The cosmological evolution of the binary black hole (BH) merger rate and the energy density of the gravitational wave (GW) background are investigated. To evaluate the redshift dependence of the BH formation rate, BHs are assumed to originate from low-metallicity stars, and the relations between the star formation rate, metallicity and stellar mass of galaxies are combined with the stellar mass function at each redshift. As a result, it is found that when the energy density of the GW background is scaled with the merger rate at the local universe, the scaling factor does not depend on the critical metallicity for the formation of BHs. Also taking into account the merger of binary neutron stars, a simple formula to express the energy spectrum of the GW background is constructed for the inspirial phase. The relation between the local merger rate and the energy density of the GW background will be examined by future GW observations.

Key words: black hole physics – galaxies: evolution – gravitational waves – stars: neutron
present the formulation of the binary BH merger rate. We also investigate the GW background from binary BH mergers and derive the relation between the merger rate at the local universe and the energy density of the GW background. Furthermore, we also consider binary NS mergers and the GW background from them. Finally, Section 4 is devoted to discussion.

2. COSMIC SFR DENSITY OF LOW-METALLICITY STARS

Heavy BHs with mass $\sim30M_\odot$ are expected to be formed in a low-metallicity environment below a critical metallicity, Z$_{\text{crit}}$. Therefore, the BH formation rate should be proportional to the cosmic star formation rate density (CSFRD) of low-metallicity stars. We investigate the dependence on Z$_{\text{crit}}$ of the CSFRD of stars with metallicity below Z$_{\text{crit}}$, as done in Abbott et al. (2016c). Note that, while Abbott et al. (2016b, 2016c) assumed Z$_{\text{crit}} = 0.5Z_\odot$, for their fiducial model, Belczynski et al. (2016) showed that the formation of the binary heavy BHs requires $<0.1Z_\odot$. In this section, we first describe our model based on the observational data of galaxies. Next, for comparison, we consider the alternative model for the CSFRD of low-metallicity stars following Abbott et al. (2016c).

2.1. Models of SFR and Metallicity Evolution of Galaxies

To derive the CSFRD of low-metallicity stars, we consider the stellar mass ($M_\ast$), SFR ($\dot{M}_\ast$), and metallicity ($Z$) of a galaxy with a mass $M_\ast$. Here, SFR is the total mass of stars formed in the galaxy per unit time. For our standard model, we adopt the redshift evolutions of the galaxy stellar mass function and the mass-dependent SFR proposed by Drory & Alvarez (2008) and the redshift-dependent mass--metallicity relation from Maiolino et al. (2008). These models were also utilized to evaluate a spectrum of supernova relic neutrinos in a previous study (Nakazato et al. 2015).

Since, at redshift $z$, $\dot{M}_\ast(M_\ast, z)$ is the SFR of a galaxy with a stellar mass of $M_\ast$, and $\rho_{\text{SMF}}(M_\ast, z) dM_\ast$ is a number density of galaxies within a stellar mass bin of $[M_\ast, M_\ast + dM_\ast]$, the total CSFRD is given by

$$\dot{\rho}_\ast(z) = \int_0^\infty \dot{M}_\ast(M_\ast, z) \rho_{\text{SMF}}(M_\ast, z) dM_\ast,$$  \hspace{1cm} (1)

with the stellar mass function $\rho_{\text{SMF}}(M_\ast, z)$. In Drory & Alvarez (2008), the stellar mass function is assumed to have a Schechter form,

$$\rho_{\text{SMF}}(M_\ast, z) = \rho_0(z) \left( \frac{M_\ast}{M_\ast^{\text{DA08}}(z)} \right)^{-1.3} \times \exp\left( -\frac{M_\ast}{M_\ast^{\text{DA08}}(z)} \right) dM_\ast,$$  \hspace{1cm} (2)

with the best-fitting parameterization

$$\rho_0(z) = 0.0031 \times (1 + z)^{-1.07} Mpc^{-3} \text{dex}^{-1}, \hspace{1cm} (3a)$$

$$\log_{10} \left( \frac{M_\ast^{\text{DA08}}(z)}{M_\odot} \right) = 11.35 - 0.22 \times \log(1 + z). \hspace{1cm} (3b)$$

As expressed in Equation (2), the stellar mass function has a sharp exponential cutoff above $M_\ast^{\text{DA08}}$. The SFR is a function of the stellar mass and redshift and is written as (Drory & Alvarez 2008)

$$\dot{M}_\ast(M_\ast, z) = \dot{M}_\ast^0(z) \left( \frac{M_\ast}{M_\ast^{\text{DA08}}(z)} \right)^{0.6} \exp\left( -\frac{M_\ast}{M_\ast^{\text{DA08}}(z)} \right). \hspace{1cm} (4)$$

with

$$M_\ast^0(z) = 1.183 \times (1 + z)^{5.5} \exp(-0.78z) M_\odot \text{yr}^{-1}, \hspace{1cm} (5a)$$

$$M_\ast^{\text{DA08}}(z) = 2.7 \times 10^{10} \times (1 + z)^{2.1} M_\odot. \hspace{1cm} (5b)$$

Note that the analytic form of Equation $5(a)$ was determined to fit not only the original data of galaxies observed by Drory et al. (2005), but the CSFRD at the local universe, $\dot{\rho}_\ast(0) = 0.02 M_\odot \text{yr}^{-1} \text{Mpc}^{-3}$ (Nakazato et al. 2015). Roughly speaking, the SFR is higher for galaxies with higher stellar mass, while the specific SFR, $\dot{M}_\ast/M_\ast$, is higher for galaxies with lower stellar mass. Nevertheless, star formation is strongly suppressed for galaxies with enough high stellar mass and $M_\ast^{\text{DA08}}$ corresponds to the mass above which the SFR begins to decline.

The CSFRD for a metallicity below Z$_{\text{crit}}$ is given by

$$\dot{\rho}_\ast^{\text{low}}(z, Z_{\text{crit}}) = \int_0^{M_\ast(z, Z_{\text{crit}})} \dot{M}_\ast(M_\ast, z) \rho_{\text{SMF}}(M_\ast, z) dM_\ast, \hspace{1cm} (6)$$

where $M_\ast(z, Z)$ is the stellar mass of a galaxy with metallicity $Z$ at redshift $z$. It is known that lower-metallicity galaxies have systematically lower stellar mass. In Maiolino et al. (2008), the
galaxy mass–metallicity relation was expressed as

$$\log_{10}\left(\frac{Z}{Z_\odot}\right) + 8.69 = -0.0864 \log_{10} M_\bullet - \log_{10} M_\bullet^{\text{MOS}} + K_0^{\text{MOS}},$$  

(7)

where we adopt the best fit values of $\log_{10} M_\bullet^{\text{MOS}}$ and $K_0^{\text{MOS}}$ at different redshifts from case $a$ in Table 5 of Maiolino et al. (2008). This relation is applicable for stellar masses of $8.5 \leq \log_{10}(M_\bullet/M_\odot) \leq 11.2$, which correspond to $8.2 \leq 12 + \log_{10}(O/H) \leq 9$ at $z = 0.7$ and $7.2 \leq 12 + \log_{10}(O/H) \leq 8.6$ at $z = 3.5$. Note that the solar metallicity ($Z_\odot = 0.02$) is assumed to correspond to the oxygen abundance of $12 + \log_{10}(O/H) = 8.69$ (Allende Prieto et al. 2001), i.e.,

$$\log_{10}\left(\frac{Z}{Z_\odot}\right) = 12 + \log_{10}(O/H) - 8.69.$$  

(8)

Using Equation (7), we calculate $M_\bullet(z, Z)$ and, thereby, $\rho_{\text{low}}(z, Z_{\text{crit}})$.

We show the total CSFRD and CSFRD of low-metallicity stars in Figure 2. The resultant total CSFRD is lower than that in Hopkins & Beacom (2006), which has often been cited. In contrast, the theoretical models predict a much lower CSFRD (e.g., Kobayashi et al. 2013) and our CSFRD model lies between them for $z \lesssim 2$. Incidentally, in the redshift range of $0 \leq z \leq 2$, the difference from the recent model of Madau & Dickinson (2014) is not large; our CSFRD model is 10%–30% higher. The total CSFRD has a peak near the redshift $z = 3$, or equivalently the lookback time of 11 Gyr, and declines toward the present epoch. In Figure 2, we also show the fraction of stars formed with metallicity below $Z_{\text{crit}}$, $\rho_{\text{low}}(z, Z_{\text{crit}})/\rho_{\text{high}}(z)$, as a function of the lookback time. For $Z_{\text{crit}} \lesssim 0.5Z_\odot$, the fraction has not varied much over the last 8 Gyr (i.e., redshift $z \lesssim 1$), while the total CSFRD is decreasing. This trend originates from the models of galaxy evolution adopted in this study. First, the stellar mass function of Drory & Alvarez (2008) does not evolve significantly in this period. Second, when the SFR is drawn as a function of the stellar mass, the slope in the low-mass range does not depend on the redshift, and the peak mass, $M_\bullet^{\text{DAR08}}$, becomes higher at a high redshift (see the top panel of Figure 3 in Drory & Alvarez 2008). On the other hand, in Maiolino et al. (2008), the galaxy mass–metallicity relation moves toward higher masses, but its shape is preserved at a high redshift, which is also described in Savaglio et al. (2005).

As a result, these two shifts balance out and the fraction of stars formed in a low-metallicity environment is almost constant for $z \lesssim 1$. 

Figure 2. Total CSFRD and CSFRD of stars with metallicity below $Z_{\text{crit}}$ as a function of redshift (left) and lookback time (center), and the fraction of stars formed with metallicity below $Z_{\text{crit}}$ as a function of lookback time (right) for our standard model. The solid lines represent the total CSFRD in the left and central panels. The other lines correspond, from bottom to top, to $Z_{\text{crit}} = 0.1Z_\odot$, $0.5Z_\odot$, and $Z_\odot$.

Figure 3. Same as Figure 2, but for the alternative model given in Section 2.2.
2.2. Alternative Model for Fraction of Low-metallicity Stars

Here, we construct the alternative model for the CSFRD of low-metallicity stars, following Abbott et al. (2016c), which was also adopted in Callister et al. (2016). It is based on the mean metallicity at the redshift \( z \), which is written as

\[
Z_{\text{mean}}(z) = Z_{\text{mean}}(0) \times \frac{\int_{z}^{z_{\text{max}}} dz' \frac{\rho_\alpha(z')}{(H_0(1 + z') + \sqrt{\Omega_m(1 + z')^3 + \Omega_\Lambda})}}{\int_{0}^{z_{\text{max}}} dz' \frac{\rho_\alpha(z')}{(H_0(1 + z') + \sqrt{\Omega_m(1 + z')^3 + \Omega_\Lambda})}}
\]

(9)

with cosmological constants \( H_0 = 70 \text{ km s}^{-1}\text{Mpc}^{-1} \), \( \Omega_m = 0.3 \), and \( \Omega_\Lambda = 0.7 \) (Madau & Dickinson 2014). We set \( z_{\text{max}} = 20 \) (Belczynski et al. 2016). For the mean metallicity at \( z = 0 \), we adopt the value of \( Z_{\text{mean}}(0) = 0.5Z_\odot \) (Vangioni et al. 2015). Note that Abbott et al. (2016c) adopted the mean metallicity-redshift relation of Madau & Dickinson (2014) but rescaled it to account for local observations (Vangioni et al. 2015; Belczynski et al. 2016). For convenience of comparison, we use the same function for the total CSFRD, \( \rho_\alpha(z) \), as that derived in Section 2.1. At each redshift, the metallicity is assumed to have a log-normal distribution with a standard deviation of 0.5 dex around the mean. Incidentally, this is the metallicity dispersion of the interstellar medium. Therefore, we take into account the metallicity dispersion of the interstellar medium.

3. MERGER RATE AND GW BACKGROUND

In this section, we investigate the merger rate of binary BHs formed in a low-metallicity environment. The merger rate is calculated as a function of redshift utilizing the models of metallicity evolution in Section 2. For this purpose, the delay time between the binary BH formation and the merger is needed. In particular, we investigate the dependence on the minimum delay time, while Abbott et al. (2016c) assumed \( t_{\text{min}} = 50 \text{ Myr} \) for their fiducial model. Using the derived merger rate, we study the contribution of binary BH mergers to the GW background. Here, we focus on the relation between the local merger rate and the energy density of the GW background. Furthermore, we consider the contribution of binary NSs. While we again utilize the models of metallicity evolution in Section 2, NSs are formed in both low- and high-metallicity environments. Therefore, we take into account the difference in the formation rate between low- and high-metallicity environments.

3.1. Cosmological Evolution of Merger Rate

We consider the merger rate of binary BHs originating from a low-metallicity environment with metallicity below \( Z_{\text{crit}} \) as in Abbott et al. (2016c). It is determined by convolving the binary BH formation rate \( R^\text{BH}_{\text{in}}(z) \) with the delay time distribution \( P(r_d) \) (e.g., Nakar 2007; Abbott et al. 2016c):

\[
R^\text{BH}_{\text{in}}(z_m) = \int_{t_{\text{min}}}^{t_{\text{max}}} R^\text{BH}_{\text{in}}(z_f) P(r_d) \, dt_d,
\]

(10)

where the redshift at the formation time \( z_f \) is related to the redshift at the merger time \( z_m \) and delay time \( t_d \) as \( T(z_f) = T(z_m) + t_d \), denoting the lookback time at redshift \( z \) as \( T(z) \). We assume that the delay time distribution follows \( P(t_d) \propto 1/t_d \) for \( t_d > t_{\text{min}} \) (Abbott et al. 2016c; Belczynski et al. 2016), where \( t_{\text{min}} \) is the minimum delay time for a binary BH to evolve until merger. The maximum delay time \( t_{\text{max}} \) is set to the Hubble time. Note that delay time distribution of \( 1/t_d \) is usually assumed for isolated binaries. Incidentally, the merger rate of binaries formed dynamically in clusters is inversely proportional to the age of the cluster after the depletion of binary formation (O’Leary et al. 2006).

The binary BH formation rate is assumed to be proportional to the CSFRD of low-metallicity stars below \( Z_{\text{crit}} \) as

\[
R_f^\text{BH}(z) = \zeta^\text{BH} P_h(z, Z_{\text{crit}}),
\]

(11)

with conversion coefficient \( \zeta^\text{BH} \). While some uncertainties regarding the formation process of a binary BH, such as the binary formation rate, binary evolution model, and BH formation rate, are encapsulated into \( \zeta^\text{BH} \), in this paper we do not evaluate \( \zeta^\text{BH} \) explicitly. In other words, we focus on the physics free from these uncertainties.

In Figure 4, the merger rates of binary BHs for our standard model are plotted as a function of the lookback time. They are shown as a ratio to the values at the local universe \( (z = 0) \) to be independent of the conversion coefficient \( \zeta^\text{BH} \), which is not evaluated. It can be seen that the evolution of the merger rate depends on the minimum delay time \( t_{\text{min}} \); the rise time of the merger rate becomes later and the ratio of the peak to local merger rates decreases for the models with a longer delay time. On the other hand, the scaled merger rate is insensitive to the critical metallicity, especially for \( Z_{\text{crit}} \lesssim 0.5Z_\odot \). This is because, for redshift \( z \lesssim 1 \), the fraction of stars formed below \( Z_{\text{crit}} \) is almost time independent, and hence the evolution of the scaled formation rate of low-metallicity stars is insensitive to \( Z_{\text{crit}} \).

The merger rates for the alternative model are shown in Figure 5. In this case, the merger rate varies with \( Z_{\text{crit}} \). In particular, for a case with a lower critical metallicity, binary BHs are formed mainly at a high redshift and the evolution of the merger rate follows the delay time distribution. Therefore, the ratio of the peak to local merger rates is larger for the models with a lower \( Z_{\text{crit}} \) and/or a shorter delay time.

3.2. GW Background from Binary Inspirals

The GWs emitted from the orbital motion of merging binaries create a GW background. Its energy density spectrum
is given by (Abbott et al. 2016c)
\[ \Omega_{\text{GW}}^{\text{BH}}(f) = \frac{f}{\rho_*} \int_{0}^{z_{\text{min}}} dz \frac{R_{m}^{\text{BH}}(z)}{H_0(1+z)\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \times dE_{\text{GW}}^{\text{BH}}(f') \frac{df'}{df'} , \]

where the critical energy density is given by \( \rho_* = 3H_0^2c^2/8\pi G \) with velocity of light \( c \) and gravitational constant \( G \) (see also Zhu et al. 2011; Wu et al. 2012). The frequency on the Earth, \( f \), is related to that at redshift \( z, f' \), as \( f' = (1+z)f \). The spectral energy density \( dE_{\text{GW}}^{\text{BH}}(f)/df \) originates from each merging binary. Here, bearing in mind the sensitive frequency band of detectors (10–50 Hz), we use the following approximation for the spectral energy density:
\[ \frac{dE_{\text{GW}}^{\text{BH}}(f)}{df} \approx \frac{(G\pi)^2/3}{3} M_{c,\text{BH}}^{5/3} f^{-1/3} , \]

where \( M_{c,\text{BH}} \) is the chirp mass of the binary. This formula closely approximates the spectrum below \( \sim 100 \) Hz, where the contribution from the inspiral phase is dominant. Substituting Equation (13) into Equation (12), we obtain
\[ \Omega_{\text{GW}}^{\text{BH}}(f) \approx \frac{(G\pi)^2/3}{3\rho_*H_0} M_{c,\text{BH}}^{5/3} R_{m}^{\text{BH}}(0) \times \left\{ \int_{0}^{z_{\text{max}}} dz \frac{R_{m}^{\text{BH}}(z)/R_{m}^{\text{BH}}(0)}{(1+z)^4/3\sqrt{\Omega_m(1+z)^3 + \Omega_\Lambda}} \right\} f^{2/3} . \]

Note that, to compute the integral over redshift \( z \), the merger rate is scaled with the local value, because the conversion coefficient \( \zeta_{\text{BH}} \) is not evaluated again.

We rewrite Equation (14) as
\[ \Omega_{\text{GW}}^{\text{BH}}(f) \approx F_{25}^{\text{BH}} \left( \frac{M_{c,\text{BH}}}{M_0} \right)^{5/3} \times \left( \frac{R_{m}^{\text{BH}}(0)}{\text{Gpc}^{-1} \text{yr}^{-1}} \right) \left( \frac{f}{25 \text{ Hz}} \right)^{2/3} , \]

and determine the factor \( F_{25}^{\text{BH}} \). As is the case for the merger rate, the critical metallicity \( Z_{\text{crit}} \) and the minimum delay time \( t_{\text{min}} \) are needed to determine \( F_{25}^{\text{BH}} \). For our standard model, we show their dependences of \( F_{25}^{\text{BH}} \) in Figure 6. Since the scaled merger rate does not depend on \( Z_{\text{crit}} \), \( F_{25}^{\text{BH}} \) also does not depend on \( Z_{\text{crit}} \).
for \( Z_{\text{crit}} \leq 0.5Z_\odot \). In contrast, for increasing \( t_{\text{min}} \), the value of \( F_{25}^{\text{BH}} \) decreases or, equivalently, the energy density of the GW background becomes lower. This is because the merger rate of the past universe relative to the local universe decreases for the models with a longer delay time. Furthermore, we find that, for \( Z_{\text{crit}} \leq 0.5Z_\odot \), the minimum delay time dependence of \( F_{25}^{\text{BH}} \) can be fitted by

\[
F_{25}^{\text{BH}} = c_0^{\text{BH}} + c_1^{\text{BH}} \log_{10}
\left( \frac{t_{\text{min}}}{50 \text{ Myr}} \right)
+ c_2^{\text{BH}} \left[ \log_{10}
\left( \frac{t_{\text{min}}}{50 \text{ Myr}} \right) \right]^2,
\]

with \( c_0^{\text{BH}} = 3.2 \times 10^{-13} \), \( c_1^{\text{BH}} = -6.6 \times 10^{-14} \) and \( c_2^{\text{BH}} = -2.6 \times 10^{-14} \). Note that, with \( M_c^{\text{BH}} = 28M_\odot \), \( R_m^{\text{BH}}(0) = 16^{+13}_{-8} \text{Gpc}^{-3} \text{yr}^{-1} \), \( Z_{\text{crit}} = 0.5Z_\odot \) and \( t_{\text{min}} = 50 \text{ Myr} \), our standard model gives \( \Omega_{\text{GW}}^{\text{NS}}(f = 25 \text{ Hz}) = 1.3_{-1.1}^{+1.2} \times 10^{-9} \), which is close to the value in Abbott et al. (2016c) with the same input parameters. In contrast, as shown in Figure 7, \( F_{25}^{\text{BH}} \) is a function of not only \( t_{\text{min}} \), but \( Z_{\text{crit}} \) for the alternative model. Incidentally, the alternative model gives \( \Omega_{\text{GW}}^{\text{NS}}(f = 25 \text{ Hz}) = 1.3_{-1.1}^{+1.2} \times 10^{-9} \) with the above inputs.

3.3. Contribution of Binary NSs

All known binary NSs are systems, which contain at least one radio pulsar and they are also the target of GW astronomy (e.g., Hulse & Taylor 1973; Kim et al. 2015). Here, we consider their merger rate and contribution to the GW background within our framework of metallicity evolution. Since binary NSs are formed in both low- and high-metallicity environments, we write their formation rate as

\[
R_f^{\text{NS}}(z) = \zeta_{\text{NS}}^{\text{low}} \rho_*(z) + \zeta_{\text{NS}}^{\text{high}} (\rho_*(z) - \rho_{\text{low}}^{\text{NS}}(z, Z_{\text{crit}})) \tag{17},
\]

with the conversion coefficients \( \zeta_{\text{NS}}^{\text{low}} \) and \( \zeta_{\text{NS}}^{\text{high}} \). When we assume that some massive stars become not NSs, but heavy BHs in a low-metallicity environment, the conversion factor in a high-metallicity environment is larger than that in a low-metallicity environment, i.e., \( \zeta_{\text{NS}}^{\text{low}} \leq \zeta_{\text{NS}}^{\text{high}} \). Therefore, they are related by a parameter \( x \) (0 ≤ \( x \) ≤ 1) as

\[
\zeta_{\text{NS}}^{\text{low}} = x \zeta_{\text{NS}}^{\text{high}}, \tag{18}
\]

and we obtain

\[
R_f^{\text{NS}}(z) = \zeta_{\text{NS}}^{\text{low}} (\rho_*(z) + (x - 1) \rho_{\text{low}}^{\text{NS}}(z, Z_{\text{crit}})). \tag{19}
\]

If binary NSs are the main population rather than binary BHs even in a low-metallicity environment, the parameter \( x \) is close to 1. Note that the formation rate of binary NSs is proportional to \( \zeta_{\text{NS}}^{\text{high}} \), which is not investigated explicitly in this study.

The merger rate of binary NSs is given by

\[
R_m^{\text{NS}}(z_m) = \int_{t_{\text{min}}}^{t_{\text{max}}} R_f^{\text{NS}}(z_f) P(t_{df}) dtd_f, \tag{20}
\]

where the meanings of \( z_f, z_m \) and \( P(t_{df}) \) are the same as in Equation (10). Furthermore, the energy density spectrum of the GW background from binary NSs is given by

\[
\Omega_{\text{GW}}^{\text{NS}}(f) = \frac{f}{\rho_c} \int_0^{t_{\text{max}}} dz \left( \frac{R_m^{\text{NS}}(z)}{H_0(1+z)} \sqrt{\Omega_m(1+z)} + \Omega_\Lambda \right)
\times \frac{dE_{\text{NS}}(f')}{df'}, \tag{21}
\]

For the spectral energy density \( dE_{\text{NS}}^{\text{GW}}(f) / df \), we again assume the approximation of the inspiral phase with a chirp mass \( M_c^{\text{NS}} \), and we rewrite Equation (21) as

\[
\Omega_{\text{GW}}^{\text{NS}}(f) \approx F_{25}^{\text{NS}} \left( \frac{M_c^{\text{NS}}}{M_\odot} \right)^{5/3}
\times \left( \frac{R_m^{\text{NS}}(0)}{\text{Gpc}^{-3} \text{yr}^{-1}} \right)^{2/3} \left( \frac{f}{25 \text{ Hz}} \right)^{2/3}, \tag{22}
\]

with a factor \( F_{25}^{\text{NS}} \). To determine \( F_{25}^{\text{NS}} \), we need not only the critical metallicity \( Z_{\text{crit}} \) and the minimum delay time \( t_{\text{min}} \), but the parameter \( x \). Their dependences of \( F_{25}^{\text{NS}} \) for our standard model are shown in Figure 8. For the case with \( Z_{\text{crit}} \leq 0.5Z_\odot \) and 0.5 ≤ \( x \) ≤ 1, \( F_{25}^{\text{NS}} \) does not depend on \( Z_{\text{crit}} \) and \( x \), because the term associated with low-metallicity stars, \( (x - 1) \rho_{\text{low}}^{\text{NS}}(z, Z_{\text{crit}}) \), is small compared with the total CSFRD, \( \rho_*(z) \), in Equation (19). As in Equation (16), for \( Z_{\text{crit}} \leq 0.5Z_\odot \) and 0.5 ≤ \( x \) ≤ 1, the \( t_{\text{min}} \) dependence of \( F_{25}^{\text{BH}} \) is fitted by

\[
F_{25}^{\text{NS}} = c_0^{\text{NS}} + c_1^{\text{NS}} \log_{10}
\left( \frac{t_{\text{min}}}{50 \text{ Myr}} \right)
+ c_2^{\text{NS}} \left[ \log_{10}
\left( \frac{t_{\text{min}}}{50 \text{ Myr}} \right) \right]^2, \tag{23}
\]
with $c_0^{\text{NS}} = 2.5 \times 10^{-13}$, $c_1^{\text{NS}} = -4.5 \times 10^{-14}$ and $c_2^{\text{NS}} = -2.4 \times 10^{-14}$.

In Figure 9, we show the factor $F_{25}^{\text{NS}}$ for the alternative model. Also for this model, as $Z_{\text{crit}} \to 0$ and/or $x \to 1$, the value of $F_{25}^{\text{NS}}$ converges to the same limit as our standard model given in Equation (23). This is because the same model for the total CSFRD is adopted in both cases and the merger rate of binary NSs is mainly determined by the total CSFRD in this limit. Therefore, if the critical metallicity is sufficiently low and/or the parameter $x$ is close to 1, the factor $F_{25}^{\text{NS}}$ does not depend on the metallicity distribution of star formation.

4. DISCUSSION

In this section, we discuss the implications of this study for future GW astronomy. In the following, we only take into account the standard model for the CSFRD of low-metallicity stars described in Section 2.1.

In the previous sections, we used the approximation for the GW spectrum in the inspiral phase given in Equation (13). In actuality, the spectrum has a cutoff at the high-frequency end, which roughly corresponds to the frequency at the merger. The cutoff frequency is higher for lower-mass mergers. Here, to study the validity of this approximation, we utilize the spectrum

Figure 7. Same as Figure 6, but for the alternative model given in Section 2.2.

Figure 8. Factor $F_{25}^{\text{NS}}$ defined in Equation (22) as a function of parameter $x$ defined in Equation (18). The left, central, and right panels correspond to the models with the minimum delay time $t_{\text{min}} = 50$, 500, and 5 Gyr, respectively. The dotted–dashed, dashed, and dotted lines correspond to the critical metallicities $Z_{\text{crit}} = 0.1Z_{\odot}$, $0.5Z_{\odot}$, and $Z_{\odot}$, respectively.

Figure 9. Same as Figure 8, but for the alternative model given in Section 2.2. Thin horizontal lines represent the limit as $Z_{\text{crit}} \to 0$. 

\[ \text{Figure 7. Same as Figure 6, but for the alternative model given in Section 2.2.} \]

\[ \text{Figure 8. Factor } F_{25}^{\text{NS}} \text{ defined in Equation (22) as a function of parameter } x \text{ defined in Equation (18). The left, central, and right panels correspond to the models with the minimum delay time } t_{\text{min}} = 50, 500, \text{ and 5 Gyr, respectively. The dotted–dashed, dashed, and dotted lines correspond to the critical metallicities } Z_{\text{crit}} = 0.1Z_{\odot}, 0.5Z_{\odot}, \text{ and } Z_{\odot}, \text{ respectively.} \]

\[ \text{Figure 9. Same as Figure 8, but for the alternative model given in Section 2.2. Thin horizontal lines represent the limit as } Z_{\text{crit}} \to 0. \]
of binary BH merger proposed by Ajith et al. (2011) for comparison. For illustration, we adopt the critical metallicity $Z_{\text{crit}} = Z_{\odot}/\sqrt{5}$ from the heavy-BH formation model in Nakazato et al. (2013, 2015), while the choice of $Z_{\text{crit}}$ does not affect the result for the GW spectrum. Furthermore, we assume the distribution of binary chirp mass based on the results of the first observational run of the Advanced LIGO detectors (Abbott et al. 2016f). The event-based merger rates are evaluated to be $R_{\text{GW150914}} = 3.4 \text{ Gpc}^{-3} \text{yr}^{-1}$, $R_{\text{LVT151012}} = 9.4 \text{ Gpc}^{-3} \text{yr}^{-1}$, and $R_{\text{GW151226}} = 37 \text{ Gpc}^{-3} \text{yr}^{-1}$ for chirp masses of $M_{\text{chirp, GW150914}} = 28.1M_{\odot}$, $M_{\text{chirp, LVT151012}} = 15.1M_{\odot}$; and $M_{\text{chirp, GW151226}} = 8.9M_{\odot}$, respectively. Hereafter, fixing the ratio of these rates, we consider the dependence on the total merger rate at the local universe, $R_{\text{BH}}(0)$. For simplicity, we ignore the spins of BHs, or equivalently, the effective spin parameter is set to zero.

In Figure 1, we compare the GW background spectrum computed with the model proposed by Ajith et al. (2011) and
the approximation of Equation (15). Since the estimated range of the local merger rate is \( R_{\text{BH}}^m(0) = 9 - 240 \text{ Gpc}^{-3} \text{ yr}^{-1} \) (Abbott et al. 2016f) and the investigated range of the minimum delay time in this study is \( t_{\text{min}} = 0.50 \text{ Myr} - 5 \text{ Gyr} \), we show spectra in the cases with \( (R_{\text{BH}}^m(0), t_{\text{min}}) = (240 \text{ Gpc}^{-3} \text{ yr}^{-1}, 50 \text{ Myr}) \) for the maximum and \( (R_{\text{BH}}^m(0), t_{\text{min}}) = (9 \text{ Gpc}^{-3} \text{ yr}^{-1}, 5 \text{ Gyr}) \) for the minimum. For both models, we find that the difference in energy density of the GW background between the model from Ajith et al. (2011) and approximation (15) is at most 15% in the frequency range of \( f < 150 \text{ Hz} \). According to the statistical study of Callister et al. (2016), Advanced LIGO can hardly distinguish the spectral difference from a simple \( f^{2/3} \) power law for the GW background. As shown in Figure 1, compared to the case assuming that all binary BHs have the same masses as in GW150914 (e.g., fiducial model of Abbott et al. 2016c), the GW background spectrum has a lower energy density and additional power at high frequencies due to low-mass BHs. It is consistent with the result of Dvorkin et al. (2016), who calculated the mass distribution of binary BHs based on the initial mass function of the progenitor stars and the relation between the initial mass and BH mass. Incidentally, while the spectrum based on Ajith et al. (2011), shown in Figure 1, has a lower energy than the approximation of Equation (15) in this range, it can be either higher or lower depending on the BH spin. The expected sensitivity curves of the network of Advanced LIGO and VIRGO (Abbott et al. 2016c) are also shown in Figure 1. If the event rate of binary BH mergers is found to be as high as \( \sim 100 \text{ Gpc}^{-3} \text{ yr}^{-1} \) through the direct detection of the GW signal, the GW background may also be observed even for a case with a long delay time.

Binary NS mergers also contribute to the GW background. In Figure 10, we plot the total energy density of the GW background at \( f = 25 \text{ Hz} \), \( \Omega_{\text{GW}}(25 \text{ Hz}) = \Omega_{\text{GW}}^\text{BH}(25 \text{ Hz}) + \Omega_{\text{GW}}^\text{NS}(25 \text{ Hz}) \), on the \( R_{\text{BH}}^m(0) \) versus \( R_{\text{NS}}^m(0) \) plane for various values of the minimum delay time, \( t_{\text{min}} \). Using Equations (15) and (22), we adopt the mass distribution from the results of the first observational run of the Advanced LIGO detectors (Abbott et al. 2016f) for the chirp mass of BH binaries again and we set the chirp mass of NS binaries to \( M_{\text{NS}} = 1.2 M_\odot \), which corresponds to an equal-mass binary with masses \( 1.4 M_\odot + 1.4 M_\odot \). Since the energy density of the GW background is proportional to the chirp mass to the power 5/3, BH binaries may be the dominant sources of the GW background in spite of their lower merger rate. According to a recent theoretical estimation, the local merger rate of binary NSs is \( R_{\text{NS}}^m(0) = 52 - 162 \text{ Gpc}^{-3} \text{ yr}^{-1} \) and their detection rate for a network of advanced GW detectors is expected to be several events per year (Dominik et al. 2015). In contrast, the NS merger rate in our Galaxy has been estimated to be 21 Myr \(^{-1} \) from the observation of a double pulsar system (Kim et al. 2015), which corresponds to \( R_{\text{NS}}^m(0) = 250 \text{ Gpc}^{-3} \text{ yr}^{-1} \). Horotkezaka et al. (2015) evaluated the event rate of the \( r \)-process sources to be \( \lesssim 90 \text{ Myr}^{-1} \) in our Galaxy. If the sites of the \( r \)-process events are NS mergers, the local merger rate will be \( R_{\text{NS}}^m(0) \leq 1000 \text{ Gpc}^{-3} \text{ yr}^{-1} \).

Roughly speaking, the direct GW detection rate is proportional to the chirp mass to the power 5/2. We draw lines denoting \( (R_{\text{BH}}^m(0), R_{\text{NS}}^m(0)) = (5/2 R_{\text{BH}}^m(0), 5/2 R_{\text{NS}}^m(0)) \) in Figure 10. Then, below these lines, the event rate of binary BH merger is higher than that of binary NS merger. In Figure 10, we also plot the points where the ratios of contributions from binary NSs and binary BHs to the GW background are \( \Omega_{\text{GW}}^\text{NS}/\Omega_{\text{GW}}^\text{BH} = 10, 1, \) and 0.1. We can see the existence of a region where the binary NS merger has a lower event rate for the direct GW detection, but a larger contribution to the GW background than the binary BH merger.

Anyway, GW astronomy will enable us to verify the consistency between the local merger rate and the energy density of the GW background in the near future. Nevertheless, there are other possible sources of the GW background. There could be binary systems of heavy BHs and NSs (Dominik et al. 2015; Belczynski et al. 2016), which have not been observed yet. It is possible to apply our study to NS-BH binaries; Equations (15) and (16) hold for NS-BH binaries, because their formation rate is proportional to that of heavy BHs, i.e., low-metallicity stars. The local merger rates of NS-NS, BH-BH, and NS-BH binaries can be measured individually by direct GW detection, and their integrated spectrum can be observed as the GW background. In contrast, the metallicity dependence of the initial mass function is beyond the scope of this study. In particular, the first stars (Population III stars) are thought to have a top-heavy initial mass function due to the absence of metals, while whether their contribution to the GW background is efficient (Inayoshi et al. 2016) or negligible (Dvorkin et al. 2016; Hartwig et al. 2016) is still an open question. We hope that this study will provide a first step toward understanding the GW background on the basis of the metallicity evolution of galaxies.

This work was partially supported by Grants-in-Aids for the Scientific Research (No. 24105008, No. 26104006 and No. 26870615) from MEXT in Japan and a University Research Support Grant from the National Astronomical Observatory of Japan (NAOJ).

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