What does the $W$ transverse momentum distribution say about the $W + 1$ jet/$W + 0$ jet ratio?

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Abstract

We compute the $W$ transverse momentum distribution at large transverse momentum at fixed next-to-leading order (i.e. $\mathcal{O}(\alpha_s^2)$) and find good agreement with the preliminary measurements reported by DØ. We find that the $W$ transverse momentum distribution is typically significantly larger than the exclusive one-jet rate for a given value of $p_T^W$ or $E_T^{\text{jet}}$, mainly because two or more jet events are excluded. As a consequence, we find that theoretically the $W + 1$ jet to $W + 0$ jet ratio $R_{10}$ is smaller than the analogous quantity defined in terms of $p_T^W$, $R^W$. This hierarchy is preserved under changes of renormalisation/factorisation scales, strong coupling constant and jet algorithm. However, this appears to be in contradiction with the preliminary experimental results which suggest $R_{10} > R^W$. 
During the past year the DØ collaboration has reported preliminary measurements of the ratio $R^{10}$ defined by

$$R^{10}(E_{T}^{\text{min}}) = \frac{\sigma(W + 1 \text{ jet})}{\sigma(W + 0 \text{ jet})},$$  

(1)

where the jets are defined with transverse energy above some $E_{T}^{\text{min}}$. At present the data appears to indicate larger values of $R^{10}$ than those obtained in next-to-leading order QCD predictions for the same quantity \cite{ref1, ref2, ref3, ref4, ref5}. There have already been several suggested explanations for this excess. For example, Balázs and Yuan \cite{ref6} have considered the effect of soft gluon resummation on the related quantity $R^{W}$,

$$R^{W}(p_{T}^{W, \text{min}}) = \frac{\sigma(W, p_{T}^{W} > p_{T}^{W, \text{min}})}{\sigma(W, p_{T}^{W} < p_{T}^{W, \text{min}})}.$$

(2)

New physics effects are also possible and Choudhury et al. \cite{ref7} have considered the effect that a massive vector boson with the quantum numbers of both a $W$ boson and a gluon would have on the observed value of $R^{10}$. A more mundane explanation is that an increase in the gluon parton distribution at medium Bjorken $x$ values would boost the $W + 1$ jet rate, which receives contributions from $qg$ scattering, while having little effect on the zero jet rate \cite{ref8}. Here we discuss the relationship between the large transverse momentum distribution of the $W$ boson, for which the DØ collaboration recently reported a preliminary measurement \cite{ref9}, and the $R^{10}$ ratio. We also consider the extent to which we can accurately theoretically predict each of these quantities.

At leading order in perturbative QCD the large $p_{T}$ distribution of the $W$ boson is given by the $W$ recoiling against a single parton. Typically, the parton will hadronize into an observed jet, with the jet $E_{T}$ balancing the $W$ transverse momentum, so that,

$$R^{W}(p_{T}^{W, \text{min}}) \simeq R^{10}(E_{T}^{\text{min}} = p_{T}^{W, \text{min}}).$$

(3)

Beyond lowest order this relationship is no longer strictly true at the parton level, however it makes the quantity $R^{W}$ worth studying in the context of the measured DØ excess in $R^{10}$. At the moment DØ make no direct measurement of the $R^{W}$ ratio. However, for a different analysis \cite{ref10} they have made a preliminary measurement of the normalised transverse momentum distribution of the $W, 1/\sigma d\sigma/dp_{T}^{W}$, and from this measurement we can construct $R^{W}$,

$$R^{W}(p_{T}^{W, \text{min}}) = \frac{\int_{p_{T}^{W, \text{min}}}^{\infty} dp_{T}^{W} \frac{1}{\sigma} \frac{d\sigma}{dp_{T}^{W}}}{1 - \int_{p_{T}^{W, \text{min}}}^{\infty} dp_{T}^{W} \frac{1}{\sigma} \frac{d\sigma}{dp_{T}^{W}}}.$$

(4)

The first question we wish to address is the following. Does the measured DØ $W$ $p_{T}$ distribution show a similar excess to $R^{10}$ when compared with perturbative QCD predictions? In figure \cite{ref10} we show the measured DØ $p_{T}^{W}$ distribution \cite{ref10} for $p_{T}^{W} > 20$ GeV as well as the fixed next-to-leading order (i.e. $\mathcal{O}(\alpha_{s}^{2})$) theoretical predictions using the parton level Monte Carlo program DYRAD \cite{ref3, ref4}. Experimentally, the $W$ is tagged in its decay into an electron
Figure 1: a) The DØ measured $p_T^W$ distribution, and the $\mathcal{O}(\alpha_s^2)$ predictions for the three choices of scale $\mu = M_W$, $E_T^W$, $p_T^W$; with the cuts described in the text. b) The ratio of the theoretical predictions, and the preliminary DØ measurement, to the theoretical prediction with $\mu = M_W$. Note that in comparing with the DØ data, we have integrated over the appropriate range of $p_T^W$. 
and neutrino, where the neutrino is detected though the presence of a significant amount of missing $E_T$. To match onto the experimental analysis, we apply the cuts,

$$E_T^e > 25 \text{ GeV}, \quad |\eta^e| < 1.1, \quad E_T > 25 \text{ GeV},$$

(5)
to trigger on the event. Jets are defined with a consize $\Delta R_{\text{jet}} = 0.7$ and may lie in the rapidity range, $|\eta_{\text{jet}}| < 3.5$. To simulate the experimental jet algorithm, we cluster all pairs of partons that lie within $R_{\text{sep}} \Delta R_{\text{jet}}$ of each other to form a proto jet, then test that all clustered partons lie within $\Delta R_{\text{jet}}$ of the proto jet. As a default parameter, we set $R_{\text{sep}} = 1.3$. The jet direction and transverse energy is constructed using the DØ recombination procedure. The electron must also be isolated from significant amounts of hadronic energy and we simulate the isolation criteria by imposing,$$
\Delta R(e, \text{jet}) > 0.4 \quad \text{for} \quad E_T^{\text{jet}} > 10 \text{ GeV}.$$ (6)
Throughout we use the CTEQ4M parton densities with $\alpha_s(M_Z) = 0.116$ unless stated otherwise and set the renormalisation scale, at which $\alpha_s$ is evaluated, equal to the factorisation scale, at which the parton densities are evaluated. For this joint scale we show $1/\sigma d\sigma/dp_T^W$ for the three choices $\mu = M_W$, $E_T^W$ and $p_T^W$ corresponding to a medium, hard and soft scale. We see that the theoretical prediction is only mildly sensitive to the choice of scale, which gives us confidence in the prediction for $1/\sigma d\sigma/dp_T^W$. Furthermore, it is clear that the $O(\alpha_s^2)$ prediction gives an adequate description of the preliminary DØ data. This is surprising for two reasons. First, the effect of smearing either the jet $E_T$ or $p_T^W$ is not taken into account. Such smearing can either arise from multiple soft gluon emission or from experimental mismeasurement of the energy of hadrons. Second, given the excess of the observed $R^{10}$ ratio over the DYRAD predictions, we would naively have expected the theoretical predictions to fall short for the $W$ transverse momentum distribution as well.

Of course, the $R^{10}$ ratio is directly related to the one-jet exclusive $E_T$ distribution rather than the $W$ boson transverse momentum distribution. We show both of these quantities in Fig. 2. For illustrative purposes, we also show the one-jet inclusive and second-jet $E_T$ distributions. As we might expect, we see that the inclusive one-jet transverse energy distribution is very similar to the inclusive $W$ transverse momentum distribution. At leading order, these two distributions are identical and this relationship is largely preserved at next-to-leading order. The one-jet exclusive distribution is suppressed relative to the one-jet inclusive rate because in some events the second parton is identified as a jet. This difference is described by the $E_T$ distribution of the second jet (short dashed line). From this plot, we anticipate that from the theoretical point of view, the $R^{10}$ ratio constructed from the single jet exclusive distribution will be somewhat smaller than the $R^W$ ratio determined from the $W$ boson $p_T$ distribution for given $E_{T_{\text{min}}} = p_{T_{\text{min}}}^W$ values less than around 120 GeV. As mentioned earlier, in the theoretical DYRAD prediction the effect of soft gluon radiation or hadron measurements on either the jet $E_T$ or the $W p_T^W$ is not taken into account. Such smearing will

1We systematically calculate the total cross-section, $\sigma$, at next-to-leading order (i.e. $O(\alpha_s)$) and with a scale $\mu = M_W$.

2At the large transverse momenta considered here, the effect of soft gluons is roughly to increase the boson transverse momentum by $O(2 \text{ GeV})$, which is much less than the bin width (5 GeV) used in the theoretical calculation.
Figure 2: The transverse energy distributions for the one-jet inclusive (solid), one-jet exclusive (long dashed) and second-jet inclusive (short dashed) rates, together with the $W$ boson transverse momentum distribution (dotted). The joint renormalisation/factorisation scale is $\mu = M_W$. 
generally lead to an increase in the jet $E_T$ and $W \, p_T^W$ and hence on both the $R^{W}$ and $R^{10}$ ratios. However because $R^{W}$ is sensitive to measurements on all hadrons/gluons in the scattering, whilst $R^{10}$ is only affected by hadrons/gluons that lie within the jet, we might expect the effect of smearing to increase the $R^{W}$ distribution more than the $R^{10}$ distribution.

In order to make the difference between the preliminary DØ measurements for $R^{10}$ and $1/\sigma \, d\sigma/dp_T^W$ more transparent, we convert the measured transverse momentum distribution into $R^{W}$ using equation 3. To estimate the experimental errors on $R^{W}$ we make the assumption that there are no common systematic errors and merely add the errors in the $1/\sigma \, d\sigma/dp_T^W$ distribution in quadrature. We show the extracted $R^{W}$ data in Fig. 3, as well as the DØ measurement for $R^{10}$ and the next-to-leading order predictions for both quantities. Note that in extracting $R^{10}$, events may only contribute to the $W + 1$ jet cross section when there is exactly one jet observed with $E_T$ above $E_T^{\text{min}}$. Similarly, the $W + 0$ jet cross section is constructed from events with no jets observed with an $E_T$ greater than $E_T^{\text{min}}$. In fact, the denominators for both $R^{10}$ and $R^{W}$ are identical at next-to-leading order. As expected from our discussion of the raw $W$ transverse momentum distribution, the theoretical prediction for $R^{W}$ is consistent with the extracted experimental value, although possibly slightly low because of the cumulative effect of events at higher $p_T^W$. We also see that, for a given $E_T^{\text{min}} = p_T^{W,\text{min}}$, the measured $R^{10}$ value lies above the experimental $R^{W}$ value. On the other hand, the corresponding next-to-leading order prediction for $R^{10}$ systematically lies below...
the $R^W$ value. This is the observed discrepancy between the measured $R^{10}$ value and the DYRAD prediction. In the case of the theoretical prediction the difference between $R^{10}$ and $R^W$ can be traced back to the fact that the numerator of $R^{10}$ in the DØ definition \((11)\) is the one-jet exclusive rate. While the experimental uncertainties in $R^{10}$ and $R^W$ are somewhat different, the exclusive nature of $R^{10}$ ensures that the next-to-leading order predictions will lie beneath those of $R^W$ for $E_{T}^{\text{min}} = p_{T}^{W,\text{min}} \leq 100$ GeV. Increasing the gluonic content of the proton or adding new heavy objects \([9]\) will affect both $R^{10}$ and $R^W$ but will not change this hierarchy. We note in passing that if the numerator of $R^{10}$ is replaced with the $W + 1$ jet inclusive rate, then the DYRAD predictions for $R^{10}$ and $R^W$ are almost identical.

We now turn to the question of how reliable the predictions for $R^{10}$ and $R^W$ are under variations of the renormalisation/factorisation scale. In Fig. 4 we show the relative change compared to $\mu = M_W$ for a soft scale, $\mu = E_T^{\text{jet}}$ or $\mu = p_T^W$ respectively, a hard scale, $\mu = E_T^W$ and simple multiples of moderate scales $\mu = 0.5M_W$ and $\mu = 2M_W$. We see that changes of $\pm 10\%$ are possible which are clearly not enough to account for the difference between the DYRAD prediction for $R^{10}$ and the DØ measurement. Furthermore, both $R^{10}$ and $R^W$ have a similar behaviour under these scale changes, and the difference between $R^{10}$ and $R^W$ is relatively insensitive to the changes of scales.

Given that the prediction for $R^W$ does fit the experimental measurement so well, we should search for the ways in which the $R^{10}$ measurement differs from the $R^W$ measurement. The only major difference between $R^{10}$ and $R^W$ is that for the $R^{10}$ measurement experimental jets must be formed, whereas this is not necessary for $R^W$. As DYRAD gives a next-to-leading order prediction for $R^{10}$ it includes configurations where two partons get

Figure 4: The scale dependence for the theoretical prediction for (a) $R^{10}$ and (b) $R^W$ relative to that for $\mu = M_W$. 
clustered into a single jet, and this gives it a non trivial dependence on the jet definition. Experimentally jets are formed from many hadrons, whereas inside DYRAD jets are made from at most two partons, and the difference between these can lead to a mismatch between jet algorithms at the hadron and parton levels. For example an experiment might choose to cluster all hadrons within some $\eta$-$\phi$ distance $\Delta R_{\text{jet}}$ from the centre of a jet. However, if we theoretically only cluster partons that lie within an $\eta$-$\phi$ distance of $\Delta R_{\text{jet}}$ of each other then the theoretical jet will never contain partons up to a distance $\Delta R_{\text{jet}}$ away from the centre of the jet. On the other hand, we could cluster all pairs of partons to form a proto jet, and then test if the partons clustered lie inside a distance of $\Delta R_{\text{jet}}$ of the proto jet direction for the proto jet to remain a genuine jet. This again is far from the experimental procedure, as there is no “seed tower” in the direction around which the proto jet is first formed. In order to understand this difference between the experimental and theoretical definition of a jet we use two extreme jet clustering algorithms in addition to the default algorithm described earlier with $R_{\text{sep}} = 1.3$.

1. **Narrow**: Cluster two partons if they lie within some $\Delta R_{\text{jet}}$ of each other corresponding to $R_{\text{sep}} = 1$.

2. **Wide**: Cluster all pairs of partons to form a proto jet and then test that all clustered partons lie within some $\Delta R_{\text{jet}}$ of the proto jet direction corresponding to $R_{\text{sep}} = 2$.

Figure 5: The dependence of the DYRAD prediction for $R^{10}$ on the jet clustering algorithm and on $\alpha_s$. We show the predictions normalised to that for $R_{\text{sep}}$ clustering and $\alpha_s(M_Z) = 0.116$. In all cases we have chosen $\mu = M_W$. 
The narrow and wide jet clustering definitions correspond to the narrowest and widest theoretical implementation of the experimental jet definition. In figure 5 we show the dependence of the $R^{10}$ prediction on the parton level jet clustering. Different jet clusterings vary the theoretical prediction by only a few percent which is certainly not enough to be able to explain the measured $R^{10}$ excess. The $R^W$ ratio shows very little dependence on the type of jet clustering chosen, arising only from the isolation cut on the observed electron.

Finally we show the dependence of the $R^{10}$ ratio on the value of $\alpha_s$ used in figure 5. As DØ have noted the dependence is less than expected due to a cancellation between the $\alpha_s$ dependence of the parton distributions and the hard scattering. Here we only note in addition that the uncertainty coming from the experimental definition of a jet is typically similar to the variation in the rate coming from $\alpha_s$, and so without an accurate understanding of exactly how to model jet clustering we do not expect measurements of $R^{10}$ to be useful in constraining $\alpha_s$. In this sense we feel that measurements of $R^W$ (or better still the analogous ratio from $Z$ boson events) would be a more reliable theoretical method for measuring $\alpha_s$ as it is largely free from the ambiguities of defining jets.

In summary, we have found good agreement between the $W$ boson $p_T$ distribution as reported by DØ and fixed order perturbative calculations. On the other hand, we find no way to reconcile the $R^{10}$ ratio measured by the DØ collaboration with the same theoretical calculation. Furthermore, because the $R^{10}$ ratio is exclusive in the number of jets, we see that for a given value of $E_T^{\min} = p_T^{W,\min}$, the predicted value for $R^{10}$ always lies beneath that for $R^W$ in the currently measured range. This appears to be in contradiction with the preliminary experimental results.

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