Numerical simulation of plastic deformation and mechanical response of strip rolled aluminium alloys

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Abstract. During the optimal design and simulation of machining or forming processes, detailed simulation of structural response is typically required for use in Finite Element Analysis (FEA). In this study, the bulk temperature and rate dependent resistance to deformation of strip formed aluminium alloys is modelled using the Mechanical Threshold Stress model. The model is characterised to AA5182 alloy data and used in the FEA simulation of a strip rolling process. The effects of element choice, dynamic or quasi-static simulations and the use of particular modelling algorithms on the computational cost and overall accuracy associated with each simulation have to be considered. Given the correctly implemented material tangent, an implicit analysis is illustrated to allow larger stable time-steps. Dynamic and quasi-static solutions are very similar for the simulated process meaning inertia effects are negligible. It is further demonstrated that great care be given to the maximum allowable time step size in order to capture the expected interaction between temporal and spatial discretisation in a fully Lagrangian FEA simulation.

1. Introduction

During the optimal design and simulation of machining or forming processes, a better understanding and detailed simulation of alloy deformation mechanisms or structural response are typically required for use in detailed Finite Element Analysis (FEA). If the effect on energy input and reaction forces or moments are required given a specific processing rate and temperature, a continuum model that models the correct bulk temperature and strain rate dependent mechanical response is needed \cite{1,2}. The purpose of this study is to inspect the development of steady state deformation so that the resulting forces and moments involved in the continuous roll may be assessed as an analogue for required energy input. The accurate modelling of bulk temperature and rate dependent resistance to deformation of strip formed aluminium alloys is investigated using the dislocation density based Mechanical Threshold Stress (MTS) material model \cite{3}. The model is first implemented into the ABAQUS FEA package \cite{4} and then characterised to experimental temperature and rate dependent stress-strain response data of an AA5182 aluminium alloy. Given a model that captures the correct bulk mechanical response, considerations for use in a Lagrangian FEA simulation of strip formed bulk material is then investigated. The effects of dynamic or quasi-static simulations and the use of various modelling or contact algorithms and friction considerations have a further effect on the computational
cost and overall accuracy associated with each simulation. These are important aspects to consider if the intent is a study on the effects of, or optimising for, specific processing parameters.

2. The Mechanical Threshold Stress Model

The Mechanical Threshold Stress (MTS) model is a continuum plasticity model that captures the dominant bulk temperature \(T\) and strain rate \(\dot{\varepsilon}\) dependent post-yielding behaviour of metals. Dislocation density \(\rho\) is introduced as a microstructural variable of state through its relationship to the stress-like variable \(2\sigma = M\mu\varepsilon\sqrt{\rho}\) using the average Taylor factor measure of polycrystallinity \(M\), a reference elastic shear modulus \(\mu\), and length of the Burger’s vector \(b\) [3]. In the MTS model, a material has a theoretical maximum flow stress (the mechanical threshold) \(\dot{\varepsilon} = 0\) K.

Any yield stress value \(\sigma_y\) is a scaled version of the mechanical threshold to accommodate for rate and temperature dependence. The effective yield stress may be calculated as a function of various threshold contributions. The standard MTS formulation calculates a yield stress value using three contributions [3]:

\[
\sigma_y = \sigma_0 + S_i(\dot{\varepsilon}, T)\sigma_i + S_e(\dot{\varepsilon}, T)\sigma_e
\]

where \(\sigma_0\) is an athermal, constant contribution to the yield stress with \(\sigma_i\) a constant scalable initial threshold value and \(\sigma_e\) an evolving scalable contribution to the yield stress. The scaling function \(S_i(\dot{\varepsilon}, T)\) takes the form [5,6]

\[
S_i(\dot{\varepsilon}, T) = \frac{\mu(T)}{\mu_e} \left[ 1 - \left( \frac{T}{\gamma_{0i} \mu(T)} \ln \frac{\dot{\varepsilon}_{0i}}{\dot{\varepsilon}} \right)^{\frac{1}{p_i}} \right]^{1/p_i}
\]

with \(\dot{\varepsilon}_{0i}\) a reference maximum strain rate while \(p_i\) and \(q_i\) are statistical constants \((0 \leq p_i \leq 1, \quad 1 \leq q_i \leq 2)\) that characterise the shape of obstacles in the path of mobile dislocations. \(\gamma_{0i}\) is a convenient grouping of physical lattice parameters and normalised activation energy into a single material parameter value [5,6]. \(\mu(T)\) is a temperature dependent shear modulus with \(\mu_e\) the maximum reference value.

The parameters are chosen so that the scaling function is between 0 and 1 for any expected value of strain rate \(\dot{\varepsilon}\) and temperature \(T\). The other scaling function \(S_e(\dot{\varepsilon}, T)\) is similarly calculated using \(\gamma_{0e}\), \(\dot{\varepsilon}_{0e}\), \(q_e\) and \(p_e\) values instead so that the evolving mechanical threshold component is allowed a different scale compared to that of the initial value. The temperature dependent shear modulus is modelled using [7]

\[
\mu(T) = \mu_e - \frac{D_r}{\exp \left( \frac{T}{T_r} \right) - 1}
\]

in which \(T_r\) and \(D_r\) are further reference parameter constants.

The evolution of \(\dot{\varepsilon}\) is given in rate form using the modified Voce hardening law [8]

\[
\dot{\varepsilon} = \frac{d}{d\varepsilon} \dot{\varepsilon} = \theta_0 \left( 1 - \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{0S}} \right) \]

where \(\theta_0\) is the constant hardening due to dislocation accumulation while \(\dot{\varepsilon}_{0S}\) is the saturation stress. A power law exponent \(\kappa\) chosen as unity exhibits saturating behaviour while \(\kappa > 1\) mimics a non-saturating final hardening rate.

The saturation threshold stress \(\dot{\varepsilon}_{0S}\) is a function of both strain rate and temperature, through [3]

\[
\dot{\varepsilon}_{0S}(\dot{\varepsilon}, T) = \dot{\varepsilon}_{0S} \exp \left( \frac{T}{\gamma_{0S} \mu(T)} \ln \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{0S}} \right)
\]

where \(\dot{\varepsilon}_{0S}, \gamma_{0S}\) and \(\dot{\varepsilon}_{0S}\) are further empirical material constants.

The MTS material model was built around physically observed deformation mechanisms in fcc metals. A desirable benefit to using this physically motivated material model is that it can therefore reasonably interpolate and extrapolate material response for temperatures and strain rates slightly outside of the experimental data used to calibrate it.
2.1. Material parameter values for AA5182

A point integration implementation of the MTS model was coded and characterised to temperature and rate dependent stress-strain data of an AA5182 alloy. The response data and certain MTS parameter values were extracted from the work by Chen et al. [8].

Of the 18 total MTS material parameter values, \( \alpha_0l, p_l, q_l, \alpha_0e, p_e, q_e, \alpha_{0eS}, \kappa, \mu_r, T_r \) and \( D_r \) were fixed while the remaining seven parameters are fine-tuned so that the model best replicates the experimentally observed response. Numerical optimisation is used to fine tune the material parameter values as follows:

Given a candidate set of parameter values \( \mathbf{x} = \{ \tilde{\sigma}_a, \tilde{\alpha}_l, \gamma_{0l}, \gamma_{0e}, \gamma_{0eS}, \tilde{\sigma}_{0eS}, \theta_0 \} \), an objective function is constructed by first evaluating the model predicted values \( \sigma_{\text{model}}^{(k)}(\mathbf{x}) \) corresponding to each of the \( (k) \) experimental data points \( \sigma_{\text{experiment}}^{(k)} \). The smallest objective function value

\[
\text{obj}(\mathbf{x}) = \sum_{(k)} \frac{\sigma_{\text{model}}^{(k)}(\mathbf{x})}{\sigma_{\text{experiment}}^{(k)}} - 1
\]

is then sought using the downhill simplex algorithm [9]. The optimal material parameter values are reported in table 1 with a visual comparison of experimental and model predicted response at selected temperatures and rate of deformation is presented in Figure 1.

Table 1. Mechanical Threshold Stress parameter values used to model the temperature and rate dependent response of AA5182.

| Equation | Parameter | Value | Unit | Equation | Parameter | Value | Unit |
|----------|-----------|-------|------|----------|-----------|-------|------|
| (1) : \( \sigma_y \) | \( \tilde{\sigma}_a \) | 4.86 | MPa | (4) : \( \theta \) | \( \theta_0 \) | 4993.8 | - |
| (2) : \( S_i \) | \( \gamma_{0l} \) | 0.757 | K/MPa | (2) : \( S_e \) | \( \gamma_{0e} \) | 3.083 | K/MPa |
| | \( \alpha_{0l} \) | \( 10^7 \) | s\(^{-1} \) | | \( \alpha_{0e} \) | \( 10^7 \) | s\(^{-1} \) |
| | \( q_l \) | 1 | - | | \( q_e \) | 1 | - |
| | \( p_l \) | 1 | - | | \( p_e \) | 1 | - |
| (3) : \( \mu \) | \( \mu_r \) | 28815 | MPa | (5) : \( \tilde{\sigma}_{0eS} \) | \( \tilde{\sigma}_{0eS} \) | 730.6 | MPa |
| | \( D_r \) | 3440 | MPa | | \( \gamma_{0eS} \) | 0.243 | K/MPa |
| | \( T_r \) | 215 | K | | | | |

![Graphs showing temperature and rate dependent response](image_url)
Figure 1. AA5182 experimental stress-strain data (markers / left) and MTS response (solid lines / right) using parameter values in table 1.

3. Simulating strip form reduction
The characterised MTS model is now used to investigate different considerations in simulating a roll pass reduction of AA5182. A 300mm thick slab at 300°C is rolled to a true (logarithmic) reduction of 10%. The rollers are 1.2m in diameter with a peripheral velocity of 1m/s ($\omega = 1.6667s^{-1}$). Given symmetry in the roll process, only half of the slab and a single roller is modelled.

Only 1m length of the slab is modelled so that there is enough material to move past any initial edge effects. The half slab is initially pushed into the roller at 0.3m/s. Once sufficient contact is made with the roller at a constant $\omega = 1.6667s^{-1}$, the half slab is pulled through as a result of friction between the two surfaces assuming a friction coefficient of 0.3. The simulations are done in Abaqus Standard and Explicit [10] using user coded Fortran subroutines for the MTS model.

The different simulation considerations investigated included the use of reduced integration quadratic plane strain elements (CPE8R) and reduced integration linear plane strain elements (CPE4R) with enhanced hourglass stiffness treatment. The reduced integration elements are considered to relax the pressure variational and prevent volumetric locking as a result of an overly stiff element during plastic deformation. The other big consideration is the use of implicit compared to explicit solvers as well as including or excluding inertia effects (i.e. dynamic versus quasi-static FEA simulations).

The simulation using quadratic elements with inertia effects in an implicit solver with small time steps results in likely the most accurate simulation. This is expected given that (1) higher order accuracy elements are used, (2) all of the physics is included and (3) solutions, states and variables are solved simultaneously instead of only being a function of the previous time step values. Some resulting contours for this baseline roll reduction simulation using a maximum allowable step size of 0.0001 seconds is given in Figure 2.

In Figure 2(a) the von Mises equivalent stress contours illustrate a maximum stress concentrated just below the surface. The steady state residual stresses downstream from the roll (at the far right of the figure) indicate that the highest residuals are below the surface with a slightly higher residual in the middle of the slab than a third into its reduced thickness. The roll process forces material through a complex shear deformation history as illustrated by the contours in Figure 2(b). In 2(c) the importance of a rate dependent plastic deformation material model is illustrated given a highly non-linear plastic strain rate distribution. The rates of deformation vary from above $3s^{-1}$ just in front of the roll to around $1s^{-1}$ at the centre and orders of magnitude variation around the deformation zone.

Figure 2. Results of a Finite Element Analysis to simulate AA5182 roll pass reduction at 300°C using CPE8R elements in a dynamic implicit analysis. Contours show the steady state developed and through thickness variation of internal material state. (a) von Mises equivalent stress $\sigma_{VM}$
In Figure 3, the evolving internal state variable $\bar{\sigma}_e$ contours for the same simulation (apart from maximum allowable step size) is illustrated. Figure 3(a) shows the evolving threshold stress $\bar{\sigma}_e$ associated with the same analysis in figure 2 using a maximum allowable step size of 0.0001 seconds. The contours in Figure 3(b) are as a result of the same simulation using a maximum allowable automatic time step size of 0.011s. If the slab and each element of length 0.01m moves at the roll peripheral velocity of 1m/s, a 0.011 second time step results in information skipping over the entire element length. In this case spurious cyclic behaviour is transferred into the material state evolution.

The reaction forces and moments at the centre of the roller are extracted for each simulation for comparison in Figure 4. From the baseline dynamic implicit result using quadratic CPE8R elements, the reaction forces exhibit a reasonably steady oscillating behaviour around 0.5 seconds after initial contact. The detail reaction moment and force plots in Figure 4 show the results extracted from five different FEA simulations: (1) the baseline dynamic implicit simulation using quadratic CPE8R elements, (2) a dynamic implicit simulation using linear CPE4R elements, (3) a dynamic explicit simulation using CPE4R, (4) a quasi-static implicit simulation using CPE4R and (5) a dynamic implicit CPE4R simulation with a larger allowable automatic time step size. In the first four simulations a maximum allowable automatic time step size of 0.0001 seconds is prescribed while in simulation (5) the allowable step size is 0.011.
4. Discussion of results and conclusion

Given the Lagrangian FEA results, the benefit of an implicit rather than explicit analysis is evident in the number of time steps required. The implicit analysis makes use of solutions and sensitivities at the time step of interest to allow larger stable step sizes while the explicit analysis required close to two orders of magnitude more time steps.

The use of automatic time stepping with a large allowable step size gives incorrect estimates on the bulk reactions. If a step size is so large that an element does not go through the expected strain history, the through thickness material state becomes increasingly cyclic. A maximum step size should be chosen so that the effects of a non-linear deformation history does not skip through any entire element.

There is very little difference observed between the bulk response in the dynamic and quasi-static simulations. This means that though the quasi-static case simulates less physics, the inertia effects are so limited that it could be neglected altogether without any major implication on the accuracy of the solution.

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**Figure 4.** Reaction forces and moments experienced at the roll centre for different modelling considerations. Left: Time history using a reduced integration quadratic element to simulate full physics (dynamic) using the implicit solver. Right: Detail of cyclic response in the steady state forming process as a result of the interaction between temporal and spatial discretisation.
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