New development in radiative neutrino mass generation

Julio
Fisika LIPI, Kompleks Puspiptek, Tangerang 15310, Indonesia
E-mail: julio@lipi.go.id

Abstract. We present a simple and predictive model of radiative neutrino masses. It is a special case of the Zee model with a family-dependent $Z_{\text{4}}$ symmetry acting on the leptons. A variety of predictions follow: The hierarchy of neutrino masses must be inverted; the lightest neutrino mass is extremely small and calculable; one of the neutrino mixing angles is determined in terms of the other two; the phase parameters take CP-conserving values with $\delta_{CP} = \pi$; and the effective mass in neutrinoless double beta decay lies in a narrow range, $m_{\beta\beta} = (17.6-18.5)$ meV.

The ratio of vacuum expectation values of the two Higgs doublets, $\tan \beta$, is determined to be either 1.9 or 0.19 from neutrino oscillation data. Flavor-conserving and flavor-changing couplings of the Higgs doublets are also determined from neutrino data. The non-standard neutral Higgs bosons, if they are moderately heavy, decay significantly into $\mu$ and $\tau$ with prescribed branching ratios. Observable rates for the decays $\mu \rightarrow e\gamma$ and $\tau \rightarrow 3\mu$ are predicted if these scalars have masses in the range of 150-500 GeV.

1. Introduction
The fact that neutrinos have masses and mix brings attentions to mechanisms that can explain it. The most popular way is seesaw mechanism in which right-handed neutrinos (which are singlets under Standard Model gauge groups) are integrated out resulting in dimension-five operator $LLHH/\Lambda$ with $L$ and $H$ being lepton and Higgs doublets, and $\Lambda$ being right-handed neutrino mass. Once the Higgs acquires vacuum expectation value (VEV), this becomes neutrino mass term. The value of $\Lambda$, in accordance with neutrino oscillation data, is of order $10^{14}$ GeV. Such high scale is well beyond the reach of LHC, thus making it difficult to be tested experimentally.

There exists, however, class of models in which neutrino masses are generated at loop level. Because of loop and chirality suppressions, the new physics scale can be lower. Thus, they may be testable at the LHC as well as in lepton flavor violation processes. A well-known example is Zee model [1]. This model features two Higgs doublets $H_a (1, 2, -1/2)$ ($a = 1, 2$) and scalar singlet $\eta^+ (1, 1, +1)$. The gauge-invariant Yukawa interactions in lepton sector can be written as

$$
\mathcal{L}_{\text{Yuk}}^{(l)} = \sum_{i,j=1,2,3} Y_{ij}^a L_i e_j^c H_a + \sum_{i,j=1,2,3} f_{ij} L_i L_j \eta^+ + h.c.
$$

(1)

where $i, j = 1–3$ indicate family indices. Fermi statistics implies that $f = -f^T$. In addition, there is also cubic term in scalar potential

$$
V = \left\{ \mu H_1 H_2 \eta^+ + h.c. \right\} + ...
$$

(2)
with “...” indicating potential terms not relevant for our discussion. The simultaneous presence of $f_{ij}L_iL_j\eta^+$ and $\mu H_1H_2\eta^+$ terms breaks lepton number by two units. The neutrino masses arise at one loop as shown in Fig. 1.

In its general form, both Higgs doublets are allowed to couple to all SM fermions. Consequently, the corresponding Yukawa matrices cannot be simultaneously diagonalized. This may lead to potentially dangerous tree-level flavor-changing neutral currents (FCNCs). On top of that, the structure of neutrino mass matrix is arbitrary in this case. Thus, it is hard to make a prediction for neutrino sector.

A more interesting approach was given by Wolfenstein in 1980. He imposed a $Z_2$ symmetry so that only one Higgs doublet could couple to fermions [2]. In contrast to the general Zee model, there is no FCNC induced at tree level here. One, therefore, can choose a basis where charged lepton Yukawa couplings are diagonal. The flavor structure of neutrino mass matrix suggests that all its diagonal elements must vanish identically. This implies that the neutrino mass hierarchy must be inverted and $\theta_{12} \simeq \pi/4$. However, this predicted value of $\theta_{12}$ is now ruled out by current solar and KamLAND data. These experiments prefer smaller value of $\theta_{12}$. (See Refs. [3,4] for detailed discussions.)

It is obvious that this $Z_2$ symmetry cannot describe a realistic neutrino mass and mixing. In this paper, we shall consider other discrete symmetry (i.e., $Z_4$ symmetry). But unlike Wolfenstein, our discrete symmetry is family dependent. As we shall show later, this approach leads to a predictive model of neutrino masses. All neutrino masses and mixings are described in terms of four real parameters. One neutrino mixing angle is given in terms of the other two. Neutrino mass hierarchy is predicted to be inverted. Tree-level FCNCs are induced, but their rates are consistent with current experimental bounds even for Higgs boson masses about few hundreds GeV.

2. Zee model with $Z_4$ discrete symmetry

We try to modify Zee model by introducing a family-dependent $Z_4$ symmetry as follows [5]:

\begin{align}
L_i & : (-i, i, i); & e_i^c & : (-i, -i, -i); \\
H_1 & : +1; & H_2 & : -1; & \eta^+ & : -1.
\end{align}

Here $i=1–3$ is the family index. Unlike the Wolfenstein realization, there will be tree-level FCNC induced here. But as we shall see later, they are within bounds allowed by experiments.

The leptonic Yukawa interactions, consistent with gauge and $Z_4$ symmetries can be written as:

\begin{align}
L^{(\ell)}_{\text{Yuk}} = \sum_{i=2,3} \sum_{\alpha=1} Y_{i\alpha} L_i e_\alpha^c H_1 + \sum_{\alpha=1} Y_{\alpha} L_1 e_\alpha^c H_2 + f_{23} L_2 L_3 \eta^+ + h.c.
\end{align}
Note that the cubic term necessary for breaking lepton number (Eq. (2)) is invariant under $Z_4$ symmetry.

In quark sector, it is necessary that all quarks couple universally to either $H_1$ or $H_2$:

$$\mathcal{L}_{\text{Yuk}}^{(q)} = \sum_{i,j=1-3} Y^{u}_{ij} Q_i u_j^c \bar{H}_a + \sum_{i,j=1-3} Y^{d}_{ij} Q_i d_j^c H_a + h.c.,$$  \hspace{1cm} (5)

with $\bar{H}_a = i\sigma_2 H_a^+$. With this form of Yukawa couplings it can be shown [5] that $Z_4$ charge assignment of Eq. (3) is anomaly-free [6, 7].

It should be noted that one can always rotate the $L_{2,3}$ and $e_{1,2,3}^c$ fields such that $Y_{ia}$ can be brought into diagonal form with $Y_{22}$ and $Y_{33}$ being nonzero. Such rotation will not induce any mixing in leptonic weak charged currents because both left-handed charged-leptons and neutrinos are rotated simultaneously. In this basis, the charged-lepton mass matrix can be written as

$$M_{\ell} = \begin{pmatrix} Y_{1v1} & Y_{2v2} & Y_{3v2} \\ 0 & Y_{22v1} & 0 \\ 0 & 0 & Y_{33v1} \end{pmatrix},$$  \hspace{1cm} (6)

where $v_1$ and $v_2$ are vacuum expectation values (VEVs) of the two Higgses:

$$\langle H_1 \rangle = v_1, \quad \langle H_2 \rangle = v_2 = |v_2| e^{i\phi}.$$  \hspace{1cm} (7)

Here $v_1$ has been made real by an $SU(2)_L \times U(1)_Y$ rotation.

We can rotate all phases in $M_{\ell}$ so that they become real. We then define

$$Y_{1v2} = m_e^0 \sqrt{1 + x^2 + y^2}, \quad Y_{2v2} = m_\mu^0 \frac{y \sqrt{1 + x^2}}{\sqrt{1 + x^2 + y^2}}, \quad Y_{3v2} = m_\tau^0 \frac{x}{\sqrt{1 + x^2}},$$

$$Y_{22v1} = m_\mu^0 \frac{\sqrt{1 + x^2}}{\sqrt{1 + x^2 + y^2}}, \quad Y_{33v1} = m_\tau^0 \frac{\sqrt{1 + x^2}}{\sqrt{1 + x^2 + y^2}},$$  \hspace{1cm} (8)

where $x, y, m_e^0, m_\mu^0, m_\tau^0$ are real parameters. Bringing the charged lepton fields into their mass eigenstates, the $M_{\ell}$ can be diagonalized by

$$M_{\ell}^{\text{diag.}} = U_L^T M_\ell U_R \simeq \text{diag}(m_e^0, m_\mu^0, m_\tau^0),$$  \hspace{1cm} (9)

with $m_e^0, m_\mu^0, m_\tau^0$ being approximate eigenvalues.

Among the two charged scalars $H_1^+$ and $H_2^+$, one combination forms a Goldstone boson $G^+ = (v_1 H_1^+ + v_2 H_2^+)/v$ (with $v \equiv \sqrt{v_1^2 + v_2^2}$). The orthogonal combination to it, $H^+ = (v_2 H_1^+ - v_1 H_2^+)/v$, forms the physical charged scalar. In unitary gauge, one can write the couplings of $H^+$ with leptons (before rotations to lepton mass eigenbasis are performed) as

$$Y_{Yuk}^{(H^\pm)} = \frac{1}{v} \begin{pmatrix} -\frac{v_1}{v_2} & v_2 \/

v_1 & v_2 \end{pmatrix} M_{\ell}. \hspace{1cm} (10)$$

In the mass eigenbasis of lepton fields, the matrix $Y_{Yuk}^{(H^\pm)}$ transforms into

$$\hat{Y} = \begin{bmatrix} \frac{m_e}{v} \frac{(x^2 + y^2) \tan \beta - \cot \beta}{1 + x^2 + y^2} & -\frac{m_\mu}{v} \frac{y (\tan \beta + \cot \beta)}{\sqrt{1 + x^2} (1 + x^2 + y^2)} & -\frac{m_\tau}{v} \frac{x (\tan \beta + \cot \beta)}{\sqrt{1 + x^2} \sqrt{1 + x^2 + y^2}} \\ -\frac{m_e}{v} \frac{y (\tan \beta + \cot \beta)}{\sqrt{1 + x^2} (1 + x^2 + y^2)} & \frac{m_\mu}{v} \frac{\tan \beta ((1 + x^2)^2 + x^2 y^2) - \cot \beta y^2}{(1 + x^2)^2 (1 + x^2 + y^2)} & -\frac{m_\tau}{v} \frac{xy (\tan \beta + \cot \beta)}{(1 + x^2)^2 \sqrt{1 + x^2 + y^2}} \\ -\frac{m_e}{v} \frac{x (\tan \beta + \cot \beta)}{\sqrt{1 + x^2} \sqrt{1 + x^2 + y^2}} & -\frac{m_\mu}{v} \frac{xy (\tan \beta + \cot \beta)}{(1 + x^2)^2 \sqrt{1 + x^2 + y^2}} & \frac{m_\tau}{v} \frac{\tan \beta - \cot \beta}{(1 + x^2)^2} \end{bmatrix},$$  \hspace{1cm} (11)
where $\hat{Y} = U_T^T \gamma_{Y_{\text{Yuk}}}^{(H^\pm)} U_R$ and $\tan \beta \equiv |v_2|/v_1$. Similarly, the flavor-antisymmetric matrix $f$ is rotated into

$$\hat{f} = f_{23} \begin{pmatrix} 0 & \frac{x}{\sqrt{1+x^2}} & \frac{y}{\sqrt{1+x^2+y^2}} \\ -\frac{x}{\sqrt{1+x^2}} & 0 & -\frac{1}{\sqrt{1+x^2+y^2}} \\ \frac{y}{\sqrt{1+x^2+y^2}} & \frac{1}{\sqrt{1+x^2+y^2}} & 0 \end{pmatrix}, \quad (12)$$

with $\hat{f} = U_T^T f U_L$.

The couplings of neutral scalar bosons to the leptons arise in analogous way. First, note that the pseudoscalar Higgs boson $A^0 = [\epsilon Y_{\ell j} c_{\psi} A^0 \sqrt{2}]$ would couple to the physical leptons as $\tilde{Y}_j c_{\psi} A^0 / \sqrt{2}$, where $\tilde{Y}$ is the same Yukawa coupling matrix as in Eq. (11). For the other scalars, their couplings to lepton will involve another mixing angle $\alpha$ defined through $H^0 = \cos \alpha \text{Re}H^0_1 + \sin \alpha \text{Re}H^0_2$, $h^0 = -\sin \alpha \text{Re}H^0_1 + \cos \alpha \text{Re}H^0_2$. For the special choice $\alpha = \beta - \pi/2$, the neutral field $h^0$ will behave like the Standard Model Higgs field (i.e., its coupling to lepton fields are flavor diagonal), while $H^0$ couplings to lepton fields is nothing but $\hat{Y}$. This choice of $\alpha$ is realized in the decoupling limit, where the second Higgs doublet mass takes large values compared to $v$.

3. Phenomenology of neutrino mass

The neutrino masses are generated through one-loop diagram shown in Fig. 1. From that diagram, it is straightforward to determine the structure of neutrino mass matrix

$$M_\nu = \kappa (\hat{f} M_\ell^{\text{diag}} \hat{Y}^T + \hat{Y} M_\ell^{\text{diag}} \hat{f}). \quad (13)$$

Here, $M_\ell^{\text{diag}}$ is the diagonal charged-lepton mass matrix, while $\hat{Y}$ and $\hat{f}$ are matrices given in Eqs. (11) and (12) respectively. The overall factor $\kappa$ accounts for the loop integral,

$$\kappa = \frac{\sin 2\gamma}{16\pi^2} \log \left( \frac{M_2^2}{M_1^2} \right). \quad (14)$$

Note the main difference between our version and Wolfenstein model [2]. In the latter, $\hat{Y}$ in Eq. (13) is nothing but $M_\ell^{\text{diag}}/v$, leading to vanishing of all $M_\nu$ diagonal elements.

Ignoring terms that are proportional to $m_\tau^2$ and $m_\mu^2$ followed by diagonalizing the neutrino mass matrix, we obtain

$$U^T M_\nu U = M_\nu^{\text{diag}} = \text{diag} \left\{ \left( \frac{m_\tau^2 \gamma_2}{v} \right) \frac{\sqrt{\tan^2 \beta + x^2 \cot^2 \beta}}{1+x^2}, -\left( \frac{m_\mu^2 \gamma_2}{v} \right) \frac{\sqrt{\tan^2 \beta + x^2 \cot^2 \beta}}{1+x^2}, 0 \right\} \quad (15)$$

where the matrix $U \equiv U_{\text{PMNS}}$, which is the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix, is found to be

$$U_{\text{PMNS}} = \begin{pmatrix} \frac{1}{\sqrt{2}}(C_\chi C_\psi + S_\psi) & \frac{1}{\sqrt{2}}(C_\chi C_\psi - S_\psi) & -S_\chi C_\psi \\ \frac{1}{\sqrt{2}}(C_\chi S_\psi - C_\psi) & \frac{1}{\sqrt{2}}(C_\chi S_\psi + C_\psi) & -S_\chi S_\psi \\ \frac{S_\chi}{\sqrt{2}} & \frac{S_\chi}{\sqrt{2}} & C_\chi \end{pmatrix}. \quad (16)$$

Here $S_\chi = \sin \chi$, $C_\chi = \cos \chi$ and $S_\psi = \sin \psi$, $C_\psi = \cos \psi$, with

$$S_\psi = \frac{y}{\sqrt{1+x^2+y^2}}, \quad S_\chi = -\frac{\tan \beta - x^2 \cot \beta}{\sqrt{1+x^2}\sqrt{\tan^2 \beta + x^2 \cot^2 \beta}}. \quad (17)$$
The results obtained in Eqs. (15)-(16) suggest the following predictions. First, the hierarchy of neutrino masses must be inverted. This follows from the fact that (in the limit of $m_\mu = 0$) two neutrino mass eigenvalues are degenerate (up to signs) and another one is zero. Turning on the muon mass will remove the degeneracy and in the same time will induce the correct solar-mass splitting. Second, the Dirac phase $\delta_{CP}$ is predicted to be 0 or $\pi$. It comes from the reality property of neutrino mass matrix. Third, from the relation $|U_{e1}| = |U_{e2}|$ given in Eq. (16) it is found that $\theta_{23}$ can be expressed in terms of other mixing angles. Considering that $\theta_{23}$ must lie in the first quadrant and its value must be nearly 45 degrees, the only consistent solution is

$$t_{23} = \frac{1 + t_{12}}{1 - t_{12}} \quad \text{and} \quad \delta_{CP} = \pi.$$  

Here $t_{23} = \tan \theta_{23}$, etc. Thus the model predicts $\delta_{CP} = \pi$.

We plot the relation given in Eq. (18) in Fig. (2) in two planes: $\sin^2 \theta_{23}$ versus $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ versus $\sin^2 \theta_{23}$. As inputs we use $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ obtained from a global fit of neutrino data: $\sin^2 \theta_{12} = 0.302 \pm 0.0125$ and $\sin^2 \theta_{13} = 0.0227 \pm 0.0023$ both taken from [8]. In Fig. 2, we also show best fit to the mixing angles along with their error bars (shown in red). We can see that the prediction of the model is in very good agreement with the data although it is preferable for both $\sin^2 \theta_{12}$ and $\sin^2 \theta_{13}$ to be slightly above their central values by one sigma.

![Figure 2](https://example.com/figure2.png)

**Figure 2.** Predicted value of $\sin^2 \theta_{23}$ as functions of $\sin^2 \theta_{12}$ (left panel) and $\sin^2 \theta_{13}$ (right panel).

The solar-mass splitting is generated by terms proportional to muon mass. Treating them as perturbations, we find

$$\Delta m_{\text{solar}}^2 = 4 \left( \frac{m_\mu^2}{m_\tau^2} \right) \frac{xy \tan \beta \sqrt{\tan^2 \beta + x^2 \cot^2 \beta}}{(1 + x^2 + y^2)(\tan^4 \beta + x^2)} \left( \frac{m_\tau^2 \kappa f_{23}}{v} \right)^2,$$

and by using Eq. (15), we also get

$$\Delta m_{\text{atm}}^2 \equiv m_3^2 - m_1^2 = -\frac{\tan^2 \beta + x^2 \cot^2 \beta}{(1 + x^2)^2} \left( \frac{m_\tau^2 \kappa f_{23}}{v} \right)^2.$$  

Now, the lightest neutrino mass $m_3$ is found to be $m_3 = \Delta m_{\text{solar}}^2 / 2|\Delta m_{\text{atm}}^2|^{1/2} \approx 7.5 \times 10^{-4} \text{ eV}$. The effective mass parameter $m_{\beta\beta}$ for neutrinoless double beta decay, therefore, can be determined from this relation. For this purpose, we use the atmospheric and solar mass splittings as input, obtained from the global fit: $\Delta m_{\text{atm}}^2 = (2.47 \pm 0.07) \times 10^{-3} \text{ eV}^2$ and
\( \Delta m_{\text{solar}}^2 = (7.5 \pm 0.19) \times 10^{-5} \text{eV}^2 \). By varying the input parameters within their 1 sigma range, we found \( m_{\beta\beta} \equiv |\sum_{i=1-3} U_{ei}^2 m_i| = (17.6-18.5) \text{meV} \). Here we also used the fact that the Majorana phases are zero, and that \( m_1 \) and \( m_2 \) have opposite CP eigenvalues. We also demand that the value of \( \sin^2 \theta_{23} \) resulting from the model prediction is within 1 sigma of the best fit value. As comparison, we plot the range of \( m_{\beta\beta} \) (in IH case) as a function of the smallest neutrino mass in Fig. 3.

To determine the parameters of the model, we choose inputs values for \( R = \Delta m_{\text{solar}}^2/|\Delta m_{\text{atm}}^2|, |U_{e2}| \) and \( |U_{e3}| \). Using the relation for \( R \) from Eqs. (19)–(20), and the relations \( |U_{e2}| = \frac{1}{\sqrt{2}} (C_X C_\psi - S_\psi) \) and \( |U_{e3}| = | - S_X C_\psi | \), for a given input choice we solve for \( (x, y, \tan \beta) \). The third mixing angle \( \theta_{23} \) is determined through Eq. (18). We find two separate solutions for the parameters (up to signs):

\[
(i) \quad (x, y, \tan \beta) = (0.038, 4.24, 0.19), \\
(ii) \quad (x, y, \tan \beta) = (4.6, 21.2, 1.9). \\
\tag{21}
\]

Solution (i) is obtained by using the analytic expressions with input values \((m_0^e, m_0^\mu, m_0^\tau) = (m_e, m_\mu, m_\tau), |U_{e2}|^2 = 0.32, |U_{e3}|^2 = 0.0227 \) and \( R = 1/32.9 \). We compared this result with the exact values determined numerically and found agreement within 1% for this case. For solution (ii), we improve the results obtained from analytic expressions iteratively by numerical methods. As input we choose \((m_0^e, m_0^\mu, m_0^\tau) = (1.001 m_e, 1.041 m_\mu, 0.9605 m_\tau), |U_{e2}|^2 = 0.324, |U_{e3}|^2 = 0.0202 \) and \( R = 1/32.3 \). Both solutions fit neutrino oscillation data quite well. These values can now be used to compute the Higgs Yukawa coupling matrix \( \hat{Y} \), which would determine the structure of flavor changing neutral currents.

For the two solutions we find this matrix to be

\[
(i) \quad \hat{Y} = \begin{pmatrix}
-2.95 \times 10^{-7} & -0.00074 & -0.00049 \\
-3.60 \times 10^{-6} & -0.003 & -0.002 \\
-1.40 \times 10^{-7} & -0.0001 & 0.0018
\end{pmatrix}, \\
(ii) \quad \hat{Y} = \begin{pmatrix}
5.57 \times 10^{-6} & -1.29 \times 10^{-5} & -0.0011 \\
-6.25 \times 10^{-8} & 0.0011 & -0.0046 \\
-3.22 \times 10^{-7} & -2.75 \times 10^{-4} & -0.0044
\end{pmatrix}. \\
\tag{22}
\tag{23}
\]

**Figure 3.** The range of double-beta-decay effective mass as function of the \( m_3 \) in inverted hierarchy (blue). The red points represent our prediction of \( m_{\beta\beta} = 17.6-18.5 \text{meV} \). The zoomed region is shown in the right panel.
With these coupling matrices, we can now determine FCNC rates, which we address in the next section.

The overall coefficient of neutrino mass matrix from Eq. (20) is found to be \( \kappa f_{23} = \{9.8 \times 10^{-9}, 2.1 \times 10^{-8}\} \) (for cases (i) and (ii)). The smallness of \( \kappa f_{23} \) may be explained by choosing \( \kappa \) and/or \( f_{23} \) small. A small \( \kappa \) is realized if the cubic scalar coupling coefficient \( \mu \) in Eq. (2) is small, or if the mass of one of the charged scalar \( \eta^+ \) or \( H^+ \) is large. As an illustration, choose \( \mu = 1 \text{ GeV}, f_{23} = 0.01, m_{\eta^\pm} = 1.5 \text{ TeV} \) and \( m_{H^\pm} = 500 \text{ GeV} \). This would yield \( \kappa f_{23} = 2.1 \times 10^{-8} \), consistent with solution (ii). Clearly, other choices are also possible. This shows that the smallness of neutrino masses can be explained in the present framework without much tuning, even when the scale of new physics is near the TeV.

4. Lepton flavor violation and Higgs phenomenology

From the structure of \( \hat{Y} \) given above, it is clear that there exist tree-level lepton flavor violation processes mediated by \( H^0 \) and \( A^0 \) scalars. In the limit \( m_{H^0} = m_{A^0} \), we find the branching ratio of \( \tau \rightarrow 3\mu \) to be

\[
(i) \quad \text{BR}(\tau \rightarrow 3\mu) = 1.6 \times 10^{-12} \left(150 \text{ GeV}/m_{A^0}\right)^4, \tag{24}
\]

\[
(ii) \quad \text{BR}(\tau \rightarrow 3\mu) = 1.2 \times 10^{-12} \left(150 \text{ GeV}/m_{A^0}\right)^4. \tag{25}
\]

Other lepton flavor violation rates in this class are much suppressed.

On the other hand, radiative decays \( \ell_i \rightarrow \ell_j + \gamma \) arise through one-loop diagrams mediated by the neutral Higgs bosons \( H^0 \) and \( A^0 \) as well as the charged Higgs bosons \( H^\pm \) and \( \eta^\pm \). Ignoring the small \( H^+ - \eta^+ \) mixing (see the discussion in the last paragraph of Section 3) and taking \( m_{H^0} = m_{A^0} \), we find their rates to be

\[
\Gamma(\ell_i \rightarrow \ell_j \gamma) = \frac{\alpha_e m_{\ell_i}^5}{(96\pi^2)^2} \left[ \frac{1}{4} \left(\hat{\mathcal{Y}}^T \hat{\mathcal{Y}}\right)_{ij} \right]^2 + \frac{1}{8} \left(\hat{\mathcal{Y}} \hat{\mathcal{Y}}^T\right)_{ij} - r^2 \left(\hat{f} \hat{f}^T\right)_{ij} \right]^2 \right], \tag{26}
\]

where \( r \equiv m_{A^0}^2/m_{\eta^+}^2 \) and we have taken \( m_{H^0} = m_{A^0} \). After substituting \( (x, y, \tan \beta) \) values determined from neutrino oscillation data, it turns out that the last two terms interfere constructively. So, our estimates will be the lower limits of these processes. For \( \mu \rightarrow e\gamma \), we find

\[
(i) \quad \text{BR}(\mu \rightarrow e\gamma) \geq 2.0 \times 10^{-15} \left(150 \text{ GeV}/m_{A^0}\right)^4, \tag{27}
\]

\[
(ii) \quad \text{BR}(\mu \rightarrow e\gamma) \geq 5.1 \times 10^{-15} \left(150 \text{ GeV}/m_{A^0}\right)^4. \tag{28}
\]

These are consistent with present limit and possibly within reach of MEG and other proposed experiments.

In quark sector, the Yukawa interactions when all quarks couple to \( H_1 \) are given by (in the limit \( \alpha = \beta - \pi/2 \))

\[
L_{\text{Yuk}}^{(q)} = -\sum_q \frac{m_q}{v\sqrt{2}} \bar{q} h^0 + \tan \beta \sum_q \left[ -\frac{m_q}{v\sqrt{2}} \bar{q} H^0 + i \frac{m_q}{v\sqrt{2}} \bar{q} \gamma_5 q \right], \tag{29}
\]

When quarks couple to \( H_2 \) one just needs to replace \( \tan \beta \rightarrow \cot \beta \), \( A^0 \rightarrow -A^0 \), and \( H^+ \rightarrow -H^+ \). Considering perturbativity, solution (i) can only be realized when all quarks couple to \( H_1 \) or else the charged Higgs coupling to top quark will be unperturbative. Conversely, solution (ii) suggests that all quarks must couple to \( H_2 \).
Now, we can determine partial decay rates for $H^0$, $A^0$, and $H^\pm$ into SM particles. Their values are found to be (for $m_{A^0} = m_{H^0} = 300$ GeV):

**Solution (i):** $H_1$ couples to quarks:

$$\text{BR}(H^0 \to t^* t) + \text{BR}(H^0 \to t t^*) = 0.15; \quad \text{BR}(H^0 \to b \bar{b}) = 0.34; \quad \text{BR}(H^0 \to \tau^+ \tau^-) = 0.089;$$

$$\text{BR}(H^0 \to \mu^+ \mu^-) = 0.24; \quad \text{BR}(H^0 \to \mu^+ \tau^-) + \text{BR}(H^0 \to \tau^+ \mu^-) = 0.11; \quad (30)$$

**Solution (ii):** $H_2$ couples to quarks:

$$\text{BR}(H^0 \to t^* t) + \text{BR}(H^0 \to t t^*) = 0.23; \quad \text{BR}(H^0 \to b \bar{b}) = 0.52; \quad \text{BR}(H^0 \to \tau^+ \tau^-) = 0.1;$$

$$\text{BR}(H^0 \to \mu^+ \mu^-) = 0.006; \quad \text{BR}(H^0 \to \mu^+ \tau^-) + \text{BR}(H^0 \to \tau^+ \mu^-) = 0.11; \quad (31)$$

Notice that there are no $H^0 \to W^+ W^-, ZZ$ decays since the relevant couplings vanish in the limit of $\beta - \alpha = \pi/2$. In principle, $H^0$ could also decay into a pair of $h^0$. However, that coupling depends on a combination of quartic couplings which is unknown, and which may be very suppressed. We assume that this decay has a negligible rate. The charged-Higgs on the other hand, decays almost 100% of the time into $t\bar{b}$ in both solutions (i) and (ii).

In the present model, the cross section for $H^0$ (and $A^0$) production is suppressed by a factor of $(\tan \beta)^2$ in solution (i) and by a factor of $(\cot \beta)^2$ in solution (ii), compared to the cross section for a SM Higgs boson of identical mass. The cross sections for a SM Higgs boson of mass 300 GeV is $2.6 \text{ pb (3.6 pb)}$ at $\sqrt{s} = 7 \text{ TeV (8 TeV)}$ [11]. Using this value, we find the model predictions for $pp \to (A^0, H^0) \to \mu^+ \mu^-$ to be $\sigma B = (0.046 \text{ pb}, 0.008 \text{ pb})$ for solutions (i) and (ii) at $\sqrt{s} = 7 \text{ TeV}$ and $\sigma B = (0.062 \text{ pb}, 0.012 \text{ pb})$ for the two solutions at $\sqrt{s} = 8 \text{ TeV}$. As a comparison, ATLAS collaboration has published their results on a search for heavy particles produced in $pp$ collisions that decay into muon pairs. A limit of $\sigma B < 0.04 \text{ pb}$ has been obtained at 90% CL for the cross section times branching ratio of a sequential $Z'$ decaying into muon pair, in an analysis using $5 \text{ fb}^{-1}$ of data collected at $\sqrt{s} = 7 \text{ TeV}$ [12]. While this limit will be modified somewhat for the case of a scalar resonance, we anticipate the limit to be not very different for the case of $H^0$ and $A^0$.

5. Conclusions

We have presented a special case of the general Zee model. We employed a family-dependent $Z_1$ symmetry that resulted in a total of four real parameters explaining the entire neutrino oscillation data. There is a variety of predictions in the neutrino sector. The CP violating parameter $\delta_{CP}$ is predicted to be $\pi$. Most interestingly, one of the neutrino oscillation angles is determined in terms of the other two angles. This nontrivial relation is found to be consistent with current data. The model employs two Higgs doublets and a charged singlet. A crucial prediction is the VEV ratio $\tan \beta$. We are able to determine its value from neutrino oscillation data. We found that $\tan \beta = 0.19$ or 1.9. The branching ratios of the neutral Higgs bosons of the model into fermions are then completely determined. We found that leptonic decays, involving the muon, can be significant, which can potentially raise the reach for such particles at the LHC. The charged and neutral Higgs bosons also mediate leptonic flavor violation, with $\mu \to e\gamma$ possibly within reach of proposed experiments. The decay $\tau \to 3\mu$, which arise at the tree-level is also significant. These lepton flavor violation with prescribed branching ratio could serve as another test for the model.

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