Spin Hall conductance in Y-shaped junction devices

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Abstract. We study the spin Hall effect in Y-shaped junction devices in presence of Rashba spin orbit coupling (RSOC). The voltage and the net spin current registered at one of the arms of the Y-junction are seen to increases although the spin Hall conductance (SHC) diminishes as the strength of the RSOC is increased. This implies the voltage increases further than that of the current, thereby causing a loss of the RSOC. Various other characteristic features obtained from our study include, a perfectly antisymmetric behaviour of the spin current and the SHC with respect to the zero bias, while the voltage shows a symmetric character. Finally a large RSOC completely destroys the SHC, owing to a complete disappearance of the local density of states, thereby reinforcing our earlier claim that RSOC emulates the effect of disorder on the quantum conductance of junction device.

1. Introduction

Nowadays dissipationless spin current \cite{1} has been a central focus of active investigation in mesoscopic physics. Various mechanisms, such as spin field effect transistor (spin-FET) \cite{2}, spin-transfer torque \cite{3, 4}, spin pumping \cite{5} and (inverse) spin Hall effect \cite{6, 7} have been proposed to generate, control and detect the spin current. The spin current generated by these means is in general dissipative in nature. On the other hand, a dissipationless spin current can be generated by an external electric field via intrinsic spin Hall effect \cite{1, 8}. The intrinsic spin Hall effect can be achieved in presence of spin orbit coupling such as Rashba spin orbit coupling (RSOC).

The spin-orbit coupling produces a transverse force on a moving electron and this force tends to form a transverse spin current. In the simplest form, a spin current is the flow of spin-up electrons in one direction, say +\textit{x}, accompanied by the flow of an equal number of spin-down electrons in the opposite direction, −\textit{x}. The total charge current in the \textit{x} direction is therefore zero, that is \textit{I}^\textit{q} = e(I^\textit{↑} + I^\textit{↓}) = 0. The total spin current is finite, that is \textit{I}^\textit{s} = e(I^\textit{↑} − I^\textit{↓}) ≠ 0.

In a two-dimensional electron gas (2DEG) lacking structure inversion symmetry of the confining potential, the Rashba spin orbit coupling (RSOC) can be important. The Hamiltonian for the system in presence of RSOC is written as,

$$H = \frac{\mathbf{p}^2}{2m^*} + \alpha (\sigma^x p_y - \sigma^y p_x)$$  \hfill (1)

where the second term is the Rashba spin-orbit coupling (RSOC). \textit{\sigma^{x/y}} is the \textit{x/y} component of the Pauli matrices, and \textit{\alpha} is the coupling parameter which denotes the strength of RSOC.

A three terminal structure is believed to be a suitable candidate for studying the spin Hall effect \cite{9}. In this paper, we have studied the behaviour of a Y-shaped three terminal structure
with different angular separation between the arms of the Y-shaped geometry as shown in Fig.1. These Y-shaped junction devices can be fabricated experimentally. The technique by which we make these devices for a theoretical investigation is presented in the following section. Specifically, we have studied the conductance properties in order to observe the effects of the angular separation between the two arms of the Y-shaped junction.

We organize our paper as follows. The theoretical formalism leading to the expressions for the spin Hall and longitudinal conductances using Landauer Büttiker formula are presented in section II. Section III includes an elaborate discussion of the results obtained for the spin Hall and longitudinal conductances in presence of spin orbit interaction.

2. Theoretical formulation

We begin our discussion by a prescription of fabricating a Y-shaped junction device which can be interesting from an experimental perspective.

We choose a three-probe measuring set-up as shown in Fig.1 to observe the spin Hall effect. Here the three ideal semi-infinite leads are attached to the central conducting region, which in our case is the Y-shaped device formed via a square lattice geometry and includes spin orbit interaction. In this work, we have taken three different Y-shapes by changing the angle between the two arms of the Y, we call it $\theta_Y$ as shown in Fig.1. The leads shown as 1,2 and 3 are semi-infinite in nature. The voltages applied at the leads are $V_1$, $V_2$ and $V_3$ respectively. The width of the scattering region is $d$, while the arms have width $d/2$.

Fig.2 provides a technique how we can fix the angle, $\theta_Y$. According to Fig.1, $\theta_Y$ is twice the angle as shown in Fig.2. For the Y-shape shown in Fig.1(a), we add the lattice sites in the arms of Y as depicted in Fig.2(a). First we add 3 sites along $x$-axis with spacing ‘$a$’ and then add another site along $y$-axis just above the third site along $x$. We repeat the same procedure to build up the rest of the arm of the Y-geometry. Since we need 3 sites along $x$-axis and 2 sites along $y$-axis we call it a (3,2) scheme. The calculation of the angle in now straightforward from the geometry. From Fig.2, $b$ is the base of the triangle and $h$ is the height. In the (3,2) scheme, $b = 2a$ and $h = a$. Hence, the angular separation between the arms of the Y will be twice the calculated angle and the angle is, $\theta_Y = 2 \tan^{-1} \frac{a}{2h} = 53.13^\circ$.

Similarly, corresponding to $\theta_Y = 90^\circ$, we need the (2,2) scheme, for which 2 sites along $x$-axis and 1 site along $y$-axis above the second site along $x$ are required, as shown in Fig.2(b). In the given case, $\theta_Y = 2 \tan^{-1} \frac{a}{b} = 90^\circ$. Again from Fig.2(c), we should now have (2,3) scheme. The angle is, $\theta_Y = 2 \tan^{-1} \frac{2a}{h} = 128.87^\circ$.

Now an unpolarized charge current is allowed to pass through the longitudinal leads along lead-1 and lead-3 (see Fig.1) inducing spin Hall current along lead-2. Assuming a tight binding approximation with nearest neighbour hopping, Eq.(1) reads as,

$$ H = \epsilon \sum_{i,\sigma} c_{i \sigma}^{\dagger} c_{i \sigma} + t \sum_{\langle ij \rangle, \sigma} c_{i \sigma}^{\dagger} c_{j \sigma} + V_R \sum_i \left[ c_{i \uparrow}^{\dagger} c_{i+\delta_x \downarrow} - c_{i \downarrow}^{\dagger} c_{i+\delta_x \uparrow} \right] - i \left( c_{i \uparrow}^{\dagger} c_{i+\delta_y \downarrow} + c_{i \downarrow}^{\dagger} c_{i+\delta_y \uparrow} \right) \quad (2) $$

Here $\epsilon$ is the on-site potential and $t$ is the hopping strength, $V_R = \frac{2e}{a}$ is the modified Rashba coupling strength with $a$ to be the lattice constant. $\delta_{x/y}$ is the unit vector along $x/y$ direction.

For the three terminal case, the longitudinal and spin Hall conductances are defined as [10],

$$ G_L = \frac{I_3^2}{V_3 - V_1} \quad \text{and} \quad G_{SH} = \frac{\hbar}{2e} \frac{I_2^2}{V_2 - V_1} \quad (3) $$

where $I_3^2$ and $I_2^2$ are the charge and spin currents flowing through the lead-3 and lead-2 respectively. $V_m$ is the potential at the $m$-th lead.

The calculation of the electric and spin currents is based on the Landauer-Büttiker multi-probe formalism [11]. The charge and spin currents flowing through lead $m(=1,2,3)$ with potential, $V_m$ can be written in terms of the spin resolved transmission probability as [9],

$$ I_m^c = \int_{-\infty}^{\infty} \mathcal{F}(E) T_{\uparrow \downarrow}(E, E_m) \frac{dE}{\hbar} $$

$$ I_m^s = \int_{-\infty}^{\infty} \mathcal{F}(E) T_{\downarrow \uparrow}(E, E_m) \frac{dE}{\hbar} $$

where $\mathcal{F}(E)$ is the Fermi function and $T_{\sigma \sigma'}(E, E_m)$ is the transmission probability of the spin-resolved channel from energy $E$ to energy $E_m$. For the lead, the transmission probability is given by the Landauer-Büttiker formula,

$$ T_{\sigma \sigma'}(E, E_m) = \frac{\sqrt{\Gamma_r^{\sigma \sigma'}(E)} \sqrt{\Gamma_l^{\sigma \sigma'}(E_m)}}{\Gamma_r^{\sigma \sigma'}(E) \Gamma_l^{\sigma \sigma'}(E_m)} $$

where $\Gamma_r^{\sigma \sigma'}(E)$ and $\Gamma_l^{\sigma \sigma'}(E_m)$ are the source and drain density of states.
\[ I_m^q = \frac{e^2}{h} \sum_{n \neq m, \sigma, \sigma'} \left( T_{nm}^\sigma V_m - T_{mn}^{\sigma'} V_n \right) \] (4)

and,

\[ I_m^\sigma = \frac{e^2}{h} \sum_{n \neq m, \sigma'} \left[ \left( T_{nm}^{\sigma} - T_{nm}^{-\sigma'} \right) V_m + \left( T_{mn}^{-\sigma} - T_{mn}^{\sigma'} \right) V_n \right] \]

\[ = \frac{e^2}{h} \sum_{n \neq m} \left[ T_{nm}^{\text{out}} V_m - T_{mn}^{\text{in}} V_n \right] \] (5)

where, we have defined two useful quantities as follows,

\[ T_{pq}^{\text{in}} = T_{pq}^{\uparrow\uparrow} + T_{pq}^{\uparrow\downarrow} - T_{pq}^{\downarrow\uparrow} - T_{pq}^{\downarrow\downarrow} \]

\[ T_{pq}^{\text{out}} = T_{pq}^{\uparrow\uparrow} + T_{pq}^{\uparrow\downarrow} - T_{pq}^{\downarrow\uparrow} - T_{pq}^{\downarrow\downarrow} \] (6)

Physically, the term \( \frac{e^2}{h} \sum_{n \neq m} T_{nm}^{\text{out}} V_m \) is the total spin current flowing from the \( m \)-th lead with potential \( V_m \) to all other \( n \) leads, while the term \( \frac{e^2}{h} \sum_{n \neq m} T_{mn}^{\text{in}} V_n \) is the total spin current flowing into the \( m \)-th lead from all other \( n \) leads having potential \( V_n \).

The zero temperature conductance, \( C_{pq}^{\sigma \sigma'} \) that describes the spin resolved transport measurements, is related to the spin resolved transmission coefficient by [12, 13],

\[ C_{pq}^{\sigma \sigma'} = \frac{e^2}{h} T_{pq}^{\sigma \sigma'} (E) \] (7)

The transmission coefficient can be calculated as [14, 15],

\[ T_{pq}^{\sigma \sigma'} = \text{Tr} \left[ \Gamma_p^\sigma G_R \Gamma_q^{\sigma'} G_A \right] \] (8)

\( \Gamma_p^\sigma \)'s are the coupling matrices representing the coupling between the central region and the leads, and they are defined by the relation [16],

\[ \Gamma_p^\sigma = i \left[ \Sigma_p^\sigma - (\Sigma_p^\sigma)^\dagger \right] \] (9)

Here \( \Sigma_p^\sigma \) is the retarded self-energy associated with the lead \( p \). The self-energy contribution is computed by modeling each terminal as a semi-infinite perfect wire [17].
The retarded Green’s function, $G_R$ is computed using

$$G_R = \left( E - H - \sum_{p=1}^{4} \Sigma_p \right)^{-1}$$

(10)

where $E$ is the electron Fermi energy and $H$ is the model Hamiltonian for the central conducting region. $G_A$ is the advanced Green’s function and is given by, $G_A = G_R^{\dagger}$.

Figure 2: Measurement of the angle between the two arms of the Y-shaped device is shown. $b$ is the base of the triangle and $h$ is the height. $a$ is the lattice constant. According to Fig.1, this angle is half of $\theta_Y$ as depicted in the given figure.

Now, following the spin Hall phenomenology, in our set-up since lead-2 is a voltage probe, $I_2^q = 0$. Also, as the currents in various leads depend only on voltage differences among them, we can set one of the voltages to zero without any loss of generality. Here we set $V_1 = 0$ and $V_3 = 1$. With the help of these conditions, from Eq.(4), one can determine the voltage, $V_2$ and the spin current flowing through terminal 2,

$$V_2 = \frac{T_{23}}{T_{12} + T_{32}} \quad \text{and} \quad I_2^s = \frac{e^2}{\hbar} \left[ \left( T_{12}^{\text{out}} + T_{32}^{\text{out}} \right) V_2 - T_{23}^{\text{in}} \right]$$

(11)

Finally, from Eq.(3) the expressions for the longitudinal and spin Hall conductances become,

$$G_L = \frac{e^2}{h} \left[ (T_{12} + T_{23}) - \frac{T_{23}T_{32}}{T_{12} + T_{32}} \right]$$

(12)

$$G_{SH} = \frac{e}{4\pi} \left[ \left( T_{12}^{\text{out}} + T_{32}^{\text{out}} \right) - \frac{T_{12}^{\text{in}}T_{12} + T_{32}}{T_{23}} \right]$$

(13)

3. Results and Discussion

We have investigated the effects of the angle variation of the Y-shaped junction in presence of Rashba spin orbit coupling on the experimentally measurable quantities such as the longitudinal conductance ($G_L$) and the spin Hall conductance ($G_{SH}$).

We briefly describe the values of different parameters used in our calculation. Throughout our work, we have considered for the Y-shaped system, $d = 20$ (see Fig.1), onsite term, $\epsilon = \epsilon_L = 0$, hopping term, $t = t_L = t_C = 1$. All the energies are measured in unit of $t$. Further we choose a unit where $c = h = e = 1$. The longitudinal conductance, $G_L$ is measured in unit of $\frac{e^2}{\hbar}$. The spin Hall conductance, $G_{SH}$ is measured in unit of $\frac{e}{4\pi}$. Also the lattice constant is taken to be unity.

For most of our numerical calculations we have used KWANT [18]. At the onset, $V_R$ is chosen to have a representative value of 0.5. In this work, we have taken three different angles for the Y-shaped device. However, we have been able to generate a number of other values for the angular...
discreteness of the energy states available for conduction. Another important observation is
except for some fluctuations shows step-like nature in absence of the RSOC demonstrating the
three different angles of the Y-shaped device. In Fig.3(a), for \( \theta_Y = 90^\circ \), \( G_L \) on an average,
except for some fluctuations shows step-like nature in absence of the RSOC demonstrating the
discreteness of the energy states available for conduction. Another important observation is
that, the magnitude of \( G_L \) at \( E = 0 \) decreases with increase in the value of \( \theta_Y \). This can
be explained in the following way. \( G_L \) is measured at terminal-3 and the charge current, \( I_3^L \)
is flowing from terminals 3 and 1. Now it is easily understandable that, as we increase the
angle between terminal-1 and terminal-3, electrons are likely suffer more scattering in reaching
terminal-3. Accordingly, \( G_L \) decreases as \( \theta_Y \) is increased.

Fig.3(b) shows the variation of \( G_L \) as a function of \( E \) in presence of RSOC with strength,
\( V_R = 0.5 \). In presence of RSOC, though the step-like nature is lost but the decreasing feature of
\( G_L \) continues with increasing \( \theta_Y \). An interesting observation from Fig.3(b) is that, for \( \theta_Y = 90^\circ \),
there exist a zero mode state, that is at \( E = 0 \) the longitudinal conductance vanishes. This can
be explained from the definition of \( G_L \) as given in Eq.(3) and Eq.(4). Since we have assumed
the voltage at terminal-3, \( V_3 = 1 \) and the voltage at terminal-1, \( V_1 = 0 \), by Eq.(3) \( G_L \) is nothing
but the charge current flowing through terminal-3. Now from Eq.(4) we can say that there is a
cancellation for the charge current, \( I_3^L \) between the two given quantities, resulting in vanishing of
\( G_L \).

![Figure 3](image-url)

Figure 3: (Color online) (a) \( G_L \) is plotted as a function of energy, \( E \) for three different angle of
the Y-shaped device when RSOC is absent. \( G_L \) shows step-like nature. (b) \( G_L \) is plotted as a
function of \( E \) in presence of RSOC with strength \( V_R = 0.5 \). The step-like nature of \( G_L \) is partly
lost.

Now let us compute the characteristics of the spin Hall conductance. An useful quantity in
this regard, is the spin Hall voltage \( V_2 \) measured at terminal-2. Fig.4 shows the variation of the
voltage, \( V_2 \) at terminal-2 as a function of energy, \( E \) for three different \( \theta_Y \) with RSOC strength,
\( V_R = 0.5 \). In Fig.4(a), (b) and (c), the voltage \( V_2 \) in each case shows a symmetric behaviour as
a function of the energy, \( E \) with respect to \( E = 0 \). Fig.4(a) is the plot for \( V_2 \) for \( \theta_Y = 53.13^\circ \),
Fig.4(b) for \( \theta_Y = 90^\circ \) and Fig.4(c) for \( \theta_Y = 128.87^\circ \). \( V_2 \) is larger at the edges of the energy band
and as we come closer to \( E = 0 \), \( G_L \) decreases. Especially we note that for larger values of \( \theta_Y \),
namely for \( \theta_Y = 90^\circ \) and \( 128.87^\circ \), \( G_L \) almost vanishes around \( E = 0 \).

Further we have studied the variation of spin current flowing through terminal-2, \( I_2^S \) as a
function of energy, $E$ for three different $\theta_Y$ in presence of RSOC as shown in Fig.5. Fig.5(a) shows the variation of $I_s^2$ versus $E$ for $\theta_Y = 53.13^\circ$. Fig.5(b) and Fig.5(c) correspond to $\theta_Y = 90^\circ$ and $128.87^\circ$ respectively. $I_s^2$, as expected has an antisymmetric behaviour as a function of $E$ about $E = 0$. As a result, $I_s^2$ is exactly zero at $E = 0$, a fact well known.

Finally, the effect of the angular separation between the arms of the Y-shaped device on the spin Hall conductance, $G_{SH}$ is shown in Fig.6. Here $G_{SH}$ is plotted as a function of energy, $E$ in presence of RSOC. From Fig.4 and Fig.5, we notice that, $V_2$ has a symmetric nature as a function of $E$ while $I_s^2$ is antisymmetric. As a result, $G_{SH}$ is antisymmetric as a function of $E$, an important feature of the spin Hall conductance [14, 19, 20]. It can be noticed that for different values of $\theta_Y$ the variation of $G_{SH}$ is different, all the while retaining the antisymmetric character.

The main point that would be of interest to the community is that a Y-shaped device can be made, which in presence of RSOC, demonstrates that spin Hall effect is realizable in experiments. It may be noted that in our calculations, the spin Hall conductance (in units of $e/4\pi$) is moderately large (see Fig.6). Thus experiments involving magneto-optical Kerr rotation spectroscopy and other methods [21, 22] should be able to detect the spin Hall effect discussed here. Further, near perfect electrical switching operation is shown to achievable using a CNT Y-shaped junction under an a.c. bias [23]. Also, we have shown that SHE is sensitive to the angular separation between the arms of the Y-shaped device.
4. Summary and Conclusions

In summary, in the present work we have studied the effect of the angular separation of a three terminal Y-shaped junction device in presence of Rashba spin orbit coupling on the longitudinal and spin Hall conductances by Landauer-Büttiker formalism.

Both in presence of and absence of RSOC, at $E = 0$, $G_L$ decreases as $\theta_Y$ increases in a Y-shaped junction device. On the other hand, no systematic variation of the spin Hall properties emerge as a function of the angular separation of the device. In particular, the voltage at terminal-2 has symmetric nature as a function of energy, while the spin current flowing through terminal-2, $I^s_2$ and the spin Hall conductance, $G_{SH}$ having a large fluctuation, have antisymmetric properties as a function of energy, $E$ with respect to $E = 0$. For different values of $\theta_Y$, the variation of $G_{SH}$ is finite and different, all the while retaining the antisymmetric character. Experimental verification of our results should be possible with improved fabrication techniques of the realizing the mesoscale and nanoscale junction devices.

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