Search and Matching for Adoption from Foster Care

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To find families for the more than 100,000 children in need of adoptive placements, most United States child welfare agencies have employed a family-driven search strategy in which prospective families respond to announcements made by the agency. However, some agencies have switched to a caseworker-driven search strategy in which the caseworker directly contacts families recommended for a child. We introduce a novel search-and-matching model to capture essential aspects of the adoption process and compare the two approaches through a game-theoretical analysis. We show that the search equilibria induced by threshold strategies form a lattice structure under either approach. Our main theoretical result establishes that the equilibrium outcomes in family-driven search can never Pareto dominate the outcomes in caseworker-driven search, but it is possible that each caseworker-driven search outcome Pareto dominates all family-driven search outcomes. We also find that when families are sufficiently impatient, caseworker-driven search is better for all children. We illustrate numerically that most agents are better off under caseworker-driven search for a wide range of parameters. Finally, we provide empirical evidence from an agency that switched to caseworker-driven search and achieved a three-year adoption probability that outperformed a statewide benchmark by 17%.

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1. Introduction
Child welfare systems worldwide face the challenge of finding families for children in need of adoption. For example, the United States foster care system serves over 600,000 children annually, with approximately 400,000 children in temporary foster care arrangements at any time. While
the goal for most of these children is to reunite them with their parents or close relatives, roughly 120,000 children are waiting for permanent adoptive placements (Children’s Bureau 2022). Finding an adoptive family for these children has become a public policy priority due to high levels of incarceration, homelessness, unemployment, and teen pregnancy observed in the population of children “aging out” of the child welfare system without a permanent family relationship (Barth 1990, Triseliotis 2002, Kushel et al. 2007). In this paper, we study the search and matching process for children waiting in the child welfare system for adoptive placement and compare two prominent search methods that differ in whether families or the children’s caseworkers drive the search process. We provide structural insights by analytically and numerically analyzing a game-theoretic model and empirically validate our findings by studying outcomes achieved by a Florida child welfare agency that switched its search strategy in 2018.

The search for an adoptive placement officially begins once a judge issues a termination of parental rights order. A caseworker represents the child’s interests throughout this process to find an adoptive family if adoption by a relative or foster care family is not possible. Identifying a family willing to adopt the child and capable of caring for the child can be a difficult task, and the challenge varies greatly according to the child’s demographic characteristics and special needs. While relatively little research has studied best practices for search operations — either through empirical studies of how caseworkers find families or prescriptive studies for how search should be conducted — states invest significant resources in trying to help the most vulnerable children find permanency; for example, in Florida’s 2022-2023 fiscal year, the state’s child welfare agencies spent $28 million just to recruit adoptive homes and improve adoption outcomes for a population of children eligible for adoption that numbered 3,787 on July 1, 2022 (Florida Department of Children and Families 2023).

Different states, counties, and agencies adhere to different paradigms for the practice of search and matching. Families first register with an adoption agency and provide a home study evaluation. In the predominant approach, caseworkers then announce children via email to a set of registered families, each of whom has the opportunity to express interest in a child. The caseworker receives these inquiries and works to identify the family that best fits the child’s needs. We label this approach family-driven search, as families direct the search process by expressing interest in available children. Hanna and McRoy (2011) shows how the child welfare literature focuses almost exclusively on this family-driven search process. One important downside of this search process is that some children may attract hundreds of interested families, all of which the caseworker has to consider simultaneously.

These interactions can be time-consuming and emotionally stressful for all parties involved, and some states mandate that agencies engage with every family that has expressed interest. For example, Florida Administrative Code Rule 65C-16.003 states:
Once the potential adoptive families have been identified, the staffing team will rate each family based on the family’s ability to meet the identified needs of the child based on information documented in [the Florida Safe Families Network information system], the Child Study and the adoptive parent’s home study. The documentation must include a key of the rating scale used by the team.

An adoptions manager for a multi-county agency in Florida directed us to this rule to emphasize the imperative that the agency must respond to every family that expresses interest in a child. Failure to respond to families can result in complaints to the governor’s office or negative comments on social media. The manager also commented that the agency dreaded announcing the availability of a “cute” young child, who would attract dozens or even hundreds of responses from families that required thorough consideration and individual responses. This is especially problematic because caseworkers often deal with twice as many cases as they should ideally handle (Yamatani et al. 2009), an issue intensifying since the COVID-19 pandemic (Lushin et al. 2023). On the other hand, other children may attract very few interested families, and nearly 20,000 children age out of foster care each year (Children’s Bureau 2022).

We have also heard families share their frustration with the emotional and time costs that they incurred while participating in a family-driven search system. In a video shared on social media, one adoptive father described his journey that began as a prospective parent engaged with an adoption agency that announced available children and had families respond to express interest in particular children:\footnote{https://www.linkedin.com/posts/adoption-share_by-inverting-the-outreach-for-adoption-matching-activity-7131265613947138048-YCM1}

Basically, you put your life on hold, and you have your hopes set on this one particular child that you’ve fallen in love with their profile and their picture. You dream every day and night and go to sleep hoping that this kid is the one, and every time that happens for me, 30-45 days later I would find out that I wasn’t chosen. So, there’s this tremendous sense of disappointment, rejection, and “why didn’t they choose me?” I went through that process over and over and over again for about 2+ years until I was finally connected with [a caseworker-driven search platform]. What I loved about the concept was that I wouldn’t have to go through that process. This time it was my profile, and I’d just have to wait for the caseworker or my forever match to find me. I’m happy to report that a couple weeks later I did find my forever match, and I have my son now.

In reaction to these challenges, some child welfare agencies have recently sought to improve outcomes by switching to caseworker-driven search (Riley 2019). In this approach, caseworkers sequentially contact specific prospective families to share details about the child. Caseworkers use
their expertise to decide which families to contact based on the child’s and the families’ characteristics. Optionally, technological tools may also aid their decisions. This approach removes the burden of having to engage with a large number of families simultaneously and allows caseworkers to target compatible families for children with very specific needs.

However, even though these are clear advantages of caseworker-driven search, it is important to note that changing the search paradigm may change the incentives of families to be interested in different types of children. For a match to occur in either discipline, both the caseworker and the prospective family need to agree. This can for example mean that if caseworker-driven search leads to a reduction in search costs for families, they may stop being interested in certain children, leaving those children unmatched. As such, it is unclear whether case-worker-driven search leads to more desirable outcomes for the entire population of children requiring adoptive placements.

To assess how different search strategies affect outcomes, we pursue two complementary approaches in this paper. First, we analyze the strategic behavior of agents under both search disciplines in a game-theoretic search-and-matching model. Second, we provide empirical evidence for the performance of caseworker-driven search compared to classic search approaches by studying outcomes of a real-world child welfare agency.

Our analytical contributions start with introducing a new search-and-matching model (Section 3), which captures critical features: First, as both approaches to adoption matching are inherently dynamic, we assume children and families (simply agents from now on) may enter and depart the system at any time. However, to keep the model tractable, we assume the distribution of agent types in the system remains stable over time. Second, we allow for uncertainty regarding whether a child-family pair would actually be compatible. Third, our model captures the heterogeneous preferences of agents, which is a key distinction between our paper and most earlier literature. While we focus specifically on analyzing adoption systems, our work is also relevant to more general search-and-matching theory. To the best of our knowledge, we are the first to formally analyze economic effects in the child welfare domain while taking heterogeneity of preferences into account.

We perform a game-theoretic comparison of the two approaches within our model. In Section 4.2, we establish that equilibria are guaranteed to exist under both search technologies (i.e., family-driven search and caseworker-driven search). Furthermore, we find that equilibria form a lattice reminiscent of the structure of the set of stable matchings in standard two-sided matching markets. We then present our main theoretical result: Family-driven search equilibrium outcomes can never be Pareto improvements over caseworker-driven search equilibrium outcomes, but there are instances where each caseworker-driven search equilibrium outcome is a Pareto improvement over all family-driven search equilibrium outcomes (Section 5.1). This holds because caseworker-driven search can reduce wasted search efforts. Thus, agents can worry less about accumulating search
costs when they express interest in a child. However, because of multiplicity of equilibria and the lattice structure over these equilibria, all children can be strictly better off in either system. The same holds for families. Even when both family-driven search and caseworker-driven search only admit a unique equilibrium each, an agent can be better off in either setting (Section 5.2). This may be surprising, given that caseworker-driven search reduces wasteful search efforts on both sides of the market. We therefore explore the conditions under which caseworker-driven search usually leads to more agents being better off. In Section 6.1, we show that all children will be better off in caseworker-driven search if families are sufficiently impatient. Furthermore, increasing family supply in family-driven search can have a negative effect on children’s utilities, but not in caseworker-driven search (Section 6.2). We find numerically that caseworker-driven search leads to more desirable outcomes for a wide variety of model parameter choices (Section 7).

To supplement this analytical and numerical insight, we then analyze case-level data from a technology platform that a multi-county child welfare agency in Florida used for the majority of its search efforts since 2018 (Section 8). We compare the outcomes for hundreds of children in need of adoptive placements to a benchmark from statewide case-level data that accounts for a child’s demographic information and disabilities. Besides dramatically reducing the search efforts and skill required by caseworkers, we show that the probability of adoption within three years was 17% higher than the benchmark from statewide data.

2. Related Literature

Recently, there has been increased interest from market designers and operations researchers in the challenges faced by historically underserved and disadvantaged communities. Researchers have, for example, studied refugee resettlement (Andersson et al. 2018, Bansak et al. 2018, Delacrétaz et al. 2020), the improvement of teacher quality at disadvantaged schools (Combe et al. 2022), and the management of volunteer workforces for non-profit organizations (Berenguer et al. 2023).

As part of this new focus on service systems for vulnerable populations, our paper contributes to the literature that studies child welfare systems from operations or market design perspectives — a challenge first articulated by (Spindler 1970). Slaugh et al. (2016) investigated how the Pennsylvania Statewide Adoption & Permanency Network could utilize a match recommendation tool to improve their process of matching children to prospective parents. Their spreadsheet-based tool can be seen as a simplified version of the previously mentioned data-driven software system. Robinson-Cortés (2019) worked with a foster care data set to analyze placements of children in foster homes. His model predicts that allowing placements across administrative regions would be beneficial for children. MacDonald (2019) studied a dynamic matching problem where children and families can either form reversible matches (foster placements) or irreversible matches (adoptions).
In her model, children are heterogeneous in the sense that there are children with disabilities and children without disabilities, while families are homogeneous. Baccara et al. (2014) estimate families' preferences over children available for adoption from a data set documenting the operations of adoption agencies. To the best of our knowledge, we are the first to formally analyze the economic effects of search and matching in the child welfare domain while taking heterogeneity of preferences into account. While we focus specifically on analyzing adoption systems, our work is also relevant for search-and-matching theory in general.

Within the child welfare literature, relatively little research has investigated the effectiveness of search strategies for children in need of adoptive placements. Some research has reported positive impacts from intensive multi-faceted search efforts by case workers: Vandivere et al. (2015) show that children served by recruiters from the Wendy’s Wonderful Kids organization were 1.7 times as likely to have an adoptive placement than a control group in an experiment with over 1,000 children. In a similar context focusing on hard-to-place youth in New York, Feldman et al. (2016) showed that a program of enhanced casework improved outcomes for children. The program utilized a variety of channels to promote 88 children and search for them. The search methods in both experiments require extensive work from skilled caseworkers funded by grants, while the platform we study provides an example of technology assisting caseworkers. Avery et al. (2009) study national photolisting service AdoptUSKids and use a hazards model to show better outcomes for children based on activity on AdoptUsKids. However, photolistings have drawn increased scrutiny since the early 2000s; Roby and White (2010) describes risks for exploitation and bullying for children publicly listed online.

Even though child adoption matching markets bear similarities to other two-sided matching markets such as centralized labor markets (Roth 1984, 1991) or ride-sharing platforms (Ma et al. 2020), there are important differences that necessitate new models and analyses. Adoption matching is inherently dynamic, and there is no centralized clearinghouse that determines final matches. Matches are only ever proposed, and both sides of the market have to invest search efforts to identify a match candidate. Purely random decentralized matching models have been widely studied under search frictions and homogeneous preferences (Eeckhout 1999, Shimer and Smith 2000, 2001, Atakan 2006) and more rarely under heterogeneous preferences (Adachi 2003). More recent studies have combined directional search rather than random search as an important feature (Lauermann and Nöldeke 2014, Lauermann et al. 2020, Cheremukhin et al. 2020).

One matching market similar to adoption from foster care is online dating (Hitsch et al. 2010a,b, Lee and Niederle 2015, Rios et al. 2021, Kanoria and Saban 2021). However, online dating markets are completely decentralized despite their dynamic and recommendation-based features. Individuals in search of a romantic partner can, at any time, decide to browse a dating platform and reach
out to other individuals who appeal to them. In contrast, the approaches we analyze in this paper follow a *centralized protocol* despite the dynamic decentralized search component: Caseworkers perform the search for a family on behalf of children, and they do so in approximately regular time intervals. Caseworkers therefore play an essential role throughout the process since they act as an intermediary to protect vulnerable children. Crucially, this introduces an asymmetry that we have not observed in any other previously studied matching market. As a consequence, we develop a new model that allows us to capture the features of the two different search technologies in one model.

Our work is also related to the literature on dynamic matching regarding the effects of congestion (Arnosti et al. 2021, Leshno 2022) and different practical policies (Ünver 2010, Akbarpour et al. 2020b, Sönmez et al. 2020, Akbarpour et al. 2020a, Kerimov et al. 2023) in various market-design environments, including settings intended to help vulnerable populations (Baccara et al. 2020, Kasy and Teytelboym 2020). Different studies investigate how matching platforms should be designed so that desirable outcomes can be achieved (see, e.g., Lee and Niederle 2015, Fradkin 2017, Akbarpour et al. 2020a, Altinok and MacDonald 2023, Dierks et al. 2024). The research in this area most closely related to our work is Shi (2023), in which the author explores which side of the market should drive the search process depending on which side’s preferences can easily be expressed or satisfied. The setting, however, is quite different from our work since we allow agents on both sides of the market to arrive over time and let them face an optimal stopping problem as they can decide to remain unmatched until better future match opportunities arise.

3. Preliminaries

In this section, we first introduce the notation that is used in both *family-driven search (FS)* and *caseworker-driven search (CS)*. We then describe the processes of the two different search technologies and derive characterizations of agents’ utilities.

3.1. Model

In our model, agents (children and families) have observable characteristics. Agents with the same characteristics are said to be of the same *type*. We treat sibling sets of children who should be placed together as a single child. Furthermore, we see the caseworker as a direct representative of the child, i.e., we think of a child-caseworker pair as one child agent. We let \( C = \{ c_1, \ldots, c_n \} \) and \( F = \{ f_1, \ldots, f_m \} \) denote the set of all \( n \) child types and the set of all \( m \) family types, respectively. Individual agents are indifferent between agents of the same type, i.e., their preferences are over agent types: A child of type \( c \) has a value \( v_c(f) \in \mathbb{R} \) for family type \( f \), and a family of type \( f \) has a value \( v_f(c) \in \mathbb{R} \) for child type \( c \). Preferences are assumed to be strict, i.e., \( v_c(f) \neq v_c(f') \) if \( f \neq f' \) and \( v_f(c) \neq v_f(c') \) if \( c \neq c' \). Agents’ valuations are summarized by a list of vectors
Given valuations \( v \), we let \( \bar{v} \) denote the maximal value of all \( v_c(f) \) and \( v_f(c) \).

There are infinitely many discrete time steps. At any time step, there is at most one agent of each type present in the system. Thus, we can use \( c \in C \) and \( f \in F \) to refer to either individual agents and agent types without ambiguity. We refer to an agent (or agent type) from either set as \( i \in A := C \cup F \). From now on, we will simply say that agent \( i \in A \) is active if they are present at the current time step.

At the beginning of each time step, all active agents are determined as follows: For each family type, one family of that type is active with probability \( \lambda \in (0, 1) \). This is determined independently for each family type. We call parameter \( \lambda \) the market thickness indicator, as it determines the expected number of active families at each time step: For small values of \( \lambda \), there will be few active families in expectation. For large values of \( \lambda \), it is quite likely that a family of each type will be active. Further, exactly one child (type) is selected uniformly at random to be active. This feature is motivated by search processes used by adoption agencies: A caseworker works on the case of one child at a time, which we assume she selects randomly.

There is uncertainty regarding whether a specific child \( c \) and a specific family \( f \) are compatible: With probability \( p \in (0, 1) \), \( f \) is a suitable match for \( c \) and unsuitable with probability \( 1 - p \). Whether a match is suitable or not is determined independently at random for child-family pairs. We refer to \( p \) as the match success probability. Parameter \( p \) captures the following aspect of adoption markets: When a family shows interest in a child, the family’s decision is based on limited reported information, such as the sex, ethnicity, age of the child, and known disabilities. However, there are many other important characteristics of a child that determine whether the child is actually a good match for the family and whether there is mutual attraction. The same holds for a child (or his caseworker) showing interest in a family. Only if a family \( f \) is a suitable match for child \( c \) can a match between \( c \) and \( f \) be formed. If \( c \) and \( f \) form a match they both obtain a value of \( v_c(f) \) and \( v_f(c) \), respectively. Determining the suitability of a match is costly for both sides, as it is a time-consuming process. Children and families incur search costs \( \kappa_C \in \mathbb{R}_+ \) and \( \kappa_F \in \mathbb{R}_+ \), respectively, each time the suitability of a match including them is determined. All agents discount the future, however, they only discount time steps in which they are active. Children’s and families’ discount factors are \( \delta_C \in [0, 1) \) and \( \delta_F \in [0, 1) \), respectively.

\(^2\)Our model does not endogenize the number of agents present in the system. This kind of instant replacement is a standard large-market assumption in the search-and-matching literature, and it is necessary to keep our model tractable.

\(^3\)An alternative interpretation is to think of children and families leaving the process before the next time step when they are active with probability \( 1 - \delta_C \) and \( 1 - \delta_F \), respectively.
We model adoption matching as a dynamic process that we assume to be stationary. An instance $(v, \delta_C, \delta_F, \kappa_C, \kappa_F, p, \lambda)$ together with a search technology (which we will introduce) induce a game. To reduce notation, we assume that instance $(v, \delta_C, \delta_F, \kappa_C, \kappa_F, p, \lambda)$ is fixed unless stated otherwise.

Agents’ strategies in this game are captured as follows: Child $c$ is either interested in a family of type $f$ or not. Similarly, each family $f$ is either interested in a child of type $c$ or not. We assume that all agents of the same type play the same strategy, and agents don’t change their strategies in different time steps. Therefore, we can represent a strategy for a child $c$ as a vector $s_c \in \{0, 1\}^m$, where $s_c(f) = 1$ if $c$ is interested in matching with a family of type $f$. Similarly, a strategy for a family $f$ is given by a vector $s_f \in \{0, 1\}^n$, where $s_f(c)$ indicates whether $f$ is interested in children of type $c$.

A strategy profile is a tuple of vectors $s = (s_{c1}, \ldots, s_{cn}, s_{f1}, \ldots, s_{fm})$, while we let $S$ denote the finite set of all possible strategy profiles. For $i \in A$ we let $s_{-i}$ denote the tuple of all agents’ strategies in $s$, except that agent $i$ is excluded. We say that $c$ and $f$ are mutually interested in each other under $s$ if $s_c(f) = 1$ and $s_f(c) = 1$. The set $M(s) = \{(c, f) \in C \times F \mid s_c(f) = s_f(c) = 1\}$ is called the matching correspondence of $s$. We use $M_i(s)$ to denote the set of agents that agent $i$ is mutually interested in under $s$.

In the remainder of this section, we describe the two different search technologies. We analyze the dynamic stochastic games induced by an instance and a search technology in a full information environment.

**Family-driven Search (FS):** At the beginning of each time step, after a child $c$ is randomly chosen to be active, each family that is active and interested in $c$ lets the caseworker of the child know of their interest. This corresponds to families responding to an email announcement made by a caseworker. The caseworker immediately discards any families without mutual interest in $c$, i.e., where $s_c(f) = 0$ or $s_f(c) = 0$. The caseworker then investigates all remaining families to determine whether they would actually be a suitable match. Recall that each investigated family is a suitable match for $c$ with probability $p$—which is determined independently for each family—and that each agent incurs cost for each investigation including them. After all families have been processed by the caseworker, $c$ either matches with the most-preferred choice from all those families that were identified as suitable matches or remains unmatched if no such family exists. We move on to the next time step.

**Caseworker-driven Search (CS):** After a child $c$ is randomly chosen to be active at the beginning of a time step, the child’s caseworker is presented with a list of active families ordered in decreasing order of $v_c(f)$. Families are then sequentially processed. The caseworker skips any family in which $c$ is not mutually interested. If there is mutual interest between $c$ and $f$, the suitability of a match between $c$ and $f$ is investigated. If the match turns out to be suitable, $c$’s search is over, $c$ and $f$ are matched and leave. We then move on to the next time step. Otherwise, the caseworker
continues the search by selecting the next family in the list. If all families have been processed, the child remains unmatched and we move on to the next time step. Proposition 10 in online Appendix A.1 shows that $c$’s utility is maximized if the caseworker processes families in decreasing order of $v_c(f)$.

3.2. Utilities

We define agents’ utilities at a time step and characterize their flow utilities in both FS- and CS-induced stochastic games. Assume child $c$ is active at the current time step and that active families are yet to be determined. Let $f$ be an arbitrary family. For any $s \in S$, let $b_{cf}(s) = |\{ f' \in M_c(s) \mid v_c(f') > v_c(f) \}|$ denote the number of families in $M_c(s)$ that $c$ likes better than $f$. Further, let $\beta_{cf}(s)$ denote the probability that $c$ will not match with any other family that $c$ prefers over $f$ at the current time step. Noting that for any child $c$ the probability that a mutually interested family $f'$ is active at the current time step and a suitable match is $\lambda p$, it follows immediately that $\beta_{cf}(s) = (1 - \lambda p) b_{cf}(s)$ for both FS and CS. In FS and CS, the expected immediate utility that active child $c$ will obtain at an arbitrary time step is then given by

$$\bar{u}^{FS}_c(s) = \lambda \sum_{f' \in M_c(s)} \beta_{cf'}(s) p v_c(f') - \kappa_C$$

and

$$\bar{u}^{CS}_c(s) = \lambda \sum_{f' \in M_c(s)} \beta_{cf'}(s) (p v_c(f') - \kappa_C)$$

respectively. Similarly, for a family $f$, we can express the expected immediate utilities at an arbitrary time step (conditional on $f$ being active) by

$$\bar{u}^{FS}_f(s) = \frac{1}{n} \sum_{c' \in M_f(s)} \beta_{c'f}(s) p v_{c'}(f) - \kappa_F$$

for FS and

$$\bar{u}^{CS}_f(s) = \frac{1}{n} \sum_{c' \in M_f(s)} \beta_{c'f}(s) (p v_{c'}(f) - \kappa_F)$$

for CS. From this, the crucial difference between FS and CS in our model becomes apparent: In FS, search costs are always incurred if there is mutual interest between agents. In CS, however, the sequential nature of the search means that search costs are only incurred if there is mutual interest and all previous match attempts at the time step have been unsuccessful.

We assume that each agent is risk-neutral and maximizes their expected (overall) utility, which is the expected discounted value of their eventual match minus the total discounted search costs they incur.\(^4\) We use $(z)^+$ as shorthand notation for $\max\{z, 0\}$. $\mathbf{1}[\cdot]$ is the indicator function, which

\(^4\)In practice, while caseworkers have multiple cases assigned to them, their goal with each case is to maximize the utility of the child belonging to that case. That is, based on our interviews with domain experts, the caseworker considers all assigned cases separately, and each child-caseworker pair is considered a separate self-interested agent.
has value 1 if its argument is true and value 0 otherwise. We denote the expected (overall) utility of agent \(i\) under strategy profile \(s\) by \(u_i^{FS}(s)\) in FS and \(u_i^{CS}(s)\) in CS. Whenever it is clear from context whether we are referring to FS or CS we will simply write \(u_i(s)\). Proposition 1 characterizes children’s and families’ utilities in FS and CS via balance equations.

**Proposition 1.** Given strategy profile \(s\), c’s utility in FS is the unique value \(u_c^{FS}(s)\) that satisfies

\[
u_c^{FS}(s) = \delta_C u_c^{FS}(s) + \lambda \sum_{f \in M_c(s)} \left( \beta_{cf}(s) p(v_c(f) - \delta_C u_c^{FS}(s)) - \kappa_C \right).
\]

(5)

Similarly, f’s utility in FS is the unique value \(u_f^{FS}(s)\) that satisfies

\[
u_f^{FS}(s) = \delta_F u_f^{FS}(s) + \frac{1}{n} \sum_{c \in M_f(s)} \left( \beta_{cf}(s) p(v_f(c) - \delta_F u_f^{FS}(s)) - \kappa_F \right).
\]

(6)

In CS, c’s utility in CS is the unique value \(u_c^{CS}(s)\) that satisfies

\[
u_c^{CS}(s) = \delta_C u_c^{CS}(s) + \lambda \sum_{f \in M_c(s)} \beta_{cf}(s) \left( p(v_c(f) - \delta_C u_c^{CS}(s)) - \kappa_C \right).
\]

(7)

Similarly, f’s utility in CS is the unique value \(u_f^{CS}(s)\) that satisfies

\[
u_f^{CS}(s) = \delta_F u_f^{CS}(s) + \frac{1}{n} \sum_{c \in M_f(s)} \beta_{cf}(s) \left( p(v_f(c) - \delta_F u_f^{CS}(s)) - \kappa_F \right).
\]

(8)

A formal proof can be found in online Appendix B.1. One difference between FS and CS is immediately apparent from the above balance equations. In CS, search costs only incur if previous match attempts have been unsuccessful. In FS, however, costs incur with certainty if there is mutual interest and the family is active.

**4. Equilibria**

We use a tie-breaking assumption, which allows us to exclude degenerate equilibria later on. After stating this assumption, we introduce two classes of strategies—one for CS and one for FS—and show that these classes capture agents’ best responses. This will be helpful for obtaining results later on. We show that for both search technologies equilibria always exist and that equilibria form a lattice.

**4.1. Threshold Strategies**

For both search technologies, we make the following tie-breaking assumption: If agent \(i\)’s utility would (weakly) increase from mutual interest with agent \(j\), then \(i\) will be interested in \(j\)—even if \(j\) is not interested in \(i\). Similarly, if agent \(i\)’s utility would decrease from mutual interest with agent \(j\), then \(i\) will not be interested in \(j\). This assumption allows us to exclude degenerate equilibria (e.g.,
no agent being interested in any other agent) later on without restricting agents in their endeavor to maximize their utility.

We now introduce threshold strategies for FS and CS. As we will see, our tie-breaking assumption implies that agents’ best responses belong to the class of threshold strategies. Note that a best response always exists, because $S$ is finite.

**Definition 1.** Child $c$ plays a CS threshold strategy (CS-TS) with threshold $y_c \in \mathbb{R}$ in $s$, if

$$s_c(f) = \frac{1}{p(v_c(f) - \delta_C y_c)} \geq \kappa_C$$

for all $f \in F$.  \hfill (9)

Family $f$ plays a CS-TS with threshold $y_f$ in $s$, if

$$s_f(c) = \frac{1}{p(v_f(c) - \delta_F y_f)} \geq \kappa_F$$

for all $c \in C$. \hfill (10)

Child $c$ plays an FS threshold strategy (FS-TS) with threshold $y_c$ in $s$, if

$$s_c(f) = \frac{1}{\beta_{cf}(s)v_c(f) - \delta_C y_c)} \geq \kappa_C$$

for all $f \in F$. \hfill (11)

Family $f$ plays an FS-TS with threshold $y_f$ in $s$, if

$$s_f(c) = \frac{1}{\beta_{cf}(s)v_f(c) - \delta_F y_f)} \geq \kappa_F$$

for all $c \in C$. \hfill (12)

In a CS-TS or an FS-TS, threshold $y_i$ can be interpreted as $i$’s reservation utility. Let $u_i^{FS}(s_{-i})$ and $u_i^{CS}(s_{-i})$ denote $i$’s utility from a best response to $s_{-i}$ in FS and CS, respectively. As before, we simply write $u_i^*(s_{-i})$ if there is no ambiguity. Proposition 2 shows that agents’ best responses always have the form of a threshold strategy.

**Proposition 2.** Let $i \in A$ and $s_{-i}$ be an arbitrary strategy profile of all agents excluding $i$. In both FS and CS, a best response of $i$ to $s_{-i}$ corresponds to a threshold strategy with threshold $u_i^*(s_{-i})$.

A formal proof can be found in online Appendix B.2. To derive results later on, it will prove useful to switch between thresholds and strategies. We therefore provide the following definition.

**Definition 2.** In CS, a strategy profile induced by threshold profile $y \in \mathbb{R}^{n+m}$ is denoted by $s^{CS}(y)$ and satisfies for each $i \in A$, $s_i^{CS}(y)$ is a CS-TS with threshold $y_i$ in $s^{CS}(y)$. In FS, a strategy profile induced by threshold profile $y \in \mathbb{R}^{n+m}$ is denoted by $s^{FS}(y)$ and satisfies for each $i \in A$, $s_i^{FS}(y)$ is a FS-TS with threshold $y_i$ in $s^{FS}(y)$.

\footnote{Note that our threshold strategies do not correspond to standard simple threshold strategy (where an agent is willing to match with another agent if and only if their match value is above a certain threshold). This is because we find that agents cannot always maximize their utility by playing simple threshold strategies, as Proposition 11 in online Appendix A.2 illustrates. Informally this follows because families’ search costs in FS can outweigh the expected benefit from being matched with a very desirable child because of the competition from other families that this child prefers.}
We again omit the superscript if this does not lead to ambiguity. It is trivial to obtain $s_{CS}(y)$ by inserting $y$ in the corresponding equations in Definition 1. Algorithm 1 from online Appendix C can be used to compute $s_{FS}(y)$. From now on, we use $\beta_{cf}^{FS}(y)$ and $\beta_{cf}^{CS}(y)$ as shorthand-notation for $\beta_{cf}(s_{FS}(y))$ and $\beta_{cf}(s_{CS}(y))$, respectively. Whenever it is clear from the context, we simply write $\beta_{cf}(y)$.

4.2. Equilibrium Existence and Lattice Structure

In this section, we show that Nash equilibria always exist under both search technologies. We say that strategy profile $s$ is an equilibrium in $FS$ (FSE) if $s$ is a Nash equilibrium in the game induced by FS. Analogously, strategy profile $s$ is an equilibrium in $CS$ (CSE) if $s$ is a Nash equilibrium in the game induced by CS. We use $S^{FS}$ to denote the set of FSE, and let $Y^{FS} = \{(u_{i}(s))_{i \in A} | s \in S^{FS}\}$ be the corresponding set of equilibrium threshold profiles in FS. For CS, those sets are defined analogously. Before we can prove that these sets are never empty, we need to define a partial order $\leq_{C}$ on $Y = [0, \bar{v}]^{n+m}$. Note that if agents only play individually rational strategies their utility is always lower bounded by 0 and upper bounded by $\bar{v}$.

**Definition 3.** Let $\leq_{C}$ be the partial order on $Y$, where for all $y, y' \in Y$ it holds that $y \leq_{C} y'$ if and only if $y_{c} \leq y'_{c}$ for all $c \in C$ and $y_{f} \geq y'_{f}$ for all $f \in F$.

Having defined partial order $\leq_{C}$, we can now prove that equilibria always exist in both settings. We state this result in Proposition 3.

**Proposition 3.** The set of FS and CS equilibrium threshold profiles $Y^{FS}$ and $Y^{CS}$ is non-empty and both $(Y^{FS}, \leq_{C})$ and $(Y^{CS}, \leq_{C})$ form complete lattices.

A formal proof can be found in online Appendix B.3. The proof proceeds by defining a best-response mapping and showing fixed-point, and therefore equilibrium, existence using Tarski’s fixed point theorem.

In general, there can be more than one FSE or CSE for a fixed instance $(v, \delta_{C}, \delta_{F}, \kappa_{C}, \kappa_{F}, p, \lambda)$. Proposition 3 not only guarantees that equilibria always exist in both settings, but also highlights that there is a special ordering over equilibria: there exists a child-optimal equilibrium that children unanimously prefer over all other equilibria; i.e., their utility is weakly higher compared to any other equilibrium. Similarly, there exists a family-optimal equilibrium that families prefer. From now on, we let $s^{co-CS}$ denote the child-optimal CSE (co-CSE), $s^{co-FS}$ the child-optimal FSE (co-FSE), $s^{fo-CS}$ the family-optimal CSE (fo-CSE), and $s^{fo-FS}$ the family-optimal FSE (fo-FSE). This is reminiscent of the structure of the set of stable matchings in standard two-sided matching markets (Knuth 1997).

6 Technically, we have a stochastic game model, and therefore, these are Nash equilibria of stochastic games. They correspond to the Markov-perfect Nash-equilibrium selection among subgame-perfect Nash equilibria if we consider the problem as a repeated game.
5. Comparison of Family-driven Search and Caseworker-driven Search

In this section, we investigate the impact of the two search technologies on equilibrium outcomes. Here, we present our main theoretical result: An FSE can never Pareto dominate a CSE, as any increase in utility for one agent can only arise if another agent lowers their interest threshold, corresponding to a decrease in that agent’s utility. There exist instances, however, where each CSE is a Pareto improvement over all FSEs. We further find that no approach is always preferable for either children or families.

5.1. Pareto Comparison

A natural way to determine which equilibrium outcomes are preferable is to check whether one equilibrium is a Pareto improvement over the other. We first formalize the Pareto dominance relationship for strategy profiles in our model.

**Definition 4.** Strategy profile $s \in S$ is a Pareto improvement over strategy profile $s' \in S$ if $u_i(s') \leq u_i(s)$ for all $i \in A$ and there exists $j \in A$, such that $u_j(s') < u_j(s)$.

Note that $u_i(s)$ either denotes $u_i^{CS}(s)$ or $u_i^{FS}(s)$, depending on whether we refer to $s$ as a CSE or an FSE. We find that FSEs can never Pareto dominate CSEs, but there are instances where each CSE Pareto dominates all FSEs. Before we can formally show this, we need to state two lemmas. The first lemma is useful for understanding why FSEs cannot Pareto dominate CSEs. Lemma 1 shows that if there is a pair with mutual interest in FS that is not present in CS, then at least one of the two agents in the pair must be strictly worse off in FS compared to CS.

**Lemma 1.** Let $s^{FS} \in S^{FS}$ and $s^{CS} \in S^{CS}$. If there exists $c \in C$ and $f \in F$, such that $(c, f) \in M^{FS}(s)$ and $(c, f) \notin M^{CS}(s)$, then either $u_c(s^{FS}) < u_c(s^{CS})$ or $u_f(s^{FS}) < u_f(s^{CS})$.

A formal proof can be found in online Appendix B.4. Intuitively, if two agents are not mutually interested in each other in CS but are in FS, then at least one of them had to lower their interest threshold in FS—which means that their optimal reservation utility is lower in FS. However, since the optimal reservation utility corresponds to the once-discounted utility, this agent must be strictly worse off.

The next lemma almost immediately follows from Lemma 1 and is used for the proof of Theorem 1 as well as for later results.

**Lemma 2.** Let $s^{FS} \in S^{FS}$, $s^{CS} \in S^{CS}$, and $c \in C$. If $M_c(s^{FS}) \subseteq M_c(s^{CS})$, then $u_c(s^{FS}) \leq u_c(s^{CS})$.

**Proof.** If $c$ responds to $s^{CS}_c$ with $s^{FS}_c$, $c$ is mutually interested in the same families as under $s^{FS}$ since $M_c(s^{FS}) \subseteq M_c(s^{CS})$. Further, $c$’s expected costs are weakly lower in CS compared to FS. Hence, there exists a strategy for $c$ in CS where $c$’s utility is weakly higher than $u_c(s^{FS})$. The fact that $c$ plays a best response in $s^{CS}$ completes the proof.
It is quite intuitive that a child \( c \) cannot be worse off under CS if all families that are mutually interested in \( c \) under FS are also interested in \( c \) under CS. This allows us to finally show the following theorem.

**Theorem 1.** An FSE can never be a Pareto improvement over a CSE. On the other hand, there exists an instance where all CSEs are Pareto improvements over all FSEs.

A formal proof can be found in online Appendix B.5. The main intuition for why an FSE can never be a Pareto improvement over a CSE is that the only way an agent can be better off in FS compared to CS is to have a higher chance of matching with someone they like. But with Lemma 1, this implies that some other agent had to lower their interest threshold, which means that their utility decreased. There are two reasons why a CSE can Pareto dominate an FSE. First, CS can save agents search costs. Second, because search costs are only incurred in CS if previous match attempts at the current time step have been unsuccessful, agents do not have to worry about accumulating search costs that much in CS unlike in FS. Therefore, agents are incentivized to express interest in more potential match candidates in CS compared to FS.

### 5.2. No Approach Dominates the Other

Even though CSEs can be Pareto improvements over FSEs, we find that CS is not always better for everyone compared to FS. In fact, a CSE might yield arbitrarily higher (or lower) utility for all children or families compared to an FSE.

**Proposition 4.** For any \( L > 0 \) and \( 0 < \epsilon < L \), there exists an instance where

1. the child-optimal equilibrium, which we denote as \( s^{co} \), is the same in both CS and FS, and similarly, the family-optimal equilibrium, which we denote as \( s^{fo} \), is the same in both CS and FS,
2. \( u_c(s^{co}) = L \) for all \( c \in C \) and \( u_f(s^{co}) \leq \epsilon \) for all \( f \in F \), and
3. \( u_c(s^{fo}) \leq \epsilon \) for all \( c \in C \) and \( u_f(s^{fo}) = L \) for all \( f \in F \).

A formal proof can be found in online Appendix B.6. It proceeds by constructing examples where child and family utilities are mismatched, causing the utility gap between child-optimal and family-optimal equilibria to be arbitrarily large in both FS and CS. Thus, depending on which equilibria are realized, both approaches can be arbitrarily better for either side of the market.

Both a single child and a single family can be arbitrarily worse off in equilibrium under CS, even if FS and CS admit only one equilibrium each. This is highlighted by the following two propositions.

**Proposition 5.** There exists an instance where a child is strictly worse off under the unique CSE compared to the unique FSE.
A formal proof can be found in online Appendix B.7. It proceeds by constructing an example with two child and family types where child $c_1$ is significantly preferred over $c_2$ by all families, while all children slightly prefer $f_1$ over $f_2$. This implies that family $f_2$ is only matched with child $c_1$ if no $f_1$ family is currently present. The higher search costs in FS then can make $f_2$ lose interest in $c_1$, regardless of patience levels, causing $f_2$ to settle for $c_2$. This allows $c_2$ to be matched. Conversely, in CS, family $f_2$ does not incur high search costs for waiting until they are matched with a $c_1$. If they are patient enough, $f_2$ therefore prefers waiting for their preferred choice $c_1$. This leaves $c_2$ without any family interested in them and, therefore, unmatched.

Similarly, a family can be worse off in CS when FS and CS each only admit one equilibrium.

**Proposition 6.** There exists an instance where a family is strictly worse off under the unique CSE compared to the unique FSE.

A formal proof can be found in online Appendix B.8. Just as children can benefit from families that decide to settle for a less preferred child, so can other families. It can be the case that a family $f$ is interested in a child $c$ in a CSE but $f$ is not interested in $c$ in an FSE, because the associated expected costs would be too high. Not having $f$ as competition might be enough incentive for another family $f'$ to be interested in $c$ under the FSE. As a result, $f'$ can be strictly better off in FS.

### 6. Effects of Model Parameters

We showed that FSEs cannot be Pareto improvements over CSEs, but CSEs can be Pareto improvements over FSEs. Additionally, we found that some agents can be better off under an FSE compared to a CSE. In order to better understand the conditions under which one of the two approaches might be preferable, we explore the effects that different parameters have on equilibrium outcomes in FS and CS. We provide two more results in favor of CS: First, we show that as families’ patience decreases, at some point all children will be weakly better off in any CSE compared to any FSE. Second, increasing supply on the family side, i.e., increasing the market thickness indicator $\lambda$, can negatively affect children’s utilities in FS but not in CS. Finally, as a sanity check, we investigate the effect of certain parameters or parameter combinations in the limit; all of these latter results can be found in Section D.

#### 6.1. Discount Factors

For this subsection, let $S^{CS}(\delta_F')$ and $S^{FS}(\delta_F')$ denote the set of CSEs and FSEs when $\delta_F = \delta_F'$, respectively. We now show that as families’ patience decreases below a certain threshold, all children will always be better off in CS compared to FS.
Proposition 7. For each instance there exists $\delta_F \in [0, 1)$, such that for all $\delta_F' \in [0, \delta_F]$ it holds that $u_c(s^{FS}) \leq u_c(s^{CS})$ for all $c \in C$, $s^{CS} \in S^{CS}(\delta_F')$, $s^{FS} \in S^{FS}(\delta_F')$.

A formal proof can be found in online Appendix B.9. Intuitively, the statement follows because in CS, families being interested in a very unlikely match incurs them lower search costs than in FS. While patient families may still not be interested in some children in CS that they are interested in under FS (which drives Proposition 5), any sufficiently impatient family will be unwilling to wait.

However, as can be seen in the proof of Proposition 6, an analogous statement for families’ utilities and children’s patience level does not hold. Intuitively, a family $f$ might be worse off in CS, because another family $f'$ is not shying away from $c$, as $f'$ does not have to worry about accumulating wasted search efforts in CS.

6.2. Market Thickness
Adoption agencies might intuitively prefer to have a larger pool of available families to choose from. Here, we present another result which suggests that this might be generally good in CS but not always in FS when it comes to children’s utilities. Increasing supply on the family side, i.e., increasing the market thickness indicator $\lambda$, can negatively affect children’s utilities in FS but not in CS. For the remainder of Section 6.2, assume that all instance parameters are fixed except for $\lambda$. Let $s^{co-CS,\lambda}$ denote the child-optimal CSE given market thickness indicator $\lambda$. Definitions for $s^{fo-CS,\lambda}$, $s^{co-FS,\lambda}$, and $s^{fo-FS,\lambda}$ are analogous. Proposition 8 shows that increasing $\lambda$ can lead to some children being worse off in FS.

Proposition 8. There exists an instance with a child $c \in C$ and $\lambda, \lambda' \in (0, 1]$ with $\lambda < \lambda'$, such that $u_c(s^{co-FS,\lambda}) > u_c(s^{co-FS,\lambda'})$.

A formal proof can be found in online Appendix B.10. Effectively, what is happening is that if multiple families are interested in a child, then increased market thickness $\lambda$ increases competition for the child and therefore the search costs for the less preferred families. If the child is close to indifferent between families, but some families lose interest due to the higher cost, then the resulting decrease in the child’s utility can be larger than the increase caused by a higher chance to match with a slightly more preferred family.

In CS, on the other hand, increasing $\lambda$ can only have a positive effect on children’s utilities in equilibrium outcomes.

Proposition 9. Let $\lambda, \lambda' \in (0, 1]$, such that $\lambda \leq \lambda'$. Then $u_c(s^{co-CS,\lambda}) \leq u_c(s^{co-CS,\lambda'})$ and $u_c(s^{fo-CS,\lambda}) \leq u_c(s^{fo-CS,\lambda'})$ for all $c \in C$.

A formal proof can be found in online Appendix B.11. The reason why this holds in CS is that, unlike in FS, families will not shy away from children in whom they are interested just because
the probability of matching with them decreases. This result is reminiscent of a similar result in standard two-sided matching markets, as the number of agents in one side increases, the other side agents become all unambiguously better off under side-optimal stable matchings (Gale and Sotomayor 1985). However, it only holds for caseworker-driven search and only for the children’s welfare.

7. Numerical Evaluation
We previously established that CSEs can be Pareto improvements over FSEs while FSEs cannot be Pareto improvements over CSEs, and that agents can be better off in either approach (see Theorem 1 and Proposition 4). Additionally, we have shown that all children will be better off in CS compared to FS if families are sufficiently impatient. Here, we present numerical results to further investigate under which conditions children and families will be better off in CS or FS. Our results suggest that CS is almost always preferable for both sides of the market. Only when agents’ preferences are perfectly correlated and families are very patient, we find that on average there are more children types better off in FS compared to CS in equilibrium.

Section 7.1 describes how our numerical experiments are set up. We then explain how equilibria are computed in Section 7.2. In Section 7.3, we compare FS and CS in terms of their Pareto dominance relationship. We further quantify how many agents are typically better off in either approach.

7.1. Setup
We now describe the setup of our numerical evaluation. We set the number of agent types on each side to be \( n = m = 50 \).

**Valuations:** For the generation of agents’ valuations, we follow other approaches from the matching literature (Abdulkadiroğlu et al. 2015, Mennle et al. 2015). Each agent type \( i \in A \) is uniformly assigned a “quality” \( q_i \) at random from \([0, 1] \). Then, for each child-family pair \((c, f) \in C \times F \), idiosyncratic values \( \eta_c(f) \) and \( \eta_f(c) \) are randomly drawn from \([0, 1] \). For a given value \( \alpha \in [0, 1] \), we obtain the preliminary valuations \( v'_c(f) = \alpha q_f + (1 - \alpha) \eta_c(f) \) and \( v'_f(c) = \alpha q_c + (1 - \alpha) \eta_f(c) \). Note that as \( \alpha \) increases, agents’ preferences become more similar and end up being identical (vertical) for \( \alpha = 1 \). Final valuations \( v \) are obtained by normalizing \( v' \), such that the minimal and maximal value that each agent has for a match is 0 and 1, respectively.

**Data:** We generated 200 quality-value pairs \((q^{(1)}, \eta^{(1)}), \ldots, (q^{(200)}, \eta^{(200)}) \) as described above. Parameters \( p \) and \( \lambda \) are chosen to be \( p = \lambda = 0.5 \), and we let \( \delta := \delta_C = \delta_F \) and \( \kappa := \kappa_C = \kappa_F \). For each pair \((q^{(k)}, \eta^{(k)}) \), we computed the child-optimal CSE/FSE and the family-optimal CSE/FSE for each combination of \( \alpha, \delta, \) and \( \kappa \), where \( \alpha \in \{0, 1/3, 2/3, 1.0\} \), \( \delta \in \{0.8, 0.9, 0.975, 0.99\} \), and \( \kappa \in \{0.01, 0.02, 0.05, 0.1\} \). Thus, we consider \( 200 \cdot 4 \cdot 4 \cdot 4 = 12800 \) different instances and compute a total of 51200 equilibria.
7.2. Equilibrium Computation

The mapping $T$ defined in the proof of Proposition 3 can be used to find the child-optimal and family-optimal equilibria. The following procedure converges to an equilibrium threshold profile:

Start from the $\leq C$-minimal element in $Y$ and recursively apply $T$ to it. This produces a sequence $y^0, y^1, y^2, \ldots$ of threshold profiles, which converges to the fo-FSE. Starting from the $\leq C$-maximal element yields the co-FSE. In order to terminate after a finite number of steps, we force the procedure to stop once $|y^k_i - y^{k+1}_i| \leq \epsilon$ for all $i \in A$ for some previously chosen small parameter $\epsilon > 0$. The threshold profiles obtained by this procedure can then be mapped to the corresponding strategy profiles. In our numerical experiments, we have performed additional checks to confirm that the computed strategy profiles are in fact equilibria.

7.3. Results

Before comparing FS and CS, we first note that family and child optimal equilibria in FS coincide roughly 97% of the time. The same holds for CS. For simplicity, we only consider family-optimal equilibria in our analysis. As equilibria are almost always unique within each search technology, results for child-optimal equilibria do not differ markedly even with substantial differences between FS and CS. Of all cases considered, the CSE and the FSE only coincide once in the sense that the same agents are mutually interested in each other.

Consistent with Theorem 1, the family-optimal FSE never represents a Pareto improvement upon the corresponding family-optimal CSE. However, for approximately 22% of all instances, the CSE Pareto dominates the corresponding FSE. Figure 1 shows the distribution of cases in which the CSE dominates an FSE for different discount factors and levels of correlation among preferences. Two insights emerge from this analysis: First, as agents become more impatient, CSEs more frequently constitute Pareto improvements over FSEs. As indicated by Proposition 7, once families become sufficiently impatient, any CSE will Pareto dominate all FSEs. Second, when preferences exhibit high correlation, CSEs rarely Pareto dominate FSEs. The case of vertical preferences—i.e., $\alpha = 1$—helps to explain this effect. If agents are patient enough, a family $f$ in the CS regime might wait for an opportunity to match with a high-type child $c$, even if $f$ is not $c$’s first choice and $f$ must wait a long time until getting matched. In FS, however, if there are enough other families that $c$ prefers over $f$, $f$ or $c$ might shy away from being interested in order to avoid accumulating search costs for such an “unlikely” match. In that case, $f$ might settle for another low-type child or multiple low-type children instead (see the example from the proof of Proposition 5). These low-type children now benefit from FS, while $f$ will be worse off in FS compared to CS.

The previous Pareto comparison only allows for a very high-level comparison of FS and CS. In order to better understand the conditions under which certain agents benefit from FS or CS,
we evaluate how many agents are better off in CS and how many agents are better off in FS. Our numerical experiments show that all families are almost always better off in CS. We refer the reader to Section E.1 of the e-companion for more details on families’ statistics. For children, the combination of model parameters affects which approach appears more appealing. Figure 2 shows how many children are (strictly) better off (in terms of utilities) in CS and FS for different parameter combinations.

CS provides higher utility than FS for almost all children when agents are sufficiently impatient (e.g., $\delta = 0.8$) because CS allows agents to express interest in more potential match partners without risking wasted search efforts. Being interested in more agents increases the probability of getting matched at each time step, which is especially valuable to children when patience is low. On the other hand, FS incentivizes agents to focus on a smaller set of match candidates due to higher expected total search costs. When $\delta = 0.8$, families will on average be interested in 35.9
and 43.5 child types in FS and CS, respectively. Interestingly, more children benefit from FS than CS when agents are extremely patient and agents’ preferences are almost completely aligned. This explains why CSEs are less frequently Pareto improvements over FSEs under these conditions, as we previously saw in Figure 1.

![Figure 3](image)

**Figure 3** The ratio of children who are on average (strictly) better off in terms of (expected discounted) match value in either approach in the family-optimal equilibrium for different combinations of agents’ patience and the level of preference correlation.

Figure 3 shows that CS not only reduces wasted search efforts in many cases but also enables children to match with more preferred families. We calculate a child’s match value as the child’s utility ignoring the expected search costs, which might be less relevant to a policymaker trying to improve child outcomes.

8. **Empirical Evidence from a Field Implementation**

To validate our model and understand the real-world implications of switching from an FS to a CS approach, we analyze children’s outcomes for a multi-county region in Florida that implemented a CS approach on July 1, 2018, by adopting a technology platform developed by a nonprofit organization. The agency’s previous search approach relied on regular email announcements to a pool of registered families to advertise children in need of adoptive families. In the new approach, the platform allows prospective adoptive parents to complete a questionnaire. Its singular role is to recommend families for a child’s caseworker to consider. The region, one of 20 “circuits” in Florida, is administered by a non-profit agency under contract with the state. Out of frustration with the difficulties in finding adoptive placements for children, the leadership of the non-profit child welfare agency for this circuit decided to implement the technology platform and follow a caseworker-driven search strategy.

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7 The probability of matches occurring is another metric of interest to stakeholders. Our findings on match probabilities can be found in Section E.2.
This analysis compares the performance of the region’s outcomes for children from the Adoption and Foster Care Analysis and Recruiting System (AFCARS) from FY2015-FY2021, which is the most recently available data on both foster care cases and finalized adoptions. To assess performance, we aggregate individual-level predictions and outcomes for the agency’s children listed on the platform. While the agency listed all or nearly all children on the platform — which is unique among agencies implementing that platform — a small number of children also found placement through other channels, such as word of mouth within the agency, Florida’s online photolisting websites, and other contracted recruitment efforts for specific children. Including these placements helps to understand whether the population of waiting children is better served once an agency implements the platform.

Florida experienced at least two dramatic shocks to its child welfare system in the years over which the platform was implemented. As shown in Figure 4, the statewide population of children legally free for adoption — which also includes children on a path to adoption by relatives and foster parents — increased by nearly 50% from 2015 to 2018, possibly due to social challenges such as the opioid epidemic (Florida Department of Children and Families 2019, 2023). Despite this increase in children needing adoptive placements, the state’s reported metric of the percentage of children eligible for adoption on June 30 of some year and adopted by July 1 of the following year remained above 55% from 2017 until 2019. However, that metric dropped below 50% in the years since the coronavirus pandemic, and the last reported value of 46.7% for 2022-2023 is the lowest of all years considered. Thus, maintaining pre-pandemic outcome levels may constitute success for the region implementing the platform.
8.1. Data

Our analysis relies on multiple data sets: the AFCARS Foster Care 6-month File (Children’s Bureau, Administration on Children, Youth and Families 2023b), the AFCARS Adoption File (Children’s Bureau, Administration on Children, Youth and Families 2023a), and case history data from the platform. Regrettably, and despite significant efforts, the agency implementing the platform could not extract information from Florida’s statewide case management system and assemble its own comparable case history dataset, although we provide validation in Section 8.3 that uses circuit-level data in state reports. Filtering children based on case goals and the relationships with adoptive families proved to be the biggest obstacles to using the agency’s data. However, the agency was helpful in manually tracking down the outcomes of children who left the platform without an adoptive placement. From 766,527 foster care 6-month update records, we identified 10,284 children as legally free and clear for adoption with cases starting after October 1, 2014, which is the closest federal fiscal year cut-off for the data in Figure 4. Children’s timelines begin with the termination of parental rights (TPR) order and end with the last status update, which could be the adoption finalization date. To qualify for this analysis, children had to have a case goal of adoption in their final record or previously had a case goal of adoption with (a) a resulting non-relative adoption, (b) a final case goal of “emancipation”, or (c) a discharge reason of “emancipation”. We excluded children listed in the Adoption File as being adopted by a relative, step-parent, or foster parent. Because the most recent Adoptions File only covers adoptions through September 30, 2021, we excluded case data past that date as we would not be able to tell if adoptions resulted from relative, step-parent, or foster placements. We also excluded any cases with a duration less than 120 days — a 30-day appeal window after TPR before a search can start plus a legal minimum of 90 days between placement with the adoptive family and adoption finalization — or over 18 years.

We note that the AFCARS datasets’ inability to explicitly identify children in need of adoptive resources results in a conservative benchmark on the platform’s performance; i.e., it will make the platform appear less helpful than it actually was. Specifically, some children included in the AFCARS dataset may quickly find placements with non-relatives, such as teachers, church members, or neighbors. Such children have short times until adoption but are typically never listed on the platform. While we filter out cases with implausibly short durations of 120 days or fewer, some of these cases with durations exceeding 120 days may still be included in the AFCARS benchmark model, as we cannot identify them separately. Child welfare professionals have also mentioned the possibility that users of the AFCARS reporting system may inadvertently classify adoptions by foster parents as non-relative adoptions. These potential limitations show the importance of more explicitly identifying children for whom an active search is being conducted in child welfare data sources to aid in evaluating performance.
We received case data from the platform about 369 children in need of adoptive placements who were listed by the agency. The platform provided its first matches around July 1, 2018. Of these children listed on the platform, 176 had finalized adoptions by February 1, 2023. Of the finalized adoptions, the adoptive parents for 143 children were found through the platform. We excluded an additional 66 children who were listed at one time on the platform but achieved permanency through adoptive placement with relatives or foster care parents or were reunified. Our data only includes activity and case updates through February 1, 2023; children adopted after that date are not counted as adopted in our dataset. For example, on February 1, 2023, 25 children were in placements intended to lead to an adoption, but that had not yet been finalized.

8.2. Results

| Attribute | AFCARS (N=10,284) | Platform (N=279) |
|-----------|------------------|------------------|
| Case Duration (years) | Mean (SD) | Mean (SD) |
| Adopted before End of Data Horizon | 1.47 (1.17) | 1.60 (0.92) |
| Age at TPR (years) | Age | 7.71 (5.16) | 9.05 (4.89) |
| Sex | Female | 48.9% | 42.3% |
| | Male | 51.1% | 57.7% |
| Race (may be multiple) | | |
| American Indian or Alaskan Native | 0.3% | 0.0% |
| Asian | 0.6% | 0.0% |
| Black or African American | Black | 36.1% | 23.3% |
| Native Hawaiian/Other Pacific Islander | 0.2% | 0.0% |
| White | 70.5% | 65.2% |
| Other | N/A | 11.5% |
| Hispanic or Latino Ethnicity | Hispanic | 14.6% | 3.6% |
| Clinical Disability Diagnosis | Disability | 32.7% | 37.6% |

Like Avery et al. (2009), we model children’s time to placement using a Cox proportional hazards model (Cox 1972). The Cox proportional hazards model includes a baseline hazards function for how the likelihood of a placement changes over time and a parameter for each covariate that affects the baseline hazard. The following characteristics were available in both AFCARS and the platform data, allowing us to control for them in the hazards model (for discrete variables, one of the categories is omitted for statistical identification):

1. female (versus male, the omitted category);
2. Black or African-American (versus neither Black nor African-American, the omitted category);
3. Hispanic or Latino ethnicity designation (versus no such designation, the omitted category);
4. clinical disability diagnosis (versus no diagnosis, the omitted category);
5. age in years upon termination of parental rights; and
6. federal fiscal year (e.g., October 1, 2014, to September 30, 2015, for FY2015) of the TPR order, with the fiscal year from October 1, 2020 to September 30, 2021 or FY2021 as the omitted category.

Table 1 provides summary statistics for these variables. Because the most recent AFCARS files are only available through September 30, 2021, we additionally excluded 90 cases that began on the platform after that date due to our inability to estimate their entry-year effect from the AFCARS data. The platform’s population of children tends to be older, more male, and more likely to have a clinically diagnosed disability — all factors associated with greater difficulty in placing children. A lower percentage of children on the platform tended to be white, although discrepancies exist in how the AFCARS data and the platform categorize race. To be conservative in our estimation of the platform’s performance, we accommodate race through a variable for children identified as Black; i.e., counting some of the children served by the platform in the “Other” category as part of a racial group associated in other research with lower rates of permanency would lower the performance benchmark for the platform.

Table 2 shows how basic demographic characteristics affect the time to placement using the AFCARS dataset for Florida children. One model includes controls for the fiscal year in which TPR occurred, and a second simpler model only includes demographic characteristics. Due to its heightened predictive power, we choose to use Model 1 for predicting child outcome probabilities and establishing the AFCARS benchmark. Controlling for the fiscal year of the start of the search, covariates associated with a faster placement are being female and having Hispanic ethnicity; factors associated with a slower time to or diminished likelihood of adoption are being older, having a disability, and being Black. Using this model, we collected the necessary data on each child served by the platform, and we predicted the likelihood of a finalized adoption at monthly intervals up to the duration for which the child was in need of an adoption placement. Children listed before the COVID years, FY2020 and the control year FY2021, were likely to be adopted faster than those with TPR in FY2020 and FY2021. If a child was adopted, the predictions continued assuming the child would have been active until February 1, 2023, or the child turned 18, whichever came first.

By aggregating the predicted adoption probabilities of all children served by the platform, we can establish a benchmark against which to compare the actual adoptions of children on the platform. Let \( C \) represent the set of children. For any child \( i \) with attributes \( X_i, i \in C \), we can predict the survival probability using the adoption probability function \( \hat{\pi}(X_i, t) \) that child \( i \) has a finalized adoption within \( t \) years. The child’s time until adoption finalization, unsuccessful search conclusion, or February 1, 2023, is recorded as \( \tau_i \), and \( \tau_i^e \) extends \( \tau_i \) to include an extended search horizon until the child turns 18 for children who are adopted. We also have a delay value \( \tau_i^d \) for the time between TPR and registration on the platform. This time duration is estimated as the platform
allows caseworkers to register children and select time buckets — for which we use the mid-point — from the time since TPR instead of a specific date. We provide an alternate analysis in which we assume $\tau^d_i = 0$ in online Appendix F. Thus, we calculate a conditional survival probability for child $i$ as

$$\tilde{\pi}(X_i, t, \tau^d_i) := \frac{\hat{\pi}(X_i, t) - \hat{\pi}(X_i, \tau^d_i)}{1 - \hat{\pi}(X_i, \tau^d_i)},$$

which provides the AFCARS benchmark $\mu(t)$ for the number of expected matches by time $t$:

$$\mu(t) := \sum_{i \in C} \tilde{\pi}(X_i, \min\{t, \tau^r_i\}, \tau^d_i).$$

Figure 5 shows the adoptions achieved through the platform compared to this benchmark from the AFCARS proportional hazards model. The results show that the platform has outperformed the commonly used two-year and three-year search window benchmarks. For children listed on the
platform before October 1, 2021, 138 adoptions were finalized within two years — 123 enabled by the platform and 15 through other channels — while the predicted number for children on the platform was only 124.5. At the three-year mark, this difference between adoptions achieved and the predicted number extends to an extra 24.3 adopted children, or a 17% increase over the benchmark. However, the platform’s performance fell short of the AFCARS benchmark one year after the search began, even though the platform not only made up for this gap but surpassed the benchmark at the two-year and three-year marks. The difference in performance may be attributed to a limitation of the AFCARS datasets mentioned above: some cases included in the benchmark might not have actually required a search for a family but rather had a potentially faster and easier non-relative adoptive placement. Despite this disadvantage, the platform’s over-performance over longer time horizons may indicate that the technology helps caseworkers to be more persistent in their search efforts for hard-to-place children. When Avery (2000) investigated the longest-waiting children in New York, she found case workers to be pessimistic about the children’s chances at adoption and that the case workers failed to use search tools available to them. However, this case study shows that providing leads of potential adoptive families may encourage caseworkers to persist in finding families.

The agency has shared other benefits of implementing the platform. First, it enabled caseworkers to work more efficiently by avoiding the process of asking families to respond to announcements and saved time and emotional energy for all participants. Other programs, such as the diligent recruitment program described by Feldman et al. (2016), that showed improvement in children’s outcomes required advanced skills and high levels of labor from caseworkers, who are part of a
workforce suffering from high burnout and turnover amidst increasing pressures on Florida’s child welfare system. With technology like the platform that we study, overburdened caseworkers could reallocate time spent reading dozens or hundreds of home studies from prospective families who expressed interest in a particular child. Second, the platform also helped to protect the privacy of the children in need of adoptive placements. Without the caseworker-driven search practice, their status as children in need of adoption would have been announced much more widely.

8.3. Validation from State Reports
To validate this analysis — especially to understand how the circuit that implemented the platform compares to statewide averages before and after implementation — we use the “Adoption Incentive” annual reports published by the Florida Department of Children and Families (2019, 2023). The analysis using AFCARS data implicitly assumes that the agency is representative of statewide patterns; if the agency already outperformed statewide averages before 2018, the analysis would overestimate the effect of the platform. The Adoption Incentive reports provide annual statistics for how the 20 circuits in Florida perform on various measures. Of the provided statistics, adoption success is best measured by the “number of children who were eligible for adoption on 7/1 who were adopted by 6/30,” which is displayed in Figure 4. In this case, eligibility refers to children for whom a termination of parental rights order has been granted. We estimate that the children listed on the platform comprise between one-quarter and one-third of all eligible children. The remaining children likely already had a path to adoption identified through a foster parent or relative and would be expected to have a faster path to adoption.

In Table 3, we compare the circuit’s average performance against the statewide average for the four years before implementing the platform and the four years after implementation. Because the monthly case creation peaked after July 1, 2018, as the platform’s usage gradually ramped up in 2018, we disregard the annual report for July 1, 2018, to June 30, 2019, as a transition period. Thus, we compare the mean across annual statistics from July 1, 2014, until June 30, 2018, against July 1, 2019, until June 30, 2023. Before implementation, the agency’s performance was only 3% higher than the statewide average, corresponding to an average ranking among all circuits of 9.25 out of 20. Thus, we expect the statewide AFCARS case data used for benchmarking to be accurate.

Considering the averages over the four years since implementation, the circuit has outperformed statewide averages. We note that the statewide average for the percentage of eligible children adopted decreased compared to before 2018, which could reflect higher acuity in children’s needs or increased difficulties in casework and judicial processes from the COVID-19 pandemic. The mean ratio for the circuit metric compared to the statewide metric increased to 13%. While it is difficult to directly link the percentage of eligible children adopted over a one-year time frame to the outcomes
explored in Section 8.2, this indication that the circuit has improved its performance in relation to the state as a whole lends credence to the value of our previous analysis using benchmarks from statewide AFCARS data.

| Table 3 | Percent of eligible children adopted each year for the circuit that implemented a caseworker-driven search platform compared to state averages. |
|---------|---------------------------------------------------------------------------------------------------------------------------------|
|         | Comparison Period | Before 7/1/2014-6/30/2018 | After 7/1/2019-6/30/2023 |
| Circuit Annual Mean % of Children Adopted | 57% | 57% |
| Statewide Annual Mean % Adopted | 55% | 51% |
| Circuit-Statewide Ratio Mean | 103% | 113% |
| Mean Rank (of all 20 circuits) | 9.25 | 7.00 |

9. Discussion and Conclusion

We are the first to apply market design principles and a formal game theoretical model to study the process by which children in the child welfare system are adopted. We introduced a novel search-and-matching model to compare two competing search paradigms. First, we analyzed the Nash equilibria of the games induced by our model. We show that agents follow novel threshold strategies as best responses, subject to a tie-breaking assumption. Also, Nash equilibria form a non-empty complete lattice in either setting.

Search frictions caused by the nature of the adoption process present one of the biggest obstacles to efficient matching efforts. We found that decreasing wasted search efforts leads to generally better outcomes, which are realized in caseworker-driven search. We explored the nuances of this finding to identify circumstances for which this intuition fails to hold.

We have shown that caseworker-driven search can Pareto dominate family-driven search, but no equilibrium of family-driven search can ever Pareto dominate any caseworker-driven search equilibrium. Despite less wasted search efforts, caseworker-driven search equilibria do not always Pareto dominate those of family-driven search. However, we numerically investigated a wide range of underlying model parameters to observe that caseworker-driven search is better for most agents.

Conversely, we also identified environments in which caseworker-driven search would not be desirable. When agents are very patient and preferences are highly correlated, we have numerically shown that a majority of children are worse off in caseworker-driven search. However, such a level of patience is unlikely to be prevalent in practice. Not only is it in children’s best interest to minimize the time in foster care, but we know from domain experts that most families’ patience is also limited: their wish to adopt a child is often very strong after completing the strenuous process of a home study investigation. We have also shown that in such cases — i.e., if families are sufficiently
impatient — all children are unambiguously better off in caseworker-driven search compared to family-driven search, regardless of correlation in preferences. Moreover, agents’ preferences are far from being highly correlated in practice. Domain experts report idiosyncratic preferences of families based on different characteristics of children such as ethnicity, gender, or age.

These results suggest that adoption agencies should consider using search technologies that allow caseworkers to perform a targeted search, rather than a strategy of broad announcements to which families respond. This is validated in our case study of an adoption agency in Florida, which, since switching to caseworker-driven search, has outperformed a state-wide benchmark for adoptions at the three-year mark by 17%. Our findings show the potential for replacing the main search paradigm in adoption, family-driven search, with newer caseworker-driven search paradigm.

Our work also motivates future empirical research to understand the operations of a child welfare matching system at a granular level. Little is known about the efficacy of different search methods and how caseworkers employ them. Understanding how caseworkers perform their jobs and how technology can help them becomes increasingly important as the workforce faces challenges of burnout and turnover. Furthermore, studying families’ interactions with the platform and the trajectories of their engagement over time represent additional important subjects of analysis. Finally, studying how different search practices affect children who participate in them represents another critical way to increase our understanding of how to serve and protect this vulnerable population.
References

Abdulkadiroğlu A, Che YK, Yasuda Y (2015) Expanding “Choice” in School Choice. *American Economic Journal: Microeconomics* 7(1):1–42.

Adachi H (2003) A search model of two-sided matching under nontransferable utility. *Journal of Economic Theory* 113(2):182–198.

Akbarpour M, Combe J, He Y, Hiller V, Shimer R, Tercieux O (2020a) Unpaired kidney exchange: Overcoming double coincidence of wants without money. *Proceedings of the 21st ACM Conference on Economics and Computation*, 465–466.

Akbarpour M, Li S, Gharan SO (2020b) Thickness and information in dynamic matching markets. *Journal of Political Economy* 128(3):783–815.

Altinok A, MacDonald D (2023) Designing the menu of licenses for foster care. *Available at SSRN 4466506* .

Andersson T, Ehlers L, Martinello A (2018) Dynamic Refugee Matching. Technical report, Lund University, Lund, Sweden.

Arnosti N, Johari R, Kanoria Y (2021) Managing congestion in matching markets. *Manufacturing & Service Operations Management* 23(3):620–636.

Atakan AE (2006) Assortative Matching with Explicit Search Costs. *Econometrica* 74(3):667–680.

Avery RJ (2000) Perceptions and practice: Agency efforts for the hardest-to-place children. *Children and Youth Services Review* 22(6):399–420.

Avery RJ, Butler J, Schmidt EB, Holtan BA (2009) AdoptUsKids national photolisting service: Characteristics of listed children and length of time to placement. *Children and Youth Services Review* 31(1):140–154.

Baccara M, Collard-Wexler A, Felli L, Yariv L (2014) Child-Adoption Matching: Preferences for Gender and Race. *American Economic Journal: Applied Economics* 6(3):133–158.

Baccara M, Lee S, Yariv L (2020) Optimal dynamic matching. *Theoretical Economics* 15(3):1221–1278.

Bansak K, Ferwerda J, Hainmueller J, Dillon A, Hangartner D, Lawrence D, Weinstein J (2018) Improving refugee integration through data-driven algorithmic assignment. *Science* 359(6373):325–329.

Barth RP (1990) On their own: The experiences of youth after foster care. *Child & Adolescent Social Work Journal* 7(5):419–440.

Berenguer G, Haskell WB, Li L (2023) Managing volunteers and paid workers in a nonprofit operation. *Management Science, Forthcoming* .

Cheremukhin A, Restrepo-Echavarria P, Tutino A (2020) Targeted search in matching markets. *Journal of Economic Theory* 185:104956.
Children’s Bureau (2022) The AFCARS report: Preliminary FY2021 estimates. URL https://www.acf.hhs.gov/sites/default/files/documents/cb/afcars-report-29.pdf, accessed: 2024-02-28.

Children’s Bureau, Administration on Children, Youth and Families (2023a) Afcars adoption file 2021 [Data set]. URL https://doi.org/10.34681/psb7-a026, accessed: 2024-02-28.

Children’s Bureau, Administration on Children, Youth and Families (2023b) AFCARS Foster Care File, 6-month periods (FY2016A - 2022B). URL https://doi.org/10.34681/yjxr-zz92, accessed: 2024-02-28.

Combe J, Dur U, Tercieux O, Terrier C, Ünver MU (2022) Market design for distributional objectives in (re)assignment: An application to improve the distribution of teachers in schools. Technical report, Boston College, Department of Economics.

Cox DR (1972) Regression models and life-tables. *Journal of the Royal Statistical Society: Series B (Methodological)* 34(2):187–202.

Delacrétaz D, Kominers SD, Teytelboym A (2020) Matching Mechanisms for Refugee Resettlement. Technical report, University of Oxford, Oxford, UK.

Dierks L, Slaugh V, Ünver MU (2024) Child welfare platform design to improve outcomes for children with disabilities. *Available at SSRN 4778791*.

Eeckhout J (1999) Bilateral Search and Vertical Heterogeneity. *International Economic Review* 40(4):869–887.

Feldman SW, Price KM, Ruppel J (2016) Not too late: Effects of a diligent recruitment program for hard to place youth. *Children and Youth Services Review* 65:26–31.

Florida Department of Children and Families (2019) Adoption incentive annual report. Office of Child Welfare, URL https://www.myflfamilies.com/sites/default/files/2023-02/2019%2520Adoption%2520Incentive%2520Report.pdf, accessed: 2024-02-28.

Florida Department of Children and Families (2023) Adoption incentive annual report. Office of Child and Family Well-Being, URL https://www.myflfamilies.com/sites/default/files/2023-11/Adoption_Incentive_Annual_Report_2022-23.pdf, accessed: 2024-02-28.

Fradkin A (2017) Search, Matching, and the Role of Digital Marketplace Design in Enabling Trade: Evidence from Airbnb. Technical report, Boston University Questrom School of Business.

Gale D, Sotomayor M (1985) Ms. Machiavelli and the stable matching problem. *The American Mathematical Monthly* 92(4):261–268.

Hanna M, McRoy R (2011) Innovative Practice Approaches to Matching in Adoption. *Journal of Public Child Welfare* 5(1):45–66.

Hitsch GJ, Hortaçsu A, Ariely D (2010a) Matching and Sorting in Online Dating. *American Economic Review* 100(1):130–163.
Hitsch GJ, Hortacsu A, Ariely D (2010b) What makes you click?—Mate preferences in online dating. *Quantitative Marketing and Economics* 8(4):393–427.

Kanoria Y, Saban D (2021) Facilitating the Search for Partners on Matching Platforms. *Management Science*.

Kasy M, Teytelboym A (2020) Adaptive Combinatorial Allocation. *arXiv:2011.02330 [econ, stat] ArXiv: 2011.02330*.

Kerimov S, Ashlagi I, Gurvich I (2023) On the optimality of greedy policies in dynamic matching. *Operations Research*.

Knuth DE (1997) *Stable marriage and its relation to other combinatorial problems: An introduction to the mathematical analysis of algorithms*, volume 10 (American Mathematical Soc.).

Kushel MB, Yen IH, Gee L, Courtney ME (2007) *Homelessness and health care access after emancipation: Results from the Midwest Evaluation of Adult Functioning of Former Foster Youth*, volume 161 (American Medical Association).

Lauermann S, Nöldeke G (2014) Stable marriages and search frictions. *Journal of Economic Theory* 151:163–195.

Lauermann S, Nöldeke G, Tröger T (2020) The Balance Condition in Search-and-Matching Models. *Econometrica* 88(2):595–618.

Lee S, Niederle M (2015) Propose with a rose? Signaling in internet dating markets. *Experimental Economics* 18(4):731–755.

Leshno JD (2022) Dynamic matching in overloaded waiting lists. *American Economic Review* 112(12):3876–3910.

Lushin V, Katz CC, Julien-Chinn FJ, Lalayants M (2023) A burdened workforce: Exploring burnout, job satisfaction and turnover among child welfare caseworkers in the era of covid-19. *Children and Youth Services Review* 148:106910.

Ma H, Fang F, Parkes DC (2020) Spatio-Temporal Pricing for Ridesharing Platforms. Technical report, Columbia Business School, 3022 Broadway, Uris Hall 423, New York, NY, 10027, USA.

MacDonald DE (2019) Foster Care: A Dynamic Matching Approach. Technical report, Arizona State University, Phoenix, AZ, USA.

Mennle T, Weiss M, Philipp B, Seuken S (2015) The Power of Local Manipulation Strategies in Assignment Mechanisms. *Proceedings of the 24th International Conference on Artificial Intelligence*, 82–89, IJCAI'15, ISBN 978-1-57735-738-4.

Riley NS (2019) Adoptions powered by algorithms. *Wall Street Journal* URL https://www.wsj.com/articles/adoptions-powered-by-algorithms-11546620390, accessed: 2024-02-28.
Rios I, Saban D, Zheng F (2021) Improving Match Rates in Dating Markets through Assortment Optimization. *Proceedings of the 22nd ACM Conference on Economics and Computation*, 788–789, EC ’21 (New York, NY, USA: Association for Computing Machinery).

Robinson-Cortés A (2019) Who Gets Placed Where and Why? An Empirical Framework for Foster Care Placement. Technical report, California Institute of Technology, 315 Baxter Hall, Pasadena, CA 91125, USA.

Roby JL, White H (2010) Adoption activities on the internet: A call for regulation. *Social Work* 55(3):203–212.

Roth AE (1984) The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory. *Journal of Political Economy* 92(6):991–1016.

Roth AE (1991) A Natural Experiment in the Organization of Entry-Level Labor Markets: Regional Markets for New Physicians and Surgeons in the United Kingdom. *The American Economic Review* 81(3):415–440.

Shi P (2023) Optimal matchmaking strategy in two-sided marketplaces. *Management Science* 69(3):1323–1340.

Shimer R, Smith L (2000) Assortative Matching and Search. *Econometrica* 68(2):343–369.

Shimer R, Smith L (2001) Matching, Search, and Heterogeneity. *Advances in Macroeconomics* 1(1).

Slaugh VW, Akan M, Kesten O, Ünver MU (2016) The Pennsylvania Adoption Exchange Improves Its Matching Process. *Interfaces* 46(2):133–153.

Sönmez T, Ünver MU, Yenmez MB (2020) Incentivized kidney exchange. *American Economic Review* 110(7):2198–2224.

Spindler A (1970) Social and rehabilitation services: A challenge to operations research. *Operations Research* 18(6):1112–1124.

Triseliotis J (2002) Long-term foster care or adoption? The evidence examined. *Child & Family Social Work* 7(1):23–33.

Vandivere S, Malm KE, Zinn A, Allen TJ, Mcklindon A (2015) Experimental evaluation of a child-focused adoption recruitment program for children and youth in foster care. *Journal of Public Child Welfare* 9(2):174–194.

Weitzman ML (1979) Optimal Search for the Best Alternative. *Econometrica* 47(3):641.

Yamatani H, Engel R, Spjeldnes S (2009) Child welfare worker caseload: What’s just right? *Social Work* 54(4):361–368.

Ünver MU (2010) Dynamic Kidney Exchange. *Review of Economic Studies* 77(1):372–414, ISSN 00346527.
A. Additional Lemmas and Propositions

A.1. Proposition 10

Here, we show that processing families in decreasing order of $v_c(f)$ is optimal for children in CS.

**Proposition 10.** In CS, $c$’s utility is maximized if the caseworker processes families in decreasing order of $v_c(f)$.

**Proof.** Assume child $c$ is active at the current time step. Note that families without interest in $c$ do not affect $c$’s utility in any way. For the remaining families, $c$ faces a Pandora’s box problem (Weitzman 1979) where $c$ receives a payoff of $v_c(f)$ with probability $p$ and a payoff of 0 with probability $1-p$ when the box corresponding to family $f$ is opened. Notice that the reservation value of the box corresponding to family $f$ is higher than for family $f'$ if and only if $v_c(f) > v_c(f')$.

A.2. Proposition 11

Strategy $s_f$ is a simple threshold strategy with threshold $z \in \mathbb{R}$ for family $f$ if $s_f = 1[v_f(c) \geq z]$, $\forall c \in C$. Here, we show that there exist instances where families cannot best respond with a simple threshold strategy.

**Proposition 11.** In FS, there exists an instance with a family $f$ and other agents’ strategies $s_{-f}$, such that no simple threshold strategy $s_f$ is a best response for $f$.

**Proof.** Consider the following example: Let $C = \{c_1, c_2\}$, $F = \{f_1, f_2\}$ and let valuations be according to the following tables for some $\epsilon > 0$.

| $v_c(f)$  | $f_1$ | $f_2$ |
|-----------|-------|-------|
| $c_1$     | 1     | $1-\epsilon$ |
| $c_2$     | $1-\epsilon$ | 1     |

| $v_f(c)$  | $f_1$ | $f_2$ |
|-----------|-------|-------|
| $c_1$     | 1     | 1     |
| $c_2$     | $1-\epsilon$ | $1-\epsilon$ |

Suppose strategy profile $s$ is such that $s_c(f) = s_f(c) = 1$ for all $c \in C$, $f \in F$. If $\epsilon$ is small enough and $(1-\lambda p)p < \kappa_F \leq p$, then it is optimal for $f_2$ to only be interested in $c_2$, even though $f_2$ strictly prefers $c_1$.

B. Remaining Proofs

B.1. Proof of Proposition 1

In FS, one way to express $c$’s utility is as follows:

$$u_c^{FS}(s) = \left[1 - \lambda p \sum_{f \in M_c(s)} \beta_{cf}(s)\right] \delta_C u_c^{FS}(s) + \lambda \sum_{f \in M_c(s)} \left[\beta_{cf}(s)pv_c(f) - \kappa_c\right]. \quad (15)$$

The probability of a match forming between $c$ and $f$ at the current time step is $\lambda p \beta_{cf}(s)$ if there is mutual interest, in which case $c$ obtains a value of $v_c(f)$ and leaves the process. For any active
family that showed interest, $c$ incurs search costs $\kappa_C$. If $c$ remains unmatched, then $c$ receives $\delta_C u_c^{FS}(s)$. By pulling $\delta_C u_c^{FS}(s)$ out of the sums, we get

$$u_c^{FS}(s) = \delta_C u_c^{FS}(s) + \lambda \sum_{f \in M_c(s)} \left( \beta_c f (s) p(v_c(f) - \delta_C u_c^{FS}(s)) - \kappa_C \right).$$ (16)

The proof for families’ utilities in FS and agents’ utilities in CS is analogous and omitted.

### B.2. Proof of Proposition 2

First of all, notice that whether agent $i$ is interested in some agent $j$ or not does not affect agent $i$’s utility if $j$ is not interested in $i$. Further, for an arbitrary family $f$ in FS, $\beta_c f (s)$ does not depend on $s_f$. By slightly modifying Equation (6), we can see that when $f$ plays a best response in $s$ it must hold that

$$u_c^{FS}(s_{-f}) = \delta_F u_c^{FS}(s_{-f}) + \frac{1}{n} \sum_{c \in M_f(s)} \left( \beta_c f (s)p(v_c(f) - \delta_F u_c^{FS}(s_{-f})) - \kappa_F \right).$$ (17)

By Equation (17) it must hold for all $c \in C$ that

$$s_f(c) = 1\left[ \beta_c f (s)p(v_c(f) - \delta_F u_c^{FS}(s_{-f})) \geq \kappa_F \right]$$ (18)

when there is mutual interest between $c$ and $f$, as $s_f$ would otherwise not be a best response. That is, because all children $c$ contribute non-negatively to $f$’s utility if and only if $\beta_c f (s)p(v_c(f) - \delta_F u_c^{FS}(s_{-f})) \geq \kappa_F$. By our tie-breaking assumption, the claim of the proposition follows for families in FS. The proof for families in CS is analogous and thus omitted.

In the remainder of the proof, we show that the statement holds for children in FS. The proof for CS is again analogous and therefore omitted. Let $c$ be an arbitrary child. For a best response $s_c$ to $s_{-c}$ we have that

$$u_c^{FS}(s_{-c}) = \delta_C u_c^{FS}(s_{-c}) + \lambda \sum_{f \in F} s_c(f)s_f(c) \left( \beta_c f (s)p(v_c(f) - \delta_C u_c^{FS}(s_{-c})) - \kappa_C \right).$$ (19)

As for families, it must be the case that

$$s_c(f) = 1\left[ \beta_c f (s)p(v_c(f) - \delta_C u_c^{FS}(s_{-c})) \geq \kappa_C \right]$$ (20)

because for all $f, f' \in F$

$$\beta_c f (s) \geq \beta_c f'(s) \iff v_c(f) \geq v_c(f'),$$ (21)

and otherwise $s_c$ would not be a best response to $s_{-c}$. Again, by our tie-breaking assumption the claim of the proposition follows for children in FS.
B.3. Proof of Proposition 3

For FS, define a mapping $T^{FS}: Y \to Y$ as follows: $T^{FS} = (T^{FS}_i)_{i \in A}$, where

$$T^{FS}_c(y) = \delta_{c}y_{c} + \lambda \sum_{f \in F} \mathbb{1} [\beta_{cf}(y)p(v_{f}(c) - \delta_{F}y_{f}) \geq \kappa_{F}] \left( \beta_{cf}(y)p(v_{c}(f) - \delta_{C}y_{c}) - \kappa_{C} \right)^{+}$$

(22)

for all $c \in C$ and

$$T^{FS}_f(y) = \delta_{F}y_{f} + \frac{1}{n} \sum_{c \in C} \mathbb{1} [\beta_{cf}(y)p(v_{c}(f) - \delta_{F}y_{f}) \geq \kappa_{C}] \left( \beta_{cf}(y)p(v_{f}(c) - \delta_{C}y_{c}) - \kappa_{F} \right)^{+}.$$  

(23)

for all $f \in F$. Note that any fixed point of $T^{FS}$ (i.e., any $y$ with $T^{FS}(y) = y$) is an equilibrium threshold profile in FS. We now show that $T^{FS}$ is monotonically increasing according to $\leq_{C}$. Let $c \in C$, $y, y' \in Y$, and $y \leq_{C} y'$. We have

$$T^{FS}_c(y) = \delta_{c}y_{c} + \lambda \sum_{f \in F} \mathbb{1} [\beta_{cf}(y)p(v_{f}(c) - \delta_{F}y_{f}) \geq \kappa_{F}] \left( \beta_{cf}(y)p(v_{c}(f) - \delta_{C}y_{c}) - \kappa_{C} \right)^{+}$$

(24)

$$\leq \delta_{c}y'_{c} + \lambda \sum_{f \in F} \mathbb{1} [\beta_{cf}(y')p(v_{f}(c) - \delta_{F}y'_{f}) \geq \kappa_{F}] \left( \beta_{cf}(y')p(v_{c}(f) - \delta_{C}y'_{c}) - \kappa_{C} \right)^{+}$$

(25)

$$= T^{FS}_c(y').$$

(26)

The inequality holds because of the following reason: Suppose a family $f$ is interested in $c$ under $s(y)$ but not under $s(y')$. Since $f$ is weakly less selective in $s(y')$, it must be the case that there exists another family $f'$ with $v_{c}(f') > v_{c}(f)$ that is not mutually interested in $c$ under $s(y)$ but under $s(y')$. Note that for any family that loses interest in $c$ under $s(y')$, there must exist such a unique family that replaces it and is preferred by $c$.

Since each child $c$ is weakly more selective under $s(y')$ than $s(y)$, we have for all $f \in F$

$$T^{FS}_f(y) = \delta_{F}y_{f} + \frac{1}{n} \sum_{c \in C} \mathbb{1} [\beta_{cf}(y)p(v_{c}(f) - \delta_{F}y_{f}) \geq \kappa_{C}] \left( \beta_{cf}(y)p(v_{f}(c) - \delta_{F}y_{f}) - \kappa_{F} \right)^{+}$$

(27)

$$\geq \delta_{F}y'_{f} + \frac{1}{n} \sum_{c \in C} \mathbb{1} [\beta_{cf}(y')p(v_{c}(f) - \delta_{F}y'_{f}) \geq \kappa_{C}] \left( \beta_{cf}(y')p(v_{f}(c) - \delta_{F}y'_{f}) - \kappa_{F} \right)^{+}$$

(28)

$$= T^{FS}_f(y').$$

(29)

Note that $T^{FS}$ maps elements from $Y$ to $Y$ and $(Y, \leq_{C})$ is a complete lattice. By Tarski’s fixed point theorem, the claim follows for FS.

For CS, we define a mapping $T^{CS}: Y \to Y$ as follows: $T^{CS} = (T^{CS}_i)_{i \in A}$, where

$$T^{CS}_c(y) = \delta_{c}y_{c} + \lambda \sum_{f \in F} \mathbb{1} [p(v_{f}(c) - \delta_{F}y_{f}) \geq \kappa_{F}] \beta_{cf}(y) \left( p(v_{c}(f) - \delta_{C}y_{c}) - \kappa_{C} \right)^{+}$$

(30)

for all $c \in C$ and

$$T^{CS}_f(y) = \delta_{F}y_{f} + \frac{1}{n} \sum_{c \in C} \mathbb{1} [p(v_{c}(f) - \delta_{C}y_{c}) \geq \kappa_{C}] \beta_{cf}(y) \left( p(v_{f}(c) - \delta_{F}y_{f}) - \kappa_{F} \right)^{+}.$$  

(31)
for all $f \in F$. Note that any fixed point of $T^{\text{CS}}$ is an equilibrium threshold profile in CS. We now show that $T^{\text{CS}}$ is monotonically increasing according to $\leq_C$. Let $c \in C$, $y, y' \in Y$, and $y \leq_C y'$. It holds that

$$T^{\text{CS}}_c(y) = \deltaCy_c + \lambda \sum_{f \in F} 1[p(v_f(c) - \deltaFy_f) \geq \kappa_F]\beta_{cf}(y)\left(p(v_c(f) - \deltaCy_c) - \kappa_C\right)^+$$  \quad (32)

$$\leq \deltaCy'_c + \lambda \sum_{f \in F} 1[p(v_f(c) - \deltaFy'_f) \geq \kappa_F]\beta_{cf}(y')\left(p(v_c(f) - \deltaCy'_c) - \kappa_C\right)^+$$  \quad (33)

$$= T^{\text{CS}}_c(y'),$$  \quad (34)

because each family is weakly less selective under $s(y')$ than $s(y)$.

Similarly, since each child is weakly more selective under $s(y')$ than $s(y)$, we have for all $f \in F$

$$T^{\text{CS}}_f(y) = \deltaFy_f + \frac{1}{n} \sum_{c \in C} 1[p(v_c(f) - \deltaCy_c) \geq \kappa_C]\beta_{cf}(y)\left(p(v_f(c) - \deltaFy_f) - \kappa_F\right)^+$$  \quad (35)

$$\geq \deltaFy'_f + \frac{1}{n} \sum_{c \in C} 1[p(v_c(f) - \deltaCy_c) \geq \kappa_C]\beta_{cf}(y')\left(p(v_f(c) - \deltaFy'_f) - \kappa_F\right)^+$$  \quad (36)

$$= T^{\text{CS}}_f(y').$$  \quad (37)

Note that $T^{\text{CS}}$ maps elements from $Y$ to $Y$ and $(Y, \leq_C)$ is a complete lattice. By Tarski’s fixed point theorem, the claim for CS follows.

### B.4. Proof of Lemma 1

Proof. If $(c, f) \in M(s^{\text{FS}})$ and $(c, f) \notin M(s^{\text{CS}})$, then it holds that $\beta_{cf}(s^{\text{FS}})p(v_c(f) - \deltaCu_c(s^{\text{FS}})) \geq \kappa_C$ and $\beta_{cf}(s^{\text{FS}})p(v_f(c) - \deltaFu_f(s^{\text{FS}})) \geq \kappa_F$. Further, either $p(v_c(f) - \deltaCu_c(s^{\text{CS}})) < \kappa_C$ or $p(v_f(c) - \deltaFu_f(s^{\text{CS}})) < \kappa_F$. If the former is true we have

$$\deltaCu_c(s^{\text{FS}}) \leq v_c(f) - \frac{\kappa_C}{\beta_{cf}(s^{\text{FS}})p} \leq v_c(f) - \frac{\kappa_C}{p} < \deltaCu_c(s^{\text{CS}}),$$  \quad (38)

since $0 < \beta_c(f, s^{\text{FS}}) \leq 1$. Otherwise, we get

$$\deltaFu_f(s^{\text{FS}}) \leq v_f(c) - \frac{\kappa_F}{\beta_{cf}(s^{\text{FS}})p} \leq v_f(c) - \frac{\kappa_F}{p} < \deltaFu_f(s^{\text{CS}}).$$  \quad (39)

### B.5. Proof of Theorem 1

Proof. It is easy to see why all CSEs can be Pareto improvements over all FSEs: Consider the following example: Let $C = \{c\}$, $F = \{f_1, f_2\}$ and let valuations be according to the following tables for some $\epsilon > 0$.

| $v_c(f)$ | $f_1$ | $f_2$ | $v_f(c)$ | $f_1$ | $f_2$ |
|---|---|---|---|---|---|
| $c_1$ | 1 | $1 - \epsilon$ | $c_1$ | 1 | 1 |
Assume that $p \geq \max\{\kappa_C, \kappa_F\}$, and $\epsilon$ and $\delta_C$ are sufficiently small. If $p(1 - \lambda p) \geq \kappa_F$ then the matching correspondences of the unique FSE and the unique CSE are identical, and $f_2$ only incurs search costs in CS if the match between $f_1$ and $c_1$ is not suitable. Thus, an agent can be made strictly better off in CS compared to FS without making anyone else worse off by saving wasted search efforts. If $p(1 - \lambda p) < \kappa_F$, then in the unique FSE there is only mutual interest between $c$ and $f_1$ while in the unique CSE, all agents have again mutually in each other. Furthermore, the CSE is a Pareto improvement over the FSE.

Let $s^{FS} \in S^{FS}$ and $s^{CS} \in S^{CS}$. We now show that if there exists an agent $i \in A$, such that $u_i(s^{FS}) > u_i(s^{CS})$, then there exists another agent $j \in A$ with $u_j(s^{FS}) < u_j(s^{CS})$. Suppose $i \in C$. If $u_i(s^{FS}) > u^C_i(s^{CS})$, we get by Lemma 2 that $M_c(s^{FS}) \not\subseteq M_c(s^{CS})$. Then, by Lemma 1, the claim immediately follows. Now suppose $i \in F$. Further, for the sake of contradiction, assume $M_c(s^{FS}) \subseteq M_c(s^{CS})$ for all $c \in C$, as otherwise by Lemma 1 the claim would immediately follow. Therefore, $f$’s increase in $u_f(s^{FS})$ can only come from the fact that there exists a child $c \in M_f(s^{FS})$ and another family $f' \in F$ with $v_c(f') > v_f(f)$ that is not interested in $c$ under $s^{FS}$ but under $s^{CS}$. Suppose $c$ responds to $s^{CS}_c$ with $s^{FS}_c$ in CS. By Lemma 2, this would imply $u_c(s^{FS}) \leq u_c((s^{FS}_e, s^{CS}_c))$. However, note that $c$’s utility would strictly increase if $c$ would be interested in $f'$ instead of $f$ since $v_c(f') > v_c(f)$. Hence, $c$ does not play a best response in $s^{CS}$, a contradiction.

B.6. Proof of Proposition 4

Proof. Consider the following example: Let $C = \{c_1, c_2\}$, $F = \{f_1, f_2\}$ and let valuations be according to the following tables.

| $v_c(f)$ | $f_1$ | $f_2$ | $v_f(c)$ | $f_1$ | $f_2$ |
|----------|-------|-------|----------|-------|-------|
| $c_1$    | $L$   | $\epsilon$ | $c_1$    | $\epsilon$ | $L$   |
| $c_2$    | $\epsilon$ | $L$  | $c_2$    | $L$   | $\epsilon$ |

If agents are patient enough (i.e., $\delta_C$ and $\delta_F$ are close enough to 1) and search costs are small enough, there exist only two equilibria, independent of search technology. In the child-optimal equilibrium, $s^{co}$, children are only interested in their preferred choice (i.e., the agent type for which they have a match value of $L$) and in the family-optimal equilibrium $s^{fo}$ families are only interested in their preferred choice. Note that if each child is mutually interested in at most one family in strategy profile $s$, then $u^C_i(s) = u^F_i(s)$ for all $i \in A$. As $p, \delta_F, \delta_C \to 1$, children’s utilities will be weakly less than $\epsilon$ under $s^{fo}$ while being positive and converging to $L$ under $s^{co}$. Similarly, families’ utilities will be weakly less than $\epsilon$ under $s^{co}$ while being positive and converging to $L$ under $s^{fo}$.
B.7. Proof of Proposition 5

Proof. Consider the following example: Let \( C = \{c_1, c_2\} \), \( F = \{f_1, f_2\} \) and let valuations be according to the following tables for some \( \epsilon > 0 \).

\[
\begin{array}{ccc}
  & f_1 & f_2 \\
v_{c}(f) & c_1 & 1 & 1 - \epsilon \\
c_2 & 1 & 1 - \epsilon & c_1 & 1 & 1 \\
  & c_2 & 0 & \kappa F/p
\end{array}
\]

If \( \epsilon > 0 \) is small enough and \( p(1 - \lambda p) < \kappa F \), then in the unique FSE \( s^{FS} \), only \( c_1 \) and \( f_1 \) will be mutually interested in each other and \( c_2 \) and \( f_2 \). Family \( f_2 \) will not be interested in \( c_1 \) in FS independent of how patient \( f_2 \) is. That is, because the probability of actually matching with \( c_1 \) while facing competition from \( c_1 \) is too small, yet \( f_2 \) would have to incur search costs every time \( c_1 \) is active. However, if \( \delta_F \) is sufficiently large, then the unique CSE matching correspondence is \( M(s^{CS}) = \{(c_1, f_1), (c_1, f_2)\} \). In CS, \( c_2 \), the “low-type” child, will remain unmatched.

B.8. Proof of Proposition 6

Proof. Consider the following example: Let \( C = \{c\} \), \( F = \{f_1, f_2, f_3\} \) and let valuations be according to the following tables for some \( \epsilon > 0 \).

\[
\begin{array}{cccc}
  & f_1 & f_2 & f_3 \\
v_{c}(f) & c & 1 & 1 - \epsilon & 1 - 2\epsilon \\
v_{f}(c) & c & 1 & 1 & \kappa F/p + \epsilon
\end{array}
\]

Choose \( \delta_C = \delta_F = 0 \). If both \( \epsilon \) and \( \kappa_C \) are sufficiently small and further \( \kappa_F < p \) we get the following: In the unique FSE \( s^{FS} \), child \( c \) will be mutually interested in both \( f_1 \) and \( f_3 \). In the unique CSE, all agents will have mutual interest in each other, and therefore \( f_3 \)’s utility strictly decreases.

B.9. Proof of Proposition 7

Proof. Suppose all instance parameters except for \( \delta_F \) are fixed. Let \( \mathcal{M} \) be the set of all child-family pairs \( (c, f) \) for which there exists \( \delta_F \in [0, 1) \), such that \( c \) and \( f \) are mutually interested in each other under some FSE \( s^{FS} \). Further, let \( v_{\text{min}} \) denote the smallest value a family in \( \mathcal{M} \) has for a mutually interested child, i.e., \( v_{\text{min}} = \min_{(c, f) \in \mathcal{M}} v_f(c) \). For \( \delta_F \), such that

\[
\delta_F \leq \frac{v_{\text{min}} - \kappa F/p}{\bar{v}},
\]

it follows that \( p(v_f(c) - \delta_F \bar{v}) \geq \kappa F \) for all \( (c, f) \in \mathcal{M} \). Since \( \bar{v} \) is an upper bound for agents’ utilities, for any such \( \delta_F \) and any \( (c, f) \in \mathcal{M} \), being interested in \( c \) is (weakly) advantageous for \( f \) under CS, independent of all other strategies. As in the proof of Lemma 2, this implies that \( c \) must be weakly better off in CS compared to FS.
B.10. Proof of Proposition 8

Proof. Consider the following example: Let $C = \{c_1, c_2\}$, $F = \{f_1, f_2\}$ and let valuations be according to the following tables for some $\epsilon > 0$.

| $v_i(f)$ | $f_1$ | $f_2$ | $v_f(c)$ | $f_1$ | $f_2$ |
|----------|-------|-------|----------|-------|-------|
| $c$      | 1     | $1-\epsilon$ | $f$      | 1     | 1     |

Choose $p$ and $\kappa_F$, such that $p(1-\lambda'p) < \kappa_F \leq p(1-\lambda p)$. If $\kappa_C$, $\delta_C$, and $\delta_F$ are sufficiently small, child $c$ will be mutually interested in $f_1$ and $f_2$ in the unique FSE for $\lambda$. But in the unique FSE for $\lambda'$ only $c$ and $f_1$ will be mutually interested in each other. However, if the difference between $\lambda$ and $\lambda'$ is very small, $c$'s utility can be smaller in the case where the market thickness indicator is $\lambda$, as the increased probability of $f_1$ being active might not compensate for the loss of $f_2$'s interest.

B.11. Proof of Proposition 9

Let $u = (u_i(s^{CS,\lambda}))_{i \in A}$ denote the vector of agents’ utilities under $s^{CS,\lambda}$ and let $Y' = [u_c, \bar{v}]^n \times [0, u_f]^m \subseteq [0, \bar{v}]^{n+m} = Y$. We now define a mapping $T^\lambda : Y \rightarrow Y$ as follows: $T^\lambda = (T^\lambda_i)_{i \in A}$, where

$$T^\lambda_c(y) = \delta_C y_c + \lambda \sum_{f \in F} 1[p(v_f(c) - \delta_F y_f) \geq \kappa_F] \beta_{cf}(y) \left(p(v_c(f) - \delta_C y_c) - \kappa_C\right)^+$$

for all $c \in C$ and

$$T^\lambda_f(y) = \delta_F y_f + \frac{1}{n} \sum_{c \in C} 1[p(v_c(f) - \delta_C y_c) \geq \kappa_C] \beta_{c,f}(y) \left(p(v_f(c) - \delta_F y_f) - \kappa_F\right)^+. \tag{42}$$

As we have seen in a previous proof, $T^\lambda$ is $\leq_C$-monotone on $Y$. We now show that $T^{\lambda'}$ maps from $Y'$ to $Y'$, which by Tarski’s fixed-point theorem yields the result. Since $T^\lambda$ is $\leq_C$-monotone and $u$ is the $\leq_C$-minimal element of $Y'$, it is sufficient to show that $T^{\lambda'}(u) \in Y'$.

Because $u$ is an equilibrium threshold profile, we have that $T^\lambda(u) = u \in Y'$. Further, it can easily be verified that $T^\lambda(y) \leq_C T^{\lambda'}(y)$ for all $y \in Y$. Hence, $T^{\lambda'}(u) \in Y'$, which completes the proof.

C. Algorithm to Compute FS-TSs from Threshold Profiles

Algorithm 1 can be used to compute $s^{FS}(y)$ for a given threshold profile $y \in \mathbb{R}^{n+m}$. Since $\beta_{cf}(s)$ only depends on families $f' \in F$ with $v_{c}(f') > v_{c}(f)$ and for each child families are processed in decreasing order of $v_c(f)$, the final strategy profile satisfies the equations for FS from Proposition 1.

D. Limit Results

Here, we provide a collection of limit results that all illustrate how the differences between FS and CS disappear as certain parameters take on extreme values.
Algorithm 1: Thresholds to strategy profile

Input: $y \in \mathbb{R}^{n+m}$

Output: Strategy profile $s \in S$

$s_c(f) := 0$ and $s_f(c) := 0$ for all $c \in C$, $f \in F$

for $c \in C$ do
    $U := F$
    while $U \neq \emptyset$ do
        $f := \arg \max_{f' \in U} v_c(f')$
        if $\beta_{cf}(s) p(v_f(c) - \delta_F y_f) \geq \kappa_F$ then
            $s_f(c) := 1$
        end
        if $\beta_{cf}(s) p(v_c(f) - \delta_C y_c) \geq \kappa_C$ then
            $s_c(f) := 1$
        end
        $U := U \setminus \{f\}$
    end
end

D.1. Negligible Search Costs

The next proposition shows that the games induced by CS and FS become identical as search costs become negligible.

**Proposition 12.** As $\kappa_C \to 0$ and $\kappa_F \to 0$, $|u_i^{FS}(s) - u_i^{CS}(s)| \to 0$ for all $s \in S$, $i \in A$.

*Proof.*** Assume that $\kappa_C = \kappa_F =: \kappa$ and let $s \in S$. For each family $f$, define $T_{s,f}^{FS}: Y \to \mathbb{R}$ and $T_{s,f}^{CS}: Y \to \mathbb{R}$ as follows:

\[ T_{s,f}^{FS}(y) = \delta_f y_f + \frac{1}{n} \sum_{c \in C} s_c(f) s_f(c) \left( \beta_{cf}(s) p(v_f(c) - \delta_F y_f) - \kappa \right) \]  \hspace{1cm} (43)

and

\[ T_{s,f}^{CS}(y) = \delta_f y_f + \frac{1}{n} \sum_{c \in C} s_c(f) s_f(c) \beta_{cf}(s) \left( p(v_f(c) - \delta_F y_f) - \kappa \right). \]  \hspace{1cm} (44)

Similarly, for all $c \in C$, let

\[ T_{s,c}^{FS}(y) = \delta_c y_c + \lambda \sum_{f \in F} s_c(f) s_f(c) \left( \beta_{cf}(s) p(v_c(f) - \delta_C y_c) - \kappa \right) \]  \hspace{1cm} (45)

and

\[ T_{s,c}^{CS}(y) = \delta_c y_c + \lambda \sum_{f \in F} s_c(f) s_f(c) \beta_{cf}(s) \left( p(v_c(f) - \delta_C y_c) - \kappa \right). \]  \hspace{1cm} (46)
Notice that the unique fixpoints \(y^{FS}, y^{CS} \in Y\) of \(T^{FS}_s(y)\) and \(T^{CS}_s(y)\) define agents’ utilities under \(s\) in FS and CS, respectively. For all \(y \in Y\) and \(f \in F\) it holds that

\[
\lim_{\kappa \to 0} \left( \delta_F y_f + \frac{1}{n} \sum_{c \in C} s_c(f)s_f(c) \left( \beta_{cf}(s)p(v_f(c) - \delta_F y_f) - \kappa \right) \right) = \delta_F y_f + \frac{1}{n} \sum_{c \in C} s_c(f)s_f(c) \left( \beta_{cf}(s)p(v_f(c) - \delta_F y_f) \right)
\]

\[
= \lim_{\kappa \to 0} \left( \delta_F y_f + \frac{1}{n} \sum_{c \in C} s_c(f)s_f(c) \beta_{cf}(s) \left( p(v_f(c) - \delta_F y_f) - \kappa \right) \right) .
\]

Similarly, for all \(y \in Y\) and \(c \in C\) we have that

\[
\lim_{\kappa \to 0} \left( \delta_C y_c + \lambda \sum_{f \in F} s_c(f)s_f(c) \left( \beta_{cf}(s)p(v_c(f) - \delta_C y_c) - \kappa \right) \right) = \delta_C y_c + \lambda \sum_{f \in F} s_c(f)s_f(c) \left( \beta_{cf}(s)p(v_c(f) - \delta_C y_c) \right)
\]

\[
= \lim_{\kappa \to 0} \left( \delta_C y_c + \lambda \sum_{f \in F} s_c(f)s_f(c) \beta_{cf}(s) \left( p(v_c(f) - \delta_C y_c) \right) \right) ,
\]

and hence \(\lim_{\kappa \to 0} |u^{FS}_i(s) - u^{CS}_i(s)| = 0\) for all \(i \in A\).

FS and CS do not necessarily become identical if only one side has negligible search costs. In both cases—i.e., if only \(\kappa_C \to 0\) or only \(\kappa_F \to 0\)—we can create instances where the sets of equilibria differ from each other in the two approaches.

**D.2. High Match Success Probability and Market Thickness**

Match success probability and market thickness indicator are strongly connected in our model, such that we cannot make any insightful statements about the limit behavior if only one of them approaches 1. However, if it is certain that a family of each type will be present at each time step and that each family would be a suitable match, we observe once more that FS and CS become equivalent in a slightly different way.

**Proposition 13.** As \(\lambda p \to 1\), \(|u^{FS}_i(s^{FS}(y)) - u^{CS}_i(s^{CS}(y))| \to 0\) for all \(y \in Y\), \(i \in A\).

**Proof.** For each family \(f\), define \(T^{FS}_f: Y \to \mathbb{R}\) and \(T^{CS}_f: Y \to \mathbb{R}\) as follows:

\[
T^{FS}_f(y) = \delta_F y_f + \frac{1}{n} \sum_{c \in C} 1[\beta^{FS}_c(y)p(v_c(f) - \delta_C y_c) \geq \kappa_C] \left( \beta^{FS}_c(y)p(v_f(c) - \delta_F y_f) - \kappa_F \right)^+
\]

and

\[
T^{CS}_f(y) = \delta_F y_f + \frac{1}{n} \sum_{c \in C} 1[p(v_c(f) - \delta_C y_c) \geq \kappa_C] \beta^{CS}_c(y) \left( p(v_f(c) - \delta_F y_f) - \kappa_F \right)^+ .
\]

Similarly, for all \(c \in C\), let

\[
T^{FS}_c(y) = \delta_C y_c + \lambda \sum_{f \in F} 1[\beta^{FS}_c(y)p(v_f(c) - \delta_F y_f) \geq \kappa_F] \left( \beta^{FS}_c(y)p(v_c(f) - \delta_C y_c) - \kappa_C \right)^+
\]

(55)
and
\[ T^C_S(y) = \delta_C y_c + \lambda \sum_{f \in F} 1[p(v_f(c) - \delta_F y_f) \geq \kappa_F] \beta^C_S(y) \left( p(v_c(f) - \delta_C y_c) - \kappa_C \right)^+. \] (56)

Notice that the unique fixpoints \( y^{FS}, y^{CS} \in Y \) of \( T^{FS}(y) \) and \( T^{CS}(y) \) define agents’ utilities under an FS-TS and CS-TS with threshold \( y \) in FS and CS, respectively. For all \( y \in Y \) and \( f \in F \), it holds that
\[
\lim_{\lambda p \to 1} \left( \delta_F y_f + \frac{1}{n} \sum_{c \in C} 1[\beta^F_S(y)p(v_c(f) - \delta_C y_c) \geq \kappa_C] \left( \beta^F_S(y)p(v_f(c) - \delta_F y_f) - \kappa_F \right) \right) = \delta_F y_f + \frac{1}{n} \sum_{c \in C} 1 \left[ f = \arg\max_{f' \in F:v_f(c) - \delta_F y_f > F.v_c(f') - \delta_C y_c > \kappa_C} v_c(f') \right] \left( v_f(c) - \delta_F y_f - \kappa_F \right)
\] (57)

\[
= \delta_F y_f + \frac{1}{n} \sum_{c \in C} 1[\beta^C_S(y)p(v_c(f) - \delta_C y_c) \geq \kappa_C] \beta^C_S(y) \left( p(v_f(c) - \delta_F y_f) - \kappa_F \right)^+. \] (59)

Similarly, for all \( y \in Y \) and \( c \in C \) we have that
\[
\lim_{\lambda p \to 1} \left( \delta_C y_c + \lambda \sum_{f \in F} 1[\beta^F_S(y)p(v_f(c) - \delta_F y_f) \geq \kappa_F] \left( \beta^F_S(y)p(v_c(f) - \delta_C y_c) - \kappa_C \right) \right) = \delta_C y_c + \lambda \sum_{f \in F} 1 \left[ f = \arg\max_{f' \in F:v_f(c) - \delta_F y_f > F.v_c(f') - \delta_C y_c > \kappa_C} v_c(f') \right] \left( v_f(c) - \delta_C y_c - \kappa_C \right)
\] (60)

\[
= \lim_{\lambda p \to 1} \left( \delta_C y_c + \lambda \sum_{f \in F} 1[p(v_f(c) - \delta_F y_f) \geq \kappa_F] \beta^C_S(y) \left( p(v_f(c) - \delta_C y_c) - \kappa_C \right)^+ \right). \] (62)

and hence \( \lim_{\lambda p \to 1} |u_i(s^S_i(y)) - u_i(s^{CS}_i(y))| = 0 \). This concludes the proof for families. For children, the proof is analogous and omitted.

In order to see why Proposition 13 holds, notice that the probability of child \( c \) matching with his first choice from \( M_c(s) \) goes to 1 as \( \lambda p \to 1 \). Thus, the contribution of all other families from \( M_c(s) \) goes to zero. As only these first choices contribute to utilities, the difference between FS and CS again disappears.

**D.3. Low Market Thickness**

As the supply on the family side becomes very small, the differences between CS and FS disappear in equilibrium.

**Proposition 14.** As \( \lambda \to 0 \), \( |u^S_i(s) - u^{CS}_i(s)| \to 0 \) for all \( s \in S, i \in A \).

**Proof.** Notice that the unique fixpoints \( y^{FS}, y^{CS} \in Y \) of \( T^{FS}_s(y) \) and \( T^{CS}_s(y) \) (recall definitions from the proof of Proposition 12) define agents’ utilities under \( s \) in FS and CS, respectively. For all \( y \in Y \) and \( f \in F \) it holds that
\[
\lim_{\lambda \to 0} \left( \delta_F y_f + \frac{1}{n} \sum_{c \in C} s_c(f)s_f(c) \left( \beta^F_S(s)p(v_f(c) - \delta_F y_f) - \kappa_F \right) \right) = \delta_F y_f + \frac{1}{n} \sum_{c \in C} s_c(f)s_f(c) \left( \beta^F_S(s)p(v_f(c) - \delta_F y_f) - \kappa_F \right) \to 0 \] (63)
Here, we first need to revisit classical matching markets. The matching market induced by Dierks, Olberg, Seuken, Slaugh and Unver:

\[
\beta_g \rightarrow 1 \text{ as } \lambda \rightarrow 0. \text{ Similarly, for all } y \in Y \text{ and } c \in C \text{ we have that }
\]

\[
\lim_{\lambda \to 0} \left( \delta_C y_c + \lambda \sum_{f \in F} s_c(f)s_f(c) \beta_{cf}(s) \left( p(v_c(f) - \delta_C y_c) - \kappa_C \right) \right)
\]

\[
= \delta_C y_c
\]

\[
= \lim_{\lambda \to 0} \left( \delta_C y_c + \lambda \sum_{f \in F} s_c(f)s_f(c) \beta_{cf}(s) \left( p(v_c(f) - \delta_C y_c) - \kappa_C \right) \right)
\]

and hence \(\lim_{\lambda \to 0} |u^i_{FS}(s) - u^i_{CS}(s)| = 0\) for all \(i \in A\).

The reason why the statement holds is that for very small \(\lambda\) families do not have to worry about competition in FS, as it is likely that there is no other family active at any given time step.

**D.4. Patient Agents**

Here, we first need to revisit classical matching markets. The matching market induced by 

\((v, \delta_C, \delta_F, \kappa_C, \kappa_F, p, \lambda)\) is a tuple \((C, F, \succ)\), where \(f \succ_c f'\) if and only if \(v_c(f) > v_c(f')\) and \(c \succ_f c'\) if and only if \(v_f(c) > v_f(c')\). Further, \(f \succ_c c\) if and only if \(pv_c(f) \geq \kappa_C\) and \(c \succ_f f\) if and only if \(pv_f(c) \geq \kappa_F\). For the remainder of this section, we assume that \(\delta_C = \delta_F =: \delta\).

**Proposition 15.** There exists \(\delta \in [0, 1)\), such that for all \(\delta \in [\delta, 1)\) the set of equilibrium matching correspondences are identical in FS and CS and coincide with the set of stable matchings in the induced marriage market.

For small \(p\) and large search costs, the statement above is trivial: If search costs become too large or the probability of a match being suitable becomes too small, no agent will have an incentive to be interested in any potential match candidate in either FS or CS. Hence, no matches will form in any equilibrium.

We only prove the statement for iii) here, as i) and ii) are trivial. For this proof, we first need to revisit classical matching markets. The matching market induced by \((v, \delta_C, \delta_F, \kappa_C, \kappa_F, p, \lambda)\) is a tuple \((C, F, \succ)\), where \(f \succ_c f'\) if and only if \(v_c(f) > v_c(f')\) and \(c \succ_f c'\) if and only if \(v_f(c) > v_f(c')\). Further, \(f \succ_c c\) if and only if \(pv_c(f) \geq \kappa_C\) and \(c \succ_f f\) if and only if \(pv_f(c) \geq \kappa_F\).

Let \((C, F, \succ)\) be a matching market where agents have strict preferences.

**Definition 5.** A pair of functions \(g = (g_C, g_F)\) is called a pre-matching if \(g_C : C \to A\) and \(g_F : F \to A\), such that if \(g_C(c) \neq c\) then \(g_C(c) \in F\) and if \(g_F(f) \neq f\) then \(g_F(f) \in C\).
We say that a pre-matching \( g \) induces a matching \( w \) if the function \( w : A \to A \) defined by \( w(i) = g(i) \) is a matching. Consider the following set of equations:

\[
g_C(c) = \max \{ \{ f \in F \mid c \succeq_f g_F(f) \} \cup \{ c \} \}, \quad c \in C, \tag{69}
\]

\[
g_F(f) = \max \{ \{ c \in C \mid f \succeq_c g_C(c) \} \cup \{ f \} \}, \quad f \in F, \tag{70}
\]

where the maxima are taken with respect to agents’ preferences.

**Lemma 3 (Adachi (2003)).** If a matching \( w \) is stable, then the pre-matching \( g \) defined by \( w \) solves the above equations. If a pre-matching \( g \) solves the above equations, then \( g \) induces a stable matching \( w \).

With the above lemma, we can now prove Lemma 4. Assume that \( \delta_C = \delta_F =: \delta \).

**Lemma 4.** There exists \( \delta \in [0, 1) \), such that for all \( \delta \in [\tilde{\delta}, 1) \) the set of equilibrium matching correspondences are identical in FS and CS and coincide with the set of stable matchings in the induced marriage market.

**Proof.** Let \( s \in S^{FS} \) and \( c \in C \). \( c \)'s utility under \( s \) is the unique value \( u_c^{FS}(s) \) that satisfies

\[
u_c(s) = \delta u_c(s) + \lambda \sum_{f \in F} s_f(c) \left( \beta_{cf}(s) p(v_f(c) - \delta u_c(s)) - \kappa_C \right)^+ \tag{71}
\]

\[
(1 - \delta)u_c(s) = \lambda \sum_{f \in F} s_f(c) \left( \beta_{cf}(s) p(v_f(c) - \delta u_c(s)) - \kappa_C \right)^+. \tag{72}
\]

Therefore, as \( \delta \to 1 \) we get

\[
\frac{1}{n} \sum_{f \in F} s_f(c) \left( \beta_{cf}(s) p(v_f(c) - \delta u_c^{FS}(s)) - \kappa_C \right)^+ \to 0. \tag{73}
\]

For the sake of contradiction, assume that \( |M_c(s)| \geq 2 \). Let \( f^* = \arg \max_{f \in F : s_f(c) = 1} v_c(f) \) and \( f \in M_c(s) \setminus \{ f^* \} \). Because of Equation (73), it must hold that

\[
\beta_{cf}(s) \left( v_c(f) - \delta u_c(s) - \kappa_C/p \right)^+ \to 0. \tag{74}
\]

However, since \( v_c(f^*) > v_c(f) \) and \( \beta_{cf^*}(s) > \beta_{cf}(s) \) we get that

\[
\beta_{cf^*}(s) \left( v_c(f^*) - \delta u_c(s) - \kappa_C/p \right)^+ \to \epsilon \tag{75}
\]

for some \( \epsilon > 0 \), a contradiction. Hence, \( M_c(s) = \{ f^* \} \) if \( pv_c(f^*) \geq \kappa_C \) or \( M_c(s) = \emptyset \) otherwise. Notice that this is equivalent to the expression of Equation (69). Now that we have established that children will be mutually interested in at most one family, the same can similarly be shown for families, which completes the proof for FS. The proof for CS is analogous and thus omitted.
E. Numerical Evaluation: Supplementary Material

E.1. Families

Figure 6 The ratio of families that are on average (strictly) better off in either approach in the family-optimal equilibrium for different combinations of agents’ patience and the level of preference correlation.

From Figure 6, we can see that the majority of families achieves a higher utility under CS, and almost no families achieve a higher utility under FS.

E.2. Match Probabilities

Figure 7 shows that on average children are more likely to get matched in CS than FS at any given time step.

Figure 7 Children’s average match probability (averaged over all children and over all instances) in the family-optimal equilibrium for different combinations of agents’ patience and the level of preference correlation.

F. AFCARS Benchmark Analysis without Conditional Survival Probabilities

While the analysis of Section 8.2 uses the TPR date as the starting point of the comparison, an alternate benchmark may rely on the time until finalized adoption with the start of the hazards analysis time horizon being the registration with the platform; i.e., we assume $\tau^r_i = 0$ for all children.
Figure 8  Actual adoptions by the agency using the platform and other channels compared to Florida AFCARS benchmark model assuming the platform case creation date as the start of the hazards model time horizon.

We observe similar results from this comparison to the AFCARS benchmark, which is shown in Figure 8. For children listed on the platform before October 1, 2021, he predicted number of finalized adoptions within two years was 124.9. However, 138 adoptions were finalized within two years. Within three years of listing, this difference extends to an extra 23.7 adopted children, or a 17% increase over the benchmark.