Molding 3D curved structures by selective heating

Harsh Jain, Shankar Ghosh, and Nitin Nitsure

1 Department of Condensed Matter Physics and Materials Science
2 School of Mathematics, Tata Institute of Fundamental Research, Mumbai 400005, India

It is of interest to fabricate complicated curved shapes in three dimensions from easily available homogeneous material in the form of flat sheets or strips. This is different from the more traditional concern to fabricate complex planar patterns using flat sheets, possibly with multiple layers, as in printed circuit boards. The interest in curved surfaces has stimulated the development of some new methods to produce them, which include 3-dimensional printing, an approach based on folding a flat sheet in the style of origami, and an approach based on differential contraction of a pre-fabricated composite material. These methods do not scale up well in length, and they do not have the ability to rapidly form the desired final shape by an easy modification of a homogeneous flat sheet. Here we demonstrate a simple and quick method, which has the above two properties. One can call this as a ‘tailoring method’, as it begins with flat homogeneous material and suitably tailors it to produce the desired curved shapes. The method is rapid, reliable, and scalable, and it can even be used for folding sheets along closed curves.

Common materials such as steel, paper, plastic and cloth are usually produced as flat sheets. More complicated curved and folded shapes have to be fashioned out of such flat sheets of materials [1–6]. For example, dresses are tailored for the human form out of a cloth which is flat, or globes of the earth with maps are fashioned from printed flat sheets. The standard tailoring method to produce an approximately curved surface from a cloth is to cut out and discard curvilinear triangular wedges (‘darts’) from the cloth and then to stitch together the edges [7]. Instead of cutting out the wedges, one can sometimes form folds in the style of origami [8, 9], or ‘pleats’ as in tailoring [7], to achieve a somewhat similar result, but this leaves us with undesirable folds.

Instead of cutting out wedges and stitching together the edges of the cuts, a qualitatively similar effect would be achieved by a method which selectively shrinks the material in the wedges. We demonstrate the feasibility of this method by molding various curved surfaces out of a thermoresponsive polymer sheet commercially known as Shrinky Dink, a material that contracts when heat is applied [10, 11]. This shrinking analogous to the tailoring method should also apply to other suitable materials [13–22].

An obvious disadvantage of a tailoring approach is the inherent patchiness of the end result. The shrinking-tailoring analogue produces smoother results, as the boundary behaviour continuously interpolates between the shrinking of the material in the wedges and the original form of the material on the outside. This is the effect of long range forces in the material produced by selective shrinking, which is in contrast to the usual method of tailoring clothes by cutting out some material, where such forces are not generated. A more smoothly varying curvature can be achieved by shrinking a larger number of smaller wedges. We use the word curvature here in the sense of Gaussian curvature. The ratio of circumference to radius of a small circle on a curved surface is equal to, greater than or less than $2\pi$ depending on whether the local curvature is zero, negative or positive [23]. Consider any small circular disk on the material sheet, centered at a chosen point. The pattern of selective contraction can be so chosen as to affect the concentric circular regions around the center of the disk differently, and this changes the ratio of perimeter to the radius. If there is more shrinking away from the center of the disk than close to the center then the sheet will locally acquire a positive curvature while if there is more shrinking near the center of the disk and less near the perimeter then the sheet will locally acquire a negative curvature around the center of the disk.

We now describe our implementation of the shrinking-tailoring method, and show the shapes that were molded. By printing wedges with an office laser printer on a piece of Shrinky Dink (a thermo responsive 0.25 mm thick polymer sheet) and then exposing the piece to the radi-
FIG. 2. The panels (a) and (b) show that the deformations do not penetrate much into the closed regions whose interior is kept white. The inset diagrams in the panels show the black and white regions drawn on a flat disk of Shrinky Dink (diameter of 6 cm). (c) The figure shows the twisting of a 1 cm wide strip of Shrinky Dink on which the pattern in the inset is printed. While the printed strip is achiral, on heating it gets twisted into a structure in which the sense of the twist (marked by the circular arrows) changes along the length.

The solution to the above problems is to break up and spread out the large black regions. It is natural to imag-
FIG. 4. The panels (a) and (b) show a positively and a negatively curved surface obtained by printing a flat sheet with a pattern of black dots which has a suitably adjusted density. The inset to the figure shows the printed pattern for the two cases. (c) The figure shows a sequences of similar molded shapes of increasing sizes. The listed lengths are the diameters of the circumcircles of the plastic triangles on which the petal designs which are scaled versions of the inset drawing were printed.

ine that instead of drawing large wedges, a smoother result might be achieved by printing a sheet with a pattern of small black patches or shading which has a suitably adjusted density (number of dots per area or proportion of blackened area). If the density varies continuously, and if long-range stresses are negligible, then the molded curved surface would be conformal to the original disk under the identity map on physical points. A continuous, locally (up to first order) isotropic contraction of the sheet will change its metric tensor from $ds^2 = dx^2 + dy^2$ to $f(x,y)^2 ds^2$ where $f(x,y)$ is the local contraction factor as a function of the position $(x,y)$ (such an $f$ exists if and only if the contraction is locally isotropic). While we know that a fully black piece of material contracts linearly by a factor $f = 0.4$, making a quantitatively precise estimate of contraction for a complicated design is not easy. This is because heat flows by conduction from blackened portions to the neighbouring white areas, and also, the black material anchors at its edges to the neighbouring white material, which distorts the contraction spoiling its local isotropy. This loss of local isotropy is evident in Fig.3(c). There is another practical problem. In our experiment, the heating responded non-linearly to the degree of shading intensity, with a negligible response below a certain threshold. This made it more convenient to use a tiled pattern of black and white regions instead of smoothly varying shading, in which the black regions are not too small so that they do not lose all the generated heat to the white surrounding region, and so undergo the desired contraction.

On the other hand, the effect of having a large black region surrounded by a white region is that the middle of the black region thins on heating, with the material migrating to the boundary. This is because the temperature in the central part of the black region becomes higher making the material there softer, and therefore susceptible to the contracting pull exerted by the boundary which is anchored to the surrounding colder and hence more rigid white region. The thermal images and the temperature profiles which bear the above point are shown in Fig.3. An extreme example of this phenomenon is that when subjected to overheating induced by prolonged exposure, a mechanical tear develops in the middle of a black region which is surrounded by white region. This can be seen in Fig.3(c), in which the black material has moved closer to the nearest edge, leading to the creation of multiple thick and thin regions. Prior to overheating, the sheet develops a small negative curvature, which disappears when the centre region develops some tears.

The temperature of the sheet decreases as we move away from the black region. However, the mechanical response of the material is such that its metric remains nearly unaltered if the temperature always remains below $90^\circ$C. Hence we have a boundary region characterised by a contraction factor on area that varies approximately between $\approx 1$ and $\approx (0.4)^2$. The remaining white region has a constant area contraction factor of $\approx 1$ while the remaining black region has a constant area contraction factor of $\approx (0.4)^2$.

An advantage of having only large patches is that the area of the boundary region of the patches (the region where the contraction is varying and where local isotropy is broken) is small in comparison to the large black and white areas where the contraction factor is known and lo-
cal isotropy is maintained, giving a predictable outcome.

To achieve smoother results by our method, we find that black patches having a size of about 4 mm work best. Smaller black patches lose too much heat to the surroundings by conduction so do not achieve the desired effect, while larger black patches undergo thinning at their centres and even tears as already mentioned. The insets of Fig. 4 (a) and (b) show two such tiling patterns which respectively lead to positively and negatively curved surfaces (see the main panels of Fig. 4 (a) and (b)).

If homogeneity of the infrared illumination is maintained over large areas, this method of molding can be scaled and applied from a few millimetre upwards, simply by adhering to the design requirement that individual black or white regions should not be too large or too small. That is, if we want to make much larger objects of the kind depicted in Fig. 4 (a) and (b), then instead of just scaling up the inset designs as in Fig. 4 (c), we will have to further break up the black regions and spread these among the white regions, so that the individual black or white regions do not become too large. The thickness of the material that we presently use makes it difficult to go below sizes smaller than a few mm but this is not fundamental. Indeed, thinner thermo-responsive materials could be used after solving the problem of how to deposit the needed heat-responsive patterns.

It may be desirable to have a heating mechanism based on spatial light modulation, which will directly heat designated portions of a sheet, molding it into a desired 3-dimensional shape. The input to a machine based on such a mechanism will be a pixelated digital pattern of light and dark spots, and the output will be the desired 3-dimensional shape, avoiding altogether the present use of painting the surface with carbon black to create temperature gradients more crudely. It is desirable to transfer heat very rapidly (‘flash heating’), which has the twin advantages that the change of shape which happens more slowly compared to the time scale of rapid heating does not interfere with the scheme of heating by radiation, and the white (non-radiated) region remain cold, which would otherwise have heated up by conduction during a longer process of heating by lower intensity radiation. One should note that a curved object with a different global topology than that of a flat sheet will have to be made by cutting and gluing together individual curved pieces molded by the above method.

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7 sghosh@tifr.res.in
8 nitsure@math.tifr.res.in

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