Crossover from Intermittent to Continuum Dynamics for Locally Driven Colloids

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We simulate a colloid with charge $q_d$ driven through a disordered assembly of interacting colloids with charge $q$ and show that, for $q_d \approx q$, the velocity-force relation is nonlinear and the velocity fluctuations of the driven particle are highly intermittent with a $1/f$ characteristic. When $q_d \gg q$, the average velocity drops, the velocity force relation becomes linear, and the velocity fluctuations are Gaussian. We discuss the results in terms of a crossover from strongly intermittent heterogeneous dynamics to continuum dynamics. We also make several predictions for the transient response in the different regimes.

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An individual particle driven through an overdamped medium exhibits a linear velocity vs applied force relation. When quenched disorder is added to the system, a critical threshold driving force must be applied before the particle can move, and once motion begins, the velocity-force curves can be nonlinear. Examples of overdamped systems with quenched disorder that exhibit threshold forces and nonlinear velocity force curves include driven vortices in type-II superconductors or Josephson junctions [1], sliding charge density waves [2,3], driven magnetic bubble arrays [4], and charge transport in assemblies of metallic dots [5]. These systems have been studied extensively and exhibit a variety of dynamic phases as well as power law velocity force curves.

Another system that has been far less studied is an overdamped particle moving in the absence of quenched disorder but in the presence of a disordered two-component background of non-driven particles. Since there is no quenched disorder, simple overdamped motion with an increased damping constant might be expected. Instead, a critical threshold force $F_{th}$ for motion exists and the velocity force curves are nonlinear with the power law form $V = (F - F_{th})^p$, as shown in recent simulations [6] and experiments [7] for individual colloids driven through a background of non-driven colloids. Unlike systems with quenched disorder, for $F < F_{th}$ the entire system must move along with the driven particle. For $F > F_{th}$ the driven particle is able to shear past its neighbors. The simulations give $p = 1.5$ for colloids interacting with a screened Coulomb potential. The experiments were performed with lightly charged colloids where steric interactions are important, and give $p = 1.5$ in the lower density limit [7]. A particle moving through a viscous fluid can be regarded as interacting with many much smaller particles. In this limit, the surrounding medium can be replaced with a continuum and the velocity force curves are linear. In the case of a single driven colloid, it is not known when and how the dynamics change from nonlinear to linear, since it is expected that the system passes to the continuum limit when the colloids in the surrounding medium are small. This change could be a sharp transition, or it could occur through a continuously changing exponent.

In this work we consider a single colloid of charge $q_d$ driven through a $T = 0$ disordered two-component assembly of other colloids with average charge $q$. We find that when $q \approx q_d$, the velocity force curves have a power law form with $\beta > 1$ that is robust over two decades and for different system sizes. In this regime the velocity fluctuations of the driven colloid are highly intermittent and the colloid velocity $V$ frequently drops nearly to zero when background colloids trap the driven colloid until rearrangements release it and $V$ jumps back to a higher value. The fluctuations in $V$ have a highly skewed distribution and $1/f$ noise fluctuation properties. As $q_d$ increases for fixed drive $F_d$, the average velocity $V$ drops, the velocity fluctuations become Gaussian, and $\beta$ is reduced. When $q_d \gg q$, the driven colloid interacts with a large number of surrounding colloids and forms a circular depletion region, while for $q_d \ll q$, the background colloids act as stationary disorder and the velocity force curves are linear. Our system can be experimentally realized for dielectric colloids driven with optical traps or magnetic colloids driven with external magnetic fields. In systems where it is difficult to vary the charge on individual particles, $q_d$ could be increased by capturing a large number of particles in a single optical trap and dragging the assembly through the background. Other related systems include dragging different sized particles through granular media [8]. There have been several proposals to use individual particle manipulation as a new microrheology method for examining frequency responses in soft matter systems [9]. It would be valuable to understand under what conditions the individual particle is in the continuum overdamped regime or in the nonlinear regime in the dc limit.

We consider a substrate-free, zero temperature, two-dimensional system with periodic boundary conditions in the $x$ and $y$ directions. A binary mixture of $N = N_c - 1$ background colloids, charged with a ratio $q_1/q_2 = 1/2$, interact with a repulsive screened Yukawa potential, $V(r_{ij}) = (q_d q_i/|r_i - r_j|) \exp(-\kappa|r_i - r_j|)$, where $q_{(j)}$ is the charge and $r_{(j)}$ is the position of colloid $i(j)$ and $1/\kappa$
is the screening length which is set to 2 in all our simulations. Throughout the paper we refer to the average background charge \( q = (q_1 + q_2)/2 \). The initial disordered configuration for the two-component background of colloids is obtained by annealing from a high temperature. An additional driven colloid with charge \( q_d \) is placed in the system and a constant driving force \( \mathbf{F}_d = q_d \mathbf{x} \) is applied only to that colloid. The overdamped equation of motion for colloid \( i \) is

\[
\frac{d\mathbf{r}_i}{dt} = \mathbf{F}^{cc}_i + \mathbf{F}_d + \mathbf{F}_T
\]

where \( \mathbf{F}^{cc}_i = -\sum_{j \neq i} N \nabla_i V(r_{ij}) \), \( \eta = 1 \), and the thermal force \( \mathbf{F}_T \) comes from random Langevin kicks. We have considered various temperatures and discuss the \( T = 0 \) case here. We have previously used similar Langevin dynamics for colloids under nonequilibrium and equilibrium conditions [10]. The interaction range is assumed much larger than the physical particle size, and in this low volume fraction limit, hydrodynamic interactions can be neglected and may be strongly screened [11]. To generate velocity-force curves, we set \( F_d \) to a fixed value and measure the average velocity of the driven colloid \( \langle V \rangle \) in the direction of the drive for several million time steps to ensure that a steady state is reached. The drive is then increased and the procedure repeated. Near the depinning threshold \( F_d \gtrsim F_{th} \), the relative velocity fluctuations are strongly enhanced. In the absence of any other particles, the driven colloid moves at the velocity of the applied drive giving a linear velocity force curve. In this work we consider system sizes of \( L = 24, 36, \) and \( 48 \) with a fixed colloid density of 1.1. This is four times denser than the system considered in Ref. [6].

In Fig. 1 we show images from the two limits of our system. In Fig. 1(a) the driven colloid (large black dot) has \( q_d/q = 1.33 \) and is similar in charge to the colloids forming the surrounding disordered medium (smaller black dots). Fig. 1(b) illustrates the case \( q_d/q = 67 \), where a large depletion zone forms around the driven colloid.

To illustrate the scaling in the velocity force curves, we plot representative \( V \) vs \( F_d - F_{th} \) curves in Fig. 2 for varied \( q_d/q \) and different system sizes. Here \( F_{th} \) is the threshold velocity and the charge of the driven colloid increases from the top curve to the bottom. All of the curves have a power law velocity force scaling of the form

\[
V \propto (F_d - F_{th})^\beta
\]

This scaling is robust over two decades in driving force. To test for finite size effects, we conducted simulations with systems of size \( L = 24, 36, \) and \( L = 48 \), indicated by different symbols in Fig. 2, and we find that the same scaling holds for all the system sizes. The scaling of the velocity force curves for the small charge \( q_d/q = 0.25 \) is linear, as seen in previous simulations [6]. In this regime the driven colloid does not cause any distortions in the surrounding media as it moves. This situation is very similar to a single particle moving in a quenched background, where it is known that the velocity force curves scale linearly or sublinearly [2]. As the charge of the driven colloid increases, the scaling exponent initially rises, as shown for the case of \( q_d/q = 1.33 \) with \( \beta = 1.54 \), but the exponent decreases again for the more highly charged driven colloids, since \( q_d/q = 13 \) gives \( \beta = 1.28 \) and \( q_d/q = 67 \) gives \( \beta = 1.13 \). As \( q_d/q \) increases, the average velocity at fixed \( F_d - F_{th} \) decreases when more background colloids become involved in the motion.

We plot the changes in the scaling exponent \( \beta \) with varying \( q/q_d \) from a series of simulations in Fig. 3, which shows three regions. For low \( q_d/q \), the motion is mainly elastic with \( \beta \) near 1. As \( q_d/q \) approaches 1, the driven colloid charge becomes of the same order as that of the surrounding medium and the motion becomes plastic with \( \beta \approx 1.5 \). As \( q_d/q \) increases further, \( \beta \) decreases approximately logarithmically toward 1, indicating that the motion of the surrounding medium is becoming more continuum-like. The maximum value of
\(\beta\) falls at higher \(q_d/q\) for less dense systems, such as that in Ref. [6], and at lower \(q_d/q\) for denser systems.

For driven colloids with \(q_d/q \ll 1\), the background colloids act as a stationary disorder potential. The driven colloid passes through this potential, deviating around background colloids as necessary, but the background colloids do not respond to the presence of the driven particle and remain essentially fixed in their locations. In contrast, when \(q_d/q \gg 1\), the driven colloid strongly distorts the background and forms a depletion zone which moves with the driven colloid. Thus a large number of background colloids must rearrange in order to allow the driven colloid to pass, producing a continuum-like behavior. Between these two limits, when \(q_d/q \approx 1\), no depletion zone forms, but when the driven colloid moves, it distorts the background which deforms plastically in order to allow the driven colloid to pass. Here we find intermittent motion in which the driven colloid sometimes slips past a background colloid similar to the \(q_d/q \ll 1\) case, but at other becomes trapped behind a background colloid and pushes it over some distance, similar to the \(q_d/q \gg 1\) case. It is in the regime of this complex motion, when all of the charges are similar in magnitude, that the strongest deviation from linear response, \(\beta \approx 1.5\), occurs.

We next consider the velocity fluctuations of the driven colloid in the regime where \(\beta \approx 1.5\) as well as in the high \(q_d/q\) regime where \(\beta\) starts to approach 1. In Fig. 4(a) we plot a segment of the time series of the instantaneous driven colloid velocity for the case of \(q_d/q = 1.33\) at a drive producing an average velocity of \(V = 0.0425\). The motion is highly intermittent and at times the colloid temporarily stops moving in the direction of the drive. When the driven colloid is trapped, strain accumulates in the surrounding media until one or more of the surrounding colloids suddenly shifts by a distance larger than the average interparticle spacing and the driven particle begins to move again. As the drive is further increased, the length of the time intervals during which the driven colloid is stopped decreases.

In Fig. 4(b) we show \(V(t)\) for a system with \(q_d/q = 67\) where the applied force gives the same average velocity as in Fig. 4(a). Here the amplitude of the velocity fluctuations is much smaller than the \(q_d/q = 1.33\) case and there are no intermittent stall periods. This strongly charged driven colloid is interacting with a much larger number of surrounding colloids than the weakly charged driven colloid would, and as a result, it cannot be trapped behind a single background colloid, giving much smoother motion. We note that we find no intermittent behavior for the strongly charged driven colloid even at the lowest applied forces. We also measured the variance of the transverse velocity fluctuations \(V_y\), and find that it decreases with increasing \(q_d/q\) roughly as a power law with an exponent of -1.7. This decrease occurs since a larger number of background colloids are contributing to the fluctuations experienced by the driven particle, leading to a smoother signal.

In Fig. 4(c) we plot the histogram of the velocity fluctuations \(P(V)\) for the time series shown in Fig. 4(a). The fluctuations are non-Gaussian and are heavily skewed toward the positive velocities with a spike at \(V = 0\) due to the intermittency. We note that in simulations of vortex systems where nonlinear velocity force scaling occurs, bimodal velocity distributions are also observed when individual vortices are intermittently pinned for a period of time before moving again [12]. Non-Gaussian velocity fluctuations are also found in sheared dusty plasmas [13].
For higher drives, we find that the average velocity increases; however, the histograms remain highly skewed for all charge and drive regimes where the scaling in the velocity force curves give a large $\beta \sim 1.5$. For comparison, in Fig. 4(d) we plot $P(V)$ for the large $q_d/q$ system shown in Fig. 4(b). Here the histogram has very little skewness and fits well to a Gaussian distribution. For other drives at this charge ratio we observe similar Gaussian distributions of the velocity. In general, we find decreasing skewness in the velocity distributions as $q_d/q$ increases. The Gaussian nature of the fluctuations is also consistent with the interpretation that, as $q_d/q$ becomes large, the system enters the continuum limit.

In Fig. 4(e), we show that the power spectrum $S(\nu)$ for the time series in Fig. 4(a) has a $1/\nu^\alpha$ form with $\alpha = 0.8$. Throughout the $\beta \approx 1.5$ regime we find similar spectra with $\alpha = 0.5$ to 1.1, indicative of intermittent dynamics. For comparison, in Fig. 4(f) we show that $S(\nu)$ for the high $q_d/q$ case has a white velocity spectrum characteristic, indicative of the absence of long time correlations in the velocity. In general, $\alpha$ decreases with increasing $q_d/q$. For the small charge regime of $q_d/q \ll 1$, where the driven colloid moves without distorting the background, we also observe a white noise spectrum. Here the velocity fluctuations are determined by the static configuration of the background particles, and $P(V)$ does not show any intermittent periods of zero velocity because if the colloid becomes trapped in this regime, there can be no rearrangements of the surrounding medium to untrap it.

To explore the transient behavior of the system, we consider the effect of a suddenly applied subthreshold drive of $F_d/F_{th} = 0.8$. In Fig. 5(a) we show the transient velocity response for a system with $q_d/q = 1.33$. Here the velocity relaxation is consistent with a power law decay, $V(t) \propto t^{-1.1}$. The driven colloid translates by several lattice constants before coming to rest with respect to the surrounding medium. The large velocity oscillations that appear at longer times are due to the local plastic rearrangements of the surrounding colloids as the driven colloid passes. We find a power law decay in the transient response for values of $q_d/q$ that give $\beta > 1.28$. If the suddenly applied subthreshold drive is small enough that no local rearrangements of the surrounding medium are possible, then an exponential decay of the velocity occurs instead. In Fig. 5(b) we show the transient response for the case of $q_d/q = 67$. Here the relaxation is fit to $V(t) \propto \exp(-t)$. The large velocity fluctuations that appeared for the smaller $q_d$ are absent. For all the large values of $q_d/q$ we observe an exponential velocity relaxation, and we also find exponential relaxation for very small charges $q_d/q \ll 1.0$.

In summary, we have studied a single colloid with varying charge driven through a disordered background of other colloids in the absence of quenched disorder. When the charge of the driven colloid is close to the same as that of the surrounding colloids, a nonlinear power law velocity force curve appears with an exponent near $\beta = 1.5$. In this regime, the time dependent velocity is intermittent with a highly skewed velocity distribution and $1/f^{\alpha}$ noise fluctuations. As the charge of the driven colloid is increased, the velocity force characteristic becomes more linear while the effective damping from the background colloids increases. The number of colloids that interact with the driven colloid increases and the velocity fluctuations become Gaussian with a white noise spectrum. We predict that in the nonlinear regime, the transient velocity responses are of a power law form, while in the linear regime the transient velocity is exponentially damped. We interpret our results as a crossover in the response of the background colloids from intermittent dynamics when the driven and background colloids are similarly charged to continuum dynamics for a highly charged driven colloid.

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