DEUTERON FORMATION IN NUCLEAR MATTER WITHIN THE FADDEEV APPROACH

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Abstract

We consider deuteron formation in heavy ion collisions at intermediate energies. The elementary reaction rates \((Nd \rightarrow NNN\text{ etc.)}\) in this context are calculated using rigorous Faddeev methods. To this end an in-medium Faddeev equation that consistently includes the energy shift and Pauli blocking effects has been derived and solved numerically. As a first application we have calculated the life-time of deuteron fluctuations for nuclear densities and temperatures typical for the final stage of heavy ion collisions. We find substantial differences between using the isolated and the in-medium rates.

1 Introduction

Nuclear matter is an example of a strongly correlated many particle system. One prominent consequence is the formation of bound states (clusters, fragments) observed in heavy ion reactions. Here we address the formation of deuterons at intermediate energies, i.e. for \(E/A \leq 200\text{ MeV/u}\). Within the quantum statistical approach to describe the complicated dynamics we employ the Green function method [1]. The cluster mean field approximation [2] decouples the hierarchy and leads to rigorous few-body equations for the two-, three-, four-particle correlations. This method is tiedly connected to the self consistent RPA approach extended to finite temperatures [3].

Deuteron formation is directly related to the \(Nd \rightarrow NNN\) break-up cross section. Photoinduced reactions have also been considered [4]. Because of the energies considered, pion induced reactions can be neglected.

Treating deuteron formation within the cluster Hartree-Fock approximation allows us to consistently include all medium modifications as they appear in the respective two- and three-body equations. These are the self energy and the Pauli blocking effects.

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2 Reactions

The quantity of interest in the quantum statistical approach is the generalized quantum Boltzmann equation for the nucleon $f_N$, deuteron $f_d$, etc. distributions. Here, we consider the collision integral to show the relevance of three-body reaction rates. The feeding of the nucleon density is driven by the collision integral (see e.g. Ref. [5] for an application to heavy ion collisions)

$$J_N(p, t) = J_N^{\text{out}}(p, t)f_N(p, t) - J_N^{\text{in}}(p, t)\bar{f}_N(p, t),$$

where we have used $\bar{f}_N = 1 - f_N$. To be more explicit we give

$$J_N^{\text{out}}(p, t) = \int d^3 k \int d^3 k_1 d^3 k_2 |\langle kp|T|k_1 k_2\rangle|^2 \bar{f}_N(k_1, t)\bar{f}_N(k_2, t)f_N(k, t)$$

$$+ \int d^3 k \int d^3 k_1 d^3 k_2 d^3 k_3 |\langle kp|U_0|k_1 k_2 k_3\rangle|^2 \times \bar{f}_N(k_1, t)\bar{f}_N(k_2, t)f_d(k, t)$$

$$+ \ldots$$

where dots stand for other possible contributions mentioned before. The quantity $U_0$ appearing in (2) is the break-up transition operator for $Nd \rightarrow NNN$.

For the isolated three-body problem $U_0$ determines the break-up cross section $\sigma_0$ via

$$\sigma_0 = \frac{1}{|v_d - v_N|} \frac{1}{3!} \int d^3 k_1 d^3 k_2 d^3 k_3 |\langle kp|U_0|k_1 k_2 k_3\rangle|^2$$

$$\times 2\pi\delta(E_f - E_i) (2\pi)^3\delta^{(3)}(k_1 + k_2 + k_3).$$

So far the strategy has been to implement the experimental cross section into the above equation. This has then been solved, for a specific heavy ion collision [5]. Using experimental cross sections respectively isolated cross sections may not be sufficient in particular in the lower energy regime. The cross section itself depends on the medium, e.g. blocking of internal lines or self energy corrections of the respective three-body Green functions. To this end we have derived a three-body Faddeev type equation [6, 7]. We use the AGS formalism [8] for the three-body algebra and solve the respective equations numerically. The equation for the three-particle Green function derived within the cluster Hartree-Fock approximation reads

$$G_3(z) = \frac{\bar{f}_1 \bar{f}_2 \bar{f}_3 + f_1 f_2 f_3}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3} + \frac{(\bar{f}_1 \bar{f}_2 - f_1 f_2)V(12) + \text{cycl.perm.}}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3} G_3(z),$$

(4)
where \( \varepsilon \) denotes the quasi particle energies evaluated in Hartree-Fock approximation and \( f_1 = f(\varepsilon_1) \) the Fermi functions \( f(\varepsilon) = (\exp[\beta(\varepsilon - \mu)] + 1)^{-1} \) with the inverse temperature \( \beta \) and the chemical potential \( \mu \) (for the time being we assume symmetric nuclear matter). Here, we use equilibrium distributions to solve Faddeev equations. This is justified within the linear response theory, where nonequilibrium quantities are expressed through equilibrium ones, because of small fluctuations only. The question of self consistency has been addressed for the much simpler case of two-particle correlations, e.g. in Ref. [9].

Within the AGS formalism (extended here to finite temperatures and densities) the break-up operator \( U_0 \) is simply related to the elastic/rearrangement scattering amplitude \( U_{\alpha\beta} \) connecting the channels \( \beta \rightarrow \alpha \). It is therefore sufficient to present the AGS equation for the transition operator \( U_{\alpha\beta} \) only, viz.

\[
U_{\alpha\beta}(z) = \bar{\delta}_{\alpha\beta} \left[ \frac{\bar{f}_1 \bar{f}_2 \bar{f}_3 + f_1 f_2 f_3}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3} \right]^{-1} + \sum_{\alpha \neq \gamma = 1}^3 T_3^{(\gamma)} \frac{\bar{f}_1 \bar{f}_2 \bar{f}_3 + f_1 f_2 f_3}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3} U_{\gamma\beta}(z) \quad (5)
\]

with \( \bar{\delta} = 1 - \delta \) and \( \bar{f} = 1 - f \). The two-body \( t \) matrix \( T_3^{(\gamma)} \) has been solved on the same footing consistently including all medium effects, i.e. \( T_3^{(\gamma)} \) is the solution of the in-medium two-body problem, e.g. for \( \gamma = 3 \)

\[
T_3^{(3)} = (1 - f_3 + g(\varepsilon_1 + \varepsilon_2))^{-1} V_2 + V_2 \frac{\bar{f}_1 \bar{f}_2 - f_1 f_2}{z - \varepsilon_1 - \varepsilon_2 - \varepsilon_3} T_3^{(3)}, \quad (6)
\]

where \( g(\omega) = (\exp[\beta(\omega - 2\mu)] - 1)^{-1} \) is the Bose function for two nucleons.

### 3 Results

The AGS equations have been solved for a separable Yamaguchi potential. To get an impression of the quality of the calculation the isolated cross section is given in Fig. 1 along with the experimental data on neutron deuteron scattering [10].

From inspection of Fig. 2 we see that the in-medium cross section is significantly enhanced compared to the isolated on. The threshold is shifted to smaller energies, which is because the binding energy of the deuteron becomes smaller. We observe that for higher energies the medium dependence of the cross section becomes much weaker, which \textit{a posteriori} justifies the use

\[\text{The channel notation } \alpha, \beta = 1, 2, 3 \text{ labels the respective spectator in the three-body system.}\]
of isolated cross sections (along with the impulse approximation) when higher energies are considered [5].

From linearizing the Boltzmann equation it is possible to define a break-up time for small fluctuations of the deuteron distributions. For small fluctuations \( \delta f(t) = f_d(t) - f_d^0 \) from the equilibrium distribution \( f_d^0 \) linear response leads to

\[
\partial_t \delta f_d(P, t) = -\frac{1}{\tau_{bu}} \delta f_d(P, t)
\]

where the “life time” of deuteron fluctuations has been introduced [7],

\[
\tau_{bu}^{-1} = \frac{4}{3!} \int d k_N d^3 k_1 d^3 k_2 d^3 k_3 \langle (k p|U_0|k_1 k_2 k_3)^2 \rangle \bar f_1 f_2 f_3 f(k_N) 2\pi \delta(E - E_0)
\]

which can be related to the break-up cross section given in Fig. 2. For low densities the life time (as a function of the deuteron momentum \( P \)) and the inverse life time, i.e. the width, at \( P = 0 \) along with the deuteron binding energy for comparison is shown in Fig. 3. These times have to be compared to the approximate duration of the heavy ion collision of about 200 fm.

Another important time scale is the chemical relaxation time for small fluctuations of the deuteron density \( \delta n_d(t) = n_d(t) - n_d^0 \) from the equilibrium distribution \( n_d^0 \). Using detailed balance and linearized rate equations the relaxation time is given through

\[
\frac{d}{dt} \delta n_d(t) = -\frac{1}{\tau_{rel}} \delta n_d(t).
\]

Figure 1: A comparison of the total, elastic, and break-up cross sections \( nd \rightarrow nd, nd \rightarrow nnp \) with the experimental data of Ref. [10].

Figure 2: In-medium break-up cross section at \( T = 10 \) MeV. Isolated cross section is shown as solid line, other lines show different nuclear densities.
The basic quantity driving the time scale is again the break-up cross section

\[ \frac{1}{\tau_{\text{rel}}} = \int d^3p_N \, d^3p_d \, f(k_N)g(p_d)\left|v_d - v_N\right| \sigma_0(E) \frac{n_N^0 + 4n_d^0}{n_N^0 n_d^0} \]  

The resulting relaxation time as a function of the uncorrelated nuclear density is given in Fig. 4.

4 Conclusion and Outlook

Our results show that medium dependent cross sections in the respective collision integrals lead to shorter reaction time scales. Chemical processes become faster. This also effects the elastic rates that are related to thermal equilibration.

The basis of this result is the cluster Hartree-Fock approach that in our approximation includes correlations up to three particles in a consistent way. The equations driving the correlations are rigorous. The respective one-, two- and three-body equations are solved, in particular for the three-particle case Faddeev/AGS type equations have been derived in Ref. [6, 7]. The AGS approach is particularly appealing since it allows generalizations to \( n \)-particle equations in a straightforward way. Results for the three-body bound states
in medium will be published elsewhere [11]. As expected from the deuteron case, the triton binding energy changes with increasing density up to the Mott density, where $E_t = 0$.

The production rates, spectra etc. of light charged particles in heavy ion collisions at intermediate energies may change because of the much smaller time scales induced through the medium dependence compared to the use of free cross sections (respectively experimental cross sections). To this end some notion of the relevant densities and temperatures during the heavy ion collision (during the final stage) should be achieved.

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