A historical perspective on Modified Newtonian Dynamics

R.H. Sanders

Abstract: I review the history and development of Modified Newtonian Dynamics (MOND) beginning with the phenomenological basis as it existed in the early 1980s. I consider Milgrom’s papers of 1983 introducing the idea and its consequences for galaxies and galaxy groups, as well as the initial reactions, both negative and positive. The early criticisms were primarily on matters of principle, such as the absence of conservation laws and perceived cosmological problems; an important step in addressing these issues was the development of the Lagrangian-based non-relativistic theory of Bekenstein and Milgrom. This theory led to the development of a tentative relativistic theory that formed the basis for later multi-field theories of gravity. On an empirical level the predictive success of the idea with respect to the phenomenology of galaxies presents considerable challenges for cold dark matter. For MOND the essential challenge remains the absence of a generally accepted theoretical underpinning of the idea and, thus, cosmological predictions. I briefly review recent progress in this direction. Finally I discuss the role and sociology of unconventional ideas in astronomy in the presence of a strongly entrenched standard paradigm.

PACS Nos.: 01.65.+g, 95.35.+d, 98.62.-g, 98.80.-k

1. Introduction

In the context of the present cosmological paradigm, ΛCDM, there are two major constituents of the Universe for which the only evidence is astronomical. There is dark energy, perhaps represented by a cosmological constant in Einstein’s equations. This medium, comprising 70% of the energy density of the Universe, causes the observed present accelerated expansion evidenced by supernovae in distant galaxies and makes up the energy difference necessary for closure of the Universe. And then there is cold dark matter – hypothetical particles, beyond the standard model of particle physics, comprising 25% of the Universe and interacting with baryonic matter (the remaining 5%) primarily through the force of gravity. Because it is cold (i.e., non-relativistic at the time it decouples from the photons and other relativistic particles in the early universe) this medium promotes the early formation of structure on all scales via gravitational collapse and explains the discrepancy between the directly observed baryonic matter (stars and gas) and the traditional dynamical mass of bound gravitational systems such as galaxies and clusters of galaxies. Here the cosmological paradigm impinges upon the dynamics of these well-observed local systems and should, in principle, be testable. And here it fails.

The evidence supporting the standard cosmological paradigm is said to be so overwhelming that there is little room for doubt. This is in spite of the fact that the most well-motivated dark matter particles – supersymmetric partners – should be detectable in terrestrial experiments via the rare scattering of atomic nuclei. In fact such events have never been seen in spite of considerable effort and expense.
invested in particle dark matter search experiments. But efforts continue because ΛCDM has become something of an official religion – a doctrine outside of which there is no salvation, beyond which there is only damnation. Yet there is a leading heresy that has attracted a relatively small but growing number of adherents: modified Newtonian dynamics, MOND, which, as will be argued here, is epistemologically “more correct” than CDM.

Viewed simply, MOND is an algorithm that, with one additional fundamental parameter having units of acceleration, allows calculation of the distribution of the effective gravitational force in astronomical objects from the observed distribution of baryonic matter – and it works remarkably well. This is evidenced primarily by use of the MOND algorithm in the determination of rotation curves of disk galaxies where the agreement with observed rotation curves is often precise, even in details. The existence of such an algorithm is problematic for CDM because this is not something that dissipationless dark matter on the scale of galaxies can naturally do; it would seem require a coupling between dark matter and baryonic matter which is totally at odds with the perceived properties of CDM.

Moreover, MOND subsumes the Tully-Fisher law, the observed near-perfect correlation between the baryonic mass of galaxies and the asymptotic constant rotation velocity, as a aspect of fundamental physics – a Kepler’s law for galaxies. This correlation appears unnatural in the context of dark matter because the asymptotic rotation velocity is a property of the dark matter halo that extends far beyond the relatively puny concentration of baryons in the center. In the context of dark matter it is explained as resulting from aspects of galaxy formation. But it remains difficult to understand how such a precise correlation can emerge from a process that must be inherently quite random, with each galaxy having its own unique history of formation, merging, feedback and dynamical evolution.

The success of the MOND algorithm has deeper implications as a modification either of gravity (general relativity) or of the way in which particles respond to an applied force at low accelerations (inertia). The idea will not be generally accepted until these implications are understood in the context of a more general theory, but the phenomenological success should, in itself, be sufficient to raise serious doubts about existence of cold dark matter and thus the prevailing cosmological paradigm.

Here my purpose to discuss MOND from a historical perspective, beginning with the phenomenological roots and philosophical basis as outlined in the original papers of Milgrom some thirty years ago [1, 2, 3]. I will describe the predictive successes of the idea and the initial and ongoing criticisms. I will outline the suggested physical bases of MOND while emphasizing that there is not yet a generally accepted physical theory and, therefore, cosmology. I will be brief because many of these points will be discussed in detail in the various articles in this compendium, but I will conclude with a discussion of the criteria that distinguish crazy from sensible ideas in astronomy and of the dangers to the creative process in science presented by the unquestioning acceptance of a standard dogma.

2. The phenomenological roots of MOND

The first rotation curves of spiral galaxies measured in neutral hydrogen and extending well-beyond the visible disk were published by the mid 1970s [4, 5]. However, to establish the fact that these extended rotation curves did not decline in the expected Keplerian fashion as well as the perception of this fact as a serious anomaly took 10 years more. An important development in this realization was the demonstration by Kalnajs [6] that for several galaxies with rotation curves measured in optical emission lines, the shapes of the curves were well matched by assuming that the mass was distributed like the visible light from stars – in the context of Newtonian gravity. This was confirmed by Kent [7] for a sample of 37 bright galaxies observed in optical emission lines by Rubin et al. [8]. The match, however, between observed and calculated rotation curves did not extend beyond the visible disk where the gas kinematics were measured only in the 21 cm line emission from tenuous neutral hydrogen [9, 10]. Here, there was a serious discrepancy between the traditional dynamical mass and the directly observable mass in stars and gas. By 1985 the existence of this discrepancy had became irrefutable [11] (see my book [12] on the dark matter problem for a general discussion of these developments).
The initial reaction of galactic dynamicists was to ascribe this phenomenon to unseen or dark matter in a surrounding spheroidal halo, a construct already in place as a means of taming the instability of rotationally supported disk systems [13]. The tendency of rotation curves to be asymptotically flat suggested that the dark matter halo in galaxies had the density law of an isothermal spheroid, i.e., density falling like $1/r^2$. Perhaps this dark matter was of the same sort that contributed to the virial discrepancy in clusters of galaxies – a discrepancy first identified by Zwicky [14] four decades earlier.

The nature of the dark matter was an issue for speculation (as it still is), but initially low luminosity baryonic objects were preferred – low mass stars, dead stars (white dwarfs or neutron stars), planetary mass objects, low mass black holes, snow balls. But increasingly, it became appreciated that some sort of universal non-baryonic particle matter was needed to promote the formation of the observed structure in an expanding Universe of finite lifetime – massive neutrinos, supersymmetric partners (weakly-interacting-massive particles, WIMPS), or axions became the preferred choice for the cosmological as well as the cluster and galactic dark matter.

At the same time that the existence of a discrepancy in galaxies was beginning to be appreciated, certain regularities in galactic photometry and kinematics were being revealed. The most important of these was the Tully-Fisher relation, a correlation between the luminosity of spiral galaxies and the width of the global 21 cm line emission of individual galaxies, a measure of the rotation velocity [15]. The form of the correlation is a power law, $L \propto (\Delta V)^\alpha$ where $L$ is the luminosity of the galaxy in a particular photometric band and $\Delta V$ is the line width corrected for inclination of the galaxy disk. The exponent $\alpha$ depended upon the color in which $L$ was measured and ranged from 2 in the blue light (B-band) to 4 in the near-infrared. Because of its potential use as a distance indicator, considerable effort was spent upon calibration of the relationship, and, by 1980 it had been established that the near infrared H band ($\approx 1.6 \mu m$) is the preferred color for measuring the luminosity. This is near the peak of the black body radiation from old stars making up the bulk of the stellar population and, unlike the B-band emission, is relatively free of the effects recent star formation and dust absorption [16]. The scatter about the mean relation is much reduced in the near-infrared, and, because the near-infrared luminosity is most nearly proportional to the stellar mass (near constant mass-to-light ratio), this implies that the true relation is between stellar mass and rotation velocity.

Faber and Jackson [17] had discovered a similar relation for elliptical galaxies, systems supported not so much by systematic rotation but by random motion of the stars. Here the relation is between the luminosity and the velocity dispersion of the stellar component and has the form $L \propto \sigma^\beta$ where it is also the case that $\beta \approx 4$. The observed relationship has greater scatter, but is also an indication of a more fundamental correlation between mass and velocity dispersion – a relation going beyond the Newtonian virial theorem with three parameters.

A regularity in the photometric properties of spiral galaxies had been discovered even earlier. In 1970 Freeman [18] noted that the distribution of surface brightness in the disks of spiral galaxies can be generally described as having an exponential form and that the mean surface surface brightness within an exponential scale length appears to have a characteristic value of about 21.6 magnitudes per square arc second in the B band corresponding to roughly 150 solar luminosities per square parsec. Later it was argued [19] that this was actually an upper limit and not an average value; that is to say, there are fainter (lower surface brightness) galaxies but not brighter ones. Again for elliptical galaxies there is a similar characteristic central surface brightness averaged within an effective radius [20] – that radius containing half the luminosity.

Initial spectroscopic measurements of the radial velocities in several stars in very low surface brightness systems, such as the dwarf spheroidal companions of the Milky Way, were carried out in the early 1980s. There were early indications, but only hints by 1983 [21], that the dynamical mass to light ratio in such systems might be large. By 1982 several low surface brightness spirals had been discovered [22]; initial 21 cm line observations indicated that these objects might have a somewhat larger mass-to-light ratio than high surface brightness galaxies, but there was no suggestion of a sur-
face brightness dependence of the dark matter content of galactic systems.

This is where the observational situation stood around 1982. In flat rotation curves, extending well beyond the visible disk, there was emerging evidence for a discrepancy between the visible and classical dynamical mass – a discrepancy that grew in the outer regions and, if described by dark matter, required a $1/r^2$ density relation in a dark halo (or a $1/r$ surface density if described by a dark disk). There were, in addition, scaling laws and photometric regularities of galactic systems: the Tully-Fisher law for spirals and Faber-Jackson for ellipticals; a characteristic surface brightness for spirals and for ellipticals, but there was no clear indication of a surface brightness dependence to the discrepancy in galaxies. This set the stage for the appearance of MOND as an alternative to dark matter, but before describing this development, it is of interest to discuss a diversion along this path.

3. A modification at large length scale

The discrepancy is apparent in large astronomical systems – the outer regions of spiral galaxies and clusters of galaxies. On the scale of the solar system, there is no evidence at all for any deviation from Newton’s law. Therefore, an obvious modification is to change the law of attraction beyond a length scale, $r_0$, of galactic dimensions. This would appear to be consistent with a larger discrepancy in larger systems – clusters of galaxies, for example, would have more missing mass than individual galaxies, which seemed to be the case before the discovery of hot gas in clusters.

In 1963 Finzi [23] emphasized the ubiquity of the mass discrepancy in large astronomical systems. He pointed out that in the Milky Way the total mass appeared to grow with scale (this from work on the kinematics of globular clusters [24]). Early 21 cm line observations of the rotation curve of the Andromeda galaxy, indicated the same [25]. In the largest bound gravitational systems, the great clusters of galaxies, the mass-to-light ratio appeared to approach several hundred. Finzi first stressed the apparent universality of these phenomena and noted that these observations could be explained with a modification of Newtonian attraction which fell more as $1/r^1.5$ rather than $1/r^2$ beyond a scale of about 1 kpc.

By the mid-1980s it had become generally apparent that rotation curves of spiral galaxies were asymptotically flat. Flat rotation curves will result if the gravitational acceleration about a point mass varies as

$$f = \nu(r/r_0)GM/r^2$$

(1)

where $\nu(x)$ is a function with the asymptotic behavior such that $\nu(x) \to 1$ when $x <<< 1$ and $\nu(x) = x$ when $x >> 1$. Then in the limit of large distance ($r > r_0$), $f = GM/(rr_0)$. Equating this to the acceleration of circular motion yields a constant rotation velocity

$$V^2 = GM/r_0.$$  

(2)

Such modifications were discussed by Tohline [26] and Kuhn and Kruglyac [27]. I proposed a specific example in which the gravity force was the sum of two components, an attractive force of infinite extent and a repulsive component mediated by a particle of finite mass and hence finite extent [28]. In this case the gravitational potential would be

$$U(r) = \frac{GM}{r}(1 - e^{-r/r_0}).$$

(3)

If $\alpha \approx 0.9$ then this would yield a plateau of constant rotation velocity from $0.5r_0$ to $2.5r_0$ followed by a return to $1/r^2$ attraction and a Keplerian decline on larger scale. I felt that $1/r^2$ attraction on largest scale would lead to a more sensible cosmology because it would permit a maximum discrepancy as well as the usual Newtonian derivation of the Friedmann equation.

The essential problem, which should have been already obvious at this point, is that modifications of this sort are in direct contradiction to the observed Tully-Fisher relation. Eq. 2 implies a relation of the
form $M \propto V^2$ whereas the exponent in the luminosity-rotation velocity relation observed at that time was certainly greater than three. One might suggest that the mass-to-light ratio varies systematically with galaxy mass, i.e., $M/L \propto V^{-2}$; that is to say, more massive galaxies have a smaller $M/L$, but the tendency is just the opposite. It appears that, if anything, more massive galaxies have a larger $M/L$ because these tend to be earlier type bulge-dominated systems. Moreover, in the near-infrared where $M/L$ is relatively constant, the exponent is most nearly 4 and the scatter is lowest.

In addition, any such modification would imply that smaller galaxies should have a smaller discrepancy and larger galaxies a larger discrepancy. Even by 1980, it had become obvious that this was not the case. In a 1979 review Faber and Gallagher [29] tabulated the dynamical masses and mass-to-light ratios of 50 nearby galaxies; no correlation between size (as measured by photometric radius) and $M/L$ was noted. This is evident in the left hand panel of Fig. 2 which is based upon more recent data [30]. This shows the dynamical $M/L$ determined via the full 21 cm line rotation curves plotted against the galaxy size (the radius at the last measured point of the rotation curve) for a sample of galaxies in the Ursa Major cluster.

Thus the early evidence did not support a modification of gravity beyond a length scale, although this fact did not prevent myself and others from proposing such hypotheses – and this after the idea of MOND had been published.

4. An acceleration-based modification

Modified Newtonian dynamics is solely the invention of Mordehai (Moti) Milgrom. The idea of an acceleration-based modification of dynamics or gravity would have probably occurred to someone else sooner or later, but it is safe to say that in the early 1980s no one but Milgrom had considered such a possible modification as an alternative to astrophysical dark matter. It was a brilliant stroke of insight to realize that astronomical systems were not only characterized by large scale but also by low internal accelerations and that this could account for the known systematics in the kinematics and photometry of galactic systems. However, the idea was hardly greeted with overwhelming enthusiasm.

Before 1980 Milgrom (Weizmann Institute, Israel) had worked primarily in high-energy astrophysics and was well-known for his highly successful kinematic model of SS 433, a compact object with precessing high velocity jets. During a sabbatical year at the Princeton Institute for Advanced Study, he decided to consider the dynamics of galaxies and galaxy systems in general and the dark matter problem in particular, taking advantage of the considerable expertise on this subject at Princeton. He felt that so long as the only evidence for dark matter on astronomical scales was its putative global gravitational or dynamical effects, then its presumed existence is not independent of the assumed law of gravity or dynamics on those scales – that so long as no candidate dark matter objects or particles had been identified, then it was legitimate to look for alternative solutions to the discrepancy in modifications of Newtonian gravity or dynamics. Such a point-of-view seems hardly radical at all but an entirely reasonable scientific approach.

Looking at the existent observations, Milgrom realized that a distance-based modification could not work for those reasons noted above. He also realized that an acceleration-based modification could account for these observations, in particular the systematics of galaxy photometry and kinematics. Thus he proposed a simple rule that can be realized as a modification of the law of inertia: the inertial reaction of a particle of mass $m$ to an applied force $F$ is

$$F = \frac{m}{m} a \mu(a/a_0)$$

where $a$ is the resulting acceleration and $a_0$ is a new fixed parameter with units of acceleration. The function $\mu(x)$ is not specified but must have the asymptotic behavior such that $\mu(x) = 1$ in the limit where $x \gg 1$ (to retain Newtonian dynamics) but $\mu(x) = x$ where $x \ll 1$. The rule may also be
Fig. 1. On this logarithmic plot the acceleration resulting from an applied force (per unit mass) is shown for Newtonian dynamics (dashed curve) and for modified Newtonian dynamics (solid curve). These only differ significantly below an acceleration of $10^{-10} \text{cm/s}^2$. The indicated region on the right corresponds to accelerations in the Solar System and that on the left to accelerations in the outer regions of bright galaxies. Solar System accelerations are deep in the Newtonian regime, but galaxy scale accelerations are typically in the modified regime. Here it is assumed that $\mu(x) = x/(1 + x)$.

described as a modification of the Newtonian gravitational acceleration $g_N$; the true gravitational acceleration $g$ is given by

$$g\mu(g/a_0) = g_N.$$  \hspace{1cm} (5)

With respect to particles in orbital motion in a galaxy the two formulations are equivalent but, obviously, there are differences in principle since $g$ refers to specifically the gravitational acceleration but $F$ can be any applied force. The relationship between the applied force per unit mass and the resulting acceleration is shown in Fig. 1; at accelerations prevailing in the Solar System, Newton and MOND are indistinguishable.

The hypothesis is that the discrepancy should begin at an acceleration below the critical acceleration $a_0$. There is very convincing evidence from 21 cm line rotation curves observed in the late 1990s [30] that this is the case. In Fig. 2, right panel, we see the same sample of galaxies from the Ursa Major cluster [31] but this time with the Newtonian dynamical mass-to-light plotted against centrifugal acceleration ($V^2/r$) in units of $10^{-10} \text{m/s}^2$ at the last measured point of the rotation curve. Keeping in mind that the stellar mass-to-light ratio (in solar units) of spiral galaxies in the near-infrared is on the order of unity, we see that the discrepancy is clearly correlated with acceleration in the sense that the dynamical mass grows below accelerations of one in these units, i.e., $a_0 \approx 10^{-10} \text{m/s}^2$. It is important to recall that observations of this quality did not exist when Milgrom made his proposal.

In the case of circular motion in a Newtonian force field in the low acceleration limit it follows directly from eqs. 4 or 5 that

$$V^4 = GMa_0.$$  \hspace{1cm} (6)
In other words, the rotational velocity is constant and related to the mass as $V^4 \propto M$. Thus flat rotation curves and the observed Tully-Fisher law are subsumed by the MOND algorithm.

These two aspects of MOND, flat rotation curves and a Tully-Fisher relation, were part of the observed phenomenology before MOND was written down, so it might asked if they can they rightly be called predictions. I believe that it is fair to characterize them as such because, in the context of MOND, these attributes, following from physical law, are absolute: every rotation curve of an isolated spiral galaxy must be asymptotically flat (the prediction of asymptotic flatness is important because by 1982 no one claimed this as a property of rotation curves). And every galaxy, without ambiguities introduced by uncertainties of inclination, distance or tidal disturbance, must lie on the Tully-Fisher relation, so long as the asymptotic velocity is plotted against the detectable baryonic mass. That is to say, Tully-Fisher is not a relation between the maximum or an averaged rotation velocity and the luminosity in some appropriate color band, but between the asymptotic rotation velocity and the baryonic mass of the galaxies [33]. When these are the plotted properties, there should be no intrinsic scatter, and this, in fact, is a powerful prediction.

The results of modern observations are shown in Figs. 3 and 4. Here the observed rotation curve (Fig. 3) demonstrates the well known asymptotic behavior beyond the visible object. The solid curve is the rotation velocity predicted by MOND with $a_0 = 10^{-10} \text{ m/s}^2$, and the dashed and dotted curves are the Newtonian rotation curves resulting from the stars and gas respectively. The baryonic Tully-Fisher relation is shown in Fig. 4, and its normalization, given by $Ga_0$ again implies $a_0 = 10^{-10} \text{ m/s}^2$ [33]. The interesting aspect of this plot is that in the left-hand panel only the mass of the stars (estimated from population synthesis models) is plotted against the asymptotic rotation velocity. The lower mass systems are dominated by gas and they fall below the relation determined by the higher mass, star dominated systems. When the gas mass is included, as in the right-hand panel, the low mass systems lie on the same TF relation demonstrating vividly that the true relation is between baryonic mass and rotation velocity.

As Milgrom pointed out, the numerical value of the critical acceleration is $\approx cH_0/6$. This is cer-
Fig. 3. The rotation curve of NGC 2403. The points are the observed rotation curve, the dashed and dotted curves are the Newtonian rotation curves of the baryonic components (stars and gas respectively), and the solid curve is the MOND rotation curve [32].

Fig. 4. The baryonic Tully-Fisher relation. The panel on the left is the mass in stars plotted against the asymptotic rotation velocity of spiral galaxies. On the right, the mass in gas is included [33].
tainly not just coincidental and suggests a connection of local dynamics with cosmology, perhaps via a cosmological derivation of Mach’s Principle. It is also of interest that that $a_0$ is the maximum gravitational acceleration on a subatomic scale; it would be the acceleration of gravity at one Compton radius from a particle with a mass of several hundred MeV. Combined with the proximity of $a_0$ to $cH_0$ yields the famous Dirac cosmological numerical coincidences.

Apart from these aspects of galaxy phenomenology that follow directly from the MOND hypothesis there are additional observational properties that are clear predictions. The critical acceleration, $a_0$, can be expressed as a surface density, $\Sigma_0 = a_0/G$, which emerges as a characteristic value. With a mass-to-light ratio on the order of unity, this can be translated into a critical surface brightness that is comparable to that of Freeman and of Fish [18, 20]. Given the well-known instability of rotationally supported Newtonian disks [13] this implies the critical surface brightness should appear as an upper limit (disks with lower surface brightness are not Newtonian). Moreover, objects such as globular clusters or elliptical galaxies with higher surface brightness, should exhibit little discrepancy between the observed and classical dynamical mass within the bright visible object. On the other hand, low-surface brightness objects are entirely in the low acceleration regime and should exhibit a large discrepancy. So there should be a strong surface brightness dependence to the apparent dark matter content of systems. Now this is known to be the case, but in 1982 it was a bold prediction.

There was an additional prediction concerning the general shapes of rotation curves: in high surface brightness spiral galaxies that are near Newtonian in the bright inner regions, the rotation curve should decline to the asymptotic constant value beyond the bright disk; in low surface brightness spirals however, the rotation curve should slowly rise to the constant value in the outer parts. Again in 1983 these systematics had never before been observed; this property of rotation curves was pointed out explicitly in 1991 [34].

Binary galaxies, small groups, and clusters of galaxies, should, in so far as they are in the low acceleration regime, also present large discrepancies. It is now known that the central regions of rich clusters of galaxies are in the high acceleration regime, so here MOND predicts more mass than is directly observed (the dynamical discrepancy is not removed in the central regions), but in 1980 this was not evident (more on this issue below).

In 1983, Milgrom published three papers in the Astrophysical Journal that fully described the idea of MOND and these observational consequences [1-3]. The first was a short paper that introduced the idea; the second dealt with galaxy phenomenology and the third with groups and clusters of galaxies. Getting these papers past critical referees was not an easy task. Milgrom had previously submitted the introductory paper to Nature and to Astronomy and Astrophysics Letters without success. The final three published papers were received by the Astrophysical Journal in February of 1982 which implies that the work had been done in 1981 or earlier – truly remarkable considering the tentative state of the data at that point.

Most of the criticisms were of matters of taste or principle rather than phenomenological. Typical is the objection of one referee of the short paper originally submitted to Astronomy and Astrophysics Letters: “In judging the success of any such proposed modification one must draw up a balance sheet of gains and losses. In this theory there are very considerable losses of accurately checked phenomena in order to achieve an interpretation of phenomena that are not well understood while maintaining that ‘most of what there is, can be seen’ and so dispensing with hidden matter.” The referee goes on to say that to obtain this dubious gain one loses various cherished principles such as equivalence, relativistic invariance, etc. “My personal opinion is that such speculations are fun around the coffee table, but I don’t want to see them in the literature until they are significantly further developed.” In other words, before a more complete theory can be presented, forget about publication.

In his response to this argument, Milgrom mentioned the Bohr atom. “It was not even a theory but a set of assumptions on what an electron does. It gave answers to a very small number of questions, etc., etc. It took 13 years and many good men to bring quantum mechanics to the stage of the Schroedinger
equation, which was not even relativistic, which disposed of the notion that mechanics is deterministic and other such well established principles.” So, following Milgrom, if a theory is successful in explaining diverse phenomena, as is MOND (but actually not the Bohr atom!), then it should be taken seriously even if it abandons cherished principles. It would not be the first time that “fundamental principles” have been abandoned (or enlarged) in progressing to new physics.

But the referee makes an additional point: the phenomena that Milgrom’s theory attempts to explain are not well-established, unlike the principles he abandons. It is actually true that these systematics — the universality of asymptotically flat rotation curves, the Tully-Fisher relation and its preferred exponent of four, the existence of a preferred surface brightness and the presence of a large discrepancy in lower surface brightness objects — were not generally accepted or known or appreciated when Milgrom wrote down his theory. But this is where he showed a particular prescience — the ability to glean from the data those systematics that are significant and that would be verified by coming observations, and the connection of these systematics through a single preferred universal value of acceleration.

This referee and several others pointed out that the original algorithm did not conserve the linear or angular momentum of an isolated compound system. To many, the idea of a binary star accelerating away by itself, seemed like distinctly bad physics. Milgrom was, from the beginning, aware of this deficiency and, with Bekenstein, set about demonstrating that it was not a necessary property of this sort of theory. The complaint, however, remained for several years (Felton [35] in 1984 was the first to point this out in print). In the same vein, the motion of a compound object in an external field was murky. A star, for example, in the outer galaxy is moving in a low acceleration external field. Yet the internal motion of the gas particles comprising the star feel an acceleration well in excess of $a_0$. How does the star move? Is it the acceleration of the center-of-mass that counts, or is it the acceleration of the individual particles?

Not everyone was hostile to Milgrom’s idea. There were several eminent scientists, who, if not actual supporters, had some appreciation of his proposal, who thought it was a legitimate avenue for research, and who thought that the idea should be published. Among these were Ed Salpeter of Cornell and Scott Tremaine of Princeton. But both Tremaine and Salpeter pointed out a substantial phenomenological problem: galactic or open star clusters in the plane of the Milky Way have accelerations below $a_0$ and that should put them in the low acceleration limit. But these objects have no significant measurable discrepancy; there is no missing mass problem for galactic star clusters. This criticism lead Milgrom to the realization of presence of the external field effect; that is to say, it is the total acceleration, internal plus external, that must be included in eq. 4 or 5. As a modification of gravity eq. 5 should read

$$\mu\left(\frac{g_i + g_e}{a_0}\right)(g_i + g_e) = g_{N_i} + g_{N_e}. \quad (7)$$

where $g_e$ is the modified external field, $g_i$ is the modified internal field, $g_{N_i}$ is the Newtonian internal field and $g_{N_e}$ is the Newtonian external field.

In other words the internal dynamics of a system is influenced by the acceleration of any external field beyond the usual tidal effect; if the external field has an acceleration greater than $a_0$, then the system is not in the regime of modified dynamics. It is important to note that this was not a new assumption but a natural consequence of the non-linearity of the formalism. The form of the algorithm implies that the underlying theory of MOND violates the strong statement of equivalence principle (although the weaker version, the universality of free-fall, is of course respected). And, as is obvious, that theory could not be General Relativity since GR embodies strong equivalence.

I show, in the appendix, a letter written in 1982 by Milgrom to John Bahcall, at that time the head of the astrophysics group at the Institute for Advanced Study and an early critic of the idea. Milgrom’s letter makes a clear and insightful exposition of the philosophy and phenomenology behind the idea and presages much of what had been written about MOND since that time. It is a remarkably up-to-date exposition.
5. And then there were two

Early on, in 1982, MOND received its first active supporter. That was Jacob Bekenstein, a respected theoretical physicist who was famous for proposing that black holes possessed the property of entropy. Together, Bekenstein and Milgrom worked out the first field theoretic version of MOND as a modification of Newtonian gravity [36]. They wrote down a non-relativistic Lagrangian which possessed space-time translational and rotational invariance (published in 1984). Therefore the resulting theory respects the laws of conservation of energy and linear and angular momentum (this addressed issues raised by several of referees of Milgrom’s initial papers). The field equation has the form of a non-linear modified Poisson equation, i.e.,

\[ \nabla \cdot \left[ \mu \left( \frac{1}{a_0} \right) \nabla \phi \right] = 4\pi G\rho \tag{8} \]

where \( \mu \) has the asymptotic behavior described earlier. Bekenstein and Milgrom demonstrated that the external field effect was embodied by such a theory as was the center-of-mass motion of a compound object in an external field.

Of course, the theory did not yield precisely the same gravitational field as the simplified MOND algorithm described by eq. 5; only in cases of high symmetry, spherical or cylindrical, did the Bekenstein-Milgrom field equation provide the same acceleration field, \( g \). The non-linear equation was difficult to solve, but, later on, it was demonstrated that for disk galaxies the simple version provided a reasonable approximation to the solutions of the modified Poisson equation [37].

Although non-relativistic, the Bekenstein-Milgrom theory pointed the way toward a possible relativistic extension as a modified scalar-tensor theory. Here, gravity is mediated by two fields – the usual metric tensor of general relativity plus a scalar field with an unconventional kinetic Lagrangian (also an aspect of what would become known as k-essence). Given that \( l_0 \) is a length scale on the order of the Hubble radius (with \( a_0 = c^2 / l_0 \)), and defining the dimensionless scalar field invariant as

\[ \chi = l_0^2 \phi,_{\alpha} \phi^{,\alpha} \tag{9} \]

the scalar action becomes

\[ S = \frac{1}{2l_0^2} \int F(\chi)d^4x \tag{10} \]

with \( F(\chi) = \omega \chi \) in the limit of large \( \chi \), the usual scalar invariant where field gradients are large. Here \( \omega \) is a large number that is equivalent to the Brans-Dicke parameter in the Solar System. When \( \chi \) is small, the low acceleration limit, then \( F(\chi) = \frac{2}{3} \chi^{3/2} \). Thus in the weak field limit, the scalar field equation is equivalent to eq. 8 above with \( \mu(x) = dF/d\chi \). This is combined with the usual Poisson equation for the Newtonian potential to yield the complete theory in the weak field limit.

To preserve the universality of free fall (the weak statement of the Equivalence Principle), the scalar field couples to matter as a conformal factor multiplying the Einstein metric, as in Brans-Dicke theory. Thus particles follow the geodesics of this "physical metric" that is conformally related to the Einstein metric.

A possible problem was immediately identified by Bekenstein and Milgrom: in the low gradient limit scalar waves propagate with a velocity in excess of that of light \( (\sqrt{2}c) \) in a direction parallel to the field gradient. Whether or not this is an actual problem leading to causal anomalies has been a matter of debate, but there was another, more serious, possible observational problem: it is evident that null-geodesics of conformally related metrics coincide. This means the scalar field has no influence on photons or other relativistic particles implying that there would be no enhanced deflection of photons by the scalar field – MOND would not provide extra bending of the path of light (photons would not “see dark matter”) in galaxies or clusters of galaxies for example. Later observations would show that this was clearly not the case.
Finally, the theory left the issue of cosmology hanging in the air. In the cosmological limit, where \( \chi \) changes signs, the form of \( F(\chi) \) must be modified further. Moreover, the constant \( l_0 \) does not follow naturally but must be put in by hand. Thus, the apparent significance of \( a_0 \approx cH_0 \) is lost.

In spite of these issues, the suggestion of possible relativistic extension of MOND was significant. It demonstrated that such an extension may be realized as a multi-field theory of gravity, and this led, after 20 years, to the first consistent covariant theory, TeVeS [38].

6. The predictive power of MOND

I became an active supporter in 1985 when confronted with the high quality 21 cm line rotation curve data then being provided by the Groningen group [39, 11] using the Westerbork radio interferometer (WSRT). In principle, the MOND algorithm permits the calculation of any rotation curve using the observed distribution of baryonic matter (stars and gas). There is, of course, the uncertainty presented by the unknown form of the interpolating function \( \mu \), but in practice, the results turn out to be rather insensitive to the abruptness of the transition between modified and Newtonian dynamics. Characterizing the function as

\[
\mu(x) = \frac{x}{(1 + x^n)^{1/n}}
\]  

(11)

it has been found that \( n = 1 \) or \( n = 2 \) work about equally well, although there is a difference in the implied mass-to-light ratio of the stellar disk. Moreover, a number of low surface brightness gas-dominated galaxies are in the deep MOND regime and therefore independent of the form of \( \mu \) as well as the stellar mass-to-light ratio; the distribution of baryonic matter is given almost entirely by the directly observed gas surface density distribution.

The procedure is to estimate the distribution of baryonic matter by surface photometry of the visible disk, preferably in the near-infrared. The gas is included by taking the measured surface density multiplied by a factor (typically 1.3) to include the primordial helium. The stars and gas are assumed to be distributed in a thin disk, apart from those cases where there is direct evidence for a spheroidal bulge. The distribution of the Newtonian gravitational acceleration is determined by solving the Poisson equation and then modified to the MONDian (“true”) gravitational acceleration by applying the MOND algorithm (eq. 5). The mass-to-light of the disk is adjusted so that the fit is optimal.

The results of applying this procedure are shown in two cases in Fig. 5 [32]. These are 21 cm line rotation curves of a LSB, low-surface-brightness, (NGC 1560) and a HSB, high surface brightness galaxy, (NGC 2903), where the points show the observations, the dotted and dashed curves are the Newtonian rotation curves of the observed baryonic components (stars and gas), and the solid curves are the MOND curves resulting from the simple algorithm (with \( a_0 = 1.2 \times 10^{-10} \) m/s\(^2\)). One should note that the rotation curves have the general forms for LSB and HSB predicted by Milgrom in 1983: in the LSB galaxy the curve slowly rises to its asymptotic form and in the HSB the curve falls in a near Keplerian fashion to the final value. The required mass-to-light ratios (shown on the figure) are quite reasonable for a gas dominated and a star dominated galaxy. There are more than 100 such rotation curves now in the literature [31, 40] and in roughly 90% of these, the agreement is of comparable quality (not all rotation curves are expected to be perfect because of uncertainties in the fundamental parameters – distance, inclination, warps, non-circular motions).

In principle there is one free parameter per galaxy: M/L (\( a_0 \) must be the same for every galaxy). For the uniform sample of spiral galaxies in the Ursa Major cluster [30] the fitted mass-to-light ratios agree well with those implied by population synthesis modeling [41] as is seen in Fig. 6. Indeed with improvements in stellar population synthesis modeling and in gas dominated galaxies it has become possible to make zero-parameter “fits”, i.e., true predictions.

The remarkable aspect is that, in many cases, details in the rotation curves are matched by the MOND rotation curves. This explains an aspect of the observations noted by Sancisi [42]: Every feature in the observed distribution of matter is matched by a corresponding feature in the rotation curve, and
Fig. 5. The 21 cm line rotation curves of two spiral galaxies, high and low surface brightness (HSB and LSB), determined with the MOND algorithm from the observed distribution of observable matter, stars and gas. The points are the observed values, the dashed curve is the Newtonian rotation curve of the gaseous disk, the dotted curve is that of the stellar disk and the solid curve is the predicted MOND curve. The general form of the rotation curves was predicted by Milgrom in 1983: the rotation velocity in the HSB declines in a near Keplerian fashion to the asymptotic constant rotation velocity, and the LSB the rotation curve gradually rises to this asymptotic value [32].
vice versa, even in the presence of a large conventional discrepancy. If there is a cusp in the light distribution, there is a cusp indicated by the rotation curve as in Fig. 7. If there is a fluctuation in the observed surface density distribution, as for the dwarf galaxy in Fig. 5, there is a corresponding fluctuation in the rotation curve. In the context of dark matter this would appear to be quite unnatural. It would require a coupling between the dark and visible matter that is not implied in presumed nature of dissipationless dark matter. These results convinced me that there must be something fundamentally correct about MOND. Dark matter, as it is thought to be, could not possibly match this predictive success.

And recall that the same value of $a_0$ which provides these fits, 1.0 to 1.2 in units of $10^{-10}$ m/s$^2$, also follows as the normalization of the Tully-Fisher relationship. This fact impressed early supporters such as Stacy McGaugh. The same Tully-Fisher law was present for galaxies of low and high surface brightness [43]. When McGaugh plotted the baryonic mass against asymptotic rotation velocity from the measured rotation curves (not a global 21 cm line width), the relation only improved (Fig. 4) [33].

There were other subsequent observations that had been predicted by MOND: the ubiquity of a large discrepancy in low surface brightness systems [31]; the absence of a discrepancy in high surface brightness systems such as luminous elliptical galaxies [44, 45] and globular clusters [46]; the presence of a preferred internal acceleration in pressure supported systems ranging from molecular clouds in the Galaxy to the giant clusters of galaxies – an internal acceleration within a factor of three of $a_0$ [31]; the fact that systems ranging from globular clusters to clusters of galaxies lie on the same $M \propto \sigma^4$ relationship [47]; the extension of Tully-Fisher (or Faber-Jackson) to very large distance (i.e., small accelerations) indicated by weak gravitational lensing [48]. And all of this was accomplished with a single value of $a_0$ having the cosmologically preferred value near $cH_0$. Moreover, there have been no subsequent modifications of MOND. It has not been necessary to tweak the theory to fit new and better data; in fact, one has the impression that as the data improves, so does the agreement with MOND.

©2014 NRC Canada
Fig. 7. Top: the mass surface density distribution in stars and gas (dotted and dashed curves) as a function of radius for the dwarf galaxy, UGC 6406. Bottom: the corresponding Newtonian and the MOND rotation curves (dotted, dashed, solid). The points are the observed curve. Note that there is a cusp in the light distribution with a corresponding spike in the central rotation velocity. The rotation curve slowly rises in the outer regions, not because of a halo, but, in the context of MOND, due to the increasing contribution of the neutral gas in this low acceleration region. From unpublished observations by Zwaan, Bosma and van der Hulst (2005).
Fig. 8. The panel on the left is a log-log plot of the Newtonian dynamical mass against the directly observed baryonic mass (mostly in the form of hot gas). The units are $10^{14} \ M_\odot$. The panel on the right is the same for the MOND dynamical mass. Note that MOND reduces the discrepancy but does not remove it.

7. Criticisms and challenges

In his original paper on clusters [3] Milgrom, using the MOND algorithm for pressure supported systems, estimated the mass-to-light ratios of 15 clusters of galaxies. He noted that in about half the cases the mass-to-light ratio remained unreasonably high ($\approx 20$ rescaling to $H_0 = 75$). Milgrom pointed out several possible causes of systematic error including the substantial contribution of hot gas to the total mass which had not generally been appreciated at that time.

In 1988, The and White [49] in a more detailed analysis of the Coma cluster, including the contribution of the gas, noted that MOND did not resolve the entire discrepancy; that to remove the need for unseen mass $\alpha_0$ had to be more than a factor of two times larger than that required to fit galaxy rotation curves. This can also be interpreted as a statement that MOND requires more matter in the cluster than can be directly detected. Over the next 10 years it became clear that the problem was more general – that most clusters of galaxies, when analyzed with MOND, contained more mass than was directed detected [50]. In particular I found [51] that there was typically a factor of two to three remaining discrepancy in a large sample of X-ray emitting clusters (see Fig. 8).

This has been trumpeted as a failure of MOND on scales greater than that of galaxies. These claims of doom culminated with multi-frequency observations of the famous "bullet" cluster [52] in which the stellar components of two colliding clusters have apparently passed through one another leaving behind the collisional X-ray emitting gas. The putative dark matter, detected by weak gravitational lensing of background galaxies, has also passed through and coincides in position with the cluster galaxies.

This observation does present a challenge for MOND but no greater than the older problem of the remaining virial discrepancy. Formally, the observed remaining discrepancy is not a falsification of MOND; it would be if MOND predicted less matter than is actually seen. In fact, one could view this a bold prediction of MOND: more mass will be detected in clusters.

As pointed out by Milgrom [53] there are more than enough undetected baryons in the Universe to make up the difference. If this constitutes the undetected cluster mass, then bullet cluster result implies that the baryons must be in some, effectively, dissipationless form – small, compact, cold clouds, for
example. Another possibility is that of massive neutrinos, either the standard three [54] or a new sterile neutrino [55]. If the three standard neutrinos have, for example, masses on the order of one to two electron volts, then, due to phase space constraints, they can accumulate on the scale of clusters but not that of galaxies. For standard neutrinos this issue will be settled soon because direct beta-decay measurements of the masses (or relevant limits) are currently underway [56].

From the beginning MOND has been criticized for its absence of a relativistic extension. As a non-relativistic theory, MOND (or the Bekenstein-Milgrom theory), makes no predictions with respect to relativistic phenomena, such as gravitational lensing or an alternative cosmology. Again this shortcoming is an incompleteness, but it is not a falsification.

Felten, in his paper on dynamical problems with the original MOND formulation [35], first described the potential problems with a MONDian cosmology, at least with the quasi-Newtonian treatment. He pointed out that the expansion in a region dominated by MOND cannot be uniform; separations cannot be expressed in terms of a universal scale factor. This means that any such region will eventually re-collapse regardless of its initial expansion velocity or density (Felten did note a positive aspect of this property: structure formation is hierarchical from bottom up and that the size of region now separating from the Hubble flow would be on the order of 30 Mpc, comparable to the scale of large scale structure). Felten’s argument does not represent a failure of MOND, but more likely the absence of a Birkhoff theorem. This theorem, arising in the context of general relativity, justifies the application of Newtonian dynamics to a uniform spherical expanding region in deriving the Friedmann equations. In the deeper theory of MOND a missing Birkhoff theorem would seem to be consistent with the external field effect.

Applying the MOND non-relativistic formalism only to density fluctuations does permit a standard cosmology but with a much enhanced growth rate for the inhomogeneities [57, 58]. This all remains tentative, however, in the absence of a relativistic theory. Is, for example, the acceleration parameter constant or does it vary with cosmological time?

Thus, the essential challenge for MOND has been and remains the absence of a more basic theoretical underpinning of the concept – a relativistic theory. There are various candidate theories, to be discussed further in this compendium, but I will summarize several historical developments (see also the up-to-date review in [40]).

Most of the trial theories, following the original Bekenstein-Milgrom relativistic toy theory, fall in the category of multi-field theories. Here the gravity force is mediated by two fields – the usual tensor field of general relativity and a second field, most often, a scalar field. This is illustrated in Fig. 9 where the force from the usual GR-Newtonian field is compared with that of a non-standard scalar field. The two fields become equal in strength at the MOND radius, $r_0 = \sqrt{GM/a_0}$, but the acceleration due to the scalar field must return to $1/r^2$ attraction at higher accelerations in order to satisfy Solar System constraints.

In 1994 Bekenstein and I [59] addressed the problem posed by enhanced gravitational lensing about clusters of galaxies, as was evident in the observations by that time [60]. As mentioned above, a traditional scalar-tensor theory fails to produce lensing beyond that of general relativity (without dark matter) due to the conformal relation between the physical and Einstein metrics. A conformal transformation takes the geometry of a particular space-time and expands or contracts it isotropically in a space-time dependent way. Bekenstein (1993) had earlier realized that a disformal relation between the metrics could be a way around the problem of lensing [61]. A disformal transformation picks out one direction in a preferred frame as special for additional contraction or expansion, and null geodesics of the original and transformed metric do not coincide. In 1997 I proposed a specific theory that realized such a transformation [62] – a theory with a non-dynamical vector field pointing in the direction of cosmological time, thus breaking the Lorentz Invariance of gravitational phenomena as well as time-reversal invariance (this was actually a modified form of historical alternatives to General Relativity known as “stratified theories” [63]). I demonstrated that, with a particular form for the disformal trans-
Fig. 9. A log-log plot of the run force per unit mass due to the traditional Newton-GR field (solid curve) and due to a non-standard scalar field that produces MOND phenomenology (dashed curve). The two forces become equal at the MOND radius $r_m = \sqrt{GM/a_0}$. At high accelerations the scalar force must also develop a $1/r^2$ dependence to satisfy Solar System constraints on deviations from inverse square attraction.

formation, one could provide a scalar-tensor theory for MOND that yielded the same relation between deflection of photons and the total weak field force (including the MOND force) as in General Relativity; that is to say, with respect to lensing the theory was equivalent to GR with dark matter.

The problem was that the non-dynamical vector field – a field that acts upon matter and other fields but is not acted upon – violates the spirit and letter of covariance with bad consequences for conservation principles. This deficiency was corrected several years later by Bekenstein in his fully covariant theory TeVeS (tensor-vector-scalar theory) [38]. As in the stratified theory, TeVeS makes use of two fields in addition to the usual metric tensor of General Relativity – a scalar field to provide the MOND force and a vector field to provide a disformal relationship between the Einstein and physical metrics. But the vector field is fully dynamical and its own field equation and source. Although the theory is not without problems, phenomenological and conceptual, it demonstrates that a fully covariant theory of gravity leading to MOND in the weak field limit can be constructed. [One such conceptual problem is that the theory does not reduce to General Relativity as the acceleration constant, $a_0$, approaches zero.]

There have been a number of successors to TeVeS which attempt to address some of the perceived problems. For example, I introduced a bi-scalar vector-tensor theory to provide a cosmological interpretation of $a_0$ along with cosmological dark matter in the form of long wavelength bosons that do not cluster on the scale of galaxies [64]. Einstein-Aether theories (vector-tensor theories) [65] can be adapted as a relativistic theory of MOND; here a function of the vector field invariant is added to that of the tensor field to promote MOND phenomenology [66]. One advantage of EA theories is that a cosmological term naturally appears with a value on the order of $a_0^2$. Bimetric theory has also been suggested [67]; here the difference of the two Levi-Civita connections (also a tensor) appears directly as an acceleration, normalized by $a_0$, and is added to the action that includes the Ricci scalars of both metrics. It can be shown that this theory also produces a cosmological term of order $a_0^2$ and reduces to Einstein-Aether theory when the two metrics are disformally related via a unit vector. I mention also
non-local single metric theories [68] and dipolar dark matter theories [69].

In an entirely different vein there is also work building upon the interpretation by Verlinde [70] of the Newtonian attraction as an entropic force due to microscopic degrees of freedom on a holographic screen. When a maximum scale corresponding to the de Sitter horizon is introduced for the screen, the gravitational force is modified in a form equivalent to MOND with \( a_0 \) being identified with the Unruh temperature of the horizon (see [71] and references therein). This is an interesting suggestion but has not yet been fully developed.

It is clear that there is at present no shortage of theories, but at most one of these is correct. Until this is resolved by confrontation with cosmological observations, MOND cannot progress further as an alternative to the present paradigm on the largest scale.

8. Conclusions: Crazy ideas and MOND

As a field, astronomy is replete with unconventional characters who propose and become obsessed with bizarre theories (often after a distinguished career as a conventional scientist). One example is provided by Victor Ambartsumian, a well-known Armenian astrophysicist who, in the mid-1950s, began speculating about the role of galactic nuclei in the formation and evolution of galaxies [72]. He suggested that small galaxies are born complete from larger galaxies – emerging from the nucleus like Athene springing full grown from the head of Zeus. Clusters of galaxies are not gravitationally bound systems but recently born galaxies expanding away from the large parent galaxy in the center; given a few billion years, the whole configuration will dissipate into intergalactic space – thus solving the mass discrepancy problem in clusters. Although there were international meetings in which the idea was discussed, most astronomers did not take Ambartsumian’s proposal seriously. It was just too “crazy”. [It interesting to note, however, that Ambartsumian’s suggestion that the nuclei of galaxies have a dominant effect on the evolution and morphology of galaxies has re-emerged recently in a different guise with a different vocabulary – “feedback” in which activity of the central massive black hole limits the growth of the visible object.]

Every astronomer is aware of other examples, particularly in cosmology, and given the abundance of such bizarre proposals, it is an effect that we should be wary of. So in the context of the present discussion the question arises: does MOND fall into the category of crazy ideas? From the strong emotions that the mere mention of the word evokes at times, it would certainly seem as though some distinguished scientists think so. But here, I will argue that, while MOND is unconventional and inconsistent with the current cosmological paradigm, it is by no means in the category of crazy ideas. And we should recall that many constructs of modern physics, such as quarks, were at an early stage considered crazy and condemned quite viciously by renown scientists. [George Zweig, one of the creators of the concept of quarks wrote: “The reaction of the theoretical physics community was generally not benign.... When the physics department of a leading university was considering an appointment for me, their senior theorist, one of the most respected spokesmen for all of theoretical physics, blocked the appointment at a faculty meeting by passionately arguing that the model was the work of a charlatan.” (quoted by Harold Frisch [73]).]

Robert Ehrlich, in his book *Nine Crazy Ideas in Science* [74] has listed several questions to be answered in order to tell if a crazy idea just might be true? I paraphrase these here as a list of criteria for an idea to be taken seriously.

1. The new hypothesis should make some contact with familiar physics. Well established physical principles or some reasonable extension of those principles should be respected. It is not, for example, easy to make such a connection for the idea of non-cosmological redshifts of quasars. But we have seen that MOND makes plausible modifications of current physical law in a regime where this law has never been tested and, certainly in its Lagrangian form, does embody cherished principles.

2. The proposer of the idea should be a knowledgeable and respected scientist, although he/she may come from outside that particular field. The world is full of gifted amateurs who imagine that they have

©2014 NRC Canada
found the key to a particular problem (or the theory of everything), but almost never are they near the truth. Again MOND and its creator, along with the several respectable scientist who have contributed to its development, meet this condition.

3. The proposer should not be overly attached to the idea. I would not rank this condition so highly because, actually, it is difficult not to become attached to an idea that one believes is correct and not to feel somewhat defensive when the idea is ignored by a majority of the relevant community. More relevant is for the proposer not to be dismissive of observations or data that does not support the idea. I believe that the supporters of MOND in general have not tried to sweep anything under the rug, although perhaps they have been overly dismissive of cosmological observations that are now quite precise.

4. Statistics should be applied in an honest way. The use of *a posteriori* statistics with respect to non-cosmological redshifts is an example of misapplied statistical arguments, but this is not so relevant to the problem of the mass discrepancy in astronomical objects. Of course, scaling relations such as Tully-Fisher are statistical in nature, but the most stringent and relevant selection criteria have been applied to data such as that plotted in Figure 4; for example it is the asymptotically constant value of the rotation velocity, beyond the visible disk, that is plotted rather than the width of a global line profile.

5. The proposer should have no agenda going beyond the science of the issue. This is more relevant to fields with political or economic impact, such as the reality (or not) of global warming; it does not apply here.

6. The theory should not have many free parameters. MOND has exactly one new fixed free parameter – $a_0$ the critical acceleration which has, coincidentally, a cosmologically interesting value.

7. The idea should be backed up by references to other independent work. Here again, the tests of MOND are generally based on data – kinematic and photometric – taken by independent, objective observers with no personal stake in the theory. In fact, they are most often negative about the idea.

8. The idea should not try to explain too much or too little. Most of us have received emails from amateurs who attempt to explain dark matter, quasars, the Big Bang, solar neutrinos and the frequency of earthquakes with one grand theory. Such theories cannot actually calculate anything or make definite predictions. On the other hand, if the theory is too narrowly focussed on, for example dwarf spheroidal galaxies, then it lacks the generality appropriate to a physical theory but could more properly be described as a model. MOND, I would say, strikes the right balance in addressing the mass discrepancy in astronomical systems, although its implications would certainly be more general.

9. The supporters should be open about their data and methods. With respect to MOND, nothing is hidden. The data, photometric and kinematic, are published by others and generally available. The methods are simple, clear and reproducible by anyone.

10. The theory should provide the simplest explanation of the phenomena. There should not be too many epicycles necessary to save the phenomena. MOND comes out very well indeed under this criterion. It has never been modified with new constructs or parameters in order to explain those phenomena which it addresses, even though observations have become more precise and the data have improved considerably. The dark matter hypothesis, on the other hand, has required numerous tweaks and adjustments to explain the well-known discrepancies – the cusp-core problem, the missing satellites, the galaxy mass function.

To this list I would add two more points: First of all, it is significant if the idea is falsifiable. Are there observations that can disprove the theory in question? For MOND this is certainly the case; if, for example, the algorithm predicted less matter in clusters than is actually observed, this would be a certain falsification. For galaxy rotation curves, if a number of these required mass-to-light ratios that were negative, then this would be a falsification. Secondly, it is significant if the number of proponents increases with time, especially if the new converts are younger scientists. It does not bode well for a theory if its principal supporters comprise a declining group of embittered old men. This is not the case.

©2014 NRC Canada
for MOND, where the number of advocates, especially younger scientists, has doubled or tripled over
the past 10 years.

So MOND meets most, if not all, of Ehrlich’s (and my) criteria for an idea to be taken seriously. Why
then has it languished for more than 30 years outside of the mainstream? After all, quarks became
an accepted concept within a few years of being proposed.

This is not due to some grand conspiracy. There was, however, from the first appearance of MOND
an alternative and dominant paradigm that claims to account for the same phenomena. There are strong
social factors that maintain support of the prevailing paradigm: an overriding tendency for scientists
to work within the established framework and to select data that reinforce rather than challenge it (an
effect that is supported by competition for academic positions and grants); and significantly, in this
case, there is the general reluctance of most astronomers to tamper with historically established laws
of nature – in part a reaction against the plethora of crazy ideas in astronomy.

But beyond this there is a tendency for astronomers to regard cosmology as the queen of sciences
– a reductionist undercurrent that gives cosmology priority over mere galaxy phenomenology. Data
such as the pattern of anisotropies in the CMB are very well-fit by the standard cosmological model,
albeit with a somewhat strange combination of six parameters. Given the precision of the fit, then the
theory must be correct, even in its implications for galaxies. This makes most cosmologists dismissive
of galaxy phenomenology and its wealth of regularities. These are problems that will be understood in
the context of more detailed computations of the baryonic processes in galaxy formation and evolution
– problems for the future.

But viewed strictly as an epistemological issue – without prioritizing classes of data – the CMB
anisotropies are defined by a single curve, the angular power spectrum, which can be fit by a six par-
parameter model. But there are at least 100 well observed galaxy rotation curves which, when combined
with population synthesis models of the stellar populations of galaxies and thus color related mass-to-
light ratios, can be fit by a theory having one fixed universal parameter.

For MOND, the absence of a more basic theoretical underpinning of the idea remains the essential
weakness. This means that cosmological calculations and predictions must be deferred. Ideally, one
would like to have a theoretical determination of the function $\mu$ that interpolates between the Newtonian
and MONDian regime – at least for the phenomenon of rotation curves. The near coincidence of the
acceleration parameter $a_0$ with $cH_0$ must be significant but there is not yet a convincing explanation.
Does $a_0$ vary with cosmic time, as $cH_0$? Or is it an aspect of a fixed cosmological constant $\Lambda$? (In some
theories, such as the Einstein-Aether version, $\Lambda$ would appear to have a scale on the order of $a_0^{-2}$.) Is
MOND more properly a modification of gravity or of inertia? In general relativity these are two sides
of the same coin, but is that equivalence broken at low accelerations?

I have argued that the success of MOND on the scale of galaxies strongly challenges the concept
of a dissipationless dark fluid that clusters on these scales. But are there other problems with the stan-
dard cosmology – problems that manifest themselves on cosmological scales and times? It is certainly
impressive that, more or less, the same set of parameters emerge from different observations: the CMB
anisotropies, the recent expansion history as traced by distant supernovae, the power spectrum of matter
fluctuations. But rather than the precision with which the parameters of a model Universe are deter-
mined, it is the peculiar composition and the remarkable coincidences embodied by the concordance
model that call for deeper insight. These motivations for questioning a paradigm are not unprecedented;
such worries led to the inflationary paradigm that has had profound impact on cosmological thinking
over the past 30 years.

A more practical difficulty is the absence of an independent detection of dark matter particles. In
the context of $\Lambda CDM$ they should be abundant locally – they cluster in the Milky Way. Where are
they? Because the properties of hypothetical particles are limited only by the human imagination, the
concept of dark matter is fundamentally not falsifiable, but at some point continued non-detection must
become a worry. This is an issue that is at least as problematic for dark matter as the absence of a
cosmology is for MOND. Without independent detection the concept of cold dark matter clustering on
galaxy scales remains hypothetical, and the standard cosmological paradigm, upon which it is based,
is a pipe dream.

All of this illustrates the dangers to the creative process in science presented by dogma too widely
and too deeply accepted. In the context of the standard paradigm most work on galaxies – for example
semi-analytic galaxy formation models – is built around patching up the standard model to make it
work rather than challenging it. Indeed, it is difficult to imagine that a fundamental challenge could
emerge from techniques characterized by numerous adjustable parameters or effects newly added as
needed. MOND presents a different, and more traditional, sort of science in which definite predictions
are made and verified – or not. This is why the idea remains and gains support.

I thank Moti Milgrom and Stacy McGaugh for helpful comments on the manuscript.

References

1. M. Milgrom, Astrophys.J. 270, 365 (1983a).
2. M. Milgrom, Astrophys.J. 270, 371 (1983b).
3. M. Milgrom, Astrophys.J. 270, 384 (1983c).
4. G.S. Shostak & D.H. Rogstad, Astron.Astrophys 24, 405 (1973).
5. M.S. Roberts & R.N. Whitehurst, Astrophys.J., 201, 327 (1975).
6. A. Kalnajs, IAU Symp. 100: Internal Kinematics and Dynamics of Galaxies, ed. E. Athanassoula, Reidel
   (Dordrecht), p. 87 (1983).
7. S.M. Kent, Astron.J. 91, 1301 (1986).
8. V.C. Rubin, W.K. Fort, N. Thonnard, Astrophys.J. 238, 471 (1980).
9. A. Bosma, Astron.J. 86, 1825 (1981).
10. S.M. Kent, Astron.J. 93, 816 (1987).
11. T.S. van Albada, J.N. Bahcall, K. Begeman, R. Sancisi, Astrophys.J. 295, 305 (1985).
12. R.H. Sanders, The dark matter problem: a historical perspective, Cambridge Univ. Press, Cambridge
   (2010).
13. J.P. Ostriker and P.J.E. Peebles, Astrophys.J. 186, 467 (1973).
14. F. Zwicky, Act.Helv.Phys. 6, 110 (1933).
15. R.B. Tully and J.R. Fisher, Astron.Astrophys. 54, 661 (1977).
16. M. Aaronson, J. Huchra, J. Mould, Astrophys.J., 229, 1 (1979).
17. S.M. Faber and R.E. Jackson, Astrophys.J. 204, 668 (1976).
18. K.C. Freeman, Astrophys.J. 160, 811 (1970).
19. R.J. Allen and F.H. Shu, Astrophys.J., 227, 67 (1979).
20. R.A. Fish, Astrophys.J. 139, 284 (1964).
21. M. Aaronson, Astrophys.J. 266, L11 (1983).
22. W. Romanishin, N. Krumm, E.E. Salpeter, G.Knapp, K.M. Strom, S.E. Strom, Astrophys.J., 263, 94
   (1982).
23. A. Finzi, Mon.Not.RAS 127, 21 (1963).
24. R. Kurth, Zeit.f.Astrophys 28, (1950).
25. H.C. van de Hulst, E. Raimond, H. van Woerden, Bull.Astr.Inst.Neth. 14, 1 (1957).
26. J.E. Tohline, IAU Symp. 100: Internal Kinematics and Dynamics of Galaxies, ed. E. Athanassoula,
   Reidel (Dordrecht), p.205 (1983).
27. J.R. Kuhn and L. Kruglyak, Astrophys.J. 313, 1 (1987).
28. R.H. Sanders, Astron.Astrophys. 136, L21 (1984).
29. S.M. Faber and J. Gallagher, Ann.Rev.Astron. Astrophys. 17, 135 (1979).
30. R.H. Sanders and M.A.W. Verheijen, Astrophys.J. 503, 97 (1998).
31. R.H. Sanders and S.S. McGaugh, Ann.Rev.Astron.Astrophys. 40, 263 (2002).
32. K.G. Begeman, A.H. Broels, R.H. Sanders, Mon.Not.RAS 249, 523 (1991).
33. S.S. McGaugh, Astrophys.J. 632, 859 (2005).
34. S. Casertano and J.H. van Gorkom, Astron.J. 101, 1231 (1991).
35. J.E. Felten, Astrophys.J. 286, 3 (1984).
Appendix

Below I reprint, with permission of Moti Milgrom, his response to a letter from John Bahcall concerning Milgrom’s three preprints circulated in 1982. Bahcall had doubts about the timeliness of these papers because he felt that there was no crisis with the hidden matter hypothesis. This is one of several issues that Milgrom discusses in this very insightful letter. His points are philosophical and phenomenological and presage comments by myself and others made some years later on. I make no further comment upon this remarkable document.

©2014 NRC Canada
Many thanks for your frank letter of Febr. 26. I was disappointed to read that you have not read at least the more detailed paper on galaxies. I do not think it is possible to appreciate the work after reading only the shorter paper.

I did not write the shorter paper as an independent piece. In it, I meant to discuss mostly matters of principle and mention only briefly the results of the other papers, in the hope that it will serve as an appetizer. Obviously, for you it did not.

As I already wrote to you, the flatness of galaxy rotation curves was the first hint for me, some 3 years ago, that hidden mass may not be a satisfying explanation. Since then, during my stay in Princeton and after that, I have studied the data very carefully. If you could see through the dust (that no doubt exists), as I think I can, I think you would be much more positive. If one prefers to wait until much of the dust settles down, one will see what there is to see with everybody else.

I think I am very careful with the data. I have not invented any of the regularities which I take to exist in the data and I think I have a good eye for things in the data which call for an explanation. Knowing what I know about the data, I feel very strongly that a new explanation is needed.

If I understood correctly, you think that the time has not yet come to consider alternatives to the Newtonian dynamics as you see no crisis emerging from the data as you now understand it. At the time of Copernicus you could argue that the time has not come to consider alternative to the Ptolemyan system as it explained the data very well if you were willing to take enough epicycles. Whether the time has come or not is very much a matter of taste. I am sure you will find many that are deeply dissatisfied with the hidden mass, as I personally am.

It should be realized that to say that the dynamics in galaxies and systems of galaxies are explained by hidden mass, which cannot be detected otherwise, is really saying practically nothing. I have not proved this as a theorem, but practically any set of dynamic measurements is describable in terms of some mass distribution, in the framework of Newtonian dynamics (as long as you need more mass than you see). What I mean is that you will never be able to rule out the HM ("hidden mass") hypothesis from dynamical measurements. You may be able to rule out certain forms of matter because they may have this or that extra effect, but you can always find shelter in unknown type of mass, etc.

What this also means is that it will be extremely difficult for the HM hypothesis in its general form to make any predictions and in fact, to the best of my knowledge, it does not make any testable predictions (other than those which may have to do with the nature of its constituents).

In addition, it can be said that none of the observed properties of galaxies or systems of galaxies are explained by the HM, in the sense that none follows in any way from this hypothesis and perhaps other known properties of galaxies (not the flat rotation curves, nor Tully-Fisher, nor the Oort discrepancy or the universal surface brightness).

What would make me seriously consider abandoning the HMH would be an alternative which a. does not clearly conflict with any known experiment or observation, b. explains many of the properties which the HMH does not. Namely that these results follow from the alternative theory and c. makes well defined predictions which can be tested.

Now, I think that my scheme has these properties. I do not expect one to accept it after reading my papers (certainly not before reading them), but I think that I can expect that it will be put to further consideration and test.

Whether the time has come to consider alternatives is not a good question to ask in this situation. I am suggesting an alternative that is a result of careful thinking and a lot of work. The question to ask is whether it stands the test or not.

You say in your letter that one could think of many modifications of the present physics which will explain the data. I do not think that this is so. Again I thought about this a lot and I believe the data is more than enough
to narrow down the possibilities very strongly. It is true that the particular form of the modification I use is one of a few possible ones that I can think of, but they all share a few features which are the basis of my suggestion and from which most of the results follow. They are: 1. a breakdown of Newtonian dynamics (second law and/or gravity). 2. the quantity which matters is the acceleration (kinetic or gravitational). 3. there exists an acceleration constant which determines the border between the Newtonian and non-Newtonian regime. 4. in the extreme non-Newtonian regime the square of the acceleration is proportional to the usual gravitational force. From these follow the flat rotation curves, Tully-Fisher, preferred value of the surface brightness, a discrepancy in the z-dynamics, etc. Details such as the exact shape of the rotation curves, the exact value of the Oort discrepancy, etc. depend on details of the modification.

You also complain in your letter that I do not really give a theory. You could in the same vein complain to Bohr for not suggesting the Schrödinger equation together with his model for the atom (to remind you it took 13 years and many people much better than myself to get from one to the other). I could give many more examples although I do not really think that they mean much, as one can always bring counter examples. I think that each case should be considered on its own.

The important things are that I do make predictions and I do find relations between various properties of galaxies, and that known properties of galaxies follow unambiguously from my proposed scheme. Take for example $a_0$. It clearly has life of its own independent of my proposal. It appears directly from the observations as 1. a universal average acceleration in spirals and ellipticals (as reflected in the preferred value of the surface brightness). 2. It appears as the proportionality factor in the Tully-Fisher relation for discs and ellipticals. Looking at things my way I noticed that this parameter of galaxies is also within the uncertainties the typical cosmic acceleration $cH_0$. Maybe this is a coincidence but surely it is an interesting result.

To make my point clearer perhaps, suppose that my predictions are born out, namely using an expression similar to mine with some $a_0$, rotation curves of individual galaxies are obtained correctly, the discrepancy in the z-dynamics is described correctly, deduced masses of systems of galaxies are equal to the observed ones, etc. One may still want to maintain the HM, as it too produces the observations if the HM is distributed properly. He will, however, have to agree that I found a single formula with one parameter which describes the distribution of the hidden mass in all galaxies and systems of galaxies (including the Oort-type mass) in terms of the observed mass distribution. In addition the parameter $a_0$ happens to equal $cH_0$. I would consider such a view an absurd one, and would much prefer to say that the formula really describes a modification of the physics and that there is really not much hidden mass.

I hope to see you in May and discuss matters in detail.

Best regards,

M. Milgrom

*************************************************************************

©2014 NRC Canada