Butane dihedral angle dynamics in water is dominated by internal friction

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Edited by Dmitrii E. Makarov, University of Texas, Austin, TX, and accepted by Editorial Board Member Peter J. Rossky April 4, 2018 (received for review December 22, 2017)

The dihedral dynamics of butane in water is known to be rather insensitive to the water viscosity; possible explanations for this involve inertial effects or Kramer’s turnover, the finite memory time of friction, and the presence of so-called internal friction. To disentangle these factors, we introduce a method to directly extract the friction memory function from unconstrained simulations in the presence of an arbitrary free-energy landscape. By analysis of the dihedral friction in butane for varying water viscosity, we demonstrate the existence of an internal friction contribution that does not scale linearly with water viscosity. At normal water viscosity, the internal friction turns out to be eight times larger than the solvent friction and thus completely dominates the effective friction. By comparison with simulations of a constrained butane molecule that has the dihedral as the only degree of freedom, we show that internal friction comes from the six additional degrees of freedom in unconstrained butane that are orthogonal to the dihedral angle reaction coordinate. While the insensitivity of butane’s dihedral dynamics to solvent viscosity is solely due to the presence of internal friction, inertial effects nevertheless crucially influence the resultant transition rates. In contrast, non-Markovian effects due to the finite memory time are present but do not significantly influence the dihedral barrier-crossing rate of butane. These results not only settle the character of dihedral dynamics in small solvated molecular systems such as butane, they also have important implications for the folding of polymers and proteins.

Significance

The interpretation of rates of reactions that take place in a solvent is complicated because of the entanglement of free-energy and history-dependent friction effects. In this context, the dihedral dynamics of butane has played a paradigmatic role since it is simple yet relevant for conformational transitions in polymers and proteins. We directly extract the friction that governs the dihedral dynamics in butane from simulations. We show that ~89% of the total friction cannot be described as solvent friction and is caused by degrees of freedom that are orthogonal to the dihedral reaction coordinate. This shows that the hydrodynamic estimate of friction severely fails, even in the simplest molecular reaction.
situation). Direly needed are models which allow to check for the presence of internal friction independently of any theoretical assumptions that relate friction to reaction times, as well as methods to extract friction and memory functions directly from simulations instead of inferring friction effects indirectly from measured reaction times.

In this work, we introduce methods to meet both challenges. We consider butane, since it is the simplest molecule that shows a nontrivial conformational transition in a solvent and since it has been a testing ground for theoretical and experimental developments (31–42). In fact, dihedral isomerization rates are quite insensitive to the solvent viscosity (19–22, 24, 43–46), which was first theoretically demonstrated for butane by comparing the reactive fluxes in liquid and frozen organic solvents (36). The origin of this insensitivity was discussed intensely, and it was argued that both inertial and memory effects are relevant (19, 47, 48). We first simulate a single butane molecule in water and compare two scenarios: the free scenario, where all four carbons can freely move, subject to bond length and bond angle constraints, and the constrained scenario, where three carbons are fixed in space and only one terminal carbon can move. While the free-energy landscape for the dihedral is the same in both scenarios, the transition times differ for high water viscosities (which we modify in our simulations by changing the water mass) by a factor of 10. This unequivocally demonstrates that the additional butane degrees of freedom (which are orthogonal to the dihedral angle) in the free scenario significantly change the effective friction along the reaction coordinate. Secondly, we introduce a generalized method to extract the friction memory kernel that couples to the reaction coordinate (26, 49), in our case the dihedral angle, from unconstrained simulation trajectories. A memory kernel accounts for the fact that friction on the molecular scale is not instantaneous but rather depends on the system’s history in a non-Markovian manner. Our calculated memory kernels reveal that indeed the friction substantially differs between the constrained and free butane scenarios. The friction coefficients, which follow by an integral over the memory kernels, are used to predict the transition times of the free and constrained butane scenarios in quantitative agreement with direct simulation results; for this, we need to use reaction rate theory that accounts for inertial effects. This shows that our theoretical framework, which simultaneously yields reaction times as well as friction effects, is consistent. Finally, the internal friction contribution is determined by a fit of the extracted total friction versus the water viscosity: For the constrained butane, the internal contribution is negligible, as expected, while for the free butane, the internal contribution overwhelms the solvent contribution by a factor of eight, which explains why the butane dihedral reaction is rather insensitive to an increase of the water viscosity.

We unambiguously show that the dihedral angle dynamics of a butane molecule is dominated by internal friction, which stems from the coupled dynamics of the four carbons and the solvent. This demonstrates that internal friction exists already for the simplest molecular system that possesses a conformational transition, in line with previous works where dihedral angle isomerization has been argued to be a source of internal friction in protein folding (9, 20–22, 45, 46). In fact, the internal friction contribution for free butane is produced by the rotational and translational butane degrees of freedom, as is further discussed in Conclusions. According to traditional terminology, we call the friction part that does not scale linearly with solvent viscosity the internal friction contribution.

Results and Discussion

Butane Dihedral Barrier-Crossing Times. In our simulations, we place a single butane in a water box. We systematically vary the mass of water molecules \( m_w \) while keeping the butane mass fixed. This modifies all intrinsic water time scales and in particular also the water viscosity according to \( \eta \propto \sqrt{m_w} \), but leaves all equilibrium distribution functions invariant (18). We use a united-atom force field for butane that neglects the hydrogens and approximates butane by four Lennard–Jones beads that are subject to fixed bond lengths and fixed bond angles; for water, we use the extended simple point charge (SPC/E) model (Materials and Methods). We compare the free scenario, where all four butane carbons can move, with the constrained scenario, where three carbons are fixed in space and only one terminal carbon can rotate; see Fig. 1. A and B for an illustration. The only degree of freedom in the constrained scenario is the dihedral angle, while in the free scenario, one has six additional degrees of freedom, three translational and three orientational. The free-energy profiles in the free and constrained scenarios in Fig. 1C perfectly overlap, as expected based on the translational and orientational invariance of the problem, and they reflect the presence of the aqueous solvent (32, 33, 39, 41).

The mean first-passage times \( t_{MFP} \) for the cis-to-trans transition of the dihedral, as defined in Fig. 1C and extracted from the simulation trajectories shown in Fig. 1D, are depicted as a function of the rescaled water viscosity \( \eta/\eta_0 \) in Fig. 2 for the free and constrained scenarios. Here, \( \eta_0 \) denotes the bulk viscosity of water with the normal mass. \( t_{MFP} \) for free butane is rather insensitive to \( \eta \), in agreement with previous results (19, 36). Constrained butane behaves differently for \( \eta > \eta_0 \) and shows
a linear increase of $\gamma_{\text{MFP}}$ with $\eta$ (indicated by a broken straight line), while for $\eta < \eta_0$ the results for the free and constrained scenarios are rather similar and depend only weakly on $\eta$, which will later be explained by inertial effects (i.e., Kramers turnover). The stark deviation between the free and constrained scenarios for $\eta > \eta_0$, amounting to a difference in the reaction times by a factor of 10 for the highest viscosity $\eta = 10\eta_0$, can only be caused by the six additional degrees of freedom for free butane that are orthogonal to the dihedral angle coordinate. Since the dihedral free energy is the same for both scenarios, we conclude that the friction is different in the two scenarios and that this friction difference is caused by the additional degrees of freedom that are present in the free scenario and absent in the constrained scenario. We will later show that the difference in the total friction between the free and constrained scenarios is accompanied by an internal friction contribution for the free case.

Memory Kernels and Friction Coefficients. To quantify the friction that acts on the dihedral angle, we map the dynamics of the butane dihedral angle $\chi$ onto the generalized Langevin equation (GLE)

$$m\ddot{x}(t) = -\int_0^t dt' \Gamma(t') \dot{x}(t - t') - \nabla U[x(t)] + F_p(t),$$

where $\Gamma(t)$ denotes the memory kernel. The random force $F_p(t)$ obeys the fluctuation-dissipation theorem and satisfies $\langle F_p(t) F_p(t') \rangle = k_B T \Gamma(t - t')$. For vanishing potential, the GLE has been derived by linear projection techniques (4, 5). The mass $m$ is an effective one and follows directly from the simulated dihedral angle trajectory $x(t)$ via the equipartition theorem $m(\ddot{\chi})^2 = k_B T$ ($SI$ Appendix). The potential $U(x)$ in the GLE is in fact a free energy and follows from the simulated equilibrium probability density along the reaction coordinate, $p(x)$, as $U(x) = -k_B T \log p(x)$ and is shown in Fig. 1C. To extract $\Gamma(t)$ from simulation trajectories in the presence of a finite potential $U(x)$, we extend previous methods (26, 49–51). It is crucial that we extract $\Gamma(t)$ without applying constraints along the reaction coordinate $x$, as the pronounced differences in the barrier-crossing dynamics between free and constrained butane show that constraints decisively influence the system dynamics. In fact, recent simulation work demonstrated that friction is modified by positional constraints even for the simple system of a methane molecule that diffuses in water (52).

$\Gamma(t)$ in Eq. 1 does not depend on the reaction coordinate $x$, which is an approximation that should be rather accurate judged from the weak dependence of calculated memory kernels for butane frozen in different dihedral angle positions (53), in particular compared with the pronounced differences between the kernels for free and constrained butane that will be presented below. To proceed, we multiply Eq. 1 by $\dot{x}(0)$ and average over the noise to obtain

$$m \langle \dot{x}(0) \ddot{x}(t) \rangle = -\int_0^t dt' \langle \dot{x}(0) \ddot{x}(t - t') \rangle - \langle \dot{x}(0) \nabla U[x(t)] \rangle,$$

where we used that the random force is not correlated with the initial velocity, i.e., $\langle \dot{x}(0) F_p(t) \rangle = 0$ (4). Discretizing all functions as $\Gamma_{i} = \Gamma(\Delta t)$ with a time step $\Delta t$ we obtain the iteration equation

$$\Gamma_{i} = -\frac{1}{\omega_{i,i} \Delta t C_0^2} \left( \sum_{j=0}^{i-1} \omega_{i,j} \Delta t C_{i-j}^2 + m C_{i}^{xx} + C_{i}^{\chi \nabla U} \right),$$

where we defined the correlation function $C_{i}^{xx} = \langle \dot{x}(0) \dot{x}(\Delta t) \rangle$ (and similarly $C_{i}^{\chi \nabla U}$ and $C_{i}^{\chi \nabla U}$) and the integration weight $\omega_{i,j} = 1 + \delta_{i,j}/2 - \delta_{i+1,j}/2$. The correlation function $C_{i}^{\chi \nabla U}$ is obtained by cubic spline interpolation of $U(x)$. In $SI$ Appendix, we demonstrate the numerical robustness of our method.

The extracted memory kernels $\Gamma(t)$ for free butane in Fig. 3B are quite similar for different water viscosities, while they differ strongly for constrained butane in Fig. 3D. In particular, for free butane, the long time tail of $\Gamma(t)$, which is mostly responsible for the effective friction, is almost independent of $\eta$, and oscillations appear that we associate with the presence of orthogonal degrees of freedom. In qualitative accordance with our results in Fig. 2 for the barrier-crossing time, we can say that for free butane, the effective friction is less sensitive to solvent viscosity compared with constrained butane.

In Fig. 4, we show the friction coefficient $\gamma$ for free and constrained butane as a function of water viscosity, which follows from an integral over the memory function according to $\gamma = \int_0^\infty dt \Gamma(t)$. For numerical integration, we fit the long time decay of $\Gamma(t)$ by an exponential function ($SI$ Appendix). The friction for constrained butane is linearly proportional to the solvent viscosity, as expected based on simple hydrodynamics: We denote the translational friction coefficient of a methyl group by $\gamma_{\text{trans}} = 57 \eta R_{CH_3}$. For a methyl group of radius $R_{CH_3} \approx 0.2$ nm that rotates at a fixed bond angle $\alpha = 111^\circ$ and C-C bond length $l_b = 0.15$ nm around a fixed point in space, we estimate the dihedral friction constant $\gamma_{\text{trans}} = (2 \pi / 360)(l_b \sin(\alpha/2))^2 \gamma_{\text{trans}} = 0.01 - (\eta/\eta_0) \text{nm}^2/\text{deg}^2$ ps, not so different from what we extract from the simulations in Fig. 4 for constrained butane. In contrast, the dynamics of free butane is characterized by a friction coefficient that very weakly depends on the water viscosity, in stark contrast to simple hydrodynamics. This failure of hydrodynamics was first inferred from simulations of free butane in an organic solvent (36).
Internal Versus Solvent Friction. We include empirical fits according to refs. 7, 9, and 12

$$\gamma = (\eta/\eta_0) \gamma_{\text{vol},0} + \gamma_{\text{int}}$$

into Fig. 4 as solid lines. The fits are very good, which validates the assumption of additive solvent and internal contributions. For constrained butane, we obtain \( \gamma_{\text{int}} = 1.8 \times 10^{-4} \text{ u n m}^2/\text{deg}^2 \text{ ps}^2 \) and \( \gamma_{\text{vol},0} = 3.9 \times 10^{-3} \text{ u n m}^2/\text{deg}^2 \text{ ps}^2 \), which corresponds to a ratio of \( \gamma_{\text{int}}/\gamma_{\text{vol},0} = 0.05 \) and shows that internal friction is negligible in this case. A small spurious internal friction contribution is in fact expected even for constrained butane, based on the finite difference between the translational friction coefficient of immobilized and free solutes, as recently demonstrated in simulations of a single methane molecule in water (52). In contrast, for free butane, we find \( \gamma_{\text{int}} = 5.2 \times 10^{-4} \text{ u n m}^2/\text{deg}^2 \text{ ps}^2 \) and \( \gamma_{\text{vol},0} = 6.7 \times 10^{-3} \text{ u n m}^2/\text{deg}^2 \text{ ps}^2 \), and thus a ratio \( \gamma_{\text{int}}/\gamma_{\text{vol},0} = 7.7 \). Hence, the dynamics of free butane is dominated by internal friction effects for normal water viscosity \( \eta_0 \). The substantial reduction of the solvent friction contribution \( \gamma_{\text{vol},0} \) in the free case compared with the constrained case can be rationalized by the fact that the dihedral angle for free butane is a relative coordinate that depends on the motion of all four carbons; it is thus governed by a relative diffusion constant for the constrained molecular motion that results from the weighted sum of the individual carbon atom diffusion constants.

It remains to be checked whether the friction coefficients we extract from simulation trajectories in Fig. 4 explain the dihedral barrier-crossing times in Fig. 2. This is nontrivial in the present case, since, as mentioned earlier, memory and inertia effects invalidate the simple Kramers prediction \( \tau_{\text{MFP}} \sim \gamma \). To proceed, it is useful to introduce the characteristic time scales of the system. These are the inertial time \( \tau_{\text{in}} = m/\gamma_{\text{int}} \), that is, the time at which ballistic motion crosses over to diffusive motion; the memory time \( \tau_{\text{M}} = \gamma/\Gamma(0) \), that is, the decay time of the memory kernel; and the diffusive time \( \tau_{\text{D}} = L^2/\gamma/(k_B T) \), that is, the free-diffusion time to advance over a characteristic angle of \( L = 60^\circ \). In Fig. 3C we demonstrate that \( \tau_{\text{in}} < \tau_{\text{D}} \) and \( \tau_{\text{in}} < \tau_{\text{D}} \) hold for all simulation data, in which case Kramers’ formula for the mean first passage time in the medium to strong friction case (6),

$$\tau_{\text{MFP}} = \frac{2\pi \omega_{\text{max}} / \omega_{\text{min}}}{(\gamma^2/4m^2 + \omega_{\text{max}}^2)^{3/2}} \exp \left( \frac{\Delta U}{k_B T} \right),$$

is expected to be approximately valid (30), in line with previous theoretical estimates (34–37). For the barrier height, we extract \( \Delta U = 3.7 k_B T \) from the free energy in Fig. 1C, \( m\omega_{\text{max}}^2 = 6 \times 10^{-3} k_B T/\text{deg}^2 \) and \( m\omega_{\text{min}}^2 = 9 \times 10^{-3} k_B T/\text{deg}^2 \) are the curvatures of the free energy at the maximum and minimum. The results from Eq. 5 for free and constrained butane are included as open data points in Fig. 2; the comparison with the simulation data is quite good without adjustable parameters. Looking more closely at the comparison, we see that the simulation data in the constrained case show a slightly shorter barrier-crossing time than expected based on the Kramers formula, whereas for free butane we see the opposite. Both trends can be explained based on memory effects (30): For intermediate memory time \( \tau_{\text{M}}/\tau_{\text{D}} \approx 0.001 \), as realized for constrained butane (Fig. 3C), memory effects significantly accelerate barrier crossing, while a longer memory time \( \tau_{\text{M}}/\tau_{\text{D}} > 0.1 \), as realized for free butane (Fig. 3C), increases the barrier-crossing time.

The saturation of \( \tau_{\text{MFP}} \) for the constrained case in the low-viscosity limit in Fig. 2 in fact is solely due to inertia effects.

![Figure 3](https://www.pnas.org/cgi/doi/10.1073/pnas.1722327115)

**Fig. 3.** (A and B) Memory kernels \( \Gamma(t) \) for different rescaled water viscosities \( \eta/\eta_0 \) extracted from simulation trajectories via Eq. 3 for constrained (A) and free (B) butane, where \( \eta_0 \) denotes the SPC/E water viscosity. (C) Inertial and memory timescale ratios \( \tau_{\text{in}}/\tau_{\text{D}} \) and \( \gamma/\tau_{\text{D}} \) calculated from the memory kernels of free and constrained butane for different viscosities, where \( \eta_0 \) denotes the characteristic diffusion time (same color coding as in B).

![Figure 4](https://www.pnas.org/cgi/doi/10.1073/pnas.1722327115)

**Fig. 4.** Friction coefficient \( \gamma \) extracted from the memory kernels in Fig. 3 A and B as a function of the rescaled water viscosity \( \eta/\eta_0 \) for free and constrained butane. Empirical fits according to Eq. 4 (denoted by lines) yield internal-to-solvent friction ratios of \( \gamma_{\text{int}}/\gamma_{\text{vol},0} = 7.7 \) for free and \( \gamma_{\text{int}}/\gamma_{\text{vol},0} = 0.05 \) for constrained butane.
and thus reflects Kramers turnover. This follows from the fact that the friction $\gamma$ for the constrained case in Fig. 4 is roughly linear in $\eta$ over the entire range of water viscosities, so any deviation from the overdamped Kramers prediction $\gamma_{0} \propto M_{\mathrm{FP}}$ for the free case can only be explained by a combination of inertia and internal friction effects. This shows that the present simulation strategy, which compares the free and constrained scenarios and at the same time extracts memory functions, is necessary and useful.

Conclusions

The dihedral barrier-crossing dynamics of a constrained butane molecule, where only one carbon atom is allowed to move and thus the dihedral angle is the only degree of freedom (besides solvent degrees of freedom), is shown to be very different from the dynamics of a free butane, where a total of seven spatial degrees of freedom are present. This unambiguously demonstrates that friction generated by degrees of freedom that are coupled but orthogonal to the reaction coordinate (in our case, the dihedral angle) is dominant in solvated butane. By monitoring the friction, which we directly extract from the memory kernel, as a function of the solvent viscosity, we show that orthogonal degrees of freedom significantly modify the solvent friction contribution and also produce an additional contribution which does not vanish in the limit of vanishing solvent viscosity. This contribution, defined by the empirical additive formula Eq. 4, we denote, in analogy to protein folding experiments (7, 9, 12), as internal friction, without implying that this contribution is caused by degrees of freedom that are spatially internal and thereby hidden from the solvent. Rather, the internal friction contribution in butane stems from the dynamic partitioning of energy over all degrees of freedom that are orthogonal to the reaction coordinate, which, in addition to the six spatial (consisting of three translational and three orientational degrees of freedom), also include six conjugate momentum degrees of freedom. In SI Appendix, we present simulations where only one carbon atom of butane is fixed and thus translational degrees of freedom are eliminated, while rotations are still possible. These results demonstrate that both translational and orientational degrees of freedom contribute rather equally to the internal friction and at the same time reduce the solvent friction contribution. This is very different from previous studies on the barrier-crossing dynamics of butane in vacuum, since in vacuum, translational degrees of freedom are decoupled because of momentum conservation (40). This means that solvent degrees of freedom strongly couple translational and orientational butane degrees of freedom to the reaction coordinate, in line with previous findings and arguments (36). It is interesting to note that the effect of orthogonal degrees of freedom is rather ambivalent in that it creates internal friction, but at the same time reduces the solvent friction contribution.

Based on our finding that already for the simplest molecule that allows for barrier crossing, internal friction dominates the dynamics, we expect that for larger and more complex molecules, which possess more orthogonal degrees of freedom, internal friction will play an even more important role for the dynamics. For macromolecular conformational transitions, for which the rate-limiting step involves dihedral angle isomerization (20, 24, 25, 54, 55), our findings constitute a mechanism for the emergence of internal friction effects. But other mechanisms, for example, based on interactions between molecular subunits, certainly also exist. Beyond these applications to polymers and proteins, dihedral isomerization of butane is also interesting in its own right and has been studied by 2D infrared spectroscopy (42). The experimental dihedral isomerization time of a butane derivative solvated in CCl$_4$ was found to be in the 10 ps range, which agrees with predictions from classical molecular dynamics (MD) simulations (36) and is similar to the simulation results we obtain here. Our analysis thus reveals that in such experiments, the internal friction, which for normal water viscosity makes up $\sim$89% of the total friction, dominates the dynamics, a fact that does not transpire from the simulations per se.

It would be desirable to derive the empirical Eq. 4, which decomposes the effective friction coefficient that governs a reaction into solvent and internal contributions, from more basic considerations. In the absence of such a derivation, the good comparison between Eq. 4 and the simulation data in Fig. 4 validates the linear additivity thus only in a heuristic sense and could break down for more complicated systems.

Materials and Methods

Simulations are carried out by using the GROMACS (Version 5.1) (56, 57) simulation package with double precision. Butane is parameterized by the GROMOS (58) united atom force field; for water, we use the SPCE (59) model. All bond angles and bond lengths of water and butane are constrained by using the SHAKE (60) algorithm. The presence of bond angle and bond length vibrations, which we neglect, has been shown to modify the equilibrium dihedral distribution of solvated butane (32) and to also lead to interesting dynamic effects for butane in vacuum (40). Clearly, the proper account of molecular vibrations goes beyond classical simulations; the reason for freezing these internal molecular degrees of freedom in our study is to make the free-energy profiles for constrained and free butane exactly the same. We perform constant number of particles, volume, and temperature (NVT) MD simulations and vary the water molecule mass $m_w$ to change the water viscosity. For water mass larger or equal to the normal water mass, we use a time step of 2 fs; for lighter water mass, we lower the time step by a factor $\sqrt{m_w}/ \sqrt{m_w}$. The total trajectory length in normal and light water are 40 ns and are elongated in heavy water by a factor 3. The temperature $T = 300$ K is controlled by the velocity rescaling (61) thermostat, which is coupled only to the solvent with a time constant of $\gamma_0/\eta_0 = 1$ ps. In SI Appendix, we compare results from NVT and constant number of particles, volume, and energy simulations and demonstrate that the ensemble and thus the thermostat have no influence on our results.

Acknowledgments

This work was supported by Deutsche Forschungsgemeinschaft Grants SFB 1078 and SFB 1114.

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