A new approach to axial coupling constants in the QCD sum rule

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Abstract

We derive new QCD sum rules for the axial coupling constants by considering two-point correlation functions of the axial-vector currents in a one-nucleon state. The QCD sum rules tell us that the axial coupling constants are expressed by nucleon matrix elements of quark and gluon operators which are related to the sigma-terms and the moments of parton distributions. The results for the iso-vector axial coupling constants and the 8th component of the SU(3) octet are in good agreement with experiment.

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The axial coupling constants are defined by the nucleon matrix elements of the axial-vector currents at zero momentum transfer. The iso-vector axial coupling constant $g_A^{(3)}(0)$ and the 8th component of the SU(3) octet $g_A^{(8)}(0)$ are known to be expressed by the SU(3) parameters $F$ and $D$: $g_A^{(3)}(0) = F + D$, $g_A^{(8)}(0) = 3F - D$. In the naive parton model the singlet axial coupling constant $g_A^{(0)}(0)$ is expressed by the fraction of the nucleon spin carried by the $q$ quark, $\Delta q$, as $g_A^{(0)}(0) = \Delta u + \Delta d + \Delta s$, while $g_A^{(3)}(0) = \Delta u - \Delta d$, and $g_A^{(8)}(0) = \Delta u + \Delta d - 2\Delta s$. These coupling constants yield an important information on the spin structure of the valence and the sea quarks in the nucleon. The parameters $F$ and $D$ are precisely known from the measurements in neutron and hyperon $\beta$-decay experiments, while an unexpected small value of $g_A^{(0)}(0)$ was found from the EMC data: The quarks contribute only a small fraction to the proton’s spin, and $g_A(0)$, therefore, attracted much attention [1].

The investigations of the axial coupling constants by QCD sum rules have been done so far by the authors in Refs. [2-6]. Belyaev and Kogan [2] calculated $g_A^{(3)}(0)$ by considering a two-point correlation function of nucleon currents in an external axial-vector field. Ioffe [5] also calculated $g_A^{(3)}(0)$ and $g_A^{(8)}(0)$ with the same correlation function. Their method has some difficulties: In the operator product expansion (OPE), there appear new vacuum expectation values of quark-gluon composite operators induced by the external field. In addition, the spectral function of the correlation function has double and single-pole terms. The residue of the former is proportional to $g_A(0)$, and the latter corresponds to the transitions of a nucleon state to excited states through the interaction with the external field, but their residues are not known. The authors used a method to obtain the values of $g_A(0)$ and the residue of the single pole term from a $\chi^2$ fitting procedure, and obtained a good agreement with experiment.

In this paper, we propose a new method to construct QCD sum rules for the axial coupling constants from two-point correlation functions of the axial-vector currents in a one-nucleon state. With the method, the correlation functions are expressed by nucleon matrix elements of quark-gluon composite operators. The spectral function has only a single pole term, whose residue is related to $g_A(0)$. As will be shown later, the axial coupling constants $g_A^{(3)}(0)$ and $g_A^{(8)}(0)$ are related to experimentally or theoretically well-known quantities such as the $\pi$-$N$ and $K$-$N$ sigma-terms and the moments of parton distributions. For the singlet axial coupling constant $g_A^{(0)}(0)$ we need to fully take into account the chiral anomaly, but it seems to be difficult within the ordinary framework of QCD sum rules, and we leave it as a future work.

We first consider a correlation function of axial-vector currents:

$$\Pi^{(i)}_{\mu\nu}(q; P) = i \int dx^4 e^{iqx} \langle T[j^{(i)}_{\mu5}(x), j^{(i)}_{\nu5}(0)] \rangle_N,$$

where the superscript $i$ is the $SU(3)_f$ index and $q^\mu \equiv (\omega, \mathbf{q})$. In Eq.(1), the nucleon matrix
element is defined by
\[ \langle \ldots \rangle_N \equiv \frac{1}{2} \sum_S [\langle N(PS) | \ldots | N(PS) \rangle - \langle \ldots \rangle_0 \langle N(PS) | N(PS) \rangle], \] (2)
where \( P^\mu \equiv (E, \mathbf{P}) \) is the nucleon momentum \( (P^2 = M^2, \; M \) is the nucleon mass), \( S \) the nucleon spin, \( \langle \ldots \rangle_0 \) the vacuum expectation value, and the one-nucleon state to be normalized as \( \langle N(PS) | N(PS') \rangle = (2\pi)^3 \delta^3(\mathbf{P} - \mathbf{P'}) \delta_{SS'} \). The axial-vector currents are defined as
\[ j^{(3)}_\mu(x) = \frac{1}{2} \eta_{\mu\nu} \left[ \bar{u}(x) \gamma^\nu \gamma_5 u(x) - \bar{d}(x) \gamma^\nu \gamma_5 d(x) \right], \] (3)
\[ j^{(8)}_\mu(x) = \frac{1}{2\sqrt{3}} \left[ \bar{u}(x) \gamma_\mu \gamma_5 u(x) + \bar{d}(x) \gamma_\mu \gamma_5 d(x) - 2\bar{s}(x) \gamma_\mu \gamma_5 s(x) \right], \] (4)
where \( u, \; d \) and \( s \) are the up, down and strange quark fields, respectively. In Eq.(3), \( \eta_{\mu\nu} \equiv q_\mu q_\nu/q^2 - g_{\mu\nu} \) is introduced to make the current conserved and suppress, simultaneously, the pion contribution to the current \[ \rho_{\mu\nu}(\omega, \mathbf{q}; P) \equiv -\frac{1}{\pi} \text{Im} \Pi_{\mu\nu}(\omega + i\epsilon, \mathbf{q}; P). \] (6)
Following Refs.[8,9], we split \( \Pi_{\mu\nu} \) into even and odd parts as \( \Pi_{\mu\nu}(\omega^2)_{\text{even}} + \omega \Pi_{\mu\nu}(\omega^2)_{\text{odd}} \), and make a Borel transform on both sides of each Lehmann representation:
\[ \hat{B}[\Pi_{\mu\nu}(\omega^2; \mathbf{q}; P)_{\text{even}}] = -\int_{-\infty}^{\infty} d\omega' \omega' \exp \left( -\frac{\omega'^2}{s} \right) \rho_{\mu\nu}(\omega', \mathbf{q}; P), \] (7)
\[ \hat{B}[\Pi_{\mu\nu}(\omega^2; \mathbf{q}; P)_{\text{odd}}] = -\int_{-\infty}^{\infty} d\omega' \exp \left( -\frac{\omega'^2}{s} \right) \rho_{\mu\nu}(\omega', \mathbf{q}; P), \] (8)
where \( s \) is a squared Borel mass and \( \hat{B} \) the Borel transformation:
\[ \hat{B} \equiv \lim_{\substack{\omega^2 \to \infty \\ n! \to \infty}} \left[ -\frac{d}{d(-\omega^2)} \right]^n. \] (9)
In Eqs.(7) and (8) the left hand sides are evaluated by OPE’s, which give rise to the Borel transformed QCD sum rules.

Let us now consider the physical content of the spectral function. Among the intermediate states of the spectral function, the lowest one is a one-nucleon state. The continuum state consists of meson-nucleon states, excited nucleon states and so on. There is an energy gap between the pole of the one-nucleon state and the threshold of the continuum states.
The contribution of the one-nucleon state to the spectral function is expressed as

$$\rho_{\mu\nu}^{(i)}(\omega, q; P) = -\frac{M}{E_+} \delta(\omega + E - E_+) \frac{1}{2} \sum_{S,S'} \langle N(PS)|j_{\mu5}^{(i)}(0)|N(P+S')\rangle \langle N(P+S')|j_{\nu5}^{(i)}(0)|N(PS)\rangle \delta(\omega - E + E_-) \frac{1}{2} \sum_{S,S'} \langle N(PS)|j_{\mu5}^{(i)}(0)|N(P-S')\rangle \langle N(P-S')|j_{\nu5}^{(i)}(0)|N(PS)\rangle,$$

where $P_\pm \equiv (E_\pm, P_\pm) = (E \pm \omega, P \pm q)$ are the four-momenta of the one-nucleon states. From the definitions of the axial coupling constants, the matrix elements in the right hand side of Eq.(10) are written as

$$\langle N(PS)|j_{\mu5}^{(3)}(0)|N(P'S')\rangle = \eta_{\mu\nu} \bar{u}(PS) \frac{1}{2} \left[g_A^{(3)}(q^2)\gamma^\nu\gamma_5\right] u(P'S'),$$

$$\langle N(PS)|j_{\mu5}^{(8)}(0)|N(P'S')\rangle = \bar{u}(PS) \frac{1}{2} \left[g_A^{(8)}(q^2)\gamma_\mu\gamma_5 + h_A^{(8)}(q^2)q_\mu\gamma_5\right] u(P'S'),$$

where $u(PS)$ is a Dirac spinor, $\lambda$ the usual Gell-Mann matrix and $q = P' - P$. Note that the pseudo-coupling term disappears owing to $\eta_{\mu\nu}$ in Eq.(11).

The continuum contribution becomes small in the Borel transformed QCD sum rules, since it is exponentially suppressed compared to the one-nucleon state because of the energy gap. Therefore, it is allowed to use a rough model of the continuum: The form of the continuum is approximated by the step function with the coefficient being the imaginary part of the asymptotic form of the correlation function in the OPE \[10,11\]. In the present case, however, the continuum contribution to the spectral function is absent within the approximation, because from the definition of the correlation function the perturbative part is subtracted. This means that the continuum contribution may be very small at least in the high energy region. In the following we therefore neglect the continuum contribution to the spectral function.

Hereafter we consider the currents in which the Lorentz indices are contracted, and the correlation function in the rest frame of the initial and final nucleon states, $P = 0$. Thus we simplify our notation as follows: $\Pi(\omega, q) = \Pi_{\mu}(\omega, q; M, 0)$, $\rho(\omega, q) = \rho_{\mu}(\omega, q; M, 0)$. Then the spectral functions become

$$\rho^{(3)}(\omega, q) = -\frac{1}{2} \left[\frac{1}{M\sqrt{M^2 + |q|^2}} g_A^{(3)}(q^2)^2 \left(q^2/2 - 2M^2\right) \right.$$

$$\times \left[\delta(\omega + M - \sqrt{M^2 + |q|^2}) - \delta(\omega - M + \sqrt{M^2 + |q|^2})\right],$$

$$\rho^{(8)}(\omega, q) = -\frac{1}{2\sqrt{3}} \left[\frac{1}{M\sqrt{M^2 + |q|^2}} \left|g_A^{(8)}(q^2)^2 \left(q^2/2 - 3M^2\right) + g_A^{(8)}(q^2)h_A^{(8)}(q^2) \cdot Mq^2 - h_A^{(8)}(q^2)^2(q^4/4)\right] \right.$$

$$\times \left[\delta(\omega + M - \sqrt{M^2 + |q|^2}) - \delta(\omega - M + \sqrt{M^2 + |q|^2})\right].$$
Because of crossing symmetry, Eqs. (13) and (14) are even functions of $\omega$, so that the Lehmann representations of Eq. (8) is not necessary. From Eqs. (7), (13) and (14) we obtain

\[
\hat{B} \left[ \Pi^{(3)}(\omega, q)_{\text{even}} \right] = \left( \frac{1}{2} \right)^2 \left( \frac{1}{M} - \frac{1}{\sqrt{M^2 + |q|^2}} \right) \exp \left[ - \left( \sqrt{M^2 + |q|^2} - M \right)^2 / s \right]
\times |g^{(3)}_A(q^2)|^2 (q^2 - 4M^2),
\]

\[
\hat{B} \left[ \Pi^{(8)}(\omega, q)_{\text{even}} \right] = \left( \frac{1}{2\sqrt{3}} \right)^2 \left( \frac{1}{M} - \frac{1}{\sqrt{M^2 + |q|^2}} \right) \exp \left[ - \left( \sqrt{M^2 + |q|^2} - M \right)^2 / s \right]
\times \left[ g^{(8)}_A(q^2)^2 (q^2 - 6M^2) + 2Mq^2 g^{(8)}_A(q^2) h^{(8)}_A(q^2) - \frac{q^4}{2} |h^{(8)}_A(q^2)|^2 \right].
\] (15)

We expand the right hand sides of Eqs. (13) and (14) in powers of $|q|^2$. There is no constant term in the contribution of the one-nucleon state to the spectral function. The coefficients of $|q|^2$ in Eqs. (13) and (14) are proportional to $|g^{(3)}_A(0)|^2$ and $|g^{(8)}_A(0)|^2$, respectively. Note that $h^{(8)}_A(0)$ contributes to higher order terms because $h^{(8)}_A(q^2)$ has no singularity at $q^2 = 0$.

Taking the first derivative with respect to $|q|^2$ we obtain the desired QCD sum rules at $|q|^2 = 0$:

\[
\frac{\partial}{\partial |q|^2} \hat{B} \left[ \Pi^{(3)}(\omega^2, q)_{\text{even}} \right] \bigg|_{|q|^2 = 0} = - \left( \frac{1}{2} \right)^2 \frac{2}{M} |g^{(3)}_A(0)|^2,
\]

\[
\frac{\partial}{\partial |q|^2} \hat{B} \left[ \Pi^{(8)}(\omega^2, q)_{\text{even}} \right] \bigg|_{|q|^2 = 0} = - \left( \frac{1}{2\sqrt{3}} \right)^2 \frac{3}{M} |g^{(8)}_A(0)|^2.
\] (17) (18)

Let us now turn to the OPE of $\Pi^{(3)}$ and $\Pi^{(8)}$. In the OPE of Eq. (1), operators of the leading terms are of dimension 4. In this work we take into account the terms up to dimension 6. The result for the iso-vector channel correlation function is in the following:

\[
\Pi^{(3)}(q) = \left( \frac{1}{2} \right)^2 \frac{6}{q^2} \left[ m_u \langle \bar{u}u \rangle_N + m_d \langle \bar{d}d \rangle_N \right] - \frac{1}{2q^2} \overline{\left< \frac{\alpha_s}{\pi} G^2 \right>_N}
- \frac{8q^\mu q^\nu}{q^4} \left< \bar{u} S \gamma_\mu D_\nu u \right>_N + i \left< \bar{d} S \gamma_\mu D_\nu d \right>_N
- \frac{6\pi\alpha_s}{q^4} \left< \bar{u} \gamma_\mu \lambda_\alpha u - \bar{d} \gamma_\mu \lambda_\alpha d \right)_N^2
+ \frac{8\pi\alpha_s q^\mu q^\nu}{q^6} \left< S \left( \bar{u} \gamma_\mu \lambda_\alpha u - \bar{d} \gamma_\mu \lambda_\alpha d \right) (\mu \rightarrow \nu) \right>_N
- \frac{4\pi\alpha_s}{3q^4} \left< \bar{u} \gamma_\mu \lambda_\alpha u + \bar{d} \gamma_\mu \lambda_\alpha d \right> \sum_{q=u,d,s} \bar{q} c_{\mu} \lambda_\alpha q \right>_N
+ \frac{2\pi\alpha_s q^\mu q^\nu}{q^6} \left< S \left( \bar{u} \gamma_\mu \lambda_\alpha u + \bar{d} \gamma_\mu \lambda_\alpha d \right) \sum_{q=u,d,s} \bar{q} c_{\nu} \lambda_\alpha q \right>_N
+ \frac{32q^\mu q^\nu q^\rho q^\sigma}{q^8} \left( i \left< \bar{u} S \gamma_\mu D_\nu D_\lambda D_\sigma u \right>_N + i \left< \bar{d} S \gamma_\mu D_\nu D_\lambda D_\sigma d \right>_N \right). \tag{19}
\]
Similarly, the result for the 8th component of the SU(3) octet is given by

\[
\Pi^{(8)}(q) = \left(\frac{1}{2\sqrt{3}}\right)^2 \left[\frac{10}{q^2} \left( m_u \langle \bar{u}u \rangle_N + m_d \langle \bar{d}d \rangle_N + 4 m_s \langle \bar{s}s \rangle_N \right) - \frac{3}{2q^2} \langle \frac{\alpha_s}{\pi} G^2 \rangle_N \right.
- \frac{8q^\mu q^\nu}{q^4} \left( i \langle \bar{u}S\gamma_\mu D_\nu u \rangle_N + i \langle \bar{d}S\gamma_\mu D_\nu d \rangle_N + 4i \langle \bar{s}S\gamma_\mu D_\nu s \rangle_N \right)
- \frac{6\pi\alpha_s}{q^4} \left\langle \left( \bar{u}\gamma_\mu \lambda^a u + \bar{d}\gamma_\mu \lambda^a d - 2\bar{s}\gamma_\mu \lambda^a s \right)^2 \right\rangle_N
+ \frac{8\pi\alpha_s q^\mu q^\nu}{q^6} \left\langle S \left( \bar{u}\gamma_\mu \lambda^a u + \bar{d}\gamma_\mu \lambda^a d - 2\bar{s}\gamma_\mu \lambda^a s \right) \left( \mu \to \nu \right) \right\rangle_N
- \frac{4\pi\alpha_s}{3q^4} \left\langle \left( \bar{u}\gamma_\mu \lambda^a u + \bar{d}\gamma_\mu \lambda^a d + 4\bar{s}\gamma_\mu \lambda^a s \right) \sum_{q=u,d,s} Q_\gamma \mu \lambda^a \right\rangle_N
+ \frac{2\pi\alpha_s q^\mu q^\nu}{q^6} \left\langle S \left( \bar{u}\gamma_\mu \lambda^a u + \bar{d}\gamma_\mu \lambda^a d + 4\bar{s}\gamma_\mu \lambda^a s \right) \sum_{q=u,d,s} Q_\gamma \mu \lambda^a \right\rangle_N
+ \frac{32q^\mu q^\nu q^\lambda q^\sigma}{q^8} \left[ i \langle \bar{u}S\gamma_\mu D_\nu D_\lambda D_\sigma u \rangle_N + i \langle \bar{d}S\gamma_\mu D_\nu D_\lambda D_\sigma d \rangle_N \right]
+ \frac{8q^\mu q^\nu q^\lambda q^\sigma}{q^8} \left[ i \langle \bar{s}S\gamma_\mu D_\nu D_\lambda D_\sigma s \rangle_N \right].
\]

where \(D_\mu\)’s are covariant derivatives, \(G^2 \equiv G^a_{\mu\nu} G^{a\mu\nu}\), and \(S\) denotes a symbol which makes the operators symmetric and traceless with respect to the Lorentz indices.

We now discuss about the nucleon matrix elements in Eqs. (19) and (20). It is known well that \(m_q \langle \bar{q}q \rangle_N\) is related to the \(\pi-N\) or \(K-N\) sigma-term as

\[
(m_u + m_d) \langle \bar{u}u \rangle_N + \langle \bar{d}d \rangle_N = 2\Sigma_{\pi N},
\]

\[
(m_u + m_u) \langle \bar{s}s \rangle_N + \langle \bar{u}u \rangle_N = 2\Sigma_{K N}.
\]

\(\langle (\alpha_s/\pi)G^2 \rangle_N\) is expressed by the nucleon mass and \(m_q \langle \bar{q}q \rangle_N\) through the QCD trace anomaly:

\[
\langle \frac{\alpha_s}{\pi} G^2 \rangle_N = -\frac{8}{9} \left( M - \sum_{q=u,d,s} m_q \langle \bar{q}q \rangle_N \right).
\]

The matrix elements which contain covariant derivatives are related to the parton distributions as

\[
\langle S\bar{q}\gamma_\mu_1 D_\mu_2 \cdots D_\mu_n q(\mu^2) \rangle_N = (-i)^{n-1} A_n^\mu(\mu^2) T_{\mu_1 \cdots \mu_n},
\]

where \(A_n(\mu^2)\) is the \(n\)-th moment of the parton distributions at scale \(\mu^2\), and \(T_{\mu_1 \cdots \mu_n} = S [P_{\mu_1} \cdots P_{\mu_n}]\). For the matrix elements of four quark operators, we apply the factorization hypothesis: In the vacuum, four quark condensates are factorized by the hypothesis which assumes that the vacuum contribution dominates in the intermediate states: \(\langle O_1 O_2 \rangle_0 \approx \langle O_1 \rangle_0 \langle O_2 \rangle_0\) \([11, 12]\). Similarly, for the nucleon matrix elements, we assume that the contribution from one nucleon state dominates in the intermediate states \([8, 13]\):
\[ \langle O_1 O_2 \rangle_N \approx \langle O_1 \rangle_N \langle O_2 \rangle_0 + \langle O_1 \rangle_0 \langle O_2 \rangle_N. \] We apply this hypothesis to the following type of the nucleon matrix elements, which appear in Eqs. (19) and (20): 
\[ \langle \bar{q} f \gamma_\mu \lambda^a q f' \bar{q} f' \gamma_\nu \lambda^a q f' \rangle_N = -\frac{(8/9)g_{\mu\nu}(\bar{q} f q f)_0(\bar{q} f q f)_N \delta_{f, f'}}, \] where \( f \) and \( f' \) are flavor indices.

We substitute Eqs. (19) and (20) into the left hand sides of Eqs. (17) and (18), respectively. Averaging over the iso-spin states, we obtain the QCD sum rules for \( |g_A^{(3)}(0)|^2 \) and \( |g_A^{(8)}(0)|^2 \) as follows:

\[
|g_A^{(3)}(0)|^2 = -\frac{M}{2} \left\{ \frac{\Sigma_{\pi N}}{s} \left[ \frac{50}{9} + \frac{4}{9} \frac{m_s}{m_u + m_d} \right] - \frac{\Sigma_{KN}}{s} \left[ \frac{8}{9} \frac{m_s}{m_s + m_u} \right] \right. \\
- \left. \frac{M}{s} \left[ \frac{4}{9} + 7 \left( A_2^u(\mu^2) + A_2^d(\mu^2) \right) \right] \right\}, \tag{25}
\]

\[
|g_A^{(8)}(0)|^2 = -\frac{M}{3} \left\{ \frac{\Sigma_{\pi N}}{s} \left[ \frac{26}{3} - \frac{116}{3} \frac{m_s}{m_u + m_d} \right] + \frac{\Sigma_{KN}}{s} \left[ \frac{232}{3} \frac{m_s}{m_s + m_u} \right] \right. \\
- \left. \frac{M}{s} \left[ \frac{4}{3} - 7 \left( A_2^u(\mu^2) + A_2^d(\mu^2) + 4A_1^u(\mu^2) \right) \right] \right\} \\
- \frac{4\pi\alpha_s(\bar{q}q)_0}{s^2} \left[ \frac{352}{27} \frac{\Sigma_{\pi N}}{m_u + m_d} - \frac{\Sigma_{KN}}{s^2} \left[ \frac{704}{27} \left( \frac{2\Sigma_{KN}}{m_s + m_u} - \frac{\Sigma_{KN}}{m_u + m_d} \right) \right] \right] \\
+ \frac{15M^2}{s^2} \left[ A_1^u(\mu^2) + A_1^d(\mu^2) + 4A_1^u(\mu^2) \right], \tag{26}
\]

respectively, where \( \langle \bar{q}q \rangle_0 \equiv \langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0. \) In Eqs. (25) and (26) we assume \( m_u = m_d. \) From these equations, we find that the axial coupling constants are related to the sigma-terms and the moments of parton distributions. Since the sigma-terms and the moments are well known, we can estimate \( |g_A^{(3)}(0)| \) and \( |g_A^{(8)}(0)|. \)

We show in Fig.1 the squared Borel mass \( s \) dependence of \( |g_A^{(3)}(0)| \) in Eq. (25) and \( |g_A^{(8)}(0)| \) in Eq. (26). In plotting the curve in Fig.1, we used the following values of the constants in the OPE. The \( \pi-N \) sigma-term is taken from Ref. [13], which are \( \Sigma_{\pi N} = 45 \text{ MeV}. \) The quark masses are taken to be \( m_u = m_d = 7 \text{ MeV}, m_s = 110 \text{ MeV}. \)

Using the above values and the ratio \( 2 \langle \bar{s}s \rangle_N / (\langle \bar{u}u \rangle_N + \langle \bar{d}d \rangle_N) = 0.2 \) given in Ref. [13], we can calculate the \( K-N \) sigma term averaged over the iso-spin states and the result is \( \Sigma_{KN} = 226 \text{ MeV}. \) Following Ref. [14] and adopting the LO scheme in Ref. [13], we calculated the moments of parton distributions: \( A_2^u(1 \text{ GeV}^2) + A_2^d(1 \text{ GeV}^2) = 1.1, A_1^u(1 \text{ GeV}^2) + A_1^d(1 \text{ GeV}^2) = 0.13, A_2^u(1 \text{ GeV}^2) = 0.03, A_1^u(1 \text{ GeV}^2) = 0.002. \) The vacuum condensates are taken from Ref. [4], which are \( \langle \bar{q}q \rangle_0 = (-225 \text{ MeV})^3 \) and \( \langle \bar{s}s \rangle_0 = 0.8 \langle \bar{q}q \rangle_0. \) Among the above constants in the OPE, the most dominant contribution comes from \( A_2^u(1 \text{ GeV}^2) + A_2^d(1 \text{ GeV}^2). \)

From Fig.1 we see good stability of the \( s \) dependence for both \( g_A^{(3)}(0) \) and \( g_A^{(8)}(0). \) We see that the variations of the curves in a larger \( s \) region are small in spite of not including the continuum contribution. This fact implies that the continuum contribution is suppressed, and the spectral function is allowed to be approximated by the lowest state.
Fig. 1. The squared Borel mass $s$ dependence of $|g_A^{(3)}(0)|$ in Eq. (25) and $|g_A^{(8)}(0)|$ in Eq. (26). The solid line corresponds to $|g_A^{(3)}(0)|$ and the dashed line to $|g_A^{(8)}(0)|$.

Our estimations of $|g_A^{(3)}(0)|$ and $|g_A^{(8)}(0)|$ taken from the stabilized region are

$$|g_A^{(3)}(0)| \simeq 1.2, \quad |g_A^{(8)}(0)| \simeq 0.6.$$  \hspace{1cm} (27)

The obtained value of $|g_A^{(3)}(0)|$ gives a good agreement with the world average $g_A^{(3)}(0) = 1.260 \pm 0.002$ \cite{14}. The value of $|g_A^{(8)}(0)|$ is also close to $g_A^{(8)}(0) = 0.59 \pm 0.02$ \cite{17} found from the data on the baryon octet $\beta$-decays under the assumption of the SU(3) flavor symmetry.

In summary, we have considered two-point correlation functions of axial-vector currents in one-nucleon state and found that the lowest state in the spectral function of the correlation function is expressed by the axial coupling constant. We have calculated the iso-vector and the 8th component of the SU(3) octet axial coupling constants in the framework of QCD sum rules. The results show that the axial coupling constants are expressed by the nucleon matrix elements of quark and gluon operators which are related to the sigma-terms and the moments of parton distributions. Since the nucleon matrix elements are known well experimentally or theoretically, the axial coupling constants are calculated with a small ambiguity, and the obtained results are in a good agreement with experiment.
Finally, we mention about the singlet axial coupling constant, $g_A^{(0)}(0)$, which is considered to be the nucleon spin carried by quarks. The unexpected small value of $g_A^{(0)}(0)$ found by EMC has raised a number of understanding of the dynamics of the nucleon spin [1]. Within the same framework as those for $g_A^{(3)}(0)$ and $g_A^{(8)}(0)$, we find that the calculated $g_A^{(0)}(0)$ is about 0.8, which is not so small as the value found by EMC. In this calculation, however, the effect of the chiral anomaly is not taken into account.

The authors in Ref.[6] have calculated $g_A^{(0)}(0)$ fully taking into account the anomaly relation. They have considered a three-point function of nucleon interpolating fields and the divergence of the singlet axial-vector current. The form factors, $g_A^{(0)}(q^2)$, are related to the vacuum condensates of the quark-gluon composite operators through the double dispersion relation. To know $g_A^{(0)}(q^2)$ at $q^2 = 0$ one must evaluate the correlation function at the zero momentum. Although the method to evaluate it is known [18], it involves large uncertainty. The calculation of $g_A^{(0)}(0)$ in our approach by taking into account the chiral anomaly will be reported elsewhere [19].

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References

[1] M. Anselmino, A. Efremov and E. Leader, Phys. Reports 261 (1995),1
[2] V. M. Belyaev and Ya. I. Kogan, JETP Lett. 37 (1983),20.
[3] V. M. Belyaev, B. L. Ioffe and Ya. I. Kogan, Phys. Lett. B151 (1985),290.
[4] B. L. Ioffe and A. Yu. Khodzhamiryan, Sov. J. Nucl. Phys. 55 (1992),1701
[5] B. L. Ioffe, talk given at St.Petersburg Winter School on Theoretical Physics, hep-ph/9804238.
[6] A. V. Belitsky and O. V. Teryaev, Phys. Atom. Nucl. 60 (1997),455
[7] L. J. Reinders, H. R. Rubinstein, and S. Yazaki, Phys. Reports 127 (1985),1.
[8] Y. Kondo and O. Morimatsu, Phys. Rev. Lett. 2855 (1993),71.
[9] Y. Kondo, O. Morimatsu and Y. Nishino, Phys. Rev. C53 (1991),1927.
[10] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979), 385.
[11] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Nucl. Phys. B147 (1979), 448.
[12] Y. Kondo and O. Morimatsu, Prog. Theor. Phys. 1 (1998),100.
[13] J. Gasser, H. Leutwyler and M. E. Sainio, Phys. Lett. B253 (1991),252.
[14] T. Hatsuda and S. H. Lee, Phys. Rev. C34 (1992),R46.
[15] M. Glück, E. Reya and A. Vogt, Z. Phys. C53 (1992),127.
[16] R. M. Barnett et al., Particle Data Group, Phys. Rev. D54 (1996),1.
[17] S. Y. Hsueh et al., Phys. Rev. D38 (1988),2056.
[18] Ya. Ya. Balitsky, A. V. Kolesnichenko and A. V. Yung, Sov. J. Nucl. Phys. 41 (1985),178.
[19] T. Nishikawa, S. Saito and Y. Kondo, in preparation.