Comparative research on various turbulence closure models for unsteady cavitation flows in the ultra-low specific speed centrifugal pump

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Abstract. This paper is aimed to develop a numerical framework for simulating unsteady cavitating flows in the ultra-low specific speed centrifugal pump. Various turbulence models, including RNG (Re-Normalization Group) $k$-$\varepsilon$ model, SST (Shear Stress Transport) $k$-$\omega$ model and FBM (Filter-Based model), were modified by density correction and applied to predict explicit performances, cavitating and pressure fluctuation characteristics of an ultra-low specific speed centrifugal pump. Afterwards, comparisons were made with experimental visualization and measurement results and the numerical results among those models. It can be concluded that the modified turbulence viscosity can fit well with the hydraulic performances measured by the experiment, but the unsteady characteristics of the cavity shapes and pressure fluctuation were predicted well only by filter-based density corrected model (FBDCM), which take the local meshing resolution into consideration. Lastly, a global sensitivity analysis was conducted to assess the limitation of the filter size $\lambda$. The results indicate that when $\lambda$ is less than 3 times the largest grid size, the numerical framework has a reasonable overall predictive capability, and it is expected to choose large value of $\lambda$ for the computational efficiency.

1. Introduction

Cavitation flow occurs when the absolute pressure falls below the pressure threshold at a local thermodynamic state in various fluid machineries, typically causing undesired results such as performance degradation, noise, vibration and erosion etc [1].

From computational aspects, turbulence models play a significant role in high Reynolds numerical flows for understanding the dynamic evolution and the accompanying acoustic and vibration characteristics [2]. Two actively pursued manners to model time dependent flows are direct numerical simulation (DNS) and large eddy simulation, but the extreme request for computational resource hinders their application [3]. The Reynolds-Averaged Navier–Stokes (RANS), such as RNG (Re-Normalization Group) $k$-$\varepsilon$ model and SST (Shear Stress Transport) $k$-$\omega$ model, were initially adopted in practice due to the robustness and reasonable engineering accuracy [4, 5]. However, RANS models have been applied widely in many fields, the over-prediction of the eddy viscous and inaccuracy of the unsteady simulation restrict the use of RANS models in precise numerical simulation in cavitation flow. For this reason, different corrections concerning the eddy viscous have been reported. One is taking the eddy viscous as a function of the local liquid volume fraction $\alpha_l$, named density corrected model (DCM) [4], and another is via filter-based modification [5], which aims to blend LES and RANS models. DCM mainly modifies the viscosity that correspond to highly compressible areas due to a significant density ratio change, whereas FBM puts emphasis on improving the resolution capacity to capture more refined flow structures as the filter size is set smaller than the length scales gotten by RANS models. Some research has been done to assess the validity of the two modified model in cavitation flow [6, 7]. Besides, filter-based density corrected model (FBDCM), which was
proposed by Huang et al. [8], performances as a hybrid model which combines the FBM and DCM. Recently, the model has been accepted by more research [9], but the selected empirical parameters values in the model usually need some experience.

The specific speed, \(n_s\), of an ultra-low specific-speed centrifugal pump is less than 30, and thus impeller passages are typically designed to be narrow and long to satisfy low flow rate and high head requirements. It is often accompanied by complex unsteady flow including cavitation and separation flow, which is urgent to improve the prediction accuracy of the numerical model.

The objectives of this paper are aim to: (1) comprehensive assess the predictive performance of time dependent cavitation flow with various modified turbulence closure models, including the hydraulic performance, cavitation performance, pressure fluctuation and internal flow features; (2) conduct a global sensitivity analysis on the filter size of FBM to reduce the time consumption.

2. Numerical model

2.1. Zwart-Gerber-Belamri cavitation model

The source and sink terms are described as followed:

\[
\frac{\partial m^v}{\partial t} = -C_{\text{vap}} \frac{3 \alpha_{\text{nuc}} (1-\alpha_n) \rho_v}{R_B} \sqrt{\frac{2}{3}} \frac{p_v - p}{\rho_l}, \quad p \leq p_v \\
\frac{\partial m^c}{\partial t} = C_{\text{cond}} \frac{3 \alpha_n \rho_v}{R_B} \sqrt{\frac{2}{3}} \frac{p - p_c}{\rho_l}, \quad p > p_v
\]  

(1)

Here, \(\alpha_{\text{nuc}}\) indicates the nucleation volume fraction, \(R_B\) is the bubble radius, \(p_v\) and \(p\) denote the saturated vapor pressure and the local flow field pressure, respectively. \(C_{\text{vap}}\) and \(C_{\text{cond}}\) are the rate constant for vapor generation and vapor condensation, which are named as evaporation coefficient and condensation coefficient, respectively.

2.2. Summary of modified turbulence models

The primary problems of original turbulence models are that the turbulent eddy viscosity in the rear part of the cavity tends to be over predicted and large deviations are happening during the unsteady simulation. In this work, two modifications, DCM and FBDCM, of the turbulent eddy viscosity for two turbulence models (RNG (Re-Normalization Group) \(k-\varepsilon\) model and SST (Shear Stress Transport) \(k-\omega\) model) will be introduced in detail as below.

2.2.1. Original RNG \(k-\varepsilon\) model and SST \(k-\omega\) model. RNG \(k-\varepsilon\) model is derived from the instantaneous Navier-Stokes equations through the so-called “Re-Normalization Group” approach. SST \(k-\omega\) model is developed by Menter et al. [10], which synthesizes the advantages of \(k-\omega\) models in the near-wall region and \(k-\varepsilon\) model in the mainstream region.

2.2.2. Modifications in the turbulent eddy viscosity. To avoid the over prediction of the turbulent eddy viscosity, some modifications have been done. One of those modifications attempts to take the influence of the local compressibility effect into account, and another combines the local meshing resolution and the compressibility effect together. Substitutions of the turbulent eddy viscosity (\(\mu_T\)) in two turbulence models by DCM and FBDCM, \(\mu_{T,\text{DCM}}\) and \(\mu_{T,\text{hybrid}}\), are represented as:

\[
\mu_{T,\text{DCM}} = f_{\text{DCM}} \left( \frac{C_{\mu_{\text{RING}}} \rho m k^2}{\varepsilon} \right) = f_{\text{DCM}} \left( \frac{C_{\mu_{\text{SST}}} \rho m k}{\omega} \right) \\
\mu_{T,\text{hybrid}} = f_{\text{hybrid}} \left( \frac{C_{\mu_{\text{RING}}} \rho m k^2}{\varepsilon} \right) = f_{\text{hybrid}} \left( \frac{C_{\mu_{\text{SST}}} \rho m k}{\omega} \right)
\]

(3)

(4)
hybrid FBM DCM

FBM 1 5 0 5
1
min 1, or
m l m l
..
f f f
f kk

\chi (\rho_m / \rho_l) [1 - \chi (\rho_m / \rho_l)] f_{DCM}

f_{FBM} = \min \left( 1, \frac{\lambda \cdot \varepsilon}{k^{1.5}} \text{or} \frac{\lambda \cdot \omega}{k^{0.5}} \right)

Where,

C_{\mu\text{RNG}} = 0.085, \quad C_{\mu\text{SST}} = 0.09, \quad \lambda > \left( \frac{\Delta x \Delta y \Delta z}{\Delta x} \right)_{\text{max}}^{1/3}

\chi (\rho_m / \rho_l) = 0.5 + \tanh \left[ \frac{2.4 (\rho_m / \rho_l) - 0.8}{0.32} \right] / 1.999

Here, the largest grid size is around 3.89mm. So, it is reasonable to set \lambda to 4mm, which is 0.08 times of the inlet diameter (d_0).

3. Numerical model

3.1. Physical model and numerical settings

The structure of the ultra-low specific-speed centrifugal pump applied in the work is shown in Figure 1. The specific speed, n_s, is 23.3, the design discharge, Q_0, is 6.25 m³/h and the rotational speed, n, is 1450 r/min. The simulations are conducted by using the CFD code ANSYS CFX. The pressure inlet and mass flow outlet conditions are given for the calculation. RNG k-\varepsilon and SST k-\omega models modified with DCM and FBDCM are added in the simulation, respectively. For the transient simulation, the time step is set as 2.299 \times 10^{-4} s, which is corresponding to the time that turns 2°.

Grid generation adjacent to the blades is a significant factor. For a better convergence, the refined grids are applied in near wall regions, such as blades of the impeller and the inducer, as shown in Figure 2. The grid independent tests and \textit{y}⁺ are presented in Table 1. Considering the precision and the time-consuming, the Case 2 with 3,035,214 elements is employed in the following calculation.

![Figure 1](image1.png)

![Figure 2](image2.png)

**Figure 1.** Computational configuration of the pump model.  
**Figure 2** Mesh of main computation domains.

**Table 1.** The grid independent tests and \textit{y}⁺.

| Scheme | Grid element | Mean \textit{y}⁺ | Convergence criteria | Head \textit{H} (m) | Efficiency \eta (%) |
|--------|--------------|------------------|----------------------|--------------------|--------------------|
| Case 1 | 2,574,645    | 126.65           | 10⁻⁵                | 17.51              | 37.49              |
| Case 2 | 3,035,214    | 56.89            | 10⁻⁵                | 17.76              | 37.86              |
| Case 3 | 3,789,562    | 48.02            | 10⁻⁵                | 17.81              | 38.17              |
| Case 4 | 4,513,741    | 39.88            | 10⁻⁵                | 17.83              | 38.19              |

3.2. Test rig and experimental setup
Hydraulic tests are carried out to validate the accuracy of the simulation. The test rig is set up as a closed-loop cavitation tunnel type in Beijing Key Laboratory of Process Fluid Filtration and Separation of China University of Petroleum-Beijing.

![Figure 3](image)

**Figure 3.** (a) Schematic diagram of the hydraulic test set-up; (b) shooting region and arrangement of model pump and high speed camera.

4. Numerical model

4.1. Comparisons of external characteristic results

The two significant dimensionless parameters appeared in external characteristic profiles are the discharge coefficient $\psi$ and cavitation number $\sigma$, which are expressed in equation (7).

$$\psi = \frac{Q}{Q_0}, \quad \sigma = \frac{P - P_v}{\rho m g H}$$  \hspace{1cm} (7)

Where $Q$ and $H$ mean volume flux and pump head, respectively.

Hydraulic performance tests were conducted at seven discharge coefficient ($\psi=0.72$–1.36), whereas cavitation performance tests were carried out under three conditions ($\psi=0.8$, 1.0 and 1.28). The curve diagrams are shown in figure 4 and 5 in the order of smallest to largest in discharge. It is note that critical cavitation coefficient is an important index to quantify the influence of cavitation coefficient on the pump performance, which is defined as the cavitation coefficient when the pump head decreases by 3% compared with that at non-cavitating condition.

As shown in figure 4 (a) and (b), each turbulence model coincides well with experimental results, and the maximum errors of head and efficiency are 2.36% and 3.98%. However, there are obvious differences in the prediction ability of each model for the cavitation evolution, especially for the critical cavitation in figure 5 (a) ~ (c). For the eddy viscosity modified by DCM, there is a certain suppressive effect in the high vapor fractions areas, but calculation errors are still large at off-design conditions. Whereas the new FBDCM method combines the compressibility effects and filter size effect. When $\psi$ rises from 0.8 to 1.28, the errors never exceed 5%. In addition, SST $k-\omega$ model shows better predictability than RNG $k-\varepsilon$ model. Therefore, in order to study the sensitivity of the filter size, the following will only use SST $k-\omega$ model modified by FBDCM (SST $k-\omega_{FBDCM}$) to find a filter size range that are both accurate and efficient.
Figure 4. Comparisons of energy characteristics.

Figure 5. Comparisons of cavitation performance ((a) $\psi=0.8$; (b) $\psi=1.0$; (b) $\psi=1.28$).

With three dynamic pressure transducers, transient pressure fluctuation has been recorded and compared with simulation results under critical cavitation condition. Note that $f$ is frequency and $f_r$ means rotating frequency. It is seen in figure 6 that one frequency with a remarkably high amplitude is identifiable in each frequency spectrum, which is denoted as the blade passing frequency of the inducer ($3f_r$) and the volute ($6f_r$). The amplitudes of the simulation with SST $k-\omega$ FBDCM are very consistent with these of the experiment as far as the amplitudes at the dominant frequency are concerned. However, because of the small size of the filter, it takes a lot of time to accomplish convergence. It is necessary to find the most economical and accuracy-guaranteed filter size segment.
4.2. Optimization of filter size segment

To investigate the sensitivity of the filter size $\lambda$, four filter sizes are evaluated here: $0.08d_{in}$, $0.16d_{in}$, $0.25d_{in}$ and $0.78d_{in}$. It is immediately evident from equation (16) that reducing $\lambda$ results in a decrease of the eddy viscosity. However, FBM strongly relies on direct numerical simulation (DNS) on the threshold of filter size. figure 7 shows the comparison of space-time evolution of the cavity shapes between simulations with different filter sizes and visualization experiment at $\psi=0.8$. Four typical captured images are chosen to show the effect of filter sizes on prediction accuracy for cavitation evolution. When $f_{FBM}$ gradually tends to 1 for a larger $\lambda$, some process, such as shedding, is not obvious.

Besides, the largest cavity length on the suction surface obtained by simulations are compared here with the experiment. The cavity length ratio is defined as $L_c/L$, where $L_c$ is the length of the attached cavity, and $L$ is the blade length. When $\lambda$ is less than $0.25d_{in}$, the errors are within 5%.

Figure 7. Time evolution of the vapor volume fraction in the same section (1st column: Exp. results; 2nd column: $\lambda=0.08d_{in}$; 3rd column: $\lambda=0.16d_{in}$; 4th column: $\lambda=0.25d_{in}$; 5th column: $\lambda=0.78d_{in}$).

In order to understand the relationship between cavity dynamics and FBDCM with different filter sizes, eddy viscosity, and vorticity profiles at four special time are chosen, as shown in figure 8.
The graphics of eddy viscosity correspond to the cavity shedding are displayed in figure 8 (left). The different eddy viscosity distributions will lead to different cavity shapes, particularly, in the cavity edge regions. It is concluded that FBDCM with a larger filter size overpredicts the eddy viscosity in cavity edge regions. Indeed, it can hinder the cavity fracture and shedding progress when the filter size is larger than 0.25\(d_{in}\).

When it comes to the vortex production, it is noted that the baroclinic torque created by the density change and pressure gradients in the cavitating region induce the vortex production. On the isosurface with a Q criterion of 0.01, core regions of vortex induced by cavitation are colored by vapor volume fraction for identification. As shown in figure 8 (right), when the filter size is less than 0.25\(d_{in}\), the vortex core in the cavitation region is fractured, which corresponds to the occurrence of the cavity shedding.

Time consumption is also one of the important factors affecting selection of the filter size with equal computing resources. In this paper, the time step is chosen as the time that the shaft turns 2° with five coefficient inner loops, and the total iterative time is equal to four rotation periods. All root mean square (RMS) residuals can be reduced below 1e-4 after calculation, but the calculation with the filter size equal to 0.25\(d_{in}\) saves 16% time (about 4 hours) of that with the filter size equal to 0.08\(d_{in}\).

In summary, the small filter size is effective in reduction the eddy viscosity, in spite of the low vapor volume fraction in the shedding cavity, but it is time-consuming. Whereas when the filter size is larger than 0.25\(d_{in}\), FBDCM will tend to DCM, which is not reasonable in low vapor volume fraction regions.

**Figure 8.** Eddy viscosity diagrams (left) and vortex distribution (right) at the instant of the cavity shedding.

**Conclusions**

The following conclusions can be drawn:

(1) As compared to the explicit performance tests, accurate head and efficiency data can be obtained in steady-state calculations of four modified models.

(2) FBDCM blends the compressibility effects and filter size effect that makes it a more precise model to make up the numerical framework.

(3) The amplitude characteristics of the frequency spectrum at monitoring points are recorded using the most accurate SST \(k-\omega\) FBDCM.

(4) SST \(k-\omega\) FBDCM with a larger filter size overpredicts the eddy viscosity in cavity edge regions and obstructs the separation of the cavitation-induced vortex core region, which both illustrate the cavity fracture and shedding is hindered.
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