1. Introduction

Cellular automata (CA) were initiated in the early 1950s to develop complex structures of capable self-reproduction and self-repair. Since then, many researchers have taken interest in the study of CA. Initially, the CA concept was first introduced by von Neumann (1966) for the proposal of modeling biological self-reproduction. His primary interest was to derive a computationally universal cellular space with self-reproduction configurations. Afterward, a new phase of activities was started by Wolfram (1983; 1984), who pioneered the investigation of CA as a mathematical model for self-organizing statistical systems. Wolfram was proved that the randomness of the patterns generated by maximum-length CA is significantly better than other widely used methods, such as linear feedback shift registers. The intensive interest in this field can be attributed to the phenomenal growth of the VLSI technology that permits cost-effective realization of the simple structure of local-neighborhood CA (Wolfram, 1986).

The PRNGs based on CA have been studied for the last decade and a variety of CA PRNGs have been proposed (Guan & Tan, 2004; Guan & Zhang, 2003; Seredynski et al., 2004; Tan & Guan, 2007; Xuelong et al., Oct. 2005). Recent interest has been focused on the two-dimensional (2-D) CA PRNGs with genetic algorithm (Chowdhury et al., 1993; Guan et al., 2004; Quieta & Guan, 2005; Tomassini et al., 2000), since, statistically, the point has been established that the quality of randomness of a 2-D CA is significantly better than a 1-D CA.

In this paper, an efficient PRNG based on hybrid between one-dimension (1-D) and two-dimension (2-D) CA is proposed. In the phase of the evolution of 2-D CA cells, the proposed CA PRNG is based on von Neumann neighborhood, in that this method refers to the five cells and new control values (rule decision input value & linear control input value) to decide a rule. In the phase of the evolution of 1-D CA cells, on the other hand, <90, 150> rule combination is used because this rule combination is better than the others (Hortensius et al., 1989). In the meantime, the proposed CA PRNG is compared with previous works (Guan et al., 2004; Tomassini et al., 2000) to check the quality of randomness. The proposed CA PRNG could generate a good quality of randomness because the proposed CA PRNG is better than
Table 1. The examples of the state transition for rules with 2-state, 3-neighborhood

| Neighborhood state | 111 110 101 100 011 010 001 000 | rule number |
|--------------------|----------------------------------|-------------|
| Next state         | 0 1 0 1 1 0 1 0                 | 90          |
| Next state         | 0 0 0 1 1 1 1 0                 | 30          |
| Next state         | 1 0 0 1 0 1 1 0                 | 150         |
| Next state         | 1 1 0 0 1 0 1 0                 | 202         |

Table 2. The combination logic gates correspond with rule numbers

| Rule number | Combination Logic gate |
|-------------|------------------------|
| 30 (= 2⁴ + 2³ + 2² + 2¹) | \(s_{i-1}(t) \oplus (s_i(t) \lor s_{i-1}(t))\) |
| 90 (= 2⁶ + 2⁴ + 2³ + 2¹) | \(s_{i-1}(t) \oplus s_{i+1}(t)\) |
| 150 (= 2⁷ + 2⁴ + 2² + 2¹) | \(s_{i-1}(t) \oplus s_i(t) \oplus s_{i+1}(t)\) |
| 202 (= 2⁷ + 2⁶ + 2³ + 2¹) | \(s_{i-1}(t) \land (s_i(t) \oplus s_{i+1}(t))\) |

where \(\oplus\), \(\lor\) and \(\land\) are the Boolean operations XOR, OR and AND, respectively.

2. The related works

In this section, the concepts of cellular automata will be introduced.

2.1 One-dimension CA

A CA is a dynamical system in which space and time are discrete. A CA consists of an array of cells, each of which can be in one of a finite number of possible states, updated synchronously in discrete time steps, according to a local identical interaction, a rule. The next state of a cell is assumed to depend on itself and on its neighbors. The cells evolve in discrete time steps according to some deterministic rule that depends only on local neighbors.

The next-state function for a three-neighborhood CA cell in one-dimension can be expressed as following equation (1):

\[ s_i(t + 1) = f(s_{i-1}(t), s_i(t), s_{i+1}(t)) \]  

where \(f\) denotes the local function realized with a combinational logic, such as AND, OR, XOR, and NOT, \(i\) is the position of an individual in the one-dimensional array of the cell, \(t\) is the \(t\)th time step, \(s_i(t)\) is the output state of the \(i\)th cell at the \(t\)th time step, and \(s_i(t + 1)\) is the output state of the \(i\)th cell at the \((t + 1)\)th time step.

In a two-state, three-neighborhood CA there can be a total of \(2^3\) (from 000 to 111) distinct neighborhood configurations. For such a CA there can be a total of \(2^{23}(= 256)\) distinct mappings from all these neighborhood configurations to the next state. Each mapping is called a rule of CA. If the next-state function of a cell is expressed, then the decimal equivalent of the output is conventionally called the rule number for the cell Wolfram (1986).
Table 1 expresses the state transition for arbitrary rules with 2-state, 3-neighborhood. In Table 1, the top row gives all eight possible states of the three neighboring cells (the left, right neighbor of the ith cell, the ith cell itself) at the time \( t \). The second to fifth rows give the corresponding states of the ith cell at time \( t + 1 \) for four CA rules. Table 2 shows that the combination logic gates correspond with rule numbers. In Table 2, rule numbers consist of sum which is converted decimal number of neighborhood states from 111 (= MSB) to 000 (= LSB). For rule 30, the next state of 111 (= \( 2^7 \)), 110 (= \( 2^6 \)), 101 (= \( 2^5 \)), 100 (= \( 2^4 \)), 011 (= \( 2^3 \)), 010 (= \( 2^2 \)), 001 (= \( 2^1 \)) and 000 (= \( 2^0 \)) are 0, 0, 0, 1, 1, 1 and 0, respectively. Hence, \( 2^7 \times 0 + 2^6 \times 0 + 2^5 \times 0 + 2^4 \times 0 + 2^3 \times 1 + 2^2 \times 1 + 2^1 \times 1 + 2^0 \times 0 = 30 \).

A CA is said to be a Null Boundary CA (NBCA) if the left (or right) neighbor of the leftmost (or rightmost) terminal cell is connected to the logic 0-state, or a Periodic Boundary CA (PBCA) if the extreme cells are adjacent to each other. Figure 2 shows the features of each boundary CA.

Fig. 2. The features of each boundary CA: (a) Null boundary CA, (b) Periodic boundary CA

### 2.2 Two-dimension CA

A two-dimension (2-D) CA is a generalization of a 1-D CA, where the cells are arranged in a two-dimensional grid with connections among the neighboring cells. For a 2-D CA, types of cellular neighborhoods are usually considered: five cells, consisting of one cell along with its four immediate non-diagonal neighbors (also known as the von Neumann neighborhood); and nine cells, consisting of one cell along with its eight surrounding neighbors (also known
as the Moore neighborhood). In this paper, a type of von Neumann neighborhood is used which considers five-neighborhoods that these consist of self, top, bottom, left and right.

![Fig. 3. The geometric representation of the von Neumann neighborhood.](image)

Figure 3 shows the type of von Neumann neighborhood with a five-neighborhood dependency.

The next state $s_{i,j}(t+1)$ of a 2-D CA is given by

$$s_{i,j}(t+1) = f(s_{i-1,j}(t), s_{i,j-1}(t), s_{i,j}(t), s_{i,j+1}(t), s_{i+1,j}(t)).$$ (2)

Since $f$ is a Boolean function of five variables, there are $2^5 = 32$ distinct neighborhood configurations. Hence to express a transition rule of a 2-D CA in a manner similar to a 1-D CA, 32-bits are considered which is almost impossible to manage practically.

2.3 The previous works

In this section, previous CA PRNGs are described. Tomassini et al. have been proposed 2-D CA PRNG based on an $8 \times 8$ structure Tomassini et al. (2000) and Guan et al. have proposed 2-D CA PRNGs based on asymmetric neighborhood Guan et al. (2004). In the final phase, these PRNGs are constructed by using the evolutional method which is called a genetic algorithm Steeb (2001). A genetic algorithm is an iterative procedure that involves a constant-size population of individuals, each one represented by a finite string of symbols (known as the genome), encoding a possible solution in a given problem space. A genetic algorithm has operators that are a crossover, and mutation and a has fitness function. Iterating this procedure, the genetic algorithm may eventually find an acceptable solution. Hence, a genetic algorithm provides a high randomness quality, but has problems that include high time complexity for the evolution of CA cells.

In Tomassini et al.’s PRNG Tomassini et al. (2000), every cell of the evolving CA assigns a fitness function value and rule number which decides the result of the evolved cell by the fitness function value. Also, to raise the quality of randomness, they used a genetic algorithm. Guan et al.’s PRNGs Guan et al. (2004) are constructed from a asymmetric neighborhood ($5 \times 10$) CA structure for reducing the number of cells. Similarly, they used a genetic algorithm. To evaluate the randomness quality of the proposed CA PRNG, in this paper, the sequence of the proposed PRNG (PRNS) will be compared with above two CA PRNS.
3. The proposed hybrid CA PRNG

In this section, the proposed scheme is explained.

3.1 The method of deciding the rules for 1-D & 2-D

First of all, the rule of CA cell for 2-D are considered. A hybrid CA PRNG based on the von Neumann neighborhood method is proposed. The number of rules in this method is $2^{32}$ because there are 32 distinct neighborhood configurations. In this paper, the XOR logic is considered because the frequencies of occurrence of 0 for the operation of XOR is 2, in four cases given in the Table 2. From this fact, it can be guessed that the binomial distribution for the probability of XOR is

$$X \sim B(n, \frac{1}{2})$$

where $n$ is independent Bernoulli trials. That is, the expected value of the binomial distribution of the frequencies of occurrence of 0 for a XOR is decided by $n$.

Let $s_{ij}(t)$ be a state value of the cell at row $i$, column $j$, and time $t$. Its state value at the next step, $s_{ij}(t+1)$ is then computed as the following equation (3):

$$s_{ij}(t+1) = X \oplus (C \land s_{ij}(t)) \oplus (N \land s_{i-1,j}(t)) \oplus (W \land s_{i,j-1}(t)) \oplus (E \land s_{i,j+1}(t)) \oplus (S \land s_{i+1,j}(t))$$

where $\oplus$ and $\land$ are the Boolean operations XOR and AND, respectively. $X$, $C$ (center), $N$ (north), $S$ (south), $W$ (west), and $E$ (east) are binary variables (that is, 0 or 1). $C$, $N$, $S$, $W$, and $E$ denote whether the respective neighboring cell state is taken into account (a value of 1) or not (a value of 0). The binary variable $X$ distinguishes between linear ($X = 0$) and nonlinear ($X = 1$) rules. For example, rule 14 (001110), that is, $(X||C||N||W||E||S)$ is represented as follows:

$$s_{ij}(t+1) = s_{i-1,j}(t) \oplus s_{i,j-1}(t) \oplus s_{i,j+1}(t)$$

Next, the rule of CA cell for 1-D with 2-state, 3-neighborhood is considered. A rule is chosen which is $<90, 150>$ in this paper. The logic of this rules consists of XOR which has $X \sim B(n, \frac{1}{2})$. Moreover, the rule combination of $<90, 150>$ has been proved to have a high randomness quality by Hortensius et al.’s paper Hortensius et al. (1989). Figure 4 shows the structure of rule $<90, 150>$ for a cell in 1-D CA.

The Boundary is considered. Generally, a PBCA is frequently applied, resulting in a circular grid for 1-D case and in a doughnut for the 2-D case. A NBCA can also be used, in which the grid is surrounded by an outer layer of cells of zero. In this paper, a PBCA is chosen to raise the quality of randomness.

3.2 Hybrid between 1-D and 2-D CA PRNG

Figure 4, 5 and 6 show the process & structure of the proposed CA PRNG, respectively. One round consists of four phases. The process step-by-step is summarized as follows.

| Cells | XOR logic |
|-------|-----------|
| 0     | 0         |
| 0     | 1         |
| 1     | 0         |
| 1     | 1         |

Table 3. The result of operation of XOR logic for two cells
Fig. 4. The structure of rule \(90, 150\)

Fig. 5. The process of the proposed CA PRNG

The start of processing

Initialization

Updating CA cells at 2-D

Transform 2-D into 1-D

Updating CA cells at 1-D

Transform 1-D into 2-D

If \(i > R\) then Yes

\[ G = G^{(1)} || G^{(2)} || \ldots || G^{(R)} \]

The start of processing

Fig. 5. The process of the proposed CA PRNG
3.2.1 Initialization

The 192-bit initial seed is randomly generated. The seed, then, divides into three parts, from 0th to 63th (=IV1), from 64th to 127th (=IV2) and from 128th to final bit (=IV3). The initial values IV1, IV2, and IV3 are stored in GS (GS: Global state, consists of the 8×8 2-D CA cells and the first row is from 0th to 7th, the second row is from 8th to 14th, ..., the final row is from 57th to 63th), RDC (rule decision control unit, consists of 64-bit 1-D CA cells) and LC (linear control unit, consists of 8×8 2-D CA cells and the first row is from 0th to 7th, the second row is from 8th to 14th, ..., the final row is from 57th to 63th), respectively.

3.2.2 Updating at 2-D

The rule \( r_{i,j}(t) \) is decided by \( RDC(t) \) which is decided by self, left 2 bits, and right 2 bits (that is, \( s_{i-2}(t), s_{i-1}(t), s_i(t), s_{i+1}(t), \) and \( s_{i+2}(t) \) at \( t \)-th time step). Then, generate the next state \( GS(t+1) \) by \( r_{i,j}, LC(t) \), and \( GS(t) \) and represents the equation (4):

\[
GS(t+1) = f(r_{i,j}, LC(t), GS(t)),
\]

where \( R \) is a repeating counter for producing PRNS of a demanded cycle length, \( 0 \leq t \leq R, \) \( 0 \leq i, j \leq 7.\)

3.2.3 Transform 2-D into 1-D

The generated state \( GS(t+1) \) transforms 2-D into 1-D because of applies to the 1-D CA rule <150,90>. The method is that each row in \( GS(t+1) \) concatenates 64-bits stream as in the following equation (concatenation is denoted “||”) (5):

\[
TGS(t+1) = \bigcup_{i=0,j=0}^{7} s_{i,j}(t+1) = s_{0,0}(t+1)||s_{0,1}(t+1)||...||s_{7,7}(t+1),
\]

where \( s_{i,j}(t+1) \) indicates each cell in \( GS(t+1) \) and \( TGS(t+1) \) (TGS: Transformed GS) indicates transform 2-D into 1-D in \( GS(t+1) \).
Table 4. The results of the operation of rule 90 & rule 150 for three cells in 1-D

|      | si−1(t) | si(t) | si+1(t) | rule 90 | rule 150 |
|------|---------|-------|---------|---------|---------|
| 0    | 0       | 0     | 0       | 0       | 0       |
| 0    | 0       | 1     | 1       | 1       | 1       |
| 0    | 1       | 0     | 0       | 1       | 1       |
| 0    | 1       | 1     | 1       | 1       | 0       |
| 1    | 0       | 0     | 1       | 1       | 1       |
| 1    | 0       | 1     | 0       | 0       | 0       |
| 1    | 1       | 0     | 1       | 0       | 0       |
| 1    | 1       | 1     | 0       | 1       | 1       |

### 3.2.4 Updating at 1-D

In order to raise the quality of randomness, the combination <90, 150> is used. This rule provides a high randomness quality Hortensius et al. (1989). This method uses the following equation (6):

\[
ATGS^{(t+1)} = f_{<90, 150>} (s_{i-1}(t+1), s_i(t+1), s_{i+1}(t+1)),
\]

(6)

where \( s_i(t+1) \) is cell at \( TGS^{(t+1)} \) in 1-D, \( ATGS^{(t+1)} \) is applied to the \( TGS^{(t+1)} \) by function \( f \) based on rule 90 and 150.

### 3.2.5 Transform 1-D into 2-D

The updated state transforms 1-D into 2-D because of the application of the 2-D rule. The 64-bit streams are divided into each row by modulo 8 as the following equation (7):

\[
GS^{(t+1)} = \bigcup_{i=0, j=0}^{7} s_{ij}(t+1),
\]

(7)

where \( s_{ij}(t+1) = s_i(t+1), i \) is that values of 0~7, 8~15, ..., 57~63 replace 0, 1, ..., 7, respectively. And \( j = i \mod 8, (0 \leq i \leq 63) \) in 1-D CA state. Then, \( RDC \) and \( LC \) replace the new values. The method is represented as the following equation.

\[
\begin{align*}
LC^{(t+1)} &= RDC^{(t)} \oplus LC^{(t)}, \\
RDC^{(t+1)} &= GS^{(t)} \oplus GS_{\text{updated at 1-D CA}}.
\end{align*}
\]

(8)

Lastly, \( t \) is increased by 1. If \( t \) is greater than \( R \), the phase is finished. Otherwise, go back to the beginning of the 2-D phase.

### 4. Analysis

In this section, the high quality of randomness of the hybrid CA PRNG is proved. The proposed CA PRNG was evolved by rule <90, 150> in 1-D. Abovementioned rule 90 & 150 consist of combinations of XOR logic, that is, \( s_{i-1}(t) \oplus s_{i+1}(t) \) & \( s_{i-1}(t) \oplus s_{i}(t) \oplus s_{i+1}(t) \), respectively. The frequencies of occurrence of 0 or 1 for operation of rule 90 or rule 150 is 4, in eight cases in Table 4. Table 4 shows the results of the operation of a rule 90 & a rule 150 for three cells which are \( s_{i-1}(t), s_i(t) \) and \( s_{i+1}(t) \). Hence, \( E(X) \) (expectation value) of the binomial distribution for a rule 90 or a rule 150 is \( \frac{n}{2} \), where \( n \) is the number of evolutions in...
Next, the rules of CA cell for 2-D are considered. The proposed CA PRNG used von Neumann neighborhood method and evolved by \( RDC(t) \) and \( LC(t) \) for next state. \( RDC(t) \) consists of five cells \((s_{i-1,j}(t), s_{i,j-1}(t), s_{i,j}(t), s_{i,j+1}(t) \text{ and } s_{i+1,j}(t))\) and combinations of XOR logic. Hence, the rule of 2-D CA cell consists of 1 bit for linear control bit, 5 bits for rule decision control and combinations of XOR logic, and expresses as equation (3). Table 5 shows the results of the rule operation of all case for six cells in 2-D. Meanwhile, a rule 0 and a rule 128 are not consider in the proposed CA PRNG because the occurrence rate of these rules are very few. Hence, \( E(X) \) (expectation value) of the binomial distribution for the rule of 2-D CA cell is \( \frac{n}{2} \), where \( n \) is the number of evolutions in 2-D. Similarly, the meaning of this proposition shows that provide a high quality randomness in terms of 1-D CA PRNG.

### Table 5. The results of the rule operation of all case for six cells in 2-D

| Test No. | Test Name                | The average value | Tomassini et al. | Guan et al. | Proposed |
|----------|--------------------------|-------------------|------------------|-------------|----------|
| 1        | Entropy (Close to 8.0)   | 7.99971           | 7.99983          | 7.99991     |
| 2        | Chi-square (Close to 1.0)| 0.989412          | 0.992002         | 0.999283    |
| 3        | SCC (Close to 0.0)       | 0.000227          | 0.000171         | 0.000062    |

### Table 6. The average values of ENT test

1-D. Therefore, the meaning of this proposition shows that rule 90 & rule 150 provide a high quality randomness in terms of 1-D CA PRNG.

Next, the rules of CA cell for 2-D are considered. The proposed CA PRNG used von Neumann neighborhood method and evolved by \( RDC(t) \) and \( LC(t) \) for next state. \( RDC(t) \) consists of five cells \((s_{i-1,j}(t), s_{i,j-1}(t), s_{i,j}(t), s_{i,j+1}(t) \text{ and } s_{i+1,j}(t))\) and combinations of XOR logic. Hence, the rule of 2-D CA cell consists of 1 bit for linear control bit, 5 bits for rule decision control and combinations of XOR logic, and expresses as equation (3). Table 5 shows the results of the rule operation of all case for six cells in 2-D. Meanwhile, a rule 0 and a rule 128 are not consider in the proposed CA PRNG because the occurrence rate of these rules are very few. Hence, \( E(X) \) (expectation value) of the binomial distribution for the rule of 2-D CA cell is \( \frac{n}{2} \), where \( n \) is the number of evolutions in 2-D. Similarly, the meaning of this proposition shows that provide a high quality randomness in terms of 1-D CA PRNG.

## 5. Experimental results

In this section, the efficiency of the hybrid CA PRNG is analyzed. To analyze, ENT Walker (Oct., 1998) and DIEHARD Marsaglia (1998) test suites are used. ENT test is useful for evaluating pseudorandom number generators for encryption and statistical sampling applications, compression algorithms, and other applications where the information density of a file is of interest Walker (Oct., 1998). ENT test is a collective term for three tests which are the Entropy test, Chi-square test, and Serial correlation coefficient (SCC) test.

The DIEHARD test suite is important because it seems to be the most powerful and difficult test suite to pass. This test consists of 18 different and independent statistical tests. The result of each test is called \( p \)-value. Most of the tests return a \( p \)-value in DIEHARD, which should be uniform if the input file contains truly independent random bits. For any given test, a smaller \( p \)-value means a better result.

The proposed CA PRNG produces a 64-bit output sequence at each round. The DIEHARD test suite requires a minimum of 10 MB of random number sequences Marsaglia (1998).
Table 7. The results of DIEHARD test in p-value pass rate ≥ 90% for PBCA

| Test No. | Test Name      | The results of tests (Pass or Fail) |
|----------|----------------|--------------------------------------|
| 1        | Birthday Spacing | Pass       | Pass       | Pass       | Pass       |
| 2        | Over 5-Per.     | Pass       | Fail       | Fail       | Pass       |
| 3        | Binary Rank 31×31 | Pass       | Pass       | Pass       | Pass       |
| 4        | Binary Rank 32×32 | Fail       | Pass       | Pass       | Pass       |
| 5        | Binary Rank 6×8  | Pass       | Pass       | Pass       | Pass       |
| 6        | Bitstream       | Pass       | Pass       | Pass       | Pass       |
| 7        | OPSSO           | Pass       | Fail       | Fail       | Pass       |
| 8        | OQSSO           | Pass       | Fail       | Fail       | Pass       |
| 9        | DNA             | Pass       | Pass       | Pass       | Pass       |
| 10       | Count-The-1’s 01 | Fail       | Fail       | Pass       | Pass       |
| 11       | Count-The-1’s 02 | Fail       | Pass       | Fail       | Pass       |
| 12       | Parking Lot     | Pass       | Pass       | Pass       | Pass       |
| 13       | Minimum Distance | Pass       | Pass       | Pass       | Pass       |
| 14       | 3DS Spheres     | Pass       | Pass       | Pass       | Pass       |
| 15       | Squeeze         | Pass       | Pass       | Pass       | Pass       |
| 16       | Overlapping Sums | Pass       | Pass       | Pass       | Pass       |
| 17       | Runs            | Fail       | Fail       | Pass       | Pass       |
| 18       | Craps           | Pass       | Pass       | Pass       | Pass       |

Therefore, the proposed PRNG needs \((10^7 \times 8)/64\) time steps for the DIEHARD test. On the other hand, the ENT test suite requires fewer numbers, but the test is executed with the same 10 MB sequence for convenience and the next DIEHARD test. A total of 100 experiments are performed for the ENT and DIEHARD test suite. Table 6 and 7 show the average values of ENT test and the results of DIEHARD test. A pass is considered when all the \(p\)-values are passed by more than 90% at the 0.05 level. As the results show in Tables, it is obvious that the quality of randomness of the proposed scheme is better than other different scheme. Additionally, we have performed the boundary CA tests for 1-D & 2-D, and the results of these tests show Table 8 & 9. In Table 8, PB and NB indicate PBCA and NBCA, respectively. The randomness quality of the rule 90 & 150 (\(= <90, 150>\)) are better than rule 30, 45. Also, The randomness quality of the PBCA is better than the NBCA. Similarly, The randomness quality of the PBCA is better than the NBCA in 2-D.

6. Conclusion

In this paper, an efficient PRNG was proposed based on the hybrid between 1-D and 2-D CA. To better randomness quality, the \(RDC\) and the \(LC\) units were used. As a result, the various types of von Neumann neighborhood and \(<90, 150>\) rule combination provide few correlation coefficients for each cell in state \(G_S^{(t)}\) and a high randomness quality by ENT and DIEHARD test suites (average pass rate more than 90% at the 0.05 level).

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| Test No. | Test Name                | The results of tests (Pass or Fail) | rule 30(PB) | rule 45(PB) | <90, 150>(PB) | <90, 150>(NB) |
|---------|-------------------------|-------------------------------------|-------------|-------------|---------------|---------------|
| 1       | Birthday Spacing        | Pass                                | Pass        | Pass        | Pass          | Pass          |
| 2       | Over. 5-Per.            | Pass                                | Pass        | Pass        | Pass          | Pass          |
| 3       | Binary Rank 31×31       | Pass                                | Pass        | Pass        | Pass          | Pass          |
| 4       | Binary Rank 32×32       | Pass                                | Pass        | Pass        | Pass          | Pass          |
| 5       | Binary Rank 6×8         | Pass                                | Pass        | Pass        | Pass          | Pass          |
| 6       | Bitstream               | Pass                                | Pass        | Pass        | Pass          | Pass          |
| 7       | OPPO                   | Fail                                | Fail        | Pass        | Fail          | Fail          |
| 8       | OQSO                   | Fail                                | Fail        | Pass        | Fail          | Fail          |
| 9       | DNA                    | Pass                                | Fail        | Pass        | Fail          | Fail          |
| 10      | Count-The-1’s 01       | Fail                                | Fail        | Pass        | Pass          | Pass          |
| 11      | Count-The-1’s 02       | Fail                                | Fail        | Pass        | Pass          | Pass          |
| 12      | Parking Lot            | Pass                                | Pass        | Pass        | Pass          | Pass          |
| 13      | Minimum Distance       | Pass                                | Pass        | Pass        | Pass          | Pass          |
| 14      | 3DS Spheres            | Pass                                | Fail        | Fail        | Fail          | Fail          |
| 15      | Squeeze                | Pass                                | Fail        | Pass        | Fail          | Fail          |
| 16      | Overlapping Sums       | Fail                                | Pass        | Pass        | Pass          | Pass          |
| 17      | Runs                   | Pass                                | Fail        | Pass        | Pass          | Pass          |
| 18      | Craps                  | Pass                                | Pass        | Pass        | Pass          | Pass          |

Table 8. The results of DIEHARD test in $p$-value pass rate $\geq 90\%$ for rule 90, 150, and $<90, 150>$ with PBCA(PB) & NBCA(NB) in 1-D

| Test No. | Test Name                | The results of tests (Pass or Fail) |
|---------|-------------------------|-------------------------------------|
| 1       | Birthday Spacing        | Pass                                |
| 2       | Over. 5-Per.            | Pass                                |
| 3       | Binary Rank 31×31       | Pass                                |
| 4       | Binary Rank 32×32       | Pass                                |
| 5       | Binary Rank 6×8         | Pass                                |
| 6       | Bitstream               | Pass                                |
| 7       | OPPO                   | Pass                                |
| 8       | OQSO                   | Pass                                |
| 9       | DNA                    | Pass                                |
| 10      | Count-The-1’s 01       | Pass                                |
| 11      | Count-The-1’s 02       | Pass                                |
| 12      | Parking Lot            | Pass                                |
| 13      | Minimum Distance       | Pass                                |
| 14      | 3DS Spheres            | Pass                                |
| 15      | Squeeze                | Pass                                |
| 16      | Overlapping Sums       | Pass                                |
| 17      | Runs                   | Pass                                |
| 18      | Craps                  | Pass                                |

Table 9. The results of DIEHARD test in $p$-value pass rate $\geq 90\%$ for uniform(U) and nonuniform(NU) 2-D CA with PBCA(PB) & NBCA(NB)
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