Measuring the P-odd Pion-Nucleon Coupling $h_{\pi NN}^{(1)}$ in $\pi^+$-Photoproton Production Near the Threshold

Jiunn-Wei Chen and Xiangdong Ji
Department of Physics, University of Maryland, College Park, MD 20742-4111

We show that $\pi p \to \pi^+ n$ in the threshold region is an excellent candidate for measuring the leading parity-violating pion-nucleon coupling $h_{\pi NN}^{(1)}$, to an uncertainty of 20% if it has a natural size from dimensional analysis. The conclusion is based on a large unpolarized cross section, a new low-energy theorem for the photon polarization asymmetry at the threshold $A_{\gamma|th} = \sqrt{2}\mu^* / \mu_N h_{\pi NN}^{(1)} / g_{AM NN} \sim h_{\pi NN}^{(1)}/2$, and its strong dominance at forward and backward angles in the threshold region.

PACS numbers: 11.30.Er, 11.30.Rd, 13.60.Le.

Parity-violating, or P-odd, hadronic observables provide crucial information about the physics of nonleptonic weak interactions in hadronic structures and reactions. At low-energy, parity-violating hadronic interactions can be systematically classified in the framework of effective field theories [1–3]. At the leading order in chiral power counting, the most important is the isovector P-odd pion-nucleon coupling $h_{\pi NN}^{(1)}$ which is responsible for the longest range part of the parity-violating $\Delta S = 1 NN$ forces [1–3]. In quantum chromodynamics (QCD), its value is dominated by the s-quark contribution through neutral current interaction [3]. A precise knowledge of $h_{\pi NN}^{(1)}$ not only is critical for understanding the P-odd $NN$ force but will also shed important light on how parity violation takes place in nonleptonic systems.

For many years, serious attempts have been made to measure $h_{\pi NN}^{(1)}$ from parity-violating processes (see [3,7] for reviews). In many-body systems, parity-violating effects can be enhanced by strong correlations and have been detected experimentally. However, the theoretical analyses have not yet been fully reliable. The disagreement in the extraction of $h_{\pi NN}^{(1)}$ from $^{18}$F and $^{133}$Cs systems could be a reflection of poor understanding of many-body physics. In few-body systems, the theory is under better control; but the P-odd effects are generally small. While previous measurements could not reach the required precision [7], new experiments under way are expected to improve significantly. These include $\pi^+ p \to \gamma n$ at LANSCE [12], $\gamma d \to np$ at Jefferson Lab (JLab) [13], and the rotation of polarized neutrons in helium at NIST [11]. Finally, in the single nucleon systems, new P-odd observables in Compton scattering on the proton were recently proposed to determine $h_{\pi NN}^{(1)}$ [14]. The process is theoretically “clean”, however the experimental feasibility is marginal because of the small total cross section and P-odd asymmetries.

In this paper, we show that the polarized photon asymmetry in $\pi^+ p \to n\pi^+$ at the threshold region is an excellent candidate to measure $h_{\pi NN}^{(1)}$. We derive a low-energy theorem for the asymmetry at the pion-production threshold in the chiral limit: $A_{\gamma|th} = \sqrt{2}\mu^* / \mu_N h_{\pi NN}^{(1)}/g_{AM NN} \sim h_{\pi NN}^{(1)}/2$. A leading-order (LO) calculation in heavy-baryon chiral perturbation theory (HB$\chi$PT) shows that the result is modified only mildly by higher partial waves, particularly at forward and backward angles, and chiral corrections from the finite pion mass and momentum in the threshold region up to photon energy $E_\gamma \sim 200$ MeV. With a total cross section $\sim 100\mu b$ and the expected asymmetry $\sim 2 \times 10^{-7}$, the experiment is feasible at existing laboratories such as JLab. Theoretical studies of the same process have been carried out before by Woloshyn [15] and by Li, Henley and Hwang [16] in the framework of meson exchange models. In particular, Ref. [16] has already noted the dominance of the $h_{\pi NN}^{(1)}$-type P-odd coupling in the asymmetry near the threshold. The present analysis sharpens the finding by deriving the low-energy theorem and defending its dominance in the threshold region using the modern theoretical tool—HB$\chi$PT [1,2].

We are interested in the following two-body process,

$$\pi^+(q^\mu) + p(P^\mu) \to \pi^+(k^\mu) + n(P_f^\mu), \quad (1)$$

where $q^\mu = (\omega, q), P^\mu = (\omega, P), k^\mu = (\omega, k),$ and $P_f^\mu$ are the center-of-mass four-momenta of photon, proton, pion and neutron, respectively, and $e^\mu$ is the photon polarization vector. In the threshold region, the pion and photon as well as the nucleon momenta are much smaller than the chiral symmetry breaking scale $\Lambda_c \sim 4\pi f_\pi \sim 1$ GeV; therefore chiral perturbation theory ($\chi$PT) is a useful tool in making theoretical analyses [3]. When the nucleon is explicitly involved, a natural scheme for systematic power counting is to treat its mass as a heavy scale as $\Lambda_N$, and thus HB$\chi$PT [2]. In addition, since the delta-resonance is only 300 MeV heavier than the nucleon (order $1/N_c$ in QCD with a large number of $N_c$ colors) and is strongly coupled to the latter through electromagnetic excitations, it is sensible to extend HB$\chi$PT to include the resonance as dynamical degrees of freedom and to treat the mass difference $\Delta = m_\Delta - m_N$ as a small
parameter \[^{17}\]. The SU(2)\(_L\) \(\times\) U(1) symmetry structure of electroweak interactions can be incorporated with the weak boson exchange described by contact interactions and the photon kept as dynamical degrees of freedom.

The unpolarized \(\gamma p \rightarrow \pi^+ n\) reaction at the threshold represents a classical example of the successes of effective theory ideas. Simply relying on the symmetry properties of the strong interactions, Kroll and Ruderman made a prediction in 1954 on the s-wave scattering length in the chiral limit \[^{18}\]. Away from this limit, the corrections have been successfully studied using effective field theories. A first analysis of the reaction in \(\chi PT\) was made by Bernard et al. \[^{19}\], who found that the one-loop correction to the tree-order threshold s-wave amplitude (\(E_{0+}\)) is insignificant. A more detailed study of partial waves in the framework of \(HB\chi PT\) has recently been made by Fearing et al. \[^{20}\], who found that the p-wave multipoles at the threshold are well described by the leading (\(O(p)\)) plus next-to-leading (\(O(p^2)\)) order calculations. For example, \(M_{1+}, M_{1-}\), and \(E_{1+}\) multipoles are \(-4.7, 9.4,\) and \(4.7\) in unit \(10^{-3}/m^3\) at \(O(p)\). At order \(O(p^2)\), the results are \(-7.7, 5.6,\) and \(5.1\) which compare favorably with \(-9.6, 6.1,\) and \(4.9\) from a dispersion-theory analysis of experimental data \[^{21}\].

For the process to be useful in studying nonleptonic parity-violating interactions, the cross section must be able with variations themselves) within 10\% up to \(E_\gamma = 200\) MeV. The difference indicates the size of the higher-order corrections expected of \(HB\chi PT\) and the level of convergence of the chiral expansion. According to the figure, we define the threshold region in terms of the laboratory photon energy from the threshold to 200 MeV. In the lower graph, we show the angular distributions of the pions in the center-of-mass frame and the data which show the largest deviation from the theory by about 20\% at 200 MeV and backward angles.

Now we turn to parity-violating effects in the process. To calculate P-odd observables, we need to extend chiral perturbation theory to include non-leptonic weak interactions. A systematic construction of the P-odd effective chiral lagrangian has been undertaken in Ref. \[^{1}\]. To \(O(p^0)\) (we choose to ignore the weak coupling in power counting), it has one term,

\[
\mathcal{L}^{PV} = -ih^{(1)}_{\pi NN} \pi^+ p^\dagger n + h.c. + \cdots ,
\]

where the ellipses denote terms with more pion fields and derivatives, and the phase convention is taken from Refs. \[^{2}\]. By matching onto four-quark interactions, \(h^{(1)}_{\pi NN}\) was found to be dominated by s-quark contributions.

\[ (\mu b) \]

\[ (b/sr) \]

\[ (\mu b) \]

\[ 0^\pi^- (\text{deg}) \]

\[ E_\gamma \text{ (MeV)} \]

\[ \frac{d\sigma}{d\Omega} \]

\[ \theta^\pi^- \text{ (deg)} \]
FIG. 2. Feynman diagrams contributing to the parity-violating amplitudes at LO ($O(1)$) and NLO ($O(p)$) in $\gamma p \to \pi^+ n$.

$$h^{(1)}_{\pi NN} \sim G_F F_\pi A_\chi/\sqrt{2} \sim 5 \times 10^{-7} \ [1]$$. This estimation is consistent with the “best value” obtained in Ref. [1] and close to a result [2] from QCD sum rules. On the other hand, a recent calculation in the SU(3) Skyrme model yields $h^{(1)}_{\pi NN} \sim 0.8-1.3 \times 10^{-7}$ [3].

To the next-to-leading order (NLO) ($O(p)$) in chiral expansion, the relevant Feynman diagrams for the P-odd $\gamma p \to \pi^+ n$ process are shown in Fig. 2. The resulting $T$-matrix can be expressed in terms of two amplitudes,

$$T^{PV} = N^\dagger \begin{bmatrix} i F_1 \hat{k} \cdot \epsilon + F_2 \epsilon \times \hat{q} \end{bmatrix},$$  \hspace{1cm} (5)

where

$$F_1 = \frac{-e h^{(1)}_{\pi NN} |k|}{q \tilde{k}}, \quad F_2 = \frac{e h^{(1)}_{\pi NN}}{2 m_N} \left[ \mu_p - \left( \frac{\omega}{\omega_\pi} \right) \mu_n \right].$$  \hspace{1cm} (6)

P-odd observables can now be constructed from the interference between $T^{PV}$ and $T^{PC}$. The leading single-spin asymmetry arises from the interference between $A_{1-3}$ and $F_1$, and is dependent on the proton polarization. Because of technical difficulties with a large volume, high-density polarized hydrogen target, an experimental measurement of this asymmetry is not within sight. Therefore, in the following we focus on the photon helicity-flip asymmetry which comes in at NLO from the interference between $A_1$ and $F_1$, and $A_4$ and $F_1$. $A_4$ in HB$\chi$PT is found nonvanishing at NLO and is

$$A_4 (\omega, \theta) = \frac{e g_A f_\pi m_N}{2 \sqrt{2} f_\pi m_N} \left[ \mu_p - \left( \frac{\omega}{\omega_\pi} \right) \mu_n \right]$$

$$+ \frac{2 e g_{\pi N \Delta} G_1 |k|}{9 g_A m_N} \left( \frac{\omega}{\omega - \Delta} + \frac{\omega}{\omega_\pi + \Delta} \right),$$  \hspace{1cm} (7)

where the delta-resonance contribution has been included explicitly. $G_1$ is the M1 transition moment between the nucleon and delta, and $g_{\pi N \Delta}$ is the $\pi-N-\Delta$ coupling.

More explicitly, the photon helicity asymmetry $A_\gamma (\omega, \theta) = (d\sigma(\lambda_\gamma = +1) - d\sigma(\lambda_\gamma = -1))/(d\sigma(\lambda_\gamma = +1) + d\sigma(\lambda_\gamma = -1))$ at the leading order in HB$\chi$PT is

$$A_\gamma (\omega, \theta) = \frac{\sqrt{2} f_{\pi NN} f_\pi}{g_A m_N g} \left\{ \left[ \mu_p - \left( \frac{\omega}{\omega_\pi} \right) \mu_n \right] \left( 1 - \frac{\sin^2 \theta k^2}{q \tilde{k}} \right) + \frac{2 e g_{\pi N \Delta} G_1 \sin^2 \theta k^2}{9 g_A q \tilde{k}} \right\} \left( \frac{\omega}{\omega - \Delta} + \frac{\omega}{\omega_\pi + \Delta} \right)$$  \hspace{1cm} (8)

where $G$ is given in Eq. (3). Although the result formally depends on the NLO amplitude $A_4$, it is dominated in the threshold region by the “beat” between the parity-violating amplitude $F_2$ and the leading-order parity-conserving amplitudes $A_{1,2,3}$ which have already been tested in Fig. 1. Right at the threshold $|k| = 0$, only the s-wave $\pi^+ n$ final-state contributes; we find the equivalent of the Kroll-Ruderman theorem for the P-odd photon-helicity asymmetry,

$$A_\gamma (\omega, \theta) = \frac{\sqrt{2} f_{\pi NN} (\mu_p - \mu_n)}{g_A m_N} h^{(1)}_{\pi NN},$$  \hspace{1cm} (9)

The solid line is the low-energy theorem in Eq. (9), and the short-dashed and dash-dotted lines are for $E_\gamma = 180$ and 200 MeV, respectively.

FIG. 3. The photon-helicity asymmetry $A_\gamma$ in unit $h^{(1)}_{\pi NN}$. For the vector asymmetry is dominated by the leading-order amplitude $A_4$, and the leading-order parity-conserving amplitudes $A_{1,2,3}$ have already been tested in Fig. 1. Right at the threshold $|k| = 0$, only the s-wave $\pi^+ n$ final-state contributes; we find the equivalent of the Kroll-Ruderman theorem for the P-odd photon-helicity asymmetry,

$$A_\gamma (\omega, \theta) = \frac{\sqrt{2} f_{\pi NN} (\mu_p - \mu_n)}{g_A m_N} h^{(1)}_{\pi NN},$$  \hspace{1cm} (9)

where $G$ is given in Eq. (3). Although the result formally depends on the NLO amplitude $A_4$, it is dominated in the threshold region by the “beat” between the parity-violating amplitude $F_2$ and the leading-order parity-conserving amplitudes $A_{1,2,3}$ which have already been tested in Fig. 1. Right at the threshold $|k| = 0$, only the s-wave $\pi^+ n$ final-state contributes; we find the equivalent of the Kroll-Ruderman theorem for the P-odd photon-helicity asymmetry,

$$A_\gamma (\omega, \theta) = \frac{\sqrt{2} f_{\pi NN} (\mu_p - \mu_n)}{g_A m_N} h^{(1)}_{\pi NN},$$  \hspace{1cm} (9)

where $G$ is given in Eq. (3). Although the result formally depends on the NLO amplitude $A_4$, it is dominated in the threshold region by the “beat” between the parity-violating amplitude $F_2$ and the leading-order parity-conserving amplitudes $A_{1,2,3}$ which have already been tested in Fig. 1. Right at the threshold $|k| = 0$, only the s-wave $\pi^+ n$ final-state contributes; we find the equivalent of the Kroll-Ruderman theorem for the P-odd photon-helicity asymmetry,

$$A_\gamma (\omega, \theta) = \frac{\sqrt{2} f_{\pi NN} (\mu_p - \mu_n)}{g_A m_N} h^{(1)}_{\pi NN},$$  \hspace{1cm} (9)
here and all couplings except \( h_{\pi NN}^{(1)} \) are known. The size of the correction will follow the canonical power counting, i.e. of order \( O(\epsilon/m_N) \), where \( \epsilon \) stands for \( m_{\pi}, \omega, \omega_{\pi}, \) and \( \Delta \). The last class involves an interference between \( \text{LO } T^{PC} \) and \( \text{NNLO } T^{PV} \) amplitudes; the latter contains one-loop integrals as well as tree contributions from new P-odd effective couplings. The following is an example of P-odd interactions at NNLO,

\[
\mathcal{L}^{PV} = \frac{e h_{\pi NN}}{m_N^2} \bar{\pi} S^\mu S^\nu \pi^+ n F_{\mu\nu} + i e\tilde{G} \frac{m_N^2}{m_N} \Delta \gamma^\mu \nu F_{\mu\nu} p. 
\]

(10)

While the one-loop integrals are not expected to yield large corrections, the magnitude of the new couplings is unknown. Since an unnatural size of couplings in effective theory usually arises from new physics, we do not expect this to happen here from our experience with the corresponding parity-conserving amplitudes. This of course can be tested by the \( \theta \) dependence of the asymmetry. In short, we expect the higher-order corrections to Eq. (8) is \( O(\epsilon/m_N) \), namely, about 20%.

Finally, we briefly comment on the experimental feasibility for measuring the polarization asymmetry in \( \gamma p \rightarrow \pi^+ n \). To overcome statistics, a large number of events (\( \sim 10^{14} \)) are needed. This requires a luminosity of order \( 10^{37}/(\text{cm}^2 \text{sec}) \) which is reasonable with the current technology and facilities such as JLab. With a total cross section \( \sim 100 \mu b = 10^{-28} \text{cm}^2 \), the \( \pi^+ \) production rate is \( 10^8/\text{sec-rad} \). Thus \( \sim 10^6 \) sec of beam time will yield the required number of events. The challenge, however, could be \( 10^8 \pi^+/\text{sec detection} \).

In conclusion, we have shown that parity-violating \( \gamma p \rightarrow \pi^+ n \) is a theoretically clean and experimentally feasible process to measure \( h_{\pi NN}^{(1)} \). Near the threshold region, the size of the photon helicity asymmetry is estimated to be \( \sim 2 \times 10^{-7} \) for an expected magnitude of \( h_{\pi NN}^{(1)} \). Assuming a luminosity of \( 10^{37}/(\text{cm}^2 \text{sec}) \), \( h_{\pi NN}^{(1)} \) can be measured to an accuracy of \( 10^{-7} \) in a few months of running. Similar results for pion electroproduction will be published separately [26].

Note added in proof: After this paper was submitted for publication, Zhu et al. published a preprint presenting a calculation of the NLO corrections [27]. We have submitted a comment about their paper to e-archive [27].

ACKNOWLEDGMENTS

We thank D. Beck, E. Beise, E. Henley, R. McKeown, R. Stuleiman, and B. Wojtsekhowski for discussions on the experimental issues. This work is supported in part by the U.S. Dept. of Energy under grant No. DE-FG02-93ER-40762.

[1] D.B. Kaplan and M.J. Savage, Nucl. Phys. A 556, 653 (1993).
[2] E. Jenkins and A. V. Manohar, Phys. Lett. B 255, 558 (1991).
[3] V. Bernard, N. Kaiser, and Ulf-G. Meissner, Int. J. Mod. Phys. E 4, 193 (1995).
[4] B. Desplanques, J.F. Donoghue and B.R. Holstein, Ann. Phys. (N.Y.) 124, 449 (1980).
[5] E.G. Adelberger and W.C. Haxton, Ann. Rev. Nucl. Part. Sci. 35, 501 (1985).
[6] J. Dai, M.J. Savage, J. Liu and R. Springer, Phys. Lett. B 271, 403 (1991).
[7] W. Haeberli, B.R. Holstein, in Symmetries and Fundamental Interactions in Nuclei, ed. W.C. Haxton and E.M. Henley (World Scientific, Singapore, 1995), p.17.
[8] W.T.H. van Oers, Int. J. Mod. Phys. E 8, 417 (1999).
[9] S.A. Page et al., Phys. Rev. C 35, 1119 (1987); M. Bini, T. F. Fazzini, G. Poggi, and N. Taccetti, Phys. Rev. C 38, 1195 (1988).
[10] C.S. Wood et al., Science 275, 1759 (1997); W.C. Haxton, Science 275, 1753 (1997); V.V. Flambaum and D.W. Murray, Phys. Rev. C 56, 1641 (1997); W.S. Willburn and J.D. Bowman, Phys. Rev. C 57, 3425 (1998).
[11] V.A. Knyazkov et al., Nucl. Phys. A 417, 209 (1984); J.F. Cavagnac, B. Vignon and R. Wilson, Phys. Lett. B 67, 146 (1997); D.M. Markoff, Ph.D. Thesis, University of Washington (1997).
[12] W.M. Snow et al., Nucl. Inst. Meth. A 440, 729 (2000).
[13] JLab LOI 00-002, W. van Oers and B. Wojtsekhowski, spokesmen.
[14] P.F. Bedaque and M.J. Savage, Phys. Rev. C 62, 018201 (2000); J.W. Chen, T.D. Cohen and C.W. Kao, nucl-th/0009031.
[15] R.M. Woloshyn, Can. J. Phys. 57, 809 (1979).
[16] S.P. Li, E.M. Henley and W-Y. P. Hwang, Ann. Phys. 143, 372 (1982).
[17] T.R. Hemmert, B.R. Holstein and J. Kambor, Phys. Lett. B 395, 89 (1997).
[18] N.M. Kroll and M.A. Ruderman, Phys. Rev. 93, 233 (1954).
[19] V. Bernard, N. Kaiser, J. Gasser and U.G. Meissner, Phys. Lett. B 268, 291 (1991).
[20] H.W. Forring, T.R. Hemmert, R. Lewis, C. Unkmeir, hep-ph/0005213.
[21] O. Hanstein, D. Drechsel and L. Tiator, Phys. Rev. A 632, 561 (1998).
[22] F.A. Berends and D.L. Weaver, Nucl. Phys. B 30, 575 (1971); M. MacCormick et al., Phys. Rev. C 53, 41 (1996); J. Ahrens et al., Phys. Rev. Lett. 84, 5950 (2000).
[23] C.M. Maekawa and U. van Kolck, Phys. Lett. B 478, 73 (2000); S.-L. Zhu, S.J. Puglia, B.R. Holstein and M.J. Ramsey-Musolf, hep-ph/0005281.
[24] E.M. Henley, W.-Y.P. Hwang and L.S. Kisslinger, Phys. Lett. B 367, 21 (1999); Erratum-ibid. B 440, 449 (1998).
[25] U.G. Meissner and H. Weigel, Phys. Lett. B 447, 1 (1999).
[26] J.W. Chen and X. Ji, nucl-th/0011100 and in preparation.
[27] S.-L. Zhu, S. J. Puglia, B. R. Holstein, M. J. Ramsey-Musolf, hep-ph/0012253; J. W. Chen and X. Ji, hep-ph/0101290.