We present the ETMC results for the bag parameters describing the neutral kaon mixing in the Standard Model and beyond and preliminary results for the bag parameters controlling the short distance contributions in the $D^0 - \bar{D}^0$ oscillations. We also present preliminary results for the $B_{d/s}, B_{s/s} / B_{d/s}$ and $\xi$ parameter controlling $B_{d/s}^0 - \bar{B}_{d/s}^0$ oscillations in the Standard Model employing the so-called ratio method. Using $N_f = 2$ maximally twisted sea quarks and Osterwalder-Seiler valence quarks we achieve both $O(a)$-improvement and continuum like renormalization pattern. Simulations are performed at three-values of the lattice spacing and several values of quark masses in the light, strange, charm region and above charm up to $\sim 2.5m_c$. Our results are extrapolated to the continuum limit and extrapolated/interpolated to the physical quark masses.

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1. Introduction

Flavour Changing Neutral Currents (FCNC) and CP violation may provide relevant information on the impact of beyond the Standard Model (BSM). The lattice computation of the relevant matrix elements appearing in $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing in combination with the experimental value of $\epsilon_K$, $\Delta M_{bd}$ and $\Delta M_{bs}$ offers the opportunity to constrain the BSM model parameters. $D^0 - \bar{D}^0$, at variance with $K^0 - \bar{K}^0$ and $B^0 - \bar{B}^0$ mixing, involves valence up-type quarks so it is sensitive to a different sector of New Physics (NP). Although long distance effects dominate over short distance effects in the D system, it is still possible to put significant constrains on the NP parameter space. The most general BSM $\Delta F = 2$ effective Hamiltonian of dimension-six operators contributing to neutral meson mixing is

$$H_{\text{eff}}^{\Delta F = 2} = \sum_{i=1}^{5} C_i(\mu) Q_i$$  \hspace{1cm} (1.1)$$

where the operators $Q_i$ involving light $(l)$ and non-light $(h)$ quarks read

$$Q_1 = \left[ \bar{h}^a \gamma_\mu (1 - \gamma_5) l^a \right] \left[ \bar{h}^b \gamma_\mu (1 - \gamma_5) l^b \right]$$
$$Q_2 = \left[ \bar{h}^a (1 - \gamma_5) l^a \right] \left[ \bar{h}^b (1 - \gamma_5) l^b \right]$$
$$Q_3 = \left[ \bar{h}^a (1 - \gamma_5) l^a \right] \left[ \bar{h}^b (1 + \gamma_5) l^b \right]$$
$$Q_4 = \left[ \bar{h}^a (1 + \gamma_5) l^a \right] \left[ \bar{h}^b (1 - \gamma_5) l^b \right]$$

2. Lattice setup

Our lattice computations has been performed at three values of the lattice spacing using the $N_f = 2$ dynamical quark configurations produced by ETMC. In the gauge sector, the tree-level Symanzik improved action has been used while the dynamical sea quarks have been regularized employing a twisted mass doublet at maximal twist [1] which provides automatic $O(a)$ improvement [2]. The fermionic action for the light doublet $\psi$ in the sea reads in the so-called physical basis

$$S_{\text{sea}}^{\text{Mim}} = \sum_x \bar{\psi}(x) \left[ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla_\mu^\dagger) - i \gamma_5 \tau^3 \left[ M_{ct} - \frac{a}{2} \sum_\mu \nabla_\mu^\dagger \nabla_\mu \right] + \mu_{\text{sea}} \right] \psi(x)$$ \hspace{1cm} (2.1)$$

In addition, both $O(a)$ improvement and continuum-like renormalization pattern for the four-fermion operators are achieved by introducing an Osterwalder-Seiler [3] valence quark action allowing for a replica of the heavy $(h, h')$ and the light $(l, l')$ quarks [4]. The valence quark action reads

$$S_{\text{val}}^{\text{OS}} = \sum_x \sum_{f=l, h, h'} \bar{q}_f \left[ \frac{1}{2} \gamma_\mu (\nabla_\mu + \nabla_\mu^\dagger) - i \gamma_5 r_f \left[ M_{ct} - \frac{a}{2} \sum_\mu \nabla_\mu^\dagger \nabla_\mu \right] + \mu_f \right] q_f(x)$$ \hspace{1cm} (2.2)$$

where the Wilson parameters are conveniently chosen such that $r_h = r_l = r_{h'} = -r_{l'}$.

In table 1 we give the details of the simulation and the values of the sea and the valence quark masses at each value of the gauge coupling. The smallest sea quark mass corresponds to a pion of about 280 MeV for the case of $\beta = 3.90$. We simulate three heavy valence quark masses $\mu_{\tau, c}$ around the physical strange one to allow for a smooth interpolation, further three $\mu_{c, c'}$ around the physical charm mass followed by a sequence of heavier masses in the range $(m_c, 2.5 m_c)$. For the
the gauge links in the interpolating fields \( \text{by the formulae} \)

\[
\langle T^0 \rangle |Q_1(\mu)| P^0 \rangle = C_1 B_1(\mu) m_{\bar{p},f_P} f_{\bar{p}}^2
\]

\[
\langle T^0 \rangle |Q_i(\mu)| P^0 \rangle = C_i B_i(\mu) \left[ \frac{m_{\bar{p},f_P}}{m_i(\mu)} \right]^2
\]

where \( C_i = \frac{8}{3}, -\frac{5}{3}, \frac{1}{3}, \frac{2}{3}, \frac{2}{3}, i = 1, \ldots, 5 \). The renormalisation constants (RCs) of the relevant four- and two-fermion operators has been computed non-perturbatively in the RI’-MOM scheme [9]. Due to the OS-tm mixed action setup, the renormalized values of the bag parameters are given by the formulae [4] [10] [11]

\[
\hat{B}_1 = \frac{Z_{11}}{Z_{AA} Z_V} B_1, \quad \hat{B}_i = \frac{Z_{ij}}{Z_{A} Z_i} B_j \quad i, j = 2, \ldots, 5
\]

4. \( K^0 - \overline{K}^0 \)

For long time separation \( y_0 \ll x_0 \ll y_0 + T_{\text{sep}} \) the plateau of the following ratio estimates

\[
E[B_1](x_0) = \frac{C_1(x_0)}{C_{AP}(x_0) C'_{AP}(x_0)}, \quad E[B_j](x_0) = \frac{C_j(x_0)}{C_{PP}(x_0) C'_{PP}(x_0)}
\]

provides the estimate of bare \( B_i \). The involved correlators are

\[
C_j(x_0) = \sum \langle \mathcal{O}_{y_0 + T_{\text{sep}}}^j(x) \rangle \mathcal{O}_{y_0}^{21} \]

\[
C_{PP}(x_0) = \sum \langle \mathcal{P}_{y_0}^j(x_0) \mathcal{P}_{y_0}^{21} \rangle, \quad C_{AP}(x_0) = \sum \langle A_{y_0}^{12}(x_0) \mathcal{O}_{y_0}^{21} \rangle
\]

\[
C'_{PP}(x_0) = \sum \langle \mathcal{O}_{y_0 + T_{\text{sep}}}^j(x) \mathcal{P}_{y_0}^{34} \rangle, \quad C'_{AP}(x_0) = \sum \langle \mathcal{P}_{y_0 + T_{\text{sep}}}^j(x_0) A_{y_0}^{34} \rangle
\]

\(^1\)see the last two columns in table 1
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| $B_1$  | $B_2$  | $B_3$  | $B_4$  | $B_5$  |
|--------|--------|--------|--------|--------|
| 0.52(02) | 0.54(03) | 0.94(08) | 0.82(05) | 0.63(07) |

Table 2: Continuum limit results for $B_i$ parameters of the $K^0$–$\bar{K}^0$ system renormalized in the $\overline{MS}$ scheme of [12] at 2GeV using M1-type RCs defined in [11].

\[
\begin{align*}
\mathcal{P}^{21}_{y_0} &= \sum_{\tilde{y}} \tilde{q}_2(\tilde{y}, y_0) \psi q_1(\tilde{y}, y_0), & \mathcal{P}^{34}_{y_0} &= \sum_{\tilde{y}} \tilde{q}_4(\tilde{y}, y_0 + T_{\text{sep}}) \psi q_3(\tilde{y}, y_0 + T_{\text{sep}}) \quad \text{and} \quad P^{ij} = \tilde{q}_i \bar{\psi} q_j,
\end{align*}
\]

with $A^{ij} = \tilde{q}_i y_0 \bar{\psi} q_j$. In table 2 we gather our final continuum results for $B_i$ in the $\overline{MS}$ scheme. For details about the analysis, results and its phenomenological implications we refer to [10] and [11].

5. $D^0$–$\bar{D}^0$

The bag parameters for the $D^0$–$\bar{D}^0$ oscillations can be determined following the same strategy as in [10] and outlined in the previous section, now with two of the OS valence quarks representing the up quark while the other two will be identified with the charm quark, i.e $l \sim u$ and $h \sim c$. Physical values are obtained by interpolating data in $\mu_{\text{phys}}$ to the physical value $\mu_c$, while chiral and continuum extrapolations are carried out simultaneously. The physical values for the charm quark mass have been previously derived and can be found in [13].

Exploratory studies show that using Gaussian smeared sources and choosing a time separation between meson sources smaller than $T/2$ (we set $T_{\text{sep}} = 16$ at $\beta = 3.8$, $T_{\text{sep}} = 18$ at $\beta = 3.9$, $T_{\text{sep}} = 22$ at $\beta = 4.05$) is crucial for quark masses around the physical charm and above. This improvement allows us to extract the ground state with more confidence and precision in a wider time interval. From figure 1a the benefit of Gaussian smearing compared to local source and sink is evident. For illustration in figure 1b we display the quality plateau of $B_i$ at the smallest lattice spacing, $\beta = 4.05$, and for the smallest value of the light quark. Preliminary results are collected in table 3.

6. $B^0$–$\bar{B}^0$

Since discretization errors on current lattices are expected to be large at the physical value of b-quark mass, our strategy for the computation of the bag-parameters in the B sector is based on the

\[\text{sources are local for K-meson and smeared for D- and B- mesons}\]
ratio method approach proposed in [14] by introducing suitable ratios with exactly known static limit and interpolating them to the b-quark mass.

A similar strategy to the one employed in [15] for the computation of the $B_d$ an $B_s$ decay constants is followed here for the $B_{Bd}$ and $B_{Bs}$ parameters. According to HQET, the ratio of renormalized bag parameters evaluated in QCD is expected to approach unity as $1/\hat{\mu}_h \to 0$. The leading deviation are predicted to be of order $1/\log(\hat{\mu}_h)$ and are expected to be tiny in the $\hat{\mu}_h$ range of our data. We consider the following chiral and continuum extrapolated scalar ratios, where the mass and the difference between them will be eventually assigned as a systematic error. In contrast, quark mass as shown in figures

\begin{equation}
\omega_d \left( \hat{\mu}_h^{(n)} \right) = \lim_{\mu_{\text{sea}} \to \hat{\mu}_h^{(n)}} \lim_{\mu_r \to 0} \omega_d^L \left( \hat{\mu}_h^{(n)} ; \hat{\mu}_{\text{sea}}, \hat{\mu}_1, a \right),
\end{equation}

with

\begin{equation}
\omega_d^L \left( \mu_h^{(n)} ; \mu_{\text{sea}}, \hat{\mu}_1, a \right) = C \left( \hat{\mu}_h^{(n)} ; \hat{\mu}_{\text{sea}}, \hat{\mu}_1, a \right) B_{Bd/s} \left( \hat{\mu}_h^{(n)} ; \hat{\mu}_{\text{sea}}, \hat{\mu}_1, a \right)
\end{equation}

The C-factors ratio contains the information on the $1/\log(\hat{\mu}_h)$ corrections. At tree-level (TL) $C \left( \hat{\mu}_h^{(n)} ; \hat{\mu}^*, \mu \right) = 1$, while at leading-log (LL) it is given by

\begin{equation}
C \left( \hat{\mu}_h^{(n)} ; \hat{\mu}^*, \mu \right) = \left[ \alpha(\hat{\mu}^*)/\alpha(\mu_h^{(n)}) \right]^{-\beta_1^L/2\hat{\beta}_0} \left[ \alpha(\mu_h^{(n)}) / \alpha(\mu_h) \right]^{-\beta_1^L/2\hat{\beta}_0} \left[ \alpha(\hat{\mu}^*) / \alpha(\mu_h^{(n)}) \right] \gamma_h / \hat{\beta}_0
\end{equation}

and at NLL (not included in this preliminary analysis) the HQET mixing of $O_1$ with $O_{2,3}$ should be considered. In the formula above $\gamma_h$ and $\beta_1^L$ characterize the anomalous dimension (AD) of the axial $\hat{h}l$-current and the “11” element of the AD-matrix of the $hhlh$ operator in HQET.

In order to have better control on the chiral extrapolation, we consider the double ratio $B_{Bs}/B_{Bd}$

\begin{equation}
\zeta_{L0}^L \left( \mu_h^{(n)} ; \mu_{\text{sea}}, \hat{\mu}_1, a \right) = \frac{\omega_d^L \left( \mu_h^{(n)} ; \mu_{\text{sea}}, \hat{\mu}_1, a \right)}{\omega_d^L \left( \hat{\mu}_h^{(n)} ; \hat{\mu}_{\text{sea}}, \hat{\mu}_1, a \right)}
\end{equation}

which also tends to 1 in the continuum and chiral limit. The ratio method is also applied to the phenomenologically interesting quantity $\xi = (f_{Bs}/f_{Bd}) \sqrt{B_{Bs}/B_{Bd}}$ by forming the ratios $\xi_{L}^L$ analogous to Eq.(6.3) at successive values of the heavy quark mass.

The quantities $\omega_d^L$, $\omega_d^L$, $\zeta_{L0}^L$ and $\zeta_{L}^L$ have a smooth chiral and continuum extrapolation, showing no significant dependence on $\mu_1$ and small cutoff effects. The results turn out to be well described by a linear dependence in $\mu_1$ and $a^2$. For instance in figure 2a we show the chiral and continuum extrapolation of $\zeta_{L0}^L$ at the fourth of the considered masses.

Finally, we study the dependence of the ratios $\omega_d$, $\omega_s$, $\zeta_{L0}$ and $\zeta_{L}$ on the inverse of the heavy quark mass as shown in figures 3a, 3b and 3c for $\omega_d$, $\zeta_{L0}$ and $\zeta_{L}$. For $\omega_d$ and $\omega_s$ we perform a linear ($\omega_{d/s} = 1 - b(\lambda)/\hat{\mu}_h$) or a quadratic ($\omega_{d/s} = 1 - b(\lambda)/\hat{\mu}_h + c(\lambda)/\hat{\mu}_h^2$) interpolation to the b-quark mass and the difference between them will be eventually assigned as a systematic error. In contrast,
\[ \zeta_\omega \text{ and } \zeta_\xi \text{ show very weak dependence on the heavy quark mass, thus in these cases we perform either a quadratic or linear interpolation or we fix the ratio equal to its asymptotic heavy-quark mass limit } \zeta_\omega = 1 \text{ and } \zeta_\xi = 1. \]

From the structure of Eq.(6.2) we derive the iterative formulae

\[
\omega_{d,s} \left( \hat{\mu}_h^{(2)} \right) \omega_{d,s} \left( \hat{\mu}_h^{(3)} \right) \ldots \omega_{d,s} \left( \hat{\mu}_h^{(K+1)} \right) = \frac{C \left( \hat{\mu}_h^{(K+1)} \right)}{C \left( \hat{\mu}_h^{(1)} \right)} \frac{B_{Bs}/B_{Bd} \left( \hat{\mu}_h^{(K+1)} \right)}{B_{Bs}/B_{Bd} \left( \hat{\mu}_h^{(1)} \right)}, \]

relating the quantity at the heavy quark mass \( \hat{\mu}_h^{(K+1)} \) with its triggering value at \( \hat{\mu}_h^{(1)} \) and the fit values of \( \omega_d, \omega_s, \zeta_\omega \text{ and } \zeta_\xi \), where the value of \( \hat{\mu}_h^{(1)} \) together with the \( \lambda \) factor and the number of steps to arrive to the b quark mass point have been previously tuned in such a way that after a finite number of steps (K) the heavy-light meson mass assumes the experimental value of \( M_B \) [17]. The value of the quantities in Eq.(6.4) at the triggering mass point can be accurately measured in the chiral and continuum limit since \( \mu_h^{(1)} \) lies in the well accessible charm quark mass region. As example, in figure 2b we display the continuum and chiral extrapolation of the ratio \( B_{Bs}/B_{Bd} \) at triggering mass, which shows small cutoff effects. Finally, Eq.(6.4) leads to the following preliminary estimates:

\[
B_{Bd} = 0.87(05), \quad B_{Bs} = 0.90(05) \quad (6.5)
\]

where the error is the sum in quadrature of statistical and systematic uncertainties coming from the variation of the result if we use a linear or quadratic fit in the heavy quark mass and the truncation of the C-factors estimated as the difference between TL and LL matching. C-factors cancel out in the ratios \( B_{Bs}/B_{Bd} \) and \( \xi \), while the implementation of a constant, a linear or a quadratic fit in \( \hat{\mu}_h \) turns out to provide essentially identical results. The preliminary estimates are

\[
B_{Bs}/B_{Bd} = 1.03(2), \quad \xi = 1.21(06) \quad (6.6)
\]

where in the second one we added in quadrature a systematic error due to the fit ansatz at the triggering mass point.

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Figure 2: Double ratio $\zeta_\omega$ for the fourth analysed mass (a) Chiral and continuum extrapolation of the ratio $B_{Bs}/B_{Bd}$ computed at the triggering point (b). Vertical lines represent the position of the physical point.

Figure 3: Heavy quark mass dependence of the ratio $\omega_\Lambda$ (a) and the double ratios $\zeta_\omega$ (b) and $\zeta_\xi$ (c) extrapolated to the physical values of the light and strange masses and to the continuum limit.

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