Pulsar tests of the graviton mass

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In Einstein’s general relativity (GR), gravity is described by a massless spin-2 metric field, and the extension of GR to include a mass term for the graviton has profound implication for gravitation and cosmology. Besides the gravity experiments carried out in the Solar System and those recently with gravitational waves (GWs), pulsar timing observations provide a complementary means to test the masslessness of graviton. In this contribution, I overview three methods in probing the mass of graviton from precision timing of binary pulsars via the modified gravitational radiation (hence, the observed damping rate of the orbital period), as well as from the pulsar-timing-array (PTA) experiments via the modified Hellings-Downs angular-correlation curve. These tests probe different aspects of gravitation in its kinematics and dynamics, complementing tests of other kinds and providing valuable information to the fundamental theory of gravity.

KEYWORDS: gravitation – pulsars: general – relativity

1 INTRODUCTION

Modern physics is built upon two fundamental pillars, the Standard Model of particle physics and the General Relativity (GR) of gravitation. The former is nicely spoken in the language of quantum field theory (see e.g., Weinberg, 2005), while the latter is communicated in the narrative of differential geometry (see e.g., Misner, Thorne, & Wheeler, 1973). Together, they account for the four fundamental forces in the Nature, namely, the electromagnetic force, the strong force, the weak force, and the gravitational force. The incentive to unify quantum field theory and GR into a fundamental theory, the often so-called quantum gravity, is the driving motivation for most of the past decades’ investigation in fundamental physics.

Among the four fundamental forces, gravity is quite unique from the other three. It might be holding the key towards new physics beyond what we currently know of. Therefore, theoretical and experimental studies in gravitational physics have been constantly carried out with great enthusiasm (Will, 2018b). In Einstein’s theory, gravity is described by a massless spin-2 metric field, and the extension to include a mass term for the graviton has profound implication for gravitation and cosmology (de Rham, 2014). In this contribution, we focus on the possibility to include a mass term, and overview the methods to bound it utilizing the precision timing of radio pulsars, which is one of the most precise strong-field experiments in the field of experimental gravity (Kramer, 2016; Shao & Wex, 2016; Taylor, 1994; Wex, 2014).

From the theoretical side, the mass of graviton was firstly perceived by Fierz & Pauli (1939) in late 1930s. After fighting for decades with pathologies like the van Dam-Veltman-Zakharov discontinuity and Boulware-Deser ghosts, some version of healthy massive gravity theory was developed by the gravity community (see e.g., de Rham, 2014 for a review). From the experimental side, various observations were used to bound the mass of graviton, including—just to name a few—the propagation of gravitational waves (GWs) (Will, 1998) and the perihelion advance rate of planets in the Solar System (Will, 2018a). Interested readers are referred to de Rham.
Deskins, Tolley, & Zhou (2017) for a recent comprehensive review.

In this short contribution, I will overview some investigations using the precision timing of radio pulsars in bounding the mass of graviton. In particular, I will cover the following studies.

(I) The Finn-Sutton method (Finn & Sutton 2002) was used to bound the graviton mass to be $m_g \lesssim 10^{-21}$ eV/c² (Miao, Shao, & Ma 2019) in a dynamic regime for a Fierz-Pauli-like gravity action.

(II) The scheme developed by de Rham, Tolley, & Wesley (2013) was used to bound the graviton mass to be $m_g \lesssim 10^{-28}$ eV/c² (Shao, Wex, & Zhou 2020) in the cubic Galileon theory.

(III) In the near future, the Hellings-Downs angular-correlation curve will be used to bound the graviton mass to be $m_g \lesssim 10^{-22}$ eV/c² (Shklovskii, 1970). These tests are of different nature and when performing comparison among them, I strongly argue to specify the context concerning the underlying physics and assumptions. Unless otherwise stated, I use units where $G = c = 1$ in the manuscript.

## 2 | THE FINN-SUTTON TEST

Finn & Sutton (2002) considered a phenomenological Fierz-Pauli-like action for linearized gravity with a mass term for the transverse tensor modes,

$$ S \sim \int d^4x \left[ \partial_\mu h_{\mu\nu} \partial^\nu h - 2 \partial_\mu h_{\mu\nu} \partial^\nu h + 2 \partial_\mu h_{\mu\nu} \partial^\nu h \right. $$

$$ \left. - \partial_\mu \partial_\nu \partial^\mu h - 32\pi h_{\mu\nu} T_{\mu\nu} + m_g^2 \left( h_{\mu\nu} h_{\mu\nu} - \frac{1}{2} h^2 \right) \right]. \tag{1} $$

The first few terms are from the linearized version of GR (Mashiner et al. 1973), while the last term is the mass term of specific interests here. The mass term is unique when requiring, (i) a standard Klein-Gordon-like wave equation for $h_{\mu\nu}$, and (ii) a recovery of GR if the graviton mass $m_g$ goes to zero. Though the simple model (1) contains ghosts and instabilities (Boyle & Deser 1972), it is nevertheless a valuable strawman target to study massive gravity as an illustration. Nevertheless, it should not be taken as a full and sophisticatedly designed theory at the end.

Assuming slow motion for a Keplerian binary orbit as a reasonable approximation for binary pulsars, Finn & Sutton (2002) showed that there is a correction to the orbital decay rate as predicted by GR,

$$ \frac{P_b - P_{b,GR}}{P_{b,GR}} = \frac{5}{24} \left( \frac{1}{1 + \frac{73}{24} e^2 + \frac{37}{96} e^4} \right) m_g^2, \tag{2} $$

where $P_b$ is the orbital period, and $e$ is the orbital eccentricity. The orbital decay rate in GR, due to the emission of GWs, is,

$$ \frac{\dot{P}_b}{P_b} = -\frac{192\pi}{5} \frac{1 + \frac{23}{24} e^2 + \frac{37}{96} e^4}{(1 - e^2)^{7/2}} \left( \frac{2\pi}{P_b} \right)^{5/3} \frac{m_1 m_2}{(m_1 + m_2)^{1/3}}, \tag{3} $$

for a binary of component masses $m_1$ and $m_2$ (Peters & Mathews, 1963).

For a handful of binary pulsars, the masses can be derived via measuring the post-Keplerian parameters (Damour & Taylor 1992; Taylor 1994), while at sometimes, in combination with optical phase-resolved spectroscopic observation of the companion (Wex 2014; Özel & Freire 2016). They can be used to derive the theoretical orbital decay rate in GR via Eq. (3). On the other hand, we can derive the value of $\dot{P}_b$ directly from the pulsar timing data. However, this value, $\dot{P}_b^{\text{obs}}$, in general is contaminated by astrophysical contribution of various sources (Lorimer & Kramer 2005). One has to subtract these contributions, in order to get the intrinsic orbital decay rate

$$ \dot{P}_b^{\text{intr}} = \dot{P}_b^{\text{obs}} - \dot{P}_b^{\text{acc}} - \dot{P}_b^{\text{Shk}}, \tag{4} $$

where we have denoted the two most significant ones, namely the contribution $\dot{P}_b^{\text{acc}}$ caused by the difference of accelerations of the binary pulsar and the barycenter of the Solar System projected along the line of sight to the pulsar (Damour & Taylor 1991), and the “Shklovskii” contribution $\dot{P}_b^{\text{Shk}}$ caused by the relative kinematic motions of the binary pulsar with respect to the barycenter of the Solar System (Shklovskii 1970).

After obtaining the intrinsic orbital decay rate, a meaningful bound on the graviton mass can be derived via using Eq. (2). Miao et al. (2019) carefully chose a few best timed binary pulsars, and performed a thorough study. In their Bayesian analysis that combines all these binary pulsars, they obtained,

$$ m_g < 5.2 \times 10^{-21} \text{ eV/c}^2, \quad (90\% \text{ C.L.}). \tag{5} $$

Though failing to compete with some other graviton mass bounds under different contexts (de Rham et al. 2017)—like using the static Yukawa potential or the modified dispersion relation of GWs—the bound in Eq. (5) is the currently best limit on the mass term in action (1) from binary pulsars in a dynamic regime. It encodes two-body dynamics instead of pure kinematics or static Yukawa-type suppression.

## 3 | THE CUBIC-GALILEON TEST

The Lovelock theorem is a useful guide to classify modified gravity theories (Berti et al. 2013). According to it, in a...
4-dimensional spacetime the only diffeomorphism invariant, divergence-free, symmetric rank-2 tensor, that is constructed solely from the metric and its derivatives up to the second order, is the Einstein tensor plus a cosmological term \( \mathcal{L}_{\Lambda} \). Consequently, unlike what is in Eq. (1), a full massive gravity often introduces extra scalar degrees of freedom (de Rham, 2014). Salient features of various massive gravity theories are captured by Galileon models, and here we will discuss the cubic Galileon theory (Luty, Porrati, & Rattazzi, 2003), which is the simplest one of them. The cubic Galileon is often taken as a proxy to all of the Lorentz-invariant massive gravity models in some appropriate limits (de Rham, 2014; Nicolis, Rattazzi, & Trincherini, 2009).

For the orbital dynamics of binary pulsars, we consider the action \( S \sim \int d^4x \left[ -\frac{1}{4} \mathcal{G}_{\mu \nu} \mathcal{G}^{\mu \nu} + \frac{\mathcal{G}_{\mu \nu} \mathcal{T}^{\mu \nu}}{2M_{\text{Pl}}} - \frac{3}{4} (\partial \phi)^2 \left( 1 + \frac{1}{3\Lambda^2} \phi \right) + \frac{\phi \mathcal{T}}{2M_{\text{Pl}}} \right] \), (6)

where the first two terms are from the linearized GR, \( \pi \) is the scalar field with Galileon symmetry, and \( \Lambda \) is the strong coupling energy scale related to the graviton mass by \( \Lambda^3 = m_g^2 M_{\text{Pl}} \) with \( M_{\text{Pl}} \) the Planck mass. The Vainshtein screening radius \( r_\star \) (Vainshtein, 1972) is given via \( r_\star^3 = M_{\text{Pl}}^4 / 16 m_g^2 \).

\( \text{de Rham et al.} \) (2013) discovered that, in binary systems, the suppression factor in the extra gravitational radiation channels due to the Galileon mode is less than that in the static fifth force. For this reason, it is interesting to check with the radiative tests in binary pulsars. These authors worked out the explicit expressions for extra monopole, dipole, and quadrupole radiations in the cubic Galileon theory, in addition to what is predicted from GR in Eq. (3). These extra contributions are proportional to \( m_g^\text{mono} \) instead of proportional to \( m_g^2 \) as in Eq. (2) for the Fierz-Pauli-like theory; explicit equations can be found in \( \text{de Rham et al.} \) (2013) and \( \text{Shao et al.} \) (2020).

Similarly to the linearized Fierz-Pauli theory, extra gravitational radiations lead to a faster orbital period damping rate, by an amount of

\[ \dot{P}_b^\pi = \dot{P}_b^\text{mono} + \dot{P}_b^\text{dipo} + \dot{P}_b^\text{quad} , \]

which can be confronted with experiments via the measurement of the intrinsic \( \dot{P}_b^\text{int} \) in Eq. (4). The dependence of the Galileon contributions on the orbital period, orbital eccentricity, and binary component masses is complicated, and a full discussion can be found in \( \text{Shao et al.} \) (2020).

A dozen of well-timed binary pulsar systems were carefully chosen to confront with Eq. (7), including the famous Hulse-Taylor pulsar PSR B1913+16 (Weisberg & Huang, 2016) and the Double Pulsar PSR J0737–3039 (Kramer, 2016). Bounds on the graviton mass from individual pulsars are illustrated in Fig. 1. Because the graviton mass is a universal quantity in these tests, a combination is possible. Combining the bounds with a Bayesian analysis, \( \text{Shao et al.} \) (2020) gives,

\[ m_g < 2 \times 10^{-28} \text{eV}/c^2, \quad \text{(95% C.L.)} \]

when a uniform prior on \( \ln m_g \) for \( m_g \in (10^{-29}, 10^{-27}) \text{eV}/c^2 \) is adopted. The bound (8) is specific to the cubic Galileon model (6).

4 | TESTS VIA PULSAR TIMING ARRAYS

Since the early 2000s, dozens of well-timed millisecond pulsars have been used to detect GWs at the nano-hertz frequency. A stochastic GW background imprints an angular-dependent correlation in pulsar timing residuals for pulsars distributed across our Milky Way (Hellings & Downs, 1983). Therefore, it is possible to use PTAs to extract the angular correlation in pulse signals and derive the physical information of GWs (Hobbs et al., 2010).

When GWs are not described by the GR, the Hellings-Downs correlation is changed accordingly. For a massive graviton satisfying the Lorentz-invariant dispersion relation,

\[ E^2 = \rho^2 c^2 + m_g^2 c^4, \]

\( \text{Lee et al.} \) (2010) derived the changes to the canonical case. Most importantly, the shift in the frequency of pulsar timing radio signals, by a monochromatic plane GW with a frequency
\( \omega_g \) and a wave vector \( k_g \), is given by,

\[
\frac{\Delta \omega(t)}{\omega} = \sum_{ij} \frac{\hat{n}_i \cdot \hat{n}_j \left[ h_{ij}(t, 0) - h_{ij}(t - |D|, D) \right]}{2 \left[ 1 + \left( k_g / \omega_g \right) \cdot \hat{n} \right]},
\]

(10)

where \( \hat{n} \) is the direction from the Earth (at location \( r = 0 \)) to the pulsar (at location \( r = D \)), and \( h_{ij} \) is the strain of GWs (Lee et al. 2010). Then the change in the timing residual is obtained via \( R(t) = \int_0^t \Delta \omega(t) / \omega_0 \, dr \). In addition to the modification of the timing residuals, the presence of a massive graviton removes all GW radiating powers at frequencies below the cutoff frequency which is defined by the mass of graviton.

Extensive simulation shows that, with a large sample of stable pulsars, the mass of the graviton will be bound to about \( 10^{-22} \text{eV/c}^2 \) with realistic PTAs (Lee et al. 2010). Inclusion of extra polarization modes (Eardley, Lee, & Lightman 1973), other than the canonical “plus” and “cross” ones, will not change much of the projected limit on \( m_g \) (Lee 2013).

5 | SUMMARY

When there are apparent conflicts between GR and quantum field theory, it is motivated to look for new physics beyond the standard paradigm. The gravity might be holding the key to a breakthrough in the field (Berti et al. 2015; Will 2018b). Various experimental examination was carried out to different catalogs of alternative gravity theories, including the massive gravity (de Rham 2014; de Rham et al. 2017).

This contribution overviews three methods in the field that uses the precision timing of radio pulsars to bound the graviton mass: two of them use the gravitational radiation backreaction in binary pulsars (de Rham et al. 2013; Finn & Sutton 2002; Miao et al. 2019; Shao et al. 2020), while the other one uses the angular-correlation curve in PTAs (Lee et al. 2010). The above derived/projected bounds on the graviton mass from radio pulsars, together with those from other experiments—as comprehensively reviewed in de Rham et al. (2017)—are all of value to the field of experimental gravity, since they are based on different assumptions about the (unknown) theory of massive gravity. They have different powers at probing the hypothesis of a massive graviton. Comparison between them is meaningful only when the underlying theory and assumptions are made clear (de Rham et al. 2017).

The tests are to be sharpened to a new level with new instruments and continued observations, in particular with the demonstrated capability of the South African MeerKAT radio telescope (Bailes et al. 2018) and the Chinese Five-hundred-meter Aperture Spherical Telescope (FAST; Jiang et al. 2019; Lu, Lee, & Xu 2020), and ultimately with the Square Kilometre Array (SKA; Kramer et al. 2004; Shao et al. 2015; Weltman et al. 2020). Radio pulsars will continue to provide interesting gravity tests, in complement to tests from other fields (Sathyaprakash et al. 2019; Shao, Sennett, Buonanno, Kramer, & Wex 2017; Wex 2014; Yunes, Yagi, & Pretorius 2016).

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Conflict of interest

The author declares no potential conflict of interests.

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