Manifestation of cluster effects in collective octupole and superdeformed states of heavy nuclei.

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Abstract. The effects of reflection-asymmetric deformation on the properties of the low-lying negative-parity collective states and superdeformed states of heavy nuclei are analyzed basing on dinuclear model. The results of consideration of the alternating parity bands in actinides and the superdeformed bands in $^{60}$Zn, Pb and Hg isotopes are discussed.

1. Introduction

There are several approaches to treat the collective motion related to reflection-asymmetric degrees of freedom. One of them is based on the assumption that the most suitable conditions for manifestation of softness with respect to octupole vibrations or even octupole deformation are realized in nuclei with a single particle level scheme in which two single particle levels with $\Delta l = \Delta j = 3$ are very close together and close to the Fermi surface. One of them is located below and the other above Fermi surface [1]. These two single particle states are connected by a strong E3 transition matrix element. This effect is stronger if this situation is realized in both proton and neutron systems.

At the same time the low-lying negative parity states are known in light nuclei, for instance, in $^{20}$Ne where $\Delta l = \Delta j = 3$ pair of the single particle levels is characterized by a too large excitation energy. It is commonly assumed that octupole collectivity in light nuclei is related to the $\alpha$-clustering [2,3]. Several Nilsson-Strutinsky-type calculations for light nuclei demonstrate that nuclear configurations corresponding to the minima of the potential energy are characterized by density profiles clearly demonstrating cluster structure.

It was also suggested in the framework of the Interacting Boson Model that the collective low-lying negative-parity states in heavy nuclei like Ra and Th isotopes are related to manifestation of $\alpha$-clustering in these nuclei [4]. Later it was indicated on the correlations between the $\alpha$-decay hindrance factor and the excitation energy of the lowest $1^-$ state in heavy nuclei [5].

2. Dinuclear system concept

At the end of 60th deep inelastic collisions (DIC) of heavy ions have been discovered [6,7]. The characteristic feature of these reactions is large variation of mass (charge) distribution. To interpret the experimental data on DIC it was assumed that during the collision a configuration of two touching nuclei (clusters) which keep their individuality is formed. This configuration was named by V.V.Volkov [8] as a dinuclear system. Such system has two main degrees of freedom which govern its dynamics: the relative motion between the nuclei and a multinucleon transfer.
leading to dependence of dynamics on mass and charge asymmetries. A multinucleon transfer is described by the mass and charge asymmetry coordinates

$$\eta = \frac{A_1 - A_2}{A_1 + A_2}$$  \hspace{1cm} (1)

and

$$\eta_Z = \frac{Z_1 - Z_2}{Z_1 + Z_2}$$  \hspace{1cm} (2)

These coordinates can be assumed as continuous or discrete quantities. For \(\eta = \eta_Z = 0\) we have a symmetric clusterization with two equal nuclei, and if \(\eta\) approaches the values \(\pm 1\) or if \(A_1\) or \(A_2\) is equal to zero a mononucleus is formed. These coordinates have been originally introduced by W. Greiner [9] and applied for description of mass distribution in fission [10]. In [11] it was suggested to apply dinuclear system concept to description of the reflection-asymmetric low-lying collective states.

3. Ground state alternating parity bands. Parity splitting

The degrees of freedom chosen to characterize a dinuclear system are related to description of rotation of a dinuclear system as a whole, and a transfer of nucleons between the fragments. The Hamiltonian of the model can be presented in the form

$$H = -\frac{\hbar^2}{2m} \frac{d}{d\eta} B(\eta) \frac{d}{d\eta} + U(\eta, I)$$  \hspace{1cm} (3)

The method of calculation of the inertia coefficient \(B(\eta)\) has been suggested in [12]. Our calculations have shown that \(B(\eta)\) is a smooth function of the mass number \(A\). As a consequence, we take nearly the same value of \(B(\eta) = 20 \times 10^4 m \cdot fm^2\) for almost all considered actinide nuclei. Here \(m\) is the nucleon mass. The potential energy \(U(\eta, I)\) is determined by the binding energies \((B_{1,2})\) of nuclei forming dinuclear system and their nuclear \(V_N\) and Coulomb \(V_{Coul}\) interaction, and by the rotational energy of a dinuclear system as a whole \(V_{rot}\)

$$U(\eta, I) = B_1(\eta) + B_2(\eta) - B + V(R = R_m, \eta, I),$$  \hspace{1cm} (4)

$$V(R, \eta, I) = V_N(R, \eta) + V_{Coul}(R, \eta) + V_{rot}(R, \eta, I).$$  \hspace{1cm} (5)

Above \(R\) is a distance between the centers of mass of nuclei forming dinuclear system and \(R_m\) is a position of the minimum of the radial potential energy.

It was further shown that among different cluster configurations the only system which gives a significant contribution to the formation of the low-lying states is the \(\alpha\)-cluster configuration. The energies of the \(\alpha\)-cluster configurations are about 15 MeV larger than the binding energies of the considered mononuclei. Therefore, for small excitations only oscillations in \(\eta\) are of interest which lie in the vicinity of \(|\eta| = 1\), i.e., only cluster configurations up to Li clusters need to be considered.

It is seen from the expression of the Hamiltonian (1) that concrete information about a nucleus under consideration is presented only by the binding energies of nuclei forming dinuclear system. Since in all cases \(\alpha\)-particle is presented as a light fragment of a dinuclear system a specific information which vary from nucleus to nucleus is a binding energy of a heavy fragment of a system. However, a binding energy of the heavy fragment is given by the experimental data and, therefore, in dinuclear model evolution of the properties of the low-lying negative parity
states is determined by the experimental data on binding energies. This circumstances give us a possibility to check a correctness of the basic concept of dineuclear model in its application to description of the properties of the low-lying collective negative-parity states. It was especially interesting to check how well dineuclear model describes evolution of the excitation energy of the lowest 1− state with the mass number in the chains of the isotopes of some actinide nuclei where this excitation energy varies significantly.

The results of application of dineuclear system model to description of the excitation energies of the lowest 1− states \( E(1^-) \) in Ra and Th isotopes were quite successful. The model describes quite well evolution of the values of \( E(1^-) \) with the mass number. For instance, the experimental values of the excitation energies of the \( 1^- \) states in Th isotopes decrease from 714 keV in \(^{232}\)Th to 230 keV in \(^{226}\)Th and then keep the values 251 keV and 250 keV in \(^{224}\)Th and \(^{222}\)Th, respectively. The calculated values are: 693 keV in \(^{232}\)Th, 254 keV in \(^{226}\)Th, 204 keV in \(^{224}\)Th and 195 keV in \(^{222}\)Th. The excitation energies of the negative-parity states with higher angular momenta have been also calculated and the results obtained demonstrate a good description of the experimental data.

The results of calculations of the wave functions of the states of the ground state alternating parity bands have shown that the weight of the \( \alpha \)-cluster component in the wave functions of these states increases with angular momentum in the light Ra and Th isotopes from approximately 5% at \( I=1 \) to 40 % at around \( I=20 \). In contrast, in \(^{232}\)Th, which has the largest energy of the \( 1^- \) state the weight of the \( \alpha \)-cluster component at \( I \approx 20 \) does not exceed 15%. The value of the weight of the \( \alpha \)-cluster component at \( I=1 \) equal to 5% is in a correspondence with the spectroscopic factor for \( \alpha \)-decay of the light Ra and Th isotopes. The results obtained also show that the weight of the \( \alpha \)-cluster component in the wave functions of negative-parity states is higher than in the neighboring positive-parity states. This fact is manifested in the staggering of the weight of the \( \alpha \)-cluster component with smooth increase of the angular momentum.

We have also calculated the E1 transition matrix elements. The results obtained show significant increase of the value of \( \langle I\mid Q(E1)\mid I+1 \rangle \) with angular momentum. This is a manifestation of an increase with angular momentum in the wave function of the weight of the \( \alpha \)-cluster component compare to the mononucleus component. Since Z to N ratio in \( \alpha \)-particle is quite different from the Z to N ratio in the heavy cluster (\( A-4, Z-2 \)) the relative motion of the \( \alpha \)-particle and a rest of a nucleus creates strong electric dipole transitional moment. This fact is important for analysis of predictions of different models describing octupole motion in heavy nuclei at high angular momenta.

4. Cluster effects in the structure of the superdeformed band in \(^{60}\)Zn

In many light nuclei clustering is a very prominent feature in a large number of states. Typical examples are \(^{16}\)O and \(^{20}\)Ne where \( ^{12}\)C + \( \alpha \) and \( ^{16}O + \alpha \) structures are particularly stable. It is interesting that clustering in \(^{20}\)Ne is realized in the ground state band, but in \(^{16}\)O cluster structure has the excited band built on 6.06 MeV intruder state. In heavier nuclei \( \alpha \)-cluster structures have been predicted to be stable in \(^{40}\)Ca and \(^{44}\)Ti. In the pairs of nuclei \(^{16}\)O–\(^{20}\)Ne and \(^{40}\)Ca–\(^{44}\)Ti one of the partners is a spherical double magic nucleus and the other is a double magic nucleus plus \( \alpha \)-particle. The next pair of this kind is \(^{56}\)Ni–\(^{60}\)Zn. In \(^{56}\)Ni the excited deformed rotational band is known up to \( I^\pi = 12^+ \). In \(^{60}\)Zn the threshold for the \( \alpha \)-decay is only 2.7 MeV higher than the ground state. Therefore it is quite possible that the ground state band of \(^{60}\)Zn contains the \( \alpha \)-cluster component. However, in \(^{60}\)Zn there is also a superdeformed band. At angular momenta higher than \( I \) only E2 transitions between the states of the superdeformed band have been seen. In the angular momentum region \( I=8−12 \) the superdeformed band decay into the states of the ground state band. Decay of the superdeformed rotational bands into normally deformed or spherical states is one of the interesting nuclear structure phenomena.

In the superdeformed band of \(^{60}\)Zn the moment of inertia depending on \( I \) takes the values
(692-795)m·fm$^2$, where $m$ is the nucleon mass. These values are close to the sticking moment of inertia of the $^{52}$Fe+$^8$Be cluster configuration which is 750$m·fm^2$. We mention also that the threshold energy for decay of $^{60}$Zn into $^{52}$Fe+$^8$Be (10.8 MeV) and $^{48}$Cr+$^{12}$C (11.2 MeV) are close to the extrapolated values of the superdeformed band head, which is approximately 7.5 MeV. Thus, it is quite possible that the superdeformed band in $^{60}$Zn corresponds to the Be cluster configuration. Our calculations for $^{60}$Zn have shown that dinuclear configuration with $\alpha$-cluster as a light cluster has a potential energy smaller than the energy of the mononucleus at $|\eta|=1$. With respect to the ground state energy the potential energy at the $\alpha$-minimum is $-4.5$ MeV. Next important minima correspond to $^8$Be and $^{12}$C cluster configurations with the following values of the potential energy at the minima with respect to the energy of the mononucleus: $5.1$ MeV and $9.0$ MeV, respectively. At the values of $\eta$ corresponding to Li- and B-clusters potential energy has maxima.

We have calculated [13] the branching ratios of the E2 $\Delta I=2$ intensities of the $\gamma$-transitions. In the experiment for the $^{18}_{sd}^+$, $^{16}_{sd}^+$ and $^{14}_{sd}^+$ states only decay into the superdeformed states have been observed. The calculated branching ratios for these transitions are

$$\frac{I(18^+_{sd} \rightarrow 16^+_{gs})}{I(18^+_{sd} \rightarrow 16^+_{sd})} = 0.02,$$

$$\frac{I(16^+_{sd} \rightarrow 14^+_{gs})}{I(16^+_{sd} \rightarrow 14^+_{sd})} = 0.07,$$

$$\frac{I(14^+_{sd} \rightarrow 12^+_{gs})}{I(14^+_{sd} \rightarrow 12^+_{sd})} = 0.18.$$  

For the ratio $I(12^+_{sd} \rightarrow 10^+_{gs})/I(12^+_{sd} \rightarrow 10^+_{sd})$ the calculated value is 0.42. For the ratio $I(10^+_{sd} \rightarrow 8^+_{gs})/I(10^+_{sd} \rightarrow 8^+_{sd})$ the calculated value is 0.63. The calculated value of the $I(8^+_{sd} \rightarrow 6^+_{gs})$ is 0.19 W.u. which is large enough to explain an absence of the $8^+_{sd} \rightarrow 6^+_{sd}$ transition.

5. Decay out of superdeformed bands in the mass region $A \approx 190$ within dinuclear model approach

Over two hundred superdeformed bands have been investigated in different mass regions of the nuclide chart. While the rotational transitions between the superdeformed states are easy to detect with modern Ge arrays, it is hard to localize the superdeformed bands in excitation energy, angular momentum and parity and to link them to the normal deformed bands. This is because of the remarkable feature of the superdeformed states: the intraband E2 rotational transitions follow the band down with practically constant intensity and drop sharply at some angular momentum. This phenomenon is referred to as the decay out of the superdeformed band.

Analysis of the potential energy of dinuclear system has shown that the important minima of the potential energy are: mononucleus, $\alpha$, $^8$Be and $^{12}$C cluster configurations. Dinuclear system with $\alpha$-cluster has a potential energy at $I=0$ which is close to the energy of a mononucleus at $|\eta|=1$. As a consequence, the states of the normal deformed bands have a significant contribution of the $\alpha$-cluster component. The states of the lowest and excited superdeformed bands are described mainly as $^8$Be and $^{12}$C cluster configurations, respectively.

The wave functions of the normal deformed and superdeformed states are well separated at low angular momenta. Only a very small tail of the wave functions of the lowest superdeformed
states in the normal deformed minimum supplies a small coupling of the superdeformed state with a complex spectrum of the compound states in the normal deformed minimum. Since different cluster configurations have different moments of inertia, the potential energy depends on the angular momentum of the system. This is an origin of the angular momentum dependence of the mixing between the superdeformed and the normal deformed states. At large angular momenta the energies of the superdeformed states are lower than the energies of the neighboring collective normal deformed states connect ed to the mass-asymmetry degree of freedom. With decreasing angular momentum the energy interval between these states decreases and at some angular momentum $I$ between 6 and 14 the collective normal deformed state become energetically lower than the superdeformed state with the same angular momentum. In all nuclei belonging to the $A \approx 190$ region considered in the framework of dinuclear model the calculated angular momentum is close to the observed angular momentum at which decay out happens. Thus, the decay out occurs through the normal deformed doorway state. In dinuclear model this normal deformed state is the excited vibrational state corresponding to the motion in mass asymmetry degree of freedom. At high excitation energy in the normal deformed potential well, the normal deformed doorway state is spread among the sea of the dense compound states. This spreading leads to a large width of a doorway state. So, statistical mixing with highly excited normal deformed states is one of the reason for decay out of the superdeformed band. The sudden decay out takes place near a crossing of the superdeformed band with the nearest neighboring excited collective normal deformed band where a weight of the normal deformed doorway state increases. Even at the near band crossing point the normal deformed admixture in the superdeformed state is relatively small but decay out occurs due to the large width of doorway state with respect to the width of the superdeformed state. The maximal normal deformed admixture of the superdeformed states were found to be in a range of a few percent [14].

6. Conclusions
We have shown that there are correlations between an increase of collectivity of octupole mode and manifestation of $\alpha$-clustering. Dinuclear model and driving potential calculated following the rules of the model qualitatively explain evolution of the characteristics of the low-lying octupole states with the atomic mass and charge numbers.

Properties of the superdeformed bands are also qualitatively described in the framework of dinuclear model.

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7. References
[1] Butler P and Nazarewicz W 1996 Rev. Mod. Phys. 68 349
[2] Rae W D M 1988 Int. J. Mod. Phys. A 3 1343
[3] Freer M and Merchant A C 997 J. Phys. G 23 261
[4] Iachello F and Jackson A D 1982 Phys. Lett. B 108 151
[5] Sheline R K and Riley M A 2000 Phys. Rev. C 61 057301
[6] Wilczyński J, Volkov V V, Dozovski P 1967 Yadernaya Fizika 5 942
[7] Gridnev G F, Volkov V V, Wilczyński J 1970 Nucl. Phys. A 142 385
[8] Volkov V V 1986 Izv. AN SSSR ser. fiz. 50 879
[9] Fink H J, Maruhn J, Scheid W and Greiner W 1974 Z. Phys. 268 321
[10] Maruhn J and Greiner W 1974 Phys. Rev. Lett. 32 548
[11] Shneidman T M, Adamian G G, Antonenko N V, Jolos R V 2003 Phys. Rev. C 67 014313
[12] Adamian G G, Antonenko N V, Jolos R V 1995 *Nucl. Phys. A* **584** 205
[13] Adamian G G, Antonenko N V, Jolos R V, Palchikov Yu V, and Scheid W 2003 *Phys. Rev. C* **67** 054303
[14] Adamian G G, Antonenko N V, Jolos R V, Palchikov Yu V, Scheid W, and Shneidman T M 2004 *Phys. Rev. C* **69** 054310