AMPLITUDE OF PRIMEVAL FLUCTUATIONS FROM COSMOLOGICAL MASS DENSITY RECONSTRUCTIONS

Uroš Seljak and Edmund Bertschinger
Department of Physics, MIT, Cambridge, MA 02139 USA

ABSTRACT

We use the POTENT reconstruction of the mass density field in the nearby universe to estimate the amplitude of the density fluctuation power spectrum for various cosmological models. We find \( \sigma_8 \Omega_m^{0.6} = 1.3_{-0.3}^{+0.4} \), almost independently of the power spectrum. This value agrees well with the COBE normalization for the standard CDM model, while some alternative models predict an excessive amplitude compared with COBE. Flat low \( \Omega_m \) models and tilted models with spectral index \( n < 0.8 \) are particularly discordant.

Subject headings: cosmic microwave background — cosmology: large-scale structure of the universe

1. Introduction

A central problem in cosmology today is the nature and abundance of the matter in the universe. While a given matter content uniquely determines the shape of the power spectrum of density fluctuations (up to variations in the primeval shape), the amplitude of mass fluctuations remains unspecified theoretically and its value must be sought from observations. After the COBE discovery of primeval fluctuations (Smoot et al. 1992) it became possible to use temperature fluctuations at the surface of last scattering to compute the small scale normalization \( \sigma_8 \) (the rms relative mass fluctuation in a sphere of radius \( 8 h^{-1} \) Mpc, \( H_0 = 100 \) h km s\(^{-1}\) Mpc\(^{-1}\)) for any particular model (Wright et al. 1992; Efstathiou, Bond, & White 1992; Adams et al. 1993). This allowed researchers to shift...
their emphasis from the amplitude determination to the study of the spectral shape. Any additional independent amplitude determinations would enable one to discriminate between different models of structure formation. The COBE results are able to provide a constraint on the shape of the primordial power spectrum, but not on the matter content of the universe, because on the scales probed by COBE different matter contents produce similar temperature fluctuations. Small scale CMB anisotropy experiments are just beginning to be useful for determining the spectral shape (e.g., Bond et al. 1991; Dodelson & Jubas 1993; Górski, Stompor & Juszkiewicz 1993).

An alternative approach is to use measured redshift-distance samples to determine the amplitude of mass fluctuations. This method has the advantage that the reconstructed peculiar velocities are directly sensitive to the underlying mass distribution. Most work to date has been based on estimates of the bulk flow averaged over large volumes (Bertschinger et al. 1990; Courteau et al. 1993). These statistics have several limitations. First, the statistical distribution of spectral amplitude estimates based on the bulk flow is broad ($\chi^2$ with only 3 degrees of freedom) and consequently bulk flows can only weakly constrain the models. Second, bulk flows are particularly sensitive to the systematic errors introduced by nonuniform sampling (the sampling gradient bias of Dekel, Bertschinger, & Faber 1990, hereafter DBF). Finally, the scales contributing to the bulk flow estimates are large ($\gtrsim 40–60 \, h^{-1} \, \text{Mpc}$), whereas most of the peculiar velocity measurements come from smaller distances ($10–30 \, h^{-1} \, \text{Mpc}$). It should be possible to place additional independent model constraints on the smaller scales alone. Several groups have combined different peculiar velocity data to probe models on a range of scales (e.g., Del Grande & Vittorio 1992; Muciaccia et al. 1993; Tormen et al. 1993), but all except Tormen et al. use bulk flow estimates with their associated uncertainties.

In this paper we estimate the amplitude of mass fluctuations on intermediate scales by applying the maximum-likelihood method directly to the POTENT reconstruction of density perturbations from the peculiar velocities (Bertschinger & Dekel 1989; DBF). This analysis assumes that the measured velocities are a fair tracer of the underlying velocity field, which was induced by the underlying gravitational field. Use of a large sample of peculiar velocity data reduces both the statistical and systematic errors, provided that care is exercised concerning nonlinear corrections and Malmquist bias effects. We test several models: standard cold dark matter (CDM), CDM plus a cosmological constant (CDM+$\Lambda$), and CDM plus massive neutrinos (CDM+HDM), with several different choices for the Hubble constant and the primeval spectral index. Because we use the relatively sparse 1990 dataset we do not attempt to discriminate between the models using the velocity data alone. Instead, we compare our results to the COBE normalization to derive some conclusions about the viability of the models. We also briefly comment on the agreement
with other methods of amplitude estimation.

## 2. Method and data analysis

Assuming a potential flow the present velocity field can be extracted from observed radial peculiar velocities of galaxies (e.g. Lynden-Bell et al. 1988) by integrating along radial rays. This is the essence of the POTENT reconstruction method (Bertschinger & Dekel 1989; DBF). Furthermore, in linear perturbation theory there is a simple relation between velocity and density fields (Peebles 1980): \( \delta(r) = -(H_0 f)^{-1} \nabla \cdot \mathbf{v}(r) \equiv f^{-1} \tilde{\delta}(r) \), where \( \delta(r) \) is the density fluctuation at position \( r \), \( H_0 \) is the Hubble constant and \( f \) is the growing mode logarithmic growth rate. For CDM and CDM+Λ models at low redshifts \( f \) is well approximated by \( f(\Omega_m) = \Omega_m^{0.6} \) (Peebles 1980; Lahav et al. 1991), where \( \Omega_m \) is the total matter content in the universe. For the CDM+HDM model \( f \) is in general a function of the wavenumber \( k \) (see section 3.2). We introduce \( \tilde{\delta}(r) \) as the measured quantity to be compared with the theoretical predictions.

The input data and their treatment are described in Bertschinger et al. (1990). We discard all points with \( r > 50h^{-1} \) Mpc and with the distance to the fourth nearest neighbor \( R_4 > 10h^{-1} \) Mpc, thus keeping only the points with small sampling and measurement errors. At the end we are left with \( N = 111 \) data points \( \tilde{\delta}_i \) on a grid of spacing \( 10h^{-1} \) Mpc (with gaussian smoothing radius \( 12h^{-1} \) Mpc), which we use for comparison with the theoretical predictions. Despite the many data points there are only about 10 independent degrees of freedom in the sample (Dekel et al. 1993), which currently prevents us from extending the analysis beyond the amplitude determination.

Given the data one can estimate the power spectrum parameters using the maximum likelihood method. The initial density perturbations are assumed to constitute a gaussian random field in all the models that we study here. Nonlinear effects and nontrivial coupling between signal and noise (sampling and measurement errors) in the data affect the distributions and in general the resulting data are not normally distributed. However, one can still define a gaussian likelihood function and use its maximum as a statistic to estimate the unknown parameters. Moreover, if the deviations from a normal distribution are small, the increase in variance owing to the use of the wrong likelihood function should be small. We use Monte Carlo simulations with the correct distributions to estimate the bias and variance of the estimated parameters.
We define the likelihood function as

\[ L(\sigma_8) = \frac{1}{\sqrt{2\pi}^N |M|} \exp \left[ -\frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} M_{ij}^{-1}(\tilde{\delta}_i - \langle \tilde{\delta}_i \rangle)(\tilde{\delta}_j - \langle \tilde{\delta}_j \rangle) \right] , \quad (1) \]

where \( M_{ij}^{-1} \) and \( M \) are the inverse and determinant of the correlation matrix \( M_{ij} = \langle (\tilde{\delta}_i - \langle \tilde{\delta}_i \rangle)(\tilde{\delta}_j - \langle \tilde{\delta}_j \rangle) \rangle \). Here \( \langle \rangle \) denotes averaging over the random field ensemble (signal) and distance errors (noise). For a given theoretical model, the correlation matrix depends on the parameters of the power spectrum, most notably on its amplitude. An estimate of the amplitude \( \sigma_8 \) is given by the value that maximizes \( L \). In general this estimate is biased, in part because the maximum likelihood estimator is only asymptotically unbiased, but also because of our assumption of normality and owing to the presence of sampling gradient and other biases in the \( \{ \tilde{\delta}_i \} \). We estimate the statistical bias by Monte Carlo simulations and correct for it as described below.

To apply the method in practice, one needs to compute the correlation matrix \( M_{ij} \). The matrix has contributions from both the noise and the true underlying signal. The signal contribution is given by

\[ M_{ij}^s = \int d^3k \ f^2(\Omega_m, k) W^2(k) P(k) e^{ik \cdot (r_i - r_j)} , \quad (2) \]

where \( P(k) \) is the density fluctuation power spectrum and \( W(k) \) is the smoothing window function in \( k \)-space. Note that we have retained a possible \( k \) dependence of \( f \).

Measurement noise arises from errors in the galaxy distance estimates and from the sparse spatial sampling of the data. A detailed analysis, performed in the appendix of DBF, shows that the noise contribution is correlated with the signal contribution in \( M_{ij} \). Thus, one cannot compute each of the two contributions separately and then add them together with the appropriate amplitude, rather, one must compute \( M_{ij} \) directly for each amplitude. We computed \( M_{ij} \) using Monte Carlo simulations as described by DBF and Bertschinger et al. (1990), with input peculiar velocities given by a random sample of the linear velocity field from a particular gaussian theory (e.g., CDM). We evaluated the radial velocities at the positions of real galaxies and added noise assuming a lognormal distribution of distance errors (in contrast with the normal distribution used in the earlier work). For each model and for a dense grid of amplitudes, 500 POTENT reconstructions were averaged over the signal and noise ensembles to compute \( \langle \tilde{\delta}_i \rangle \) and \( M_{ij} \).

Before discussing the results we have to address the possible effects of bias \( \langle \tilde{\delta}_i \rangle \). As discussed in DBF, the two main contributions are from the sampling gradient bias and the Malmquist bias. The first one is adequately taken into account by the Monte Carlo simulations, because they use similar sampling of space as the real data. On the other
hand, Malmquist bias computed by the Monte Carlo simulations is not exactly the real bias present in the data. Since the real data had the homogeneous Malmquist bias subtracted, the only bias left is the density gradient bias \((\partial \ln n / \partial \ln r)(\sigma^2/r)\). However, the bias in the Monte Carlo samples is \((3.5 + \partial \ln n P / \partial \ln r)(\sigma^2/r)\), where \(P(r)\) is a poorly known selection function. Without knowledge of \(P\), one cannot properly correct the simulations for the Malmquist bias. To test the importance of inhomogeneous Malmquist bias on the parameters we wish to estimate, we compare the results for two simple cases. In first case, we correct the real data for the Monte Carlo bias, while in the second we do not. This is a correct treatment of the bias if the density gradient bias is negligible in the first case or the volume and selection function contributions to the bias exactly cancel in the second case. We find that the final mass amplitudes differ from each other by about 10%. This is significantly smaller than the statistical errors of our amplitude estimates.

Another possible source of error are nonlinear effects, which tend to change the densities and velocities compared to their linear values. The real velocities are nonlinear while our simulations are strictly linear. Due to the large smoothing radius, the differences are small in the case studied here, even if the unsmoothed density fluctuations are large. To test this one can also compute the true density field by applying the following correction in the quasi-linear regime \((-0.8 \lesssim \delta \lesssim 4.5)\): \(f^{-1}\delta = \delta/(1 + 0.18\delta)\) (Nusser et al. 1991). This changes our final estimated amplitudes by 5–10%, depending on the value of \(f\). Note that inhomogeneous Malmquist bias tends to increase the amplitude of the measured peculiar velocities, while nonlinear effects decrease the linear \(\delta\). Because both effects are small (\(\sim 10\%\)) and opposite in sign, we will neglect them.

3. Results

Our analysis was restricted to the simplest generalizations of standard CDM model that decrease the power on small scales relative to that on large scales and have been proposed recently as viable models of large scale structure. We computed transfer functions for the models by integrating the coupled linearized relativistic Einstein, Boltzmann, and fluid equations for baryons, CDM, photons, and neutrinos. In all cases, we fix the baryon contribution to \(\Omega_B = 0.0125h^{-2}\), as given by the nucleosynthesis constraint (Walker et al. 1991).

For each model considered, we estimated the maximum-likelihood value of the amplitude, which we denote \(\sigma_{8,v}\). The error distribution of estimated \(\sigma_{8,v}\) from the Monte Carlo samples is somewhat asymmetric with longer tails toward larger values (fig. [ ]). For
all the models, the relative one-sigma errors are well approximated by \((+0.3/−0.25)\). To this one must add the sum of systematic errors due to the Malmquist bias in the data and to residual nonlinear effects, whose sign is unclear but whose magnitude is, at most, about 10%.

The results for different models are summarized in Table 1. Our estimates of \(\sigma_{8,v}\) from POTENT include a small statistical bias correction \((\sim 5\%)\). In addition, in Table 1 we include \(\sigma_{8,l=2}\), the COBE normalization based on assuming a value of \(Q_{\text{rms–PS}} = 15.7 \exp[0.46(1−n)]\) µK (Smoot et al. 1992; Seljak & Bertschinger 1993). We also include the age of the universe as a possible additional constraint for these models. Despite the complicated window function, we find that there is only a weak dependence of \(\sigma_{8,v}\) on the shape of the power spectrum. This is not surprising considering that the POTENT sample is sparse at large distances and has been smoothed with a gaussian of radius \(12h^{-1}\)Mpc. We find that \(\sigma_{8,v}\) depends mainly on \(\Omega_{m}\) through \(f(\Omega_{m})\). An approximate value valid through most of the parameter space is thus

\[
\sigma_{8,v} \Omega_{m}^{0.6} = 1.3^{+0.4}_{-0.3}.
\]

95% confidence limit intervals give \(\sigma_{8} \Omega_{m}^{0.6} \sim 0.7−2.3\). For a given model (and thus, for a given value of \(n\)), the relative 1σ uncertainty of \(\sigma_{8,l=2}\) is only 17% (Seljak & Bertschinger 1993; Scaramella & Vittorio 1993), which is 2 times smaller than the uncertainty of \(\sigma_{8,v}\). As a first approximation one can thus neglect the uncertainty of \(\sigma_{8,l=2}\) when comparing it with \(\sigma_{8,v}\).

### 3.1. CDM model

The first model we tested is the CDM model. We performed Monte Carlo simulations for the standard case with \(h = 0.5\). The \(\sigma_{8,v}\) and \(\sigma_{8,l=2}\) amplitudes agree well with each other (see Table 1). In fact, the amplitude predicted by the standard CDM model is consistent with the COBE normalization over most of the allowed range of \(h\). Thus, for example, \(\Omega_{m} = 1\) and \(h = 0.75\) (not shown in the Table) gives \(\sigma_{8,l=2} = 1.5\), which is still compatible within the uncertainties with \(\sigma_{8,v} = 1.3\). The CDM model cannot be ruled out based on the comparison between the velocity data and COBE. This conclusion agrees with the previous comparisons based on the bulk flow estimates on somewhat larger scales (Bertschinger et al. 1990; Efstathiou et al. 1992; Courteau et al. 1993), but the present analysis gives smaller uncertainty in the amplitude and thus places more stringent limits on the models.
3.2. CDM+Λ models

We studied a family of generalized CDM models adding a cosmological constant Λ with \( \Omega_m + \Omega_\Lambda = 1 \). Monte Carlo simulations were performed for the model with \( h = 0.8 \) and \( \Omega_\Lambda = 0.8 \). This model is a representative of the models which match the recent determinations of large scale galaxy clustering power (Maddox et al. 1990; Kofman, Gnedin & Bahcall 1993) and the value of \( h \) (Jacoby et al. 1992), yet is compatible with globular cluster ages. Our values for \( \sigma_{8,l=2} \) are lower than the corresponding values given by Efstathiou et al. (1992) owing to a different baryon content (which affects the density transfer function) and because we include the contribution from the time derivative of the potential integrated along the line of sight when \( \Lambda \neq 0 \) (Kofman & Starobinsky 1985; Görski, Silk, & Vittorio 1992). The agreement with the low \( \Omega_m \) model is not very good. While decreasing \( \Omega_m \) increases \( \sigma_{8,v} \) [due to the \( f(\Omega_m) \) factor], the time derivative of the potential integrated along the line of sight tends to decrease \( \sigma_{8,l=2} \) for a given \( \Delta T/T \) quadrupole. Decreasing \( h \) even further decreases \( \sigma_{8,l=2} \). We find that the results strongly constrain these models toward small \( \Omega_\Lambda \) values. 95% upper limits on \( \Omega_\Lambda \) are 0.6 for \( h = 0.8 \) and 0.4 for \( h = 0.5 \).

3.3. CDM+HDM models

A mixed dark matter model with \( \Omega_{CDM+B} = 0.7 \) and \( \Omega_\nu = 0.3 \) has recently emerged as one of the best candidates to explain the large-scale structure measurements (Schaefer, Shafi, & Stecker 1989; Davis, Summers, & Schlegel 1992; Taylor & Rowan-Robinson 1992; Klypin et al. 1993). In this model, the growth factor \( f(k) \) depends on wavenumber because free-streaming damps small wavelengths; \( f(k) \) ranges between 1 on large scales to \( \frac{1}{4}[(1 + 24\Omega_{CDM+B})^{1/2} - 1] \) on small scales (Bond, Efstathiou, & Silk 1980; Ma 1993). For \( h = 0.5 \), which corresponds to \( m_\nu = 7 \) eV, \( f = 1 \) on large scales and \( f \approx 0.8 \) on small scales, with a transition at \( k \approx 1 \) Mpc\(^{-1} \). Because this is a relatively small scale, the effect on our estimate of \( \sigma_{8,v} \) is small.

The results for this model in Table 1 imply that the CDM+HDM model is not strongly constrained, although the estimated \( \sigma_{8,v} \) is somewhat high compared to the \( \sigma_{8,l=2} \). Decreasing the \( \Omega_\nu \) contribution or increasing the value of \( h \) increases \( \sigma_{8,l=2} \) and thus reduces the discrepancy. We find excellent agreement between the two normalizations for a particular model of \( \Omega_\nu = 0.2 \) and \( h = 0.75 \), but of course other parameter values with smaller \( \Omega_\nu \) and/or larger \( h \) give acceptable results as well. In general, the difference
between CDM and CDM+HDM power spectra is small on large scales ($> 8h^{-1}\text{Mpc}$) and consequently the COBE normalized $\sigma_{8,l=2}$ differs little for the two models.

3.4. Tilted models

A third way to decrease small scale power relative to that on large scales is to tilt the primordial power spectrum $P(k) \propto k^n$ by decreasing $n$ (Adams et al. 1993; Muciaccia et al. 1993). The power spectrum for tilted models differs from its standard CDM counterpart ($n = 1$) on all scales, not just on small scales as for the CDM+HDM power spectra. Because the COBE scale is about 3 orders of magnitude larger than the $\sigma_8$ scale, a given $Q_{\text{rms-PS}}$ normalization drastically changes $\sigma_{8,l=2}$ even for modest changes in $n$. This is true despite the fact that the best-fit value of $Q_{\text{rms-PS}}$ increases when $n$ is decreased (Seljak & Bertschinger 1993). For the CDM transfer function with $h = 0.5$ we find

$$\sigma_{8,l=2} = 1.05 (1 \pm 0.17)e^{-2.48(1-n)},$$

similar to the expression based on the $10^\circ$ COBE normalization (Adams et al. 1993).

Low values of $n$ generally imply an excessively small value of $\sigma_{8,l=2}$ relative to $\sigma_{8,v}$. For the particular case $n = 0.75$ and $h = 0.5$, the discrepancy between $\sigma_{8,l=2}$ and $\sigma_{8,v}$ is more than a factor of 2 (see Table 1; the COBE normalization assumes no gravitational wave contribution). Including a possible gravitational wave contribution (Lucchin, Matarrese, & Mollerach 1992; Davis et al. 1992) further decreases $\sigma_{8,l=2}$, by a factor of $\sqrt{(3 - n)/(14 - 12n)}$. Higher values of $h$ allow somewhat lower values of $n$, but in general $n$ cannot differ from 1 by more than $\sim 0.1$-$0.2$. 95% confidence limits give $n > 0.85$ with no gravitational wave contribution and $n > 0.94$ with a gravitational wave contribution, assuming $h = 0.5$.

4. Summary

The analysis presented here gives the amplitude of mass fluctuations $\sigma_{8,v} \approx 1.3 \Omega_m^{0.6}$ in different models. Comparing the $\sigma_{8,v}$ with the COBE normalized $\sigma_{8,l=2}$, one can constrain different models of structure formation, due to the fact that the two normalizations work on very different scales. In general, all the usual extensions of CDM, which decrease the power
on small scales relative to that on large scales, decrease the $\sigma_{8,l=2}$ value relative to the standard CDM value, but leave $\sigma_{8,v}$ almost unchanged. While the estimated $\sigma_{8,v}$ agrees well with the COBE value for standard CDM, the agreement becomes worse for the extensions of CDM. This particularly strongly challenges the non-zero cosmological constant models and the tilted models. It also points to a somewhat lower value of $\Omega_\nu$ or a higher value of $h$ than have been assumed by most workers studying the mixed dark matter models.

Recently, using the constraints from the masses and abundances of rich clusters, Efstathiou, White, & Frenk (1993) obtained $\sigma_8 = 0.57 \Omega_m^{-0.56}$. This result is inconsistent with our result at more than the 2$\sigma$ level, although including all the sources of systematic errors could bring the two results into better agreement. Nevertheless, there is increasing evidence that the estimate of $\sigma_8$ from cluster abundances give lower values than estimates based on peculiar velocities (Lilje 1992; Henry & Arnaud 1991; Evrard 1989). The discrepancy may point to a significant systematic error in either of the two methods. Further investigations are needed to resolve this issue.

We thank Avishai Dekel for allowing us the use of the POTENT results. This work was supported by grants NSF AST90-01762 and NASA NAGW-2807.

REFERENCES

Adams, F.C., Bond, J.R., Freese, K., Frieman, J.A., & Olinto, A.V. 1993, Phys. Rev. D, 47, 426

Bertschinger, E., & Dekel, A. 1989, ApJ, 336, L5

Bertschinger, E., Dekel, A., Faber, S. M., Dressler, A., & Burstein, D. 1990, ApJ, 364, 370

Bond, J. R., Efstathiou, G., & Silk, J. 1980, Phys. Rev. Lett. 45, 1980

Bond, J. R., Efstathiou, G., Lubin, P. M., & Meinhols, P. R. 1991, Phys. Rev. Lett., 66, 2179

Courteau, S., Faber, S. M., Dressler, A., & Willick, J. A. 1993, ApJ, 412, L51

Davis, R. L., Hodges, H. M., Smoot, G. F., Steinhardt, P. J., & Turner, M. S. 1992, Phys. Rev. Lett. 69, 1856

Davis, M., Summers, F. J., & Schlegel, M. 1992, Nature, 359, 393
Dekel, A., Bertschinger, E., & Faber, S. M. 1990, ApJ, 364, 349 (DBF)
Dekel, A., Bertschinger, E., Yahil, A., Strauss, M., & Davis, M. 1993, ApJ, 412, 1
Del Grande, P., & Vittorio, N. 1992, ApJ, 397, 26
Dodelson, S. & Jubas, J. 1993, Phys. Rev. Lett., 70, 2224
Efstathiou, G., Bond, J. R., & White, S. D. M. 1992, MNRAS, 258, 1p
White, S. D. M., Efstathiou, G., & Frenk C. S. 1993, MNRAS, 262, 1023
Evrard, A. E., 1989, ApJ. 341, L71
Górski, K., Silk, J., & Vittorio, N. 1992, Phys. Rev. Lett., 68, 733
Górski, K., Stompor R. & Juszkiewicz, R. 1993, ApJ, 410, L1
Jacoby, G. H. et al. 1992, PASP, 104, 599
Henry, J. P., & Arnaud, K. A., 1991, ApJ, 372, 410
Klypin, A., Holtzman, J. A., Primack, J., & Regos, E. 1993, ApJ, in press
Kofman, L. & Starobinsky, A. 1985, Sov. Astron. Lett., 11, 271
Kofman, L. A., Gnedin, N. Y., & Bahcall, N. A. 1993, ApJ, 413, 1
Lahav, O., Lilje, P. B., Primack, J. R., & Rees, M. J. 1991, MNRAS, 251, 128
Lilje, P. B., 1992, ApJ, 386, L33
Lucchin, F., Matarrese, S., & Mollerach, S. 1992, ApJ, 401, L49
Lynden-Bell, D., Faber, S. M., Burstein, D., Davies, R. L., Dressler, A., Terlevich, R. J., & Wegner, G. 1988, ApJ, 326, 19
Ma, C.-P. 1993 MIT Ph.D. Thesis
Maddox, S. J., Efstathiou, G., Sutherland, W. J. & Loveday, J. 1990, MNRAS, 242, 43p
Muciaccia, P. F., Mei, S., de Gasperis, G., & Vittorio, N. 1993, ApJ, 410, L61
Nusser, A., Dekel, A., Bertschinger, E., & Blumenthal, G. R. 1991, ApJ, 379, 6
Peebles, P. J. E. 1980, The Large-Scale Structure of the Universe (Princeton: Princeton University Press)
Scaramella, R., & Vittorio, N. 1993, MNRAS, 263, L17
Schaefer, R.K., Shafi, Q., & Stecker, F. W. 1989, ApJ, 347, 575
Seljak, U., & Bertschinger, E. 1993, ApJ (Letters), in press
Smoot, G. F. et al. 1992, ApJ, 396, L1
Taylor, A. N., Rowan-Robinson, M. 1992, Nature 359, 396
Tormen, G., Moscardini, L., Lucchin, F., & Matarrese, S. 1993, ApJ, 411, 16
Walker, T. P., Steigman, G., Schramm, D. N., Olive, K. A., & Kang, H. 1991, ApJ, 376, 51
Wright, E. L. et al. 1992, ApJ, 396, L13
| $\Omega_{CDM+B}$ | $\Omega_\Lambda$ | $\Omega_\nu$ | $h$ | $n$ | $\sigma_{8,\nu}$ | $\sigma_{8,l=2}$ | age[Gyr] |
|-----------------|-----------------|-------------|-----|----|----------------|----------------|---------|
| 1.0             | 0.0             | 0.0         | 0.5 | 1.0| 1.4            | 1.05           | 13.1    |
| 0.2             | 0.8             | 0.0         | 0.8 | 1.0| 3.0            | 0.67           | 13.2    |
| 0.7             | 0.0             | 0.3         | 0.5 | 1.0| 1.3            | 0.66           | 13.1    |
| 0.8             | 0.0             | 0.2         | 0.75| 1.0| 1.3            | 1.20           | 8.7     |
| 1.0             | 0.0             | 0.0         | 0.5 | 0.75| 1.3            | 0.55           | 13.1    |

Table 1: $\sigma_{8,\nu}$, $\sigma_{8,l=2}$ and age of the universe for various models discussed in the text. Relative errors are $^{+0.4}_{-0.3}$ for $\sigma_{8,\nu}$ and $\pm0.17$ for $\sigma_{8,l=2}$. 
Fig. 1.— Distribution of $\sigma_8$ estimates from the 500 Monte Carlo simulations of the standard CDM power spectrum. The input amplitude value is $\sigma_8 = 1.5$. The statistical bias correction to $\sigma_8$ given by the above distribution is $-0.1$. Other models and amplitudes give similar error distributions.