Competitive Accretion in Clusters and the IMF

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Abstract. Observations have revealed that most stars are born in clusters. As these clusters typically contain more mass in gas than in stars, accretion can play an important role in determining the final stellar masses. Numerical simulations of gas accretion in stellar clusters have found that the stars compete for the available reservoir of gas. The accretion rates are highly nonuniform and are determined primarily by each star’s position in the cluster. Stars in the centre accrete more gas, resulting in initial mass segregation. This competitive accretion naturally results in a mass spectrum and is potentially the dominant mechanism for producing the initial mass function. Furthermore, accretion on to the core of a cluster forces it to shrink, which may result in formation of massive stars through collisions.

1. Introduction

One of the most important goals of a general theory of star formation is to explain the origin of the initial mass function (IMF). In order to do this, we need to understand the differences between low-mass and high-mass star formation. A stellar cluster is the natural size-scale to investigate these differences as they contain the full mass range of stars. In this paper, we review how competitive accretion in clusters can form the basis of a theory for the IMF. Competitive accretion arises when a group of stars compete for a finite mass-reservoir (Zinnecker 1982). If this accretion contributes a large fraction of the final stellar mass, then the competition process determines the overall distribution of stellar masses.

Surveys of star forming regions have found that the majority of pre-main sequence stars are found in clusters (e.g. Lada et. al. 1991; Lada, Strom & Myers 1993; see also Clarke, Bonnell & Hillenbrand 2000). The fraction of stars in clusters depends on the molecular cloud considered but generally varies from 50 to \( \gtrsim 90 \) per cent. These clusters contain anywhere from tens to thousands of stars with typical numbers of around a hundred (Lada et. al. 1991; Phelps & Lada 1997; Clarke et. al. 2000). Cluster radii are generally a few tenths of a parsec such that mean stellar densities are of the order of \( \approx 10^3 \) stars/pc\(^3\) (c.f. Clarke et. al. 2000) with central stellar densities of the larger clusters (e.g. the ONC) being \( \gtrsim 10^4 \) stars/pc\(^3\) (McCaughrean & Stauffer 1994; Hillenbrand & Hartmann 1998; Carpenter et. al. 1997).
Furthermore, young clusters are usually associated with massive clumps of molecular gas (Lada 1992). Generally, the mass of the gas in the youngest clusters is larger than that in stars (Lada 1991), with up to 90% of the cluster mass in the form of gas. Gas can thus play an important role in the dynamics of the clusters and affect the final stellar masses through accretion.

Surveys of the stellar content of young (ages $\approx 10^6$ years) clusters (e.g. Hillenbrand 1997) reveal that they contain both low-mass and high-mass stars in proportion as you would expect from a field-star IMF (Hillenbrand 1997). Furthermore, there is a degree of mass segregation present in the clusters with the most massive stars generally found in the cluster cores.

### 2. Mass Segregation

Young stellar clusters are commonly found to have their most massive stars in or near the centre (Hillenbrand 1997; Carpenter et al. 1997). This mass segregation is similar to that found in older clusters but the young dynamical age of these systems offers the chance to test whether the mass segregation is an initial condition or due to the subsequent evolution. We know that two-body relaxation drives a stellar system towards equipartition of kinetic energy and thus towards mass segregation. In gravitational interactions, the massive stars tend to lose some of their kinetic energies to lower-mass stars and thus sink to the centre of the cluster.

Numerical simulations of two-body relaxation have shown that while some degree of mass segregation can occur over the short lifetimes of these young clusters, it is not sufficient to explain the observations (Bonnell & Davies 1998). Thus the observed positions of the massive stars near the centre of clusters like the ONC reflects the initial conditions of the cluster and of massive star formation that occurs preferentially in the centre of rich clusters.

Forming massive stars in the centre of clusters is not straightforward due to the high stellar density. For a star to fragment out of the general cloud requires that the Jeans radius, the minimum radius for the fragment to be gravitationally bound,

$$R_J \propto T^{1/2} \rho^{-1/2}, \quad (1)$$

be less than the stellar separation. This implies that the gas density has to be high, as you would expect at the centre of the cluster potential. The difficulty arises in that the high gas density implies that the fragment mass, being approximately the Jeans mass,

$$M_J \propto T^{3/2} \rho^{-1/2}, \quad (2)$$

is quite low. Thus, unless the temperature is unreasonably high in the centre of the cluster before fragmentation, the initial stellar mass is quite low. Equation (2) implies that the stars in the centre of the cluster should have the lowest masses, in direct contradiction with the observations. Therefore, we need a better explanation for the origin of massive stars in the centre of clusters.
3. The dynamics of accretion in clusters

Young stellar clusters are commonly found to be gas-rich with typically 50\% to 90\% of their total mass in the form of gas (e.g. Lada 1991). This gas can interact with, and be accreted by, the stars as both move in the cluster. If significant accretion occurs, it can affect both the dynamics and the masses of the individual stars (e.g. Larson 1992).

Fragmentation models of multiple systems and of stellar clusters show that the fragmentation is inherently inefficient with a small fraction of the total mass in the initial fragments (e.g. Larson 1978; Boss 1986; Bonnell et.al. 1992; Boss 1996; Klessen, Burkert & Bate 1998). The remaining gas is accreted by the fragments on the gas free-fall timescale. This occurs as the gas is self-gravitating and is the dominant mass component. The free-fall timescale is roughly the crossing or dynamical time of a stellar cluster as both are related to the total mass in the cluster which is mostly in the form of gas. Thus the gas is accreted on the same timescale on which the stars move. An exception to this is if the pre-fragmented cluster is highly structured (e.g. Klessen et.al. 1998), then the initial dynamical time can be significantly longer than the local gas free-fall timescale. Such clusters should have little gas by the time they have relaxed to a quasi-spherical distribution. Thus, for the remainder of this paper, we assume that the initial cluster is approximately spherical and that the gas and stars have similar distributions.

Simulations of accretion in clusters have been performed for clusters of 10 to 100 stars using a combination SPH and N-body code. These simulations found that accretion is a highly non-uniform process where a few stars accrete significantly more than the rest (Bonnell et.al. 1997; Bonnell et.al. 2000). Individual stars’ accretion rates depend primarily on their position in the cluster (see Figure 1. The accretion rates versus radius for a cluster containing 100 stars and 90\% of its mass in gas.)
Fig. 2. The position of the 6 most massive stars (with $M_\star \gtrsim 4M_{\mathrm{med}}$, filled triangles) in a cluster of 100 stars. The right-hand panel is a blow-up of the cluster core. The masses of the most massive stars are due to the competitive accretion process.

Fig. 1) with those in the centre accreting more gas than those near the outside. Stars near the centre accrete more gas than do others further out due to the effect of the cluster potential which funnels the gas down towards the deepest part of the potential. The accretion rates can also be relatively large when the gas is the dominant component such that the final masses of the more massive stars are due to the accretion process. In contrast, many of the stars do not accrete significant amounts of gas and their final masses are a closer reflection of any initial mass distribution in the cluster due to the fragmentation.

Accretion in stellar clusters naturally leads to both a mass spectrum and mass segregation. Even from initially equal stellar masses, the competitive accretion results in a wide range of masses with the most massive stars located in or near the centre of the cluster. Furthermore, if the initial gas mass-fraction in clusters is generally equal, then larger clusters will produce higher-mass stars and a larger range of stellar masses as the competitive accretion process will have more gas to feed the few stars that accrete the most gas.

4. Modelling Competitive Accretion

The accretion process outlined above is termed “competitive accretion” (Zinnecker 1982) as each star competes for the available gas reservoir. In order to investigate the possible mass functions from this process, we need to consider larger clusters, or numbers of clusters, to get statistically significant numbers. This cannot be done by the above simulations as numerical resolution is presently inadequate to follow the accretion process with more than 100 stars. One option is an analytical or semi-analytical approach to competitive accretion which can then be applied to larger clusters (Bonnell et.al. 2000).
Gas accretion by a star is given by the general formula

\[ \dot{M}_* \approx \pi \rho v R_{\text{acc}}^2, \]  

(3)

where \( \rho \) is the gas density and \( v \) is the relative gas-star velocity and \( R_{\text{acc}} \) is the accretion radius. In a cluster model, we have the velocities and densities. Thus, in order to parametrise the accretion process, we need a description of the accretion radius \( R_{\text{acc}} \).

Accretion by a star was first explored by Bondi and Hoyle (Bondi & Hoyle 1944; Bondi 1952) in terms of an isolated star in a uniform, non-self-gravitating medium. In this model, the accretion radius is given by

\[ R_{\text{BH}} = \frac{GM_*}{(v^2 + c_s^2)^{1/2}}, \]  

(4)

where \( M_* \) is the stellar mass, and \( c_s \) is the gas sound speed. This approach neglects the self-gravity of the gas, the presence of other stars, the cluster potential and how these affect the accretion.

An alternative to Bondi-Hoyle accretion is to consider a tidal accretion radius where gas can only be accreted onto a star if it is more bound to it than to other stars or the cluster as a whole. Accretion in this context is then similar to the Roche-lobe overflow problem. Taking the Roche-lobe radius to be the accretion radius we have

\[ R_{\text{roche}} = 0.5 \left( \frac{M_*}{M_{\text{enc}}} \right)^{1/3} R_*, \]  

(5)

where \( M_{\text{enc}} \) is the mass enclosed in the cluster at the star’s position \( R_* \). This approach is consistent with the tidal effects of the cluster but does not consider whether the gas is bound to the star when considering it’s thermal and kinetic energies.

Support for using such a model comes from studies of accretion in binary systems (Bate 1997, Bate & Bonnell 1997). These studies found that the accretion of cold gas was well represented when the accretion radius was taken to be the Roche-lobe of the individual stars.

The simulations of accretion in clusters were used to test which model best represented the accretion process (Bonnell et.al. 2000). Figure 3 shows a comparison of the SPH determined accretion rate versus that estimated from Bondi-Hoyle and from Roche-lobe accretion for a cluster of 30 stars embedded in cold gas. We see that the Bondi-Hoyle accretion is too high early on in the evolution when the SPH accretion rate is low and that overall there is little correspondence between the Bondi-Hoyle accretion rate which is nearly constant and the SPH determined accretion rate. In contrast, the Roche-lobe accretion follows the SPH determined accretion rate from the early low values to the much higher values that occur towards the end of the simulation.

That the Roche-lobe accretion works better when the Bondi-Hoyle accretion gives a higher accretion rate makes sense as the Bondi-Hoyle radius is then larger than the Roche-lobe radius and the effective accretion radius would be the minimum of the two. In contrast, it is surprising (at first) that the Roche-lobe accretion works better even when the Bondi-Hoyle radius is smaller than the
Roche radius. After closer inspection, it is apparent that the star is carrying an envelope of gas with it through the cluster and that this envelope approximately fills the Roche-lobe. This envelope forms while the stars are initially moving subsonically and subsequently acts to dampen the relative high velocity gas so that it can be bound to the star. Simulations where the stars are initially moving supersonically and are devoid of an envelope are found to be less well modelled by Roche-lobe accretion, in agreement with this interpretation. In general, we expect that all stars will form with circumstellar envelopes and thus the Roche-lobe accretion should be a good estimate of the accretion rates.

5. Accretion and the IMF

Using the above formulation for Roche-lobe accretion in a stellar cluster, we can estimate plausible IMFs considering simple cluster models. Hillenbrand & Hartmann (1998) showed that the ONC can be approximated by a King model which has a stellar density profile $n \propto r^{-2}$. In this case, the number of stars at a given radius is constant, $dn \propto dr$. The gas density is somewhat more tricky but two possible distributions are $\rho \propto r^{-2}$, similar to the stellar distribution, or $\rho \propto r^{-3/2}$, corresponding to an accretion solution (e.g. Shu 1977; Foster & Chevalier 1993). Considering these two possibilities and that the stellar velocities are in virial equilibrium with the dominant gas distribution, we can calculate $M_*(r)$ and thus an IMF.
Figure 4. Mass functions for an analytic model of competitive accretion in clusters using Roche-lobe accretion. Different lines show the evolution from an initial distribution containing only very low-mass stars. The mass functions are summed over 28 clusters of 1000 stars each. The diagonal line at the right of the graph denotes the Salpeter slope of -2.35. The stars do not move in this model.

In the first case where $\rho \propto r^{-2}$, we find that $M_* \propto r^{-2}$ and that the resulting mass function is

$$dn \propto m^{-3/2} dm.$$  \hspace{1cm} (6)

For the second case where $\rho \propto r^{-3/2}$, then $M_* \propto r^{-3/4}$, and the IMF is then

$$dn \propto m^{-7/3} dm.$$  \hspace{1cm} (7)

In both cases, we see that we expect the massive stars to be segregated towards the centre of the cluster.

A plausible evolutionary picture would have the gas density evolving from a $\rho \propto r^{-2}$ initial distribution to a $\rho \propto r^{-3/2}$ as material is depleted due to the accretion and is replaced with gas inflowing from larger radii. As this occurs from the inside out, stars near the centre of the cluster, which are the more massive stars, would have the steeper mass-profile. An illustration of the type of mass function that results from this process is shown in figure 4. Limitations of this approach is that it neglects the stellar dynamics of the cluster and assumes a simple prescription for the gas and thus for the stellar velocities.

An alternative is to follow both the stellar and gas-dynamics simultaneously but with a prescription of the accretion to lessen resolution requirements (Bonnell et.al. 2000). Figure 5 shows the results of four clusters of 1000 stars each undergoing competitive accretion using the Roche-lobe formalism. All four clusters result in reasonable IMFs with exponents in the range of $-1.5$ to $\approx -2.5$ where the Salpeter IMF is $-2.35$. The four models shown in figure 5 span the range of possible gas temperatures (virialised, cold) and equations of state.
Figure 5. Mass functions for simulated clusters of 1000 stars accreting gas using a Roche-lobe accretion approximation. The gas is initially 90 per cent of the cluster mass and the different curves represent different thermal temperatures of the gas (cold, virialised) and either an isothermal or adiabatic equation of state. The three diagonal lines at the right of the graph denote slopes of -1.5, -2 and the Salpeter slope of -2.35.

(isothermal, adiabatic). The one major limitation for these models is the lack of any feedback from the stars and the possibility of turbulence in the gas.

From these models we see that competitive accretion gives reasonable IMFs and fulfills a basic requirement of producing the more massive stars near the centre of the cluster. Unfortunately, there is an added complication in forming very massive stars, $M_\star > \sim 10M_\odot$.

6. Formation of Massive Stars

The formation of massive stars is problematic not only for their special location in the cluster centre, but also due to the fact that the radiation pressure from massive stars is sufficient to halt the infall and accretion (Yorke & Krugel 1977; Yorke 1993). This occurs for stars of mass $\gtrsim 10 M_\odot$.

A secondary effect of accretion in clusters is that it can force it to contract significantly. The added mass increases the binding energy of the cluster while accretion of basically zero momentum matter will remove kinetic energy. If the core is sufficiently small that its crossing time is relatively short compared to the accretion timescale, then the core, initially at $n \approx 10^4$ stars pc$^{-3}$, can contract to the point where, at $n \approx 10^8$ stars pc$^{-3}$, stellar collisions are significant (Bonnell, Bate & Zinnecker 1998). Collisions between intermediate mass stars ($2M_\odot \lesssim m \lesssim 10M_\odot$), whose mass has been accumulated through accretion in the cluster core, can then result in the formation of massive ($m \gtrsim 50M_\odot$) stars. This model
for the formation of massive stars predicts that the massive stars have to be significantly younger than the mean stellar age due to the time required for the core to contract (Bonnell et al. 1998).

Preliminary studies of possible IMFs that would result from a merger process (Bailey 1999) show that, plotted as a cumulative distribution to lessen the effects of small statistics, the mass function is compatible with the high mass function of Kroupa, Tout & Gilmore (1990) where \( dn \propto m^{-2.5} dm \).

7. Summary

Competitive accretion in young, gas rich stellar clusters is an appealing mechanism to explain the origin of the IMF. This one simple physical process can explain both the initial mass segregation in stellar clusters and potentially the exact mass-distribution. Stellar dynamical effects such as two-body relaxation are not able to explain the mass segregation found in clusters such as the ONC due to their extreme youth.

The accretion in a cluster environment is found to be better represented by a tidal or Roche-lobe accretion radius than by Bondi-Hoyle accretion. This occurs as the tidal radius determines when the gas is bound to the star compared to the cluster as a whole. Furthermore, this radius represents the maximum extent of any circumstellar envelope which can act to sweep up the intracluster gas.

Gas accretion in a stellar cluster is highly competitive and uneven. Stars near the centre of the cluster accrete at significantly higher rates due to their position where they are aided in attracting the gas by the overall cluster potential. This competitive accretion naturally results in both a spectrum of stellar masses, and an initial mass segregation even if all the stars originate with equal masses.

Simple analytical models of the cluster and the competitive accretion yield IMFs which range from \( \gamma = -3/2 \) when the gas is assumed to be in the form \( \rho \propto r^{-2} \) to \( \gamma = -7/3 \) when the gas is in a \( \rho \propto r^{-3/2} \) distribution as in an accretion flow. These two power-laws could be expected to represent the low-mass stars in the outer part of the cluster and the higher-mass stars in the inner parts of the cluster, respectively, as the accretion flow would grow from the inside-out. Simulations of clusters undergoing the Roche-lobe prescription for accretion produce mass functions which are compatible with these limits.

Finally, massive stars may form through stellar collisions in the centre of dense clusters. The necessary density for collisions would result due to the accretion process of adding mass without significant momentum which then forces the core to contract to higher densities. Such a collisional model for the formation of massive stars evades the problem of accreting onto massive stars.

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