The viscosity bound in string theory

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Abstract
The ratio of shear viscosity to entropy density \( \frac{\eta}{s} \) of any material in nature has been conjectured to have a lower bound of \( \frac{1}{4\pi} \), the famous KSS bound. We examine string theory models for evidence in favour of and against this conjecture. We show that in a broad class of models quantum corrections yield values of \( \frac{\eta}{s} \) just above the KSS bound. However, incorporating matter fields in the fundamental representation typically leads to violations of this bound. We also outline a program to extend AdS/CFT methods to RHIC phenomenology.

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1. Introduction

The AdS/CFT correspondence is one of the most profound developments not only in string theory but also in the study of quantum field theories in recent times. This holographic framework allows one to use simple gravitational calculations to understand the strong coupling behaviour of conformal gauge theories. Using this correspondence, Kovtun, Son and Starinets showed that the ratio of the shear viscosity to entropy density \( \frac{\eta}{s} \) for a large class of strongly coupled relativistic fluids satisfied

\[
\frac{\eta}{s} = \frac{1}{4\pi}.
\]  

(1)

They went further to conjecture that this value actually represents a lower bound on this ratio for any fluid in nature, the KSS bound \([1]\). While any known fluid respects this bound (by more than an order of magnitude), RHIC data seems to indicate that the strongly coupled quark-gluon plasma has \( \frac{\eta}{s} \sim 1/4\pi \) \([2]\). This triggered a great deal of work to better understand the bound and put it on a firmer footing. The KSS bound is frequently referred to as the 'quantum bound' \([3]\), since this leading-order result corresponds to a regime where the 't Hooft coupling \( \lambda = g^2_{YM} N_c \) and the number of colours \( N_c \) are both infinite. These limits are necessary in order to keep the curvature corrections and string-loop corrections small in the gravity calculations. Hence, it may seem rather curious that the RHIC plasma seems to yield a value for \( \frac{\eta}{s} \) quite close to (1) but it may point to some type of universal behaviour. Of course, the obvious question becomes can we compute \( 1/\lambda \) and \( 1/N_c \) corrections – which we will refer to as quantum corrections – to \( \frac{\eta}{s} \)?

This talk summarizes what is known from string theory about quantum corrections to \( \frac{\eta}{s} \). The punch-line is that in a wide class of models where (1) is the leading result, the quantum corrections are positive and so the bound is respected. However, a common feature to all of these models is the absence of matter fields in the fundamental representation, i.e., ‘no quarks.’ It is also found that in many models with fundamental matter, the leading corrections produce violations of the KSS bound. The implications of these findings will be discussed in the conclusion. This talk is based on \([5]-[8]\). For a more complete list of references, please refer to \([3]-[8]\).
2. Quantum corrections to $\eta/s$

For a 4d conformal gauge theory, the dual gravity action may be written schematically as

$$I = \frac{1}{2\ell_p^2} \int d^3 x \sqrt{-g} \left( \frac{12}{L^2} + R + L^2 \lambda_1 W^2 + L^4 \lambda_2 W^3 + L^6 \lambda_3 W^4 + \cdots \right). \quad (2)$$

Here, $R$ and $W$ denote the Ricci scalar and Weyl tensor, respectively for the 5d metric $g_{\mu\nu}$. Implicitly, we are working in a perturbative expansion with respect to $\ell_p/L$, the ratio the 5d Planck length to the AdS$_5$ curvature scale. The dimensionless parameters controlling the higher curvature corrections are expected to be $|\lambda_n| \sim (\ell_p/L)^{2n} \ll 1$. To describe a situation with a nonvanishing chemical potentials, one would also have to introduce a gauge field in the action. Further, there could be scalar fields as well and in principle, one needs to examine how these could affect the results. For the purpose of this talk, these fields will be ignored, as may be rigorously justified in certain approximations – see [5, 6]. The precise tensor contractions in $W^n$ as well as the values of $\lambda_n$’s are string theory inputs. When the (zero temperature) gauge theory is supersymmetric, $\lambda_2$ vanishes. Similarly, without fundamental matter, the $\lambda_1$ term is absent. In this case, the leading correction begins with the $\lambda_3$ term. From string theory, we find that $\lambda_3 = \zeta(3)/8L^{3/2}$. It was shown rigorously in [4, 7] that starting with the full ten-dimensional string theory, one indeed gets an action of the form (2) with $\lambda_1 = \lambda_3 = 0$. It was further confirmed that there are no other fields in the full 10d solution which alter the existing result [4] which is

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 + \frac{15\zeta(3)}{\lambda^{3/2}} \right). \quad (3)$$

It was also argued in [5] that the leading corrections in the $1/N_c$ expansion appear at order $A^{1/2}/N_c^2$ and are also positive (again to emphasise, only in the absence of fundamental matter).

To make a better comparison to real world QCD, one should also put in fundamental matter and recalculate the leading correction. In string theory, this requires more involved constructions involving D7-branes and orientifold planes. These objects, in particular the D7-branes, are known to produce a $W^2$ term in (2). While a precise derivation of the effective 5d action becomes much more complicated, we can invoke an indirect argument [6] as follows: Any four-dimensional conformal field theory can be characterized by two central charges, $c$ and $a$, related to the trace anomaly. The trace anomaly can be calculated using holographic methods, yielding $L^3/\ell_p^3 \approx c/\pi^3$ and $\lambda_1 \approx (c-a)/8c$ – see [6] for more details. Thus if $c$ and $a$ are given by some knowledge of the gauge theory, we can engineer the appropriate holographic action (2) which must arise from the full string theory. Then given the gravity action, we can easily calculate [10]

$$\frac{\eta}{s} = \frac{1}{4\pi} \left( 1 - 8\lambda_1 + O(\lambda_3, \lambda_1^2) \right). \quad (4)$$

(See [8] for an efficient method of calculating the leading corrections to $\eta/s$.) Recall that we are assuming a perturbative expansion with $|\lambda_n| \ll 1$ in the gravity calculations. Thus if and only if $\lambda_1 < 0$ will the leading correction be positive and the KSS bound be respected. At this point, we can turn the question around and ask if there are any superconformal gauge theories for which we have both $c-a < 0$ with $|c-a|/c \ll 1$. Furthermore, for the next-to-leading order correction proportional to $\lambda_3$ to be subdominant, we must also require $\lambda \gg N_c^{4/3}$. One may have naively expected that there should be very many such theories. However, in [6], we searched a whole class of CFT’s and found that not a single one satisfied these criteria! All the examples
we studied had \( c - a > 0 \) indicating that for these CFT’s, \( \eta/s \) would violate the KSS bound. For example, all the models with gauge group \( SU(N_c) \) and \( |c-a|/c \ll 1 \) as \( N_c \to \infty \), are shown in the following table. We should point out that this violation is \( O(1/N_c) \) and hence very small in the

| \( (n_{ad}, n_{asym}, n_{sym}, n_f) \) | \( c - a \) | \( \delta = (c - a)/c \) |
|-----------------|---|---|
| (a) (3,0,0,0)   | 0 | 0  |
| (b) (2,1,0,2)   | \( \frac{N_c+1}{48} \) | \( \frac{1}{N_c} + O(N_c^{-2}) \) |
| (c) (1,2,0,4)   | \( \frac{N_c+1}{24} \) | \( \frac{1}{N_c} + O(N_c^{-2}) \) |
| (d) (1,1,1,0)   | \( \frac{7}{24} \) | \( \frac{1}{6N_c} + O(N_c^{-1}) \) |
| (e) (0,3,0,6)   | \( \frac{N_c+1}{16} \) | \( \frac{1}{N_c} + O(N_c^{-2}) \) |
| (f) (0,2,1,2)   | \( \frac{N_c+1}{16} \) | \( \frac{1}{N_c} + O(N_c^{-2}) \) |

Table 1: \( \delta \) for \( SU(N_c) \) models

large \( N_c \) limit (where the gravity calculations are reliable). Note that in model (d), the correction from \( \lambda_1 \) is \( 1/N_c^2 \) and hence subdominant compared to \( \lambda_3 \). Therefore this example falls into the category of theories preserving the bound, along with those with no fundamentals, i.e., \( n_f = 0 \).

This brings us to the question as to what happens to this bound violation if we added a chemical potential. In supergravity models, one easy way to add a chemical potential is to turn on a R-charge. This corresponds to adding a Maxwell term \(-1/4F_{ab}F^{ab}\) and certain higher derivative \( RF^2 \) and \( F^4 \) terms to (2), as was considered in [8]. There, it was demonstrated that turning on a R-charge chemical potential only worsens the violations of the KSS bound.

3. Conclusion

While we have found counter-examples to the proposed KSS bound, it is premature to conclude that there is no bound at all. In [9], it was argued that with Gauss-Bonnet gravity, where \( W^2 \) is replaced by the Gauss-Bonnet term which makes the equations of motion second order and removes any ghosts, one may consider the coefficient \( \lambda_1 \) to be finite. However, demanding that the dual gauge theory respect causality puts constraints on this coefficient and it was found that with these constraints in place [9],

\[
\frac{\eta}{s} \geq \frac{16}{25} \frac{1}{4\pi} \tag{5}
\]

for this class of models. It seems unlikely that this result represents a fundamental bound. Hence it is interesting to speculate as to whether a bound actually exists or whether \( \eta/s \) can be systematically reduced towards zero. In any event, holographic constructions seem to provide an interesting new regime of fluid dynamics.

What remains of course curious is the fact that the RHIC plasma has a value for \( \eta/s \) in the vicinity of \( 1/4\pi \). The current upper bound sits at \( \eta/s < 0.2 \) [2]. Hence an interesting question is if we can extend the AdS/CFT methods to do quantitative phenomenology relevant for physics at RHIC or the LHC. Any progress in this direction will be very worthwhile. One simple approach is the following: Let us assume that there is fundamental matter and the leading correction in (2) begins with \( \lambda_1 \). We can show that the holographic energy density normalized by the free field value [6] is given by

\[
\frac{\varepsilon}{\varepsilon_0} = \frac{3}{4} \left( 1 + \frac{1}{4} \frac{c-a}{c} \right) \equiv \frac{3}{4} \left( 1 + \frac{1}{4} \delta \right), \tag{6}
\]

3
\[ \eta = \frac{1}{4\pi} \left( 1 - \frac{c-a}{c} \right) = \frac{1}{4\pi} (1 - \delta). \]  

(7)

Current lattice calculations indicate that \( \varepsilon / \varepsilon_0 \) is roughly between 0.85 – 0.90. Plugging this into (6) yields \( \delta \sim 0.53 - 0.80 \), which using (7) then produces \( \eta / s \sim 0.016 - 0.037 \). This ratio is clearly quite a bit lower than (1). However, quite remarkably the consistency of the CFT’s in general requires that \(|\delta| \leq 0.5 \) [9, 11]. Thus, this exercise shows us that in order to extend the AdS/CFT methods to RHIC phenomenology, we will need to work harder. The next to leading correction which begins with \( \lambda_3 \) in \( \lambda \) for supersymmetric theories will be equally important and to go beyond just qualitative results, these need to be incorporated into the theory. When this is done, we will have two parameters, \( \lambda_1 \) and \( \lambda_3 \), which we will phenomenologically fix. This means we require more inputs from the lattice and/or experiment to constrain the holographic model. Then using these values, we will calculate \( \eta / s \) to see if it yields a sensible result or not. If it does, then using the same phenomenological lagrangian, we will go on to calculate other physical quantities, \( e.g. \), the relaxation time, and see if these can be seen to agree with experiments as well. This will be one way of extending AdS/CFT methods to phenomenology. Of course, for consistency as a string theory calculation, we will also need \(|\lambda_n|\)'s to be small compared to unity. If we are lucky, then the number of corrections to be put in will be manageable. So the bottom line is that any numerology without incorporating higher derivative corrections at least up to \( \lambda_3 \) in (2) should be taken with a grain of salt. It does seem like hard work to get something sensible out of this endeavor.

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