Can neutrino mass be measured in low-energy electron capture decay?

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Abstract

The standard kinematic method for determining neutrino mass from the beta decay of tritium or other isotope is to measure the shape of the electron spectrum near the endpoint. It has been known for 30 years that a similar distortion of the “visible energy” remaining after electron capture is caused by neutrino mass. There has been a resurgence of interest in using this method with $^{163}$Ho. Recent theoretical analyses offer reassurance that there are no significant theoretical uncertainties. We show that the situation is, however, more complicated, and that the spectrum shape is presently not well enough understood to permit a sensitive determination of the neutrino mass in this way.
INTRODUCTION

The fact that neutrinos have mass was established by the discovery of neutrino oscillations in atmospheric [1], solar [2], and reactor [3] neutrinos. The minimal standard model does not include right-handed fields for neutrinos, and therefore predicts the mass is zero. How neutrinos acquire their small masses is consequently a matter of great theoretical interest, and may be evidence of new physics at very high mass scales. Oscillation data provide only the differences between the squares of masses, but do constrain the average mass of the 3 species to be at least 0.02 eV because no squared mass can be less than zero. Laboratory measurements of the beta spectrum of tritium [4, 5] yield an upper limit on the absolute scale of neutrino mass of less than 2 eV. Given that the mass must then lie in this range, new, sensitive laboratory measurements are being pursued [6–8] to shed further light on the mechanism for neutrino mass generation.

Neutrinos are also an abundant ingredient of the universe, created in numbers comparable to photons during the big bang. The combination of direct laboratory measurements and neutrino oscillation data show that neutrino mass is too small for active neutrinos to be the dark matter that makes up some 27% of the energy density of the universe, but their mass may influence large-scale structure and other observables. A laboratory measurement of the mass at an improved level of sensitivity would be valuable in helping to constrain cosmological parameters that are correlated with it, such as the equation of state of dark energy and the fluctuation amplitude of the matter power spectrum [9].

Among the ideas being investigated for a laboratory measurement of neutrino mass is one originally proposed more than 30 years ago [10, 11], a measurement of the energy retained following electron capture in $^{163}$Ho, a nucleus with a particularly low Q-value [12] for the decay to the ground state of $^{163}$Dy. In this note we raise a concern that, technological progress notwithstanding, the theoretical description of the spectrum is insufficiently understood yet to permit an eV-scale determination of the neutrino mass.
ELECTRON-CAPTURE DECAY

In its simplest form, electron-capture decay is the capture by the nucleus of a bound atomic electron with the release of an electron neutrino. The neutrino’s energy is the Q-value minus the electron binding energy, and thus consists of several mononertgetic lines.

\[ ^A Z \rightarrow ^{A(Z-1)}i + \nu_e + Q_i \]  \hspace{1cm} (1)

where \( A \) and \( Z \) are an atomic mass and number, respectively, and \( Q_i \) refers to the Q-value for the particular atomic final state \( i \). In this form there is very little sensitivity to neutrino mass because the neutrino is always relativistic. However, in the early 1980s De Rújula and Lusignoli \cite{10, 11} recognized that the lines are in fact not monoenergetic because atomic vacancies have short lifetimes and therefore non-negligible widths. The tails of the lines extend to the energy limit imposed by the ground-state Q-value, and at that limit are sensitive to the modification of phase space caused by neutrino mass, just as in beta decay. The existence of an electron-capture isotope, \( ^{163}\text{Ho} \), with a very low Q-value \cite{12} in the vicinity of 2.5 keV heightened the interest in this approach and a number of experimental groups have explored the possibility with a variety of techniques \cite{13–18}. In a calorimetric experiment one is indifferent to the details of how the vacancy refills, whether by radiation or electron ejection, and records a spectrum of \( E_c \), the “visible energy” \((i.e., \text{that not carried away by the neutrino})\) converted to heat. Advances in the art of microcalorimetry have spurred a resurgence of interest, as very high resolution spectra from large arrays of detectors become a possibility \cite{19–24}.

In order to make a convincing case about the neutrino mass from this kind of experiment, it is important to understand what the spectrum would look like without it. As there is no way to set the mass to zero experimentally, there is no recourse but to rely on theory.

Ascribing a Breit-Wigner line shape to each vacancy and imposing a phase-space and energy-conservation envelope, De Rújula and Lusignoli calculate the spectrum to be expected \cite{10, 11, 25}. Expanding the neutrino flavor eigenstate in the mass basis, and following \cite{10} and
one can obtain the spectrum in the following form:

\[
\frac{d\lambda_{EC}}{dE_c} = \frac{G_F^2 \cos^2 \theta_C}{2\pi^3} (Q - E_c) \times \\
\sum_i |U_{ei}|^2 \left[ \frac{1}{2} \left( (Q - E_c)^2 - m_{\nu_i}^2 \right) \right]^{1/2} \times \\
\sum_j \beta_j^2 C_j |M_{j0}|^2 \frac{\Gamma_j}{4(E_c - E_j)^2 + \Gamma_j^2},
\]

(2)

where \(G_F\) is the Fermi coupling constant and \(\theta_C\) is the Cabibbo angle, \(U\) is the neutrino mixing matrix and \(m_{\nu_i}\) is an eigenmass, \(\beta_j\) is the amplitude of the electron wave function at the origin, \(C_j\) is the nuclear shape factor, and \(E_j\) and \(\Gamma_j\) are the excitation energy and natural width of atomic configuration \(j\). The quantity \(M_{j0}\) is an overlap (monopole) electronic matrix element between the ground state of the decaying atom and state \(j\) of the daughter atom. Exchange effects \[26\] and orbital occupancies are here absorbed into \(M_{j0}\). In the specific case of \(^{163}\)Ho, the index \(j\) runs over the 7 occupied orbitals from which capture can occur: (3s), (3p1/2), (4s), (4p1/2), (5s), (5p1/2), and (6s).

Recent high-resolution calorimetric data \[22, 27\] confirm this expectation: there are sharp lines corresponding to the energy released and thermalized as the vacancies refill. The statistics are insufficient yet to reveal the wings in any detail, although the strong \((4s)^{-1}\) NI line has shoulders broader and with more structure than theory predicts. Nevertheless, De Rújula argues that far from peaks the spectrum shape is determined only by phase space, and variation of the matrix element cannot be large enough to be relevant. If so, the spectrum given in Eq. 2 can be used with confidence to predict the zero-mass shape near the endpoint and thereby derive experimental values for the neutrino masses.

Treating the capture in the simplified way described above, with \(j\) running over 7 single-particle orbitals, is standard, but it is an approximation. In what isotope is the vacancy formed, \(^{163}\)Ho or \(^{163}\)Dy? An inner-shell electron has been absorbed suddenly in the nucleus, the nuclear charge has changed, and the index \(j\) should range over the complete set of states energetically allowed in the 66-electron final-state Dy atom. Included in that basis are many configurations of neutral Dy with two or more inner-shell vacancies and electrons in bound but normally unoccupied valence levels, or in the continuum. It might be thought that the
probability of multiple vacancies must be very small. On the contrary, for these rare earths, all final states have at least two atomic-orbit occupancies that are different from the ground-state configuration. The ground states of Ho and Dy differ by a single (4f7/2) electron, but only (s) and (p1/2) orbitals have sufficient amplitude at the origin for electron capture. Hence the final state consists of at least an inner-shell vacancy and an extra (4f) electron. While this particular circumstance modifies the spectrum only slightly [28], more significant modifications result from additional vacancies in other shells.

De Rújula presents an estimate [25] that the rate for populating a Dy configuration with simultaneous (3s)_−1 and (4s)_−1 vacancies is 10^{-5} compared to a single (3s)_−1 vacancy and therefore negligible. However, while the probabilities may decrease strongly with more complicated configurations, it is the total intensity near the endpoint that is relevant. Population of the (3s)_−1(4s)_−1 configuration peaks where the single (3s)_−1 tail has become very weak and can even dominate the spectrum in that region. The complex multi-vacancy configurations of neutral Dy include some that are ‘resonant’ in the sense defined by De Rújula: they have relatively narrow widths. The vacancies refill by single-particle electromagnetic transitions, and we therefore assign them widths that are the same as the width of the primary (most deeply bound) vacancy, in the absence of experimental data. The continuum shakeoff process is included with shakeup in the Carlson-Nestor (CN) theory [29] adopted for the present analysis. Shakeoff features are not as narrow as shakeup, but still give rise to enhancements at threshold with a higher-energy tail that falls off on a scale of tens of eV (see, for example, Ref. [30]). They are, therefore, also quite sharply defined spectral features. When the atom is part of a solid, valence and continuum excitations of a still more complex nature become possible.

**CALCULATION**

This argument can be made more quantitative by considering the available configurations in this shakeup process and assigning energies to each based on single-particle estimates. The ground state of Dy I is ([Xe]4f^{10}6s^{2}). Multivacancy configurations can be populated
in electron capture subject to the monopole selection rule, namely that the operator in the
matrix element is the unit operator. This restricts the types of single-particle excitations
that can be present. Restricting the space further to configurations that have only one or two
vacancies and the extra (4f) electron, one can then construct a spectrum. Table I contains
the elements needed. Single-vacancy capture probabilities are adopted from Lusignoli and
Vignati (LV) \[31\] which are in good agreement with the more recent results of Faessler et
al. \[26\], with the inclusion of overlap and exchange corrections. The relative populations
of satellite shakeup configurations are taken from the calculations by Carlson and Nestor \[29\]
for Xe and given with a common intensity normalization in the first column. The excitation
energy of each configuration is assembled from the single-particle binding energies in Ho and
Dy \[32\]. The innermost vacancy is taken to have a binding energy appropriate to Dy while
less-bound shells are assigned binding energies appropriate to Ho, thereby allowing for the
missing inner electron. Both are simplifying approximations that can be expected to lead
to energy errors of a few eV. The energies and line widths are listed in the second and third
columns. Each subsequent column identifies an accessible configuration with an entry of \(-1\)
for a hole in a normally filled shell, and 1 for a normally incomplete shell containing an extra
electron.

The resulting spectrum is shown in Fig. 1. It is seen to be quite complex even in the
relatively restricted space considered. The appearance of a shakeup peak very close to the
endpoint for the chosen Q-value, 2.5 keV, is accidental but underscores the difficulty in
determining precisely the underlying spectrum, as would be required in order to make a
definitive statement about neutrino mass from \(^{163}\)Ho electron capture.

There is evidence already in the data that shakeup satellites are present. An unidentified
peak is observed on the upper shoulder of the \((4s)^{-1}\) line in the high-resolution calorimetric
study by Ranitzsch et al. \[22, 27\]. In Fig. 2 this region is compared with the theoretical
spectrum including shakeup and shakeoff satellites. The unidentified satellite has the correct
energy to be the \((4s)^{-1}(5s)^{-1}\) double vacancy line.

The satellite spectrum presented here is only indicative, rather than quantitative. The
shakeup and shakeoff calculations of Carlson and Nestor were carried out for photoionization
TABLE I. Energies, intensities, and occupation number differences for configurations populated in $^{163}$Ho electron capture. The single-particle binding energies $E_b$ are in eV [32].

| $E_b$ | 3s1/2 | 3p1/2 | 3p3/2 | 3d3/2 | 3d5/2 | 4s1/2 | 4p1/2 | 4p3/2 | 4d3/2 | 4d5/2 | 4f5/2 | 5s1/2 | 5p1/2 | 5p3/2 |
|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| Dy    | 2047  | 1842  | 1676  | 1333  | 1292  | 333.5 | 293.2 | 153.6 | 153.6 | 8     | 4.3   | 49.9  | 26.3  | 26.3  |
| Ho    | 2128  | 1923  | 1741  | 1392  | 1351  | 432.4 | 343.5 | 308.2 | 160   | 160   | 8.6   | 5.2   | 49.3  | 30.8  | 24.1  |

| $E_c$% | $E_c$ eV | $\Gamma$ | Rel. intens. | |
|--------|----------|---------|--------------|---|
| 100    | 2041.8   | 13.2    | -1           | 1 |
| 0.075  | 2474.2   | 13.2    | -1           | 1 |
| 0.11   | 2385.3   | 13.2    | -1           | 1 |
| 0.25   | 2350.0   | 13.2    | -1           | 1 |
| 1.05   | 2201.8   | 13.2    | -1           | 1 |
| 1.62   | 2201.8   | 13.2    | -1           | 1 |
| 1.36   | 2091.1   | 13.2    | -1           | 1 |
| 3.12   | 2072.6   | 13.2    | -1           | 1 |
| 7.31   | 2065.9   | 13.2    | -1           | 1 |
| 5.26   | 1836.8   | 6       | -1           | 1 |
| 0.004  | 2269.2   | 6       | -1           | 1 |
| 0.006  | 2180.3   | 6       | -1           | 1 |
| 0.014  | 2145.0   | 6       | -1           | 1 |
| 0.057  | 1996.8   | 6       | -1           | 1 |
| 0.087  | 1996.8   | 6       | -1           | 1 |
| 0.072  | 1886.1   | 6       | -1           | 1 |
| 0.165  | 1867.6   | 6       | -1           | 1 |
| 0.386  | 1860.9   | 6       | -1           | 1 |
| 23.29  | 409.0    | 5.4     | -1           | 1 |
| 0.001  | 841.4    | 5.4     | -2           | 1 |
| 0.004  | 752.5    | 5.4     | -1           | 1 |
| 0.01   | 717.2    | 5.4     | -1           | 1 |
| 0.077  | 569.0    | 5.4     | -1           | 1 |
| 0.123  | 569.0    | 5.4     | -1           | 1 |
| 0.254  | 458.3    | 5.4     | -1           | 1 |
| 0.629  | 439.8    | 5.4     | -1           | 1 |
| 1.502  | 433.1    | 5.4     | -1           | 1 |
| 1.19   | 328.3    | 5.3     | -1           | 1 |
| 0.0001 | 671.8    | 5.3     | -2           | 1 |
| 0.0005 | 636.5    | 5.3     | -1           | 1 |
| 0.004  | 488.3    | 5.3     | -1           | 1 |
| 0.006  | 488.3    | 5.3     | -1           | 1 |
| 0.013  | 377.6    | 5.3     | -1           | 1 |
| 0.031  | 359.1    | 5.3     | -1           | 1 |
| 0.076  | 352.4    | 5.3     | -1           | 1 |
| 3.45   | 44.7     | 3       | -1           | 1 |
| 0.15   | 21.1     | 3       | -1           | 1 |
FIG. 1. The visible energy in a calorimeter following electron capture in $^{163}$Ho. The simpler spectrum (blue online) is calculated in the customary single-vacancy approximation. The more complex spectrum (red online) includes configurations with 2 vacancies and an extra (4f7/2) electron. The energies are calculated using primary vacancy energies in $^{163}$Dy and secondary vacancy energies in $^{163}$Ho. The shakeup probabilities for the satellite peaks are taken from calculations for Xe by Carlson and Nestor [29]. Vacancies in the (6s) shell have not been considered.

of Xe in a non-relativistic treatment. In photoionization, the electron is ejected from the atom, while in electron capture it is captured in the nucleus. Thus in photoionization it is the outermost orbitals that are subjected to the largest change in effective charge, whereas in electron capture it is the innermost. It is therefore not surprising that the CN calculation applied to electron capture would overestimate shakeup from orbitals that do not have significant amplitude at the nucleus, nor, conversely, is it surprising that the sole double-vacancy state visible in the data so far is $(4s)^{-1}(5s)^{-1}$ at an intensity underestimated by the CN theory.

A less important mismatch is that for comparable excitations in Dy promotions into the (6s)
FIG. 2. Expanded view of the vicinity of the NI line in the $^{163}$Ho spectrum recorded calorimetrically by Ranitzsch et al. [27] (solid line, red online). The calculated spectrum (blue dotted line) exhibits satellites on the high-energy side of the line, and the location of the $\text{(4s)}^{-1}(5s)^{-1}$ double vacancy at 458.3 eV corresponds to the observed satellite peak. However, the intensity predicted with CN theory is lower, and other satellites in the vicinity do not appear at the predicted intensity (see text). The experimental resolution is given in [27] as 8.3 eV; the theory is shown only with the assumed natural width of 5.4 eV.

shell are blocked, reducing the phase space available compared to Xe. Similarly, the increased binding of $(5s5p)$ electrons in Dy compared to Xe can be expected to inhibit shakeup from those orbitals. Non-relativistic treatments of shakeup and shakeoff in photoionization of Xe are found to be fairly satisfactory [33]. Many of the shortcomings could be addressed in a more advanced and specific theoretical treatment, but the conclusion that the spectrum is much more complex than has been assumed heretofore is one that does not depend on such refinements.
Electron capture in $^{163}$Ho measured calorimetrically offers a potentially attractive method for measuring neutrino mass. A quantitative understanding of the shape of the underlying spectrum with zero neutrino mass is essential for drawing reliable conclusions experimentally about the actual value of the mass. An indication of the complexity of the spectrum has been presented. Considering vacancy multiplicities of only 1 or 2, the spectrum is dense with line and edge features up to a $Q$-value of about 2.5 keV, but for larger $Q$-values up to about 3 keV the spectrum is featureless near the endpoint in this approximation. This may offer an avenue for experiments if the $Q$-value is confirmed to be in the vicinity of 2.8 keV, as is indicated by recent studies [22, 27]. A more detailed calculation could reveal if higher-order satellites or shakeoff features also populate that region. Coherent interference between the tails of resonances and inner bremsstrahlung is another likely complication. For the larger $Q$-values, the continuum phase space at the endpoint becomes so small [24] that the experimental measurement itself is very challenging. Nevertheless, if the steadily improving experimental sensitivity is matched by new, more quantitative relativistic theoretical calculations, the precise agreement between theory and experiment necessary for a neutrino mass measurement may yet emerge.

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[1] Y. Fukuda et al. (Super-Kamiokande Collaboration), Phys.Rev.Lett. **81**, 1562 (1998), arXiv:hep-ex/9807003 [hep-ex]

[2] Q. R. Ahmad et al. (SNO), Phys. Rev. Lett. **87**, 071301 (2001), arXiv:nucl-ex/0106015

[3] K. Eguchi et al. (KamLAND Collaboration), Phys.Rev.Lett. **90**, 021802 (2003), arXiv:hep-ex/0212021 [hep-ex]

[4] C. Kraus et al., Eur. Phys. J. **C40**, 447 (2005), hep-ex/0412056

[5] V. Aseev et al. (Troitsk Collaboration), Phys.Rev. **D84**, 112003 (2011)
[6] G. Drexlin, V. Hannen, S. Mertens, and C. Weinheimer, Adv. High Energy Phys. **2013**, 293986 (2013) [arXiv:1307.0101 [physics.ins-det]].

[7] B. Monreal and J. A. Formaggio, Phys. Rev. **D80**, 051301 (2009) [arXiv:0904.2860 [nucl-ex]].

[8] E. Andreotti *et al.*, Nucl. Instrum. Meth. **A572**, 208 (2007).

[9] N. Palanque-Delabrouille, C. Yche, J. Lesgourgues, G. Rossi, A. Borde, *et al.*, (2014), [arXiv:1410.7244 [astro-ph.CO]].

[10] A. De Rujula, Nucl.Phys. **B188**, 414 (1981).

[11] A. De Rujula and M. Lusignoli, Physics Letters B **118**, 429 (1982).

[12] J. Kopp and A. Merle, Phys.Rev. **C81**, 045501 (2010) [arXiv:0911.3329 [hep-ph]].

[13] F. Hartmann and R. Naumann, Nucl.Instrum.Meth. **A313**, 237 (1992).

[14] S. Yasumi, G. Rajasekaran, M. Ando, F. Ochiai, H. Ikeda, *et al.*, Phys.Lett. **B122**, 461 (1983).

[15] S. Yasumi, M. Ando, H. Maezawa, H. Kitamura, T. Ohta, *et al.*, Phys.Lett. **B181**, 169 (1986).

[16] S. Yasumi, H. Maezawa, K. Shima, Y. Inagaki, T. Mukoyama, *et al.*, Phys.Lett. **B334**, 229 (1994).

[17] J. Andersen, G. Beyer, G. Charpak, A. De Rujula, B. Elbek, *et al.*, Phys.Lett. **B113**, 72 (1982).

[18] B. Jonson, J. Andersen, G. Beyer, G. Charpak, A. De Rujula, *et al.*, Nucl.Phys. **A396**, 479 (1983).

[19] P. Meunier and C. Salvo (Cryogenic Detector Group of Genoa), Phys.Lett. **B398**, 415 (1997).

[20] P. Meunier, Nucl.Phys.Proc.Suppl. **66**, 207 (1998).

[21] L. Gastaldo, P. Manfrinetti, F. Gatti, G. Gallinaro, D. Pergolesi, *et al.*, Nucl.Instrum.Meth. **A520**, 224 (2004).

[22] P. C.-O. Ranitzsch, J.-P. Porst, S. Kempf, C. Pies, S. Schaefer, D. Hengstler, A. Fleischmann, C. Enss, and L. Gastaldo, J.Low.Temp.Phys. **167**, 1004 (2012).

[23] G. Kunde *et al.* (NuMECS Collaboration), “Towards measuring the neutrino mass via holmium electron capture spectroscopy,” (2014), http://fsnutown.phy.ornl.gov.

[24] A. Nucciotti, Eur. Phys. J. C (2014), in press, arXiv:1405.5060 [physics.ins-det].

[25] A. De Rujula, (2013), arXiv:1305.4857 [hep-ph].

[26] A. Faessler, L. Gastaldo, and M. Simkovic, (2014), arXiv:1407.6504 [nucl-th].
[27] P. C. O. Ranitzsch, C. Hassel, M. Wegner, S. Kempf, A. Fleischmann, et al., (2014), arXiv:1409.0071 [physics.ins-det].

[28] It improves the accuracy of the calculated peak energies; compare the present results with ref. [27].

[29] T. A. Carlson and C. W. Nestor, Phys. Rev. A 8, 2887 (1973).

[30] D. L. Wark, R. Bartlett, T. J. Bowles, R. G. H. Robertson, D. S. Sivia, W. Trela, J. F. Wilkerson, G. S. Brown, B. Crasemann, S. L. Sorensen, S. J. Schaphorst, D. A. Knapp, J. Henderson, J. Tulkki, and T. Åberg, Phys. Rev. Lett. 67, 2291 (1991).

[31] M. Lusignoli and M. Vignati, Physics Letters B 697, 11 (2011).

[32] R. C. Weast, ed., CRC Handbook of Chemistry and Physics 69th Ed. (CRC Press Inc., Boca Raton, 1989).

[33] K. Zhang, E. A. Stern, J. J. Rehr, and F. Ellis, Phys. Rev. B 44, 2030 (1991).