Abstract—In this paper, a cooperative localization algorithm is proposed that considers the existence of obstacles in mobility-assisted wireless sensor networks (WSNs). In this scheme, a mobile anchor (MA) node cooperates with static sensor nodes and moves actively to refine location performance. The localization accuracy of the proposed algorithm can be improved further by changing the transmission range of mobile anchor node. The algorithm takes advantage of cooperation between MAs and static sensors while, at the same time, taking into account the relay node availability to make the best use of beacon signals. For achieving high localization accuracy and coverage, a novel convex position estimation algorithm is proposed, which can effectively solve the localization problem when infeasible points occur because of the effects of radio irregularity and obstacles. This method is the only range-free based convex method to solve the localization problem when the feasible set of localization inequalities is empty. Simulation results demonstrate the effectiveness of this algorithm.

I. INTRODUCTION

Localization algorithms for wireless sensor networks (WSNs) have been designed to find sensor location information, which is a key requirement in many applications of WSNs. Generally speaking, based on the type of information required for localization, protocols can be divided into two categories: (i) range-based and (ii) range-free protocols [1]. Range-based techniques require special hardware for estimating the distance between anchors and sensors, which may become prohibitively expensive for large networks [2]. Range-free techniques, on the other hand, do not impose such complexity as the anchor informs other sensors about its own position through message passing [3]. After finishing the distance-from-anchor estimation process, a regular sensor can determine its own position, through a variety of methods, such as multilateration, triangulation, etc. If necessary, an optional step is performed, in which regular sensors exchange messages among themselves to refine their locations. Due to the hardware limitations and power constraints of sensors, solutions of range-free localization are often preferable and can be considered as cost-effective options when compared with more expensive and energy-consuming range-based schemes [4]-[7]. In this paper, we focus on the investigation of range-free localization algorithms for mobility-assisted WSNs.

An obstacle can be dynamically formed due to unbalanced deployment, failure or power exhaustion of sensor nodes, animus interference, or physical obstacles such as mountains or buildings. In this paper, we consider only physical obstacles. Most previous algorithms cannot work well in anisotropic networks, where obstacles appear among sensor nodes. However, anisotropic networks with obstacles are more likely to exist in practice for several reasons. Firstly, a uniform node distribution cannot always be achieved because of random deployment of sensor nodes, which may cause some regions to not contain any sensor node. Secondly, unbalanced power consumption among nodes results in some regions without functionality of sensing and communication. Thirdly, physical obstacles such as mountains or buildings will naturally exist in many networks.

In this work, we propose a multi-power level mobile anchor assisted range-free algorithm for WSNs with obstacles. By using a relay node, our scheme can effectively reduce the effects of obstacles on node localization. Furthermore, our scheme can calculate the positions of infeasible points caused by a complex radio transmission environment, which is recognized as a problem when the feasible set for localization inequalities is empty.

II. COLLABORATIVE LOCALIZATION USING A MOBILE ANCHOR

In this section, we propose a collaborative node localization approach using an MA. We first introduce the technical preliminaries of our algorithm in subsection A and then formulate the localization problem as an optimization problem in subsection B. We propose an algorithm for decreasing the impact of obstacles in subsection C.

A. Background

In WSNs, a node can determine whether it is in the transmission radius of an anchor node according to the beacon signal received from the one-hop anchor. The anchor node can adjust its transmission radius by tuning the transmission power [9]. For example, the TelosB mote is equipped with an IEEE 802.15.4 compliant Chipcon CC2420 radio, which has 31 transmission power levels between -25 and 0 dBm.
We have conducted experiments on a testbed composed of TelosB motes. As shown in Fig. 1, the experiments demonstrate that the transmission radius of a sensor can be efficiently changed by tuning the transmission power level.

We assume an anchor node has \( M \) levels of transmission power, and the related transmission range is \( R_i, \ i = 1, 2, \ldots, M \). Normally, the MA is assumed to have a global positioning system (GPS) receiver and knows its position. During the moving period, the MA transmits beacon signals at varying power levels consecutively including its ID, current position, transmission power and transmission radius. After receiving these beacon signals, an unknown-position sensor can construct an effective constraint on its position.

For example, we assume that the current position for the MA is \( a \) and its transmission radius is \( R \). If the unknown-position sensor, at position \( x \), receives the beacon signal, we can conclude that the distance between both nodes satisfies

\[
\|x - a\| \leq R. \tag{1}
\]

Otherwise,

\[
\|x - a\| > R. \tag{2}
\]

Using the various transmission power levels of the MA in different positions, the unknown-position sensor can obtain a set of inequalities on \( x \):

\[
r_i < \|x - a_i\| \leq R_i, \quad i = 1, 2, \ldots, M \tag{3}
\]

where \( a_i \) is the position of the MA at time \( i \), \( r_i \) (it might be zero) and \( R_i \) are valid radii for that time. Herein, the valid constraint radii denote the related lower and upper bounds, for the tightest constraint among all of the constraints that are constructed by all of the transmission powers for the mobile anchor node at position \( a_i \).

Hence, the localization problem based on an MA with variable transmission power can be successfully converted into the problem of solving a set of quadratic inequalities

\[
\sum_{i=1}^{M} \left(\|x - a\| - r_i\right) + \left(\|x - a\| - R_i\right)^2 = 0.
\]

Some of the location algorithms (eg, [6] and [16]) are also based on the solution of a set of quadratic inequalities. However, their methods all assume that the set of quadratic inequalities \( (3) \) must have solutions. Nevertheless, because of the complicated transmission environment, there are two different location scenarios as shown in Fig. 2: the set of quadratic inequalities has a solution (i.e., the feasible set is nonempty) for the first case; the set of quadratic inequalities have no solution (i.e., the feasible set is empty) for the second case. The dots on the figure represent the anchors and the squares represent the unknown-position sensor.

For the above two different scenarios, we propose a novel localization algorithm based on convex optimization to solve the problem when the feasible set is empty. To the best of our knowledge, our proposed method is the only range-free algorithm using convex optimization to solve the problem when the feasible set is empty.

### B. Localization Algorithm Using Convex Optimization

In real environments, the actual transmission radius varies in different directions of radio propagation because of the non-isotropic properties of the propagation medium and the heterogeneous properties of devices. It is possible that there is no communication between two nodes although their relative distance is within their ideal transmission radius. On the other hand, two nodes may also be able to communicate although their relative distance is larger than their transmission radius. Thus, with the effects of radio irregularity and obstacles, a localization algorithm might not be able to guarantee full coverage and an infeasible case would occur [10]-[14].

In order to deal with the case with an empty feasible set, we propose a novel convex position estimation algorithm, which can provide good position estimation accuracy in both the feasible case and the infeasible case. As shown in Fig. 3, for the single constraint case \(( r < \|x - a\| < R )\), it is easy to see that the optimal position estimate lies on the circle with center \( a \) and radius \( \frac{R - r}{2} \). In the figure, the square indicates the possible position for the optimal position estimate and the black dot denotes the anchor node with position \( a \).

The position estimate can be found by minimizing the following expression:

\[
\left(\|x - a\| - r\right)^2 + \left(\|x - a\| - R\right)^2.
\]
A case can still be handled by using the same convex relaxation technique. Bound and inequality A into a convex problem by using a convex relaxation technique.

Although, the problem (5) is still nonconvex, we can turn it via convex relaxation techniques, we transform it to the following problem

\[
\min_x \sum_i \left( \|x - a_i\|^2 - r_i^2 \right) + \left( \|x - a_i\| - R_i \right)^2 \tag{4}
\]

Obviously, the problem (4) is nonconvex. Moreover, this problem cannot be directly approximated by using some convex relaxation techniques like that of [15]. To approximately solve the problem via convex relaxation techniques, we transform it to the following problem

\[
\min_x \left\{ \sum_i \left[ \left( \|x - a_i\|^2 - r_i^2 \right)^2 + \left( \|x - a_i\|^2 - R_i^2 \right)^2 \right] \right\} \tag{5}
\]

Although, the problem (5) is still nonconvex, we can turn it into a convex problem by using a convex relaxation technique. Firstly, the problem (5) can be transformed to the epigraph [8], namely, the sum of the products of corresponding elements of two matrices.

Using (7), the equivalent form for the problem (6) is obtained as follows:

\[
\begin{align*}
\min_{Y,v,t} & \quad t \\
\text{s.t.} & \quad ||v|| \leq t \\
& \quad Y_{2d+1,2d+1} = I_2 \\
& \quad A_i \cdot Y - v_{i1} = r_i^2 \\
& \quad A_i \cdot Y - v_{i2} = R_i^2, \quad \forall i \\
y & = ||x||^2.
\end{align*} \tag{8}
\]

In (8), \( ||v|| \leq t \) is a second-order cone. However, the problem (5) is still not convex due to the nonlinear equality constraint. Herein, we relax the equality \( y = ||x||^2 \) to \( y \geq ||x||^2 \) which is equivalent to requiring that \( Y \) is a positive semidefinite matrix by the Schur complement theorem [8]. Hence, we have the following convex optimization problem:

\[
\begin{align*}
\min_{Y,v,t} & \quad t \\
\text{s.t.} & \quad ||v|| \leq t \\
& \quad Y \geq 0 \\
& \quad Y_{2d+1,2d+1} = I_2 \\
& \quad A_i \cdot Y - v_{i1} = r_i^2 \\
& \quad A_i \cdot Y - v_{i2} = R_i^2, \quad \forall i \\
y & = ||x||^2.
\end{align*} \tag{9}
\]

where \( Y \geq 0 \) indicates that \( Y \) is a positive semidefinite matrix. The resulting problem is a convex cone programming problem which can be solved by using efficient interior-point algorithms [8]. After obtaining the value of \( Y \), we can further calculate the position estimate for the unknown-position sensor \( x \), namely, \( x = Y_{2:3,1:1} \), where \( Y_{2:3,1:1} \) denotes the vector constructed from the elements of the second and third rows for the first column of the matrix \( Y \).

C. Algorithm for Decreasing the Impact of Obstacles

In this paper, we assume that boundary nodes around the obstacle have been discovered by some boundary recognition algorithms [17], so that each sensor node knows whether it is a boundary node or not. Only boundary nodes can participate in contention for relaying beacons from the MA because their rebroadcasts may cover some blind areas as shown in Fig. 4. Hearing a beacon from the MA, boundary nodes will compete to relay this location information through a distributed contention process. The probability that a candidate node wins the contention depends on the node’s remaining energy and the number of neighboring sensors. A node with greater remaining energy and greater number of neighbors has higher priority to be the optimal relay node. The proposed selection scheme for the optimal relay node is concluded as follows:

Receiving a beacon from the MA, a boundary node sets a backoff timer that defines the amount of time that the node...
must wait before rebroadcasting the location information. The backoff time $\delta$ is calculated as

$$\delta = (\alpha(\text{used-energy/initial-energy}) + \beta/\text{num-neighbors}) \times \text{max-delay}$$

where $\alpha$ and $\beta$ are coefficients that provide different weights for different parameters. The specific values of $\alpha$ and $\beta$ can be set depending on which property is more important for users: energy balance or coverage efficiency. In total, $\alpha + \beta = 1$. We can see that a greater remaining energy and a greater number of neighbors will lead to a shorter backoff time. If a candidate boundary node does not hear any beacon signal from other sensors during its backoff time, it will rebroadcast the beacon signal and other boundary nodes will cancel their contentions if they receive the rebroadcast of the beacon. As a result, the node with the highest priority will rebroadcast first and win the competition to serve as the relay for the MA’s beacon signal. In this way, we can deliver the MA’s location information to some areas that cannot receive the MA’s direct communication. Similarly to (3), the unknown-position sensor in these special areas can obtain a set of inequality constraints on $x$:

$$r_i < ||x - a_i|| \leq R_i + R_{\text{relay}}, \quad i = 1, 2, ..., n$$

where $R_{\text{relay}}$ is the current transmission radius for the relay node. We can also use the proposed convex localization algorithm to solve the problem (11). Based on this scheme, we can efficiently decrease the impact of the obstacle on node localization and improve the location accuracy.

III. NUMERICAL RESULTS

In this section, simulation results are presented and analyzed. The performance evaluation focuses on the position estimation accuracy of the proposed algorithm. We consider a 2-dimensional region with a size of 100 m x 100 m. We assume the MA has two level transmission power with the transmission radii $r$ and $R = 2r$, respectively. The method of [16] is also evaluated with our proposed algorithm for performance comparisons. First, we deploy 100 sensor nodes randomly and the transmission radius $r$ is set to 15 meters. In subsection A, we simulate our algorithm in the ideal situation. The effects of degree of irregularity (DOI) and obstacle on localization performance will be discussed in subsection B. All simulation results are averaged over 100 network scenarios. The average localization error is used to evaluate the performance for our localization algorithm. Localization error is defined as follows:

$$\text{error} = \frac{1}{N} \sum_{i=1}^{N} ||x_i - \hat{x}_i|| \times \frac{1}{r},$$

where $x_i$ is the actual position for node $i$ and $\hat{x}_i$ is the estimated position of node $i$. Note that we normalize the absolute localization error using radio range. For instance, an error of 20% means that the localization error is 20% of the radio range.

A. Performance in the Ideal Environment

In this subsection, we give the simulation results for different algorithms in the ideal situation, namely, when there is no obstacle in the sensing area. We use the DOI to indicate the radio irregularity characteristic. Fig. 5(a) and 5(b) shows the simulation results in the ideal situation, where the true nodes are denoted by circles, the position estimates are denoted by asterisks, and the lines that link the true nodes and the estimates represent the estimation errors. It is clear from Fig. 5(a) and 5(b) that our algorithm works better than the algorithm of [16] in terms of the average localization error.

B. Performance in the Non-ideal Environment

For the next set of experiments, we use a fading coefficient (f) that represents the percentage of total mobile beacon points that cannot be heard by the sensor at any given time. This models the obstacles encountered in the sensing area that limit the number of mobile beacon points that can be heard at any point. As Fig. 6(a) and 6(b) illustrates, our algorithm outperforms the algorithm of [16] in terms of the average localization error in this non-ideal environment.

IV. CONCLUSIONS

We have presented a new cooperative localization scheme that can achieve high localization accuracy in mobility-assisted wireless sensor networks when obstacles exist. Considering the complex localization scenario, namely, the feasible set is empty, a convex localization algorithm has been presented to address the effects of non-ideal transmission of radio signals. It has been shown in the simulation results that the proposed cooperative localization scheme can significantly improve the localization accuracy by including a mobile element. In future work, we intend to verify and improve the proposed cooperative localization scheme using real sensors in a mobility-assisted wireless sensor networks.

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Fig. 5. Performance comparison: (a) Localization error of our method (DOI=0.2, error = 11.68%); (b) Localization error of method [16] (DOI=0.2, error = 13.7%)

Fig. 6. Performance comparison in the non-ideal environment: (a) Localization error of our method (DOI=0.2, f=0.1, error = 12.95%); (b) Localization error of method [16] (DOI=0.2, f=0.1, error = 14.89%)

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