Applying Extensive Air Shower Universality to Ground Detector Data

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Abstract: Air shower universality states that the electromagnetic part of hadron-induced extensive air showers (EAS) can be completely described in terms of the primary energy and shower age. In addition, simulations show that the muon part is well characterized by an overall normalization which depends on the primary particle and hadronic interaction model. We investigate the consequences of EAS universality for ground arrays, which sample EAS at large core distances, and show how universality can be used to experimentally determine the muon content as well as the primary energy of cosmic ray air showers in a model-independent way.

A ground array detector samples the particles in an Extensive Air Shower (EAS) at a limited number of points at different distances $r$ from the shower axis. From this sample, an observable has to be defined to estimate the shower size. To avoid the large fluctuations in the signal integrated over all distances caused by fluctuations in the shower development, Hillas [1] proposed to use the signal at a given distance, $S(r)$, to determine the shower size. The optimal distance $r_{\text{opt}}$ [2] where experimental uncertainties in the signal determination are minimized is mainly determined by the experiment geometry (spacing between ground array detectors). In this paper, we consider the signal in water Cherenkov detectors as employed by the Auger Observatory ($r_{\text{opt}} = 1000$ m). Similar calculations can be done for any ground array detectors.

Using Monte Carlo simulations, $S(r_{\text{opt}})$ is related with the energy of the incoming cosmic ray. This calibration suffers from large systematics due to uncertainties in the hadronic models and the assumptions that have to be made about the primary cosmic ray composition. In this work, we propose a new method to determine the calibration in a model independent way. Furthermore, this method allows us to determine the number of muons produced in air showers.

The method is based on what we will call air shower universality [3]: to a remarkable degree of precision, EAS can be characterized by only three parameters: the primary energy $E_0$, the depth of shower maximum $X_{\text{max}}$, and the overall normalization of the muon component $N_\mu$. The parameters $X_{\text{max}}$ and $N_\mu$ are linked to the mass of the primary particle, ranging from proton to iron, and are subject to significant shower-to-shower fluctuations. All composition and model dependence is distilled in these two parameters with clear physical interpretation. In addition to determining a model-independent energy estimator, they can be compared with simulations to infer the cosmic ray composition and place constraints on hadronic interaction models.

Previous studies have demonstrated that the energy spectra and angular distributions of electromagnetic particles [4, 5], as well as the lateral distribution of energy deposit close to the shower core [6] are all universal, i.e. they are functions of $E_0$, $X_{\text{max}}$, and the atmospheric depth $X$ only (the dependence on $X$ and $X_{\text{max}}$ is commonly put in terms of the shower age $s$).

By exploiting shower universality, we show that it is possible to separate the known shower properties, including the electromagnetic particle flux on ground and the average depth of shower maximum ($X_{\text{max}}$), from the unknown, the surface detector energy scale and the normalization of the muon signal at $r_{\text{opt}}$ which is tightly correlated with the
overall number of muons in a shower, \( \langle X_{\text{max}} \rangle \) as a function of energy has been measured with good precision by fluorescence detectors, and can also be inferred from surface detector variables.

**EAS Universality at large core distances**

In this section we will test shower universality in terms of the shower plane signal, i.e. the signal generated by particles in a fiducial flat detector parallel to the shower plane (orthogonal to the shower axis). By avoiding geometric projection effects, this allows us to compare showers at different zenith angles. We have assumed a cylindrical detector with a top area of 10 m\(^2\) and 1.2 m height (similar to the ones used in the Pierre Auger Observatory). The response of the detector, simulated using Geant 4, is expressed in units of VEM (the signal of a vertical, central muon).

We have generated a library of showers that span a zenith angle range of 0° to 70° and an energy range of \(10^{17}\) eV to \(10^{20}\) eV. Showers of proton and iron primaries were generated using CORSIKA 6.500/6.502 [7] and the hadronic interaction models QGSJetII-03 [8] and Fluka [9]. In addition, we simulated proton/iron showers at \(10^{19}\) eV and different zenith angles using other hadronic interaction models (QGSJetII-03/Gheisha2002 [10] and Sibyll 2.1/Fluka [11, 12]).

The shower-plane signals were separated into signals from electromagnetic particles and muons. We include the signal from the electromagnetic decay products of muons (\(\sim 15\%\) of the muon signal) in the muon component, the remaining signal being the ‘pure’ electromagnetic component \(S_{\text{em}}\).

Fig. 1 shows the electromagnetic signal for a core distance of 1000 m (circles, proton and iron showers) as a function of \(DG = X_{\text{ground}} - X_{\text{max}}\), the distance from the shower maximum to the detector measured along the shower axis (in g/cm\(^2\)). Note that this plot contains showers from all zenith angles. Apparently, the signals from proton and iron are very similar, though there is a slight shift in the overall normalization. This is in violation of shower universality, which states that showers of the same energy at the same evolutionary stage (given by \(DG\)) should have the same electromagnetic component.

Fig. 2 shows the electromagnetic signal for different models and primaries, relative to a reference (proton QGSJetII/Fluka). Note that the different model predictions for a given primary are within 5% of each other. There is, however, a systematic offset of about 13% between proton and iron signals. We also found that the systematic differences in the number density of particles are smaller, about 8%. This effect persists also when comparing signals at the same shower age instead of \(DG\).

Fig. 1 also shows the muon signal \((S_\mu, \text{triangles})\) as a function of \(DG\) for the same proton and iron
shower dependence on the primary mass (~40% between proton and iron) as well as the hadronic model is well known. It should be stressed that the difference is mostly in the normalization, not in the functional dependence on \( DG \). This is shown clearly by the muon signals plotted relative to proton-QGSJetII, Fig. 3.

### Determining the muon normalization and energy scale

The universality of the electromagnetic ground signal as well as of the evolution of the muon signal can be used to parameterize the total ground signal in a model- and primary-independent way. The signal at a fixed core distance is then only a function of primary energy, distance to ground \( DG \), zenith angle \( \theta \), and the overall muon normalization. The slight primary-dependence of the electromagnetic signal enters as a systematic uncertainty in the method. Given the measured average depth of shower maximum \( \langle X_{\text{max}} \rangle \) as a function of energy, either from a fluorescence detector (on site or a separate experiment) or from ground observables, the distance to ground can be directly determined from the zenith angle for each shower: \( DG = X_0 / \cos \theta - \langle X_{\text{max}} \rangle \), where \( X_0 \) stands for the vertical depth of the atmosphere at the experiment site. We parametrize the electromagnetic and muon signal as separate Gaisser-Hillas type functions in \( DG \), leaving a normalization factor free for the muon signal:

\[
S(E, \theta) = S_{\text{em}}(E, \theta, \langle X_{\text{max}} \rangle) + N_{\mu}(E) \cdot S_{\mu}(\theta, \langle X_{\text{max}} \rangle) \tag{1}
\]

Here, \( S_{\mu} \) denotes a reference muon signal, which we take to be proton-QGSJetII at \( 10^{19} \) eV, and \( N_{\mu}(E) \) is the relative muon normalization at this energy. Hence, the energy \( E \) and \( N_{\mu}(E) \) are the remaining unknowns, which however cannot be disentangled for individual events in a ground array.

Fig. 4 (upper panel) shows the zenith angle dependence of the signal (Eq. (1)) for a fixed energy of \( 10^{19} \) eV and different values of \( N_{\mu} \). It is evident that the smaller the \( N_{\mu} \), the steeper the \( \theta \) dependence is. We can now use the fact that, within statistics, the arrival directions of high energy cosmic rays are isotropic. Therefore, we divide the ground detector data set in equal exposure bins in zenith angle (\( \sin^2 \theta \) bins). Given a muon normalization, we calculate the number of events in each bin above a given reference energy.

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**Figure 3**: Muon ground signals (in the shower plane, \( r = 1000 \) m) relative to that of proton-QGSJetII vs. distance to ground for different primaries and hadronic interaction models.

**Figure 4**: Upper panel: the signal parametrization Eq. (1) vs. \( \sec \theta \) for different \( N_{\mu} \) (black/solid − 1.1, red/dashed − 0.5, blue/dotted − 2.0). Lower panel: histograms of number of events above the parametrized signal in equal exposure bins, obtained for the same \( N_{\mu} \) as shown in the upper panel from a Monte Carlo data set (see text).
(here \(E_{\text{ref}}=10^{19}\) eV), using Eq. (1). We then adjust \(N_{\mu}(E_{\text{ref}})\) in the signal parametrization Eq. (1) to that value which gives an equal number of events \(N(>S(E_{\text{ref}}, \theta))\) in each zenith angle bin (lower panel in Fig. 4). For a range of \(N_{\mu}\) values, we calculate the \(\chi^2/\text{dof}\) of the event histogram relative to a flat distribution in \(\sin^2 \theta\). This determines the experimental value of \(N_{\mu}\) and its errors. Once \(N_{\mu}\) is determined, Eq. (1) can be used to set the energy scale of the experiment.

In order to prove the feasibility of this method, we have simulated 1,000 realizations of a ground array data set with \(\sim 2,000\) events above \(10^{19}\) eV, distributed according to the observed cosmic ray spectrum and for different primary compositions (pure proton, iron, or mixed composition). The zenith angle of each shower is sampled from a flat distribution in \(\sin^2 \theta\) distribution, while \(X_{\text{max}}\) is obtained from the distributions predicted by QGSJetII for each primary and energy. \(N_{\mu}\) is fluctuated according to the model predictions. Note that the magnitude of fluctuations in \(X_{\text{max}}\) and \(N_{\mu}\) are only dependent on the primary particle, not the hadronic model.

Eq. (1) is then used to calculate the signal at 1000 m from the shower core, \(S(1000)\), which is also smeared with an experimental reconstruction accuracy (10% for high signals, and increasing rapidly at signals less than 10 VEM). We then applied the method described above to calculate the muon normalization for each simulated data set. We found that \(N_{\mu}\) is systematically slightly overestimated, with the bias mainly depending on composition, and only weakly on the detector resolution. For pure proton composition, the bias at \(10^{19}\) eV was found to be around 14% of the true \(N_{\mu}\) value, while for pure iron, it only amounts to a few percent, due to the much smaller fluctuations of iron showers. This bias can then be subtracted from the determined \(N_{\mu,\text{exp}}\) to obtain an estimate of the true \(N_{\mu}\), the uncertainty in the bias entering as an additional contribution to the systematic error. Note however that a knowledge of \(\langle X_{\text{max}}\rangle\) already places strong constraints on the composition.

Taking into account this knowledge, and assuming the observed universality violation and an error of \(\langle X_{\text{max}}\rangle\) of \(\sim 15\) g/cm², we found that the total systematic uncertainty of \(N_{\mu}\) achievable is less than 10%, roughly the statistical error of \(N_{\mu}\) for this data set.

Conclusions

Assuming that air shower universality holds, the method presented allows for a measurement of the muon content of air showers to better than 10% for existing experiments. With similar precision, it also determines a converter of signal at ground to energy, i.e. a model-independent ground detector energy scale. In addition, the measurement of \(N_{\mu}\) can be performed at any energy accessible to the experiment. The measured evolution of \(N_{\mu}(E)\) is a further observable of relevance to hadronic models and composition. This method has been applied to data from the Pierre Auger Observatory [13] yielding results that constrain hadronic interaction models.

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