Probing Planckian corrections at the horizon scale with LISA binaries

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Several quantum-gravity models of compact objects predict microscopic or even Planckian corrections at the horizon scale. We discuss two model-independent, smoking-gun effects of these corrections in the gravitational waveform of a compact binary, namely the absence of tidal heating and the presence of tidal deformability. For events detectable by the future space-based interferometer LISA, we show that the effect of tidal heating dominates and allows one to constrain putative corrections down to the Planck scale, up to redshift $z \sim 9$. Furthermore, the measurement of the tidal Love numbers with LISA can constrain the compactness of an exotic compact object down to microscopic scales in conservative scenarios, and down to the Planck scale in the case of a highly spinning binary at $1 \sim 10$ Gpc. Our analysis suggests that spinning, supermassive binaries provide unparalleled tests of quantum-gravity binaries at the horizon scale.

Introduction. Gravitational waves (GWs) are the most direct probes of compact objects down to the horizon scale and can shed light on one of the outstanding open issues in gravitational astronomy: the nature of compact, dark and massive objects. It has been tacitly assumed that the latter must be black holes (BHs) for a number of compelling reasons: BHs form from classical gravitational collapse of stars, while there are no known, equally-well motivated alternatives which are sufficiently compact to explain observations, especially the recent GW detections \cite{Abbott:2016blz,Abbott:2016nmj}. Nonetheless, over the years several arguments have been put forward, suggesting that new physics at the horizon scale might set in during gravitational collapse, possibly halting or altering the formation of BHs \cite{Abbott:2016nmj,Abbott:2017gyy,Abbott:2017mdw}. While the end product of the collapse in these scenarios is essentially unknown or model dependent \cite{Cutler:2000im,Berti:2018-scalable,Barausse:2012bd}, the exotic compact objects (ECOs) that might form share two common features: they are extremely compact and horizonless. Regardless of the viability of these objects and the mechanisms behind them, we face for the first time the free option of testing these scenarios with GWs.

It was recently argued that ECOs can be detected or ruled out through GW measurements in two different regimes: the postmerger ringdown phase of a coalescence – where putative corrections at the horizon scale will produce GW echoes \cite{Abbott:2016blz,Abbott:2017mdw} (see also Ref. \cite{Zenginoglu:2016epl} for an earlier study, and Refs. \cite{Barausse:2015uwa,Barausse:2016kux} for a debate on the evidence of this effect in aLIGO data) – and the late-time inspiral of the coalescence – through the measurement of the tidal deformability of the two objects \cite{Barausse:2012bd,Barausse:2016kux} and of their spin-induced quadrupole moment \cite{Zenginoglu:2016epl}.

Here, we discuss another effect that can be used to distinguish ECOs from BHs, namely the absence of tidal heating if the binary components do not possess a horizon. For supermassive binaries detectable by the future Laser Interferometer Space Antenna (LISA) \cite{Sathyaprakash:2009xs}, we show that this effect can be used to constrain putative corrections near the horizon down to the Planck scale, even for binaries at cosmological distances. In addition, we show that the effects of both the tidal heating and the tidal deformability in the GW signal are enhanced for unequal-mass binaries as those detectable by LISA.

GW tests of the nature of compact objects. Consider a compact binary, of masses $m_1$ (i = 1, 2), total mass $m = m_1 + m_2$, mass ratio $q = m_1/m_2 \geq 1$, and dimensionless spins $\chi_i$. In a post-Newtonian (PN) expansion, dynamics is driven by energy and angular momentum loss, and particles are endowed with a series of multipole moments and with finite-size tidal corrections. The nature of the inspiralling objects is encoded, loosely speaking, in (i) the way they respond to their own field – i.e., on their own multipolar structure, (ii) the way they respond when acted upon by the external gravitational field of their companion – through their tidal Love numbers (TLNs) \cite{Arun:2008cg}, and (iii) on the amount of radiation that they possibly absorb, i.e. on tidal heating \cite{Scheel:2013pqa,Barausse:2016kux}. These effects are all included in the waveform produced by the inspiral, and can be incorporated in the (Fourier-transformed) GW signal as (we use $G = c = 1$ units)

$$\tilde{h}(f) = A(f)e^{i(\psi_{PP} + \psi_{TH} + \psi_{TD})},$$

where $f$ and $A(f)$ are the GW frequency and amplitude, $\psi_{PP}(f)$ is the “pointlike” phase, whereas $\psi_{TH}(f), \psi_{TD}(f)$ are the contribution of the tidal heating and the tidal deformability, respectively.

1 We assume a standard $\Lambda$CDM flat universe with $H_0 \sim 67.7$, $\Omega_M \sim 0.26$ and $\Omega_\Lambda \sim 0.69$ \cite{Planck:2015xua}. All masses quoted in the text are assumed to be the redshifted masses. With this choice, the waveform \cite{Abbott:2017gyy} is independent of the redshift $z$.\[\text{[arXiv:1703.10612v1 [gr-qc]] 30 Mar 2017}\]
Spin-orbit and spin-spin interactions are included in \( \psi_{PP} \), the latter also depending on the spin-induced quadrupole moment. This property has been recently used to constrain \( \mathcal{O}(\chi^2) \) deviations from the Kerr geometry \cite{17}. However, in known models of rotating ECOs – e.g., gravastars \cite{25, 26} and strongly-anisotropic, incompressible neutron stars \cite{27} – the multipole moments approach those of a Kerr BHs in the high-compactness limit. This suggests that the distinction between ultra-compact objects and BHs can only be done, realistically, using finite-size corrections, \( \psi_{TH}(f) \) and \( \psi_{TD}(f) \).

BHs are very special objects in general relativity and it is no surprise that finite-size effects are different for ECOs than for BHs. If the inspiralling objects are BHs, a small contribution to the dynamics is provided by dissipation of energy and angular momentum at the horizon, through tidal heating (akin to the tidal acceleration in the Earth-Moon system \cite{22, 42, 43}). To leading PN order, the energy flux associated to tidal heating reads, \( \bar{E}_{\text{TH}} \), (henceforth we assume circular orbits and spins aligned with the orbital angular momentum)

\[
\bar{E}_{\text{TH}} = -E_{GW} \sum_{i=1,2} \frac{v^5}{4} \left( \frac{m_i}{m} \right)^3 (1 + 3 \chi_i^2) \times \left\{ \chi_i - 2 \left[ 1 + \sqrt{1 - \chi_i^2} \right] \frac{m_i}{m} \psi^3 \right\},
\]

where \( E_{GW} \) is the leading order GW flux, whereas \( v = (\pi m f)^{1/3} \) is the orbital velocity. Angular momentum flux is subleading so the spins remain roughly constant during the evolution \cite{30}. The above term is proportional to the superradiant combination \( (\Omega - \Omega_H) \) \cite{22, 30, 34}, so that when the orbital frequency \( \Omega \) is smaller than the angular velocity at the event horizon \( \Omega_H \), the tidal-heating flux is negative and slows down the inspiral \cite{24}. The GW phase \( \psi \) is governed by \( d^2\psi/df^2 = 2\pi(dE/df)/\bar{E} \), where \( \bar{E} \) is the binding energy of the binary. To the leading order, this yields

\[
\psi_{TH}(f) = F(\chi_i, q) \log(\pi f m) + G(q) \pi f m \left[ 1 - \log(\pi f m) \right],
\]

where \( F(\chi_i, q) \) and \( G(q) \) are simple functions of their arguments, which grow linearly in \( q \) as \( q \gg 1 \), and \( F \sim \chi_i + \mathcal{O}(\chi_i^2) \). Thus, absorption at the horizon introduces a 2.5PN (4PN) correction to the GW phase of spinning (nonspinning) binaries, relative to the leading term. For all known matter, GW absorption is negligible: tidal heating is therefore a good discriminator for the existence of horizons. It is therefore convenient to introduce an absorption coefficient \( \gamma \) such that, \( \psi_{TH} = \gamma \psi_{TH} \), with \( \gamma = 0 \) for ECOs with a perfectly reflecting surface or whose interior does not absorb GWs, \( \gamma = 1 \) for BHs, and \( 0 < \gamma < 1 \) for partial absorption.

In addition, while the TLNs of BHs are zero \cite{35, 40}, those of ECOs are small but finite \cite{18, 29, 26, 41, 42}. In line with neutron star binaries \cite{22, 42, 43}, the leading tidal deformation term for ECO binaries reads,

\[
\psi_{TD}(f) = -\frac{\Lambda}{6m^5} (m \pi f)^{5/3} \left[ 1 + q^2 \right]^2, \tag{4}
\]

where \( \Lambda = (1 + 12/q) m_i^2 k_1 + (1 + 12q) m_2^2 k_2 \) is the weighted tidal deformability, and \( k_i \) is the (dimensionless) TLN of the \( i \)-th object. Thus, tidal deformability introduces a 5PN correction to the GW phase relative to the leading-order GW term, whereas spin-tidal couplings are subleading \cite{39, 41, 45} and will be neglected.

The TLNs of a nonspinning ECO of mass \( M \) and radius \( r_0 = 2M(1 + \epsilon) \) (with \( \epsilon \ll 1 \)) in Schwarzschild coordinates vanish logarithmically in the BH limit \cite{18}, \( k \sim -1/\log \epsilon \), opening the way to probe horizon scales. As depicted in Fig. 1, any measurement of the TLN translates into an estimate of the distance of the ECO surface from its Schwarzschild radius,

\[
\delta := r_0 - 2M \sim 2Mc^{-1/k}.
\]

For a supermassive object with \( M = 10^6 M_\odot \), \( \delta \) is of the order of the Planck length, \( \ell_P = 1.6 \times 10^{-33} \text{ cm} \), when \( k \approx 0.005 \) [cf. Fig. 1]. Therefore, future GW observations should aim at reaching the level of accuracy necessary to measure TLNs as small as \( k \approx 0.005 \). Below, we show that this will be achievable with LISA.

To summarize, BHs and ECOs produce orthogonal finite-size effects in the inspiral waveform. The former
have zero TLNs ($\psi_{TD} = 0$) but introduce a nonzero tidal heating ($\gamma = 1$), whereas the latter have (logarithmically small) TLNs ($\Lambda \neq 0$) but zero tidal heating ($\psi_{TH} = 0$).

**Error analysis.** We use the TaylorF2 approximant of the GW signal in the frequency domain, including spin-orbit and spin-spin terms [20]. As we discussed, we assume the quadrupole moment to be that of a Kerr BH. Tidal heating is included to leading order, i.e., to 2.5PN (4PN) relative order for spinning (nonspinning) binaries. We include tidal-deformability terms up to 2PN corrections [17] and neglect subleading spin-tidal couplings.

We employ a Fisher matrix analysis, accurate at large signal-to-noise ratios [49], as those discussed here. For a given set of parameters $\vec{\xi}$, the error associated with the measurement of parameter $\xi^a$ is $\sigma_a = \sqrt{\Sigma_{aa}}$, where the covariance matrix $\Sigma^{ab}$ is the inverse of the Fisher matrix, $\Gamma_{ab} = (\partial \xi^a/\partial h^b)_{\vec{\xi}=\vec{\xi}_0}$; $\vec{\xi}_0$ are the injected values of the parameters $\vec{\xi}$, and the inner product is defined as

$$
(g|h) = 2\int_{f_{\text{min}}}^{f_{\text{max}}} df \frac{\hat{h}(f)\tilde{g}^*(f) + \hat{h}^*(f)\tilde{g}(f)}{S_h(f)},
$$

where $S_h(f)$ is the detector’s noise spectral density. In our analysis, we worked with the recently proposed LISA sensitivity curve [20] with an observing time $T_{\text{obs}} = 4$ yr, sky-averaging the GW signal [50]. The relevant parameters are $\vec{\xi} = (\ln A, \psi_c, t_c, \ln M, \ln \nu, \chi_1, \chi_2)$ where $\psi_c$ and $t_c$ are the phase and time at the coalescence, plus possibly $\gamma$ and $\ln \Lambda$, depending on the system under consideration. We numerically integrate Eq. (6) within the frequency range $f_{\text{min}} = \text{Max}[10^{-5} \text{ Hz}, 4.149 \times 10^{-5}(10^{-6}M/M_\odot)^{-5/6} (T_{\text{obs}}/\text{yr})^{-3/8} \text{ Hz}]$ [51] and $f_{\text{max}} = \text{Min}[1 \text{ Hz}, f_{\text{ISCO}}]$, where $M = (m_1 m_2)^{3/5}/m_1^{1/5}$ is the chirp mass and $f_{\text{ISCO}}$ is the GW frequency at the innermost-stable circular orbit (ISCO) of the Kerr metric, including corrections due to the self-force and the spin of the less massive object [52]. For signals such as those produced by binary coalescences, a sharp cut-off $f \leq f_{\text{max}}$ which abruptly terminates the GW template does not alter the parameter covariance [53].

As the inclusion of spin often leads to badly conditioned $\Gamma_{ab}$, to numerically compute $\Sigma^{ab}$ we use a singular value decomposition, by zeroing out the pieces of $\Gamma_{ab}$ which are very small, and hence effectively unmeasurable [53]. In our analysis, we remove only the largest of the Fisher eigenvalues, which corresponds to the pivot $t_c$, leading to a more precise inversion without affecting the correlations on the remaining parameters.

**GW constraints on ECOs.** Focus first on the effects of tidal heating, by setting $\Lambda = 0$. The error $\sigma_\gamma$ is shown in Fig. 2 as a function of the mass ratio $q$ for different systems. The dashed horizontal line marks the threshold $\sigma_\gamma = 1$: measurements of the tidal heating parameter $\gamma$ below the threshold have less than 100% uncertainty and would discriminate between ECOs and BHs at least at 1$\sigma$ level. While nonspinning, equal-mass binaries are not distinguishable, the accuracy improves drastically for larger mass ratios and, even more dramatically, for larger spin. For example, BH coalescences up to luminosity distance of 10 Gpc with mass ratio $q \in (50, 5000)$ and spins $\chi_1 = \chi_2 = 0.9$ can be confidently distinguished from ECO binaries on the basis of the presence of tidal heating. The enhancement with spin is expected, given that tidal heating enters the GW phase at 2.5PN order [cf. Eq. (3)] and that $f_{\text{ISCO}}$ increases with the spin. On the other hand, the enhancement at large mass ratios is due to the linear growth of the phase with $q$, whereas the amplitude decreases as $A \sim q^{-1/2}$, the Fisher matrix $\Gamma_{ab} \sim q$ and therefore the error decreases as $\sigma_\gamma \sim q^{-1/2}$. When $q \gtrsim 10^3$, this effect is compensated by the smaller frequency range in Eq. (6), since $f_{\text{min}} \to f_{\text{max}}$ as $q$ grows. These results are consistent with previous studies, where a significant enhancement of tidal effects with spin were observed for EMRIs [24]. Note that the PN series converges very slowly in the extreme mass ratio limit [55], so our results are only indicative when $q \gtrsim 10^3$. In some favorable scenarios [$q \in (50, 5000)$ and $\chi_1 \sim 0.99$], even BH binaries at luminosity distance of 100 Gpc (roughly corresponding to redshift $z = 9$) can be distinguished from ECO binaries. For large mass ratios, these results are independent of the spin of the small object.

The second case of interest is orthogonal to the above: with ECO binaries in mind we now set $\gamma = 0 = \psi_{TH}$ but a nonvanishing TLN. We are interested in estimating whether a measurement of $\Lambda$ is incompatible with zero

\[\text{We remark that } \sigma_\alpha \text{ scales with the inverse luminosity distance.}\]
and therefore whether ECOs can be distinguished from BHs on these grounds. This case was preliminarily studied in Ref. [18] only for equal-mass binaries and neglecting spin. Figure [3] shows that the constraints on $\Lambda$ are orders of magnitude more stringent for spinning binaries, whereas the dependence on $q$ is spin-dependent and non-monotonic at large spins, due to correlations in the Fisher matrix. In particular, for $\chi_i = 0.9$ and $q \sim 1$ or $q \gtrsim 500$, a constraint on the TLN as stringent as $k \lesssim 0.005$ can be obtained for a coalescence within 1 Gpc, whereas for $\chi_i = 0.99$ the same constraint can be achieved for a coalescence roughly within 10 Gpc. These findings are only mildly dependent on $\chi_2$. For several ECO models [18], a bound $k \lesssim 0.005$ on the TLN translates into an impressive constraint on the compactness of ECOs down to the Planck scale near the horizon, i.e. $\delta / M \sim 10^{-45}$ for a supermassive object [cf. Fig. 1].

Thus, finite-size effects open up the tantalizing possibility to know, through GW observations, if BHs do actually exist, and are in fact complementary to searches for new physics at the horizon scale through the detection of GW echoes [11, 12, 14, 10].

**Effective absorption by ECOs.** It might be argued that there should be a continuous transition from BHs to ultracompact horizonless objects, whereas we have assumed that tidal heating of ECOs is negligible. In fact, an ultracompact object can trap radiation within its photosphere efficiently [11, 12, 55], thus mimicking the effect of a horizon. This mechanism is interesting per se at a theoretical level and deserves (and requires) a separated study. Nonetheless, we can estimate it as follows. In order for the absorption to affect the orbital motion, it is necessary that the (arrival) time radiation takes to reach the companion, $T_{\text{arr}}$, be much longer than the radiation-reaction time scale due to heating, $T_{\text{RR}} \sim E / \dot{E}_{\text{heating}}$. From Eq. [2], the latter reads $T_{\text{RR}} \sim m v_{\text{ISCO}}^{13} / (\chi_i + 3 \chi_3^3)$, in the most conservative case, namely $q = 1$ and $\chi_i \neq 0$.

For BHs, $T_{\text{arr}} \rightarrow \infty$ because of time dilation, so that the condition $T_{\text{arr}} \gg T_{\text{RR}}$ is always satisfied. For ECOs in the $\epsilon \rightarrow 0$ limit, $T_{\text{arr}}$ turns out to be just the GW echo delay time of the $i$-th object, $T_{\text{arr}} \sim -m_i \log \epsilon$ [11, 12]. For a given compactness, the condition $T_{\text{arr}} \gg T_{\text{RR}}$ implies that an effective heating can take place only when $f \gg f_{\text{crit}}$. Including spins [14] and generic mass ratios in the computation, we find,

$$f_{\text{crit}} \sim \left[ \frac{5 m Q \Delta_i}{2 \pi m^2} \right]^{3/13} \left[ m_i (\chi_i + 3 \chi_3^3) (1 + \Delta_i) \log (r_{+}^{(i)} / \delta) \right],$$

where $\Delta_i = \sqrt{1 - \chi_i^2}$, $Q = (1 + q) q / (q[1 + q(q - 1)])$, and $r_{+}^{(i)} = m_i (1 + \Delta_i)$. When $\chi_i \rightarrow 0$, the subleading term in Eq. [2] becomes dominant, but its effect is negligible.

As expected, $f_{\text{crit}}$ decreases as the compactness increases. However, even for $\delta \sim \ell_p$, $f_{\text{crit}} \gtrsim f_{\text{ISCO}}$ when $\chi_i < 0.9$ or when $q \gg 1$ for any spin. In the least favorable case $0.9 < \chi_i < 0.99$ and $q \sim 1$, $f_{\text{crit}} \approx 0.4 f_{\text{ISCO}}$. Therefore, in the entire region where the PN expansion is valid the “effective” tidal heating of a Planck-scale ECO can be neglected.

Another possible caveat concerns GW dephasing due to mode excitation. The fundamental QNMs of an ECO have small frequency and are extremely long lived. Because $\omega_{\text{QNM}} \sim 2 \pi / T_{\text{arr}} \sim -[M \log \epsilon]^{-1}$ [11, 12], the QNMs can be possibly excited only when $\Omega \sim 2 \pi / T_{\text{arr}}$, which, for Planck-scale ECOs, occurs only near the ISCO and with extremely narrow resonances that should absorb a negligible amount of energy.

In summary, our results seem robust even when relaxing some of the assumptions: mode excitation and tidal heating can be expected to be negligible for ECOs, validating our analysis.

**Discussion and Prospects.** The future space-based GW detector LISA [20] will be able to distinguish whether supermassive dark objects in a binary coalescence have a horizon or not by measuring two distinct and complementary finite-size effects on the waveform: tidal heating and tidal deformability. We have shown that these effects become stronger for highly spinning, unequal-mass binaries, and allow us to constrain the location of the ECO surface down to Planck scales, even for high-redshift coalescences. This is a truly spectacular potential. GW observations will possibly provide the most impressive tests of near-horizon physics, and will challenge our understanding of quantum gravity backre-
This work is intended as a proof-of-principle; including eccentricity, spin misalignment, higher-order PN terms, developing the full ECO coalescence waveform analytically (e.g. using hybrid waveforms, or within an effective-one-body framework [54, 57, 65] for BH binaries), and performing a Bayesian analysis are certainly relevant extensions.

The rates of these events for LISA are uncertain, especially when involving intermediate-mass objects. However, several observations [59–63] suggest that dark objects with masses $\sim 10^2 - 10^3 M_\odot$, could form in stellar clusters near the centers of galaxies. N-body simulations [64] predict that most of them eventually merge with the supermassive dark object at the center of the host galaxy, at a rate $\sim 0.1 \text{Myr}^{-1}$ and with $q \sim 10^5$. Given the density of spiral galaxies, $\sim 2 \times 10^6 \text{Gpc}^{-3}$ [65], this yields tens of mergers per year within a distance of 2 – 4 Gpc. If these models are correct, we can estimate several events per year for which Planck corrections at the horizon scale can be confidently ruled out or detected through GW finite-size effects. Since even a single detection of such systems during the LISA lifetime is enough to impose concise constraints, we claim that highly spinning, supermassive binaries may be wonderful probes of putative quantum-gravity effects at the horizon scale.

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