Secondary Neutrinos from Tau Neutrino Interactions in Earth

Sharada Iyer Dutta, Mary Hall Reno and Ina Sarcevic
1 Department of Physics, SUNY Stony Brook, Stony Brook, NY 11794
2 Department of Physics and Astronomy, University of Iowa, Iowa City, Iowa 52242
3 Department of Physics, University of Arizona, Tucson, Arizona 85721

The energy dependence of “secondary” neutrinos from the process (ντ → τ → νµ → ¯ντ) for two input tau neutrino fluxes (Fντ → Eντ−1 and Eντ−2), assumed to have been produced via neutrino oscillations from extragalactic sources, is evaluated to assess the impact of secondary neutrinos on upward muon rates in a km3 detector. We show that the secondary fluxes are considerably suppressed for the steeper flux, and even for fluxes ∼ Eντ−1, the secondary flux will be difficult to observe experimentally.

Evidence of neutrino oscillations from measurements of the atmospheric νe and νµ fluxes leads one to the conclusion that νµ → ντ over distances characterized by the radius of the Earth for neutrino energies ∼ 1 GeV. The results of a two-flavor analysis yield Δm2 = 2.5 × 10−3 eV2 and bi-maximal mixing, sin2 2θ = 1. Three flavor analyses are consistent with this result.

Atmospheric neutrinos come from cosmic ray interactions with air nuclei, yielding hadrons, especially mesons, which decay to neutrinos. Neutrinos may also be produced at the sources of cosmic rays, where energetic protons interact with nucleons and photons at the source. The pions and kaons produced at these distances beyond the galactic nuclei and gamma ray bursters are parents of neutrino fluxes. Active galactic nuclei and gamma ray bursters are two proposed astrophysical sources are parents of neutrino fluxes. Active galactic nuclei and gamma ray bursters are two proposed astrophysical sources.

Bi-maximal mixing, in the context of astrophysical neutrino sources, results in flavor oscillation from the source ratio of fluxes: νe : νµ : ντ = 1 : 2 : 0 to flux ratios at the Earth of νe : νµ : ντ = 1 : 1 : 1, independent of the neutrino energy, given that astronomical distances are so large.

Halzen and Saltzberg pointed out in Ref. that high energy νe flux attenuation in the Earth differs from νµ flux attenuation due to the fact that the τ produced in charged-current (CC) interactions with nucleons decays before it loses energy. For each ντ lost in CC interactions, another ντ appears following each τ decay, albeit at a lower energy. Detailed evaluations of ντ flux attenuation in the Earth appear in Refs. and . Depending on the incident flux, the ντ flux shows a degree of “pile-up” as neutrinos of sufficiently high energy interact in the Earth and yield neutrinos at lower energies.

In a recent paper, Beacom, Crotty and Kolb have suggested that in addition to a pile-up of tau neutrinos, the signal of extragalactic tau neutrinos will be enhanced by the appearance of “secondary” neutrinos. These secondary neutrinos come from purely leptonic decays of τs. The idea is that while νℓ (ℓ = e, µ) fluxes starting, for example, at nadir angle 0 are extinguished for sufficiently high energies, they will be regenerated by the B = 0.18 branching fraction for τ → νℓ( ¯ντ), the τ being produced by ντ → τ CC interactions. The flux of ντ is less likely to be extinguished due to the shorter path-length through the Earth (the ντ already had to travel its “interaction distance,”) and its lower energy due to the combined energy loss in the CC process and the decay of the τ.

The evaluations in Ref. were for mono-energetic neutrinos. In this paper, we consider a few energy dependences for the ντ fluxes. We show that the secondary fluxes are considerably suppressed for steeply falling fluxes and even for fluxes ∼ Eντ−1, the secondary flux will be difficult to observe experimentally.

Neutrino attenuation in the Earth is governed by a coupled set of partial differential equations. To illustrate, we write the coupled equations for the ντ flux. For energy dependent flux Fντ (neutrinos/(cm2s sr GeV)),

\[ \frac{\partial F_{\nu\tau}(E, X)}{\partial X} = - \frac{F_{\nu\tau}(E, X)}{L_{\nu\tau}^{int}(E)} \]

\[ + \int_{E'}^{E} dE_y [G^{\nu\tau\to\nu\tau}(E, E_y, X) + G^{\nu\tau\to\nu\tau}(E, E_y, X)] \]

\[ \frac{\partial F_{\tau\tau}(E, X)}{\partial X} = \frac{F_{\tau\tau}(E, X)}{L_{\tau\tau}^{int}(E)} - \frac{F_{\tau\tau}(E, X)}{L_{\tau\tau}^{dec}(E)} \]

\[ + \int_{E'}^{E} dE_y [G^{\tau\to\tau}(E, E_y, X) + G^{\nu\tau\to\tau}(E, E_y, X)] . \]

Here, \( L_{\nu\tau}^{int} = 1/N_A \sigma_{\nu\tau} N \) and similarly for the τ interaction length, and \( L_{\tau\tau}^{dec} = \gamma c \tau / \rho \) for density ρ and Lorentz factor \( \gamma = E_\tau / m_\tau \). The quantity X is the column depth (in g/cm2), and for example,

\[ G^{\nu\tau\to\nu\tau}(E, E_y, X) = \frac{F_{\nu\tau}(E_y, X)}{L_{\nu\tau}^{int}(E)} d\sigma_{\nu\tau}^{NC}(E_y, E) . \]

The cross section normalized energy distribution of neutrinos with incident energy \( E_y \) and final energy E is represented by \( d\sigma_{\nu\tau}^{NC}/dE \) in Eq. (2).

There are similar equations for \( \bar{\nu}_\tau \) and \( \bar{\tau} \) fluxes. In our discussion of flux ratios in the introduction, we didn’t distinguish between neutrinos and anti-neutrinos. Models predict that \( F_\nu = F_{\bar{\nu}} \) to a good approximation. For our discussion in this section, we distinguish between particles and antiparticles since the secondary neutrinos are actually anti-neutrinos (\( \bar{\nu}_\tau \)) for incident \( \nu_\tau \). One must
keep in mind that there are equal incident neutrino and antineutrino fluxes of all three flavors in the context of neutrino oscillations. In the event rates evaluated below, we sum both neutrino and antineutrino contributions.

For nadir angle 0, $\mathcal{L}^{\nu\mu}_{\nu\mu}$ equals the Earth’s diameter when $E_\nu \simeq 40$ TeV. Tau neutrino pile-up and associated secondary anti-neutrino production is relevant at this energy and higher. At very high energies, we have shown that the effect of attenuation for a surface detector viewing at zero nadir angle is significant even for $\nu_\tau$. For example, for a $1/\mathcal{E}_\nu$ flux (with a smooth cutoff at $E_\nu = 10^8$ GeV), the attenuated flux of $\nu_\tau$ at $E_\nu = 10^6$ GeV is only 6% of the incident flux at that same energy. The $\nu_\tau$ and $\nu_\mu$ fluxes are about 1% of the incident flux at the same energy. Steeper fluxes have an even more significant attenuation and thus will have lower relative detection rates at high energies, whether it be $\nu_\tau$, $\nu_\mu$, or $\nu_\tau$ incident fluxes.

Guided by the falling fluxes and the increased attenuation, we confine our attention to the energy range of $E_\nu = 10^7 - 10^8$ GeV. For energies below $10^8$ GeV, $\mathcal{L}^{\nu\mu}_{\nu\mu} > \mathcal{L}^{\nu\nu}_{\nu\nu}$. Starting at $E \sim 10^8$ GeV, $G^{\tau\rightarrow\tau}$ becomes important, however, we neglect it here and confine our attention to lower energies. We also neglect the contribution of $\mathcal{L}^{\nu\mu}_{\nu\mu}$ in Eq. (1). With the fluxes considered below, the error in this approximation should be small for rates evaluated with minimum neutrino energies of $10^4 - 10^5$ GeV. With these approximations, the set of differential equations is simplified and solved using a modification of the iterative method detailed by Naumov and Perrone in Ref. 1. For $\nu_\mu$ fluxes, $G^{\mu\rightarrow\mu}$ from electromagnetic muon energy loss effectively eliminates any return of $\nu_\mu$ from CC interactions followed by $\mu$ decay in the energy range of interest.

The $\tau$ flux solution to Eq. (1) is responsible for generating the secondary neutrino flux. In the energy range of interest, one can write the differential equation for the $\bar{\nu}_\tau$ flux as:

$$\frac{\partial F_{\bar{\nu}_\tau}(E, X)}{\partial X} = -\frac{F_{\bar{\nu}_\tau}(E, X)}{\mathcal{L}^{\nu\mu}_{\nu\mu}(E)}$$  

$$+ \int_E^{\infty} dE_y \left[ G^{\nu\rightarrow\bar{\nu}_\mu}(E_y, X) + G^{\tau\rightarrow\bar{\nu}_\tau}(E_y, X) \right].$$  

In what follows, we set $G^{\nu\rightarrow\nu} = G^{\mu\rightarrow\mu}$. At sufficiently high energies, neutrino and antineutrino interaction rates are equal because the cross sections are dominated by the sea quark distributions. The energy at which the interaction length equals the column depth increases with nadir angle, so the approximation is best at larger nadir angles. Because the pile-up comes from higher energy neutrino interactions, this approximation is not unreasonable, as discussed below.

A second approximation to obtain the secondary flux is to take:

$$G^{\tau\rightarrow\bar{\nu}_\tau} \simeq B \cdot G^{\tau\rightarrow\nu_\tau},$$  

for $B = 0.18$, the branching fraction for $\tau \rightarrow \ell$. In fact, the $\bar{\nu}_\tau$ spectrum from $\tau$ decay is a little softer than the $\nu_\tau$ spectrum, so this is an approximation that will slightly overestimate the secondary flux.

With these two approximations, one finds that the combination $\Delta(E, X) = F_{\nu_\tau}(E, X) - F_{\bar{\nu}_\nu}(E, X)/B$ satisfies the same transport equation as $F_{\nu_\tau}(E, X)$. At $X = 0$, $\Delta(E, 0) = F_{\nu_\tau}(E, 0) = F_{\nu_\mu}(E, 0)$, so we can write:

$$F_{\nu_\tau}(E, X) \simeq B \cdot (F_{\nu_\tau}(E, X) - F_{\nu_\mu}(E, X)).$$  

Eq. (5) will be used in what follows to approximate the secondary antineutrino flux for two different incident spectra: $E_{\nu}^{-1}$ and $E_{\nu}^{-2}$.

In Fig. 1, we show the ratio of the attenuated flux to the incident flux at nadir angle $\theta = 0$, 30 and 60 degrees for

$$F_0^0 = F_\nu(E, X = 0) = N_1/E \cdot 1/(1 + E/E_0)^2$$  

where $E_0 = 10^8$ GeV and $N_1$ is a normalization factor, and for $F_0^1 \propto 1/E^2$. The dashed curve is the $\nu_\tau$ flux, the dotted curve shows the attenuation of the other two neutrino species. The solid curve, the result of Eq. (5), is the secondary $\bar{\nu}_\ell$ flux, were $\ell = e$ or $\mu$. Except for the smallest nadir angles for the $E_{\nu}^{-1}$ spectrum, the secondary antineutrino flux is a small correction to the primary attenuated electron neutrino or muon neutrino flux. Even at nadir angle zero, the secondary flux is negligible compared to the transmitted primary flux for the $1/E^2$ spectrum.

One should note that even though the $\nu_\tau$ flux dominates the primary and secondary $\nu_\mu$ fluxes, it does not dominate the contributions to the muon event rate because of the branching fraction $B = 0.18$ of $\tau \rightarrow \mu$ together with the effect of energy loss as the $\nu_\tau$ converts to a $\tau$ which then decays to a $\mu$.

As a quantitative illustration of the implications of the secondary neutrino flux, we consider the muon event rate from the $F_0^0 \nu_\tau \simeq N_1/E_\nu$ case (Eq. (6)) with $N_1 = 10^{-14}/(\text{GeV cm}^2 \text{ s sr})$ for each neutrino flavor. The normalization factor $N_1$ is chosen to be in line with the Waxman and Bahcall gamma ray burst flux of Ref. 4, in which $N_1 = 4 \times 10^{-13}$ (in the same units) for the sum of all neutrino species and for $E_\nu < 10^5$ GeV. The event rates shown below are for an underground detector of 1 km$^2$ effective area, for example, the proposed IceCube detector 5. Details of the calculation appear in Ref. 6. Our integrals over neutrino energies were performed from the minimum neutrino energy $E_\mu > 10^4$ or $10^5$ GeV, up to a maximum neutrino energy of $10^8$ GeV.

In Fig. 2, we include the contributions from $\nu_\mu \rightarrow \mu$ and $\nu_\tau \rightarrow \tau \rightarrow \mu$ (shown with the dashed lines) and in addition, the corresponding antineutrino induced antineutinos from $\bar{\nu}_\tau \rightarrow \bar{\nu}_\rightarrow \bar{\nu}_\mu \rightarrow \bar{\mu}$ (solid line). Antineutrino induced muons and antineutinos are also included. We note that for the $\nu_\mu \rightarrow \mu$ rates, doing the neutrinos and antineutrinos separately results in at most a $\sim 10\%$.
FIG. 1. The ratio of the attenuated neutrino flux for $\nu_\tau$ (dashed), $\nu_\mu = \nu_e$ (dotted) and secondary $\bar{\nu}_\ell$ (solid) to incident flux $F^{0}_\nu \propto E^{-1}$ (Eq. (6)) and $F^{0}_\nu \propto E^{-2}$. 
correction to the event rates at the energies shown here. This small correction is an indication that our approximations to obtain the secondary flux of neutrinos are not unreasonable.

The secondary neutrino contribution to the muon event rate has its largest relative contribution at nadir angle zero, with an enhancement over the $\nu_\tau \to \mu$ plus $\nu_\tau \to \tau \to \mu$ rate of 50-60% for the $1/E_\nu$ flux. At this angle, the ratio of rates of the secondary contribution to the muon event rate relative to the event rate of muons from tau decays is quite large, about 1.5 for $E_\mu > 10^4$ GeV and 2.6 for $E_\mu > 10^5$ GeV. Unfortunately, this is where the event rate is smallest and statistics are low. By a nadir angle of $\sim 60^\circ$ (1 rad), where the event rate is roughly a factor of 10-20 larger, depending on the minimum muon energy, the enhancement in the overall muon rate is about 25%. At this nadir angle, the secondary $\nu$ produced muons are equal to the tau decay muon rate for $E_\mu > 10^5$ GeV. The crossover occurs at $\theta \sim 0.7$ rad for $E_\mu > 10^4$ GeV. At the larger nadir angles, the $\nu_\mu \to \mu$ contribution to the muon event rate is dominant.

The normalization of the isotropic $1/E_\nu$ flux has been guided by the Waxman-Bahcall gamma ray burster flux of Ref. [4]. With $E_\nu = 10^8$ GeV in Eq. (6), our flux violates the Waxman-Bahcall bound [4] above $E_\nu \sim 10^6$ GeV, so the event rates in Fig. 2 may be optimistic. If the normalization $N_1$ is reasonable, then the secondary contribution to the muon rate will be difficult to observe. Low statistics will make it hard to have a meaningful comparison between the small and large nadir angle rates. Compounding the problem is that one does not expect to know the input flux energy dependence or normalization exactly.

For more rapidly falling fluxes, the contribution of secondary neutrinos to the event rates is smaller. The attenuated fluxes shown in Fig. 1 for $1/E_\nu^2$ yield secondary enhancements which are quite small. The secondary muon rate is only about 10-15% of the primary $\nu_\mu \to \mu$ rate at nadir angle zero. Typical theoretical neutrino fluxes have spectra that lie somewhere between the $1/E_\nu$ and $1/E_\nu^2$ cases in this energy range [4].

In summary, the energy dependence of the incident tau neutrino flux is crucial in evaluations of the implications of $\nu_\tau$ interactions to regenerate $\nu_\tau$ and secondary $\bar{\nu}_\mu$ and $\bar{\nu}_\tau$. Neutrino fluxes are attenuated due to their passage through the Earth, even the tau neutrinos, thus moderating tau neutrino contributions to secondary neutrino fluxes. Our evaluation has relied on approximations including Eq. (4), setting $E_\nu^\text{int} = 0$ and $G^\nu_{\nu'\nu} = G^{\nu'\bar{\nu}}$. If muon rates are determined to be large, even near nadir angle zero, then a more detailed evaluation of the secondary flux may be in order. With our current theoretical expectations for flux normalizations, however, the secondary neutrinos coming from $\tau$ decays will be difficult to observe experimentally, as they contribute significantly to a muon excess only at small nadir angles where the fluxes are already strongly attenuated and for spectra like $1/E_\nu$.

We thank J. Beacom for discussions. The work of S.I.D. has been supported in part by NSF Grant 0070998. The work of I.S. has been supported in part by NSF Grant No. PHY-9802403 and DOE under contract FG02-91ER40664. M.H.R. thanks the Fermilab Theory Group for its hospitality.

\[\begin{align*}
\text{FIG. 2. The muon event rate for muons with energy above} & \quad E_\mu = 10^4 \text{ and } 10^5 \text{ GeV originating from neutrinos with} \quad E_\nu < 10^8 \text{ GeV from } (\nu_\mu \to \mu) + (\nu_\tau \to \tau \to \mu) \text{ plus antiparticles (dashed) and additionally from secondaries via} \quad (\nu_\tau \to \tau \to \bar{\nu}_\mu \to \mu) \text{ plus antiparticles (solid) for the incident flux as in Eq. (6) and } N_1 = 10^{-13}/\text{GeV cm}^2 \text{ s sr} \text{ for each incident neutrino plus antineutrino flavor.}
\end{align*}\]

\[\theta [\text{radians}]\]

\[\begin{align*}
\text{FIG. 2. The muon event rate for muons with energy above} & \quad E_\mu = 10^4 \text{ and } 10^5 \text{ GeV originating from neutrinos with} \quad E_\nu < 10^8 \text{ GeV from } (\nu_\mu \to \mu) + (\nu_\tau \to \tau \to \mu) \text{ plus antiparticles (dashed) and additionally from secondaries via} \quad (\nu_\tau \to \tau \to \bar{\nu}_\mu \to \mu) \text{ plus antiparticles (solid) for the incident flux as in Eq. (6) and } N_1 = 10^{-13}/\text{GeV cm}^2 \text{ s sr} \text{ for each incident neutrino plus antineutrino flavor.}
\end{align*}\]

\[\theta [\text{radians}]\]

\[\begin{align*}
\text{FIG. 2. The muon event rate for muons with energy above} & \quad E_\mu = 10^4 \text{ and } 10^5 \text{ GeV originating from neutrinos with} \quad E_\nu < 10^8 \text{ GeV from } (\nu_\mu \to \mu) + (\nu_\tau \to \tau \to \mu) \text{ plus antiparticles (dashed) and additionally from secondaries via} \quad (\nu_\tau \to \tau \to \bar{\nu}_\mu \to \mu) \text{ plus antiparticles (solid) for the incident flux as in Eq. (6) and } N_1 = 10^{-13}/\text{GeV cm}^2 \text{ s sr} \text{ for each incident neutrino plus antineutrino flavor.}
\end{align*}\]

\[\theta [\text{radians}]\]
[9] F. Becattini and S. Bottai, *In Salt Lake City 1999, Cosmic ray, vol. 2* 249-252; F. Becattini and S. Bottai, Astropart. Phys. 15, 323 (2001) [arXiv:astro-ph/0003179].

[10] J. F. Beacom, P. Crotty and E. W. Kolb, Phys. Rev. D 66, 021302 (2002) [arXiv:astro-ph/0111482].

[11] R. Gandhi, C. Quigg, M. H. Reno and I. Sarcevic, Phys. Rev. D 58, 093009 (1998); Astropart. Phys. 5, 81 (1996), updated to use the CTEQ5 parton distribution functions of H. L. Lai et al. [CTEQ Collaboration], Eur. Phys. J. C 12, 375 (2000) [arXiv:hep-ph/9903282].

[12] S. I. Dutta, M. H. Reno, I. Sarcevic and D. Seckel, Phys. Rev. D 63, 094020 (2001) [arXiv:hep-ph/0012350].

[13] V. A. Naumov and L. Perrone, Astropart. Phys. 10, 239 (1999) [arXiv:hep-ph/9804301].

[14] E. Waxman and J. N. Bahcall, Phys. Rev. Lett. 78, 2292 (1997) [arXiv:astro-ph/9701231]; E. Waxman and J. N. Bahcall, Phys. Rev. D 59, 023002 (1999) [arXiv:hep-ph/9807282].

[15] See, for example, URL [http://icecube.wisc.edu/](http://icecube.wisc.edu/).