A New Generalization of Quasi Gamma Distribution with Properties and Applications

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Abstract: we have introduced weighted technique for quasi gamma to convert known distribution into new model called weighted quasi gamma distribution. Finally, newly proposed distribution is examined with an application.

Keywords: Quasi gamma, Order statistics, Statistical measures, Weighted model.

I. INTRODUCTION

New Probability models used by researchers for the purpose of modelling or transforming known as weighted models proposed by Fisher in 1934, then after that in 1965 Rao makes significant changes in it and made it in other terms when he observe that the standard distributions are not suitable for modelling. For survival data analysis, Jing (2010) introduced the inverse version for both weighted Weibull and beta Weibull as a new lifetime models. Kilany (2016), have obtained the weighted version of lomax distribution. Ajami and Jahanshahi (2017) introduced weighted Rayleigh distribution as a new generalization of Rayleigh distribution and discussed its parameter estimation in broad. Para and Jan (2018) introduced the Weighted Pareto type II Distribution as a new model for handling medical science data and studied its statistical properties and applications. Khan et al. (2018) discussed the weighted modified weibull distribution. Recently, Ganaie, Rajagopalan and Rather (2019), discussed A new extension of Ram Awadh distribution.

The two parametric quasi gamma distribution was introduced by shanker (2018). Two parametric quasi gamma distribution also has better flexibility in handling real lifetime data over one parameter exponential, quasi exponential and two parameters gompertz, weibull and gamma distribution.

II. WEIGHTED QUASI GAMMA DISTRIBUTION (WQGD)

The distribution of quasi gamma having probability density function

\[ f(x; \theta, \alpha) = \frac{2\theta^\alpha}{\Gamma(\alpha)} e^{-\theta x^2} x^{2\alpha - 1}; \quad x > 0, \theta > 0, \alpha > 0 \]

and cumulative function for weighted quasi gamma distribution is obtained as

\[ F_w(x; \theta, \alpha, c) = \frac{\Gamma(2\alpha + c + 1)}{\Gamma(2\alpha + c + 1)} \left[ \frac{x^{2\alpha + c - 1}}{\theta} \right]^{-\theta} x^2 dx \]

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Put \( \theta x^2 = t \) \( \Rightarrow 2\theta x dx = dt \) \( \Rightarrow dx = \frac{dt}{2\theta x} \), \( \therefore dx = \frac{1}{\theta} \), \( \Rightarrow 2x = \frac{1}{\theta} \)

Also \( \theta x^2 = t \) \( \Rightarrow x^2 = \frac{t}{\theta} \) \( \Rightarrow x = \left( \frac{t}{\theta} \right)^{\frac{1}{2}} \)

As \( x \to 0, t \to 0 \) and \( x \to x, t \to \theta x^2 \)

After simplification, we obtain the cumulative function for weighted quasi gamma distribution

\[
F_w(x; \theta, c, \alpha) = \frac{1}{\Gamma(2\alpha + c + 1)} \left( \frac{1}{\theta^2} \Gamma \left( \frac{2\alpha + c + 1}{2}, \theta x^2 \right) \right)
\]

(6)

\[
R(x) = 1 - \frac{1}{\Gamma(2\alpha + c + 1)} \left( \frac{\theta^2}{2} \Gamma \left( \frac{2\alpha + c + 1}{2}, \theta x^2 \right) \right)
\]

B. Hazard function

\[
h(x) = \frac{f_w(x; \theta, c, \alpha)}{R(x)}
\]

\[
h(x) = \frac{2x^2 \alpha + c - 1}{\Gamma(2\alpha + c + 1)} \frac{2\alpha + c + 1}{\theta^2} e^{-\theta x^2}
\]

C. Reverse hazard function

\[
h^r(x) = \frac{f_w(x; \theta, c, \alpha)}{F_w(x; \theta, c, \alpha)}
\]

\[
h^r(x) = \frac{2x^2 \alpha + c - 1}{\Gamma(2\alpha + c + 1)} \frac{2\alpha + c + 1}{\theta^2} e^{-\theta x^2}
\]

III. RELIABILITY ANALYSIS

A. Reliability function

The reliability function of newly obtained distribution is

\[
R(x) = 1 - F_w(x; \theta, c, \alpha)
\]
IV. STATISTICAL MEASURES

In this Part, different statistical properties of weighted quasi gamma distribution including its moments, Harmonic mean, moment generating function and characteristics function are explained.

A. Moments

Let $X$ denotes the random variable of Weighted quasi gamma distribution, the $r$th order of weighted quasi gamma is

$$E(X^r) = \int_0^\infty x^r f_W(x; \theta, c, \alpha) \, dx$$

After simplification, we obtain from equation (7)

$$E(X^r) = \frac{2}{\theta^2 \Gamma(2\alpha + c + 1)} \left( \frac{\theta}{\Gamma(2\alpha + c + 1)} \right)^{-r}$$

Moments of weighted quasi gamma distribution are obtained by substituting values 1, 2, 3 & 4 in (8)

$$E(X) = \mu'_1 = \frac{\Gamma(2\alpha + c + 2)}{\theta^2 \Gamma(2\alpha + c + 1)}$$

$$E(X^2) = \mu'_2 = \frac{\Gamma(2\alpha + c + 3)}{\theta^2 \Gamma(2\alpha + c + 1)}$$

$$E(X^3) = \mu'_3 = \frac{\Gamma(2\alpha + c + 4)}{\theta^2 \Gamma(2\alpha + c + 1)}$$

B. Harmonic mean

The Harmonic mean is the reciprocal of the arithmetic mean of the reciprocals. The harmonic mean for the proposed weighted quasi gamma distribution can be obtained as

$$H.M = E\left(\frac{1}{X}\right) = \int_0^\infty \frac{1}{x} f_W(x; \theta, c, \alpha) \, dx$$

After simplification, we obtain from equation (9)

$$H.M = \frac{1}{\theta} \left( \frac{\theta}{\Gamma(2\alpha + c + 1)} \right)^{1/2}$$

Put $\theta x^2 = t \Rightarrow x^2 = \frac{t}{\theta} \Rightarrow x = \left( \frac{t}{\theta} \right)^{1/2}$

Also $2\theta dx = dt \Rightarrow dx = \frac{dt}{2\theta x} = \frac{dt}{2}$$

After simplification, we obtain from equation (9)

$$H.M = \frac{1}{\theta} \left( \frac{\theta}{\Gamma(2\alpha + c + 1)} \right)^{1/2}$$
C. Moment Generating Function
The moment generating function is the expectation of a function of the random variable. Generating function of moment is given by

\[ M_x(t) = E(e^{tx}) = \int_0^\infty e^{tx} f_w(x; \theta, c, \alpha) \, dx \]

\[ = \sum_{j=0}^\infty \frac{t^j}{j!} \int_0^\infty \frac{j^j}{j!} \left( \frac{\Gamma(2\alpha + c + j + 1)}{2} \right) \frac{1}{\theta^2} \frac{1}{2} \left( \frac{\Gamma(2\alpha + c + 1)}{2} \right)^2 \]

\[ M_x(t) = \frac{1}{\Gamma(2\alpha + c + 1)} \sum_{j=0}^\infty \frac{(it)^j}{j!} \left( \frac{\Gamma(2\alpha + c + j + 1)}{2} \right) \]

D. Characteristics function
In probability theory and statistics characteristic function is defined as the function of any real-valued random variable completely defines the probability of a random variable.

\[ \varphi_X(t) = M_x(it) \]

\[ M_x(it) = \frac{1}{\Gamma(2\alpha + c + 1)} \sum_{j=0}^\infty \frac{(it)^j}{j!} \left( \frac{\Gamma(2\alpha + c + j + 1)}{2} \right) \]

V. ORDER STATISTICS
Order statistics are sample values placed in order of increasing just as like x1, x2, ..., xn then rth order statistics has probability density function

\[ f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_x(x) \left( F_x(x) \right)^{r-1} \left( 1 - F_x(x) \right)^{n-r} \]

Using equation (5) and (6) in equation (10), we obtain rth weighted model of quasi gamma distribution which is given by

\[ f_{x(r)}(x) = \frac{n!}{(r-1)!(n-r)!} f_x(x) \left( F_x(x) \right)^{r-1} \left( 1 - F_x(x) \right)^{n-r} \]

VI. LIKELIHOOD RATIO TEST
the sample of size n randomly drawn from the distribution of quasi gamma or distribution of weighted quasi gamma. For testing purpose setting up hypothesis

\[ H_o : f(x) = f(x; \theta, c) \quad VS \quad H_1 : f(x) = f_w(x; \theta, c, \alpha) \]

For testing, likelihood ratio function is applied whether the randomly selected sample comes for quasi gamma or weighted quasi gamma

\[ \Delta = \frac{L_1}{L_o} = \prod_{i=1}^n \frac{f_w(x; \theta, c, \alpha)}{f(x; \theta, c)} \]
\[
\Delta = \frac{L_1}{L_0} = \prod_{i=1}^{n} \left( \frac{\Gamma(\frac{c+1}{2})}{\Gamma(\frac{2(\alpha+c)+1}{2})} \right) \left( \frac{2}{\theta^2 \Gamma(\frac{2(\alpha+c)+1}{2})} \right)
\]

Null hypothesis is not accepted if

\[
\Delta = \left( \frac{\Gamma(\frac{2(\alpha+c)+1}{2})}{\Gamma(\frac{c+1}{2})} \right)^n \left( \frac{\prod_{i=1}^{n} x_i^c}{\prod_{i=1}^{n} x_i} \right) > k
\]

Equivalently, null hypothesis is also rejected when

\[
\Delta^* = \prod_{i=1}^{n} x_i^c > k^*, \quad \text{where } k^* = k \left( \frac{\Gamma(\frac{2(\alpha+c)+1}{2})}{\Gamma(\frac{c+1}{2})} \right)^n \left( \frac{2}{\theta^2 \Gamma(\frac{2(\alpha+c)+1}{2})} \right)
\]

Further, null hypothesis should not be accepted, when the value of probability is

\[
p(\Delta^* > \gamma^*) \text{ and } \gamma^* = \prod_{i=1}^{n} x_i^c \text{ is minimum than a particular level of significance and } \prod_{i=1}^{n} x_i^c \text{ is the experimental value of the Statistic } \Delta^*.
\]

\[\text{VII. BONFERRONI AND LORENZ CURVES}\]

The proposed measure of weighted quasi gamma is

\[
B(p) = \frac{1}{p \mu_1} \int_0^q x f_{w}(x; \theta, c, \alpha) dx
\]

and

\[
L(p) = p B(p) = \frac{1}{\mu_1} \int_0^q x f_{w}(x; \theta, c, \alpha) dx
\]

Where \[\mu_1 = \frac{1}{\theta^2 \Gamma(\frac{2(\alpha+c)+1}{2})} \quad \text{and} \quad q = F^{-1}(p)\]

\[\text{VIII. MAXIMUM LIKELIHOOD ESTIMATION AND FISHER'S INFORMATION MATRIX}\]

Parameters of the distribution of weighted quasi gamma are calculated by using estimation of maximum likelihood, and then corresponding function of likelihood is

\[
L(x; \theta, c, \alpha) = \prod_{i=1}^{n} f_{w}(x_i; \theta, c, \alpha)
\]

\[
L(x; \theta, c, \alpha) = \frac{2^{2c} \Gamma(\frac{2(\alpha+c)+1}{2})}{\Gamma(\frac{2(\alpha+c)+1}{2})} \left( \frac{2^{\alpha+c+1}}{\Gamma(\frac{2(\alpha+c)+1}{2})} \right) e^{\theta x_i^2}
\]

\[
L(x; \theta, c, \alpha) = \frac{2^{n} \theta \Gamma(\frac{2(\alpha+c)+1}{2})}{\Gamma(\frac{2(\alpha+c)+1}{2})} \left( \prod_{i=1}^{n} x_i^2 \Gamma(\frac{2(\alpha+c)+1}{2}) \right)
\]
The function of log likelihood is
\[ \log L = n \log 2 + n \left( \frac{2\alpha + c + 1}{2} \right) \log \theta - \frac{n}{2} \sum x_i^2\]
\[ n \log \left( \Gamma \left( \frac{2\alpha + c + 1}{2} \right) \right) + (2\alpha + c - 1) \frac{n}{2} \sum x_i - \theta \frac{n}{2} \sum x_i^2 \]

Equation (12) is differentiated with parameters. The normal equations are
\[ \frac{\partial \log L}{\partial \alpha} = \frac{n(2\alpha + c + 1)}{2} - \frac{n}{2} \sum x_i = 0 \]
\[ \frac{\partial \log L}{\partial \theta} = n \log \theta - n \psi' \left( \frac{2\alpha + c + 1}{2} \right) + 2 \frac{n}{2} \sum x_i = 0 \]
\[ \frac{\partial \log L}{\partial c} = \frac{n}{2} \log \theta - n \psi' \left( \frac{2\alpha + c + 1}{2} \right) + \sum x_i = 0 \]
\[ \psi(.) \text{ is the function of digamma} \]

Because of complicated form of likelihood equation, algebraically it is very difficult to solve the system of nonlinear equations. Therefore for estimating the required parameters by using R and wolfram mathematica.

To obtain confidence interval, we use the asymptotic normality results. We have that, if \( \hat{\gamma} = (\hat{\theta}, \hat{\alpha}, \hat{c}) \) denotes the MLE of \( \gamma = (\theta, \alpha, c) \), we can state the result as
\[ \sqrt{n}(\hat{\gamma} - \gamma) \rightarrow N_{3}(0, I^{-1}(\gamma)) \]

Where \( I(\gamma) \) is the Fisher's Information matrix.

Fisher’s 3x3 Information matrix is given below as

\[ I(\gamma) = \frac{1}{n} \left( \begin{array}{ccc} E \frac{\partial^2 \log L}{\partial \theta^2} & E \frac{\partial^2 \log L}{\partial \theta \partial \alpha} & E \frac{\partial^2 \log L}{\partial \theta \partial c} \\ E \frac{\partial^2 \log L}{\partial \alpha \partial \theta} & E \frac{\partial^2 \log L}{\partial \alpha^2} & E \frac{\partial^2 \log L}{\partial \alpha \partial c} \\ E \frac{\partial^2 \log L}{\partial c \partial \theta} & E \frac{\partial^2 \log L}{\partial c \partial \alpha} & E \frac{\partial^2 \log L}{\partial c^2} \end{array} \right) \]

Where
\[ E \left( \frac{\partial^2 \log L}{\partial \theta^2} \right) = -n(2\alpha + c + 1) \frac{1}{2\theta^2} \]
\[ E \left( \frac{\partial^2 \log L}{\partial \alpha^2} \right) = -n \psi' \left( \frac{2\alpha + c + 1}{2} \right) \]
\[ E \left( \frac{\partial^2 \log L}{\partial c^2} \right) = -n \psi' \left( \frac{2\alpha + c + 1}{2} \right) \]
\[ E \left( \frac{\partial^2 \log L}{\partial \theta \partial \alpha} \right) = \frac{n}{\theta} \]
\[ E \left( \frac{\partial^2 \log L}{\partial \theta \partial c} \right) = \frac{n}{2\theta} \]

IX. DATA EVALUATION

In the application portion analysis, we have used data sets for indicating that weighted quasi gamma distribution fits better over quasi gamma, exponential and one parameter lindley distribution. The following two data sets are given below as

**Table 1:** Strength of data by Naylor and Smith (1987) regarding 1.5cm glass fibers

| Data | 1.50 | 1.51 | 1.52 | 1.53 | 1.54 | 1.55 | 1.56 | 1.57 | 1.58 | 1.59 | 1.60 |
|------|------|------|------|------|------|------|------|------|------|------|------|
| Value | 1.62 | 1.65 | 1.68 | 1.70 | 1.72 | 1.74 | 1.76 | 1.78 | 1.80 | 1.82 | 1.84 |

**Table 2:** Data regarding relief times (in minutes) of patients by Gross and Clark (1975)

| Data | 1.1 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 |
|------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Value | 1.5 | 1.6 | 1.7 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.7 | 2.8 | 2.9 |

In order to compare the weighted quasi gamma distribution with quasi gamma, exponential and one parameter lindley distribution, we are using the criterion values like AIC, BIC and AICC. The distribution is good if it has minimum AIC, BIC, AICC and -2logL values than the others which we have mentioned for comparing. The formulas for calculation of values are

AIC = 2k - 2logL, AICC = AIC + \frac{2k(k+1)}{n-k-1}

and BIC = k log n - 2logL

the number of parameters is k, the size of sample is n and maximised function of log-likelihood is -2logL. The parameters of the distribution are obtained by the estimation of maximum likelihood. It is found from results in table 3 given below that weighted quasi gamma have lower criterion values than quasi gamma, exponential and one parameter lindley distribution which clearly indicates that the weighted quasi gamma distribution fits better than the quasi gamma, exponential and one parameter lindley distribution for the two data sets given. Finally, our conclusion reached that the distribution of weighted quasi gamma fits good than quasi gamma, exponential and one parameter lindley distribution.
Table-3: Comparison of the weighted quasi gamma distribution Vs quasi gamma, Exponential and one Parameter Lindley distribution.

| Data sets | Distribution | MLE | S.E | -2logL | AIC | BIC | AICC |
|-----------|--------------|-----|-----|--------|-----|-----|------|
| 1         | Weighted quasi gamma | $\hat{a} = 0.001$ | $\hat{a} = 0.27$ | 9.3 | 15.3 | 21.7 | 15.7 |
|           | $\hat{\theta} = 0.842$ | $\hat{\theta} = 0.07$ | 582.2 | 124.2 | 042.5 | 09 | 99 | 13 | 81 | 11 | 526.0 | 19 | 87 | 490.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 |
|           | $\hat{c} = 2.582$ | $\hat{c} = 0.40$ | 582.2 | 124.2 | 042.5 | 09 | 99 | 13 | 81 | 11 | 526.0 | 19 | 87 | 490.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 |
| 2         | Quasi gamma | $\hat{a} = 5.042$ | $\hat{a} = 0.87$ | 41.6 | 45.6 | 49.9 | 45.8 |
|           | $\hat{\theta} = 2.124$ | $\hat{\theta} = 0.38$ | 582.2 | 124.2 | 042.5 | 09 | 99 | 13 | 81 | 11 | 526.0 | 19 | 87 | 490.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 |
|           | $\hat{c} = 0.663$ | $\hat{c} = 0.08$ | 41.6 | 45.6 | 49.9 | 45.8 |
|           | Exponential | $\hat{\theta} = 0.99$ | $\hat{\theta} = 0.09$ | 162.5 | 164.5 | 166.7 | 164.6 |
|           | Lindley | $\hat{\theta} = 0.99$ | $\hat{\theta} = 0.09$ | 162.5 | 164.5 | 166.7 | 164.6 |
| 2         | Weighted quasi gamma | $\hat{a} = 0.001$ | $\hat{a} = 0.47$ | 29.66 | 35.66 | 38.64 | 37.16 |
|           | $\hat{\theta} = 0.490$ | $\hat{\theta} = 0.07$ | 582.2 | 124.2 | 042.5 | 09 | 99 | 13 | 81 | 11 | 526.0 | 19 | 87 | 490.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 |
|           | $\hat{c} = 2.582$ | $\hat{c} = 0.72$ | 29.66 | 35.66 | 38.64 | 37.16 |
| 2         | Quasi gamma | $\hat{a} = 2.347$ | $\hat{a} = 0.69$ | 38.3 | 42.3 | 44.3 | 43.0 |
|           | $\hat{\theta} = 0.575$ | $\hat{\theta} = 0.19$ | 582.2 | 124.2 | 042.5 | 09 | 99 | 13 | 81 | 11 | 526.0 | 19 | 87 | 490.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 | 663.0 | 582.2 | 490.0 | 001.0 | 575.0 | 347.2 | 0.0 |
|           | Exponential | $\hat{\theta} = 0.526$ | $\hat{\theta} = 0.11$ | 41.6 | 45.6 | 49.9 | 45.8 |
|           | Lindley | $\hat{\theta} = 0.81$ | $\hat{\theta} = 0.13$ | 41.6 | 45.6 | 49.9 | 45.8 |
|           | $\hat{\theta} = 0.81$ | $\hat{\theta} = 0.13$ | 41.6 | 45.6 | 49.9 | 45.8 |

X. CONCLUSION

In the present manuscript, weighted technique is applied and taking the two parameter quasi gamma distribution as the base distribution called as weighted quasi gamma and also determines its parameters. The distribution which is introduced newly demonstrated with application. Then after that the results are compared over quasi gamma, exponential and one parameter Lindley distribution and finally weighted quasi gamma proved and gives best results over quasi gamma, exponential and one parameter Lindley distribution.

REFERENCES

1. Khan, M. N., Saeed, A., & Alzaatreh, A. (2018). Weighted Modified Weibull distribution. Journal of Testing and Evaluation, 47(5), 20170370.
2. Ganaie, R.A., Rajagopalan, V. and Rather, A.A. (2019), A new extension of Ram Awdh distribution, Journal of Information and Computational Science, Vol.9, Issue.11, pp. 938–953.
3. Clark, V.A. and Gross, A.J., (1975), Survival Distributions: Reliability Applications in the Biometrical Sciences, John Wiley, New York.
4. Para, B.A., & Jan, T. R. (2018). On Weighted three Parameters Pareto Type II Distribution: Properties and Applications in Medical Sciences. Applied Mathematics and Information Sciences Letters, 6 (1), 13-26.
5. Jing (2010). Inverse of Weighted Weibull and Beta Weibull Distribution. Georgia Southern University Digital Commons®@Georgia Southern.
6. Fisher, R.A. (1934). The effects of methods of ascertainment upon the estimation of frequencies. Ann. Eugenics, 6, 13-25.
7. Rao, C. R. (1965), On discrete distributions arising out of method of ascertainment, in classical and Contagious Discrete, G.P. Patiled; Pergamum Press and Statistical publishing Society, Calcutta. 320-332.
8. Shanker, R., Shukla, K.K., Sharma, S. and Shanker, R. (2018), Quasi gamma distribution, International journal of Statistics and Applied Mathematics, Vol.3, Issue. 4, pp. 208-217.
9. Ajami, M., & Jahanshahi, S. M. (2017). Parameter Estimation in Distribution of Weighted Rayleigh. Journal of Modern Applied Statistical Methods, 16(2), 256-276. doi:10.22237/jmasm/1509495240
10. Kilany, N. M. (2016). Weighted Lognormal distribution. SpringerPlus, 5(1). doi:10.1186/s40064-016-34892
11. Smith RL, Naylor JC. A comparison of Maximum likelihood and Bayesian estimators for the three parameter Weibull distribution. Applied Statistics. 1987;36:358–369

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