Application of Closed Magnetic Circuit Method for Evaluating the Stress Distribution along Depth Direction

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Abstract: Electromagnetic nondestructive evaluation method is feasible in assessing stress or residual stress levels in ferromagnetic structures, for there is a strong inherent correlation between the permeability and the stress. This study is based on the closed magnetic circuit method for testing stress distribution along depth. Firstly, a theory is proposed for describing the propagation of electromagnetic energy inside ferromagnetic materials. A layered model is established considering the electromagnetic field experienced different attenuation at different depths. And each layer is assumed to have the identical physical state. Then, multi-frequency eddy current method is proposed to testing the in-depth permeability variation and consequently the stress-depth profile. For simply using the convolution integral, a novel inversion algorithm is given to resolve the convolution integral. To apply the proposed method for testing the stress-depth profile, multi-frequency eddy current experiments are conducted through a four-point bending device; with the specimen material employed is 45# steel. Finally, map the experimental data fitting curve to the theoretical calculation curve, and hence can verify the proposed theory and the inversion algorithm.

Keywords: Multi-frequency eddy current, permeability, stress-depth profile, magnetic circuit.

1. Introduction
The residual stress is generated inside materials after heat treatment or machining being applied to the components [1]-[3]. The residual stress is self-equilibrated inside materials. However, the stress state changes during service period of the components. The tensile stress inside steel structures demonstrates hazards to the load-bearing structures [4]-[6]. Evaluating the distribution of applied stress and residual stress along the depth direction is of importance to avoid sudden failure of the structures. Destructive stress testing methods such as hole-drilling way have high measurement accuracy, but the parts or the components cannot be repaired as original after testing. Nondestructive stress testing methods such as x-ray diffraction and neutron diffraction are expensive and inconvenient for in-situ implementation.

Electromagnetic NDT method is widely used in mechanical properties testing of ferromagnetic materials for inexpensiveness and convenience. The physical principle of electromagnetic NDT is that there are strong inherent correlations between the electromagnetic properties and the microstructures, also the stress state inside materials [7]-[10]. The magnetic property varieties originate from
interactions between magnetic domains motion and microstructures such as grain boundary, precipitate, etc., and the stress is also a significant factor influencing the magnetizations [11]-[13]. Many researchers focus on developing analytical models to reveal the connections between stresses and magnetic properties. Some attempts have been made to provide a model for stress-depth profile based on spectra and depth-dependent eddy current damping of Barkhausen emissions [14]. Due to high frequency characteristics of Barkhausen noise, most energy generated inside materials is attenuated when reaches the specimen surface, therefore, available testing depth is very limited [15]-[18]. For the stress measurement in components with larger thickness, eddy current method is a promising option. By varying excitation frequencies in the eddy current inducer, the penetration depth of eddy current field can be adjusted. Therefore, multi-frequency eddy current signal carries in-depth permeability information. Due to there is a strong correlation between the change in permeability and the variation in stress, therefore, it is possible to evaluate stress distribution along depth by multi-frequency eddy current method.

In this study, a multi-frequency eddy current closed magnetic circuit model based on convolution integral is employed for illustrating the propagation of electromagnetic field inside materials within stress. To obtain converged solutions of the convolution integral, hypothesis of layered structures along depth is introduced to the model. It is assumed at each layer the permeability (also the stress) is distributed uniformly. Then the proposed inversion algorithm is given for calculating the stress-depth profile. Because the permeability is different at different excitation frequencies, in order to eliminate the difference in permeability caused by frequencies, the variation between the permeability without stress and the permeability under stress is used as the object parameter. Finally, under the existing laboratory condition the four-point bending experiment is implemented, and the test specimen material is of 45# steel. The closed magnetic circuit method based on multi-frequency eddy current is successfully applied to test the stress-depth profile generated by four-point bending experiment. The final experimental results and the theoretical calculated results show a good linear relationship.

2. Methodology

According to the classical electromagnetism theory (1) and (2), which are second-order linear partial differential equations describing the propagation behaviour of the electromagnetic field in free space.

$$\nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (1)$$

$$\nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{B}}{\partial t^2} \quad (2)$$

Where $\mathbf{E}$ and $\mathbf{B}$ denote the electric field strength vector and the magnetic induction vector, respectively; the parameters $\mu_0$ and $\varepsilon_0$ represent vacuum permeability and vacuum permittivity, respectively. The electromagnetic field propagation behavior carries information of vacuum permeability and vacuum permittivity. Similarly, if the case comes to the ferromagnetic materials, the electromagnetic field propagating through materials can also reflect the electromagnetic properties of the materials.

Considering the electromagnetic field propagating inside materials is similar to plane wave, and will be attenuated with both distance and frequency. The skin depth expression (3) means when the field goes through distance $\delta_z$, the energy will be reduced to $1/e$ of its original value.

$$\delta_z = \frac{2\rho}{\omega_\varepsilon\mu} \quad (3)$$

Where $\omega_\varepsilon$ is angular frequency of the applied field; $\rho$ is the resistivity of ferromagnetic materials, which is considered to be homogeneous and isotropic; $\mu$ is the permeability. For simplicity, the ferromagnetic material without stress is treated to be magnetic isotropy. The magnetization depth is
deeper than the skin depth theoretically; therefore, it is feasible to represent the testing depth using the skin depth.

Figure 1 shows the tested material is divided into \( n \) layers model; considering each layer of the tested model has the identical stress distribution, thus the \( i \)th layer has the same permeability \( \mu_i \). As sketched in the figure, the different angular frequencies \( \omega_{ei} \) correspond to the different testing depths \( z_i \), and \( \Delta z_i \) represents the layer thickness. The electromagnetic fields attenuated energy \( E_i \) originating from each layer can be estimated as follows (4).

\[
\begin{align*}
E_1 &= E_0 e^{-\frac{\omega_{e1}}{2\mu_i} \Delta z_1} \\
E_2 &= E_0 e^{-\frac{\omega_{e2}}{2\mu_i} \Delta z_2} \\
&\vdots \\
E_i &= E_0 e^{-\frac{\omega_{ei}}{2\mu_i} \Delta z_i} \\
&\vdots \\
E_n &= E_0 e^{-\frac{\omega_{en}}{2\mu_i} \Delta z_n}
\end{align*}
\]

Where, \( E_0 \) indicates the linear energy density, which is a constant. The detected electromagnetic field total energy \( E_{\text{energy}} \) on sample surface is the accumulation results of the attenuated electromagnetic fields energy coming from all layers, the expression is as follows (5).

\[
E_{\text{energy}} = \sum_{i=1}^{n} e^{-\frac{\omega_{ei}}{2\mu_i} \Delta z_i} ; \ i = 1, 2, ..., n; \ j = 1, 2, ..., i
\]

In reality, the permeability \( \mu(z) \) of materials is a continuous and smooth function of the depth \( z \). Considering the situation of \( \Delta z_i \to 0 \), the integral form can be given as (6).

\[
E_{\text{energy}} = \int e^{-\frac{\omega_{ei}}{2\mu_i} z} d\mu
\]
Figure 2 shows the eddy current sensor diagram, employing an excitation coil wound onto a U-shape ferrite core and a testing coil. If electromagnetic field propagates through materials, the integral form of Faraday’s law and the integral form of Ampere-Maxwell’s law are as follows (7) and (8).

\[ \oint E \cdot dl = -\frac{d}{dt} \int_s B \cdot n \, da \]  
(7)

\[ \oint H \cdot dl = I_{\text{free, enc}} + \frac{d}{dt} \int_s D \cdot n \, da \]  
(8)

Where, the magnetic field strength vector \( H \) is equal to \( (B/\mu_0 - M) \); \( M \) is the magnetization vector of materials. The displacement vector \( D \) is defined as \( (\varepsilon_0 E + P) \); \( P \) is the electrode strength vector of materials. \( I_{\text{free, enc}} \) represents the single current. If taking the Ampere integration loop along main flux loop, the derivative of displacement vector flux which is perpendicular to the Ampere integration loop plane is zero.

The magneto-motive force \( \varepsilon_m \) along a closed magnetic circuit is \( \varepsilon_m = \sigma R_m \), here \( \sigma \) is magnetic flux through cross section. \( R_m = L/\mu S \) denotes magneto-resistance calculation formula; \( L \) and \( S \) represent the length and the cross sectional area of circuit, respectively. The magneto-resistance expression originated from (7) and (8) is given as (9).

\[ \varepsilon_m = N I_{\text{free, enc}} \]  
(9)

**Figure 2.** Diagram of the eddy current sensor used, an excitation coil, an induction coil, and a yoke are included.

When an current \( I_0(t) = I_0 \cos \omega t \) is fed into the excitation coil, the voltage induced in induction coil can be obtained as (10).

\[ V(t) = \frac{n N I_0}{R_{m, \text{total}}} \sin \omega t \]  
(10)

Where, \( R_{m, \text{total}} \) denotes the total magneto-resistance of magnetic circuit. According to the continuous convolution formula and electromagnetic propagation attenuation characteristic, letting \( I(t) = I_0 \sin \omega t \), the voltage is obtained as (11).

\[ V(t) = \mu_0 n N \omega I_0 (t) / (L_{1 \mu \omega t} + L_{2 \mu \omega t} + \frac{L_{3 \mu_1 \mu_2 \omega t}}{2 \omega \rho} \int \frac{e^{-\mu_1 \mu_2 \omega t}}{2 \rho} \, dz) \]  
(11)

Where, \( n \) is testing coil turns; \( N \) is excitation coil turns; \( L_1, L_2, \) and \( L_3 \) represent the magnetic circuit length in yoke, the air gap length, and the magnetic circuit length in sample respectively; \( w \) and \( t \) represent the width and thickness of magnetic circuit respectively; \( \mu_1 \) and \( \mu_2 \) represent the relative permeability of iron core and air. A discretized inversion algorithm is proposed here to find the
solution of $\mu(z)$. For the cases with different testing depths at different frequencies, the convolution integral is simplified as (12).

$$\int e^{-\frac{1}{2}\sqrt{\frac{\rho_0 \mu(z)}{\rho_1}} \cdot dz} = \sqrt{\frac{2\rho_1}{\rho_0 \mu_{i-1}}} (1 - e^{-\frac{1}{2}\sqrt{\frac{\rho_0 \mu(z)}{\rho_1}} \cdot dz})$$  \hspace{1cm} (12)

Where, define $\bar{\mu}_{i-1}$ as the averaged permeability of materials from layers 1 to $i$. Consequently, by solving the corresponding permeability variation at different frequencies based on the proposed discretized inversion algorithm, and using the curve fit tool a continuous solution function $\mu(z)$ can be obtained.

3. Experimental setup and results
The multi-frequency eddy current experimental testing system is shown in Figure 3, including PXI host (built-in signal generation card and data acquisition card), power amplifier, AD/DA modules, the system is controlled by LabVIEW® program. The excitation current can be acquired by connecting 1 $\Omega$ sampling resistor in series. Figure 4 shows the geometric size of the U-shaped yoke, with the material employed is silicon steel sheet. The specimen used to generate the stress gradient by the four-point bending device is of 45# steel, with a thickness of 6 mm, here the elastic modulus is about 200 GPa. In this study, five different excitation frequencies are used; ensure the magnetization depth is greater than the testing depth. Table 1 presents the parameters of the closed magnetic circuit.

![Figure 3. The multi-frequency eddy current experimental testing system setup.](image)

![Figure 4. The geometric size of the U-shaped magnetic core, with the material used is silicon steel sheet.](image)
To obtain the measuring signals within the elastic limit, the maximum loading deflection of the sample is 0.25 mm. In a single experiment, synchronously load and record the testing data, and then unload; three measurement signals are collected each. In total, three parallel experimental cycles are implemented, and nine sets of signals are obtained. This study is based on the change of permeability with and without loading; therefore, the experimental results are obtained through the following formula (13).

\[ \Delta \mu_i \Delta z_i = \int_{0}^{z_i} \Delta \mu(z) \, dz - \int_{z_i}^{z_i+1} \Delta \mu(z) \, dz \]

Where, \( \Delta z_i \) represents the \( i \)th layer thickness, \( \Delta \mu_i \) denotes the average permeability variation of the \( i \)th layer, and \( \Delta \mu(z) \) indicates the permeability variation between the loading and the unstressed state.

**Table 1. Parameters of the Closed Magnetic Circuit**

| Symbol | Quantity and Unit | Additional Information |
|--------|-------------------|------------------------|
| \( n \) | 300 | Testing coil turns |
| \( N \) | 150 | Excitation coil turns |
| \( L_1 \) | 80 mm | Length of the magnetic circuit in yoke |
| \( L_2 \) | 0.18 mm | Length of the air gap |
| \( L_{3, \text{1200Hz}} \) | 15 mm | Magnetic circuit length in sample under 1200 Hz |
| \( L_{3, \text{150Hz}} \) | 16 mm | Magnetic circuit length in sample under 150 Hz |
| \( L_{3, \text{50Hz}} \) | 18 mm | Magnetic circuit length in sample under 50 Hz |
| \( L_{3, \text{10Hz}} \) | 30 mm | Magnetic circuit length in sample under 10 Hz |
| \( L_{3, \text{4Hz}} \) | 48 mm | Magnetic circuit length in sample under 4 Hz |
| \( t \) | 8 mm | Thickness of the magnetic circuit |
| \( w \) | 14 mm | Width of the magnetic circuit |
| \( z \) | 6 mm | Thickness of the sample |
| \( \rho \) | 2.0\times10^{-7} \, \Omega \cdot m | The resistivity of the sample |
| \( \mu_0 \) | 4\pi\times10^{-7} \, H/m | The vacuum permeability |
| \( \mu_1 \) | 8000 | Relative permeability of the iron core |
| \( \mu_2 \) | 1 | Relative permeability of the air |

The final experimental results are shown in the following figures. Figure 5 shows the magnetic permeability under different excitation frequencies without and with loading. Figure 6 draws the corresponding permeability variation relative to the unloading situation, and Figure 7 presents the testing depth under different frequencies. More generally, according to the aforementioned algorithm, \( N(N+1)/2 \) values can be inverted for \( N \) different frequencies. Thence 15 permeability variation values can be inverted for 5 frequencies as shown in Figure 8, which the green solid line indicates the theoretical stress gradient, the red circles represent 15 inversion values with error limits, and the black dashed line shows the fitting curve of the inversion values.
**Figure 5.** The magnetic permeability under different excitation frequencies without and with loading.

**Figure 6.** The permeability variation values under different excitation frequencies.

**Figure 7.** The testing depth values under different excitation frequencies.
4. Discussion

In this study, based on the magnetic permeability of ferromagnetic materials changes with stress or residual stress, a new layered model is established and a multi-frequency eddy current sensor to test the stress gradient is designed. The final experimental results present the data fitting curve can map the theoretical calculation curve, and hence can verify the aforementioned theory model and the proposed inversion algorithm. This study provides an effective means for qualitative analysis and approximate quantitative calculation for testing the stress-depth profile.

Using the magnetic circuit method for testing stress distribution along the depth is just an approximate estimation method. Because the iron core permeability is considered to be constant in testing. Due to the magnetic circuit distribution in specimen under different frequencies is extremely complicated; it is almost impossible to use only one-dimensional magnetic circuit length to describe the magnetic circuit distribution in specimen, so the circuit lengths in sample are estimated values. The length of the air gaps is also difficult to measure in each experiment. And the inversion algorithm proposed is only an approximate method. In summary, these all affect the measured permeability values. Therefore, if this assessment method is applied to the actual project, there is still a lot of work to be done.

5. Conclusion

In this research, a layered structure is established for describing the propagation of the electromagnetic energy, and then the multi-frequency eddy current testing principle is described. Assuming each layer has the same physical state, therefore in each layer the permeability is assumed to be identical. According to the hypothetical model, an inversion algorithm is given for simply using the convolution integral. Finally, under the existing condition in laboratory, the four-point bending experiment are carried out. From the experimental results, the data fitting curve can map the theoretical calculation curve. This study provides an effective means for qualitative analysis and approximate quantitative characterization for testing the stress-depth profile. Although using the magnetic circuit method is just an approximate estimation method, this work opens the possibility of testing the stress or residual stress as a function of the depth.

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