An intelligent financial portfolio trading strategy using deep Q-learning

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Abstract

A goal of financial portfolio trading is maximizing the trader’s utility by allocating capital to assets in a portfolio in the investment horizon. Our study suggests an approach for deriving an intelligent portfolio trading strategy using deep Q-learning. In this approach, we introduce a Markov decision process model to enable an agent to learn about the financial environment and develop a deep neural network structure to approximate a Q-function. In addition, we devise three techniques to derive a trading strategy that chooses reasonable actions and is applicable to the real world. First, the action space of the learning agent is modeled as an intuitive set of trading directions that can be carried out for individual assets in the portfolio. Second, we introduce a mapping function that can replace an infeasible agent action in each state with a similar and valuable action to derive a reasonable trading strategy. Last, we introduce a method by which an agent simulates all feasible actions and learns about these experiences to utilize the training data efficiently. To validate our approach,

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we conduct backtests for two representative portfolios, and we find that the
intelligent strategy derived using our approach is superior to the benchmark
strategies.

**Keywords:** Portfolio trading, Reinforcement learning, Deep Q-learning,
Deep neural network, Markov decision process

1. Introduction

A goal of financial portfolio trading is maximizing the trader’s utility by
allocating capital to assets in a portfolio over the periods during the investment
horizon. Portfolio trading is also the process of determining capital allocations
across different financial assets according to time-varying market variables. A
portfolio trading strategy is needed to do portfolio trading, which can be derived
by optimizing an objective. Several objectives are commonly used for deriving
a trading strategy, such as expected returns and Sharpe ratio (i.e., risk-adjusted
returns). In addition, a trading strategy should satisfy a reasonable turnover
rate to be applicable in a real financial environment.

Traditionally, a portfolio trading strategy is derived using stochastic programming-
based models [12, 13, 23]. Because of the computational burden of applying such
models to complex financial environment, many heuristics methods have been
devised [6, 8, 9, 15, 24, 33, 39, 40], and these heuristics methods are still used
as mainstream tools in the relevant research fields. However, because the fi-
nancial environment contains substantial noise and complex correlations, the
traditional models face limitations in that humans must define the correlations
and reflect them into the model. Thus, many recent studies focus on deriving
portfolio trading strategies using artificial intelligence methods [3, 4, 17, 20, 24].
In particular, studies using reinforcement learning (RL) methods have been
conducted [1, 2, 5, 7, 13, 16, 28, 29, 30, 31, 32, 34, 39] and have received signifi-
cant attention. More recently, studies using deep reinforcement learning (DRL)
methods to approximate the value function or policy of an agent using a deep
neural network (DNN) have been introduced [14, 21, 22]. Such methods allow
agents to learn about the financial environment through their experience within the environment and then optimize their trading strategies based on this experience. These methods have the important advantage that learning agents can update their trading strategies based on their experiences on future trading days. Instead of simply maintaining trading strategies derived from historical data, learning agents can adapt their strategies using their observed experiences on each real trading day [38].

The financial environment contains substantial noise and has many complex correlations, making it difficult for people to understand the underlying mechanisms and derive trading strategies. By applying DRL to portfolio trading, a learning agent can understand a complex financial environment and derive an intelligent trading strategy from this complex environment. In this respect, these related studies are important and deserve significant attention. As advanced DRL methodologies are developed, the trading strategies derived using DRL methods will become flawless, and thus, the study of trading strategy derivations using DRL will be more important in the future.

Previous studies have been conducted to apply different DRL algorithms to various portfolio trading problem settings. Although the methods used by previous studies can work well in each setting, some limitations remain. First, previous studies’ methods can easily be applied to single asset trading, but applying them to multi-asset portfolio trading is more difficult because of the increasing complexity of the problem. Second, portfolio trading strategies derived using previous studies’ methods have low practicality because previous studies defined inapplicable action spaces and did not consider transaction costs.

The purpose of our study is to suggest an approach for deriving a multi-asset portfolio trading strategy using deep Q-learning (DQL), one of the most well-known DRL methods. Our approach can overcome the limitations of previous studies. In our study, we define a discrete action space to increase the strategy’s practicality. In the action space, each action includes trading directions corresponding to each asset in a portfolio, and each trading direction comprises either holding each asset or buying or selling each asset in a unit quantity. Al-
though a recent study argued that optimizing a trading strategy based on a
discrete action space has a negative effect [34], we can use our discrete action
space modeling to derive a trading strategy that has a lower turnover rate and
is more practical than strategies derived using continuous action space model-
ing. However, applying this discrete action space may lead to infeasible actions,
and, thus, we may derive an unreasonable trading strategy (i.e., a frequent and
pointless portfolio weight-changing strategy that only leads to more transaction
cost payments) as a result of handling these infeasible actions. To address this
issue, we introduce a mapping function that enables us to handle the selection
of unreasonable actions by mapping infeasible actions onto similar and valuable
actions. By applying this mapping function, we can derive a reasonable trading
strategy. In addition, although a large amount of data is required to apply DRL,
we introduce an efficient technique that allows an agent to fully utilize training
data to overcome this data shortage issue.

The rest of this paper is organized as follows. In Section 2, we first re-
view the related literature and present the differences between our study and
previous studies. Section 3 describes the definition of our problem, and Sec-
tion 4 introduces our approach for deriving an appropriate trading strategy. In
Section 5, we provide experimental results to validate the advantages of our
approach. Finally, we conclude in Section 6 by providing relevant implications
and identifying directions for future research.

2. Literature Review

This section describes previous studies related to portfolio trading. These
studies are grouped by methodology, from traditional methods to recent trends.

2.1. Stochastic programming-based models

Early studies on portfolio trading and, sometimes, management used stochas-
tic programming-based models. Stochastic programming models formulate a
sequence of investment decisions over time that can maximize a portfolio man-
ger’s expected utility at the end of the investment horizon. Golub et al. [15]
modeled an interest rate series as a binomial lattice scenario using Monte Carlo procedures to solve a money management problem with stochastic programming. Kouwenberg [23] solved an asset liability management problem using the event tree method to generate random stochastic programming coefficients. Consigli and Dempster [12] used scenario-based stochastic dynamic programming to solve an asset liability management problem. However, stochastic programming-based models have the limitation of needing to generate numerous scenarios to solve a complex problem, such as understanding a financial environment, resulting in a large computational burden.

2.2. Heuristics methods

Because of this limitation of stochastic programming-based models, many studies have devised heuristics methods (i.e., trading heuristics). One of the most famous such methods is technical analysis for asset trading. This method provides a simple and sophisticated way to identify hidden relationships between market features and asset returns through the study of historical data. Using these identified relationships, investments are made in assets by taking appropriate positions. Brock et al. [6] conducted backtests with real and artificial data using moving average and trading range strategies. Zhu and Zhou [40] considered theoretical rationales for using technical analysis and suggested a practical moving average strategy to determine a portion of the investments. Chourmouziadis and Chatzoglou [11] suggested an intelligent stock-trading fuzzy system based on rarely used technical indicators for short-term portfolio trading. Another popular heuristics method is the pattern matching (i.e., charting heuristics) method, which detects critical market situations by comparing the current series of market features to meaningful patterns in the past. Leigh et al. [24] developed a trading strategy using two types of bull flag pattern matching. Chen and Chen [9] proposed an intelligent pattern-matching model based on two novel methods in the pattern identification process. The other well-known heuristics method is a metaheuristics algorithm that can find a near optimal solution in an acceptable computation time. Derigs and Nickel [15] developed a decision
support system generator for portfolio management using simulated annealing, and Potvin et al. [35] applied genetic programming to generate trading rules automatically. Chen and Yu [8] used a genetic algorithm to group stocks with similar price series to support investors in making more efficient investment decisions. However, these heuristics methods have a limited ability to fully search a very large feasible solution space because they are inflexible. Thus, we need to be careful about the reliability of obtaining an optimal solution using these methods.

2.3. Artificial intelligence-based methods

Recently, some studies have used artificial intelligence techniques that enable searches of high-dimensional environments. Machine learning-based methods are used to derive a model to predict asset prices by statistically analyzing and learning from meaningful features of financial data. Then, portfolio trading is conducted based on the derived prediction model. For example, Armano et al. [3] developed a prediction model using a genetic algorithm and an artificial neural network and then conducted asset trading. Cho [10] suggested a multi-level, interactive stock market investment system with a consensus prediction model that received prediction results with multiple predictors as inputs. Baralis et al. [4] not only used a regression model to predict the variation in stock prices but also used sequence mining to identify the group of top-k best-performing stocks the following day.

In addition to using simple machine learning algorithms, more recent studies have tried to understand and predict complex financial mechanisms using deep learning to extract meaningful features through high-dimensional and nonlinear mapping. Gunduz et al. [20] conducted a study to determine the correlations and time relationships between technical indicators using a convolution neural network (CNN) to predict stock price movements. Fischer and Krauss [17] applied long short term memory (LSTM), a DNN model suitable for learning long time series data to predict numerous stock price movements. Long et al. [25] proposed a multi-filter neural network that can be combined with a CNN
and a recurrent neural network to predict stock price movements. Although such studies are frequently conducted and provide good predictions in the financial environment, an issue arises in that trading module must be incorporated in the next stage of the prediction model. Thus, the trading process consists of a first stage, in which the prices of the assets in the portfolio are predicted, and a second stage, in which trading takes place based on these predictions. This two-stage model leads to information loss and results in sub-optimal performance because the trading module can only reflect the output of the prediction model and because the stages have two different objectives (i.e., an intermediate objective for the prediction model and an ultimate objective for the trading model) [29].

2.4. Reinforcement learning-based methods

Another recent research direction is optimizing a trading strategy using RL such that a learning agent develops a policy while interacting with the financial environment. Unlike a supervised learning process, such as deep learning, RL learns from experience, leading to a kind of unlabeled data gained from interactions with the environment.

In the earliest such studies, Neuneier [30, 31] optimized multi-asset portfolio trading using Q-learning, a model-free and value-based RL. In other early studies, Moody et al. [29] and Moody and Saffell [28] used Direct RL with Recurrent RL as a base algorithm and derived a multi-asset long-short portfolio trading strategy and a single asset trading rule, respectively. Direct RL is policy-based RL, which optimizes an objective function by adjusting policy parameters, and Recurrent RL is an RL algorithm in which the last action is received as an input. These studies introduced several measures, such as profits and the differential Sharpe ratio, as objective functions and compared the trading strategies derived using different objectives. Casqueiro and Rodrigues [7] derived a single asset trading strategy using Q-learning, which can maximize the differential Sharpe ratio. Dempster and Leemans [13] developed an automated foreign exchange trading system using an adaptive learning system with a base algorithm of RRL by dynamically adjusting a hyper-parameter depending on the market.
situation. O et al. [32] proposed a Q-learning-based local trading system that
categorized an asset price series into four patterns and applied different trading rules. Bertoluzzo and Corazza [5] suggested a single asset trading system using Q-learning with linear and kernel function approximations. Eilers et al. [16] developed a trading rule for an asset with a seasonal price trend using Q-learning. Zhang et al. [39] derived a trading rule generator by using extended classifier systems, which combined with RL and genetic algorithm. Almahdi and Yang [1] suggested a Recurrent RL-based trading decision system that enabled multi-asset portfolio trading and compared the performance of the system when several different objective functions were adopted. Pendharkar and Cusatis [34] suggested an indices trading rule derived using two different RL methods, on-policy (SARSA) and off-policy (Q-learning) methods, and compared the performance of these two methods, and it also compared the performances of discrete and continuous agent action space modeling. Almahdi and Yang [2] used a hybrid method that combined Recurrent RL and particle swarm optimization to derive a portfolio trading strategy that considers real-world constraints.

More recently, DRL, which combines deep learning and RL algorithms, was developed, and, thus, studies have suggested using DRL-based methods to derive portfolio trading strategies. DRL methods enable an agent to understand a complex financial environment through deep learning and to learn a trading strategy by automatically applying an RL algorithm. Jiang et al. [22] used a deep deterministic policy gradient, an advanced method of combining policy-based and value-based RL, and introduced various DNN structures and techniques to trade a portfolio consisting of cash and several cryptocurrencies. Deng et al. [14] derived an asset trading strategy using a Recurrent RL-based algorithm and introduced a fuzzy deep recurrent neural network that used fuzzy representation to reduce uncertainty in noisy asset prices and used a deep recurrent neural network to consider the previous action and utilize high-dimensional nonlinear features. Jeong and Kim [21] derived an asset trading rule that determined actions for assets and the number of shares for the actions taken. To learn this trading rule, an agent used a deep Q-network (DQN) with a novel DNN
structure consisting of two branches, one of which learned action values while the other learned the number of shares to take to maximize the reward. Jeong and Kim [21] also defined three steps for training this two-branched network structure.

The above studies used various RL-based methods in different problem settings. All of the methods performed well in each setting, but some issues limit the applicability of these methods to the real world. First, some problem settings did not consider transaction costs [5, 16, 21, 32, 34]. A trading strategy developed without assuming transaction costs is likely to be impractical for application in the real world. The second issue is that some strategies consider trading for only one asset [1, 5, 7, 13, 16, 14, 21, 28, 39]. A trading strategy of investing in only one risky asset may have high risk exposure because it has no risk diversification effect. Finally, in previous studies deriving multi-asset portfolio trading strategies using DRL, the agent’s action space was defined as the portfolio weights in the next period [1, 2, 22, 29]. The action spaces of these studies do not provide portfolio traders with a direct guide that is applicable to a real-world trading scenario that includes transaction costs. Also, there exist many different ways to change from the current portfolio weight to the next portfolio weight. Thus, previous studies using portfolio weights as the action space required finding a way to minimize transaction costs in each rebalancing moment. Resolving a rebalancing way that reduces both transaction costs and dispersion from the next target portfolio is not an easily solved problem [19]. In addition, a portfolio trading strategy derived based on the action spaces of the previous studies may be difficult to apply to real-world trading because the turnover rate is likely to be high. An action space that determines portfolio weights can result in frequent asset switching because the amount of asset changes has no upper bound. Thus, we contribute to the literature by deriving a portfolio trading strategy that has no such issues.
3. Problem definition

In this study, we consider a portfolio consisting of cash and several risky assets. All assets in the portfolio are bought using cash, and the value gained from selling assets is held in cash. That is, the agent cannot buy an asset without holding cash and cannot sell an asset without holding the asset. This type of portfolio is called a long-only portfolio, which does not allow short selling. Our problem setting also has a multiplicative profit structure in that the portfolio value accumulates based on the profits and losses in previous periods. We consider proportional transaction costs that are charged according to a fixed proportion of the amount traded in transactions involving buying or selling. In addition, we allow the agent to partially buy or sell assets (e.g., the agent can buy or sell half of a share of an asset).

We set up some assumptions in our problem setting. First, transactions can only be carried out once a day, and all transactions in a day are made at the closing price in the market at the end of that day. Second, the liquidity of the market is high enough that each transaction can be carried out immediately for all assets. Third, the trading volume of the agent is very small compared to the size of the whole market, so the agent’s trades do not affect the state transition of the market environment.

To apply RL to solve our problem, we need a model of the financial environment that reflects the financial market mechanism. Using the notations summarized in Table 1, we formulate a Markov decision process (MDP) model that maximizes the portfolio return rate in each period by selecting sequential trading actions for the individual assets in the portfolio according to time-varying market features.

3.1. State space

The state space of the agent is defined as the weight vector of the current portfolio before the agent selects an action and the tensor that contains the market features (technical indicators) for the assets in the portfolio. This type
| Decision variables | Set and indices | Parameters | Features |
|-------------------|----------------|------------|---------|
| \( a_t = (a_{t,1}, a_{t,2}, ..., a_{t,I}) \) | \( i = 0, 1, 2, ..., I \) | \( n \) | \( k_{t,i} \) |
| \( \) agent’s actions at the end of period \( t \) \{ \( a_t \in \mathbb{Z}^I : a_{t,i} \in \{-1, 0, 1\} \ \forall i \} \) | portfolio asset index (\( i = 0 \) represents cash) | size of the time window containing recent previous market features | rate of change of the closing price of asset \( i \) in period \( t \) \( \left( = \frac{p_{t,i} - p_{t-1,i}}{p_{t-1,i}} \right) \) |
| \( t \) | \( t \) | \( P_t \) | \( k_{t,i}^o \) |
| time period index | \( \) | portfolio value changed by the action at the end of period \( t \) | ratio of the opening price in period \( t \) to the closing price in period \( t - 1 \) for asset \( i \) \( \left( = \frac{a_{t,i} - p_{t-1,i}}{p_{t-1,i}} \right) \) |
| \( S^- (a_t) \) | \( S^+ (a_t) \) | \( P_t' \) | \( k_{t,i}^h \) |
| set of index \( i \) if \( a_{t,i} = -1 \) | set of index \( i \) if \( a_{t,i} = 1 \) | portfolio value at the end of period \( t \) when the agent takes no action at the end of the previous period \( t - 1 \) (static portfolio value in period \( t \)) | ratio of the closing price to the highest price of asset \( i \) in period \( t \) \( \left( = \frac{p_{t,i}}{h_{t,i}} \right) \) |
| \( w_{t,i} \) | \( w'_{t,i} \) | \( P_t' \) | \( k_{t,i}^l \) |
| proportion of asset \( i \) changed by the action at the end of period \( t \) | auxiliary parameter used to derive \( w_{t,i} \) | portfolio value before the agent takes an action at the end of period \( t \) | ratio of the closing price to the lowest price of asset \( i \) in period \( t \) \( \left( = \frac{p_{t,i}}{l_{t,i}} \right) \) |
| \( c_t \) | \( c^- \) | \( \delta \) | \( u_{t,i} \) |
| decay rate of transaction costs at the end of period \( t \) | commission rate for selling \( \left( 0 < \delta < \frac{P_t'}{T} \right) \) | unit quantity for selling or buying \( \) | rate of change of the volume of asset \( i \) in period \( t \) \( \left( = \frac{v_{t,i} - v_{t-1,i}}{v_{t-1,i}} \right) \) |
| \( \rho_t \) | \( \rho_t \) | return rate of the portfolio in period \( t \) \( \left( = \frac{P_t - P_{t-1}}{P_{t-1}} \right) \) | \( \) |
of state space is similar to that used in a previous study [22]. That is, the state in period $t$ can be represented as below (Equations (1)-(3)):

$$s_t = (X_t, w'_t),$$

$$w'_t = (w'_{t,0}, w'_{t,1}, w'_{t,2}, ..., w'_{t,I})^T,$$

$$X_t = [K_t, K_o^h, K_h^h, K_l^l, U_t],$$

where $w'_t$ denotes the weight vector of the current portfolio and $X_t$ represents the technical indicator tensor for the assets in the portfolio. For this tensor, we use five technical indicators for the assets in the portfolio, as below (Equations (4)-(8)):

$$k_t = (k_{t,1}, k_{t,2}, ..., k_{t,I})^T,$$

$$k_o^h = (k_{o,1}^h, k_{o,2}^h, ..., k_{o,I}^h)^T,$$

$$k_h^h = (k_{h,1}^h, k_{h,2}^h, ..., k_{h,I}^h)^T,$$

$$k_l^l = (k_{l,1}^l, k_{l,2}^l, ..., k_{l,I}^l)^T,$$

$$u_t = (u_{t,1}, u_{t,2}, ..., u_{t,I})^T,$$

Every set of five technical indicators can be expressed as a matrix (Equations (9)-(13)), where the rows represent each asset in the portfolio and the columns represent the series of recent technical indicators in the time window. Here, if we set a time window of size $n$ (considering $n$-lag autocorrelation) and a portfolio of $I$ assets, the technical indicator tensor is an $(I, n, 5)$-dimensional tensor, as in Figure 1.

$$K_t = [k_{t-n+1}, k_{t-n+2}, ..., k_{t}],$$

$$K_o^h = [k_{o,1}^h, k_{o,2}^h, ..., k_{o,I}^h],$$

$$K_h^h = [k_{h,1}^h, k_{h,2}^h, ..., k_{h,I}^h],$$

$$K_l^l = [k_{l,1}^l, k_{l,2}^l, ..., k_{l,I}^l],$$

$$U_t = [u_{t-n+1}, u_{t-n+2}, ..., u_{t}].$$
3.2. Action space

We define the action space to overcome the limitations of the action spaces in previous studies. Agent actions determine which assets to hold and which assets to sell or buy by prespecifying a constant quantity (i.e., unit quantity: the value amount of buying or selling of each asset, which is different from the value of a share of an asset). For example, if a portfolio includes two assets and the unit quantity is 10,000 USD, then the agent can select the action of buying 10,000 USD of asset1 and selling 10,000 USD of asset2. The action space includes the trading directions of buying, selling, or holding each asset in the portfolio, so the action space contains $3^l$ different actions. These actions are expressed in a vector form that includes trading directions for each asset in a portfolio. In addition, each trading direction (sell, hold, buy) is encoded as ($-1$, $0$, $1$), respectively. For example, an action that involves selling asset1 and buying asset2 can be encoded into the vector ($-1$, $1$).

Because the trading actions for individual assets are carried out in fixed unit quantities, this action space is modeled as a discrete type. Although this discrete action space may not be able to derive a trading strategy that outperforms trading strategies derived using a continuous action space [34], this action space can provide a direct trading guide that a portfolio trader can follow in the real world. Furthermore, this discrete action space can derive a portfolio trading strategy with lower turnover relative to the strategies developed in previous studies. In previous studies, if a portfolio with a very large amount of capital is changed by a small amount in portfolio weight then the trader may pay significant transaction costs. In addition, the losses from these transaction costs
can be very high because portfolio weight changes have no upper bound. In contrast, our action space has an upper bound for portfolio weight changes, and, thus, the issue of massive changes in portfolio weights and the resulting large losses from transaction costs do not arise. Our agent action space has these advantages, and the only disadvantage of the fixed trading amount is similar to the restrictions of hedge funds that allow portfolio traders to trade below a certain amount each day. Thus, our discrete agent action space is not too unrealistic to apply to real-world trading.

In our action space, some actions are infeasible in some states (e.g., the agent cannot buy assets because of a cash shortage or cannot sell assets because of a shortage of held assets). To handle infeasible actions, we first set the action values (i.e., Q-values) of infeasible actions to be very low so that the agent does not select these actions. Thus, we devise a way to select the best action among the feasible actions. The details of this method are explained in Section 4.1.

3.3. MDP modeling

With the state space and action space defined in the previous subsections, we can define the MDP model as follows. The financial market environment operates according to this model during the investment horizon. To define the transitions in the financial market environment (i.e., the system dynamics in the MDP model), we need to define following parameters:

\[ w_t = (w_{t,0}, w_{t,1}, w_{t,2}, ..., w_{t,I})^T, \]  
\[ w_t \cdot \vec{1} = w_t' \cdot \vec{1} = 1 \quad \forall t, \]  
\[ P_t' = P_{t-1} w_{t-1} \cdot \phi(k_t) \quad \forall t, \]  
\[ w_t' = \frac{w_{t-1}}{w_{t-1} \cdot \phi(k_t)} \quad \forall t, \]

where \( w_t \) denotes the portfolio weight after the agent takes an action at the end of period \( t \) (Equation (14)). Equation (15) provides the constraint that the portfolio weight elements sum to one in all periods. Equations (16) and (17)
represent the change in the portfolio value and the change in the proportions of the assets in the portfolio given the changes in the value of each asset in the portfolio, respectively. Here, ⊙ represents the elementwise product of two vectors, and $\vec{1}$ is a vector of size $I+1$ with all elements equal to one. $\phi(\cdot)$ is an operator that not only increases a vector’s dimension by positioning zero as the first element but also adds it to the $\vec{1}$ vector ($\phi : (e_1, e_2, ..., e_I)^T \rightarrow (1, e_1 + 1, e_2 + 1, ..., e_I + 1)^T$).

Now, we can define the state changes after the agent takes an action as follows:

$$c_t = \delta P'_t \left( c^-|S^-(a_t)| + c^+|S^+(a_t)| \right) \quad \forall t, \quad (18)$$

$$P_t = P'_t (1 - c_t) \quad \forall t, \quad (19)$$

$$\hat{w}'_t = (\hat{w}'_{t,0}, \hat{w}'_{t,1}, \hat{w}'_{t,2}, ..., \hat{w}'_{t,I})^T, \quad (20)$$

$$\hat{w}'_{t,i} = \begin{cases} 
  w'_{t,i} - \delta & \text{if } i \in S^-(a_t), \\
  w'_{t,i} + \delta & \text{if } i \in S^+(a_t), \\
  w'_{t,i} & \text{otherwise} 
\end{cases} \quad \forall i = 1, ..., I, \quad (21)$$

$$\hat{w}'_{t,0} = w'_{t,0} + \delta \left( (1 - c^-)|S^-(a_t)| - (1 + c^+)|S^+(a_t)| \right), \quad (22)$$

$$w_t = \frac{\hat{w}'_t}{\hat{w}'_t \cdot \vec{1}}. \quad (23)$$

After the agent takes an action, transaction costs arise, and the portfolio value is then decayed (Equations (18)-(19)). Here, $|S|$ is the size of set $S$. $\hat{w}'_{t,i}$ denotes the auxiliary weight of the portfolio that is needed to connect the change in the portfolio weights before and after the agent takes an action at the end of period $t$ (Equation (20)). The procedure by which the action selected by the agent is handled for trading in the financial environment is as follows. The auxiliary weight of an asset in the portfolio increases (or decreases) as a proportion of the unit quantity when buying (or selling) the asset. On the contrary, the auxiliary weights of the assets do not change when the agent holds the assets (Equation (21)). As a result of selling asset, the proportion of cash increases by the proportion of the unit quantity discounted by the selling commission rate.
As a result of buying asset, the proportion of cash decreases by the proportion of the unit quantity multiplied by the buying commission rate (Equation (22)). To ensure that the sum of the portfolio weight elements equals one after the agent takes an action, a process for adjusting the auxiliary weights is required (Equation (23)). In summary, the financial market environment transition is illustrated by Figure 2.

Figure 2: Financial environment transition

Last, the reward in the MDP model should reflect the contribution of the agent’s action to the portfolio return. This reward can be simply defined as the portfolio return. However, if the portfolio return is defined only as a reward, then different reward criteria can be given depending on the market trend. For example, when the market trend is sufficiently improving, then no matter how poor the agent’s action is, a positive reward is provided to the agent. In contrast, if the market trend is sufficiently negative, then no matter how helpful the agent’s action is, a negative reward is provided to the agent. Thus, the reward must be defined as the rate of change in the portfolio value by which the market trend is removed. Therefore, we define the reward as the change in the portfolio value at the end of the next period relative to the static portfolio value (Equation (24)). The static portfolio value is the next portfolio value when the agent takes no action at the end of the current period (Equation (25)).

\[
\begin{align*}
    r_t &= \frac{P'_{t+1} - P^s_{t+1}}{P^s_{t+1}}, \\
    P^s_{t+1} &= P_t' w_t' \cdot \phi(k_{t+1}),
\end{align*}
\]
4. Methodology

In this section, we introduce our proposed approach for deriving the portfolio trading strategy using DQL. In our action space, some issues may prohibit a DQL agent from deriving an intelligent trading strategy. We first explain how to resolve these issues by introducing some techniques and applying existing methodologies. Then, we describe our DQL algorithm with these techniques.

4.1. Mapping function

In our action space, we need to define a rule for selecting the appropriate action from the remaining actions when infeasible actions are excluded. In the simplest way, we can define this rule as selecting the action that has the largest Q-value, excluding infeasible actions. However, if we adopt this simple rule, then it may lead to an agent deriving an unreasonable trading strategy. For example, when an agent’s strategy selects the action of selling both asset1 and asset2 but this action is infeasible owing to a lack of asset2, the action of buying both asset1 and asset2, which is the largest Q-value action in the remaining action space, is selected. Because learning the similarity between actions is difficult for an RL agent, the agent will take this action without any doubt even though this selected action is the opposite of the original action determined by the agent’s strategy. This issue leads to the selection of unreasonable actions, which degrades the trading performance. Thus, a mapping rule is required to map infeasible actions to similar and valuable actions in feasible action set. Thus, we resolve this problem by introducing such a mapping function.

The mapping function is a type of RL constraint that allows the agent to derive a reasonable trading strategy by mapping infeasible actions to similar and valuable actions in feasible action set. However, it is difficult to apply this constraint to handle the dynamic action space of mapping rules in the learning phase. Thus, we implement the mapping function by applying the constraint to a separate module from the learning phase. As a result, this technique works as if the mapping rule constraint is applied in the learning phase.
The mapping function has several different rules for each infeasible action case, and we call these rules mapping rules. Our mapping function has two mapping rules, each of which is required for mapping infeasible actions, that are divided into two cases. In the first case, the amount of cash is not sufficient to take an action that involves buying assets. In this case, a similar action set is derived by holding rather than buying a subset of the asset group to be bought in the original action. Thereafter, infeasible actions are mapped to the most valuable feasible actions in the similar action set. For example, if the action of buying both asset1 and asset2 is infeasible owing to a cash shortage, this action is mapped to the most valuable feasible action within the set of similar actions, which includes the action of buying asset1 and holding asset2, the action of holding asset1 and buying asset2, and the action of holding both asset1 and asset2. In the second case, an action that involves selling assets is infeasible because of a shortage of the assets. In this case, the original action is simply mapped to an action in which the assets that are not enough to sell are held. These examples are illustrated in Figure 3.

We provide the details of the two mapping rules and the mapping function in the following pseudocode in Algorithms (1) and (2). In Algorithm (1), the last part (i.e., Lines (21)-(22)) of the mapping rule for the second case (Rule2) is necessary. Because, in the second case, converting the original action of selling assets that cannot be sold into an action that holds the selling assets which cannot be sold, then the cash amount gained from selling assets is removed, causing the first infeasible action case to arise. Furthermore, this part of the code can handle the special case in which an asset shortage and a cash shortage occur simultaneously. Next, the RL flow chart with the mapping function technique is shown in Figure 4.

4.2. DQN algorithm

We optimize the multi-asset portfolio trading strategy by applying the DQN algorithm. DQN is the primary algorithm for DQL. Mnih et al. [26] developed the DQN algorithm, and Mnih et al. [27] later introduced additional techniques.
and completed this algorithm. The base algorithm for DQN, Q-learning, is value-based RL, which is a method that approximates an action value (i.e., a Q-value) in each state. Further, Q-learning is a model-free method such that even if the agent does not have knowledge of the environment, the agent can develop a
Algorithm 1 Mapping rule for two cases

1: \( s_t \): state of the agent
2: \( a_t \): infeasible action in state \( s_t \)
3: \( Q(s_t, a_t) \): Q-value for state action pair \((s_t, a_t)\)

4: procedure \( \text{Rule1}(s_t, a_t) \)
5: \( \text{MAXQ} \leftarrow \text{inf} \)
6: subset of buying asset index: \( S = \{C_1, \ldots\} \)
7: for each subset \( C \) in \( S \) do
8:   replicate action \( \hat{a}_t \leftarrow a_t \)
9:   for each asset \( j \) in \( C \) do
10:      \( \hat{a}_{t,j} \leftarrow 0 \)
11:      if converted action \( \hat{a}_t \) is feasible in state \( s_t \) then
12:         if \( Q(s_t, \hat{a}_t) > \text{MAXQ} \) then
13:            \( \text{MAXQ} \leftarrow Q(s_t, \hat{a}_t) \)
14:            \( a_{best} \leftarrow \hat{a}_t \)
15:      end if
16:   end for
17: return \( a_{best} \)

18: procedure \( \text{Rule2}(s_t, a_t) \)
19: for asset \( i = 1, 2, \ldots \) do
20:   if action \( a_t \) to asset \( i \) in state \( s_t \) infeasible then
21:      \( a_{t,i} \leftarrow 0 \)
22: end if
23: if converted action \( a_t \) is infeasible in state \( s_t \) then
24:      \( a_t \leftarrow \text{Rule1}(s_t, a_t) \)
25: end if
26: return \( a_t \)

policy using repeated experience by exploring. In addition, Q-learning is an off-policy algorithm, that is, the action policy for selecting the agent’s action is not the same as the update policy for selecting an action on the target value. An algorithm based on Q-learning that approximates the Q-function using DNN is the basis of DQN \[26\]. To prevent DNN from learning only through the
Algorithm 2 Mapping function

1: \( s_t \): state of the agent
2: \( a_t \): infeasible action in state \( s_t \)
3: procedure \( \text{MAP}(s_t, a_t) \)
4: \( \text{if} \) asset shortage for action \( a_t \) in state \( s_t \) \( \text{then} \)
5: \( a_{map} \leftarrow \text{RULE2}(s_t, a_t) \)
6: \( \text{else if} \) cash shortage for action \( a_t \) in state \( s_t \) \( \text{then} \)
7: \( a_{map} \leftarrow \text{RULE1}(s_t, a_t) \)
8: \( \text{return} \ a_{map} \)

experience of a specific situation, experience replay was introduced to sample a general experience batch from memory. Additionally, the DQN algorithm used two separate networks: a Q-network that approximates the Q-function and a target network that approximates the target value needed for the Q-network updated to follow a fixed target [27]. In addition to this algorithm, we introduce several techniques to support the derivation of an intelligent trading strategy using DQN.

The existing DQN algorithm updates the Q-network with experience by allowing the agent to take only one action in each stage. Because the agent has no information about the environment, only one action is taken then proceeding to the next state. Thus, it is impossible to take multiple actions in the existing DQN. However, for this problem, we use historical technical indicator data of the assets in the portfolio as training data. Thus, our agent can take multiple actions in one state in each stage and observe all of their experiences based on those actions. To utilize this advantage, we introduce a technique that simulates all feasible actions in one state at each stage and updates the trading strategy by using the resulting experiences from conducting these simulations.

However, the historical technical indicator data may not be sufficient for the agent to learn because the application of RL requires sufficient training data. Motivated by Tan et al. [37], we utilize a simulation technique that takes all
feasible actions virtually to force the agent learns about many experiences efficiently for deriving a fully searched trading strategy. Thus, this technique can relax the data shortage issue. Although simulating all feasible actions can result in a huge computational burden, using multi-core parallel computing can prevent this computational burden from greatly increasing. Moreover, even if the agent takes multiple actions in the current state, the next state only depends on the action selected by the action policy (epsilon-greedy) with the mapping rule. The application of this technique requires a change in the data structure of the element in replay memory for storing a list of experiences in a state. The concepts related to this technique are illustrated in Figure 5. In this figure, $a_{jt}$ means that the $j$ - $th$ action of the agent is taken at the end of period $t$. $r_{jt}$ is the reward obtained by taking action $a_{jt}$, and $s_{jt+1}$ is the next state that results from taking action $a_{jt}$.

In DQN, a multiple output neural network is commonly adopted as the Q-network structure. In this network structure, the input of the neural network is the state, and the output is the Q-value of each action. Using the above technique, we can approximate the Q-value of all feasible actions by updating this multiple output Q-network in parallel with the experience list. To maintain Q-values of infeasible actions, the current Q-value of an infeasible state-action pair is assigned to the target value of the Q-network output of the corresponding infeasible action to set a temporal difference error of zero. Furthermore, as in DQN, several experience lists are sampled from replay memory, and the Q-network is updated using the experience list batch. A detailed description of the process for updating the Q-network is shown in Figure 6.

In addition, to apply RL, learning episodes must be defined for the agent to explore and experience the environment. Rather than defining all of the training data, which cover several years, as one episode, we divide the training data into several episodes. If we define a much longer training episode than the investment horizon of the test data that will be used to backtest the trading strategy, this difference in the lengths of the training and test data can produce negative results. For example, in our experiment, the training and testing processes
begin with the same portfolio weights. In this case, the farther the agent is from the beginning of the long training episode, the farther the agent is from the initial portfolio weights. Thus, it is difficult for the agent to utilize the critical experience obtained from the latter half of the long episode in the early testing process. Therefore, we divide training data into sets of the same length as the investment horizon of the test data (i.e., one year, as the investment horizon of the test data is a year in our experiment). Thus, the criteria for dividing the training data are defined in yearly units so that the episodes do not overlap (e.g., episode1 contains data from 2016, and episode2 contains data
from 2015). In each training epoch, the agent explores and learns in an episode sampled from the training data.

It is well known that more recent historical data have more explainable for predicting future data than less recent historical data have. Thus, it is reasonable to assign higher sampling probabilities to episodes that are closer to the test data period [22]. We use a truncated geometric distribution to assign higher sampling probabilities to episodes that are closer to the test period. This truncated geometric sampling distribution is expressed in Equation (26). Here, \( y \) is the year of the episode, \( y_t \) is the year of the test data, and \( N \) is the number of total training episodes. \( \beta \) is a parameter for this sampling distribution that ranges from zero to one. If this parameter is closer to one, episodes closer to the test period are sampled frequently.

\[
g_\beta(y) = \frac{\beta(1 - \beta)^{y_t - y - 1}}{1 - (1 - \beta)^N}, \tag{26}
\]

To implement DQN, we need to model the neural network structure for approximating the Q-function of an agent’s state and action. We construct a hybrid LSTM-DNN neural network that enables us to approximate the Q-value of an agent’s action in our predefined state and action space. First, we use LSTM, a deep learning model suitable for long-term time series pattern learn-
ing, to encode the technical indicator sequences for assets in the portfolio. The technical indicator sequence of each asset in the portfolio shares the same LSTM layer to be encoded in the low-dimensional encoding vector. It is known that a single deep learning model is more effective for learning the price patterns of different assets than multiple deep learning models that learning individual assets [36]. Because this LSTM layer encodes a multivariate timeseries of technical indicators for each asset into a low-dimensional latent vector, we refer to this layer as the pattern encoder. Using a sigmoid as the activation function for the output layer of the pattern encoder, we set the same scale (0~1) for another input, the portfolio weights. Then, the encoded outputs for each asset are concatenated to create the intermediate output, and this intermediate output is then combined again with the current portfolio weights to use as the input to the DNN. Through this DNN layer, we can obtain the Q-value of each action of the agent. Because this DNN layer extracts meaningful features through non-linear mapping using a multi-layer neural network and conducts a regression for the Q-value, we refer to this layer as the DNN regressor. The overall Q-network structure is as shown in Figure 7. In summary, the overall DQN algorithm for our approach for deriving the portfolio trading strategy is as follows (Algorithm 3).

4.3. Adaptive learning

Adaptive learning is a learning method that can be applied after deriving a trading strategy based on historical data using our algorithm. Even during the test period in which the trading strategy is applied, the trading strategy can be updated by learning about already observed experiences with test data [34]. In testing, the situation in the next period is highly correlated with the current observed state. Thus, by learning about the current observed test experience and updating the trading strategy, this adaptation enables the agent to respond to the next uncertain period. However, unlike learning during the training episode, the trading strategy is updated after only one current observed experience (it is impossible to take multiple actions in the test data) rather than through batch
learning. This one-sample learning is similar to online learning, which updates the trading strategy with a bias toward the current observed experience in the current test period. Learning that is biased toward the current experience can make the trading strategy more responsive to the situation in the next test period. In other words, adaptive learning allows the agent to update the trading strategy using more and important experiences and to update the trading strategy flexibly during real trading.

5. Experimental results

In this section, we demonstrate that the DQN strategy (i.e., the trading strategy derived using our proposed DQN algorithm for portfolio trading) can outperform in real-world trading. We conduct a trading simulation for two different portfolio cases using both our DQN strategy and traditional trading strategies as benchmarks, and we verify that the DQN strategy is relatively su-
Algorithm 3 DQN algorithm for portfolio trading

1: $F(s)$ : feasible action set in state $s$
2: Initialize replay memory $D$
3: Initialize weights of Q-network $\theta$ randomly
4: Initialize weights of target network $\theta' \leftarrow \theta$
5: for episode is sampled by sampling distribution $y \leftarrow g_\beta(\cdot)$ do
6: Initialize state $s_0$
7: for period $t=0...T$ in episode $y$ do
8: With probability $\epsilon$ select random $a_t \in F(s_t)$ 
otherwise, $a_t = \begin{cases} \arg\max_a Q(s_t, a; \theta) 
\text{ if } \arg\max_a Q(s_t, a; \theta) \in F(s_t), \\
\text{MAP}(s_t, \arg\max_a Q(s_t, a; \theta)) \ o/w \end{cases}$
9: Take action $a_t$ and then observe reward $r_t$ and next state $s_{t+1}$
10: Simulate all actions $a \in F(s_t)$, then observe experience list $L$
11: Store $L$ in replay memory $D$
12: Sample random batch of experience list $K$ from $D$
13: $(s_t, a_t, r_t, s_{t+1})$ is element of experience list in batch, 
update Q-network from current prediction $Q(s_t, a_t; \theta)$ to target
$z_t = \begin{cases} r_t + \gamma \max_{a'} \hat{Q}(s_{t+1}, a'; \theta') 
\text{ if } \max_{a'} \hat{Q}(s_{t+1}, a'; \theta') \in F(s_{t+1}), \\
\gamma \max_{a'} \hat{Q}(s_{t+1}, \text{MAP}(s_{t+1}, \max_{a'} \hat{Q}(s_{t+1}, a'; \theta')); \theta') \ o/w \end{cases}$
14: Update $\theta$ by minimizing the loss:
$L(\theta) = \frac{1}{|K|} \sum_{L \in K} \sum_{j \in L} (z_j - Q(s_j, a_j; \theta))^2$
15: $\theta' \leftarrow \theta$

prior to the other benchmark strategies based on several common performance measures.

5.1. Performance measures

We use three different output performance measures to evaluate trading strategies. The first measure is the cumulative return based on the increase in the portfolio value at the end of the investment horizon relative to the initial
portfolio value, as defined as Equation (27):
\[ CR = \frac{P_{t_f} - P_0}{P_0} \times 100(\%), \] (27)
where \( t_f \) is the final date of the investment horizon and \( P_0 \) is the initial portfolio value.

The second measure is the Sharpe ratio, as defined in Equation (28):
\[ SR = \frac{E[\rho_t - \rho_f]}{std(\rho_t)} \times \sqrt{D}, \] (28)
where \( std(\rho_t) \) is the standard deviation of the daily return rate, \( \rho_f \) is the daily risk-free rate (assumed to be 0.01%), and \( D \) is the number of transaction days in the investment horizon. This ratio is a common measure of the risk-adjusted return, and it is used to evaluate not only how high the risk premium is but also how small the variation in the return rate is.

For the last measure, we use the customized average turnover rate defined as in Equation (29):
\[ AT = \frac{1}{2t_f} \sum_{t=0}^{t_f} \sum_{i=1}^{I} |\hat{w}_{t,i} - w_{t,i}'| \times 100(\%). \] (29)
The average turnover measures the average rate of change of the portfolio weight vector during the investment horizon. We do not have to consider changes in the cash proportion, so we customize this measure by excluding the change in the weight on cash before and after the agent takes an action. This rate can evaluate the change in the proportions of asset investments. Considering transaction costs, this measure should be low to better apply the trading strategy in the real world.

5.2. Data summary

We experiment with two different three-asset portfolios. The first consists of three exchange traded funds (ETFs) in the US market that track the S&P500 index, the Russell 1000 index, and the Russell Microcap Index. This type of portfolio was tested in a previous study [1]. The second portfolio is a Korean portfolio consisting of the KOSPI 100 index, the KOSPI midcap index, and the
KOSPI microcap index. More information for these test portfolios is provided in Table 2.

Table 2: Test portfolios

| Assets           | Portfolio                              |
|------------------|----------------------------------------|
| US Portfolio (US-ETF) | Korean Portfolio (KOR-IDX)             |
| Asset 1          | SPDR S&P 500$^1$                        |
| Asset 2          | iShares Russell 1000 Value$^2$         |
| Asset 3          | iShares Microcap$^3$                    |
|                  | KOSPI 100 index                         |
|                  | Midcap KOSPI index                      |
|                  | Microcap KOSPI index                    |

$^1$ ETF tracks the S&P500 index

$^2$ ETF tracks the Russell 1000 (mid- and large-cap US stocks) index

$^3$ ETF tracks the Russell microcap index

We obtain data on the three US ETFs from Yahoo Finance and data on the Korean indices from Investing.com. Both cases are tested in 2017. The trading strategy for the US portfolio is derived by training on data from 2010 to 2016, and the trading strategy for the Korean portfolio is derived by training on data from 2012 to 2016.

5.3. Experiment setting

Through several rounds of tuning, we derive appropriate hyper-parameters. In particular, the time window size ($n$) is the most important hyper-parameter, and we adopt the value of 20 among the candidates (5, 20, 60, 120). This time window size and the other tuned hyper-parameters are summarized in Table 3.

In the experiment, we also need to set trading parameters, such as the initial portfolio value and the unit trading quantity. We set the initial portfolio value as one million in both portfolio cases (e.g., 1M USD for the US portfolio and 1M KRW for the Korean portfolio). Similarly, we set the unit quantity as ten thousand in both portfolio cases (e.g., 10K USD trading quantity for the US portfolio case and 10K KRW for the Korean portfolio case). We set the commission rate for buying and selling in both the US and Korean markets as 0.25%. In both cases, the initial portfolio is set up as an equally weighted portfolio, in which every asset and cash has the same proportion.
| hyper-parameter                  | value | hyper-parameter                  | value |
|----------------------------------|-------|----------------------------------|-------|
| time window size \((n)\)         | 20    | replay memory size               | 2000  |
| learning rate \((\alpha)\)       | 1e-7  | initial epsilon                  | 1.0   |
| distribution parameter \((\beta)\) | 0.3   | number of epochs                 | 500   |
| discount factor \((\gamma)\)      | 0.9   | batch size                       | 16    |
| DNN input dimension              | 64    | dropout rate                      | 0.5   |
| DNN layer                        | 2     | LSTM layer                        | 3     |
| DNN 1st layer dimension          | 64    | LSTM unit dimension              | 128   |
| DNN 2nd layer dimension          | 32    | LSTM output dimension            | 20    |

Table 3: hyper-parameter summary

5.4. Benchmark strategy

To evaluate our DQN strategy, we compare it to some traditional portfolio trading strategies. The first strategy is a buy-and-hold strategy \((B&H)\) that does not take any action but rather holds the initial portfolio until the end of the investment horizon. The second strategy is a randomly selected strategy \((RN)\) that takes action within the feasible action space randomly in each state. The third strategy is a momentum strategy \((MO)\). This strategy buys assets whose values increased in the previous period and sells assets whose values decreased in the previous period. However, if it cannot buy all assets with increased values, it gives buying priority to assets whose values increased more. If it is unable to sell assets whose values decreased, it simply holds the assets. The last strategy is a reversion strategy \((RV)\), which is the opposite of the momentum strategy. This strategy sells assets whose values increased in the previous period and buys assets whose values decreased in the previous period. However, if it cannot buy all of the assets whose values decreased, it gives buying priority to the assets whose values decreased more. If it is unable to sell the assets whose values increased, then it simply holds the assets.
5.5. Result

We derive a trading strategy for both portfolio cases using DQN. For both cases, we identify the increase in the cumulative return over the investment horizon of the test period as episode learning continues. Figure 8 shows the trend in the cumulative return performance over the learning episodes in both cases.

Figure 9 shows the portfolio value trend when applying DQN and the benchmark strategies in the US portfolio case. In this figure, we observe that the DQN strategy outperforms the benchmark strategies for most of the test period. The final portfolio value of the DQN strategy is 15.69% higher than that of the B&H strategy, 23.46% higher than that of the RN strategy, 21.81% higher than that of the MO strategy, and 114.47% higher than that of the RV strategy.

Figure 10 shows the portfolio value trend when applying DQN and the benchmark strategies in the Korean portfolio case. Likewise, we observe that the DQN strategy outperforms the benchmark strategies for most of the test period. The final portfolio value of the DQN strategy is 25.52% higher than that of the B&H strategy, 114.35% higher than that of the RN strategy, 13.22% higher than that of the MO strategy, and 247.91% higher than that of the RV strategy.

Table 4 summarizes the output performance measure results when using DQN and the benchmark strategies in both portfolio cases. This table shows that the DQN strategy has the best cumulative return and Sharpe ratio performances for the US portfolio, and this strategy has the lowest turnover rate except for the B&H strategy, which has no turnover rate. In the Korean portfolio case, the DQN strategy also has the best cumulative return and Sharpe ratio performances. Moreover, the DQN strategy has the lowest turnover rate except for the B&H strategy. Given that the B&H strategy does not incur any transaction costs during the investment horizon, it is a remarkable achievement that the DQN strategy outperforms the B&H strategy in terms of the cumulative return and Sharpe ratio.
Figure 8: Cumulative return rate as the learning episode continues (a) US portfolio, (b) Korean portfolio
6. Conclusion

The main contribution of our study is applying the DQN algorithm to derive a multi-asset portfolio trading strategy. However, applying DQN to portfolio trading has some challenges. To overcome these challenges, we introduce an ac-
Figure 10: Comparative portfolio value results for the DQN and benchmark strategies for the Korean portfolio

Our approach involves a discrete action space and several techniques. First, we define a discrete action space that can be applied to individual assets in a portfolio, and the resulting derived trading strategy has a low turnover rate and can provide a direct trading guide to a portfolio trader. Second, we introduce a mapping function for handling infeasible actions to derive a reasonable trading strategy. Trading strategies derived from RL agents can be unreasonable to apply in the real world. Thus, we apply a domain knowledge rule to develop a trading strategy with an infeasible action mapping constraint. As a result, this function works well, and we can derive a reasonable trading strategy. Third, we relax the data shortage issue in RL by introducing a technique that simulates all feasible actions and then updating the trading strategy based on the experiences of these simulated actions. The experimental results show that the DQN strategy outperforms most benchmark strategies in terms of overall performance in the two portfolio cases. We also find that the DQN strategy performs relatively well under general transaction cost.
levels. Thus, the DQN trading strategy can be applied to real-world trading.

However, as shown in Figure 8 in a certain training range, the cumulative return performance trend tends to decrease as learning goes on. In the US portfolio case, the cumulative return performance trend is decreasing in the early training phase, but it recovers to an increasing trend. In addition, in the Korean portfolio case, the cumulative return trend is decreasing in the latter half of training phase. However, this decreasing trend is not significant when the decrease is considered in the context of the smoothing trend of the cumulative return. These flaws are tolerable and are not critical compared to the advantage of applying DQN to portfolio trading. Thus, this DQN strategy derived using our approach is worth introducing.

In future work, we will compare the performance of the DQN strategy to that of a portfolio trading strategy derived using the deep RL method of previous studies, and we verify how the lower turnover rate of our strategy compares to those of previous strategies. This comparative verification is difficult to do in our study because our setting is not the same as those of previous studies. Thus, we cannot compare the performance measures of trading strategies numerically. In the following study, we will therefore implement the methods of previous studies to derive a trading strategy using DRL in our problem setting, and we will compare our DQN strategy numerically to the performance results of previous strategies. In addition, in our current study, we use a long-only portfolio, but we will extend the analysis to a long-short portfolio setting. Furthermore, in our study, the reward of the MDP model is optimized only for returns and not for risk. We can extend to the risk management portfolio by adding a penalty term for risk, such as the variance of the return rate or the conditional value at risk. In addition, advanced DQN methodologies have been developed, and we plan to apply these methods to derive a portfolio trading strategy in future research.
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