Variation of Fine Structure Constant from Non-Universal Gravity

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We relate the reported variation in the value of the fine structure constant to a possible non-universality of the gravitational interaction with respect to different particle generations.

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Recent measurement of the fine structure constant in the remote, yet on cosmological scale recent past yielded a 4σ deviation \( \Delta \alpha / \alpha = -(0.72 \pm 0.18) \times 10^{-5} \) from laboratory value within the red shift domain \( z \simeq 0.6-2 \) \([1]\). This poses the question about the origin of such an effect \([2,3]\), identified as either temporal (general presumption) or spatial variation. Since the experimental effect which is observed is seen within a scheme of a systematic change in absorption lines by nebula in line of sight of quasar illumination of laboratory sensors, the effect could be originating in properties of the nebula. Such spatial interpretation of the data analysis is more compatible with other limits on variation of \( \alpha \) \([4]\).

As recent flurry of activity has shown that it is quite difficult to find a self consistent interpretation of this experimental result. We propose here a quite different approach:

a) We interpret the change in \( \alpha \) as being due to fermion masses \( m_f \) which enter vacuum polarization loops.

b) We will argue that the effect cannot involve electrons, thus only the heavy fermions such as the muon contribute.

c) Assuming that heavy fermion masses have varied by about -0.15–0.45% this experimental observation can be understood.

d) There is no consistent interpretation assuming time variation of \( m_f \) (or \( \alpha \)) since there is no sensitivity to the time evolution of the late Universe we live in.

e) When the effect is associated with spatially localized gravitational mass it requires non-universal gravitational coupling.

f) We show how a theory of gravity is formulated in principle which combines different ratios of gravitational to inertial mass for the three fermion generations. This is not in contradiction with the Eötvös experimental evidence for universality of the ratio of inertial to gravitational mass, and all principal results of Einstein gravity and cosmology are retained.

g) The 1000-fold enhanced strength of gravity for 2nd and/or 3rd generation, which is required to understand \( \Delta \alpha / \alpha \), also explains the dark matter in terms of the more strongly gravitating cosmic background 2nd and/or 3rd generation neutrinos.

h) We do not present here a theoretical model how a gauge universal, gravity non-universal, theory could arise.

Conventional theoretical framework: Within the current paradigm of a unified structure of all interactions, the value of \( \alpha \), the electromagnetic coupling in the infrared large distance \( r \to \infty \) limit is not fundamental, the determining value is the strength of the universal interaction at the unification scale, which in case of ‘Grand Unification’ is \( M_{GUT} \simeq 3 \times 10^{16} \) GeV. As energy scale diminishes, broken symmetries emerge and individual interactions separate, and their strength evolves with the scale. Electromagnetic (EM) interaction emerges when the electro-weak (EW) interactions separate at the scale of \( \mathcal{O}(100 \text{ GeV}) \), and where \([4, 5]\),

\[
\alpha(M_Z = 91.2 \text{GeV}) = \frac{1}{128.936(46)}. \tag{1}
\]

Unification of interactions poses a problem for an apparent time variation of \( \alpha \), since it implies a much larger associated variation in strong interaction coupling parameters, implicating significant changes in quantities such as the nucleon mass and nuclear interactions \([6, 7]\), and indeed leading to possible and probable disagreement with latest limits on cosmological variation of e.g. the proton to electron mass \([8]\). In addition we need to resolve the question, how can sensitivity to the structure of the universe arise at a relatively late stage of its evolution, or if spatial interpretation is preferred, in localized nebula.

Introduction of relatively large compactified extra dimensions reduces the unification scale considerably, to the TeV range \([9]\). Association of the time dependence of \( \alpha \) or as we argue here \( m_f \) with large extra dimensions has the inherent flaw that these dimensions do not evolve beyond the early, formative period of the Universe \([10]\).

The case is quite strong today, in view of the many ramifications \([11]\) of a time-changing strong interaction strength accompanying the change in \( \alpha \), that it is necessary to search for other possible sources of, on cosmological scale ‘nearby’, time, or spatial, dependence of \( \alpha \). Regarding the forthcoming discussion of the three particle generations we recall that it is not understood today how the masses of the ‘heavy’ fermion families are gen-
erated from a unified theory involving radically different scales (hierarchy problem). It is expected that in a unified theory which comprises the quantum gravity, not only the interactions, but also the three particle family generations will be naturally explained.

The fundamental generation which is at the origin of the matter around us, also provides the measuring sticks in the Universe. The unified theory than explains the mass ratios such as the muon to electron mass is $m_{\mu}/m_e = 206.77$. Furthermore, if electron mass (along with other lepton masses) were to vary, in its units the nuclear world around us would evolve, and this is not safe from contradictions with precision experiments. One can only the interactions, but also the three particle family

generations will be naturally explained.

Here we assume that all the variation occurs within the energy scale at which the interaction is stable as function of internal degeneracy, such as color of quarks (=3). The scale evolution of strong interactions is dominated by gluons, a similar argument is applicable in a very good approximation. Thus we cannot introduce a variation of mass of (any of the) three leptons $e, \mu, \tau$ and five quarks $u, d, s, c, b$ is sufficient to generate the observed effect. Restricting our attention to the 2nd and 3rd generation and distributing the effect uniformly, we need less than -0.1% variation in mass for each of the 5 fermions.

Within this chain of arguments we cannot expect that the variation of the masses of second and third families is found in the conventional interactions, as these effects would also alter the fundamental generation:

1. Masses, like coupling strengths, evolve with the interaction energy scale, driven by variation of the coupling constant. It is customary to write, within QED,

$$\frac{M}{m} \frac{d m}{d M} = \gamma(\alpha(M)).$$

(4)

Given a universal function $\gamma(\alpha)$ for leptons, it is easy to see that the variation of all lepton masses is exactly the same and thus the ratio of lepton masses is scale independent. For quark mass ratios, where the scale evolution of strong interactions is dominated by gluons, a similar argument is applicable in a very good approximation. Thus we cannot introduce a variation of mass of 2nd and 3rd generation only by manipulating the scale running of masses.

2. Considering the case that the masses of fermions are derived from the coupling to the Higgs field, the mass ratios are ratios of coupling strengths. It is less than obvious how a fraction of percent variability of mass can arise within the Higgs scheme solely for the 2nd and 3rd generation.

**Gravity for 2nd and 3rd generation:** To explain the observed effect, we would like to see a variation of

section results [4], rather than through the use of Eq. (2). The lepton contribution is $\Delta \alpha_e^{-1} = 4.3164$ (including a very small higher order effect) while the hadron contribution is $\Delta \alpha_h^{-1} = 3.823 \pm 0.054$. In comparison, the reported cosmological variation is [1] can be expressed as

$$\delta(1/\alpha) = \frac{-1}{\alpha} \frac{\delta \alpha}{\alpha} \simeq 0.001,$$

which is 50 times smaller than the uncertainty remaining in the understanding how the quark degrees of freedom influence the evolution of EM interaction strength, see Eq. (1).

From Eq. (2) we obtain how $\alpha(0)$ responds to the variation of fermion masses, assuming that $\alpha(M)$ is constant:

$$\delta \left( \frac{1}{\alpha(0)} \right) = \frac{-2}{3\pi} \sum_f Q_f^2 N_f \delta m_f / m_f$$

(3)

In consequence, a compound variation by $-0.72 \times \alpha^{-1} 3\pi/2 = -0.46\%$ in mass of (any of the) three leptons $e, \mu, \tau$ and five quarks $u, d, s, c, b$ is sufficient to generate the observed effect. Restricting our attention to the 2nd and 3rd generation and distributing the effect uniformly, we need less than -0.1% variation in mass for each of the 5 fermions.

**Relation of $\alpha$ to fermion masses:** The scale evolution of the EM coupling strength $\alpha \equiv \alpha(M = 0) = 1/137.03599976(50)$, arises from the interaction of photons with the virtual fluctuations of charged fermi particle pairs of mass $m_f$ with mass thresholds within the range of energy scale considered. Quantum electrodynamics allows to evaluate this vacuum polarization effect for leptons with great precision. For our consideration it is sufficient to recall the lowest order in electromagnetic interaction, and the large $M/m$ behavior:

$$\Delta \alpha^{-1} = \frac{1}{\alpha(M)} - \frac{1}{\alpha(0)}$$

$$\approx -\frac{2}{3\pi} \sum_f Q_f^2 N_f \left( \ln \frac{M}{m} - \frac{5}{6} + O \left( \frac{m_f}{M} \right) \right).$$

Here $Q_f$ is the charge of the fermion $f$ and $N_f$ is the internal degeneracy, such as color of quarks (=3). $M$ is the scale at which the interaction is stable as function of time, and/or environment. In our present considerations we assume that all the variation occurs within the energy scale domain below $M_Z$, in which EM emerged as separate interaction. This does not restrict the validity of our principal argument.

Eq. (2) can be applied to evaluate the contribution of the electron, muon and tau $f = e, \mu, \tau$ leptons, and to estimate the effect of strongly interaction quarks, $u, d, s, c, b$ to the ‘running’ of $\alpha(M)$. The contribution of quarks which also interact strongly, is more precisely determined combining dispersion methods with experimental cross
some of the masses, while keeping the fundamental generation masses constant. Thus by necessity we need an interaction
1) which breaks the fermion generation universality, and 2) which is sensitive to the conditions prevailing in the late Universe.

We propose a novel and speculative solution that each fermion generation may have a different ratio of inertial to gravitational mass, as is expressed by gravitational coupling. We recall that the Eötvös type experiment establishes the ratio of gravitational and inertial mass solely for the “stable” first generation which surrounds us.

To be very specific we propose a natural and very slight generalization of the Einstein general relativity equations:

\[ R_{\mu \nu} - \frac{1}{2} g_{\mu \nu} R = \frac{\delta}{\delta g_{\mu \nu}} (G_1 I_1 + G_2 I_2 + G_3 I_3) \]  

(5)

Here, \( R \) is the Ricci curvature tensor, \( G_i = 8\pi g_i / c^2 \) with \( g_1 = G \) the usual gravitational constant and \( G_2, G_3 \) are the gravitational constants of the 2nd and 3rd generation of particles, while \( I_i \) is the action of the generation \( i \) derived from the study of inertial forces. Variation with respect to the metric \( g_{\mu \nu} \) does not yield a global inertial energy-momentum tensor \( T_{\mu \nu} \), rather we find for each generation its \( T^i_{\mu \nu} \)-component weighted with different strength of the non-universal gravitational interaction. Since each of the components, \( T^i_{\mu \nu} \), has the same transformation properties, the right hand side of Eq. (5) transforms like the inertial energy-momentum tensor.

We have not included gauge fields in the above. A priori their universality poses a technical challenge in our proposed approach. What needs to be derived to make our approach correctly rooted in e.g. the brane theory is a brane configuration which maintains gauge universality, while yielding gravity non-universality with respect to the different generations \([10]\). However, within the brane theory more common is to find a configuration leading to gauge non-universal, gravity universal theory \([17]\).

The inertial matter action \( I_i \) is as given by Weyl, Fock and Iwanenko in late 20’s:

\[ I_i = \int d^4 x \sqrt{-g} [\bar{\psi}_i (\gamma^\mu i D_\mu - m_i) \psi_i]. \]  

(6)

Where \( \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2 g^{\mu \nu} \) defines the Dirac-\( \gamma \) general relativistic matrices and \( D^\mu = \partial / \partial x^\mu + \Gamma^\mu \) with:

\[ \Gamma^\mu = \frac{1}{4!} \gamma^\nu \left( \partial \gamma^\nu / \partial x^\mu + C^\nu_{\lambda \mu} \gamma^\lambda \right), \]

and the Christoffel symbol is:

\[ C^\nu_{\lambda \mu} = \frac{1}{2} g^{
u \beta} \left( \partial g_{\lambda \beta} / \partial x^\mu + \partial g_{\beta \mu} / \partial x^\lambda - \partial g_{\lambda \mu} / \partial x^\beta \right). \]

Cosmological solutions of field equations arising from Eq. (4) have the usual structure. Presence of matter with different gravitational couplings (e.g. cosmic background neutrinos) does not alter the structure of the Einstein equations. In the dust model for the energy-momentum tensor the effective equations of state are different, for the case of mass less (neutrino) gas the Einstein equations including the 2nd and 3rd generation appear with effective degeneracy \( g_{2,3}^{\text{eff}} = g_{2,3} G_{2,3}/G_1 \), there seems to be more gravitating matter in the Universe, just as required. Stellar objects containing 2nd and 3rd generation neutrinos would still be defined by the asymptotic Schwarzschild mass, we could not even tell which matter is within the star, short of locally probing the flavor and inertia of each component. Important for our later consideration of mass threshold is that the background Schwarzschild metric attaches to the mass of a particle as

\[ m_i \rightarrow m_i \sqrt{1 - \frac{2M g_i}{r}} \rightarrow m_i \left(1 - \frac{M g_i}{r}\right). \]  

(7)

We thus see that the local gravitational potential attaches to the gravitational charge \( g_i \) of particle ‘i’. Accordingly the mass defect is more significant for particles with greater coupling strength.

**Consequences of non-universal gravity:** There is lack of any direct experimental information on the gravity action on 2nd and 3rd family. A very weak upper limit of the strength of the interaction \( G_{3,2} \) arises noting that massive particles in these families should not be beyond Planck mass limit, which requires \( G_{3,2}/G_1 < 10^{17–10^{19}} \). A stronger limit can be derived from loop contributions of virtual particles to physics on Earth, which begins to play a noticeable role when \( G_{3,2}/G_1 \approx 10^{20} \). An experimental study of muonium \( \mu^+ e^- \) fall in the gravitational field of the Earth could reveal a better limit on \( G_2 \). During muons lifespan of \( 2\mu s \) a particle falls 20 Å, thus a measurable effect requires \( G_2 \gg G_1 \). Short of approaching the Planck mass (see above) the enhanced gravitational interaction remains weak.

Turning to cosmological consequences we note that even if the inertial masses of neutrinos are at the (small) level of the mass differences seen in neutrino oscillations, \( m_i \approx 10^{-2} \text{ eV} \) \([12]\), a 1000 fold enhancement of second and third neutrino generation gravitational coupling allows the two neutrinos in cosmological background to contribute a gravitational mass-energy equivalent of nearly 10 eV \( G_\text{C} \) each, and thus to describe the ‘dark’ gravitating mass of the Universe. The presence of more strongly gravitating component in the background of our Universe is likely to help the dynamical evolution of density fluctuations into the present day large inhomogeneities. The dynamics of the Universe expansion would need to be reexamined in this new context.

One should not before a thorough exploration of the family specific gravitational interaction see the above factor \( 10^3 \) as an upper limit on gravitational interaction enhancement. Namely, the neutrino oscillation phe-
nomenon combined with the large gravitational fields in early Universe can greatly alter this naive limit.

The required change of the fermion mass by -0.1–0.5% within the nebulae which we are looking for is understood to be the gravitational mass defect. Our Sun generates for normal gravity a mass defect \(\delta m_1/m_1 = -0.3 \times 10^{-5}\). Thus coincidental with the explanation of the dark matter, the non-universal gravity with \(G_{3.2}/G_1 \approx 10^3\) yields the expected -0.3% mass defect for the 2nd and 3rd generation fermions. Note that the sign of the effect is correct, which aside of checking all equations can be seen as follows: background gravity reduces the effective threshold for creation of a pair of particles, and this reduction in effective mass allows the finite structure constant to ‘run’ faster and thus from a fixed unification value toward a smaller value, as is reported experimentally to be present within the nebula.

Once we interpret the variation of \(\alpha\) as a spatial effect, we could go back to check if it would not be possible that all fermions are subject to the mass defect. This would require that nebula appear to have a gravitational interactions 1000 stronger compared to our sun also for regular matter. This seems not to be possible.

Enhanced gravitational coupling has the capacity to resolve some riddles in stellar structure. For example, the accretion of background neutrinos and resulting concentration of more strongly gravitating neutrinos in large stellar objects \([13]\) releases greatly enhanced amount of gravitational energy as e.g. is required to fuel the quasar phenomenon. Since there are two different additional gravitational couplings possible, and there is a finite supply of cosmic neutrinos it may be possible to explain both quasars and active galactic nuclei in terms of accretion of neutrinos, and to limit the range of cosmological time during which these phenomena occur. These remarks are consistent with appearance of event horizons (black holes) in such stellar objects, indeed the non-universal gravity of some of the accreted (neutrino) particles would lead to faster formation of such a gravitational singularity. Naturally, this singularity is the same for all test particles of any of the three generations.

**Final remarks:** We conclude that the small deviation of \(\alpha(0)\) from the established values could be result of a somewhat larger (by factor \(645 = 137 \cdot 3\pi/2\)) -0.46% compound total mass variation of unstable second and third generation fermions. It seems that a spatial rather than temporal variation of fermi masses, due to a gravitational mass defect, is the source of this effect. The nebula which act as source of the absorption lines are required to generate gravitational fields causing this mass defects. An enhanced anomalous gravitational coupling of heavy fermi generations is our interpretational scheme. A factor 1000 enhancement in non-universal gravity allows interpretation of dark matter as being due to \(\nu_\mu, \nu_\tau\) cosmic background, while the gravitational mass defect of stable matter in absorbing nebula is at \(10^{-5}\), just a factor 5 greater than the mass defect of the sun.

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