Interband transition in narrow gap InSb spherical layer quantum dot in the presence of electric field

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Abstract. We perform the theoretical investigation of interband dipole transitions in a narrow-gap InSb spherical layer quantum dot. We consider the transitions from the light hole and heavy hole states to the electron state of the conduction band. The dispersion law for electron and light hole is approximated using a two-band Kane model, while the heavy hole states are described in the parabolic approximation. The effect of electric field on interband transitions is investigated.

1. Introduction
The ring-shaped and layered nanostructures, where the radial motion of charge carriers is confined from two sides on interior and exterior boundaries, form the interesting class of zero-dimensional structures with nontrivial geometry [1-3]. It is clear that this property allows us to carry out flexible manipulations of the energy levels and of the optical absorption spectrum of quantum layered structures. This is an important argument allowing to use such systems as the elements of new generation semiconductor devices (see, e.g. [4,5]). Hence, the results of theoretical studies of the optical properties of layered nanostructures can be directly used in the instrumentation technologies.

In the present work we perform the theoretical investigation of the interband transitions in narrow-gap InSb spherical layer quantum dot in homogeneous electric field. The dispersion laws for electron and light hole (lh) are approximated using a two-band Kane model [6], while the heavy hole (hh) states are described in the parabolic approximation. Then the electron dispersion law will be as in case of the relativistic one [6] \( E = \sqrt{p^2 s^2 + m^2 s^2 - ms^2} \), where \( s \) is the parameter for non-parabolicity and \( m \) is the effective mass of the electron in InSb.

2. Theory
Let us consider the narrow-gap InSb spherical quantum layer with inner radius \( R_1 \) and outer radius \( R_2 \). The confinement potential of the spherical layer quantum dot has the following form:
An equation of the electron and lh envelope wave function in a layer is analogous to the Klein – Gordon equation, while the hh wave function is described by Schrödinger equation with the corresponding effective mass. This equation can be presented in a common form
\[ \frac{\hbar^2}{2\mu} \hat{\Psi} + V_{\text{conf}}(r) \Psi^{(0)} = \varepsilon \Psi^{(0)} \].

Here it is supposed that for the electron and for the lh
\[ \mu = \mu^{(\text{lh})}, \quad \varepsilon^{(\text{lh})} = \frac{(E^{(\text{lh})} + \mu^{(\text{lh})} S^2)^2 - (\mu^{(\text{lh})} S^2)^2}{2\mu^{(\text{lh})} S^2} \quad \text{and} \quad \Psi^{(0)} = \Psi^{(0)(\text{lh})} \],

and for the hh
\[ \mu = \mu^{(\text{hh})}, \quad \varepsilon^{(\text{hh})} = E^{(\text{hh})} \quad \text{and} \quad \Psi^{(0)} = \Psi^{(0)(\text{hh})} \].

According to [7] the wave function can be represented as follows,
\[ \Psi^{(0)(\text{lh})}_{n,l,m}(r, \theta, \phi) = \frac{\pi k}{2r} [C_l J_{(l+0.5)}(kr) + C_{-l} J_{-(l+0.5)}(kr)] Y_{l,m}(\theta, \phi), \]

where \( C_l \) and \( C_{-l} \) are the normalization constants.

Eventually we obtain the following expression for the wave functions
\[ \Psi^{(0)(\text{hh})}_{n,l,m}(r, \theta, \phi) = \frac{\pi k}{2r} \delta(\Delta - E_{n,l} - E^{(\text{hh})}_{n',l'}) \]

where \( \Delta = \omega - E_{n,l} \) is the frequency of the incident light. As a result, the expression for the absorption coefficient of the system as well.

The absorption coefficient for the transitions from lh and hh states to the electron states in conduction band in the dipole approximation looks as follows
\[ K^{\text{hh\rightarrow e}}(\omega, R_1, R_2) = A_{CV} \sum_{n_{l,m}, l,l',m'} \int |\Psi^{(0)(\text{hh})}_{n_l,l',m'}(r, \theta, \phi)|^2 |\Psi^{(0)(\text{lh})}_{n_l,l,m}(r, \theta, \phi)|^2 dV \delta(\Delta - E_{n,l} - E^{(\text{hh})}_{n',l'}) \].

Here \( A_{CV} \) is the matrix element constructed on Bloch amplitudes of the valence and conduction bands, \( n_{l,m}, l,l',m' \) and \( \Delta = \omega - E_{n,l} \) is the width of the forbidden band of InSb.

In order to calculate the absorption coefficient for any array of non-interacting spherical layers we have to multiply the absorption coefficient on the concentration of the layer and then integrate over the radius \( R_1 \) or \( R_2 \).
\[ K(\omega, R_1) = \int_{R_1}^\infty P(R_i)K(\omega, R_i, R_2)dR_i, \quad K(\omega, R_2) = \int_0^{R_2} P(R_i)K(\omega, R_i, R_2)dR_i. \]  

(10)

Now, let us consider the behavior of the system in electric field \( \vec{E} \), by adding to the Hamiltonian the potential
\[ V(\vec{r}) = -e\vec{E} \cdot \vec{r} = -p(F_x \sin \theta \cos \phi + F_y \sin \theta \sin \phi + F_z \cos \theta). \]  
where \( p = er \) is the dipole momentum.

We will consider the influence of the electric field within perturbation theory. For the matrix elements of the perturbative potential \( V(\vec{r}) \) one has
\[ \langle n', l', m' | V | n, l, m \rangle = \int R_{n', l', m'}(\rho, \theta, \phi)V(r, \theta, \phi)R_{n, l, m}(\rho, \theta, \phi)d\Omega. \]  

(12)

For the given value of \( l \) and \( m \) from all the integrals over \( \theta \) and \( \phi \) only six integrals with \( l' = l, l-1 \) and \( m' = m-1, m, m+1 \) remain to be non-zero [9]. That’s why the first order corrections vanish, so that we have to calculate the second-order ones.

We proceed now to the calculation of wave function in the first-order approximation, which can be presented in the form

\[ \Psi^{(1)}_{n, l, m}(r, \theta, \phi) = \sum_{n', l', m'} \Psi^{(0)}_{n', l', m'}(r, \theta, \phi) \frac{\langle n', l', m' | V | n, l, m \rangle}{E^{(0)}_{n, l, m} - E^{(0)}_{n', l', m'}}. \]  

(13)

Also, we get the following expression for the matrix element of interband transition

\[ M_{cv} \sim \int \left[ \Psi^{(0)}_{c, v}(r, \theta, \phi) + \Psi^{(1)}_{c, v}(r, \theta, \phi) \right] \left[ \Psi^{(0)}_{v, v}(r, \theta, \phi) + \Psi^{(1)}_{v, v}(r, \theta, \phi) \right] d\Omega. \]  

(14)

In order to find the selection rules, we consider matrix element of interband transition from initial state \((n_i, l_i, m_i)\) to final state \((n_f, l_f, m_f)\). Substituting eq. (13) into eq. (14) and ignoring the second-order perturbation terms, we get the following selection rules for both lh–electron and hh–electron transitions: \( n_f = n_i, l_f = l_i \pm 1, m_f = m_i + m_c \pm 1 \).

3. Discussion

For the quantitative analysis of the obtained results let us consider the spherical layer from InSb for which \( s \sim 10^8 \text{ cm/s} \), \( \mu_{lh} = 0.015m_e \) and \( \mu_{hh} = 0.5m_e \) (here \( m_e \) denotes the mass of the free electron).

![Figure 1](image1.png)  

Figure 1. The dependence of threshold energy on inner radius \( R_i \) at the fixed value of outer radius \( R_2 = 900E \).

![Figure 2](image2.png)  

Figure 2. The dependences of threshold energies for the fixed values \( R_1 = 800E, R_2 = 900E \) for the lh → electron transition.
In figure 1 the dependence of the threshold energy upon the inner radius \( R_1 \) and fixed outer radius \( R_2 = 900 \text{E} \) for lh and hh is presented. As it is seen from the figure, threshold energy increases monotonically for lh and hh, upon the increase of \( R_1 \). This is connected with the increase of the role of size quantization (layer thickness \( L = R_2 - R_1 \) decreases). Notice, that the level of the threshold energy at fixed value of \( R_2 \) grows slower with the increase of \( R_1 \) for the parabolic dispersion of hh, than that for the non-parabolic dispersion law of lh. The slower growth of the absorption threshold frequency in \( hh \rightarrow e \) transitions is caused by larger effective mass of a heavy hole in comparison with the mass of light hole. Hence, upon the change of layer parameters, the energy levels of the heavy hole have less energy shifts than ones of the light holes. So, the electron levels possess the same energy shift, as the light hole. Similar conclusions can be made if consider the dependence of absorption threshold frequencies on \( R_2 \).

In the figure 2 the dependence of the threshold energy on the electric field for the lh–electron transition is presented for fixed values of \( R_1 = 800 \text{E} \) and \( R_2 = 900 \text{E} \). For the hh–electron transitions the perturbation theory is acceptable only for small values of the field (\( F_z \leq 50 \text{V/cm} \)), due to the large value of the hh effective mass. It is seen from the figure that with the increase of \( F_z \) the threshold energy goes down due to lowering of the well bottom. Also, let us mention, that the transitions from hole ground state to electron ground state are the most sensitive to the field. The reason is that the ground states are more sensitive to the changes of band borders.

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