The superfluid helium flow in the channel with porous insert at the presence of longitudinal heat flux

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Abstract The processes of heat and mass transfer during the flow of helium-II (He-II) in a channel with porous backfilling placed in a particular section of its length are studied. Heat flux is directed along the axis of the channel in such a way that on one side of the backfilling vapor plug is formed. Calculation of steady-state transport processes at vapor–He-II interfaces is carried out using methods of molecular-kinetic theory. The normal fluid flow in the pores in laminar and turbulent regimes is described by equations taking into account features of heat and mass transfer in superfluid helium. The relationships between the length of the porous insert and the velocities of fluids for different flow regimes are formulated. The results of the calculations are analyzed by comparison with previous data for the flow of He-II in the individual capillary.

1. Introduction
Development of cryogenic systems for special purposes associated with a high level of heat fluxes at helium temperature level, including space applications, makes it necessary to investigate the dynamics of helium-II within the porous body. The processes of heat transfer at the superfluid helium (He-II) flow in a porous body are studied in [1] with respect to a new type of superconducting cable insulation. These studies related to the improvement of the Large Hadron Collider, the increase in the power of magnets and a corresponding increase in heat capacity. Authors investigated the problem of heat transfer through a porous medium filled with superfluid helium assuming that the two-fluid model of Landau describes the behavior of He-II at low heat flux or small velocity in a porous medium. The method of volume averaging has been applied to the pore-scale equations. The permeability was calculated on an array of simple unit cells in the temperature range from 1.4K to 2.1K and was found to be equal to the intrinsic permeability (Stokes’ flow), independently of the heat flux and temperature. Heat flow through superfluid helium contained in porous media is examined in [2]. In particular, heat transfer experiments were performed on He-II contained in a bed of polyethylene spheres of uniform size arranged in random packs. Measured results include the steady-state temperature drops across the three random packs of spheres (35, 49, and 98 μm diameter) and the associated steady heat inputs. Bath temperatures range from 1.7 to 2.1 K to help grasp the superfluid effects. Two pure flow regimes (laminar and turbulent) are decipherable from the heat flux dependence of the temperature gradient. Laminar permeability and shape factor results are compared to past studies of He-II in porous media and in channel flows.
The main aim of this paper is the investigation of heat and mass transfer processes for superfluid helium dynamics in porous media. Thus, the new possibilities for interpreting the effects in a quantum liquid and development of new equipment for various scientific applications will be opening up.

2. Problem

The problem of the helium-II flow in the vapor-filled capillary at the presence of the longitudinal heat flux was considered in paper [3]. It was found that the helium-II column moves to the heater if the length of this liquid helium plug in the capillary more than certain “critical” length, not away from heater, unlike ordinary liquids [4]. If the helium-II column length is “critical” length than the helium-II flow velocity is zero. This critical length does not depend on the prescribed heat flux for the laminar regime of the normal fluid and superfluid flows. For other fluids flows regimes [5] the helium-II column movement to the heater is possible if certain conditions related to the length of the helium-II column and the heat flux density (which dependent on the capillary diameter).

The experimental data confirming the theoretical model were obtained early in paper [6]. In the present investigation a single capillary is replaced by a system of a capillary channels formed by the monodisperse backfilling of metallic balls in a pipe (porous insert). In this case the hydraulic diameter of the capillary channels within the porous media is definitely less than the diameter of the capillary considered in [3, 6]. The change of the heat and mass transfer processes characteristics in a quantum liquid may be expected in this case.

A porous insert (monodisperse backfilling of metallic spheres of diameter of \(d\)) is placed in the channel (pipe) of diameter of \(d_c\). The porous insert length is \(L_2\). The channel is filled with helium, i.e. the superfluid helium column of known length, \(L_0\), is placed in this channel (see figure 1). A heat flux, whose density, \(q_w\), is assumed to be constant thereafter, is delivered to the left-hand interface. The side surface of a channel is adiabatically insulated. The vapor pressure, \(P_b\), is maintained constant in immediate proximity (in macroscopic sense) from the right-hand interface (and the vapor–He-II interface temperature, \(T_b\), corresponding saturation pressure is constant). It is required to define the relationship between the velocity of liquid helium flow in the capillary and structural characteristics of the monodisperse backfilling (porosity of \(m\), metallic spheres diameter of \(d\), length of \(L_2\)). The authors consider only steady-state flow because they believe that the unsteady-state flow problem requires a special analysis. Generally speaking, the steady-state flow is impossible in this case as length of a liquid helium plug decreases. However, quasi-stationary flow is possible at small intensity of evaporation. For such flow regime the same method of description as for steady-state flow is applicable as a first approximation. The length of the liquid helium plug, \(L_0\), is accepted constant. The problem is axisymmetric, gravity is neglected.

![Figure 1. The channel with porous backfilling](image)

It is known that in accordance with two-liquid model of Landau the heat is transferred in He-II by normal fluid flow. Equations describing flow of superfluid helium are derived in the beginning and then required velocities are obtained from the system of these equations.
3. Equations system

Let’s write equation which is the relationship between the pressure, \( P'' \), in vapor volume near the heater and the heat flux density, \( q_w \). The solution of steady-state Boltzmann kinetic equation for problems of vaporization-condensation in linearized statement, i.e. at \( q_w \left( P_s(T_1) \sqrt{2RT_1} \right)^{-1} \ll 1 \) gives the relationship (from paper [7]):

\[
P'' = P_s(T_1) + \frac{\sqrt{\pi}}{4} \frac{q_w}{\sqrt{2RT_1}},
\]

where \( R \) is helium gas constant, \( T_1 \) is temperature of interface surface, \( P_s(T_1) \) is saturation pressure at the temperature \( T_1 \). It is necessary to note that the relation (1) is valid only for problems where the mass flux through an interface is equal to zero during steady stage (in a macroscopic time scale). But the vapor pressure differs from the pressure value in a similar problem where interface is unpenetrable for a mass flow. This fact is confirmed by the numerical solutions [8].

On the right-hand interface the solution of steady-state Boltzmann kinetic equation in linearized statement gives the following result:

\[
P_b = P_s(T_4) - 0.6 \frac{q_w \sqrt{2\pi RT_4}}{\Lambda},
\]

where \( P_s(T_4) \) is saturation pressure at the temperature \( T_4 \), \( q_w/\Lambda=j \) is mass flux density, \( \Lambda \) is specific evaporation heat, \( \rho_b'' \) is vapor density along a saturation curve. It is necessary to note that the equations (1) and (2) are written for a case when a condensation coefficient is equal to 1.

The pressure in a liquid near left-hand interface, \( P_1 \), connects with the pressure in a vapor, \( P'' \), by the conservation equation for impulse which has following form: \( P_1 + 4\sigma / d_c = P'' \), where \( \sigma \) is surface-tension coefficient at vapor-liquid interface. In the same time the pressure in a liquid near right-hand interface, \( P_4 \), connects with the pressure in a vapor, \( P_b \), by the following impulse conservation equation: \( P_4 + 4\sigma / d_c = P_b + \rho_b'' / \rho_b'' \) (if vapor density is constant), where \( \rho_b'' \) is vapor density near interface. As the terms \( 4\sigma / d_c \) in the impulse conservation equations writing above practically compensate each other and term \( \rho_b'' \) is negligible at small evaporation intensity then it is possible to accept that pressures in a liquid near the interfaces are equal to vapor pressures: \( P_1=P'' \) and \( P_4=P_b \).

Using formulae (1), (2) and also Clapeyron-Clausius equation the pressure difference \( P_1 - P_4 \) can be written as

\[
P_1 - P_4 = q_w \frac{\sqrt{2\pi RT}}{\Lambda} \left( 0.6 + \frac{\Lambda}{8RT} \right) + \frac{\Delta P' \rho'(T_1 - T_4)}{T (\rho' - \rho^*)},
\]

where \( \Delta \) is specific evaporation heat. In formula (3) and further it is necessary that \( T \) is the average liquid temperature, at the same time it is accepted that \( T_1/T_4 \approx 1 \). The main hydraulic resistance of the channel consists of resistances of a several sections along the channel length (a local resistances is neglected):

\[
P_1 - P_4 = \frac{\xi \rho' V_n^2}{2} \frac{L_0 - L_2}{d_k} + \frac{\eta' V_n}{k_p} L_2 + b_\rho \rho n m^2 V_n^2 L_2 + b_\rho \rho n m^2 V_s^2 L_2,
\]

where \( \eta' \) is a normal component viscosity, \( \rho' \) is liquid density, \( \rho_n \) is a normal component density, \( V_n \) is the normal fluid velocity in the channel, \( \xi \) is hydraulic resistance coefficient for the free sections of the channel length, \( k_p \) is permeability coefficient of a porous insert. The equation members \( b_\rho \rho n m^2 V_n^2 \) and \( b_\rho \rho n m^2 V_s^2 \) are entered by S.W. Van-Sciver in [9] for the description of a pressure gradient in the
porous medium at turbulent normal fluid flow and vortex superfluid flow, where \( b_n \) and \( b_s \) are the corresponding empirical coefficients.

In the superfluid helium the heat flux connects with the normal fluid velocity. This relationship follows from the equations of two-fluid Landau hydrodynamics. In accordance with these equations the expression for a heat flux in the He-II for the considered problem has following form (see [10]):

\[
q_w = \rho^\ast ST (V_n - V),
\]

where \( S \) is specific entropy of helium-II, \( V \) is the velocity of the liquid helium plug as whole.

The equation describing relationship between a temperature difference in helium-II and a pressure difference is derived from the two-fluid hydrodynamics equations and formulae of Gorter-Mellink theory:

\[
\text{grad} P = \rho^\ast S \cdot \text{grad} T + \rho^\ast S \cdot \overline{f}_{GM} (T) \cdot q^3,
\]

where \( \overline{f}_{GM} (T) \) is Gorter-Mellink function.

### 4. The flows regimes

For the cases of the laminar normal fluid and nonvortex superfluid flows and also the laminar superfluid and turbulent normal fluids flows the critical length values are found below. For other combinations of the normal fluid and superfluid flows regimes the authors plan to find the critical length values in future.

From the equation (5) for nonvortex superfluid flow regime the following relationship between a temperature difference and a pressure difference is obtained:

\[
P_1 - P_4 = \rho^\ast S \cdot (T_1 - T_4).
\]

(7)

Temperature difference \( T_1 - T_4 \) from formula (7) is substituted in equation (3) and then pressure difference \( P_1 - P_4 \) is derived. Further this pressure difference is substituted in equation (4). Taking into consideration the fact that normal fluid flow regime in pores of monodisperse backfilling is laminar we can write:

\[
\frac{q_w \sqrt{2 \pi RT}}{\Lambda \left( 1 - \frac{\Delta \rho^*}{TS (\rho^* - \rho^\ast)} \right)} \left( 0.6 + \frac{\Lambda}{8RT} \right) = \frac{32\eta V_n}{d_c^2} \left( L_0 - L_2 \right) + \frac{\eta V_n L_2}{k_p L_0}.
\]

(8)

The liquid velocity is defined from the equation (4):

\[
V = V_n - \frac{q_w}{\rho^\ast ST}.
\]

(9)

If the porous insert is absent \( (L_2=0) \) then critical length, \( L_0_c \), deduced in (8) becomes the solution obtained early in [3]. At \( L_0 > L_0_c \), the liquid velocity is the negative, i.e. liquid helium plug moves to the heater when the heat flux is a constant. This result confirms the main conclusion of the works [3] where the helium-II flow in individual capillary is considered. Obviously, at \( V=0 \) and normal fluid flow velocity, \( V_n \), determined from eq. (8) at \( L_2=0 \) the equation (9) gives the following expression for critical length, \( L_0_c \):

\[
L_{0_{cr}} = \frac{\rho^\ast ST \sqrt{2 \pi RT}}{\Lambda \left( 1 - \frac{\Delta \rho^*}{TS (\rho^* - \rho^\ast)} \right)} \left( 0.6 + \frac{\Lambda}{8RT} \right) \frac{d_c^2}{32\eta}.
\]

(10)

The formula (10) does not differ from the solution presented in [3]. Having estimated values of the members of equation in a right part of the equation (8), we can see that the main hydraulic resistance of the channel is resistance of a porous insert. This result is explained by mainly strong influence of
the permeability, \( k_p \). The values, \( k_p \), for various porous mediums are equal to \( 10^{-9} - 10^{-12} \) m\(^2\). Therefore the equation (8) is reduced to

\[
\frac{q_w \sqrt{2\pi RT}}{\Lambda \left( 1 - \frac{\Lambda \rho^*}{8RT (\rho^*-\rho^0)} \right) \left( 0.6 + \frac{\Lambda}{8RT} \right)} = \frac{\eta \nu}{k_p} L_2
\]

Critical length is not the length of liquid helium plug but only a porous insert length (i.e. \( L_2 \)) as we see from eq. (11). Taking into account that \( V=0 \) in accordance with (9) we obtain \( V_n = q_w / \rho^* ST \) and the value, \( L_{2cr} \), is expressed as

\[
L_{2cr} = \frac{\rho^* ST \sqrt{2\pi RT}}{\Lambda \left( 1 - \frac{\Lambda \rho^*}{8RT (\rho^*-\rho^0)} \right) \left( 0.6 + \frac{\Lambda}{8RT} \right)} \frac{k_p}{\eta}.
\]

It is seen that the critical length of a porous insert does not depend on channel diameter \( d \). The liquid column velocity is easily derived from eq. (9) with use eq. (8) and eq. (12):

\[
V = \frac{q_w}{\rho^* ST} \left( \frac{L_{2cr}}{L_2} - 1 \right).
\]

One can see in formula (14) that the liquid column velocity is negative, i.e. helium column moves to the heater if a porous insert length, \( L_2 \), is more than, \( L_{2cr} \) (of course, only in the presence of a vapor cavity near the heater). The permeability coefficient, \( k_p \), for a monodisperse backfilling is defined by the sphere diameter of \( d \) and porosity of \( m \) (depending on a spheres packing way) [11]:

\[
k_p = 5.97 \times 10^3 \frac{m^3}{(1-m)^2} d^2.
\]

The critical length of the liquid helium plug, \( L_{0cr} \), for diameter of 100 microns is 0.6 m (at a temperature of liquid of 2K). For monodisperse backfilling if spheres diameter is 100 micron and porosity is 0.4 the porous insert critical length \( L_{2cr} \) is equal to 21 cm. In details the dependences of critical length of \( L_{2cr} \) on diameter of the particles (spheres) and the critical length, \( L_{0cr} \), on the individual capillary diameter is shown in figures 2 and 3 respectively.

**Figure 2.** The dependence of critical length calculated from (12) on particles diameter. 1 – \( T_b=2K \), 2 – \( T_b=1.8K \).

**Figure 3.** The dependence of critical length calculated from (10) on channel diameter. 1 – \( T_b=2K \), 2 – \( T_b=1.8K \).

In paper [9] it is noted that the pressure gradient in counterflow He-II followed to the form which implies the superfluid contribution to the turbulent friction factor is “negligible, if not non-existent”.

At \( L_2=L_{2cr} \), the liquid helium velocity \( V=0 \) and there is helium-II counterflow in the capillary channels of the porous medium. Therefore we believe that \( b_s=0 \). Then for nonvortex superfluid flow and turbulent normal fluid flow the porous insert critical length may be found:
In difference of a case of the laminar normal fluid and nonvortex superfluid flows in this case the critical length of \( L_{2\text{cr}} \) depends on the heat flux density. The coefficient \( b_n \) is defined as \( b_n = 1.75 \frac{1-m}{m \cdot d} \). This coefficient is presented in paper [9] and \( b_n \) is a dimensional value. The dependence of the critical length, \( L_{2\text{cr}} \), on a heat flux in the considered case is shown in figure 4.

\[
L_{2\text{cr}} = \frac{\rho' ST\sqrt{2\pi RT}}{8RT} \left( 0.6 + \frac{\Lambda}{8RT} \right) \left[ \frac{1 - \frac{\Lambda p^*}{ST(p^*-p^n)}}{\frac{\rho_w b_n p_m m^2}{\rho' ST} + \frac{\eta_f'}{k_p}} \right]
\]

(14)

5. Conclusion

The analysis of the different helium-II flow regimes in the channel filled with a vapor at the presence of the porous insert is shown that the liquid helium plug moves to the heater when the monodisperse backfilling length is more than determinate “critical” length. Thus, non-ideal thermomechanical effect is observed in considered system: in porous media the helium-II moves to the heat souse although the heat flux is supplied to the He-II through a vapor plug and the average pores diameter more than 1 micrometer in differ from the widely known experiments (see [12, 13]).

The critical length values of the porous insert for the different normal fluid and superfluid flows regimes are obtained. It is shown that the critical length depends on the structural characteristics of the monodisperse backfilling and also the heat flux density (but not for case when normal fluid and superfluid flows are laminar both). The dependences of the critical length on the spheres diameter of monodisperse backfilling and the heat flux are presented.

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