An impedance approach to the response of matter

S L Vesely¹, C A Dolci² and S R Dolci²

¹ ITB-CNR, via Fili Cervi 93, Milan, I
² UNIMI-VESPA, via dell‘Università 6, Lodi, I
E-mail: sara.vesely@cnr.it

Abstract. At the dawn of the research on waveguides the propagation by electrical conduction through transmission lines was compared with the general transmission theory of plane electromagnetic waves. After the invention of lasers we wonder whether the impedance concept can allow a seamless shift from the geometric rendering of received electromagnetic signals to the understanding of simple arguments on power transfer. Perhaps, the impedance concept could help noticing the occurrence of radiation-matter interactions and give hints as to how some phenomena could be enhanced by exposing matter to specific non-ionizing radiations.

1. Introduction
Over time radiation has been fitted with a number of attributes, among which energy stands out. Current mathematical theories, primarily electromagnetism, the theory of relativity, and QED, are mainly concerned with two types of tasks. Firstly, the representation of the distribution and propagation of the electromagnetic (EM) field in physical space, which is faced by drawing on geometry, and secondly, the investigation of the interaction of radiation with matter. In a matter-dominated universe natural radiations are perceived as only slightly affecting matter, perhaps except for ionizing radiations. This attitude transpires from the analysis of quantum effects. It doesn’t imply that solar radiation elicits small effects on matter, but rather that those don’t pop up stamped with the ℏω radiation-matter interaction seal. While up until WWII nobody felt the need to extend the theoretical framework beyond the perturbative approximation, researchers gradually started pointing out nonlinear interaction effects after the technical achievements in the generation, manipulation, and detection of radiation. In the end all types of phenomena appear to show a degree of individual variability. Thus, small effects are being isolated, that were previously overlooked or unnoticed, and whose explanation demands some kind of nonlinear approach. Emergence usually needs either a more detailed case-by-case description, or supplementary ad hoc assumptions, and attempts are ongoing at explanations based on statistical inference and the extant probabilistic theories. However, more often than not alongside ‘nonlinearity’ radiation-matter interactions feature specificity. Alternatively, one can deem variability to be worth a second look when availing of directional sources, power amplifiers, sensitive receivers, and computer assisted data collection and analysis. Perhaps, phenomena that on Earth take place under the influence of sunlight manifest differently in other environments, and can be enhanced under artificial conditions. Illumination is relevant to our life, and understanding how it may affect matter can open new avenues. In the following we’ll discuss a few points related to an impedance approach to the response of matter.
2. Electromagnetic (EM) field as a property of space

Human eyes receive in the so-called visible range \((2.5 - 5) \times 10^{15} \text{ rad}\). Although the luminous efficiency is not uniform in this range, for luminance levels in the \(10 - 10^7 \text{ lux}\) range the adapted photopic response of the normal eye can be described by a linear function \([1]\). In this range radiations are ineffable as they reveal bodies without being themselves visible. Furthermore, the eyes are not for direct viewing of light sources. Rather, most seen images consist of passively scattered radiations by lit bodies pertaining to the outer world, that happen to be received by the sense organs. Although our eyeballs function like EM signal receivers, our visual perception is far from linear. Humans tend to focus attention on the details of interest with the purpose of recognizing them and interpreting information, decidedly brain activities. Notwithstanding this, while standing still the eye movements allow sensing a ‘whole space’ around the particular that initially caught the eye. And, intriguingly enough, we perceive the visualized feature as located in space. The ancients formulated geometry in order to deal with shapes in space. Owing to Euclid’s involvement in optics we know that to him mirrors likewise focus on details. In contrast, when diaphragms and vignetting can be disregarded our optical image forming instruments, above all lenses and mirrors, limit to render what we call ‘space’. Sizes and aspects of all items of surveyed space change considerably when recorded from different locations, or when differently lit. Images also keep changing under minor optical alignment adjustments. This is shown in Figure 1. On closer attention, those changes appear to be due to the inherent inability of optical instruments (microscopes, radars, receiver antennas, and whatnot) to select and follow the detail of interest. Provided we clarify what mathematical framework we mean by ‘received space’, and achieve a reasonable rendering of it, visual geometric representations can enable a coherent interpretation of the perceived objects across the whole domain of instrumental imaging. This, in turn, can ease the detection of nonlinear traits.

![Figure 1](image-url)

**Figure 1:** A scene (in the foreground) lit by different sources, and some of its mirror images. A) Light source is a hand held led torch. B) Same torch covered by a soft cloth fabric (white nylon) as light diffuser. C) Red laser not aligned with the camera is pointed at the silver thing with handle. Possibly, all bright spots result from secondary scattered light. D) Long acquisition time while the pointer is moved around to light up the scene. The shot forms the whole image at once rather than scanning each point individually.

The EM field that came to be understood as the locus of EM phenomena evolved from Faraday’s magnetic lines of force. Faraday recorded the action of magnetic forces on test bodies (TBs) at different locations disregarding the functioning of the detectors, namely, the position of iron filings on a paper sheet or the impulse response of a coiled conductor wire (Faraday induction). Experimentally, the EM response depends on the source of the field, e.g. the magnet, and on the receiver impedance \(Z_t\). At worst, it includes a non-quantifiable contribution of the interposed medium. When mapping the spatial distribution of the ‘stuff’ constituting the magnetic field, we are the terminal field sensor locating the TB in space in daylight, and thus \(Z_{TB} \neq Z_t\). Experimental data points can be analyzed in terms of the small-signal equivalent electric circuit (EEC) of the TB, filing or coil, characterized by a time-invariant EM impedance.
\( Z_{TB} \). Let’s resort to a TB consisting of a RLC series circuit, and draw on hydrodynamic analogies. The impedance \( Z_{TB} = \frac{V}{I} \), a pressure/flux analog, belongs to the EEC diagram of the TB whose voltage and current are respectively \( V \) and \( I \). Moreover, let’s neglect that Faraday observed free induction decays and dwell instead on the oscillatory behavior of the RLC connected to an electromotive force (EMF). In the real domain \( \mathbb{R} \) the time response of the EEC is obtained by applying Kirchhoff’s voltage law \( V_R(t) + V_L(t) + V_C(t) = E(t) \), i.e. by solving the complete second order ordinary differential equation (ODE) with real constant coefficients \( R \) (resistance), \( L \) (inductance), and \( C \) (capacity) that models the TB. The steady state impedance \( Z_{TB} \) is a complex function even when the circuit is driven by the EMF \( E(t) = V_0 e^{i\omega t} \), where \( \omega \) is the angular frequency.\(^1\) It is split either into real an imaginary parts \( Z_{TB} = \Re[Z] + i\Im[Z] = R + i (\omega L - \frac{1}{\omega C}) \), or alternatively into magnitude \( |Z_{TB}| = \frac{V}{I} = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2} \) and phase \( \varphi_s = \arctan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right) \). Calling \( \omega_s = \sqrt{\frac{1}{LC}} \) the undamped oscillation of the EEC, and \( \alpha = \frac{R}{2L} \) the damping coefficient,\(^2\) we write \( Z_{TB} = \frac{\omega_s^2}{2} \left[ 2\alpha + i (\omega^2 - \omega_s^2) \right] \). If the EMF has the more general form \( V_{in}(t) = V_0 (\cos(\omega t) + i \sin(\omega t)) \), and the circuit is meant as a voltage divider, at steady state the potential difference across \( R \) can be written as \( V_{out} = \frac{R}{Z_{TB}} V_{in} = H(\omega) V_{in} \). Particular solutions provide the time-response to the equation for a second-order filter circuit and sinus-wave applied voltages. The transfer function \( H(\omega) \) of the EEC in this capacity satisfies the symmetry relation \( H(\omega) = \frac{4\alpha^2 \omega^2}{4\alpha^2 \omega^2 + (\omega^2 - \omega_s^2)^2} - \frac{i 2\alpha \omega (\omega^2 - \omega_s^2)}{4\alpha^2 \omega^2 + (\omega^2 - \omega_s^2)^2} \). Since integrals over odd terms vanish, if an inverse Fourier transform \( h(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} H(\omega) e^{i\omega t} d\omega \) existed in this case, it would be a real function. The energy interpretation of the EM field analyzes the collected TB data in terms of EEC by drawing on a hydrodynamic analogy. Qualifying the TB data as properties of space takes advantage of our spatial perception.

3. Resonance, a nonlinear coupling condition

What do we perceive, or measure with an image forming device, when a TB tightly couples to a radiation source, i.e. when the source is tuned to it? The principle of maximum power transfer tells that, for source and TB to couple, the allegedly time-invariant \( Z_{TB} \) of the two-port EEC of the TB ought to match the source. Writing \( R \) for the voltage reflection coefficient, and \( n \) for the refractive index, the TB’s reflectance is \( |R|^2 = \frac{|Z_{TB} - Z_{source}|^2}{|Z_{TB} + Z_{source}|^2} = \frac{n_{source} - n}{n_{source} + n} \).\(^3\) 0 under matching conditions. Ideally, its interface and internal structure may be deeply affected without scattering light, and we see nothing. For an example, at very high frequencies (ionizing radiations) \( n \rightarrow 1 \) for all matter, and matched conditions possibly occur. Similar conditions apply to the sun as a light source, and cause seen objects to look like ‘mismatched’ loads. Still, the source itself cannot be modeled as an EEC, being it an active element. Moreover, we expect the TB to respond nonlinearly, too, when a matching condition is steadily approached, and specifically to start flickering.\(^4\) In fact, engineers often choose the source impedance of a driving stage \( Z_1 \) much smaller than the load just to avoid affecting the voltage transferred to the receiver stage. The linear signal approach is flawless as long as flicker contributes a minor EM response, and the final receiver doesn’t itself couple. Yet, resonance is a nonlinear phenomenon. Although a scant power may suffice for efficient coupling, to uphold the matched condition an artificial source ought to follow the time-fluctuations in \( Z_{TB} \).

---

1. Standard initial conditions are \( E(0) = V_0 \) and \( V(0^-) = I(0^-) = 0 \).
2. The natural frequency \( \omega_0 = \sqrt{\frac{1}{LC}} \) of the RLC, pertains to the homogeneous ODE.
3. The modulation effect is not a linear ping pong of energy. A strong impulse readily produces saturation effects.

---

3
4. Lasing sources and geometry

Poynting likely addressed the problem of the geometric representation of the EM energy transfer in the context of the ‘null system’ [2]. Figures 1C-D show that no directional light readily lends itself to a geometric interpretation. Rather, the attendant things need to diffuse to reveal the outer world. Lasers are reckoned to be directional sources on a par with direct sunlight. Here we won’t digress to consider how the sensed shapes depend on illumination, and just stress that the transfer efficiency of sources depends on impedance matching. EM source-object coupling famously occurs through an interposed void, as opposed to what happens with mechanical analogies. This is why individual fine-tuning can fool statistical predictions and energy interpretations. In principle, an impedance approach is suited to discuss diffuse and directional illumination. To that end, let’s briefly retrace the cross-fertilization of engineering and theoretical developments that makes infinite transmission line and field equations look the same [3]. An ideal transmission line is ruled by equations \[ i \frac{dV}{dx} = -ZI \text{ and } i \frac{dI}{dx} = -YV. \]

Its electric parameters \( R, L, G, C \) allow writing series impedance \( Z = R + i \omega L \), shunt admittance \( Y = G + i \omega C \) and characteristic impedance \( Z_0 = \sqrt{\frac{\gamma}{\eta}} = \frac{\gamma}{\eta} \) besides other related functions, such as the reflection and transmission coefficients. As with the two-port RLC network and in optics it is possible to pair output and input functions via Fourier/Laplace transform. Other types of transforms have been developed in the top-down network synthesis of RLC-filters. To discuss the propagation along the line in terms of a linearly polarized TEM wavefront parametrized by the fields \( E = \hat{\mathbf{x}} E_0 e^{-\gamma z} \) and \( H = \hat{\mathbf{y}} H_0 e^{-\gamma z} \) Schelkunoff replaces the line equations by \[ i \frac{dE}{dx} = -i \mu H_y \text{ and } i \frac{dH}{dx} = -(g + i \omega \varepsilon) E_x, \]

respectively. The propagation constant \( \gamma \) and the intrinsic impedance \( \eta \) are written out in terms of the wave parameters permittivity \( \varepsilon \), conductivity \( \sigma \) (ideally, \( \sigma = 0 \)), and permeability \( \mu \). They become \( \gamma = \sqrt{i \omega \mu \cdot (g + i \omega \varepsilon)} \) and \( \eta = \sqrt{\frac{i \omega \mu}{g + i \omega \varepsilon}} = \frac{\gamma}{\sqrt{\eta}} \). Once this is done, \( Z_0 \) neither transforms as a vector, nor satisfies the addition rule. This problem has been tackled by various authors [4]. However, if \( Z \) is related to streamlines and equiphasic surfaces, it allows for transformations over \( \mathbb{C} \) in ‘space’ as an entire analytic function [5]. Starting with the so-called stereographic projection, \( \mathbb{C} \) the representation of \( \mathbb{C} \)-numbers as points can be interchangeably represented either on the Argand plane, that as a field can be made isomorphic to \( \mathbb{R}^2 \), or on the Riemann sphere \( S^2 \) (in \( \mathbb{R}^3 \)) [6]. So can level curves of entire analytic functions over \( \mathbb{C} \). Signal and power transfer can both be dealt with in terms of \( Z \) surfaces/spots without reifying the fields. In given experimental conditions the energy issues can be rated with respect to conversion yields.

References
[1] Soffer B H and Lynch D K 1999 Am. J. Phys. 67(11) 946-53
[2] Poynting J H 1884 Phylso. Trans. R. Soc. 175 343–61
[3] Schelkunoff S A 1937 Proc. Inst. Radio Eng. 25(11) 1457–92
[4] Riesz M, Bolinder E F and Lounesto P 1993 Clifford Numbers and Spinors (NL: Springer)
[5] Markushevich A I 1977 Theory of Functions of a Complex Variable (NY (USA): Chelsea)

4 It isn’t self-evident how energy moves across space. Perhaps, Ball’s screws were Poynting’s guiding theme for the geometric transformations of the electric and magnetic forces.

5 The parameters are series resistance \( R \) and inductance \( L \), shunt capacity \( C \) and conductance \( G \). \( \gamma = \pm \sqrt{\varepsilon \mu} = \alpha + i \beta \) is the propagation constant along the \( z \)-axis. \( \alpha \) is the attenuation constant, and \( \beta \) the phase constant. The line parameters depend on the material with which the line is made, but are distinct from the characteristic values of the material. In the present case, they can be either understood as ‘vacuum parameters’ or referred to a source-object transfer function.

6 Apart from the name, this is a central perspective rather than a projective space transformation.

7 Points can be identically mapped from \( S^2 \setminus \{ \infty \} \) on \( \mathbb{C} \).