Two-dimensional gridless super-resolution method for ISAR imaging

Mohammad Roueinfar and Mohammad Hossein Kahaei*
Iran University of Science and Technology, School of Electrical Engineering, Tehran, Iran

ABSTRACT. We are focused on improving the resolution of images of moving targets in inverse synthetic aperture radar (ISAR) imaging. This can be achieved by recovering the scattering points of a target that have stronger reflections than other target points, resulting in a higher radar cross section of a target. However, these points are sparse and moving targets cannot be correctly detected in ISAR images. To increase the resolution in ISAR imaging, we propose the fast reweighted trace minimization (FRWTM) method to retrieve frequencies of sparse scattering points in both range and azimuth directions. This method is a two-dimensional gridless super-resolution method that does not depend on fitting the scattering point on the grids. Using computer simulations, the proposed algorithm is compared with fast reweighted atomic norm minimization (FRANM), sparse Bayesian learning (SBL), and \( \text{SL}_0 \) algorithms in terms of mean squared error (MSE). The results show that FRWTM performs better than the other methods, especially SBL and \( \text{SL}_0 \) at low signal-to-noise ratio (SNR) and fewer samples.

Keywords: inverse synthetic aperture radar imaging; gridless super-resolution; optimization; alternating direction method of multipliers

1 Introduction
Inverse synthetic aperture radar (ISAR) is an imaging radar for moving targets that can form a two-dimensional image of a target by adding a second dimension as azimuth and benefiting from dual processing gain for the range and azimuth directions. This structure, wherein the radar is fixed and the targets are moving, can identify and distinguish moving targets from each other. In ISAR imaging, azimuth direction, i.e., the second dimension, is obtained based on the relative motion between the radar antenna and the target. Then, a two-dimensional (2D) image is obtained by collecting the scattering points of a target from different viewing angles that are generated by the relative motion between the radar and the target. It is well known that the resolution in the direction of the range depends on the bandwidth of the transmitted signal and the azimuth direction is determined by the coherent processing interval (CPI). However, due to the limitations in the transmitted signal bandwidth and the CPI, it is not always possible to increase the resolution in each direction. In order to overcome these limitations, some super-resolution methods have been introduced to increase the resolution with no need for increasing the signal bandwidth and the CPI. These methods can be categorized as

- Higher-resolution spectrum estimation
- Sparse Bayesian Learning (SBL)

*Address all correspondence to Mohammad Hossein Kahaei, kahaei@iust.ac.ir

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Compressive sensing (CS)\textsuperscript{3,15,16} and Gridless sparse methods\textsuperscript{17}

Higher-resolution spectrum estimation methods often require some initial information, such as the number of sources and the Nyquist rate sampling.\textsuperscript{16} The SBL method requires prior information such as the probability distribution of previously observed data.\textsuperscript{18} Due to the sparse nature of the target scattering points, CS methods have been developed.\textsuperscript{3,15,16} The goal of these methods is to recover unknown scattering points of a target from a limited number of samples. These points generate strong radar cross sections (RCSs) at certain frequencies and the more they are retrieved, the higher the resolution of ISAR images will be achieved. However, a major difficulty with the CS approach is that these methods are designed based on a predefined grid, which in practice may not coincide with the scattering points. In fact, as the target location is not predictable, the scattering points with high RCSs would not necessarily place on these grids. In this case, to get a higher resolution, it is necessary to increase the number of grids, which leads to more complexity of the algorithm. To cope with this problem, as an alternative, gridless-based methods have been introduced. These methods may be interpreted as a continuous version of CS approaches whose solution involves convex optimization problems using atomic-norm minimization (ANM). However, since gridless methods do not need to fit the scattering points on grids, their complexities are lower than the CS methods.\textsuperscript{19,20} A gridless-based method referred to as the reweighted atomic-norm minimization (RAM) is introduced in Ref. 21, which is accordingly developed for ISAR imaging in Ref. 17. This algorithm executes a convex optimization problem based on ANM using an appropriate weighting matrix. It has been shown that RAM can enhance the sparsity and thus the resolution. In Ref. 22, ANM has been expanded for transmitting frequency-stepped chirp signals (FSCS), which is solved by a semidefinite programming (SDP). Then, the scattering points are recovered using off-the-shelf SDP solvers. Also in Ref. 23, a super-resolution method called fast reweighted atomic norm minimization (FRANM) is presented by including sparse apertures in the problem.

In this work, we propose the fast reweighted trace minimization (FRWTM) algorithm based on reweighted trace minimization (RWTM)\textsuperscript{20} and alternating direction method of multipliers (ADMM)\textsuperscript{24} to retrieve the 2D-frequencies of sparse scattering points of a target. This method unlike the other high-resolution methods\textsuperscript{4–11} does not require any initial information. Also, like SBL,\textsuperscript{12–14} it does not require the prior probability distribution of the previously observed data. Moreover, as opposed to the CS methods,\textsuperscript{3,15,16} we do not need a predefined fine grid, which increases the computational complexity of the algorithm. In comparison to the FRANM,\textsuperscript{23} the proposed FRWTM increases the resolution of ISAR images by assuming sparsity of samples in both range and azimuth directions, which leads to better performance at low signal-to-noise ratios (SNRs) with lower mean squared errors (MSEs).

The paper is organized as follows. In Sec. 2, the signal model is given. Section 3 is devoted to the FRWTM method and Sec. 4 presents the experimental results. Finally, Sec. 5 concludes the paper.

2 Signal Model

According to the principles of ISAR imaging, the relative movement between radar and a target can lead to two types of motion: (a) translational and (b) rotational. The translational motion occurs when the entire target body is displaced, which generally leads to darkening. But in the other type of motion, only rotational and maneuvering motions of a target in its place are considered. Here, we assume that the translational and the rotational motions are compensated. There are different methods for motion compensation in ISAR imaging, but in practice, the latter assumption is achievable, which facilitates reducing computational complexity. However, a major problem with ISAR images is that they are basically reconstructed from the pulses received from different angles of a target, which for any possible reasons may be missed during transmission, which leads to blurred images. In Fig. 1, an ISAR radar observes only the signals from the angles $\theta_0$, $\theta_2$, and $\theta_4$, from six possible viewing angles and misses the others. Also, we consider the FSCS, which has a relatively low instantaneous bandwidth compared to the total processing bandwidth. This signal is made up of smaller and discrete components called bursts, each one consisting of a number of pulses modulated by linear frequency modulation (LFM).
It is assumed that the target is composed of \( K \) scattering points with coefficients \( \sigma_k \), which have high RCS. Then, the received signal after dechirping is obtained as

\[
S_R(\theta_n, f_m) = \sum_{k=0}^{K-1} \sigma_k \exp \left[ -j2\pi f_m \frac{2(R_0 - x_k \sin(\theta_n)) + y_k \cos(\theta_n)}{c} \right],
\]

where \( f_m \) is the central frequency of the \( m \)'th pulse, \( \theta_n \) presents the \( n \)'th viewing angle, \( (x_k, y_k) \) show the coordinates of the \( k \)'th scattering point, \( R_0 \) is the instantaneous distance between the radar and target, and \( c \) is the light speed. Assuming a small viewing angle, which leads to the approximations \( \sin(\theta_n) \approx \theta_n \), \( \cos(\theta_n) \approx 1 \), and compensating for the translational motion, the received signal is simplified as

\[
S_R(\theta_n, f_m) = \sum_{k=0}^{K-1} \sigma_k \exp \left[ -j4\pi f_m \frac{x_k \theta_n}{c} \right] \exp \left[ -j4\pi f_m \frac{y_k}{c} \right].
\]

By defining \( f_{x,n} = 4\pi f_m \theta_n/c \) and \( f_{y,n} = 4\pi f_m / c \) in Eq. (2) and according to \( f_m = f_0 + m\Delta f \) in an FSCS signal, where \( f_0 \) is the fundamental carrier frequency in a burst and \( \Delta f \) is the frequency step for each pulse relative to the next pulse and assuming \( f_0 \) is larger than \( \Delta f \) and \( \theta_n \), we have \( f_{x,n} = 4\pi f_0 \theta_n/c \). Thus, Eq. (2) can be written as

\[
S_R(\theta_n, f_m) = \sum_{k=0}^{K-1} \sigma_k \exp[j(f_{x,n} x_k + f_{y,n} y_k)].
\]

Also, considering \( N \) viewing angles and \( M \) pulses in a burst of the transmitted signal, we have different values of \( m \) and \( n \) for \( f_{x,n} \) and \( f_{y,m} \), and we can present the received signal in matrix form as

\[
S = \begin{bmatrix}
S_R(\theta_0, f_0) & \cdots & S_R(\theta_0, f_{M-1}) \\
\vdots & \ddots & \vdots \\
S_R(\theta_{N-1}, f_0) & \cdots & S_R(\theta_{N-1}, f_{M-1})
\end{bmatrix} \in \mathbb{C}^{N \times M},
\]

or in vector form as

\[
s_{n,m} = \text{vec}(S^T) \in \mathbb{C}^{NM}.
\]


### 3 Developing Two-Dimensional ISAR Imaging Algorithm

In order to increase the resolution of ISAR images, we develop the FRWTM algorithm by retrieving the scattering points of a target in two dimensions based on the RWTM algorithm\(^\text{20}\). To do so, Eq. (3) can be presented using the Kronecker product of 2D atoms in the direction of range and azimuth as

\[
\mathbf{s}_{n,m} = \sum_{k=1}^{K} \sigma_k \mathbf{a}(f_{x,i}) \otimes \mathbf{a}(f_{y,j}),
\]

where \(\otimes\) shows the Kronecker product, \(\sigma_k\) is the RCS of scattering points, and \(K\) is the number of scattering points. Also, \(\mathbf{a}(f_{x,i})\) and \(\mathbf{a}(f_{y,j})\) are the atoms in the direction of range and azimuth, respectively, defined as

\[
\mathbf{a}(f_{x,i}) = \frac{1}{N} [1, \exp[-jf_{x,i}], \ldots, \exp[-j(N-1)f_{x,i}]]^T, \quad (7)
\]

\[
\mathbf{a}(f_{y,j}) = \frac{1}{M} [1, \exp[-jf_{y,j}], \ldots, \exp[-j(M-1)f_{y,j}]]^T. \quad (8)
\]

Also, the received signal vector can be shown as

\[
\mathbf{s}_{n,m} = \sum_{k=1}^{K} \sigma_k \mathbf{c}(\mathbf{f}_i), \quad (9)
\]

where \(\mathbf{f}_i = (f_{x,i}, f_{y,j})\) shows a 2D frequency for the \(i\)th scattering point, \(\mathbf{c}(\mathbf{f}_i) = \mathbf{a}(f_{x,i}) \otimes \mathbf{a}(f_{y,j}) \in \mathbb{C}^{N \times M}\) is a 2D-normalized complex atom, and \(\mathcal{A}\) is a set of 2D atoms defined as

\[
\mathcal{A} = \{ \mathbf{c}(\mathbf{f}_i) | \mathbf{f}_i \in [0.1] \times [0.1] \}. \quad (10)
\]

In other words, the received signal is a linear combination of the atoms of \(\mathcal{A}\). Then, based on \(\mathcal{A}\), the sparse scattering points of a target received by ISAR can be recovered by solving the following ANM optimization problem:

\[
\| \mathbf{s}_{n,m} \|_{\mathcal{A}} \rightarrow \inf_{\mathbf{f}_i \in [0.1] \times [0.1]} \left\{ \sum_{i} \| \sigma_k \| \mathbf{s}_{n,m} = \sum_{i=1}^{K} \sigma_k \mathbf{c}(\mathbf{f}_i) \right\}. \quad (11)
\]

Then, the sparse scattering points of a target are recovered as

\[
\hat{\mathbf{s}}_{n,m} = \arg \min_{\mathbf{s}_{n,m}} \| \mathbf{s}_{n,m} \|_{\mathcal{A}} \quad \text{s.t.} \quad \mathbf{S} = \mathbf{S}_{\Omega}. \quad (12)
\]

where \(\mathbf{S}\) is the complete data matrix (or full observations) as given by Eq. (4), \(\mathbf{S}_{\Omega}\) shows the measured (or incomplete) data matrix, and \(\Omega \in \{0, \ldots, N - 1\} \times \{0, \ldots, M - 1\}\) is a subset of the pulse and viewing angle indices. For example, in Fig. 1, with six viewing angles and eight pulses for each viewing angle \((N = 6, M = 8)\) and three missing signals at \(\theta_1, \theta_3,\) and \(\theta_5,\) the missing frequencies \(f_1, f_3, f_4, f_5, f_7\) of the first, third, fourth, fifth, and seventh pulse at each viewing angle, \(\mathbf{S}_{\Omega}\) is represented as

\[
\mathbf{S}_{\Omega} = \begin{bmatrix}
S_{00} & 0 & S_{02} & 0 & 0 & 0 & S_{06} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
S_{20} & 0 & S_{22} & 0 & 0 & 0 & S_{26} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
S_{40} & 0 & S_{42} & 0 & 0 & 0 & S_{46} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \quad (13)
\]

where \(S_{nm} = S_B(\theta_n, f_m).\) Due to the complexity of solving Eq. (12), an approximated solution based on SDP is used. This can be done using the data matrix and defining the two-level (2L) Toeplitz matrices\(^\text{19,20}\). In this work, we apply the Vandermonde decomposition to a 2L Toeplitz matrix and use the ADMM algorithm to retrieve the scattering points from the received signal, as will be explained in the sequel.
3.1 2L Toeplitz and Vandermonde Decomposition

According to Eqs. (3), (6)–(8), the received signal shown in matrix form in Eq. (4) can be decomposed to

\[ S = YDZ. \]  

(14)

where \( Y, D, Z \) are given as

\[ Y = [a(f_{x,1}), \ldots, a(f_{x,K})] \in \mathbb{C}^{NK}, \]  

(15)

\[ D = \text{diag}(d_1, \ldots, d_K) \in \mathbb{C}^{K \times K}, \]  

(16)

\[ Z = [a(f_{y,1}), \ldots, a(f_{y,K})] \in \mathbb{C}^{MK}. \]  

(17)

Then, a \((2N - 1) \times (2M - 1)\) Toeplitz matrix is defined as

\[
T = \begin{bmatrix}
    s_{-N,-M} & \cdots & s_{-N,0} & \cdots & s_{-N,M} \\
    \vdots & \ddots & \vdots & \ddots & \vdots \\
    s_{0,-M} & & s_{0,0} & & s_{0,M} \\
    \vdots & & \vdots & \ddots & \vdots \\
    s_{N,-M} & \cdots & s_{N,0} & \cdots & s_{N,M}
\end{bmatrix} \in \mathbb{C}^{(2N-1)\times(2M-1)},
\]  

(18)

based on which a 2L Toeplitz matrix is given by Ref. 19

\[
T' = \begin{bmatrix}
    T_0 & T_{-1} & \cdots & T_{-(N-1)} \\
    T_1 & T_0 & \cdots & T_{-(N-2)} \\
    \vdots & \vdots & \ddots & \vdots \\
    T_{N-1} & T_{N-2} & \cdots & T_0
\end{bmatrix} \in \mathbb{C}^{NM \times NM},
\]  

(19)

where each submatrix \(T_i\) is an \(M \times M\) Toeplitz matrix defined as

\[
T_i = \begin{bmatrix}
    s_{i,0} & s_{i,-1} & \cdots & s_{i,-(M-1)} \\
    s_{i,1} & s_{i,0} & \cdots & s_{i,-(M-2)} \\
    \vdots & \vdots & \ddots & \vdots \\
    s_{i,M-1} & s_{i,M-2} & \cdots & s_{i,0}
\end{bmatrix}.
\]  

(20)

Then, according to Theorem 2 in Ref. 20 and the Vandermonde decomposition, we can present \(T'\) as

\[
T_i = \Lambda V \Lambda^* ,
\]  

(21)

where \(V = [c(f_1), \ldots, c(f_K)]\) is the Vandermonde matrix and \(\Lambda = \text{diag}(\sigma_1, \ldots, \sigma_K)\). According to Theorem 1 in Ref. 20, if the rank of \(T'\) is \(k < \min(N, M)\), then \(T'\) can be decomposed as Eq. (21). Then, the atomic \(l_0\) norm of the received signal can be defined by the following non-convex rank minimization problem \(20\)

\[
\min_{i,s_{n,m}:T'} \text{rank}(T') \quad \text{s.t.} \quad \left[ \begin{array}{cc}
    t & s^H_{n,m} \\
    s_{n,m} & T'
\end{array} \right] \succeq 0,
\]  

(22)

where \((\cdot)^H\) is a Hermitian operator and \(\succeq\) shows a positive semidefinite matrix. To solve Eq. (22), we use a convex relaxation to convert it to a Trace minimization problem provided that \(T'\) admits a Vandermonde decomposition \(20\) as

\[
\min_{i,s_{n,m}:T'} (t + \text{tr}(T')) \quad \text{s.t.} \quad \left[ \begin{array}{cc}
    t & s^H_{n,m} \\
    s_{n,m} & T'
\end{array} \right] \succeq 0,
\]  

(23)

where \(\text{tr}(\cdot)\) is the trace of a matrix.
3.2 Reweighted Trace Minimization

A smoothed approximation of Eq. (23) as a metric for \( s_{n,m} \) is defined as\(^2^0\)

\[
L''(s_{n,m}) = \min_{t,T'} (t + \ln(|T'| + \mu|I|)) \quad \text{s.t.} \quad \begin{bmatrix} t & s_{n,m}^H T' \end{bmatrix} \geq 0,
\]

(24)

where \( \mu \) is the regularization factor. To solve Eq. (24), by defining \( W \) as a weighting factor, we use the iterative algorithm RWTM, in which the \( i \)th iteration of \( T' \) denoted by \( T'_i \) locally converges to an optimal point\(^2^0\) as

\[
\min_{t,s_{n,m},T'} (t + \text{tr}(T'_i + \mu|I|^{-1}T')) \quad \text{s.t.} \quad \begin{bmatrix} t & s_{n,m}^H T'_i \end{bmatrix} \geq 0,
\]

(25)

where the weighting factor is defined as

\[
W = (T'_i + \mu|I|)^{-1}.
\]

(26)

Assuming that the signal \( s_{n,m} \) is contaminated with AWGN, the received signal is given as

\[
s = s_{n,m} + n.
\]

(27)

where \( n \) shows the noise. Next, based on Refs. 19 and 25, Eq. (25) can be rewritten as

\[
\min_{s,t} \frac{1}{2} ||s - s_{n,m}||^2_2 + \frac{\tau}{\sqrt{2NM}} (t + \text{tr}(WT')) \quad \text{s.t.} \quad \begin{bmatrix} t & s_{n,m}^H T'_i \end{bmatrix} \geq 0,
\]

(28)

where \( \tau \) is a regularizer parameter. This complex optimization problem can be solved using ADMM, as will be explained in the sequel.

3.3 Proposed FRWTM Algorithm

We develop the FRWTM algorithm to retrieve the frequencies of sparse scattering points in both range and azimuth directions. According to Fig. 1, the target consists of a number of sparse scattering points. We assume that some transmitted pulses from different viewing angles of the target are not received. According to Eq. (9), the received signal vector consists of the RCSs of the scattering points and the 2D normalized complex atoms. Then, based on Eq. (18) and using the received signal vector in Eq. (9), we can form a Toeplitz matrix, which can also be changed to a 2L Toeplitz matrix using Eq. (19). Although RWTM can be used to solve Eq. (28), it is undesirably very time-consuming in practice. To increase the speed of this algorithm, we develop the FRWTM algorithm based on the ADMM. To do so, we first write Eq. (28) in an SDP form and then present it as an augmented Lagrange function.

Next, after initialization, by applying the ADMM at the \((l + 1)\)th iteration, we obtain Eqs. (29) and (31) as

\[
T'_i(u^{l+1}) = Z'_0 + \frac{1}{\rho} \left( A'_0 - \frac{\tau}{2\sqrt{NM}} W_i \right),
\]

(29)

\[
u^{l+1} = T'_i \left( Z'_0 + \frac{1}{\rho} A'_0 - \frac{\tau}{2\rho\sqrt{NM}} W_i \right),
\]

(30)

\[
s^{l+1}_{n,m} = \frac{1}{1 + 2\rho} (s_{n,m} + 2A'_1 + 2\rho Z'_1).
\]

(31)

where \( u \in \mathbb{C}^{N_u} \) in Eq. (30) is an auxiliary vector with \( N_u = (N - 1)(2M - 1) + M \), \( * \) is complex conjugation, \( \rho \) is a penalty parameter, \( s_{n,m} \in \mathbb{C}^{NM} \), \( t \in \mathbb{R} \), \( Z \in \mathbb{C}^{NM\times NM} \), \( \Lambda \) is an arbitrary matrix, \( A'_0 \in \mathbb{C}^{NM\times NM} \), \( Z'_0 \in \mathbb{C}^{NM\times NM} \), \( A'_1 \in \mathbb{C}^{NM\times 1} \), and \( Z'_1 \in \mathbb{C}^{NM\times 1} \). In order to update \( Z \), we use the following equation:\(^2^5\)

\[
Z^{l+1} = \arg\min_{Z \in \mathbb{C}^{NM\times NM}} ||Z - \begin{bmatrix} t^{l+1} & s^{H(l+1)} \end{bmatrix} \begin{bmatrix} s_{n,m}^H T' \end{bmatrix} + \frac{1}{\rho} \Lambda' ||_F^2,
\]

(32)

where \( || \cdot ||_F \) shows the Frobenious norm and \( \Lambda' \) is a Hermitian matrix. Then, we project this hermitian matrix on the positive semidefinite cone,\(^2^4\) which is performed by eigenvalue decomposition of this matrix and setting negative eigenvalues to zero.\(^2^5\) The details of
derivations are presented in Appendix A and the implementation steps of the proposed method are shown in Algorithm 1.

4 Experimental Results

We examine the performance of the FRWTM algorithm for the Yak-42 and MiG-25 aircraft with real data. The ISAR specifications are denoted in Table 1. It is assumed that the data are received incompletely in the range direction due to missing some radar pulses and in the azimuth direction due to missing the viewing angles. Figure 2 shows Yak-42 ISAR images with real data for different percentages of the received data at 10 dB SNR. As seen, the target image is almost recovered by receiving only 15% of the samples. Obviously, the more samples we get, the better resolution we obtain.

In the next experiment, we investigate the performance of FRWTM against AWGN for Yak-42. The results are shown in Fig. 3 for 65% of the received data for the SNR range of

| Table 1 | ISAR specifications. |
|---|---|
| Radar type | Pulse radar |
| Frequency band | X band |
| Center frequency | 9 GHz |
| Bandwidth | 100 MHz |
| Transmitter signal | FSCS |
| PRF | 6.2 kHz |
| Pulse length | 50 μs |
Fig. 2 ISAR imaging of Yak-42 by FRWTM at 10 dB SNR based on receiving (a) 90%, (b) 75%, (c) 60%, (d) 45%, (e) 30%, and (f) 15% of samples.

Fig. 3 ISAR imaging of Yak-42 by FRWTM using 65% of received data at SNR = (a) $-15$, (b) $-10$, (c) $-5$, (d) 0, (e) 5, and (f) 10 dB.
−15 to 10 dB. It is seen that the target is almost recognized as an aircraft from SNR = −5 dB and the image resolution improves by increasing the SNR. We repeat computer simulations for the MiG-25 aircraft, which has different scattering points compared to the Yak-42. The recovered ISAR images are shown in Fig. 4 for the SNR range of −15 to 10 dB. As can be observed, we achieve a similar performance by the proposed FRWTM as we observed for Yak-42.

In the third experiment, we compare the FRWTM with FRANM as a super-resolution-based method, $SL_{0}$ as a CS-based method, and SBL as a Bayesian-based method for

![Fig. 4 ISAR imaging of MiG-25 by FRWTM using 65% of received data at SNR= (a) −15, (b) −10, (c) −5, (d) 0, (e) 5, and (f) 10 dB.](image)

![Fig. 5 ISAR imaging of Yak-42 for (a) and (e) FRANM, (b) and (f) FRWTM, (c) and (g) SBL, and (d) and (h) $SL_{0}$ (top row for −5 and bottom row for 0 dB SNR).](image)
Yak-42 at −5 and 0 dB SNRs. As shown in Fig. 5, both FRANM and FRWTM algorithms generate more distinguishable images of the target than the SBL and $SL_0$. Failure of SBL is due to its dependence on the primary information, and in $SL_0$ is due to the need for placing scattering points on the predetermined grids.

In the last experiment, we compare the MSEs of algorithms based on receiving 65% of samples and for the SNRs from −10 to 10 dB. The results are averaged over 100 independent trials. Figure 6 shows that for Yak-42, the FRWTM is superior to FRANM, SBL, and $SL_0$ at all SNRs. Also, similar to Fig. 5, $SL_0$ and SBL give lower performance by generating higher MSEs than FRANM and FRWTM. Similarly, MSEs of different algorithms are compared in Fig. 7 for MiG-25 based on receiving 65% of data samples for the SNR from −10 to 10 dB. These results also confirm those of Fig. 6, showing the effectiveness of FRWTM.

Next, we compare the running times in Table 2. As seen, although the running time of FRANM is slightly lower than that of the proposed FRWTM, its MSE in Fig. 6 is higher especially at low SNRs. In other words, the FRWTM can better recover ISAR images at low SNRs with lower MSE and fewer samples.
5 Conclusion

We proposed the FRWTM algorithm for ISAR imaging in order to increase the resolution of the retrieved images. This super-resolution algorithm was developed based on retrieving sparse scattering points of a target to generate stronger RCSs. This was performed using the definition of 2D atoms under the practical assumption of receiving incomplete data from both range and azimuth directions. To solve the problem, we first formed a 2L Toeplitz matrix from the received signal. Then, using the Vandermonde decomposition of the latter matrix, the target scattering points were recovered. Due to the high computational burden of this method, we developed the ADMM method in two dimensions to reduce the calculations and speed up the computation process. Simulation results for ISAR imaging of Yak-42 and MiG-25 aircraft showed that FRWTM can successfully retrieve the frequencies of scattering points in both ranges and azimuth directions and increase the resolution of retrieved images. It was shown in terms of the MSE criterion that the proposed FRWTM outperforms the FRANM, SBL, and SL0 at all SNRs using fewer samples.

6 Appendix A: Derivation of Proposed FRWTM

We first rewrite Eq. (28) in an SDP form as

\[
\{\hat{u}, \hat{s}, \hat{t}, \hat{Z}\} = \arg\min_{u, s, t, Z} \frac{1}{2} \|s - s_{n,m}\|^2 + \frac{\tau}{\sqrt{2NM}} \left(t + \text{tr}(WT'(u)) \right) \quad \text{s.t.} \quad Z = \begin{bmatrix} t & s_{n,m}^H \end{bmatrix} \succeq 0,
\]

(33)

where \(\hat{u}, \hat{s}, \hat{t}, \) and \(\hat{Z}\) are the estimates of \(u, s, t,\) and \(Z\), respectively. Then, Eq. (33) is rewritten as an augmented Lagrangian function

\[
\mathcal{L}(u, s, t, Z, \Lambda) = \frac{1}{2} \|s - s_{n,m}\|^2 + \frac{\tau}{\sqrt{2NM}} \left(t + \text{tr}(WT'(u)) \right) + \frac{\rho}{2} \|Z - \begin{bmatrix} t & s_{n,m}^H \end{bmatrix} \|^2, \quad (34)
\]

where \(\Lambda \in \mathbb{C}^{(NM+1)\times(NM+1)}\) is the Lagrangian multiplier, \(\rho\) is a penalty parameter, and \(\langle \cdot, \cdot \rangle\) shows the inner product. By initializing \(\Lambda_0 = 0\) and \(Z_0 = 0\), Eq. (34) is updated by applying the ADMM at the \((l+1)\)th iteration as

\[
\{t^{l+1}, u^{l+1}, s^{l+1}\} = \arg\min_{t, u, s, Z} \mathcal{L}(t, u, s, Z, \Lambda^l), \quad (35)
\]

\[
Z^{l+1} = \arg\min_{Z \succeq 0} \mathcal{L}(t^{l+1}, u^{l+1}, s^{l+1}, Z, \Lambda^l), \quad (36)
\]

\[
\Lambda^{l+1} = \Lambda^l + \rho \left(Z^l - \begin{bmatrix} t^{l+1} & s_{n,m}^{(l+1)H} \end{bmatrix} T'(u^{l+1}) \right), \quad (37)
\]

where \(\Lambda^l\) and \(Z^l\) are, respectively, given as

\[
\Lambda^l = \begin{bmatrix} \Lambda_0^l & \Lambda_1^l & \Lambda_2^l \end{bmatrix}, \quad (38)
\]

and

Table 2 Running times of different algorithms.

| Method          | Time (s) |
|-----------------|----------|
| FRANM           | 8.69     |
| RWTM            | 7443     |
| FRWTM (proposed)| 9.8      |

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\[
Z' = \begin{bmatrix}
Z'_{11} \in \mathbb{C}^{NM \times NM} & Z'_{12} \in \mathbb{C}^{NM \times 1} \\
Z'_{21} \in \mathbb{C}^{l \times NM} & Z'_{22} \in \mathbb{C}^{l \times 1}
\end{bmatrix}.
\] (39)

By obtaining partial derivatives of Eq. (35) with respect to \( t, T'(u) \), and \( s \), we have
\[
\frac{\partial \mathcal{L}(t, u, s, Z', A')}{\partial t} = 0, \tag{40}
\]
\[
\frac{\partial \mathcal{L}(t, u, s, Z', A')}{\partial T'(u)} = 0, \tag{41}
\]
\[
\frac{\partial \mathcal{L}(t, u, s, Z', A')}{\partial s} = 0. \tag{42}
\]

By solving Eqs. (41) and (42), we obtain Eqs. (29) and (31) in Sec. 3.3, respectively, which completes the proof.

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Mohammad Roueinfar is currently pursuing his PhD in communication engineering in the School of Electrical Engineering, Iran University of Science and Technology. His research is focused on radar, space imaging, super-resolution theory, compressive sensing, and signal denoising.

Mohammad Hossein Kahaei received his BSc degree in electrical engineering from Isfahan University of Technology, Iran, in 1986, his MSc degree in adaptive signal processing from the University of the Ryukyus, Japan, in 1994, and his PhD in signal processing from QUT, Australia, in 1998. Since 1999, he has been with the School of Electrical Engineering, IUST, Iran, where he is currently an associate professor. His research interests include estimation theory, deep learning, compressed sensing, and tracking.