Principles, Progress and Problems in Inflationary Cosmology

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(June 1, 2018)

Inflationary cosmology has become one of the cornerstones of modern cosmology. Inflation was the first theory within which it was possible to make predictions about the structure of the Universe on large scales, based on causal physics. The development of the inflationary Universe scenario has opened up a new and extremely promising avenue for connecting fundamental physics with experiment. This article summarizes the principles of inflationary cosmology, discusses progress in the field, focusing in particular on the mechanism by which initial quantum vacuum fluctuations develop into the seeds for the large-scale structure in the Universe, and highlights the important unsolved problems of the scenario. The case is made that new input from fundamental physics is needed in order to solve these problems, and that thus early Universe cosmology can become the testing ground for trans-Planckian physics.

I. INTRODUCTION

With the recent high-accuracy measurements of the spectrum of the cosmic microwave background (CMB) (see Fig. 1), cosmology has become a quantitative science. There is now a wealth of new data on the structure of the Universe as deduced from precision maps of the cosmic microwave background anisotropies, from cosmological redshift surveys, from redshift-magnitude diagrams of supernovae, and from many other sources. Standard Big Bang (SBB) cosmology provides the framework for describing the present data. The interpretation and explanation of the existing data, however, requires us to go beyond SBB cosmology and to consider scenarios like the Inflationary Universe in which space-time evolution in the very early Universe differs in crucial ways from what is predicted by the SBB theory. Since inflationary cosmology at later times smoothly connects with the SBB picture, we must begin this article with a short review of the framework of SBB cosmology.

Standard big bang cosmology rests on three theoretical pillars: the cosmological principle, Einstein’s general theory of relativity, and the assumption that matter is a classical perfect fluid. The cosmological principle concerns the symmetry of space-time and states that on large distance scales space is homogeneous and isotropic. This implies that the metric of space-time can be written in Friedmann-Robertson-Walker (FRW) form. For simplicity, we consider the case of a spatially flat Universe:

$$ds^2 = dt^2 - a(t)^2 [dr^2 + r^2(d\theta^2 + \sin^2 \vartheta d\varphi^2)] .$$

Here, $t$, $r$, $\theta$ and $\varphi$ are the space-time coordinates, and $ds$ gives the proper time between events in space-time. The coordinates $r$, $\theta$ and $\varphi$ are “comoving” spherical coordinates, and $t$ is the physical time coordinate. Space-time curves with constant comoving coordinates correspond to the trajectories of particles at rest. If the Universe is expanding, i.e. the scale factor $a(t)$ is increasing, then the physical distance $\Delta x_p(t)$ between two points at rest with fixed comoving distance $\Delta x_c$ grows as $\Delta x_p = a(t)\Delta x_c$.

The dynamics of an expanding Universe is determined by the Einstein equations, which relate the expansion rate to the matter content, specifically to the energy density $\rho$ and pressure $p$. For a Universe obeying the cosmological principle (and neglecting the possible presence of a cosmological constant) the Einstein equations reduce to the Friedmann-Robertson-Walker (FRW) equations

$$\left(\frac{\dot{a}}{a}\right)^2 - \frac{k}{a^2} = \frac{8\pi G}{3}\rho ,$$

$$\dot{\rho} = -3H(\rho + p) ,$$

where $H = \dot{a}/a$ is the Hubble expansion rate, and an overdot denotes the derivative with respect to time $t$. 

FIG. 1. Compilation of the spectrum of the CMB. In the region around the peak (the region probed with greatest precision by the COBE satellite) the error bars are smaller than the size of the data points.
The third key assumption of standard cosmology is that matter is described by a classical ideal gas with some equation of state which is conveniently parametrized in the form $p = w\rho$ which some constant $w$. For cold matter, pressure is negligible and hence $w = 0$. For radiation we have $w = 1/3$. In standard cosmology, the Universe is a mixture of cold matter and radiation, the former dominating at late times, the latter dominating in the early Universe. In this case, the FRW equations can be solved exactly, with the result that the Universe had to be born in a “Big Bang” singularity.

SBB cosmology explains Hubble’s redshift-distance relationship, and it explains the abundances of the light elements Helium, Deuterium and Lithium. These nucleosynthesis predictions of the SBB model depend on a single free parameter (which is the present ratio of the number densities of baryons to photons), and thus the fact that the abundances of more than one element can be matched by adjusting this ratio is remarkable. Most importantly, SBB cosmology predicts the existence and black-body spectral distribution of a microwave cosmic background radiation, the CMB. The precision measurement of the spectrum of the CMB is a triumph for SBB cosmology (see Fig. 1).

However, the triumph of the CMB also leads to one of several major problems for SBB cosmology: within the context of this theory, there is no explanation for the high degree of isotropy of the radiation. As is illustrated in Fig. 2, at the time the microwave radiation last scattered (which occurred when the temperature was about a factor $10^3$ of the present temperature), the maximal distance which light could have communicated information starting at the Big Bang (the forward light cone) is much smaller than the distance over which the microwave photons are observed to have the same temperature (the past light cone). This is the famous “horizon problem” of SBB cosmology. Within the context of SBB cosmology it is also a mystery why the Universe today is observed to be approximately spatially flat, since a spatially flat Universe is an unstable fixed point of the FRW equations in an expanding phase. This problem is called the “flatness problem”. Finally, as illustrated in Fig. 3, within standard cosmology there is no causal mechanism which can explain the nonrandom distribution of the seed inhomogeneities which develop into the present-day large-scale structure. This constitutes the “formation of structure problem”. Under the assumption that only gravity is responsible for the development of inhomogeneities on cosmological scales, the seeds for fluctuations must have been correlated at the time $t_{eq}$, the time when the energy densities in cold matter and radiation were equal (which occurred when the Universe was about $10^{-3}$ of its present size) when inhomogeneities can first start to grow by gravitational instability. But, at that time, the forward light cone was smaller than the separation between the seeds.

Standard Big Bang cosmology is also internally inconsistent as a theory of the very early Universe. We know that at very high energy densities which the theory predicts in the initial stages, the description of matter as a classical ideal gas is invalid. Since quantum field theory is a better description of matter at high energies, this naturally leads us to consider quantum field theory as the description of matter which must take over in the very early Universe, and this realization led to the discovery of the inflationary scenario. What follows is a brief overview of principles, progress and problems in inflationary cosmology. For more details, the reader is referred to [4].

II. THE INFLATIONARY UNIVERSE SCENARIO

The inflationary Universe scenario is based on the simple hypothesis that there was a time interval in the early Universe beginning at some time $t_i$ and ending at a later time $t_R$ (the “reheating time”) during which the scale factor is exponentially expanding. Such a period is
FIG. 4. Sketch (physical distance $x_p$ vs. time $t$) of the solution of the homogeneity problem. During inflation, the forward light cone $L_f(t)$ is expanded exponentially when measured in physical coordinates. Hence, it does not require many e-foldings of inflation in order that $L_f(t)$ becomes larger than the past light cone $L_p(t)$ at the time $t_{rec}$ of last scattering. The dashed line is the forward light cone without inflation.

called “de Sitter” or “inflationary”. The success of Big Bang nucleosynthesis demands that this time interval was long before the time of nucleosynthesis. It turns out that during the inflationary phase, the energy of matter is stored in some new form (see below). At the time $t_R$ of reheating, all this energy is released as thermal energy. This is a nonadiabatic process during which the entropy of the Universe increases by a large factor.

Independent of any specific realization, the above simple hypothesis of inflation leads immediately to possible solutions of the horizon, flatness and formation of structure problems. Fig. 4 is a sketch of how a period of inflation can solve the horizon problem. During inflation, the forward light cone increases exponentially compared to a model without inflation, whereas the region over which isotropy is observed is not affected. Hence, provided inflation lasts sufficiently long, the forward light cone at the time of last scattering of CMB photons can be made larger than the region from which the microwave photons are reaching us today.

Inflation also can solve the flatness problem. The key point is that at the time of reheating, the entropy of the Universe increases by a large factor, and this drives the Universe towards spatial flatness, as can be seen from the FRW equations. In fact, one of the key predictions of inflationary cosmology is that the Universe should be spatially flat to great accuracy (although it is possible to construct special models of inflation which produce any given deviation from spatial flatness).

Most importantly, inflation provides a mechanism which in a causal way generates the primordial perturbations required to explain the nonrandom distribution of matter on the scales of galaxies and galaxy clusters, and
to produce small-amplitude anisotropies in the CMB. At any given time, microphysics can act coherently on scales up to the Hubble radius $H^{-1}(t)$. The key point is that during the inflationary period the Hubble radius is constant, whereas the physical length scale associated with fluctuations which have fixed comoving scale increases exponentially. Thus, as is depicted in Fig. 5, provided that the inflationary period is sufficiently long, all comoving scales of cosmological interest today had a physical wavelength smaller than the Hubble radius in the early stages of inflation. Thus, it is possible without violating causality to have a mechanism which generates microscopic-scale fluctuations during the period of inflation whose wavelengths then get stretched exponentially so that they can become the seeds for structure in the present Universe.

As will be discussed in Section 4, the density perturbations produced during inflation are due to quantum fluctuations in the matter and gravitational fields. These fluctuations are continuously generated during the period of inflation. Once the physical wavelength equals the Hubble radius, the vacuum oscillations freeze out, the quantum state is squeezed, and the highly squeezed state at late time appears as a state with a large number of particles, representing the density fluctuations at late times. The amplitude of these fluctuations is given by the Hubble expansion rate $H$. Since $H$ is approximately constant during the period of inflation, this mechanism of quantum vacuum fluctuations freezing out and becoming classical density fluctuations leads to the prediction that the spectrum of fluctuations should be scale invariant, i.e. the physical measure of the amplitude of fluctuations at
late times should be independent of the wavenumber $k$:

$$k^3|\delta_k|^2 = \text{const},$$

(4)

where $\delta_k$ is the fractional energy density fluctuation in momentum space. Thus, independent of the specific mechanism which drives inflation, as long as the expansion is almost exponential, the spectrum of fluctuations is predicted to be almost scale-invariant. As will be discussed in Section 4, this quantitative prediction of inflationary cosmology is confirmed by observations. However, as will be mentioned in Section 5, this prediction may depend on unstated assumptions about the trans-Planckian physics.

III. HOW TO OBTAIN INFLATION

Because in the year 1980, when inflationary cosmology was being developed, quantum field theory was the best available description of matter at high energies such as must have occurred close to the Big Bang, it in retrospect seems obvious to turn to it for a possible implementation of the inflationary Universe scenario. Since scalar matter fields are supposed to play an important role in high energy physics, in particular for implementing the spontaneous breaking of internal gauge symmetries, it is necessary to consider the role of such fields in cosmology.

If we assume that Einstein’s equations remain valid in the early Universe, then it follows from the FRW equations that an equation of state with negative pressure $p \approx -\rho$ is required to obtain exponential inflation. In order to obtain an accelerated expansion (“generalized inflation”), an equation of state with $p < -(1/3)\rho$ is needed. In the context of renormalizable quantum field Lagrangians, it is only scalar fields which provide the possibility to obtain inflation. From the action of a scalar quantum field $\varphi$ (in an expanding space-time) it immediately follows that the energy density and pressure of such a field are given by

$$\rho = \frac{1}{2}(\dot{\varphi})^2 + \frac{1}{2a^2}(\nabla \varphi)^2 + V(\varphi)$$

(5)

$$p = \frac{1}{2}(\dot{\varphi})^2 - \frac{1}{6a^2}(\nabla \varphi)^2 - V(\varphi),$$

(6)

where $V(\varphi)$ is the potential energy density. It thus follows that if the scalar field is homogeneous and static, but the potential energy positive, then the equation of state $p = -\rho$ necessary for exponential inflation results. This is the idea behind potential-driven inflation.

Guth’s initial hope was that the same scalar field responsible for the spontaneous symmetry breaking of a unified gauge symmetry could be the inflaton, the field responsible for inflation. However, this hope cannot be realized since the potential of such a scalar field is too steep in order to provide a period of more than a few Hubble times $H^{-1}$ during which the field kinetic energy remains negligible. This follows almost immediately by considering the variational equation of motion for $\varphi$, which in the absence of spatial gradients becomes

$$\ddot{\varphi} + 3H\dot{\varphi} = -V'(\varphi),$$

(7)

where a prime denotes the derivative with respect to $\varphi$.

Although at the present time there are many models for inflation, there is no single convincing one. Chaotic inflation is a prototypical model. It assumes the existence of a new scalar field, the “inflaton” with a smooth potential (see Fig. 6). The inflaton is so weakly coupled that it does not start out in thermal equilibrium in the early Universe. Most of the phase space of initial conditions consists of values of $\varphi$ which are large (compared to the Planck scale). It follows from the equation of motion that for such initial conditions the scalar field will roll slowly (in the sense that $\dot{\varphi} \ll 3H\dot{\varphi}$) for a long (compared to $H^{-1}$) time, thus yielding inflation. Once $\varphi$ drops much below the Planck scale, the kinetic energy begins to dominate, the field oscillates around its minimum and releases its energy to regular matter via interaction terms in the Lagrangian.

IV. PROGRESS IN INFLATIONARY COSMOLOGY

Simple prototypical models of inflation predict a scale-invariant spectrum of adiabatic fluctuations (adiabatic in this context means that the energy density fluctuations in all components of matter are proportional), and this will arise naturally in the context of inflation if during the phase of inflationary expansion a single scalar field dominates the dynamics). This prediction of the inflationary scenario has recently been confirmed to high accuracy by the measurements of CMB anisotropies. Microwave anisotropies are usually quantified by expanding the temperature maps into spherical harmonics and
calculating the power at different values of the angular quantum number \( l \). Fig. 7[1] shows a compilation of the recent measurements. The predictions of a theory based on a scale-invariant spectrum of adiabatic fluctuations with a background cosmology which is taken to be dominated today by a remnant cosmological constant which contributes 70% to the density required for a spatially flat Universe (most of the other 30% is believed to be made up of cold dark matter) fits the data very well, whereas a model with a background dominated by cold dark matter does not reproduce the observed details of the spectrum in the region of the oscillations of the spectrum.

Note the oscillations in the spectrum at values of \( l \) larger than 100. These “acoustic” oscillations are an imprint of the coherence of the primordial fluctuations. Refer to Fig. 8 for a sketch of the mechanism by which density fluctuations induce CMB anisotropies. If we imagine the inhomogeneities as a superposition of plane waves (which evolve independently in the early Universe since their amplitudes are small), then inflation predicts that these waves start oscillating with the same phase at the time when the wavelength equals the Hubble radius. Thus, when measured at the time of last scattering, the phase of the wave is a periodically varying function of \( l \). Maxima and minima of the waves result in maxima of the temperature anisotropies on the corresponding angular scales, nodes of the waves result in minima (see e.g. for a detailed discussion of the physics of the acoustic oscillations).

The theoretical understanding of the theory by which quantum vacuum fluctuations on microscopic scales evolve to give rise to classical inhomogeneities on cosmological wavelengths is one of the main areas of progress in inflationary cosmology. Since it is necessary to propagate the fluctuations on scales larger than the Hubble radius, the Newtonian theory of cosmological perturbations obviously is inapplicable, and a general relativistic analysis is needed. On these scales, matter is essentially frozen in comoving coordinates. However, space-time fluctuations can still increase in amplitude. In principle, it is straightforward to work out the general relativistic theory of linear fluctuations. One linearizes the Einstein and scalar matter field equations about an expanding FRW background cosmology. Tediuous but straightforward algebra gives a set of coupled linear differential equations for the metric and matter perturbations.

At the level of linear fluctuations, perturbations with different wavelengths decouple. Hence, the analysis becomes simple if we work in momentum space. Furthermore, there are several independent degrees of freedom. First, there are gravitational waves, space-time perturbations which do not couple to matter. Next, there are vector perturbations which correspond to rotational degrees of freedom. Finally, there are the scalar metric fluctuations, space-time inhomogeneities produced by matter perturbations. The scalar metric fluctuations are the most difficult to analyze, in particular since one must be careful to isolate the physical degrees of freedom from gauge artefacts, modes which correspond to space-time coordinate transformations. For matter described by a single scalar field, as is the case in many prototypical inflationary models, there is only one matter fluctuating degree of freedom. According to the Einstein equations, matter fluctuations are coupled to corresponding scalar metric perturbations. There can be no scalar metric fluctuations without matter inhomogeneities. Thus, restricting attention to scalar metric fluctuations of one particular wavelength, there is only one physical degree of freedom. This implies that the analysis of scalar metric fluctuations is reducible to the theory of a single free scalar field (free because we are dealing with linear fluctuations) in a time-dependent background (the time-dependence is set by the background cosmology).

As mentioned earlier, the primordial fluctuations in an inflationary cosmology result from quantum fluctuations. At the linearized level, the equations describing both gravitational and matter perturbations can be quantized in a consistent way (for a detailed review see[10]). The first step of this analysis is to expand the gravitational and matter action to quadratic order in the metric and matter fluctuation variables. Focusing on the scalar metric perturbations, it turns out that one can express the resulting action in terms of a variable \( v \) which is the coordinate-invariant scalar matter field fluctuation[11].

FIG. 7. Compilation of recent data on the angular power spectrum of CMB anisotropies. The green (-) data points are a compilation of the data prior to March 2000; the others represent data from the following experiments: Boomerang (2001 release), blue (diamond); Maxima (2001 release), purple (triangle); DASI, red (x), CBI, black (square).
The surface of last scattering (solid line LS) is a surface of constant temperature. Thus, photons $c_1$ and $c_2$ from different directions in the sky are redshifted by different amounts before reaching us (we are at the point along the t-axis where the lines labelled $c_1$ and $c_2$ hit), and therefore arrive with different temperatures.

$$v = a(\delta \varphi + \frac{\varphi(0)'}{\mathcal{H}}\Phi). \quad (8)$$

Here, $\varphi(0)$ is the background scalar matter field, $\delta \varphi$ is the scalar field fluctuation, a prime denotes the derivative with respect to conformal time $\eta$ ($\eta$ being defined by $dt = ad\eta$), $\mathcal{H} = a'/a$, and $\Phi$ is the generalized Newtonian gravitational potential (in a coordinate system in which the metric tensor - including fluctuations - is diagonal, $2\Phi$ is the perturbation of the time-time component of the metric). It can then be shown that the action $S_2$ for the fluctuations reduces to the action of a single gauge invariant free scalar field (namely $v$) with a time dependent mass [12,13] (the time dependence reflects the expansion of the background space-time)

$$S_2 = \frac{1}{2} \int dt d^3x (v'^2 - (\nabla v)^2 + \frac{z''}{z}v'^2), \quad (9)$$

where the function

$$z = \frac{a\varphi_{,0}'}{\mathcal{H}}, \quad (10)$$

a function of the background fields, determines the effective mass for the field $v$. The action $S_2$ has the same form as the action for a free scalar matter field in a time dependent gravitational or electromagnetic background, and we can use standard methods to quantize this theory (see e.g., [14]). Each momentum mode of the field obeys a harmonic oscillator equation with time dependent mass. The time dependence of the mass is reflected in the non-trivial form of the solutions of the mode equations. Let us now follow the evolution of a particular mode from the beginning of inflation until late times. The mode starts out in its vacuum state when its wavelength is smaller than the Hubble radius. While the wavelength remains smaller than the Hubble radius, the mode functions oscillate since the time dependence of the mass is negligible, as can be seen from (8). However, once the mode crosses the Hubble radius, the spatial gradient term becomes negligible and the mass term begins to dominate. The mode function thus no longer oscillates, but its amplitude begins to grow as $v \sim z$. This corresponds to squeezing of the vacuum state and corresponds to the generation of fluctuations. This calculation can be followed up to the time the mode re-enters the Hubble radius at late times. It turns out (see Section 5) that the amplitude of the resulting density generically is predicted to be several orders of magnitude larger than what is compatible with current observations, unless some parameter in the quantum field theory Lagrangian is set to a value much smaller than would be inferred from dimensional analysis.

V. PROBLEMS OF INFLATIONARY COSMOLOGY

In spite of the spectacular success of the inflationary scenario in predicting a spectrum of microwave anisotropies and large-scale density fluctuations in excellent agreement with the recent observation, scalar field-driven inflationary models suffer from some serious conceptual problems.

The main success of inflationary cosmology is that it provides a causal theory for the generation of large-scale cosmological fluctuations. However, this success directly leads to a major problem for most realizations of scalar field-based models of inflation studied up to now. It concerns the amplitude of the density perturbations which are induced by quantum fluctuations during the period of accelerated expansion as discussed in the previous section. Unless a parameter in the scalar field potential is set to have a value several orders of magnitude smaller than what would be given by dimensional analysis, the models predict an amplitude of the fluctuation spectrum several order of magnitude larger than the predicted amplitude. For example, in a model with a single inflaton field with quartic potential, the quartic coupling constant $\lambda$ must be of the order of $10^{-12}$ in order that the resulting amplitude of fluctuations agrees with observations. This situation is clearly unsatisfactory for a cosmological scenario motivated by the desire to eliminate cosmological fine-tunings. There have been many attempts to justify such small parameters based on specific particle physics models, but no single convincing model has emerged. However, this is probably the least serious of the problems mentioned here.

In many models of inflation, in particular in chaotic inflation, the period of inflation is so long that comoving scales of cosmological interest today correspond to a physical wavelength much smaller than the Planck length at the beginning of inflation. In extrapolating the evolution of cosmological perturbations according to linear
theory to these very early times, one is implicitly making the assumption that the theory remains perturbative to arbitrarily high energies, and that the classical theory of general relativity remains the appropriate framework for describing space-time. Both of these assumptions are clearly not justified. It has recently been shown that some (admittedly quite violent) changes to the physics on length scales smaller than the Planck length can lead to a spectrum of density fluctuations totally different from what the usual theory predicts [13]. This can be called the “trans-Planckian problem” for inflationary cosmology. On the other hand it shows that in an inflationary Universe, the spectrum of fluctuations can potentially be used to explore Planck-scale physics, thus turning the “problem” into a “window” of opportunity. Planck-scale physics may not only alter the spectrum of fluctuations, it can also dramatically alter the background cosmology, as is seen in “Pre-big-bang Cosmology” [16], a string-motivated dilaton-gravity model which undergoes a dilaton-dominated phase of super-exponential expansion.

Scalar field-driven inflation does not eliminate singularities from cosmology. Although the standard assumptions of the Penrose-Hawking theorems (the theorems which is the context of Einstein gravity coupled to classical fluid matter show that an initial cosmological singularity is inevitable) break down if matter has an equation of state with negative pressure, as is the case during inflation, nevertheless it can be shown that an initial singularity persists in inflationary cosmology [17]. This implies that the theory is incomplete. In particular, the physical initial value problem is not defined.

The Achilles heel of our current inflationary models is without doubt the “cosmological constant problem”. There is some as of yet unknown mechanism which prevents the bare cosmological constant, which in theories with quantum fields is predicted to be at least 62 orders of magnitude larger than the observational limit (this number comes from assuming the cancellation of vacuum energies on scales larger than the supersymmetry breaking scale taken to be about 1 TeV), from gravitating. How do we know that this unknown mechanism does not also lead the transient “cosmological constant” given by the potential energy of the scalar field to be gravitationally inert, thus eliminating the basis of scalar field-driven inflation?

VI. FUTURE DIRECTIONS

In the light of the problems of scalar field potential-driven inflation discussed in the previous sections, many cosmologists have begun thinking about new avenues towards early Universe cosmology which, while maintaining (some of) the successes of inflation, address and resolve some of its difficulties. In the same way that inflationary cosmology builds on and transcends standard big

bang cosmology by making use of a new theory of matter (quantum field theory in the case of inflation), it is quite likely that a resolution of the problems of inflationary cosmology will once again come from an improved description of matter at high energies. The main candidate at the moment for such a theory is string theory. String theory, in fact, will also lead to a modified picture of space-time at short distances, and may allow a unified quantum description of both the background space-time and of the cosmological fluctuations.

There are at least two ways in which an improved cosmological model could come about. It is possible that a better understanding of string theory will lead to a convincing realization of inflation which does not suffer from the problems mentioned in the previous section, and that many of the cosmological predictions of inflation for present day observations would remain unchanged. However, it is also possible that string theory will provide alternatives to inflationary cosmology, a possibility which would lead to clear observational signatures.

There are several reasons to hope that string theory might lead to an improved realization of inflation. One reason is that the vacuum space of string theory is a complicated moduli space, and the individual directions in this space (e.g. radii of extra dimensions) can be associated with scalar fields with potentials which vanish at the perturbative level. It is hoped that nonperturbative corrections might generate in a natural way the flat potentials required for inflation. The realization that string theory contains higher dimensional fundamental objects called branes has led to speculations that our space-time might be a brane in a higher dimensional space-time. These “brane-world” scenarios have opened new avenues to realizing inflation in the context of string theory (see e.g. [18] for a recent attempt and for references to earlier work).

However, it is also possible that string theory will generate an alternative to inflationary cosmology which maintains most of the successes of inflation, but at the same time gives rise to predictions with which this new theory can be distinguished from inflation. One approach which has received a lot of recent attention is pre-big-bang cosmology [19], a theory in which the Universe starts in an empty and flat dilaton-dominated phase which leads to super-exponential expansion. A nice feature of this theory is that the mechanism of acceleration is completely independent of a scalar field potential and thus independent of the cosmological constant issue. Pre-big-bang cosmology in its simplest realizations does not give rise to a spectrum of scale-invariant adiabatic fluctuations. Rather, the spectrum is isocurvature (see e.g. [19] for a recent review and original references). Another recent attempt to provide an alternative to inflation in the context of string theory is the “Eckpyrotic Universe” scenario [20], a nonsingular cosmological model designed to solve the problems of SBB cosmology mentioned in Section 1 and to provide a spectrum of scale-invariant adiabatic perturbations without any period of accelera-
tion (see, however, [21–23] for criticisms of this scenario).

It is, however, also possible that some of the problems of inflationary cosmology mentioned in Section 5 can be addressed within the context of more conventional physics (general relativity plus quantum field theory). In particular, it is possible that some of the problems are consequences of neglecting the intrinsically non-linear structure of general relativity (see e.g. [24] for some speculations along these lines).

ACKNOWLEDGMENTS

I am grateful to Douglas Scott for providing Figures 1 and 7. This work was supported in part by the US Department of Energy under Contract DE-FG02-91ER40688, Task A.

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