A Beyond Mean Field Approach to Yang-Mills Thermodynamics

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Abstract. We propose a beyond mean field approach to evaluate Yang-Mills thermodynamics from the partition function with n-body gluon contribution, in the presence of a uniform background Polyakov field. Using a path integral based formalism, we obtain, unlike the previous mean field studies within this model framework, physically consistent results with good agreement to the lattice data throughout the temperature range.

Keywords: Pure gauge, effective model, equation of state

1 Introduction

Thermodynamic quantities are some of the fundamental observables to understand the properties of the strongly interacting medium and the nature of the associated phase transition. For SU(3) pure gauge theory the center symmetry, a global Z(3) symmetry, is spontaneously broken at high temperatures resulting in a first-order phase transition from the confined phase to the plasma phase [1]. The order parameter for this phase transition is the average of Polyakov loop, defined by,

$$L(\vec{x}) = \frac{1}{3} \text{Tr} \left( T \exp \left[ i g \int_0^\beta d\tau A_0(\vec{x}, \tau) \right] \right)$$  \hspace{1cm} (1)$$

where, $A_0(\vec{x}, \tau)$ is the temporal component of the gluonic field and $\tau$ is the Euclidean time.

To study SU(3) pure gauge system, we considered a partition function of thermal gluons in the presence of a background Polyakov field [2,3], but instead of the mean field approximation, which may lead to the unphysical results [2,3], we took a beyond mean field approach based on the path integral formalism to achieve a physically consistent model framework with gluonic distribution function. In Sec.(2) we describe our formalism followed by results and discussion in Sec.(3).
2 Formalism

The thermodynamic description of a gluonic quasiparticle system with a background Polyakov field can be formulated using the partition function [2,3],

\[ Z = \int \prod_x d\theta_3(x) d\theta_8(x) \text{Det}_{VdM} \exp \left( -2V \int \frac{d^3p}{(2\pi)^3} \ln \left( 1 + \sum_{n=1}^{8} a_n e^{-\frac{\left| \vec{p} \right|}{T}} \right) \right) \] (2)

where \( \theta_3 \) and \( \theta_8 \), are two independent parameters that characterize the SU(3) group elements and can be associated with two diagonal generators \( T_3 \) and \( T_8 \). The coefficients \( a_n \) for \( n = 1 \cdots 8 \), are the following,

\[ a_1 = a_7 = 1 - 9\bar{\Phi}\Phi; \quad a_2 = a_6 = 1 - 27\bar{\Phi}\Phi; \quad a_3 = a_5 = -2 + 27\bar{\Phi}\Phi - 81(\bar{\Phi}\Phi)^2 \]
\[ a_4 = 2[-1 + 9\bar{\Phi}\Phi - 27(\bar{\Phi}^3 + \Phi^3) + 81(\bar{\Phi}\Phi)^2]; \quad a_8 = 1. \] (3)

Where, \( \Phi \) and \( \bar{\Phi} \) are the normalised characters defined as,

\[ \Phi = \frac{1}{N_c} \text{Tr}\hat{L}_F; \quad \bar{\Phi} = \frac{1}{N_c} \text{Tr}\hat{L}_F^\dagger, \] (4)

with, \( \hat{L}_F \), the Polyakov line in the fundamental representation, given as,

\[ \hat{L}_F = \text{diag}(e^{i\theta_3}, e^{i\theta_8}, e^{-i(\theta_3 + \theta_8)}) . \] (5)

\( \text{Det}_{VdM} \) is the Vandermonde determinant [2,3], given by,

\[ \text{Det}_{VdM} = 64\sin^2 \left( \frac{\theta_3 - \theta_8}{2} \right) \sin^2 \left( 2\frac{\theta_3 + \theta_8}{2} \right) \sin^2 \left( \frac{\theta_3 + 2\theta_8}{2} \right) . \] (6)

Next, to obtain the thermodynamic observables, instead of evaluating the infinite dimensional integration in Eqn. (2), one may use the saddle point approximation. Here, solving the following equations,

\[ \frac{\partial \Omega}{\partial \Phi} = 0; \quad \frac{\partial \Omega}{\partial \bar{\Phi}} = 0 , \] (7)

one can obtain the mean fields \( \Phi_{mf} \) and \( \bar{\Phi}_{mf} \) and all the thermodynamic observables in terms of those fields. However, the thermodynamic quantities evaluated from this \( \Omega = \Omega(\Phi_{mf}, \bar{\Phi}_{mf}) \) show unphysical behaviour below the transition temperature [43], which we claim to be an artefact of using the mean field approximation. Noting that the thermodynamic potential, in terms of the Polyakov loop, is an oscillating function of \( \theta_3 \) and \( \theta_8 \), we propose a beyond mean field approach to include the significant contribution from the configurations away from the mean field. As the Polyakov loop fields in Eqn. (2) are spatially uniform, we consider the configuration space to consist of \( N \to \infty \) points and defining,

\[ z = \int d\theta_3 d\theta_8 \text{Det}_{VdM} \exp \left( -2\delta v \int \frac{d^3p}{(2\pi)^3} \ln \left( 1 + \sum_{n=1}^{8} a_n e^{-\frac{\left| \vec{p} \right|}{T}} \right) \right) , \] (8)
where \( \delta v \) is a parameter with the dimension of volume, we write Eqn. (2) as,
\[
Z = z^N
\]  
(9)

The thermodynamic variables then follow simply from the pressure which is given as,
\[
p = \frac{T}{V} \ln[Z] = \frac{T}{N \delta v} \ln z = \frac{T}{\delta v} \ln z .
\]  
(10)

The expectation values of the local operators are obtained as,
\[
\langle O[\Phi(x), \bar{\Phi}(x)] \rangle = \frac{1}{z} \int d\theta_3(x) d\theta_8(x) \text{Det}_{V_d M} O[\Phi(x), \bar{\Phi}(x)] \exp \left( -2\delta v \int \frac{d^3 p}{(2\pi)^3} \ln \left( 1 + \sum_{n=1}^{8} a_n e^{-\frac{\pi |\vec{p}|}{T}} \right) \right). 
\]  
(11)

**3 Result and Discussion**

**Parameters:** We choose \( \delta v = (0.5 T_d)^{-3} \) where \( T_d \) is the deconfinement temperature and the obtained pressure shows physically consistent behaviour through out the complete temperature range. However, to have a quantitative agreement to the lattice simulation we introduce an effective gluon mass of the form,
\[
m_g(T)/T = \alpha + \beta \ln(\gamma T/T_d), \text{ for } T/T_d > 1
\]  
(12)
\[
= \zeta (T_d/T)^2, \text{ for } T/T_d < 1,
\]  
(13)

where the parameters are fitted to reproduce the lattice result for the pressure of the SU(3) pure gauge system \([4]\) and are given by, \( \alpha = 0.564; \beta = 0.176657; \gamma = 1.08526 \) and \( \zeta = 2.70066. \) In the Fig. (1), we show the thermal variation of pressure scaled by \( T^4 \) for both without (left) and with (right) the mass term. Other thermodynamic quantities, derived with the mass term, also show a good agreement with the lattice results (Fig. 2).

![Fig. 1. Thermal behaviour of pressure without (left) and with (right) mass parameter. Lattice source \([4]\).](image-url)
In the Fig. (3), the thermal behaviour of $\langle \Phi \rangle$, shows a discontinuity at transition point, signaling the first order phase transition.

We thus now have a complete model description for the gluonic medium that can reproduce the symmetry properties and the thermodynamic observables of an SU(3) pure gauge system.

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