RADIATIVE HYDRODYNAMICS IN THE HIGHLY SUPER ADIABATIC LAYER OF STELLAR EVOLUTION MODELS

F.J. Robinson\textsuperscript{1}, P. Demarque\textsuperscript{1}, S. Sofia\textsuperscript{1}, K.L. Chan\textsuperscript{2}, Y.-C. Kim\textsuperscript{3}, and D. B. Guenther\textsuperscript{4}

\textsuperscript{1}Yale University, New Haven CT USA
\textsuperscript{2}Hong Kong University of Science & Technology, Hong Kong, China
\textsuperscript{3}Yonsei University, Seoul, South Korea
\textsuperscript{4}St Mary’s University, Halifax, N.S., Canada

ABSTRACT

We present results of three dimensional simulations of the uppermost part of the sun, at 3 stages of its evolution. Each model includes physically realistic radiative-hydrodynamics (the Eddington approximation is used in the optically thin region), varying opacities and a realistic equation of state (full treatment of the ionization of H and He). In each evolution model, we investigate a domain, which starts at the top of the photosphere and ends just inside the convection zone (about 2400 km in the sun model). This includes all of the super-adiabatic layer (SAL). Due to the different positions of the three models in the \[ \log(g) \] vs \[ \log T_{\text{eff}} \] plane, the more evolved models have lower density atmospheres. The reduction in density causes the amount of overshoot into the radiation layer, to be greater in the more evolved models.

Key words: Radiative hydrodynamics, compressible turbulence.

1. INTRODUCTION

In most of the convection zone, heat transport can be modelled quite well by using the mixing length theory. Due to vigorous turbulent mixing, the entropy is almost constant. This well mixed layer has an extremely small super-adiabatic temperature gradient \( (\nabla - \nabla_{\text{ad}} \approx 10^{-8}) \) and is practically opaque (the optical depth is greater than \( 10^4 \)). In such a layer, radiative transport accounts for a very small fraction of the heat flow. This changes further up. Near the top of the convection zone the gas density is very low, so the enthalpy flux is not big enough to carry all of the outward flowing heat. Radiation must carry most of the heat flux. To enable radiation to transport this heat flux, the temperature gradient must increase significantly. This region, in which \( \nabla - \nabla_{\text{ad}} \) is of order unity, is known as the super adiabatic layer (SAL).

The SAL is important for the following reasons:

- It is the site of maximum amplitude of the p-mode oscillations (particularly the high frequency modes).
- As most of the entropy change occurs within this layer, it plays an important role in determining the radius of stellar models (Larson 1974). (Note \( dS/d\ln P = c_p(\nabla - \nabla_{\text{ad}}) \).
- The amount of overshoot into the enveloping photosphere depends on the structure of the SAL.

In this poster we will describe three simulations of stellar atmospheres each centered around the SAL. We consider the sun at three different stages of its evolution, namely the ZAMS, present sun and subgiant. These are shown as three points in the theoretical HR diagram, figure 1.
2. MATHEMATICAL MODEL

2.1. Hydrodynamic equations

In the outer layers of the sun the Mach number, \((v/v_s)^2\) can be of order unity (Cox and Giuli 1968) (\(v\) is the flow velocity and \(v_s\) is the sound speed). In such an environment, the governing hydrodynamic equations are the fully compressible Navier Stokes equations (cf. Kim et al 1995). These are modified to include radiative energy transport by the insertion of the \(Q_{\text{rad}}\) term (described later) into the energy equation. The full set of governing equations are:

\[
\begin{align*}
\frac{\partial p}{\partial t} & = -\nabla \cdot \rho \mathbf{v} \\
\frac{\partial \rho \mathbf{v}}{\partial t} & = -\nabla \cdot \rho \mathbf{v} \mathbf{v} - \nabla P + \nabla \cdot \mathbf{\Sigma} + \rho \mathbf{g} \\
\frac{\partial E}{\partial t} & = -\nabla \cdot [(E + P)\mathbf{v} - \mathbf{v} \cdot \mathbf{\Sigma} + f] + \rho \mathbf{v} \cdot \mathbf{g} + Q_{\text{rad}}
\end{align*}
\]

where \(E = e + \rho v^2/2\) is the total energy density and \(p, v, P, e\) and \(\mathbf{g}\) are the density, velocity, pressure, specific internal energy and acceleration due to gravity, respectively. Ignoring the coefficient of bulk viscosity (Becker 1968), the viscous stress tensor for a Newtonian fluid is \(\Sigma_{ij} = \mu (\partial v_i/\partial x_j + \partial v_j/\partial x_i) - 2\mu/3(\nabla \cdot \mathbf{v}) \delta_{ij}\). In LES, the dynamic viscosity \(\mu\) is increased so that it represents the effects of Reynolds stresses on the unresolved or sub-grid scales (Smagorinsky 1963),

\[
\mu = \rho (c_\mu \Delta)^2 (2\sigma : \sigma)^{1/2}.
\]

The colon inside the brackets denotes tensor contraction of the rate of strain tensor \(\sigma_{ij} = (\nabla_i v_j + \nabla_j v_i)/2\). The SGS eddy coefficient \(c_\mu\), is set to 0.2, the value for incompressible turbulence, and \(\Delta\) is an estimate of the local mesh size. The present formulation ensures that the grid Reynolds number \(\Delta \times v/\nu\) is of order unity everywhere. To handle shocks, \(\mu\) is multiplied by \(1 + C \cdot (\nabla \cdot \mathbf{v})^2\), where the constant \(C\) is made as small as possible to maintain numerical stability. As \(\mu\) is dependent on the horizontal divergence, any large horizontal velocity gradients are smoothed out by the increased viscosity. The diffusive flux \(f = -(\mu/Pr)T \nabla S\), where the horizontal mean of \(\nabla S \geq 0\) i.e. the convection zone, and \(f = -(\mu v_p/Pr) \nabla T\) where the horizontal mean of \(\nabla S \leq 0\) i.e. the radiation zone. The Prandtl number \(Pr = \nu/\kappa\) (\(\nu\) is the kinematic viscosity and \(\kappa\) is the thermal diffusivity) is 1/3. Due the inclusion of radiative energy transport, the effective \(Pr\) is actually much smaller and not constant.

2.2. Radiative energy transport

In the deeper part of the domain, radiative transfer is treated by the diffusion approximation,

\[
Q_{\text{rad}} = \nabla \cdot \left[ \frac{4acT^3}{3\kappa \rho} \nabla T \right],
\]

where \(\kappa\) is the Rosseland mean opacity, \(a\) is the Boltzmann constant and \(c\) is the speed of light.

In the shallow region, the photon mean free path is at least one tenth of the depth of the atmosphere so the diffusion approximation may not apply. Instead \(Q_{\text{rad}}\) is computed as

\[
Q_{\text{rad}} = 4\kappa \rho (J - B)
\]

where mean intensity \(J\) is computed by using the generalised three-dimensional Eddington approximation (Unno and Spiegel 1966),

\[
\nabla \cdot \left( \frac{1}{3\kappa \rho} \nabla J \right) - \kappa \rho J + \kappa \rho B = 0.
\]

This is exact for isotropic radiation, and in non-equilibrium the Eddington approximation describes the optically thick and thin regions exactly.

3. NUMERICAL METHODS

The simulation domain is a small box of aspect ratio 1.5, which includes the photosphere and top part of convection zone. In the sun model this represents \(6.936 \times 10^{10}\text{cm} < R < 6.960 \times 10^{10}\text{cm}\). Each of the three models span about 8 pressure scale heights.

The governing equations are discretised in cartesian coordinates on \(80^3\) uniformly spaced grid points. Using a code developed by Chan and Wolff (1982), an implicit scheme (the Alternating Direction Implicit Method on a Staggered grid or ADISM) relaxes the fluid to a self consistent thermal equilibrium. In the fully relaxed layer, the energy flux leaving the top of the box is within 5 \% of the input flux at the base. Next a second order explicit method (Adams Bashforth time integration) gathers the statistics of the time averaged state. The statistical integration time is about 2500 seconds of solar surface convection, and requires about a million time steps. On an 667 MHz Alpha processor, each integration step requires about 10 seconds of CPU time. Consequently each simulation takes at least 3 months to run.

3.1. Modelling a stellar atmosphere

A Standard Solar model, calculated with the Yale Stellar Evolution model (Guenther & Demarque 1997), is used to compute the initial stratification i.e. run of pressure, temperature, density, internal energy, for the box. The model uses realistic physics, Alexander low temperature opacities and the OPAL opacities and Equation of State (Hydrogen and helium ionisation zones are included). The hydrodynamical simulations use identical opacities and e.o.s, as used in the 1-d stellar model (Kim & Chan 1998). The horizontal boundaries are periodic, while the vertical boundaries are stress free. A constant heat flux flows through the base, and the top is a perfect
To ensure mass, momentum and energy are fully conserved, we use impenetrable (closed) top and bottom boundaries.

4. RESULTS AND INFERENCES

4.1. Turbulent quantities

Each parameter, which consists of a mean and a fluctuating part, is computed from the 3-d statistically averaged flow. For a given parameter $X$, the turbulent part is approximated by the variance,

$$x = \sqrt{X^2 - \overline{X}^2},$$

where the overbar denotes horizontal and temporal averaging, and $X$ is the total quantity (mean plus fluctuating). The autocorrelation between two fluctuating quantities $X'_1$ and $X'_2$ is computed as

$$C[X'_1 X'_2] = (X_1 - \overline{X}_1)(X_2 - \overline{X}_2)/x_1x_2.$$

4.2. Structure of the SAL in the three models

Figures 2, 3 and 4 show the entropy gradient and the autocorrelation between vertical velocity fluctuation and temperature, $C[V'_z T']$, vs depth, using $\ln P$ as the unit of depth.

The SAL peak is higher and sharper in the subgiant model compared to the ZAMS model. This is because density is much lower in the SAL region for the subgiant, than it is for the ZAMS. Convection is less efficient, and thus $\nabla - \nabla_{ad}$ (or similarly $dS/d\ln P$) needs to be steeper and higher, so that radiation can carry more heat. Generally, the SAL regions are radially further outwards in progressively more evolved models.

The difference between the top of the convection zone, as defined by $dS/dz = 0$ (Schwarzschild criterion), and the point where the enthalpy flux, $\rho c_p C[V'T']\omega t$, is zero, is one measure of the amount of overshoot into the photosphere. The figures (which exclude the two outermost levels) show that the overshoot increases slightly between the ZAMS and sun models, while extending high up into the lower density region in the subgiant model. It is important to note that because the grid points are uniformly spaced, the resolution of the SAL is worse in the subgiant than in the ZAMS. As the SAL occurs higher up in the subgiant where the pressure scale height is smaller, there are less grid points between $\ln P$ equals 11 and 12, than between 12 and 13. Clearly it is very important to have enough grid points to model the SAL in more evolved stellar models.
4.3. Turbulent velocities

The turbulent velocities $u$, $v$ and $w$, computed as described in section [1] are shown in figures 5, 6 and 7. The closeness of the turbulent horizontal velocities ($u$ and $v$) indicates the degree of statistical convergence. The ZAMS and sub-giant models are fully converged, while the sun is close to convergence.

ACKNOWLEDGEMENT

This research was supported in part by NASA grant NAG5-8406 to Yale University. Support from the Creative Research Initiative Program of The Korean Ministry of Science and Technology (Y.-C. Kim) and from the National Science and Engineering Research Council of Canada (D. B. Guenther) are also gratefully acknowledged. F.J.Robinson acknowledges helpful comments from H. Ludwig in improving this poster.

REFERENCES

Becker, E., 1968, Gas Dynamics, Academic Press, p.229
Chan K. L. & Wolff, C. L. J. Comp. Physics f 47, 109 1982.
Cox & Giuli Principles of Stellar Structure 1968 (Gordon and Breach, New York)
Guenther, D.B. & Demarque, P. 1997, ApJ, 484, 937
Kim, Y.-C., Fox, P.A., Sofia, S., and Demarque, P., (1995), ApJ 442, 422
Kim, Y.-C. & Chan, K.L. 1998, ApJ, 496, L121.
Larson, R.B. 1974, Fund. Cosmic Phys.,1,1
Smagorinsky, J.S., 1963, Mon. Weather. Rev., 91, 99
Unno, W., & Spiegel, E. A. 1966, PASJ, 18, 85