Abstract
The $K^-pp$ system is investigated using a variational approach with realistic two-body interactions: the Argonne v18 $NN$ potential and an energy dependent $\bar{K}N$ effective interaction derived from chiral SU(3) coupled-channel dynamics. Uncertainties in subthreshold extrapolations of the $\bar{K}N$ interaction are considered. A weakly bound $K^-pp$ state is found, with a binding energy $B = (19 \pm 3)$ MeV substantially smaller than suggested in previous calculations. The decay width $\Gamma(K^-pp \rightarrow \pi \Sigma N)$ is estimated to range between about 40 and 70 MeV.

1. Introduction
In the continuing quest for possible antikaon-nuclear quasibound states, the $K^-pp$ system figures as an important prototype [1]. It represents the simplest configuration in which the strong two-body $K^-p$ attraction might be amplified to form a tightly bound three-body cluster.

The FINUDA measurements with stopped $K^-$ on Li and C targets [2] seemed to suggest an interpretation in terms of strongly bound $K^-pp$ clusters, with a binding energy larger than 100 MeV and a width around 70 MeV. Ever since this hypothesis was launched [2] and subsequently criticized [3], there have been active developments towards realistic calculations of such $\bar{K}N\bar{N}$ systems.

Two complementary approaches have so far been used in such investigations: the variational method [4] and three-body coupled-channel Faddeev calculations [5]. The $K^-pp$ system emerging from these computations was found to be quasibound with binding energies ranging from about 50 to 80 MeV. At these energies the $K^-pp \rightarrow \pi \Sigma N$ channel is still open. Consequently relatively large widths, between 60 and 100 MeV, were suggested.
All these calculations were based on parametrized interactions constrained by $\bar{K}N$ scattering data close to threshold and by information about the $\Lambda(1405)$ as a $\bar{K}N$ quasibound state. While simple local potentials were employed in the variational approach, separable approximations for the coupled-channel interactions were used in the Faddeev calculations. Additional constraints from the leading chiral SU(3) (Tomoza-Weinberg) interaction terms were implemented in Ref. [6]. At the same time the strong sensitivity to details of the range and energy dependence of the subthreshold $\bar{K}N$ interaction was pointed out in Ref. [7]. In fact, the limited predictive power in all those exploratory calculations can primarily be traced to ambiguities in performing off-shell subthreshold extrapolations of the $\bar{K}N$ interactions into regions not yet controlled by observables.

In the strongly coupled $\bar{K}N \leftrightarrow \pi\Sigma$ system, the $\Lambda(1405)$ emerges as an $I = 0$ $\bar{K}N$ quasibound state embedded in a resonant $\pi\Sigma$ continuum, and a detailed examination of the relevant coupled-channel dynamics is required in order to set constraints on the input for the $K^- pp$ studies. With this in mind, our present investigation is thus based on a new effective $\bar{K}N$ interaction [8] systematically derived from the full chiral SU(3) coupled-channel approach. An important issue in this context is the identification of the $\bar{K}N$ “binding” energy scale associated with the $\Lambda(1405)$. The observed $\pi\Sigma$ mass spectrum, by its strongly asymmetric form, already indicates that there are subtleties involved. They relate to the two-pole analytic structure of the coupled-channel amplitudes: a weakly bound $\bar{K}N$ state with a pole located just a few MeV below threshold gets dynamically entangled with a broad $\pi\Sigma$ resonance. The resulting $\pi\Sigma$ mass spectrum does not directly reflect the position of the $\bar{K}N$ quasibound state. The real part of the subthreshold $\bar{K}N$ amplitude ends up being located at a center-of-mass energy $\sqrt{s} \simeq 1420$ MeV. This defines the binding energy scale to be used when constraining the $\bar{K}N$ amplitude, rather than the maximum of the $\pi\Sigma$ mass spectrum which is indeed at $\sqrt{s} \simeq 1405$ MeV, the position traditionally identified with the $\Lambda^*$ resonance.

Consequently, the binding energy of the $\Lambda(1405)$ is not to be adjusted at 27 MeV as it is frequently done in simple potential models. It should instead be fixed around 12 MeV. This significantly weaker binding translates from the effective $\bar{K}N$ interaction into the antikaon-nuclear few-body systems, as we will demonstrate.

In the following section a Hamiltonian with realistic $\bar{K}N$ and $NN$ interactions as input is constructed. This Hamiltonian is used together with a variational ansatz for the $K^- pp$ wave function to minimize the energy and find the lowest quasibound state. The width of this state is estimated taking the expectation value of the imaginary part of the $\bar{K}N$ effective interaction. Further details about the $\bar{K}N$ effective interaction, inasmuch as they relate to the present calculation, are described in Section 3. Calculational details are described in Section 4. Results are presented and discussed in Section 5, followed by concluding remarks in Section 6.

2. Framework and formalism

2.1. Hamiltonian

The present calculation starts from the following non-relativistic Hamiltonian of the interacting $\bar{K}NN$ system:

$$\hat{H} = \hat{T} + \hat{V}_{NN} + \text{Re} \hat{V}_{\bar{K}N} - \hat{T}_{CM} .$$

(1)
Here $\hat{T}$ is the total kinetic energy:

$$\hat{T} = \sum_{i=1,2} \frac{\hat{p}_i^2}{2M_N} + \frac{\hat{p}_K^2}{2m_K}.$$  \hspace{1cm} (2)

The energy of the center-of-mass motion,

$$\hat{T}_{CM} = \frac{(\hat{p}_1 + \hat{p}_2 + \hat{p}_K)^2}{2(2M_N + m_K)},$$  \hspace{1cm} (3)

is subtracted.

As a realistic two-nucleon interaction $\hat{V}_{NN}$ we choose the Argonne v18 (Av18) potential [9]. This interaction reproduces $NN$ scattering phase shifts, fits deuteron data and includes the proper short-distance $NN$ repulsion as an important ingredient. We are searching for a bound $\bar{K}NN$ state with total isospin $T = 1/2$ because this configuration (unlike the one with $T = 3/2$) makes use of the strong attraction in the $I = 0 \bar{K}N$ channel. The dominant $K^-pp$ ground state configuration is supposed to have the two nucleons in a spin-singlet state ($S_N = 0$) with isospin $T_N = 1$. The driving low-energy $\bar{K}N$ interaction does not change the nucleon spin, so the $NN$ pair can be in singlet-even (${^1E}$) or singlet-odd (${^1O}$) states (with singlet-even dominating). The important parts of the $NN$ interaction at work in the present context are therefore the central pieces of the Av18 potential, namely $v_{ST,NN}^i(r)$ as given in Eq. (20) of Ref. [9]:

$$\hat{V}_{NN} = v_{1E}(r_{12}) \hat{P}({^1E}) + v_{1O}(r_{12}) \hat{P}({^1O}),$$  \hspace{1cm} (4)

with projectors $\hat{P}$ onto the respective channels. For practical calculational purposes we use a representation of these potentials in terms of Gaussian forms optimally fitted to the original Av18 potentials:

$$v_{1E}(r) = \left[3.605 e^{-(r/0.42\text{fm})^2} - 0.571 e^{-(r/0.74\text{fm})^2} - 0.012 e^{-(r/1.95\text{fm})^2}\right] \text{GeV},$$  \hspace{1cm} (5)

$$v_{1O}(r) = \left[0.90 e^{-(r/0.42\text{fm})^2} + 0.25 e^{-(r/0.74\text{fm})^2}\right] \text{GeV}.$$  \hspace{1cm} (6)

The effective $\bar{K}N$ interaction

$$\hat{V}_{\bar{K}N} = \hat{\upsilon}(\bar{K}N_1) + \hat{\upsilon}(\bar{K}N_2)$$  \hspace{1cm} (7)

is given in the form of an energy dependent, complex $s$-wave potential derived in Ref. [8] from the full chiral SU(3) coupled-channel approach. Its components in the $\bar{K}N$ channels with isospins $I = 0, 1$ are represented as Gaussians with a common range $a_s$:

$$\hat{\upsilon}(\bar{K}N_i) = \sum_{I=0,1} v_{\bar{K}N}^I(\sqrt{s}) \exp \left[-(r_{\bar{K}N_i}/a_s)^2\right] \hat{P}_I(\bar{K}N_i),$$  \hspace{1cm} (8)

where $r_{\bar{K}N_i} = |\vec{r}_K - \vec{r}_i|$ is the distance between the antikaon and each nucleon, and $\hat{P}_I(\bar{K}N_i)$ denotes the isospin projectors in the $\bar{K}N$ subsystems. The complex potential strength $v_{\bar{K}N}^I(\sqrt{s})$ is a function of the CM energy $\sqrt{s}$ in the $\bar{K}N$ subsystem. The imaginary part, $\text{Im} v_{\bar{K}N}^I(\sqrt{s})$, describes the open $\bar{K}N \to \pi\Sigma$ channels. Since the behaviour and properties of the $\bar{K}N$ effective interaction and its role in the $\bar{K}NN$ system is a key issue in the present work, we reserve a separate Section 3 for its more detailed presentation.
2.2. Model wave function

The lowest energy state of the $K^-pp$ system is found by performing a variational calculation,

$$\delta \langle \Psi | \hat{H} - E | \Psi \rangle = 0 ,$$

with $|\Psi\rangle$ represented by a suitably parametrized variational wave function.

In the present study, the energetically most favourable $K^-pp$ configuration is assumed to have total angular momentum and parity $J^P = 0^-$ and total isospin $(T, T_3) = (1/2, 1/2)$. The parity assignment includes the intrinsic negative parity of the antikaon. We describe this state $|\Psi\rangle$ by the following two-component model wave function:

$$|\Psi\rangle = N^{-1} [ |\Phi_+\rangle + C |\Phi_-\rangle ],$$

where $N^{-1}$ is the normalization constant and $C$ is a mixing coefficient. The components $|\Phi_+\rangle$ and $|\Phi_-\rangle$ have the following form:

$$|\Phi_+\rangle \equiv \Phi_+(r_1, r_2, r_K) \ |S_N = 0\rangle \ \left[ |NN|_{T_3=1/2} \ K_T = 1/2 \ , \right.$$ (11)

$$|\Phi_-\rangle \equiv \Phi_-(r_1, r_2, r_K) \ |S_N = 0\rangle \ \left[ |NN|_{T_3=1/2} \ K_T = 1/2 \right],$$ (12)

The first, second and third terms correspond to the spatial wave function, the spin state of the two nucleons and the isospin state of the total system, respectively. In both components, the spin of the nucleon pair is assumed to be zero ($S_N = 0$). The state $|\Phi_+\rangle$ corresponds to the dominant $K^-pp$ part with inclusion of $\bar{K}^0np$ through charge exchange. The mixing amplitude $(\Phi_+ | \bar{V}_{\bar{K}N} | \Phi_-)$ is proportional to the corresponding matrix element involving the difference $v^{I=0}_{\bar{K}N} - v^{I=1}_{\bar{K}N}$, with $v^{I=0}_{\bar{K}N}$ typically twice as strong as $v^{I=1}_{\bar{K}N}$. The admixture of the $|\Phi_-\rangle$ component turns out to be small, typically less than 5%. The detailed ansatz for the spatial wave functions is as follows:

$$\Phi_\pm(r_1, r_2, r_K) \equiv F_N(r_1) F_N(r_2) F_K(r_K) \times G(r_1, r_2) \ [H_1(r_1, r_K) H_2(r_2, r_K) \pm H_2(r_1, r_K) H_1(r_2, r_K)].$$ (13)

Here, $F_N(r_i) \ (i = 1, 2)$ and $F_K(r_K)$ are trial functions describing the localization of the nucleon and the kaon, respectively. Their forms are assumed to be single Gaussians:

$$F_N(r_i) \equiv \exp[-\mu r_i^2], \quad F_K(r_K) \equiv \exp[-\gamma r_K^2] .$$ (14)

We introduce the $NN$ correlation function $G(r_1, r_2)$ and $KN$ correlation functions $H_\alpha(r_i, r_K)$ (with $\alpha = 1, 2$ and $i = 1, 2$) of the following form:

$$G(r_1, r_2) \equiv 1 - \sum_{n=1}^{N_N} f_n^{NN} \exp \left[-\lambda_n^{NN} (r_1 - r_2)^2 \right],$$ (15)

$$H_\alpha(r_i, r_K) \equiv 1 + \sum_{n=1}^{N_K} f_{\alpha,n}^{KN} \exp \left[-\lambda_n^{KN} (r_i - r_K)^2 \right].$$ (16)

The $NN$ correlation function properly accounts for the strong short-distance repulsion in the $NN$ interaction by keeping the two nucleons apart. The $KN$ correlation functions can flexibly adjust themselves to the antikaon-nucleon attractive interaction so as to variationally determine the configuration which minimizes the energy.
Note that the spatial wave functions $\Phi_{\pm}(r_1, r_2, r_K)$ of Eq. (13) are even or odd under exchange of two nucleons:

$$\Phi_{\pm}(r_2, r_1, r_K) = \pm \Phi_{\pm}(r_1, r_2, r_K).$$

(17)

The two nucleons in $|\Phi_+\rangle$ are thus in a singlet-even state, while in $|\Phi_-\rangle$ they are in a singlet-odd state.

The model wave functions (10-16) have real-valued variational parameters: $C$ in Eq. (10), $\mu$ and $\gamma$ in Eq. (14), $\{f_{n n}^{NN}, \lambda_{n n}^{NN}\} (n = 1, ..., N_N)$ in Eq. (15), and $\{f_{a,n}^{KN}, \lambda_{a,n}^{KN}\} (n = 1, ..., N_K)$ in Eq. (16). The variational principle (9) then determines the optimal parameter set which minimizes the expectation value of the total Hamiltonian.

3. Effective $\bar{K}N$ potential

Here we discuss the $\bar{K}N$ potential used in Eq. (8). The strangeness $S = -1$ meson-baryon scattering and the properties of the $\Lambda(1405)$ resonance are well described by the chiral SU(3) coupled-channel approach [10,11,12,13]. In Ref. [8], two of the present authors have derived the effective $\bar{K}N$ interaction based on chiral SU(3) dynamics. First, the coupled-channel framework is translated into the equivalent single $\bar{K}N$ channel problem with a complex and energy-dependent interaction kernel, $V_{\text{eff}}(|\sqrt{s}|)$ defined in Ref. [8], which fully incorporates the dynamics of the eliminated channels. Starting from this effective interaction, a local $\bar{K}N$ potential $U^I(r, \sqrt{s})$ is then constructed in each isospin channel, to be used in the $\bar{K}N$ two-body Schrödinger equation

$$-\frac{1}{2\mu} \frac{d^2u(r)}{dr^2} + U^I(r, \sqrt{s}) u(r) = -B u(r),$$

(18)

where $u(r)$ is the radial $\bar{K}N$ s-wave function and $B$ is the binding energy. We adopt a Gaussian form for the spatial distribution of the $\bar{K}N$ potential:

$$U^I(r, \sqrt{s}) = v_{\bar{K}N}^I(\sqrt{s}) \exp \left[-(r/a_s)^2\right],$$

(19)

where $r$ is the relative coordinate in the $\bar{K}N$ system and $a_s$ is the range parameter of the potential.

The potential strength is related to the single-channel $\bar{K}N$ effective interaction kernel $V_{\text{eff}}^I(\sqrt{s})$, derived from the coupled-channel approach, as

$$v_{\bar{K}N}^I(\sqrt{s}) = -\frac{4\pi}{\pi^{3/2} a_s^3} \frac{V_{\text{eff}}^I(\sqrt{s})}{2\tilde{\omega}},$$

(20)

where $\tilde{\omega}$ is the reduced energy of the $\bar{K}N$ two-body system. The potential so obtained is complex and energy dependent, reflecting the elimination of the dynamics of the $\pi\Sigma$ and other (less important) channels. The center-of-mass energy $\sqrt{s}$ is related to the binding energy of the two-body system as $B = M_N + m_K - \sqrt{s}$. We choose the range parameter $a_s$ such that the resonance structure in the $I = 0$ channel below the $\bar{K}N$ threshold matches the result of the full chiral SU(3) dynamics calculation. Around $\bar{K}N$ threshold, the scattering amplitudes of both $I = 0$ and $I = 1$ are well reproduced by this potential.

However, approximating the full $\bar{K}N$ effective interaction $V_{\text{eff}}(\sqrt{s})$ by a local potential $U^I(r, \sqrt{s})$ can obviously work only in a limited energy range. It was found indeed that a simple extrapolation of the potential (20) to the deep subthreshold region, $\sqrt{s} < 1400$ MeV, significantly overestimates the scattering amplitude in comparison with the full
chiral dynamics result [8]. Extra energy dependence is required in the approximate local potential to repair this deficiency. A correction is applied, modifying the energy dependent strength of the real part of the potential such that the scattering amplitude of the full chiral dynamics calculation is reproduced all the way down to $\sqrt{s} = 1300$ MeV. The energy dependence of these “corrected” potentials is parametrized by polynomials as

$$v_{\bar{K}N}(\sqrt{s}) = K_0^I + K_1^I s^{1/2} + K_2^I s + K_3^I s^{3/2},$$

with coefficients $K_i^I$ given in Ref. [8]. These improved local potentials are then used in the three-body variational calculation.

In order to estimate systematic theoretical uncertainties, we adopt as in Ref. [8] four different versions of chiral SU(3) dynamics approaches to construct the equivalent local potentials, ORB [14], HNJH [15], BNW [16], and BMN [17], all of which reproduce the experimental data of the $\bar{K}N$ scattering and the properties of the $\Lambda(1405)$. The values of their range parameters are shown in the second row of Table 1.

Let us examine more closely the structure of the two-body $\bar{K}N$ system with $I = 0$ which features the $\Lambda(1405)$ as a quasibound state below $\bar{K}N$ threshold. Given the small imaginary part of the potential [8], we can solve the Schrödinger equation (18) starting with $\text{Re } U_{I=0} = 0$ and study the structure of this quasibound state. The solution is found self-consistently, with the energy dependence of the potential fully taken into account. The results for the $\bar{K}N$ binding energies $B$ in the $I = 0$ channel are summarized in Table 1 together with the mean $\bar{K}N$ distance of the bound state wave functions. We find $B \sim 10 - 13$ MeV and $\sqrt{\langle r^2 \rangle} \sim 1.7 - 2.0$ fm. Based on the calculations reported in Ref. [8], we assign an estimated additional $3 - 4$ MeV uncertainty to these values of $B$ from dispersive effects induced by the imaginary part of the $\bar{K}N$ potential.

Note that the small binding energy $B$ has its correspondence in the zero of the real part of the corresponding subthreshold (off-shell) $\bar{K}N$ scattering amplitude. This zero is consistently located around $\sqrt{s} \simeq 1420$ MeV for all four variants of chiral SU(3) models, not at 1405 MeV as one would naively expect. At the same time, the maximum of the calculated $\pi \Sigma$ invariant mass spectrum is indeed located around $\sqrt{s} \simeq 1405$ MeV.

In chiral SU(3) coupled-channel dynamics, these important features are understood as originating from the strong $\pi \Sigma$ interaction and discussed in detail in Ref. [8].

The results just mentioned should be compared with those of the phenomenological model [4] in which a local $K^-p$ potential, unconstrained by chiral SU(3), is tuned to yield $B = 27$ MeV and thus produces a smaller size of the quasibound state, $\sqrt{\langle r^2 \rangle} = 1.36$ fm. Given the substantially weaker $\bar{K}N$ attraction in our chiral SU(3) dynamics approach, it is then perhaps not surprising that, in the present work, the binding energy of the $K^-pp$ cluster will end up not far from twice the binding energy of the individual $K^-p$ state, at about 20 MeV as we shall demonstrate, more than a factor of two lower than the $K^-pp$ binding energy predicted in Ref. [4].

4. Calculational procedure

Given the explicit energy dependence of the $\bar{K}N$ potential, a self-consistent solution of the variational Eq. (19) must be found. This is done in the same way as described in our previous work [7]. In this procedure, an auxiliary antikaon “binding energy” $B_K$ is introduced and defined as follows:
Table 1

Binding energies $B$ and mean $\sqrt{r^2}$ of the quasibound $\bar{K}N$ state (the $\Lambda(1405)$) calculated using Eq. (18) with equivalent local $\bar{K}N$ potentials derived from four variants of the chiral SU(3) coupled-channel approach. The range parameters $a_s$ of the potentials are collected in the second row.

| ORB | HNJH | BNW | BMN |
|-----|------|-----|-----|
| $a_s$ [fm] | 0.52 | 0.47 | 0.51 | 0.41 |
| $B$ [MeV] | 11.77 | 11.47 | 9.97 | 13.31 |
| $\sqrt{\langle r^2 \rangle}$ [fm] | 1.87 | 1.86 | 1.99 | 1.72 |

$-B_K \equiv \langle \Psi | \hat{H} | \Psi \rangle - \langle \Psi | \hat{H}_N | \Psi \rangle$.

Here $\hat{H}_N$ is the Hamiltonian of the two-nucleon subsystem:

$$\hat{H}_N = \hat{T}_N + \hat{V}_{NN} - \hat{T}_{CM,N},$$

$$\hat{T}_N = \frac{\hat{p}_1^2 + \hat{p}_2^2}{2M_N}, \quad \hat{T}_{CM,N} = \frac{(\hat{p}_1 + \hat{p}_2)^2}{4M_N}.$$ (24)

Obviously $B_K$ is not an observable since $\langle \Psi | \hat{H}_N | \Psi \rangle$ is not an observable either, but it is a useful variable to control the energy $\sqrt{s}$ of the $\bar{K}N$ subsystem as it enters the potential $\bar{V}_{\bar{K}N}$.

The relation between $\sqrt{s}$ and $B_K$ is not a priori fixed since $\sqrt{s}$ is the energy of a two-body subsystem within the three-body system. In general,

$$\sqrt{s} = M_N + m_K - \eta B_K,$$ (25)

where $\eta$ is a parameter describing the balance of the antikaon energy between the two nucleons of the $\bar{K}N\bar{N}$ three-body system. One expects $1/2 \leq \eta \leq 1$. The upper limit ($\eta = 1$) corresponds to the case in which the antikaon field collectively surrounds the two nucleons, a situation encountered in the limit of static (infinitely heavy) nucleon sources. In the lower limit ($\eta = 1/2$) the antikaon energy is split symmetrically half-and-half between the two nucleons. We will investigate both cases and label them “Type I” and “Type II”, respectively:

Type I : $$\sqrt{s} = M_N + m_K - B_K,$$ (26)

Type II : $$\sqrt{s} = M_N + m_K - B_K/2.$$ (27)

The actual calculation now proceeds as follows. First, assume a trial starting value $B_K^{(0)}$ and determine $\sqrt{s}$ with either the Type I or the Type II option. This specifies $\sqrt{s}$ in the input $\bar{K}N$ potential. Then perform the variational calculation to determine the minimum energy of the system. With the resulting wave function, calculate the improved antikaon binding energy $B_K^{(1)}$ according to Eq. (22). Examine whether $B_K^{(1)}$ coincides with $B_K^{(0)}$. If not, iterate this procedure until $B_K^{(n)} \simeq B_K^{(n-1)}$ is satisfied at a prescribed level of accuracy.

1 In practice, the input $B_K^{(0)}$ is optimized by hand as outlined in Ref. [7].
The $K^-pp$ bound state $|\Psi\rangle$ is calculated variationally using the real part $\text{Re}\hat{V}_{KN}$ of the $KN$ potential. The decay width $\Gamma$ for $K^-pp \to \pi\Sigma N$ is then estimated in leading order perturbation theory as

$$\Gamma(K^-pp \to \pi\Sigma N) = -2 \langle \Psi | \text{Im}\hat{V}_{KN}|\Psi\rangle.$$  

(28)

Such an estimate is justified by the fact that $\text{Im}\hat{V}_{KN} \ll |\text{Re}\hat{V}_{KN}|$ [8]. However, the detailed balance between kinetic and potential energy terms finally produces a weakly bound, short-lived state whose binding energy is smaller than the width, so that this estimate of $\Gamma$ should only be taken for qualitative orientation.

5. Results

We now present results of our variational $K^-pp$ calculation. All four variants of $KN$ potentials derived from chiral SU(3) dynamics (“ORB”, “HNJH”, “BNW” and “BMN”, as explained in Section 3) have been used in order to estimate theoretical uncertainties. Both versions for the splitting relation between $\sqrt{s}$ and $B_K$, Type I (Eq. (26)) and Type II (Eq. (27)), have been employed in comparison. The results of the self-consistent calculations are collected in Table 2. They all predict weak $K^-pp$ binding, considerably weaker than what was found in previous computations. The total $K^-pp$ binding energy (BE) ranges from 15.6 MeV to 17.7 MeV for the Type I scenario and from 19.6 to 21.6 MeV for Type II. The detailed energy dependence of the $KN$ effective interactions obviously matters, and there is an indication that the Type II configuration may be energetically favored over Type I. The decay width for $K^-pp \to \pi\Sigma N$ induced by the imaginary part of $\hat{V}_{KN}$ is in intervals $39-53$ MeV for Type I and $54-72$ MeV for Type II. Altogether we combine both the Type I and Type II scenarios in a conservative estimate of uncertainties, resulting in an predicted binding energy range $B = 19 \pm 3$ MeV and a width ranging between 40 and 70 MeV.

The calculated average distance between the two nucleons in the $K^-pp$ bound state is $R_{NN} \simeq 2.2$ fm. This is obviously not a very dense system. The average between the antikaon and a given nucleon is $R_{\bar{K}N} \simeq 1.9$ fm, not far from the mean $KN$ distance of the isolated $\Lambda(1405)$ quasibound state (see Table 1).

It is instructive to examine the detailed decomposition of the total $K^-pp$ energy into kinetic and potential energy pieces of the $NN$ and $KN$ subsystems. This is shown in Table 3 for the energetically favored Type II case. One notes that the nucleons are the "slow" movers in this system, with small kinetic energies per nucleon around 20 MeV in all models considered. The $\bar{K}$-nuclear potential energy is large and negative, but it wins over the total kinetic energy of the system by only a few MeV. The additional binding is then provided by the moderate average $NN$ potential energy of about $-15$ MeV.

6. Summary and concluding remarks

The present variational calculation of the $ppK^-$ system, as a prototype for antikaon-nuclear quasibound states, has been performed with the aim to satisfy two important minimal requirements, namely the use of
Table 2
Results of the self-consistent variational $K^-pp$ calculations using effective $\bar{K}N$ interactions based on chiral SU(3) coupled-channel dynamics as explained in the text. Upper row: range $a_s$ of the effective potential (19). Shown are the total $K^-pp$ binding energy (BE) and the $K^-pp \to \pi\Sigma N$ decay width ($\Gamma$) for Type I and Type II configurations (26,27).

|      | ORB | HNJH | BNW | BMN |
|------|-----|------|-----|-----|
| $a_s$ [fm] | 0.52 | 0.47 | 0.51 | 0.41 |
| Type I | | | | |
| BE [MeV] | 17.7 | 15.9 | 17.1 | 15.6 |
| $\Gamma$ [MeV] | 53.2 | 47.1 | 60.9 | 39.2 |
| Type II | | | | |
| BE [MeV] | 21.6 | 19.8 | 19.6 | 20.8 |
| $\Gamma$ [MeV] | 64.5 | 58.6 | 71.7 | 53.7 |

Table 3
Detailed decomposition of the total $K^-pp$ binding energy (BE) for the Type II configuration: binding energy ($B_K$) of the $\bar{K}N$ subsystem (see Eq. (22)), energy $E_N = \langle \Psi|\hat{H}_N|\Psi\rangle$ and kinetic energy $T_N = \langle \Psi|\hat{T}_N - \hat{T}_{CM,N}|\Psi\rangle$ of the two-nucleon subsystem, total kinetic energy $T_{tot} = \langle \Psi|\hat{T} - \hat{T}_{CM}|\Psi\rangle$, and potential energies $V(NN) = \langle \Psi|\hat{V}_{NN}|\Psi\rangle$, $V(KN) = \langle \Psi|\text{Re} \hat{V}_{\bar{K}N}|\Psi\rangle$.

|      | BE [MeV] | $B_K$ [MeV] | $E_N$ [MeV] | $T_N$ [MeV] | $T_{tot}$ [MeV] | $V(NN)$ [MeV] | $V(KN)$ [MeV] |
|------|----------|-------------|-------------|-------------|----------------|--------------|--------------|
| ORB  | 21.6     | 46.1        | 24.5        | 40.1        | 136.0          | -15.6        | -142.0       |
| HNJH | 19.8     | 45.2        | 25.4        | 40.3        | 141.5          | -14.9        | -146.4       |
| BNW  | 19.6     | 43.2        | 23.6        | 38.5        | 132.1          | -14.9        | -136.8       |
| BMN  | 20.8     | 49.6        | 28.8        | 43.4        | 160.0          | -14.6        | -166.1       |

– a realistic nucleon-nucleon interaction (here: the Av 18 $NN$ potential);
– a realistic $\bar{K}N$ interaction (here: the subthreshold effective $\bar{K}N$ interaction based on chiral coupled-channel dynamics).

The variational $\bar{K}NN$ wave function has been constructed so as to handle the strong short-distance $NN$ interaction which keeps the two nucleons apart. The effective $\bar{K}N$ interaction incorporates essential features of the $\bar{K}N \leftrightarrow \pi\Sigma$ coupled-channel dynamics. In particular, it accounts for the important fact that the zero in the real part of the $I = 0$ subthreshold $\bar{K}N$ amplitude and the maximum of the $\pi\Sigma$ invariant mass spectrum do not coincide: the quasibound $\bar{K}N$ state is located around $\sqrt{s} \simeq 1420$ MeV whereas the $\pi\Sigma$ mass spectrum peaks at $\sqrt{s} \simeq 1405$ MeV. This implies weaker $\bar{K}N$ attraction than naively anticipated. As a consequence, the predicted $K^-pp$ binding energy found in the present calculations is

$$B(K^-pp) \simeq (19 \pm 3) \text{ MeV}$$

where the uncertainty measure is based entirely on using four different versions of chiral SU(3) coupled-channel models as input. Additional systematic uncertainties, such as the dispersive shift induced by the imaginary part of the $\bar{K}N$ potential, are under analysis (19).

The $K^-pp \to \pi\Sigma N$ decay width is estimated to be, roughly,
\[ \Gamma(K^-pp \rightarrow \pi\Sigma N) \sim (40 - 70) \text{ MeV} \quad \text{(30)} \]

This suggests that \( K^-pp \) clusters, even if quasibound, would be difficult to identify experimentally. The width is in fact expected to increase even more through the non-mesonic decay \( K^-pp \rightarrow Y\bar{N} \) into a hyperon-nucleon pair [15]. Detailed studies of this and further corrections (such as the influence of spin-dependent \( NN \) correlations and the role of \( p \)-wave \( KN \) interactions involving the \( \Sigma^+(1385) \)) are in progress and will be reported elsewhere [19].

The \( K^-pp \) binding energy found in the present calculation is significantly smaller than corresponding values reported from variational [1,4] and Faddeev [5,6] calculations. While the difference with respect to the previous variational results is understood in terms of the improved chiral \( \bar{K}N \) interaction used in the present approach, a direct comparison with the Faddeev results (which explicitly incorporate coupled-channel dynamics, though with separable potentials) is not so obvious and requires further detailed studies.

In any case it is found that the \( K^-pp \) binding energy turns out to be very sensitive to details of the off-shell, subthreshold extrapolation of the \( \bar{K}N \) interaction. This extrapolation relies so far on constraints from threshold scattering and kaonic hydrogen measurements, together with the available low-statistics data of the \( \pi\Sigma \) invariant mass spectrum. Stronger constraints are expected to be imposed once kaonic hydrogen and deuterium precision measurements become available. A better determination of the \( \pi\Sigma \) mass spectrum would also be highly welcome, as well as exclusive data on the final states from decaying antikaon-nuclear systems in order to clarify their dynamics.

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