On the duality in CPT-even Lorentz-breaking theories

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Abstract

In this paper, we generalize the duality between self-dual and Maxwell-Chern-Simons theories for the case of a CPT-even Lorentz-breaking extension of these theories. The duality is demonstrated with use of the gauge embedding procedure, both in free and coupled cases, and with the master action approach. The physical spectra of both Lorentz-breaking theories are studied. The massive poles are shown to coincide and to respect the requirements for unitarity and causality at tree level. The extra massless poles which are present in the dualized model are shown to be nondynamical.
I. INTRODUCTION

The duality is a very important phenomenon in quantum field theories, allowing for mutual mapping of physical theories whose actions are essentially different. In [1], the concept of duality has been discussed as a generic feature of a wide class of field theory models. The duality was first established in three spacetime dimensions in the paradigmatic example of the dual correspondence between the free self-dual and Maxwell-Chern-Simons theories [2].

Afterwards, different methods have been elaborated to establish and study the duality in many cases (see [3] for a nice review). One approach to determine the physical equivalence between two theories is the master action method [4], whose essence consists in determining the action involving two vector fields, so that integration over one of them yields the self-dual action, and over other one, the Maxwell-Chern-Simons action. On the other hand, the gauge embedding method [5] is based on the transformation of the self-dual model into the gauge theory by adding on mass shell vanishing terms. These methods have been shown to be efficient tools for studying different field theory models, allowing, in particular, to find new couplings for vector fields. Also in [5], the self-dual theory minimally coupled to the spinor matter has been shown to generate, through gauge embedding, the magnetic (nonminimal) coupling and the Thirring-like current-current coupling. Further, the duality has been established between nonlinear generalizations of self-dual and Maxwell-Chern-Simons theories (the last one yields a Born-Infeld-Chern-Simons theory) [6], higher-rank tensor generalizations of these theories [7] and their higher-derivative extensions [8]. Noncommutative extensions of the duality have been discussed in [9].

Since in the recent years great attention has been devoted to Lorentz-breaking theories (for a general review on this issue, see [10] and references therein), the natural question consists in the implementation of the duality concept for Lorentz-breaking theories. In [11], the duality methodology has been applied to the CPT-odd Lorentz-breaking extension of the self-dual theory. Further, in [12] and [13] the duality was promoted to the four-dimensional case. At the same time, the study of purely CPT-even Lorentz-breaking theories is very interesting. Many issues related to classical solutions in these theories have been discussed (a very incomplete list is given in [14, 15]). In [16] the dual embedding for a four-dimensional Proca-like theory with a CPT-even Lorentz-breaking mass term was performed, where the resulting theory was shown to involve higher derivatives terms.

Therefore, a natural task would consist in constructing a case which has not been considered yet: a CPT-even Lorentz-breaking self-dual model, with the obtaining of its dual and the verification of
the duality through different procedures. In this paper, we verify this duality with the use of the
gauge embedding approach and master action method, both in free and coupled cases, and show
the physical equivalence of the spectra of the two models.

The paper is organized as follows: section II is dedicated to the presentation of the self-dual (SD)
model, the determination of the corresponding dual Maxwell-Chern-Simons-like (MCS-like) theory
by means of the gauge embedding technique and the confirmation of duality through the analysis
of the equations of motion; section III is devoted to the confirmation of this duality with the use
of the master action formalism; in section IV, we discuss the physical consistency of both models
through the analysis of their spectra, obtained from the propagators. The physical equivalence of
the models is also shown from the physical spectra viewpoint; the concluding remarks are presented
in section V.

II. GAUGE EMBEDDING

Let us consider the following generalization of the 3D self-dual (SD) model \[2\],
\[
\mathcal{L} = -\frac{m}{2} \varepsilon^{abc} f_a \partial_b f_c + \frac{1}{2} m^2 h_{ab} f^b + f_a j^a,
\]
where we added a interaction term, being \( j_a \) a current (for example, the spinor one, \( j^a = \bar{\psi} \gamma^a \psi \)),
and \( h_{ab} \) is a tensor which includes Lorentz-violating terms. We next will proceed with the gauge
embedding of the model in order to obtain the corresponding modified Maxwell-Chern-Simons
(MCS) theory which is the dual of our model. Let us then calculate the first variation of our
Lagrangian density,
\[
\delta \mathcal{L}[f_a] = \left\{ -m \varepsilon^{abc} \partial_b f_c + m^2 h_{ab} f^b + j^a \right\} \delta f_a,
\]
where we may recognize the Noether current as
\[
K^a = -m \varepsilon^{abc} \partial_b f_c + m^2 h_{ab} f^b + j^a.
\]
The first iterated Lagrangian is constructed by introducing an auxiliary field \( B \),
\[
\mathcal{L}^{(1)} = \mathcal{L} - K^a B_a,
\]
with \( \delta B_a = \delta f_a = \partial_a \eta \), so that we get
\[
\delta \mathcal{L}^{(1)} = -(\delta K_a) B^a.
\]
Using
\[ \delta K^a = m^2 h^{ab} \delta f_b, \]  
we have
\[ \delta \mathcal{L}^{(1)} = -m^2 B_a h^{ab} \delta f_b. \]  
The second iterated Lagrangian is defined by
\[ \mathcal{L}^{(2)} = \mathcal{L}^{(1)} + \frac{m^2}{2} B^a h_{ab} B^b, \]  
so that if we use the variation of \( B_a \) and (7), we get that the total variation vanishes,
\[ \delta \mathcal{L}^{(2)} = 0. \]  
Let us write down the explicit form of this action,
\[ \mathcal{L}^{(2)} = -\frac{1}{2} m \varepsilon^{abc} f_a \partial_b f_c + \frac{m^2}{2} f^a h_{ab} f^b + f_a j^a \]
\[ -K_a B^a + \frac{m^2}{2} B^a h_{ab} B^b. \]  
After carrying out the variation of this action in \( B_a \), we get the following equation of motion:
\[ K_a - m^2 h_{ab} B^b = 0. \]  
Plugging this back into (9), we arrive at the following gauge invariant MCS-like action:
\[ \mathcal{L}_{MCS} = \frac{m}{2} F^{a} A_a - \frac{1}{2} F^a (h^{-1})_{ad} F^d - \frac{1}{2m^2} G^a (h^{-1})_{ad} G^d + \frac{1}{m} F^a (h^{-1})_{ad} G^d, \]  
where we renamed the field \( f \) as \( A \) and the current \( j \) as \( G \) and used \( F^a = \varepsilon^{abc} \partial_b A_c \). In this paper we will concentrate on the specific case where the Lorentz-violating tensor is given by
\[ h_{ad} = \eta_{ad} - \beta b_a b_d, \]  
where \( b_a \) is a Lorentz-breaking constant background field which selects a preferred direction in the 3D spacetime and \( \beta \) is a dimensionless parameter. So, we have
\[ (h^{-1})_{ad} = \eta_{ad} + \alpha b_a b_d, \]  
with \( \alpha = \beta/(1 - \beta b^2) \). The MCS-like Lagrangian density will be given by
\[ \mathcal{L}_{MCS} = -\frac{1}{4} F_{ab} F^{ab} + \frac{m}{2} \varepsilon^{abc} A_a \partial_b A_c - \frac{\alpha}{8} \left( \varepsilon_{abc} b^a F^{bc} \right)^2 \]
\[ -\frac{1}{2m^2} G^a (h^{-1})_{ad} G^d + \frac{1}{m} F^a (h^{-1})_{ad} G^d. \]
We note that the term quadratic in the Levi-Civita symbol, which emerged from the gauge embedding procedure, is nothing but the sum of a Maxwell term with a constant multiplier and the aether term [15]:

\[(\epsilon_{abc}b^a F^{bc})^2 = 2b^2 F_{ab} F^{ab} - 4b_a F^{ac} b^b F_{bc}.\]  

(15)

Besides, we note that this Lagrangian involves a Thirring-like current-current interaction and a magnetic coupling, as in [5].

The equations of motion for the self-dual and MCS fields, after some simple transformations, respectively, look like

\[mf_k - \epsilon^{nlm} (h^{-1})_{kn} \partial_l f_m = - \frac{1}{m} j^n (h^{-1})_{kn};\]  

\[mF^m - \epsilon^{man} (h^{-1})_{ab} \partial_m F^b = - \frac{1}{m} \epsilon^l mn (h^{-1})_{md} \partial_l G^d.\]  

(16)

We see that \(f_n\) and \(\tilde{F}_p = F^k (h^{-1})_{kp}\) satisfy similar equations:

\[[mh^{bn} - \epsilon^{bmn} \partial_m] f_n = - \frac{1}{m} j^b;\]  

\[[mh^{bn} - \epsilon^{bmn} \partial_m] \tilde{F}_n = - \frac{1}{m} \epsilon^{abc} (h^{-1})_{cd} \partial_a G^d.\]  

(17)

It is clear that in the Lorentz-invariant case (so, \(h_{bk} = \eta_{bk}\)), the known result from [5] is reproduced. The mapping of currents, \(j^b \rightarrow \epsilon^{abc} (h^{-1})_{cd} \partial_a G^d\), together with the mapping \(\tilde{F}_m \rightarrow f_m\) confirm the duality between our extended SD and MCS theories.

III. MASTER ACTION APPROACH

It is interesting to establish the duality discussed in the last section in the framework of a master action. We would like to show that there is a consistent master action which generates the two actions studied in the gauge embedding approach. Indeed, let us consider the following Lagrangian density describing the dynamics of two vector fields, \(f_a\) and \(A_a\), and verify that it originates the free parts of the two dual models of the previous section:

\[\mathcal{L}_M = \frac{m^2}{2} f^a h_{ab} f^b + m f_a \epsilon^{abc} \partial_b A_c + \frac{m}{2} \epsilon^{abc} A_a \partial_b A_c + \frac{p}{2} (\partial \cdot A)^2.\]  

(18)

The last part represents a gauge-fixing term. First, we find the equations of motion for \(f_a\). We get

\[f_a = - \frac{1}{m} (h^{-1})_a^n \epsilon_{nbc} \partial^b A^c \equiv - \frac{1}{m} (h^{-1})_{an} F^n.\]  

(19)
We eliminate the $f_a$ with this equation from the action (18) and get
\[
\mathcal{L}_A = -\frac{1}{2} F^a (h^{-1})_{ab} F^b + \frac{m}{2} F^a A_a + \frac{\rho}{2} (\partial \cdot A)^2, \tag{20}
\]
which is nothing but the free lagrangian density (11) with the gauge-fixing term.

We now obtain the equation of motion for $A_a$, which yields
\[
-m \varepsilon^{abc} \partial_b f_a - m \varepsilon^{abc} \partial_b A_a - \rho \partial^c (\partial \cdot A) = 0, \tag{21}
\]
and will give us
\[
A_a = m (\Delta^{-1})_{ac} \varepsilon^{mbc} \partial_b f_m, \tag{22}
\]
where
\[
\Delta^{ac} = m \varepsilon^{abc} \partial_b - \rho \partial^c \partial^d. \tag{23}
\]
Inverting $\Delta^{ac}$, we get
\[
(\Delta^{-1})_{ac} = -\frac{\partial_a \partial_c}{\rho \Box} - \frac{1}{m |\varepsilon_{acd}|} \varepsilon_{acd} \partial^d. \tag{24}
\]
Substituting this expression in (22), one finds the relation between $f_a$ and $A_a$:
\[
A_a = -f_a + \frac{\partial_a (\partial \cdot f)}, \tag{25}
\]
which fixes $\partial \cdot A = 0$. We eliminate $A_a$ from (18) with this equation, and get
\[
\mathcal{L}_f = \frac{m^2}{2} f^a h_{ab} f^b - \frac{m}{2} f_a \varepsilon^{abc} \partial_b f_c, \tag{26}
\]
which is the self-dual Lagrangian density (1). We thus have shown that (18) is a master action (under integration) for our Lorentz-violating self-dual and MCS models.

IV. PROPAGATORS AND STRUCTURE OF THE POLES

In the previous sections we have established the duality between the Lorentz-violating self-dual (SD) and MCS models. First we obtained the MCS model by means of the gauge embedding procedure and then compared the two equations of motion, finding a mapping between the two vector fields $f_a$ and $A_a$. In the sequence we have shown that there exists a master action which generates the two models. However, it is necessary a further investigation. It was observed in [12] and shown in [17] that, although dual models share the same physical spectrum, the gauge invariant
model obtained through gauge embedding (also called Noether Dualization Method) exhibits new nonphysical poles. In this section we will study the propagators and show that the two models share the same physical spectrum and, besides, that the new poles which appear in the MCS model have no dynamics.

In the analysis below, we will consider only the quadratic part of the Lagrangian densities, which, under partial integration, are written in the form

$$\mathcal{L} = \frac{1}{2} u^a \mathcal{O}_{ab} u^b,$$

where $u_a$ represents the corresponding vector field. The propagator is given by $i (\mathcal{O}^{-1})_{ab}$. We will perform the calculations with the use of the following set of spin operators:

$$\theta_{ab} = \eta_{ab} - \frac{\partial_a \partial_b}{\Box}, \quad \omega_{ab} = \frac{\partial_a \partial_b}{\Box}, \quad S_{ab} = \varepsilon_{abc} \partial^c, \quad \Lambda_{ab} = b_a b_b,$$

$$\Sigma_{ab} = b_a \partial_b, \quad A_{ab} = \Sigma_{ab}, \quad B_{ab} = \tilde{\Sigma}_{a} \partial_b,$$  

where $\tilde{\Sigma}_a = \varepsilon_{abc} \Sigma^{bc}$ ($\lambda$ stands for $\Sigma_a^a = b_a \partial^a$). The Lorentz algebra of these operators is shown in Table 1:

|   | $\theta$ | $\omega$ | $S$ | $\Lambda$ | $\Sigma$ | $\Sigma^T$ | $A$ | $A^T$ | $B$ | $B^T$ |
|---|---------|---------|---|---------|---------|---------|---|-------|---|-------|
| $\theta$ | $\theta$ | $\omega$ | $S$ | $\Lambda - \frac{\lambda}{\Box} \Sigma^T$ | $\Sigma - \lambda \omega$ | $0$ | $A$ | $A^T - \frac{\lambda}{\Box} B^T$ | $B$ | $0$ |
| $\omega$ | $0$ | $\omega$ | $0$ | $\frac{\lambda}{\Box} \Sigma^T$ | $\lambda \omega$ | $\Sigma^T$ | $0$ | $\lambda A^T$ | $0$ | $B^T$ |
| $S$ | $S$ | $0$ | $-\Box \theta$ | $A$ | $B$ | $0$ | $-\Box \Lambda + \lambda \Sigma^T$ | $C$ | $\Box (\lambda \omega - \Sigma)$ | $0$ |
| $\Lambda$ | $\Lambda - \frac{\lambda}{\Box} \Sigma$ | $\frac{\lambda}{\Box} \Sigma$ | $-A^T$ | $b^2 A$ | $b^2 \Sigma$ | $\lambda \Lambda$ | $0$ | $b^2 A$ | $0$ | $\lambda A^T$ |
| $\Sigma$ | $0$ | $\Sigma$ | $0$ | $\lambda \Lambda$ | $\lambda \Sigma$ | $\Box \Lambda$ | $0$ | $\lambda A^T$ | $0$ | $\Box A^T$ |
| $\Sigma^T$ | $\Sigma^T - \lambda \omega$ | $\lambda \omega$ | $-B^T$ | $b^2 \Sigma^T$ | $b^2 \Box \omega$ | $\lambda \Sigma^T$ | $0$ | $b^2 B$ | $0$ | $\lambda B^T$ |
| $A$ | $A - \frac{\lambda}{\Box} B$ | $\frac{\lambda}{\Box} B$ | $-C$ | $b^2 A$ | $b^2 B$ | $\lambda \Lambda$ | $0$ | $b^2 C$ | $0$ | $\lambda C$ |
| $A^T$ | $A^T$ | $0$ | $\Box \Lambda - \lambda \Sigma$ | $0$ | $0$ | $0$ | $(b^2 \Box - \lambda^2) \Lambda$ | $0$ | $(b^2 \Box - \lambda^2) \Sigma$ | $0$ |
| $B$ | $B$ | $0$ | $\lambda \Lambda$ | $\lambda B$ | $\Box A$ | $0$ | $\lambda C$ | $0$ | $\Box C$ |
| $B^T$ | $B^T$ | $0$ | $\Box (\Sigma^T - \lambda \omega)$ | $0$ | $0$ | $0$ | $(b^2 \Box - \lambda^2) \Sigma^T$ | $0$ | $(b^2 \Box - \lambda^2) \Box \omega$ | $0$ |

Table 1: Multiplicative table fulfilled by $\theta$, $\omega$, $S$, $\Lambda$, $\Sigma$, $\Sigma^T$, $A$, $A^T$, $B$ and $B^T$. The products are supposed to obey the order “row times column”.

In the table above, we have

$$C_{ab} = (b^2 \Box - \lambda^2) \theta_{ab} - \lambda^2 \omega_{ab} - \Box \Lambda_{ab} + \lambda (\Sigma_{ab} + \Sigma_{ba}).$$  

(29)
A. Self-dual model

The quadratic part of the Lagrangian density of the self-dual model is given by

$$ L_f = -\frac{m^2}{2} \varepsilon^{ab} f_a \partial_n f_b + \frac{1}{2} m^2 f^a h_{ab} f^b, \quad (30) $$

with \( h_{ab} = \eta_{ab} - \beta b_a b_b \). To find the propagator, we should invert the operator

$$ K_{ab} = m^2 \theta_{ab} + m^2 \omega_{ab} + m S_{ab} - \beta m^2 \Lambda_{ab}. \quad (31) $$

The inverse of this wave operator is obtained with the use of the algebra of table 1 and is given by

$$ (K^{-1})_{ab} \equiv G_{ab} = \frac{1}{R + m^2} \left\{ \theta_{ab} + \frac{1}{m^2} \left( R + m^2 + \alpha \lambda^2 \right) \omega_{ab} - \frac{1}{m} S_{ab} + \alpha \Lambda_{ab} - \frac{\alpha}{m} (A_{ab} - A_{ba}) \right\}, \quad (32) $$

where

$$ R = \left( 1 + \alpha b^2 \right) (\Box - \beta \lambda^2), \quad (33) $$

We use the signature (+−−), so, it means that in our self-dual theory the dispersion relation looks like

$$ -E^2 + p^2 + m^2 - \alpha \left[ b^2 (E^2 - p^2) - (b_0 E - b \cdot p)^2 \right] = 0. \quad (34) $$

We are now in position to study this dispersion relation and the physical spectrum of the model. We are interested in two situations for the background vector \( b_\mu \), namely the cases in which it is spacelike or timelike.

1. \( b_\mu \) spacelike

We use a representative background vector given by \( b^\mu = (0, 0, t) \). In this case, we have

$$ b \cdot p = t p_2 \quad \text{and} \quad b^2 = -t^2, \quad (35) $$

and the dispersion relation yields

$$ E^2 - p^2 - (1 + \beta t^2)(m^2 + \alpha t^2 p_2^2) = 0, \quad (36) $$

which ensures positive definite energy particles for a little deviation from Lorentz symmetry \((t^2 << 1)\), complying with causality.

Our present task consists in checking the features of the pole of the propagator for \( b_\mu \) spacelike. In order to investigate the physical nature of the simple pole, we need to calculate the eigenvalues
of the residue matrix for this pole. Let us argue that the momentum $p^a$ is actually a Fourier-integration variable, so that we are allowed to pick a representative momentum whenever $p^2 > 0$. We carry out our analysis of the residue by taking $p^a = (p^0, 0, p^2)$. In this analysis, we are interested in checking the degrees of freedom of this mode and if it respects physical requests such as unitarity and causality. These properties are not expected to vary with the spatial direction of propagation of the electromagnetic wave. Moreover, the physics described by the Standard Model Extension (SME) is Lorentz invariant from the observer point of view. For a general passive Lorentz transformation, as long as the vectors $p^a$ and $b^a$ are not proportional to each other, they will not remain parallel in the pure spatial sector. For the choices we have made, the propagator is the following

$$
i G_{ab} = - i \frac{(1 + \beta t^2)}{p_0^2 - m_1^2} \left\{ \theta_{ab} + \frac{1}{m^2} (R + m^2 - \alpha t^2 p_2^2) \omega_{ab} - \frac{i}{m} (p_0 \varepsilon_{ab0} - p_2 \varepsilon_{ab2}) + \alpha t^2 \delta^a_a \delta^b_b + i \frac{\alpha t^2}{m} p_0 (\delta^1_a \delta^2_b - \delta^1_b \delta^2_a) \right\},$$

where

$$m_1^2 = (1 + \beta t^2) (m^2 + p_2^2)$$

is the unique pole of the present propagator. We find the following residue matrix:

$$\mathcal{R} = \begin{pmatrix} (1 + \beta t^2) \frac{p_2^2}{m^2} & -i(1 + \beta t^2) \frac{p_2}{m} & \frac{m_1 p_2}{m^2} \\ i(1 + \beta t^2) \frac{p_2}{m} & (1 + \beta t^2) & \frac{i m_1}{m} \\ \frac{m_1 p_2}{m^2} & -i \frac{m_1}{m} & \frac{m^2 + p_2^2}{m^2} \end{pmatrix},$$

with eigenvalues

$$\lambda_1 = 0,$$

$$\lambda_2 = 0,$$

$$\lambda_3 = (2 + \beta t^2) \frac{(m^2 + p_2^2)}{m^2}.$$  

As it can be seen, for a tiny Lorentz symmetry violation ($t^2 << 1$), this pole is to be associated with one physical degree of freedom, since we have one non-null positive eigenvalue.

2. $b_a$ timelike

We use a representative background vector given by $b^\mu = (t, 0, 0)$, which will give us

$$b \cdot p = tp_0 \quad \text{and} \quad b^2 = t^2,$$

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and the dispersion relation yields

\[ E^2 - (1 + \alpha t^2) p^2 - m^2 = 0, \quad (44) \]

which again furnishes us positive definite energy particles for \( t^2 << 1 \). Once more using the Fourier momentum as \( p^a = (p^0, 0, p^2) \), the propagator will yield the pole

\[ m_1'^2 = (1 + \alpha t^2)p_2^2 + m_1^2. \quad (45) \]

The residue matrix will have only one non-null eigenvalue, which is positive and is given by

\[ \lambda_3 = 1 + \frac{m_1'^2 + (1 + \alpha t^2)p_2^2}{m_1^2}. \quad (46) \]

We see that at tree level the model predicts a mode which complies with unitarity (positive norm particle) and causality (positive pole) for \( t^2 << 1 \) for both spacelike and timelike \( b_a \).

**B. Maxwell-Chern-Simons model**

The quadratic part of the Lagrangian density of the MCS-like Lorentz-violating model is written as

\[ \mathcal{L}_A = -\frac{1}{2} F^a (h^{-1})_{ab} F^b + \frac{m}{2} F^a A_a, \quad (47) \]

In this Lagrangian, one can fix the gauge by adding the rescaled usual gauge-fixing term \(-\frac{1}{2}(1 + \alpha b^2)(\partial \cdot A)^2\). Afterwards, one gets after partial integrations

\[ \mathcal{L}_A = \frac{1}{2} A^a \Delta_{ac} A^c, \quad (48) \]

with

\[ \Delta_{ab} = R\theta_{ab} + R\omega_{ab} - m S_{ab} - \alpha \Box \Lambda_{ab} + \alpha \lambda (\Sigma_{ab} + \Sigma_{ba}). \quad (49) \]

This wave operator should be inverted in order to obtain the propagator. With the help of the algebra of table 1, we get

\[ (\Delta^{-1})_{ab} \equiv \tilde{G}_{ab} = \frac{1}{R + m^2} \left\{ \theta_{ab} + \frac{1}{\Box} \left[ (1 - \beta b^2) (R + m^2) + \alpha \lambda^2 \right] \omega_{ab} + \frac{m}{R} S_{ab} + \alpha \Lambda_{ab} \right. \]

\[ - \frac{\alpha \lambda}{\Box} (\Sigma_{ab} + \Sigma_{ba}) + \frac{\alpha m}{R} (A_{ab} - A_{ba}) - \frac{\alpha m \lambda}{\Box R} (B_{ab} - B_{ba}) \left\} , \quad (50) \]

Before carrying out the same analysis performed for the self-dual model, with the study of the residues at the poles, let us remark that we have now, besides the pole \( m_1^2 \) found in the self-dual
model, two more poles which appear in some sectors of the propagator, due to the denominator factors $R$ and $\Box$. Let us argue that these poles are actually nondynamical. Let us saturate the propagator with conserved currents,

$$SP \equiv J^a i\tilde{G}_{ab} J^b,$$

such that $\partial_a J^a = 0$. So, it is clear that $J^a \omega_{ab} J^b = 0$, $J^a (\Sigma_{ab} + \Sigma_{ba}) J^b = 0$ and $J^a (B_{ab} - B_{ba}) J^b = 0$. We rest with

$$SP \equiv J^a i\tilde{G}_{ab} J^b = i J^a \frac{1}{R + m_1^2} \left\{ \theta_{ab} + \frac{m}{R} S_{ab} + \frac{\alpha m}{R} (A_{ab} - A_{ba}) \right\} J^b. \quad (52)$$

The remaining terms which are proportional to $\frac{1}{R}$, that is, those ones involving $S_{ab}$ and $A_{ab} - A_{ba}$, however, can be treated as analogues of the terms involving massless poles for the usual MCS theory. Indeed, in the Lorentz-invariant limit $b_a = 0$, the usual MCS propagator is recovered. Those terms with massless poles are well known to yield no physical dynamics [18]. Indeed, it is well known that in the three-dimensional gauge theories, there is only one degree of freedom. At the same time, it was noted in [11] (see also the references therein), that in dual theories only physical dispersion relations should coincide. Since namely the massive pole is common for the propagators (52) and (50), we conclude that just this pole is physical. Another argument in favour of this is that in the Lorentz-invariant limit $b_a \to 0$, only the massive pole is reduced to the usual physical pole of the MCS theory. Therefore, despite the existing massless pole, we conclude that the unique degree of freedom corresponding to the denominator $R + m_1^2$ is physical. Thus, we only have to check the residues for the pole $m_1^2$ corresponding just to this denominator.

1. $b_a$ spacelike

Adopting the same choices for $b_a$ and $p_a$ we have done before ($b^a = (0, 0, t)$ and $p^a = (p^0, 0, p^2)$), the propagator yields

$$i\tilde{G}_{ab} = -i \frac{(1 + \beta t^2)}{p_0^2 - m_1^2} \left\{ \theta_{ab} + \frac{1}{p^2} (p_0^2 - m_1^2 + \alpha t^2 p_2^2) \omega_{ab} + \frac{im}{R} (p_0 \varepsilon_{ab0} - p_2 \varepsilon_{ab2}) ight.$$ 

$$+ \frac{\alpha t^2 \delta_a^2 \delta_b^2}{p^2} + \frac{\alpha t^2 p_2^2}{p^2} (\varepsilon_a^2 p_b + \varepsilon_b^2 p_a) - \frac{im\alpha t^2 R}{R} (p_0 (\delta_a^1 \delta_b^2 - \delta_b^1 \delta_a^2))$$

$$- \frac{im\alpha t^2 p_2 p_0}{p^2 R} (\delta_a^1 p_b - \delta_b^1 p_a). \quad (53)$$
The residue in the pole \( m_1^2 \) will give us the matrix,

\[
\mathcal{R} = (1 + \beta t^2) \begin{pmatrix}
\frac{m^2 p^2}{(m_1^2 - p_2^2)} & -i \frac{m p_2}{(m_1^2 - p_2^2)} & m_1 p_2 m^2 \\
-i \frac{m p_2}{(m_1^2 - p_2^2)} & 1 & 1 \\
m_1 p_2 m_1^2 & -i \frac{m_1 m}{(m_1^2 - p_2^2)} & m_2^2 m_1^2
\end{pmatrix},
\]

(54)

with eigenvalues

\[
\lambda_1 = 0, \\
\lambda_2 = 0, \tag{55} \\
\lambda_3 = (1 + \beta t^2) \left( 1 + \frac{m^2(m_1^2 + p_2^2)}{(m_1^2 - p_2^2)^2} \right). \tag{56}
\]

Again, we verify that in a situation with a tiny Lorentz symmetry-breaking the MCS-like model respects, at tree level, unitarity and causality.

2. \( b_a \) timelike

We repeat here the choices for \( b^a \) and \( p^a \) used in the analysis of the self-dual model. The propagator will be given by

\[
i \tilde{G}^{ab} = -i \frac{1}{p_0^2 - m_1^2} \left\{ \theta^{ab} - \frac{1}{p^2} \left[ (1 - \beta t^2)(R + m^2) - \alpha t^2 p_0^2 \right] \omega_{ab} + i \frac{m}{R} \epsilon_{abc} p^c \\
+ \alpha t^2 \delta^0 a \delta^0 b - \frac{\alpha t^2 p_0}{p^2} \left( \delta^0 a p_b + \delta^0 b p_a \right) + i \frac{m \alpha t^2}{R} p_2 \left( \delta^1 a \delta^0 b - \delta^0 b \delta^1 a \right) \\
- i \frac{m \alpha t^2 p_0 p_2}{p^2 R} \left( \delta^1 a p_b - \delta^0 b p_a \right) \right\}. \tag{58}
\]

The residue in the pole \( m_1^2 \) yields two null eigenvalues and a positive one, given by

\[
\lambda_3 = 1 + \frac{m^2(m_1^2 + p_2^2)}{(m_1^2 - p_2^2)^2} > 0. \tag{59}
\]

A peculiarity that should be observed is the fact that, for timelike \( b_a \), both models respect unitarity at tree level for all values of \( t \). For the spacelike \( b_a \), on the other hand one obtains positive definite norm particles only for tiny Lorentz-breaking \((t^2 \ll 1)\).

V. CONCLUDING REMARKS

We presented CPT-even extended versions of the 3D self-dual and MCS models which violate Lorentz symmetry. This violation is accomplished by the addition of a Lorentz-breaking mass
term to the self-dual model. The corresponding MCS-like lagrangian density was obtained by means of the gauge embedding procedure. The duality was confirmed through the study of the two equations of motion, since it was found a mapping between the two vector fields $f_a$ and $A_a$, together with a mapping between the currents. The dualized model involves a Thirring-like current-current interaction and a magnetic coupling, as in [3].

In the sequence we have shown that there exists a master action which generates the two models. A further investigation was carried out to check the equivalence of the two spectra. Although the MCS-like model includes new poles, these new excitations have been shown to be restricted to nondynamical sectors of the propagator.

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