Black Holes, Information Loss, and Hidden Variables

Antony Valentini

Perimeter Institute for Theoretical Physics, 35 King Street North, Waterloo, Ontario N2J 2W9, Canada.

We consider black-hole evaporation from a hidden-variables perspective. It is suggested that Hawking information loss, associated with the transition from a pure to a mixed quantum state, is compensated for by the creation of deviations from Born-rule probabilities outside the event horizon. The resulting states have non-standard or ‘nonequilibrium’ distributions of hidden variables, with a specific observable signature – a breakdown of the sinusoidal modulation of quantum probabilities for two-state systems. Outgoing Hawking radiation is predicted to contain statistical anomalies outside the domain of the quantum formalism. Further, it is argued that even for a macroscopic black hole, if one half of an entangled EPR-pair should fall behind the event horizon, the other half will develop similar statistical anomalies. We propose a simple rule, whereby the relative entropy of the nonequilibrium (hidden-variable) distribution generated outside the horizon balances the increase in von Neumann entropy associated with the pure-to-mixed transition. It is argued that there are relationships between hidden-variable and von Neumann entropies even in non-gravitational physics. We consider the possibility of observing anomalous polarisation probabilities, in the radiation from primordial black holes, and in the atomic cascade emission of entangled photon pairs from black-hole accretion discs.

1 email: avalentini@perimeterinstitute.ca
1 Introduction

In classical physics, when a star undergoes gravitational collapse to form a black hole, the final spacetime geometry in the exterior region depends only on the total mass, charge, and angular momentum. Other details of the collapsing matter are completely erased. Once the material falls behind the event horizon, information concerning its detailed structure is screened off from the exterior.

According to quantum field theory on the resulting classical spacetime background, the black hole radiates like a body at temperature \( T = \frac{1}{8\pi M} \) (in the non-rotating case) \[1\]. In conventional units

\[
T = \frac{\hbar c^3}{8\pi GkM} \approx (10^{-7} \text{ K})(M_\odot/M)
\]

(where \( M_\odot \) is the mass of the Sun). The outgoing Hawking radiation modes are entangled with ingoing modes crossing the horizon. Upon tracing over the latter degrees of freedom, the field in the asymptotically flat region is found to be in a mixed state \( \hat{\rho}_{\text{ext}} \) corresponding to thermal radiation at the temperature \[1\]. Heuristically, the process may be visualised in terms of the creation of entangled pairs of particles near the horizon, with one member of each pair falling behind the horizon and the other escaping to infinity. Like the classical exterior geometry, this thermal emission is independent of the details (apart from the total mass) of what fell behind the horizon.

The emitted radiation decreases the mass of the hole. For slow quasistatic changes, \( T \propto \frac{1}{M} \) and the rate of energy emission is \( \propto \frac{1}{M^2} \). The black hole evaporates on a timescale \[2\]

\[
t_{\text{bh}} \sim \frac{G^2 M^3}{\hbar c^4} \sim (10^{64} \text{ yr})(M/M_\odot)^3
\]

Hawking has argued that the formation and evaporation of black holes allows a closed system to evolve from a pure to a mixed quantum state, in violation of the usual rules of quantum theory \[3\]. Specifically, if matter in an initial pure state \( |\Psi\rangle \) undergoes gravitational collapse, then once the black hole has evaporated the final mixed state \( \hat{\rho}_{\text{ext}} \) becomes the state of the whole system. According to this argument, the formation and evaporation of the black hole results in a pure-to-mixed evolution, represented by a non-unitary map \( |\Psi\rangle \rightarrow \hat{\rho}_{\text{ext}} \). Further, because \( \hat{\rho}_{\text{ext}} \) describes thermal radiation that depends on the initial mass of the hole but is independent of the details of \( |\Psi\rangle \), it follows that given the final state \( \hat{\rho}_{\text{ext}} \) it is impossible to retrodict the initial state \( |\Psi\rangle \) – a situation that is commonly referred to as ‘information loss’, since the details of what originally fell behind the horizon seem to have been erased.

Hawking’s argument for information loss is controversial, and a number of well-known proposals have been made to avoid the conclusion of pure-to-mixed evolution. (For reviews see, for example, refs. \[4\] \[5\] \[6\] \[7\] \[8\].) It is sometimes suggested that evaporation stops as the hole approaches the Planck mass, leaving a remnant such that the total state is still pure; however, since the initial mass can be arbitrarily large, such low-mass remnants would have to have an arbitrarily large number of internal states, arguably leading to unbounded production rates for remnants in other (soft) processes \[6\]. Alternatively, it has
been suggested that the emitted radiation contains extra correlations over time such that the final radiated state is actually pure; but to produce such correlations seems to require nonlocal interactions operating across the horizon [9]. Another possibility is that during evaporation a new universe is formed, causally disconnected from our own, such that the joint state of the two universes is still pure; this scenario, too, has its difficulties [3, 4, 5, 10, 11].

The desire to avoid pure-to-mixed transitions has been a strong motivation for the holographic hypothesis, according to which the fundamental degrees of freedom are defined on a boundary of conventional spacetime [11]. Holography, and the closely-related idea of black-hole complementarity, make it seem possible that the information that apparently disappears behind the horizon is actually encoded on the horizon (from where it can be transferred to the outgoing radiation). A possible realisation of this is through AdS/CFT duality [12], which relates a string theory in ten dimensions to a gauge field theory in four dimensions (the local gauge-theory observables being mapped to boundary conditions on the string spacetime). According to this framework, the presumed unitarity of the dual gauge description guarantees that the formation and evaporation of black holes will be strictly unitary, with appropriate correlations in the outgoing Hawking radiation; though it remains to be shown explicitly – in terms of the gravitational variables – where the (semiclassical) argument for a pure-to-mixed transition breaks down [13, 14].

Other authors conclude that the argument for information loss may signal a genuine failure of quantum theory [4]. Hawking [3] has proposed that quantum theory should be generalised to include pure-to-mixed evolution. It was suggested that such evolution would imply currently-observable violations of either locality or energy-momentum conservation [15]; but in fact this is not necessarily the case [16]. Penrose [17] has argued (on the basis of thermal fluctuations for black holes in equilibrium with radiation in a large container) that information loss should be balanced by a gravitationally-induced collapse of the state vector, which might be observable for massive bodies [18]. It has been suggested by ’t Hooft [19] that information loss in black holes might be clarified in a deterministic but dissipative local hidden-variables theory (that is, in a local and deterministic completion of quantum theory with effective irreversibility at the hidden-variable level); though it is unclear how a local hidden-variables theory can be reconciled with the observed violations of Bell’s inequalities [20]. Recently, it has been suggested that entangled states might in principle be used to test for nonlinear evolution [21] and for (‘superquantum’) cloning or deleting [22] behind the horizon, by allowing part of the state to fall into the hole and monitoring the exterior.

It is sometimes suggested that Hawking’s argument points to the need for some form of nonlocal information flow from behind the horizon, throughout the evaporation process (even when the black hole is macroscopic) [23]. Holography and black-hole complementarity are often regarded as effective instantiations of this, whereby locality breaks down not merely microscopically but over macroscopic distances comparable to the size of the hole. For example, a failure of commutativity over spacelike separations (for observables inside and outside the
horizon) might arise from strings stretched between spacelike-separated points [24, 25]. According to holography, the fundamental degrees of freedom are nonlocal, and locality in ordinary spacetime emerges only in some approximation; but again, the precise nonlocal mechanism that leads to purity of the outgoing state is not easy to discern in the semiclassical regime, because of the difficulty in translating the dual picture into the higher-dimensional picture [13, 14].

In this paper, we shall assume that the pure-to-mixed transition $|\Psi\rangle \rightarrow \hat{\rho}_{\text{ext}}$ for quantum states really does occur during the formation and evaporation of black holes. Nevertheless, we shall argue that information need not be lost (in the sense that retrodiction might still be possible) if one allows deviations from the standard Born rule for quantum probabilities. For entangled states, such deviations may depend nonlocally on processes in the interior, so that information does indeed escape from behind the horizon.

To motivate our proposal, consider an analogy with classical statistical thermodynamics. When an ideal gas reaches thermal equilibrium, the resulting macroscopic state contains no memory of how the state was formed; the approach to statistical equilibrium has the remarkable effect of erasing information about the past, at least at the macroscopic level. Furthermore, once the gas has reached thermal equilibrium, its macroscopic state may be specified by a very small number of parameters, such as volume and temperature. And all statistical information about the gas may be obtained from the partition function $Z = \sum_{E} e^{-E/kT}$, again regardless of how the equilibrium state was prepared.

Now, in classical physics this ‘thermal information loss’ would be regarded as an artifact of averaging over microscopic degrees of freedom. At any finite time, the precise positions and velocities of the gas molecules contain more detailed information from which the details of the past preparation may in principle be recovered (assuming a deterministic and time-reversible dynamics).

With this in mind one might consider that, in the case of black holes, information about what fell behind the horizon is actually stored in extra degrees of freedom that are usually disregarded (or averaged over). Our proposal is that these extra degrees of freedom are nonlocal hidden variables – parameters of a nonlocal deterministic theory which quantum theory averages over (much as classical statistical thermodynamics averages over the deterministic dynamics of gas molecules).

Deviations from the Born rule are a natural possibility in deterministic hidden-variables theories. For example, in the pilot-wave theory of de Broglie and Bohm [29–38], the quantum state of an individual system is supplemented by a (hidden) deterministic trajectory in configuration space, with velocity given by the gradient of the phase of the wave function. For an ensemble of systems with wave function $\psi$, it is usually assumed that the configurations are distributed according to the Born rule $\rho = |\psi|^2$. This assumption (made at some initial time) guarantees empirical agreement with the statistical predictions of quantum theory. But there is no reason – within the theory – why one could not consider more general, ‘nonequilibrium’ distributions $\rho \neq |\psi|^2$ (just as in classi-
cal physics one may consider ensembles that depart from thermal equilibrium). These lead to predictions that deviate from standard Born-rule probabilities for outcomes of quantum measurements; and, it is found that the marginal statistics at one wing of an entangled state generally depend nonlocally on what happens at the other wing [39].

More generally, deterministic hidden-variables theories contain extra degrees of freedom $\lambda$ that are averaged over some ‘quantum equilibrium’ distribution $\rho_{\text{eq}}(\lambda)$ to yield the statistical predictions of quantum theory [40]. Under reasonable assumptions, Bell’s theorem requires that all such theories be nonlocal [41]. For generic ‘quantum nonequilibrium’ distributions $\rho(\lambda) \neq \rho_{\text{eq}}(\lambda)$, the outcomes of quantum measurements violate the Born rule [42], and the underlying nonlocality is visible at the statistical level [43, 44].

At present there is of course no evidence for deviations from the Born rule, and in discussions of hidden-variables theories attention is usually restricted to the quantum equilibrium state. This state appears to be stable, at least in non-gravitational physics.

We suggest that quantum equilibrium is disturbed in the interior of black holes (perhaps near the singularity). For entangled quantum states that straddle the horizon, deviations from equilibrium in the interior may (as we shall see) be transmitted to the exterior region, resulting in observable deviations from standard quantum probabilities outside the black hole. This provides a possible mechanism by which information from behind the horizon could indeed reach the exterior by nonlocal effects, though in a way that is quite different from previous proposals.

We shall argue, then, that nonlocal hidden variables provide additional degrees of freedom in which information may be stored about what fell behind the event horizon of an evaporating black hole, and that via entanglement this information may leak out to the exterior region. In effect, the hidden variables provide an additional entropy reservoir that is usually filled (in equilibrium), and becomes ‘unfrozen’ in nonequilibrium.

It will be suggested that the transition $|\Psi\rangle \rightarrow \hat{\rho}_{\text{ext}}$ from a pure to a mixed quantum state is accompanied by the creation of anomalous nonequilibrium distributions $\rho(\lambda) \neq \rho_{\text{eq}}(\lambda)$ of hidden variables outside the horizon. Such distributions carry a specific signature, in the form of a breakdown of the sinusoidal modulation of quantum probabilities for two-state systems [42]. A simple rule will be proposed, whereby the relative entropy of the nonequilibrium distribution generated outside the horizon balances the increase in von Neumann entropy associated with the transition $|\Psi\rangle \rightarrow \hat{\rho}_{\text{ext}}$, yielding quantitative predictions that could (at least in principle) be tested experimentally.

According to our proposal, outgoing Hawking radiation will contain statistical anomalies outside the domain of the quantum formalism – anomalies corresponding to non-standard distributions of hidden variables. In particular, photons will have anomalous polarisation probabilities, deviating from the $\cos^2 \Theta$ modulation of transmission through a pair of polarisers set at a relative angle $\Theta$. Further, we shall argue that similar anomalies could be created by allowing one half of an entangled EPR-pair to fall behind the event horizon of
a macroscopic black hole. As we shall see, it is not impossible that these effects could be observed in radiation from primordial black holes, and in photons from atomic cascade emissions in black-hole accretion discs.

The prediction of thermal radiation from black holes has long been thought to point to a deep connection between quantum theory, gravitation, and statistical physics. And Hawking’s argument, that the formation and evaporation of black holes can induce a transition from a pure to a mixed quantum state, still stands as a challenge (albeit a controversial one) to the basic principles of quantum theory. According to the reasoning given below, the connection with gravitation involves not just ordinary statistical physics, but the statistical physics of nonlocal hidden variables.

In section 2, we review the notion of statistical equilibrium in classical and pilot-wave dynamics, and in deterministic hidden-variables theories generally. It is shown that quantum equilibrium has special properties, such as locality, which arise from an effective erasure of underlying degrees of freedom. We also summarise what is currently known about gravitation in the context of hidden-variables theories. It is emphasised that while hidden-variables theories with a stable equilibrium state may easily be constructed on a globally-hyperbolic spacetime (an example is given), it is not known whether this can be done in the non-globally-hyperbolic case.

In section 3, we consider a thought experiment with black holes and entangled states. We make the hypothesis that for a quantum state entangled across the horizon, the external part of the state evolves away from quantum equilibrium, in accordance with a simple rule. A model is provided, showing how an entangled state can provide a channel for the nonlocal propagation of nonequilibrium across the horizon. It is noted that nonequilibrium may be conveniently detected in the form of anomalous polarisation probabilities for photons, and that the expected degree of nonequilibrium is related in a simple way to the maximal violation of Bell’s inequality associated with the entangled state.

In section 4, the hypothesis of section 3 is extended to the entangled field modes of ingoing and outgoing Hawking radiation, resulting in a constraint on the quantum nonequilibrium distribution for particles emitted by black holes.

In section 5, we discuss how the seemingly disparate concepts of hidden-variable and von Neumann entropies are related even in non-gravitational physics. In particular, systems in quantum nonequilibrium (with non-zero hidden-variable entropy) may be used to separate non-orthogonal quantum states for ordinary systems, resulting in an anomalous evolution of the von Neumann entropy.

In section 6, we consider the possibility of observing the proposed processes experimentally, in the Hawking radiation from primordial black holes (perhaps left over from the early universe), and in entangled photons from black-hole accretion discs. The latter possibility in particular is considered in some detail. We discuss current observations of iron emission lines in the vicinity of the event horizons of macroscopic black holes, and argue that the identification of an atomic cascade generating entangled photon pairs close to a horizon would enable the experiment to be performed, at least in principle.
2 Thermal and Quantum Equilibrium

In this section, we first review the notion of statistical equilibrium in deterministic theories – in classical and pilot-wave dynamics, and in general (deterministic) hidden-variables theories. We note some special properties of equilibrium. In particular, we show how equilibrium erases information about underlying degrees of freedom. Finally, we sketch what is currently known about the role of gravitation in hidden-variables theories.

2.1 Classical and Pilot-Wave Dynamics

In classical dynamics, the phase-space trajectory \((q(t), p(t))\) of an individual system is determined by Hamilton’s equations \(\dot{q} = \partial H/\partial p\) and \(\dot{p} = -\partial H/\partial q\), given the initial conditions \((q_0, p_0)\). For an ensemble with the same Hamiltonian, the velocity field \(\dot{X} \equiv (\dot{x}, \dot{p})\) determines the evolution of any distribution \(\rho(q, p, t)\) via the continuity equation (with \(\nabla \equiv (\partial/\partial q, \partial/\partial p)\))

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \dot{X}) = 0 \tag{3}
\]

(more usually written as \(\partial \rho/\partial t + \{\rho, H\} = 0\)). Because \(\nabla \cdot \dot{X} = 0\), a uniform initial distribution \(\rho(q, p, 0) = \text{const.} \) (on the energy surface) remains uniform, \(\rho(q, p, t) = \text{const.}\). This steady state is just thermal equilibrium. For an arbitrary initial state \(\rho(q, p, 0)\), the evolution \(\rho(q, p, t)\) is obtained – in principle – from integration of (3).

The deviation of \(\rho\) from thermal equilibrium may be quantified in terms of the classical \(H\)-function

\[
H_{\text{class}} = \int \int dqdp \rho \ln \rho \tag{4}
\]

Under Hamiltonian evolution, Liouville’s theorem states that \(d\rho/dt = 0\) along trajectories, so that the exact \(H_{\text{class}}\) is constant in time, \(dH_{\text{class}}/dt = 0\). But in appropriate circumstances, the coarse-grained \(H\)-function

\[
\bar{H}_{\text{class}} = \int \int dxdp \bar{\rho} \ln \bar{\rho} \tag{5}
\]

does decrease, corresponding to thermal relaxation on a coarse-grained level. Assuming there is no initial fine-grained microstructure in \(\rho\) at \(t = 0\), we have the classical coarse-graining \(H\)-theorem \cite{15, 10} \(\bar{H}_{\text{class}}(t) \leq \bar{H}_{\text{class}}(0)\), where \(\bar{H}_{\text{class}}\) is minimised by \(\bar{\rho} = \text{const.}\). The \(H\)-theorem formalises the idea of Gibbs \cite{17} – that an initial non-uniform distribution tends to develop fine-grained structure, becoming more uniform on a coarse-grained level.

In pilot-wave theory, the dynamics takes place in configuration space. The trajectory \(q(t)\) of an individual system is determined by the wave function \(\psi(q, t)\) via the de Broglie guidance equation

\[
\frac{dq}{dt} = \frac{j}{|\psi|^2} \tag{6}
\]
where $\psi$ obeys the usual Schrödinger equation
\[ i \frac{\partial \psi}{\partial t} = \hat{H}\psi \] (7)
in configuration space (units $\hbar = 1$), and where $j = j[\psi] = j(q, t)$ is the conserved current derived from (7), satisfying
\[ \frac{\partial |\psi|^2}{\partial t} + \nabla \cdot j = 0 \] (8)
(where here $\nabla \equiv \partial/\partial q$). For example, for a system of $n$ nonrelativistic particles with positions $x_i(t)$ and masses $m_i$, we have $q = (x_1, x_2, ..., x_n)$ and (8) generally takes the form
\[ \frac{dx_i}{dt} = \frac{1}{m_i} \text{Im} \frac{\nabla_i \psi}{\psi} = \frac{\nabla_i S}{m_i} \] (9)
where $\psi = |\psi| e^{iS}$.

Mathematically, $j/|\psi|^2$ is the ratio of the quantum probability current to the quantum probability density. Physically, however, $\psi$ (and hence $j[\psi]$) is here regarded as an objective physical field (in configuration space) guiding the motion of an individual system.

The equations (6) and (7) define a deterministic dynamics for individual systems. Given the initial field (or wave function) $\psi(q, 0)$, (7) determines $\psi(q, t)$ at all times. And given the initial configuration $q(0)$, (6) then determines the trajectory $q(t)$ at all times. Note that $\psi$ has no a priori connection with probabilities: it is a physical field on configuration space, driving the dynamics of an individual system.

For an ensemble of independent systems, each with the same wave function $\psi(q, t)$, we may define a distribution $\rho(q, t)$ of actual configurations $q$ at time $t$. In principle, the ensemble distribution $\rho(q, t)$ need have no relation to $|\psi(q, t)|^2$.

The guidance equation (6) defines a velocity field $\dot{q}$, which determines the evolution of any distribution $\rho(q, t)$ via the continuity equation
\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \dot{q}) = 0 \] (10)
(where again $\nabla \equiv \partial/\partial q$).

Rewriting (5) as
\[ \frac{\partial |\psi|^2}{\partial t} + \nabla \cdot (|\psi|^2 \dot{q}) = 0 \] (11)
it follows that an initial distribution $\rho(q, 0) = |\psi(q, 0)|^2$ evolves into $\rho(q, t) = |\psi(q, t)|^2$. This is the state of quantum equilibrium, analogous to thermal equilibrium.

Given the velocity field $\dot{q}$, (10) determines the evolution of an arbitrary initial distribution $\rho(q, 0) \neq |\psi(q, 0)|^2$. 

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The deviation of $\rho$ from quantum equilibrium may be quantified in terms of the hidden-variable $H$-function \[32, 48, 49\]

$$H_{hv} = \int dq \rho \ln(\rho/|\psi|^2)$$ \hspace{1cm} (12)

From (10) and (11), the ratio $f = \rho/|\psi|^2$ is preserved along trajectories, $df/dt = 0$ (the analogue of Liouville’s theorem), so that the exact $H_{hv}$ is constant in time, $dH_{hv}/dt = 0$. However, as in the classical case, in appropriate circumstances the coarse-grained $H$-function

$$\bar{H}_{hv} = \int dq \bar{\rho} \ln(\bar{\rho}/|\psi|^2)$$ \hspace{1cm} (13)

does decrease, corresponding to relaxation to quantum equilibrium on a coarse-grained level. Assuming there is no initial fine-grained microstructure in $\rho$ and $|\psi|^2$ at $t = 0$, we have the coarse-graining $H$-theorem \[32, 48, 49\] $\bar{H}_{hv}(t) \leq \bar{H}_{hv}(0)$, where $\bar{H}_{hv} \geq 0$ for all $\bar{\rho}$, $|\psi|^2$ and $\bar{H}_{hv} = 0$ if and only if $\bar{\rho} = |\psi|^2$ everywhere. This version of the $H$-theorem formalises the idea that $\rho$ and $|\psi|^2$ behave like two ‘fluids’ which are ‘stirred’ by the same velocity field $\dot{q}$, so that $\rho$ and $|\psi|^2$ tend to become indistinguishable on a coarse-grained level.

Note that the ‘hidden-variable entropy’

$$S_{hv} = -\int dq \rho \ln(\rho/|\psi|^2)$$ \hspace{1cm} (14)

is just the relative entropy of $\rho$ with respect to $|\psi|^2$, and is a natural measure of the difference between $\rho$ and $|\psi|^2$.

Significant relaxation $\rho \to |\psi|^2$ (on a coarse-grained level) occurs only if the velocity field varies rapidly over the coarse-graining cells. For very simple systems, such as a single particle in an energy eigenstate (for which the velocity field \[50\] vanishes), there is no relaxation at all – just as there is none for an ensemble of classical particles bouncing back and forth perpendicular to the walls of a box. For systems whose wave functions are a superposition of many energy states, the velocity field varies rapidly and numerical simulations confirm the expected relaxation, on timescales that agree with the estimate obtained from time-derivatives of $\bar{H}_{hv}(t)$ near $t = 0$ \[49, 50\].

A statistical mixture of wave functions $\psi_\alpha(q,t)$, weighted by probabilities $p_\alpha$, corresponds to a mixed quantum state with density operator

$$\hat{\rho} = \sum_\alpha p_\alpha |\psi_\alpha\rangle\langle \psi_\alpha|$$

For each pure subensemble with wave function $\psi_\alpha(q,t)$, one may define a distribution $\rho_\alpha(q,t)$ (which need not equal $|\psi_\alpha(q,t)|^2$) and an $H$-function

$$H^\alpha_{hv} = \int dq \rho_\alpha \ln(\rho_\alpha/|\psi_\alpha|^2)$$
satisfying the above theorem. For the whole ensemble, with distribution
\[ \rho(q,t) = \sum_\alpha p_\alpha \rho_\alpha(q,t) \]
one may define a mean \( H \)-function
\[ H_{hv} = \sum_\alpha p_\alpha H_{hv}^\alpha \]
and again the coarse-grained \( H_{hv} \) will satisfy the theorem (for a closed system with constant \( p_\alpha \)). In equilibrium, \( H_{hv} = 0 \) implies \( H_{hv}^\alpha = 0 \) and \( \rho_\alpha = |\psi_\alpha|^2 \)
(for all \( \alpha \)), the distribution for the whole ensemble then being equal to \( \rho(q,t) = \langle q|\hat{\rho}(t)|q \rangle \). Thus, for mixed states the hidden-variable entropy is
\[ S_{hv} = -\sum_\alpha p_\alpha \int dq \rho_\alpha \ln(\rho_\alpha/|\psi_\alpha|^2) \] (16)

Note that in pilot-wave theory, mixed quantum states are interpreted as statistical mixtures of physically real pilot waves, which correspond to a preferred decomposition of \( \hat{\rho} \).

The above statistical mechanics of hidden variables is conceptually similar to its classical counterpart. In the time-reversal invariant dynamics defined by (6) and (7), for every initial state that evolves towards equilibrium one can construct a time-reversed ‘initial’ state that evolves away from equilibrium. Such reversed initial states will, however, contain fine-grained microstructure. Thus, relaxation to equilibrium requires an assumption about initial conditions, and any such assumption is arguably related to questions of cosmology [46, 49, 50, 51, 52, 53].

It has been suggested that the universe began in a state of quantum nonequilibrium, the relaxation \( \rho \to \langle q|\hat{\rho}|q \rangle \) taking place during the violence of the big bang, and that remnants of early nonequilibrium might be found in relic particles that decoupled at sufficiently early times [32, 38, 48, 49, 54, 55]. An alternative view [56] takes \( |\psi|^2 \) to be the natural measure of probability or ‘typicality’ for initial configurations of the whole universe (with \( \psi \) the universal wave function), resulting in quantum probabilities for all subsystems at all times.

For the purposes of the present paper, we may leave aside the question of initial conditions. The key point is that one can in principle consider nonequilibrium distributions – within the framework of de Broglie-Bohm theory, and indeed in any deterministic hidden-variables theory (just as it is possible to consider thermal nonequilibrium in classical physics). And the natural measure of quantum nonequilibrium is the hidden-variable entropy (14), or (16) for mixed states.

As a simple example of quantum nonequilibrium, consider an ensemble of free nonrelativistic particles represented by the quantum momentum state \( |p \rangle \), where the particles have energy \( E = p^2/2m \) and are confined to a large normalisation volume \( V \). The state \( |p \rangle \) has wave function \( \psi(x,t) = e^{ip\cdot x} e^{-iEt}/\sqrt{V} \). Should
such an ensemble be prepared experimentally, and should the particle positions
then be measured, quantum theory predicts a uniform distribution $|\psi|^2 = 1/V$
of measured results. In pilot-wave theory there is no reason (in principle) why
the measured particle distribution could not, for example, be confined to one half
of the volume $V$. Similarly, a plane wave $\psi_{inc}$ incident on a two-slit screen yields
the usual interference pattern at the backstop, provided the incident particles
have distribution $\rho_{inc} = |\psi_{inc}|^2$. It is a well-known feature of pilot-wave theory
that incoming particles on one side of the symmetry axis (perpendicular to the
screen) hit the backstop on the same side of the symmetry axis [31, 33]. Thus
if, for example, the incident distribution $\rho_{inc}$ has support on only one side of
the symmetry axis, all particles will land on only one half of the backstop, and
only one half of the interference pattern will appear.

These examples make it clear that, at least in principle, pilot-wave theory
with $\rho = |\psi|^2$ is a special case of a wider theory (just as classical physics in
thermal equilibrium is a special case of a wider theory). For nonequilibrium
distributions $\rho \neq |\psi|^2$ at $t = 0$, the outcomes of subsequent quantum measure-
ments over an ensemble will have a distribution that departs from quantum
predictions (assuming relaxation $\rho \rightarrow |\psi|^2$ has not occurred by the time the
measurements have taken place).

### 2.2 General (Deterministic) Hidden-Variables Theories

The situation is conceptually the same in any deterministic hidden-variables
theory. Consider, for example, a two-state system in the standard Bloch-sphere
representation. A quantum measurement of the observable $\hat{\sigma} \equiv \hat{m} \cdot \hat{\sigma}$ (which
might be spin along an axis $\hat{m}$ in space, in units of $\hbar/2$) can yield outcomes
$\sigma = \pm 1$. In a deterministic hidden-variables theory, the outcome $\sigma$ is determined
in advance by the setting $\hat{m}$ of the measuring apparatus together with some
unknown parameters $\lambda$ (defined at some initial time $t = 0$, say at preparation).
In other words, there is a deterministic mapping

$$\sigma = \sigma(\hat{m}, \lambda)$$

from the conditions $\hat{m}, \lambda$ to the outcome $\sigma = \pm 1$.

Over an ensemble, with fixed measurement axis $\hat{m}$ and variable $\lambda$, quantum
theory can hold only if there exists a distribution $\rho_{eq}(\lambda)$ of hidden variables such
that the mean outcome

$$\langle \sigma(\hat{m}, \lambda) \rangle_{eq} \equiv \int d\lambda \rho_{eq}(\lambda)\sigma(\hat{m}, \lambda)$$

is equal to the quantum mean

$$\langle \hat{m} \cdot \hat{\sigma} \rangle = \text{Tr}(\hat{m} \cdot \hat{\sigma}) = \hat{m} \cdot \text{Tr(}\hat{\rho}\hat{\sigma}) \equiv \hat{m} \cdot \hat{P} = \hat{P} \cos \theta$$

$^3$It is customary to write as if $\lambda$ were a continuous variable. The integral sign is really a
generalised sum, and no assumptions are made about $\lambda$. 

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where $\hat{\rho}$ is the quantum density operator associated with the preparation, $P$ is the mean polarisation, and $\theta$ is the angle (on the Bloch sphere) between $m$ and $P$. With just two possible outcomes $\sigma = \pm 1$, the mean fixes the outcome probabilities

$$p^{\pm}_{eq}(m) = \frac{1}{2} (1 \pm P \cos \theta) \quad (19)$$

which depend sinusoidally on $\theta$, for any pure or mixed state (as long as $P \neq 0$). If the ensemble is completely polarised, $P = 1$ and $p^{\pm}_{eq}(m) = \cos^2(\theta/2)$.

Thus, any deterministic hidden-variables theory is specified by two conceptually distinct components. The first refers to individual systems: the mapping (17) determines the outcome $\sigma$, given the conditions $m, \lambda$. The second refers to an ensemble: the distribution $\rho_{eq}(\lambda)$ specifies how the individual hidden variables $\lambda$ are distributed over the ensemble.\(^4\)

Conceptually, this is the same as in pilot-wave theory: (6) and (7) determine the outcome of any quantum experiment, given the ‘hidden variables’ $q(0)$, $\psi(q,0)$ together with the experimental arrangement (specified by the external potential in 7); while the choice $\rho(q,0) = |\psi(q,0)|^2$ for the initial ensemble distribution guarantees that the outcome probabilities agree with quantum theory.

Clearly, just as one may contemplate non-standard ensemble distributions $\rho(q,0) \neq |\psi(q,0)|^2$ in pilot-wave theory, so one may equally contemplate non-standard ensemble distributions $\rho(\lambda) \neq \rho_{eq}(\lambda)$ in any deterministic hidden-variables theory. By retaining the same mapping (17) for individual systems, one then obtains a theory that is wider than quantum theory but includes it as a special case: quantum probabilities are obtained for the ‘equilibrium’ distribution $\rho(\lambda) = \rho_{eq}(\lambda)$, but not for general ‘nonequilibrium’ distributions $\rho(\lambda) \neq \rho_{eq}(\lambda)$.

Generically, the nonequilibrium ensemble mean will deviate from the quantum prediction,

$$\langle \sigma (m, \lambda) \rangle \equiv \int d\lambda \rho(\lambda)\sigma (m, \lambda) \neq P \cos \theta \quad (20)$$

and the outcome probabilities

$$p^{\pm}(m) = \frac{1}{2} (1 \pm \langle \sigma (m, \lambda) \rangle) \neq \frac{1}{2} (1 \pm P \cos \theta) \quad (21)$$

will not be sinusoidal in $\theta$.\(^4\)

The essential point is that in any deterministic theory (including classical or pilot-wave dynamics) there is a clear conceptual distinction between the dynamical equations for an individual system and the distribution of initial conditions over an ensemble. One is free to retain the former while changing the latter.

In a general hidden-variables theory, with nonequilibrium distributions $\rho(\lambda) \neq \rho_{eq}(\lambda)$, it is difficult to see how one could construct an argument for relaxation

\(^4\)The uncertainty principle – that is, the unavoidable statistical dispersion over quantum ensembles – originates from the dispersion of $\rho_{eq}(\lambda)$.\(^{12}\)
\( \rho(\lambda) \rightarrow \rho_{eq}(\lambda) \), in the absence of specific dynamical equations. Nevertheless, the key point is generally true: any deterministic hidden-variables theory with \( \rho(\lambda) = \rho_{eq}(\lambda) \) is a special case of a wider theory with generic nonequilibrium \( \rho(\lambda) \neq \rho_{eq}(\lambda) \). And the natural measure of nonequilibrium is the hidden-variable entropy

\[
S_{hv} = -\int d\lambda \, \rho \ln(\rho/\rho_{eq})
\]

(22)

(the relative entropy of \( \rho(\lambda) \) with respect to \( \rho_{eq}(\lambda) \)).

2.3 Properties of Equilibrium. Local Statistics, Memory Loss, and Information Compression

In quantum theory, for an entangled state of two widely-separated systems, the marginal statistics of outcomes at each wing do not depend on the measurement setting at the other distant wing. This locality of statistics is remarkable from a hidden-variables perspective, given that any reasonable hidden-variables theory has to be fundamentally nonlocal.

Specifically, consider a pair of two-state systems at points \( A \) and \( B \) in space. Quantum measurements of the local observables \( \hat{\sigma}_A \equiv m_A \cdot \hat{\sigma}_A \), \( \hat{\sigma}_B \equiv m_B \cdot \hat{\sigma}_B \) yield possible outcomes \( \sigma_A, \sigma_B = \pm 1 \), for arbitrary axes \( m_A, m_B \). Over an ensemble represented by the singlet state

\[
|\Psi\rangle = \frac{1}{\sqrt{2}} (|+\rangle - |-\rangle)
\]

Bell’s theorem \[41\] shows that to reproduce the quantum correlation

\[
\langle \Psi | \hat{\sigma}_A \hat{\sigma}_B | \Psi \rangle = -m_A \cdot m_B = -\cos \theta_{AB}
\]

between \( A \) and \( B \), any hidden-variables theory must take the nonlocal form

\[
\sigma_A = \sigma_A(m_A, m_B, \lambda), \quad \sigma_B = \sigma_B(m_A, m_B, \lambda)
\]

(23)
in order to obtain

\[
\langle \sigma_A \sigma_B \rangle_{eq} = \int d\lambda \, \rho_{eq}(\lambda) \sigma_A \sigma_B = -\cos \theta_{AB}
\]

for some distribution \( \rho_{eq}(\lambda) \) of hidden variables. The individual outcomes \( \sigma_A, \sigma_B \) at \( A \) and \( B \) must depend nonlocally on the distant settings.\(^5\) And yet, the marginal statistics of outcomes at \( A \) and \( B \) do not depend on the distant settings.

This ‘washing out’ of nonlocality is a peculiarity of equilibrium. Generically, for an arbitrary distribution \( \rho(\lambda) \neq \rho_{eq}(\lambda) \) − but retaining the same nonlocal mappings \[46\] from individual conditions \( m_A, m_B, \lambda \) to individual outcomes \( \sigma_A, \sigma_B \) − the marginal statistics at one wing do depend instantaneously on

\(^5\)More precisely, there must be a nonlocal dependence in at least one direction.
the choice of measurement axis at the distant wing. This is true not only in pilot-wave theory [39], but in any deterministic hidden-variables theory [43, 44].

Thus, the locality property of quantum theory – that entangled states cannot be used for nonlocal signalling – is a contingent feature of quantum equilibrium, and is generically violated for nonequilibrium ensembles.

One might think that, in nonequilibrium, nonlocal signals would necessarily create causal paradoxes. However, these can be evaded by modifying the causal structure of spacetime. Specifically, one may assume that in nonequilibrium there is a preferred foliation by spacelike hypersurfaces, labelled by a time parameter $t$ that defines a fundamental causal sequence, as in fact is the case in pilot-wave field theory (see section 2.4 below). Nonlocal, nonequilibrium signals then define an absolute simultaneity.$^6$

In equilibrium, the underlying details of the hidden-variable dynamics are washed out. The resulting ‘erasure of information’ leads to the emergent property of statistical locality, which is contingent on $\rho(\lambda) = \rho_{eq}(\lambda)$. An analogy may be drawn with the state of classical thermodynamic heat death (or global thermal equilibrium), in which the lack of differences of temperature makes it impossible to convert heat into work $^{32}$ $^{15}$ $^{54}$. Such a limitation is an artifact of the state, not a fundamental law of physics.

The analogy with thermal equilibrium goes further and deeper. As noted in the Introduction, thermal equilibrium has the remarkable effect of erasing information about the past. For a gas in thermal equilibrium, the macroscopic state contains no memory of how the state was formed. Further, the equilibrium state may be described by a small number of parameters, and its statistical properties are completely specified by the partition function $Z$. These remarkable features of thermal equilibrium may be summarised as ‘memory loss’ and ‘information compression’. Similar features emerge in quantum equilibrium.

In quantum theory, once a quantum state has been prepared, the predicted probabilities carry no trace of how the state was prepared. That this is indeed remarkable has been emphasised by Peres $^{58}$, who adopts a postulate of ‘statistical determinism’, according to which quantum probabilities for a pure state do not depend on the details of the preparation procedure.$^7$ Peres also points out another remarkable fact, that for mixed states a density operator $\hat{\rho}$ may be prepared in an infinite number of macroscopically-distinct ways, and yet no information distinguishing these preparations may be recovered from $\hat{\rho}$, which contains all the statistical information about the prepared ensemble.$^8$

The quantum probabilities for all possible quantum measurements are described by the operator $\hat{\rho}$, and may be encoded into a small number $K = N^2$ of ‘fiducial probabilities’, where $N$ is the (quantum) dimension of the system $^{59}$.

Thus, there is a quantum ‘memory loss’ with respect to state preparation,$^6$

$^6$ On this view, ‘back-in-time’ effects created by Lorentz boosts are fictitious, because moving clocks are incorrectly synchronised if one assumes isotropy of the speed of light in all frames $^{32}$ $^{55}$ $^{57}$.

$^7$ This is a component of Peres’ postulate A (ref. $^{55}$, p. 30). As Peres notes (p. 31), ‘all the past history of the selected quantum systems becomes irrelevant’ (italics in original).

$^8$ See ref. $^{55}$, p. 75. This property is expressed in Peres’ final postulate K (p. 76).
analogous to the memory loss in equilibrium thermodynamics. Further, there is a quantum ‘information compression’, in which the statistics of outcomes may be encoded into a small number \( K \) of parameters and described by a single mathematical object \( \hat{\rho} \), analogous to the parameters \( V, T \) (labelling thermodynamic states) and the partition function \( Z \).

These features of quantum theory are – from a hidden-variables perspective – contingencies of quantum equilibrium \( \rho(\lambda) = \rho_{eq}(\lambda) \). For nonequilibrium ensembles, the quantum state alone is not sufficient to specify the probabilities for outcomes of quantum measurements, as we saw in Sect. 2.1. In other words, \( \hat{\rho} \) is generally not a complete description, and additional details about the past need not be erased. Different preparations of the same density operator \( \hat{\rho} \) can be distinguished in nonequilibrium. As noted by Peres [58], the ability to do so leads to nonlocal signalling. Measurements at one wing of an entangled pure state may be regarded as preparing a mixed state at the other distant wing, and as we have already noted, in nonequilibrium the marginal statistics at the distant wing will generally depend on what measurements were performed far away – or equivalently, on how the mixed state was prepared [60]. Thus, the reduced density matrix at the distant wing is not a complete description, and information about the preparation is not lost.

The incompleteness of the reduced density matrix will be crucial in understanding how Hawking information loss may be avoided, since a key assumption of Hawking’s argument is that the reduced density matrix in the exterior region provides a complete description post-evaporation.

Note also that, in pilot-wave theory, the true decomposition of the density operator cannot be determined in equilibrium, but the fundamental dynamics does depend on it, and in nonequilibrium the true decomposition would be apparent from the detailed behaviour of the trajectories.

It may be shown that the number \( K \) of fiducial probabilities is larger than \( N^2 \) in nonequilibrium, an effect which may be traced to the extra information about hidden variables that is revealed in nonequilibrium [61]. Similar effects occur, of course, in thermal nonequilibrium, where information about microscopic variables is no longer screened off, resulting in a breakdown of what we have called memory loss and information compression.

The fundamental message is that, in general, equilibrium erases information. In particular, quantum equilibrium erases information about hidden variables (or de Broglie-Bohm trajectories), about nonlocal interactions, and about how the system was prepared.

### 2.4 The Role of Gravitation

In non-gravitational physics, quantum equilibrium appears to be stable, in the sense of being preserved in time under standard processes and interactions. The Born rule continues to hold, for instance, in high-energy collisions (as probed by scattering cross-sections). An example of a hidden-variables theory with a stable quantum equilibrium state at high energies is provided by the pilot-wave theory of fields in flat spacetime [30, 32, 33, 34, 35, 38, 54, 61, 57, 62, 63, 64, 65].
scalar field $\phi$ (for example) one may write quantum field theory in the functional Schrödinger picture, in terms of a wave functional $\Psi[\phi, t]$, and one may assume that the velocity $\partial \phi(x, t)/\partial t$ of the actual field configuration $\phi(x, t)$ is given by the functional derivative $\delta S/\delta \phi(x)$ of the phase $S$ of $\Psi$ (or more generally, by the ratio of the quantum probability current $J$ in field configuration space to the quantum probability density $|\Psi|^2$). This is the natural generalisation of pilot-wave dynamics to continuous degrees of freedom. The configuration $q(t)$ is now the field configuration $\phi(x, t)$, and the wave function $\psi(q, t)$ is now the wave functional $\Psi[\phi, t]$. (The construction requires a fundamental time parameter $t$, with respect to which nonlocal effects occur instantaneously [57, 32].) Standard quantum field theory, together with ordinary local Lorentz symmetry, is recovered for ensembles of fields in quantum equilibrium, that is, for fields distributed according to $P[\phi, t] = |\Psi[\phi, t]|^2$. As usual, this equilibrium distribution need be given at some initial time only. Quantum equilibrium is therefore stable with respect to high-energy collisions, at the energies probed so far, in at least one hidden-variables theory.

Pilot-wave field theory may be readily extended to a curved spacetime that is globally hyperbolic [38]. Let us sketch the construction. Any globally hyperbolic spacetime may be foliated (in general nonuniquely) by spacelike hypersurfaces $\Sigma$ labelled by a global time function $\tau$ [66]. The classical spacetime metric may be written as

$$d\tau^2 = (4)^{\mu\nu}dx^\mu dx^\nu = N^2dt^2 - g_{ij}dx^i dx^j$$

We have set the shift vector $N^i = 0$, so that lines $x^i = \text{const.}$ are normal to $\Sigma$ (as may always be done as long as the lines $x^i = \text{const.}$ do not run into singularities). The lapse function $N(x^i, t)$ measures the proper time lapse normal to $\Sigma$ per unit of coordinate time $t$. Now, restricting ourselves to a massless and minimally-coupled scalar field $\phi$, the Lagrangian density $L = \frac{1}{2} \sqrt{-g} \frac{\partial \mu}{\partial \nu} \partial \phi^i \partial \phi^j$ (with action $\int dtd^3x L$) may be written as

$$L = \frac{1}{2} N \sqrt{g} \left( \frac{\dot{\phi}^2}{N^2} - g^{ij} \partial_i \phi \partial_j \phi \right)$$

which implies a canonical momentum density $\pi = \partial L / \partial \dot{\phi} = (\sqrt{g}/N) \dot{\phi}$ and a Hamiltonian

$$H = \int d^3x \frac{1}{2} N \sqrt{g} \left( \frac{\dot{\phi}^2}{g} + g^{ij} \partial_i \phi \partial_j \phi \right)$$

The wave functional $\Psi[\phi, t]$ then satisfies the Schrödinger equation (with units $\hbar = c = 1$)

$$i \frac{\partial \Psi}{\partial t} = \int d^3x \frac{1}{2} N \sqrt{g} \left( -\frac{1}{g} \frac{\delta^2}{\delta \phi^2} + g^{ij} \partial_i \phi \partial_j \phi \right) \Psi \quad (24)$$

which implies the continuity equation

$$\frac{\partial |\Psi|^2}{\partial t} + \int d^3x \frac{\delta}{\delta \phi} \left( |\Psi|^2 \frac{N}{\sqrt{g}} \frac{\delta S}{\delta \phi} \right) = 0 \quad (25)$$
and a de Broglie velocity field
\[ \frac{\partial \phi}{\partial t} = \frac{N}{\sqrt{g}} \frac{\delta S}{\delta \phi} \] (26)

(where \( \Psi = |\Psi| e^{iS} \)). According to (26), the field velocity \( \dot{\phi} \) at a point \( x \) on the hypersurface \( \Sigma \) depends instantaneously (with respect to \( t \)) on field values at distant points \( x' \neq x \) on \( \Sigma \), if the wave functional is entangled with respect to the fields at those points. To ensure physical consistency we assume, as in the flat case, that the theory is constructed using a preferred foliation associated with a specific lapse function \( N(x^i, t) \) (which then plays the role of an additional physical field affecting the rate of macroscopic clocks [38]). As before, (24) and (26) determine the evolution \( \phi(x, t) \) of an individual field (given the initial configuration \( \phi(x, 0) \) and wave functional \( \Psi[\phi] \)). The time evolution of an arbitrary ensemble distribution \( P[\phi, t] \) is then given by
\[ \frac{\partial P}{\partial t} + \int d^3x \delta(\delta \phi) \left( P \frac{N}{\sqrt{g}} \frac{\delta S}{\delta \phi} \right) = 0 \] (27)

Comparing (26) and (27), it follows as usual that \( P[\phi, t] = |\Psi[\phi, t]|^2 \) is an equilibrium state. And in equilibrium, one may ignore the details of the trajectories defined by (26), and consider only the probabilities obtained from the (modulus-squared of the) wave functional governed by (24), which will agree with ordinary quantum field theory on curved spacetime. Thus, again, even in the presence of gravitation, the quantum equilibrium state is stable in at least one hidden-variables theory – provided the spacetime is globally hyperbolic.9

In contrast, for a non-globally-hyperbolic spacetime – such as that generated by the formation and (complete) evaporation of a black hole – even standard quantum field theory has been developed to only a very limited degree. And a pilot-wave analogue has not been developed at all.

According to Hawking’s argument, an initial pure state \( |\Psi\rangle \) may be defined on some (global) initial spacelike hypersurface \( \Sigma_1 \), before the hole forms (treating spacetime as a classical background). Once the horizon has formed, and evaporation begins, the Hilbert space may be written as a product \( H_{\text{int}} \otimes H_{\text{ext}} \) over the degrees of freedom interior and exterior to the hole. On a hypersurface \( \Sigma_2 \) that crosses the horizon, the quantum state of the system is still the pure state \( |\Psi\rangle \) (in the Heisenberg picture). However, the quantum state in the exterior region is mixed, and is represented by the reduced density operator \( \hat{\rho}_{\text{ext}} = Tr_{\text{int}}(|\Psi\rangle\langle\Psi|) \), obtained by tracing over the interior degrees of freedom. After the black hole has evaporated, the mixed state \( \hat{\rho}_{\text{ext}} \) is the state of the whole system, defined on a final hypersurface \( \Sigma_3 \).

To construct quantum field theory on such a spacetime is not straightforward. The standard quantisation procedure uses canonical commutation relations on a Cauchy surface, so that the wave equation has a well-posed initial

---

9 We are of course glossing over the question of the rigorous definition of the functional Schrödinger equation, which is used extensively in (for example) cosmological inflationary models. We are implicitly assuming some sort of regularisation, such as an analytical continuation of the number of space dimensions away from 3 [67].
value formulation \[\text{[9]}\]; it relies on quantising a well-posed Hamiltonian dynamics for classical fields, and is therefore applicable only to globally-hyperbolic spacetimes. An algebraic approach to quantum field theory on non-globally-hyperbolic spacetimes has been developed by Yurtsever \[\text{[68]}\] and applied to simple, flat (two-dimensional) examples. In this construction, it is insufficient to specify the (algebraically-defined) quantum state on an initial spacelike hypersurface; the state must be specified on the entire spacetime, with boundary conditions at naked singularities (if any).

It remains to be seen if Yurtsever’s approach can be extended to a hidden-variables theory. Extant deterministic theories require a preferred hypersurface along which nonlocality acts, as in equation \(\text{[26]}\). Even in flat spacetime, attempts to write down a fundamentally Lorentz-invariant theory of (hidden) particle trajectories run into problems associated with nonlocality, so that the quantum equilibrium distribution must be defined on a preferred hypersurface \[\text{[69, 70, 71, 72]}\].\(^{11}\) In the absence of any Cauchy hypersurface whatever, there might be a fundamental difficulty in defining a quantum equilibrium state for a nonlocal hidden-variables theory. In particular, the de Broglie-Bohm construction depends on the existence of a local quantum probability current in configuration space, and it is not clear that such a current will exist; though this remains to be studied.\(^{12}\)

In the usual pilot-wave dynamics of a single system, there is a deterministically evolving, well-defined wave function (or functional) at all times, generating a velocity field in configuration space. Mixed states may, as we have mentioned, be interpreted in terms of ordinary statistical mixtures of such (physically real) wave functions. A hypothetical transition from a pure to a mixed state, however, would in de Broglie-Bohm theory amount to a denial of the deterministic evolution of the pilot wave. Possibly, one could extend ordinary pilot-wave theory by introducing a stochastic element in the evolution of the wave function, to account for pure-to-mixed transitions; or otherwise define a local current by a more general prescription that is not tied to the wave function. If this could be done, the resulting theory might exhibit an equilibrium distribution that agrees with quantum probabilities throughout the transition.\(^{13}\) Otherwise, if one begins with a pure quantum state \(\Psi\) and a quantum equilibrium distribution \(P = |\Psi|^2\) at some initial time, then in the absence of a detailed theory there is no guarantee that upon transition to a mixed state \(\rho\) the fields will end with

\(^{10}\) The derivation of Hawking radiance is based on quantum field theory on a background globally-hyperbolic spacetime. Once the radiation rate has been derived, it is assumed that the mass of the hole steadily decreases to compensate, and assuming complete evaporation results in a non-globally-hyperbolic spacetime. The pure-to-mixed transition is intimately related to the failure of global hyperbolicity \[\text{[9]}\].

\(^{11}\) See, however, ref. \[\text{[73]}\] for an attempt to circumvent this.

\(^{12}\) Note that the de Broglie-Bohm version of canonical quantum gravity (which has been applied to Hawking evaporation \[\text{[74]}\]) is not relevant here, because the canonical formalism – in which general relativity is written as the theory of a 3-geometry evolving in time (with respect to an arbitrary foliation) – has physical significance only if spacetime is globally hyperbolic.

\(^{13}\) Maroney \[\text{[75]}\] considers a de Broglie-Bohm-type theory in which the density operator plays the role of a physical guiding field for individual systems.
the quantum distribution $\langle \phi | \hat{\rho} | \phi \rangle$.

At present, then, there is no known example of a hidden-variables theory exhibiting a stable state of quantum equilibrium on a non-globally-hyperbolic spacetime or in the presence of a pure-to-mixed transition. And even if it proves possible to construct such a theory, one might still consider the possibility that in such situations – and in particular in the spacetime generated by the formation and evaporation of a black hole – the quantum equilibrium state becomes unstable.

From a hidden-variables perspective, it is natural to ask if there exist some physical processes in which quantum nonequilibrium can be generated from an initial equilibrium state. Possibly, the quantum equilibrium state becomes unstable at very high (hitherto-unprobed) energies or in strong gravitational fields, where quantum theory might break down. However, rather than simply postulating arbitrary new effects, one would like theoretical reasons for why nonequilibrium might be generated in specific conditions.

Gravitation seems the natural arena in which to search for such effects, since it is not really known how gravitation fits in with quantum theory. Further, Hawking’s argument for information loss already suggests that gravitation may force a significant departure from the usual quantum formalism.

We shall now argue that hidden-variable degrees of freedom can be ‘unfrozen’ by gravitational effects. In particular, we suggest that black holes can throw systems out of quantum equilibrium, if those systems are entangled with other systems that have fallen behind the event horizon of the black hole. Hidden variables provide an additional entropy reservoir that is usually filled (in equilibrium with $S_{hv} = 0$), and remains filled in all known processes. And because details of the hidden variables are erased in equilibrium, they may be ignored in ordinary (quantum) physics. However, if information is allowed to flow into and out of this reservoir of extra degrees of freedom (so that $S_{hv}$ changes), then new phenomena can arise that lie outside the scope of quantum physics.

### 3 A Thought Experiment with Black Holes and Entangled States

Consider the following thought experiment. Outside the event horizon of a black hole, a source emits EPR-pairs – specifically, pairs of entangled two-state systems (labelled $A$ and $B$) prepared in the (pure) singlet state

$$|\Psi\rangle = \frac{1}{\sqrt{2}} (|+\rangle - |\rangle)$$

The states $|+\rangle$, $|\rangle$ could for example be orthogonal states of spin, polarisation, momentum or energy. The ensemble of EPR-pairs is represented by a density operator $\hat{\rho}^{in} = |\Psi\rangle \langle \Psi |$, and the von Neumann entropy is zero,

$$S_{vonN}^{in} = -Tr(\hat{\rho}^{in} \ln \hat{\rho}^{in}) = 0$$
Now, if one half of each pair falls behind the event horizon, then the reduced density operator for the ensemble of systems in the exterior region will be

$$\hat{\rho}_{\text{ext}} = \frac{1}{2} |+\rangle \langle +| + \frac{1}{2} |\rangle \langle -|$$ (28)

At this stage, the total state is still pure, and the mixed state in the exterior region is merely the result of tracing over degrees of freedom that have fallen behind the horizon. However, if we now wait for the black hole to evaporate completely, the interior region disappears and the remaining two-state systems are left in a strictly mixed state

$$\hat{\rho}^{\text{out}} = \hat{\rho}_{\text{ext}}$$ (29)

Of course, these systems will now be accompanied by a bath of thermal radiation. But if we assume that the systems in the exterior region were carried far away from the hole, and (or) appropriately isolated, they will not have interacted with the emerging Hawking radiation: thus, they will not be entangled with any of the emitted particles, and their quantum state will indeed be just (29), with a von Neumann entropy

$$S^{\text{out}}_{\text{vonN}} = -Tr(\hat{\rho}^{\text{out}} \ln \hat{\rho}^{\text{out}}) = \ln 2$$

The increase in entropy by $\ln 2$ quantifies the 'information loss' associated with the transition from a pure to a mixed quantum state.

We now make the following hypothesis: that during the pure-to-mixed transition, the ensemble of systems outside the horizon evolves away from quantum equilibrium, so that the total entropy $S_{\text{hv}} + S^{\text{out}}_{\text{vonN}}$ is conserved.

According to this hypothesis, the ensemble of systems post-evaporation will be equivalent to an ensemble that was subjected to a preparation procedure represented by (28), but whose distribution of hidden variables differs from quantum equilibrium, $\hat{\rho}^{\text{out}}(\lambda) \neq \rho_{\text{eq}}(\lambda)$, where $\hat{\rho}^{\text{out}}(\lambda)$ is such that the change in hidden-variable entropy balances the change in von Neumann entropy:

$$S^{\text{out}}_{\text{hv}} + S^{\text{out}}_{\text{vonN}} = S^{\text{in}}_{\text{hv}} + S^{\text{in}}_{\text{vonN}}$$ (30)

If the initial quantum state is pure, and if the initial ensemble is in quantum equilibrium, then $S^{\text{in}}_{\text{hv}} = S^{\text{in}}_{\text{vonN}} = 0$ and

$$S^{\text{out}}_{\text{hv}} = -\ln 2$$ (31)

This provides a quantitative prediction for the amount of nonequilibrium generated.

Note that in (30) we are comparing two very different kinds of entropy, $S_{\text{hv}}$ and $S^{\text{out}}_{\text{vonN}}$. These may in fact be related even in non-gravitational processes, as we shall discuss in section 5.

Further, the assumed conservation rule (30) is merely a simple hypothesis. If a process of this type really does occur, one hopes that (30) will be correct at
least as an order-of-magnitude estimate. In any case, as we shall see in section
6, the estimated change (31) might be susceptible to experimental test.

A detailed theory of the proposed process will not be attempted here. How-
ever, intuitively we envisage the mechanism to be along these lines. The entan-
gled systems interior and exterior to the horizon are nonlocally connected at the
hidden-variable level (presumably along some unknown spacelike hypersurface,
at least prior to complete evaporation). As the interior system approaches the
singularity, something happens to it that we do not currently understand, and
this is communicated nonlocally to the exterior system, causing the latter (over
an ensemble) to fall out of quantum equilibrium. It might be the case, for exam-
ple, that in a black hole the internal degrees of freedom close to the singularity
are far from quantum equilibrium, and that when the infalling system interacts
with these it communicates the nonequilibrium to the exterior system.

This last possibility may be illustrated with a simple model based on de
Broglie-Bohm theory. Consider two nonrelativistic particles moving in one
spatial dimension, with positions \( x_A \) and \( x_B \). Let their initial wave function
\( \psi_0(x_A, x_B) \) be entangled, and assume that (over an ensemble) the particles are
in quantum equilibrium, with a joint distribution \(|\psi_0(x_A, x_B)|^2\) of positions.
Now let particle \( A \) interact locally with a third particle, with position \( y \) and ini-
tial wave function \( \phi_0(y) \), via the Hamiltonian \( \hat{H} = a y \hat{p}_A \) (where \( a \) is a coupling
constant and \( \hat{p}_A \) is conjugate to \( \hat{x}_A \)). It is straightforward to show that if \( y \) has
a nonequilibrium distribution, then provided \( \psi_0(x_A, x_B) \) is entangled the local
interaction between \( y \) and \( x_A \) drives the marginal distribution of \( x_B \) away from
equilibrium. For let the coupling \( a \) be so large that the Hamiltonians of the
particles themselves may be neglected. Then the wave function \( \Psi(x_A, x_B, y, t) \)
obey the Schrödinger equation

\[
\frac{i}{\hbar} \frac{\partial \Psi}{\partial t} = -i a y \frac{\partial \Psi}{\partial x_A}
\]  

(32)

and the initial quantum state \( \Psi_0(x_A, x_B, y) = \psi_0(x_A, x_B)\phi_0(y) \) evolves into

\[
\Psi(x_A, x_B, y, t) = \psi_0(x_A - a y t, x_B)\phi_0(y)
\]

From (32) one obtains the continuity equation

\[
\frac{\partial |\Psi|^2}{\partial t} + a y \frac{\partial |\Psi|^2}{\partial x_A} = 0
\]

which implies that the (hidden-variable) positions have velocities \( \dot{x}_A = a y \),
\( \dot{x}_B = 0 \), \( \dot{y} = 0 \) and trajectories

\[
x_A(t) = x_A(0) + a y(0) t, \quad x_B(t) = x_B(0), \quad y(t) = y(0)
\]

An arbitrary distribution \( P(x_A, x_B, y, t) \) then evolves according to the same
continuity equation

\[
\frac{\partial P}{\partial t} + a y \frac{\partial P}{\partial x_A} = 0
\]
and an initial distribution $P_0(x_A, x_B, y) = |\psi_0(x_A, x_B)|^2 \pi_0(y)$ with $\pi_0(y) \neq |\phi_0(y)|^2$ (that is, with $x_A, x_B$ in equilibrium and $y$ in nonequilibrium) evolves into

$$P(x_A, x_B, y, t) = |\psi_0(x_A - ayt, x_B)|^2 \pi_0(y)$$

This is to be compared with the equilibrium result

$$P_{eq}(x_A, x_B, y, t) = |\psi_0(x_A - ayt, x_B)|^2 |\phi_0(y)|^2$$

In particular, for $\pi_0(y) \neq |\phi_0(y)|^2$ the marginal distribution $\rho(x_B, t)$ at $B$

$$\rho(x_B, t) = \int dx_A \left( \int dy \ |\psi_0(x_A - ayt, x_B)|^2 \pi_0(y) \right)$$

will generally differ from the equilibrium marginal

$$\rho_{eq}(x_B, t) = \int dx_A \left( \int dy \ |\psi_0(x_A - ayt, x_B)|^2 |\phi_0(y)|^2 \right)$$

(Note, however, that for a product state $\psi_0(x_A, x_B) = \alpha(x_A)\beta(x_B)$ both (33) and (34) reduce to $|\beta(x_B)|^2$ (since $\int dx_A |\alpha(x_A - ayt)|^2 = 1$), and the system at $B$ remains in equilibrium.) Thus, an entangled state of ordinary quantum systems $A$ and $B$ can provide a channel for the nonlocal propagation of nonequilibrium from $A$ to $B$, if we allow a nonequilibrium system to interact locally with $A$.

In practice, each infalling system is likely to undergo different interactions with the interior, and the ensemble distribution for the exterior systems will be obtained by averaging over such interactions. It might then be thought that even if there were nonequilibrium degrees of freedom behind the horizon, such averaging would result in an equilibrium distribution for the exterior ensemble. However, a ‘thermodynamic’ constraint of the form (30) (if such exists) would prevent this.

Because we do not have a detailed theory, we do not know along which spacelike hypersurface the nonlocality acts. Thus, we cannot say how long an experimenter outside the horizon would have to wait in order to see the exterior systems fall out of quantum equilibrium. However, it seems unlikely that the process should occur only in the final stages of evaporation. For consider a black hole that had been formed entirely from the collapse of one half of an EPR-ensemble. In the late stages of evaporation, the other half of the EPR-ensemble (assumed to be far away from the hole and shielded from the outgoing radiation) will be essentially mixed, because a Planck-sized remnant cannot store the quantum information required to maintain overall purity (without having an unreasonably large number of internal states). If information loss is to be avoided, the nonequilibrium transition must take place earlier, even when the hole is macroscopic.

We shall therefore assume that the process takes place whenever one half of an EPR-pair falls behind the event horizon, even for a macroscopic black hole.
And, we shall assume that the process occurs in a ‘reasonably accessible time’, with respect to a distant experimenter.

Note that (31) entails a large deviation from quantum probabilities. For example, let $|\pm\rangle$ be momentum states $|\pm p\rangle$ of a free (nonrelativistic) particle of energy $E = p^2 / 2m$ in some normalisation volume $V$. In pilot-wave theory, the quantum state (28) consists of a statistical mixture of objectively-existing wave functions, which we take to be (for example)

$$
\psi_+(x,t) = \frac{1}{\sqrt{V}} e^{ip\cdot x} e^{-iEt}, \quad \psi_-(x,t) = \frac{1}{\sqrt{V}} e^{-ip\cdot x} e^{-iEt}
$$

As discussed in section 2.1, for each pure subensemble – with wave function $\psi_\pm$ and distribution $\rho_\pm$ – we may define a hidden-variable entropy

$$
S_{hv}^\pm = - \int d^3x \rho_\pm \ln(\rho_\pm / |\psi_\pm|^2)
$$

and for the whole ensemble the (mean) hidden-variable entropy is $S_{hv} = \frac{1}{2} S_{hv}^+ + \frac{1}{2} S_{hv}^-$ (where in quantum equilibrium $S_{hv} = 0$, while out of equilibrium $S_{hv} < 0$). Taking for simplicity $\rho_+ = \rho_- = \rho$, (31) becomes

$$
\int d^3x \rho \ln(\rho V) = \ln 2 \quad (35)
$$

A simple distribution $\rho$ satisfying this equality is obtained by dividing the volume $V$ into two halves, and setting $\rho = 0$ in one half and $\rho = 2/V$ in the other. There are, of course, an infinite number of nonequilibrium distributions satisfying (35). Possibly, a detailed theory with a specific mechanism for generating nonequilibrium would yield a definite prediction for the final-state distribution $\rho$; this remains to be seen. In any case, it is clear that the required degree of nonequilibrium is rather large.

According to our argument, then, throwing one half of an EPR-pair into a black hole will drive the hidden variables for the remaining half away from quantum equilibrium (over an ensemble). Subsequent experiments with these systems will then yield non-standard (non-quantum) probabilities.

This process may be viewed in terms of a nonlocal leakage of information from behind the horizon. For example, let the initial state consist of a pair of photons whose polarisations are entangled. Allowing one photon to fall behind the horizon, let the other now be directed towards a polariser, followed by a second polariser at angle $\Theta$ with respect to the first. According to quantum theory, any photons passing through the first polariser are prepared in a quantum state of definite (linear) polarisation, and for these photons the probability of transmission through the second polariser will be

$$
p_{eq}^+(\Theta) = \cos^2 \Theta \quad (36)
$$

Whatever the state of photons entering the first polariser, the fraction transmitted through the second must vary as $\cos^2 \Theta$ as the angle $\Theta$ is varied. But we
saw in section 2.2 that, in any deterministic hidden-variables theory, a nonequilibrium distribution \( \rho(\lambda) \neq \rho_{eq}(\lambda) \) will generically imply a non-sinusoidal transmission probability \( p^+(\Theta) \neq \cos^2 \Theta \) (37)

(where here the values \( \sigma = \pm 1 \) of the observable \( \hat{\sigma} = \mathbf{m} \cdot \hat{\sigma} \) correspond respectively to polarisation parallel or perpendicular to an axis in 3-space with angle \( \Theta = \theta/2 \)). In effect, the non-sinusoidal distribution \( p^\pm(\Theta) \) would contain information from behind the horizon. Heuristically speaking, the usual one-way membrane is ‘pierced’ by the entangled state, which provides a channel along which information may leak out, in the form of non-standard polarisation probabilities.\(^{14}\)

The above discussion for the singlet state is easily generalised. Any pure state for the pair of two-state systems may be written by Schmidt decomposition in the form

\[
|\Psi\rangle = \sqrt{p_1}|A^+,B^+\rangle + \sqrt{p_2}|A^-,B^-\rangle
\]

(38)

where \( p_1, p_2 \) are real and non-negative with \( p_1 + p_2 = 1 \), and \( |A^\pm\rangle \) and \( |B^\pm\rangle \) are orthonormal bases for systems \( A \) and \( B \) respectively. The reduced density operator (for either system, suppressing the labels \( A \), \( B \) in the kets) takes the form

\[
\hat{\rho}_{A,B} = p_1 |+\rangle \langle +| + p_2 |-\rangle \langle -|
\]

with a von Neumann entropy

\[
S_{vonN}^{A,B} = -p_1 \ln p_1 - p_2 \ln p_2
\]

(39)

If one half of each pair falls behind the horizon, the conservation hypothesis (30) implies that the external ensemble will acquire a (nonequilibrium) hidden-variable entropy given by

\[
S_{out}^{hv} = p_1 \ln p_1 + p_2 \ln p_2
\]

(40)

Note that for any pure state (38), the amount of nonequilibrium generated – that is, the value of \( |S_{out}^{hv}| \) – does not depend on which half (\( A \) or \( B \)) of the ensemble falls behind the horizon. This is because the von Neumann entropy (39) for the reduced state is the same at each wing.

The entanglement entropy (39) is a good measure of entanglement for the state (38). It is equal to \( \ln 2 \) times the number of maximally-entangled states – or Bell states – into which \( |\Psi\rangle \) may be interconverted (by local operations at each wing and classical communication alone) \( ^{[76]} \). The amount of nonequilibrium generated is therefore in direct proportion to the degree of entanglement

\(^{14}\)We are assuming, of course, that nonequilibrium is produced in the exterior photon before it enters the first polariser (while it is still entangled with the photon behind the horizon). Note also that there is no particular reason why passage through the first polariser should cause the photon to relax back to equilibrium (though this might happen in some hidden-variables theories). For example, in de Broglie-Bohm theory, the division of a nonequilibrium particle ensemble by a Stern-Gerlach spin measurement generally results in nonequilibrium within each of the final (separated) wave packets.

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of the initial state, $|S^\text{out}_{\text{hv}}\rangle = S^A_{\text{vonN}}$. The nonequilibrium generated may also be related to the maximal violation of Bell’s inequality obtainable from quantum measurements at each wing of the state \( S_{\text{vonN}} \). In appropriate units, the eigenvalues of the states $|A\pm\rangle, |B\pm\rangle$ take values $a, b = \pm 1$, and in any local hidden-variables theory the expectation value $E(a, b)$ of the product $ab$ satisfies a Bell inequality (in the form due to Clauser et al. [77])

$$|E(a, b) + E(a', b) + E(a, b') - E(a', b')| \leq 2$$

(41)

where the primes denote a different orthonormal basis (corresponding to a different measurement setting, for example measurement of polarization along a different axis). In quantum theory, for a state $S_{\text{vonN}}$ the left hand side of (41) attains a maximum value [78]

$$f_{\text{max}} = 2\sqrt{1 + 4p_1p_2}$$

(42)

Both (39) and (42) are minimised by $p_1 = 0$ (or $p_2 = 0$) and maximised by $p_1 = 1/2$; further, both are monotonically increasing functions of $p_1$ (or $p_2$) on the interval $[0, 1/2]$. Thus, the amount of nonequilibrium generated $|S^\text{out}_{\text{hv}}\rangle = S^A_{\text{vonN}}$ grows monotonically with the maximal violation of Bell’s inequality $f_{\text{max}}$. This accords with our intuitive picture of the exterior system being thrown out of equilibrium by nonlocal effects from behind the horizon.

4 Nonequilibrium Hawking Radiation

As mentioned in the Introduction, outgoing modes for Hawking radiation are entangled with ingoing modes. Mode-by-mode, then, the situation is conceptually similar to the thought experiment just described, in which one half of an EPR-pair falls behind the horizon. One may then similarly posit an evolution away from quantum equilibrium for the outgoing radiation, resulting in emitted particles whose statistical behaviour deviates from the quantum formalism.

To illustrate the entanglement involved, for simplicity we shall first consider the (rather artificial) case of the ‘eternal’ black hole restricted to two dimensions [79]. The standard Schwarzschild line element

$$d\tau^2 = \left(1 - \frac{2M}{r}\right)dt^2 - \left(1 - \frac{2M}{r}\right)^{-1}dr^2$$

may be rewritten in terms of Kruskal null coordinates $\bar{u}$, $\bar{v}$ as

$$d\tau^2 = \frac{2M}{r}e^{-r/2M}d\bar{u}d\bar{v}$$

Extending $\bar{u}$, $\bar{v}$ over the full range $(-\infty, +\infty)$, one obtains the maximally-extended Kruskal manifold. This contains two asymptotically-flat regions I and II that are causally disconnected.

Quantising a massless scalar field $\phi$ on this spacetime, the wave equation has natural basis modes $\propto e^{-i\omega\bar{u}}, e^{-i\omega\bar{v}}$ which are regular on the whole manifold.
These have an associated vacuum state $|0\rangle_K$ (the Hartle-Hawking vacuum), where the modes have positive frequency with respect to the time coordinate $\bar{t} = \frac{1}{2}(\bar{u} + \bar{v})$. By relating these modes to modes defined in regions I and II only, the state $|0\rangle_K$ may be written in terms of particle states $|n_k\rangle_I$ and $|n_k\rangle_{II}$ for regions I and II respectively:

$$|0\rangle_K = \prod_k \frac{1}{\cosh \alpha_\omega} \left( \sum_{n_k=0}^{\infty} e^{-4\pi M n_k \omega} |n_k\rangle_I \otimes |n_k\rangle_{II} \right)$$  \hspace{1cm} (43)

where $\omega = |k|$ and $\tanh \alpha_\omega \equiv e^{-4\pi M \omega}$.

While there is no entanglement between different modes in (43), each mode is entangled between the causally-disconnected regions I and II. For experiments performed in region I, all quantum probabilities are given by the reduced density operator

$$\hat{\rho}_I = \prod_k \hat{\rho}^k_I$$  \hspace{1cm} (44)

where for each mode

$$\hat{\rho}^k_I = \sum_{n_k=0}^{\infty} \frac{e^{-n_k \omega/T}}{Z_k} |n_k\rangle \langle n_k| $$  \hspace{1cm} (45)

with $T = 1/8\pi M$ and

$$Z_k = \sum_{n_k=0}^{\infty} e^{-n_k \omega/T} = (1 - e^{-\omega/T})^{-1}$$

The state (44) corresponds to a thermal mixture at temperature $T = 1/8\pi M$. The Hartle-Hawking vacuum $|0\rangle_K$ represents a black hole in thermal equilibrium with a bath of blackbody radiation [80]. A more realistic choice of vacuum on the extended Kruskal manifold, due to Unruh [81], yields a purely outgoing thermal flux.

The von Neumann entropy associated with $\hat{\rho}_I$ is a sum

$$S_{\text{vonN}}^I = \sum_k S_{\text{vonN}}^{1,k}$$  \hspace{1cm} (46)

of entropies of the individual modes

$$S_{\text{vonN}}^{1,k} = -T \text{Tr}(\hat{\rho}^k_I \ln \hat{\rho}^k_I) = -\sum_{n_k=0}^{\infty} p(n_k) \ln p(n_k)$$

where $p(n_k) = e^{-n_k \omega/T}/Z_k$. We have

$$S_{\text{vonN}}^{1,k} = \frac{\omega}{T} (e^{\omega/T} - 1)^{-1} - \ln(1 - e^{-\omega/T})$$  \hspace{1cm} (47)

(The total entanglement entropy (40) diverges in the continuum limit, and is rendered finite by an appropriate high-frequency cutoff.)
For the case of actual gravitational collapse, one begins with the Minkowski vacuum \( |0\rangle_M^\text{in} \) in the distant past (assuming the infalling matter to be arbitrarily tenuous at arbitrarily early times). In the distant future, after the horizon has formed, one may define an asymptotic Minkowski vacuum \( |0\rangle_M^\text{out} \). One finds that \( |0\rangle_M^\text{in} \) is a thermal mixture of particle states defined with respect to \( |0\rangle_M^\text{out} \) (in the exterior region, outside the horizon). In other words, the in-state is a thermal state with respect to the asymptotic outgoing modes. The ingoing and outgoing field modes (representing pair creation near the horizon) are entangled, and the reduced density operator for the asymptotically Minkowski region is obtained by tracing over the ingoing modes, resulting in a thermal mixture of outgoing radiation \[79\].

Each mode \( k \) (in a two-dimensional model) of the outgoing Hawking radiation is represented by a thermal mixture \( \hat{\rho}^\text{out}_k \) of the form \[45\], with a von Neumann entropy \( S^\text{out}_k \) given by \[47\]. Post-evaporation, the pure in-state becomes a mixture

\[
|0\rangle_M^\text{in} \rightarrow \hat{\rho}^\text{out} = \prod_k \hat{\rho}^\text{out}_k
\]

(48)

As in the thought experiment of section 4, we propose that during the pure-to-mixed transition \[48\], the emitted particles evolve away from quantum equilibrium, by an amount such that the total entropy \( S^\text{hv} + S^\text{vonN} \) is conserved.

An appropriate hidden-variable entropy \( S^\text{hv} \) may be defined in pilot-wave field theory. Assuming the state \( \hat{\rho}^\text{out} \) to be a thermal mixture of wave functionals \( \Psi_{n_k n_{k'}} \ldots \) for Fock states \( |n_k n_{k'} \ldots \rangle \), each pure subensemble with wave functional \( \Psi_{n_k n_{k'} \ldots} \) has a field distribution \( P^\text{out}_{n_k n_{k'} \ldots} [\phi] \) and a hidden-variable entropy

\[
S^\text{out}_{n_k n_{k'} \ldots} = - \int D\phi P^\text{out}_{n_k n_{k'} \ldots} [\phi] \ln(\langle \Psi_{n_k n_{k'} \ldots} | [\phi] \rangle^2) / \langle \Psi_{n_k n_{k'} \ldots} | [\phi] \rangle^2
\]

For the whole ensemble, the mean hidden-variable entropy is

\[
S^\text{out}_{hv} = \sum_{n_k n_{k'} \ldots} p(n_k, n_{k'}, \ldots) S^\text{out}_{n_k n_{k'} \ldots}
\]

where

\[
p(n_k, n_{k'}, \ldots) = p(n_k) p(n_{k'}) \ldots
\]

If the initial state is pure and in quantum equilibrium (so that \( S^\text{in}_{hv} = S^\text{in}_{hv} = 0 \)), imposing the conservation rule \[30\] implies

\[
\sum_{n_k n_{k'} \ldots} p(n_k, n_{k'}, \ldots) S^\text{out}_{n_k n_{k'} \ldots} = - \sum_k \left( \frac{\omega}{T} (e^{\omega/T} - 1) - \ln(1 - e^{-\omega/T}) \right)
\]

(49)

(where the sums are taken over outgoing modes only). This gives a constraint on the subensemble distributions \( P^\text{out}_{n_k n_{k'} \ldots} [\phi] \) and on the total (nonequilibrium) distribution

\[
P^\text{out} [\phi] = \sum_{n_k n_{k'} \ldots} p(n_k, n_{k'}, \ldots) P^\text{out}_{n_k n_{k'} \ldots} [\phi]
\]
As discussed in section 3, it is expected that the generation of quantum nonequilibrium is driven by some unknown phenomenon taking place inside the hole (perhaps near the singularity), which is communicated to the exterior – at the nonlocal hidden-variable level – via the entangled state.

Note that in pilot-wave field theory, the outcomes of quantum measurements depend on the initial field configuration $\phi$; and the probabilities for the outcomes depend on the probability distribution $P[\phi]$ for $\phi$. In general, a nonequilibrium distribution of fields will imply a non-quantum distribution of outcomes of quantum measurements. (For some simple examples of measurement in pilot-wave field theory, see ref. [63].) Thus, the electromagnetic field in quantum nonequilibrium will yield anomalous polarisation probabilities for single photons, as discussed in sections 2.2 and 3.

According to our argument, then, particles radiated by black holes will show statistical anomalies outside the formalism of quantum theory. These deviations from standard quantum probabilities may be interpreted as information leaking nonlocally from behind the event horizon, as discussed above.

5 Relating Hidden-Variable and von Neumann Entropies

We have proposed that the pure-to-mixed transition envisaged by Hawking is accompanied by the generation of quantum nonequilibrium. And, as a simple quantitative hypothesis, we have assumed that the associated decrease in hidden-variable entropy $S_{hv}$ balances the increase in von Neumann entropy $S_{vonN}$. As we have noted, this simple hypothesis involves a comparison of two very different forms of entropy. Here we outline how, even in non-gravitational physics, there are relationships (whose details remain to be developed) between $S_{hv}$ and $S_{vonN}$ – the two kinds of entropy are not completely independent.

Consider a statistical mixture of pure quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ for some system, with density operator

$$\hat{\rho}_{sys} = \frac{1}{2} |\psi_1\rangle \langle \psi_1| + \frac{1}{2} |\psi_2\rangle \langle \psi_2|$$

If the states are orthogonal, $\langle \psi_1 | \psi_2 \rangle = 0$, then according to quantum theory it is possible to separate the mixed ensemble into pure subensembles with density operators $\hat{\rho}_1 = |\psi_1\rangle \langle \psi_1|$ and $\hat{\rho}_2 = |\psi_2\rangle \langle \psi_2|$. For example, an appropriate coupling to an apparatus in an initial pure state $|g_0\rangle$ leads to the evolution

$$\hat{\rho}_{total}^{in} = \hat{\rho}_{sys} \otimes |g_0\rangle \langle g_0| \rightarrow \hat{\rho}_{total}^{out} = \frac{1}{2} |\psi_1 g_1\rangle \langle \psi_1 g_1| + \frac{1}{2} |\psi_2 g_2\rangle \langle \psi_2 g_2|$$

where $\langle g_1 | g_2 \rangle = 0$. Conditionalising on the final readings of the apparatus, we are left with two pure subensembles $\hat{\rho}_1$, $\hat{\rho}_2$ labelled by those readings. (The von Neumann entropy for the whole ensemble is preserved in this process, being equal to $\ln 2$ throughout; while the sum of the reduced von Neumann entropies
for the system and apparatus – associated with their reduced density operators – increases from \( \ln 2 \) to \( 2 \ln 2 \), as a result of ignoring the correlations generated by the interaction.)

If on the other hand \( \langle \psi_1 | \psi_2 \rangle \neq 0 \), no quantum process can separate the subensembles, since no unitary (norm-preserving) evolution can lead to both \( |\psi_1 g_0\rangle \rightarrow |\psi_1 g_1\rangle \) and \( |\psi_2 g_0\rangle \rightarrow |\psi_2 g_2\rangle \) with \( \langle g_1 | g_2 \rangle = 0 \). As is well-known, in quantum theory it is not possible to distinguish non-orthogonal states without disturbing them \[76\].

Such a process can exist, however, in nonequilibrium de Broglie-Bohm theory (for example). Given a nonequilibrium ensemble of systems whose statistical dispersion is less-than-quantum, these systems may be used to perform 'subquantum measurements' on ordinary (equilibrium) quantum systems, allowing non-orthogonal quantum states to be resolved without disturbing them \[82\]. The key point is that de Broglie-Bohm trajectories generally differ for non-orthogonal states, and in nonequilibrium we can have more information about the trajectories than quantum theory allows.

Let us illustrate this with a simple example. Take the system to be a free (nonrelativistic) particle in one dimension with coordinate \( x \), and with alternative initial states

\[
|\psi_1\rangle = \frac{1}{\sqrt{L}} e^{ipx}, \quad |\psi_2\rangle = \frac{1}{\sqrt{2L}} (e^{ipx} + e^{-ipx})
\]

(where \( L \) is a normalisation length). The apparatus pointer has coordinate \( y \) and initial wave function

\[
g_0(y) = (2\pi \Delta^2)^{-1/4} e^{-y^2/4\Delta^2} \quad (50)
\]

We assume that over an ensemble \( y \) has an initial (presumed known) nonequilibrium distribution \( \pi_0(y) \neq |g_0(y)|^2 \). Further, for simplicity we consider the extreme case of an essentially dispersion-free distribution \( \pi_0(y) \), for which the values of \( y \) are concentrated arbitrarily closely to \( y_0 = 0 \). We shall show that, for an appropriate interaction between \( x \) and \( y \), after an arbitrarily short time a mixture

\[
\hat{\rho}_{\text{total}}^\text{in} = \left( \frac{1}{2} |\psi_1\rangle \langle \psi_1| + \frac{1}{2} |\psi_2\rangle \langle \psi_2| \right) \otimes |g_0\rangle \langle g_0| \quad (51)
\]

(with \( \langle \psi_1 | \psi_2 \rangle \neq 0 \)) separates into pure subensembles – with density operators \( \hat{\rho}_1 = |\psi_1\rangle \langle \psi_1| \) and \( \hat{\rho}_2 = |\psi_2\rangle \langle \psi_2| \) – labelled by distinct values of the pointer coordinate \( y \). This separation generates an effective decrease in von Neumann entropy, from an initial positive value to a final value of 0.

To see this, at \( t = 0 \) switch on an interaction \( \hat{H} = a \hat{p}_x \hat{p}_y \) between \( x \) and \( y \). As in section 3, \( a \) is a coupling constant and we neglect the Hamiltonians

---

\[15\] We are assuming, for theoretical purposes, that the nonequilibrium distribution is known. How such an ensemble could be discovered in practice is a separate issue. One possibility, already mentioned, is that nonequilibrium relic particles might be left over from the early universe \[49\] – where the nonequilibrium distribution of the parent population could be deduced from measurements made on a random sample \[82\].
of $x$ and $y$. An initial pure state $\Psi_0(x, y) = \psi_0(x)\pi_0(y)$ evolves into $\Psi(x, y, t)$ according to the Schrödinger equation

$$i\frac{\partial \Psi}{\partial t} = -a \frac{\partial^2 \Psi}{\partial x \partial y}$$

The associated continuity equation

$$\frac{\partial |\Psi|^2}{\partial t} + \frac{\partial}{\partial x} \left( |\Psi|^2 \frac{a}{\partial y} \right) + \frac{\partial}{\partial y} \left( |\Psi|^2 \frac{a}{\partial x} \right) = 0$$

implies the de Broglie guidance equations\(^{16}\)

$$\dot{x} = a \frac{\partial S}{\partial y}, \quad \dot{y} = a \frac{\partial S}{\partial x}$$

for the hidden-variable trajectories. For $\psi_0(x) = \psi_1(x, 0)$ we have

$$\Psi_1(x, y, t) = \frac{1}{\sqrt{L}} e^{ipx} \pi_0(y - apt)$$

while for $\psi_0(x) = \psi_2(x, 0)$ we have

$$\Psi_2(x, y, t) = \frac{1}{\sqrt{2L}} e^{ipx} \pi_0(y - apt) + \frac{1}{\sqrt{2L}} e^{-ipx} \pi_0(y + apt)$$

To leading order in $at$, the phase $S = \text{Im} \ln \Psi$ is either

$$S_1 = px$$

or

$$S_2 = \frac{apy}{2\Delta^2} \tan px$$

The solution for the trajectory $y(t)$ (with initial condition $y_0 = 0$) is then either

$$y_1(t) = apt$$

or

$$y_2(t) = 0$$

After an arbitrarily short time the quantum density operator $\hat{\rho}_{\text{total}}$ (following the usual unitary evolution) will still be given by (51), so that $\hat{\rho}_{\text{out}} = \hat{\rho}_{\text{in}}$, to arbitrary accuracy. And yet, no matter how small $t$ may be, if $\pi_0(y)$ is sufficiently peaked around $y = 0$, the ensemble will have divided into subensembles labelled by distinct pointer positions $y_1 = apt$ and $y_2 = 0$, with system wave functions $\psi_1$ and $\psi_2$ respectively. (The key point here is that, for small $at$, while the total quantum state of system and apparatus is hardly affected, for sufficiently narrow $\pi_0(y)$ the small change in $y(t)$ at the hidden-variable level can

\(^{16}\)Note that for this Hamiltonian the roles of $x$ and $y$ are reversed with respect to the gradient of phase.
be large enough to provide unambiguous information about the initial quantum state of the system.

The initial quantum state \( |\psi_1\rangle\) has a total von Neumann entropy

\[
S_{\text{vonN}}^\text{in} = -\lambda_+ \ln \lambda_+ - \lambda_- \ln \lambda_-	ag{52}
\]

where \(\lambda_\pm = \frac{1}{2} \pm \frac{1}{2} |\langle \psi_1 | \psi_2 \rangle| \) are the eigenvalues of \(\hat{\rho}_{\text{total}}^\text{in}\). Mathematically, the quantity \(-Tr(\hat{\rho}_{\text{total}} \ln \hat{\rho}_{\text{total}})\) is preserved by unitary evolution (even over finite times), and at the end of the above (brief) interaction what one usually calls the von Neumann entropy is strictly speaking still given by \(S_{\text{vonN}}^\text{in}\). Physically, however, the ensemble has separated into pure subensembles identifiable by distinct pointer positions, so that in effect the total von Neumann entropy has really become \(S_{\text{vonN}}^\text{out} = 0\).

This finite decrease in an arbitrarily short time is possible if the initial apparatus ensemble has an arbitrarily narrow nonequilibrium distribution \(\pi_0(y)\), corresponding to a hidden-variable entropy \(S_{\text{hv}}^\text{in}\) of arbitrarily large magnitude. For example, with the Gaussian wave function \(\psi(y)\) of width \(\Delta\), a Gaussian distribution \(\pi_0(y)\) of width \(w\) has a relative entropy

\[
S_{\text{hv}}^\text{in} = \frac{1}{2} \left( 1 - \frac{w^2}{\Delta^2} \right) + \ln \left( \frac{w}{\Delta} \right)
\]

and \(S_{\text{hv}}^\text{in} \to -\infty\) as \(w \to 0\) (for fixed \(\Delta\)).

For the realistic case of finite \(w < \Delta\), the initial hidden-variable entropy \(S_{\text{hv}}^\text{in}\) is finite, and the above separation into pure subensembles could take place only to some approximation (in some finite time \(t\)). The magnitude of \(S_{\text{hv}}^\text{in}\) might be regarded as the ‘cost’ of (approximate) non-orthogonal state separation. But the details of the efficiency of such separation – for example, how large \(S_{\text{hv}}^\text{in}\) must be to separate a given mixture of non-orthogonal states with a given accuracy – remain to be developed.

What is required here is the development of a ‘thermodynamics’ of quantum nonequilibrium systems interacting with ordinary quantum equilibrium systems. In a specific model such as de Broglie-Bohm theory, this should be possible. By reasoning along the above lines, one may for example study the interconversion of hidden-variable and von Neumann entropies, and elucidate the relations between them. At present, such an extended ‘thermodynamics’ is in its infancy. But it is already clear that the presence of quantum nonequilibrium, with \(S_{\text{hv}} \neq 0\), results in anomalous behaviour for \(S_{\text{vonN}}\).

The von Neumann entropy \(-Tr(\hat{\rho} \ln \hat{\rho})\) is usually identified with thermodynamic entropy. However, von Neumann’s original proof \cite{83} really shows that \(-Tr(\hat{\rho} \ln \hat{\rho})\) is the entropy of a mixture of eigenstates of \(\hat{\rho}\) with probabilities equal to the corresponding eigenvalues.\(^\text{17}\) It is then assumed that \(-Tr(\hat{\rho} \ln \hat{\rho})\) is still the entropy for any mixture with the same density operator \(\hat{\rho}\); that is, it is assumed that the entropy depends only on \(\hat{\rho}\). This assumption is justified in quantum theory, where \(\hat{\rho}\) provides complete statistical information about the

\(^{17}\)The proof proceeds by separating the (orthogonal) components of the mixture by means of semipermeable membranes.
ensemble. But as we saw in section 2.3, in quantum nonequilibrium $\hat{\rho}$ fails to provide complete statistical information. There is therefore no reason to assume that ensembles with the same $\hat{\rho}$ will have the same physical properties, and no reason to assume that the entropy depends on $\hat{\rho}$ alone. It therefore seems likely that the definition of thermodynamic entropy will have to be generalised in quantum nonequilibrium.

A similar conclusion was drawn by Weinberg [84] in the context of nonlinear quantum theory, in response to Peres’ claim [85] that the (hypothetical) nonlinear evolution of quantum states would spontaneously decrease the entropy of a closed system. Note also that Peres [85] (following von Neumann [83]) has considered a thermodynamic cycle, involving a gas of photons with non-orthogonal polarisation states, and hypothetical semi-permeable membranes capable of separating the non-orthogonal states. The apparent effect of the cycle is to convert heat into work at a single temperature, in violation of the second law of thermodynamics. As noted by Peres, the conclusion that the second law is violated rests on the assumption that von Neumann entropy is equivalent to thermodynamic entropy. But there is no reason why this should be true in a hypothetical ‘postquantum’ theory (such as nonequilibrium de Broglie-Bohm) that allows such membranes to exist.

In the context of black holes and pure-to-mixed transitions, it is possible that considerations of black-hole thermodynamics will lead to a natural candidate for a generalised notion of thermodynamic entropy, which includes contributions from hidden variables as well as from the usual quantum degrees of freedom. Whether or not a generalised second law would imply the production of quantum nonequilibrium by black holes remains to be seen.

In the context of deterministic hidden variables, it seems natural to assume that any nonequilibrium distribution generated by a black hole should contain the ‘lost’ information regarding the initial quantum state. That is, it seems natural to assume that there exists a retrodictive mapping from the final nonequilibrium mixed state to the initial equilibrium pure state. Imposing the existence of such a mapping may provide a guide to the construction of an appropriate generalised entropy. This remains to be studied.

In our discussion of black-hole evaporation, we made the simple assumption that $S_{hv} + S_{vonN}$ is conserved. This assumption has no firm theoretical basis at present, and may well prove to be wrong, even if black holes really do generate quantum nonequilibrium. On the other hand, it is already clear from non-gravitational physics that there are good physical reasons for positing a connection between $S_{hv}$ and $S_{vonN}$. The two entropies are not as unrelated as they might appear at first sight, making it not implausible that the evaporation of black holes effectively converts hidden-variable entropy into von Neumann entropy, as we have suggested. In any case, the assumption that $S_{hv} + S_{vonN}$ is conserved might be amenable to experimental test, as we shall now discuss.
6 Possible Experimental Tests

As discussed in section 3, a characteristic feature of nonequilibrium photons would be a breakdown of the quantum modulation (36) of transmission through a pair of polarisers. If the polarisers are set at a relative angle Θ, quantum theory predicts a transmission probability \( \cos^2 \Theta \) through the second polariser, independently of the state of the incoming photons. Generically in nonequilibrium, there will be deviations from \( \cos^2 \Theta \) as the angle Θ is varied [42].

Experiments were carried out by Papaliolios [86], with ordinary laboratory photons, to test for deviations from \( \cos^2 \Theta \) transmission for successive polarisation measurements over very short timescales. Such deviations were predicted by Bohm and Bub [87], in a hidden-variable model where measurements generate quantum nonequilibrium for short times. (The model is not of the type being considered in this paper, where initial equilibrium remains in equilibrium in all non-gravitational processes.) It was found that successive measurements within times of order \( 10^{-13} \) s led to agreement with the quantum \( \cos^2 \Theta \) modulation to within 1% [86].

The prediction that Hawking radiation will be in quantum nonequilibrium might in principle be tested, if such radiation is ever detected from the evaporation of microscopic black holes. A more promising experiment, however, which might be feasible now, involves the prediction that if one half of an entangled pair falls into a macroscopic black hole then quantum nonequilibrium will be generated in the other half. This might be tested using entangled photons generated by atomic cascade emission in black-hole accretion discs.

It might be thought that nonequilibrium would in any case be smeared out by the finite size of the emitting region. If photons are emitted with a nonequilibrium distribution \( \rho(\lambda) \neq \rho_{eq}(\lambda) \) that depends on the spatial location of the emission, then the ensemble of received photons will have a distribution \( \bar{\rho}(\lambda) \) consisting of a spatial average of \( \rho(\lambda) \), and it might be that \( \bar{\rho}(\lambda) \) is very close to \( \rho_{eq}(\lambda) \) even if \( \rho(\lambda) \) is not. While this might occur, it is not necessary. And such averaging will have no effect at all if \( \rho(\lambda) \) is uncorrelated with spatial location. For example, in de Broglie-Bohm theory, the outcome of a spin measurement for a nonrelativistic spin-1/2 particle is determined by the initial position inside the two-component wave packet [31], and a nonequilibrium distribution will generally yield spin probabilities deviating from quantum theory. If the position of each particle is defined relative to its packet, any averaging over the positions of the packets will not affect the deviation from quantum probabilities. More generally, the hidden variables (whatever they might be) determining the outcome of a photon polarisation measurement could be (at least effectively) independent of where the photon is generated in space – in which case spatial averaging over the source will not affect any nonequilibrium that may be present.

6.1 Hawking Radiation from Microscopic Black Holes

Black holes with a range of masses may have formed in the early universe [88, 89]. The evaporation timescale [2] is of order the current age of the universe for
microscopic black holes with $M \sim 10^{15}$ g or $M \sim 10^{-18} M_\odot$. Thus, if primordial black holes with mass $\lesssim 10^{15}$ g are sufficiently abundant, it might be possible to detect Hawking radiation from them today. The radiated power scales with time as $\gtrsim (-t)^{-2/3}$, diverging at $t = 0$, when the hole is assumed to disappear in an explosion, whose products depend on the high-temperature behaviour of matter.

For black holes of mass $M \sim 10^{15}$ g, a significant fraction of the luminosity is expected to be in the form of $\gamma$-rays peaked at $\sim 100$ MeV [90]. However, from measurements of the diffuse $\gamma$-ray background, the mean density of primordial black holes with mass $\lesssim 10^{15}$ g is constrained to be no more than a fraction of order $10^{-8}$ of the critical density of the Universe, making direct detection difficult [91, 92, 93, 94]. However, if the black holes are sufficiently clustered in our own Galaxy, they might be directly detectable [95]. It has in fact been claimed that the observed anisotropic component of the $\gamma$-ray background is caused by primordial black holes clustered inside our Galactic halo [96]. Further, it has been suggested that observations of a class of very short-time $\gamma$-ray bursts are consistent with an interpretation as primordial black holes evaporating in our Galaxy [97]; though the consistency of this scenario with the primordial density fluctuation spectrum has been disputed [98].

It remains to be seen if present or future $\gamma$-ray satellites will lead to a definitive detection of radiation from primordial black holes. The INTEGRAL mission is currently providing a wealth of new data, as in the recent resolution of compact sources responsible for the soft $\gamma$-ray glow from the Milky Way [99]. Three new missions are imminent (Astro E II, GLAST and Swift).

Should $\gamma$-rays from the evaporation of primordial black holes ever be definitively detected, we suggest that their polarisation probabilities be probed for deviations from the standard $\cos^2 \Theta$ modulation. Polarimetry for $\gamma$-rays is currently at an early stage of development, however. The polarisation of $\gamma$-rays may be measured by Compton scattering. According to the theoretical differential cross-section (the Klein-Nishina formula), linearly-polarised $\gamma$-rays Compton scatter preferentially in the plane perpendicular to their axis of polarisation. This technique has recently been applied to measure the linear polarisation of the $\gamma$-ray burst GRB021206 (at energies $\sim 1$ MeV) [100], and is being developed further for future $\gamma$-ray telescopes which will be able to measure polarisation at energies up to 20 MeV [101]. Note that our experiment requires two successive polarisation measurements (or a preparation followed by a measurement) at a relative angle $\Theta$. Because Compton polarimetry does not destroy but merely scatters the incoming photon, there seems no reason in principle why this could not be done: if the (absorbing) photon detector is removed from the first device, the second may be configured so as to accept only a particular polarisation output ($\gamma$-rays scattered in a particular direction) from the first.

In theories with large extra dimensions [102], the Planck scale is of order 1 TeV, raising the possibility that microscopic black holes might be produced in collisions at the TeV scale. It is expected that such holes will evaporate, or thermally decay, primarily into standard-model particles (including hard photons) [103]. Such events could be observed at the Large Hadron Collider [104], or in
collisions between cosmic rays and atmospheric nucleons. Anomalous Centauro-like events in cosmic rays have in fact been interpreted in terms of exploding microscopic black holes of mass $\sim 1$ TeV. Again, if particles from the Hawking decay of microscopic black holes were definitively identified, we would suggest that their polarisation probabilities be tested for deviations from quantum behaviour.

### 6.2 Entangled Photons from Black-Hole Accretion Discs

While microscopic black holes have not been definitively detected, the existence of macroscopic black holes is well established. It is believed that most (if not all) galactic nuclei contain a supermassive black hole, of mass in the range $\sim 10^6 - 10^{10}$ solar masses, and that our Galaxy is populated with stellar-mass black holes. (For a review of the evidence, see for example ref. [108].) It is expected that most astrophysical black holes are accompanied by thin accretion discs. The strong gravity region close to the hole is characterised by the production of X-rays, and the profile of an X-ray emission line from the inner region of the disc may be used to probe the spacetime structure at the location of the radiating material [109].

Observation of the active galaxy MCG–6-30-15 by the X-ray satellite ASCA detected a broadened and skewed K-shell X-ray (fluorescent) emission line of iron, with a profile showing an extended red wing consistent with the effect of gravitational redshift close to the horizon [110, 111]. The line is believed to come from the inner region of the (thin) accretion disc, the surface of which is irradiated by a continuum of X-rays (originating in a hot corona above the disc), leading to photoionisation of iron. The transition with the largest cross-section results in the ejection of a K-shell ($n = 1$) electron. An L-shell ($n = 2$) electron can then drop into the K-shell with the emission of a Kα line (fluorescent) photon at 6.4 keV. The line is intrinsically narrow, but photons emitted at different distances from the horizon suffer different gravitational redshifts, resulting in a broad and skewed profile. Detailed calculations (including relativistic Doppler effects) result in a line profile that agrees strikingly with observation. Similarly broadened iron emission lines have been detected from other (Seyfert) galaxies, as well as from black holes in our Galaxy. (For reviews of fluorescent iron lines as probes of black-hole systems, see refs. [112, 113].) Broadened lines from oxygen, nitrogen and carbon have also been reported [114, 115]; though this interpretation has been disputed [116, 117].

Now, a two-photon cascade emission (an atomic decay through an intermediate state) generates a pair of photons with entangled polarisations. This effect was used in the classic early tests of Bell’s inequality, based on cascades in atomic calcium and mercury [77, 118, 119, 120]. If such a cascade could be identified sufficiently close to the horizon in a black-hole accretion disc, then this naturally-occurring situation might be used to realise the thought experiment considered in section 3. For there will be a significant probability that one of the photons is captured while the other is detected on Earth (or on an orbiting satellite). The polarisation probabilities of the detected photons could
then be tested for deviations from the standard $\cos^2 \Theta$ modulation. In practice, however, a number of difficulties must be addressed.

**Cascade Emission Close to the Event Horizon**

We require a two-photon cascade so close to the horizon that there is a significant probability that one half of the entangled pair is actually captured. It is helpful to first consider just how close to the horizon the observed Kα iron line originates from.

The iron line profile from MCG–6-30-15 was initially thought to be consistent with a Schwarzschild black hole, with emission from the surface of an accretion disc extending inwards to about $r = 6M$, with the disc inclined at about 30° to the line of sight [110, 111]. However, subsequent analysis and observation favour a near-extremal Kerr black hole ($a/M > 0.94$, where $a \equiv J/M$ is the specific angular momentum), with line emission from radii as small as $r \approx 2M$ [121, 122, 123, 124, 125, 126] – very close to the horizon at $r = M + \sqrt{M^2 - a^2} \lesssim 1.3M$. (Similar results have been obtained for Galactic black holes [127, 128].)

Note that it is usually assumed that the accretion disc does not extend all the way to the event horizon, but has an inner edge close to or at the ‘radius of marginal stability’ $r_{ms}$ (the radius of the last stable circular orbit for massive test particles). And in most models, the observed X-ray line emission cannot come from inside $r = r_{ms}$ [113].

If the interpretation of these observations is correct, the extreme red wing of the Kα iron line originates from radii within a factor of 2 of the horizon. For these photons, any entangled partners (should such exist) will have a large probability of being captured by the black hole – because in a cascade the directions of the emitted photon momenta are not strongly correlated (owing to atomic recoil), so that any partners could have been emitted over a wide range of angles. (For photons coming from infinity the capture cross section for a black hole of mass $M$ is $\sim 20\pi M^2$ [2].)

As for identifying a cascade close to the horizon, let us consider further the observed Kα iron line. The detected photons are produced by an L-shell ($n = 2$) electron dropping into the vacant K-shell ($n = 1$), leaving a vacancy in the L-shell, amounting to a vacancy transition $1s \rightarrow 2p$. A further radiative transition can then follow: for example, an M-shell ($n = 3$) electron can fall into the L-shell, emitting a second (Lα) photon, amounting to a vacancy transition $2p \rightarrow 3d$. According to the detailed calculations of Jacobs *et al.* [129], for an initial K-shell (1s) vacancy created in neutral iron, the probabilities for the radiative vacancy transitions $1s \rightarrow 2p$ and $2p \rightarrow 3d$ are respectively $P(1s \rightarrow 2p) = 0.28$ and $P(2p \rightarrow 3d) = 0.18 \times 10^{-2}$. (See table I of ref. [129]. The only other possible transition after $1s \rightarrow 2p$ is $2p \rightarrow 3s$, whose probability of $0.13 \times 10^{-3}$ may be ignored here.) The probability for the vacancy cascade $1s \rightarrow 2p \rightarrow 3d$ is then just $P(1s \rightarrow 2p \rightarrow 3d) = P(2p \rightarrow 3d) \approx 2 \times 10^{-3}$, as $2p \rightarrow 3d$ can occur only after $1s \rightarrow 2p$. Thus, for an initial ensemble of iron atoms with a K-shell (1s) vacancy, about 0.2% will undergo $1s \rightarrow 2p \rightarrow 3d$, emitting a pair
of photons. Further, of the subensemble that emits a Kα photon, a fraction

$$P(2p \rightarrow 3d|1s \rightarrow 2p) = P(2p \rightarrow 3d)/P(1s \rightarrow 2p) \approx 6 \times 10^{-3}$$

will subsequently emit an accompanying Lα photon. In other words, of the Kα photons currently being observed from black-hole accretion discs, about 0.6% are accompanied by Lα cascade photons.\(^{18}\) This figure is small, but significant.

Thus, there appears to be no difficulty in identifying cascade photons emitted sufficiently close to the horizon. Given that relativistically broadened lines are beginning to be reported from other elements besides iron, the situation is likely to improve in the near future. (Broadened line emission from oxygen in NGC 4051 has been reported to require emission down to \(r < 1.7M\), for a near-extremal Kerr black hole \([115]\).)

In principle, the proposed experiment might be attempted now with the observed iron Kα photons. But even if the effect we are looking for exists, and even if the cascade partners are highly entangled (which we have not established), the effect will be greatly diluted because most (99.4%) of the Kα photons are not accompanied by cascade partners and are therefore not susceptible to the suggested deviations from quantum probabilities. Further, there is a difficulty with performing the required polarisation measurements for X-ray photons. An efficient X-ray polarimeter (based on the direction of electron emission in the photoelectric effect) has been developed for astrophysical observations in the 2 – 10 keV band \([130]\), and might be deployed in the proposed XEUS mission to study supermassive black holes \([131]\). This device could certainly be used to measure the polarisation of the Kα iron line (at 6.4 keV). However, our experiment requires two successive measurements at a relative angle \(\Theta\), and to achieve this an alternative method for measuring X-ray polarisation would have to be developed – for unlike the \(\gamma\)-ray Compton polarimeter mentioned above, a polarimeter based on the photoelectric effect has the unfortunate feature of destroying the measured photon (which is absorbed by an atom, resulting in the ejection of a photoelectron).

**Maximising Quantum Information Loss and Bell Inequality Violation**

The proposed conservation rule \([60]\) suggests that the effect we are looking for will be large when the von Neumann entropy generated by tracing over the infalling photons is large – that is, when the quantum information lost behind the horizon is large. To ensure that this is the case raises some practical difficulties, to which we now turn.

For a mixed two-photon state with nonzero (total) von Neumann entropy, the relevant quantity is the difference between the reduced and total von Neumann entropies. In the simple (if rather unrealistic) pure case, the relevant

\(^{18}\)We have quoted the transition probabilities for the case of neutral iron only. Results for other ionisation states are listed in ref. \([129]\). The assumption that iron is neutral is thought to be a good approximation for a wide range of accretion discs in active galactic nuclei, where the temperature of the (surface of the) disc is expected to be low (\(kT \sim 10\) eV) – in contrast with galactic black holes whose discs are expected to have much higher temperatures (\(kT \sim 1\) keV) \([113]\).
quantity is just the reduced von Neumann entropy, which is also a good measure of entanglement for pure states. As we noted in section 3, for pure states the amount of nonequilibrium predicted by (30) grows monotonically with the maximal violation of Bell’s inequality, in accord with our intuitive picture of nonlocal information flow from behind the horizon. Previous work on correlation experiments designed to violate Bell’s inequality is then a useful guide.

The theory of two-photon polarisation correlations in atomic cascades was worked out in great detail by Fry [132]. The results were applied to select an appropriate atomic transition for use in tests of Bell’s inequality. In a typical experiment performed in the laboratory, photon detectors are placed collinearly on each side of the source, and for ideal polarisers the (normalised) coincidence rate is calculated to be

$$\frac{1}{4} (1 + F(\delta) \cos 2\phi)$$

Here, $\delta$ is the half-angle subtended by the detectors and $\phi$ is the relative angle between the polariser axes; $F$ depends on the details of the transition, and is a measure of the degree of correlation between the polarisations of the photons. The correlations are large enough to violate Bell’s inequality if and only if

$$|F(\delta)| \geq 2^{-1/2}$$  \hspace{1cm} (53)

Because $|F(\delta)|$ is a monotonic decreasing function [132], we require

$$|F(0)| \geq 2^{-1/2}$$  \hspace{1cm} (54)

As already noted, because of atomic recoil the directions of the emitted momenta are not strongly correlated. If $\delta$ is increased, so as to accept non-antiparallel pairs of momenta, $|F(\delta)|$ decreases and the polarisation correlation is reduced.

Fry has calculated and tabulated values of $F(0)$ for many cascade transitions [132]. With appropriate preparation of the populations of the initial states, there are many cascades satisfying (54). However, as shown by Fry, if the initial states are isotropically populated (with respect to the atomic angular momentum), then of the many possible cascades only 5 satisfy (54). These all have zero nuclear spin: otherwise the hyperfine structure weakens the polarisation correlations to a level that can be explained locally. Examples are the $0 - 1 - 0$ cascade of atomic calcium and the $1 - 1 - 0$ cascade of atomic mercury used in the experiments cited above.\(^{19}\) At first sight it might seem likely that in an accretion disc the atomic states would be isotropically populated – however, magnetic fields in the disc might generate a non-isotropy.

Note that (in ideal conditions) the cascade $0 - 1 - 0$ induces a perfect polarisation correlation by virtue of conservation of angular momentum and of parity – provided the detectors are placed so that the accepted photons have anti-parallel (‘back-to-back’) momenta. Schematically (for the pure case, and ignoring the atom), the emitted photon pair may be represented by an entangled

\(^{19}\)An initial excited atomic state has total electron angular momentum $J_i$. A transition $J_i - J - J_f$ consists of decay through an intermediate state $J$ to a final state $J_f$. \hspace{1cm} 38
state of the form
\[ \sum_{k, k'} c(kr, k'r') |kr⟩ \otimes |k'r⟩ \]
where \( k, k' \) are the emitted momenta and \( r, r' \) the polarisations. For the 0−1−0 cascade, the coefficients \( c(kr, k'r') \) contain terms of the form \( (ε_{kr} \cdot k)(ε_{k'r'} \cdot k) \) (where the linearly-independent polarisation vectors \( ε_{kr}, r = 1, 2 \), are orthogonal to the momenta \( k \)). For small acceptance angles the relevant \( k, k' \) are nearly antiparallel; \( (ε_{kr} \cdot k)(ε_{k'r'} \cdot k) \) is then small, while the presence of \( (ε_{kr} \cdot ε_{k'r'}) \) constrains the polarisation state to take the form
\[ \frac{1}{\sqrt{2}} (|\hat{x}⟩ \otimes |\hat{x}⟩ + |\hat{y}⟩ \otimes |\hat{y}⟩) \] (55)
where \( |\hat{x}⟩, |\hat{y}⟩ \) respectively denote polarisation along the \( x-, y- \)-axes (taking \( k, k' \) respectively along \( +z, −z \)). If the detectors accept non-antiparallel pairs \( k, k' \), terms in \( (ε_{kr} \cdot k)(ε_{k'r'} \cdot k) \) reduce the polarisation correlation.

The required restriction on the angle depends on how quickly the function \( |F(δ)| \) decreases. For a 0−1−0 cascade, \( \delta \lesssim 70° \) (assuming perfect polarisers), while for other cascades the required range can be much smaller (for example for \( \frac{1}{2} − \frac{1}{2} − \frac{1}{2} \) the range is \( \delta \lesssim 15° \)), and the largest range is obtained for the case 1−1−0, for which \( \delta \lesssim 95° \). (See the plots in Fig. 3 of ref. 132.) In the experiments performed by Aspect et al. (with a 0−1−0 cascade in calcium), \( δ = 32° \) and \( F(δ) = 0.984 \), showing that large angles need not significantly decrease the correlation 134. Clearly, at least for some cascades, a large range of emission angles does not diminish the correlations below the bound required to violate Bell’s inequality. It suffices that the momenta \( k, k' \) be only approximately oppositely-directed.

The above concerns an experiment with collinear detectors on each side of the source. In the situation at hand, we have just one detector at one wing of the entangled state. The discussion in section 3 suggests that our proposed effect will be large in a situation in which Bell’s inequality is potentially violated by a large amount – potentially, that is, if an additional experimenter behind the horizon were actually to measure the polarisations of the infalling photons. The question is whether such a situation (without the additional experimenter) could arise in a black-hole accretion disc.

The single detector (on Earth or a satellite) is so far away from the source that the half-angle \( δ \) subtended is virtually zero. However, in general the detected photons will have partners whose momenta are not necessarily even approximately oppositely directed. (Here, by ‘oppositely directed’ we mean as defined in the local Lorentz rest frame of the emitting atom.) Should one wish to restrict the experiment to photons with approximately oppositely-directed momenta – a case that is particularly well understood and is known to provide strong polarisation correlations – then this may be arranged if the line of sight from emission to Earth runs close to the plane of the accretion disc (so that our view of the disc is edge-on). Of the photons detected on Earth, some will have partners that actually fell behind the horizon, and the momenta of the partners...
will be approximately oppositely directed. The rest of the detected photons will have partners that did not fall behind the horizon, and the momenta of the partners need not be at all oppositely directed; these detected photons are expected to have standard polarisation probabilities, and their presence will merely dilute the sought-for effect. Note that X-ray flares \[135\] and infrared flares \[136\] have been observed from the supermassive black hole at the centre of our Galaxy – and in view of our location in the Galactic plane, our Galaxy would be the ideal system with which to arrange a line of sight parallel to the accretion disc (assuming the disc to be co-aligned with the Galactic plane).

It might be thought that the natural conditions in an accretion disc would be too uncontrolled to produce the required entanglement. By comparison, in the later Bell experiments with atomic cascades the atoms were excited by lasers \[120\]. However, this was only for the sake of efficiency: in the earlier Bell experiments, the atoms were excited by the (filtered) continuum output from an arc lamp \[115\] and by electron bombardment \[119\]. Irradiation by a broad continuum of frequencies – such as the continuum of X-rays striking the surface of an accretion disc – will excite some of the atoms to appropriate states, leading to the cascade emission of entangled photon pairs. Note also that, for a \(0 \rightarrow 1 \rightarrow 0\) cascade and for approximately oppositely-directed pairs, conservation of angular momentum and of parity fixes the relative phase between the terms in the polarisation state \(|\psi\rangle\), so that the entanglement is necessarily phase coherent. (A mixture of pure entangled states with random relative phases would of course be equivalent to a mixture with no entanglement.)

An edge-on view of the disc raises the question of the possible effect of scattering along the line of sight. Entanglement is surprisingly robust against scattering \[137\] \[138\]; though the momentum spread of the scattered states (generated by for example elastic scattering in a random medium) does diminish the polarisation entanglement \[139\] \[140\]. Of greater concern is that significant scattering might destroy any quantum nonequilibrium that may have been generated in the exterior photons: by the time they reach Earth, scattering along the line of sight may have caused their hidden variables \(\lambda\) to relax back to the quantum distribution \(\rho_{eq}(\lambda)\). In the de Broglie-Bohm model, for example, scattering terms in the wave function create perturbations in the velocity field that can drive a system back to quantum equilibrium, if the terms are sufficiently large \[35\]. However, whether or not such relaxation occurs will depend on the details of the hidden-variables theory, as well as on the extent of scattering. Scattering by dust in the plane of the galaxy in question could be avoided by using a cascade that emits infrared photons (or possibly X-rays).

Alternatively, one might consider a situation where the relevant photons do not have approximately oppositely-directed momenta. This would be the case if the accretion disc were viewed face-on. From the point of view of observation this would be an advantage, because according to current models the accretion discs in active galactic nuclei are surrounded by a co-aligned dusty torus, so that a clear line of sight to the central engine is obtained only face-on \[141\]. However, this experiment would require a cascade emission in which approximately orthogonal photon momenta \(k, K\) nevertheless yield strong polarisation
correlations. This possibility needs to be studied further.

In practice, polarisation correlations produced by atomic cascades are usually considered for photon pairs with oppositely-directed momenta, but in principle the results of Fry [132] may be applied more generally, as well as to general atomic populations. In an accretion disc, the atomic level population will of course be determined by local conditions such as the temperature and the presence of magnetic fields. Again, this is a matter for future research.

Note that in curved spacetime parallel transport is required to define relations between spin directions at a distance [142]. The choice of measurement axes required for a maximal violation of Bell’s inequality must be adjusted appropriately, and by this means the standard correlations and maximal violations may be obtained even beyond the event horizon [143]. The need to adjust the axes is not relevant here, however, where we are considering measurements at one wing only; it is only the potential violation of Bell’s inequality that is desirable (were appropriate measurements at the second wing actually carried out), and the maximum possible violation is the same as in flat spacetime.

Note also that our scenario requires that the infalling photons do not interact strongly with the infalling material, otherwise such interactions might destroy the entanglement with the outgoing photons.

Finally, note that in the above discussion we have conflated two issues: the degree of quantum information lost behind the horizon, and the degree of (potential) violation of Bell’s inequality. These are directly related in a simple way for pure states (as noted in section 3), whereas for mixed states the relationship is currently obscure.

**Further Remarks**

In any real experiment, polarisation measurements will always show deviations from \( \cos^2 \Theta \) resulting from ordinary noise and experimental errors. This could be distinguished from the sought-for effect by switching the input back and forth from the astronomical source to a similar source (for example of iron atoms irradiated by X-rays) prepared in the laboratory. Further, our discussion suggests that, if the proposed effect exists, it will have a distinctive signature: the deviations from \( \cos^2 \Theta \) should be bigger for those photons that are closer to the red end of the emission line, as these are emitted closer to the horizon and are therefore more likely to have partners that were actually captured.

Recent developments in X-ray interferometry achieve an angular resolution which will, in the foreseeable future, make it possible to image supermassive black holes directly [144] (as envisioned in the projected MAXIM mission [145]). Though as we have noted, to perform two successive polarisation measurements for X-ray photons requires a measurement technique that does not destroy the measured photon. Further, if the experiment were indeed performed at X-ray wavelengths, it would have to take place above the Earth’s atmosphere (which is opaque to X-rays). A ground-based experiment might be possible in the infrared, to which our atmosphere is transparent in certain wavelength windows.

One should beware that, in principle, quantum nonequilibrium (if it exists) could invalidate some of the standard techniques used to identify emission lines.
For example, diffraction-grating spectroscopy assumes that single photons obey standard quantum scattering laws. If the nonequilibrium photon statistics are sufficiently anomalous, instead of bunching in the strong central maximum of the relevant beam (corresponding to a particular spectral order), the photons might be scattered primarily into the weak side lobes— that is, in directions that do not correspond to their true frequency.

Throughout our discussion, we have been assuming that the proposed effect alters only the statistical distribution of outcomes of quantum measurements—and not the allowed values of those outcomes. (That is, we allow the ensemble distribution $\rho(\lambda)$ of hidden variables to be anomalous, but the mapping from $\lambda$ to outcome for a single system is unchanged.) If, for instance, the effect should generate an unexpected shift in photon frequencies, then this too could in principle lead to confusion as to what has been observed.

Clearly there are a number of experimental difficulties and uncertainties, and it is possible that even if the effect exists we would not detect it. On the other hand, a positive detection would be of great interest. The observation of anomalous polarisation probabilities (deviations from $\cos^2 \Theta$ transmission through two successive polarisers) for photons from a distant source would be remarkable, regardless of any uncertainties about the origin or nature of the photons. If such an effect were observed, it may well prove possible to find an alternative explanation, and further experiments would be required to decide between the alternatives.

The identification of an appropriate cascade is a task for the future. An experiment could in principle be performed now with the Kα iron line. But a practical experiment will probably need to await the detection of other relativistically broadened emission lines, with a larger fraction of photons accompanied by entangled partners, and in a frequency band more convenient for accurate measurements of polarisation.

7 Discussion and Conclusion

The hypothesis advanced in this paper is that, during the formation and evaporation of a black hole, the extra degrees of freedom associated with hidden-variables theories become unfrozen, resulting in deviations from standard (Born-rule) quantum probabilities. In particular, we have suggested that deviations from quantum theory will occur for particles outside the horizon that are entangled with particles inside the horizon. Specifically, in the case of photons we have proposed a search for anomalous polarisation probabilities, in the form of deviations from the $\cos^2 \Theta$ modulation of transmission through a pair of polarisers at a relative angle $\Theta$. We have suggested that the effect might be observable experimentally in Hawking radiation from primordial black holes, and in entangled photons from atomic cascade emission in black-hole accretion discs. For the latter in particular, we have considered a number of practical difficulties, none of which seem insurmountable.

As we have noted, the generation of quantum nonequilibrium by black holes
opens up the possibility that the initial state could be retrodicted, so that no information is lost. The initial pure state that collapses to form a black hole might be recoverable, in principle, from the final nonequilibrium Hawking radiation. This seems natural in the fundamentally deterministic theories we have been considering. However, to know whether the process really is reversible would require the construction of a detailed theory.

Today, we see thermal nonequilibrium because in the early universe gravitation amplified the primordial inhomogeneities in energy density, leading to the formation of large-scale structure [40]. Were it not for this peculiarity of gravitation, our universe would be in a state of global thermal equilibrium. In contrast, with respect to hidden variables we do see global equilibrium today: all systems we have access to obey the Born rule (at least to high accuracy). Presumably then, there is no hidden-variable analogue of the gravitational amplification of fluctuations. However, according to our arguments, there are circumstances in which gravitation can drive an ensemble of systems out of quantum equilibrium — namely, in the presence of black holes. If the proposed process does exist, then in the early universe the formation and evaporation of primordial (microscopic) black holes will ensure that quantum nonequilibrium is present at early times, as has been suggested elsewhere [32, 38, 43, 49, 54, 55].

If hidden variables do exist, one expects the Bekenstein bound on the number of distinct states of a spatially bounded system to be merely a feature of quantum equilibrium (another example of the ‘information compression’ noted in section 2.3). Out of equilibrium, extra degrees of freedom will be unleashed. In the case of de Broglie-Bohm theory, the extra (currently hidden) parameters are continuous configurations, which can in principle store an unlimited amount of information.

For 30 years Hawking radiation has stood as a clue to some underlying connection between quantum theory, gravitation and statistical physics. According to the arguments given in this paper, the true nature of the connection has hitherto not come to light because a crucial ingredient was missing: the connection involves not ordinary statistical physics, but the statistical physics of nonlocal hidden variables. If there are additional variables outside the domain of quantum theory, then there is an additional entropy reservoir that has not been taken into account. As we have seen, this offers a new perspective on Hawking information loss: far from being lost, information about systems behind the horizon can leak out in the form of anomalous distributions at the hidden-variable level.

Should it prove possible in practice to identify atomic cascade photons that form one half of an entangled ensemble, the other half having fallen into a black hole, then the theoretical considerations of this paper suggest that it would be worthwhile to test the received photons for deviations from the Born rule — for example, by searching for anomalous polarisation probabilities. Even leaving aside the motivations put forward here, such an experiment would be worthwhile on general grounds, as a test of quantum theory in new and extreme conditions.

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