SU(2N_F) hidden symmetry of QCD

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Recently a global SU(4) ⊃ SU(2)_L × SU(2)_R × U(1)A symmetry of the confining Coulombic part of the QCD Hamiltonian has been discovered with N_F = 2. This global symmetry includes both independent rotations of the left- and right-handed quarks in the isospin space as well as the chiralspin rotations that mix the left- and right-handed components of the quark fields. It has been suggested by lattice simulations, however, that a symmetry of mesons in the light quark sector upon the quasi-zero mode truncation from the quark propagators is actually higher than SU(4), because the states from a singlet and a 15-plet irreducible representations of SU(4) are also degenerate. Here we demonstrate that classically QCD, ignoring irrelevant exact zero mode contributions, has a SU(2N_F) symmetry. If effects of dynamical chiral symmetry breaking and of anomaly are encoded in the same near-zero modes, then truncation of these modes should restore a classical SU(2N_F) symmetry. Then we show in a Lorentz- and gauge-invariant manner emergence of a bilocal SU(4) × SU(4) symmetry in mesons that contains a global SU(4) as a subgroup upon truncation of the quasi-zero modes. We also demonstrate that the confining Coulombic part of the QCD Hamiltonian has this bilocal symmetry. It explains naturally a degeneracy of different irreducible representations of SU(4) observed in lattice simulations.

I. INTRODUCTION

A number of unexpected QCD phenomena has been observed recently in lattice simulations. A response of mesons with spins J = 0, 1 to an artificial subtraction of the lowest-lying modes of the overlap Dirac operator from valence quarks has been studied in dynamical N_F = 2 simulations [1, 2]. A surprising large degeneracy of mesons, that is larger than the SU(2)_L × SU(2)_R × U(1)A symmetry of QCD within the perturbation theory (ignoring the U(1)A anomaly) has been discovered in this paper we always drop a U(1)A symmetry, that is essential for our discussion. Apriori one expects that such a procedure would remove the chiral symmetry breaking dynamics from hadrons, because the quark condensate of the vacuum is directly related to a density of the near-zero modes of the Dirac operator via the Banks-Casher relation. Obviously, this truncation deforms QCD and the quark field becomes nonlocal in configuration space. The gluodynamics is kept intact. Such truncation is a gauge-invariant procedure and does not violate the Lorentz-invariance, because each eigenvalue of the Dirac operator in a given gluonic background is a Lorentz scalar and one does not affect the Lorentz transformation properties of the quark field.

One expects that after truncation correlators of operators that are connected with each other via the SU(2)_L × SU(2)_R transformations would become identical. If hadrons survive this "surgery", then masses of chiral partners should be equal. It has turned out that a very clean exponential decay of correlators was observed in all J = 1 mesons. This implies that confined bound states survive the truncation [1]. In the J = 0 sector, while all J = 0 correlators become identical, the ground states disappear from the spectrum, because there is no exponential decay of the corresponding correlators: The quasi-zero modes are crucially important for the very existence of the (pseudo) Goldstone bosons, which is not surprising.

It has also been found that the truncation restores in hadrons not only SU(2)_L × SU(2)_R symmetry, that is broken in QCD dynamically via the quark condensate, but also a U(1)A symmetry, which is broken anomalously. From this fact one concludes that the same lowest-lying modes of the Dirac operator are responsible for both SU(2)_L × SU(2)_R and U(1)A breakings which is consistent with the instanton-induced mechanism of both breakings [3–8].

However, not only a degeneracy within the SU(2)_L × SU(2)_R and U(1)A multiplets was detected. A larger degeneracy that includes all possible chiral multiplets of the J = 1 mesons was observed, which was completely unexpected. This larger degeneracy implies a symmetry that is larger than SU(2)_L × SU(2)_R × U(1)A.

A symmetry group that drives this degeneracy has been reconstructed in Ref. [9], that is SU(4) ⊃ SU(2)_L × SU(2)_R × U(1)A. Transformations of this group "rotate" the fundamental vector (u_L, u_R, d_L, d_R)^T and include both independent rotations of the left- and right-handed quarks in the isospin space as well as rotations in the chiralspin space that mix the left- and right-handed components of the quark fields. This symmetry implies that there are no magnetic interactions between quarks and a meson represents a dynamical quark-antiquark system connected by a confining electric field. Such a system was interpreted as a dynamical QCD string and the SU(4) symmetry was identified to be a symmetry of confinement.

The SU(4)-transformation properties of the ¯qq operators have been studied in Ref. [10]. It has also been found where this symmetry is hidden - the γ_0-part of the quark-
The gluon interaction term is a \( SU(4) \)-singlet. This part of the Lagrangian is responsible for interaction of quarks with the chromo-electric field. Interactions of quarks with the chromo-magnetic field are not \( SU(4) \)-invariant and transform as a 15-plet of \( SU(4) \). It is a generic property of any local gauge-invariant theory. With \( N_F \) light flavors the symmetry group is obviously \( SU(2N_F) \).

The role of this symmetry in hadrons can be seen using the Hamilton language \([10]\). In Coulomb gauge the QCD Hamiltonian \([11]\) consists of a few terms: a gauge field dynamics, a quark field term, an interaction between the quark field and the chromo-magnetic field, and the "Coulombic" part that represents an instantaneous chromo-electric interaction between the color-charge densities located at different spatial points. The color-charge density operator includes both the charge density of the gluonic field, that is obviously independent from quark isospin and its chirality, as well as the quark charge density. Since the latter is a \( SU(4) \)-singlet, the whole "Coulombic" part of the QCD Hamiltonian is also a \( SU(4) \)-singlet. Note that while the Hamiltonian description in Coulomb gauge is not a covariant description, the observable color-singlet gauge-invariant quantities are Lorentz-invariant. The Hamiltonian provides a very convenient description of a system in its rest-frame.

With the static color sources the Coulombic part of the Hamiltonian implies a confining linear potential \([12]\). The \( SU(4) \) symmetry can be viewed as a symmetry of a confining dynamical QCD string in the light quark sector.

There is another implication from this. The interaction of quarks with the chromo-magnetic field, that explicitly breaks the \( SU(4) \) symmetry, is located in the confinement regime only in the near-zero mode zone and is responsible for all symmetries \( SU(2)_L \times SU(2)_R, U(1)_A \) and \( SU(4) \) breakings. A truncation of the near-zero modes filters out a confining dynamics in the system.

Meanwhile emergence of \( SU(4) \) has been confirmed in the \( J = 2 \) meson sector \([13]\) and in baryons \([14]\).

The global \( SU(4) \) symmetry cannot explain, however, a degeneracy of the \( J = 1 \) states \( \rho, \rho', \omega, \omega', a_1, h_1, b_1 \), that form a \( SU(4) \) 15-plet, and of a singlet \( f_1 \). It has been suggested in Ref. \([10]\) that this degeneracy, if confirmed, should imply existence of a larger symmetry that includes \( SU(4) \) as a subgroup. Cohen has found very recently that no higher symmetry is phenomenologically acceptable, that would connect local quark bilinears from the 15-plet and singlet of \( SU(4) \) within one and the same irreducible representation of some higher group \([15]\).

Here we show that classically QCD has, excluding irrelevant exact zero mode contributions, a \( SU(2N_F) \) symmetry. Chiral symmetry spontaneous breaking is encoded in the near-zero modes of the Dirac operator, as it follows from the Banks-Casher relation. If effects of anomaly are also encoded in the same near-zero modes, then truncation of the near-zero modes should restore a classical \( SU(2N_F) \) symmetry. Given this symmetry we demonstrate in a gauge- and Lorentz-invariant manner emergence at \( N_F = 2 \) of a bilocal \( SU(4) \times SU(4) \) symmetry in mesons and of a trilocal \( SU(4) \times SU(4) \times SU(4) \) symmetry in baryons upon elimination of the quasi-zero modes of the Dirac operator. We also show that the confining part of the QCD Hamiltonian is actually invariant not only under global \( SU(4) \) transformations, but is also a singlet under bilocal \( SU(4) \times SU(4) \) transformations. An irreducible dim=16 representation of \( SU(4) \times SU(4) \) combines both the \( SU(4) \)-singlet and the 15-plet into one irreducible representation, which explains a degeneracy of \( f_1 \) with the 15-plet mesons. This symmetry implies invariance upon independent instantaneous \( SU(4) \) transformations of the quark fields at different space points \( \mathbf{x} \) and \( \mathbf{y} \) and is intrinsically nonlocal and cannot be represented by local composite operators.

## II. Global and Space-Local Symmetries of the Coulombic Interaction for Massless Quarks

Consider the quark-gluon interaction part of the QCD Lagrangian with \( N_F \) massless flavors in Minkowski spacetime:

\[
\bar{\Psi}(\mathbf{1}_F \otimes i\gamma^\mu D_{\mu})\Psi = \bar{\Psi}(\mathbf{1}_F \otimes i\gamma^0 D_0)\Psi + \bar{\Psi}(\mathbf{1}_F \otimes i\gamma^i D_i)\Psi .
\]

The first term describes an interaction of the quark charge density \( \rho(x) = \bar{\Psi}(x)\gamma^0\Psi(x) \) with the chromo-electric part of the gluonic field. The second term contains an interaction of the spatial current density with the chromo-magnetic field.

The chromo-electric part of the interaction Lagrangian is invariant under a global and space-local (\( \mathbf{x} \)-dependent) \( SU(2)_{CS} \supset U(1)_A \) and \( SU(2N_F) \) transformations of the quark field. The \( SU(2)_{CS} \supset U(1)_A \) transformations are defined as

\[
\Psi \rightarrow \Psi' = e^{i(\mathbf{1}_F \otimes \xi_5)}\Psi .
\]

with the following generators

\[
\Sigma = \{ \gamma^0, i\gamma^5\gamma^0, -\gamma^5 \} ,
\]

that form an \( SU(2) \) algebra

\[
[\Sigma^i, \Sigma^j] = 2i\epsilon^{ijk} \Sigma^k .
\]

An imaginary three-dimensional space in which these rotations are performed is referred to as the chiral spin (CS) space \([9,11]\). Upon the chiralspin rotations the right- and left-handed components of the fermion fields get mixed. It is similar to the well familiar concept of the isospin space: The electric charges of particles are conserved.

\footnote{The latter degeneracy requires, however, a confirmation, because an effective mass plateau for the \( f_1 \) state is not convincing \([2]\).}
quantities, but rotations in the isospin space mix particles with different electric charges.

If the rotation vector $\varepsilon$ is space-time independent, then this transformation is global. If the rotation parameters are different at different space points $x$, $\varepsilon(x)$, then we refer to such a rotation as a space-local transformation. The first term in eq. (1) is invariant with respect to both global and space-local $SU(2)_{CS}$ transformations.

When we combine the $SU(2)_{CS}$ rotations with the chiral $SU(N_F)_L \times SU(N_F)_R$ transformations into one larger group, then we arrive at a $SU(2N_F)$ group. For example, in case of two flavors the $SU(4)$ transformations

$$\Psi \rightarrow \Psi' = e^{i\varepsilon \cdot \mathbf{T}/2} \Psi,$$

are defined through the following set of 15 generators:

$$\{ (\tau^a \otimes 1_D), (1_F \otimes \Sigma^i), (\tau^a \otimes \Sigma^i) \}.$$  

With the space-time independent $(2N_F)^2 - 1$-dimensional rotation vector $\varepsilon$ the corresponding symmetry is global, while with the space-dependent rotation $\varepsilon(x)$ it is space-local.

The magnetic part of the interaction Lagrangian does not admit this higher symmetry and is invariant only with respect to global $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$ chiral transformations.

The higher symmetry of the electric part of the interaction Lagrangian is generic for any local gauge-invariant theory and has significant implications for confinement in QCD.

### III. Zero Modes of the Dirac Operator and Symmetries of Euclidean QCD

Enlarged symmetry, reviewed in the Introduction, is obtained in lattice simulations upon subtraction of the near-zero modes of the Dirac operator. This means that this symmetry should be encoded in the Euclidean QCD once the near zero modes are removed. In this section we discuss symmetry properties of the nonperturbatively defined QCD in Euclidean space-time and demonstrate that indeed once the near zero modes, that are responsible for both spontaneous and anomalous chiral symmetries breakings, are subtracted the QCD partition function is $SU(2N_F)$ symmetric. As an introduction to the Euclidean field theory we recommend the textbook [10].

The Lagrangian in Euclidean space-time with $N_F$ degenerate massive quarks in a given gauge configuration is:

$$L = \bar{\Psi}(x)(\gamma_{\mu}D_{\mu} + m)\Psi(x),$$

with

$$D_{\mu} = \partial_{\mu} + ig \frac{t^a}{2} A^a_{\mu},$$

where $A^a_{\mu}$ is the gluon field configuration and $t^a$ are the $SU(3)$-color generators.

In Euclidean space the Grassmannian fields $\Psi(x)$ and $\bar{\Psi}(x)$ are completely independent from each other. Different parts of the Lagrangian [14] have different symmetries. For example, the mass term $\bar{\Psi}(x)\Psi(x)$ is invariant under a $U(1)_A$ transformation

$$\Psi(x) \rightarrow e^{i\alpha} \Psi(x); \quad \bar{\Psi}(x) \rightarrow \bar{\Psi}(x)e^{-i\alpha}.$$  

At the same time the kinetic term, $\bar{\Psi}(x)(\gamma_{\mu}D_{\mu})\Psi(x)$, breaks this symmetry, because the $\gamma_5$ matrix anticommutes with all $\gamma_\mu$ matrices, $\gamma_5\gamma_\mu + \gamma_\mu\gamma_5 = 0$. The same is true with respect to the axial part of the $SU(N_F)_L \times SU(N_F)_R$ transformation.

What are symmetry properties of both parts under the $SU(2)_{CS}$ and $SU(2N_F)$ transformations? The Euclidean $\gamma_4$ coincides with the Minkowski $\gamma_0$ and $\gamma_5$ matrices in both spaces are equal. Then we can define Euclidean $SU(2)_{CS}$ transformations through generators that satisfy a $SU(2)$ algebra,

$$\Sigma = \{ \gamma^4, i\gamma^5 \gamma^4, -\gamma^5 \}.  

Combining the $SU(2)_{CS}$ generators with the $SU(N_F)$ flavor generators into a larger algebra like in eq. (6) we arrive at the Euclidean $SU(2N_F)$ transformations.

It is then obvious that the kinetic term in [14] breaks both symmetries. It is invariant only under flavor transformation $SU(N_F)$. It is important to understand the underlying reason why these symmetries are missing in the kinetic term. The reason is that these symmetries are absent for a quark that is "on-shell". The "on-shell" quark is described by the Dirac equation,

$$\gamma_{\mu}D_{\mu}\Psi_0(x) = 0.$$  

3 This definition of the chiral transformation in Euclidean space is consistent with the Lorentz $(SO(4))$- transformation properties of the $\Psi$ field, see for details ref. [12], and is used in the literature [13]. Very often another definition of the chiral transformation is given:

$$L = \bar{\Psi}(x)(\gamma_{\mu}D_{\mu} + m)\Psi(x),$$

$$\Psi(x) \rightarrow e^{i\alpha_5} \Psi(x); \quad \bar{\Psi}(x) \rightarrow \bar{\Psi}(x)e^{i\alpha_5},$$

which is inconsistent, however, with the $(SO(4))$-rotation properties of the field $\Psi(x)$, that transforms as $\Psi^\dagger(x)$. When calculating the fermion determinant the field $\Psi(x)$ is implicitly substituted through $\Psi^\dagger(x)$, so the fermion determinant and generating functional have correct Lorentz transformation properties, see for a transparent exposition ref. [12]. Consequently all real Euclidean lattice results are correct since they do not depend on a semantical issue what part of the Lagrangian above should be called chirally symmetric and what part - chiral symmetry breaking.
Its solution is traditionally called a zero mode. Zero modes are solutions of the Dirac equation with the gauge configurations of a nonzero topological charge. They are absent in gauge configurations with \( Q = 0 \). The difference of numbers of the left-handed and right-handed zero mode solutions is according to the Atiyah-Singer theorem fixed by the topological charge \( Q \) of the gauge configuration:

\[
    n_L - n_R = Q. \tag{12}
\]

For example, with a gauge configuration of \( Q = 1 \) there is only a left-handed zero mode and there is no right-handed zero mode solution \([19, 20]\). Some \( SU(2)_C \) transformations rotate the right-handed solution into the left-handed solution and vice versa. Consequently, the zero mode explicitly violates the \( SU(2)_C \) and \( SU(2N_F) \) symmetries. Similar analysis can be done for any \( SU(N) \) symmetries. Similar analysis can be done for any \( SU(N) \) symmetries. Some \( SU(3)_C \) transformations rotate the right-handed solution into the left-handed solution and vice versa. Consequently, the zero mode explicitly violates the \( SU(3)_C \) and \( SU(6F) \) symmetries. Similar analysis can be done for any \( SU(N) \) symmetries. Some \( SU(N) \) symmetries.

The Grassmannian fields \( \Psi(x) \) and \( \Psi^\dagger(x) \) are defined in the following standard way. The hermitian Dirac operator, \( i\gamma^\mu D_\mu \), in a given gluonic background has in a finite volume \( V \) a discrete spectrum with real eigenvalues \( \lambda_n \):

\[
    i\gamma^\mu D_\mu \Psi_n(x) = \lambda_n \Psi_n(x). \tag{14}
\]

The nonzero eigenvalues come in pairs \( \pm \lambda_n \), because

\[
    i\gamma^\mu D_\mu \gamma_5 \Psi_n(x) = -\lambda_n \gamma_5 \Psi_n(x). \tag{15}
\]

We can expand fields \( \Psi(x) \) and \( \Psi^\dagger(x) \) over a complete and orthonormal set \( \Psi_n(x) \):

\[
    \Psi(x) = \sum_n c_n \Psi_n(x), \quad \Psi^\dagger(x) = \sum_k \bar{c}_k \Psi^\dagger_k(x), \tag{16}
\]

where \( \bar{c}_k, c_n \) are Grassmannian numbers. For all eigenvectors with the nonzero eigenvalues \( \lambda_n \neq 0 \) we can replace

\[
    \gamma^\mu D_\mu \Psi_n(x) \rightarrow -i\lambda_n \Psi_n(x). \tag{17}
\]

This substitution effectively eliminates the \( \gamma^\mu D_\mu \) operator and replaces it with the Lorentz-scalar \( \lambda_n \). Then the partition function \([19]\) takes the following form

\[
    Z^V_{Q=0} = \int \prod_k d\bar{c}_k d c_n e^{\sum_k \sum_n \int d^4x \bar{c}_k c_n (\lambda_n + i m) \Psi^\dagger_k(x) \Psi_n(x)}. \tag{18}
\]

Now we can directly read-off symmetry properties of this partition function. Since this functional contains only a superposition of terms \( \Psi^\dagger_k(x) \Psi_n(x) \) it is precisely \( SU(2)_C \) and \( SU(2N_F) \) symmetric, because

\[
    (U \Psi_k(x))^\dagger U \Psi_n(x) = \Psi^\dagger_k(x) \Psi_n(x), \tag{19}
\]

where \( U \) is any unitary transformation from the groups \( SU(2)_C \) and \( SU(2N_F) \), \( U^\dagger = U^{-1} \). The exact zero modes, for which the equation \([19]\) does not hold, do not contribute to this partition function. We conclude that classically the Euclidean QCD in a finite volume \( V \) without the exact zero modes, i.e. in the \( Q=0 \) sector, is invariant with respect to both global and space-local \( SU(2N_F) \) transformations.

Quantization implies an integration over fields \( \Psi(x) \) and \( \Psi^\dagger(x) \). In this case the \( SU(1)_A \) symmetry is broken anomalously, because the measure \( D\Psi D\Psi^\dagger \) is not invariant upon a local \( U(1)_A \) transformation \([21]\). Since the \( U(1)_A \) is a subgroup of \( SU(2)_C \), the anomaly also breaks the \( SU(2)_C \) symmetry.

In the thermodynamic limit \( V \rightarrow \infty \) the otherwise finite lowest eigenvalues \( \lambda \) condense around zero, \( \lambda \rightarrow 0 \), and according to the Banks-Casher relation,

\[
    \lim_{m \rightarrow 0} <0|\bar{\Psi}(x)\Psi(x)|0> = -\pi \rho(0), \tag{20}
\]

provide a nonvanishing quark condensate in Minkowski space. Here a sequence of limits is important. First an infinite volume limit must be taken and only then a chiral limit. The quark condensate in Minkowski space-time breaks all \( SU(2)_C \), \( SU(2N_F)_L \times SU(2N_F)_R \) and \( SU(2N_F) \) symmetries to the vector flavor symmetry \( SU(N)_V \). Consequently the new \( SU(2)_C \) and \( SU(2N_F) \) symmetries are broken both by the condensate and anomalously.

Effect of dynamical chiral symmetry breaking is encoded in the near-zero modes of the Dirac operator. If effects of anomaly are also encoded in the near-zero modes, as suggested e.g. by the instanton mechanism of both breakings \([22]\), then removal on lattice of eigenmodes with lowest \( \lambda \) should restore consequently not only chiral \( SU(N)_L \times SU(N)_R \) and \( U(1)_A \) symmetries, but also a larger \( SU(2N_F) \) symmetry, in agreement with observations reviewed in the Introduction.

In the thermodynamic limit, \( V \rightarrow \infty \), the \( Q = 0 \) partition function coincides, up to an inessential normalization factor, with the total QCD partition function taken at zero theta-angle, \( \theta = 0 \) \([23]\). It approaches the full QCD partition function rather fast, as \( 1/V \) \([24]\).
What we have described above is a symmetry of QCD in the $Q=0$ sector. What about symmetries in all other sectors? They all contribute to the QCD partition function at $\theta = 0$. We can work in any of them. In this case the partition function will contain the zero mode contributions. For example, in the case $Q=1$ there will appear terms that contain a zero mode eigenfunction: $\Psi_0^L(x)\Psi_0^{-1}(x)$ and $\Psi_0^R(x)\Psi_0^{-1}(x)$. They all explicitly break both $SU(2)_{CS}$ and $SU(2N_F)$ symmetries, because in this case the relation (19) does not hold. However, these zero mode contributions are completely irrelevant in the Green functions and observables in the thermodynamic limit since they vanish as $1/V$. Consequently, in the limit $V \to \infty$ we will approach the same result in any sector.

Summarizing, if effects of anomalous and dynamical chiral symmetry breakings are encoded in the same near-zero modes, then removal of these modes should restore a hidden classical $SU(2N_F)$ symmetry.

We have discussed in this section a symmetry of QCD defined nonperturbatively. With the noninteracting fermions or within the perturbation theory there is no $SU(2)_{CS}$ symmetry, because for on-shell massless quarks chirality is a conserved quantum number. Any Feynman diagram can be continued from Minkowski to Euclidean space via the Wick rotation. Consequently, a symmetry of QCD within the perturbation theory ignoring the $U(1)_A$ anomaly is only $SU(N_F)_L \times SU(N_F)_R \times U(1)_A$.

**IV. SU(2N_F) × SU(2N_F) AND SU(2N_F) × SU(2N_F) × SU(2N_F) EMERGENT SYMMETRIES IN MESONS AND BARYONS**

We have considered above emergence of global and space-local $SU(2N_F)$ symmetries of a quark in a given gauge background. What symmetries should we expect in hadrons upon the quasi-zero modes elimination?

Hadron spectra are obtained from the correlation functions calculated with the gauge-invariant source operator. At each time slice "t" a meson correlator contains minimum the lowest Fock component consisting of valence quark and antiquark located at different space points $x$ and $y$. Both quark and antiquark interact with the same gauge configuration according to eq. (19). Consequently, all arguments of the previous section apply independently for both quark and antiquark. Since the $SU(2N_F)$ invariance is space-local, we can perform independent $SU(2N_F)$ rotations at points $x$ and $y$ with different rotation parameters. One then concludes that the meson correlation function with the quark-antiquark valence content has a bilocal $SU(2N_F) \times SU(2N_F)$ symmetry. A symmetry of higher Fock components is larger, but the whole correlator has a symmetry of the lowest quark-antiquark component. Obviously, averaging over gauge configurations will not change this symmetry property.

The same arguments apply to baryons and in this case we can expect a trilocal $SU(2N_F) \times SU(2N_F) \times SU(2N_F)$ symmetry.

**V. SU(2N_F) × SU(2N_F) BILOCAL SYMMETRY OF CONFINEMENT IN COULOMB GAUGE**

In previous sections we discussed a Lorentz- and gauge-invariant derivation of emergence of bilocal and trilocal symmetries of hadrons upon the quasi-zero mode elimination. A clear connection with confinement physics is missing in those derivations, however. To get an insight consider the QCD Hamiltonian in Coulomb gauge in Minkowski space [11].

$$H_{QCD} = H_E + H_B$$

$$+ \int d^3x \bar{\Psi}^L(x)[-i\alpha \cdot \nabla + \beta m] \Psi^L(x) + H_T + H_C, \quad (21)$$

where the transverse (magnetic) and Coulombic interactions are:

$$H_T = -g \int d^3x \bar{\Psi}^L(x) \alpha \cdot \frac{i}{\alpha} A^a(x) \Psi^L(x), \quad (22)$$

$$H_C = \frac{g^2}{2} \int d^3x \ d^3y \ J^{-1} \rho^a(x) F^{ab}(x, y) J \rho^b(y), \quad (23)$$

with $J$ being Faddeev-Popov determinant, $\rho^a(x)$ is a color-charge density and $F^{ab}(x, y)$ is a confining Coulombic kernel.

The fermionic and transverse (magnetic) parts of the Hamiltonian have at $m \to 0$ $SU(N_F)_L \times SU(N_F)_R$ and $U(1)_A$ global chiral symmetries. A symmetry of the confining Coulombic part is higher, however. It is not only invariant under $SU(N_F)_L \times SU(N_F)_R$ and $U(1)_A$ global chiral transformations, but is also a singlet with respect to bilocal $SU(2N_F) \times SU(2N_F)$ transformations. Indeed, the color charge density $\rho^a(x)$ at a space point $x$ is a singlet with respect to $SU(2N_F)$ rotations. The same is true for the color charge density at a space point $y$. However, the rotations of the fermion field at the space points $x$ and $y$ can be completely independent from each other, with different rotation parameters. Consequently, the Coulombic part of the QCD Hamiltonian is not only a singlet with respect to global $SU(2N_F)$ transformations, but is also invariant under independent $SU(2N_F)$ transformations of the fermion field at points $x$ and $y$.

We conclude that the Coulombic confining part of the QCD Hamiltonian has a bilocal $SU(2N_F) \times SU(2N_F)$ symmetry.
VI. IMPLICATIONS

An irreducible representation of the $SU(4) \times SU(4)$ group is 16-dimensional and is a direct sum of the 15-plet and singlet of $SU(4)$. Consequently, a direct prediction of this symmetry is a degeneracy of the $SU(4)$-singlet and of the $SU(4)$ 15-plet, in agreement with the lattice observations reviewed in the Introduction. Since this symmetry is bilocal it cannot be represented by local composite operators and our result is consistent with conclusions of Ref. 16.

A $N_F = 2$ model with the Hamiltonian $H_{\Sigma}$ has been solved in the past in Ref. 20. In that work a confining linear instantaneous potential has been assumed as a confining kernel $F^{ab}(x, y)$ and a large $N_c$ meson spectrum has been obtained upon solution of the gap and Bethe-Salpeter equations. At high meson spins, where effects of chiral symmetry breaking in the vacuum became irrelevant, the spectrum was observed to be highly degenerate and all parity-chiral multiplets of states of the same spin, namely $(0, 0), (0, 0), (1/2, 1/2)_a, (1/2, 1/2)_b, (0, 1) + (1, 0)$ had the same mass. This result was obtained both numerically and analytically. This degeneracy is nothing else but a $SU(4) \times SU(4)$ symmetry discussed above, because a dimension-16 irreducible representation of $SU(4) \times SU(4)$ is a direct sum $(0, 0) + (0, 0) + (1/2, 1/2)_a + (1/2, 1/2)_b + (0, 1) + (1, 0)$ of irreducible representations of the parity-chiral group. This result does not necessarily mean that confinement in QCD in the light quark sector is reduced to an instantaneous linear potential, but it does illustrate a generic symmetry property and implications of the Coulombic Hamiltonian. Namely, when the chiral symmetry breaking dynamics is switched off a spectrum reveals a bilocal $SU(4) \times SU(4)$ symmetry of the Hamiltonian.

VII. INTERPOLATION BETWEEN THE HEAVY QUARK AND CHIRAL LIMITS

In the heavy quark limit a matrix element of the charge density operator is diagonal in flavor and spin spaces and the Coulombic Hamiltonian has a nonrelativistic $SU(2N_F)_{SF} \times SU(2N_F)_{SF}$ spin-flavor symmetry. A spectrum is $SU(2N_F)_{SF} \times SU(2N_F)_{SF}$ symmetric. It is a symmetry of the nonrelativistic quark model.

In the chiral limit the charge density operator is diagonal in flavor and chirality spaces and the Coulomb Hamiltonian has a $SU(2N_F) \times SU(2N_F)$ symmetry discussed in previous sections. The same Coulomb Hamiltonian has in the opposite limits of QCD two different symmetries.

VIII. CONCLUSIONS

We have demonstrated the following points.

1. Classically QCD has, excluding irrelevant exact zero mode contributions, $SU(2)c_S$ and $SU(2N_F)$ symmetries. Since these symmetries are not present at the level of the QCD Lagrangian and are visible only when we subtract exact zero modes, we call them as hidden symmetries of QCD. These symmetries are broken in the real world by the axial anomaly and by the condensate.

2. Truncation of the near-zero modes that encode both dynamical chiral symmetry breaking and anomaly restores hidden classical symmetries of QCD.

3. In a Lorentz- and gauge-invariant manner we have shown that elimination of the near-zero modes of quarks leads to $SU(2N_F) \times SU(2N_F)$ and $SU(2N_F) \times SU(2N_F)$ symmetries in hadrons.

4. The confining Coulombic part of the QCD Hamiltonian in Coulomb gauge has a bilocal $SU(2N_F) \times SU(2N_F)$ symmetry. This symmetry implies a $SU(2N_F) \times SU(2N_F)$-symmetric spectrum in mesons and $SU(2N_F) \times SU(2N_F)$ symmetry in baryons in case when other parts of the Hamiltonian that break this symmetry become inessential. This property has been illustrated with a solvable relativistic confining model.

5. This bilocal symmetry of confinement is consistent with degeneracy of the $SU(4)$ singlet $f_1$ correlator with the $SU(4)$ 15-plet $\rho, \rho', \omega, \omega', h_1, a_1, b_1$ correlators observed on the lattice after subtraction of the quasi-zero modes of the Dirac operator.

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