CP violation in $\mu - \tau$ symmetric models

D.C. Rivera-Agudelo $^{a,1}$ and A. Pérez-Lorenzana $^{a,2}$

$^a$ Departamento de Física, Centro de Investigación y de Estudios Avanzados del I.P.N. Apdo. Post. 14-740, 07000, México, Distrito Federal, México
E-mail: $^1$ drivera@fis.cinvestav.mx, $^2$ aplorenz@fis.cinvestav.mx

Abstract. The observed masses and mixings in the neutrino sector suggest an approximate $\mu - \tau$ symmetry in the neutrino mass matrix. We explore the corresponding parameter space linked to a small breaking of this symmetry to show that in such a scenario neutrino masses must be quasi degenerated, whereas Majorana CP phases shall be non zero and strongly correlated to the Dirac CP phase.

1. Introduction

Global fits of the available data of different neutrino oscillation experiments obtained in the three flavor scheme indicate that [1], within the $3\sigma$ range, the mixing angles are given by solar $\sin^2 \theta_{12} \equiv \sin^2 \theta_{\odot} \approx 0.308^{+0.064}_{-0.055}$, atmospheric $\sin^2 \theta_{23} \equiv \sin^2 \theta_{\text{ATM}} \approx 0.437^{+0.189}_{-0.063}$ ($0.455^{+0.186}_{-0.075}$), and reactor $\sin^2 \theta_{13} \approx 0.0234^{+0.0061}_{-0.0058} (0.0240^{+0.0058}_{-0.0068})$, for the normal (inverted) hierarchy. The squared mass differences are given by the so-called solar scale $\Delta m_{32}^2 = m_3^2 - m_1^2 \equiv \Delta m_{\text{sol}}^2 \approx 7.54 \times 10^{-5} \text{eV}^2$ and the atmospheric scale $|\Delta m_{21}^2| = \Delta m_{\text{ATM}}^2 \approx 2.4 \times 10^{-3} \text{eV}^2$. However, the sign in $\Delta m_{21}^2 = m_2^2 - m_1^2$, and thus the neutrino mass hierarchy pattern, is still unknown. A very recent global analysis of neutrino oscillation data was provided by [2].

The mixing angles measured in oscillation experiments are large, except for $\theta_{13}$, which was found to be rather small but certainly non-zero. In the standard parametrization, the mixings are given by [3, 4],

$$U_{\text{mix}} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{23} & s_{12}s_{23}c_{13}e^{i\delta_{\text{CP}}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta_{\text{CP}}} & -c_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta_{\text{CP}}} & c_{23}s_{13} \end{pmatrix} \cdot K,$$  \hspace{1cm} (1)

where $c_{ij}$ and $s_{ij}$ denote $\cos \theta_{ij}$ and $\sin \theta_{ij}$, respectively, and $\delta_{\text{CP}}$ is the Dirac $CP$-violating phase, whereas $K = \text{Diag}(e^{-i\delta_1/2}, e^{-i\delta_2/2}, 1)$ is a diagonal matrix containing two Majorana $CP$-violating phases.

The neutrino mass terms are given by the effective operator $\tilde{\nu}_{\alpha L}(M_\nu)_{\alpha \beta} \nu_{\beta L}^c + h.c.$ Let us consider a basis where the weak flavor eigenstates of the charged lepton are identified with their masses eigenstates, then the mixing matrix also becomes that which diagonalizes the mass terms, and $U_{\text{mix}} = U_{\nu} \cdot K$. Therefore, the neutrino mass matrix can be written in terms of a diagonal matrix, as

$$M_\nu = U_{\nu}^\dagger \cdot M_{\text{diag}} \cdot U_{\nu}^\dagger,$$  \hspace{1cm} (2)

where $M_{\text{diag}} = \text{Diag}(m_1, m_2, m_3)$, and we have denoted $m_j \equiv |m_j|e^{-i\delta_j}$, for $j = 1, 2$. 

Published under licence by IOP Publishing Ltd

Content from this work may be used under the terms of the Creative Commons Attribution 3.0 licence. Any further distribution of this work must maintain attribution to the author(s) and the title of the work, journal citation and DOI.
When $\theta_{13}$ and $\theta_{23}$ take the critical values, zero and $\pi/4$, respectively, the $M_{\text{mix}}$ matrix in Eq. (1) becomes the one that holds the equality $|U_{e1}| = |U_{\mu1}|$, and gives rise to the so called $\mu - \tau$ symmetry [5]. As a consequence $|m_{e\tau}| = |m_{e\tau}|$ and $|m_{\mu\tau}| = |m_{\mu\tau}|$ in Eq. (2). Nowadays, we know that the value $\theta_{13} = 0$ is disfavored, but $\theta_{23} = \pi/4$ is still allowed at the 1$\sigma$ level. Then, the observed values of these angles could be understood as a result of the breaking of the $\mu - \tau$ symmetry. That is why we claim that there must be an approximate $\mu - \tau$ symmetry behind the observed pattern of the mixing matrix.

In this study, we provide a detailed analysis of the magnitude of the $\mu - \tau$ symmetry breaking using the current data for neutrino mixing parameters, and we identify the mass spectrum and $CPVP$ (CP violating phases) required to obtain a small symmetry breaking.

2. $\mu - \tau$ symmetry breaking parameters

Without loss of generality, any neutrino mass matrix can be written out as $M_{\nu} = M_{\mu_{\tau}} + \delta M$, where $M_{\mu_{\tau}}$ posses the $\mu - \tau$ exchange symmetry, and $\delta M$ is defined by only two non-zero elements,

$$\delta M = \begin{pmatrix} 0 & 0 & \delta \\ 0 & 0 & 0 \\ \delta & 0 & \epsilon \end{pmatrix},$$

where $\delta = m_{e\tau} - m_{e\mu}$ and $\epsilon = m_{\mu\tau} - m_{\mu\mu}$ are considered the breaking parameters. Two dimensionless parameters can be defined as $\hat{\delta} \equiv \frac{\delta}{m_{e\mu}}$ and $\hat{\epsilon} \equiv \frac{\epsilon}{m_{\mu\mu}}$, and using Eq. (2) we get

$$\hat{\delta} = \frac{\sum_i(U_{e\tau}^\ast U_{\mu\tau}^\ast - U_{e\mu}^\ast U_{\mu\mu}^\ast)m_i}{\sum_i U_{e\mu}^\ast U_{\mu\mu}^\ast m_i},$$
$$\hat{\epsilon} = \frac{\sum_i(U_{e\mu}^\ast U_{\mu\mu}^\ast - U_{e\mu}^\ast U_{\mu\mu}^\ast)m_i}{\sum_i U_{e\mu}^\ast U_{\mu\mu}^\ast m_i},$$

where $i = 1, 2, 3$. The absolute masses can be rewritten as

$$|m_2| = \sqrt{m_0^2 + \Delta m^2_{\text{sol}}} \quad \text{and} \quad |m_3| = \sqrt{m_0^2 + \Delta m^2_{\text{ATM}}} \quad \text{for NH.}$$
$$|m_1| = \sqrt{m_0^2 + |\Delta m^2_{\text{ATM}}|} \quad \text{and} \quad |m_2| = \sqrt{m_0^2 + |\Delta m^2_{\text{ATM}}| + |\Delta m^2_{\text{sol}}|} \quad \text{for IH.}$$

In the equations above, $m_0$ is the lightest neutrino mass that becomes $|m_1|$ for the normal mass hierarchy (NH) and $m_3$ for the inverted mass hierarchy (IH). Thus, $\hat{\delta}$ and $\hat{\epsilon}$, depend on nine observables: three mixing angles, two mass squared differences, the lightest neutrino mass, and three $CPVP$. The neutrino oscillation experiments have already provided accurate values for the first five. Hence, the breaking parameter space $\hat{\delta} - \hat{\epsilon}$ that can accommodate neutrino mixings and oscillation mass scales within experimental uncertainties should, in principle, remain undetermined due to the arbitrariness of the $m_0$ and the $CPVP$. However, we suggest that specific predictions for the lightest neutrino mass and CP phases can be extracted by assuming small values of $|\hat{\delta}|$ and $|\hat{\epsilon}|$. We infer that the resulting bounded parameter space will not be consistent with any arbitrary values of the neutrino observables. In order to obtain some insights into the expected conditions required for small symmetry breaking we write the Eq. (4) in the analytical form

$$\hat{\delta} = \frac{y_+ f s_{13} - y_-}{1 + f s_{13} \tan \theta_{23}}, \quad \hat{\epsilon} = \frac{g \cos \theta_{23} s_{13} h}{1 + g s_{23}^2 s_{13} h/2},$$

where

$$y_\pm = \frac{c_{23} \pm s_{23}}{c_{23}}.$$
We again note that gives the presented in the following.

\[
\begin{align*}
\beta \\

\text{Thus, we can see that implies that}
\end{align*}
\]

Using the expressions above, let us analyse the conditions for \(|\delta|,|\epsilon| \ll 1\), in the three possible approaches given by the hierarchies and using the central values for the mixing parameters [1].

For NH, where \(|m_1| \ll |m_2| \approx \sqrt{\Delta m^2_{sol}} \ll m_3 \approx \sqrt{\Delta m^2_{ATM}}\), we have

\[
f \approx \frac{e^{i(\delta_{CP}+\beta_2)}}{s_{12} c_{12}} \sqrt{\frac{\Delta m^2_{ATM}}{\Delta m^2_{sol}}} \left(1 - O\left(\frac{\Delta m^2_{sol}}{\Delta m^2_{ATM}}\right)\right), \quad |f| \sim 12.5,
\]

that implies that \(|\delta| \sim 3.26\). Thus, in NH, the breaking of \(\mu - \tau\) symmetry by \(\delta\) is always large.

For IH, we have \(|m_1| \approx \sqrt{\Delta m^2_{ATM}} \), \(|m_2| \approx \sqrt{\Delta m^2_{sol} + \Delta m^2_{ATM}} \gg m_3\), which gives

\[
f \approx \frac{-e^{-i\delta_{CP}} (c_{12}^2 e^{-i(\beta_1-\beta_2)} + s_{12}^2 + \frac{s_{12}^2 \Delta m^2_{sol}}{2 \Delta m^2_{ATM}})}{s_{12} c_{12} \left(1 - e^{-i(\beta_1-\beta_2)} + \frac{1}{2} \frac{\Delta m^2_{sol}}{\Delta m^2_{ATM}}\right)},
\]

We can see that \(|f|\) is very large when \(\beta_1 - \beta_2 = 0\), whereas, if \(\beta_1 - \beta_2 = \pm \pi\) then \(|f| \sim 1\), which gives \(|\delta| \sim 0.1\). Also, from Eq. (6)

\[
g \approx \frac{(c_{12}^2 s_{13}^2 - s_{12}^2) e^{-i(\beta_1-\beta_2)} + (s_{12}^2 s_{13}^2 - c_{12}^2)}{s_{12}^2 e^{-i(\beta_1-\beta_2)} + c_{12}^2}, \quad h \approx \frac{(e^{-i(\beta_1-\beta_2)} - 1) \sin 2\theta_{23} \sin 2\theta_{12} e^{-i\delta_{CP}}}{s_{12}^2 e^{-i(\beta_1-\beta_2)} + c_{12}^2}.
\]

Thus, \(\beta_1 - \beta_2 = \pm \pi\) gives \(g \approx -1\) and \(h \approx 4 e^{-i\delta_{CP}}\), then \(|\epsilon| \sim 0.6\). Hence, the largest contribution to \(\mu - \tau\) breaking in this case comes from \(|\epsilon|\) rather than \(|\delta|\).

For the degenerate hierarchy (DH), where \(|m_1| \approx |m_2| \approx m_3\), we obtain

\[
f \approx \frac{-e^{-i\delta_{CP}} (c_{12}^2 e^{-i(\beta_1-\beta_2)} + s_{12}^2)}{s_{12} c_{12} (1 - e^{-i(\beta_1-\beta_2)})},
\]

\[
g \approx \frac{(c_{12}^2 s_{13}^2 - s_{12}^2) e^{-i(\beta_1-\beta_2)} + (s_{12}^2 s_{13}^2 - c_{12}^2) + c_{12}^2 e^{-i\beta_1}}{s_{12}^2 e^{-i(\beta_1-\beta_2)} + c_{12}^2},
\]

\[
h \approx \frac{(e^{-i(\beta_1-\beta_2)} - 1) \sin 2\theta_{23} \sin 2\theta_{12} e^{-i\delta_{CP}}}{s_{12}^2 e^{-i(\beta_1-\beta_2)} + c_{12}^2}.
\]

We again note that \(|f|\) is strongly enhanced for \(\beta_1 - \beta_2 = 0\), which implies that \(|\delta| > 1\), e.g., for \(\beta_1 = \beta_2 = \pi\) and \(\delta_{CP} = -\pi/2\), we obtain \(|f| \sim 68\), which leads to \(|\delta| \sim 2\). However, other values of \(\text{CPVP}\) may lead to small breaking parameters. As an example, for \(\beta_1 = \pi, \beta_2 = \pi/2\) and \(\delta_{CP} = -\pi/2\), we obtain \(|f| \sim 0.6, |g| \sim 2,\) and \(|h| \sim 1.7\), such that \(|\delta| \sim 0.2\) and \(|\epsilon| \sim 0.1\), which is desirable.

An interesting conclusion of the last analytical approach is that the case of equal Majorana phases is strongly disfavored in any hierarchy. It is also consistent with the numerical analysis presented in the following.
In Fig. 1, we show the allowed region for the symmetry breaking parameters obtained by varying CPVP in the $(0 - 2\pi)$ interval, and taking the mixing parameters within $3\sigma$ level. We depict the $\text{Max}[|\hat{\delta}|,|\hat{\epsilon}|]$, as a function of $m_0$. According to this figure, we can see that for $m_0 \sim 0$ eV in the NH, the breaking parameters are very large regardless of the CPVP values, as found in the previous analysis. In contrast, in the IH the breaking is at least of 30%. Finally, we can see that for $m_0 \sim 0.05$ eV, which corresponds to the DH, the breaking can be less than 10%. This indicates that DH is the best hierarchy for regarding $\mu - \tau$ as a good approximate symmetry, which is consistent with the results obtained in [6] using a different approach.

If we impose the phenomenological requirement of $\text{Max}[|\hat{\delta}|,|\hat{\epsilon}|] < 0.1$, or any other selected small value, we can examine the CPVP that match with this condition. We will study this point of view in the following.

3. Majorana phases and $\mu - \tau$ symmetry

We have shown that the requirement for small symmetry breaking favors the DH. Accordingly, we pick up $m_0 = 0.1$ eV in the following and use the exact expressions in Eqs. (6) and (7) for our numerical analysis. Once we have selected $m_0$, $\hat{\delta}$ and $\hat{\epsilon}$ only depend on 3 CPVP. Thus, by selecting a suitable value of the Dirac phase, $\delta_{CP}$, any two given values for the Majorana phases will provide a unique set of absolute breaking parameters, and vice versa. Therefore, only a well-defined allowed region in the Majorana CP phases space will be consistent with our symmetry breaking requirement. This is shown in Fig. 2, where the left-hand side shows the allowed region for the two Majorana phases which match with the requisite that $\text{Max}[|\hat{\delta}|,|\hat{\epsilon}|] \leq 0.25$. We select $\delta_{CP} = -\pi/2$, which is near to the best fit value reported by [1, 2]. The other mixing parameters are varied up to 1 $\sigma$ and 3 $\sigma$ from their central values, respectively. We can see that there is a strong correlation between the two Majorana phases, which can be inferred from the expressions in Eqs. (11–13), where the relative phase $\beta_1 - \beta_2$ appears. This figure shows that the values of $\beta_1 = \pi$ and $\beta_2 = \pi/2$ analysed in the previous section, which lead to $\text{Max}[|\hat{\delta}|,|\hat{\epsilon}|] \approx 0.2$, are contained well within the allowed regions.

A similar study with different values for the Dirac CP phase shows a strong correlation of the allowed Majorana phases with the former.
Figure 2. In the left-hand plot, the green color (cross signs) corresponds to the mixing parameters varied up to 1σ, the red color (plus signs) corresponds to the 3σ, and the blue color (stars) corresponds to the central values (cv) of the mixing parameters, which match with Max[|δ|, |ε|] ≤ 0.25. In the right-hand plot, the green color (cross signs) corresponds to Max[|δ|, |ε|] ≤ 0.1 and the red color (plus signs) corresponds to Max[|δ|, |ε|] ≤ 0.25, where the experimental values are varied within 3σ.

4. Concluding remarks
As it is shown in Fig. 2, null Majorana phases are completely excluded at the 3σ level, which is consistent with the analysis presented in sec. 2. The allowed region is not modified greatly when we move the experimental values from 1σ to 3σ deviation limits, as we can see from the left plot in Fig. 2. Thus, the correlation is only slightly sensitive to small mixing angle variations. By contrast, in the right hand side of the same plot, when we restrict the upper bound condition even more on Max[|δ|, |ε|], the allowed area is reduced greatly. Therefore, we can conclude that the size of the breaking of $\mu - \tau$ symmetry is highly sensitive to the Majorana phases and vice-versa. Finally, we can make two important conclusions. First, if we regard $\mu - \tau$ as a good approximate symmetry, then the Majorana phases will definitely be non-zero. Second, a future determination of the Dirac phase can restrict the Majorana phases that are compatible with the condition of a weak breaking of $\mu - \tau$. It is interesting that a recent global analysis provided by [2] confirmed the previous preference for $\delta_{CP}$ near $-\pi/2$, which brings our attention to the robust prediction of the Majorana phases obtained for this value. More detailed information about this work can be found in [7].

Acknowledgments
DCRA thanks the organizing committee of the XXX Annual Meeting of the Division of Particles and Fields of the Mexican Physical Society, for the economical support to attend the event. This work was partially supported by CONACyT, México, under grant No.237004.

References
[1] Capozzi F et al. 2014 Phys. Rev. D89 093018.
[2] Capozzi F et al. 2016 Nucl. Phys. B908 218.
[3] Pontecorvo B 1957 J. Exptl. Theoret. Phys. 33 549.
[4] Maki Z, Nakagawa M, and Sakata S 1962 Prog. Theor. Phys. 28 870.
[5] Xing Z and Zhao Z 2016 Rep. Prog. Phys. 79 076201.
[6] Gupta S, Joshipura A, and Patel K 2013 J. High Energy Phys. 09 035.
[7] Rivera-Agudelo D and Pérez-Lorenzana A 2016 Phys. Lett. B760 153.