On Multiple Images in Closed Universes

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Abstract

Universes with multiconnected spatial sections predict multiple images of cosmic sources. A confusing terminology exists in the naming of these images as real ones vs. ghosts. Here an attempt is made to clarify the situation.

I. INTRODUCTION

In a cosmological model with closed, nontrivial space topology, a distant astronomical source is predicted to produce several images, corresponding to the different paths its radiation can take to reach the observer on Earth. See Lachièze-Rey & Luminet [1] for a review.

Theoretically these multiconnected spatial sections are manifolds $M$ of the form $\tilde{M}/\Gamma$, where $\tilde{M}$ is the universal covering space of $M$, and $\Gamma$ is a discrete group of isometries acting freely on $\tilde{M}$. This action tessellates $\tilde{M}$ into cells which are copies of a fundamental polyhedron (FP). The FP has an even number of faces, which are pairwise congruent; $M$ may be represented by the FP, with members of these pairs identified (more colloquially, they are “glued together”; more technically, see Massey [2]).

In practice one looks at the FP as the physical space where the observer and the sources exist, and at its copies as the apparent space of repeated cosmic images. Hence the tendency to consider those images within the FP as ‘real,’ those in other cells as ‘ghosts.’ But it also appears reasonable to call real the nearest image of an object, the other ones being the ghosts. It is the purpose of this paper to show that these two criteria for classifying multiple images are not always consistent with each other.

II. A TWO-DIMENSIONAL EXAMPLE

One of the possible universe models with Einstein-de Sitter metric

$$ds^2 = c^2 dt^2 - \left(\frac{t}{t_0}\right)^{4/3}(dx^2 + dy^2 + dz^2)$$

has as spatial section the orientable manifold $E^2$ in Fig. 17 of [1]. A Klein bottle surface $K^2$ is imbedded in $E^2$, so we may use this nonorientable surface as our simplified model of cosmic space. It turns out that the FP (here fundamental polygon) is not unique; it depends on an arbitrary basepoint in its definition: The FP of a manifold $M = \tilde{M}/\Gamma$ with basepoint $x \in \tilde{M}$ is the set of points $\{y \in \tilde{M}; \text{distance}(y, x) \leq \text{distance}(y, \gamma x), \forall \gamma \in \Gamma\}$. 

\[1\]
Let $M = K^2$ be the Klein bottle obtained from the square $ABCD$ of side length $= 6$ in Fig. 1, with coordinates $(x, y)$ and basepoint $bp0 = (0, 0)$. This fundamental polygon will be called FP0. Group $\Gamma$ is generated by the motions
\[ \gamma_1 : (x, y) \mapsto (x + 6, -y) \]
\[ \gamma_2 : (x, y) \mapsto (x, y + 6), \]
and $\tilde{M}$ is the Euclidean plane. The identified pairs of sides are $AD \Leftrightarrow \gamma_1(AD) = CB$, $AB \Leftrightarrow \gamma_2(AB) = DC$.

Now we choose another basepoint, $bp1 = (2, 2)$, and find the corresponding FP as indicated in Fig. 1. The six images of $bp1$ in the figure $(im1 - im6)$ are sufficient to determine the irregular hexagon $EFGHIJ$ as FP1. The points on line $EF$ are equidistant from $bp1$ and $im1$, and thus belong to the border; and similarly for the other sides. The coordinates of the vertices are $E = (-1/3, -1), F = (13/3, -1), G = (17/3, 1), H = (13/3, 5), I = (-1/3, 5)$, and $J = (-5/3, 1)$, and the identifications are $EJ \Leftrightarrow \gamma_1(EJ) = GF$, $EF \Leftrightarrow \gamma_2(EF) = IH$, $GH \Leftrightarrow \gamma_2\gamma_1^{-1}(GH) = IJ$.

III. AMBIGUITY OF THE NAMES ‘REAL’ AND ‘GHOST’ IMAGES

In Fig. 2, let the observer’s position be $bp1$, and two images of a cosmic source be located at $p = (-2, -1)$ and $q = \gamma_1p = (4, 1)$. The nearest image is $q$, and if the observer is using FP1, then by both criteria at the end of Sec. I $q$ would be the real image and $p$ a ghost. But if she or he is using FP0, which is much easier to handle mathematically despite the fact that the observer is not at its center, then by first criterion the real image is $p$ and $q$ is a ghost, with the opposite holding by the second rule.

Therefore, in order that the terminology ‘real’ vs. ‘ghost’ (or ‘source’ vs. ‘ghost’) could be used consistently, one would have to always work with an FP where the basepoint is at the observer’s position. A further advantage of thus having the nearest images labeled as sources would be in the comparison with astrophysical data, which are usually richer for nearby objects.

However, the calculations are much simpler in the more symmetrical FP like FP0 above, or in those FP’s which mathematicians say have “maximum injectivity radius” - see Weeks [3], for example. This is what was done by the author in [4], where the FP is a regular icosahedron with basepoint at its center, far from the observer’s position.

I suggest calling the nearest images just that, while the second nearest would be second image, and so on.

I am grateful to Conselho Nacional de Desenvolvimento Científico e Tecnológico (CNPq - Brazil) for partial financial support.
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