Helicity amplitude method for two electron positron pairs photoproduction.

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Abstract

We present the matrix element of $\gamma\gamma \to 4l$ process with four charged particles in final state. The constructions are performed in frame of Standard Model using the helicity amplitude method. Every possible polarization state of initial and final particles are considered.

1 Introduction

In designed and constructed future linear colliders besides of $e^-e^-$ and $e^+e^-$ interactions the realization $\gamma\gamma$ and $\gamma e$ modes are planned. In the last case high energy photon beams are created by Compton backscattering of initial photons on high energy electrons. This possibility will allow a detailed study of non-abelian nature of electroweak interaction, gauge boson coupling as well as the couplings of gauge bosons with Higgs particles if it is light enough to be produced. Since $W^\pm$ and Higgs bosons decay within detector they can be obtained via their decay products, for instance four leptons in final state.

For successful realization of such kind of experiments an exact calculation of all backgrounds and luminosity of initial beams value is required. These data can be obtained using consideration of $\gamma\gamma \to 4l$, $\gamma\gamma \to 2l$ and $\gamma\gamma \to 2l + \text{photons}$ processes.

Total cross sections of such interactions have been already calculated and analyzed in refs. [1]-[3] about 30 years ago and were found to be large enough:

$$\sigma = 6500\text{nb} \quad (\gamma\gamma \to 2e^-2e^+),$$
$$\sigma = 5.7\text{nb} \quad (\gamma\gamma \to e^+e^-\mu^+\mu^-),$$
$$\sigma = 0.16\text{nb} \quad (\gamma\gamma \to 2\mu^-2\mu^+).$$

However these calculations have used the low energy approximation, and obtained results are not applicable to analyze the result of high energy experiments.

The matrix element of $\gamma\gamma \to 4l$ process has been constructed also in ref. [4]. It was done in implicit form at limit of small polar angles of pair productions. However at that paper neither calculation of cross section no numerical analyze are present. So one could not perform any numerical congruence.

Besides that, processes of $\gamma\gamma$ scattering have considered in ref. [5]. There was applied the algorithm ALPHA for automatic computations of scattering amplitude. However modern high energy experiments require calculation of the cross section at definite polarization states of initial and final particles that ALPHA method couldn’t provide.

This process was also analysed in ref. [6], where several numerical calculations were performed, and the dependence of total cross section from the energy of initial beam was investigated.

Present paper is devoted to construction of the matrix element of $\gamma\gamma$ electroweak interaction with production of four leptons using helicity amplitude method [7]-[10]. Numerical integration of obtained amplitude under kinematics of future projects will be described in the next paper.
2 Construction and calculations

There are six topographically different Feynman diagrams of electroweak interaction describing process $\gamma \gamma \rightarrow 4l$ (see fig.1). Using C-, P- and crossing symmetries one can build 40 different diagrams.

![Feynman Diagrams](image)

*Fig.1.*

The diagrams containing charged current exchange are excluded because only process with four charged leptons in final state is considered. Matrix elements for remaining diagrams (1)-(3) have the following form:

\[
M_1 = \frac{-ie^4}{(k_1 - p_1 - p_2)^2} \overline{u}(p_1) \gamma^\mu(p_1) \xi(k_1) \left( \frac{p_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \gamma^\nu(p_2) \overline{u}(p_3) \gamma^\nu \hat{k}_2 - \hat{p}_4 + m \right) \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \xi(k_2) v(p_4) -
\]

\[
\times \frac{ie^2}{2\cos(\theta_W)} D_{\mu\nu}(k_1 - p_1 - p_2) \overline{u}(p_1) \xi(k_1) \left( \frac{p_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \right) \gamma^\mu(p_2) \xi(k_2) v(p_2) \overline{u}(p_3) \gamma^\nu \hat{k}_2 - \hat{p}_4 + m \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \xi(k_2) v(p_4),
\]

\[
M_2 = \frac{-ie^4}{(p_3 + p_4)^2} \overline{u}(p_1) \gamma^\mu(p_1) \xi(k_1) \left( \frac{p_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \gamma^\nu(p_2) \overline{u}(p_2) \gamma^\nu \hat{k}_2 - \hat{p}_4 + m \right) \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \xi(k_2) v(p_4) -
\]

\[
\times \frac{ie^2}{2\cos(\theta_W)} D_{\mu\nu}(p_1 + p_2) \overline{u}(p_1) \xi(k_1) \left( \frac{p_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \right) \gamma^\mu(p_2) \xi(k_2) v(p_2) \overline{u}(p_3) \gamma^\nu \hat{k}_2 - \hat{p}_4 + m \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \xi(k_2) v(p_4),
\]

\[
M_3 = \frac{-ie^4}{(p_1 + p_2)^2} \overline{u}(p_3) \gamma^\mu(p_1) \frac{p_1 + \hat{p}_2 + \hat{p}_3 + m}{(p_1 + p_2 + p_3)^2 - m^2} \xi(k_1) \left( \frac{p_1 - \hat{k}_1 + m}{(p_1 - k_1)^2 - m^2} \right) \gamma^\nu(p_2) \overline{u}(p_2) \gamma^\nu(p_2) \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \xi(k_2) v(p_2) -
\]

\[
\times \frac{ie^2}{2\cos(\theta_W)} D_{\mu\nu}(p_1 + p_2) \overline{u}(p_3) \gamma^\mu(p_2) \xi(k_1) \left( \frac{p_1 + \hat{p}_2 + \hat{p}_3 + m}{(p_1 + p_2 + p_3)^2 - m^2} \right) \gamma^\nu(p_2) \overline{u}(p_2) \gamma^\nu \frac{\hat{k}_2 - \hat{p}_4 + m}{(k_2 - p_4)^2 - m^2} \xi(k_2) v(p_2).\]
Here $\vec{p}_1 = p_1^\mu \gamma_\mu$, where $p_1^\mu$ is $\mu$-component of four momentum $p_1$; $\varepsilon(k_1) = \varepsilon^\mu(k_1) \gamma_\mu$, where $\varepsilon^\mu(k_1)$ is $\mu$-component of polarization vector of photon with four momentum $k_1$, $D_{\mu\nu}(q)$ — propagator of $Z^0$–boson with momentum $q$.

Corresponding cross section has the form:

$$\sigma = \frac{1}{4(k_1k_2)} \int \left| M \right|^2 d\Gamma,$$

where

$$d\Gamma = \frac{d^3p_1}{(2\pi)^32p_1^0} \frac{d^3p_2}{(2\pi)^32p_2^0} \frac{d^3p_3}{(2\pi)^32p_3^0} \frac{d^3p_4}{(2\pi)^32p_4^0} (2\pi)^4 \delta(k_1 + k_2 - p_1 - p_2 - p_3 - p_4)$$

is phase space element.

In this paper squared matrix elements are constructed using helicity amplitude method (see, for example, refs. [7, 10]). It allows to calculate cross section directly for each definite polarization state of initial and final particles. Matrix element constructed by this method consists of invariants without any bispinor, so many difficulties are excluded in squaring and numerical integrating.

We present here the final form of matrix elements of the diagrams (1) - (3) (see fig.1.) in case of electromagnetic $\gamma\gamma$–interactions 3 T. Shishkina, I. Sotsky.

Matrix elements (3) - (4) correspond to the first diagram and the main polarization states. Whole set of rearrangements of initial and final particles are considered in every equation (3) - (12).

For example:

\begin{align}
M_1(+, -, +, -) &= \left\{ -4(2e)^4 D(k_3^2) N_1(p_1, p_2) N_2(p_3, p_4) \left[ 2\sqrt{(k_1p_3)(k_1p_2)} \times \sqrt{(p_1p_2)(p_1p_3) e^{i(\Delta\varphi_1)} - (p_1p_2)(p_1p_3) - (k_1p_3)(k_1p_2)e^{i(\Delta\varphi_2)} \right] \sqrt{(p_1p_2)(p_3p_4)} \right\} - \left\{ (p_1 \leftrightarrow p_3) \right\} e^{i(\Delta\varphi_3)} - \left\{ (p_2 \leftrightarrow p_4) \right\} e^{i(\Delta\varphi_4)} + \left\{ (p_1 \leftrightarrow p_3, p_2 \leftrightarrow p_4) \right\} e^{i(\Delta\varphi_{1+})}, \\
M_1(+, -, -, +, -) &= \left\{ 4(2e)^4 D(k_3^2) N_1(p_1, p_2) N_2(p_3, p_4) \left[ 2\sqrt{(k_1p_3)(k_1p_1)} \times \sqrt{(p_1p_2)(p_3p_2)e^{i(\Delta\varphi_5)} - (p_1p_2)(p_3p_2) - (k_1p_3)(k_1p_1)e^{i(\Delta\varphi_6)} \right] \sqrt{(p_1p_2)(p_3p_4)} \right\} + \left\{ (k_1 \leftrightarrow k_2, p_1 \leftrightarrow p_2, p_3 \leftrightarrow p_4) \right\} e^{i(\Delta\varphi_7)}, \\
M_1(+, +, -, -) &= \left\{ 4(2e)^4 D(k_3^2) N_1(p_1, p_2) N_2(p_3, p_4) \left[ (p_1k_1) + (p_2k_1) - (p_1p_2) \right] \times (p_1p_2)(p_3p_4) \sqrt{(p_1p_2)(p_3p_4)} \right\} + \left\{ (k_1 \leftrightarrow k_2) \right\} e^{i(\Delta\varphi_{1+})} - \left\{ (p_1 \leftrightarrow p_3) \right\} e^{i(\Delta\varphi_8)} - \left\{ (p_1 \leftrightarrow p_3, k_1 \leftrightarrow k_2) \right\} e^{i(\Delta\varphi_8)} e^{i(\Delta\varphi_{1+})}, \\
M_1(+, +, -) &= \left\{ 4(2e)^4 D(k_3^2) N_1(p_1, p_2) N_2(p_3, p_4) \left[ (k_1p_1) + (k_1p_2) - (p_1p_2) \right] \times (p_1p_2)(p_3p_4) \sqrt{(p_1p_2)(p_3p_4)} \right\} + \left\{ (k_1 \leftrightarrow k_2) \right\} e^{i(\Delta\varphi_{1+})}.
\end{align}

Equations (7) and (8) describe the second diagram:
\[ M_2(+, -, +, - , +, - ) = \left\{ -4(2e)^4 D(k_2^2) N_1(p_1, p_2)N_2(p_1, p_2) \left[ (p_1p_2) \sqrt{(k_1p_4)(k_1p_1)} \times \sqrt{(p_1p_3)(p_3p_4)} e^{i(\Delta \varphi_{15})} \right] \right\} \]

\[ + \left\{ (p_2 \leftrightarrow p_4) e^{i(\Delta \varphi_{13})} + \left\{ (p_1 \leftrightarrow p_3, p_2 \leftrightarrow p_4) e^{i(\Delta \varphi_{2+})} \right\} e^{i(\Delta \varphi_{2+})}, \right\} \]

\[ M_2(+, +, +, - , +, - ) = 0, \]

\[ M_2(+, +, - , +, - , - ) = 0. \]

Matrix elements (9) - (12) correspond to the third diagram. Basic polarization states of interacting particles are considered:

\[ M_3(+, +, + , - , +, - ) = \left\{ 4(2e)^4 \left[ D(k_2^2) N_1(p_1, p_2)N_2(p_1, p_2) \left[ (p_1p_2) \sqrt{(k_1p_4)(k_1p_1)} \times \sqrt{(k_1p_3)(p_3p_4)} e^{i(\Delta \varphi_{15})} \right] \right] \right\} \]

\[ + \left\{ (p_2 \leftrightarrow p_4) e^{i(\Delta \varphi_{12})} + \left\{ (p_1 \leftrightarrow p_3, p_2 \leftrightarrow p_4) e^{i(\Delta \varphi_{2+})} \right\} e^{i(\Delta \varphi_{2+})}, \right\} \]

\[ M_3(+, +, - , +, - , - ) = \left\{ 4(2e)^4 \left[ D(k_2^2) N_1(p_1, p_2)N_2(p_1, p_2) \left[ (k_2p_1) \sqrt{(p_1p_3)(p_1p_4)} \times \sqrt{(k_2p_4)(p_3p_4)} e^{i(\Delta \varphi_{22})} \right] \right] \right\} \]

\[ + \left\{ (p_1 \leftrightarrow p_3, p_2 \leftrightarrow p_4) e^{i(\Delta \varphi_{25})} \right\} e^{i(\Delta \varphi_{25})}, \right\} \]
The following notations were used in equations (3) - (12):

- \( p \) - four momentum of a particle
- \( \gamma \) - virtual photon
- \( M_1, M_2, M_3 \) - matrix elements
- \( k_1, k_2, k_3, k_4 \) - four momenta of particles
- \( N_1, N_2 \) - normalization factors
- \( \lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 \) - polarization indices
- \( (\pm k) = p_0 k_0 - \vec{p} \vec{k} \) - scalar production of four momentum \( p \) and \( k \)
- \( D(k_3^2) \) - boson propagator, where \( k_3 \) is four momentum of virtual particle (\( \gamma \) or \( Z \))

The list of all used phase factors is presented in appendix.

Squared matrix element for each certain polarization state of interacting particles has the form:

\[ |M(\pm, +, +, +, +, -)|^2 = |M_1(\pm, +, +, +, +, -) + M_2(\pm, +, +, +, - +, -) + M_3(\pm, +, +, +, - +, -)|^2. \]

Using symmetry one can obtain the following relations:

\[
\begin{align*}
|M(\pm, +, +, +, +, -)|^2 &= |M(\pm, +, +, +, +, -)|^2_{p_1 \leftrightarrow p_2, p_3 \leftrightarrow p_4}, \\
|M(\pm, +, +, +, +, -)|^2 &= |M(\pm, +, +, +, +, -)|^2_{p_1 \leftrightarrow p_3, p_2 \leftrightarrow p_4}, \\
|M(\pm, +, +, +, - +, +)|^2 &= |M(\pm, +, +, +, - +, +)|^2_{p_1 \leftrightarrow p_3, p_2 \leftrightarrow p_4}, \\
|M(\pm, +, - +, +, +, -)|^2 &= |M(\pm, +, - +, +, +, -)|^2_{k_1 \leftrightarrow k_2}, \\
|M(\pm, +, - +, +, - +, -)|^2 &= |M(\pm, +, - +, +, - +, -)|^2_{p_1 \leftrightarrow p_3}, \\
|M(\pm, +, - +, - +, +)|^2 &= |M(\pm, +, - +, - +, +)|^2_{p_2 \leftrightarrow p_4}, \\
|M(\pm, +, - +, - +, -)|^2 &= |M(\pm, +, - +, - +, -)|^2_{p_1 \leftrightarrow p_3}, \\
|M(\pm, +, - +, - +, +)|^2 &= |M(\pm, +, - +, - +, +)|^2_{p_2 \leftrightarrow p_4}, \\
|M(\pm, +, +, - +, - +, -)|^2 &= |M(\pm, +, +, - +, - +, -)|^2_{p_1 \leftrightarrow p_3}, \\
|M(\pm, +, +, - +, - +, +)|^2 &= |M(\pm, +, +, - +, - +, +)|^2_{p_2 \leftrightarrow p_4}.
\end{align*}
\]
\[ |M(-, +, -, +, +)|^2 = |M(+, -, +, +, +)|^2, \]
\[ |M(-, +, -, +, +)|^2 = |M(+, +, +, -, +)|^2, \]
\[ |M(-, -, +, +, +)|^2 = |M(+, +, +, +, -)|^2, \]
\[ |M(-, +, +, +, -)|^2 = |M(+, +, -, +, +)|^2, \]
\[ |M(-, +, +, +, -)|^2 = |M(+, +, - +, +)|^2, \]
\[ |M(-, - +, +, +)|^2 = |M(+, +, +, - +)|^2, \]
\[ |M(-, +, +, +, +)|^2 = |M(+, +, +, +, +)|^2, \]
\[ |M(-, -, +, +, +)|^2 = |M(+, +, +, - +)|^2, \]
\[ |M(-, +, +, +, +)|^2 = |M(+, +, +, +, +)|^2, \]
\[ |M(-, -, +, +, +)|^2 = |M(+, +, +, - +)|^2, \]
\[ |M(-, +, +, +, +)|^2 = |M(+, +, +, +, +)|^2, \]
\[ |M(-, -, +, +, -)|^2 = |M(+, +, +, - +)|^2, \]
\[ |M(-, +, +, +, -)|^2 = |M(+, +, +, - +)|^2, \]
\[ |M(-, -, +, +, -)|^2 = |M(+, +, +, - +)|^2, \]
\[ |M(-, +, +, +, +)|^2 = |M(+, +, +, +, +)|^2. \]

(15)

to simplify real calculations.

3 Conclusion

Matrix element and differential cross section of process $\gamma \gamma \rightarrow l^+l^-l^+l^-$ are calculated using helicity amplitude method at every possible polarization state. Obtained formulas don’t contain any spinor. It allows to square and integrate these amplitudes effectively. Numerical calculation of differential and total cross sections under different kinematics cuts as well as the main problems of integration will be discussed in the next paper.

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4 Appendix

The whole list of used phase factors:

\[ e^{i\Delta \varphi_1} = \frac{Tr((\hat{k}_1\hat{p}_2\hat{p}_1\hat{p}_3)(1-\gamma_5))}{8\sqrt{(p_1p_3)(p_1p_2)(k_1p_3)}}; \]

\[ e^{i\Delta \varphi_2} = \frac{((p_1p_3)(k_1p_2) - (p_2p_3)(k_1p_1))Tr((\hat{k}_1\hat{p}_2\hat{p}_1\hat{p}_3)(1-\gamma_5)) - ((p_1p_2)(k_1p_3))Tr((\hat{k}_1\hat{p}_2\hat{p}_3\hat{p}_1)(1-\gamma_5))}{8(p_1p_3)(p_1p_2)(k_1p_3)}; \]

\[ e^{i\Delta \varphi_3} = Tr((\hat{p}_3\hat{p}_2\hat{p}_1\hat{p}_4)(1-\gamma_5)) \times (\varepsilon_1^- (p_3, p_2) \varepsilon_1^+(p_1, p_2)) \times (\varepsilon_2^+ (p_1, p_4) \varepsilon_2^- (p_3, p_4)); \]

\[ e^{i\Delta \varphi_4} = Tr((\hat{p}_3\hat{p}_2\hat{p}_1\hat{p}_4)(1-\gamma_5)) \times (\varepsilon_1^- (p_1, p_4) \varepsilon_1^+(p_1, p_2)) \times (\varepsilon_2^+ (p_1, p_2) \varepsilon_2^- (p_3, p_4)); \]

\[ e^{i\Delta \varphi_5} = \frac{Tr((\hat{k}_1\hat{p}_2\hat{p}_3)(1-\gamma_5))}{8\sqrt{(p_1p_2)(p_3p_2)(k_1p_3)}}; \]

\[ e^{i\Delta \varphi_6} = \frac{((p_1p_2)(k_1p_3) - (p_1p_3)(k_1p_2))Tr((\hat{k}_1\hat{p}_2\hat{p}_3\hat{p}_1)(1-\gamma_5)) - ((p_2p_3)(k_1p_1))Tr((\hat{k}_1\hat{p}_2\hat{p}_1\hat{p}_3)(1-\gamma_5))}{8(p_1p_2)(p_3p_2)(k_1p_3)}; \]

\[ e^{i\Delta \varphi_7} = \frac{Tr((\hat{p}_3\hat{p}_2\hat{p}_1\hat{p}_4)(1-\gamma_5))}{8\sqrt{(p_1p_4)(p_3p_2)(p_1p_2)(p_3p_4)}} \times \frac{Tr((\hat{p}_3\hat{p}_2\hat{p}_1\hat{p}_4)(1-\gamma_5))}{8\sqrt{(p_1p_4)(p_3p_2)(p_1p_2)(p_3p_4)}} \times e^{i\Delta \varphi_{1-}}; \]

\[ e^{i\Delta \varphi_8} = \frac{Tr((\hat{p}_3\hat{p}_2\hat{p}_1\hat{p}_4)(1-\gamma_5))}{8\sqrt{(p_3p_4)(p_3p_2)(p_1p_4)(p_1p_2)}} \times (\varepsilon_1^- (p_3, p_2) \varepsilon_1^+(p_1, p_2)) \times (\varepsilon_2^- (p_1, p_4) \varepsilon_2^+ (p_3, p_4)); \]

\[ e^{i\Delta \varphi_9} = \frac{Tr((\hat{k}_1\hat{p}_4\hat{p}_3\hat{p}_1)(1-\gamma_5))}{8\sqrt{(p_1p_3)(p_3p_4)(k_1p_1)}}; \]

\[ e^{i\Delta \varphi_{10}} = \frac{((p_3p_4)(k_1p_1) - (p_1p_4)(k_1p_3))Tr((\hat{p}_2\hat{k}_1\hat{p}_3\hat{p}_1)(1-\gamma_5)) - ((p_1p_3)(k_3p_1))Tr((\hat{p}_2\hat{k}_1\hat{p}_4\hat{p}_1)(1-\gamma_5))}{8(p_1p_3)\sqrt{(p_2p_1)(k_2p_3)(k_1p_3)(k_1p_4)(p_3p_4)}}; \]

\[ e^{i\Delta \varphi_{11}} = (\varepsilon_2^+ (p_3, p_4) \varepsilon_2^- (p_1, p_2)); \]

\[ e^{i\Delta \varphi_{12}} = \frac{Tr((\hat{p}_3\hat{p}_2\hat{p}_1\hat{p}_4)(1-\gamma_5))}{8\sqrt{(p_1p_4)(p_3p_2)(p_1p_2)(p_3p_4)}} \times (\varepsilon_1^- (p_3, p_2) \varepsilon_1^+(p_1, p_2)) \times (\varepsilon_2^+ (p_1, p_4) \varepsilon_2^- (p_1, p_2)); \]

\[ e^{i\Delta \varphi_{13}} = \frac{Tr((\hat{p}_3\hat{p}_2\hat{p}_1\hat{p}_4)(1-\gamma_5))}{8\sqrt{(p_1p_4)(p_3p_2)(p_1p_2)(p_3p_4)}} \times (\varepsilon_1^- (p_1, p_4) \varepsilon_1^+(p_1, p_2)) \times (\varepsilon_2^+ (p_1, p_2) \varepsilon_2^- (p_1, p_2)); \]

\[ e^{i\Delta \varphi_{2-}} = (\varepsilon_1^- (p_3, p_4) \varepsilon_1^+(p_1, p_2)) \times (\varepsilon_2^- (p_3, p_4) \varepsilon_2^+ (p_1, p_2)); \]

\[ e^{i\Delta \varphi_{2+}} = (\varepsilon_1^- (p_3, p_4) \varepsilon_1^+(p_1, p_2)) \times (\varepsilon_2^- (p_3, p_4) \varepsilon_2^+ (p_1, p_2)); \]
\[ e^{i\Delta_{\varphi_{14}}} = \frac{\text{Tr}((\hat{p}_2\hat{k}_2\hat{p}_4\hat{p}_3)(1 + \gamma_5))}{8\sqrt{(p_3p_4)(p_3p_2)(k_2p_4)(k_2p_1)}}, \quad e^{i\Delta_{\varphi_{15}}} = \frac{\text{Tr}((\hat{k}_2\hat{p}_3\hat{p}_2\hat{p}_4)(1 + \gamma_5))}{8\sqrt{(p_3p_4)(p_1p_2)(k_2p_3)(k_2p_1)}}; \]

\[ e^{i\Delta_{\varphi_{16}}} = \frac{\text{Tr}((\hat{p}_4\hat{p}_3\hat{p}_2\hat{p}_1)(1 + \gamma_5))}{8\sqrt{(p_1p_4)(p_3p_4)(p_1p_2)(p_3p_2)}}, \quad e^{i\Delta_{\varphi_{17}}} = \frac{\text{Tr}((\hat{p}_4\hat{p}_3\hat{p}_2\hat{p}_1)(1 - \gamma_5))}{8\sqrt{(p_3p_4)(p_1p_2)(p_3p_4)(p_1p_4)}} \times (\varepsilon_2^+(p_3, p_4)\varepsilon_2^-(p_1, p_2)); \]

\[ e^{i\Delta_{\varphi_{18}}} = \frac{\text{Tr}((\hat{p}_1\hat{k}_1\hat{p}_3\hat{p}_4)(1 - \gamma_5))}{8\sqrt{(k_1p_1)(k_1p_3)(p_3p_4)(p_1p_4)}}, \quad e^{i\Delta_{\varphi_{19}}} = \frac{\text{Tr}((\hat{p}_2\hat{k}_2\hat{p}_4\hat{p}_3)(1 - \gamma_5))}{8\sqrt{(k_2p_2)(k_2p_4)(p_3p_4)(p_3p_2)}}; \]

\[ e^{i\Delta_{\varphi_{20}}} = \frac{\text{Tr}((\hat{p}_2\hat{k}_1\hat{p}_3\hat{p}_4)(1 - \gamma_5))}{8\sqrt{(k_1p_2)(k_1p_1)(p_1p_3)(p_3p_2)}} \times \frac{\text{Tr}((\hat{p}_4\hat{p}_3\hat{k}_2\hat{p}_1)(1 - \gamma_5))}{8\sqrt{(p_2p_4)(k_2p_2)(k_2p_1)(p_1p_4)}}; \]

\[ e^{i\Delta_{\varphi_{21}}} = \frac{(p_1p_4)\text{Tr}((\hat{k}_2\hat{p}_2\hat{p}_3\hat{p}_4)(1 - \gamma_5)) - (p_1p_4)\text{Tr}((\hat{k}_2\hat{p}_2\hat{p}_4\hat{p}_3)(1 - \gamma_5))}{8\sqrt{(k_2p_2)(p_1p_4)(p_3p_4)(p_1p_3)(k_2p_1)}} \times i\Delta_{\varphi_{11}}; \]

\[ e^{i\Delta_{\varphi_{22}}} = \frac{\text{Tr}((\hat{p}_1\hat{k}_3\hat{p}_2\hat{p}_4)(1 - \gamma_5))}{8\sqrt{(k_2p_1)(k_2p_4)(p_3p_4)(p_1p_3)}}, \quad e^{i\Delta_{\varphi_{23}}} = \frac{\text{Tr}((\hat{p}_4\hat{p}_3\hat{k}_1\hat{p}_3)(1 + \gamma_5))}{8\sqrt{(k_1p_2)(k_1p_3)(p_2p_4)(p_3p_4)}}, \]

\[ e^{i\Delta_{\varphi_{24}}} = \frac{\text{Tr}((\hat{p}_2\hat{k}_3\hat{p}_4\hat{p}_1)(1 - \gamma_5))}{8\sqrt{(p_2p_4)(k_2p_2)(k_2p_1)(p_1p_4)}} \times \frac{\text{Tr}((\hat{k}_2\hat{p}_3\hat{p}_4\hat{p}_1)(1 + \gamma_5))}{8\sqrt{(p_3p_4)(k_2p_3)(k_2p_1)(p_1p_4)}} \times e^{i\Delta_{\varphi_{11}}}; \]

\[ e^{i\Delta_{\varphi_{25}}} = \frac{\text{Tr}((\hat{p}_4\hat{p}_1\hat{k}_2\hat{p}_3)(1 + \gamma_5))}{8\sqrt{(p_1p_4)(p_1p_3)(k_2p_2)(k_2p_3)}}, \quad e^{i\Delta_{\varphi_{26}}} = \frac{\text{Tr}((\hat{k}_1\hat{p}_2\hat{p}_3\hat{p}_1)(1 - \gamma_5))}{8\sqrt{(p_2p_3)(p_1p_3)(k_2p_1)(k_2p_1)} \times e^{i\Delta_{\varphi_{24}}}}; \]

\[ e^{i\Delta_{\varphi_{27}}} = \frac{\text{Tr}((\hat{p}_2\hat{k}_1\hat{k}_1\hat{p}_2)(1 - \gamma_5))}{8\sqrt{(k_2p_1)(k_1k_2)(k_2p_2)(p_1p_2)}}, \quad e^{i\Delta_{\varphi_{28}}} = (\varepsilon_2^-(p_3, p_4)\varepsilon_2^+(p_1, p_2)); \]

\[ e^{i\Delta_{\varphi_{29}}} = \frac{\text{Tr}((\hat{p}_4\hat{p}_3\hat{p}_2\hat{p}_8)(1 - \gamma_5))}{8\sqrt{(p_3p_4)(p_3p_2)(p_1p_4)(p_1p_2)}} \times (\varepsilon_2^-(p_3, p_2)\varepsilon_2^+(p_1, p_2)) \times (\varepsilon_2^-(p_3, p_2)\varepsilon_2^+(p_1, p_2)); \]

\[ e^{i\Delta_{\varphi_{30}}} = \frac{\text{Tr}((\hat{p}_4\hat{p}_3\hat{p}_2\hat{p}_1)(1 - \gamma_5))}{8\sqrt{(p_3p_4)(p_3p_2)(p_1p_4)(p_1p_2)}} \times (\varepsilon_2^-(p_1, p_4)\varepsilon_2^+(p_1, p_2)) \times (\varepsilon_2^-(p_1, p_4)\varepsilon_2^+(p_1, p_2)); \]

\[ e^{i\Delta_{\varphi_{31}}} = \frac{\text{Tr}((\hat{k}_2\hat{k}_1\hat{p}_2\hat{p}_1)(1 + \gamma_5))}{8\sqrt{(k_1k_2)(k_1p_2)(p_1p_2)}}, \quad e^{i\Delta_{\varphi_{32}}} = \frac{\text{Tr}((\hat{p}_4\hat{p}_3\hat{k}_2\hat{k}_1)(1 + \gamma_5))}{8\sqrt{(k_1k_2)(p_3p_4)(k_2p_3)(k_1p_4)}}. \]

Here the following notations are used:

\[ \varepsilon_1^+(p_1, p_2)\varepsilon_1^+(p_1, p_2) = -N_1(p_1, p_k)N_1(p_1, p_2)\text{Tr}(p_k p_1 p_1 p_2 k_1(1 + \gamma_5)), \]

\[ \varepsilon_2^+(p_1, p_k)\varepsilon_2^+(p_3, p_4) = -N_2(p_1, p_k)N_2(p_3, p_4)\text{Tr}(p_k p_2 p_3 p_4 k_2(1 + \gamma_5)), \]

\[ N_1(p_1, p_k) \text{ and } N_2(p_1, p_k) \] have been defined by equation (13).