Chiral symmetry and the constituent quark model:
A null-plane point of view

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In order to clarify the connection between current and constituent quarks (of u,d,s flavors), several authors have used the lightlike chiral $SU(3) \otimes SU(3)$ algebra as a central concept. This literature is reviewed here with the goal of offering an introduction to the subject within a convenient, unified framework. It is shown that the null-plane Hamiltonian for free massive fermions is chirally symmetric, provided only that the particles have equal masses ($SU(3)$ limit). In the free quark model, hadrons can be classified by a chiral $SU(3) \otimes SU(3)$ algebra, the generators of which are current lightlike charges. Naturally, QCD interactions break chiral symmetry, and the axial nonsinglet charges are not conserved. This remains true in the 'chiral limit' (zero quark masses), signalling the spontaneous breakdown of chiral symmetry. The actual generators of the $SU(3) \otimes SU(3)$ classification of physical hadrons are obtained from the current charges by means of a unitary transformation. Under this transformation, quarks of the same flavor and of opposite helicities mix to form a constituent quark. The functional form of this unitary transformation can be strongly constrained on the basis of symmetry arguments. This analysis is potentially rich in phenomenological applications, and a few are presented here. The author offers also some new results: 1) it is shown that the null-plane axial currents are different from their space-time counterparts, even in the $SU(3)$ limit; 2) when defining charges, the effect of boundary terms at infinity in the longitudinal direction is taken into consideration. No prior knowledge of light-cone formalism on the part of the reader is assumed.

CONTENTS
I. Introduction
II. Free Fermions
   A. Generalities
   B. Chiral Symmetries
   C. Chiral Charge and Fermion Number
   D. Flavor Symmetries
I. INTRODUCTION

“When you think about light hadrons, what picture comes naturally to your mind?” The answer to this question may be quite different from one physicist to another depending on his/her subspecialty. Indeed we have been using for many years two successful, but quite distinct, pictures of hadronic structure: Quantum Chromodynamics (QCD), and the Constituent Quark Model (CQM).

QCD emerged in the mid-70’s from Current Algebra (which gave birth to modern Chiral Perturbation Theory) on one hand, and from the Parton Model (itself a product of the SLAC experiments of the late 60’s) on the other. In Current Algebra, one makes use of the Partially Conserved Axial-Current hypothesis (PCAC), which states that light hadrons would be subjected to a fermionic symmetry called ‘chiral symmetry’ if only the pion mass was zero. If this were the case, the symmetry would be spontaneously broken, and the pions and kaons would be the corresponding Goldstone bosons. As seen by PCAC, the real world slightly misses this state of affairs by effects quantifiable in terms of the pion mass and decay constant. This violation can be expressed in terms of explicit symmetry breaking due to the nonzero masses of the fundamental fermion fields (quarks of three light flavors), and typically one assigns values of 4 MeV for the up-quark, 7MeV for the down-quark and 130 MeV for the strange-quark (Gasser and Leutwyler, 1975; Shifman et al., 1979). In a related approach, the MIT bag model, one neglects the masses of the up- and down-quarks, and uses a value of about 280 MeV for the strange-quark (Hasenfratz
and Kuti, 1978).

In the other picture, the Constituent Quark Model (which can be traced back to the Eightfold Way), one always describes mesons as made of a quark and an anti-quark, and baryons as made of three quarks (or three anti-quarks). These constituents are bound by some phenomenological potential which is tuned to account for hadrons’ properties such as masses, decay rates or magnetic moments.

Unfortunately, there exist severe contradictions between these two pictures:

• The QCD vacuum is an infinite sea of quark-antiquark pairs; since the nonrelativistic Constituent Model ascribes low momenta to the quarks, these constituents should be strongly coupled to the sea quarks; but then how can one identify two, or three, valence quarks in the sea?

• Since the coupling is strong, one expects that vacuum polarization effects should be important; but the Constituent Model cannot give account of these effects since there are no gluons, and the number of constituents is fixed.

• The Constituent Quark Model does not display any visible manifestation of spontaneous chiral symmetry breaking; actually, it totally prohibits such a symmetry since the constituent masses are large on a hadronic scale, typically of the order of one-half of a meson mass or one-third of a baryon mass; standard values are 330 MeV for the up- and down-quark, and 490 MeV for the strange-quark (Georgi, 1982), very far from the ‘current’ masses quoted above (even the ratio of the up- or down-quark mass to the strange-quark mass is vastly different).

• If one attempted to incorporate a bound gluon into the model, one would have to assign to it a mass at least of the order of magnitude of the quark mass in order to limit its impact on the classification scheme; but a gluon mass violates the gauge-invariance of QCD.

This long-standing incompatibility cannot be resolved in a nonrelativistic framework. This observation has provided motivation for a variety of relativistic CQM’s, which, at least in the case of light quark systems, certainly bring us closer to the phenomenology of QCD than the nonrelativistic models. Nonetheless, all of these models take as ansatz the existence of constituent quarks whose masses, after fitting to the observed spectrum, turn out to be much larger than current masses.

It is however possible to relate in a natural way the relativistic Constituent Quark...
Model to the underlying field theory, provided one sits in a null-plane frame rather than in space-time. We shall elaborate in great detail about why this choice of metric is key to exposing the connection we are seeking between current and constituent quarks\(^1\).

The idea of deriving a Null-Plane Constituent Model from QCD actually dates from the early seventies, and there is a rich literature on the subject (Buccella et al., 1970; De Alwis, 1973; Eichten et al., 1973; Bell, 1974; De Alwis and Stern, 1974; Leutwyler, 1974b, 1974c; Melosh, 1974; Osborn, 1974; Carlitz et al., 1975; Ida, 1975b, 1975c; Carlitz and Tung, 1976). The purpose of this article is to review in varying degrees of detail the theories that have been proposed, with the goal of presenting a self-consistent framework rather than trying to cover the subject exhaustively. Along the way, we clarify some obscure or little-known aspects, and offer some new results. Among the latter, we will show that the space-time and null-plane axial currents are distinct; this remark is at the root of the difference between the chiral properties of QCD in the two frames.

The emphasis of the present work will be on concepts rather than on calculational techniques, with special attention devoted to the contrasting implications of flavor symmetries in the two frames. Lengthy derivations will be offered only in a few cases where (to our knowledge) no proof has been provided in the literature. Otherwise, we will guide the interested reader to the relevant sources.

The organization of the article is a follows. In part II we present the vector and axial properties of free fermions, first for a single flavor, then in the framework of approximate SU(3) symmetry. In part III we discuss how QCD behaves under chiral flavor transformations, progressively working our way to the properties of real-world hadrons. The connection between current and constituent quarks is finally elucidated. In part IV, we summarize our results, and discuss a few unresolved issues.

II. FREE FERMIONS

A. Generalities

\(^1\) No prior knowledge of light-cone formalism on the part of the reader is assumed, and all necessary definitions are provided in the text.
In our metric, we use
\[ x^\pm \equiv \frac{x^0 \pm x^3}{\sqrt{2}} , \]
where \( x^+ \) is the 'light-cone time' and \( x^- \) is the 'longitudinal' coordinate.

Consider to begin with the free theory of fermions of a single flavor. From the Lagrangian density
\[ \mathcal{L} = \bar{\psi}(\frac{i}{2} \not{\partial} - m)\psi , \]
one derives the Dirac equation
\[ (i \not{\partial} - m)\psi = 0 , \quad \bar{\psi}(\not{i \partial} + m) = 0 , \]
and the energy-momentum tensor
\[ T^{\mu \nu} = \bar{\psi} \left[ \frac{i}{2} \gamma^\mu \not{\partial}^\nu + g^{\mu \nu} (m - \frac{i}{2} \not{\partial}) \right] \psi . \]

In a space-time frame, the energy-momentum operator is
\[ P^\mu = \int d^3x \ T^{\mu 0} . \]
In particular, the Hamiltonian is
\[ P^0 = \int d^3x \ T^{00} , \]
where
\[ T^{00} = \bar{\psi}(-i\gamma \cdot \not{\partial} + m)\psi . \]

In a null-plane frame, the 'energy-momentum' operator is
\[ P^\mu = \int d^3\tilde{x} \ T^{\mu +} , \]
where
\[ d^3\tilde{x} \equiv dx^- d^2\mathbf{x}_\perp . \]
In particular, the energy operator is
\[ P^- = \int d^3\tilde{x} \ T^{-} , \]
where
\[ T^{-+} = \frac{i}{2} \psi \gamma^- \partial_- \psi \] (10)
The matrices
\[ \Lambda_\pm \equiv \frac{\gamma^+ \gamma^+}{2}, \] (11)
where
\[ \gamma^\pm \equiv \frac{\gamma^0 \pm \gamma^3}{\sqrt{2}} , \] (12)
are Hermitian projection operators, viz.,
\[ (\Lambda_\pm)^2 = \Lambda_\pm , \quad \Lambda_\pm \Lambda_\mp = 0 , \quad \Lambda_+ + \Lambda_- = 1 . \] (13)

Their action on Dirac spinors yields
\[ \psi_\pm \equiv \Lambda_\pm \psi . \] (14)

The reason for splitting the Dirac field \( \psi \) into \( (\psi_+ + \psi_-) \) is that the Dirac equation shows that \( \psi_- \) is a dependent field. Indeed with, say, antiperiodic boundary conditions at \( x^-\) infinity, Eq. (2) can be solved for
\[ \psi_-(x) = -\frac{i}{4} \int dy^- \epsilon(x^- - y^-) (i\gamma_\perp \cdot \vec{\partial}_\perp + m)\gamma^+ \psi_+(y) , \] (15)
where \( y_\perp = x_\perp \). This means that the physical fermion states are built out of \( \psi_+ \). Substituting Eq. (15) into Eqs. (9) and (10), one finds after an integration by parts
\[ P^- = \frac{i\sqrt{2}}{4} \int d^3x \int dy^- \epsilon(x^- - y^-) \psi_+^\dagger(y) (m^2 - \Delta_\perp) \psi_+(x) . \] (16)

To any given transformation of the fermion field we associate a current
\[ \frac{\delta L}{\delta (\partial_\mu \psi)} \frac{\delta \psi}{\delta \theta} = i\bar{\psi} \gamma^\mu \frac{\delta \psi}{\delta \theta} , \] (17)
where \( \delta \psi \) is the infinitesimal variation parametrized by \( \theta \). For example the vector transformation is defined in space-time by
\[ \psi \mapsto e^{-i\theta} \psi \quad \text{,} \quad \delta \psi = -i\theta \psi \] (18)
whence the current
\[ j^\mu = \bar{\psi} \gamma^\mu \psi \]  \hspace{1cm} (19)

In a null-plane frame, in view of the constraints structure, the vector transformation will be defined as

\[ \psi_+ \mapsto e^{-i\theta} \psi_+ \hspace{0.5cm}, \hspace{0.5cm} \delta \psi_+ = -i\theta \psi_+ \hspace{0.5cm}, \hspace{0.5cm} \delta \psi = \delta \psi_+ + \delta \psi_- \]  \hspace{1cm} (20)

where \( \delta \psi_- \) is calculated using Eq. (15). The distinction in the case of vector U(1) is of course academic:

\[ \delta \psi_- = -i\theta \psi_- \hspace{1cm} \Rightarrow \hspace{1cm} \delta \psi = -i\theta \psi \]  \hspace{1cm} (21)

therefore

\[ \bar{j}^\mu = j^\mu \]  \hspace{1cm} (22)

In contrast, we shall see in the next section that the axial-vector currents (defined respectively in a space-time and in a null-plane frame) are not equal. Finally, using Eq. (2), one checks easily that the vector current is conserved:

\[ \partial_\mu j^\mu = 0 \]  \hspace{1cm} (23)

therefore the space-time and null-plane vector charges (which measure fermion number)

\[ Q \equiv \int d^3x \ j^0(x) \hspace{0.5cm}, \hspace{0.5cm} \tilde{Q} \equiv \int d^3\tilde{x} \ j^+(x) \]  \hspace{1cm} (24)

are equal (McCartor, 1988) (we show in Appendix A that the boundary term at \( x^- \)-infinity vanishes).

**B. Chiral symmetries**

The space-time chiral transformation is defined by

\[ \psi \mapsto e^{-i\theta \gamma_5} \psi \hspace{0.5cm}, \hspace{0.5cm} \delta \psi = -i\theta \gamma_5 \psi \]  \hspace{1cm} (25)

where \( \gamma_5 \equiv i\gamma^0\gamma^1\gamma^2\gamma^3 \). From Eqs. (5) and (6), one sees that the space-time theory with nonzero fermion masses is not chirally symmetric.

The null-plane chiral transformation is

\[ \psi_+ \mapsto e^{-i\theta \gamma_5} \psi_+ \hspace{0.5cm}, \hspace{0.5cm} \delta \psi_+ = -i\theta \gamma_5 \psi_+ \]  \hspace{1cm} (26)
This is a symmetry of the null-plane theory, as seen from Eq. (16), without requiring zero bare masses.

The space-time axial-vector current associated to the transformation Eq. (25) is

$$ j^\mu_5 = \bar{\psi} \gamma^\mu \gamma_5 \psi \ . \ (27) $$

Using Eq. (2), one obtains

$$ \partial_\mu j^\mu_5 = 2i m \bar{\psi} \gamma_5 \psi \ . \ (28) $$

As expected, this current is not conserved for nonzero fermion mass. The associated charge is

$$ Q_5 \equiv \int d^3 x \ j_5^0 = \int d^3 x \ \bar{\psi} \gamma^0 \gamma_5 \psi \ . \ (29) $$

Inserting Eq. (26) in Eq. (15), and using \{\gamma^\mu, \gamma_5\} = 0, one finds

$$ \delta \psi_-(x) = -\theta \gamma_5 \int dy - \frac{\epsilon(x^- - y^-)}{4} (i \vec{\gamma}_\perp \cdot \vec{\partial}_\perp - m) \gamma^+ \psi_+(y) \ . \ (30) $$

This expression differs from

$$ -i \theta \gamma_5 \psi_- = -\theta \gamma_5 \int dy - \frac{\epsilon(x^- - y^-)}{4} (i \vec{\gamma}_\perp \cdot \vec{\partial}_\perp + m) \gamma^+ \psi_+(y) \ (31) $$

(again, for nonzero masses), therefore \( \bar{j}^\mu_5 \neq j^\mu_5 \) (except for the plus component, due to \( (\gamma^+)^2 = 0 \)). To be precise,

$$ \bar{j}^\mu_5 = j^\mu_5 + i m \bar{\psi} \gamma^\mu \gamma_5 \int dy - \frac{\epsilon(x^- - y^-)}{2} \gamma^+ \psi_+(y) \ . \ (32) $$

Using Eqs. (28) and (32), and the identity

$$ 0 \int dy - \frac{\epsilon(x^- - y^-)}{4} \gamma^+ \psi_+(y) = \psi_+(x) + \int dy - \frac{\epsilon(x^- - y^-)}{4} \vec{\gamma}_\perp \cdot \vec{\partial}_\perp \gamma^+ \psi_+(y) \ , \ (33) $$

a straightforward calculation shows that

$$ \partial_\mu \bar{j}^\mu_5 = 0 \ (34) $$

as expected. Finally the null-plane chiral charge is

$$ \bar{Q}_5 \equiv \int d^3 \bar{x} \ \bar{j}^+_5 = \int d^3 \bar{x} \ \bar{\psi} \gamma^+ \gamma_5 \psi \ (35) $$

8
(one can write here $\psi$ or $\psi_+$ indifferently due to the $\gamma^+$ factor).

C. Chiral charge and fermion number

From the canonical anti-commutator

$$\{\psi(x), \psi^\dagger(y)\}_{x^0-y^0} = \delta^3(x - y),$$

one derives

$$[\psi, Q_5] = \gamma_5\psi \implies [Q, Q_5] = 0,$$

so that fermion number, viz., the number of quarks minus the number of anti-quarks is conserved by the chiral charge. However, the latter are not conserved separately. This can be seen by using the momentum expansion of the field

$$\psi(x) = \int \frac{d^3p}{(2\pi)^3/2} \sum_{s=\pm 1} \left[ u(p, s)e^{-ipx}b(p, s) + v(-p, -s)e^{+ipx}d^\dagger(p, s) \right],$$

where $px \equiv p^0x^0 - p \cdot x$, $p^0 = \sqrt{p^2 + m^2}$, and

$$\{b(p, s), b^\dagger(q, s')\} = 2p^0\delta^3(p - q)\delta_{ss'} = \{d(p, s), d^\dagger(q, s')\},$$

$$\sum_{s=\pm 1} u(p, s)\bar{u}(p, s) = \rho + m, \quad \sum_{s=\pm 1} v(-p, s)\bar{v}(-p, s) = \rho - m.$$

Inserting Eq. (38) in Eq. (29) yields indeed

$$Q_5 = \int \frac{d^3p}{2p^0} \sum_{s=\pm 1} s \left[ \left| \frac{p}{p^0} \right| \left( b^\dagger(p, s)b(p, s) + d^\dagger(p, s)d(p, s) \right) ight]$$

$$+ \frac{m}{p^0} \left( d^\dagger(-p, s)b^\dagger(p, s)e^{2ip^0t} + b(p, s)d(-p, s)e^{-2ip^0t} \right).$$

This implies that when $Q_5$ acts on a hadronic state, it will add or absorb a continuum of quark-antiquark pairs (the well-known pion pole) with a probability amplitude proportional to the fermion mass and inversely proportional to the energy of the pair. Because of that, $Q_5$ is most unsuited for classification purposes.
In contrast, the null-plane chiral charge conserves not only fermion number (electric charge), but also the number of quarks and anti-quarks separately. In effect, the canonical anti-commutator is
\[
\{\psi_+(x), \psi_+^\dagger(y)\}_{x^+=y^+} = \frac{\Lambda_+}{\sqrt{2}} \delta^3(\vec{x} - \vec{y}), \quad (42)
\]
hence the momentum expansion of the field reads
\[
\psi_+(x) = \int \frac{d^3\vec{p}}{(2\pi)^3/2\gamma^4} \sqrt{p^+} \sum_{h=\pm \frac{1}{2}} \left[ w(h)e^{-ipx}b(\vec{p}, h) + w(-h)e^{ipx}d^\dagger(\vec{p}, h) \right], \quad (43)
\]
where \( px \equiv p^-x^+ + p^+x^- - p_\perp \cdot x_\perp \), \( p^- = \frac{p_3^2 + m^2}{2p^+} \), and
\[
\{b(\vec{p}, h), b^\dagger(\vec{q}, h')\} = 2p^+\delta^3(\vec{p} - \vec{q})\delta_{hh'} = \{d(\vec{p}, h), d^\dagger(\vec{q}, h')\}, \quad (44)
\]
\[
\sum_{h=\pm \frac{1}{2}} w(h)w^\dagger(h) = \Lambda_+. \quad (45)
\]
(In the rest frame of a system, its total angular momentum along the z-axis is called ‘null-plane helicity’; the helicity of an elementary particle is just the usual spin projection; we label the eigenvalues of helicity with the letter ‘h’.) It is easiest to work in the so-called ‘chiral representation’ of Dirac matrices, where
\[
\gamma_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}, \quad w(+\frac{1}{2}) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad w(-\frac{1}{2}) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \Rightarrow \quad w^\dagger(h)\gamma_5w(h') = 2h\delta_{hh'}. \quad (46)
\]
Inserting Eq. (43) into Eq. (35), one finds
\[
\tilde{Q}_5 = \int \frac{d^3\vec{p}}{2p^+} \sum_h 2h \left[ b^\dagger(\vec{p}, h)b(\vec{p}, h) + d^\dagger(\vec{p}, h)d(\vec{p}, h) \right]. \quad (47)
\]
This is just a superposition of fermion and anti-fermion number operators, and thus our claim is proved. This expression also shows that \( \tilde{Q}_5 \) annihilates the vacuum, and that it simply measures (twice) the sum of the helicities of all the quarks and anti-quarks (‘constituents’) of a given state. Indeed, in a null-plane frame, the handedness of an individual fermion is automatically determined by its helicity. To show this, note that
\[
\gamma_5w(\pm \frac{1}{2}) = \pm w(\pm \frac{1}{2}) \quad \Rightarrow \quad \frac{1 \pm \gamma_5}{2}w(\pm \frac{1}{2}) = w(\pm \frac{1}{2}), \quad \frac{1 \pm \gamma_5}{2}w(\pm \frac{1}{2}) = 0. \quad (48)
\]
Defining as usual
\[ \psi_{+R} \equiv \frac{1 + \gamma_5}{2} \psi_+ , \quad \psi_{+L} \equiv \frac{1 - \gamma_5}{2} \psi_+ , \]
(49)

it follows from Eqs. (43) and (48) that \( \psi_{+R} \) contains only fermions of helicity \( +\frac{1}{2} \) and anti-fermions of helicity \( -\frac{1}{2} \), while \( \psi_{+L} \) contains only fermions of helicity \( -\frac{1}{2} \) and anti-fermions of helicity \( +\frac{1}{2} \). Also, we see that when acted upon by the right- and left-hand charges
\[ \tilde{Q}_R \equiv \frac{\tilde{Q} + \tilde{Q}_5}{2} , \quad \tilde{Q}_L \equiv \frac{\tilde{Q} - \tilde{Q}_5}{2} , \]
(50)

a chiral fermion (resp. anti-fermion) state may have eigenvalues \( +1 \) (resp. \( -1 \)) or zero.

In a space-time frame, this identification between helicity and chirality applies only to massless fermions.

D. Flavor symmetries

We proceed now to the theory of three flavors of free fermions \( \psi_f \), where \( f = u, d, s \). Based on Eqs. (5) and (6), the space-time Hamiltonian is
\[ P^0 = \sum_f \int d^3x \bar{\psi}_f ( -i \vec{\gamma} \cdot \vec{\partial} + m_f ) \psi_f \]
\[ = \int d^3x \bar{\psi} ( -i \vec{\gamma} \cdot \vec{\partial} + M ) \psi , \]
(51)

where now
\[ \psi \equiv \begin{bmatrix} \psi_u \\ \psi_d \\ \psi_s \end{bmatrix} , \quad M \equiv \begin{bmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{bmatrix} . \]

The vector, and axial-vector, flavor nonsinglet transformations are defined respectively as
\[ \psi \mapsto e^{-i \frac{\lambda^\alpha}{2} \theta^\alpha} \psi , \quad \psi \mapsto e^{-i \frac{\lambda^\alpha}{2} \gamma^\alpha \gamma_5} \psi , \]
(52)

where the summation index \( \alpha \) runs from 1 to 8. \( P^0 \) is invariant under vector transformations if the quarks have equal masses (‘\( SU(3) \) limit’), and invariant under chiral transformations if all masses are zero (‘chiral limit’).

From Eq. (16), the null-plane Hamiltonian is
\[ P^- = \sum_f \frac{i \sqrt{2}}{4} \int d^3 \bar{x} \int dy^- \epsilon(x^- - y^-) \psi^\dagger_{f+}(y) (m_f^2 - \Delta_\perp) \psi_{f+}(x) \\
= \frac{i \sqrt{2}}{4} \int d^3 \bar{x} \int dy^- \epsilon(x^- - y^-) \psi^\dagger_{+}(y) (M^2 - \Delta_\perp) \psi_{+}(x) . \]
(53)
Naturally, $P^-$ is not invariant under the vector transformations

$$\psi_+ \mapsto e^{-i \frac{\lambda^\alpha}{2} \sigma^\alpha} \psi_+$$  \hspace{1cm} (54)$$

unless the quarks have equal masses. But if they do, then $P^-$ is also invariant under the chiral transformations

$$\psi_+ \mapsto e^{-i \frac{\lambda^\alpha}{2} \sigma^\alpha \gamma_5} \psi_+$$  \hspace{1cm} (55)$$

whether this common mass is zero or not.

Writing the Dirac equation for the three flavors in compact form:

$$(i \not\partial - M)\psi = 0 \quad , \quad \bar{\psi}(i \not\partial + M) = 0 \quad ,$$  \hspace{1cm} (56)$$

one finds that the space-time currents

$$j^{\mu\alpha} = \bar{\psi} \gamma^\mu \frac{\lambda^\alpha}{2} \psi \quad , \quad j_5^{\mu\alpha} = \bar{\psi} \gamma^{5\mu} \frac{\lambda^\alpha}{2} \psi \quad ,$$  \hspace{1cm} (57)$$

have the following divergences:

$$\partial_\mu j^{\mu\alpha} = i\bar{\psi} \left[ M, \frac{\lambda^\alpha}{2} \right] \psi \quad , \quad \partial_\mu j_5^{\mu\alpha} = i\bar{\psi} \gamma_5 \left\{ M, \frac{\lambda^\alpha}{2} \right\} \psi \quad .$$  \hspace{1cm} (58)$$

These currents have obviously the expected conservation properties.

Turning to the null-plane frame, we rewrite Eq. (15) as

$$\psi_-(x) = -\frac{i}{4} \int dy^- \epsilon(x^- - y^-) (i\gamma_{\perp} \cdot \vec{\partial}_{\perp} + M) \gamma^+ \psi_+(y) \quad .$$  \hspace{1cm} (59)$$

Then we find

$$\tilde{j}^{\mu\alpha} = j^{\mu\alpha} - i\bar{\psi} \left[ M, \frac{\lambda^\alpha}{2} \right] \gamma^\mu \int dy^- \frac{\epsilon(x^- - y^-)}{4} \gamma^+ \psi_+(y) \quad .$$  \hspace{1cm} (60)$$

So $\tilde{j}^{\mu\alpha}$ and $j^{\mu\alpha}$ may be equal for all $\mu$ only if the quarks have equal masses. The vector, flavor nonsinglet charges in each frame are two different octets of operators, except in the $SU(3)$ limit.

For the null-plane current associated with axial transformations, we get

$$\tilde{j}_5^{\mu\alpha} = j_5^{\mu\alpha} - i\bar{\psi} \left\{ M, \frac{\lambda^\alpha}{2} \right\} \gamma_5 \gamma^\mu \int dy^- \frac{\epsilon(x^- - y^-)}{4} \gamma^+ \psi_+(y) \quad .$$  \hspace{1cm} (61)$$
Hence $\bar{j}_5^{\mu \alpha}$ and $j_5^{\mu \alpha}$ are not equal (except for $\mu = +$), even in the $SU(3)$ limit, unless all quark masses are zero. Finally, one obtains the following divergences:

$$\partial_{\mu} \bar{j}_5^{\mu \alpha} = \bar{\psi} \left[ M^2, \frac{\lambda^{\alpha}}{2} \right] \int dy^- \epsilon(x^- - y^-) \frac{\gamma^+ \psi(y)}{4},$$

$$\partial_{\mu} j_5^{\mu \alpha} = -\bar{\psi} \left[ M^2, \frac{\lambda^{\alpha}}{2} \right] \gamma_5 \int dy^- \epsilon(x^- - y^-) \frac{\gamma^+ \psi(y)}{4}.$$  \hspace{1cm} (62)

As expected, both null-plane currents are conserved in the $SU(3)$ limit, without requiring zero masses. Also note how null-plane relations often seem to involve the masses squared, while the corresponding space-time relations are linear in the masses. The integral operator

$$\int dy^- \frac{\epsilon(x^- - y^-)}{2} \equiv \frac{1}{\partial^-}$$

compensates for the extra power of mass.

E. Lightlike chiral algebra

The associated null-plane charges are

$$\tilde{Q}^{\alpha} \equiv \int d^3 \bar{x} \bar{\psi} \gamma^+ \frac{\lambda^{\alpha}}{2} \psi, \quad \tilde{Q}_5^{\alpha} \equiv \int d^3 \bar{x} \bar{\psi} \gamma^5 \frac{\lambda^{\alpha}}{2} \psi.$$  \hspace{1cm} (63)

Using the momentum expansion of the fermion triplet Eq. (43), where now

$$b(\tilde{p}, h) \equiv \begin{bmatrix} b_u(\tilde{p}, h) \\ b_d(\tilde{p}, h) \\ b_s(\tilde{p}, h) \end{bmatrix}, \quad \text{and} \quad d(\tilde{p}, h) \equiv \begin{bmatrix} d_u(\tilde{p}, h) \\ d_d(\tilde{p}, h) \\ d_s(\tilde{p}, h) \end{bmatrix},$$

one can express the charges as

$$\tilde{Q}^{\alpha} = \int \frac{d^3 \tilde{p}}{2p^+} \sum_h \left[ b^\dagger(\tilde{p}, h) \frac{\lambda^{\alpha}}{2} b(\tilde{p}, h) - d^\dagger(\tilde{p}, h) \frac{\lambda^{\alpha} T}{2} d(\tilde{p}, h) \right],$$  \hspace{1cm} (64)

$$\tilde{Q}_5^{\alpha} = \int \frac{d^3 \tilde{p}}{2p^+} \sum_h 2h \left[ b^\dagger(\tilde{p}, h) \frac{\lambda^{\alpha}}{2} b(\tilde{p}, h) + d^\dagger(\tilde{p}, h) \frac{\lambda^{\alpha} T}{2} d(\tilde{p}, h) \right],$$  \hspace{1cm} (65)

where the superscript $T$ denotes matrix transposition. Clearly all sixteen charges annihilate the vacuum (Jersák and Stern, 1968, 1969; Leutwyler, 1968, 1969; Ida, 1975a; Sazdjian and Stern, 1975).
As $\tilde{Q}^\alpha$ and $\tilde{Q}^5_\alpha$ conserve the number of quarks and anti-quarks separately, these charges are well-suited for classifying hadrons in terms of their valence constituents, \textit{whether the quarks masses are equal or not} (De Alwis and Stern, 1974). Since the charges commute with $P^+$ and $P_\perp$, all hadrons belonging to the same multiplet have same momentum. But this common value of momentum is arbitrary, because in a null-plane frame one can boost between any two values of momentum, using only kinematic operators.

Using Eq. (42) and the $SU(3)$ commutation relations, one finds that these charges generate an $SU(3) \otimes SU(3)$ algebra:

$$
[\tilde{Q}^\alpha, \tilde{Q}^\beta]\ = \, i \, f_{\alpha\beta\gamma} \tilde{Q}^\gamma , \quad [\tilde{Q}^\alpha, \tilde{Q}^5_\beta]\ = \, i \, f_{\alpha\beta\gamma} \tilde{Q}^\gamma_5 , \quad [\tilde{Q}^5_\alpha, \tilde{Q}^5_\beta]\ = \, i \, f_{\alpha\beta\gamma} \tilde{Q}^\gamma , \quad (66)
$$

and the corresponding right- and left-hand charges generate two commuting algebras denoted $SU(3)_R$ and $SU(3)_L$ (Jers´ ak and Stern, 1968, 1969; Leutwyler, 1968, 1969, 1974b, 1974c; Buccella \textit{et al.}, 1970; De Alwis, 1973; Eichten \textit{et al.}, 1973; Feinberg, 1973; Bell, 1974; De Alwis and Stern, 1974; Ida, 1974, 1975a, 1975b, 1975c; Melosh, 1974; Osborn, 1974; Carlitz \textit{et al.}, 1975; Sazdjian and Stern, 1975; Carlitz and Tung, 1976)\textsuperscript{1}.

Since

$$
[\psi_+, \tilde{Q}^5_\alpha]\ = \, \gamma_5 \frac{\lambda^\alpha}{2} \psi_+ , \quad (67)
$$

the quarks form an irreducible representation of this algebra. To be precise, the quarks (resp. anti-quarks) with helicity $+\frac{1}{2}$ (resp. $-\frac{1}{2}$) transform as a triplet of $SU(3)_R$ and a singlet of $SU(3)_L$, the quarks (resp. anti-quarks) with helicity $-\frac{1}{2}$ (resp. $+\frac{1}{2}$) transform as a triplet of $SU(3)_L$ and a singlet of $SU(3)_R$. Then for example the ordinary vector $SU(3)$ decuplet of $J = \frac{3}{2}$ baryons with $h = +\frac{3}{2}$ is a pure right-handed $(10,1)$ under $SU(3)_R \otimes SU(3)_L$. The octet ($J = \frac{1}{2}$) and decuplet ($J = \frac{3}{2}$) with $h = +\frac{1}{2}$ transform together as a $(6,3)$. For bosonic states we expect both chiralities to contribute with equal probability. For example, the octet of pseudoscalar mesons arises from a superposition of irreducible representations of $SU(3)_R \otimes SU(3)_L$:

$$
|J^{PC} = 0^{-+} > = \frac{1}{\sqrt{2}} \left| \begin{array}{c} (8, 1) - (1, 8) \end{array} \right> , \quad (68)
$$

\textsuperscript{1} Most of these papers in fact study a larger algebra of lightlike charges, namely $SU(6)$, but the subalgebra $SU(3)_R \otimes SU(3)_L$ suffices for our purposes.
while the octet of vector mesons with zero helicity corresponds to

$$|J^{PC} = 1^- > = \frac{1}{\sqrt{2}} |(8, 1) + (1, 8) > , \quad (69)$$

and so on. These low-lying states have $L_z = 0$, where

$$L_z = -i \int d^3 \bar{x} \bar{\psi} \gamma^\perp (x^1 \partial_2 - x^2 \partial_1) \psi \quad (70)$$

is the orbital angular momentum along $z$.

In the realistic case of unequal masses, the chiral charges are not conserved. Hence they generates multiplets which are not mass-degenerate — a welcome feature. The fact that the invariance of the vacuum does not enforce the 'invariance of the world' (viz., of energy), in sharp contrast with the order of things in space-time (Coleman’s theorem), is yet another remarkable property of the null-plane frame.

F. Conclusions

In contrast with the space-time picture, free null-plane current quarks are also constituent quarks because:

- They can be massive without preventing chiral symmetry, which we know is (approximately) obeyed by hadrons.
- They form a basis for a classification of hadrons under the lightlike chiral algebra.

III. QUANTUM CHROMODYNAMICS

A. Explicit breaking of chiral symmetry

In the quark-quark-gluon vertex $gj^\mu A_\mu$, the transverse component of the vector current is

$$j^\perp(x) = \cdots + \frac{im}{4} \int dy^- \epsilon(x^- - y^-) \left[ \bar{\psi}_+(y) \gamma^\perp \bar{\gamma}_\perp \psi_+(x) + \bar{\psi}_+(x) \gamma^\perp \bar{\gamma}_\perp \psi_+(y) \right] , \quad (71)$$

where the dots represent chirally symmetric terms, and where color, as well as flavor, factors and indices have been omitted for clarity. The term explicitly written out breaks chiral symmetry for nonzero quark mass. Not surprisingly, it generates vertices in which the two quark lines have opposite helicity.
The canonical anticommutator Eq. (42) for the bare fermion fields still holds in the interactive theory (for each flavor). The momentum expansion Eq. (43) gets rewritten as

\[ \psi_+(x) = \int \frac{d^3\tilde{p}}{(2\pi)^{3/2}2^{3/4}\sqrt{p^+}} \sum_{h=\pm\frac{1}{2}} \left[ w(h)e^{-i\tilde{p}\tilde{x}}b(\tilde{p}, h, x^+) + w(-h)e^{i\tilde{p}\tilde{x}}d^\dagger(\tilde{p}, h, x^+) \right], \]

(72)

where

\[ \tilde{p}\tilde{x} \equiv p^+x^- - \mathbf{p}_\perp \cdot \mathbf{x}_\perp, \]

(73)

and

\[ \{b(\tilde{p}, h, x^+), b^\dagger(\tilde{q}, h', y^+)\}_{x^+=y^+} = 2p^+\delta^3(\tilde{p} - \tilde{q})\delta_{hh'} = \{d(\tilde{p}, h, x^+), d^\dagger(\tilde{q}, h', y^+)\}_{x^+=y^+}. \]

(74)

The momentum expansions of the lightlike charges remain the same as in Eqs. (64)-(65) (keeping in mind that the creation and annihilation operators are now unknown functions of ‘time’). Hence the charges still annihilate the Fock vacuum, and are suitable for classification purposes.

We do not require annihilation of the physical vacuum (QCD ground state). The successes of CQM’s suggest that to understand the properties of the hadronic spectrum, it may not be necessary to take the physical vacuum into account. This is also the point of view taken by the authors of a recent paper on the renormalization of QCD (Glazek et al., 1994). Their approach consists in imposing an ‘infrared’ cutoff in longitudinal momentum, and in compensating for this suppression by means of Hamiltonian counterterms. Now, only terms that annihilate the Fock vacuum are allowed in their Hamiltonian \( P^- \). Since all states in the truncated Hilbert space have strictly positive longitudinal momentum except for the Fock vacuum (which has \( p^+ = 0 \)), the authors hope to be able to adjust the renormalizations in order to fit the observed spectrum, without having to solve first for the physical vacuum.

B. Dynamical breaking of chiral symmetry

Making the standard choice of gauge: \( A_- = 0 \), one finds that the properties of vector and axial-vector currents (Eqs. (57)-(62)) are also unaffected by the inclusion of QCD interactions, except for the replacement of the derivative by the covariant derivative
in Eq. (59). The divergence of the renormalized, space-time, nonsinglet axial current is anomaly-free (Collins, 1984). As \( j_5^{\mu\alpha} \) and \( \tilde{j}_5^{\mu\alpha} \) become equal in the chiral limit, the divergence of the null-plane current is also anomaly-free (and goes to zero in the chiral limit). The corresponding charges, however, do not become equal in the chiral limit. This can only be due to contributions at \( x^-\)-infinity coming from the Goldstone boson fields, which presumably cancel the pion pole of the space-time axial charges\(^1\).

From soft pion physics we know that the chiral limit of \( SU(2) \otimes SU(2) \) is well-described by PCAC. Now, using PCAC one can show that in the chiral limit \( Q_5^\alpha \) (\( \alpha = 1, 2, 3 \)) is conserved, but \( \tilde{Q}_5^\alpha \) is not (Ida, 1974; Carlitz et al., 1975). In other words, the renormalized null-plane charges are sensitive to spontaneous symmetry breaking, although they do annihilate the vacuum. It is likely that this behavior generalizes to \( SU(3) \otimes SU(3) \), viz., to the other five lightlike axial charges. Its origin, again, must lie in terms at infinity/zero modes.

In view of this 'time'-dependence, one might wonder whether the null-plane axial charges are observables. From PCAC, we know that it is indeed the case: their matrix elements between hadron states are directly related to off-shell pion emission (Feinberg, 1973; Carlitz et al., 1975). For a hadron \( A \) decaying into a hadron \( B \) and a pion, one finds

\[
< B|\tilde{Q}_5^\alpha(0)|A> = -\frac{2i(2\pi)^3\tilde{p}_A^+}{m_A^2-m_B^2} < B, \pi^\alpha|A> \delta^3(\tilde{p}_A - \tilde{p}_B).
\]

(75)

Note that in this reaction, the mass of hadron \( A \) must be larger than the mass of \( B \) due to the pion momentum.

C. Physical multiplets

Naturally, we shall assume that real hadrons fall into representations of an \( SU(3) \otimes SU(3) \) algebra. In Sec. II.E, we have identified the generators of this algebra with the lightlike chiral charges. But this was done in the artificial case of the free quark model. It remains to check whether this identification works in the real world.

Of course, we already know that the predictions based on isospin (\( \alpha = 1, 2, 3 \)) and hypercharge (\( \alpha = 8 \)) are true. Also, the nucleon-octet ratio \( D/F \) is correctly predicted

\(^1\) Equivalently, if one chooses periodic boundary conditions, one can say that this effect comes from the longitudinal zero modes of the fundamental fields.
to be $3/2$, and several relations between magnetic moments match well with experimental data.

Unfortunately, several other predictions are in disagreement with observations (Close, 1979). For example, $G_A/G_V$ for the nucleon is expected to be equal to $5/3$, while the experimental value is about 1.25. Dominant decay channels such as $N^* \rightarrow N\pi$, or $b_1 \rightarrow \omega\pi$, are forbidden by the lightlike current algebra. The anomalous magnetic moments of nucleons, and all form factors of the rho-meson would have to vanish. De Alwis and Stern (1974) point out that the matrix element of $\tilde{j}^{\mu\alpha}$ between two given hadrons would be equal to the matrix element of $\tilde{j}_5^{\mu\alpha}$ between the same two hadrons, up to a ratio of Clebsch-Gordan coefficients. This is excluded though because vector and axial-vector form factors have very different analytic properties as functions of momentum-transfer.

In addition there is, in general, disagreement between the values of $L_z$ assigned to any given hadron. This comes about because in the classification scheme, the value of $L_z$ is essentially an afterthought, when group-theoretical considerations based on flavor and helicity have been taken care of. On the other hand, at the level of the current quarks, this value is determined by covariance and external symmetries. Consider for example the $L_z$ assignments in the case of the pion, and of the rho-meson with zero helicity. As we mentioned earlier, the classification assigns to these states a pure value of $L_z$, namely zero. However, at the fundamental level, one expects these mesons to contain a wave-function $\phi_1$ attached to $L_z = 0$ (antiparallel $q\bar{q}$ helicities), and also a wave-function $\phi_2$ attached to $L_z = \pm 1$ (parallel helicities). Actually, the distinction between the pion and the zero-helicity rho is only based on the different momentum-dependence of $\phi_1$ and $\phi_2$ (Leutwyler, 1974b, 1974c). If the interactions were turned off, $\phi_2$ would vanish and the masses of the two mesons would be degenerate (and equal to $(m_u + m_d)$).

We conclude from this comparison with experimental data, that if indeed real hadrons are representations of some $SU(3) \otimes SU(3)$ algebra, then the generators $G_\alpha$ and $G_5^\alpha$ of this classifying algebra must be different from the current lightlike charges $\tilde{Q}^\alpha$ and $\tilde{Q}_5^\alpha$ (except however for $\alpha = 1, 2, 3, 8$). Furthermore, in order to avoid the phenomenological discrepancies discussed above, one must forego kinematical invariance for these generators; that is, $G_\alpha(\tilde{k})$ and $G_5^\alpha(\tilde{k})$ must depend on the momentum $\tilde{k}$ of the hadrons in a particular irreducible multiplet.
Does that mean that our efforts to relate the physical properties of hadrons to the underlying field theory turn out to be fruitless? Fortunately no, as argued by De Alwis and Stern (1974). The fact that these two sets of generators (the $\tilde{Q}$’s and the $G$’s) act in the same Hilbert space, in addition to satisfying the same commutation relations, implies that they must actually be unitary equivalent (this equivalence was originally suggested by Dashen, and by Gell-Mann, 1972a, 1972b). There exists a set of momentum-dependent unitary operators $U(\tilde{k})$ such that

$$G^\alpha(\tilde{k}) = U(\tilde{k})\tilde{Q}^\alpha U^\dagger(\tilde{k}) \quad G_5^\alpha(\tilde{k}) = U(\tilde{k})\tilde{Q}_5^\alpha U^\dagger(\tilde{k}) \quad (76)$$

Current quarks, and the real-world hadrons built out of them, fall into representations of this algebra. Equivalently (e.g., when calculating electroweak matrix elements), one may consider the original current algebra, and define its representations (namely, the ones constructed in Sec. II.E) as ‘constituent’ quarks and ‘constituent’ hadrons. These quarks (and antiquarks) within a hadron of momentum $\tilde{k}$ are represented by a ‘constituent fermion field’

$$\chi_+^\alpha(x)\big|_{x^+=0} \equiv U(\tilde{k})\psi_+(x)\big|_{x^+=0} U^\dagger(\tilde{k}) \quad (77)$$

on the basis of which the physical generators can be written in canonical form:

$$\tilde{G}^\alpha \equiv \int d^3\tilde{x} \bar{\chi} \gamma^+ \frac{\lambda^\alpha}{2} \chi \quad \tilde{G}_5^\alpha \equiv \int d^3\tilde{x} \bar{\chi} \gamma^+ \gamma^5 \frac{\lambda^\alpha}{2} \chi \quad (78)$$

From Eq. (77), it follows that the constituent annihilation/creation operators are derived from the current operators via

$$a^\dagger_\tilde{k}(\tilde{p},h) \equiv U(\tilde{k})b(\tilde{p},h)U^\dagger(\tilde{k}) \quad c^\dagger_\tilde{k}(\tilde{p},h) \equiv U(\tilde{k})d^\dagger(\tilde{p},h)U^\dagger(\tilde{k}) \quad (79)$$

Due to isospin invariance, this unitary transformation cannot mix flavors, it only mixes helicities. It can therefore be represented by three unitary $2 \times 2$ matrices $T^f(\tilde{k},\tilde{p})$ such that

$$a^\dagger_f(\tilde{p},h) = \sum_{h'=\pm\frac{1}{2}} T^f_{hh'}(\tilde{k},\tilde{p}) b_f(\tilde{p},h') \quad c^\dagger_f(\tilde{p},h) = \sum_{h'=\pm\frac{1}{2}} T^f_{hh'}(\tilde{k},\tilde{p}) d_f(\tilde{p},h') \quad (80)$$

one for each flavor $f = u,d,s$. Since we need the transformation to be unaffected when $\tilde{k}$ and $\tilde{p}$ are boosted along $z$ or rotated around $z$ together, the matrix $T$ must actually be a
function of only kinematical invariants. These are
\[
\xi \equiv \frac{p^+}{k^+} \quad \text{and} \quad \kappa_\perp \equiv p_\perp - \xi k_\perp, \quad \text{where} \quad \sum_{\text{constituents}} \xi = 1, \quad \sum_{\text{constituents}} \kappa_\perp = 0. \quad (81)
\]
Invariance under time reversal \((x^+ \mapsto -x^+)\) and parity \((x^1 \mapsto -x^1)\) further constrain its functional form, so that finally
\[
T_f(k, \bar{p}) = \exp \left[ -i \frac{k_\perp}{|\kappa_\perp|} \cdot \sigma_\perp \beta_f(\xi, \kappa_\perp^2) \right] \quad (82)
\]
(Leutwyler, 1974b, 1974c).

Thus the relationship between current and constituent quarks is embodied in the three functions \(\beta_f(\xi, \kappa_\perp^2)\), which we must try to extract from comparison with experiment. (In first approximation it is legitimate to take \(\beta_u\) and \(\beta_d\) equal since \(SU(2)\) is such a good symmetry.)

Based on some assumptions abstracted from the free-quark model (as suggested by Fritzsch and Gell-Mann, 1972), Leutwyler (1974a, 1974b, 1974c) has derived a set of sum rules obeyed by mesonic wave-functions. Implementing then the transformation described above, Leutwyler finds various relations involving form factors and scaling functions of mesons, and computes the current quark masses. For example, he obtains
\[
F_\pi < F_\rho, \quad F_\rho = 3 F_\omega, \quad F_\rho < \frac{3}{\sqrt{2}} |F_\phi|, \quad (83)
\]
and the \(\omega/\phi\) mixing angle is estimated to be about 0.07 rad. Leutwyler (1974a) also shows that the average transverse momentum of a quark inside a meson is substantial \((|p_\perp|_{\text{rms}} > 400 \text{ MeV})\), thus justifying \(a \text{ posteriori}\) the basic assumptions of the relativistic CQM (e.g., Fock space truncation and relativistic energies). This large value also provides an explanation for the above-mentioned failures of the \(SU(3) \otimes SU(3)\) classification scheme (Close, 1979).

On the negative side, it appears that the functional dependance of the \(\beta_f\)'s cannot be easily determined with satisfactory precision.

IV. SUMMARY AND OUTLOOK

In this review, we have studied the properties of vector and axial-vector nonsinglet charges, and compared their space-time with their null-plane realization. We have shown
that the free-quark model in a null-plane frame is chirally symmetric in the $SU(3)$ limit, whether the common mass is zero or not. The difference between space-time and null-plane chiral properties clearly shows up at the level of the axial currents; this feature has not been exhibited before this work.

In QCD, chiral symmetry is broken both explicitly and dynamically. This is reflected in a null-plane frame by the fact that the axial charges are not conserved, even in the 'chiral limit'. Vector and axial-vector charges annihilate the Fock vacuum, and so are bona fide operators. They form an $SU(3) \otimes SU(3)$ algebra, and conserve the number of quarks and anti-quarks separately when acting on a hadron state. Hence they classify hadrons, on the basis of their valence structure, into multiplets, which are not mass-degenerate.

This classification however turns out to be phenomenologically deficient. The remedy to this situation is a unitary transformation between the charges and the (postulated) physical generators of the classifying $SU(3) \otimes SU(3)$ algebra. The latter generators are built out of 'constituent' quark fields produced by the unitary transformation from the fundamental 'current' fields. The functional form of this transformation can be strongly constrained based on symmetry arguments. Since it depends necessarily on the momenta of the hadron multiplets, the transformation does not commute with the null-plane 'energy', and the constituent 'masses' are themselves momentum-dependent. Given that the CQM’s have been quite successful while using of course fixed masses, one expects that the variation of constituent masses over a typical range of momenta might be small compared to their average value.

That quark masses are allowed to vary, is not surprising in view of the fact that the constituent quarks are not elementary particles, but composite systems containing current quarks of both helicities, as displayed in Eq. (80). We observe that the null-plane picture of a constituent quark (of a given flavor) is much simpler than the space-time description, since the latter must include both current quarks and current antiquarks, of all three flavors, in addition to the expected spin mixing (Weinberg, 1990; Fritzsch, 1993).

Further research is needed on several key aspects of the approach to hadron phenomenology which has been reviewed in this paper. In Sec. III.C, we have discussed applications to the meson sector; but these methods have not yet (to our knowledge) been applied to baryons. It would also be gratifying if one could devise a calculation of
constituent masses within this framework, and find the correct order of magnitude. Such a derivation could, by the way, only benefit from a better knowledge of the functions $\beta_f(\xi, \kappa_\perp^2)$ which parametrize the relationship between current and constituent quarks.

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APPENDIX: BOUNDARY TERM FOR FREE-FERMION NUMBER

The system is confined in a 'box' whose boundaries extend from: $z = -L\sqrt{2}$ to $z = +L\sqrt{2}$ at $t = 0$, from $x^- = -L$ to $x^- = +L$ at $x^+ = 0$, from $x^+ = -L$ to $x^+ = 0$ at $x^- = +L$, and from $x^- = 0$ to $x^+ = +L$ at $x^- = -L$.

We are concerned here with the last two hypersurfaces. We wish to prove that

$$B \equiv \lim_{L \to \infty} \int d^2 x_\perp \left[ \int_{-L}^{0} dx^+ \, j^- (x) \bigg|_{x^- = +L} + \int_{0}^{L} dx^+ \, j^- (x) \bigg|_{x^- = -L} \right] , \quad (A1)$$

(where $j^- = \bar{\psi} \gamma^- \psi$) is zero.

First, we insert in this expression the momentum-space expansion of the fermion field. The system is confined in a longitudinal 'box', hence $p^+$ is discretized. However, since we are going to take the limit $L \to \infty$ at the end of this calculation, we may use right away the infinite-volume expansion. Using Eq. (15) and Eqs. (43)-(45), one finds

$$\psi (x) = \int \frac{d^2 p_\perp}{(2\pi)^{3/2}} \int_{0}^{+\infty} \frac{dp^+}{\sqrt{2p^+}} \sum_{h = \pm \frac{1}{2}} \left[ u(\bar{p}, h) e^{-ipx} b(\bar{p}, h) + v(\bar{p}, h) e^{ipx} d^4 (\bar{p}, h) \right] , \quad (A2)$$

where the creation and annihilation operators are the same as in Eq. (44), and

$$\sum_{h = \pm \frac{1}{2}} u(\bar{p}, h) \bar{u}(\bar{p}, h) = \not{p} + m , \quad \sum_{h = \pm \frac{1}{2}} v(\bar{p}, h) \bar{v}(\bar{p}, h) = \not{p} - m . \quad (A3)$$
Inserting Eq. (A2) in the expression of the current yields
\[
\left. j^-(x) \right|_{x^-=\Lambda} = \int \frac{d^2 p \, dq}{(2\pi)^3} \int_0^{+\infty} dp^+ dq^+ \sum_{h,\eta=\pm} \left[ e^{+iq^+\Lambda} e^{i(q^- + p_\perp \cdot \mathbf{x}_\perp)} \bar{u}(\bar{q}, \eta) b^\dagger(\bar{q}, \eta) + e^{-iq^+\Lambda} e^{-i(q^- + p_\perp \cdot \mathbf{x}_\perp)} \bar{v}(\bar{q}, \eta) d(\bar{q}, \eta) \right] \gamma^- \quad (A4)
\]
\[
\left[ e^{-ip^+\Lambda} e^{-i(p^- + p_\perp \cdot \mathbf{x}_\perp)} u(p, h) b(p, h) + e^{+ip^+\Lambda} e^{i(p^- + p_\perp \cdot \mathbf{x}_\perp)} v(p, h) \right] d^\dagger(p, h) .
\]

Integrating over \(\mathbf{x}_\perp\), and making use of the spinor identities
\[
\left. \bar{u}(\bar{q}, \eta) \gamma^- u(p, h) \right|_{\mathbf{q}_\perp = p_\perp} = 2p^- \sqrt{\frac{p^+}{q^+}} \delta_{h\eta} = \left. \bar{v}(\bar{q}, \eta) \gamma^- v(p, h) \right|_{\mathbf{q}_\perp = p_\perp} , \quad (A5)
\]
\[
\left. \bar{u}(\bar{q}, \eta) \gamma^- v(p, h) \right|_{\mathbf{q}_\perp = -p_\perp} = -2p^- \sqrt{\frac{p^+}{q^+}} \delta_{h, -\eta} = \left. \bar{v}(\bar{q}, \eta) \gamma^- u(p, h) \right|_{\mathbf{q}_\perp = -p_\perp} ,
\]

one gets
\[
\int d^2 \mathbf{x}_\perp \left. j^-(x) \right|_{x^-=\Lambda} = \int \frac{d^2 p \, dq}{2\pi} \int_0^{+\infty} dp^+ dq^+ \left( \frac{p^-}{q^+} \right) \sum_{h=\pm} \left[ e^{i\Lambda(q^- + p^-)} e^{i(x^+ - q^- - p^-)} b^\dagger(\bar{q}, h) b(\bar{q}, h) + e^{-i\Lambda(q^- + p^-)} e^{-ix^+ + (q^- - p^-)} d(\bar{q}, h) d^\dagger(\bar{q}, h) \quad (A6) \right. 
\]
\[
- e^{i\Lambda(q^- + p^+)} e^{i(x^+ + p^-)} b^\dagger(\bar{q}', -h) d^\dagger(\bar{q}', -h) - e^{-i\Lambda(q^- + p^+)} e^{-ix^+ - (q^- + p^-)} d(\bar{q}', -h) b(\bar{q}, h) \right] ,
\]

where \(\bar{q} \equiv (q^+, p_\perp)\), and \(\bar{q}' \equiv (q^- + p_\perp)\). Inserting Eq. (A6) into (A1), one obtains
\[
B = \int \frac{d^2 p \, dq}{2\pi} \int_0^{+\infty} dp^+ dq^+ \left( \frac{p^-}{q^+} \right) \sum_{h=\pm} \left[ T(\bar{p}, \bar{q}) b^\dagger(\bar{q}, h) b(\bar{p}, h) + T^*(\bar{p}, \bar{q}) d(\bar{q}, h) d^\dagger(\bar{p}, h) \right.
\]
\[
- T'(\bar{p}, \bar{q}') b^\dagger(\bar{q}', -h) d^\dagger(\bar{p}, h) - T''(\bar{p}, \bar{q}) d(\bar{q}', -h) b(\bar{p}, h) \right] , \quad (A7)
\]

where
\[
T(\bar{p}, \bar{q}) \equiv \lim_{L \to \infty} \left[ e^{+iL(q^+ + p^-)} \int_{-L}^0 dx^+ e^{ix^+ + (q^- - p^-)} + e^{-iL(q^+ + p^-)} \int_{-L}^L dx^+ e^{ix^+ + (q^- - p^-)} \right] ,
\]
\[
T'(\bar{p}, \bar{q}') \equiv \lim_{L \to \infty} \left[ e^{+iL(q^+ + p^+)} \int_{-L}^0 dx^+ e^{ix^+ + (q^- - p^-)} + e^{-iL(q^+ + p^+)} \int_{-L}^L dx^+ e^{ix^+ + (q^- + p^-)} \right] . \quad (A8)
\]
Next, we show that both $T$ and $T'$ are identically zero, thus obviously making $B$ zero, as advertised. Consider the first term within the square brackets in the expression of $T$:

\[
e^{+iL(q^+-p^+)} \int_{-L}^{0} dx e^{ix^+(q^--p^-)} = e^{+iL(q^+-p^+)} \frac{1 - e^{-iL(q^--p^-)}}{i(q^- - p^-)}
\]

\[
= \frac{1}{i(q^- - p^-)} \left[ (e^{iL(q^+-p^+)} - 1) - (e^{iL(q^+-p^-+q^-+p^-)} - 1) \right]
\]

\[
= \frac{1}{i(q^- - p^-)} \left[ i(q^+-p^+) \int_{0}^{L} dy e^{iy(q^+-p^+)} - i(q^+-p^+-q^-+p^-) \int_{0}^{L} dy e^{iy(q^+-p^-+q^-+p^-)} \right]
\]

\[
= -\frac{q^+}{p^-} \int_{0}^{L} dy e^{iy(q^+-p^+)} + \left(1 + \frac{q^+}{p^-} \right) \int_{0}^{L} dy e^{iy(q^+-p^-+q^-+p^-)} . \] (A9)

In the last step, we exploited the fact that

\[
q^- - p^- = p^- \left[ \frac{q^-}{p^-} - 1 \right] = p^- \left[ \frac{\left(\frac{p^+}{2p^+} + m^2 \right)^2}{\left(\frac{p^+}{2p^+} + m^2 \right)^2} - 1 \right] = p^- \left( \frac{p^+}{q^+} \right) - 1 = \frac{p^-}{p^+} (p^+ - q^-) . \] (A10)

Similarly, the second term in $T$ is

\[
e^{-iL(q^+-p^+)} \int_{0}^{L} dx e^{ix^+(q^-p^-)}
\]

\[
= -\frac{q^+}{p^-} \int_{-L}^{0} dy e^{iy(q^+-p^+)} + \left(1 + \frac{q^+}{p^-} \right) \int_{-L}^{0} dy e^{iy(q^+-p^-+q^-+p^-)} . \] (A11)

Putting Eqs. (A9) and (A11) into Eq. (A8) yields

\[
T = -\frac{q^+}{p^-} \int_{-\infty}^{\infty} dy e^{iy(q^+-p^+)} + \left(1 + \frac{q^+}{p^-} \right) \int_{-\infty}^{\infty} dy e^{iy(q^+-p^-+q^-+p^-)} . \] (A12)

Finally,

\[
T = -\frac{q^+}{p^-} \int_{-\infty}^{\infty} dy e^{iy(q^+-p^+)} + \frac{q^+}{p^-} \left(1 + \frac{p^-}{q^+} \right) \int_{-\infty}^{\infty} dy e^{iy\left(1+\frac{p^-}{q^+}\right)(q^+-p^+)}
\]

\[
= -\frac{q^+}{p^-} \int_{-\infty}^{\infty} dy e^{iy(q^+-p^+)} + \frac{q^+}{p^-} \int_{-\infty}^{\infty} du e^{iu(q^+-p^+)} = 0 . \] (A13)

A similar massage on $T'$ gives

\[
T' = \frac{q^+}{p^-} \int_{-\infty}^{\infty} dy e^{iy(q^++p^+)} + \left(1 - \frac{q^+}{p^-} \right) \int_{-\infty}^{\infty} dy e^{iy(q^++p^+)-q^-+p^-} . \] (A14)
analogous to Eq. (A12). Since \((q^+ + p^+)\) is always positive, the first integral does not contribute, and we are left with

\[
T' = -\frac{q^+}{p^-} \left(1 - \frac{p^-}{q^+}\right) \int_{-\infty}^{\infty} dy \ e^{iy(q^++p^+)(1-\frac{p^-}{q^+})} \\
= -\frac{q^+}{p^-} \left(1 - \frac{p^-}{q^+}\right) \frac{1}{q^++p^+} \int_{-\infty}^{\infty} dz \ e^{iz(1-\frac{p^-}{q^+})} \\
= -\frac{2\pi q^+}{p^- (q^+ + p^+)} \left(1 - \frac{p^-}{q^+}\right) \delta \left(1 - \frac{p^-}{q^+}\right),
\]

(A15)

which is zero as claimed.

REFERENCES

Bell, J.S., 1974, Acta Phys. Austriaca, Suppl. 13, 395.

Buccella, F., E. Celeghin, H. Kleinert, C.A. Savoy and E. Sorace, 1970, Nuovo Cimento A 69, 133.

Carlitz, R., D. Heckathorn, J. Kaur and W.-K. Tung, 1975, Phys. Rev. D 11, 1234.

Carlitz, R., and W.-K. Tung, 1976, Phys. Rev. D 13, 3446.

Collins, J.C., 1984, *Renormalization* (Cambridge U.P., New York).

Close, F., 1979, *An Introduction to Quarks and Partons* (Academic, New York).

De Alwis, S.P., 1973, Nucl. Phys. B 55, 427.

De Alwis, S.P., and J. Stern, 1974, Nucl. Phys. B 77, 509.

Eichten, E., F. Feinberg and J.F. Willemsen, 1973, Phys. Rev. D 8, 1204.

Feinberg, F.L., 1973, Phys. Rev. D 7, 540.

Fritzsch, H., 1993, “Constituent Quarks, Chiral Symmetry and the Nucleon Spin”, talk given at the Sep. 1993 Leipzig Workshop on Quantum Field Theory Aspects of High Energy Physics (CERN preprint TH. 7079/93).

Fritzsch, H., and M. Gell-Mann, 1972, in *Proc. 16th. Int. Conf. on HEP, Batavia, IL*.

Gasser, A., and H. Leutwyler, 1975, Nucl. Phys. B 94, 269.

Gell-Mann, M., 1972a, Lectures given at the 1972 Schladming Winter School (CERN preprint TH. 1543/72).

Gell-Mann, M., 1972b, Acta Phys. Austriaca, Suppl. 9, 733.

Georgi, H., 1982, *Lie Algebras in Particle Physics* (Benjamin-Cummings, Reading, M.A.).

Glazek, S.D., A. Harindranath, R.J. Perry, T. Walhout, K.G. Wilson, and W.-M. Zhang,
1994, “Nonperturbative QCD: A Weak-Coupling Treatment on the Light-Front”, Ohio State Univ. preprint.

Hasenfratz, P., and J. Kuti, 1978, Phys. Rep. C 40, 75.
Ida, M., 1974, Progr. Theor. Phys. 51, 1521.
Ida, M., 1975a, Progr. Theor. Phys. 54, 1199.
Ida, M., 1975b, Progr. Theor. Phys. 54, 1519.
Ida, M., 1975c, Progr. Theor. Phys. 54, 1775.
Jersák, J., and J. Stern, 1968, Nucl. Phys. B 7, 413.
Jersák, J., and J. Stern, 1969, Nuovo Cimento 59, 315.
Leutwyler, H., 1968, Acta Phys. Austriaca, Suppl. 5, 320.
Leutwyler, H., 1969, in Springer Tracts in Modern Physics, No. 50, (Springer, New York), p. 29.
Leutwyler, H., 1974a, Phys. Lett. B 48, 45.
Leutwyler, H., 1974b, Phys. Lett. B 48, 431.
Leutwyler, H., 1974c, Nucl. Phys. B 76, 413.
McCartor, G., 1988, Z. Phys. C 41, 271.
Melosh, H.J., 1974, Phys. Rev. D 9, 1095.
Osborn, H., 1974, Nucl. Phys. B 80, 90.
Sazdjian H., and J. Stern, 1975, Nucl. Phys. B 94, 163.
Shifman, M.A., A.I. Vainshtein and V.I. Zakharov, 1979, Nucl. Phys. B 147, 385.
Weinberg, S., 1990, Phys. Rev. Lett. 65, 1181.