Remarks on Gribov mechanism on N=1 Supersymmetric 3D theories and the possibility of obtaining Gribov from one ABJM like Theory.

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Abstract

Some remarks on Gribov mechanism on N=1 Supersymmetric 3D theories are presented. The two point correlation function is analysed and the possibility of obtaining the confining Gribov regime is discussed. Also the possibility of obtaining Gribov behaviour in ABJM due to a symmetry breaking is presented.
1 Introduction

Three-dimensional Yang-Mills theory is one important model in which it is possible to analyse problems such as color confinement. The theory has local degrees of freedom and the coupling constant is dimensionful. This properties indicates that this theory can be seen as an approximation to the high temperature phase of QCD with the mass gap serving as the magnetic mass. One more interesting possibility is in order to study the deconfining one is the introduction of a Chern-Simons term, the addition of the topological Chern-Simons term has the effect of generating a de-confined massive excitation. It is also important to note that due to the supersymmetric extension is also possible to introduce fermions in a natural way. Of course this fermions are the gauge partners and are not in the fundamental representation as quarks. In spite of that they can be useful in order to understand the behaviour of confining fermions and the possibility of relations under Gribov and supersymmetric theories.

So in this paper we investigate the Super-Yang-Mills Chern-Simons (SYM-CS) theories \((N = 1, D = 3)\) with superfields formalism \([1]\) addressing the Gribov problem \([2, 3, 4, 5, 6]\) and the Gribov two-point correlator

\[
G(p^2) = \frac{p^2}{p^2 + \gamma^4}
\]

as well as a modified Gribov Zwanziger (GZ) \([7]\) type correlator

\[
G(p^2) = \frac{p^2 + M^2}{p^4 + (M^2 + m^2)p^2 + (M^2m^2 + \gamma^4)},
\]

thus obtaining information on how the theory behaves in the Gribov regime and how to obtain the Gribov regime in a closest relation to the ABJM scenario.

The paper is organized as follows: in Section 2, the SYM-CS theory in superspace \(D = 3, N = 1\) is presented, Landau gauge fixing is performed and the Gribov problem is analysed. In section 3 the Gribov Zwanziger local action is presented and a mathematical analysis of the value of the Gribov parameter is presented. In section 4 a possible symmetry breaking mechanism in order to obtain a Gribov behaviour in a ABJM type theory is presented.

2 Superfield approach to Gribov problem, \(N = 1, D = 3, \) SYM-CS theory.

2.1 \(N = 1, D = 3, \) Euclidean SYM-CS theory

In three-dimensional Minkowski space-time the Lorentz group is \(SL(2, R)\) (instead of \(SL(2, C)\)) and the corresponding fundamental representation acts on a two components real spinor (Majorana). So to formulate the superspace, we started with the introduction of spinorial coordinates \(\theta^\alpha\) (with \(\alpha = 1, 2\)) that are transformed under \(SO(1, 2)\). In the case of Euclidean \(D = 3\), the two components spinor shall be transformed under \(SO(3)\) and as is well known \([8, 9, 10]\) one can not have the usual Majorana condition. It’s the same question we are in \(D = 4\) \([11]\). In the same way we follow the approach of generalizing the concept of complex conjugation of Grassmann algebra \([12]\). The notations and conventions are in Appendix A.

In \(D = 3\) it is possible to add an additional gauge invariant term beyond the YM, the term CS \([13, 14]\), which is a topological mass term for the gauge field. Thus the pure supersymmetry \(N = 1\) version of this action must have both terms. Let us take the Euclidean version of this superspace action of SYM-CS \([15]\):

\[
S_{SYMCS} = S_{SYM} + S_{SCS},
\]

with,

\[
S_{SYM} = \frac{1}{2} \int d^3x d^2 \theta W^a \Gamma^a \theta
\]

and

\[
S_{SCS} = im \int d^3x d^2 \theta \left[ (D^\alpha \Gamma^a \theta)(D_\beta \Gamma^a \theta) + 2 \frac{g}{3} f^{abc} \Gamma^a \Gamma^b \Gamma^c (D_\alpha \Gamma^c) - \frac{1}{6} \theta^2 f^{abc} f^{cde} \Gamma^a \Gamma^b \Gamma^c \right].
\]
The field strength is given by:

\[ W^a_\alpha = D^\beta D_\alpha \Gamma^a_\beta + ig f^{abc} \Gamma^b_\gamma D^c_\delta \Gamma^a_\alpha - \frac{1}{3} g^2 f^{abc} f^{\gamma de} \Gamma^b_\beta \Gamma^d_\delta \Gamma^a_\gamma, \]  

(6)

and superspace derivative:

\[ D_\alpha = \frac{\partial}{\partial \theta^\alpha} + i \sigma^\mu \gamma^\alpha \epsilon^\gamma_\beta \theta^\beta \partial^\mu. \]  

(7)

The supermultiplet of gauge fields is given by the components of the spinor superfield, in Wess-Zumino gauge:

\[ \Gamma^a_\alpha(x, \theta) = i \sigma^\mu \gamma^\alpha \epsilon^\gamma_\beta \theta^\beta A^a_\mu(x) + i \theta^2 \lambda^a_\alpha(x). \]  

(8)

They belong to the adjoint representation of the gauge group SU(N).

2.2 Gauge-fixing

In order to quantize the theory correctly we have to fix the gauge and we can do covariantly using the usual procedure of Faddeev-Popov (FP) \[1,15,16\].

In the supersymmetric Landau gauge we must implement the conditions \( D^\alpha \Gamma^a_\alpha = 0 \). And following the usual procedure we ended with the action of gauge fixing

\[ S_{gf} = \frac{1}{4} s \{ d^3 x d^2 \theta (c^a D^\alpha \Gamma^a_\alpha) \}, \]  

(9)

where the Faddeev-Popov ghost fields will be scalar superfield \( c^a \) and \( c^\alpha \) are the antighost and the ghost respectively. And \( s \) is the BRST nilpotent operator \( (s^2 = 0) \).

The total action \( S = S_{SYM} + S_{gf} \) is invariant under the BRST transformations \[15\]:

\[ s \Gamma^a_\alpha = (\nabla_\alpha c)^a, \]

\[ sc^a = -i \frac{1}{2} g f^{abc} b^c, \]

\[ sc^a = b^a, \]

\[ sb^a = 0, \]  

(10)

with \( s \) carrying ghost number 1.

The ghost part of gauge fixing action becomes:

\[ S_{FP} = -\frac{1}{4} s \{ d^3 x d^2 \theta (c^a \nabla^a_\alpha c^b), \]  

(11)

with superspace covariant derivative:

\[ \nabla^a_\alpha = \delta^a_\beta D_\alpha + g f^{abc} \Gamma^c_\alpha \]

In order to calculate the propagator for the gauge superfield \( \Gamma_\alpha \), we need only the bilinear of S. So, for the bilinear part, we have:

\[ S_{SYM} = \frac{1}{2} \int d^3 x d^2 \theta (D^\beta D^a_\alpha \Gamma^a_\beta)(D^\gamma D_\delta \Gamma^a_\gamma) \]

\[ = \frac{1}{2} \int d^3 x d^2 \theta \Gamma^a_\beta D^\beta D^\gamma D_\delta \Gamma^a_\gamma, \]

and using \[80,88,91,92\):

\[ S_{SYM} = \int d^3 x d^2 \theta \Gamma^a_\beta D^\beta D^\gamma D^\beta \Gamma^a_\gamma, \]  

2
and for Chern-Simons:

\[
S_{SCS2} = im \int d^3x d^2\theta \left[ (D^a \Gamma^{a\beta})(D_\beta \Gamma^a_\alpha) \right] = im \int d^3x d^2\theta \left[ \Gamma^a_\beta D^\gamma D^\delta \Gamma^a_\gamma \right].
\]

So, with \( A = D^2 + im \):

\[
(AD^\gamma D^\beta + \frac{1}{\xi} D^\beta D^\gamma)(a_1 D_\beta D_\lambda + a_2 D_\lambda D_\beta) = \delta^\gamma_\lambda,
\]

where \( \xi \) is a gauge parameter to be set to zero after having evaluated the gauge propagator. Using \([88, 92, 80, 91]\) and \([89]\), (in Landau gauge \( \xi \rightarrow 0, a_2 = 0 \)) we get the inverse

\[
\frac{1}{2A\partial^2} D_\beta D_\lambda,
\]

so the massive gauge propagator for SYM-CS:

\[
< \Gamma^a_\alpha(1) \Gamma^b_\beta(2) > = \frac{\delta^{ab}}{\partial^2(-\partial^2 + m^2)}(D^2 - im)D_\beta D_\alpha \delta_S(1, 2),
\]

with \( \delta_S(1, 2) \) given by \([94]\).

### 2.3 Gribov problem

In order to clarify the understanding of the Gribov problem let us start by giving a briefly overview of the GZ framework in \( D = 4 \) \([2, 3, 4, 6]\). The Euclidean \( SU(N) \) Yang-Mills action in the Landau gauge is given by:

\[
S_{YM} = \int d^4x \left( \frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} + ib^a \partial_\mu A^a_\mu + \epsilon^a \partial_\mu D^{ab}_\mu c^b \right),
\]

where

\[
F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu,
\]

and

\[
D^{ab}_\mu = \delta^{ab} \partial_\mu - gf^{abc} A^c_\mu.
\]

Here, \( A^a_\mu \) is the gauge field, \( b^a \) is a Lagrange multiplier enforcing the Landau gauge, \( \partial_\mu A^a_\mu = 0 \), \( (\epsilon^a, c^a) \) are a pair of anti-commuting scalar fields known as the Faddeev-Popov ghost fields, and \( g \) is the coupling constant of the theory. The labels \( (a, b, c, \ldots) \) run to \( 1 \) to \((N^2 - 1)\) and \( f^{abc} \) are the totally anti-symmetric structure constant of the Lie algebra of the generator of \( SU(N) \). Also, this action is left invariant under the following nilpotent BRST transformations:

\[
s A^a_\mu = -D^{ab}_\mu c^b, \quad sc^a = \frac{g}{2} f^{abc} c^b c^c, \quad se^a = ib^a, \quad sb^a = 0.
\]

Although the gauge be fixed by the Faddeev-Popov method, Gribov showed in \([2]\) that there are still field configurations obeying the Landau gauge linked by gauge transformations, \( i.e. \) there are still equivalent configurations, or copies, being taken into account into the Feynman path integral. In other words, the gauge is not completely fixed and the remaining ambiguity is allowed due to the existence of normalizable zero-modes of the Faddeev-Popov operator,

\[
\mathcal{M}^{ab} = -\partial_\mu D^{ab}_\mu.
\]

Gribov also showed that to eliminate these copies the domain of integration of the functional integral should be restricted to a certain region \( \Omega \), the so-called Gribov region, that is defined as the set of field configurations performing the Landau gauge condition, for which the Faddeev-Popov operator is strictly positive, namely

\[
\Omega := \{ A^a_\mu | \partial_\mu A^a_\mu = 0, \mathcal{M}^{ab}(A) > 0 \}.
\]

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1. See also \([17]\) for a pedagogical review.
Its boundary, $\partial \Omega$, where the first vanishing eigenvalue of the Faddeev-Popov operator shows up, is known as the Gribov horizon.

As in the region $\Omega$ the Faddeev-Popov operator is positive than its inverse must diverge when approaching the horizon, due to the existence of a zero mode. So the restriction to the first Gribov region is implemented requiring that

$$G(p^2, A) = \frac{\delta^{ab}}{N^2 - 1} \langle p | (\partial_{\mu} D_{\mu}^{ab})^{-1} | p \rangle, \quad (19)$$

which is the normalized trace of the ghost connected two point function in momentum space, has no pole for a given nonvanishing value of the momentum $p$, except for the singularity at $p = 0$, corresponding to the first Gribov horizon. At $p \approx 0$ one can write

$$G(p^2, A) \approx \frac{1}{p^2} \frac{1}{1 - \sigma(p^2, A)}, \quad (20)$$

$$\sigma(p^2, A) = \frac{N}{N^2 - 1} \frac{1}{p^2} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(p - q)^2} A^a_{\mu}(-q) A^a_{\nu}(q). \quad (21)$$

From the above expression (21), it follows that the no-pole condition at finite nonvanishing $p$ is

$$\sigma(p^2, A) < 1. \quad (22)$$

As $\sigma(p^2, A)$ decreases as $p^2$ increases one can also take

$$\sigma(0, A) = \frac{1}{4} \frac{N}{N^2 - 1} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^2} (A^a_{\mu}(-q) A^a_{\mu}(q)) \leq 1. \quad (23)$$

It is important to emphasize here that we work with the trace of the ghost propagator to find the restriction to the Gribov region. This is a particularity of the Gribov mechanism in the Landau gauge. This is related to the convexity of the Gribov region in the Landau gauge. Other gauges, like the maximal Abelian gauge, does not necessarily present the same property.

In order to perform the restriction to the Gribov region into the partition function, $Z$, the final step is to introduce the no-pole condition with the help of a Heaviside function:

$$Z = \int \mathcal{D} A \delta(\partial A) \theta(1 - \sigma(0, A)) \exp^{-S_{YM}}. \quad (24)$$

This will give rise to a propagator for the gauge field of the type

$$\langle A^a_{\mu}(-q) A^b_{\nu}(q) \rangle = \delta^{ab} \frac{q^2}{q^4 + \gamma^4} \left( \delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right), \quad (25)$$

Note that the only allowed singularity at $p^2 = 0$, whose meaning is that of approaching the horizon, where $G(p^2, A)$ is singular due to the appearance of zero modes of the Faddeev-Popov operator. Thus we have to take $\sigma[2]$

$$\sigma(0, A) = 1. \quad (26)$$

And thus the Gribov parameter $\gamma$ is fixed by the gap equation

$$\frac{3N g^2}{4} \int \frac{d^4 q}{(2\pi)^4} \frac{1}{q^4 + \gamma^4} = 1. \quad (27)$$

It is clear that the Gribov approach is only the first step in order to consistently treat the problem of zero modes and the Gribov copies in a gauge fixed Yang-Mills theory. The second step is the GZ theory [5][6], which consists in a renormalizable and local way to implement the restriction to the first Gribov region. In fact, Zwanziger observed that the restriction could be implemented by adding the following term in the action [13]:

$$S_{GZ} = S_{YM} + \gamma^4 H(A), \quad (28)$$

\[2\text{In the maximal Abelian gauge we take only the trace of diagonal ghost propagator.}\]
where, $H(A)$ is the so-called horizon function,

$$H(A) = g^2 \int d^4 x \, d^4 y \, f^{abc} A^b_\mu (x) [M^{-1}]^{ad}(x, y) f^{dec} A^e_\mu . \quad (29)$$

In the Zwanziger approach, the parameter $\gamma$ is fixed by the equation

$$\langle H(A) \rangle = 4V (N^2 - 1), \quad (30)$$

where $V$ is the Euclidean space volume. Notice that the Gribov form factor (23) coincides with the first order of the horizon function \(^3\):

$$H(A) = 4V (N^2 - 1) = \sigma(0, A) + O(A^3). \quad (31)$$

It is clear that the horizon function is nonlocal, but it can be localized with the help of a suitable set of auxiliary fields. In order to ensure that those extra fields do not introduce extra degrees of freedom they are introduced in the form of a BRST quartet \(^4\):

$$s\bar{\omega}^{ab} = \bar{\varphi}^{ab}, \quad s\bar{\varphi}^{ab} = 0, \quad s\omega^{ab} = 0, \quad s\varphi^{ab} = 0, \quad (32)$$

where $(\bar{\varphi}, \varphi)$ are a pair of complex commuting fields, while $(\bar{\omega}, \omega)$ are anti-commutating ones. Now, the local version of the GZ action is then given by:

$$S_{local}^{GZ} = S_{YM} + \int d^4 x \left[ \bar{\varphi}^{abc} M^{bc}_{\mu \nu} \varphi^{abc} - \bar{\omega}^{abc} M^{abc}_{\mu \nu} \omega^{abc} + \gamma^2 g f^{abc} (\varphi^{abc} - \bar{\varphi}^{abc}) A^{c}_\mu \right]. \quad (33)$$

Similarly to YM we explicitly have shown that Gribov problem also exists in SYM in Landau gauge \([11]\). In the case of YM-CS the Gribov problem has been studied in original semi-classical approach of Gribov in \([23]\).

We note that the usual Faddeev-Popov operator is the component $\theta^2$ of the supersymmetric FP operator that appears \([11]\), so we can get directly:

$$M^{abc} = D^2 D^a \nabla^b c \quad (34)$$

as the natural supersymmetric generalization in $D = 3$ of the usual FP operator. So we can generalize the horizon function \([33]\) taking $[M^{-1}]^{ad}$ to be the inverse of the operator \([34]\)

$$H(\Gamma^a) = \int d^2 \theta \int d^3 x \, d^3 y \, f^{abc} \Gamma^b (x) \left[ \frac{\varepsilon^{a\beta}}{D^2 D^a \nabla^c_{\beta}} \right]^{ad} (x, y) f^{dec} \Gamma^e (y) . \quad (35)$$

Thus following the usual GZ procedure, addressing the problem of Gribov, in our case, evolves insert in the action the nonlocal term above what can be done consistently as we shall see in the next section.

### 3 Gribov-Zwanziger local action on superspace

To localize the operator \([34]\) we consider introducing auxiliary superfields in the form of one quartets of BRST, with spinor superfield:

$$sw'_\alpha = u'_\alpha, \quad su_\alpha = w_\alpha \quad (36)$$

At this point is important for our construction to show the ultraviolet dimension and ghost number of all fields and operators which are in Table \([11]\):
Thus, keeping in mind the non-supersymmetric approach, we propose the super GZ action:

$$S_{SGZ} = tr \int d^4 x d^2 \theta \left[ s(w'_\alpha \bar{\epsilon}^\gamma \bar{\gamma}^\beta D^2 D^a \nabla_a u_{\beta}) + 4 \gamma^2 \Gamma_{\gamma}^\epsilon \gamma^\beta (u'_{\beta} - u_{\beta}) \right],$$

where $\gamma^2$ is a mass parameter, which should be determined by the theory, shown below, must be nonzero. And the total action is:

$$S = S_{SYMCS} + S_{\beta f} + S_{SGZ}.$$  \hspace{2cm} (38)

With GZ action generalization at our disposal we can now calculate the propagators and ensure they have the expected behavior that occurs in confining YM theories and analyze other features it adds to SYM.

Before starting the calculation of the super propagator it is important to emphasize here that the Yang-Mills-Chern-Simons is a renormalizable gauge theory and the BRST breaking in the Gribov procedure is a soft breaking that does not spoil the renormalizability, explicitly proven in $D = 4$ 

Due to power counting, the Gribov procedure must also be renormalizable in $D = 3$.

### 3.1 The super gauge propagator

First we calculate the propagator for gauge superfield $\Gamma_\alpha$. To calculate the gauge propagator we need only the bilinear of $S$ like to calculate \((12)\). Thus, for $S_{SGZ}$, we have (with \((12)\):

$$S_{SGZ2} = tr \int d^4 x d^2 \theta (-u'_{\alpha} \bar{\epsilon}^\gamma \bar{\gamma}^\beta \partial^2 u_{\beta} + w'_{\alpha} \bar{\epsilon}^\gamma \partial^2 w_{\beta} + 2 \gamma^2 \Gamma_{\gamma}^\epsilon \gamma^\beta (u'_{\beta} - u_{\beta})).$$

With give one contribution to bilinear term

$$S_{SGZ2} = tr \int d^4 x d^2 \theta \Gamma_{\gamma}^\epsilon \gamma^\beta \frac{2 \gamma^4}{\partial^2} \bar{\epsilon}^\gamma \gamma^\beta.$$  \hspace{2cm} (39)

Similar to SYM-CS propagator calculus, with $A = D^2 + im$ and $B = \frac{2 \gamma^4}{\partial^2}$:

$$(AD^\gamma D^\beta + \frac{1}{\xi} D^\beta D^\gamma + B\epsilon^{\gamma\beta})(a_1 D_\beta D_\lambda + a_2 D_\lambda D_\beta) = \delta_\lambda^\gamma,$$

and we get the inverse:

$$\frac{1}{2A\partial^2 + BD^2} D_\beta D_\lambda,$$

or

$$- \frac{1}{2} \left[ (\partial^4 + \gamma^4) + im\partial^2 D^2 \right] D^2 D_\beta D_\lambda.$$  \hspace{2cm} (43)

So the gauge propagator for SYM-CS-GZ:

$$< \Gamma^\alpha_\alpha(1) \Gamma^\beta_\beta(2) >= \frac{1}{2} \delta^{ab} \left[ \frac{(\partial^4 + \gamma^4) + im\partial^2 D^2}{-(\partial^4 + \gamma^4) + m^2(\partial^2)^2} \right] D^2 D_\beta D_\alpha \delta_S(1, 2),$$

with $\delta_S(1, 2)$ given by \([14]\).

To see how the introduction of $S_{SGZ}$ brings light on confinement of both bosons as fermions and to compare with literature, we shall observe the propagators in field components.

| fields and operators | $\theta^\alpha$ | $D_\alpha$ | $\Gamma_\alpha$ | $c'$ | $c$ | $b'$ | $w'$ | $w$ | $u'$ | $u$ | $g$ |
|---------------------|----------------|------------|----------------|-----|-----|-----|-----|-----|-----|-----|-----|
| UV dimension        | $-\frac{1}{2}$ | $\frac{3}{2}$ | $0$            | $\frac{1}{2}$ | $\frac{3}{2}$ | $-\frac{1}{2}$ | $\frac{3}{2}$ | $-\frac{1}{2}$ | $\frac{3}{2}$ | $\frac{3}{2}$ | $-\frac{1}{2}$ |
| Ghost number        | $0$            | $0$        | $0$            | $-1$ | $1$  | $0$  | $-1$ | $1$  | $-1$ | $1$  | $0$  |

Table 1: Quantum numbers of fields and operators.
Using (49) and taking components from (3) we can project the propagator for the gauge field $A_\mu$: 

$$<A_\mu^a(x_1)A_\nu^b(x_2)> = \delta^{ab} \left[ \left( \partial^4 + \gamma^4 \right) \left( \partial^2 - m^2 \right) \right] (\delta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\partial^2}) - \frac{2im\partial^2 \varepsilon_{\mu\nu\alpha\beta} \partial_\sigma}{(\partial^4 + \gamma^4)^3} \delta^3(x_1 - x_2),$$  

(45)

and gaugino $\lambda^\alpha$: 

$$<\lambda_\alpha^a(x_1)\lambda_\beta^b(x_2)> = \frac{1}{4} \delta^{ab} \left[ \left( \partial^4 + \gamma^4 \right) \left( \partial^2 - m^2 \right) \right] (\partial^2 \partial_{\beta\alpha} - \frac{im(\partial^2)^3 \varepsilon_{\beta\alpha}}{(\partial^4 + \gamma^4)^3}) \delta^3(x_1 - x_2).$$  

(46)

And we found that both show behavior in limit $m = 0$ as occurs for gauge field in non-supersymmetric theories (1). However as we'll see in the next section the Gribov parameter $\gamma$ can be determined as a function of coupling constant $g$ such that these propagators will be function of the two parameters, $g$ and $m$ (in the case $m \neq 0$), and so it is possible the study of phases involving this theory. The phases for this type of propagator was studied in (23) through the analysis of the poles in this propagator.

3.2 Ghost propagators and $\gamma$ parameter

Since the action (38) only makes sense if the $\gamma$ parameter is nonzero, we will now explicitly show that it is not independent in this theory. Its determination is closely linked to the restriction of the functional integration to the first Gribov region, which we will discuss some details here.

First, it is noteworthy that in the literature dealing with the Gribov problem in YM theories there are recent consensus on the scenario of dominance of configurations on the Gribov horizon on the Landau gauge (29), so that the restriction to the first Gribov region is, in practice, to take the configurations on the horizon, ie where occur the zeros modes of the FP operator. Second, and as we have pointed out in the introduction of super GZ, calculate the propagator of the ghosts is to take the inverse of these operators. So we focus on these calculus to one loop order to establish the one loop gap equation Gribov style.

In order to characterize the integration in the first Gribov region it is important to remember that the two point ghost function is essentially the inverse of the Faddeev-Popov operator and the zero eigenvalue of the Gribov equation corresponds to a exactly to the Gribov frontier. In these sense the two point ghost function goes to infinity at the Gribov frontier. These condition is the most simpler way to obtain the gap equation for $\gamma$. These procedure is explained in details in (2) and is easily extended to the $N = 1$ supersymmetric case (11). First we need to calculate the two point ghost function. Using perturbation theory these is at first order of the form:

\[ \Delta_{cc}^{(1,2)} = \frac{1}{p^2} \delta^{ab} D_1^2 \delta^2(\theta_1 - \theta_2). \]

(47)

And we can define in momentum space, the one loop corrected ghost propagator as

\[ G_{cc}^{ab} = (G_{cc}^{ab} + G_{cc}^{ab}). \]

(48)

according to diagram above. With $G_{cc}^{ab}$ given from (47).

Using the Feynman rules and D algebra from (11) (and due $f^{abcd} f^{bcd} = N \delta^{ab}$):

\[ G_{cc}^{ab} = (2\pi)^3 g^2 N \delta^{ab} \int d^2 \theta_3 \int d^2 \theta_4 \frac{1}{p^2} D_1^2 \delta^2(\theta_1 - \theta_3) \]

\[ D_1^2 \left\{ \int \frac{d^3 k}{(2\pi)^3} \left( k^4 + \gamma^4 \right) \frac{1}{(p - k)^2} \left( 1 + \frac{imk^2 D_2^2}{(k^4 + \gamma^4)} \right) D_3^2 D_3 D_3 \delta^2(\theta_3 - \theta_4) D_3^2(p - k) \delta^2(\theta_3 - \theta_4) \right\} \]

\[ \frac{1}{p^2} D_1^2 \delta^2(\theta_4 - \theta_2). \]

(49)
And after delta functions and D derivatives manipulations [31], we have:

\[ G^{ab}_{c'c} = -4(2\pi)^3 g^2 N \delta^{ab} \frac{1}{p^2} D^2_1 \delta^2(\theta_1 - \theta_2) \int \frac{d^3 k}{(2\pi)^3} \left( \frac{(k^4 + \gamma^4)}{(k^4 + \gamma^4)^2 + m^2 k^6} \right) \frac{k^2}{(p - k)^2}. \]  

(50)

Next we define:

\[ \sigma(\gamma^2, p^2, m^2) = 2(2\pi)^3 g^2 N \int \frac{d^3 k}{(2\pi)^3} \left( \frac{(k^4 + \gamma^4)}{(k^4 + \gamma^4)^2 + m^2 k^6} \right) \frac{k^2}{(p - k)^2}. \]  

(51)

Therefore, from (48):

\[ G^{ab}_{c'c} = -2\delta^{ab} \frac{1}{p^2} D^2_1 \delta^2(\theta_1 - \theta_2)(1 + \sigma). \]  

(52)

Re-summing the one-particle irreducible diagrams gives:

\[ G^{ab}_{c'c} = -2\delta^{ab} \frac{1}{p^2} D^2_1 \delta^2(\theta_1 - \theta_2) \frac{1}{(1 - \sigma)}. \]  

(53)

Now, as we are interested in the low momentum behavior we analyze the behavior of \((1 - \sigma)\) we get:

\[ \sigma(\gamma^2, 0, m^2) = 2(2\pi)^3 g^2 N \int \frac{d^3 k}{(2\pi)^3} \left( \frac{1}{k^4 + \gamma^4} \right). \]  

(54)

According to the above discussion of the scenario of dominance of configurations on the Gribov horizon, i.e. the ghost propagator (the inverse of FP operator) going to infinity, \((1 - \sigma) = 0\), we have to get the greatest value of the above integral which is with \(m = 0\) as we can see in the integral graph shown in Figure 1:

Figure 1: Graph of integral presented in (54)

So we should take

\[ \sigma(\gamma^2, 0, 0) = 2(2\pi)^3 g^2 N \int \frac{d^3 k}{(2\pi)^3} \left( \frac{1}{k^4 + \gamma^4} \right). \]  

(55)

And so we are able to define the one loop gap equation:

\[ 2(2\pi)^3 g^2 N \int \frac{d^3 k}{(2\pi)^3} \left( \frac{1}{k^4 + \gamma^4} \right) = 1. \]  

(56)

Thus the \(\gamma\) parameter is not independent, being defined as a function of the coupling constant \(g\):

\[ \gamma = \sqrt{2\pi^2 N g^2}. \]  

(57)

It is clear that in close analogy to the Gribov-Zwanziger procedure [4, 5, 6] it is also possible to work directly with the gap equation \(\frac{d\Gamma}{d\gamma} = 0\). The results will be the same as in the more simple method explained in these section. The behaviour of the gauge and gaugino correlators is the same as presented in [23].

8
4 Some aspects of $\mathbb{N} = 1$ Chern-Simons-Matter Theories

Now we will consider only the Chern-Simons sector. With interest in BLG and ABJM theories. Many works about BLG and ABJM exists in the literature like [32, 33, 34] and recently about BRST breaking in ABJM theory [35]. So we would focus on the possibility of a replica model [36] for confinement using the two Chern-Simons sectors of theories of that type. So considering only the Chern-Simons sector, we have that the ultraviolet mass dimension of gauge superfield $\tilde{\Gamma}^a_\alpha$ becomes $\frac{1}{2}$. In this case we rewrite the action (5) as

$$S_{SCS} = ik \int d^3x d^2\theta \left( (D^\alpha \tilde{\Gamma}^a_\beta)(D_\beta \tilde{\Gamma}^c_\alpha) + \frac{2}{3} if^{abc} \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}^{b\beta}(D_\beta \tilde{\Gamma}^c_\alpha) - \frac{1}{6} f^{abc} f^{cde} \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}^{b\beta} \tilde{\Gamma}^{d\gamma} \tilde{\Gamma}^{e\delta} \right).$$

(58)

As we are interested in $\mathbb{N} = 1$ supersymmetric gauge field theory with the gauge group $G \times G$, we write a second action for another gauge superfield $\tilde{\Gamma}^a_\alpha$

$$\tilde{S}_{SCS} = ik \int d^3x d^2\theta \left( (D^\alpha \tilde{\Gamma}^a_\beta)(D_\beta \tilde{\Gamma}^c_\alpha) + \frac{2}{3} if^{abc} \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}^{b\beta}(D_\beta \tilde{\Gamma}^c_\alpha) - \frac{1}{6} f^{abc} f^{cde} \tilde{\Gamma}^{a\alpha} \tilde{\Gamma}^{b\beta} \tilde{\Gamma}^{d\gamma} \tilde{\Gamma}^{e\delta} \right).$$

(59)

And we take the following total Chern-Simons action with matter:

$$S = S_{SCS} - \tilde{S}_{SCS} + S_{matter}. \quad (60)$$

With the matter action given by

$$S_{matter} = \int d^3x d^2\theta tr \left( \nabla^\alpha X^I \nabla_\alpha X_I + V \right), \quad (61)$$

with the matter superfield $X$ in the bi-fundamental representation of the gauge group, i.e the superspace covariant derivatives for matrix-valued complex scalar superfields $X^I$ and $X_I$ are defined by

$$\nabla_\alpha X^I = D_\alpha X^I + i \Gamma_\alpha X^I - i X^I \tilde{\Gamma}_\alpha,$$

$$\nabla_\alpha X_I = D_\alpha X_I - i X_I \Gamma_\alpha + i \tilde{\Gamma}_\alpha X_I,$$

(62)

and $V$ is the potential term given by

$$V = \frac{1}{k} \epsilon^{IJKL}[X_I X^K X_J X^L]. \quad (63)$$

The classical Action (60) remains invariant under the following gauge transformation

$$\delta \Gamma_\alpha = \nabla_\alpha \Lambda, \quad \delta \tilde{\Gamma}_\alpha = \tilde{\nabla}_\alpha \tilde{\Lambda},$$

$$\delta X^I = i(\Lambda X^I - X^I \Lambda), \quad \delta X_I = i(\tilde{\Lambda} X_I - X_I \tilde{\Lambda}), \quad (64)$$

where $\Lambda = \Lambda^A T_A$ and $\tilde{\Lambda} = \tilde{\Lambda}^A \tilde{T}_A$ are parameters of transformations.

In terms of the field components this action can represents the gauge part of the ABJM or BLG : with gauge group $G = SU(2)$, we can have a decomposition of BLG theory (It is possible to decompose the gauge symmetry generated by $SO(4)$ into $SU(2) \times SU(2)$) and with $G = U(N)$, we can have ABJM theory [32, 33, 34, 35]. In both cases we have $G(N)_k \times G(N)_{\pm k}$, with $k$ the Chern-Simons level. We will consider this theory with $G = SU(2)$ in a regime with levels $\pm k$, $SU(2)_k \times SU(2)_{-k}$, namely $S = S_{SCS} - (\tilde{S}_{SCS})_{-k} + S_{matter}$, such that we will work the action

$$S = S_{SCS(k)} + \tilde{S}_{SCS(k)} + S_{matter}. \quad (65)$$

The gauge invariance of this theory reflects that the theory have some spurious degrees of freedom. In order to quantize the theory correctly we need to fix the gauge. And we can do with the Faddeev-Popov method already studied, ending with an action of gauge fixing for each superfield $\Gamma^a_\alpha$ and $\tilde{\Gamma}^a_\alpha$, equation (9) [35].

We know that the solution of the Gribov problem for a Chern-Simons theory is trivial because the theory does not have metric [23]. But now we will see that in this case with two Chern-Simons interacting with bi-fundamental matter, we can have a spontaneous symmetry breaking such that insert a metric, but leading to Gribov confinement...
scenario at level of propagator, namely, we get a Gribov type propagator after the break. So, from \(61\), in SU(2), the bilinear part of coupling term is

\[
S_{\text{matter}2} = \frac{1}{2} \int d^4x d^2 \theta \left( f_{abc} X_b^{\gamma} \Gamma_c^{\alpha} - f_{abc} \tilde{\Gamma}_c^{\beta} X^c \right) \left( -f_{abc} \Gamma_{\alpha}^{\beta} X^c + f_{abc} X_b^{\dag} \tilde{\Gamma}_c^{\alpha} \right).
\]  

By giving an expectation value to the scalar \( X \) it is possible to spontaneously break the gauge group to its diagonal subgroup \(35\), and we choose a break in the direction 3 in this way

\[
<X >^1 = i \rho \delta_3^a,
\]

\[
<X >^a = -i \rho \delta_3^a.
\]

Where \( \rho^2 \) has the dimension of mass, and we get

\[
S_{\text{matter}2} = \frac{1}{2} \int d^4x d^2 \theta \left( \rho^2 \Gamma^{\gamma\alpha} \Gamma_c^{\alpha} - \rho^2 \Gamma^{3\alpha} \Gamma_3^{\alpha} + \rho^2 \tilde{\Gamma}_c^{\beta} \tilde{\Gamma}_c^{\alpha} - \rho^2 \Gamma^{3\alpha} \tilde{\Gamma}_c^{\alpha} + 2 \rho^2 \Gamma^{\gamma\alpha} \Gamma_3^{\alpha} - 2 \rho^2 \Gamma^{3\alpha} \tilde{\Gamma}_c^{\alpha} \right).
\]  

So we get for full bilinear part of the total action in the directions \( i=1,2 \)

\[
\int d^4x d^2 \theta \left[ ik(D^\alpha \Gamma^{i\beta})(D\beta \Gamma_{\alpha}^i) + ik(D^\alpha \tilde{\Gamma}^{i\beta})(D\beta \tilde{\Gamma}_{\alpha}^i) + \frac{1}{2} \left( \rho^2 \Gamma^{1\alpha} \Gamma_3^{\alpha} + \rho^2 \tilde{\Gamma}_c^{\beta} \tilde{\Gamma}_c^{\alpha} + 2 \rho^2 \Gamma^{1\alpha} \tilde{\Gamma}_c^{\alpha} \right) \right]
\]

\[
= \int d^4x d^2 \theta \left[ ik(D^\alpha \Gamma^{i\beta})(D\beta \tilde{\Gamma}_{\alpha}^i) + ik(D^\alpha \tilde{\Gamma}^{i\beta})(D\beta \tilde{\Gamma}_{\alpha}^i) + \frac{1}{2} \left( \rho^2 \varepsilon^{\gamma\alpha\beta} \Gamma_3^{\gamma} \Gamma_{\alpha}^{\beta} + \rho^2 \varepsilon^{\alpha\beta\gamma} \tilde{\Gamma}_c^{\gamma} \tilde{\Gamma}_{\alpha}^{\beta} + 2 \rho^2 \varepsilon^{\alpha\beta\gamma} \Gamma_3^{\gamma} \tilde{\Gamma}_{\alpha}^{\beta} \right) \right].
\]

The variation with respect to \( \Gamma_{\alpha}^i \) and \( \tilde{\Gamma}_{\alpha}^i \) give

\[
\begin{align*}
&ikD^\alpha \Gamma^{i\beta}(D\beta \Gamma_{\alpha}^i) + \frac{1}{2} \rho^2 \varepsilon^{\gamma\alpha\beta} \Gamma_3^{\gamma} \Gamma_{\alpha}^{\beta} = 0, \\
&ikD^\alpha \tilde{\Gamma}^{i\beta}(D\beta \tilde{\Gamma}_{\alpha}^i) + \frac{1}{2} \rho^2 \varepsilon^{\alpha\beta\gamma} \tilde{\Gamma}_c^{\gamma} \tilde{\Gamma}_{\alpha}^{\beta} = 0.
\end{align*}
\]  

By isolating \( \tilde{\Gamma}_{\alpha}^i \) in equation (70) and substituting in (71), we obtain the operator that we need to invert to obtain the propagator for \( \Gamma_{\alpha}^i \) (and similarly for \( \tilde{\Gamma}_{\alpha}^i \))

\[
(k^2 D^2 + i \rho^2) D^\alpha D^\beta - \frac{3}{4} \rho^4 \varepsilon^{\alpha\beta}.
\]

By inverting this operator in the same way of \(41\) and \(42\) we get the propagator in momentum space of the type of \(44\)

\[
\begin{align*}
&<\Gamma_{\alpha}^i(1)\Gamma_{\beta}^j(2)> = - \frac{1}{2} \delta^{ij} \left[ \frac{\left( k^2 \rho^2 + \frac{3}{8} \rho^4 \right) + i \rho^2 D^2}{k^4 (p^2 + \frac{\rho^2}{4})^2 + \frac{1}{16} \rho^4} \right] D^2 D_\beta D_\alpha \delta_S(1,2). \\
&<\Gamma_{\alpha}^i(1)\tilde{\Gamma}_{\beta}^j(2)> = - \frac{1}{2} \delta^{ij} \left[ \frac{p^2 + \frac{3}{8} \rho^4}{(p^2 + \frac{\rho^2}{4})^2 + \frac{1}{16} \rho^4} \right] D^2 D_\beta D_\alpha \delta_S(1,2).
\end{align*}
\]

In the strong coupling regime (taking \( k=1 \)) this propagator has two parts, one of Yang-Mills type and other of Chern-Simons, and the pole is of Gribov type, more precisely one found by the refined Gribov-Zwanziger theory \(2\)

\[
<\Gamma_{\alpha}^i(1)\Gamma_{\beta}^j(2)> = - \frac{1}{2} \delta^{ij} \left[ k^2 \rho^2 D^2 \left( \frac{1}{(p^2 + \frac{\rho^2}{4})^2 + \frac{1}{16} \rho^4} \right) \right] D^2 D_\beta D_\alpha \delta_S(1,2).
\]

It is important to stress that this propagator has a refined Gribov-Zwanziger behaviour related to the choice of the regime \(SU(2)_k \times SU(2)_{-k}\). This particular regime gives rise to \( S = S_{SCS} - (\tilde{S}_{SCS})_k \) for the pure gauge sector. One different choice, like \( S = S_{SCS} - (\tilde{S}_{SCS})_k \) with imaginary \( k \), is responsible for a massive particle usually obtained in the SYM without Gribov. And can be calculated with the same procedure as above, namely

\[
<\Gamma_{\alpha}^i(1)\Gamma_{\beta}^j(2)> = \frac{1}{2} \delta^{ij} \left[ \frac{1}{k^2 \rho^2} \right] D^2 D_\beta D_\alpha \delta_S(1,2).
\]

The meaning of this case have only a term of type SYM and still need \( k \) imaginary, demands further research for better understanding.
5 Conclusions

In this work we have studied the Super-Yang-Mills Chern-Simons theory, \( N = 1 \), considering the Gribov problem present in YM theories as well as a generalization of the Gribov-Zwanziger approach with auxiliary superfields in superspace. A local supersymmetric Gribov-Zwanziger sector is presented providing the starting point in order to implement the restriction to the first Gribov region beyond one-loop order. Also a possible mechanism in order to obtain a Gribov like propagator from a ABJM theory is presented by making use of a spontaneous symmetry breaking mechanism similar to the one proposed in [37]. This mechanism offers a possibility of seeing Gribov as a phase in ABJM theory, offering a link between confinement in Gribov scenario and confinement as seeing by the brane scenario. It is clear that this is only the first small step in order to study this possibility. We expect to study the implications of this relation in future works.

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A Notation, conventions and some useful formulas

We work with Euclidean metric: \( \text{diag}(+++) \). So we choose the gamma matrices being the Pauli matrices \( \sigma_i \): 

\[
\gamma^\mu \equiv (\sigma_\mu)_\alpha^\beta 
\]

(76)

which are OS self-conjugate and:

\[
\{\sigma^\mu, \sigma^\nu\} = 2\delta^{\mu\nu} I, \tag{77}
\]

\[
[\sigma^\mu, \sigma^\nu] = 2i\varepsilon^{\mu\nu\sigma} \sigma^\sigma. \tag{78}
\]

The invariant anti-symmetric tensor is defined as

\[
\varepsilon^{-+} = \varepsilon_{-+} = +1, \tag{79}
\]

\[
\varepsilon^\gamma_\beta \varepsilon_\beta^\alpha = -\delta^\gamma_\alpha, \tag{80}
\]

and are used to raise and lower indices as conversion:

\[
\psi^\alpha = \varepsilon^{\alpha\beta} \psi_\beta, \tag{81}
\]

\[
\psi_\alpha = \psi^\beta \varepsilon_\beta^\alpha. \tag{82}
\]

In this way is possible to find the representation of differential operator of the generators of super algebra in D=3, with the concept of graded Majorana [38]:

\[
Q_\alpha = -\partial_\alpha + \partial_{\alpha\beta} \theta^\beta, \tag{83}
\]

with

\[
\partial_{\alpha\beta} = i\sigma^\mu_\alpha \varepsilon_{\gamma\beta} \partial_\mu. \tag{84}
\]

As well as the superspace derived:

\[
D_\alpha = \partial_\alpha + \partial_{\alpha\beta} \theta^\beta, \tag{85}
\]

with the following relations:

\[
\{D_\alpha, D_\beta\} = 2\partial_{\alpha\beta}, \tag{86}
\]

\[
[D_\alpha, D_\beta] = -2\varepsilon_{\alpha\beta} D^2, \tag{87}
\]

\[
D_\alpha D_\beta = \partial_{\alpha\beta} - \varepsilon_{\alpha\beta} D^2, \tag{88}
\]

\[
D^3 D_\alpha D_\beta = 0. \tag{89}
\]

And it is easy to verify that

\[
[Q_\alpha, D_\beta] = 0. \tag{90}
\]

Another useful relations:

\[
\partial_{\alpha\beta} \partial^{\alpha\gamma} = \partial^2 \delta^\gamma_\beta, \tag{91}
\]

\[
(D^2)^2 = -\partial^2, \tag{92}
\]

\[
\int d^2\theta = -\frac{1}{4} D^2, \tag{93}
\]

\[
\delta_S(1, 2) = D^2_1 \delta^2(\theta_1 - \theta_2) \delta^3(x_1 - x_2) = -e^{i(\theta_1^a \theta_2^a) \partial_0} (\theta_1 - \theta_2)^2 \delta^3(x_1 - x_2). \tag{94}
\]