Nonlocal black-hole thermodynamics and massive remnants

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Abstract

To alleviate the black-hole (BH) information problem, we study a holographic-principle-inspired nonlocal model of Hawking radiation in which radiated particles created at different times all have the same temperature corresponding to the instantaneous BH mass. Consequently, the black hole loses mass not only by continuously radiating new particles, but also by continuously warming previously radiated particles. The conservation of energy implies that the radiation stops when the mass of the black hole reaches the half of the initial BH mass, leaving a massive BH remnant with a mass much above the Planck scale.

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There are two (not necessarily independent) problems related to the information contained in black holes. The first one is to explain why the information contained in the black hole is proportional to its surface, rather than to its volume. The second one is to reconcile the process of Hawking radiation with the principle of unitary evolution, according to which information cannot be destroyed.

A paradigm introduced as a theoretical framework for dealing with the first problem is the holographic principle [1, 2, 3], according to which the information that can be stored inside the black hole is determined by its boundary - the black-hole (BH) horizon. One immediate consequence of the holographic principle is nonlocality - in some way, the degrees of freedom inside the black hole should know about the boundary. To explain the holographic principle, one must go beyond local quantum field theory. There are indications that string theory possesses certain nonlocal features (see e.g. [4, 5, 6, 7]), but the fact is that there is not yet a general well-understood theory of holography. Instead,
the holographic principle often serves merely as a guiding principle in construction of physical models.

A possible solution of the second problem is a BH remnant scenario, according to which the process of Hawking radiation stops before the black hole evaporates completely, so that all the BH information can be contained in the BH remnant. A problem with the BH remnant scenario is to find a physical mechanism that stops the Hawking radiation. If this mechanism is an effect of quantum gravity, one generically expects that the significant deviation from semiclassical gravity occurs at the Planck scale, so that the mass and radius of the remnant are of the order of the Planck mass and Planck distance, respectively. Such a light remnant that can contain a huge amount of information is problematic [8, 9, 10] because light objects that can exist in a huge number of different states are expected to be often produced in various physical processes, which is not seen in nature. Thus, a massive remnant [11] (i.e., a remnant with a mass much larger than the Planck mass) seems to be a more attractive possibility, but the problem is to find a physical mechanism that makes remnants so massive. One possibility is that the light remnant cannot exist without the Hawking radiation entangled with the BH interior [12], so that the total system - the black hole together with its Hawking radiation - is not light at all. However, it would be more appealing if the BH remnant itself would remain massive.

The purpose of the present work is to suggest a possible nonlocal physical mechanism that could stop the process of Hawking radiation much before reaching the Planck scale, thus leaving a massive BH remnant that can store the BH information. The mechanism we suggest is inspired by the holographic principle and partially by the suggestion [13] that there should exist a nonlocal connection between the degrees of freedom in the BH interior and those outside of the black hole. The holographic principle suggests that the degrees of freedom in the BH interior are determined by its boundary - the BH horizon. On the other hand, if there is also a nonlocal relation between some exterior degrees of freedom with those in the BH interior, then these exterior degrees of freedom could also be determined by the same BH boundary. However, it does not seem reasonable to expect that all exterior degrees of freedom are determined by a single BH boundary. (There may be a lot of black holes in the universe, so one particular black hole cannot play a preferred role for all exterior degrees of freedom.) Instead, neglecting the interaction of the Hawking radiation with other exterior degrees of freedom, we assume that only the Hawking radiation radiated from this particular black hole remains in a nonlocal contact with its boundary. The horizon relevant to the process of Hawking radiation is the apparent horizon [14, 15], which, in the case of Hawking radiation, is a time-dependent object. The time-dependent temperature \( T(t) \) associated with the time-dependent apparent horizon is determined by the time-dependent BH mass \( M(t) \), through the relation [16, 17]

\[
T = \frac{1}{8\pi M},
\]

where we use units \( \hbar = c = G_N = k_B = 1 \). In the standard semiclassical analysis of the process of BH radiation, when particles are radiated away from the black hole, then the temperature of these particles does not change with subsequent BH evolution. Instead, if these particles do not interact with other exterior degrees of freedom, their temperature
remains the same, despite the fact that later BH temperature may change. Indeed, this is a consequence of locality, according to which radiated particles cannot know about possible later changes of the BH temperature. However, if radiated particles remain in a nonlocal contact with its source - the evolving apparent horizon - then it seems reasonable to assume that this nonlocal contact could manifest as a nonlocal thermodynamic system in which all radiated particles have the same temperature given by the instantaneous BH mass $M(t)$. Thus, in addition to the standard backreaction of Hawking radiation on the black hole (owing to which the BH mass decreases, such that the total energy measured by a distant observer is conserved), there is an additional nonlocal backreaction of the horizon on the previously radiated particles, owing to which the temperature of these radiated particles increases.

Now let us see how such a nonlocal backreaction leads to massive BH remnants. The energy of all radiated particles at the time $t$ is

$$E = N\epsilon,$$

where $N$ is the total number of radiated particles at the time $t$ and $\epsilon$ is the average energy per particle. According to our assumption, all radiated particles have the same temperature $T(t)$ at a given time $t$, so, for massless particles in a thermal equilibrium,

$$\epsilon = bT,$$

where $b \sim 1$ is a dimensionless parameter that depends on the spin of the particles. Since the energy must be conserved, at each time $t$ there must be

$$M + N\epsilon = M_0,$$

where $M_0$ is the initial BH mass corresponding to $N = 0$. Inserting (3) and (1) into (4), we obtain

$$N = \frac{(M_0 - M)M}{b'},$$

where $b' \equiv b/8\pi$. Eq. (5) can also be viewed as a quadratic equation for $M$, with the general solution

$$M = \frac{M_0}{2} \pm \sqrt{\left(\frac{M_0}{2}\right)^2 - b'N}.$$

The upper sign is the correct one consistent with the requirement that $M = M_0$ when $N = 0$. During the evolution in time, $N$ increases while $M$ decreases, until the number of radiated particles attains the critical value

$$N_{\text{crit}} = \frac{M_0^2}{4b'},$$

which corresponds to the vanishing square root in (6). At this moment, the mass of the black hole attains the critical value

$$M_{\text{crit}} = \frac{M_0}{2}.$$
Can the mass further decrease after reaching this critical value? Mathematically, it would be possible only by taking the lower sign in (6) at times after the mass has reached the critical value. However, in this case, after this critical moment of time, \( N \) should start to decrease. This would correspond to an inverted process of Hawking radiation, in which the outgoing Hawking particles suddenly reverse their direction of motion, thus becoming ingoing particles that eventually become destroyed when they approach the horizon. A sudden inversion of the direction of motion does not seem to be physical, so we conclude that such a further decrease of mass is unphysical. In other words, the critical mass (8) is the smallest possible mass of the black hole. The final state of BH radiation is a massive remnant with the mass equal to the half of the initial BH mass. Clearly, such a massive remnant solves the problem of unitary evolution without leading to overproduction of light objects that can store a huge amount of information. In addition, such massive BH remnants could be responsible for the existence of dark matter in the universe.

Now let us study the time evolution of such a radiating black hole. The change of the radiation energy \( E \) is equal to the negative change of the BH mass \( M \), i.e. \( dE = -dM \). Therefore, (2) implies

\[
-dM = \epsilon dN + N d\epsilon.
\]

Note that the second term on the right-hand side of (9) is absent in the standard approach, because, in the standard approach, one assumes that the temperature of the fixed number of radiated particles does not change, so that \( d\epsilon = b dT = 0 \) in (2). In our approach, from (3) and (1) we find

\[
N d\epsilon = \frac{-b' N}{M^2} dM.
\]

The first term on the right-hand side of (9) can be calculated in the same way as in the standard scenario. Assuming that the black hole radiates as a perfect black body, we use the Stefan-Boltzmann law

\[
\epsilon dN = \sigma A T^4 dT,
\]

where \( \sigma = \pi^2/60 \) is the Stefan-Boltzmann constant and \( A = 4\pi R^2 = 16\pi M^2 \) is the BH surface. Thus, (11) with (1) can be written as

\[
\epsilon dN = \frac{\sigma'}{M^2} dt,
\]

where \( \sigma' \equiv \sigma/256\pi^3 \). Eq. (12) with (3) and (1) can be written as the differential equation

\[
\frac{dN}{dt} = \frac{\sigma'}{b' M}.
\]

Similarly, by inserting (10) and (12) into (9), we obtain another differential equation

\[
\frac{dM}{dt} = \frac{-\sigma'}{M^2 - b' N}.
\]

Eqs. (13) and (14) represent a system of two coupled equations that determine the functions \( N(t) \) and \( M(t) \), with the initial conditions \( N(0) = 0 \) and \( M(0) = M_0 \). From
We see that $dM/dt$ diverges at the critical values of $N$ and $M$. For a numerical analysis of the system of equations (13) and (14), it is convenient to introduce the rescaled variables

$$
\tau = \frac{\sigma'}{M_0^3} t, \quad m = \frac{M}{M_0}, \quad n = \frac{b'}{M_0^2} N.
$$

Eqs. (13) and (14) then become

$$
\frac{dn}{d\tau} = \frac{1}{m}, \quad \frac{dm}{d\tau} = -\frac{1}{m^2 - n},
$$

with the initial conditions $n(0) = 0$, $m(0) = 1$, which does not involve numerical constants that are not of the order of unity. Eqs. (13) and (14) can also be solved analytically. From (13) and (14) one obtains the differential equation $dN/dM = (N - M^2/b')/M$, the solution of which is given by (5). Thus, by inserting (5) into (14), we obtain a decoupled differential equation

$$
\frac{dM}{dt} = -\frac{\sigma'}{2M^2 - M_0M}.
$$

This is easily integrated to give

$$
\frac{2M^3}{3} - \frac{M_0M}{2} - \frac{M_0^3}{6} = -\sigma't,
$$

where the requirement $M(t = 0) = M_0$ is incorporated. This, together with (8), determines the time needed for the black hole to approach the critical mass:

$$
t_{\text{crit}} = \frac{5}{24} \frac{M_0^3}{\sigma'}. \tag{19}
$$

Let us compare the results above with those in the standard semiclassical scenario of BH radiation. In this case, there is no second term on the right hand side of (9), so (14) is replaced by a simpler equation $dM/dt = -\sigma'/M^2$. Thus, instead of (18), we obtain a simpler solution

$$
M = 3\sqrt[3]{M_0^3 - 3\sigma't}.
$$

The critical point at which $dM/dt$ diverges corresponds to $M_{\text{crit}} = 0$ and $t_{\text{crit}} = (1/3)M_0^3/\sigma'$. The number of radiated particles satisfies (13) which is easily integrated to give

$$
N = \frac{M_0^2 - (M_0^3 - 3\sigma't)^{2/3}}{2b'},
$$

leading to $N_{\text{crit}} = M_0^2/2b'$. Comparing it with (7), we see that the number of particles produced in the massive remnant scenario is equal to the half of that in the standard scenario. Combining (20) with (21) one obtains

$$
M = \sqrt{M_0^2 - 2b'N},
$$

which is to be compared with (6). The time evolutions of the BH mass in the two scenarios (Eqs. (18) and (20)) are compared in Fig. 1.
As we have seen, the assumption that, at each time, all radiated particles have the same temperature given by the instantaneous BH mass leads to massive BH remnants. Such remnants offer a solution to the problem of unitarity of BH evolution. The main problem with such a scenario is to find an independent justification of such an assumption that clearly contradicts the principle of locality. In particular, this assumption requires a preferred notion of the time coordinate, but note that this preferred time coordinate is the same preferred time coordinate as the one needed to define particles in the standard description of BH radiation [17], which, in an ideal case, can be identified with the Killing time. (Note also that the definition of particles in quantum field theory is intrinsically nonlocal [17], even when formulated in terms of local particle currents [18].) We have argued that such a nonlocal thermodynamic behaviour could be related to the holographic principle, but a clear derivation of this relation is missing. Nevertheless, the fact that such a simple assumption leads to such a simple (and surprising!) solution to the problem of BH unitarity leads us to suspect that this assumption could be on the right track. Hopefully, the future research could reveal a more compelling explanation of our assumption.

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