Research article

Cascade computing model to optimize energy exchanges in prosumer communities

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ABSTRACT

Recently, the increasing availability of renewable energy plants has changed the market of electrical energy. The concept of energy community enables prosumers to exploit and exchange the energy produced locally and reduce the need for external energy sources. This can help to obtain significant cost savings and increase the percentage of green energy. In this paper, we present the Cascade model, which aims to achieve a twofold goal: compute an energy schedule that satisfies the needs of single prosumers, and maximize the energy sharing at the community level, thus minimizing the overall cost. The Cascade model partitions the prosumers in groups: at each step, an optimization problem is solved for all the users of a group. The solution enables defining a super-user that summarizes the energy requirements of the groups considered before. Then, a new group is considered in the next step, and so on, until all the groups have been processed. This approach enables preventing the exponential increase in computing complexity that is inevitable when all the prosumers are considered together, using the model referred to as Unified. Experimental results show that the Cascade model leads to a great reduction of computing time, while the overall cost closely approximates the optimal solution ensured by the Unified model.

1. Introduction

Most of the current demand of energy, which is continually rising worldwide, is met by non-renewable energy sources, like coal, petroleum, and natural gas. In today’s world, a major trend aims to induce users to reduce their household energy consumption, and shift to energy produced from renewable sources, such as sun, water, and wind. Furthermore, users are encouraged to generate green energy themselves, and to either store the surplus for future usage or share it with other users [1]. The reasons for this trend include strong social attitudes aiming to alleviate negative climatic impacts of carbon emissions, various government regulations, including generous feed-in tariff schemes, and the desire to decrease electricity costs: individual prosumers are often excluded from the cheaper costs ensured by the wholesale energy market due to their perceived inefficiency and unreliability [2].

To fulfil the mentioned goals, new strategies are needed to manage the interaction of the prosumers with the utility grid more efficiently, and foster the sharing of energy among them. In this respect, the vision of goal-oriented prosumer community groups (PCGs) is a concrete application of the concept of “virtual community” [3]. A prosumer community group enables the virtual aggregation of a number of prosumers, possibly at different locations, who share similar or complementary energy behaviors. As underlined in [4], the members of a prosumer community group can assume different roles concurrently. They can: (i) consume and store energy; (ii) produce renewable energy locally, and sell or share it within the community; (iii) negotiate directly with the energy provider, and access to wholesaler energy prices. The adoption of a PCG helps to minimize the overall cost for the community, since the consumers can obtain energy at a lower cost from the prosumers of the same community, while the prosumers can sell energy to local users more profitably than to the external grid. The advantages of PCGs are discussed in Section 2 in more detail.

In [5, 6], we presented a model for the efficient management of energy in a prosumer community, the “Unified model”. With this model, prosumers are not modeled separately, instead, a unique model is developed for an entire prosumer community group, with the goal of maximizing the energy sharing and balance the energy consumption at the community level. The Unified model is formalized in terms of a Mixed Integer Linear Programming (MILP) problem, which considers the energy needs and production profiles of the prosumers, and their

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energy exchanges both with the grid and among themselves. The Unified model is able to compute the optimal solution, i.e., the one that minimizes the energy cost of the community as a whole, but its main limitation is that the execution time becomes excessive as the number of prosumers, and therefore the number of variables of the optimization problem, increase. In practice, this model can become useless in large communities, with hundreds of users or more.

To tackle the mentioned complexity issue, in this paper we present a novel approach to solve the energy community problem. When a PCG has a large number of users, instead of solving a single big optimization problem, as it would occur with the Unified model, a new model is adopted, which will be referred to as Cascade computing model or simply Cascade model. This model divides the problem into a number of smaller problems, each related to a subset of users. The smaller problems are solved in a sequential fashion: each one includes a group of users along with a “super-user”, i.e., a virtual user that represents the requirements and characteristics of all the users considered before. The objective function aims to minimize the overall cost of energy obtained by considering, for each user and for the super-user, the costs derived from the use of external energy and the revenues obtained by selling the energy produced locally. The last sub-problem involves the last group of users and the super-user that summarizes the solutions derived previously for all the other users. Therefore, the solution of the last problem defines an admissible energy profile for all the users of the community.

The main drawback of the Cascade model is that the solution can be sub-optimal. Indeed, each sub-problem is solved separately, therefore the energy exchanges can only occur among the users of the same group and between them and a single super-user that represents all the users considered before. The optimal solution can be achieved only if all possible energy exchanges are considered in the same problem, which is the case of the Unified model.

A number of experiments were performed to assess the savings in computing time and the possible increase of the energy costs incurred by the community due to the implementation of a sub-optimal solution. We compared the results obtained with the Unified and the Cascade model, for different numbers of users and different proportions between simple consumers and prosumers. For completeness, the comparison is also extended to the Separated model, which refers to the base situation in which no community energy is organized, and the users exchange energy directly with the external grid. Furthermore, the Cascade model was combined with three different strategies for the partitioning of users into groups.

The results confirm that the savings in computing time, achieved with the Cascade model, are big and increase with the size of the community. On the other hand, the increase in the overall cost – due to the non-optimal solution of the optimization problem, as mentioned before – is moderate. This increase becomes nearly negligible when the partitioning of users into groups is performed in such a way that the proportion between consumers and prosumers, within each group, mirrors or approximates the proportion that is observed in the entire community.

Our main contributions in this paper can be summarized as follows:

- we present a novel approach for the efficient management of energy in a PCG, the Cascade model, which is based on the partitioning of users into groups and on the iterative computation of solutions for the different groups, until a global sub-optimal solution is achieved for the entire community;
- we report a set of experiments – for communities containing up to 1024 users – and show that the Cascade model enables a notable reduction of computing time, which becomes linear with respect to the number of users, while it is exponential when all the users are included in the same optimization problem, using the Unified model. This speed up is obtained because each problem involves a reduced number of variables and a limited amount of memory;
- through the experimental results, we also show that the increase in the overall cost, due to the sub-optimal solution, is moderate;
- we compared three different strategies for the composition of groups, and found that the solution is improved – and therefore the increase in the overall cost is minimized – when each group mirrors the proportion between consumers and producers that is observed in the whole community;

The paper is organized as follows: Section 2 discusses the related work in this area, and highlights the benefits deriving from the definition and exploitation of prosumer community groups; Section 3 describes the Cascade architecture, and in particular reports the definition of groups and super-users; Section 4 provides the mathematical details on the optimization problem solved by the Cascade model; Section 5 illustrates the case study used to test the Cascade model; Section 6 reports experimental results in terms of computing time and overall cost; Section 7 concludes the paper, gives information about the limits of the proposed model, and introduces some avenues for future work.

2. Related work

The concept of Prosumer Community Group (PCG) upgrades the existing paradigms of Virtual Power Plant (VPP) and micro-grids, and strengthens the socio-economic aspects of the smart grid [7]. The major difference between a PCG and a VPP or micro-grid is that VPPs and microgrids are usually connected through a peer-to-peer electricity grid whereas the PCG members are interconnected through the main utility grid: they are virtually joined together, based on their energy behaviors, and stimulated to achieve a common goal.

The adoption of a PCG enables an efficient sharing of energy among the members of the community, both consumers and producers, and allows them to reduce the costs when buying energy and increase the revenues when selling energy. One more benefit is that the consumers that are included in a PCG can directly buy the energy from the local community or other communities that are geographically close, which helps avoiding energy interruptions from the main utility grid. This local approach to power distribution minimizes energy losses as the energy travels a shorter distance to reach the consumers while providing a higher selling price for the prosumers [8]. Furthermore, a prosumer community group can be driven to fulfill the energy demand of its own members, even when they are disconnected from the main grid. This encourages local producers to anticipate the demand for energy expressed by the overall community thus eliminating electricity waste and maximizing energy sharing. PCGs are particularly advantageous in remote areas that do not have abundant energy resources and that experience notable costs and difficulties in transporting the amount of energy that is needed to satisfy the energy needs of local users. In such situations, a strong interaction among the prosumers, consumers, and the utility grid will induce individuals to work together in order to manage their electricity usage more efficiently [9]. PCGs can be formed by clustering the prosumers based on the homogeneity of energy sharing behaviors. In this way, disagreements among the members are minimized, leading to more stable groups. Ultimately, this also results in a more sustainable energy sharing in the long term [10].

The authors of [10, 11, 12] cover several topics regarding the management of prosumer communities, including the analysis of prosumers’ energy behavior profiles, the development of a framework to manage multiple goals, and a methodology to form the PCGs. In [13], innovative scalable and privacy-preserving optimization methods are proposed, which allow large-scale energy communities to offer ancillary services through the sharing of residential PV-battery systems. In [14], the authors focus on the issue of optimally exploiting the energy storage systems deployed in a PCG, where the objective is to reduce the energy consumption and the carbon emissions. They formulate an optimization problem that aims to find an emission-aware schedule of distributed energy storage and helps to evaluate how dis-
tributed storage enables utilities to reduce their reliance on inefficient and carbon-intensive power plants, thereby reducing the overall carbon emissions. In [15], the authors observe that the engagement of local energy systems can help to defer new investments in power lines, reducing the system peaks and distributing the load more evenly during a day or a week. The widespread availability of flexible generation and energy storage facilities will enable these communities to provide higher flexibility to the national energy system. In this way, supply and demand will cooperatively optimize system operations while reducing the overall costs, improving the security of supply chains, and ultimately helping to achieve climate policy objectives.

As anticipated in the introductory section, in [5, 6] we presented and evaluated an optimization model, namely the Unified model, which is able to integrate the needs and constraints of all the users of a community. The main features of the Unified model are: (i) the optimal solution minimizes the energy cost of the community as a whole; (ii) the fraction of self-consumption i.e., of the amount of energy that is produced and consumed locally, is maximized; (iii) the peaks of power demand are shaved when possible.

The main limitation of the Unified model is that the execution time needed to solve the optimization problem can become excessive as the number of prosumers, and therefore the number of variables, increase. In MILP problems, some variables are integer-valued and others are real-valued, and both the objective function and the constraints are linear. In these problems, since the integrality constraints on the variables can be expressed as nonlinear functions, the problem itself is nonlinear, despite the linearity of the functions. Two of the most common algorithms employed for solving MILPs are the Cutting Plane and the Branch and Bound, which have worst-case exponential time complexity [16, 17].

In detail, the MILP problem is at least as complex as the Integer Linear Programming (ILP), which, in turn, is at least as complex as the 0-1 integer program. The last problem is convertible to the SAT optimization problem, whose complexity is NP-hard [18]. As a consequence of this chain, a MILP problem is also NP-hard and requires computational times and resources that can become unacceptable as the size of the problem grows.

In light of the advantages that large PCG can offer when managed efficiently, public institutions can profitably operate with strategies aiming to drive the deployment of smart grids: indeed, they can change the economic and regulatory context that orientates the interactions among grids and end-users [19]. In particular, public policies can promote investments in smart-grid technologies and architectures, which are widely recognized as a key factor for the development of smart, reliable, and secure power grid applications [20, 21]. For example, distributed ledger technologies and smart contracts can help to overcome the lack of transparency and trustworthiness in the process of energy distribution and trading [22].

Public institutions can also establish feed-in tariffs, net-metering rules, and other incentives that can stimulate users to create and join energy communities and to participate in demand-response programs. The objective of minimizing carbon emissions can be pursued through incentives for building local renewable energy systems and requirements for green building practices. Policies can also lower the financing costs or offer tax advantages for users and companies that are willing to develop new platforms for delivering energy services or invest in smart grid assets and renewable energy sources [23]. Ultimately, in the next few years, the diffusion of smart energy communities can be fostered effectively by public procurement rules [24].

3. The Cascade model

This section describes the Cascade model, a novel optimization model for the optimal energy management in energy communities composed of a large number of users. Before discussing the model, it is useful to introduce some notation as follows:

- $N$ is the number of users, and a single user is denoted with the integer $u$, $1 \leq u \leq N$;
- $G$ is the number of prosumer community groups, and each group is denoted with the integer $g$, $1 \leq g \leq G$;
- $U_g$ is the set of users that belong to the group $g$;
- $U_g^+$ is the set of users belonging to all the groups $g' : g' \leq g$;
- $\hat{U}$ is the size of the groups, and the last group can have a lower size if $\hat{U}$ is not a divisor of $N$. As an example, a community of $N = 87$ users can be partitioned into $G = 9$ groups, each containing 10 users except the last group that contains 7 users;
- $S_g$ is a super-user that represents all the users included in $U_g^+$.

Each user sets its own energy needs, while a central coordinator supplies the energy tariffs and the energy production forecasts related to the following day as an input to the model. Then, the model finds, for each prosumer, the optimal way of scheduling controllable loads, local power plants, and storage systems. As anticipated in the introductory section and shown in Fig. 1, the users are partitioned into groups in order to simplify the problem and speed up the solution process. The central coordinator communicates with the different groups and takes the responsibility to orchestrate the cascade computation process, discussed below in this section. The solution consists in the optimization of the electrical energy management of the entire energy community: the objective is to schedule the loads, production and storage of a set of electric devices, in order to minimize the cost associated with the supply of electricity.

The model takes into account various types of loads (both schedulable and non-schedulable) and different types of power generation systems (using traditional and renewable sources, programmable and non-programmable). In detail, a schedulable load is a load that can be activated at any time within a user-defined time interval, whereas a non-schedulable load must be activated at a fixed time of the day.

The Cascade model solves $G$ optimization problems in a sequential and iterative fashion. In Fig. 2, we describe the case in which the approach is applied to a community composed of $G$ groups, each with size $\hat{U}$. The central coordinator receives the parameters of the computation, in particular, the requirements of the users and the production data of the local plants, and solves the optimization problem for the different groups. At the first iteration, the problem is solved for the first group, $U_1$. The super-user $S_1$ summarizes the energy exchanges between the group as a whole and the external grid, and $U_1^+$ is set to $U_1$. At the $g$-th iteration, with $g \geq 2$, the model considers the energy exchanges computed in the previous iterations.
In order to summarize the energy exchanges between the group $U_{k-1}^*$ and the external grid, the super-user $S_{k-1}$ is modeled as a virtual user having a fixed production profile and a non-schedulable load profile. More in detail:

- the production profile is given by the sum of the energy quantities exported by the users of the group $U^*_{k-1}$ to the external grid;
- the load profile is given by the sum of the energy quantities imported by the users of the group $U^+_{k-1}$ from the external grid;
- the super-user does not have any own schedulable load, storage system, or controllable plant.

The production and load profiles of $S_{k-1}$ are given as an input for the problem of the $g$-th iteration, along with the characteristics and energy requirements of the users of the set $U^*_g$, i.e., the users belonging to the group $g$. In other words, at the $g$-th iteration of the procedure, the super-user $S_{k-1}$ is added to the users of the set $U^*_g$, and an optimization problem is solved for $\tilde{U} + 1$ users, the $\tilde{U}$ users of $U^*_g$ plus the super-user $S_{k-1}$. The solution of the problem at the $g$-th iteration optimizes the energy exchanges of all these users. Then, the new super-user $S_{g}$ is defined, which is given as an input for the iteration $g+1$. This super-user represents all the users belonging to the set $U^*_g$, where $U^*_g = U^*_{g-1} \cup U^*_g$, and has a production (load) profile that is computed as the sum of the production (load) profiles of all the users of $U^*_g$.

As the Unified model [5], the Cascade model aims to match the users’ energy needs and achieve the minimum global energy cost, obtained by summing the energy costs and revenues of the users that join the community. Differently from the Unified model, however, the Cascade model allows the problem to be solved in a time that is linear with respect to the number of users $N$. Indeed, if $T_N$ is the average time needed to solve the problem with $n$ users, the time to solve the Cascade model is approximately $G \cdot T_{N+1} = N/\tilde{U} \cdot T_{\tilde{U}+1}$ (recalling that the subproblems, with the only exception of the last one, involve $\tilde{U} + 1$ users). If $N$ increases, and $\tilde{U}$ is kept constant, the average time increases linearly with $N$. On the other hand, if all the $N$ users are considered in the same problem, as happens with the Unified model, the computing time $T_N$ increases exponentially with $N$, since the MILP problem is known to be NP-hard [25]. This advantage in complexity, however, comes with a possible drawback: the Cascade model does not lead to the optimal solution, but to a sub-optimal one, since it does not considers all the user parameters, but exploits a super-user to summarize the profiles of a number of users.

4. Mathematical model of the prosumer problem

This section describes the mathematical model that solves the prosumer problem. We first report the sets, variables and constants that are used in the optimization model, then the objective function and, finally, the constraints that need to be matched and the upper and lower bounds of variables.

4.1. Sets, variables and constants

At a generic iteration, the model considers a set of $\tilde{U} + 1$ users, i.e., the users of $U^*$ and the super-user $S_{k-1}$. In the following, the subscript $u$ is used to represent either a single user or the super-user.

- Sets
  - $H$ set of the hours of a day
  - $A_u$ set of schedulable loads of user $u$
  - $B_u$ set of non-schedulable loads of user $u$

- Variables
  - $E_{\text{imp}, u}$ energy imported from the community at hour $h \in H$ [kWh]
  - $E_{\text{exp}, u}$ energy exported to the community at hour $h \in H$ [kWh]
  - $E_{\text{imp}, g}$ energy imported from the grid at hour $h \in H$ [kWh]
  - $E_{\text{exp}, g}$ energy exported to the grid at hour $h \in H$ [kWh]
  - $\gamma_h^{\text{sta}, a}$ state of the schedulable load $a \in A_u$ at hour $h \in H$ [1 = on; 0 = off]
  - $E_{\text{store}, u}$ energy stored in the energy storage system during hour $h \in H$ [kWh]
  - $E_{\text{dis}, u}$ energy drawn from the energy storage system during hour $h \in H$ [kWh]

- Constants
  - $c_h$ cost to import a kWh from the community at hour $h \in H$ [€]
  - $p_h$ price to export a kWh to the community at hour $h \in H$ [€]
  - $P_{\text{U/N}, h}$ cost to import a kWh from the external grid at hour $h \in H$ [€]
  - $P_{Z, h}$ price to export a kWh to the external grid at hour $h \in H$ [€]
  - $a_h^n, b_h^n$ start and end time range for scheduling the load $a \in A$ [h]
  - $s_u^{\text{cha}}$ working time of the schedulable load $a \in A$ [h]
  - $s_u^{\text{dis}}$ rated power of the load $a \in A$ [kW]
  - $s_{b, h}^{\text{cha}}$ consumption forecast for non-schedulable load $b \in B$ at hour $h \in H$ [kWh]
  - $E_{\text{PV}}$ production forecast for PV plant at hour $h \in H$ [kWh]
  - $E_{\text{max, Grid}}$ maximum hourly energy imported from the grid [kWh]
  - $\eta_{\text{cha}}, \eta_{\text{dis}}$ charging and discharging efficiency factors of the storage system of user $u$
$SOC_{\text{max}}^u$ maximum percentage of the state of charge of the storage system
$SOC_{\text{min}}^u$ minimum percentage of the state of charge of the storage system
$E_{\text{max,Cha}}^h$ maximum hourly charging amounts of energy of the storage system [kWh]
$E_{\text{max,Dis}}^h$ maximum hourly discharging amounts of energy of the storage system [kWh]
$C_u$ maximum capacity of the storage system [kWh]
$E^h_{\text{STO}}$ residual energy of the day before stored in the storage system [kWh]

4.2. Objective function

The objective function of the optimization model at the $g$-th iteration aims to minimize the global energy cost obtained by summing the energy costs (positive values) and the revenues (negative values) of the users of $U^*_g$:

\[
\min \sum_{u \in U^*_g} \sum_{h \in H} \left( \chi^u \cdot E_{\text{imp}}^h + p^h \cdot E_{\text{exp}}^h + PU \cdot N^h \cdot E_{\text{imp,G}}^h \right) - PZ^h \cdot E_{\text{exp,G}}^h \]

The expression in (1) is defined as the sum of four terms for each user $u \in U^*_g$:
- $\chi^u \cdot E_{\text{imp}}^h$: the cost incurred by user $u$ at hour $h$ to import energy from the community;
- $p^h \cdot E_{\text{exp}}^h$: the revenue obtained by user $u$ at hour $h$ by selling energy to the community;
- $PU \cdot N^h \cdot E_{\text{imp,G}}^h$: the cost incurred by user $u$ at hour $h$ to import energy from the external grid;
- $PZ^h \cdot E_{\text{exp,G}}^h$: the revenue obtained by user $u$ at hour $h$ by selling energy to the external grid.

4.3. Constraints

The model includes a set of constraints. In detail, equation (2) expresses the balance of energy for each user $u$ and at each hour $h$:

\[
E_{\text{imp}}^u \cdot E_{\text{imp,G}}^h - E_{\text{exp}}^u \cdot E_{\text{exp,G}}^h + \eta_{\text{dis}}^g \cdot E_{\text{dis}}^u \cdot \frac{1}{\eta_{\text{cha}}^h} - \sum_{a \in A_u} y_{a}^u \cdot E_{\text{cha}}^h = 0 \quad \forall h \in H, \forall u \in U^*_g
\]

The balance of energy considers the following components: the energy imported from the other prosumers ($E_{\text{imp}}^a$), the energy imported from the grid ($E_{\text{imp,G}}^h$), the energy supplied by $u$ to the community ($E_{\text{exp}}^u$), the energy injected to the grid ($E_{\text{exp,G}}^h$), the energy supplied by the storage system of $u$ ($\eta_{\text{dis}}^g \cdot E_{\text{dis}}^u$, where $\eta_{\text{dis}}^g$ is the discharging efficiency factor), the energy charged in the storage system of $u$ ($\frac{1}{\eta_{\text{cha}}^h} \cdot E_{\text{cha}}^h$), the sum of the products of the variables $y_{a}^u$ and the rated powers ($E_{\text{cha}}^h$), expressed as energy/hour, can be considered as an amount of energy since the time interval is equal to one hour) $E_{\text{cha}}^h$, the sum of the forecast energy quantities consumed by non-schedulable loads $b \in B_u$ ($y_{b}^u \cdot E_{\text{cha}}^h$) and the forecast energy produced by the local photovoltaic (PV) generators ($E_{\text{PV}}^h$).

Equation (3) balances the energy exchanges within the community:

\[
\sum_{a \in A_u} (E_{\text{imp}}^a - E_{\text{exp}}^a) = 0 \quad \forall h \in H
\]

Equations (4) and (5) force the activation of load $a$ to occur at a single hour $h \in [\alpha_a^u, \beta_a^u - \theta_a^u + 1]$ and ensure that the working time of the load $a$ ends before $\beta_a^u$. The auxiliary variable $z_{a}^h$ is set to 1 when the schedulable load $a$ is activated, and to 0 when the load is not activated.

\[
\sum_{a \in A_u} z_{a}^h = 1 \quad \forall u \in U^*_g, \forall a \in A_u
\]

\[
z_{a}^h = 0 \quad \forall h \in H \setminus [\alpha_a^u, \beta_a^u - \theta_a^u + 1] \quad \forall u \in U^*_g, \forall a \in A_u
\]

Equation (6) ensures that the load $a$ of user $u$ is activated exactly for $\theta_a^u$ hours inside the $[\alpha_a^u, \beta_a^u]$ interval.

\[
\sum_{h=\alpha_a^u}^{\beta_a^u-\theta_a^u} z_{a}^h = \theta_a^u \quad \forall u \in U^*_g, \forall a \in A_u
\]

Inequalities (7) force every uninterruptible load $a \in A_u$ to operate during its working time, without interruptions.

\[
y_{a}^h \cdot \theta_u^h \cdot \theta_a^u \cdot \theta_u^h \leq y_{a}^h \cdot \theta_u^h \cdot \theta_a^u \quad \forall h \in [\alpha_a^u, \beta_a^u], \forall u \in U^*_g, \forall a \in A_u
\]

Inequalities (8) and (9) express the constraint that the stored energy at hour $h$ is within the allowed minimum and maximum values.

\[
E_{\text{STO}}^u + \sum_{h=1}^{h} E_{\text{cha}}^h - \sum_{h=0}^{h} E_{\text{dis}}^h \leq SOC_{\text{max}}^u \cdot C_{\text{max}}^u \quad \forall h \in H, \forall u \in U^*_g
\]

\[
E_{\text{STO}}^u + \sum_{h=1}^{h} E_{\text{cha}}^h - \sum_{h=0}^{h} E_{\text{dis}}^h \geq SOC_{\text{min}}^u \cdot C_{\text{max}}^u \quad \forall h \in H, \forall u \in U^*_g
\]

4.4. Upper and lower bounds of variables

Inequalities (10) and (11) force the hourly energy exported by user $u$ to the community ($E_{\text{exp}}^u$) and to the grid ($E_{\text{exp,G}}^h$) to be positive values.

\[
0 \leq E_{\text{exp}}^u \quad \forall h \in H, \forall u \in U^*_g
\]

\[
0 \leq E_{\text{exp,G}}^h \quad \forall h \in H, \forall u \in U^*_g
\]

Inequalities (12) and (13) force the hourly energy imported by user $u$ from the community ($E_{\text{imp}}^u$) and from the grid ($E_{\text{imp,G}}^h$) to be positive values, lower or equal than the maximum operation power ($E_{\text{max,Grid}}$).

\[
0 \leq E_{\text{imp}}^u \leq E_{\text{max,Grid}} \quad \forall h \in H, \forall u \in U^*_g
\]

\[
0 \leq E_{\text{imp,G}}^h \leq E_{\text{max,Grid}} \quad \forall h \in H, \forall u \in U^*_g
\]

The inequalities (14) and (15) define the maximum amounts of energy drawn and stored by the storage systems:

\[
0 \leq E_{\text{cha}}^h \leq E_{\text{max,Cha}} \quad \forall h \in H, \forall u \in U^*_g
\]

\[
0 \leq E_{\text{dis}}^h \leq E_{\text{max,Dis}} \quad \forall h \in H, \forall u \in U^*_g
\]

5. Case study

In order to evaluate the performance of the Cascade model, a large number of simulation experiments have been carried out, with up to 1024 users. A percentage of these users are simple consumers, while the remaining ones are real prosumers, i.e., both consumers and producers, equipped with local plants powered by renewable energy sources. The parameters of each user have been extracted from probability distributions, so as to differentiate them and achieve results that are not dependent from a specific configuration of the prosumers. The extremes of these distributions have been taken from a set of real users at the University of Calabria campus, in Italy.

In detail, the parameter values of each prosumer are set as follows, using uniform probability distributions:
the number of schedulable loads is comprised between 0 and 4; each schedulable load has a rated power comprised between 0.5 kW and 2.5 kW; the working time of each schedulable load varies from 1 to 5 hours, and the load is scheduled casually within the 24 hours; the schedulable loads are interruptible with a 50% probability;

- each user has a non-schedulable load profile with a power that can vary, during the day, from 0.1 kW to 0.3 kW;
- the maximum operation power is set to 3 kW, 4.5 kW, or 6 kW;
- the installed power of PV plants can vary between 2 kW and 6 kW and must be a multiple of 0.5 kW;
- a prosumer is equipped, with a 50% probability, with a storage system whose capacity value is twice the installed PV power.

The electrical energy tariffs are defined by the energy market. As an example, Fig. 3 reports the prices and costs of the energy in a specific day, that is, February 14, 2020, in Italy. The figure shows the trends of the wholesaler energy prices, \( P_{Z^h} \) and \( P_{UN^h} \), and the prices applied within the community, i.e., \( c^h \) and \( p^h \). The last two prices are set to zero in order to encourage the energy sharing among the prosumers of the community.

6. Results and discussion

This section reports the results obtained by solving the optimization problem with the Cascade model described in Section 3. We mentioned that the Cascade model can provide significant advantages in terms of the computing time, possibly at the expense of achieving a suboptimal value for the objective function, i.e., the overall cost. Therefore, time and cost are the two indices discussed in this section. The experiments consider two different cases regarding the proportion between prosumers and simple consumers: in the first case, 25% of the users are prosumers and 75% consumers, while, in the second case, there are 50% prosumers and 50% consumers.

To offer a deep insight into the behavior of the Cascade model, we used three different approaches for the composition of user groups. With the sequential approach, the groups are built with users of the same kind, all prosumers or all consumers. Specifically, the first groups are composed of prosumers, and when all the prosumers have been considered, the remaining groups are composed of consumers. With the alternate approach, the groups are composed of, alternatively, all prosumers and all consumers. Finally, with the mixed approach, each group contains both prosumers and consumers, in the same proportion as the whole set of users.

The results obtained with these three approaches are compared with each other, in order to see if the way the groups are composed has an impact on the performance, and how big is the impact. The results are also compared to those achieved with the Unified model, in which all the users are processed at once in the optimization problem, and with the Separated model, in which no energy community is built and a separate problem is solved for each user. We varied the overall number of users, up to 1024, always maintaining the same proportion between prosumers and simple consumers, and we varied also the size of groups. Actually, when using the Unified model, we had to stop to 256 users: with more users the computing time continues to grow exponentially, as discussed at the end of Section 3, and the memory is filled up. This marks a clear advantage of the Cascade model, since it allows the prosumer problem to be solved even for large communities for which the problem is unmanageable with the Unified model.

We first focus on the results obtained with the 75%-25% proportion between consumers and prosumers, which is the one actually observed in the University of Calabria campus. Afterwards, we will see what happens when the percentage of prosumers increases to 50%, a proportion that is expected to occur more likely in the future, as more and more users are equipped with local plants.

Figs. 4 and 5 report, respectively, the computing time and the value of the overall cost, when using the Cascade model with the mixed approach, the Unified model and the Separated model. The number of users ranges from 4 to 1024 and the size of the groups, in the Cascade model, is set to four different values: 4, 8, 16 and 32. Fig. 4 shows that the computing time has a nearly exponential trend with the Unified model, while the trend is approximately linear, as expected, with the Cascade model and the Separated model. It is also observed that small groups are preferable, which is consistent with the considerations on complexity made in Section 3. As reported in Fig. 5, the remarkable saving in computing time is not paid in terms of the overall cost: the values of this index obtained with the Cascade and the Unified model
Fig. 6. Comparison, in log scale, among the three models in the time/cost plane. The notation \((N; C)\) is used for the Cascade model, mixed approach, and specifies that the community has \(N\) users and the size of groups is set to \(C\). The notation \((N)\) specifies the number of users when adopting the Unified and the Separated model.

Fig. 7. Computing time: comparison among Cascade (with sequential approach), Unified and Separated models.

are comparable. On the other hand, the Separated model shows a small advantage in terms of the computing time (see Fig. 4), but at the expense of much larger values of the overall cost (see Fig. 5), which is a sign that the solution of the optimization problem is far from being optimal when using this model. When restricting our attention to the Cascade model, we see that no remarkable difference, in terms of the overall cost, is observed when varying the size of the groups.

Fig. 6 gives a different perspective on the results of the same experiments, with the aim to highlight a possible Pareto frontier, i.e., individuate the values of the group size that are preferable both in terms of computing time and cost. The figure shows that, with a given number of users, the Cascade model with group size equal to 4 appears to be the best choice: larger values of the group size lead to a significantly longer computing time, while the overall cost is nearly independent from the group size.

As mentioned before, we tested two more approaches for the partitioning of users into groups when adopting the Cascade model: the sequential and alternate approaches. In Figs. 7 and 8 we report, respectively, the computing time and the overall cost experienced with the sequential approach, along with the results obtained with the Unified and the Separated model. When comparing these figures with those related to the mixed approach, i.e., Figs. 4 and 5, we can notice that: (i) no remarkable difference is observable in terms of the computing time; (ii) the overall cost is higher with the sequential approach than with the mixed approach, which leads to a small gap between the Cascade and the Unified model, as seen in Fig. 8 (to notice this gap, we recall that the results related to the Unified model have been obtained with up to 256 users). A similar analysis of the alternate approach led to the conclusion that the mixed approach is preferable: the composition of groups is more effective when it mirrors the same proportion of consumers and producers as the one observed in the whole energy community.

Figs. 9 and 10 report a direct comparison among the three approaches: sequential, alternate and mixed, with the group size set to 16. Fig. 9 confirms that the computing time is hardly affected by the choice of the approach, whereas Fig. 10 marks an advantage in terms of the overall cost when using the mixed approach. Similar considerations can be made when choosing other values for the size of user groups.

Figs. 11–13 report the results achieved when setting the proportion of consumers and prosumers to 50% and 50%. Following the considerations made above, we chose the mixed approach when adopting the Cascade model. When comparing Fig. 11 with Fig. 4, we observe no remarkable difference in terms of the computing time: with both proportions of consumers and producers, the times experienced with the Cascade model are considerably lower than those needed with the Unified model. Conversely, the comparison between Figs. 5 and 12 shows that the increased availability of prosumers enables a remarkable reduction of overall costs, since a larger amount of energy is produced locally and does not need to be retrieved from the external grid. Moreover, with the 50%-50% proportion, the Cascade model increases its advantage with respect to the Separated model in terms of the energy costs. Finally, Fig. 13 confirms the same achievements discussed with reference to the 75%-25% proportion (see Fig. 6), i.e., small group sizes.
6. Conclusions and future work

This paper has presented and assessed the Cascade computing model, a novel strategy for the efficient management of energy communities, based on the definition of MILP optimization problems. The model aims to maximize the energy sharing among the members of a community, and consequently reduce the use of external, and more expensive, energy sources, while tackling the main issue experienced by most methods based on optimization problems, i.e., the remarkable increase of computational complexity when the size of the community increases.

To assess the performance of the Cascade model, we run a set of experiments with up to 1024 users, some of which are simple consumers and others are prosumers equipped with local plants. Furthermore, we adopted three different approaches for the composition of user groups, which differ for the way in which consumers and prosumers are combined in the same group.

Experimental results have confirmed that the Cascade model enables significant savings in terms of the computing time both with respect to the Unified model, which processes all the users in a unique resolution stage, and the Separated model, which does not exploit the benefits of energy communities. A possible limitation of the Cascade model is that the solution is sub-optimal, and the overall cost can be higher than the one obtained with the Unified model. The experiments have shown that this drawback becomes negligible when the groups are composed so that, in each group, the proportion of prosumers and consumers mirrors the one that is observed in the whole community. However, this penalty can become significant when this type of group composition cannot be applied, because the groups are formed in accordance with other criteria, for example the physical location of users or their belonging to other types of communities.

Some possible avenues for future work are: (i) the investigation of the issue mentioned before, i.e., the performance of the Cascade model when the groups are determined by external criteria; (ii) the design of a different model that, instead of solving the prosumer problems in a sequential fashion, solves them in parallel, in order to reduce the computing time further. In this case, the architecture can profitably

are preferable since they enable a reduction of the computing time, while the costs are hardly affected by the group size.

As a final comment, the results reported in this section confirm that: (i) Prosumer Community Groups contribute to a more efficient use and sharing of local energy, which is testified by the comparison between the Separated model and the other two models; (ii) the Cascade model speeds up the process, since the computing time becomes linear with respect to the size of the community, while no significant penalty is paid in terms of the energy cost; (iii) when using the Cascade model, the partitioning of the users into small groups is the best choice, and the composition of groups should respect the proportion between producers and consumers that is observed in the whole community. Furthermore, results suggest that the benefits of the Cascade model are kept or even increase when the percentage of producers increases, which is the expected trend in the coming years.
exploit the edge computing paradigm, since the solution of the problem regarding a group can be committed to the group itself, in particular to one of the prosumers of the group.

**Declarations**

**Author contribution statement**

Carlo Mastroianni, Andrea Giordano: Conceived and designed the experiments; Analyzed and interpreted the data; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Luigi Scarcello: Conceived and designed the experiments; Performed the experiments; Contributed reagents, materials, analysis tools or data; Wrote the paper.

Giandomenico Spezzano: Conceived and designed the experiments; Analyzed and interpreted the data; Wrote the paper.

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**Data availability statement**

Data will be made available on request.

**Declaration of interests statement**

The authors declare no conflict of interest.

**Additional information**

No additional information is available for this paper.

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