The chiral condensate in holographic models of QCD

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Bottom-up holographic models of QCD, inspired by the AdS/CFT correspondence, have shown a remarkable degree of phenomenological success. However, they rely on a number of bold assumptions. We investigate the reliability of one of the key assumptions, which involves matching the parameters of these models to QCD at high 4D momentum $q^2$ and renormalization scale $\mu^2$. We show that this leads to phenomenological and theoretical inconsistencies for scale-dependent quantities such as $\langle \bar{q}q \rangle$.

There are still no systematic analytic tools to study the strong-coupling dynamics of QCD, except for models that probe certain limited classes of observables. Thus, for example, one can use chiral perturbation theory for some low-energy observables, but for more general ones one is forced to resort to more phenomenological approaches such as chiral soliton models. In recent years, ‘bottom-up’ holographic models of QCD have emerged as another approach to the low energy phenomenology of QCD, and have attracted considerable interest [1, 2].

In these models, QCD in the large $N_c$ limit is taken to be dual to a classical 5D theory in a curved space, and the parameters of the 5D model are matched to the corresponding values in large $N_c$ QCD [20, 21], with the field content of the 5D models chosen to match the low energy chiral symmetry of QCD. In contrast to approaches like chiral perturbation theory, these models allow the computation of meson spectra and couplings, at least in principle. Even very simple 5D models seem to show a remarkable phenomenological success when compared to data (see, for instance, ref. [1, 2]).

There are, however, some phenomenological puzzles with the models. For example, quark masses as extracted by ref. [1] and ref. [3] are different by a factor of four. This inconsistency, which we address in this Letter, is in fact symptomatic of issues which are more serious than they may first appear. The resolution of these issues, associated with the treatment of operators which are scale dependent in QCD in holographic models, will shed light on the the matching of the 5D models to QCD.

Holographic models of QCD are motivated by the conjectured dualities between some types of gauge theories and higher-dimensional gravity theories. The most well-known such duality is the AdS/CFT correspondence [22–24], where a conformal field theory, $\mathcal{N} = 4$ $SU(N_c)$ super Yang-Mills theory in the large $N_c$ limit, is dual to a type IIB string theory on $AdS_5 \times S^5$, where $AdS_5$ is 5D Anti-DeSitter space, and $S^5$ is the 5-sphere. When the ’t Hooft coupling of the field theory is large, the $AdS_5 \times S^5$ physics is described by weakly-curved classical supergravity. The CFT lives on the boundary of $AdS_5$, in the sense that each operator $O$ of the CFT is identified with a bulk field $\phi^O$ in $AdS_5 \times S^5$, the boundary values of the bulk fields $\phi^O$ are the sources for the CFT operators, and the supergravity partition function $Z_{SG}$ is identified as the generating functional of the CFT correlation functions:

$$Z_{SG}[\phi^O] = \int D\phi^O \exp \left( -S_{SG}[\phi^O] \right) = \left( \frac{e^{-iS_{AdS}[\phi^O]}}{Z_{CFT}} \right),$$

(1)

CFT correlation functions in the strongly-coupled domain can be calculated by evaluating the supergravity action $S_{SG}$ on the classical solution for fields $\phi^O$ that approaches a specified boundary value $\phi^O_0$ and taking functional derivatives with respect to $\phi^O_0$ [23, 24].

The bottom-up approach to holographic QCD generally consists of guessing a 5D background and field content that captures some aspects of large $N_c$ QCD for some observables of interest. In AdS/CFT, the conformal invariance of the CFT corresponds to the fact that coordinate rescaling is an isometry of AdS. QCD is approximately conformal at high energies (due to asymptotic freedom), but not at low energies (due to confinement). This means that an unmodified $AdS_5$ background can not capture the essential features of QCD, and the $AdS_5$ space must be modified in some way. Generally, the holographic QCD models on the market use an asymptotically $AdS_5$ background to reflect the fact that QCD is approximately conformal at high energies. The deep bulk region is then modified (that is, cut off in some way) in order to model confinement. For example, in hard wall models [1, 2, 4, 6, 12, 17], confinement is simulated by cutting off the AdS space at some finite radius by hand. In soft-wall models, a dilaton field is turned in the bulk and tuned to smoothly cut off the AdS space to produce a linear Regge meson mass spectrum [10, 17, 19], opposed to the quadratic spectrum that is generally seen in hard-wall models. Other models take into account the back-reaction of the bulk fields on the metric, which can dynamically cut off the AdS space [14, 15].

A well-known general theoretical issue with all such models is that in the regime where QCD is approximately conformal, it is weakly coupled due to asymptotic freedom. As a result, it is far from obvious that the use of a classical, weakly curved 5D background is justified, since one generally expects that a systematic holographic dual of QCD should be a string theory on some highly curved
space (as noted for instance in ref. [10]). However, despite this possible problem, it is interesting to try to investigate whether a classical 5D background might serve as a phenomenologically useful ad hoc approximation to a holographic dual of QCD, and many bottom-up models appear to show a remarkable agreement with experimental data.

The AdS/CFT dictionary [23, 24] is used in bottom-up models to dictate the bulk field content: a p-form 4D QFT operator $O$ with scaling dimension $\Delta$ corresponds to a p-form bulk field with mass $m^2 = (\Delta - p)(\Delta + p - 4)$. Conserved currents in the QFT correspond to gauge fields in the bulk, with $m^2 = 0$. In bottom-up holographic models, it is assumed that this dictionary remains valid even when one modifies the 5D background away from $AdS_5$. It is not obvious that this should be justified, because for instance while the scaling dimension of operators is well-defined in a CFT, in QCD the scaling dimensions of most operators receive scale-dependent corrections proportional to $\alpha_s$. That is, most operators in QCD have scale-dependent anomalous dimensions. An exception is operators that correspond to conserved currents, i.e., those associated with vector and axial currents, and the stress-energy tensor, whose anomalous dimensions are zero. However, in an exploratory spirit holographic models of QCD assume that the dictionary remains valid.

In order to model some observables of interest, one must choose a set of 4D operators and corresponding 5D fields that probe the relevant physics. Of course, QCD has an infinite number of operators with the same quantum numbers, and due to operator mixing and the lack of any obvious suppression scale, an arbitrarily large subset of them can contribute to any given process [23]. As an ad hoc approximation, to obtain tractable holographic models one simply chooses a minimal set of lowest-dimension operators to probe the observables one is interested in. The parameters of the resulting 5D models are matched using the AdS/CFT dictionary to large $N_c$ QCD in the UV regime, where the asymptotic freedom of QCD allows reliable perturbative calculations.

Given the many bold assumptions that are necessary to construct holographic models of QCD, their phenomenological success is remarkable, and may suggest that the assumptions are more reliable than might be expected. It is natural to wonder if there is anything in these models that can test the reliability of the assumptions. In this Letter, we show that the ad hoc treatment of scale-dependent operators in holographic models of QCD can lead to serious phenomenological and theoretical problems. For definiteness, we work in the simple hard-wall model of ref. [11] for chiral symmetry breaking, and focus on the behavior of the quark condensate $\langle \bar{q}q \rangle$ and its source $m_q$. We show that to match the large $N_c$ scaling of QCD quantities, the operator-field mapping for the quark condensate needs to be properly normalized, and this normalization affects the value of the quark mass and the quark condensate. Next, we argue that the values of the quark mass and the chiral condensate obtained in the model from matching to data are inconsistent with the matching of the 5D models to QCD in the asymptotically UV region, and discuss the implications of the analysis for the treatment of scale dependent operators in bottom-up holographic models of QCD.

| 4D Operators | 5D Fields | $p$ | $\Delta$ | $(m^2)$ |
|--------------|-----------|-----|--------|--------|
| $\bar{q}L\gamma^\mu t^a qL$ | $A^L_{\mu} \gamma_5$ | 2 | 3 | 0 |
| $\bar{q}R\gamma^\mu t^a qR$ | $A^R_{\mu} \gamma_5$ | 2 | 3 | 0 |
| $\bar{q}L qL qL$ | $\frac{2}{3} \phi^{ab}$ | 2 | 0 | -3 |

TABLE I: Field content and dictionary of the model.

I. THE MODEL

Our analysis is in the simple model of ref. [11], which we describe in this section. QCD has an $SU(N_f)_L \times SU(N_f)_R$ flavor symmetry in the chiral limit, and the model focuses on the lowest-dimensional operators important to chiral dynamics. These are the left and right handed quark currents $\bar{q}L\gamma_\mu t^a qL$ and $\bar{q}R\gamma_\mu t^a qR$, where $t^a$ are the generators of $SU(N_f)$. They are associated with the 5D gauge fields $A^L_\mu$ and $A^R_\mu$ using the AdS/CFT dictionary. We will work in the $N_f = 2$ limit in this Letter.

To model chiral symmetry breaking, we include the operator $\bar{q}R qL$, which acquires a vacuum expectation value, and associate it with a massive bulk scalar field $\phi$. The dictionary between $\bar{q}R qL$ and $\phi$ includes a factor of $1/\zeta$ due to the dimension of $\bar{q}R qL$.) The field content of the model and the dictionary are given in Table 1. Note that in assigning these 5D masses it is assumed that the scaling dimensions of the operators are purely classical. Note that this is not the case for $\bar{q}R qL$ in QCD, so that this is an ad hoc approximation. The model uses an AdS space with a hard-wall cutoff in the IR as the holographic 5D background. The metric is

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = L^2 \left(-dz^2 + dx^4 dx_1\right),$$

(2)

where $L$ is the radius of the AdS space, which will be set to unity in the rest of this Letter, and $z \in [0, z_m]$. Holographic calculations generally use $z = \epsilon$ as a UV regulator [24], with $\epsilon \to 0$ at the end of the computations. We use the convention that Latin indices take values 0, 1, 2, 3, and Greek indices take values $z, 0, 1, 2, 3$. This choice of metric corresponds to the assumption that QCD, which lives on the $z = 0$ boundary, remains conformal until confinement suddenly sets in at an energy scale of order $1/z_m$. While this is clearly a drastic oversimplification, it does not affect our general conclusions.

In terms of the bulk fields that we have defined above, the classical bulk action can be written as

$$S = \int d^5x \sqrt{\gamma} \text{Tr} \left\{ |D\phi|^2 + 3|\phi|^2 - \frac{1}{4\kappa^2} (F^L_\mu F^L_{\mu} + F^R_\mu F^R_{\mu}) \right\},$$

(3)
with \( D_\mu \phi = \partial_\mu \phi - iA_\mu L X + iX A_\mu R \), \( A_{L,R} = A^a_{L,R} T^a \), and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i[A_\mu, A_\nu] \). The gauge coupling \( g_5 \) can be determined from matching the two-point vector correlation function to the leading term in the QCD operator product expansion for the vector two-point function. To do this, one can evaluate the 5D action on solutions of the equations of motion of the vector field \( (V = A_L + A_R) \), assuming that the 5D side can be treated classically, and take two derivatives with respect to the boundary sources to obtain the vector two-point function. The vector two-point correlation function is

\[
\Pi_V(q^2) = \frac{-1}{2g_5^2} \log (q^2) ,
\]

(4)

where \( q \) is the 4D momentum. This can be matched to the corresponding QCD expression, coming from a quark bubble and valid at high \( q^2 \), where QCD is weakly coupled due to asymptotic freedom:

\[
\Pi_V(q^2) = -\frac{N_c}{24\pi^2} \log (q^2) .
\]

(5)

Matching the holographic model to QCD implies that \( g_5^2 = 12\pi^2/N_c \).

To incorporate chiral symmetry breaking, it is necessary to connect the scalar field \( \phi \) with its dual \( \bar{q}Rq_L \) to the chiral condensate \( \langle \bar{q}Rq_L \rangle = \Sigma = \sigma 1 \) and the quark mass \( M = m_q \), which is the source of \( \bar{q}Rq_L \) in QCD. There is a straightforward prescription for this in AdS/CFT. As shown by Klebanov and Witten \[27\], the near-boundary solution of the equations of motion for a field \( \phi^O \) dual to an operator \( O \) with dimension \( \Delta \) is given by

\[
\phi^O(z,q) = z^{4-\Delta} \phi^O_0(q) + z^\Delta \frac{\langle O \rangle}{2\Delta - 4} ,
\]

(6)

where \( \phi^O_0(q) \) is the source for \( O \), and \( \langle O \rangle = \delta \log Z/\delta \phi^O_0(q) \) is the one-point correlation function.

Before applying this to our problem, we must address a slightly subtle normalization issue in the association of operators with fields in holographic models. In a field theory with an operator \( O \) and a source \( J \), one always has the trivial freedom to redefine \( O \to a O \) and \( J \to J/a \), so that \( J O \) is unchanged. In holographic models defined by actions such as Eq. [1] this amounts to the freedom to take \( O \to a O \) and \( \phi_0^O \to \phi_0^O/a [28] \). Many treatments of holographic models of QCD implicitly take \( a = 1 \). However, as we are about to show, this is not generally correct, and the issue of operator/source normalization turns out to have significant implications for holographic models such as the one in this Letter.

We specialize the discussion above our case with \( O = \bar{q}Rq_L \) and \( \Delta = 3 \), keeping the normalization parameter \( a \) explicit in the equations. Using the dictionary \( \lim_{z \to 0} \frac{2}{a} \phi(z, q) = a M \), we can write the \( q = 0 \) classical solution for \( \phi \) as

\[
\phi(z) = \frac{1}{2} a M z + \frac{\Sigma}{2a} z^3 ,
\]

(7)

where \( M = m_q \) and \( \Sigma = \langle \bar{q}Lq_R \rangle = \sigma 1 \).

It is now clear how the seemingly trivial issue of source and operator normalization becomes important. If we take \( a = 1 \), as was done in ref. [1] implicitly, then Eq. [7] has incommensurate \( N_c \) scaling: the first term scales as \( N_c^0 \), while the second scales as \( N_c^1 \). It is also not hard to show that with \( a = 1 \) in this model, the \( \rho/a_1 \) mass splitting scales \( N_c^{1/6} \), in direct contradiction with large \( N_c \) QCD. To be consistent with large \( N_c \) QCD, we must take \( a \sim N_c^{1/2} \) rather than \( a = 1 \), so that the \( \rho/a_1 \) mass splitting scales as \( N_c^0 \). Of course, this should not be surprising: in large \( N_c \) QCD, the normalization of operators explicitly depends on \( N_c \), so the same must be true in the holographic approach.

To complete the description of the holographic model, one must specify the values of \( \sigma, m_q \) and \( a \). The chiral condensate \( \sigma \) comes from IR physics, and in this model it can be taken to be an input parameter. In fact, the structure of the model fixes \( \sigma/a \) and \( m_q \). To determine \( \sigma/a \), one can use the relation [1]

\[
f^2_\pi = \frac{1}{g_5^2} \frac{\partial A(z)}{\partial z} \bigg|_{z=\epsilon} ,
\]

(8)

where \( A(z) \) is the transverse part of the axial vector current \( (A = A_L + A_R) \) at \( q = 0 \), which satisfies the equation

\[
\partial_z \left( z^{-1} \partial_z A(z) \right) + \frac{g_5^2 \sigma^2 z^3}{a^2} A(z) = 0 ,
\]

(9)

which is shown in the chiral limit \( m_q = 0 \) for simplicity. Since this model obeys the Gell-Mann-Oakes-Renner (GOR) relation [1] \( m_q^2 f_\pi^2 = 2m_q \sigma^2 \), it is possible to calculate \( m_q \) and \( \sigma/a \) by fitting \( m_q^2 \) and \( f_\pi^2 \) to data.

To complete the specification of the model, it is necessary to determine the normalization parameter \( a \). We will compute \( a \) by matching \( a \) to QCD explicitly, as was first done in ref. [3], and show that this leads to the correct \( N_c \) scaling. To do this, we will compare the two-point functions for \( \bar{q}Rq_L \) in the holographic models and in the asymptotic regime in QCD, using an identical procedure to the one that was followed for \( g_5^2 \) in the vector sector. We will see that this brings up a troubling issue involving the matching of the holographic model to QCD.

II. MATCHING OF THE SCALAR SECTION TO QCD

The equation of motion of the scalar field for general 4D momentum \( q \) is

\[
\partial_z(z^{-3} \partial_z \phi(z, q)) + \frac{3 + z^2 q^2}{z^5} \phi(z, q) = 0 ,
\]

(10)
The solution involves Bessel functions, and can be matched for small $q$ to Eq. (1). To compute the two point correlation function, we evaluate the scalar field action on a solution $\phi_{cl}$ of Eq. (10) which leaves a boundary term:

$$S[\phi_{cl}] = \int d^4x \left(z^{-3}\phi_{cl}(z,q)\partial_z\phi_{cl}(z,q)\right)_{z=\epsilon}.$$  \hspace{1cm} (11)

To find the two point correlation function, we can now take two functional derivatives with respect to the source $\lim_{z \to 0} \phi(z,q) = aM$, and in the large $q$ limit we find that

$$\int d^4xe^{ipx} \langle \bar{q}q(x)\bar{q}q(0) \rangle = \frac{a^2}{2} q^2 \log(q^2L^2) + \ldots ,$$  \hspace{1cm} (12)

where we have suppressed contact terms. This can be compared to the QCD result for large euclidean momentum $q^2$ and renormalization scale $\mu$:

$$\int d^4xe^{ipx} \langle \bar{q}q(x)\bar{q}q(0) \rangle = \frac{N_c}{8\pi^2} q^2 \log(q^2/\mu^2) + \ldots .$$  \hspace{1cm} (13)

This implies that $a = \sqrt{N_c/2\pi}$, which matches the $N_c$ scaling we expected on general grounds. This identification does not depend on any matching between $\mu$ and $L$, since both of them can be reabsorbed into the subleading terms in the equations above. Following the fitting procedure of ref. [1], but using $a$ as extracted above yields $m_q = 8.3$ MeV and $\sigma = (213 \text{ MeV})^3$, which differs from the results one obtains with $a = 1$, where $m_q = 2.29$ MeV and $\sigma = (327 \text{ MeV})^3$ [1].

Of course, even though $m_q$ and $\sigma$ are determined by matching to experimental data, they have a very specific interpretation in the holographic model that can be checked against QCD: $M = m_q 1$ is the source of the QCD operator $\langle \bar{q}q \rangle$. This presents a serious theoretical problem, since in QCD the value of $m_q$ and $\sigma$ are scale dependent. However, the quantities $\alpha m_q$ and $\alpha \sigma$ in the holographic model are fixed by matching fitting to the GOR relation and relating $f_2^2$ and $\sigma$. The computation of $a$ above then determines $m_q$ and $\sigma$, and since $a$ does not depend on a renormalization scale, neither do $m_q$ and $\sigma$, in conflict with the identification of $m_q$ as the source of the QCD operator $\langle \bar{q}q \rangle$.

This situation is actually rather common in phenomenological models of QCD, where one probes various observables that are scale-dependent in QCD in models where they do not depend on the scale in any systematic way. Examples of this include the treatment of structure functions in bag models, and the treatment of chiral condensates in Nambu-Jona-Lasinio (NJL) models [3]. The phenomenological models are generally taken to be at some “natural scale,” generally about 1 GeV, and the observables computed in the models are taken to correspond to QCD quantities evaluated at that scale. This is the interpretation taken in ref. [3] for $m_q$ and $\sigma$.

However, while this phenomenological approach may be reasonable in models such as NJL model, it is not consistent with the general structure of holographic models of QCD. One of the greatest attractions of holographic models of QCD is that they can be matched to QCD. The features and parameters of the holographic models are generally taken from the AdS/CFT dictionary, so that the 5D model is matched to QCD on the AdS boundary. To match the parameters of the holographic models to QCD, it is necessary to compute correlation functions in the bulk, and then match them on the AdS boundary at high 4D momentum $q^2$ to the equivalent QCD parameters evaluated at the same high $q^2$ and renormalization scale $\mu^2$. However, in the matching procedure for $a$ above, which is used to find $m_q$, the renormalization scale $\mu$ does not appear explicitly on the 5D side, since the 5D model is classical, so that the scale to which the holographic computations are supposed to correspond is not obvious.

Of course, in holographic models it is generally assumed that $1/z$ plays the role of $\mu$. However, this does not resolve the issues with scale dependent quantities like $m_q$ and $\sigma$. The issue is due to the fact that the AdS/CFT dictionary relates fields on the AdS $z = 0$ boundary to QCD quantities in the UV. The matching to QCD is done at asymptotically high scales, where it is weakly coupled, and $\mu \to \infty$, in accordance with the identification $1/z \sim \mu$ on the holographic side. However, in QCD [29], as $\mu \to \infty$ the quark mass $m_q$ runs to zero and chiral condensate $\sigma$ runs to infinity. Consistency with QCD then implies that in the holographic model $m_q$, which is fixed on the $z = 0$ boundary, should also be zero, while $\sigma$ in the model must diverge because of the GOR relation.

This is clearly inconsistent with the phenomenology of the model, which requires that $\sigma \neq \infty$ in order to have a finite splitting between the $\rho$ and the $a_1$ mesons. We note, moreover, that this problem does not go away in the chiral limit of $m^2_\pi = 0$, since $\sigma$ still diverges.

**III. DISCUSSION**

Our analysis above should not be surprising: the construction of the holographic model required a number of ad hoc assumptions that are clearly connected to this issue. The behavior of scale-dependent quantities like the chiral condensate is an explicit probe of the self-consistency of the assumptions. Clearly, if one wants to match the key features of large $N_c$ QCD in a consistent way, it is essential to capture the scale dependence of QCD on the 5D side of the model. This amounts to trying to improve on the the ad hoc approximations involved in the construction of the 5D model.

It is well known that the $\langle \bar{q}q \rangle$ has a scale-dependent anomalous dimension $\delta(\mu)$ proportional to $\alpha_s$ to leading order. Presumably, incorporating such an an anomalous dimension would change the 5D mass of the scalar field. Since the anomalous dimension depends on the scale $\mu \sim 1/z$, it is reasonable to make the 5D mass in a holographic model depend on $z$, $m^2_\pi = m^2_\pi(z)$, with the constraint that $\lim_{z \to 0} m^2_\pi(z) = -3$. Matching the 5D
model to QCD for $m_q$ would then amount to demanding that $m_q$ in the holographic model obey the same renormalization group equation as the quark mass in QCD \cite{11}. The anomalous dimension is proportional to the running coupling $\alpha_s$, suggesting that one must also modify the 5D background to allow the $\alpha_s$ to run \cite{14}. Also, one can include a more general potential in the action for the field $\phi$ \cite{32], at the price of increasing the number of parameters in the model.

Although we have focused our analysis on the simple model of ref. \cite{1}, the problems with scale dependence apply rather broadly to bottom-up holographic models of QCD, which as yet have not treated scale-dependence of quantities like $m_q$ and $\sigma$ consistently. It is an open question as to whether position-dependent 5D masses and more realistic 5D geometries could make the treatment of the chiral condensates in holographic models consistent with QCD.

The analysis here explicitly demonstrates that potentially serious consequences arise for scale-dependent quantities when one attempts to match holographic models to QCD in the weakly coupled asymptotic region. One can try to avoid the issues of matching to QCD in its weakly coupled regime by imposing a UV cutoff on the holographic models, as for instance in refs. \cite{33,34}. However, refs. \cite{33,34} showed this that a fit to data with the UV cutoff as an additional parameter then gives models that are defined on a rather short slice of AdS space, raising questions as to whether it is still reasonable to use the AdS/CFT dictionary directly.

Since the reasons for worrying about the consistency of the matching to the weakly coupled region are rather general, it is not implausible that problems may also arise even for conserved currents. Specifically, there are some subtleties associated with vector currents that we will discuss in a forthcoming publication.

In holographic models that do not systematically deal with scale-dependence, the natural way to evade the problems discussed in this Letter is to give up on matching the holographic models to QCD in the asymptotic regime, and simply fit all of the parameters of the 5D models phenomenologically. In the analysis above, this would correspond to fitting $m_q$ and $\sigma/a$ to data, without computing $a$ separately, and giving up on the identification of $m_q$ as the source of the QCD operator $\bar{q}Rq_L$. This has the disadvantage of losing many of the theoretical connections to QCD. This scenario is not ideal, but it is not obviously inconsistent. While it is conceivable that it may be possible to treat scale-dependence consistently by modifying the holographic models, it is clear that the issues that we have discussed in this Letter must be addressed in bottom-up holographic models of QCD.

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