Properties of the Prompt Optical Counterpart Arising from the Cooling of Electrons in Gamma-Ray Bursts

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Abstract

This work extends a contemporaneous effort to study the properties of the lower-energy counterpart synchrotron emission produced by the cooling of relativistic gamma-ray burst (GRB) electrons through radiation (synchrotron and self-Compton) emission and adiabatic losses. We derive the major characteristics (pulse duration, lag time after burst, and brightness relative to the burst) of the prompt optical counterpart (POC) occurring during or after the GRB. Depending on the magnetic field lifetime, duration of electron injection, and electron transit time $\Delta t_\gamma$ from hard X-ray (GRB) to optical-emitting energies, a POC may appear during the GRB pulse (of duration $\delta t_\gamma$) or after (delayed OC). The signature of counterparts arising from the cooling of GRB electrons is that POC pulses ($\Delta t_\gamma < \delta t_\gamma$) last as long as the corresponding GRB pulse ($\delta t_\gamma \approx \delta t_\gamma$), while delayed OC pulses ($\Delta t_\gamma > \delta t_\gamma$) last as long as the transit time ($\delta t_\gamma \approx \Delta t_\gamma$). If OC variability can be measured, then another signature for this OC mechanism is that the GRB variability is passed on to POCs but not to delayed OCs. Within the GRB electron cooling model for counterparts, POCs should be on average dimmer than delayed ones (consistent with the data), and harder GRB low-energy slopes $\beta_{\text{LE}}$ should be associated more often with the dimmer POCs. The latter sets an observational bias against detecting POCs from (the cooling of electrons in) GRBs with a hard slope $\beta_{\text{LE}}$, making it more likely that the detected POCs of such bursts arise from another mechanism.

Unified Astronomy Thesaurus concepts: Non-thermal radiation sources (1119); Gamma-ray bursts (629); Radiative processes (2055); Plasma astrophysics (1261)

1. Introduction

The prompt counterpart is defined here as emission at an energy below the hard X-ray/gamma-ray of the main burst, and whose short variability timescale suggests a common origin. Simultaneity with the burst is not a requirement, as some prompt counterparts can appear during the burst (and are thus truly prompt), while others can appear after the burst. The latter will be called delayed counterparts, to avoid the oxymoron of “delayed prompt counterpart,” and should not be confused with the afterglow, whose lack of short timescale variability indicates that it arises from a different mechanism. Given that a slowly varying afterglow emission may exist even during the burst (e.g., gamma-ray burst, GRB 050820A, whose OC was decomposed by Vestrand et al. 2006 into a brighter afterglow-like emission and a dimmer component varying synchronously with the GRB), the identification of the prompt counterpart (as defined above) should be based on its short variability timescale and, when possible, on its correlation with GRB pulses.

A short-lived prompt optical counterpart (POC) emission has been detected during or after the prompt phase in many GRBs (e.g., sample listed by Kopac et al. 2013) and could arise from the following mechanisms:

(i) the reverse-shock propagating in the GRB ejecta (Mészáros & Rees 1997; Panaitescu & Mészáros 1998) could produce a delayed bright POC occurring after the GRB pulse if its synchrotron (SY) emission peaks around the optical, as proposed for the optical flash (OF) of GRB 991023 by Mészáros & Rees (1999) and Sari & Piran (1999);

(ii) optical emission arriving before/during the GRB pulse could be SY emission from relativistic electrons, which is upscattered (synchrotron self-Compton) to produce the GRB emission (Papathanassiou & Mészáros 1996; Mészáros & Rees 1997), as proposed for the POC/OF of GRB 990123 by Panaitescu & Kumar (2007); and

(iii) prompt/delayed OCs could arise from the pairs (produced by gigaelectronvolt photons emitted during the GRB phase) that are accelerated by the forward shock, as proposed for the OF of GRB 130427A by Vurm et al. (2014), or that form in the shocked medium, as proposed for the same OF by Panaitescu (2015).

Here, we study only the SY emission from cooled GRB electrons as a mechanism for POCs. A POC dimmer than the burst results if the magnetic field lives shorter than the radiative cooling timescale $t_{\text{rad}}$ of the typical GRB electron. If injection of the GRB electrons stops before $t_{\text{rad}}$ but their cooling continues until they radiate SY emission in the optical, then the POC peak flux will be the same as that of the burst (assuming a constant magnetic field). However, if the electron injection lasts longer than the electron transit time from gamma-to optical emission, then a bright POC can be produced.

We are interested in identifying temporal properties of POCs resulting from the cooling of GRB electrons that can be used as identifiers of this mechanism for POCs, albeit there are few criteria to discriminate the POCs arising from the other three mechanisms above, most noteworthy being that

(i) POCs produced by the reverse-shock and pairs formed from the gigaelectronvolt prompt emission should and could, respectively, occur after the GRB, and

(ii) POCs produced in the synchrotron self-Compton model for GRBs could occur before the prompt burst emission.

Table 1 lists the notations most often used here.


2. Prompt Counterpart Properties

The analytical formalism for calculating the counterpart pulse light curve \( f(t) \) resulting from the cooling of GRB electrons is presented in a companion paper (Panaitescu & Vestrand 2022—PV22). Those light curves allow the calculation of the counterpart peak flux and its brightness relative to that of the GRB, quantified by the effective GRB-to-counterpart spectral slope \( \beta_{\gamma_e} \).

The counterpart peak epoch \( t_p \) depends on

(i) the time \( \delta t_\gamma \) that it takes electrons “to cool through” the observing energy, which is often equal to the time \( t_{\gamma_e} \) that it takes GRB electrons to migrate from gamma-ray emitting energies to the energy \( \epsilon \) at which the counterpart is observed,

(ii) the lifetime \( t_B \) of the magnetic field and the duration \( t_f \) of electron injection, and, if the emitting surface is of uniform brightness, and

(iii) the spread in photon-arrival time \( t_{\text{ang}} = R/(2c\Gamma) \) (in the source frame, with \( R \) the source radius across the ejecta surface of opening \( \Gamma^{-1} \) (the inverse of the source Lorentz factor) from which the observer receives a relativistically enhanced emission.

For pulses that are not too peaky or too stretched, the counterpart peak epoch \( t_p \) is also a good measure for the counterpart pulse duration \( \delta t_\gamma \). Another quantity of interest is the time lag \( \Delta t_e \) between the GRB and the counterpart peak epochs, which depends on the transit time \( t_e \) and the two timescales \( t_B \) and \( t_f \) for the magnetic field lifetime and electron injection, but is independent on the angular spreading timescale \( t_{\text{ang}} \).

For ease of access, we reiterate here some of the analytical results for the above counterpart properties of interest.

If the pulse duration is set by radiative cooling (SY or inverse Compton, iC, through scatterings in the Thomson regime having a cooling power \( P_{\gamma_e} \sim \gamma_e^2 \), as for SY), then the \( \gamma \)-to-\( \epsilon \) electron transit time \( t^{(\gamma\epsilon)}_{\gamma_e} \) is

\[
t^{(\gamma\epsilon)}_{\gamma_e} = \left( \frac{\epsilon}{E_{\gamma_e}} \right)^{-1/2} t_{\gamma_e,i}.
\]

Here, \( t_{\gamma_e,i} \) is the SY-cooling timescale of the typical GRB \( \gamma_e \) electrons that radiate SY emission at the peak energy \( E_{\gamma_e} \) of the \( \epsilon F_\epsilon \) GRB spectrum:

\[
t_{\gamma_e}(\gamma_e) = \frac{\gamma_e m_e c^2}{P_{\gamma_e}(\gamma_e)} = \frac{8.10^8 \text{ s}}{B^2 \gamma_e}. \tag{2}
\]

For iC cooling through scatterings at the Thomson–Klein–Nishina transition, characterized by a cooling power \( P_{\gamma_e} \sim \gamma_e^{-2/3} \), the electron transit time is

\[
t_{\gamma_e}(\gamma_e) = t_{\gamma_e,i} \left[ 1 - \left( \frac{\epsilon}{E_{\gamma_e}} \right)^{1/6} \right]
\]

where \( t_{\gamma_e,i} \) is the iC-cooling timescale of the GRB \( \gamma_e \)-electrons.

For adiabatic-dominated cooling and a power-law electron injection rate \( R_e \sim t^\theta \)

\[
t^{(\text{ad})}_{\gamma_e} \simeq \left( \frac{\epsilon}{E_{\gamma_e}} \right)^{-3/4} \left[ \frac{t_f}{t_o} y < 1 \right] \left[ \frac{t_f}{t_o} y > 1 \right]
\]

where \( t_o \) is the epoch (ejecta age) when electron injection began.

2.1. Prompt Counterpart Timing

Equations (A8) and (A20) of PV22 for radiative electron cooling show that the pulse-peak epoch is at \( t_p = t_{\gamma_e} \) if the electron injection lasts shorter than the transit time \( (t_f < t_{\gamma_e}) \) and for an exponent \( n > 1 \) (as for SY and iC cooling in the Thomson regime) of the electron cooling power \( P(\gamma_e) \sim \gamma_e^n \); at \( t_p = t_{\gamma_e} \) if \( t_{\gamma_e} < t_f \) for \( n > 1 \), and at \( t_p = t_{\gamma_e} + t_f \) for \( n < 1 \). Thus, \( t_p = t_{\gamma_e} + t_f \) is a good approximation for any ordering of \( t_{\gamma_e} \) and \( t_f \), a result that can be extended to AD cooling with \( t_{\gamma_e} \equiv t^{(\gamma\epsilon)}_{\gamma_e} \) for \( y < 1 \) and \( t_{\gamma_e} \equiv t^{(\text{ad})}_{\gamma_e} \) for \( y > 1 \) (but only if \( t^{(\gamma\epsilon)}_{\gamma_e} > t_f \) in the latter case).

The above results are valid if the magnetic field lives \( t_B > t_p \), i.e., if the electrons that yield the pulse peak cool to below the observing energy. Conversely, for a short-lived magnetic field with \( t_B < t_p \), the pulse-peak epoch is set by \( t_B \).
Note. Countparts are “very prompt” if \( \Delta t_e \ll t_{\gamma} \), prompt if \( \Delta t_e \lesssim t_{\gamma} \), and delayed if \( \Delta t_e > t_{\gamma} \). Counterpart brightness is relative to that of the GRB, with an “average” counterpart brightness corresponding to \( \beta_{\gamma} = 0 \), i.e., a counterpart that is as bright as the burst (e.g., magnitude \( R = 16 \) for a typical GRB peak flux of 1 mJy). There is no clear correlation between the counterpart type and the counterpart-GRB brightness ratio. Note: for AD electron cooling, \( t_p \) sets the duration of SY emission but has no effect on electron cooling. Furthermore, because the AD-cooling timescale is the current time, \( t_{\gamma} \) becomes \( t_p \); thus, only cases shown in boldface apply to AD cooling. Note that the angular time spread \( t_{\text{ang}} \) associated with the emission from a spherically curved surface increases all timescales (including \( t_p^{(\gamma)} \) and \( t_p^{(p)} \)) by \( \sim 50\% \); thus, the type of OC does not change when going from a bright spot to a spherical emitting surface. However, given that only electron cooling and angular integration introduce the observed GRB pulse-duration increase with increasing energy, the GRB pulses whose duration \( \delta t_{\gamma} \) is set by \( t_p \) or \( t_B \) should be inconsistent with that trend if the SY emission arose from a bright spot. (i) Pulse-peak epochs \( \delta t_{\gamma}^{(p)} \) and \( \delta t_{\gamma}^{(\gamma)} \) (ii) GRB-to-low-energy peak lag \( \Delta t_{\gamma} \equiv \delta t_{\gamma}^{(p)} - \delta t_{\gamma}^{(\gamma)} \) (which is independent of \( t_{\text{ang}} \)) and (iii) low-energy-to-GRB Slope \( \beta_{\gamma} \) for radiative-dominated electron cooling (dimmest counterpart is for \( \beta_{\gamma} = 1/3 \), brightest for \( \beta_{\gamma} = -(n - 1)/2 \); \( \beta_{\gamma} \approx 0 \) means that any value between these extremes is possible), for various orderings of the relevant timescales: cooling \( t_{\text{ang}} \) of the GRB typical electrons, duration \( t_p \) of electron injection, lifetime \( t_B \) of magnetic field, electron transit time \( t_{\gamma} \), and time delay \( t_{\text{ang}} \) from GRB emission to observing energy \( \epsilon \).

Thus, in general, the pulse-peak time is

\[
\Delta t_{\gamma} = t_p^{(\gamma)} - t_p^{(p)} = \min \{ t_{\gamma} + t_{\gamma}, t_B \} - \min \{ t_{\text{ang}} + t_B \}
\]

between the burst and the lower-energy pulse-peak epochs is independent of the angular time spread \( t_{\text{ang}} \). Here, \( t_{\gamma} \) is the radiative cooling timescale of the typical GRB \( \gamma \) electron:

\[
t_{\gamma} = t_{\gamma, \text{int}} + t_{\gamma, \text{ic}}.
\]

Table 2 summarizes the temporal features expected for the GRB and lower-energy \( \epsilon \) pulses resulting from the adiabatic cooling or the radiative cooling with \( n > 1 \) of GRB electrons, for various orderings of the relevant timescales \( t_B, t_p, t_{\gamma} \), and \( t_{\text{ang}} \), and if the angular time spread \( t_{\text{ang}} \) does not set the pulse duration.

For a short-lived magnetic field \( t_B < t_{\gamma} + t_{\text{ang}} \), pulses peak at \( t_B \) (cases 1–3), and the GRB-to-low-energy lag time \( \Delta t_{\gamma} = 0 \). For an electron injection duration satisfying \( t_{\gamma} < t_{\gamma} < t_B \), pulses peak at \( t_B \) (case 4), yielding \( \Delta t_{\gamma} = t_{\gamma} < t_B \). In all of these cases, the counterpart is truly prompt, defined by its peak occurring during the GRB pulse (\( \Delta t_{\gamma} < t_{\gamma} \)).

Delayed counterparts, defined by the pulse peak appearing after the GRB pulse (\( \Delta t_{\gamma} > t_{\gamma} \)), occur when the electron injection lasts \( t_B \) shorter than the transition time \( t_{\gamma} \) and when the magnetic field \( B \) is sufficiently long-lived \( t_B > \max (t_{\gamma}, t_{\text{ang}}) \). If SY emission stops before the transit time (i.e., \( t_B < t_{\gamma} \), cases 5–6), then the peak time delay \( \Delta t_{\gamma} \) is the lifetime \( t_B \) of the magnetic field. If SY emission is produced until after the transit time (i.e., \( t_B > t_{\gamma} \), cases 7–8), then the peak time delay \( \Delta t_{\gamma} \) is approximately the transit time \( t_{\gamma} \) given in Equation (1) for SY cooling, in Equation (4) for AD cooling, and the \( t_{\gamma}^{(ic)} \) in Equation (3) for iC cooling.

If the angular time spread \( t_{\text{ang}} \) sets the pulse duration, then the duration of the GRB pulse \( t_{\gamma} \) will be larger than that given in Table 2 by at most a factor of 2. However, the angular time spread \( t_{\text{ang}} \) does not affect the pulse-peak lag \( \Delta t_{\gamma} \) because the GRB and low-energy pulse-peak epochs are delayed by the same duration \( t_{\text{ang}} \); thus, the spherical curvature of the uniformly emitting surface can change some delayed counterparts into prompt ones.

As shown in Table 2, prompt counterparts (\( \Delta t_{\gamma} < t_{\gamma} \)) should satisfy \( \Delta t_{\gamma} < t_{\gamma} \), i.e., prompt counterpart and GRB pulse durations are comparable, and delayed counterparts (\( \Delta t_{\gamma} > t_{\gamma} \)) should satisfy \( \Delta t_{\gamma} < t_{\gamma} < t_{\text{ang}} \), i.e., a delayed counterpart lasts longer than the GRB pulse.

Table 3 summarizes the temporal features of counterparts expected when the cooling of GRB electrons is dominated by iC scatterings at the T-KN transition, for which the cooling index is \( n = 2/3 \). This cooling process yields POCs pulses satisfying \( t_p > t_{\gamma} = t_{\gamma, ic} + t_f \) (Equation (3)), which does not imply \( t_p \approx t_{\gamma} \); thus, the counterpart pulse duration \( \Delta t_{\gamma} \) may not be comparable to its pulse-peak time \( t_p^{(\gamma)} \).

2.2. Prompt Counterpart Brightness (Relative to GRB)

The pulse light curves derived so far can be used to calculate the (peak) flux at the counterpart pulse peak and to convert the ratio of counterpart-to-GRB peak fluxes (at different peak times, separated by \( \Delta t_{\gamma} \)) to an effective counterpart/optical-to-GRB spectral slope

\[
\beta_{\gamma} = \frac{\log f_{\gamma}^{(p)}}{\log \frac{f_{\gamma}^{(p)}}{f_{\gamma}^{(p)}}}, \quad \beta_{\gamma} = -0.2 \log \frac{f_{pk} (1 \text{ eV})}{f_{pk} (100 \text{ keV})}.
\]

If the electron cooling is SY dominated (or dominated by iC scatterings in the Thomson regime and with an index \( n = 2 \)),

| Case | \( t_p^{(\gamma)} \) (= \( t_{\gamma} \)) | \( t_p^{(p)} \) (= \( t_{\gamma} \)) | \( \Delta t_{\gamma} \) | Counterpart Type | \( \beta_{\gamma} \) | Counterpart Brightness |
|------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1    | \( t_B < t_{\gamma} \) | \( t_B \) | \( t_B \) | 0 | very prompt | 1/3 | dimmest |
| 2    | \( t_{\gamma} < t_B < t_{\gamma} \) | \( t_B \) | \( t_B \) | 0 | very prompt | ~0 | any |
| 3    | \( t_{\gamma} < t_B < t_{\gamma} \) | \( t_B \) | \( t_B \) | 0 | very prompt | ~0 | any |
| 4    | \( t_{\gamma} < t_B < t_{\gamma} \) | \( t_B \) | \( t_B \) | 0 | very prompt | ~0 | any |
| 5    | \( t_B < t_{\gamma} < t_{\gamma} \) | \( t_{\gamma} \) | \( t_B \) | 0 | very prompt | ~0 | any |
| 6    | \( t_{\gamma} < t_B < t_{\gamma} \) | \( t_B \) | \( t_B \) | 0 | very prompt | ~0 | any |
| 7    | \( t_{\gamma} < t_B < t_{\gamma} \) | \( t_B \) | \( t_B \) | 0 | very prompt | ~0 | any |
| 8    | \( t_B < t_{\gamma} < t_{\gamma} \) | \( t_{\gamma} \) | \( t_B \) | 0 | very prompt | ~0 | any |
Because the transit time $t_\gamma$ is shorter than the iC-cooling timescale $t_{ic}$ of the typical GRB electron, the ordering of $t_{ic}, t_\gamma,$ and $t_B$ is not relevant. There is no correlation between the counterpart type and the counterpart-to-GRB brightness ratio. The counterpart-to-GRB slope is likely to be $\beta_\gamma \gtrsim 0$; thus, these counterparts arising from an electron cooling of exponent $n \lesssim 1$ are expected to be dimmer, on average, than those for an electron-cooling exponent $n > 1$ (Table 2).

Then the counterpart pulse light curves given in Equations (3)–(22) of PV22 lead to the OC-to-GRB slope

$$\beta_{\gamma} = \begin{cases} \frac{1}{3} - \frac{1}{3} \log \frac{t_B}{t_{ic}} \quad (1) \\ \frac{1}{3} - \frac{1}{3} \log \frac{t_B}{t_{ic}} - \frac{1}{3} \log \frac{t_B}{t_{ic}} \quad (2) \\ \frac{1}{3} - \frac{1}{3} \log \frac{t_B}{t_{ic}} - \frac{1}{3} \log \frac{t_B}{t_{ic}} \quad (3)(4) \\ \frac{1}{3} - \frac{1}{3} \log \frac{t_B}{t_{ic}} - \frac{1}{3} \log \frac{t_B}{t_{ic}} + \frac{1}{3} \log \frac{t_B}{t_{ic}} \quad (5)(6) \\ \frac{1}{3} - \frac{1}{3} \log \frac{t_B}{t_{ic}} - \frac{1}{3} \log \frac{t_B}{t_{ic}} + \frac{1}{3} \log \frac{t_B}{t_{ic}} \quad (7)(8) \\ \end{cases}$$

for the cases listed in Table 2.

This equation shows that both prompt and delayed OCs can be either dim or bright relative to the GRB, depending on the ratios $t_B/t_{ic}$ and $t_B/t_{ic}$, without any expected correlation between the POC type and the POC-to-GRB brightness ratio. Furthermore, for POCs with a slope $\beta_{\gamma} \sim 0$ (i.e., between the extreme values 1/3 and −1/2), a measured slope $\beta_{\gamma}$ constrains the ratio $t_B/t_{ic}$, but the resulting constraint is unclear for delayed OCs.

When the electron cooling is AD dominated, Equations (B6) and (B8) of PV22 yield

$$\beta_{\gamma} = \begin{cases} \frac{1}{3} + \frac{1}{5} \log \left(1 - \frac{t_B}{t_{ic}}\right) \quad (1) \\ \frac{1}{5} \quad (2) \\ 0 \quad (3) \\ \frac{1}{3} + \frac{1}{5} \log \left(1 - \frac{t_B}{t_{ic}}\right) \quad (4) \\ \frac{1}{3} + \frac{1}{5} \log \left(1 - \frac{t_B}{t_{ic}}\right) \quad (5) \\ 0 \quad (6) \end{cases}$$

where $t_{ic}$ is the iC-cooling timescale of the $\gamma$-electron and with the corresponding cases of Table 3 identified. These results are valid for $t_B, t_B < t_{ic}$. Similar to $n > 1$ electron cooling, for POCs, a measured slope $\beta_{\gamma}$ represents a constraint on the ratio $t_B/t_{ic}$.

### 2.3. GRB-to-prompt Counterpart Lag Time

From Equation (6), for a magnetic field with a lifetime longer than the transit time $t_B > t_\gamma$, the pulse-peak lag time $\Delta t_{e} = t_\gamma - t_{rad} \lesssim t_\gamma$ is close to the transit time $t_\gamma$ from the GRB to the observing energy, given in Equation (1) for SY cooling, Equation (4) for AD cooling, and Equation (3) for iC cooling at the T-KN transition.

Corrections occur when the transit time $t_\gamma$ is sufficiently long that the electron cooling departs from the cooling laws used above. As shown in Appendix C of PV22, if the cooling of the typical GRB $\gamma$-electron is initially SY dominated ($t_{ic} < 1.5 t_{ic}$), it becomes AD dominated after a critical time $t_{crit}$,
but the evolution of the electron energy changes from the SY solution to the one-third-SY solution after a time equal to the initial ejecta age $t_o$. Thus, for $t^{(sy)} > t_o$, the correct transit time $t_c^{(sy)}$ can be up to three times shorter than that given in Equation (1).

If the cooling of the GRB $\gamma_e$-electron is initially AD dominated $(t_{sy,i} < 1.5 t_o)$, then cooling remains AD dominated at all times, yet a change in the electron cooling occurs at the epoch $t$ of Equation (C12) of PV22, when the electron cooling switches from the AD solution to the one-third-SY cooling, leading to a more substantial correction for the transit time $t_c^{(ad)}$ if $t_c^{(ad)} > t$. This is of relevance for an electron injection rate $R_t \sim t^{-3}$ with $y > 1$, when the peak-epoch is set by passage of the lowest-energy electrons.

For an electron injection rate with $y < 1$, the peak-epoch is set by passage of the highest-energy electrons, whose cooling begins at time $t_o$; thus, the initial cooling regime is set by the parameter $2t_{sy,i}/3 t_f = t_{sy,i}/t_{ad}(t_f)$, and with a corresponding $t_o \rightarrow t_f$ substitution in the definition of the switch time $t$. The calculation of the correct transit time is further complicated if the critical energy $\gamma_{c,i} (t) \approx (2/3) \gamma_{c,i}(t + t_o)$, where the AD- and SY-cooling timescales are equal, crosses the observing energy $\epsilon$ before the high-end energy $\epsilon_{gs}$ of the SY spectrum from the cooling tail. As shown in Figure 3 of PV22, the cooling tail peaks below $\gamma_{c,i}$ (but that peak is very shallow and broad) and the pulse-peak epoch corresponds to the time when the SY characteristic energy for the $\gamma_{c,i}$-electrons crosses the observing energy $\epsilon$:

$$t_{c}^{(cr)} = \frac{2}{3} \left( \frac{E_{\gamma}}{\epsilon} \right)^{1/2} t_{sy,i} = \frac{2}{3} t_{c}^{(sy)}.$$

(13)

Putting together all the above corrections, the lag time between the GRB peak and the pulse-peak time at energy $\epsilon < E_{\gamma}$ is

$$(AD + SY: R_t \sim t^{-3}) \quad Z = \frac{t_{sy,i}}{t_{ad}(t = 0)}$$

$$\Delta t_c = \begin{cases} 
    t_{c}^{(sy)} & Z \ll \left( \frac{\epsilon}{E_{\gamma}} \right)^{1/2} \\
    \frac{1}{3} t_{c}^{(sy)} & \left( \frac{\epsilon}{E_{\gamma}} \right)^{1/2} \ll Z < 1 \\
    \frac{2}{3} t_{c}^{(sy)} & (y > 1) \quad 1 < Z \ll \left( \frac{\epsilon}{E_{\gamma}} \right)^{1/4} \\
    t_{c}^{(ad)} & (y > 1) \quad \left( \frac{E_{\gamma}}{\epsilon} \right)^{1/4} \ll Z \\
    t_{c}^{(ic)} & (y < 1) \quad \left( \frac{E_{\gamma}}{\epsilon} \right)^{1/2} \ll 1 + \frac{2}{t_{sy,i}} \\
    t_{c}^{(ad)} & (y < 1) \quad \frac{2}{t_{sy,i}} < Z \ll \left( \frac{E_{\gamma}}{\epsilon} \right)^{1/4} \ll 1 + \frac{2}{t_{sy,i}} \\
    \frac{4}{3} t_{c}^{(ic)} & (y < 1) \quad \frac{2}{t_{sy,i}} < Z < \left( \frac{E_{\gamma}}{\epsilon} \right)^{1/2} \ll 3 \left( 1 + \frac{2}{t_{sy,i}} \right) \\
    \frac{4}{3} t_{c}^{(ic)} & (y < 1) \quad \frac{2}{t_{sy,i}} < Z \ll \left( \frac{E_{\gamma}}{\epsilon} \right)^{1/2} \ll \frac{t_{sy,i}}{2\epsilon} + \frac{3}{2} + 6
\end{cases}$$

(14)

assuming that electron injection lasts longer than the initial AD-cooling timescale ($t_f > t_o$). The above branches lack continuity whenever the result shown is asymptotic.

By identifying the GRB-to-1-keV transit time with those cases above that set a lower limit on the observing energy $\epsilon$, one obtains the GRB-to-X-ray pulse-peak lag

$$\Delta t_c = \begin{cases} 
    t_{c}^{(sy)} & \left( \frac{E_{\gamma}}{\epsilon} \right)^{1/2} \ll Z < 0.07 E_{\gamma}^{1/2} \\
    \frac{3}{3} t_{c}^{(ad)} & (y > 1) \quad Z \gg 6 E_{\gamma}^{1/4} \\
    \frac{7}{3} t_{c}^{(sy)} & (y < 1) \quad 1 < Z \ll \frac{2}{t_{sy,i}} \approx 10 E_{\gamma}^{1/2} \\
    t_{c}^{(ic)} & (y < 1) \quad \frac{2}{t_{sy,i}} < Z \ll \frac{3}{2} + 6
\end{cases}$$

(15)

and those cases above that set an upper limit on $\epsilon$ should be identified with the GRB-to-POC pulse-peak lag

$$\Delta t_c = \frac{1}{3} t_{c}^{(ad)} \approx 100 E_{\gamma}^{1/2} t_{sy,i} \equiv t_{c}^{(sy)}.$$

(16)

Thus, for all cases, the pulse-peak lag is one-third of the SY transit time $t_{c}^{(sy)}$ and

$$0.01 E_{\gamma}^{-1/2} < \frac{t_{sy,i}}{t_{ad}} \ll 50 E_{\gamma}^{1/4}$$

(17)

is satisfied, with the ratio above being close to $t_{sy,i}/t_{ad}$, where $t_{ad} = t_o$ (initial age) for $y > 1$ (fast decreasing electron injection rate) and $t_{ad} = t_f$ (electron injection duration) for $y < 1$.

If the SY-cooling timescale $t_{sy,i}$ of the GRB electrons is smaller than the lower limit above, then electrons are still in the SY-cooling regime by the time they radiate in the optical; thus, the transit time $t_{c}^{(sy)}$ will be three times longer than that in Equation (16). If $t_{sy,i}$ is longer than the higher limit above, then electrons are still in the AD-cooling regime when they reach optically emitting energies. The large gap of five decades between optical and GRB energies implies that, for GRB SY-cooling times $t_{sy,i}$ ranging over almost 4 orders of magnitude, the electron cooling should be in the one-third-SY-cooling regime when the electron has cooled enough to radiate in the optical. Owing to its proximity to GRB energies, the peak lag $\Delta t_c$ to soft X-rays (1 keV) has a more complex dependence on the initial cooling timescales (Equation (15)).

The above results did not include iC cooling. When electron cooling is iC dominated by scatterings in the Thomson regime, the electron cooling tail has a falling spectrum $f_c \sim \epsilon^{-1/2}$ and, as discussed in Appendix A1 of PV22, the POC pulse peaks at epoch $t_{c,i}$ when the lowest-energy electrons radiate in the optical. Thus, the gamma-to-optical transit time is:

$$(iC - Thomson) t_{c}^{(ic)} = (10^5 E_{\gamma}^{-3})/t_{c,i} \approx \frac{t_{c}^{(sy)}}{Y}$$

(18)

where $Y > 1$ is the Compton parameter of the $\gamma_i$ electrons, and $t_{c}^{(sy)}$ is the gamma-to-optical transit time for SY-dominated electron cooling (Equation (16)). This result is accurate if $t_{c,i}$ and $Y$ are constant, which is true if the condition for the growth of a power-law cooling tail is satisfied. Otherwise, the Compton parameter above is an average during the electron cooling.

When electron cooling is iC dominated by scatterings at the T-KN transition, the cooling tail has a rising spectrum $f_c \sim \epsilon^{1/6}$, and Appendix A2 of PV22 shows that the POC pulse peaks when the high-energy end of the cooling tail falls below the optical (due to electron cooling after the end of electron
injection at $t_I$) at epoch

$$t_{\text{fs}}^{(iC)} \lesssim t_I + t_{2i} = t_I + \frac{t_{2i}}{300 E_{i,3}^{1/2} Y}.$$  \hspace{1cm} (19)

Equations (16), (18), and (19) give the GRB-to-POC pulse-peak delay for either the emission from a bright spot or a uniformly bright, spherically curved surface because, in the latter case, all observer-frame timescales are stretched by the same angular spread in photon-arrival time $t_{\text{ang}}$; thus, the difference between the pulse-peak epochs at two different observing energies should be unaffected by the time spread $t_{\text{ang}}$.

Over the visible surface of angular opening $\Gamma^{-1}$, the photons emitted from the edge (at angle $\theta = \Gamma^{-1}$ relative to radial direction) arrive at the observer later by a duration $t_{\text{ang}}$ than the photons emitted directly toward the observer (at angle $\theta = 0$) and have an energy in observer-frame that is twice as small. This softening of the received emission due to the curvature of the emitting surface will delay pulse peaks at lower energies, but the delay should be much less than $t_{\text{ang}}$ because the reduction in the relativistic boost by a factor of 2 across the $\Gamma^{-1}$ region is raised to the third power in the received spectral flux (or flux density); thus, the peak pulse at a lower energy will occur well before $t_{\text{ang}}$ after the peak at a higher energy.

Using Equation (16), the GRB-to-POC peak delay should be unaffected by the photon energy and arrival time spreads over the $\Gamma^{-1}$ visible region if $t_{\text{ang}}^{(iC)} = 100 t_{2i,e} \gtrsim t_{\text{ang}}/\text{few}$, which leads to $t_{2i,e} > 10^{-3} t_o$ after using Equation (17). Thus, the result of Equation (16) provides a robust estimate of the GRB-to-POC peak lag $\Delta t_o$ for a sufficiently long-lived magnetic field ($t_B > \Delta t_o$) and if electron iC cooling is not dominant. Otherwise, the peak-lag time is that given in Equations (18) and (19).

3. Observational Constraints on Model Parameters

The GRB model has five basic parameters: GRB ejecta Lorentz factor $\Gamma$, typical electron energy $\gamma e_{0} m c^2$, magnetic field $B$, total number of injected electrons $N_e$ (in the bright spot or in the visible $\Gamma^{-1}$ region), and source radius $R$, and two temporal parameters; the timescales of electron injection $t_I$ and magnetic field $t_B$.

3.1. Basic Model Parameters

The five fundamental parameters ($\Gamma$, $B$, $N_e$, $\gamma e_0$, and $R$) determine the following observables:

(1) the peak energy of the GRB spectrum

$$E_{\gamma} = \frac{2}{z + 1} B^\Gamma \gamma e_0^2 (\text{eV});$$  \hspace{1cm} (20)

thus, a measurement of $E_{\gamma}$ sets this constraint on the basic model parameters

$$B^\Gamma \gamma e_0^2 = 5 \times 10^{12} (z + 1) E_{\gamma,5}.$$  \hspace{1cm} (21)

(2) the GRB-pulse duration given in Equation (5), with the transit time $t_\text{c}$, being the radiative cooling timescale $t_{\text{rad}}$. After taking into account that

(i) the angular spread timescale $t_{\text{ang}}$ is negligible in the case of a bright spot (of angular opening $\delta \theta \ll \Gamma^{-1}$ much less than that of the area of maximal relativistic Doppler boost),

(ii) for the emission from a uniform-brightness surface, the angular spread $t_{\text{ang}} = R/(2c\Gamma) = t_o/2$, which is one-third of the AD-cooling timescale,

(iii) the AD-cooling timescale is $t_{\text{ad}} = 1.5 t_o$,

where $t_\text{c}$ is the comoving-frame epoch corresponding to the end of electron injection or the disappearance of the magnetic field, the duration of the GRB pulse can be written as

$$\delta t_\gamma = \begin{cases} 
\min\{t_I + t_{\text{rad}}(B; N_e, R), t_B\} & (\delta \theta \ll \Gamma^{-1} + \text{Rad}) \\
\min\{t_I + t_{\text{rad}}, t_B\} + t_{\text{ang}} & (\delta \theta = \Gamma^{-1} + \text{Rad}) \\
\min\{2.5 t_I, 1.5 t_B\} \approx 2(t_o - t_\gamma) & (\delta \theta \ll \Gamma^{-1} + \text{AD}) \\
\min\{3 t_I, 2 t_B\} \approx 3(t_o - t_\text{c}) & (\delta \theta = \Gamma^{-1} + \text{AD}) 
\end{cases}$$  \hspace{1cm} (22)

for either radiative (SY and iC) or adiabatic-dominated electron cooling.

For SY-dominated cooling, the radiative cooling timescale is $t_{\text{rad}} \equiv t_{\text{SY},i}$ (Equation (2)); for iC-dominated cooling, $t_{\text{rad}} = t_{\text{SY},i} = t_{\text{SY},i}/Y$ depends also on the number of electrons $N_e$ and source radius $R$ because they determine the electron scattering optical depth $\tau_e \sim N_e/R^2$ and the Compton parameter $Y(\gamma_e, N_e, R) \sim \gamma_e^2 \tau_e \sim \gamma_e^2 N_e/R^2$.

From the first line of Equation (22), it can be inferred that the GRB-pulse duration $\delta t_\gamma$ is equal to the cooling timescale $t_{\text{rad}}$ of the typical GRB electrons for the emission from a bright spot, for a radiative-dominated electron cooling, and for cases 5 and 8 of Table 2. In these cases, the constraint derived from the measured GRB pulse duration $\delta t_{\text{GRB}} = (z + 1)\delta t/\Gamma$ is (bright-spot $\delta \theta \ll \Gamma^{-1} + \text{Rad}$; case 5,8—Table 2):

$$B^2 \Gamma^\gamma (Y + 1) = 8 \times 10^5 \frac{z + 1}{\delta t_{\text{GRB}}}.$$  \hspace{1cm} (23)

For a uniform-brightness surface and $\min(t_I, t_B) < t_{\text{ang}}$, the GRB-pulse duration $\delta t_{\gamma}$ (second line of Equation (22)) is set by the angular timescale $t_{\text{ang}}$, thus, a measured $\delta t_{\gamma}$ implies

$$\text{(unif. – surface } \delta \theta = \Gamma^{-1} + \text{Rad or AD – cooling):} \frac{R}{\Gamma^2} = (1 - 6) \times 10^{10} \frac{\delta t_{\text{GRB}}}{z + 1}.$$  \hspace{1cm} (24)

a constraint that is also valid for the last two lines of Equation (22), corresponding to AD-dominated electron cooling, if the comoving-frame source age $t_o = R/c\Gamma$ is larger than the age $t_o$ when electron injection began.

(3) the average/peak SY flux $F_\gamma$ at the GRB peak energy $E_{\gamma}$:

$$F_\gamma = \frac{100 \text{ Jy}}{4\pi D_l^2/(z + 1)} B^\Gamma N_e \min\left\{1, \frac{t_{\text{rad}}}{\min(t_I, t_B)}\right\}$$  \hspace{1cm} (25)

where $D_l$ is the luminosity distance. The last term above accounts for the cooling of electrons below the peak $E_{\gamma}$ (of the $\epsilon F_\gamma$ power-per-decade); for $t_{\text{rad}} < \min(t_I, t_B)$, only a fraction $t_{\text{rad}}/\min(t_I, t_B) < 1$ of the total injected electrons $N_e$ radiate at $E_{\gamma}$; and only a fraction $(Y + 1)^{-1}$ is released as SY emission (that factor is ignored). The last term above exists only for a radiative electron cooling because the AD-cooling timescale is the current time, which implies $t_{\text{rad}} = t_I$ (with $t_B$ being irrelevant for electron cooling).
The constraint provided by a measurement of the GRB peak flux is

\[ B \Gamma^3 N_e = 10^{52} \frac{D_{28}^2}{z + 1} \frac{F_p \max \left\{ t_{\text{min}}(t_I, t_B), t_{\text{rad}}(B; N_e, R) \right\}}{t_{\text{rad}}(B; N_e, R)} \]  

(26)

where \( D_{28} \) is the luminosity distance in units of \( 10^{28} \) cm. Note that the GRB peak flux \( F_p \) does not depend on the temporal parameters \( t_I, t_B \) only if \( \min(t_I, t_B) < t_{\text{rad}} \), which corresponds to cases 1, 5, and 8 of Table 2, all cases of Table 3, or if the electron cooling is AD dominated.

(4) The GRB-to-POC peak delay \( \Delta t_o \) only for cases 4, 7, and 8 of Table 2 and for cases 3 and 6 of Table 3, when \( \Delta t_o \) is equal to the gamma-to-optical transit time \( t_o \), which is proportional to the cooling time \( t_{\text{rad}}(B; N_e, R) \) of the typical GRB electron. This is true whether the emission arises from a bright spot or from a uniformly bright surface, because, for the latter, the angular time spread \( t_{\text{ang}} \) delays equally both pulse-peak epochs. Furthermore, for SY-, AD-, and \( n = 2 \) IC-dominated cooling, Equations (16) and (18) can be combined to obtain the observer-frame GRB-to-POC lag time \( \Delta t_o^{\text{obs}} = (z + 1) \Delta t_o / \Gamma \) (case 4, 7, 8 – Table 2)

\[ \Delta t_o^{\text{obs}} \approx 100 E_\gamma^{1/2} \left( \frac{z + 1}{\Gamma} + 1 \right) \]  

(27)

leading to

\[ B^2 \Gamma \gamma(Y + 1) \approx 10^{11}(z + 1) \frac{E_\gamma^{1/2}}{\Delta t_o^{\text{obs}}} \]  

(28)

(5) the GRB-to-optical effective slope \( \beta_{\gamma,\text{eff}} \) between the peak fluxes of the GRB and POC pulses, in the cases indicated in Equations (8)-(12), by setting the cooling timescale \( t_{\text{rad}} \) of the typical GRB electron.

3.2. Radiative or Adiabatic Cooling?

For the above cases (4, 7, and 8 of Table 2, and 3 and 6 of Table 3) where the GRB-to-POC lag time \( \Delta t_o \) provides a direct measurement (Equation (27)) of the radiative timescale \( t_{\text{rad}} = t_{\text{sy},i}(Y + 1) \) of the \( \gamma \)-electrons, one can identify the radiative regime of those electrons using the AD-cooling timescale determined from the GRB duration (Equation (22)).

But first, for a bright-spot emission and radiatively cooling GRB electrons (first line of Equation (22)) and for case 8 of Table 2, the GRB pulse duration is equal to the electron radiative cooling timescale, \( \delta t_r = t_{\text{rad}} \), thus,

\[ \frac{\Delta t_o^{\text{obs}}}{\delta t_r^{\text{obs}}} = 100 E_\gamma^{1/2} . \]  

(29)

Such long-delayed OCs are not found among the 10 POCs of Table 4. According to Table 2, delayed OCs with \( \Delta t_o = t_o \) are expected to have an average or above-average brightness. Then, the lack of long-delayed OCs could mean that radiatively cooling electrons, a bright-spot emission, and \( t_I < t_{\text{rad}} < t_B \) occur in less than 10% of POCs. However, if the cooling timescale \( t_{\text{rad}} \) of the GRB electrons is very short, then it would be difficult to identify the 10 ms GRB pulse corresponding to a POC peaking only 1 s after it.

For a uniform-brightness surface and radiatively cooling GRB electrons (second line of Equation (22)), the GRB pulse duration is larger than the angular timescale \( (\delta t_r > t_{\text{ang}}) \), and the condition for radiative electron cooling \( (t_{\text{rad}} < t_{\text{ang}}) \) implies

\[ \frac{\Delta t_o^{\text{obs}}}{\delta t_r^{\text{obs}}} \approx 100 E_\gamma^{1/2} t_{\text{rad}} / t_{\text{ang}} < 300 E_\gamma^{1/2} \]  

(30)

which is satisfied by all POCs of Table 4. Thus, all POCs identified here could be from radiatively cooling electrons (but that is not necessarily so).

For an AD-dominated cooling of the \( \gamma \)-electrons at the end of the GRB pulse, the GRB pulse duration (given in the last two lines of Equation (22)) is larger than the AD-cooling timescale: \( \delta t_r = (2 - 3) \gamma t_o = (1.3 - 2) t_{\text{rad}} \). As discussed in Appendix C of PV22 and shown by some cases in Equation (14), depending on the \( t_{\text{sy},i}/t_{\text{rad}}(t = 0) > 1 \) ratio, the cooling-law of electrons may become a one-third-SY solution well before the AD-cooling electrons radiate in the optical, so that the gamma-to-optical transit time \( t_o \), and the GRB-to-POC time lag \( \Delta t_o \) are set by the SY-cooling timescale \( t_{\text{sy},i} \) of the GRB \( \gamma \)-electrons. In that case, the condition for AD-dominated electron cooling \( (t_{\text{rad}}(\gamma) > t_{\text{rad}} = 1.5 t_o) \) implies

\[ \frac{\Delta t_o^{\text{obs}}}{\delta t_r^{\text{obs}}} = 100 E_\gamma^{1/2} t_{\text{rad}} / (4 - 5) t_{\text{ang}} > (50 - 75) E_\gamma^{1/2} . \]  

(31)

Such long-delayed OCs are not found in Table 4, which strengthens the previous suggestion that GRB electrons are cooling radiatively in at least 90% of bursts, with the caveat that a long-delayed OC may be dimmer than indicated in Table 2 (cases 7 and 8 for AD cooling); i.e., there could be an observational bias against detecting them.

It is important to note that the above assessments are restricted to those cases where the radiative cooling timescale \( t_{\text{rad}} \) of the typical GRB electron can be measured from the GRB-to-POC time lag \( \Delta t_o \) (Equation (27)) and where the adiabatic timescale \( t_{\text{rad}} \) at the end of the GRB pulse (which is either the disappearance of the magnetic field at \( t_B \) or the end of electron injection at \( t_I \)) can be constrained/determined from the duration \( \delta t_r \), of the GRB pulse, using the second or the last two lines of Equation (22), respectively.

3.3. Temporal Model Parameters

The two temporal parameters \( t_I \) and \( t_B \) for the duration of electron injection and magnetic field life determine:

(i) the GRB pulse duration \( \delta t_r = \min(t_I, t_B) \) (as shown for several cases in Tables 2 and 3), only if the SY emission arises from a bright spot and the electron cooling is radiative dominated (first line of Equation (22)). In these cases, the \( \delta t_r \) provides a direct measurement of either \( t_B \) or \( t_I \) (but the source radius \( R \) remains unconstrained);

(ii) the GRB peak (or average) flux \( F_p \) only if \( t_{\text{rad}} < \min(t_I, t_B) \) (cases 2–7 in Table 2). In these cases, the \( F_p \) constrains the ratio \( t_{\text{rad}}/\min(t_I, t_B) \) (Equation (25)); and

(iii) the POC brightness relative to the GRB, quantified by the effective POC-to-GRB slope \( \beta_{\gamma,\text{eff}} \) (Equations (8)-(12), for cases 2, 5, 6, and 7 in Table 2 and cases 1, 4, and 5 in Table 3). In most of these cases, the \( \beta_{\gamma,\text{eff}} \) constrains either \( t_{\text{rad}}/t_B \) or \( t_{\text{rad}}/t_I \).

Additionally, the magnetic field lifetime \( t_B \) sets the GRB-to-POC lag time \( \Delta t_o \) for cases 5 and 6 in Table 2 and cases 4 and 5 in Table 3. In these cases, the \( \Delta t_o \) provides a direct measurement of \( t_B \), but there is no overlap with a direct determination of \( t_B \) from the GRB pulse duration \( \delta t_r \) that could lead to a test of this model.
Table 4: Temporal and Spectral Properties of Some GRBs with Prompt Optical Counterparts

| GRB  | Pulse | \( t_{p}^{(1)} \) | \( \delta t_{\gamma} \) | \( \delta t_{o} \) | \( \Delta t_{o} \) | \( \beta_{LE} \) | \( \beta_{\gamma} \) | References |
|------|-------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| 060526 | 2 (BAT) | 255 | 30 | 35 | 10 | –0.5 | 0.0 | Thone et al. (2010), Kopac et al. (2013) |
| 061121 | 3 (KW) | 72 | 8 | 10 | 5 | –0.3 | 0.1 | Page et al. (2007) |
| 110205 | 3 (BAT) | 210 | 20 | 15 | 5 | –0.5 | 0.2 | Cucchiara et al. (2011), Gendre et al. (2012) |
| 120711 | 2 (IBIS) | 90 | 25 | 30 | 0 | –0.4 | 0.5 | Martin-Carillo et al. (2014) |
| 130925 | 2 (KW) | 2650 | 280 | 300 | 290 | –0.4 | 0.0 | Greiner et al. (2014) |

Note. GRB pulse number and GRB instrument are indicated (KW = Konus-Wind). The \( t_{p}^{(1)} \) is the GRB peak epoch measured from trigger (not from the beginning of the pulse) and is of no use, but is listed here for identifying the GRB pulse. GRB pulse duration \( \delta t_{p} \) and optical pulse duration \( \delta t_{o} \) are the width at half-maximum read from the light curves found in the References; \( \Delta t_{o} \) is the GRB-to-POC pulse delay/\( \gamma \)-time delay; and their uncertainties are 10%. The GRB low-energy slope \( \beta_{LE} \) is taken from Gamma-Ray Coordinates Network circulars and has an uncertainty of at least 0.1. The effective slope \( \beta_{o} \) between the optical and \( \gamma \)-ray pulse-fluxes (separated by \( \Delta t_{o} \)) was calculated from peak fluxes found in the References, and has an uncertainty less than 0.1. POCs are defined by \( \Delta t_{o} < 3 \times 10^{-2} \) (optical peak occurring during the GRB pulse), delayed OCs are defined by \( \Delta t_{o} > \delta t_{o} \) (optical peak occurring after the GRB pulse). One expects that \( \delta t_{p} \approx \delta t_{o} \) for POCs and \( \delta t_{o} \approx \Delta t_{o} \) for delayed OCs. For either type, it is also expected that \( \beta_{LE} \approx \beta_{o} \), with the equality taking place for \( \beta_{LE} \approx \beta_{o} = 1/3 \) or \( \sim 1/2 \). POCs are counterparts with an optical emission brighter than the expectation for the cooled GRB electrons (in a constant magnetic field) and satisfy \( \beta_{LE} > \beta_{o} \). As shown in Figure 1, most POCs do not display the above temporal correlations expected for POCs (if \( \Delta t_{o} < \delta t_{o} \), then \( \delta t_{o} \approx \delta t_{o} \) or for delayed POCs (if \( \Delta t_{o} > \delta t_{o} \), then \( \delta t_{o} \approx \Delta t_{o} \)). Evidently, there is an observational bias in favor of detecting OFs using robotic telescopes over following up dim POCs with \( \beta_{o} = 1/3 \), which implies that the measured slopes \( \beta_{o} \) are sometimes softer than expected for the cooling of GRB electrons.

Lastly, intermediate GRB low-energy slopes \( \beta_{LE} \sim 0 \) require field lifetimes \( t_{b} \) that are just above the cooling timescale \( t_{rad} \) of the typical GRB electrons and should exhibit a good \( t_{b} \sim \beta_{LE} \) correlation if \( t_{b} \in (1, 5) t_{rad} \) for radiative electron cooling (Equation (30) of PV22), and if \( t_{b} \in (5 t_{o}, 5 t_{i}) \) for adiabatic cooling (Equation (50) of PV22). These correspond to the very POC of case 2 in Tables 2 and 3, for which the GRB-to-POC time lag \( \Delta t_{o} \sim 0 \) does not constrain the electron-cooling timescale \( t_{rad} \).

For these intermediate GRB low-energy slopes \( \beta_{LE} \sim 0 \), the power-law low-energy spectrum is not fully developed. Numerical calculations of that low-energy spectrum and fits to it with the Band function can quantify the \( t_{b}/t_{rad} - \beta_{LE} \) correlation and provide an observational constraint on the ratio \( t_{b}/t_{rad} \).

For the GRB low-energy slope, which could also be included in the determination of model parameters by assuming that the SY spectrum below the GRB peak energy \( E_{p} \) is a perfect power law of exponent \( \beta_{LE} \), turning to a one-third slope below the minimal energy \( \varepsilon_{m} \) reached by electron cooling after a duration \( t_{b} \), and by using POC measurements during the GRB pulse (these POCs are very prompt). For example, for SY-dominated electron cooling, Equation (1) leads to

\[
\log \left( 1 + \frac{t_{p}}{t_{b,0}} \right) = \frac{2.5 - 3 \beta_{o}}{1 - 3 \beta_{LE}}. \tag{32}
\]

which is consistent with the second line of Equation (8) for \( t_{b} \in (1, 300) t_{o,0} \).

This result quantifies the \( \beta_{LE} - t_{p} \) correlation of Equation (20) in PV22 but includes POC measurements. Consequently, it does not represent a substitute for the correlation, which would be inferred from GRB observations alone (as described above). Instead, it is only a refinement of the second line of Equation (8), which assumed a GRB low-energy slope \( \beta_{LE} = -1/2 \), and which is now set to the measured slope \( \beta_{LE} \).

We note that, in the framework of POCs arising from the cooling of GRB electrons, the duration \( \delta t_{o} \) of a POC follows from the duration \( \delta t_{o} \) of the GRB pulse and that of a delayed OC follows from the GRB-to-POC peak delay \( \Delta t_{o} \); thus, the measured POC pulse duration \( \delta t_{o} \) does not provide a constraint on the model parameters; instead, it can only serve as a test of the POC origin in the cooling of GRB electrons.

Summarizing the above, we conclude that the GRB pulse duration \( \delta t_{o} \) and GRB-to-optical time lag \( \Delta t_{o} \) may constrain directly the model temporal parameters \( t_{b} \) and \( t_{b} \), and the GRB low-energy slope \( \beta_{LE} \) and GRB-to-optical slope \( \beta_{\gamma} \); constrain the ratio of the temporal parameters to the electron cooling timescale \( t_{rad} \). For the cases listed in Tables 2 and 3, the observables \( \Delta t_{o} \) and \( \beta_{o} \) provide up to two constraints on the model parameters, observable \( \beta_{LE} \) provides another constraint only in case 2 (either table) and a semiconstraint for
all other cases, and observable $\delta t_\gamma$ yields another constraint in most cases but only if the GRB emission arises from bright spots and if electron cooling is radiative dominated.

We note that the equality of the two temporal parameters, $t_B = t_I$ (in a model where the production of magnetic fields and the acceleration of relativistic particles are interdependent), selects cases shown in Tables 2 and 3 for which $t_B^{\text{GRB}} = t_I^{\text{GRB}}$, i.e., only POCs. Thus, the existence of delayed OCs shows that the magnetic field lifetime $t_B$ and the duration of electron injection $t_I$ are not always strictly equal.

### 3.4. Constraints on Basic Model Parameters

In the final tally, observations provide up to six constraints: $E_\gamma, \delta t_\gamma, F_p, \Delta t_\gamma, \beta_0, \gamma_\text{eff}$, for both basic and temporal parameters, and the last for temporal parameters) for seven model parameters: five basic ($\Gamma, \gamma_i, B, N_\epsilon, \text{and} \ R$) and two temporal ($t_B$ and $t_I$).

Even if one focused only on the conditions under which the temporal parameters $t_B$ and $t_I$ do not determine the GRB pulse duration $\delta t_\gamma$, the GRB peak flux $F_p$ and the GRB-to-POC peak delay $\Delta t_\gamma$, one (i.e., the emission from a bright spot and case 8 of Table 2, or the emission from AD-cooling electrons), one would still have only four constraints (Equations (21), (24), (26), and (28)) for five model parameters. This system of four equations can be solved after choosing a free model parameter to parameterize the remaining four model parameters. Using the source Lorentz factor $\Gamma$ for that parameterization leads to

$$\gamma_i = 3 \times 10^6 \frac{E_\gamma}{100 \text{ keV}} \frac{1}{(z + 1)} \left( \frac{\Delta t_\gamma^{\text{obs}} / 10 \text{ s}}{\Gamma / 100} \right)^{1/3}$$

$$B = 50 \frac{z + 1}{(\Gamma / 100)(\Delta t_\gamma^{\text{obs}} / 10 \text{ s})^2} (\text{G})$$

for cases 4, 7, and 8 of Table 2 and for $Y < 1$, and

$$R \lesssim (1 - 6) \times 10^{15} \left( \frac{\Gamma}{100} \right)^2 \frac{\delta t_\gamma^{\text{obs}}}{10(z + 1) \text{ s}} (\text{cm})$$

for the emission from a uniform surface ($\delta \theta = \Gamma^{-1}$) and radiatively cooling electrons or for AD-dominated cooling.

Constraints from the X-ray (1–10 keV) counterpart cannot break the degeneracy among the five model parameters because, for counterparts arising from the cooling of GRB electrons in a constant magnetic field, the prompt X-ray counterpart pulse duration, peak flux, and peak delay after the GRB should follow from the corresponding POC features. Conversely, the prompt X-ray and optical counterpart temporal and spectral properties not being consistent with each other would either constrain the evolution of the magnetic field or would indicate that the two emissions arise from different mechanisms.

Thus, the full determination of the five basic model parameters requires the addition of another strong observational constraint. Below, we discuss some weaker constraints that are either inequalities or that rely on assuming some parameters for the afterglow dynamics.

#### 3.4.1. Semiconstraints

**From transparency to SY self-absorption.** The condition that the optical is above the SY self-absorption frequency (so that the optical continuum is not a hard $\beta_o = 2$) can be used to set a lower limit on the Lorentz factor:

$$\Gamma > (16 - 63) \left( \frac{D_p^{28} (F_p / \text{mJy})^3}{(z + 1)^2 \Delta t_\gamma^{\text{obs}} / 10 \text{ s}} \right)^{1/8} \quad (36)$$

with the lowest value for SY-dominated electron cooling and emission from a uniform surface, and the highest value for AD-dominated electron cooling.

**From afterglow dynamics.** This constraint follows from the expectation that the GRB emission is produced before the deceleration of the post-GRB ejecta by their interaction with the ambient medium. Given that the dynamics of decelerating relativistic blast waves, i.e., their radius $R_{\text{rec}}(t)$ and Lorentz factor $\Gamma_{\text{rec}}(t)$, are set by their kinetic energy and by the density of the circumburst medium. Kumar et al. (2007) obtained upper limits on the radius $R$ and lower limits on the Lorentz factor $\Gamma$ of the GRB source using the timing of the first afterglow measurements.

**From escape of gigaelectronvolt photons.** The escape of 10–100 GeV photons requires a sub-unitary optical thickness to pair formation on the megaelectronvolt burst photons (e.g., Abdo et al. 2009); thus, Fermi/LAT measurements of the gigaelectronvolt emission accompanying a GRB may set a stringent lower limit $\Gamma_{\text{min}}$ on the source Lorentz factor, with the source whose radius $R$ is determined from the GRB-pulse duration, as in Equation (24). Evidently, this method provides an accurate lower limit $\Gamma_{\text{min}}$ only if the GRB and gigaelectronvolt emissions arise from the same source, i.e., if the gigaelectronvolt emission occurs during the burst (temporal consistency) and if the gigaelectronvolt measurements lie on the extrapolation of the burst megaelectronvolt spectrum above the peak energy $E_\gamma$, or have a spectral energy distribution consistent with that of the burst (spectral consistency).

### 4. Discussion

#### 4.1. Temporal Correlations between GRB and POC

By comparing the GRB-to-POC pulse-peak delay/lag $\Delta t_\gamma$ with the GRB pulse duration $\delta t_\gamma$, one can identify two types of POCs. (1) Prompt POCs, defined by $\Delta t_\gamma < \delta t_\gamma$ (i.e., POC pulse peak occurs during the GRB pulse), arise when the pulse-peaks (or pulse durations) given in Equation (5) are both determined either by the duration of electron injection $t_I$ or by the lifetime $t_B$ of the magnetic field. (2) Delayed POCs, defined by $\Delta t_\gamma > \delta t_\gamma$ (i.e., POC pulse peak occurs after the GRB pulse), occur whenever the two pulse durations are determined by different factors that introduce different timescales.

The GRB and POC temporal features of Tables 2 and 3 refer to the case when the spread $t_{\text{ang}}$ in the photon-arrival time over the curved emitting surface does not set the pulse duration. If the electron cooling is dominated by a radiative process (i.e., $t_{\text{rad}} < t_{\text{ang}}$), then the assumption is correct only if the emitting region is a bright spot of angular extent less than the $\Gamma^{-1}$ region (moving toward the observer) of maximal Doppler boost. The same assumption is correct if the electron cooling is dominated by AD losses, because, in that case, the comoving-frame angular timescale $t_{\text{ang}}$ is less than the comoving-frame current age, which is comparable to the dominant timescale appearing in Equation (5) for the pulse-peak epoch and pulse duration.

For a radiative electron cooling ($t_{\text{rad}} < t_{\text{ang}}$) and an uniformly emitting surface, the peak epochs and pulse durations are increased by the angular time spread $t_{\text{ang}}$, but the pulse-peak lag
time $\Delta t_o$ (Equation (6)) remains unchanged. Consequently, including the angular time spread $t_{ang}$ in the peak epoch and in the pulse duration will change some $t_{ang}$-free delayed OCs with $\Delta t_o < \delta t_\gamma$, as the GRB pulse duration is increased.

With that limitation for the identification of POC types, one can search for correlations between the temporal/spectral properties of GRBs/POCs for these two types of POCs.

Figure 1, which shows the OCs listed in Table 4, illustrates the temporal correlation between the POC type and optical-to-GRB pulse duration ratio $\delta t_i / \delta t_o$ discussed in Section 2.1, which can be summarized as

$$\begin{cases} 
\text{prompt OC: } \Delta t_o < \delta t_o \simeq \delta t_\gamma \\
\text{delayed OC: } \delta t_\gamma < \delta t_o \simeq \Delta t_o.
\end{cases} \quad (37)$$

That the POCs shown in Figure 1 display the above features derived from $t_p \simeq \delta t_\gamma$ indicates that their electron cooling is not dominated by IC scatterings at the T-KN transition ($n < 1$).

For the brightest OCs, robotic telescopes may measure the OC variability associated with the GRB pulse variability. If the latter arises from (large-scale) fluctuations in the magnetic field, then the GRB and optical flux should fluctuate synchronously. However, such fluctuations should be easier to evidence in the OC only around its peak time; thus, there will be a bias in detecting temporally correlated GRB and OC fluctuations mostly in prompt OCs.

Figure 2 illustrates the diversity of POCs (from a truly prompt OC to a delayed OC), which is obtained by adjusting temporal factors (here, the duration $t_I$ of electron injection) that determine the GRB pulse duration.

Figure 2 also shows that GRB fluctuations (here, resulting from a variable electron injection rate $R_t$) will “survive” electron cooling (in the sense that they will be displayed by a variable POC light curve) if the injection rate variability timescale (which sets the GRB variability timescale $\delta t_o$) is longer than the GRB-to-OC transit time $t_o$ (which sets the GRB-to-optical pulse-peak lag $\Delta t_o$ for a full electron cooling); otherwise, the cooled electrons of consecutive injection episodes reach optically emitting energies in short succession, separated by a time interval $\delta t_\gamma < t_o$, and integration over the curved emitting surface and over the synchrotron function will wipe out any OC variability of timescale $\delta t_\gamma$.

Thus, electron cooling allows the “propagation” of the GRB variability to the OC only if $\Delta t_o < \delta t_\gamma$, i.e., only for POCs.

### 4.2. Spectral Correlations between GRB and POC

For a constant magnetic field, the electron cooling (through any process) yields an effective optical-to-GRB slope harder than the GRB low-energy slope, $\beta_{\gamma o} \geq \beta_{LE}$, with the equality resulting if either electrons do not cool significantly during the magnetic field lifetime $t_B$, leading to $\beta_{LE} = \beta_{\gamma o} = \beta_o = 1/3$ with $\beta_o$ being the optical continuum slope, or if electrons cool below optical during $t_B$, leading to $\beta_{LE} = \beta_{\gamma o} = \beta_o = -(n - 1)/2$. If either the magnetic field lifetime or electron injection duration satisfy $t_B < t_o$, $t_f < t_o$, then the SY emission integrated spectrum peaks between optical and gamma-rays and $\beta_{\gamma o} > \beta_{LE} - (n - 1)/2$ ($\beta_o = 1/3$ or $\beta_o = -(n - 1)/2$).

For a minimal electron cooling ($t_B < t_{rad}$, $\beta_{LE} = 1/3$), one expects (i) a very POC (case 1 in Tables 2 and 3) because the magnetic field lifetime sets the pulse duration and peak epoch at any energy, and (ii) a POC dimmer than the GRB by a factor $(E_G / 1 \text{ eV})^{1/3} \simeq 40$ or about 4 mag. For a typical GRB average flux of 1 mJy, the POC should be of magnitude $R = 20$; for a bright GRB with an average flux of 10 mJy, one gets $R = 18$. Even for the latter case, the POC, occurring during the burst, is too dim to be monitored by robotic telescopes at such early times; thus, there will be a bias against following up POCs arising from the cooling of electrons in GRBs with a low energy slope. When such POCs are detected (as for GRB 990123—Table 4), it is more likely that that optical emission arises from another mechanism, a possibility that can be tested (Section 4.3).

If electrons cool well below gamma-rays ($t_B > t_{rad}$, $\beta_{LE} = -1/2$ for SY-dominated electron cooling), the POC may be either prompt or delayed, and the POC may be brighter than the GRB by a factor up to $(E_G / 1 \text{ eV})^{1/2} \simeq 300$, or about 6 mag. For an average GRB flux of 1 mJy, the maximal POC brightness is $R = 10$, while brighter bursts, with an average flux of 10 mJy, could yield a POC of $R = 7.5$; thus, the cooling of GRB electrons may account for the OFs of bright bursts such as GRB 090123, 080319B, or 130427A (Table 4).

The above considerations suggest a correlation between:

(i) POC type and GRB low-energy slope $\beta_{LE}$, induced by GRBs with a short magnetic field lifetime $t_B < t_{rad}$: harder
\( B = \text{constant} \cdot R_\lambda(t \leq t_o) \cdot \sin^2(2\pi t/t_i) \)

Figure 2. Metamorphosis of a delayed optical counterpart (DOC) into a (truly) POC obtained by increasing the duration \( \Delta t \) of electron injection (which sets the duration \( \Delta t \) of the GRB pulse), from well below to well above the GRB-to-optical transit time \( t_{\text{po}} = 300 t_{\text{v}} \) (which sets the GRB-to-optical peak time delay \( \Delta t \)). Parameters include magnetic field \( B = 100 \text{ G} \), electrons are injected above energy \( \gamma_i = 3 \times 10^6 \), with a \( p = 3 \) power-law distribution with energy, and source Lorentz factor \( \Gamma = 100 \). The peak energy of the \( t \) \( f \) spectrum is \( E_\gamma \approx 200 \text{ keV} \), the observer-frame SY-cooling time is \( t_{\text{sy}} = t_{\text{v}} / (2t) = 13 \text{ ms} \), and the GRB-to-optical transit time \( t_{\text{po}} = 4 \text{ s} \). The GRB peak flux is normalized at unity (but 1 mJy is a typical/average peak flux). The electron injection has two sinusoidal pulses, and the electron cooling is SY dominated. For \( t_i \gg t_{\text{v}} \), the GRB pulse has two peaks at \( t_i/4 \) and \( 3t_i/4 \) (slightly delayed by the angular integration), and the optical pulse peaks are delayed by \( t_{\text{po}} \). The legend quantifies the optical-to-GRB peak-flux ratio and the relationship between \( \Delta t_i \) and \( \Delta t \), with GRB and optical pulses of the same \( t_i \) shown with the same color. That \( \Delta t_i = t_i \) and \( \Delta t = t_i \) implies that: (i) a DOC (blue), defined by \( \Delta t_i < \Delta t \), is obtained for \( t_i < t_{\text{v}} \); (ii) an intermediate DOC-POC (green), defined by \( \Delta t_i \approx \Delta t \), results for \( t_i \approx t_{\text{v}} \); and (iii) a POC (red), defined by \( \Delta t_i \ll \Delta t \), occurs for \( t_i > t_{\text{v}} \), with the peak-flux ratio reaching maximal value. The variability timescale \( t_i/2 \) of the sinusoidal electron injection rate \( R_i \) that is displayed by the GRB flux also appears in the optical if the GRB-to-optical transit time \( t_{\text{po}} \) is shorter than the injection variability timescale \( t_i/2 \), which means that the GRB variability is preserved for POCs (because \( t_t \approx t_i \approx t_{\text{v}} \), and \( t_{\text{po}} < t_i/2 \) imply \( \Delta t_i < \Delta t \)). For \( t_i \approx t_{\text{v}} \), the two sides of injected electrons cool and reach optical-emitting energies separated by \( t_i/2 < \Delta t_i = \Delta t \), with integration over the synchrotron function and the angular opening of the ejecta ironing out the initial variability, which means that GRB variability is lost for DOCS (similar to above, \( t_i < t_{\text{v}} \) implies \( \Delta t_i < \Delta t \)). Thus, delayed OCs lose the GRB variability, but prompt OCs retain it. The indicated OC flux power laws are those of Equations (21) in PV22: \( f_i(t < t_i/2) \sim t_i^3 \), \( f_i(t_i/2 < t < t_i) \sim t_i^{15/3} \) with \( t_i/2 \) marking the end of the first injection episode, and are close to those displayed by the rise of the OC flux calculated numerically despite that the first analytical result (for \( t_i/t_i/2 \)) was derived for a constant injection rate. The OC power-law falling flux is the large-angle emission given in Equation (26) in PV22 for the slope \( \beta = -1/2 \) of the integrated spectrum of the SY emission from a quasi-monoenergetic cooled electron distribution.

Slopes \( \beta_\text{LE} \) should be associated more often with POCs than with delayed OCs. All POCs in Table 4 have soft low-energy slopes \( \beta_\text{LE} \in (-0.5, -0.3) \) in a narrow range; thus, this correlation cannot be tested with the POCs identified here.

(ii) Optical-to-GRB \( \beta_\gamma \) and GRB low-energy \( \beta_\text{LE} \) slopes, induced by their dependence on the magnetic field lifetime \( t_B \), which can be quantified using Equations (30), (34), and (48)–(50) of PV22 for the slope \( \beta_\text{LE} \) and Equations (8)–(12) for the effective slope \( \beta_\gamma \). For SY-dominated electron cooling, the slope \( \beta_\gamma \) on the second and fourth branches of Equation (8) show that a magnetic field lifetime \( t_B \in (0, 10^{10} t_{\text{v}}) \) yields correlated slopes \( \beta_\gamma \in (0, 1/3) \) and \( \beta_\text{LE} \in (-1/2, 1/3) \), while for \( t_B > 10^{10} t_{\text{v}} \), the resulting range of slopes \( \beta_\gamma \in (-1/2, 0) \) is uncorrelated with the only slope \( \beta_\text{LE} = -1/2 \).

In general, this correlation can be written as a condition: \( \beta_\text{LE} \geq \beta_\gamma \), meaning that GRBs with harder low-energy slopes are associated more often with dimmer POCs. If the POC emission from cooling GRB electrons is not overshadowed by another mechanism, then the observational bias against following such dimmer POCs will lead to a paucity of GRBs with high low-energy slopes and POCs, which accounts for softness of the GRBs with the POCs listed in Table 4.

(iii) POC type and POC-to-GRB relative brightness (quantified by the effective slope \( \beta_\gamma \)), with POCs being dimmer on average than delayed ones. This correlation is supported by the POCs of Table 4, with the average slope \( \beta_\gamma \) of the former being harder than for the latter.

Optical extinction and reddening by dust in the host galaxy. Correlations involving the POC brightness may be weakened by the hard-to-determine accurately dust extinction in the host galaxy, with an extinction \( A_{V_{\text{ph}}} \) in the host galaxy frame reducing the POC brightness by about \((1 + z)A_{V_{\text{ph}}} \) magnitudes (for a linear reddening curve). Thus, dust extinction hardens the slope \( \beta_\gamma \) by \( \delta \beta_\gamma = 0.4(z + 1)A_{V_{\text{ph}}}/5 \). For \( A_{V_{\text{ph}}} = 1 \text{ mag} \) and a redshift \( z = 2 \), the resulting hardening \( \delta \beta_\gamma = 1/4 \) is about one-third of the entire expected range \( \beta_\gamma \in (-1, 1/2, 1/3) \).

Furthermore, dust extinction is accompanied by a softening of the optical continuum slope \( \beta_\gamma \) by \( \delta \beta_\gamma = -0.4(z + 1)A_{V_{\text{ph}}} \), i.e., \( \delta \beta_\gamma = -1.2 \) for \( z = 2 \) and \( A_{V_{\text{ph}}} = 1 \text{ mag} \); thus, this hard intrinsic optical slope \( \beta_\gamma^{\text{intr}} \) is \( 1/3 \) (expected for a magnetic field that lives shorter than the GRB-to-POC transit time, \( t_B < t_{\text{v}} \) could become a softer measured slope \( \beta_\gamma^{\text{dust}} \)).

4.3. Identification of POCs from Cooling of GRB Electrons

Thus, the temporal correlation between POC type and the optical-to-GRB pulse duration ratio \( t_B/t \), and the spectral condition \( \beta_\gamma > \beta_\text{LE} \) between the optical-to-GRB effective
slope and the GRB low-energy slope are two criteria to identify the POCs originating from the cooling of GRB electrons. Because the spectral criterion is only an inequality, that criterion is rather weak and could yield many “false positives,” as POCs arising from other mechanisms may satisfy it.

Figure 1 and Table 4 show that most OFs, defined by \( \beta_{\gamma} > \beta_{\text{LE}} \) (i.e., POCs that are brighter than expected from the cooling of GRB electrons in a constant magnetic field), do not satisfy the temporal correlation expected for POCs from electron cooling (\( \delta t_o \approx \delta t_e \), etc., for POCs, \( \Delta t_m \approx \Delta t_o \) for delayed OCs).

We note that an increasing magnetic field could account for the higher brightness of OFs even when they arise from the cooling of GRB electrons, and that such an increasing magnetic field should not invalidate the expected temporal correlations for each POC type because those correlations arise from the fact that the electron cooling time at energy \( \epsilon \) is comparable to the electron transit time to energy \( \epsilon \), which is correct even for a variable \( B \). This implies that the temporal criterion for the identification of POCs from the cooling of GRB electrons works even for a variable magnetic field, while the spectral criterion is useful only for a constant magnetic and may miss true POCs from GRB electron cooling if the magnetic field is increasing.

Thus, the temporal criterion is clearly superior to the spectral one in selecting POCs that arise from the cooling of GRB electrons, but that does not mean that the spectral criterion is useless, because the temporal criterion has a limitation that is alleviated by adding the spectral criterion. Because the temporal correlations induced by electron cooling arise solely from the pulse-peak epoch being comparable to the pulse duration (\( \delta t_o \approx \delta t_e \)), they are not sensitive to the origin of the cooling electrons and cannot discriminate among various mechanisms that produce optical emission (pairs produced by gigaelectronvolt photons, reverse-shock, SY emission in synchrotron self-Compton GRBs) if the timing of POC pulse is set by cooling of electrons.

If the POC arises from another mechanism overshining the emission from cooling GRB electrons, then the POC will not satisfy the spectral condition \( \beta_{\gamma} > \beta_{\text{LE}} \). Consequently, adding the spectral condition to the temporal criterion is a trade-off, as it increases the probability that an POC satisfying both criteria arises from the cooling of GRB electrons, at the risk of missing some true POCs from GRB electrons cooling in an increasing magnetic field.

Putting together the considerations in Section 4.1, one can conclude that GRB variability is passed on the OC only for POCs, with that being either an observational bias (for GRB variability arising from fluctuations in the magnetic field) or a consequence of electron cooling (for GRB variability from fluctuations in the injection rate). To the extent that OC variability can be measured by robotic telescopes, this conclusion provides another criterion for identifying OCs arising from the cooling of GRB electrons.

5. Conclusions

The aim of this work is to investigate what new information can be extracted from the properties of the POCs resulting from the cooling of GRB electrons.

Starting from the durations \( \delta t_o \) and \( \delta t_e \) of GRB and POC pulses (which are set by the corresponding electron cooling times for the emission from a bright spot, the angular time spread \( t_{\text{ang}} \) for the emission from a uniform-brightness surface or for AD cooling, or the two temporal parameters \( t_B \) and \( t_I \) for the duration of magnetic field and electron injection) and the lag time \( \Delta t_o \) between the peaks of the GRB and POC pulses (which is set by the gamma-to-optical transit time \( t_o \), or by the timescale \( t_B \)), it can be shown that the POCs arising from the cooling of GRB electrons are of two types: “prompt,” for which \( \Delta t_o < \delta t_o = \delta t_m \), and “delayed,” for which \( \delta t_o < \Delta t_o = \delta t_m \).

The preceding inequalities are definitions, and the following equalities represent a test for the electron-cooling model for POCs. Adding the condition \( \beta_{\gamma} \geq \beta_{\text{LE}} \) between the POC-to-GRB effective spectral slope \( \beta_{\gamma} \) and the GRB low-energy slope \( \beta_{\text{LE}} \) strengthens this temporal criterion for identifying POCs arising from the cooling of GRB electrons by discriminating the POCs arising from other mechanisms (involving electron cooling or not), although that may miss some POCs from cooling if GRB electrons in the magnetic field were to increase.

Adding that POCs should be associated more often with GRBs with a harder low-energy slope \( \beta_{\text{LE}} \); these two correlations imply that POCs should be dimmer (on average) than delayed ones. This correlation finds support in a set of 10 POCs, with the average POC-to-GRB brightness ratio \( \beta_{\gamma} \) being harder for POCs.

If GRB electrons do not cool significantly during the magnetic field lifetime, then the burst should have a hard low-energy slope \( \beta_{\text{LE}} = 1/3 \), and the POC should be prompt and dimmer by a factor (100 keV/1 keV)\(^{1/3} \) (about 4 mag) than the burst. Thus, a burst with an average flux of 1 (10) mJy will be accompanied by a POC of magnitude \( R = 20 \) (18), which implies that some GRBs with a hard low-energy slope \( \beta_{\text{LE}} \) will be accompanied by a prompt and dim intrinsic POC emission (from cooling of GRB electrons), which is difficult to detect with robotic telescopes. That sets a bias against detecting the intrinsic POCs of hard GRBs.

If the GRB electrons cool to optical energies, then the burst should have a soft low-energy slope \( \beta_{\text{LE}} = -1/2 \) (for SY-dominated electron cooling), and the POC may be brighter than the burst by a factor up to (100 keV/1 keV)\(^{1/2} \) (6 mag). Thus, the POC of a burst with a soft slope \( \beta_{\text{LE}} \) and an average flux of 1 (10) mJy could be as bright as magnitude \( R = 10 \) (7.5), which is comparable to the brightness of the OFs accompanying GRBs 990123, 803019B, and 130427A (Table 4). However, all of those bursts have a hard low-energy slope \( \beta_{\text{LE}} > 0 \), which implies an incomplete electron cooling and yields a dim intrinsic POC emission (from the cooled GRB electrons). Thus, their OFs must have arisen from another mechanism (emission from pairs formed from the gigaelectronvolt prompt emission, external reverse-shock, SY emission in the synchrotron self-Compton model for GRBs).

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