Effect of Relic Neutrino on Neutrino Pair Emission from Metastable Atoms

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ABSTRACT

A possiblity of measuring the cosmic neutrino temperature $\sim 1.9K$ and other important quantities such as the chemical potential $\mu$ and the de-coupling temperature $T_d$ is discussed, using the recently proposed process of photon irradiated neutrino pair emission from metastable atoms. The Pauli blocking effect of relic neutrinos reduces the rate by a large factor $\approx (1 + m_1/T_d)/4$ at the threshold of the lightest neutrino pair (of mass $2m_1$). Correction of linear order in $\mu$ near the mass thresholds can be used to improve the constraint on the lepton asymmetry.
Observation of the cosmic neutrino background is a direct indication of the hot early universe, three minutes after the big bang. One cannot overemphasize its importance. It would be very exciting to explore a possibility of measuring the relic cosmic neutrino expected to have the Fermi-Dirac distribution of nearly zero chemical potential and temperature \( \sim (4/11)^{1/3} T_0 \approx 1.9K \), with \( T_0 \sim 2.7K \) the cosmic microwave background temperature \([1]\).

In this work we study a possibility of using the recently proposed laser (or microwave, which is not discussed in the present work) irradiated neutrino pair emission from metastable atoms \([2]\) in order to indirectly detect the relic neutrino. It is an indirect detection because the presence of relic neutrinos is felt only by the Pauli blocking effect in this proposal. A great merit of atomic transition is obvious; closeness of the atomic energy level difference to neutrino masses. The energy difference is also made close to the cosmic neutrino temperature; \( k_B T \sim 0.166 \text{meV}(T_B/1.9K) \). We examine the pair emission rate affected by the Pauli blocking. In the rest of this paper we take the natural unit; \( \hbar = 1, c = 1, k_B = 1 \).

The photon irradiated neutrino pair (all 6 Majorana pair channels \( \nu_i \nu_j \) added) emission from a metastable atom proceeds as depicted in Figure 1. The intermediate atomic state \( |n\rangle \) is taken close in the energy to the initial metastable state \( |i\rangle \) — the mass of the neutrino pair (neutrino eigenmasses arranged by \( m_1 < m_2 < m_3 \)), thus \( E_i - E_n \approx m_i + m_j \), along with the laser tuning condition to the final excited state \( |f\rangle \), \( \omega \approx E_f - E_n \). Thus, all 6 thresholds corresponding to \( \nu_i \nu_j \) pair emission appear at laser energies, \( \omega = E_f - E_i + m_i + m_j \equiv \Delta f_i + m_i + m_j \) for different combinations of \( ij \). It is necessary to vacate the level \( |n\rangle \) in order to use the population in the final state \( |f\rangle \) lifted by laser as an experimental signature of the pair emission. It is also highly desirable for unambiguous detection of the weak process, \( \gamma + |i\rangle \rightarrow \nu_i + \nu_j + |f\rangle \) to measure a parity violating (PV) quantity such as
Figure 1: Atomic level structure and laser irradiated neutrino pair emission

the rate difference due to different circular polarization of laser.

To explore all six neutrino pair thresholds from one metastable atom we
need to have an atom of the level structure of the corresponding complexity. A strategy for determination of unknown neutrino parameters shall be
described elsewhere. For our purpose of the relic neutrino detection, observa-
tion of the pair emission including the lightest neutrino, \( m_1 + m_i \) \((i = 1, 2, 3)\)
near their mass thresholds is most important. Thus, effect of the neutrino
form factor as given by Fourier transformed atomic wavefunction overlap,
\( \langle n|e^{i(\vec{p}_1 + \vec{p}_2)\cdot \vec{x}}|i\rangle \), is not important, since the most significant region for the relic neutrino effect appears in the small momentum region, \(|\vec{p}| \ll \) the inverse of
atomic size. In the present work we shall also ignore PV effect, anticipating
that the PV quantity is comparable to parity-conserving quantity.

The rate via resonance is given by [2]

\[
\Gamma^M(\omega ; T_\nu) = \frac{4G^2_F F_0 \gamma_r}{\pi \omega^2 \Delta \omega \gamma} \sum_{ij} \theta(\omega - \Delta_{fi} - m_i - m_j) \times
\]

\[
|f\rangle
\]
\[
|n\rangle
\]
\[
|i\rangle
\]
\[
\nu_i
\]
\[
\nu_j
\]
\[
\int_{m_i}^{\omega_f - m_j} dE_1 I(E_1) (1 - f_i(E_1)) (1 - f_j(\omega_f - E_1)) \, . \quad (1)
\]

\[
I(E_1) = k_0^{ij} E_1 (\omega_f - m_j) \sqrt{(E_1^2 - m_j^2)(\omega_f - E_1)^2 - m_j^2}
+ k_{M}^{ij} \delta_{ij} m_i m_j \sqrt{(E_1^2 - m_i^2)(\omega_f - E_1)^2 - m_j^2} \,. \quad (2)
\]

Here \(F_0\) is the photon number flux of laser light of frequency resolution \(\Delta\omega\). Effect of the relic neutrino appears in the Pauli blocking factor of \((1 - f_i)(1 - f_j)\), where \(f_i\) is the Fermi-Dirac (FD) momentum distribution function for the neutrino \(\nu_i\). We assumed the vanishing chemical potential in this formula; the case of a finite chemical potential is discussed later.

The precise form of the distribution function after decoupling follows time evolution equation in the expanding universe with the Hubble rate \(H(t) = \dot{a}(t)/a(t)\):

\[
\left( \frac{\partial}{\partial t} - \frac{\dot{a}(t)}{a(t)} p \frac{\partial}{\partial p} \right) f(p; t) = 0 \, ,
\]

which has the solution of the form, \(f(p; t) = f(pa(t)/a(t_d)) = f(p(z_d + 1)/(z + 1))\), with \(t_d\) the time of decoupling. From this one concludes that the FD distribution in the present epoch \(t = t_0\), when written as a function of energy, takes the form of

\[
f_i(E) = \frac{1}{e^{\sqrt{E^2 - m_i^2 + m_i/(z_d + 1)^2}/T_{\nu} + 1}} \, , \quad (4)
\]

with \(z_d\) the redshift factor at the neutrino decoupling. The constants \(k_{a}^{ij}\) at each threshold are functions of the neutrino mass matrix elements whose explicit forms are in \([2]\). In the case of the Dirac neutrino pair emission the rate formula is modified and given by deleting the term \(\propto k_{M}^{ij}\) in the above formula, which is the interference term of identical Majorana fermions.

Assuming a commercially available laser, one can take the laser power (denoted above by \(\omega F_0\)) of order \(1 W\) and the laser frequency resolution
\( \Delta \omega / \omega \sim 10^{-9} \). The threshold rise at \( m_i + m_j \) is of order,

\[
\Gamma_{ij}^M(\omega ; 0) \sim \frac{G_F^2 F_0 \gamma_r}{\omega^2 \Delta \omega \gamma}(m_i m_j)^{3/2}(\omega - \Delta_{fi} - m_i - m_j)^2(k_0^{ij} + k_M^{ij})
\]

\[
\approx 2.4 \times 10^{-22} \text{s}^{-1}(k_0^{ij} + k_M^{ij}) \frac{P}{W \text{mm}^{-2}} \frac{(\text{eV})^3}{\omega} \frac{10^{-9} \omega 10^9 \gamma_r}{\Delta \omega \gamma} \frac{(m_i m_j)^{3/2}(\omega - \Delta_{fi} - m_i - m_j)^2}{(0.1\text{eV})^5},
\]

(6)

disregarding the relic effect, and

\[
\Gamma_{ij}^M(\omega ; 0) \sim \frac{G_F^2 F_0 \gamma_r}{30 \omega^2 \Delta \omega \gamma}(\omega - \Delta_{fi})^5(k_0^{ij} + k_M^{ij})
\]

\[
\approx 8.1 \times 10^{-19} \text{s}^{-1}(k_0^{ij} + k_M^{ij}) \frac{P}{W \text{mm}^{-2}} \frac{(\text{eV})^3}{\omega} \frac{10^{-9} \omega 10^9 \gamma_r}{\Delta \omega \gamma} \frac{(\omega - \Delta_{fi})^5}{1\text{eV}},
\]

(8)

far away from the threshold. Here \( \gamma = \sqrt{\gamma_i^2 + \gamma_n^2} \) is the width associated with initial and intermediate atomic levels, both assumed metastable, for instance \( 1/\gamma > 1 \text{sec} \), while \( \gamma_r \) is E1 width of the final level \( |f \rangle \) of order \( 1\text{ns} \). The angle factor is for instance \( k_0^{11} + k_M^{11} = 2 \cos^2 \theta_{12} \cos^2 \theta_{13} \sim 1.3 \) at \( 2m_1 \) threshold.

Effects of the relic neutrino are maximal near the laser energy threshold of \( \omega = \Delta_{lf} + m_i + m_j \). At this threshold both momenta of two emitted neutrinos vanish, and the Pauli blocking factor becomes

\[
(1 - f_i)(1 - f_j) = \frac{1}{(1 + e^{-m_i/T_d})(1 + e^{-m_j/T_d})} \sim \frac{1}{4} + \frac{m_i + m_j}{8T_d}.
\]

(9)

Here

\[
\frac{m_i}{T_d} \approx 5 \times 10^{-10} \frac{m_i}{1\text{meV}} \frac{2\text{MeV}}{T_d},
\]

with \( T_d \) the neutrino decoupling temperature of order \( 2\text{MeV} \) \( \Box \). The theoretically calculated ratio

\[
r(\omega ; T_\nu) \equiv \frac{\Gamma_{ij}^M(\omega ; T_\nu)}{\Gamma_{ij}^M(\omega ; 0)},
\]

(10)
thus approaches $\approx \frac{1}{4} + \frac{m_i + m_j}{T_d}$ at the $m_i + m_j$ threshold. The threshold rate is eq.(5) times this ratio.

An experimental strategy is then as follows. For a very small $m_i/T_d$ one derives as a function of $\omega - \Delta f_i$ the ratio of experimental data to the theoretical value $\Gamma^M(\omega; 0)$, which is meant to be the theoretical rate without the Pauli blocking. The theoretical function $\Gamma^M(\omega; 0)$ contains both mixing angle factors as an overall factor and the mass $m_i$. One can determine both of these parameters internally from an experiment by fitting this ratio normalized to $\approx 1/4$ at the threshold. With an extreme precision one may even hope to measure the decoupling temperature $T_d$. We note that precision measurement of mixing angles is not a prerequisite in this approach.

In order to discuss the magnitude of relic neutrino effect, we assume in the present work that all neutrino masses are known with a good precision. Numerical results for the rate $\Gamma^M(\omega; 1.9K)$ in the laser energy range including $2m_1 \sim 2m_3$ are shown in Figure 2 for the Majorana case assuming the standard neutrino temperature $T_\nu = 1.9K$. We took for the neutrino parameters, $m_1 = 1meV$ and $\sin^2 \theta_{13} = 0.032$, the maximal allowed value, and

Figure 2: Event rate with relic effect included
Figure 3: Ratio with to without relic effect for several neutrino masses; 0.1 ∼ 5 meV

other parameters constrained by neutrino oscillation data. The Dirac case can be dealt with in a similar way.

The distinction of Majorana and Dirac neutrinos is possible at higher thresholds above $m_1 + m_2$, and shall be explained elsewhere. Here we shall assume that all neutrinos are of the Majorana type.

The ratio $r(\omega; 1.9K)$ near the $2m_1$ threshold region is shown in Figure 3 assuming the Majorana neutrino of the mass range $m_1 = 0.1 meV \sim 5 meV$. For smaller values of the neutrino mass $m_1$ a larger region in the laser energy exits for visible relic neutrino effect.

In Figure 4 we plot the ratio $r(\omega; T_\nu)$ for different neutrino temperatures $T_\nu = (1 \sim 3)K$. It appears that if high statistics data become available, the temperature determination at the level of 10 % is possible for smaller $m_1$ masses.

The region at higher thresholds is also interesting, because the event rate is much larger, for instance by $O[10^3]$ at $m_1 + m_2$, than at $\omega - \Delta f_i = 3m_1$ near $2m_1$ threshold ($m_1 = 1 meV$ taken). This is shown in Figure 5 near
Figure 4: Difference of relic effect for temperature variation taking 3 neutrino masses; 0.1, 1, 5 meV

Figure 5: Ratio with to without relic effect in $m_1 + m_2$ mass region for indicated $m_1$ values
$m_1 + m_2$ threshold, where one needs a precision of $O[10^{-4}]$ for detection of the relic effect.

We shall now turn to implications of determination of the relic neutrino effect. We discuss two issues; (1) limit on the chemical potential, hence the lepton asymmetry, (2) restriction on extra species of particles.

The difference between particles and anti-particles for the Dirac neutrino, and two helicity states for the Majorana case, is reflected by a finite chemical potential $\mu$, which is related to the lepton asymmetry. For a small chemical potential the lepton asymmetry is of order $\mu/T$,

$$n_L/n_\gamma \approx \pi^2/12\zeta(3) \frac{\mu}{T_\nu} \approx 0.68 \frac{\mu}{T_\nu}. \tag{11}$$

It is natural to expect the asymmetry of order, $\mu/T_\nu = O[10^{-10}]$, the same order as the baryon asymmetry. In leptogenesis scenario [3] the lepton asymmetry $L$ of this order has a definite relation to the baryon asymmetry $B$; $L = -51B/28$, taking 3 generations and 1 Higgs doublet for the standard model [4]. Hence observation of the lepton asymmetry is an unambiguous test of leptogenesis scenario. Although a measurement of $\mu/T_\nu = O[10^{-10}]$ effect is extremely difficult, it would be a rewarding challenge to verify or falsify the leptogenesis scenario.

From nucleosynthesis, one has a crude limit on the chemical potential of all neutrino flavors $\alpha$ (considering the neutrino oscillation is important in this respect [5]), of order $|\mu_\alpha/T_\nu| \leq 0.04$, much larger than the expectation of leptogenesis. Thus, it would be interesting to improve the bound on the chemical potential $\mu$ from neutrino pair emission experiments.

The FD function in $z_d \to \infty$ limit takes the form for different helicity $h$ states,

$$f(p; \mu) = \frac{1}{e^{(p+h\mu)/T_\nu} + 1}. \tag{12}$$
Figure 6: Effect of finite chemical potential

To leading $\mu$ order,

$$1 - f(p; \mu) \approx 1 - f(p; 0) + h \frac{\mu}{T_\nu} f(p; 0) (1 - f(p; 0)),$$

hence in neutrino helicity sums the linear term in helicity $h$ is relevant. From corresponding formulas of [6] the leading linear term in the chemical potential is thus derived as

$$\delta \Gamma^{M,D}(\omega; T_\nu) = \frac{\mu}{T_\nu} \frac{G_F^2 F_0}{2 \pi^2 \Delta \omega} \frac{\gamma}{\gamma} \sum_{ij} \theta(\omega - \Delta f_i - m_i - m_j) \times$$

$$\int_{m_i}^{\omega - \Delta f_i - m_j} dE_1 I_{\mu}(E_1)(1 - f_i)(1 - f_j),$$

$$I_{\mu}(E_1) = k_{ij} \sqrt{(E_1^2 - m_i^2)/((\omega - \Delta f_i - E_1)^2 - m_j^2)} \times$$

$$\left(f_i(\omega - \Delta f_i - E_1)\sqrt{E_1^2 - m_i^2} + f_j E_1 \sqrt{(\omega - \Delta f_i - E_1)^2 - m_j^2}\right),$$
threshold rate of this quantity is calculated as
\[
\delta \Gamma^{M,D}(\omega; T\nu) \sim \frac{\sqrt{2}}{30\pi} \frac{\mu}{T\nu} \frac{G^2_F F_0}{\omega^2} \frac{\gamma_r}{\Delta\omega} \times \kappa_{ij}^0 m_i m_j (\sqrt{m_i} + \sqrt{m_j}) (\omega - \Delta f_i - m_i - m_j)^{5/2}.
\]

There is no difference between the Majorana and Dirac cases. As an illustration, we show in Figure 6 effect of a finite chemical potential plotting the quantity \(\delta \Gamma^{M,D}(\omega; 1.9K)T\nu/(\Gamma^M(\omega; 1.9K)\mu)\) in the energy range \(2m_1 \sim m_1 + m_2\).

The neutrino temperature is also a sensitive probe for physical processes after the neutrino decoupling. We are content here to discuss a trivial implication once the neutrino temperature is determined with a precision. Suppose that hypothetical light \(\Delta N_{\text{eff}}\) species of particles, either bosons or fermions, with weight factors 1 and 7/8 respectively, exist, and are thermally coupled to \(e^\pm\) and nucleons (or light nuclei) in the cosmic temperature range between the neutrino decoupling and some freeze-out temperature prior to \(e^\pm\) pair annihilation. The present neutrino temperature is then modified from \((4/11)^{1/3} T_0\) to \((4/11)^{1/3} (1 + 2\Delta N_{\text{eff}}/11)^{-1/3} T_0\), with \(T_0 \approx 2.7K\) the cosmic microwave temperature. This way one may derive a constraint on the number of extra light species \(\Delta N_{\text{eff}}\).

In summary, we discussed a challenging proposal of measuring the cosmic temperature of relic neutrino, and mentioned how to experimentally test the leptogenesis scenario.

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[6] In relevant formulas in [2], eqs.(16), (17),(19),(20), (21), helicities were inadvertently flipped; correct formulas are obtained by replacing $h_2$ there by $-h_2$. 

12