Towards the minimal seesaw model for the prediction of neutrino CP violation

Kenta Takagi (Hiroshima Univ.)

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Y. Shimizu (Hiroshima Univ.), KT, M. Tanimoto (Niigata Univ.)
- Introduction
  --- Background and motivation

- Model
  --- Setup for minimal seesaw model

- Prediction of Dirac CP violating phase

- Baryon asymmetry in the Universe (BAU) and CP violation

- Summary and discussions
Background and motivation

CP violating interaction is necessary for the Baryon Asymmetry in the Universe (BAU).

Sakharov’s three conditions

- Baryon number violation
- C and CP violation
- Interact out of thermal equilibrium era

Kobayashi-Maskawa model:
Mixing among three flavors can violate CP symmetry (quark sector)

\[
V_{CKM} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\
-s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\
s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23}
\end{pmatrix}
\]

The origin of CP violation closely relates to the flavor structure
CP violating phase in the lepton sector $\delta_{CP}$:

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}c_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 \\ e^{i\alpha} \\ e^{i\beta} \end{pmatrix}.$$ 

CP conservation ($\delta_{CP} = 0, \pm \pi$) is excluded in $2\sigma$ C.L.

$$\delta_{CP} = -\frac{\pi}{2}$$ may be favored?

Is there something symmetric structure?

$2\sigma$ C.L.:  
Normal Hierarchy (NH) $[-171^\circ, -34.4^\circ]$  
Inverted Hierarchy (IH) $[-88.2^\circ, -68.2^\circ]$
How to predict CP violating phase

\[ U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}c_{23} - s_{12}s_{13}s_{23}e^{i\delta_{CP}} & c_{13}s_{23} \\ s_{12}s_{23} - c_{12}s_{13}c_{23}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}s_{13}c_{23}e^{i\delta_{CP}} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} 1 \\ e^{i\alpha} \\ e^{i\beta} \end{pmatrix} \]

PMNS mixing matrix is derived from neutrino mass matrix. 9 parameters contribute to the PMNS mixing matrix at most

Global experimental data of neutrino oscillation

NuFIT 3.2 (2018), JHEP 01 (2018) 087

| 3\sigma interval | Normal Hierarchy | Inverted Hierarchy |
|------------------|------------------|--------------------|
| \( \Delta m^2_{12} \) | [6.80, 8.02] \( \times 10^{-5} \) [eV\(^2\)] | [6.80, 8.02] \( \times 10^{-5} \) [eV\(^2\)] |
| \( \Delta m^2_{13} \) | [2.399, 2.593] \( \times 10^{-3} \) [eV\(^2\)] | \(- [2.369, 2.562] \times 10^{-3} \) [eV\(^2\)] |
| \( \sin^2 \theta_{12} \) | [0.272, 0.346] | [0.272, 0.346] |
| \( \sin^2 \theta_{23} \) | [0.418, 0.613] | [0.435, 0.616] |
| \( \sin^2 \theta_{13} \) | [1.981, 2.436] \( \times 10^{-2} \) | [2.006, 2.452] \( \times 10^{-2} \) |

5 parameters are available
Approaches to $\delta_{CP}$ -- reduce model parameters --

(A). 2 right-handed (RH) Majorana neutrinos
   -- The lightest neutrino becomes massless.

(B). Flavor symmetry ($A_4, S_4, A_5$, etc.)
   -- control Yukawa couplings in the Lagrangian.
   -- introduce gauge singlet scalars (called as “flavons”).

(C). Texture zeros
   -- put zeros in some elements of the neutrino mass matrix.
   -- can not construct the Lagrangian.

Our model is a combination of the three methods
(--). First setting (without loss of generality)

-- Diagonal basis of charged lepton mass matrix

\[
M_l = \begin{pmatrix}
    m_e & m_\mu & m_\tau
\end{pmatrix}
\]

\[
U_{PMNS} = U_l^\dagger U_\nu = U_\nu
\]

(A). 2 right-handed ( RH ) Majorana neutrinos

\[
M_R = \begin{pmatrix}
    M_1 & 0 \\
    0 & M_2
\end{pmatrix} = M_2 \begin{pmatrix}
    p^{-1} & 0 \\
    0 & 1
\end{pmatrix} \quad p = \frac{M_2}{M_1}
\]

We can take diagonal basis of \( M_R \) in the seesaw mechanism

\[
M_\nu = -M_D M_R M_D^T
\]
(B). Flavor symmetry ($A_4$ or $S_4$ are implied)
--Assume tri-maximal mixing

$$U_{PMNS}^{TM_1} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & e^{i\sigma}\sin\theta \\ 0 & -e^{-i\sigma}\sin\theta & \cos\theta \end{pmatrix}$$

We focus on $TM_1$ here

$$U_{PMNS}^{TM_2} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \cos\theta & 0 & e^{i\sigma}\sin\theta \\ 0 & 1 & 0 \\ -e^{-i\sigma}\sin\theta & 0 & \cos\theta \end{pmatrix}$$

$TM_2$ will be discussed in numerically...

tri-bimaximal (TBM) mixing

$$V_{TBM} = \begin{pmatrix} 2 & 1 & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

Harrison, Perkins, Scott
Phys. Lett. B 458, (1999) 79

named by W. Rodejohann et al.
**TM*\textsubscript{1}** realization

We obtain the following Dirac mass matrix:

\[
M_D = v \begin{pmatrix}
  b + c & e + f \\
  2 & 2 \\
  b & e \\
  c & f
\end{pmatrix}
\]

This leads to TM*\textsubscript{1}** mixing with NH.

\[v \sim 174.1 \text{ GeV} : \text{Higgs doublet vacuum expectation value}\]

**Assume** the relative phase between \(b\) and \(c\) to be 0 or \(\pi\). \(\rightarrow b/c\) is real.

(C). **Texture zeros** -- finalize the model minimization --

- impose a 0 in the Dirac mass matrix.

\[
M_D = v \begin{pmatrix}
  0 & e + f \\
  2 & 2 \\
  b & e \\
  -b & f
\end{pmatrix}
\]

**Case I**

\[
M_D = v \begin{pmatrix}
  b & e + f \\
  2 & 2 \\
  b & e \\
  0 & f
\end{pmatrix}
\]

**Case II**

\[
M_D = v \begin{pmatrix}
  c & e + f \\
  2 & 2 \\
  0 & e \\
  c & f
\end{pmatrix}
\]

**Case III**

*Excluded from 3\(\sigma\) interval (off the edge but near)*
Symmetry realization by $S_4$

Dirac mass term:  \[
\mathcal{L}_D = \frac{y_1}{\Lambda} \phi_1 L H_u \nu_{R1}^c + \frac{y_2}{\Lambda} \phi_2 L H_u \nu_{R2}^c
\]

**TM$_1$ with NH**

\[
M_D = v \begin{pmatrix}
\frac{b+c}{2} & \frac{e+f}{2} \\
\frac{2}{b} & \frac{2}{e} \\
\frac{2}{c} & \frac{2}{f}
\end{pmatrix}
\]

\[
\langle \phi_1 \rangle \sim \begin{pmatrix}
\frac{b+c}{2} \\
\frac{2}{c} \\
\frac{2}{b}
\end{pmatrix}
\]  \[\langle \phi_2 \rangle \sim \begin{pmatrix}
\frac{e+f}{2} \\
\frac{2}{f} \\
\frac{2}{e}
\end{pmatrix}
\]

$SU^+ \langle \phi_1 \rangle = \langle \phi_1 \rangle$

$SU^+ \langle \phi_2 \rangle = \langle \phi_2 \rangle$

-- residual $Z_2$ symmetry from $S_4$

generators of $S_4 : S, T, U^\pm$

\[
S = \frac{1}{3} \begin{pmatrix}
-1 & 2 & 2 \\
2 & -1 & 2 \\
2 & 2 & -1
\end{pmatrix}, \quad T = \begin{pmatrix}
1 & \omega^2 \\
\omega & \omega
\end{pmatrix}, \quad U^\pm = \mp \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & 1 \\
0 & 1 & 0
\end{pmatrix}
\]

$-$ : for 3  
$+$ : for 3'$
Profile of case I

The neutrino mass matrix (seesaw mechanism) \( M_\nu = -M_D M_R M_D^T \)

in the TBM basis:

\[
M_\nu^{TBM} \equiv V_{TBM}^T M_\nu V_{TBM} = -\frac{f^2 v^2}{M_2} \begin{pmatrix}
0 & 0 & 0 \\
0 & \frac{3}{4} (k + 1)^2 & -\frac{\sqrt{3}}{2\sqrt{2}} (k^2 - 1) \\
0 & -\frac{\sqrt{3}}{2\sqrt{2}} (k^2 - 1) & \left\{ 2|B|^2 p e^{2i\phi_B} + \frac{1}{2} (k - 1)^2 \right\}
\end{pmatrix}
\]

\( k \equiv e/f \)

\( |B| e^{i\phi_B} \equiv b/f \)

\( k \) can be make real by freedom of the phase redefinition.

- 3 model parameters in the mixing matrix : \( \{k, \ |B|' (\equiv |B|\sqrt{p}), \ \phi_B \} \)

- Jarlskog invariant :

\[
J_{CP} = -\frac{3}{8} \frac{f^{12}}{M_0^6} (|B|\sqrt{p})^6 (k + 1)^4 (k^2 - 1) \sin 2\phi_B \frac{v^{12}}{\Delta m^2_{13} - \Delta m^2_{12}} \frac{\Delta m^2_{13} \Delta m^2_{12}}{\Delta m^2_{13} \Delta m^2_{12}} \propto \sin \delta_{CP}
\]

This factor determines the sign of \( \sin \delta_{CP} \).
Profile of TM$_1$ with IH

\[ M_{\nu}^{TBM} = -\frac{\nu^2}{M_2} \begin{pmatrix} 6b^2 p & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \frac{f^2\nu^2}{M_2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{3}{4}(ke^{i\phi_k} + 1)^2 & -\frac{\sqrt{3}}{2\sqrt{2}}(k^2e^{2i\phi_k} - 1) & 0 \\ 0 & -\frac{\sqrt{3}}{2\sqrt{2}}(k^2e^{2i\phi_k} - 1) & \frac{1}{2}(ke^{i\phi_k} - 1)^2 & 0 \end{pmatrix} \]

Profile of TM$_2$ with NH or IH

\[ M_{\nu}^{TBM} = -\frac{\nu^2}{M_2} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 3b^2 p & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} - \frac{f^2\nu^2}{M_2} \begin{pmatrix} \frac{3}{2}(ke^{i\phi_k} + 1)^2 & 0 & \sqrt{3} \frac{1}{2}(k^2e^{2i\phi_k} - 1) \\ 0 & 0 & 0 & 0 \\ \sqrt{3} \frac{1}{2}(k^2e^{2i\phi_k} - 1) & 0 & \frac{1}{2}(ke^{i\phi_k} - 1)^2 \end{pmatrix} \]

- 2 model parameters in the mixing matrix: \{k, \phi_k\}
## Numerical Results

| 3σ interval | Normal Hierarchy | Inverted Hierarchy |
|-------------|------------------|-------------------|
| $\Delta m_{12}^2$ | $[6.80, 8.02] \times 10^{-5}[eV^2]$ | $[6.80, 8.02] \times 10^{-5}[eV^2]$ |
| $\Delta m_{13}^2$ | $[2.399, 2.593] \times 10^{-3}[eV^2]$ | $-[2.369, 2.562] \times 10^{-3}[eV^2]$ |
| $\sin^2 \theta_{12}$ | $[0.272, 0.346]$ | $[0.272, 0.346]$ |
| $\sin^2 \theta_{23}$ | $[0.418, 0.613]$ | $[0.435, 0.616]$ |
| $\sin^2 \theta_{13}$ | $[1.981, 2.436] \times 10^{-2}$ | $[2.006, 2.452] \times 10^{-2}$ |

We show these minimal models

**Case I**

$$M_D = v \begin{pmatrix} 0 & e + f \\ b & 2 \\ -b & f \end{pmatrix}$$

**TM$_1$ with IH**

$$M_D = v \begin{pmatrix} -2b & e + f \\ b & 2 \\ b & e \end{pmatrix}$$

**TM$_2$ (common in NH and IH)**

$$M_D = v \begin{pmatrix} b & e + f \\ b & 2 \\ b & f \end{pmatrix}$$

NuFIT 3.2 (2018), JHEP 01 (2018) 087
Predictions of Dirac CP violating phase

Numerical Results (case I)

\[ \delta_{CP} : \pm [71.4^\circ, 97.9^\circ] \ (1\sigma) \]
\[ : \pm [57.5^\circ, 112^\circ] \ (3\sigma) \]
\[ : \pm [77.8^\circ, 101^\circ] \ (k = -3) \]

Blue : 3\sigma plot
Green : 1\sigma plot
Magenta : 3\sigma plot with \( k = -3 \)
Red (horizontal) : 2\sigma interval by T2K \([-171^\circ, -34.4^\circ]\)
Red (vertical) : 3\sigma interval by NuFIT

\[ k = e/f \]
Numerical Results (TM$_1$ with IH)

\[ \delta_{CP}: \pm [69.9^\circ, 84.7^\circ] \ (1\sigma) \]
\[ : \pm [56.8^\circ, 107^\circ] \ (3\sigma) \]

Blue : 3\sigma plot
Green : 1\sigma plot
Magenta : 3\sigma plot with \( k = -3 \)
Red (horizontal) : 2\sigma interval by T2K \([-88.2^\circ, -68.2^\circ]\)
Red (vertical) : 3\sigma interval by NuFIT
Numerical Results (TM$_2$)

NH

\[ \delta_{CP}: \pm [36.2^\circ, 180^\circ] \] (3σ)

IH

\[ \delta_{CP}: \pm [51.0^\circ, 180^\circ] \] (3σ)

\[ k = e/f \]
Predictions of Dirac CP violating phase

back to case I

The predicted $\delta_{CP}$ is sensitive to $k$.

But the sign of $\delta_{CP}$ is not determined...

This result (case I) indicates

$$\text{Sign}[J_{CP}] = \text{Sign}[\delta_{CP}]$$

*Recall

$$J_{CP} = -\frac{3f^{12}}{8M_0^6}(|B|\sqrt{p})^6(k + 1)^4(k^2 - 1)\sin 2\phi_B\frac{\nu^{12}}{\left(\Delta m_{13}^2 - \Delta m_{12}^2\right)\Delta m_{13}^2\Delta m_{12}^2}$$
Leptogenesis in our models

B—L asymmetry in the comoving volume \((M_1 \ll M_2)\)

\[
Y_{B-L} \equiv \frac{n_{B-L}}{s} = -\varepsilon_1 \kappa Y_{N1}^{eq}(T \gg M_1)
\]

is relevant to CP asymmetry of the lighter RH neutrino \(N_1\) decay.

\[
\varepsilon_1 \sim -\frac{3}{16\pi} \frac{\text{Im} \left[ (Y_D^\dagger Y_D)_{21}^2 \right]}{\left( Y_D^\dagger Y_D \right)_{11}} \frac{1}{p} = \frac{M_2}{M_1}
\]

The heavier RH neutrino decay is relevant at \(M_1 \geq 10^{14}\) [GeV]

Here, we assume \(M_1 \ll 10^{14}\) [GeV] for simplicity.
Baryon asymmetry in the Universe and CP violation

CP asymmetry in 1 loop decay of $N_1$

\[ \epsilon_1 = -\frac{3}{16\pi} \frac{\text{Im} \left[ (Y_D^\dagger Y_D)_{21} \right]}{(Y_D^\dagger Y_D)_{11}} \frac{1}{p} \]

**Case I**

\[ Y_D^\dagger Y_D = \begin{pmatrix} 2|b|^2 & b^* (e - f) \\ b(e - f)^* & \frac{|e + f|^2}{4} + |e|^2 + |f|^2 \end{pmatrix} \]

\[ \epsilon_1 = -\frac{3}{16\pi} \frac{1}{2} |f|^2 (k - 1)^2 \sin 2\phi_B \frac{1}{p} \]

**TM$_1$ with IH**

\[ Y_D^\dagger Y_D = \begin{pmatrix} 6|b|^2 & 0 \\ 0 & \frac{|e + f|^2}{4} + |e|^2 + |f|^2 \end{pmatrix} \]

\[ \epsilon_1 = 0 \]

**No leptogenesis**

**TM$_2$**

\[ Y_D^\dagger Y_D = \begin{pmatrix} 3|b|^2 & 0 \\ 0 & |e + f|^2 + |e|^2 + |f|^2 \end{pmatrix} \]

\[ \epsilon_1 = 0 \]

**No leptogenesis**
Numerical results (3σ)

Demanded Baryon asymmetry for the nucleosynthesis:

\[ \eta_B \equiv \frac{n_B}{n_\gamma} = 7.04Y_B = [5.8, 6.6] \times 10^{-10} \text{ (95% C.L.)} \]

[PDG] Chin. Phys. C 40 (2016) 10, 10001

The sign of $\delta_{CP}$ is split by $k$

\[ M_2 = 10^{14} \text{ [GeV]} \]
Sign of $\delta_{CP}$

$B$ and $B - L$ asymmetry are related (sphaleron transition at $T > T_{EW} \sim 100$[GeV]):

$$Y_B \equiv \frac{n_B}{s} = \frac{8N_{flavor} + 4N_{Higgs}}{22N_{flavor} + 13N_{Higgs}} \frac{Y_{B-L}}{Y_{B-L}} = \frac{28}{79} Y_{B-L}$$

(Suppose 3 flavors and 1 Higgs)

S.Yu. Khlebnikov, M.E. Shaposhnikov, Nucl. Phys. B 308, 885 (1998)

Demanded Baryon asymmetry for the nucleosynthesis:

$$\eta_B \equiv \frac{n_B}{n_Y} = 7.04 Y_B = [5.8, 6.6] \times 10^{-10} \ (95\% \ C.L.)$$

[PDG] Chin. Phys. C 40 (2016) 10, 10001

$$-\epsilon_1 \kappa \ Y_{N1}^{eq} = \frac{1}{7.04} \frac{79}{28} \eta_B > 0$$
For simple analysis

\[ \frac{1}{\kappa} \sim \frac{3.3 \times 10^{-3} \text{[eV]}}{\tilde{m}_{1}} + \left( \frac{\tilde{m}_{1}}{0.55 \times 10^{-3} \text{[eV]}} \right)^{1.16} > 0 \quad \text{valid for } M_{1} \ll 10^{14} \text{[GeV]} \]

where \( \tilde{m}_{1} \equiv \frac{v^{2}(Y_{D}Y_{D}^{+})_{11}}{M_{1}} = 2|f|^{2}|B|^{2} \frac{v^{2}}{M_{2}} \) for case I

fitted by G.F. Giudice, A. Notari, M. Raidal, A. Riotto and A. Strumia, [Nucl. Phys. B 685 (2004) 89]

The number density of \( N_{1} \) in thermal equilibrium

\[ Y_{N1}^{eq} = \frac{135\zeta(3)}{4\pi^{4}g_{*}} > 0 \quad g_{*} = 106.75 \text{ (SM)} \]

Nucl. Phys. B 685 (2004) 89

\[ -\epsilon_{1} = \frac{3}{16\pi} \frac{1}{2} |f|^{2}(k - 1)^{2} \sin 2\phi_{B} \frac{1}{p} > 0 \]
Mass hierarchy of RH neutrinos

\[ p = \frac{M_2}{M_1} \]

Consistent with our assumptions:

\[ M_1 \ll M_2 \]

\[ M_1 \ll 10^{14} \text{[GeV]} \]
Summary and discussions

Minimal seesaw model with

2 RH neutrinos & tri-maximal mixing $T_M^1$ and $T_M^2$

$$M_D = v \begin{pmatrix} 0 & e + f \\ b & 2 \\ -b & e \\ f & f \end{pmatrix}$$

--- Normal hierarchy
--- distinct the sign of $\delta_{CP}$
--- include $(\theta_{23}, \delta_{CP}) = (\frac{\pi}{4}, -\frac{\pi}{2})$

explain neutrino oscillation & BAU

Symmetry realization by $S_4$

Dirac mass term:

$$\frac{\gamma_1}{\Lambda} \phi_1 L H_u \nu_{R1}^c + \frac{\gamma_2}{\Lambda} \phi_2 L H_u \nu_{R2}^c$$

$$SU \begin{pmatrix} e + f \\ 2 \\ e \\ f \end{pmatrix} = \begin{pmatrix} e + f \\ 2 \\ e \\ f \end{pmatrix}$$

-- residual $Z_2$ symmetry from $S_4$

$s, t, u :$ generators of $S_4$

Is it consist with quark sector?

$\delta_{CKM}^{CP} \sim +70^\circ$ can be realized with our model?
THANK YOU
Global experimental data of neutrino oscillation

|                   | Normal Ordering (best fit)           | Inverted Ordering ($\Delta \chi^2 = 4.14$) |
|-------------------|-------------------------------------|------------------------------------------|
|                   | bfp ±1σ                             | bfp ±1σ                                  |
|                   | 3σ range                            | 3σ range                                 |
| $\sin^2 \theta_{12}$ | $0.307^{+0.013}_{-0.012}$          | $0.307^{+0.013}_{-0.012}$                |
|                   | $0.272 \rightarrow 0.346$          | $0.272 \rightarrow 0.346$                |
| $\theta_{12}/^\circ$ | $33.62^{+0.78}_{-0.76}$          | $33.62^{+0.78}_{-0.76}$                |
|                   | $31.42 \rightarrow 36.05$          | $31.43 \rightarrow 36.06$                |
| $\sin^2 \theta_{23}$ | $0.538^{+0.033}_{-0.069}$          | $0.554^{+0.023}_{-0.033}$                |
|                   | $0.418 \rightarrow 0.613$          | $0.435 \rightarrow 0.616$                |
| $\theta_{23}/^\circ$ | $47.2^{+1.9}_{-3.9}$               | $48.1^{+1.4}_{-1.9}$                    |
|                   | $40.3 \rightarrow 51.5$            | $41.3 \rightarrow 51.7$                 |
| $\sin^2 \theta_{13}$ | $0.02206^{+0.00075}_{-0.00075}$   | $0.02227^{+0.00074}_{-0.00074}$         |
|                   | $0.01981 \rightarrow 0.02436$      | $0.02006 \rightarrow 0.02452$           |
| $\theta_{13}/^\circ$ | $8.54^{+0.15}_{-0.15}$             | $8.58^{+0.14}_{-0.14}$                  |
|                   | $8.09 \rightarrow 8.98$            | $8.14 \rightarrow 9.01$                 |
| $\delta_{CP}/^\circ$ | $234^{+43}_{-31}$                  | $278^{+26}_{-29}$                       |
|                   | $144 \rightarrow 374$              | $192 \rightarrow 354$                   |
| $\frac{\Delta m^2_{21}}{10^{-5} \text{ eV}^2}$ | $7.40^{+0.21}_{-0.20}$            | $7.40^{+0.21}_{-0.20}$                  |
|                   | $6.80 \rightarrow 8.02$            | $6.80 \rightarrow 8.02$                 |
| $\frac{\Delta m^2_{3\ell}}{10^{-3} \text{ eV}^2}$ | $+2.494^{+0.033}_{-0.031}$         | $-2.465^{+0.032}_{-0.031}$              |
|                   | $+2.399 \rightarrow +2.593$        | $-2.562 \rightarrow -2.369$             |

NuFIT 3.2 (2018), JHEP 01 (2018) 087