Application of multiple linear regression (MLR) analysis for concentration of chromite tailings by the flotation

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Abstract: In this study, the concentration of chromite minerals by amine flotation of a chromite tailing (slime) with content 23.84% Cr$_2$O$_3$ from Yeşilova-Burdur (Turkey) was investigated. In experimental studies, firstly, some operating parameters of the rougher flotation observed for a low-grade chromite tailing were investigated. Secondly, multiple linear regression (MLR) analyses were performed to determine the effects of some operating parameters on the performance of the rougher amine flotation. From the experimental results, multiple linear regression equations were developed to predict the recovery and grade of the chromite concentrate, and the regression coefficients between experimental and predicted values were found to be quite good ($R^2$ values of 0.772 and 0.917, respectively). Additionally, it was found that the conditioning time and low pH value using H$_2$SO$_4$ showed an important effect on the recovery and the grade of the chromite concentrate.

Keywords: chromite, tailing, flotation, separation, multiple linear regression analysis

1. Introduction

Chrome is an important metal in especially stainless steel production. The most important ore in the production of chrome is chromite ore. The main mineral components of the chromite ore are chromite, olivine and serpentine minerals. It is desirable that Cr$_2$O$_3$ content should be over 40% for saleable chromite ores in the market. If chromite content is under 40% Cr$_2$O$_3$ then it must be concentrated. Gravity, magnetic and flotation methods are generally used to concentrate low-grade chromite minerals from gangue minerals such as olivine and serpentine. It is very important to know the physical, chemical and mineralogical properties of chromite ores in the determination of concentration type. Additionally, the reserve and grade of chromite ore also affect determination of beneficiation method (Deniz, 1992; Deniz et al., 2001; Deniz, 2019).

Chromite production is performed in various regions of Turkey, primarily Adana, Kayseri, Bursa, Elazig, Denizli and Burdur. There are more than 50 chromite concentrators in Turkey. These concentrator plants use mostly gravity-based concentration methods such as jiggling and shake tables (Deniz et al., 2001; Guney et al., 2001; Deniz, 2019). During the concentration, a significant amount of fines and chromite minerals are produced as tailings at the mineral processing plants, and this situation not only causes resource losses due to the valuable chromite mineral content but also serious environmental hazards.

In recent years, more researchers have focused on the concentration of chromite minerals from the tailings in terms of environmental protection and economics. Gravity methods are generally applied for the beneficiation of chromite due to economic reasons, but generally their very fine-sized fractions (less than 0.1 mm) are discarded as gangue. The loss of fines from different gravity plants in the World reaches millions of tons. As with the concentration of all other minerals, the very fine chromite tailings (slimes) should also be evaluated with high recoveries. Therefore, the flotation method is very useful as a necessary process for the concentration of the very fine chromite particles (slimes).
Industrial-scale flotation is rarely applied to chromite ores in the world. Due to the high-quality chromite ore reserves in Turkey, plant scale concentration studies have not been done on a chromite plant tailing in Turkey until now, and chromite tailings are always stored in heaps. In subsequent years, chromite tailings will have to be evaluated due to not only decreasing the quality of chromite deposits in Turkey but also in terms of environmental pollution. Therefore, this study will serve as an important task in shedding light on the use of the flotation method in chromite concentration to be carried out on the plant scale in the following years.

Many studies have been done on the flotation of chromite with anionic collectors such as petroleum sulphonate, oleic acid and Na-oleate which is one of the oleic acid salts (Havens, 1946; Palmer et al., 1975; Kotlyar et al., 1995; Sysila et al., 1996; Guney and Atak, 1996; Seifelnasr and Tammam, 2011). In these studies, generally, anionic flotation of chromite mineral has been carried out successfully by using fatty acids in an alkaline medium. However, especially, Na-Oleate flotation of a chromite ore containing serpentine instead of olivine is very difficult because both minerals exhibit almost similar surface properties with chromite mineral (Gallios et al., 2007). In very few previous studies (Smith, 1981; Smith et al., 1981; Andrews, 1990; McDonald, 1990), the effects of some cationic collectors on chromite flotation, especially at low pH, were discussed.

Even though the recovery and the grade determination in the mineral processing are not costly, they are routine application items in mineral processing plants. Therefore, prediction of the recovery and grade of the concentrate using a mathematical method can help in the operation of the mineral processing plants. Many different statistical methodologies (linear and nonlinear regressions) have been applied to predict the recovery and grade of the concentration fractions.

Recently, several modeling approaches have been put forward for the optimization of multiple variables. Regression analysis is an important statistical method to investigate the relationship between two or more variables. The linear regression model supposes that there is a linear relationship between the dependent variable and each independent variable. If there are two and more independent variables, the linear regression model is called a multiple linear regression model (Coruh et al., 2013).

In this study, firstly, a comparative study was conducted to determine the effects of some operating parameters on the performance of the rougher amine flotation of chromite tailing. Then, it was aimed to estimate the recovery and grade of the chromite concentrate by multiple linear regression (MLR) method depending on some operating parameters of the rougher amine flotation. The mathematical models for the recovery and grade of the chromite concentrate fraction were determined by using a statistical software package (SPSS).

As a contribution to the originality of this study, while chromite flotation is generally done with anionic collectors in the basic medium, as a difference in this study it was performed with an anionic collector at low pH. Additionally, the effect of acid type (HCl or H₂SO₄) for pH adjustment was found to be very important in determining the acidity in the flotation.

2. Materials and methods

2.1. Material

The sample used in this study were, first, taken by drilling from different sections of the heap of chromite tailings (slimes) with the amount of the 200,000 Mg located in Yesilova town of Burdur (Turkey). Next, these samples were mixed to obtain, a representative sample to be used for experimental studies. Then, this sample was sieved to different size fractions to determine the Cr₂O₃% content in sample (Table 1). As shown in Table 1, while the particle size decreased, the chromite grade also decreased. These results indicated that gangue minerals (serpentine, olivine etc.) were a little more brittle than chromite minerals.

Additionally, the mineralogical analysis of the tailing sample was performed to determine the texture of chromite and gangue minerals from thin and polished sections by using Canada balsam. The results of mineralogical analyses of the sample (Fig. 1) showed that the chromite tailing contained chromite and serpentine which formed as a result of alteration of serpentine group minerals such as olivine, chrysotile and talc. As seen in Fig. 1a, the sample was completely serpentinized. Also, olivine remains can be relatively defined as fine particles within the serpentine matrix (Fig. 1b). Sieve texture developed predominantly in the serpentinized areas (Fig. 1c). Chromite grains can be seen as cataclastic...
texture (Fig. 1d). The chromite tailing contained approximately 20-30% of chromite, 60-70% of serpentine and 5-10% of olivine.

Table 1. Particle size distribution and Cr₂O₃% content of the chromite tailing as a function of sieve size

| Sieve Size | Weight (%) | Cr₂O₃ (%) | Distribution (%) | Weight (%) | Cr₂O₃ (%) | Distribution (%) |
|------------|------------|-----------|------------------|------------|-----------|------------------|
| -75+63     | 13.00      | 26.91     | 14.67            | 13.00      | 26.91     | 14.67            |
| -63+45     | 14.25      | 26.41     | 15.79            | 27.25      | 26.65     | 30.46            |
| -45+38     | 18.95      | 24.23     | 19.26            | 46.20      | 25.05     | 49.72            |
| -38        | 53.80      | 22.28     | 50.28            | 100.00     | 23.84     | 100.00           |
| Σ          | 100.00     | 23.84     | 100.00           |

Fig. 1. Views of the texture of chromite and gangue minerals from thin and polished sections of the chromite tailing, XN 10x10; (a) Grain of serpentine embedded in a chromite matrix, (b) Completely transformation into serpentine in micro fractures of olivine grains, (c) Development of serpentine into sieve texture of olivine, (d) Cataclastic texture in a chromite mineral

2.2. Methods

The general flotation method for chromite ores was developed by Havens (1946). Chromite minerals can be floated using long-chain fatty acid collectors (Oleic acid and its salt) and petroleum sulphonate between pH= 1.5 and 5.5. The Havens method generally results in higher chromite recoveries, but a decrease in concentrate grade occurs. Na-Oleate, one of the oleic acid salts, is widely used at high pH (pH= 8-10) values for flotation of oxide minerals (hematite, alumina, etc.) (Fuerstenau, 1970).

In flotation of the oxide minerals by anionic or cationic collectors, they can adsorb onto the oxide minerals and then float the oxide minerals depending on the pH associated with the charge value of zero point of charge (ZPC) of the minerals. Determining the charge sign (negative or positive) on the oxide mineral surfaces is necessary to ascertain the response of minerals by using cationic or anionic collectors.
collectors in the flotation (Fuerstenau, 1970). As an example, anionic collectors can be used only below pH= 9.0 (zpc value for alumina) in the flotation of alumina, while cationic collectors can be used above pH=9.0. This behavior explains the electrostatic model of the flotation (Jordan et al., 1996).

However, inorganic ions can affect the flotation behavior of oxidized minerals, such as the effect of a depressant. An example of the depressing effect can be given the effects of Cl$^-$ and SO$_4^{2-}$ on the alumina flotation at pH= 6.0 with sodium dodecyl sulfate (SDC), an anionic collector. That is, both Cl$^-$ and SO$_4^{2-}$ ions inhibit the flotation. This is attributed to the adsorption of Cl$^-$ and SO$_4^{2-}$ on the positively charged alumina surface at pH= 6.0. It should be noted that the effect of SO$_4^{2-}$ is much larger than Cl$^-$. Since SO$_4^{2-}$ is double charged compared to Cl$^-$ anion, it has a greater affinity and makes to specific adsorption of alumina surface. Alumina can be floated at pH 6.0 using a cationic collector and SO$_4^{2-}$ as an activator (Fuerstenau, 1970; Jordan et al., 1996).

Smith (1981) said that if the main gangue mineral of chromite ore is serpentine, it could easily float with an amine collector, and if the main gangue mineral is olivine, it could not float. Additionally, if it is used H$_2$SO$_4$ or HCl as a pH modifier, H$_2$SO$_4$ has a beneficial effect on chromite flotation with an acidic pH. That is, sulfuric acid containing SO$_4^{2-}$ anions is more effective than HCl.

The flotation experiments of chromite tailing (slimes) were carried out using a Denver laboratory flotation machine. At first, an experimental program was prepared for the rougher amine flotation with five different process variables; the pH value, the collector dosage, the conditioning time, the solid ratio of feed pulp and the flotation time. The ranges of operating values for each parameter tested are shown in Table 2.

| Experimental No | Operating variables |
|----------------|---------------------|
|                | pH | Collector dosage (g/Mg) | Conditioning time (minute) | Pulp solid ratio of feed (Wt, %) | Flotation time (minute) |
| 1              | 2  | 1500 | 25 | 30 | 2.0 |
| 2              | 3  | 1500 | 25 | 30 | 2.0 |
| 3              | 4  | 1500 | 25 | 30 | 2.0 |
| 4              | 5  | 1500 | 25 | 30 | 2.0 |
| 5              | 6  | 1500 | 25 | 30 | 2.0 |
| 6              | 2  | 500  | 25 | 30 | 2.0 |
| 7              | 2  | 1000 | 25 | 30 | 2.0 |
| 8              | 2  | 1500 | 25 | 30 | 2.0 |
| 9              | 2  | 2000 | 25 | 30 | 2.0 |
| 10             | 2  | 2500 | 25 | 30 | 2.0 |
| 11             | 2  | 1500 | 5  | 30 | 2.0 |
| 12             | 2  | 1500 | 10 | 30 | 2.0 |
| 13             | 2  | 1500 | 20 | 30 | 2.0 |
| 14             | 2  | 1500 | 25 | 30 | 2.0 |
| 15             | 2  | 1500 | 30 | 30 | 2.0 |
| 16             | 2  | 1500 | 30 | 20 | 2.0 |
| 17             | 2  | 1500 | 30 | 30 | 2.0 |
| 18             | 2  | 1500 | 30 | 40 | 2.0 |
| 19             | 2  | 1500 | 30 | 50 | 2.0 |
| 20             | 2  | 1500 | 30 | 60 | 2.0 |
| 21             | 2  | 1500 | 30 | 20 | 1.0 |
| 22             | 2  | 1500 | 30 | 20 | 1.5 |
| 23             | 2  | 1500 | 30 | 20 | 2.0 |
| 24             | 2  | 1500 | 30 | 20 | 2.5 |
| 25             | 2  | 1500 | 30 | 20 | 3.0 |
The laboratory-scale flotation procedure was performed through the following steps:
1. Preparation of pulps of the chromite tailings at 20% to 60% solid ratios. The slime sample is easily dispersed in suspension with only a mixing during the conditioning process, without the need for a comminution process before the flotation experiments.
2. Adjusting pH of the pulp with H$_2$SO$_4$ and conditioning from 5 to 30 min, monitoring the pH, and adding H$_2$SO$_4$ if necessary.
3. Adding an amine collector (Armac-C) from 500 to 2500 g/Mg as water-soluble acetate salt followed by a 1 to 3 min rougher flotation time.
4. In floating chromite in a rougher stage, constant pH was maintained during the flotation.
5. In cleaning of the rougher concentrate, in each three stages, the amine collector was not added to cleaner flotation except pH modifier.
6. Combining cleaner tailings with new rougher feed and repeating steps from two to four.

3. Results and discussion
3.1. Rougher flotation experiments
Firstly, the effects of three different collectors for the slime sample, a grade with 23.84% Cr$_2$O$_3$, were investigated. The first collector was Na-Oleate, one of the oleic acid salts, a long-chain fatty acid collector. The second collector was a mixture of $\frac{1}{2}$A801 + $\frac{1}{2}$A825, a petroleum sulphonate produced by Cytec company, and the third collector was Armac-C (primary coco amine acetate), an amine type cationic collector. The flotation procedure for each collector was performed as specified by other researchers (Havens, 1946; Fuerstenau, 1970; Smith, 1981). In the experiments on collector type, the experimental conditions for each collector type used the solid ratio of feed pulp of 30% and collector dosage of 1500 g/Mg. Also, the other conditions for Armac-C were used a pH value of 2.5, conditioning time of 25 min and flotation time of 2 min. The other conditions for Na-Oleat were used a pH value of 8.5, conditioning and flotation time of 5 minute. The other test conditions for $\frac{1}{2}$A801 + $\frac{1}{2}$A825 were used a pH value of 8.5, conditioning and flotation time of 5 min. In the evaluation of the effects on the recovery and grade of the chromite concentrate for three different collectors, the amine-type collector (Armac-C) gave the best result as a similar to the results of the other researcher. Rougher flotation using Armac-C provided 80.52% recovery in a 31.09% Cr$_2$O$_3$ product (Fig. 2a). Both concentrate recovery and selectivity were very high in amine flotation at pH 2.0. In particular, the concentrate grade (Cr$_2$O$_3$) increased significantly in the cleaning stage at pH 2.0. The use of known silicate depressants used for chromite ore showed no effect on selectivity.

Deniz (1992) showed that the zero point of charge (zpc) for the Yesilova-Burdur (Turkey) chromite tailings generally was to be between pH 5 to 6. Therefore, adsorption of anionic collector on the chromite mineral surface at pH 8.0 was associated with the electrostatic attraction between the positive amine collector and the negative chromite surface. On the other hand, when the chromite surface was positively charged at pH 2.0, and the amine-type collector is positively charged, then it appears that it must be charged with an ion that will negative charge on the chromite surface.

Fuerstenau (1970) and Jordan et al., (1988) indicated that SO$_4$ ions in H$_2$SO$_4$ specifically adsorbed as on the chromite surface, and chromite surface gained a negative charge. Therefore, the flotation of chromite mineral at pH 2 can be attributed to the SO$_4$ ions in H$_2$SO$_4$ which are acting as a bridge between the positive chromite surface and the positive amine.

Serpentine is the most important silicate mineral in the Yesilova-Burdur chromite tailings. Results of thin and polished section investigations showed that there were very few olivine minerals in the concentrate, while most of the serpentine was found to be in the tailing. Due to the low olivine content of the Yesilova-Burdur chromite tailings, it is not necessary to use a selective olivine depressant to increase the selectivity of chromite in amine flotation.

The effects of some operation parameters (the pH value, the collector dosage, the conditioning time, the solid ratio of feed pulp and the flotation time) on the concentration of the chromite tailings were analysed by the flotation experiments, and they are shown in Fig. 2b-f.

In the experiments, firstly, the previous researchers (Fuerstenau, 1970; Smith, 1981; Smith et al., 1981; Jordan et al., 1996) reported that the pH value is very important for flotation with amines of chromite ores containing serpentine as the main gangue mineral. With the 23.84% Cr$_2$O$_3$ tailings, in the expe-
Fig. 2. Chromite grade-recovery trend depending on some operating parameters for flotation experiments at varying the pH from 2.0 to 6.0 by keeping of the other operating conditions constant showed that optimum chromite flotation occurred at pH 2.0 (Fig. 2b). The reason for this result is that both the degree and the recovery of the chromite concentrate may be increased due to the cleaning of mineral surfaces using H$_2$SO$_4$ at pH 2.0.

Secondly, the optimum collector dosage was determined by flotation experiments for the different collector dosages. Varying the Armac-C addition from 500 to 2500 g/Mg by keeping of the other operating conditions constant showed that optimum chromite flotation occurred at 1500 g/Mg Armac-C. The results given in Fig. 2c show that with the increase in the collector dosage, a significant decrease in the grade of concentrate was obtained, while a maximum in the chromite recovery to 1500 g/Mg Armac-C was obtained, then a decrease in the recovery of concentrate for the collector dosages above 1500 g/Mg was obtained.
The conditioning time was one of the most important parameters of operating conditions. Thirdly, the conditioning time for pH 2.0 was changed from 5 min to 30 min, and the results as given in Fig. 2d showed that there was the best result of conditioning time of 30 min. With the increase in the conditioning time, a significant increase in not only the chromite grades but also the recovery of the chromite concentrate was obtained. The reason for this situation can be caused by cleaning of mineral surfaces with long-term conditioning time using H_2SO_4. A similar situation was observed when the pure chromite mineral was long-conditioned with HCl acid before flotation by using the dodecyl amine collector (DAH) (Smith, 1981; Smith et al., 1981).

Fourth, the solid ratio of the feed dough ranged from 20% to 60%, keeping constant of the other conditions, and the solid ratio of feed pulp of 20% produced the best result as shown in Fig. 2e. In the experiments of the solid ratio of feed pulp, a similar situation was observed in the experiments made for the pH value. With the increase in the solid ratio of feed pulp, a significant decrease was obtained not only in the chromite grades but also in the recovery of concentrate.

Finally, the flotation time was varied between 1 min and 3 min by keeping constant of the other conditions, and the flotation time of 3 minute produced the best result as shown in Fig. 2f. It can be seen that Cr_2O_3 grade of concentrate decreased when the flotation time increased, but the recovery of concentrate increased.

According to all experimental results, the best results were obtained at the results of the experiment which used a pH value of 2.0, 1500 g/Mg collector dosage, 30 min conditioning time, 20% solid ratio of feed pulp and 3 min flotation time.

From the result, the increasing the flotation time leads to higher concentrate recovery but lower grades. On the other hand, with the increasing the conditioning time, not only higher recovery but also higher concentrate chromite grades were obtained. Additionally, with the increasing the solid ratio of feed pulp and pH value, not only lower recovery but also lower concentrate chromite grades were obtained. Finally, with the increasing in the collector dosage, a significant decrease in the grade of concentrate was obtained, while a maximum in the chromite recovery to 1500 g/Mg Armac-C was obtained, and then a decrease in the recovery of concentrate for the collector dosage above 1500 g/Mg was obtained.

The concentration of low grade chromite tailings taken from Yesilova-Burdur of Turkey, containing 23.84% Cr_2O_3, was investigated using amine flotation method. The maximum achievable chromite grade and recovery by the rougher flotation were 31.20% and 95.69%, respectively.

Preliminary laboratory scale experiments were carried out on a rough product containing 31.20% Cr_2O_3 using a flotation procedure similar to the one described for the locked-cycle tests. The three-stage flotation results showed that it could be produced a flotation cleaner concentrate containing 47.33% Cr_2O_3 at a chromite recovery of 88.09%.

The presence of excessive very fine particles in the pulp increased the adsorption of H_2SO_4 and the dissolution rate of the minerals. The formation of soluble magnesium sulfate at low pH generally was due to the dissolution of magnesium silicate minerals. In this study, any dissolution minerals such as magnesium silicate minerals showed no negative effect on the flotation of chromite mineral by repeating 3 cleaning steps.

The results showed that the chromite tailing containing serpentine mineral can more easily float than chromite tailing containing olivine as the primary gangue mineral by amine flotation at low pH. Conditioning with H_2SO_4, used to maintain the pH of the pulp, increased both chromite grade and the flotation recovery.

The amine flotation technique was more effective than a slime shaking table method to avoid escape into a tailing with a high Cr_2O_3. On the other hand, the consumption of H_2SO_4 was high due to the presence of very fine particles. To determine the economic feasibility of H_2SO_4 consumption in terms of industrial application, it will be evaluated in future studies.

3.2. Statistical analysis

By using multiple linear regressions (MLR), researchers try to model the relationship between a dependent variable and two or more independent variables by fitting a linear equation (Kutner et al.,
When there are \( n \) independent variables \( X_1, X_2, \ldots, X_n \); a general form of the multiple linear regression equation is given below:

\[
Y = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \cdots + \beta_n X_n \tag{1}
\]

where \( Y \) is the dependent variable; \( X_1, X_2, \ldots, X_n \) are the independent variables (explanatory variables); \( \beta_0 \) is the constant, where the regression line intercepts the \( Y \)-axis, representing the amount the dependent variable \( Y \) will be when all the explanatory variables are 0; \( \beta_1, \beta_2, \ldots, \beta_n \) are the regression coefficients, describing the amount the dependent variable \( Y \) changes when the independent variable changes 1 unit (Zhou and Li, 2011).

When the investigators want to investigate a relationship between independent and dependent variables, the experimental data are applied to multiple linear regression analysis. The purpose of multiple regression analysis is to use the independent variables with values known to estimate the only one dependent value chosen by the investigator. Multiple linear regression analysis can be performed by using computer software packages (SPSS etc.) because they contain very complex calculations (Hair et al., 1998; Mahdevari et al., 2012). \( R^2 \) is the most important factor which determines how much the determined regression model fits the existing data. This statistical analysis represents how much of the variance in the independent variable is explained by the weighted combination of independent variables. The closer the \( R^2 \) value is to 1, the better the model fits (Coruh et al., 2013).

A specific set of operating variables of the flotation using the empirical models was required for determining the recovery and grade of the concentrate fraction. The aim of multiple linear regression (MLR) analysis is to define with two or more independent variables that definition of a dependent variable. In this study, the importance analysis of an independent variable (the recovery or the grade of chromite) was performed by using SPSS.15.0, assuming that effect on the independent variables such as the pH value, the collector dosage, the conditioning time, the pulp solid ratio of feed and the flotation time.

In predicting of the results, the dependent variables, which were related to the recovery (\( \% \)) and the grade (\( \text{Cr}_2\text{O}_3, \% \)) of the chromite concentrate, were indicated with independent variables, which were \( X_1 \) (the pH value), \( X_2 \) (the collector dosage, g/Mg), \( X_3 \) (the conditioning time, min), \( X_4 \) (the pulp solid ratio of feed, \( \% \)) and \( X_5 \) (the flotation time, min). The most reliable regression equations that can be obtained by multiple linear regression in determining the recovery and grade of concentrate as a result of the concentration of the chromite tailings with flotation were the following Eqs. (2)-(3).

\[
\text{Recovery} = 48.43 - (8.641 \times X_1) + (0.014 \times X_2) + (0.81 \times X_3) - (0.718 \times X_4) + (13.076 \times X_5) \quad R^2=0.772 \tag{2}
\]

\[
\text{Grade} = 42.629 - (2.019 \times X_1) - (0.002 \times X_2) + (0.263 \times X_3) - (0.243 \times X_4) - (2.352 \times X_5) \quad R^2=0.917 \tag{3}
\]

The relationships between the predicted values and the experimental results obtained using these model equations are given in Fig. 3. Consequently, the predicted grade and the recovery of the chromite concentrate derived by using the above equations were in good relation with experimental values. The
$R^2$ values between the experimental and the predicted values of the recovery and the grade of the chromite concentrate were 0.772 and 0.917, respectively.

To see whether or not there were any multiple relations between variables, the simple correlations, which are variance increase factors (VIF) and tolerance values, were examined. The findings concerning these values are presented in Tables 3 and 4.

### Table 3. Multiple relations coefficients for recovery

|       | $B$     | Std. Error | $\beta$ | $t$     | $p$     | Zero-order | Partial | Part | Tolerance ($T$) | VIF |
|-------|---------|------------|---------|---------|---------|------------|---------|------|----------------|-----|
| Const.| 42.629  | 16.839     | 2.881   | .010    |         |            |         |      |                |     |
| $X_1$ | -8.461  | 1.658      | -.573   | -5.212  | .000    | -.585      | -.767   | -.573 | .998           | 1.002|
| $X_2$ | .014    | .005       | .282    | 2.568   | .019    | .282       | .508    | .282  | 1.000          | 1.000|
| $X_3$ | .810    | .279       | .319    | 2.900   | .009    | .341       | .554    | .319  | .998           | 1.002|
| $X_4$ | -.718   | .189       | -.418   | -3.800  | .001    | -.418      | -.657   | -.418 | 1.000          | 1.000|
| $X_5$ | 13.076  | 5.343      | .269    | 2.447   | .024    | .269       | .490    | .269  | 1.000          | 1.000|

### Table 4. Multiple relations coefficients for grade

|       | $B$     | Std. Error | $\beta$ | $t$     | $p$     | Zero-order | Partial | Part | Tolerance ($T$) | VIF |
|-------|---------|------------|---------|---------|---------|------------|---------|------|----------------|-----|
| Const.| 42,629  | 2.446      | 17.427  | .000    |         |            |         |      |                |     |
| $X_1$ | -2.019  | .241       | -.554   | -8.369  | .000    | -.570      | -.887   | -.553 | .988           | 1.002|
| $X_2$ | -.002   | .001       | -.164   | -2.485  | .022    | -.164      | -.495   | -.164 | 1.000          | 1.000|
| $X_3$ | .263    | .041       | .427    | 6.461   | .000    | .449       | .829    | .427  | .988           | 1.002|
| $X_4$ | -.243   | .027       | -.585   | -8.847  | .000    | -.585      | -.897   | -.585 | 1.000          | 1.000|
| $X_5$ | -2.352  | .777       | -.200   | -3.025  | .007    | -.200      | -.570   | -.200 | 1.000          | 1.000|

It is known that there will be less reliable regression models if the Variance Increase Factor (VIF) values are excessive and the Tolerance ($T$) values are too low. If an individual VIF is higher than 10.0 and the average of VIFs is higher than 3.0, this indicates a problem. In other words, the variables in the model are independent of each other. If the $T$ values are less than 0.10, there is a cause for concern. $T$ values approaching zero indicate that the variable is highly predicted with other prediction variables (Hair et al., 1998; Paulson, 2007). As shown in the Tables 3 and 4, VIF and $T$ values were the same for both dependent variables (recovery and grade) because they reveal whether or not there was an intercorrelation between the five independent variables. Therefore, VIF values for all variables were considerably smaller than 10.0. In addition, all $T$ values were higher than 0.10. Besides, all VIF and $T$ values were approximately 1.00, indicating that an independent variable was not completely affected by other independent variables. Both values indicated a complete lack of multicollinearity.

The standardized regression coefficients or the absolute values of $\beta$ (beta) in Tables 3 and 4 show the importance order of the independent variables. The higher the absolute value of $\beta$, the more important the independent variable is in the relationship (Hair et al., 1998). When the contributions of the independent variables in the models for the flotation tests were examined from Tables 3 and 4, it was found that the end of the term scores received from measurement and the pH value ($X_1$) made the biggest contribution with the values of $\beta = -0.573$ for the recovery of the chromite concentrate. Addition, it was found that the solid ratio of feed pulp ($X_4$) made the biggest contribution with the values of $\beta = -0.585$ for the grade of the chromite concentrate.
Zero-order, partial and part correlations are bi-variation correlation between the independent variable and the dependent variables. The higher zero-order, partial and part correlations, the more important as a direct effect of the independent variable, and ignores the effect of other independent variables that may/may-not be influencing the dependent variable (Hair et al., 1998). In this study, it was determined that within all three correlations (zero-order, partial and part), the solid ratio of feed pulp \(X_4\) given the greatest contribution on the chromite grade, while the pH \(X_3\) given the greatest meaning on the chromite recovery.

In Tables 3 and 4, the “t” values are the t-statistic. The t value is the ratio of the mean of the difference to the standard error of the difference. In addition, the “p” values are the two-tailed p-value computed using the t distribution. If a p-value is <0.050, it provides evidence that the coefficient is different to 0. If the independent value is significant, there is a signification of the relationship between independent and dependent variables (Hair et al., 1998). In this study, p values of \(X_1\) (p<0.001), \(X_2\) (p = 0.019), \(X_3\) (p<0.009), \(X_4\) (p = 0.001) and \(X_5\) (p = 0.024) were significant of all predictors of the chromite recovery (Table 3). Additionally, in Table 4, p values of \(X_1\) (p<0.001), \(X_2\) (p = 0.022), \(X_3\) (p<0.001), \(X_4\) (p<0.001) and \(X_5\) (p = 0.007) were significant of all predictors of the chromite grade. Therefore, there was a signification of the relationship between the grade and recovery of chromite concentrate and dependent variables (the pH value, the collector dosage, the conditioning time, the pulp solid ratio of feed and the flotation time).

4. Conclusions

For the best results of the flotation experiments, the effects of the pH value \(X_3\), the collector dosage \(X_4\), the conditioning time \(X_5\), the pulp solid ratio of feed \(X_6\) and the flotation time \(X_7\) were examined. The best results were obtained at a pH value of 2.0, the collector dosage of 1500 g/Mg, the conditioning time of 30 minute, the solid ratio of feed pulp of 20% and the flotation time of 3 min. It was obtained from these results that, pH value and conditioning time were determined to have a significant effect on flotation.

Rougher flotation of slime samples containing 23.84% Cr₂O₃ increased to the grade of 31.20% Cr₂O₃ with a chromite recovery of 95.69%. When 3 stage cleaner flotations was performed on the rougher concentrate without any collector, a grade of 47.33% Cr₂O₃ and recovery of 88.09% could be obtained.

The developed multiple linear equations can be used for predicting the recovery and the grade of the chromite concentrate of the rougher amine flotation, and the \(R^2\) values between the experimental and predicted values of the recovery and grade of the chromite concentrate were 0.772 and 0.917, respectively. Unlike other physical concentration methods (gravity, magnetic and electrostatic methods etc.), the flotation method is difficult to modelling due to chemical and physicochemical processes, hence this success in the multiple linear regression (MLR) was an important result.

From statistical results, while the solid ratio of feed pulp \(X_4\) had major effect on the grade of concentrate fraction, the pH value \(X_3\) had major effect on the recovery due to the cleaning of mineral surfaces by using H₂SO₄ at low pH value.

Further studies of cleaner and scavenging processes should be applied to determine whether the amine flotation is suitable for enrichment or not in beneficiation of fine size chromite tailings. In addition, the effect of other operating conditions (amine type etc.) must also be investigated.

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