Outage Performance of Multi-Antenna Mobile UAV-Assisted NOMA Relay Systems Over Nakagami-\(m\) Fading Channels

TRAN MANH HOANG\(^{1}\), BA CAO NGUYEN\(^{2}\), LE THE DUNG\(^{3}\), (Member, IEEE),
AND TAEJOON KIM\(^{3}\), (Member, IEEE)

\( ^{1}\)Faculty of Telecommunications Services, Telecommunications University, Nha Trang 650000, Vietnam
\( ^{2}\)Faculty of Basic Techniques, Telecommunications University, Nha Trang 650000, Vietnam
\( ^{3}\)School of Information and Communication Engineering, Chungbuk National University, Cheongju 28644, South Korea

Corresponding author: Taejoon Kim (ktjcc@chungbuk.ac.kr)

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ABSTRACT This paper investigates the outage performance of the communications between two pairs of source−destination in a non-orthogonal multiple access (NOMA) system where a multi-antenna mobile unmanned aerial vehicle (UAV) acts as the relay for these communications. We first characterize the signal-to-interference-plus-noise ratio (SINRs) of uplink and downlink, taking into account the influences of power allocation, successive interference cancellation (SIC) capability, and the number of antennas at the UAV. We then derive the exact closed-form expression of the outage probability (OP) of each source−destination communication pair. Monte-Carlo simulations are conducted to verify the correctness of our mathematical analysis. Numerical results show that the outage performance of the proposed NOMA-UAV relay system greatly depends on the line-of-sight (LoS) probability, the SIC capability, the velocity, and the number of antennas of the UAV. The best outage performance is obtained when the UAV is in the middle of the source and destination or when suitable power allocation coefficients are assigned to the uplink and downlink.

INDEX TERMS NOMA relay system, multi-antenna UAV, successive interference cancellation, outage probability.

I. INTRODUCTION

Unmanned Aerial Vehicles (UAVs) are becoming more and more attractive as they can provide robust and reliable communications in disaster and remote areas thanks to their advantages such as extensive coverage, ready deployment, and eviction flexibility. On the other hand, UAVs are low cost and have a high probability of line-of-sight (LoS) connections [1]. Consequently, the research on UAVs is a hot topic in both academy and industry.

The applications of the UAV cover various fields, e.g., wireless power transfer, data collection, data transmission, and hot-spot offloading [2], [3]. In terms of antenna configuration, the UAV can be equipped with a single antenna [4], [5] or multiple antennas [6]. When being used as relay in wireless systems, the UAV can be stationary [6] or mobile [4], [5] and use amplify-and-forward (AF) protocol [5] or decode-and-forward (DF) protocol [4], [6] to transmit signals from source to destination. Additionally, optimizing the trajectory and transmission power is applied to minimize the outage probability (OP) of the UAV-assisted relay systems [5]. Considering the case that the UAV is able to perform energy harvesting, the authors in [7] derived the approximate performance of a two-phase UAV assisted DF relay network including path-loss, shadowing, and Nakagami-\(m\) fading. However, the drawback of [7] is that the communication of only one source−destination pair with stationary UAV was considered.

All aforementioned UAV-assisted relay systems only employed orthogonal multiple access (OMA) technique. However, due to the increase in data traffic demand and massive connectivity in the fifth-generation (5G) and beyond 5G (B5G) wireless systems, non-orthogonal multiple access (NOMA) is considered to be a promising technique [8], [9]. Notably, the usage of NOMA enhances not only the spectrum efficiency but also an efficient approach for
improving the multi-user channel capacity [10]. In contrast, the usage of OMA technique is inefficient in spectrum utilization because each user is assigned an orthogonal spectrum resource. Regarding the UAV-assisted NOMA relay systems, the authors of [11] proposed a pairs of strategies, i.e., UAV-centric strategy and the user-centric strategy, in NOMA-UAV communications in large-scale cellular networks. In [12], a UAV was utilized to assist with emergency communications in a heterogeneous Internet of Things (IoT), and a distributed NOMA scheme was proposed without the requirement of successive interference cancellation (SIC). However, both [11] and [12] only considered the downlink from a single-antenna UAV to ground users. The authors in [13] introduced downlink NOMA into an UAV-enabled mobile relay system and investigated a scenario where an UAV flies in a circular trajectory to serve as a mobile DF relay. The physical-layer security in an UAV-enable mobile relaying system was considered in [14]. Specifically, the authors investigated the secrecy energy efficiency maximization problem of that relaying system, where a high-mobility UAV flies at a fixed altitude from a given initial location to a final location to assists forwarding confidential information. The UAV-aided wireless communication systems with multiple single-antenna UAVs were studied in [15]–[17]. These UAVs acted as the relays for two-hop communication [15], [16] or multi-hop communication [17]. However, the major issue of these works is that controlling many UAVs simultaneously may be very difficult and consumes much energy.

Motivated by the above observations, in this paper, we propose and analyze the outage performance of a multi-antenna mobile UAV-assisted NOMA relay system. The contributions of this paper are summarized as follows:

- Unlike previous studies on UAV-assisted wireless communication systems having only uplink [3], [18], [19] or downlink [6], [11], [13], or multiple UAV with single antenna [15]–[17], this paper proposes and analyzes an UAV-assisted NOMA relay system where a mobile multi-antenna UAV participates in signal forwarding of two source–destination pairs.
- We firstly characterize the signal-to-interference-plus-noise ratios (SINRs) of uplink and downlink, considering the effects of power allocation, SIC capability, and the number of antennas at the UAV. Then, based on these SINRs, we derive the exact closed-form expressions of the OPs of two source–destination communication paths under the effects of LoS and NLoS probabilities, and validate these derived expressions through various evaluating scenarios. We also compare the outage performance of NOMA-UAV and OMA-UAV relay systems.
- We show that system parameters such as the UAV’s location and velocity, fading severity, SIC capacity, and the number of antennas at the UAV greatly impact the outage performance of the proposed NOMA-UAV relay system. Moreover, selecting suitable power allocation coefficients for uplink and downlink can provide the best system outage performance.

The remainder of this paper is structured as follows. Section II describes the system model of the proposed multi-antenna mobile UAV assisted NOMA relay system. The OP expressions of two source-destination pairs are derived in Section III. Section IV provides numerical results verified by Monte-Carlo simulations. Finally, Section V concludes the paper.

II. SYSTEM MODEL

We consider a $K$-antenna mobile UAV, serving as the relay for the communications between two pairs of single-antenna source and destination, as illustrated in Fig. 1. It is noted that using a multi-antenna UAV is more energy-efficient than using multiple UAVs because the communication-related energy is usually much smaller than the UAV’s propulsion energy [20]. Moreover, using multi-antenna UAV reduces the system complexity thanks to fewer efforts in control and non-payload communications. Moreover, since sources and destinations locate in different areas, it is feasible for the UAV to use NOMA technique in power domain. Specifically, two sources $S_i$, $i = \{1, 2\}$, simultaneously transmit signals to the UAV, then the UAV forwards these signals to two destinations $D_i$ at the same time with different power levels.

It is assumed that the UAV flies with a speed of $v$ at a constant altitude $H$. A two-dimensional coordinate is used to describe the locations of two $S_i - D_i$ pairs. Let us denote the coordinate of a ground terminal $(G_i)$, $G_i = \{S_i, D_i\}$, as $(x_{G_i}, 0)$ and the coordinate of the UAV $(U)$ as $(x_U, y_U)$. Moreover, the UAV flies from the middle point to $S_2$ or $D_2$ in a duration $T$. During each time step $\delta t$, UAV flies from $x_U(t)$ to $x_U(t + \delta t)$ with a length of $v\delta t$, where $v$ is the speed of the UAV. Then, the distance between them is $d_{UG_i} = \sqrt{H^2 + [x_U(t) + v\delta t - x_{G_i}]^2}$ with an elevation angle $\theta_{UG_i} = \frac{\pi}{2} - \arcsin\left(\frac{H}{d_{UG_i}}\right)$.

The wireless channels from $S_i$ to the UAV and from the UAV to $D_i$ are influenced by the line-of-sight (LoS), non-LoS (NLoS), and Doppler effect due to the mobility of UAV. The combined effects are presented by large-scale fading and small-scale fading. Similar to [21], the large-scale fading of

![FIGURE 1. System model of the proposed multi-antenna mobile UAV-assisted NOMA relay system.](image-url)
each link can be presented as
\[
h(t) = \frac{\beta_0}{\sqrt{\left[H^2 + |x_U| + \nu d\right]}}.
\] (1)
where \( \beta_0 \) denotes the channel power gain at the reference distance \( d_0 = 0 \), \( \alpha(\theta_{UG}) = \alpha(\varphi) = \alpha(0) + \alpha(0) \) is the path-loss coefficients of \( S \) minus \( U \) and \( U + D \) channels with \( \omega = 1 \) for the case of LoS communication and \( \omega < 1 \) for the case of NLoS communication [19], [22].

The probabilities of having LoS and NLoS communications can be presented as functions of \( \theta_{UG} \), [23], i.e.,
\[
P_{\text{LoS}}(\theta_{UG}) = c_1 - \frac{c_1 - c_2}{1 + \left(\frac{\theta_{UG} - c_3}{c_4}\right)^\alpha}.
\] (2)
\[
P_{\text{NLoS}}(\theta_{UG}) = 1 - P_{\text{LoS}}(\theta_{UG}),
\] (3)
where \( c_r, e = \{1, \ldots, 5\} \), is constant and its value depends on the communication environments as shown in Table 1.

It is worth noticing that the altitude of the UAV affects the communication condition between ground terminals and the UAV, i.e., larger \( \theta_{UG} \) (the UAV is at high altitude) leads to a higher probability of having LoS communication between \( S \) and UAV but also higher path loss.

On the other hand, the small-scale fading coefficient from \( S \) to the \( k \)th antenna of the UAV is subject to Nakagami-\( m \) distribution. It is modeled as \( g_{k,i} \sim \mathcal{G}(m, \frac{m}{\bar{\Omega}_k}) \), \( k = \{1, \ldots, K\}, i = \{1, 2\}, \) where \( m \) is the standardized variance of \( g_{k,i} \) and \( \bar{\Omega}_k = \mathrm{E}[|h(t)|^2] \) is the average channel gain in the case of LoS or NLoS. Similarly, \( p_{k,i} \sim \mathcal{G}(m, \frac{m}{\bar{\Omega}_k}) \) is small-scale fading coefficient from the UAV to \( D \). Generally, the Rayleigh distribution is commonly used to characterize the small-scale fading for two reasons: (i) it is simpler for analyzing the system performance, and (ii) more importantly, if a wireless system influenced by Rayleigh fading can fulfill the performance requirements, it works well under other fading conditions. Other more complex but more general models for small-scale fading in ground-UAV and UAV-ground communications are Rician or Nakagami-\( m \) distributions. These distributions are used because they take into account the light-of-sight (LoS) components. Specifically, individual multi-path components with very similar time delay are grouped and modeled by the Nakagami-\( m \) distribution. Therefore, \( m \) represents the fading severity, i.e., higher \( m \) means a higher possibility of having LoS communications.

Additionally, it should be noted that a one-to-one mapping exists between the \( m \) parameter of the Nakagami-\( m \) distribution and the \( L \) parameter of the Rician distribution, allowing the Nakagami-\( m \) distribution to approximate the Rician distribution [24].

Denoting \( X_k = |g_{k,i}|^2 \) and \( Y_k = |p_{k,i}|^2 \), the CDF and PDF of \( Z = \{X_k, Y_k\} \) are, respectively, expressed as [25]
\[
F_Z(z) = 1 - \exp\left(-\frac{mz}{\Omega_k}\right)^{m-1}, \quad x \geq 0.
\] (4)
\[
f_Z(z) = \left(\frac{m}{\Omega_k}\right)^{m-1} \frac{z^{m-1}}{\Gamma(m)} \exp\left(-\frac{mz}{\Omega_k}\right), \quad k = 1, \ldots, K.
\] (5)

For the sake of simplicity in mathematical notations, the channel matrices between \( S \) and the UAV and between the UAV and \( D \) are, respectively, presented as \( h_i = h(t)g_{i,k} \in \mathbb{C}^{K \times 1} \) and \( f_i = h(t)p_{i,k} \in \mathbb{C}^{K \times 1} \), where \( g_i = \{g_{1,i}, g_{2,i}, \ldots, g_{K,i}\}^T \) and \( p_i = \{p_{1,i}, p_{2,i}, \ldots, p_{K,i}\}^T \) whose elements are the channel coefficients and \([\cdots]^T\) denotes the transposition of a matrix. The additive white Gaussian noise (AWGN) at the receiver (\( U \) or \( D \)) is \( w \sim \mathcal{CN}(0, \sigma^2 I_K) \), with \( \sigma^2 \) noise variance and \( I_K \) the identity matrix. The channel state information (CSI) is assumed to be perfect at each terminal.

It is assumed that the UAV operates in half-duplex mode. Thus, the communications from \( S \) to the UAV and from the UAV to \( D \) take two time slots. Particularly, in the first time slot, \( S_1 \) and \( S_2 \) transmit two independent, unit-power data symbols \( x_1 \) and \( x_2 \) simultaneously to the UAV through the uplink. Consequently, the received signal at the UAV is expressed as
\[
r_U = h_1 \sqrt{a_1} P_S x_1 + h_2 \sqrt{a_2} P_S x_2 + w_U,
\] (6)
where \( a_1 \) and \( a_2 \) are the power allocation coefficients for \( x_1 \) and \( x_2 \) assigned by sources \( S_1 \) and \( S_2 \), respectively.

According to the principle of NOMA uplink communication [26], the UAV detects \( x_1 \) having strong signal power first while treating \( x_2 \) as noise, then decodes \( x_2 \) by using SIC algorithm to subtract \( x_1 \). Therefore, the SINRs associated with \( x_1 \) and \( x_2 \) at the UAV in the case of imperfect SIC are
\[
\gamma_U^{x_1} = \frac{a_1 P_S |h_1|^2}{a_2 P_S |h_2|^2 + \sigma^2_U},
\] (7)
\[
\gamma_U^{x_2} = \frac{a_2 P_S |h_2|^2}{\xi_1 a_1 P_S |h_1|^2 + \sigma^2_U},
\] (8)
where \( \xi_1, 0 \leq \xi_1 \leq 1 \), represents the efficiency of SIC for \( x_1 \) at the UAV. The cases \( \xi_1 = 0 \) and \( \xi_1 = 1 \) correspond to perfect SIC and without SIC, respectively.

In the second time slot, \( x_1 \) and \( x_2 \) are decoded by the UAV and then beamformed to \( D \) by using the maximal ratio transmission (MRT) scheme. We assume that the UAV is able to successfully decode both \( x_1 \) and \( x_2 \), re-encodes them to become \( \hat{x}_1 \) and \( \hat{x}_2 \), and then transmits \( x_U = \sqrt{b_1} \hat{P}_U \hat{x}_1 + \sqrt{b_2} \hat{P}_U \hat{x}_2 \) to both \( D_1 \) and \( D_2 \) in one time slot, where \( b_1 \) and \( b_2 \) are the power allocation coefficients for \( \hat{x}_1 \) and \( \hat{x}_2 \), respectively. Thus, the received signal at \( D_i \) is expressed as
\[
y_{D_i} = f_i(\sqrt{b_1} \hat{P}_U \hat{x}_1 + \sqrt{b_2} \hat{P}_U \hat{x}_2) + w_{D_i}.
\] (9)

### Table 1. Path-loss coefficients in different environments.

| Environment  | \( c_1 \) | \( c_2 \) | \( c_3 \) | \( c_4 \) | \( c_5 \) |
|--------------|---------|---------|---------|---------|---------|
| Suburban     | 101.6   | 0       | 0       | 3.25    | 1.241   |
| Urban        | 102.0   | 0       | 0       | 24.30   | 1.729   |
| Dense Urban  | 187.3   | 0       | 0       | 82.10   | 1.478   |
Since near user $D_1$, i.e., the user having higher received signal power, applies SIC technique, the SINRs needed to do SIC for $\hat{x}_1$ and $\hat{x}_2$ at $D_1$ are, respectively, given by
\[
\gamma_{D_1}^{\hat{x}_1} = \frac{b_1 P_U ||f_1||^2}{\xi_2 b_2 P_U ||f_1||^2 + \sigma^2_{D_1}}, \tag{10}
\]
\[
\gamma_{D_1}^{\hat{x}_2} = \frac{b_2 P_U ||f_2||^2}{\xi_2 b_1 P_U ||f_2||^2 + \sigma^2_{D_1}}. \tag{11}
\]
where $0 \leq \xi_2 \leq 1$, represents the efficiency of SIC for $x_2$ at $D_1$.

In contrast, $D_2$ decodes its desired signal $\hat{x}_2$ by considering $\hat{x}_1$ as interference. Thus, the SINR at $D_2$ is
\[
\gamma_{D_2}^{\hat{x}_2} = \frac{b_2 P_U ||f_2||^2}{\xi_2 b_1 P_U ||f_2||^2 + \sigma^2_{D_2}}. \tag{12}
\]

Remark: We should remind that the channel matrices from $S_1$ and $S_2$ to $K$ antenna of UAV is $h_i = h(r_i)g_i$, and the channel matrices from $D_1$ to $D_2$ is $f_i = h(r_i)p_i$, where $g_i = [g_{1,i}, g_{2,i}, \ldots, g_{K,i}]^T$ and $p_i = [p_{1,i}, p_{2,i}, \ldots, p_{K,i}]^T$. Moreover, the channel coefficients $|g_{k,i}|$ and $|p_{k,i}|$ follow Nakagami-m distributions. We can see that equations presenting the received signals and the SINRs at the UAV and two destinations $D_1$ and $D_2$ such as (6), (7), (8), and other equations contain $h_i$ and $f_i$. When $K$ varies, the sizes of $h_i$ and $f_i$ are changed. Thus, these equations are also affected.

III. OUTAGE PROBABILITY ANALYSIS

A. OUTAGE PROBABILITY OF RECEIVING $x_1$ BY $D_1$

Signal $x_1$ cannot be received successfully by $D_1$ when the following cases occur: (i) UAV fails to decode $x_1$ or (ii) $D_1$ fails to do SIC for $x_2$, and thus cannot decode $x_1$. Mathematically, the probability of this outage event (hereafter called as the OP of $D_1$) is $Pr(y_{x_1}^{e_1}|_{\omega} \leq \gamma_{th} \cup y_{x_2}^{\hat{e}_2}|_{\omega} \leq \gamma_{th} \cup y_{x_1}^{\hat{e}_1}|_{\omega} \leq \gamma_{th}$ and is equivalent to the end-to-end SINR $y_{x_2}^{e_2}$ as $Pr(y_{x_2}^{e_2}|_{\omega} \leq \gamma_{th})$ [27], where $y_{x_2}^{e_2} = \min\{y_{U_1}^{\hat{e}_1}, y_{x_2}^{\hat{e}_2}, y_{D_2}^{\hat{e}_2}\}$. Thus, the OP of $D_1$ under the effects of LoS and NLoS propagation is calculated as
\[
OP_{o_1} = Pr(y_{x_2}^{e_2}|_{\omega} \leq \gamma_{th}) = 1 - Pr\left(\min\{y_{U_1}^{\hat{e}_1}, y_{x_2}^{\hat{e}_2}, y_{D_2}^{\hat{e}_2}\}|_{\omega} > \gamma_{th}\right). \tag{13}
\]
From (13), the closed-form expression of $OP_{o_1}$ is given in the following Proposition 1.

Proposition 1: The OP of the proposed NOMA-UAV relay system is calculated as
\[
OP_{o_1} = 1 - (I_1 + I_2)(I_3 + I_4), \tag{14}
\]
where $I_1$ and $I_2$ are given in (15) and (16), as shown at the bottom of the page, respectively, with $\mu_1 = \frac{m_11_{\gamma_{th}}}{\Omega_1 a_1 P_S} + \frac{m_2}{\Omega_2}$ and $\mu_2 = \frac{m_11_{\gamma_{th}}}{\omega \Omega_1 a_1 P_S} + \frac{m_2}{\Omega_2}$, $\Gamma(\cdot)$ denotes Gamma function [28], and the power allocation coefficients $b_1$ and $b_2$ satisfy the condition $\xi_2 \gamma_{th} \leq b_1/b_2 \leq 1/\gamma_{th}$.

\[
I_3 = P_{LoS} \sum_{k=0}^{Km_1-1} \left(\frac{m_11_{\gamma_{th}}}{\Omega_1 a_1 P_S} \cdot \frac{m_2}{\Omega_2}\right)^{Km_2} \exp\left(-\frac{m_11_{\gamma_{th}}}{\Omega_1 a_1 P_S}\right) \frac{(a_2 P_S)^n}{\Gamma(Km_2 + n)} \frac{\Gamma(Km_2 + n)}{\mu_1^{Km_2+n}}. \tag{15}
\]

\[
I_4 = P_{NLoS} \sum_{k=0}^{Km_1-1} \left(\frac{m_11_{\gamma_{th}}}{\omega \Omega_1 a_1 P_S} \cdot \frac{m_2}{\Omega_2}\right)^{Km_2} \exp\left(-\frac{m_11_{\gamma_{th}}}{\omega \Omega_1 a_1 P_S}\right) \frac{(a_2 P_S)^n}{\Gamma(Km_2 + n)} \frac{\Gamma(Km_2 + n)}{\mu_2^{Km_2+n}}. \tag{16}
\]
TABLE 2. System parameters used in evaluating scenarios.

| Parameter | Value |
|-----------|-------|
| $H, r_1, r_2, \beta_0, \delta_0$ | 100 m, 50 m, 100 m, 20 dB, 0.5 s |
| $a_1, a_2; b_1, b_2$ | 0.6, 0.4; 0.3, 0.7 |
| $\alpha(\xi), \alpha(0), \sigma^2_{\text{LeV}}, \sigma^2_{\text{LoS}}$ | 2, 3.5, 1 W, 1 W |
| $R_{\text{th}}, \xi_1 = \xi_2 = \xi$ | 0.5 b/s/Hz, 0.1 |

FIGURE 2. The three dimensions of the proposed multi-antenna mobile UAV-assisted NOMA relay system.

to validate the derived mathematical expressions. The system parameters used in evaluating scenarios are summarized in Table 2. For the sake of clarity, the three dimensions of the proposed NOMA-UAV relay system is illustrated in Fig. 2. Similar to [18], [29], and [30], we also consider the case where all terminals locate on a straight line. Specifically, the locations of sources and destinations are $S_1(-50, 0, 0), S_2(-100, 0, 0), D_1(50, 0, 0)$, and $D_2(100, 0, 0)$. The UAV initially locates at $(0, 0, 100)$ and can fly to $(-100, 0, 100)$ or $(100, 0, 100)$. The power allocation coefficients are fixed because adaptive power allocation requires a large amount of communication overheads.

Fig. 3 plots the LoS probability versus the x-coordinate $x_U$ of the UAV for $H = 100$ m, suburban environment.

$J_1 = P_{\text{LoS}} \sum_{k=0}^{K_{m1}-1} \frac{1}{k!} \left( \begin{array}{c} m_2 \gamma_{\text{th}} \\ \Omega_2 a_1 P_S \end{array} \right)^k \left( \begin{array}{c} m_2 \gamma_{\text{th}} \\ \Omega_2 \end{array} \right)^{K_{m2}} \exp \left( -\frac{m_1 \gamma_{\text{th}}}{\Omega_1 a_1 P_S} \right) \frac{(a_2 P_S)^n}{\Gamma(K_{m2} + n)} \frac{\Gamma(K_{m2} + n)}{\mu_1^{K_{m2+n}}} , \quad (21)$

$J_2 = P_{\text{NLoS}} \sum_{k=0}^{K_{m1}-1} \frac{1}{k!} \left( \begin{array}{c} m_2 \gamma_{\text{th}} \\ \Omega_2 a_2 P_S \end{array} \right)^k \exp \left( -\frac{m_2 \gamma_{\text{th}}}{\Omega_2 a_2 P_S} \right) \frac{(a_1 P_S)^n}{\Gamma(K_{m1} + n)} \frac{\Gamma(K_{m1} + n)}{\mu_3^{K_{m1+n}}} , \quad (22)$

$J_3 = P_{\text{LoS}} \sum_{k=0}^{K_{m2}-1} \frac{1}{k!} \left( \begin{array}{c} m_2 \gamma_{\text{th}} \\ \Omega_2 a_2 P_S \end{array} \right)^k \exp \left( -\frac{m_2 \gamma_{\text{th}}}{\Omega_2 a_2 P_S} \right) \frac{(a_1 P_S)^n}{\Gamma(K_{m1} + n)} \frac{\Gamma(K_{m1} + n)}{\mu_3^{K_{m1+n}}} , \quad (23)$

$J_4 = P_{\text{NLoS}} \sum_{k=0}^{K_{m2}-1} \frac{1}{k!} \left( \begin{array}{c} m_2 \gamma_{\text{th}} \\ \Omega_2 a_2 P_S \end{array} \right)^k \exp \left( -\frac{m_2 \gamma_{\text{th}}}{\Omega_2 a_2 P_S} \right) \frac{(a_1 P_S)^n}{\Gamma(K_{m1} + n)} \frac{\Gamma(K_{m1} + n)}{\mu_3^{K_{m1+n}}} , \quad (24)$

$J_5 = P_{\text{LoS}} \exp \left( -\frac{m_4 \gamma_{\text{th}}}{\Omega_4 P_U(b_2 - b_1 \gamma_{\text{th}})} \right) \sum_{k=0}^{K_{m4}-1} \frac{1}{k!} \left( \frac{m_4 \gamma_{\text{th}}}{\Omega_4 P_U(b_2 - b_1 \gamma_{\text{th}})} \right)^k , \quad (25)$

$J_6 = P_{\text{NLoS}} \exp \left( -\frac{m_4 \gamma_{\text{th}}}{\Omega_4 P_U(b_2 - b_1 \gamma_{\text{th}})} \right) \sum_{k=0}^{K_{m4}-1} \frac{1}{k!} \left( \frac{m_4 \gamma_{\text{th}}}{\Omega_4 P_U(b_2 - b_1 \gamma_{\text{th}})} \right)^k , \quad (26)$

FIGURE 3. LoS probabilities of two source—destination pairs versus the x-coordinate $x_U$ of the UAV for $H = 100$ m, suburban environment.

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Fig. 4, we can see that increasing velocity \( v \) reduces the outage performance of \( D_1 \) because increasing \( v \) makes the communication between the UAV and source/destination unstable. Another observation is that the error floor appears in the high SINR regime because the co-channel interference of NOMA uplink scales with the transmission power of the source.

Fig. 5 shows the comparison between the OP of the NOMA-UAV relay system with the OMA-UAV relay system versus the average SINR in dB for different numbers of antennas \( K \). It is noticed that Fig. 5 provides a fair comparison between the OMA-UAV and NOMA-UAV relay systems. For the OMA-UAV relay system, we assume that two sources with different frequencies transmit simultaneously, and their decoded symbols at UAV are also forwarded at the same time. We let the outage thresholds \( \gamma_{th} = 2^{2K} - 1 \) for both NOMA-UAV and OMA-UAV relay systems. We can see in Fig. 5 that the OP of the NOMA-UAV relay system is better than that of the OMA-UAV relay system in the low SINR regime. However, the NOMA-UAV relay system performs worse than the OMA-UAV relay system in the high SINR regime. It is because the interference among destinations' signals exists in the NOMA-UAV relay system. On the other hand, we chose a large imperfect SIC coefficient (i.e., \( \xi = 0.1 \)) to plot Fig. 5. If this imperfect SIC coefficient were smaller, the OP of the NOMA-UAV relay system would be better. In terms of the system throughput, it was demonstrated in [31]–[33] that NOMA systems achieve high bandwidth efficiency, thus improving the system throughput.

Fig. 6 illustrates the OP of the proposed NOMA-UAV relay system versus the x-coordinate \( x_U \) of the UAV for different communication environments. It is assumed that the UAV maintains an altitude of 100m while flying horizontally. We can see that, at \( x_U = 0 \), the OP of \( D_1 \) in each communication environment is minimum. It is because when the UAV stays in the middle of sources and destinations, the distance of uplink and downlink are equal; thus, the total probability of having LoS both uplink and downlink is largest.

Fig. 7 presents the OPs of \( D_1 \) and \( D_2 \) in NOMA-UAV relay system versus the altitude \( H \) of UAV for different communication environments. We can see that the OPs of both \( D_1 \) and \( D_2 \) decrease and then increase as \( H \) becomes higher. In other words, there exists an optimal \( H \) that minimizes
the OP. Moreover, the gap between the OPs of D₁ and D₂ is remarkable when the UAV is at optimal H but is insignificant when H is high. The reason is that increasing the altitude of UAV leads to an increase in the probability of having LoS link, resulting in higher channel gain. However, when H increases further, the communication distance is longer, thus the path loss is higher. Especially, when H = 0, the OPs corresponding to dense urban communication environment are nearly one because the probability of having NLoS links is high.

Fig. 8 plots the OPs of D₁ and D₂ versus the probability ω of having LoS communications for different fading severity m of urban environment. It is shown that the outage of D₁ and D₂ always happens when ω < 0.7. The OPs of D₁ and D₂ rapidly reduce when ω > 0.7, but the OP of D₂ is still higher than the OP of D₁. This gap is reduced as m increases and ω approaches one.

Fig. 9 depicts the impact of SIC capability ξ on the OPs of NOMA-UAV and OMA-UAV relay systems. Since OMA-UAV system does not apply SIC, its OP remains unchanged. In contrast, the OP of the NOMA-UAV relay system increases significantly with ξ. It is worth noticing that, for the same ξ, the influences of imperfect SIC on the UAV and D₁ are different. The reason behind this feature is that the power levels for x₁ and x₂ allocated by S₁ and the UAV are different.

Fig. 10 presents OP = min (OP₁, OP₂) of NOMA-UAV relay system versus the power allocation coefficients a₁ and b₁. It is noticed that the power allocation coefficients must satisfy a₁ + a₂ = 1 and b₁ + b₂ = 1. From two above conditions, i.e., b₁ > ξ₂γb₂ and b₂ > γb₁, we have γb₂(1 + γb₂) < b₁ < 1/(1 + γb₂). Then, based on the parameter settings in Table 2, we determine b₁ ∈ [0.031, 0.731]. As observed from Fig. 10, there is a pair (a₁, b₁) that minimizes the OP of NOMA-UAV relay system, i.e., OP = 0.22 when a₁ = 0.95 and b₁ = 0.43. Moreover, increasing the number of antennas K significantly reduces this OP, i.e., OP = 0.03 when a₁ = 0.6 and b₁ = 0.43.

V. CONCLUSION

In this paper, we have analyzed the outage performance of our proposed multi-antenna mobile UAV-assisted NOMA relay system. Notably, we derived the closed-form expression of the OP, taking into account the effects of UAV’s location and velocity, fading severity, SIC capability, and the number of antennas at the UAV. We also compared the OPs of the NOMA-UAV and OMA-UAV relay systems. Numerical results show that these network parameters significantly influence the outage performance of the proposed NOMA-UAV relay system. Moreover, the best outage performance can be obtained when the UAV is in the middle of the source and destination, or using suitable power allocation coefficients for uplink and downlink. The proposed NOMA-UAV relay system can be applied in monitoring the traffic and security of smart cities. As future work, we will consider full-duplex communication at the UAV to improve the spectrum utilization.
APPENDIX A

The following lemma will be used to prove the exact closed-form expressions of \( \text{OP}_A \) and \( \text{OP}_B \) in Proposition 1 and Proposition 2, respectively.

**Lemma 1:** For two given Gamma random variables (RVs) \( X = \sum_{k=1}^{K} X_k \) and \( Y = \sum_{k=1}^{K} Y_k \) with parameters \( (m, \Omega_x) \) and \( (m, \Omega_y) \), respectively, the exact closed-form expression of the complement cumulative density function (CCDF), \( \Pr(W > w) = \tilde{F}_W(w) \) with \( W = \frac{aX}{by+1} \), is

\[
\tilde{F}_W(w) = \sum_{k=0}^{Km-1} \binom{k}{m} \frac{b^m}{\Gamma(Km)} \exp\left(-\frac{mw}{a\Omega_x}\right) \binom{m}{\Omega_y}^{Km-1} \left(1 - \frac{w}{by+1}\right)^m.
\]

Based on the conditional probability, we can rewrite \( \mathcal{O}_1 \) as

\[
\mathcal{O}_1 = \int_0^\infty \Pr \left( X > \frac{\gamma_0(a_2 P_S Y + 1)}{a_1 P_S} \right) f(y) dy, \tag{34}
\]

where \( X = \|\mathbf{h}_1\|^2 \) and \( Y = \|\mathbf{h}_2\|^2 \).

Since the propagation of each link depend on the LoS or NLoS propagation as [35, Eq. (18)], (34) becomes

\[
\int_0^\infty \Pr \left( X > \frac{\gamma_0(a_2 P_S Y + 1)}{a_1 P_S} \right) f(y) dy = P_{\text{LoS}} \int_0^\infty \Pr \left( X > \frac{\gamma_0(a_2 P_S Y + 1)}{a_1 P_S} \right) f(y) dy \bigg| \text{LoS} \frac{dy}{I_1} + P_{\text{NLoS}} \int_0^\infty \Pr \left( X > \frac{\gamma_0(a_2 P_S Y + 1)}{a_1 P_S} \right) f(y) dy \bigg| \text{NLoS} \frac{dy}{I_2}.
\]

(35)

\[
\mathcal{I}_1 = P_{\text{LoS}} \int_0^\infty \exp\left(-\frac{m_1 \gamma_0(a_2 P_S Y + 1)}{a_1 P_S}\right) dy + P_{\text{NLoS}} \int_0^\infty \exp\left(-\frac{m_2 \gamma_0(a_2 P_S Y + 1)}{a_1 P_S}\right) dy.
\]

(36)

After some manipulations, we can rewrite (36) as

\[
\mathcal{I}_1 = P_{\text{LoS}} \sum_{k=0}^{Km_1-1} \frac{1}{k!} \left(\frac{m_1 \gamma_0(a_2 P_S Y + 1)}{a_1 P_S}\right)^k \exp\left(-\frac{m_2 \gamma_0(a_2 P_S Y + 1)}{a_1 P_S}\right) dy + P_{\text{NLoS}} \sum_{k=0}^{Km_2-1} \frac{1}{k!} \left(\frac{m_1 \gamma_0(a_2 P_S Y + 1)}{a_1 P_S}\right)^k \exp\left(-\frac{m_2 \gamma_0(a_2 P_S Y + 1)}{a_1 P_S}\right) dy.
\]

(37)

Using [28, Eq. (3.381.4)] for solving the integral in (37), we obtain the closed-form expression of \( \mathcal{I}_1 \) as (15). Similar to the calculations of \( \mathcal{I}_1 \), we have closed-form expression of \( \mathcal{I}_2 \) as (16).
Next, we can express $O_2$ in (33) under the effects of LoS and NLoS propagation as
\[
\Pr \left( \|\mathbf{f}_1\|^2 > \Psi_{\text{max}} \right) = \Pr \left( \|\mathbf{f}_1\|^2 > \Psi_{\text{max}} | \text{LoS} \right) I_3 + \Pr \left( \|\mathbf{f}_1\|^2 > \Psi_{\text{max}} | \text{NLoS} \right) I_4.
\]
(38)

From (28) the closed-form expressions of $I_3$ and $I_4$ are, respectively, given in (17) and (18).

**APPENDIX C**

Putting (7), (8), and (12) into (19), we have
\[
\text{OP}^{t_2}_{\omega} = 1 - \Pr \left( \frac{a_1 P_S \|\mathbf{h}_1\|^2}{a_2 P_S \|\mathbf{h}_2\|^2 + 1} > \gamma_h \right) \frac{O_1}{O_3} \times \Pr \left( \frac{a_2 P_S \|\mathbf{h}_2\|^2}{\xi_1 a_1 P_S \|\mathbf{h}_1\|^2 + 1} > \gamma_h \right) \frac{O_2}{O_5} \times \Pr \left( \|\mathbf{f}_2\|^2 > \frac{\gamma_h}{P_U(b_2 - \gamma_b)} \right). \tag{39}
\]

To derive the closed-form expression of (39), we need calculate $O_3$, $O_4$, and $O_5$.

Based on conditional probability, we can compute $O_3$ as
\[
O_3 = \int_0^\infty \Pr \left( X > \frac{\gamma_h(a_2 P_S y + 1)}{a_1 P_S} \right) f_y(y)dy.
\]
\[= \int_0^\infty \Pr \left( Y > \frac{\gamma_h (a_2 P_S y + 1)}{a_1 P_S} | \text{LoS} \right) f_y(y)dy \tag{40}
\]

Applying Lemma 1, we obtain the closed-form expressions of $J_1$ and $J_2$ as (21) and (22), respectively.

For the second component $O_4$ of (39), we can calculate as
\[
O_4 = \Pr \left( \frac{a_2 P_S \|\mathbf{h}_2\|^2}{\xi_1 a_1 P_S \|\mathbf{h}_1\|^2 + 1} > \gamma_h \right) = \int_0^\infty \Pr \left( Y > \frac{\gamma_h (\xi_1 a_1 P_S x + 1)}{a_2 P_S} \right) f_y(x)dy \tag{41}
\]

Using the conditional probability for two random variables in (41) and taking into account the effects of LoS or NLoS propagation, we rewrite (41) as
\[
O_4 = \int_0^\infty \Pr \left( Y > \frac{\gamma_h (\xi_1 a_1 P_S x + 1)}{a_2 P_S} | \text{LoS} \right) f_y(x)dy \tag{42}
\]

\[+ \int_0^\infty \Pr \left( Y > \frac{\gamma_h (\xi_1 a_1 P_S x + 1)}{a_2 P_S} | \text{NLoS} \right) f_y(x)dy \tag{43}
\]

\[
J_3 = \int_0^\infty \tilde{F}_Y \left( \frac{\gamma_h (\xi_1 a_1 P_S x + 1)}{a_2 P_S} | \text{LoS} \right) f_y(x)dy \tag{44}
\]

\[+ \int_0^\infty \tilde{F}_Y \left( \frac{\gamma_h (\xi_1 a_1 P_S x + 1)}{a_2 P_S} | \text{NLoS} \right) f_y(x)dy \tag{45}
\]

where $\tilde{F}_Y (y | \text{LoS})$ and $\tilde{F}_Y (y | \text{NLoS})$ are the complement CDFs of $Y$ in the cases of LoS or NLoS propagation, respectively.

Using the CDF and PDF in (28) and (29), and after some manipulations, we can express $J_3$ as
\[
J_3 = \int_0^\infty \Pr \left( Y > \frac{\gamma_h (\xi_1 a_1 P_S x + 1)}{a_2 P_S} \right) f_y(x)dy \tag{46}
\]

Finally, we compute the third component $O_5$ of (39) as
\[
O_5 = \Pr \left( \|\mathbf{f}_2\|^2 > \frac{\gamma_h}{P_U(b_2 - b_1)} \right) \text{LoS} = \int \Pr \left( \|\mathbf{f}_2\|^2 > \frac{\gamma_h}{P_U(b_2 - b_1)} \right) f_x(x)dx \tag{47}
\]

\[+ \int \Pr \left( \|\mathbf{f}_2\|^2 > \frac{\gamma_h}{P_U(b_2 - b_1)} \right) f_x(x)dx \tag{48}
\]

From (28), we have the closed-form expressions of $J_5$ and $J_6$ as (25) and (26), respectively.

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LE THE DUNG (Member, IEEE) received the B.S. degree in electronics and telecommunication engineering from the Ho Chi Minh City University of Technology, Ho Chi Minh City, Vietnam, in 2008, and the M.S. and Ph.D. degrees in electronics and computer engineering from Hongik University, Seoul, South Korea, in 2012 and 2016, respectively. From 2007 to 2010, he joined Signet Design Solutions Vietnam, as a Hardware Design Engineer. He has been a Postdoctoral Research Fellow with Chungbuk National University, since May 2016. He has more than 50 papers in refereed international journals and conferences. His major research interests include routing protocols, network coding, network stability analysis and optimization in mobile ad hoc networks, cognitive radio ad hoc networks, and visible light communication networks. He was a recipient of the IEEE IS3C2016 Best Paper Award.

TAEJOON KIM (Member, IEEE) received the B.S. degree in electronics engineering from Yonsei University, Seoul, South Korea, in 2003, and the Ph.D. degree in electrical engineering from the Korea Advanced Institute of Science and Technology (KAIST), Daejeon, South Korea, in 2011. From 2003 to 2005, he was a Researcher with LG Electronics, Seoul, South Korea. From 2011 to 2013, he was a Senior Researcher with the Electronics and Telecommunications Research Institute (ETRI), Daejeon. He is currently an Associate Professor with the School of Information and Communication Engineering, Chungbuk National University, Chungju, South Korea. His research areas include communication theory and analysis, and optimization of wireless networks.