are charge and spin components of the scattering amplitude $A(\pi)$, respectively. At large $x$, $f_s(x) \approx 3c(3)$, and the last term in the r.h.s. of (13) vanishes. This obviously implies that at a large field, only the charge the longitudinal spin components of the scattering amplitude contribute to $\Delta \gamma(T)$.

We next consider the 3D case. The polarization operators at small $q$ are given by [2]:

$$\Pi^{\uparrow\downarrow}(\vec{q}, \Omega_n, T) = \frac{m k_F \Omega_n}{2\pi^2 v_F q} \arctan \left( \frac{\Omega_n - i\delta \mu}{v_F q} \right) + ...$$

$$\Pi^{\uparrow\uparrow}(\vec{q}, \Omega_n, T) = \frac{m k_F \Omega_n}{2\pi^2 v_F q} \arctan \left( \frac{\Omega_n}{v_F q} \right) + ...$$

As before, dots stand for analytic terms, expandable in powers of $\Omega_n^2$ or $q^2$, and $\delta \mu = \mu_T - \mu_0 = 2\mu_B H$. Differentiating the thermodynamic potential, Eq. (4), with respect to $H$, we obtain

$$\Delta M = -\frac{\partial \Delta \Phi}{\partial H} = -\frac{\mu_B (mU k_F)^2}{4\pi^2 v_F^4} T \sum_{n=1}^{\infty} \Omega_n^2 \arctan \left( \frac{2\mu_B H}{\Omega_n} \right).$$

Differentiating further with respect to $H$, we obtain

$$\Delta \chi_s(T, H) = -\frac{\mu_B (mU k_F)^2}{2\pi^2 v_F^2} \left[ T \sum_{n=1}^{M} \Omega_n + 4(\mu_B H)^2 T \sum_{n=1}^{M} \frac{\Omega_n}{\Omega_n^2 + 4\mu_B H^2} \right],$$

where $M \sim E_F/T$ is the upper cutoff in the summation over frequency. The first term in the r.h.s of Eq. (16) is the susceptibility at zero field. By power counting, one might expect the spin susceptibility $\chi_s(T)$ in 3D to scale as $T^2 \log T$. However, the Matsubara sum $T \sum_{n=1}^{M} \Omega_n$ only contains a $T$-independent term, of order $E_F^2$, and a term $-1/(6\pi^2)T^2$. This last term is universal, but it is analytic in $T$. As a result, $\Delta \chi_s(T, 0) \propto T^2$ and is analytically and essentially irrelevant as the analytic in $T$ contributions to $\chi_s(T)$ are already present in the Lindhard function for free fermions. The absence of the non-analytic-temperature correction to the spin susceptibility in 3D was first noticed in Ref. [19], (see also [2]). The second term in the r.h.s. of (16) is the extra contribution in a finite field. Evaluating the Matsubara sum we find that this contribution scales as $H^2 \log \{ \max(T, \mu_B H) \}$. We see therefore that in a finite magnetic field, $\chi_s(T)$ does indeed become non-analytic. Casting $\Delta \chi_s(T, H)$ into the scaling form, we obtain

$$\Delta \chi_s(T, H) = \chi_0 \left( \frac{mU k_F^2}{2\pi^2} \right)^2 \left( \frac{T}{E_F} \right)^2 g \left( \frac{\mu_B H}{T} \right),$$

where $\chi_0 = \mu_B^2 k_F^3/(2\pi^2 E_F)$ is Pauli susceptibility, and to a logarithmic accuracy,

$$g(x) = x^2 \log \left[ \frac{E_F}{T \max\{x, 1\}} \right].$$

The $H^2 \log H$ dependence of $\chi(H)$ was earlier reported by Misawa [10]. However, his prefactor is different from the one we obtained.

Differentiating the thermodynamic potential twice over $T$, we also obtain field dependence of the specific heat coefficient. The field dependence in 3D parallels the one for 2D systems. Namely, at zero field,

$$\Delta \gamma(T, 0) = \frac{3}{20} \left( \frac{m k_F}{T} \right)^2 \left( \frac{T}{E_F} \right)^2 \ln \left[ \frac{E_F}{T} \right].$$

In a finite field, the charge part is not affected, while in the spin part, the logarithmic factor $3 \log \frac{E_F}{T}$ is replaced by $\log \frac{E_F}{T} + 2 \log \frac{E_F}{\max(T, \mu_B H)}$. As a result, at $\mu_B H \gg T$, $\Delta \gamma(T, H)$ still behaves as $T^2 \log T$, but the prefactor gets smaller.

To summarize, in this paper we analyzed non-analytic terms in the magnetization, the spin susceptibility and the specific heat of 2D and 3D Fermi liquids, placed into an external magnetic field $\mu_B H \ll E_F$. We obtained the non-analytic terms in the forms of scaling functions of $\mu_B H/T$. We found that at $\mu_B H \gg T$, the spin susceptibility scales as $|H|$ in 2D and as $H^2 \log |H|$ in 3D. The specific heat in a field preserves the same temperature dependence as in the absence of a field, but the prefactor changes between small and large $\mu_B H/T$.

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