Optimization Of Queueing Model

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Abstract: In the paper, we are considering the single server queueing system have interdependent arrival of the service processes having bulk service. In this article, we consider that the customers are served \( k \) at any instance except when less then \( k \) are in the system & ready to provide service at which time customers are served.

Keyword: -Interdependent queueing models, arrival process, service process, waiting line system, mean dependence.

1. OPTIMIZATION M/M\([k]\)/1 QUEUEING MODEL WITH VARYING BATCH SIZE :-

In this type of systems, the interdependence could be induced by considering the dependent structure with parameters \( \lambda \), \( \mu \) and \( \epsilon \) as marginal arrival rate, service rate and mean dependence rate respectively.

Let \( P_0 (t) \) be the probability when there are \( n \) customers in system at time \( t \). The difference – differential equations of above model may have written as,

\[
P'_0 (t) = -\left( \lambda + \mu - 2 \epsilon \right) P_0 (t) + (\lambda - \epsilon) P_{n-1} (t) + P_{n-k} (t); \quad n \geq 1
\]

\[
P'_n (t) = -\left( \lambda - \epsilon \right) P_0 (t) + (\mu - \epsilon) \sum_{i=1}^{k} P_i (t)
\]

Let us consider that, the system achieved the steady state, therefore the transition equations of considered model are,

\[
-(\lambda + \mu - 2 \epsilon) P_n + (\lambda - \epsilon) P_{n-1} + (\mu - \epsilon) P_{n-k} = 0 \quad ; \quad n \geq 1
\]

\[
-(\lambda - \epsilon) P_0 + (\mu - \epsilon) \sum_{i=1}^{k} P_i = 0
\]

Applying heuristic arguments of “Gross and Harris” (1974). One can obtain the solution of mentioned equations as,

\[
P_n = Cr^n \geq 0 \quad , \quad 0 < r < 1
\]

Where, \( r \) is the root of equations which lie in \((0, 1)\) of the characteristic equation.

\[
[(\mu - \epsilon)D^{k+1} - (\lambda + \mu - 2 \epsilon)D + (\lambda - \epsilon)]P_n = 0
\]

Here \( D \) represents the operator.

2. MEASURES OF EFFECTIVENESS: -

The probability that the system is empty is,

\[
P_0 = (1 - r)
\]

Where \( r \) is as given in equation (3).

For different values of \( \epsilon \& k \), for the given values of \( \lambda \) and \( \mu \), we are able to compute \( P_0 \) values & are given in table (5.1). The values of \( P_0 \) for the fixed \( k \), \( \lambda \) and \( \mu \) mentioned in the table (5.2).

From tables (5.1), (5.2) and equation number---(5), we observe that for fixed of \( \lambda \), \( \mu \) and \( \epsilon \), the value of \( P_0 \) increases with respect to increase \( k \). As the dependence parameter \( \epsilon \) increases the value of \( P_0 \) increases for fixed values of \( \lambda \) and \( \mu \). The value of \( P_0 \) decreases for fixed values of \( \mu, k \) and \( \epsilon \) as \( \lambda \) increases. As \( \mu \) increases the value of \( P_0 \) increases for fixed values of the \( \mu \), \( k \) and dependence parameter \( \epsilon \). If the mean dependence rate, is zero then the value of \( P_0 \) is also same as in the M/M\([k]\)/1 – model.

The average no. of customers in the system can obtained as

\[
L = \frac{r}{1 - r}
\]

and mean number, of customers in the queue are

\[
L_q = \frac{r^2}{1 - r}
\]
where \( r \) is as given in equation (3).

The value of \( L \) and \( L_q \) has been computed and given in table 5.3 and table 5.5 for provided values of \( \lambda, \mu \) and for different values \( \varepsilon \) and \( k \) respectively. The values of \( L \) and \( L_q \) for fixed values of \( \varepsilon \) and \( k \) and \( \lambda \) also given in tables 5.4 and 5.6.

By equations 6 and 7, also for the corresponding tables we observe, that as \( \varepsilon \) increases, the values of \( L \) and \( L_q \) are decreasing and also as \( k \) increases the values of \( L \) and \( L_q \) are decreasing for fixed values of other parameters. As the arrival rate increases, the values of \( L \) and \( L_q \) are increasing for fixed values of \( \mu, k \) and \( \varepsilon \). As \( \mu \) increases the values of \( L \) and \( L_q \) are decreasing for fixed values of \( \lambda, k \), and \( \varepsilon \). When the dependence parameter \( \varepsilon = 0 \) then the average queue length is same as that of \( M/M^{1[k]}/1 \) model. When \( k = 1 \) this is same as \( M/M/1 \) interdependence model.

The variability of this model can be obtained as

\[
    V = \frac{r}{(1 - r)^2} \quad \text{……………… (8)}
\]

where \( r \) is as given in equation (3).

The coefficient of variation of the model is

\[
    C. V = \frac{\sqrt{V}}{L} \times 100 \quad \text{……………… (9)}
\]

Where \( L \) & \( V \) are provided as in equations (6) and (7).

The values of ‘variability of system’ and ‘coefficient of variation’ for various values of \( k, \varepsilon \) for fixed values of \( \lambda, \mu \) are computed which are given in tables 5.7 & 5.9. The values of ‘variability of the system’ and ‘coefficient of variation’ for fixed values of \( k, \varepsilon \) and for various values of \( \lambda, \mu \) are provided in tables (5.8) and (5.10).

From equation 9 a& from the corresponding table we can observe that as \( \mu \) increases the “variability of the system size” decreases and “coefficient of variation” increases. As \( \lambda \) increases and for fixed values of \( \mu, \varepsilon \) and \( k \), the ‘variability of the system size’ increases & the ‘coefficient of variation decreases. We may observe that as \( \varepsilon \) increases the ‘variability of the system size’, decreases and ‘coefficient of variation’ increases for fixed values of \( \lambda, \mu \) and \( k \). As \( k \) increases, the ‘variability of the system’ decreases and the ‘coefficient of variation increases’.

For this model \( \varepsilon = 0 \) and \( k = 1 \) reduces to \( M/M/1 \) classical model. The mean-queue length & ‘variability of the system size’ of this model are less than that of the classical. When \( k = 1 \), this model becomes \( M/M/1 \) independent model for \( \varepsilon = 0 \), this model is same as \( M/M^{1[k]}/1 \) model.

**TABLE 1.1**

VALUES OF \( P_0 \)

\( \lambda = 3 \), \( \mu = 5 \)

| \( K/\varepsilon \) | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|-------------------|-----|-----|-----|-----|-----|
| 1.                | 0.4000 | 0.4167 | 0.4348 | 0.4545 | 0.4762 |
| 2.                | 0.5780 | 0.5871 | 0.5971 | 0.6081 | 0.6203 |
| 3.                | 0.6106 | 0.6182 | 0.6264 | 0.6357 | 0.6459 |
| 4.                | 0.6201 | 0.6270 | 0.6347 | 0.6433 | 0.6529 |
| 5.                | 0.6214 | 0.6300 | 0.6374 | 0.6458 | 0.6551 |

**TABLE 1.2**

“VALUES OF \( P_0 \)” for \( K = 2 \& \varepsilon = 5 \)

| \( \mu/\lambda \) | 1   | 2   | 3   | 4   | 5   |
|-------------------|-----|-----|-----|-----|-----|
| 1.                | 0.9024 | 0.7681 | 0.6548 | 0.5551 | 0.4649 |
| 2.                | 0.9161 | 0.7983 | 0.6975 | 0.6081 | 0.5269 |
| 3.                | 0.9265 | 0.8214 | 0.7305 | 0.6493 | 0.5752 |
| 4.                | 0.9345 | 0.8397 | 0.7568 | 0.6823 | 0.6141 |
| 5.                | 0.9410 | 0.8545 | 0.7783 | 0.7094 | 0.6461 |
### TABLE 1.3
VALUES OF $L$

| $K/\xi$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|--------|-----|-----|-----|-----|-----|
| 1      | 1.5000 | 1.3998 | 1.2999 | 1.2002 | 1.1000 |
| 2      | 0.7301 | 0.7033 | 0.6748 | 0.6445 | 0.6121 |
| 3      | 0.6377 | 0.6176 | 0.5964 | 0.5731 | 0.5482 |
| 4      | 0.6126 | 0.5949 | 0.5755 | 0.5545 | 0.5316 |
| 5      | 0.6093 | 0.5873 | 0.5689 | 0.5485 | 0.5295 |

### TABLE 1.4
VALUES OF $L$

| $K = 2$ | $\xi = 0.4$ |
|--------|--------------|
| $\lambda/\mu$ | 1 | 2 | 3 | 4 | 5 |
| 1      | 0.1082 | 0.3019 | 0.5272 | 0.8015 | 0.1510 |
| 2      | 0.0916 | 0.2527 | 0.4337 | 0.6445 | 0.6121 |
| 3      | 0.0793 | 0.2174 | 0.3689 | 0.5401 | 0.7385 |
| 4      | 0.0701 | 0.1909 | 0.3214 | 0.4656 | 0.6284 |
| 5      | 0.0627 | 0.1703 | 0.2849 | 0.4096 | 0.5477 |

### TABLE 1.5
VALUES OF $L_q$

| $K/\xi$ | 0.0 | 0.2 | 0.4 | 0.6 | 0.8 |
|--------|-----|-----|-----|-----|-----|
| 1      | 0.9000 | 0.8165 | 0.7347 | 0.6547 | 0.5762 |
| 2      | 0.3081 | 0.2904 | 0.2228 | 0.2526 | 0.2324 |
| 3      | 0.2483 | 0.2219 | 0.2228 | 0.2088 | 0.1941 |
| 4      | 0.2327 | 0.2219 | 0.2102 | 0.1978 | 0.1845 |
| 5      | 0.2307 | 0.2173 | 0.2063 | 0.1943 | 0.1816 |

### TABLE 1.6
VALUES OF $L_q$

| $K = 2$ | $\xi = 0.4$ |
|--------|--------------|
| $\lambda/\mu$ | 1 | 2 | 3 | 4 | 5 |
| 1      | 0.700 | 0.1820 | 0.3566 | 0.6159 |
| 2      | 0.0077 | 0.510 | 0.1321 | 0.2526 | 0.4248 |
| 3      | 0.0058 | 0.0388 | 0.0994 | 0.1941 | 0.3137 |
| 4      | 0.0046 | 0.0306 | 0.0782 | 0.1479 | 0.2425 |
| 5      | 0.0037 | 0.0248 | 0.632 | 0.1190 | 0.1938 |

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