Counterfactual errors and state reduction in relativistic quantum physics

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Abstract

We use the laws of relativistic physics to show that classically motivated counterfactual statements are inadequate when discussing the principles of quantum physics and that EPR style arguments against state reduction are incorrect.

In classical logic, counterfactual statements are statements of the form: “if $P$ were true (which is in fact known not to be the case) then $Q$ would be true”. Such statements are often acceptable in classical mechanics because one of its cardinal unstated principles is that a classical, objective reality exists regardless of whether or not anything has been observed.

When it comes to quantum mechanics, however, the greatest care must be taken to adopt the right way of thinking. Physicists generally have sound intuition when it comes to classical mechanics, but it is all too easy to use this intuition inappropriately when discussing quantum mechanics. One such argument concerns state reduction and is the subject of this note. State reduction is somewhat unfashionable currently, particularly from the point of view of decoherence theory, which asserts that only continuous in time Schrödinger unitary evolution occurs, contrary to the state reduction postulate of von Neumann \[.\] Clearly, it is of importance to our understanding of the universe to have a sound view of this issue, because it has implications at every level of physics, ranging from sub-atomic physics to cosmology. If we accept incomplete or misleading arguments against state reduction, there remains the possibility that we have misunderstood the nature of quantum reality.
Perhaps the most famous example of inadmissible classical thinking applied to quantum mechanics is to be found in the Einstein-Podolsky-Rosen (EPR) paper [2], which attempted to give an argument for the incompleteness of quantum mechanics. Although the EPR argument was refuted immediately by Bohr [3], the sort of thinking used by EPR has persisted, with the consequence that basic errors continue to be made in the interpretation of quantum mechanics. One of the difficulties in accepting Bohr’s counterattack is that it requires us to think of entangled quantum systems holistically. This is classically counter-intuitive because it involves non-locality, which is anathema in classical mechanics. Since there still appears to be confusion in the recent literature surrounding the interpretation of the EPR “paradox”, the aim of this note is to point out the dangers of applying classical counterfactual reasoning to quantum mechanics. Specifically, we shall refute the Stanford (Encyclopedia of Philosophy) argument against state reduction [4] by using its own line of reasoning and the principles of relativistic physics. There is no need to appeal to recent developments in quantum information theory.

As with the Stanford argument, we discuss a particular “thought experiment”. In our case, however, we are confident that it could be carried out in actuality, because we avoid making any counterfactual statements, which, by definition, can never be proved. The experiment is equivalent to the one discussed in the EPR paper [2] and in [4], confronting nonlocality in relativity (i.e., separation in space) with the von Neumann state reduction postulate [1] in quantum mechanics (i.e., wave function collapse). It involves a spin-zero state $\Psi$ of an electron-positron system created by some apparatus $O$ (see Figure 1). Considering only the spins of the particles, $\Psi$ is an entangled element of a tensor product Hilbert space $H = \mathcal{H}_e \otimes \mathcal{H}_p$, where $\mathcal{H}_e$ and $\mathcal{H}_p$ represent the spin degrees of freedom of the electron and positron respectively. A spin zero state is of the form

$$\Psi = e_n^+ p_n^- - e_n^- p_n^+,$$

where $n$ is any unit three-vector in physical space. Here $e_n^+$ is an eigenstate of electron spin measured along the direction $n$ with eigenvalue $+1$, $p_n^-$ is an eigenstate of positron spin along $n$ with eigenvalue $-1$, and so on. The state $\Psi$ is invariant to spatial rotations and therefore we are at liberty to choose any direction in physical space for the unit vector $n$.

Suppose now that $A$ and $B$ are two spatially separated observers at rest in some inertial frame $F$, such that $A$ tests for electron spin whilst $B$ tests for positron spin. These tests are carried out over sufficiently small intervals of space and time so that we can identify them with events (points in spacetime),
Figure 1: A tests for electron spin whereas B tests for positron spin. C and D are excluded by charge conservation from testing for positron spin given B, whereas E is a possible second test for positron spin.

also labelled by A and B respectively. In frame F, mutually orthogonal spatial axes have been previously chosen and denoted by the unit vectors i, j and k respectively, and there is no ambiguity about this concerning A or B. The test $\Sigma_k^e$ employed by A tests for electron spin along the k-direction and therefore has two possible outcomes, $e_k^+$ and $e_k^-$, whilst the test $\Sigma_j^p$ employed by B tests for positron spin along the j direction and also has two possible outcomes, $p_j^+$ and $p_j^-$.

The argument for the inconsistency of the state reduction postulate given in [4] appears quite convincing and goes as follows. The tests carried out by A and B on their respective particles are conducted with a spacelike interval between them, so they cannot interfere with each other. Suppose at time $t_A$, A finds the electron to be in state $e_k^+$. Then by conservation of total
angular momentum, $A$ would deduce that, for any subsequent time $t > t_A$, the positron state has collapsed into the anti-correlated state $p_k^-$. Essentially, $A$’s measurement of the electron’s spin is equivalent to an indirect ideal measurement of the positron’s spin.

But suppose now that $B$ had actually tested for positron spin at time $t_B$ just before time $t_A$, according to observers in frame $F$, i.e., just before $A$ had performed their test $\Sigma_k^e$ on the electron. $B$’s test throws the positron into an eigenstate of $\Sigma_j^p$, i.e., into either $p_j^+$ or else $p_j^-$, according to the reduction postulate. Because of the spacelike interval between them, Einstein locality (the principle of local causes) implies that the two tests $\Sigma_k^e, \Sigma_j^p$ could not possibly interfere with each other. But this means that, at any time $t > t_A$, the positron is in state $p_k^-$ because of $A$’s test, whereas it is in either of the states $p_j^+, p_j^-$ because of $B$’s test. Since the positron cannot be in two definite states at once, the Stanford conclusion is that state reduction is inconsistent.

This argument is equivalent to the original EPR argument and is incorrect for the same reasons. The error lies with the unfettered application of counterfactual statements to quantum mechanical situations. In the EPR case, direct tests of position and momentum cannot be carried out on the same particle simultaneously. In the case of the Stanford experiment, $A$’s assertion that the positron is in the state $p_k^-$ cannot be verified by $B$, because $B$ has already tested the positron (with outcome $p_j^+$ or $p_j^-$). In quantum physics, it very much matters what is actually done, rather than speculated on. As it stands, $A$’s belief about the positron’s state after $A$’s observation of the electron is a counterfactual one (with the twist that it relates to a non-existent future experiment by $B$, rather than to a non-existent past one) and therefore it does not have the logical status of $B$’s valid knowledge about the positron’s state after time zero.

In quantum mechanics, it is meaningless to talk about a state without any reference to any test of that state. $A$’s statement about the positron spin would have to be tested by a second test for positron spin, over and above the test at $B$, in order for it (i.e., $A$’s statement) to have physical significance. However, any additional test for positron spin at events such as $C$ or $D$ which are outside the forwards lightcone of $B$ could not be completed, because this would violate charge conservation, which is believed to hold absolutely (there would be at least one inertial frame in which two positrons existed at different places simultaneously in such a case). The only possible region of spacetime where a second test of the positron’s spin could be carried out consistent with known physics is at some event $E$ which is in the region of overlap of the forwards lightcones from both $A$ and $B$. But then it could not be argued that $E$ was testing only the state of the positron as prepared indirectly by $A$, because $B$ could causally influence $E$. Therefore, $A$’s view
of the positron state is inadmissible and the Stanford argument against wave function collapse itself collapses.

It is easy to see that the probability of $E$ finding the positron in state $p_k^+$, given that $A$ had found the electron in state $e_k^+$ and that $B$ had tested for positron spin along $\mathbf{j}$, is one half and not the value zero as predicted by $A$ on the basis of angular momentum conservation. The conclusions we draw from this is that the greatest care must be taken in the interpretation of quantum mechanics, particularly where state reduction is concerned. We conjecture that the state reduction concept will never lead to a paradox in quantum physics, provided proper care is taken to eliminate counterfactual errors.

Finally, we remark that the state reduction concept need not be invoked if only one test (or measurement) of a quantum system is being discussed. In such a case Schrödinger (unitary) evolution can be used, up to the point of measurement, plus the Born probability interpretation, without reference to state reduction. That is why decoherence calculations are valid under such circumstances. However, if a second test is to be performed subsequently, as in the case of event $E$ in our thought experiment above, then state reduction prior to that second test must be invoked.

References

[1] J. Von Neumann, *The Mathematical Foundations of Quantum Mechanics* (Princeton University Press, 1955).

[2] B. Podolsky, A. Einstein and N. Rosen, Phys. Rev. 47, 777-780 (1935).

[3] N. Bohr, Phys. Rev. 48, 696-702 (1935).

[4] H. Krips, *Measurement in Quantum Theory* (Stanford Encyclopedia of Philosophy (online), 1-9, 1999).

[5] A. Peres, *Quantum Theory: Concepts and Methods*. (Kluwer Academic Publishers, 1993)