Cosmological limit on the neutrino mass

Steen Hannestrand
NORDITA, Blegdamsvej 17, DK-2100 Copenhagen, Denmark
(Dated: March 4, 2022)

We have performed a careful analysis of constraints on the neutrino mass from current cosmological data. Combining data from the cosmic microwave background and the 2dF galaxy survey yields an upper limit on the sum of the three neutrino mass eigenstates of $\sum m_\nu \leq 3$ eV (95\% conf.), without including additional priors. Including data from SNIa observations, Big Bang nucleosynthesis, and HST Hubble key project data on $H_0$ tightens the limit to $\sum m_\nu \leq 2.5$ eV (95\% conf.). We also perform a Fisher matrix analysis which illustrates the cosmological parameter degeneracies affecting the determination of $\sum m_\nu$.

PACS numbers: 14.60.Pq,95.35.+d,98.80.-k

I. INTRODUCTION

The absolute value of neutrino masses are very difficult to measure experimentally. On the other hand, mass differences between neutrino mass eigenstates, $(m_1, m_2, m_3)$, can be measured in neutrino oscillation experiments. Observations of atmospheric neutrinos suggest a squared mass difference of $\delta m^2 \sim 3 \times 10^{-3}$ eV$^2$ [1,2]. While there are still several viable solutions to the solar neutrino problem the so-called large mixing angle solution gives by far the best fit with $\delta m^2 \sim 5 \times 10^{-5}$ eV$^2$ [3,4].

In the simplest case where neutrino masses are hierarchical these results suggest that $m_1 \sim 0$, $m_2 \sim \delta m_{\text{solar}}$, and $m_3 \sim \delta m_{\text{atmospheric}}$. If the hierarchy is inverted one instead finds $m_3 \sim 0$, $m_2 \sim \delta m_{\text{atmospheric}}$, and $m_1 \sim \delta m_{\text{atmospheric}}$. However, it is also possible that neutrino masses are degenerate [11,12,13,14,15,16,17,18,19,20,21], $m_1 \sim m_2 \sim m_3 \gg \delta m_{\text{atmospheric}}$, in which case oscillation experiments are not useful for determining the absolute mass scale.

Experiments which rely on kinematical effects of the neutrino mass offer the strongest probe of this overall mass scale. Tritium decay measurements have been able to put an upper limit on the electron neutrino mass of 2.2 eV (95\% conf.) [22]. However, cosmology at present yields an even stronger limit which is also based on the kinematics of neutrino mass. Neutrinos decouple at a temperature of 1-2 MeV in the early universe, shortly before electron-positron annihilation. Therefore their temperature of 1-2 MeV in the early universe, shortly before electron-positron annihilation. Therefore their temperature of 1-2 MeV in the early universe, shortly be-

$\Omega_\nu h^2 = \frac{\sum m_\nu}{92.5 \text{ eV}}, \tag{2}$

calculated for a present day photon temperature $T_0 = 2.728$K. Here, $\sum m_\nu = m_1 + m_2 + m_3$. However, because they are so light these neutrinos free stream on a scale of roughly $k \sim 0.03 m_\nu \Omega_m^{1/2} h$ Mpc$^{-1}$ [26,27,28]. Below this scale neutrino perturbations are completely erased and therefore the matter power spectrum is suppressed, roughly by $\Delta P/P \sim -8 \Omega_\nu/\Omega_m$ [29].

This power spectrum suppression allows for a determination of the neutrino mass from measurements of the matter power spectrum on large scales. This matter spectrum is related to the galaxy correlation spectrum measured in large scale structure (LSS) surveys via the bias parameter, $b^2 \equiv P_g(k)/P_m(k)$. Such analyses have been performed several times before most recently using data from the 2dF galaxy survey [31]. This investigation finds an upper limit of 1.8-2.2 eV for the sum of neutrino masses. However, this result is based on a relatively limited cosmological parameter space.

It should also be noted that, although massive neutrinos have little impact on the cosmic microwave background (CMB) power spectrum, it is still necessary to include CMB data in any analysis in order to determine other cosmological parameters.

In the present paper we perform an extensive analysis, carefully discussing the issue of parameter degeneracies. The next section is devoted to a Fisher matrix analysis of the problem which establishes possible parameter degeneracies and yields a general idea of the precision with which the neutrino mass can be measured. Section III describes a full numerical likelihood analysis of data from CMB and LSS which yields a robust limit on the neutrino mass. Finally, section IV contains a discussion and conclusion.
II. FISHER MATRIX ANALYSIS

Measuring neutrino masses from cosmological data is quite involved since for both CMB and LSS the power spectra depend on a plethora of different parameters in addition to the neutrino mass. Furthermore, since the CMB and matter power spectra depend on many different parameters one might worry that an analysis which is too restricted in parameter space could give spuriously strong limits on a given parameter. Therefore, it is desirable to study possible parameter degeneracies in a simple way before embarking on a full numerical likelihood analysis.

It is possible to estimate the precision with which the cosmological model parameters can be extracted from a given hypothetical data set. The starting point for any parameter extraction is the vector of data points, $x$. This can be in the form of the raw data, or in compressed form, typically the power spectrum ($C_l$ for CMB and $P(k)$ for LSS).

Each data point has contributions from both signal and noise, $x = x_{\text{signal}} + x_{\text{noise}}$. If both signal and noise are Gaussian distributed it is possible to build a likelihood function from the measured data which has the following form [32]

$$\mathcal{L}(\Theta) \propto \exp \left(-\frac{1}{2} x^T [C(\Theta)^{-1}] x \right), \quad (3)$$

where $x$ is the data vector, $C(\Theta)$ is the covariance matrix, and $\mathcal{L}(\Theta)$ is the likelihood function.

The second term in the second derivative can be expected to be very small because $(x_{k,\text{obs}} - x_k)$ is in essence just a random measurement error which should average out.

The Fisher matrix is closely related to the precision with which the parameters, $\theta_i$, can be determined. If all free parameters are to be determined from the data alone without any priors then it follows from the Cramer-Rao inequality [33] that

$$\sigma(\theta_i) = \sqrt{(F^{-1})_{ii}} \quad (8)$$

for an optimal unbiased estimator, such as the maximum likelihood [34].

In order to estimate how degenerate parameter $i$ is with another parameter, $j$, one can calculate how $\sigma(\theta_i)$ changes if parameter $j$ is kept fixed instead of free in the analysis. Starting from the $2 \times 2$ sub-matrix

$$S_{ij} = (F^{-1})_{ij}, \quad (9)$$

one then finds

$$\sigma_j \text{ fixed}(\theta_i) = \sqrt{\frac{1}{(S^{-1})_{ii}}} \quad (10)$$

We therefore define the quantity

$$r_{ij} = \frac{\sigma_j \text{ fixed}(\theta_i)}{\sigma(\theta_i)} \leq 1 \quad (11)$$

as a measure of the degeneracy between parameters $i$ and $j$.

In order to perform an actual calculation we use the most present data from CMB and LSS.

CMB data set — Several data sets of high precision are now publicly available. In addition to the COBE [35] data for small $l$ there are data from BOOMERANG [36].
and therefore neutrino masses will be degenerate with atmospheric neutrino observations. Such an impact on CMB and matter spectra their masses must be much larger than the mass splitting inferred from at

cosmological models. In Section III we find that the best fit for other additional parameters. Deviations from the slow-roll prediction of a simple power-law initial spectrum could be present. However, such effects only appear in cosmological models which are more complicated than the "standard" \( \Lambda \)CDM model. The parameters we use fully describe the features of the simplest working model.

In the end one can check the consistency of the numerical parameter extraction by calculating the \( \chi^2 \) per degree of freedom. In Section III we find that the best fit is consistent with expectations and therefore it is unlikely that there are other parameters significantly affecting the power spectra.

Fig. 1 shows the matrix \( r_{ij} \), calculated for the WTZ + 2dF data set errors, around a reference cosmological model with parameters \( \Omega_m = 0.3, \Omega_\Lambda = 0.7, \Omega_b h^2 = 0.020, H_0 = 70 \text{ km s}^{-1} \text{ Mpc}^{-1}, n_s = 1.0, \text{ and } \tau = 0 \), i.e.

\[
\frac{\Delta P}{P} \simeq -8 \frac{\Omega_\nu}{\Omega_m},
\]

i.e. it is scale independent and therefore indistinguishable from changing the bias, \( b \). This degeneracy can be broken by precision measurements around the free-streaming scale where the break in the power spectrum occurs. A good example of how the mass limit on neutrinos can be tightened if bias is fixed comes from Ref. \[30\]. Here the mass limit comes from comparing the overall normalization of the spectra at COBE scales with those on cluster scales. However, we believe that, at present, keeping bias as a free parameter yields a much more robust constraint. To a much lesser extent the neutrino

**LSS data set** — At present, by far the largest survey available is the 2dF \[41\] of which about 147,000 galaxies have so far been analysed. Tegmark, Hamilton and Xu \[42\] have calculated a power spectrum, \( P(k) \), from this data, which we use in the present work. The 2dF data extends to very small scales where there are large effects of non-linearity. Since we only calculate linear power spectra, we use (in accordance with standard procedure) only data on scales larger than \( k = 0.2 h \text{ Mpc}^{-1} \), where effects of non-linearity should be minimal. Making this cut reduces the number of power spectrum data points to 18.

For calculating the theoretical CMB and matter power spectra we use the publicly available CMBFAST package \[43\]. As the set of cosmological parameters we choose \( \Omega_m \), the matter density, \( \Omega_b = 1 \Omega_m - \Omega_\Lambda - \Omega_\nu \), the curvature parameter, \( \Omega_b \), the baryon density, \( H_0 \), the Hubble parameter, \( n_s \), the scalar spectral index of the primordial fluctuation spectrum, \( \tau \), the optical depth to reionization, \( Q \), the normalization of the CMB power spectrum, \( b \), the bias parameter, and \( \Omega_\nu \), the neutrino density. In all cases we take the number of massive neutrinos to be 3. The reason is that if neutrinos are to have an impact on CMB and matter spectra their masses must be much larger than the mass splitting inferred from atmospheric neutrino observations \( (\delta m \sim 0.05 - 0.1 \text{ eV}) \), and therefore neutrino masses will be degenerate with \( m_1 \sim m_2 \sim m_3 \gg \delta m_{\text{atmospheric}} \).

In principle one might include even more parameters in the analysis, such as \( r \), the tensor to scalar ratio of primordial fluctuations. However, \( r \) is most likely so close to zero that only future high precision experiments may be able to measure it. The same is true for other additional parameters. Deviations from the slow-roll prediction of a simple power-law initial spectrum \[44, 45, 46, 47\] or additional relativistic energy density \[48, 49, 50, 51, 52, 53, 54, 55, 56\] could be present. However, such effects only appear in cosmological models which are more complicated than the "standard" \( \Lambda \)CDM model. The parameters we use fully describe the features of the simplest working model.

In the end one can check the consistency of the numerical parameter extraction by calculating the \( \chi^2 \) per degree of freedom. In Section III we find that the best fit is consistent with expectations and therefore it is unlikely that there are other parameters significantly affecting the power spectra.

**FIG. 1:** Values of the parameter \( r_{ij} \), defined in Eq. (11).
mass is also degenerate with the Hubble parameter.

It should be noted that there is little degeneracy with \( n_s \), the spectral index. In Refs. \[26, 31\] a significant degeneracy between \( \Omega_\nu \) and \( n_s \) was found when only LSS data is considered. However, this is broken when CMB data is included (as is also noted in Ref. \[26\]), the reason being that changing \( n_s \) affects both CMB and matter power spectra, not just the matter spectrum.

Clearly, it would be desirable to fix the parameters with which the neutrino mass is most degenerate, \( \Omega_m \), \( b \), and \( H_0 \). As for \( \Omega_m \) one can use the SNIa result \( \Omega_m = 0.28 \pm 0.14 \) which applies to a flat universe \[25\]. However, this value is not much more restrictive than what is found from the CMB+LSS data alone. Fixing the bias is much more difficult since it is not a physically well understood parameter. In Elgaroy et al. \[31\] bias was kept as a free parameter, and we follow this line. \( H_0 \) has been determined precisely by the HST Hubble key project to be \( H_0 = 70 \pm 7 \, \text{km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) \[59\].

If all the above parameters are included in the Fisher matrix analysis the estimated 1\( \sigma \) precision on \( m_\nu \) is 1.8 eV, equivalent to a 95% confidence limit of 3.6 eV.

### III. NUMERICAL RESULTS

The Fisher matrix analysis can only give a general idea of the constraints which can be found from a given data set. In reality the likelihood is non-Gaussian and away from the best fit point the formalism breaks down. In order to get reliable estimates it is necessary to perform a full numerical likelihood analysis over the space of cosmological parameters.

In this full numerical likelihood analysis we use a slightly restricted parameter space with the following free parameters: \( \Omega_m \), \( \Omega_b \), \( H_0 \), \( n_s \), \( Q \), \( b \), and \( \tau \). We restrict the analysis to flat models, \( \Omega_k = 0 \). This has very little effect on the analysis because there is little degeneracy between \( m_\nu \) and \( \Omega_k \). In order to study the effect of the different priors we calculate three different cases, the priors for which can be seen in Table I. The BBN prior on \( \Omega_b h^2 \) comes from Ref. \[10\]. The actual marginalization over parameters other than \( \Omega_b h^2 \) was performed using a simulated annealing procedure \[11\].

Fig. 2 shows \( \chi^2 \) for the three different cases as a function of the \( m_\nu \). The best fit \( \chi^2 \) values are 24.81, 25.66, and 25.71 for the three different priors respectively. In comparison the number of degrees of freedom are 34, 35, and 36, meaning that the fits are compatible with expectations, roughly within the 68% confidence interval.

We identify the 95% confidence limit on \( m_\nu \) with the point where \( \Delta \chi^2 = 4 \). These limits are shown in Table II. For the most restrictive prior we find a 95% confidence upper limit of \( \sum m_\nu \leq 2.47 \text{ eV} \). This is compatible with the findings of Ref. \[31\] who find that \( \sum m_\nu \leq 1.8 - 2.2 \text{ eV} \) for a slightly more restrictive parameter space.

Based on the present analysis we consider \( \sum m_\nu \leq 3 \text{ eV} \) (95% conf.) a robust upper limit on the sum of the neutrino masses. This corresponds roughly to the value found for the CMB+LSS data alone without any additional priors. Even though this value is significantly higher than what is quoted in Ref. \[31\], it is still much more restrictive than the value \( \sum m_\nu \leq 4.4 \text{ eV} \) \[39\] found from CMB and PSCz data. As is also discussed in Ref. \[31\] the main reason for the improvement is the much greater precision of the 2dF survey, compared to the PSCz data \[62\].

![Fig. 2: \( \chi^2 \) as a function of \( \Omega_b h^2 \), plotted for the three different priors. The dotted curve is for CMB+LSS, the dashed for CMB+LSS+BBN+\( H_0 \), and the full curve for CMB+LSS+BBN+\( H_0 \)+SNIa.](image_url)
TABLE II: Best fit $\chi^2$ and upper limits on $\sum m_{\nu,\text{max}}$ for the three different priors.

| prior type                        | best fit $\chi^2$ | $\sum m_{\nu,\text{max}}$ (eV) (95% conf.) |
|----------------------------------|-------------------|--------------------------------------------|
| CMB + LSS                        | 24.81             | 2.96                                       |
| CMB + LSS + BBN + $H_0$          | 25.66             | 2.65                                       |
| CMB + LSS + BBN + $H_0$ + SNIa   | 25.71             | 2.47                                       |

IV. DISCUSSION

We have studied cosmological constraints on the neutrino masses from present CMB and LSS data. Initially a Fisher matrix analysis was performed which illustrates the main degeneracies of $m_\nu$ with other cosmological parameters used in the analysis. From this simplified analysis it was estimated that the precision on $\sum m_{\nu}$ should be roughly 3.6 eV at 95% confidence from CMB+LSS data alone.

However, in order to obtain reliable estimates we performed a full numerical likelihood analysis. Using reasonable priors on $\Omega_m$, $\Omega_b h^2$ and $H_0$ we obtained a limit of $\sum m_{\nu} \leq 2.47$ eV at 95% confidence, while using no priors on the CMB+LSS data yielded $\sum m_{\nu} \leq 3$ eV, again at 95% confidence. We believe this to be a robust upper limit.

Our analysis shows, not surprisingly, that priors are extremely important for parameter estimation of $m_{\nu}$. Our most restrictive prior yields a result similar to that found by Ref. [31], while our no-prior case yields a significantly looser constraint.

The Fisher matrix analysis showed that the parameter most degenerate with $m_{\nu}$ is the bias parameter, $b$. In order to obtain much stronger limits one must either determine $b$ precisely in an independent way or obtain better LSS power spectrum statistics around the scale corresponding to the free-streaming scale for neutrinos, $k \approx 0.02 \sim 0.03 \, h \, \text{Mpc}^{-1}$. The Sloan Digital Sky Survey [32] will measure the power spectrum shape with higher precision on the relevant scale, and this data, combined with CMB data from the MAP experiment [33], will either push the limit by a factor of at least a few or indeed detect a non-zero neutrino mass directly. It was estimated in Ref. [29] that $\sum m_{\nu} \lesssim 0.65$ eV can be reached.

Finally, we note that the present cosmological limit is significantly stronger than current laboratory limits. The Mainz tritium experiment [22] currently quotes a 95% upper limit to the $\nu_e$ mass of 2.2 eV, which translates to a sum of roughly 6.5-7 eV for the three mass eigenstates. As is also noted in Elgaroy et al. [31], the cosmological limit is compatible with the very controversial detection of neutrinoless double beta decay by the Heidelberg-Moscow experiment [15]. If this finding is confirmed it would imply a sum of masses of order 1 eV, within range of the MAP+SDSS data.

[1] N. Fornengo, M. C. Gonzalez-Garcia and J. W. Valle, Nucl. Phys. B 580 (2000) 58 [arXiv:hep-ph/0002147].
[2] S. Fukuda et al. [Super-Kamiokande Collaboration], Phys. Rev. Lett. 85, 3999 (2000) [arXiv:hep-ex/0009001].
[3] J. N. Bahcall, M. C. Gonzalez-Garcia and C. Pena-Garay, arXiv:hep-ph/0204314.
[4] See the SNO homepage http://sno.phy.queensu.ca.
[5] V. A. Kostelecky and S. Samuel, Phys. Lett. B 318, 127 (1993).
[6] G. M. Fuller, J. R. Primack and Y. Z. Qian, Phys. Rev. D 52, 1288 (1995) [arXiv:astro-ph/9502089].
[7] D. O. Caldwell and R. N. Mohapatra, Phys. Lett. B 354, 371 (1995) [arXiv:hep-ph/9503516].
[8] S. M. Bilenky, C. Giunti, C. W. Kim and S. T. Petcov, Phys. Rev. D 54, 4432 (1996) [arXiv:hep-ph/9604336].
[9] S. F. King and N. N. Singh, Nucl. Phys. B 596, 81 (2001) [arXiv:hep-ph/0007243].
[10] H. J. He, D. A. Dicus and J. N. Ng, arXiv:hep-ph/0203233.
[11] A. Ioannisian and J. W. Valle, Phys. Lett. B 332, 93 (1994) [arXiv:hep-ph/9402333].
[12] P. Bannert and C. P. Burgess, Phys. Lett. B 329, 289 (1994) [arXiv:hep-ph/9402229].
[13] R. N. Mohapatra and S. Nussinov, Phys. Lett. B 346, 75 (1995) [arXiv:hep-ph/9411274].
[14] H. Minakata and O. Yasuda, Phys. Rev. D 56, 1692 (1997) [arXiv:hep-ph/9609270].
[15] F. Vissani, arXiv:hep-ph/9708483.
[16] H. Minakata and O. Yasuda, Nucl. Phys. B 523, 597 (1998) [arXiv:hep-ph/9712291].
[17] J. R. Ellis and S. Lola, Phys. Lett. B 458, 310 (1999) [arXiv:hep-ph/9904278].
[18] J. A. Casas, J. R. Espinosa, A. Ibarra and L. Navarro, Nucl. Phys. B 556, 3 (1999) [arXiv:hep-ph/9904392].
[19] J. A. Casas, J. R. Espinosa, A. Ibarra and L. Navarro, Nucl. Phys. B 569, 82 (2000) [arXiv:hep-ph/9905381].
[20] E. Ma, J. Phys. G 25, L97 (1999) [arXiv:hep-ph/9907404].
[21] R. Adhikari, E. Ma and G. Rajasekaran, Phys. Lett. B 486, 134 (2000) [arXiv:hep-ph/0004193].
[22] J. Bonn et al., Nucl. Phys. Proc. Suppl. 91, 273 (2001).
[23] S. Hannestad and J. Madsen, Phys. Rev. D 52, 1764 (1995) [arXiv:astro-ph/9506015].
[24] A. D. Dolgov, S. H. Hansen and D. V. Semikoz, Nucl. Phys. B 503, 426 (1997) [arXiv:hep-ph/9703317].
[25] G. Mangano, G. Miele, S. Pastor and M. Peloso, arXiv:astro-ph/0111405.
[26] W. Hu, D. J. Eisenstein and M. Tegmark, Phys. Rev.
