FERMION MASSES WITHOUT YUKAWA COUPLINGS

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ABSTRACT
Radiatively generated fermion masses without tree level Yukawa couplings are re-analyzed within supersymmetric models. Special emphasis is given to the possible appearance of color and charge breaking vacua. Several scenarios in which the radiative mechanism can be accommodated for the first, second, and third generation fermion masses are presented. Some of these require a low scale of supersymmetry breaking.

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1 Introduction

It is often thought that fermion masses cannot be generated radiatively within supersymmetric models, unless their generation is due to fermion–sfermion misalignment. The radiative mechanism requires large trilinear couplings. These are generically obtained through the $F$–vacuum expectation value of spurions which parametrize the breaking of supersymmetry as well as that of chiral flavour symmetries. The large values required for these trilinear couplings are believed to produce vacuum instabilities.

Starting from the most general trilinear structure which low–energy supersymmetric models allow, we reexamine this issue in some detail. We find that the conventional trilinear couplings, indeed, are not likely candidates for radiatively generating fermion masses. In contrast, the “wrong” Higgs trilinear couplings, which are absent in minimal models, may lead to a successful implementation of the radiative mechanism. For some flavours, a very particular type of supersymmetry breaking is selected, one in which the scale of breaking, $M_{SU SY}$, is one or two order of magnitude above the electroweak scale. Other options involve mirror fermions at TeV scales.

The paper is organized as follows. In Sect. 2, we discuss the trilinear couplings which give rise to fermion masses and yukawa couplings, for which we give explicit expressions in Sect. 3. In Sect. 4, we give a sufficient condition to avoid unwanted minima. In Sect. 5, we classify possible scenarios consistent with charge and color conservation, in which the mechanism of radiative
generation of masses can be implemented. Phenomenological implications for low–energy and collider physics are discussed elsewhere.

2 Classification of operators

In the absence of tree–level Yukawa couplings, chiral flavor symmetries can be broken by trilinear terms in the scalar potential,

\[ V = \sum_i m_i^2 \phi_i^2 + \left[ B_{ij} \phi_i \phi_j + A_{ijk} \phi_i \phi_j \phi_k + h.c. \right] + \lambda_{ij} \phi_i^2 \phi_j^2. \]  

The chiral flavor symmetries in the fermion sector are then broken at the quantum level. Gauge loops proportional to \( A \) or \( A' \), which dress the fermion propagator, generate fermion masses as well as effective couplings fermion–fermion–Higgs and fermion–sfermion–Higgsino.

The flavor symmetries of the high–energy theory can be chosen in such a way to forbid certain fundamental Yukawa couplings but allow for either operators in the superpotential of the type \((i)\) \( ZH \Phi_L \Phi_R / M \), or operators in the Kähler potential \((ii)\) \( ZZ^\dagger H^\dagger \Phi_L \Phi_R / M^3 \). The chiral superfield \( Z = z + \theta^2 F_Z \) parametrizes here the supersymmetry breaking sector, and \( \langle F_Z \rangle = M^2_{\text{SUSY}} \) signals supersymmetry breaking at a scale \( M_{\text{SUSY}} \). If the scalar component \( \langle z \rangle \) vanishes and the auxiliary component \( \langle F_Z \rangle \) does not, then no Yukawa couplings arise but only soft supersymmetry breaking trilinear terms \( \propto \langle F_Z \rangle^2 \) in the scalar potential. The operators \((i)\) and \((ii)\) lead respectively to \( A \)– and \( A' \)–type terms,

\[ (i) \, AH \phi_L \phi_R, \quad (ii) \, A'H^\ast \phi_L \phi_R, \]  

which are not proportional to any Yukawa couplings. The symmetries of the models typically allow for only one type of operators for a given flavor. Note that a sufficiently large \( A' \sim M^2_{\text{SUSY}} / M^3 \) requires that the supersymmetry breaking scale, \( M_{\text{SUSY}} \), and the scale that governs the dynamics in the Kähler potential, \( M \), are both relatively low–energy scales. Such a situation could arise, for example, if there is strong dynamics at the scale \( M \).

3 Masses and Higgs Couplings

The one–loop sfermion–gaugino exchange which dresses the fermion propagator generates a finite contribution to the fermion mass. It is given by

\[ m_f = -m_{LR}^2 \left\{ \frac{\alpha_s}{2\pi} C_f m_3 I(m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{g}}^2) + \frac{\alpha'}{2\pi} m_B I(m_{\tilde{f}_1}, m_{\tilde{f}_2}, m_{\tilde{B}}^2) \right\}, \]  

where \( C_f = 4/3 \), \( 0 \) for quarks and leptons, respectively, \( m_{LR}^2 = A(H) \) or \( A'(H^\ast) \), and \( \tilde{f}_1, \tilde{f}_2 \) are the two mass eigenstates superpartners of \( f \). The first
and second terms correspond to the gluino (˜g) and bino (˜B) contributions, respectively. In the second term, corrections due to possible ˜B–W3 mixing are omitted. The function \( I(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_\lambda^2) \) is such that

\[
I(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_\lambda^2) \times \max(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_\lambda^2) \simeq \mathcal{O}(1),
\]

where \( m_\lambda \) denotes generically a gaugino mass. If \( A \) (or \( A' \)), \( m_{\tilde{f}} \), and \( m_\lambda \), are all of the same order of magnitude, the radiatively generated fermion mass is not sensitive to the superpartners scale and does not vanish even when this is rather large.

In the approximation \( \tan \beta \sim 1 \) (with \( \tan \beta \) the ratio of the two vacuum expectation values \( v_2/v_1 \)), eq. (3) implies, for a typical sfermion mass scale \( m_{\tilde{f}} \),

\[
a \equiv \frac{A}{m_{\tilde{f}}} \sim \frac{m_q(M_{\text{weak}})}{(1.5 - 3 \text{ GeV})} \quad \text{for quarks}
\]

\[
\sim \frac{m_l(M_{\text{weak}})}{(50 - 100 \text{ MeV})} \quad \text{for leptons}
\]

(and similarly for \( a' \equiv A'/m \)). Hence, the maximal magnitude of the trilinear parameters that can be realized consistently determines which fermion masses can be generated radiatively.

Effective Yukawa couplings Higgs–fermion–fermion, as well as couplings Higgsino–sfermion–fermion, are obtained by the corresponding loop diagrams, induced by the two types of operators \( A' H^* \phi_L \phi_R \) and \( A H \phi_L \phi_R \). It is interesting to notice that, in general, the Higgsino–sfermion–fermion couplings are suppressed with respect to the Higgs–fermion–fermion ones by factors of order \( \alpha_2/\alpha_s \) or \( \alpha'/\alpha_s \). If fermion masses are generated through such a radiative mechanism, large deviations from the usual hard supersymmetric relations among these couplings have to be expected. Details can be found in Ref. 4.

It is straightforward to discuss the effective vertex Higgs–fermion–fermion with on–shell fields, as in the decay \( H \to f f \), where \( H \) is here generically one of the physical Higgs states. We denote the relative coupling by \( \bar{y}_f \). It depends on masses internal and external to the loop which generates it. In the case of a light Higgs boson \( h^0 \), when the approximation \( m_{h^0}/m_{\tilde{f}}, m_{h^0}/m_\lambda \to 0 \) can be used \( \bar{y}_f \) has the form

\[
\bar{y}_f = \frac{m_{\tilde{f}}}{\langle H \rangle} \left\{ \sin^2 2\theta_f \left[ \frac{1}{2} \frac{\sum_i I(m_{\tilde{f}_i}^2, m_{\tilde{f}_i}^2, m_\lambda^2)}{I(m_{\tilde{f}_1}^2, m_{\tilde{f}_2}^2, m_\lambda^2)} - 1 \right] + 1 \right\},
\]
where \( \sin 2\theta_f \) is the sfermion mixing angle. One observes that the radiative Yukawa coupling can deviate by a significant percentage in comparison to the case of a tree–level fermion mass. Most importantly, it should be stressed that this deviation is always an enhancement, which increases with the mass splitting between the sfermion eigenstates. This remains true also in the case of a massive external Higgs boson (see Ref. 4). Note that the projecting factors between the physical and interaction Higgs eigenstates were omitted in eq. (6).

In the case of \( A' \)–type operators these factors are different than in the usual case of tree–level couplings. For \( h^0 \), this is irrelevant in the limit of decoupling of the heavy Higgs bosons, which applies to most of the parameter space. These factors, however, may affect even further the couplings of heavier Higgs bosons to fermions. They may, indeed display larger deviations from the couplings relative to the usual case of tree–level fermion masses.

Other phenomenological consequences of this radiative scenario can be found in Ref. 4. We concentrate in the following on a more fundamental aspect, i.e. whether the possibly large trilinear scalar operators \( AH\phi_L\phi_R \) and \( A'H^*\phi_L\phi_R \) produce vacuum instabilities.

4 Stability analysis of the scalar potential

The low–energy realization of the \( F_Z \)–spurion framework relies on the presence of substantial dimensionful trilinear couplings \( A \) or \( A' \) in the scalar potential. One can typically constrain the magnitude of such couplings from above by analyzing the vacuum of the theory. It is instructive for our purposes to examine the stability of the vacuum along an equal field direction corresponding to the relevant trilinear operator (2).

At the tree–level the problem can be partially addressed analytically. Following Refs. 6,7, we consider a scalar potential of the form

\[
V(\phi) = m^2\phi^2 - \gamma\phi^3 + \lambda\phi^4,
\]

where \( \phi \) here corresponds to the field along the equal field direction \( \phi = \phi_L = \phi_R = H_\alpha, (\alpha = 1, 2) \). More generally, the parameters \( m^2, \gamma, \) and \( \lambda \) can depend on various angles, which we ignore here. It is then possible to derive a condition for color and charge conservation,

\[
\gamma^2 \leq 4\lambda m^2. \tag{8}
\]

Condition (8) ensures that the deepest minimum along the equal field direction is at the origin, and hence, the global minimum of the theory conserves color and charge. It is a sufficient, but not necessary condition for color and charge
conservation. Thus, conclusions derived using the sufficient condition (8) could, in principle, be weakened. The dimensionful coefficients are given by

\[ m^2 = m_{f_L}^2 + m_{f_R}^2 + m_{H_\alpha}^2, \]  

(9)

and

\[ \gamma = 2A \text{ (or } 2A') \]  

(10)

where a choice of phase corresponding to the deepest possible minimum was made. If \( m_{\tilde{f}} \) represents an average sfermion mass scale, \( m_{\tilde{f}_L} \sim m_{\tilde{f}_R} \sim m_{\tilde{f}} \), then \( m^2 \sim 3m_{\tilde{f}}^2 \) and \( \gamma \) can be expressed in terms of the dimensionless variable \( a \) (or \( a' \)) in (8), \( \gamma \sim 2am_{\tilde{f}} \) (or \( \gamma \sim 2a'm_{\tilde{f}} \)). It should be noticed that the squared Higgs mass contribution in eq. (9) sums over a soft supersymmetry breaking and a supersymmetry conserving (usually denoted by \( \mu^2 \)) mass parameters. Hence, \( m^2 \gg 3m_{\tilde{f}}^2 \) is in principle possible when a large supersymmetric mass parameter \( \mu^2 \gg m_{\tilde{f}}^2 \) is present. Such a large value, however, would correspond to an increased degree of fine tuning in the electroweak symmetry breaking mechanism. In fact, minimal tuning usually implies \( |m_{H_\alpha}^2| \sim (1/2)M_Z^2 < m_{\tilde{f}}^2 \), and therefore \( 2m_{\tilde{f}}^2 \lesssim m^2 \lesssim 3m_{\tilde{f}}^2 \). Aside from these possible deviations in \( m_{H_\alpha}^2/m_{\tilde{f}}^2 \), one can approximate the condition (8) with

\[ |a| \lesssim \sqrt{3}\lambda, \]  

(11)

and similarly for \( a' \). Whether or not this condition is satisfied depends on the details of the quartic coupling \( \lambda \) in each specific model considered.

The coupling \( \lambda \) receives tree–level contributions from \( F \)–terms

\[ V_F = \sum_i \left| \frac{\partial W}{\partial \Phi_i} \right|^2 \sim h^2 \phi^4 \]  

(12)

and from gauge \( D \)–terms

\[ V_D = \frac{1}{2} g^2 \left| \sum_{i,j} \phi_i^a T^a_{ij} \phi_j \right|^2 \sim \frac{1}{2} g^2 (\text{Tr} Q_i)^2 \phi^4. \]  

(13)

It would also receive additional supersymmetry breaking contributions if a heavy sector of the theory, which mixes with the light fields, is integrated out.

It is possible that charge and colour breaking global minima appear when values of \( |a| \) larger than those satisfying condition (8) are required to generate radiatively certain masses. In this case, the universe may still be on a
metastable vacuum with a sufficiently long lifetime. This somewhat less desirable situation and the resulting weaker constraints on $A$, $A'$ are discussed in Ref. 4.

5 Possible Models

Below, we will consider and classify possible contributions to the quartic coupling $\lambda$. The different contributions distinguish among the different models.

5.1 Minimal $AH\phi_L\phi_R$ operator model

Such operators correspond to gauge invariant holomorphic operators, and as such are associated with flat $D$–terms along the equal field direction, $V_D \propto (\text{Tr} Q_i)^2 = 0$. In the absence of tree–level Yukawa couplings $V_F$ contains only the supersymmetric mass contribution to the Higgs fields, and hence, $\lambda = 0$ at tree–level. The trilinear tree–level scalar potential is unbounded from below. This situation persists at one–loop where negative quartic couplings $\lambda^{OL} \sim -a^4/96\pi^2$ are generated.

The scalar potential is presumably stabilized at very large field values due to the physics at those scales. Nevertheless, the vacuum cannot be assumed to conserve color and charge, and hence, we are forced into a metastable vacuum. As mentioned above, one could tolerate such a situation if the tunneling amplitude to the true vacuum is sufficiently suppressed. It would typically require $a \lesssim 1.4$.

Only very light fermions can be generated radiatively in this type of models: the electron, the $u$– and $d$–quark, for which it is sufficient to have $a \sim 10^{-3}$.

5.2 Minimal $A'H^*\phi_L\phi_R$ operator model

Operators of this type do not correspond to gauge invariant holomorphic directions and are not necessarily associated with $D$–flatness. In particular, non–flat is the hypercharge $D$–term since $Y(H) = (Y(\phi_L) + Y(\phi_R))$. We obtain in this case (independently of flavor labels):

$$\lambda \sim \frac{1}{2} g^2 \sim 0.06. \quad (14)$$

The potential is now bounded from below. Substituting (14) in condition (11) gives $a' \lesssim 0.4$.

Thus, the $c$– and $s$–quark masses, which require respectively $a' = 0.2$–0.5 and 0.1, can be easily accommodated in this model.
### Table 1: The quartic coupling along the equal field direction in the different scenarios.

| Model                  | $\lambda$ | Limits       | Comments                  |
|------------------------|-----------|--------------|---------------------------|
| Minimal $A$            | $-\frac{1}{96\pi^2}a^4$ | $a \approx 1$ | a metastable vacuum       |
| Minimal $A'$           | $\frac{g'^2}{2}$ | $a' \lesssim 0.4$ |                           |
| Hybrid $h_t - A'$      | $h_t^2$ | $a' \lesssim \sqrt{3}$ | relevant for $b$          |
| Mirror matter          | $h_t^2(\tilde{m}^2/(\tilde{m}^2 + \mu^2))$ | $a, a' \lesssim a$ few | assumes a multi–TeV scale |

5.3 A hybrid model $W \sim h_t H_2 QU$ and $V \sim A'H_2^* QD$

Here we will consider a specific example motivated by the sharp distinction between the $t$–quark Yukawa couplings $h_t \sim 1$ and all other low–energy Yukawa couplings $h_f \ll 1$ (for $\tan \beta \ll 50$). The presence of the tree level supersymmetric operator $h_t H_2 QU$ carries important consequences for the scalar potential along the equal field direction associated with the supersymmetry breaking operator $A'H_2^* QD$, which could be a source for the $b$–quark mass. (Note that we do not distinguish in our notation between a standard matter chiral superfield and its scalar component.)

The $F$–terms contains the quartic term $|\partial W/\partial U|^2 = h_t^2 H_2^2 Q^2$ and hence,

$$\lambda \approx h_t^2 + \frac{1}{2} g'^2 \sim h_t^2 \sim 1.$$

In the case of the $b$–quark mass one needs $a' \approx \sqrt{3}$, which can be easily accommodated in this model. Of $O(1)$ is also the $a(a')$ needed for the muon mass. Such a large value excludes the radiative generation of this mass in the models described in Secs. 5.1 and 5.2. A mechanism similar to that described in this section can work, if there exist a term in the superpotential involving second generation lepton fields with a large coupling. This can then play the same stabilizing role that $h_t$ has in the case of the $b$–quark mass, as it will be discussed in the next section.

5.4 Models with mirror matter

Vector–like (mirror) pairs of chiral superfields exist in many extensions of the standard model near or above the weak scale. Often such fields are expected to have the same transformation properties under the SM gauge group as the SM fields. They may transform differently under additional symmetries, in particular, flavor symmetries.

It is natural to expect some mixing between the two sectors, which lead to a lower bound on the mass scale of the exotic matter. The new scale could be set by either supersymmetric mass terms for the vector–like pairs or by
soft supersymmetry breaking parameters. It is also reasonable to expect that
the mirror matter fields transform differently than the SM fields under the
flavor symmetries, and therefore Yukawa operators that mix SM and exotic
fields may be allowed with large couplings. Clearly, the role played by the SM
superfield $U$ in the previous example can be played in this case by an exotic
field. Furthermore, there are no restrictions, in this case, on the flavor labels
or on the type of operator.

Motivated by recent discussions of decoupling in supersymmetric models,$^8$
we will assume, for simplicity, that all soft supersymmetry breaking $\tilde{m}^2$ and
supersymmetry conserving mass parameters $\mu^2$ for the exotic fields (and per-
haps for some SM fields) are multi TeV parameters. After integrating out the
heavy fields one obtains for the light fields

$$
\lambda = h^2 \left( \frac{\tilde{m}^2}{\tilde{m}^2 + \mu^2} \right),
$$

(16)

where $h$ is the relevant Yukawa coupling between the Higgs, SM and exotic
heavy superfields, $hH\phi_{SM}\phi_{heavy}$. The integration of the heavy fields leads
to a supersymmetry breaking contribution to $\lambda$. (In fact, one has $\lambda = 0$
in the supersymmetric limit.) The usual $h^2$ term is now modified by an a priori
arbitrary factor of $O(1)$. Since a few of these $O(1)$ contributions may be
present, it is possible to have in this case $a$, $a' \lesssim$ a few.

The muon mass can then be generated radiatively in models of this type.
The far-reaching phenomenological consequences for the process $\mu^+\mu^- \rightarrow
H \rightarrow f\bar{f}$ and for the muon magnetic moment are discussed in Refs. 4,5.

6 Conclusions

We have re–analyzed the problem of radiative generation of fermion masses
through trilinear soft operators. These are obtained from holomorphic and/or
non–holomorphic operators in which a spurion field acquires a vacuum expecta-
tion value only in the $F$–component, breaking simultaneously supersymmetry
and chiral fermion symmetries. In general, large values of these trilinear cou-
plings are needed for the radiative generation of second and third generation
fermion masses. We have found several consistent scenarios in which fermion
masses for light and heavy flavours could be generated consistently and with
stable vacua. Some of these scenarios seem to point to a scale of supersymme-
try breaking not far above the electroweak scale.
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References

1. speaker at the Workshop
2. F. del Aguila, M. Dugan, B. Grinstein, L.J. Hall, G.G. Ross, and P. West, \textit{Nucl. Phys.} B250, 225 (1985); T. Banks, \textit{Nucl. Phys.} B303, 172 (1988); E. Ma, \textit{Phys. Rev. D} 39, 1922 (1989); V.A. Krasnikov, \textit{Phys. Lett. B} 302, 59 (1993).
3. N. Arkani–Hamed, C.–H. Cheng, and L.J. Hall, \textit{Phys. Rev. D} 54, 2242 (1996).
4. F.M. Borzumati, G.R. Farrar, N. Polonsky, and S. Thomas, \textit{Soft Yukawa couplings in supersymmetric theories}, ZU-TH 23/97, RU-97-56, SU-ITP 97-45.
5. F.M. Borzumati, G.R. Farrar, N. Polonsky, and S. Thomas, \textit{Precision Measurements at The Higgs Resonance: A Probe of Radiative Fermion Masses}, [hep–ph/9712428]
6. J.F. Gunion, H.E. Haber, and M. Sher, Nucl. Phys B306 (1988) 1.
7. See Appendix B in P. Langacker and N. Polonsky, Phys. Rev. D 50 (1994) 2199.
8. For example, see A.G. Cohen, D.B. Kaplan, and A.E. Nelson, Phys. Lett. B 388 (1996) 588.