Block to granular-like transition in dense bubble flows

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Abstract. – We have experimentally investigated 2-dimensional dense bubble flows underneath inclined planes. Velocity profiles and velocity fluctuations have been measured. A broad second-order phase transition between two dynamical regimes is observed as a function of the tilt angle $\theta$. For low $\theta$ values, a block motion is observed. For high $\theta$ values, the velocity profile becomes curved and a shear velocity gradient appears in the flow.

Introduction. – The dense granular flow of identical particles is a complex phenomenon which involves multiple processes such as friction, correlated motion and inelastic collisions \cite{1–3}. Various flow regimes have been identified in different experiments. Three major flow regimes are emphasized in Figure 1. Block motion occurs for example when particles form some jammed phases \cite{4–6}. In granular flows on a pile, only the surface grains are in motion. This rolling phase has typically a thickness of 10-20 grains and the velocity $v_h$ as a function of the height $h$ is roughly linear for that surface grains \cite{7–9}.

Fig. 1 – Typical velocity profiles $v_h$ encountered in dense flows. From left to right: block motion, surface flow and Poiseuille flow.

A lot of efforts has been devoted to the understanding of dense flows of particles. Some studies \cite{10, 11} address the problem of contact force networks, i.e. the presence of arches, which plays an important role \cite{12}. Fluctuations and correlations of the grain motion as a function of the shear rate have been also studied. Collective behaviors have been put into evidence such as the motion of clusters of grains \cite{6, 13}. Different theoretical approaches have been developed for describing dense flows. Among them, one can cite the Saint-Venant
approach which considers some conservation equations in the rolling phase [14]. Pouliquen and coworkers [2] have also proposed that the granular flows can be considered as quasistatic flows, the global motion resulting from fractures. Nevertheless, no general theory can predict the occurrence of the different flow regimes and corresponding velocity profiles from the various physical parameters such as the density, the shear rate and the friction.

In 1947, Bragg and coworkers proposed a simple experiment [15] in order to put into evidence some crystallographic ordering. This experiment is also famous because it was quoted in the Feynman’s lectures [16]. The experiment consisted in producing small identical bubbles at the surface of a liquid. Hexagonal packed structures appeared and crystallographic domains separated by grain boundaries were observed. Recent experiments [17,18] were devoted to the study of defects in such bubble rafts.

We propose to modify the Bragg’s experiment in order to study dense flows of identical bubbles. The flow of bubbles is of physical interest since contacting bubbles are characterized by a nearly zero friction [19]! Our experimental system corresponds thus to an “ideal” dense flow. We show in this letter that this system of bubbles presents a phase transition between two distinct dense flow regimes depending on the kinetic energy of the incoming bubbles.

![Fig. 2 – A transparent inclined plane is immersed into water and tilted by an angle $\theta$. Small air bubbles are injected from below at the bottom of the plane and rise towards the top where they aggregate on a linear rough obstacle. A sketch of the experimental setup (left) and typical bubble packing taken by the CCD camera placed above the setup (right) are illustrated. The angle is $\theta = 1^\circ$. Arrows indicate the motion of bubbles.](image)

Experimental setup. – Our experiment is analogous to the filling of a 2D-silo by continuous feeding of granular material from a central hopper. The experimental setup consists in a transparent inclined plane which is immersed into water [see Fig. 2]. The tilt angle $\theta$ is small and ranges from 0° to 3° in our experiments. The error on $\theta$ is 0.1°. Spherical air bubbles are injected from below at the bottom of the tilted plane. The bubble production rate is kept constant at 5 bubbles by one second. The size of the bubbles can be controlled by the air pump. The bubble radius is typically $R = 0.75$ mm in the present study. In order to avoid the coalescence of the bubbles, a small quantity of commercial soap (based on SDS surfactants [20]) is added to the water. The mixture has a surface tension $\gamma = 0.03$ N/m and a viscosity $\eta = 0.001$ kg m$^{-1}$ s$^{-1}$. Due to buoyancy $B$, the bubbles rise beneath the inclined plane. A rough rigid object has been placed at the top of the plane such that bubbles aggregate there. The obstacle roughness is typically one bubble diameter and avoids
purely translational motions of the bubble stack [22]. One should note that in the direction perpendicular to the plane, the bubble stack is only one layer of bubbles thick. We thus have a really 2D bubble pile. A CCD camera captures top views of the packing of bubbles through the transparent tilted plane.

The unique parameter to be investigated is the tilt angle $\theta$ of the plane. Indeed, the angle $\theta$ controls the buoyancy
\[ B = \frac{4\pi R^3 \Delta \rho g}{3} \sin \theta, \] (1)
where $\Delta \rho \approx 1000 \text{ kg m}^{-3}$ is the difference between air and liquid densities. One should also note that the bubble motion is mainly limited by the viscous drag force
\[ f \sim \eta R v_i. \] (2)

The bubble/plane friction is small with respect to the drag force $f$ as we will see below. Force equilibrium $B = f$ gives the terminal incoming velocity $v_i$ of the bubbles which is a few millimeters per second since $\sin \theta \approx 10^{-2}$ in our experiments. We are thus in a laminar regime ($Re \approx 1$).

**Experimental results.** – Bubbles tend to aggregate in an ordered hexagonal packing since the bubble size is monodisperse. The packing is continuously fed by new bubbles. As a consequence, the packing evolves and a dense flow of bubbles is seen in both tails. As for usual granular flow, a thin layer of moving grains is observed [21]. Moreover, smaller motions are also observed deeper inside the packing, even for low $\theta$ values.

Figure 2 presents a typical packing of bubbles for $\theta = 1^\circ$. The shape of the pile is clearly rounded. If the incoming bubble flow is interrupted, the bubbles slide on each other under the influence of buoyancy and the angle of the bubble stack tends to zero. The pile collapses completely. This indicates that the friction between contacting bubbles is quite low, as discussed in the introduction. As contacting bubbles are sliding on each other, small shape deformations are also observed at the bottom of the pile. Thus, the bubble pile is not a strict granular medium in the sense that no arch is created.

Fig. 3 – Part of the packing. Bubbles tend to aggregate in a hexagonal packing. A dislocation is seen in the center of the picture.

In the packing, the bubble motion is mainly due to moving dislocations [see Fig. 3]. Those punctual defects are created at the surface of the packing and propagate along bubble lines at various speeds. One should note that the mean dislocation speed is larger than the bubble
speed. This is due to the cooperative motion of the bubbles. For larger values of θ, several dislocations are seen in the different parts of the pile. The majority of the bubbles is in motion.

In order to quantify and to measure the bubble flow, we have tracked the motion of every bubble in the tail of the bubble pile. Movies of the evolution of different parts of the bubble stack have been recorded at a frame rate of 12 fps. The recorded areas contains typically 30 × 30 bubbles. A precision of 0.03 mm on bubble positions is obtained. We have then measured the bubble paths and deduced velocity profiles using a particle tracking algorithm. Figure 4 presents two typical instantaneous (∆t = 0.083 s) velocity fields in the tail of the bubble pile, for respectively θ = 0.5° and θ = 1.5°. One can observe slow bubble motions for a small θ value (left) and moving layers of bubbles over a quasi-static phase for a larger θ value (right). Velocity measurements presented hereafter result from averages over long movies (more than 30 s).

Fig. 4 – Instantaneous velocity fields of a part of the tail of the bubble pile for (left) θ = 0.5° < θc and (right) θ = 1.5° > θc. The bubbles are observed to move from right to left. A block motion is observed for θ < θc, while a velocity profile decreasing with the depth h is found for θ > θc. Sketches of the velocity profiles are also illustrated.

Figure 5 presents the mean velocity ⟨v⟩ in the bubble pile as a function of sin(θ). We propose a quadratic behavior of the mean velocity as emphasized by the fit in Figure 5. The mean velocity ⟨v⟩ would then be controlled by the tilt angle θ and would be proportional to the kinetic energy (∼v_i^2) of individual bubbles before they reach the packing. In order to characterize the bubble flow, we have also performed measurements of the velocity fluctuations, similarly to the granular ‘temperature’ [7]. The velocity fluctuations amplitude σ is then defined by

$$\sigma = \sqrt{\langle v^2 \rangle - \langle v \rangle^2},$$

i.e. the second moment of the velocity distribution in both x and y directions [7]. Those velocity fluctuations σ are plotted as a function of sin(θ) in Figure 6. One can observe a ‘jump’ of the velocity fluctuations σ for tilt angles above sin(θc) ≈ 0.025. This suggests the presence of a broad second-order phase transition as a function of the tilt angle θ. The critical angle corresponds to θc ≈ 1.4°. Around this critical angle, two dense flow regimes should be distinguished. They will be characterized below.

For low angle values (θ < θc), the dislocations are created at the surface of the pile. They propagate in diagonal directions and are reflected at the bottom of the bubble packing. As a consequence, we have observed the motion of giant triangular domains of bubbles. Above the critical angle θc, we observe numerous dislocations. The density of dislocations is so high
that they are interacting by pairs. Indeed, we have observed the collision, the merging, and
the annihilation of dislocation pairs. The cooperative motion of the dislocations lead to the
motion of small clusters of bubbles.

The above behaviors imply distinct velocity profiles. Velocity profiles have been measured
as a function of the vertical position $h$ reduced by the bubble diameter $2R$ and counted
from the obstacle. Three typical velocity profiles are given in Figure 4. Velocity profiles $v_h$
reveal two components: (i) a mainly constant profile which dominates for low $\theta$ values and
(ii) a curved profile (a gradient) seen for high $\theta$ values. The curvature above $\theta_c$ is similar
to the velocity profile of granular systems [see Fig. 1]: a velocity profile decreasing abruptly
in the more static phase (e.g. deeper in the pile) and growing in the flowing layer [6]. The
velocity profile penetrates as deeply in the pile as the tilt angle $\theta$ value is large, as in granular
systems [6].

The ‘vertical’ profiles at low $\theta$ values can be explained in terms of block motions of the

Fig. 6 – Velocity fluctuations $\sigma$ defined by Eq. 4 as a function of $\sin(\theta)$. A broad transition is seen
at $\sin(\theta_c) \approx 0.025$ or $\theta_c \approx 1.4^\circ$. Each dot represents an average over 1500 pictures and over $1.3 \times 10^6$
bubbles. Error bars are indicated.
giant triangular domains of bubbles. Domains are sliced by fast moving dislocations. As the kinetic energy of the incoming bubbles increases, the size of the bubble domains decreases. The density of dislocations increases and the bubble stack becomes more 'fluid'. At this stage, interacting dislocations which are created at the surface of the flow are observed to annihilate deep in the dense flow. The study of the interaction/annihilation between dislocations could explain the particular curvature of our velocity profiles. This needs more theoretical work and is outside the scope of this letter.

The 'jump' in the velocity fluctuations $\sigma$ suggests that we have a transition between block and granular flows depending on the density of dislocations in the packing. Varying the bubble size $R$ or the bubble production rate does not affect qualitatively the results reported herein. Moreover, the bubble motions and dislocation mechanisms reported hereabove are also observed when changing the soap-water mixture.

In the future, the analogy between granular and bubble flows could be developed by changing the geometry of the bubble flow and considering “chute flow”. By tilting the rough obstacle relatively to the incoming bubble flow, one would get a “chute” flow of bubble on an inclined substrate. Tuning the bubble rate production and the tilt angle, could control the thickness of the flowing layer.

Summary. – We have studied bubbles moving and aggregating on inclined planes. We have underlined the similarities between dense bubble flows and granular flows. Dense bubble flows can be considered as ‘ideal’ granular flows with nearly zero friction. We have emphasized a broad second-order transition between two dense flow regimes as a function of the tilt angle $\theta$ of the plane below which the bubbles are injected.

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