Influence of wavelet characteristics on single-phase grounding fault line selection

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Abstract. It is difficult to select the fault line in distribution network because of the small single-phase grounding fault current. Wavelet transform is especially suitable for fault transient analysis because of its unique time-frequency localization performance. However, the characteristics of different wavelets have a direct impact on the line selection results. Improper selection of wavelets will directly lead to misjudgment. Firstly, the transient characteristics of single-phase grounding fault and the relationship between wavelet transform modulus maxima and singularity are analysed, and the line selection criterion of wavelet modulus maxima is given. Then, the influence of wavelet vanishing moment order, support length, symmetry, and orthogonality on the detection effect of signal singularity is analysed, and four principles for selecting wavelet in fault line selection are proposed. The larger the order of vanishing moment and support length, the more conducive to detection. The influence of vanishing moment order is much greater than that of support length. The 10kV system simulation model is built, five representative wavelets are selected to decompose the zero-sequence current of the fault line in four scales respectively, and the different modulus maxima corresponding to the sudden change point are compared. The experimental results prove the reliability of the wavelet selection principles proposed in this paper.

1. Introduction

After a single-phase grounding fault occurs in the distribution network, the non-fault phase voltage will rise \( \sqrt{3} \) times, endangering the system insulation, seriously developing into phase to phase short circuit, resulting in the expansion of the fault range. Therefore, it is of great practical significance to find out the fault line quickly and reliably [1]. China's distribution network mostly adopts neutral indirect grounding. When a single-phase grounding fault occurs, the grounding fault current is weak, so the problem of line selection has not been well solved. Many literatures have conducted in-depth research on this problem [2-10]. Since the transient quantity is several times of the steady-state quantity and has obvious characteristics, the transient quantity line selection method is adopted to effectively improve the reliability.

Wavelet transform is especially suitable for the analysis of unstable transients. Wavelet transform provides adjustable time-frequency window, which automatically narrows the time window of high-frequency signal and widens the time window of low-frequency signal. The unique zoom feature of wavelet transform can decompose the transient signal more finely. At the same time, the modulus
maxima of wavelet transform on different scales represent the mutation characteristics of the signal. Wavelet transform can better extract the characteristics of singular mutation points of the signal. The generating function used in the traditional Fourier transform is the only sine and cosine function. However, the selection of wavelet generating function is diverse. At present, there is no recognized principle to select wavelet generating function, which is often determined by qualitative analysis combined with experimental comparison. Because the wavelet function is not unique, different literatures use different wavelets at present. Literature [11] uses sym6 wavelet, Literature [12] uses DB4 wavelet, Literature [13] uses db5, and Literature [14] uses cubic B-spline function. However, these literatures do not deeply study the influence of wavelet characteristics on line selection results and do not explain the specific selection principles. Different wavelets have different characteristics. The difference of wavelet characteristics will lead to different modulus maxima detection results, which directly affects the accuracy of line selection.

To solve this problem, the effects of the vanishing moment order, support length and symmetry of wavelet on fault line selection are studied, and four principles for selecting wavelet in fault line selection are proposed, which are proved to be correct by simulation experiments.

2. Fault transient characteristics and line selection criteria

After a single-phase grounding fault occurs in the neutral indirect grounding system, the fault phase to ground voltage suddenly decreases to zero, resulting in the discharge of the fault phase to ground capacitor. The discharge current flows to the fault point through the bus, decays rapidly, and the oscillation frequency is as high as thousands of Hz. This is because the resistance and inductance of the discharge circuit are small, and the oscillation frequency mainly depends on the parameters of the line, location of fault point and value of transition resistance. However, due to the sudden increase of $\sqrt{3}$ times of the voltage to the ground of the non-fault line, the charging current is caused. Because the charging circuit needs to pass through the power supply and the inductance of the whole circulation circuit is large, the attenuation of the charging current is slow and the oscillation frequency is low, only a few hundred hertz. The transient analysis of single-phase grounding fault is introduced in the textbook. It will not be repeated here, but only the conclusion is given. After single-phase grounding fault, the first half wave of the first cycle of transient zero sequence current of fault line is much larger than that of non-fault line, and the direction is opposite [15].

Transient current contains important information of fault mutation. Singularity can be detected and key information can be extracted by using wavelet modulus maxima. There is the following theorem between modulus maxima and singularity of wavelet transform [16]. Let $n$ be a strictly positive integer, $\theta(t)$ a smoothing function, $\psi$ a compactly supported $n$-degree continuously differentiable wavelet function, and $\psi = (-1)^n \theta^{(n)}$. Set $f(t) \in L^1[a,b]$, if there is $s_0 > 0$, so that $|\mathcal{W}_f(s,u)|$ has no local maximum for any $s < s_0$ and $u \in [a,b]$. Then for any $\epsilon > 0$, $f$ is consistent Lipschitz $n$ on $[a+\epsilon, b-\epsilon]$. Wavelet transform has the property of spatial localization, that is, the wavelet transform of the signal at a certain point is completely determined by the local information near the point on a small scale. Wavelet transform can better analyze the position and singularity of the signal singular point. The position of the singular point can be detected by tracking the modulus maximum curve of wavelet transform on a small scale. The strength of signal singularity can be characterized by the attenuation of modulus maxima with scale parameters.

The practical criterion of line selection can be obtained: the signal mutation point at the time of fault corresponds to the modulus maximum point of wavelet transform, the modulus maximum of fault line is the largest, and its polarity is opposite to that of other non-fault lines.
3. Wavelet characteristics and selection principles

3.1. Wavelet characteristics analysis
Vanishing moment order: If the wavelet function $\psi(t)$ has $K$-order vanishing moment, the following conditions are satisfied:

$$\int_{-\infty}^{\infty} t^k \psi(t) dt = 0, \quad k = 0,1,\cdots, K - 1.$$ 

The characteristics of wavelet vanishing moment enable the wavelet function to eliminate the smooth part below order $k$ of the signal, that is, the wavelet transform only reflects the singularity above order $k$ of the signal. In this way, higher-order changes can be studied and higher-order singular points can be detected. The higher the order of vanishing moment, the stronger the concentration of energy after wavelet transform, the stronger the localization ability in frequency domain and the stronger the singularity detection ability. Generally, with the increase of vanishing moment order, the number of modulus extremum points of corresponding decomposition scale will also increase, and the transformation equation becomes more complex, which increases the amount of calculation, so it needs to be considered comprehensively in practice.

Support length: The compact support of wavelet function refers to the characteristic that the wavelet function is not 0 in an interval near 0 and is 0 outside this interval. The support length is the interval length when the base wavelet converges to 0. As the time or frequency tends to infinity, the wavelet function and scale function converge gradually, and the support length indicates the speed of convergence from finite value to zero. The filter length and localization characteristics are reflected by the support length. The shorter the support length, the better the local analysis effect for the time-domain signal.

Symmetry: Set $f(t) \in L^2(\mathbb{R})$, if $f(a + t) = f(a - t)$ is satisfied, $f(t)$ has symmetry. If $f(a + t) = -f(a - t)$ is satisfied, $f(t)$ has anti-symmetry. Symmetry can ensure that the filter has linear phase, which can effectively suppress the problems of frequency aliasing and phase distortion in the process of signal analysis.

Orthogonality: Generally speaking, if the basis $\{\varphi_i\}$ in a space satisfies the orthogonal condition, that is, when $i \neq j$, the inner product of $\varphi_i$ and $\varphi_j$ is zero, it is called the orthogonal basis in this space [17]. If the normalization condition is more satisfied, that is, the inner product of $\varphi_j$ itself is 1, it is called normal orthogonal basis. The subband data obtained by multi-scale decomposition of orthogonal wavelet are distributed in mutually orthogonal subspaces, so that the coefficients on the time-scale plane are not correlated with each other, and then the interaction between adjacent time signals is eliminated. The stronger the orthogonality is, the lower the redundancy is.

3.2. Selection principles
Based on the analysis of wavelet characteristics in Section 3.1, the selection of wavelet in fault line selection shall follow the following principles.

(1) Higher vanishing moment order. When the wavelet $\psi(t)$ has high vanishing moment, the series generated by the wavelet or scale function approximates the smooth function, which can achieve good results. From the perspective of numerical analysis, high vanishing moment can make the calculated matrix sparser; From the signal detection observation, in order to effectively detect singular points, the vanishing moment of wavelet must also have a certain order.

(2) Longer support length. In fault line selection, more attention is paid to the frequency domain information after wavelet decomposition, and the requirements for time domain are not high. The support length affects the localization characteristics. The longer the support length, the more suitable for the frequency domain local analysis of signals.

(3) The influence of vanishing moment order is much greater than that of support length. The support length of a function is independent of its vanishing moment order. If the signal has few isolated singular points and is smooth between singular points, the wavelet with high vanishing
moment should be selected to make the amplitude of many wavelet coefficients very small. The vanishing moment characteristic of wavelet is very important in signal singularity detection.

(4) Symmetry is not required, and orthogonality can reduce redundancy. The line selection based on wavelet transform uses transient information and has no special requirements for symmetry. Orthogonality can reduce redundancy, but it has no direct impact on the result of line selection using modulus maximum criterion.

4. Experiments and analysis

4.1. Experimental model and flow chart
The 10kV system simulation model is shown in Figure 1. The over compensation degree is 10%, and the lengths of four feeders are 25, 7, 8 and 6km respectively. In order to better reflect the multi feeder situation of the actual substation, a longer line 25km is set in the simulation model. Distribution line parameters: $R_1 = 0.1273 \, \Omega/km$, $R_0 = 0.3864 \, \Omega/km$, $L_1 = 0.9337 \, mH/km$, $L_0 = 4.1264 \, mH/km$, $C_1 = 12.74 \, nF/km$, $C_0 = 7.751 \, nF/km$, where subscript "1" represents positive sequence parameters and subscript "0" represents zero sequence parameters. According to the symmetrical component method, the zero sequence current is collected by adding three current measurement modules at the head end of each line, and the zero sequence voltage is collected by adding three voltage measurement modules at the bus, which can be used as the starting signal of single-phase grounding fault alarm of the system. Timer is used to control breaker to simulate fault closing angle, and to workspace module realizes waveform output for wavelet analysis. The detailed experimental flow chart is shown in Figure 2. The $u_0$ setting value can be set as the system unbalanced voltage during normal operation.

![Figure 1. System simulation model.](image1)

![Figure 2. Experimental flow chart.](image2)

4.2. Experimental records and comparative analysis
When a single-phase direct grounding short circuit fault occurs in phase B of line 4 at $t=0.041$s, the zero-sequence voltage and three-phase voltage waveforms are shown in Figure 3.

Figure 3(1) shows that before $t=0.041$s, the system operates symmetrically without zero-sequence component. In case of short circuit, single-phase grounding leads to three-phase asymmetry, which will produce a large zero-sequence voltage, which can be used as the starting signal of single-phase grounding fault alarm. Figure 3(3) shows that the voltage of phase B of ground fault phase drops to 0 after attenuation oscillation. Figures 3(2) and 3(4) show that the voltage of phase A and C of non-fault phase rises to $\sqrt{3}$ times after fault.
Take the grounding transition resistance as 3000 Ω to simulate the high resistance grounding fault. In Figures 4-8, "s" is the original zero-sequence current waveform of fault line 4, and "d1" is the waveform corresponding to d1 scale after four-layer wavelet decomposition of "s".

(1) Experiment 1: Comparison of increasing vanishing moment order and support length

Waveform analysis: It can be seen from Figure 4 that the corresponding values between sampling points 2000 and 2300 are very messy, the modulus maxima after Haar wavelet decomposition is not obvious, and the position of modulus maxima does not exactly correspond to the sudden change time of fault, which will lead to wrong judgment. In Figure 5, the modulus maxima are clearly visible, the value is about 1.2, and other values between the interval [2000,2300] are very small or directly zero, which means that even if there is interference, the judgment of modulus maxima will not be affected. The position of modulus maxima also corresponds to the sudden change time of fault, so accurate line selection can be realized.

The vanishing moment order and support length of Haar wavelet are 1, the vanishing moment order of db10 wavelet is 10 and the support length is 19. The experimental waveform proves the selection principles (1) and (2) in Section 3.2: the higher the vanishing moment order, the better the singularity detection effect of catastrophe point modulus maxima; The longer the support length, the stronger the local analysis ability of frequency domain signal.

Experimental conclusion: the higher the order of vanishing moment and the longer the support length, the greater the number of modulus maxima, that is, the better the effect and the higher the reliability of line selection.

(2) Experiment 2: Comparison of influence degree between vanishing moment order and support length

Waveform analysis: Comparing Figure 6 and Figure 7, there is no modulus maxima at the corresponding fault time for the waveforms decomposed by bior1.1 and bior1.5 wavelet, so it is impossible to correctly judge and select the faulty line. The vanishing moment order of bior1.1 and bior1.5 wavelets is 0, the support length of bior1.1 wavelet is 3, and the support length of bior1.5 wavelet is 11. When the vanishing moment is the same, the change of support length has no effect on the detection results.

Comparing Figure 6 and Figure 8, the modulus maxima of the waveform after bior3.1 wavelet decomposition appears at the corresponding fault time. Figure 8 shows that the value is about 1.2. The
Extreme value is prominent and the position is obvious, which is easy to realize correct line selection. Bior1.1 and bior3.1 wavelet have the same support length of 3, but their vanishing moment order is different. The vanishing moment of bior1.1 wavelet is 0 and bior3.1 wavelet is 2. Their support length is the same. With the increase of vanishing moment order, the detection effects of bior1.1 and bior3.1 wavelet are very different. Obviously, the detection effect of bior3.1 is better, this proves the selection principle (3) in Section 3.2: the influence of vanishing moment order is much greater than that of support length.

![Figure 6. Waveform after Bior1.1 wavelet decomposition.](image)

![Figure 7. Waveform after Bior1.5 wavelet decomposition.](image)

![Figure 8. Waveform after Bior3.1 wavelet decomposition.](image)

Experimental conclusion: compared with the support length, the order of vanishing moment is the key factor affecting the detection result of modulus maxima.

3) Experiment 3: the influence of orthogonality and symmetry

Waveform analysis: Comparing Figure 4 and Figure 8, referring to the above analysis, the detection effect of bior3.1 wavelet is significantly better than Haar wavelet. The characteristics of the two wavelets are further analyzed. Haar wavelet is symmetric, bior3.1 wavelet is asymmetric, Haar wavelet has orthogonality, and bior3.1 wavelet has no orthogonality. This proves the selection principle (4) in Section 3.2: the symmetry of wavelet has little effect on the singularity detection effect, and the orthogonality of wavelet does not play a decisive role.

The vanishing moment order of Haar wavelet is 1 and that of bior3.1 wavelet is 2. Although bior3.1 wavelet has no orthogonality, its detection effect is better because its vanishing moment order is greater than that of Haar wavelet. Compared with orthogonality, the order of vanishing moment is still the key factor affecting the correctness of line selection.

Experimental conclusion: line selection has no special requirements for wavelet symmetry.

5. Conclusions

The influence of vanishing moment order, support length, orthogonality, and symmetry of wavelet on single-phase grounding fault line selection is studied. The principles of wavelet selection in fault line selection are given, that is, higher vanishing moment order, longer support length, no requirement for symmetry, orthogonality can reduce redundancy, but it has no direct impact on the results, and the impact of vanishing moment order is much greater than that of support length. A four feeder 10kV system simulation model is built to simulate high resistance grounding fault. Haar, db10, bior1.1, bior1.5 and bior3.1 wavelet are selected for four scale decomposition. The comparison results of modulus maxima of d1 scale prove the correctness of the principles proposed in this paper.
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