Quark and Lepton Mass Matrix Model with Only Six Family-Independent Parameters

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Abstract

We propose a unified mass matrix model for quarks and leptons, in which sixteen observables of mass ratios and mixings of the quarks and neutrinos are described by using no family number-dependent parameters except for the charged lepton masses and only six family number-independent free parameters. The model is constructed by extending the so-called “Yukawaon” model to a seesaw type model. As a result, once the six parameters is fixed by the quark mixing and the mass ratios of quarks and neutrinos, no free parameters are left in the lepton mixing matrix. The results are in excellent agreement with the neutrino mixing data. We predict $\delta_{CP} = -68^\circ$ for the leptonic $CP$ violating phase and $\langle m \rangle \approx 21$ meV for the effective Majorana neutrino mass.

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1 Introduction: It is a big concern in the flavor physics to investigate the origin of the observed hierarchical structures of masses and mixings of quarks and leptons. In the present paper, we attempt to describe the quark and neutrino mass matrices in terms of the charged lepton masses as only family number-dependent parameters with the help of the smallest number of possible family number-independent parameter. We report in this paper that such an ambitious attempt has succeeded, and what is more surprising, the number of family number-independent free parameters of the model is only six for sixteen observables. It should be noted that even in the latest Yukawaon model \cite{1} we needed ten family number-independent parameters. The success suggests that hierarchical structures in all the quark and lepton mass matrices are caused by one common origin. This result will bring new light to understand the origin of flavors.

In the so-called Yukawaon model \cite{2}, the Yukawa coupling constants are considered to be effective coupling constants $Y_f^{\text{eff}}$ which are given by vacuum expectation values (VEVs) of scalars (“Yukawaons”) $Y_f$ with $3 \times 3$ components for each flavor $f$:

$$(Y_f^{\text{eff}})^{i,j} = (y_f / \Lambda) \langle Y_f \rangle^i_j \quad (f = u, d, \nu, e),$$

where $\Lambda$ is an energy scale of the effective theory. Although the Yukawaon model is a kind of flavon models \cite{3}, all the flavons in the Yukawaon model are expressed by $3 \times 3$ components.
The Yukawaon model is based on the basic concepts that (i) the fundamental flavor basis is a basis in which charged lepton mass matrix $M_e$ is diagonal and (ii) fundamental parameters in the quarks and leptons are $(\sqrt{m_e}, \sqrt{m_{\mu}}, \sqrt{m_{\tau}})$ [not $(m_e, m_{\mu}, m_{\tau})$]. These concepts have been motivated by a phenomenological success of the charged lepton mass relation \[ m_e + m_{\mu} + m_{\tau} = (2/3)(\sqrt{m_e} + \sqrt{m_{\mu}} + \sqrt{m_{\tau}})^2. \]

2 VEV relations: At first, we propose a model in the present paper, in which VEVs of the Yukawaons $Y_f$ (would-be Dirac mass matrices) take a universal seesaw form as given by

$$\langle \hat{Y}_f \rangle_i^j = k_f \langle \Phi_0 f \rangle_i^\alpha \langle (S_f)^{-1} \rangle_\alpha^\beta \langle \Phi_0 f \rangle_\beta^j \quad (f = u, d, \nu, e),$$

where VEVs of the fields $\Phi_0 f$, $S_f$, $\Phi_0$, $P_f$, and so on are defined by

$$\langle \Phi_0 f \rangle_i^\alpha = (1/\Lambda) \langle \Phi_0 \rangle_{ik}^\alpha \langle P_f \rangle^{k\alpha},$$

$$\langle P_a \rangle = v P \text{ diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}), \quad \langle P_d \rangle = v P 1, \quad \langle P_e \rangle = v P 1, \quad \langle P_e \rangle = v P 1,$$

$$\langle S_f \rangle = v S_f (1 + b f e^{i\beta_f} X_3).$$

Here $(m_{e1}, m_{e2}, m_{e3}) \equiv (m_e, m_{\mu}, m_{\tau})$. $1$ and $X_3$ are defined by

$$1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (7)$$

Here, we have assumed $U(3) \times U(3)'$ family-symmetries, and indexes $i, j, \cdots$ and $\alpha, \beta, \cdots$ denote those of $U(3)$ and $U(3)'$, respectively. The VEV form of $S_f$, Eq.(6), breaks the symmetry $U(3)'$ into a discrete symmetry $S_3$. The factor $S_f^{-1}$ in Eq.(2) comes from a seesaw scenario discussed in detail in Sec.3. Especially, $U(3)'$ plays an essential role in considering a family gauge boson model \[3], in which masses of family gauge bosons $A_i^j$ are given by VEV of $(\Phi_0)_i^\alpha$, so that the model can avoid a severe constraint from the observed $K^0-\bar{K}^0$ mixing.

In this model, a VEV form which explicitly breaks $U(3)$ family symmetry is only the form $\langle \Phi_0 \rangle$, Eq.(5). In this paper, we do not discuss the origin of the values $(z_1, z_2, z_3)$ given by Eq.(5), which is a basic assumption in the Yukawaon model.

A neutrino mass matrix is assumed, by adopting the conventional seesaw mechanism, as

$$(M_{\nu}^{\text{Majorana}})_{ij} = (\hat{Y}_\nu)_i^k (\hat{Y}_R^{-1})_{kl} (\hat{Y}_\nu^T)_l^j.$$

Here, the following VEV structure of the $Y_R$ (the Majorana mass matrix of the right-handed neutrinos $\nu_R$) is assumed, according to the previous Yukawaon model \[4]:

$$\langle \hat{Y}_R \rangle_{ij} = k_R \frac{1}{\Lambda} \left[ (\langle \Phi_0 \rangle)^i_k (\hat{Y}_u)_k^j + (\hat{Y}_T^e)_i^k (\Phi_0^T)_k^j \right] + \frac{\xi_R}{\Lambda} (\hat{Y}_e^T)_i^k (\hat{E})^k_l (\hat{Y}_e)_l^j, \quad (9)$$
where $\langle E \rangle = v_E 1$. Here, the last term ($\xi_R$ term) has been introduced in order to give a reasonable value for a neutrino mass squared difference ratio $R_\nu = (m^2_{\nu_2} - m^2_{\nu_1})/(m^2_{\nu_3} - m^2_{\nu_2})$. Exactly speaking, three terms $\hat{E}^{ik}(\hat{Y}_e)_{k}^i (\hat{Y}_e)^{ij}_{k} \hat{E}^{kl}(\hat{Y}_e)_{l}^j$, and $(\hat{Y}_e)^{ij}_{k} (\hat{Y}_e^T)^{ki}_{l} \hat{E}^{lj}$ are possible as the $\xi_R$ term. However, since we have considered $\langle E \rangle = v_E 1$, we have denoted only one term of the three terms in Eq.(9) for convenience.

Since we deal with mass ratios and mixings only, the common coefficients $k_f, v_{SF}$, and so on does not affect the numerical results, so that hereafter we omit such coefficients even it those have dimensions.

Let us state some remarks on our new Yukawaon model in order:

(a) The VEV form (5) is a fundamental postulation in the Yukawaon model. We assume that the VEV form of $\Phi_0$ is in the flavor basis in which $\langle S_f \rangle$ take a form “unit matrix plus democratic matrix”. We do not ask the origin of the values of $z_1$ for the moment.

(b) The structures of the quark and Dirac neutrino mass matrices $\langle \hat{Y}_f \rangle$ are essentially determined by the parameters $b_f e^{i\beta_f}$. Since we take a superpotential

$$W_S = \mu_1 S \mathrm{Tr}[S_u S_v] + \mu_2 S \mathrm{Tr}[S_\nu \mathrm{Tr}[S_\nu]],$$

(10)

which leads to $\langle S_u \rangle = v_S 1$ and $\langle S_\nu \rangle = v_S 1$, we obtain $b_e = b_\nu = 0$ in the lepton sector. So that $\langle \hat{Y}_e \rangle$ and $\langle \hat{Y}_\nu \rangle$ are given by a common form $(\Phi_0)/\langle \Phi_0 \rangle$. (However, this does not mean that $\hat{Y}_e$ and $\hat{Y}_\nu$ are a common one flavon.) Of course, here, we assume $R$ charges, $R(S_\nu) + R(S_e) = 2$.

(c) VEV relations are derived from the supersymmetric vacuum conditions. The possible combinations among those flavons are selected by $R$ charges in the SUSY scenario. See, for instance, Ref.[6]. For example, the forms (4) are derived superpotential terms

$$W_{P_q} = (1/\Lambda) \left( \lambda_1 P_q \mathrm{Tr}[P_u P_d P_d] + \lambda_2 P_q \mathrm{Tr}[P_u P_u] \mathrm{Tr}[P_d P_d] \right),$$

$$W_{P_\ell} = (1/\Lambda) \left( \lambda_1 P_\ell \mathrm{Tr}[P_\nu P_\nu P_\nu] + \lambda_2 P_\ell \mathrm{Tr}[P_\nu P_\nu] \mathrm{Tr}[P_\nu P_\nu] \right),$$

(11)

which lead to VEV relations $\langle P_f \rangle/\langle \tilde{P}_f \rangle = v^2_f 1$ ($f = u, d, \nu, e$). We regard the VEV form (4) as one of special solutions in the general relation $\langle P_f \rangle/\langle \tilde{P}_f \rangle = v^2_f 1$. (Here, we have taken $R$ charge relations $R(P_u) + R(P_d) = 1$ and $R(P_\nu) + R(P_\nu) = 1$.)

(d) The parameters $\phi_i$ ($i = 1, 2, 3$) in Eq.(4) look like typical family number-dependent parameters. However, in the previous Yukawaon model[1], we have proposed a mechanism that the parameters $\phi_i$ can always be expressed in terms of the charged lepton mass parameters $m_{\ell i}$ with the help of two family number-independent parameters. The reason is as follows: when we put $(\phi_1, \phi_2, \phi_3) = (\phi_0 + \tilde{\phi}_1, \phi_0 + \tilde{\phi}_2, \phi_0)$, the phase values $(\tilde{\phi}_1, \tilde{\phi}_2)$ are observables in fitting of the Cabibbo-Kobayashi-Maskawa (CKM) [2] mixing parameters, but $\phi_0$ is not observable. Therefore, we can always relate the values $(\phi_1, \phi_2, \phi_3)$ to the values $(m_\tau, m_\mu, m_\tau)$ by adjusting $\phi_0$. Therefore, for convenience, we count the parameters $(\tilde{\phi}_1, \tilde{\phi}_2)$ as family number-independent parameters. (For the details, see Ref.[1].)
except for $\langle \bar{P} \rangle = \langle P \rangle^*$ with $\phi_i \neq 0$. However, in the present model, the corresponding VEV form is $\bar{P}_f \Phi_0 S_f \Phi_0 P_f$, not $\bar{P}_f \Phi_0 S_f \Phi_0 P_f$, so that we cannot obtain a similar result as in Ref.\[1\]. Therefore, in the present paper, the relations ($\beta_u = 0, P_u \neq 1$) and ($\beta_d \neq 0, P_d = 1$) are only assumptions. This is a task in future.

Finally, we summarize the parameters in the present model. We have only six free parameters $b_u, b_d, \beta_d, (\tilde{\phi}_1, \tilde{\phi}_2), \xi_R$ for sixteen observable quantities (four quark mass ratios, two neutrino mass ratios, four CKM mixing parameters and four plus two Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing parameters). (Hereafter, for convenience, we denote $(\tilde{\phi}_1, \tilde{\phi}_2)$ as $(\phi_1, \phi_2)$ simply.) Note that the number of parameters is surprisingly small. The parameter $b_u$ gives the up-quark mass ratios $m_u/m_c$ and $m_c/m_t$, so that we have one prediction. The parameters $b_d$ and $\beta_d$ are fixed by the observed down-quark mass ratios. Therefore, four parameters in the CKM mixing matrix are described only by two parameters ($\phi_1, \phi_2$). The parameter $\xi_R$ adjusts the neutrino mass ratio $R_{\nu}$. If the value $\xi_R$ is fixed, four plus two lepton mixing parameters are predicted with no free parameters. However, since the neutrino mass ratio has not been so precisely measured at present, we will give predictions of two neutrino mass ratios and four PMNS mixing parameters (and with two Majorana phase) by adjusting one parameter $\xi_R$ as a practical matter. (See Fig.2.)

3 Seesaw-type mass matrix model: Let us discuss the origin of the VEV structure in Eq. (2). The mass matrix model given in Eq. (2) is a sort of seesaw type mass matrix model for all the flavor. The form (2) has been suggested by a block diagonalization of a $6 \times 6$ mass matrix term in a universal seesaw model given by

$$
\begin{pmatrix}
\bar{f}_L^i F_L^\alpha \\
\langle \bar{\Phi}_0 \rangle_i^j & \langle \Phi_0 \rangle_i^j \\
-\langle S_f \rangle_i^j & \langle S_f \rangle_i^j \\
\end{pmatrix}
\begin{pmatrix}
f R_j \\
F R \beta
\end{pmatrix}
$$

(12)

Here $f_{L(R)}$ and $F_{L(R)}$ are, respectively, left (right) handed light and heavy fermions fields. Exactly speaking, we have to read $\bar{f}_L$ in Eq.(13) as $\bar{f}_L H_{u/d}/\Lambda$. However, for convenience, we have denoted those as $\bar{f}_L$ simply. The mass matrix model with the heavy fermion mass matrix $M_F = -S_f$ with the form (6) is known as the so-called “democratic seesaw” model$[9]$. The authors in Ref.$[9]$ have considered that the observed top quark mass enhancement originates in a condition $\det M_F = 0$ in the seesaw mass matrix (12). In the seesaw approximation form (2), the condition $\det M_F \rightarrow 0$ gives $m_3 \rightarrow \infty$ for one of the mass eigenvalues $(m_1, m_2, m_3)$. However, they found that the exact diagonalization of $6 \times 6$ mass matrix (12) gives $m_3 \sim \Lambda_{\text{weak}}$, for one of the mass eigenvalues $(m_1, m_2, m_3, m_4, m_5, m_6)$, where $\Lambda_{\text{weak}}$ is a breaking scale of the electroweak symmetry. The reason is quite simple: The matrix (6) with $f = u$ takes
$S_u = \text{diag}(1, 1, 0)$ in the limit of $b_u \to -1$ (i.e. det$S_u \to 0$), so that the seesaw suppression affects only the first and second generations of up-quarks, so that the third generation quark $t$ takes mass of the order of $\Lambda_{\text{weak}}$ without the seesaw suppression. Furthermore, the model can give $m_u \sim m_d \sim m_e$ insensitively to the values of the parameters $b_u$ and $b_d$ as seen in Sec.4 later. In spite of such successful description of quark masses and mixing, the authors in Ref.[9] failed to give reasonable neutrino masses and mixing. On the other hand, the Yukawaon model have succeeded in giving not only reasonable quark mass ratios and mixing but also neutrino masses and mixing. However, the Yukawaon model needs a lot of parameters. In the present paper, we have applied this seesaw model to our Yukawaon model.

However, note that the $3 \times 3$ mass matrix between $\bar{f}_L$ and $f_R$ is absent in the form (12). If we assume the seesaw mechanism plus the Yukawaon model,

$$
\begin{pmatrix}
\bar{f}_i^L & F_L^\alpha
\end{pmatrix}
\begin{pmatrix}
(\hat{Y}_f)_{ij} & (\Phi_0f)^i_j & (\Phi_0f)^i_\alpha & (\Phi_0f)^i_\beta \\
(\Phi_0f)^j_i & -S_i^j & S_i^j & 0 \\
\end{pmatrix}
\begin{pmatrix}
\bar{f}_j^R & F_R^\beta
\end{pmatrix},
$$

then, we obtain a $3 \times 3$ mass matrix between $\bar{f}_L$ and $f_R'$,

$$
M_f \simeq \hat{Y}_f + \Phi_0fS_f^{-1}\Phi_0f,
$$

after the block diagonalization. (Here and hereafter, for convenience, we sometimes omit the notations “(” and “)” which denote VEV matrices.) However, note that, in Eq.(14), the first term $\hat{Y}_f$ is independent of the second term $\Phi_0fS_f^{-1}\Phi_0f$. In order to obtain the relation (2), we put the following two assumptions:

[Assumption 1] The VEV value $\hat{Y}_f$ and the VEV value $M_F = -S_f$ take the same scale transformation (we denote the scale transformation as a parameter $\zeta_f$):

$$
M_f = \zeta_f\hat{Y}_f + (1/\zeta_f)\Phi_0fS_f^{-1}\Phi_0f.
$$

[Assumption 2] The VEV value $\hat{Y}_f$ is taken so that $M_f$ takes a locally minimum value under the $\zeta_f$ transformation:

$$
\frac{\partial M_f}{\partial \zeta_f} = \hat{Y}_f - \frac{1}{\zeta_f^2}\Phi_0fS_f^{-1}\Phi_0f = 0.
$$

Then, we obtain

$$
\hat{Y}_f = (1/\zeta_f^2)\Phi_0fS_f^{-1}\Phi_0f, \quad \text{i.e.} \quad M_f = (2/\zeta_f)\Phi_0fS_f^{-1}\Phi_0f = 2\zeta\hat{Y}_f.
$$

In Ref.[9], the up-quark masses $m_{ui}$ have been estimated by diagonalizing $6 \times 6$ mass matrix (12) with the input value $b_u = -1$ with $\beta_u = 0$. However, in this paper, for convenience, we use the approximate expression (17), although the seesaw approximation (17) is not valid for
$b_u = -1$. Therefore, instead of $b_u = -1$, the parameter value $b_u$ is fixed by the observed value of the up-quark mass ratio $m_c/m_t$. (We will obtain $b_u = -1.011$ as seen in Sec.4.) We use the relation (17) for Dirac masses of all quarks and leptons. (Hereafter, we put simply $\zeta_f = 1$.)

4. Numerical predictions: We summarize our mass matrices for the numerical analysis as follows:

$$
\hat{Y}_u = \Phi_0 \tilde{P}_u(1 + b_u X_3)^{-1} P_u \Phi_0, \quad \hat{Y}_d = \Phi_0 (1 + b_d e^{i\beta_d} X_3)^{-1} \Phi_0, \quad \hat{Y}_e = \Phi_0 \Phi_0,
$$

(18)

$$
M^\text{Majorana}_\nu = \hat{Y}_\nu \hat{Y}_R^{\dagger} \hat{Y}_\nu^T, \quad \hat{Y}_R = \Phi_0 \hat{Y}_u + \hat{Y}_d \Phi_0 + \xi_R \hat{Y}_e, \quad \hat{Y}_\nu = \Phi_0 \Phi_0.
$$

(19)

For convenience of numerical fitting, we re-define all VEV matrices of flavons as dimensionless matrices, i.e. $\tilde{P}_u = \text{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, $\Phi_0 = \text{diag}(z_1, z_2, z_3)$, and so on. For the input values $m_{ei}$, we use the values at $\mu = m_Z$. For the parameter $\xi_R$ defined in $\hat{Y}_R$, we redefine $\xi_R/\Lambda$ as $\xi_R$.

The parameter value of $b_u$ can be determined from the observed up-quark mass ratio $m_c/m_t$ at $\mu = m_Z$. we determine $b_u = -1.011$, which leads to the up-quark mass ratios

$$
r_{12}^u \equiv \sqrt{m_u/m_c} = 0.061, \quad r_{23}^u \equiv \sqrt{m_u/m_c} = 0.060,
$$

(20)

which are good in agreement with the observed up-quark mass ratios at $\mu = m_Z$ [10], $r_{12}^u = 0.045_{-0.010}^{+0.013}$ and $r_{23}^u = 0.060 \pm 0.005$. Here, we have used the value of $r_{23}^u$ as an input value in determining $b_u$ because the light quark masses have large errors. Although the predicted value $r_{12}^u$ in Eq.(20) is somewhat large compared with the observed value, we consider that this discrepancy is acceptable, since the purpose of the present paper is to give an overview of quark and lepton masses and mixings with free parameters as few as possible.

For down-quark mass ratios, we take parameter values $b_d = -3.3522$ and $\beta_d = 17.7^o$ which gives down-quark mass ratios

$$
r_{12}^d \equiv m_d/m_s = 0.049, \quad r_{23}^d \equiv m_s/m_b = 0.027.
$$

(21)

The observed down-quark mass ratios at $\mu = m_Z$ are a little controversial: $r_{12}^d = 0.053_{-0.003}^{+0.005}$ and $r_{23}^d = 0.019 \pm 0.006$ (Xing, et al. [10]) and also $r_{12}^d = 0.050 \pm 0.010$ and $r_{23}^d = 0.031 \pm 0.004$ (Fusaoka-Koide [11]). Our model cannot give the observed values by Xing el al. Our best fit parameter values are near to the values given in Ref.[11]. The fitting of $b_d$ and $\beta_d$ have been done with keeping in mind that it also leads to consistent CKM mixings, since the CKM mixings also depend on $b_d$ and $\beta_d$.

The explicit mass eigenvalues are as follows: $(m_u, m_c, m_t) = (0.000398, 0.1064, 29.74)m_0$ and $(m_d, m_s, m_b) = (0.000725, 0.01467, 0.5365)m_0$, where $m_0 = (v_0 v_P/\Lambda)^2/v_S$ and $v_{Su} = v_{Sd} \equiv v_S$, so that we can obtain reasonable $m_d/m_u$ ratio, $m_d/m_u = 1.8$, which well agrees with the observed ratio [11] $m_d/m_u = 2.01_{-0.40}^{+0.47}$. Though we obtain $(m_e, m_\mu, m_\tau) = m_0(0.000263, 0.0555, 0.9442)$, the ratio $m_e/m_u \sim 0.66$ is not agree with the observe value $(m_e/m_u)^{\text{obs}} \sim 0.38$. We think that the common coefficient $m_0$ should be distinguished between $(m_0)_{\text{quark}}$ and $(m_0)_{\text{lepton}}$, and the difference is originated in the difference of $v_{Sq} \equiv v_S(\text{quark})$ and $v_{Sl} \equiv v_S(\text{lepton})$ in Eq.(6). It is interesting that this discrepancy suggests $v_{Sq} \simeq v_{Sl}/\sqrt{3}$. 

6
Next, let us try to fitting CKM mixing parameters. Since the parameters $b_u$, $b_d$ and $\beta_d$ have been fixed by the observed quark mass rations, the CKM mixing matrix elements $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$ are functions of the remaining two free parameters $\phi_1$ and $\phi_2$. In Fig. 1, we draw contour curves of the CKM mixing matrix elements in the $(\phi_1, \phi_2)$ parameter plane which are obtained from the observed constraints of the CKM mixing matrix elements, with taking $b_u = -1.011$, and $b_d = -3.3522$, $\beta_d = 17.7^\circ$. As shown in Fig. 1, all the experimental constraints on CKM parameters are satisfied by fine tuning of the parameters $\phi_1$ and $\phi_2$ as $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$, which predicts

$$|V_{us}| = 0.2257, \quad |V_{cb}| = 0.03996, \quad |V_{ub}| = 0.003701, \quad |V_{td}| = 0.009173, \quad \delta^\ell_{\text{CP}} = 80.99.$$  

(22)

![Contour curves in the $(\phi_1, \phi_2)$ parameter plane of the observed CKM mixing matrix elements of $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$. We draw the three contour curves, which corresponds to the center, upper, and lower values of the observed constraints for each the CKM mixing matrix elements, with taking $b_u = -1.011$, and $b_d = -3.3522$, $\beta_d = 17.7^\circ$. We find that the parameter set around $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$ indicated by a star (⋆) is consistent with all the observed values.]

Now let us present the result for the neutrino sector. Substantial differences between the present and previous papers appear in the parameter fitting of the PMNS lepton mixing. We have already fixed the four parameters $b_u$, $b_d$, $\beta_d$, $\phi_1$, and $\phi_2$ from the quark mass ratios and CKM mixing. Therefore, the PMNS mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, CP violating Dirac phase parameter $\delta^\ell_{\text{CP}}$, and the neutrino mass squared difference ratio $R_\nu \equiv \Delta m^2_{21}/\Delta m^2_{32}$ are turned out to be functions of the remaining only one parameter $\xi_R$. In Fig. 2, we draw the curves as functions of $\xi_R$ with taking $b_u = -1.011$, and $b_d = -3.3522$, $\beta_d = 17.7^\circ$, and $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$. As seen in Fig. 2, we find that the predicted value of the
Figure 2: $\xi_R$ dependence of the lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared difference ratio $R_\nu$. We draw curves of those as functions of $\xi_R$ for the case of $b_u = -1.011$ and $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$.

$\sin^2 2\theta_{23}$ is almost constant as $\sin^2 2\theta_{23} \simeq 1$ and the value $\sin^2 2\theta_{12}$ is not so sensitive to the parameter $\xi_R$ as $\sin^2 2\theta_{12} = 0.8 - 1.0$. By using the observed value of $R_\nu$ as input, we determine the value $\xi_R = 2039.6$ in the unit of $v^2_{eE}/v_uv_u\Lambda$, which gives the predictions

$$\sin^2 2\theta_{12} = 0.8254, \quad \sin^2 2\theta_{23} = 0.9967, \quad \sin^2 2\theta_{13} = 0.1007, \quad \delta^\ell_{\text{CP}} = -68.1^\circ, \quad R_\nu = 0.03118.$$  

These predictions are in good agreement with the observed values [12] given in Table 1.

Now, we predict neutrino masses with a normal hierarchy, which are consistent with the observed oscillation data, as

$$m_\nu_1 \simeq 0.038 \text{ eV}, \quad m_\nu_2 \simeq 0.039 \text{ eV}, \quad m_\nu_3 \simeq 0.063 \text{ eV},$$  

by using the input value [12] $\Delta m^2_{32} \simeq 0.00244 \text{ eV}^2$. We also predict the effective Majorana neutrino mass [13] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = \left| m_\nu_1(U_{e1})^2 + m_\nu_2(U_{e2})^2 + m_\nu_3(U_{e3})^2 \right| \simeq 21 \times 10^{-3} \text{ eV}.$$  

The predictions of our model are listed in Table 1. The process for fitting parameters is summarized in Table 2.

5 Concluding remarks: We have proposed a model which combines the Yukawaon model [11] with the democratic seesaw scenario [9], and have demonstrated that the observed masses and mixings of quarks and neutrinos can be described only by the observed charged lepton mass values and
Table 1: Predicted values vs. observed values.

|       | $|V_{us}|$ | $|V_{cb}|$ | $|V_{ub}|$ | $|V_{td}|$ | $\delta_{CP}^d$ | $r_{12}^u$ | $r_{23}^u$ | $r_{12}^d$ | $r_{23}^d$ |
|-------|-----------|-----------|-----------|-----------|-------------|-----------|-----------|-----------|-----------|
| Pred  | 0.2257    | 0.03996   | 0.00370   | 0.00917   | 81.0$^\circ$| 0.061     | 0.060     | 0.049     | 0.027     |
| Obs   | 0.22536   | 0.0414    | 0.00355   | 0.00886   | 69.4$^\circ$| 0.045     | 0.060     | 0.053     | 0.019     |
|       | $\pm 0.00061$ | $\pm 0.0012$ | $\pm 0.00015$ | $\pm 0.00033$ | $\pm 3.4^\circ$ | $\pm 0.013$ | $\pm 0.005$ | $\pm 0.005$ | $\pm 0.006$ |

|       | $\sin^2 \theta_{12}$ | $\sin^2 \theta_{23}$ | $\sin^2 \theta_{13}$ | $R_\nu [10^{-2}]$ | $\delta_{CP}^\nu$ | $m_{\nu_1}$ [eV] | $m_{\nu_2}$ [eV] | $m_{\nu_3}$ [eV] | $\langle m \rangle$ [eV] |
|-------|----------------------|----------------------|----------------------|-------------------|-------------------|------------------|------------------|------------------|------------------|
| Pred  | 0.8254               | 0.9967               | 0.1007               | 3.118             | $-68.1^\circ$     | 0.038            | 0.039            | 0.063            | 0.021            |
| Obs   | 0.846                | 0.999                | 0.093                | 3.09              | -                 | -                | -                | -                | $< O(10^{-1})$   |
|       | $\pm 0.021$          | $\pm 0.001$          | $\pm 0.018$          | $\pm 0.008$       | $\pm 0.15$        |                  |                  |                  |                  |

Table 2: Process for fitting parameters.

| Step | Inputs     | $N_{input}$ | Parameters | $N_{parameter}$ | Predictions                                      |
|------|------------|-------------|------------|-----------------|--------------------------------------------------|
| 1st  | $m_c/m_t$  | 1           | $b_u$      | 1               | $m_u/m_c$                                        |
|      | $m_d/m_s$, $m_s/m_b$ | 2           | $a_d$, $\beta_d$ | 2               | $m_d/m_u$                                        |
| 2nd  | $|V_{us}|$, $|V_{cb}|$ | 2           | $(\phi_1, \phi_2)$ | 2               | $|V_{ub}|$, $|V_{td}|$, $\delta_{CP}^\nu$ |
| 3rd  | $R_\nu$    | 1           | $\xi_R$    | 1               | $\sin^2 \theta_{12}$, $\sin^2 \theta_{23}$, $\sin^2 \theta_{13}$, $\delta_{CP}^\nu$ |
| option | $\Delta m_{32}^2$ |               | $m_{\nu_3}$ |                   | 2 Majorana phases, $m_{\nu_1}$, $m_{\nu_2}$, $m_{\nu_3}$, $\langle m \rangle$ |
| $\sum N_{total}$ | 6           |             | 6           |                 |                                                 |
six family number-independent parameters. The model provides an interesting picture that the observed hierarchical structures of the masses and the mixings of quarks and neutrinos are brought by a common origin which comes from the charged lepton mass spectrum ($m_e, m_\mu, m_\tau$). The model also predicts $\delta_{CP} = -68^\circ$ for the leptonic CP violating Dirac phase, which will be checked by neutrino oscillation experiments in the near future. The prediction $\langle m \rangle \simeq 21$ meV is also within the reach of neutrinoless double beta decay experiments in the near future.

For a full table of our flavons together with their $R$ charges and full expressions of all the superpotential terms, we will give them elsewhere.

References

[1] Y. Koide and H. Nishiura, Phys. Rev. D 91, 116002 (2015).

[2] Y. Koide, Phys Rev. D 79, 033009 (2009); Phys. Lett. B 680, 76 (2009).

[3] C. D. Froggatt and H. B. Nelsen, Nucl. Phys. B 147, 277 (1979). For recent works, for instance, see R. N. Mohapatra, AIP Conf. Proc. 1467, 7 (2012); A. J. Buras et al., JHEP 1203 088 (2012).

[4] Y. Koide, Lett. Nuovo Cim. 34, 201 (1982); Phys. Lett. B 120, 161 (1983); Phys. Rev. D 28, 252 (1983).

[5] Y.Sumino, Phys. Lett. B 671, 477 (2009); Y. Koide and T. Yamashita, Phys. Lett. B 711, 384 (2012); Y. Koide, Phys. Lett. B 736, 499 (2014).

[6] Y. Koide and H. Nishiura, Phys. Rev. D 90, 016009 (2014).

[7] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973).

[8] B. Pontecorvo, Zh. Eksp. Teor. Fiz. 33, 549 (1957) and 34, 247 (1957); Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).

[9] Y. Koide and H. Fusaoka, Z. Phys. C 71, 459 (1996).

[10] Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Rev. D 77, 113016 (2008).

[11] H. Fusaoka and Y. Koide, Phys. Rev. D 57, 3986 (1998).

[12] K. A. Olive et al. (Particle Data Group), Chinese Phys. C, 38, 090001 (2014).

[13] M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Phys. Lett. B 103, 219 (1981) and B 113, 513 (1982).