A Photon Splitting Cascade Model of Soft Gamma-Ray Repeaters

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The spectra of soft gamma-ray repeaters (SGRs), with the exception of the March 5, 1979 main burst, are characterized by high-energy cutoffs around 30 keV and low-energy turnovers that are much steeper than a Wien spectrum. Baring [1] found that the spectra of cascades due to photon splitting in a very strong, homogeneous magnetic field can soften spectra and produce good fits to the soft spectra of SGRs. Magnetic field strengths somewhat above the QED critical field strength \(B_{cr} = 4.413 \times 10^{13} \text{ G}\), is required to produce cutoffs at 30-40 keV. We have improved upon this model by computing Monte Carlo photon splitting cascade spectra in a neutron star dipole magnetic field, including effects of curved space-time in a Schwarzschild metric. We investigate spectra produced by photons emitted at different locations and observer angles. We find that the general results of Baring hold for surface emission throughout most of the magnetosphere, but that emission in equatorial regions can best reproduce the constancy of SGR spectra observed from different bursts.

I. INTRODUCTION

The association of supernova remnants with at least two of the three known SGRs (2-4) (SGR1806-20 and SGR0525-66, the Mar 5, 1979 source) is now well established. The third, SGR1900+14, is near a ROSAT source that is possibly a supernova remnant [5]. These associations strongly indicate that this class of \(\gamma\)-ray bursts is linked to relatively young galactic neutron stars. Furthermore, it has been suggested [6] that SGRs are a special class of neutron stars, known as “magnetars”, that have extremely strong magnetic fields. Those neutron stars that are born with periods of several ms can acquire high fields during a short period of vigorous convection that follows core collapse. The convection generates a rapid dynamo which can generate dipole fields as high as \(10^{15-16} \text{ G}\). It has been pointed out that there are several attractive features of very high fields in accounting for observations of SGRs. Among these are (i) an explanation of the 8 sec periodicity of the Mar 5 event as dipole

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spin down in the $\sim 10^4$ yr age of the N49 supernova remnant (ii) a reduction of the Compton scattering opacity below the cyclotron fundamental, allowing a photon flux $\sim 10^4$ above the Eddington limit (7).

Yet another advantage of very strong magnetic fields in models for SGRs (1) is the effectiveness of photon splitting in producing the soft spectra of SGRs. Photon splitting, $\gamma \rightarrow \gamma\gamma$, attenuates $\gamma$-ray photons, degrading them to lower energies, where they split repeatedly until they escape the high-field region. Baring (1) showed that the emerging spectra of such photon splitting cascades could account for both the shape and the softness of observed SGR spectra if the field in the emission region $B \gtrsim 4 B_{cr}$. The splitting cascade spectra were computed by the solution of a kinetic equation for the photons, assuming a uniform field in a region of size $R = 2 \times 10^6$ cm. This approach, while important in demonstrating the effectiveness of photon splitting in modelling SGR spectra, neglected the dipole field geometry and the strong gravitational field of a neutron star magnetosphere. We have improved upon the model of Baring (1) by including these effects in a Monte Carlo calculation of photon splitting cascades near strongly magnetized neutron stars.

II. PHOTON SPLITTING CASCADE SPECTRA

Photon splitting is forbidden in field-free regions but is allowed in neutron star magnetic fields. The splitting rate (8) for a photon of energy $\epsilon$ in units of $mc^2$, $T_{sp}(\epsilon) \propto \epsilon^5 (B/B_{cr})^6 \sin^6 \theta_kB$, where $\theta_kB$ is the angle between the photon momentum and the magnetic field, can be large if $B$ is near $B_{cr}$. The above rate is valid in the nondispersive limit, where $\epsilon \sin \theta_kB \ll 2$ and $B \ll B_{cr}$. It is dependent on the two polarization states of the photons in the birefringent, magnetized vacuum: $||$ or $\perp$, where the photon’s electric vector is parallel or perpendicular to $k \times B$. The modes $|| \rightarrow \perp ||$, $\perp \rightarrow \perp \perp$ and $\perp \rightarrow || ||$ are the only ones that are non-zero through CP invariance. Since there is a difference in the rates of these modes which depends on field strength (1), polarized photons emerge from the emission region. In our calculations, we have included high field corrections to the above formula for splitting (8) that cause the attenuation coefficient to saturate somewhat above $B \sim 4B_{cr}$.

Photon splitting cascade spectra will turn over roughly at the escape energies of the photons, above which photons undergo at least one splitting generation, but below which the optical depth is always $\ll 1$ and photons can escape the magnetosphere. The existence of such an escape energy is a consequence of the $r^{-3}$ decay of the dipole field. Our previous analysis (8) showed that escape energies for photons emitted at the neutron star surface and propagating through a dipole magnetic field are quite sensitive to the propagation angle $\theta_k$ of the photon to the dipole axis at the magnetic pole ($\theta = 0^0$), but nearly independent of this angle near the equator ($\theta = 90^0$). This effect is a function of the dipole field curvature: near the pole, the field lines are diverging rapidly, and different emission directions sample very dif-
FIG. 1. a) Ratio of the energies below which photons escape a dipole field without splitting in curved to flat space as a function of magnetic field. b) Photon splitting cascade spectra in curved and flat space for different surface field strengths. \( \theta \) and \( \phi \) are the colatitude and azimuth angles of the photon emission point in the dipole field and \( \theta_k \) is the photon emission angle to the dipole axis in the local inertial frame.

Different field orientations, but at the equator the field looks nearly the same in all directions.
In the present work we have included the general relativistic effects of curved spacetime in a Schwarzschild metric, following the treatment of Gonthier & Harding (10) who studied the effects of general relativity on photon attenuation via magnetic pair production. Those effects are the curved spacetime photon trajectories, the magnetic dipole field in a Schwarzschild metric and the gravitational redshift of the photon energy as a function of distance from the neutron star. We have taken a neutron star mass, \( M = 1.4 \, M_\odot \), and radius, \( R = 10^6 \, \text{cm} \) in these calculations. The effects of curved spacetime decrease the escape energies by a factor of about 2 compared to flat spacetime (9), the largest contributions coming from an increase in the dipole field strength (by 1.4 at the pole) and the correction for the gravitational redshift, which increases the photon energy by roughly a factor of 1.2 in the local inertial frame at the neutron star surface. The qualitative behavior of escape energies as a function of \( \theta_k \) in curved space is the same as in flat space: in the polar emission case, there is still a substantial variation in escape energy with emission direction \( \theta_k \) for both initial polarizations, but almost no variation in the case of equatorial emission. The ratio of curved to flat space escape energies as a function of magnetic field are shown in Figure 1a. At low field strengths, one is seeing the combined effects of the increase in the dipole field strength (which varies from pole to equator) and the increase in local frame photon energy; in high fields, only the increase in photon energy affects the escape energy, giving a factor of 1.2 decrease, due to the saturation of the splitting with increasing field strength.

Figure 1b shows Monte Carlo cascade spectra resulting from monoenergetic photons injected with \( \epsilon = 2 \) at the neutron star surface at the magnetic pole. The photons split many times before escape, each time dividing their energy into two photons with a distribution that peaks at half the parent photon energy, in the non-dispersive limit (8,9). The cascade spectra peak just below the escape energy for that field strength. Compared to those of Baring, these spectra show the same \( \epsilon^2 \) power law below the peak, but a more rapid decrease above the peak than the \( \epsilon^{-7} \) found by Baring, due to the fact that the Monte Carlo spatial injection occurs as a delta function at the surface, while the kinetic equation solution assumes an exponential injection. The peak in the flat space spectra in Fig. 1 are also a factor of around 2 higher than the homogeneous field case of Baring, due to the fall off in strength of the dipole field. The cascade saturates at \( B \sim 30 \, B_{cr} \), due to the saturation of the cross section in high fields, but the saturation energy as well as the peaks of all spectra are lower when curved space corrections are included.

The effect of varying the emission location on the neutron star surface on the cascade spectrum is shown in Figure 2. Fig. 2a illustrates that the spectra are distinctly different for two emission angles at the magnetic pole, while they are almost identical for different observer angles at the equator. Thus, the behavior of the cascade spectrum is almost entirely determined by the escape energies at the initial photon injection point.
III. DISCUSSION

The generation of pure splitting cascades is contingent upon operation of at least two polarization modes of splitting so that polarization exchange is effected. Adler (8) demonstrated that in the dispersive magnetized vacuum, only one of the modes of splitting considered here, namely $\perp \rightarrow \parallel$, satisfies kinematic selection rules imposed by four-momentum conservation. This result was derived assuming weak dispersion, i.e. a refractive index close to unity. While Shabad (11) has extensively looked at the regime of strong vacuum dispersion, mostly near and above pair creation threshold, numerical computation of the refractive index and kinematic selection rules for supercritical fields well below pair threshold remains to be explored; this is a major goal of our future research.

Should the selection rules extend to the strongly dispersive regime sampled by the SGR problem, cascade continuation could be effected by a polarization-switching process such as Compton scattering. The cross-section for this is suppressed below the cyclotron resonance (12), and effective polarization state switching ($\parallel \rightarrow \perp$) will occur if the radiation is somewhat beamed along the field in the rest frame of the scattering electrons. Significant scattering opacities are quite plausible in the luminous environment of SGRs, and the complications of scattering in a dipole field geometry, which will tend to broaden the output cascade spectrum and smear its polarization spectrum, are deferred to future work.

These calculations of photon splitting cascades in neutron star magnetospheres lend support to the idea that the splitting mechanism in strong mag-
netic fields could be the cause for the softness and quasi-thermal shape of SGR spectra. The insensitivity of the cascade spectra to observer angle near the equator is very important for modeling SGR spectra. It implies that the observed spectra would not vary from burst to burst, even if the neutron star orientation changes (i.e. the star rotates), as long as the emission occurs at large magnetic colatitudes. This is consistent with the lack of observed spectral variation in bursts from SGR1806-20 [13] and in the phase-resolved spectroscopy of the periodic soft tail of the Mar 5, 1979 event [14].

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