The derivation of the dual superconductor theory from the Maximal Center projected $SU(3)$ – gluodynamics

M.A. Zubkov$^a$

$^a$ ITEP, B.Cheremushkinskaya 25, Moscow, 117259, Russia

Abstract

We consider the Center projected $SU(3)$ gluodynamics and rewrite it as a dual superconductor theory. The center monopole field plays the role of Higgs field in the dual superconductor theory. The center monopole creation operator is constructed.
1 Introduction

The investigation of the confinement problem is now one of the important subjects of QCD. One of the most popular schemes of confinement is the dual superconductor mechanism. According to this mechanism the quark-antiquark string appears in the dual representation of the theory in the way similar to the appearance of the Abrikosov string in Ginsburg-Landau theory. But now it is not clear how the analogue of the Ginsburg-Landau theory can be derived from the QCD.

The relativistic analogue of Ginsburg-Landau theory is the abelian Higgs model. Thus the dual superconductor theory should be the nonlocal variant of abelian Higgs model. There exists the point of view that the field correspondent to the Maximal Abelian monopoles [1] should play the role of Higgs field. The fact that the condensate of that monopoles is the order parameter and is not equal to zero in the confinement phase partially justifies this hypothesis. This picture seems to be quite natural in the SU(2) theory, but in the SU(3) theory two maximal abelian monopoles appear and the dual superconductor mechanism becomes too complex.

Recently the approach alternative to the Maximal Abelian projection was investigated. That approach is the Maximal Center projection. It occurs that the Center dominance also takes place as the Abelian dominance [2]. And furthermore in the Maximal Center projected theory also one can construct the monopoles which are condensed in the confinement phase [3] [4]. The pleasant feature of that monopole is that there is only one center monopole in the SU(3) theory.

In this work we derive the representation of the Maximal Center projected gluodynamics which has the form of the dual superconductor theory. The field of the center monopole plays the role of the Higgs field. The quark-antiquark string appears as the topological defect in this theory. Also we construct the monopole creation and annihilation operators.

2 The Maximal Center Projection.

The Maximal Center projection is the procedure of partial gauge fixing. The gauge ambiguity is used to make all the link variables \( U \in SU(3) \) as close as possible to the elements of the center \( Z_3 \) of \( SU(3) \):

\[
Z_3 = \{ \text{diag}(e^{2\pi i/3}N, e^{2\pi i/3}N, e^{2\pi i/3}N) \}, \text{ where } N \in \{1, 0, -1\}.
\]
There is a lot of versions of the Maximal Center projection. That versions differ from each other by the choice of the gauge fixing potential $O(U)$. The projection is achieved by the minimizing of the potential $O(U)$ with respect to the gauge transformations $U_{xy} \rightarrow g_x U_{xy} g_y^{-1}$.

This gauge condition is invariant under the central subgroup $Z_3$ of $SU(3)$. Thus the Maximal center projected theory is nonlocal $Z_3$ gauge theory. The well-known center vortices are defined as follows. After fixing the Maximal Center gauge we define the integer-valued link variable $N$:

\[ N_{xy} = \begin{cases} 0 & \text{if } (\text{Arg}(U_{11}) + \text{Arg}(U_{22}) + \text{Arg}(U_{33}))/3 \in ] -\pi/3, \pi/3], \\ 1 & \text{if } (\text{Arg}(U_{11}) + \text{Arg}(U_{22}) + \text{Arg}(U_{33}))/3 \in ]\pi/3, \pi], \\ -1 & \text{if } (\text{Arg}(U_{11}) + \text{Arg}(U_{22}) + \text{Arg}(U_{33}))/3 \in ] -\pi, -\pi/3]. \tag{1} \]

In other words $N = 0$ if $U$ is close to 1, $N = 1$ if $U$ is close to $e^{2\pi i/3}$ and $N = -1$ if $U$ is close to $e^{-2\pi i/3}$. Next we define the plaquette variable:

\[ \sigma_{xywz} = N_{xy} + N_{yw} - N_{zw} - N_{xz} \tag{2} \]

In terms of the calculus of the differential forms on the lattice this equation looks like

\[ \sigma = dN \tag{3} \]

Then we introduce the dual lattice and define the variable $\sigma^*$ dual to $\sigma$: if plaquette $^*\Omega$ is dual to plaquette $\Omega$, then $\sigma^*_{^*\Omega} = \sigma_\Omega$. One can easily check that the variable $\sigma$ represents a closed surface. This surface is known as the worldsheet of the center vortex.

We express the $SU(3)$ gauge field $U$ as the product of $\exp((2\pi i/3)N)$ and $V$, where $V$ is the $SU(3)/Z_3$ variable $(\text{Arg}(V_{11}) + \text{Arg}(V_{22}) + \text{Arg}(V_{33}))/3 \in ] -\pi/3, \pi/3]$. Then $U = \exp((2\pi i/3)N)V$.

After that we represent the action of the Wilson loop $C$ as follows:

\[ W_C = \Pi_C U = \exp((2\pi i/3)L(C, \sigma))\Pi_C V \tag{4} \]

The term $(2\pi i/3)L(C, \sigma)$ is known as the Aharonov - Bohm interaction term. The quantity $L(C, \sigma)$ is the linking number of the loop $C$ and the closed surface $\sigma^*$.

The center dominance means that after the Maximal Center projection the Aharonov - Bohm interaction term causes confinement and produces the full string tension.
The center monopole is just the $Z_3$ analogue of the monopole in $U(1)$ theory. Let us recall that monopoles in $U(1)$ theory are constructed as loops on which the force lines of the gauge field end. It is well known that in electrodynamics the Maxwell equations $dF = 0$ restrict the existence of magnetic charges. But in the compact theory values of $F$ which differ from each other by $2\pi$, are equivalent. Thus the correct field strength is $F \mod 2\pi$ and $*d(F \mod 2\pi) = 2\pi j_m$, where $j_m$ is the monopole current.

In the $Z_3$ theory $\sigma = dN$ is the analogue of the field strength $F = dA$. The Aharonov - Bohm interaction between the center vortex and the quark depends only on $[\sigma] \mod 3$. The variable $[\sigma] \mod 3$ represents the surface with boundary. This boundary is a closed line. We assume that this line represents the world trajectory of the particle, which we call a center monopole:

$$3j_m = *d([\sigma] \mod 3) = \delta([\sigma^*] \mod 3).$$

(5)

3 The derivation of the dual superconductor theory

We start with the Wilson loop average in the $SU(3)$ theory.

$$<W(C)> = \int DU \exp(-\sum_{plaq} \beta(1 - 1/3ReTrU_{plaq}ReTr\Pi_C U)$$

(6)

To consider the Maximal Center projection we have to use the Faddeev-Popov unity

$$1 = \lim_{\alpha \to \infty} \int Dg exp(-\alpha O(g_x U_{xy} g^{-1}_y)) \Delta_{FP}(U, \alpha)$$

(7)

Here the gauge condition is that we assume $U$ to provide a minimum of the functional $O(U)$.

This functional is the given measure of the distance in the functional space between the matrix function $U$ and the set of functions, which attach one of the center elements of $SU(3)$ to each link. It is obvious, that the functional $O$ is invariant under the remaining $Z_3$ symmetry.

As in the previous section we consider the following representation of the projected $U$;

$$U = e^{2\pi i N/3} V$$

(8)
where \( V \) is closer to 1 than to other center elements.

It follows from the invariance of \( O \) under the \( Z_3 \) symmetry, that the Faddeev-Popov determinant does not depend upon \( N \).

Thus we have

\[
<W(C)> = \lim_{\alpha \to \infty} \int_{D_V \in SU(3)/Z_3} \sum_{N=1,0,-1} \exp(-\sum_{\text{plaq}} \beta(1 - 1/3 \text{Re}Tr e^{2\pi dN/3} V_{\text{plaq}}))
- \alpha O(V) + (2\pi i/3) (N,C) \Delta_{FP}(V,\alpha) \text{Re}Tr \Pi_C V
\]  

(9)

The center dominance means, that we can omit the expression \( \text{Re}Tr \Pi_C V \) to calculate the correct string tension. Thus we are considering the following expression for the \( Z_3 \) Wilson loop:

\[
<Z(C)> = \lim_{\alpha \to \infty} \int_{D_V \in SU(3)/Z_3} \sum_{N=1,0,-1} \sum_{m=1,0,-1} \sum_{\delta j=0} \exp(-S(dN mod 3) + (2\pi i/3)(N,C)) \delta(m - dN - 3n) \delta(3^*j - dm)
\]  

(10)

After the integration over short-ranged field \( V \) we obtain the resulting \( Z_3 \) theory with nonlocal action

\[
<Z(C)> = \sum_{N=1,0,-1} \exp(-S(dN mod 3) + (2\pi i/3)(N,C))
\]  

(11)

It seems that this theory behaves like an usual \( Z_3 \) theory in the confinement phase.

Now we are going to transfer ourselves into the dual representation of the above theory to understand how the superconductor appears. First let us remember, that the center monopoles are defined as \( j = 1/3^*d(dN mod 3) \).

Then we apply the dual transformation.

\[
<Z(C)> = \sum_{N=1,0,-1} \sum_{m=1,0,-1} \sum_{j,n,\delta j=0} \exp(-S(dN mod 3) + (2\pi i/3)(N,C)) \delta(m - dN - 3n) \delta(3^*j - dm)
\]  

(12)
We use the formulas
\[ \sum_{k=0,1,-1} e^{(2\pi i/3)(Z,k)} = \sum_n \delta(Z - 3n); \]
\[ \int_{-\pi}^\pi dh e^{ihZ} = \delta(Z), \]  
(13)
to obtain:
\[ <Z(C)> = \sum_{N,m,k=1,0,-1} \sum_{j,\delta j=0} \int_{-\pi}^\pi Dh \exp(-S(dN mod3)) \]
\[ + (2\pi i/3)(N,C) + (m - dN, k)2\pi i/3 + i(h, 3^*j - dm)) \]  
(14)

Then we use the expression \( C = \delta A[C] \), where \( A \) is some surface, spanned on the quark loop.

\[ <Z(C)> = \sum_{N,m,k=1,0,-1} \sum_{\delta j=0} \int_{-\pi}^\pi Dh \exp(-S(m)) \]
\[ + (m, -\delta h + (2\pi/3)(A[C] + k)) - (2\pi i/3)(N, \delta k) + i(h, 3^*j)) \]  
(15)

We can perform the summation over \( N \) to obtain the constraint \( \delta k = 3l \)
for some integer \( l \). Also we can perform the summation over \( m \), obtaining
\[ \exp(-Q(f)) = \sum_{m=1,0,-1} \exp(-s(m) + i(m, f)) \]  
(16)

It’s obvious, that \( Q(f) \) is periodic with the period \( 2\pi \). Thus we get
\[ <Z(C)> = \sum_{k=1,0,-1; \delta k=3l; i,j} \int_{-\pi}^\pi Dh \exp(-Q(-\delta h + (2\pi/3)(A[C] + k))) \]
\[ \exp(i(h, 3^*j)) \]  
(17)

We can solve the constraint \( \delta k = 3l : k = 3A[l] + \delta z \). Due to the periodicity of \( Q, l \) is eliminated. Then we redefine \( h \rightarrow (h + 2\pi/3\delta z) mod 2\pi \), and finally get:
\[ <Z(C)> = \sum_{\delta j=0} \int_{-\pi}^\pi Dh \exp(-Q(-\delta h + (2\pi/3)A[C]) + i(h, 3^*j)) \]  
(18)
Thus we have obtained that the theory dual to the original $Z_3$ projected gluodynamics is just the nonlocal $U(1)$ gauge theory with additional summation over the worldlines of the center monopoles, which carry the charge 3 with respect to the mentioned $U(1)$ gauge field.

We can rewrite the summation over the worldlines of the monopoles as the integral over the Higgs field of charge 3:

$$\sum_j \exp(-i(H,3j)) = \int D\Phi_c \Phi_c \exp(-\sum_{xy} \Phi_x e^{3iH_{xy}} \Phi_y^+ - V(|\Phi|)), \quad (19)$$

where the potential $V$ is infinitely deep, and the vacuum average of $|\Phi|$ is infinitely large. So $V(r) = a(r^2 - b)^2$, where $a, b \to \infty$. Here we denoted $^*h = -H$.

Finally we have

$$<Z(C)> = \int^{-\pi}_{\pi} DH \int D\Phi_c \Phi_c \exp(-Q(dH + (2\pi/3)^*A[C])$$

$$- \sum_{xy} \Phi_x e^{3iH_{xy}} \Phi_y^+ - V(|\Phi|)) \quad (20)$$

The last representation is the dual superconductor representation of the $SU(3)$ theory. Here the field $\Phi$ is the field of our center monopole. When the monopole is condensed, the Abrikosov-Nielsen Olesen strings appear. That strings carry the magnetic flow $2\pi/3$, thus connecting the quarks, which play the role of monopoles here. Also the usual monopoles, existing due to the periodicity of the action, create the Abrikosov-Nielsen-Olesen strings. But that strings should carry the magnetic flow $2\pi$, which is clear from the above expression. Thus that monopoles indeed create 3 strings and do not influence the confinement mechanism.

4 The center monopole creation operator

We define the monopole creation operator in the way similar to that of in the $U(1)$ theory representing the Maximal Abelian projected $SU(2)$ gluodynamics. Following [4], we obtain the vacuum average of the monopole - antimonopole correlator in the dual theory:

$$<\Phi(z1)\Phi^*(z2)> = \int^{-\pi}_{\pi} DH \int D\Phi_c \Phi_c \exp(-Q(dH)$$

7
\[ -\sum_{xy} \Phi_x e^{3iH_{xy}} \Phi_y^+ - V(|\Phi|)\Phi(z1)\Phi^*(z2)\exp(i(D(z1) - D(z2), H)), \] (21)

where \( \delta D(z) = -3\delta_z \).

After coming back to the representation throw the worldlines of the monopoles we get

\[ <\Phi(z1)\Phi^*(z2)> = \sum_{\delta j = \delta z1 - \delta z2} \int_{-\pi}^{\pi} DH \exp(-Q(dH) + i(H, 3j - (D(z1) - D(z2)))) \] (22)

Repeating the steps back to the original representation we obtain

\[ <\Phi(z1)\Phi^*(z2)> = \lim_{\alpha \to \infty} \int_{D \in SU(3)/Z3} V \sum_{N=1,0,1} \exp(-\sum_{\text{plaq}} \beta(1 - 1/3 \Re Tr e^{2\pi(dN)/3} V_{\text{plaq}}) - \alpha O(V)) \Delta_{FP}(V, \alpha) Q(\ast d([dN]_{mod3}) - (3j_{z1,z2} - D(z1) + D(z2)))) \] (23)

Here \( j_{z1,z2} \) is the line connecting points \( z1 \) and \( z2 \) of the dual lattice. And

\[ Q(x) = \sum_{\delta j = 0} \Pi_{\text{links}} \sin(\pi(x - j))/(x - j), \] (24)

where the summation is over the closed integer 1-forms on the dual lattice. In other words

\[ <\Phi(z1)\Phi^*(z2)> = \int DU \exp(-\sum_{\text{plaq}} \beta(1 - 1/3 \Re Tr U_{\text{plaq}})) Q(3\ast j_{Z3} - (3j_{z1,z2} - D(z1) + D(z2)))) \] (25)

where \( j_{Z3} \) is the center monopole trajectory extracted from the field configuration \( U \).

5 Conclusions

In this work we made an attempt to extract the kind of superconductor theory from the \( SU(3) \) gluodynamics in the Maximal Center Gauge. We obtained
the theory, which contains $U(1)$ gauge field and the scalar field charged with respect to that $U(1)$ field. The action is essentially nonlocal. The usual quarks play the role of monopoles in this theory. The scalar field is condensed in the confinement phase and is not condensed in the deconfinement phase. The worldlines of the particle correspondent to that scalar field are just the worldlines of the center monopoles.

Of course, the analogous representation one can obtain for the gluodynamics in the Maximal Abelian Gauge. But in that case two scalar fields and two $U(1)$ fields appear. Thus the dual superconductor mechanism becomes too complex and unnatural.

**Acknowledgments**

The author is grateful to B.L.G. Bakker, J. Greensite, and A. Veselov for useful discussions.

This work was supported by the JSPS Program on Japan–FSU scientists collaboration, by the grants INTAS-RFBR-95-0681 and RFBR-97-02-17491.

**References**

[1] A.S. Kronfeld, M.L. Laursen, G. Scheierholz, U.J. Wiese Phys. Lett. B 198 (1987) 516

[2] M. Faber, J. Greensite, S. Olejnik, [hep-lat/9810008](http://www.arxiv.org/abs/hep-lat/9810008)

[3] B.L.G. Bakker, A.I. Veselov, M.A. Zubkov, [hep-lat/9902010](http://www.arxiv.org/abs/hep-lat/9902010)

[4] M.N. Chernodub, M.I. Polikarpov, A.I. Veselov, M.A. Zubkov, [hep-lat/9809158](http://www.arxiv.org/abs/hep-lat/9809158)

[5] M.I. Polikarpov, U.J. Wiese, M.A. Zubkov, Phys. Lett. B 309 (1993) 133

[6] M.N. Chernodub, M.I. Polikarpov, A.I. Veselov Phys.Lett. B399 (1997) 267

[7] T. G. Kovacs, E.T. Tomboulis, [hep-lat/9808046](http://www.arxiv.org/abs/hep-lat/9808046)