Research Article

Fractal Ion Acoustic Waves of the Space-Time Fractional Three Dimensional KP Equation

M. A. Abdou,1,2 Saud Owyed,3 S. Saha Ray,4 Yu-Ming Chu5,6 Mustafa Inc7,8 and Loubna Ouahid1

1Physics Department, College of Science, University of Bisha, Bisha 61922, P.O. Box 344, Saudi Arabia
2Theoretical Research Group, Physics Department, Faculty of Science, Mansoura University, 35516 Mansoura, Egypt
3Mathematics Department, College of Science, University of Bisha, Bisha 61922, P.O. Box 344, Saudi Arabia
4Department of Mathematics, National Institute of Technology, Rourkela, 769008 Odisha, India
5Department of Mathematics, Huzhou University, Huzhou 313000, China
6Hunan Provincial Key Laboratory of Mathematical Modeling and Analysis in Engineering, Changsha University of Science and Technology, Changsha 410114, China
7Department of Mathematics, Science Faculty, Firat University, 23119 Elazig, Turkey
8Department of Medical Research, China Medical University Hospital, China Medical University, Taichung, Taiwan

Correspondence should be addressed to Yu-Ming Chu; chuyuming2005@126.com and Mustafa Inc; minc@firat.edu.tr

Received 3 August 2020; Revised 29 August 2020; Accepted 22 September 2020; Published 17 October 2020

1. Introduction

Nonlinear propagation of electrostatic excitations in electron-positron ion plasmas and nonthermal distribution of electrons is an important research area in astrophysical and space plasmas [1–6].

Many important phenomena such as the effective behavior of the ionized matter, magnetic field near the surfaces of the sun and stars, emission mechanisms of pulsars, the origin of cosmic rays and radio sources, dynamics of magnetosphere, and propagation of electromagnetic radiation through the upper atmosphere required the study of plasma physics. Equations such as Korteweg de Vries (KdV), Burgers, KdV-Burgers, and Kadomtsev-Petviashvili (KP) were highly used models in the description of plasma systems.

We study the physical phenomena for space-time fractional KP equation with the aid of fractional calculus and examine the resulting solutions in detail. The fractional calculus [7–13] has a wide range of applications and is deeply rooted in the field of probability, mathematical physics, differential equations, and so on. Very recently, fractional differential equations have got a lot of consideration as they define many complex phenomena in various fields. Several fractional-order models play very important roles in different areas including physics, engineering, mechanics and dynamical systems, signal and image processing, control theory, biology, and materials [14–18].

The paper is summarized as follows. Definitions and properties of conformable derivatives are discussed. In Section 2, a discussion about the two algorithms method,
namely, fractional subequation method and sine-Gordon expansion method for solving FPDEs arising in plasma physics are given. In Section 3, two schemes are employed for some new exact solutions for the FKPE. We presented a graphical description of some of the solutions with a fixed value of fractal order \( \alpha \) in a brief conclusion at the end of the article.

**Definition 1.** Let \( \psi : (0, \infty) \rightarrow R \). Some definitions, useful properties, and a theorem about conformable derivatives are given as follows:

\[
U_a(\psi)(t) = \lim_{\varepsilon \to 0} \frac{\psi(t + \varepsilon t^\alpha) - \psi(t)}{\varepsilon}, \quad t > 0, \quad 0 < \alpha < 1,
\]

\[
U_a(b \psi + c h) = BU_a(\psi) + CU_a(h), \quad B, C \in R,
\]

\[
U_a(\psi h) = hU_a(\psi) + U_a(h),
\]

\[
U_a\left(\frac{\psi}{h}\right) = \frac{hU_a(\psi) - \psi U_a(h)}{h^\alpha}.
\]

(1)

If \( \psi \) is differentiable, then \( U_a(\psi)(t) = t^{1-\alpha}(d\psi/dh) \).

**Theorem 2.** Let \( \psi : (0, \infty) \rightarrow R \) be a differentiable function. Then,

\[
U_a(\psi \ast h) = t^{1-\alpha}h'(t)\psi'(h(t)).
\]

(2)

### 2. Solution Method

#### 2.1. Extended Fractional Subequation Method.

For a given nonlinear FPDE as

\[
\chi(u_1, u_2, D^\alpha u_1, D^\beta u_1, D^\gamma u_1, D^\delta u_1, D^\epsilon u_1) = 0, \quad 0 < \alpha < 1.
\]

in which \( \chi \) is a polynomial of \( u \). Using wave transformation as

\[
u(t, x_1, x_2, \cdots x_n) = U_i(\xi), \quad \xi = ct + \cdots + k_n x_n,
\]

Eq. (3) reads

\[
\phi(U_1, \cdots U_M, \partial^\alpha U_1, \cdots \partial^\epsilon U_1, k_1^\alpha \partial^\alpha U_1, \cdots k_\alpha^\alpha \partial^\alpha U_1, \cdots, k_\epsilon^\epsilon \partial^\epsilon U_1) = 0, \quad 0 < \alpha < 1.
\]

Thus,

\[
U_i(\xi) = \sum_{i=0}^{M} a_i \left( \frac{D_i^\alpha w(\xi)}{w(\xi)} \right)^i,
\]

(6)

where \( w = w(\xi) \) satisfies

\[
A(\xi)D_i^\alpha w(\xi) - B(\xi)D_i^\alpha w(\xi) - C \left[ D_i^\alpha w(\xi) \right]^2 - Eu(\xi)^2 = 0,
\]

(7)

where \( D_i^\alpha w(\xi) \) is a RL fractional operator of order \( \alpha \). To solve Eq. (7), assume \( w(\xi) = z(\eta) \), with the fractional complex transformation, then

\[
Az(\eta)z''(\eta) - Bz(\eta)z'(\eta) - C \left[ z'(\eta) \right]^2 - Ez^2(\eta) = 0.
\]

Since \( D_i^\alpha w(\xi) = D_i^\alpha z(\eta) \). The general solutions Eq. (7) is as follows:where \( \nu_1 = B^2 + 4E(A - C), \nu_2 = E(A - C), L_1 \), and \( L_2 \) are arbitrary constants and \( \eta = \xi^\alpha/\Gamma(1 + \alpha) \). Inserting Eq. (6) into (5) knowing Eq.(7), collecting the same order terms \( (D_i^\alpha w(\xi)/w(\xi)) \), then equating it to zero, \( k \) and \( c \) are obtained. As long as the solutions are obtained with the general expression \( [D_i^\alpha w(\xi)/w(\xi)] \), admits several solutions of Eq. (3).

**Family 1.** As long as \( B = 0, \nu_1 > 0 \), admits to

\[
[D_i^\alpha w(\xi) / w(\xi)] = \frac{B}{2(A - C)} + \frac{\nu_1}{2(A - C)} \left[ L_1 \sin \left( \sqrt{\nu_2/4} h(2(A - C)) \right) + L_2 \cos \left( \sqrt{\nu_2/4} h(2(A - C)) \right) \right].
\]

(9)

**Family 2.** Limiting case \( B \neq 0, \nu_1 < 0 \) gains

\[
[D_i^\alpha w(\xi) / w(\xi)] = \frac{B}{2(A - C)} + \frac{\nu_1}{2(A - C)} \left( L_1 \cos \left( \sqrt{\nu_2/4} h(2(A - C)) \right) + L_2 \sin \left( \sqrt{\nu_2/4} h(2(A - C)) \right) \right).
\]

(10)

**Family 3.** For \( B = 0, \nu_1 = 0 \),

\[
[D_i^\alpha w(\xi) / w(\xi)] = \frac{B}{2(A - C)} + \frac{L_1}{L_1 + L_2 \eta}.
\]

(11)

**Family 4.** When \( B = 0, \nu_2 > 0 \),

\[
[D_i^\alpha w(\xi) / w(\xi)] = \frac{\sqrt{\nu_2}}{2(A - C)} \left[ L_1 \sin \left( \sqrt{\nu_2/4} h(2(A - C)) \right) + L_2 \cos \left( \sqrt{\nu_2/4} h(2(A - C)) \right) \right].
\]

(12)

**Family 5.** When \( B = 0, \nu_2 < 0 \), then

\[
[D_i^\alpha w(\xi) / w(\xi)] = \frac{\sqrt{\nu_2}}{2(A - C)} \left( -L_1 \sin \left( \sqrt{\nu_2/4} h(2(A - C)) \right) + L_2 \sin \left( \sqrt{\nu_2/4} h(2(A - C)) \right) \right).
\]

(13)

#### 2.2. Analysis of the Fractional Sine-Gordon Expansion (FSGE) Method.

Let us first consider the fractional sine-Gordon equation as

\[
V_{xx} - D_i^{2\alpha} V = m^2 \sin (V),
\]

(14)

where \( m \) is constant.
By using the transformation \( V = V(\xi), \xi = a(x - v(t^a/a)) \).

Then Eq. (14) yields
\[
\frac{d^2 (V/2)}{d\xi^2} = \frac{m^2}{a^2 (1 - v^2)} \sin^2 (V/2) + C, \tag{15}
\]
where \( C \) is an integration constant to be zero. Setting \( \chi(\xi) = V(\xi)/2, \) \( b^2 = m^2/a^2 (1 - v^2) \). Then Eq. (15) reads
\[
\frac{d(\chi u(\xi))}{d\xi} = b \sin (\chi), \tag{16}
\]
Setting \( b = 1 \), we have
\[
\sin (\chi(\xi)) = \frac{2d\chi}{d^2\xi + 1} = \sec h(\xi), \quad \cos (\chi(\xi)) = \frac{d^2\xi - 1}{d^2\xi + 1} = \tan h(\xi), \tag{17}
\]
\[
\sin (\chi(\xi)) = \csc h(\xi), \quad \cos (\chi(\xi)) = \cot h(\xi), \quad d = 1. \tag{18}
\]

In view of this method, we assume the trial solutions by
\[
V(\xi) = \sum_{j=1}^{N} \tan h^{-1}(\xi) \left[ B_j \sec h(\xi) + A_j \sec h(\xi) \right] + A_0, \tag{19}
\]
\[
V(\xi) = \sum_{j=1}^{N} \cot h^{-1}(\xi) \left[ B_j \csc h(\xi) + A_j \cot h(\xi) \right] + A_0. \tag{20}
\]

Making use of Eq. (18), then Eq. (19) can be rewritten as follows
\[
V(\xi) = \sum_{j=1}^{N} \cos^{-1}(\xi) \left[ B_j \sin (\xi) + A_j \cos (\xi) \right] + A_0, \tag{21}
\]
where \( N \) can be obtained by balancing principle. Inserting Eq. (21) into (15) and collecting the same power of \( \cos^i(\xi) \sin^j(\xi) \) admitting the system of algebraic equation, by solving them by Maple, the coefficient values \( A_j, B_j, v \) can be determined. Inserting these values into Eq. (19), the exact solutions of Eq. (14) are determined.

3. New Applications

In this part of our research, we apply a novel computational approach mentioned above to illustrate the advantages for finding analytical solutions of \((3 + 1)\)-dimension space-time FKPE which is as follows
\[
D_t^a D_x^a u(x, y, z, t) + d D_x^a u(x, y, z, t) D_y^a u(x, y, z, t) + v D^2_{x\alpha\alpha} u(x, y, z, t) + \beta D^2_{z\alpha\alpha} u(x, y, z, t) + \nu D^2_{z\beta\beta} u(x, y, z, t) = 0, \tag{22}
\]
where \( u(\xi) = U(x, y, z, t) \) is the field function, \( v, \delta, d, \) and \( \beta \in \mathbb{R} \). Let \( u(x, y, z, t) = U(\xi), \) where \( \xi = kx + ct + ly + mz + \xi_0, k, l, c, m, \xi_0 \) then
\[
D_x^a U = D_x^a U(\xi) = (D_x^a U)(\xi) = k^a D_x^a U, \tag{23}
\]
\[
D_{y}^a U = D_y^a U(\xi) = (D_y^a U)(\xi) = \nu D_y^a U. \tag{24}
\]

Then, Eq. (22) reduces to
\[
\nu k^a D^a_{\alpha\alpha} u(\xi) + dk^2a D^{2a}_{\alpha\alpha} [U(\xi) D_{\alpha\alpha} U(\xi)] + v k^{4a} D^4_{\beta\beta} u(\xi) + \delta k^2a D^{2a}_{\alpha\alpha} U(\xi) + m^2a \beta D^4_{\beta\beta} U(\xi) = 0. \tag{25}
\]

Now, we assume the solution of Eq. (24) as
\[
U(\xi) = \sum_{i=0}^{M} a_i \left( \frac{D^i_{\alpha\alpha} u(\xi)}{u(\xi)} \right), \tag{26}
\]
where \( \omega = u(\xi) \). Using the proposed algorithm for Eq. (24), we have \( M = 2 \). Then,
\[
U(\xi) = a_0 + a_1 \left( \frac{D^1_{\alpha\alpha} u(\xi)}{u(\xi)} \right) \tag{27}
\]
\[
+ a_2 \left( \frac{D^2_{\alpha\alpha} u(\xi)}{u(\xi)} \right) \tag{28}
\]

Inserting (26) into (24) and collecting the terms with a similar degree of \((D_{\alpha\alpha}^i u(\xi)/u(\xi))\), equating it to zero, we have two values of \( a_i \) \((i = 0, 1, 2, \ldots, k', c, l, \) and \( m \)
\[
a_0 = \frac{(-k^2a A_m^i + k^2a A_m^i \delta^2 + k^2a A_m^i \epsilon^k - 8k^2a v E A + k^2a v B^2 + 8k^2a v E C)}{dA^2}. \tag{29}
\]

From Eqs. (28) and (26), we gain
\[
U(\xi) = \frac{-k^2a A_m^i + k^2a A_m^i \delta^2 + k^2a A_m^i \epsilon^k - 8k^2a v E A + k^2a v B^2 + 8k^2a v E C)}{dA^2} \tag{30}
\]
\[
+ \left[ \frac{12(-C + A) \nu k^{4a} B}{dk^2a A^2} \right] \left( \frac{D^1_{\alpha\alpha} u(\xi)}{u(\xi)} \right) + \left[ \frac{-12(C^2 - 2AC + A^2) \nu k^{4a}}{dk^2a A^2} \right] \left( \frac{D^2_{\alpha\alpha} u(\xi)}{u(\xi)} \right) \tag{31}
\]

In view of Family 1–5 in (26), we obtain the following
Family 6. When $B \neq 0$, $\nu_1 > 0$

\[
U_1(\xi) = -\left( k^{-2a} A^2 m^2 \beta + k^{-2a} A^2 \delta l^{2a} + k^{-2a} A^2 \alpha \xi^a - 8k^2 \nu_2 vE + k^2 \nu_2 B^2 + 8k^2 \nu_2 vE \right) \frac{dA^2}{dA^2} + \left[ \frac{12(-C + A) \nu k^{2a} B}{dk^{2a} A^2} \right] \left[ \frac{B}{2(A - C)} + \frac{\nu_1}{2(A - C)} \left[ L_1 \sin\left( \nu_1 \eta/2(A - C) \right) + L_2 \cos\left( \nu_1 \eta/2(A - C) \right) \right] \right] + \left[ \frac{-12(C^2 - 2AC + A^2) \nu k^{2a}}{dA^2} \right] \left[ \frac{B}{2(A - C)} + \frac{\nu_1}{2(A - C)} \left[ L_1 \sin\left( \nu_1 \eta/2(A - C) \right) + L_2 \cos\left( \nu_1 \eta/2(A - C) \right) \right] \right]^2. \tag{30}
\]

Family 7. When $B \neq 0$, $\nu_1 < 0$

\[
U_2(\xi) = -\left( k^{-2a} A^2 m^2 \beta + k^{-2a} A^2 \delta l^{2a} + k^{-2a} A^2 \alpha \xi^a - 8k^2 \nu_2 vE + k^2 \nu_2 B^2 + 8k^2 \nu_2 vE \right) \frac{dA^2}{dA^2} + \left[ \frac{12(-C + A) \nu k^{2a} B}{dk^{2a} A^2} \right] \left[ \frac{B}{2(A - C)} + \frac{\nu_1}{2(A - C)} \left[ -L_1 \sin\left( \nu_1 \eta/2(A - C) \right) + L_2 \cos\left( \nu_1 \eta/2(A - C) \right) \right] \right] + \left[ \frac{-12(C^2 - 2AC + A^2) \nu k^{2a}}{dA^2} \right] \left[ \frac{B}{2(A - C)} + \frac{\nu_1}{2(A - C)} \left[ -L_1 \sin\left( \nu_1 \eta/2(A - C) \right) + L_2 \cos\left( \nu_1 \eta/2(A - C) \right) \right] \right]^2. \tag{31}
\]

Family 8. When $B \neq 0$, $\nu_1 = 0$

\[
U_3(\xi) = -\left( k^{-2a} A^2 m^2 \beta + k^{-2a} A^2 \delta l^{2a} + k^{-2a} A^2 \alpha \xi^a - 8k^2 \nu_2 vE + k^2 \nu_2 B^2 + 8k^2 \nu_2 vE \right) \frac{dA^2}{dA^2} + \left[ \frac{12(-C + A) \nu k^{2a} B}{dk^{2a} A^2} \right] \left[ \frac{B}{2(A - C)} + \frac{L_2}{2(A - C) + L_2 \eta} \right] + \left[ \frac{-12(C^2 - 2AC + A^2) \nu k^{2a}}{dA^2} \right] \left[ \frac{B}{2(A - C)} + \frac{L_2}{2(A - C) + L_2 \eta} \right]^2. \tag{32}
\]

Family 9. For $B = 0$, $\nu_2 > 0$

\[
U_4(\xi) = -\left( k^{-2a} A^2 m^2 \beta + k^{-2a} A^2 \delta l^{2a} + k^{-2a} A^2 \alpha \xi^a - 8k^2 \nu_2 vE + 8k^2 \nu_2 vE \right) \frac{dA^2}{dA^2} + \left[ \frac{-12(C^2 - 2AC + A^2) \nu k^{2a}}{dA^2} \right] \left[ \frac{\nu_2}{2(A - C)} \left[ L_1 \sin\left( \nu_2 \eta/2(A - C) \right) + L_2 \cos\left( \nu_2 \eta/2(A - C) \right) \right] \right]^2 \tag{33}
\]
Family 10. In case of $B = 0$, $\nu_2 < 0$

$$U_5(\xi) = \frac{-(k^{-2a}A^2m^2\beta + k^{-2a}A^2\delta^{2a} + k^{-2a}A^2c^2k^2 - 8k^{2a}vEA + 8k^{2a}vEC)}{dA^2} + \frac{-12(C^2 - 2AC + A^2)v^2}{dk^2A^2} [\frac{\sqrt{-\nu_2}}{2(A - C)}] \left[\frac{-L_1 \sin \left(\sqrt{-\eta}/2(A - C)\right) + L_2 \cos \left(\sqrt{-\nu_2}/2(A - C)\right)}{L_1 \cos \left(\sqrt{-\nu_2}/2(A - C)\right) + L_2 \sin \left(\sqrt{-\nu_2}/2(A - C)\right)}\right]^2,$$  \hspace{1cm} (34)

where $\nu_1 = 4E(A - C)$, $\nu_2 = E(A - C)$, $\eta = \xi^2/\Gamma(\alpha + 1)$, and $\xi = kx + ct + mz + ly + \xi_0$. It is clearly seen that the solutions depend on $\alpha$, and when $\alpha = 1$, we have the solutions that are obtained for normal derivative. The results introduce free parameters. Hence, five solutions are essential in handling initial and boundary problems. To solve the reduced Eq. (24) by the sine-Gordon expansion (FSGE) method, assume the solution of Eq. (24) as

$$U(\xi) = A_0 + B_1 \sin \left(\xi\right) + A_1 \cos \left(\xi\right) + A_2 \cos^2 \left(\xi\right) + B_2 \cos \left(\xi\right) \sin \left(\xi\right).$$  \hspace{1cm} (35)

Inserting Eq. (35) into (24) and collecting the same power of $\cos^i(\chi(\xi)) \sin^j(\chi(\xi))$, admitting the system of algebraic equation, by solving them by Maple, admits to

Set 1.

$$k^a = k^a, \hspace{0.5cm} c^a = c^a, \hspace{0.5cm} A_1 = 0, \hspace{0.5cm} B_1 = 0, \hspace{0.5cm} B_2 = \pm \frac{6ik^{2a}v}{d}, \hspace{0.5cm} A_2 = \frac{-6vk^4s}{dk^{2a}}, \hspace{0.5cm} A_0 = \frac{\left(5k^{2a}v - k^{-2a}\delta L^{2a} - k^{-2a}c^2k^2 - k^{-2a}m^2\beta\right)}{d}.$$  \hspace{1cm} (36)

Set 2.

$$k^a = k^a, \hspace{0.5cm} c^a = c^a, \hspace{0.5cm} A_1 = 0, \hspace{0.5cm} B_1 = 0, \hspace{0.5cm} B_2 = 0, \hspace{0.5cm} A_2 = \frac{-12vk^4s}{dk^{2a}}, \hspace{0.5cm} A_0 = \frac{8k^{2a}v - k^{-2a}c^2k^2 - k^{-2a}\delta L^{2a} - k^{-2a}m^2\beta}{d}.$$  \hspace{1cm} (37)

Inserting Set 1 into (35), we obtain the exact solution of Eq. (22) as

$$u_4(\xi) = \left[\frac{(5k^{2a}v - k^{-2a}\delta L^{2a} - k^{-2a}c^2k^2 - k^{-2a}m^2\beta)}{d}\right] - \frac{6vk^4s}{dk^{2a}} \tan h^2(\xi) \pm \frac{6ik^{2a}v}{d} \tan h(\xi) \sech(\xi).$$  \hspace{1cm} (40)

where $\xi = kx + ct + ly + mz + \xi_0$. Knowing Set 2 and Eq. (35), we gain the exact solution of Eq. (22) as follows:

$$u_4(\xi) = \left[\frac{(5k^{2a}v - k^{-2a}\delta L^{2a} - k^{-2a}c^2k^2 - k^{-2a}m^2\beta)}{d}\right] - \frac{12vk^4s}{dk^{2a}} \tan h^2(\chi(\xi)).$$  \hspace{1cm} (41)

where $\xi = kx + ct + ly + mz + \xi_0$. It is to be noted that, the 3D graph represent the obtained solutions with fixed $y = z = 1$ of Eqs. (38) and (40) are shown graphically (see Figures 1–7) for fixed parameter with a different choice of fractal order $\alpha$.

4. Concluding Remarks

In this article, the extended fractional subequation method and sine-Gordon expansion (FSGE) method have been proposed for finding exact solutions of fractional partial differential equations (FPDEs) in the sense of conformable derivative. This paper studies $(3 + 1)$-dimensions space-time FKPE which appears in plasma physics in the sense of conformable derivatives via two algorithms, namely, the extended fractional subequation method and FSGE method to obtain sets of exact solutions. Using suitable wave transform, the equations are reduced to some ODEs. Then, the admissible solutions are substituted into the resultant ODE. Equating the coefficients of $(D^2_t\tilde{w}(\xi)/\tilde{w}(\xi))$ in extended
The presence of parameters makes our results useful for the IVBVP with fractional order. For $\alpha = 1$, our solutions go back to that previously obtained solution. The performance of the fractional subequation method and cosine and sine functions and their multiplications in FSGE method to zero leads to some algebraic system of equations. Solving this system gives the relations among the parameters. Some 3-D solution graphs are presented in some finite domains to comprehend the effects of $\alpha$.
of these approaches shows the ability for applying on various space-time fractional nonlinear equations in nonlinear science.

Data Availability
No any data availability

Conflicts of Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgments
The work was supported by the Natural Science Foundation of China (Grant Nos. 61673169, 11701127, 11626101, and 11601485).

References
[1] R. Sabry, W. M. Moslem, P. K. Shukla, and H. Saleem, “Cylindrical and spherical ion-acoustic envelope solitons in multicomponent plasmas with positrons,” Physical Review E, vol. 79, no. 5, article 056402, 2009.
[2] H. Schamel, “Stationary solitary, snoidal and sinusoidal ion acoustic waves,” Journal of Plasma Physics, vol. 14, no. 10, pp. 905–924, 1972.
[3] M. A. Zahran, E. K. El-Shewy, and H. G. Abdelwahed, “Dust-acoustic solitary waves in a dusty plasma with dust of opposite polarity and vortex-like ion distribution,” Journal of Plasma Physics, vol. 79, no. 5, pp. 859–865, 2013.
[4] E. K. El-Shewy, “Linear and nonlinear properties of electron-acoustic solitary waves with non-thermal electrons,” Chaos, Solitons and Fractals, vol. 31, no. 4, pp. 1020–1023, 2007.
[5] A. A. Kilbas, H. M. Srivastava, and J. J. Trujillo, Theory and Applications of Fractional Differential Equations, vol. 204, Elsevier, Amsterdam, The Netherlands, 2006.
[6] D. Baleanu, K. Diethelm, E. Scalas, and J. J. Trujillo, Fractional Calculus Models and Numerical Methods, World Scientific, Series on Complexity Nonlinearity and Chaos. Boston, 2012.
[7] A. Jajarmi, A. Yusuf, D. Baleanu, and M. Inc, “Theory and application for the system of fractional Burger equations with Mittag leffler kernel,” Physica A, vol. 547, p. 123860, 2020.
[8] R. Khalil, M. Al Forani, A. Yusuf, and M. Sababheh, “A new definition of fractional derivative,” Journal of Computational Applied Mathematics, vol. 264, pp. 65–70, 2014.
[9] R. Cimpioiaus and R. Constaintinescu, “The inverse symettry problem for a 2D generalized second order evolutionary equation,” Nonlinear Analysis: Theory, Methods & Applications, vol. 57, pp. 147–154, 2010.
[10] S. Saha Ray, “New exact solutions of nonlinear fractional acoustic wave equations in ultrasound,” Computers and Mathematics with Applications, vol. 71, no. 3, pp. 859–868, 2016.
[11] Z. Korpinar, M. Inc, and M. Bayram, “Theory and application for the system of fractional Burger equations with Mittag leffler kernel,” Applied Mathematics and Computation, vol. 367, p. 124781, 2020.
[12] S. Sahoo and S. S. Ray, “Improved fractional sub-equation method for (3+1)-dimensional generalized fractional KdV-Zakharov–Kuznetsov equations,” Computers and Mathematics with Applications, vol. 70, no. 2, pp. 158–166, 2015.
[13] M. A. Abdou and A. A. Soliman, “New exact travelling wave solutions for space-time fractional nonlinear equations describing nonlinear transmission lines,” Results in Physics, vol. 9, pp. 1497–1501, 2018.
[14] M. A. Abdou, “An analytical method for space time fractional nonlinear differential equations arising in plasma physics,” Journal of Ocean Engineering and Science, vol. 2, no. 4, pp. 288–292, 2017.
[15] S. Sahoo and S. S. Ray, “New exact solutions of fractional Zakharov—Kuznetsov and modified Zakharov—Kuznetsov equations using fractional sub-equation method,” Communications in Theoretical Physics, vol. 63, no. 1, pp. 25–30, 2015.
[16] Z. Odibat and D. Baleanu, “Numerical simulation of initial value problems with generalized Caputo-type fractional derivatives,” Applied Numerical Mathematics, vol. 156, pp. 94–105, 2020.
[17] J. Singh, D. Kumar, Z. Hammouch, and A. Atangana, “A fractional epidemiological model for computer viruses pertaining to a new fractional derivative,” Applied Mathematics and Computation, vol. 316, pp. 504–515, 2018.
[18] S. Uçar, E. Uçar, N. Özdemir, and Z. Hammouch, “Mathematical analysis and numerical simulation for a smoking model with Atangana-Baleanu derivative,” Chaos, Solitons and Fractals, vol. 118, pp. 300–306, 2019.