Zero-field superfluid density in \(d\)–wave superconductor evaluated from the results of muon-spin-rotation experiments in the mixed state

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We report on measurements of the in-plane magnetic penetration \(\lambda_{ab}\) in the optimally doped cuprate superconductor \((\text{BiPb})\_2(\text{SrLa})_2\text{CuO}_{4+\delta}\) (OP Bi2201) by means of muon-spin rotation (\(\mu\)SR). We show that in unconventional \(d\)–wave superconductors (like OP Bi2201), \(\mu\)SR experiments conducted in various magnetic fields allow to evaluate the zero-field magnetic penetration depth \(\lambda_0\), which relates to the zero-field superfluid density in terms of \(\rho_s \propto \lambda_0^{-2}\).

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Muon-spin-rotation (\(\mu\)SR) measurements in the mixed state of type-II superconductors provide valuable information on the superconducting properties. An important advantage of this method is that the muons probe the \textit{bulk} of the material, and the results are not complicated by surface imperfections. The quantitative parameters extracted from \(\mu\)SR experiments depend, however, on the details of the model applied to reconstruct the internal magnetic field distribution in the superconductor in the mixed state. So far, field distributions measured by means of \(\mu\)SR were analyzed within the framework of analytical models based on London and Ginzburg-Landau (GL) theories, which can be applied, in general, to conventional superconductors with a single isotropic energy gap.\(^\text{12,13}\) The situation becomes much more complicated in the case of unconventional superconductors, like cuprates, MgB\(_2\), etc. It was found, in particular, that the effective magnetic field penetration depth \(\lambda_{eff}\) extracted from \(\mu\)SR measurements depends on the applied magnetic field (see e.g. Refs. \(^\text{14,15,16}\)), which is unexpected within the GL theory. Here \(\lambda_{eff}\) refers to a quantity evaluated from \(\mu\)SR experiments conducted in a superconductor in the mixed state, in contrast to \(\lambda_0\) as obtained from Meissner state experiments (\(H \ll H_{c1}\), \(H_{c1}\) denotes the lower critical field). In addition, it was observed that not only the absolute value, but also the shape of \(\lambda_{eff}(T)\) changes with field.\(^\text{12,13}\) In this respect the question concerning the relation of \(\lambda_{eff}\) to \(\lambda_0\), which is generally assumed to be proportional to the superfluid density (\(\lambda_0^{-2} \propto \rho_s\), becomes very important.

In this paper we report on the results of a \(\mu\)SR study of the in-plane magnetic penetration depth \(\lambda_{ab}\) in optimally doped \((\text{BiPb})_2(\text{SrLa})_2\text{CuO}_{4+\delta}\), \(\lambda_{eff}(T, H)\) was obtained from the measured temperature dependence of the \(\mu\)SR linewidth by using numerical calculations of Brandt.\(^\text{12}\) The temperature dependence of \(\lambda_0^{-2}\) was further evaluated from \(\lambda_{eff}(T, H)\) considering the nonlinear and the nonlocal response of a superconductor with nodes in the energy gap to the applied magnetic field. It was found that only at relatively low magnetic fields \(B/B_{c2}(0) \lesssim 10^{-5}\), \(B_{c2}(0)\) is the zero-temperature value of the upper critical field \(\lambda_{eff}\) is a good measure of \(\lambda_0\). The high field data, however, need to be evaluated by taking into account both the nonlinear and the nonlocal corrections.

Details on the sample preparation of optimally doped \((\text{BiPb})_2(\text{SrLa})_2\text{CuO}_{4+\delta}\) (OP Bi2201) single crystals can be found elsewhere.\(^\text{12,13}\) Field-cooled magnetization (\(M_{FC}\)) measurements of OP Bi2201 were performed with a SQUID magnetometer at \(\mu_0 H = 1\) mT, applied parallel to the \(c\) axis, for temperatures ranging from 5 K to 50 K. The transition temperature \(T_c = 34.8\) K was obtained as the intersect of the linearly extrapolated \(M_{FC} (T)\) curve in the vicinity of \(T_c\) with the \(M = 0\) line [see Fig. 1(a)].

![FIG. 1: (Color online) (a) Field-cooled magnetization \(M_{FC}(T)\) of OP Bi2201. The field \(\mu_0 H = 1\) mT was applied parallel to the crystallographic \(c\) axis. (b) The Magnetic field distribution \(P(B)\) of OP Bi2201 taken at \(T = 1.6\) K, \(\mu_0 H = 0.04\) T. The lines represent the best fit within a two-Gaussian approach.](image-url)
ing from 5 mT to 0.64 T. The magnetic field was applied parallel to the c axis and transverse to the muon-spin polarization. The typical counting statistics were \( \sim 15 – 18 \) million muon detections per data point. The experimental data were analyzed within the same scheme as described in Refs. 11,14. This is based on a two-component Gaussian fit of the \( \mu \)SR time spectra which allows to describe the asymmetric local magnetic field distribution \( P(B) \) in the superconductor in the mixed state [see Fig. 1 (b)]. The magnetic field penetration depth \( \lambda \) was derived from the second moment of \( P(B) \) as \( \sigma^2 \propto \lambda^{-1} \).

The superconducting part of the square root of the second moment \( \langle \Delta B^2 \rangle = 4 \sigma^2 - \sigma_{nm}^2 \) is the upper critical field. For a superconductor \( \lambda_{sc}(T,H) \) can occur as a function of temperature and/or magnetic field \((0) = 50 \) T. The temperature dependence \( \lambda_{sc}(T,H) \) is constant for 1.6 K \( \leq T \leq 26 \) K and smaller than the ideal value of 1.2, which is probably caused by distortions of the VL due to pinning effects. The sharp change of \( \lambda_{sc} \) at \( T \approx 30 \) K is similar to what was observed in Bi2212, where it was attributed to VL melting15,16. Therefore, we conclude that for temperatures 1.6 K \( \leq T \leq 30 \) K the temperature variation of \( \sigma_{sc} \) in OP Bi2201 studied in the present work reflects the intrinsic behavior of the in-plane magnetic penetration depth \( \lambda_{ab}(T) \).

From the measured \( \sigma_{sc}(T,H) \) we reconstructed \( \lambda_{c,ff}^2(T,H) \) by using the procedure described in Ref. 17. A correction between \( \sigma_{sc} \) and \( \lambda_{c,ff}^2 \) was considered according to:

\[
\sigma_{sc}(b)[\mu s^{-1}] = A(b)\lambda_{c,ff}^2[\text{nm}^{-2}],
\]

which accounts for decreasing of the field variance within the VL with increasing magnetic field. The correction factor \( A(b) \) depends only on the reduced field \( b = B/B_{c,2} \) (\( B_{c,2} \) is the upper critical field). For a superconductor with a Ginzburg-Landau parameter \( \kappa = \lambda/\xi \geq 5 \) measured in fields ranging from 0.25/\( \kappa^{1.3} \leq b \leq 1 \), \( A(b) \) can be obtained analytically as \( A(b) = 4.83 \times 10^4(1-b)[1 + 1.21(1 - \sqrt{b})^3] \mu s^{-1} \text{nm}^2 \) (see Ref. 13).

The reconstructed \( \lambda_{c,ff}^2(T,H = \text{const}) \) curves are shown in Fig. 2. The corresponding \( A(b) \) dependences are displayed in the inset. The calculations were made for \( B_{c,2}(0) = 50 \) T.15 The temperature dependence of \( B_{c,2} \) was assumed to follow the Werthamer-Helfand-Hohenberg (WHH) prediction.18 Below \( T \approx 20 \) K, \( \lambda_{c,ff}^2 \) is linear in \( T \), as is expected for superconductor with nodes in the energy gap. Fig. 3 also implies that in the whole temperature region (from \( T \approx 1.6 \) K up to \( T_c \), \( \lambda_{c,ff}^2(T,H) \) decreases with increasing field. This contrasts the results obtained by using a similar procedure for the ternary boride Li2Pd3B and electron-doped Sr0.9La0.1CuO2.17,20 For these two compounds the
\( \lambda_{eff}^2(T, H) \) curves were found to collapse onto a single curve. Since Li_{2}Pd_{3}B and Sr_{10}La_{0.1}CuO_{2} are supposed to be fully gaped, we may conclude that the field dependence of \( \lambda_{eff}^2(T) \), shown in Fig. 3, is caused by the presence of nodes in the superconducting energy gap of OP Bi2201.

As shown in Refs. 26 and 27, the magnetic field dependence of \( \lambda_{eff} \) arises from the nonlocal and the nonlinear response of a superconductor with nodes in the energy gap to the applied magnetic field. The nonlocal correction to \( \lambda_{ab} \) appears due to the magnetic field induced quasiparticle excitation over the gap nodes. According to Volovik, the density of the delocalized states increases proportionally to \( \sqrt{b} \). The nonlocal correction to \( \lambda_{ab} \) appears from the response of electrons with momenta on the Fermi surface close to the gap nodes. This is because the coherence length \( \xi \), being inversely proportional to the gap, becomes very large close to the nodes and, formally, diverges at the nodal points. Thus there exist areas on the Fermi surface where \( \lambda / \xi \lesssim 1 \), and the response of a superconductor to an applied magnetic field becomes highly nonlocal.

In order to reconstruct the temperature dependence of the superfluid density in zero magnetic field we used the general assumption that the proportionality factor relating \( \lambda_{eff} \) to \( \lambda_0 \) is a function of the reduced magnetic field \( b \) only, so that:

\[
\lambda_{eff}(b, T) = C(b)\lambda_0(T).
\]  (2)

This statement is correct, at least, in case of nonlinear corrections which scale with \( \sqrt{b} \).

The fact that the field and the temperature dependences of \( \lambda_{eff} \) are described by separate terms [see Eq. (3)] allows to reconstruct \( \lambda_0(T) \). In order to demonstrate this, we refer to the inset in Fig. 4 which shows the dependence of \( \lambda_{eff} \) on the magnetic field for some selected temperatures. It is seen, e.g., that the reduced field \( b \) for \( \lambda_{eff} \) measured at \( T = 10 \) K, \( \mu_0H = 0.4 \) T (lower point in the oval selection) is almost the same as the one for the point at \( T = 23 \) K, \( \mu_0H = 0.2 \) T (upper point). This implies that the coefficients \( C(b) \) for these two points are nearly equal and that the difference in the absolute values of \( \lambda_{eff}(10 \) K, 0.4 T) and \( \lambda_{eff}(23 \) K, 0.2 T) is due to different values of \( \lambda_0 \). The reconstruction procedure was performed in the following way. First, from \( \lambda_{eff}^2(T, H) \) plotted in Fig. 3 \( \lambda_{eff}(b) \) was reconstructed for various constant temperatures (see inset in Fig. 4). Second, the resulting values of \( \lambda_{eff}(T = const, b) \) were scaled in order to have them collapsing on a single curve (see Fig. 4). According to Eq. (2) this curve corresponds to \( C(b) = \lambda_{eff}(T, b) / \lambda_0(T) \), while the scaling factor, in turn, corresponds to \( \lambda_0(T) \). The solid line represents the result of the fit by means of the relation:

\[
\lambda_{eff}(b) / \lambda_0 = C(b) = (1 - K \sqrt{b})^{-1/2},
\]  (3)

which takes into account the nonlinear correction to \( \lambda_0 \) for a superconductor with \( d \)-wave energy gap. Here the parameter \( K \) depends on the strength of the nonlinear effect. It is obvious that the "nonlinear" curve describes the experimental \( C(b) = \lambda_{eff}(b) / \lambda_0 \) dependence reasonably well. In particular, it reproduces the curvature at \( b \lesssim 0.01 \) and the linear increase of \( C(b) \) for
0.01\lesssim b\lesssim 0.05. We believe, however, that the whole \(\lambda_{0}^{-2}(b)/\lambda_{0}\) curve must be a combination of both nonlinear and nonlocal correction effects, similar to the results of Ref. \[2\].

The temperature dependence of \(\lambda_{0}^{-2}\) and the comparison of \(\lambda_{0}^{-2}(T)\) with \(\lambda_{sc}(T)\) and \(\lambda_{eff}^{-2}(T)\) measured at \(\mu_{0}H = 0.04\) T and 0.64 T are presented in Fig. \[3\]. Both \(\sigma_{sc}(T)\) and \(\lambda_{eff}^{-2}(T)\) measured at \(\mu_{0}H = 0.04\) T almost coincide with each other as well as with \(\lambda_{0}^{-2}(T)\). This implies that the two sets of corrections, namely, the first, accounting for decrease of the second moment of the \(\sigma_{sc}\) line with increasing field \([\text{Eq. (1)}\) and the inset in Fig. \[3\)] and the second, arising due to the nonlocal and the nonlinear response of a superconductor with nodes in the gap to the applied magnetic field \([\text{Eq. (2)}\) and Fig. \[4\)] are not really important at this relatively low field. Consequently, the second moment of \(\mu_{0}\)SR line \(\sigma_{sc}\) measured at \(\mu_{0}H = 0.04\) T is still a good measure of the zero-field superfluid density \(\rho_{s} \propto \lambda_{0}^{-2}\). On the other hand, \(\sigma_{sc}(T)\) and \(\lambda_{eff}^{-2}(T)\) at \(\mu_{0}H = 0.64\) T differ substantially from each other and from the resulting \(\lambda_{0}^{-2}(T)\). This implies that for high fields both above mentioned corrections have to be taken into account.

To conclude, muon-spin rotation measurements were performed on the optimally doped cuprate superconductor (BiPb)\(_{2}(\text{SrLa})\(_{3}\)CuO\(_{6+\delta}\). It was demonstrated that in unconventional \(d^{-}\)-wave superconductors (like OP Bi2201) \(\mu\)SR experiments taken in various magnetic fields allow a reliable evaluation of the zero-field superfluid density \(\rho_{s} \propto \lambda_{0}^{-2}\). The deviation of the effective penetration depth \(\lambda_{eff}\) from \(\lambda_{0}\) observed for higher fields was explained by the nonlinear and the nonlocal response of the superconductor with nodes in the energy gap to the applied magnetic field. The dependence of \(\lambda_{eff}/\lambda_{0}\) on the reduced magnetic field \(b = B/B_{c2}\) follows a \((1 - K \sqrt{b})^{-1/2}\) behavior, accounting for the nonlinear correction to \(\lambda_{0}\) for a \(d^{-}\)-wave superconductor.

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