Band Calculations for Ce Compounds with AuCu$_3$-type Crystal Structure on the basis of Dynamical Mean Field Theory I. - CePd$_3$ and CeRh$_3$ -

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Band calculations for Ce compounds with the AuCu$_3$-type crystal structure were carried out on the basis of dynamical mean field theory (DMFT). The auxiliary impurity problem was solved by a method named NCAf$^2$vc (noncrossing approximation including the $f^2$ state as a vertex correction). The calculations take into account the crystal-field splitting, the spin-orbit interaction, and the correct exchange process of the $f^1 \rightarrow f^0, f^2$ virtual excitation. These are necessary features in the quantitative band theory for Ce compounds and in the calculation of their excitation spectra. The results of applying the calculation to CePd$_3$ and CeRh$_3$ are presented as the first in a series of papers. The experimental results of the photoemission spectrum (PES), the inverse PES, the angle-resolved PES, and the magnetic excitation spectra were reasonably reproduced by the first-principles DMFT band calculation. At low temperatures, the Fermi surface (FS) structure of CePd$_3$ is similar to that of the band obtained by the local density approximation. It gradually changes into a form that is similar to the FS of LuPd$_3$ as the temperature increases, since the 4f band shifts to the high-energy side and the lifetime broadening becomes large.

KEYWORDS: dynamical mean field theory, band theory, ARPES, magnetic excitation, CePd$_3$, CeRh$_3$, Cauchy integral

1. Introduction

Nonempirical band calculations for strongly correlated electron systems have been extensively developed on the basis of dynamical mean field theory (DMFT). The 4f electrons in Ce compounds are typical strongly correlated electrons. Recently, a DMFT band calculation scheme for Ce compounds was developed in refs. 6 and 7. In the present work, it is applied to Ce compounds with the AuCu$_3$-type crystal structure, which show a wide variety of 4f electronic states from the most itinerant limit to the localized limit.

The 4f state splits into the $j = 5/2$ ground multiplet and the $j = 7/2$ excited multiplet with a separation of about 0.3 eV owing to the spin-orbit interaction (SOI). The multiplet shows crystal-field splitting (CFS) of the order of 100 K. In cubic crystals, the $j = 5/2$ multiplet splits into the $j = 5/2$ and $j = 3/2$ states, respectively. It is important to take account of the SOI and CFS effects in 4f systems. In DMFT, the correlated band electron problem is mapped onto the calculation of the single-particle excitation spectrum of the auxiliary impurity Anderson model in an effective medium. Reliable methods of solving the impurity Anderson model with CFS and the SOI effect are needed in the DMFT band calculation for 4f compounds. A theory named NCAf$^2$vc (noncrossing approximation including the $f^2$ state as a vertex correction) has been developed and combined with the linear muffin-tin orbital (LMTO) method to carry out the DMFT band calculation. NCAf$^2$vc can include CFS and the SOI effect, and also the correct exchange process of the $f^1 \rightarrow f^0, f^2$ virtual excitation. The calculation gives an accurate order of the Kondo temperature ($T_K$).

The DMFT band calculation will be applied in a series of studies to Ce compounds with the AuCu$_3$-type structure: CePd$_3$, CeRh$_3$, CeIn$_3$, and CeSn$_3$. Each of these materials is classified as a typical example of strongly correlated 4f systems. CePd$_3$ is a typical heavy Fermion system with $T_K$ of about 250 K, and has been studied extensively by various methods. It has a nonmagnetic Fermi liquid (FL) ground state at low temperatures. A controversy existed in the experimental works on inelastic magnetic excitation, but it has recently been resolved by a detailed study of the wave number vector (wave vector) dependence of spectra. The wave-vector-integrated-spectrum of the magnetic excitation has a broad peak at approximately 55 meV. In the single-particle excitation, a broad peak with the 4f character was observed on the inverse photoemission spectrum (IPES) side (i.e., in the energy region above the Fermi energy ($E_F$)). This seems to be consistent with the high $T_K$ of this compound. However, a strong peak structure has never been observed on the photoemission spectrum (PES) side (i.e., in the energy region below $E_F$), contradicting other physical properties. Recent careful study using the 3d-4f high-resolution resonant photoemission spectrum (RPES) revealed that the bulk component of the PES of this compound shows strong intensity at $E_F$ consistently with high $T_K$. The angle-resolved PES (ARPES) has also been studied recently. It may be worthwhile whether to confirm these recent results of studies are reproduced or not by the first-principles DMFT band calculation. CeRh$_3$ is known as one of the compounds having the most itinerant 4f states.
CeSn$_3$ is also known to show the nonmagnetic FL ground state with a high characteristic temperature.\textsuperscript{37,38) CeRu$_3$ has an antiferromagnetic ground state with a Neel temperature of $T_N = 11$ K. Recently, it was found that $T_N$ decreases to zero under the pressure of 2.5 GPa, in addition, the transition to superconductivity occurs at $T_{SC} = 0.2$ K.\textsuperscript{39–41) It will be interesting to study the change of the band structure under pressures by the DMFT calculation.

These compounds commonly have the AuCu$_3$-type crystal structure, which is classified into the simple cubic (sc) lattice. The DMFT band structure of these compounds will be reported in two papers. In CePd$_3$ and CeRh$_3$, the hybridization of 4$f$ states with 4$d$ states of the transition metal is very strong. The 4$d$ states almost sink to below $E_F$ in CePd$_3$ whereas they are located near $E_F$ in CeRh$_3$.\textsuperscript{42) In this paper, calculations for these 4$d$ compounds will be reported. Calculations for CeSn$_3$ and CeIn$_3$ will be given in a subsequent paper. Their 4$f$ states hybridize with broad 5$p$ states of ligand ions.

Calculated results of CePd$_3$ and CeRh$_3$ generally show reasonable agreement with experimental results of the PES, IPES, ARPES, and inelastic magnetic excitation by neutrons. However, the calculation gives a higher $T_K$ than that expected from experiments when it is examined in detail. The DMFT band calculation for CePd$_3$ gives a Fermi surface (FS) structure similar to that obtained by the local density approximation (LDA) calculation at very low temperatures. When the temperature increases, 4$f$ bands shift to the high-energy side and their lifetime broadening increases. This leads to the change of the FS structure into one that is similar to the FS of LaPd$_3$. At $T = 150$ K, the FS has a different form from both FSs of LaPd$_3$ and the LDA band of CePd$_3$ as an intermediate stage of the change. The lifetime broadening overcomes the fine wave vector dependence of the 4$f$ spectrum at $T = 300$ K. In CeRh$_3$, the 4$f$ band is located at about 0.9 eV above $E_F$, and the dispersion of the DMFT band is almost identical to that of the LDA band in the energy region near $E_F$. The density of states (DOS) has an appreciable value slightly below $E_F$ in the ARPES. This low-binding energy part shows a weak wave vector dependence, though no flat bands do not exist in the vicinity of the Fermi energy.

In §2, we briefly give the formulation on the basis of the LMTO method. Results of the application to CePd$_3$ are shown in §3, and results for CeRh$_3$ are given in §4. A summary is given in §5. In the appendices, notes on the calculation of the total electron number are given. An efficient method of calculating the Cauchy integral using the spline interpolation scheme is also presented. This integral is frequently used in the DMFT calculation.

2. Formulation

The method of calculation is described briefly because its details have been given in previous papers.\textsuperscript{6,7) We consider the excitation spectrum of the following Hamiltonian:

$$\mathcal{H} = \mathcal{H}_{\text{LDA}} + \frac{U}{2} \sum_i \left( \sum_{\Gamma \gamma} \varepsilon^{\Gamma}_{i \Gamma \gamma} c_{i \Gamma \gamma}^+ c_{i \Gamma \gamma} - n_{i \Gamma}^{\text{LDA}} \varepsilon_{i \Gamma}^{\text{LDA}} \right)^2. \quad (1)$$

Here, $c_{i \Gamma \gamma}^+$ is the annihilation operator for the atomic localized state $\phi^2_{i \Gamma} (r)$ at site $i$ with the $\gamma$ orbital of the $\Gamma$-irreducible representation. The quantity $n_{i \Gamma}^{\text{LDA}}$ is determined using the occupation number of the atomic 4$f$ electron per Ce ion in the LDA calculation. We assume that the local Coulomb interaction acts only on the orbital $\phi^2_{i \Gamma} (r)$.

The excitation spectrum is expressed by introducing the self-energy terms,\textsuperscript{6) $\mathcal{H}_{\text{DMFT}} = \mathcal{H}_{\text{LDA}}$}

$$+ \sum_i \sum_{\Gamma \gamma} \left( \Sigma_{\gamma}(\varepsilon + i\delta) + \varepsilon_{i \Gamma}^{\text{LDA}} \right) |\phi^2_{i \Gamma \gamma} > < \phi^2_{i \Gamma \gamma}| , \quad (2)$$

where $\varepsilon_{i \Gamma}^{\text{LDA}}$ is the single-electron energy level of the 4$f$ state, and $\Sigma_{\gamma}(\varepsilon + i\delta)$ is the energy level in the LDA calculation. The self-energy $\Sigma_{\gamma}(\varepsilon + i\delta)$ is calculated by solving the auxiliary impurity problem with the use of NCA $f^2 G_{vc}$; its outline is described in the Appendix of ref. 7.

In later calculations we will approximate the localized 4$f$ state $\phi^a$ by the band center orbital $\phi(-)$ because its localization is good for the 4$f$ state. $\phi(-)$ has the logarithmic derivative $-\ell - 1$ on the muffin-tin surface.\textsuperscript{12,13) In the LMTO method, the Hamiltonian $\mathcal{H}_{\text{LDA}}$ is diagonalized using the LMTO bases, $\psi(\ell, m, \alpha, qL)$, and the spin $(\alpha)$ at site $q$ in the unit cell, where $L \equiv (\ell, m, \alpha)$.\textsuperscript{13) The explicit expression of $\chi_{qL}(r)$ has been given in a previous paper.\textsuperscript{6) Note that they are not orthogonal to each other, but the eigenvectors $\psi(\ell, m, \alpha, qL)$ are orthonormal.

The DMFT band structure is calculated in the following way: (A) first the LDA part of the Hamiltonian $\mathcal{H}_{\text{LDA}}$ is diagonalized for a given $\mathbf{k}$, (B) then the matrix equation of the Greenian is prepared in the manifold of the eigenvectors $\psi(\ell, m, \alpha, qL)$. The Greenian equation for the given $\mathbf{k}$ is written as

$$[zI - D_{\text{LDA}}(\mathbf{k}) - \Sigma(z)]G(z; \mathbf{k}) = I, \quad (4)$$

where $I$ is the unit matrix and $D_{\text{LDA}}(\mathbf{k})$ is the diagonal matrix of the eigenenergies of $\mathcal{H}_{\text{LDA}}$ with $\mathbf{k}$. The matrix elements of $\Sigma(z)$ are given by calculating the self-energy operator term of eq. (2) based on eq. (3).

The DOS on the atomic 4$f$ state is given by

$$\rho_{\Gamma}^{(\text{band})} (\varepsilon; \mathbf{k}) = \frac{1}{\pi} \text{Tr} \left[ \mathbf{O}_{\Gamma} G(\varepsilon + i\delta; \mathbf{k}) \right] , \quad (5)$$

where $\mathbf{O}_{\Gamma} = \text{diag} (\Gamma_1, \Gamma_2, \Gamma_3)$.
where the projection operator is defined as
\[ \hat{O}_{\Gamma} = \sum_{i, \gamma} |\phi_{i}(\Gamma, \gamma)\rangle \langle \phi_{i}(\Gamma, \gamma)|. \]  

The local DOS in the DMFT band calculation is obtained by summing \( \rho_{\Gamma}(\text{band})^{(\text{imp})}(\varepsilon; \mathbf{k}) \) over \( \mathbf{k} \) in the Brillouin zone:
\[ \rho_{\Gamma}(\text{band})(\varepsilon) = \frac{1}{N} \sum_{\mathbf{k}} \rho_{\Gamma}(\text{band})(\varepsilon; \mathbf{k}). \]  
Here, \( N \) is the total number of unit cells.

The auxiliary impurity problem is solved by the NCA\( f^{2}\text{vc} \) method. The splitting of the self-energy due to the SOI and CFS effects is considered. As shown in ref. 7, this method gives an accurate order of the Kondo temperature when the result is compared with that of the more correct numerical renormalization group (NRG) calculation\(^7,44\) in a simple model case.

Since the method of the self-consistent calculation in the DMFT has been described previously\(^6,7\) we exclude the detailed explanation from this paper. First of all, we calculate the self-consistent LDA band by the LMTO method, and potential parameters, except for the \( f \) levels, are fixed to those in the LDA calculation. (I) We calculate the atomic \( 4f \) density of states \( \rho_{\Gamma}^{(\text{imp})}(\varepsilon) \) (\( 4f \) DOS) for the auxiliary impurity Anderson model by the NCA\( f^{2}\text{vc} \) method with a trial energy dependence of the hybridization intensity (HI) and \( 4f \) levels,\(^45\) then calculate the local self-energy. (II) The DMFT band calculation is carried out using the self-energy term, and the local \( 4f \) DOS in the DMFT band is calculated. The calculation is iterated so that the \( 4f \) DOS of the local auxiliary impurity model and the DMFT band satisfy the self-consistent conditions.\(^46,47\)

The \( 4f \) level is adjusted in the DMFT self-consistent iterations under the condition that the \( 4f \) occupation number has a given target value, \( n_{f}(\text{rsl.target}) \), which is estimated from the LDA band calculation. The temperature dependence of the Fermi energy, \( E_{F} \), is neglected by fixing it at a value determined at a low temperature. It is estimated using the occupation number of the renormalized band (RNB) calculation, in which the self-energy is approximated by an expansion form up to the linear term in the energy variable at \( E_{F} \) (see Appendix A for the calculation of the total occupation number). The target \( 4f \) electron number \( n_{f}(\text{rsl.target}) \) is the occupation number calculated directly using the resolvents to stabilize the self-consistency iterations. The occupation number obtained by the integration of the \( 4f \) DOS, \( n_{f}(\text{intg.}) \), has a deviation within 1.0\% from \( n_{f}(\text{rsl.target}) \) because many intermediate calculation process are included.

3. CePd\(_{3}\)

3.1 Density of states

In Fig. 1, we show the DOS of the \( 4f \) component (\( 4f \) DOS) for CePd\(_{3}\) at \( T = 37.5 \) K. The solid line shows the total \( 4f \) component of the PES (\( 4f \) PES). The dashed line is the DOS of the \( \Gamma_{7} \) component and the dot-dash line is the DOS of the \( \Gamma_{8} \) component. The two-dots-dash line is the DOS of the \( j = 7/2 \) component. The CFS of the self-energy in the excited \( j = 7/2 \) multiplet is neglected. The vertical dot-dash line indicates the Fermi energy \( E_{F} = 0.6143 \) Ry. The inset shows spectra in the energy region near \( E_{F} \). The \( 4f \) DOS has a large peak at 0.642 Ry, about 0.028 Ry (0.38 eV) above \( E_{F} \). This has mainly the \( j = 7/2 \) character. The spin-orbit splitting on the IPES side is usually enhanced in Ce systems. The spectral intensity on the PES side consists mostly of the \( \Gamma_{8} \) component. The PES has a sharp peak at \( E_{F} \) with a steep tail up to 0.094 Ry (1.3 eV) below \( E_{F} \). It also has a long tail with small structures reflecting the DOS of \( 4d \) states of Pd on the high-binding-energy side. In the steep tail region, shoulders appear at binding energies of 0.004 Ry (0.05 eV) and 0.024 Ry (0.33 eV). These may correspond, respectively, to the CFS and the SOI side band. The sharp peak at \( E_{F} \) with the steep and the long tail has been observed in high-resolution experiments by Kasai et al.\(^30\) The shoulder due to the SOI seems to be
Table I. Various quantities obtained in the DMFT calculation for CePd$_3$ at $T = 18.75$ K. $n_f^{(\text{imp})}$ is the DMFT occupation number in the auxiliary impurity problem of the effective HI of the DMFT calculation. $\varepsilon_f^\text{LDA}$ is the energy level in Ry. $\rho_f'(E_f)$ is the partial DOS at $E_f$ in Ry$^{-1}$. $Z_f^{-1}$ is the mass renormalization factor of the 4$f$ band. $\varepsilon_f$ is the effective energy of the renormalized band in Ry, and $\Gamma_f$ is the imaginary part of the self-energy in Ry. The effective energy levels are measured from the Fermi energy $E_F = 0.6143$ Ry. The lattice constant is $a = 7.80079$ a.u., and the spin-orbit interaction constant is $\xi_{4f} = 7.2302 \times 10^{-3}$ Ry. The 4$f$ level in the band calculation is $\varepsilon_f^{4f\text{LDA}} = 0.6754$ Ry. The electrostatic CFS of $(j = 5/2)$ for the $\Gamma_7$ is set to be $\Delta E_{\Gamma_7} = 27$ meV (310 K). The ratio does not change so greatly even when the temperature is raised: 0.089/0.723 (0.12) at 150 K and 0.108/0.707 (0.15) at 300 K. Moreover, the occupation of the $j = 7/2$ component, 0.185, is not small, and is almost independent of $T$.

The occupation number of the 4$f$ state, $n_f$, has been tentatively chosen to be 0.98 in the present calculation. This value is small compared with the 4$f$ occupation number, 1.045, of the LDA band calculation for CePd$_3$. When we carry out the LDA calculation for compounds in which Ce ions are replaced by La ions, the occupation number on the 4$f$(La) state usually amounts to 0.1. We use the occupation number as reduced to 94% of the LDA value as the atomic occupation number on the Ce 4$f$ state. Kanai et al. concluded that 4$f$ occupancy in CePd$_3$ is expected to be 0.92 on the basis of the results of resonant inverse photoemission (RIPES) experiments. The DMFT band calculation by setting $n_f^\text{LDA}$ to be 0.92, $T_K$ determined from the magnetic excitation spectrum is expected to be 500 K. On the other hand, $T_K$ is greatly reduced to about 10 K when we choose 1.05.

In Fig. 2, we show the trial HI$^{44}$ obtained in the DMFT band calculation at 37.5 K. It has very large peaks corresponding to the 4$d$ band of Pd, but these peaks are located in the energy region deep below $E_F$. The HI is not high in the energy region near $E_F$. In particular, the HI of the $\Gamma_7$ component is low though it is high in the deeper energy region. The overall features of the HI in the DMFT band are similar to those calculated directly using the LDA band, but the HI in DMFT is increased in the vicinity of $E_F$ to about twice the LDA value for the $\Gamma_7$ and $\Gamma_8$ components, and is decreased in the deep energy region. Moreover, the DMFT calculation causes fine structures of the HI in the vicinity of $E_F$, which are shown in the inset of the figure. This contrasts with the HI in the LDA, which has a weak energy dependence in this region. The HI of the $\Gamma_8$ component in DMFT has a small peak at $E_F$, while those of the $\Gamma_7$ and $j = 7/2$ components have small peaks above or on both sides of $E_F$. The reason for this different behavior is not clear at present, but we note that the peaks of the 4$f$ DOS for the latter two cases are located above

Identified.

The Kondo temperature $T_K$ and the CFS excitation energy are, respectively, estimated to be about 27 meV (310 K) and 41 meV (480 K) from magnetic excitation spectra, as will be shown in Fig. 4 in the next subsection. We note that spectra do not show appreciable change even when the temperature is decreased to 18.75 K in the calculation.

The parameters and the calculated values are given in Table I. The LMTO band parameters for states except for the $f$ component are fixed to those of the LDA calculation. $E_F$ is fixed to the value determined by the occupied state in the RNB, as is discussed in a later section. The relative occupation number of the $\Gamma_7$ component to the $\Gamma_8$, 0.084/0.719 (0.12) is small compared with the 0.5 expected from the ratio of the degeneracy, but is very large compared with the value expected from the simple model of the CFS for an isolated ion with $\varepsilon_{\Gamma_7} - \varepsilon_{\Gamma_8} = 0.00155$ Ry (240 K). The ratio does not change so greatly even when the temperature is raised: 0.089/0.723 (0.12) at 150 K and 0.108/0.707 (0.15) at 300 K. Moreover, the occupation of the $j = 7/2$ component, 0.185, is not small, and is almost independent of $T$.

A simple picture of the CFS for an isolated ion cannot be applied.

Usually, the electrostatic potential causes cubic CFS in 4$f$ electron systems with a higher energy level of about 150 K for the $\Gamma_7$ state.\textsuperscript{48,49} This is included in the present calculation. Even when it is neglected, the 4$f$ DOS and the magnetic excitation spectrum are not greatly changed because the hybridization effect causes large CFS, of greater than 300 K.\textsuperscript{50}
3.2 Magnetic excitation

When we do a calculation in the fictitious case that $\Gamma_7$ is mainly occupied by assigning a low energy level to it, the HI of $\Gamma_7$ has a small peak at $E_F$. However, this result should not be used as a general rule, because the modification of HI in DMFT is delicately dependent on details of the band structures. The Kondo temperature is increased in the DMFT calculation in the CePd$_3$ case. We have obtained the Kondo temperature of 10 K in the single impurity calculation using the HI of the LDA band.

The total DOS at $T = 37.5$ K is shown by the dashed line in Fig. 3. The large peaks at about 0.3 and 0.45 Ry have the 4$d$ character of Pd, and that at 0.8 Ry has the 5$d$ character of Ce. These peaks are also obtained by the LDA calculation.\textsuperscript{55} The sharp 4$f$ peaks slightly above $E_F$, which are called the $f^1$ peak in IPES, are also obtained in the LDA calculation.\textsuperscript{42} Their intensity in DMFT is reduced compared with that in the LDA because a part of it is transferred to the intensity of the $f^2$ peak. In the present calculation, the width of the $f^2$ peak is not large because the multiplet splitting of the $f^2$ final state is neglected.

In the analysis of RIPES experiments, the ratio of the $f^1$ peak to the total RIPES intensity has been given as 0.22,\textsuperscript{27} whereas it is estimated to be about 0.2 in the present calculation. The present DMFT calculation seems to give results that emphasizes the hybridization effects strongly (i. e. the higher $T_K$).

3.2 Magnetic excitation

![Fig. 4. Magnetic excitation spectrum of CePd$_3$. The solid line shows the spectrum at $T = 37.5$ K, the dot-dash line is the spectrum at $T = 150$ K, and the two-dots-dash line is the spectrum at $T = 300$ K. The dashed line is the spectrum in a fictitious case where matrix elements of the magnetic moment are restricted within the intra-\Gamma$_8$ manifold of space.](image)

In Fig. 4 we show the magnetic excitation spectrum. The total magnetic excitation spectrum at $T = 37.5$ K is shown by the solid line. It has a peak at about $E = 0.003$ Ry (41 meV). The dashed line depicts the spectrum for a fictitious case in which the matrix elements of the magnetic moment are nonzero only in the manifold of $\Gamma_8$, and thus it may correspond to the excitation spectrum within the $\Gamma_8$ manifold. It has a peak at about $E = 0.002$ Ry (27 meV), and leads to a faint shoulder in the solid line. The CFS excitation energy seems to be slightly larger than $T_K$ in the present calculation. The calculated magnetic susceptibility is $1.9 \times 10^{-3}$ emu/mol, whereas the experimental value is $1.5 \times 10^{-3}$ emu/mol at $T \sim 0$ K.\textsuperscript{56,57} We show magnetic excitation spectra at $T = 150$ K and at $T = 300$ K by the dot-dash line and the two-dots-dash line, respectively. The spectrum at $T = 300$ K has a broad peak centered at about 0.003 Ry (41 meV). The shoulder shifts to $E \sim 0$ and becomes a peak. The overall features do not change so greatly when we neglect the electrostatic CFS of 150 K. The magnetic excitation spectrum observed in the wave-vector-integrated case has a peak at about 55 meV at $T = 10$ K, and the peak shifts to the low-energy side as $T$ increases.\textsuperscript{21} The calculated results seem to be generally consistent with those of the experiment. However, the peak at $E \sim 0$ is higher at $T = 300$ K in the experiments.\textsuperscript{21} The present DMFT calculation seems to give stronger HI compared with the value in the experiments. The detailed calculation of physical quantities using finely tuned parameters will be given in the future.

In ref. 22, excitation peaks with an energy of 15 meV and less than 3 meV are indicated. Low-energy peaks are not expected in the present calculation of the wave-vector dependence of the magnetic excitation spectra because $T_K$ is not low. One possibility of the origin of the low-energy peaks may be the wave vector dependence of the magnetic excitation spectra, as noted in ref. 23.

3.3 RNB, and wave number-vector-dependent DOS

In Fig. 5, we show the RNB dispersion at $T = 37.5$ K. The energy shift (the real part of the self-energy at $E_F$: $\Re \Sigma_F(E_F)$) and the mass renormalization factor $(1 - \partial \Re \Sigma_F(\varepsilon)/\partial \varepsilon|_{E_F})$, which are given in Table I, are taken into account in this calculation. Narrow bands with the $j = 5/2$ character appear slightly above $E_F$, and those with the character of $j = 7/2$ appear around the energy 0.645 Ry which is near the energy of the $j = 7/2$ peak in the 4$f$ DOS shown in Fig. 1.

The lowest 4$f$ band sinks to below $E_F$ near the $\Gamma$ and R points, and is located above $E_F$ in the other regions. The dispersion of RNB corresponds well with the behavior of the $k$ -dependent density of states (k-DOS). For example, we show the k-DOS when $k$ moves from the $\Gamma$ (bottom) to the R (top) point along the A line in Fig. 6. A peak of the DOS with mainly the 4$f$ character is located below $E_F$ at the $\Gamma$ and R points. Starting from the peak below $E_F$ at the $\Gamma$ point, one of ridge lines runs above the Fermi energy across $E_F$, and then connects to the peak below $E_F$ at the R point. Another runs to the low-energy side of $E_F$ up to 0.611 Ry at the halfway, and then turns back to connect to the peak below $E_F$ at the R point. This “hanging” branch does not cross the Fermi energy.

Note that we have depicted the total spectral intensity, not the $f$-component, in Fig. 6. In the energy region...
shown in the figure, the spectral intensity has mainly the 4f character. On the other hand, very sharp spectral peaks of a non-4f character appears in the energy region outside of the figure. The hanging band on the Λ axis has a stronger non-4f character. This branch is a hybridization band between the 4f of Ce and a conduction band that has the character of the sp-free electron band and the 5d of Ce.

The dispersion of the RNB is qualitatively similar to the result of the band structure determined by Hasegawa and Yanase by the LDA calculation, although the width of the 4f band with $j = 5/2$ is about 0.005 Ry in the RNB, whereas that of the LDA band is about 0.02 Ry. Both calculations give electron pockets at the Γ and R points, and hole pockets centered on the T axis.

The main features of the band dispersion near the Fermi energy are formed by the hybridization of the narrow 4f bands and the wide sp bands. The 4f bands have dispersion characterized by the LMTO (linear combination of atomic orbitals) tight-binding bands of the sc lattice. Although the 4f band width is reduced in DMFT, the qualitative features of the dispersion of the 4f-sp hybridized bands are not changed because the number of participating sp bands is small and their dispersion is very rapid compared with that of 4f bands.

Here, we must note a weak point of the NCA$f^2$vc method, that is it does not automatically ensure the Fermi liquid sum rule, i.e., the integral of the total DOS below $E_F$, $N(\text{total}; \rho) = \int d\varepsilon \frac{1}{\pi} \sum_{k} \left( -\frac{1}{\pi} \Im G(\varepsilon + i\delta; k) \right) f(\varepsilon)$, is not equal to the occupation number of electrons calculated by the volume of the occupied states in the $\mathbf{k}$ space. The quantity $N(\text{total}; \rho)$ is a smaller value in CePd$_3$, and thus we obtain a higher Fermi energy. If we use it in the RNB calculation, the volume of the occupied states in the $\mathbf{k}$ space becomes large. In the case of CePd$_3$, which has an even total electron number, the balance between the electron and hole states is lost. In this study, we tentatively use the Fermi energy determined using the quantity $N(\text{total}; \text{RNB}) = \sum_{\mathbf{k}} f(E_{\text{RNB}}(\mathbf{k}))$ to ensure the electron-hole balance in the RNB band, where $E_{\text{RNB}}(\mathbf{k})$ is the energy obtained by the RNB calculation. (For more details, see Appendix B.)

When we calculate the RNB dispersion at $T = 150$ K, the 4f band shifts slightly to the high-energy side. The 4f state at the Γ point nears $E_F$, and the 4f state at...
the R point is located on $E_F$. Therefore, a part of the hanging band on the Λ line rises above $E_F$. Hole pockets on the T axis grow into larger hole regions around the R point. As the temperature increases further, the 4f state at the R point shifts up to above $E_F$, and a large hole surface enclosing the R point appears. In other words, we have a connected electron Fermi surface (FS) that contains the X and M points inside it.

The primary structure of the FS of the RNB at high temperatures is similar to that of the FS of the LDA band of LaPd$_3$, but their fine topologies will differ. At $T = 150$ K, the main part of the hanging band on the Λ line is still located below $E_F$. The hole sheets around the Γ and R points are separated by an electron region on the Λ axis. On the other hand, this entire hanging branch is located above $E_F$ in LaPd$_3$, and thus the separation by the electron region does not occur.

We should, of course, note that the RNB picture has only limited meaning at high temperatures because the imaginary part is large. The k-DOS at $T = 150$ K for $k$ along the Λ line is shown in Fig. 7. The widths of peaks become large, but we can see that the peak at the R point is located almost at $E_F$, and the peak at Γ nears $E_F$. The trace of broad peaks shows a shift corresponding to that of the RNB dispersion. We can recognize that the ridge of the peaks of the hanging band crosses the Fermi energy, as noted in the previous paragraph. At $T = 300$ K, the width of peaks becomes so large that it surpasses the fine $k$ dependence of the spectra. However, the 4f peak near the Fermi energy still exists in the 4f DOS calculation, similar to that shown in Figs. 1 and 3. It moves slightly to the high-energy side with increasing width at 300 K.

The present calculation has been performed with fixed $E_F$ and $n_f$(rls.target). The calculated total occupation number in the DMFT band increases to $N$(total; RNB) = 34.304 at $T = 300$ K, although the energies of the 4f bands shift upward. We point out that even when we move $E_F$, the energy of 4f bands relative to the Fermi energy will not change greatly because the Kondo resonance peak usually shifts following the change of the Fermi energy.

Recently, the wave vector dependence of the PES has been observed under the $4d \rightarrow 4f$ resonant condition. The experiment was carried out for the (111) surface of a thin film by sweeping the $k$ vector along Γ-K-M-K-Γ in the surface Brillouin zone (BZ). This may correspond to the following sweeping of $k$ in the 3-dimensional sc BZ: the component parallel to the surface runs as Γ-(Σ)-M-(Σ)-Γ with averaging over its components normal to the surface. This means that the sweeping of the parallel component R-(S)-X-(S)-R is also included.

In Fig. 8, we show the PES when the representative wave vector moves from the Γ (bottom) point to the M (top) point with average intensities for $k$ normal to the surface. The total intensity is plotted in the figure, but the intensity above the energy of 0.612 Ry has mainly the 4f character. The spectra have a peak for $k$ near Γ, and this corresponds well to the experimental results. The intensity below the energy of 0.611 Ry has a non4f character. As noted previously, the contribution from the R point is also included at the representative Γ point. At these points, the 4f component is located below $E_F$ at $T = 37.5$ K. In Fig. 9, we show spectra at $T = 150$ K. The peak of the intensity at Γ nears $E_F$ because of the shifting up of the 4f band. The thermal distribution effect and the very high intensity of the 4f DOS above $E_F$ also have affect this spectral shape. The change of the peak can be checked in experiments.

4. CeRh$_3$

In Fig. 10, we show the 4f DOS of CeRh$_3$ at $10^3$ K. The 4f spectrum has a sharp and large peak at an energy about of 0.76 Ry, which is 0.066 Ry (0.9 eV) above the Fermi energy. This separation of energy from $E_F$ is slightly smaller than that of the 4f band in the LDA, about 0.009 Ry (1.2 eV). The result seems to be consistent with those of RIPES experiments and their detailed analysis. The PES has a relatively large peak at
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Fig. 10. 4f DOS of CeRh$_3$ at $T = 10^3$ K. For the definition of lines, see the caption of Fig. 1. $E_F = 0.6940$ Ry is indicated by the vertical dot-dash line.

Fig. 11. Total DOS (dashed line) and 4f DOS (solid line) of CeRh$_3$ at $T = 10^3$ K. See the caption of Fig. 3.

about 0.1 Ry (1.4 eV) below $E_F$ and a peak at $E_F$. The qualitative behavior of the present PES is similar to the result obtained by Harima in the LDA. However, the binding energy of the peak at 0.1 Ry below $E_F$ is slightly lower and the intensity is higher than that of the LDA. The peak at $E_F$ is also slightly sharper. In the experiment, the sharp peak at $E_F$ was observed, but the peak at 0.1 Ry below $E_F$ has not been identified at present.

In Fig. 11, we show the total and 4f DOS of CeRh$_3$ at $T = 10^3$ K. The 4d band of Rh exhibits strong peaks of the DOS at about 0.4 and 0.6 Ry. The 4f component also has a small peak at about 0.6 Ry, as noted previously. The Fermi energy is located in the top region of the 4d band, and the hybridization intensity in this region is high. Dispersions of the RNB are almost identical to those of the LDA in the energy region very near $E_F$, but the width of the 4f band, which is located at 0.76 R, is about 2/3 that of the LDA. A $f^2$ satellite peak appears in DMFT at about 1.2 Ry on the high-energy side. The ratio of the intensity of peak at 0.76 Ry ($f^1$ peak) to the total IPES intensity is estimated to be about 0.4 in the present calculation, while a slightly larger value, 0.6, was obtained in the experiment. Uozumi et al. predicted the 4f occupation number to be 0.86, but we tentatively used $n_f$(rsl.target) = 0.94, which is 94% of the LDA value. The difference between these values is not small, but will not cause extreme differences in the calculation of the very strong hybridization limit. The mass enhancement factor is expected to be about 2, as given in Table II.

In Fig. 12, we show the PES for the (111) surface when the representative wave vector moves from the $\Gamma$ point (bottom) to the M point (top). In the energy region between $E_F$ and 0.6 Ry, the relative intensity of the 4f component is about 10% of the total. The spectra have fine peaks as if some flat bands exist slightly below $E_F$. However, we note that no flat bands exist very near $E_F$ in the RNB dispersion. Let us denote as $k^*$ the representative $k$ corresponding to the second solid line from the top (i.e., the $k$ point on the $\Sigma$ axis at a distance of about 0.3 x length of the $\Sigma$ axis from the M point). The spectral intensity slightly below $E_F$ is relatively large for $k$ near $k^*$. For the wave vectors near $k^*$, several bands stay...
Table II. Various quantities obtained in the DMFT calculation for CeRh$_3$ at $T = 10^3$ K. For the definition of notation, see the caption of Table I. The Fermi energy is $E_F = 0.6940$ Ry and $\Delta E_F = 0$ K. The lattice constant is $a = 7.6005$ a.u., and the spin-orbit interaction constant is $\xi_{ff} = 7.3374 \times 10^{-3}$ Ry. The 4$f$ level in the band calculation is $\xi_{4f} = 0.7639$ Ry. The target 4$f$ occupation number is $n_f$ (sl.target) = 0.94, and the resultant occupation number calculated using the resolvent is 0.940. The 4$f$ electron number calculated by the integrating the spectra is $n_f$ (intg) = 0.943, and the obtained total band electron number is $N$(total; RNBP) = 30.094, $n_f$(Ce, LDA) = 0.996.

| $n_f$ (intg.) | $\xi_{ff}$ (Ry) | $\mu_f$ ($E_F$) (Ry$^{-1}$) | $Z_f^{-1}$ | $\Gamma_f$ (Ry) | $\Gamma_f$ (Ry) | $E_{\text{melt}}$ |
|---------------|-----------------|---------------------------|-----------|----------------|----------------|--------------|
| 0.178         | -0.26923        | 0.27                      | 2.2       | 0.1218         | 4.6 $\times 10^{-3}$ |
| 0.319         | -0.26921        | 1.40                      | 1.9       | 0.1216         | 12.6 $\times 10^{-3}$ |
| 0.446         | -0.24426        | 0.99                      | 1.9       | 0.1646         | 5.8 $\times 10^{-3}$ |
| 0.46          |                 |                           |          |                |                 |

slightly below $E_F$ around the M and X points when the normal components are varied. Calculated results show quantitatively similar behaviors to experimental results, but the careful separation of the surface and bulk components is necessary to enable a detailed comparison. The magnetic excitation spectra are shown in Fig. 13. They have a steep increase at the excitation energy of about 0.03 Ry. This energy may correspond to the energy from $E_F$ to the low-energy edge of the peak at 0.76 Ry of the 4$f$ DOS. The low-energy end of the spectrum mainly originates from the excitation within the $j = 5/2$ components, but contribution of the $j = 7/2$ components is not small even in the low-excitation-energy region. The $j = 7/2$ components also join the Kondo effect. We may expect the Kondo temperature of this system to be about 0.03 Ry (4700 K). The calculated value of the magnetic susceptibility is $0.7 \times 10^{-3}$ emu/mol. The experimental magnetic susceptibility ($\chi \sim 4 \times 10^{-4}$ emu/mol)$^{[37]}$ may indicate a high $T_K$ of several thousand K, but the calculated value of 4700 K seems to be too high. DOSs, both of the total and of the 4$f$ component, do not have any gap, as seen in Fig. 10, but the magnetic excitation spectrum has a shape indicating the existence of a pseudogap. We note that, in Fig. 13, the excitation spectra of only intra-4$f$ components are shown. A broad continuous 5$d$ component will be superposed on these spectra.

The HI of CeRh$_3$ in the LDA calculation has a spectrum shape similar to that of the partial DOS on the 5$d$ state of Rh, i.e., the spectral shape given by subtracting the 4$f$ and 5$d$ parts of the Ce ion from the total DOS in Fig. 11. The HI in DMFT is almost equal to that of the LDA in the high-energy region, but it has a steep dip at an energy slightly above $E_F$. Similar behavior has been seen in the HI of the $j = 7/2$ component of CePd$_3$ which has shown in Fig. 2.

5. Summary and Discussion

We have studied the electronic structures of CePd$_3$ and CeRh$_3$ on the basis of the DMFT calculation. The auxiliary impurity problem was solved by a method named NCA$^2$vc, which includes the correct exchange process of the $f^1 \rightarrow f^0$ and $f^1 \rightarrow f^2$ virtual excitation. The splitting of the self-energy owing to the SOI and CFS effects was also considered.

The DMFT band calculation gives Fermi surface structures similar to those obtained by the LDA calculation in CePd$_3$ at very low temperatures. Electron pockets appear at the bottom of the 4$f$ band at the $\Gamma$ and $R$ points. Hole pockets appear that are centered on the $T$ symmetry axis.

The 4$f$ band shifts to the high-energy side relative to the Fermi energy as the temperature increases. At the same time, the lifetime width of the 4$f$ states increases. The band structures produced by the band overlap between the 4$f$ and non-4$f$ components shift up to the high energy side of the Fermi energy. Therefore the primary structures of the band in the vicinity of the Fermi energy approaches to those of the sp free-electron-like band of LaPd$_3$. However, some characteristic features of the LDA band of CePd$_3$ remain at higher temperatures. For example, a region that electrons occupy will appear on the $\Lambda$ axis in CePd$_3$ at $T \sim 150$ K, which is not expected in LaPd$_3$. In CePd$_3$, the lifetime broadening overcomes the $k$ dependence of the 4$f$ spectrum at room temperature, thus, the 4$f$ state becomes a broad dispersive state located above $E_F$.

The ARPES of CePd$_3$ shows strong intensity near the representative $\Gamma$ point at low temperatures because the 4$f$ band is located below the Fermi energy at the $\Gamma$ and $R$ points. This result seems to be consistent with the recent experimental results. The observed 4$f$ component will be greatly reduced at room temperature because of the shift of the 4$f$ band to the high-energy side.

The $k$-integrated magnetic excitation spectrum has a peak at 41 meV and a faint shoulder structure at about 27 meV at low temperatures. The temperature dependence of the excitation spectrum generally seems to be consistent with the results of experiments. The magnetic CFS excitation energy is estimated to be about 41 meV, while the Kondo temperature is slightly smaller, 320 K ($27$ meV).

The calculated PES shows good correspondence with the bulk component obtained in recent high-resolution experiments. The intensity ratio of the $f^1$ peak of the IPES to the total IPES is estimated to be about 0.2, while it was predicted to be 0.22 by the recent experiment analysis. The HI is enhanced near the Fermi energy in the DMFT band compared with that of the LDA calculation. The present DMFT calculation seems to give electronic structures with slightly stronger HI than that expected from experiments for CePd$_3$.

The DMFT calculation for CeRh$_3$ gives an almost identical band structure to that obtained by the LDA calculation. However, the energy of the $f^3$ peak of the IPES in the former is slightly lower than that in the latter. In addition, the $f^2$ satellite peak appears in the DMFT band calculation. The calculated intensity ratio of the $f^3$ peak to the total IPES, 0.4, is comparable to, but smaller than the experimental value of 0.6. The calculated PES has a sharp peak at the Fermi energy. A peak reflecting the 4$d$(Rh) DOS also appears, simi-
larly to the result of the LDA calculation. The former has been observed, but the latter has not been identified in experiments.\(^{(35,36)}\)

The Fermi energy is located in the energy region of the 4\(d\) band of Rh in CeRh\(_3\), therefore the HI near the Fermi energy is strong, about three times greater than that of CePd\(_3\). The 4\(f\) band width of CeRh\(_3\) is about twice that of CePd\(_3\) in the LDA calculation. In the DMFT calculation, the characteristic energy scales are drastically different from each other, 300 and 5000 K.

In CeRh\(_3\), the ARPES shows an appreciable DOS slightly density below the Fermi energy and is weakly dependent on the wave vector, although no flat 4\(f\) bands exist near the Fermi energy in the RNB dispersion. The spectra have stronger intensity halfway along the \(\Sigma\) axis from the \(\Gamma\) to \(M\) points of the Brillouin zone. This seems to be similar to the results of experiments, but further studies to separate the surface effects are necessary.\(^{(63)}\)

General features of the experimental results for CePd\(_3\) and CeRh\(_3\) are reproduced by the DMFT band calculation with the LMTO+NCA\(^2\)vc scheme. However, the present DMFT calculation gives a higher Kondo temperature than that in the results of the detailed analysis of experiments. Moreover, there is some arbitrariness in the choice of the target value of the 4\(f\) occupation number, \(n_f(rsl\text{.target})\). According to calculations for AuCu\(_3\)-type Ce compounds, accurate calculated results seem to be obtained when a 4\(f\) occupation number between 90% and 95% of the LDA value is used. We have used 94% in the present calculation, and we obtained 310 K (27 meV) for \(T_K\) of CePd\(_3\) \((n_f(rsl\text{.target}) = 0.98)\) and 4700 K (0.03 Ry) for \(T_K\) of CeRh\(_3\) \((n_f(rsl\text{.target}) = 0.94)\) from the magnetic excitation spectrum. When we perform calculations using 90%, the \(T_K\) are 400 K for CePd\(_3\) \((n_f(rsl\text{.target}) = 0.94)\) and 6300 K for CeRh\(_3\) \((n_f(rsl\text{.target}) = 0.90)\). These values are not greatly different from the previous values because these compounds belong to a group of materials having high \(T_K\). For materials with lower \(T_K\), \(T_K\) drastically depends on the choice of \(n_f(rsl\text{.target})\). Careful treatment of the target value is necessary in such cases.

The effectiveness and some of the weaknesses (for example, the correct calculation of the occupied electron number) of the present DMFT scheme are recognized. Calculations of the 4\(f\) band state in CeIn\(_3\) and CeSn\(_3\), and also various Ce compounds will be carried out in the near future.

At the end of this paper, we refer the very early and recent application of methods similar to NCA\(^2\)vc to the DMFT band calculation. Lægsgaard and Svane calculated the band structure of Ce pnictides in 1998.\(^{(64)}\) Recently, Haule \textit{et al.} studied the \(\alpha \rightarrow \gamma\) transition of Ce metal.\(^{(65)}\) and Shim \textit{et al.} studied the electronic band structure of CeIrIn\(_5\).\(^{(66)}\) The CFS of the self-energy was not considered in those studies.

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Appendix A: Occupation number

The calculation based on the NCA $f^2\nu c$ method usually does not satisfy the FL sum rule, i.e., the integral of the total DOS below the Fermi energy, $N(\text{total}; \rho) = \int d\epsilon \sum_{\Lambda} \langle \epsilon \mid \sum_{\Lambda} (-\frac{1}{\omega} \Delta G(\epsilon + i\delta; k)) f(\epsilon) \rangle$, is not equal to the value $N(\text{total}; \ln G) = \int d\epsilon \sum_{\Lambda} (-\frac{1}{\omega} \text{ln det } G(\epsilon + i\delta; k))(-\frac{\partial f(\epsilon)}{\partial \epsilon})$, where $f(\epsilon)$ is the Fermi distribution function at $T = 0$. In the RNB calculation, in which the self-energy term is approximated by $\Sigma(\epsilon) \approx \text{Re } \Sigma(\epsilon) + \frac{\partial \Sigma(\epsilon)}{\partial \epsilon}(\epsilon - \epsilon_F)$, we obtain real eigen energies $E_{\text{RNB}}(\lambda k)$ where $\lambda$ is the band index. The number of occupied states in the RNB band, $N(\text{total}; \text{RNB}) = \sum_{\lambda} \rho(E_{\text{RNB}}(\lambda k))$, is expected to agree with $N(\text{total}; \ln G) = 0$ if the imaginary part can be neglected near the Fermi energy. However, the imaginary component has a considerable magnitude in NCA $f^2\nu c$ even at very low temperatures; this is partly because $T$ must maintain the condition $T > 0.1 T_K$. For example, $N(\text{total}; \rho)$, $N(\text{total}; \ln G)$, and $N(\text{total}; \text{RNB})$ are 33.59, 34.16, and 34.01, respectively, at $T = 18.75$ K in CePd$_4$. The difference between these values becomes serious when the detailed structure of the Fermi surface is discussed. In this study, we use $N(\text{total}; \text{RNB})$ to determine the Fermi energy $E_F$ at $T = 0$ because this quantity is directly related to the occupation number of electrons calculated from the volume of the occupied states in the $k$ space.

Appendix B: Cauchy integral of using spline interpolation

The Cauchy integral appears in various places in the DMFT calculation. Therefore, an efficient and accurate numerical calculation of the Cauchy integral is needed. We briefly explain a method of using the spline interpolation for the DOS. Let us assume that numerical data of DOS $\{y_i\}$ at energy points $\{x_i\}$ are given. In the cubic spline interpolation, the DOS in the interval $[x_i, x_{i+1}]$ is expressed as

$$\rho_{3,i}(x) = \frac{1}{6h_i} \left\{ (x_{i+1} - x)^3 M_i + (x - x_i)^3 M_{i+1} \right\}$$

$$+ (y_i - \frac{h_i^2 M_i}{6}) \frac{x_{i+1} - x}{h_i} + (y_{i+1} - \frac{h_i^2 M_{i+1}}{6}) \frac{x - x_i}{h_i},$$

where $h_i = x_{i+1} - x_i$. The quantity $M_i$ is the second derivative of the DOS at $x = x_i$, and is given by the usual procedure of the spline interpolation.

The integral of the interval $[x_i, x_{i+1}]$ is calculated as

$$\int_{x_i}^{x_{i+1}} \frac{\rho_{3,i}(x)}{z - x} dx = \frac{(x_{i+1} - z)^3 M_i}{6} + \frac{(x - x_i)^3 M_{i+1}}{6}$$

$$+ (x_{i+1} - z) \frac{M_i h_i}{12} - (x - x_i) \frac{M_{i+1} h_i}{12}$$

$$+ (y_i - \frac{h_i^2 M_i}{9}) - (y_{i+1} - \frac{h_i^2 M_{i+1}}{9})$$

$$- \rho_{3,i}(z) \ln \frac{z - x_{i+1}}{z - x_i}.$$  

The total integral is given by summing the contribution from each interval. Equation (B-2) is expressed as a combination of powers of quantities $(x_{i+1} - z)$ and $(z - x_i)$; therefore it is applicable for $z$ when $|z - x_{i+1}|$ is not much larger than $h_i$. When $z$ approaches the edge of the integration, i.e., $z \to x_i$ or $x_{i+1}$, the singularity of the logarithm term is removed by the counter contribution of the neighboring $[x_{i-1}, x_i]$ or $[x_{i+1}, x_{i+2}]$ region. The round-off error due to the subtraction of logarithm terms of neighboring regions is not serious even when $z$ is extremely near the edge point, because the divergence of the logarithm is very weak. The integral (B-2) is expected to be $O(z - x_{i+1})^{-1}$ when $|z - x_{i+1}| \gg h_i$. Therefore, mutual cancellation occurs among terms in (B-2) in this limit. We find that the integral is re-expressed by a compact form in this case

$$\int_{x_i}^{x_{i+1}} \frac{\rho_{3,i}(x)}{z - x} dx$$

$$= - \sum_{\nu=1}^{\infty} \left( \frac{h_i}{x_{i+1} - z} \right)^\nu \left( \frac{y_i}{\nu + 1} - \frac{h_i^2 M_i}{3(\nu + 3)(\nu + 1)} \right)$$

$$- \left( \frac{h_i}{z - x_i} \right)^\nu \left( \frac{y_{i+1}}{\nu + 1} - \frac{h_i^2 M_{i+1}}{3(\nu + 3)(\nu + 1)} \right).$$

This equation gives a highly accurate estimation of the integral even when terms are truncated up to $\nu \sim 10$.

In most cases, it is convenient to use the linear interpolation scheme with a very fine mesh for the DOS,

$$\rho_{1,i}(x) = m_i (x - \frac{x_i + x_{i+1}}{2}) + \frac{y_i + y_{i+1}}{2},$$

where $m_i = \frac{y_{i+1} - y_i}{h_i}$. The Cauchy integral is expressed by the following forms

$$\int_{x_i}^{x_{i+1}} \frac{\rho_{1,i}(x)}{z - x} dx = - m_i h_i - \rho_{1,i}(z) \ln \frac{z - x_{i+1}}{z - x_i},$$

and for $|z - x_{i+1}| \gg h_i$,

$$\int_{x_i}^{x_{i+1}} \frac{\rho_{1,i}(x)}{z - x} dx = - \frac{1}{2} \sum_{\nu=1}^{\infty} \left( \frac{h_i}{x_{i+1} - z} \right)^\nu \left( \frac{y_{i+1} - m_i h_i}{\nu + 1} \right)$$

$$- \left( \frac{h_i}{z - x_i} \right)^\nu \left( \frac{y_i}{\nu + 1} + \frac{m_i h_i}{\nu + 1} \right).$$

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46) The impurity 4f Green function is given using the 4f DOS \( \rho_{\text{imp}}^{(\text{h})}(\varepsilon) \) as the Cauchy integral: 
\[
\rho_{\text{imp}}^{(\text{h})}(\varepsilon) = \frac{1}{\pi} \int d\varepsilon' \frac{\rho_{\text{imp}}^{(\text{h})}(\varepsilon')}{\varepsilon - \varepsilon'}
\]
It is also expressed as \( \rho_{\text{imp}}^{(\text{h})}(\varepsilon) = 1/(\varepsilon - \varepsilon_a - \Sigma_{\text{imp}}^{(\text{h})}(\varepsilon) - \Sigma^{(\text{h})}(\varepsilon)) \), where \( \varepsilon_a \) is the impurity level, and \( \Sigma_{\text{imp}}^{(\text{h})}(\varepsilon) \) and \( \Sigma^{(\text{h})}(\varepsilon) \) are, respectively, the self-energies due to the Coulomb interaction and the hybridization. The self-energy \( \Sigma_{\text{imp}}^{(\text{h})}(\varepsilon) \) is calculated as \( \Sigma_{\text{imp}}^{(\text{h})}(\varepsilon) = -\Sigma_{\text{imp}}^{(\text{h})}(\varepsilon) - \Sigma^{(\text{h})}(\varepsilon) \) by solving the impurity model with the HI shown in Fig. 2 is defined as \( \Gamma = \Sigma_{\text{imp}}^{(\text{h})}(\varepsilon + i\delta) \). The self-consistent condition on the 4f DOS in DMFT is \( \rho_{\text{imp}}^{(\text{h})}(\varepsilon) = \rho_{\text{imp}}^{(\text{h})}(\varepsilon) \). Under the self-consistent condition, the self-energy due to the hybridization is given by \( \Sigma_{\text{imp}}^{(\text{h})}(\varepsilon) = -\Sigma_{\text{imp}}^{(\text{h})}(\varepsilon) - \Sigma^{(\text{h})}(\varepsilon) \).
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50) When the CFS of the electrostatic term is not included, a large number of iteration loops are needed to reach a solution for large CFS because the calculation starts with a condition on the 4f occupation in LaPd\(_3\) with the lattice constant of CePd\(_3\) as \( 0.1 \). This large occupation is induced through the strong hybridization between the 4f state and the 4d(Pd) state located in the energy region deep below \( E_F \). Some parts of the 4f occupation in LaPd\(_3\) may be ascribed to the tail of the 4d(Pd) extending to the Ce region.
51) Kasai et al. predicted the 4f occupation number to be 0.79 from their bulk PES analysis. However, they analyzed the PES using the NCA calculation and obtained a small occupation number because the \( j^2 \) configuration was neglected. For NCA, see, for example, refs. 11 and 53.
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60) The $4f$ DOS calculated by DMFT is shown in Fig. 1. The spectrum intensity in the deep region from $E_F$ ("the incoherent component" in the terminology of band theory and "the $f^0$ excitation part" in the terminology of the local Kondo problem) has a considerably high intensity. However, this deep energy component is disregarded in RNB, but its intensity is recovered as the full value of the intensity of the "coherent part" in the calculation of the occupied state. Only the band dispersion in the neighborhood of $E_F$ and the count of the occupied $k$ states at very low temperatures compared with the Kondo temperature have meaning in the RNB calculation.
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