Extracting Lifted Mutual Exclusion Invariants from Temporal Planning Domains

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Abstract

We present a technique for automatically extracting mutual exclusion invariants from temporal planning instances. It first identifies a set of invariant templates by inspecting the lifted representation of the domain and then checks these templates against properties that assure invariance. Our technique builds on other approaches to invariant synthesis presented in the literature, but departs from their limited focus on instantaneous actions by addressing temporal domains. To deal with time, we formulate invariance conditions that account for the entire structure of the actions and the possible concurrent interactions between them. As a result, we construct a significantly more comprehensive technique than previous methods, which is able to find not only invariants for temporal domains, but also a broader set of invariants for non-temporal domains. The experimental results reported in this paper provide evidence that identifying a broader set of invariants results in the generation of fewer multi-valued state variables with larger domains. We show that, in turn, this reduction in the number of variables reflects positively on the performance of a number of temporal planners that use a variable/value representation by significantly reducing their running time.

Keywords: Automated Planning, Temporal Planning, Mutual Exclusion Invariants, Automatic Domain Analysis

1. Introduction

This paper presents a technique for synthesising mutual exclusion invariants from temporal planning domains expressed in PDDL2.1 [Fox and Long, 2003]. A mutual exclusion invariant over a set of ground atoms means that at most one atom in the set is true at any given moment. A set with this property can intuitively be seen as the domain of a multi-valued state variable\(^1\). For

\(^1\)To be precise, a set of mutually exclusive atoms is the domain of an implicit state variable only when augmented with a catch-all null value, which can be manufactured on demand. This is because a mutual exclusion invariant encodes the concept of “at most one”, whereas a state variable encodes the concept of “at least one”. The null value is then used when no atom is true, if these situations exist.

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instance, consider the Floortile domain from the 8th International Planning Competition (IPC8 - see Appendix A). A mutual exclusion invariant for this domain states that two ground atoms that indicate the position of a robot can never be true at the same time. Intuitively, this means that a robot cannot be at two different locations simultaneously. To give a concrete case, consider a planning problem for the Floortile domain with one robot r1 and three locations, t1, t2 and t3. We can create a state variable that indicates the position of r1 with a domain of three values: robot-at_r1_t1, robot-at_r1_t2 and robot-at_r1_t3.

Although a number of approaches to invariant synthesis have been proposed so far (Gerevini and Schubert, 2000; Rintanen, 2000, 2008; Fox and Long, 1998; Helmert, 2009), they are limited in scope because they deal with non-temporal domains only. Recently, Rintanen (2014) has proposed a technique for temporal domains, but this technique does not scale to complex problems because it requires to ground the domain. Our approach solves both these problems at the same time. We find invariants for temporal domains by applying an algorithm that works at the lifted level of the representation and, in consequence, is very efficient and scales to large instances.

Our invariant synthesis builds on Helmert (2009), which presents a technique to translate the non-temporal subset of PDDL2.2 (Edelkamp and Hoffmann, 2004) into FDR, a multi-valued planning task formalism used by Fast Downward (Helmert, 2006). Since finding invariants for temporal tasks is much more complex than for tasks with instantaneous actions, a simple generalisation of Helmert’s technique to temporal settings does not work. In the temporal case, simultaneity and interference between concurrent actions can occur, hence our algorithm cannot check actions individually against the invariance conditions, but needs to consider the entire set of actions and their possible intertwinnings over time. In capturing the temporal case, we formulate invariance conditions that take into account the entire structure of the action schemas as well as the possible interactions between them. As a result, we construct a significantly more comprehensive technique that is able to find not only invariants for temporal domains, but also a broader set of invariants for non-temporal domains.

Our technique is based on a two-steps approach. First, we provide a general theory at the ground level and propose results that insure invariance under two types of properties: safety conditions for individual instantaneous and durative actions as well as collective conditions that prevent dangerous intertwinnings between durative actions. Then, we lift these results at the level of schemas so that all checks needed for verifying invariance can be performed at this higher level, without the need of grounding the domain. Complexity of such checks are of linear or low polynomial order in terms of the number of schemas and literals appearing in the domain.

1.1. Motivations

Automated planning is a well-established field of artificial intelligence and, over more than fifty years since its appearance, several paradigms have emerged. One fundamental difference between these paradigms is whether time is treated implicitly or explicitly. While classical planning focuses on the causal evolution of the world, temporal planning is concerned with the temporal properties of the world. In classical planning, actions are considered to be instantaneous, whereas in temporal planning actions have durations and can be executed concurrently. Another important difference between planning paradigms relates to whether the world is modelled by adopting a Boolean propositional representation or a representation based on multi-valued state variables. Traditionally, the majority of the work in planning has been devoted to classical

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2We call IPC3-IPC8 the competitions for temporal planning held in 2002, 2004, 2006, 2008, 2011 and 2014.
planning with domains expressed through propositional languages, and in particular PDDL (McDermott, 2000) and its successors (Fox and Long, 2003), the language of the IPC. However, in parallel with the development of classical propositional planning, a number of temporal planning systems have been proposed for coping with practical problems, especially space mission operations (Frank and Jónsson, 2003; Chien et al., 2000; Ghallab and Laruelle, 1994; Muscettola, 1994; Fratini et al., 2008). Typically, these systems use variable/value representations. Table 1 shows a classification of several well-known planners based on these different characteristics.

Recently, a few techniques have been proposed for translating propositional representations into variable/value representations (Helmert, 2006; Bernardini and Smith, 2008b; Rintanen, 2014). A central task of all these techniques is the generation of state variables from propositions and actions. The basic procedure to do this (which we use as the baseline in our experiments) relies on generating one state variable with two values, true and false, for each proposition. Naturally, such translation produces no performance advantage. A more sophisticated strategy, which produces compact and optimised encodings, rests on extracting mutual exclusion invariants from propositional domains and using such invariants to generate multi-valued state variables. This is the focus on our work.

These translation techniques are important as they allow fair testing of planners developed for variable/value representations on PDDL benchmarks (which are propositional). The practical issue is that planners that permit variable/value representation need this feature to be well-exploited and perform competitively. Since translation between the two different representations can be cheaply automated, there is no reason to reject providing the richer representations to those planners that accept them (if the translation was expensive, one might reasonably argue about the fairness of this process). In consequence, these translating techniques are extremely useful for comparing alternative planning paradigms and for promoting cross-fertilisation of ideas between different planning communities, which is our primary motivation.

However, the importance of these translation techniques goes beyond the engineering of a bridge between different input languages. In transforming propositional representations into state variable representations, they generate new domain knowledge, where new means accessible in this context. Effectively, these techniques turn into internal mini theorem provers since, rather than merely translating, they firstly selectively explore the deductive closure of the original theory to find theorems that permit optimising the representation and secondly execute those
optimisations. We will show that the cost of performing these optimisations is worth it because it is very fast and can be amortised over many problems.

Mutual exclusion invariants are also beneficial in pruning the search space for search methods such as symbolic techniques based on SAT (Kautz and Selman [1999] Huang et al. [2010]) and backward chaining search (Blum and Furst [1997]). In addition, as invariants help to reduce the number of variables required to encode a domain, they are used in planning systems based on binary decision diagrams (BDDs) (Edelkamp and Helmert [2001]), constraint programming (Do and Kambhampati [2001]), causal graph heuristics (Helmert [2006]) and pattern databases (Haslum et al. [2007]).

Finally, from a knowledge engineering perspective, the invariant synthesis presented in this paper can be used as a powerful tool for debugging temporal planning domains, although we do not focus on this specifically in this paper. As shown in Cushing et al. (2007), several temporal planning tasks developed for the various IPC editions are bugged with the consequence that the planners take a long time to solve them, when they actually manage to do so. As invariants capture intuitive properties of the physical systems described in the domains, it is easy for a domain expert to identify modelling mistakes by inspecting them. Discrepancies between the invariants found by the automatic technique and those that the expert expects to see for a given domain indicate that the domain does not encode the physical system correctly. In consequence, the expert can revise the domain and repair it. For example, considering the Rover domain, we expect that a store could be either full or empty at any time point. However, the invariant synthesis does not produce an invariant with the atoms full and empty. It can be shown that this is because the action drop is not properly modelled. Our technique not only alerts the expert that the system is not properly modelled, but also refers the expert to the action that is not encoded correctly. This is a useful feature to fix modelling errors quickly and safely.

1.2. Contributions of the paper

In brief, the contributions of this paper are the following.

From a theoretical point of view:

• We give the first formal account of a mutual exclusion invariant synthesis for temporal domains that works at the lifted level of the representation. Our presentation of this topic is rigorous and comprehensive and our theory is general and not tailored around IPC domains as with related techniques.

• Our technique is based on inferring general properties of the state space by studying the structure of the action schemas and the lifted relations in the domain, without the need to ground it. This is generally an hard task. Our theoretical framework is sophisticated, but it results in practical tools that have high efficiency and low computational cost.

From a practical point of view:

• We provide a tool for optimising the generation of state variables from propositions and actions. This results in compact encodings that benefit the performance of planners, as we will show in our experimental results (see Section 10).

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3One might argue that optimising is a more technically precise term than translating
We offer a technique that can be used as a debugging tool for temporal planning domains. As this type of domains are particularly challenging to encode, especially when large and complex, a rigorous debugging process is crucial in producing correct representations of the systems under consideration.

1.3. Organisation of the paper

This paper is organised as follows. After presenting PDDL and PDDL2.1, our input languages, in Section 2, we formally introduce the notion of invariance in Section 3. Sections 4, 5 and 6 are devoted to a detailed analysis of actions at the ground level. In particular, Section 4 focuses on instantaneous actions: the fundamental concept of strong safety is introduced and analysed and a first sufficient result for invariance, Corollary 24, is established. Section 5 analyses sequences of actions and, in particular, durative actions (seen as a sequence of three instantaneous actions) for which two new concepts of safety are proposed and investigated: individual and simple safety. Our main technical results are presented in Section 6 and consist of Theorem 49, Theorem 51 and Corollary 57; these results insure invariance under milder safety requirements on the durative actions than Corollary 24. This is obtained by adding requirements that prevent the intertwining of durative actions that are not strongly safe.

Sections 7 and 8 are devoted to lift the concepts and results obtained in the previous sections to the level of action schemas. In particular, Section 7 deals with the problem of lifting the concept of strong safety for instantaneous schemas, while Section 8 considers durative action schemas and presents the lifted version of our main results, Corollaries 95, 96 and 97. These results are the basic ingredients of our algorithm to find invariants, which is proposed in Section 9. Section 10 reports an extensive experimental evaluation of our approach against the domains of the last three IPCs. Sections 11 and 12 conclude the paper with a description of related works and closing remarks. There are three appendices: Appendix A and B contain the specifications of the planning domains used as the running examples in the paper; Appendix C contains all the technical proofs.

2. Canonical Form of Planning Tasks

In this work, we consider planning instances that are expressed in PDDL2.1 (Fox and Long, 2003). However, before applying our algorithm to find invariants, we manipulate the domain to enforce a regular structure in the specification of the action schemas. In what follows, we first give an overview of this canonical form that we use and then describe how such a form can be obtained starting from a domain expressed in PDDL2.1.

2.1. PDDL Canonical Form

A planning instance is a tuple $I = (D, P)$, where $D$ is a planning domain and $P$ a planning problem. The domain $D = (R, A^i, A^d, arity)$ is a 4-tuple consisting of finite sets of relation symbols, instantaneous actions, durative actions, and a function arity mapping all of these symbols to their respective arities. $P = (O, Init, G)$ is a triple consisting of the objects in the domain, the initial logical state and the goal logical state specifications.

The ground atoms of the planning instance, $Atms$, are the (finitely many) expressions formed by applying the relations in $R$ to the objects in $O$ (respecting arities). A logical state is any subset of $Atms$ and $S = 2^{Atms}$ denotes the set of logical states. The initial state $Init \in S$ and the goal $G \subseteq S$ is a subset of logical states (typically defined as a conjunction of literals). A state is a
tuple in \( \mathbb{R} \times \mathcal{S} \), where the first value is the time of the state and the second value (logical state) is a subset of \( \mathcal{A}mss \). The initial state for \( I \) is of the form \( (t_0, \text{Init}) \).

The set \( \mathcal{A} \) is a collection of instantaneous action schemas. An instantaneous action schema \( \alpha \) is composed of the following sets:

- \( V_\alpha \), i.e. the free schema’s arguments;
- \( Pre_\alpha = Pre_\alpha^+ \cup Pre_\alpha^- \), where \( Pre_\alpha^+ \) are the positive preconditions and \( Pre_\alpha^- \) the negative preconditions;
- \( E f f_\alpha = E f f_\alpha^+ \cup E f f_\alpha^- \), where \( E f f_\alpha^+ \) are the add effects and \( E f f_\alpha^- \) the delete effects.

Preconditions and effects are sets of literals \( l \) of the form: \( \forall v_1, \ldots, v_k : q \) where \( q \) is a non-quantified positive literal of the form \( r(v'_1, \ldots, v'_k) \), where \( r \in \mathcal{R} \), \( \text{arity}(r) = n \), \( \{v_1, \ldots, v_k\} \subseteq \{v'_1, \ldots, v'_k\} \) is the set of quantified arguments, \( \{v'_1, \ldots, v'_k\} \setminus \{v_1, \ldots, v_k\} \subseteq V_\alpha \) is the set of free arguments. The universal quantification can be trivial (i.e. quantification over zero arguments) and, in this case, it is omitted. We indicate the set of the positions of the free and the quantified arguments, respectively, as \( \text{Var}[l] \) and \( \text{Varq}[l] \), and the pair \( (r, a) \), where \( r \) is the relation symbol that appears in literal \( q \) and \( a \) is its arity, as \( \text{Rel}(l) \). Given a position \( i \), we indicate the corresponding argument as \( \text{Arg}[i, l] \).

The set \( \mathcal{A}' \) is a collection of durative action schemas. A durative action schema \( Da \) is a triple of instantaneous action schemas \( \alpha = (\alpha^{st}, \alpha^{in}, \alpha^{end}) \) such that \( V_\alpha^{st} = V_\alpha^{in} = V_\alpha^{end} \) (this common set is denoted \( V_{Da} \)). We indicate as \( [Da] \) the set \( \{\alpha^{st}, \alpha^{in}, \alpha^{end}\} \).

We call \( \mathcal{A} \) the set of all the instantaneous actions schemas in the domain, including those induced by durative actions: \( \mathcal{A} = \mathcal{A}' \cup \bigcup_{Da \in \mathcal{A}'} \{Da\} \). Given any two action schemas \( \alpha_1 \) and \( \alpha_2 \) in \( \mathcal{A} \) such that it does not exist a durative action \( Da \) with both \( \alpha_1 \) and \( \alpha_2 \) in \( \{Da\} \), we assume that the free arguments of \( \alpha_1 \) and \( \alpha_2 \) are disjoint sets, i.e. \( V_{\alpha_1} \cap V_{\alpha_2} = \emptyset \).

Given an action schema \( \alpha \in \mathcal{A}' \) with free arguments \( V_\alpha \), consider an injective grounding function \( gr : V_\alpha \rightarrow O \) that maps the free arguments in \( \alpha \) to objects \( O \) of the problem. The function \( gr \) induces a function on the literals in \( \alpha \) as follows. Given a literal \( l \) that appears in \( \alpha \), we call \( gr(l) \) the literal that is obtained from \( l \) by grounding its free arguments according to \( gr \) and \( gr(l) \) the set of ground atoms obtained from \( gr(l) \) by substituting objects in \( O \) for each quantified argument in \( l \) in all possible ways. Note that, when there are no quantified arguments, \( gr(l) = gr(l) \) and they are singletons. Given a set \( L \) containing literals \( l_1, \ldots, l_n \), we call \( gr(L) = gr(l_1) \cup \ldots \cup gr(l_n) \) and \( gr(L) = gr(l_1) \cup \ldots \cup gr(l_n) \). We call \( gr(\alpha) \) the action schema obtained from \( \alpha \) by grounding each literal \( l \) that appears in \( \alpha \) according to \( gr \) and \( gr(\alpha) \) the ground action that is obtained from \( gr(\alpha) \) by replacing the quantified arguments with the set of ground atoms formed by substituting objects in \( O \) for the quantified arguments in all possible ways.

Given a durative action schema \( Da \in \mathcal{A}' \) and a grounding function \( gr \), the ground durative action \( gr(Da) \) is obtained by applying \( gr \) to the instantaneous fragments of \( Da \): \( gr(Da) = (gr(\alpha^{st}), gr(\alpha^{in}), gr(\alpha^{end})) \). Note that we cannot apply different grounding functions to different parts of a durative action schema.

Given a ground action \( a \), we indicate its positive and negative preconditions as \( Pre_a^+ \) and its add and delete effects as \( E ff_a^+ \). We call \( G\mathcal{A} \), \( G\mathcal{A}' \), respectively, the set of instantaneous, and durative ground actions. Finally, we call \( G\mathcal{A} \) the set of all ground actions in \( I \) (obtained from grounding all schemas in \( \mathcal{A} \)).
A ground action \( a \) is applicable in a logical state \( s \) if \( \text{Pre}_a^+ \subseteq s \) and \( \text{Pre}_a^- \cap s = \emptyset \). The result of applying \( a \) in \( s \) is the state \( s' \) such that \( s' = (s \setminus \text{Eff}_a^-) \cup \text{Eff}_a^+ \). We call \( \xi \) this transition function: \( s' = \xi(s, a) \).

The transition function \( \xi \) can be generalised to a set of ground actions \( A = \{a_1, \ldots, a_n\} \) to be executed concurrently: \( s' = \xi(s, A) \). However, in order to handle concurrent actions, we need to introduce the so-called no moving targets rule: no two actions can simultaneously make use of a value if one of the two is accessing the value to update it. The value is a moving target for the other action to access. This rule avoids conflicting effects, but also applies to the preconditions of an action: no concurrent actions can affect the parts of the state relevant to the precondition tests of other actions in the set (regardless of whether those effects might be harmful or not). In formula, two ground actions \( a \) and \( b \) are non-interfering if:

\[
\text{Pre}_a \cap (\text{Eff}_a^+ \cup \text{Eff}_b^-) = \text{Pre}_b \cap (\text{Eff}_b^+ \cup \text{Eff}_a^-) = \emptyset
\]

\[
\text{Eff}_a^- \cap \text{Eff}_b^- = \text{Eff}_b^+ \cap \text{Eff}_a^+ = \emptyset
\]

If two actions are not non-interfering, they are mutex.

In this work, whenever we consider a set of concurrent actions \( A = \{a_1, \ldots, a_n\} \), we implicitly assume that the component actions are pairwise non-interfering. In this case, given a state \( s \) such that each \( a_i \in A \) is applicable in \( s \), the transition function \( s' = \xi(s, A) \) is defined as follows:

\[
s' = (s \setminus \bigcup_{a \in A} \text{Eff}_a^+) \cup \bigcup_{a \in A} \text{Eff}_a^+
\]

The following useful result shows that the application of a set of actions can always be serialised.

**Proposition 1** (Serialisability). Given a set of actions \( A = \{a_1, \ldots, a_n\} \) and a state \( s \) in which \( A \) is applicable, consider the sequence of states recursively defined as \( s_0 = s \) and \( s_k = \xi(s_{k-1}, a_k) \) for \( k = 1, \ldots, n \). Then,

(i) The sequence \( s_k \) is well defined: \( a_k \) is applicable in \( s_k \) for every \( k = 1, \ldots, n \);

(ii) \( s_n = \xi(s, A) \).

An instantaneous timed action has the following syntactic form: \( (t, a) \), where \( t \) is a positive rational number in floating point syntax and \( a \) is a ground instantaneous action. A durative timed action has the following syntactic form: \( (t, Da[t']) \), where \( t \) is a rational valued time, \( Da \) is a ground durative action and \( t' \) is a non-negative rational-valued duration. It is possible for multiple timed actions to be given the same time stamp, indicating that they should be executed concurrently.

A simple plan \( \pi \) for an instance \( I \) is a finite collection of instantaneous timed actions and a plan \( \Pi \) consists of a finite collection of (instantaneous and durative) timed actions. The happening time sequence \( [t_{\Pi}]_{t=0}^{\infty} \) for a plan \( \Pi \) is:

\[
\{t_0\} \cup \{t(t, a) \in \Pi \ or (t, Da[t']) \in \Pi \ or (t - t', Da[t']) \in \Pi\}
\]

Note that the last disjunct allows the time corresponding to the end of execution of a durative action to be included as a happening time.

Given a plan \( \Pi \), the induced simple plan for \( \Pi \) is the set of pairs \( \Pi \) containing:

(i) \( (t, a) \) for each \( (t, a) \in \Pi \), where \( a \) is an instantaneous ground action;
(i) \((t, a^w)\) and \((t + t', a^{end})\) for all pairs \((t, Da[t'])\) \(\in \Pi\), where \(Da\) is a durative ground action; and

(ii) \(((t_1 + t_{i+1})/2, a^{inv})\) for each pair \((t, Da[t'])\) \(\in \Pi\) and for each \(i\) such that \(t \leq t_i < (t + t')\), where \(t_i\) and \(t_{i+1}\) are in the happening sequence for \(\Pi\).

The happening at time \(t\) of the plan \(\pi\) is defined as \(A_t = \{a \in G|A| \exists (t, a) \in \pi\}.\) Note that in \(\pi\) we have formally lost the coupling among the start and end fragments of durative actions. Since in certain cases this information is necessary, we set a definition: a durative action \(Da\) is said to happen in \(\pi\) in the time interval \([t, t + t']\) whenever this holds true in the original plan \(\Pi\), namely when \((t, Da[t'])\) \(\in \Pi\).

A simple plan \(\pi\) for a planning instance \(I\) is executable if it defines a happening sequence \(\{t_i\}_{i=0}^k\) and there is a sequence of logical states \(\{s_i\}_{i=0}^k\) such that \(s_0 = Init\) and for each \(i = 0, \ldots, k, s_{i+1}\) is the result of executing the happening at time \(t_i\) in \(\pi\). Formally, we have that \(A_{init}\) is applicable in \(s_i\) and \(s_{i+1} = \xi(s_i, A_{init})\). The state \(s_k\) is called the final logical state produced by \(\pi\). The sequence of times and states \(\{s_i = (t_i, s_i)_{i=0}^k\}\) is called the (unique) trace of \(\pi, \text{trace}(\pi)\). Two simple plans are said to be equivalent if they give rise to the same trace.

The following result holds, from the definition of mutex, induced plan and executability:

**Remark 2.** Given a ground durative action \(Da = (a^w, a^{inv}, a^{end})\) and a ground instantaneous action \(a'\), if \(a'\) and \(a^{inv}\) are mutex, then there is no executable simple plan that contains the timed actions \((t_1, a^w), (t_2, a')\) and \((t_3, a^{end})\) with \(t_1 < t_2 < t_3\).

A simple plan for a planning instance \(I\) is valid if it is executable and produces a final state \(s_k \in G\).

We call Plans all the valid (induced and not) simple plans for \(I\) and \(S_r\) the union of all the logical states that appear in the traces associated with the plans in Plans: \(S_r = \{s | \exists \pi \in Plans \text{ and } (t, s) \in \text{trace}(\pi)\}\). Note that \(S_r \subseteq S\). We call the states in \(S_r\) reachable states.

### 2.2. From PDDL2.1 to Canonical PDDL

We build the canonical form described above starting from PDDL2.1 instances, which are characterised by metric and temporal information (Fox and Longi, 2003). Numeric variables can be seen as already in the variable/value form and so we do not handle them. We could potentially exploit metric information in order to find additional state variables, but currently we do not do that. Instead, we assume that numeric variables are already in the right form and ignore them and numeric constraints when we look for logical state variables.

Temporal information are handled in PDDL2.1 by means of durative actions. They can be either discretised or continuous, but we focus on discretised durative actions only here. They have a duration field and temporally annotated conditions and effects. The duration field contains temporal constraints involving terms composed of arithmetic expressions and the dedicated variable duration. The annotation of a condition makes explicit whether the associated proposition must hold at the start of the interval (the point at which the action is applied), the end of the interval (the point at which the final effects are asserted) or over all the interval (open at both ends) from the start to the end (invariant over the duration of the action). The annotation of an effect makes explicit whether the effect is immediate (it happens at the start of the interval) or delayed (it happens at the end of the interval). No other time points are accessible. Logical changes are considered to be instantaneous and can only happen at the accessible points. To build our canonical form, we transform durative actions into triples of instantaneous actions. We do this in such a way that we do not change the set of plans that can be obtained for any goal.
specification. Plans with durative actions, in fact, are always given a semantics in terms of the semantics of simple plans (Fox and Long 2003), as explained in the previous section.

Let us see now in more detail how we obtain the PDDL canonical form from PDDL2.1 instances.

A PDDL2.1 instance looks the same as a canonical instance, except for the set of action schemas in the domain. In particular, in a PDDL2.1 domain, in place of the sets $\mathcal{A}^c$ and $\mathcal{A}^d$, we find a set $\mathcal{A}^d$ that contains both instantaneous and durative action schemas, which have the following characteristics. Durative action schemas have temporally annotated conditions and effects, which we indicate as $\text{Pre}^\alpha$ and $\text{Eff}^\alpha$, where $\alpha$ is in the set \{st, inv, end\}. Given an action schema in $\mathcal{A}^d$ (durative or not), the condition formula can be a relation, a negation, a conjunction or disjunction of relations or a quantified formula on relations. The effect formula can be a relation, a negation or a conjunction of relations, a universally quantified formula on relations or a conditional effect formula, which is a tuple formed by a precondition formula and an effect formula. We manipulate the action schemas in $\mathcal{A}^d$ to obtain $\mathcal{A}^c$ and $\mathcal{A}^d$, where each action schema in these sets has the canonical form described in Section 2.1.

First, we eliminate conditional effects and existentially quantified formulae through an operation referred to as flattening (see Fox and Long 2003 for details). Since these features are syntactic sugar, they can be eliminated by applying simple syntactic transformations. The resulting schemas are equivalent to the original ones.

Given a flatten action schema $\alpha$, we take the formulas (temporally annotated or not) in its conditions and effects and normalise them by using the algorithm introduced by Helmert (2009) (we refer the interested reader to this paper for a full description of the normalisation process 4). After normalisation, all action schema conditions and effects become sets of universally quantified first-order literals $l$ of the form $\forall v_1, \ldots, v_k : q$, where $q$ is a non-quantified literal and the universal quantification can be trivial. We indicate by $\text{Pre}^\alpha_{st}$ and $\text{Eff}^\alpha_{st}$ the set of positive literals that appear positive in $\alpha$ and by $\text{Pre}^\alpha_{inv}$ and $\text{Eff}^\alpha_{inv}$ the set of positive literals that appear negative in $\alpha$.

Note that we consider illegal durative action schemas $Da$ such that it exists a literal $l$ that satisfies one of the following conditions:

- $l \in \text{Pre}^{\text{inv}}_{Da}$ and $l \in (\text{Pre}^{\alpha st}_{Da} \cup \text{Eff}^{\alpha st}_{Da} \cup \text{Eff}^{\alpha inv}_{Da})$;
- $l \in \text{Pre}^{\alpha inv}_{Da}$ and $l \in (\text{Pre}^{\alpha st}_{Da} \cup \text{Eff}^{\alpha st}_{Da} \cup \text{Eff}^{\alpha inv}_{Da})$;
- $l \in \text{Pre}^{\text{inv}}_{Da}$ and $l \in \text{Pre}^{\alpha end}_{Da}$;
- $l \in \text{Pre}^{\alpha inv}_{Da}$ and $l \in \text{Pre}^{\alpha end}_{Da}$

We assume that $\mathcal{A}^d$ contains no durative action schemas of such types.

After flattening and normalisation, we transform the durative action schemas in $\mathcal{A}^d$ in triples of instantaneous action schemas. Given a durative action $Da \in A^d$, we create two instantaneous action schemas that correspond to the end points of $Da$, $\alpha^{\text{st}}$ and $\alpha^{\text{end}}$, and one that corresponds to the invariant conditions that must hold over that duration of $Da$, $\alpha^{\text{inv}}$. More formally, given a durative action schema $Da$ we create $\alpha^{\text{st}}$, $\alpha^{\text{inv}}$ and $\alpha^{\text{end}}$ as follows:

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4Our normalisation differs from Helmert (2009) only in that we preliminarily eliminate conditional effects by applying the flattening operation before normalisation and we keep universal quantification in the preconditions. We also apply normalisation not only to formulas appearing in instantaneous actions as in Helmert (2009), but also to temporally annotated formulas in durative actions. We normalise the formulas and leave the temporal annotation unchanged.
\[ \begin{array}{lll}
\alpha^{st} & \alpha^{inv} & \alpha^{end} \\
Pre^{\alpha^{st}} = Pre^{\alpha^{st}}_{D\alpha} & Pre^{\alpha^{inv}} = Pre^{\alpha^{inv}}_{D\alpha} & Pre^{\alpha^{end}} = Pre^{\alpha^{end}}_{D\alpha} \\
Pre^{\alpha^{inv}} = Pre^{\alpha^{inv}}_{D\alpha} & Eff^{\alpha^{inv}} = \emptyset & Eff^{\alpha^{end}} = \emptyset \\
Eff^{\alpha^{st}} = Eff^{\alpha^{st}}_{D\alpha} & Eff^{\alpha^{inv}} = \emptyset & Eff^{\alpha^{end}} = \emptyset \\
\end{array} \]

Table 2: Transformation of durative action schemas in triples of instantaneous action schemas.

At this point, we are ready to construct \( A_i \) and \( A_d \) from \( A_a \). We add each flatten and normalised instantaneous action in \( A_a \) to \( A_i \). For each durative action \( D\alpha \in A_a \), after applying flattening and normalisation, we create the corresponding tuple \((\alpha^{st}, \alpha^{inv}, \alpha^{end})\) and add it to \( A_d \).

Given a planning instance \( I \) in canonical form obtained from a PDDL2.1 instance \( I' \) and a valid plan \( \Pi \) for \( I \), \( \Pi \) can be converted into an equivalent valid plan \( \Pi' \) for \( I' \).

2.3. Running Example: the Floortile Domain

We use the Floortile domain as our running example. It has been introduced in the IPC-2014 and then reused in 2015. The full PDDL2.1 specification is available in Appendix A. The domain describes a set of robots that use different colours to paint patterns in floor tiles. The robots can move around the floor tiles in four directions (up, down, left and right). Robots paint with one color at a time, but can change their spray guns to any available color. Robots can only paint the tile that is in front (up) and behind (down) them, and once a tile has been painted no robot can stand on it.

We have the following relations in this domain: \( R = \{ \text{up, down, right, left, robot-at, robot-has, painted, clear, available-color} \} \). They have arity two, except for the last two, which have arity one. \text{clear} indicates whether a tile is still unpainted, \text{available-color} whether a color gun is available to be picked by a robot and \text{up, down, right, left} indicate the respective positions of two tiles.

The set of instantaneous action schemas \( \mathcal{A}^i \) is empty, while the set of durative action schemas \( \mathcal{A}^d \) is the following: \( \mathcal{A}^d = \{ \text{change-color, paint-up, paint-down, up, down, right, left} \} \).

As an example, the durative action schema \text{paint-up} corresponds to the following triple: \( (\text{paint-up}^{st}, \text{paint-up}^{inv}, \text{paint-up}^{end}) \), where the single instantaneous action schemas have the following specifications:

\[
\begin{array}{llll}
\alpha^{st} & \alpha^{inv} & \alpha^{end} \\
V_a & \{r.y,x,c\} & \{r.y,x,c\} & \{r.y,x,c\} \\
Pre_a & \{\text{robot - at}(r,x)\} & \{\text{robot - has}(r,c)\} & \emptyset \\
& \text{clear}(y) & \text{up}(y,x) & \emptyset \\
Eff_a & 0 & 0 & 0 \\
\end{array}
\]

Table 3: Durative action schema \text{paint-up} seen as a triple of instantaneous action schemas.
Note that the triple of single instantaneous action schemas in canonical form is obtained from the following PDDL2.1 specification:

```
( :durative-action paint-up
  :parameters (?r - robot ?y - tile ?x - tile ?c - color)
  :duration (= ?duration 2)
  :condition (and (over all (robot-has ?r ?c))
               (at start (robot-at ?r ?x))
               (over all (up ?y ?x))
               (at start (clear ?y)))
  :effect (and (at start (not (clear ?y)))
             (at end (painted ?y ?c)))
```

3. Mutual Exclusion Invariants and Templates

In this section, we formally introduce the concepts of invariant and mutual exclusion invariant and give examples of them.

**Definition 3 (Invariant).** An invariant of a PDDL2.1 planning instance is a property of the world states such that when it is satisfied in the initial state $Init$, it is satisfied in all reachable states $S_r$.

For example, given the Floortile domain, a trivial invariant says that for each object $x$, if $x$ is a robot, then $x$ is not a tile. Similar invariants hold for each type defined in the domain. A more interesting invariant says that, for any two objects $x$ and $y$, if $up(x, y)$ holds, then $down(y, x)$ holds too, but $down(x, y)$ does not. It is possible to identify several invariants for the Floortile domain, ranging from trivial invariants such as those involving type predicates to very complex invariants.

In this paper, we focus on mutual exclusion invariants, which state that a set of ground atoms can never be true at the same time.

**Example 1 (Floortile domain).** A mutual exclusion invariant for this domain states that two ground atoms indicating the position of a robot identified as $rbt1$, such as $robot-at(rbt1, tile1)$ and $robot-at(rbt1, tile2)$, can never be true at the same time. Intuitively, this means that $rbt1$ cannot be in two different positions simultaneously. Another more complex invariant states that, given a tile $tile1$, a robot $rbt1$ and a colour $clr1$, atoms of the form $clear(tile1)$, $robot-at(rbt1, tile1)$ and $painted(tile1, clr1)$ can never be true at the same time. This means that a tile can be in one of three possible states: not yet painted (clear), occupied by a robot that is painting it or already painted.

**Definition 4 (Mutual Exclusion Invariant).** Given a planning instance $I$, let $Z$ be a set of ground atoms in $2^{Atms}$. A mutual exclusion invariant is an invariant stating that at most one element of $Z$ is true in any reachable state. We refer to a set $Z$ with this property as a mutual exclusion invariant set.

In what follows, we refer to mutual exclusion invariants and mutual exclusion invariant sets as simply invariants and invariant sets for the sake of brevity.

Although we aim to find sets of mutually exclusive ground atoms, we often work with relations and action schemas to control complexity. A convenient and compact way for indicating several invariant sets at the same time involves using invariant templates, which are defined below, after introducing a few preliminary definitions.
**Definition 5 (Component).** A component $c$ is a tuple $<r,a,p>$, where $r$ is a relation symbol in $\mathcal{R}$, $a$ is a number that represents the arity of $r$, i.e. $a = \text{arity}(r)$, and $p \in \{0,\ldots,a\}$ is a number that represents the position of one of the arguments of $r$, which is called the counted argument. We put $p = a$ if there are no counted arguments. The set of the labelled fixed arguments of $c$ is $F_c = \{(c,i) | i = 0,\ldots,(a-1); i \neq p\}$.

Given a set of components $C = \{c_1,c_2,\ldots,c_n\}$, we define the set of fixed arguments of $C$ as $F_C = \bigcup_{c \in C} F_c$.

**Definition 6 (Admissible Partition).** Given a set of components $C$ and a set of fixed arguments $F_C$, an admissible partition of $F_C$ is a partition $\mathcal{F}_C = \{G_1,\ldots,G_k\}$ such that $|G_j \cap F_c| = 1$ for each $c \in C$.

Given two elements $(c_1,i)$ and $(c_2,j)$ of $F_C$ that belong to the same set of the partition $\mathcal{F}_C$ we use the notation: $(c_1,i) \sim_{\mathcal{F}_C} (c_2,j)$.

**Remark 7.** Note that the existence of an admissible partition of $F_C$ implies that all the components in $C$ have the same number of fixed arguments, which is also the number of the sets in the partition. In the special case in which the number of fixed arguments in each component is equal to one, there is just one admissible (trivial) partition $\mathcal{F}_C = \{F_C\}$.

**Definition 8 (Template).** A template $\mathcal{T}$ is a pair $(C,\mathcal{F}_C)$ such that $C$ is a set of components and $\mathcal{F}_C$ is an admissible partition of $F_C$. We simply write $\mathcal{T} = (C)$ when the partition is trivial, i.e. $\mathcal{F}_C = \{F_C\}$.

By previous considerations, the set of relations appearing in the set of components $C$ of a template, up to a permutation of the position of the arguments, will always have the following form:

$$\{r_i(x^i_1,\ldots,x^i_l,v^i) | i = 1,\ldots,n_1\} \cup \{r_j(x^j_0,\ldots,x^j_k) | i = n_1 + 1,\ldots,n_1 + n_2\}$$

where $v^i$s indicate the counted arguments, $x^i_j$s the fixed arguments and $x^j_i \sim_{\mathcal{F}_C} x^j_j$ for every $i,j$.

**Definition 9 (Template’s Instantiation).** Given a template $\mathcal{T}$, an instance $\gamma$ of $\mathcal{T}$ is a function that maps the elements in $F_C$ to the objects $O$ of the problem $\mathcal{P}$ such that $\gamma(c_1,i) = \gamma(c_2,j)$ if and only if $(c_1,i) \sim_{\mathcal{F}_C} (c_2,j)$.

**Definition 10 (Template’s Instantiation).** The instantiation of $\mathcal{T}$ according to instance $\gamma$, $\gamma(\mathcal{T})$, is the set of ground atoms in $2^{\text{Atoms}}$ obtained as follows: for each component $c = <r,a,p>$ of $\mathcal{T}$, take the relation symbol $r$, for each element $(c,i) \in F_C$ bind the argument in position $i$ according to $\gamma((c,i))$ and the counted argument in position $p$ to all the objects $O$ of the problem $\mathcal{P}$.

Given how we construct $\gamma(\mathcal{T})$, it is easy to see that its elements will look as follows: $\gamma(\mathcal{T}) = \{r_i(\gamma(x^i_1),\ldots,\gamma(x^i_l),v^i) | i = 1,\ldots,n_1\} \cup \{r_j(\gamma(x^j_0),\ldots,\gamma(x^j_k)) | i = n_1 + 1,\ldots,n_1 + n_2\}$.

Instances are interesting because they can be used to reason about their (exponentially larger) instantiations without, in fact, constructing those instantiations.

Given a template $\mathcal{T}$ and an instance $\gamma$, if the ground atoms in the instantiation of $\mathcal{T}$ according to $\gamma$ are mutually exclusive in the initial state $\text{Init}$ and remain such in any state reachable $s \in S_r$, then $\gamma(\mathcal{T})$ is (by definition) a mutual exclusion invariant set. A template with this property for each possible instantiation $\gamma$ is called invariant template.

**Definition 11 (Invariant Template).** A template $\mathcal{T}$ is an invariant template if, for each instance $\gamma$, the instantiation of $\mathcal{T}$ according to $\gamma$ is a mutual exclusion invariant set.
Given an invariant template $T$, we can create one state variable for each of its instances. The domains of these variables are the corresponding mutual exclusion invariant sets with an additional null value, which is used when no element in the mutual exclusion invariant set is true.

Before describing in what situations we can feasibly prove that a template is invariant, we introduce a final concept:

**Definition 12 (Template Instance’s Weight).** Given an instance $\gamma$ of a template $T$, its weight in a state $s$, $w(T, \gamma, s)$, is the number of ground atoms in its instantiation $\gamma(T)$ that are true in $s$:  
$$w(T, \gamma, s) = \left| s \cap \gamma(T) \right|$$

**Proposition 13.** A template $T$ is an invariant template if and only if, for each instance $\gamma$ and each state $s \in S$, $w(T, \gamma, s) \leq 1$.

**Proof.** It follows from Definitions 11 and 12.

**Example 2 (FloorTile domain).** A template for this domain is $T_{fi} = \{(c_1, c_2, c_3)\}$, where:

- $c_1 = \{\text{robot-at}(0, 0)\}$ is the first component. It includes the relation robot-at that has an arity of two (i.e. the relation robot-at(robot, tile) has two arguments) and the argument in position zero, i.e. robot, is the counted argument. The remaining argument, tile, which is in position one, is the fixed argument: $F_{c_1} = \{(c_1, 1)\}$.
- $c_2 = \{\text{painted}(0, 1)\}$ is the second component with $F_{c_2} = \{(c_2, 0)\}$.
- $c_3 = \{\text{clear}(0, 1)\}$ is the last component with $F_{c_3} = \{(c_3, 0)\}$.

Note that, since the three components have one fixed argument, all components are in the same equivalent class (trivial partition).

Assume that we have a problem $P$ with two robots rbt1 and rbt2, three tiles, tile1, tile2 and tile3 and one colour black. Consider one possible instance $\gamma_1$ such that $\gamma_1((c_1, 1)) = \gamma_1((c_2, 0)) = \gamma_1((c_3, 0)) = \text{tile1}$. The instantiation of the template $T_{fi}$ according to $\gamma_1$ is: $\gamma_1(T_{fi}) = \{\text{robot-at}(\text{rbt1, tile1}), [\text{robot-at}(\text{rbt2, tile1})]+, [\text{painted}(\text{tile1, black})], \text{clear}(\text{tile1})\}$.

The weight of the instance $\gamma_1$ in a state $s$ is the number of ground instantiations of $[\text{robot-at}]+, [\text{robot-at}]+, [\text{painted}(\text{tile, colour})]+$ and $[\text{clear}(\text{tile})]+$ that are true in $s$, where the variable tile has been instantiated as tile1. If we have a state $s$ in which no instantiations of robot-at(robot, tile) and painted(tile, colour) are true, but clear(tile) is true, the weight in $s$ is one.

We will see that we can actually prove that $T_{fi}$ is an invariant, which states that a tile can be clear, already painted or in the process of being painted by a robot. Hence, for the problem $P$, we can create a state variable that represents each of the three tiles, whose values are the possible configurations of such tiles and the null value. It can also be proved that at least and at most one element of the mutual exclusion invariant set need to be true in any reachable state and, in consequence, the null value can be removed from the domain of the state variable. Hence we have: $SV_{tile1} = \{\text{robot-at}(\text{rbt1, tile1}), \text{robot-at}(\text{rbt2, tile1}), \text{painted}(\text{tile1, black}) \text{ clear}(\text{tile1})\}$ and similarly for $SV_{tile2}$ and $SV_{tile3}$. 

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4. Safe Instantaneous Ground Actions

In this and in the following sections, given a planning instance \( I = (\mathcal{D}, \mathcal{P}) \) and a template \( \mathcal{T} \), we discuss the conditions that \( \mathcal{T} \) needs to satisfy to be an invariant. We make the standing assumption that the initial state \( \text{Init} \) satisfies the weight condition \( w(\mathcal{T}, \gamma, \text{Init}) \leq 1 \) for every instance \( \gamma \) and determine sufficient conditions on the families of instantaneous and durative actions in \( \mathcal{D} \) that ensure that \( \mathcal{T} \) is an invariant. In this section, in particular, we work out a concept of safety for instantaneous actions that guarantees that, when a safe action is executed, the weight bound is not violated.

4.1. Safe instantaneous ground actions

We assume a template \( \mathcal{T} \) to be fixed as well an instance \( \gamma \). Consider a set of concurrent pairwise non-interfering ground actions \( A \subseteq \mathcal{G}A \). The set of states \( s \in \mathcal{S} \) on which \( A \) is applicable is denoted by \( \mathcal{S}_A \). We start with the following definition:

Definition 14 (Strongly safe actions). The set of actions \( A \) is strongly \( \gamma \)-safe if, for each \( s \in \mathcal{S}_A \) such that \( w(\mathcal{T}, \gamma, s) \leq 1 \), the successor state \( s' = \xi(s, A) \) is such that \( w(\mathcal{T}, \gamma, s') \leq 1 \).

The study of strong \( \gamma \)-safety for an action set \( A \) can be reduced to the study of the state dynamics on the template instantiation \( \gamma(\mathcal{T}) \). This is intuitive and is formalised below.

Remark 15. Given an action \( a \in \mathcal{G}A \) define \( a_{\gamma} \) and \( a_{\neg \gamma} \) as the actions, respectively, specified by

\[
\begin{align*}
\text{Pre}_{a_{\gamma}} &= \text{Pre}_a \cap \gamma(\mathcal{T}), \\
\text{Pre}_{a_{\neg \gamma}} &= \text{Pre}_a \cap \neg \gamma(\mathcal{T}), \\
\text{Eff}_{a_{\gamma}} &= \text{Eff}_a \cap \gamma(\mathcal{T}), \\
\text{Eff}_{a_{\neg \gamma}} &= \text{Eff}_a \cap \neg \gamma(\mathcal{T}).
\end{align*}
\]

(where \( A^c \) denotes the set complement of \( A \)). Accordingly, we define, given an action set \( A \), the action sets \( A_{\gamma} = \{a_{\gamma} | a \in A\} \) and \( A_{\neg \gamma} = \{a_{\neg \gamma} | a \in A\} \). Split now the states in a similar way: given \( s \in \mathcal{S} \), put \( s_{\gamma} = s \cap \gamma(\mathcal{T}) \) and \( s_{\neg \gamma} = s \cap \neg \gamma(\mathcal{T}) \). Then, it is immediate to see that given a state \( s \) we have that \( s \in \mathcal{S}_A \) if and only if \( s_{\gamma} \in \mathcal{S}_{A_{\gamma}} \) and \( s_{\neg \gamma} \in \mathcal{S}_{A_{\neg \gamma}} \) and it holds that:

\[
s' = \xi(s, A) \Leftrightarrow \begin{cases} \\
    s'_{\gamma} = \xi(s_{\gamma}, A_{\gamma}) \\
    s'_{\neg \gamma} = \xi(s_{\neg \gamma}, A_{\neg \gamma}) \end{cases}
\]

This leads to the following simple but useful result.

Proposition 16. For a set of actions \( A \), the following conditions are equivalent:

(i) \( A \) is strongly \( \gamma \)-safe;

(ii) \( A_{\gamma} \) is strongly \( \gamma \)-safe;

(iii) For every \( s \in \mathcal{S}_{A_{\gamma}} \) such that \( s \subseteq \gamma(\mathcal{T}) \) and \( w(\mathcal{T}, \gamma, s) \leq 1 \), it holds that the successor state \( s' = \xi(s, A_{\gamma}) \) is such that \( w(\mathcal{T}, \gamma, s') \leq 1 \).

Definition 17 (Classification of Ground Actions). A set of ground actions \( A \) is:

- \( \gamma \)-unreachable if \( |\text{Pre}_{a_{\gamma}}| \geq 2 \);
- \( \gamma \)-heavy if \( |\text{Pre}_{a_{\gamma}}| \leq 1 \) and \( |\text{Eff}_{a_{\gamma}}| \geq 2 \);
- \( \gamma \)-irrelevant if \( |\text{Pre}_{a_{\gamma}}| \leq 1 \) and \( |\text{Eff}_{a_{\gamma}}| = 0 \).
• \(\gamma\)-relevant for \(T\) if \(|\text{Pre}_{A^*_\gamma}| \leq 1\) and \(|\text{Eff}_{A^*_\gamma}| = 1\).

It is immediate to see that each \(A \subseteq GA\) belongs to one and just one of the above four disjoint classes. The following result clarifies their relation with strong safety.

**Theorem 18.** Let \(A\) be a set of ground actions. Then,

1. if \(A\) is \(\gamma\)-unreachable or \(\gamma\)-irrelevant, \(A\) is strongly \(\gamma\)-safe;
2. if \(A\) is heavy, \(A\) is not strongly \(\gamma\)-safe.

As the next example shows, relevant action sets may be strongly safe or not.

**Example 3.** Consider a template \(T\) and an instance \(\gamma\) such that \(\gamma(T) = \{q, q', q''\}\), where \(q, q'\) and \(q''\) are three ground atoms, and \(A = \{a\}\), where \(a\) is a ground action such that \(\text{Eff}_{A^*_\gamma} = \{q\}\). Since \(|\text{Eff}_{A^*_\gamma} \cap \gamma(T)| = 1\), \(a\) is \(\gamma\)-relevant. Consider a state \(s \in S_A\) such that \(w(T, s) = 1\).

• Suppose that \(\text{Pre}_{A^*_\gamma} = \{q'\}\) and \(\text{Eff}_{A^*_\gamma} = \{q'\}\) as shown in Figure 1 left. In this case, \(a\) is strongly \(\gamma\)-safe. In fact, \(q' \in s\) and in consequence \(w(T, \gamma, s) = 1\). Given \(s' = \xi(s, a)\), \(q' \notin s'\), but \(q \in s'\) and therefore \(w(T, \gamma, s') = 1\).

• Suppose that \(\text{Pre}_{A^*_\gamma} = \emptyset\) and \(\text{Eff}_{A^*_\gamma} = \{q\}\) as shown in Figure 1 right. In this case, \(a\) is \(\gamma\)-relevant, but is not strongly \(\gamma\)-safe. In fact, suppose that \(q'' \in s\) and therefore \(w(T, \gamma, s) = 1\). Since \(q'' \notin \text{Eff}_{A^*_\gamma}\) and \(q \in \text{Eff}_{A^*_\gamma}\), \(s' = \xi(s, a)\) is such that \(w(T, \gamma, s') = 2\).

\[
\begin{align*}
\gamma(T) &= \{q, q', q''\} \\
\text{Pre}_{A^*_\gamma} &= \{q'\} \\
\text{Eff}_{A^*_\gamma} &= \{q\} \\
a &= \{a\}
\end{align*}
\]

![Figure 1: Right: action \(a\) is \(\gamma\)-relevant and strongly \(\gamma\)-safe. Left: action \(a\) is essentially \(\gamma\)-relevant. Note that, in action specifications, only preconditions and effects different from the empty sets are represented.](image)

We now propose the following classification of relevant actions.

**Definition 19** (Classification of Relevant Actions). A \(\gamma\)-relevant set of ground actions \(A\) is:

- balanced if \(|\text{Pre}_{A^*_\gamma}| = 1\) and \(\text{Pre}_{A^*_\gamma} \subseteq \text{Eff}_{A^*_\gamma} \cup \text{Eff}_{A^*_\gamma}^c\);
- unbalanced if \(|\text{Pre}_{A^*_\gamma}| = 1\) and \(\text{Pre}_{A^*_\gamma} \cap (\text{Eff}_{A^*_\gamma} \cup \text{Eff}_{A^*_\gamma}^c) = \emptyset\);
- bounded if \(|\text{Pre}_{A^*_\gamma}| = 0\) and \(\text{Pre}_{A^*_\gamma} \cup \text{Eff}_{A^*_\gamma} = \gamma(T)\);
- unbounded if \(|\text{Pre}_{A^*_\gamma}| = 0\) and \(\text{Pre}_{A^*_\gamma} \cup \text{Eff}_{A^*_\gamma} \neq \gamma(T)\).

Again it is obvious that every relevant set of ground actions \(A\) belongs to one and just one of the above four disjoint classes. The following results completes the analysis of strong safety.
Theorem 20. Let \( A \) be a \( \gamma \)-relevant set of ground actions. Then,

1. if \( A \) is balanced or bounded, \( A \) is strongly \( \gamma \)-safe;
2. if \( A \) is unbalanced or unbounded, \( A \) is not strongly \( \gamma \)-safe.

Immediate consequence of Theorems 18 and 20 is the following result:

Corollary 21. Let \( A \) be a set of ground actions. Then,

1. if \( A \) is either \( \gamma \)-unreachable, \( \gamma \)-irrelevant, \( \gamma \)-relevant balanced, or \( \gamma \)-relevant bounded, \( A \) is strongly \( \gamma \)-safe;
2. if \( A \) is either \( \gamma \)-heavy, \( \gamma \)-relevant unbalanced, or \( \gamma \)-relevant unbounded, \( A \) is not strongly \( \gamma \)-safe.

Finally, this result shows that strong \( \gamma \)-safety can always be checked at the level of single actions.

Proposition 22. Let \( A \) be a set of actions. Then, \( A \) is strongly \( \gamma \)-safe if \( a \) is strongly \( \gamma \)-safe for all \( a \in A \).

Example 4 (Floortile domain). Consider a template \( T = (\{c\}, \{(c,0)\}) \), where \( c = \langle \text{painted}, 2, 1 \rangle \). Take two instances \( \gamma_1(c,0) = \text{tile1} \) and \( \gamma_2(c,0) = \text{tile2} \) and the ground action \( a = \text{paint-up}(\text{black}, \text{tile1}, \text{tile3}, \text{red}) \) (Table 4). This action is strongly \( \gamma_2 \)-safe since it is irrelevant, but it is \( \gamma_1 \)-relevant and not strongly \( \gamma_1 \)-safe. This is because, given a state \( s \in S_A \) such that, for example, \( \text{painted}(\text{tile4}, \text{black}) \in s \), \( s' = \xi(s, a) \) is such that \( \text{painted}(\text{tile4}, \text{black}), \text{painted}(\text{tile1}, \text{black}) \in s', \) with \( w(T, \gamma_1, s') = 2 \).

We conclude now with a definition and a first result that expresses a sufficient condition for a template to be invariant.

Definition 23. Given a template \( T \), a set of actions \( A \subseteq GA \) is strongly safe if it is strongly \( \gamma \)-safe for every instance \( \gamma \).

We have the following result:

Corollary 24. Given a template \( T \), \( T \) is invariant if for each \( a \in GA \), \( a \) is strongly safe.

Proof. It follows from Remark 22, Definition 23 and Proposition 13.

The condition expressed in Corollary 24 cannot be inverted in general. Indeed, a template can be invariant even if not all actions are strongly safe. We will see when this happens in the following section.

5. Safe action sequences and safe durative actions

A template can be invariant even if not all ground actions are strongly safe. This happens for two reasons. On the one hand, since the set of reachable states \( S_r \) is in general smaller than \( S \), it may be that all the states that are responsible for the lack of strong safety are unreachable, i.e. they are not in \( S_r \). On the other hand, in domains with durative actions, some instantaneous
actions are temporally coupled because they are the start and end fragments of the same durative action. This coupling imposes constraints on the states where the end part can be applied, which might prove helpful to establish that a template is invariant. While in this paper we will not analyse the first case as it would require an analysis of the set of reachable states $S_r$, which is practically unfeasible, we now elaborate suitable simple concepts of safety for durative actions, which are weaker than strong safety. This extension is of great importance to apply our technique to real-world planning domains. In fact, they often present durative actions that have a non strongly safe end fragment, but that nonetheless never violate the weight condition when appearing in a plan. We propose a definition of safety for durative actions that captures this case. However, given that in a plan a durative action may intertwine with other actions that happen in between its start and end points, we need to work out a concept of safety for more general sequence of actions than just durative ones.

Below, we consider general sequences of ground action sets $A := (A^1, A^2, \ldots, A^n)$. Note that any valid simple plan $\pi$ naturally induces such a sequence. Indeed, if $\text{trace}(\pi) = \{ s_i = (t_i, s_i) | 0 \leq i \leq k \}$ and $A_h$ are the relative happenings, we can consider the so called happening sequence of $\pi$: $A_\pi = (A_{\pi_0}, \ldots, A_{\pi_k})$. $A_\pi$ contains all the information on the plan $\pi$ except the time values at which the various actions happen.

To study the invariance of a template, we break the happening sequence of each plan into subsequences determined by the happenings of durative actions. More precisely, we consider subsequences determined by the happenings of durative actions. More precisely, we consider values at which the various actions happen.

5.1. Safe ground action sequences

Given a sequence of ground action sets $A := (A^1, A^2, \ldots, A^n)$, we denote with $S_A$ the set of state sequences $(s^0, \ldots, s^n) \in S^{n+1}$ such that

$$A^i \text{ is applicable in } s^{i-1} \text{ and } s^i = \xi(s^{i-1}, A_i) \forall i = 1, \ldots, n$$

If $(s^0, \ldots, s^n) \in S_A$, we say that $(s^0, \ldots, s^n)$ is a state sequence compatible with $A$. Given an instance $\gamma$, we also define $S_A(\gamma)$ as the set of compatible state sequences $(s^0, \ldots, s^n)$ such that $w(T, \gamma, s^0) \leq 1$. We use the following notation for subsequences of $A$: $A^i_k = (A^0, A^{i+1}, \ldots, A^k)$.

We now fix a template $T$ and an instance $\gamma$ and propose the following natural definition of safety for a sequence.
Definition 25 (Individually safe actions). A sequence of ground action sets $A := (A^1, A^2, \ldots, A^n)$ is individually $\gamma$-safe if for every sequence of states $(s^0, \ldots, s^n) \in S_{A}$ we have that

$$w(T, \gamma, s^0) \leq 1 \Rightarrow w(T, \gamma, s^i) \leq 1 \forall i = 1, \ldots, n$$

The invariance of a template can now be expressed in terms of individual safety for the happening sequences.

Proposition 26. Let $T$ be a template. Suppose that for every valid simple plan $\pi$, the sequence $A_\pi$ is individually $\gamma$-safe for every instance $\gamma$. Then, $T$ is invariant.

Below are elementary properties of individual $\gamma$-safety for subsequences of $A$.

Proposition 27. Consider a sequence of ground action sets $A := (A^1, A^2, \ldots, A^n)$. The following properties hold:

(i) if, for some $k$ and $h$ such that $k \geq h - 1$, $A^k = (A^1, A^2, \ldots, A^k)$ and $A^h = (A^h, \ldots, A^n)$ are both individually $\gamma$-safe, then also $A$ is individually $\gamma$-safe;

(ii) if $A$ is individually $\gamma$-safe and $A^k$ and $A^{k+1}$ are non-interfering, then $A' = (A^1, A^2, \ldots, A^k \cup A^{k+1}, \ldots, A^n)$ is individually $\gamma$-safe;

(iii) if $A$ is individually $\gamma$-safe and $B^j$, for $j = 1, \ldots, n$ are action sets such that $\bigcap_j B^j = \emptyset$, then, $A' = (A^1, B^1, A^2, \ldots, B^n, A^n)$ and $A'' = (A^1 \cup B^1, \ldots, A^n \cup B^n)$ are individually $\gamma$-safe.

The following is a useful consequence of the previous results: it asserts that if individual safety holds locally in a sequence, then it also holds globally.

Corollary 28. For a sequence of ground action sets $A := (A^1, A^2, \ldots, A^n)$, the following conditions are equivalent:

(i) the sequence $A$ is individually $\gamma$-safe;

(ii) for each $j = 1, \ldots, n$, there exists a subsequence $A_{j-r}^{j+s}$, with $r, s \geq 0$, that is individually $\gamma$-safe.

Proof. (i)⇒(ii) is trivial and (ii)⇒(i) follows from an iterative use of (i) of Proposition 27.

Individual safety is a weak property since it is not robust with respect to the insertion of other actions, even when these actions are irrelevant but possess delete effects. This is connected to the fact that, while individual safety has this nice local to global feature illustrated in Corollary 28, it does not possess the opposite feature: subsequences of individual safe sequences may not be individually safe. The following example shows both these phenomena.

Example 5. Consider a template $T$ and an instance $\gamma$ such that $\gamma(T) = \{q, q'\}$. The set of state sequences compatible with $A := (a^1, a^2)$ (Figure 2 - top diagram) is: $S_{A} = \{(s^0, s^1, s^2) | q \notin s^0, s^1 = s^0 \cup \{q'\}, s^2 = s^1\}$. Note that $q \notin s^0$ because, by hypothesis, $a^2$ is applicable in $s^1$ and $s^1 = s^0 \cup \{q'\}$. $A$ is individually $\gamma$-safe since $w(T, \gamma, s^1) \leq 1$ for every state $s'$ that appears in $S_{A}$. Note that $a^1$ is $\gamma$-relevant unbounded and thus not strongly $\gamma$-safe.

Now consider the sequence $\tilde{A} := (a^1, b, a^2)$ (Figure 2 - bottom diagram) where a $\gamma$-irrelevant action $b$ is inserted between $a^1$ and $a^2$. The new set of state sequences compatible with $\tilde{A}$ is: $S_{\tilde{A}} = \{(s^0, s^1, s^2, s^3) | s^1 = s^0 \cup \{q'\}, s^2 = s^1 \setminus \{q\}, s^3 = s^2\}$. Note that now $q$ can be in $s^3$ since it is
the action \( b \) that ensures the applicability of \( a^2 \). If \( q \in s^0 \), since \( a^1 \) adds \( q' \) to \( s^0 \), \( w(T, \gamma, s^1) = 2 \). Clearly, this new sequence is not individually \( \gamma \)-safe. The insertion of a \( \gamma \)-irrelevant action has failed the individual \( \gamma \)-safety of the sequence \( A \).

For proving some of our results, the concept of individual safety is not sufficient. Below we present a stronger definition of safety for an action sequence that is robust with respect to the insertion of irrelevant actions in it. First, we define the simple concepts of executable and reachable sequences.

**Definition 29** (Executable and reachable actions). The sequence \( A = (A^1, A^2, \ldots, A^n) \) is called:

- executable if \( S_A \neq \emptyset \);
- \( \gamma \)-(un)reachable if \( S_A(\gamma) \neq \emptyset \) (\( S_A(\gamma) = \emptyset \)).

**Remark 30.** Note the following chain of implications:

\[ \text{non-executable } \Rightarrow \text{\( \gamma \)-unreachable } \Rightarrow \text{individually \( \gamma \)-safe} \]

Note that if \( \pi \) is a valid simple plan with happening sequence \( A_\pi \), then \( A_\pi \) is \( \gamma \)-reachable for every \( \gamma \) due to the standing assumption that \( w(T, \gamma, \text{Init}) \leq 1 \) for every \( \gamma \). Moreover, every subsequence \( A \) of \( A_\pi \) is executable. If a subsequence \( A \) of \( A_\pi \) is \( \gamma \)-unreachable, the weight will surely exceed 2 at some point of the plan \( \pi \) and thus the template \( T \) will not be invariant.

In the special case of a sequence of length 2, executability and reachability admit very simple characterisations. We report them below as we will need them later. First define, for a generic set of actions \( A \), the subsets

\[
\Gamma^-_A := (Pre^+_A \setminus Ef f^-_A) \cup Ef f^+_A, \quad \Gamma^+_A := (Pre^-_A \setminus Ef f^+_A) \cup Ef f^-_A
\]

We have the following result:

**Proposition 31.** Given a sequence of two ground action sets \( A = (A^1, A^2) \), the following conditions are equivalent:

(i) \( A \) is executable;

(ii) \( \Gamma^+_A \cap Pre^+_A = \emptyset = \Gamma^-_A \cap Pre^-_A \).
Proposition 32. Given a sequence of two ground action sets $A = (A^1, A^2)$, the following conditions are equivalent:

(i) $A$ is $\gamma$-reachable;

(ii) $A$ is executable and $|\text{Pre}_{A^1}^+ \cup (\text{Eff}_{A^1}^+ \setminus \text{Eff}_{A^2}^+)| \leq 1$.

The following are immediate properties of executability and unreachability:

Proposition 33. Consider a sequence $A = (A^1, A^2, \ldots, A^n)$ that is executable or $\gamma$-reachable. Then,

(i) if $B^j \subseteq A^j$ are such that $\text{Eff}_{B^j} = \emptyset$ for every $j = 1, \ldots, n-1$, then also $A' = (A^1 \setminus B^1, A^2 \setminus B^2, \ldots, A^n \setminus B^n)$ is, respectively, executable or $\gamma$-reachable.

(ii) if $A^j = A^j \cup A''^j$ for some $j = 1, \ldots, n$, then also $A' = (A^1, A^2, \ldots, A^j, A''^j, \ldots, A^n)$ is, respectively, executable or $\gamma$-reachable.

Here is our stronger notion of safety:

Definition 34 (Safe actions). A sequence of ground action sets $A := (A^1, A^2, \ldots, A^n)$ is $\gamma$-safe if it is executable and $A^k_1$ is individually $\gamma$-safe for every $k = 1, \ldots, n$.

Note how the sequence $A := (a^1, a^2)$ considered in Example 5 is indeed not $\gamma$-safe, since $a^1$ is not individually $\gamma$-safe. The next example shows instead the reason why executability is required.

Example 6. Consider a template $T$ and an instance $\gamma$ such that $\gamma(T) = \{q, q', q''\}$. The sequence $A := (a^1, a^2)$ (Figure 5 - top diagram) is individually $\gamma$-safe because $S_A = \emptyset$ ($\neg q''$ is required false by $a^2$, but it is asserted true by $a^1$).

Now consider the sequence $\tilde{A} := (a^1, b, a^2)$ (Figure 5 - bottom diagram) where a $\gamma$-irrelevant action $b$ is inserted between $a^1$ and $a^2$. This insertion makes $S_{\tilde{A}} \neq \emptyset$. Since $q, q' \in s^3$, $w(T, \gamma, s^3) = 2$ and therefore $A$ is not individually $\gamma$-safe.

$\gamma(T) = \{q, q'\}$

$A = \{a^1, a^2\}$

\[
\begin{array}{ccc}
A = \{a^1, a^2\} & a^1 & a^2 \\
& s^0 & s^1 & s^2 \\
& \text{Eff}^7: q, q'' & \text{Eff}^7: q' \\
\end{array}
\]

$A^\sim = \{a^1, b, a^2\}$

\[
\begin{array}{ccc}
A^\sim = \{a^1, b, a^2\} & b & a^2 \\
& s^0 & s^1 & s^2 & s^3 \\
& \text{Eff}^7: q, q'' & \text{Eff}^7: q'' & \text{Eff}^7: q' \\
\end{array}
\]

Figure 3: Lack of robustness for individually $\gamma$-safe actions.
Remark 35. If $A = (A^1, A^2, \ldots, A^n)$ is $\gamma$-safe, the first action set $A^1$ must necessarily be strongly $\gamma$-safe. On the other hand, if $A$ is executable and every $A^j$ for $j = 1, \ldots, n$ is strongly $\gamma$-safe then, $A$ is $\gamma$-safe.

This motivates the following definition.

**Definition 36 (Strongly and simply safe actions).** A sequence of ground action sets $A = (A^1, A^2, \ldots, A^n)$ is:

- strongly $\gamma$-safe if it is executable and every $A^j$ for $j = 1, \ldots, n$ is strongly $\gamma$-safe;
- simply $\gamma$-safe if it is $\gamma$-safe but not strongly $\gamma$-safe.

The following result shows that heavy or relevant unbalanced actions cannot be part of safe reachable sequences.

**Proposition 37.** Suppose $A = (A^1, A^2, \ldots, A^n)$ is a $\gamma$-safe and $\gamma$-reachable sequence of ground action sets. Then, for every $j = 1, \ldots, n$, $A^j$ is not $\gamma$-heavy and is no $\gamma$-relevant unbalanced.

The property of $\gamma$-reachability is necessary for the previous result to hold, as the following example shows.

**Example 7.** Consider a template $T$ and an instance $\gamma$ such that $\gamma(T) = \{q, q'\}$. The sequence $A := (a^1, a^2)$ (Figure 4) is $\gamma$-safe because it is executable and the subsequences $(a^1)$ and $(a^1, a^2)$ are both individually safe given that $a^1$ is $\gamma$-unreachable. In this case, Proposition 37 does not hold since $a^2$ is $\gamma$-heavy.

In studying the two safety properties for a sequence $A$ introduced so far, we can essentially restrict ourselves to study the state dynamics on the template instantiation $\gamma(T)$ as we did for strong $\gamma$-safety of instantaneous actions (see Remark 15).

Given the sequence $A := (A^1, A^2, \ldots, A^n)$, we denote by $A_\gamma := (A^1_\gamma, A^2_\gamma, \ldots, A^n_\gamma)$ and $A_{\neg\gamma} := (A^1_{\neg\gamma}, A^2_{\neg\gamma}, \ldots, A^n_{\neg\gamma})$ the corresponding restricted sequences. We have the following result.

**Proposition 38.** Given the sequence $A := (A^1, A^2, \ldots, A^n)$,

1. $A$ is executable if and only if $A_\gamma$ and $A_{\neg\gamma}$ are both executable;
2. $A$ is $\gamma$-reachable if and only if $A_\gamma$ is $\gamma$-reachable and $A_{\neg\gamma}$ is executable;
3. $A$ is individually $\gamma$-safe if and only if $A_\gamma$ is individually $\gamma$-safe.
4. $A$ is $\gamma$-safe if and only if $A_\gamma$ is $\gamma$-safe and $A_{\neg\gamma}$ is executable.
We are now ready to state and prove the following fundamental result, which ensures that the concept of safe sequence is robust to the insertion of irrelevant actions.

**Theorem 39.** Consider a $\gamma$-safe sequence $A := (A^1, A^2)$ and $\gamma$-irrelevant ground action sets $B^1, B^2, \ldots, B^n$. Then, the sequence $\tilde{A} := (A^1, B^1, \ldots, B^n, A^2)$ is either non executable or $\gamma$-safe.

We conclude this section with a last definition:

**Definition 40.** Given a template $T$, a sequence of ground action sets $A$ is, respectively, safe or strongly safe if it is, respectively, $\gamma$-safe or strongly $\gamma$-safe, for every instance $\gamma$. It is simply safe if it is safe but not strongly safe.

### 5.2. Safe ground durative actions

We now restrict our attention to ground durative actions $Da = (a^{st}, a^{inv}, a^{end})$. If we interpret $Da$ as a sequence of three actions, we can consider for it the properties defined for general sequences such as $\gamma$-safety and strong $\gamma$-safety. We propose an explicit characterisation of these properties in this case, which will be useful later on.

First, let us focus on the specific way in which durative actions appear in the happening sequence of a plan. Consider a simple induced plan $\pi$ having $\text{trace}(\pi) = \{S_i = (t_i, s_i)_{i=0}^{\infty}\}$ and happenings $A_i$. Let $A_s$ be the corresponding happening sequence. If a durative action $Da$ happens in $\pi$ in the time interval $[t_{i+1}, t_i]$, we clearly have that $a^{st} \in A_{i+1}$ and $a^{end} \in A_i$. Moreover, by the way $\pi$ is constructed from the original plan, we also have that $j - i$ is odd and for every even $h = 2, 4, \ldots, j - i - 1$, $A_{i+h}$ consists of $\{a^{inv}\}$ and, possibly, preconditions of other durative actions happening in the original plan $\Pi$ simultaneously or intertwined with $Da$. This motivates the following definition.

**Definition 41 (Admissible actions).** A sequence $A := (A^1, A^2, \ldots, A^n)$ is:

- admissible if, for any durative action $Da'$, it holds
  \[ a^{st} \in A^i \Rightarrow a^{inv} \in A^{i+1} \quad a^{end} \in A^i \Rightarrow a^{inv} \in A^{i-1} \]

- $Da$-admissible, for some durative action $Da$, if it is admissible and the following conditions are satisfied:
  
  1. $a^{inv} \in A^i$ and $a^{end} \in A^n$;
  2. $n$ is odd and for every $j = 2, 4, \ldots, n - 1$, $A^j$ consists of $\{a^{inv}\}$ and, possibly, preconditions of other durative actions.

Any subsequence of the happening sequence of a simple plan is admissible and if its starting and its ending coincide with, respectively, the start and the end of a durative action $Da$, it is $Da$-admissible.

To study the safety of a $Da$-admissible sequence, we can, in many cases, reduce the analysis of the durative action $Da$ to the analysis of an auxiliary sequence of just two actions $Da_s = (a^{st}, a^{end})$, where $a^{st}$ and $a^{end}$ are instantaneous actions such that:

\[
\begin{align*}
E f f_{a^{st}}^* &= E f f_{a^{st}}^*, \\
Pre_{a^{st}}^* &= Pre_{a^{st}}^* \cup (Pre_{a^{st}} \setminus E f f_{a^{st}}^*) \\
E f f_{a^{end}}^* &= E f f_{a^{end}}^*, \\
Pre_{a^{end}}^* &= Pre_{a^{end}}^* \cup Pre_{a^{end}}
\end{align*}
\]

The relation between the two sequences $Da$ and $Da_s$ is clarified by the following result. Assume, as always, that a template $T$ and an instance $\gamma$ have been fixed.
Proposition 42. The following facts hold true:

\[ \begin{align*}
&(i) \ (s^0, s^1, s^2) \in S_{\omega^0, \omega^1} \text{ if and only if } s^1 = s^2 \text{ and } (s^0, s^1) \in S_{\omega^1}; \\
&(ii) \ (s^0, s^1, s^2) \in S_{\omega^0, \omega^1} \text{ if and only if } s^0 = s^1 \text{ and } (s^1, s^2) \in S_{\omega^1}; \\
&(iii) \ (s^0, s^1, s^2, s^3) \in S_{Da} \text{ if and only if } s^1 = s^2 \text{ and } (s^0, s^1, s^3) \in S_{Da}; \\
&(iv) \ (a^u, a^{inv}) \text{ is individually } \gamma\text{-safe if and only if } a^u \text{ is strongly } \gamma\text{-safe}; \\
&(v) \ (a^{inv}, a^{end}) \text{ is individually } \gamma\text{-safe if and only if } a^{end} \text{ is strongly } \gamma\text{-safe}; \\
&(vi) \ Da \text{ is individually } \gamma\text{-safe if and only if } Da \text{ is individually } \gamma\text{-safe.}
\end{align*} \]

The next result studies the effect of exchanging the start and end of a durative action \( Da \) with those of the auxiliary sequence \( Da_s \), in a \( Da\)-admissible sequence.

Proposition 43. Consider a durative action \( Da = (a^u, a^{inv}, a^{end}) \) and a \( Da\)-admissible sequence of actions \( A = ((a^u), A^2, \ldots, A^{n-1}, (a^{end})) \). Put \( A_s = ((a^u_s), A^2_s, \ldots, A^{n-1}_s, (a^{end}_s)) \). Then \( S_A = S_{A_s} \).

In particular, \( A \) is individually \( \gamma\)-safe if and only if \( A_s \) is individually \( \gamma\)-safe.

The last proposition implies that, in analysing the state dynamics in a valid plan, we can replace the start and end of each durative action \( Da \) with the corresponding ones of the auxiliary sequence \( Da_s \), if such start and end happen isolated from other actions. This is useful for two reasons. On the one hand, there are cases in which \( Da_s \) is strongly safe even if \( Da \) is not. On the other hand, we can directly apply Theorem 39 to \( Da_s \), since it is of length 2.

As we shall see later, our sufficient results for the invariance of a template always require safety (strong or simple) of the auxiliary actions \( Da_s = (a^u_s, a^{end}_s) \). The check for strong safety can be done by considering the single components of \( Da_s \) and referring back to the analysis that we carried out in previous chapter. Below, we propose a full characterisation of simple safety for auxiliary actions.

Note first that if \( Da_s = (a^u_s, a^{end}_s) \) is simply \( \gamma\)-safe (Definition 36), necessarily \( Da_s \), is executable, \( a^u_s \) is strongly \( \gamma\)-safe and \( a^{end}_s \) is not strongly \( \gamma\)-safe. If, besides these three properties, \( Da_s \) is \( \gamma\)-unreachable, then, \( Da_s \) is simply \( \gamma\)-safe because of Remark 30. If we instead assume that \( Da_s \) is simply \( \gamma\)-safe and \( \gamma\)-reachable, then, because of Proposition 37, we have that \( a^{end}_s \) is \( \gamma\)-relevant unbounded. The following result completely characterises simple \( \gamma\)-safety for such actions.

Proposition 44. Assume that \( Da_s = (a^u_s, a^{end}_s) \) is a \( \gamma\)-reachable sequence such that \( a^u_s \) is strongly \( \gamma\)-safe and \( a^{end}_s \) is relevant unbounded. Then, \( Da_s \) is simply \( \gamma\)-safe if and only if one of the following mutually exclusive conditions are satisfied:

\[ \begin{align*}
&(a) \ a^u \ \gamma\text{-irrelevant, } |Pre^+_a| = 1, Pre^+_{a^u} \subseteq Ef \int_{a^u}; \\
&(b) \ a^u \ \gamma\text{-irrelevant, } |Pre^+_a| = 1, Pre^+_{a^u} \not\subseteq Ef \int_{a^u}, Pre^+_{a^u} \subseteq Ef \int_{a^u} \cup Ef \int^{+}_{a^u}; \\
&(c) \ a^u \ \gamma\text{-irrelevant, } |Pre^+_a| = 0, Pre^+_{a^u} \cup Ef \int_{a^u} \cup Ef \int^{+}_{a^u} \cup Ef \int^{++}_{a^u} = \gamma(T); \\
&(d) \ a^u \ \gamma\text{-relevant, } Ef \int^{+}_{a^u} \subseteq Ef \int_{a^u} \cup Ef \int^{+}_{a^u}.
\end{align*} \]
Remark 45. If Condition (a) of Proposition holds, this implies that the same conditions needs to be satisfied by \( a'' \), namely it holds: \( |\text{Pre}_{a''}^f| = 1, \text{Pre}_{a''}^f \subseteq E_{\gamma}^f \).

Definition 46 (Safe durative actions). We say that \( Da \) is simply \( \gamma \)-safe of type \( x \) where \( x \in \{a, b, c, d\} \) if it is \( \gamma \)-reachable, \( a'' \) is strongly \( \gamma \)-safe, \( a''_end \) is \( \gamma \)-relevant unbounded, and, finally, \( Da \) satisfies the condition (x) of Proposition.

Example 8. Consider a template \( T \) and an instance \( \gamma \) such that \( \gamma(T) = \{q, q'\} \). Figure shows possible instances of actions of types (a)-(d).

![Figure 5: Examples of actions of types (a)-(d).](image)

When the start or the end of a durative action \( Da \) happen simultaneously with other actions, the reduction of \( Da \) to \( Da' \) cannot be performed in general as shown in the following example.

Example 9. Consider a template \( T \) and an instance \( \gamma \) such that \( \gamma(T) = \{q, q', q''\} \). Figure shows that, when the durative actions \( Da \) and \( Da' \) are considered in isolation, both \( a''_st \) and \( a''_end \) are strongly safe since they are \( \gamma \)-unreachable. Since \( a''_st \) and \( a''_end \) are irrelevant, \( Da \) and \( Da' \) are strongly safe. However, if we now consider the case in which \( Da \) and \( Da' \) happen simultaneously, giving rise to the sequence \( A = (A_1 = \{a''\}, A'' = \{a''_st, a''_end\}) \), we see that \( A \) is not individually \( \gamma \)-safe. In fact, if we put \( s^1 = (q'') \) with \( w(T, \gamma, s^1) = 1 \), we have that \( s^1 = \xi(s^1, A^1) = \{q, q'', q'''\} \) with \( w(T, \gamma, s^1) = 3 \), which violates the definition of individual \( \gamma \)-safety.

Note that, in the previous example, the two durative actions are \( \gamma \)-unreachable. The following result shows that such pathological phenomena can only happen in that case and will be instrumental for the results of the next section.

Proposition 47. Let \( Da \) be a \( \gamma \)-reachable durative action such that \( a'' \) is not strongly \( \gamma \)-safe, while \( a''_st \) is strongly \( \gamma \)-safe. Then,

(i) \( a''_st \) is \( \gamma \)-relevant bounded;

(ii) for every ground action sets \( A^1 \) such that \( (\{a''\} \cup A^1, a''_st) \) is executable, \( (\{a''\} \cup A^1, a''_st) \) is individually \( \gamma \)-safe.
\( \gamma(T) = \{q, q', q''\} \)

\[
\begin{array}{cccc}
\text{Da} & a^\text{st} & a^\text{inv} & a^\text{end} \\
\text{Eff}^+: q & \text{Pre}^-: q, q' & \text{Pre}^-: q, q'' \\
\text{Da'} & a^\text{st} & a^\text{inv} & a^\text{end} \\
\text{Eff}^+: q' & \text{Pre}^-: q, q'' \\
\end{array}
\]

\( A = (A^1, A^2) \)

\( A^1 = \{a^\text{inv}, a'^\text{inv}\} \)

\( A^2 = \{a^\text{end}, a'^\text{end}\} \)

No similar results hold for the end parts of durative actions as next example shows.

**Example 10.** Consider a template \( T \) and an instance \( \gamma \) such that \( \gamma(T) = \{q, q'\} \). When the durative actions \( Da \) and \( Da' \) (Figure 5.2) are considered in isolation, both \( a^\text{end} \) and \( a'^\text{end} \) are strongly \( \gamma \)-safe since they are \( \gamma \)-bounded. Since \( a^\text{st} \) and \( a'^\text{st} \) are irrelevant, \( Da \) and \( Da' \) are strongly safe. However, if \( Da \) and \( Da' \) happen simultaneously, giving rise to the sequence \( A = (A^1 = \{a^\text{inv}, a'^\text{inv}\}, A^2 = \{a^\text{end}, a'^\text{end}\}) \), \( A \) is not individually \( \gamma \)-safe. If we put \( s^0 = \emptyset \) with \( w(T, \gamma, s^0) = 0 \), we have that \( s^1 = \xi(s^0, A^1) = \emptyset \) and \( s^2 = \xi(s^1, A^2) = \{q, q'\} \) with \( w(T, \gamma, s^2) = 2 \), which violates the definition of individual \( \gamma \)-safety.
6. Conditions for the invariance of a template

Any plan \( \pi \) where all instantaneous ground actions are strongly safe, all durative ground actions are safe and take place in isolation, i.e. with no other actions happening in between them, yields a safe happening sequence \( A_\pi \), as a consequence of Corollary \[28\]. The difficulty, in general, is that durative actions can in principle start or end together and be intertwined with other instantaneous or durative actions. Safety of durative actions must therefore be accompanied by suitable hypothesis guaranteeing that dangerous intertwinnements or simultaneous happenings cannot take place in valid plans. In this way, we can work out sufficient conditions for the invariance of a template, which will be useful in analysing concrete examples.

In this section, we present two results that give sufficient conditions for the invariance of a template. The first deals with the particular case when all instantaneous actions are strongly safe and all durative actions \( Da \) are such that \( Da^* \) is strongly safe. The second result considers a more general case when there are durative actions \( Da \) for which \( Da^* \) is only simply safe. We recall our standing assumption that \( \nu(T, \gamma, \text{Init}) \leq 1 \) for every \( \gamma \).

Given a template \( T \) and an instance \( \gamma \), we denote by \( GA^d(\gamma) \) the collection of durative actions which are not strongly \( \gamma \)-safe and with \( GA^i(\gamma) \) and \( GA^{end}(\gamma) \), respectively, the collection of their start and end fragments. The following property prevents the simultaneous end of durative actions that could yield unsafe phenomena.

**Definition 48 (Relevant right isolated actions).** Given a template \( T \), the set of ground durative actions \( GA^d \) is said to be relevant right isolated if, for every instance \( \gamma \) and every \( Da_1, Da_2 \in GA^d(\gamma) \), one of the following conditions is satisfied:

(i) \( |Eff_{a_1}^{+} + Eff_{a_2}^{+}| \leq 1 \);

(ii) at least one of the two pairs \( \{a_1^{\text{end}}, a_2^{\text{end}}\} \) or \( \{a_1^{\text{inv}}, a_2^{\text{inv}}\} \) is mutex;

(iii) if they are both non-interfering, \( (\{a_1^{\text{inv}}, a_2^{\text{inv}}\}, \{a_1^{\text{end}}, a_2^{\text{end}}\}) \) is \( \gamma \)-unreachable.

**Theorem 49.** Consider a template \( T \) and suppose that the set of instantaneous actions \( GA^i \) and that of durative actions \( GA^d \) satisfy the following properties:

(i) every \( a \in GA^i \) is strongly safe;

(ii) for every instance \( \gamma \) and every \( Da \in GA^d(\gamma) \), \( Da_\gamma \) is \( \gamma \)-reachable and strongly \( \gamma \)-safe;

(iii) \( GA^d \) is relevant right isolated.

Then, \( T \) is invariant.

Note that assumption (iii) in the statement of Theorem \[49\] is to exclude the simultaneous end of durative actions; if such phenomena can be excluded a-priori, the hypothesis can be removed.

When there are durative ground actions \( Da \) for which \( Da_\gamma \) is not strongly \( \gamma \)-safe, further hypotheses are needed in order to guarantee that the template \( T \) is invariant. The main point is that, in this case, not only simultaneity can be harmful, but also any intertwinnement between such a durative action and other actions. The following examples show the type of phenomena that can happen and that any theorem extending Theorem \[49\] needs to prevent.
Example 11. Consider a template $T$ and an instance $\gamma$ such that $\gamma(T) = \{q, q', q''\}$. Both the durative actions $Da$ and $Da'$ (Figure 8) are $\gamma$-safe. However, they can intertwine in such a way to give rise to a sequence that is individually unsafe: $A = (A_1 = \{a_{st}\}, A_2 = \{a'_{st}\}, A_3 = \{a_{end}\}, A_4 = \{a''_{end}\})$. If we put $s^0 = \{q\}$ with $w(T, \gamma, s^0) = 1$, we have that $s^4 = \{q', q''\}$ with $w(T, \gamma, s^4) = 2$.

![Figure 8: Schemas $Da$ and $Da'$ can intertwine in such a way to give rise to a sequence that is individually unsafe.](image)

The following definition describes a set of durative actions for which such phenomena cannot take place. It consists of three requirements acting, for each instantiation $\gamma$, on the subset of dangerous durative actions $G.A^d(\gamma)$. The first prevents the simultaneous happening of two start fragments of such durative actions. The second states that, between two successive start fragments of durative actions in $G.A^d(\gamma)$, there must be the end of a third action also in $G.A^d(\gamma)$. Finally, the third requirement prevents $\gamma$-relevant actions to happen in between a durative action in $G.A^d(\gamma)$.

**Definition 50** (Relevant non intertwining actions). Given a template $T$, the set of ground durative actions $G.A^d$ is said to be relevant non intertwining if, for every instance $\gamma$, every $Da \in G.A^d(\gamma)$ and for every $\gamma$-reachable $Da$-admissible sequence of actions $A = ((a^n) \cup A^1, A^2, \ldots, A^{n-1}, (a_{end}) \cup A^n)$, the following conditions are satisfied:

(i) $A^1 \cap G.A^d(\gamma) = \emptyset$;

(ii) If $A^1 = \emptyset$ and $b \in A^j \cap G.A^d(\gamma)$ for $j < n$, then there exists $b' \in A^j \cap G.A^d(\gamma)$ for some $0 < j' \leq j$;

(iii) If $A^1 = \emptyset$ and $A^j \cap (G.A^d(\gamma) \cup G.A^d(\gamma)) = \emptyset$ for every $j = 2, \ldots, n - 1$, then each $A^j$ is $\gamma$-irrelevant for $j = 2, \ldots, n - 1$. 

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We are now ready to state and prove the main result of this section, that expresses a sufficient condition for a template to be invariant.

**Theorem 51.** Consider a template $T$ and suppose that the set of instantaneous actions $G\mathcal{A}$ and that of durative actions $G\mathcal{A}^d$ satisfy the following properties:

(i) every $a \in G\mathcal{A}$ is strongly safe;

(ii) for every $Da \in G\mathcal{A}^d$, $Da$, is safe;

(iii) the set $G\mathcal{A}^d$ is relevant non-intertwining.

Then, $T$ is invariant.

![Figure 9: Structure of a plan $\tilde{\pi}$ as constructed in Theorem 51. Strongly safe actions are indicated in green, relevant in red and irrelevant in grey.](image)

The properties that the set of durative actions $G\mathcal{A}^d$ needs to satisfy to be relevant non intertwining, which are expressed in Definition 50, are in general difficult to check as they require to consider sequences of actions of possibly any length. Below we propose a sufficient condition that guarantees such properties to hold, which is much simpler and suitable to be later analysed at the lifted level of action schemas.

We start with two definitions. The first is a left version of the relevant right isolated property. It prevents dangerous durative actions to start simultaneously. It is needed to insure condition (i) of Definition 50 of relevant non intertwining. The second definition allows us to reformulate conditions (ii) and (iii) of Definition 50.

**Definition 52** (Relevant left isolated actions). Given a template $T$, the set of ground durative actions $G\mathcal{A}^d$ is said to be relevant left isolated if, for every instance $\gamma$ and for every $Da^1, Da^2 \in G\mathcal{A}^d(\gamma)$, one of the following conditions is satisfied:

(i) at least one of the two pairs $\{a^{1st}, a^{2st}\}$ or $\{a^{1inv}, a^{2inv}\}$ is mutex;

(ii) if they are both non-interfering, $\{(a^{1st}, a^{2st}), (a^{1inv}, a^{2inv})\}$ is $\gamma$-unreachable.

**Definition 53** (Irrelevant unreachable actions). Consider a template $T$ and an instance $\gamma$. A pair of actions $(a, a')$ is $\gamma$-irrelevant unreachable if any sequence of actions

$A = ([a], A^2, \ldots, A^{n-1}, [a'])$

such that $A^2, \ldots, A^{n-1}$ are $\gamma$-irrelevant, is $\gamma$-unreachable.

The next result expresses a sufficient condition for the set $G\mathcal{A}^d$ to be relevant non intertwining.
Proposition 54. Consider a template $T$. The set $\mathcal{G}A^{l}$ is relevant non intertwining if the following conditions are satisfied:

(i) $\mathcal{G}A^{l}$ is relevant left isolated;

(ii) for every instance $\gamma$, for every $Da \in \mathcal{G}A^{l}(\gamma)$, and for every $a' \in \mathcal{G}A \setminus \mathcal{G}A^{end}(\gamma)$ that is not $\gamma$-irrelevant or $a' \in \mathcal{G}A^{u}(\gamma)$, $\{a^{inv}, a'\}$ is mutex or the pair $(a'', a')$ is $\gamma$-irrelevant unreachable.

The property of $\gamma$-irrelevant unreachable, though conceptually simpler than the original properties required in the definition of relevant non-intertwining actions, is still too complex for practical implementation, as it requires to verify properties over sequences of undefined length. We now propose a stronger version of it that is instead of simple computational complexity (linear in the number of ground actions).

Definition 55 (Strongly irrelevant unreachable actions). A pair of actions $(a, a')$ is strongly $\gamma$-irrelevant unreachable if any of the following conditions are satisfied:

(i) there exists $q \in \Gamma^{+}_{a} \cap \text{Pre}_{a}$ such that, for every $a''$ that is $\gamma$-irrelevant, $q \notin E \text{ff}_{a''}$;

(ii) there exists $q \in \Gamma^{-}_{a} \cap \text{Pre}_{a'}$ such that, for every $a''$ that is $\gamma$-irrelevant, $q \notin E \text{ff}_{a''}$;

(iii) $|\text{Pre}_{a}^{+} \cup (\text{Pre}_{a'}^{-} \setminus E \text{ff}_{a'})| > 1$.

The first condition essentially says that the application of the action $a$ leads to a state containing a ground atom $q$ that needs to be false in order to then apply $a'$ and that there is no $\gamma$-irrelevant action that can make this atom false. The second condition is the analogous of the first, but exchanges the role of true and false atoms. Finally, the third condition is equivalent to require that $(a, a')$ is a $\gamma$-unreachable pair, assuming that it is executable.

Proposition 56. If a pair of actions $(a, a')$ is strongly $\gamma$-irrelevant unreachable, it is also $\gamma$-irrelevant unreachable.

Based on the previous results, we conclude with a simple sufficient condition for the invariance, which is very useful in analysing concrete cases.

Corollary 57. Consider a template $T$ and suppose that, for every instance $\gamma$,

- every $Da \in \mathcal{G}A^{l}(\gamma)$ is such that $Da_{a}$ is simply $\gamma$-safe of type $(a)$;
- every $a \in \mathcal{G}A \setminus (\mathcal{G}A^{l}(\gamma) \cup \mathcal{G}A^{end}(\gamma))$ is either $\gamma$-irrelevant or $\gamma$-relevant balanced.

Then, $T$ is invariant.

7. Safety of Action Schemas for a Template

In Section 6, we have established two results guaranteeing the invariance of a template, Theorems 49 and 51. To be applied, they both need to check that all instantaneous and durative ground actions satisfy a safety condition as well as that other extra properties, which prevent potentially dangerous simultaneous happenings or intertwimenations among actions, hold true. Since we aim to find invariants off-line quickly and efficiently, our algorithm does not work at the level of
ground actions. Instead, it reasons at the lifted level and uses the structure of the action schemas, i.e. their conditions and effects, to decide whether the ground instantiations of these schemas are safe or not. Our main goal in this section is to obtain lifted versions of Theorems 49 and 51 and Corollary 57.

In general, we call **liftable** a property $P$ of ground actions if, given an action schema $\alpha$, if one instantiation $\alpha' = gr'(\alpha)$ satisfies $P$, then all instantiations $\alpha = gr(\alpha)$ satisfy $P$. In this case, we say that the action schema $\alpha$ satisfies property $P$.

The results presented in this and the next sections achieve two main goals. On the one hand, they show that the properties of safety introduced for instantaneous and durative ground actions in Sections 4 and 5 are liftable as well the non-intertwining properties, even if in a weaker sense, behind the formulation of Theorems 49 and 51. On the other hand, they will give efficient characterisations of such properties at the lifted level, which we use in our algorithmic implementation (see Section 9).

In the remaining part of this section, we analyse instantaneous action schemas and their ground instantiations. We show that strong safety is liftable and work out a complete characterisation of this property at the lifted level. Next section is devoted to lifting properties for durative actions.

### 7.1. Structure and properties of action schemas

We start with the following definition that introduces the key concept of matching. It couples an action schema $\alpha$ and a template $T$ introduced above. First, it is useful to work out more concrete representations for literals: this is the content of next Remark.

**Definition 58** (Matching). Given a template $T = (C, F_C)$ and an action schema $\alpha \in \mathcal{A}$, a literal $l$ that appears in $\alpha$ such that it exists a template’s component $c = (r, a, p) \in C$ with $\text{Rel}(l) = (r, a)$ and, if $l$ is universally quantified, $\text{Var}(l) = \{p\}$ is said to match $T$ via the component $c$. Given two literals $l$ and $l'$, we say that they are $T$-coupled (and we write $l \sim_T l'$) if the following two conditions hold:

1. $l$ and $l'$ individually match $T$ via the components $c$ and $c'$;
2. if $(c, i) \sim_{F_C} (c', j)$, $\text{Arg}[i, l] = \text{Arg}[j, l']$.

We now fix a template $T$ and an action schema $\alpha$ and study the properties of the relation $\sim_T$ on the literals of $\alpha$ that match $T$, introduced above. First, it is useful to work out more concrete representations for literals: this is the content of next Remark.

**Remark 59.** Suppose that $l$ is a literal in the action schema $\alpha$ that matches the template $T$ via the component $c$. The corresponding relation $r$ will necessarily have the structure $r(x_1, \ldots, x_v)$ where $x_j$’s denote the fixed arguments, $v$ the counted argument (which could also be absent) and $l = r(a_1, \ldots, a_k, v)$ or $l = r(a_1, \ldots, a_k, a_{k+1})$ depending if, respectively, $l$ is universally quantified or simple, and where $a_1, \ldots, a_k, a_{k+1}$ are free arguments. Suppose now that $l_1$ and $l_2$ are two literals in the action schema $\alpha$ that match the template $T$ via $c_1$ and $c_2$, respectively. Up to a permutation of the position of the fixed arguments, the corresponding relations $r_1$ and $r_2$ can be written as, $r_i(x_1', \ldots, x_k', v')$ for $i = 1, 2$ where the fixed arguments satisfy the relations $x_j^1 \sim_{F_C} x_j^2$ for every $j$. If, moreover, $l_1 \sim_T l_2$, the two literals will have the form $l_i = r_i(a_1, \ldots, a_k, a_{k+1})$ (or $l_i = \forall v : r_i(a_1, \ldots, a_k, v)$), where $a_1, \ldots, a_k, a_{k+1}$ are the free arguments.

**Proposition 60.** Given a template $T$ and an action schema $\alpha$, $\sim_T$ is an equivalence relation.
Proof. The only property to be checked is transitivity and this is evident from the equivalent description of the relation $\sim_T$ proposed in Remark 59.

An equivalence class of literals with respect to $\sim_T$ is called a $T$-class.

We now consider a grounding function $gr$ for $\alpha$ and an instance $\gamma$ for $T$. We recall the standing assumption that both $gr$ and $\gamma$ are injective maps (this will be often used in what follows). If $l$ is a literal in $\alpha$ that matches $T$, the subset of ground atoms $gr(l)$ is either a subset of $\gamma(T)$ or it must have empty intersection with $\gamma(T)$. This is simply because, both sets are closed under any modification of the assignment of the counted argument. This motivates the following definition:

Definition 61 (Coherence). $gr$ and $\gamma$ are coherent over $l$ if $gr(l) \subseteq \gamma(T)$.

Coherence is more concretely described in the following Remark.

Remark 62. Suppose that $l$ matches $T$ via the component $c$ whose relation is $r$. It follows from the considerations in Remark 59 that, depending if $r$ posses a counted variable or not and if $l$ is simple or universally quantified, $r$, $l$ and $gr(l)$ take the following forms:

\[
\begin{align*}
    r(x_1, \ldots, x_k) & \quad l = r(a_1, \ldots, a_k) & \quad gr(l) = \{r(gr(a_1), \ldots, gr(a_k))\} \\
    r(x_1, \ldots, x_k, v) & \quad l = r(a_1, \ldots, a_k, a_{k+1}) & \quad gr(l) = \{r(gr(a_1), \ldots, gr(a_k), gr(a_{k+1}))\} \\
    r(x_1, \ldots, x_k, v) & \quad l = \forall v : r(a_1, \ldots, a_k, v) & \quad gr(l) = \{r(gr(a_1), \ldots, gr(a_k), a), \ | a \in O\}
\end{align*}
\]

Note that, in all cases, the coherence condition $gr(l) \subseteq \gamma(T)$ is equivalent to require that:

\[gr(a_j) = \gamma(x_j), \ \forall j = 1, \ldots, k\]  \hspace{1cm} (3)

The following result is immediate from the conditions (3):

Proposition 63. Let $l$ be a literal of the action schema $\alpha$. Then, for every grounding function $gr$, it is possible to find an instance $\gamma$ such that $gr$ and $\gamma$ are coherent over $l$ and viceversa.

Lemma 64. Assume that $gr$ and $\gamma$ are coherent over a literal $l_1$ of $\alpha$ and let $l_2$ be another literal in $\alpha$ that matches $\gamma$. Then, $gr$ and $\gamma$ are coherent over $l_2$ if and only if $l_2 \sim_T l_1$.

The following result immediately follows from the definition of coherence and Lemma 64.

Proposition 65. Suppose that $M$ is a subset of literals appearing in $\alpha$. Then, $gr(M) \cap \gamma(T) = gr(M \cap \alpha)$ where $\alpha$ is the $T$-class of literals of $\alpha$ on which $gr$ and $\gamma$ are coherent.

Proposition 65 has an important practical consequence. Once $gr$ and $\gamma$ have been fixed, only the part of $\alpha$ made of literals in the class $\alpha$ where $gr$ and $\gamma$ are coherent affect the part of state dynamics concerning the set $\gamma(T)$. Precisely, if $a = gr(\alpha)$, it follows from the definition of $a_\gamma$ (see Remark 15) that:

\[
\begin{align*}
    Pre_\gamma^+ &= gr(Pre_\alpha^+ \cap L), \quad Ef f_\gamma^+ = gr(Eff_\alpha^+ \cap L)
\end{align*}
\]

Considering that, by Proposition 16, $a$ is strongly $\gamma$-safe if and only if $a_\gamma$ is also strongly safe, the property of strong safety of an action schema $\alpha$ does not depend on the literals in $\alpha$ that do not match $T$. Hence, in principle, such a property should be analysed by studying the restrictions of $\alpha$ to the different $T$-classes $\alpha$ of matching literals. This intuition leads to the following definition.
Definition 66 (Pure Action Schemas). Given a template $\mathcal{T}$, an action schema $\alpha$ and a $\mathcal{T}$-class $L$ of literals in $\alpha$, we define $\alpha_L$ to be the action schema where we only consider literals belonging to $L$. More precisely, $\alpha_L$ is the action schema such that

$$\text{Pre}_{\alpha_L}^\omega = \text{Pre}_{\alpha}^\omega \cap L, \quad \text{Eff}_{\alpha_L}^\omega = \text{Eff}_{\alpha}^\omega \cap L$$

We call $\alpha_L$ a pure action schema.

Example 12 (Floortile domain). Consider the template $\mathcal{T}_{ft}$ given in Example 2 and the action schema $\alpha$ = paint-up$^H$: $\text{Pre}_{\alpha}^\omega = \{\text{robot-at}(r,x) \mid \text{clear}(y)\}$. $\text{Eff}_{\alpha}^\omega = \{\text{clear}(y)\}$.

Note that both literals $\text{robot-at}(r,x)$ and $\text{clear}(y)$ in $\alpha$ match $\mathcal{T}_{ft}$, and they form two different $\mathcal{T}$-classes because they do not satisfy condition (ii) in Definition 58. $L_1 = \{\text{robot-at}(r,x)\}$ and $L_2 = \{\text{clear}(y)\}$.

Consider the instance $\gamma_1$ that associates $\text{tile}_1$ to each fixed argument in the components of $\mathcal{T}_{ft}$ and grounding function $gr(r) = \text{rbl}, gr(x) = \text{tile}_1$ and $gr(y) = \text{tile}_2$. In this case, $gr$ and $\gamma_1$ are coherent on the $\mathcal{T}$-class $L_1$.

We have two pure action schemas corresponding to $\alpha$: $\alpha_{L_1}$ and $\alpha_{L_2}$. $\alpha_{L_1}$ has the following specification: $\text{Pre}_{\alpha_{L_1}}^\omega = \{\text{robot-at}(r,x)\}$ and $\alpha_{L_2}$: $\text{Pre}_{\alpha_{L_2}}^\omega = \{\text{clear}(y)\}$, $\text{Eff}_{\alpha_{L_2}}^\omega = \{\text{clear}(y)\}$.

7.2. Pure Action Schema Classification

We now carry out a detailed analysis of pure action schemas, showing in particular how the check for strong safety for a ground action $\alpha = gr(\alpha)$ can be efficiently performed at the lifted level working with the different pure action schemas $\alpha_L$.

We fix an action schema $\alpha$ and a $\mathcal{T}$-class $L$ of its literals. First, we introduce a concept of weight at the level of literals in $L$ that allows us to distinguish between simple and universally quantified literals. Precisely, given $l \in L$, we put $w_l = 1$ if $l$ is simple, while $w_l = \omega$ if $l$ is universally quantified and where $\omega = |\mathcal{Q}|$. Given a subset $A \subseteq L$, we define $w(A) = \sum_{l \in A} w_l$. Note that $w(\cdot)$ simply coincides with the notion of cardinality in case all literals in $L$ are simple. If we consider a grounding function $gr$ for $\alpha$, then for every subset $A \subseteq L$, it holds:

$$|gr(A)| = w(A)$$

(4)

Similarly, if $c$ is a component of $\mathcal{T}$, we define $w_c$ equal to 1 or to $\omega$ if $c$, respectively, does not have or does have a counted variable.

We need a last concept:

Definition 67 (Coverage). Given a component $c \in \mathcal{T}$, we let $L_c$ to be the subset of literals in $L$ that match $\mathcal{T}$ through the component $c$. A subset of literals $M \subseteq L_c$ is said to cover the component $c$, if $w(M \cap L_c) \geq w_c$. $M$ is said to cover $\mathcal{T}$, if $M$ covers every component $c \in \mathcal{T}$.

Remark 68. If we consider a component $c \in \mathcal{T}$, we have that all ground atoms generated by $c$ are in $gr(M)$ if and only if $M$ covers $c$. In particular, $\gamma(\mathcal{T}) = gr(M)$ if and only if $M$ covers $\mathcal{T}$.

We now propose a classification of the pure action schemas $\alpha_L$, formally analogous to the one introduced for action sets in Definitions 17 and 19. We simply replace preconditions and effects of $\alpha_y$ with those of $\alpha_L$ and the concept of cardinality with that of weight.

Definition 69 (Classification of Pure Action Schemas). The pure action schema $\alpha_L$ is:

- unreachable for $\mathcal{T}$ if $w(\text{Pre}_{\alpha_L}^\omega) \geq 2$;
• heavy for $T$ if $w(\text{Pre}^+_\alpha) \leq 1$ and $w(\text{Eff}^+_\alpha) \geq 2$;
• irrelevant for $T$ if $w(\text{Pre}^+_\alpha) \leq 1$ and $w(\text{Eff}^+_\alpha) = 0$;
• relevant for $T$ if $w(\text{Pre}^+_\alpha) \leq 1$ and $w(\text{Eff}^+_\alpha) = 1$.

**Definition 70** (Classification of Relevant Action Schemas). The pure relevant action schema $\alpha_L$ is:

- balanced for $T$ if $w(\text{Pre}^+_\alpha) = 1$ and $\text{Pre}^+_\alpha \subseteq \text{Eff}^+_\alpha \cup \text{Eff}^-_\alpha$;
- unbalanced for $T$ if $w(\text{Pre}^+_\alpha) = 1$ and $\text{Pre}^+_\alpha \cap (\text{Eff}^+_\alpha \cup \text{Eff}^-_\alpha) = \emptyset$;
- bounded for $T$ if $w(\text{Pre}^+_\alpha) = 0$ and $L$ covers $T$;
- unbounded for $T$ if $w(\text{Pre}^+_\alpha) = 0$ and $L$ does not cover $T$.

The following result clarifies the relation with the corresponding ground actions.

**Proposition 71.** Consider an action schema $\alpha$, a $T$-class $L$ of its literals, a grounding function $\text{gr}$ and an instance $\gamma$ coherent over $L$. Put $a = \text{gr}(\alpha)$. Then, $\alpha_L$ satisfies any of the properties expressed in Definitions 69 and 70 if and only if $a$ satisfies the corresponding $\gamma$-property as defined in Definitions 17 and 19.

**Proof.** Immediate consequence of the fact that $a_\gamma = \text{gr}(\alpha_L)$, of equation (4), and of Remark 68.

We are now ready to propose the following final result concerning strong safety of general action schemas. It shows how strong safety can be seen as a property of an action schema and can be studied by analysing its pure parts.

**Corollary 72.** Strong safety is a liftable property. Moreover, an action schema $\alpha$ is strongly safe if and only if, for every $T$-class of literals $L$ of $\alpha$, $\alpha_L$ is unreachable, irrelevant, relevant balanced or relevant bounded.

**Example 13** (FloorTile domain). Consider the template $T_{ft}$ and the action schema $\alpha = \text{paint-up}^*$ given in Example 12. The two pure action schemas $\alpha_L^1$ and $\alpha_L^2$ are both irrelevant and hence strongly safe. Hence, $\alpha$ is strongly safe.

Now consider the action schema $\alpha' = \text{paint-up}^\text{end}$ with specification: $\text{Eff}^+_\alpha = \{\text{painted}\{y, c\}\}$. This is a pure action schema. It is relevant unbounded and thus not strongly safe.

An immediate consequence of Corollary 24 is:

**Corollary 73.** Given a template $T$, $T$ is invariant if for each $\alpha \in \mathcal{A}$, $\alpha$ is strongly safe.

### 8. Durative action schemas

Our goal now is to work out proper lifted versions of the properties of durative actions given in Section 5 in particular those involved in the statement of our main results, Theorems 49 and 51. Some of these properties concern just one durative action (e.g. safety), while others involve more actions (e.g. non-interfering, irrelevant unreachable). We start analysing the first type of
properties, presenting, in particular, an explicit characterisation of safety for durative actions at the lifted level.

We always use the following notation. Given a durative action schema $D\alpha = (a^{st}, a^{inv}, a^{end})$ and a grounding function $gr$ for $D\alpha$, we put $D\alpha = gr(D\alpha)$, where $D\alpha = (a^{st}, a^{inv}, a^{end})$ with $a^{st} = gr(a^{st}), a^{inv} = gr(a^{inv})$, and $a^{end} = gr(a^{end})$. Also, we define the auxiliary durative action schema $Da_\alpha = (a^{st}_\alpha, a^{end}_\alpha)$ where $a^{st}_\alpha$ and $a^{end}_\alpha$ are the instantaneous action schema such that:

$$
\begin{align*}
E f f^{+}_{a^{st}} &= E f f^{+}_{a^{st}_\alpha}, & Pre^{+}_{a^{st}} &= Pre^{+}_{a^{st}_\alpha} \cup (Pre^{-}_{a^{inv}} \setminus E f f^{+}_{a^{st}_\alpha}) \\
E f f^{+}_{a^{end}_\alpha} &= E f f^{+}_{a^{end}_\alpha}, & Pre^{+}_{a^{end}_\alpha} &= Pre^{+}_{a^{end}_\alpha} \cup Pre^{-}_{a^{inv}}
\end{align*}
$$

$Da_\alpha = gr(D\alpha_\alpha)$ is the corresponding ground auxiliary action already defined in Section 5.2

8.1. Safety of durative action schemas

We now fix a template $T$ and start to analyse safety. We consider a durative action schema $D\alpha$, its auxiliary action schema $D\alpha^*$ and its groundings $D\alpha = gr(D\alpha)$ and $Da_\alpha = gr(D\alpha_\alpha)$. Strong safety for durative actions reduces to check strong safety of its components and it is thus a lifting property. We can thus talk about the strong safety of $D\alpha$ or $D\alpha_\alpha$: this is equivalent to the strong safety of all its groundings, $D\alpha = gr(D\alpha)$ or, respectively, $Da_\alpha = gr(D\alpha_\alpha)$. Check of such property at the lifted level can be done using Corollary [72] for the starting and ending fragments.

We now want to characterise simple safety of the auxiliary durative action $Da_\alpha = gr(D\alpha_\alpha)$ at the lifted level. First we consider executability.

Define, for a generic action schema $\alpha$, the subsets

$$
\Gamma^+_\alpha := (Pre^{+}_\alpha \setminus E f f^{+}_{\alpha}) \cup E f f^{+}_{\alpha}, \quad \Gamma^-_\alpha := (Pre^{+}_\alpha \setminus E f f^{+}_{\alpha}) \cup E f f^{+}_{\alpha}
$$

We have the following result:

**Proposition 74.** Executability of auxiliary durative actions is a lifted property. Precisely, $D\alpha_\alpha$ is executable if and only if

$$
\Gamma^+_\alpha \cap Pre^{+}_{\alpha^{end}_\alpha} = \emptyset = \Gamma^-_\alpha \cap Pre^{+}_{\alpha^{end}_\alpha} \tag{5}
$$

**Proof.** Immediate consequence of Proposition [31] □

Assume now $D\alpha_\alpha$ to be executable. Fix an instance $\gamma$ and let $L$ be the the $T$-class of literals in $D\alpha$ on which $gr$ and $\gamma$ are coherent. Put $D\alpha_{\gamma L} = (a^{st}_{\gamma L}, a^{inv}_{\gamma L}, a^{end}_{\gamma L})$ and $Da_\alpha_{\gamma L} = (a^{st}_{\alpha L}, a^{end}_{\alpha L})$. Note that $D\alpha_{\gamma L} = gr(D\alpha_{\gamma L})$. Therefore, since simple $\gamma$-safety of $D\alpha_\alpha$ only depends on $D\alpha_{\gamma L}$ (since executability has already been assumed), we expect that such property can be formulated in terms of the pure auxiliary durative action schema $D\alpha_{\gamma L}$. To this aim, we now propose, for such durative schemas, the same classification introduced for ground durative actions in Definition [16].

First, we need a further concept:

**Definition 75 (Reachable action schemas).** $D\alpha_{\gamma L}$ is said to be reachable if it is executable and

$$
w(Pre^{+}_{\alpha^{st}_{\gamma L}} \cup (Pre^{+}_{\alpha^{end}_{\gamma L}} \setminus E f f^{+}_{\alpha^{end}_{\gamma L}})) \leq 1
$$

**Proposition 76.** If $gr$ and $\gamma$ are coherent over $L$ and $Da_\alpha = gr(D\alpha_\alpha)$, we have that $D\alpha_{\gamma L}$ is $\gamma$-reachable if and only if $D\alpha_{\gamma L}$ is reachable.
Proof. Immediate consequence of Propositions 32 and 65, and of equation (4). \hfill\square

Definition 77 (Safe durative action schemas). When \( Da_{sL} \) is such that

(i) \( Da_{sL} \) is reachable;
(ii) \( \alpha^{st}_{sL} \) is strongly safe;
(iii) \( \alpha^{end}_{sL} \) is relevant unbounded;
(iv) \( Da_{sL} \) satisfies any of the conditions below:

(a) \( \alpha^{st}_{sL} \) irrelevant, \( w(Pre_{a_{st}^+}) = 1 \), \( Pre_{a_{st}^+} \subseteq Eff_{a_{st}^-} \);
(b) \( \alpha^{st}_{sL} \) irrelevant, \( w(Pre_{a_{st}^+}) = 1 \), \( Pre_{a_{st}^+} \nsubseteq Eff_{a_{st}^-} \), \( Pre_{a_{st}^+} \subseteq Eff_{a_{st}^-} \cup Eff_{a_{st}} \);
(c) \( \alpha^{st}_{sL} \) irrelevant, \( w(Pre_{a_{st}^+}) = 0 \), \( Pre_{a_{st}^+} \cup Eff_{a_{st}^-} \cup Eff_{a_{st}} \cup Eff_{a_{st}} \) covers \( T \);
(d) \( \alpha^{st}_{sL} \) relevant, \( Eff_{a_{st}^+} \subseteq Eff_{a_{st}} \cup Eff_{a_{st}} \).

we say that \( Da_{sL} \) is simply safe of type \( x \) where \( x \in \{a, b, c, d\} \).

Corollary 78. Safety for durative auxiliary actions is a liftable property. \( Da_s = gr(Da_s) \) is safe if and only if:

- \( Da_s \) is executable;
- For every \( T \)-class \( L \) of literals in \( Da \), one of the following conditions hold:
  - \( Da_{sL} \) is strongly safe;
  - \( \alpha^{st}_{sL} \) is strongly safe and \( Da_{sL} \) is unreachable;
  - \( Da_{sL} \) is simply safe of type \( x \) where \( x \in \{a, b, c, d\} \).

Proof. Immediate consequence of previous definitions and Proposition 44. \hfill\square

Example 14 (Floortile domain). Consider our usual template:

\[ T_{fi} = \{ (\text{robot-at}, 2, 0), (\text{painted}, 2, 1), (\text{clear}, 1, 1) \} \]

and the action schema:

\[ Da = \text{paint} - \text{up} : (\text{paint} - \text{up}^a, \text{paint} - \text{up}^b, \text{paint} - \text{up}^c) \]

where the single instantaneous action schemas have the specifications as in Table 2.

In this action schema, we have three literals that match \( T_{fi} \): \[ \text{robot-at} (r, x) \text{ clear} (y) \] and \[ \text{painted} (y, c) \]. They form two \( T \)-classes: \( L_1 = \{ \text{robot-at} (r, x) \} \) and \( L_2 = \{ \text{clear} (y) \} \) and \[ \text{painted} (y, c) \]. Note that in this case \( \text{paint-up}^a_{L_1} \) is equal to \( \text{paint-up}^a_{L_2} \) for \( i = 1, 2 \) and the same holds for \( \text{paint-up}^b_{L_i} \) and \( \text{paint-up}^c_{L_i} \).

The pure action schemas \( \text{paint-up}^a_{L_i} \), \( \text{paint-up}^b_{L_i} \), and \( \text{paint-up}^c_{L_i} \) are strongly safe because they are irrelevant. The pure schema \( \text{paint-up}^a_{L_2} \) is relevant unbounded.

The pure durative action schema \( \text{paint-up}_{L_1} \) is strongly safe because \( \text{paint-up}^a_{L_1} \) and \( \text{paint-up}^c_{L_1} \) are strongly safe since they are irrelevant.

The pure schema \( \text{paint-up}_{L_2} \) is simply safe of type (a) since:
8.2. Lifting properties of multiple actions

In this section, we study how properties that involve more than one action (e.g. mutex) can be lifted. This requires to work simultaneously with different groundings and, for this reason, additional concepts are need.

Consider two action schemas $a^1$ and $a^2$ (instantaneous or durative) with set of free arguments $V_{a^1}$ and $V_{a^2}$, respectively. Whenever we consider two groundings $gr^1$ and $gr^2$ for $a^1$ and $a^2$, respectively, the pairwise properties of the two actions $a' = gr^a(a')$ (e.g. properties regarding the sequence $(a^1, a^2)$ or the set $(a^1, a^2)$) are non liftable, as in general they may depend on the specific groundings chosen. A key aspect is the possible presence, in the two action schemas, of pairs of free arguments $v' \in V^{a'}$ such that $gr^1(v') = gr^2(v')$: this may cause the same ground atom to appear in the two actions $a^1$ and $a^2$, which in principle can affect the validity of certain properties, such as non-interference. To cope with this complexity at the lifted level, we introduce a concept of reduced union of the two sets $V_{a^1}$ and $V_{a^2}$ to be used as a common set of free arguments for the two schemas.

We define a matching between $a^1$ and $a^2$ as any subset $M \subseteq V_{a^1} \times V_{a^2}$ such that:

- If $(v^1, v^2), (w^1, w^2) \in M$, then $v^1 = w^1$;
- If $(v^1, v^2), (v^1, w^2) \in M$, then $v^2 = w^2$.

We now define the set $V_{a^1} \cup_M V_{a^2}$ obtained by $V_{a^1} \cup V_{a^2}$ by reducing each pair of arguments $v^1 \in V_{a^1}$ and $v^2 \in V_{a^2}$ such that $(v^1, v^2) \in M$ to a new argument, denoted as $v^1 v^2$. Note that in the case when $M = \emptyset$, no reduction takes place and $V_{a^1} \cup_M V_{a^2} = V_{a^1} \cup V_{a^2}$.

Given a matching $M$, we have natural maps $\pi^i_M : V^{i'} \rightarrow V_{a^1} \cup_M V_{a^2}$ associating to each argument $v'$ in its literals with $\pi^i_M(v')$. If $F$ is a literal of $a'$, we denote by $\pi^i_M(F)$ the literal obtained with this substitution. Similarly, if $A'$ is a set of literals of $a'$, we put $\pi^i_M(A') = \{\pi^i_M(F) \mid F \in A'\}$.  

| $a$       | paint-up$^+$       | paint-up$^{end}$       | paint-up$^+$       |
|-----------|--------------------|------------------------|--------------------|
| $Pre^+$   | [robot - at(r,x)]  | [robot - has(r,c)]    | $\emptyset$       |
|           | clear(y)]          | up(y,x)]               |                    |
| $Eff^+$   | $\emptyset$       | $\emptyset$           | [$\text{painted}(y,c)$] |
| $Eff^-$   | $\{\text{clear}(y)\}$ | $\emptyset$          | $\emptyset$       |

Table 4: Durative action schema $\text{paint-up}$ (abbreviated specification).
On the literals of the two schemas, expressed in the common argument set \( V_{a^1} \cup_M V_{a^2} \), we can jointly apply set theoretic operators. If \( l^i \) is a literal of \( a^i \) and \( A^i \) is a set of literals of \( a^i \), for \( i = 1, 2 \), we will use the notation \( l^i =_M l^i \) for \( \pi^1_M(l^i) = \pi^2_M(l^i) \) and \( l^i \in_M A^j \) for \( \pi^1_M(l^i) \in \pi^2_M(A^j) \). Similarly, we put \( A^1 \ast_M A^2 = \pi^1_M(A^1) \ast \pi^2_M(A^2) \) where \( \ast \in \{ \cup, \cap, \setminus \} \).

We now investigate the relation between matchings and specific groundings of the two schemas.

**Definition 79** (Coherent grounding functions). Consider two action schemas \( a^1 \) and \( a^2 \) and a matching \( M \) between them. Two grounding functions \( gr^1 \) and \( gr^2 \) for \( a^1 \) and \( a^2 \), respectively, are said to be \( M \)-adapted if given \( v^i \in V_{a^i} \) for \( i = 1, 2 \), it holds \( gr^i(v^1) = gr^2(v^2) \) if and only if \( (v^1, v^2) \in M \).

**Remark 80.** Note that, given two groundings \( gr^1 \) and \( gr^2 \), if we consider \( M = \{(v^1, v^2) | gr^1(v^1) = gr^2(v^2)\} \) we clearly have that \( M \) is a matching (recall that maps \( gr^i \) are injective) and \( gr^1 \) and \( gr^2 \) are \( M \)-adapted.

Coherent groundings can clearly be factorised through the reduced set \( V_{a^1} \cup_M V_{a^2} \):

**Proposition 81.** Consider two action schemas \( a^1 \) and \( a^2 \), a matching \( M \) between them, and grounding functions \( gr^i \) for \( a^i \), \( i = 1, 2 \). The following conditions are equivalent:

(i) \( gr^1 \) and \( gr^2 \) are \( M \)-adapted;

(ii) there exists an injective function \( gr : V_{a^1} \cup_M V_{a^2} \to O \) such that, \( gr^i = gr \circ \pi^i_M \) for \( i = 1, 2 \).

Suppose that \( gr^1 \) and \( gr^2 \) are two \( M \)-adapted groundings of \( a^1 \) and \( a^2 \). If \( A^i \) is a set of literals of \( a^i \), for \( i = 1, 2 \), for any set theoretic operation \( \ast \in \{ \cup, \cap, \setminus \} \) it holds that:

\[
gr^1(A^1) \ast gr^2(A^2) = gr(\pi^1_M(A^1)) \ast gr(\pi^2_M(A^2)) = gr(A^1 \ast_M A^2)
\]

(6)

This follows from Proposition 81 and the fact that \( gr \) is injective. An iterative use of (6) shows that any set theoretic expression on the two ground actions \( gr(a^i) \) is in bijection (through \( gr \)) with a corresponding expression on the two schemas \( a^i \) expressed in the common reduced set \( V_{a^1} \cup_M V_{a^2} \). As a consequence, any property of ground actions (with the standing assumption of \( M \)-adapted groundings) that can be expressed by set theoretic operations on their literals can be reformulated by rewriting these literals in the new alphabet \( V_{a^1} \cup_M V_{a^2} \). This is the key observation in order to lift properties of pairs of actions. To be more concrete, we consider the example of non-interfering actions, which will be needed in what follows.

**Definition 82** (Mutex simple action schemas). We say that two action schemas \( a^1 \) and \( a^2 \) are \( M \) non-interfering if for \( i \neq j \)

\[
E f f^+(a^i) \cap_M E f f^-(a^j) = \emptyset
\]

\[
[Pre^+(a^i) \cup_M Pre^-(a^j)] \cap_M [E f f^+(a^j) \cup_M E f f^-(a^i)] = \emptyset
\]

If \( a^i \) and \( a^j \) are not \( M \) non-interfering, they are called \( M \)-mutex.

**Proposition 83.** Suppose that \( a^1 \) and \( a^2 \) are \( M \)-mutex and suppose that \( gr^1 \) and \( gr^2 \) are two \( M \)-adapted grounding functions for \( a^1 \) and \( a^2 \), respectively. Then, the two ground actions \( a^i = gr(a^i) \) are mutex.

**Proof.** Immediate consequence of (6).
Remark 84. Note that certain properties that depend on the matching $M$ have a monotonic behaviour, i.e. if they are true for a matching $M$, they remain true for a larger matching $M' \supseteq M$. This is the case, for instance, of properties that can be expressed in terms of identities between literals of type $l^1 =_M l^2$, such as the $M$-mutex property.

To cope with properties related to a template and its instantiations, it is useful to introduce a family of matchings induced by the presence of literals in the two schemas matching in a template. Precisely, consider now a template $T = (C, T_C)$ and two action schemas $\alpha^1$ and $\alpha^2$. Consider $T$-classes $L'$ of literals of $\alpha^i$ for $i = 1, 2$. There is a natural way to associate a matching to $L^1$ and $L^2$ as follows. Pick literals $l^i \in L'$ for $i = 1, 2$ and consider components $c^i \in C$ such that $l^i$ matches $T$ through $c^i$. Put

$$M_{L^1, L^2} := \{(\text{Arg}[h, l^1], \text{Arg}[k, l^2]) | (c^1, h) \sim_{T_C} (c^2, k)\} \quad (7)$$

It immediately follows from the definition of $T$-coupled pairs of literals (Definition 58) that $M_{L^1, L^2}$ does not depend on the particular literals $l^i$ chosen, but only on the $T$-classes $L'$.

Essentially, in $M_{L^1, L^2}$, we are rewriting arguments in the literals of $L^1$ and $L^2$ that correspond to $T_C$-equivalent variables in the template $T$. The next proposition shows the role played by such a matching.

Proposition 85. Consider two groundings $gr^1$ and $gr^2$ for $\alpha^1$ and $\alpha^2$, respectively, which are $M$-adapted. Then the following facts hold:

(i) given an instance $\gamma$ for $T$, if $L^1$ are the $T$-classes of literals of $\alpha^i$ on which $gr^i$ and $\gamma$ are coherent. Then, $M_{L^1, L^2} \subseteq M$;

(ii) given $T$-classes of literals $L^1$ of $\alpha^i$, if $M_{L^1, L^2} \subseteq M$, there exists just one instance $\gamma$ of $T$ such that $gr^i$ and $\gamma$ are coherent on $L^1$.

Proof. Immediate consequence of the definition (7) and of Remark 62. \qed

We are now ready to lift the properties used in Section 4. We start with unreachability.

Definition 86 (Unreachable durative action schemas). Given two durative action schemas $Da^1$, $Da^2$ and corresponding $T$-classes of literals $L^1$ and $L^2$, we say that $((\alpha^{1\text{inv}}, \alpha^{2\text{inv}}), (\alpha^{1\text{end}}, \alpha^{2\text{end}}))$ is $(L^1, L^2)$-unreachable, if, denoted $M = M_{L^1, L^2}$, at least one of the following conditions is satisfied

(i) $Pre^{+}_{a^{1\text{inv}}} \cap M Pre^{-}_{a^{2\text{end}}} \neq \emptyset$;

(ii) $Pre^{+}_{a^{1\text{inv}}} \cap M Pre^{-}_{a^{2\text{end}}} \neq \emptyset$;

(iii) $w((Pre^{+}_{a^{1\text{inv}}} \cup Pre^{+}_{a^{2\text{inv}}}) \cup M (Pre^{+}_{a^{1\text{end}}} \cup Pre^{+}_{a^{2\text{end}}}))) \geq 2$.

Proposition 87. Suppose that $Da^1$, $Da^2$ are two durative action schemas and $gr^1$, $gr^2$ two corresponding grounding functions. Put $Da' = gr(Da')$ and consider an instance $\gamma$. Let $L'$ be the $T$-class of literals of $Da'$ on which $gr'$ and $\gamma$ are coherent. If $((\alpha^{1\text{inv}}, \alpha^{2\text{inv}}), (\alpha^{1\text{end}}, \alpha^{2\text{end}}))$ is $(L^1, L^2)$-unreachable, then $((\alpha^{1\text{inv}}, \alpha^{2\text{inv}}), (\alpha^{1\text{end}}, \alpha^{2\text{end}}))$ is $\gamma$-unreachable.

We now propose the lifted version of relevant right isolated.
Definition 88 (Relevant right isolated schemas). Given a template $\mathcal{T}$, the set of durative action schemas $\mathcal{A}^d$ is said to be relevant right isolated if, for every $Da^1, Da^2 \in \mathcal{A}^d$, corresponding $\mathcal{T}$-classes $L^1, L^2$ of literals of each of them such that $Da^1_L$ are both not strongly safe, one of the following conditions is satisfied (we use the notation $M = M_{L^1, L^2}$):

(i) $|Ef f_{a^1_{L^1}}^+ \cup_M Ef f_{a^2_{L^2}}^-| \leq 1$;

(ii) at least one of the two pairs $\{a^1\text{end}, a^2\text{end}\}$ or $\{a^1\text{inv}, a^2\text{inv}\}$ is $M$-mutex;

(iii) $\{(a^1\text{inv}, a^2\text{inv}), (a^1\text{end}, a^2\text{end})\}$ is $(L^1, L^2)$-unreachable.

Proposition 89. Given a template $\mathcal{T}$, suppose that the set of durative action schemas $\mathcal{A}^d$ is relevant right isolated. Then $\mathcal{G}\mathcal{A}^d$ is also relevant right isolated.

Similarly, we can lift the property of relevant left isolated expressed in Definition 52 by analogously defining:

Definition 90 (Relevant left isolated schemas). Given a template $\mathcal{T}$, the set of durative action schemas $\mathcal{A}^d$ is said to be relevant left isolated if, for every $Da^1, Da^2 \in \mathcal{A}^d$, corresponding $\mathcal{T}$-classes $L^1, L^2$ of literals of each of them such that $Da^1_L$ are both not strongly safe, one of the following conditions is satisfied (we use the notation $M = M_{L^1, L^2}$):

(i) at least one of the two pairs $\{a^1\text{start}, a^2\text{start}\}$ or $\{a^1\text{inv}, a^2\text{inv}\}$ is $M$-mutex;

(ii) $\{(a^1\text{inv}, a^2\text{inv}), (a^1\text{start}, a^2\text{start})\}$ is $(L^1, L^2)$-unreachable.

Similarly, it holds:

Proposition 91. Given a template $\mathcal{T}$, suppose that the set of durative action schemas $\mathcal{A}^d$ is relevant left isolated. Then $\mathcal{G}\mathcal{A}^d$ is also relevant left isolated.

The last property we want to lift is that of strong irrelevant unreachable expressed in Definition 55. This is important since it can be efficiently implemented at the algorithmic level.

Definition 92 (Strongly irrelevant unreachable schemas). Consider a template $\mathcal{T}$, a pair of action schemas $a^1, a^2 \in \mathcal{A}$, and corresponding $\mathcal{T}$-classes of literals $L^1$ and $L^2$. We say that the pair $(a^1, a^2)$ is strongly $(L^1, L^2)$-irrelevant unreachable if, denoted $M = M_{L^1, L^2}$, any of the following conditions is satisfied:

(i) there exist $l^1 \in \Gamma_{a^1}, l^2 \in \text{Pre}^+_{a^2}$ with $l^1 =_M l^2$ such that, for every action schema $a$, for every $\mathcal{T}$-class $L$ of literals of $\alpha$ for which $a_L$ is irrelevant, and for every matching $M$ between $a^1$ and $a$ for which $M \supseteq M_{L^1, L}$, we have that $l^1 \notin_M Ef f_{a^1}$;

(ii) there exist $l^1 \in \Gamma_{a^1}, l^2 \in \text{Pre}^+_{a^2}$ with $l^1 =_M l^2$ such that, for every action schema $a$, for every $\mathcal{T}$-class $L$ of literals of $\alpha$ for which $a_L$ is irrelevant, and for every matching $M$ between $a^1$ and $a$ for which $M \supseteq M_{L^1, L}$, we have that $l^1 \notin_M Ef f_{a^1}$;

(iii) $\text{w}(\text{Pre}^+_{a^1} \cup_M (\text{Pre}^+_{a^2} \setminus_M Ef f_{a^1})) > 1$. 

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Proposition 93. Consider a template \( T \), a pair of action schemas \( \alpha_1, \alpha_2 \in \mathcal{A} \) and relative groundings \( \text{gr}^1 \) and \( \text{gr}^2 \). Put \( a' = \text{gr}(\alpha') \) and consider an instance \( \gamma \). Let \( L' \) be the \( T \)-class of literals of \( a' \) on which \( \text{gr}^1 \) and \( \gamma \) are coherent. If \((\alpha_1, \alpha_2)\) is strongly \((L_1, L_2)\)-irrelevant unreachable, then \((a_1, a_2)\) is strongly \( \gamma \)-irrelevant unreachable.

Denote by \( \mathcal{A}^d(T) \) the durative action schemas that are not strongly safe with respect to the template \( T \) and with \( \mathcal{A}^m(T), \mathcal{A}^\text{end}(T) \) the corresponding start and ending fragments.

Proposition 94. Consider a template \( T \). The set \( \mathcal{G} \mathcal{A}^d \) is relevant non intertwining if the following conditions are satisfied:

(i) \( \mathcal{A}^d \) is relevant left isolated;

(ii) for every \( D \alpha_1 \in \mathcal{A}^d \) and \( T \)-class \( L_1 \), and for every \( \alpha_2 \in \mathcal{A} \) and \( T \)-class \( L_2 \) such that \( \alpha_1^{L_1} \in \mathcal{A}^m(T), \alpha_2^{L_2} \in \mathcal{A}^\text{end}(T) \) and is not irrelevant or \( \alpha_2^{L_2} \in \mathcal{A}^\text{st}(T) \), we have that \((\alpha_1^{L_1}, \alpha_2^{L_2})\) is strongly \((L_1, L_2)\)-irrelevant unreachable.

We are now ready to propose the lifted versions of our main Theorems 49, 51. Proofs are straightforward consequences of our previous definitions and results.

Corollary 95. Consider a template \( T \) and suppose that the set of instantaneous action schemas \( \mathcal{A} \) and that of durative action schemas \( \mathcal{A}^d \) satisfy the following properties:

(i) every \( \alpha \in \mathcal{A} \) is strongly safe;

(ii) for every \( D \alpha \in \mathcal{A}^d \) and every \( T \)-class \( L \) such that \( D \alpha \in \mathcal{A}^{d}(T), D \alpha \) is reachable and strongly safe;

(iii) \( \mathcal{A}^d \) is relevant right isolated. \( \mathcal{T} \) is invariant.

Corollary 96. Consider a template \( T \) and suppose that the set of instantaneous action schemas \( \mathcal{A} \) and that of durative action schemas \( \mathcal{A}^d \) satisfy the following properties:

(i) every \( \alpha \in \mathcal{A} \) is strongly safe;

(ii) for every \( D \alpha \in \mathcal{A}^d \), \( D \alpha \) is safe;

(iii) \( \mathcal{A}^d \) satisfies the conditions expressed in Proposition 94.

Then, \( \mathcal{T} \) is invariant.

Finally, it is useful to consider a lifted version of Corollary 57.

Corollary 97. Consider a template \( T \) and suppose that the set of instantaneous action schemas \( \mathcal{A} \) and that of durative action schemas \( \mathcal{A}^d \) satisfy the following properties:

(i) for every \( D \alpha \in \mathcal{A}^d \) and every \( T \)-class \( L \), if \( D \alpha \in \mathcal{A}^{d}(T), D \alpha \) is simply safe of type \((a)\);

(ii) for every \( \alpha \in \mathcal{A} \) and every \( T \)-class \( L \), if \( \alpha \in \mathcal{A}^{d}(T) \cup \mathcal{A}^{\text{end}}(T) \), then, \( \alpha \) is either irrelevant or relevant balanced.
Then, $T$ is invariant.

We end this section by presenting two examples from the IPCs in which we apply Corollaries 97 and 95 to demonstrate the invariance of the templates under consideration. Corollary 96 is the most general one and can be used in more complex cases.

Example 15 (Floortile domain). Consider our usual template:

$$T_{ft} = \{(\text{robot-at}(r,2), \text{paint}(2,1), \text{clear}, 1, 1)\}$$

The actions schemas in the domains are:

$$\mathcal{A} = \{\text{change-color, paint-up, paint-down, up, down, up-down, right, left}\}$$

The schemas $\text{paint-up}$ and $\text{paint-down}$ are symmetrical and differ only on literals not in the components of $T_{ft}$. They have the same $T$-classes $L_1 = \{\text{Robot-at}(r, x)\}$ and $L_2 = \{\text{clear}(y)\}$.$\text{paint}(y, c)\}$. As seen in Example 14, the pure schemas $\text{paint-up}_{1L}$ and $\text{paint-up}_{2L}$ are irrelevant and $\text{paint-up}_{4L}$ is simply safe of type (a). The same holds for $\text{paint-down}_{1L}$ and $\text{paint-down}_{2L}$.

The schemas $\text{up}$, $\text{down}$, $\text{right}$, and $\text{left}$ are also symmetrical and differ only on literals not in the components of $T_{ft}$. They have the same $T$-classes $L_3 = \{\text{robot-at}(r, x)\}$, $L_4 = \{\text{robot-at}(r, y), \text{clear}(y)\}$. The schemas $\text{up}_{1L}$, $\text{down}_{1L}$, $\text{right}_{1L}$, and $\text{left}_{1L}$, with $i = 3, 4$, are all simply safe of type (a).

The schema $\text{change-color}$ has no equivalence classes and its start and end fragments are both irrelevant.

By Corollary 97 the template template $T_{ft}$ is invariant.

Example 16 (Depot domain). Consider the domain Depot (see Appendix B) and the template:

$$T_{dp} = \{(\text{lifting}(2,1), \text{available}, 1, 1)\}$$

Invariants of this template mean that, given a hoist, it can be in two possible states: lifting a crate or available. The actions schemas in the domains are all durative:

$$\mathcal{A} = \{\text{drive, lift, drop, load, unload}\}$$

We indicate them as $\text{Da}^1, \ldots, \text{Da}^5$ respectively and, given $\text{Da}^i$, its arguments as $x_i, y_i, \ldots.

To demonstrate that $T_{dp}$ is invariant, we want to apply Corollary 95. We start with condition (ii) since $\mathcal{A}$ is empty.

The action $\text{Da}^1 = \text{drive}$ has no literals that match the template so it is strongly safe. The other schemas have respectively $T$-classes $L_i = \{\text{lifting}(x_i, y_i), \text{available}(x_i)\}$. There are only two fragments of the durative actions that are not strongly safe as they are relevant unbounded: $\alpha^\text{start}_{2L}$ and $\alpha^\text{end}_{2L}$. However, their auxiliary versions $\alpha^\text{start}_{2L}$ and $\alpha^\text{end}_{2L}$ are strongly safe since they are balanced (when the over all condition $\text{lifting}(x_3, y_3)$ is added to the end effects, it matches the delete effect of $\text{lifting}(x_3, y_3)$ and balances the add effect $\text{available}(x_3)$). Similar considerations hold for $\alpha^2$. Reachability for $\alpha^\text{start}_{2L}$ and $\alpha^\text{end}_{2L}$ is a straightforward check. In consequence, condition (ii) holds.

We now need to verify condition (iii) of Corollary 95, i.e. $\mathcal{A}$ is relevant right isolated. Under the re-writing $\mathcal{M}_{\text{lift}, \text{load}}$, we have that $x_3 = x_4$ and $y_3 = y_4$ and therefore $\text{Eff}_{\text{load}}^{\text{start}} \cup \mathcal{M}_{\text{lift}, \text{load}} \text{Eff}_{\text{load}}^{\text{end}} = \{\text{available}(x_3) = \text{available}(x_4)\}$. Hence condition (i) of Definition 88 is satisfied.

We can conclude that $T_{dp}$ is an invariant template.
9. Guess, Check and Repair Algorithm

As with related techniques (Gerevini and Schubert 2000; Rintanen 2000; Helmer 2009), our algorithm for finding invariants implements a guess, check and repair approach. It starts by generating a set of initial simple templates. For each template $T$, it then applies the results stated in the previous sections to check its invariance. If $T$ is invariant, the algorithm outputs it. On the other hand, if the algorithm does not manage to prove the invariance of $T$, it discards it. Before rejection, however, the algorithm tries to fix the template by generating a set of new templates that are guaranteed not to fail for the same reasons as $T$. In turn, these new templates need to be checked against the invariance conditions as they might fail due to other reasons.

9.1. Guessing initial templates

When we create the set of initial templates, we ignore constant relations, i.e. relations whose ground atoms have the same truth value in all the states (for example, type predicates). In fact, they are trivially invariants and so are typically not interesting.

For each modifiable relation $r$ with arity $a$, we generate $a + 1$ initial templates. They all have one component and zero or one counted argument (which can be in any position from 0 to $(a - 1)$): $(r, a, a)$ (no counted argument) and $(r, a, p)$ with $p \in \{0, \ldots, (a - 1)\}$. Since the templates have one component, there is only one possible admissible partition $\mathcal{F}_C$, with $C = \{c\}$. Hence, we construct the template $T = (C, \mathcal{F}_C)$.

**Example 17 (Floortile domain).** Consider the components $c_1 = \langle \text{robot-at}, 2, 1 \rangle$. Put $\mathcal{F}_C = \{F_1\}$ where $F_1 = \{(c_1, 0)\}$. An initial template is $T_1 = ((c_1), F_1))$. Intuitively, invariants of $T_1$ mean that a robot can occupy only one position at any given time and our algorithm validates it as an invariant. Another initial template is built by considering the component $c_2 = \langle \text{robot-at}, 2, 0 \rangle$ and the partition $\mathcal{F}_C = \{F_2\}$ where $F_2 = \{(c_2, 1)\}$. We have another initial template: $T_2 = ((c_2), F_2))$. Invariants of this template mean that a tile cannot be occupied by more than one robot, which is not true in general, and our algorithm correctly discards it. Finally, consider the component $c_3 = \langle \text{robot-at}, 2, 2 \rangle$ and the partition $\mathcal{F}_C = \{F_3\}$ where $F_3 = \{(c_3, 0), (c_3, 1)\}$. Another initial template is $T_3 = ((c_3), F_3))$. This is also not an invariant and is rejected.

If we repeat this process with every modifiable relation $r$ in the Floortile domain, we obtain the full set of initial templates.

9.2. Checking conditions for invariance

Given a template, we apply the results stated in the previous sections to check its invariance. In particular, we apply our most operative results: Corollary 73 and Corollaries 95 - 97. Remarkably, all these results work at the level of action schemas, not ground actions.

We first need to verify if all the instantaneous action schemas $\mathcal{A}$ in the domain, both the native ones and those obtained from the fragmentation of durative actions, respect the strong safety conditions. We then check safety conditions that only involve durative action schemas that are not strongly safe. Finally, we validate additional conditions that avoid the intertwining of potentially dangerous durative actions. Given the different computational complexity of our results (see considerations below), our algorithm checks the applicability of them in the following order: first, Corollary 73 which involves only conditions for instantaneous schemas, then Corollary 97 which considers safety conditions for individual action schemas, and finally Corollaries 95 and
which verify conditions involving pairs of durative action schemas. To implement this
procedure, we apply the decision tree shown in Figure 10 to the set of action schemas $\mathcal{A}$. The leaves
labelled as *Possibly Not Invariant* arise when our sufficient results do not apply. In this case, we
cannot assert anything about the invariance of the template.

![Decision Tree](image)

**Figure 10:** Decision Tree for deciding the invariance of a template $T$. 

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Our checks involve the analysis of all \( T \)-classes in each action schema \( \alpha \) in the domain. Since the \( T \)-classes form a partition of the set of literals in the schema that match the template, the maximum number of \( T \)-classes is equal to the number of such literals. We can estimate this term with the product \( \omega \cdot |C| \) where \( \omega \) is the maximum number of literals in any schema that share the same relation and \( |C| \) is the cardinality of the template’s component set C. We deduce that all safety checks for individual schemas (both the instantaneous and the durative ones) have a computational complexity of the order of \( M \cdot |A| \cdot \omega \cdot |C| \), where \( M \) is the maximum number of the literals appearing in any schema and \( |A| \) is the total number of schemas. The check of right and left relevant isolated properties involve instead a pair of schemas and \( T \)-classes and, in consequence, the computational complexity is of the order of \( M^2 \cdot |A|^2 \cdot \omega^2 \cdot |C|^2 \). The check that two schemas are strongly irrelevant unreachable, as required in Corollary 96, involves considering a third action schema and a family of matchings, \( \tilde{M} \). The complexity relating to the check of the matchings can be shown to be reducible to a check of complexity of the order of the maximum number \( N \) of arguments in any literal in the domain. Therefore, the total complexity of the check for strongly irrelevant unreachable schemas is of the order \( N \cdot M^3 \cdot |A|^3 \cdot \omega^3 \cdot |C|^3 \).

In our experiments, we have found no cases in which this check needs to be applied.

**Example 18 (Floortile domain).** The parameters for the computational complexity analysis are as follows:

\[
|A| = 14, \ M = 4, \ \omega = 1, \ |C| = 3, \ N = 2
\]

### 9.3. Repairing templates

When, in analysing an action schema \( \alpha \), we reach a failure node in our decision tree, we discard the template \( T \) under consideration since we cannot prove its invariance. This might be because of two reasons: either \( T \) is not an invariant or our sufficient conditions are not powerful enough to capture it. Given that we cannot assume that the template is not invariant with certainty, before discarding it, we try to fix it in such a way to obtain new templates for which it might be possible to prove invariance under our conditions. In particular, based on the schema \( \alpha \), we enlarge the set of components of the template by adding suitable literals that appear in the preconditions and negative effects of \( \alpha \) since they can be useful to prove that \( \alpha \) is simple or strong safety.

More precisely, if the algorithm rejects \( T \) because it finds an instantaneous schema that is heavy or unbalanced (first step in the decision tree), no fixes are possible for \( T \). Since \( \alpha \) leads to a weight greater or equal to two for at least an instance of \( T \), enlarging the set of components of \( T \) cannot help in repairing the template. Similarly, if there are durative schemas that are non-executable or unreachable, no fixes are possible since these properties cannot be changed by adding components. However, when a failure node is reached in the presence of unbounded schemas, enlarging the set of components might prove useful in making them simply or strongly safe schemas. We operate as follows: for each unbounded schema \( \alpha \), we try to turn it into a balanced action schema and, when \( \alpha \) is the end fragment of a durative action \( D\alpha \), we alternatively attempt to make \( D\alpha \) a simply safe schema, as defined in Definition 77.

Given a template \( T = (C, \tilde{T}_C) \) that has been rejected by the algorithm, put \( k \) the number of fixed arguments for \( T \) and \( m \) the number of its components. Consider an unbounded schema \( \alpha \) with relevant literal \( I \). We look for another literal \( I' \) in \( \alpha \) with the following characteristics:

(i) \( \text{Rel}[I'] = \langle r', a' \rangle \), where \( a' = k \) or \( a' = (k + 1) \);
(ii) There exists a bijection β from the set of free arguments of l to the set of free arguments of $l'$ such that $Arg[i, l] = Arg[i, l']$ for every $i \in I$.

(iii) $l' \in Pre_{P_e}^e \cap Eff_{P_d}^d$.

If α is the end fragment of a durative action $Da$, then condition (iii) can be substituted with one the alternative following conditions:

(iv) $l' \in Pre_{P_e}^e \cap Eff_{P_d}^d$

(v) $l' \in Pre_{P_e}^e \cap Eff_{P_d}^d$

For each literal $l'$ that satisfies conditions (i), (ii) and one between conditions (iii), (iv) and (v), we create a new component $c' = (r', a', p')$, where $p' \in \{0, \ldots, a'\}$, and one new template $T' = (C', F'_C)$, where $C' = C \cup \{c'\}$ and $F'_{C}$ is an admissible partition of $F_C$ such that for each $c_1, c_2 \in C$, we have that $(c_1, i) \sim_{F'_{C}} (c_2, j)$ if and only if $(c_1, i) \sim_{F_{C}} (c_2, j)$ and $(c, i) \sim_{F'_{C}} (c', j)$ if and only if $Arg[i, l] = Arg[j, l']$ (or, equivalently, $j = β(i)$).

If we find a literal $l'$ that satisfies condition (iii), the schema α is guaranteed to be balanced for $T'$; if the literal $l'$ satisfies condition (iv), α is guaranteed to be simply safe of type (a) for $T'$; finally, if the literal $l'$ satisfies condition (v), α is guaranteed to be simply safe of type (b) for $T'$.

**Example 19 (Floortile domain).** Consider the template $T_2 = ([c_2], [F_2])$ as indicated in Example 18 and the action schema $α = \cup_2^{end}$: $Pre_α = \emptyset$, $Eff_α = \{robot-at(r, y), clear(x)\}$. The literal $\{robot-at(r, y)\}$ matches the $T_2$ and forms a $T$-class $L_1 = \{robot-at(r, y)\}$. The pure action schema $α_{L_1}$ is relevant unbounded as well as the end parts of the other schemas that indicate movements. If we apply our decision tree to $T_2$ and the set of actions $A$, we cannot prove that $T_2$ is an invariant since the unbounded schemas are not simply safe. Before discarding $T_2$, we try to fix it. In particular, the literal $\{clear(y)\}$ satisfies conditions (i), (ii) and (iv) above. If we add it to $T_2$, we obtain a new template $T'_2 = ([c_2, c'_2], [F'_2])$ where $c'_2 = (clear, 1, 1)$ with $F'_2 = ([c_2, 1], (c'_2, 0))$ and $F'_2 = ([c_2, 1], (c'_2, 0))$. If we apply our decision tree to this new template, we can prove that $T'_2$ is an invariant since Corollary 9 can be successfully applied (all schemas are either strongly sage or simply safe of type (a)). Intuitively, invariants of this template mean that, given a tile, either it is clear or it is occupied by a robot.

### 10. Experimental Results

To evaluate the performance of our Temporal Invariant Synthesis, referred as TIS in what follows, we have performed a number of experiments on the IPC benchmarks. We implemented the TIS algorithm, reported in Section 7 in the Python language and conducted the experiments by using a 2.53 GHz Intel Core 2 Duo processor with a memory of 4 GB.

Currently, it is difficult to compare our technique for generating lifted temporal invariants to related techniques since they either handle non temporal domains only (STRIPS domains, in particular) [Fox and Long 1998; Gerevini and Schubert 2000; Rintanen 2000, 2008; Helmert 2009] or find ground temporal invariants (Rintanen 2014). The approach that appears most similar to ours is the invariant synthesis implemented within the Temporal Fast Downward (TFD) planner [Eyerich et al. 2009]. However, there is no formal account of such a technique and its soundness, and our knowledge of it is based on a manual inspection of the code.

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3TFD-0.4 code available at [http://gki.informatik.uni-freiburg.de/tools/tfd/index.html](http://gki.informatik.uni-freiburg.de/tools/tfd/index.html)
The TFD invariant synthesis is a simple extension of Helmert’s original synthesis devised to deal with temporal and numeric domains. The algorithm analyses the temporal schemas directly, without slitting them into their start, overall and end fragments. As the original technique, only the weight of one is considered safe and only this weight is checked for assessing whether an action schema is safe or not. This implies that bounded action schemas (which are strongly safe) and simply safe schemas of type (c) are always considered unsafe. Only two types of relevant durative schemas are evaluated as safe: (i) those that add and delete relevant literals at start; and (ii) those that check that one relevant literal is true at start, then delete this literal at start and finally add another relevant literal at end. Case (i) corresponds to a balanced schema in our classification. However, the TFD synthesis misses schemas that are balanced at end. Case (ii) corresponds to simply safe schemas of type (a). In all the other cases, the action schemas are labelled as unsafe and the candidate invariant is dismissed.

The TFD invariant synthesis seems to have been carefully tailored to meet the particular requirements of the domains of the most recent competitions, the IPC6 in particular, in which almost all the action schemas fall in Cases (i) and (ii) above. In consequence, the TFD synthesis succeeds in finding useful invariants for these domains, but it fails in addressing the more general problem of finding invariants in generic PDDL temporal domains. While for the IPC6 domains the TIS and the TFD synthesis produce similar results, this is not true in general. Their output is different when the domains used offer a broader variety of action schema’s types, as shown in Figure 11. This figure highlights the limited applicability of the TFD synthesis in domains not included in the IPC6. In these domains, almost all the ground atoms end up being translated as state variables with two values (true and false) and the performance of TFD suffers from this trivial encoding.

![Figure 11: Examples of the different output of the TIS and the TFD invariant syntheses for IPC domains.](image)

Given the limited scope of the TFD synthesis, in our experimental results, we propose a comparison between our TIS and another technique, which we call Simple Invariant Synthesis (SIS). We designed and used the SIS in order to analyse the impact on temporal planners’ performance.

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6We do not include experiments concerning the performance of TFD as this is outside the scope of our paper.
of different encodings, which result from synthesising state variables based on different sets of lifted invariants. The SIS is also a simple extension of Helmert’s original technique to temporal domains, but it is general in its applicability and not devised around specific IPC domains. It adopts the simplest strategy to extend the synthesis of invariants from instantaneous to durative actions: it considers safe only changes in weight that happen at the same time point. More in depth, there are two main differences between the TIS and the SIS: i) in the SIS, only a weight equal to one is considered to be legal when a template is checked, whereas in the TIS both zero and one are considered to be legal weights; and ii) in the SIS, all the potential interactions between concurrent action schemas that affect the weight of a template are considered to be problematic and so such schemas are labeled as unsafe. As a consequence of these conservative choices, the SIS algorithm considers safe only irrelevant schemas and schemas that we define as balanced in our classification, and judges unsafe all the other action schemas.

10.1. Invariants, state variables and quality of the representation

Figure 12 provides the readers with examples of the invariants that our algorithm finds when applied to IPC domains. Each set in Figure 12 corresponds to a set of components, which are separated by a comma and indicated with the relation name (arity is omitted here), the positions of the fixed arguments (not enclosed in square brackets) and the position of the counted variable (enclosed in square brackets). For example, \( \{at \ 0 \ [1], \ in \ 0 \ [1]\} \) indicates the invariant with the components \( c_1 = \langle at, 2, 1 \rangle \) and \( c_2 = \langle in, 2, 1 \rangle \). For these domains, the only admissible partition is the trivial one and so it is not indicated. The only exception is the Parkprinter-strips domain and the invariant \( \{hasimage \ 0 \ 1 \ [2], \ notprintedwith \ 0 \ 1 \ [2]\} \), in which we choose an admissible partition that connects together the fixed arguments in position 0 and those in position 1.

| elevators-strips: | matchcolor: | operstacks-all: |
|------------------|------------|----------------|
| passengers 0 [1] | (unused) 5 | (stacks-off) 0 |
| (at 0 [1], boarded 0 [1]) | light 0 | (waiting) 0 |
| (at 0 [1]) | (unused) 0 | (waiting, 0) |
| (idle) | (not-occupied) 0 | (unloading) 0 |
| currentday 0 [1] | (handfree) | (unloading, 0) |
| woodworking-numeric: | modeltrain-numeric: | pipesworld-tankage: |
| unused 0, wood 0 [1] | (head-segment 0 [1], tail-segment 0 [1]) | normal 0, crushed 0 |
| (unused) 0 | (read-train 0 [1], read-train-in-head-segment 0 [1]) | (certified 0, unoccupied 0) |
| treatment 0 [1] | (first-train-in-head-segment 0 [1]) | (uncertified 0) |
| unused 0, surface-condition 0 [1] | (light 0) | (certified 0, delivered 0, uncrushed 0) |
| (unused) 0 | (head-segment 0 [1]) | (uncertified 0, crushed 0) |
| (idle 0) | (idle) | (unoccupied 0, uncrushed 0) |
| unused 0 | (light 0) | (occupied 0, uncrushed 0) |
| currentday 0 [1] | (idle 0) | (push-updating 0, occupied 0) |
| woodworker-numeric: | next-train 0 [1], first-train-in-head-segment 0 [1] | (normal 0, push-updating 0) |
| unused 0 | (switch-exit 0 [1]) | (push-updating, 0) |
| (not-occupied) 0 | (switch-entrance 0 [1]) | (push-updating, 0) |
| treatment 0 [1] | (head-segment 0 [1]) | (push-updating, 0) |
| unused 0, surface-condition 0 [1] | (idle 0) | (push-updating, 0) |
| (unused) 0 | (light 0) | (push-updating, 0) |
| (idle 0) | (idle 0) | (push-updating, 0) |
| unused 0 | (light 0) | (push-updating, 0) |
| currentday 0 [1] | (idle 0) | (push-updating, 0) |
| woodworker-numeric: | next-train 0 [1], first-train-in-head-segment 0 [1] | (normal 0, push-updating 0) |
| unused 0 | (switch-exit 0 [1]) | (push-updating, 0) |
| treatment 0 [1] | (switch-entrance 0 [1]) | (push-updating, 0) |
| unused 0, surface-condition 0 [1] | (head-segment 0 [1]) | (push-updating, 0) |
| (unused) 0 | (idle 0) | (push-updating, 0) |
| (idle 0) | (light 0) | (push-updating, 0) |
| unused 0 | (light 0) | (push-updating, 0) |
| currentday 0 [1] | (idle 0) | (push-updating, 0) |

Figure 12: Examples of invariants for the temporal domains of the IPCs. Each invariant is enclosed in braces where the predicate names indicate the components of the invariant, the numbers not enclosed in square brackets indicate the positions of the fixed variables in the list of arguments of the corresponding predicate and numbers enclosed in square brackets indicate the position of the counted variables.
Table 5-Left reports the number of invariants (\# INV), number of invariants obtained by applying fixes (\# FIX) and run time (RT) for generating invariants for the temporal domains of the IPC-6, IPC-7, and IPC-8 when the TIS algorithm is applied. The first two columns of Table 5-Left also compares the number of invariants found by the TIS to those found by the SIS. This table shows that the computational time to compute invariants is negligible and that there is no significant delay associated with splitting each action schema in its initial and final parts and with checking a broad set of configurations in the schemas’ conditions and effects. While these features of our algorithm do not impact the computational time, they allow us to find a more comprehensive set of invariants than related techniques.

Tables 5-Right and 6 show a comparison between the number of state variables obtained by instantiating invariants for the domains of the IPC-6, IPC-7, and IPC-8 obtained by applying our TIS and the SIS as well as by simply producing a state variable with two truth values (true and false) for each atom in the domain (Basic Invariant Synthesis, BIS). In many domains, the TIS produces a significant reduction in the number of state variables in comparison with the other two techniques. In several cases (see instances of Elevators, Sokoban, Transport, Drivelog, and Parking), the reduction is greater than an order of magnitude. In addition, Tables 5-Right and 6 report the mean (M) of the number of values in the domain of each state variable (when different from 2). In the BIS, the mean is always two as each state variable can only assume two values, true and false, and it is not indicated. In the SIS, the mean is often two with some exceptions, whereas in the TIS several state variables have larger domains with many values (see, for example, Elevators, Transport, Sokoban, Parking and Drivelog).

10.2. Performance in Temporal Planners

We have performed a number of additional experiments in order to evaluate the impact of using the state variables generated by the TIS on the performance of those planners that use a variable/value representation. In particular, we focus here on the performance of two planners: Temporal Fast Downward (TFD) (Eyerich et al. 2009) and POPF-SV, a version of POPF (Coles et al. 2010) that makes use of multi-valued state variables.7

TFD is a planning system for temporal and numeric problems based on Fast Downward (FD) (Helmert 2006), which is limited to non temporal and non numeric domains. TFD uses a multi-valued variable representation called “Temporal Numeric SAS+” (TN-SAS+), which is a direct extension of the “Finite Domain Representation” (FDR) used within FD to handle tasks with time and numeric fluents. TN-SAS+ captures all the features of PDDL - Level 3 and represents planning tasks by using: i) a set of state variables, which are divided into logical and numeric state variables; ii) a set of axioms, which are used to represent logical dependencies and arithmetic sub-terms; and iii) a set of durative actions, which comprise: a) a duration variable; b) start, persistent and end conditions; and c) start and end effects.

TFD translates PDDL2.1 tasks into TN-SAS+ tasks first and then performs a heuristic search in the space of time-stamped states by using a context-enhanced additive heuristic (Helmert and Geffner 2008) extended to handle time and numeric fluents. The translation from PDDL2.1 to TN-SAS+ works in four steps. First, the PDDL instance is normalised, i.e. types are removed and conditions and effects are simplified. Then, an instance where all the literals are ground is produced through a grounding step and the invariant synthesis is applied to generate invariants.

7This version of POPF is not documented, but it has been made available to us by their authors Andrew Coles and Amanda Coles.
Table 5: Left: Number of invariants (# Inv), number of invariants obtained by repairing failed templates (# Fix) and run time (RT) for generating invariants for the temporal domains of the IPCs by using the Temporal Invariant Synthesis (TIS) and the Simple Invariant Synthesis (SIS).

Right: Number of state variables (# SV) and mean (M) of the number of the values in the domain of each state variable (when different from 2) for the temporal domains of the IPC-8. The state variables are obtained by instantiating invariants obtained by applying: (1) Basic Invariant Synthesis (BIS); (2) Simple Invariant Synthesis (SIS); and (3) Temporal Invariant Synthesis (TIS).
(the grounding and the invariant synthesis can be performed in parallel). Starting from the invariants provided by the invariant synthesis and the ground domain, a set of multi-valued state variables is generated. Finally, a set of actions is obtained starting from PDDL actions, which describe how the state variables change over time.

In our experiments, we modified TFD by substituting the original invariant synthesis with the three alternative versions: TIS, BIS and SIS. The first is our technique, which we want to evaluate. The BIS is used as a baseline for our experiments. The SIS, as we explained above, is a simple but general alternative to generate invariants.

Tables 7 display the search time (ST) in seconds for the planner TFD and the domains of the IPC-6, IPC-7 and IPC-8. The three different columns indicate the search time when state variables are obtained by applying the BIS, the SIS and the TIS. For each domain, the planner is run against the following problems: p01, p05, p10, p15, p20, p25 and p30. The dash symbol indicates that a plan is not found in 300 seconds. Problems for which a plan could not be found in 300 seconds by applying all three techniques do not appear in the table.

The tables show that in several domains having fewer state variables with larger domains is beneficial to the search. In particular, in domains such as Elevators, Sokoban, Transport,

We do not offer a direct comparison between the original TFD and TFD integrated with our TIS because on IPC6 domains the two versions have comparable performance, while in the other domains TFD does not perform well. For big instances, it often produces no plans with a time bound of 10 minutes. As already mentioned, this behaviour can be linked to the fact that the TFD invariant synthesis works well for IPC6 domains, but it is not general enough to capture invariants in other domains.

Table 6: Number of state variables (# SV) and mean (M) of the number of the values in the domain of each state variable (when different from 2) for the temporal domains of the IPC-7 and IPC-8. The state variables are obtained by instantiating invariants obtained by applying: (1) Basic Invariant Synthesis (BIS); (2) Simple Invariant Synthesis (SIS); and (3) Temporal Invariant Synthesis (TIS).

| Domains - IPC7 | # SV | ST | M |
|---------------|------|----|----|
| Elevators - p01 | 382 | 2.03 | 2.03 |
| Elevators - p10 | 380 | 2.03 | 2.03 |
| Elevators - p20 | 382 | 2.03 | 2.03 |
| Elevators - p30 | 380 | 2.03 | 2.03 |

| Domains - IPC8 | # SV | ST | M |
|---------------|------|----|----|
| Elevators - p01 | 382 | 2.03 | 2.03 |
| Elevators - p10 | 380 | 2.03 | 2.03 |
| Elevators - p20 | 382 | 2.03 | 2.03 |
| Elevators - p30 | 380 | 2.03 | 2.03 |
and TurnAndOpen, the gain is high. If we analyse Table 7 in combination with Tables 5 and 6, we see that there is a strong correlation between the mean of the cardinality of the domains of the state variables and the impact of the state variables on performance. More specifically, state variables whose value domains have mean cardinality greater or equal to three seem to produce the strongest improvements on the search, whereas state variables whose value domains have mean cardinality around two do not yield significant differences in performance. This is not particularly surprising because the use of variables with only two values resolves in producing the same state space as it is obtained without applying any invariant synthesis. On the other hand, variables with three or more values provide an actual reduction in the number of states.

We speculate that the number of the state variables and the cardinality of their domains has an impact on the calculation of the heuristic estimates. In particular, let us consider the additive heuristic plus context used within TFD in terms of its corresponding causal graph interpretation by Helmert and Geffner (2008). The causal graph heuristic gives an estimate of the number of actions needed to reach the goal from a state $s$ in terms of the estimated costs of changing the value of each state variable that appears in the goal from its value in $s$ to its value in the goal. In order to compute this estimate, the heuristic uses two structures: the domain transition graph, which describes the relations between the different values of a state variable, and the causal graph, which describes the dependencies between the different state variables. Both structures are highly influenced by the number of the state variables and the cardinality of their domains. Fewer state variables with larger domains result in a smaller number of domain transition graphs, each of which has a more complex structure. In addition, a smaller number of state variables gives rise to a much more compact and structured causal graph. We believe that, in a number of cases, these different configurations of the two types of graphs produce more effective heuristic estimates with the consequence of improving performance.

POPF-SV is a version of the forwards-chaining temporal planner POPF (Coles et al., 2010) that is capable of reading a variable/value representation and making use of it to perform some inference in a pre-processing step and also to reduce the size of states during search. In particular, POPF-SV reads a standard PDDL task along with its corresponding TN-SAS+ translation and reasons with both representations. The multi-valued state variable representation of the task is not used in the heuristic computation, but it is used for two different purposes. An inference step based on the state variables is performed to support temporal preferences. This step extracts rules that are then used during search (for example, is it possible to have action $a$ within 10 time units of action $b$). The second use of the invariant analysis aims to make the state representation more efficient. Only one proposition from each mutex group needs to be stored within a state since if one is true then the others must necessarily be false. This property results in massive savings in memory. This is particularly beneficial for POPF as memory is generally what causes the planner to fail (rather than time).

Tables 8 display the search time (ST) in seconds for the planner POPF-SV and the domains of the IPC-6 and IPC-7 (we do not show the domains of the IPC-8 as the planner has not been maintained since IPC-7 and so does not perform well on those domains). The three different columns indicate the search time when state variables are obtained by applying the BIS, the SIS and the TIS. For each domain, the planner is run against the following problems: p01, p05, p10, p15, p20, p25 and p30. Problems for which a plan could not be found in 300 seconds do not appear in the table.

The tables show that in several domains having fewer state variables with larger domains is beneficial to the search. In particular, in domains such as Elevators, Sokoban, Parking,
Table 7: Search time (ST) in seconds for the planner TFD and the domains of the IPC-6 – IPC-8 (ST does not include the translation from PDDL2.1 to temporal SAS+). Invariants are obtained by applying: (1) a Basic Invariant Synthesis (BIS); (2) a Simple Invariant Synthesis (SIS); and (3) our Temporal Invariant Synthesis (TIS). For each domain, the planner is run against problems p01, p05, p10, p15, p20, p25 and p30. The dash symbol indicates that a plan has not been found in 300 seconds. Problems for which all the techniques do not find a plan in 300 seconds do not appear in the table.

| Domains - IPC6 | TFD - ST | Domains - IPC7 | ST |
|----------------|---------|----------------|-----|
|                | BIS     | SIS            | TIS |
| Crew Planning - p01 | 0.02    | 0.02           | 0.02 |
| Crew Planning - p05 | 0.02    | 0.02           | 0.02 |
| Crew Planning - p10 | 0.14    | 0.14           | 0.14 |
| Crew Planning - p15 | 0.02    | 0.02           | 0.02 |
| Crew Planning - p20 | 0.54    | 0.54           | 0.53 |
| Crew Planning - p30 | 1.46    | 1.46           | 1.46 |
| Elevators-Num - p01 | 1.57    | 1.57           | 0.02 |
| Elevators-Num - p05 | 0.32    | 0.32           | 0.05 |
| Elevators-Num - p10 | -       | -              | 57.56 |
| Elevators-Num - p15 | -       | -              | 11.74 |
| Elevators-Num - p25 | -       | -              | 32.85 |
| Elevators-Str - p01 | 0.34    | 0.33           | 0.03 |
| Elevators-Str - p05 | 0.27    | 0.28           | 0.08 |
| Elevators-Str - p10 | -       | -              | 23.04 |
| Elevators-Str - p15 | -       | -              | 23.98 |
| Openstacks-Adj - p01 | 0.01    | 0.01           | 0.01 |
| Openstacks-Adj - p05 | 0.04    | 0.04           | 0.03 |
| Openstacks-Adj - p10 | 0.09    | 0.09           | 0.07 |
| Openstacks-Adj - p15 | 0.2     | 0.2            | 0.14 |
| Openstacks-Adj - p20 | 0.34    | 0.37           | 0.25 |
| Openstacks-Adj - p30 | 0.62    | 0.74           | 0.46 |
| Openstacks-Num - p01 | 0       | 0              | 0   |
| Openstacks-Num - p05 | 0       | 0              | 0   |
| Openstacks-Num - p10 | 0.02    | 0.02           | 0.01 |
| Openstacks-Num - p15 | 0.03    | 0.03           | 0.03 |
| Openstacks-Num - p20 | 0.07    | 0.07           | 0.06 |
| Openstacks-Num - p25 | 0.1     | 0.11           | 0.11 |
| Openstacks-Num - p30 | 0.19    | 0.2            | 0.18 |
| Parcprinter - p01  | 1.41    | 1.44           | -   |
| Parcprinter - p05  | 21.51   | 21.7           | -   |
| Pegsol - p01      | 0       | 0              | 0   |
| Pegsol - p05      | 0.51    | 0.52           | 0.41 |
| Pegsol - p10      | 1.27    | 1.3            | 1.13 |
| Pegsol - p15      | 1.11    | 1.11           | 0.15 |
| Pegsol - p20      | 1.42    | 1.45           | 1.11 |
| Sokoban-Str - p01 | 1.4     | 1.41           | 0.22 |
| Sokoban-Str - p05 | 94.55   | 95.1           | 10.73 |
| Sokoban-Str - p10 | 31.6    | 32.13          | 11.89 |
| Sokoban-Str - p15 | 3.96    | 4.02           | 0.04 |
| Transport-Num - p01 | 0       | 0              | 0   |
| Transport-Num - p05 | -       | -              | 13.64 |
| Woodworking-Num - p01 | 0.01   | 0.01           | 0.02 |
| Woodworking-Num - p05 | 0.02    | 0.02           | 0.08 |
| Woodworking-Num - p10 | 0.03    | 0.03           | 0.01 |
| Woodworking-Num - p15 | 0.08    | 0.08           | 0.06 |
| Woodworking-Num - p20 | 28.86   | 29.47          | 19.8 |
| Woodworking-Num - p30 | 28.45   | 29.17          | 0.18 |
| Mapanalyser - p01 | 119.24  | 122.14         | 0.14 |
| Mapanalyser - p05 | 5.71    | 5.88           | 0.96 |
| Mapanalyser - p10 | 173.54  | 176.14         | 2.16 |
| Mapanalyser - p15 | 3.17    | 3.06           | 3.05 |
| Matchcellar - p01 | 9.71    | 9.35           | 9.37 |
| Matchcellar - p05 | 24.49   | 23.65          | 23.64 |
| Matchcellar - p10 | 54.32   | 52.32          | 52.32 |
| Matchcellar - p15 | 95.16   | 90.57          | 90.64 |
| Parking - p01    | 11.01   | 11.04          | 0.05 |
| Parking - p05    | 0.17    | 0.18           | 0.09 |
| Parking - p10    | 0.3     | 0.31           | 0.17 |
| Parking - p15    | 1.12    | 1.14           | 0.55 |
| Satellite - p01  | 0.55    | 0.56           | 1.46 |
| Satellite - p05  | 13.57   | 13.87          | 2.93 |
| Satellite - p10  | 2.59    | 2.67           | 13   |
| Satellite - p15  | 6.3     | 6.41           | 5.19 |
| Turn&Open - p01  | 54.78   | 55.99          | 23.8 |
| Turn&Open - p10  | 65.83   | 67.64          | 55.88 |
| Turn&Open - p15  | 136.9   | 142.32         | 81.35 |

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Openstacks and TurnAndOpen, the gain is significant. If we analyse Table 8 in combination with Tables 5 and 6, we again see that there is a correlation between the number of state variables and the performance of the planner. When the reduction in the number of variables is significant, the planner works significantly better than with the SIS or the BIS. In addition, we observe a correlation between the mean of the cardinality of the domains of the state variables and the impact of the state variables on performance.

| Domains - IPC6 | ST       | BIS | SIS | TIS |
|---------------|----------|-----|-----|-----|
| Crew Planning - p01 | 0        | 0   | 0   | 0   |
| Crew Planning - p05 | 0        | 0.02| 0.02| 0.02|
| Crew Planning - p10 | 0.04     | 0.04| 0.04| 0.04|
| Crew Planning - p15 | 0.02     | 0.02| 0.02| 0.02|
| Crew Planning - p20 | 0.26     | 0.30| 0.26| 0.26|
| Crew Planning - p25 | 0.24     | 0.24| 0.22| 0.22|
| Elevators-Num - p01 | 0.84     | 0.82| 0.80| 0.80|
| Elevators-Num - p05 | 0.20     | 0.18| 0.18| 0.18|
| Elevators-Num - p10 | 0.24     | 0.22| 0.24| 0.24|
| Elevators-Num - p15 | 7.36     | 7.64| 7.14| 7.14|
| Elevators-Num - p20 | 1.16     | 1.22| 0.94| 0.94|
| Elevators-Str - p05 | 0.62     | 0.70| 0.48| 0.48|
| Elevators-Str - p10 | 4.98     | 5.02| 3.22| 3.22|
| Elevators-Str - p15 | 885.21   | -   | -   | -   |
| Elevators-Str - p20 | 468.16   | -   | -   | -   |
| Elevators-Str - p25 | -        | -   | -   | -   |
| Openstacks-Str - p01 | 1.16     | 1.22| 0.94| 0.94|
| Openstacks-Str - p05 | 0.02     | 0.02| 0.02| 0.02|
| Openstacks-Str - p10 | 0.08     | 0.08| 0.06| 0.06|
| Openstacks-Str - p15 | 0.18     | 0.18| 0.14| 0.14|
| Openstacks-Str - p20 | 0.32     | 0.34| 0.24| 0.24|
| Openstacks-Str - p25 | 0.56     | 0.58| 0.44| 0.44|
| Openstacks-Str - p30 | 0.84     | 0.86| 0.60| 0.60|
| Parcprinter-Str - p01 | 0.02    | 0.02| 0.02| 0.02|
| Parcprinter-Str - p20 | -       | -   | 21.36| 21.36|
| Pegsol-Str - p01 | 0        | 0   | 0   | 0   |
| Pegsol-Str - p05 | 0.02     | 0.02| 0.02| 0.02|
| Pegsol-Str - p10 | 0.10     | 0.10| 0.10| 0.10|
| Pegsol-Str - p15 | 0.20     | 0.20| 0.18| 0.18|
| Pegsol-Str - p20 | 0.06     | 0.06| 0.06| 0.06|
| Pegsol-Str - p25 | 0.28     | 0.28| 0.24| 0.24|
| Sokoban-Str - p01 | 1.40     | 1.50| 1.10| 1.10|
| Sokoban-Str - p05 | 58.98    | 58.98| 30.80| 30.80|
| Sokoban-Str - p10 | 11.08    | 10.60| 10.60| 10.60|
| Sokoban-Str - p15 | 11.08    | 11.08| 11.08| 11.08|
| Sokoban-Str - p20 | 2.92     | 3.22| 2.26| 2.26|
| Transport-Num - p01 | 0        | 0   | 0   | 0   |
| Woodworking-Num - p01 | 0        | 0   | 0   | 0   |
| Woodworking-Num - p05 | 0.04     | 0.04| 0.04| 0.04|
| Woodworking-Num - p10 | 0.10     | 0.08| 0.10| 0.10|
| Woodworking-Num - p15 | 3.02     | 3.48| 3.22| 3.22|
| Woodworking-Num - p20 | 0.10     | 0.10| 0.08| 0.08|
| Woodworking-Num - p25 | 0.10     | 0.10| 0.08| 0.08|
| Woodworking-Num - p30 | -       | 2.09| 2.02| 2.02|

Table 8: Search time (ST) in seconds for the planner POPP-SV and the domains of the IPC-6 and IPC-7 (ST does not include the translation from PDDL2.1 to temporal SAS+). Invariants are obtained by applying: (1) a Basic Invariant Synthesis (BIS); (2) a Simple Invariant Synthesis (SIS); and (3) our Temporal Invariant Synthesis (TIS). For each domain, the planner is run against problems p01, p05, p10, p15, p20, p25 and p30. The dash symbol indicates that a plan has not been found in 300 seconds. Problems for which all the techniques do not find a plan in 300 seconds do not appear in the table.
11. Related Work

The invariant synthesis presented in this paper builds on early work described in Bernardini and Smith (2011a). However, the theory behind the invariant synthesis presented here is significantly more comprehensive and the implementation of the technique, in reflecting this extended theory, is capable of finding more invariants and more complex ones. We work here with instantaneous action schemas instead of durative ones and we resort to them only when necessary. From a theoretical point of view, this makes our presentation cleaner and facilitates the exhibition of sound results. From a practical point of view, this makes our algorithm more efficient. The entire classification of action schemas is different from Bernardini and Smith (2011a) and, in consequence of this more sophisticated classification, our new approach can handle complex interactions between schemas that our original simplistic technique cannot. In fact, our original approach works well only in simple domains with balanced schemas, while it fails in more complex domains because the conditions imposed on potentially interfering schemas are too conservative.

Several other approaches to invariant synthesis are available in the literature. In what follows, we present these approaches more in depth by highlighting differences and similarities with our technique.

11.1. Fast Downward and Temporal Fast Downward

In Helmert (2009), Helmert present a translation from a subset of PDDL2.2 into FDR (Finite Domain Representation), a multi-valued planning task formalism used within the planner Fast Downward (Helmert, 2006). In particular, the translation only handles non-temporal and non-numeric PDDL2.2 domains, the so-called “PDDL Level 1” (equivalent to STRIPS (Fikes and Nilsson, 1971) with the extensions known as ADL (Pednault, 1986)). One of the step of this translation is the identification of mutual exclusion invariants and it is an extension of the technique presented in Edelkamp and Helmert (1999) developed for STRIPS.

When considering non-temporal domains, the invariant synthesis presented in this paper works similarly to Helmert’s one. In particular, both work at the lifted level, while all the other related techniques discussed below work at the ground level. Both techniques start from simple invariant candidates, check them against conditions that ensure invariance by analyzing the structure of the action schemas in the domain. When a candidate is rejected, they both try to refine it to create a new stronger candidate, which is then checked from scratch.

However, in contrast with our technique, Helmert’s method considers safe only the weight one instead of both the weights of one and zero. This simplified analysis results in the identification of a smaller set of invariants with respect to our technique. For example, Helmert’s invariant synthesis labels as unsafe all the action schemas that add a relevant literal without deleting that literal or another relevant one, even when the preconditions impose that the weight is zero when the action schema is applied. In this way, Helmert’s invariant synthesis misses invariants that our technique is able to find.

Chen et al. (2009) builds on Helmert’s invariant synthesis and his multi-valued domain formulation to synthesize long-distance mutual exclusions (londex), which capture constraints over actions and facts not only at the same time step but also across multiple steps. The londex has been successfully used in SAT-based planners to improve their performance. In future work, we will explore how the concept of londex can be extended to temporal domains.
Within the context of Temporal Fast Downward (TFD) (Eyerich et al., 2009), a simple extension of Helmert’s invariant synthesis is used to deal with temporal and numeric domains of the ICPs. See Section 10 for a description of such a technique.

11.2. Rintanen’s Invariant Synthesis

An algorithm for inferring invariants in propositional STRIPS domains is proposed by Rintanen (2000, 2008). It synthesises not only mutual-exclusion invariants, but also other types of invariants. The algorithm works on a ground representation of the domain and, starting from an inductive definition of invariants as formulae that are true in the initial state and are preserved by the application of every action, the algorithm is based on an iterative computation of a fix-point, which is useful for reasoning about all the invariants of a domain at the same time rather than inferring some invariants first and then use them for inferring others.

Rintanen’s algorithm uses a guess, check and repair approach but, unlike our technique, it starts from stronger invariant candidates and then progressively weaken them if they are not preserved by the actions. Thus, the repair phase consists in considering a less general invariant instead of a more general one. For example, let us consider the schema $\sigma = x \neq y \rightarrow P(x, y) \lor Q(y, z)$ as a potential invariant (all the invariants considered have this implicative form). One of the weakening operation consists of identifying two variables. In this case, if $z$ is set equal to $x$, the weaker candidate $\sigma = x \neq y \rightarrow P(x, y) \lor Q(y, x)$ is obtained and checked.

This technique has been successfully used within both Graphplan based planners (Blum and Furst, 1997), where it helps to identify unreachable subgoals, and SAT-based planners (Kautz and Selman, 1999), where it can be useful to reduce the amount of search needed. However, although its implementation is limited to invariants involving two literals at the most, it incurs a high performance penalty on large instances.

In Rintanen (2014), Rintanen extends the original algorithm presented in Rintanen (2000, 2008) in order to handle temporal domains. As the original algorithm, the temporal one works on ground domains, not using a lifted representation at any stage. The format of the invariants found is $l_1 V(r) l_2$, where $l_1$ and $l_2$ are positive or negative ground facts, $r$ is a floating point number, and the formula says that either $l_1$ is true or $l_2$ is true over the interval $[0..r]$ relative to the current time point. If $r = \text{inf}$, the formula means that if $l_1$ is false, then $l_2$ will remain true forever.

Since Rintanen’s invariant synthesis exploits the initial conditions and the ground representation of the domain, it usually finds a broader range of invariants than our technique. However, this makes the invariant synthesis suffer high computational cost. Reachability analysis on a ground representation of the planning instances is computationally very expensive and, while our algorithm takes a few seconds to run, Rintanen’s synthesis employs tens of minutes to find invariants in several domains (see Table 1 in Rintanen (2014)).

We do not directly compare our technique against Rintanen’s algorithm in Section 10 because the two techniques aim to find different types of invariants (our focuses on mutual exclusion invariants, while Rintanen’s tackles a broad range of invariant types) and they work on different representations of the problem (lifted versus ground). However, in what follows, we give examples of the output of Rintanen’s technique for completeness.

Let us consider the Crewplanning domain (IPC6 and IPC7). For each crew member $c_i$, Rintanen’s algorithm finds ground invariants of the type:

$$\text{not current\_day} - c_i - d_j V(\text{inf}) \text{ not current\_day} - c_i - d_k$$
which means that if it is day $d_j$ for the crew member $c_i$, it cannot be day $d_k$ at the same time. All these invariants correspond to the lifted invariant $\text{current\_day } 0 \{1\}$ that is found by our invariant synthesis. For the same domain, however, Rintanen’s algorithm finds additional invariants that express temporal relations between atoms. Our technique does not aim to find this type of invariants. For example, Rintanen’s method finds temporal invariants of the form:

$$\text{done\_sleep} - c_i - d_k V(255) \text{ not done\_meal} c_i - d_{(k+2)}$$

which means that, for the crew member $c_i$, the atom $\text{done\_meal}$ in day $k + 1$ becomes true 255 time units after the atom $\text{done\_sleep}$ was true in day $k$. In fact, in day $k$, $\text{done\_sleep}$ is made true by the end effects of the action $\text{sleep}$. From this time point, in order to make $\text{done\_meal}$ true the day after $k + 1$, two actions need to be executed: $\text{post\_sleep}$, with duration 195, and $\text{have\_meal}$ with duration 60, for a total time separation of 255 time units. For the Crewplanning domain, the run time of our algorithm is 0.29 seconds, while Rintanen’s one has a runtime of 1 minute and 23.24 seconds for hard instances. This is actually one of the best run times, since for problems such as Parcprinter, Elevators, Sokoban, Transport-numeric and other the algorithm has a run time of more than 4 hours. Given these run times, it does not seem plausible to use Rintanen’s algorithm as a pre-processing steps to improve search in planning, which is one of the most important cases of use of invariant synthesis algorithms.

11.3. DISCOPLAN

DISCOPLAN (DIScovering State CONstraints for PLANning) [Gerevini and Schubert 1998] is a technique for generating invariants from PDDL Level 1 tasks. DISCOPLAN discovers not only mutual exclusion invariants, but also other types of invariants: static predicates, simple implicative, (strict) single valuedness and n-valuedness, anti-symmetry, OR and XOR invariants.

Considering mutual exclusion invariants, DISCOPLAN uses a guess, check and repair approach similar to our approach: an hypothetical invariant is generated by analysing simultaneously the preconditions and the effects of each action to see whether an instantiation of a literal is deleted whenever another instantiation of the same literal is added. Then, this candidate is checked against all the other actions and the initial conditions. If the hypothetical invariant is not found to be valid, then all the unsafe actions are collected together and a set of possible refinements are generated. However, whereas our technique tries to refine a candidate as soon as an unsafe action is found, DISCOPLAN tries to address all the causes of unsafety at the same time while generating refinements. This approach leads to more informed choices on how to refine hypothetical invariants and can result in the identification of more invariants. However, it is more expensive from a computational point of view, which is why DISCOPLAN is often inefficient on big instances.

DISCOPLAN can be used not only for finding invariants, but also for inferring action-parameter domains. An action-parameter domain is a set including all the objects that can be used to instantiate the parameters of an action. Such sets of possible tuples of arguments are found by forward propagation of ground atoms from the initial state. This technique is related to the reachability analysis performed by Graphplan [Blum and Furst 1997], but does not implement mutual exclusion calculation.

DISCOPLAN is usually used in combination with SAT encodings of planning problems. In particular, a pre-processing step is performed over the domain under consideration in order to find invariants and parameter domains, then the domain as well as the invariants and the parameter domains are translated into SAT. Finally, a SAT-based planner is used to solve the
resulting translated domain. SAT-based planners (Kautz and Selman 1999; Huang et al. 2010) show significant speed-up when invariants and action-parameter domains are used.

11.4. Type Inference Module

TIM (Type Inference Module) (Fox and Long 1998) uses a different approach for finding invariants in PDDL Level 1 domains. More precisely, TIM is a pre-preprocessing technique for inferring object types on the basis of the actions and the initial state. Data obtained from this computation are then used for inferring invariants. TIM recognises four kinds of invariants (invariants of type 2 correspond to mutual exclusion invariants):

1. **Identity invariants** (for example, considering the domain Blockworld, two objects cannot be at the same place at the same time);
2. **Unique state invariants** (for example, every object must be in at most one place at any time point);
3. **State membership invariants** (for example, every object must be in at least one place at any time point); and
4. **Resource invariants** (for example, in a 3-blocks world, there are 4 surfaces).

The invariants found by TIM have been exploited for improving performance within the planner STAN (Fox and Long 2011).

11.5. Knowledge representation and engineering

In addition to works that address the creation of invariants directly, there are works in the literature that highlight the importance of multi-valued state variables for debugging domain descriptions and for assisting the domain designer in building correctly encoded domains (Fox and Long 1998; Bernardini and Smith 2011b; Cushing et al., 2007). In particular, Cushing et al. (2007) analyse well-studied IPC temporal and numeric domains and reveal several modelling errors that affect such domains. This analysis lead the authors to suggest better ways of describing temporal domains. They identify as a central feature to do so the direct specification of multi-valued state variables and show how this can help domain experts to write correct models.

Other works in the literature use the creation of invariants and state variables as an intermediate step in the translation from PDDL to other languages. In particular, Huang et al. (2010) introduce SASE, a novel SAT encoding scheme based on the SAS+ formalism Helmert (2009). The state variables (extracted from invariants) used by SASE play a key role in achieving its efficiency. Since our technique generates a broader set of invariants than related techniques, it gives rise to SAS+ tasks with smaller sets of state variables. We speculate that this would in turn reverberate positively on SAT-based planners that use a SASE encoding. Testing of this hypothesis is part of our future work.

12. Conclusions and Future Work

In this paper, we present a technique for automatically finding lifted mutual exclusion invariants in temporal planning domains expressed in PDDL2.1. Our technique builds on Helmert’s invariant synthesis (Helmert 2009), but generalised it and extends it to temporal domains. Synthesising invariants for temporal tasks is much more complex than for tasks with instantaneous actions only because actions can occur simultaneously or concurrently and interfere with each
other. For this reason, a simple generalisation of Helmert’s approach does not work in temporal settings. In extending the theory to capture the temporal case, we have had to formulate invariance conditions that take into account the entire structure of the actions as well as the possible interactions between them. As a result, we have constructed a technique that is significantly more comprehensive than the related ones. Our technique is presented here formally and proofs are offered that support its soundness.

Since our technique, differently from related approaches, works at the lifted level of the representation, it is very efficient. The experimental results show that its run time is negligible, while it allows us to find a wider set of invariants, which in turn results in synthesising a smaller number of state variables to represent a domain. The experiments also indicate that the temporal planners that use state variables to represent the world usually benefit from dealing with a smaller number of state variables.

Our approach to finding invariants can be incorporated in any translation from PDDL2.1 to a language based on multi-valued state variables. For example, we have used (a simplified version of) the temporal invariant synthesis described in this paper in our translator from PDDL2.1 to NDDL, which is the domain specification language of the planner EUROPA2. EUROPA2 has been the core planning technology for several NASA space mission operations. It uses a language based on multi-valued state variables that departs from PDDL2.1 in several ways. The use of our translator from PDDL2.1 to NDDL has facilitated the testing of EUROPA2 against domains of the IPCs originally expressed in PDDL2.1. This has originally motivated our work on temporal invariant synthesis.

In future work, we plan to extend our experimental evaluation by incorporating our invariant synthesis in other planners that use a multi-valued variable representation and that are not currently publicly available. This will allow us to assess more exhaustively the impact that handling fewer state variables with broader domains has on the performance of temporal planners. In addition, we plan to exploit the metric information encoded in planning domains to find a broader range of invariants. Invariants for domains with metric fluents are interesting and challenging. We envisage that there are two kinds of situations to be considered: those in which it can be shown that a linear combination of fluents is invariant (relevant to domains with linear effects on variables) and those in which metric fluents interact with propositional fluents in a more complex structure. For example, one might think of a domain encoding the act of juggling in which the number of balls in the air plus the number in the hands is a constant, but the balls in the hand might be encoded propositionally (for example, by a literal holding_left and so on), while those in the air as a count. Finding the invariant in this case is a challenging problem since it crosses the propositional and metric fluent spaces.
Appendix A: PDDL2.1 Specification of the Floortile Domain

( define (domain floor–tile)
  (:requirements :typing :durative–actions)
  (:types robot tile color – object)

  (:predicates
   (robot–at ?r – robot ?x – tile)
   (up ?x – tile ?y – tile)
   (down ?x – tile ?y – tile)
   (right ?x – tile ?y – tile)
   (left ?x – tile ?y – tile)
   (clear ?x – tile)
   (painted ?x – tile ?c – color)
   (robot–has ?r – robot ?c – color)
   (available–color ?c – color)
   (free–color ?r – robot)
  )

  (:durative–action change–color
    :parameters (?r – robot ?c – color ?c2 – color)
    :duration (= ?duration 5)
    :condition (and (at start (robot–has ?r ?c))
                 (over all (available–color ?c2)))
    :effect (and (at start (not (robot–has ?r ?c)))
               (at end (robot–has ?r ?c2)))
  )

  (:durative–action paint–up
    :parameters (?r – robot ?y – tile ?x – tile ?c – color)
    :duration (= ?duration 2)
    :condition (and (over all (robot–has ?r ?c))
                  (at start (robot–at ?r ?x))
                  (over all (up ?y ?x))
                  (at start (clear ?y)))
    :effect (and (at start (not (clear ?y)))
              (at end (painted ?y ?c)))
  )

  (:durative–action paint–down
    :parameters (?r – robot ?y – tile ?x – tile ?c – color)
    :duration (= ?duration 2)
    :condition (and (over all (robot–has ?r ?c))
                  (at start (robot–at ?r ?x))
                  (over all (down ?y ?x))
                  (at start (clear ?y)))
    :effect (and (at start (not (clear ?y)))
              (at end (painted ?y ?c)))
  )

  (:durative–action up
    :parameters (?r – robot ?x – tile ?y – tile)
    :duration (= ?duration 3)
    :condition (and (at start (robot–at ?r ?x))
                  (over all (up ?y ?x))
                  (at start (clear ?y)))
    :effect (and (at start (not (robot–at ?r ?x)))
               (at end (robot–at ?r ?y))
               (at start (not (clear ?y)))
               (at end (clear ?x)))
  )
Appendix B: PDDL2.1 Specification of the Depot Domain
\begin{verbatim}
(\textbf{durative-action} Lift
:parameters (?x - hoist ?y - crate \(\gamma\) - surface ?p - place)
:duration (= duration 1)
:condition (and (over all (at ?x ?p)) (at start (available ?x))
(at start (at ?y ?p)) (at start (on ?y ?z))
(at start (clear ?y)))
:effect (and (at start (not (at ?y ?p)))
(at start (lifting ?x ?y))
(at start (clear ?z)) (at start (not (on ?y ?z))))

(\textbf{durative-action} Drop
:parameters (?x - hoist ?y - crate ?z - surface ?p - place)
:duration (= duration 1)
:condition (and (over all (at ?x ?p)) (over all (at ?z ?p))
(over all (clear ?z)) (over all (lifting ?x ?y)))
:effect (and (at end (available ?x)) (at end (not (lifting ?x ?y)))
(at end (at ?y ?p)) (at end (not (clear ?z)))
(at end (clear ?y))(at end (on ?y ?z)))

(\textbf{durative-action} Load
:parameters (?x - hoist ?y - crate ?z - truck ?p - place)
:duration (= duration 3)
:condition (and (over all (at ?x ?p)) (over all (at ?z ?p))
(over all (lifting ?x ?y)))
:effect (and (at end (lifting ?x ?y)))
(at end (in ?y ?z))
(at end (available ?x)))

(\textbf{durative-action} Unload
:parameters (?x - hoist ?y - crate ?z - truck ?p - place)
:duration (= duration 4)
:condition (and (over all (at ?x ?p)) (over all (at ?z ?p))
(at start (available ?x)) (at start (in ?y ?z))
(at start (lifting ?x ?y)))
:effect (and (at start (in ?y ?z))) (at start (not (available ?x)))
(at start (lifting ?x ?y)))
)
\end{verbatim}

\textbf{Appendix C: Proofs}

\textit{Proof of Proposition 1.} The action \(a_1\) is applicable in \(s_0\) by definition. Assuming that \(a_j\) is applicable in \(s_{j-1}\) for \(j = 1, \ldots, k\), we now show that \(a_{k+1}\) is applicable in \(s_k\). Note that from the definition of transition function \(\xi\) for single actions \(s_k = (s \setminus \bigcup_{j=1}^{k} Eff_{a_j}^*) \cup \bigcup_{j=1}^{k} Eff_{a_j}^*\). Since \(Pre_{a_{k+1}}^s \subseteq s\) and \(Pre_{a_{k+1}}^s \cap s = \emptyset\) by assumption and \(a_{k+1}\) is not interfering with \(a_1, a_2, \ldots, a_k\), we have that \(Pre_{a_{k+1}}^s \subseteq s_k\) and \(Pre_{a_{k+1}}^s \cap s_k = \emptyset\). In addition, note that: \(s_n = (s \setminus \bigcup_{j=1}^{k} Eff_{a_j}^*) \cup \bigcup_{j=1}^{k} Eff_{a_j}^* = \xi(s,A)\).

\textit{Proof of Proposition 1.} (ii)\(\Rightarrow\)(iii) is trivial and (iii)\(\Rightarrow\)(i) is an immediate consequence of (i) and of the fact that \(w(T, \gamma, s) = w(T, \gamma, s')\) and \(w(T, \gamma', s) = w(T, \gamma', s')\).

Finally, (i)\(\Rightarrow\)(ii) follows from the following argument. Given any \(s \in S_{A}\) such that \(w(T, \gamma, s) \leq 1\), consider \(s' := s_y \cup Pre_{A_{\gamma}}^s\). Since \(s_y = s_y \in S_{A}\) and \(s' = Pre_{A_{\gamma}}^s \in S_{A_{\gamma}}\), it follows that
$s' \in S_A$. If we consider the successor states $s' = \xi(s, A_y)$ and $s'' = \xi(s', A_y)$, it follows from \ref{eq:1} that

$$s'_y = \xi(s_y, A_y) = \xi(s''_y, A_y) = s''_y$$

Therefore,

$$w(T, \gamma, s') = w(T, \gamma, s'_y) = w(T, \gamma, s''_y) = w(T, \gamma, s'') \leq 1$$

where the last equality follows from the assumption that $A$ is strongly $\gamma$-safe.

\textbf{Proof of Theorem\ref{thm:gamma_safe}} If $A$ is $\gamma$-unreachable and $A$ is applicable in the state $s$, It follows that $Pre_A^\gamma \subseteq s$ and thus $w(T, \gamma, s) \geq |Pre_A^\gamma| \geq 2$. This shows that the condition $w(T, \gamma, s) \leq 1$ is never verified and thus $A$ is strongly $\gamma$-safe.

If $A$ is $\gamma$-irrelevant and $A$ is applicable in the state $s$, we have that the successor state $s' = \xi(s, A)$ $\subseteq s$. This yields $w(T, \gamma, s') \leq w(T, \gamma, s)$. This implies that $A$ is strongly $\gamma$-safe.

Suppose $A$ is $\gamma$-heavy and consider the state $s = Pre_A^\gamma$. $A$ is applicable in $s$ and $w(T, \gamma, s) = |Pre_A^\gamma| \leq 1$. After applying $A$ in $s$, the successor state $s' = \xi(s, A)$ is such that $s' \geq Eff_A^\gamma$. This yields $w(T, \gamma, s') \geq |Eff_A^\gamma| \geq 2$ and proves that $A$ is not strongly $\gamma$-safe.

\textbf{Proof of Theorem\ref{thm:corresponding_property}} We will prove the corresponding property for $A_y$ making use of Condition (iii) of Proposition\ref{prop:gamma_safe}.

We first analyse the case when $A$ is balanced or unbalanced. Let $Pre_A^\gamma = \{q_1\}$ and $Eff_A^\gamma = \{q_2\}$. Suppose now that $A$ is balanced and fix a state $s \in \gamma(T)$ such that $w(T, \gamma, s) \leq 1$ and $A_y$ is applicable in $s$. Clearly, $q_1 \subseteq s$ so, necessarily, $s = \{q_1\}$ and $w(T, \gamma, s) = 1$. Consider the subsequent state $s' = \xi(s, A_y)$. If $q_1 = q_2$, we have that $s' = s$ so that $w(T, \gamma, s') = 1$. If instead $q_1 \in Eff_A^\gamma$, we have that $s' \subseteq (s \cup \{q_2\}) \setminus \{q_1\} = \{q_2\}$ and thus $w(T, \gamma, s') = 1$.

Suppose that $A$ is unbalanced and consider the state $s = \{q_1\}$. The subsequent state $s' = \xi(s, A_y) = \{q_1, q_2\}$ so that $w(T, \gamma, s') = 2$.

We now consider the remaining two cases. Let $Eff_A^\gamma = \{q_2\}$. Suppose now that $A$ is bounded and fix a state $s \in \gamma(T)$ such that $w(T, \gamma, s) \leq 1$ and $A_y$ is applicable in $s$. Since $A$ is $\gamma$-relevant, the subsequent state $s' = \xi(s, A_y)$ is such that $w(T, \gamma, s') \leq w(T, \gamma, s) + 1$. The only case we need to consider is thus when $w(T, \gamma, s) = 1$. Suppose that $s \in \{q_1\}$. Since, by assumption $Pre_A^\gamma \cup Eff_A^\gamma = \gamma(T)$, it follows that $q_1 \in Pre_A^\gamma \cup Eff_A^\gamma$. Clearly $q_1 \notin Pre_A^\gamma$ (otherwise $A_y$ would not be applicable on the state $s$). Therefore, necessarily, either $q_1 \in Eff_A^\gamma$ or $q_1 \in Eff_A^\gamma$. In the first case, we have that $q_1 = q_2$ and thus $s' = s = \{q_1\}$. In the second case, $s' = \{q_2\}$. In both cases, $w(T, \gamma, s') = 1$.

Finally, if $A$ is unbounded, we consider any ground atom $q_1 \in \gamma(T) \setminus (Pre_A^\gamma \cup Eff_A^\gamma)$ and we put $s = \{q_1\}$. Clearly, $A_y$ is applicable in $q_1$ since $Pre_A^\gamma = \emptyset$ and $q_1 \notin Pre_A^\gamma$, and $w(T, \gamma, s) = 1$. Since it also holds that $q_1 \notin Eff_A^\gamma$, we have that the subsequent state $s' = \xi(s, A_y) = \{q_1, q_2\}$ and $w(T, \gamma, s') = 2$.

\textbf{Proof of Proposition\ref{prop:corresponding_property}} Write $A = \{a_1, \ldots, a_n\}$ and let $s$ be state such that $A$ is applicable in $s$. Note that from Proposition\ref{prop:gamma_safe} the actions in $A$ can be serialised and the successor state $s' = \xi(s, A)$ can be recursively obtained as $s_0 = s, s_k = \xi(s_{k-1}, a_k), k = 2, \ldots, n$ and $s' = s_n$. By the assumption, it follows that $w(T, \gamma, s_i) \leq 1$ for every $i$. In particular, $w(T, \gamma, s_n) \leq 1$.

\textbf{Proof of Proposition\ref{prop:corresponding_property}} Given any instance $\gamma$ and any valid induced simple plan $\pi$ having $\text{trace}(\pi) = \{S_j = (t_i, s_i)_{i=0,\ldots,j}\}$ with happening sequence $A_y$, we have that the state sequence $(s_0, \ldots, s_k) \in S_A$. Therefore, since $w(T, \gamma, s_0) \leq 1$ (recall that $s_0 = Init$ and $w(T, \gamma, Init) \leq 1$ for every $\gamma$), the
individual $\gamma$-safety of $A$, implies that $w(\mathcal{T}, \gamma, s_j) \leq 1$ for every $j = 1, \ldots, k$. Since this holds for every $\gamma$ and every valid plan, invariance of $\mathcal{T}$ follows.

Proof of Proposition 37

(i): If $(s^0, s^1, \ldots, s^h) \in S_A$, we have that

$$(s^0, s^1, \ldots, s^h) \in S_{A^1}, \quad (s^{h-1}, s^h, \ldots, s^h) \in S_{A^1}.$$ 

Therefore, if $w(\mathcal{T}, \gamma, s^0) \leq 1$, from the fact that $A^1$ is individually $\gamma$-safe, it follows that $w(\mathcal{T}, \gamma, s^j) \leq 1$ for every $j = 1, \ldots, k$. In particular, being $k \geq h - 1$, we have that $w(\mathcal{T}, \gamma, s^{h-1}) \leq 1$. From the fact that $A^1$ is also individually $\gamma$-safe, it now follows that $w(\mathcal{T}, \gamma, s^j) \leq 1$ for every $j = h, \ldots, n$.

This implies that $w(\mathcal{T}, \gamma, s^j) \leq 1$ for every $j = 1, \ldots, n$ and proves the thesis.

(ii): Suppose $(s^0, s^1, \ldots, s^{h-1}, s^h, \ldots, s^h) \in S_A$ where $s^h = \xi(A^1, s^{h-1})$. Put $s^k = \xi(A^1, s^{h-1})$ and note that, by serialisability (see Proposition 1), $(s^0, s^1, \ldots, s^{h-1}, s^h, \ldots, s^h) \in S_A$. This implies that $w(\mathcal{T}, \gamma, s^j) \leq 1$ for every $j = 1, \ldots, n$ and proves the thesis.

(iii): If $(s^0, s^1, \ldots, s^{n-1}, s^n) \in S_A$, then, $s^{k-1} = s^k$ for every $k = 1, \ldots, n$ and $(s^0, s^1, \ldots, s^n) \in S_A$. Individual $\gamma$-safety of $A$ now yields the thesis. Regarding $A''$ thesis follows from the fact that $A'$ is individually $\gamma$-safe and previous item (ii).

Proof of Proposition 37

(i)⇒(ii): Note that if $(s^0, s^1, s^2) \in S_A$, it follows that $Pre_{A^2}^* \subseteq s^0$. Since $s^1 = (s^0 \setminus E f f_{s^0}) \cup E f f_{s^0}$ it follows that $\Gamma_{A^2} \subseteq s^1$. Analogously, using the fact that $(Pre_{A^2}^*)^c \supseteq s^0$, it follows that $(\Gamma_{A^2})^c \supseteq s^1$. Since $A^2$ must be applicable on $s^1$ conditions (ii) immediately follow.

(ii)⇒(i): Consider $s^0 = Pre_{A^1}^* \cup (Pre_{A^1}^* \setminus E f f_{s^0})$. Straightforward set theoretic computation, using conditions (ii), show that $A^1$ can be applied on $s^0$ and that $A^2$ can be applied on $s^0 = \xi(A^1, s^0)$. This proves (i).

Proof of Proposition 37

(ii)⇒(i): It follows from the proof of (ii)⇒(i) in Proposition 31 that there exists $(s^0, s^1, s^2) \in S_A$ with $s^0 = Pre_{A^1}^* \cup (Pre_{A^1}^* \setminus E f f_{s^0})$. By the assumption made $w(\mathcal{T}, \gamma, s^0) \leq 1$ and this proves (i).

(i)⇒(ii): it follows from the fact that if $(s^0, s^1, s^2) \in S_A$, necessarily $Pre_{A^1}^* \cup (Pre_{A^1}^* \setminus E f f_{s^0}) \subseteq s^0$.

Proof of Proposition 37

(i): If $(s^0, s^1, \ldots, s^{n-1}, s^n) \in S_A$, we have that $(s^0, s^1, \ldots, s^{n-1}, s^n) \in S_A$ for a suitable state $s''$. Result then follows from the definition of executability and $\gamma$-reachability.

(ii): This follows immediately from serialisability (see Proposition 1).

Proof of Proposition 37

Let $(s^0, \ldots, s^n) \in S_A(\gamma)$ and suppose that $A'$ is either $\gamma$-heavy or $\gamma$-relevant unbalanced. Then, necessarily, $w(\mathcal{T}, \gamma, s^n) \geq 2$.

Proof of Proposition 38

1.: It follows from (1) that, given any sequence of states $(s^0, \ldots, s^n) \in S^{n+1}$, we have that

$$(s^0, \ldots, s^n) \in S_A \Leftrightarrow \begin{cases} (s^0, \ldots, s_n) \in S_A, \\ (s_{n+1}^\prime, \ldots, s^n) \in S_{A^1}, \end{cases} \quad (8)$$

This immediately proves the ‘only if’ implication. On the other hand, if $s' \in S_{A^1}$ and $s'' \in S_{A^1}$, we have that $s'_n \in S_{A^1}$ and $s''_n \in S_{A^1}$ and thus $s = s'_n \cup s''_n \in S_A$ by (3).

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2. can be proven analogously to 1. and 3. follows by a straightforward extension of the arguments used to prove Proposition 16. Finally, 4. follows from the definition of strong γ-safety and previous items 1. and 2. \(\square\)

Proof of Theorem 39 Consider the sequences restricted on the instantiation γ(\(T\)) and its complement: \(A_γ, A_¬γ\), and respectively, \(¬A_γ, A_¬γ\). By virtue of Proposition 38, we have that \(A_γ\) is γ-safe and to prove the result it is sufficient to show that \(¬A_γ\) is either non-executable or γ-safe.

Assume that \(¬A_γ\) is executable and let \((s^0, s^1, s^2, \ldots, s^{n+1}, s^{n+2}) \in S_{¬A_γ}\) be such that \(w(T, γ, s^0) ≤ 1\). Since \((s^0, s^1) \in S_{A_γ}\) and \(A_γ\) is strongly safe, it follows that \(w(T, γ, s^1) ≤ 1\). Note now that 
\[
w(T, γ, s^{n+1}) = w(T, γ, s^n) ≤ \cdots ≤ w(T, γ, s^1) ≤ 1
\]
What remains to be shown is that also \(w(T, γ, s^{n+2}) ≤ 1\). To this aim, we introduce the following sets:
\[
Ω' = (\bigcup_{i=1}^n Eff_{B_i}^-) \cap Pre_{A_γ}^\prime, \quad Ω'' = (\bigcup_{i=1}^n Eff_{B_i}^-) \setminus Pre_{A_γ}^\prime
\]
Note that since \(A\) is executable, we have that \(Pre_{A_γ}^- \cap Eff_{B_i}^- = \emptyset\). Consequently, also \(Ω' \cap Eff_{B_i}^- = \emptyset\). Therefore, \((s^0 \setminus Ω', s^1 \setminus Ω') \in S_{A_γ}\). On the other hand, we also have that \(A_γ\) is applicable on the state \(s^{n+1} \cup Ω\). This implies that there exists \(\tilde{γ} \in S\) such that \((s^{n+1} \cup Ω, \tilde{γ}) \in S_{A_γ}\). Note that \(w(T, γ, s^j) ≤ w(T, γ, s^{n+1})\) since \(s^{n+1} ∪ Ω = (s^1 \cup \bigcup_{i=1}^n Eff_{B_i}^-) \cup Ω = s^1 \setminus Ω\). We deduce that \((s^0 \setminus Ω', s^1 \setminus Ω, \tilde{γ}) \in S_{A_γ}\). Since \(w(T, γ, s^0 \setminus Ω') ≤ w(T, γ, s^0) ≤ 1\), the fact that \(A_γ\) is γ-safe implies that \(w(T, γ, s^0) ≤ 1\). This also implies that \(w(T, γ, s^{n+2}) ≤ 1\) and the proof is complete. \(\square\)

Proof of Proposition 42 (i): Suppose \((s^0, s^1, s^2) \in S_{a^m, a^m′}\). Since \(a^m\) only contains preconditions, we have that \(s^1 = s^2\). Note now that \(s^1 = 0 = \xi(a^m, s^0) = (s^0 ∪ Eff_{B_i}^-) \setminus Eff_{B_i}^+\) must satisfy the conditions \(Pre_{a^m}^- \subseteq s^1 \subseteq (Pre_{a^m}^-)\). This yields \(Pre_{a^m}^- \subseteq s^0 \cup Eff_{B_i}^+\) and thus \(Pre_{a^m}^- \cap Eff_{B_i}^+ \subseteq s^0\).

Similarly, from \(s^0 \setminus Eff_{B_i}^+ \subseteq (Pre_{a^m}^-)\), we obtain that \(s^0 \subseteq (Pre_{a^m}^- \cap Eff_{B_i}^-)^\prime\). This implies that also \(a^m\) is applicable on \(s^0\) and \(s^1 = \xi(a^m, s^0)\) since \(a^m\) and \(a^0\) have the same effects. If instead \((s^0, s^1) \in S_{a^m, a^m′}\), we have that \(a^m\) is applicable on \(s^0\) (since the preconditions of \(a^m\) are also preconditions of \(a^0\)) and \(s^1 = \xi(a^m, s^0) = \xi(a^m, s^0)\). (ii) is proven similarly to (i). (iii) follows from (i) and (ii) and, finally, (iv), (v), and (vi) follow, respectively, from (i), (ii), and (iii). \(\square\)

Proof of Proposition 43 Since \(A\), differs from \(A_γ\) only for having more preconditions, it holds that \(S_A \supseteq S_{A_γ}\). Conversely, suppose \((s^0, \ldots, s^8) \in S_{A_γ}\). Then, \((s^0, s^1) \in S_{a^m, a^m′}\). Therefore, by (i) of Proposition 42, we have that \((s^0, s^1) \in S_{a^m}\). Similarly, using (ii) of Proposition 42, we obtain that \((s^{n+1}, s^n) \in S_{a^m, a^m′}\). These two facts together with \((s^0, s^1, s^2, \ldots, s^{n+1}) \in S_{A_γ} \setminus \{a^m, a^m′\}\), yield \((s^0, s^1, s^2) \in S_{A_γ}\). \(\square\)

Proof of Proposition 44 Note that \(Da_γ\), being γ-reachable and \(a^m\) strongly γ-safe, is simply γ-safe if and only if \(Da_γ\) is individually γ-safe. This last fact is equivalent to show that, given any state sequence \((s^0, s^1, s^2) \in S_{Da_γ}\) such that \(s^0 \in γ(T)\) and \(w(T, γ, s^0) ≤ 1\), it holds that \(w(T, γ, s^i) ≤ 1\) for \(i = 2\) (since for \(i = 1\) follows from the strong safety of \(a^m\)). Put 
\[
W_γ := \{s^1 \in γ(T) | \exists s^0, s^2 \in γ(T), w(T, γ, s^0) ≤ 1, (s^0, s^1, s^2) \in S_{Da_γ}\}
\]
We need to show that, for every \( s^1 \in W_\gamma \), we have that \( w(T, \gamma, s^2) \leq 1 \), where

\[
s^2 = \xi(a_{st}^{end}, s^1) = s^1 \cup E f f_{a_{st}^{\gamma}}^+ \setminus E f f_{a_{st}^{\gamma}}^-
\]

Since \( a_{st}^{end} \) is \( \gamma \)-relevant unbounded, the condition \( w(T, \gamma, s^2) \leq 1 \) is clearly equivalent to

\[
s^1 \subseteq E f f_{a_{st}^{\gamma}}^+ \cup E f f_{a_{st}^{\gamma}}^-	ag{9}
\]

Since \( a_{st}^{\gamma} \) is \( \gamma \)-reachable and strongly \( \gamma \)-safe, it follows from Theorem \[18\] that is either \( \gamma \)-irrelevant or \( \gamma \)-relevant. If \( a_{st}^{\gamma} \) is \( \gamma \)-irrelevant and \( |Pre_{a_{st}^{\gamma}}^+| = 1 \), we have that \( W_\gamma = (Pre_{a_{st}^{\gamma}}^+ \setminus E f f_{a_{st}^{\gamma}}^-) \).

Combining with (9), we thus have that in this case \( Da_\gamma \) is \( \gamma \)-safe if and only if

\[
Pre_{a_{st}^{\gamma}}^+ \setminus E f f_{a_{st}^{\gamma}}^- \subseteq E f f_{a_{st}^{\gamma}}^+ \cup E f f_{a_{st}^{\gamma}}^-\tag{10}
\]

This leads to the two possible cases (a) and (b).

Suppose now that \( a_{st}^{\gamma} \) is \( \gamma \)-relevant and \( |Pre_{a_{st}^{\gamma}}^+| = 0 \). In this case,

\[
W_\gamma = \{s^1 \subseteq \gamma(T) \mid w(T, \gamma, s^1) \leq 1, \ s^1 \cap (Pre_{a_{st}^{\gamma}}^- \cup E f f_{a_{st}^{\gamma}}^-) = \emptyset\}
\]

Combining with (9), we thus have that in this case \( Da_\gamma \) is \( \gamma \)-safe if and only if

\[
Pre_{a_{st}^{\gamma}}^- \cup E f f_{a_{st}^{\gamma}}^- \cup E f f_{a_{st}^{\gamma}}^+ \cup E f f_{a_{st}^{\gamma}}^+ = \gamma(T)\tag{11}
\]

This leads to case (c).

Finally, if \( a_{st}^{\gamma} \) is relevant we have that \( W_\gamma = (E f f_{a_{st}^{\gamma}}^+ ) \). Combining again with (9), we obtain that in this case \( Da_\gamma \) is \( \gamma \)-safe if and only if condition (d) is verified. \( \square \)

**Proof of Proposition 47.** Since \( Da_\gamma \) is \( \gamma \)-reachable, it follows from Proposition \[37\] that \( a_{st}^{\gamma} \) must necessarily be \( \gamma \)-relevant unbounded. In particular, this yields \( Pre_{a_{st}^{\gamma}} \subseteq \emptyset \). Therefore, \( Pre_{a_{st}^{\gamma}} = Pre_{a_{st}^{\gamma}}^+ \setminus E f f_{a_{st}^{\gamma}}^+ \) cannot have any intersection with \( E f f_{a_{st}^{\gamma}}^- \). This says that \( a_{st}^{\gamma} \) cannot be \( \gamma \)-relevant balanced. Since it can neither be \( \gamma \)-unreachable (since \( Da_\gamma \) is \( \gamma \)-reachable), it follows from Corollary \[21\] that \( a_{st}^{\gamma} \) must be \( \gamma \)-relevant bounded. This proves (i).

Suppose now that the sequence \((a_{st}^{\gamma+1}, A^{a_{st}^{\gamma+1}})\) is executable and let \( q \in E f f_{A_{st}^{\gamma+1}}^+ \). By (i), it follows that \( q \in E f f_{a_{st}^{\gamma}}^+ \cup E f f_{a_{st}^{\gamma}}^- \cup Pre_{a_{st}^{\gamma}}^- \). Note that \( q \) cannot either belong to \( E f f_{a_{st}^{\gamma}}^- \) or \( Pre_{a_{st}^{\gamma}}^- \) since \( a_{st}^{\gamma} \) and the actions in \( A^{a_{st}^{\gamma}} \) must be non-interfering. On the other hand, \( q \) cannot belong to \( Pre_{a_{st}^{\gamma}}^- \) otherwise the sequence would not be executable. Therefore the only possibility is that \( q \in E f f_{a_{st}^{\gamma}}^+ \).

Therefore we have that \( E f f_{A_{st}^{\gamma+1}}^+ \subseteq E f f_{a_{st}^{\gamma}}^+ \). Consider now \( A^{a_{st}^{\gamma}} \) the action set obtained from \( A^{a_{st}^{\gamma}} \) by eliminating all positive effects belonging to \( \gamma(T) \). Clearly, \( (a_{st}^{\gamma}) \cup A^{a_{st}^{\gamma}} = \{a_{st}^{\gamma}\} \cup A^{a_{st}^{\gamma}} \). Consider now the sequence \((A^{a_{st}^{\gamma}}, a_{st}^{\gamma}, A^{a_{st}^{\gamma}})\) and note that \( A^{a_{st}^{\gamma}} \) is \( \gamma \)-irrelevant, and \( (a_{st}^{\gamma}, A^{a_{st}^{\gamma}}) \) is \( \gamma \)-individually safe because of (iv) of Proposition \[42\]. Therefore, by Proposition \[27\] also \((A^{a_{st}^{\gamma}}, a_{st}^{\gamma}, A^{a_{st}^{\gamma}})\) is individually \( \gamma \)-safe, and thus also \((a_{st}^{\gamma}) \cup A^{a_{st}^{\gamma}} \). \( \square \)

**Proof of Theorem 49.** Fix any valid simple plan \( \pi \) with happening sequence \( A_{\pi} = (A_{\pi}, \ldots, A_{\pi}) \) and any instance \( \gamma \). We prove that \( A_{\pi} \) is individually \( \gamma \)-safe.

We split happenings as follows: \( A_{\pi} = A_{\pi}^{st} \cup A_{\pi}^{\gamma} \cup A_{\pi}^{end} \) where

- \( A_{\pi}^{st} \) is either empty or consists in the start fragments of durative actions in \( G(A_{\pi}(\gamma)) \);
• \( A^\text{end}_t \) is either empty or consists in the ending fragments of durative actions in \( \mathcal{GAR}^t(\gamma) \);

• \( A^\text{end}_t = A_t \setminus (A^{\mu}_t \cup A^{\text{end}}_t) \) consists of strongly \( \gamma \)-safe actions (either instantaneous or possibly the starting and ending of durative ones in \( \mathcal{GAR}^t \setminus \mathcal{GAR}^t(\gamma) \)).

Note that if \( A^\mu_t \neq \emptyset \), it either consists of all strongly \( \gamma \)-safe actions and is thus strongly \( \gamma \)-safe, or there exists a durative action \( D_a \in \mathcal{GAR}^t(\gamma) \) such that \( a^{\mu} \) is not strongly safe and \( a^{\mu} \in A^\mu_t \). Note that \( A^\mu_t \) simply consists of \( \{a^{\mu}\} \) possibly together with other overall fragments of durative actions. Consequently, since \( (A^\mu_t, A_{t,\gamma}) \) is executable, it is also executable \( (A^\mu_t, a^{\mu}) \) (see (i) of Proposition \([33]([33])\)). By hypothesis, \( D_a \) is \( \gamma \)-reachable and \( a^{\mu} \) is individually \( \gamma \)-safe, we can thus apply Proposition \([47]([47])\) and conclude that \( (A^\mu_t, a^{\mu}) \) is individually \( \gamma \)-safe. Using (ii) of Proposition \([27]([27])\) we obtain that \( (A^\mu_t, A_{t,\gamma}) \) is individually \( \gamma \)-safe. Therefore, in any case, if \( A^\mu_t \neq \emptyset \), \( (A^\mu_t, A_{t,\gamma}) \) is individually \( \gamma \)-safe.

Similarly, if \( A^\text{end}_t \neq \emptyset \), it either consists of all strongly \( \gamma \)-safe actions and is thus strongly \( \gamma \)-safe, or there exists a durative action \( D_a \in \mathcal{GAR}^t(\gamma) \) such that \( a^{\text{end}} \) is not strongly safe and \( a^{\text{end}} \in A^\text{end}_t \). Suppose that it exists another durative action \( D_a' \in \mathcal{GAR}^t(\gamma) \) such that \( a^{\text{end}} \in A^\text{end}_t \) and \( \{a^{\text{end}}, a^{\text{end}}\} \) is \( \gamma \)-heavily. Then, since \( \mathcal{GAR}^t \) is right relevant isolated and the two pairs \( \{a^{\mu}, a^{\text{end}}\} \), \( \{a^{\text{end}}, a^{\text{end}}\} \) are both non-interfering, the sequence \( \{(a^{\mu}, a^{\text{end}}), (a^{\text{end}}, a^{\text{end}})\} \) is \( \gamma \)-unreachable. Since \( A_{t,\gamma} \) only consists of actions with no effects, it then follows from Proposition \([33]([33])\) that also the sequence \( (A^\mu_t, A^\text{end}_t) \) is \( \gamma \)-unreachable. The other possibility is that \( \text{Eff}_{A^\text{end}_t} = \text{Eff}_{a^{\text{end}}} \). Consider in this case \( A^\text{end}_t \) to be the action set obtained from \( A^\text{end}_\gamma \) by eliminating all positive effects belonging to \( \gamma(T) \). Clearly, \( A^\text{end}_t = \{a^{\text{end}}\} \cup A^\text{end}_t \). Note now that \( \{a^{\mu}, a^{\text{end}}\} \) is individually \( \gamma \)-safe by (v) of Proposition \([42]\). Considering that \( A^\mu_t \) only contains preconditions and \( A^\text{end}_t \) is strongly \( \gamma \)-safe, a repeated application of the different items of Proposition \([27]([27])\) implies that \( (A^\mu_t, A^\text{end}_t) \) is individually \( \gamma \)-safe.

Note that, given each happening time \( t_h \), there are four possibilities:

• \( A^\mu_t = \emptyset, A^\text{end}_t = \emptyset \): in this case \( A_t = A^\mu_t \) is strongly \( \gamma \)-safe by definition;

• \( A^\mu_t \neq \emptyset, A^\text{end}_t = \emptyset \): in this case, since \( A^\mu_t \) and \( (A^\mu_t, A_{t,\gamma}) \) are individually \( \gamma \)-safe, using (i) and (ii) of Proposition \([27]([27])\) we obtain that also \( (A^\mu_t, A^\mu_t, A_{t,\gamma}) \) and \( (A^\mu_t, A^\mu_t) = (A^\mu_t \cup A^\mu_t, A^\mu_t) \) are individually \( \gamma \)-safe.

• \( A^\mu_t = \emptyset, A^\text{end}_t \neq \emptyset \): arguing analogously to the case above we obtain that \( (A^\mu_t, A_t) \) is individually \( \gamma \)-safe.

• \( A^\mu_t \neq \emptyset, A^\text{end}_t \neq \emptyset \): arguing analogously to the case above we obtain that \( (A^\mu_t, A^\text{end}_t, A_{t,\gamma}) \) is individually \( \gamma \)-safe.

Using Corollary \([28]\) we obtain that \( A^\mu_t \) is individually \( \gamma \)-safe. \(\square\)

**Proof of Theorem** \([27]\). Fix any valid (possibly induced) simple plan \( \pi \) with happening sequence \( A_\pi = (A_h, \ldots, A_{h_k}) \) and any instance \( \gamma \). We prove that \( A_\pi \) is individually \( \gamma \)-safe.

Suppose that we can prove that if \( D_a \in \mathcal{GAR}^t(\gamma) \) appears in \( \pi \) on the time window \( [t_h, t_h] \) (namely, \( a^{\mu} \in A_h \) and \( a^{\text{end}} \in A_h \)), the corresponding action sequence \( A = (A_h, \ldots, A_k) \) satisfies the following conditions:

(a) for every \( i \in (h, k) \), \( A_i \) consists exclusively of \( \gamma \)-irrelevant actions;
(b) for every $i \in [h, k)$, $A_i$ does not contain actions in $\mathcal{G}\mathcal{A}^d(\gamma)$.

Note that if (b) holds true for every $Da \in \mathcal{G}\mathcal{A}^d(\gamma)$, we also have automatically that,

(c) for every $i \in (h, k]$, $A_i$ does not contain actions in $\mathcal{G}\mathcal{A}^end(\gamma)$.

Assuming this to hold, we now proceed as in the proof of Theorem 39 and we split each happening $\tilde{A}_h$ in the following way. We put $\tilde{A}_h = A^n_h \cup A^i_h \cup A^end_h$ where:

- $A^n_h$ is either empty or consists in a start fragment in $\mathcal{G}\mathcal{A}^n(\gamma)$;
- $A^i_h$ is either empty or consists in an ending fragment in $\mathcal{G}\mathcal{A}^{end}(\gamma)$;
- $A^i_h = A_h \setminus (A^i_h \cup A^end_h)$.

We now consider the new plan $\tilde{\pi}$ given by

$$\tilde{\pi} = \{(t, a) \in \pi | a \in A^n_h \} \cup \{(t - \epsilon, a) \in \pi | a \in A^i_h \} \cup \{(t + \epsilon, a) \in \pi | a \in A^i_h \}$$

where $\epsilon > 0$ is chosen in such a way that $\epsilon < t_{i+1} - t_i$ for every $i = 0, \ldots, \tilde{k} - 1$.

It follows from Proposition 1 on serializability that plan $\tilde{\pi}$ is also valid. We denote its happening sequence as $\tilde{A}_h = (\tilde{A}_{h, 1}, \ldots, \tilde{A}_{h, \tilde{k}})$. For the sake of notation simplicity, happening times are denoted as those in $\pi$ even if in general they differ and form a larger set. Note now that the happening times in $\tilde{\pi}$ can be split into singletons $t_i$ such that $\tilde{A}_i$ only consists of strongly $\gamma$-safe actions, and intervals $[t_{i+1}, t_i]$ such that there exists a durative action $Da \in \mathcal{G}\mathcal{A}^d(\gamma)$ happening in that interval. In this case we have that the subsequence $A = (\tilde{A}_{h, 1}, \ldots, \tilde{A}_{h, \tilde{k}})$ is $Da$-admissible. Put $A = (\tilde{A}_{h, 1} \cup (a^i), \ldots, \tilde{A}_{h, \tilde{k}} \cup (a^i))$. Note that, since $A$ is executable (as it appears in a valid plan), also $A$, is executable by Proposition 43. Since, by assumption (ii), $Da$ is $\gamma$-safe, it follows from Theorem 39 that $A$, is also $\gamma$-safe. Using again Proposition 43 we finally obtain that $A$ is individually $\gamma$-safe.

We have thus proven that each happening time $t_i$ in the new plan $\tilde{\pi}$ stays inside an individually $\gamma$-safe sequence (possibly of length 1). By Corollary 25 this implies that $A_\pi$ is individually $\gamma$-safe.

A repetitive use of (ii) of Proposition 27 now yields that $A_\pi$ is also individually $\gamma$-safe.

We are thus left with proving that every durative action $Da \in \mathcal{G}\mathcal{A}^d(\gamma)$ happening in $\pi$ satisfies properties (a) and (b) stated above. Suppose this is not true and let $Da$ be the first (as starting time) to happen in $\pi$ (in the time window $[t_h, t_{h+1}]$) and to violate either condition (a) or (b). Note that all durative actions in $\mathcal{G}\mathcal{A}^d(\gamma)$ happening in $\pi$ and starting strictly before time $t_h$, will necessarily end at a time $t \leq t_h$ by the way $Da$ has been chosen. Moreover, all such durative actions will satisfy properties (a) and (b). We can then proceed as before and consider the splitting $A_i = A^n_i \cup A^i_i \cup A^end_i$ for every $i \leq h$ (note that in $t_h$ there could be, in principle, more than one starting actions in $A^n_i$). Consider now the auxiliary plan $\tilde{\pi}$ constructed exactly like before for $t \leq t_h$ and coinciding with $\pi$ for $t > t_h$. As before, we denote its happening sequence as $A_h = (\tilde{A}_{h, 1}, \ldots, \tilde{A}_{h, \tilde{k}})$ using the same notation for the happening times as in $\pi$ and we assume that $\tilde{A}_h = A^n_h$ (this is for simplicity of notation considering that it would be instead $\tilde{A}_{h, \epsilon} = A^i_h$). Arguing as above,
we obtain that $(\bar{A}_{t_0}, \ldots, \bar{A}_{t_n})$ is individually $\gamma$-safe. If we take any $(\bar{s}_{t_0}, \ldots, \bar{s}_{t_n}) \in S_{\bar{A}_t}(\gamma)$, we thus have that $w(T, \gamma, \bar{s}_{t_{n+1}}) \leq 1$. Consider now $A = (\bar{A}_{t_0}, \ldots, \bar{A}_{t_n}) = (A^\alpha_{t_0}, A^\alpha_{t_1}, \ldots, A^\alpha_{t_n})$ and note that, $(\bar{s}_{t_{n+1}}, \ldots, \bar{s}_{t_1}) \in S_{\bar{A}_t}(\gamma)$ so that $A$ is $\gamma$-reachable. It then follows from the relevant non-interwining property (ii) of Definition 50 that $A^\alpha_{t_0} = \{a^\alpha\}$. Suppose now that property (b) stated above is not satisfied and let $l \in (h, k)$ be the first index such that $A_l \cap \bar{G}A^\alpha(\gamma) \neq \emptyset$. By (ii) of Definition 50 it follows there exists a durative action $Da^\prime \in \bar{G}A^\alpha(\gamma)$ such that $a^\alpha_{t_0} \in A^\alpha_{t_0}$ for some $l \in (h, k)$. Note that such durative action cannot start, in the plan $\pi$ and thus also in the plan $\bar{\pi}$, before time $t_0$ for the way $Da$ was chosen, it cannot either start at time $t_0$ by previous considerations and neither in the interval $(t_0, t_1)$ by the way $l'$ has been chosen. This proves that property (b) must be satisfied. Note that this also shows that $A_l$ does not contain actions in $\bar{G}A^\alpha(\gamma)$ for any $i \in (h, k)$ (as the corresponding start fragment cannot happen neither before or after time $t_0$). Finally, $A_l$ is $\gamma$-irrelevant for every $i \in (h, k)$ because of (iii) of Definition 50. Therefore $A$ satisfies properties (a) and (b) contrarily to the assumptions made on $Da$. Proof is now complete.

Proof of Proposition 56 Consider $Da \in \bar{G}A^\alpha(\gamma)$ and a $\gamma$-reachable $Da$-admissible sequence

$$A = ([a^\alpha] \cup A^1, A^2, \ldots, A^{n-1}, \{a^\alpha_{t_0} \cup A^n\}).$$

If $a^\alpha_{t_0} \in \bar{G}A^\alpha(\gamma) \cap A^1$, necessarily, $a^\alpha_{t_0}$ and $a^\alpha_{t_0}$ are non-interfering, and, by Proposition 33 the sequence $([a^\alpha_{t_0}], A^2, \ldots, A^{n-1}, \{a^\alpha_{t_0}\})$ is $\gamma$-reachable contradicting assumption (i). Therefore $\bar{G}A^\alpha(\gamma) \cap A^1 = \emptyset$. This proves (i) in Definition 50.

Suppose now that $A^1 = \emptyset$ and suppose that (ii) in Definition 50 does not hold true for $A$. Let $j > 1$ be the first index for which (ii) is violated. Let $a^\alpha_{t_0} \in A^j \cap \bar{G}A^\alpha(\gamma)$. Since $([a^\alpha_{t_0}], A^2, \ldots, A^{j-1}, \{a^\alpha_{t_0}\})$ is $\gamma$-reachable and for sure the pair $[a^\alpha_{t_0}, a^\alpha_{t_0}]$ is non-interfering, it follows from assumption (ii) that there must exist $0 < j' < j$ such that $A^j$ is not $\gamma$-relevant. Let $j'$ be the first index for which this happens and let $b \in A^j$ be an action which is not $\gamma$-irrelevant. Note that $b \notin \bar{G}A^\alpha(\gamma)$ (otherwise $a^\alpha_{t_0}$ would not violate (ii)). This however contradicts assumption (ii). Therefore this proves (ii) in Definition 50.

Suppose now that $A^1 = \emptyset$ and $A^j \cap (\bar{G}A^\alpha(\gamma) \cup \bar{G}A^\alpha(\gamma)) = \emptyset$ for every $j = 2, \ldots, n-1$. If (iii) in Definition 50 does not hold true for $A$, consider $j > 1$ to be the first index for which (iii) is violated, namely $A^j$ is not $\gamma$-irrelevant, and let $b \in A^j$ be any action which is not $\gamma$-irrelevant. Since $([a^\alpha_{t_0}], A^2, \ldots, A^{j-1}, \{b\})$ is $\gamma$-reachable, it follows from assumption (ii) that there must exist $0 < j' < j$ such that $A^j$ is $\gamma$-relevant but this contradicts the choice of $j$. Proof is thus complete.

Proof of Proposition 56 Assume that, by contradiction, there exists a $\gamma$-reachable sequence

$$A = ([a] \cup A^2, \ldots, A^{n-1}, \{a^\prime\})$$

such that $A^2, \ldots, A^{n-1}$ are $\gamma$-irrelevant set of actions. Consider $(s^1, \ldots, s^n) \in S_A(\gamma)$.

Suppose condition (i) is satisfied. Clearly, $q \in s^1$ and, because of the assumption made, it follows that $q \notin Ef f_{A^j}$ for every $j = 2, \ldots, n-1$. Therefore, $q \in s^{n-1}$. Since $q \in Pre_{a^\prime}$, this is a contradiction.

A similar arguments can be used if instead condition (ii) is satisfied.

Finally, assume that condition (iii) is satisfied. Note that, since $A^2, \ldots, A^{n-1}$ are $\gamma$-irrelevant, $Pre_{a^\prime} \cup (Pre_{a^\prime} \setminus Ef f_{A^j}) \subset s^0$ and this contradicts the fact that $(s^0, \ldots, s^n) \in S_A(\gamma)$.
Proof of Corollary 87. It is clear that condition (i) and (ii) of Theorem 57 are satisfied. In order to check that $G_{\mathcal{A}'}$ is relevant non-intertwining, we show that the properties (i) and (ii) of Proposition 54 are satisfied. Fix any instance $\gamma$.

Consider $Da^i, Da^j \in G_{\mathcal{A}'}(\gamma)$. It follows from the fact that $Da^i$ and $Da^j$ are both simply $\gamma$-safe of type (a) (see Remark 45) that

$$Pre^*_{a^i} = \{q^i\} \subseteq Eff_{a^i}, \quad i = 1, 2$$

If $a^{1, i}$ and $a^{2, i}$ are non-interfering, it follows that $q^1 \neq q^2$ and, in this case, $\{a^{1, i}, a^{2, i}\}$ is $\gamma$-unreachable. This proves (i).

Consider now $Da \in G_{\mathcal{A}'}(\gamma)$ and $a' \in G_{\mathcal{A}} \setminus G_{\mathcal{A}'}(\gamma)$ that is $\gamma$-relevant or $a' \in G_{\mathcal{A}'}(\gamma)$. Then, by assumptions (i) and (ii) we have that

$$Pre^*_{a^i} = \{q\} \subseteq Eff_{a^i}, \quad Pre^*_{a'} = \{q'\}$$

If $q = q'$, we have that $q \in \Gamma_{a^i} \cap Pre^*_{a}$ and, since $q \in \gamma(T)$, for sure $q \not\in Eff_{a^i}$ for any $a''$ which is $\gamma$-irrelevant. This implies that condition (ii) of Definition 55 is satisfied. If instead $q \neq q'$, we have that the condition (iii) is also satisfied. In any case this says that the pair $(a^i, a')$ is strongly $\gamma$-irrelevant unreachable and thus also, because of Proposition 56 $\gamma$-irrelevant unreachable.

Proof of Lemma 64. We use the formalism introduced in Remarks 59 and 62. Assume that, for $i = 1, 2$, $l_i$ matches $T$ via the component $c_i$ whose corresponding relation has the form $r_i(x_1, \ldots, x_k, v)$ so that $l_i = r(a_1^i, \ldots, a_{k+1}^i)$ or $l_i = \forall v : \ r(a_1^i, \ldots, a_k^i, v)$ for free arguments $a_1^i, \ldots, a_k^i, a_{k+1}^i$. We have that

$$gr(a_j^i) = \gamma(x_j^i), \quad \forall j = 1, \ldots, k$$

(12)

where the first equality follows from the assumption of coherence over $l_1$, while the second follows from the definition of an instance. Now, if $l_2 \sim_T l_1$, we have that $a_j^i = a_j^j$ for every $j$. It thus follows from (12) that

$$gr(a_j^i) = \gamma(x_j^i), \quad \forall j = 1, \ldots, k$$

(13)

which says that $gr$ and $\gamma$ are coherent over $l_2$. On the other hand, if $l_2 \not\sim_T l_1$, it follows that $a_j^i \neq a_j^j$ for some $j$ and, since $gr$ is injective, we also have $gr(a_j^i) \neq gr(a_j^j) = \gamma(x_j^j)$ which says that $gr$ and $\gamma$ are not coherent over $l_2$.

Proof of Corollary 72. Suppose that $a = gr(a)$ for some $gr$ and let $\gamma$ be an instance. Then, $a_\gamma = gr(a_\gamma)$ where $L$ is the $T$-class on which $gr$ and $\gamma$ are coherent. Result is now a straightforward consequence of Proposition 71 and Corollary 21.

Proof of Proposition 87. Note first of all that each of the conditions (i), (ii), (iii) expressed in Definition 86, if true for $M = M_{\mathcal{A}, L_2}$ is also true for any matching $M \supseteq M_{\mathcal{A}, L_2}$: this is evident for properties (i) and (ii) (see Remark 54) while for (iii) follows from the following argument. Condition (iii), for $M = M_{\mathcal{A}, L_2}$, holds true if, either, $w(Pre^*_{a^i} \cup Pre^*_{a^j}) \geq 2$ for $i = 1$ or $2$ (and this does not depend on $M$), or if there exist two unquantified literals $l^i \in Pre^*_{a^i} \cup Pre^*_{a^j}$ for $i = 1, 2$ such that $l^i \neq \lambda_{M_{\mathcal{A}, L_2}} l^j$. This implies that, necessarily, $l^i, l^j$ match $T$ through components
c^1 = \langle r^1, a^1, p^1 \rangle \text{ and } c^2 = \langle r^2, a^2, p^2 \rangle \text{ with } r^1 \neq r^2. \text{ This yields } \text{Rel}[l^1] \neq \text{Rel}[l^2] \text{ and, as a consequence, } l^1 \notin \mathcal{M}^\mathcal{L} \text{ with respect to any possible matching } \mathcal{M}.

Let \( \mathcal{M} \) be the matching such that \( \mathcal{g}^1 \) and \( \mathcal{g}^2 \) are \( \mathcal{M} \)-adapted. It follows from Proposition\(^85\) that \( \mathcal{M} \subseteq \mathcal{M}_\mathcal{L} \). Consequently we know that at least one of the conditions (i), (ii), (iii) expressed in Definition\(^88\) holds true for such \( \mathcal{M} \). It then follows from (6) that at least one of the following conditions hold

(i) \( \text{Pre}_{\mathcal{g}^1}^+ \cap \text{Pre}_{\mathcal{g}^2}^- \neq \emptyset; \)

(ii) \( \text{Pre}_{\mathcal{g}^1}^- \cap \text{Pre}_{\mathcal{g}^2}^+ \neq \emptyset; \)

(iii) \( |\text{Pre}_{\mathcal{g}^1}^+ \cup \text{Pre}_{\mathcal{g}^2}^+ \cup \text{Pre}_{\mathcal{g}^1}^- \cup \text{Pre}_{\mathcal{g}^2}^-| \geq 2. \)

By virtue of Propositions\(^31\) and \(^32\) this implies that \((\alpha^{1inv}, \gamma^{2inv}, \alpha^{1end}, \alpha^{2end})\) is \( \gamma \)-unreachable.

**Proof of Proposition\(^93\).** Fix any instance \( \gamma \) and consider \( D\mathcal{a}^1, D\mathcal{a}^2 \in \mathcal{G}\mathcal{R}(\gamma) \). Let \( D\mathcal{a}^i \) and \( \mathcal{g}^i \), for \( i = 1, 2 \), durative schems and groundings such that \( D\mathcal{a}^i = \mathcal{g}^i(D\mathcal{a}^i) \). Let \( L^i \) be the \( \mathcal{T} \)-class of literals of each schema \( D\mathcal{a}^i \) such that \( \mathcal{g}^i \) and \( \gamma \) are coherent over \( L^i \) for \( i = 1, 2 \). Therefore, \( D\mathcal{a}^1 \) and \( D\mathcal{a}^2 \) must satisfy one of the conditions (i) to (iii) in the Definition\(^88\). Let \( \mathcal{M} \) be the matching respect to which \( \mathcal{g}^1 \) and \( \mathcal{g}^2 \) are adapted (in the sense of Remark\(^80\)). We know from Proposition\(^85\) that \( \mathcal{M} \supseteq \mathcal{M}_\mathcal{L} \). Note now that if condition (i) holds true, it also holds true for such larger \( \mathcal{M} \) (Remark\(^84\)) and this yields condition (i) of Definition\(^88\). Similarly, condition (ii) yields the same condition with this new \( \mathcal{M} \) (Remark\(^84\)) from which condition (ii) in Definition\(^88\) follows using Proposition\(^85\). Finally, if condition (iii) holds true, then condition (iii) in Definition\(^88\) follows by using Proposition\(^87\). Therefore, by Definition\(^88\) we have that the two durative action schems \( D\mathcal{a}^1 \) and \( D\mathcal{a}^2 \) must satisfy one of the conditions (i) to (iii) in the definition. From the fact that \( \mathcal{g}^1 \) and \( \mathcal{g}^2 \) are \( \mathcal{M} \)-adapted, it follows that conditions (i) of Definition\(^88\) yields condition (i) of Definition\(^48\) Condition (ii) and (iii) in Definition\(^88\) finally follow conditions (ii) and (iii) in Definition\(^88\) using Propositions\(^83\) and \(^87\).

**Proof of Proposition\(^92\).** Let \( \mathcal{M} \) be the matching respect to which the two groundings \( \mathcal{g}^1 \) and \( \mathcal{g}^2 \) are \( \mathcal{M} \)-adapted. By Proposition\(^85\), we have that \( \mathcal{M} \supseteq \mathcal{M}_\mathcal{L} \). Arguing like in the proof of Proposition\(^87\) we obtain that one of the conditions (i) to (iii) of Definition\(^92\) must hold true for such a matching \( \mathcal{M} \). Suppose (i) holds and put \( q = \mathcal{g}^1(l^1) = \mathcal{g}^2(l^2) \in \Gamma_{\mathcal{g}^1}^+ \cap \text{Pre}_{\mathcal{g}^2}^- \). Consider now any ground action \( a \) which is \( \gamma \)-irrelevant and let \( a \) be an action schema such that \( a = \mathcal{g}(a) \) for some grounding \( \mathcal{g} \). Let \( L \) be the \( \mathcal{T} \)-class of literals of \( \alpha \) on which \( \mathcal{g} \) and \( \gamma \) are coherent. It follows that \( a_{\mathcal{g}} \) is irrelevant. Consider now the matching \( \mathcal{M} \) between \( a^1 \) and \( \mathcal{g}^1 \) with respect to which \( \mathcal{g}^1 \) and \( \mathcal{g}^2 \) are \( \mathcal{M} \)-adapted. We have that \( \mathcal{M} \supseteq \mathcal{M}_{a^1, \mathcal{L}} \). Then, by (i) we have that \( l^1 \notin EEff_{\mathcal{g}} \) which implies that \( q \notin EEff_{\mathcal{g}} \). This shows that condition (i) of Definition\(^55\) is satisfied. Similarly, one can prove that condition (ii) of Definition\(^92\) yields condition (ii) of Definition\(^55\). Finally the fact that (iii) of Definition\(^92\) yields condition (iii) of Definition\(^55\) follows from a repeated application of relation (6).
To this aim, fix an instance $\gamma$ and $a^{1st} \in G_{A_i}(\gamma)$ and $a^2 \in G_A \setminus G_{A_i}(\gamma)$ that is not $\gamma$-irrelevant. Let $D a^1 \in L^1$ and $a^2 \in G^2$ and $gr^2$ be action schemas and groundings such that $gr^2(D a^1) = D a^1$ and $a^2 = gr^2(a^2)$. Let $L^i$, for $i = 1, 2$, be $T$-classes of literals of $D a^1$ and $a^2$, respectively, such that $gr^2$ and $\gamma$ are coherent over $L^i$. We have that $a^{1st}_{L^1} \in G_{A_i}(T)$, $a^2 \notin G_{A_i}(T)$ and is not irrelevant. By assumption (ii) it then follows that $a^{1st}$, $a^2$ is strongly $(L^1, L^2)$-irrelevant unreachable and thus, by Proposition 93, $(a^{1st}, a^2)$ is strongly $\gamma$-irrelevant unreachable. In the case when instead $a^2 \in G_{A_i}(\gamma)$ proof is analogous.

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