Spin susceptibility in small Fermi energy systems: effects of nonmagnetic impurities

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Abstract. In small Fermi energy metals, disorder can deeply modify superconducting state properties leading to a strong suppression of the critical temperature $T_c$. In this paper, we show that also normal state properties can be seriously influenced by disorder when the Fermi energy $E_F$ is sufficiently small. We calculate the normal state spin susceptibility $\chi$ for a narrow band electron-phonon coupled metal as a function of the non-magnetic impurity scattering rate $\gamma_{\text{imp}}$. We find that as soon as $\gamma_{\text{imp}}$ is comparable to $E_F$, $\chi$ is strongly reduced with respect to its value in the clean limit. The effects of the electron-phonon interaction including the nonadiabatic corrections are discussed. Our results strongly suggest that the recent finding on irradiated MgB$_2$ samples can be naturally explained in terms of small $E_F$ values associated with the $\sigma$-bands of the boron plane, sustaining therefore the hypothesis that MgB$_2$ is a nonadiabatic metal.

PACS. 74.25.-q General properties; correlations between physical properties in normal and superconducting states – 71.28.+d Narrow-band systems; intermediate-valence solids – 74.62.Dh Effects of crystal defects, doping and substitution

1 Introduction

Scattering from weak disorder or diluted non magnetic impurities plays a marginal role on many thermodynamics quantities of conventional metals. Most peculiar is the absence of any reduction on the critical temperature $T_c$ in conventional isotropic $s$-wave superconductors as stated by the Anderson’s theorem and as confirmed by several experimental measurements.

This insensitivity stands out in particular in comparison with $d$-wave superconductors where the strong anisotropy of the order parameter leads to a suppression of $T_c$. In that case for instance the reduction on $T_c$ upon disorder can give qualitative information of the microscopic characteristic on the pairing ($d$- vs. $s$- wave symmetry, local vs. long-ranged interaction, etc...) of.

A conventional role of nonmagnetic impurities is recently questioned in some high-$T_c$ superconductors, as MgB$_2$ and fullerene compounds. In these materials a notable reduction of $T_c$ upon disorder has been reported in spite of the $s$-wave symmetry of both of them. Quite remarkable is also the reduction of the density of states (DOS) inferred by NMR measurements of the nuclear spin-lattice relaxation rate $T_1 [1/T_1 T \propto N_2(T)]$ where $N_0$ is the electronic density of states at the Fermi level and $T$ is the temperature. In Ref. a reduction of 62 % of $1/T_1 T$ upon disorder was reported by $^{11}$B NMR measurements in contrast with the conventional theory of non magnetic impurity scattering which would predict no effect of the magnetic susceptibility. An additional puzzling feature is the discrepancy between spin-lattice relaxation rate measurements performed on $^{11}$B NMR and on $^{25}$Mg. No reduction of $1/T_1 T$ was indeed observed on magnesium atoms. The authors of Ref. speculate this difference could be related to the different nature of the electronic states: magnesium atoms would mainly probe the $\pi$ bands of MgB$_2$ through the hybridization of B(2$p_x$) orbitals with Mg(s) states, while the spin-lattice relaxation rate on the boron is expected to be very sensitive to the twodimensional $\sigma$ bands formed by B(2$p_x$2$p_y$).

The evidences of anomalous effects of disorder and non magnetic impurities in these systems prompt thus some intriguing open questions: i) which is the origin of the suppression of $T_c$ in $s$-wave systems as MgB$_2$ and fullerenes? ii) which is the origin of the reduction of the density of states as probed by spin-lattice relaxation rate $1/T_1 T$ measurements? iii) which is the origin of the different behaviour of Mg and B NMR measurements?

The point (i) was previously addressed in Ref. in the context of a nonadiabatic theory of superconductivity where impurity effects in the nonadiabatic channels were shown to suppress $T_c$ even for purely isotropic $s$-wave superconductors. In this paper we extent our analysis to the spin susceptibility. In particular we show that a possible unifying explanation of all this complex anomalous
scenario could come from taking into account in a coherent way the small Fermi energy nature of these materials. This is clearly unavoidable in $C_{60}$ compounds where the narrow bandwidth of the $t_{1u}$ bands (but also the $h_u$ bands for hole doped $C_{60}$) results in a Fermi energy $E_F \sim 0.25$ eV [1]. This is also true in MgB$_2$ where the low hole filling of the 2D $\sigma$ bands leads to $E_F^\sigma \sim 0.4-0.6$ eV [11,13]. These values of $E_F$ are at least one order of magnitude less than in common metals and in conventional superconductors. The discrepancy between Mg and B NMR measurements can be thus related to the probing of different bands (π on Mg, $\sigma$ on B), and, in the last analysis, to the different magnitude of the Fermi energies ($E_F^\pi \sim 5$ eV $\gg E_F^\sigma$).

On microscopic grounds, small Fermi energy effects are operative as soon as $E_F$ becomes of the same order of the other relevant energy scales. For an electron-phonon system in the presence of non magnetic impurities as we consider here, $E_F$ should be thus compared with the characteristic phonon energy scale $\omega_{ph}$ and with the impurity scattering rate $\gamma_{imp}$. The breakdown of the adiabatic hypothesis ($E_F \gg \omega_{ph}$) in a small Fermi energy system implies the onset of new channels of electron-phonon interaction which need to be taken into account. On the other hand the finiteness of the ratio $\gamma_{imp}/E_F$ gives rise to anomalous impurity effects which have to be analyzed in the presence of the same electron-phonon interaction since electron, phonon and impurity energy scales could be all of the same magnitude: $E_F \sim \omega_{ph} \sim \gamma_{imp}$.

### 2 The model

In this section, we derive the electron spin susceptibility by employing the Baym-Kadanoff technique which permits to derive, within a conserving theory, higher order response functions as functional derivatives of the single particle Green’s function in the presence of an external field [14]. This approach is thus an appropriate starting point to study small Fermi energy systems where the violation of the Migdal’s theorem valid for $E_{ph} \gg \omega_{ph}$ requires a generalization of the conventional theory in the nonadiabatic regime.

Objects of our investigation is the non magnetic impurity effects on the spin susceptibility in small Fermi energy systems in the presence of a sizable electron-phonon interaction. NMR techniques can probe the electron density of states by means of different ways. Most direct is the evaluation of the static uniform limit $\chi$ of the generalized electron spin susceptibility $\chi(q, \omega)$:

$$\chi = \lim_{q \to 0} \lim_{\omega \to 0} \chi(q, \omega)$$

(1)

which, for a non interacting system with large Fermi energy, is simply $\chi \propto N_0$. Electron-electron exchange interaction gives rise however to the so called Stoner enhancement: $\chi \propto N_0/(1-I)$ (I being the Stoner factor). Experimentally the static uniform limit $\chi$ of electron spin susceptibility can be measured by a proper analysis of the Knight shift after the orbital contribution is subtracted.

Similar information are obtained by spin-lattice relaxation rate $T_1$, which can be also mainly related, after subtraction of orbital terms, to the electron spin susceptibility through the relation:

$$\frac{1}{T_1T} \propto \lim_{\omega \to 0} \sum_q A^2(q) \frac{\chi(q, \omega)}{\omega},$$

(2)

where $A(q)$ is the form factor relative to the particular nucleus. As pointed out in the introduction, $1/T_1T \propto N_0^2$ in large Fermi energy systems.

In the following we focus on the static uniform spin susceptibility $\chi$ which permits a more direct comparison with the density of states and which is only slightly affected by different form factors $A(q)$. As it will be clear in the following the anomalous effects of non magnetic impurities are essentially related to the similar energy scales of $\gamma_{imp}$, $E_F$ and $\omega_{ph}$. In this situation impurity scattering leads to an effective renormalization of the generalized spin susceptibility which is expected to appear in similar way both in the static uniform limit $\chi$ and in the spin-lattice relaxation rate. In this perspective the reduction of the magnetic susceptibility upon disorder pointed out by NMR technique should be read more as an anomalous renormalization effect appearing in small Fermi energy system than as a real reduction of the density of states.

In Quantum Field Theory the electron spin susceptibility is usually related to the one particle Green’s function $G$ through the relation: [13]

$$\chi(T) = -2\mu^2 T \sum_n \sum_k G(k, n)^2 \Gamma(k, n),$$

(3)

where $G(k, n)$ is the electron propagator at finite temperature expressed in Matsubara frequencies

$$G^{-1}(k, n) = i\omega_n - \epsilon(k) + \mu - \Sigma(k, n),$$

(4)

and $\Gamma(k, n)$ is the spin vertex function. The Baym-Kadanoff formalism provides a powerful technique to related the spin vertex function $\Gamma(k, n)$ to $G(k, n)$. Following the standard procedure we generalize the Green’s function in Eq. (1) in the presence of an external magnetic field $h$:

$$G_{\sigma}^{-1}(k, n) = i\omega_n - \epsilon(k) + \mu + h\sigma - \Sigma_{\sigma}(k, n).$$

(5)

The spin vertex function is thus obtained as functional derivative of the Green’s function $G_{\sigma}$ in the presence of the external magnetic field for $h \to 0$ [14]:

$$\Gamma(k, n) = \frac{1}{2} \sum_\sigma \sigma \left[ \frac{dG_{\sigma}^{-1}(k, n)}{dh} \right]_{h=0}$$

$$= 1 - \frac{1}{2} \sum_\sigma \Sigma_{\sigma}(k, n) \left[ \frac{dh}{dh} \right]_{h=0}.$$  

(6)

The set of Eqs. (1)-(3) defines a self-consistent method to obtain the spin susceptibility from the knowledge of the self-energy. The complex nature of the interactions in the systems is thus hidden in the specific form of the self-energy which needs to be explicitly provided.
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the nonadiabatic theory which accounts for the additional

interaction channels arising when $E_F \sim \omega_{ph}$. The formal

derivation of the nonadiabatic theory has been already

presented in several papers where we refer for more de-

tails. Here we focus on the role of non magnetic

impurities. In the spirit of the Baym-Kadanoff theory our

starting point will be the self-energy which is diagrammatic-

ally depicted in Fig. 1. The first two diagrams represent

the electron-phonon interaction in nonadiabatic regime

($E_F \sim \omega_{ph}$) including the first order vertex processes;

the third diagram is the self-energy in Born approximation

for impurities of density $n_{imp}$ interacting with electrons

via a scattering potential $v_{imp}$. The last diagram is the

exchange electron-electron interaction: this term is just a

constant and does not play any role in the self-energy, but

it gives rise to the Stoner factor in the spin susceptibility.

Fig. 1 defines in an unambiguous way the self-energy

and the one particle properties of the system. Standard

procedure in isotropic materials is to replace the self-en-

ergy $\Sigma(k, n)$ with its Fermi surface average: $\Sigma(k, n) \rightarrow

\Sigma(n) \equiv \langle \langle \Sigma(k, n) \rangle \rangle_{FS}$. It is convenient to take into account

self-energy effects in the electronic Green’s function is to

introduce the renormalized Matsubara frequencies defined

as $W_n = \omega_n - \Sigma(n)$. In addition, for sake of simplicity we

consider a half-filled band with bandwidth $E$ and constant

density of states $N(e) = N_0 [-E/2 \leq e \leq E/2]$. The parameter $E/2$ represents thus the Fermi energy $E_F = E/2$. Within these assumptions the analytic expression of the renormalized Matsubara frequencies $W_n$ involving the self-energy depicted in Fig. 1 reads:

$$W_n = \omega_n - 2T \sum_{m} V(n, m) \arctan \left( \frac{E_F}{W_m} \right)$$

$$+ 2\gamma_{imp} \arctan \left( \frac{E_F}{W_n} \right),$$

where

$$V(n, m) = \lambda D(n - m)[1 + \lambda P(Qc; n, m)]$$

is the nonadiabatic electron-phonon kernel appearing in

the self-energy equation and where we have neglected the

electron-electron exchange interaction which leads just to

a constant term. In Eq. 3, $D(n - m)$ is the phonon prop-

agator which for a single Einstein mode $\omega_{ph} = \omega_0$

reduces simply to $D(n - m) = -\omega_0^2/[(\omega_n - \omega_m)^2 + \omega_0^2]$ and

$P(Qc; n, m)$ is the vertex function [11,12].

$$P(Qc; n, m) = -T \sum_l D(n - l) \left\{ B(n, m, l) \right.$$} 

$$+ \frac{A(n, m, l) - B(n, m, l)(W_l - W_{l-n+m})^2}{(2E_FQ_c^2)^2}$$

$$\times \left[ R(Qc; n, m, l) - 1 - \log \left( \frac{1 + R(Qc; n, m, l)}{2} \right) \right] \right\},$$

(9)

where

$$A(n, m, l) = (W_l - W_{l-n+m}) \left[ \arctan \left( \frac{E_F}{W_l} \right) \right.$$}

$$- \arctan \left( \frac{E_F}{W_{l-n+m}} \right) \right],$$

(10)

$$B(n, m, l) = \frac{E_F W_{l-n+m}}{E_F^2 + W_{l-n+m}^2},$$

(11)

$$R(Qc; n, m, l) = \sqrt{1 + \left( \frac{4E_FQ_c^2}{W_l - W_{l-n+m}} \right)^2}. $$

(12)

The dimensionless parameter $Q_c = q_c/2k_F$, where $k_F$ is the Fermi momentum, takes into account the upper cut-off $q_c$ for the momentum transfer in the electron-phonon interaction. This cutoff has been introduced to simulate a momentum dependent renormalization due to possible strong electronic correlations [13]. For weak correlated metals $Q_c \approx 1$, while $Q_c \ll 1$ when correlation is strong. As we are going to see, the parameter $Q_c$ plays only a marginal on the static spin susceptibility, whereas it strongly affects the superconducting critical temperature $T_c$.

In the formula for $W_n$, Eq. 3, $\gamma_{imp}$ is the impurity

scattering rate which in the Born approximation reduces to $\gamma_{imp} = \pi n_{imp}N_0 v_{imp}^2$ [14]. This expression holds true

for low values $n_{imp}$ of impurity concentrations and weak

scattering potential $v_{imp}$. An expression of $\gamma_{imp}$ valid also for strong, but diluted, impurity interactions is provided

by the $T$-matrix approximation: $\gamma_{imp} = \pi n_{imp}N_0 v_{imp}^2/[1 + (\pi N_0 v_{imp})^2]$. Using the Baym-Kadanoff formalism we are able to obtain also an analytic expression for the spin vertex function $\Gamma(k, n)$. The diagrammatic expression of $\Gamma(k, n)$ corresponding to the self-energy depicted in Fig. 1 is shown in Fig. 2. In the isotropic case we have considered here we can replace also the spin vertex function $\Gamma(k, n)$ with its Fermi surface average $\Gamma(n)$. The momentum average of the spin vertex implies that the momentum correlations
between spin up electrons and spin down holes are taken into account only at a mean level through the parameter $Q_c$. This will be a poor approximation when the dispersion of collective modes is investigated, while it is expected to not affect in a qualitative way the static uniform spin susceptibility. Disregarding the momentum dependence of $\Gamma(k, n)$ and we get thus:

$$\Gamma(n) = 1 + T \sum_m \left[ I + V_T(n, m) \right] \frac{2E_F}{W_m^2 + E_F^2} \Gamma(m)$$

$$- \gamma_{\text{imp}} \frac{2E_F}{W_m^2 + E_F^2} \Gamma(n),$$

where the quantity $I = N_0 U$ is the Stoner factor arising from the electron-electron exchange interaction and the last term comes from the impurity scattering processes. Moreover

$$V_T(n, m) = \lambda D(n - m) \left[ 1 + 2\lambda P(Q_c; n, m) \right]$$

$$+ \lambda^2 C(Q_c; n, m)$$

(14)

describes the electron-phonon processes in nonadiabatic regime which include electron-phonon vertex $P(Q_c; n, m)$ given by Eqs. (13) and the cross diagrams:

\begin{align*}
C(Q_c; n, m) &= T \sum_l D(n - l) D(l - m) \\
& \times \left\{ 2B(n, m, l) + \arctan \left( \frac{4E_F Q_c^2}{|W_l - W_{n+m-l}|} \right) \right. \\
& \left. \times \frac{A(n, m, l) - B(n, m, l)(W_l - W_{n+m-l})^2}{2E_F Q_c^2 |W_l - W_{n+m-l}|} \right\},
\end{align*}

(15)

where $A(n, m, l)$ and $B(n, m, l)$ are given by equations (10) and (11), respectively. Note that $V_T(n, m)$ is deeply different from $V(n, m)$ since the first describes electron-phonon scattering in the spin electron-hole channel, and the second one the electron-phonon interaction in the single particle propagator.

\section{3 Results and discussion}

Eqs. (7)-(15) can be solved in a self-consistent iterative way to obtain $W_n$ and $\Gamma(n)$. Eq. (3), in its isotropic form:

$$\chi(T) = \chi_0 T \sum_n \frac{2E_F}{W_n^2 + E_F^2} \Gamma(n),$$

(16)

provides finally the spin susceptibility as function of generic impurity scattering rate $\gamma_{\text{imp}}$, electron-phonon coupling constant $\lambda$, adiabatic ratio $\omega_0/E_F$ and momentum cut-off $Q_c$. Here $\chi_0$ is the free electron Pauli spin susceptibility $\chi_0 = 2\mu_B^2 N_0$.

In order to point out the role of small Fermi energy in the impurity scattering effects on the spin susceptibility, we consider for the moment the simple case of no electron-phonon interaction ($\lambda = 0$). In this case the only energy scales in the system are $\gamma_{\text{imp}}$ and $E_F$. From Eq. (16), equation (13) has thus the simple self-consistent solution as function of the spin susceptibility itself:

$$\Gamma(n) = \frac{1 + I(\chi/\chi_0)}{1 + \gamma_{\text{imp}} \frac{2E_F}{W_n^2 + E_F^2} \chi}$$

(17)

and the spin susceptibility $\chi$ recovers the usual Stoner-like expression:

$$\chi = \frac{\chi_0}{1 - I(\chi_0/\chi_T)},$$

(18)

where the bare spin susceptibility $\chi_0$ is now affected by the non magnetic impurity scattering:

$$\chi_0 = \chi_0 T \sum_n \frac{2E_F}{W_n^2 + E_F^2 + \gamma_{\text{imp}} 2E_F}.$$

(19)

For large Fermi energy systems, $E_F \gg \gamma_{\text{imp}}$, equation (19) reduces to the Pauli spin susceptibility $\chi_0 = \chi_T$. It is thus clear the non magnetic impurity effects can appear only if the Fermi energy is small enough to be comparable with $\gamma_{\text{imp}}$. Note that in the presence of electron-phonon interaction an additional energy scale is provided by $\omega_{\text{ph}}$, so that additional anomalous impurity effects are ruled by the additional parameter $\gamma_{\text{imp}}/\omega_{\text{ph}}$.

In Fig. 3 we plot the behaviour of the static spin susceptibility $\chi$ in a small Fermi energy system ($\omega_0/E_F = 0.7$) as function of the impurity scattering rate $\gamma_{\text{imp}}$. The data are normalized with respect to the “pure” limit $\gamma_{\text{imp}} \rightarrow 0$. Left panel refer to a weak coupling electron-phonon case ($\lambda = 0.4$), right panel to strong coupling ($\lambda = 1.0$). In both cases a Stoner factor $I = 0.4$ was considered. Solid lines represent the nonadiabatic vertex corrected theory with different values of $Q_c$ (from the top to the bottom: $Q_c = 0.1, 0.3, 0.5, 0.7, 0.9$) and the dashed line the non crossing approximation where only finite bandwidth effects were retained $|P(Q_c; n, m) = C(Q_c; n, m) = 0$ in Eqs. (8)-(14)). Fig. 3 shows a strong reduction of $\chi$ due to the impurities scattering with respect to a large Fermi energy case ($E_F \gg \omega_0$, dotted line). We observe only a weak dependence on the electron-phonon coupling (right
Fig. 3. Spin susceptibility $\chi$ as function of the impurity scattering rate $\gamma_{\text{imp}}$ in the presence of electron-phonon interaction ($\lambda = 0.4$, left panel; $\lambda = 1.0$, right panel, $\omega_0/E_F = 0.7$ and electron-electron exchange repulsion $I = 0.4$. Solid lines: nonadiabatic vertex corrected theory with different values of $Q_c$ (from the top to the bottom: $Q_c = 0.1, 0.3, 0.5, 0.7, 0.9$); dashed line: non crossing approximation.

Fig. 4. Spin susceptibility $\chi$ as function of the impurity scattering rate $\gamma_{\text{imp}}$ for a small Fermi energy ($\omega_0/E_F = 0.7$, left panel) and for a large Fermi energy system ($\omega_0/E_F = 0.1$, right panel). Solid lines correspond to different values of $\lambda$: (from bottom to the top) $\lambda = 0.2, 0.4, 0.6, 0.8, 1$. Other parameters: $Q_c = 0.4$ and $I = 0.4$. Dotted line: infinite Fermi energy case.

We are now in the position to re-address the open questions arisen in the introduction, concerning namely: the origin of the reduction of the density of states as probed by NMR susceptibility measurements; the discrepancy between the different behaviour of Mg and B NMR measurements. In particular we suggest that the reduction of the spin-lattice relaxation rate $1/T_1T$ upon induced disorder could reflect the small Fermi energy nature of the electronic structure probed by the experiments. Within this context the insensitivity to disorder scattering of the spin-lattice relaxation rate $1/T_1T$ probed on $^{25}$Mg in contrast to the marked reduction of $^{11}$B NMR measurements acquires a natural explanation related to the different Fermi energy scales involved in the two cases. $^{25}$Mg NMR measurements mainly probe the $\pi$ band structures with high Fermi energy $E_F \sim 5$ eV. Using a typical phonon frequency $\omega_0 \approx 70$ meV [17] we estimate $\omega_0/E_F \sim 0.014$ which yields a negligible dependence of the spin susceptibility on the amount of disorder. On the other hand NMR on the $^{11}$B boron nucleus is strongly coupled with the inplane $\sigma$ orbitals with Fermi energy $E_F \sim 0.4 - 0.6$ eV. The same phonon frequency scale gives thus $\omega_0/E_F \sim 0.12 - 0.14$, where visible impurity scattering effects are expected. We conclude that the small Fermi energy of the $\sigma$ bands is the major responsible for the reduction of the magnetic susceptibility when probed on $^{11}$B nuclei compared with NMR measurements on the same quantity on $^{25}$Mg.

Note that the usual two band model within the Migdal-Eliashberg framework, with different electron-phonon coupling for $\sigma$ and $\pi$ bands ($\lambda_\sigma \sim 1$, $\lambda_\pi \sim 0.2$), can not alone explain the discrepancy between the reduction rate probed by NMR. Indeed: a) electron-phonon interaction does not
affect impurity scattering for high Fermi energy systems \((E_F \gg \gamma_{\text{imp}}, \omega_0)\); b) the larger electron-phonon coupling constant in the \(\sigma\) bands would predict a smaller reduction rate of \(\chi\) as compared with the smaller \(\lambda\) of the \(\pi\) bands.

The above discussion suggests that the reduction of \(\chi\) upon disorder is a possible tool to point out nonadiabatic effects in small Fermi energy materials where \(\omega_0 \sim E_F\). In this perspective it is interesting to compare the simultaneous reduction of \(\chi\) and \(T_c\) as function of the impurity scattering rate \(\gamma_{\text{imp}}\). In this way in principle one can trace out the effects of non magnetic impurity scattering in small Fermi energy systems as functions of physical measurable quantities as \(\chi\) and \(T_c\) avoiding the use of the unaccessible parameter \(\gamma_{\text{imp}}\).

A conserving derivation of the superconducting equations in a fully consistent way with the evaluation of the spin susceptibility follows once again the Baym-Kadanoff theory based on Fig.\[4\] written in Nambu notation. A formal derivation of those equations and some technicalities about the numerical calculations of \(T_c\) were discussed in Ref.\[6\] where we refer for more details. In Fig.\[4\] we plot \(\chi\) and \(T_c\) as functions of \(\gamma_{\text{imp}}\) for \(I = 0.2\) and the couples of parameters \((\lambda = 1.0, \omega_0/E_F = 0.2), (\lambda = 0.2, \omega_0/E_F = 0.02)\). These cases should be qualitatively representative of the MgB\(_2\) \(\sigma\) and \(\pi\) bands which are respectively: strong coupled with small Fermi energy; and weak coupled with large Fermi energy. Note that a significant dependence on \(\gamma_{\text{imp}}\), for both \(T_c\) and \(\chi\) is observed only for \((\lambda = 1.0, \omega_0/E_F = 0.2)\) which represents the case of \(\sigma\) bands. The \(T_c\) vs. \(\chi\) plot is shown in Fig.\[4\] for \((\lambda = 1.0, \omega_0/E_F = 0.2, I = 0.2)\). A reduction of \(T_c\) of the order of 30 – 80% is predicted for a reduction of \(\chi\) of \(\sim 20\%\), depending on the parameter \(Q_c\). The general trend is thus in agreement with the experimental data reported in Ref.\[4\].

In MgB\(_2\), where the electronic correlation is thought to be negligible, there is no reason to expect a significant momentum selection and \(Q_c\) is expected to be \(Q_c \sim 1\). In this situation our analysis would underestimate the suppression of \(T_c\) \((\Delta T_c/T_c \sim 30\%)\) and \(\chi\) \((\Delta \chi/\chi \sim 20\%)\) when compared with the experimental scenario, although some care should be used to extrapolate from the static magnetic susceptibility \(\chi\) to \(1/TT_1\).

It is clear however that additional ingredients are required to be taken into account for a quantitative analysis of the experimental data of \(T_c\) vs. induced disorder. In particular the discussion in terms of two separated \(\sigma\) and \(\pi\) bands is expected to be a poor description for the superconducting properties of a complex multiband system as MgB\(_2\). On this basis we conclude that further investigation is needed to account in a fully satisfactory way for the anomalous dependence of \(T_c\) on the amount disorder. On the other hand the reduction of the spin susceptibility upon disorder and non magnetic impurities in MgB\(_2\) could be qualitatively understood within the present analysis. In particular our results suggest that a primary role could be played by the small Fermi energy effects driven in MgB\(_2\) by the closeness of the chemical potential to the top of the \(\sigma\) band. In this framework the different behaviour of B and Mg NMR data receives a natural explanation.

Interesting perspectives are also opened in regards to the fullerene based materials. The analysis is indeed simplified in these compounds as a single Fermi energy is present. The extreme smallness of \(E_F\) is fullerenes (\(E_F \simeq 0.25\) eV) suggests that disorder or nonmagnetic impurity effects could lead to even more marked reduction of \(T_c\) and \(\chi\) than in MgB\(_2\). A suppression of \(T_c\) upon induced disorder as previously been reported in Ref.\[4\]. At our knowledge no measurements of magnetic susceptibility as function of disorder or impurity amount as been at the present performed. Experimental work along this line is thus encouraged.

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