Pion Production in High-Energy Neutrino Reactions with Nuclei

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Background: A quantitative understanding of neutrino interactions with nuclei is needed for precision era neutrino long baseline experiments (MINOS, NOvA, LBNE) which all use nuclear targets. Pion production is the dominant reaction channel at the energies of these experiments.

Purpose: Investigate the influence of nuclear effects on neutrino-induced pion production cross sections and compare predictions for pion-production with available data.

Method: The Giessen Boltzmann–Uehling–Uhlenbeck (GiBUU) model is used for the description of all incoherent channels in neutrino-nucleus reactions.

Results: Differential cross sections for charged and neutral pion production for the MINERνA neutrino and antineutrino flux are calculated. An estimate for the coherent cross section is obtained from a comparison of data with theoretical results for incoherent cross sections. The invariant mass (W) distribution of the Δ resonances produced is analyzed.

Conclusions: Final state interactions affect the pion kinetic energy spectra significantly. The data for charged pion production at MINERνA are compatible with the results of calculations using elementary data taken from an old Argonne National Laboratory experiment. Remaining differences for charged pion production can be attributed to coherent production; the data for antineutrino induced neutral pion production, where no coherent contribution is present, are reproduced quite well. The analysis of W-distributions shows that sharp cuts on experimentally reconstructed invariant masses lead to shape-distortions of the true W distributions for nuclear targets.

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I. INTRODUCTION

The planned long-baseline neutrino experiment (LBNE) will operate in a neutrino beam the properties of which are similar to the presently used NUMI beam at Fermilab. Already ongoing and planned experiments in that beam offer the possibility for precision measurements of the differential cross sections for quasielastic scattering (QE) and single- and multi-pion production, as a function of (reconstructed) neutrino energy. These cross sections are essential for an understanding of the interactions of the incoming neutrinos with the nuclear targets (C, O, Ar) used in these experiments. A quantitative understanding of these cross sections is essential for the energy reconstruction and - consequently - for a precise extraction of oscillation parameters from observed event rates \[1\]. This extraction has to rely on generators and thus the cross sections measured now in experiments such as, e.g., MINER\(\nu\)A could be a basis for a fine-tuning of event generators \[2\] for use at the LBNE.

All events following from a neutrino-nucleus interaction can be grouped into two general classes. The first one is that of quasielastic scattering (QE), including many-body interactions. The second class is connected with pion production, either through resonances, \(t\)-channel background processes or deep inelastic scattering (DIS). At the presently running MINER\(\nu\)A experiment the pion production channels make up for about 2/3 of the total inclusive cross section while the QE events account only for about 1/3 \[3\]. Both event types are entangled with each other and cannot be separated by purely experimental means. Separation is only possible with the help of a event generator and, therefore, is necessarily model-dependent. Recent calculations of the inclusive QE response of nuclei \[4–8\] in their comparison with experiment thus have to rely on the accuracy with which pion production is described by the generator used in extracting the data.

It is, therefore, essential to obtain a quantitative understanding of the dominant reaction channels at MINER\(\nu\)A and LBNE; these are those connected with pion production. It is the purpose of the present paper to present results of calculations for pion production at the MINER\(\nu\)A experiment. While first results were already contained in \[9, 10\] in the present paper we now perform a detailed comparison with MINER\(\nu\)A data using their flux and invariant mass cuts. Over the last few years valuable insight into the neutrino-induced pion production on nuclei at lower energies has been obtained mainly from the MiniBooNE experiment \[11–14\]. We, therefore, also discuss the consistency of its data with those obtained at the higher energies of the MINER\(\nu\)A experiment. Finally, we discuss some problems connected with kinematical cuts in the experiment.

II. METHOD

The calculations are performed within the transport theoretical framework GiBUU \[15\]. GiBUU approximately factorizes the reaction into a very first interaction of the incoming neutrino with a bound and Fermi-moving nucleon and the following final state interactions (fsi) \[1\]. The latter are described by a numerical implementation of the Kadanoff-Baym equations in the gradient approximation \[16\], using the Botermans-Malfliet approximation for off-shell transport \[17\]. More details about this treatment of final state interactions can be found in Ref. \[18\].

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1 We denote by 'final state interactions' only the secondary and following collisions. This is different from the nomenclature often used in studies of inclusive cross sections for QE where fsi denote the potentials felt by the outgoing nucleon in the final state of the initial reaction.
The groundstate of the target nucleus is assumed to be that of a local relativistic Fermi gas with the nucleons being bound in a coordinate- and momentum-dependent potential that has been fitted to equation of state and effective mass data \[^19, 20\]. The hole spectral function is given by

\[ P_h(p, E) = g \int \frac{d^3 r}{\text{nucleus}} \Theta(p_f(r) - |p|) \Theta(E) \delta \left( E - m^*(r, p) + \sqrt{p^2 + m_e^2(r, p)} \right) \]  \hspace{1cm} (1)

here \( p_f(r) \) is the local Fermi momentum given by the local Thomas Fermi model and \( g \) is a degeneracy factor. In this spectral function all effects of the nucleon potential are assumed to be contained in the effective mass \( m^* \) \[^18\] which depends on location and momentum of the nucleon. The corresponding momentum distribution approximates that obtained in state-of-the-art nuclear many-body theory calculations quite well; see Fig. 4 in \[^21\]. The initial interaction rates are calculated for a nucleon at rest; the results are then boosted to the local rest frame of the Fermi-moving target nucleon.

The single-pion production cross section at fixed neutrino energy is then given by (cf. \[^22\] where all the details can be found)

\[ d\sigma^{\nu A \to \ell' X\pi} = g \int \frac{d^3 r}{\text{nucleus}} \int \frac{d^3 p}{(2\pi)^3} \Theta(p_f(r) - |p|) f_{\text{corr}} d\sigma^{\text{med}} P_{\text{PB}}(r, p) F_\pi(q, r) . \]  \hspace{1cm} (2)

Here \( f_{\text{corr}} \) is a flux correction factor \( f_{\text{corr}} = (k \cdot p)/(k^0 p^0) \); \( k \) and \( p \) denote the four-momenta of the neutrino and nucleon momentum, respectively. \( P_{\text{PB}}(r, p) \) describes the Pauli-blocking and the factor \( F_\pi(q, r) \) in (2) describes the effects of all the fsi contained in GiBUU. For pions the latter involve elastic and inelastic scattering as well as pion absorption through 2-body and 3-body processes of the types \( N^+ + N \rightarrow N + N \) and \( N^+ + N + N \rightarrow N + N + N \); here \( N^* \) stands for the \( \Delta \) resonance and higher excitations of the nucleon. In GiBUU these resonances are treated explicitly; they become excited and then are being propagated until they decay or collide with other nucleons; this is described in some detail in \[^18\]. The time-development of the \( \pi - N - \Delta \) dynamics in the nucleus is determined by the resonance widths and collision rates alone. In the resonance region there is no room for a further free parameter, such as a formation time used in other generators \[^23\]. The use of formation times during which interactions of the produced particle are prohibited introduces an arbitrariness into the description of pion production. Only in the DIS part, above invariant nucleon masses of about 2 GeV, the concept of a formation time makes sense since it accounts for the widths of high lying, no longer separable excitations of the nucleon. Even then, cross sections of the produced particles should rise with time until the final hadron has fully been formed \[^24\].

The cross section \( d\sigma^{\text{med}} \) in Eq. (2) stands for the pion production inside the nuclear medium. Pions can be produced either through nucleon resonances or through DIS. The latter, denoted in the following by \( d\sigma_{\text{DIS}} \), is obtained from the string-fragmentation model PYTHIA \[^25\]. We smoothly switch over been these two pictures around an invariant mass of the nucleon of 2 GeV. Below this energy we treat all nucleon resonances with their correct pion decay branches taken from the PDG. In this mass region the pion production cross section consists of a resonance and of a background contribution which have to be added coherently in order to obtain the full cross section. For the background contribution we assume a form taken from an effective field theory treatment of pion production up the \( \Delta \) resonance region \[^26, 27\]. We do not take any background contributions for the higher resonances into account. Since transport (or any Monte Carlo code) cannot handle the coherence of resonance and background amplitudes we split the coherent sum up into a resonance cross section and one for the background which contains both the squared background amplitude and the interference term. Background pions are then produced locally, without any
time-delay. Because GiBUU describes only incoherent processes it does not contain a coherent pion-production cross section; the latter would require a quantum mechanical description.

The resonance production cross section is given by
\[
\frac{d\sigma_{\text{med}}}{d\omega d\Omega'} = \frac{|k'|}{32\pi^2} \frac{A_{\text{med}}(p')}{[(k \cdot p)^2 - m^2 M^2]^{1/2}} |M_R|^2.
\]

Here \( M \) is the nucleon mass, \( p \) denotes the nucleon’s four-momentum, \( p' \) that of the outgoing resonance and \( k \) and \( k' \) that of the initial and final state lepton, resp. The quantities \( \omega \) and \( \Omega' \) give the energy transfer and the scattering angle of the outgoing lepton, resp. The in-medium spectral function of the resonance is denoted by \( A_{\text{med}}(p') \).

The cross section for resonance formation contains the square of an invariant matrix element \( M_R \) that is obtained by contracting the lepton tensor with the hadron tensor. For the latter we have
\[
H^{\mu\nu} = \frac{1}{2} \text{Tr} \left[ p + M \right] \Lambda_{\alpha\beta} \Gamma^{\alpha\beta\nu}
\]
where \( \Lambda_{\alpha\beta} \) is (for the \( \Delta \)) the spin-3/2 projector and the vertex factor \( \Gamma^{\alpha\mu} \) is given by
\[
\Gamma^{\alpha\mu} = [V^{\alpha\mu} - A^{\alpha\mu}] \gamma^5
\]
for a positive parity resonance. The vector part \( V \) is taken from the MAID analysis of electron scattering data so that the data for electro excitation of nucleon resonances are reproduced by construction. For the axial part \( A \) the spin-3/2 transition current contains in principle four independent axial form factors \( C_A \), but the presently available data do not allow to determine them separately. We, therefore, assume
\[
A^{\alpha\mu} = -\left( \frac{C_4^A(Q^2)}{M^2} (g^{\alpha\mu} q \cdot p' - q^{\alpha} p'^{\mu}) + C_5^A(Q^2) g^{\alpha\mu} + \frac{C_6^A(Q^2)}{M^2} q^{\alpha} q^{\mu} \right) \gamma^5
\]
with the further simplification \( C_4^A = -C_5^A/4 \) and \( C_6^A = C_5^A M^2/(Q^2 + m^2) \). More details can be found in Ref. [22].

The absolute strength of the resonance contributions determined by \( C_6^A(0) \) is obtained by fitting the available pion production data on an elementary target. The two datasets available are those obtained at Argonne National Lab (ANL) and Brookhaven National Lab (BNL). These two datasets differ in the relevant energy regime, with the BNL dataset being higher than the ANL one by about 25%. This introduces a corresponding uncertainty into the calculations for nuclear targets. Earlier it had been argued that this difference is due to flux-uncertainties. The authors of [27], in a detailed study of the consistency of the various isospin channels and the measured \( d\sigma/dQ^2 \), concluded that the BNL data were too high. This has very recently been verified by the authors of [32]. In a reanalysis of the old data that fixes the flux with the help of the QE cross section it was shown that at least for the \( \pi^+ \) channel the ANL data were preferable and also consistent with other data from CERN. The reanalysis of Wilkinson et al [32] seems to settle the question for the correct elementary cross section. However, most recently a new theoretical calculation of pion production on Deuteron has shown that even in this small system fsi can play a significant role. This then affects the extraction of cross sections for \( p \) and \( n \) targets from data obtained with a \( D \) target. There is still some uncertainty left on the elementary pion production cross section.

The extensive pion production data from the MiniBooNE experiment obtained on a \( CH_2 \) target are consistently higher than the ones calculated within GiBUU, both for the ANL and \( - \) less so \( - \) for the BNL input cross sections. Motivated by the reanalysis by Wilkinson et al we are now using the ANL dataset as default input into GiBUU. In order to illustrate the sensitivity of the results for a nuclear target to the remaining uncertainties in the nucleon cross section we show, however, also some results obtained with the BNL input.
To obtain the pion production cross section we multiply the resonance formation cross section with the branching ratio for decay into the $\pi N$ channel

$$\frac{d\sigma_{\text{med}}}{d\omega d\Omega_{\pi}} = \frac{1}{4\pi} \frac{d\sigma_{\text{med}}}{d\omega d\Omega'} \frac{\Gamma_{R\rightarrow N\pi}}{\Gamma_{\text{tot}}} \tag{7}$$

Here we have assumed that the decay of the resonance happens isotropically in the rest frame of the resonance. This is not a good approximation for an isolated $\Delta$ resonance. Therefore, for the fsi process $\pi + N \rightarrow \Delta \rightarrow \pi + N$ we use a more sophisticated parametrization of the $p$-wave behavior of the $\Delta$ decay as explained in detail in [34]. Although most of the primary reactions populate higher lying states via DIS or higher resonances, the $\Delta$ resonance plays quite an essential role in the observable pion spectrum as will be discussed later.

The in-medium effects for pion production are contained both in the spectral function $\mathcal{A}$ in (3) as well as in the branching ratio for the resonance decay into $\pi N$ where the final nucleon state may be Pauli-blocked. The final state nucleon is bound in a momentum and coordinate dependent potential which – through energy- and momentum-dependence – affects the decay width. The spectral function contains a collisional broadening which we take from intensive investigations of $\Delta$ properties inside the nuclear medium by the Valencia group [35]. To obtain the correct spectral distribution of pions it is essential to maintain consistency between this collisional broadening and the actual collision rates embedded in the collision cross sections encoded in the generator.

The theory described so far has been extensively tested with the help of photon- [36] and electron-induced [37] pion production data.

For the comparison with the experimental neutrino data we fold the fixed-energy cross sections described above with the appropriate flux

$$\langle d\sigma \rangle = \int_{1.5\text{GeV}}^{10\text{GeV}} dE_\nu \phi(E_\nu) \left( d\sigma_{\text{med}}(E_\nu) + d\sigma_{\text{DIS}}(E_\nu) \right) \tag{8}$$

where $\phi(E_\nu)$ is the incoming energy distribution (the ’flux’), normalized to 1. The integration boundaries 1.5 and 10 GeV are those of the experimental analysis. All results shown in this paper were obtained with the MINER$\nu$A flux for a $CH$ target. Cross sections are given per nucleon, i.e. the total cross section for $CH$ is divided by 13.

III. PION PRODUCTION AT MINER$\nu$A

A. Charged Pion Production in the Neutrino Beam

In Fig. 1 the calculated kinetic energy distribution for the sum of $\pi^+$ and $\pi^-$ is shown in comparison with the data [38]. The solid curve, calculated with the ANL input, follows the shape reasonably well, but lies below the data. The biggest disagreement shows up for kinetic energies $T_\pi < 0.1$ GeV; for the higher energies the calculation lies at the lower end of the error bars so that the disagreement there may not be significant.

Before discussing the difference between theoretical and experimental kinetic energy distributions further we show in Fig. 2 the calculated angular distribution in comparison with the data. It is evident now that the disagreement noticed for the kinetic energy distribution is localized at forward angles $\theta < 50^\circ$. At about $30^\circ$ the calculated cross section amounts to only about $1/2$ of the measured one while it describes the data quite well for $\theta > 50^\circ$.

The major systematic error in the calculated results comes from the uncertainty in the elementary cross section. Figs. 1 and 2 also show in the dash-dotted curves the distributions calculated with the BNL input. As discussed earlier
FIG. 1. Kinetic energy distributions per nucleon of incoherently produced charged pions on a CH target in the MINERνA neutrino flux. The solid curve gives the results obtained with the ANL cross sections as elementary input. The dashed-dotted curve gives the same distribution calculated with the BNL elementary cross sections. The data are from Ref. [38].

FIG. 2. Angular distribution of incoherently produced charged pions in the MINERνA experiment (data from [38]). The solid curve gives results of a calculation with the ANL input, the dash-dotted curve has been obtained with the BNL input.
this is probably an overestimate of the true cross section and is shown here only in order to illustrate the effects of a change of input. Now the agreement between theory and experiment for the kinetic energy distribution (Fig. 1) is clearly better (calculated cross section within all error bars). However, the angular distribution now becomes considerably worse (see Fig. 2) with a clear overshooting at the intermediate angles around 60°.

1. Experiment-Theory discrepancies: Coherent Production

In order to investigate the discrepancies between theory and experiment further we show in Fig. 3 the difference between a smoothed curve through the data for the kinetic energy distribution and the calculated cross section. The error bars shown in that figure are only those given by Eberly et al. [38] for the data; no systematic error for the GiBUU calculation has been added. The difference in the kinetic energy distribution amounts to roughly $0.1 \cdot 10^{-38}$

![Graph showing the kinetic energy distribution and angular distribution](image)
cm²/GeV for energies above about 0.1 GeV; at the energies below that the difference rises up to about $0.5 \cdot 10^{-38}$ cm²/GeV at 0.04 GeV. For the angular distribution (lower part of Fig. 3) the difference between the calculated and the experimental values peaks at about $\theta = 18^\circ$.

Assuming that GiBUU describes the incoherent pion production quite well (cf. the discussion in Sect. III.C.1), the excess at low kinetic energies and small angles could be due to the coherent production. Most generators use PCAC based models employing the Adler relations [39]; it has been argued, however, that these break down for coherent excitations on nuclei [40]. Indeed, the experimental analysis in [41] finds that these models implemented in standard generators do not describe the data.

MINERνA has also measured the coherent cross section [41], but without the $W_{\text{rec}} < 1.4$ GeV cut so that these data are not directly comparable with the ones extracted here. The measured value without the cut amounts to about $3.5 \times 10^{-39}$ cm² [41]; the integral over the difference shown in Fig. 3 amounts to about $1.9 \times 10^{-39}$ cm² and is thus quite reasonable considering the different $W$-ranges. A reanalysis of the coherent data [41] using the same cuts as those employed by the same experimental collaboration in [38] would help to clarify the situation.

2. Comparison with MiniBooNE CC pion production

The MiniBooNE pion production data required input data even higher than the BNL values so that these data are obviously not compatible with the ones obtained by MINERνA. For comparison we show in Fig. 4 also the calculated pion spectrum for the MiniBooNE experiment as the lowest, dotted curve; for a more detailed presentation of results for pion production in MiniBooNE we refer to [13]). The calculated MINERνA cross section is roughly by a factor 1.5 larger than that for the MiniBooNE experiment whereas the data obtained by both experiments essentially lie on top of each other for kinetic energies above about 150 MeV; at the lower energies the MINERνA values are considerably higher. This has recently also been pointed out by Sobczyk and Zmuda [42] in a detailed comparison of both data sets. These authors noted a disagreement both in absolute height and in shape between the data from these two experiments and a generator prediction. We speculate that a coherent contribution explains the difference, not in height, but in shape, between the pion spectra obtained at MiniBooNE and MINERνA where the latter show a clear surplus at forward angles. The coherent contribution is concentrated at small $T_\pi$ and forward angles and increases with increasing beam energy [41]. Therefore, while at the lower MiniBooNE energies the coherent contribution is negligible it can be observable at the MINERνA energies.

3. Invariant mass cuts

All the data in Figs. 1 and 2 were obtained with a cut on the reconstructed invariant-mass $W_{\text{rec}} < 1.4$ GeV in the analysis [38]. Here $W_{\text{rec}}$ is defined as

$$W_{\text{rec}}^2 = M^2 + 2 M \omega - Q^2 ,$$

where $\omega$ is the energy transfer and $Q^2$ the four-momentum transfer. The experiment does not measure these latter two quantities directly but has to reconstruct them from a calorimetrically reconstructed incoming neutrino energy [38]. The

$^2$ It would be interesting to know how much of the final energy is actually measured and how much has to be reconstructed by using a generator.
FIG. 4. (Kinetic energy spectra for incoherent single charged pion production in CC reactions. The solid lines give the results for neutrino with the ANL input and a cut $W_{\text{rec}} < 1.4$ GeV. The dashed, uppermost line shows the single-pi production spectrum without any $W_{\text{rec}}$-cut. The dotted, lowest curve shows the results for the MiniBooNE flux and a CH$_2$ target, again without a $W_{\text{rec}}$-cut.

cut has been used in [38] to make the higher energy data obtained at MINER$\nu$A comparable to those obtained earlier at lower energies in the MiniBooNE experiment [11] where the population of higher invariant masses was energetically suppressed.

In order to illustrate the effect of the cut on $W_{\text{rec}}$ we show in Fig. 4 also the kinetic energy spectrum without any such cut (topmost dashed line). This uncut spectrum is larger by about a factor of 1.7 at the maximum. The cut on $W_{\text{rec}}$ thus has a considerable influence on the published experimental cross section. This large effect is worrying since $W_{\text{rec}}$ has no direct physical meaning for bound nucleons.

$W_{\text{rec}}$ is the invariant mass for an interaction with a free, unbound nucleon at rest. For a bound and Fermi-moving nucleon the correct invariant mass is given by

$$W^2 = (E_N + \omega)^2 - (p_N + q)^2,$$

where $E_N$ is the energy of the bound nucleon, $p_N$ its momentum and $q$ the three-momentum transfer. Also $W$ is experimentally not directly accessible.

Both $W_{\text{rec}}$ and $W$ represent entrance channel properties. For pion production relevant are also the invariant mass distributions of the final $\pi N$ pairs

$$W_{\pi N}^2 = (E_N + E_\pi)^2 - (p_N + p_\pi)^2.$$  

Here $E_\pi$ and $p_\pi$ are the energy and the momentum of the pion. For values up to about 1.5 GeV $W_{\pi N}$ contains information on the $\Delta$ spectral function and is experimentally directly accessible.

In order to investigate these different definitions of an invariant mass in some more detail we show in Fig. 5 various $W$-distributions calculated with GiBUU. There is, first, the true $W$ distribution (Eq. 10), without any cut, shown by
FIG. 5. (color online) Invariant mass distributions of incoherent events in which first a $\Delta$ resonance was excited. The calculations were performed for a $C$ target with the MINERvA flux. Shown are the true $W$-distribution without (solid, green curve) and with (dashed, red curve) cut. Also shown is the reconstructed $W_{\text{rec}}$ distribution (red dash-dotted curve) and the distributions of the $\pi N$ invariant mass $W_{\pi N}$ without (dotted green curve) and with cut (red, dot-dot-dashed curve).

a solid green line. This curve has been obtained from a calculation without any cut. The same true $W$ distribution for a calculation in which the cut $W_{\text{rec}} < 1.4$ GeV has been used is shown by the dashed red line. This curve shows no sharp cutoff at 1.4 GeV, but instead it loses strength on the high $W$ side, starting from the maximum\(^3\) and upwards towards higher $W$. The cut on $W_{\text{rec}}$, therefore, does not eliminate the pions from high-mass excitations, but instead distorts the spectral shape of the $\Delta$.

The distribution of $W_{\text{rec}}$ (red, dashed-dotted curve) looks very different from the true $W$ distributions discussed so far. It is considerably lower in its peak cross section, broader and shifted to higher masses until it is cut off at 1.4 GeV. It shows no resemblance to the $\Delta$ spectral function.

Finally shown are also, by the green dotted and the red dot-dot-dashed lines, the invariant mass distributions of the final $\pi N$ pairs. Again, the red dot-dot-dashed curve obtained in a calculation with a cut on $W_{\text{rec}}$ misses strength from 1.15 GeV on upwards. No sharp cut in $W_{\pi N}$ appears. Since the events in both distributions are subject to strong final state interactions they are lower than all the others.

### B. Charged Pion Production in the Antineutrino Beam

For completeness we show in Fig. 6 the charged pion spectrum (sum of $\pi^-$ and $\pi^+$) in an antineutrino beam. These results were obtained with a flux $1.5$ GeV $< E_\nu < 10$ GeV and no invariant mass cut. The shape is very similar

\(^3\) The peak in Fig. 5 appears at a somewhat lower mass than the free $\Delta$ mass; this shift is due to the binding of the $\Delta$ inside the nucleus.
to that obtained for the neutrino beam (topmost curve in Fig. 4), but is lower by about a factor of 2 due to the different $V - A$ interference for antineutrinos. In Fig. 7 the angular distribution is given for the incoherent single pion production in the MINERV\(\nu\)A antineutrino beam.

![Kinetic energy spectra for incoherent single charged pion production in CC reactions with the MINER\(\nu\)A antineutrino beam between 1.5 and 10 GeV. No $W_{\text{rec}}$-cut has been used.](image1)

**FIG. 6.** Kinetic energy spectra for incoherent single charged pion production in CC reactions with the MINER\(\nu\)A antineutrino beam between 1.5 and 10 GeV. No $W_{\text{rec}}$-cut has been used.

![Angular distribution of incoherently produced charged pions in the MINER\(\nu\)A experiment with an antineutrino beam between 1.5 and 10 GeV. No $W_{\text{rec}}$-cut has been used.](image2)

**FIG. 7.** Angular distribution of incoherently produced charged pions in the MINER\(\nu\)A experiment with an antineutrino beam between 1.5 and 10 GeV. No $W_{\text{rec}}$-cut has been used.

production in the MINERV\(\nu\)A antineutrino beam.
C. Neutral Pion Production in the Neutrino and Antineutrino Beams

We have also calculated the $\pi^0$ production both for neutrino and antineutrino induced reaction with the ANL input. These results were also obtained with a flux $1.5\text{ GeV} < E_\nu < 10\text{ GeV}$ and no invariant mass cut (Fig. 8). The two upper curves give the results for neutrino-induced and the two lower ones for antineutrino-induced pion production. The effects of fsi are remarkable: the cross section after fsi are significantly larger than those before, in particular around 0.1 GeV kinetic energy. Furthermore the shape is significantly distorted by fsi. The strong overshoot at 0.08 GeV is followed by an undershoot at around 0.24 GeV. For somewhat higher kinetic energies the cross section is again increased by the fsi and only from about 1 GeV on upwards there is a slight attenuation by final state interactions. The cross section after fsi shows a net increase compared to that before fsi. This increase is due to charge-transfer reactions from the dominant $\pi^+$ channel in the $\nu$-induced production and the $\pi^-$ channel in the $\bar{\nu}$ induced one. The importance of this charge transfer was already pointed out in [22]; here we note that GiBUU has been extensively tested against pion charge-exchange reactions on nuclei [34, 43].

The region around $T_\pi = 0.24\text{ GeV}$ reflects the strong pion absorption through the pion-less decay of the $\Delta$ resonance. This indicates that even at the high energies of the incoming neutrino beam in the GeV region, the $\Delta$ resonance plays an essential role in the fsi. Initially produced energetical pions cascade down through a sequence of elastic or inelastic scattering, possibly connected with charge transfer. The pions that finally end up in the $\Delta$ region can then be absorbed. Those pions that are slowed down even further can no longer be absorbed into a $\Delta$ thus causing the strong peak at 0.08 GeV.

The angular distributions for $\pi^0$ production shown in Fig. 9 look very similar to those for charged pion production.

1. Neutral Pion Production at MINERνA

We now turn to a comparison of the results for CC antineutrino-induced neutral pion production data [44] that became available about 1 week after after a first version of the present paper paper had been uploaded to the arXiv [45]. The main difference to the charged pion data, besides the different flavor of the incoming beam, consists in selecting the incoming neutrino energy to be between 1.5 and 20 GeV (instead of 10 GeV for the charged pions) and the absence of any $W$ cut. The absence of a $W$-cut has a more significant effect on the cross sections than the raising of the upper neutrino energy; the latter has about a 10% effect. We have, therefore, also performed calculations for these experimental specifications. The results are shown in Figs. 10 and 11. The comparison with the data shows again an underprediction at the lowest kinetic energies at around 40 and 80 MeV. On the other hand, for kinetic energies above about 0.3 GeV the data are lower than the theoretical prediction. In contrast, the angular distribution, which sums over all the kinetic energies, is now described quite well, even at the forward angles.

This behavior lends some support to the earlier discussions of a coherent contribution to the measured charged pion cross sections in Sect. III A 1. There is no coherent contribution to the CC neutral pion production channel because of charge conservation. This explains why in this case there is no small-angle enhancement in the experimental angular distribution. On the other hand, the overshoot of the kinetic energy distribution at higher energies, combined with the underprediction at the lowest energies, may indicate an underestimate of fsi charge exchange reactions for the pion or problems in the experimental background subtraction.
FIG. 8. (color online) Kinetic energy spectra of incoherently produced single \( \pi^0 \) for CC reactions on a CH target using the MINER\( \nu \)A neutrino and antineutrino fluxes between 1.5 and 10 GeV. The solid lines give the results for neutrino (upper, green) and for antineutrino (lower, red) beams with fsi included. The dashed lines, labeled with 'incl', give the corresponding results before fsi. No \( W_{\text{rec}} \) cut has been used.

FIG. 9. Angular distribution of incoherently produced single \( \pi^0 \) in the MINER\( \nu \)A experiment with the neutrino and antineutrino fluxes between 1.5 and 10 GeV. No \( W_{\text{rec}} \) cut has been used.
FIG. 10. (color online) Kinetic energy spectra of incoherently produced single $\pi^0$ for CC reactions on a CH target using the MINER$\nu$A antineutrino flux between 1.5 and 20 GeV. Data are from [44], converted into $d\sigma/dT_\pi$.

FIG. 11. Angular distribution of incoherently produced single $\pi^0$ in the MINER$\nu$A experiment with the antineutrino flux between 1.5 and 20 GeV. Data are from [44].
IV. SUMMARY AND CONCLUSION

In the energy regime of the planned LBNE experiment pion production, either through resonances or DIS, represents the dominant reaction channel. It is, therefore, absolutely mandatory to obtain a quantitative understanding of this process. Quasi-elastic scattering, which has received a lot of theoretical attention recently, is necessarily entangled with pion production in the actual observables and can not be separated without the help of an event generator. Thus, the quality of the QE data can never be better than the quality of the generator used to remove the pion contamination. On the other hand, pion production initiated by neutrino interactions is interesting in itself since it may provide information on the axial couplings of resonances.

In this paper we have primarily analyzed the recent MINERνA data on charged pion production [38]. Unfortunately, the crucial input to such calculations, the pion production cross section on isolated nucleons, is still somewhat uncertain. While the discrepancy between two older data sets, from ANL and BNL, has recently found a reasonable explanation in terms of flux uncertainties in the BNL experiment [32], there are still lingering problems with the extraction of nucleon cross sections from experiments using Deuterium targets [33]. This latter problem clearly deserves more theoretical work, but ultimately a dedicated experiment using Hydrogen targets is needed to clarify this point.

The calculations for charged pion production reproduce the measured kinetic energy spectra quite well, except for the lowest kinetic energies around 100 MeV, where the calculation comes out significantly lower than the experiment. This discrepancy can be localized in events connected with small scattering angles. At forward angles, the calculation significantly underestimates the experimental cross sections. In this paper we have analyzed the discrepancies, both in kinetic energy and angle, and have shown that they are compatible with contributions expected from a coherent excitation process. A reanalysis of the experimental MINERνA data on coherent pion production [41], using exactly the same cuts as the Eberly et al data [38], would help to verify this explanation. This explanation also finds some support in the observation that the angular distribution of the neutral pion production data in the antineutrino beam [44], where no coherent component can be present, is reproduced quite well by the present calculations.

In an earlier paper [10] we had already noted that the flux cut used in the MINERνA analyses introduces a model dependence into the comparison of theory with data. The cuts used in theoretical calculations are those for true energies whereas the experimental cuts can only be done for reconstructed energies. We have shown in [10] that the flux cuts used by the MINERνA experiment can significantly distort the event rates, requiring a fairly large and generator-dependent correction through energy-migration matrices. Here we have now analyzed the effects of cuts imposed on the reconstructed invariant mass $W_{\text{rec}}$ in obtaining the pion production data. The cut has a major influence on the total pion production cross section. This was to be expected since a large part of the pions is produced by DIS events. A cut at 1.4 GeV, as employed in the experiment, was intended to remove these contributions and enrich the $\Delta$ resonance contribution. This, however, is not what the $W_{\text{rec}}$ cut actually does. Instead it cuts off strength over a large, high-mass part of the $\Delta$ spectral function, starting already around its peak value. The explanation for this behavior lies in the Fermi-motion and binding energy of the nucleons inside the nuclear target. A comparison of pion cross sections obtained with this cutoff on $W_{\text{rec}}$ with data in other experiments and at lower energies is then difficult. In addition, this cut – being an entrance channel cut – has to rely on the reconstruction of neutrino energy which introduces a model dependence into the data. A cut on the $\pi - N$ invariant mass in the final state would be free of this problem and is thus preferable.
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