COMPENSATION PLAN, PRICING AND PRODUCTION DECISIONS WITH INVENTORY-DEPENDENT SALVAGE VALUE, AND ASYMMETRIC RISK-AVERSE SALES AGENT

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ABSTRACT. In this paper, we investigate the joint decision on production and pricing, and the compensation strategy of a supply chain, where the manufacturer relies on a risk-averse sales agent to sell the products. The sales outcome is determined by the sales agent’s selling effort and the product price. Most of the previous research about salesforce assumes that the risk attitude to an agent is known to each other and the salvage value is a constant. In this study, we have considered that the salvage value is a function of inventory, and both of the sales agent’s selling effort and risk attitude are their private information on the general framework of dual information asymmetric. With the help of revelation principle and principal-agent theory, we have been able to derive the optimal compensation contracts, and joint decision on production and pricing for the manufacturer. Analyzing them and comparing to the symmetric scenario, we found that only the optimal production strategy and the manufacturer’s profit depended on the variation rate of salvage value. When the manufacturer comes across asymmetric risk-averse sales agents its profit decreases, whereas the sales agent with private information obtains higher income but exerts less effort, which implies the value of information. The results also mean that the manufacturer should not only focus on offering a lower commission rate to the more risk-averse sales agent, but also on screening his risk information.

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1. Introduction. With the refined social division of labor, manufacturers depend heavily on sales agents to sell their products to the customers, increase sales quantity, and become more profitable. More sales need more sales effort which is costly to the sales agent, resulting in conflicting goals for the manufacturer and the sales agent. A great number of firms spent a great amount of resources on salesforce compensation. It is reported that for companies involved in B2B operations the compensation cost of their salesforce is about 40% of their total sales spending. As discussed by Zoltners et al. [26], salesforce compensation amounted to 800 billion dollars by US firms in 2006 alone, which was three times of the advertising expenses. An extensive focus on research was diverted to the salesforce compensation design and its disciplines like marketing, economics and operations. As a result, salesforce compensation designs has been a classic research stream in various disciplines, including marketing, economics, and operations. For example, many automobiles as well as the electronics manufacturers sell their products in aboard through sales agents. Price is always considered as a key factor, affecting the manufacturers, dealers, customers and market future. A sensible pricing policy is, therefore, important to maintain manufacturer’s interest, mobilizing the enthusiasm for dealers, attracting customers, winning the competitors, developing and consolidation of the market. The manufacturer fixes sales price decision to reflect fairness to the end customer. A question, therefore, arises on how to deal the problem of designing compensation plans and setting sensible sales price, which always happen to be the biggest challenges to the manufacturer.

Sales compensation plans, their strategies relating to pricing and production/order have attracted much attention as the historic models were based on risk neutrality, where decision makers made decisions to maximize their expected profits. In the last decade, researchers adopted risk-averse models for supply chain management, to reflect the risk preferences in decision making, in order to represent more realistic settings (see [5], [8], [14], [15], [16], [22]). In these models, the risk attitude is usually modeled as a parameter in the utility function, and it is assumed to be common knowledge (see [5], [22]). Many pieces of literature proposed that decision makers have risk preference; it is further acknowledged that, estimating the exact utility function or risk attitude to decision makers is difficult, i.e., the degree of risk aversion is their private information which other participants cannot observe (see [8], [9]). Pavlov and Katok [20] studied a model of coordinating contracts considering the fairness behavior, found that the inability of theoretically-coordinated supply chains to obtain coordination was due to incomplete information, and explained many problems in contract empirical research, such as rejection, low efficiency, etc. Katok and Pavlov [12] further investigated the causes of three factors (inequality aversion, bounded rationality, and incomplete information) on the inefficiency of coordinating a simple supplier-retailer channel. The main results are that incomplete information about the retailer’s degree of inequality aversion plays a more important role than bounded rationality in explaining the suppliers' behavior. Therefore, in this paper, we consider the sales agent’s degree of risk aversion as his private information.

In addition, most of the literature on salesforce assumes that the salvage value is a constant. Aviv and Pazgal [1] proposed that customers exhibit strategic behavior, in doing so, they may decide to postpone their purchases, if they believe that a later purchase at a lower price may bring a higher expected surplus than what they can gain by an immediate purchase. However, in real life, retailers often adjust the price
at the end of sales season, and sell it based on the surplus of inventory. In general, the more the surplus inventory in clearance period, the lower the salvage value, i.e., it is decreasing as the surplus inventory increases. Therefore, it is necessary to understand on how to design effective sales incentive contract with incomplete information and inventory-dependent salvage value, and further study the joint effect of asymmetric information and the variation rate of salvage value of the supply chain system.

We consider a salesforce compensation, pricing and production planning problems where the sales agent has private risk-averse information and makes sales effort, unobservable to the manufacturer. The manufacturer makes optimal pricing and production decisions, and offers a menu of compensation contracts to allow the sales agent to self-select under inventory-dependent salvage value. We have designed a linear incentive contract to induce the risk-averse sales agent, to select and maximize the manufacturer’s expected profit. The analytical results have shown that only the optimal production strategy and the manufacturer’s profit depended on the variation rate of salvage value, the manufacturer however was observed to offer a lower commission rate to a sales agent who is more risk averse. When the manufacturer faces the asymmetric risk-averse sales agents, its profit decreases while the sales agent with private information, obtains higher income but exerts less effort, in other words, information plays an important role.

The contribution of this paper is to formally introduce various aspects of our model; firstly, our model is a combination of moral hazard and adverse selection. Besides, the sales agent’s selling effort is unobservable, his degree of risk aversion is also his private information, and the manufacturer’s compensation plan and joint decision on production and pricing are based upon his own knowledge and probability. Secondly, we have modeled the compensation plan using inventory-dependent salvage value. Thirdly, our model indicates that the manufacturer should hire a lower risk-averse agent, and try to make use of his actual degree of risk-averse information. We considered a different setting with joint pricing and production planning, and salesforce compensation, then examined the impacts of the private risk as well as the variation rate of salvage value. We observed that only the manufacturer’s profit and the optimal production strategy depend on the variation in the rate of salvage value. A linear compensation contract is designed by the manufacturer for the sales agents to reveal their private degree of risk aversion and simultaneously maximizing their expected profit. We obtained the pricing and production plan, and designed a closed-form linear contract to reveal the private degree of risk aversion considering inventory-dependent salvage value.

The rest of this paper is organized as follows. We reviewed in Section 2 the different streams of literature relevant to our research and describe the problem in Section 3. In Section 4, the benchmark model is presented. Section 5 analyzed the joint decision on production and pricing, and the contract design problem from the sales agent’s perspective and studied the characteristics. In Section 6, we provided analytical results to show the impact of the asymmetric risk attitude and the variation rate of salvage value, and the results of symmetric scenario are briefly presented for comparison purpose. Then some numerical examples were presented. Finally, we gave concluding remarks and suggestions for future research in Section 7. All proofs are presented in the Appendix.
2. Literature review. Our work is closely related to the extensive literature on the salesforce incentive and pricing and production/ordering decisions, principal-agent theory and asymmetric information. In the following, we briefly review this stream of literature.

The modeling of compensation on salesforces has been one of the marketing management research issues since the early work of Basu et al. [2]. A comprehensive review of the topic can be found in Coughlan [7]. Salesforce management has been vastly studied in the analytical marketing literature. Chao et al. [4] studied a dynamic inventory and pricing optimization problem in a periodic review inventory system with setup cost and finite ordering capacity in each period. Huang et al. [11] modeled a dual-channel supply chain that experiences a disruption in demand and examined changes in the pricing and production quantity decisions. Oh et al. [18] studied coordinated pricing and production decisions in an assemble-to-order system in which a firm makes pricing and production decisions in an ATO system over $T$ time periods. Qin et al. [21] considered the pricing and lot-sizing problem for products with quality and physical quantity deteriorating simultaneously. These studies are based on full information. Many researchers studied the compensation incentives to induce a sales agent to disclose what he knows about the hidden cost (see [3], [6], [13], [19], [25]), hidden market condition (see [5], [14], [15], [22]), or hidden demand (see [16]). Gonik [10] reported a clever scheme under which it is in the salesperson’s interest to forecast accurately and to work hard. Kaya and Ozer [13] combined the moral hazard and asymmetric cost information in an OEM-CM relationship. Liu et al. [17] investigated the online dual channel supply chain system and its joint decision on pricing and production under asymmetric cost information. Xu et al. [24] studied the effects of the presence of a contingent urgent supplier with private cost information on the performance of both the prime supplier and the manufacturer. Ozer and Gal [19] investigated the information about the supplier’s asymmetric production cost affecting the profits and contracting decision. Chen et al. [6] considered a supply chain in which a CM assembles a product for a large OEM and at the same time produces a different product for a smaller OEM under asymmetric cost information. Cao et al. [3] studied the incentive contracts that can improve the supply chain performance when the cost information in the dual-channel supply chain is asymmetric. Zhang et al. [25] studied the problem of designing contracts in a closed-loop supply chain when the cost of collection effort is the retailer’s private information, and analyzed the impact of information on the equilibrium results of supply chain members. All of the above literatures are closely related to the hidden cost. Lee and Yang [15] employed a screening model to examine the problem of supply chain contracting involving one retailer and two suppliers under asymmetric demand information. Many studies have employed compensation plan to induce salespeople to disclose the market condition information. Chen [5] investigated how a firm can provide incentives to its salesforce so that it is in their interest to truthfully disclose their information about the market and to work hard. Kung and Chen [14] studied a three-layer supply chain in which a manufacturer relies on a salesperson to sell the products to the consumers, and jointly study the manufacturer’s partner selection problem and the resellers’ salesforce compensation problem. The sales outcome is determined by a random market condition and the salesperson’s service level, both of which are privately observed by the salesperson. Lee and Yang [16] investigated the optimal compensation scheme involving one firm and two competing salespersons deployed in different territories under asymmetric
market condition information. Saghafian and Chao [22] studied the dependence of the operational decisions of production/inventory management and the design of salesforce incentives, and considered the problem of joint salesforce incentive design and inventory/production control with both moral hazard and adverse selection, due to the sales agent’s private market condition. Different from the above salesforce research, Dai and Chao [8] studied the salesforce incentive and inventory planning problem where the exact value of the sales agent’s risk attitude is the sales agent’s private information, and indicates that the firm should offer a lower commission rate to a salesperson that is more risk averse. Our paper is most closely related to hidden risk aversion.

Traditional supply chain models are based on risk neutrality, where decision makers make decisions to maximize their expected profits (see [4], [6], [13]). While it is generally accepted that decision makers are risk-averse, it is also known that estimating the exact utility function of a decision maker is not easy, if possible at all. In this paper, the source of asymmetric information comes from the individual decision maker’s risk attitude. Different from Dai and Chao [8], we have considered the joint decision on pricing and production, and inventory-dependent salvage value. There is a considerable literature devoted to risk-aversion supply chain management. Most of the literature assumes that the salvage value is a constant and not decided by the surplus inventory. Aviv and Pazgal [1] proposed that customers exhibit strategic behavior, in that they may decide to postpone their purchases, if they believe that a later purchase at a lower price may bring a higher expected surplus than what they can gain by an immediate purchase. We considered the joint effect of asymmetric information of sales agent’s risk attitude and inventory-dependent salvage value on the general framework of salesforce compensation plan. In these models, the degree of risk aversion is usually modeled as a parameter in the utility function and it is assumed to be a known information. Katok and Pavlov [12] compared inequality aversion, bounded rationality, and incomplete information on the inefficiency of supply chain. They went on to establish that incomplete information about the retailer’s degree of inequality aversion played a more important role than bounded rationality while explaining the suppliers’ behavior.

In this paper, the source of asymmetric information comes from the individual decision maker’s degree of risk aversion. Different from the papers mentioned above, we consider a supply chain with risk-averse sales agent who keeps the exact value of its degree of risk aversion as personal information. In addition, most of the literature on pricing and inventory decisions assumes the salvage value is a constant, we consider the joint effect of private risk-averse information and inventory-dependent salvage value on the general framework of sales force. In particular, the salvage value depends on leftover inventory at the end of a selling season. Our paper contributes to this aspect of research by investigating an inherent problem within contract design while applied on a joint decision on pricing and production, when the sales agent’s degree of risk aversion falls under asymmetric information and inventory-dependent salvage value.

3. Problem formulation. We considered a supply chain in which a manufacturer (she) hires a sales agent (he) to sell products. Suppose that the sales quantity, or demand, is random and depends on the sales effort $e$ exerted by the sales agent as well as the selling price $p$. We adopt the following linear demand function (see [9], [22]) to model the sales volume for the product in the following additive form:
$X = a - bp + e + \theta,$ \hfill (1)

where the sales parameter $a > 0$ is the average sales quantity in the case without sales effort, $b > 0$ represents the price elasticity of demand, both are known. Here $\theta$ is a normally distributed random noise with mean 0 and variance $\sigma^2$. Let $F(\cdot)$ and $f(\cdot)$ represent the cumulative distribution function and probability density function of $\theta$, respectively.

At the beginning of a selling season, a decision about production quantity $Q$ and selling price $p$ are made by the manufacturer, and the production cost per unit is $c$. When production does not match with the demand, additional costs are incurred. In the case of oversupply, the excess supply is salvaged at $s'$ per unit (net salvage value); and in the case of undersupply, the excess demand must be satisfied via an emergency production at a cost of $c'$ per unit. To avoid triviality, we assume that $s' < c < c' < p$ (see [8, 22]). Moreover, we assume that $\theta$ is sufficiently large such that the probability of $X$ being negative is negligible, $a - bc > 0$ holds, as the expected market demand $a - bp$ is positive only when $a - bc > 0$.

The cost of sales effort is assumed to be $C(e) = ke^2/2$, which is increasing convex in $e$, and $k > 0$ is the effort cost parameter. We assume $2bk > 1$ holds, the assumption ensures that when price sensitivity $b$ is low, the cost of sales effort should be high enough (high $k$) to prevent the manufacturer from setting an infinite effort level $e$ and making an infinite profit (see [19]).

In the previous studies, the unit salvage value $s'$ is assumed to be a constant, and not dependent on the inventory (see [1]). However, in reality, when the product misses the sales season, retailers often adjust the price and sell it based on the amount of inventory. In general, the more the surplus inventory in clearance period, the lower the salvage value the retailers adjust. Thus, in this paper, we assume that the unit salvage value $s'$ is dependent on the final inventory $\max(Q - X, 0)$, and $s' = s - \gamma \max(Q - X, 0)$, where $s$ is the initial unit salvage value, $Q$ is the production quantity, $\gamma$ is a constant which measures the variation rate of salvage value. Here $s'$ is random and changes with the demand $X$ and production $Q$.

We use $s(X)$ to denote the compensation that sales agent receives from the manufacturer, which depends on his total sales $X$, we restrict our attentions to the class of linear contracts as $s(X) = \alpha + \beta X$, because of the prevalence in practice. Specifically, we use $(\alpha, \beta)$ to denote the contract signed by the manufacturer and the sales agent, where $\alpha$ is the base salary and $\beta \geq 0$ is the commission rate.

While the manufacturer is risk-neutral and maximizes her expected profit, therefore, the manufacturer’s net profit function can be written as follows:

$$\pi_M = pX - cQ + s'(Q - X)^+ - c'(X - Q)^+ - s(X)$$
$$= pX - cQ + [s - \gamma(Q - X)^+](Q - X)^+ - c'(X - Q)^+ - s(X)$$
$$= pX - cQ + s(Q - X)^+ - c'(X - Q)^+ - \gamma((Q - X)^+)^2 - s(X),$$ \hfill (2)

where $x^+ = \max(0, x)$, and the sales agent’s net income is given by

$$\pi_S = s(X) - C(e) = \alpha + \beta(a - bp + e + \theta) - ke^2/2.$$ \hfill (3)

The sales agent is risk averse and maximizes his expected utility, his risk attitude is represented by a negative exponential utility function $U = -\exp(-r\pi_S)$, where $\pi_S$ is his net income and $r > 0$ is the coefficient of absolute risk aversion. The sales agent has superior information, besides the effort level $e$, the risk attitude $r$ is also his private information, the manufacturer treats $r \in [r, \overline{r}]$ as random with
distribution \( G(r) \) and density \( g(r) \). We assume that \( G(r) \) satisfies the Increasing Failure Rate property (IFR), that is, the inverse failure rate \( H(r) = G(r)/g(r) \) increases in \( r \), and \( G(\infty) = 0, G(\infty) = 1 \). Here \(-U_0\) denotes the sales agent’s reservation utility, representing the best outside opportunity for the agent, and the corresponding certainty equivalence is \( \pi = -\ln U_0/r \) (see [6]). Thus, for the agent to accept a contract, the contract has to maximize his expected utility among his choices and the expected utility value has to be at least \(-U_0\), this is equivalent to

\[
E[-\exp(-r\pi_S)] \geq -U_0. \tag{4}
\]

The Individual Rationality (IR) constraint (4) ensures the participation of the sales agent.

The above model assumptions, including the linear payment structure, the negative exponential utility, and the normally distributed randomness, together referred to as the LEN (Linear-Exponential-Normal) assumption, are commonly used in the agency literature for tractability (see [8], [14], [22]). The above assumptions are common knowledge to all the parties concerned. We first investigate the manufacturer’s optimal contract design, joint production and pricing problem with symmetric risk attitude as the benchmark case, further we consider the sales agent’s risk attitude as his private information, and analyze the impact of both asymmetric risk attitude and the variation rate of salvage value on the decisions and income of both sides. Throughout the paper, for the sake of convenience, the traditional factor \( A = kr\sigma^2 \) is defined as the product of the effort cost parameter \( k \), the risk aversion level \( r \) and actual demand variance \( \sigma^2 \). We use the following notation: Using the \( E[\cdot] \) to represent the mathematical expectation, \( \Phi(\cdot) \) is the cumulative probability function for the standard normal distribution, and \( \Phi^{-1}(\cdot) \) is its inverse function. The subscripts “M”, “S” and “C”, respectively, denote the Manufacturer, the Sales agent and the salvage value is Constant scenario, and the superscript “\( \ast \)” denotes the optimal cases.

4. Benchmark model. To serve as a benchmark, this section studies the selling scheme in which the sales agent’s risk attitude \( r \) is common knowledge. In this case, the manufacturer faces only moral hazard problem (without adverse selection). According to the assumptions and Equation (2) in Section 3, contracting with the symmetric risk-averse sales agent, the expected utility of the risk-neutral manufacturer can be written as follows:

\[
E(\pi_M) = E[pX - cQ + s(Q - X)^+ - c'(X - Q)^+ - \gamma((Q - X)^+)^2 - s(X)]. \tag{5}
\]

From Equation (3), we have the sales agent’s expected utility corresponding to his net profit as \( E[-\exp(-r(a + \beta(a - bp + e + \theta) - ke^2/2))] \), by the certainty equivalence principle, the sales agent’s certainty equivalence corresponding to the expected utility is given by:

\[
E(\pi_S) = \alpha + \beta(a - bp + e) - ke^2/2 - r\sigma^2\beta^2/2. \tag{6}
\]

Similar to a principal-agent framework, the sales agent acts as the follower, and the manufacturer, as a leader designs the incentive contract \((\alpha, \beta)\), and sets the product price \( p \) and production quantity \( Q \) to maximize her expected utility, while satisfying the sales agent’s Individual Rationality (IR) and Incentive Compatibility
(IC) constraints. The manufacturer’s decision problem is given as follows:

\[
\max_{\alpha, \beta, p, Q} E[pX - cQ + s(Q - X)^+ - c'(X - Q)^+ - \gamma((Q - X)^+)^2 - s(X)],
\]

\[
\text{s.t. (IR) } E(U_s) = \alpha + \beta(a - bp + e) - ke^2/2 - r\sigma^2\beta^2/2 \geq \bar{\pi},
\]

\[
\text{(IC) } e^* = \arg\max_{e \geq 0} \left[ \alpha + \beta(a - bp + e) - ke^2/2 - r\sigma^2\beta^2/2 \right].
\]

The IR constraint ensures the participation of the sales agent, because of exceeding the reservation profit. Equation (9) is IC constraint, assuring that the sales agent does not pretend to choose the other effort level. The sales agent aims to maximize his expected utility, and his IC constraint can be replaced by first best effort levels as:

\[
e^* = \beta/k.
\]

Hence, we obtain Theorem 4.1.

**Theorem 4.1.** Under symmetric risk information, the optimal commission rate \(\beta^*\), sales agent’s effort \(e^*\), and pricing strategies \(p^*\) are given by

\[
\begin{align*}
\beta^* &= \frac{k(a - bc)(1 + A) - c}{2bk(1 + A) - 1}, \\
e^* &= \frac{\beta^*}{k}, \\
p^* &= \frac{k(a + bc)(1 + A) - c}{2bk(1 + A) - 1},
\end{align*}
\]

and the optimal production quantity \(Q^*\) satisfies

\[
(c' - s)F(Q^* - m(\beta^*)) + 2\gamma \int_{-\infty}^{Q^* - m(\beta^*)} F(x)dx = c' - c.
\]

Correspondingly, the optimal base salary is given by

\[
\alpha^* = \bar{\pi} + \frac{r\sigma^2\beta^2 - (a - bc)\beta^*}{2}.
\]

The manufacturer’s optimal expected net profit is given by

\[
E^*(\pi_M) = \frac{k(a - bc)^2(1 + A)}{4bk(1 + A) - 2} - \bar{\pi} + \pi(\beta^*),
\]

where

\[
\pi(\beta^*) = (c' - c)(Q^* - m(\beta^*)) - [2\gamma(Q^* - m(\beta^*)) + c' - s] \times \\
\int_{-\infty}^{Q^* - m(\beta^*)} F(x)dx + 2\gamma \int_{-\infty}^{Q^* - m(\beta^*)} xF(x)dx,
\]

\[A = kr\sigma^2, a - bc > 0, 2bk - 1 > 0 \text{ and } m(\beta^*) = (a - bc)/2 + \beta^*/2k.\]

The proof of Theorem 4.1 and the other proofs are given in the Appendix.

Equations (11) and (12) indicate that the initial unit net salvage value \(s\), the variation rate of salvage value \(\gamma\), and the unit emergency production cost \(c'\) only affect the manufacturer’s production quantity \(Q^*\) and her expected profit \(E^*(\pi_M)\). It is easy to verify that the manufacturer’s expected profit is strictly increasing in \(c'\) and \(s\), which is consistent with the traditional newsvendor model with lost sale penalty cost.

From Theorem 4.1, we deduce two observations as follows:
Observation 1. When the agent’s risk information is symmetric, the manufacturer’s optimal production quantity is given by
\[ Q^* = T + \frac{(a - bc)}{2} + \frac{\beta^*}{2k}. \] (16)

The existence and uniqueness of \( T \) are guaranteed and dependent on Equation (17):
\[ (c' - s)F(T) + 2\gamma \int_{-\infty}^{T} F(t) dt = c' - c. \] (17)

or
\[ (2\gamma T + c' - s)\Phi(T/\sigma) + \sqrt{2/\pi}\gamma\sigma \exp(-x^2/2\sigma^2) = c' - c, \] (18)
and \( \pi(\beta^*) = (c' - c)/T - (2\gamma T + c' - s) \int_{-\infty}^{T} F(x) dx + 2\gamma \int_{-\infty}^{T} x F(x) dx, \) or
\[ \pi(\beta^*) = (c' - c)T - \gamma F(T)(T^2 + \sigma^2) - (\gamma T + c' - s) \exp(-T^2/2\sigma^2)\sigma/\sqrt{2\pi} 
- (c' - c)TF(T). \] (19)

Observation 2. If the salvage value is a constant (\( \gamma = 0 \)), the optimal production quantity \( Q^*_C \) and the manufacturer’s optimal expected profit \( E^*(\pi_M) \) are given by
\[ Q^*_C = \frac{a - bc}{2} + \frac{\beta^*}{2k} + F^{-1}\left(\frac{c' - c}{c' - s}\right), \] (20)
\[ E^*(\pi_M) = \frac{k(a - bc)^2(1 + A)}{4bk(1 + A) - 2} - \frac{\sigma(c' - s)}{\sqrt{2\pi}} \exp\left(-\frac{\Phi^{-1}\left(\frac{c' - c}{c' - s}\right)}{2}\right). \] (21)

where \( \Phi(\cdot) \) is the cumulative probability function for the standard normal distribution \( N(0,1) \), and \( \Phi^{-1}(\cdot) \) is its inverse function.

Conducting sensitivity analysis of \( r \), we have the following results.

Proposition 1. Under symmetric risk information,
(i) the optimal commission rate \( \beta^* \) is decreasing in \( r \); and
(ii) the optimal sales effort \( e^* \) is decreasing in \( r \); and
(iii) the optimal price \( p^* \) is decreasing in \( r \); and
(iv) the optimal production quantity \( Q^* \) is decreasing in \( r \); and
(v) the manufacturer’s optimal expected profit \( E^*(\pi_M) \) is decreasing in \( r \).

From the proof of Proposition 1, we know that under symmetric risk attitude, the sales effort \( e^* \), the manufacturer’s optimal expected profit \( E^*(\pi_M) \), the commission rate \( \beta^* \), the price \( p^* \) and the production quantity \( Q^* \) are all decreasing in \( A = kr\sigma^2 \), i.e., they are all decreasing in the cost parameter \( k \), the degree of risk aversion \( r \) and the demand variance \( \sigma^2 \).

5. A model with asymmetric risk attitude. This analysis is distinct from previous studies as it treats the sales agent’s risk attitude as his private information. We assume that the sales agent has his own private information about his risk attitude \( r \in [\overline{r}, \overline{r}] \). The sales agent surely knows the actual \( r \), while the manufacturer has only a subjective assessment according to her probability distribution \( G(r) \) and probability density function \( g(r) \). In this case, the manufacturer faces a mixture of adverse selection and moral hazard. The goal of the manufacturer is to design a menu of incentive contracts, and determine the decisions on production and pricing, so as to maximize his expected profit based on revelation principle.
The manufacturer designs a menu of compensation contracts and plans the production and pricing decisions, the sequence of events is as follows. (1) The sales agent’s risk attitude information is observed by the manufacturer. (2) The manufacturer offers a menu of compensation contracts \((\alpha(r), \beta(r))\) for the sales agent to self-select. (3) the sales agent decides whether or not to participate and, if so, which contract to sign based on his private risk attitude. (4) under a signed contract, the manufacturer determines the joint decision on pricing and the production quantity, and the sales agent makes the effort decision. (5) the sales outcome is realized, and the sales agent is compensated.

From Equation (3), the sales agent’s utility is \(-\exp(-r\pi_S)\), for normal \(Y\) with mean \(\mu_Y\) and variance \(\sigma_Y^2\), it holds that \(E[-\exp(-r\pi_S)] = -\exp[-CE_Y(Y)]\), where \(CE_Y(Y) = r(\mu_Y - r\sigma_Y^2/2)\). Thus the maximization of the sales agent’s utility is equivalent to maximizing \(CE_Y(Y)\), the sales agent’s reservation utility is \(-U_0\), the corresponding certainty equivalence is \(\pi = -\ln U_0/r\), and the sales agent’s certainty equivalence satisfies \(CE_Y(Y) \geq \pi\).

Under the incentive contract \((\alpha(r), \beta(r))\), the sales agent’s expected utility corresponding to his net profit \(\pi_S = \alpha(r) + \beta(r)(a - bp + e + \theta) - ke^2/2\) is given by

\[
E[-\exp(-r(\alpha(r) + \beta(r)(a - bp + e + \theta) - ke^2/2))].
\] (22)

From the certainty equivalence principle, we obtain the certainty equivalence, namely

\[
CE_Y = r[\alpha(r) + \beta(r)(a - bp + e) - ke^2/2 - r\sigma^2\beta^2(r)/2].
\] (23)

To characterize the optimal compensation design problem, we use backward induction and start with the sales agent’s problem. Suppose the sales agent’s actual risk attitude is \(r\), but he has chosen the contract \((\alpha(r'), \beta(r'))\) related to \(r'\). Thus, the sales agent’s Certainty Equivalent (CE) is given by

\[
CE_Y(r, r'|e) = r[\alpha(r') + \beta(r')(a - bp + e) - ke^2/2 - r\sigma^2\beta^2(r')/2].
\] (24)

From Equation (4), for the sales agent to accept a contract \((\alpha(r'), \beta(r'))\), the expected utility value has to at least satisfy the sales agent’s reservation utility, this is, equivalent to \(CE_Y(r, r'|e) \geq \pi\). Because the exponential function is monotonic, maximizing the expected utility is equivalent to maximizing the CE by choosing

\[
e(r) = \arg\max_{e \geq 0} CE_Y(r, r'|e) = \beta(r')/k.
\] (25)

Substituting Equation (25) into Equation (24), the sales agent’s maximum CE is given by

\[
CE_Y(r, r') = \max_{e \geq 0} CE_Y(r, r'|e) = r[\alpha(r') + \beta(r')(a - bp) + (1 - k\sigma^2)\beta^2(r')/2k].
\] (26)

Let \(CE_Y(r) \equiv CE_Y(r, r)\), thus

\[
CE_Y(r) = r[\alpha(r) + \beta(r)(a - bp) + (1 - k\sigma^2)\beta^2(r)/2k].
\] (27)

For convenience of analysis, without loss of generality, we normalize the sales agent’s certainty equivalence to \(U_S(r) \equiv CE_Y(r)/r = \alpha(r) + \beta(r)(a - bp) + (1 - k\sigma^2)\beta^2(r)/2k\); obviously, \(U_S(r)\) is an affine transformation of \(CE_Y(r)\).

In equilibrium, the manufacturer induces the sales agent to disclose his risk attitude truthfully by choosing the contract \((\alpha(r), \beta(r))\), then the manufacturer’s
expected profit is \( E_r(\Pi_M) \), namely
\[
\int_{\mathbb{R}} \Pi_M dG(r),
\]
where \( \Pi_M = E[pX - cQ + s(Q - X)^+ - c'(X - Q)^+ - \gamma((Q - X)^+)^2 - s(X)] \).

Therefore, under asymmetric risk information, the manufacturer’s decision problem can be expressed as follows:
\[
\max_{\alpha(r), \beta(r), \rho(r), \gamma(r)} E_r(\Pi_M),
\]
subject to
\[
\begin{align*}
& \text{(TC) CE}_S(r) \geq CE_S(r, r'), \\
& \text{(IR) CE}_S(r) \geq \mathbb{E}, \\
& \text{(IC) } e(r) = \beta(r)/k, \ r, r' \in [\underline{r}, \overline{r}].
\end{align*}
\]

Equation (29) is the Truth-Telling (TC) constraint under adverse selection assuring that the sales agent with risk attitude \( r \) selects the contract designed for him; the IR constraint (30) ensures the participation of the sales agent; Equation (31) is the IC constraint under moral hazard to prevent the sales agent from shirking. Under dual information asymmetry of the mixture of adverse selection and moral hazard, the manufacturer safeguards her interests through the menu of contract. The following theorem characterizes the optimal menu of contracts and decisions.

**Theorem 5.1.** Under asymmetric risk information, the manufacturer offers menu of contracts \((\alpha^*(r), \beta^*(r))\), with the optimal menu, it induces the effort level \( e^*(r) \), the optimal pricing \( p^*(r) \) and production decisions \( Q^*(r) \) constitute the unique Bayesian Nash equilibriums as
\[
\begin{align*}
\alpha^*(r) &= \pi + \frac{r\sigma^2 \beta^2(r) - (a - bc)\beta^*(r) + \sigma^2 \int_r^\infty \beta r^2(d\tau)}{2}, \\
\beta^*(r) &= \frac{k(a - bc)}{2bkM(r) - 1}, \\
e^*(r) &= \frac{\beta^*(r)}{k} \in [\underline{r}, \overline{r}].
\end{align*}
\]
and \( Q^*(r) \) satisfies
\[
(c' - s)F(Q^*(r) - m(\beta^*(r))) + 2\gamma \int_{-\infty}^{Q^*(r) - m(\beta^*(r))} F(x)dx = c' - c.
\]
The optimal base salary is given by
\[
\alpha^*(r) = \pi + \frac{r\sigma^2 \beta^2(r) - (a - bc)\beta^*(r) + \sigma^2 \int_r^\infty \beta r^2(d\tau)}{2}.
\]
The sales agent’s optimal certainty equivalence is given by
\[
U^*_S(r) = \pi + \frac{\sigma^2}{2} \int_r^\infty \beta r^2(d\tau).
\]

With the optimal menu, the manufacturer receives an expected profit \( E_r^*(\Pi_M) \), and
\[
\Pi_M = \frac{k(a - bc)^2 M(r)}{4bkM(r) - 2} - \pi + \pi(\beta^*(r)),
\]
where \( \pi(\beta^*(r)) = (c' - c)(Q^*(r) - m(\beta^*(r))) - [2\gamma(Q^*(r) - m(\beta^*(r))) + c' - s] \times \int_{-\infty}^{Q^*(r) - m(\beta^*(r))} F(x)dx + 2\gamma \int_{-\infty}^{Q^*(r) - m(\beta^*(r))} xF(x)dx, M(r) = 1 + k\sigma^2 + k\sigma^2 H(r), m(\beta^*(r)) = (a - bc)/2 + \beta^*(r)/2k, a - bc > 0, 2bk - 1 > 0, A = k\sigma^2, H(r) = G(r)/g(r), \text{ and } E_r(\cdot) \text{ is the mathematical expectation.}
Equation (35) states that the sales agent obtains additional information rent \( \frac{\sigma^2}{2} \int_0^T \beta^2(r) dt \) due to the private risk information, rather than the reservation profit \( \pi \). Equation (32) indicates that the unit net salvage value \( s \), unit emergency production cost \( c' \) and the variation rate of salvage value \( \gamma \) only affect the manufacturer’s production quantity \( Q^*(r) \) and her expected profit \( E_r^*(\Pi_{MC}) \). It is easy to verify that the manufacturer’s expected profit \( E_r^*(\Pi_{MC}) \) and production quantity \( Q^*(r) \) are strictly increasing in \( c' \) and \( s \), which is also consistent with the traditional newsvendor model with lost sale penalty cost.

From Theorem 5.1, we deduce two observations as follows:

**Observation 3.** Under asymmetric risk information, the optimal production quantity \( Q^*(r) \) satisfies Equation (37):

\[
Q^*(r) = T + \frac{(a - bc)}{2} + \frac{\beta^*(r)}{2k},
\]

the existence and uniqueness of \( T \) are guaranteed and dependent on the following equation:

\[
(c' - s)F(T) + 2\gamma \int_{-\infty}^{T} F(t) dt = c' - c,
\]
or equivalently

\[
(2\gamma x + c' - s)\Phi(T/\sigma) + \sqrt{2/\pi\gamma}\sigma \exp(-T^2/2\sigma^2) = c' - c,
\]

and

\[
\pi(\beta^*(r)) = (c' - c)T - (2\gamma T + c' - s) \int_{-\infty}^{T} F(x) dx + 2\gamma \int_{-\infty}^{T} x F(x) dx,
\]
or

\[
\pi(\beta^*(r)) = (c' - c)T - \gamma F(T)(T^2 + \sigma^2) - (\gamma T + c' - s) \exp(-T^2/2\sigma^2)\sigma/\sqrt{2\pi} - (c' - c)TF(T).
\]

See the proof of Observation 1.

Comparing Observations 1 and 3, we find that, whether the sales agent’s risk attitude is symmetric or not, \( T = Q^*(r) - \beta^*(r)/2k - (a - bc)/2 \) is a constant, \( T \) only depends on the unit net salvage value \( s \), unit emergency production cost \( c' \), the variation rate of salvage value \( \gamma \) and the unit production cost \( c \), doesn’t depend on the sales agent’s risk attitude \( r \).

**Observation 4.** If the salvage value is a constant \( (\gamma = 0) \), the optimal production quantity \( Q^*_C(r) \) and the manufacturer’s optimal expected profit \( E_r^*(\Pi_{MC}) \) satisfies

\[
Q^*_C(r) = \frac{a - bc}{2} + \frac{\beta^*(r)}{2k} + F^{-1}\left(\frac{c' - c}{c' - s}\right),
\]

\[
\Pi_{MC} = \frac{k(a - bc)^2M(r) - 2}{4bkM(r) - 2} - \frac{\sigma(c' - s)}{\sqrt{2\pi}} \exp\left(-\frac{\Phi^{-1}\left(\frac{c' - c}{c' - s}\right)^2}{2}\right),
\]

where \( M(r) = 1 + k\sigma^2 + ka^2H(r) \), \( a - bc > 0 \), \( 2bk - 1 > 0 \), \( A = k\sigma^2 \), \( H(r) = G(r)/g(r) \), \( \Phi(\cdot) \) is the cumulative probability function for the standard normal distribution \( N(0,1) \), \( \Phi^{-1}(\cdot) \) is its inverse function, and \( E_r(\cdot) \) is the mathematical expectation.

The proof is straightforward. Consistent with the proof of Observation 2, replacing the quantities in Equation (33), we obtain \((c' - s)F(Q^*(r) - m(\beta^*(r))) = c' - c\), where \( m(\beta^*(r)) = (a - bc)/2 + \beta^*(r)/2k \).
Comparing Observations 2 and 4, we find that, when the salvage value is a constant ($γ = 0$), $E^*(Π_{MC}) ≥ E^*_r(Π_{MC})$ holds. Obviously, $E^*_r(Π_{MC})$ is decreasing in $M(r)$, if the risk attitude $r$ is symmetric, $H(r) = 0$, $M(r)$ degrades into $1 + krσ^2$.

Conducting sensitivity analysis of $r$, we have the following results.

**Proposition 2.** Under asymmetric risk information,

(i) the optimal commission rate $β^*(r)$ is decreasing in $r$; and
(ii) the optimal sales effort $e^*(r)$ is decreasing in $r$; and
(iii) the optimal price $p^*(r)$ is decreasing in $r$; and
(iv) the optimal production quantity $Q^*(r)$ is decreasing in $r$; and
(v) the manufacturer’s optimal expected profit $E^*_r(Π_M)$ is decreasing in $r$; and
(vi) the optimal certainty equivalence $U^*_S(r)$ is decreasing in $r$; and
(vii) if either one of the following conditions is satisfied: (a) $kb ≥ 1$, or (b) $1 ≥ kb ≥ 0.5$ and $\frac{2kb-1}{2kσ^2(1-kb)} > r > 0$, the optimal base salary $α^*(r)$ is increasing in $r$.

The proof of Proposition 2 indicates that, under asymmetric risk attitude, the sales agent’s effort $e^*(r)$, the manufacturer’s optimal expected profit $E^*_r(Π_M)$, the commission rate $β^*(r)$, joint pricing $p^*(r)$ and the production $Q^*(r)$ decisions are all decreasing in the cost parameter $k$, risk averse $r$ and the demand variance $σ^2$, which is consistent with the symmetric case in Proposition 1.

Proposition 2 indicates that, as the increasing of the sales agent’s degree of risk aversion, which means that the sales agent is not willing to take more risks or exert more sales effort to gain more returns. The decreased effort results in the decrease in the overall income, and leads to decrease in the manufacturer’s profit, i.e., the increasing of the sales agent’s risk aversion will result in the decreasing of the profit of both sides, the results also show that the manufacturer who acts as the game leader is willing to hire the less risk-averse sales agent.

Proposition 2 also states that, when the sales agent becomes more risk averse, the manufacturer will offer him a lower commission rate and set a lower selling price and a lower production quantity, while the base salary is dependent on the effort cost and price sensitivity, which is not guaranteed to be higher.

6. **Analytical results.** In this section, we proceed to compare the optimal results derived from Sections 4 and 5. We mainly compare the optimal decisions under asymmetric risk-averse information and symmetric cases.

6.1. **Sensitivity analysis and comparison.** Based on the equilibrium results in Section 5, the following results can be derived.

**Proposition 3.** For a menu of contracts $(α^*(r), β^*(r))$ the manufacturer offered under asymmetric risk information, the risk-averse sales agent has no motivation to hide his true risk information $r$, which implies the truth-telling behavior of the sales agent.

**Proposition 4.** When the variation rate of salvage value is fixed, comparing the optimal results under asymmetric risk and symmetric risk information, the following observations can be made: (i) $β^*(r) ≤ β^*$, $e^*(r) ≤ e^*$, $Q^*(r) ≤ Q^*$ and $p^*(r) ≤ p^*$; (ii) $E^*_r(Π_M) ≤ E^*(Π_M)$ and $E^*_S(r) ≥ E^*_S(Π_S)$.

Proposition 4 indicates that if the manufacturer is uncertain about the sales agent’s risk attitude, the commission rate, the sales effort, the production quantity and price will be smaller; and her expected profit must be less than that in
the certain case; the sales agent not only obtains the reservation profit \( \pi \), but also the additional strictly nonnegative information rent \( \sigma^2 \int_r^\tau \beta^2(\tau) d\tau \), and the information rent is decreasing in \( r \) (see the proof of Proposition 4). When the risk attitude is common knowledge, the sales agent only obtains the reservation profit. Which also implies the value of information, the asymmetric information made the manufacturer relatively conservative.

From Proposition 4, we also know that when the manufacturer faces asymmetric risk information, she screens the hidden information and designs a menu of compensation contracts according to the probability distribution of agent’s risk attitude in her mind, there exists deviation relative to the incentive contract under the actual risk attitude, the profit of the manufacturer declines due to the asymmetric information. The conclusion of Proposition 4 implies that the manufacturer should strengthen communication and cooperation with sales agent, to share the risk attitude information in time and improve the incentive strategy according to the obtained information.

Combining \( \beta^*(r) \leq \beta^* \), Equations (20) and (38), we can obtain \( Q^*_C(r) \leq Q^*_C \) holds.

**Proposition 5.** The variation rate of salvage value \( \gamma \) only affects the manufacturer’s optimal expected profit and the optimal production quantity.

From the proof of Proposition 5, there is no difference between the impact of the variation rate of salvage value \( \gamma \) on the production quantity of the symmetric risk-averse and asymmetric case, as well as the manufacturer’s expected profit. The variation rate of salvage value \( \gamma \) doesn’t affect the manufacturer’s pricing decision, compensation plan, or the sales agent’s decision and income.

**Observation 5.** The manufacturer’s optimal expected profit and the optimal production quantity are lower than the constant salvage value scenario, i.e., the following inequalities hold: \( Q^*_C \geq Q^* \), \( Q^*_C(r) \geq Q^*(r) \), \( E^*(\pi_{MC}) \geq E^*(\pi_M) \) and \( E^*(\Pi_{MC}) \geq E^*(\Pi_M) \).

Observation 5 states that the manufacturer’s expected profit and production decisions are less than the constant scenario. Observation 5 also implies that the manufacturer makes the production decisions with reservations because of the variation rate of salvage value, which results in the decrease of her profit.

### 6.2. Numerical examples.

In this subsection, we provide some numerical examples based on Sections 4 and 5. The purpose is two-fold. First, the examples are used to illustrate the model developed in previous sections to make further investigation. Second, since the manufacturer’s optimal profit and the production quantity cannot be explicitly written as a function of the risk attitude \( r \) and the variation rate of salvage value \( \gamma \), it is difficult to obtain a closed-form expression of the manufacturer’s optimal profit and the production quantity, although we obtain the closed-form expression of the other optimal decisions. Thus we need to use the numerical examples in investigating the characteristic of the asymmetric risk attitude and the variation rate of salvage value, and provide several key managerial insights.

In the following numerical examples, we assign these parameters as follows: \( a = 7 \), \( b = 1 \), \( \pi = 0 \), \( k = 1 \), \( \sigma^2 = 2 \), \( \gamma \geq 0 \), \( A = 2r \), \( r \) is uniformly distributed, and \([\underline{r}, \overline{r}] = [0, 1] \), \( s = 1 \), \( c = 2 \), \( c' = 3 \), \( H(r) = r \), \( M(r) = 1 + 4r \). From Theorem 4.1, when the sales agent’s risk attitude is common knowledge of both sides, we
obtain \( e^* = \frac{5}{4r+1} \), the optimal contract is \( \beta^* = \frac{5}{4r+1} \), \( \alpha^* = r\beta^* - 2.5\beta^* \); the optimal price and production quantity are \( p^* = \frac{14r+5}{4r+1} \), \( Q^* = T + 2.5 + \frac{2.5}{4r+1} \), and the manufacturer’s expected profit is

\[
E^*(\pi_M) = \frac{50r + 25}{8r + 2} + T - 2(\gamma T + 1) \int_{-\infty}^{T} F(x)dx + 2\gamma \int_{-\infty}^{T} xF(x)dx.
\]

From Observation 1, if the salvage value is a constant, we can derive \( Q^*_C = 2.5 + \frac{2.5}{4r+1} \), \( E^*_C(\pi_M) = \frac{50r + 25}{8r+2} - 1.128 \).

When the sales agent’s risk attitude is his private information, from Theorem 5.1, we obtain the optimal effort as \( e^*(r) = \frac{5}{8r+2} \); the optimal menu of contracts
are $\beta^*(r) = \frac{5}{8r+1}, \alpha^*(r) = r\beta^2(r) - 2.5\beta^*(r) + \int_1^r \beta^2(\tau) d\tau$; the optimal price is $p^*(r) = \frac{28r+5}{8r+1}$, the optimal production quantity is $Q^*(r) = T + 2.5 + \frac{2.5}{8r+1}$, the manufacturer’s optimal expected profit $E^*_R(\Pi_M)$ satisfies

$$\Pi_M = \frac{100r + 25}{16r + 2} + T - 2(\gamma T + 1) \int_{-\infty}^{T} F(x) dx + 2\gamma \int_{-\infty}^{T} x F(x) dx,$$

and the sales agent’s certainty equivalence is $U^*_S(r) = \int_r^1 \frac{25}{8r+1} d\tau = \frac{25(1-r)}{8(8r+1)}$, where $T$ is decided by $F(x) + \gamma \int_{-\infty}^{x} F(t) dt = 0.5$.

From Observation 3, we can obtain $Q^*_C(r) = 2.5 + \frac{2.5}{8r+1}, \Pi_{MC} = \frac{100r+25}{16r+2} - 1.128$. 

**Figure 3.** The impact of the sales agent’s risk attitude on the price.

**Figure 4.** The impact of the sales agent’s risk attitude and the variation rate of salvage value on the production quantity.
Figure 5. The impact of the sales agent’s risk attitude and the variation rate of salvage value on the manufacturer’s expected profits.

Figure 6. The impact of the sales agent’s risk attitude on her expected profits.

Other parameters are kept constant, while sales agent’s degree of risk aversion $r$ is in $[0, 1]$. We depict the impact of the degree of risk aversion $r$ and the variation rate of salvage value $\gamma$ on sales effort $e^*$ and $e^*(r)$, the commission rate $\beta^*$ and $\beta^*(r)$, the price $p^*$ and $p^*(r)$, the production quantity $Q^*$ and $Q^*(r)$, the manufacturer’s optimal expected profit $E^*(\pi_M)$ and $E^*(\Pi_M)$, and the sale agent’s certainty equivalence $U^*_r(r)$ as summarized in Figs. 1-6.

From Figs. 1-5, we find that the sales effort, the commission rate, joint decision on pricing and production and the manufacturer’s income are all decreasing in $r$, and are relatively less under asymmetric risk information. From Figs. 4 and 5, we find that both the manufacturer’s expected profits and the production quantity are
decreasing in $\gamma$. From Fig. 6, under asymmetric risk information, the sales agent not only obtains the reservation profit, he also obtains the additional information rent which is decreasing in $r$.

7. Conclusions. Significantly large investment is observed by many firms, as reported in a Harvard Business Review study [23], where US companies were shown spending a huge sum of 800 billion dollars annually on salesforce compensation. In this paper, we studied a supply chain with a manufacturer hiring a risk-averse sales agent, it is distinct from previous studies as it treats the sales agent’s risk attitude as the private information, and considers inventory-dependent salvage value. The purpose of this paper is to investigate how a manufacturer make the joint decision on production and pricing, which will affect the random market demand, and provide a menu of incentive contracts to the sales agent, so that he works hard to sell the product and disclose his private risk attitude information. To solve the above problem, the principal-agent model is developed, and the impact of the sales agent’s risk attitude and asymmetric information, as well as the variation rate of salvage value on the decisions and profit of both sides are depicted. The obtained results are further compared to the symmetric risk attitude scenario where some interesting results are likewise found. First, the sales agent’s effort level, the manufacturer’s pricing and production decisions, the corresponding expected profit of both sides are all decreasing in the risk-averse degree, while the base salary’s monotonicity is not guaranteed. Second, under the same salvage value, when the manufacturer is uncertain about the sales agent’s risk attitude, its expected profit decreases while the agent with private information exerts less sales effort but obtains additional information rent, which implies the value of information. Next, only the manufacturer’s income and the production decisions are dependent on the variation rate of salvage value. The results show that the manufacturer should not only focus on hiring the lower risk aversion sales agent but also on screening his risk attitude information.

The scope of this paper has necessarily been limited, and its coverage could be fruitfully extended in a number of interesting areas in future studies. First, in this study we have only proposed a linear compensation plan and linear inventory-dependent salvage value. It still remains for future research to consider other compensation schemes and more reasonable salvage value, such as nonlinear or quota-based compensation plan, and the nonlinear salvage value scenario. Second, the current study only considers single sales agent, one useful extension could be, to discuss the condition that is to compensate multiple competing sales agents, and the payment function of each sales agent depends on the sales volumes generated by all sales agents. Third, the study only provides analysis for the case that the sales agent is risk averse. It still remains for future research to consider the case that both the manufacturer and the sales agent are risk averse and bilateral information asymmetry. Finally, we have only discussed the compensation plan with the complete rationality and ignore the behavior characteristics, thus, future work may also include extending the primal model to behavioral economics context, behavioral operation research especially the fairness or overconfidence supported strongly by experiments.

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Appendix.

Proof of Theorem 4.1. Substituting Equation (10) into Equation (8), the IR condition is binding at optimality

\[ E(U_s) = \alpha + \beta(a - bp + e) - \frac{ke^2}{2} - r\sigma^2\beta^2 / 2 = \bar{\pi}, \]

from the binding IR constraint (5), we obtain \( \alpha = \bar{\pi} + (kr\sigma^2 - 1)\beta^2 / 2 - \beta(a - bp) \), to substitute Equation (10) and the above \( \alpha \) into the optimal problem (7), we can get the manufacturer’s equivalence problem as:

\[
\max_{\beta, p, Q} E(\pi_M) = p(a - bp + e) - cQ - \bar{\pi} - (1 + kr\sigma^2)\beta^2 / 2k \\
+ E[s(Q - X)^+ - c'(X - Q)^+ - \gamma((Q - X)^+)^2] \\
= p(a - bp + \beta/k) - cQ - \bar{\pi} - (1 + kr\sigma^2)\beta^2 / 2k \\
+ E[s(Q - X)^+ - c'(X - Q)^+ - \gamma((Q - X)^+)^2].
\]

Owing that

\[
E[s(Q - X)^+ - c'(X - Q)^+ - \gamma((Q - X)^+)^2] \\
= s \int_{-\infty}^{\infty} [Q - a + bp - \beta/k - x]dF(x) \\
-c' \int_{-\infty}^{\infty} [x - (Q - a + bp - \beta/k)]dF(x) \\
-\gamma \int_{-\infty}^{\infty} [Q - a + bp - \beta/k - x]^2dF(x) \\
= (s - c')BF(B) + c'B - \gamma B^2F(B) + (2\gamma B - s) \int_{-\infty}^{B} xdF(x) \\
-c' \int_{-\infty}^{B} xdF(x) - \gamma \int_{-\infty}^{B} x^2dF(x),
\]

where \( B = Q - a + bp - \beta/k \).

We can replace the above equation and transform the manufacturer’s problem into

\[
\max_{\beta, p, Q} E(\pi_M) = p(a - bp + \beta/k) - cQ - \bar{\pi} - (1 + kr\sigma^2)\beta^2 / 2k \\
+ (s - c')BF(B) + c'B - \gamma B^2F(B) + (2\gamma B - s) \int_{-\infty}^{B} xdF(x) \\
-c' \int_{-\infty}^{B} xdF(x) - \gamma \int_{-\infty}^{B} x^2dF(x).
\]

Taking the second-order partial derivatives of \( E(\pi_M) \) with respect to \( Q, p \) and \( \beta \) respectively, we have

\[
\frac{\partial^2 E(\pi_M)}{\partial Q^2} = (s - c')f(B) - 2\gamma F(B),
\]

\[
\frac{\partial^2 E(\pi_M)}{\partial Q \partial p} = \frac{\partial^2 E(\pi_M)}{\partial p \partial Q} = b(s - c')f(B) - 2\gamma bF(B),
\]

\[
\frac{\partial^2 E(\pi_M)}{\partial p^2} = \frac{\partial^2 E(\pi_M)}{\partial \beta^2} = \frac{\partial^2 E(\pi_M)}{\partial \beta^2} = \frac{\partial^2 E(\pi_M)}{\partial Q^2} = (s - c')f(B) - 2\gamma F(B).
\]
distribution (19). Here \( \Phi(\cdot) \) using the routine approach of change of order of integration, we have Equation (18). From Equation (18) we obtain Equation (19). Here \( \Phi(\cdot) \) is the cumulative probability function for the standard normal distribution \( N(0,1) \). Therefore, this Observation is proved.

\[
\frac{\partial^2 E(\pi_M)}{\partial Q \partial \beta} = \frac{\partial^2 E(\pi_M)}{\partial \beta \partial Q} = \frac{(c' - s)f(B) + 2\gamma b F(B)}{k},
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial p^2} = -2b + b^2(s - c')f(B) - 2\gamma b^2 F(B),
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial p \partial \beta} = \frac{\partial^2 E(\pi_M)}{\partial \beta \partial p} = \frac{b(s - c')f(B) + 1 + 2\gamma b F(B)}{k},
\]
\[
\frac{\partial^2 E(\pi_M)}{\partial \beta^2} = \frac{(s - c')f(B) - 2\gamma F(B)}{k^2} - \frac{1 + kr\sigma^2}{k}.
\]

We obtain the Hessian matrix \( H_1 \) by second-order derivatives. Due to \( a - bc > 0 \) and \( 2bk - 1 > 0 \), \( H_1 \) is negative definite concavity, \( E(\pi_M) \) is strictly jointly concave in \( Q, p \) and \( \beta \), that is, the first-order derivatives are therefore sufficient. Based on the first-order necessary condition,
\[
\frac{\partial E(\pi_M)}{\partial \beta} = 0, \quad \frac{\partial E(\pi_M)}{\partial p} = 0, \quad \frac{\partial E(\pi_M)}{\partial Q} = 0,
\]
we can obtain
\[
\begin{cases}
\frac{c' - c}{k} + \frac{(c' - s)f(B)}{k} - \frac{1 + kr\sigma^2}{k} \beta + \frac{2\gamma f_{\infty}^B F(x)dx}{k} = 0, \\
a - 2bp + \frac{\beta}{k} + c' + b(s - c')f(B) - 2\gamma b \int_{-\infty}^{B} F(x)dx = 0, \\
(s - c')f(B) + c' - c - 2\gamma \int_{-\infty}^{B} F(x)dx = 0,
\end{cases}
\]
hence, the optimal strategies are given by
\[
\beta^* = \frac{k(a - bc)}{2bk(1 + kr\sigma^2) - 1}, \quad p^* = \frac{k(a + bc)(1 + kr\sigma^2) - c}{2bk(1 + kr\sigma^2) - 1},
\]
and \( Q^* \) satisfies
\[
(c' - s)F[Q^* - \beta^*/k - (a - bc)/2] + 2\gamma \int_{-\infty}^{Q^* - \beta^*/k - (a - bc)/2} F(x)dx = c' - c,
\]
or
\[
Q^* = a - bp^* + \beta^*/k + \frac{1}{F}\left(\frac{c' - c - 2\gamma \int_{-\infty}^{Q^* - \beta^*/k - (a - bc)/2} F(x)dx}{c' - s}\right).
\]

We obtain \( \alpha^* \) by replacing \( \beta^* \) in the binding IR constraint, to substitute the above \( e^*, \beta^*, p^* \) and \( Q^* \) into Equation (7), the maximum objective value \( E^*(\pi_M) \) can be calculated. Therefore, this Theorem is proved.

Proof of Observation 1. Let \( K(x) = (c' - s)f(x) + 2\gamma \int_{-\infty}^{x} F(t)dt - (c' - c) \), obviously, \( K(+\infty) = (c - s) + 2\gamma \int_{-\infty}^{\infty} F(t)dt > 0 \), \( K(-\infty) = c - c' < 0 \), and \( K'(x) = (c' - s)f(x) + 2\gamma F(x) > 0 \) implies \( K(x) \) is increasing in \( x \), owing that \( K(+\infty)K(-\infty) < 0 \), \( T \)'s existence and uniqueness are guaranteed.

It is easy to verify that \( \int_{-\infty}^{x} F(t)dt = xF(x) + \exp(-x^2/2\sigma^2)\sigma/\sqrt{2\pi} \) (using the routine approach of change of order of integration), and \( F(x) = \Phi(x/\sigma) \), we obtain Equation (18). From \( \int_{-\infty}^{\infty} F(t)dt = x^2 F(x)/2 - \sigma^2 F(x)/2 + \exp(-x^2/2\sigma^2)\sigma/\sqrt{2\pi} \) (using the routine approach of change of order of integration), we have Equation (19). Here \( \Phi(\cdot) \) is the cumulative probability function for the standard normal distribution \( N(0,1) \). Therefore, this Observation is proved.
Proof of Observation 2. Substituting $\gamma = 0$ into Equation (12), we have $(c' - s)F[Q_C^* - m(\beta^*)] = c' - c$, and $T = F^{-1}(\frac{c' - c}{c' - s})$. Thus $Q_C^* - m(\beta^*) = F^{-1}(\frac{c' - c}{c' - s})$, and

$$
\pi(\beta^*) = (c' - c)[Q_C^* - m(\beta^*)] - (c' - s) \int_{-\infty}^{Q_C^* - m(\beta^*)} F(x) dx
$$

$$
= (c' - c) F^{-1}(\frac{c' - c}{c' - s}) - (c' - s) \int_{-\infty}^{F^{-1}(\frac{c' - c}{c' - s})} F(x) dx
$$

$$
= (c' - s) \int_{-\infty}^{F^{-1}(\frac{c' - c}{c' - s})} x dF(x) = -(c' - s) \int_{F^{-1}(\frac{c' - c}{c' - s})}^{\infty} x dF(x).
$$

We obtain $F^{-1}(\frac{c' - c}{c' - s}) = \sigma \Phi^{-1}(\frac{c' - c}{s - c})$, because $\theta$ follows the normal distribution $N(0, \sigma^2)$, then

$$
\int_{F^{-1}(\frac{c' - c}{c' - s})}^{\infty} x dF(x) = \int_{\sigma \Phi^{-1}(\frac{c' - c}{s - c})}^{\infty} x \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx
$$

$$
= \frac{\sigma}{\sqrt{2\pi}} \exp\left(-\frac{(\Phi^{-1}(\frac{c' - c}{s - c}))^2}{2}\right).
$$

Here $\Phi(\cdot)$ is the cumulative probability function for the standard normal distribution $N(0, 1)$, and $\Phi^{-1}(\cdot)$ is its inverse function. Therefore, this Observation is proved.

Proof of Proposition 1. From Equation (11) in Theorem 1, we have, $\beta^*$ is decreasing in $A$. $e^* = \beta^*/k$, $p^* = (a + bc + e^*)/2b$ and $Q^* = T + (a - 2bc)/2 + \beta^*/2k$ imply that $e^*, p^* \text{ and } Q^*$ are all decreasing in $A$ and $r$ ($A = kr\sigma^2$ is increasing in $r$).

Obviously, $\frac{k(a - bc)^2(1 + A)}{4bk(1 + A)^2 - 2}$ is decreasing in $A$ and $r$. $\pi(\beta^*) = (c' - c) T - (2\gamma T + c' - s) \int_{-\infty}^{\infty} F(x) dx + 2\gamma \int_{-\infty}^{\infty} x F(x) dx$ doesn’t have $A$ and $r$, thus, $E^*(\pi_M)$ is decreasing in $A$ and $r$. Therefore, this Proposition is proved.

Proof of Theorem 5.1. It follows from the first-order necessary condition that $\frac{dU_s(r)}{dr} = -\sigma^2 \beta^2(r)/2 < 0$ for all $r$, which implies $U_s(r)$ is decreasing in $r$. The IR constraint (30) implies that $U_s(r) = \min U_s(r) = \bar{\pi}$ at the optimal solution. Consequently

$$
U_s(r) = U_s(\bar{\pi}) - \int_{\bar{\pi}}^{r} U_s'(\tau) d\tau = \bar{\pi} + \frac{\sigma^2}{2} \int_{\bar{\pi}}^{r} \beta^2(\tau) d\tau,
$$

and the binding IR constraint leads to

$$
\alpha(r) = \bar{\pi} + \frac{\sigma^2}{2} \int_{\bar{\pi}}^{r} \beta^2(\tau) d\tau - \frac{(1 - kr\sigma^2)\beta^2(r)}{2k} - \beta(r)(a - bp(r)),
$$

replace the $\alpha(r)$ and IC constraint (31) in the objective function, we reduce the manufacturer’s objective to
\[ \Pi_M = p(r)(a - bp(r) + e) - cQ(r) - \alpha(r) - \beta(r)(a - bp(r) + e) \\
+ E[s(Q(r) - X)^+ - c'(X - Q(r)) + \gamma((Q(r) - X)^+)^2] \\
= p(r)(a - bp(r) + \beta(r)/k) - cQ(r) - \pi - \frac{\sigma^2}{2} \int_r^\tau \beta^2(\tau)d\tau - \frac{(1 + kr\sigma^2)\beta^2(r)}{2k} \\
+ E[s(Q(r) - X)^+ - c'(X - Q(r)) + \gamma((Q(r) - X)^+)^2]. \]

Then, due to \( G(\tau) = 0, \ G(\tau) = 1 \), and to use the routine approach of change of order of integration:

\[
\int_{-\infty}^\tau \int_r^\tau \beta^2(\tau)d\tau G(r) = \int_{-\infty}^\tau \beta^2(r)G(r)/g(r)dG(r).
\]

We can obtain

\[
E[s(Q(r) - X)^+ - c'(X - Q(r)) + \gamma((Q(r) - X)^+)^2] \\
= s \int_{-\infty}^{Q(r) - a + bp(r) - \beta(r)/k} \left[ Q(r) - a + bp(r) - \beta(r)/k - x \right]dF(x) \\
- c' \int_{-\infty}^{Q(r) - a + bp(r) - \beta(r)/k} \left[ x - (Q(r) - a + bp(r) - \beta(r)/k) \right]dF(x) \\
- \gamma \int_{-\infty}^{Q(r) - a + bp(r) - \beta(r)/k} \left[ Q(r) - a + bp(r) - \beta(r)/k - x \right]^2dF(x) \\
= (s - c')B(r)F(B(r)) + c'B(r) - \gamma B^2(r)F(B(r)) \\
+ (2\gamma B(r) - s) \int_{-\infty}^{B(r)} xdF(x) - c' \int_{B(r)}^{\infty} xdF(x) - \gamma \int_{-\infty}^{B(r)} x^2dF(x),
\]

where \( B(r) = Q(r) - a + bp(r) - \beta(r)/k \).

We transform the manufacturer’s problem to:

\[
\max_{\beta(r), p(r), Q(r)} \int_{-\infty}^\tau R(\beta, p, Q, r) dG(r),
\]

where

\[
R(\beta, p, Q, r) = p(a - bp(r) + \beta(r)/k) - cQ - \pi - \frac{1 + kr\sigma^2 + k\sigma^2H(r)}{2k}\beta^2(r) \\
+ (s - c')B(r)F(B(r)) + c'B(r) - \gamma B^2(r)F(B(r)) \\
+ (2\gamma B(r) - s) \int_{-\infty}^{B(r)} xdF(x) - c' \int_{B(r)}^{\infty} xdF(x) - \gamma \int_{-\infty}^{B(r)} x^2dF(x).
\]

Taking the second-order partial derivatives of \( R(\beta, p, Q, r) \) with respect to \( \beta \), \( p \) and \( Q \) respectively, we have
\[
\frac{\partial^2 R}{\partial Q^2} = (s - c')f(B(r)) - 2\gamma F(B(r)),
\]
\[
\frac{\partial^2 R}{\partial Q \partial p} = \frac{\partial^2 R}{\partial p \partial Q} = b(s - c')f(B(r)) - 2\gamma bF(B(r)),
\]
\[
\frac{\partial^2 R}{\partial Q \partial \beta} = \frac{\partial^2 R}{\partial \beta \partial Q} = \frac{(c' - s)f(B(r)) + 2\gamma bF(B(r))}{k},
\]
\[
\frac{\partial^2 R}{\partial p \partial \beta} = \frac{\partial^2 R}{\partial \beta \partial p} = \frac{b(s - c')f(B(r)) + 1 + 2\gamma bF(B(r))}{k},
\]
\[
\frac{\partial^2 R}{\partial p^2} = -2b + b^2(s - c')f(B(r)) - 2\gamma b^2 F(B(r)),
\]
\[
\frac{\partial^2 R}{\partial \beta^2} = \frac{(s - c')f(B) - 2\gamma F(B(r))}{k} - 1 + kr\sigma^2 + k\sigma^2 H(r).
\]

We obtain the Hessian matrix \(H_2\) by second-order derivatives. Due to \(a - bc > 0\) and \(2bb' - 1 > 0\), it is easy to verify that \(H_2\) is negative definite concavity, \(R(\beta, p, Q, r)\) is strictly jointly concave in \(\beta, p\) and \(Q\), that is, the first-order derivatives are therefore sufficient. Based on the first-order necessary condition,
\[
\frac{\partial R}{\partial \beta} = 0, \quad \frac{\partial R}{\partial p} = 0, \quad \frac{\partial R}{\partial Q} = 0,
\]
we can obtain
\[
\begin{cases}
\frac{p - c}{k} + \frac{(c' - s)f(B(r))}{k} + \frac{1 + kr\sigma^2 + k\sigma^2 H(r)}{k} \beta(r) + \frac{2\gamma J_{B(r)}^B F(x)dx}{k} = 0, \\
\frac{a - 2bp(r) + \beta(r)/k + c' + b(s - c')F(B(r)) - 2\gamma b J_{B(r)}^B F(x)dx = 0, \\
(s - c')F(B(r)) + c' - c - 2\gamma J_{-\infty}^{B(r)} F(x)dx = 0.
\end{cases}
\]

Hence, the optimal strategies are given by
\[
\beta^*(r) = \frac{k(a - bc)}{2bk(1 + kr\sigma^2 + k\sigma^2 H(r)) - 1},
\]
\[
p^*(r) = \frac{k(a + bc)(1 + kr\sigma^2 + k\sigma^2 H(r)) - c}{2bk(1 + k\sigma^2 + k\sigma^2 H(r)) - 1},
\]
\[
e^*(r) = \beta^*(r)/k = \frac{a - bc}{2bk(1 + k\sigma^2 + k\sigma^2 H(r)) - 1},
\]
and \(Q^*(r)\) satisfies
\[
(c' - s)F[Q^*(r) - \beta^*(r)/2k - (a - bc)/2] + 2\gamma \int_{-\infty}^{Q^*(r) - \beta^*(r)/2k - (a - bc)/2} F(x)dx = c' - c,
\]
or
\[
Q^*(r) = a - bp^*(r) + \beta^*(r)/k + F^{-1}(\frac{c' - c - 2\gamma \int_{-\infty}^{Q^*(r) - \beta^*(r)/2k - (a - bc)/2} F(x)dx}{c' - s}).
\]

We can obtain \(\alpha^*(r)\) by replacing \(\beta^*(r)\), and the maximum objective value \(E^*_e(\Pi_M)\) can be calculated by plugging \(e^*(r)\), \(\beta^*(r)\), \(p^*(r)\) and \(Q^*(r)\) back. Therefore, this Theorem is proved.

**Proof of Proposition 2.** With the asymmetry \(r\) case, obviously, \(\beta^*(r) = \frac{k(a - bc)}{2bkM(r) - 1}\) is decreasing in \(M(r) = 1 + kr\sigma^2 + k\sigma^2 H(r)\), from the assumption that \(H(r)\) is increasing in \(r\), \(M(r)\) is increasing in \(A\) and \(r\), we know \(\beta^*(r)\) is decreasing in \(A\).
and \( r \), therefore, \( e^*(r) = \beta^*(r)/k \), \( p^*(r) = (a + bc + \beta^*(r))/2b \), \( Q^*(r) = T + (a - bc)/2 + \beta^*(r)/2k \) are all decreasing in \( A \) and \( r \), \( \pi(\beta^*(r)) \) doesn’t contain \( A \) or \( r \), and \( \frac{\partial M}{\partial M(r)} = -\frac{k(a - bc)^2}{2k M(r) - \Pi_M^2} = -\frac{\beta^2(\gamma)}{2k} < 0 \), where \( \Pi_M = \frac{k(a - bc)^2 M(r)}{4k M(r) - 2} - \frac{\pi(\beta^*(r))}{T} \), thus, \( E^*_p(\Pi_M) \) is decreasing in \( A \) and \( r \).

\[
\frac{dU^*_p(r)}{dr} = -\sigma^2 \beta^2(r)/2 < 0 \implies U^*_p(r) \text{ is decreasing in } r.
\]

Then we discuss the base salary \( \alpha^*(r) \), as \( \frac{d\alpha^*(r)}{dr} = [\sigma^2 \beta^*(r) - (a - bc)/2] \frac{d\beta^*(r)}{dr} \), and \( \frac{d\beta^*(r)}{dr} < 0 \) holds, we have: if \( r\sigma^2 \beta^*(r) - (a - bc)/2 < 0 \) holds, \( \frac{d\alpha^*(r)}{dr} \geq 0 \), i.e., \( 2k(b - 1 + 2k \sigma^2 ((bk - 1) r) + bk H(r)) \geq 0 \), the sufficient condition are \( kb > 1 \) or \( 1 > kb \geq 0.5 \), and \( 2k(b - 1 + 2k \sigma^2 r) > r > 0 \). Therefore, this Proposition is proved.

**Proof of Proposition 3.** For the sales agent with private risk attitude \( r \), from the menu contracts in Theorem 2, its optimal certainty equivalence is given by

\[
U^*_p(r) = \frac{2}{\sigma^2} \int_0^r \beta^*(\tau) d\tau,
\]

if he chooses the contract \((\alpha^*(r'), \beta^*(r'))\), his expected utility modified by the certainty equivalent principle is

\[
U^*_S(r') = \alpha^*(r') + \beta^*(r') (a - bp(r') + e) + b' \epsilon^2 / 2 - r \sigma^2 \beta^2(r')/2
\]

\[
= \alpha^*(r') + \beta^*(r') (a - bc)/2 + \sigma^2 \beta^2(r')/2
\]

\[
= \frac{2}{\sigma^2} \int_0^r \beta^*(\tau) d\tau + (r' - r) \sigma^2 \beta^2(r')/2,
\]

we obtain

\[
\Delta U = U^*_p(r') - U^*_p(r) = \frac{2}{\sigma^2} \int_0^r \beta^2(\tau) d\tau - \beta^2(r')/2,
\]

since \( \beta^2(r') \) is decreasing in \( r \) (see the proof of Proposition 2), thus, either \( r' < r < r' \) holds, \( \Delta U \geq 0 \) holds, which implies that the sales agent does not pretend to hide the actual risk attitude information. Therefore, this Proposition is proved.

**Proof of Proposition 4.** Obviously, \( \beta^*(r) \) is decreasing in \( M(r) \), and \( M(r) \geq 0 \), when the sales agent’s risk attitude is symmetric and \( H(r) = 0 \), \( M(r) \) degrades into \( 1 + k r \sigma^2 \), \( \beta^*(r) = \beta' \), thus \( \beta^*(r) \leq \beta^* \) holds; and \( \beta^*(r)/k \leq \beta^*/k \), i.e., \( e^*(r) \leq e^* \) holds. From \( p^*(r) = (a + bc + \beta^*(r)/2b \) and \( p^* = (a + bc + \beta^*)/2b \), we get \( p^*(r) \leq p^* \). From Observations 1 and 3, we have \( Q^* - m(\beta^*) = Q^*(r) - m(\beta^*(r)) = T, Q^*_0 = T + (a - bc)/2 + \beta^*/2k, Q^*_q = T + (a - bc)/2 + \beta^*/2k \) implies \( Q^*_q \leq Q^* \), holds.

From the proof of Proposition 2, we have

\[
\frac{k(a - bc)^2 M(r)}{4k M(r) - 2} \leq \pi(\beta^*) \leq \frac{k(a - bc)^2 (1 + A)}{4k (1 + A) - 2} \implies E^*_p(\Pi_M) \leq E^*(\pi_M) \text{ holds. Therefore, this Proposition is proved.}
\]

**Proof of Proposition 5.** From the Equations (12) and (33), combining with the implicit function differentiation rule, we have

\[
\frac{\partial Q^*(r)}{\partial r} = -\frac{2}{\sigma^2} \int_0^r \beta^*(\tau) d\tau/2k (a - bc)/2 + 2\gamma \int_0^r x F(x) dx < 0 \text{ and } \frac{\partial Q^*_p}{\partial r} = -\frac{2}{\sigma^2} \int_0^r \beta^*/2k (a - bc)/2 + 2\gamma \int_0^r x F(x) dx < 0 \text{ hold. We note only } \pi(\beta^*(r)) \text{ and } \pi(\beta^*) \text{ have } \gamma, \text{ and } \frac{\partial \pi(\beta^*)}{\partial r} = -2 \int_0^r (Q^* - m(\beta^*)) (Q^* - m(\beta^*)) F(x) dx < 0,
Proof of Observation 5. From the proof of Proposition 5, we know that $Q^*$ and $Q^*(r)$ are unique, both are equal to $T$, and decided by Equation (17) (Observations 1 and 3). Therefore, this Proposition is proved.

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