3-D reconstruction of phase grating via digital micro-holography

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Abstract. Digital micro-holography technique, a combination of digital holography with microscopy, can be used to measure and analyse the micro-structure and status information of samples when the digital hologram registered is reconstructed numerically by some numerical methods based on the principle of scalar diffraction or coherent image. By summarizing the development of digital micro-holography, Fresnel approximation reconstruction algorithm and two-wavelength technique are introduced to study the three dimensional phase reconstruction of phase grating. Experimental results show good agreement with true parameters of phase grating. It also analyses the aberration of the image reconstructed by Fresnel approximation approach and two-wavelength technique.

1. Introduction

Scanning Probe Microscopy [1], Confocal Laser Scanning Microscopy [2], micro-interference profiler [3] and digital micro-holography are the main techniques developed in recently years for quantitative analysis of micro-structures. Digital micro-holography is an amalgamation technology of digital holography and microscopy which can be used to promote the development and application of digital micro-holography.

Both scanning and non-scanning [4] Confocal laser scanning microscopes, are high precision microscopes with such drawbacks as the necessity to add fluorescence to samples and time-consuming set-up. Scanning Probe measurement can be used to achieve an atomic resolution, but it has the drawbacks, such as easy pollution of probe and damage of sample by probe. And they are all mainly used for static measurements. Micro-interference profiler provides high precise measurement with better hardware equipments. Its axial resolution gets to angstrom level and the lateral resolution is limited by numerical aperture of micro-objective. Digital micro-holography has its advantages over the above-mentioned techniques including easy setup non-contact measurement, and no need for strict mechanical adjustment, time-consuming scan and pre-processing of samples. And what is more, digital micro-holography has its own superiority in dynamically testing the biological samples,
particles, Micro ElectroMechanical System (MEMS)/Nano ElectroMechanical System (NEMS), insect’s shape and micro-organs.

With rapid development of Personal Computers, image sensor and digital signal processing techniques, digital micro-holography can be used to numerically reconstruct the intensity and phase information of micro-objects, including transparent objects and reflecting objects, by single hologram\[^5\]. Its applications had been extended to for example, inspection of particulate positions in space and etched nuclear tracks\[^6,7\], measurement of profiles or deformation of micro-optics and micro-components of MEMS\[^8,9\], investigation of biologic samples and living cells\[^10\], determination of material parameters, such as Poisson-ratio, Young’s modulus or thermal expansion coefficient\[^11\].

As far as reconstruction is concerned, aberration of reconstructed image is one of the major concerns. The 3-dimensional reconstruction of phase grating by digital micro-holography is presented together with the analyses of the aberration of image reconstructed by Fresnel approximation approach and the effect of two-wavelength technique on the aberration of reconstructed image.

2. Principle of image reconstruction via digital micro-holography

3-dimensional phase information of phase grating is obtained using Fresnel approximation approach and two-wavelength technique in this paper.

2.1 Fresnel approximation reconstruction algorithm.

Fresnel approximation algorithm and convolution algorithm are used to numerically reconstruct object waves via digital holography. One advantage of convolution algorithm is the unlimited registered distance, but two Fourier transformations and one inverse Fourier transformation are time-consuming. The second characteristic is its small reconstructed size which determines this algorithm is only suitable for small size objects. For application of simple Fresnel approximation, the recording distance must satisfy Fresnel approximation conditions and suits for big scale objects. Although they are main algorithms used in micro-holography, the convolution algorithm is usually modified in numerical reconstruction processing\[^12\], or the registered object wave is pre-magnified by a microscope and then projected to the Charge Coupled Device (CCD) if Fresnel approximation algorithm is applied. In this paper, we use the latter algorithm.

Fresnel diffraction expression is given by:

\[
\tilde{E}(x', y') = \frac{\exp(ikd')}{i\lambda d'} \left[ \int \int \tilde{E}(\xi, \eta) \exp\left\{ \frac{ik}{2d'} \left[ (x' - \xi)^2 + (y' - \eta)^2 \right] \right\} d\xi d\eta \right]
\]

(1)

Where \((\xi, \eta)\) is the diffraction aperture plane, \((x', y')\) is the diffraction image plane, \(d'\) is the distance between diffraction aperture and image planes and \(\lambda\) is wavelength of light source.

Using equation(1), the expression of diffraction of hologram can be given by:

\[
\tilde{E}(x', y') = \frac{\exp(ikd)}{i\lambda d} \left[ \int \int H(\xi, \eta) R(\xi, \eta) \exp\left\{ \frac{ik}{2d} \left[ (x' - \xi)^2 + (y' - \eta)^2 \right] \right\} d\xi d\eta \right]
\]

Where \(H(\xi, \eta), R(\xi, \eta)\) are hologram and reconstruction reference wave respectively, \(k = 2\pi\lambda^{-1}\), \(\tilde{E}(x', y')\) is the corresponding diffraction field of object wave.

After quantification,

\[
E(n, m) = z(n, m) \cdot F^{-1}\{H(k, l) \cdot R(k, l) \cdot w(k, l)\}
\]

(2)

with

\[
z(n, m) = \exp\left\{ \frac{i\pi d'}{N \Delta \xi} \left( \frac{n^2}{\Delta \xi^2} + \frac{m^2}{\Delta \eta^2} \right) \right\},
\]

\[
w(k, l) = \exp\left\{ \frac{i\pi d}{N^2 \Delta \xi^2} \left( k^2 \Delta \xi^2 + l^2 \Delta \eta^2 \right) \right\}
\]

where \((\Delta x', \Delta y'), (\Delta \xi, \Delta \eta)\) are pixel size of reconstructed image and CCD respectively, \(d'\) and \(\lambda\) are the reconstruction distance and the wavelength of reference wave used in reconstruction process respectively and \(N\) is the pixel number along axes \(x\) and \(y\) of CCD.
Thus, the whole reconstruction process is the inverse fast Fourier transform. In order to use fast Fourier transform, the following is satisfied automatically:

\[
\frac{k}{d'} \Delta \xi' = \frac{2\pi}{N}, \quad \frac{k}{d'} \Delta \eta' = \frac{2\pi}{N}
\]

That is

\[
\Delta x' = \frac{\lambda d''}{N \Delta \xi'}, \quad \Delta y' = \frac{\lambda d''}{N \Delta \eta'}
\]

Equation (4) also indicates the determinant factors of the lateral resolution of the reconstructed image and the spacial resolution of the system.

2.2 Basic principle of two-wavelength technique.

It is the wrapped phase by calculating \( \arctan[E(n,m)] \) by Equation (2). And the continuous phase distribution can be obtained by unwrapping when the phase difference of adjacent points does not exceed \( 2\pi \), Or else, it can not get the accurate phase value. In this paper, two-wavelength technique is applied to extend the measurement depth.

\( \lambda_1, \lambda_2 \) are defined as two different wavelengths, \( \phi_1 \) and \( \phi_2 \) describe the phases of wave front of hologram recorded by \( \lambda_1 \) and \( \lambda_2 \), respectively. \( \text{OPD}_1, \text{OPD}_2 \) describe the corresponding optical path differences. The relation among the three factors is as follows:

\[
\phi_1 = \frac{2\pi}{\lambda_1} \text{OPD}_1, \quad \phi_2 = \frac{2\pi}{\lambda_2} \text{OPD}_2
\]

When the wavelength difference is small enough and the refractive indexes in phase medium are approximatively equal for different wavelengths, Equation (5) can be written as:

\[
\phi_1 - \phi_2 = \left( \frac{2\pi}{\lambda_1} - \frac{2\pi}{\lambda_2} \right) \text{OPD}
\]

Then \( \Delta \phi = 2\pi \Lambda^{-1} \text{OPD} \), With \( \Lambda = (\lambda_1 - \lambda_2) (\lambda_2 - \lambda_1)^{-1} \), is the synthetic wavelength.

In general, the synthetic wavelength \( \Lambda >> \lambda_1 \) (or \( \lambda_2 \)) because the wavelength difference is very small. Thus the two-wavelength technique can be interpreted as a wavelength scaling process used to measure some samples with big depth. In this paper the two holograms are recorded and stored separately at two wavelengths. The numerical reconstruction can be carried out to get wrapped phases \( \phi_1 \) and \( \phi_2 \), then depth distribution of phase grating can be calculated using the two-wavelength technique mentioned above.

3. Experiment of phase grating

The experiment is performed to record the hologram of phase grating and reconstruct the 3-dimensional profile. The phase grating is 50line/mm and its step is about 0.248\( \mu \)m.

3.1 Experimental set-up.

A Mach-Zender interference system for transparent samples is used in this paper. Before registering the holograms, the phase grating is magnified by Micro-Objectives (MO) with 40 magnification.

The primary elements include the tunable diode laser (NewFocus Inc., mode: Velocity 6308, tuning range 668-678 nm, coarse tuning Resolution 0.02 nm), MO (40 ×, 0.65 NA), CCD (CoolSNAP Inc., mode: CoolSNAP cf with Frame Grabber, 1392 × 1040 pixels, 4.65 × 4.65 \( \mu \)m). Figure 1 shows the sketch map of the experimental set-up based on the Mach-Zender system, and Figure 2 shows its photo with 1. tunable diode laser 2. beam splitter 3. reflector 4. grating 5. objective 6. expanding –collimation setting 7. tunable diaphragm 8. reflector 9. beam splitter 10. CCD 11. Frame Grabber, PC 12. laser controller and 13. vibration isolating platform.
The laser source is separated into two parts by beam splitter 2, one branch is reference wave, the other forms object wave after sample and micro-objective. The interference between object wave and reference wave by beam splitter 9 creates hologram recorded by CCD and transmitted to computer.

3.2 Experimental results.
With the phase grating of 50 line/mm used as sample, the recorded length is about 0.1 mm. The 3-dimensional continuous height distribution is gotten using Fresnel approximation approach and two-wavelength technique. The experimental results show a good agreement with the true parameters of phase grating. Figures 3 and 4 show the digital holograms recorded with $\lambda_1 = 664.5 \text{nm}$, $\lambda_2 = 664.4 \text{nm}$. Figure 5 shows the 3D height distribution of grating.

4. The aberration analysis of image reconstructed

4.1 Aberration analysis of image reconstructed using Fresnel approximation algorithm.
For Fresnel approximation algorithm, Equation (4) shows that the lateral resolution of reconstructed image changes with the reconstruction distance when the parameters of CCD and the wavelength of reference wave remain unchanged. So, the 3-dimensional reconstructed image produces the aberration due to the lateral resolution difference on the different reconstructed planes. The aberration can be neglected when the whole pixel size difference along axial direction is less than half of one pixel of reconstructed image. Because the hologram recorded is interference with sample magnified and reference wave, the aberration of image reconstructed which is reduced should be discussed.

Supposing $(d_1, d_2')$ is true depth of phase grating. The pixel size difference of reconstructed original image which is not image magnified is can be given by Equation (4):

\[
\frac{(\Delta x_1 - \Delta x_2')}{40} = \frac{\lambda (d_1 - d_2')}{N \Delta \xi}, \quad \frac{(\Delta y_1 - \Delta y_2')}{40} = \frac{\lambda (d_1 - d_2')}{M \Delta \eta} \quad (6)
\]
Then

\[
\begin{align*}
N(\Delta x'_1 - \Delta x'_2) / 40 &= \frac{\lambda (d'_1 - d'_2)}{2} < \frac{1}{2} \Delta x' / 40, \left(\frac{1}{2} \Delta x'_2 / 40\right) \\
M(\Delta y'_1 - \Delta y'_2) / 40 &= \frac{\lambda (d'_1 - d'_2)}{2} < \frac{1}{2} \Delta y' / 40, \left(\frac{1}{2} \Delta y'_2 / 40\right)
\end{align*}
\]

(7)

With 40 is MO magnification. Equation(7) shows that the bigger the depth value of samples, the severer the aberration.

Supposing \(\lambda = 664.5\text{nm}, d'_2 = 135\text{mm}, d'_1 - d'_2 = 0.248\mu\text{m}, N = 1392, M = 1040\) (the same values as the experimental system above), and corresponding values are calculated as follows:

\[
\Delta x'_1 / 40 = 0.35\mu\text{m, } \Delta y'_1 / 40 = 0.46\mu\text{m, } \frac{\lambda (d'_1 - d'_2)}{\Delta \xi} = \frac{\lambda (d'_1 - d'_2)}{\Delta \eta} = 0.035\mu\text{m}
\]

Thus, the aberration can be neglected in this work. But it should be considered if phase grating height exceeds about \(1.2\mu\text{m}\), while the same working parameters are used for the experimental system above.

4.2 Aberration analysis of image reconstructed using two-wavelength technique

Because the depth of sample was also magnified before it interfered with the reference wave, two-wavelength is selected to reconstruct the height information of the magnified phase grating. For the two-wavelength approach, different wavelengths produce aberration in the same plane of the reconstructed image.

Considering:

\[
\begin{align*}
N(\Delta x'_1 - \Delta x'_2) &= \frac{(\lambda_1 - \lambda_2)d'_1}{\Delta \xi} < \frac{1}{2} \Delta x'_1 \left(\frac{1}{2} \Delta x'_2\right) \\
M(\Delta y'_1 - \Delta y'_2) &= \frac{(\lambda_1 - \lambda_2)d'_1}{\Delta \eta} < \frac{1}{2} \Delta y'_1 \left(\frac{1}{2} \Delta y'_2\right)
\end{align*}
\]

(8)

Equation(8) shows that the bigger the wavelength difference, the bigger the aberration.

Applying the same analysis described in section 4.1, when the wavelength difference exceeds about \(0.24\text{nm}\), the aberration should be considered. In this work, the wavelength difference is \(0.1\text{nm}\), thus, the aberration will not affect the reconstructed image.

5. Conclusions

This paper studies the 3-dimensional phase reconstruction of a phase grating via digital micro-holography. The experimental results show good agreement with actual parameters of phase grating. Then it analyzes the aberration of image reconstructed using Fresnel approximation approach and two-wavelength technique. The aberration of reconstructed image should be considered seriously for Fresnel approximation approach when the depth of samples exceeds a certain range. And the range of wavelength difference should also be taken into account when two-wavelength technique is used.

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