Determination of net premium rates on Bonus-Malus system based on frequency and severity distribution

A S Pratama, S Nurrohmah and M Novita
Department of Mathematics, Faculty of Mathematics and Natural Sciences (FMIPA), Universitas Indonesia, Kampus UI Depok, Depok 16424, Indonesia

Corresponding author’s e-mail: snurrohmah@sci.ui.ac.id

Abstract. Vehicle insurance companies in many countries use the Bonus-Malus System to determine the policyholder's net premium. The determination of net premiums on the Bonus-Malus System is based solely on the frequency of claims and ignores the severity of claims. This is unfair to policyholders who have small claims. To overcome this problem, the net premium determination method in Bonus-Malus System was developed taking into account both the frequency and severity of claims. Frequency and severity are assumed to be independent. In determining the net premium, a posterior distribution of parameters of the frequency and severity distribution is required. In the case of frequency and severity independent, the determination of the posterior distribution for frequency and severity is performed separately. This thesis discusses the determination of net premium based on frequency distribution and severity distribution for frequency and severity independent.

Keywords: Bonus-Malus System, Frequency distribution, Net premium, Severity distribution.

1. Introduction
The number of vehicles in Indonesia continues to increase. According to data published by the Badan Pusat Statistik (BPS), since 1988 to 2014 the number of vehicles in Indonesia has always increased every year. The increased number of vehicles is often the cause of increased accidents on the highway so drivers have to spend more to repair their vehicles. Therefore, public awareness of the importance of vehicle insurance continues to increase so that more and more people insure their vehicles.

Insurance companies offer various systems in the determination of net premiums to potential policyholders [1]. One such system is the Bonus-Malus System. The Bonus-Malus System punishes policyholders who perform one or more claims with a premium increase (malus) and reward the policyholder who does not claim with a premium decrease (bonus) [2]. In this way, the Bonus-Malus System encourages policyholders to be more careful in driving to reduce the frequency of claims.

2. Methods
The Bonus-Malus System has been implemented in many countries. Table 1 is the Bonus-Malus System used in Brazil [3]. In determining the net premium, all existing Bonus-Malus Systems only consider the number of claims [4]. A policyholder who claim with small benefit will be treated the same as a policyholder who experienced an accident with big benefits. It's certainly not fair for policyholders with little benefit.

In this paper, the authors will determine the net premium of the Bonus-Malus System, which not only takes into account the distribution of the number of claims but also the distribution of claim size. The distribution of the number of claims is also called the frequency distribution and the distribution of claim size is also called severity distribution. Frequency and severity can be independent or dependent. The Bonus-Malus System that takes into account both the frequency distribution and severity distribution is expected to give fairness to every policyholder.
Table 1. Brazil Bonus-Malus System

| Class | Premium (Percentage) | Frequency of Claim |
|-------|----------------------|--------------------|
|       |                      | 0  | 1  | 2  | 3  | 4  | 5  | ≥ 6 |
| 7     | 100                  | 6  | 7  | 7  | 7  | 7  | 7  | 7   |
| 6     | 90                   | 5  | 7  | 7  | 7  | 7  | 7  | 7   |
| 5     | 85                   | 4  | 6  | 7  | 7  | 7  | 7  | 7   |
| 4     | 80                   | 3  | 5  | 6  | 7  | 7  | 7  | 7   |
| 3     | 75                   | 2  | 4  | 5  | 6  | 7  | 7  | 7   |
| 2     | 70                   | 1  | 3  | 4  | 5  | 6  | 7  | 7   |
| 1     | 65                   | 1  | 2  | 3  | 4  | 5  | 6  | 7   |

3. Results and discussion

In the independent case, the determination of posterior distribution for claim frequency and claim severity is done separately. Furthermore, the authors look for posterior distribution of mean of claim frequency and posterior distribution of mean of claim severity. Net Premium is derived from the expected posterior distribution of the mean of claim frequency and the expected posterior distribution of the mean of claim severity.

3.1. Posterior distribution of mean of claim frequency

Let a random variable \( \lambda \) represents the mean of claim frequency with \( \lambda > 0 \). The mean of claim frequency among policyholder is allowed to vary so that \( \lambda \) has a certain distribution.

Suppose a random variable \( N|\lambda \) represents the claim frequency depending on the value of \( \lambda \) in which its possible values are a data count. The Poisson distribution is used to random, independent and rare events within a certain time interval or in a particular region such as the policyholder’s claim within a given time period so that the Poisson distribution is used as the distribution of \( N|\lambda \). The pdf of \( N|\lambda \) is

\[
p(k|\lambda) = \frac{e^{-\lambda k} k^k}{k!}
\]

(1)

where \( k \) is the value of the random variable \( N \) [5].

\( \lambda \) is assumed to be gamma distributed because the gamma distribution is the prior natural conjugate of the Poisson distribution. The pdf of \( \lambda \) [5] is

\[
u(\lambda) = \frac{\lambda^{\alpha-1}e^{-\lambda}}{\Gamma(\alpha)}
\]

(2)

the posterior distribution of \( \lambda \) will be sought. Suppose the total frequency of claims made by a policyholder during the age of the policy \( t \) is expressed by a random variable \( K \), where \( K = \sum_{i=1}^{t} k_i \) and \( k_i \) are random variables that state the claim frequency made by the policyholder in the \( i^{th} \) year, \( i = 1, 2, \ldots, t \).

By using Bayes' theorem [6], the posterior pdf of \( \lambda \) is obtained as follows

\[
u(\lambda|k_1, k_2, \ldots, k_t) = \frac{1}{\Gamma(\alpha+K)(t+\tau)^{-(\alpha+K)}} \lambda^{\alpha+K-1} e^{-\lambda (t+\tau)}
\]

(3)

That is pdf of gamma distribution with parameter \( \alpha + K \) and \( \frac{1}{t+\tau} \) so that posterior expectation of \( \lambda \) is

\[
E[\lambda|k_1, k_2, \ldots, k_t] = \frac{\alpha+K}{t+\tau}
\]

(4)

The values of \( \alpha \) and \( \tau \) are estimated from the claim frequency data by the maximum likelihood method by first obtaining an unconditional pdf of \( N \) [7]. By mixing between \( N|\lambda \) and \( \lambda \), obtained pdf of \( N \) which is a pdf of negative binomial distribution [8] with parameters \( \alpha \) and \( \tau \) [5].
3.2. Posterior distribution of mean of claim severity

Suppose a random variable $\theta$ denotes the mean of claim severity with $\theta > 0$. Each policyholder is allowed to have mean of claim severity that vary so that $\theta$ has a certain distribution.

Let $X|\theta$ be a random variable denotes the claim severity depending on the value of $\theta$ in which the value space of $X|\theta$ is continuous data. The choice of distribution for claim severity was also selected from distributions with continuous random variables and has a positive value space. One of the distributions that satisfies both requirements is exponential distribution. $X|\theta$ which is exponentially distributed with parameter $\theta$ has pdf

$$f(x|\theta) = \frac{1}{\theta} e^{-\frac{x}{\theta}}, x > 0, \theta > 0$$

(5)

the prior natural conjugate distribution of the exponential distribution is inverse gamma distribution so that $\theta$ follows the inverse gamma distribution. The pdf of $\theta$ is

$$g(\theta) = \frac{1}{m} e^{-\frac{m}{\theta}} \Gamma(s)$$

(6)

does the next the posterior distribution of the mean of claim severity will be determined. Suppose a person becomes a policyholder for $t$ years. The claim frequency made in the $i^{th}$ year is expressed by a random variable $k_i$ where $i = 1, 2, \ldots, t$. The random variable $K$ where $K = \sum_{i=1}^t k_i$ represents the total claim frequency and $x_k$ is a random variable denotes the claim severity for the $k^{th}$ claim.

The policyholder's claim severity is written in the form $x_1, x_2, ..., x_K$ and the total of policyholder's claim severity during $t$ year(s) the policy issued is $\sum_{k=1}^K x_k$. The posterior pdf of $\theta$ is obtained by using Bayes’ theorem:

$$g(\theta|x_1, x_2, ..., x_K) = \frac{1}{(m+\sum_{k=1}^K x_k)^s} e^{-\frac{m+\sum_{k=1}^K x_k}{\theta}} \frac{\theta^{s+K}}{(m+\sum_{k=1}^K x_k)^{s+K+1}} \Gamma(s+K)$$

(7)

that is an inverse gamma pdf with parameters $s + K$ and $m + \sum_{k=1}^K x_k$. The posterior expectation of $\theta$ is obtained as follows

$$E[\theta|x_1, x_2, ..., x_K] = \frac{m + \sum_{k=1}^K x_k}{s + K - 1}$$

(8)

the values of $s$ and $m$ are estimated from the claims severity data using the maximum likelihood method. To use the maximum likelihood method, we need an unconditional pdf of $X$. By using mixing, the unconditional pdf of $X$ is obtained as follows

$$h(x) = \frac{sm^s}{(x+m)^{s+1}}$$

(9)

that is a Pareto pdf with parameters $s$ and $m$. The estimated values of $s$ and $m$ are determined by the maximum likelihood method.

3.3. Determination of net premium

Net premiums with frequency and severity independent are analogous to the relationship of the net premium principle [9] and the collective risk model such that net premiums are obtained as follows

$$P = \frac{\alpha + K}{t + \tau} \frac{m + \sum_{k=1}^K x_k}{s + K - 1}$$

(10)

3.4. Properties of net premium

The net premiums on the Bonus-Malus System with frequency and severity independent obtained have the following properties:

- Fairly, policyholders pay a net premium compatible with the frequency and severity of claims.
• In the first year, each policyholder pays the same net premium rate of $P = \frac{\alpha}{\tau} \frac{m}{s-1}$.
• The more the frequency of claims and the larger the severity of claims, the higher the net premium.
• The net premium always decreases when no claims are made.
• The phenomenon of bonus hunger will decrease.

4. Conclusions
In the case of independent, net premiums are proportional to the total claim frequency and total claims severity and are inversely proportional to the age of the policy. This means that the more frequent the claim frequency and the larger the severity of the claim, the higher the net premium while the larger the policy age, the smaller the net premium. Policies that have a large claim severity are charged net premiums that are more expensive than policies that have a small claim severity. This shows that Net earned premiums are fairer than the Net premiums on the classic Bonus-Malus System.

References
[1] Lemorenty M 2013 Program aplikasi dengan menggunakan Windows Visual Basic dalam menentukan premi pada asuransi jiwa seumur hidup (whole life) dan asuransi jiwa berjangka (term life) (Bandung: Jurusan Pendidikan Matematika, Universitas Pendidikan Indonesia)
[2] Tzougas G, Vrontos S D and Frangos N E 2014 ASTIN Bull. 44 417–44
[3] Kusmiari R 2009 Elasticity of A Bonus-Malus System (Depok: Universitas Indonesia) Undergraduate Thesis
[4] Kaas R, Goovaerts M, Dhaene J and Denuit M 2008 Modern Actuarial Risk Theory (Berlin: Springer-Verlag)
[5] Klugman S A, Panjer H H and Willmot G E 2012 Loss Models: From Data to Decisions 4th ed (Hoboken: John Wiley & Sons)
[6] Gelman A, Carlin J B, Stern H S, Dunson D B, Vehtari A and Rubin D B 2009 Bayesian Data Analysis 3rd ed (Boca Raton: CRC Press)
[7] Hogg R V, McKean J W and Craig A T 2012 Introduction to Mathematical Statistics 7th ed (Boston: Pearson Education)
[8] Frangos N E and Vrontos SD 2001 ASTIN Bulletin 31 1–22
[9] Dickson D C M, Hardy M R and Waters H R 2009 Actuarial Mathematics for Life Contingent Risks (New York: Cambridge University Press)