Finite Element Analysis of Steel Columns Subjected to Bi-directional Cyclic Loads

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Abstract: Steel columns are very useful in highway bridge pier construction as they offer flexible space requirement and have speedy construction time. Behaviour of steel columns under earthquake-induced loads is rather complicated as earthquakes occur in oblique direction. However, modern seismic design philosophies have been based on the behaviour of structures under independent actions of uni-directional loading in orthogonal directions. In this study, inelastic cyclic behaviour of steel columns subjected to axial force together with simultaneous bi-directional cyclic loads is investigated using an advanced finite element analyses procedure. Several types of linear and non-linear idealized loading patterns are employed to check the strength and ductility. The effects of important structural parameters on the behaviour under different loading patterns are also examined using the proposed procedure.

Keywords: Steel columns, cyclic analysis, finite element analysis, seismic performance

1. Introduction

The highway network of Sri Lanka is currently undergoing considerable upgrading with the introduction of motorways and grade separated crossing which need viaducts and flyovers. Although concrete is primarily used, steel also would offer many applications in such structures especially with respect to construction time. Present seismic design guidelines for steel columns have been based on numerous analytical and experimental investigations conducted under constant axial load plus uni-directional lateral loads. The superposition of independent action of uni-directional design seismic motion in orthogonal directions or the behaviour in the most critical direction is being considered in the present seismic capacity checks. However, a reasonable issue exists whether to incorporate the bi-axial effects in seismic designs. Several experimental studies have been so far carried out to investigate the effect of bi-directional cyclic loads on the behaviour of steel and concrete columns [1-7]. Nevertheless, those tests were found to be very costly and the results were Inadequate to make firm conclusions. This strongly suggests the important of having a reliable analytical procedure. In this study a finite element analysis procedure is used to examine the behaviour of stiffened steel columns using several idealized bi-directional cyclic loading patterns. An advanced general purpose finite element program called ABAQUS [8] was employed to this end. The analyses included both material and geometrical non-linearity. The analytical and test results were compared and it was shown that the proposed analytical procedure is very reliable in predicting seismic resisting performance of steel columns that undergo inelastic cyclic deformations.

2. Analytical Procedure

Finite element analysis procedure is very effective in determining the seismic resisting capacity of concrete and steel structures. The reliability of such an analysis mainly depends on the modelling technique and the type of elements, boundary condition, type of material model, etc. In this section, analytical procedure is explained in view of geometrical details of column, element mesh, loading procedure including loading patterns and the material model.
2.1 Geometrical details of columns

Stiffened steel columns were considered in the analysis since they have much better seismic resisting characteristics than unstiffened section columns. The study was conducted in two stages: (1) analysing previous test specimen under linear loading paths, (2) conducting parametric study using non-linear loading paths. A side view of the stiffened steel column used in the analysis is shown in Figure 1. Figure 2 shows the actual cross section and its equivalent unstiffened section. The equivalent unstiffened section is employed to model the upper part of the column because it allows significant simplification in the element mesh without sacrificing the accuracy of the analysis. The dimensions $B_b$ and $D_b$ of the equivalent section are determined in such a way that both sections have the same cross sectional area and second moment of inertia.

The dimensions of the column used in the first stage of the analysis are: $h=2420$ mm; $B(=D)=450$ mm; $t=5.8$ mm; $b_s=53.0$ mm; $t=5.8$ mm; $D_b=B_b=436.2$ mm; and $t_b=7.45$ mm (see Figures 1 and 2 for the descriptions of the notations). The cross sectional area $A$, and the second moment of inertia $I$, of the section are $1.28 \times 10^4$ mm$^2$ and $3.92 \times 10^8$ mm$^4$, respectively. These columns were taken from a previous experiment where specimens have been tested under various bi-directional cyclic loading patterns [5].

The structural parameters that play important role in earthquake resisting performance of stiffened steel column are the width-thickness ratio parameters $R_R$ and $R_f$, slenderness ratio parameter $A$, and the stiffness rigidity ratio $\gamma/\gamma^*$ (Chen and Duan [9]). The values of $R_R (=R_f$ for square section), $A$, and $\gamma/\gamma^*$ of the test column were found to be 0.61, 0.39, and 2.42, respectively. In the second stage of the analysis, three column models namely B35-35 ($R_R=0.35$, $A=0.35$), B35-50 ($R_R=0.35$, $A=0.50$), and B46-35 ($R_R=0.46$, $A=0.35$) were designed aiming at a parametric study. The value of $\gamma/\gamma^*$ of all three models was 3.0. The dimensions of the models are given in Table 2.

### Table 1: Dimensions of analytical models

| Model | $h$ | $B=D$ | $b$ | $D_b=B_b$ | $t_b$ |
|-------|-----|-------|-----|-----------|-------|
| B35-35 | 5551 | 1043  | 179 | 997       | 28.4  |
| B35-50 | 8160 | 1043  | 105 | 1019      | 24.8  |
| B46-35 | 7559 | 1364  | 113 | 1336      | 24.0  |

All dimensions are in millimetres.

2.2 Element mesh

The modelling and the analysis were carried out using advanced finite element programme ABAQUS [8]. The finite element mesh is shown in Figure 3. The upper part of the column, that is the portion above the third lateral diaphragm, was modelled using thick beam-column element
type B31 available in the program while the lower part (i.e., first three panels from the base) was modelled using doubly curved shell element type S4R. The base panel was modelled using comparatively finer mesh. The interface between shell and beam-column elements was simulated using rigid beam elements. The loads were applied at the column top in the sequence of axial load and simultaneous incremental cyclic lateral loads. The lateral loads were given in terms of repeated displacements defined by the multiples of yield displacement of the column.

2.3 Loading procedure

The cyclic loading path and the loading history of unidirectional loading are shown in Figure 4. The yield displacement $\delta_y$ was calculated using Equation 1 where $M$ = yield moment of the section; $h$ = column height; $E$ = Young’s modulus; $I$ = moment of inertia; $P$ = axial load; and $P_y$ = axial yield load of the section. In lateral cyclic analysis axial load is applied prior to lateral loads. In this study, three types of linear loading patterns were considered in the first stage of the analysis. They are: (a) unidirectional loading (UNI); (b) loading along 26.7 degrees inclined to the major axis (BI-L27); and (c) loading along 45 degrees inclined to the major axis (BI-L45), as shown in Figure 5.

\[ \delta_y = \frac{H \cdot h^3}{3EI} \]  

in which

\[ H = \frac{M}{h} \left(1 - \frac{P}{P_y}\right) \]

In the second stage, three models were subjected to three types of non-linear loading patterns namely BI-CI, BI-OV, and BI-SQ, as shown in Figure 6. The analyses were carried out to examine the effects of width thickness ratio parameters and the slenderness ratio parameter on the behaviour of column.

2.4 Material model

Modern seismic design specifications allow steel structures to deform up to a certain displacement level in inelastic range, which involves both material and geometrical non-linearity. In non-linear analysis, the accuracy of the material model has a large effect on the reliability of predictions. Modified two surface plasticity model (2SM), which has been proved to be very accurate in simulating cyclic behaviour of steel (Shen et al. [10]), was used in the analysis.

The material properties such as yield stress $\sigma_y$, Young’s modulus $E$, and Poisson ratio $\nu$ of models are listed in Table 2.

| Model | $\sigma_y$ (MPa) | $E$ (GPa) | $\nu$ |
|-------|----------------|-----------|-------|
| UNI, BI-L27, BI-L45 | 412 | 206 | 0.276 |
| B35-35, B35-50, B46-35 | 315 | 200 | 0.300 |

3. Analytical Results

Analytical results of linear and non-linear loading patterns are separately presented in this section.
3.1 Linear loading patterns

The analyses were carried out using three loading patterns described in Figure 5 to check the effect of bi-axial cyclic bending. In loading pattern UNI, incremental cyclic lateral displacements were applied along the X-direction only. The same loading history as used in the test was employed in the analysis in order to have valid comparison between test and the analysis. Figure 7 shows the comparison of test and the analytical results of the loading type UNI. The envelope curves are also included in the same figure. Shown in Figure 8 and Figure 9 are the comparison of analytical and test results under loading patterns BI-L27 and BI-L45. The comparison of buckling modes between analysis and test are shown in Figure 10. The photos were taken at the end of sixth cycle in the cases of UNI and BI-L27 and fourth cycle in the case of BI-L45. As clearly seen in Figures 7 to 10, the analytical and test results match very well in all the cases, hence the proposed procedure can be considered to be accurate enough for reliable predictions. The seismic behaviour is normally measured in terms of strength and ductility.

![Image of test and analysis comparison](image-url)

**Figure 7:** Load displacement curves of test and analysis: Loading type UNI

**Figure 8:** Load displacement curves of test and analysis: Loading type BI-L27
The strength of the column is defined by index $H_m/H_y$ where $H_m$ is the peak load. There are two ways to defined the ductility [11]: (1) ratio of lateral displacement at the peak load to yield displacement $\delta_{y}$ (i.e., $\delta_m/\delta_{y}$); and (2) ratio of lateral displacement at 95% of the peak load to yield displacement $\delta_{y}$ (i.e., $\delta_{95}/\delta_{y}$). The resultant lateral displacement $\delta_1$ and lateral load $H_1$ for the cases BI-L27 and BI-L45 are calculated using following two equations.

$$H_1 = H_y \cos \theta + H_x \sin \theta$$  \hspace{0.5cm} (3)

$$\delta_1 = \delta_y \cos \theta + \delta_\delta \sin \theta$$  \hspace{0.5cm} (4)

where, $\theta$ is the angle between the loading direction and the major axis of the section (i.e., X-axis). The values of strength and ductility indices $H_m/H_y$, $\delta_m/\delta_{y}$ and $\delta_{95}/\delta_{y}$ are calculated in terms of resultant load $H_1$ and resultant displacements $\delta_1$ (i.e., $H_m/H_y$, $\delta_m/\delta_{y}$, and $\delta_{95}/\delta_{y}$) using the test and analytical results and given in Table 3. It is seen here that the $H_m/H_y$ of uni-directional loading case (UNI) is higher than $H_m/H_y$ of the BI-L27 and BI-L45 cases. On the other hand, values of $\delta_m/\delta_{y}$ and $\delta_{95}/\delta_{y}$ of unidirectional loading case are quite lower than the $\delta_m/\delta_{y}$ and $\delta_{95}/\delta_{y}$ of the other two cases. This means that when bi-directional loading effects are considered, unidirectional analysis gives overestimated strength predictions, while
the ductility is underestimated. The results clearly prove that the bi-directional loads affect considerably the seismic behaviour of columns. As a result, investigating the effects of parameters such as width thickness ratio parameters (RR and RF) and slenderness ratio parameter \(X\), on the strength and ductility performance of columns when they subject to multi-directional cyclic loads has significant practical importance.

### Table 3: Test and analytical results

| Loading patterns | \(H_{m}/L/H_y\) | \(\delta_{m}/\delta_y\) | \(\delta_{mL}/\delta_y\) |
|------------------|-----------------|-----------------|-----------------|
| UNI              | 1.23            | 2.23            | 2.87            |
| BI-L27           | 1.15            | 3.35            | 3.76            |
| BI-L45           | 1.11            | 2.46            | 3.37            |

3.2 Non-linear loading patterns

The column models specifically designed in order to check the effects of parameters \(R_y\), \(R_F\) and \(\lambda\), were analysed using three types of non-linear loading patterns as shown in Figure 6. The relevant load-displacement envelope curves of each specimen are shown in Figure 11. The corresponding strength and ductility indices obtained in the X-direction are given in Table 4.

### Table 4: Results of non-linear loading patterns in X-direction

| Model          | Loading type | \(H_{m}/H_y\) | \(\delta_{m}/\delta_y\) | \(\delta_{mL}/\delta_y\) |
|----------------|--------------|----------------|-----------------|-----------------|
| B35-35         | UNI          | 1.65           | 6.00            | 6.40            |
|                | BI-OV        | 1.55           | 2.67            | 4.27            |
|                | BI-CI        | 1.42           | 2.43            | 2.87            |
|                | BI-SQ        | 1.64           | 4.00            | 4.36            |
| B35-50         | UNI          | 1.55           | 4.00            | 4.50            |
|                | BI-OV        | 1.57           | 3.24            | 4.77            |
|                | BI-CI        | 1.48           | 2.43            | 3.13            |
|                | BI-SQ        | 1.47           | 3.00            | 3.22            |
| B46-35         | UNI          | 1.47           | 3.01            | 3.40            |
|                | BI-OV        | 1.47           | 2.67            | 3.14            |
|                | BI-CI        | 1.30           | 1.81            | 2.03            |
|                | BI-SQ        | 1.36           | 2.00            | 2.36            |

It has been revealed from these results that the strength and ductility of columns having the same width thickness ratio parameters and slenderness ratio parameter are different when they are subject to different loading patterns.

Also, as expected, the predictions were different with different width thickness and slenderness ratio parameters for a particular loading pattern. The minimum strength and ductility were found to occur under circular (BI-CI) loading pattern and the maximum were under uni-directional loading. As seen in Figure 11, the BI-CI loading path has caused the highest post-peak strength degradation. It is understood from results in Table 4 that the values of \(H_{m}/H\) of B35-35 (1.42) and B35-50 (1.57) under loading type BI-CI do not differ much. The corresponding values under loading type BI-OV (1.55 and 1.57) are also nearly the same. This means that the effect of \(\lambda\) on the strength is not significant under circular loading type. On the other hand under the loading type BI-SQ the values of \(H_{m}/H\) of B35-35 (1.61) and B35-50 (1.47) differ about 8
percent. Thus, it seems that the effect of A on the strength varies with the loading type. Moreover, similar kind of comparisons revealed that the effects of parameters R\textsubscript{R} and R\textsubscript{p} on the strength are different with different loading types. Similar to the strength, the degree of the effects of parameters on the ductility also significantly varies with the type of loading type.

Another important measure of earthquake resisting performance of structures is the energy absorption capacity. The area of a loop of load-displacement curve gives the energy dissipation through material yielding and local buckling in a complete one cycle of loading. It is seen from the hysteretic curves of the three models that the BI-CI loading path dissipates minimum cumulative energy.

4. Conclusions

The finite element modelling procedure for analysing steel columns subjected to bi-directional cyclic loads are presented in this paper. The analytical procedure was verified by analysing previous test specimens. Several columns were designed in view of identifying the effects of structural parameters such as width to thickness ratio and slenderness ratio parameters on the behaviour when columns undergo different bi-directional loading paths. The main conclusion of analytical results is that the strength predictions under bi-directional loading paths are smaller than those of the uni-directional loading path. However, the ductility could be more. Therefore, it is suggested that the bi-directional loading effects should be incorporated adequately into present seismic design procedures.

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