Multipartite nonlocality swapping

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Nonlocality swapping of bipartite binary correlated boxes can be realized by a coupler ($\chi$) in nonsignaling models. By studying the swapping process we find that the previous bipartite coupler can be applied to the swapping of two multipartite boxes, and then generate a multipartite box with more users than that of any of the boxes before swapping. Here quantum bound still appears in the scheme. The bipartite coupler also can be applied to a hybrid scheme of generating a multipartite extremal box from many PR boxes. As the analogue of multipartite entanglement swapping, we generalize the nonlocality swapping of bipartite binary boxes to multipartite binary boxes by using a multipartite coupler $\chi_N$, and get the probability of success by connecting the coupler to the generalized Svetlichny inequality. The multipartite coupler acting on many multipartite boxes makes multipartite nonlocality swapping be a more efficient device to manipulate nonlocality between many users. The results show that Tsirelson’s bound for quantum nonlocality emerges only when two of the $n$ boxes involved in the coupler process are noisy ones.

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I. INTRODUCTION

Violation of local realism is a fascinating phenomenon of quantum mechanics (QM). Since the paradox was presented by Einstein, Podolsky and Rosen[1] in 1935, QM was at one time under suspicion until Bell[2] showed that there existed certain settings for physical experiments that contradicted ‘common sense’ views of reality. Bell’s theorem was proved by the experiment[3] in 1982 and other more later. Nowadays, quantum nonlocality is getting the attention it deserves as another special aspect of the quantum correlations-entanglement.

To get more insight of quantum mechanics, it makes sense to have a study on the connection between quantum entanglement and quantum nonlocality. Like Von Neumann entropy and concurrence[4] for entanglement, various criteria were put forward to give a convenient test of nonlocality. The earliest one-Bell inequality[2] which comes from Bell’s theorem gives a criterion that classic local correlation must obey. Numbers of Bell-type inequalities[5–7] were presented for different systems afterwards. The violation of Bell-type inequality gives a straightforward impression about quantum nonlocal correlations (QNC). A common feature for all these inequalities is that QM’s violation can’t reach the maximum value. Naturally, Popescu and Rohrlich[8] showed a correlation gives a maximal violation 4 of Clauser-Horne-Shimony-Holt (CHSH) inequality[5], while the maximal violation in QM domain, i.e. Tsirelson’s bound[9], $B_Q = 2\sqrt{2}$ by QNC. Similarly, an algebraic maximum $8[10]$ and quantum bound $4\sqrt{2}$ by Svetlichny inequality[7] exist in tripartite system. With the addition of the correlations over quantum bound, postquantum correlations (PQC), nonlocal correlations were discussed in generalized nonsignaling models, as nonsignaling correlated boxes[11] decided by joint probability distribution[11]. Strong information-theoretic capabilities were revealed in the study of PQC[12], such as secure cryptography[13] and the reduction of communication complexity[14,15].

Properties of entanglement, such as monogamy, distillation, and swapping, were proved to be capable for nonlocality in nonsignaling theory[11,16,17], but distinctions still exist between entanglement and nonlocality. Local operations and classic communication (LOCC) were essential in entanglement distillation processes, but nonlocality distillation occurs without CC[16]. Only the operations on the input and output of nonlocal boxes are crucial to the distillation of nonlocality. Dramatically, the device operating on the outside of the boxes failed to swap the correlation between nonlocal boxes[18]. Skrzypczyk et al.[16] raised the concept of genuine boxes and coupler as analogue of the quantum joint measurement for nonlocality swapping. Clauser-Horne (CH) inequality[6] as an appropriate measure of nonlocality was applied to obtain the possibility of successful swapping. The fact that only PQC can be successfully swapped in isotropic resources makes the emergence of Tsirelson’s bound under CH expression. Then, Skrzypczyk and Brunner[19] considered all theoretically possible couplers, ranging from perfect to minimal couplers, for limited nonlocality, and quantum bound still appeared in their study.

Zukowski et al.[20] proposed the first entanglement swapping scheme, and it was soon generalized to the case of generating a three-particle Greenberger-Horne-Zeilinger (GHZ) state from three Bell pairs[21]. Bose et al.[22] generalized the procedure of entanglement swapping to the multipartite case. A natural question whether multipartite nonlocal correlations can be swapped raises to us. Exploring the analogy of multipartite entanglement swapping, we’ll try to design a generalized multipartite coupler for realizing multipartite non-
locality swapping.

Replacing the CH inequality by CHSH inequality[5] and the generalized Svetlichny inequality[23], we find that the existing bipartite coupler provides an adequate dynamical process for two correlated nonlocal boxes, including multipartite ones. But this bipartite-coupler-based swapping consumes much resources and seems less efficient in the case of swapping of multi nonlocal boxes. Then we will show a generalized coupler, which can be applied on many multipartite boxes, makes the swapping of many multipartite boxes succeed in a more efficient way.

The paper is organized as follows. In Sec.II we briefly review the quantum nonlocality swapping of two bipartite binary correlated boxes and show the key points therein. In Sec.III we define the general multipartite binary correlated boxes after presenting the form of the generalized Svetlichny inequality for nonsignal systems. In Sec.IV we generalize the nonlocality swapping on two bipartite boxes to the case of two multipartite boxes by using the bipartite coupler \( \chi \) and give a discussion on the bound of quantum nonlocal correlations. In Sec.V we attempt to achieve the nonlocality swapping on three bipartite boxes and then we show the multipartite coupler \( \chi_N \) for nonlocality swapping on arbitrary number of multipartite boxes. Conclusions and remarks are presented in Sec.VI.

II. FRAMEWORK

Firstly, we’ll give a brief review on Skrzypczyk’s nonlocality swapping scheme[17]. Bob shares two nonlocal boxes with another two spatially separated users Alice and Charlie (See FIG.1).

![FIG. 1: Nonlocality swapping of two isotropic boxes. The coupler is the analog of quantum joint measurement, and quantum bound (Tsirelson’s bound) emerges in this process.](image)

Bob carries out the bipartite coupler \( \chi \) on two bipartite isotropic boxes \( P^{PR}_x(ab_1 | x y_1) \) and \( P^{PR}_y(b_2c | y_2z) \), which are superposition of Popescu-Rohrlich (PR) box[10] and the fully mixed box \( \mathbb{I} \)[14].

\[
P^{PR}_x = \xi P^{PR} + (1 - \xi) \mathbb{I}.
\]

Where \( \xi \in [-1, 1] \), and when \( \xi = -1 \), the box is ‘anti-PR’ box \( P^{PR}_x \) given by \( a \oplus b = x y \oplus 1 \).

The output \( b' = 0 \) indicates that the correlation between Alice and Charlie was created successfully. Thus, the whole procedure is expressed as

\[
P^{PR}_x(ab_1 | x y_1) P^{PR}_y(b_2c | y_2z) \xrightarrow{\chi} P^{PR}_x(ac | xz),
\]

(2)

Some characteristics for the swapping process were summarized as follows:

A coupler working on the genuine part[17] of the ‘quantum’ box but not on the measurement devices, is different from the action in Ref.[18].

The successful probability \( p(b' = 0) = \frac{2}{3} \mathbf{CH} \cdot \mathbf{CH}_R(b_1b_2 | y_1y_2) = \frac{2}{3}[P(11|00) + P(00|10) + P(00|01) - P(00|11)] \) is proportional to the nonlocalities of the boxes which the coupler is applied on, i.e., the box \( P(b_1b_2 | y_1y_2)'s \) violation of CH inequality[6]. The coupler doesn’t always successfully swap the correlations, with an exception that the correlation which coupler acted on is PR correlation. Generally, \( P(b_1b_2 | y_1y_2) = \mathbb{I} \) and the optimal probability of success is \( \frac{1}{3} \).

A successful coupling process is a linear transformation. By this property, we can get the results as follows immediately:

\[
P^{PR}_x(ab_1 | x y_1) P^{PR}_y(b_2c | y_2z) \xrightarrow{\chi} P^{PR}_x(ac | xz),
\]

(3a)

\[
\mathbb{I}(ab_1 | x y_1) P^{PR}_y(b_2c | y_2z) \xrightarrow{\chi} \mathbb{I}(ac | xz),
\]

(3b)

\[
P^{PR}_x(ab_1 | x y_1) P^{PR}_y(b_2c | y_2z) \xrightarrow{\chi} P^{PR}_x(ac | xz).
\]

(3c)

Notice that Eq.(1) with the coefficient \( \xi \in [-1, 1] \) is the correlation to be swapped and \( \xi \in [0, 1] \) is the allowed correlations where the coupler works fine.

When \( \xi = 1 \), we got

\[
P^{PR}_x(ab_1 | x y_1) P^{PR}_y(b_2c | y_2z) \xrightarrow{\chi} \begin{cases} P^{PR}_x(ac | xz) & \text{if } b' = 0, \\ \frac{1}{2} P^{PR}_x(ac | xz) & \text{if } b' = 1. \end{cases}
\]

(4)

Coincidently, the coupler would always succeed when applied on PR box and fail on the failure box \( P^f = \frac{1}{P(b' = 0)} \mathbb{I} - P(b' = 0) P^{PR} \).

When \( \xi = \frac{1}{\sqrt{2}} \), we could see a connection between QNC and PQC in the transformation:

\[
P^{PR}_{1/\sqrt{2}}(ab_1 | x y_1) P^{PR}_{1/\sqrt{2}}(b_2c | y_2z) \xrightarrow{\chi} P^{PR}_{1/2}(ac | xz).
\]

(5)

The final correlation are nonlocal(\( \mathbf{CH} \cdot \mathbf{CH}_R > \xi^2 + 1/2 = 1 \)) if and only if the initial correlation is in the set of PQC(\( \mathbf{CH} \cdot \mathbf{CH}_R > \xi + 1/2 = 1/\sqrt{2} + 1/2 \)).

In this scheme, CH inequality is the measure of nonlocality. We find that replacing the CH inequality with CHSH inequality[5] is feasible for swapping joint probability-symmetric boxes, including isotropic boxes. The characteristics are all tenable in the sense of CHSH expression.
successful probability is
\[ p(b' = 0) = \frac{1}{6} \text{CHSH} \cdot \tilde{P}(b_1 b_2 | y_1 y_2) + \frac{1}{3} \]  \hspace{1cm} (6)
Where \( \text{CHSH} \cdot \tilde{P}(ab | xy) = E_{xy} + E_{xy} + E_{\bar{y}z} - E_{\bar{y}z} \) with \( E_{xy} = P(a = b | xy) - P(a \neq b | xy) \). The final correlation are nonlocal (\( \text{CHSH} \cdot \tilde{P} > 4\xi^2 = 2 \)) if and only if the initial correlation is in the set of PQC (\( \text{CHSH} \cdot \tilde{P} > 4\xi = 2\sqrt{2} \)).

III. GENERALIZED SVETLICHNY INEQUALITY FOR MULTIPARTITE CORRELATED BOXES

Before generalizing the swapping of bipartite nonlocalities to the multipartite case, we need to make some theoretical preparations.

In tripartite system, a famous inequality for nonlocality is Svetlichny inequality, and for nonsignaling box \( P(abc | xyz) \) it has the form[10]:
\[ E_{xyz} + E_{xy} + E_{\bar{y}z} - E_{\bar{y}z} - E_{\bar{y}z} - E_{\bar{y}z} \leq 4, \]  \hspace{1cm} (7)
where \( E_{xyz} \) is defined as
\[ E_{xyz} = \sum_{a,b,c} (-1)^{a+b+c} P(abc | xyz). \]  \hspace{1cm} (8)
The box with probability distribution
\[ P(abc | xyz) = \begin{cases} 1/4, & \text{if } a \oplus b \oplus c = xy \oplus xz \oplus yz, \\ 0, & \text{otherwise}. \end{cases} \]  \hspace{1cm} (9)
gives the algebraic maximal violation 8, namely, the Svetlichny box (SB), and the GHZ state (\( \frac{1}{\sqrt{2}} |000 \rangle + |111 \rangle \)) achieves the quantum maximal violation \( 4\sqrt{2} \) of Svetlichny inequality with an appropriate set of measurements. From the joint probability distribution matrix of GHZ we know that it is a linear superposition of Svetlichny box and tripartite fully mixed box.

Now, we’ll present the form of \( n \)-particle Svetlichny inequality[23] in nonsignaling models, namely, the generalized Svetlichny inequality (GSI).

For a \( n \)-partite binary input and output correlated box \( P(a_1 a_2 \cdots a_n | x_1 x_2 \cdots x_n) \), the expectation values of measurements are defined as
\[ E_{x_1 x_2 \cdots x_n} = \sum_{a_1 a_2 \cdots a_n = 0} (-1)^{\sum_{i=1}^n a_i} P(a_1 a_2 \cdots a_n | x_1 x_2 \cdots x_n), \]  \hspace{1cm} (10)
and \( E_{x_1 x_2 \cdots x_n} \in [0, 1] \).

The generalized Svetlichny inequality has the form:
\[ \text{GSI}^n = \sum_{x_1 x_2 \cdots x_n = 0} C_{x_1 x_2 \cdots x_n} \cdot E_{x_1 x_2 \cdots x_n} \leq 2^{n-1}, \]  \hspace{1cm} (11)
The coefficients are defined as:
\[ C_{x_1 x_2 \cdots x_n} = \sqrt{2} \cos \left( \frac{\pi}{2} \right) \left( \sum_{i=1}^n x_i \right) \text{ mod } 4 - \frac{\pi}{4}, \]  \hspace{1cm} (12)
where \( X \text{ mod } n \) is addition modulo \( n \) of \( X \), and \( C_{x_1 x_2 \cdots x_n} \in \{-1, 1\} \).

When \( n = 2 \), the GSI will be reduced to CHSH inequality and \( n = 3 \) as the Svetlichny inequality.

The generalized Svetlichny box (GSB) is characterized by the following probability distribution:
\[ P_n^{\text{GSB}}(a_1 a_2 \cdots a_n | x_1 x_2 \cdots x_n) = \begin{cases} 2 \| a_j \text{ mod } 2, & \text{if } \oplus_{i=1}^n a_i = \oplus_{j,k=1}^n x_j x_k, \\ 0, & \text{otherwise}. \end{cases} \]  \hspace{1cm} (13)
Here the symbol \( \oplus \) means addition modulo 2 of summation.
The generalized Svetlichny box violates the GSI up to its algebraic maximum \( 2^n \).

If a box has the probability distribution
\[ P_n(a_1 a_2 \cdots a_n | x_1 x_2 \cdots x_n) = \prod_{i=1}^n P(a_i | x_i) = \prod_{i=1}^n [P(a_i | x_i)]^n = (1/2)^n, \]  \hspace{1cm} (14)
for all \( a_i \) and \( x_i \), it is the \( n \)-partite fully mixed box \( \mathbb{I}^n(a_1 a_2 \cdots a_n | x_1 x_2 \cdots x_n) \) (written as \( \mathbb{I} \) for simple later), and it has a violation of zero in expression of GSI.

The \( n \)-partite isotropic boxes are defined as mixtures of \( n \)-partite GSB and \( n \)-partite fully mixed box:
\[ P^{\text{GSB}}_\xi = \xi P^{\text{GSB}}_n + (1 - \xi) \mathbb{I}, \]  \hspace{1cm} (15)
Where \( \xi \in [-1, 1] \). When \( \xi = 1/\sqrt{2} \), \( B_\xi = \text{GSI} \cdot \tilde{P}^{n/\sqrt{2}} = 2^n - 1/2 \) is \( n \)-partite Tsirelson’s bound, a generalized quantum bound. The quantum state which can reach this bound is \( |GHZ\rangle^n = \frac{1}{\sqrt{2^n}} (|00 \rangle^\otimes n + |11 \rangle^\otimes n) \).

After defining the resources of multipartite nonlocality swapping, we will begin the generalization of nonlocality swapping from the two bipartite cases to the more general cases.

IV. SWAPPING TWO MULTIPARTITE CORRELATED BOXES

A. Nonlocality swapping of two multipartite boxes

In the previous scenario[17], Bob shares bipartite nonlocal box with both Alice and Charlie. Nonlocal correlations will be swapped with a certain probability when Bob applies the coupler \( \chi \) on his two boxes. Here we present a generalized nonlocality swapping on two multipartite nonlocal correlated boxes.

Bob applies the bipartite coupler \( \chi \) on two GSBs and swaps the correlations between two groups of users (FIG. 2). Besides
Bob, there are \( m - 1 \) and \( n - 1 \) users in group A and C, respectively, which have inputs \( x_1 x_2 \cdots x_{m-1} z_1 z_2 \cdots z_{n-1} \) and outputs \( a_1 a_2 \cdots a_{m-1}, c_1 c_2 \cdots c_{n-1} \). The coupler acting on two GSBs implements a linear transformation:

\[
\begin{align*}
    p_{m}^{\text{GBS}}(a_1 a_2 \cdots a_{m-1} b_1 | x_1 x_2 \cdots x_{m-1} y_1) &= p_{n}^{\text{GBS}}(b_2 c_1 c_2 \cdots c_{n-1} | y_2 z_1 z_2 \cdots z_{n-1}) \\
    \chi &\rightarrow P(a_1 a_2 \cdots a_{m-1} b'_1 c_1 c_2 \cdots c_{n-1} | x_1 x_2 \cdots x_{m-1} z_1 z_2 \cdots z_{n-1}) \\
    &= \begin{cases} 
        p(b' = 0) \chi_{m+n-2}^{\text{GBS}}(a_1 a_2 \cdots a_{m-1} c_1 c_2 \cdots c_{n-1} | x_1 x_2 \cdots x_{m-1} z_1 z_2 \cdots z_{n-1}), \\
        p(b' = 1) \chi_{m+n-2}^{\text{GBS}}(a_1 a_2 \cdots a_{m-1} c_1 c_2 \cdots c_{n-1} | x_1 x_2 \cdots x_{m-1} z_1 z_2 \cdots z_{n-1}),
    \end{cases}
\end{align*}
\]

(V. SWAPPING MANY MULTIPARTITE NONLOCALITY)

(A. Multipartite nonlocality swapping)

After presenting the generalization in section IV, we now introduce a more general nonlocality swapping scheme. Bob shares nonlocal boxes with three or more groups of users, and his new purpose is swapping nonlocal correlation between all of the groups. Toward this new goal, Bob makes a feasible try by using bipartite coupler \( \chi \). He will show the probability to swap nonlocal correlation between groups of users by demonstrating the simplest example where he shares PR boxes with other three users Alice(A), Charlie(C) and Danny(D).

As depicted in FIG. 3, Bob has two copies of PR box with A, C and D each. First, Bob applies the coupler on the boxes he shares with A and C, C and D, A and D separately, and then he will get three outputs after his actions. The process succeeds if the three outputs are all bit 0. Second, some corresponding local operations (LO) must be applied on A, C and D’s boxes. After that, an Svetlichny correlation is generated between A, C and D.

Suppose that Bob’s correlation \( P(b_1 b_2 b_3 | y_1 y_2 y_3) \) is fully mixed correlation, and the transformation of this scheme

\[
\begin{align*}
    p_{m,\xi}^{\text{GBS}} p_{n,\xi}^{\text{GBS}} \frac{\chi(b' = 0)}{\chi_{m+n-2}^{\text{GBS}}} &= p_{m+n-2,\xi^2}^{\text{GBS}} = \xi^2 \chi_{m+n-2}^{\text{GBS}} + (1 - \xi^2) \mathbb{I}.
\end{align*}
\]

By setting the coefficient \( \xi = 1/\sqrt{2} \), we can get the result that only the postquantum type initial correlations can make the final box nonlocal, which shows the generalized Tsirelson’s bound \( B_{Q} = 2^{(n' - 1/2)} \) (here \( n' = m + n - 2 \)) again.

B. Emerge of quantum bound

After swapping the extremal nonlocal boxes, we’ll consider the swapping of noisy correlations. Group A and C now have the multipartite isotropic boxes \( p_{m,\xi}^{\text{GBS}} = \xi P_{m}^{\text{GBS}} + (1 - \xi) \mathbb{I} \) and \( p_{n,\xi}^{\text{GBS}} = \xi P_{n}^{\text{GBS}} + (1 - \xi) \mathbb{I} \) instead, and Bob will apply the coupler on his two boxes to check whether the quantum bound emerges or not. Fortunately, the answer is positive, the coupling process will be like this:

\[
\begin{align*}
    p_{m,\xi}^{\text{GBS}} p_{n,\xi}^{\text{GBS}} \frac{\chi(b' = 0)}{\chi_{m+n-2}^{\text{GBS}}} &= p_{m+n-2,\xi^2}^{\text{GBS}} = \xi^2 P_{m+n-2}^{\text{GBS}} + (1 - \xi^2) \mathbb{I}.
\end{align*}
\]
In this scheme, only one generalized multipartite coupler is needed, and the probability of success is the same as Skrzypczyk’s scheme.

\[ \chi_N^{\text{GBS}} (A_1 b_1 | X_1 y_1) \prod_{i=2}^{n(N)} (A_2 b_2 | X_2 y_2) \cdots \prod_{i=N}^{n(N)} (A_N b_N | X_N y_N) \]

\[ p(b' = 0) \prod_{i=1}^{n(N)} (A_i X_i), \]

\[ p(b' = 1) P^N_{\text{LO}} (A X) \]

Where \( A_i \) means the \( i \)th box’s all outputs \( a_1 a_2 \cdots a_{n(i)-1} \) except Bob’s \( b_i \), and \( X_i \) means the \( i \)th box’s inputs \( x_1 x_2 \cdots x_{n(i)-1} \) except Bob’s \( y_i \). The \( A \) and \( X \) without subscript mean all users’ input bits and output bits except Bob’s. One \( \sum_{i=1}^{N} n(i) - N \) - partite correlation will be generated after a successful coupling in Bob’s location.

Suppose that a perfect coupler works on the region of nonsignaling polytope depicted in FIG. 5 and then the rest users will get a local correlation which violates the GSI by \(-2^{n-1}\) when swapping fails; a GSB will be generated when the coupler returns a single bit. The output is deterministic 1
when applying the coupler on the failure correlation $P^f(A|X)$ and 0 on generalized Svetlichny correlation. Then the generalized coupler acting on any allowed box gives a successful probability:

$$p(b' = 0) = \frac{1}{3 \cdot 2^{n-1}} GS1 \cdot \vec{P} + \frac{1}{3}.$$  

(20)

For a natural correlation $I$, the optimal probability of success is also $1/3$ as before.

Comments on the generalization: Nonlocality swapping makes it possible to generate multipartite nonlocal correlation from many nonlocal correlations. For instance, we could generate a 4-partite Svetlichny box from one tripartite Svetlichny box and two PR boxes by a generalized coupler $\chi_3$.

The generalized swapping process is also a linear transformation. Let $N = 2$, the scheme becomes the one we presented in Sec. IV. And further, let $n(1) = n(2) = 2$, it is the same as Skrzypczyk’s scheme.

When the noisy condition is considered, Tsirelson’s bound for quantum nonlocality doesn’t always appear in the swapping process. Basing on the linearity of the swapping, if all group’s correlations are noisy like $F^G_{\xi} = \xi F^G_{\xi n(i)} + (1 - \xi) I$, the final correlation will be $F^G_{\xi} = \xi N F^G_{\xi n(i) - N} + (1 - \xi N) I$. The quantum bound only appears in the case where two boxes are noisy or the coupler is $\chi_2$.

VI. CONCLUSIONS

We found that CHSH inequality is also pragmatic for previous nonlocality swapping scheme. Then we focused on the Svetlichny inequality in tripartite system, and showed the form of $N$-Particle Svetlichny inequality in multipartite nonsignaling system. Basing on this generalized Bell-type inequality, we defined the extremal multipartite nonlocal boxes for our generalization of swapping process in the paper.

Using the same coupler applied in Skrzypczyk’s swapping scheme, we first presented a generalized swapping of two arbitrary multipartite nonlocal correlations, which could generate a multipartite correlation with more users than the initial ones. Later, through a combined scheme, we illustrated that multipartite correlation could also be generated from swapping many bipartite boxes. For the sake of efficiency, we finally presented a more general multipartite nonlocality swapping scheme with a generalized multipartite coupler, an analogue of quantum joint measurement in multipartite entanglement swapping. The multipartite coupler builds an efficient device to generate multipartite nonlocal correlation between many users.

The generalized quantum bound always emerges in the scheme of swapping two multipartite isotropic boxes, and occasionally emerges in the process of swapping three or more multipartite boxes. Judged by appearance, the emergence of quantum bound is merely a numerical coincidence, but the mathematical relation between nonlocal criterion $(2^{n-1})$, quantum bound $(2^{n-1/2})$ and extremal violation $(2^n)$ is a clue to get a deep understanding of quantum nonlocality.

Many open problems are presented in front of us. The fact that nonlocal correlation only can be generated from swapping the postquantum boxes makes not only an emergence of quantum bound but also a gap between QNC and PQC. The coupler, as the most important part in swapping process, beyond the device for poor dynamics applied outside the nonsigned boxes after measurement, is very likely valuable for some other dynamical processes, individually or combined as the scheme depicted in Fig. 3. How to generalize the swapping to high dimensional systems (multipartite multi-nary boxes) with suitable inequality is worth to study, too.

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