Spin and spin-spin correlations in
chargino pair production at future linear
$e^+e^-$ colliders

V. Lafage$^1$, T. Ishikawa$^1$, T. Kaneko$^2$, T. Kon$^3$, Y. Kurihara$^1$, H. Tanaka$^4$

$^1$High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305–0801, Japan
$^2$Meiji-Gakuin University, Yokohama, Kanagawa 244–8539, Japan
$^3$Seikei University, Musashino, Tokyo 180–8633, Japan
$^4$Rikkyo University, Toshima-ku, Tokyo 171–8501, Japan

Abstract

A possibility to measure the spin and spin-spin correlations of a chargino pair is investigated in the process $e^+e^- \rightarrow \tilde{\chi}_1^+\tilde{\chi}_1^- \rightarrow (\tilde{\chi}_0^1q\bar{q}') (\tilde{\chi}_0^0q''\bar{q'''})$ at future linear-collider energies. The total and the differential cross sections are calculated by the GRACE system which allows for the full spin correlation. Experimental sensitivity of the measurements are examined by assuming the limited detector resolution, the initial state radiation and the beam-beam effect (beamstrahlung). It is found that generally the spin-spin correlation can only be measured with a lower sensitivity than the chargino spin itself. The dependence of the correlation measurements on the relevant SUSY parameters can be seen for a light $\tilde{\nu}_e$ case, but the situation becomes worse for a heavier $\tilde{\nu}_e$. 
1 Introduction

If Nature has chosen a supersymmetric scenario (SUSY) to build the universe as many physicists expect, the chargino pair-production process should be one of the first SUSY signals observed in the near future at LEP2 or linear $e^+e^-$ colliders. Charginos are mixed states of the spin-1/2 partners of the $W$ boson and the charged Higgs boson, and form two mass-eigenstates ($\tilde{\chi}^{\pm}_{1,2}$). In many SUSY models, the lighter chargino, $\tilde{\chi}^{+}_{1}$, is thought to be the next-to-lightest SUSY particle while the neutralino $\tilde{\chi}^{0}_{1}$ (a mixed state of SUSY partners of the photon, $Z$ boson and neutral Higgs bosons), is the lightest SUSY particle. Then the main decay-mode of the chargino is $\tilde{\chi}^{\pm}_{1} \rightarrow \tilde{\chi}^{0}_{1} f \bar{f}'$, where $f$ denotes a quark or a lepton.

Since the neutralino is invisible in the detector, the experimental signature of the chargino pair-production is four jets or two jets and one isolated lepton with large missing transverse momentum. If the mass difference between chargino and neutralino is greater than the $W$ boson mass, a real $W$ boson can be created and the two-body decay, $\tilde{\chi}^{\pm}_{1} \rightarrow \tilde{\chi}^{0}_{1} W$, should dominate. In this case the additional signature of the $W$ boson production can also be used for the event selection. The mass of $\tilde{\chi}^{\pm}_{1}$ can be easily measured from a sharp rise of the total cross section at its threshold. The mass difference, $M_{\tilde{\chi}^{\pm}_{1}} - M_{\tilde{\chi}^{0}_{1}}$, can be measured from the maximum energy of $q\bar{q}'$ system [1].

If one wants to find other SUSY parameters beyond the measurements of SUSY particle masses, one encounters some difficulties\footnote{In ref. [1], they also pointed out that the electron-beam polarization is helpful to separate the higgsino component from gaugino one because the right-handed beam does not couple to the gaugino component.}. Since two invisible particles escape from the detection, a complete reconstruction of the event kinematics is not possible. The same situation happens in the $\tau$ pair-production: $\tau$ decays into $\nu_{\tau}$ (invisible) and $q\bar{q}'$ system through virtual $W$. After the discovery of $\tau$ lepton, it has been discussed [2] how to extract its weak properties. As a result it has been pointed out that there are three angles which can be reconstructed unambiguously from the experimental observables. They are $\cos \theta^{*}_{\pm}$, and $\cos \Delta \phi^{*}$, where $\theta^{*}_{\pm}$ are the polar angle of the $q\bar{q}'$ pair with respect to the momentum of a mother particle ($\tilde{\chi}^{\pm}_{1}$ or $\tau^{\pm}$) measured in the rest frame of the mother particle, and $\Delta \phi^{*}$ is the azimuthal angle between the decay planes of the two mother particles. This implies in turn that the spin and spin-spin correlations can be measured from the limited number of experimental observables.
In order not to lose the precious azimuthal information, one must keep track of the decaying chargino polarisation (full spin correlation), as done in [3] for application to angular distributions of outcoming leptons. Recently Choi et al. [4] argued that the spin information from the chargino decays directly reflects the chargino mixing angles. They concluded that the chargino mixing angles and the fundamental SUSY parameters could be determined from the measurement of the total cross section of the chargino pair-production, their spin, and spin-spin correlations.

In this report, we will discuss quantitatively the possibility to measure those SUSY parameters from the spin and spin-spin correlations in the chargino pair-production based on the detailed simulation which also includes some detector effects and the smearing of colliding energies due to the initial state radiation as well as the beam-beam effect at future linear $e^+e^-$ colliders.

2 SUSY parameters

2.1 Chargino description

Charginos do not conserve fermion number. Therefore the fermion number of charginos is not determined by interactions but is a matter of convention. In GRACE [3] we adopt the convention that the positively charged charginos are Dirac-particles. The two charginos are made of four Weyl spinors, $\lambda^+, \lambda^-, \tilde{H}^-_1$ and $\tilde{H}^+_2$. The corresponding physical states with mass $m_{\tilde{\chi}^\pm_1}$ and $m_{\tilde{\chi}^\pm_2}$ are given by

$$\Psi(\tilde{\chi}^+_i) = \begin{pmatrix} \lambda^-_{1R} \\ \lambda^+_L \end{pmatrix}, \quad \Psi(\tilde{\chi}^-_i) \equiv \Psi(\tilde{\chi}^+_i)^c = \begin{pmatrix} \lambda^+_L \\ \lambda^-_{1R} \end{pmatrix}, \quad i = 1, 2 \quad (1)$$

with

$$\begin{pmatrix} \lambda^+_{1L} \\ \lambda^+_{2L} \end{pmatrix} = \begin{pmatrix} \cos \phi_R & \sin \phi_R \\ -\sin \phi_R & \cos \phi_R \end{pmatrix} \begin{pmatrix} \lambda^- \tilde{H}^-_1 \\ \tilde{H}^+_2 \end{pmatrix},$$

$$\begin{pmatrix} \lambda^-_{1R} \\ \lambda^-_{2R} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \epsilon_L \end{pmatrix} \begin{pmatrix} \cos \phi_L & \sin \phi_L \\ -\sin \phi_L & \cos \phi_L \end{pmatrix} \begin{pmatrix} \lambda^+ \tilde{H}^+_2 \\ \tilde{H}^-_1 \end{pmatrix}. \quad (2)$$

The two orthogonal matrices in (2) diagonalize the mass matrix,

$$\mathcal{M}_C \equiv \begin{pmatrix} M_2 \\ \sqrt{2} M_W \sin \beta \mu \end{pmatrix} \begin{pmatrix} \sqrt{2} M_W \cos \beta \\ \mu \end{pmatrix}$$

3
as

\[
\begin{pmatrix}
\cos \phi_L & \sin \phi_L \\
-\sin \phi_L & \cos \phi_L
\end{pmatrix}
\mathcal{M}_C
\begin{pmatrix}
\cos \phi_R & -\sin \phi_R \\
\sin \phi_R & \cos \phi_R
\end{pmatrix}
= \begin{pmatrix}
m_{\tilde{c}_1} & 0 \\
0 & m_{\tilde{c}_2}
\end{pmatrix}
\quad \text{(4)}
\]

(the indexing of charginos is as follows: \(|m_{\tilde{c}_1}| < |m_{\tilde{c}_2}|\).

\(\mu\) is the Higgs mixing parameter. In our convention, it is real and can take any sign. It is one of the superpotential parameters. As for \(M_2\), it is a soft SUSY breaking parameter related to the mass of the SU(2) gaugino. The \(\beta\) angle is related to the vacuum expectation values, \(v_1\) and \(v_2\), through

\[
\tan \beta = \frac{v_2}{v_1}, \quad \cos \beta = \frac{v_1}{\sqrt{v_1^2 + v_2^2}}, \quad \sin \beta = \frac{v_2}{\sqrt{v_1^2 + v_2^2}}.
\quad \text{(5)}
\]

The diagonal matrix with \(\epsilon_L\) in (2) is to take care of the possible negative eigenvalue for \(m_{\tilde{c}_2}\). (We can always choose the mixing parameters \(\phi_R\) and \(\phi_L\) such that \(m_{\tilde{c}_1} > 0\).) Practically, from (3) we find

\[
\epsilon_L = \text{sign}(M_2\mu - M_W^2 \sin 2\beta) = \text{sign} \left( \det (\mathcal{M}_C) \right).
\quad \text{(6)}
\]

The physical masses of charginos are given by

\[
m_{\tilde{\chi}^\pm_1} = m_{\tilde{c}_1}, \quad m_{\tilde{\chi}^\pm_2} = \epsilon_L m_{\tilde{c}_2},
\quad \text{(7)}
\]

with \(\tilde{\chi}^\pm_1\) lighter than \(\tilde{\chi}^\pm_2\).

The chargino spectrum is then related to the electroweak symmetry breaking sector through \(\mu\) and \(\beta\), and to the SUSY soft-breaking sector through \(M_2\).

The couplings to \(Z^0\) are determined by the sine and cosine of mixing angles \(\phi_R\) and \(\phi_L\), as well as \(\cos^2 \theta_W\).

### 2.2 SUSY spectrum evaluation

Very few parameters are involved in the \(2 \to 2\) chargino pair-production process: besides those involved in the chargino spectrum (\(\mu\), \(\beta\) and \(M_2\)), we only need the \(t\)-channel related sneutrino \(\tilde{\nu}_e\) mass (\(m_{\tilde{\nu}_e}\)). However for the simulation of the experimentally more realistic \(2 \to 6\) process, a large part of the SUSY parameters must be known: the neutralino spectrum and mixing matrix, the sfermions spectrum and couplings.
In order not to span the 80-dimensional SUSY parameters space, one needs simplifying assumptions. The minimal supergravity (mSUGRA) is a popular way to restrict this space. mSUGRA keeps only 4 parameters: the common mass for scalar $m_0$, the common mass for gaugino $M_{1/2}$, the common (rescaled) trilinear parameter $A_0$ and $\tan \beta$, as well as the possible choice of the sign of $\mu$ ($\text{sign}(\mu)$), its absolute value being fixed by the constraint of radiative electroweak symmetry breaking. These parameters being determined at the grand unification scale ($\Lambda_{GUT}$), except $\tan \beta$, one needs to get their running values at the electroweak scale ($M_Z$) through the Renormalization Group Equation (RGE). Further complications occurs due to the fact that this equation have a two-boundary condition (some parameters are known at $M_Z$ and others at $\Lambda_{GUT}$) and that the Yukawa’s coupling are not computed from the pole mass but from the running mass. Some programs deal with all the detail of this computation. In this study, the utility MUSE [5], is used for this purpose.

In order to investigate the possibilities to measure the gaugino-mixing angles, we tried to prepare three typical parameter sets as proposed in [4]: gaugino-like ($M_2 = 81$ GeV, $\mu = -215$ GeV), higgsino-like ($M_2 = 215$ GeV, $\mu = -81$ GeV) and mixed ($M_2 = 92$ GeV, $\mu = -93$ GeV). However, the constraints imposed by mSUGRA are quite restrictive. The run down of $\tilde{m}_{H_2}^2$ (mass parameter of the 'up-sector' Higgs) is strongly pulled toward negative values by $M_{1/2}^2$. In order to get electroweak symmetry breaking, one needs a large enough negative value for $\tilde{m}_{H_2}^2$. So once $M_{1/2}^2$ is fixed by the constraint on $M_2$, the only remaining degree of freedom is $m_0$ ($A_0$ has not so much effect).

mSUGRA allows to get gaugino-like configurations, but not mixed or higgsino-like. Moreover once $m_0$ is fixed, $m_{\tilde{\nu}_e}$ is also determined. Consequently, we chose an extended approximation with three common scalar mass (one for the Higgs bosons, one for the squarks and one for the sleptons), and we put $\mu$ ‘by hand’ instead of computing it with the radiative symmetry breaking constraint. The sneutrino mass for the mixed scenario in the light sneutrino case had also to be put by hand. The resulting spectra are summarized in Table-1 and Table-2.

3 Calculation method
3.1 Exact calculation

All the cross sections given in this report, irrespective of various levels of approximation used (see below), are numerically calculated using the automatic calculation system GRACE \cite{6}, based on the helicity amplitude formalism. It includes the minimal SUSY standard model \cite{7}. The cross section and any kinds of distributions can be obtained with the aid of the multi-dimensional phase space integration package BASES \cite{8}. Light fermion masses are also taken into account.

The chargino pair-production in $e^+e^-$ collisions can be expressed by the three Feynman diagrams shown in Fig. 1 (first row). It is followed by the subsequent decay of $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_1^0 q\bar{q}'$, which has six diagrams in unitary gauge. In this report, the decay channels of $\tilde{\chi}_1^{+(-)} \rightarrow \tilde{\chi}_1^0 u\bar{d}(s\bar{c})$ are chosen as benchmark processes. Among those diagrams the contribution from Higgs-exchange and $\tilde{u}_2(\approx \tilde{u}_R)$ diagrams is found to be much smaller than the statistical errors of the numerical integration (less than 0.5%) and thus safely omitted. Then three diagrams shown in Fig. 1 (second row for $\tilde{\chi}_1^+$ and third row for $\tilde{\chi}_1^-$) are taken into account in our study. When we look at the final state of $\tilde{\chi}_1^0 u\bar{d}\tilde{\chi}_1^0 s\bar{c}$ via the chargino pair-production, 54 diagrams $= 3 \times 3 \times 3 \times 2$ contribute to the process. Besides those specified there remain a huge number of diagrams giving the same final state but not through the chargino pair. Even with an automatic system like GRACE the calculation with the complete set of diagrams ($\sim 30,000$) is hopeless within a reasonable CPU time. To estimate the magnitude of the contribution from those background diagrams, we have calculated the cross section of the process $e^+e^- \rightarrow \tilde{\chi}_1^\pm \tilde{\chi}_1^0 u\bar{d}$ with full diagrams. This process has 292 diagrams in unitary gauge. Among them there are 9 diagrams which comes through the chargino pair-production. The cross section above the threshold of the pair-production has shown that the contribution from the background diagrams was smaller than the statistical error of the numerical integration. Hence it is confirmed that taking only the 54 diagrams related to the chargino pair-production for the $\tilde{\chi}_1^0 f\bar{f}'\tilde{\chi}_1^0 f\bar{f}'$ process is accurate enough up to the precision of 0.5%. It is worth mentioning that this calculation treating the amplitude of the six-body final state can reproduce the full spin information including the spin-spin correlation between charginos.

The decay width of the chargino are calculated by summing up five possible decay channels with keeping fermion masses using GRACE system. Numerical results for the six parameter-sets are summarized in Table 3.
total cross sections of six-particle final-state based on the 54 diagrams are checked against those of the chargino pair-production multiplied by the decay branching ratio at the center of mass system (CMS) energy of 250 GeV. The results are consistent one with each other as shown in Table 4 (first and second column).

To see the effect of the interference arising from the exchange of two identical neutralinos, the cross section for 54 diagrams (considering the statistical factor 1/2) is compared with that for 27 diagrams omitting neutralino exchange. Both results completely agree within the statistical error of the integration as shown in Table 4 (second and third column). This fact together with the other fact that the decay width of the chargino is very narrow allows us to employ the narrow width (on-shell) approximation to evaluate amplitudes of this process.

### 3.2 Narrow width approximation with full spin correlation

The simplest approximation of the full amplitude would be to take the amplitude of the 2-body production process followed by the isotropic decays. However, it is obvious that this approximation is senseless when one talks about the spin measurement as it ignores any spin information of the chargino and the vector boson. It is not a sufficient approximation even for the estimation of the experimental acceptance of the detectors.

The best approximation to calculate the cascade decays of the SUSY particles should be as follows: connect the production amplitude for $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-$ and the decay amplitudes for $\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_0^\pm q\bar{q}'$ tracing the helicity of each particle. The cross sections is then expressed as

$$
\sigma = \frac{1}{C} \sum_{h_i} \int \int \int \left| \sum_{h_{+-},h_{-+}} A_{e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-} \cdot A_{\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_0^0 u\bar{d}} \cdot A_{\tilde{\chi}_1^- \rightarrow \tilde{\chi}_0^0 e\bar{e}} \right|^2 d\Omega_{2 \rightarrow 2} \times \frac{d\Omega_{1 \rightarrow 3}}{2m_{\tilde{\chi}_1^\pm} \Gamma_{\tilde{\chi}_1^\pm}^2} \frac{d\Omega_{1 \rightarrow 3}}{2m_{\tilde{\chi}_1^\pm} \Gamma_{\tilde{\chi}_1^\pm}^2}
$$

where $A_{e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-}$ and $A_{\tilde{\chi}_1^\pm \rightarrow \tilde{\chi}_0^0 u(s)\bar{d}(c)}$ are respectively the amplitudes of the chargino pair-production and decay, $h_i$ the helicity for the initial and the final particles, $h_{+-}$ the helicities of $\tilde{\chi}_1^\pm$, $d\Omega_{1 \rightarrow j}$ the phase space for the production and the decay, $m_{\tilde{\chi}_1^\pm}$ the mass of the chargino, $\Gamma_{\tilde{\chi}_1^\pm}$ the decay width of
chargino, and $C = (\text{spin average factor}) \times (\text{flux factor})$. In this method the spin summation is taken not only over the diagonal part but also over the off-diagonal one. This is equivalent to deal with the full spin density matrix for the chargino decay.

When integrating over the whole phase-space (without cuts), the off-diagonal part disappears (but one must keep it when studying differential distributions), and the total cross sections can be expressed as:

$$\sigma = \sigma_{e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-} \cdot \frac{\Gamma_{\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 u\bar{d}}}{\Gamma_{\tilde{\chi}_1^+}} \cdot \frac{\Gamma_{\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 s\bar{c}}}{\Gamma_{\tilde{\chi}_1^-}}$$

$$= \sigma_{e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-} \cdot Br(\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 u\bar{d}) \cdot Br(\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 s\bar{c}),$$

This approximation is precise enough and much faster to reproduce the cross section of the exact calculation based on the full 54 diagrams as one can see in Table.4. Furthermore, the distributions relevant to the chargino spin and spin-spin correlations are also reproduced very accurately as shown in Fig.2 and Fig.3. In the figures, ‘missing $p_T$’ means the missing transverse momentum carried by the neutralinos, ‘$p_T^{\text{jet}}$’ the transverse momentum of the quarks (parton level information of each quark), $\theta^*$ the polar angle of the $s\bar{c}$ system with respect to the momentum of $\tilde{\chi}^-$ in its rest frame, and $\phi^*$ its azimuthal angle measured from the chargino pair-production plane. Though $\phi^*$ is not an experimental observable, it is shown that the event topology is correctly described by this approximation. In the following sections we will present the results of the simulation study based on this approximation.

Sometimes a similar approximation is used without the off-diagonal part in the spin summation as in [8]:

$$\sigma = \frac{1}{C} \sum_{h_i} \int \int \int \sum_{h^+, h^-} |A_{e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^-}|^2 \cdot |A_{\tilde{\chi}_1^+ \rightarrow \tilde{\chi}_1^0 u\bar{d}}|^2 \cdot |A_{\tilde{\chi}_1^- \rightarrow \tilde{\chi}_1^0 s\bar{c}}|^2 \, d\Omega_2 \cdot \frac{d\Omega_1 \cdot d\Omega_1}{2m_{\tilde{\chi}_1^+} \Gamma_{\tilde{\chi}_1^+}}$$

This approximation gives the same total cross sections as those with full spin-correlation. However, it might give some deviation in differential distributions, especially those of azimuthal angle correlation between $\tilde{\chi}_1^+$ and $\tilde{\chi}_1^-$. (See Figs.2 and 3.) The approximation with diagonal spin-correlation could be used for limited studies, such as the detector acceptance estimation or the measurements of the SUSY particle masses. It should, however, not be used for the detailed studies of spins.
4 Spin and spin-spin correlation measurements

4.1 Detector effects

Even though there are two missing particles in the final state, three angles can be obtained unambiguously from the experimental observables as mentioned previously, \( \cos \theta^*_\pm \), and \( \cos \Delta \phi^* \). They are given by [3]:

\[
\cos \theta^*_\pm = \frac{1}{\beta \gamma} \left( E\gamma \mp E\gamma^* \right),
\]

\[
\sin \theta^*_+ \sin \theta^*_- \cos \Delta \phi = \frac{P^+ P^-}{P^+ P^-} \cos \theta_{X+X^-} + \frac{(E_+ - E_+^*/\gamma)(E_- - E_-^*/\gamma)}{\beta^2 P^+ X^+ P^- X^-}.
\]

The direct experimental observables are:

- \( E_{\pm} \): the energy of the \( q\bar{q}' \) system in the laboratory frame,

- \( P_{\pm} = \sqrt{E_{\pm}^2 - M_{\pm}^2} \): the corresponding momentum, where \( M_{\pm} \) is the invariant mass of the \( q\bar{q}' \) pair, is also an experimental observable,

- \( \theta_{X+X^-} \): the opening angle between two \( q\bar{q}' \) systems in the laboratory frame.

On the other hand the following variables require the independent informations such as the nominal beam-energy, the chargino and neutralino masses:

- \( \beta, \gamma \): the velocity and the Lorentz factor of the chargino calculated from the nominal beam energy and the chargino mass,

- \( E_{\pm}^* \): the energy of the \( q\bar{q}' \) system in the rest frame of the mother chargino, which is calculated from the chargino and the neutralino masses and the measured \( q\bar{q}' \) invariant mass,

- \( P_{\pm}^* \): the corresponding momentum.

One cannot avoid the experimental errors in measuring the above variables. There are two kinds of sources for these errors. The first is the limited resolution of the detectors. To estimate the effect we use PYTHIA [10] to hadronize quarks and to make them to decay into stable particles. The obtained energies of the particles are smeared by an assumed experimental
resolution of the electro-magnetic (hadron) calorimeters: these resolutions are supposed to be $\sigma_E/\sqrt{E} = 15%/\sqrt{E}$ plus 1% constant term ($\sigma_E/\sqrt{E} = 40%/\sqrt{E}$ plus 2% constant term) assuming a Gaussian distribution. The second source stems from the incomplete knowledge of the colliding beam energies due to the initial state radiation (ISR) and the beamstrahlung (BS). Though smearing of the beam energy does not affect the energy measurement of the particles directly, this must give some uncertainty to $\beta$ and $\gamma$ factors of the charginos and also to the direction of the momentum of the charginos, which defines the quantization axis of the chargino spin. Hence the chargino pair is not necessarily produced in back-to-back configuration in the laboratory frame. This means the quantization axis cannot be taken common for both charginos in the laboratory frame. For the estimation of these effects we use the simple structure function [11] for ISR and the utility Luminos [12] for BS. The TESLA reference parameters at the CMS energy of 500 GeV listed in Table 5 are used for the BS simulation in the Luminos.

The calculated distributions of the experimental observable necessary for the spin and spin-spin correlation measurements are shown in Fig. 4. The parameter set for the ‘gaugino region’ with a light $\tilde{\nu}_e$ mass is used for the benchmark. The generated distributions are deformed due to the effects mentioned above as shown in Fig. 4. One can see that the hadronization affects the heavy quark side ($\tilde{\chi}_0^1\bar{s}\tilde{c}$) largely. The ISR and BS give more deformation to the distributions than to the hadron energy measurement. Even after including the experimental effects, one still has the possibility to measure the spin and spin-spin correlations.

4.2 Mixing angle dependence

The cross section of the chargino pair-production depends on the chargino mass, neutralino mass, the $\tilde{\nu}_e$ mass, and the chargino mixing angles. The chargino and neutralino masses can be determined from the threshold energy of the production and the energy distribution of the quark pairs. In order to extract the rest of parameters, the total cross section of the chargino pair-production itself is helpful. The total cross section at the CMS energy of 250 GeV are summarized in Table 7 for the six different parameter sets. However, the information is not enough to derive the other unknown parameters such as the $\tilde{\nu}_e$ mass and the chargino mixing angles.

As proposed in ref. [4], let us look at the spin and spin-spin correlations on the angular distributions at the CMS energy of 250 GeV. There are two
angular observables in the chargino spin measurement, \( \cos \theta^*_+ \) and \( \cos \theta^-_+ \), and two observables for the spin-spin correlation measurement, \( \cos \theta^*_+ \cos \theta^*_+ \) and \( \sin \theta^*_+ \sin \theta^*_- \cos \Delta \phi \).

First let us consider the light \( \tilde{\nu}_e \) mass case. As shown in Fig. 5 (the left column) the generated distribution on \( \cos \theta^*_\pm \) shows a clear difference among three SUSY parameter sets, ‘gaugino region’, ‘higgsino region’, and ‘mixed region’. (In order to see only the difference of the distributions but not their absolute values, all distributions in Figs. 5-7 are normalized to unity as indicated by A.U. (arbitrary unit).)

It can be seen that the experimental effects modify the distribution to a rather large extent. However, even with these experimental uncertainties, these three parameter regions can be distinguished as seen in Fig. 5 (the right column). The error bars in the figures show the expected (statistical) experimental error after accumulating 10 fb\(^{-1}\) integrated luminosity\(^2\). The limited knowledge of the beamstrahlung would give the biggest uncertainty to the measurements. If the energy spectrum of the colliding beams are measured precisely enough, it will be able to measure the chargino spin and to distinguish three typical regions of the chargino mixing angles. On the other hand as shown in Fig. 6 the distributions for the spin-spin correlation measurement overlap and cannot be separated even in the generator level without any smearing. The sensitivity to the spin-spin correlation is lower than to the spin measurement itself. These sensitivity can be described in terms of analyzing-powers appearing in the cross section formula: \( \kappa_\pm (\leq 1) \) is the spin analyzing-power of \( \tilde{\chi}^\pm \), and \( \kappa_c = \kappa_+ \times \kappa_- \) is the analyzing-power of the spin-spin correlation. Then the analyzing power of the spin-spin correlation is smaller than those of the spin measurements.

For a heavy \( \tilde{\nu}_e \) mass case the situation is rather worse than in the previous case. Even for the spin measurements the sensitivity is not enough to distinguish three typical chargino mixing-angle regions, as shown in Fig. 6. In this case the contribution from the \( \tilde{\nu}_e \) exchange diagrams is greatly suppressed and this kills the sensitivity to distinguish the gaugino and higgsino regions. The same studies at the CMS energy at 500 GeV have also been performed. It is found that the situation does not change much from that at 250 GeV concerning the sensitivities for the measurement of the spin and spin-spin correlations.

\(^2\) All hadronic channels are summed up and 50% of detection efficiency is assumed.
5 Conclusions

The total and the differential cross sections of the process, $e^+e^- \rightarrow \tilde{\chi}_1^+ \tilde{\chi}_1^- \rightarrow (\tilde{\chi}_1^0 q\bar{q}')(\tilde{\chi}_1^0 q\bar{q}')$, have been calculated numerically. By comparing the narrow width approximation with the exact cross sections based on the numerical helicity amplitudes with the full 54 diagrams, it was confirmed that this approximation with full spin correlation, could reproduce the details of the distributions very accurately. The possibilities have been investigated to distinguish experimentally three typical sets of the chargino-mixing angles by measuring the spin and spin-spin correlation. It was found that the light $\tilde{\nu}_e$ mass case could distinguish three cases (gaugino, higgsino and mixed region) in the chargino spin measurements once the data of 10 fb$^{-1}$ is accumulated, even the distributions are largely distorted by the experimental effects. However, the measurements of the spin-spin correlation for the light $\tilde{\nu}_e$ mass case and even the spin measurements for the heavy $\tilde{\nu}_e$ mass case turned out to be difficult.

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For the program, please contact ‘toomi@kekvax.kek.jp’.
Table 1: Three parameter sets with a light $\tilde{\nu}_e$. 

| parameter          | light $\tilde{\nu}_e$ case |
|--------------------|-------------------------------|
|                    | gaugino region | higgsino region | mixed region |
| $\tan \beta$       | 2              | 2               | 2            |
| $\mu$ (GeV)        | $-215.0$       | $-81.0$         | $-93.0$      |
| $M_2$ (GeV)        | 81.4           | 215.0           | 92.0         |
| $\cos \phi_L$     | 0.914          | 0.056           | 0.437        |
| $\sin \phi_L$     | 0.405          | $-0.998$        | 0.900        |
| $\cos \phi_R$     | 0.998          | 0.405           | 0.908        |
| $\sin \phi_R$     | 0.055          | $-0.914$        | $-0.419$     |
| $M_{\tilde{\chi}^\pm_1}$ (GeV) | 95.1            | 94.7            | 94.3         |
| $M_{\tilde{\chi}^0_1}$ (GeV) | 44.6            | 74.5            | 50.8         |
| $M_{\tilde{\nu}_e}$ (GeV) | 139.7       | 140.0           | 144.7        |
| $M_{\tilde{U}_{1(R)}}$ (GeV) | 284.9          | 689.2           | 315.2        |
| $M_{\tilde{U}_{2(L)}}$ (GeV) | 279.2          | 668.6           | 308.3        |
| $M_{\tilde{D}_{1(R)}}$ (GeV) | 291.5          | 691.9           | 321.2        |
| $M_{\tilde{D}_{2(L)}}$ (GeV) | 280.4          | 666.9           | 309.1        |
| parameter          | heavy $\tilde{\nu}_e$ case |                  |                  |
|--------------------|------------------------------|------------------|------------------|
|                    | gaugino region               | higgsino region  | mixed region     |
| $\tan\beta$        | 2                            | 2                | 2                |
| $\mu$ (GeV)        | $-215.0$                     | $-81.0$          | $-93.0$          |
| $M_2$ (GeV)        | $81.0$                       | $215.0$          | $92.0$           |
| $\cos \phi_L$     | $0.914$                      | $0.056$          | $0.436$          |
| $\sin \phi_L$     | $0.405$                      | $-0.998$         | $0.899$          |
| $\cos \phi_R$     | $0.998$                      | $0.405$          | $0.908$          |
| $\sin \phi_R$     | $0.056$                      | $-0.914$         | $-0.418$         |
| $M_{\tilde{\chi}^\pm}$ (GeV) | $94.7$             | $94.7$           | $94.3$           |
| $M_{\tilde{\chi}^0}$ (GeV)   | $44.3$                       | $74.5$           | $74.5$           |
| $M_{\tilde{\nu}_e}$ (GeV)    | $403.2$                      | $396.9$          | $396.9$          |
| $M_{\tilde{U}_{1(R)}}$ (GeV)  | $284.5$                      | $680.1$          | $392.6$          |
| $M_{\tilde{U}_{2(L)}}$ (GeV)  | $279.0$                      | $529.6$          | $249.0$          |
| $M_{\tilde{D}_{1(R)}}$ (GeV)  | $291.2$                      | $765.9$          | $487.9$          |
| $M_{\tilde{D}_{2(L)}}$ (GeV)  | $280.2$                      | $682.9$          | $397.4$          |

Table 2: Three parameter sets with a heavy $\tilde{\nu}_e$.  

| light $\tilde{\nu}_e$ case | gaugino region | higgsino region | mixed region |
|-----------------------------|----------------|-----------------|--------------|
|                             | $3.00 \times 10^{-5}$ | $9.08 \times 10^{-6}$ | $2.35 \times 10^{-5}$ |
| heavy $\tilde{\nu}_e$ case | $2.56 \times 10^{-5}$ | $8.96 \times 10^{-6}$ | $2.41 \times 10^{-5}$ |

Table 3: The total width (in GeV unit) of chargino corresponding to the six sets of SUSY parameters.

| 2-body×Br. | 6-body exact | Narrow-width Approx. |
|------------|--------------|-----------------------|
|            | 54 diag.     | 27 diag.              |
|            | full-spin    | diagonal              |
| $58.68(1)$ | $58.6(1)$    | $58.6(1)$             |
|            | $58.66(6)$   | $58.67(6)$            |

Table 4: The total cross sections (in fb unit) of the process $e^+e^- \rightarrow \tilde{\chi}_1^0 u \bar{d} \tilde{\chi}_1^0 s \bar{c}$ with the parameter set ‘gaugino region, light $\tilde{\nu}_e$’ at the CMS energy of 250 GeV. The number in parenthesis is the statistical error of the numerical integration on the last digit.
Beam parameters

|                           | \(2 \times 10^{10}/\text{bunch}\) |
|---------------------------|-----------------------------------|
| Number of particles      |                                   |
| beam size \(\sigma_x\)   | 553nm                             |
| \(\sigma_y\)             | 5nm                               |
| \(\sigma_z\)             | 0.4mm                             |
| beam momentum spread     | 1\%                               |

Table 5: The beam parameters used to calculate the beamstrahlung, taken from the TESLA reference parameter set for the CMS energy of 500 GeV. These parameters are used for the CMS energy of 250 GeV also.

Table 6: The total cross sections (in fb unit) with the six parameter sets including ISR and BS with the parameter set ‘gaugino region, light \(\tilde{\nu}_e\)’ at the CMS energy of 250 GeV. The number in parenthesis is the statistical error of the numerical integration on the last digit.
Figure 1: Feynman diagrams of a chargino pair-production (first row) and its decays (second row for $\tilde{\chi}_1^+$ and third row for $\tilde{\chi}_1^-$)
Figure 2: Distributions obtained from the exact calculations (solid lines), narrow-width approximation with full spin-correlation (dashed lines), and it with diagonal spin-correlation (dotted lines) with the parameter set ‘gaugino region, light $\tilde{\nu}_e$’ at the CMS energy of 250 GeV. The definitions of the variables are given in the text.
Figure 3: The correlation of azimuthal angles of the two decay planes of the chargino decays obtained from (a) the 54 diagrams exact calculation (upper-left), (b) the narrow-width approximation with full spin correlation (upper-right), and (c) it with diagonal spin correlation (down-left) with the parameter set ‘gaugino region, light $\tilde{\nu}_e$’ at the CMS energy of 250 GeV.
Figure 4: The effects of the detector resolution, ISR, and BS on the experimental observable used in the spin measurements with the parameter set ‘gaugino region, light $\tilde{\nu}_e$’ at the CMS energy of 250 GeV. Solid lines show the original distribution, dashed lines with smearing due to the limited resolution of the calorimeters, dotted lines with ISR and BS, and dot-dashed lines with all experimental effects. The parameter set of ‘gaugino region’ with light $\tilde{\nu}_e$ is used.
Figure 5: The distributions used for the chargino spin measurement with parameters of the ‘light $\tilde{\nu}_e$ case’ at the CMS energy of 250 GeV. The figures at left side show the original distribution and those at right side show those with the experimental effects. Solid lines show those with parameters of ‘gaugino region’, dashed lines with ‘higgsino region’, and dotted lines with ‘mixed region’. The error bars shown in the figures indicate the expected statistical errors after accumulating a luminosity of 10 fb$^{-1}$. All distributions are normalized to unity.
Figure 6: The distributions used for the chargino spin-spin correlation measurement with parameters of the ‘light $\tilde{\nu}_e$ case’ at the CMS energy of 250 GeV. The figures at left side show the original distribution and those at right side show those with the experimental effects. Solid lines show those with parameters of ‘gaugino region’, dashed lines with ‘higgsino region’, and dotted lines with ‘mixed region’. The error bars shown in the figures indicate the expected statistical errors after accumulating a luminosity of 10 fb$^{-1}$. All distributions are normalized to unity.
Figure 7: The distributions used for the chargino spin measurement with parameters of the ‘heavy $\tilde{\nu}_e$ case’ at the CMS energy of 250 GeV. The figures at left side show the original distribution and those at right side show those with the experimental effects. Solid lines show those with parameters of ‘gaugino region’, dashed lines with ‘higgsino region’, and dotted lines with ‘mixed region’. The error bars shown in the figures indicate the expected statistical errors after accumulating a luminosity of 10 fb$^{-1}$. All distributions are normalized to unity.