The principle of measurement is outlined in Fig. 1. A single scattering element is illuminated by a beam composed of multiple wavelengths; each wavelength is scattered to produce a unique speckle pattern. Provided the wavelengths of the Components are sufficiently separated, the resultant speckle patterns are a simple intensity sum of the speckles produced by each wavelength in isolation. A calibration dataset is acquired to train PCA to recognize how the speckle changes with wavelength.

To demonstrate the training method, we simulate (using paraxial wave theory, see [11] for details) the propagation of two co-polarized, co-incident and co-propagating Gaussian laser beams of equal power and identical spatial distribution. The light propagates through five equally-spaced planes (separated by one Rayleigh length) at which the phase is randomized. The refractive index difference to air is small (Δn = 0.001) to ensure most scattering is in the forward direction. After further free-space propagation of two Rayleigh lengths after the final randomization, the resulting speckled intensity is sampled on a 256 × 256 pixel grid with a bit-depth of 8, to approximate the acquisition by a camera. A series of 1200 speckle patterns are accumulated, where wavelengths λ_1 and λ_2 of the two lasers are centered around 780.000 nm and 780.014 nm. They are both sinusoidally modulated with a 1 pm-amplitude but with different periods of oscillation (such that they undergo three and ten oscillations in the measurement period, respectively), as

FIG. 1. Principle of multi-wavelength measurement in a speckle wavemeter. Laser beams are overlapped and illuminate a single scattering medium, generating a speckle pattern. The speckle pattern is uniquely determined by the precise values of each wavelength, so can be used as a marker to recover the wavelengths.
FIG. 2. Principal Component Analysis of simulated speckle patterns produced by wavelength variations of two overlapped lasers. (a) Sinusoidal modulations applied to the wavelengths of two lasers $\lambda_1$ and $\lambda_2$. (b) Principal Components 1-3 of the image set. The first Principal Component (PC1) captures a mixed signal of both wavelength modulations, while PC2 and PC3 show responses dominated by $\lambda_2$ and $\lambda_1$ respectively. Parametric plots of (c) $\lambda_1$ vs $\lambda_2$ and (d) PC3 vs PC2 show that the combined modulations are faithfully recorded in PC-space. A small rotation angle between (c) and (d) highlights mixing of the two wavelength Components across the two PCs.

shown in Fig. 2(a). At each time interval, the multi-wavelength speckle pattern is obtained by summing the intensities of the speckle distribution of each wavelength in isolation, i.e. neglecting interference between the two beams. PCA is then performed on the time-series of multi-wavelength speckle patterns. The Principal Components (PCs) are the projections of the data onto the eigenbasis of the covariance matrix of the training set, i.e. by design they measure the maximal variations in the dataset. The largest three PCs (PC1, PC2 and PC3, shown in Fig. 2(b)), contain 96% of the variations in the data. The non-commensurate modulation rates for the two beams ease the identification of the contribution from each wavelength. The first Principal Component, PC1, shows modulation of the speckle pattern at both of the applied modulation rates. This is associated with intensity fluctuations due to speckles moving in and out of the field of view of the camera. However, the separate modulations are dispersed across the next two Principal Components (PC2 and PC3 in Fig. 2(b)), in analogy with the wavelength-dependent dispersion produced in a grating-based spectrometer. Retrieval of these two PCs in isolation is sufficient to characterize the independent wavelengths: the parametric relationship between $\lambda_1$ and $\lambda_2$ is illustrated in Fig. 2(c), and the same parametric relationship is shown to exist between PC2 and PC3 in Fig. 2(d). A small rotation angle between the two parametric plots signifies cross-talk between the two measurement channels, i.e. PC2 is strongly dependent on $\lambda_2$ and weakly dependent on $\lambda_1$ while PC3 is strongly dependent on $\lambda_1$ and weakly dependent on $\lambda_2$. We find that this cross-talk can be minimized by using wavelength modulations of equal amplitude, but regardless it does not effect the accuracy of the PCA, as the two wavelengths are always uniquely identified by the measurement of these two PCs. The link between PCs and wavelength is established by a linear fitting of this training set. A speckle pattern produced by an unknown combination of wavelengths within the training range can subsequently be projected into this PC-space to retrieve the wavelengths.

We experimentally verify the method using the apparatus shown in Fig. 3 to generate tunable, multi-wavelength spectra. Light from an external cavity diode laser (Topica DL-100, LD-0785-P220), stabilized to the $^{87}$Rb D$_2$ line ($F = 2 \rightarrow F = 2$ $\times$ 3 crossover) with saturated absorption spectroscopy and current modulation, is separated into two beams using a polarizing beam splitter. The wavelength of each beam is shifted by independent acousto-optic modulators (AOMs) (Crystal Technologies 3110-120) in cat-eye double-pass configuration, with a modulation range for each beam of 20 fm. The two beams are recombined and co-polarized using further polarizing beam splitters and a half-waveplate. The light is coupled into an angle-cleaved single-mode fiber (SMF) (ThorLabs P5-780PM-FC-10) to ensure each beam has
the same spatial profile, and delivered to a multi-mode fiber (MMF) speckle wavemeter. Laser speckle is generated by multiple scattering and modal interference in the 1 m-long step-index MMF, which has 105 μm core diameter and NA = 0.22 (ThorLabs FG105LCA). After exiting the MMF, the light propagates for 5 cm and is captured by a fast CMOS camera (Mikrotron EoSens 4CXP). Images of 240 × 240 pixels were recorded at 2,000 fps with an exposure time of 10 μs and a power of 150 μW per beam. The multi-wavelength speckle image at each time interval is independently normalized by the total intensity. The speckle correlation limit of this system is ~ 320 pm, which is determined as the HWHM of the Pearson correlation coefficient of the speckle patterns at different wavelengths.

Fig. 4 shows the measurement of two wavelengths with an average separation of 22 fm, which is four orders of magnitude below the speckle correlation limit and for which the speckle patterns acquired at each wavelength have a structural similarity index > 0.97. Training was performed by acquiring the speckle patterns over a 1 s interval for a 8.5 fm-amplitude sinusoidal wavelength modulation to each beam, with incommensurate periods of 125 ms and 37.5 ms. After the training phase, an exponential decay of the amplitude of the wavelength modulation is introduced. The standard deviation between the set wavelength and that measured by the speckle wavemeter is 0.37 fm for the slowly modulated beam and 0.29 fm for the fast modulated beam. The accuracy of the measurement of each wavelength is limited by high-frequency modulations of the laser wavelength introduced by the lock-in electronics for wavelength stabilization, and are in agreement with those reported in [12] for measurements of a single wavelength.

When the wavelength separation is large, PCA accurately recovers the wavelength. However, if the wavelengths converge, PCA is incapable of correctly analyzing the speckle pattern, giving large values of the PCs which do not correspond to the expected wavelengths. The erratic values for close approach are due to interference between the beams causing the speckle pattern on the camera to flicker. When the beat-note frequency of this interference-induced flicker is fast compared to the exposure time of the camera, PCA gives reliable wavelength estimation. Using the camera settings above, we measured the wavelengths of two beams to an accuracy (standard deviation of measured and set wavelength over 1 s) of 0.21 fm and 0.19 fm when the wavelength separation was 1.0 fm. In principle, decreasing the measurement rate and using longer exposure times should allow for an improvement in spectral resolution, while the issue may be avoided in the measurement of separate laser sources.

In addition to wavelength separation, we also investigated the role of other potential issues with our approach. For modest power ratio between the beams, the two wavelengths are always uniquely determined by PC2 and PC3. However, when this power ratio is large, e.g. 500 μW and 50 μW, the less intense beam is instead dispersed into PC4, therefore further PCs must be considered to track multiple wavelengths in this regime. When the lasers have different linewidth, the differing Rayleigh distributions of the resultant speckle patterns will aid the discrimination of the contributions of each laser to the speckle.

Simultaneous measurements of more than two wavelengths are also possible. Fig. 4(a) shows simulated wavelength modulation of three separate beams, with mean separations of 14 pm, which are combined as before. As in the two-wavelength case, the wavelength modulations are dispersed across PC-space (97% of the variation is described by the first four PCs). PC1 shows a mixture of all three modulations, while PC2 to PC4 are respectively dominated by \( \lambda_1 \) to \( \lambda_3 \) (Fig. 4(b)). Mixing of the spectral channels is again observed: the 3-dimensional parametric plot of wavelength (Fig. 4(c)) is related to the parametric plot of PC2, PC3 and PC4 (Fig. 4(d)) by a 3-dimensional rotation. The mixing of spectral channels can also be seen in the transformation matrix \( T_{\lambda,PC} \) which defines the linear transformation between PCs and wavelength, i.e.

\[
\begin{pmatrix}
PC_2 \\
PC_3 \\
PC_4
\end{pmatrix} =
\begin{pmatrix}
T_{\lambda_1,PC_2} & T_{\lambda_1,PC_3} & T_{\lambda_1,PC_4} \\
T_{\lambda_2,PC_2} & T_{\lambda_2,PC_3} & T_{\lambda_2,PC_4} \\
T_{\lambda_3,PC_2} & T_{\lambda_3,PC_3} & T_{\lambda_3,PC_4}
\end{pmatrix}
\begin{pmatrix}
\lambda_1 \\
\lambda_2 \\
\lambda_3
\end{pmatrix}.
\]

\( T_{\lambda,PC} \) is established in the training phase by multiplication of the matrix containing the time series of the PCs and the inverse of the matrix containing the time series of the corresponding training wavelengths, and is plotted in Fig. 4(e). It shows that the mean dependence of PCi on \( \lambda_i \) is 83.1%, with an average contribution of 12.1% from the nearest neighboring wavelength(s). The wavelengths present in any individual unknown speckle pattern can be measured by multiplying the matrix inverse of \( T_{\lambda,PC} \) with the PCs extracted for that image.

The transformation matrix representation is necessary...
to examine the correlations between higher numbers of beams. As shown in Fig. 6, a similar transformation matrix can be established for a system of 10 distinct lasers, where the ten wavelengths are dispersed across PC2 to PC11. In this simulation, the ten wavelengths were evenly separated by 1 pm, and undergo incommensurate sinusoidal modulations of 200 fm amplitude over 400 frames. The period of oscillation of $\lambda_i$ was set so that it undergoes $2p_i$ oscillations in the training phase, where $p_i$ is the $i$th prime integer. The PCA finds a basis in which 74% of the variance is contained in the first eleven PCs. We note that the variance captured in higher PCs in this case follows a step-like trend in groups of ten PCs: continuously falling by 50% within the group but discontinuously dropping by 50% between the last PC of one group and the first PC of the next. Ignoring these higher terms and projecting test data into the 10-dimensional PC space comprising PC2 to PC11 recovers the wavelength to within 20 fm. As can be seen in Fig. 6, there is greater mixing between the spectral channels in this ten-wavelength measurement, with the diagonal elements of $T_{\lambda, PC}$ having a mean value of 31.3% and a standard deviation of 9.1%.

In this letter, we have demonstrated that the wavelengths of multiple lasers can be measured simultaneously using a speckle wavemeter with Principal Component Analysis. The procedure projects a speckle pattern generated by $n$ wavelengths into an $n$-dimensional Principal Component space. In the experiment, we demonstrated simultaneous recovery of the wavelengths of two lasers, separated by as little as 1 fm with an accuracy of 0.2 fm, limited by the stabilization electronics of the laser. The approach is limited in spectral range, requiring that the Principal Components vary monotonically with wavelength. However, for single wavelength measurements, PCA has been shown to be complimentary to the transmission matrix method, which operates over a much larger range but with lower resolution [11]. We suggest such a tandem approach will also be possible for the measurement of multiple wavelengths. The method is likely to find application in the development of portable quantum technologies, where robust methods to lock multiple lasers for atom cooling are sought. Courtrier, et al, have shown that such stabilization can be achieved using a commercial (Fizeau) wavemeter and a multi-mode fiber switch, but report fluctuations of the
atomic fluorescence due to the switching \cite{15}. Stabilization of a single laser using speckle was demonstrated in \cite{11}, and we suggest that the simultaneity of measurements of multiple wavelengths with speckle may obviate the switching limitation. In future work, the training phase could be extended to include variable powers of the beams, which would allow for the recovery of sparse spectra with variable mode intensities, which may be applicable to areas such as chemical analysis.

We acknowledge funding from Leverhulme Trust (RPG-2017-197) and EPSRC (EP/R004854/1) and thank D Cassettari and P Rodríguez-Sevilla for technical assistance and discussions.

[1] W. Freude, C. Fritzche, G. Grau, and L. Shan-da, Journal of Lightwave Technology 4, 64 (1986)
[2] T. W. Kohlgraf-Owens and A. Dogariu, Opt. Lett. 35, 2236 (2010)
[3] I. Alexeev, J. Wu, M. Karg, Z. Zalevsky, and M. Schmidt, Appl. Opt. 56, 7413 (2017)
[4] M. Mazilu, A. Mourka, T. Vettenburg, E. M. Wright, and K. Dholakia, Appl. Phys. Lett. 100, 231115 (2012)
[5] B. Redding, S. F. Liew, R. Sarma, and H. Cao, Nature Photonics 7, 746 (2013)
[6] B. Redding, M. Alam, M. Seifert, and H. Cao, Optica 1, 175 (2014)
[7] M. Chakrabarti, M. L. Jakobsen, and S. G. Hanson, Opt. Lett. 40, 3264 (2015)
[8] H. Cao, J. Opt. 19, 060402 (2017)
[9] U. Kürüm, P. R. Wiecha, R. French, and O. L. Muskens, Opt. Express 27, 20965 (2019)
[10] M. Mazilu, T. Vettenburg, A. Di Falco, and K. Dholakia, Opt. Lett. 39, 96 (2014)
[11] N. K. Metzger, R. Spesyvtsev, G. D. Bruce, B. Miller, G. T. Maker, G. Malcolm, M. Mazilu, and K. Dholakia, Nature Communications 8, 15610 (2017)
[12] G. D. Bruce, L. O’Donnell, M. Chen, and K. Dholakia, Opt. Lett. 44, 1367 (2019)
[13] L. O’Donnell, K. Dholakia, and G. D. Bruce, arXiv: 1909.00665 (2019)
[14] R. K. Gupta, G. D. Bruce, S. J. Powis, and K. Dholakia, arXiv: 1910.10702 (2019)
[15] L. Couturier, I. Nosske, F. Hu, C. Tan, C. Qiao, Y. Jiang, P. Chen, and M. Weidemüller, Rev. Sci. Instrum. 89, 043103 (2018)