Crossover from marginal Fermi liquid to Luttinger liquid behavior in carbon nanotubes

S. Bellucci 1 and J. González 2

1INFN-Laboratori Nazionali di Frascati, P. O. Box 13, 00044 Frascati, Italy
and INFN-Dipartimento di Fisica, Università di Roma Tor Vergata, Via della Ricerca Scientifica 1, 00133 Roma, Italy.
2Instituto de Estructura de la Materia. Consejo Superior de Investigaciones Científicas. Serrano 123, 28006 Madrid, Spain.

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We study graphene-based electron systems with long-range Coulomb interaction by performing an analytic continuation in the number of dimensions. We characterize in this way the crossover between the marginal Fermi liquid behavior of a graphite layer and the Luttinger liquid behavior at \( D = 1 \). The former persists for any dimension above \( D = 1 \). However, the proximity to the \( D = 1 \) fixed-point strongly influences the phenomenology of quasi-one-dimensional systems, giving rise to an effective power-law behavior of observables like the density of states. This applies to nanotubes of large radius, for which we predict a lower bound of the corresponding exponent that turns out to be very close to the value measured in multi-walled nanotubes.

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Recently there has been much interest in the search of unconventional electron behavior deviating from the Fermi liquid picture \([1]\). Besides this, the other paradigm that is well-established on theoretical grounds is the Luttinger liquid behavior of one-dimensional (1D) electron systems \([2,3]\). There have been suggestions that this behavior could be extended to two-dimensional (2D) systems, in the hope that it may explain some of the features of the copper-oxide materials \([4]\). However, at least for the Luttinger model, the analytic continuation in the number \( D \) of dimensions has shown that the Luttinger liquid behavior is lost as soon as one departs from \( D = 1 \) \([5]\).

Several authors have also analyzed the possibility that singular interactions could lead to the breakdown of the Fermi liquid picture \([6]\). With regard to real low-dimensional systems, such as carbon nanotubes, the main electron interaction comes actually from the long-range Coulomb potential \( V(|\mathbf{r}|) \sim 1/|\mathbf{r}| \). This is also the case of the 2D layers in graphite, which have a vanishing density of states at the Fermi level. Quite remarkably, a quasi-particle decay rate linear in energy has been measured experimentally in graphite \([7]\), pointing at the marginal Fermi liquid behavior in such 2D layers. Due to the singular Coulomb interaction, the imaginary part of the electron self-energy in the 2D system behaves at weak \( g \) coupling like \( g^2 \omega \) \([6]\). It is crucial, though, the fact that the effective coupling scales at low energy as \( g \sim 1/\log(\omega) \). This prevents the logarithmic suppression of the quasi-particle weight, which gets corrected by terms of order \( g^2 \log(\omega) \sim 1/\log(\omega) \) \([8]\).

In this letter we investigate whether the long-range Coulomb interaction may lead to the breakdown of the Fermi liquid behavior at any dimension between \( D = 1 \) and 2. The issue is significant for the purpose of comparing with recent experimental observations of power-law behavior of the tunneling conductance in multi-walled nanotubes \([11]\). These are systems whose description lies between that of a pure 1D system and the 2D graphite layer. It turns out, for instance, that the critical exponent measured for tunneling into the bulk of the multi-walled nanotubes is \( \alpha \approx 0.3 \). This value is close to the exponent found for the single-walled nanotubes \([12,13]\). However, it is much larger than expected by taking into account the reduction due to screening (\( \sim 1/\sqrt{N} \)) in a wire with a large number \( N \) of subbands, what points towards sensible effects of the long-range Coulomb interaction in the system.

We develop the analytic continuation in the number of dimensions having in mind the low-energy modes of metallic nanotubes, which have linear branches crossing at the Fermi level. From this picture, we build at general dimension \( D \) a manifold of linear branches in momentum space crossing at a given Fermi point. We consider the Hamiltonian

\[
H = v_F \int_0^\Lambda d\mathbf{p} |\mathbf{p}|^{D-1} \int \frac{d\Omega}{(2\pi)^D} \Psi^+(\mathbf{p}) \sigma \cdot \mathbf{p} \Psi(\mathbf{p}) + e^2 \int_0^\Lambda d\mathbf{p} |\mathbf{p}|^{D-1} \int \frac{d\Omega}{(2\pi)^D} \rho(\mathbf{p}) \frac{c(D)}{|\mathbf{p}|^{D-1}} \rho(-\mathbf{p})
\]

where the \( \sigma_i \) matrices are defined formally by \( \{\sigma_i, \sigma_j\} = 2\delta_{ij} \). Here \( \rho(\mathbf{p}) \) are density operators made of the electron modes \( \Psi(\mathbf{p}) \), and \( c(D)/|\mathbf{p}|^{D-1} \) corresponds to the Fourier transform of the Coulomb potential in dimension \( D \). Its usual logarithmic dependence on \( |\mathbf{p}| \) at \( D = 1 \) is obtained by taking the 1D limit with \( c(D) = \Gamma((D - 1)/2)/(2\sqrt{\pi})^{3-D} \).

The dispersion relation \( \epsilon(\mathbf{p}) = \pm |\mathbf{p}| \) is of Dirac fermions, with a vanishing density of states at the Fermi level above \( D = 1 \). This ensures that the Coulomb interaction remains unscreened in the analytic continuation. At \( D = 2 \) we recover the low-energy description of the electronic properties of a graphite layer, dominated by the presence of isolated Fermi points with conical dispersion relation at the corners of the Brillouin zone \([14]\).

In the above picture, we are neglecting interactions that mix the two inequivalent Fermi points common to

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the low-energy spectra of graphite layers and metallic nanotubes. In the latter, such interactions have been considered in Refs. 15 and 16, with the result that they have smaller relative strength (∼ 0.1/N, in terms of the number N of subbands) and remain small down to extremely low energies. More recently, the question has been addressed regarding the interactions in the graphite layer, and it also turns out that phases with broken symmetry cannot be realized, unless the system is doped about half-filling 17 or it is in a strong coupling regime 18.

We will accomplish a self-consistent solution of the model by looking for fixed-points of the renormalization group transformations implemented by the reduction of the cutoff Λ 19. As usual, the integration of high-energy modes at that scale leads to the cutoff dependence of the parameters in the low-energy effective theory. We will see that the Fermi velocity v_F grows in general as the cutoff is reduced towards the Fermi point. On the other hand, the electron charge e stays constant as Λ → 0. This electron charge e cannot be realized, unless the system is doped regarding the interactions in the graphite layer 20.

The polarizability is then given by

\[ \Pi(k, \omega) = b(D) \frac{v_F^2 - D k^2}{|v_F k^2 - \omega|^2 (3-D)/2}, \]

where \( b(D) = \frac{2}{\sqrt{\pi}} \Gamma((D+1)/2)^2/(\Gamma(D+1)/2). \)

The dependence of \( \omega = \sqrt{\omega^2 + (3-D)/2} \) on \( \Lambda \) at \( D = 2 \) implies an incomplete cancellation between self-energy and vertex corrections to the polarizability. The dressed polarizability depends therefore on the effective Fermi velocity \( v_F(\Lambda) \). The renormalized value of \( v_F \) is determined by fixing it self-consistently to the value obtained in the electron propagator \( G \) corrected by the self-energy contribution

\[ \Sigma(k, \omega) = -e^2 \int_0^\Lambda dp |p|^{D-1} \int \frac{d\Omega}{(2\pi)^D} \int \frac{d\omega_p}{2\pi} \left( \frac{v_F^2 - D k^2}{|v_F p^2 - \omega_p|^2 (3-D)/2} \right). \]

The fixed-points of the renormalization group in the limit \( \Lambda \rightarrow 0 \) determine the universality class to which the model belongs. At \( D = 2 \), we are bound to obtain the low-energy fixed-point at vanishing coupling of the model of Dirac fermions with Coulomb interaction 16. On the other hand, at \( D = 1 \) there has to be a fixed-point corresponding to Luttinger liquid behavior. We note, however, that no solution of the model has been obtained yet without carrying dependence on the transverse scale needed to define the 1D logarithmic potential. Our dimensional regularization overcomes the problem of introducing such external parameter, which prevents a proper scaling behavior of the model 21.

At general \( D \), the self-energy 18 shows a logarithmic dependence on the cutoff at small frequency \( \omega_k \) and small momentum \( k \). This is the signature of the renormalization of the electron field scale and the Fermi velocity. In the low-energy theory with high-energy modes integrated out, the electron propagator becomes

\[ \frac{1}{G} = \frac{1}{G_0} - \Sigma \approx Z^{-1}(\omega_k - v_F \sigma \cdot k) \]

\[ -Z^{-1} f(D) \sum_{n=0}^\infty (-1)^n g^{n+1} \left( \frac{n(3-D)}{n(3-D)+2 \omega_k} \right) \]

\[ + \left( 1 - \frac{n(3-D)+1}{D(n(3-D)+2)} \right) v_F \sigma \cdot k \ h_n(D) \log(\Lambda), \]

where \( g = b(D)c(D)e^2/v_F \), \( f(D) = \frac{2}{2^{2-D}D^2/(2\pi)^D} \), and \( h_n(D) = \frac{\Gamma(n(3-D)/2+1/2)}{\Gamma(n(3-D)/2+1)} \). The quantity \( Z^{1/2} \) represents the scale of the bare electron field compared to that of the renormalized electron field for which \( G \) is computed.

The renormalized propagator \( G \) must be cutoff-independent, as it leads to observable quantities in the quantum theory. This condition is enforced by fixing the dependence of the effective parameters \( Z \) and \( v_F \) on \( \Lambda \) as more states are integrated out from high-energy shells. We get the differential renormalization group equations

\[ \Lambda \frac{d}{d\Lambda} \log Z(\Lambda) = -f(D) \sum_{n=0}^\infty (-1)^n g^{n+1} \frac{n(3-D)}{n(3-D)+2} h_n(D), \]

\[ \frac{d}{d\Lambda} v_F(\Lambda) = -v_F f(D) \sum_{n=0}^\infty (-1)^n g^{n+1} \left( \frac{1}{D} - \frac{n(3-D)(2-D)+2}{n(3-D)+2} \right) h_n(D). \]

At \( D = 2 \), the right-hand-side of these equations can be summed up to the functions that have been found previously in the renormalization of the graphite layer 16. Furthermore, they also provide meaningful expressions in the 1D limit. At \( D = 1 \), the right-hand-side of Eq. 18 vanishes identically as a function of the variable \( g \). Therefore, the 1D model has formally a line of fixed-points, as it happens in the case of a short-range interaction. The scaling of the electron wavefunction can be read from the right-hand-side of Eq. 18, which becomes \( (2 + g)/(2\sqrt{1 + g}) - 1 \) at \( D = 1 \). This coincides with the anomalous dimension that is found in the solution of the Luttinger model, which provides an independent check of the renormalization group approach to the 1D system.

We have therefore a model that interpolates between marginal Fermi liquid behavior, that is known to characterize the 2D model, and non-Fermi liquid behavior at \( D = 1 \). As the electron charge e is not renormalized for \( D < 3 \), the scaling of the effective coupling \( g = b(D)c(D)e^2/v_F \) is given by Eq. 18 by

\[ \Lambda \frac{d}{d\Lambda} g(\Lambda) = f(D) \sum_{n=0}^\infty (-1)^n g^{n+2} \left( \frac{1}{D} - \frac{n(3-D)(2-D)+2}{n(3-D)+2} \right) h_n(D). \]

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The right-hand-side of Eq. (7) is a monotonous increasing function of $g$, for any dimension between 1 and 2, as observed in Fig. 1.

Fig. 1. Plot of the $\beta$ function at the right-hand-side of Eq. (7) for different dimensions. From top to bottom, the curves correspond to the values $D = 2.0, 1.7, 1.4, 1.3, 1.2$ and 1.1.

We find that, away from $D = 1$, there is only one fixed-point of the renormalization group at $g = 0$. The scale dependence of the effective coupling $e^2/v_F$ is displayed for different values of $D$ in Fig. 2, where the flow to the fixed-point is seen. Consequently, the scale $Z$ of the wavefunction is not renormalized to zero in the low-energy limit, and the quasiparticle weight remains finite above $D = 1$. We conclude then that, even in a model that keeps the Coulomb interaction unscreened, the breakdown of the Fermi liquid behavior only takes place formally at $D = 1$.

Fig. 2. From top to bottom, curves of the effective coupling as a function of the energy scale for $D = 1.1, 1.2, 1.3, 1.4, 1.7$ and 2.0.

The subtlety concerning the long-range Coulomb interaction is that the function $c(D)$ diverges in the limit $D \to 1$. This is actually what transforms the power-law dependence of the potential into a logarithmic dependence at $D = 1$. We observe that the 1D limit and the low-energy limit $\Lambda \to 0$ do not commute. If we stick to $D = 1$, we obtain a divergent coupling $g$ for the Coulomb interaction as well as a divergent electron scaling dimension. At any dimension slightly above $D = 1$, however, the fixed-point is at $g = 0$, with its corresponding vanishing anomalous dimension.

In order to understand whether the 1D model has any stable fixed-point for finite values of $e^2/v_F$, one can study the model by performing an expansion in powers of $g^{-1}$. The Fermi velocity is renormalized by terms that are analytic near the point $g = \infty$, and that lead to the scaling equation

$$\Lambda \frac{d}{d\Lambda} v_F(\Lambda) = v_F f(D) \left(3 - D - \frac{2}{D}\right) \frac{\Gamma(D/2 - 1)}{\Gamma((D + 1)/2)}$$

up to terms of order $O(g^{-1})$. In the limit $D \to 1$, $g \to \infty$, the right-hand-side of Eq. (8) vanishes identically. This confirms, on nonperturbative grounds, that the 1D model with the Coulomb interaction has a line of fixed-points covering all values of $e^2/v_F$.

In the vicinity of $D = 1$, the presence of such critical line becomes sensible, and a crossover takes place to a behavior with a sharp reduction of the quasiparticle weight. This can be seen in the renormalization of the electron field scale $Z$, displayed in Fig. 3. For values of $D$ above $\approx 1.2$, we have a clear signature of quasiparticles in the value of $Z$ at low energies. For lower values of $D$, the picture cannot be distinguished from that of a vanishing quasiparticle weight for all practical purposes. The drastic suppression of the electron field scale $Z$ takes place over a variation of only two orders of magnitude in the energy scale.

Fig. 3. Plot of the electron field scale $Z$ for a value of the bare coupling $e^2/(\pi^2 v_F) = 4.0$. The different curves correspond, from top to bottom, to $D = 2.0, 1.7, 1.4, 1.3, 1.2$ and 1.1.
The above picture allows us to make contact with the experiments carried out in multi-walled nanotubes. In the proximity of the $D = 1$ fixed-point, the density of states displays an effective power-law behavior, with an increasingly large exponent. Moving to the other side of the crossover, the density of states approaches the well-known behavior of the graphite layer, $n(\varepsilon) \sim |\varepsilon|$. In Fig. 4 we give the representation of the density of states

$$n(\varepsilon) \sim Z(\varepsilon)|\varepsilon|^{D - 1}$$

(9)

for several dimensions approaching $D = 1$.

A value $\varepsilon^2/(\pi^2 v_F) = 0.5$ for the bare coupling is appropriate for typical multi-walled nanotubes, as it takes into account the reduction due to the interaction with the inner metallic cylinders. We observe that the exponents of $n(\varepsilon)$ at different dimensions are always larger than a lower bound $\alpha \approx 0.26$. This is in agreement with the values measured experimentally. Our analysis stresses the need of an appropriate description of the dimensional crossover between one and two dimensions, showing that the picture of a thick nanotube as an aggregate of 1D channels does not allow to obtain the correct values of the critical exponents.

To summarize, we have studied the renormalization of the Coulomb interaction in graphene-based structures. We have made a rigorous characterization of the different behaviors, as we have proceeded by identifying the fixed-points of the theory. We have seen that the Fermi liquid behavior persists formally for any dimension above $D = 1$, as it also happens in the case of a short-range interaction. On the other hand, the proximity to the 1D fixed-point influences strongly the phenomenology of real quasi-onedimensional systems, giving rise to an effective power-law behavior of observables like the tunneling density of states. This is the case of the multi-walled nanotubes, for which we predict a lower bound for the corresponding exponent that turns out to be very close to the value measured experimentally.

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