Generalised representation of multi-terminal VSC-HVDC systems for AC–DC power flow studies

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Abstract: This study presents a generalised representation of voltage source converter (VSC) based high voltage direct current (HVDC) systems appropriate for power flow studies using the Newton–Raphson method. To reach this aim, the active loads and ideal synchronous machines are employed in order to incorporate both converter losses and power balance, respectively. Also, considering different aspects of computer implementation, the proposed solution method uses the conventional Newton–Raphson method. The proposed representation considers practical restrictions, switching and conduction losses of semiconductors, and different control strategies for VSC-HVDC stations. Moreover, the proposed generalised representation of VSC-HVDC systems can be easily extended to incorporate the multi-terminal VSC-HVDC grids in an efficient manner. To investigate the application of the proposed representation for VSC-HVDC systems and load flow solution, three test systems including the standard IEEE 30 bus and IEEE two area RTS-96 networks are used and discussion on results is provided. Results show that the proposed algorithm is able to solve AC–DC power flow problems very efficiently with considerably less time in comparison to other existing algorithms.

Nomenclature

Constants

\[ Y_{ij} \quad \text{admittance matrix elements} \]
\[ \theta_{ij} \quad \text{angle of admittance matrix elements} \]
\[ R_{ij}, X_{ij} \quad \text{leakage resistance/reactance from } i \text{ to } j \]
\[ \rho \quad \text{resistant characteristic at the rated voltage and current} \]
\[ I_{VSC} \quad \text{VSC terminal current connected to bus } i \]
\[ P_i^\text{M}, Q_i^\text{M} \quad \text{scheduled active/reactive power of bus } i \]
\[ \alpha, \beta, \gamma \quad \text{constant parameters to calculate VSC power loss} \]

Functions

\[ P_i^M \quad \text{active power of ideal synchronous machine connected to bus } i \text{ which operating as motor} \]
\[ P_i^G \quad \text{active power of ideal synchronous machine connected to bus } i \text{ operating as generator} \]
\[ P_{\text{SW}} \quad \text{switching losses} \]
\[ P_{\text{Loss}} \quad \text{converter losses} \]
\[ P_{i,j}^Q, Q_{i,j} \quad \text{transferred active/reactive power between buses } i \text{ and } j \]
\[ P_{i,j}^Q, Q_{i,j} \quad \text{active/reactive power of bus } i \text{ before installing VSC} \]
\[ P_{i,j}^Q, Q_{i,j} \quad \text{active/reactive power of bus } i \text{ after installing VSC} \]
\[ \Delta P_i, \Delta Q_i \quad \text{active/reactive power mismatch of bus } i \]

Variables

\[ \delta \quad \text{amplitude modulation ratio} \]
\[ I_{\text{SW}} \quad \text{required current at DC side to for switching losses} \]
\[ G_{\text{SW}} \quad \text{conductance at the DC side} \]
\[ I_{VSC} \quad \text{actual VSC terminal current at bus } i \]
\[ \delta_i \quad \text{angle of bus } i \]

1 Introduction

Increasing of electrical power demand and restrictions in construction of new power lines have forced utilising of the existing transmission grids more efficiently [1]. High voltage direct current (HVDC) transmission system is a promising solution for transmitting a large amount of power over long distances. The HVDC transmission system is the only practical way for the integration of remote offshore wind farms to the main grids due to the low transmission losses, lower filter use, active and reactive powers control considering that wind park does not operate necessarily at 50 or 60 Hz [2]. Moreover, since the HVDC systems have no skin effect and the DC current is distributed uniformly in the conductor, these systems require less conductors compared to the HVAC transmission lines with the same length which leads to the less active power loss [3].

In the conventional HVDC transmission systems, line commutated converters (LCCs) are used in which a separate voltage source is required to provide a power source for thyristor valves. Since these power converters operate with the AC voltage leading the current, the process requires the reactive power which is generally about 50% of the transmitted active power. In addition, since the DC-current flow is unidirectional, the power direction can be changed by reversing the polarity of DC-voltage in which the complicated switching arrangements are required as discussed in [4, 5].

In contrast to the LCC-HVDC transmission systems, the VSC-HVDC system uses the voltage source converters (VSCs). If the DC side of VSC is equipped with the active power energy source, generation and absorption of both active and reactive powers, can be provided [6, 7]. To exchange the reactive power with the AC grid, the pulse width modulation (PWM) shifts the output current to the lead or lag the terminal voltage based on the system requirements. Several research papers have discussed basic features on the performance and characteristics of the VSC-HVDC technology [8–10]. Moreover, the recent developments of VSC-HVDC systems such as less switching losses and reduction of harmonic generation have been reported in [11, 12].

Modelling of the VSC-HVDC systems has been presented in [13, 14]. Proposed model of [14] includes two ideal variable AC
different approaches in order to solve the load flow study in the HVDC based combined HV AC–HVDC grid has been compared in the AC and DC systems are solved sequentially, i.e. one is HVDC has been modelled using a controllable voltage source needs the reconstruction of power flow equations. integrated HV AC–HVDC and asynchronous AC grids connected flow approach for the integrated HV AC–HVDC grids has been shown that these voltage sources do not operate independently. This modelling approach is simple; however, the DC side voltage and PWM scheme restrictions are not considered. In [8], VSC-HVDC has been modelled using a controllable voltage source behind an impedance. However, the control modes and phase shifting of PWM are not considered and the concept of power transfer through DC grids is not very clear.

A review of the literature in this field shows that there are two different approaches in order to solve the load flow study in the HVDC-HVAC systems: the sequential and simultaneous techniques [10, 11]. In the sequential technique, the load flow of the AC and DC systems are solved sequentially, i.e. one is determined after another; while, in simultaneous method, the load flow of the entire system is solved altogether. It should be noted that as the main drawback of simultaneous approach, it requires modification of the existing AC power flow since the entire program implementation procedure needs to be reformed to solve the AC and DC system together. The performance of sequential and simultaneous algorithms for power flow studies in the VSC-HVDC based combined HVAC–HVDC grid has been compared in [15]. The VSC-HVDC station model reported in [9], includes the phase-shifting property and scaling nature of the PWM control. It also considers the account switching and ohmic losses. The proposed model is accurate; however, it suffers from the complexity of the solution approach and uses the simultaneous approach to perform power flow. In [16], a multi-option power flow approach for the integrated HVAC–HVDC and asynchronus AC grids connected via a common DC-link. The proposed approach is actually an extension of the simultaneous method's application and therefore needs the reconstruction of power flow equations.

This paper deals with developing a generalised representation and formulation of the VSC-HVDC systems suitable for the load flow analysis considering different aspects by using sequential technique. As the most salient feature of this paper, hybrid active load and ideal synchronous machine representation are introduced for the VSC-HVDC appropriate for well-known conventional Newton–Raphson (NR) based load flow technique. Proposed representation includes different control schemes and practical limitations and it also considers the DC side parameters such as PWM limitations and switching losses. Active load and synchronous machines are used to include the switching losses and provide power balance, respectively. The proposed solution technique is general and describes the concept of power transfer efficiently and can be applied to different kinds of multi-terminal VSC-HVDC systems. It also provides the use of the prevalent NR load flow method without any change.

2 Proposed equivalent circuit for VSC-HVDC representation

2.1 General representation for VSC-HVDC station

Fig. 1a illustrates the schematic representation of the VSC-HVDC station and Fig. 1b shows the proposed equivalent circuit for the VSC-HVDC station from the AC side point of view. This representation includes an ideal synchronous machine, active load (P_L) and coupling impedance (R_L + jX_L). It is worth noting that the voltage magnitude of the virtual bus in the proposed equivalent circuit for VSC is a function of the DC side voltage and amplitude modulation ratio. In the proposed representation, an ideal synchronous machine is employed to provide the active power balance which is required to be transmitted through the VSC-HVDC system (P). It also provides generation or absorption of the required reactive power based on the used control strategy like the voltage control mode or the reactive power control mode (Q) which will be discussed. In summary, for power flow studies, active and reactive powers of the synchronous machine should be determined according to the required active power which needs to be transmitted and the used control strategy, respectively. It is worth noting that the active load is used to model the switching losses, which is a function of the VSC terminal current [8, 9]. According to this representation for VSC-HVDC, it is concluded that no change is required to be applied to prevalent the NR power flow solution.

2.2 Equivalent circuit for the VSC-HVDC model

As shown in Fig. 2, the proposed VSC-HVDC representation consists of two virtual buses (j and n) and each virtual bus is connected to coupling impedance (R_L + jX_L and R_L + jX_L). active load (P_L and P_L) for the switching losses modelling and ideal synchronous machine (P + jQ) and (P + jQ) for providing the power balance. It is worth noting that in this figure, it is assumed that VSC connected to bus absorbs active power (VSC acts as the ideal synchronous motor) and VSC connected to bus n injects active power (VSC acts as an ideal synchronous generator). The voltage magnitude of the virtual bus is a function of amplitude modulation ratio as follows [9]:

\[ V = \theta V_{dc} \]  \hspace{1cm} (1)

For the sake of simplicity, as mentioned earlier, one of the ideal synchronous machine operates as a motor and other one as a generator. In summary, synchronous motor models the absorbed active power at sending end which needs to be transferred through the VSC-HVDC and synchronous generator is used to model the active power at receiving end. The active loads are included to model the switching losses of each converter. It should be noted that the transferred active power through the DC link could be combined to active loads; however, for providing better interpretation ideal synchronous machines are employed.

3 VSC-HVDC switching and ohmic losses

As mentioned earlier, the active load connected to the virtual bus is employed to model switching losses of VSC while the ohmic losses can be modelled by adding resistance in series with the

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**Fig. 1 VSC-HVDC station**

(a) Schematic representation, (b) Model

**Fig. 2 Proposed equivalent circuit of a VSC-HVDC installed between buses i and m of an AC/DC grid**
coupling impedance of each VSC-HVDC station. The proposed representation can easily consider different models for converter power loss. Several formulas are proposed and discussed in the literature for the converter losses [9, 14]. In [9], switching losses are taken into account by adding a conductance to the DC side of the converter which incorporates both the terminal current and DC side voltage as shown in the following equation:

\[ p_{SW} = V_d G = V_d (G_{SW} V_d) = G_{SW} V_d \]

where \( G_{SW} \) is related to the terminal current of VSC and it is determined by using \( \rho (\alpha \beta)^2 \). In [14], a loss formula is proposed in which the converter losses are quadratically dependent on the terminal current as shown in the following equation:

\[ p = \alpha + \beta i_{VSC}^2 + \gamma i_{VSC}^2 \]

Considering this equation, it can be seen that the last term can be applied to the proposed representation by adding resistance by the value of \( \gamma \) in series with the coupling impedance. Other terms can be considered in the active load as shown in Fig. 1, as switching losses. However, (3) can be directly included in the proposed representation as the sum of switching and ohmic losses.

4 Modelling of multi-terminal VSC-HVDC grids and determining the related parameters of ideal synchronous machines

Load flow of the DC grid of a multi-terminal VSC-HVDC can be analysed considering one converter as the constant voltage bus and other buses by constant active power. It should be noted that due to the similarity of this bus to slack bus in the AC power flow algorithms, this converter is considered as the DC grid slack converter. Same as the AC power flow, active power injection of DC slack converter is assumed unknown and depends on the DC grid losses. In the proposed representation for VSC-HVDC systems, the active power of each synchronous machine must be determined according to the scheduled active power which is required to be transmitted through each DC link. Considering the DC side, it is assumed that in each DC grid, one converter acts as a DC slack bus and other converters operate in power control mode. However, it should be noted that based on the requirements, the DC bus voltage control mode can be easily considered. There is one DC slack converter in each multi-terminal direct current (MTDC) system that adjusts their power to maintain the DC grids voltages and consider the DC system losses.

To implement power flow analysis, the total system is divided into two main sub networks as discussed in the following:

(i) Equivalent DC systems in which voltage and active power of each DC bus in each DC grid is calculated in order to determine the required parameters for the synchronous machines. Power flows in DC grids is dictated by differences in the DC buses’ voltage magnitude.

(ii) Equivalent AC system which uses the results of the equivalent DC system to form Jacobian Matrix (JM) and solve load flow problem.

It should be noted that both sub networks should be solved separately. First, in each DC grid of the system, after forming the admittance matrix based on the system transmission lines, prevalent NR load flow solution is performed to determine the DC side voltage of each VSC-HVDC station assuming that the generated/absorbed active power of each station is known. A typical presentation of multi-terminal HVDC grid is illustrated in Fig. 3.

For instance, from the AC system point of view, the converter connected to bus 2 operates as a synchronous motor and then its absorbed active power is equal to \( P_2 \) and the converter connected to bus \( n \) operates as a synchronous generator and its generated active power is equal to \( P_n \). Then, load flow of the AC system is performed based on the control strategy of each station (as will be discussed in the following sections) and determined active power for each VSC-HVDC station which is obtained according to the DC grid limitations like terminal current and modulation index associated to each converter are checked in each iteration. Fig. 4 summarises the flow chart of the proposed solution method for the MT VSC-HVDC systems. As it can be seen in this figure, first, the DC side load flow is performed and calculated voltage and power of each VSC-HVDC station is determined. Then, the AC side load flow is conducted considering determined parameters of each VSC-HVDC resulted from DC side load flow along with the control mode. However, it should be noted that if its assumed power flow of each converter from AC side point of view is known, then, first AC system is solved and after that DC grids are analysed. It should be noted that the proposed solution method can be used in the simultaneous approach as well. However, as the integrated JM of AC–DC system should be reconstructed in this approach, computer implementation, in this case, is more complex compared to the sequential method. It is worth mentioning that it is assumed that all the calculations are performed and shown in p.u.

5 Power flow equations in presence of VSC-HVDC

A review of the literature shows that power flow equations in the presence of HVDC systems is greatly affected. For instance, in [9],
for the developed power flow model a 10 × 10 matrix is required for each VSC-HVDC station which increases the computational burden. One of the main salient features of the proposed representation is that no additional variables need to be defined to form the power-flow equations in the presence of VSC-HVDC, except those required for the internal virtual buses which are easily included. Since the used virtual buses in the proposed model are only connected to the terminal buses, their associated equations can be simply applied to the JM and can consider different operation modes.

This section first represents a general NR power flow model for VSC-HVDC and then different control strategies are applied to form the JM associated to each operation mode. Since VSCs operate independently in the reactive power flow, following equations can be easily applied to each converter station. According to the power flow balance, active and reactive powers of the bus $i$ before installing VSC are as follows:

$$P_i = \sum_{k=2}^{n} \sum_{j=2}^{n} |V_i||V_j|Y_{ij}\cos(\delta_i - \delta_j + \theta_{ij})$$

$$Q_i = -\sum_{k=2}^{n} \sum_{j=2}^{n} |V_i||V_j|Y_{ij}\sin(\delta_i - \delta_j + \theta_{ij})$$

Considering each virtual bus, general form of NR power flow model can be shown as (5):

$$\begin{align*}
\Delta P_i &= \left[ \frac{\partial P_i}{\partial \delta_i} \frac{\partial P_j}{\partial \delta_i} \frac{\partial P_j}{\partial \delta_j} \frac{\partial Q_j}{\partial \delta_i} \frac{\partial Q_k}{\partial \delta_j} \right] \Delta \delta_i \Delta \delta_j \\
\Delta Q_i &= \left[ \frac{\partial Q_j}{\partial \delta_i} \frac{\partial Q_i}{\partial \delta_i} \frac{\partial Q_j}{\partial \delta_j} \frac{\partial Q_k}{\partial \delta_i} \frac{\partial Q_k}{\partial \delta_j} \right] \Delta \delta_i \Delta \delta_j
\end{align*}$$

In the general form, both the active load and synchronous machine connected to the virtual bus are included in the power flow equations. Based on the used control mode, (5) can be simplified for each as discussed by providing further details in the following sections.

### 5.1 Voltage control mode

As mentioned before, the VSC station can control the reactive power injection to the AC grid. This feature allows the VSC to affect the AC grid voltage profile in a wide range. In the voltage control operation mode, considering Fig. 1b, the VSC station injects the required reactive power ($Q^s$) to maintain the voltage magnitude of bus $i$ at the specified value. Equation (6) shows the partial derivation terms which form the Jacobian elements with respect to bus $j$ variables while $Y_{ik} = 0$ for $k = 2, 3, ..., n$ and $k \neq i, j$

$$\frac{\partial P_i}{\partial \delta_j} = 0, \quad k = 2, 3, ..., n, \quad k \neq i, j$$

$$\frac{\partial Q_i}{\partial \delta_j} = 0, \quad k = 2, 3, ..., n, \quad k \neq i, j$$

It is worth noting that since the virtual bus $j$ is only connected to bus $i$, then derivative terms of $P$ and $Q$ of all buses expect for bus $i$ are zero to one. By excluding the row related to injected reactive power and the column related to voltage magnitude, the modified version of (4) and (5) under the voltage control mode are obtained as (7) and (8), respectively. In comparison with (4), it can be concluded that the proposed approach has just led to the changes in equations associated to the active and reactive powers of bus $i$

$$\begin{align*}
\Delta P_i &= \left[ \frac{\partial P_i}{\partial \delta_i} \frac{\partial P_j}{\partial \delta_i} \frac{\partial P_j}{\partial \delta_j} \frac{\partial Q_i}{\partial \delta_i} \frac{\partial Q_k}{\partial \delta_j} \right] \Delta \delta_i \\
\Delta Q_i &= \left[ \frac{\partial Q_j}{\partial \delta_i} \frac{\partial Q_i}{\partial \delta_i} \frac{\partial Q_j}{\partial \delta_j} \frac{\partial Q_k}{\partial \delta_i} \frac{\partial Q_k}{\partial \delta_j} \right] \Delta \delta_i
\end{align*}$$

5.2 Reactive power control mode

If reactive power control mode is used, $Q^s$ shown in Fig. 1b is directly controlled to reach a specified value. This parameter is the reactive power injected to bus $i$ of the AC system and can be used in order to provide power factor modification. In this case, JM is obtained by substituting $Q_i$ by $Q^s$ in (5). Expanding (9) for the depicted buses, in Fig. 1, injected reactive power is obtained as shown in the following equation:

$$Q^s = |V_i||V_j|\sin(\theta_{ij}) - |V_i||V_j|\sin(\delta_i - \delta_j + \theta_{ij})$$

### 5.3 Amplitude modulation ration control mode

In this mode of operation, the virtual bus acts such as a $PV$ bus and thus the row associated to $Q_i$ and the column related to $|V_j|$ are omitted form (5). The obtained NR flow model is as (13). It should be noted the mismatch power terms in this case are similar to that of the voltage control mode

$$\begin{align*}
\Delta P_i &= \left[ \frac{\partial P_i}{\partial \delta_i} \frac{\partial P_j}{\partial \delta_i} \frac{\partial P_j}{\partial \delta_j} \frac{\partial Q_j}{\partial \delta_i} \frac{\partial Q_k}{\partial \delta_j} \right] \Delta \delta_i \\
\Delta Q_i &= \left[ \frac{\partial Q_j}{\partial \delta_i} \frac{\partial Q_i}{\partial \delta_i} \frac{\partial Q_j}{\partial \delta_j} \frac{\partial Q_k}{\partial \delta_i} \frac{\partial Q_k}{\partial \delta_j} \right] \Delta \delta_i
\end{align*}$$

5.4 Implementation aspects of the proposed method

5.4.1 State variables and increments: As mentioned before, there is no need for the additional variables to describe the
operation of VSC, except those of the virtual bus \(|V_j|, \delta_j\). It is worth noting that increments of the state variables at iteration \((k)\) associated with the voltage and reactive power control modes are reported in (14)

\[
\Delta|V|^k_j = |V|^k_j - |V|^{k-1}_j \\
\Delta\delta^k_j = \delta^k_j - \delta^{k-1}_j
\]

(14)

Moreover, increments of the state variables at iteration \((k)\) associated to amplitude modulation ratio control just deals with (15)

\[
\Delta\delta^k_j = \delta^k_j - \delta^{k-1}_j
\]

(15)

5.4.2 Practical limitations, power flow initialisation and convergence criteria: Considering limitations of both the voltage and current, the minimum/maximum producible/absorbable power of each VSC converter must meet the acceptable margin. In addition, the VSC overcurrent may cause thermal instability either in inductive or capacitive modes; this constraint may be applied to the VSC power while its terminal voltage is supposed to be a constant value. When the reactive power reaches its boundary, the switching loss of each converter is determined according to (2) in which \(\rho\) and \(I_{sc}^0\) are 0.01 and 1, respectively [9]. All generators maintain the terminal voltage magnitude at 1 p.u. In this case, the converter connected to bus 2 acts as the DC slack bus with the voltage value of \(\sqrt{2}\) p.u. Equivalent DC grid system along with load flow results are presented in Fig. 6. Total active loss in DC grid is 0.005 p.u. and power flow converges in six iterations to an accuracy of 4.012 \times 10^{-11}. Based on the active power value and operation mode of each converter, JM is formed according to the proposed representation and control mode. Load flow solutions from AC side view point are depicted in Fig. 7 for each converter.

As it can be seen in this figure, the converter connected to bus 2 absorbs the active power while converters connected to buses 4, 6 inject the active power to the AC grid. In this case, all converters inject the reactive power to the AC grid in order to maintain the voltage at the specified value.

In Table 1, results of the proposed model and results of the presented model in [9] are compared. As expected, results are equal. As mentioned before, in the proposed model, all DC side elements are transferred to the AC side considering the power transfer concept. Therefore, reduction in matrix dimensions from

6 Application of different control modes

Considering different control modes described in the previous sections, following details can be summarised.

(i) If the aim is to control the voltage at the AC grid, the voltage control mode is used. In this mode, the injected reactive power is controlled to provide the constant voltage at the VSC station terminal.

(ii) If the aim is to provide power factor correction or achieving specified voltage is not possible due to the practical limitations, injected reactive power is set to the desired value by using reactive power control mode.

(iii) If it is desired to limit the modulation ratio to specified values to prevent over modulation, for example, modulation ratio control mode is used.

It is worth noting that in the first two control modes, first, both DC bus and virtual bus voltages are determined by performing load flow for DC and AC grids and then modulation ratio is calculated. However, in the third operation mode, after the DC bus voltage, virtual bus voltage is determined and then this bus acts like a PV bus in load flow of AC grids.

7 Tests and results

In this section, the proposed model for VSC-HVDC is used and solution approach for the combinational AC/DC power flow algorithm is implemented on three test systems including the modified IEEE 30 bus and modified IEEE two area RTS-96 networks.

7.1 Test case 1

In the first test case, the VSC-HVDC link is used to interconnect three independent AC grids. Notice that in this case, since the back-to-back VSC-HVDC provides an asynchronous interconnection of the three AC sub-systems, then each AC subsystem requires its own slack bus.

Fig. 5 represents the multi-terminal VSC-HVDC test system in which the VSC stations connected to buses 2, 4 and 6 are acting in the voltage control operation mode in order to maintain the terminal voltage at 1.05, 1.03 and 1.03 p.u., respectively. This operation mode is used in order to compare the results with the proposed model in [9] since only the voltage control mode is considered in this reference. It is assumed that the AC side transmission lines have an impedance of 0.05 + j0.1 p.u., coupling impedance of each VSC station is 0.01 + j0.1 p.u. and resistance of each DC cable is 0.02 p.u.

During the system operation, no active or reactive power exchange occurs between AC side and DC side. As it can be seen in this figure, the converter connected to bus 2 absorbs the active power while converters connected to buses 4, 6 inject the active power to the AC grid. In this case, all converters inject the reactive power to the AC grid in order to maintain the voltage at the specified value.

In Table 1, results of the proposed model and results of the presented model in [9] are compared. As expected, results are equal. As mentioned before, in the proposed model, all DC side elements are transferred to the AC side considering the power transfer concept. Therefore, reduction in matrix dimensions from

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the AC side point of view is expected. In this case, the required simulation time for the proposed model is 0.319 s while using the proposed model in [9], the required time is 0.338 s.

7.2 Test case 2

In this section, a modified IEEE 30-bus test system [18] is used in order to investigate the benefits of the proposed method to analyse the multi-terminal VSC-HVDC systems. In this test case, the multi-terminal VSC-HVDC system is used in order to connect buses 2, 4, and 6 as depicted in Fig. 8. It is assumed that the DC side transmission lines have a half resistance of AC transmission lines. In order to consider different control strategies, the VSC-HVDC station connected to bus 2 operates in the reactive power control mode and injects 0.20 p.u. reactive power. VSC's connected to buses 4 and 6 operate in voltage control mode and maintain the terminal voltage at 1.020 and 1.060 p.u., respectively. It is also assumed that the impedance of each VSC-HVDC station is combined with the connecting transformer and it has value of 0.0016 + j0.0310 p.u. It should be noted that since the VSC-HVDC station connected to bus 6 is assumed to operate in voltage control mode in order to improve the voltage profile of downstream buses. Moreover, since VSC-HVDC station connected to bus 2 operates in reactive power control mode, it is assumed that generator connected to bus 2 regulates the terminal voltage at 1.03 p.u. in order to decrease the absorbed reactive power by the slack generator connected to bus 1.

It is assumed that in the MTDC grid, the VSC stations connected to buses 4 and 6 absorb 0.255 and 1.294 p.u. active power, respectively. Equivalent DC grid along with load flow results are depicted in Fig. 9. It should be mentioned that the DC side load flow converges in four iterations to an accuracy of $9.290 \times 10^{-6}$. Table 1 summarises operation mode of each VSC-HVDC according to Fig. 10. Total active loss in the DC grid is 0.018 p.u. and load flow converges in six iterations to an accuracy of $1.802 \times 10^{-11}$.

After solving the DC power flow problem based on the scheduled active power transfer, AC side load flow is performed considering the operation mode of each VSC-HVDC station (Table 2). It should be mentioned that the AC side load flow converges in eight iterations to an accuracy of $3.094 \times 10^{-5}$. Voltage profile resulted from power flow is shown in Fig. 8. In this figure, the voltage profile of the original IEEE 30-bus system is also included in order to provide a comparison between two cases. According to Fig. 11, the voltage profile improves significantly by employing the VSC-HVDC system so that the average voltage magnitude increases for 1.090%. Power flow solution of each converter is shown in Table 3.

In the presence of VSC-HVDC system, the generated active power reduces for 0.279% and absorbed reactive power by slack generator reduces for 97.033%. The active and reactive powers' loss of AC transmission system reduces for 20.57 and 36.67%, respectively. As mentioned in the previous sections, the proposed solution approach uses additional variables, so that the required simulation time for the included test case by using the proposed method is 0.4109 s while using the approaches described in [8, 9] take 0.9302 and 1.5219 s, respectively.

7.3 Test case 3

In this section, in order to show that the representation is general, results of the proposed model are compared with those reported in [8]. In this test system, results are presented on a modified version...
of the IEEE two area RTS-96 network [14] which is shown in Fig. 12. In [8], first, AC system power flow is solved since it is assumed that active power at the converter terminal is known. Details regarding the characteristics of the used converters and modified network are provided in [8]. It is worth noting that in this test case, the loss formula of each converter is described by 
\[ P_{loss} = a + b \times I_c + c \times I_c^2 \]
in which \( I_c \) is converter current.

In this system, there are three asynchronous AC systems with different voltage levels depicted in Fig. 12 by using dashed lines.

DC 1 with 1 p.u. DC side voltage is assumed to act as slack DC bus in the first MTDC with DC link rated voltage equal to ±150 kV. It is worth noting that generator connected to bus 302 is assumed to be offshore wind farm and available active power at bus 301 is 150 MW. Also, active power of bus 203 is controlled to be 75 MW. In this MTDC system, in the DC grid, DC 3 injects active power while both DC 1 and DC 2 absorb active power. Also, from the AC system point of view, DC 1 and DC 3 operate in reactive power control mode with \( Q^{inj} = 0 \) while DC 2 operates in voltage control mode and keeps the terminal voltage at 1 p.u.

In the second MTDC grid with the rated voltage of ±300 kV, DC 4 acts as DC slack bus. Same as the first MTDC grid, DC side voltage of the slack bus is 1 p.u. In this case, from AC side point of view, DC 5 and DC 6 operate as synchronous motors and absorb 50 and 135 MW, respectively, while DC 7 acts as a synchronous generator and injects 50 MW. Moreover, all converters operate in reactive power control mode with \( Q^{inj} = 0 \).

Considering the operation from the AC side point of view, since AC grids are asynchronous, load flow is performed for each AC grid separately. In AC grid 1, load flow converges in three iterations to an accuracy of \( 3.205 \times 10^{-11} \). In AC grid 2, load flow converges in four iterations to an accuracy of \( 9.904 \times 10^{-9} \). Finally, the load flow solution of AC grid 3 converges in four iterations to an accuracy of \( 1.482 \times 10^{-8} \).

After the load flow of AC grids, load flow of each DC grid is performed. According to the obtained results, active power loss of each converter along with DC side active power is calculated. Power flow of the first MTDC grid converges in three iterations to an accuracy of \( 4.320 \times 10^{-8} \). Power flow solution of each converter in the first MTDC is shown in Table 4.

It is worth noting that considering the DC grid, converters 1, 2 and 3 absorb 68.47 MW, absorb 76.76 MW and inject 146.16 MW, respectively (as listed in Table 4). Moreover, DC side voltages of DC 2 and DC 3 in per unit are 0.999 and 1.006, respectively. In Table 4, converter loss and transformer loss are separated and transformer loss can be calculated from Table 4. For instance, considering DC 1, transformer loss is equal to (68.47–66.84 MW) – (0.200 MW) = 75 MW. In this MTDC system, in the DC grid, DC 3 injects active power while both DC 1 and DC 2 absorb active power. Also, from the AC system point of view, DC 1 and DC 3 operate in reactive power control mode with \( Q^{inj} = 0 \) while DC 2 operates in voltage control mode and keeps the terminal voltage at 1 p.u.

The same procedure is followed in order to analyse MTDC grid 2. Power flow solution of the MTDC grid 2 converges in five iterations to an accuracy of \( 4.320 \times 10^{-11} \). Results are shown in Table 5. As expected, reactive power injection to AC grids at the terminal is equal to zero for all converters. It is worth noting that the results of this section, completely match results obtained and reported in [8] which shows that the proposed method can easily include previous models with different loss equations.

### 7.4 Discussion

According to the obtained results in previous sections, the following can be concluded:

1. Proposed representation for VSC-HVDC system can easily describe the concept of power transfer. This is done by using synchronous machines to describe injected power to the AC grid (synchronous generator) or absorbed power from the AC grid (synchronous motor).

### Table 2  Operation mode of each converter

| VSC number | Operation mode | DC side voltage, p.u. | Active power, p.u. |
|------------|----------------|----------------------|-------------------|
| 1          | synchronous motor | 1.414                | 1.567             |
| 2          | synchronous generator | 1.399                | 0.255             |
| 3          | synchronous generator | 1.397                | 1.294             |

### Table 3  Operation mode of each converter

| VSC number | Control mode | Terminal active power, p.u. | Terminal reactive power, p.u. | Active power loss, p.u. | Terminal voltage | Terminal angle |
|------------|--------------|-------------------------------|--------------------------------|------------------------|-----------------|----------------|
| 1          | P–Q          | 1.573                         | 0.200                          | 0.006                  | 1.030           | −6.629         |
| 2          | P–V          | 0.257                         | 0.088                          | 0.002                  | 1.020           | −4.683         |
| 3          | P–V          | 1.302                         | 0.425                          | 0.008                  | 1.060           | −11.267        |
Converter loss is included in the proposed representation by using active power. In general, the value of this active power depends on current (or other parameters) as discussed in Section 3.

Different operation modes are considered according to the developed equations. Based on system requirements, terminal voltage or injected reactive power to the AC grid can be controlled as discussed in Section 5.

No new variable is added to the system except virtual bus voltage and therefore prevalent NR solution method without any modification is used. This is one of the main salient features of the proposed representation since Jacobian elements are not modified.

Previous models, for instance, advance models proposed in [8, 9], can be easily described according to the proposed representation.

In this paper, generalised representation of the VSC-HVDC stations appropriate for AC–DC load flow studies has been proposed and analysed. Utilisation of active load and the ideal synchronous machine has been proposed to model the switching losses and required active and reactive powers based on the control strategy, respectively. Furthermore, DC side load if exists could be combined with the switching losses or transferred active power as a constant term. This paper also described a comprehensive load flow solution for different VSC-HVDC station operation modes considering the NR load flow algorithm which is able to solve the large and complex hybrid AC–DC grid power flow equations in a reliable and fast manner. The eligibilities of proposed generalised representation as well as load flow solution have been analysed and confirmed by the numeric examples.

### 8 Conclusion

In this paper, generalised representation of the VSC-HVDC stations appropriate for AC–DC load flow studies has been proposed and analysed. Utilisation of active load and the ideal synchronous machine has been proposed to model the switching losses and required active and reactive powers based on the control strategy, respectively. Furthermore, DC side load if exists could be combined with the switching losses or transferred active power as a constant term. This paper also described a comprehensive load flow solution for different VSC-HVDC station operation modes considering the NR load flow algorithm which is able to solve the large and complex hybrid AC–DC grid power flow equations in a reliable and fast manner. The eligibilities of proposed generalised representation as well as load flow solution have been analysed and confirmed by the numeric examples.

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**Table 4** Power flow of each converter used in the MTDC grid 1

| VSC number | Control mode | Terminal active power, MW | DC side active power, MW | Terminal reactive power, MW | Active power loss, MW | Terminal voltage, p.u. | Terminal angle, ° |
|------------|--------------|---------------------------|--------------------------|-----------------------------|----------------------|------------------------|-------------------|
| DC 1       | P–Q          | 66.84                     | 68.47                    | 0                           | 1.56                 | 1.025                  | −8.93             |
| DC 2       | P–V          | 75                        | 76.76                    | 5.86                        | 1.67                 | 1.000                  | −4.88             |
| DC 3       | P–Q          | 150                       | 146.16                   | 0                           | 3.52                 | 1.050                  | −0.08             |

**Table 5** Power flow of each converter used in the MTDC grid 2

| VSC number | Control mode | Terminal active power, MW | DC side active power, MW | Terminal reactive power, MW | Active power loss, MW | Terminal voltage, p.u. | Terminal angle, ° |
|------------|--------------|---------------------------|--------------------------|-----------------------------|----------------------|------------------------|-------------------|
| DC 4       | P–Q          | 124.43                    | 127.49                   | 0                           | 2.81                 | 1.020                  | 0                 |
| DC 5       | P–Q          | 50                        | 48.8                     | 0                           | 1.36                 | 1.050                  | 10.11             |
| DC 6       | P–Q          | 135                       | 131.67                   | 0                           | 3.04                 | 1.014                  | 10.25             |
| DC 7       | P–Q          | 50                        | 51.37                    | 0                           | 1.3                  | 1.039                  | 14.60             |
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