Hierarchy of kinetic freeze-out parameters in low energy heavy-ion collisions

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We study the mass dependent hierarchy of kinetic freeze-out parameters of hadrons in low energy heavy-ion collisions. For this purpose, the transverse momentum and rapidity spectra of the identified hadrons produced in central Pb+Pb collisions, available at SPS energies ranging from \( E_{\text{Lab}} = 20\text{A} - 158\text{A}\) GeV, are analyzed within a generalized non boost-invariant blast wave model. We consider separate simultaneous fits for light hadrons \((\pi^-, K^\pm)\) and heavy strange hadrons \((\Lambda, \bar{\Lambda}, \phi, \Xi^\pm, \Omega^\pm)\), for which the transverse momentum spectra as well as rapidity spectra are available. We also perform a separate fit to transverse momentum spectra of charmonia \((J/\Psi, \Psi')\) at 158A GeV collisions. We find a clear mass dependent hierarchy in the fitted kinetic freeze-out parameters. Further, we study the rapidity spectra using analytical Landau flow solution for non-conformal systems. We find that the fitted value of sound velocity in the medium also shows a similar hierarchy.

I. INTRODUCTION

Relativistic heavy-ion collisions are a suitable tool to produce and study hot and dense strongly interacting matter in the laboratory [1–3]. By varying the collision energy, the nuclear matter can be created over a wide range of temperatures and densities, which facilitates the systematic investigation of the large parts of the QCD phase diagram. The ultra-relativistic nuclear collisions at Relativistic Heavy Ion Collider (RHIC) [4, 5] and Large Hadron Collider (LHC) [6–8] predominantly produces a partonic medium at high temperature and vanishingly small baryon chemical potential, thermodynamic properties of which are most suitably studied using lattice QCD (lQCD) simulations [9–13].

In relativistic nuclear collisions at lower energies, nuclear matter is created at high net baryon densities and moderate temperature, where scope of application of lQCD is rather limited. On experimental front, this has led to renewed interest in collision at moderate energies, manifested in the ongoing and upcoming experimental programs at RHIC [14], SPS [15, 16], FAIR [17, 18] and NICA [19]. An optimum utilization of the future facilities demands a coherent interpretation of the available data sets in the similar energy range collected by the previous generation fixed target experiments at AGS and SPS accelerator facilities. Of particular interest is the determination of freeze-out conditions of the fireball at various collision energies.

During chemical freeze-out the inelastic scatterings cease, leading to the stabilization of the particle chemistry in the fireball. On the other hand, at kinetic or thermal freeze-out hadrons stop to interact with each other and their momentum distribution does not undergo further change. In the so called “standard model” of heavy-ion collisions, chemical freeze-out occurs earlier than kinetic freeze-out due to larger mean free path of inelastic collisions. Usually, the yields and transverse momentum \((p_T)\) spectra of the produced hadrons are analyzed to extract the parameters of chemical and kinetic freeze-out. In Ref. [20] the authors advocated for a multiple chemical freeze-out scenario, with strange hadrons fixing their chemical composition earlier than the non-strange light hadrons, due to smaller inelastic cross-sections. An interesting question to ask is whether a similar hierarchical structure is also present in the case of kinetic decoupling. One may expect a mass dependent hierarchy of kinetic freeze-out as the medium induced momentum change of heavy hadrons would be smaller compared to lighter hadrons. Therefore, as the temperature of the fireball decreases, one would expect an earlier kinetic decoupling of heavy hadrons. In the present article, we have made an attempt to look for the possible hierarchy in thermal freeze-out, in low energy nuclear collisions.

Kinetic freeze-out conditions are commonly studied in hydrodynamics inspired blast-wave model framework [21]. In our previous work [22], we investigated the kinetic freeze-out conditions of the light (bulk) hadrons \((\pi^-, K^\pm, p)\) by analyzing their transverse and longitudinal spectra in the beam energy range of \(E_{\text{Lab}} = 2\text{A} - 158\text{A}\) GeV by employing a non boost-invariant blast-wave model formulated in Ref. [23]. In the original blast wave model, the hydrodynamical results for particle spectra are approximated by emission from a cylindrically symmetric and longitudinally boost-invariant fireball [24]. In the non boost-invariant extension, the symmetry is explicitly broken by introducing a dependence of the transverse size of the fireball on the space-time rapidity. This is particularly useful for low energy collisions where the longitudinal boost invariance is absent in the measured rapidity \((y)\) distributions of the hadrons.

In this article, we employ non boost-invariant blast-wave model to study the mass dependent hierarchy in kinetic freeze-out parameters of hadrons produced in central Pb+Pb collisions at SPS energies. To this end, we analyze the \(p_T\)-spectra and rapidity spectra of the identified hadrons at collision energies ranging from \(E_{\text{Lab}} = 20\text{A} - 158\text{A}\) GeV. We consider separate simultaneous fits
for light hadrons ($\pi^-$, $K^\pm$) and heavy strange hadrons ($\Lambda$, $\bar{\Lambda}$, $\phi$, $\Xi^\pm$, $\Omega^\pm$), for which the transverse momentum spectra, as well as rapidity spectra, are available. We do not consider protons in the fits as the rapidity spectra of protons are not available at SPS energies. For heavy strange hadrons, our analysis results indicate a relatively low kinetic freeze-out temperature in the range of $90 - 110$ MeV, with a rather strong mean transverse velocity of collective expansion of about $0.4c - 0.5c$. We also perform a separate fit to transverse momentum spectra of charmed hadrons ($J/\Psi$, $\Psi'$) at 158 A GeV collisions. We find a clear mass dependent hierarchy in the fitted kinetic freeze-out parameters. Further, we study the rapidity spectra using analytical Landau flow solution for non-conformal systems. We find that the fitted value of sound velocity in the medium also shows a similar hierarchy.

In the present work, we perform for the first time, a systematic analysis of the heavy strange hadrons produced in the low energy nuclear collisions using non boost-invariant blast-wave model. Note that the application of blast-wave dynamics to study the transverse spectra of heavy hadrons have been attempted earlier. In Ref. [25], the authors have analyzed the $p_T$ spectra of $J/\Psi$, $\psi'$ mesons and $\Omega$ baryon within the longitudinal boost-invariant blast-wave model, with the hypothesis that for these heavy hadrons, the rescattering effects in the hadronic phase is negligible and they leave the fireball at hadronization. However, to the best of our knowledge, a thorough analysis of $p_T$ and $y$ distributions of all varieties of multi-strange hadrons produced in the low energy domain has never been attempted before using a non boost-invariant blast wave model.

The paper is organized as follows. In section II, the essential features of the non boost-invariant model are described in a nutshell. The analysis results are presented in section III. In section IV we summarize our main results and conclude.

II. A BRIEF DESCRIPTION OF THE MODEL

Details of the non boost-invariant blast wave model and its adopted version employed for the present analysis can be respectively found in Ref. [23] and Ref. [22]. Within this model, the thermal single particle spectrum for central collisions, in terms of transverse mass $m_T(\equiv \sqrt{p_T^2 + m^2})$ and rapidity $y$ are given by
TABLE I: Details of the data sets from different experiments at different accelerator facilities along with energy (E_{Lab}), beam rapidity (y_{beam}) in lab frame, System, Centrality, Phase space and Hadron species, used for this blast wave analysis.

| Facility | Experiment | E_{Lab} (A GeV) | y_{beam} | System | Centrality | Phase space | Hadron Species |
|----------|------------|-----------------|----------|---------|------------|-------------|----------------|
| SPS      | NA49       | 20              | 3.75     | Pb+Pb   | 0 - 7.2%   | 0 - 7%      | A (Å) [26], φ [28], Ξ^± [26] |
|          |            |                 |          |         |            |             | 0 - 7% -0.4 < y_{c.m.} < 0.4 (Λ, Ā) |
|          |            |                 |          |         |            |             | 0 - 7% -0.5 < y_{c.m.} < 0.5 (Ξ^±) |
| SPS      | NA49       | 30              | 4.16     | Pb+Pb   | 0 - 7.2%   | 0 - 7%      | A (Ā) [26], φ [28], Ξ^± [26] |
|          |            |                 |          |         |            |             | 0 - 7% -0.4 < y_{c.m.} < 0.4 (Λ, Ā) |
|          |            |                 |          |         |            |             | 0 - 7% -0.5 < y_{c.m.} < 0.5 (Ξ^±) |
| SPS      | NA49       | 40              | 4.45     | Pb+Pb   | 0 - 7%     | 0 - 7%      | A (Ā) [26], φ [28], Ω^± [27], Ξ^± [26] |
|          |            |                 |          |         |            |             | 0 - 7% -0.4 < y_{c.m.} < 0.4 (Λ, Ā) |
|          |            |                 |          |         |            |             | 0 - 7% -0.5 < y_{c.m.} < 0.5 (Ξ^±) |
| SPS      | NA49       | 80              | 5.12     | Pb+Pb   | 0 - 7%     | 0 - 7%      | A (Ā) [26], φ [28], Ξ^± [26] |
|          |            |                 |          |         |            |             | 0 - 7% -0.4 < y_{c.m.} < 0.4 (Λ, Ā) |
|          |            |                 |          |         |            |             | 0 - 7% -0.5 < y_{c.m.} < 0.5 (Ξ^±) |
| SPS      | NA49       | 158             | 5.82     | Pb+Pb   | 0 - 10%    | 0 - 7%      | A (Ā) [26], φ [28], Ω^± [27], Ξ^± [26] |
|          |            |                 |          |         |            |             | 0 - 7% -0.4 < y_{c.m.} < 0.4 (Λ, Ā) |
|          |            |                 |          |         |            |             | 0 - 7% -0.5 < y_{c.m.} < 0.5 (Ξ^±) |

TABLE II: Summary of the fit results of p_T spectra of heavy strange hadrons at different energies ranging from 20A to 158A GeV at SPS.

| E_{Lab} (A GeV) | η_{max} | ⟨β_T⟩ | T_{kin} (MeV) | χ^2/N_{dof} |
|-----------------|---------|-------|--------------|-------------|
| 20              | 1.255 ± 0.020 | 0.443 ± 0.003 | 93.12 ± 0.19 | 1.98 |
| 30              | 1.631 ± 0.022 | 0.449 ± 0.003 | 95.78 ± 0.17 | 2.28 |
| 40              | 1.779 ± 0.017 | 0.454 ± 0.003 | 98.83 ± 0.13 | 3.61 |
| 80              | 1.977 ± 0.020 | 0.448 ± 0.002 | 106.43 ± 0.12 | 3.46 |
| 158             | 1.944 ± 0.023 | 0.469 ± 0.002 | 109.25 ± 0.11 | 3.14 |

related to the collective transverse fluid velocity, β_T, via the relation β_T = tanh(ρ). Assumption of the instantaneous common freeze-out of the fireball makes freeze-out time τ_F independent of the transverse co-ordinate r_⊥. In the spirit of a Hubble like expansion of the fireball in the transverse plane, the radial dependence of the transverse fluid velocity is assumed to be of the form:

$$β_T(r_⊥) = β^0_T \left( \frac{r_⊥}{R} \right).$$

where β^0_T is the transverse fluid velocity at the surface of the fireball. The transverse flow vanishes at the center

$$\frac{dN}{mdtdy} = \frac{g}{2π} m_T τ_F \int_{-η_{max}}^{η_{max}} dη \cosh(y-η)$$

$$\times \left( \frac{pr \sinh(ρ(r_⊥))}{T} \right) \left( \frac{p \cosh(y-η) \cos(ρ(r_⊥))}{T} \right).$$

where g denotes the degeneracy of particle species and η = tanh\(^{-1}\)(z/t) is the space-time rapidity. In the transverse plane, the flow rapidity (or transverse rapidity) ρ is
and assumes maximum value at the edges. For such a linear parametrization, the average transverse flow velocity is given by \( \langle \beta_T \rangle = \frac{1}{4} \beta_0 T \).

Considering reflection symmetry about the center of mass, the freeze-out volume is constrained in the region \(-\eta_{\text{max}} \leq \eta \leq \eta_{\text{max}}\), to account for the limited available beam energy. In the transverse plane, the fireball is considered to have an elliptic shape, and the transverse size is parameterized as

\[
R(\eta) = R_0 \sqrt{1 - \frac{\eta^2}{\eta_{\text{max}}^2}},
\]

where \( R_0 \) is the transverse size of the fireball at \( \eta = 0 \).

Not much data on strange hadrons are available in Au+Au collisions at AGS energies, except the measurements of \( \Lambda \) \cite{29} and \( \phi \) \cite{30} at 11.5 and 11.7 AGeV and with different kinematic coverage, from E877 and E917 experiments respectively. Nonetheless, we confine ourselves only to the SPS energy domain. Data on \( p_T \) distribution of a variety of strange hadron species from STAR Collaboration \cite{31} at RHIC beam energy scan (BES) program are preliminary at the moment \cite{32}. Therefore, we have not considered it for the analysis at the moment. Moreover, the corresponding \( y \) distributions have also not been reported yet. The analysis of the data above SPS energies is beyond the scope of this work.

### III. RESULTS AND DISCUSSIONS

The results of our analysis have been presented in this section. The measured \( p_T \) and \( y \) spectra of all the available heavy strange hadrons produced in central Pb+Pb collisions from NA49 Collaboration \cite{26–28} at SPS, in the beam energy range \( E_{\text{lab}} = 20A - 158A \) GeV, are analysed for this purpose. Not much data on strange hadrons are available in Au+Au collisions at AGS energies, except the measurements of \( \Lambda \) \cite{29} and \( \phi \) \cite{30} at 11.5 and 11.7 AGeV and with different kinematic coverage, from E877 and E917 experiments respectively. Nonetheless, we confine ourselves only to the SPS energy domain. Data on \( p_T \) distribution of a variety of strange hadron species from STAR Collaboration \cite{31} at RHIC beam energy scan (BES) program are preliminary at the moment \cite{32}. Therefore, we have not considered it for the analysis at the moment. Moreover, the corresponding \( y \) distributions have also not been reported yet. The analysis of the data above SPS energies is beyond the scope of this work.

Details of the data sets of heavy strange hadrons under investigation are summarized in Table I. The lightest hadron in our chosen set is thus \( \phi \) meson, having a mass of 1.02 GeV. Therefore contributions from hadronic resonance decays are expected to be small and hence ignored. The model fits are done by minimizing the value \( \chi^2/N_{\text{dof}} \), where \( N_{\text{dof}} \) denotes the number of degrees of freedom that is the number of data points minus the number of fitting parameters. The MINUIT2 \cite{33} package as available in ROOT framework \cite{34}, is employed for the
FIG. 4: The (partial) expansion history of the fireball created in central Pb+Pb collisions at 158A GeV. The points indicate the temperature ($T_{\text{kin}}$) and mean transverse collective flow velocity ($\langle \beta_T \rangle$) of the system at the time of charm kinetic freeze-out (filled circle), heavy strange kinetic freeze-out (filled square) and light hadron kinetic freeze-out (filled triangle). Errors are within the marker size.

minimization procedure in our analysis.

For light hadrons, we consider results from Ref. [22] for simultaneous fits to only $\pi^-$ and $K^\pm$. In the present work, we do not consider protons in the fits as the rapidity spectra of protons are not available at SPS energies. Moreover, due to stopping at low energies, all observed protons may not be thermally produced. However, we have checked that the main message of the present work remains unaltered irrespective of whether we include proton in light hadron or heavy hadron set. At 158A GeV, data are also available for $p_T$-spectra of $J/\Psi$ and $\Psi'$ for which we perform a simultaneous fit as separate set.

We start with analysis of $p_T$ spectra of the heavy strange hadrons. At a given collision energy, the $p_T$ distribution of all the available heavy strange hadrons are fitted simultaneously using Eq. (1). To minimize
tablab

| $E_{\text{Lab}}$ (A GeV) | Hadrons                | squared sound velocity ($c_s^2$) | $\chi^2/N_{\text{dof}}$ |
|--------------------------|------------------------|----------------------------------|--------------------------|
| 20                       | Heavy strange          | 0.1602 ± 0.0006                  | 2.4                      |
|                          | Light                  | 0.0755 ± 0.0000                  | 331.7                    |
| 30                       | Heavy strange          | 0.2156 ± 0.0009                  | 2.2                      |
|                          | Light                  | 0.121 ± 0.0000                   | 238.7                    |
| 40                       | Heavy strange          | 0.2215 ± 0.0007                  | 2.1                      |
|                          | Light                  | 0.1682 ± 0.0001                  | 38.4                     |
| 80                       | Heavy strange          | 0.2234 ± 0.0005                  | 2.9                      |
|                          | Light                  | 0.2136 ± 0.0000                  | 22.2                     |
| 158                      | Heavy strange          | 0.2511 ± 0.0003                  | 3.1                      |
|                          | Light                  | 0.2276 ± 0.0001                  | 26.5                     |

TABLE III: Summary of the fit results (squared speed of sound ($c_s^2$) and $\chi^2/N_{\text{dof}}$ values) of Rapidity spectra of heavy strange and light hadrons at different energies from SPS using Non-conformal Landau model.

the free parameters in the fit, the freeze-out time $\tau_F$, degeneracy factor $g$ and the fugacity (chemical potential) are coupled together into a single normalization constant $Z = \frac{1}{2\pi} \tau_F \exp(\mu/T)$, which is adjusted separately for each particle species. Since chemical freeze-out fixes the value of chemical potential, its absorption inside the normalization would not affect the thermodynamic conditions at kinetic freeze-out. As mentioned earlier, the dependence on $R_0$ factors out leading to a volume factor $\tau_F R_0^3$ which can also be absorbed inside the overall normalization. Thus we are left with three free parameters, namely $T_{\text{kin}}$, $\eta_{\text{max}}$ and $\beta_T^0$. Simultaneous fits of the $p_T$ spectra at 20A, 30A, 40A, 80A and 158A GeV are performed for all available heavy strange hadrons. The best fit results are shown in Fig. 1. The fit to the data is well described by the single $\eta_{\text{max}}$, $\langle \beta_T \rangle$ and $T_{\text{kin}}$ values as can be observed from the $\chi^2/N_{\text{dof}}$ values given in Table II.

In Fig. 2, we plot the extracted best fit parameters namely, average transverse velocity ($\langle \beta_T \rangle$) and kinetic...
freeze-out temperature \(T_{\text{kin}}\) and \(\eta_{\text{max}}\) as a function of the beam energy \(E_{\text{Lab}}\). All three quantities show increasing trend as a function of beam energy \(E_{\text{Lab}}\).

Moreover, at all collision energies, the extracted temperatures are larger than those for light hadrons, obtained in \([22]\). Also the corresponding smaller \(\langle \beta_T \rangle\) and \(\eta_{\text{max}}\) values indicate the heavy strange particles decouple from the fireball earlier in time compared to the light hadrons. Thus the kinetic freeze-out also seem to exhibit a hierarchical structure, with more massive particles leaving the medium earlier in time.

As mentioned earlier, in Ref. \([25]\), the \(m_T\) spectra of \(J/\psi\), \(\psi'\) and \(\Omega\) produced in 158A GeV central Pb+Pb collisions were analyzed within boost-invariant blast-wave dynamics. Based on the hypothesis that these heavy hadrons are produced via statistical coalescence and undergo freeze-out during hadronization, due to their small rescattering cross-sections in hadronic phase, an average transverse collective flow velocity of \(\langle \beta_T \rangle \approx 0.2\) was extracted from simultaneous fit to the spectra, restricting \(T_{\text{kin}} = 170\) MeV, from analysis of hadron multiplicities.

For us it would be worth analyzing the available transverse distribution of charmonia in 158A GeV Pb+Pb collisions, measured by NA50 Collaboration \([35]\), within the present model framework. Instead of fixing \(T_{\text{kin}}\), we keep it free with other two parameters. Simultaneous fitting of \(J/\psi\) and \(\psi'\) \([36]\) \(p_T\) distributions in rapidity range \(0 \leq y_{\text{c.m.}} \leq 1\) shown in Fig. 3, gives the following values of the parameters: \(T_{\text{kin}} = 164\) MeV, \(\eta_{\text{max}} = 1.70\) and \(\langle \beta_T \rangle \approx 0.2\), indicating the emission of these heavy resonances from the fireball much earlier in time. The NA60 Collaboration \([37]\) has measured \(J/\psi\) production in 158A GeV In+In collisions. However, the corresponding transverse distributions have not been published yet. Note that we exclude \(\Omega\) baryon, as it is a member of our heavy strange set at 80A and 158A GeV and much lighter than the charmonium family.

In Fig. 4, we show the freeze-out points extracted from the measured transverse spectra of hidden charm, heavy strange and light hadrons at 158A GeV, defining the path of the expanding system in the \(T_{\text{kin}}-\langle \beta_T \rangle\) plane (left panel), \(T_{\text{kin}}-\eta_{\text{max}}\) plane (center panel) and \(\langle \beta_T \rangle-\eta_{\text{max}}\) plane (right panel). Results show a monotonous behavior which support a clear existence of a mass dependent hierarchy in thermal freeze-out of hadrons. This hierarchy of kinetic freeze-out is expected as the medium induced momentum change of heavy hadrons would be smaller compared to lighter hadrons. Hence, as the temperature of the fireball decreases, one would expect an earlier kinetic decoupling of heavy hadrons. Therefore, with a systematic investigation of the freeze-out parameters of different hadron species one can in principle trace (partially) the expansion history of the fireball produced in nuclear collisions. Till date no charm data are available in heavy-ion collisions below top SPS energy. The upcoming NA60+ experiment at SPS \([16]\), aims at the measurement of charmonia in 20A – 158A GeV Pb+Pb collisions. Data once available at lower energies will enable to concretely establish this mass dependent hierarchy in thermal freeze-out.

Before we move forward, it might be interesting to note that the possible existence of hierarchy in the kinetic freeze-out parameters of the produced particles has been studied earlier at RHIC and LHC energies. In Ref. \([38]\), the authors have analyzed the \(p_T\) spectra of the identified hadrons in \(\sqrt{s_{NN}} = 2.76\) TeV Pb+Pb collisions, using a so-called longitudinal boost-invariant single freeze-out model, which describe both the particle spectra and particle ratios with a single value of the temperature. Their results indicated a flavour dependent kinetic freeze-out scenario, with strange hadrons leaving the fireball earlier in time than the non-strange hadrons. In Refs. \([39-41]\), the authors have also analyzed the \(p_T\) spectra of different particle species measured at mid-rapidity in p+p and A+A collisions at various collision energies at RHIC and LHC, using different variants of Tsallis distribution. The freeze-out temperature is found to increase with the increase in particle mass, exhibiting an evidence of mass dependent multiple kinetic freeze-out scenario.

After fitting the \(p_T\) distributions, we now shift our focus to the \(y\) distributions of the heavy strange hadrons. Integrating Eq. (1) with respect to \(m_T\), we obtain the rapidity distributions which are compared with the available data at SPS. No data on rapidity distributions of heavy strange hadrons are available from RHIC-BES program. The same \(\eta_{\text{max}}\) values as listed in Table II are used to reproduce the rapidity density distributions. However the corresponding \(T_{\text{kin}}\) values are not chosen from Ta-
Since the $p_T$ spectra are fitted only in mid-rapidity region. Based on the fact that rapidity spectra are rather insensitive to the underlying temperature [24] and following the same analysis strategy as described in [22], we fix $T_{\text{kin}}$ to be 120 MeV at all collision energies, to get a reasonable description of the rapidity spectra as shown in Fig. 5. However, such high values of $T_{\text{kin}}$ fail to provide a reasonable fit to the $p_T$ spectra. Note that we refrain from fitting the $\Lambda$ rapidity distributions at 80A and 158A GeV because due to incomplete stopping at these energies and the fact that $\Lambda$ carry significant fraction of total net baryon number, its rapidity distributions are flat [26, 42].

We also investigate the effect of longitudinal flow on the observed rapidity distribution of the heavy strange hadrons. We have seen earlier [22] that isotropic emission from static thermal model cannot describe the measured rapidity distribution of light hadrons at all beam energies. Collective expansion in the longitudinal direction is essential to reproduce the data. An illustrative comparison to understand how the longitudinal motion influence the rapidity distribution of the heavy strange hadrons is presented in Fig. 6. The rapidity distribution of $\phi$ mesons measured in 80A GeV central Pb+Pb collisions is contrasted with that from a static thermal model as well as from the present blast-wave model calculations. The rapidity distribution as obtained from static isotropic thermal source falls much faster than the data, a feature that is common for all heavy strange hadrons and at all investigated energies.

It is also interesting to look at the collective motion of the fireball in the longitudinal direction, as obtained from the strange analysis. In absence of longitudinal boost-invariance, the longitudinal and transverse motion of the thermal source are coupled. Assuming the space-time rapidity to be equal to the fluid rapidity, the mean longitudinal velocity of the medium can be estimated as [22]

$$\langle \beta_L \rangle = \frac{\int_{0}^{\eta_{\text{max}}} d\eta \tanh(\eta)}{\int_{0}^{\eta_{\text{max}}} d\eta} = \frac{\ln(cosh\eta_{\text{max}})}{\eta_{\text{max}}}.$$  (4)

As the boost invariance is restored, $\langle \beta_L \rangle \rightarrow 1$. As expected $\langle \beta_L \rangle$ is smaller for heavy strange hadrons than the light hadrons. It shows monotonic increase as function of the beam energy as shown in Fig. 7. The saturative trend at higher energies indicates the recovery of longitudinal boost invariance.

This essentially completes our study of kinetic freeze-out conditions for heavy strange hadrons within non boost-invariant blast wave model. However, before we close, it might be useful to take a deeper look at the longitudinal dynamics particularly so due to the absence of boost-invariance at low energy collisions. Hence mov-
which and plotted in as a function of beam energy. We observe that \( E_{lab} = 158 \text{ AGeV} \) and 4 GeV.

where, \( y \) stored by putting conformal solution of Landau hydrodynamics can be re-
given in Eq. (3). We develop a non-conformal solution of Landau hydro-
dynamics is given by 

\[
\chi^2/y_c = \ln\left[\frac{1 + c_s^2}{2c_s} \sqrt{y_b^2 - y^2}\right], \tag{5}
\]

where, \( c_s^2 \) is the squared sound velocity in the medium, \( y_b' \equiv \frac{1}{2} \ln\left[\frac{1 + c_s^2}{2c_s} \right] + y_b \), with \( y_b = \ln(\sqrt{s_{NN}}/m_p) \) being the beam rapidity and \( m_p \) the proton mass. The conformal solution of Landau hydrodynamics can be re-
stored by putting \( c_s^2 = 1/3 \) [44]. In Fig. 8, we compare the available data with predictions from different dynamical models. We find that the rapidity spectra from conformal solution falls off too slowly and does not give good agreement with the data. On the other hand, both blast-wave as well as the non-conformal solution of Landau hydro-
dynamics explains the data really well.

We perform simultaneous fits to the available rapidity spectra using the non-conformal Landau distribution given in Eq. (5). We obtain reasonably good fits, as shown in Fig. 9, with good \( \chi^2/N_{dof} \) and the extracted values of \( c_s^2 \) are shown in Table III. Here, \( c_s^2 \) is a common parameter for all species and only the overall normalization constant is allowed to be different.

Eq. (5) is also used to fit the rapidity spectra of light hadrons at SPS energies as shown in Fig. 10. Resultant values of the \( c_s^2 \) are illustrated in Table III and plotted in Fig. 11 as a function of beam energy. We observe that \( c_s^2 \) increases monotonically as a function of beam energy for both cases implying that \( c_s^2 \) increases with temperature as the average temperature of the fireball increases with increase in beam energy.

This effect can also be observed in the relative hier-
archy in the values of \( c_s^2 \) for light and heavy strange hadrons. Since heavy strange hadrons freeze-out at higher temperature, the average temperature experi-
enced by them is larger compared to the light hadrons for same beam energy. This is in accordance with the fitted value of \( c_s^2 \) which is consistently larger for heavy strange hadrons as shown in Fig. 11. Moreover, this result is also in line with the expectation from Fig. 4 which support a clear existence of a mass dependent hierarchy in thermal freeze-out of hadrons. However, note that for rapidity spectra of both light hadrons and heavy strange hadrons at low energies, we do not get a good fit with non-conformal Landau solution given in Eq. (5) resulting in large values of \( \chi^2/N_{dof} \). This may be attributed to

FIG. 10: Fitted rapidity distribution of \( \pi^- \) and \( K^\pm \) using non-conformal Landau distribution in central Pb+Pb collisions from SPS at (a) 20A GeV, (b) 30A GeV, (c) 40A GeV, (d) 80A GeV and (e) 158A GeV beam energies. Error bars indicate available statistical error.
the fact that due to complete stopping at low energy collisions, the baryon chemical potential is large which has been ignored while obtaining the non-conformal Landau solution.

Before we close, one may note that within blast-wave framework, the macroscopic thermodynamic parameters are directly extracted by fitting the certain phase-space density distribution of experimentally measured hadrons. Recently the kinetic freeze-out stage has been explored in central Au+Au collisions at energies ranging from $\sqrt{s_{NN}} = 2.4$ GeV to $\sqrt{s_{NN}} = 200$ GeV, using the microscopic UrQMD model and the corresponding macroscopic parameters are calculated via coarse graining approach [45]. Results indicate the kinetic freeze-out as a continuous process, leading to a distribution of the freeze-out parameters at different collision energies. The corresponding average kinetic freeze-out temperatures at different beam energies are higher than those obtained by us in our previous work for bulk hadrons.

IV. SUMMARY

In conclusion, we have made an attempt to study the hierarchy in the kinetic freeze-out conditions of different hadrons in central Pb+Pb collisions at different SPS energies using non boost invariant blast wave model. Transverse momentum spectra of these hadrons fitted simultaneously to obtain the $\eta_{max}$, $\langle \beta_T \rangle$ and $T_{kin}$ values which explain the data really well. We found a clear mass dependent hierarchy in the fitted kinetic freeze-out parameters. This hierarchy of kinetic freeze-out parameters is expected as the medium induced momentum change of heavy hadrons would be smaller compared to lighter hadrons. Therefore, as the temperature of the fireball decreases, one would expect an earlier kinetic decoupling of heavy hadrons. The results indicate that $T_{kin}$ values are in the range 90 – 110 MeV with $\langle \beta_T \rangle$ of about 0.4 to 0.5c. The temperature values are rather higher than the light particles discussed in [22] which indicate early thermal freeze-out of heavy strange hadrons. The extracted $\eta_{max}$ also explains the corresponding rapidity spectra reasonably well. We found that the extracted freeze-out parameters for charmed hadrons also corroborates this mass dependent hierarchy. The values of $\eta_{max}$, $\langle \beta_T \rangle$ and $T_{kin}$ was found to increase monotonously as a function of beam energy.

Moreover, the rapidity spectra of light hadrons as well as heavy strange hadrons are tested with a different model prescription than blast-wave in order to explore the longitudinal properties of the medium. For this, a non-conformal Landau description of rapidity distributions from recent work [43] is used. This prediction explains the heavy strange hadrons spectra nicely however, reasonably well for light hadrons. We found that the fitted value of sound velocity in the medium also exhibit a similar hierarchy which is obtained from fits to $p_T$-spectra. We advocate that our findings are essential to provide predictions for upcoming experiments at FAIR and NICA accelerator facilities.

Looking forward, it will be interesting to repeat this exercise with charmed hadrons for lower energy collisions, when the data become available. This would be possible with the future measurements at SPS. As mentioned earlier, the NA60+ experiment [16] at SPS aims at the measurement of charmonia in Pb+Pb collisions in the beam energy range $E_{lab} = 20A - 158A$ GeV. In addition, the upgraded version of NA61/SHINE experiment at SPS plans to measure the open charm mesons ($D$ meson) via their hadronic decay channel, in Pb+Pb collisions at beam energies 40A and 150A GeV [46]. A large statistics data set at 150A GeV has already been collected which is presently being analyzed. The existence of mass hierarchy in kinetic decoupling at low energy collisions, can be tested more robustly, if in addition to charmonia, transverse spectra of $D$ mesons are also made available, since their rest mass is closer to that of multi-strange hadrons. We leave this analysis for future.

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