Effects of the surface crack shape on $J$ values along the front of an elliptical crack

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Abstract

Surface cracks are one of common forms of flaws in thin-walled structures such as pressure vessels, oil, and gas pipelines. Accurate evaluation of the growth driving force of such surface cracks is important for integrity analyses of these structures. In this study, the combined effect of the depth and the length of a given surface crack under tension was analyzed using combined elastoplastic finite element analysis (EPFEA) and crack propagation experiments with selected crack shapes. Based on the consideration of the distribution profile of the $J$ integral as the crack growth driving force along crack front, the crack growth stability with different crack shapes was analyzed. Finite element analysis (FEA) results showed that the growth from partial to through-wall penetration is affected by the shape of the initial crack. As shown by the distribution profile of the $J$ integral along the crack front, the location(s) of the maximum $J$, that is, the highest crack driving force, is (are) found to vary with the crack shape and development.

KEYWORDS

crack growth, elastoplastic finite element analysis (EPFEA), $J$ integral, surface crack

1 | INTRODUCTION

Pressurized components for storage or transportation of static or circulating fluids are commonly used in the industry, such as the nuclear, chemical, oil, and gas sector. These critical components are sometimes utilized up to their design capacity during services. Operators may also look into potentials to extend the service life of ageing installations. The structural integrity assessment of these components is thus required...
to ensure adequate safety margins can be maintained at all time.

Substantial research has been carried out on surface cracks subjected to tension or bending loads. One of the earliest studies investigating the evolution of surface cracks on pressure vessels was presented by Irwin in 1960s. Since then, a number of researches have been carried out, looking at both surface cracks and through-wall cracks in plates and pressure pipes. For instance, an idealized through-wall crack with straight crack sides perpendicular to the inner surface of a pipe is typically postulated by several codes and standards. Any crack growth is assumed to happen by extending the crack length but maintaining the same crack shape. However, in reality, a surface crack may change its shape while growing through the wall thickness and break through at its deepest point. This simplified fixed crack shape would sometimes overestimate the leak rate. Numerical simulations show that the shape of a semi-elliptical crack could affect the stability of a leaking crack in pressure pipes.

The Raju–Newman (R-N) solution based on analytical expressions of stress intensity factor (SIF) has been developed and applied to evaluate critical pressurized components. FEA has also become readily available and widely accepted for the analysis of mixed-mode cracks using a singular structural element. In comparison, SIF-based calculation lacks consideration of plasticity of the material and thus is not applicable for cases where the plastic zone in front of the crack tip cannot be treated as negligible. In such cases, the J integral developed by Cherepanov and Rice is more appropriate to calculate the crack driving force with nonnegligible plastic deformation.

More recently, different shapes of semi-elliptic cracks were analyzed numerically for the evolution of SIF and \( J \) integral, but these investigations need to be further verified by experiments. A recent study on environmentally assisted cracking (EAC) through combined experiments and FEA on crack growth in light-water reactor components shows that it is feasible to obtain mechanical properties along a semi-elliptic crack front under complex loading conditions, though less emphasis has been put on the growth of the initial crack, or on the driving force at the crack tip during the evolution of the crack shape. Several recent studies have predicted the effect of crack shape on SIF and crack opening areas along the crack front, and results show that finite element solutions for the SIF associated with crack shapes could be used to perform fatigue life calculation.

There are still uncertainties in the effect of the evolving shape of a surface crack on the shift of the maximum SIF or \( J \) location along its crack front. To further understanding the link between the crack growth driving force and the crack shape, and for potential reduction of unnecessary conservatively in structure design and maintenance, a more thorough investigation into the crack shape evolution through the plate thickness will be beneficial. This study has looked into detailed profiles of the crack driving force along the whole crack front, including its deepest point and surface points, as well as the point of the highest \( J \) integral, which could be moving between the deepest point and the surface points. Both experimental and numerical studies were performed to investigate these features as the crack shape evolves.

Fatigue tests on deeply notched plates with two different aspect ratios (short and long in terms of the crack depth over its surface length) were carried out. Strains on the back surface of the specimens were recorded using strain gauges. Cracked section surfaces were then examined using scanning electron microscopy (SEM). Finite element (FE) analyses were first validated with the experimental results, and further analyses were then carried out to investigate the \( J \) integral profile over a range of crack shapes. The outcome helps to improve understanding of the development of cracks of different shapes.

## 2 | EXPERIMENTS

### 2.1 | Material and uniaxial tensile tests

P690 high strength steel was selected for crack experiments. The chemical compositions of the material are shown in Table 1.

All tests were carried out in accordance with the BS EN ISO 6892-1:2009 B at the National Structural Integrity Research Centre based at TWI Ltd (Cambridge, UK). The round tensile samples were machined from raw material plates. The schematic of sample and the mechanical response of P690 steel are shown in Figure 1.

### 2.2 | Experiment procedures

Dumbbell-shaped specimens (400 \( \times \) 150 \( \times \) 22 mm) were machined with a semi-elliptical surface crack. The crack was produced by notching to a depth of approximately 80% of the plate thickness using electro-discharge machining (EDM). Dimensions of machined specimens

| C  | Si  | Mn  | P   | S   | Cr  | Mo  | Ni  |
|----|-----|-----|-----|-----|-----|-----|-----|
| 0.2 | 0.31 | 1.43 | 0.007 | 0.005 | 0.22 | 0.53 | 0.59 |
and details of the notch are shown in Figure 2A,B. The edges of specimens have been rounded to avoid undesired fatigue initiation or secondary cracking. Two different surface crack geometries were produced and from this point onwards, are described as the “short” and “long” cracks, respectively. For the “short” crack, the length of the deepest point of the crack $a$ is 11.1 mm, and the crack surface length $2c$ is 58.9 mm. For the “long” crack, the length of the deepest point of the crack $a$ is 11.5 mm, and the crack surface length $2c$ is 24 mm. The sample thickness $t$ in both cases is 15 mm.

Strain gauges were attached on the free surface of the plates to record changes of the strain field. One 6-mm uniaxial strain gauge (TML FLA-6-11) and sixteen 1-mm uniaxial strain gauges (TML FLA-1-11) were attached to Specimen S-01, as shown in Figure 2C,D,

- Gauge 1, and 7 to 14 are along the crack surface length in the X-axis, with Gauge 1 being at the location corresponding to the crack center,
- Gauge 2 to 6 are along the Y-axis, perpendicular to the plane of the crack
- Gauge 15 to 17 are along a 45-degree diagonal axis to the plane of the crack

All gauges were put in the Y direction, measuring $\varepsilon_{yy}$. For Gauges 1 to 14, the center-to-center distance was kept at 10 mm, and Gauges 15 to 17 along the 45-degree line were put in locations with the $x$-$y$ coordinates corresponding to other gauges in the $X$ and $Y$-axes.

The specimens were subject to cyclic loading in the $Y$ direction using a servo-hydraulic test machine. Tests were conducted using the load control with a constant sinusoidal fluctuating tensile mean load of 135 kN and a load ratio $R = 0.1$. The maximum loading was 150 kN or 100 MPa, which is approximately 15% of the material’s yield strength. Tests were terminated when the through-wall crack had extended to the vicinity of Gauge 13 or/and 14. The specimen was then removed from the machine and broken apart to examine the fracture surface.

3 | FINITE ELEMENT ANALYSIS

3.1 | Material model

The mechanical properties of the material used in the numerical analyses are as follows: the Young’s modulus is 217 GPa, the yield strength is 638.2 MPa, and the Poisson’s ratio is 0.3. It should be noted that the value of Young’s modulus is calculated use the stress–strain data from the extensometer. The true stress–strain data from the uniaxial tensile test were used as the plastic parameters in the simulation process.

3.2 | Geometric model

A 3D model containing a surface crack was analyzed using ABAQUS. A quarter axisymmetric model of the specimen was analyzed using the sub-model technique with a refined mesh based on interpolation of the solution from an initial relatively coarse global model. This is to obtain an accurate, detailed solution in the crack.
region for the strain field and the $J$ integral along the crack tip.\textsuperscript{22,23}

The geometric and load parameters of the experiments were first modeled to compare with the experimental results. Once the model was verified, different geometric parameters of the surface crack were studied as illustrated in fatigue experiment with different dimensions of cracks for better understanding of crack growths. Details are given in Table 2, including the two experimental cases of short and long cracks. These crack types have been classified in terms of the depth and the surface length, respectively. The thickness of the model was kept at 15 mm, the same as the tested specimens.

### 3.3 The mesh and element choice

The mesh of the model is shown in Figure 3, where the $Z$-axis is in the opposite direction of the crack growth through the thickness, and the $Y$-axis is normal to the crack plane. Both the element type and mesh density were selected based convergence tests of $J$ integral. Elements used in both the global model and sub-model are C3D20 (20-node quadratic brick). And the final global model and the sub-model have 14,596 and 40,952 elements, respectively. The contour integral method was used to determine the $J$ integral along the crack front.\textsuperscript{23} The number of contours used in calculation was 10, and the $J$ integral was determined upon convergence.
**TABLE 2** Parameters of the crack shape (mm)

| Crack depth variation group | No.   | S-01 | S-02 | S-03 | S-04 | S-05 | S-06 | S-07 (tested) |
|-----------------------------|-------|------|------|------|------|------|------|---------------|
| a                           | 3.5   | 5.5  | 6.5  | 7.0  | 7.5  | 9.5  | 11.5 |
| c                           | 12    |      |      |      |      |      |      |               |
| No.                         | M-01  | M-02 | M-03 | M-04 | M-05 | M-06 |
| a                           | 5.5   | 6.5  | 7.5  | 8.0  | 8.5  | 9.5  |
| c                           | 20    |      |      |      |      |      |      |               |
| No.                         | L-01  | L-02 | L-03 | L-04 | L-05 (tested) | L-06 |
| a                           | 11.1  |      |      |      | 29.45 |      |
| c                           | 12    | 16   | 20   | 24   | 33    |      |

**FIGURE 3** Mesh of the 3D model. (A) Global mesh and (B) sub-model [Colour figure can be viewed at wileyonlinelibrary.com]

## 4 EXPERIMENTAL RESULTS

Fatigue tests were terminated when the through-wall cracks reached one of the last strain gauges along the X-axis, that is, Gauge 13 or 14. The specimen was then removed from the machine and broken apart to examine the fracture surface. The fracture surface from Specimens L-05 and S-07 are shown in Figure 4A,B, respectively. The crack lengths from the blue line to the red line in Figure 4A,B were measured by using ruler. The average crack growth rates for the short and long cracks at the deepest (A) and surface (B and C) points from initial shape to penetration are listed in Table 3.

Table 3 appears to indicate that the growth rate at the deepest point (A) of the long crack is higher than those at the surface points (B and C), and that for the short crack is comparable to, or slightly lower than those at the surface points. However, it needs to point out that this is based on only one test of each of the two crack profiles, and repeated tests are needed to draw more definite conclusions. It should be noted that the Paris law with “C” being $1.36 \times 10^{-7}$ and “m” 2.25 for P690 steel.

Fracture surfaces were cleaned before further examination by SEM. The application of a soap water solution during the test to mark fatigue rings has unfortunately oxidized the fracture surface, and the corrosion may have weakened the physical features of the fatigue crack growth, but these features are still clearly visible. Figure 5A illustrates the locations where SEM photographs were taken. Four distinct regions on the fractured section surface have been identified:

1. Close to the deepest crack tip (Specimen L-05)
2. Close to the break-through location (Specimen S-07)
3. Further away from the break-through location (Specimen L-05)
4. Post break-through fractured area (Specimen S-07)

The SEM photos of each area in Figure 5A are given in Figure 5B–E. Figure 5B shows the interface between the initial notch and the fatigue crack. Typical initiation of fatigue crack can be observed along the boundary. The presence of numerous pits indicated corrosion attack due to soap water. Figure 5C shows the area adjacent to the break-through point on the free surface. Three distinct regions have been observed. Classic fatigue patterns could be seen until the last stage of propagation. Once the crack broke through,
another region of transition began and followed by the formation of a lip before the final rupture. Figure 5D shows the transition area. A transition consisting of classic fatigue, crack blunting, and corrosion was also evident. Figure 5E was taken at a region in the “far field” showing the post breakthrough crack development. The propagation was dominated by classic fatigue.
5 | SIMULATION RESULTS AND ANALYSES

5.1 | Comparison of strains from experiments and simulations

To verify the correctness of the FEA model, strains on the free surface of FEA model were extracted, and an

| Point                  | Long crack (L-05) | Short crack (S-07) |
|------------------------|-------------------|--------------------|
| Deepest points-A       | $2.8 \times 10^{-5}$ mm/cyc | $4.9 \times 10^{-6}$ mm/cyc |
| Surface points-B       | $5.0 \times 10^{-6}$ mm/cyc | $8.0 \times 10^{-6}$ mm/cyc |
| Surface points-C       | $1.3 \times 10^{-5}$ mm/cyc | $5.2 \times 10^{-6}$ mm/cyc |

FIGURE 5  Schematic of the location and SEM analysis photographs. (A) Schematic of the location, (B) location 1(L), (C) location 2(S), (D) location 3(L), and (E) location 4(S) [Colour figure can be viewed at wileyonlinelibrary.com]
average value of simulated strains was calculated corresponding to the local area covered by individual strain gauges. For both short and long surface cracks, a good agreement can be seen in Figure 6A–F. Strain distributions along the X-axis exhibit a W-shaped profile (Figure 6A,B). On the other hand, distributions along the Y-axis show a M-shaped profile (Figure 6C,D). Figure 6E, F gives strains along the diagonal axis, reaching the lowest value at the crack center.

On average, simulated results are approximately 8% lower than the strain gauge reading. The difference is likely due to variations in the dimension of the specimens, including the prefabricated crack, compared to FE models where idealized geometries were employed. Possible variations in material properties may also contribute to the difference.

5.2 | J integral from FEA

The J integral, as the driving force of a crack, represents the potential or likelihood of the crack growth. Values of

![Figure 6](https://wileyonlinelibrary.com)
the $J$ integral along the crack front were extracted from simulations to understand the trend of crack propagation.

5.2.1 $J$ integral in tested specimens

Figure 7 shows the calculated value of the $J$ integral along the whole crack front for both tested specimens. It can be seen that the distribution of $J$ exhibits different profiles. For the long crack in Specimen L-05 in Figure 7A, the $J$ profile exhibits a local minimum at the crack center location (A), then along the X-direction, gradually increases in value in a concave (U-shaped) profile, reaching a symmetric local maximum at about 19 mm (Location B) from the crack center. It then reduces in value rapidly towards the sides of the crack. The difference in the $J$ values between the local minimum (A) and the maxima (B) is approximately 10%.

For the short crack shown in Figure 7B of Specimen S-05, the $J$ integral has a minimum (A) at the crack center, then along the X-direction increases in a concave profile (U-shaped), reaching the symmetric maximum (B) at the crack end with a sudden cliff-typed drop. Apart from the differences in the distribution profile, the $J$ integral of the short crack is noticeably lower than that of the long one even with the same crack depth.

The two distinct profiles of the $J$ integral in Figure 7 appear to explain the different crack development patterns observed in the fractured section surface of the two tested specimens: the long crack propagates primarily in the thickness direction with two maxima between the crack center and the sides. The low value of $J$ towards the crack’s two ends yields in little increase in the crack

**Figure 7** The $J$ integral across the crack front for two types of crack shapes. (A) Long crack and (B) short crack [Colour figure can be viewed at wileyonlinelibrary.com]
In contrast, the maxima at the two ends in the short crack lead to a noticeable increase in the crack length, together with the growth in depth.

To further explore the influence of the crack geometrical effect, simulation results are discussed in following sections on different values of the aspect ratio \( a/c \), that is, the crack depth to the half of the crack surface length, in different combination groups of fixed depth versus various lengths, and fixed length versus various depths. It should be noted that the calculated values of \( J \) are unique to materials of this stress–strain behavior, and also specific to the thickness specified in the model.

### 5.2.2 \( J \) integral of cracks of a fixed depth \( a = 11.1 \) mm

Figure 8A illustrates the \( J \) profile of cracks of a fixed depth of 11.1 mm. Apart from the increase in \( J \) with respect to the crack length, the change of the \( J \) distribution remains broadly similar, with the local minimum remaining at the crack center, and the maxima close to the crack sides, but moving from the very ends towards the center in terms of the crack length increase.

The relative location of the local \( J \) maxima with respect to the crack length is shown in Figure 8B. The term \( d_{J\text{max}} \) is introduced here, defined as the distance in the X-direction between the \( J \) maximum location and the crack center. The decreasing \( d_{J\text{max}}/c \) indicates that the \( J \) maximum location moves from the side of the crack \((d_{J\text{max}}/c = 1)\) towards the center \((d_{J\text{max}}/c < 1)\), which can be interpreted as that longer cracks have more driving forces to grow in the depth, and less in the length, in confirmation with the observation of the tested sample.

Figure 8B also shows that longer cracks have higher \( J \) values.

### 5.2.3 \( J \) integral of short cracks with a fixed length \( c = 12 \) mm

Figure 9A shows the \( J \) profiles of short cracks with a fixed length of 12 mm. In terms of the crack depth, the \( J \) profiles vary from convex (upside down U shape with the maximum at the crack center) to concave (U shape with the minimum at the crack center).

A key difference to Figure 8A is that the depth change yields a plateau profile when the crack depth is around 7.5 mm, which is about half of the plate thickness, where \( J \) remains a constant value along most of the central length of the crack. Then, for deeper cracks, the maximum locations almost occur at (jump to) the side of the crack. There are no cases of a clear local maximum \( J \) occurring at locations between the crack center and the sides.

The swift change of the maximum \( J \) location from the crack center to the end (through a plateau stage) appears to indicate that when the crack depth is less than half of the plate thickness, the crack grows mainly in depth, but when the depth is more than half of the plate thickness, the crack tends to have more growth in length.

Figure 9B shows the relative location of the local \( J \) maximum with respect to the crack depth. It can be clearly seen that the local maximum remains at the crack center \((d_{J\text{max}}/c = 0)\) until the crack depth is close to half of the plate thickness where the \( J \) shows a long plateau profile across most of the center part of the crack, then with the maximum \( J \) emerging close to the sides \((d_{J\text{max}}/c < 1)\).
\( c = 1 \) as the crack grows deeper. This is very different from that in Figure 8B. It also reveals that deeper cracks have higher \( J \) values.

### 5.2.4 \( J \) integral of a long crack (\( c = 20 \) mm) of different depths

Compared with Figure 9A, Figure 10A shows the \( J \) distribution of a long crack (\( c = 20 \) mm) of different depths. As the depth increases, the \( J \) profile changes from convex (upside down U) with a single maximum at the crack center to two maxima near the ends (M-shape). A plateau profile across an extent of the central part of the crack can be observed when the crack depth is between 8 and 8.5 mm.

Comparison between Figures 9B and 10B shows similar changes in \( J \) distributions in long and short cracks of fixed lengths, correspondingly. The position of the maximum \( J \) gradually moves from the crack center towards the two sides in terms of the crack depth, and always through a plateau stage. This suggests that shallow cracks are more likely to grow in depth, and deep cracks grow in length.

### 5.3 \( J \) integral and the crack ligament

The ratio of the crack ligament (the plate thickness minus the crack depth) to the half-length of crack \( (t-a)/c \) is introduced to define the crack shape change here, and the maximum value of the \( J \) integral is used to

![Figure 9](image1.png)  
**Figure 9** \( J \) integral of short cracks with a fixed length \( c = 12 \) mm. (A) Distribution of \( J \) integral along the crack front for cracks of different depths and (B) relationship between the maximum \( J \) integral value, the maximum \( J \) integral position for cracks of various depths [Colour figure can be viewed at wileyonlinelibrary.com]

![Figure 10](image2.png)  
**Figure 10** \( J \) integral of short cracks with a fixed length \( c = 20 \) mm. (A) Distribution of \( J \) integral along the crack front for cracks of different depths and (B) relationship between the maximum \( J \) integral value, the maximum \( J \) integral position for cracks of various depths [Colour figure can be viewed at wileyonlinelibrary.com]
characterize the driving force. Figure 11 illustrates that \((t-a)/c\) is linearly related to the maximum \(J\) integral value in logarithm scales. The slope of the line is approximately \(-0.75\).

The linear fitting shown in Figure 11 may possibly be used to assess the potential of crack growth in terms of the crack depth, as the maximum \(J\) is proportional to the ligament length in a power law.

5.4 Comparison of SIF from ABAQUS and R-N solution

The SIF is a widely used parameter to characterize the driving force of crack propagation, particularly in industrial codes and standards, such as BS7910,24 an industrial guide for assessment of metallic components with defects.

In this work, for comparison purposes, \(K_J\) denotes values obtained by converting the \(J\) value from ABAQUS to SIF using the following equation when the crack front under different stress state. Though \(J\) calculation in FEA was based on an elastoplastic fracture theory, the conversion of \(J\) into SIF will treat the analysis under an assumption of linear-elastic fracture:

\[
K_J = \begin{cases} \sqrt{EJ}, & \text{plane stress} \\ \sqrt{EJ / (1 - \nu^2)}, & \text{plane strain} \end{cases}
\] (1)

where \(E\) is the Young’s modulus, \(J\) the \(J\) integral from ABAQUS, and \(\nu\) the Poisson’s ratio.

The \(R-N\) model is a widely adopted analytical solution based on linear-elastic fracture to characterize the driving force of crack propagation.11 The SIF at the crack tip under tension loading is given as

\[
K_{RN} = S_t \sqrt{\pi \left(\frac{a}{Q}\right)} F_S \left(\frac{a}{t}, \frac{a}{c}, \frac{c}{b}, \phi\right)
\] (2)

for the condition \(0 < a/c \leq 2.0, 0 \leq a/t < 1.0, 0 \leq \phi \leq \pi\) and \(c/b < 0.8\).11,24

The remote uniform-tension stress is \(S_t\). Other parameters are given as

\[
Q = \begin{cases} 1 + 1.464\left(\frac{a}{c}\right)^{1.65} & (0 < \frac{a}{c} \leq 1) \\ 1 + 1.464\left(\frac{a}{c}\right)^{1.65} & (1 < \frac{a}{c} \leq 2) \end{cases}
\] (3)

\[
F_S = \left[ M_1 + M_2 \left(\frac{a}{t}\right)^2 + M_3 \left(\frac{a}{t}\right)^4 \right] f_{\phi} f_w
\] (4)

For \(a/c \leq 1.0:\)

\[
M_1 = 1.13 - 0.09 \left(\frac{a}{c}\right)
\] (5)

\[
M_2 = -0.54 + \frac{0.89}{0.2 + a/c}
\] (6)

\[
M_3 = 0.5 - \frac{1}{0.65 + a/c} + 14 \left(1 - \frac{a}{c}\right)^{24}
\] (7)

The relationship between maximum \(J\) integral and \((t-a)/c\) [Colour figure can be viewed at wileyonlinelibrary.com]
Figure 12 shows the calculated $K$ (from ABAQUS with elastic material properties only), $K_J$ and $K_{RN}$ for specimens L-05 and S-07, respectively. For L-05, the distribution profiles of the two cracks in Figure 12A are different. $K_{max}$ from R-N solution (or $K_{RNmax}$) is notably higher than $K_{Jmax}$ from FEA. And their positions are also different. It is interesting to see that the values of $K_{RN}$ on the crack's two end are not far from that of the $K_{Jmax}$, which is neither at the crack surface point nor the deepest one, but between. R-N solution appears to be on the conservative side compared with FEA results, especially at the crack's surface points. The fatigue test shows that crack grow relatively slowly at the surface points (see Figure 4A), but the R-N solution indicates high SIF, that is, large driving force, there.

The results for Specimen S-07 of a shorter crack are better matched, both in terms of the magnitude and the location of $K_{max}$, though $K_{Jmax}$ appears to be 2% higher than $K_{RNmax}$ on the surface points. $K_{RN}$ distribution among the crack front almost coincides with that of $K_J$, both with the highest value at the surface points, which are consistent with the fatigue test showing crack length growth (see Figure 4B).

**Figure 12** Comparison of $K$ from $J$ integral and Raju–Newman solution. (A) Long crack (L-05) and (B) short crack (S-07) [Colour figure can be viewed at wileyonlinelibrary.com]
FIGURE 13  Comparison of $K$ from $J$ integral and Raju–Newman solution. (A) L-01, (B) L-02, (C) L-03, (D) L-04, (E) L-05, and (F) L-06 [Colour figure can be viewed at wileyonlinelibrary.com]
Figure 12 illustrates that the calculated value of $K_J$ (plane strain) is virtually identical to that of $K$ with elastic material properties only for both long and short cracks, while the $K_J$ (plane stress) is close to but lower than of $K$. This shows that the state of the crack front is closer to plane strain under the specified load conditions. Therefore, the conversion between $J$ and $K$ was carried out using the formula in the plane strain state in this study.

To further compare the SIFs from FEA and $R-N$ solution, more crack shapes with different depths and lengths were studied.

5.4.1 $K_{RN}$ and $K_J$ of cracks of a fixed depth $a = 11.1$ mm

Figure 13A to G illustrates the SIF profile of cracks in Table 1 of a fixed depth of 11.1 mm (L-series). The results for Specimen L-01 are similar. However, overall, $K_{RN_{max}}$ are generally higher than those from FEA, and the highest $K$ positions are different. It is interesting to see that positions of $K_{J_{max}}$ from FEA are neither at the surface point nor the deepest one, but somewhere between. $R-N$ results, on the other hand,
FIGURE 15  Comparison of $K$ from $J$ integral and Raju-Newman solution. (A) M-01, (B) M-02, (C) M-03, (D) M-04, (E) M-05, and (F) M-06 [Colour figure can be viewed at wileyonlinelibrary.com]
always appear either at the surface point or the deepest one.

5.4.2  $K_{RN}$ and $K_J$ of cracks of a fixed length $c = 12$ mm

Figure 14A to G illustrates the SIF profile of cracks in Table 1 of a fixed length of 12 mm (S-series). There are some differences in the profile of $K$ distribution, and the $R-N$ model appears to be more conservative in $K_{max}$. Overall, differences are not significant.

5.4.3  $K_{RN}$ and $K_J$ of cracks of a fixed length $c = 20$ mm

Figure 15A to G illustrates the SIF profile of cracks in Table 1 of a fixed length of 20 mm (M-series). Again, it shows the $R-N$ model being more conservative in terms of the maximum SIF value. It is interesting to see that in some cases (e.g., M-04, M-05, and L-03) $K_{RN}$ remains constant over the entire crack front including at the surface points, $K_J$ from FEA showing sharp drops.

6  CONCLUSIONS

A three-dimensional finite element analysis has been carried out on semi-elliptical cracks in a thick plate. The model was verified by fatigue tests, and $J$ integrals along the crack tip were calculated in terms of the crack depth and length. It is also important to point out that $R-N$ solutions are based on linear-elastic fracture theory thus has specific conditions on geometry (Equation 2). In comparison, simulations based on elastoplastic FEA provide full $J$ analyses with no similar constraints.

Main conclusions are as follows:

1. The location of the highest $J$ is not fixed and varies, depending on the depth and length of the crack. The profile of the $J$ value along the crack can be of a upside down U-shape with a single maximum at the deepest point of the crack, or an M-shape with two symmetric highest $J$ at the crack surface points or between the surface points and the deepest crack point.

2. The ratio of the crack ligament ($t-a$) to its side length $c$ is negatively linear to the maximum $J$ integral value in logarithm scales.

3. $R-N$ solution is generally more conservative for the maximum $K$ compared with that from FEA (ABAQUS).

4. Based on the limited tests of this study, fatigue crack propagation may be dependent on the aspect ratio/geometry of the initial flaw size. The measured average crack growth rates at the deepest and the side points seem to indicate that long cracks may propagate more in depth and less in length, whereas short cracks will grow in both depth and length, possibly more in the latter. Clearly more tests are needed to further investigate this aspect.

For purpose of structural integrity analyses, many holders of industrial codes and standards are taking efforts to reduce unnecessary conservatism in the guidance to engineering practices. Comparisons between the $R-N$ model and $J$ solution from FEA may help to achieve this purpose by providing a better understanding on the prediction of the crack driving force.

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CONFLICT OF INTEREST

The authors declare that there is no conflict of interest regarding the publication of this manuscript. We confirm that the mentioned funding in the “Acknowledgment” section did not lead to any conflict of interests regarding the publication of this manuscript, and there is no other possible conflict of interests in the manuscript.

AUTHOR CONTRIBUTIONS

Conceptualization: Bin Wang and Yin-Jin Janin. Data curation: Shuai Wang and Renaud Bourga. Formal analysis: Shuai Wang and Renaud Bourga. Funding acquisition: Bin Wang and He Xue. Investigation: Shuai Wang and Renaud Bourga. Methodology: Bin Wang and Yin-Jin Janin. Project administration: Bin Wang and He Xue. Writing—original draft: Shuai Wang. Writing—review and editing: Bin Wang and Yin-Jin Janin.

DATA AVAILABILITY STATEMENT

All data and models used during the study appear in the submitted article.

NOMENCLATURE

$\alpha$  crack depth
$b$  Half-width of cracked plate
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