EVOLUTION OF REVENUE PREFERENCE FOR COMPETING Firms WITH NONLINEAR INVERSE DEMAND

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Abstract. This paper studies evolutionarily stable preferences of competing firms across independent markets. Two models are considered according to whether firms’ preferences are discrete or continuous. When preferences are discrete, firms have two marketing strategies: profit maximization and revenue maximization. We find that, whether pure and mixed strategies are evolutionarily stable depends on the spectrum of pricing capability. When the pricing capability is moderate, the mixed strategy is an evolutionarily stable strategy. Revenue maximization is evolutionarily stable under relatively high pricing capability, whereas, in case of low pricing capability, firms opt to maximize their profits. Further, the stability of revenue preference is also examined under continuous preferences. We derive the conditions, under which a unique evolutionarily stable revenue preference appears as well as it is continuously stable. Our main results still hold when we extend our model to a general framework.

1. Introduction. In classical economics, economists assume that people are selfish, and the unique objective of a firm is seeking to maximize the owner’s material interest. However, some cases show that human behaviors are not always driven by his material self-interest. Rudis [32] investigates 658 Chief Executive Officers (CEOs) across the world and finds that 37.5% of these CEOs regard the sustainable and stable growth of operations revenue as a primary challenge. This proportion amounts to 53.8% in Chinese CEOs. For example, in recent years, electric vehicle firms NIO, Tesla, etc., sell more and more products, and yet their profits are negative. To some extent, seeking revenue maximization may be the first objective, because what the firms concern about is to survive in recent years. In addition, early studies indicate that revenue maximization may acquire higher profits compared to using profit

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maximization [42, 43, 44]. A common theme is that revenue maximization can have more aggressive without considering the operations cost in decision-making [44]. However, this literature is limited in that attention is restricted to a linear demand in discussing the evolution of revenue maximization. For some products, such as mobile phone, automobile, electronic, their demand functions are convex but do not tend to infinity as prices tend to zero [2, 19]. The nonlinear inverse demand function has been used in other duopoly models and in the experimental economics dealing with learning and expectations formation [27, 29]. How do firms adjust their marketing objective for acquiring long-term competitive advantage under nonlinear demand? In addition to explaining revenue or profit maximization behaviors, we also want to address why firms sometimes have revenue preferences.

We employ a directly evolutionary game in independent markets with competing firms, to discuss the strategic choice of marketing objective. We further investigate the effects of some vital factors like unit production cost, price cap on the choice. The evolutionary approach adopts the concept of bounded rationality, and defines firms as a set of essential skills based on their learning ability in a complex environment [28]. We consider that firms have discrete preferences: revenue or profit. The marketing objective of a firm with revenue preference is to maximize revenue, while a firm with profit preference seeks for maximizing profit. The strategic choice of marketing objective for firms is inevitably affected by their competitors’ decisions.

The indirectly evolutionary game theory [21] has been adopted to study whether revenue preference may have evolved in firms through a process of natural or cultural selection. The degree of revenue preference is different among firms, denoted by a preference parameter describing how a firm cares for its revenue. In competition, each firm rationally chooses the degree of revenue preference to achieve its objective. As a result, in equilibrium a firm’s effective success depends on the revenue attitudes of all the involved firms. This allows to compare the success of firms with different degrees of preferences. Firms with higher expected success are less likely to be eliminated in the evolution. Revenue preference is said to be evolutionarily stable if it survives evolutionary selection. By assuming rational behavior and applying the concept of evolutionary stability to preferences rather than to strategies, we endogenously determine preferences [3].

In our article, the following questions are discussed: (1) the conditions for the evolutionary stability of Nash equilibria (pure-strategy and mixed strategy equilibria); (2) the evolutionary/dynamic stability of revenue preference; (3) the influence of the demand function’s power on the evolution of revenue preference. Our results indicate that price cap and the unit production cost can affect the firms’ strategic choice and the evolution of preference, and the main result can be extended to a more general framework. In Section 3, we establish our basic model, and find that the equilibria depend on the product’s price cap.

In Section 4, with discrete preference, we analyze the evolutionarily stable strategy for firms when the equilibrium state is disturbed in the long-term using the directly evolutionary game theory. Whether a symmetric pure strategy profile/the mixed strategy is evolutionarily stable depends on the price cap (pricing capability). When the pricing capability is moderate, the mixed strategy is an evolutionarily stable strategy.

Furthermore, considering the continuous preference, we discuss the evolution of revenue preference by modeling and solving an indirectly evolutionary game in
Section 5. We find that though the pure profit strategy profile can be an evolutionarily stable strategy, the zero revenue preference is not an evolutionarily stable preference. There exists a unique positive evolutionarily stable preference, which is also continuously stable according to the adaptive dynamics. Finally, Section 6 discusses the extended model and analyzes the impact of demand function’s power on the revenue preference evolution.

2. Literature review. Our paper focuses primarily on firms’ behaviors and preferences evolution, such that we can review the existing literature on exogenous behavior evolution, and endogenous preference evolution.

The first strand of literature lies in exogenous behavior evolution, which is characterized by modeling a directly evolutionary game. For example, Chai et al. [6], Chai and Xiao [7] indicate that firms are willing to choose social responsibility or stakeholder strategy in the long-term competition with replicator dynamics. Kopel et al. [22] propose a mixed duopoly with profit maximization and socially concerned firms, and find that firms considering both profit and consumer surplus can obtain larger market shares and profits than their rivals with profit maximization. The evolution of firms’ behaviors under Nash and best reply players have been investigated in the second half of the paper. Rhode and Stegeman [31] study a differentiated duopoly and find the evolution of marketing objective towards revenue maximization if firm’s choice under the Darwinian process. Schaffer [33] points out profit maximization may be not the best survivor strategy when firms have market power. Xiao and Yu [42] discuss strategic choice of retailers (profit maximization or revenue maximization) in duopoly market with differentiated or homogeneous goods, and show that the ESS of retailers depends on the relative unit production cost, market scale, etc. in differentiated duopoly, demand and raw material supply disruptions in homogeneous duopoly. Yi and Yang [43, 44] account for that, if firms’ choices follow replicator dynamics, the revenue maximization is evolutionarily stable with a high strength network externality. These works focus on firms’ choice between revenue maximization, or consumer concerned and profit maximization under linear demand. We study the effect of firms’ revenue preference on firms’ decisions and evolutionary dynamics of firms’ strategy in a duopoly homogenous market with convex inverse demand.

Taking endogenous preference as the subject of evolution is a popular topic in the evolutionary game theory. The indirectly evolutionary approach is initiated by Güth and Yaari [17] to examine the evolution of endogenous preferences. Chai and Xiao [7] analyze the evolution of social responsibility preference by establishing an indirectly evolutionary game, and obtain a unique continuously stable social responsibility preference parameter. Gale et al. [15], Huck and Oechssler [20], and Shirata [34] show that the dynamics of fairness in the mini ultimatum game using the indirectly evolutionary approach or replicator dynamics under different match types. Bester and Güth [3] investigate the evolutionarily stable altruistic parameters, and conclude that altruism is context dependent. Based on the theory in Bester and Güth [3], this paper studies the static evolution solution of an indirectly evolutionary game about firms’ revenue preference in a continuous space using the indirectly evolutionary approach.

The static evolution solution of an indirectly evolutionary game cannot present the evolutionary process of revenue preference in firms. Adaptive dynamics is used to analyze the dynamic evolution of game when the strategies of the game are continuous functions of parameters [30]. Hofbauer and Sigmund [18] ensure that the
stable point of an adaptive dynamics must (essentially) be an ESS. In fact, Chai et al. [5], Friedman and Sinervo [14] show that the stability (convergence stable) in an adaptive dynamics is stronger than evolutionary stability, because evolutionary stability is locally stable while convergence stability is globally stable. There are other researchers discuss the relationship between evolutionary stability and adaptive dynamics [1, 11, 16, 24]. The canonical equation proposed by Dieckmann and Law [12] is an important method to analyze the stability of dynamics in a continuous space. Champagnat et al. [8] explain the canonical equation in a mathematical view. Le Galliard et al. [23] explore the evolution of altruism in spatially heterogeneous populations using canonical equation. To the best of our knowledge, few papers study endogenous preference evolution using adaptive dynamics.

Our paper is different from the previous literature in several points. Firstly, many papers about firms’ choice focus on the linear inverse demand, whereas we discuss the evolutionary dynamics of firms under nonlinear inverse demand. Secondly, the literature on the preference evolution considers the locally evolutionary stability (static evolution solution of an evolutionary game). Differently, we combine the adaptive dynamics theory with managerial practice, study globally stability of firms’ preference using an adaptive dynamics, and try to find the branch points in the monomorphic population. The overall flowchart of our paper is given by Figure 1.

Figure 1. The overall flowchart for the proposed methodology.

3. Basic model and one-shot duopoly games. Consider an economic system consists of many independent markets, in one of which two firms compete to sell homogeneous goods [4, 40, 42]. These markets are completely distinct of each other because there are different physical locations. All firms are identical so that the interaction between competing firms in a given market is described by a symmetric game. In this game, one firm is labelled as firm 1, the other is labelled as firm 2. We assume that the inverse market demand for the two firms is \( p = a - \sqrt{q_1 + q_2} \) [13, 26, 27], where \( q_i \) is the market demand for firm \( i \) \((i=1,2)\), \( a \) is the price cap (i.e., the highest marketing price, reflecting pricing capability). The function is convex, but does not tend to infinity as \( p \to 0 \). In fact, \( a^2 \) represents the maximum amount of output that can be brought to the market. The unit production cost for two firms is \( c \) \((0 < c < a)\).

Each firm chooses the marketing strategy between profit maximization \((P)\) and revenue maximization \((R)\). So there are four strategy profiles: (1) both firms choose profit maximization \((PP)\); (2) both firms choose revenue maximization \((RR)\); (3) firm 1 chooses profit maximization and firm 2 chooses revenue maximization \((PR)\);
(4) firm 1 chooses revenue maximization and firm 2 chooses profit maximization \((RP)\). The objective function of a profit maximization firm is its profit function \(\pi_i = (a - \sqrt{q_1 + q_2} - c_i)q_i\); for a revenue maximization firm, the objective function is its revenue function \(R_i = (a - \sqrt{q_1 + q_2})q_i\), \(i = 1, 2\). By analyzing, the material payoff bi-matrix can be established, as Table 1 shows. The row denotes firm 1 and column denotes firm 2. We extract the common factors \(\frac{1}{2}\), the profits of firms do not include the common factors in the next analysis of Sections 3 and 4.

|   | \(R\)                  | \(P\)                  |
|---|-------------------------|-------------------------|
| \(R\) | \(2a^2(a - 5c),\)      | \((a - 3c)(2a - c)(a + 2c),\) |
| \(P\) | \((a - 3c)^2(2a - c),\) | \(2(a - c)^3,\)       |
|     | \((a - 3c)(2a - c)(a + 2c)\) | \(2(a - c)^3\)     |

Table 1. The material payoff matrix

To ensure that all above equilibrium profits under four strategy profiles are non-negative, the products’ price cap should satisfy the condition: \(a > 5c\), which can also be interpreted as initial condition for firms’ participation constraint. We use \(\pi^{RP} = 2a^2(a - 5c)\) to denote the profit of firm 1 when it selects strategy \(R\) against firm 2 using strategy \(R\). Similarly, we define the profit of firm 1 by \(\pi^{j1}, j = P, R\), and \(l = P, R\), where firms 1 and 2 select strategy \(j\) and \(l\), respectively.

By comparing the profits made by two firms using different strategies, Proposition 1 holds.

**Proposition 1.** When the price cap \(a\) is larger than \(5c\),

1. Given firm 2’s strategy \(P\), the threshold for firm 1 to select strategy \(R\) or \(P\) is \(a_1 = \frac{c(17 + \sqrt{153})}{6}\). If \(a > a_1\), then \(\pi^{RP} > \pi^{PP}\); otherwise, \(\pi^{RP} < \pi^{PP}\).

2. Given firm 2’s strategy \(R\), the threshold between strategy \(R\) and strategy \(P\) selected by firm 1 is set to \(a_2 = (4 + \sqrt{13})c\). If \(a > a_2\), then \(\pi^{RR} > \pi^{PP}\); and vice versa.

**Proof.** (1) When firm 2 chooses strategy \(P\), the profit made by firm 1 (adopting strategy \(R\) and \(P\)) is \(\pi^{RP} = (a - 3c)(2a - c)(a + 2c)\) and \(\pi^{PP} = 2(a - c)^3\), respectively.

By calculating the difference between these two values, \(\pi^{RP} - \pi^{PP} = c(3a^2 - 17ac + 8c^2)\) is obtained, such that, when the value of this equation equals zero, the threshold \(a_1 = \frac{c(17 + \sqrt{153})}{6}\) is acquired. Therefore, when the price cap \(a > a_1\), \(\pi^{RP} > \pi^{PP}\), that is, firm 1 chooses strategy \(R\). When \(a < a_1\), \(\pi^{RP} < \pi^{PP}\) and therefore firm 1 selects strategy \(P\).

Similarly, we can prove Proposition 1(2).

Then Proposition 1 is true. \(\square\)

Because firms 1 and 2 are symmetric, the results obtained in Proposition 1 are also applicable to firm 2. As a consequence, the discussion does not distinguish between firm 1 and firm 2. Proposition 1 shows that when one firm has chosen a strategy, the rival selects the optimal strategy based on the threshold of the price cap in the market. When the price cap \(a\) is higher than a threshold \((\max{\{a_1, a_2\}})\), choosing strategy \(R\) can make a higher profit for the rival in comparison with strategy \(P\). When the price cap \(a\) is below a threshold \((\min{\{a_1, a_2\}})\), it is more profitable for the rival to select strategy \(P\) than strategy \(R\). Meanwhile, because
\( \frac{\partial a_1}{\partial c} \) is larger than zero, the thresholds \( a_1 \) and \( a_2 \) increase with the growth of the unit production cost \( c \). This indicates that the unit production cost can affect the strategic choices of firms through exerting an influence on the threshold of the price cap. Strategy \( R \) may be more suitable than strategy \( P \) for a high cost firm. Taking the electric vehicle as an example, its cost is higher than that of a traditional gasoline vehicle; as a result, electric vehicle firms choose revenue maximization to occupy the market share in the early stage of development.

Equilibrium state is important in an economic system, as it provides a rule how a firm acts in the short-term competition. However, in the real world, the equilibrium state of an one-shot game is almost disturbed. How does the system change once the equilibrium state is disturbed? Can it return to the equilibrium state? Whether the revenue maximization strategy in the early stage should be chosen in the long-term competition? Next, we will discuss the evolutionary stability of the equilibrium state.

4. **Evolutionarily stable strategy.** With the evolution of the market environment, the behavior with low material payoffs is eliminated by the behavior with high material payoffs, while the material payoff of firms is related to the choice of marketing objective. Firms place orders based on their strategies, followed by the revision of these strategies according to the actions of competitors. Through this kind of repeated dynamic adjustment, firms make more material payoffs through selecting different strategies. In this section, we discuss the evolutionary stability of the two marketing strategies: profit maximization and revenue maximization.

Let the share of firms choosing strategy \( R \) be \( x \), \( 0 \leq x \leq 1 \). Thus, the share of firms choosing strategy \( P \) is \( 1 - x \). The average payoffs of strategies \( R \) and \( P \) are as follows, respectively,

\[
\begin{align*}
    w^R &= x \pi_{RR} + (1 - x) \pi_{RP}, \\
    w^P &= x \pi_{PR} + (1 - x) \pi_{PP}.
\end{align*}
\]

The payoff difference between strategy \( R \) and strategy \( P \) is

\[
\Delta w = w^R - w^P = c[3a^2 - 17ac + 8c^2 + (c^2 - 7ac)x].
\]

As \( \frac{c(17 + \sqrt{13})}{6} < a < (4 + \sqrt{13})c \), a unique solution for \( \Delta w = 0 \) with respect to \( x \) is interior \( x^* = \frac{3a^2 - 17ac + 8c^2}{(7a - c)c} \).

When \( a = \frac{c(17 + \sqrt{13})}{6} \), the point \( x = x^* \) converts to \( x = 0 \); when \( a = (4 + \sqrt{13})c \), the point \( x = x^* \) converts to \( x = 1 \). Hence there exist possible three Nash equilibria. Which strategy should be chosen in the long-term interaction? Which equilibrium can not be invaded when the equilibrium is disturbed or which strategy is stable in the evolutionary process? We give the results in Proposition 2.

**Proposition 2.** The evolutionary stability of three equilibria depends on the price cap. Specifically,

1. When \( \frac{c(17 + \sqrt{13})}{6} < a < (4 + \sqrt{13})c \), the unique interior equilibrium \( (x^*, 1 - x^*) \) is locally asymptotically stable, i.e., the mixed strategy is an evolutionarily stable strategy (ESS), where \( x^* = \frac{3a^2 - 17ac + 8c^2}{(7a - c)c} \);
2. When \( 5c < a \leq \frac{c(17 + \sqrt{13})}{6} \), strategy \( P \) (equilibrium \( x = 0 \)) is an evolutionarily stable strategy;
3. When \( a \geq (4 + \sqrt{13})c \), strategy \( R \) (equilibrium \( x = 1 \)) is an evolutionarily stable strategy.
we find that the unit production cost also affects the equilibrium. Differently, if the firm with low pricing capability, it needs to pay more attention to its profit, and adopts the profit maximization strategy. Furthermore, its profits. Differently, if the firm with low pricing capability, it needs to pay more attention to its profit, and adopts the profit maximization strategy. Hence, a firm should adopt the revenue maximization strategy if it has high pricing capability. This is because the price level that consumers can accept is very high, and the products do not have the problem of not being able to sell, so the firm does not need to pay more attention to its profits. Differently, if the firm with low pricing capability, it needs to pay more attention to its profit, and adopts the profit maximization strategy. Furthermore, we find that the unit production cost also affects the equilibrium x*. Because \( \frac{\partial x^*}{\partial c} \) is less than zero when the price cap a is larger than 5c, the share of firms adopting strategy R decreases with the growth of the unit production cost.

**Proof.** (1) From \( \frac{\partial \Delta u}{\partial a} < 0 \), we know that the interior \( x^* \) is a downcrossing. So the interior equilibrium \( x^* \) is locally asymptotically stable [14]. According to Cressman [9], Taylor and Jonker [38], TraulsenHauert [39], the mixed strategy is an evolutionarily stable strategy.

(2) From \( \pi((0,1),(0,1)) - \pi((x,1-x),(0,1)) = -cx(3a^2 - 17ac + 8c^2) \). When \( a \leq \frac{c(17 + \sqrt{193})}{6} \), we get \( \pi((0,1),(0,1)) \geq \pi((x,1-x),(0,1)) \). Solving \( \pi((0,1),(0,1)) = \pi((x,1-x),(0,1)) \) for \( a \), we have \( \hat{a}_1 = \frac{c(17 + \sqrt{193})}{6} \). Further, when \( a = \hat{a}_1 \), we have \( \pi((0,1),(x,1-x)) - \pi((x,1-x),(x,1-x)) = \frac{c^3}{6}(113 + 7\sqrt{193}) > 0 \). According to the original definition of ESS [35, 36, 41], symmetric strategy P \( (x = 0) \) is an ESS.

(3) Because \( \pi((1,0),(1,0)) - \pi((x,1-x),(1,0)) = 3c(1-x)(a^2 - 8ac + 3c^2) \geq 0 \) for \( a \geq \frac{c(4 + \sqrt{13})}{6} \), we solve \( \pi((1,0),(1,0)) = \pi((x,1-x),(1,0)) \) for \( a \), and get \( \hat{a}_2 = \frac{c(4 + \sqrt{13})}{6} \). When \( a = \hat{a}_2 \), we have \( \pi((1,0),(x,1-x)) - \pi((x,1-x),(x,1-x)) = (27 + 7\sqrt{13})c^3(1-x)^2 > 0 \). Then, according to the definition of ESS, symmetric strategy R \( (x = 1) \) is an ESS.

Hence, Proposition 2 is true.

From Proposition 2, the evolutionary stability of equilibria depends on the price cap; that is, the three strategies are evolutionarily stable under some conditions. In explaining the results in Proposition 2 clearly, we illustrate the dynamics of Proposition 2 through Figures 2-4. From Figures 3 and 4, we know that, firms should use revenue maximization in the long-term interaction only if the price cap is high enough (Figure 4); otherwise, profit maximization is supposed to prevail in the population. According to Figure 2, when the price cap is moderate, then firms should choose a mixed strategy; in other words, firms should care about not only its revenue maximization, but also its profit maximization. Hence, a firm should adopt revenue maximization strategy if it has high pricing capability. This is because the price level that consumers can accept is very high, and the products do not have the problem of not being able to sell, so the firm does not need to pay more attention to its profits. Differently, if the firm with low pricing capability, it needs to pay more attention to its profit, and adopts the profit maximization strategy. Furthermore, we find that the unit production cost also affects the equilibrium x*. Because \( \frac{\partial x^*}{\partial c} \) is less than zero when the price cap a is larger than 5c, the share of firms adopting strategy R decreases with the growth of the unit production cost.
5. Evolution of continuous revenue preference.

5.1. Equilibrium with continuous revenue preference. In previous sections, we research the equilibrium and the evolutionary stability of strategies when firms choose their marketing strategies from profit maximization and revenue maximization. We use the theory that firms adjust their preferences based on their own and their competitors’ profits in interaction, to indicate that how the firms’ preferences evolve in interaction. Next, we discuss the question that why the firms have revenue preference.

A firm has revenue preference when its preferences reflect some concern for its revenue. Assume that firms have different degree of revenue preference, and the degree of revenue preference is expressed by a preference parameter \( k \), describing how much a firm cares for its revenue, and \( k \) varies in the closed set \([0,1]\), i.e., \( k \in [0,1] \). We describe such preferences by

\[
\begin{align*}
    u_1(q_1, q_2) &= [(1-k_1)(a - \sqrt{q_1 + q_2 - c})q_1 + k_1(a - \sqrt{q_1 + q_2})q_1], \\
    u_2(q_1, q_2) &= [(1-k_2)(a - \sqrt{q_1 + q_2 - c})q_2 + k_2(a - \sqrt{q_1 + q_2})q_2].
\end{align*}
\]

(3)

Accordingly, the concern that firms 1 and 2 express for their own revenue is represented by the weights \( k_1 \) and \( k_2 \), respectively. In interaction with others, each firm rationally adopts a strategy to maximize its preference. Obviously, the degree of revenue preference indirectly affects the strategic interactions between the two firms because their behavior depends on the parameters \( k_1 \) and \( k_2 \). Let \( \pi_1(k_1, k_2) \) denote the payoff of firm 1 when its degree of revenue preference is \( k_1 \), and the degree of revenue preference of its rival is \( k_2 \).

Thus, we get the indirectly evolutionary game \( \Gamma \equiv ((Q_1, K_1, u_i, \pi_i)_{i=1,2}) \), where \( Q_i \in [0, q_1 + q_2] \) denotes the quantity chosen by firm \( i \), \( K_i \in [0, 1] \) denotes the type of firm \( i \), i.e., the degree of revenue preference of firm \( i \), \( u_i \) is the utility function of firm \( i \), and \( \pi_i \) is the evolutionary success function of firm \( i \).

By calculating, the equilibrium decisions \( (q_1^*, q_2^*) \) of the indirectly evolutionary game are

\[
\begin{align*}
    q_1^*(k_1, k_2) &= \frac{4}{25}[(a-c)^2 + (7k_1 - 3k_2)(a - c) + (3k_1 - 2k_2)(k_1 + k_2)c^2], \\
    q_2^*(k_1, k_2) &= \frac{4}{25}[(a-c)^2 + (7k_2 - 3k_1)(a - c) + (3k_2 - 2k_1)(k_1 + k_2)c^2],
\end{align*}
\]

(4)

respectively.

Since the interaction between firms results in the equilibrium \( (q_1^*, q_2^*) \), we get the indirectly evolutionary success payoff for \( k_1 \)-firm is

\[
\pi(k_1, k_2) = \frac{4[a - (1 + 2k_1 + 2k_2)c]}{125} \frac{1}{[2(a-c)^2 + c^2(a-c)(7k_1 - 3k_2) + c^2(3k_1 - 2k_2)(k_1 + k_2)].}
\]

(5)
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From \( \frac{\partial^2 \pi(k_1,k_2)}{\partial k_1^2} = \frac{8c^2}{125} [11a - 11c + 18c(k_1 + k_2)] < 0 \), and \( \frac{\partial^2 \pi(k_1,k_2)}{\partial k_2^2} = -\frac{4c}{125} [7(a - c)^2 + c(a - c)(7k_1 - 8k_2) + 4c^2(2k_1 - 3k_2)(k_1 + k_2)] < 0 \), we find that the payoffs of firms are concave in its own revenue preference, decreasing with the rival’s revenue preference. Figure 5 illustrates this situation, where the values of parameters are \( a = 4, c = 1 \), and \( k_2 = 0.1 \) or \( k_1 = 0.45 \).

![Figure 5](image)

Figure 5. Firms’ payoffs change with the degree of revenue preference.

According to Figure 5, the revenue preference of a firm reduces the indirectly evolutionary success payoff (profit) of its rival, while at the same time first increasing its own indirectly evolutionary success payoff (profit) and then decreasing it. Hence, there should exist an appropriate revenue preference for a firm with revenue preference so that the firm could get the best profit in the long-term.

**Proposition 3.** Suppose \( a < c(1 + 2k_1 + 2k_2) \). Then in the interaction between two firms, the more revenue preference motivated firm is less successful than its rival, i.e., \( \pi(k_1,k_2) > \pi(k_2,k_1) \), for all \( k_1 < k_2 \).

**Proof.** Owing to Equation (5), it follows \( \pi(k_1,k_2) - \pi(k_2,k_1) = \frac{4c(k_1 - k_2)}{25} [2(a - c) + c(k_1 + k_2)] [a - c - 2c(k_1 + k_2)] \). When \( a < c(1 + 2k_1 + 2k_2) \), we have \( \pi(k_1,k_2) - \pi(k_2,k_1) > 0 \) for all \( k_1 < k_2 \).

So Proposition 3 is true.

When the price cap of the products is moderate, a firm with revenue preference is willing to reduce its own success for the sake of the market share. Therefore, one might conclude that a firm with profit maximization has a higher survival value than a firm with revenue preference. However, Proposition 3 presents only one factor that is important for evolutionary selection. Based on Equation (5), a firm with revenue preference will perform better than a firm with profit maximization in the interaction when the price cap is larger than a critical value \( a > c(1 + 2k) \), \( k \) is the degree of revenue preference for a firm with revenue preference. Is revenue preference \( k^* = 1 \) evolutionarily stable? We will address the question using two methods: indirect evolution and adaptive dynamics.

5.2. **Indirect evolution.** Indirectly evolutionary method is introduced by Güth and Yaari [17], a popular foundation to analyze the preference evolution. The basic assumption of indirect evolution is that all players behave rationally for given preferences but that their preferences change with an evolutionary process [3, 20].
In an environment where evolutionary selection favors the more successful firms, firms with lower profits will become extinct. In this way, preferences are chosen for their capacity to generate profits. In this subsection, applying the concept of evolutionary stability [37] to preferences rather than to strategies [3], we discuss the evolutionary stability of the revenue preference using the indirectly evolutionary method, i.e., whether a monomorphic population of firms with revenue parameter \( k^* \) is immune against invading mutant firms with a different revenue preference. For analyzing the static evolution solution of the indirectly evolutionary game \( \Gamma \), we give the definition of an evolutionarily stable preference.

**Definition 1.** [3] A preference \( k \in [0, 1] \) is called evolutionarily stable if

\[
\pi(k^*, k^*) \geq \pi(k, k^*) \quad \text{for all} \quad k \in [0, 1];
\]

and

\[
\pi(k^*, k) \geq \pi(k, k) \quad \text{whenever} \quad \pi(k^*, k^*) = \pi(k, k^*).
\]

Bester and Guth [3] have explained that this definition is line with the original definition of ESS [35, 36, 41].

**Proposition 4.** When \( c < a < \frac{35c}{3} \), there is a unique evolutionarily stable revenue preference \( k^* = \frac{3(a-c)}{32c} \) \( (> 0) \); when \( a \geq \frac{35c}{3} \), \( k^* = 1 \) is a unique evolutionarily stable preference.

**Proof.** Since \( \pi(k^*, k^*) - \pi(k, k^*) = \frac{(3a-3c-32ck)^2(103a-103c+48ck)}{256000} > 0 \) for all \( k \neq k^* \), according to Definition 1, the revenue preference \( k^* \) is evolutionarily stable, and \( k^* < 1 \) for \( a < \frac{35c}{3} \). When \( a \geq \frac{35c}{3} \),

\[
\pi(1, 1) - \pi(k, 1) = \frac{4c(1-k)}{125} (3a^2 - 24ac - 11ack + 9c^2 - 3c^2k - 6c^2k^2)
\]

\[
= \frac{4c(1-k)}{125} \left[ a - \frac{c(24+11k+\sqrt{368+564k+193k^2})}{6} \right] \left[ a - \frac{c(24+11k-\sqrt{468+564k+193k^2})}{6} \right]
\]

\[
\geq \frac{4c(1-k)}{125} \left[ a - \frac{35c}{3} \right] \left[ a - \frac{c(24+11k-\sqrt{468+564k+193k^2})}{6} \right]
\]

\[
\geq 0 \quad \text{for all} \quad 0 \leq k < 1.
\]

So Proposition 4 is true. \( \square \)

From Proposition 4, the evolutionarily stable preference \( k^* > 0 \), this means that there exists some degree of revenue preference in firms. The result implies that as long as the price cap of goods is higher than its cost, a firm will pay attention to its revenue in some degree. Like the mobile phone Mi11, its price cap is relatively high, so the firm focuses primarily on how to occupy the market to compete with other brands. The level of stable preference depends on price cap \( a \) and unit cost \( c \). Specifically, it is positively related to \( a \), while is negatively related to \( c \).

Why cannot an egoistic mutant with \( k > k^* \) invade a population of \( k^* \)-firms when \( c < a < \frac{35c}{3} \)? Actually, Proposition 3 shows that such a mutant has a higher material payoff than the firm of the population with whom it interacts. However, the low material payoff of a \( k^* \)-firm against an invading mutant is less important for evolutionary considerations. For a firm in the population, the probability of interacting with a mutant is small. Mostly they interact with each other and so their expected level of material payoff is relatively high.

Chai et al. [5] discuss the evolution of social responsibility preference in the case of linear demand. They obtain that the evolutionarily stable parameter is related to
the substitution of the products, and has nothing to do with the price cap and cost. Therefore, the demand of the products affects the evolution of preference. We will discuss the issue in more details in Section 6. Whether there will be evolutionary branches in the long-term competition; in other words, whether the evolutionarily stable preference is globally stable is also the core issue in this paper. Next, we study the global stability of the preference using the adaptive dynamics.

5.3. Adaptive dynamics. Adaptive dynamics is a long-run approach to frequency-dependent selection [14, 25]. The analysis focuses on whether an infinitesimal fraction of a particular nearby mutant strategy can gain share under replicator dynamics. Specially, suppose that increasing revenue preference enhances success of firms over some range \([0, r]\), but the cost lowers overall success beyond \(r\). Then adaptive dynamics would model how the firms’ degree of revenue preference approaches to \(r\) in long run evolutionary time, and analyze the dynamic stability of \(r\). Based on the result in Section 5.2, we know that there is a unique evolutionarily stable revenue preference. But it does not show how to get this unique preference. Compared with the analysis of indirect evolution, the results with adaptive dynamics are more comprehensive. After culminating in the canonical equation, we will present whether there is an evolutionary branch in the monomorphic population, that is, here we analyze the global stability of the revenue preference.

To avoid messy mixes of short term and long term analysis, we assume that the firm is mostly monomorphic: at almost all times \(t\) there is a single strategy \(k\in[0,1]\) used by 100\% of the firm [5, 14]. The argument of \(k^*\) is not offered in Definition 1. For explaining the preference \(k^*\), firstly, we define the material payoff function \(H(m,k)\) as follows:

\[
H(m,k) = \frac{4c(m-k)((a-c)(3a-3c-18k-11m)-2c^2(2k+3m)(3k-m)-12c^2m^2)}{125}.
\]  

(6)

Obviously, we have \(H(k,k) = 0\). For slight mutants \(m\) (\(|m-k|\) is small), the selection gradient is

\[
g(k) = \frac{\partial H(m,k)}{\partial m} \bigg|_{m=k} = \frac{4c[3(a-c)^2-ck(29a-29c+32ck)]}{125}.
\]  

(7)

In order to get \(H(m,k) \approx (m-k)g(k) > 0 = H(k,k)\), if the selection gradient is positive, then mutant \(m\) needs to be slightly higher than \(k\); if the selection gradient is negative, then mutants need to be slightly lower than \(k\). So long-run steady states can only occur at a critical point of the selection gradient, where \(k^*\) is the root of selection gradient \(g(k)\).

Based on Friedman and Sinervo [14]¹, an evolutionarily stable point is a local payoff maximum, which is immune to invasion from nearby mutants. So, it is dynamically stable in the short-term sense. Obviously, the point \(k^*\) satisfies Definition 1.

For analyzing the stability in the long-term of the revenue preference, we get the canonical equation of revenue preference [12], as follows,

\[
\frac{dk}{dt} = \frac{4c[3(a-c)^2-ck(29a-29c+32ck)]}{125}.
\]  

(8)

It is easy to verify that the equilibrium for the canonical equation is the root of the selection gradient \(g(k)\).

¹The critical point \(k^*\) is evolutionarily stable if \(\frac{\partial^2 H(m,k)}{\partial m^2} \bigg|_{m=k=k^*} < 0\) [14].
According to the definition of evolutionarily stable and convergence stable, we know the difference between the definition of convergence stable and evolutionarily stable: convergence stable requires that the total derivative of the selection gradient is negative, whereas, evolutionarily stable requires the first partial derivative of the selection gradient is negative. But it should notice that there is no relationship between convergence stable and evolutionarily stable.

**Proposition 5.** When $c < a < \frac{35c}{3}$, the revenue preference $k^* = \frac{3(a-c)}{32c}$ is continuously stable; when $a \geq \frac{35c}{3}$, the preference $k^* = 1$ is continuously stable.

**Proof.** According to Proposition 4, the revenue preference $k^*$ is evolutionarily stable.

Next, we prove $k^*$ is convergence stable.

From $\frac{\partial^2 H(m,k)}{\partial m^2} \bigg|_{m=k} = \frac{-8c^2(11a-11c+26c=k)}{125}$ and $\frac{\partial^2 H(m,k)}{\partial k^2} \bigg|_{m=k} = \frac{-8c^2(29a-29c+64c=k)}{125}$, we have

$$\frac{\partial^2 H(m,k)}{\partial m^2} \bigg|_{m=k} < \frac{\partial^2 H(m,k)}{\partial k^2} \bigg|_{m=k}. \quad (9)$$

For $H(k,k) = 0$, differentiating the equation with respect to $k$ twice by using the implicit function theorem, we get

$$\frac{\partial^2 H(m,k)}{\partial m^2} \bigg|_{m=k} + 2 \frac{\partial^2 H(m,k)}{\partial m \partial k} \bigg|_{m=k} + \frac{\partial^2 H(m,k)}{\partial k^2} \bigg|_{m=k} = 0. \quad (10)$$

Combining Equations (9) and (10), we have

$$2 \frac{\partial^2 H(m,k)}{\partial m^2} \bigg|_{m=k} + 2 \frac{\partial^2 H(m,k)}{\partial m \partial k} \bigg|_{m=k} < 0. \quad (11)$$

Hence,

$$g'(k^*) = \frac{\partial^2 H(m,k)}{\partial m^2} \bigg|_{m=k=k^*} + \frac{\partial^2 H(m,k)}{\partial m \partial k} \bigg|_{m=k=k^*} < 0. \quad (12)$$

According to Friedman and Sinervo [14], $k^*$ is convergence stable, and $k^* < 1$ for $c < a \leq \frac{35c}{3}$. Similarly, we can prove the result when $a \geq \frac{35c}{3}$.

So Proposition 5 is true. \qed

In fact, condition (9) is the sufficient and necessary condition under which $k^*$ is convergence stable rather than only sufficient condition, for details one can refer to Chai et al. [5].

From Proposition 5, if the pricing capability for firms is moderate, the revenue preference $0 < k^* \leq 1$ is globally stable in the long-term. This explains the existence of firm’s revenue preference even without enforcement. So when the pricing capability for a product is high, like the initial stage for a new product, the firm pays attention to its revenue; when the market enters the normal state, where the competition becomes stronger, and the pricing capability decreases, the firm will balance its revenue and profit. To clearly explain the result, we simulate the dynamics in Figure 6, where the parameters are used as: $a = 4, c = 1$.

Figure 6 shows that whatever the initial values of the firm’s revenue preference, the firm’s revenue preference will arise and keep in a stable state. Specifically, the firm’s revenue preference will tend to $k^*$ over time. Combining Propositions 4 and 5, the revenue preference is both evolutionarily stable and continuously stable.

2 A monomorphism $k^* \in [0,1]$ is called to be convergence stable if it is a downcrossing of the selection gradient, i.e., $g'(k^*) < 0$ [14].

3 A critical point $k^*$ is continuously stable if it is both evolutionarily stable and convergence stable [14].
This implies that there is no branch point in our economic system, where the firms cannot be invaded. In other words, as long as the stable state is reached in this population, it will remain in this state for a long time and not separate into two distinct subpopulations.

6. Extended model. In previous sections, we have employed the parametric specification of the power in the inverse demand function to study the evolutionary stability of revenue preference. Here, we generalize the power in the inverse demand function, and discuss the impact of the demand function on the evolution of revenue preference. Here, the power is $b$. We extend our main model to a more general framework with $p = a - (q_1 + q_2)^b$, $b > 0$. Then the utility function of firm $i (i = 1, 2)$ with revenue preference $k$ is

$$u_{ie}(q_{1e}, q_{2e}) = (1 - k)[a - (q_{1e} + q_{2e})^b - c]q_{ie} + k[a - (q_{1e} + q_{2e})^b]q_{ie}.$$  (13)

We get the optimal quantities of firms 1 and 2, where firm 1 has the revenue preference $k_1$ and its opponent firm 2 has revenue preference $k_2$

$$q^*_1(k_1, k_2) = \frac{b(a - c - ck_1) + c(k_1 - k_2)}{2b(a - c) + bc(k_1 + k_2)} \left[\frac{2(a - c) + c(k_1 + k_2)}{2 + b}\right]^{1/b},$$  (14)

$$q^*_2(k_1, k_2) = \frac{b(a - c - ck_1) - c(1 + b)(k_1 - k_2)}{2b(a - c) + bc(k_1 + k_2)} \left[\frac{2(a - c) + c(k_1 + k_2)}{2 + b}\right]^{1/b},$$

respectively. Thus, the material payoff of $k_1$-firm at the equilibrium ($q^*_1, q^*_2$) is

$$\pi_e(k_1, k_2) = \frac{b(a - c - c(k_1 + k_2)) b(a - c + ck_1) + c(k_1 - k_2)}{b(2 + b)} \left[\frac{2(a - c) + c(k_1 + k_2)}{2 + b}\right]^{1/b}.  \quad (15)$$

Based on the material payoff $\pi_e(k_1, k_2)$, we assume that the material payoff function is $H_e(m, k)$ for mutant preference $m \in [0, 1]$ when the share of the resident preference $k \in [0, 1]$ is 1.

$$H_e(m, k) = 2[b(a - c) - c(m + k)][b(a - c + cm) + c(m - k)][2(a - c) + c(m + k)]^{1/b}$$

$$- b[2(a - c) + c(m + k)][b(a - c) - 2ck][2(a - c + ck)]^{1/b}.  \quad (16)$$

Obviously, $H_e(k, k) = 0$, that is, material payoff is measured as the long-run expected growth rate of the preference's share, which is zero when the share remains at 1.
The evolutionarily stable critical point is a local material payoff maximum, and it is dynamically stable in the short run sense that it is immune to invasion from nearby mutants. When we consider the long-run adaptive process, the stability analysis takes a different twist. Although immune to local invasion, there is no guarantee that long run evolutionary dynamics would actually take us to an evolutionarily stable preference if we start with a nearby monomorphism \[14\]. Based on the adaptive dynamics literature, we define these long run dynamics via the canonical equation

\[ \dot{k} = \frac{\partial H_\varepsilon(m, k)}{\partial m} \bigg|_{m=k} = c[a(b+1) - c(b^2 + 6k + 4bk)][2(a - c + ck)]^{1/b}. \] (17)

Thus, we have the results as follows.

**Proposition 6.** When \( 0 < b < \frac{-a + 5c + \sqrt{a^2 + 14ac + c^2}}{2(a-c)} \) (\( \hat{b} \)), the revenue preference \( k^*_e = \frac{b(1+b)(a-c)}{2(3+2b)c} \) is evolutionarily stable; when \( b \geq \hat{b} \), preference \( k^*_e = 1 \) is continuously stable.

**Proof.** For the case of \( 0 < b < \hat{b} \), we prove \( k^*_e = \frac{b(1+b)(a-c)}{2(3+2b)c} \) is evolutionarily stable. According to the expression of \( H_\varepsilon(m, k) \), we have

\[ \frac{\partial H_\varepsilon(m, k)}{\partial m} \bigg|_{m=k} = -c^2 \left( ab(7+5b) + c(8b^2 k + 10k + 18bk - 5k^2 - 7b) \right) \left[ 2(a - c + ck) \right]^{1/b}. \] (18)

Then \( \frac{\partial H_\varepsilon(m, k)}{\partial m} \bigg|_{m=k=k^*_e} = \frac{-(13 + 15b + 4b^2)c^2}{3+b} \left( \frac{6+5b+b^2}{3+2b} \right)^{1/b} < 0. \)

Therefore, based on Friedman and Sinervo \[14\], \( k^*_e \) is evolutionarily stable. Next, we prove \( k^*_e \) is convergence stable.

\[ \frac{\partial k}{\partial t} \bigg|_{k=k^*_e} = \frac{\partial^2 H_\varepsilon(m, k)}{\partial m^2} \bigg|_{m=k=k^*_e} + \frac{\partial^2 H_\varepsilon(m, k)}{\partial m \partial k} \bigg|_{m=k=k^*_e} \]

\[ = -2(3 + 2b)c^2 \left( \frac{6+5b+b^2}{3+2b} \right)^{1/b} < 0. \] (19)

Again, based on Friedman and Sinervo \[14\], \( k^*_e \) is continuously stable. Furthermore, because of \( 0 < b < \hat{b} \), we have \( k^*_e < 1 \). Similarly, we can prove the preference \( k^*_e = 1 \) is continuously stable when \( b \geq \hat{b} \).

Thus, Proposition 6 is true. \( \square \)

From the result, we find that evolutionarily stable preference must exhibit some degree of revenue preference. When \( 0 < b < \hat{b} \), the level of revenue preference is positively related to the price cap \( a \), and negatively related to the unit production cost \( c \). If the price cap \( a \) and the unit production cost \( c \) are constant, revenue preference becomes more important when the parameter \( b \) is relatively high. In fact, \( b \geq \hat{b} \) implies \( k^*_e = 1 \). Thus, the main result that revenue preference for a firm is context dependent holds in the general framework.

7. **Conclusions.** In this paper, we discuss the evolutionary stability of revenue preference for competing firms who sell homogenous products. Two kinds of preferences are considered: discrete and continuous. When the preference is discrete, our discussion of individual interactions reveals two insights. Firstly, the choice of firm's strategy depends on the price cap. If the price cap is high enough, a firm can get more material payoff by selecting revenue maximization than profit maximization. Secondly, the study in the population dynamics shows that profit maximization, revenue maximization, and even the mixed strategy may be evolutionarily stable,
and the stability of strategies depends on the price cap. Specifically, if the price cap in this market is moderate, the mixed strategy profile is an evolutionarily stable strategy; when the price cap is high enough, it is more profitable for both firms to choose revenue maximization; otherwise, profit maximization is evolutionarily stable.

Furthermore, we consider why some firms have revenue preference. Here, assume that the endogenous preference is continuous, and firms have different degrees of revenue preference. The degree of revenue preference is expressed by a preference parameter $k$. We address the evolution of revenue preference using the indirect evolution method and adaptive dynamics. The evolutionary process selects the more successful strategy; that is, the strategy with a higher material payoff will be selected in the evolutionary process. Similarly, the preference increasing firm’s material payoff will be selected [3, 5, 10]. We find that there exists a unique evolutionarily stable revenue preference, and the preference is also continuously stable. The result gives an explanation for the existence of the firm’s revenue preference in the market for a long time. In our analysis, the revenue preference is common knowledge, so our results reveal why the famous firms are more willing to show their revenue preferences, such as Xiaomi, Tesla, and so on, in their early stage of development.

Finally, we extend our analysis to the general inverse demand. We get that the level of revenue preference depends on the power of the inverse demand function. Specially, the larger the power, the higher the level of revenue preference will be. When the power is high enough, the stable revenue preference equals one; in other words, all firms compete in revenue preference and are totally indifferent to their profits.

In future research, several directions can be discussed. First, as mentioned above, we assume that the firm’s preference is common knowledge, which indicates that the game in our analysis is complete information. We are also interested in the case that the firm’s preference is private information. Second, the impact of some uncertain factors, such as market demand and cost, risk is also an interesting topic. Furthermore, we may use the empirical method to test our results. Finally, in our model, we consider the homogeneous population; as a result, the evolution of preferences in heterogeneous populations is also an interesting issue.

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