Angular magnetoresistance oscillations in bilayers in tilted magnetic fields

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Abstract

Angular magnetoresistance oscillations (AMRO) were originally discovered in organic conductors and then found in many other layered metals. It should be possible to observe AMRO to semiconducting bilayers as well. Here we present an intuitive geometrical interpretation of AMRO as the Aharonov-Bohm interference effect, both in real and momentum spaces, for balanced and imbalanced bilayers. Applications to the experiments with bilayers in tilted magnetic fields in the metallic state are discussed. We speculate that AMRO may be also observed when each layer of the bilayer is in the composite-fermion state.

Key words: Magnetoresistance oscillations, Bilayers, Interlayer tunneling, Aharonov-Bohm effect, Composite fermions

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The so-called angular magnetoresistance oscillations (AMRO) were originally discovered in the quasi-two-dimensional (Q2D) organic conductors of the (BEDT-TTF)\textsubscript{2}X family \cite{1,2}. Upon rotation of a magnetic field \(B\), electrical resistivity oscillates periodically in \(\tan \theta\), where \(\theta\) is the angle between \(B\) and the normal to the layers. The oscillations are very strong and the most pronounced in the interlayer resistivity \(\rho_z\). AMRO are distinct from the Shubnikov-de Haas (SdH) oscillations, where resistivity oscillates as a function of the magnetic field magnitude for a fixed orientation. In AMRO, resistivity has maxima at certain angles \(\theta\), often called the “magic angles”, that are independent of the magnetic field magnitude. AMRO typically persist to substantially higher temperatures than the SdH oscillations, so the two effects can be clearly separated experimentally. Theory explained that the period of AMRO in \(\tan \theta\) is inversely proportional to \(k_F d\), where \(d\) is the interlayer distance, and \(k_F\) is the in-plane Fermi wave vector. Thus, AMRO can be utilized to determine \(k_F\) and to map out Fermi surfaces of Q2D materials with anisotropic \(k_F\). This was done first in \(\beta\)-(BEDT-TTF)\textsubscript{2}IBr\textsubscript{2} \cite{3}, and then in a variety of organic conductors (see reviews \cite{4,5,6}). AMRO were also observed in many other layered materials, such as intercalated graphite \cite{7}, \(\text{Sr}_2\text{RuO}_4\) \cite{8}, \(\text{Tl}_2\text{Ba}_2\text{CuO}_6\) \cite{9,10}, and the GaAs superlattices \cite{11,12,13}.

The first theory of AMRO was presented by Yamaji \cite{14}, who pointed out that the amplitude of the SdH oscillations should be maximal at the magic angles determined by zeroes of the Bessel function \(J_0(k_F d \tan \theta)\). Yagi et al. \cite{15} calculated angular oscillations of the interlayer conductivity \(\sigma_z(\theta)\) from the Boltzmann equation using semiclassical electron trajectories on the cylindrical 3D Fermi surface. It was assumed that a periodic crystal with many layers and a 3D Fermi surface is necessary for observation of AMRO. However, it was also recognized \cite{15} that AMRO exist already in the limit of infinitesimal interlayer tunneling amplitude \(t_\perp \to 0\). Using the Landau wave functions, Kurihara

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In this paper, we would like to make a connection between AMRO and the Landau filling factors, whereas Q2D metals were studied experimentally in parallel \[23,24\] and tilted \[25,26\] magnetic fields. On the theory side, Hu and MacDonald \[27\] calculated \(\tilde{t}_\parallel\) in a tilted field using the Landau wave functions, and Lyo et al. \[28,29,30\] studied conductivity using the Kubo formula. They found vanishing \(\tilde{t}_\parallel\) for certain angles \(\theta\) \[27\] and oscillatory dependence of \(\sigma_z\) on the magnetic field component \(B_\parallel\) parallel to the layers for a fixed perpendicular component \(B_\perp\) \[29\]. However, these papers (also \[31\]) focused on the low Landau filling factors, whereas Q2D metals were studied for the high filling factors, so a relation between AMRO and the Fermi surface is relevant for conduction. For balanced bilayers, \(k_F\) is the same in both layers.

The gauge phase \(\phi\) in Eq. \(\eqref{eq:hamiltonian}\) leads to interference between electron tunneling at different points along the cyclotron orbit, and the effective tunneling amplitude \(\tilde{t}_\perp\) is obtained by phase averaging \[19,20\]:

\[
\tilde{t}_\perp = t_\perp \left< e^{\frac{i e B_\perp x(t) d}{\hbar c}} \right>_t = t_\perp \text{J}_0(k_F d \tan \theta). \tag{2}
\]

Here the brackets represent averaging over time \(t\) for the cyclotron motion \(x(t) = R_c \cos(\omega_c t)\), \(\text{J}_0\) is the Bessel function, and \(\tan \theta = B_\perp / B_{\perp}\). Since the interlayer tunneling conductivity \(\sigma_z\) is proportional to \(t_\perp^2\), the organic conductors community will be stimulating for further studies of oscillatory phenomena in semiconductor bilayers (for the Q1D case see \[32\]).
Eq. (2) gives $\sigma_z(\theta)/\sigma_z(0) = J_0^2(k_F d \tan \theta)$, which is shown by the curve (c) in Fig. 2. From the asymptotic expression $J_n(x) \propto \cos[(x - \pi/4)/\sqrt{2}]$, we find that $\tilde{t}_\perp$ and $\sigma_z$ oscillate periodically in $\tan \theta$ and vanish at the “magic angles”

$$\frac{B_\parallel}{B_\perp} = \tan \theta_n = \pi(n - 1/4)/k_F d,$$

where $n$ is an integer. This is the AMRO effect discussed in the introduction. In [27,29,31], the effective tunneling amplitude $\tilde{t}_\perp$ was obtained as a matrix element of the Hamiltonian (1) between the Landau wave functions and expressed in terms of the Laguerre polynomials. However, as pointed out in Refs. [16,17,18], the Laguerre polynomials reduce to the Bessel function $J_n$ for the high Landau levels, so the quasiclassical expression (2) agrees with the quantum calculation [27,29,31].

Vanishing of $\tilde{t}_\perp$ at the magic angles not only results in minima of $\sigma_z$, but also in disappearance of beating in the SdH oscillations. Generally, the symmetric and antisymmetric electron states in a density-balanced bilayer are split in energy by $\tilde{t}_\perp$, which results in two slightly different SdH frequencies. However, at the magic angles, the energy split and the beating of the SdH oscillations should disappear, because $\tilde{t}_\perp \to 0$. This effect is observed in organic conductors [5] and was explained theoretically by Yamaji [14]. In bilayers, it was observed [25] that the SdH beating period increases with the increase of $B_\parallel$, in qualitative agreement with the argument presented above. However, the ratio $B_\parallel/B_\perp$ was not big enough to reach a magic angle and to observe disappearance of the SdH beating.

AMRO can be interpreted geometrically as a particular manifestation of the Aharonov-Bohm effect. Let us look at the bilayer along the layers, as shown in Fig. 1b. The gauge phase in Eq. (1) is proportional to the area contained between the layers up to the point of electron tunneling. The lines of the length $2R_e$ represent the side view of the cyclotron orbits. Electrons spend more time at the extremal turning points denoted as the dots, which naturally define the shaded area $2R_e d$. The magnetic flux $\Phi$ through this area results in destructive interference between electron tunneling at the opposite turning points and vanishing of $\tilde{t}_\perp$ when $\Phi = 2R_e d B_\parallel = \phi_0 (n + C)$, where $\phi_0 = 2\pi h c/e$ is the flux quantum, and $C$ is an appropriate constant.

Inserting the expression for $R_e$, we recover Eq. (3). Notice that one dimension $d$ of the Aharonov-Bohm area is fixed by the bilayer structure, but the other dimension $2R_e$ is adjustable and is proportional to $B_\perp^{-1}$. This results in the condition (3) on the ratio of $B_\parallel$ and $B_\perp$.

AMRO can be also interpreted as a result of interference in the momentum space, as illustrated in Fig. 3. Suppose that only the $B_\parallel$ component is applied. Then, according to Eq. (1), the in-plane electron momentum changes by $\Delta k_\parallel = eB_\parallel d/\hbar$ upon tunneling between the layers [23,24,25], so the Fermi surfaces of the two layers are shifted relative to each other as shown in Fig. 3. Thus, electrons can tunnel only at the points $k_1$ and $k_2$, where the conservation laws of both energy and momentum are satisfied. When the $B_\perp$ component is turned on, it causes interference between the two trajectories $a$ and $b$ connecting the points $k_1$ and $k_2$. The phase difference between the two trajectories is proportional to the shaded area $S$ between them in momentum space. In the balanced case shown in Fig. 3a, $S = 2k_F \Delta k_\parallel$, where we assumed that $\Delta k_\parallel \ll k_F$, which is a typical condition for Q2D metals. The interference between the two momentum-space trajectories is destructive when the condition $B_\perp = \phi_0 S/(n + C)(2\pi)^2$ is satisfied, which reproduces Eq. (3).

In the imbalanced case, the interference oscillations develop between the parallel trajectories that involve the momentum-space areas $S_1$ and $S_2$ in Fig. 3b. The frequencies of these oscillations are given by the SdH-like formula $B_\perp = \phi_0 S_{1,2}/(n + C)(2\pi)^2$, where the areas $S_1$ and $S_2$ depend on $B_\parallel$. Notice that these interference oscillations are different from the SdH oscillations. The later are the consequence of the energy quantization originating from closed orbits, whereas the former result from quantum interference between parallel orbits that do not form a closed loop and do not produce energy quantization. Magnetoresistance oscil-
lations due to the momentum-space interference are known in some metals\cite{12} and organic conductors\cite{13}. The in-plane resistivity $\sigma_z$ of an imbalanced bilayer in tilted magnetic fields was measured in Ref. [26]. The oscillations originating from the areas $S_1$ and $S_2$ can probably be found in the Fourier spectrum shown in Fig. 4 of Ref. [26]. However, this paper focused only on the SdH oscillations originating from closed orbits, but not on the interference oscillations from parallel orbits. In Fig. 3 of this paper, one can recognize a pattern of oscillations originating from the areas $\omega$ and $\omega'$. However, for $\omega < \omega'$, probably be found in the Fourier spectrum shown in Ref. [26].

A finite lifetime $\tau$ of quasiparticles results in loss of phase coherence, which can be described phenomenologically by an exponentially decaying factor in the Kubo formula for $\sigma_z$\cite{15,19,20}:

$$\sigma_z \propto t_z^2 \left\langle e^{-\frac{Bt_zd}{\hbar}} e^{i(k_Fd \tan \theta)} e^{-\frac{t_z}{t}} dt \right\rangle.$$ \hspace{1cm} (4)

Doing the integral in Eq. (4), one finds\cite{15,19,20}

$$\frac{\sigma_z(B)}{\sigma_z(0)} = J_0^2(k_Fd \tan \theta) + 2 \sum_{j=1}^\infty J_j^2(k_Fd \tan \theta) \frac{1}{1 + (j\omega_c \tau)^2}. \hspace{1cm} (5)$$

For $\omega_c \tau \gg 1$, Eq. (4) gives $\sigma_z \propto t_z^2 \tau$, and Eq. (5) reproduces AMRO. However, for $\omega_c \tau \ll 1$, electrons lose coherence before they complete a cycle, so the interference effect is washed out, and $\sigma_z$ reduces to $\sigma_z(0) \propto t_z^2 \tau$. Fig. 2 shows $\sigma_z(\theta)$ calculated from Eq. (5) for several values of $\omega_c \tau$. When $B$ is increased at a fixed angle $\theta$, resistivity $\rho_z = 1/\sigma_z$ increases and saturates at a finite value in the limit $\omega_c \tau \rightarrow \infty$ for generic angles. However, for the magic angles, $\rho_z$ increases without saturation, because $\sigma_z \rightarrow 0$ at $\omega_c \tau \rightarrow \infty$. Notice that observation of AMRO requires $\omega_c \tau > 1$, whereas, according to the Lifshitz-Kosevich formula, observation of the SdH oscillations requires $\hbar \omega_c > T$, where $T$ is temperature. These are different conditions, and, typically, AMRO are still visible at elevated temperatures, where the SdH oscillations have already disappeared. For example, in GaAs superlattices\cite{12}, AMRO are clearly visible at 25 K, whereas the SdH oscillations dominate at 1.5 K.

Finally, we briefly discuss a possibility of observing AMRO in the case where each layer of a bilayer is in the composite-fermion state with the filling factor $\nu$ close to 1/2. The composite fermions experience the effective magnetic field $B_{\perp}^* = B_{\perp} (1 - 2\nu)$ and execute cyclotron motion with the radius $R_{\perp}^* = k_F^* \phi_0 / 2\pi B_{\perp}^*$, where $k_F^* = \sqrt{2} k_F$ is their effective Fermi wave vector. By analogy, we would expect to see AMRO in the interlayer conductivity with the magic angles given by Eq. (3) with the substitution $B_{\perp} \rightarrow B_{\perp}^*$ and $k_F \rightarrow k_F^*$. Unfortunately, the interlayer tunneling is greatly suppressed, because the composite fermions need to decompose and recombine for tunneling\cite{35}. However, the interlayer conductivity may increase at higher temperatures and help to observe AMRO. A systematic attempt to observe AMRO would provide useful information about the nature of the composite-fermion state.

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