Nuclear Physics of Double Beta Decay

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Abstract

Study of the neutrinoless $\beta\beta$ decay allows us to put a stringent limit on the effective neutrino Majorana mass, a quantity of fundamental importance. To test our ability to evaluate the nuclear matrix elements that govern the decay rate, it is desirable to be able to describe the allowed two-neutrino decay. It is argued that only low-lying virtual intermediate states are important for that process, and thus it appears that the large-scale shell model evaluation, free from the various difficulties of QRPA, is the preferred method. In the $0\nu$ decay, it is shown that there is a substantial cancellation between the paired and broken pair pieces of the nuclear wave function. Thus, despite the popular claim to the contrary, one expects that the $0\nu$ rate is also sensitive to the details of nuclear structure. This is reflected in the spread of the theoretical values, leading to the uncertainty of about factor 2-3 in the deduced $\langle m_\nu \rangle$ limit. Finally, brief comment about the $0\nu$ decay with the exchange of a very heavy neutrino are made.

1 Introduction

Double beta decay has been long recognized as a powerful tool for the study of lepton conservation. The theoretical description of $\beta\beta$ decay involves particle physics and nuclear structure physics. The latter is the topic of the present work. After a brief introduction and a review of the experimental situation I will describe three distinct set of problems:

- $2\nu$ decay: the physics of the Gamow-Teller amplitudes.
- $0\nu$ decay - exchange of light massive Majorana neutrinos: no selection rules on multipoles, role of nucleon correlations, sensitivity to nuclear models.
- $0\nu$ decay - exchange of heavy neutrinos: physics of the nucleon-nucleon states at short distances.

Since the lifetimes of $\beta\beta$ decay are so long, the experimental search has spawned a whole field of experiments requiring very low background. During the last decade there has been enormous progress in the experimental study of the double beta decay. The $2\nu$ decay, with lifetimes of $10^{19} - 10^{21}$ y, has been now observed in ten cases, often repeatedly and by different

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groups [1]. The halflives are typically determined to better than 10% accuracy. Thus, the 2ν decay has become a tool, rather than an exotic curiosity.

The main experimental effort is concentrated, naturally, on the search for the 0ν decay. There, multikilogram sources with very low background are the state of the art at present. Various techniques are being pursued: two large experimental efforts (Heidelberg-Moscow collaboration in Gran Sasso and the IGEX collaboration in Canfranc) employ enriched 76Ge in sets of detectors operated deep underground. The latest result [2] is based on 41.55 kg y of exposure and halflife limit $T_{1/2} > 1.3 \times 10^{25}$y (90% CL). Alternatively, the analysis of the 24.16 kg y of data with pulse shape measurement, and hence improved background suppression, leads to even better limit of $T_{1/2} > 1.6 \times 10^{25}$y (90% CL). Note, that in the latter case the actual number of observed events is smaller than the expected background. When taking advantage of this, the authors of [2] report yet much improved limit of $T_{1/2} > 5.7 \times 10^{25}$y (90% CL). The latest IGEX limit is $T_{1/2} \geq 0.8 \times 10^{25}$y (90% CL)[3]. I will discuss the interpretation of this result in terms of the neutrino Majorana mass below in Section 3.

Other techniques which allow expansions to multikilogram sources involve gas TPC (136Xe in the Gotthardt tunnel [4]), electron tracking detectors combined with calorimeters (NEMO [5], ELEGANTS [6]), and cryogenic bolometers [7]. These experiments will make it possible (or currently already achieve) to reach neutrino mass limit well below 1 eV.

In addition, there are plans for improvements of the limit to the 0.1 eV range by scaling up the source mass to about one ton quantities. Such experiments, with either enriched 76Ge (GENIUS) or with large amount of cooled TeO$_2$ (CUORE) will require very large capital expenditures and running times of about five years. The field of ββ decay then becomes competitive, in complexity, manpower, and price, to large accelerator particle physics experiments.

The Majorana neutrino mass $\langle m_\nu \rangle$ below 1 eV is an important landmark. This has been stressed often, e.g., in the work of Georgi and Glashow [8]. There, the authors quote as “established facts” the discovery by the SuperKamiokande collaboration of the $\nu_\mu \rightarrow \nu_\tau$ neutrino oscillations with $\Delta m^2 \simeq 10^{-3}$ eV$^2$ and mixing angle $\sin^2 2\theta \simeq 1$, and the solar neutrino deficit, confirmed in five experiments, which involves $\nu_e \rightarrow \nu_x$ oscillations with $\Delta m^2 \leq 10^{-5}$ eV$^2$. The “tentative facts” are based on theoretical prejudices and need experimental confirmation. They are the existence of precisely three massive Majorana neutrinos, and the assumption that these neutrinos are responsible for the hot dark matter, leading to the conclusion that $m_1 + m_2 + m_3 \sim 6$ eV. Taking all of that together one is forced to the wholly unexpected and somewhat bizarre conclusion that the three neutrino flavors are essentially degenerate with masses of 2 eV each (albeit with a sizable uncertainty in this value).

It is now clear that if the study of the 0ν ββ decay can without doubt establish that $\langle m_\nu \rangle < 1$ eV, this finding has a profound consequences for the structure of the neutrino mixing matrix. In particular, as Georgi and Glashow [8] argue, it would lead to maximum mixing involving electron neutrinos also.
2 Two neutrino decay

This decay, characterized by the transformation of two neutrons into two protons with the emission of two electrons and two $\bar{\nu}_e$, does not violate any selection rules. Since the energies involved are modest, the allowed approximation should be applicable, and the rate is governed by the double Gamow-Teller matrix element

$$M_{2\nu}^{GT} = \sum_m \frac{\langle f | \sigma \tau^+ | m \rangle \times \langle m | \sigma \tau^+ | i \rangle}{E_m - (M_i + M_f)/2},$$

where $i, f$ are the ground states in the initial and final nuclei, and $m$ are the intermediate $1^+$ (virtual) states in the odd-odd nucleus. The first factor in the numerator above represents the $\beta^+$ (or $(n, p)$) amplitude for the final nucleus, while the second one represents the $\beta^-$ (or $(p, n)$) amplitude for the initial nucleus. Thus, in order to correctly evaluate the $2\nu$ decay rate, we have to know, at least in principle, all GT amplitudes for both $\beta^-$ and $\beta^+$ processes, including their signs. The difficulty is that the $2\nu$ matrix element exhausts a very small fraction ($10^{-5} - 10^{-7}$) of the double GT sum rule [9], and hence it is sensitive to details of nuclear structure.

![Graph](image)

Figure 1: The $\beta^-$ strength (upper panel), and the contributions to the $2\nu$ matrix element, eq. (1) (lower panel). Both as the function of the excitation energy in $^{48}\text{Sc}$.

Various approaches used in the evaluation of the $2\nu$ decay rate have been reviewed recently by Suhonen and Civitarese [10]. The Quasiparticle Random Phase Approximation
(QRPA) has been the most popular theoretical tool in the recent past. Its main ingredients, the repulsive particle-hole spin-isospin interaction, and the attractive particle-particle interaction, clearly play a decisive role in the concentration of the $\beta^-$ strength in the giant GT resonance, and the relative suppression of the $\beta^+$ strength and its concentration at low excitation energies. Together, these two ingredients are able to explain the suppression of the $2\nu$ matrix element when expressed in terms of the corresponding sum rule.

Yet, the QRPA is often criticized. Two “undesirable”, and to some extent unrelated, features are usually quoted. One is the extreme sensitivity of the decay rate to the strength of the particle-particle force (often denoted as $g_{pp}$). This decreases the predictive power of the method. The other one is the fact that for a realistic value of $g_{pp}$ the QRPA solutions are close to their critical value (so-called collapse). This indicates a phase transition, i.e., a rearrangement of the nuclear ground state. QRPA is meant to describe small deviations from the unperturbed ground state, and thus is not fully applicable near the point of collapse. Numerous approaches have been made to extend the range of validity of QRPA, see, e.g., Ref. [10]. The description of all these generalizations is beyond the scope of this talk.

Table 1: Experimental halflives of the $2\nu$ decay, and the ratios of calculated to experimental halflives (see text).

| Nucleus | QRPA | Shell model | $T_{1/2}$ (y) |
|---------|------|-------------|---------------|
| $^{48}$Ca | – | 0.91 | $4.3 \times 10^{19}$ |
| $^{76}$Ge | 0.71 | 1.44 | $1.8 \times 10^{21}$ |
| $^{82}$Se | 1.5 | 0.46 | $8.0 \times 10^{19}$ |
| $^{100}$Mo | 0.6 | – | $1.0 \times 10^{19}$ |
| $^{128}$Te | 0.27 | 0.25 | $2.0 \times 10^{24}$ |
| $^{130}$Te | 0.27 | 0.29 | $8.0 \times 10^{20}$ |
| $^{136}$Xe | < 1.5 | < 3.7 | $>5.6 \times 10^{20}$ |

At the same time, detailed calculations show that the sum over the excited states in Eq.(1) converges quite rapidly [11]. In fact, a few low-lying states usually exhaust the whole matrix element. Thus, it is not really necessary to describe all GT amplitudes; it is enough to describe correctly the $\beta^+$ and $\beta^-$ amplitudes of the low-lying states, and include everything else in the overall renormalization (quenching) of the GT strength. The situation is illustrated in Fig. [12] modified from Ref. [12].

Nuclear shell model methods are presently capable of handling much larger configuration spaces than even a few years ago. Thus, for many nuclei the evaluation of the $2\nu$ rates
within the $0\hbar\omega$ shell model space is feasible. (Heavy nuclei with permanent deformation, like $^{150}$Nd and $^{238}$U remain, however, beyond reach.) Using the interacting shell model avoids, naturally, the above difficulties of QRPA. At the same time, the shell model is capable to predict, with the same method and the same residual interaction, a wealth of spectroscopic data, allowing a much better test of the predictive power.

To judge the degree of understanding of the $2\nu$ decay I show in Table 1 the comparison with experiment of the initial Caltech QRPA calculation [13] and the modern shell model [15]. One can see that both methods are able, at least in these cases, explain the $2\nu$ decay rates reasonably well, even though in the case of Te both methods underestimate the halflife by a factor of about four.

3 Neutrinoless decay: light Majorana neutrino

In the neutrinoless decay the two electrons are the only leptons emitted, and consequently their sum energy is just the sharp nuclear mass difference. This feature makes the experimental recognition of the $0\nu$ decay much easier; it also results in a more favorable phase space factor.

If one assumes that the $0\nu$ decay is caused by the exchange (virtual) of a light Majorana neutrino between the two nucleons, then several new features arise: a) the exchanged neutrino has a momentum $q \sim 1/r_{nn} \approx 50 - 100$ MeV ($r_{nn}$ is the distance between the decaying nucleons). Hence, the dependence on the energy in the intermediate state is weak and the closure approximation is applicable. Also, b) since $qR > 1$ ($R$ is the nuclear radius), the expansion in multipoles is not convergent, unlike in the $2\nu$ decay. In fact, all possible multipoles contribute by a comparable amount. Finally, c) the neutrino propagator results in a neutrino potential of a relatively long range.

Thus, in order to evaluate the rate of the $0\nu$ decay, we need to evaluate only the matrix element connecting the ground states $0^+$ of the initial and final nuclei. Again, we can use the QRPA or the shell model. Both calculations show that the features enumerated above are indeed present. In addition, the QRPA typically shows less extreme dependence on the particle-particle coupling constant $g_{pp}$, since the contribution of the $1^+$ multipole is relatively small. The calculations also suggest that for quantitatively correct results one has to treat the short range nucleon-nucleon repulsion carefully, despite the long range of the neutrino potential.

Does that mean that the calculated matrix elements are insensitive to nuclear structure? An answer to that question has obviously great importance, since unlike the $2\nu$ decay, we cannot directly test whether the calculation is correct or not.

For simplicity, let us assume that the $0\nu \beta\beta$ decay is mediated only by the exchange of a light Majorana neutrino. The relevant nuclear matrix element is then the combination
$M_{0\nu}^{GT} - M_{0\nu}^{F}$, where the GT and F operators change two neutrons into two protons, and contain the corresponding operator plus the neutrino potential. One can express these matrix elements either in terms of the proton particle - neutron hole multipoles (i.e. the usual beta decay operators) or in the multipoles coupling of the exchanged pair, $nn$ and $pp$.

![Figure 2: The cumulative contribution of the pair states with the natural parity multipolarity to the $0\nu$ nuclear matrix element combination $M_{0\nu}^{GT} - M_{0\nu}^{F}$. The full line is for $^{76}$Ge and the dashed line for $^{48}$Ca.](image)

When using the decomposition in the proton particle - neutron hole multipoles, one finds that all possible multipoles (given the one-nucleon states near the Fermi level) contribute, and the contributions have typically equal signs. Hence, there does not seem to be much cancellation. However, perhaps more physical is the decomposition into the exchanged pair multipoles. There one finds, first of all, that only natural parity multipoles ($\pi = (-1)^I$) contribute noticeably. And there is a rather severe cancellation. The biggest contribution comes from the $0^+$, i.e., the pairing part. All other multipoles, related to higher seniority states, contribute with an opposite sign. The final matrix element is then a difference of the pairing and higher multipole (or broken pair $\equiv$ higher seniority) parts, and is considerably smaller than either of them. This is illustrated in Fig. 2 where the cumulative effect is shown, i.e. the quantity

$$M(I) = \sum_J I \left[ M_{0\nu}^{GT}(J) - M_{0\nu}^{F}(J) \right]$$

(2)

is displayed for $^{76}$Ge (from [14]) and $^{48}$Ca (from [12]). Thus, the final result depends sensitively on both the correct description of the pairing and on the admixtures of higher seniority configurations in the corresponding initial and final nuclei.

Since there is no objective way to judge which calculation is correct, one often uses the spread between the calculated values as a measure of the theoretical uncertainty. This is illustrated in Table 2. There, I have chosen two representative QRPA sets of results, the
highly truncated “classical” shell model result of Haxton and Stephenson [17], and the result of more recent shell model calculation which is convergent for the set of single particle states chosen (essentially $0\hbar\omega$ space).

For the most important case of $^{76}$Ge, the calculated rates differ by a factor of 6-7. Since the effective neutrino mass $\langle m_\nu \rangle$ is inversely proportional to the square root of the lifetime, the experimental limit of $1.6 \times 10^{25}$ y translates into limits of about 1 eV according to [13, 15] and about 0.4 eV according to [16, 17]. On the other hand, if one would accept the more stringent limit of $5.7 \times 10^{25}$ [2], even the more pessimistic matrix elements restrict $\langle m_\nu \rangle < 0.5$ eV, hence the scenario discussed by Georgi and Glashow [8] is confirmed. Needles to say, a more objective measure of the theoretical uncertainty would be highly desirable.

### Table 2: Halflives in years calculated for $\langle m_\nu \rangle = 1$ eV by various representative methods.

|                  | QRPA [16]   | QRPA [13]   | SM [17]     | SM [15]     |
|------------------|-------------|-------------|-------------|-------------|
| $^{76}$Ge        | $2.3 \times 10^{24}$ | $1.4 \times 10^{25}$ | $2.4 \times 10^{24}$ | $1.7 \times 10^{25}$ |
| $^{82}$Se        | $6.0 \times 10^{23}$ | $5.6 \times 10^{24}$ | $8.4 \times 10^{23}$ | $2.4 \times 10^{24}$ |
| $^{100}$Mo       | $1.3 \times 10^{24}$ | $1.9 \times 10^{24}$ | $-$          | $-$          |
| $^{130}$Te       | $4.9 \times 10^{23}$ | $6.6 \times 10^{23}$ | $2.3 \times 10^{23}$ | $-$          |
| $^{136}$Xe       | $2.2 \times 10^{24}$ | $3.3 \times 10^{24}$ | $-$          | $1.2 \times 10^{25}$ |

### 4 Neutrinoless decay: very heavy Majorana neutrino

The neutrinoless $\beta\beta$ decay can be also mediated by the exchange of a heavy neutrino. The decay rate is then inversely proportional to the square of the effective neutrino mass [18]. In this context it is particularly interesting to consider the left-right symmetric model proposed by Mohapatra [19]. In it, one can find a relation between the mass of the heavy neutrino $M_N$ and the mass of the right-handed vector boson $W_R$. Thus, the limit on the $\beta\beta$ rate provides, within that specific model, a stringent lower limit on the mass of $W_R$.

The process then involves the emission of the heavy $W_R^-$ by the first neutron, the vertex $W_R^- \rightarrow e^- + \nu_N$ followed by $\nu_N \rightarrow e^- + W_R^+$ with the absorption of the $W_R^+$ on the second neutron. Since all exchanged particles between the two neutrons are very heavy, the corresponding “neutrino potential” is of essentially zero range. Hence, when calculating the nuclear matrix element, one has to take into account carefully the short range nucleon-nucleon repulsion.

As long as we treat the nucleus as an ensemble of nucleons only, the only way to have a nonvanishing nuclear matrix elements for the above process is to treat the nucleons as finite
size particles. In fact, that is the standard way to approach the problem \([18]\); the nucleon size is described by a dipole form factor with the cut-off parameter \(\Lambda \simeq 0.85\) GeV.

However, another way of treating the problem is possible, and already mentioned in \([18]\). Let us recall how the analogous situation is treated in the description of the parity-violating nucleon-nucleon force \([20]\). There, instead of the weak (i.e., very short range) interaction of two nucleons, one assumes that a meson \((\pi, \omega, \rho)\) is emitted by one nucleon and absorbed by another one. One of the vertices is the parity-violating one, and the other one is the usual parity-conserving strong one. The corresponding range is then just the meson exchange range, easily treated. The situation is schematically depicted in the left-hand panel of Fig. 3. The analogy for \(\beta\beta\) decay is shown in the right-hand graph. It involves two pions, and the “elementary” lepton number violating \(\beta\beta\) decay then involves a transformation of two pions into two electrons. Again, the range is just the pion exchange range. To my knowledge, no detailed evaluation of the corresponding graph was ever made (see, however, Ref.\([21]\)). It would be interesting to see if it would lead to a more or less stringent limit on the mass of the \(W_R\) than the treatment with form factors.

![Feynman graph](image)

Figure 3: The Feynman graph description of the parity-violating nucleon-nucleon force (left graph) and of the \(\beta\beta\) decay with the exchange of a heavy neutrino mediated by the pion exchange.

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