On the Impact of Attachment Strategies for Payment Channel Networks

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Abstract—Payment channel networks, such as Bitcoin’s Lightning Network, promise to improve the scalability of blockchain systems by processing the majority of transactions off-chain. Due to the design, the positioning of nodes in the network topology is a highly influential factor regarding the experienced performance, costs, and fee revenue of network participants. As a consequence, today’s Lightning Network is built around a small number of highly-connected hubs. Recent literature shows the centralizing tendencies to be incentive-compatible and at the same time detrimental to security and privacy. The choice of attachment strategies therefore becomes a crucial factor for the future of such systems. In this paper, we provide an empirical study on the (local and global) impact of various attachment strategies for payment channel networks. To this end, we introduce candidate strategies from the field of graph theory and analyze them with respect to their computational complexity as well as their repercussions for end users and service providers. Moreover, we evaluate their long-term impact on the network topology.

Index Terms—Payment Channel Networks, Lightning Network, Autopilot, Attachment Strategies, Join Strategies

I. INTRODUCTION

Payment channel networks, such as Bitcoin’s Lightning Network [1], are second-layer solutions that aim to improve the scalability, performance, and privacy aspects of blockchain networks by taking the majority of transactions off-chain. They allow nodes to open bilateral payment channels by depositing money in a shared multisig address. By doing this, parties can negotiate state changes locally in a secure and rapid fashion. As each node may establish multiple channels, a network of payment channels is created, which enables payments to non-adjacent nodes. While such multihop payments are routed over intermediary nodes, the protocol ensures not only that the payments are settled atomically, but also that intermediaries are compensated for their service through transaction fees. In this way, nodes are incentivized to lock up funds in order to provide the payment routing infrastructure.

The Lightning Network currently exhibits a high degree of centralization [2]–[4], which has shown to be detrimental to the security [4], [5] and privacy [6], [7] properties of the network. Moreover, since the Lightning Network uses a source-routed best-effort routing protocol to conduct multihop payments, payment reliability is not guaranteed but highly depends on the connectivity of involved nodes [8]. Likewise, the position of routing nodes in the network topology is highly correlated with their fee revenue. Therefore, the question arises which connection points are preferable for nodes joining the network with respect to their connectivity or revenue. Prior works studying this question from a theoretical perspective indicate that profit-optimal join strategies tend to promote network centralization [9]–[11]. These results therefore not only suggest the existence of a fundamental trade-off between the network’s goals of efficiency and decentralization, but also a conflict of interest between the local egoistical point of view of an individual node and the global long-term development of the network topology.

In this paper, we therefore present an empirical analysis on the local and global impact of attachment strategies for payment channel networks. We survey the field of graph theory for strategies that aim to increase the joining node’s connectivity and routing revenue, i.e., we propose strategy candidates from the perspectives of end-users and service providers. Each strategy is analyzed with respect to its local impact on the performance of an individual joining node based on network simulations of the Lightning Network. Moreover, the computational complexities and resource requirements of all strategies are evaluated in order to classify them according to their real-world practicability. In contrast, we study the global long-term impact of the discussed attachment strategies on the Lightning Network topology under the assumption of mass adoption. To this end, we study how the network’s centralization and performance metrics change in dependence of a given attachment strategy. We show that, while in the short term centrality-based strategies perform best in some scenarios, they in the long term result in suboptimal network-wide transaction success rates and fee costs. To this end, we identify two candidate strategies with the potential to combine local short-term and global long-term interests.

The remainder is structured as follows and includes the following contributions. Section II provides more detailed background information about the Lightning Network and introduces the model and notations serving as a basis for our research. In Section III, we introduce and discuss attachment strategies for the Lightning Network, matching different usage scenarios. The strategies are empirically analyzed from the user and hub perspectives in Section IV and their long-term impact is evaluated in Section V. After that, Section VI gives an overview of related work, before Section VII concludes the paper.


II. PRELIMINARIES

Payment channel networks (PCNs) establish an overlay network on top of a cryptocurrency. Instead of storing the details of every transaction on the blockchain, a PCN offers the possibility to open payment channels and to process payments bilaterally off-chain. While different designs of payment channels have been introduced so far [12]–[15], the most popular PCN is the Lightning Network [1]. By the beginning of May 2020, it consisted of more than 4,300 nodes and around 25,000 payment channels exhibiting a combined capacity of more than 785 bitcoins (more than USD 8 million)\(^1\). Lighting enables potentially infinite payments between two users with only two on-chain transactions, the first for opening and the second for closing the payment channel. The opening transaction allows both parties to securely deposit money in a shared multisig address on the Bitcoin blockchain. After that, both users can send bitcoins through the channel by renegotiating the balance allocation between them. In order to close the channel, its latest state is published on the blockchain, whereby the final balances are returned to the involved parties. Lightning, and PCNs in general, provide mechanisms for resolving conflicts and attempts of fraud.

A. Network Model

We model the Lightning Network as a directed multigraph \( G = (V, E) \), where the vertex set \( V \) constitutes the Lightning nodes and the multiset \( E \) the payment channels. Every bidirectional channel is represented by two directed edges in order to separately store the individual capacities and channel policies of both channel endpoints. Accordingly, the edge \((u, v)\) stores how high \( u \)'s share of the total channel balance is and which settings \( u \) chose for the channel. As each channel locks funds and each on-chain transaction involves costly transaction fees, opening many channels on the Lightning Network can be expensive. In order to reduce the number of required channels, the Lightning Network offers multihop routing, which enables the sending of payments to non-adjacent nodes in the network. In this case, the payment is routed over intermediate nodes along the payment path, which is determined by the payment’s sender and secured by Hashed Time-Locked Contract (HTLC) protocols. For a more detailed description of the HTLC construction the reader is referred to [1], [17]. Moreover, privacy is improved by applying an onion routing scheme based on the Sphinx [18] mix packet format.

The payment’s sender typically selects the most suitable route by running an adapted version of Dijkstra’s shortest path algorithm [19] that considers channel capacities, fees and locking duration in the edge weight calculation. That is, the algorithm first discards all candidate edges with insufficient capacities and then selects the path with minimal aggregated edge weights based on the intermediate nodes’ fee policies and maximum lock-time. Such a weight-based algorithm is for example utilized by the popular LND implementation, which accounts for more than 90% of today’s network nodes [5]. For the calculation of the respective transaction fees, each edge in the public network graph stores the routing fee policies, which are composed of a base fee \( f^B \) and a proportional fee \( f^P \). The base fee \( f^B \) is a fixed amount that has to be paid to the routing node for every forwarded payment; the default value is 1 satoshi (= \( 1 \cdot 10^{-8} \) BTC). The default value of proportional fee \( f^P \) is \( 1 \cdot 10^{-6} \) satoshi, which is multiplied with the transaction amount \(|tx|\) of each payment. Therefore, routing higher value payments generates higher fees for the routing nodes. Concisely, the fee \( f_u(v, |tx|) \) that has to be paid to the routing node \( u \) for forwarding a transaction with amount \(|tx|\) to \( v \) can be calculated accordingly as

\[
 f_u(v, |tx|) = f^B_u(v) + f^P_u(v) \cdot |tx|.
\]

In order to account for the weight-based routing algorithm, parts of our graph analysis is based on the fee graph \( G_{F,|tx|} \), which we obtain through a transformation on \( G \). This transformation allows the network analysis to account for Lightning’s routing behavior in an approximative fashion, even when applying standard weight-based graph algorithms. In particular, \( G \) is reduced to \( G_{F,|tx|} \) by excluding all edges of insufficient capacities with respect to a transaction amount \(|tx|\). The weights for each edge \((u, v)\) in \( G_{F,|tx|} \) are set to \( f_u(v, |tx|) \), i.e., they denominate the routing fees that would arise from transferring \(|tx|\) through this channel.\(^2\)

B. Joining the Network

Due to the costs associated with channel establishment, a node joining the network should follow a certain set of rules for choosing its initial connection points according to an optimization goal. We call such an algorithm returning a candidate node set \( C \subseteq V \) an attachment strategy \( S(G, k, \text{cap}) \rightarrow C \), which takes as parameters the public network graph \( G \), the number of channels to be opened \( k = |C| \), and the capacity \( \text{cap} \) (in satoshi) that each of the channels should hold.

The respective optimization goal depends on the motivation for joining the network. We consider the attachment strategies from the point-of-view of three distinct perspectives:

- **End-users** join the network to conduct cheap, reliable, and fast payments and therefore are interested in strategies that improve their local connectivity to the network.
- **Service providers** participate as routing nodes in the network in order to earn transaction fees. They are therefore interested in optimizing their local node’s channel selection in order to receive maximal profit.
- **The network** perspective regards the global impact of a particular strategy and considers its impact on the network’s overall connectivity and reliability over time.

As these view points follow partly conflicting interests, they may not easily be reconciled, but expose a fundamental trade-off between short-term egoistical efficiency and the long-term

\(^1\)According to the network snapshots [16] provided by [4].

\(^2\)Note that this approximative approach is only applied when necessary for general graph analysis or as part of the attachment algorithms. In contrast, the simulation framework used for the evaluation of the proposed strategies follows a payment protocol that closely resembles the real-world behavior, as will be discussed in Section IV-A.
development of the network (cf. [8]). However, as different attachment strategies fall on different points in the spectrum of this trade-off, we empirically investigate their usefulness regarding these three view points.

The performance of each strategy of course highly depends on the user’s behavior: if we for example assume an end-user would conduct frequent payments to only a single service provider, the optimal connectivity-oriented strategy would be to establish a direct payment channel to it. However, so far no reliable data source on user behavior in payment channel networks is publicly available to the research community, which necessitates the introduction of a number of assumptions with regard to the payment model. To this end, we refrain from introducing overly complex assumptions that may act as confounding factors to our analysis. In particular, we assume for the sake of simplicity that the user plans to send payments to destinations all over the network. Moreover, we assume that the capacity $cap$ is the same for all $k$ channels and that initial balances are split equally between the channel endpoints. We also assume that every node in the network agrees to open a channel, which may not be the case in the real network, in particular since recent research found such optimistic behavior to entail security risks [20]. Finally, we assume new channels to be established with the default fee settings. Note that in current Lightning implementations attachment strategies are used in the so-called autopilot feature that allows the client software to automatically choose and establish new channels.

III. NETWORK ATTACHMENT STRATEGIES

In the following, we introduce candidate strategies for nodes joining payment channel networks. We also provide a first assessment of their applicability as well as their complexity in dependence of the number of nodes $n = |V|$ and number of edges $m = |E|$.

A. Random

The Random strategy is the simplest attachment strategy, in which the attachment points are determined by uniform random sampling from the node set $V$. This strategy can be quickly computed in $O(n)$ and, while it mainly serves as a baseline for comparison, it counteracts centralizing tendencies since it does not prefer any particular connection point.

B. Highest Degree

The Highest Degree strategy sorts all nodes $V$ according to their degree, and returns the $k$ nodes with the highest degrees. As the number of different neighbors is presumably more meaningful than the total number of channels a node $v$ has, its degree $deg(v)$ is determined in the fee graph $G_F$, since it disregards multi-edges. The candidate set can be computed quickly with this strategy because $deg(v)$ can be retrieved from the adjacency lists and sorting can be done in $O(n \log n)$.

Connecting to nodes with highest degrees is an extreme form of preferential attachment which is known to induce a “rich-gets-richer” effect that yields scale-free networks [21], and is likely responsible for the highly centralized substructures found in the Lightning Network today. In fact, highest-degree attachment strategies were deployed in prior versions of LND’s autopilot feature and have been critically discussed in the community [22].

C. Betweenness Centrality

The notion of betweenness centrality [23] indicates how many shortest paths in the network graph $G$ a node $v$ is part of. More specifically, $bc(v) = \sum_{s,t \in V} \frac{\sigma_{st}(v)}{\sigma_{st}}$, where $\sigma_{st}$ is the total number of shortest paths from $s$ to $t$ and $\sigma_{st}(v)$ is the number of shortest paths from $s$ to $t$ via $v$.

In context of Lightning, nodes exhibiting a high betweenness centrality implies that they are often chosen by the weight-based routing algorithm and therefore are part of many payment paths. Since a large share of the network can be reached via these nodes with minimal distance in terms of fees, they are in return promising candidates for node attachment. Note that this often also corresponds to overall shorter payment paths, which improves reliability.

Consequently, the Betweenness attachment strategy elects the $k$ nodes with the highest betweenness centrality values, which are calculated via the weighted Brandes’ algorithm [24] based on the fee graph $G_F$. As the weighted version of the algorithm has a runtime complexity in $O(mn + n^2 \log n)$, our implementation additionally employs the optimizations from [25], which speed up the calculation of betweenness centralities (but do not change the algorithmic complexity). Connecting to nodes with the highest betweenness centralities is another form of preferential attachment and likely results in further network centralization.

D. $k$-Center

The $k$-Center strategy is based on the assumption that the joining node can improve its overall connectivity to the network by establishing channels to $k$ nodes such that the highest distances between them and any other node in the network are minimized. Ideally, this would lead to nodes in different parts of the network being chosen as the $k$ new neighbors in order to minimize the length of the longest shortest payment path. This likely results in faster and cheaper transactions due to fewer nodes being part of the routes. Reducing the number of nodes and channels contained in a payment route can also decrease the risk that a transaction fails as there are less points of failure.

The idea for this strategy is based on the $k$-center problem [26], which is defined as follows.

**Definition 1:** Given a complete undirected graph $G = (V, E)$ in a metric space and an integer $k$, a $k$-center is a subset of nodes $C \subseteq V$ with $|C| \leq k$ such that $\max_{v \in V} d(v, C)$ is minimized, with $d(v, C)$ being the shortest distance of $v$ to the closest node in $C$.

It was previously proven that this problem is NP-complete and that it is NP-hard even for an $\epsilon$-approximation with $\epsilon < 2$ [26]. This means that 2-approximation algorithms, which return a solution that is within twice the optimal solution value
in polynomial time, are the best possible algorithms for the $k$-center problem, unless $P = NP$. Due to the fact that distances in the fee graph $G_F$ are not necessarily symmetric, the Lightning Network unfortunately cannot be modeled as a weighted fee graph in metric space. We therefore use the greedy $k$-center algorithm introduced in [27] on a generated complete distance graph to minimize the number of hops on the longest shortest path and disregard fees or channel capacities. To this end, the joining node first establishes a connection to the network’s highest degree node and then executes a single-source shortest path (SSSP) search to retrieve the distances for the $k$-center algorithm. This results in a total time complexity of $O(k(m + n))$.

As the $k$-Center strategy aims to interconnect the network centers, it should improve the network’s robustness and facilitate decentralization.

E. $k$-Median

Besides looking at the longest shortest path to any other node in the network, a promising strategy is to minimize the average shortest path distance to all other nodes. Assuming that the joining node sends a transaction to any other node with the same probability, it is very likely favorable to require a minimal average number of hops to any other node in order to reduce transaction fees, latencies, and failures. Hence, we have to solve a problem that is known as the Single-Source Average Shortest Path Distance Minimization (SS-ASPDM) problem [28]. It was previously proven that an optimal solution to the SS-ASPDM problem for a node $v$ can be found by only adding edges incident to $v$ [28]. Thus, the problem can be utilized in our use case of the Lightning Network since a joining node may only influence the opening of channels which are incident to itself. Adopting the approach to only add edges incident to the source node $v$, the SS-ASPDM problem corresponds to the $k$-median problem [28].

In a graph context, the $k$-median problem can be formulated as follows.

Definition 2: Given a complete undirected graph $G = (V,E)$ in a metric space and an integer $k$, the $k$-median problem strives to find a subset of nodes $C \subseteq V$ with $|C| \leq k$ such that $\sum_{v \in V} d(v,C)$ is minimized, with $d(v,C)$ being the shortest distance of $v$ to the nearest node in $C$.

Again, the problem is NP-hard [29] and only an approximate solution can be found within polynomial time, unless $P = NP$. For solving the $k$-median problem in a distance graph, we establish an initial connection to the highest degree node and then utilize the “forward” greedy algorithm presented in [29], which results in an overall time complexity of $O(kn(n + m) \log n)$ when applied to the weighted fee graph.

Similarly to the $k$-Center approach, the $k$-Median strategy promises to improve network robustness and reduce centralization.

F. Maximum Betweenness Improvement (MBI)

A node that joins the network with the intent to act as a service provider or routing node strives for financial profit from participating in the Lightning Network. To this end, a routing node $v$ should rather focus on optimizing its own betweenness centrality $bc(v)$ than connecting to central nodes.

Therefore, it has to solve a problem known as Maximum Betweenness Improvement (MBI) [30], which is defined as follows.

Definition 3 (MBI): Given a directed graph $G$, a node $v$, and an integer $k$, which set of edges $S$ incident to $v$, with $|S| \leq k$, should be added to $G$ in order to maximize $bc(v)$?

The MBI problem has been proven to be NP-hard, but a greedy algorithm that provides an approximate solution exists [30]. This MBI strategy temporarily opens any channel that node $v$ could set up, calculates $bc(v)$, and closes the channel again. This is repeated for all possible channels and in the end the channel generating the highest betweenness improvement for the joining node is elected. This channel is then established and the procedure is repeated until all $k$ candidates are found, leading to an overall high time complexity in $O(kn^3)$.

Note that this strategy is similar to the approach found in [11], which however also optimizes the node’s fee settings. As this results in a further increased computational complexity over the already high resource requirements of Bergamini et al.’s algorithm, we in lieu of these optimizations follow the more feasible MBI strategy.

IV. Empirical Analysis

In the following, we empirically analyze the performance of attachment strategies for payment channel networks from a local perspective, i.e., from the view of a single end-user or service provider aiming to join the network.

A. Network Simulator, Setup, and Methodology

As a basis for the empirical analysis, we developed a time-discrete event simulator that implements the network multi-graph model (cf. Section II-A) and allows to simulate payment processing as well as nodes joining the network according to a given attachment strategy. The simulator initially reads the network graph from a snapshot of the Lightning Network and simulates path finding through a weight-based route selection algorithm similar to the one found in LNND. While some aspects of the real-world payment procedure—such as the HTLC protocol negotiations—are omitted by our simulation model for the sake of simplicity, the simulator was carefully implemented to approximate the real-world behavior. To this end, transaction processing is simulated by checking and adjusting the available balances along the payment path. During this phase, the arising fee revenues are calculated based on the provided fee policies and the remaining transaction value for each hop along the way. Note that consequentially and just as in the real network, transaction success is not guaranteed even if a path is found, as the path finding algorithm does not operate on the private balances, but the public capacities.

We base our further analysis on a snapshot of the Lightning Network from May 1, 2020 at 10am that was taken from the

3 The simulator code base is publicly available in our companion repository at https://gitlab.tu-berlin.de/rohrer/pcn-attachment-data.
dataset [16] provided by [4]. At this time, the largest connected component of the network consisted of more than 4,300 nodes connected by nearly 25,000 channels, which held an overall capacity of more than 785 BTC.

In order to analyze their performance, we simulated the joining of individual nodes according to the given attachment strategy, every time establishing $k \in \{1, \ldots, 15\}$ channels with sufficient capacity and default fee settings. We then evaluated the connectivity and fee revenue of the joined node through two sets of simulated payments: one set of 1,000 transactions with the joined node as a fixed source and the destination selected by uniform random sampling, and another set of 1,000 transactions for which both source and destination were chosen randomly. The simulations were conducted under the assumption of three different transaction volumes: micro payments of 100 sats, medium payments of 10,000 sats, and macro payments of 1,000,000 sats (see also [11]). If not stated otherwise, our analysis is based on the most relaxed assumption of 100 sats. For every strategy, transaction value, and every value of $k$, the simulations were furthermore repeated 30 times with different seed value inputs for the utilized random number generator. This results in a five-digit sample size ensuring the statistical significance of the results.

**B. Transaction Success**

In order to assess the impact of the attachment strategies on the connectivity of the joining node, we analyzed the average transaction success rate, i.e., the share of all transaction that actually succeeded. In Figure 1, the average success rate is shown in dependence of the number of transaction amounts and channels $k$ that were established corresponding to the respective strategies. Moreover, the a priori network-wide average success rate is shown for comparison, which was determined by simulating 10,000 transactions with randomly chosen sources and destinations in the initial graph configuration.

We observe that generally node connectivity improves with the number of established channels and that all but the Random strategy tend to result in an average success rate higher than the network average. Moreover, strategies that prefer central connection points, such as the Betweenness strategy, fare better than strategies that connect the periphery of the network, such as $k$-Center. This is likely the case because connecting to very central points in the network reduces the average path length and thereby also the probability of routing failures due to unavailable balances.

This is supported by the fact that the assumed transaction volume has a big impact on the average success rate: while the network average for micro payments is around 83% (Figure 1a), it drops below 34% for medium payments (Figure 1b), and even to less than 4% for macro payments (Figure 1c). This observation is of course in line with previous literature, in which Lightning’s limited available capacity and the resulting low success rates for higher-volume payments have been discussed for some time [4], [8], [31]. Our results underline that currently only a small number of central nodes hold enough capacity to be able to route any high-volume payments. While the heavily skewed capacity distribution results in overall very low transaction success rates, we observe that strategies that preferably connect to these few central nodes—such as Betweenness, Highest Degree, and $k$-Best—can increase their lead in such high-volume payment scenarios. However, in order to limit the impact the current capacity constraints found in the Lightning Network have on our results, we continue our further analysis of attachment strategies under the most relaxed assumption of micro payments.

**C. Transaction Fees**

End-users joining the Lightning Network likely want to optimize their connection point with regards to the result fees that arise from sending payments. In Figure 2a the fees paid by the connecting node are shown in dependence of the number of channels and with respect to the chosen strategy. Again, generally all strategies result in fee costs lower than the network average, which is even true for Random for more than $k = 5$ channels. As the fees in most cases improve linearly with the number of established channels,
D. Service Provider Revenue

Service providers join the network with the intent to earn the maximum amount of profit. To this end, we analyze which attachment strategy can help to improve their fee revenue. The share of routed transactions (which in our case directly corresponds to the fee revenue) is shown in Figure 2b.\(^4\)

Independently of the strategy, the share of routed transactions improves with the overall connectivity of the joining nodes, i.e., it increases with the number of established payment channels \(k\), but tends to favor strategies that improve path diversity. However, the MBI strategy is clearly superior in this regard, allowing the joining node even to route close to 6% of all payments conducted in the network by establishing \(k = 15\) channels. This comes to no surprise as this strategy is specifically focused on maximizing the number of payment paths routed through the joining node, and previous work showed the benefits of such an approach [11]. Apart from this, the \(k\)-Median strategy is a promising candidate, as it able to secure the service provider a routing share of close to 3% of all payments in the case of \(k = 15\).

\(^4\)Note that our analysis compares the proportional fee revenues gained from routing in the Lightning Network and does not consider any costs for running a routing node, such as the on-chain fees associated with channel establishment. In order to estimate the net profit of a node operator, such cost would have to be known and subtracted from the revenue.

E. Runtime Analysis

In order for an attachment strategy to be an actual candidate to be implemented in the autopilot functionality of a Lightning client implementation, it should deliver its results in a viable amount of time. Therefore, we measured the run times of discussed strategies under real-world conditions. To this end, we deployed our strategy implementations on an \texttt{t2.xlarge} instance (4 vCPUs based on Intel Xeon 3.3 GHz, 16 GB memory) on Amazon Elastic Compute Cloud (EC2) running Ubuntu Server 18.04. We then measured the execution time it took the algorithms to return the respective candidate sets.

The results shown in Table I generally concur with our complexity analysis given in Section III: while the Highest Degree, Betweenness and \(k\)-Center strategies remain roughly constant runtimes, \(k\)-Median and especially MBI grow in a linear fashion with the number of established payment channels \(k\).

This is of particular significance, since it takes MBI between 2,000 and 2,500 seconds longer to finish for each additional channel. As this amounts to an overall runtime of around seven hours for \(k = 10\), the practicability of this strategy is heavily put under question, potentially even given its performance benefits in terms of fee revenue.

V. Evaluating the Long-term Impact

So far, we analyzed the discussed attachment strategies with respect to their local short-term impact, i.e., from the point of view of an egoistical node joining the network. In the following, we assess the global long-term impact of the discussed attachment strategies for payment channel networks.

A. Simulation Setup

In order to evaluate the global long-term impact, we utilized the time-discrete event-based network simulator from Section IV-A to model the process of 5,000 nodes sequentially joining the Lightning Network, which corresponds to more than doubling the network size. Each node joins the network with \(k = 10\) channels that are established according to the given strategy, which is roughly the network average node degree. While the future network development will probably not exactly follow these assumptions, this approach allows us to compare the advantages and drawbacks of each strategy without considering additionally interfering and confounding factors. This simulation-based analysis was conducted for all but the MBI strategy. Due to MBI’s significantly higher computational requirements (cf. Section IV-E), we had to

Table I: Algorithm runtimes in dependence of chosen attachment strategy and number \(k\) of established channels (in sec.).

|                | 1    | 2    | 3    | 4    | 5    | 6    | 7    | 8    | 9    | 10   |
|----------------|------|------|------|------|------|------|------|------|------|------|
| Highest Degree | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 | 0.25 |
| Betweenness    | 440.00 | 440.00 | 440.00 | 440.00 | 440.00 | 440.00 | 440.00 | 440.00 | 440.00 | 440.00 |
| \(k\)-Median   | 2.70 | 4.70 | 6.40 | 8.50 | 9.70 | 11.30 | 13.30 | 15.00 | 17.10 | 18.20 |
| \(k\)-Center   | 0.51 | 0.59 | 0.63 | 0.66 | 0.67 | 0.72 | 0.75 | 0.81 | 0.88 | 0.81 |
| MBI            | 2,784.00 | 4,834.00 | 6,965.00 | 9,232.00 | 11,429.00 | 13,938.00 | 16,478.00 | 19,371.00 | 22,294.00 | 24,634.00 |
refrain from including it in the long-term evaluation. As before, all randomized transactions were repeated 1,000 and all simulations 30 times to ensure statistically significant results.

**B. Impact on the Network’s Topology**

In order to analyze the impact each attachment strategy has on network centralization over time, we analyzed the network topology in intervals of 500 joining nodes and recorded essential network metrics. Figure 3a shows the Gini coefficient of the node degree, which quantifies the inequality of the degree distribution for an increasing number of added nodes. As expected, the network exhibits initially a high Gini value of nearly 0.75, which underlines the high degree of inequality currently exhibited by Lightning’s network topology. Furthermore, the results show that strategies following a preferential attachment pattern, such as the **Highest Degree** or **Betweenness** strategies only marginally decrease the centralization over time, while strategies that also connect the fringes of the network, such as **k-Center** and **Random** have a strong positive impact on centralization. Interestingly, we observe that **k-Median** tends to elect the same set of \(k\) nodes over time. While these \(k\) nodes increase their connectivity, it does not result in a significant improvement with respect to the degree inequality.

Figure 3b shows the average network diameter, i.e., the longest shortest path in the network, which is an indicator for the worst-case routing complexity. Again, **Random** and **k-Center** perform best and are able to immensely reduce the initial network diameter of 13 already after attaching 500 nodes. Notably, the **k-Center** strategy quickly allows all network nodes to reach all other nodes in just four hops. In order to get an understanding of how the participation in routing is impacted over time, we analyzed the inequality of betweenness centralities and the central point dominance. The results generally concur with our observations for node degrees. They also show that our current choice of establishing the initial connection of the **k-Center** and **k-Median** strategies to the single highest degree node results in an increased central point dominance. While this is an implementation detail, its impact requires further investigation in the future.

**C. Impact on the Network’s Performance**

In order to evaluate the performance of the network in dependence of each attachment strategy, we analyzed the average success rate and the arising fees by regularly simulating transactions in the network. To this end, we executed batches of 1,000 micro transactions with randomly chosen sources and destinations after the addition of every 500 nodes.

As can be seen in Figure 3c, the average network success rate generally improves with an increasing number of nodes and the additionally provided routing capacity. The evaluation moreover shows that again the decentralizing strategies **Random** and especially **k-Center** benefit the overall network connectivity the most, letting the success rate quickly rise to close to 100%.

While this pattern is generally also reflected in the average paid transaction fees, as shown in Figure 3d, the results highlight that a high degree of centralization can be beneficial for fee costs. In particular, while the **Highest Degree** strategy does generally not offer many benefits, it does result in rather low average fee costs. This is likely due to the short average path lengths and high efficiency of star sub-structures (cf. [9], [10]). However, again the **k-Center** strategy proves to be the most promising candidate to minimize fee costs for the end-user in the long term, with **k-Median** being a close second.

**D. Discussion**

Throughout our analysis, it became apparent that the Lightning Network currently is heavily restricted by its overall limited capacity and its concentration on a few central service providers. We therefore found that the provided quality of service and user experience would immensely benefit from any kind of higher-volume and higher-connectivity adoption.

We also found that from an egotistical perspective, strategies selecting central attachment points seem to provide the best short-term performance, with the exception of transaction fees, in which case the **k-Median** strategy showed to be the most promising candidate. From the global point of view, however, decentralizing strategies proved to provide the best long-term benefits for the network overall. With regard to this conflict of interest, we empirically confirm the trade-off between efficiency and decentralization [8], [9].
However, our analysis showed two strategies to be feasible and potentially capable of combining local short-term and global long-term interests: $k$-Center and $k$-Median. While these strategies may not be the absolute optimum from the egoistical point of view, they benefit the long-term network development the most. It therefore remains an open question whether users would accept non-optimal short-term strategies, if they benefit them and the whole network in the long-term.

In order to balance this trade-off, real-world implementations should consider to employ a set of different well-chosen strategies to establish their channels. However, the exact choices and the share of connections established through a particular strategy are up to further analysis. Our implementation of the $k$-Center and $k$-Median strategies currently builds on an initial centralized connection. We therefore also deem the potential of such “mixed” strategies, i.e., strategies that further randomize and distribute these connection types, a promising subject for future research. Furthermore, while we generally hold the inherent conflicts of interest to be hard to reconcile, we think they should further be discussed and addressed in the community.

VI. RELATED WORK

Most research on payment channel networks focuses on aspects such as the channel design [1], [12]–[15], the network’s topology [2], [4], [31], and routing algorithms [32], [33]. While most of these entries take the network topology as a given, few entries study how the network structure emerges and which algorithms for creation are preferable. To this end, Avarikioti et al. [9] follow a game-theoretic approach and show that centralized structures can make the network more efficient and stable. In particular, the authors show that a star graph, i.e., a graph with one central hub, poses a social optimum as well as a Nash equilibrium in terms of efficiency and stability. These results are supported by Sali and Zohar [10], who show the efficiency of centralized hub structures. Interestingly, Rincon et al. [34] come to the conclusion that it is nonetheless not disadvantageous for smaller nodes, i.e., nodes with few channels and a limited budget, to connect to other small nodes. These connection types were even proven to positively impact the network’s robustness and efficiency, although connections to larger or richer nodes were shown to improve the efficiency even further.

Most related to our work, Ersoy et al. [11] study attachment strategies and have been the first to observe that profit maximization of service providers in the Lightning Network is connected to their position in the network. To this end, the authors introduce the maximum reward improvement (MRI) problem, which, in contrast to the MBI problem, additionally aims to optimize the joining node’s fee policies. While the authors reduce MRI to MBI and show that it is also NP-hard, they provide an approximation algorithm. The results underline that improving the betweenness is a successful strategy to increase a node’s fee revenue. As the approximation algorithm proposed in [11] is still very costly, we deliberately refrain from optimizing fee policies in our work. Instead, we opt to implement a profit-oriented strategy based on Bergamini et al.’s [30] approximation algorithm under the assumption of default fee policies.

While many prior entries highlight that, in theory, attachment strategies optimizing for profit and efficiency tend to favor the creation of highly centralized topologies, studies on the security [4], [5], [35], [36] and privacy [6], [7], [37]–[39] of payment channel network emphasize the risk associated with network centralization. These contradictory results indicate an inherent trade-off between the network efficiency and decentralization, also observed by Waugh and Holz [8]. As this conflict of interest is not easily resolved to one side or the other, additional insight on the long-term effects of certain design decisions becomes necessary. To this end, we provide the first empirical study on the long-term impact of attachment strategies for payment channel networks.

Li et al. [40] present an algorithm that allows to calculate the optimal distribution of initial channel balances assuming a certain budget in order to satisfy payment demands. Similarly, channel rebalancing protocols [41], [42] aim to optimally redistribute the allocation of funds in order to ensure frictionless payment processing. We deem the integration of such capacity planning algorithms into our model promising future work.

VII. CONCLUSION

In this work, we provided an empirical study on the impact of attachment strategies for payment channel networks that once more exposes the fundamental trade-off between efficiency and decentralization. While we were able to identify two candidate strategies with the potential to combine local short-term and global long-term interests, we deem the question of attachment strategies an important avenue for future research and discussion on payment channel networks.

REFERENCES

[1] J. Poon and T. Dryja, “The bitcoin lightning network: Scalable off-chain instant payments,” 2016.
[2] I. A. Sérès, L. Gulyás, D. A. Nagy, and P. Burcsi, “Topological analysis of bitcoin’s lightning network,” in Mathematical Research for Blockchain Economy. Springer International Publishing, 2020, pp. 1–12.
[3] J. Lin, K. Primicerio, T. Squarini, C. Decker, and C. J. Tessone, “Lightning network: a second path towards centralisation of the bitcoin economy,” CoRR, vol. abs/2002.02819, 2020. [Online]. Available: https://arxiv.org/abs/2002.02819
[4] E. Rohrer, J. Malliaris, and F. Tschorsch, “Discharged payment channels: Quantifying the lightning network’s resilience to topology-based attacks,” in 2019 IEEE European Symposium on Security and Privacy Workshops, EuroS&P Workshops 2019, Stockholm, Sweden, June 17-19, 2019. IEEE, 2019, pp. 347–356.
[5] S. Tochner, S. Schmid, and A. Zohar, “Hijacking routes in payment channel networks: A predictability tradeoff,” CoRR, vol. abs/1909.06890, 2019. [Online]. Available: http://arxiv.org/abs/1909.06890
[6] G. Kappos, H. Youssaf, A. Piotrowska, S. Kanjalkar, S. Delgado-Segura, A. Miller, and S. Meiklejohn, “An empirical analysis of privacy in the lightning network,” arXiv preprint arXiv:2003.12470, 2020.
[7] E. Rohrer and F. Tschorsch, “Counting down thunder: Timing attacks on privacy in payment channel networks,” in AFT ’20: Proceedings of the second ACM conference on Advances in Financial Technologies, 2020.
[8] F. Waugh and R. Holz, “An empirical study of availability and reliability properties of the bitcoin lightning network,” CoRR, vol. abs/2006.14358, 2020. [Online]. Available: https://arxiv.org/abs/2006.14358
