ELECTROMAGNETIC SPINDOWN OF A TRANSIENT ACCRETING MILLISECOND PULSAR DURING QUIESCEENCE

A. MELATOS AND A. MASTRANO

School of Physics, University of Melbourne, Parkville, VIC 3010, Australia; amelatos@unimelb.edu.au, alpham@unimelb.edu.au

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ABSTRACT

The measured spindown rates in quiescence of the transient accreting millisecond pulsars IGR J00291+5934, XTE J1751–305, SAX J1808.4–3658, and Swift J1756.9–2508 have been used to estimate the magnetic moments of these objects assuming standard magnetic dipole braking. It is shown that this approach leads to an overestimate if the amount of residual accretion is enough to distort the magnetosphere away from a force-free configuration through magnetospheric mass loading or crushing, so that the lever arm of the braking torque migrates inside the light cylinder. We derive an alternative spindown formula and calculate the residual accretion rates where the formula is applicable. As a demonstration we apply the alternative spindown formula to produce updated magnetic moment estimates for the four objects above. We note that based on current uncertain observations of quiescent accretion rates, magnetospheric mass loading and crushing are neither firmly indicated nor ruled out in these four objects. Because quiescent accretion rates are not measured directly (only upper limits are placed), without more data it is impossible to be confident about whether the thresholds for magnetospheric mass loading or crushing are reached or not.

Key words: accretion, accretion disks – stars: magnetic field – stars: neutron – stars: rotation – X-rays: binaries

1. INTRODUCTION

Four accreting millisecond pulsars (MSPs), the transient systems IGR J00291+5934, XTE J1751–305, SAX J1808.4–3658, and Swift J1756.9–2508, are observed to spin down during the quiescent interval between accretion episodes (Hartman et al. 2008; Patruno 2010; Patruno et al. 2010; Riggio et al. 2011). Hitherto, the deceleration has been interpreted as arising from a standard magnetic dipole torque just as in isolated rotation-powered pulsars (Melatos 1997; Contopoulos & Spitkovsky 2006; Spitkovsky 2006). Under this interpretation, the measured spindown rate can be inverted to infer the dipole magnetic field strength at the stellar surface, \( B_\star = 2\mu / R_\star^3 \), where \( \mu \) and \( R_\star \) denote the magnetic dipole moment and stellar radius, respectively. The available X-ray timing data imply \( 0.9 < B_\star < 10^8 \text{ G} \) \( \lesssim 6 \) for the above four objects, consistent with the recycling scenario in general and the magnetocentrifugal spin up in particular (Wijnands & van der Klis 1998).

Magnetic dipole braking, as traditionally conceived, requires a rotation-powered pulsar to have a properly developed, force-free, electron–positron magnetosphere extending out to the light cylinder at cylindrical radius \( R_L = c / \Omega \), where \( \Omega \) is the angular speed. Under force-free conditions (Michel 1991; Beskin 2010) the light cylinder coincides with the lever arm, where the stellar magnetic field lines are swept back and the electromagnetic torque is effectively exerted.\(^1\) In this context, “properly developed” means that the magnetosphere hosts exactly the right charge and current distributions to sustain the so-called “oblique rotator solution” supplied by electron–positron pairs created in vacuum gap cascades near the polar cap and/or \( R_L \) (Melrose 1996; Spitkovsky 2006). It is doubtful that this structure can be sustained over the long term in an accreting system. Even during quiescence, residual high-density plasma from the accretion process is expected to leak into the magnetosphere until force-free conditions cease to apply (Illarionov & Sunyaev 1975; Cheng 1985; Luo & Melrose 2007; Cordes & Shannon 2008). A neutron star with a mass-loaded magnetosphere still spins down, of course, but the inertial forces are significant (Cheng 1985) and the lever arm is effectively the Alfvén radius \( R_A < R_L \) rather than \( R_L \) (Bucciantini et al. 2006). In other words, the size of the corotating magnetosphere (and hence the lever arm) is set by the ram pressure of the residual accretion flow from the previous accretion episode, not by the self-consistent conduction and displacement currents in the force-free solution. Mass loading is also expected to switch off the pulsar radio emission by shorting out (“poisoning”) the parallel electric fields that power the vacuum gap cascades, but poisoning is a separate physical process that does not affect the braking torque directly (Cheng 1985; Cordes & Shannon 2008).

In this paper we quantify the maximum residual accretion rate that can be tolerated before the force-free approximation breaks down and inertial effects become important in an accreting environment during quiescence. We consider two mechanisms that modify the magnetic dipole braking torque away from its standard form: magnetospheric mass loading (Section 2) and “crushing” by accretion ram pressure (Section 3). The results are compared with theoretical and observational limits on the residual accretion rate from a remnant disk during quiescence in Section 4. It is found that current observationally inferred upper limits of residual accretion rates do not conclusively rule out magnetospheric mass loading or crushing in four particular accreting pulsars: IGR J00291+5934, XTE J1751–305, SAX J1808.4–3658, and Swift J1756.9–2508. In Section 5 we present an alternative braking formula that should be used when the standard magnetic dipole picture cannot be applied. We reanalyze the data from IGR J00291+5934, XTE J1751–305, SAX J1808.4–3658, and Swift J1756.9–2508 to provide updated limits on \( B_\star \).

\(^1\) The lever arm has length \( R_L \), whether the dipole is point-like (Ostriker & Gunn 1969) or extended (Melatos 1997), but the spindown law is modified in the latter case.
2. MAGNETOSPHERIC MASS LOADING

A rotation-powered pulsar magnetosphere is force-free provided that two conditions are met (Michel 1991; Beskin 2010): (i) the mechanical energy density is much less than the electromagnetic energy density and (ii) the charge density equals the Goldreich–Julian value required to sustain the motional electric field, \(2\pi \rho_0 \Omega \cdot B\), where \(\Omega\) is the angular velocity and \(B\) is the local magnetic field strength, everywhere except in thin “vacuum gaps” in the inner and/or outer magnetosphere. Without accretion, pair production in the vacuum gaps guarantees that there is enough plasma to satisfy condition (ii) in the systems of interest here. The force-free magnetosphere carries a self-consistent, relativistic conduction current density \(2\pi \rho_0 \Omega \cdot Bc\), which is comparable to the displacement current density at \(R_L\) and spins down the star via a magnetic dipole braking torque \(\propto B^2_B R^4_B R^2_S \Omega^2 \propto B^2_B R^4_B \gamma^2\), with the effective lever arm \(R_N = R_L\) (Bucciantini et al. 2006).

The dominance of electromagnetic stresses (condition (i)) can be expressed in terms of the magnetization parameter (Michel 1991)

\[
\sigma = \frac{eB_s R^3_S \Omega^2}{4mc^3 \left(\frac{n}{n_{GJ}}\right)^{-1}},
\]

where \(n\) denotes the plasma number density and \(n_{GJ} = 2\pi \rho_0 \Omega \cdot B/e\) is the Goldreich–Julian value. Force-free conditions apply for

\[
\sigma > \gamma > 1,
\]

where \(\gamma\) is the Lorentz factor of the gap-accelerated magnetospheric plasma. The left-hand side of Equation (2) equals \(eB^2 / \mu_0 \rho r^3 mc^3\), the ratio of the Poynting flux to the mechanical energy flux, up to a factor of order unity. Given the standard scaling \(\sigma / \gamma \propto r^{-3}\) as a function of radius \(r < R_A\), with \(n \propto B \propto r^{-3}\) and \(\gamma \approx\) constant, it is enough to have \(\sigma / \gamma > 1\) at \(r = R_L\) in order for the magnetosphere to be force-free everywhere in the region \(r < R_A\).

Now suppose that residual accretion loads the magnetosphere all the way down to the surface with excess plasma, with number density \(n_a\). Three-dimensional simulations show that leakage into the magnetosphere is facilitated by tongue-like accretion streams and two-stream instabilities, which occur even when the system is in the magnetocentrifugal (propeller) regime (Romanova et al. 2008). The rate of leakage increases with magnetic obliquity (Romanova et al. 2003). Equations (1) and (2) then imply that the force-free approximation breaks down and inertial forces become important for

\[
\frac{n_a}{n_{GJ}(R_L)} > \frac{eB_s R^3_S \Omega^2}{4mc^3 \left(\frac{n}{n_{GJ}}\right)^{-1}}.
\]

Making the standard simplifying assumption (Ghosh & Lamb 1979) that residual accretion occurs spherically at roughly the free-fall speed, \(v \approx (GM_p/r)^{1/2}\) (where \(M_p\) is the neutron star mass), we can relate the accretion rate \(\dot{M}_a\) (mass per unit time) to the number density at radius \(r\) approximately by

\[
\dot{M}_a = 4\pi r^2 m_p n_a(r)(GM_p/r)^{1/2},
\]

where \(m_p\) is the proton mass. Then the maximum residual accretion rate that can be tolerated before force-free conditions break down, \(\dot{M}_f\), is given by

\[
\frac{\dot{M}_f}{M_E} = \frac{\epsilon_0 eB^2_S R_a \sigma_\gamma}{2\gamma m_p} \left(\frac{R_L}{R_a}\right)^{-3/2} \left(\frac{G M_p}{R_a c^2}\right)^{-1/2}
\]

\[
= 1.6 \times 10^{-10} \left(\frac{\Omega}{10^3 \text{ rad s}^{-1}}\right)^{9/2} \left(\frac{B_s}{10^8 \text{ G}}\right)^2 \left(\frac{\gamma}{10^6}\right)^{-1},
\]

where \(\sigma_\gamma\) is the Thomson cross-section, Equation (6) follows from (5) for the canonical values \(M_a = 1.4 M_\odot\) and \(R_a = 10 \text{ km}\), and we normalize by the Eddington rate with unit radiative efficiency, \(M_E = 4\pi GM_p m_p/\sigma T\). Equation (6) defines a relatively low accretion rate. We ask how low in terms of plausible models of residual accretion in Section 4.

The Lorentz factor \(\gamma\) in Equation (6) depends on the extent to which the parallel electric fields inside the vacuum gaps are shorted out. In turn, this depends on the detailed physics of the instabilities and diffusion processes controlling mass loading and is difficult to predict from first principles. If poisoning is effective, one obtains \(\gamma \approx 1\), i.e., weak acceleration like in the equatorial “dead zone” (Luo & Melrose 2007). If poisoning is ineffective, one obtains \(\gamma \approx \gamma_0 / \kappa\), where \(\gamma_0 \approx 10^7\) is the Lorentz factor of the primary beam and \(\kappa\) is the multiplicity of the pair production process, with \(10 \lesssim \kappa \lesssim 10^4\) for inverse-Compton- and curvature-triggered cascades (Hibschman & Arons 2001). We do not express a preference for either regime here and leave the \(\gamma^{-1}\) scaling in (6) for the reader’s convenience. We emphasize that gap poisoning enters the problem only in this indirect sense, through its effect on \(\gamma\) and hence \(\dot{M}_f\); it does not modify the spindown torque directly. We mention for completeness that poisoning can conceivably work in the opposite sense too. Mitrofanov (1990) and Mitrofanov & Sagdeev (1991) predicted that an interstellar comet passing through a dead pulsar’s magnetosphere may short-circuit the outer gaps and initiate a transient pair cascade, which may trigger a gamma-ray burst.

3. MAGNETOSPHERIC CRUSHING

The ram pressure of the residual accretion can also disrupt the operation of a rotation-powered pulsar by crushing its magnetosphere. Let the Alfvén radius \(R_A\) be the distance, where the electromagnetic momentum flux in the magnetosphere stands off the accretion flow. There are three crushing scenarios: (i) \(R_A > R_L\): the classical, force-free magnetosphere is undisturbed, the electromagnetic torque is exerted at \(R_L\), and the standard magnetic dipole braking formula applies; (ii) \(R_L < R_A < R_L\): there is still an undisturbed portion of the magnetosphere just above the stellar surface where vacuum gaps can form and a Poynting-flux-dominated wind is launched, but the outer magnetosphere (including at \(r \approx R_L\)) is distorted away from its normal structure so the effective lever arm and hence the magnetic dipole braking formula are modified; and (iii) \(R_L < R_A\): the magnetosphere is completely disrupted and the object cannot function as a rotation-powered pulsar.

Typically \(R_A\) is determined by balancing the magnetic pressure of a static dipole against the ram pressure of matter

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5. A more realistic (and difficult) calculation involving tongue-like streams implies higher \(n_a\) and more mass loading locally, so Equation (4) is conservative from the perspective of force-free breakdown.
falling at the free-fall speed, viz.

\[ R_A = \mu^{1/7}(GM_\text{a})^{-1/7}M_\text{a}^{-2/7}, \]  

(7)

up to a factor of order unity (Ghosh & Lamb 1979; Hartman et al. 2011; Melatos & Priymak 2014). To crush the magnetosphere at least partially, one requires \( R_A < R_L \) (scenarios (ii) or (iii) above), which occurs when the residual accretion rate \( M_\text{a} \) exceeds a threshold of \( M_\text{L} \) given from Equation (7) by

\[ \frac{M_\text{L}}{M_\text{E}} = \frac{3}{8} \frac{\Omega}{10^5 \text{rad s}^{-1}} \left( \frac{B_\text{c}}{10^8 \text{G}} \right)^3. \]

The question of whether the magnetosphere is crushed can be asked in another related way: is the pulsar’s spindown luminosity \( L_{\text{SD}} \) (carried predominantly by a Poynting-flux-dominated wind beyond \( R_L \)) high enough to blow away the residual accretion flow? The momentum flux transported by the wind, \( L_{\text{SD}}/(4\pi r^2c) \), scales \( \propto r^{-3} \) for \( r > R_L \), whereas the accretion ram pressure scales \( \propto r^{-3} \) hence the wind blows away the accreting gas everywhere beyond \( R_L \), provided that \( L_{\text{SD}}/(4\pi r^2c) \) exceeds the accretion ram pressure at \( R_L \). The threshold accretion rate in this scenario, \( M_{\text{SD}} \), satisfies \( M_{\text{SD}}/M_\text{L} \sim 1 \) and is a function of magnetic inclination angle as in standard oblique rotator formulae (Contopoulos & Spitkovsky 2006; Spitkovsky 2006).

In general, the threshold for crushing (9) is higher than that for mass loading (6). The reader may question how there can be enough matter in the magnetosphere for mass loading if the crushing threshold is not exceeded. In other words, a system where the accretion rate is lower than the threshold given by (9) should be in the “propeller” state and all matter should be ejected from the system. However, this is an open question. The simulations of Romanova et al. (2003, 2004, 2008) showed that matter is still accreted toward the star along “tongues” that penetrate the magnetosphere. Kelvin–Helmholtz and Rayleigh–Taylor instabilities as well as collisional cross-field diffusion (neglected in MHD models) allow leakage into the magnetosphere. In fact, any magnetic field line that connects the disk and the star can be a conduit, just like for aurorae on Earth and Jupiter. The aforementioned simulations are in a slightly different parameter regime so it is hard to be certain what happens, but mass loading without crushing is plausible.

4. RESIDUAL ACCRETION

The next task is to estimate the rate \( M_\text{s} \) at which residual accretion occurs in an accreting MSP during periods of quiescence. This comes down to examining theoretical predictions of, and observational upper limits on, the presence of a remnant disk. If \( M_\text{s} \) exceeds either \( M_{\text{fl}} \) or \( M_\text{L} \), then the standard magnetic dipole braking law is modified.

Theoretically, residual accretion disks should persist around accreting MSPs during quiescence for some of the same physical reasons that debris disks persist around isolated neutron stars after supernova fallback. The constituent asteroids of a neutron star’s fallback disk can undergo collisional migration, whereby inelastic collisions between the asteroids tend to broaden the disk and transport mass inward toward the neutron star, and Yarkovsky migration, whereby uneven heating of the asteroids by the neutron star tends to shrink the disk (Rubincam 1998; Cordes & Shannon 2008). The characteristic lifetime of a fallback disk undergoing collisional migration is \( \sim 3 \) Myr (proportional to the disk’s mass, density, radius, size of the asteroids, the root-mean-square speed of the asteroids, and the neutron star’s spin frequency), whereas the characteristic lifetime of a fallback disk undergoing Yarkovsky migration is \( \sim 10^{3.1} \) Myr (proportional to the disk radius and size of the asteroids, and inversely proportional to the neutron star’s luminosity, the inner radius of the disk, and the constituent asteroids’ drift rate \( \approx 10^{-2} \) AU Myr\(^{-1} \) near the light cylinder of a typical pulsar) (Cordes & Shannon 2008).

In addition, numerical simulations show that the accretion disk around a neutron star can be in one of three regimes: (1) \( M_\text{s}/M_{\text{co}} \gg 1 \), (2) \( M_\text{s}/M_{\text{co}} \approx 1 \), or (3) \( M_\text{s}/M_{\text{co}} \approx 0 \), where \( M_{\text{co}} = \eta \mu^2/(5\Omega R_\text{C}^5) \) is the accretion rate that puts the inner edge of the disk at \( R_{\text{co}} = (GM_\text{S}/\Omega^2)^{1/3} \), and \( \eta \lesssim 0.1 \) is the ratio between the azimuthal magnetic field strength (generated by star-disk differential rotation) and the poloidal magnetic field strength at the inner edge of the disk (D’Angelo & Spruit 2010, 2012). In regime (1) the neutron star accretes and spins up until the inner edge of the disk approaches \( R_{\text{co}} \) and the system enters regime (2). In regime (2) mass is prevented from accreting by a centrifugal barrier, but may not gain enough speed to be flung out of the system (Sunyaev & Shakura 1977; Spruit & Taam 1993). If \( M_\text{s} \) continues to fall, the system enters regime (3); on the other hand, if mass piles up in the inner regions of the disk until it overcomes the centrifugal barrier, accretion restarts and the disk radius approaches \( R_{\text{co}} \) (D’Angelo & Spruit 2010, 2011, 2012). In regime (2) the disk can vacillate between accreting and non-accreting states or it can become unstable, depending on \( M_\text{s} \) and the depth to which the star’s magnetic field penetrates the disk (D’Angelo & Spruit 2010). The disk often gets “trapped” just outside \( R_{\text{co}} \) even as \( R_{\text{co}} \) moves outwards as the star spins down, because \( M_\text{s} \) is too low (D’Angelo & Spruit 2012).

Fallback debris disks have been confirmed around neutron stars such as the magnetar 4U 0142+61 (Wang et al. 2006; but see also Wang et al. 2008 for an alternative explanation) and, more spectacularly, as a planetary system around the radio pulsar PSR B1257+12 (Wolszczan & Frail 1992). Recent searches for disks around other radio pulsars have not been successful (Wang et al. 2014). Once detected, optical and infrared (IR) spectra can be used to infer the temperature and hence the inner radius of the disk, although the latter inference relies on knowing the distance and albedo (Wang et al. 2006). The debris disk of 4U 0142+61, for example, has an inner temperature (inferred from the shape of the near-IR spectrum) that is comparable to the sublimation temperature of dust. This suggests that the inner radius of 4U 0142+61 may be set by X-ray destruction of dust, although the possibility that the radius might have been set by past interactions with the magnetosphere cannot be ruled out (Wang et al. 2006). Gas can continue to flow inwards through the sublimation radius and accrete. Circumbinary disks have also been detected in IR/near-IR around some low-mass X-ray binaries (Muno & Mauerhan 2006; Wang & Wang 2014). These disks may be remnants of fallback debris disks or may consist of matter lost from the low-mass companion (Muno & Mauerhan 2006). However, note that unambiguous evidence of the existence of accretion disks around AMSPs (e.g., double-peaked emission lines of the Balmer series; Coti Zelati et al. 2014)
has not been reported. According to radio ejection models (Burderi et al. 2003; Campana et al. 2004), a disk should not exist at all. Direct measurements of residual accretion rates for the four MSPs discussed in this paper are therefore impossible to set.

In SAX J1808.4–3658, Homer et al. (2001) proposed that the optical flux during quiescence can be used to infer a residual accretion rate of $M_{\text{res}, \text{obs}} \approx 9 \times 10^{-12} (M_\odot/0.05 M_\odot)^2 M_\odot$ yr$^{-1}$, where $M_\odot$ is the companion’s mass. However, Burderi et al. (2003), Campana et al. (2004), and Deloye et al. (2008) showed that in order to explain the amplitude of the modulation of the optical flux at the orbital period, the donor star must be irradiated by an external energy flux two orders of magnitude greater than the measured X-ray luminosity. These authors suggested that the irradiation comes from the spindown power output of SAX 1808.4–3658 itself. Furthermore, Deloye et al. (2008) and Wang et al. (2009, 2013) modeled the light curves, including the contribution of a putative residual accretion disk, and found that the disk contributes $\lesssim 30\%$ of the total optical emission. In another AMSP, XTE J1814–338, D’Avanzo et al. (2009) and Baglio et al. (2013) also found that a residual accretion disk contributes $\lesssim 20\%$ of the optical flux. Because of the difficulty of deriving tight constraints on $M_{\text{res}, \text{obs}}$ from optical flux measurements, we do not use this method.

An upper limit on the mass accretion rate of SAX J1808.4–3658 has been evaluated by Heinke et al. (2009) at $9 \times 10^{-12} M_\odot$ yr$^{-1}$ by averaging over outbursts and quiescence over 12 years, which presupposes that there is no mass loading or quenching. Alternatively, Wang et al. (2013) fitted the disk contribution to the optical emission of SAX J1808.4–3658 with a disk extending down to the light cylinder radius to obtain a temperature profile of $T(r) \approx 6.2 \times 10^4 (r/10^5 \text{ m})^{-1/2}$ K. At $r = R_d$, this gives $T = 1.8 \times 10^5$ K. Using the relation valid for a Shakura–Sunyaev accretion disk, $T_{\text{in}} = 3GMM_d/(8\pi\sigma R_d a_{\text{in}}^2)$ (Shakura & Sunyaev 1973), where $R_d$ and $T_{\text{in}}$ are the truncation radius of the accretion disk and the temperature at the truncation radius, we find an accretion rate of $\approx 7 \times 10^{-14} M_\odot$ yr$^{-1}$. In the quiescent state, the Shakura–Sunyaev solution does not hold and the accretion rate may be higher. To be conservative, we use the upper limits given by Heinke et al. (2009) (based on an average rate over outbursts and quiescence) in Table 1.

Heinke et al. (2009) analyzed X-ray data from XTE J1751–305 and obtained $M_{\text{res}, \text{obs}} \approx 6 \times 10^{-12} M_\odot$ yr$^{-1}$ as an upper limit (an average over outbursts and quiescence), although no optical spectrum has been detected to provide an independent check (Jonker et al. 2003; D’Avanzo et al. 2009).
Note. The “classic” surface polar magnetic field strength \(B_{s,\text{classic}}\) is obtained using the magnetic dipole spindown formula \(B_{s,\text{classic}} = 3.2 \times 10^{19} (\dot{\nu}/\nu^3)^{1/2}\) (lever arm \(R_s\)), whereas \(B_s\) and the accretion rate \(M\) are calculated using the magnetized accretion torque \((10)\). For comparison, we show the threshold accretion rates for magnetospheric mass loading (Section 2) and crushing (Section 3), assuming conservatively that \(B_{s,\text{classic}}\) is true. All accretion rates are normalized to the Eddington rate \(M_E = 3.1 \times 10^{-9} M_\odot \text{yr}^{-1} (M_\odot = 1.4 M_\odot)\).

## 5. Modified Braking Torque

We see from Table 1 that \(M_{a,\text{obs}}\) is higher than \(M_{\text{ff}}\) for all four transient accreting MSPs and higher than \(M_{\text{ff}}\) for Swift J1756.9–2508 (see Table 2). When the Mukherjee et al. (2015) estimate is used, \(M_{a,\text{obs}}\) is higher than \(M_{\text{ff}}\) for \(\gamma \gtrsim 10\) for all four transient accreting MSPs and higher than \(M_{\text{ff}}\) for Swift J1756.9–2508 and XTE J1751–305, with IGR J00291+5934 and SAX J1808.4–3658 on the borderline. The upper limits of quiescent accretion rates quoted in Section 4 suggest that magnetospheric mass loading and crushing are neither ruled out nor favored conclusively by observations. In particular, \(M_{a,\text{obs}}\) is at least \(\sim 10^4\) times \(M_{\text{ff}}\) for all four objects. However, without stronger observational constraints we cannot conclude that mass loading or crushing is operating in these four objects, since quiescent accretion rates can be \(\lesssim 10^3\) times lower than outburst rates (Heinke et al. 2009). Note that it is enough for either one of the mass loading and crushing thresholds to be exceeded for the force-free spindown formula to be modified. If either mass loading or crushing is activated, then the standard dipole spindown formula should not be employed when calculating \(B_s\) for these objects. The magnetosphere is typically mass-loaded and/or crushed, so the braking torque is more like that described by Ghosh & Lamb (1979) for magnetized accreting stars (lever arm \(R_s\)) than the standard magnetic dipole torque (lever arm \(R_s\)). Indeed, the fact that the four accreting MSPs in Table 1 undergo recurring outbursts is circumstantial evidence that they are near magnetocentrifugal equilibrium, as is the low upper limit on the frequency derivative of SAX J1808.4–3658 during outburst (Haskell & Patruno 2011).

Near magnetocentrifugal equilibrium, a transiently accreting MSP is thought to exist in a “quasi-propeller regime,” with \(R_A\) fluctuating in the band \(0.7 R_{\text{co}} \lesssim R_A \lesssim 1.2 R_{\text{co}}\) (Rappaport et al. 2004; Perna et al. 2006; Hartman et al. 2009, 2011). Magnetohydrodynamic simulations of the accretion disk in this state suggest that accretion onto the star either switches off completely or is inadequate to spin up the MSP, but the system does not enter the true propeller phase where accreting matter is flung out by centrifugal forces (Rappaport et al. 2004; Long et al. 2005; Perna et al. 2006). In addition, Romanova et al. (2008) showed that there is a regime of unstable accretion where the star accretes intermittently and that the star can oscillate between the unstable and stable regimes. If we assume that the star verges on the propeller phase, with \(R_A \approx 1.2 R_{\text{co}}\) (Rappaport et al. 2004), analysis of the torques operating on the star has shown that the spindown rate \(\dot{\Omega}\) is approximated by

\[
\dot{\Omega} = (1 - \omega)M_a (GM_A R_A)^{1/2}, \quad (10)
\]

where \(\omega = (R_A/R_{\text{co}})^{3/2}\) is the fastness parameter and \(I\) is the moment of inertia of the accreting star (Ghosh & Lamb 1979; Rappaport et al. 2004; Melatos & Priymak 2014). We can then solve Equation (10) simultaneously with the condition \(R_A = 1.2 R_{\text{co}}\) to find \(B_s\) and \(M_a\). This gives us not only an updated value of \(B_s\), but also a consistency check on \(M_a\) for comparison with Table 2.

We present \(M_a\) and \(B_s\) for the four transient MSPs in Table 2. We also list the spin frequency \(\nu\), the quiescent spin frequency derivative \(\dot{\nu}\), the “classically derived” surface magnetic field \(B_{s,\text{classic}}\), and \(B_s\) (assumption magnetic dipole braking with lever arm \(R_s\)), and the threshold accretion rates for the magnetospheric mass loading and crushing \((M_{\text{ff}}\) and \(M_{\text{ff}}\) respectively), assuming the surface magnetic field is indeed \(B_{s,\text{classic}}\). The quiescent frequency derivatives are hard to pinpoint with certainty with existing X-ray timing data from the Rossi X-ray Timing Explorer (RXTE). We use the estimates given by Hartman et al. (2008), Patruno (2010), Patruno et al. (2010), and Riggio et al. (2011).

Table 2 leads to two main conclusions. First, the accretion rates inferred from (10) with \(R_A = 1.2 R_{\text{co}}\) are higher than needed for poisoning and crushing to occur, so the use of Equation (10) is justified a posteriori.3 Secondly, we find \(B_s < B_{s,\text{classic}}\) for all four objects, with \(0.34 \lesssim B_s/B_{s,\text{classic}} \lesssim 0.58\). From Tables 1 and 2, \(M_a\) for IGR J00291+5934 and XTE J1751–305 are one order of magnitude higher than the quiescent \(M_{a,\text{obs}}\) estimate (Table 1). On the other hand, we find \(M_a \approx M_{a,\text{obs}}\) for Swift J1756.9–2508. \(M_a\) for SAX J1808.4–3658 is either approximately equal to \(M_{a,\text{obs}}\) or two orders of magnitude higher, depending on which estimate is used.

How easy is it to adjust the system parameters to make \(M_a\) agree with \(M_{a,\text{obs}}\)? For XTE J1751–305, one would need to set \(R_A = 2.3 R_{\text{co}}\) to give \(M_a/M_{\text{ff}} = 1.9 \times 10^{-3}\) to agree with \(M_{a,\text{obs}}\). For IGR J00291+5934, we need \(R_A = 2.7 R_{\text{co}}\) to get \(M_a/M_{\text{ff}} = 0.8 \times 10^{-3}\). Namely, such values of \(R_A\) take the system far out of the quasi-propeller regime and the star cannot be a transient accreting MSP, so we consider these values unlikely. If one uses these values of \(R_A\) in (10), one finds \(B_s = 1.3 \times 10^8\) G for XTE J1751–305 and \(B_s = 7.5 \times 10^7\) G for IGR J00291+5934, which are close to \(B_{s,\text{classic}}\). This is pure coincidence, however, since the magnetic dipole and accretion torques are fundamentally different physically, e.g., they depend differently on \(\mu\). It must be noted also that the presence of thermal components in the X-ray spectra of IGR J00291

5 Using the newly derived \(B_s\) yields lower mass loading and crushing thresholds.

| Name             | \(\nu\) (Hz) | \(\dot{\nu}\) \((10^{-15} \text{ Hz s}^{-1})\) | \(B_{s,\text{classic}}\) \((10^4 \text{ G})\) | \(B_s\) \((10^4 \text{ G})\) | \(M_a/M_{\text{ff}}\) \((10^{-3})\) | \(M_{a,\text{obs}}/M_{\text{ff}}\) \((10^{-6})\) | \(M_a/M_{\text{ff}}\) \((10^{-4})\) |
|------------------|-------------|---------------------------------|--------------------------|-----------------|-----------------|---------------------|-----------------|
| IGR J00291+5934  | 598.89      | -3.0                            | 1.2                      | 0.7             | 13              | 90                  | 9.3             |
| XTE J1751–305    | 435.32      | -5.5                            | 2.6                      | 1.4             | 21              | 100                 | 15              |
| SAX J1808.4–3658 | 401         | -5.6 \times 10^{-1}            | 0.9                      | 0.5             | 2.1             | 8.3                 | 1.4             |
| Swift J1756.9–2508 | 182        | -2.0                            | 5.8                      | 2.0             | 5.8             | 9.8                 | 3.4             |

Table 2: Modified Magnetic Moments of Transient Accreting MSPs with \(\nu\) Measured During Quiescence
+5934 and XTE J1751–305 during quiescence (taken by XMM-Newton) is uncertain (Jonker et al. 2003; D’Avanzo et al. 2007, 2009; Heinke et al. 2009).

As an alternative approach, we can estimate the quiescent $M_b$ from the lowest X-ray flux with detected pulsations, like Mukherjee et al. (2015). Now instead of solving for $M_b$ and $R_A$ in (10) with $R_A = 1.2 \ R_{\text{co}}$, we solve for $\omega$ (or, equivalently, $R_A$) and $B_x$ with the new values of $M_b$ listed in the third column of Table 1. The results are shown in Table 3. We see again that $M_{b,\text{obs}}$ is higher than the mass loading threshold for all four objects and the crushing threshold for two out of four objects, but is borderline for IGR J00291+5934 and SAX J1808.4–3658. Because the values derived by Mukherjee et al. (2015) correspond to upper limits of accretion rates, we cannot state unequivocally that this means the magnetospheres of the AMSPs are crushed or mass-loaded; we can only state that these scenarios (particularly mass loading) are not ruled out. The accretion rate upper limits calculated by Mukherjee et al. (2015) gives values of $R_A/R_{\text{co}}$ that are fairly close to 1.2, i.e., the quasi-propeller regime (Rappaport et al. 2004), except for IGR J00291+5934 where we find $R_A/R_{\text{co}} = 1.6$. We note also that our values of $B_x$ fall within the ranges inferred by Mukherjee et al. (2015).

6. DISCUSSION

In this paper we argue that the standard magnetic dipole spindown formula overestimates the field strengths of transient accreting MSPs with $\dot{\nu}$ measured during quiescence, in particular IGR J00291+5934, XTE J1751–305, SAX J1808.4–3658, and Swift J1756.9–2508. Current observational estimates for the residual accretion rates can only set (relatively lax) upper limits, which neither rule out nor favor conclusively magnetospheric mass loading or crushing. If it transpires that residual accretion during quiescence does mass load or crush the magnetosphere, then the magnetosphere is distorted away from a force-free configuration and the torque lever arm is shortened from $R_L$ to $R_A$. Under these conditions, spindown during quiescence occurs due to a Ghosh–Lamb-like magnetized accretion torque with $R_A > R_{\text{co}}$ in the quasi-propeller regime (Rappaport et al. 2004; Perna et al. 2006; Hartman et al. 2009, 2011). Assuming $R_A = 1.2 \ R_{\text{co}}$, by way of illustration, we find $B_x < B_{x,\text{classic}}$ for all four MSPs (Table 2), i.e., the standard spindown formula overestimates $B_x$ by a factor of $\lesssim 3$. The accretion rate inferred thus is consistent with estimates (averaged over outbursts and quiescence) for SAX J1808.4–3658 and Swift J1756.9–2508, and higher by about an order of magnitude for IGR J00291+5934 and XTE J1751–305. Compared to $M_{b,\text{obs}}$ estimates of Mukherjee et al. (2015), our values are slightly larger for SAX J1808.4–3658 and Swift J1756.9–2508, one order of magnitude higher for IGR J00291+5934, and three times smaller for XTE J1751–305. The inferred $B_x$ values remain consistent with recycling-related scenarios of $\mu$ reduction, such as polar magnetic burial (Payne & Melatos 2004; Priyamk et al. 2011). Note that we do not derive a new spindown formula rigorously; our goal is simply to bring attention to the effects of residual accretion on the spindown of a transient accreting MSP. Nevertheless, Equation (10) is a good approximation to $\dot{\Omega}$ for $R_{\text{lever}} \approx R_A < R_L$, when inertial effects are important. We stress again that systematic uncertainties affect both the theoretical and observational facets of the problem.

One may ask why there is accretion at all during a transient accreting MSP’s quiescence. For $R_A \approx R_{\text{co}}$ (as discussed in Section 5, for example), propeller matter is not likely to gain sufficient speed to escape the system (Spruit & Taam 1993; Rappaport et al. 2004; D’Angelo & Spruit 2010). Rappaport et al. (2004) showed that the angular momentum given to the disk by the neutron star via the magnetosphere is transported outwards, so that matter at the inner radius of the disk does not acquire enough speed to escape, leading to a buildup of matter near $R_{\text{co}}$. Effectively, the inner disk radius is located just inside $R_{\text{co}}$, even for $R_A > R_{\text{co}}$ even for small accretion rates $M_b \lesssim 10^{-11} M_{\odot}\, \text{yr}^{-1}$ (Rappaport et al. 2004; Kluzniak & Rappaport 2007). Furthermore, Lovelace et al. (1999) suggested that the effective Alfvén radius as opposed to the nominal Alfvén radius given by (7), depends on $\dot{\Omega}$ as well and wanders around $R_{\text{co}}$, stochastically or chaotically, triggered by small variations in $M_b$ or magnetic field configuration. In fact, even in the propeller regime some quasiperiodic accretion still occurs (Romanova et al. 2004). Thus, the disk-magnetosphere interaction is never enough to halt accretion completely and there is always some matter accreting inside $R_A$. In addition, as mentioned in Section 4, larger, neutral particles in the disk can undergo collisional or Yarkovsky migration into the magnetosphere (Cordes & Shannon 2008). In a slightly different physical regime than the one we discuss here, Romanova & Lovelace (2006) showed that the misalignment angle between the magnetic axis and the rotational axis affects how much matter can migrate into a solar-type protostar’s magnetosphere. Repeating their simulation for an accreting MSP may yield interesting results.

Incidentally, other mechanisms may also disrupt the magnetosphere and modify the braking torque. For example, the pulsar is encased in a conducting cage of accreting plasma. Even if the magnetosphere is not crushed (i.e., $R_A > R_L$), the cage reflects the low-frequency, large-amplitude electromagnetic or magnetohydrodynamic wave in the Poynting-flux.
dominated wind inside the accretion shock (Kundt & Krotscheck 1980; Coroniti 1990; Melatos & Melrose 1996; Skjærbaek et al. 2005; Amano & Kirk 2013). This is analogous to sealing an antenna inside a partially reflecting conducting box. If the cage has large inertia, the reflected wave bounces back onto the pulsar, modifying the spindown torque away from its standard magnetic dipole form.

In addition to residual accretion from a remnant disk, Bondi–Hoyle accretion also occurs as the pulsar travels through the interstellar medium. This serves as an important sanity check on the quenching mechanisms described in Sections 2 and 3 because we know that isolated MSPs are routinely detected as radio sources and often show other evidence for a functioning pulsar machine, e.g., the relativistic wind in the Hα bow shock nebula around PSR J0437–4715 (Bell et al. 1995). Hence vacuum gap poisoning (although not necessarily magnetospheric crushing or mass loading) by accretion of the interstellar medium is ruled out observationally in such objects. The Bondi–Hoyle accretion rate for a pulsar with speed \( V_b \ll V_{th} \), where \( V_{th} \) is the thermal speed in the interstellar medium and \( V_* \) is the sum of orbital (binary) and translational (kick) velocity components, is given by

\[
\frac{\dot{M}_{ISM}}{\dot{M}_E} = n_{ISM} R_b \sigma_T \left( \frac{V_b}{c} \right)^3 \left( \frac{GM_*}{R_{bc} c^2} \right)
\]

(11)

\[= 3.8 \times 10^{-9} \left( \frac{n_{ISM}}{1 \, \text{cm}^{-3}} \right) \left( \frac{V_b}{10^7 \, \text{km} \, \text{s}^{-1}} \right)^3,
\]

(12)

where \( n_{ISM} \) is the proton number density in the interstellar medium. Comparing (12) with (6) and (9), we see that Bondi–Hoyle accretion is unlikely to crush the magnetosphere. Interestingly, it is borderline for poisoning certain objects; see also Cordes & Shannon (2008) and references therein. We stress again that gap poisoning affects the radio emission but it does not affect the spindown torque (except indirectly through \( \gamma \) is Equations (5) and (6).

Recently, Mukherjee et al. (2015) estimated the minimum and maximum surface field strengths of 14 accreting MSPs as follows. The minimum \( B_* \) is found by setting the disk truncation radius (\( \approx R_{A,up} \) to a boundary layer correction factor of order unity) equal to \( R_{bc,up} \) with \( M_b \) given by the maximum pulsating X-ray flux in Equation (7). The maximum \( B_* \) is found by setting the truncation radius equal to \( R_{op} \), with \( M_b \) inferred from the minimum pulsating X-ray flux. This approach resembles ours leading to Equation (10) except that we match \( R_A = R_A(M_*, B_*) \) to some radius just outside \( R_{op} \) and assume that the MSP hovers between the accreting and propeller regimes (Hartman et al. 2011; D’Angelo & Spruit 2012). Our calculated values of \( B_* \) fall within the ranges obtained by Mukherjee et al. (2015) and are lower than those obtained purely from quiescent spindown as Mukherjee et al. (2015) found independently. This agreement reinforces our argument that quiescent spindown cannot be used naively to estimate \( B_* \). Note that only one mechanism (magnetospheric mass loading or crushing) needs to be activated to modify the spindown torque away from classical dipole expectation.

\[5\] If the wind transitions from a Poynting- to a kinetic-dominated outflow inside the termination shock, the shock has the capacity to emit strongly in X-rays (Kennel & Coroniti 1984; Melatos & Melrose 1996; Chatterjee et al. 2007). This possibility and its implications for pulsar wind physics deserve further study in the context of accreting millisecond pulsars like SAX J1808.4–3658.

In the future, it would be worth looking for direct observational signatures of non-force-free magnetospheres in transient accreting MSPs. However, it remains to be seen whether such signatures can be interpreted unambiguously. For example, there have already been detections during quiescence of sinusoidal modulations of the optical flux from the companion of SAX J1808.4–3658 (Deloye et al. 2008; Wang et al. 2009), whose photometric maxima occur whenever the neutron star is between the companion and the observer. Homer et al. (2001) originally interpreted the modulations as emission from a non-irradiated accretion disk truncated at the corotation radius. More recently, however, it has been argued that the neutron star switches on during quiescence as a rotation-powered pulsar whose relativistic wind irradiates one hemisphere of the companion (Burderi et al. 2003; Di Salvo & Burderi 2003), although such irradiation (by a Poynting-flux-dominated outflow) occurs whether or not the star is a pulsar. By analyzing the spin distributions of MSPs, Papitto et al. (2014) found that there is a 90% probability that accreting MSPs and eclipsing rotation-powered MSPs (rotation-powered MSPs that show irregular eclipses in their radio emission caused by matter irradiated away from the companion by the pulsar; Roberts 2013) belong to the same population. A number of MSPs in close binary systems are active as radio pulsars which emit winds that prevent the formation of accretion disks (Roberts 2013). Recently, one of these systems (IGR J18245–2452) has been observed to behave as an accreting MSP during an X-ray outburst and as a radio pulsar during quiescence (Papitto et al. 2013), indicating the tight link between AMSPs and radio pulsars. However, searches for radio pulsations in the four MSPs discussed in this paper have been carried out without success (Iacolina et al. 2010; Papitto et al. 2014). There are many reasons why this might be so, e.g., beaming. A neutron star can act like a rotation-powered pulsar electro-dynamically (with a Poynting-flux-dominated wind flowing out from the light cylinder and carrying most of the spindown luminosity) without being a magnetospheric radiation source, cf. Luo & Melrose (2007). By the same token, a neutron star can heat its companion without switching on as a rotation-powered pulsar; an accretion-dominated magnetosphere carries an outward-directed Poynting flux even when the Goldreich–Julian current system is disrupted. Hence the optical modulations observed from SAX J1808.4–3658 have several valid interpretations. More multiwavelength studies are needed to clarify the situation.

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5 No radio emission, pulsed or otherwise, has been detected during outbursts either (Tudose et al. 2008).
