Hydrodynamic flow from RHIC to LHC*

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The hydrodynamic model for the expansion of the fireball in relativistic heavy-ion collisions is presented. Calculations using relativistic hydrodynamics of a fluid with small viscosity yield a satisfactory description of the experimental data on the particle spectra, the elliptic flow or the interferometry radii.

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1. Introduction

The matter created in relativistic heavy-ion collisions is strongly interacting. The dynamics of the system can be described as the relativistic expansion of a hot fluid [1, 2]. Deviations from local equilibrium in the dynamical system lead to effects of viscosity in the hydrodynamics [3, 4]. The collective flow of the fluid is indirectly observed in the transverse momentum spectra of produced particles, in the azimuthally asymmetric directed, elliptic and triangular flows, and in the interferometry radii. The experimental results for such soft observables at the top RHIC and LHC energies can be quantitatively understood using hydrodynamic models [5, 6, 7, 8, 9, 10].

Two conclusions on the nature of the hot and dense matter created can be deduced. First, the equation of state at small baryon density has a crossover transition from the quark-gluon plasma to the hadronic phase, in agreement with lattice QCD calculations [11]. Second, the shear viscosity to entropy ratio is small $\eta/s < 0.2$, close to the estimates from strongly coupled theories [12].

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2. Hydrodynamics

Second order viscous hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0$$  \hspace{1cm} (1)

for the evolution in the proper time \(\tau = \sqrt{t^2 - z^2}\) of the energy momentum tensor are usually solved in \(2 + 1\) \([5, 13, 14, 15, 7, 8]\) (but also in \(3 + 1\) dimensions \([8, 16]\)). The energy momentum tensor contains corrections from shear viscosity \(\pi^{\mu\nu}\) and from bulk viscosity \(\Pi^{\mu\nu}\), \(\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu\), and \(u^\mu\) is the fluid velocity. The corrections are solutions of differential equations \([17]\)

$$\Delta^{\mu\alpha} \Delta^{\nu\beta} u^\gamma \partial_\gamma \pi_{\alpha\beta} = \frac{2\eta \sigma^{\mu\nu} - \pi^{\mu\nu}}{\tau_\pi} - \frac{1}{2} \pi^{\mu\nu} \eta T^{\alpha}_{\alpha} \partial_\alpha \left( \frac{\tau_\pi u^\alpha}{\eta T} \right)$$  \hspace{1cm} (2)

and

$$u^\gamma \partial_\gamma \Pi = -\zeta \partial_\gamma u^\gamma \Pi^{\gamma} - \frac{1}{2} \Pi \frac{\xi T}{\tau_\Pi} \partial_\alpha \left( \frac{\tau_\Pi u^\alpha}{\xi T} \right)$$  \hspace{1cm} (3)

with the shear \(\eta\) and bulk viscosity \(\zeta\) coefficients, and the relaxation times \(\tau_\pi, \tau_\Pi\).

We use a boost-invariant viscous hydrodynamic model with parameters adjusted to RHIC data \([17]\) \(\eta/s = 0.08\) and \(\zeta/s = 0.04\) (bulk viscosity only in the hadronic phase). The initial profile of the density in the transverse plane is taken from the Glauber model \([18, 19, 20]\). The emission of particles at the freeze-out temperature of 135MeV is performed using the event generator THERMINATOR \([21]\) including non-equilibrium corrections to the momentum distribution from shear and bulk viscosity.

3. Results

The directed flow in ultrarelativistic collisions measures the deflection of the fluid motion from the beam axis. It can be a remnant of the initial flow or could result from the early dynamics of a deformed fireball \([22, 23]\). In 3+1-dimensional perfect fluid hydrodynamics, it can be generated in the expansion of a source tilted away from the collision axis. The tilt of the source originates from the preferential emission in the forward (backward) hemisphere from participant nucleons going in the forward (backward) direction \([24]\). Hydrodynamic results in Fig. \([1]\) show that the measured directed flow in central rapidities can be explained. As observed experimentally, the flow is similar in Au-Au and Cu-Cu collisions.

The transverse momentum spectra of particles produced in Au-Au collisions at the top RHIC energies can be quantitatively described in hydrodynamic models \([2, 5, 6, 7, 8, 9, 13, 14, 15]\). The shape of the spectra results
from a convolution of the collective flow of the fluid with the thermal emission at the freeze-out. In Pb-Pb collisions at $\sqrt{s} = 2760\text{GeV}$, the transverse collective flow is stronger. One observes a significant shift of the spectra of protons towards higher $p_{\perp}$ (Fig 2). For heavier particles ($\Xi$ and $\Omega$) the flow predicted by hydrodynamic models is very strong, showing itself as an increase of the mean $p_{\perp}$ with the particle mass (Fig. 3). The observed flow of heavy baryons is not as strong, which could indicate that multistrange baryons decouple earlier. Also their chemical decoupling temperature is higher than for protons [28].

The elliptic flow generated in the hydrodynamic expansion of a fluid with small viscosity is compatible with the observations [5, 6, 7, 8, 13, 14, 15, 19]. The main uncertainty in the estimation of the viscosity coefficient from phe-
Fig. 2. Transverse momentum spectra of identified particles in Pb-Pb collisions. Preliminary data of the ALICE Collaboration ($\pi^+$, $K^+$, $p$) for centrality $0 - 5\%$ [27] and ($\Xi^-$, $\Omega^-$) for centrality $0 - 20\%$ [28] scaled by 1.3 to compare with viscous hydrodynamic results for the centrality $0 - 5\%$ [19].

Fig. 3. Average $p_{\perp}$ of identified particles from viscous hydrodynamics [19] compared to ALICE Collaboration preliminary data [27].

Nomenclature studies comes from the uncertainty in the value of the initial eccentricity of the fireball. The elliptic flow coefficient of charged particles as function of the transverse momentum is very similar at RHIC and at the LHC [29]. The same is observed in hydrodynamic calculations, where the dependence of the elliptic flow $v_2(p_{\perp})$ as function of energy saturates. The
elliptic flow of identified particles shows splitting for particles of different masses. At LHC the splitting is stronger as the transverse flow is more important (Fig. 4). The elliptic flow of protons, kaons and pions can be well described in hybrid models connecting a hydrodynamic expansion stage with a hadronic cascade afterburner [30].

The triangular flow reflects shape fluctuations in the source [31, 32, 33, 8, 34]. The collective expansion of the fireball with a triangular deformation is very sensitive to the value of the shear viscosity. A common description of the elliptic and triangular flows could constraint the initial fluctuations and the value of the parameters. In Fig. 4 is shown the triangular flow of identified particles at the LHC. In our calculation, the triangular deformation is added to the optical Glauber model density following the prediction of the Glauber Monte-Carlo model. We observe a similar splitting in the value of $v_3$ between particles of different mass as for the elliptic flow coefficient. However, the same calculations that describes the elliptic flow of identified particles cannot reproduce the values for the triangular flow.

The identical particle interferometry is an important tool for measuring the size and the life-time of the interacting system [2, 37]. Hydrodynamic calculations with a hard equation of state yield reasonable values of the extracted interferometry radii (Fig. 5) at RHIC energies [9, 10, 38, 18, 39, 40]. At the LHC, the hydrodynamic transverse flow is stronger which gives an even better agreement of the interferometry radii with the data. We notice that nonzero viscosity or the presence of the pre-equilibrium flow improve the agreement for the $R_{out}/R_{side}$ ratio.

The high multiplicity of particles created in proton-proton collisions at LHC energies would indicate that some degree of collectivity could be achieved in the interaction region. Effects of the collective expansion of
Fig. 5. Interferometry radii for Au-Au collisions at $\sqrt{s} = 200\text{GeV}$ [35] (left panel) and Pb-Pb collisions at $\sqrt{s} = 2760\text{GeV}$ [36] (right panel) compared to viscous hydrodynamic calculations [18].

the matter in such collisions have been estimated [41, 42, 43, 44]. The observation of the ridge in the two-particle correlations in high multiplicity events in proton-proton collisions by the CMS Collaboration [45] can be interpreted as the existence of the elliptic flow [46, 47]. However, it is difficult to separate it from important non-flow effects. The presence of a significant collective flow could be observed through interferometry methods applied to proton-proton collisions [48, 42, 49, 50].

4. Conclusions

The hydrodynamic expansion of a hot and dense fluid represents a realistic model of the dynamics of the fireball in relativistic heavy-ion collisions. At RHIC energies the viscous hydrodynamic model can describe the particle spectra, the elliptic flow of charged and identified particles, the
interferometry radii, and the triangular flow. Different implementations of the hydrodynamic model in the literature, especially with respect to the initial conditions and the final hadron rescattering give a better agreement for some of the soft above mentioned observables than for other.

Using the hydrodynamic model for Pb-Pb collisions at the LHC, one finds a satisfactory description of most of the available data at soft transverse momenta. The collective transverse flow is stronger and as a consequence the mean $p_{\perp}$ of produced particles and the integrated elliptic flow increases as compared to RHIC. The interferometry radii show a dependence on the momentum of the pion pair characteristic for the collective flow. The elliptic flow of charged particles as function of $p_{\perp}$ is hydrodynamically saturated and does not change significantly when $\sqrt{s}$ increases from 200 to 2760GeV. The strong collective component in the spectra is visible for pions, kaons and protons, but is less so for multistrange baryons. This may indicate an earlier decoupling of heavy strange particles. The strong flow explains the particle mass splitting for the elliptic flow but cannot explain the one for the triangular flow. The appearance of the collective expansion in high multiplicity proton-proton collisions has been suggested, but it is difficult to evidence it experimentally.

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