Low Momentum Nucleon-Nucleon Interaction and Fermi Liquid Theory

Achim Schwenk\(^{(a)}\)\(^1\), Gerald E. Brown\(^{(a)}\)\(^2\), and Bengt Friman\(^{(b)}\)\(^3\)

\(^{(a)}\)Department of Physics and Astronomy, State University of New York, Stony Brook, N.Y. 11794-3800, U.S.A.
\(^{(b)}\)Gesellschaft für Schwerionenforschung, Planckstr. 1, 64291 Darmstadt, Germany

We dedicate this paper to the memory of Sven-Olof Bäckman.

Abstract

We use the induced interaction of Babu and Brown to derive two novel relations between the quasiparticle interaction in nuclear matter and the unique low momentum nucleon-nucleon interaction \(V_{\text{low }k}\) in vacuum. These relations provide two independent constraints on the Fermi liquid parameters of nuclear matter. We derive the full renormalization group equations in the particle-hole channels from the induced interaction. The new constraints, together with the Pauli principle sum rules, define four combinations of Fermi liquid parameters that are invariant under the renormalization group flow. Using empirical values for the spin-independent Fermi liquid parameters, we are able to compute the major spin-dependent ones by imposing the new constraints and the Pauli principle sum rules. The effects of tensor forces are discussed.

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\(^{1}\) E-mail: aschwenk@nuclear.physics.sunysb.edu
\(^{2}\) E-mail: popenoe@nuclear.physics.sunysb.edu
\(^{3}\) E-mail: b.friman@gsi.de
1 Introduction

This work was motivated by the results of Bogner, Kuo and Corragio [1], who have constructed a low momentum nucleon-nucleon potential $V_{\text{low } k}$ using folded-diagram techniques. The starting point of their procedure is a realistic nucleon-nucleon interaction, which is reduced to a low momentum potential by integrating out relative momenta higher than a cutoff $\Lambda$, in the sense of the renormalization group (RG) [2]. The hard momenta larger than $\Lambda$ renormalize $V_{\text{low } k}$, such that the low momentum half-on-shell $T$ matrix and bound state properties of the underlying theory remain unchanged. Consequently, the physics at relative momenta smaller than $\Lambda$ is preserved.

Bogner et al. find that various, very different bare interactions, such as the Paris, Bonn, and Argonne potential and a chiral effective field theory model, flow to the same $V_{\text{low } k}$ for $\Lambda \lesssim 2 \text{ fm}^{-1}$ [2]. All the nuclear force models are constructed to fit the experimentally available nucleon-nucleon phase shifts up to momenta $k \sim 2 \text{ fm}^{-1}$. However, they differ substantially in their treatment of the short range parts of the interaction, since these effects cannot be pinned down uniquely by the scattering data. Therefore the work of Bogner et al. demonstrates that one can isolate the physics of the nucleons at low momenta from the effects probed by high momenta and in this way obtain a unique low momentum nucleon-nucleon potential $V_{\text{low } k}$. When one compares the low momentum part of the bare potentials with $V_{\text{low } k}$, one observes that for reasonable values of the cutoff the main effect of the RG decimation to a unique $V_{\text{low } k}$ is a constant shift in momentum space corresponding to a delta function in coordinate space [7]. This is in keeping with the ideas of effective field theory, where one projects $V_{\text{low } k}$ on one and two pion exchange terms plus contact terms, the latter resulting from the exchange of the heavy mesons. The non-pionic contact term contributions flow to “fixed point” values for $\Lambda \lesssim 2 \text{ fm}^{-1}$. Therefore, the most important feature of the unique low momentum interaction is its value at zero initial and final relative momenta $V_{\text{low } k}(0,0)$, since it directly incorporates the largest effect of the RG decimation – the removal of the model dependent short range core by a smeared delta function.

Moreover, it is seen [2] that in the $^1S_0$ channel, $V_{\text{low } k}(0,0)$ is almost independent of the cutoff $\Lambda$ for $1 \text{ fm}^{-1} \lesssim \Lambda \lesssim 3 \text{ fm}^{-1}$, while in the $^3S_1$ channel only a weak linear dependence on $\Lambda$ remains in the same range of momenta. For $\Lambda$ in this momentum range, the contribution of the short range repulsion, which is peaked around approximately $4 \text{ fm}^{-1}$ [3], is already integrated out, while the common one pion exchange long range tail remains basically unchanged until $\Lambda \sim m_\pi$. The residual dependence on the cutoff in the $^3S_1$ channel is due to

\footnote{Due to the cutoff employed, the constant shift within the model space corresponds to a smeared delta function.}
higher order tensor contributions, which are peaked at an intermediate momentum transfer of approximately 2 fm\(^{-1}\) [4]. The weak cutoff dependence of \(V_{\text{low } k}\) around \(\Lambda \sim 2 \text{ fm}^{-1}\) is characteristic of effective field theories, where the dependence on the cutoff is expected to be weak, provided the relevant degrees of freedom – here nucleons and pions – are kept explicitly. The separation of scales implied by the exchanged meson masses is complicated, however, by the higher order tensor interactions.

Diagrammatically \(V_{\text{low } k}\) sums all ladders with bare potential vertices and intermediate momenta greater than the cutoff. Subsequently, the energy dependence of the ladder sum is removed in order to obtain an energy independent \(V_{\text{low } k}\). This is achieved by means of folding, which can be regarded as averaging over the energy dependent effective interaction weighted by the low momentum components of the low energy scattering states. Therefore, it is intuitive to use \(V_{\text{low } k}\) for \(\Lambda = k_F\) as the Brueckner \(G\) matrix. This identification is approximative, since self energy insertions and the dependence on the center of mass momentum are ignored in \(V_{\text{low } k}\). However, it has been argued that the self energy insertions, which must be evaluated off-shell, are small [4]. Hence, we expect that \(V_{\text{low } k}\) reproduces the \(G\) matrix reasonably well. Furthermore, Bogner et al. [5] argue that \(V_{\text{low } k}\) may be used directly as a shell model effective interaction instead of the Brueckner \(G\) matrix, since \(V_{\text{low } k}\) includes the effects of the repulsive core and is generally smooth. They find very good agreement for the low lying states of core nuclei with two valence nucleons such as \(^{18}\text{O}\) and \(^{134}\text{Te}\).

Our second motivation is the work of Birse et al. on the Wilsonian renormalization group treatment of two-body scattering [6,7], where the existence of a unique low momentum potential is addressed. By demanding that the physical \(T\) matrix be independent of the cutoff \([7]\), they obtain a RG flow equation for the effective potential. After rescaling all dimensionful quantities with the cutoff, they find a trivial fixed point corresponding to zero scattering length and a nontrivial one corresponding to an infinite scattering length. The expansion around the nontrivial fixed point yields the effective range expansion. This demonstrates that the s-wave nucleon-nucleon potential, where the scattering length is large, must lie in the vicinity of the nontrivial fixed point. It would be of interest to clarify the role of this fixed point structure in the RG flow to \(V_{\text{low } k}\) and in particular whether this can be used to understand why a unique potential is obtained already for \(\Lambda \lesssim 2 \text{ fm}^{-1}\).

In normal Fermi systems, the low momentum quasiparticle interaction, which

\[\text{Their analysis was carried out for the reaction matrix, but a similar analysis holds for the T matrix. In [8] the RG equation for } V_{\text{low } k} \text{ is derived from the Lippmann-Schwinger equation for the half-on-shell T-matrix and it is shown that the same flow equation can be equivalently obtained from the Kuo-Lee-Ratcliff folded diagram series and the Lee-Suzuki similarity transformation.}\]
is characterized by the Fermi liquid parameters, is determined by a RG fixed point. In this paper we derive a relation between the Fermi liquid parameters of nuclear matter and the s-wave low momentum nucleon-nucleon interaction $V_{\text{low } k}(0,0)$ at $\Lambda = k_F$. This relation connects the fixed point of the quasiparticle interaction to $V_{\text{low } k}$ in the region where it depends only weakly on the cutoff. The existence of such a relation is supported by the success of the model space calculations of Bogner et al. [5], where $V_{\text{low } k}$ is used as the shell model effective interaction. These calculations are in spirit very similar to Fermi liquid theory. In both cases one uses an empirical single-particle spectrum and the energy is measured with respect to a filled Fermi sea. In the case of $^{18}$O, the zero of the energy corresponds to the ground state of $^{16}$O.

We start by giving a brief introduction of Landau’s theory of normal Fermi liquids. We then review the induced interaction introduced by Babu and Brown [9], which will be used to derive the two new constraints. We give a diagrammatically motivated heuristic derivation of the induced interaction, which demonstrates that the induced interaction generates the complete particle-hole parquet for the scattering amplitude, i.e. all fermionic planar diagrams except for particle-particle loops. The latter should be included in the driving term. We then derive two new constraints that relate the Fermi liquid parameters to the low momentum nucleon-nucleon interaction $V_{\text{low } k}$, by solving the integral equation for the scattering amplitude and the induced interaction in a particular limit simultaneously. Making contact with the RG approach to Fermi liquid theory, we derive the coupled RG equations for the particle-hole channels from the induced interaction. Within the particle-hole parquet, the particular combinations of Fermi liquid parameters that appears in these constraints, as well as the Pauli principle sum rules, are invariant under the (in medium) RG flow towards the Fermi surface. Using empirical values for the spin-independent Fermi liquid parameters, we are able to compute the major spin-dependent parameters by imposing the new constraints and the Pauli principle sum rules. Finally, we include tensor interactions in the constraints and demonstrate the necessity of a self-consistent treatment within the induced interaction.

2 Fermi Liquid Theory

Fermi liquid theory was invented by Landau [10] to describe strongly interacting normal Fermi systems at low temperatures. Landau introduced the quasiparticle concept to describe the elementary excitations of the interacting system. For low excitation energies, the corresponding quasiparticles are long lived and in a sense weakly interacting. One can think of the ground state of the system as a filled Fermi sea of quasiparticles, while quasiparticles above and quasiholes below the Fermi surface correspond to low-lying excited states. The quasiparticles can be thought of as free particles dressed by the
interactions with the many-body medium.

When quasiparticles or quasiholes are added to the interacting ground state, the energy of the system is changed by

$$\delta E = \sum_{p\sigma} \epsilon_p^{(0)} \delta n_{p\sigma} + \frac{1}{2V} \sum_{p\sigma, p'\sigma'} f_{\sigma,\sigma'}(p, p') \delta n_{p\sigma} \delta n_{p'\sigma'} + \mathcal{O}(\delta n^3), \quad (1)$$

where $V$ is the volume of the system, $\delta n_{p\sigma}$ the change in the quasiparticle occupation number and $\epsilon_p^{(0)} - \mu = v_F (p - k_F)$ the quasiparticle energy expanded around the Fermi surface. The Fermi momentum is denoted by $k_F$, the Fermi velocity by $v_F$ and the chemical potential by $\mu$. The quasiparticle lifetime in normal Fermi systems at zero temperature, is very large close to the Fermi surface ($\tau \sim (p - k_F)^{-2}$). Consequently, the quasiparticle concept is useful for describing long wavelength excitations, where the corresponding quasiparticles are restricted to momenta $|p| \approx k_F$. When studying such excitations, one can set $|p| = |p'| = k_F$ in the effective interaction $f_{\sigma,\sigma'}(p, p')$. In a rotationally invariant system, the only remaining spatial variable of $f$ is then the angle $\theta$ between $p$ and $p'$. The dependence of $f_{\sigma,\sigma'}(p, p')$ on this angle reflects the non-locality of the quasiparticle interaction.

It follows from Eq. (1) that the effective interaction is obtained from the energy by varying twice with respect to the quasiparticle occupation number. An illustrative example is the Hartree-Fock approximation, where one finds that the Landau $f$ function is simply given by the direct and exchange terms of the bare interaction:

$$\frac{\delta E_{\text{HF}}}{\delta n_{p\sigma} \delta n_{p'\sigma'}} = \frac{f_{\sigma,\sigma'}^{\text{HF}}(p, p')}{V} = \langle p\sigma, p'\sigma'|V|p\sigma, p'\sigma'\rangle - \langle p\sigma, p'\sigma'|V|p'\sigma', p\sigma\rangle - \mathcal{O}(\delta n^3). \quad (2)$$

As in effective field theories, the functional form of the spin- and isospin-dependence of the Landau function is determined by the symmetries of the system only – in the case of symmetric nuclear matter these are invariance under spin and isospin rotations \[^6\]. The dependence of $f$ on the angle $\theta$ is expanded in Legendre polynomials:

$$f(\theta) = \frac{1}{N(0)} F(\theta) = \frac{1}{N(0)} \sum_l \left( F_l + F'_l \mathbf{\tau} \cdot \mathbf{\tau}' + G_l \mathbf{\sigma} \cdot \mathbf{\sigma}' + G'_l \mathbf{\tau} \cdot \mathbf{\tau}' \mathbf{\sigma} \cdot \mathbf{\sigma}' \right) P_l(\cos \theta) + \frac{1}{N(0)} (p - p')^2 S_{12}(p - p') \sum_l \left( H_l + H'_l \mathbf{\tau} \cdot \mathbf{\tau}' \right) P_l(\cos \theta) + \mathcal{O}(A^{-1/3}). \quad (3)$$

\[^6\] When tensor forces are considered the quasiparticle interaction is not invariant under rotations in spin space, but under combined spin and spatial rotations.
\( \hat{k} - \sigma \cdot \sigma' \) is the tensor operator and we have pulled out a factor \( N(0) = \frac{2m^* k_F}{\pi^2} \), the density of states at the Fermi surface, in order to make the Fermi liquid parameters \( F_l, F'_l, G_l, G'_l, H_l \) and \( H'_l \) dimensionless. The effective mass of the quasiparticles is defined as \( m^* = k_F/v_F \). We will discuss tensor interactions in Section 6, but in order to simplify the discussion, we suppress them in the derivation of the constraints. It is straightforward to generalize the derivation and include them. Finally, since we consider infinite nuclear matter, the spin-orbit interaction can be neglected.

As in effective field theories, the Fermi liquid parameters are determined by comparison with experiments. For nuclear matter we have the following relations for the incompressibility, the effective mass and the symmetry energy [10,11]:

\[
K = \frac{3 \hbar^2 k_F^2}{m^*} (1 + F_0),
\]

\[
\frac{m^*}{m} = 1 + \frac{F_1}{3}, \text{ and}
\]

\[
E_{\text{sym}} = \frac{\hbar^2 k_F^2}{6 m^*} (1 + F'_0).
\]

In order to establish the connection between the quasiparticle interaction and the quasiparticle scattering amplitude, we consider the leading particle-hole reducible contributions to the full vertex function. We denote the bare particle-hole vertex by \( B(p, p'; q) \), where the momenta \( p, p', \) etc. and \( q \) are 4-momenta, \( p = (\varepsilon, \mathbf{p}) \) and \( q = (\omega, \mathbf{q}) \).

\[
B(p, p'; q) = \begin{array}{cc}
p + \frac{q}{2} & p - \frac{q}{2} \\
p' + \frac{q}{2} & p' - \frac{q}{2}
\end{array}
\]

There are two possible ways to join two particle-hole vertices with a particle-
In the recent literature, the first channel, Eq. (8), is referred to as the zero sound channel (ZS), while the second one is called ZS'. The ZS' diagram, Eq. (9), is the exchange diagram to the ZS graph. Landau wrote down a Bethe-Salpeter equation, which sums the particle-hole ladders in the ZS channel. This equation relates the full particle-hole vertex $\Gamma(p, p'; q)$ to the ZS particle-hole

Note that in Eq. (9) we have used the antisymmetry of the bare vertex $B(1 + 2, 3 + 4; 1 - 2) = -B(1 + 4, 3 + 2, 1 - 4)$. 

---

\[ p' + \frac{q}{2} \quad p' - \frac{q}{2} \]

\[ p + \frac{q}{2} \quad p - \frac{q}{2} \]

\[ p'' + \frac{q}{2} \quad p'' - \frac{q}{2} \]

\[ p'' + \frac{p' - p}{2} \quad p'' - \frac{p' - p}{2} \]

\[ p + \frac{q}{2} \quad p - \frac{q}{2} \]
irreducible one $\tilde{\Gamma}(p, p'; q)$:

\[
p' + \frac{q}{2} \quad p' - \frac{q}{2} \quad p' + \frac{q}{2} \quad p' - \frac{q}{2} \quad \Gamma_{\uparrow} \quad = \quad \tilde{\Gamma} \quad + \quad p'' + \frac{q}{2} \quad p'' - \frac{q}{2} \quad \Gamma_{\uparrow} \quad .
\]

The Bethe-Salpeter equation in the ZS channel reads

\[
\Gamma(p, p'; q) = \tilde{\Gamma}(p, p'; q) - i \int \frac{d^4p''}{(2\pi)^4} \tilde{\Gamma}(p, p''; q)
\times G(p'' + \frac{q}{2}) G(p'' - \frac{q}{2}) \Gamma(p'', p'; q). \quad (11)
\]

As argued above, we set $p$ and $p'$ on the Fermi surface and let $q \to 0$. In finite nuclei, typical momentum transfers $|q|$ are of the order of the inverse size of the nucleus. Therefore, on physical grounds, $|q| \sim 1/R \sim A^{-1/3}$ vanishes in nuclear matter [11]. Landau noticed that the product of propagators $G(p'' + \frac{q}{2}) G(p'' - \frac{q}{2})$ is singular in the limit $|q| \to 0$ and $\omega \to 0$ (see e.g. [12]) and therefore $\tilde{\Gamma}$ is by construction finite as $q \to 0$. The singularity is due to the quasiparticle poles in the propagators:

\[
G(p'' + \frac{q}{2}) G(p'' - \frac{q}{2}) = \frac{z}{\epsilon'' + \omega/2 - v_F (|p'' + q/2| - k_F) + i\delta_{p''+\frac{q}{2}}} \times \frac{z}{\epsilon'' - \omega/2 - v_F (|p'' - q/2| - k_F) + i\delta_{p''-\frac{q}{2}}} + \text{multi-pair background}
\]
\[
= 2\pi iz^2 \frac{v_F \hat{p}'' \cdot q}{\omega - v_F \hat{p}'' \cdot q} \delta(\epsilon'') \delta(|p''| - k_F) + \text{non-singular } \phi(p''), \quad (12)
\]

where the quasiparticle energy is measured relative to the Fermi energy $\mu$. We note that the singular part, which is due to the quasiparticle piece of the Green functions, vanishes in the limit $|q| \to 0$ and $\omega \to 0$ with $|q|/\omega \to 0$. Therefore, one can eliminate all quasiparticle-quasihole reducible contributions in a given channel by taking this limit.
The singularity of the ZS particle-hole propagator is reflected in the dependence of the coefficient of the delta functions in Eq. (12) on the order of the limits $|q| \to 0$ and $\omega \to 0$. The $|q|$ and $\omega$ limits of the particle-hole vertex are defined as:

$$\Gamma^\omega(p, p') = \lim_{\omega \to 0} \left( \Gamma(p, p'; q) \mid_{q=0} \right), \quad \text{and}$$

$$\Gamma^q(p, p') = \lim_{|q| \to 0} \left( \Gamma(p, p'; q) \mid_{\omega=0} \right).$$

In the $\omega$ limit the singular part in Eq. (12) vanishes. Thus, from Eq. (10) it follows that $\Gamma^\omega$ itself is obtained by solving a Bethe-Salpeter equation, which sums the ZS particle-hole ladders with the non-singular part $\phi$ only. Consequently, $\Gamma^\omega$ is quasiparticle-quasihole irreducible in the ZS channel.

With this at hand, one can eliminate $\bar{\Gamma}$ and the non-singular $\phi$ to obtain the following quasiparticle-quasihole analogue of Eq. (10) for $\Gamma$, at $T = 0$ [12]:

$$\Gamma_{\sigma, \sigma', \tau, \tau'}(p, p'; q) = \Gamma^\omega_{\sigma, \sigma', \tau, \tau'}(p, p') + N(0) \frac{z^2}{4} \mathrm{Tr}_{\sigma''\tau''} \int \frac{d\Omega_{p''}}{4\pi} \Gamma^\omega_{\sigma, \sigma'', \tau, \tau''}(p, p'') \frac{v_F p'' \cdot q}{\omega - v_F p'' \cdot q} \Gamma_{\sigma'', \sigma', \tau', \tau''}(p''', p''; q).$$

Diagrammatically this equation corresponds to

$$\Gamma_{\sigma, \sigma', \tau, \tau'}(p, p'; q) = \Gamma^\omega_{\sigma, \sigma', \tau, \tau'}(p, p') + \Gamma_{\sigma, \sigma', \tau, \tau'}(p, p'; q) + \Gamma_{\sigma, \sigma', \tau, \tau'}(p, p'; q).$$

where $\Gamma^\omega$ is denoted by a blob with a line across. The line is drawn perpendicular to the channel, in which $\Gamma^\omega$ is quasiparticle-quasihole irreducible [9].

We use the notation that the particle-hole propagators in diagrams with the crossed blob correspond to the singular quasiparticle-quasihole part only.
The $|q|$ limit $\Gamma^q$ corresponds to the full particle-hole vertex for $q = 0$, i.e. scattering of quasiparticles strictly on the Fermi surface with vanishing momentum transfer $|q| \to 0$. Thus, Eq. (15) can be used to relate the two limits:

$$
\Gamma^q_{\sigma\sigma',\tau\tau'} = \Gamma^\omega_{\sigma\sigma',\tau\tau'} - N(0) z^2 \frac{1}{4} \int \frac{d\Omega'}{4\pi} \text{Tr}_{\sigma''\tau''} \Gamma^\omega_{\sigma\sigma'',\tau\tau''} \Gamma^q_{\sigma''\sigma',\tau''\tau'}.
$$

(17)

The quasiparticle-quasihole irreducible vertex can be identified with the quasiparticle interaction introduced above [12], $N(0) z^2 \Gamma^\omega(p,p') = \mathcal{F}(\theta)$, while $N(0) z^2 \Gamma^q(p,p') = \mathcal{A}(\theta)$ is the quasiparticle forward scattering amplitude. By inserting this into Eq. (17) and expanding the angular dependence of $\mathcal{F}(\theta)$ and $\mathcal{A}(\theta)$ on Legendre polynomials, we arrive at a set of algebraic equations for the scattering amplitude with the solution:

$$
\mathcal{A}(\theta) = \sum_i \left( \frac{F_i}{1 + F_i/(2l + 1)} + \frac{F_i'}{1 + F_i'/(2l + 1)} \tau \cdot \tau'
\right.
\left. + \frac{G_i}{1 + G_i/(2l + 1)} \sigma \cdot \sigma' + \frac{G_i'}{1 + G_i'/(2l + 1)} \tau \cdot \tau' \sigma \cdot \sigma' \right) P_l(\cos \theta).
$$

(18)

The antisymmetry of the quasiparticle scattering amplitude implies two Pauli principle sum rules [10,13] for the Fermi liquid parameters, corresponding to scattering at vanishing relative momentum in singlet-odd and triplet-odd states:

$$
\sum_i \left( \frac{F_i}{1 + F_i/(2l + 1)} + \frac{F_i'}{1 + F_i'/(2l + 1)} 
\right.
\left. + \frac{G_i}{1 + G_i/(2l + 1)} + \frac{G_i'}{1 + G_i'/(2l + 1)} \right) = 0
$$

(19)

$$
\sum_i \left( \frac{F_i}{1 + F_i/(2l + 1)} - 3 \frac{F_i'}{1 + F_i'/(2l + 1)} 
\right.
\left. - 3 \frac{G_i}{1 + G_i/(2l + 1)} + 9 \frac{G_i'}{1 + G_i'/(2l + 1)} \right) = 0.
$$

(20)

It is important to note that the quasiparticle interaction is strictly speaking defined only in the Landau limit $q = 0$. This is reflected in the one pion exchange (OPE) contribution (direct and exchange) to $\Gamma^\omega$, where the direct tensor interaction, which is proportional to $q^2$, vanishes in the Landau limit. For later use we give the one pion exchange contribution to $\Gamma^\omega$:

$$
\Gamma_{\sigma\sigma',\tau\tau'}^{\text{OPE}}(p,p';q) = -\frac{f^2}{3 m^2} \tau \cdot \tau' \left\{ q^2 \frac{S_{12}(q)}{q^2 + m^2} - \frac{m^2_\pi \sigma \cdot \sigma'}{q^2 + m^2_\pi} \right\}
\left. + \frac{f^2}{3 m^2} \frac{3 - \tau \cdot \tau'}{2} \left\{ (p - p')^2 \frac{S_{12}(p - p')}{(p - p')^2 + m^2_\pi} - \frac{1}{2} \frac{m^2_\pi (3 - \sigma \cdot \sigma')}{(p - p')^2 + m^2_\pi} \right\}. \right.
$$

(21)
3 The Induced Interaction

The quasiparticle scattering amplitude includes particle-hole diagrams in the ZS channel to all orders. Therefore, if one were to use a finite set of diagrams for the quasiparticle-quasihole irreducible vertex $\Gamma_\omega$, e.g. the Hartree-Fock approximation, Eq. (2), then the corresponding quasiparticle scattering amplitude, obtained by solving Eq. (15), would not obey the Pauli principle. This is because the particle-hole diagrams in the ZS channel, which, as discussed above are the exchange diagrams to those in the ZS channel, are not iterated. Thus, in order to obey the Pauli principle, it is necessary to iterate the ZS channel to all orders as well. This is done by the induced interaction, which was invented by Babu and Brown [9] and applied to nuclear matter by Sjöberg [14]. Here we give a diagrammatically motivated heuristic derivation of the induced interaction.

The integral equation for $\Gamma_\omega$ must be constructed in such a way that it generates all possible ZS and ZS' joined diagrams for the quasiparticle scattering amplitude. To third order these are:

\begin{equation}
\begin{align*}
(a) & \quad (b) & \quad (c) & \quad (d) & \quad (e) \\
(f) & \quad (g) & \quad (h) & \quad (i)
\end{align*}
\end{equation}

Using an antisymmetric, particle-hole irreducible vertex function in Eq. (22) guarantees the antisymmetry of the quasiparticle scattering amplitude. We have marked the propagators $G(p'' + \frac{1}{2}) G(p'' - \frac{1}{2})$, that are generated by solving the Bethe-Salpeter equation, Eq. (10), with thick lines. The diagrams with only thin lines are contained in $\tilde{\Gamma}$. All of these can be constructed from a ZS' ladder sum, where the vertices are lower order diagrams of $\tilde{\Gamma}$ rotated by 90 degrees. To second order $\tilde{\Gamma}$ consists of the one ZS' bubble only, diagram (c),
to third order $\tilde{\Gamma}$ also includes the two ZS bubble string, diagram (i), and the diagrams (d) and (e). The latter are constructed by taking a second (lower) order diagram, the one ZS bubble, diagram (c), rotating it by 90 degrees, and then inserting it as left or right vertex into the one ZS bubble. Thus, the integral equation for $\tilde{\Gamma}$, for a system with spin only, reads:

$$
\tilde{\Gamma}_{\sigma,\sigma'}(p, p'; q) = I_{\sigma,\sigma'}(p, p'; q) - \frac{1}{2} (1 + \sigma \cdot \sigma')
\times \left\{ \frac{1}{2} \text{Tr}_{\sigma''} \int \frac{-i d^4 p''}{(2\pi)^4} \tilde{\Gamma}_{\sigma,\sigma''}(\frac{p + p' + q}{2}, p''; p - p') G(p'' + \frac{p - p'}{2})
\times G(p'' - \frac{p - p'}{2}) \tilde{\Gamma}_{\sigma'',\sigma'}(p'', \frac{p + p' - q}{2}; p - p') + \tilde{\Gamma} G^2 \tilde{\Gamma} G^2 \tilde{\Gamma} + \ldots \right\}, \quad (23)
$$

where we have denoted the antisymmetric, ZS and ZS′ particle-hole irreducible vertex with $I$. The spin operator $\sigma$ in the brackets of Eq. (23) is contracted with the spinors of the left particle-hole pair with momenta $p + q/2$ and $p' + q/2$, whereas $\sigma$ in the left hand side and in $I$ is contracted with the bottom particle-hole pair spinors with momenta $p \pm q/2$. The recoupling between the two particle-hole channels is accounted for by including the spin exchange operator $P_\sigma = 1/2 (1 + \sigma \cdot \sigma')$. Since $\tilde{\Gamma}$ is finite, we can take the limit $q \rightarrow 0$ in Eq. (23) and obtain for $p \approx p'$ [7]

$$
\tilde{\Gamma}_{\sigma,\sigma'}(p, p') = I_{\sigma,\sigma'}(p, p') - \frac{1}{2} (1 + \sigma \cdot \sigma')
\times \left\{ \frac{1}{2} \text{Tr}_{\sigma''} \int \frac{d\Omega_{p''}}{4\pi} \tilde{\Gamma}_{\sigma,\sigma''}(\frac{p + p'}{2}, p''; p - p')
\times \frac{v_F \hat{p''} \cdot (p - p')}{\varepsilon - \varepsilon'' - v_F \hat{p''} \cdot (p - p')} \tilde{\Gamma}_{\sigma'',\sigma'}(\frac{p + p'}{2}; p - p')
+ \int \frac{-i d^4 p''}{(2\pi)^4} \tilde{\Gamma}_{\sigma,\sigma''}(\frac{p + p'}{2}, p''; p - p') \phi(p'') \tilde{\Gamma}_{\sigma'',\sigma'}(p'', \frac{p + p'}{2}; p - p') \right\}
+ \tilde{\Gamma} \left( (GG)_{ZS'} + \phi \right) \tilde{\Gamma} \left( (GG)_{ZS'} + \phi \right) \tilde{\Gamma} + \ldots \right\}, \quad (24)
$$

where $(GG)_{ZS'}$ denotes the quasiparticle-quasihole part of the propagators in the ZS′ channel. To both sides of Eq. (24) we add the series $\tilde{\Gamma} \phi \tilde{\Gamma} + \tilde{\Gamma} \phi \tilde{\Gamma} + \tilde{\Gamma} \phi \tilde{\Gamma} + \ldots$ and obtain

$$
\Gamma^\omega = I + \tilde{\Gamma} \phi \tilde{\Gamma} + \tilde{\Gamma} \phi \tilde{\Gamma} + \ldots - \frac{1}{2} (1 + \sigma \cdot \sigma') \left\{ \tilde{\Gamma} \left( (GG)_{ZS'} + \phi \right) \tilde{\Gamma}
+ \tilde{\Gamma} \left( (GG)_{ZS'} + \phi \right) \tilde{\Gamma} \left( (GG)_{ZS'} + \phi \right) \tilde{\Gamma} + \ldots \right\}. \quad (25)
$$

To guarantee continuity in the forward scattering amplitude the limit $q \rightarrow 0$ has to be performed before taking $p \rightarrow p'$ [15].
By regrouping the terms we find

\[
\Gamma^\omega = I + \left(1 - \frac{1}{2} (1 + \sigma \cdot \sigma')\right) \tilde{\Gamma} \phi \frac{1}{1 - \tilde{\Gamma} \phi} \Gamma
\]

\[-\frac{1}{2} (1 + \sigma \cdot \sigma') \Gamma^\omega (\mathcal{G}\mathcal{G})_{ZS'} \frac{1}{1 - \Gamma^\omega (\mathcal{G}\mathcal{G})_{ZS'}} \Gamma^\omega. \tag{26}\]

The first term \(I_{qp} = I + \left(1 - \frac{1}{2} (1 + \sigma \cdot \sigma')\right) \tilde{\Gamma} \phi (1 - \tilde{\Gamma} \phi)^{-1} \tilde{\Gamma}\) is quasi-particle-quasi-hole irreducible both in the ZS and ZS' channel. Due to the identity \(P_\sigma (1 - P_\sigma) = -(1 - P_\sigma)\) and the antisymmetry of \(I\), \(I_{qp}\) is also antisymmetric.

For \(p = p'\) the non-singular parts of the ZS and the ZS' graphs differ only in the spin dependence. This is reflected in the factor \((1 - P_\sigma)\), which vanishes for a Fermi liquid of say spin up species only.

Eq. (26) is an integral equation for \(\Gamma^\omega\), which diagrammatically is of the form:

\[
\begin{align*}
&\begin{array}{c}
p' + \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p - \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' + \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p - \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} \\
\begin{array}{c}
p + \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p - \frac{q}{2}
\end{array} & \begin{array}{c}
p - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array}
& = & \begin{array}{c}
p' + \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p - \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} \\
\begin{array}{c}
p' + \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p - \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p'' - \frac{p''}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array}
& + & \begin{array}{c}
p' + \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p - \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p'' - \frac{p''}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} \\
\begin{array}{c}
p + \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p - \frac{q}{2}
\end{array} & \begin{array}{c}
p - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p'' + \frac{p''}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' + \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p'' + \frac{p''}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p'' - \frac{p''}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p'' - \frac{p''}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p'' - \frac{p''}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p'' - \frac{p''}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array} & \begin{array}{c}
p' - \frac{q}{2} \\
\downarrow \hspace{1cm} \uparrow \\
p + \frac{q}{2}
\end{array}
\end{align*}
\tag{27}\]

The diagram with the crossed lines denotes the driving term \(I_{qp}\), which con-
sists of all quasiparticle-quasihole irreducible diagrams (in both the ZS and ZS' channels). The series of all ZS' bubble diagrams corresponding to the remaining terms in Eq. (26) is called the induced interaction. It may be regarded as the linear response of the system to the presence of the quasiparticle. Due to the exchange of the external lines it is explicit that all diagrams in the induced interaction are irreducible in the ZS channel. The limit \( q \to 0 \) is to be taken only after the iteration of the induced interaction. In order to illustrate this, consider the one pion exchange vertex function, Eq. (21), as driving term. The momentum transfers in \( \Gamma_{\text{OPE}}^{\text{OP}}(p, p'; q) \) are \( q \) and \( p - p' \). However, due to the exchange character of the induced interaction, the corresponding momentum transfers in the vertices of the one ZS' bubble are \( p - p' \) and \( \frac{1}{2}(p + p' + q) - p'' \). Although terms proportional to \( q^2 \) in the driving term vanish in the Landau limit, they appear in the induced interaction. Thus, in general the induced interaction requires input beyond Fermi liquid theory, since the Landau parameters are defined only in the \( |q| \to 0 \) limit. In the limit \( p = p' \), the induced interaction expressed solely in terms of Landau parameters is exact. Nevertheless, applications to nuclear matter \([14,16,17]\), neutron matter \([18,19]\) and liquid \(^3\text{He}\) \([20–22]\) have shown that the induced interaction is a very powerful approximation even for non vanishing angles \( \theta \), i.e. \( p \neq p' \).

The one ZS' bubble contribution to the induced interactions is given by

\[
\Gamma_{\sigma\sigma',\tau\tau'}^{\text{ind}}(p, p') = -\frac{1}{4}(1 + \sigma \cdot \sigma')(1 + \tau \cdot \tau') N(0) \ z^2 \\
\times \frac{1}{4} \text{Tr}_{\sigma''\tau''} \int \frac{d\Omega_{\sigma''\tau''}}{4\pi} \Gamma_{\sigma''\sigma',\tau''\tau'}^{\omega}(p + p' + q, 2; p - p') \\
\times \frac{v_F \hat{p}'' \cdot (p - p')}{\varepsilon - \varepsilon'' - v_F \hat{p}'' \cdot (p - p')} \Gamma_{\sigma''\sigma',\tau''\tau'}^{\omega}(p'', 2; p - p'). \tag{28}
\]

In the extrapolation away from \( p = p' \) the initial and final momenta are treated symmetrically. This yields the correct result, e.g. for a current-current coupling. Using the bare direct and exchange interaction as driving term, i.e.

\[
p' + \frac{q}{2} \quad p' - \frac{q}{2} \\
\begin{array}{c}
\begin{array}{c}
\bigoplus
\end{array}
\end{array} \\
p + \frac{q}{2} \quad p - \frac{q}{2}
\]

\[
= \quad + \quad , \tag{29}
\]

one finds that the lowest order contributions to Eq. (28) correspond to the
We expand the angular dependence of the quasiparticle interaction $\Gamma^\omega$ on Legendre polynomials, $\Gamma^\omega = \sum_l \Gamma^\omega_l (\cos \theta)$. After inserting this in Eq. (28), we find

\[
\Gamma^{\text{ind}}_\sigma \sigma', \tau \tau', (p, p') = -\frac{1}{4} (1 + \sigma \cdot \sigma') (1 + \tau \cdot \tau') N(0) z^2 \times \frac{1}{4} \Tr_{\sigma'' \tau''} \sum_{l, l'} \Gamma^{\omega}_{\sigma \sigma', \tau \tau', l} \Gamma^{\omega}_{\sigma'' \sigma', \tau'' \tau', l'} \times \int \frac{dQ_{\sigma''}}{4\pi} \frac{P_l (\tilde{p} + \tilde{p}') P_{l'} (\tilde{p}'' + \tilde{p}'')} {\epsilon - \epsilon' - \nu_F \tilde{p}'' \cdot (p - p')} .
\]

In order to cover all possible combinations of $p$ and $p'$, the induced interaction is needed for momentum transfer $q' = p - p'$ up to $2k_F$. This is done by extrapolating the quasiparticle-quasihole propagator in Eq. (12) to large $q$ using the particle-hole propagator of a free Fermi gas with an effective mass $m^*$. Furthermore, the external quasiparticles are assumed to be on the Fermi surface, so that $\epsilon = \epsilon' = 0$. For $l, l' = 0, 1$ the resulting integrals in Eq. (31) are given in [14]. We introduce the notation $\mathcal{F}_{\text{ind}} = N(0) z^2 \Gamma_{\text{ind}}$ and decompose the induced interaction into its scalar, spin, isospin and spin-isospin components,

\[
\mathcal{F}_{\text{ind}} = F_{\text{ind}} + F'_{\text{ind}} \tau \cdot \tau' + G_{\text{ind}} \sigma \cdot \sigma' + G'_{\text{ind}} \tau \cdot \tau' \sigma \cdot \sigma' .
\]

The resulting expression for the scalar induced interaction, $F_{\text{ind}}$, including $l = 0, 1$ terms, is [14,16]

\[
4F_{\text{ind}} = 1 \cdot \left( \frac{F_0^2 \alpha_0 (q'/k_F)}{1 + F_0 \alpha_0 (q'/k_F)} + (1 - \frac{q^2}{4k_F^2}) \frac{F_1^2 \alpha_1 (q'/k_F)}{1 + F_1 \alpha_1 (q'/k_F)} \right) + 3 \cdot \left( \frac{F_0^2 \alpha_0 (q'/k_F)}{1 + F_0 \alpha_0 (q'/k_F)} + (1 - \frac{q^2}{4k_F^2}) \frac{F_1' \alpha_1 (q'/k_F)}{1 + F_1' \alpha_1 (q'/k_F)} \right) + 3 \cdot \left( \frac{G_0^2 \alpha_0 (q'/k_F)}{1 + G_0 \alpha_0 (q'/k_F)} + (1 - \frac{q^2}{4k_F^2}) \frac{G_1 \alpha_1 (q'/k_F)}{1 + G_1 \alpha_1 (q'/k_F)} \right) + 9 \cdot \left( \frac{G_0^2 \alpha_0 (q'/k_F)}{1 + G_0 \alpha_0 (q'/k_F)} + (1 - \frac{q^2}{4k_F^2}) \frac{G_1' \alpha_1 (q'/k_F)}{1 + G_1' \alpha_1 (q'/k_F)} \right) ,
\]

where $q' = |q'|$, $\alpha_0(x)$ and $\alpha_1(x)$ are the Lindhard (or density-density) and
current-current correlation functions, respectively. The factor \((1 - q^2/4k_F^2)\) guarantees that the current response vanishes for back to back scattering.

\[
\alpha_0(x) = \frac{1}{2} + \frac{1}{2} \left( \frac{x}{4} - \frac{1}{x} \right) \ln \frac{1 - x/2}{1 + x/2} \tag{34}
\]

\[
\alpha_1(x) = \frac{1}{2} \left[ \frac{3}{8} - \frac{1}{2x^2} + \left( \frac{1}{2x^3} + \frac{1}{4x} - \frac{3x}{32} \right) \ln \frac{1 + x/2}{1 - x/2} \right] \tag{35}
\]

For the spin, isospin and spin-isospin induced parts, the coefficients in (33) have to be changed according to the table below. These coefficients follow from the recoupling of spin and isospin between the two particle-hole channels.

|       | \(F\) | \(F'\) | \(G\) | \(G'\) |
|-------|-------|-------|-------|-------|
| \(F_{\text{ind}}\) | 1     | 3     | 3     | 9     |
| \(F'_{\text{ind}}\) | 1     | -1    | 3     | -3    |
| \(G_{\text{ind}}\) | 1     | 3     | -1    | -3    |
| \(G'_{\text{ind}}\) | 1     | -1    | -1    | 1     |

By construction, the induced interaction with the bare direct and exchange interaction as driving term generates the complete particle-hole parquet for the scattering amplitude. The particle-hole parquet are all planar fermionic diagrams except those that are joined by the particle-particle (BCS) channel. This corresponds to the solution to the fermionic parquet equations of Lande and Smith ignoring the coupling to the s channel [23]. The s channel diagrams are particle-hole irreducible in both the ZS and ZS' channels. Hence, they should be included in the driving term. Traditionally the driving term has been computed within Brueckner theory by varying the energy twice with respect to the occupation number and removing all contributions that are included in the induced interaction [14,24]. If two hole contributions, which are expected to be small, are neglected, one can express the driving term as the direct and exchange Brueckner \(G\) matrix multiplied by the renormalization factor \(z^2\). The factor \(z^2\) accounts for some of the higher order completely particle-hole irreducible diagrams. Diagrams involving e.g. particle-particle ladders with a screened interaction are neglected. In this work we identify the \(G\) matrix in the driving term with the low momentum nucleon-nucleon interaction \(V_{\text{low } k}\) for \(\Lambda = k_F [2]\) and consequently employ \(z^2 V_{\text{low } k}\) as the driving term. As discussed in the introduction, \(V_{\text{low } k}\) is smooth and includes the effects of the short range repulsion.
4 Relation between the Fermi Liquid Parameters and the Low Momentum Nucleon-Nucleon Interaction

For \( \mathbf{p} = \mathbf{p}' \) the induced interaction expressed in terms of Fermi liquid parameters is exact and one can derive general constraints for these parameters. In this limit the integral in Eq. (31) simplifies to \( -\delta_{l,l'}/(2l + 1) \) and all higher ZS' bubble terms are easily summed. Thus, Eq. (27) can be written as follows

\[
F_s + F_a \sigma \cdot \sigma' = F_s^d + F_a^d \sigma \cdot \sigma' + \int \frac{d\Omega_{p''}}{4\pi} \left\{ F_s(p,p'')A_s(p'',p) \left[ 1 + \frac{\sigma \cdot \sigma'}{2} \right] \right. \\
\left. + F_a(p,p'')A_a(p'',p) \left[ 3 - \frac{\sigma \cdot \sigma'}{2} \right] \right\}, \tag{36}
\]

where we again consider a system with spin only. For \( \mathbf{p} = \mathbf{p}' \) the series of ZS' bubbles is equivalent to the series of ZS bubbles summed by the scattering amplitude up to a sign and the spin exchange operator for the exchange of the external lines in the induced interaction. The equation for the scattering amplitude reads

\[
A_s + A_a \sigma \cdot \sigma' = F_s + F_a \sigma \cdot \sigma' - \int \frac{d\Omega_{p''}}{4\pi} \left\{ F_s(p,p'')A_s(p'',p) \right. \\
\left. + F_a(p,p'')A_a(p'',p) \right\}. \tag{37}
\]

We have introduced the notation

\[
\mathcal{F} = F_s + F_a \sigma \cdot \sigma' \tag{38}
\]
\[
\mathcal{A} = A_s + A_a \sigma \cdot \sigma' \tag{39}
\]
\[
\mathcal{F}_{\text{driving}} = F_s^d + F_a^d \sigma \cdot \sigma'. \tag{40}
\]

It is easy to solve the integral equations for the driving term. In the \( S = 1 \) channel the sum and in the \( S = 0 \) channel the difference of the two integral equations, Eqs. (36) and (37), leads to

\[
\mathcal{F}_{\text{driving}} (S = 1) = \mathcal{A} = 0 \tag{41}
\]
\[
\mathcal{F}_{\text{driving}} (S = 0) = 2\mathcal{F} - \mathcal{A}, \tag{42}
\]

where we have used the Pauli principle sum rule in the case \( S = 1 \). The first case, Eq. (41), projects on odd partial waves, while the second case, Eq. (42), projects on even partial waves. In symmetric nuclear matter, there are two spin-isospin states corresponding to odd partial waves (\( S = T = 0 \) and \( S = T = 1 \)) and two corresponding to even states (\( S = 0, T = 1 \) and \( S = 1, T = 0 \)). We thus obtain two new constraints on the Fermi liquid
parameters of nuclear matter:

\[
\sum_l \left\{ 2F_l - \frac{F_l}{1 + F_l/(2l + 1)} + 2F'_{l} - \frac{F'_{l}}{1 + F'_{l}/(2l + 1)} - 3 \left( 2G_l - \frac{G_l}{1 + G_l/(2l + 1)} \right) - 3 \left( 2G'_{l} - \frac{G'_{l}}{1 + G'_{l}/(2l + 1)} \right) \right\} = \mathcal{F}_{\text{driving}} (S = 0, T = 1) \tag{43}
\]

\[
\sum_l \left\{ 2F_l - \frac{F_l}{1 + F_l/(2l + 1)} - 3 \left( 2F'_{l} - \frac{F'_{l}}{1 + F'_{l}/(2l + 1)} \right) + 2G_l - \frac{G_l}{1 + G_l/(2l + 1)} - 3 \left( 2G'_{l} - \frac{G'_{l}}{1 + G'_{l}/(2l + 1)} \right) \right\} = \mathcal{F}_{\text{driving}} (S = 1, T = 0). \tag{44}
\]

These are general constraints, which however are useful only if the driving term is known. Such a constraint was first derived by Bedell and Ainsworth [21] for paramagnetic Fermi liquids, like liquid $^3$He or $^3$He–$^4$He mixtures, and employed to extract the effective scattering length. As reasoned above, we approximate the driving term with $z^2 V_{\text{low } k}$. We need the matrix elements of $V_{\text{low } k}$ in the basis of total spin $S$ and total isospin $T$. By summing over $M_S$, we project onto the central components of the forward scattering amplitude [17].

\[
\frac{1}{V} \mathcal{F}_{\text{driving}} (S, T) = \frac{z^2}{2S + 1} N(0) \sum_{M_S} \left( \langle p p' S T | V_{\text{low } k} | p p' S T \rangle - \text{exchange} \right). \tag{45}
\]

Transforming to relative momentum $\mathbf{q}' = \mathbf{p} - \mathbf{p}'$ and coupling angular momentum and total spin leads to

\[
\mathcal{F}_{\text{driving}} (S, T) = z^2 N(0) \frac{4\pi}{2S + 1} \sum_{J, l} (2J + 1) \left( 1 - (-1)^{l+S+T} \right) \times \langle k = \frac{q'}{2} l S J T | V_{\text{low } k} | k = \frac{q'}{2} l S J T \rangle. \tag{46}
\]

At vanishing relative momentum there are only s-wave contributions to the driving term due to the rotational invariance. Since the driving term is antisymmetric, these contributions are in the $S = 0, T = 1$ and $S = 1, T = 0$ channels, consistent with the two Pauli principle sum rules. Thus, with the input $z^2 V_{\text{low } k}$ for the driving term, the two relations, Eqs. (43) and (44), constrain the dimensionless Fermi liquid parameters of nuclear matter in a nontrivial way to

\[
\mathcal{F}_{\text{driving}} (S = 0, T = 1) = z^2 \frac{16 m_N k_F (1 + F_1/3)}{\pi} V_{\text{low } k}(0, 0; \Lambda = k_F, ^1S_0) \tag{47}
\]

\[
\mathcal{F}_{\text{driving}} (S = 1, T = 0) = z^2 \frac{16 m_N k_F (1 + F_1/3)}{\pi} V_{\text{low } k}(0, 0; \Lambda = k_F, ^3S_1). \tag{48}
\]
The dimension of the potential is absorbed by the density of states. As explained in the introduction and in [2], \( V_{\text{low } k} \) is obtained from a RG decimation of various nuclear force models. Bogner \textit{et al.} [2] find that the \( V_{\text{low } k} \) obtained from various bare potentials at \( \Lambda = k_F \) are identical. Moreover, when one compares the low momentum part of the bare interaction models with \( V_{\text{low } k} \), one observes that the main effect of the renormalization is a constant shift in momentum space. This correspond to a smeared delta function in coordinate space and accounts for the removal of the model dependent short range core. Thus, the two constraints, which use as dynamical input \( V_{\text{low } k}(0,0) \), connect the pivotal matrix element of the RG decimation to the unique set of Fermi liquid parameters of nuclear matter. As the Fermi liquid parameters are fixed points under the RG flow towards the Fermi surface, the constraints relate \( V_{\text{low } k} \) to these fixed points.

5 Renormalization Group with the Induced Interaction

In the microscopic derivation of Fermi liquid theory, one isolates the quasi-particle part of the full propagator from the pair background. We have shown that, for \( p \approx p' \), this is rigorously possible also when both particle-hole channels are taken into account. This is necessary in order to preserve the Pauli principle and leads to the induced interaction. Having reduced the theory to interactions among quasiparticles, we now separate the soft modes of the quasiparticle-quasihole propagators from the hard ones. To this end we introduce a momentum cutoff at \( k_F \pm \Lambda_F \). In this way we arrive at a theory of quasiparticles interacting in a model space of slow modes exclusively. For a discussion of the RG approach to Fermi liquid theory see [25–27].

In a shorthand notation we write \((GG)_{ZS} = (GG)^S_{ZS} + (GG)^H_{ZS}\) for the ZS propagators (at finite \( q \)) and with analogous expressions for the ZS' channel. The indices \( S \) and \( H \) denote integrations over the soft (inside the shell) and hard (outside) momenta, respectively. We define the vertices \( \gamma^q(p,p';q,\Lambda_F) \) and \( \gamma^\omega(p,p';q,\Lambda_F) \) by

\[
\gamma^q_{ZS}(\Lambda_F) = \gamma^q_{ZS}(\Lambda_F) + (GG)^H_{ZS} \gamma^q_{ZS}(\Lambda_F)
\]

\[
\gamma^\omega_{ZS}(\Lambda_F) = I_{qp} - \left\{ \gamma^\omega_{ZS}(\Lambda_F) (GG)^H_{ZS} \gamma^\omega_{ZS}(\Lambda_F) + \gamma^\omega_{ZS}(\Lambda_F) (GG)^H_{ZS} \gamma^\omega_{ZS}(\Lambda_F) + \ldots \right\}
\]

\[
= I_{qp} - \gamma^\omega_{ZS}(\Lambda_F) (GG)^H_{ZS} \gamma^\omega_{ZS}(\Lambda_F)
\]

\[10\]We denote with \( \Lambda_F \) the cutoff in medium, which is not to be confused with the cutoff \( \Lambda \) for \( V_{\text{low } k} \).
where $I_{qp}$ denotes the quasiparticle-quasihole irreducible driving term defined above. Furthermore, we introduce the shorthand notation $\gamma^q_{ZS} = \gamma^q(p, p'; q, \Lambda_F)$ and the exchange thereof $\gamma^q_{ZS'} = \gamma^q((p + p' + q)/2, (p + p' - q)/2; p - p', \Lambda_F)$, where for simplicity the spin- and isospin-dependence is suppressed. Analogous expressions hold for $\gamma^q_{ZS}$ and $\gamma^q_{ZS'}$. Due to phase space restrictions, the running of $\gamma^q$ and $\gamma^{\omega}$ at $T = 0$ starts at $\Lambda_F = \max(|q|/2, |p - p'|/2)$. With a weakly energy dependent driving term $I_{qp}$, we can set $\omega = 0$ in the flow equations. For $\Lambda_F = 0$, the quasiparticle scattering amplitude $\Gamma^q$ and the quasiparticle interaction $\Gamma^{\omega}$ are obtained as the $|q| \to 0$ limit of $\gamma^q_{ZS}$ and $\gamma^{\omega}_{ZS}$, respectively. On the other hand, for $\Lambda_F \geq k_F$, the particle-hole contributions vanish in the momentum range of interest $|q|, |p - p'| \leq 2k_F$, so that $\gamma^q_{ZS}(k_F) = \gamma^{\omega}_{ZS}(k_F) = I_{qp}$.

We differentiate Eqs. (49) and (50) with respect to $\Lambda_F$ and require $dI_{qp}/d\Lambda_F = 0$. This corresponds to ignoring the flow from the particle-particle (BCS) channel. The coupled RG equations then read:

$$
\frac{d\gamma^q_{ZS}}{d\Lambda_F} = \gamma^q_{ZS} \frac{d(GG)^H_{ZS}}{d\Lambda_F} \gamma^q_{ZS} + \frac{d\gamma^{\omega}_{ZS}}{d\Lambda_F} (GG)^H_{ZS} \gamma^q_{ZS} + \gamma^q_{ZS} \frac{d\gamma^{\omega}_{ZS'}}{d\Lambda_F} (GG)^H_{ZS} \gamma^q_{ZS'} + \gamma^q_{ZS} \frac{d\gamma^{\omega}_{ZS'}}{d\Lambda_F} (GG)^H_{ZS'} \gamma^q_{ZS'} \gamma^q_{ZS} \gamma^{\omega}_{ZS}
$$

(52)

$$
\frac{d\gamma^{\omega}_{ZS}}{d\Lambda_F} = - \left\{ \frac{1}{1 - \gamma^q_{ZS'}} (GG)^H_{ZS'} \gamma^q_{ZS} + \frac{1}{1 - \gamma^{\omega}_{ZS'}} (GG)^H_{ZS'} \gamma^{\omega}_{ZS} \right\}.
$$

(53)

Using the notation

$$
\delta_{ZS}(\Lambda_F) = \frac{d\gamma^{\omega}_{ZS}}{d\Lambda_F} (GG)^H_{ZS} \gamma^q_{ZS} + \gamma^q_{ZS} \frac{d\gamma^q_{ZS}}{d\Lambda_F}
$$

and the analogous expression for the $ZS'$ channel, we write the RG equations in the compact form

$$
\frac{d\gamma^q_{ZS}}{d\Lambda_F} = \gamma^q_{ZS} \frac{d(GG)^H_{ZS}}{d\Lambda_F} \gamma^q_{ZS} + \frac{d\gamma^{\omega}_{ZS}}{d\Lambda_F} + \delta_{ZS}(\Lambda_F)
$$

(55)

$$
\frac{d\gamma^{\omega}_{ZS}}{d\Lambda_F} = - \left\{ \gamma^q_{ZS'} \frac{d(GG)^H_{ZS'}}{d\Lambda_F} \gamma^q_{ZS'} + \gamma^{\omega}_{ZS'} \frac{d\gamma^q_{ZS'}}{d\Lambda_F} + \delta_{ZS'}(\Lambda_F) \right\}.
$$

(56)

In the limit $p = p'$ we can replace $\gamma^q_{ZS'}$ in Eq. (56) by $\gamma^q_{ZS}$ and obtain

$$
\frac{d\gamma^{\omega}_{ZS}}{d\Lambda_F} = - P_{\sigma} \left\{ \gamma^q_{ZS} \frac{d(GG)^H_{ZS}}{d\Lambda_F} \gamma^q_{ZS} + \delta_{ZS}(\Lambda_F) \right\},
$$

(57)
where the spin structure in the exchange channel is accounted for by $P_\sigma$. This implies that for $p = p'$ we have $d\gamma^q_{2S}|_{q=0}/d\Lambda_F = 0$ in singlet-odd and triplet-odd states, while $d(2\gamma^q_{2S} - \gamma^q_{2S})|_{q=0}/d\Lambda_F = 0$ in singlet-even and triplet-even states. Thus, the Pauli principle sum rules and the new constraints are invariant under the RG flow. The coupled RG equations, Eqs. (52) and (53), are nonperturbative. To lowest order, where $\delta$ in Eqs. (55) and (56) is neglected, these agree with the perturbative one-loop RG equations of Dupuis [27].

6 Fermi Liquid Parameters and Tensor Interactions

The aim of this section is to study whether phenomenological values for the Fermi liquid parameters are consistent with the sum rules as well as the constraints. For this purpose we approximate the quasiparticle interaction by the $l = 0$ and $l = 1$ terms. As additional input we take the phenomenological values for the scalar and isospin Fermi liquid parameters. The central spin and spin-isospin Fermi liquid parameters are then obtained from the sum rules and the constraints. By taking linear combinations of the sum rules Eqs. (19) and (20), and the constraints, Eqs. (43) and (44), the equations for $G_l$ and $G'_l$ decouple:

\[
\begin{align*}
\sum_l \left\{ \frac{F_l}{1 + F_l/(2l + 1)} + 3 \frac{G'_l}{1 + G'_l/(2l + 1)} \right\} &= 0 \quad (58) \\
\sum_l \left\{ 2F_l - \frac{F_l}{1 + F_l/(2l + 1)} - 2 \left( 2F'_l - \frac{F'_l}{1 + F'_l/(2l + 1)} \right) \\
&\quad - 3 \left( 2G'_l - \frac{G'_l}{1 + G'_l/(2l + 1)} \right) \right\} \\
&= z^2 \frac{16 m_N k_F (1 + F_l/3)}{\pi} V_{\text{low } k}(0, 0; \Lambda = k_F, \frac{1S_0 + 3 \cdot 3S_1}{4}) \quad (59) \\
\sum_l \left\{ \frac{F_l}{1 + F_l/(2l + 1)} + \frac{3}{2} \frac{F'_l}{1 + F'_l/(2l + 1)} + \frac{3}{2} \frac{G_l}{1 + G_l/(2l + 1)} \right\} &= 0 \quad (60) \\
\sum_l \left\{ 2F'_l - \frac{F'_l}{1 + F'_l/(2l + 1)} - \left( 2G'_l - \frac{G'_l}{1 + G'_l/(2l + 1)} \right) \right\} \\
&= z^2 \frac{16 m_N k_F (1 + F_l/3)}{4\pi} V_{\text{low } k}(0, 0; \Lambda = k_F, \frac{1S_0 - 3S_1}{4}) \quad . \quad (61)
\end{align*}
\]

We note that the relevant input for the spin-isospin Fermi liquid parameters $G'_l$, Eq. (59), is the spin averaged s-wave low momentum potential, whereas the one for the spin Fermi liquid parameters $G_l$, Eq. (61), is the difference of the spin singlet and spin triplet s-wave low momentum potentials. Since the $3S_1$ channel is only slightly more attractive than the $1S_0$ channel, the right hand side of Eq. (61) is small. Consequently, this constraint is not very sensitive to the precise value of the renormalization factor $z^2$. 

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The quasiparticle strength \( z \) was recently computed in a self-consistent description of the nucleon spectral functions. Roth [28] finds \( z = 0.76 \) at the Fermi surface for \( k_F = 1.35 \text{ fm}^{-1} \). However, there is a systematic uncertainty on the value of the \( z \) factor, since the relevance of experimental constraints from \((e,e'p)\) knockout reactions on the jump in the occupation number at the Fermi surface is questionable. Furnstahl and Hammer [29] have recently shown that within the rigorous effective field theory for the interacting dilute Fermi gas the occupation numbers are not observable. We use \( z = 0.8 \) and for the nucleon effective mass at the saturation point we use \( m^*/m = 0.72 \) corresponding to

\[
F_1 = -0.85. 
\]  

(62)

The empirical value for the anomalous orbital gyromagnetic ratio provides a constraint on \( F_1' \). For a proton in the Pb region [30], \( \delta g_l = 0.23 \pm 0.03 \). In Fermi liquid theory [11] \( \delta g_l = (1/3)(F_1' - F_1)/(1 + F_1/3) \), which for \( F_1 = -0.85 \) yields

\[
F_1' = 0.14. 
\]  

(63)

The incompressibility of nuclear matter is experimentally best constrained by the isoscalar giant monopole resonance and by fitting binding energies and the diffuseness of the nuclear surface. Microscopic calculations by Blaizot et al. [31] and Youngblood et al. [32] and the Thomas-Fermi equation of state of Myers and Swiatecki [33] give an incompressibility of \( K = 230 \pm 20 \text{ MeV} \). The Thomas-Fermi equation of state gives a symmetry energy of \( E_{\text{sym}} = 32.7 \text{ MeV} \). The empirical value of the symmetry energy is limited by various fits to nuclear masses, resulting in \( E_{\text{sym}} = 31 \pm 5 \text{ MeV} \) [34]. Thus, we find

\[
F_0 = -0.27 \quad F_0' = 0.71. 
\]  

(64) \hspace{1cm} (65)

In units where \( m = 1 \), the matrix elements of the low momentum nucleon-nucleon interaction are given by [2]

\[
V_{\text{low}}(0,0; \Lambda = k_F, 1S_0) = -1.95 \text{ fm} 
\]  

(66)

\[
V_{\text{low}}(0,0; \Lambda = k_F, 3S_1) = -2.51 \text{ fm}. 
\]  

(67)

These are identical for the Bonn-A, Paris, and Argonne-V18 potential as well as a chiral model.

Very similar results are obtained by Feldmeier et al. [35,36], who introduce a unitary correlation operator including central and tensor correlations. For both the Bonn-A and Argonne-V18 potentials, they find \( V_{\text{UCOM}}(0,0; 1S_0) = -1.88 \text{ fm} \) and \( V_{\text{UCOM}}(0,0; 3S_1) = -2.86 \text{ fm} \). The value in the \( 3S_1 \) channel depends on the range of the tensor correlations, which for the value quoted here is chosen to reproduce the d-state admixture when the uncorrelated deuteron
Fig. 1. The solution for the spin-dependent Fermi liquid parameters (solid lines) with error bands limited by the dashed lines. Here the effect of tensor parameters is neglected.

The trial wave function contains only an s-wave component. The dependence on the range of the tensor correlation corresponds to the cutoff dependence of $V_{\text{low } k}(0, 0; \Lambda, ^3S_1)$ discussed in the introduction.

Epelbaoum et al. [37] constructed an effective potential from a s-wave Malfliet-Tjon type potential. The transformation method of Okubo used in their work is similar to the RG decimation employed for $V_{\text{low } k}$. They find $V_{\text{eff}}(0, 0; ^1S_0) = -1.94$ fm for a cutoff of $\Lambda = 300$ MeV. Their results are in a good agreement with $V_{\text{low } k}$.

In Fig. 1 we show the solution to the Eqs. (58,59,60,61) without tensor Fermi liquid parameters. In the error estimates we include the uncertainties in the input Fermi liquid parameters, the uncertainties due to the truncation of the Legendre series as well as the uncertainties in the driving term. The latter include only the estimated error of the renormalization factor $z$, since the effects of the neglected higher order contributions are difficult to appraise. We thus find

\begin{align*}
G_0 &= 0.15 \pm 0.3 \quad (68) \\
G_1 &= 0.45 \pm 0.3 \quad (69) \\
G'_0 &= 1.0 \pm 0.2 \quad (70) \\
G'_1 &= 0 \pm 0.2. \quad (71)
\end{align*}

The relative errors of $G_0$ and $G_1$ are large, because the corresponding bands are almost parallel. Nevertheless, this calculation demonstrates that $V_{\text{low } k}$ is a very promising starting point for calculations of Fermi liquid parameters.
The mean value of $G'_0 = 1$ should be confronted with the experimental constraints imposed by the energy of the giant Gamow-Teller resonance. Since the Fermi liquid parameters embody the effective interaction in the nucleon subspace, the empirical value of $g'_{NN}$, obtained in a model that includes $\Delta$-isobar degrees of freedom, must be corrected for the screening due to $\Delta$-hole excitations. Including $\Delta$-hole excitations to all orders, we find

$$G'_0 = N(0) \left( \frac{f_{\pi NN}^2}{m^*_\pi} \right) \left( g'_{NN} - \frac{\frac{f_{\pi NN}^2}{m^*_\pi} g'_{NN} \frac{8}{9} \frac{\rho_0}{m_{\Delta} - m_N}}{1 + \frac{f_{\pi NN}^2}{m^*_\pi} g'_{\Delta\Delta} \frac{8}{9} \frac{\rho_0}{m_{\Delta} - m_N}} \right),$$

(72)

where $\rho_0$ denotes the nuclear matter density. Furthermore, $g'_{NN}$ is the short-range part of the spin-isospin dependent effective nucleon-nucleon interaction in pionic units, while $g'_{N\Delta}$ and $g'_{\Delta\Delta}$ are the corresponding $NN \rightarrow N\Delta$ and $N\Delta \rightarrow \Delta N$ interaction strengths. Kawahigashi et al. [38] find $g'_{NN} = 0.6$, $g'_{N\Delta} = 0.3$, while Körfgen et al. [39,40] obtain $g'_{N\Delta} = 0.3$, $g'_{\Delta\Delta} = 0.3$. Using these values, we find $G'_0 = 1.0$ in good agreement with our result. The $\Delta$-hole polarization reduces the value of $G'_0$ by about 10%.

The discussion presented above is easily generalized to include the effects of the tensor force. For $\mathbf{p} = \mathbf{p}'$ the tensor components of the quasiparticle interaction $F$, Eq. (3), and the quasiparticle scattering amplitude $A$ vanish and the tensor force enters only together with the spin-dependent parameters in the $\sigma \cdot \sigma'$ and $\tau \cdot \tau' \sigma \cdot \sigma'$ components of the scattering amplitude [13]. The coupling of spin and Landau $l$ to good total angular momentum $J$ was carried out by Bäckman et al. [17]. We use the tensor Fermi liquid parameters obtained from a $G$ matrix calculation using Reid’s soft core potential for $l \leq 4$ and from the one pion exchange potential for higher $l$ (see Table 2 of [17]). To account for the effects of tensor forces, we replace the spin-dependent parameters of the scattering amplitude $C_l = G_l/(1+G_l/(2l+1))$ and $C'_l = G'_l/(1+G'_l/(2l+1))$ in the constraints up to $l = 4$ with the corresponding expressions including tensor interactions [13]. Since the expansion of the tensor interaction in Landau $l$ is poorly convergent, we include terms up to $l = 4$. We have checked that the contributions of higher $l$ are negligible. We note that the tensor parameters of Ref. [17] are given for $z = 1$ and $m^*/m = 1$. Consequently, these parameters should be reduced by the factor $z^2 m^*/m$.

In Fig. 2 we show the solution to the Eqs. (58,59,60,61) including tensor Fermi liquid parameters. We have not included errors for the tensor parameters. Since the isospin tensor parameters $H'_l$ are small (one pion and one rho exchange yields $H'_l = -H_l/3$ [16]), the solution for $G'_0$ and $G'_1$ in the spin-isospin sector is basically unaffected by the presence of tensor interactions. However, the solution for the spin Fermi liquid parameters $G'_0$ and $G'_1$ is strongly modified. In fact, the error bands overlap only in a small region, when we include tensor interactions. The reason is that in this sector the tensor parameters of Ref. [17] are quite large. We note that this may change, when the contribution
of the induced interaction to the tensor parameters is included. In order to illustrate the possible effects of this type, we compute the leading contribution to the tensor Fermi liquid parameters from the one bubble polarization in the induced interaction using the one pion exchange interaction. The lowest order contribution to the tensor Fermi liquid parameters $H_l$ from the one pion exchange driving term, Eq. (21), is given by

$$H_l(\theta) = N(0) \frac{f^2}{3 m_{\pi}^2} \frac{3}{2} \frac{k_F^2}{(p - p')^2 + m_{\pi}^2}.$$  \hspace{1cm} (73)

The dominant tensor contribution from the one bubble term in the induced interaction is obtained by employing the direct tensor part $\pi_T$ of the one pion exchange potential as vertices in the induced interaction. This corresponds to the first and in part the third and the last diagrams of Eq. (30). More explicitly, we compute the diagrams

$$H_l(\theta) = N(0) \frac{f^2}{3 m_{\pi}^2} \frac{3}{2} \frac{k_F^2}{(p - p')^2 + m_{\pi}^2}.$$  \hspace{1cm} (74)

For the long range part of $G'$ we include the momentum dependence by splitting the interaction into a one pion exchange piece and a short ranged piece $^{11}$:

$$G' = N(0) \frac{f^2}{3 m_{\pi}^2} \left( \frac{m_{\pi}^2}{q^2 + m_{\pi}^2} + \Delta g' \right).$$  \hspace{1cm} (75)

$^{11}$ This is justified since the $q$ dependence of the exchanged heavy mesons is weak.
Using Eqs. (21) and (28) we then find

\[
\Gamma_{\sigma'\sigma''}\text{dir. OPE}(p, p') = -\frac{1}{4} (1 + \sigma \cdot \sigma') (3 - \tau \cdot \tau') N(0) z^2 \left( \frac{f^2}{3 m_\pi^2} \right)^2
\]

\[
\times \frac{1}{2} \text{Tr}_{\sigma''} \int \frac{d\Omega_{p''}}{4\pi} \frac{(p - p')^2}{(p - p')^2 + m_\pi^2} \left\{ \frac{S_{12''(p - p')}}{(p - p')^2 + m_\pi^2} - 2 \frac{m_\pi^2 \sigma'' \cdot \sigma'}{(p - p')^2 + m_\pi^2} - 6 \Delta q' \right\} \frac{v_F \hat{p}'' \cdot (p - p')}{\epsilon - \epsilon' - v_F \hat{p}'' \cdot (p - p')}. 
\]

(76)

The integral over \( \Omega_{p''} \) yields the Lindhard function \(-\alpha_0(q' / k_F)\). By exploiting the following identities for the tensor operator,

\[
\frac{1}{2} \text{Tr}_{\sigma''} S_{12''(p - p')} S_{2''2(p - p')} = S_{12}(p - p') + 2 \sigma \cdot \sigma' 
\]

(77)

\[
\frac{1}{2} \text{Tr}_{\sigma''} S_{12''(p - p')} \sigma'' \cdot \sigma' = S_{12}(p - p') 
\]

(78)

\[
\frac{1}{2} (1 + \sigma \cdot \sigma') S_{12}(p - p') = S_{12}(p - p'), 
\]

(79)

we finally arrive at the second order correction to the tensor Fermi liquid parameters

\[
\Delta H(\theta) = H(\theta) N(0) z^2 f^2 / 3 m_\pi^2 \alpha_0(q' / k_F) 
\]

\[
\times \left\{ \frac{(p - p')^2}{(p - p')^2 + m_\pi^2} - 2 \frac{m_\pi^2}{(p - p')^2 + m_\pi^2} - 6 \Delta q' \right\}. 
\]

(80)

In order to reproduce the empirical value for \( G_0' = 1 \) with the direct one pion exchange contribution plus \( \Delta q' \), we need \( \Delta q' = 0.5 \). The resulting corrections to the tensor Fermi liquid parameters \( H_0 = 0.35 \) and \( H_1 = 0.43 \) are \( \Delta H_0 = -0.40 \) and \( \Delta H_1 = -0.69 \). Thus, we find that the induced interaction tends to reduce the tensor Fermi liquid parameters, in agreement with the results of Dickhoff \textit{et al.} [41]. The very large effects show that the tensor interactions must be treated self consistently within the induced interaction. Finally, we note that about 60\% of the left hand side of Eq. (59) is due to the Landau parameter \( G_0' \). Thus, there is a close connection between the spin-averaged s-wave low momentum interaction \( V_{\text{low k}}(0, 0; \Lambda = k_F) \) and the local spin-isospin dependent part of the quasiparticle interaction.

\[\text{Note that the value of } g_{\text{NN}}' \text{ here is larger than in [38], because we use } z < 1. \text{ The physics is determined by } G_0', \text{ not by } g_{\text{NN}}'.\]
7 Summary and Conclusions

In this paper we presented two new algebraic constraints that relate the Landau Fermi liquid parameters in nuclear matter to the driving term of the induced interaction. By identifying the driving term with the s-wave low momentum nucleon-nucleon interaction $V_{\text{low } k}$ at $\Lambda = k_F$, including some straightforward in-medium effects, we obtained an intriguing relation between the effective interaction in vacuum and in nuclear matter.

The resulting constraints on the Fermi liquid parameters were used in conjunction with the Pauli principle sum rules to compute the major spin dependent parameters, given the phenomenological values for the spin independent parameters. We find good agreement with empirically determined parameters. The present calculation indicates that a good approximation to the driving term of the induced interaction can be obtained from $V_{\text{low } k}$ in a straightforward manner, by including minimal in-medium corrections, the wave-function renormalization factors and the nucleon effective mass in the density of states. A full calculation of the induced interaction, including a self-consistent treatment would be needed to firmly establish this identification. In such a calculation, the spin, isospin and velocity dependence of $V_{\text{low } k}$ would be reflected in the corresponding Fermi liquid parameters. A comparison with empirical parameters would then provide a test of e.g. the velocity dependence of the low momentum nucleon-nucleon interaction $V_{\text{low } k}$. In $V_{\text{low } k}$, the role of the (local) short range repulsion of the bare interactions is taken over by a non-locality, which interpolates between a weak repulsion at low energies and a stronger one at higher energies.

The effects of tensor forces are also studied, using the tensor parameters obtained in a $G$ matrix calculation. We find a fairly large effect of the tensor force on the isoscalar spin dependent parameters. However, as indicated by a simple estimate, this effect will probably be reduced when the tensor parameters are computed self consistently by including the tensor force in the induced interaction.

Moreover, we derive the flow equations for the renormalization group decimation of the quasiparticle scattering amplitude and the quasiparticle interaction in the two particle-hole channels starting from the induced interaction. A solution of these equations would provide the scattering amplitude also for non-forward scattering, which is of high interest for the calculation of superfluid gaps and transport processes, e.g. in neutron star interiors. In condensed matter systems an ab initio RG analysis of this type [42] applied to the 2D Hubbard model has successfully established the existence of d-wave superconductivity. The RG equations for the quasiparticle scattering amplitude and the quasiparticle interaction we obtained from the induced interaction are
nonperturbative. Existing RG studies in Fermi systems have been restricted to one-loop approximations.

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