Partial breaking $N = 4$ to $N = 2$: hypermultiplet as a Goldstone superfield

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Abstract

We describe the partial breaking of $N = 1$ $D = 10$ supersymmetry down to $(1, 0) d = 6$ supersymmetry within the non-linear realization approach. The basic Goldstone superfield associated with this breaking is shown to be the $(1, 0) d = 6$ hypermultiplet superfield $q^a$ subjected to a non-linear generalization of the standard hypermultiplet superfield constraint. The dynamical equations implied by this constraint are identified as the manifestly worldvolume supersymmetric equations of the Type I super 5-brane in $D = 10$. We give arguments in favour of existence of the appropriate brane extension of off-shell hypermultiplet action in harmonic superspace. Some related problems, in particular, the issue of utilizing other $(1, 0) d = 6$ supermultiplets as Goldstone ones, are shortly discussed.

1. Introduction. Spontaneous breakdown of any global symmetry is accompanied by appearance of Goldstone fields. They have quantum numbers of generators of spontaneously broken symmetries and, as the most characteristic feature, are transformed inhomogeneously under the action of these generators. Namely, their transformations start with a pure shift by the appropriate group parameter. A nice geometric meaning of Goldstone fields is revealed within the non-linear realizations theory [1] - [3]: they can be identified with parameters of the coset $G/H$ of the spontaneously broken symmetry group $G$ over its unbroken symmetry subgroup $H$, the vacuum stability subgroup. When $G$ is realized on the coset manifold $G/H$ by left shifts as its group of motions, the coset coordinates, Goldstone fields, are transformed non-linearly and inhomogeneously under the $G/H$ part of symmetries and undergo linear rotations under the action of the subgroup $H$. The theory of non-linear realizations give general recipes how to construct invariant actions of Goldstone fields and their couplings to all other, ”matter” fields.
A remarkable property of such actions is that any $H$-invariant action of matter fields can be made $G$-invariant by switching on proper couplings to the Goldstone fields. Any model with a linear realization of spontaneously broken symmetry can be rewritten in terms of the fields of the corresponding non-linear realization by means of appropriate equivalence redefinitions of the original fields. The text-book example of theories based on non-linear realizations is provided by non-linear sigma models of spontaneously broken internal symmetries.

In case of ordinary, bosonic symmetry groups (e.g., the internal symmetry groups) the Goldstone fields are obviously bosonic. Spontaneously broken supersymmetry (SUSY) necessarily requires Goldstone fermions to be present among the parameters of the corresponding cosets. One more novel feature of the supersymmetry case is extending of the notion of the vacuum stability subalgebra: besides the generators yielding homogeneous rotations (e.g., Lorentz group) it includes also those generators from the coset which have as the associated parameters the space-time coordinates (e.g., translation generators), or the superspace Grassmann coordinates, if some supersymmetries remain unbroken. The rest of the coset parameters are treated as Goldstone fields (superfields) given on this space-time (superspace). The corresponding generators are genuine spontaneously broken symmetry generators.

The case when all supersymmetries are spontaneously broken is referred to as the total spontaneous breaking of SUSY. The corresponding coset manifold is parametrized by the space-time coordinates and Goldstone fermion fields defined on this space-time. E.g., in the case of totally spontaneously broken $N = 1 \ d = 4$ Poincaré SUSY, the coset parameters are the Minkowski space coordinates $x^m$ and the Goldstinos $\psi^\alpha(x), \bar{\psi}^{\dot{\alpha}}(x)$.

The case of spontaneous partial breaking of global supersymmetry (PBGS) is tantamount to the situation when a part of SUSY generators remains unbroken. Then, in the coset approach, one is led to associate with these generators Grassmann coordinates extending the space-time to a superspace of the unbroken SUSY and to treat the coset parameters associated with the genuine spontaneously broken symmetry generators as superfields on this superspace, Goldstone superfields. E.g., in a generic case of partial breaking of $N = 2 \ d = 4$ SUSY down to $N = 1 \ d = 4$ SUSY the coset is parametrized by $N = 1$ superspace coordinates $\{x^m, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}\} \equiv \{X^M\}$ and Goldstone fermionic $N = 1$ superfields $\Psi^\alpha(X), \bar{\Psi}^{\dot{\alpha}}(X)$. The Goldstone fermionic field comes out as the first component of such a superfield.

The study of partial breaking of $N = 2 \ d = 4$ SUSY in the coset space approach in refs. [5] - [8] revealed a few peculiarities of such theories.

- The treatment of fermionic Goldstone superfields as the basic unconstrained ones does not lead to a self-consistent theory: the $N = 1$ Goldstone multiplet includes ghost degrees of freedom [6]. A way out was proposed in [8]: it consists in considering central charge-extended $N = 2$ SUSY with putting the central charge generators into the coset. Then the basic Goldstone superfields prove to be chiral bosonic $N = 1$ superfields associated with the central charge generators. The fermionic Goldstone superfields are expressed as $N = 1$ spinor derivatives of the basic ones by imposing some covariant constraints on the relevant Cartan 1-forms (the so called inverse Higgs effect [9]). Actually, $N = 1$ chirality of the central charge Goldstone superfields is also one of the consequences of the inverse Higgs constraints: these superfields are originally introduced as general $N = 1$ ones. For the basic Goldstone superfields in [8], there was obtained a self-consistent

\[1\] It is a generic feature of non-linear realizations of space-time (super)symmetries, i.e. those including the space-time group of motion (e.g., Poincaré group) as a subgroup [4].
\(N = 2\) invariant action with a non-linearly realized second supersymmetry.

- There exist several inequivalent \(N = 1\) Goldstone supermultiplets related to the partial breaking \(N = 2 \rightarrow N = 1\). The Goldstone fermionic field can be embedded into different \(N = 1\) multiplets: chiral \([6]\), vector \([7]\) and tensor ones \([8]\). These versions correspond to different theories. Moreover, it seems that they require to choose different central (or semi-central) extensions of standard \(N = 2\) SUSY as inputs for a non-linear realization.

- The \(N = 1\) superfield Goldstone actions for all these versions can be treated as gauge-fixed forms of the world-volume superfield actions of some BPS superbranes, along the line of refs. \([11, 10]\) (see also, e.g., \([12, 13]\)). The \(N = 1\) chiral Goldstone superfield action is recognized as that of the Type I super 3-brane in a flat \(D = 6\) background (the action possesses the whole set of \((1, 0)\) \(D = 6\) SUSY symmetries including the \(D = 6\) Lorentz symmetry; all these symmetries except for the worldvolume \(N = 1\) \(d = 4\) SUSY are realized non-linearly, in a Goldstone fashion). Even more interestingly, the \(N = 1\) vector Goldstone multiplet action describes a super D3-brane and yields the Born-Infeld action for the gauge vector field.

- In accord with the general property of non-linear realizations mentioned in the beginning, one can promote different \(N = 1\) matter actions to \(N = 2\) supersymmetric ones by coupling the former to Goldstone superfields.

All the actions presented in \([6, 7, 8]\) are nonlinear, "brane" generalizations of various familiar off-shell \(N = 1\) superfield actions. On the other hand, there exists a good off-shell description of theories with linearly realized \(N = 2\) \(d = 4\) SUSY, e.g. in harmonic \(N = 2\) superspace \([14]\). Then a natural question arises: whether some of these theories can be promoted to those with non-linearly realized higher SUSY, say \(N = 4\) SUSY, by constructing the formalism of partial breaking of this higher SUSY down to \(N = 2\) SUSY and identifying some of well-known \(N = 2\) superfields as the Goldstone ones accompanying this breakdown \([3]\). Related questions are as to what kind of superbranes could be associated with such theories, whether a brane generalization of the harmonic analyticity underlying ordinary \(N = 2\) theories exists, how many different Goldstone \(N = 2\) multiplets are possible, etc.

In this talk we partly answer some of these questions and make some proposals concerning other ones. We show that the hypermultiplet superfield can be regarded as a Goldstone superfield. It realizes the partial breaking of \(N = 1\) \(D = 10\) SUSY (amounting to properly central-charge extended \(N = 4\) SUSY in \(d = 4\) or \((1, 1)\) SUSY in \(d = 6\)) down to \((1, 0)\) \(d = 6\) SUSY. Using the coset space techniques, we present the explicit form of non-linear transformations of hidden symmetries, show that all the superfield coset parameters are covariantly expressed through the hypermultiplet Goldstone superfield and find a covariant nonlinear generalization of the standard hypermultiplet constraint in ordinary \((1, 0)\) \(d = 6\) superspace (or the central-charge extended \(N = 2\) \(d = 4\) superspace) \([10]\). We argue that the dynamical equation for the hypermultiplet Goldstone superfield is a gauge-fixed form of the equations of motion of the Type I super 5-brane in \(D = 10\) with manifest worldvolume \((1, 0)\) \(d = 6\) SUSY. We also adduce arguments in favour of existence of the relevant brane extension of harmonic analyticity and the harmonic superspace off-shell hypermultiplet actions \([14]\).

\(^{2}\)In a different, supergravity and string context with a linear realization of \(N = 4\) SUSY the partial breaking \(N = 4 \rightarrow N = 2\) was discussed in \([13]\).
2. *Poincaré superalgebra in the $d = 6$ notation.* Instead of dealing with $N = 4$ and $N = 2$ SUSY in $d = 4$, we choose as the starting point their higher-dimensional counterparts, $N = 1D = 10$ and $(1,0) d = 6$ Poincaré superalgebras. Our basic reasoning is the desire to consider most symmetric situation. The $d = 4$ case can be then reproduced via dimensional reduction. As we wish to construct a superfield description of partial breaking of $N = 1D = 10$ SUSY down to $(1,0) d = 6$ SUSY \(^3\), it is natural to write the full superalgebra in the $d = 6$ notation. From the $d = 6$ viewpoint the $N = 1D = 10$ SUSY algebra is a sort of central-charge extended $(1,1)$ Poincaré superalgebra. In the standard spinor notation (see, e.g. \([17] - [20]\)) it is constituted by the following set of generators

$$N = 1D = 10 \quad \text{SUSY} \quad \propto \left\{ Q^i_\alpha , P_{\alpha \beta }, S^{\beta \alpha }, Z^{ia} \right\}, \quad (1)$$

where

$$\alpha , \beta = 1, \ldots , 4 \quad , \quad i = 1, 2 \quad , \quad a = 1, 2 \quad$$

are, respectively, the $d = 6$ spinor indices and the doublet indices of two commuting automorphism $SU(2)$ groups realized on the $Q$ and $S$ supertranslations generators. The basic anticommutation relations read

$$\left\{ Q^i_\alpha , Q^j_\beta \right\} = \epsilon^{ij} P_{\alpha \beta} \quad , \quad \left\{ Q^i_\alpha , S^{\beta \alpha} \right\} = \delta^\beta_\alpha Z^{ia} \quad , \quad \left\{ S^{\beta \alpha} , S^{\gamma \delta} \right\} = \epsilon^{\alpha \beta \gamma \delta} P^{\alpha \beta}. \quad (2)$$

The $d = 6$ translation generator\(^4\) $P_{\alpha \beta} = -\gamma^{\alpha \beta} = \frac{1}{2}\epsilon_{\alpha \beta \rho \lambda} P^{\rho \lambda}$, together with the ”semi-central charge” generator $Z^{ia}$, form the $D = 10$ translation generator.

To the generators \([\Pi]\) one should also add the generators of the $D = 10$ Lorentz group $SO(1,9)$ which in the $d = 6$ notation are naturally divided into the following set

$$SO(1,9) \quad \propto \left\{ M_{\alpha \beta \gamma \delta} , \quad T^{ij} , \quad T^{ab} , \quad K^{\alpha \beta}_{ia} \right\}. \quad (3)$$

Here the generators $M$ and $T$ generate mutually commuting $d = 6$ Lorentz group $SO(1,5)$ and the automorphism (or $R$-symmetry) group $SO(4) \sim SU(2) \times SU(2)$, the generators $K$ belong to the coset $SO(1,9)/SO(1,5) \times SO(4)$. The relevant commutation relations are

$$\left[ M_{\alpha \beta \gamma \delta} , M_{\alpha' \beta' \gamma' \delta'} \right] = \epsilon_{\alpha \beta \alpha' \beta'} M_{\gamma \delta \gamma' \delta'} - \epsilon_{\gamma \delta \alpha' \beta'} M_{\alpha \beta \gamma' \delta'} - \left( \alpha' \beta' \leftrightarrow \gamma' \delta' \right)$$

$$\left[ M_{\alpha \beta \gamma \delta} , K^{ia}_{\alpha' \beta'} \right] = \epsilon_{\alpha \beta \alpha' \beta'} K^{ia}_{\gamma \delta} - \epsilon_{\gamma \delta \alpha' \beta'} K^{ia}_{\alpha \beta} ,$$

$$\left[ T^{ij} , T^{kl} \right] = \epsilon^{ij} T^{kl} + \epsilon^{kl} T^{ij} + \epsilon^{jk} T^{il} + \epsilon^{il} T^{jk} ,$$

$$\left[ T^{ab} , T^{cd} \right] = \epsilon^{ac} T^{bd} + \epsilon^{ad} T^{bc} + \epsilon^{bc} T^{ad} + \epsilon^{bd} T^{ac} ,$$

$$\left[ T^{ij} , K^{ka}_{\alpha \beta} \right] = \epsilon^{ij} K^{ka}_{\alpha \beta} + \epsilon^{jk} K^{ia}_{\alpha \beta} , \quad \left[ T^{ab} , K^{ia}_{\alpha \beta} \right] = \epsilon^{ac} K^{ib}_{\alpha \beta} + \epsilon^{bc} K^{ia}_{\alpha \beta} ,$$

$$\left[ K^{ia}_{\alpha \beta} , K^{jb}_{\gamma \delta} \right] = \epsilon^{ij} \epsilon^{ab} M_{\alpha \beta \gamma \delta} + \frac{1}{2} \epsilon_{\alpha \beta \gamma \delta} \left( \epsilon^{ij} T^{ab} + \epsilon^{ab} T^{ij} \right) , \quad (4)$$

$$\left[ M_{\alpha \beta \gamma \delta} , P_{\alpha' \beta'} \right] = \epsilon_{\alpha \beta \alpha' \beta'} P_{\gamma \delta} - \epsilon_{\gamma \delta \alpha' \beta'} P_{\alpha \beta} ,$$

$$\left[ M_{\alpha \beta \gamma \delta} , Q^i_\mu \right] = \frac{1}{2} \left( \epsilon_{\alpha \beta \gamma \mu} \delta^i_\mu + \epsilon_{\alpha \beta \mu \delta} \delta^i_\gamma - \epsilon_{\mu \beta \gamma \delta} \delta^i_\alpha \right) Q^i_\mu ,$$

$$\left[ M_{\alpha \beta \gamma \delta} , S^{\alpha \mu} \right] = -\frac{1}{2} \left( \epsilon_{\alpha \beta \mu} \delta^\delta_\mu + \epsilon_{\alpha \beta \delta} \delta^\mu_\gamma - \epsilon_{\mu \beta \gamma \delta} \delta^\mu_\alpha \right) S^{\alpha \mu} ,$$

\(^3\)We could equally choose $(0,1) d = 6$ SUSY subalgebra as the unbroken one.

\(^4\)We use the following notation $A^{\alpha \beta} = \frac{1}{2} \epsilon^{\alpha \beta} A_\gamma$ , $\epsilon_{\alpha \beta \gamma \delta} = 24$ , $V^i = \epsilon^{ij} V_j$ , $\epsilon_{ijk} = \delta^i_j$. \[\]
3. Coset space and transformations. We are going to construct a non-linear realization of \( N = 1 \) \( D = 10 \) SUSY (together with the \( D = 10 \) Lorentz group), such that \((1,0)\ d = 6\) SUSY remains unbroken. So, following the generic coset approach prescriptions, we are led to choose the vacuum stability subgroup to be

\[
H \propto \{Q^i_\alpha, P_{\alpha\beta}, T^{(ij)}, T^{(ab)}, M_{\alpha\beta\gamma\delta}\}
\]

We included into \( H \) the maximal subgroup of \( SO(1,9) \) with respect to which the algebra of \( Q^i_\alpha, P_{\alpha\beta} \) is closed, namely \( \tilde{H} = SO(4) \times SO(1,5) \propto \{M_{\alpha\beta\gamma\delta}, T^{(ij)}, T^{(ab)}\} \). This subgroup will produce purely homogeneous rotations of all involved objects and it is the genuine linear subgroup from the standpoint of the coset manifolds approach. Then we put the generators \( Q^i_\alpha, P_{\alpha\beta} \) into the coset and associate with them as the coset parameters the coordinates of \((1,0) \ d = 6\) superspace

\[
Q^i_\alpha \Rightarrow \theta^i_\alpha, \quad P_{\alpha\beta} \Rightarrow x^{\alpha\beta}.
\]

The remaining coset generators, \( S^{\alpha\alpha}, Z^{ia}, K^{ia}_{\alpha\beta} \), correspond to genuine spontaneously broken symmetries and the corresponding coset parameters are Goldstone superfields on the \((1,0) \ d = 6\) superspace \( \{x^{\alpha\beta}, \theta^i_\alpha\} \)

\[
S^{\alpha\alpha} \Rightarrow \Psi_{\alpha\alpha}(x, \theta), \quad Z^{ia} \Rightarrow q_{ia}(x, \theta), \quad K^{ia}_{\alpha\beta} \Rightarrow \Lambda^{ia}_{\alpha\beta}(x, \theta).
\]

As the next step, one should choose the appropriate parametrization of the element \( g \) of the coset space \( G/\tilde{H} \) where \( G \) is the full supergroup of \( N = 1 \) \( D = 10 \) SUSY, including the \( D = 10 \) Lorentz group. We use the exponential parametrization

\[
g = e^{x^{\alpha\beta}P_{\alpha\beta}e^{\theta^i_\alpha}Q^i_\alpha} e^{q_{ia}Z^{ia}} e^{\Psi_{\alpha\alpha}S^{\alpha\alpha}} e^{\Lambda^{ia}_{\alpha\beta}K^{ia}_{\alpha\beta}}.
\]

Acting on (11) from the left by different elements of \( G \) with constant parameters, one can determine the transformation properties of the coset coordinates and superfields.

Unbroken supersymmetry \((g_0 = \exp (a^{\alpha\beta} P_{\alpha\beta} + \eta^i_i Q^i_\alpha)):\)

\[
\delta x^{\alpha\beta} = a^{\alpha\beta} + \frac{1}{4} \left( \eta^{i\alpha} \theta_i^\beta - \eta^{i\beta} \theta_i^\alpha \right), \quad \delta \theta_i^\alpha = \eta^i_i.
\]

Broken supersymmetry \((g_0 = \exp (\eta_{\alpha\alpha} S^{\alpha\alpha})):\)

\[
\delta x^{\alpha\beta} = \frac{1}{4} a^{\alpha\beta\gamma\delta} \eta^\gamma_{\gamma} \Psi_{\alpha\delta}, \quad \delta q_{ia} = - \eta_{\alpha\alpha} \theta_i^a, \quad \delta \Psi_{\alpha\alpha} = \eta_{\alpha\alpha}.
\]

Broken \( Z \)-translations \((g_0 = \exp (c_{ia} Z^{ia})):\)

\[
\delta q^{ia} = \epsilon^{ia}
\]
Broken $K$ transformations ($g_0 = \exp (r_{ia}^{\alpha \beta} K_{a\beta})$):

\[
\begin{align*}
\delta x_{a\beta} &= -x_{ia}^{\alpha \beta} q_{ia} - r_{i}^{\alpha \gamma} \theta^{i\alpha} \Psi_{a\gamma} + \frac{1}{2} \epsilon_{\alpha \beta \mu \nu} r_{i}^{\alpha \mu} \Psi_{a\nu}, \\
\delta \theta^{i\alpha} &= 2 r_{i}^{ba\beta} \Psi_{b\beta}, \\
\delta q_{ia} &= -2 r_{i}^{\alpha \beta} x_{a\beta} + r_{a\alpha \beta} \theta^{a\alpha} \theta^{a\beta} - r_{i}^{ba\beta} \Psi_{b\alpha} \Psi_{a\beta}, \\
\delta \Psi_{a\alpha} &= 2 r_{a\alpha \beta} \theta^{a\beta}, \quad \delta \Lambda_{a\beta}^{\alpha} = r_{ia}^{\alpha \beta} + \ldots .
\end{align*}
\]

As was already mentioned, the subgroup $\tilde{H} = SO(1,5) \times SO(4)$ is realized as rotations of the $SO(1,5)$ spinor and $SU(2)$ doublet indices.

We see that the $N=1 \; D=10$ supergroup as a whole admits a realization on the coordinates of $(1,0) \; d=6$ superspace and Goldstone superfields "living" on this superspace. It is easy to check that the closure of the above infinitesimal transformations is just the $N=1 \; D=10$ superalgebra presented in the previous Section.

4. Cartan forms. Next and important step of the coset approach is the construction of the Cartan 1-forms which are used to define covariants of given non-linear realization. They are defined by the generic formula

\[
g^{-1} dg = \Omega_Q + \Omega_P + \Omega_Z + \Omega_S + \Omega_K + \Omega_{\tilde{H}},
\]

with

\[
\begin{align*}
\Omega_Z &\equiv \Omega_{Z_{ia}} \equiv \left( \left( \text{ch} \sqrt{\Phi} \right)_{ja}^{ia} \hat{d}q_{jb}^{j} \left( \frac{\text{sh} \sqrt{\Phi}}{\sqrt{\Phi}} \right)_{ja}^{ia} \right) 2 \Lambda_{\mu \nu}^{ja} \hat{d}x_{\mu \nu} Z_{ia}, \\
\Omega_P &\equiv \Omega_{P_{a\beta}} \equiv \left( \left( \text{ch} \sqrt{\phi} \right)_{\mu \nu}^{\alpha \beta} \hat{d}x^{\mu \nu} + \left( \frac{\text{sh} \sqrt{\phi}}{\sqrt{\phi}} \right)_{\mu \nu}^{\alpha \beta} \Lambda_{\mu \nu}^{ja} \hat{d}q_{ia}^{j} \right) P_{a\beta}, \\
\Omega_Q &\equiv -\Omega_{Q_{ia}} \equiv \left( - \left( \text{ch} \sqrt{v} \right)_{ja}^{ia} \hat{d}\theta_{\beta}^{j} - \left( \frac{\text{sh} \sqrt{v}}{\sqrt{v}} \right)_{ja}^{ia} \right) 2 \Lambda_{\gamma}^{ja} \hat{d}\Psi_{\alpha \gamma} Q_{ia}, \\
\Omega_S &\equiv \Omega_{S_{a\beta}} \equiv \left( \left( \text{ch} \sqrt{\omega} \right)_{a\beta}^{\alpha \gamma} \hat{d}\Psi_{\alpha \gamma} - \left( \frac{\text{sh} \sqrt{\omega}}{\sqrt{\omega}} \right)_{a\beta}^{\alpha \gamma} \right) 2 \Lambda_{j}^{a} \hat{d}\theta_{a \gamma} S_{a\beta}.
\end{align*}
\]

Here

\[
\begin{align*}
d_{\hat{x}}^{a\beta} &= d_{x}^{a\beta} - \frac{1}{4} \theta^{ia} d_{\theta}^{i} + \frac{1}{4} \theta^{ia} d_{\theta}^{i} - \frac{1}{4} \epsilon_{\alpha \beta \mu \nu} \Psi_{\mu}^{a} d\Psi_{a \nu} \\
d_{\hat{q}}_{ia} &= d_{q_{ia}} + \Psi_{ao} d_{\theta}^{a} \\
\varphi_{ja}^{ia} &\equiv 2 \Lambda_{ja}^{ia} \hat{\Lambda}_{ja}^{ia} \mu_{a \beta}^{\mu} \varphi_{ao}^{ia} \equiv 2 \Lambda_{ja}^{ia} \hat{\Lambda}_{ja}^{ia} \mu_{a \beta}^{\mu}, \\
v_{ja}^{ia} &\equiv -4 \Lambda_{ja}^{ia} \hat{\Lambda}_{ja}^{ia} \omega^{b} \gamma_{j}^{b} \omega_{a \alpha}^{b} \equiv -4 \Lambda_{ja}^{ia} \hat{\Lambda}_{ja}^{ia} \omega^{b} \gamma_{j}^{b} \omega_{a \alpha}^{b}.
\end{align*}
\]

We do not give the explicit expressions for the coset Lorentz form $\Omega_{K} = (d_{\Lambda}^{a\beta} + \ldots) K_{a\beta}$ and the inhomogeneously transforming form $\Omega_{\tilde{H}}$ on the stability subgroup as they are of no immediate relevance for our further discussion.

5. Inverse Higgs constraints and dynamical equation. By construction, the forms (15) are covariant under all transformations of $G$ realized as left shifts of $g$. They merely
undergo some induced $\tilde{H}$ rotations in their spinor and $SU(2)$ indices (these rotations are field-dependent in the case of the $G/\tilde{H}$-transformations). This fact allows us to apply the inverse Higgs procedure to eliminate all Goldstone superfields in favour of $q^{ia}(x, \theta)$. Indeed, we observe that $\Psi_{\alpha a}$ and $\Lambda_{a\beta}^\rho$ appear inside the form $\Omega_Z$ linearly as coefficients of the coordinate differentials $d\theta_i^\beta$ and $dx_{\alpha\beta}$, respectively. The Goldstone superfield $\Lambda$ linearly appears also in the form $\Omega_S$. Thus these superfields are covariantly expressible in terms of $\theta$- and $x$-derivatives of $q^{ia}(x, \theta)$.

The natural covariant constraint making this job is as follows

$$\Omega_Z = 0.$$  \hspace{1cm} (18)

It is easy to find that this constraint amounts to the following set of equations

$$\partial_{\mu\nu}q_{ia} = \frac{1}{2} \left( \delta^\rho_\mu \delta^\sigma_\nu - \delta^\sigma_\mu \delta^\rho_\nu - \frac{1}{2} \epsilon^{\rho\sigma\alpha\beta} \Psi^b_{\alpha} \partial_{\mu\nu} \Psi^b_{\beta} \right) \bar{\Lambda}_{ia\rho\sigma} \equiv E^{\rho\sigma}_{\mu\nu} \bar{\Lambda}_{ia\rho\sigma},$$  \hspace{1cm} (19)

$$D^i_\beta q_{ia} - \delta^i_\beta \Psi_{a\beta} = \frac{1}{4} \bar{\Lambda}_{ia\mu\nu} \epsilon^{\mu\nu\alpha\gamma} \Psi^b_{\alpha} D^j_\beta \Psi^b_{\gamma},$$  \hspace{1cm} (20)

where

$$\bar{\Lambda}_{ia\mu\nu} \equiv -2 \left( \frac{\th \sqrt{\varphi}}{\sqrt{\varphi}} \right)^i a \Lambda^b_j \mu\nu$$  \hspace{1cm} (21)

and $D^i_\beta$ is the ordinary flat $(1, 0)$ $d = 6$ spinor derivative

$$D^i_\beta = \frac{\partial}{\partial \theta^i_j} - \frac{1}{2} \theta^i_j \partial_{\alpha\beta}, \quad \{D^i_\alpha, D^j_\beta\} = \epsilon^{ik} \partial_{\alpha\beta}.$$  \hspace{1cm} (22)

It is easy to directly check covariance of this system under, say, the nonlinear supersymmetry transformations (11). Let us point out that there is actually no need to explicitly check the covariance as it directly stems from the manifest covariance of the constraint (18).

Looking at the equations (19), (20) we observe that the first equation and the trace part of the second one are indeed purely algebraic nonlinear relations allowing to trade $\bar{\Lambda}$ and $\Psi$ for the $x$- and $\theta$-derivatives of $q^{ia}$

$$\bar{\Lambda}_{ia\rho\sigma} = (E^{-1})^\mu\nu_{\rho\sigma} \partial_{\mu\nu}q_{ia},$$  \hspace{1cm} (23)

$$\Psi_{a\beta} = \frac{1}{2} \nabla^k_\beta q_{ka},$$  \hspace{1cm} (24)

where

$$\nabla^k_\beta \equiv D^k_\beta - \frac{1}{4} (E^{-1})_{\rho\lambda}^\mu\nu \epsilon^{\rho\lambda\alpha\gamma} (\Psi^b_{\alpha} D^k_\beta \Psi^b_{\gamma}) \partial_{\mu\nu} = D^k_\beta - \frac{1}{4} \epsilon^{\mu\nu\alpha\gamma} (\Psi^b_{\alpha} \nabla^k_\beta \Psi^b_{\gamma}) \partial_{\mu\nu}$$  \hspace{1cm} (25)

(for the time being, we are not aware of the full expression of $\Psi^b_{\alpha}$ through $q^{ia}$, only a few first terms in the iteration solution of (24) were found).

The remaining, isotriplet part of (20) yields the following constraint on $q^{ia}$:

$$\nabla^{(i}_\beta q^{k)a} = 0.$$  \hspace{1cm} (26)

We recognize it as a nonlinear generalization of the well-known hypermultiplet constraint (13)

$$D^{(i}_\beta q^{k)a} = 0.$$  \hspace{1cm} (27)
It is known that it reduces the field content of \( q^{ia}(x, \theta) \) to four bosonic and eight fermionic components
\[
q^{ia}(x, \theta) \Rightarrow \phi^{ia}(x) + \theta^{\alpha i} \psi^{a}_\alpha(x) + x\text{-derivatives} ,
\]
and simultaneously puts these fields on shell
\[
\Box \phi^{ia}(x) = 0 , \quad \partial^{\alpha \beta} \psi^{a}_\beta = 0 \quad \left( \Box \equiv \partial^{\alpha \beta} \partial_{\alpha \beta} = \frac{1}{2} \epsilon^{\alpha \beta \mu \nu} \partial_{\alpha \beta} \partial_{\mu \nu} , \quad \partial_{\alpha \beta} \partial^{\lambda \beta} = \frac{1}{4} \delta^\lambda_\alpha \Box \right) .
\]
Eq. (29) is expected to yield a non-linear generalization of the \( d = 6 \) hypermultiplet irreducibility conditions and equations of motion.

Inspecting how the spontaneously broken nonlinear (super)symmetries (11) - (13) are realized on the components of \( q^{ia}(x) \) (at the linearized level), we conclude that \( \phi^{ia}(x) \) and \( \psi^{a}_\alpha(x) \) are just Goldstone fields associated with the broken \( Z \)-translations and \( S \)-supertranslations, while the Goldstone fields accompanying the spontaneous breakdown of the \( SO(1,9)/SO(1,5) \times SO(4) \) transformations, \( \partial_{\alpha \beta} \phi^{ia}(x) \), are recognized as the coefficients of the second-order \( \theta \) monomials in the \( \theta \)-expansion of \( q^{ia}(x, \theta) \).

The conclusion is that the only essential Goldstone superfield supporting the partial spontaneous breaking of \( N = 1 \) \( D = 10 \) SUSY down to \( (1,0) \) \( d = 6 \) SUSY within the non-linear realization scheme is the hypermultiplet superfield \( q^{ia}(x, \theta) \). It is subjected to the nonlinear dynamical constraint (26) and accommodates all the Goldstone fields associated with the spontaneously broken symmetry generators including parameters of the \( D = 10 \) Lorentz group coset \( SO(1,9)/SO(1,5) \times SO(4) \).

Putting to zero the singlet part has the standard inverse Higgs motivation as the condition of the covariant elimination of the Goldstone spinor superfield \( \Psi^{a\beta} \) in favour of \( q^{ia} \). However, the same requirement for the triplet part (which produces the reduction of the field content of \( q^{ia} \) and simultaneously yields the dynamics) has no such a clear interpretation. It is not implied by the formalism of non-linear realizations, and should be regarded as a kind of dynamical postulate. In the superembedding approach to superbranes (initiated in [21], [22]) a similar postulate is known as the ”geometro-dynamical” principle (see [22] and references therein). An interplay between the superembedding approach and the non-linear realizations PBGS approach is discussed, e.g., in a recent preprint [24]. We will return to this point in the concluding section.

For further discussion it will be convenient to project all the involved quantities on the \( SU(2) \) harmonics \( u^\pm_i, u^{+i}u^-_i = 1 \) [14]
\[
\theta^\alpha_i \Rightarrow \theta^{\pm \alpha} = \theta^\alpha_i u^\pm_i , \quad \mathcal{D}^a_\alpha \Rightarrow \mathcal{D}^\pm_\alpha = \mathcal{D}^a_\alpha u^\pm_i , \quad \{ D^+_\alpha , D^-_\beta \} = -\partial^\alpha_\beta , \quad q^{ia} \Rightarrow q^{\pm a} = q^{ia} u^\pm_i .
\]  
\[\text{5}\] It is worth noting that in one of the first papers where this postulate was introduced and exploited [23] it was regarded as a sort of inverse Higgs effect.
Then eqs. (22), (24) can be written in the following concise form
\[ \nabla^+_\alpha q^{+a} = 0 , \]
\[ \Psi^\alpha_{\beta} = -\nabla^-_{\beta} q^{-a} = \nabla^+_\beta q^{-a} . \]

The covariant derivatives \( \nabla^\pm \) satisfy the following algebra
\[ \{ \nabla^+_{\alpha} , \nabla^-_{\beta} \} \equiv -\nabla^\alpha_{\beta} = -F^{\alpha \lambda}_{\beta \rho} \partial_{\rho \lambda} , \]
\[ \{ \nabla^+_{\alpha} , \nabla^+_{\beta} \} \equiv -\nabla^\alpha_{\beta} = -F^{++\rho \lambda}_{\alpha \beta} \partial_{\rho \lambda} , \]
\[ F^{\rho \lambda}_{\alpha \beta} = (E^{-1})^{\rho \lambda}_{\alpha \beta} \left[ \frac{1}{2} (\delta^\rho_{\gamma} \delta^\lambda_{\delta} - \delta^\rho_{\delta} \delta^\lambda_{\gamma}) + \epsilon^{\omega \sigma \gamma \tau} (\nabla^+_{\alpha} \Psi^\delta_{\gamma}) (\nabla^-_{\beta} \Psi^\delta_{\tau}) \right] , \]
\[ F^{++\rho \lambda}_{\alpha \beta} = (E^{-1})^{\rho \lambda}_{\alpha \beta} \epsilon^{\omega \sigma \gamma \tau} (\nabla^+_{\alpha} \Psi^\delta_{\gamma}) (\nabla^-_{\beta} \Psi^\delta_{\tau}) . \]
The remaining anticommutator \( \{ \nabla^-_{\alpha} , \nabla^-_{\beta} \} \) follows from (34), (37) via the replacement of indices \(+ \rightarrow -\).

We observe that the derivative \( \nabla^\alpha_{\beta} \), alongside with the \( SU(2) \) singlet part, which starts with \( \partial_{\alpha \beta} \) and is antisymmetric in spinor indices, contains also a non-standard triplet part which starts with a three-linear term and is symmetric in indices \( \alpha, \beta \). Just this second part appears in the r.h.s. of the anticommutator (32). This last property, at first sight, seems to obscure the consistency of the dynamical constraint (32) since it leads to the integrability condition
\[ \nabla^+_{\alpha \beta} q^+_a = 0 , \]
which could be too strong. E.g., it could imply \( q^{ia} \) to be constant. However, we have checked that, up to seventh order in \( q^{ia} \), this condition is satisfied identically as a consequence of the structure of \( \nabla^+_{\alpha \beta} \). Though we are still unable to prove this property in general, in what follows we take for granted that (33) produces no new dynamical restrictions on the superfield \( q^{ia} \) (or \( q^{+a} \)).

Using the algebra (34) - (37) and eqs. (32), (33) it is easy to find
\[ \nabla^+_{\alpha} \Psi_{\alpha \beta} = -\nabla_{\alpha \beta} q^+_a , \quad \nabla^-_{\alpha} \Psi_{\alpha \beta} = -\nabla_{\beta \alpha} q^-_a . \]

From the structure of covariant derivatives one immediately concludes that all superfields obtained by successive action of \( \nabla^\pm_{\alpha \beta} \) on \( \Psi_{\alpha \beta} \) are reduced to ordinary \( x \)-derivatives of \( q^{ia} \) and \( \Psi_{\alpha \beta} \), i.e. these two superfield projections indeed exhaust the irreducible field content of \( q^{ia}(x, \theta) \).

In the standard free hypermultiplet case an analog of the constraint (32) reads (cf. (27))
\[ D^+_{\beta} q^{+a} = 0 . \]

Hitting it by three appropriate \( D \)'s and using their anticommutator algebra, one gets
\[ D^+_{\rho} D^-_{\gamma} D^+_{\nu} D^-_{\beta} q^+_a = 0 \iff (\partial_{\rho \gamma} \partial_{\nu \beta} - \partial_{\gamma \beta} \partial_{\rho \nu} + \partial_{\rho \beta} \partial_{\nu \gamma}) q^+_a = 0 \iff \Box q^+_a = 0 , \]
i.e. (40) puts \( q^+_a \) on shell, in accord with the said earlier. To find equations of motion in the full nonlinear case, one can proceed in a similar way, replacing \( D^\pm \) by \( \nabla^\pm \) in (41). Because of the essential nonlinearity of the algebra (34) - (37), the analog of eq. (41) looks rather complicated, but it is simplified in the bosonic limit, when all fermionic components are omitted
\[ (\nabla^-_{\beta \nu} \nabla^+_{\rho \tau} + \nabla^-_{\nu \gamma} \nabla^+_{\rho \beta} - \nabla^-_{\beta \gamma} \nabla^+_{\rho \nu}) q^+_a - \{ \nabla^+_\rho , [\nabla^-_{\nu} , \nabla^-_{\beta \nu}] \} q^+_a + \{ \nabla^+_\rho , [\nabla^+_\beta , \nabla^-_{\nu}] \} q^-_a = 0 . \]
Note that the arrangement of indices in (42) is important, as the covariant vector derivatives do not commute with themselves and with $\nabla^\pm$. Besides, as we already mentioned, they contain the parts both symmetric and antisymmetric in the spinor indices. To see, what kind of dynamics is hidden in (42), we considered it up to the first non-trivial order in fields, the third order. Even in this lowest order the calculations are rather tiresome though straightforward. We found that it amounts to the following equation for $\phi^ia(x) \equiv q^ia(x, \theta)|_{\theta=0}$

$$\Box \phi^ia + \frac{1}{2} \partial^{\rho\lambda} \partial^{\mu\nu} \phi^ia (\partial_{\mu\nu} \phi \cdot \partial_{\rho\lambda} \phi) = 0 ,$$

(43)

where we omitted three-linear terms containing $\Box$ as they contribute to the next, 5th order, and used the notation $A \cdot B \equiv A^{ia}B_{ia}$. All other terms which are present in (42) in this order and have more complicated $SU(2)$ representation content have been found either to identically vanish or to contain terms $\sim \Box \phi$. 

Looking at (43), one observes that this equation just corresponds to the ”static gauge” form of the standard bosonic 5-brane Nambu-Goto action with the induced metric

$$g_{\rho\lambda \mu\nu} = \frac{1}{2} (\epsilon_{\rho\lambda\mu\nu} - \partial_{\rho\lambda} \phi \cdot \partial_{\mu\nu} \phi) \equiv \frac{1}{2} (\epsilon_{\rho\lambda\mu\nu} - d_{\rho\lambda \mu\nu}) ,$$

(44)

that is

$$S_{NG} = \text{const} \int d^6x \left( \sqrt{-\det g} - 1 \right)$$

$$\sim \int d^6x \left\{ \text{Tr} d - \frac{1}{8} (\text{Tr} d)^2 + \frac{1}{4} \text{Tr} d^4 + O(\phi^6) \right\} .$$

(45)

Though it still remains to prove that the higher-order corrections are combined into this nice geometric form, the above consideration suggests that this is very plausible. Then eq. (42) should be interpreted as a manifestly $(1,0)$ $d=6$ world-volume super symmetric PBGS form of the equations of the type I super 5-brane in $D=10$. So the non-linear realization description of the partial breaking of $N=1$ $D=10$ SUSY down to $(1,0)$ $d=6$ SUSY admits the natural brane interpretation, much in line of the previous studies [6] - [7], [24].

Note that all the relations presented so far admit simple dimensional reduction to the $d=5$ and further $d=4, ..., 1$ worldvolumes by neglecting dependence on the corresponding worldvolume coordinates. Without entering into details, one gets in this way manifestly worldvolume supersymmetric superfield equations describing super 4-brane in $D=9$, super 3-brane in $D=8$ and so on, up to a superparticle in $D=5$. In all these cases 8 supersymmetries are realized linearly in the relevant worldvolume superspaces, while the remaining 8 are realized nonlinearly.

6. Brane extension of the off-shell $q^+$ action? In the case of ordinary hypermultiplet it is well-known that just because the irreducibility constraint (27) implies equations of motion, no off-shell superfield action exists for the hypermultiplet in ordinary $(1,0)$ $d=6$ ($N=2$ $d=4$) superspace. However, the off-shell description becomes possible in the harmonic superspace [14]. There, eq. (27) in the form (33) is interpreted as the analyticity condition implying that $q^+_a$ naturally ”lives” on some analytic subspace $\zeta^M = \{x^{\alpha\beta}_A, \theta^{+\gamma}, u^+_i \}$ of the full harmonic $(1,0)$ $d=6$ superspace, i.e. $q^+_a \Rightarrow q^+_a(\zeta)$. On the other hand, the homogeneity of $q^+$ in (31) in harmonics $u^+_i$ now follows from the equation of motion derived by the analytic $q^+$ superfield action

$$S_q = \frac{1}{2} \int d\zeta^{(-4)} q^+_a D^{++} q^+_a . \quad d\zeta^{(-4)} \equiv d^6x_A d^4\theta^+ [du] .$$

(46)
Here $D^{++}$ is the analyticity-preserving harmonic derivative in the analytic basis of the harmonic $d = 6$ superspace

$$D^{++} = \partial^{++} - \frac{1}{2} \theta^{+\alpha} \theta^{+\beta} \partial_{\alpha\beta}, \quad (\partial^{++} = u^{+i} \frac{\partial}{\partial u^{-i}}). \quad (47)$$

The free $q^+$ action admits addition of self-interactions which produce, in the generic case, a general hyper-Kähler sigma model in the bosonic sector (with a four-dimensional target in the case of one hypermultiplet). Most characteristic feature of the free off-shell action (46) and its sigma model generalizations is the presence of infinite sets of auxiliary fields.

One can wonder whether a brane generalization of eq. (40), i.e. eq. (32), also admits an interpretation as the analyticity condition and whether a brane extension of the action (46) exists. By the brane extension we understand the action such that the associated equations of motion together with the analyticity conditions amount to the basic dynamical constraint (32).

The fact that the consistency condition (38) is satisfied (at least up to seventh order, as we have checked) implies that (32) indeed can be interpreted as a sort of Cauchy-Riemann conditions defining a non-linear Grassmann harmonic analyticity for $q^+$. Then, by analogy with the standard hypertmultiplet case, this analyticity can be made manifest by passing to a new basis in $(1,0)d = 6$ harmonic superspace where $\nabla^{+}_{\alpha}$ becomes ”short” on $q^+_a$, i.e. proportional to the partial derivative $\partial/\partial \theta^{-\alpha}$. Clearly, the relevant change of coordinates should be highly nonlinear in $q^ia$ and its derivatives (analogously to the relation between non-linear and manifest $N = 1$ chiralities in the case of the partial breaking $N = 2$ to $N = 1$ [6]). Unfortunately, for the time being we do not know how to construct such nonlinear ”bridges” within the nonlinear realization formalism. One way is, of course, to find them ”by brute force”, order by order in fields. But it seems there exists another way around.

Namely, let us for a moment forget about eq. (32) and deal with the manifestly analytic superfield $q^+_a(\zeta)$ having the free action (46). In the bosonic sector, after eliminating an infinite tower of auxiliary fields, it yields the free action for the physical bosons $v^ia(x)$ ($q^+a = v^ia(x)u^+_i + ...$)

$$S_q \Rightarrow \frac{1}{2} \int d^6x \ (\partial v \cdot \partial v) . \quad (48)$$

Assume that one succeeded in constructing a generalization of (46), such that in the bosonic sector it yields the 5-brane Nambu-Goto extension of (48) in the form (45). Then this extension can naturally be expected to provide the analytic basis description of the above ”brane” hypermultiplet and to be the correct Goldstone superfield action for the considered PBGS pattern. All symmetries (10) - (13) found in the central basis are expected to have their analytic basis counterparts which play the decisive role in fixing the precise structure of the ”brane” $q^+$ action.

It is remarkable that the first simplest correction to (46) which adds quadrilinear terms to the free bosonic action (48) arrange these terms just in the way required by the Nambu-Goto action!

This correction is almost uniquely fixed by the dimensionality considerations and the preservation of the harmonic $U(1)$ charge

$$S_q \Rightarrow S_q + \frac{\alpha}{4} \int d^6x_A d^8\theta [du] \ (q^{+\alpha} D^{++} - q^+_a)^2 . \quad (49)$$

In the second term integration goes over the whole $(1,0)d = 6$ harmonic superspace, $\alpha$ is a
dimensionless parameter and
\[
D^{--} = \partial^{--} - \frac{1}{2} \theta^{-\alpha} \theta^{-\beta} \partial_{\alpha\beta} + \theta^{-\alpha} \frac{\partial}{\partial \theta^+ \alpha}.
\] (50)

Passing to components and eliminating auxiliary fields (they do not become propagating as one could anticipate) yield in the fourth order in \( v^{ia}(x) \) a few terms of the fourth order in \( x \) derivatives \(^\text{6}\). At first sight, these terms have nothing in common with (45): some of them involve a few derivatives on a single \( v^{ia}(x) \), in such a way that they cannot be distributed as in (47) by integrating by parts.

Surprisingly, these unwanted terms prove to be removable by means of appropriate nonlinear redefinition of \( v^{ia}(x) \): they are cancelled by similar terms coming from the free part of the action, i.e. (48). This change of variable in the considered order in fields is uniquely fixed by requiring such a cancellation:
\[
v^{ia} = \phi^{ia} + \frac{\alpha}{3} \left\{ [\Box \phi \cdot \phi] - (\partial \phi \cdot \partial \phi) \right\} \phi^{ia} - \frac{1}{4} (\phi)^2 \Box \phi^{ia} + \frac{1}{2} (\phi \cdot \partial_{\mu\nu} \phi) \partial^{\mu\nu} \phi^{ia} \right\} + O(\phi^5). \] (51)

Finally, the bosonic part of the action (19), up to the fourth order in fields, acquires the following form
\[
S_b = \frac{1}{2} \int d^6x \left\{ (\partial \phi \cdot \partial \phi) + \frac{3\alpha}{4} \left[ (\partial_{\mu\nu} \phi \cdot \partial_{\rho\lambda} \phi) (\partial^{\mu\rho} \phi \cdot \partial^{\nu\lambda} \phi) - \frac{1}{2} (\partial \phi \cdot \partial \phi)^2 + O(\phi^6) \right] \right\}. \] (52)

It precisely coincides with (19) under the choice
\[
\alpha = \frac{1}{3}.
\]

This consideration strongly suggests the existence of the whole "brane" \( q^+ \) action yielding the full Nambu-Goto action (19) in the bosonic sector. Then the field redefinition (51) shows first terms in the bosonic part of the change of variables from the central basis in the harmonic \((1,0)\) \( d = 6 \) superspace, where the hypermultiplet is described by the superfield \( q^{+a}(Z,u) = q^{ia}(Z)u^+_a \) subjected to the dynamical constraint (22), to the analytic basis where the same hypermultiplet is represented by the manifestly analytic superfield \( q^+_a(\zeta) \) possessing highly nonlinear action the first terms of which are given by (49). Such an action, at least for the given particular case, could provide a viable alternative to the standard GS-type Lagrangian description of superbranes (25). It should be a natural generalization of the Goldstone chiral superfields action of ref. (6). It is worth mentioning that possible existence of such an off-shell brane action for hypermultiplet was anticipated in (11) based upon the superembedding considerations.

It is still unclear how to find out the analytic basis form of the nonlinear coset transformations (10) - (14) which should constitute the underlying symmetries of this hypothetical action. As eq. (51) shows, even the \( R^{1,9}/R^{1,5} \) translations \( q^{ia}(Z) \rightarrow q^{ia}(Z) + c^{ia} \) should contain

\(^6\) We normalize the Grassmann integrals over analytic and full superspaces in the following way
\[
\int d^4\theta^+ (\theta^+)^4 = \int d^4\theta^+ d^4\theta^- (\theta^+)^4(\theta^-)^4 = 1, \quad (\theta^\pm)^4 = \frac{1}{4!} \epsilon_{\alpha\beta\gamma\lambda} \theta^{\pm\alpha} \theta^{\pm\beta} \theta^{\pm\gamma} \theta^{\pm\lambda}.
\]
non-linear terms when realized on $q^{+a}(\zeta)$ (this transformation of $q^{+a}$ starts with the well-known isometry of the free $q^+$ action \[ q^{+a} \rightarrow q^{+a} + c^a u^+ \]). Good guiding principle in searching for the full brane $q^+$ action is the preservation of manifest invariance under both linearly realized mutually commuting $SU(2)$ groups, the harmonic one which is realized as the standard automorphism group of $(1,0)$ $d = 6$ SUSY and the "Pauli-Gürsey" one which acts on the index $a$ of $q^{+a}(\zeta)$. It is funny that this last symmetry playing an important role in the harmonic superspace approach comes out in the present framework as a part of the $D = 10$ Lorentz group $SO(1,9)$, on equal footing with the harmonic $SU(2)$ and the $d = 6$ Lorentz group $SO(1,5)$.

Another severe restriction is provided by the dimensionality considerations which require the second Lagrangian density in \[ (46) \] and any further corrections to it to be of dimension $-4$ (in mass units). This implies, in particular, that the next, sixth-order term (if it is needed) should contain the appropriate number of $x$ or spinor derivatives on $q$'s (the geometric dimension of $q^{+a}$ is $-1$). Of course, all such correction terms should have zero harmonic $U(1)$ charge.

Finally, we note that the second term in \[ (49) \] can be rewritten as an integral over the analytic superspace with the Lagrangian density

$$
\sim (q^{+a} \partial^{a\beta} q^{+a})(q^{+b} \partial_{a\beta} q^{+b})
$$

This term can be regarded as the appearance of some composite analytic vector vielbein $H^{++[a,\beta]} \sim q^{+a} \partial^{a\beta} q^{+a}$ in the analytic derivative $D^{++}$. This could be an indication that the full brane $q^+$ action is representable as a kind of the $q^+$ action in the background of $(1,0)$ $d = 6$ supergravity \[ [26, 20] \], with some composite superfield vielbeins built out of $q^+$'s. Though it is unlikely that the next correction terms would admit such a simple representation in the analytic superspace.

In principle, the simple action \[ (49) \] has a chance to be the sought brane $q^+$ action without any further correction terms. This potential possibility is related to the fact that the full physical bosonic part of \[ (49) \] is non-polynomial in $x$-derivatives of $\psi^{ia}$ and so can contain the whole Nambu-Goto action (this on-shell non-polynomiality emerges from solving the auxiliary fields equations, like, e.g., in the well-known Taub-NUT example \[ [27] \]). Such an opportunity would be of course very surprising.

7. **Concluding remarks.** One of the most intriguing and urgent problems for the future study is the construction of the full 5-brane extension of the free analytic $q^+$ action \[ (46) \]. Once such an action is known, one can pose the question as to what could be brane extensions of nontrivial $q^+$ actions with hyper-Kähler sigma models in the bosonic sector. The brane version of \[ (46) \] (if existing) should describe, in the bosonic sector, the 5-brane with transverse coordinates $\hat{q}^a(x)$ parametrizing a flat target manifold $R^4$ (this amounts to the splitting $R^{i,9} \rightarrow R^{i,5} \otimes R^4$). Then an analogous extension of the action of self-interacting $q^+$ is expected to give the static gauge action of 5-brane evolving on some curved $D = 10$ manifold $\sim R^{1,5} \otimes H^4$, $H^4$ being a hyper-Kähler manifold.

It is interesting to see whether other known $(1,0)$ $d = 6$ supermultiplets can play a role of Goldstone ones supporting a partial spontaneous breaking of higher SUSY. Let us examine, e.g., abelian gauge vector multiplet \[ [17] \]. The fundamental object of $(1,0)$ $d = 6$ gauge theory is the analytic harmonic prepotential $V^{++}(\zeta) \ [14, 18, 19]$ which in the WZ gauge collects the components of vector multiplet in the following suggestive way

$$
V^{++} = \theta^{+\mu} \theta^{+\nu} A_{[\mu\nu]}(x) + \theta^{+\mu} \theta^{+\nu} \theta^{+\rho} \epsilon_{\mu\rho\lambda\chi} \psi^{\lambda\chi}(x) u^\gamma_i + (\theta^+)^4 D^{(ik)}(x) u^i_k u^i_k . \tag{53}
$$
One sees that the physical fermionic field in this multiplet has the same chirality as $\theta^\lambda_i$, in contrast to the physical fermion in $q^{\lambda a}(x, \theta)$ which has the opposite chirality. Thus, if we wish to utilize the $d = 6$ vector multiplet as a Goldstone one describing partial breaking of some higher SUSY, the spontaneously broken and unbroken spinor generators of the latter should have the same chirality. In other words, this multiplet is suitable to represent the partial supersymmetry breaking $(2,0)\rightarrow (1,0)$ $d = 6$. In this notation, the breaking associated with $q^{\lambda a}(x, \theta)$ as the Goldstone multiplet corresponds to the pattern $(1,1)\rightarrow (1,0)$ $d = 6$. By analogy with the results of [3], one can expect that the theory of the vector Goldstone $d = 6$ multiplet is manifestly $(1,0)$ supersymmetric $d = 6$ Born-Infeld theory with hidden nonlinearly realized $(2,0)$ SUSY. In the brane language, such a theory should correspond to D5-superbrane. After reduction to $d = 4$ the relevant action should produce $N = 2 d = 4$ Born-Infeld action with hidden $N = 4$ SUSY.

One more possible candidate for the Goldstone multiplet is the $d = 6$ self-dual tensor multiplet with the following on-shell content [28, 29]

$$\sigma(x), \quad B_{\beta}^a(x) \left( B_{\beta}^\lambda = 0 \right), \quad \psi_{ai}. \quad (54)$$

It is capable to support the breakdown of some kind of $(1,1)\rightarrow (1,0)$ $d = 6$ SUSY down to $(1,0)\rightarrow (0,1)$ $d = 6$. Like the Goldstone hypermultiplet. It is known to be associated with the PBGS pattern $N = 1 D = 7 \rightarrow (1,0)\rightarrow (0,1)$ $d = 6$ [33, 24].

Some additional possibilities arise upon the reduction to $d = 5$ and $d = 4$.

It is also interesting to study other versions of partial spontaneous breaking of $N = 1 D = 10$ SUSY within this framework. If we limit our attention to the $1/2$ breaking, a simple analysis shows that only one self-consistent option is possible, besides the one considered here. It corresponds to breaking $N = 1 D = 10$ SUSY down to $(8,0)$ (or, equivalently, $(0,8)$) $d = 2$ SUSY. From the brane standpoint, it should provide $(8,0)\rightarrow (0,8)$ $d = 2$ worldsheet superfield PBGS description of the heterotic $N = 1 D = 10$ superstring in a flat background. All other possible $1/2$ SUSY breaking patterns can be ruled out on the physical grounds: in all of them some unbroken supersymmetry generators yield in their anticommutator broken translation generators, that is in conflict with the interpretation of these spinor charges as belonging to the vacuum stability subgroup. It is curious that such simple algebraic PBGS reasonings distinguish just two self-consistent BPS $N = 1 D = 10$ super $p$-branes, viz., super 5-brane and superstring.

As one more remark, it is noteworthy that the superfield PBGS approach can be successfully extended to the most interesting case of $N = 1 D = 11$ (Type IIA $N = 2 D = 10$) SUSY. The $(1,0)\rightarrow (0,1)$ $d = 6$ Goldstone superfield framework is well adapted for description of the $1/4$ partial breaking of this SUSY down to $(1,0)\rightarrow (0,1)$ $d = 6$ (or $(0,1)\rightarrow (1,0)$ $d = 6$) SUSY. It turns out that in this case the set of fundamental unremovable Goldstone superfields is reducible: besides $q^{\lambda a}(x, \theta)$ it includes two more superfields. These are a bosonic scalar superfield $\Phi(x, \theta)$ (it parametrizes spontaneously broken 11th direction in $R^{1,10}$) and a fermionic one $\xi^{\alpha \rho}(x, \theta)$ (it is associated with one of two extra spontaneously broken $d = 6$ supercharges present in $D = 11$ SUSY algebra in the $d = 6$ notation). The dynamical and irreducibility constraints on these superfields following from the $D = 11$ SUSY counterparts of the basic covariant constraint (18), as well as the brane interpretation of the emerging system will be presented elsewhere.

Finally, we wish to point out that it is desirable to further clarify the relationship between

\footnote{An $N = 2$ superfield extension of Born-Infeld action has been recently constructed in [28].}
the PBGS and superembedding approaches. It seems that the appropriate PBGS description exists for most of superbranes and it corresponds to choosing the static gauge with respect to local symmetries in the GS formulation, including \( \kappa \)-symmetry. It is a rather difficult task to find such symmetries and to prove, e.g., invariance of the relevant GS-type actions under them. At the same time, the PBGS approach deals with a minimal set of worldvolume superfields accommodating the superbrane physical degrees of freedom and provides a systematic way to deduce their transformation laws both under manifest and hidden symmetries. In a number of cases it also gives precise recipes or, at least, hints of how to construct the relevant manifestly worldvolume supersymmetric off-shell actions. In this sense it should be regarded as complementary to the superembedding approach. On the other hand, the power of the latter consists, in particular, in providing a possibility to classify all possible physical worldvolume supermultiplets related to various superbranes and to learn whether they are on- or off-shell as a result of imposing some basic constraints on the relevant superfields. Actually, all the minimal Goldstone supermultiplets appearing in the PBGS constructions known so far are in the list of physical superbrane worldvolume multiplets obtained by the linearized level analysis of the superembedding equations in [22]. In particular, for the \( N = 1 \, D = 10 \) Type I super 5-brane it picks out the hypermultiplet as such a physical multiplet and predicts it to be on-shell in a precise correspondence with our PBGS analysis.

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