Inflation in energy-momentum squared gravity in light of Planck2018

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Abstract We study cosmological dynamics of the energy-momentum squared gravity. By adding the squared of the matter field’s energy-momentum tensor (ζ T^2) to the Einstein–Hilbert action, we obtain the Einstein’s field equations and study the conservation law. We show that, the presence of ζ T^2 term, breaks the conservation of the energy-momentum tensor of the matter fields. However, an effective energy-momentum tensor in this model is conserved in time. By considering the FRW metric as the background, we find the Friedmann equations and by which we explore the cosmological inflation in ζ T^2 model. We perform a numerical analysis on the perturbation parameters and compare the results with Planck2018 different data sets at 68% and 95% CL, to obtain some constraints on the coupling parameter ζ. We show that, for 0 < ζ ≤ 2.1 × 10⁻⁵, the ζ T^2 gravity is an observationally viable model of inflation.

1 Introduction

The standard model of cosmology despite all successes in the early years of development of cosmology, failed to justify some observations of the universe. Considering a single canonical scalar field (inflaton) with a flat potential, leading to the slow-roll of the inflaton and causing the enough exponential expansion of the early universe, is one simplest way to solve some main problems of the standard model of cosmology. In the simple single field inflation model, we get the adiabatic, scale invariant and Gaussian dominant modes of the primordial perturbations [1–8]. However, models with non-Gaussian distributed and not exactly scale invariant perturbations have attracted a lot of attentions [8–21].

One class of the interesting models in describing the early time inflation and primordial perturbations is the one related to the modified gravity. Modified gravity, in its simplest form, is a function of the Ricci scalar (f(R)) [22–28]. A lot of work on the inflation and perturbations issue, have been done in the modified gravity and interesting results have been obtained [29–34]. Another interesting proposal in the modified gravity is to consider an arbitrary coupling between the Ricci scalar and Lagrangian density of the matter fields in the theory [35–46]. In this regard, some authors have been attracted to the models in which there is an arbitrary coupling between a function of the Ricci scalar and the trace of the energy-momentum tensor of the matter part [47–51].

In Ref. [52], the authors have considered an energy-momentum squared gravity model in which they have added ζ T^2 to the Einstein–Hilbert action which T^2 = T_{μν} T^{μν}. In this term, T_{μν} is the energy-momentum tensor of the matter part of the theory and ζ is the coupling constant parameter which its positive values lead to a viable cosmological scenario. They have shown that, the presence of this term leads to a maximum energy density corresponding to a minimum length. In this way there is a bounce in the early universe, avoiding the early time singularity. Besides, to constraint ζ, one can find the scalar spectral index, tensor spectral index and tensor-to-scalar ratio in the model and compare the results with Planck2018 data set. The constraints on the perturbation parameters n_s and r, obtained from Planck2018 TTT, EEE, TTE and EET data, is as n_s = 0.9658 ± 0.0038 and r < 0.072, respectively [53–55]. Also, Planck2018 TT, TE, EE + lowE + lensing + BK14 + BAO + LIGO and Virgo2016 data implies the constraint −0.62 < n_T < 0.53 on the tensor spectral index [53–55]. By using these released data, one can find some constraints on the model’s parameter space. An interesting work on this subject has been done by the authors of Ref. [56].

It is worth mentioning that, most of the inflationary models contain one or more scalar fields whose potential or kinetic energy is responsible for the inflation in the early universe. However, in the Energy-Momentum Squared Gravity model, we consider ordinary matter as a perfect fluid without including any scalar field. Considering the square of the energy-momentum of this perfect fluid in action, it is possible to derive the inflation phase without including any scalar or vector fields. In fact, as has been mentioned in Ref. [52], this model gives a ¨a > 0 which is reminiscent of an inflationary epoch. In this respect and in the absence of a usual scalar field, the term T^2 can be corresponding to the inflaton. In this manuscript, we will show that this model (a perfect fluid...
In the above equation, $\frac{\kappa}{\Theta_1^{\mu
u}}$ is a reasonable inflationary model which is consistent with observational data and also with $R^2$ inflation, at least in some domains of its parameter space. When we study an inflationary model containing the scalar field, some points are necessary to be considered. For instance, if we consider a model with a single canonical scalar field, the inflationary model with $\Phi^2$, $\Phi^{4/3}$ and $\Phi$ is not consistent with Planck2018 TT, TE, EE + lowE + lensing + BK14 + BAO data [53]. The single-field inflation model, only with $\Phi^{2/3}$ and natural potentials is consistent with mentioned data at 95% CL, in a very small range of its parameter space. To have an observationally viable single field inflationary model with wider ranges of the parameter space, and at both 68% CL and 95% CL, we should consider, for example, a hilltop potential [55]. Another way to have a viable inflation model containing a scalar field is to consider a non-minimal or non-minimal derivative coupling between the scalar field and gravity [17]. We also can consider an inflation model with a non-canonical scalar field [16]. However, when we consider an inflation model containing a $T^2$ term in the action, there is no need to seek some specific functions for potentials and even coupling functions to have a viable model.

In this paper, following Ref. [52], we consider the energy-momentum squared gravity model and organize the paper as follows. In Sect. 2, we obtain the main equations in the $\zeta T^2$ gravity model. By considering an additional term as $\zeta T^2$ in the Einstein-Hilbert action, we obtain the Einstein’s field equations and study the conservation law for the effective energy-momentum tensor. We show that the effective energy-momentum tensor obeys the conservation law. In Sect. 3, we explore the cosmological dynamics in the $\zeta T^2$ gravity model. In this regard, we obtain the Friedmann and the conservation equations of the model. In Sect. 4, the cosmological inflation in this model is studied. By obtaining the slow-roll parameters, we find $n_s$, $n_T$ and $r$ in terms of the model’s parameters. Then, we seek for the observational viability of the model in confrontation with Planck2018 data. In Sect. 5, we summarize the model and its results.

2 The setup of $\zeta T^2$ gravity

To study the energy-momentum squared gravity, we start with the following action

$$S = \frac{1}{2\kappa} \int \sqrt{-g} \left[ \mathcal{R} - 2\Lambda - \zeta T^2 \right] d^4x + S_M. \quad (1)$$

In this action, $g$ is determinant of metric, $\mathcal{R}$ is Ricci scalar, $\kappa = 8\pi G$ is the gravitational constant and $\Lambda$ is the cosmological constant. Note that, as mentioned in [52], the presence of the cosmological constant leads to a positive acceleration in EMSG model in form of $\ddot{a} = \frac{2\Lambda}{3} \dot{a}$, without considering any scalar field in the theory. Also, the matter part of action is defined as follows

$$S_M = \int \sqrt{-g} \mathcal{L}_M d^4x, \quad (2)$$

where $\mathcal{L}_M$ is the Lagrangian of the matter fields. By assuming that the matter Lagrangian density is only a function of metric and not its derivative, the energy-momentum tensor of the matter fields is given by

$$T_{\mu\nu} = \frac{-2}{\sqrt{-g}} \frac{\delta(\sqrt{-g}\mathcal{L}_M)}{\delta g^{\mu\nu}} = \mathcal{L}_M g_{\mu\nu} - 2 \frac{\delta\mathcal{L}_M}{\delta g^{\mu\nu}}. \quad (3)$$

By varying action (1) with respect to the metric $g_{\mu\nu}$, we obtain the Einstein’s field equations in the $\zeta T^2$ gravity model as

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{1}{2} \zeta T^2 g_{\mu\nu} = \zeta \frac{\delta T^2}{\delta g^{\mu\nu}} - \kappa T_{\mu\nu} = 0. \quad (4)$$

While [52,57]

$$\frac{\delta T^2}{\delta g^{\mu\nu}} = 2 \left( T^a_{\mu} T^{a\sigma} + T^{ab\nu \rho} \frac{\delta T_{ab}}{\delta g^{\mu\nu}} \right), \quad (5)$$

where $T^2 = T_{\mu\nu} T^{\mu\nu}$. Now, we can rewrite Eq. (4) as follows

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T_{\mu\nu} - \frac{1}{2} \zeta T^2 g_{\mu\nu} + \zeta \Theta_{\mu\nu} + \zeta T_{\mu\nu}. \quad (6)$$

In the above equation, $\Theta_{\mu\nu}$ is defined as

$$\Theta_{\mu\nu} = -\mathcal{L}_M g_{\mu\nu} + \frac{1}{2} g_{\mu\nu} \mathcal{L}_M - \frac{1}{2} T_{\mu\nu} - 2 g^{\rho\nu} \frac{\partial^2 \mathcal{L}_M}{\partial g^{\rho\nu} \partial g^{\alpha\beta}}. \quad (7)$$

Considering that all the changes appear in the right-hand side of field equations, we can introduce the following effective energy-momentum tensor

$$T^{\text{eff}}_{\mu\nu} = T_{\mu\nu} + \frac{1}{\kappa} \zeta \Theta_{\mu\nu} + \zeta T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} \zeta T^2. \quad (8)$$

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and rewrite the Einstein field equations as follows

\[ G_{\mu\nu} + \Lambda g_{\mu\nu} = \kappa T^{\text{eff}}_{\mu\nu}. \]  

(9)

Note that, these equations are also obtained in Ref. [58], where the authors have considered \( \xi (T^{\mu\nu} T_{\mu\nu})^n \). In this work, we study the case with \( n = 1 \).

Now, we seek for the conservation law in this model. In this regard, we obtain the covariant derivative of the energy-momentum tensor as

\[ \nabla^\mu T_{\mu\nu} = \frac{1}{(\kappa + \frac{1}{2} \xi)} \left[ \frac{1}{2} \xi T^2 g_{\mu\nu} + \mathcal{L}_M \xi g_{\mu\nu} \right. \]

\[ - \frac{1}{2} \xi g_{\mu\nu} \mathcal{L}_M + 2 \xi g^{\alpha\beta} \frac{\partial^2 \mathcal{L}_M}{\partial g^{\mu\nu}\partial g^{\alpha\beta}} \].

(10)

This equation shows that in this model the energy-momentum of the ordinary matter is not conserved (\( \nabla^\mu T_{\mu\nu} \neq 0 \)). This is because that in this model, due to presence of the \( T^2 \) term in the action, there are some extra non-vanishing terms (the terms with coefficient \( \xi \)) in the right-hand side of Eq. (6). This issue itself causes the non-conservation property of the “ordinary matter” energy-momentum tensor due to indistinct nature of the corresponding energy-momentum in \( T^2 \)-gravity. However, this doesn’t mean that the Bianchi identity is broken. In fact, by defining an “effective energy-momentum tensor,” and according to the Bianchi identity \( (\nabla^\mu \mathcal{G}_{\mu\nu} = 0) \), from Eq. (9) we find the conservation of the energy-momentum tensor (see also Ref. [59]). So, while the energy-momentum tensor of the ordinary matter is not conserved in this framework, the effective energy-momentum is conserved in the favor of Bianchi identity, that is,

\[ \nabla^\mu T^{\text{eff}}_{\mu\nu} = 0. \]  

(11)

Up to this point, we have obtained the main equations of the energy-momentum squared gravity model and studied the conservation law in this model. In the next section, we explore the cosmological implications of this interesting model.

3 Cosmology in \( T^2 \) gravity

Since our universe is homogeneous and isotropic in large scales, it is convenient to adopt the following Friedmann–Robertson–Walker metric as the background

\[ ds^2 = dx^\mu dx^\nu g_{\mu\nu} = -dr^2 + a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2) \right]. \]  

(12)

Considering that the observational data confirms the flat universe, we consider the FRW metric with \( k = 0 \). To obtain the Friedmann equations, it is necessary to adopt a suitable choice for the Lagrangian since the tensor \( \Theta_{\mu\nu} \) is related to the matter field’s Lagrangian via Eq. (7). It has been shown that one can choose either \( \mathcal{L}_M = p \) (where \( p \) is the pressure) or \( \mathcal{L}_M = -\rho \) (where \( \rho \) is the energy density) in Refs. [60,61]. Here, we adopt the case \( \mathcal{L}_M = p \). In this regard, we find

\[ H^2 = \frac{k}{3} \rho + \frac{\Lambda}{3} - \xi \left( \frac{1}{2} \rho^2 + \frac{4}{3} \rho p + \frac{1}{6} \rho^3 \right) \]

(13)

\[ \frac{\dot{a}}{a} = -\frac{k}{6} \left( \rho + 3 p \right) + \frac{\Lambda}{3} + \xi \left( \rho^2 + \frac{2}{3} \rho p + \frac{1}{3} \rho^3 \right) \]

(14)

As we mentioned \( \xi \) is the coupling constant and the important point about it is that, against other coupling constants in higher order theories of gravity which are dimensionless, dimension of \( \xi \) is \( \frac{1}{\ell_p^4} \) (Note that, in this paper we adopt the light speed as \( c = 1 \)). Furthermore, as mentioned in Ref. [52] this coupling constant should be both positive and small enough in order to give interesting cosmological results and pass classical gravitational tests in low energy regimes, respectively. We see in the next section that both of these conditions are satisfied in our observational analysis.

From Eqs. (13) and (14), we can define the following effective energy density and pressure

\[ \rho_{\text{eff}} = \rho - \frac{1}{2} \xi \left( \rho^2 + 3 p^2 + 8 \rho p \right), \]  

(15)

\[ p_{\text{eff}} = p - \frac{1}{2} \xi \left( \rho^2 + 3 p^2 \right), \]  

(16)

leading to the following continuity equation

\[ \dot{\rho}_{\text{eff}} + 3 H (\rho_{\text{eff}} + p_{\text{eff}}) = 0. \]  

(17)
Which dot means derivatives with respect to cosmological time. This equation means that the effective energy density is conserved in time.

After obtaining the main cosmological equation in the $\zeta T^2$ gravity model, we study the inflation and observational viability of this model in the next section.

4 Inflation and observational viability of $\zeta T^2$ gravity

By using the definition of a constant and positive equation of state as $\omega = \frac{p}{\rho}$, we can rewrite the Friedmann Eqs. (13) and (14) as follows, respectively

$$H^2 = \frac{\kappa}{3} \rho + \frac{\Lambda}{3} - \frac{1}{6} \lambda \rho^2,$$

where constants are defined as

$$\lambda = \zeta (3 \omega^2 + 8 \omega + 1),$$

$$\xi = 1 + 3 \omega,$$

$$\beta = \zeta \left( \omega^2 + \frac{2}{3} \omega + \frac{1}{3} \right)$$

By differentiating Eq. (19) with respect to cosmological time, we obtain

$$\ddot{H} + 2H \dot{H} = -\frac{\kappa}{6} \xi \dot{\rho} + 2 \beta \rho \dot{\rho}.$$

By substituting Eqs. (24) and (25) into Eq. (23), we obtain a Liénard equation as

$$\ddot{H} + A H \dot{H} + B H^3 = 0.$$

Where the parameters $A$ and $B$ are

$$A = \frac{4 \zeta}{\kappa}, \quad B = \frac{27 \zeta}{\kappa^2} + \frac{2 \Lambda}{\kappa^2}.$$

In Ref. [52] it has been shown that, to resolve the singularity of the early universe and achieve the positive acceleration expansion in the energy-momentum squared gravity model the universe should be dominated with the radiation component. In this regard, to obtain coefficients (28), we have adopted $\omega = \frac{1}{3}$. The solution of the differential equation (27) in the general form is

$$H = \left( \frac{-2 \zeta}{A} \right)^{\frac{1}{3}},$$

which is given in the parametric form. Also, the parameter $z$ is given by

$$z = C \exp \left( - \frac{\sigma}{\sigma^2 - \sigma - D} \right),$$

where $C$ is a constant and we have

$$D = \frac{-2 B}{A^2}.$$
We can use Eqs. (29) and (32)–(34) to study the inflation in the number. The result is shown in Fig. 2, which has been plotted for It is possible to write the slow-roll parameters in terms of the e-folds number. The e-folds number is defined as

\[ \zeta \]

and

\[ \epsilon \]

and

\[ \epsilon_3 \equiv \left( \frac{\dddot{H} H - 2 \dot{H}^2}{\dot{H}^2} \right)^{-1} \frac{\dot{H} \dddot{H}^2 - \dddot{H} H^2 + \dddot{H} H \dot{H}^2}{\dddot{H} H^2} - \frac{2 \dddot{H} H \dddot{H} - 2 \dot{H}^2}{\dddot{H}^2} \]

It is possible to write the slow-roll parameters in terms of the e-folds number. The e-folds number is defined as

\[ N \equiv \ln \left( \frac{a_f}{a_i} \right) = - \int_{a_i}^{a_f} H(t) \, dt, \]

where \( a_i \) and \( a_f \) show the values of the scale factor at the beginning and end of inflation era, respectively. The e-folds number in \( \zeta T^2 \) gravity model takes the following form

\[ N = \frac{2}{A \sqrt{4D + 1}} \tanh^{-1} \left( \frac{2\sigma + 1}{\sqrt{4D + 1}} \right). \]

By using the definition of the e-folds number given in Eq. (38), we obtain the following expressions for the slow-roll parameters

\[ \epsilon_1(N) \equiv \frac{-H'(N)}{H(N)}, \]

\[ \epsilon_2(N) \equiv \frac{H''(N)}{H'(N)} - \frac{H'(N)}{H(N)}, \]

and

\[ \epsilon_3(N) \equiv \left[ \frac{H(N)H'(N)}{H'(N)H(N) - H^2(N)} \right] \left[ \frac{H''(N)}{H'(N)} - \frac{H'^2(N)}{H(N)} + \frac{H''(N)}{H'(N)} \right]. \]

These parameters in our \( \zeta T^2 \) model, take the following form

\[ \epsilon_1 = \frac{\sigma A}{2}, \]

\[ \epsilon_2 = A \left( \frac{\sigma^2 - \sigma - D}{\sigma} \right), \]

\[ \epsilon_3 = A \left( \frac{-\sigma^2 + \sigma + D}{-\sigma^2 + \sigma + D} \right). \]

Now, by considering Eqs. (39) and (43)–(45), we can seek for graceful exit of the model from inflation era. In the inflation era we have \( \epsilon_1, \epsilon_2, \epsilon_3 \ll 1 \). To have graceful exit from inflation, one of the slow-roll parameters should reach unity. In this regard, we plot the parameters \( \epsilon_1 \) and \( \epsilon_2 \) versus the e-folds number for two sample values of \( \zeta \). The results are shown in Fig. 1. As figure shows, the slow-roll parameter \( \epsilon_2 \) meet unity at \( N = 60 \). This means that in our model inflation ends after 60 e-folding.

Another way to seek for inflation and its graceful exit is the study of the evolution of the Hubble parameter versus the e-folds number. The result is shown in Fig. 2, which has been plotted for \( \zeta = 2.1 \times 10^{-5} \). As this figure shows, the Hubble parameter during inflation changes very slowly until inflation ends.

Also, the perturbation parameters are defined in terms of the slow-roll parameters \[ 27,47,63–65 \]. In this regard, the scalar spectral index and its running are given by

\[ n_s \approx 1 - 2 \epsilon_1 - 2 \epsilon_2, \]
Fig. 1 The evolution of the slow-roll parameters $\epsilon_1$ and $\epsilon_2$ versus the e-folds number during the inflation for the two sample values of $\zeta$.

Fig. 2 The evolution of the Hubble parameter versus the e-folds number during inflation.

\[ \alpha_s \approx -2\epsilon_1\epsilon_2 - \epsilon_2\epsilon_3. \]  

(47)

respectively. The tensor spectral index, in terms of the slow-roll parameters, is defined as

\[ n_T \approx -2\epsilon_1. \]  

(48)

Finally, the tensor-to-scalar ratio is given by

\[ r \approx 16\epsilon_1. \]  

(49)
By substituting Eqs. (43)–(45) in Eqs. (46)–(49) we obtain the following expressions for the perturbation parameters

\[ n_s = 1 - \Lambda \sigma - 2A \left( \frac{\sigma^2 - \sigma - D}{\sigma} \right), \]

\[ \alpha_s = \left[ -\Lambda^2 \left( \frac{\sigma^2 - \sigma - D}{\sigma} \right) \right]\left[ \sigma + \frac{-\sigma^3 + \sigma + D}{-\sigma^2 + \sigma + D} \right]. \]

\[ n_T = -A \sigma, \]

and

\[ r = 8A \sigma. \]

Finally, by using Eq. (53) to eliminate the parameter \( \sigma \), we get

\[ n_s = \frac{3}{4} - \frac{r}{8} + 2A - \frac{32B}{r}. \]

\[ \alpha_s = \left( \frac{r}{8A} - 1 + \frac{16B}{A R} \right) \left[ -\frac{rA}{8} - A^2 \left( -\frac{r^3 + 64rA^2 - 1024AB}{-r^2 + 8Ar - 128B} \right) \right]. \]

and

\[ n_T = -\frac{r}{8}. \]

After obtaining the main perturbations parameters, now we explore the model numerically and compare the results with the observational data. In this regard, we can examine the observational viability of our setup and obtain some constraints on the coupling parameter \( \xi \). Note that, in Ref. [52] it has been shown that only positive values of \( \xi \) lead to the viable cosmology. Therefore, in our analysis, we consider only the positive values of this parameter. Figure 3 shows the behavior of the scalar spectral index versus the scalar spectral index, in the background of the Planck2018 TT, TE, EE + lowE + lensing data [53]. To plot this figure, we have used Eqs. (54) and (55), where the parameters \( A \) and \( B \) are given by Eq. (28). This figure and forthcoming figures have been plotted for \( 0 < \xi \leq 10^{-5} \). We can also study the behavior of the tensor-to-scalar ratio versus the scalar spectral index by using Eq. (54). The result is shown in Fig. 4, in the background of the Planck2018 TT, TE, EE+lowE+lensing+BK14+BAO data set [54]. Also, Fig. 5 shows the tensor-to-scalar ratio versus the tensor spectral index [see Eq. (56)] in the background of the Planck2018 TT, TE, EE + lowE + lensing + BK14 + BAO + LIGO and Virgo-2016 data [54]. To plot Figs. 3, 4 and 5, we have borrowed the contour plots released by Planck 208 team [53–55]. This is because, in this paper, we compare the results of the numerical analysis in our model with the Planck observational data. However, the blue regions are the numerical results of our setup which have been obtained from Eqs. (54)–(56). As these figures show, the energy-momentum squared gravity model in some ranges of the model’s parameter space is consistent with observational data. By performing the numerical analysis, we have obtained some ranges of the parameter \( \xi \) which cause the viability of the model in confrontation with different data sets. The constraints are summarized in Table 1. Not that, to obtain the constraints on the parameter \( \xi \), we have used three different data sets on the scalar spectral index, tensor spectral index and tensor-to-scalar ratio, released by the Planck2018 team. The constraints on these parameters have been given in two confidence levels as 68% CL and 95% CL [53–55]. These confidence levels are shown in our figures with dark and light red contours, respectively. When we study the perturbations parameters \( n_s, \alpha_s, n_T \) and \( r \) numerically, we are interested to those values of these perturbations parameters which lie in the observational contour plots. The values of \( n_s, \alpha_s, n_T \) and \( r \) depend on the values of the parameter \( \xi \) [see Eqs. (30) and (56)–(58)]. Some values of \( \xi \) lead to the values of the perturbations parameters which lie in 68% CL of the observational data (dark red region of the contour plots). Also, some values of \( \xi \) lead to the values of the perturbations parameters which are in 95% CL of the observational data (light red region of the contour plots). Of course, for other values of \( \xi \), the values of \( n_s, \alpha_s, n_T \) and \( r \) lie out of the contour plots and are not observationally viable. These points have been considered to obtain the confidence intervals in Table 1.

As has been mentioned in Ref. [55], \( R^2 \) inflation has the best fit with Planck2018 observational data. Therefore, it seems interesting to compare the model of this work with \( R^2 \) inflation. We can compare the marginalized probability densities of the perturbations parameters \( n_s \) and \( r \) in our model with the ones in \( R^2 \) inflation. If we consider the likelihood function of the data for a given model \( M \) as \( L(\text{data}|\hat{x}) \), where \( \hat{x} = \{ x_1, \ldots, x_N \} \) are the free parameters of the model, the probability density is given by

\[ P(\hat{x}|\text{data}, M) \propto L(\text{data}|\hat{x}, M) \cdot P(\hat{x}|M). \]

In the above definition, we have shown the data-independent prior probability density by \( P(\hat{x}|M) \). We can also perform a \( \chi^2 \) analysis between our model (\( M_1 \)) and \( R^2 \) inflation (\( M_2 \)), by using the following definition

\[ \Delta \chi^2 = 2 \ln \frac{L_{\text{max}}(M_1)}{L_{\text{max}}(M_2)}. \]

By performing numerical analysis, we find \( \Delta \chi^2 = -0.6 \) between our model and \( R^2 \) inflation. The marginalized probability densities of the scalar spectral index and tensor-to-scalar ratio in both models is shown in Fig. 6. As this figure also shows, there is some consistency between our model with Planck2018 data and \( R^2 \) inflation, at least in some ranges of the model’s parameter space.
Fig. 3 Running of the scalar spectral index versus the scalar spectral index for $0 < \zeta \leq 10^{-5}$ (blue region), in the background of Planck2018 TT, TE, EE + lowE + lensing data (red regions). The yellow arrow shows the direction in which the parameter $\zeta$ increases.

Fig. 4 Tensor-to-scalar ratio versus the scalar spectral index for $0 < \zeta \leq 10^{-5}$ (blue region), in the background of the Planck2018 TT, TE, EE + lowE + lensing + BK14 + BAO data set (red regions). The yellow arrow shows the direction in which the parameter $\zeta$ increases. The cyan line shows $r - n_s$ behavior in $R^2$ inflation for $50 \leq N \leq 60$.

Fig. 4, we have plotted $r - n_s$ of $R^2$ inflation for $50 \leq N \leq 60$. This figure shows that, for $8.37 \times 10^{-7} \leq \zeta \leq 1.17 \times 10^{-6}$, the values of the scalar spectral index and tensor-to-scalar in our model are equal to the corresponding values in $R^2$ inflation with $50 \leq N \leq 60$.

After studying the perturbation’s parameters numerically and obtaining some constraints on the model from the observational data, it seems interesting to seek the abundance of the fluid $\rho$ with $\omega = \frac{1}{3}$ (corresponding to radiation component). For this purpose, we rewrite Eq. (18) in terms of the density parameters $\Omega$ as

$$1 = \Omega_{\text{rad}} + \Omega_{\Lambda} - \Omega_{\text{rad}}^2 \Omega_{\zeta},$$

(59)

where

$$\Omega_{\text{rad}} = \frac{\kappa \rho}{3H^2}, \quad \Omega_{\Lambda} = \frac{\Lambda}{3H^2}, \quad \Omega_{\zeta} = \frac{3\lambda H^2}{2\kappa^2},$$

(60)

With rad presenting the radiation component. As our numerical analysis has shown, the strength of the energy-momentum squared gravity in our model is small ($0 < \zeta \leq 2.1 \times 10^{-5}$). This means that even for small strength of the energy-momentum squared gravity, it is possible to get the observationally viable inflationary model. In this sense, to study the abundance of the fluids in our
Fig. 5  Tensor spectral index versus the tensor spectral index for $0 < \zeta \leq 10^{-5}$ (blue region), in the background of the Planck2018 TT, TE, EE + lowE + lensing + BK14 + BAO + LIGO and Virgo2016 data (red regions). The yellow arrow shows the direction in which the parameter $\zeta$ increases.

Table 1  The ranges of the parameter $\zeta \left( \frac{\sqrt{\lambda}}{k_0} \right)$ in which the tensor-to-scalar ratio, the scalar spectral index, its running and the tensor spectral index of the $\zeta T^2$ gravity model are consistent with different data sets

|                | Planck2018 TT, TE, EE + lowE | Planck2018 TT, TE, EE + lowE + lensing | Planck2018 TT, TE, EE + lowE + lensing + BK14 + BAO + LIGO and Virgo2016 |
|----------------|-------------------------------|----------------------------------------|--------------------------------------------------------------------------|
| 68% CL         | $\zeta \leq 1.6 \times 10^{-3}$ | $0 < \zeta \leq 1.4 \times 10^{-5}$   | $0 < \zeta$                                                              |
| 95% CL         | $0 < \zeta \leq 1.6 \times 10^{-3}$ | $0 < \zeta \leq 2.1 \times 10^{-5}$   | $0 < \zeta$                                                              |

Fig. 6  Marginalized probability densities of the scalar spectral index (left panel) and the tensor-to-scalar ratio (right panel) in the background of Planck2018 data. The solid blue line and dashed black line show $R^2$ inflation and our model, respectively.
The abundance of $\Omega_r$ and $\Omega_\Lambda$ at 68% CL (red region) and 98% CL (plum region), for $\Omega_\zeta = 0.001$.

model, we adopt small value of $\Omega_\zeta$ as $\Omega_\zeta = 0.001$. Then, we find the abundance of $\Omega_r$ and $\Omega_\Lambda$ at 68% CL and 98% CL, for this adopted value of $\Omega_\zeta$. The result is shown in Fig. 7. According to our analysis at 68% CL, we have $\Omega_r = 0.908 \pm 0.003$ and $\Omega_\Lambda = 0.091 \pm 0.003$.

5 Conclusion

In this paper, we have studied the cosmological dynamics of the energy-momentum squared gravity. In this regard, we have considered an additional term in the Einstein–Hilbert action as $\zeta T^2$, where $\zeta$ is a positive constant coupling and $T^2 = T^{\mu\nu} T_{\mu\nu}$. We have presented the Einstein’s field equations in $\zeta T^2$ gravity model and also studied the conservation law via the energy-momentum tensor. We have shown that, by adding a $\zeta T^2$ term to the action, the energy-momentum conservation of the ordinary matter breaks down. However, if we consider an effective energy-momentum tensor, the conservation law would be satisfied. After that, by assuming the flat FRW metric as the background geometry, we have obtained the Friedmann equations in this setup. In this regard, we have introduced the effective energy density and the effective pressure, by which we have shown the conservation of the effective energy density in the energy-momentum squared gravity.

Then, we have studied the inflation phase in this model. By obtaining the slow-roll parameters, we have expressed the perturbation quantities in terms of the model’s parameters. By performing a numerical analysis on the scalar spectral index, its running, tensor spectral index and tensor-to-scalar ratio, we have studied the viability of the $\zeta T^2$ model in the context of the inflation. This model, which leads to the bounce at the early universe, is an observationally viable inflation model with $0 < \zeta \leq 2.1 \times 10^{-5}$.

Finally, the novel points of physics on inflation in this model are as follows:

- The modification of the Friedmann equations, resulted from the modification of the energy density and pressure, affects the dynamic of the universe in this model.
- In most inflationary models, the problem of the singularity of the early universe has been not considered (i.e., it is not resolvable in these models); however, in the EMSG model the Friedmann equations have been modified in a way to solve this problem [52].
- As claimed in defining $F(T) = \zeta T^2$, the coupling constant $\zeta$ must be both small enough and positive to pass the classical tests of gravity and avoid the unsatisfactory cosmological behavior. We see from data analysis that both claims are satisfied and $\zeta$ is small and positive.
- In this inflationary model, there is a graceful exit from inflation (needed for inflation to be connected to the standard model of cosmology for viable thermal history of the universe). Also, the perturbation parameters in our model are consistent with observational data.
- When we consider the model with a $T^2$ term in the action, as in our model, studying the abundance of the fluid components leads to interesting results. By exploring the density parameters in this model, we see that the value of $\Omega_r$ in this model is consistent with observation. Meanwhile, the value of $\Omega_\zeta$ is small. This shows that, even with very small values of $\Omega_\zeta$, corresponding to the small strength of the energy-momentum squared gravity, it is possible to have an inflation model that is consistent with Planck2018 TT, TE, EE + lowE + lensing + BK14 + BAO data.

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Data availability statement All data generated or analyzed during this study are included in this article.
Appendix

We have the following Lienard differential equation
\[ \ddot{H} + A \dot{H} + B H^3 = 0, \]
where
\[ A = \frac{4 \zeta}{\kappa} \]
and
\[ B = \frac{-27 \zeta}{\kappa} \pm \frac{2 \Lambda}{\kappa^2} \]
By assuming \( H \equiv y \), we can rewrite the above Lienard equation as follows
\[ \ddot{y} + f(y) \dot{y} + g(y) = 0, \]
where
\[ f(y) = A y, \]
and
\[ g(y) = B y^3. \]
Now, we define \( w(y) = \dot{y} \) and by which we convert our Lienard equation to the Abel differential equation of the second kind as
\[ w w' + f(y) w + g(y) = 0, \]
with \( w w' = \ddot{y} \) and \( w' = \frac{dw}{dy} \). By introducing \( z = \int F(y) dy \), with \( F(y) = -A y \) and \( G(y) = -B y^3 \), the Abel equation takes the following canonical form
\[ w w, z = w + \phi(z), \]
where “, \( z \)” demonstrates derivative with respect to \( z \), and
\[ \phi(z) = \frac{G(y)}{F(y)}. \]
Considering that
\[ z = -\frac{A y^2}{2}, \]
we find the following expression for \( \phi(z) \)
\[ \phi(z) = \frac{B}{A} y^2, \]
leading to
\[ w w, z = w + \frac{B}{A} y^2 = w + D z. \]
By defining \( k(z) = D z \), we get
\[ w w, z = w + k(z), \]
which has the following solution
\[ z = C \exp \left( -\int \frac{\sigma d\sigma}{\sigma^2 - \sigma - D} \right), \quad w = C \sigma \exp \left( -\int \frac{\sigma d\sigma}{\sigma^2 - \sigma - D} \right). \]
Now, we can obtain the Hubble parameter and its derivatives. From \( y \equiv H \) and \( z = -\frac{A y^2}{2} \), we find
\[ H = \left( \frac{-2z}{A} \right) \frac{1}{3}. \]
Using $\dot{H} = \dot{y} = w$, we obtain
\[ \dot{H} = \sigma z. \]
Also, from $\ddot{H} = w u'\prime$ and considering that $\frac{dw}{dy} = \frac{dw}{dz} \frac{dz}{dy}$, we get
\[ \ddot{H} = -A \left( \frac{\sigma + D}{\sigma} \right) \dot{H} \dot{H}. \]

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