Quark deconfinement and meson properties at finite temperature

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Abstract

A simple confining separable interaction Ansatz for the rainbow-ladder truncated QCD Dyson-Schwinger equations is used to study quark deconfinement and meson states at finite temperature. The model is fixed at $T = 0$ to implement quark confinement while preserving the Goldstone mechanism for the $\pi$. Within the Matsubara formalism, a very slow temperature dependence is found for the $\pi$ and $\rho$ meson masses $M_H(T)$ until near the deconfinement temperature $T_c = 143$ MeV. Related to rapid decrease of the dynamically-generated quark mass function for $T > T_c$, this model produces $\pi$ and $\rho$ masses that rise significantly and are better interpreted as spatial screening masses. The $T$-dependent screening mass defect $\Delta M_H = 2\pi T - M_H(T)$ is compared to results of lattice gauge theory simulations and also to those of an infrared dominant analytic model.

1 Introduction

The experimental search for the QCD deconfinement phase transition in ultrarelativistic heavy-ion collisions will enter a new stage when the relativistic heavy-ion collider (RHIC) at Brookhaven will provide data complementary to results from the CERN SPS \cite{1}. It is desirable to have a continuum field-theoretical modeling of quark deconfinement and chiral restoration at finite temperature and density (or chemical potential $\mu$) that can be extended also to hadronic observables in a rapid and transparent way. Significant steps in this direction have recently been taken through a continuum approach to QCD$_{T,\mu}$ based on the truncated Dyson-Schwinger equations (DSEs) within the Matsubara formalism \cite{2,3,4}. For a recent review see \cite{5}. A most appealing feature of this approach to modeling nonperturbative QCD$_{T,\mu}$ is that dynamical chiral symmetry breaking and confinement is embodied in the the model gluon 2-point function constrained by chiral observables at $T = \mu = 0$ and no new parameters are needed for extension to $T, \mu > 0$. Approximations introduced by a specific truncation scheme for the set of DSEs can be systematically relaxed. However due partly to the discrete Matsubara modes, the
finite $T, \mu$ extension of current realistic DSE models entails a complicated set of coupled integral equations. In the separable model we study here, detailed realism is sacrificed in the hope that a few dominant and essential features may be captured in a simple and transparent format. We simplify an existing $T = \mu = 0$ confining separable interaction Ansatz \[6\] by using a gaussian form factor. A related gaussian separable model has recently been explored for meson properties \[7\].

## 2 Model Dyson-Schwinger equations at $T = 0$

In a Feynman-like gauge where we take $D_{\mu \nu} = \delta_{\mu \nu}D(p - q)$ to be an effective interaction between quark colored vector currents, the rainbow approximation to the DSE for the quark propagator $S(p) = [i\gamma \cdot A(p) + B(p) + m_0]^{-1}$ yields in Euclidean metric

$$B(p) = \frac{16}{3} \int \frac{d^4q}{(2\pi)^4} \frac{B(q) + m_0}{q^2 A^2(q) + [B(q) + m_0]^2}, \quad (1)$$

$$[A(p) - 1]p^2 = \frac{8}{3} \int \frac{d^4q}{(2\pi)^4} \frac{(p \cdot q) A(q)}{q^2 A^2(q) + [B(q) + m_0]^2}. \quad (2)$$

We study a separable interaction given by \[8\]

$$D(p - q) = D_0 f_0(p^2)f_0(q^2) + D_1 f_1(p^2)(p \cdot q)f_1(q^2), \quad (3)$$

where $D_0, D_1$ are strength parameters and the form factors, for simplicity, are here taken to be $f_i(p^2) = \exp(-p^2/\Lambda_i^2)$ with range parameters $\Lambda_i$. It is easily verified that if $D_0$ is non-zero, then $B(p) = \Delta m f_0(p^2)$, and if $D_1$ is non-zero, then $A(p) = 1 + \Delta a f_1(p^2)$. The DSE then reduces to nonlinear equations for the constants $\Delta m$ and $\Delta a$. The form factors should be chosen to simulate the $p^2$ dependence of $A(p)$ and $B(p)$ from a more realistic interaction. We restrict our considerations here to the rank-1 case where $D_1 = 0$ and $A(p) = 1$. The parameters $D_0, \Lambda_0$ and $m_0$ are used to produce reasonable $\pi$ and $\omega$ properties as well as to ensure the produced $B(p)$ has a reasonable strength with a range $\Lambda_0 \sim 0.6 \ldots 0.8$ GeV to be realistic \[8\].

If there are no solutions to $p^2 A^2(p) + (B(p) + m_0)^2 = 0$ for real $p^2$ then the quarks are confined. If in the chiral limit ($m_0 = 0$) there is a nontrivial solution for $B(p)$, then chiral symmetry is dynamical broken. Both phenomena are present in the separable model. In the chiral limit, the model is confining if $D_0$ is strong enough to make $\Delta m/\Lambda_0 \geq 1/\sqrt{2e}$. Thus for a typical range $\Lambda_0$, confinement will typically occur with $M(p \approx 0) \geq 300$ MeV.

Mesons as $q\bar{q}$ bound states are described by the Bethe-Salpeter equation which in the ladder approximation for the present approach is

$$-\lambda_H(P^2)\Gamma_H(p, P) = \frac{4}{3} \int \frac{d^4q}{(2\pi)^4} \left\{ D(p - q)\gamma_\mu S(q + \frac{P}{2})\Gamma_H(q, P)S(q - \frac{P}{2})\gamma_\mu \right\}, \quad (4)$$
where \( P \) is the total momentum and \( H \) denotes a particular hadron. The quantity \( \lambda_H \) is an eigenvalue and at the physical mass \( M_H \) it satisfies \( \lambda_H (P^2 = M_H^2) = 1 \).

With the rank-1 separable interaction, only the \( \gamma_5 \) and the \( \gamma_5 P \) covariants contribute to the \( \pi \) \( \delta \), and here we retain only the dominant form \( \Gamma_\pi (p, P) = i \gamma_5 E_\pi (p, P) \). For the vector meson, the only surviving form is \( \Gamma_\rho (p, P) = \gamma_\rho^T (P) E_\rho (p, P) \), with \( \gamma_\rho^T (P) \) being the projection of \( \gamma_\rho \) transverse to \( P \).

The separable solutions have the form \( E_i (p, P) = f_0 (q^2) C_i (P^2) \), \( i = \pi, \rho \), where the \( C_i \) factor out from Eq. (4). The eigenvalues \( \lambda_H (P^2) \) are the only determined quantities, and for example,

\[
\lambda_\pi (P^2) = \frac{16 D_0}{3} \int \frac{d^4 q}{(2\pi)^4} f_0^2 (q^2) \left[ (q^2 - \frac{P^2}{4}) \sigma^+ \sigma^- + \sigma^\pm \sigma^\pm \right].
\]

The quark propagator amplitudes are defined by \( S(p) = -i \phi \sigma_V (p^2) + \sigma_S (p^2) \), and the \( \pm \) superscripts indicate momentum arguments \( q \pm P/2 \) respectively. For the transverse and longitudinal \( \rho \) states, there are expressions for \( \lambda_\rho^T (P^2) \) and \( \lambda_\rho^L (P^2) \) analogous to Eq. (5).

With parameters \( m_0 / \Lambda_0 = 0.0096 \), \( D_0 \Lambda_0^2 = 128 \) and \( \Lambda_0 = 0.687 \) GeV, this model yields \( M_\pi = 0.14 \) GeV, \( M_\rho = M_\omega = 0.783 \) GeV, \( f_\pi = 0.104 \) GeV, a quark condensate \( \langle \bar{q} q \rangle^{1/3} = -0.248 \) GeV, and a \( \rho - \gamma \) coupling constant \( g_\rho = 5.04 \).

In the limit where a zero momentum range for the interaction is simulated by \( f_0^2 (q^2) \to \infty \), then the expressions for the various BSE eigenvalues \( \lambda_H (P^2) \) reduce to those of the Munczek and Nemirovsky \( \delta \) model which implements extreme infrared dominance via

\[
D(p - q) = \frac{3}{16} (2\pi)^4 \eta^2 \delta^{(4)} (p - q).
\]

The correspondence is not complete because the quark DSE solution in this model has \( A(p) \neq 1 \). We use \( \eta = 1.107 \) GeV to produce the same \( M_\omega \). The finite temperature and chemical potential generalization of this infrared dominant (ID) model has been studied in \( \delta \).3

\section*{3 Finite temperature extension}

The generalisation to \( T \neq 0 \) is systematically accomplished by transcribing the Euclidean quark momentum via \( q \to (\omega_n, \vec{q}) \), where \( \omega_n = (2n + 1)\pi T \) are the discrete Matsubara frequencies. Thus the integration measure in previous expressions becomes

\[
\int \frac{d^4 q}{(2\pi)^4} \to T \sum_{n=-\infty}^{\infty} \int \frac{d^3 \vec{q}}{(2\pi)^3}.
\]

At finite temperature, the \( O(4) \) symmetry is broken and the employed mass shell condition must be specified. One expects that if there is a bound \( \bar{q} q \) meson state,
then the associated pole contribution to the relevant propagator or correlator will have a denominator proportional to

\[ 1 - \lambda_H(\Omega_m^2, \vec{P}^2) \propto \Omega_m^2 + \vec{P}^2 + M_H^2. \] (8)

We investigate the meson mode eigenvalues \( \lambda_H \) using only the lowest meson Matsubara mode (\( \Omega_m = 0 \)) and the continuation \( \vec{P}^2 \rightarrow -M_H^2 \). The masses so identified are spatial screening masses corresponding to a behavior \( \exp(-M_H x) \) in the conjugate 3-space coordinate \( x \) and should correspond to the lowest bound state if one exists.

\[ \text{Figure 1: Spatial meson masses of } \pi \text{ and } \rho \text{ for the Gaussian separable model (left panel) and for the analytic ID model (right panel).} \]

The results are displayed in the left panel of Fig. 1 along with the strength (\( \Delta m \)) of the dynamical mass function at \( p = 0 \). The deconfinement temperature is indicated. The \( \pi \) and \( \rho \) masses are only weakly dependent upon \( T \) until near the transition. For the \( \pi \) this agrees with similar behavior obtained in the DSE approach with a more realistic interaction [2]. For both the 3-space transverse and 3-space longitudinal \( \rho \), the separate but still weak temperature dependence agrees with the findings [4] from the closely related ID model. The right panel of Fig. 1 displays the \( \pi \) and transverse \( \rho \) results from the ID model in the chiral limit. In both panels of Fig. 1 we display the \( T \)-dependence obtained from the eigenmass condition \( \lambda_H = 1 \) for \( T > T_c \) in comparison with \( 2\pi T \).

We interpret the model results rising similar to \( 2\pi T \) in the following way. In the chiral limit and just above the transition, the dynamically generated mass function of the quarks has disappeared and the axial vector Ward identity that ties the pion Bethe-Salpeter amplitude in the \( \gamma_5 \) channel to this quantity indicates that there should be no solution of the pion BSE there. However the particular model used here to estimate \( \lambda_H \) assumes the existence of a bound state with an amplitude or wave function profile given by the chosen form factor \( f_0(q^2) \). The form of the
latter was chosen at $T = 0$ to simulate the qualitative features of the DSE (and hence $\pi$ BSE) solution. If bound state conditions disappear with rising temperature as interactions weaken, this approach is not well suited to detect it. The high temperature form of the denominators of the quark propagators in the loop integral for the “polarization” function $\lambda_H$ is $\omega_n^2 + (\vec{q} \pm \vec{P}/2)^2$. The dependence of the form factor on $\omega_n^2$ emphasizes the lowest mode. For $\omega_0 >> \Lambda_0$ the form factor dictates that non-zero relative momentum $q$ makes a minor contribution to the location of the lowest singularity in $\vec{P}^2$ that determines the spatial screening mass. The higher the temperature the more the model behaves as if the fixed 3-space profile of the amplitudes had support effectively concentrated at $q = 0$, that is, almost independent of relative separation of quark and antiquark. Thus an approach to a screening mass characteristic of an independent fermion pair is to be expected here. In Fig. 2 this point is emphasized via comparison with spatial screening masses taken from [9] and representing lattice simulations above the transition. The horizontal lines mark $2\pi$, which in the right panel is corrected [9] for the lattice time extent $N_t$.

![Figure 2](image-url)

Figure 2: Hadronic screening masses from the gaussian separable and ID models (left panel) compared to those of lattice gauge theory simulations (right panel, data taken from [9]).

From Figs. 1 and 2 it is evident that for $T > T_c$, the “screening mass defect” $\Delta M_H = 2\pi T - M_H(T)$ is small and decreasing rapidly in the separable model, it is apparently larger in the lattice simulations especially for the $\pi$, and it is significantly stronger with a larger temperature range in the ID model. The latter we attribute to the strong quark self-energy amplitude $A(p) > 1$ in the ID model which is known to significantly slow the approach to Stefan-Boltzmann thermodynamics [9]. Such effects should be investigated further with a finite range interaction and the rank-2 version [10] of the present separable model might help in that regard.
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