The possible $B\pi$ molecular state and its radiative decay

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Abstract

Recently, several exotic bosons have been confirmed as multi-quark states, but there are violent disputes about their inner structures, namely if they are molecular states or tetraquarks, or even mixtures of the two structures. It would be interesting to experimentally search for non-strange four-quark states with open charm or bottom which are lighter than $\Lambda_c$ or $\Lambda_b$. Reasonable arguments indicate that they are good candidates of pure molecular states $D\pi$ or $B\pi$ because pions are the lightest boson. Both $B\pi$ and $D\pi$ bound states do not decay via strong interaction. The $B\pi$ molecule may decay into $B^*$ by radiating a photon, whereas $D\pi$ molecule can only decay via weak interaction. In this paper we explore the mass spectra of $B\pi$ molecular states by solving the corresponding B-S equation. Then the rate of radiative decay $|\frac{3}{2}, \frac{1}{2}\rangle \rightarrow B^*\gamma$ is calculated and our numerical results indicate that the processes can be measured by the future experiment. We also briefly discuss the $D\pi$ case, due to the constraint of the final state phase space, it can only decay via weak interaction.

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I. INTRODUCTION

Many charmonium-like or bottomonium-like resonances, such as $X(3872)$, $X(3940)$, $Y(3940)$, $Z(4430)$, $Y(4260)$, $Z_c(4020)$, $Z_c(3900)$, $Z_b(10610)$ and $Z_b(10650)$ have been experimentally observed. The data show that there is no room in the regular representations of $O(3) \otimes SU_f(3) \otimes SU_s(2)$ to accommodate those newly observed resonances, especially some of them are charged, thus it is suggested that those exotic bosons are in multi-quark states. Since their masses are close to that of charmonia or bottomonia, thus those states should have hidden charm or bottom components. Whereas a newly observed exotic state $X(5568)$ measured at the $B^0_s\pi^\pm$ invariant mass spectrum is believed to possess four differently flavored quarks (antiquarks). If the resonance is eventually confirmed it must be a four-quark state with open bottom.

Even though so many exotic resonances are confirmed as multi-quark bosons, there is an acute dispute about their inner structure. By contrast to the regular quark-anti-quark structure, the system containing two quarks and two anti-quarks may have different combination patterns: it may reside in a molecular state, a tetra-quark or a mixing of the two structures. An intuitive opinion suggests that a narrow-width (i.e. several tens of MeV) exotic particle might be a molecular state, whereas a wide-width (i.e. several hundreds MeV) one should be a tetraquark. However definitely, this naive consideration cannot be a criterion for judging the exotic structure by merely its width, at most it provides a hint to help confirming the inner structure. As a matter of fact, so far no any exotic state has ever been firmly determined as a molecular state. Actually, if an exotic boson is confirmed to be in a molecular state, a careful study on it would be very helpful for understanding the dynamics which results in the different inner structures. Because we lack available data at present, let us theoretically construct such states which should be ideal molecular systems. We would argue that bound states of $B\pi$ and $D\pi$ should be ideal molecular systems.

The authors of Ref. argued that the newly observed $X(5568)$ contains constituents of $s ub \bar{d}$ which has an additional valence quark than $\Xi_b$ with $us b$ contents, has mass of 5619.5 MeV, it is lighter than the mass of $\Xi_b$, so that $X(5568)$ seems not to be a tetraquark of $s ub \bar{d}$, if it indeed exists. The other research works support that such tetraquark with constituents of $s ub \bar{d}$ should be heavier than $\Xi_b$.

Following this argument we would be tempted to suppose that if a non-strange four-quark state with open bottom or charm exists and is lighter than $\Lambda_b$ or $\Lambda_c$, the only possible choice is that they are pure molecular states of $B\pi$ or $D\pi$. The reason is that pion is the lightest boson and especially lighter than a valence quark. Even though the reason why pions are so light is still a not fully understood enigma yet, the fact that it is lighter than valence quarks is surely confirmed. More concretely, since the mass of $\pi$ is lighter than any constituent quark generally the molecular state of $B\pi$ or $D\pi$ should be lighter than the tetraquark state with the same quark-structure and as well as the corresponding baryons such as $\Lambda_b$ or $\Lambda_c$. Namely, we are going to experimentally search for exotic four-quark states which are lighter than $\Lambda_b$ or $\Lambda_c$ because there is a strong evidence that they are hadronic molecules. Moreover, if the bound state of $B\pi$ ($D\pi$) is experimentally confirmed, we will have all reasons to believe
that other molecular states indeed exist in nature, that is why exploration of $B\pi$ and $D\pi$ bound states is crucially important.

Obviously, molecules $B\pi$ or $D\pi$ do not decay via strong interactions, therefore, one expects to observe them only at radiative and/or weak processes. It would definitely make detection more difficult, but not impossible. Indeed the bound state $B\pi$ may decay into $B^*\gamma$ with a larger rate than weak decay modes. Thus we will more focus on the $B\pi$ bound state and its radiative decay in this paper.

In the quantum field theory at the lowest order two particles interact with each others by exchanging certain particles. For our case, the molecular state consists of two color-singlet mesons, we can derive the effective hamiltonian which corresponds to exchanging scalar (such as $\sigma$) or vector (such as $\rho$) etc. mesons between $B$ and $\pi$ (or $D$ and $\pi$).

In Ref. [27] the authors employed the Bethe-Salpeter equation to study the $K\bar{K}$ or $BK$ molecular state and their decays. In this work we follow their approach to study the molecular state of $B\pi$. Here we only concern the ground states i.e. the orbital angular momentum between the two constituent mesons is zero ($l = 0$) so the $J^P$ of the molecular state is $0^+$. Since the isospins of $B$ and $\pi$ are $1/2$ and $1$, the isospin state of $B\pi$ can reside in either $3/2$ or $1/2$ states. Different isospin states have different effective vertices for the strong-interaction which determine if the bound states can be formed as a physical object. We will solve the B-S equation first to explore the possibility of forming the bound state and obtain the corresponding B-S wave function. Then with the wave function we estimate its radiative decay rate in the same framework.

After this introduction we will present the B-S equations for the $0^+$ molecular state and derive the formula for its radiative decay rate. Then in section III we present our numerical results along while explicitly displaying all input parameters. Section IV is devoted to a brief summary. As we indicated above, in this work we concentrate on the case of $B\pi$ molecular states, then in the last section, we will briefly discuss the $D\pi$ case.
II. THE BOUND STATES OF $B\pi$ AND THEIR RADIATIVE DECAY IN THE BETHE-SALPETER FRAMEWORK

A. the molecular state of $B\pi$

Since the isospins of $B$ and $\pi$ are $1/2$ and $1$ the possible bound states of $B\pi$ should be in two isospin assignments i.e. $|I, I_3\rangle$ are $|\frac{1}{2}, \pm\frac{1}{2}\rangle$, $|\frac{3}{2}, \pm\frac{1}{2}\rangle$ and $|\frac{3}{2}, \pm\frac{3}{2}\rangle$. Let us work on the isospin states

$$|\frac{1}{2}, \frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|B^0\pi^+\rangle - \sqrt{\frac{1}{3}}|B^+\pi^0\rangle,$$

$$|\frac{3}{2}, \frac{1}{2}\rangle = \sqrt{\frac{1}{3}}|B^0\pi^+\rangle + \sqrt{\frac{2}{3}}|B^+\pi^0\rangle,$$

and

$$|\frac{3}{2}, \frac{3}{2}\rangle = |B^+\pi^+\rangle.$$  

While the states $|\frac{1}{2}, -\frac{1}{2}\rangle$, $|\frac{3}{2}, -\frac{1}{2}\rangle$ and $|\frac{3}{2}, -\frac{3}{2}\rangle$ are just the charge conjugate states of $|\frac{1}{2}, \frac{1}{2}\rangle$, $|\frac{3}{2}, \frac{1}{2}\rangle$ and $|\frac{3}{2}, \frac{3}{2}\rangle$, therefore their properties are the same.

B. The Bethe-Salpeter (B-S) equation for $0^+$ molecular state

Two mesons may form a bound state by exchanging appropriate mesons, and the scenario is depicted in Fig.1. The relative and total momenta of the bound state in the equations are defined as

$$p = \eta_2 p_1 - \eta_1 p_2, \quad p' = \eta_2 p'_1 - \eta_1 p'_2, \quad P = p_1 + p_2 = p'_1 + p'_2,$$

where $p$ and $p'$ are the relative momenta before and after the effective vertices, $p_1$ ($p'_1$) and $p_2$ ($p'_2$) are those momenta of the constituents before and after the effective vertices, $P$ is the total momentum of the bound state, $\eta_i = m_i/(m_1 + m_2)$ and $m_i$ ($i = 1, 2$) is the mass of the $i$-th constituent meson.

The corresponding B-S equation was deduced in Ref.\cite{28} as

$$\frac{E^2 - (E_1 + E_2)^2}{(E_1 + E_2)/E_1 E_2} \tilde{\chi}_p(p) = \frac{i}{2} \int \frac{d^3p'}{(2\pi)^3} K(p, p') \tilde{\chi}_p(p') F(p - p')^2,$$

where $E$ is the total energy of the bound state, $E_i = \sqrt{p^2 + m_i^2}$ and $\tilde{\chi}_p(p)$ is the B-S wave function in the three-momentum space. Therefore, the key point is to determine the kernel function $K(p, p')$.

Since the constituent mesons are not point particles, a form factor at each effective vertex should be introduced to reflect the finite-size effects of these hadrons. The form factor is assumed to be in the form:

$$F(k) = \frac{2\Lambda^2 - M^2_k}{2\Lambda^2 + k^2}, \quad k = p - p'.$$  

(3)
where $\Lambda$ is a cutoff parameter and usually fixed by fitting data. For exchanging a light vector ($\rho$ or $\omega$) between the mesons as shown in Fig. (a), the kernel is

$$K(p, p') = iC_{1,1}g_{BBV}g_{\pi\pi} \frac{(p + p')^2 + 4\eta_1\eta_2E^2 + (p^2 - p'^2)^2/M_0^2}{(p - p')^2 + M_0^2}.$$  \hspace{1cm} (4)

The Feynman diagram for exchanging $\sigma(f_0(500))$ is the same as in Fig. (a), the kernel is

$$K(p, p') = iC_{1,1}4m_Bm_\sigma g_{\pi\pi} \frac{1}{(p - p')^2 + M_\sigma^2}.$$ \hspace{1cm} (5)

While for exchanging $B^*$ the kernel (shown in Fig. (b)) is

$$K(p, p') = -iC_{1,1}g_{BBB}g_{B\pi\pi} \frac{(p - p')^2 + E^2 + \eta_1E_2 - (p^2 - p'^2)}{(p + p')^2 + M_{B^*}^2} = \frac{(p^2 - p'^2)}{(p + p')^2 - \eta_1\eta_2E^2}.$$ \hspace{1cm} (6)

Since the function $\tilde{\chi}(p)$ only depends on the norm of the three-momentum we may first integrate over the azimuthal angle in Eq. (2)

$$\frac{i}{2} \int \frac{d^3p'}{(2\pi)^3} K(p, p')F(p - p')^2,$$

to obtain a new form $U(|p|, |p'|)$ corresponding to Eq. (2), and it can be found in Ref. [27]. Then the B-S equation turns into a simplified one-dimensional integral equation

$$\tilde{\chi}(|p|) = \frac{(E_1 + E_2)/E_1}{E_2 - (E_1 + E_2)^2} \int d|p'| U(|p|, |p'|)\tilde{\chi}(|p'|).$$ \hspace{1cm} (7)

In terms of the approach given in Ref. [27, 28] the isospin factor can be obtained. For the $B^0\pi^+$ molecule, the corresponding isospin factor $C_{1,1}$ appearing in Eqs. (4) and (5) takes different values as $C_{1,1} = 1 - \sqrt{2}$, 1, 1 and 2 $- \sqrt{2}$ whereas $C_{1,1} = 1 + 2\sqrt{2}$, 1, 1 and 2 $+ \sqrt{2}$ corresponding to respectively exchanging $\rho$, $\omega$, $\sigma$ and $B^*$. Whereas for the $B^+\pi^0$ molecule, the isospin factor changes as $C_{1,1}$ being 1 $- \sqrt{2}$, 1, 1 and $- \sqrt{2}$; $C_{1,1}$ are $1 - \sqrt{2}$, 1, 1 and $2 + \sqrt{2}$, instead. Since the values of $C_1$ are different for $B^0\pi^+$ and $B^+\pi^0$ we will solve their B-S equations respectively. For the $B^+\pi^+$ system the isospin factor $C_{1,1}$ would take 1, 1, 2 and 1 corresponding to exchanging three different vector mesons $\rho$, $\omega$ and $B^*$ and $\sigma$.

In order to employ the wave function one first needs to normalize it. The normalization condition is

$$-\frac{1}{\pi^2} \int \frac{d^3p}{(2\pi)^3} \chi_p(p)^2 R - \frac{1}{4\pi^2} \int \frac{d^3p d^3p'}{(2\pi)^6} \chi_p(p)\chi_p(p')F(p - p') \frac{\partial K(p, p')}{\partial E} = 1.$$  \hspace{1cm} (8)

where

$$R = \frac{-E[-2E^2(E_1^2 - E_2^2)(E_1\eta_1 - E_2\eta_2) + E^4(E_1\eta_1 + E_2\eta_2) + (E_1^2 - E_2^2)(E_1^3\eta_1 + 3E_2E_1\eta_1 - 3E_1E_2\eta_2 - E_2^3\eta_2)]}{2E_1E_2[E^4 + (E_1^2 - E_2^2)^2 - 2E^2(E_1^2 + E_2^2)]^2}.$$ \hspace{1cm} (9)
C. Estimating the decay rate of $B\pi$ molecule to $B^* + \gamma$

It is crucial to ask a question, that as $B$ and $\pi$ constitute a bound state, i.e. a hadronic molecule, how can we identify the molecular four-quark system? As usual, to confirm the inner structure, one needs to measure its spectrum via its production and decay patterns. $B$ and $\pi$ constitute a ground state hadronic molecule, which cannot decay via strong interaction. Actually, the overwhelming decay portals of these bound states are induced by weak interaction, whose rates are small, so experimental detections are rather difficult, because the complex background, especially for the hadron colliders. Fortunately some molecular states can decay by radiating a photon, obviously such process is easier to be observed and our $B\pi$ molecule is just the case.

The Feynman diagrams for radiative decays of the $B\pi$ molecule are shown in Fig. 2. Fig. 2 (a) corresponds to exchanging $\rho$, $\omega$, or $\pi$ while Fig. 2 (b) is for exchanging $B$ or $B^*$. Following Ref. [27, 28, 30] the transition matrix elements by exchanging $\rho(\omega)$, $\pi$, $B$, and $B^*$ are

\[
M_{\rho(\omega)} = i\sqrt{2E_m}g_{BB^\ast\rho}g_{\rho\pi\pi}C_{1,I_3} \int \frac{d^4p}{(2\pi)^4} q_0p_1\epsilon_1\epsilon_{a\beta\mu\nu}q_2p_2\epsilon_2\frac{g_{\mu\nu} - q_0q_\nu/M_B^2}{M_B^2 - q^2} F(||q||^2) \chi(p)
\]

\[
M_{\pi} = i\sqrt{2E_m}g_{BB^\ast\pi}g_{\pi\pi\pi}C_{1,I_3} \int \frac{d^4p}{(2\pi)^4} q_0q_3\epsilon_1\epsilon_{a\beta\mu\nu}q_2p_2\epsilon_2\frac{g_{\mu\nu} - q_0q_\nu/M_B^2}{M_B^2 - q^2} F(||q||^2) \chi(p) \tag{11}
\]

\[
M_{B^*} = -i\sqrt{2E_m}g_{BB^\ast\pi}g_{B^*B^\ast\pi}C_{1,I_3} \int \frac{d^4p}{(2\pi)^4} q_0q_3\epsilon_1\epsilon_{a\beta\mu\nu}q_2p_2\epsilon_2\frac{g_{\mu\nu} - q_0q_\nu/M_B^2}{M_B^2 - q^2} F(||q||^2) \chi(p) \tag{12}
\]

\[
M_B = i\sqrt{2E_m}g_{BBB^\ast}g_{B^*B^\ast\pi}C_{1,I_3} \int \frac{d^4p}{(2\pi)^4} q_0q_3\epsilon_1\epsilon_{a\beta\mu\nu}q_2p_2\epsilon_2\frac{g_{\mu\nu} - q_0q_\nu/M_B^2}{M_B^2 - q^2} F(||q||^2) \chi(p) \tag{13}
\]

where $\epsilon_1$ and $\epsilon_2$ are the polarizations of $B^*$ and photon respectively. For $B^0\pi^+$ the isospin factor $C_{\frac{1}{2}, \frac{1}{2}}$ takes $\frac{2}{\sqrt{3}}$, 0, $\frac{1}{\sqrt{3}}$, $\frac{1}{\sqrt{3}}$ corresponding to exchanging $\rho$, $\omega$, $\pi$, $B$ and $B^*$, whereas for $B^+\pi^0$ the isospin factor $C_{\frac{1}{2}, \frac{1}{2}}$ are respectively $\sqrt{\frac{2}{3}}$, $\sqrt{\frac{2}{3}}$, $\sqrt{\frac{2}{3}}$, $2\sqrt{\frac{2}{3}}$, $2\sqrt{\frac{2}{3}}$. To simplify our calculation we set $p_0 = 0$ in the kinetic part of the integrand in Eqs.(9), for example, in Eq.(10), $p_0 = 0$ applies merely to $q_0p_1\epsilon_1\epsilon_{a\beta\mu\nu}(p_2 - q)q_2p_2\epsilon_2\frac{g_{\mu\nu} - q_0q_\nu/M_B^2}{M_B^2 - q^2}$,
then the integrand turns into

\[ M_\rho = i\sqrt{2E}g_1g_2C^\prime_{I,I3} \int \frac{d^3p}{(2\pi)^3} q_\mu p^\prime_1 \epsilon^{\alpha_1 \epsilon_1 \beta_1 \gamma_1} (p_2 - q)_\sigma p^\prime_2 \epsilon^{\alpha_2 \epsilon_2 \beta_2 \gamma_2} g_{\mu \nu} - q_\mu q_\nu/M^2_\rho \frac{M^2_\rho - q^2}{M^2_\rho - q^2} F'(|q|)^2 \chi(p) \]  

where the definition \( \chi(p) = \int dp^0 \chi_p(p) \) is used. Namely, in the new expression, the argument of \( \chi_p(p) \) is a three-momentum \( p \) instead of the four momentum \( p \). It is noted that this simplification is similar to the instantaneous approximation for solving the B-S equation which is usually adopted.

Generally we can define two form factors for the transition

\[ M = F_1 \epsilon_1 \cdot \epsilon_2 + F_2 \epsilon_1 \cdot P \epsilon_2 \cdot P \]  

and \( F_1 \) and \( F_2 \) can be extracted from Eq. (14) and calculated numerically.

### III. NUMERICAL RESULTS

To solve the B-S equation and numerically calculate the radiative decay rate some input parameters are needed. The mass of \( B, B^*, \rho, \omega, \pi \) are taken from the databook [29].

On the other hand we need to determine the relevant coupling constants appearing at the effective vertices. By calculating the transition \( \rho \rightarrow \pi \pi \) and comparing the result with the data [29] one can fix the coupling \( g_{\rho \pi \pi} = 5.97 \). Similarly we fix \( g_{\omega \pi \pi} = 0.175, g_{\rho \pi \gamma} = 0.417 \text{GeV}^{-1}, g_{\omega \pi \gamma} = 1.215 \text{GeV}^{-1} \). However for determining the coupling constants involving \( B^{(*)} \) mesons there are not available data, so we fix \( g_{D^*D\pi} = 8.05 \) and \( g_{D^*D\gamma} = 0.706 \text{GeV}^{-1} \) by using relations \( g_{B^*B\pi} = g_{B^*B\pi} = g_{D^*D\pi} \) and \( g_{B^*B\gamma} = g_{B^*B\gamma} = g_{D^*D\gamma} \) which are reasonable in the heavy quark limit. \( g_{BB\rho} = g_{B^*B\rho} = 3 \) is taken from Ref. [30]. The \( g_{\rho} = 0.76 \) was fixed in Ref. [31]. If one sets \( m_\sigma = 500 \text{ MeV} \) and \( \Gamma_\sigma = 550 \text{ MeV} \), \( g_{\sigma \pi \pi} = 4.09 \) is obtained. \( \Lambda \) is the cutoff parameter which will be used while searching for a solution of the B-S equation. In Ref. [32] the value of \( \Lambda \) is suggested to be 0.88 GeV to 1.1 Gev. In this work letting \( \Lambda \) span in the range from 0.8 to 1.2 GeV, we solve the B-S equation.

#### TABLE I: The \( \Lambda \) for different bound energies of \( B\pi(I = \frac{3}{2}, I_z = \frac{1}{2}) \).

| \( \Delta E \) (MeV) | -10 | -20 | -30 | -40 | -50 | -60 | -70 | -80 |
|---------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| \( \Lambda(B^0\pi^+) \) | 0.991 | 1.024 | 1.053 | 1.080 | 1.102 | 1.122 | 1.140 | 1.155 |
| \( \Lambda(B^+\pi^0) \) | 0.947 | 0.979 | 1.006 | 1.030 | 1.051 | 1.068 | 1.084 | 1.096 |

#### TABLE II: The \( \Lambda \) for different bound energies of \( B^+\pi^+(I = \frac{3}{2}, I_z = \frac{3}{2}) \).

| \( \Delta E \) (MeV) | -10 | -20 | -30 | -40 | -50 | -60 | -70 | -80 |
|---------------------|-----|-----|-----|-----|-----|-----|-----|-----|
| \( \Lambda \) | 0.878 | 0.906 | 0.930 | 0.950 | 0.967 | 0.981 | 0.993 | 1.003 |
FIG. 3: the B-S wave function of the molecular state of $B^0π^+$ ($ΔE = 40$ MeV)

We now solve the B-S equation. $|p⟩(|p′⟩)$ takes $n$ discrete values which are arranged in order from small to large and the gap between two adjacent values is $Δp$, then $χ(|p⟩)$ can constitute a column matrix and the coefficients on the right side of Eq. (7) make an $n × n$ matrix $M$. Our strategy is following. The binding energy is $ΔE = m_1 + m_2 - E$, thus we write up the determinant of $M(ΔE, Λ) - I$ ($I$ is a unit matrix) where $M(ΔE, Λ)$ is a matrix function of the binding energy $ΔE$ and parameter $Λ$. Then setting equation $|M(ΔE, Λ) - I| = 0$ which is equivalent to the secular equation in regular quantum mechanics, by varying $ΔE$ we obtain a series of solutions for $Λ$. We would check whether the obtained values of $Λ$ fall within the range of 0.8 to 1.2 GeV which is priori set. If the answer is yes, we would conclude that the bound state should exist. Moreover, with the obtained $ΔE$ and $Λ$, the B-S wave function is achieved.

When we try to solve the B-S equation for the $Bπ$ system in isospin $|1/2, 1/2⟩$ state, we find that by setting different binding energies one cannot achieve a value of $Λ$ which falls in the supposed range 0.8-1.2 GeV, so that we would determine that a $Bπ$ bound state of isospin $(1/2, 1/2)$ does not exist in Nature. By contrary, the isospin $|3/2, 1/2⟩$ $Bπ$ bound state does exist. According to the aforementioned $C_{I,I_3}$ values one can understand that the interaction between $B$ and $π$ in $|1/2, 1/2⟩$ system is not strong enough to bind the constituents but it is sufficiently large for $|3/2, 1/2⟩$. In Table II we present the $Λ$ values for the bound state of $Bπ$ state $|3/2, 1/2⟩$. The normalized wave function is depicted in Fig. 3. For the bound state $B^+π^+$ besides the strong interaction there electromagnetic interaction also applies, however comparing the electromagnetic coupling $e^2$ with the effective strong coupling $g_1g_2$, one can safely ignore the contribution of electromagnetic interaction at all.

On other aspect, even though $|3/2, 1/2⟩$ bound states do not decay via strong interaction, they decay into other hadrons by emitting a photon, i.e a radiate decay. The form factor $F_1$ and $F_2$ in the transition $M → B^+π$ are calculated numerically. The theoretically estimated
decay rates are present in table III for different binding energies.

TABLE III: The form factors and decay widths for different binding energies of $B^0\pi^+(I = \frac{3}{2}, I_z = \frac{1}{2})$.

| $\Delta E$(MeV) | -10  | -20  | -30  | -40  | -50  | -60  | -70  | -80  |
|-----------------|------|------|------|------|------|------|------|------|
| $F_1$(GeV)      | -0.287 | -0.472 | -0.630 | -0.790 | -0.971 | -1.176 | -1.455 | -1.800 |
| $F_2$(GeV)$^{-1}$ | 0.0127 | 0.0238 | 0.0364 | 0.0525 | 0.0738 | -0.0966 | -0.140 | -0.194 |
| $\Gamma$(keV)   | 2.79  | 7.07  | 12.48 | 19.98 | 30.65 | 40.09 | 59.97 | 68.12 |

IV. CONCLUSION AND DISCUSSIONS

In this work we study the bound state of $B\pi$ which seems to be identified as a pure molecular state and meanwhile as long as it is experimentally observed, we can firmly determine existence of hadronic molecules. Combining future experimental data with the results provided in this work, we would gain valuable information about the structures of the four-quark exotic states and moreover, the applying dynamics.

We suggest that by solving the B-S equation with appropriate effective interaction between two constituent hadrons one can determine whether the four-quark system can be bound as a molecular state. Since the constituents are hadrons the effective interactions can be derived in terms of field theory. Since the isospins of $B$ and $\pi$ are 1/2 and 1 respectively the bound state can be $|\frac{1}{2},\frac{1}{2}\rangle$, $|\frac{3}{2},\frac{1}{2}\rangle$ and $|\frac{3}{2},\frac{3}{2}\rangle$. Priori setting a reasonable range for the parameter $\Lambda$ within 0.8-1.2 GeV according to the suggestions given in literature, one can numerically solve the B-S equation to gain the binding energies and wavefunctions of the systems with quantum number $|\frac{3}{2},\frac{1}{2}\rangle$ and $|\frac{3}{2},\frac{3}{2}\rangle$. Our numerical results show that there is not a solution for the bound state with quantum number $|\frac{1}{2},\frac{1}{2}\rangle$.

Since the parameter cannot be determined very precisely, our prediction on the mass spectrum of the bound state is also not very accurate as the errors come with uncertainties of theoretical inputs. As solving the B-S equation of the system for different binding energies, the corresponding parameter $\Lambda$ and B-S wave function are obtained. With the wave function we can estimate the radiative decay rate of $B\pi(|\frac{3}{2},\frac{1}{2}\rangle) \rightarrow B^*\gamma$. It is found that the partial width can vary in a certain range with different input values of $\Lambda$. We lay hope on the future measurement which will tell us the measured values of the binding energy and partial width of radiative decay. The data would check our calculation and help to fix the relevant parameters. Definitely the smart experimentalists will do good jobs to measure them in the near future to determine whether the bound states exist.

Even though in this work we only deal with the $B\pi$ molecular states, the same approach can be easily applied to study the $D\pi$ molecular states. Only difference is that the evaluated $D\pi$ mass is smaller than $D^*$, so that $D\pi$ molecule cannot decay via electromagnetic interaction due to the constraint of the final state phase space, thus it only has weak decay
portals. Definitely, since the rates of weak decays are obviously small, so the measurements on such $D \pi$ molecular states become even tougher, but not impossible.

If the result of our experimental measurements is positive we would have confidence for existence of molecular states and know more about their inner structures.

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[1] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 91, 262001 (2003) [arXiv:hep-ex/0309032].
[2] K. Abe et al. [Belle Collaboration], Phys. Rev. Lett. 98, 082001 (2007) [arXiv:hep-ex/0507019].
[3] S. K. Choi et al. [Belle Collaboration], Phys. Rev. Lett. 94, 182002 (2005).
[4] S. K. Choi et al. [BELLE Collaboration], Phys. Rev. Lett. 100, 142001 (2008) [arXiv:0708.1790 [hep-ex]].
[5] B. Aubert et al. [BaBar Collaboration], Phys. Rev. Lett. 95, 142001 (2005) doi:10.1103/PhysRevLett.95.142001 [hep-ex/0506081].
[6] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 112, 132001 (2014) [arXiv:1308.2760 [hep-ex]].
[7] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 111, 242001 (2013) [arXiv:1309.1896 [hep-ex]].
[8] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 110, 252001 (2013) [arXiv:1303.5949 [hep-ex]].
[9] Z. Q. Liu et al. [Belle Collaboration], Phys. Rev. Lett. 110, 252002 (2013) [arXiv:1304.0121 [hep-ex]].
[10] B. Collaboration. [arXiv:1105.4583 [hep-ex]].
[11] V. M. Abazov et al. [D0 Collaboration], Phys. Rev. Lett. 117, no. 2, 022003 (2016) doi:10.1103/PhysRevLett.117.022003 [arXiv:1602.07588 [hep-ex]].
[12] T. Xiao, S. Dobbs, A. Tomaradze and K. K. Seth, Phys. Lett. B 727, 366 (2013) [arXiv:1304.3036 [hep-ex]].
[13] C. Deng, J. Ping and F. Wang, Phys. Rev. D 90, 054009 (2014) doi:10.1103/PhysRevD.90.054009 [arXiv:1402.0777 [hep-ph]].
[14] Z. G. Wang, Eur. Phys. J. C 74, 2963 (2014) doi:10.1140/epjc/s10052-014-2963-7 [arXiv:1403.0810 [hep-ph]].
[15] Q. Wang, C. Hanhart and Q. Zhao, Phys. Rev. Lett. 111, 132003 (2013) [arXiv:1303.6355 [hep-ph]].
[16] E. Wilbring, H.-W. Hammer and U.-G. Meiβner, Phys. Lett. B 726, 326 (2013) doi:10.1016/j.physletb.2013.08.059 [arXiv:1304.2882 [hep-ph]].
Appendix A: The effective interactions

\begin{align}
\mathcal{L}_{BB\rho} &= ig_{BB\rho}[\bar{B}^0 \partial_\mu B^\mu - \partial_\mu \bar{B}^0 B^\mu + \bar{B}^0 \partial_\mu B^0 \rho_{\mu} - \partial_\mu B^- B^+ \rho_{0\mu} + h.c.] \quad (A1) \\
\mathcal{L}_{BB\omega} &= ig_{BB\omega}[\bar{B}^0 \partial_\mu B^0 \omega_{\mu} - \partial_\mu B^- B^+ \omega_{\mu} + h.c.] \quad (A2) \\
\mathcal{L}_{\rho\pi\pi} &= ig_{\rho\pi\pi}[\partial_\mu \rho^{+}\pi^{+} - \rho_{\mu} \pi^{+} + \partial_\mu \rho^{-}\pi^{-} + \rho_{\mu} \pi^{-} - \pi^{+} \pi^{-} + h.c.] \quad (A3) \\
\mathcal{L}_{B^+B^-} &= ig_{B^+B^-}[\partial_\mu \pi^+ \bar{B}^0 B^0 - \pi^+ \bar{B}^0 B^0 \partial_\mu B^- - \partial_\mu \pi^+ \bar{B}^0 B^0 - \pi^+ \partial_\mu B^0 B^- + \pi^+ \partial_\mu B^0 B^- + h.c.] \quad (A4) \\
\mathcal{L}_{B^+B^-}' &= \frac{i g_{B^+B^-}}{\sqrt{2}}[\pi^0 \partial_\mu B^0 \bar{B}^0 - \partial_\mu \pi^0 \bar{B}^0 B^0 + \partial_\mu \pi^0 B^+ B^- - \pi^0 \partial_\mu B^+ B^- + h.c.] \quad (A5)
\end{align}
\[ \mathcal{L}_{B^*B} = \, ig_{B^*B + \varepsilon}^{\mu\nu\alpha\beta} \partial_\mu A_\nu (B_\alpha^* \partial_\beta B^\dagger \varepsilon - \partial_\beta B_\alpha^* B^\dagger + \text{h.c.}) \] (A6)

\[ \mathcal{L}_{\rho\pi} = \, ig_{\rho\pi + \varepsilon}^{\mu\nu\alpha\beta} \partial_\mu A_\nu (\rho_\alpha^0 \partial_\beta \pi^0 - \partial_\beta \rho_\alpha^0 \pi^0 + \rho_\alpha^+ \partial_\beta \pi^- - \partial_\beta \rho_\alpha^+ \pi^- + \text{h.c.}) \] (A7)

\[ \mathcal{L}_{BB} = \, e A_\mu (\partial_\mu B B^\dagger - B \partial_\mu B^\dagger) \] (A8)

\[ \mathcal{L}_{B^*B^*\pi} = -\frac{g_{B^*B^*\pi}}{m_{B^*}} \varepsilon^{\mu\nu\alpha\beta} \partial_\mu B_\nu^* B_\alpha^*^\dagger \partial_\beta \pi \] (A9)

\[ \mathcal{L}_{B^*B\rho} = -\frac{g_{B^*B\rho}}{m_{B^*}} \varepsilon^{\mu\nu\alpha\beta} (B \partial_\mu \rho_\nu \partial_\alpha B_\beta^*^\dagger + \partial_\mu B_\nu \partial_\alpha \rho_\beta B^\dagger) \] (A10)

\[ \mathcal{L}_{BB\sigma} = -2m_B g_{\sigma} \sigma B B^\dagger \] (A11)

\[ \mathcal{L}_{\sigma\pi\pi} = -2m_\sigma g_{\sigma\pi\pi} \sigma \pi \pi^\dagger \] (A12)