On the macroscopic phenomena of plastic flow localization and solids microscopic characteristics

S A Barannikova1,2,3, Yu V Li1, A M Zharmukhambetova2 and L B. Zuev1,2

1Institute of Strength Physics and Materials Science SB RAS, 10/4 Akademichesky Ave., Tomsk, 634055, Russia
2National Research Tomsk State University, 36 Lenin Ave., Tomsk, 634055, Russia
3Tomsk State University of Architecture and Building, 2 Solyanaya Sq., Tomsk, 634003, Russia

E-mail: bsa@ispms.tsc.ru

Abstract. In the paper to be submitted the notions of plastic flow localization are outlined. It is shown that a particular kind of localized plasticity pattern corresponds to a given stage of deformation hardening. In the course of plastic flow development a changeover in the types of localization patterns occurs. It is found that the emergent patterns are manifestations of the autowave nature of plastic flow localization process, each pattern corresponding to a definite type of autowave. The most intriguing localization pattern corresponds to a phase autowave which forms at the stage of linear work hardening. The following characteristics of phase autowave have been determined experimentally: propagation velocity and dispersion dependence of wavelength. Moreover, elastic-plastic strain relation is introduced which relates the elastic and plastic properties of the deforming medium. It is shown that the characteristics of autowaves follow from this relation.

1. Introduction

The earlier obtained experimental data [1-4] assume that plastic deformation is prone to localization while the solid undergoes deformation caused by the flow waves. Typically, plastic flow localization manifests itself at the macro-scale level at which the type of local strain pattern is defined by the work hardening law that is valid for a given flow stage, i.e. \( \theta \equiv E \frac{d \sigma}{d \varepsilon} = \theta(\varepsilon) \) (here \( E \) is the elastic modulus). In this case, the plastic deformation is implied to cause the auto-wave, or self-excited, process [5]. The generation of auto-waves by the plastic deformation is studied from the point of view of the gradient plasticity theory [6-11].

A large amount of experimental and theoretical results on plastic flow macro-localization was gathered heretofore [1-4], assuming that the macro-scale heterogeneities of localized plastic flow are typically about \( 10^{-2} \) m. The intrinsic picture in a sample subjected to deformation reveals the concerted movement of strained macrodomains, inducing localized plastic flow auto-waves with wavelengths \( \lambda \) close to \( 10^{-2} \) m. This allows spontaneous separation of strained body into alternating deformed and undeformed macrodomains (figure 1). According to H. Hacken [12], the unprompted appearance of plastic flow heterogeneities has to be considered in the context of self-organization processes that take place in the deforming metals.
2. Experiments and samples

The quantitative parameters of auto-wave processes were found for a variety of pure metals and alloys with single-crystal and polycrystalline structures, face-centered cubic (FCC), bulk-centered cubic (BCC) and hexagonal close packed (HCP) crystal lattices, using the available experimental data [1-4]. The mechanical properties of samples and the plastic flow curves acquired for the test samples are expected to undergo noticeable variations, depending on grain size of polycrystalline materials, chemical composition and extension axis orientation of single crystals. Below the peculiarities of all samples are discussed.

The localized plastic flow auto-waves [1-4] were experimentally studied by speckle photography [13]. This method was purposely elaborated to expedite the evaluation of displacement vector fields and the determination of plastic distortion tensor components for strained metals. Localized plastic flow macrodomains can be displayed via the spatial distributions of plastic distortion tensor components, and the kinetics of motion of macrodomains can be evaluated from their temporal evolution.

![Image](image_url)

Figure 1. Auto-wave of plastic deformation localization, spreading upon the linear work hardening stage in a tensile single crystal of alloyed Fe.

3. Experimental results

The plastic flow will generally exhibit a regular localization behavior, which is markedly pronounced at the linear work hardening stage. For $\sigma = \varepsilon$ and $\theta = \text{const}$, the localization macrodomains will move in a concerted manner at a constant rate along the tensile sample, forming a phase autowave (figure 1). The experimental data on propagation rate, dispersion and material structure response obtained for the phase autowaves are demonstrated in figure 2.

(i): The propagation rates of localized plasticity autowaves in all studied materials are in the range $10^{-5} \leq V_{aw} \leq 10^{-4}$; they depend solely upon the work hardening coefficient, $\theta_{II}$, and are given as (figure 2a)

$$V_{aw} (\theta_{II}) = V_0 + \Xi/\theta_{II} - \theta_{II}^2,$$  \hspace{1cm} (1)

where $V_0$ and $\Xi$ are empirical constants.

(ii): We have also obtained dispersion relation, $\omega(k)$ (here $\omega = 2\pi/T$ is frequency and $k = 2\pi/\lambda$ is wave number) for localized plasticity autowaves [14]. This relation has quadratic form (figure 2b)

$$\omega(k) = \omega_0 + \alpha \cdot \left( k - k_0 \right)^2.$$  \hspace{1cm} (2)

By substituting into (2) the terms $\omega = \omega_0 \cdot \tilde{\omega}$ and $k = k_0 + \frac{\tilde{k}}{\sqrt{\alpha/\omega_0}}$ (here $\tilde{\omega}$ and $\tilde{k}$ are dimensionless frequency and wave number, correspondingly), it is reduced to the canonical form $\tilde{\omega} = 1 + \tilde{k}^2$. 

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The above dispersion quadratic relation satisfies the Schrödinger nonlinear equation [15, 16] that is widely explored for the description of self-organization processes arising in active nonlinear media. It is thus proved that plastic flow localization is a phenomenon making up a part of self-organization of the medium under deformation.

Figure 2. Characteristics of localized plastic flow autowaves (a) autowave rate as a function of work hardening coefficient for all studied materials; (b) dispersion observed for γ-Fe single crystals and polycrystalline Al; representation in functional co-ordinates \( (\omega - \omega^*)/(k - k^*) - k \) (here \( \omega^* \) and \( k^* \) are arbitrary values of \( \omega(k) \) dependence); (c) autowave length as a function of grain size plotted for polycrystalline Al; representation in functional co-ordinates \( \ln[\lambda_0/(\lambda - 1)] - \delta \).

(iii): The grain size dependence of autowave length illustrated in figure 2c has the form of logistic curve

\[
\lambda(\delta) = \lambda_0 + \frac{a_1/a_2}{1 + C \cdot \exp(-a_1 \delta)}, \quad (3)
\]

where \( a_1 \) and \( a_2 \) are empirical coefficients, \( \lambda_0=5 \) mm and \( C=2.25 \). The inflection point of (3) is found from the condition \( d^2\lambda/d\delta^2 = 0 \); this corresponds to the boundary value of grain size \( \delta = \delta_0=0.15…0.2 \) mm. The dependence \( \lambda(\delta) \) has two limiting cases, i.e. \( \lambda \sim \exp(\delta/\delta_0) \) for \( \delta<\delta_0 \) and \( \lambda \sim \exp(\delta/\delta_0) \) for \( \delta>\delta_0 \). The quantity \( \lambda \) generally depends only weakly on the structural characteristics of the deforming medium. Thus variation in the grain size of polycrystalline Al from 5 \( \mu \)m to 5 mm corresponds to a 2.5-fold increase in the value \( \lambda \).

Thus, the most significant features of localized plastic flow at the linear stage of work hardening are specified herein by (1) through (3). By way of summing up our findings, we contend that in the course of plastic flow a large-scale deformation structure would form. Its elements are characterized by the nontrivial dependence \( V_{aw}(\theta) \), the quadratic dispersion law \( \omega = 1 + k^2 \) and the logistic dependence of autowave length on material structure, \( \lambda(\delta) \).
4. Discussion of results

We suggest a link between plastic flow macro-parameters and crystal lattice characteristics. For this purpose, two products are matched, i.e. $\lambda \cdot V_{aw}$ and $\chi \cdot V_{aw}$, which characterize plastic flow and elastic deformation, respectively. The quantities $\chi$ and $V_{t}$ are interplanar spacing of crystal lattice and transverse ultrasound wave velocity, respectively. Numerical analysis was performed using experimentally obtained values $\lambda$ and $V_{aw}$ as well as hand-book values $\chi$ and $V_{t}$. The data listed in table 1 allow one to write the equality

$$\frac{\lambda \cdot V_{aw}}{\chi \cdot V_{t}} = const = \alpha.$$  \hspace{1cm} (4)

The (4) holds true for all studied materials. The averaging of the value $\alpha$ was performed for seventeen materials to give $\langle \alpha \rangle = 0.68\pm0.25=2/3<1$. This result constitutes both formal and physical proofs that the elastic and plastic processes involved in the deformation are closely related. Therefore, (4) has been labeled as ‘elastic-plastic strain relation’.

| Metals | Mg | Al | Ti | V | γ-Fe | α-Fe | Ni | Cu | Zn | Zr | Nb | In | Sn | Pb |
|--------|----|----|----|---|-------|------|----|----|----|----|----|----|----|----|
| $2\lambda \cdot V_{aw}$ | 19.8 | 14.6 | 7.0 | 5.6 | 5.1 | 4.5 | 4.2 | 7.2 | 7.4 | 3.8 | 3.6 | 5.2 | 6.8 | 6.4 |
| $\chi \cdot V_{t}$ | 16.0 | 7.5 | 6.6 | 6.1 | 6.9 | 6.7 | 6.5 | 4.8 | 5.2 | 5.5 | 5.2 | 2.2 | 5.3 | 2.0 |

Table 1 Data for verification of Eqn. (4) for the “elastic-plastic strain relation”

Apparently, $V_{t}=\chi \omega_{D}$ (here $\omega_{D}$ is the Debye frequency); hence, we write

$$\lambda V_{aw} \approx \alpha \frac{V_{t}^{2}}{\omega_{D}} = \alpha \frac{G}{\omega_{D} \cdot \rho} = \alpha \frac{\partial^{2}W/\partial v^{2}}{(\omega_{D} \chi) \rho} = \alpha \frac{\partial^{2}W/\partial v^{2}}{\xi_{1}}.$$  \hspace{1cm} (5)

where $v<\chi$ is atomic displacement near interparticle potential minimum ($W$); the elastic modulus is expressed in terms of interparticle potential as $G = (\partial^{2}W/\partial v^{2})/\chi$ [17] and the value $\xi_{1} = (\omega_{D} \chi) \rho = V_{t} \rho$ is specific acoustic resistance of the medium, which is shown to be related to crystal lattice perturbation due to dislocation motion. The interparticle potential from (5) can be written as

$$W(v) = \frac{1}{2} (\partial^{2}W/\partial v^{2}) \cdot v^{2} + \frac{1}{6} (\partial^{3}W/\partial v^{3}) \cdot v^{3} = \frac{1}{2} f \cdot v^{2} - \frac{1}{3} g \cdot v^{3},$$  \hspace{1cm} (6)

where $f$ is the coefficient of quasi-elastic coupling and $g$ is anharmonicity coefficient. With the proviso that $\frac{1}{2} f \cdot v^{2} >> \left| \frac{1}{3} g v^{3} \right|$, (5) assumes the form

$$\lambda V_{aw} \approx \frac{f}{\xi_{1}} = \frac{f}{V_{t} \rho} = Z,$$  \hspace{1cm} (7)

where $Z$ is taken to be a criterion, which serves to relate elastic and plastic deformation components.

5. Conclusion

It is shown that the plastic flow in solids will exhibit a macro-localization behavior over the entire plastic flow. The types of localization patterns are limited in number, with each pattern type determined by the acting law of work hardening.
An analysis of the plastic flow suggests that the deformation process is distinguished by regular features which are manifested in all deforming solids. The kinetics of plastic flow is determined by a regular changeover in the localization patterns (types of autowaves).

Elastic-plastic strain relation is introduced to relate the processes involved in plastic and elastic deformation. It is shown that the main laws of autowave plastic deformation are corollaries of this relation.

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