Passive Localisation Using a Blind Interferometer on a Single UAV

Caiyong Hao\textsuperscript{1,a}, Luxi Zhang\textsuperscript{2,b}, Mingbing Li\textsuperscript{3,c} and Qun Wan\textsuperscript{2,d}

\textsuperscript{1} Shenzhen Station of State Radio Monitoring Center, Yintan Road 30#, Dapeng District Shenzhen, China, 518120
\textsuperscript{2} School of Information and Communication Engineering, University of Electronic Science and Technology of China, Chengdu, China, 611731
\textsuperscript{3} Southwest Institute of Electronic Technology, Chengdu, China, 610036

Corresponding author’s e-mail: \textsuperscript{a}srnchcy@hotmail.com, \textsuperscript{b}1602295537@qq.com, \textsuperscript{c}12865579@qq.com, and \textsuperscript{d}wanqun@uestc.edu.cn

Abstract. This paper considers the problem of passive location using an interferometer on a single UAV. In conventional algorithm both measurements of baseline vector and initial phase difference are necessary to locate the target, which reduces the reliability of location system for the fact that the measurements may be polluted by noise or almost unavailable in practice. Therefore, a novel target location method without using the measurements of baseline vector and initial phase difference is proposed in this paper. Simulation results show that the proposed algorithm performs better than conventional algorithm when the measurements noise is relatively large.

1. Introduction

Location plays an important role in modern military, especially passive location. The location by an observation platform is more flexible and subtle than multiple stations, and can cooperate with the station salted on the ground and the carrier-based system [1-2]. Recently, direction finding has been widely applied in location. And the interferometer [3-4] is a relatively mature direction finding system. The receiving phases of the array element are different. And direction of arrival can be determined by the phase difference. The interferometer direction finding technology is widely applied for high accuracy and good real-time performance, such as navigation, detection, aerospace direction finding in military and civil fields and so on.

The improvement of a single interferometer direction finder is introduced using a VXI receiver [5]. The angle of arrival estimation based on an interferometer and its accuracy are discussed [6]. In [7], the improvement of the fast interferometer direction finder is presented using I/Q demodulator. The bias and mean squared error of direction estimates with some cost functions are derived for the interferometer [8]. An array configuration for improving the direction-finding accuracy of an interferometer direction-finding method is presented [9]. The approach proposed in [10] offers array spacing that satisfy the maximum phase error that is required at interferometer direction-finding system by using the phase-difference and rotation matrices. For the high direction-finding accuracy in phase interferometer systems, a novel method is proposed [11]. These ways is applied the single interferometer. However, the baseline vectors of interferometer and initial phase are known or can be measured for these ways, actually, they usually are noisy or unknown. Therefore, we propose a novel
blind location method for single interferometer, called blind location, and the baseline vectors of interferometer and initial phase are estimated by observational data. Blind location performs better positioning accuracy, particularly the baseline vectors of interferometer and initial phase seriously polluted by noise.

The remainder of this paper is organized as follows. The problem is described in Section 2. Section 3 introduces tracking algorithms. Section 4 shows performance of proposed algorithm and comparison with conventional algorithm. Section 5 concludes the paper.

2. Problem Description

Consider a stationary target at position \( \mathbf{p} = (x, y, z)^T \) which emits signal with wavelength \( \lambda \). There is a moving interferometer at position \( \mathbf{p}_n = (x_n, y_n, z_n)^T \) receives the signal from the target, where \( n \) indicates the number of discrete sampling and total sampling number is \( N \).

The distance vector between the target and interferometer is \( \mathbf{r}_n = \mathbf{p}_n - \mathbf{p} \). And the phase difference of signal received by two arrays mounted on interferometer is defined by

\[
\phi_n = \frac{2\pi}{\lambda} \frac{\mathbf{r}_n^T \mathbf{a}}{\|\mathbf{r}_n\|} + \eta + w_n
\]

(1)

Where \( \mathbf{a} = [u \ v \ w]^T \) denotes the baseline vector, and \( \eta \) is initial phase difference caused by system error and \( w_n \) is zero-mean white Gaussian noise with covariance \( \sigma^2 \). The measurement sequence of phase difference is given by

\[
\Phi = \frac{2\pi}{\lambda} \mathbf{R} \mathbf{a} + \mathbf{1} \eta + \mathbf{w}
\]

(2)

where \( \mathbf{R} = \begin{bmatrix} r_1 \|r_1\| & r_2 \|r_2\| & \cdots & r_N \|r_N\| \end{bmatrix}^T \)

(3)

Given measurements of \( \lambda \), \( \mathbf{a} \) and \( \eta \), it is quite easy to define the objective function of estimating the target position as below

\[
\hat{\mathbf{p}} = \arg \min_{\mathbf{p}} \left\| \Phi - \frac{2\pi}{\lambda} \mathbf{R} \mathbf{a} \right\|
\]

(4)

However, it is often the case that the measurements of \( \lambda \), \( \mathbf{a} \) and \( \eta \) are seriously affected by noise or almost unavailable in the complex environment, thus the passive location system based on (4) will not work properly. How to locate the target using only measurements of the phase difference and the position of interferometer is the problem to be discussed in the following section.

3. Location algorithm

Due to lack of measurements of \( \mathbf{a} \) and \( \eta \), it is reasonable to regard them as parameters to be estimated and (2) can be written as

\[
\Phi = \frac{2\pi}{\lambda} \mathbf{A} \mathbf{b} + \mathbf{w}
\]

(5)

where \( \mathbf{w} \) is additive zero-mean white Gaussian noise with variance \( \sigma_0^2 \) and

\[
\mathbf{A} = \begin{bmatrix} \mathbf{R} & \mathbf{1} \end{bmatrix}
\]

(6)

\[
\mathbf{b} = \begin{bmatrix} \mathbf{a} \\ \frac{\lambda}{2\pi} \eta \end{bmatrix}
\]

(7)
where $A$ is related to the target position. Given the estimated value $\hat{p}$ and $\hat{b}$, the error vector is defined by

$$err(\hat{p}, \hat{b}) = \Phi - \frac{2\pi}{\lambda} A \hat{b}$$

(8)

To minimize the norm of error vector, it is equal to minimize

$$J = \left\| err(\hat{p}, \hat{b}) \right\|^2 = \left( \Phi - \frac{2\pi}{\lambda} A \hat{b} \right)^H \left( \Phi - \frac{2\pi}{\lambda} A \hat{b} \right)$$

(9)

$$= \Phi^H \Phi - \frac{2\pi}{\lambda} \hat{b}^H A^H \Phi - \frac{2\pi}{\lambda} \Phi^H A \hat{b} + \left( \frac{2\pi}{\lambda} \right)^2 \hat{b}^H A^H A \hat{b}$$

The derivative of $J$ with respect to $\hat{b}$ is

$$\nabla J = -2 \frac{2\pi}{\lambda} A^H \Phi + 2 \left( \frac{2\pi}{\lambda} \right)^2 A^H A \hat{b}$$

(10)

Let $\nabla J = 0$ and least square estimation of $\hat{b}$ can be presented by $A$ and $\Phi$ as follows

$$\hat{b} = \frac{\lambda}{2\pi} (A^T A)^{-1} A^T \Phi$$

(11)

Then (9) can be written as

$$J = \Phi^H \Phi - \Phi^H A (A^T A)^{-1} A^T \Phi$$

(12)

As can be seen from (12), $\lambda$ is ignored without affecting the performance. And in this way, without measurements of $\lambda$, $\alpha$ and $\eta$, using only measurements of the phase difference and the position of interferometer, the position of target can be estimated by minimizing $J$ and expressed as

$$\hat{p} = \arg \min_p J$$

(13)

4. Simulation

In comparison with conventional passive location algorithm using a single interferometer, the performance of proposed algorithm is analysed in this section. For simplicity, the proposed algorithm is called Blind location, and conventional algorithm is called Non-Blind location.

4.1. Scenario

Assuming there is a static target at position $(1500,2000,150)$ (m) which emits signal with carrier frequency $970$MHz. A UAV with initial position $(0,-1000,1000)$ (m) is moving at a constant speed $(0,100,0)$ (m/s), whose initial phase difference is $\frac{\pi}{12}$ and baseline vector is $(0.2918,-0.0620,0.0314)$. After 32s of motion, the relative change of position between target and UAV is shown in Figure 1.

4.2. Result

To solve the function presented in (10), the simplest way is to search by grid and the grid point with minimum value within the search area is the position of target. It is assumed that the measurements noise of phase difference exists and $\sigma_0 = 0.01$, and the simulation statistics are accumulated over 200 runs. As can be seen from Figure 2 and Figure 3, the closer the grid point is to the real position of target, the larger the reciprocal of error is. Due to the presence of measurements noise and the setting of grid spacing, the estimated position is near to but not exactly the same as the real position.
Figure 1. Single observer location scenario.

Figure 2. The reciprocal of error corresponding to each grid point within search area using blind location.

In non-blind location, the measurements of wavelength of received signal, baseline vector and initial phase difference are necessary which may be polluted by measurements noise. Suppose the variance of measurements noise of initial phase difference is $\sigma_1^2$ and of baseline vector is $\sigma_2^2$, both of which obey Gaussian distributions with zero mean and uncorrelated to each other.

The location result presented in Figure 4 and Figure 5 illustrate the influence of measurements noise on positioning performance. Obviously, positioning performance of non-blind location will be better when measurements noise of them is relatively small, however, it degated steeply when $\sigma_1$ or
$\sigma_1$ increases. By contrast, performance of blind location remains unaffected because these measurements are not used at all, which means higher reliability and lower cost to do localisation in practice.

**Figure 3.** The contour map of the reciprocal of error corresponding to each grid point within search area using blind location. The circle represents the real position and star represents the estimated position.

**Figure 4.** Comparison between blind location and non-blind location with different measurements noise of initial phase difference and $\sigma_2 = 0.005$. 

![Blind Location](image)

![Comparison](image)
Figure 5. Comparison between blind location and non-blind location with different measurements noise of baseline vector and $\sigma_i = 1$.

5. Conclusion
This paper proposed a passive location algorithm using a blind interferometer on a single UAV without using the measurements of initial phase difference caused by system error and baseline vector, which are necessary in conventional algorithm. Because some measurements may be seriously affected by noise or almost unavailable in the complex environment, the performance of conventional algorithm will degrade. Simulation results show that the proposed algorithm outperforms the conventional algorithm when the measurements noise is relatively large.

Acknowledgments
This work was supported in part by the National Natural Science Foundation of China under Grant U1533125, and Grant 61771108, in part by the National Science and Technology Major Project under Grant 2016ZX03001022, in part by the Fundamental Research Funds for the Central Universities under Grant ZYGX2015Z011 and Sichuan science and technology planning project (key R & D project 18ZDYF0990).

Reference
[1] Guoqing Z 1999 *The principle of radar countermeasures* (Xi’an: Xidian University Press)
[2] Yue Yang, Xunchao Cong, Keyu Long, Yongjie Luo, Wei Xie, Qun Wan 2018 MRF model-based joint interrupted SAR imaging and coherent change detection via variational Bayesian inference. Signal Processing. 151 144-154
[3] Sundaram K R, Mallik R K and Murthy U M S 2000 Modulo conversion method for estimating the direction of arrival *IEEE Transactions on Aerospace and Electronic Systems* 36 1391-1396
[4] Xiuli X 2006 The theory of interferometer direction finding *China Radio* 5 43-49
[5] Xiaobo A, Zhenghe F 2002 A single channel correlative interferometer direction finder using VXI receiver *3rd International Conference on Microwave and Millimeter Wave Technology Proceedings* (China: Beijing)
[6] Park C S, Kim D Y 2006 The Fast Correlative Interferometer Direction Finder using I/Q Demodulator Asia-Pacific Conference on Communications (South Korea: Busan)

[7] Wei H W, Shi Y G 2010 Performance analysis and comparison of correlative interferometers for direction finding International Conference on Signal Processing (China: Beijing)

[8] Yingbing W and Lirong N 2016 Studies on circular array interferometer direction finding International Symposium on Microwave, Antenna, Propagation, and Emc Technologies (China: Shanghai)

[9] Lee J H and Woo J M 2015 Interferometer Direction-Finding System With Improved DF Accuracy Using Two Different Array Configurations IEEE Antennas & Wireless Propagation Letters 14 719-722

[10] Jung-Hoon L, Jong-Hwan L and Jong-Myung W 2016 Method for obtaining three- and four-element array spacing for interferometer direction-finding system IEEE Antennas and Propagation Society 15 897–900

[11] Jung-Hoon L, Jong-Kyu K, Hong-Kyun R. 2018 Multiple Array Spacings for an Interferometer Direction Finder with High Direction-Finding Accuracy in a Wide Range of Frequencies IEEE Antennas and Propagation Society 17 563 - 566.