LC Circuits for the Direct Detection of Ultralight Dark Matter Candidates

Christopher M. Donohue,1,2,† Susan Gardner,2 ‡ and Wolfgang Korsch2 ¶

1Department of Physics, Cornell University, Ithaca, NY 14853-0001, USA
2Department of Physics and Astronomy, University of Kentucky, Lexington, KY 40506-0055, USA

Cosmological mechanisms that yield ultralight dark matter are insensitive to the intrinsic parity of a bosonic dark matter candidate, but that same quantity plays a crucial role in a direct detection experiment. The modification of electrodynamics in the presence of ultralight axion-like dark matter is well-known and has been used to realize sensitive probes of such sub-eV mass-scale dark matter, and analogous studies exist for hidden-photon dark matter as well. Here we reframe the modification of electrodynamics for ultralight dark matter of positive intrinsic parity, with a focus on the scalar case. In particular, we show that resonant LC circuit searches for axions can be modified to detect scalar dark matter particles by exploiting the large electric fields developed for use in neutron EDM experiments. Our proposed experimental set-up can improve upon previous sensitive searches for scalar particles from “light shining through a wall” experiments to probe scalar-photon couplings some three orders of magnitude smaller in the $1 \times 10^{-11} - 4 \times 10^{-8}$ eV mass ($2 \text{kHz} - 10 \text{MHz}$ frequency) range.

Introduction. Despite the preponderance of astrophysical evidence in support of the existence of dark matter, its essential nature has remained elusive. Although the dark matter candidates motivated by weak-scale supersymmetry, the WIMP [1], and by the explanation of why the strong interaction does not break P and CP symmetries, the axion [2–5], remain well-motivated, it has become apparent that yet broader possibilities exist and can act as alternative solutions to the dark matter problem [6]. In particular, a particle dark matter candidate could be much heavier or much lighter in mass than the $O(100 \text{keV})$ (“visible”) axion and the MeV-100 TeV WIMP mass range suggested by theory, and in which many searches have been made. The “visible” axion is regarded as excluded [7], although the existing experimental constraints can be evaded [8], and both heavier and lighter axions are also possible and are the targets of experimental searches [7]. It is the purpose of this article to explore the possibility of sub-eV dark matter more broadly and to show that this larger set of possibilities can be probed with existing technology.

The possibility of sub-keV mass particle dark matter candidates was first explored in the context of axion cosmology [9–11], in which it was realized, although thermally cold axions could be readily produced in the early Universe through the QCD phase transition, that if they were too light in mass, say less than some 10 $\mu$eV, too much dark matter would be produced, making the scenario incompatible with the Universe as it is observed. Although the $O(10 \mu\text{eV})$ mass range is a highly motivated search window [12], still broader mass ranges become possible in particular cosmological histories with inflation [13]. For example, the axion is associated with the spontaneous breaking of Peccei-Quinn symmetry [2–3], and the relationship between its mass and its coupling to light is essentially fixed once the energy scale $f_a$ at which the symmetry is broken is known, where we note the KSVZ [14, 15] and DFSZ [16, 17] “invisible” axion models. If $f_a$ exceeds the energy scale of inflation, then the “extra” matter can be inflated away and no longer contribute to the mass of the Universe as we observe it. Thus the viability of sub-10 $\mu$eV dark matter is tied to the cosmological history of our Universe [13], opening many more possibilities. Indeed, if the axion’s mass and coupling to light are regarded as independent parameters, making the particle, rather, axion-like, then the possible parameter space becomes enormously broader [18, 19]. In what follows we refer to particle dark matter with sub-eV mass as ultralight.

The properties of ultralight dark matter are qualitatively different from those of WIMP-like candidates, because their associated number densities are grossly different. We recall that cosmological simulations of the evolution of Milky-Way-like galaxies, as well as astrometric observations of the Milky Way itself, reveal the local dark matter density to be roughly $0.3 \text{GeV/cm}^3$ [20], so that the number density of sub-eV dark matter candidates is enormous, making their behavior wave-like and essentially quantum mechanical in nature. The ubiquity of the Pauli Principle implies that such dark matter must be bosonic in nature [21], because the Pauli repulsion between such ultralight fermions would make them no longer bound to the galaxy in which they reside, in conflict with the observed galactic rotation curves. Moreover, its minimal possible particle mass is about $10^{-22}$ eV [20], as yet lighter mass dark matter would not “fit” into dwarf galaxies [22]. Thus far axion-like and hidden-photon dark matter candidates have been considered, in favor of yet higher spin candidates, and the production mechanism need not be tied to the QCD phase transition. Rather, a misalignment mechanism [18, 19, 23, 24] is possible. After inflation, the light field can take a random nonzero value in a casually connected region of the Universe and oscillations in that field can be interpreted as particles [23]. We emphasize that these cosmological scenarios in no way select...
Thus in the spin-zero sector, to account for the 

densities associated with ordinary electromagnetism. 

where \( \rho \) becomes the 

tion term between the scalar and electromagnetic field 

teraction between the scalar and electromagnetic field 

\[ L_{0^-} = -\frac{1}{4\mu_0} g_\phi a F_{\mu\nu} \tilde{F}^{\mu\nu} \rightarrow L_{0^+} = \frac{1}{4\mu_0} g_\phi \phi F_{\mu\nu} F^{\mu\nu} \]  

(1)

and 

\[ L_{1^-} = -\frac{\varepsilon_\phi}{2\mu_0} F_{\mu\nu} F^{\mu\nu} \rightarrow L_{1^+} = \frac{\varepsilon_\phi}{2\mu_0} F_{\mu\nu} \tilde{F}^{\mu\nu}. \]  

(2)

Thus in the spin-zero sector, to account for the 

different intrinsic parity, a electromagnetic field tensor, \( F^{\mu\nu} \), 

needs to replace the dual tensor, \( \tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta}/2 \), in 

the axion-electromagnetic interaction, with an analogous 

replacement in the spin-one case. Finally the interaction 

term between the scalar and electromagnetic field 

becomes 

\[ L_{\text{int}} = -\frac{1}{4\mu_0} g_\phi \phi(x) F_{\mu\nu} F^{\mu\nu} = \frac{g_\phi}{2\mu_0} \phi(x)(E^2 - B^2). \]  

(3)

Here, \( \phi(x) \) is the scalar field, \( g_\phi \) is the scalar-photon coupling 

constant, and \( E \) and \( B \) are the electric and magnetic 

fields, respectively.

The coupling of the scalar field to electromagnetism 

implies that the inhomogeneous Maxwell equations are 

modified through \( O(g) \) as follows:

\[ \nabla \cdot \vec{E} = g_\phi \nabla \phi \cdot \vec{E} + \frac{\rho_e}{\varepsilon_o}; \]  

(4)

\[ \nabla \times \vec{B} - \frac{1}{c^2} \frac{\partial \vec{E}}{\partial t} = g_\phi \left( \nabla \phi \times \vec{B} - \frac{1}{c^2} \vec{E} \frac{\partial \phi}{\partial t} \right) + \mu_0 \vec{j}_e, \]  

(5)

where \( \rho_e \) and \( \vec{j}_e \) are the electric charge and current 

densities associated with ordinary electromagnetism.

Just as for the axion field [25], we assume the potential 

of the scalar field to be of simple harmonic form, so that 

\[ U_\phi = m_\phi^2 \phi^2/2. \]  

This potential and the interaction term imply the wave equation:

\[ \square \phi = \frac{2g_\phi}{\mu_0} (E^2 - B^2) - m_\phi^2 \phi. \]  

(6)

If \( \vec{E} \) and \( \vec{B} \) are static, then the scalar field oscillates with 

angular frequency:

\[ \omega = \frac{m_\phi c^2}{\hbar} \left( 1 + \frac{1}{2} \varepsilon^2 \right), \]  

(7)

where \( \varepsilon \) is the scalar field velocity, in the laboratory rest 

frame.

Existing Constraints. The possibility of ultralight 

calar dark matter can be probed in a similar man-

to that of a weakly coupled axion [25], as through 

tests of electrodynamics [25]. Particular constraints de-

rive from searches for (i) the possibility of “light shining 

through a wall” (LSW) [25, 28, 29], (ii) the appearance 

of new, macroscopic interactions [30, 31], or (iii) the 

lack of anomalous energy loss from stars, stellar remnants, 

or supernovae [22]. We note that the limits on new, macro-

scopic forces [33] directly limit scalar-fermion couplings 

only [30]. LSW experiments can be conducted in either 

broadband [34, 35] or resonant versions [36]. For sub-

eV scalar dark matter, the most severe limits on scalar-

photon couplings come from the LSW experiment of Bal-

lou et al. [35] and, for \( m_\phi \lesssim 10^{-15} \) eV, from searches for 

the direct impact of such relic dark matter on atomic 

spectroscopy [37, 38] and atomic clocks [39].

Direct Detection via an LC circuit. As Eq. (4) 

shows, a substantial difference between the dynamics of the scalar 

and pseudoscalar cases is that the time-dependent 

portion of the dark matter-generated current depends on 

an electric field instead of a magnetic field. Thus detection 

methods developed for axion-like dark matter can be 

adapted to the scalar case, where we note the LSW experiment of Ref. [35] as an existing example. The possibility of 

powerful empirical constraints on the scalar-photon coupling \( g_\phi \) arise if the scalar-generated magnetic field is 

amplified using a resonant LC circuit and detected with a 

SQUID magnetometer. Consequently, this leads to a 

natural adaptation of the proposal of Ref. [10]; namely, the use of a large static electric field, rather than a large 

magnetic field, to produce a scalar-generated magnetic field, with a similar expected enhancement in a supercon-

ducting LC circuit. Yet further experiments for ultralight 

axion or hidden photon dark matter are under develop-

ment [27, 41, 42] and could potentially be reframed to 

consider the even intrinsic parity dark matter candidates 

of interest to us here.

If the spatial extent of a static electric field is less than 

\( \sim m_\phi^{-1} \) and the spatial gradient in \( \phi \) is assumed neg-

ligible, then \( \partial \vec{E} / \partial t = 0 \) and \( \nabla \phi \times \vec{B} \approx 0 \) in Eq. (5). 

Therefore, in the presence of a static, uniform electric
field, $\vec{E}_\text{DM}$, Eq. (5), becomes:

$$\vec{\nabla} \times \vec{B}_\phi = -\frac{g_\phi}{c^2} \vec{E}_\omega \frac{\partial \phi}{\partial t} = \vec{j}_\phi,$$

(8)

where $j_\phi$ is the dark matter produced electric current. As in Ref. [40], pick-up loops connected to an LC circuit can amplify this magnetic field when the resonant frequency of the circuit, $\omega/2\pi = 1/\sqrt{LC}$, is near $m_\phi c^2/h$. Thereby, both the coupling constant, $g_\phi$, and the particle mass, $m_\phi$, can be detected. However, as the schematic drawing of the proposed experimental set-up in Fig. 1 shows, the superconducting pick-up loops are placed outside the electric field. This prevents voltage breakdown due to the pick-up loops. Additionally, multiple loops may be placed around a circular electric field in order to increase the total magnetic flux through the LC circuit. When the resonant frequency of the LC circuit equals that of the scalar mass, since the wire is superconducting, the current caused by $\vec{B}_\phi$:

$$I = -Q \Phi_\phi/L,$$

(9)

where $Q$ is the quality factor of the circuit, $\Phi_\phi$ is the magnetic flux of $\vec{B}_\phi$ through the pickup loops, and $L$ is the inductance of the circuit in its environment. This leads to a magnetic field detected by the SQUID magnetometer:

$$B_d \approx \mu_0 \frac{N_d}{2r_d} I = -\mu_0 \frac{Q N_d}{2L r_d} \Phi_\phi,$$

(10)

where $N_d$ is the number of turns and $r_d$ is the radius of the inductor facing the magnetometer. If the radius of the electrodes is $r_e$ with a separation distance $d$, then using Eq. (9), and Stokes’ Theorem, the scalar-generated magnetic field outside the electric field is:

$$\vec{B}_\phi = \frac{g_\phi}{2c^2} E_\omega \frac{\partial \phi}{\partial t} r_e^2 \hat{r},$$

(11)

where $(z, r, \varphi)$ are cylindrical coordinates centered at the middle of the electrodes, with $\varphi > 0$ in the counterclockwise direction. If the pick-up loops have length $l$ and are of the same height as the electrode separation distance, then the magnetic flux through a single loop is:

$$\Phi_\phi = -\frac{1}{2c^2} g_\phi E_\omega \frac{\partial \phi}{\partial t} r_e^2 d \ln (1 + l/r_e).$$

(12)

For multiple loops, this quantity would be multiplied by the number of loops, $N_\ell$.

If the scalar field is real, as is considered for the axion field in Ref. [43], then the dark matter energy density is related to the time derivative of the scalar through:

$$\rho_{DM} = \frac{1}{h c^3} \left( \frac{\partial \phi}{\partial t} \right)^2.$$

(13)

Combining Eqs. (10), (12), and (13), the detected magnetic field would be:

$$B_d \approx \mu_0 \sqrt{\frac{\hbar}{c}} Q N_d \frac{\ell}{4L r_d} g_\phi E_\omega \sqrt{\rho_{DM} r_e^2} d \ln (1 + l/r_e)$$

$$= 1.47 \times 10^{-20} T \left( \frac{Q}{10^4} \right) N_d N_\ell \left( \frac{\mu H}{L} \right) \left( \frac{\text{cm}}{r_d} \right) \left( \frac{\text{r}_e^2 d}{\text{cm}^3} \right)$$

$$\times \left( \frac{g_\phi}{10^{12} \text{GeV}^{-1}} \right) \left( \frac{E_\omega}{10 \text{ kV/cm}} \right) \sqrt{\frac{\rho_{DM}}{\text{GeV/cm}^3}} \ln (1 + l/r_e).$$

(14)

Sensitivity Estimates. The sensitivity for the experiment is determined by the signal to noise ratio $S/N$ for the current, where we note Ref. [43] for a comparison of this criterion with a likelihood analysis, as well as by the sensitivity of the SQUID magnetometer. Following Ref. [40], we assume the loci of parameters $g_\phi$ and $m_\phi$ that would generate a $S/N > 5$ would be detectable and thus could be excluded. The signal due to the induced current is:

$$I = \sqrt{\frac{\hbar}{c}} Q N_\ell g_\phi E_\omega \sqrt{\rho_{DM} r_e^2} d \ln (1 + l/r_e)$$

$$= 2.34 \times 10^{-16} A \left( \frac{Q}{10^4} \right) \left( \frac{\mu H}{L} \right) N_\ell \left( \frac{g_\phi}{10^{12} \text{GeV}^{-1}} \right)$$

$$\times \left( \frac{E_\omega}{10 \text{ kV/cm}} \right) \sqrt{\frac{\rho_{DM}}{\text{GeV/cm}^3}} \left( \frac{\text{r}_e^2 d}{\text{cm}^3} \right) \ln (1 + l/r_e).$$

(15)

The expected main sources of noise in its detection are the Johnson-Nyquist thermal noise $\delta I_T$ at temperature $T$ with circuit noise $\Delta \nu$ and the noise $\delta I_B$ associated with that in the magnetometer $\delta B$, namely [40],

$$\delta I_T = \sqrt{\frac{4 k_B T Q \Delta \nu}{L \omega}} = 2.96 \times 10^{-13} \text{ A}$$

$$\times \sqrt{\left( \frac{\text{MHz}}{\nu} \right) \left( \frac{Q}{10^4} \right) \left( \frac{\mu H}{L} \right) \left( \frac{T}{\text{mK}} \right) \left( \Delta \nu \right)},$$

(16)
\[ \delta I_B = \frac{2r_d}{N_d} \delta B = 5.03 \times 10^{-14} \Lambda \]
\[ \times \frac{1}{N_d} \left( \frac{r_d}{\text{cm}} \right) \left( \frac{B_n}{10^{-16} \text{T}} \right) \sqrt{\left( \frac{\Delta \nu}{\text{mHz}} \right)}, \quad (17) \]

where \( \delta B = B_n \sqrt{\Delta \nu / \text{Hz}} \) and \( B_n \approx 10^{-16} \text{T} \), with the former noise source being numerically larger. We note that additional RF noise was noticed in its experimental realization \[44\]. In the current case, additional, possible sources of noise or false signals come from non-uniformity or drift in the electric field. If the field is not uniform, the signal could be weakened since the scalar-generated magnetic field might not be perfectly symmetric around the electrodes under these circumstances. Also, if the electric field drifts, due to the \( \partial \vec{E} / \partial t \) in Eq. (5), it could potentially result in a magnetic field comparable to that of the dark matter signal. Finally, if \( \partial \vec{E} / \partial t \) is greater than \( g_\phi E_o \partial \phi / \partial t \), then the signal from the dark matter can be drowned out. Spatial and/or temporal variations in the ambient magnetic field could also prove to be an important source of systematic error, though the experiment should probably be conducted in a magnetically shielded environment. Nevertheless, for concrete comparison with the sensitivity of the axion dark-matter experiment proposed in Ref. [40], we compute

\[ S/N = \frac{I}{\sqrt{(\delta I_f)^2 + (\delta I_B)^2}}, \quad (18) \]

noting, too, that for a superconducting circuit \( L = N_l L_m + L_c + L_d \), with the inductance coming for a pickup coil, the coaxial cable, and the inductor facing the SQUID magnetometer, respectively. Here \( L_d = N_d^2 \xi \) with \( N_d \) coils and \( \xi = \mu_0 r_d \ln(8r_d / a_d) \) \[45\]. We eschew the optimization procedure of Ref. [40] for \( N_d \) because \( S/N \) depends on a plurality of inputs and \( S/N \) varies slowly with \( N_d \). We emphasize that the sensitivity of the experiment can be limited by the sensitivity of the magnetometer. If the magnetic field generated by \( \phi \) is so small that the magnetometer cannot detect it, then even if the signal to noise ratio is still large enough the signal will not be detected. In the few hundred kHz frequency regime we study here, magnetometer sensitivities of \( 10^{-16} \text{T} \) have been established \[46\], with increases of sensitivities possible from the coherence of the dark matter field and its integration time. Under the assumption of the standard dark matter halo model \[47\], the energy density is \( \rho_{DM} \approx 0.3 \text{ GeV/cm}^3 \), and we suppose the coherence time is \( t_c \approx 0.16 s (\text{MHz}/\nu) \) \[48\]. The magnetometer can detect a field: \( B_d = 10^{-16} \text{T}(\text{Hz})^{-1/2}(t_e)^{-1/4} \) with \( t \) being the integration time. If the integration time is \( t = 10^8 s \), then \( B_d \approx 2.8 \times 10^{-17} \text{T}(\nu/\text{MHz})^{1/4} \) \[40\]. We show the coupling associated with the smallest detectable magnetic field in Fig. 2, illustrating, for larger values of \( m_\phi \), that this factor can limit the sensitivity of the proposed experiment. If we were to employ the caustic ring halo model \[47\] instead, the coherence time would be longer, making the detection of still smaller \( B_d \) possible. For this article, though, we focus on the standard halo model.

LSW experiments from the OSQAR collaboration have constrained the coupling constant \( g < 3.2 \times 10^{-8} \text{ GeV}^{-1} \) \[35\] for \( m_\phi < 200 \mu \text{eV} \). The sensitivity of the proposed experiment in comparison to that of Ref. [40] is determined by the magnitude of the induced currents. Figure 2 shows that there is approximately a 4 order of magnitude difference between the different proposals. We assume that \( Q = 10^4 \) and \( T = 0.5 \text{ mK} \) and choose \( N_l = N_d = 1 \). Noting Ref. [40] we suppose the circuit would have a thermal noise of \( \Delta \nu = 4 \text{ mHz} \), \( L_m = 2.5 \mu \text{H} \), \( L_c = 0.5 \mu \text{H} \), \( r_d = 1 \text{ cm} \), \( a_m = 7.4 \times 10^{-4} \text{ m} \), and \( l = 15 \text{ cm} \). The dimensions of the electric field are \( r_e = 30 \text{ cm} \), \( d = 10 \text{ cm} \), and \( E_o = 75 \text{ kV/cm} \), which are based on the dimensions from the 1/5-scale test studies in Ref. [48] scaled-up to the dimensions of the neutron EDM experiment under development at Oak Ridge [49]. To realize an electric field of that strength, the experiment would need to be performed in liquid \(^4\text{He} \); the largest electric field established in vacuum is \( 30 \text{ kV/cm} \) \[43, 50\]. Our estimated limits are shown in Fig. 2 for the frequency window studied in Ref. [40]. Comparing the sensitivity of the limits proposed by Ref. [40] to those found by Ref. [44], executed at 4.2 K, and supposing a similar loss of sensitivity in the current case, we see that values of the couplings smaller than the OSQAR limits can still be probed by this proposed set-up. We emphasize that the electric field studies for the Oak Ridge experiment are performed at a temper-
ature of 300 mK; at that temperature the proposed LC circuit experiment would lose about a factor of 20 in sensitivity to the scalar-photon coupling constant. We suppose further improvements to our proposal could be made in differing ways, such as, e.g., by employing a slowly varying electric field to search for a beat frequency from the dark matter signal, or using more loops $N_f$. We can also imagine using an array of SQUID or atomic magnetometers, as employed in neutron EDM experiments, to make magnetic field measurements to set limits on ultralight scalar dark matter directly, where we note that Oak Ridge experiment plans to use exceptionally large electric fields [49]. For reference, we note an analogous study of the axion-gluon coupling from the PSI neutron EDM experiment [51]. Finally, we note the development and demonstration of a quantum-enhanced sensor of mechanical displacement and weak electric fields in a trapped ion crystal within the 10 kHz $- 10 \text{MHz}$ frequency range [52]; such a scheme could potentially be sensitive to the scalar and axial vector dark matter candidates we have noted here, as well as to the hidden photon and axion candidates they consider.

Summary. We have explored the modification of electrodynamics in the presence of ultralight dark matter of even intrinsic parity to show how, in the context of a resonant, superconducting LC circuit with a large, static electric field, this can yield a new and sensitive direct probe of its existence in the sub-$\mu$eV regime. In so doing we have exploited the large electric fields under development for neutron EDM experiments, making dark matter probes of greatly enhanced sensitivity possible within the scope of existing technology.

Acknowledgements. We acknowledge partial support from the U.S. National Science Foundation under Award Number PHY-1950795 (UK REU) and the U.S. Department of Energy under contracts DE-FG02-96ER40989 and DE-SC001462. We thank P. Sikivie for clarifying correspondence regarding Ref. [40].
