Hadron Spectrum and the Infrared Behavior of QCD Coupling

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Abstract. A relativistic quantum-field model based on analytical confinement is developed to build stable bound states of spin-half and spin-one particles. The spectra of two-quark ground states are defined by using the ladder Bethe-Salpeter equation. An analytic expression for the running coupling in QCD is obtained that can be extended to the low energy region. By comparing the obtained result with recent experimental data on meson masses we obtain a new, independent and specific infrared-finite behavior of QCD coupling below energy scale ∼ 1 GeV. Particularly, an infrared-fixed point is extracted at \( \alpha_s(0) \approx 0.757 \). We also calculate masses of some conventional mesons in the range of 0.8 – 9.5 GeV.

1. Introduction
The study of the fundamental parameters of nature, the QCD effective coupling \( \alpha_s \) at large distances or, in the infrared (IR) region below 1 GeV is an active field of research in particle physics (e.g., [1, 2]). The polarization of QCD vacuum causes a variation of the physical coupling under changes of distance ∼ 1/Q, so QCD predicts a dependence \( \alpha_s \propto g^2/(4\pi) = \alpha_s(Q) \). This dependence is described theoretically by the renormalization group equations and determined experimentally at relatively high energies [3, 4]. However, the well-established conventional perturbation theory cannot be used effectively in the IR domain. Meanwhile, there exists a phenomenological indication in favor of a smooth transition from short distance to long distance physics [5]. Actual values of \( \alpha_s \) at a given \( Q \) are measured at relatively high energies (e.g., [3, 4], see Tab.1). Recent developments on this way were summarized in a number of articles (e.g., [2, 6, 7]). Many quantities in hadron physics are affected by the IR behavior of \( \alpha_s \) in different amounts. Nevertheless, the long-distance behavior of \( \alpha_s \) is not well defined, it needs to be more specified [8, 9, 10] and correct description of QCD effective coupling in the IR regime remains one of the actual problems in particle physics.

Below, we take into account the dependence of \( \alpha_s \) on mass scale \( M \) and determine the QCD effective charge in the low-energy region by exploiting the hadron spectrum [12]. In doing so, we determine the meson masses by solving the ladder Bethe-Salpeter equations for two-quark bound states within a relativistic field model [13].

2. Model
We investigate QCD effective (running) charge in the low-energy levels by exploiting the hadron spectrum. For the spectra of two-quark bound states we develop a relativistic quantum-field
model based on analytic (or, IR) confinement and consider the model Lagrangian:

\[ \mathcal{L} = -\frac{1}{4} \left( \partial^{\mu} A^{A}_\mu - \partial^{\nu} A^{A}_\nu - g f^{ABC} A^B_\mu A^C_\nu \right)^2 + \sum_f \left( \bar{q}_f \left[ \gamma^a \partial^a - m_f + g \gamma^C A^C_\mu \right] q_f \right) \],

(1)

where \( A^{C}_\mu \) is gluon adjoint representation, \( q^a_f - \) quark field and \( \Gamma^a_C = i \gamma^a C \). The model parameters are the confinement scale \( \Lambda \) and the constituent quark masses \( m_f = \{ m_u, m_s, m_c, m_t \} \).

The Green’s functions in QCD are tightly connected to confinement and are ingredients for hadron phenomenology. However, any widely accepted and rigorous analytic solutions to these propagators are still missing. One may encounter difficulties by defining the explicit quark and gluon propagator at the confinement scale [8]. Taking into account the correct global symmetry properties and their breaking, also by introducing additional physical parameters, may be more important than the working out in detail of propagators (e.g., [11]).

In previous papers we exploited relativistic quantum-field models based on analytical confinement (AC), where quark and gluon propagators were entire analytic functions in Euclidean space [12, 13]. Following [12] we introduce the quark propagator as follows:

\[ \tilde{S}^{ab}_{m}(\hat{p}) = \delta^{ab} \{ i \hat{p} + m_f [1 \pm \gamma_5 \omega(m_f/\Lambda)] \} / (m_f \Lambda) \cdot \exp \left\{ -p^2 + m_f^2 / (2 \Lambda^2) \right\} , \]

(2)

where \( \Lambda \) is the scale of AC and \( \omega(z) = (1 + z^2 / 4)^{-1} \). In (2) chiral symmetry breaking is induced by AC. A singular behavior \( \tilde{S}_{m}(\hat{p}) \sim 1/m_f \) for \( m_f \to 0 \) is connected with the zero-mode solution (the lowest Landau level) of the massless Dirac equation and generates a nontrivial behavior \( \sim \Lambda \).

Recent theoretical results predict an IR behavior of the gluon propagator [14, 15, 16, 17, 18]. We consider a gluon propagator in Feynman gauge as follows:

\[ \hat{D}^{AB}_{\mu\nu}(p) = \delta^{AB} \delta_{\mu\nu} 1 - \exp \left( -p^2 / 2 \Lambda^2 \right) \int_0^{1 / \Lambda^2} ds e^{-sp^2} . \]

(3)

It exhibits an explicit IR-finite behavior \( \hat{D}(0) \sim 1 / \Lambda^2 \) and an IR parametrization is hidden in \( \Lambda \).

3. Two-quark Bound States

The leading-order contribution to the \((q\bar{q})\) bound states is determined by partition function:

\[ Z_{q\bar{q}} = \int \int DqD\bar{q} \exp \left\{ - (\bar{q}S^{-1}q) + \frac{g^2}{2} (\bar{q}\Gamma Aq)(\bar{q}\Gamma Aq) \right\} \], \quad \langle(q\bar{q})\rangle_D = \int DA \ e^{-\frac{1}{2} (A^D^{-1}A)(\bullet)} . \]

(4)

Below we shortly introduce the basic steps entering into our model.

First, we allocate the one-gluon exchange between colored biquark currents and isolate the color-singlet combinations. Then, perform a Fierz transformation

\[ (i \gamma_{\mu}) \delta^{\mu\nu}(i \gamma_{\nu}) = \sum_J C_J : O_J O_J , \quad J = \{ S, P, V, A, T \} , \quad O_J = \{ I, i \gamma_5, i \gamma_{\mu}, \gamma_5 \gamma_{\mu}, i [\gamma_{\mu}, \gamma_{\nu}] / 2 \} . \]

By introducing a system of orthonormalized functions \( U_Q(x) \), where \( Q = \{ n_r, l, \mu \} \) are quantum numbers, we diagonalize the one-gluon exchange term. Then, we involve a Gaussian path-integral representation for the exponentials by introducing new auxiliary meson fields \( B_{N} \) with \( N = \{ Q, J, f_1, f_2 \} \). This allows us to take explicit path integration over quark variables.

We introduce a hadronization ansatz and identify \( B_{N}(x) \) with meson fields carrying quantum numbers \( N \). Isolate all quadratic field configurations (\( \sim B_{N}^2 \)) in the ‘kinetic’ term and rewrite the partition function for mesons [13]:

\[ Z_{q\bar{q}} \rightarrow Z = \int \prod_N DB_{N} \ \exp \left\{ - \frac{1}{2} \sum_{N,N'} (B_{N} [\delta^{NN'} + \alpha_s \lambda_{NN'}] B_{N'}) \right\} - W_{res}[B_{N}] \],

(5)
where the residual part $W_{res}[B_N] \sim 0(B_N^3)$ describes interaction between mesons.

The Fourier transform of the leading-order term of the polarization operator reads

$$\lambda_{J,J'}(p,x,y) = \frac{16\pi}{9} \sqrt{C_J C_{J'} D(x)D(y)} \int \frac{d^3k}{(2\pi)^3} e^{-i k (x-y)} \text{Tr} \left[ O_J \tilde{S}_m(k + \xi \vec{p}) O_{J'} \tilde{S}_m(k - \xi \vec{p}) \right],$$

where $\text{Tr} \equiv \text{Tr}_c \text{Tr}_\gamma \sum_{\pm}: \text{Tr}_c$ and $\text{Tr}_\gamma$ are traces taken on color and spinor indices, correspondingly.

We diagonalize the polarization kernel on the orthonormal basis $\{U_N\}$:

$$\int dxdy U_N(x) \lambda_{J,J'}(p,x,y) U_{N'}(y) = \delta^{NN'} \lambda_N(-p^2)$$

that is equivalent to the solution of the corresponding ladder BSE.

In relativistic quantum-field theory a stable bound state of $n$ massive particles shows up as a pole in the $S$ matrix. Accordingly, the meson masses may be derived from the equation [12]:

$$1 + \alpha_s(M^2_N) = 0, \quad -p^2 = M^2_N. \quad (6)$$

We use the meson mass $M$ as the appropriate characteristic parameter, so the coupling $\alpha_s(M)$ is defined in a timelike domain. On the other hand, the most of known data on $\alpha_s(Q)$ are possible in spacelike domain. The continuation of the invariant charge from one to another domain is elaborated by making use of the integral relationships (e.g. [9, 19]).

4. Conventional Meson Spectrum and Running Coupling

The dependence of meson masses on $\alpha_s$ is defined by Eq. (6). The polarization kernel $\lambda_N$ is obtained real and symmetric that allows us to find a simple variational solution to this problem.

Further we exploit Eq. (6) in different ways, by solving either for $\alpha_s$ at given masses, or for $M_J$ at known values of $\alpha_s$. We adjust the model parameters by fitting experimental data for different values of confinement scale. As a particular case, first we choose $\Lambda_1 = 345$ MeV.

![Figure 1](image-url)

**Figure 1.** Our estimates of $\alpha_s(M)$ in the low-energy region at different values of confinement scale (in the left panel: dots for $\Lambda = 330$ MeV, rombs for $\Lambda = 345$ MeV, squares for $\Lambda = 360$ MeV and dotted curves are the envelope lines) compared with the three-loop analytic coupling $\alpha_s(Q)$, its perturbative counterpart and the massive one-loop analytic coupling (in the right panel: solid, dot-dashed and dashed curves, correspondingly [20]).

1) First, we derive meson mass formula and adjust the constituent quark masses \{m_{ud}, m_s, m_c, m_b\} by fitting heavy meson masses $T(9460), J/\Psi(3097), D_s^*(2112)$ and $D^*(2010)$. We fix a particular set of model parameters as follows [12]:

$$\Lambda = \Lambda_1 = 345 \text{ MeV}, \quad m_{ud} = 192.56 \text{ MeV},$$

$$m_s = 293.45 \text{ MeV}, \quad m_c = 1447.59 \text{ MeV}, \quad m_b = 4692.51 \text{ MeV}. \quad (7)$$
QCD running coupling in Euclidean and Minkowskian domains. We show that for $q^2 \to 0$, the dependence of $\hat{S}_m(p) \sim 1/m_f$ of the quark propagator for $m_f \to 0$. On the other hand, this allows us to describe correctly light meson masses including $\pi(138)$ and $K(495)$. The possibility that the QCD coupling constant features an IR-finite behavior has been extensively studied in recent years (e.g., [21, 22, 23]).

Note, the singular behavior $\hat{S}_m(p)$ is the singular behavior $\tilde{\eta}$.

1) Derive the IR-fixed point $\hat{\alpha}_s(0)$ by evaluating Eq. (6) for $M = 0$ and $m_1 = m_2 = m$:

$$
\hat{\alpha}_s = \frac{3\pi^2 \mu^2}{8} e^{\mu^2 / \Lambda^2} \left\{ \max_{0 < c < 2} \int_0^1 \frac{du dw}{\sqrt{(1/u - 1)(1/w - 1)}} \left[ \int (1 - c^2) \left( \frac{1 + (1 + c(u + w))^2}{1 + c(u + w)w} \right) \right]^{-1} \right\}.
$$

The dependence of $\hat{\alpha}_s(0)$ on $\mu = m/\Lambda$ is plotted in Figure 2.

| $J^{PC} = 0^{-+}$ | $M_P$ | $J^{PC} = 0^{-+}$ | $M_P$ | $J^{PC} = 1^{--}$ | $M_V$ | $J^{PC} = 1^{--}$ | $M_V$ |
|------------------|------|------------------|------|------------------|------|------------------|------|
| $\pi(138)$ | 138 | $\eta_c(2980)$ | 3039 | $\rho(770)$ | 770 | $D_s^*(2112)$ | 2112 |
| $K(495)$ | 495 | $B(5279)$ | 5339 | $\omega(782)$ | 785 | $J/\Psi(3097)$ | 3097 |
| $\eta(547)$ | 547 | $B_s(5370)$ | 5439 | $K^*(892)$ | 892 | $B^*(5325)$ | 5357 |
| $D(1870)$ | 1941 | $B_c(6286)$ | 6489 | $\Phi(1019)$ | 1022 | $Y(9460)$ | 9460 |
| $D_s(1970)$ | 2039 | $\eta_b(9389)$ | 9442 | $D^*(2010)$ | 2010 |

Table 1. Estimated masses $M$ of conventional mesons (in units of MeV) at $\Lambda = 345$ MeV [12].

2) Having fixed quark masses, we solve an inverse problem, to estimate $\hat{\alpha}_s(M)$ in the region below 1 GeV by exploiting masses of mesons $\pi$, $K$, $\rho$, and $K^*$. The result is plotted in Fig. 1.

3) By interpolating smoothly $\hat{\alpha}_s(M)$ results into intermediate-energy above 1 GeV region and taking into account correct asymptotical, we define $\hat{\alpha}_s$ on a wide interval 0.14 – 9.5 GeV.

4) As an application, we also calculate masses of some conventional mesons, namely, for $\eta$, $\omega$, $D$, $F$, $D_s$, $\eta_c$, $B$, $B^*$, $B_s$, $B_c$, and $\eta_b$ [12]. Our estimates of meson masses along experimental data [2] are shown in Table 1. The relative error of our estimates does not exceed 3.5 per cent.

5. IR-finite Behavior of Effective Coupling

The possibility that the QCD coupling constant features an IR-finite behavior has been extensively studied in recent years (e.g., [21, 22, 23]).

Particularly, for $m = m_{ud} = 192.56$ MeV and $\Lambda = 345$ MeV we estimate

$$
\hat{\alpha}_s(0) = 0.757, \quad \text{or} \quad \hat{\alpha}_s(0)/\pi = 0.241.
$$

To compare our result with known data on $\alpha_s(Q)$ we exploit the relationships between the QCD running coupling in Euclidean and Minkowskian domains. We show that for $q^2 \to 0$ [12]

$$
\alpha_s(0) = \hat{\alpha}_s(0).
$$
Therefore, we may conclude that our result (9) is in a reasonable agreement with often quoted estimates

\[
\begin{aligned}
\alpha_s^0/\pi &\simeq 0.19 - 0.25 & [24], \\
\alpha_s^0/\pi &\simeq 0.265 & [25], \\
\alpha_s^0/\pi &\simeq 0.26 & [26], \\
\langle \alpha_s^0/\pi \rangle_{1 \text{GeV}} &\simeq 0.2 & [5]
\end{aligned}
\]  

(11)

and phenomenological evidences [1, 20]. The obtained IR-fixed value of the coupling constant is moderate, it depends on the mass of the lowest constituent quark.

In conclusion, we do not aim to obtain the behavior of the coupling constant at all scales. At moderate \(M^2 = -p^2\) we obtain \(\alpha_s\) in coincidence with the QCD predictions. However, at large mass scale (above 10 GeV) \(\alpha_s\) decreases much faster than expected by QCD prediction. The reason is the use of confined propagators in the form of entire functions. This leads to a rapid decreasing (or, a rapid growth in Minkowski space) of physical matrix elements.

Despite its pure model origin, our approach reveals a new, independent and specific IR-finite behavior of QCD coupling. As an application, we performed estimates on intermediate and heavy meson masses and the result was in reasonable agreement with experimental data.

We demonstrate that global properties of the low-energy phenomena such as QCD running coupling and conventional meson spectrum may be explained reasonably in the framework of a simple relativistic quantum-field model of quark-gluon interaction based on analytic (or, infrared) confinement. Our guess about the symmetry structure of the quark-gluon interaction in the confinement region has been tested and the use of simple forms of propagators has resulted in quantitatively reasonable estimates.

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