Collective excitations of a trapped Bose-Einstein condensate in the presence of a 1D optical lattice

C. Fort, F. S. Cataliotti, L. Fallani, F. Ferlaino, P. Maddaloni, and M. Inguscio

LENS, Dipartimento di Fisica, Università di Firenze and Istituto Nazionale per la Fisica della Materia, via Nello Carrara 1, I-50019 Sesto Fiorentino (Firenze), Italy

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We study low-lying collective modes of a horizontally elongated $^{87}$Rb condensate produced in a 3D magnetic harmonic trap with the addition of a 1D periodic potential which is provided by a laser standing-wave along the horizontal axis. While the transverse breathing mode results unperturbed, quadrupole and dipole oscillations along the optical lattice are strongly modified. Precise measurements of the collective mode frequencies at different height of the optical barriers provide a stringent test of the theoretical model recently introduced [M. Krämer et al. Phys. Rev. Lett. 88 180404 (2002)].

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The measurement of frequencies of collective modes was immediately recognized to be a fundamental and precise tool to investigate the quantum macroscopic behaviour of atomic Bose-Einstein condensates. The observations of low-lying excitations [1-3], including the scissor mode [4], were an important step toward the characterization of these systems. Collective excitations were also studied at finite temperature [4, 5, 6, 7, 8, 9, 10] investigating frequency shifts and damping rates. All the experiments so far reported were performed on Bose-Einstein condensates magnetically trapped in pure harmonic potentials.

More recently, condensates trapped in periodic potentials have attracted much interest demonstrated by the flourishing of both theoretical and experimental papers in this field. BEC trapped in a periodic potential is particularly interesting as a possible system for the investigation of pure quantum effects. Many results have been reported e.g. interference of atomic de Broglie waves tunneling from a vertical array of BECs [11], observation of squeezed states in a BEC [12, 13], realization of a linear array of Josephson junctions [14] with BECs, observation of Bloch oscillations [15] and culminating with the observation of the quantum phase transition from a superfluid to a Mott insulator [16]. Recently, the general interest of this field has been extended to a careful study of the loading of BECs in an optical lattice [17] and to the observation of collapse and revival of the matter wave field of a BEC [18]. In this context, the characterization of collective modes of a condensate in the presence of an optical lattice motivated the development of a new theoretical treatment [19] based on a generalization of the hydrodynamic equation of superfluids for a weakly interacting Bose gas [17, 18] to include the effects of the periodic potential. The collective modes of a trapped BEC are predicted to be significantly modified by the presence of an optical lattice. Precise measurements of the low-lying collective frequency would verify the validity of the mass renormalization theory [16].

In this Letter we quantitatively investigate the modification of the low-lying excitation spectrum of a harmonically trapped BEC, due to the presence of a 1D optical lattice. In particular we measure the frequencies of the quadrupole and transverse breathing modes as a function of the optical lattice depth. The experimental observations are compared with the predictions of the model reported in [16] consisting in a frequency shift of the collective modes characterized by a motion along the lattice axis, whereas the frequency of collective modes involving an atomic flow transverse respect to the periodic potential, is expected to remain unchanged.

The experiment is performed with a $F = 1, m_F = -1$ $^{87}$Rb condensate produced in the combined potential obtained superimposing a 1D optical lattice along the axial direction of a Ioffe-Pritchard magnetic trap [14, 15]. The harmonic magnetic trap is characterized by a cylindrical symmetry with axial and radial frequencies respectively $\nu_z = 8.70 \pm 0.02$ Hz and $\nu_r = 85.7 \pm 0.6$ Hz. The 1D optical periodic potential is provided by retroreflecting along the axial direction a far detuned laser beam. The light is produced by a commercial Ti:Sapphire laser tuned at $\lambda = 757$ nm. The radial dimension of the laser beam in correspondence of the condensate is $\sim 300 \mu$m, large enough, compared to the radial size of the condensate, to neglect the effect of the optical potential on the radial dynamics. The optical potential height $V_{opt}$ has been varied in the experiment up to $5.2 E_r$, $E_r = \hbar^2/2m\lambda^2$ being the recoil energy of an atom of mass $m$ absorbing one lattice photon. The corresponding photon scattering rate, below $0.04 \text{s}^{-1}$, is negligible in the time scale of our experiment. In the experiment the lattice depth spans from the weak- to the tight-binding regime in order to investigate the generality of the theoretical predictions in [16].

After the condensate has been produced, in order to excite the collective modes, we perturb the magnetic bias field [17, 18] by applying five cycles of resonant sinusoidal modulation, thus producing a periodical perturbation of the radial frequency of the magnetic trap. We modulate the radial frequency by 10% of its value. A bigger oscillation amplitude would produce a loss of superfluidity caused by entering regions of unstable dynamics [20]. This procedure excites modes with zero angular momen-
tum along the symmetry axis of the trap. For an ellipti-
cal trap, the two lowest energy modes of this type corre-
semble the in-phase oscillations of the width along x and y
and out-of-phase along z (quadrupole mode), and an in-
phase compression mode along all directions (breathing
mode). In the Thomas-Fermi regime, for small am-
plitude oscillations and strongly elongated traps, the two
modes are characterized respectively by the frequencies
$\sqrt{5/2}\nu_z$ and $2\nu_\perp$ [17]. In this limit the two frequencies
are quite different and the axial and radial excitations
are almost decoupled. The axial width performs small
amplitude oscillations at $2\nu_\perp$ superimposed to wider am-
plitude oscillations at frequency $\sqrt{5/2}\nu_z$ (quadrupole
mode), and vice versa for the radial width (transverse
breathing mode).

After exciting the collective mode, we let the cloud
evolve for a variable time $t$, then we switch off the com-
bined trap, let the cloud expand for 29 ms and take an
absorption image of the expanded cloud along one of the
radial directions. In the regime of large optical lattice
heights the density profile after the expansion results in
an interferogram consisting of a central cloud and two
lateral peaks [19]. From the image we extract the ra-
dial ($R_\perp$) and axial ($R_z$) radii of the central peak as a
function of time $t$. In order to compensate the effect of
fluctuations in the condensates number of atoms we then
plot the aspect ratio $R_\perp/R_z$ and fit the data to obtain
the mode frequency.

**Quadrupole Mode.** Following [16], a trapped BEC in
the presence of a 1D optical lattice can still be described
by the hydrodynamic equations that take the same form
as in the absence of the lattice once defined a renor-
malized interaction coupling constant and an effective mass
$m^*$. The effective mass depends on the tunneling rate be-
between adjacent optical wells thus accounting for the mod-
ification inertia of the system along the lattice. In particular,
in the linear regime of small amplitude oscillations, the
new frequency of the collective modes is simply obtained
by replacing the axial magnetic trap frequency $\nu_z$ with
$\nu_z\sqrt{m/m^*}$. In our experimental configuration, where we
have an elongated magnetic trap ($\nu_z \ll \nu_\perp$), the dipole
mode along the periodic potential and the quadrupole
mode are both shifted and characterized respectively by
the frequencies

$$\nu_D = \sqrt{\frac{m}{m^*}} \nu_z$$

$$\nu_Q = \sqrt{\frac{5}{2}} \sqrt{\frac{m}{m^*}} \nu_z = \sqrt{\frac{5}{2}} \nu_D$$

To resonantly excite the quadrupole mode we thus need to
first measure $\nu_D$. The measurement of the dipole mode
frequency is done as in [11] where we already investigated
the dependence of the frequency of the dipole mode in
conjunction with current/phase dynamics in an array of
Josephson junctions. In particular, we induce dipole os-
cillations, by suddenly displacing the position of the mag-
netic field minimum along $z$ and observing the center-of-
mass motion as a function of time. The quadrupole mode
is then excited by modulating the magnetic bias field at a
frequency close to $\sqrt{5/2}\nu_D$. The procedure to excite the
quadrupole mode takes $\sim 700$ ms producing a significant
heating of the condensate. The final observed temperature
of $\sim 150$ nK ($0.9 T_c$) is consistent with the heating
rate of $64 \pm 7$ nK/$s$ measured in the absence of the optical
lattice. The heating results in a degradation of the inter-
fERENCE pattern visibility so that typically only the central
peak is observable also for our larger laser intensities.

A typical series of data is represented in Fig. 1 where,
in the upper part, we show images of the expanded con-
densate taken at different times after the excitation pro-
cedure and in the lower part we report the measured as-
pect ratio together with the sinusoidal fit. We repeated
the same procedure for various intensities of the laser
light creating the optical lattice. Even if the data were
taken at finite temperature, we do not observe any sig-
nificant damping rate in the time scale of our experiment
where we follow the oscillation for $\sim 200$ ms. This is
consistent with previous results for the quadrupole mode
in pure magnetic traps [10].

From Eqs. (1) $\nu_D$ and $\nu_Q$ are expected to scale in the
same way with the optical potential depth. In Fig. 2
we report the quadrupole mode frequency as a function of
the dipole mode frequency varying the optical lattice
height up to 4.1 $E_r$. Both the dipole and the quadrupole
frequency exhibit a strong dependence on the lattice po-
tential (we observed a variation of $\sim 30\%$) demonstrating

![Absorption images taken after exciting the quadrupole mode](image)

**FIG. 1:** In the upper part we show absorption images taken
after exciting the quadrupole mode of a condensate in the
combined trap with $V_{opt} = 3.4 E_r$ and waiting different evo-
olution times (10, 40, 60, 80 and 100 ms) before switching off the
trap and letting the cloud expand for 29 ms. In the lower part we show the evolution of the Aspect Ratio ($R_\perp/R_z$) of the
central interference peak obtained from the absorption images together with a sinusoidal fit to extract the frequency of
the mode.
the marked effect of the optical lattice on these modes. Furthermore, from a linear fit of the data shown in Fig. 2 we obtain a slope of $1.57 \pm 0.01$ in very good agreement with the theoretical prediction of $\sqrt{5/2} = 1.58$. Using Eqs. (1), from our data we can also extract the value of the effective mass $m^*$ as a function of $V_{\text{opt}}$. The results obtained from both the dipole mode and the quadrupole mode frequencies are reported in Fig. 3 together with the theoretical predictions reported in [16] (continuous line). Even if this theoretical curves have been obtained neglecting the mean field interaction and the magnetic confinement, the agreement with our data is very good. In fact, in the regime of $V_{\text{opt}}$ explored in our experiment, the effect of interactions is negligible [21] as also confirmed by the direct solution of the Gross-Pitaevskii equation (dashed and dotted line in Fig. 3 [22]). It would be interesting to investigate also the regime of higher optical lattice depth where the effective mass grows exponentially. On the other hand, accessing this regime without entering instability regions seems to be very difficult [20].

Transverse Breathing Mode. In a different series of experiments we also excite the transverse breathing mode modulating the bias field at a frequency close to $2 \nu_\perp$. In this case the excitation procedure takes only $\sim 30$ ms and no evident heating of the condensate is observed (from the condensed fraction of the cloud we can estimate a temperature $T \lesssim 0.8 T_c$). We repeat this measurement for different optical lattice depths and the results are summarized in Fig. 4. The experimental data are in agreement with the expected value $2 \nu_\perp$ and no dependence on the optical potential height is observed. This confirms the prediction that the dynamical behaviour of the condensate along the radial direction (perpendicular to the lattice axis) is not affected by the presence of the optical potential.

In conclusion, we have investigated the quadrupole and transverse breathing modes of a harmonically trapped condensate in the presence of a 1D optical lattice along the axial direction. The frequency of the quadrupole mode, mainly characterized by an axial oscillation of the
cloud shape, shows an evident dependence on the lattice height in agreement with the renormalized mass theory. On the contrary, the transverse breathing mode, which in our geometry predominantly occurs perpendicularly to the lattice axis, does not exhibit any dependence on the lattice intensity. Our measurements demonstrate that the transport properties of a trapped BEC in the presence of a periodic potential can be described generalizing the hydrodynamic equations of superfluids for a weakly interacting Bose gas. From the measured frequency of the quadrupole and dipole modes we extracted a value for the effective mass that is in very good agreement with the predictions obtained even neglecting in the calculation the effect of interactions.

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