Robust Generalized Sidelobe Canceller with an Eigenanalysis-Based Blocking Matrix

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Abstract. In acoustic environments with strong background noise and interferences, the generalized sidelobe canceller (GSC) suffers from signal distortion and reduced amount of noise suppression since the blocking matrix cannot produce pure noise reference due to unavoidable direction of arrival (DOA) mismatch. In this paper, a generalized sidelobe canceller robust to DOA mismatch is proposed. It employs eigenanalysis to reconstruct the interference-plus-noise covariance matrix with the possible DOA region of signal of interest (SOI). The reconstructed matrix is provided for the MaxSINR beamforming and subsequent blocking matrix construction. The designed blocking matrix can minimize the signal leakage meanwhile maximize the noise and interference power for further noise and interference suppression. Numerical simulation results show that compared with the alternatives, the proposed GSC has better interference suppression capability and lower SOI distortion in situations without precise DOA of SOI.

Introduction

In sophisticated environments, the desired signal is always concealed under the strong background noise and interferences, causing target localization and recognition to be very difficult. Adaptive beamforming as an important theory to suppress noise and extract desired signal has been researched in the past decades [1, 2]. However, it is very sensitive to model mismatch including covariance matrix mismatch and steering vector mismatch.

A very famous and robust beamforming is the GSC. Griffiths and Jim introduced the GSC as a time domain unconstrained adaptive beamformer [3]. However, facing the DOA mismatch, the blocking matrix merely subtracts pairs of time-aligned signals, thus cannot produce pure noise reference. Recently, Warsitz \textit{et al} proposed a generalized eigenvalue decomposition (GEVD) based blocking matrix aiming to minimize the signal leakage [4], and the fixed beamformer coefficients are determined using the Maximum-SINR-criterion [5]. Its robustness depends on interference-plus-noise covariance matrix estimation using pure noise periods. However, this kind of pure period is unavailable in many acoustic applications due to strong background noise. In [6], the SOI covariance matrix was reconstructed using received signal samples by building robust power ratios to identify all the eigenvectors not dominated by the SOI and subtracting them from the matrix. It shows robustness, but cannot produce the enhanced waveform.

In this paper, a robust GSC whose blocking matrix aims to produce pure noise reference is proposed. Facing the situations with DOA mismatch, it uses eigenanalysis to adaptively estimate the interference-plus-noise covariance matrix with received data samples. Then the blocking matrix can be obtained based on Maximum-SINR-criterion. It features in minimizing signal leakage meanwhile maximizing the noise and interference power. Numerical simulations confirm that this method shows stronger robustness to DOA mismatch meanwhile maintains good performance.
Signal Model

Assume that an array consisting of \( M \) sensors receives signals from \( J \) wideband sources in separated directions. The received observation vector at the \( k \)th snapshot and the frequency \( \omega \) in a single block can be modeled as

\[
X(\omega, k) = A(\omega, \phi)S(\omega, k) + N(\omega, k),
\]

(1)

where \( X(\omega, k) = [x_1(\omega, k) \cdots x_m(\omega, k) \cdots x_M(\omega, k)]^T \), \((\cdot)^T\) denotes the transpose, \( S(\omega, k) \) and \( N(\omega, k) \) are the vector of \( J \) sources and independent stationary noise, and \( A(\omega, \phi) = [a(\omega, \phi_1) \cdots a(\omega, \phi) \cdots a(\omega, \phi_{J-1})] \) consists of steering vectors for all sources.

The frequency domain covariance matrix estimated with \( K \) snapshots in a single block at frequency \( \omega \) and its eigenvalue decomposition can be expressed as

\[
R_X(\omega) = \frac{1}{K} \sum_{k=1}^{K} X(\omega, k)X^H(\omega, k) = \sum_{m=1}^{M} \lambda_m(\omega) v_m(\omega) v_m^H(\omega),
\]

(2)

where \((\cdot)^H\) denotes conjugate transpose of a vector or a matrix, \( \lambda_m(\omega) \) and \( v_m(\omega) \) denote \( m \)th eigenvalue and eigenvector of \( R_X(\omega) \), respectively.

The Proposed Algorithm

The block diagram of the proposed robust GSC is shown in Fig. 1. The structure consists of five blocks. In terms of robustness, the conventional beamformer (CBF) is chosen to be the fixed beamformer. It is used to produce an enhanced desired signal reference. The interference-plus-noise covariance matrix reconstruction block eliminates SOI components and output what the MaxSINR beamformer block needs. The following null space blocking matrix produces noise-only reference signals and the adaptive noise canceller (ANC) block attempts to cancel the residual noise in the signal reference.

Figure 1. The proposed GSC structure.
Eigenanalysis Based Interference-plus-noise Covariance Matrix Reconstruction

In order to reconstruct interference-plus-noise covariance matrix, the eigenvalues and the corresponding eigenvectors for interference and noise should remain, while the eigenvalues and the corresponding eigenvectors for SOI should be removed.

A power ratio aiming to determine which of the eigenvectors is dominated by SOI is defined as

$$PR_m = \frac{\max_{\phi_{\Theta_{SOI}}} BV_m (\omega, \phi)}{\max_{\phi \in [-90', 90']} BV_m (\omega, \phi)}, m = 1, 2, ..., M,$$

where $BV_m (\omega, \phi) = a^H (\omega, \phi)v_m (\omega)\nu_m (\omega)a(\omega, \phi)$ is the CBF spatial spectra of every eigenvector $v_m (\omega)$. It can be interpreted as the ratio between the maximum of the CBF power spectra of $v_m (\omega)$ in angular sector $\Theta_{SOI}$ where the desired signal is located and the maximum in the whole direction region [6].

Then we can obtain that,

$$\begin{cases} PR_m < \text{threshold} \Rightarrow v_m (\omega) \in U_{IPN} (\omega) \\ PR_m \geq \text{threshold} \Rightarrow v_m (\omega) \in U_{SOI} (\omega), m = 1, 2, ..., M \end{cases}$$

where $U_{SOI} (\omega)$ and $U_{IPN} (\omega)$ denote the set of eigenvectors dominated by the SOI and the set of eigenvectors dominated by the interferences and noise components, respectively.

Thus, the interference-plus-noise covariance matrix can be reconstructed as

$$R_{IPN} (\omega) = P_z (\omega)R_x (\omega)P_z^H (\omega),$$

where $P_z (\omega) = U_{IPN} (\omega)U_{IPN}^H (\omega)$ is the projection matrix.

Please note that in ideal condition, the threshold of PR is equal to one. However, in practice, especially when the number of interferences are big or the power of them are extremely huge, the power of interferences and noise components may leak into SOI subspace and the CBF power spectra will be less accurate, causing $PR_m$ of the SOI eigenvector lower than one. So threshold = 0.95 can be chosen.

MaxSINR Beamformer and Null Space Blocking Matrix

Based on the reconstructed $R_{IPN} (\omega)$, the maxSINR beamformer aims to maximize the signal to interference-plus-noise ratio at the beamformer output. The beamformer’s weight vector can be expressed as

$$F_{\max\text{SINR}} (\omega) = \arg \max_{F (\omega)} \frac{F^H (\omega)R_{SOI} (\omega)F (\omega)}{F^H (\omega)R_{IPN} (\omega)F (\omega)} = \arg \max_{F (\omega)} \frac{F^H (\omega)R_x (\omega)F (\omega)}{F^H (\omega)R_{IPN} (\omega)F (\omega)},$$

where $R_x (\omega) = R_{SOI} (\omega) + R_{IPN} (\omega)$. For a single target (i.e. $R_{SOI} (\omega)$ is a rank-1 matrix), the solution of (6) can be obtained from GEVD [5], $F_{\max\text{SINR}} (\omega) = \eta F_{\max\text{eig}} (\omega)$, where $\eta$ is a complex-valued scalar and $F_{\max\text{eig}} (\omega)$ is the generalized eigenvector corresponding to the largest generalized eigenvalue. Then the output of the beamformer is expressed as

$$Y_{\max\text{SINR}} (\omega) = F_{\max\text{SINR}}^H (\omega)X(\omega).$$
The MaxSINR beamformer doesn’t impose distortionless constrains, so it will produce a lot of distortion, thereby it can be utilized to construct blocking matrix rather than the fixed beamformer. The proposed null space-blocking matrix is designed to produce noise reference signals $U(m)$ orthogonal to the SOI. Similar to the derivation in [4], we obtain the projection vector,

$$P(m) = \frac{R_X(m)F_{\max SINR}(m)}{F_{\max SINR}^H(m)R_X(m)F_{\max SINR}(m)}.$$  (8)

We use $R_X(m)$ in (8), which is very different with [4]. In addition, it will be beneficial to the output, because the $R_{\text{IPN}}(m,t)$ estimation has unavoidable error. So, the null space-blocking matrix

$$B(m) = I_M - P(m)F_{\max SINR}^H(m)$$  (9)

ensures to minimize the signal leakage meanwhile maximize the noise and interference power for further noise and interference suppression in the ANC.

**Simulation**

We assume a uniform linear array with $M = 40$ omnidirectional sensors spaced 1.5m apart. The interest frequency band is [400Hz,500Hz], and sound speed is 1500m/s. One SOI and two interferences with plane wavefronts are stationary Gaussian white noise with bearing 5°, 10° and 30° respectively. In addition, the additive noise is Gaussian zero-mean temporally white process. The SOI-to-noise ratio (SNR) in each sensor ranges from -10 dB to 0dB; the SOI-to-interference ratio (SIR) for every interference in each sensor is equal to -10dB. As mentioned, $\text{threshold} = 0.95$ is taken for proposed GSC, the number of snapshots used is 0.5*M, the DOA error is fixed at -1°. The SOI possible DOA region is set as $[3°,5°]$ to center the nominal SOI DOA (4°) with a span 1° to include the actual DOA but to exclude interference DOAs. All algorithms are implemented in frequency domain block by block with the FFT length 1024, and the time domain output waveform is obtained using the overlap and add method [7]. And every point is measured after convergence of the ANC [4].

The proposed GSC is compared with the CBF, the traditional GSC [3], and the theoretic GSC without DOA mismatch. Four performance assessments are used, including correlation coefficients between output waveform and the pure SOI waveform, the output SINR, the signal distortion and the noise reduction $NR$. The last two are expressed as,

$$\text{distortion} = 10\log \left( \frac{\sum (s(m) - y_{\text{out}}(m))^2}{\sum s(m)^2} \right), \text{NR} = 10\log \left( \frac{\sum (n_{\text{in}}(m))^2}{\sum n_{\text{out}}(m)^2} \right),$$  (10)

where $s(m), y_{\text{out}}(m), n_{\text{in}}(m), n_{\text{out}}(m)$ denote input pure SOI waveform, output waveform, input noise-plus-interference waveform, output noise-plus-interference waveform.
The performance comparisons of GSC algorithms in terms of the input SNR variations are illustrated in Fig. 2. Though both the proposed GSC and the traditional GSC gained higher NR than the theoretic GSC at the price of distortion (theoretic distortion is -31dB over all SNRs), the proposed GSC outperformed the traditional GSC with remarkable improvements in correlation coefficients with pure SOI and output SINR. Due to DOA mismatch, the traditional GSC suffered from serious distortion, while the proposed produced less distortion, which was almost the same level of distortion as the CBF, and higher amount of noise reduction. During the period of interference-plus-noise covariance matrix reconstruction, the proposed method can preserve almost every eigenvector dominated by interference and noise and remove almost every SOI eigenvector with a robust threshold. Therefore, the distortion and noise reduction are minimized and maximized in the ANC, respectively.

Conclusion

In this paper, a generalized sidelobe canceller robust to DOA mismatch with an eigenanalysis-based blocking matrix is proposed. The key idea is to reconstruct interference-plus-noise covariance matrix from fewer array samples employing eigenanalysis to remove SOI subspace adaptively. Then the extracted covariance matrix is fed into a MaxSINR beamformer for subsequent blocking matrix construction. The blocking matrix is obtained by projecting the MaxSINR beamformer weight vector to the interference-plus-noise subspace. It can minimize the SOI leakage meanwhile maximize the noise and interference power in the noise reference, which is provided for the ANC to get the enhanced waveform. Simulation results demonstrates that when facing serious DOA mismatch, the proposed GSC gains remarkable improvements in the output SINR and correlation coefficients with the pure desired signal. In addition, it can still work even when the number of snapshots is fewer than that of the sensors.

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