On MOND, extended gravity and non-geodesic motion

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Abstract

Starting from the origin of Einstein general relativity (GR) the request of Mach on the theory’s structure has been the core of the foundational debate. That problem is strictly connected with the nature of the mass-energy equivalence. It is well known that this is exactly the key point that Einstein used to realize a metric theory of gravitation having an unequalled beauty and elegance. On the other hand, the current requirements of particle physics and the open questions within extended gravity theories request a better understanding of Equivalence Principle (EP). The MOND theory by Milgrom proposes a modify of Newtonian dynamics and a variation of the ratio $m_i/m_g$, to be tested, at least, within the solar system. In this paper we attack this important issue from the general point of view of a weak modification of GR which considers a direct coupling between the Ricci curvature scalar and the matter Lagrangian. It is shown that a non geodesic ratio $m_i/m_g$ can be fixed and that Milgrom acceleration is retrieved at low energies.

1 Introduction

It is well known that The Science of Mechanics by Ernst Mach had a strong influence on Einstein and played an important role in the development of GR
In Newton’s Philosophiae Naturalis Principia Mathematica, acceleration is considered as absolute. In the famous Gedankenexperiment of the rotating bucket filled with water, Newton deduced the existence of an absolute rotation by observing the curved surfaces on the water.

The aim of Newton was to explain the inertia through a sort of resistance to motion in the absolute space which, in this way, comes to be an agent and not a mere physical theater of coordinates, although unspecified. The first thinker to question the Newtonian reasoning was the philosopher George Berkeley in his De Motu, published in 1721 and he can be considered the precursor of Mach. Indeed, after more than 150 years, Mach proposed a radical criticism of Newton’s absolute space and he concluded that the inertia would be an interaction that requires other bodies to manifest itself, so that it would make no sense in a Universe consisting of just a single mass. According to Mach, there is a total relational symmetry and every motion, uniform or accelerated, makes sense only in reference to other bodies. Therefore, following the so called “Mach Principle”, the inertia of a body is not an intrinsic property and depends on the mass distribution in the rest of the Universe. Einstein was very fascinated by Mach reasoning but it is widely acknowledged that Mach Principle is not fully incorporated into relativistic field equations. The challenge of a Machian physics was accepted several times (though less than expected) in the context of both classical and quantum. Here we recall the Narlikar theory with variable mass derived from Wheeler-Feynman-like action at a distance theory. The Sciama’s theory requires to get the inertia as "gravitational closeness" (and the perfect equivalence) under the precise cosmological condition $G\rho \frac{r^2}{c^2} = 1$ where $r$ is the radius of the universe, $\rho$ the density, $c$ is the speed of light and $G$ the gravitational constant. In quantum contexts, and in Higgs times, the problem becomes more complex. In this paper we take into consideration the Milgrom’s MOND theory although we do not think it as the "final solution" to the dark matter problem. Here the Milgrom’s MOND Theory will be examined in relation to the foundational status of the EP in GR.

2 The Physics Under the Metric

Einstein has often stated that some Machian effects are present in GR. In particular, in the famous Lectures of 1921 he states that it showed the following effects:

1) "The inertia of a body must increase when ponderable masses are piled up in its neighbourhood".
2) "A body must experience an accelerating force when neighboring masses are accelerated and the force must be in the same direction as that acceleration".
3) "A rotating hollow body must generate inside of itself a Coriolis field which deflects moving bodies in the sense of the rotation and a radial centrifugal field as well".

Let us follow the Einstein reasoning. By considering the geodesic equation
\[
\frac{d^2 x_\mu}{ds^2} + \Gamma_{\alpha\beta}^\mu \frac{dx_\alpha}{ds} \frac{dx_\beta}{ds} = 0,
\]
Einstein worked in the weak-field approximation and the metric he found to represent the gravitational field due to a distribution of small masses corresponding to a density \( \sigma \) and having small velocities, \( \frac{dx_\alpha}{ds} \), can be written as

\[
g_{00} = 1 - \frac{2G \sigma}{c^2} \int \frac{\sigma dV}{r},
\]
\[
g_{0i} = \frac{4G \sigma}{c^2} \int \frac{dx_i \sigma dV}{r},
\]
\[
g_{ij} = -\delta_{ij} \left( 1 + \frac{2G \sigma}{c^2} \int \frac{\sigma dV}{r} \right)
\]
The equation of motion in this field becomes

\[
\frac{d}{dt} \left[ (1 + \sigma) v \right] = \nabla \sigma + \frac{\partial A}{\partial x^0} + (\nabla \times A) \times v
\]
where

\[
\sigma = \frac{G \sigma}{c^2} \int \frac{\sigma dV}{r}
\]
\[
\frac{4G \sigma}{c^2} \int \frac{\sigma dV}{r}
\]
Einstein interpreted it by saying that the inertial mass is proportional to \( 1 + \sigma \) and therefore increases when ponderable masses approach the test body

\[
m_i = m_g \left( 1 + \frac{G \sigma}{c^2} \int \frac{\sigma dV}{r} \right)
\]
Many physicists believe, according to C. Brans [12], that only the second and third effect are contained in GR. At first glance it seems that, if the Einstein interpretation is right, the EP is violated but we emphasize that all bodies with different inertial masses are still falling with the same acceleration in a gravitational field. In [13] the author analyzes what he calls Modified Mach Principle in the context of an expanding universe. He suggests the following definitions for the inertial mass within and beyond the bulge of galaxies as

\[
m_i = C \quad r \leq R_0
\]
\[
m_i = \frac{C'}{r} = m_g \frac{R_0}{r} \quad r > R_0
\]
where \( C \) and \( C' \) are constants and he calls the first one as inertial mass versus gravitational interaction within the bulge, and the second one as inertial mass versus cosmological expansion beyond the bulge. It would seem that the introduction of a genuine Mach’s principle implies a re-introduction of the distinction between inertial mass and gravitational mass, hidden under the metric of GR and the strong form of the EP. Let recall that the equivalence between \( m_i \) and \( m_g \) is the axiomatic and constructive keystone of GR. This raises the problem of the interpretation of the formalism able to establish the EP on the physical meaning of the relationship between \( m_i \) and \( m_g \). We note that the Equivalence Principle is valid at each point.
The Milgrom’s Theory

The nature of dark matter is one of the unsolved mysteries in cosmology since C. Zwicky measured the velocity dispersion of the Coma cluster of galaxies [14]. M. Milgrom, from 1983 to today, developed the MOND theory to explain a great variety of astronomical phenomena without requiring the presence of a dark matter component. Such Modified Newtonian Dynamics introduces a constant with the dimensions of an acceleration, $a_0$, and posits that standard Newtonian dynamics is a good approximation only for accelerations that are much larger than $a_0$. Here we will limit ourselves to the original formulation and its conceptual aspects connected with the foundations of GR rather than the many subsequent developments. The application of the MOND has had great success in fitting astronomical observations by using the following Milgrom’s law:

$$F = m\mu\left(\frac{a}{a_0}\right)a$$

(7)

with $\mu = 1$ for $\left|\frac{a}{a_0}\right| \gg 1$ and $\mu = \frac{a}{a_0}$ for $\left|\frac{a}{a_0}\right| \ll 1$. If we consider the Keplerian two-body problem with $a \ll a_0$, we get

$$v = \sqrt{GMa_0}$$

(8)

It has long been known that rotation curves of galaxies disagree with Newton’s law, contrariwise the previous relation (7) fits all the data if $a_0$ is about $10^{-10}$ m [15]-[23].

The first immediate result of eqs. (2) is that at a large radius around a mass $M$, the orbital speed on a circular orbit becomes independent of radius and this asymptotic rotational speed depends only on the total mass $M$ via $v^4 = GMa_0$ in agreement with the observed Tully-Fisher-type relations. The extremely small MOND accelerations make testing the hypothesis in terrestrial laboratories impossible. Indeed the acceleration of the Earth is transmitted to the test objects.

Instead, if we are in an inertial frame with a harmonic oscillator, it is governed by the well-known equation $\frac{d^2x}{dt^2} = -\frac{k}{m}$. When $x = \frac{10^{-10}}{2}$ meters, it enters the MOND regime and the motion equation becomes

$$\frac{d^2x}{dt^2}\mu\left(\frac{1}{a_0} \frac{d^2x}{dt^2}\right) = -\frac{k}{M}x$$

(9)

This equation no longer has the classical harmonic solution and therefore there exists a neighborhood of the center where we could observe a deviation from the Newtonian oscillation. In a recent and interesting paper [24], the authors show that it may be possible to test MOND with experiments using a conventional apparatus performed in locally inertial frames such as artificial satellites and other freely falling laboratories. Instead we believe that Milgrom’s approach is not verifiable in a reference frame but only between references located in regions with different gravitational fields.
4 Inertial and Gravitational Mass

Let us rewrite the following relation

\[ m_1 \frac{v^2}{r} = \frac{GM_g m_g}{r^2} \]  

(10)

where \( m_1 \) is a body that rotates around a gravitational mass \( M_g \) over a constant radius \( r \). The relation (8) is in perfect agreement with the experimental data and therefore we write

\[ v^2 = \frac{GM_g m_g}{r} m_i = \sqrt{GM_g a_0}. \]  

(11)

It follows that

\[ \frac{m_g}{m_i} = \sqrt{\frac{a_0 r^2}{GM_g}}. \]  

(12)

If we do not interpret the MOND parameter from the kinematic point of view but as a gravitational field, we can write

\[ \frac{m_g}{m_i} = \sqrt{\frac{g_0}{g}}. \]  

(13)

According to Mach and his interpreters, the inertial mass of a body arises as a consequence of its interactions with the Universe and so we assume that

\[ \frac{m_g}{m_i} = \mu(x) \]  

(14)

with \( \mu = 1 \) for \( \frac{g}{g_0} \gg 1 \) and \( \mu = \sqrt{\frac{g_0}{g}} \) for \( \frac{g}{g_0} \ll 1 \).

A possible form of \( \mu \) may be for example

\[ \mu = \sqrt{\frac{g_0 + g}{g}} \]  

(15)

where \( g \) is the field generated by nearby masses.

It is easy to verify that when \( g \gg g_u \), circular velocity decreases in Keplerian way but when \( g \ll g_u \) we obtain

\[ v^2 = \frac{GM_g}{r} \sqrt{\frac{g_u}{g}} = GM_g \sqrt{\frac{g_u}{GM_g}} = \sqrt{GM_g g_u}, \]  

(16)

and finally

\[ v = \sqrt{GM_g g_u}. \]  

(17)

Obviously the value of \( g_u \) that fits all the data of galaxies rotation curves is about \( 10^{-10} m/s^2 \). From the mathematical point of view the relations (8) and (17) coincide but from a physical point of view the situation is different. At
every point in the Universe, the second Newtonian law is still valid even for small accelerations.

Deviations between inertial and gravitational mass as stressed by eq. (12) can have an intriguing geometrical explanation in the framework of $f(R)$ theories of gravity, assuming an explicit coupling between an arbitrary function of the scalar curvature, $R$, and the Lagrangian density of matter [23]. The most simple situation is to consider a weak modification of general relativity, which could be consistent with solar system tests, which implies a direct coupling between the Ricci curvature scalar and the matter Lagrangian [26]. Following [25, 26], let us consider the action

$$S = \int d^4x \sqrt{-g} (R + RL_m + \mathcal{L}_m),$$

(18)

which only includes a coupling between the Ricci scalar and the matter Lagrangian with respect to the well known canonical Einstein - Hilbert action of standard general relativity [27]

$$S = \int d^4x \sqrt{-g} R + \mathcal{L}_m.$$  

(19)

For the sake of simplicity we set $4\pi G = 1$, $c = 1$ and $\hbar = 1$ (natural units) hereafter. Thus, the standard variation analysis in a local Lorentz frame enables to write [25, 26]

$$\delta \int d^4x \sqrt{-g} (R + RL_m + \mathcal{L}_m) = \int d^4x [\delta \sqrt{-g} (R + RL_m + \mathcal{L}_m) + g^{\mu\nu} \delta R_{\mu\nu} + ...] = \int d^4x [\sqrt{-g} (1 + \mathcal{L}_m) g^{\mu\nu} \delta R_{\mu\nu}].$$

(20)

The relation between the connections and the Ricci tensor [27] gives [25, 26]

$$g^{\mu\nu} \delta R_{\mu\nu} = g^{\mu\nu} \partial_\alpha (\delta \Gamma^\alpha_{\mu\nu}) - g^{\mu\alpha} \partial_\nu (\delta \Gamma^\nu_{\mu\nu}) \equiv \partial_\alpha X^\alpha,$$

(21)

where

$$X^\alpha \equiv g^{\mu\nu} (\delta \Gamma^\alpha_{\mu\nu}) - g^{\mu\alpha} (\delta \Gamma^\nu_{\mu\nu}).$$

(22)

Thus, the second integral in equation (20) results to be [25, 26]

$$\int d^4x \sqrt{-g} (1 + \mathcal{L}_m) g^{\mu\nu} \delta R_{\mu\nu} = \int d^4x \sqrt{-g} (1 + \mathcal{L}_m) \partial_\alpha X^\alpha =$$

$$= \int d^4x \partial_\alpha [\sqrt{-g} (1 + \mathcal{L}_m) X^\alpha] - \int d^4x \partial_\alpha [\sqrt{-g} (1 + \mathcal{L}_m)] X^\alpha.$$  

(23)

A standard assumption is that fields are equal to zero at infinity. Then we obtain [25, 26]

$$d^4x \sqrt{-g} (1 + \mathcal{L}_m) g^{\mu\nu} \delta R_{\mu\nu} = - \int d^4x \partial_\alpha [\sqrt{-g} (1 + \mathcal{L}_m)] X^\alpha.$$  

(24)

Let us calculate the quantity $X^\alpha$. In a local Lorentz frame it is [25, 26]
\[ \nabla_\beta g_{\mu\nu} = \partial_\beta g_{\mu\nu} = 0. \quad (25) \]

Thus, the well known definitions of the Christoffel connections \[27\] gives \[25, 26\]
\[ \delta \Gamma^\alpha_{\mu\nu} = \delta \left[ \frac{1}{2} g_{\beta\alpha} (\partial_\mu g_{\beta\nu} + \partial_\nu g_{\beta\mu} - \partial_\beta g_{\mu\nu}) \right] = \frac{1}{2} g^{\beta\alpha} (\partial_\mu \delta g_{\beta\nu} + \partial_\nu \delta g_{\beta\mu} - \partial_\beta \delta g_{\mu\nu}). \quad (26) \]

In analogous way one gets \[25, 26\]
\[ \delta \Gamma^\nu_{\mu\nu} = \frac{1}{2} g^{\nu\beta} \partial_\mu (\delta g_{\nu\beta}). \quad (27) \]

Using eqs. \[26\] and \[27\] we find \[25, 26\]
\[ g^{\mu\nu} (\delta \Gamma^\alpha_{\mu\nu}) = \frac{1}{2} \partial^\alpha (g_{\mu\nu} \delta g^{\mu\nu}) - \partial^\mu (g_{\beta\mu} \delta g^{\nu\beta}) \quad (28) \]
and
\[ g^{\mu\alpha} (\delta \Gamma^\nu_{\mu\nu}) = -\frac{1}{2} \partial^\nu (g_{\nu\beta} \delta g^{\mu\beta}). \quad (29) \]

Now, substituting in \[22\], we obtain \[25, 26\]
\[ X^\alpha = \partial^\alpha (g_{\mu\nu} \delta g^{\mu\nu}) - \partial^\mu (g_{\alpha\nu} \delta g^{\alpha\nu}). \quad (30) \]

In this way, equation \[24\] becomes \[25, 26\]
\[ \int d^4x \sqrt{-g} (1 + L_m) g^{\mu\nu} \delta R_{\mu\nu} = \int d^4x \partial_\alpha [\sqrt{-g}(1 + L_m)] [\partial^\mu (g_{\nu\mu} \delta g^{\nu\alpha}) - \partial^\nu (g_{\mu\nu} \delta g^{\mu\alpha})], \quad (31) \]
which also gives \[25, 26\]
\[ \int d^4x \sqrt{-g} (1 + L_m) g^{\mu\nu} \delta R_{\mu\nu} = \]
\[ = \int d^4x \{g_{\mu\alpha} \delta \partial_\alpha [\sqrt{-g}(1 + L_m)] \delta g^{\mu\nu} \} - \int d^4x \{g_{\mu\nu} \partial^\alpha \partial_\alpha [\sqrt{-g}(1 + L_m)] \delta g^{\mu\nu} \}. \quad (32) \]

Inserting eq. \[32\] in the variation \[20\] we get \[25, 26\]
\[ \delta \int d^4x \sqrt{-g} (R + RL_m + L_m) = \int d^4x [\sqrt{-g}(1 + L_m) R_{\mu\nu} - g_{\mu\nu}(R + RL_m + L_m) \delta g^{\mu\nu} + \]
\[ + \int d^4x \{g_{\mu\alpha} \partial^\alpha \partial_\alpha [\sqrt{-g}(1 + L_m)] - g_{\alpha\nu} \partial^\mu \partial_\alpha [\sqrt{-g}(1 + L_m)] \} \delta g^{\mu\nu} \} + \int d^4x (1 + R) \delta (\sqrt{-g} L_m). \quad (33) \]

This variation is equal to zero for \[25, 26\]
\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -L_m R_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) L_m + \frac{1 + R}{2} T_{\mu\nu}^{(m)}, \quad (34) \]
which are the Einstein field equations modified by direct coupling between the Ricci curvature scalar and the matter Lagrangian. In fact, the standard stress-energy tensor \[27\]
\[ T^{(m)}_{\mu\nu} = \frac{-2}{\sqrt{-g}} \delta(\sqrt{-g} \mathcal{L}_m) \delta g^{\mu\nu} \]  

(35)

has been introduced in the modified field equations (34). Writing down explicitly the Einstein tensor and introducing a “total” stress-energy tensor [25, 26, 28]

\[ T^{(\text{tot})}_{\mu\nu} \equiv \frac{1}{(1 + \mathcal{L}_m)}[(\nabla_{\mu} \nabla_{\nu} - g_{\mu\nu} \Box) \mathcal{L}_m + \frac{(1 + R)}{2} T^{(m)}_{\mu\nu} - \frac{R \mathcal{L}_m}{2} g_{\mu\nu}], \]  

(36)
eqs (34) can be put in the well known Einsteinian form

\[ G_{\mu\nu} = 8\pi T^{(\text{tot})}_{\mu\nu}, \]  

(37)
in which a curvature contribute [28] is added and mixed to the material one. In other words, the high order terms contribute, like sources, to the modified field equations and have to be considered like effective fields (see [28] for details). The condition of energy conservation [25, 26, 27]

\[ \nabla^{\mu} G_{\mu\nu} = 0 \]  

(38)
can be inserted in eqs. (37) and (36), obtaining [25, 26]

\[ \nabla^{\mu} T^{(m)}_{\mu\nu} = \frac{1}{R + 1} (g^{\mu\nu} \mathcal{L}_m - T^{(m)}_{\mu\nu}) \nabla^{\mu} R. \]  

(39)

Now, we can introduce the well known stress-energy tensor of a perfect fluid [27]

\[ T^{(m)}_{\mu\nu} \equiv (\epsilon + p) u_{\mu} u_{\nu} - pg_{\mu\nu}, \]  

(40)
in order to test the motion of test particles [25, 26], where \( \epsilon \) is the proper energy density, \( p \) the pressure and \( u_{\mu} \) the fourth-velocity of the particles. This is the simplest version of a stress-energy tensor for the matter, concerning inchoerent matter, and it is considered a good approximation in astrophysics frameworks [25, 26, 27]. Introducing the projector operator [25, 26]

\[ P_{\mu\alpha} \equiv g_{\mu\alpha} - u_{\mu} u_{\alpha}, \]  

(41)
we can apply the contraction \( g^{\alpha\beta} P_{\mu\beta} \) to equation (39), obtaining [25, 26]

\[ \frac{d^2 x^\alpha}{ds^2} + \Gamma_{\mu\nu}^{\alpha} \frac{dx^\mu}{ds} \frac{dx^\nu}{ds} = F^\alpha. \]  

(42)

Thus, we find the existence of an extra force [25, 26]

\[ F^\alpha \equiv (\epsilon + p)^{-1} P^{\alpha\nu} \left( \frac{1}{R + 1} (\mathcal{L}_m + p) \nabla_{\nu} R + \nabla_{\nu} p \right) \]  

(43)
showing that the motion of test particles is non-geodesic. This extra force is orthogonal to the four-velocity of test masses [25, 26]
By substituting (42) into (43), we obtain:
\[ F_\alpha \frac{dx_\alpha}{ds} = 0. \] (44)

The Newtonian limit in three dimensions of equation (42) reads [25, 26]
\[ \overrightarrow{a}_{\text{tot}} = \overrightarrow{a}_n + \overrightarrow{a}_{\text{ng}}. \] (45)

The total acceleration \( \overrightarrow{a}_{\text{tot}} \) is given by the ordinary Newtonian acceleration \( \overrightarrow{a}_{n} \) plus the repulsive acceleration \( \overrightarrow{a}_{\text{ng}} \) due to the extra (non-geodesic) force [25, 26].

Eq. (45) and a bit of three-dimensional geometry permit to write the Newtonian acceleration \( \overrightarrow{a}_{n} \) as [25, 26]
\[ \overrightarrow{a}_{n} = \frac{1}{2} (a_{\text{tot}}^2 - a_n^2 - a_{\text{ng}}^2) \overrightarrow{a}_n a_{\text{tot}} a_{\text{ng}} a_{\text{tot}}. \] (46)

Considering the limit in which \( \overrightarrow{a}_{\text{ng}} \) dominates (i.e. \( a_n \ll a_{\text{tot}} \)) one gets
\[ a_n \simeq \frac{a_{\text{tot}} \overrightarrow{a}_{\text{tot}}}{2a_{\text{ng}}} (1 - \frac{a_{\text{ng}}^2}{a_{\text{tot}}^2}). \] (47)

Thus, the extra acceleration is given by [25, 26]
\[ a_0 = \left[ \frac{1}{2a_{\text{ng}}} (1 - \frac{a_{\text{ng}}^2}{a_{\text{tot}}^2}) \right]^{-1}, \] (48)

and combining eq. (48) with eqs. (12) and (13) one gets
\[ \frac{m_g}{m_i} = \sqrt{g_0} \frac{g}{g} = \frac{r^2}{N_{g} (1 - \frac{a_{\text{ng}}^2}{a_{\text{tot}}^2})}, \] (49)

with
\[ g_0 = \frac{r^2}{2a_{\text{ng}} (1 - \frac{a_{\text{ng}}^2}{a_{\text{tot}}^2})}. \] (50)

Thus, we have shown that in our model the ration between gravitational and inertial mass is explained in an elegant, geometric way through a direct coupling between the Ricci curvature scalar and the matter Lagrangian which generates a non geodesic motion of test particles.

5 Conclusions

In this paper, we have analyzed the Milgrom’s approach by adopting another physical viewpoint. The assumption of an \( R \)-dependent inertial mass is in accordance with the spirit of Mach principle and Einstein himself tried to implement this hypothesis in the context of General Relativity [29]. The possible relation \( m_g/m_i \) is deduced by comparing it with Milgrom’s rotational equation that is in perfect agreement with the experimental data. However, the interpretation given here is different, and it leaves unchanged in any point the second law of dynamics.
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