A iteration-free multi-channel mismatch estimation method for parallel time-interleaved system

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Abstract. A time-interleaved (TI) architecture with slower sub-ADCs in parallel is a promising approach to avoid the conflict between sampling rate and quantization precision of single chip ADC. A major bottleneck in realizing a TI ADC is mismatch among the sub-ADCs, which may greatly degrade the receiver performance. An iteration-free method is proposed to simultaneously estimate time errors and gain errors. In our research, the frequency response vectors corresponding to a pair of frequencies are studied in terms of their rotational relationships. The method is based on the fact that the rotational matrix only depends on the time errors and the frequency spacing between the paired frequencies but is independent of gain errors. Upon combining the projection matrices corresponding to the paired frequencies, we directly obtain the time errors in terms of extracting the rotational matrix in a least squares framework. Moreover, the presented method is robust to residual offset errors and system noise. The effectiveness of the proposed approach is verified by simulated data.

1. Introduction
A time-interleaved analog-to-digital converter (TIADC) has a parallel structure where a number of sub-converters cyclically sample the input signal, and outputs are similarly taken to form a digital stream. TIADC can effectively avoid the conflict between sampling rate and quantization precision of single chip ADC. However, the spectral performance of a TIADC is limited by mismatches in electrical characteristics between sub-converters. In practice, gain, sampling time and dc offset mismatches are usually dominant.[1]

It is therefore important to remove the mismatch errors. However, calibration of an ADC system is time consuming and costly. Furthermore, the mismatch errors may change slowly with, for instance, temperature and aging. Therefore, we want to estimate the mismatch errors with less computational load. Methods for estimation of timing errors have been published in [2] and [3]. These methods require a known calibration signal, which means that the operation of the ADC must be stopped during calibration. A blind time error estimation method was presented in [4] and validated on measurements in [5]. This method works well, but gives a bias error.

In [6], we have indicated that time errors or gain errors cannot be extracted individually without the impact of each other no matter in time or frequency domain. Iteration seems to be the only way to solve the above coupling problem. In this paper, an iteration-free approach to simultaneously estimate gain and time errors is proposed. This method extracts the time errors form the rotational matrix of the steering vectors associated with the time delays of ADCs corresponding to a pair of frequency outputs,
on the basis of that the rotational matrix only depends on the time errors and the frequency spacing between the paired frequencies but is independent of gain error. In the case of rotational matrix estimation, our approach is predicated on first employing the facts that the steering matrix and the signal eigenvectors span the same subspace which is represented by its unique projection matrix. The proposed method performs stably as well as has less computational load, compared with the conventional methods.

2. Problem statement
TIADCs operate in a round-robin manner. In a system with \( M \) converters, to realize a system sampling period of \( \tau = 1/Mf_s \), each converter operates with sampling period \( T_s = 1/f_s \) and a spacing of \( \tau \) between consecutive converters, where \( f_s \) is the sampling rate of each ADC. The output of the \( m \)-th \((m = 0, 1, \ldots, M - 1)\) chip can be written as

\[
\hat{s}_m(n) = g_n \cdot s(nT_s - mt\tau_n - \Delta\tau_n) + a_n + e_n
\]

(1)

where \( s(t) \) is the analog signal to be sampled, \( n \) the sampling index, \( \Delta\tau_n \) the time error, \( a_n \) the dc offset error and \( e_n \) the additive noise. Due to the fact that the dc offset errors may be treated as the additive noise in the system [2], it can be eliminated by the statistical averaging method [3], and the residual offset errors can also be grouped into additive noise item. Performing FFT to each output data, we can obtain

\[
\hat{S}_m(f) = g_n \cdot \sum_{i=0}^{M} S_i(f) e^{-j2\pi f(iM+mt\tau_n)} + N_n(f)
\]

(2)

Obviously, the frequency \( f \) is confined within \([-f_s/2, f_s/2]\). Since TIADC systems utilize lower rate ADCs to sample wideband signal, spectra aliasing will be introduced. In (2), \( 2f \) is number of ambiguities folded at each frequency, \( S(f) = S(f + if) \), where, \( S(f) \) is the Fourier transform of \( s(t) \), and \( N_n(f) \) the Fourier transform of the grouped noise item. In addition, the linear phase exponential item in the equation is introduced by the sampling delay of each ADC. It can be noted that the linear phase mainly depends on frequency except time delay, therefore each ambiguity corresponding to different linear phase item. Employing vector notation for the outputs of the \( M \) ADCs, the data model becomes

\[
\hat{S}(f) = \Gamma \cdot P(f) \cdot S(f) + N(f)
\]

where

\[
\hat{S}(f) = [\hat{S}_0(f), \hat{S}_1(f), \ldots \hat{S}_{M-1}(f)]^T
\]

(4)

\[
P(f) = [p_0(f), p_1(f), \ldots, p_M(f)]
\]

(5)

\[
S(f) = [S_0(f), S_1(f), \ldots S_M(f)]^T
\]

(6)

\[
p(f) = [1, e^{-j2\pi f/M}, \ldots e^{-j2\pi f(M-1)/M}, \ldots, e^{-j2\pi f(M-1)/M-1}]
\]

(7)

\[
\Gamma = \text{diag}(g_0, \ldots g_M)
\]

(8)

\(^T\) denotes the vector transpose operation. According to the derivations above, it can be found that the \( i \)-th ambiguities associated to a unique linear phase vector \( p(f) \). \( \Gamma \) is termed here the sampling array response for the \( i \)th ambiguity in the presence of mismatch errors. Define \( \Delta\tau = [\Delta\tau_0, \ldots, \Delta\tau_{M-1}]^T \) as the time error vector. The objective of this paper is to estimate \( \Delta\tau \) and \( \Gamma \), respectively. In the sampling array response, \( \Delta\tau \) and \( \Gamma \) have different properties and affect the response much in different ways. The treatment of extracting \( \Delta\tau \) and \( \Gamma \) from the output vector as a conventional subspace-based estimation problem raises the issue of how to estimate one of them without the impact of the other.

3. Proposed method

3.1. Derivation
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Consider the output from frequency $f + \Delta f$, along with that from the given frequency $f$, to develop a method for estimating mismatch errors free of iteration. Here, $\Delta f$ is the frequency spacing between these two frequencies.

To simplify the following expressions, we denote the quantities corresponding to the paired frequencies $f$ and $f + \Delta f$ by ($\psi^a$ and $\psi^b$, respectively. Furthermore, we use notations $\mathbf{p}, \mathbf{P}, \hat{\mathbf{S}}, \mathbf{S}$ and $\mathbf{S}^H$ to substitute $\mathbf{p}(f), \mathbf{P}(f), \hat{\mathbf{S}}(f)$ and $\mathbf{S}(f)$, respectively.

Assuming the additive noise at each ADC is independent and taking expectation with respect to $n$, the covariance matrices of output vector from frequencies $f$ and $f + \Delta f$ can be formulated as

$$
\mathbf{R}^{f} = \mathbf{R}(f) = \mathbf{R}^{\psi^a} = \mathbf{R}^{\psi^b} = \mathbf{R}^{\psi^c} + \sigma^2 \mathbf{I}
$$

where $\sigma^2$ and $\sigma^a$ are the array element noise variances associated with $f$ and $f + \Delta f$, respectively, and $\mathbf{I}$ is the identity matrix. $\mathbf{R}^{\psi^b} = \mathcal{E}[\mathbf{S}(f)\mathbf{S}^H(f)]$ and $\mathbf{R}^{\psi^a} = \mathcal{E}[\mathbf{S}(f + \Delta f)\mathbf{S}^H(f + \Delta f)]$.

Upon inverse factorization to the phase term of $\mathbf{p}^b$, it can be rewritten as

$$
\mathbf{p}^b = \mathbf{C}[1, e^{j2\pi f(1+2\pi M+\sigma_{\psi})(m-1)+\sigma_{\psi}}] \mathbf{p}^a
$$

where

$$
\mathbf{C} = \text{diag}[1, e^{j\psi_1}, \ldots, e^{j\psi_M}]
$$

where the matrix $\mathbf{C}$ is a diagonal $M \times M$ matrix of the phase difference between the response vectors corresponding to the paired frequencies. Combining (5) and (11), we can obtain $\mathbf{P}^b = \mathbf{C} \mathbf{P}^a$. Substituting it into (10), $\mathbf{R}^b$ becomes

$$
\mathbf{R}^{\psi^b} = \mathbf{R}^{\psi^a} = \mathbf{R}^{\psi^c} + \sigma^2 \mathbf{I}
$$

Next, we are to find two sets of $2I + 1$ linearly independent vectors those are contained in $\mathfrak{g}(\mathbf{P}^a)$ and $\mathfrak{g}(\mathbf{P}^b)$, the subspaces spanned by the columns of $\mathbf{P}^a$ and $\mathbf{P}^b$, respectively. In [6], two such sets of vectors are given by $\mathbf{U}^a = [\mathbf{u}^a_1, \ldots, \mathbf{u}^a_{2I+1}]$ and $\mathbf{U}^b = [\mathbf{u}^b_1, \ldots, \mathbf{u}^b_{2I+1}]$, where $\{\mathbf{u}^a_i\}$ and $\{\mathbf{u}^b_i\}$ ($i = 1, \ldots, 2I + 1$) are the sets of eigenvectors of $\mathbf{R}^a$ and $\mathbf{R}^b$ corresponding to the $2I + 1$ largest eigenvalues, respectively. Also, in [6], the space spanned by the practical response vectors is referred to as signal subspace. Followed that, the projection matrix of the signal subspace of $\mathbf{R}^a$ is

$$
\mathbf{Q}^a = \mathbf{U}^a(\mathbf{U}^a)^H
$$

Furthermore, the practical response matrix, as has been pointed out, spans the same subspace as those $2I + 1$ eigenvectors does. Based on the fact that a subspace is represented by its unique projection matrix $\mathbf{Q}^a$ can thus be alternatively calculated by $\mathbf{R}^a$.

$$
\mathbf{Q}^a = \mathbf{R}^{\psi^a} = \mathbf{R}^{\psi^c} + \sigma^2 \mathbf{I}
$$

In the same way, the projection matrix of the signal subspace of $\mathbf{R}^b$ can be given by

$$
\mathbf{Q}^b = \mathbf{R}^{\psi^b} = \mathbf{R}^{\psi^a} + \sigma^2 \mathbf{I}
$$

Combining (15) and (16), the relationship between the two projection matrices corresponding to the paired frequencies can be expressed as $\mathbf{Q}^b = \mathbf{C} \mathbf{Q}^a \mathbf{C}^H$. Moreover, by (14) and (15), we are able to obtain the following expression

$$
\mathbf{U}^a(\mathbf{U}^b)^H = \mathbf{C} \mathbf{U}^a(\mathbf{U}^a)^H \mathbf{C}^H
$$
As has been pointed out in [5], it is a rotation that differentiates one subspace from another of the same rank. Here, $Q_p$ and $Q_q$ are the projection matrices representing two distinct signal subspaces. Just as Eq. (17) illustrate, $Q_p$ is the rotated version of $Q_q$ , provided $C$ is the rotation matrix.

According to (12), $C$ only depends on $\Delta \tau$ when $\Delta \tau'$ is given, but is independent of $\Gamma$ . It follows that $\Delta \tau$ can be directly estimated with known $C$ . Taking this idea further, the problem of determining $\Delta \tau$ is shown to be equivalent to the problem of estimating $C$ . Our approach to this problem is predicated on first learning the two projection matrices and then extracting from them the rotation matrix $C$ .

By Eq. (17), the $k_l$ th element of projection matrix $Q_p$ can be expressed as

$$[Q_p]_{kl} = [Q_p]_{kl} e^{i \omega \tau_{kl}} e^{-\psi_{kl}}$$

(18)

Since $Q_p$ and $Q_q$ can be directly estimated by eigen-decomposition of covariance matrices $R^p$ and $R^q$, respectively, as is shown by (14) and (16), it is straight forward to obtain

$$\text{angle}(Q_p^m) = \text{angle}(Q_q^m) + \psi_m - \psi_l$$

where $\text{angle}(\cdot)$ denotes taking phase operation. Now defining

$$\nu_k = \text{angle}(Q_p^m) - \text{angle}(Q_q^m)$$

(19)

(20)

Since $\nu_k$ is zero, (34) is meaningful only when $k, l$ lie on the super diagonals. Taking all such (nonredundant) relations when $k, l$ lie on the super diagonals, we get a total of $p_v = \sum_{k=1}^{M-1} k$ equations of the type (20). We can write these compactly as

$$B [\psi_1, \psi_2, \ldots, \psi_d]^T = [\nu_0, \nu_1, \ldots]^T$$

(21)

where, $B$ is a $p_v \times M$ matrix whose rows have the following form: $[\ldots, 0, 1, 0, \ldots, 0, -1, 0, \ldots]$. All entries in this row are zero except for a $1$ and $-1$ at the $k$ th and $l$ th positions, respectively.

Examining $B$ shows that the $1 \times M$ row vector $[1, 1, \ldots, 1]$ is the lone vector spanning its row null space. Then solving (35) with a least squares approach using the singular value decomposition of $B$ yields

$$[\psi_1, \psi_2, \ldots, \psi_d]^T = B^+ [\nu_0, \nu_1, \ldots]^T$$

(22)

where $B^+$ is the pseudo inverse defined in terms of the $M-1$ singular values and the corresponding left and right singular vectors of $B$ . Note that (22) yields the least-squares solution with minimum-norm. A general solution can be obtained by adding an arbitrary scalar times the null space vector. This amounts to saying that we can only determine $[\psi_1, \psi_2, \ldots, \psi_d]^T$ to within an arbitrary multiplicative constant.

Fortunately, as pointed out earlier, the first antenna is taken as the reference. Thus we can obtain the unique solution to $[\psi_1, \psi_2, \ldots, \psi_d]^T$ with the constraint $\psi_1 = 0$.

Now having found $\psi_m (m = 1, 2, \ldots, M)$, we can directly utilizing (12) to determine $\Delta \tau_m$

$$\Delta \tau_m = -\psi_m / 2 \pi m - (m-1) \tau$$

(23)

Then, we can compensate the time errors at each ADC in (3). After that, the self-calibration algorithm [4] can be employed to directly estimate gain error without the impact of time error.

To simplify the following expressions, “^” will denote the estimate of the quantity over which it appears. Then, the proposed method can be summarized as follows.

**Step 1** Select a pair of frequencies, and obtain the estimate of the covariance matrices $R^p$ and $R^q$ corresponding to those paired frequencies, respectively (see [6] for the approach to estimate the statistical covariance matrix)

**Step 2** Decompose $\hat{R}^p$ and $\hat{R}^q$ to obtain $\hat{U}^p$ and $\hat{U}^q$, and calculate $\hat{Q}^p$ and $\hat{Q}^q$, respectively.

**Step 3** Obtain $\hat{\Psi}$ according to (22), and compute time errors $\Delta \hat{\tau}_m$ by (23).

**Step 4** Compensate the time errors at each ADC, and then, employ the self-calibration algorithm to obtain the gain error estimate $\hat{G}$ .

In order to mitigate the effect of noise thus further improving the estimation accuracy, we can average the estimates obtained according to multiple pairs of frequencies.
3.2. Rules of selecting a pair of Doppler bins

In the previous derivations, we combine the outputs from a given frequency with those from another frequency, to get rid of the impact of the gain error on estimating the time errors. Notable confusion arose, however, whether the selection of the second frequency is exactly arbitrary.

For a complex quantity, as is well known to us, the principal value interval of its phase term is between $\pi - \pi$ and $\pi$. According to (12), the estimation of the rotation matrix $\Psi$ may suffer from the $\pi$ ambiguity when the absolute value of $\psi$ exceeds $\pi$. Clearly, we cannot determine $m\tau$ with an aliased $\psi$. This limitation must be carefully considered when employing the proposed method.

Besides the constants, as can be observed by (12), $\psi$ is only determined by $\tau$, $\Delta \tau$ and $\Delta f \cdot (m-1)\tau$ and $\Delta \tau$, after data preprocessing, i.e. correlation operation, can be reasonably assumed to be less than $T$. Expressed in a formula, the $\pi$ ambiguity constraint, provided $|\psi| > \pi$, thus can be written as

$$|\psi| = 2\pi\sqrt{(m-1)\tau + \Delta \tau}$$

As can be observed, only $\Delta f$ determines whether the above constraint can be satisfied. Upon simplifying (24), the $\pi$ ambiguity constraint becomes as follows

$$|\Delta f| < f_{\Delta} / 4$$

This means that we can arbitrarily select a pair of frequencies when employing the proposed method, as long as the frequency interval between them is less than a quarter of sampling rate.

4. Verification of simulated data

In this section, the simulated radar echo signal is utilized to verify the effectiveness of this method presented in this paper. Suppose that the signal transmitted by radar system is chirp signal with bandwidth of 180MH, pulse duration is 2$\mu$s, wavelength is 0.03m. The number of channels is 5 and the sampling rate of each ADC is 60MHz. And the noise is plus Gaussian white noise with SNR 20dB. Since the sampling rate is only 1/3 of echo signal bandwidth, the spectrum of the echo signal is aliased 2 times, which means $l=1$ in this paper. In such simulation context, the controlled position, gain and phase errors, namely, $\Delta \tau$, $\Delta \tau$, and $g_m$ are applied to the generated raw data of each channel. Meanwhile, the average root-mean-square error (ARMSE) of these errors estimates are used to evaluate the performance of the estimator, which is defined as $1/M \sum_{i=1}^{M} \text{RMSE}(|\hat{\Delta} \tau|)$, measured by $r$. In the following simulations, for the sake of convenience, the two methods proposed in [4] and [6], are referred to as method 1 and method 2, respectively.

In the first experiment, time and gain errors are applied to the raw data. For each of M ADCs, $\Delta \tau$ and $g_m$ are uniform random variables in $[-0.2r, 0.2r]$ and $[0.8, 1.2]$, respectively. In addition, the iteration number of method 1 is 3. Empirical results are obtained based on 100 trials. The ARMSEs of the time errors estimates versus SNR are shown in Fig. 1. We can see that the proposed position estimator performs better than method 1 and method 2, no matter SNR is low or high in that those two methods are based on the first-order Taylor series expansion of the time-error exponential function, which causes an approximation error. To further observe the performance of the proposed method in the context of larger errors, we extend the applied time errors to the range of $[0.3, 0.3]$, as is shown in Figure 2. In such situation, method 1 and method 2 may converge to the local minima, while the proposed method performs stably.

5. Conclusions

An iteration-free method for estimation of the mismatch errors for use in a TIADC system by the simultaneous processing of the outputs corresponding to dual-frequencies has been developed here. The mathematics is straightforward and results in a solution involving a rotational matrix of the two signal
subspaces. The new time-error estimator, when used in conjunction with the self-calibration algorithm for estimating gain mismatch, results in an iteration-free method that appears to perform stably well. Simulations verify the validity of the proposed method and reveal that it behaves better than the conventional method even when the SNR is low.

![Fig1. ARMSE versus SNR][0.2,0.2]

![Fig2. ARMSE versus SNR][0.3,0.3]

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