Bremsstrahlung Response of Homogeneous Magnetofactive Plasma on a Gravitational Wave

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Abstract

Numeric model of the bremsstrahlung response of homogeneous magnetoactive plasma on a gravitational wave with $e_+$ polarization was constructed. Electromagnetic response dependencies on the plasma and gravitational wave parameters were determined.

1 Introduction

The equations of the relativistic magnetohydrodynamics (RMHD) of a magnetoactive plasma in a gravitational field were formulated in paper [1] using the equality requirements for dynamic velocities of plasma and electromagnetic field. These equations were obtained on the basis of the Einstein and Maxwell equations. Also the remarkable class of exact solutions of obtained RMHD equations was found. It explains movement of a magnetoactive locally isotropic plasma in a field of plane gravitational wave (PGW). This class was called gravimagnetic shock waves (GMSW). It describes essentially nonlinear processes which do not exist in a linear approximation of the magnetohydrodynamics and essentially relativistic processes in terms of domination of massless electromagnetic component in a magnetoactive plasma.

It was shown in paper [2] that the GMSW in pulsars magnetospheres may be the highly effective detectors of gravitational waves from neutron stars. Particularly, giant pulses which sporadically appear in radiation of some pulsars may be the observation result of transferring energy from a gravitational wave to the GMSW. Estimations made in [2], [3] allow to connect giant pulses in radiation of pulsar B0581+21 with gravitational radiation in the basic mode of oscillations from this pulsar. In fact, at this moment it is sufficiently difficult to say about identification of giant pulses as the electromagnetic display of gravimagnetic shock wave evolution in pulsar magnetosphere and to connect unambiguously these pulses with pulsars gravitational radiation. Nevertheless, the idea of analyzing the influence of gravitational waves from a compact astrophysical object on its own electromagnetic radiation is highly productive for solving the problems of gravitational waves detection.

In fact, the main difficulties of gravitational waves detection in the Earth conditions are:

\footnotesize

\begin{enumerate}
\item Till 2000 Yu.G. Ignatyev wrote his name as Yu.G. Ignat’ev.
\item This requirement is completely equivalent to the condition of plasma infinite conductivity, see Ref. [1].
\end{enumerate}
1. Negligible amplitude of gravitational waves in the Earth conditions ($h \lesssim 10^{-19}$) because of significant distance between relativistic astrophysical objects and the Earth.

2. Sporadic nature of events leading to radiation of gravitational waves inside relativistic astrophysical objects with enough power. This does not allow to connect unambiguously received signal with a fact of gravitation radiation detection.

3. Impossibility to construct relativistic detectors with anomalous highly effective parameters for registration of gravitational waves in conditions of the Earth laboratory (super-strong magnetic fields, highly anisotropic working body of detector, low level of background noise, etc.).

It is possible to avoid these problems if one can transfer detector directly close to a relativistic astrophysical object. In this case one always has prepared electromagnetic signal and there is no need to convert it in other forms so it allows to do correlation analysis. If the detector’s working body is the magnetosphere of relativistic astrophysical object, the optimal for gravitational waves registration parameters of detector’s working body will be achieved automatically: super-strong magnetic fields, ultrarelativistic equation of state, highly anisotropy, etc.

The fundamental importance of the GMSW for theory as the direct conversion effect of gravity-waves energy into electromagnetic energy leads to necessity of more detailed and comprehensive researches. In Ref.[4] the strict proof of the GMSW hydrodynamic theory based on relativistic kinetic theory was given. In [1], [2], [3] was shown that GMSW realizes in essentially collisionless nonequilibrium plasma within anomalous strong magnetic fields. Isotropy of a local plasma electron distribution essentially violates in the such conditions due to strong bremsstrahlung. Therefore, an anisotropy factor of magnetoactive plasma is highly essential for effectiveness of GMSW formation mechanism. Hydrodynamic model of GMSW in anisotropic plasma with adjusted correlation between parallel and perpendicular components of plasma pressure was constructed in [5]. It was based on the general equations of RMHD. Particularly, in [5] was considered the elementary linear correlation. The research made in [5] discovered the strong dependence of GMSW effect upon the plasma anisotropy degree. That fact led to necessity of constructing the dynamic model of anisotropic magnetoactive plasma movement in a gravitational radiation field.

Process of a gravitational wave energy pumping over into the electromagnetic energy is describing with the help of the *energobalance equation* introduced in Ref.[1]-[3]. It performs the fact of total momentum conservation law inside the system “gravitational wave + magnetoactive plasma”. In paper [6] analytic research of essentially nonlinear equation was done and some features of the solution were detected. However, because of software existed in 1998 and other reasons the total research of GMSW evolution was not done and the parameters of bremsstrahlung response of magnetoactive plasma on gravitational wave were
not obtained. This paper is dedicated to these problems solution. Here we set the unit system where \( c = G = \hbar = 1 \).

## 2 Gravimagnetic shock waves

Let us reproduce the main results of the GMSW theory which are necessary for the goals of this paper. Let us set the metrics of vacuum PGW with \( e_+ \) polarization\(^3\) which propagates alone the \( Ox^1 \) axis:

\[
ds^2 = 2dudv - L^2[e^{2\beta}(dx^2)^2 + e^{-2\beta}(dx^3)^2],
\]

where \( \beta(u) \) is an arbitrary function (the PGW amplitude); the function \( L(u) \) (the PGW background factor) obeys an ordinary second order differential equation\(^4\): \( u = \sqrt{2}(t - x^1) \) is the retarded time and \( v = \sqrt{2}(t + x^1) \) is the advanced time. Let in the absence of PGW \((u \leq 0)\) be given a homogeneous magnetic field directed along the \( Ox^2 \) axis\(^5\):

\[
H_i(u \leq 0) = \delta^i_2 H_0.
\]

In general, alternating electromagnetic field which appears in the presence of gravitational wave has only spacelike magnetic component \( H^i \) in the comoving frame of reference moving with local velocity \( v^i \)\(^1\):

\[
H_i = v^k F^*_ki; \quad F^*_ki = \frac{1}{2} \eta^*_{kilm} F_{lm};
\]

\((F_{ik} \text{ - Maxwell tensor}, \ F^*_{ki} \text{ - dual Maxwell tensor}, \ \eta^*_{kilm} \text{ - discriminant tensor})\). The electric component of electromagnetic field in the comoving frame of reference equals zero \(^2\)\(^1\):\n
\[
E_i = v^k F_{ki} = 0,
\]

therefore the energy-momentum tensor (EMT) of electromagnetic field is:

\[
\frac{H}{T}_{ij} = \frac{1}{8\pi} \left( 2H^2 v_i v_j - 2H_i H_j - g_{ij} H^2 \right).
\]

Squared magnetic field strength is determined as \(^1\)\(^2\):

\[
H^2 = -(H, H) = \frac{1}{2} F_{lm} F^{lm}.
\]

Thus, the trace of the EMT of electromagnetic field is equal to zero:\n
\[
\frac{H}{T} = g^{ij} \frac{H}{T}_{ij} \equiv 0.
\]

\(^1\)The case of two polarization states will be considered in next paper.

\(^2\)See, for example, Ref.[7].

\(^3\)The general case of a magnetic field arbitrarily directed in the plane \( x^1 Ox^2 \) was considered in the cited papers.
Invariants of electromagnetic field comply with the conditions:
\[ F_{ik} F^{ik} = F_{ik} \tilde{F}^{ik} = 2H^2 > 0; \quad \tilde{F}_{ik} F^{ik} = 0. \quad (8) \]

Let further magnetoactive plasma be homogeneous but anisotropic in general in the absence of PGW. In gravitational field the EMT of anisotropic magnetoactive plasma is [5]:
\[ P_{ij} = \left( \varepsilon + p_{\perp} \right) v^i v^j - p_{\perp} g_{ij} + \left( p_{\parallel} - p_{\perp} \right) h^i h^j, \quad (9) \]

where \( h^i = H^i / H \) - spacelike unitary vector of a magnetic field \( (h, h) = -1 \), \( p_{\perp}, p_{\parallel} \) - perpendicular and parallel components of the plasma’s pressure, and according to (3):
\[ (v, h) = 0. \quad (10) \]

And the EMT (9) according to virial law complies with the condition:
\[ P = \varepsilon - p_{\parallel} - 2p_{\perp} \geq 0 \iff p_{\parallel} + 2p_{\perp} \leq \varepsilon. \quad (11) \]

Let’s further suppose a barotropic equation of state:
\[ p_{\parallel} = k_{\parallel} \varepsilon; \quad p_{\perp} = k_{\perp} \varepsilon, \quad (12) \]

where coefficients of baratrops \( k_{\parallel}, k_{\perp} \) by reason of (11) are follow the inequality:
\[ k_{\parallel} + 2k_{\perp} \leq 1. \quad (13) \]

Then in a presence of PGW the exact solution of RMHD equations is [5]:
\[ v_2 = 0; \quad v_u = \frac{1}{2v_v}; \quad (14) \]
\[ v_v = \frac{1}{\sqrt{2}} \left[ \Delta L^{k_{\parallel}} \nu e^{\beta(k_{\parallel} - k_{\perp})} \right]^{g_{\perp}}; \quad (15) \]
\[ \varepsilon = \nu \left[ \Delta L^{(1+k_{\parallel})} e^{2\beta(k_{\parallel} - k_{\perp})} \right]^{-g_{\perp}}; \quad (16) \]
\[ H = H_0 \left[ \Delta L^{(1+k_{\parallel})} e^{-\beta(1-k_{\parallel})} \right]^{-g_{\perp}}; \quad (17) \]
\[ n = \frac{1}{\sqrt{2} v_v L^2}; \quad (18) \]

where
\[ g_{\perp} = \frac{1}{1 - k_{\perp}} \in [1, 2], \quad (19) \]
\( \Delta(u) \) is the governing function of GMSW.
\[ \Delta(u) = \left[ 1 - \alpha^2 (e^{2\beta} - 1) \right], \quad (20) \]
$n$ - local charged particle density, $\alpha^2$ - dimensionless parameter:

$$\alpha^2 = \frac{H_0^2}{4\pi(\epsilon^0 + P_\perp^0)}.$$  \hspace{1cm} (21)

Variables from above marked with zero are given in the absence of PGW. The RDMD equations solution consists of the physical singularity on the hypersurface $\Sigma^*: u = u^*$:

$$\Delta(u^*) = \left[1 - \alpha^2(e^{2\beta(u^*)} - 1)\right].$$  \hspace{1cm} (22)

on which the densities of the plasma energy and of the magnetic field tend to infinity, the dynamic velocity of the plasma as a whole tends to the velocity of light in the PGW propagation direction. In this case the ratio of the magnetic field energy density to the plasma energy tends to infinity. The above singularity is the GMSW spreading in the PGW propagation direction at a subluminal velocity. According to Eq. (22) the conditions of the singularity arising are

$$\beta(u) > 0;$$  \hspace{1cm} (23)

$$\alpha^2 > 1.$$  \hspace{1cm} (24)

The extremely important fact is that, the singular condition is even possible in a weak PGW ($|\beta| \ll 1$) on the condition of a highly magnetized plasma ($\alpha^2 \gg 1$). In this case the singular condition, according to (22), arises on the hypersurfaces $u = u^*$:

$$\beta(u^*) = \frac{1}{2\alpha^2}.$$  \hspace{1cm} (25)

It follows from (14) - (17) that, by $\beta > 0$ the plasma moves in the GW propagation direction ($v^1 = \frac{2}{\sqrt{2}}(v_u - v_v) > 0$), by $\beta < 0$ - in the opposite direction.

The singularity was removed by taking into account back influence of the magnetoactive plasma on a PGW. It leads to effective absorption of a PGW energy by the plasma and to PGW amplitude restriction. The simple model of energy balance which describes that process was constructed in [3]. Gravitational wave with metrics (1) in WKB-assumption corresponds to the EMT with one nonzero component:

$$T_{uu} = \frac{1}{4\pi}(\beta')^2.$$  \hspace{1cm} (26)

Let $\beta_u(u)$ be a PGW vacuum amplitude and $\beta(u)$ be a PGW amplitude in consideration of absorption in plasma. In this case the energobalance equation in the short-wave approximation becomes:

$$\text{In this case it corresponds to the condition of } \rho \gg \lambda = c/\omega \text{ where } \rho \text{ is a space-time curvature radius.}$$
\[(\beta_*')^2 = (\beta')^2 + 4\pi \left(T_{uu} - \frac{\partial}{\partial T_{uu}}\right), \quad (27)\]

where \(T_{ik}\) is total plasma EMT. Under condition of \(\alpha^2 \gg 1\) Eq. (14) may be written in the form:
\[ \dot{q}_i^2 = \dot{q}^2 + \xi^2 V(q), \quad (28)\]
where \(q = \beta/\beta_0\) and the dot signifies a derivative in the dimensionless time variable \(s\):
\[ s = \sqrt{2\omega_u}, \quad (29)\]
\((\omega - \text{the PGW frequency}), \ V(q) - \text{potential function which in a weak PGW becomes:}\)
\[ V(q) = \Delta^{-4g_\|}(q) - 1, \quad (30)\]
where \(\xi^2\) is so-called *the first parameter of GMSW* [2]:
\[ \xi^2 = \frac{H_0^2}{4\beta_0^2\omega^2}. \quad (31)\]

Eq. (28) may be treated as an equation with respect to the variable \(q\). On the other hand, (28) completely coincides in its form with the energy conservation law of a 1-dimensional mechanical system described by the canonical variables \(\{q(s), \dot{q}(s)\}\) [9], where \(V(q)\) is the potential, \(\dot{q}^2\) is its kinetic energy and \(\dot{q}_i^2 = E_0\) is its total energy.
Let us introduce the new dimensionless parameter:
\[ \Upsilon = 2\alpha^2\beta_0 \quad (32) \]
- *the second GMSW parameter* and rewrite (22) in a weak PGW as:
\[ \Delta(q(s)) = 1 - 2\alpha^2\beta_0 q(s) = 1 - \Upsilon q(s). \quad (33)\]
It leads from (33):
\[ \dot{q} = -\frac{\dot{\Delta}(q)}{\Upsilon}. \quad (34)\]
To analyze the system behavior, let us suppose that the moment \(s = 0\) corresponds to the front edge of a GW, while:
\[ \beta_* \approx \beta_0(1 - \cos(s)) \Rightarrow q_* \approx 1 - \cos(s). \quad (35)\]
According to (33)-(35) the system starts with negative value of the governing function derivative and with function value equal to 1:
\[ \dot{\Delta}(s) \approx -\Upsilon \sin s \approx -\Upsilon s; \quad \Delta(s) \approx 1 - \Upsilon(1 - \cos s) \approx 1 - \Upsilon \frac{s^2}{2}; \quad (s \to +0). \quad (36)\]
\[ ^7\text{This provides zero PGW metrics derivatives at the moment } s = 0, \text{ i.e. } C^1 \text{ class of metric functions.}\]
The energobalance equation (28) according to (30), (34), (35) becomes:
\[ \dot{\Delta}^2 + \xi^2 \Upsilon^2 \left[ \Delta^{-4g_{\perp}} - 1 \right] = \Upsilon^2 \sin^2(s). \] (37)

Solving the Eq. (37) with respect to \( \dot{\Delta} \) we obtain:
\[ \dot{\Delta} = \mp \Upsilon \sqrt{\sin^2(s) - \xi^2 \left[ \Delta^{-4g_{\perp}}(s) - 1 \right]}. \] (38)

Integrating according to (36) first of all it’s necessary to take negative branch of the Eq. (38) but when we reach the minimum value of the governing function we should change it by the positive one. From (38) we obtain the minimum value of the governing function which is reached by \( s = \pi/2 \):
\[ \Delta_{\text{min}} = \left( \frac{1}{\xi^2} + 1 \right)^{-\gamma_{\perp}}, \] (39)
where:
\[ \gamma_{\perp} = \frac{1}{4g_{\perp}} = \frac{1 - k_{\perp}}{4} \Rightarrow \frac{1}{8} \leq \gamma_{\perp} \leq \frac{1}{4}. \] (40)

The maximum accessible density of a magnetic energy is
\[ \left( \frac{H^2}{8\pi} \right)_{\text{max}} = \frac{H_0^2}{8\pi} \sqrt{1 + \frac{1}{\xi^2}} \] (41)
and it does not depend on a plasma equation of state (12). Also plasma velocity in the GMSW does not depend on equation of state. And the maximum plasma energy density without magnetic field depends on the exponent of plasma anisotropy:
\[ \varepsilon_{\text{max}} = \varepsilon \left( 1 + \frac{1}{\xi^2} \right)^{\frac{1}{2}(1 + k_{\perp})}. \] (42)

It is maximum for the ultrarelativistic plasma with zero valuation of the parallel pressure.
Thus, the maximum value of the local response amplitude of a highly magnetized plasma \( (\alpha^2 \gg 1) \) with linear state equations does not depend on the exponent of plasma anisotropy and its equation of state.

3 Numerical analysis of the energobalance equation in Mathematica

Eq. (38) is essentially nonlinear and difficult for analyzing in spite of its apparent simplicity. Since there is no possibility to find the exact solution of the energobalance equation which has important astrophysical applications there is a need for its numerical analysis. First attempts of numerical calculation has
met significant troubles. Therefore, for the numeric integration control the analytic researches were made in [6]. They revealed that the solution has a plateau form with minimum value at the point of $\pi/2$ and after this point the solution becomes instable. Also some numerical solutions of the energobalance equation in TurboPascal were obtained in the paper.

Comprehensive analysis of the homogeneous magnetoactive plasma GMSW response on a gravitational wave within a wide range of plasma and GW parameters was not done in that time due to the software abilities. Nowadays the abilities of nonlinear differential equations numeric solution in computer algebra system (CAS) Mathematica allow to do such researches. However, the direct use of build-in numeric methods towards the energobalance equation (37) is still impossible in case that the governing function derivative changes the sign at the point $s = \pi/2$. So one can not change the step size according to the equation parameters.

Empirically was established that the second-order implicit Adams method solves the equation much faster and much correct in comparison with other explicit and implicit methods. In this research the procedure for numerical solving of the differential equation in CAS Mathematica was developed. It adapts the integration step according to the parameters $\xi^2$, $\Upsilon$. For all that the differential equation is being solving with negative value of derivative up to the point $\pi/2$ using the second-order implicit Adams method. The value of function is being taking as the initial value for the positive equation branch (38) after derivative changes the sign. The integration step is being changed and integration method is changing to Euler method which works better in the instability region.

The procedure allows to make analysis of (38) numerical solutions depending on the first and the second GMSW parameters. It also allows to construct the model of magnetoactive plasma response on a GW and to calculate plasma’s physical characteristics. Numerical researches with our procedure completely approve the analytic predictions for the governing function form. At first, the solution decreases rapidly then comes to plateau and slowly approaches the point of minimum $\pi/2$ with the function value close to the (39). After the point of minimum an instability evolves rapidly. For all that the governing function is smooth in the whole interval. In Fig.1 the results of numerical solution for the energobalance equation in the case of tiny parameter $\xi^2$ and huge parameter $\Upsilon$ are presented. In this case a process of numeric solution is the most difficult and on the other hand the predicted features of the solution are visible. The seeming fractures of function by small $s$ and by $s = \pi/2$ are fake. In fact they disappear by scaling up.

Numeric analysis of the energobalance equation allows to determine the fact that the governing function is sufficiently close to the function $\Delta_0$ in the plateau area (i.e. the small value of derivative). $\Delta_0$ nullifies a radical value in right hand side of (38):

$$
\Delta_0(s) = \left(1 + \frac{\sin(s)^2}{\xi^2}\right)^{-\gamma_\perp}.
$$

(43)
Figure 1. The governing function $\Delta(s)$ by $\xi^2 = 10^{-6}$, $\Upsilon = 100$, $\gamma_\perp = 1/6$

At the point of minimum $s = \pi/2$ this value coincides with the governing function minimum value $\Delta_0(s)$. By increasing the parameter $\Upsilon$ coincidence of the governing function $\Delta(s)$ and the function $\Delta_0(s)$ becomes by the smaller values of time $s$. By small values of the time variable $s$ the governing function is well approximated by parabolic law $\Delta_0(s)$. In Fig.2 the plots of the $\Delta(s)$ and the $\Delta_0(s)$ functions are shown:

Figure 2. The $\Delta(s)$ function (solid line), the $\Delta_0(s)$ function (dashed line), the asymptotic $\Delta_0(s)$ by small values of $s$: $1 - \Upsilon s^2/2$ (dotted line). Everywhere $\xi^2 = 0.001$, $\Upsilon = 10$, $\gamma_\perp = 1/6$.

This result allows to approximate the plasma response $H^2/H_0^2$ in the plateau area of the governing function, i.e. in the maximum response area by the expression:

$$\frac{H^2}{H_0^2} \approx \sqrt{1 + \frac{\sin^2 s}{\xi^2}}.$$  \hspace{1cm} (44)
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In a weak GW:

\( |\beta(s)| \ll 1; \quad L(s) \approx 1 \) \hspace{1cm} (45)

the exact solution of RMHD equations (14)-(17) is simplified and we obtain the following expressions for physical observed values (see also \([6]\)).

1. Magnetic field energy density in the comoving frame of reference:

\[
\frac{H^2}{8\pi} = \frac{H_0^2}{8\pi} \Delta^{-1/2\gamma_\perp}. \tag{46}
\]

2. Plasma energy density in the comoving frame of reference:

\[
\varepsilon = \varepsilon_0 \Delta^{1/4\gamma_\perp}. \tag{47}
\]

3. Physical velocity of plasma:

\[
v^1 = \frac{1 - 2v_{v}^2}{1 + 2v_{v}^2} = \frac{1 - \Delta^{1/2\gamma_\perp}}{1 + \Delta^{1/2\gamma_\perp}}. \tag{48}
\]

4. Charged particle density:

\[
n = n_0 \Delta^{-1/4\gamma_\perp}. \tag{49}
\]

5. Total observed bremsstrahlung intensity detected by resting observer:

\[
W = W_0 \Delta^{3+2k} \frac{1}{2} \left( \Delta^{1/4\gamma_\perp} + \Delta^{-1/4\gamma_\perp} \right), \tag{50}
\]

where \( W_0 \) - total bremsstrahlung intensity in the absence of a PGW \([8]\):

\[
W_0 = \frac{2e^4 H_0^2}{3m^2c^3} n_0 \left( \frac{\mathcal{E}}{mc^2} \right)^2, \tag{51}
\]

where \( \mathcal{E} \) - the kinetic energy of a charged particle. 6. Radiation spectral intensity in a high frequencies range where frequencies are comparable with the unperturbed cyclotron frequency \( \omega_c^0 \):

\[
\omega_c^0 = \frac{3eH_0}{2mc} \left( \frac{\mathcal{E}_0}{me^2} \right)^2, \tag{52}
\]

and higher. One may calculate the intensity using standard electrodynamical formulas \([8]\) and find:

\[
J = J_0 \Delta^{-3} F \left( \frac{\omega}{\omega_c^0} \Delta^{-5/2} \right), \tag{53}
\]
where

\[
J_0 = \frac{\sqrt{3} e^3 H_0 n_0}{2\pi mc^2}, \tag{54}
\]

\[
F(x) = x \int_{x}^{\infty} K_{5/3}(y) dy, \tag{55}
\]

\[K_\nu(z) - \text{modified Bessel functions of the third kind or Macdonald functions (see Ref.\[10\]):}\]

\[
K_\nu(z) = \frac{\sqrt{\pi} z^\nu}{2^\nu \Gamma(\nu + 1/2)} \int_{0}^{\infty} e^{-z \cosh t} \sinh^{2\nu} t \, dt.
\]

In Fig. 3-7 the results of a numeric solution for response of magnetoactive plasma on a gravitational wave depending on GMSW parameters are presented.

Figure 3. Influence of the second GMSW parameter \(\Upsilon\) on the relative magnetic field energy density evolution \(H^2/H_0^2\) by \(\xi^2 = 0.0045\), \(\gamma_\perp = 1/6\): \(\Upsilon = 3\) (solid line), \(\Upsilon = 10\) (dashed line), \(\Upsilon = 100\) (dotted line).

Figure 4. Influence of the second GMSW parameter \(\Upsilon\) on the plasma energy density evolution \(\varepsilon/\varepsilon_0\) by \(\xi^2 = 0.0045\), \(\gamma_\perp = 1/6\): \(\Upsilon = 3\) (solid line), \(\Upsilon = 10\) (dashed line), \(\Upsilon = 100\) (dotted line).
Figure 5. Influence of the second GMSW parameter $\Upsilon$ on the plasma drift velocity evolution $v^1/c$ by $\xi^2 = 0.0045$, $\gamma_\perp = 1/6$: $\Upsilon = 3$ (solid line), $\Upsilon = 10$ (dashed line), $\Upsilon = 100$ (dotted line).

Figure 6. Influence of the second GMSW parameter $\Upsilon$ on the total observed bremsstrahlung intensity evolution $W/W_0$ by $\xi^2 = 0.0045$, $\gamma_\perp = 1/6$: $\Upsilon = 3$ (solid line), $\Upsilon = 10$ (dashed line), $\Upsilon = 100$ (dotted line).

Numeric results allow to determine the region of the parameters $\xi^2$ and $\Upsilon$ where GMSW mechanism becomes sufficiently effective. At that the effect was treated as essential if the total observed bremsstrahlung intensity exceeds in maximum its initial value in about 2 times. As a result of essential dependence of the total observed bremsstrahlung intensity on the parameter $\xi^2$ this region is close to ellipse (Fig. 7).
Figure 7. Region of the GMSW existence against a quarter ellipse with semi-axis 4.2 and 1 background.

Figure 8. Bremsstrahlung spectral intensity time evolution in relative units by $\xi^2 = 0.0045$, $Y = 10$ and relative time: $s=0; 0.15; 0.31; 0.47; 0.63; 0.78; 0.94; 1.10; 1.25; 1.41; 1.56 \approx \pi/2$ (from bottom to top).

In Fig 8 along x-coordinate is a common logarithm of bremsstrahlung frequency in units of the unperturbed cyclotron frequency $\omega_c^0$ and along y-coordinate is relative radiation intensity $J/J_0$. Maximum of spectral intensity shifts according to the law:

$$\omega_{max} = 0,29\omega_c^0\Delta^{-5/2}(s).$$

As was mentioned before an instability rapidly evolves when the governing function passes through its minimum which corresponds to the observed values maximum. Plasma makes irreversible reversion in the direction opposite to the PGW propagation direction. This situation is clearly illustrated in Fig 5. Thus, the magnetoactive plasma reacts to a PGW by a single impulse Ref. 2, 3.
Figure 9. Influence of the anisotropy parameter $\gamma_\perp$ on the relative magnetic field energy density evolution $H^2/H_0^2$ by $\xi^2 = 0.01$, $\Upsilon = 10$: $\gamma_\perp = 1/4$ (solid line), $\gamma_\perp = 1/6$ (dashed line), $\gamma_\perp = 1/8$ (dotted line).

The numeric analysis results of the anisotropy parameter $\gamma_\perp$ influence on the observed magnetic field energy density are presented in Fig.9. One can see that resultant anisotropy factor influence is insignificant in spite of the essential dependence of the exact solution (14) - (19) on this factor.

The dependence of the total observed bremsstrahlung intensity semiwidth on the GMSW parameters was researched. One can see in Fig.10 that the GMSW impulse duration i.e. the impulse semiwidth to a high accuracy is equal to $\pi/4 \approx 0.79$ or in common units:

$$\delta \tau = \frac{T}{8},$$

where $T$ is a PGW period.

Figure 10. Dependence of the total observed bremsstrahlung intensity semiwidth $W/W_0$ on $\Upsilon$ by $\gamma_\perp = 1/6$, $\xi^2 = 0.1$, $\xi^2 = 0.01$, $\xi^2 = 0.001$, $\xi^2 = 0.0001$ (from top to bottom).
5 Conclusion

Summarizing the paper results we would like to underline that gravitational wave weakness is considered in terms of the conditions realization. In this case the value of $\alpha^2\beta$ may not be small. Therefore linearity of the theory by GW smallness in comparison with 1 (linearity of the Einstein equations left hand side by $\beta$) in general does not mean linearity of hydrodynamic theory by GW amplitude smallness. Let us notice that such situation is rather unexpected though it can be foreseen using a MHD equations exact dimensional analysis.

The research proved preliminary results of the earlier papers and helped to work out in details the GMSW behavior and to describe its evolution process in all regions of the parameters.

Numeric simulation of a magnetoactive plasma response on a gravitational wave allows to find next rules of the gravimagnetic shock wave excitation process:

1. Under realization of the GMSW origin conditions

$$\xi < 1; \ \ U > 1$$

magnetoactive plasma reacts to a PGW by a single impulse where plasma moves in the gravitational wave propagation direction. The impulse semiwidth order is 1/8 of GW period;

2. Impulse stops with plasma revers; by this appears the typical impulse form (see Fig.11). It weakly depends on the second GMSW parameter $U$ and is determined by the $\Delta_0(s)$ function.

![Figure 11. Influence of the first GMSW parameter on the magnetic field energy density evolution $H^2/H_0^2$ by $U = 100$, $\gamma_\perp = 1/6$: $\xi^2 = 0.5$ (solid line), $\xi^2 = 0.3$ (dashed line), $\xi^2 = 0.1$ (dotted line).](image)

Under conditions the second GMSW parameter $U$ affects only on front (small values of time $s$) and back (time values $s$ are close to $\pi/2$) edges of the
3. Bremsstrahlung spectrum becomes harder during a shock wave passing.

4. In the maximum of magnetoactive plasma response almost all of a gravitational wave energy transfers to plasma, magnetic field and to bremsstrahlung (see Fig.12).

Acknowledgement

The authors are grateful to Prof. D.V. Galtsov, B.E. Meierovich and N.I. Kolosnitsin for useful discussion of the results.

The authors are grateful to Dr. A.V. Matrosov, who pointed out the stiff class of the energobalance equation.

Figure 12. Influence of the second GMSW parameter Υ on the GW energy absorption $Δε_g/ε_g = (β^2_0 − β^2)/β^2_0$ by $ξ^2 = 0.0045$, $γ_⊥ = 1/6$: $Υ = 3$ (solid line), $Υ = 10$ (dashed line), $Υ = 100$ (dotted line).

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