Interacting Logarithmic Entropy-Corrected Agegraphic Chameleon-Tachyon Dark Energy

A. Ravanpak* and G. F. Fadakar
Department of Physics, Vali-e-Asr University, Rafsanjan, Iran

Abstract
Considering entropy correction terms resulting from loop quantum gravity, we investigate an interacting agegraphic dark energy model. The connection between such a model and a tachyon scalar field which couples to matter in Lagrangian is studied. We reconstruct the related potential of tachyon field and find its dynamics in the Universe. Also, we discuss the implications of this entropy-corrected agegraphic dark energy model.

Keywords: Dark energy; Chameleon; Tachyon; Agegraphic; Entropy corrected

Introduction
Recent cosmological observations demonstrate that our Universe is in an accelerated expansion phase [1-6]. We ascribe this acceleration to a mysterious matter with negative pressure called dark energy (DE). Different DE models have been proposed in literature [7-17]. Among them the holographic DE (HDE) and agegraphic DE (ADE) models are of particular interest because they arise from a more fundamental theory, dubbed quantum gravity. Choosing an appropriate length scale is one of the most important features of these models. From this point of view, ADE is preferable, because the HDE suffers from the causality problem that in turn arise from choosing event horizon as the length scale [18-19]. The energy density of ADE models is derived from the energy density of metric fluctuations of Minkowski spacetime as [20-22]

\[ \rho_D = 3n^3M_p^2t^{-2} \]  

in which \( M_p \) is the reduced Planck mass and the numerical factor \( 3n^3 \), is introduced to parameterize some uncertainties. In ADE models we have two options. One can choose the age of the Universe as the length scale in a model called original ADE (OADE) [23]. Also, in a new ADE (NADE) model, the conformal age instead of the cosmic age can be chosen [16]. Therefore there is not any causality problem here.

A key point in deriving the energy density of HDE and ADE, is the relation between entropy and area of black holes in Einstein gravity [14]. This relationship can be modified due to some quantum considerations in standard general relativity. There are two modifications, the power-law entropy corrections [24-25] and the logarithmic entropy corrections [26-27]. In the logarithmic entropy corrected ADE (LECADE) model, the energy density in Eq. (1), is modified as

\[ \rho_D = 3n^3M_p^2t^{-2} + \alpha t^{-4} \ln \left( M_p^4 t^2 \right) + \beta t^{-4} \]  

in which \( \alpha \) and \( \beta \) are dimensionless constants of order unity. It is obvious that for large \( t \), the correction terms can be negligible and the LECADE model reduces to the ordinary ADE model.
On the other hand, interaction between the dark sectors of the Universe is important, because it can resolve or at least alleviate a few cosmological problems such as coincidence problem [28-33]. So, the interaction between ADE and dark matter (DM), has been investigated in recent past [34-41].

Also, much attempts have been done to reconstruct scalar field DE models, from HDE and ADE [42-46]. In some of them the coupling between the scalar field and DM Lagrangian, called chameleon, has been considered [47-49]. Tachyon scalar field motivated from string theory is a especial scalar field which can explain the whole history of the Universe from inflation to late time acceleration [50-59]. The tachyon field reconstruction of HDE and ADE models has been studied separately in some works [60-64].

In this manuscript, an interacting ADE model with the logarithmic correction in the entropy-area relation is considered. In this context a chameleon-tachyon cosmology, i.e., a tachyon scalar field coupled to matter is reconstructed. This work is a complementary of the articles [39, 61-64].

Chameleon-Tachyon Reconstruction of Original Lecade (Olecade)

Assuming the energy density of the OLECADE of the form Eq. (2), and choosing the age of the Universe T, as the time scale, we have

$$\rho_D = 3n^2M_p^2T^2 + aT^{-4}\ln\left(M_p^2T^2\right) + \beta T^{-4}$$

(3)

where T is defined as

$$T = \frac{a}{Ha}$$

(4)

Using Eq.(3) and defining \(\Omega_D = \rho_D/(3M_p^2H^2)\), we reach to

$$\Omega_D = \frac{n^2}{T^2H^2} + \frac{\alpha}{3M_p^2H^2T^2}\ln\left(M_p^2T^2\right) + \frac{\beta}{3M_p^2H^2T^4}$$

(5)

Now, we consider the chameleon cosmology with a tachyon potential in the action given by

$$S = \int\left[\frac{M_p^2R}{2} - V(\phi)\sqrt{-g}\bar{\partial}\phi\partial\phi + f(\phi)L_m\right]
\sqrt{-g}d^4x$$

(6)

where R is Ricci scalar and V(\phi) is the tachyonic potential. The matter Lagrangian \(L_m\) is modified as \(f(\phi)L_m\), which shows the interaction between the matter content of the Universe and tachy on field.

We consider the spatially flat Friedmann-Robertson-Walker (FRW) Universe with the line element

$$ds^2 = dr^2 - a^2(\tau)(dr^2 + r^2d\Omega^2)$$

(7)

With the variational approach one can reach to the following field equations

$$3H^2M_p^2 = \rho_m f + \rho_{\text{vac}}$$

(8)

$$M_p^2(2H + 3H^2) = -p_m - p_{\text{vac}}$$

(9)

where \(H = \dot{a}/a\) is the Hubble parameter and dot means derivative with respect to time t. Also, we have assumed that a perfect fluid with the equation of state \(p_m = \frac{\rho_m}{3}\) has filled the Universe. \(\rho_m\) and \(p_m\) are the energy density and pressure of the tachyon field, respectively

$$\rho_m = \frac{V(\phi)}{\sqrt{1 - \phi^2}}, \quad p_m = -V(\phi)\sqrt{1 - \phi^2}$$

(10)

Then, we define the fractional energy densities as

$$\Omega_ch = \frac{\rho_m}{3M_p^2H^2}, \quad \Omega_{\text{vac}} = \frac{\rho_{\text{vac}}}{3M_p^2H^2}$$

(11)

where the subscript ch stands for chameleon and \(\Omega_ch = \Omega_m\). So, the Friedmann equation (8), can be written as

$$\Omega_ch + \Omega_{\text{vac}} = 1$$

(12)

In all interacting models the densities of dark matter (DM) and DE, violate the independent conservation law. The form of interaction term is an assumption and is not unique, but usually it expresses as a combination of the DM and DE densities and simultaneously is linear in terms of the Hubble parameter. Here, in one hand, we are assuming that the tachyon field plays the role of DE, as an ADE and on the other hand plays the role of DM, as a chameleon field coupled to matter. Assuming the Universe filled with cold DM (CDM), i.e. \(\rho_m = 0\), the dark sectors in our model satisfy

$$\dot{\rho}_ch + 3H\rho_ch = Q$$

(13)

and

$$\dot{\rho}_{\text{vac}} + 3H(1 + w_{\text{vac}})\rho_{\text{vac}} = -Q$$

(14)

where \(\rho_ch = \rho_m f\). The very interesting feature of our model is that the interaction term Q appears naturally and without any assumption. With some calculation one can find that \(Q = \varepsilon\rho_m f\), with \(\varepsilon = 1 - 3\gamma\), which is directly dependent to the chameleon coupling function \(f(\phi)\) and \(\rho_m\) DM density and indirectly to the Hubble parameter.

Now, we differentiate Eq.(3) with respect to time and obtain

$$\dot{\rho}_D = -2H\left\{3n^2M_p^2T^2 + 2aT^{-4}\ln(M_p^2T^2) + (2\beta - \alpha)T^{-4}\right\}\sqrt{3M_p^2\Omega_D}$$

(15)

Replacing \(\rho_{\text{vac}}\) in Eq.(14), with the expression above we obtain the equation of state parameter of the DE component as

$$w_{\text{vac}} = -1 - \frac{2}{3}\left[3n^2M_p^2 + 2aT^{-4}\ln(M_p^2T^2) + (2\beta - \alpha)T^{-4}\right]\sqrt{\frac{3M_p^2\Omega_D}{5}} - \frac{Q}{3H\rho_o}$$

(16)

in which we have used Eq.(5). On the other hand, we take the time derivative of Eq.(5) and using \(\Omega_o = \Omega_o H\) we reach to

$$\Omega_o = -2\frac{H}{T^2}\left[\frac{n^2}{3M_p^2} + \frac{\alpha}{3M_p^2T^2}\ln(M_p^2T^2) + \frac{\beta}{3M_p^2T^4}\right] + \frac{2}{3}\left[\frac{n^2}{3M_p^2} + \frac{\alpha}{3M_p^2T^2}\ln(M_p^2T^2) + \frac{\beta}{3M_p^2T^4}\right]$$

(17)

where the prime denotes differentiation with respect to the e-folding parameter, \(n\). Differentiating Eq.(8) with respect to time and using Eqs. (11), (12), (13) and (15) we obtain

$$\frac{\dot{H}}{H^2} = \frac{\sqrt{\Omega_o}}{3M_p^2H^2}\left[3n^2M_p^2T^2 + 2aT^{-4}\ln(M_p^2T^2) + (2\beta - \alpha)T^{-4}\right] + \frac{3(1 - \Omega_o)}{6M_p^2H^2} - Q$$

(18)
Substituting the expression above in Eq.(17) yields to

\[
\Omega_b = \frac{2}{3} \left( n_0 a^3 + \frac{\alpha}{3M_{TH}^4} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \frac{\beta}{3M_{TH}^4} \right) \left( \frac{\sqrt{3}}{3} \right) + \frac{Q}{6M_{TH}^4} 
+ \frac{2}{3} \left( n_0 a^3 + \alpha \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right) \right) \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

We complete our chameleon-tachyon reconstruction of OLECade model by rewriting the potential and kinetic term of tachyon scalar field as

\[
V(\phi) = \left[ 3n_0^2 M_{TH}^4 \frac{\alpha}{2} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right] + \beta \n
\]

and

\[
\phi = \frac{1}{H} \left[ \frac{2}{3} \left( n_0 a^3 + \alpha \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right) \right] \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

where we have used Eq.(16) and the relation $w_{sc}=\phi^2-1$. With attention to $\phi=\phi H$, we find

\[
\phi = \frac{1}{H} \left[ \frac{2}{3} \left( n_0 a^3 + \alpha \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right) \right] \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

Integrating the above equation results

\[
\phi(\alpha) - \phi(0) = \frac{1}{aH} \left[ \frac{2}{3} \left( n_0 a^3 + \alpha \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right) \right] \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

Here $a_0$ is the present value of the scale factor. Thus, we have organized an interacting logarithmic entropy corrected agegraphic chameleon-tachyon dark energy model and reconstructed the tachyon potential and obtained its dynamics. It is interesting to note that in the case $\alpha = \beta = 0$, our results reduce to the corresponding ones in [64].

Chameleon-Tachyon Reconstruction of New Lecade (Nlecade)

In NLECade, the length scale is assumed to be the conformal time $n$, which is introduced as

\[
\eta = \frac{1}{H a^2} \left[ \frac{2}{3} n_0^2 M_{TH}^4 \frac{\alpha}{2} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right] \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

So, Eq.(2) is written as

\[
\rho_c = 3n_0^2 M_{TH}^4 \frac{\eta}{2} + \alpha \eta^3 \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \eta^4 
\]

Also, we can obtain the corresponding dimensionless density parameter as

\[
\Omega_L = \frac{n_0^2}{\eta H^2} + \frac{\alpha}{3M_{TH}^4} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \frac{\beta}{3M_{TH}^4} \eta^4 
\]

Differentiating Eq.(25) with respect to the cosmic time $t$, one can reach

\[
\rho_c = \frac{2M_{TH}^4}{a} \left( n_0^2 M_{TH}^4 \frac{\eta}{2} + \alpha \eta^3 \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \eta^4 \right) \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

Inserting Eq.(27) into Eq.(14), we get

\[
w_0 = -\frac{1}{3} \left( \frac{3n_0^2 M_{TH}^4 \frac{\alpha}{2} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta}{3M_{TH}^4} \right) \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

On the other hand, differentiating Eq.(26) with respect to $t$, we obtain

\[
\Omega_L = \left( \frac{n_0^2}{\eta H^2} + \frac{\alpha}{3M_{TH}^4} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \eta^4 \right) \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

where we have used $\phi_0=\phi H$. Also, in a similar way to the OLECade and using Eq.(27), one can get

\[
\frac{H(t)}{H^2} = \frac{1}{3} \left( \frac{3n_0^2 M_{TH}^4 \frac{\alpha}{2} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta}{3M_{TH}^4} \right) \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

Substituting the latter in Eq.(29) and using Eq.(26), we reach to

\[
\Omega_L = \left( \frac{n_0^2}{\eta H^2} + \frac{\alpha}{3M_{TH}^4} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \eta^4 \right) \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

The reconstruction will be completed if we rewrite tachyonic potential and its kinetic term as

\[
V(\phi) = \left[ 3n_0^2 M_{TH}^4 \frac{\alpha}{2} \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right] + \beta \n
\]

and

\[
\phi = \frac{1}{aH} \left[ \frac{2}{3} \left( n_0 a^3 + \alpha \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right) \right] \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

In a similar approach to the prior section we find that

\[
\phi(\alpha) - \phi(0) = \frac{1}{aH} \left[ \frac{2}{3} \left( n_0 a^3 + \alpha \ln \left( \frac{M_{TH}^4}{n_0 a^4} \right) + \beta \right) \right] \left( \frac{3M_{TH}^4}{\sqrt{3}} \right) + \frac{Q}{6M_{TH}^4} 
\]

Again, it is easy to check that in the case $\alpha = \beta = 0$, all the above relations reduce to those discussed in [64].

Cosmological implications of Lecade Chameleon-Tachyon Models

To deal with the cosmological implications we calculate two important cosmological parameters in our model. The deceleration parameter is defined as

\[
q = -\frac{\ddot{a}}{aH^2} = -1 - \frac{\dot{H}}{H^2} 
\]
Substituting Eqs. (18) and (30) in the above relation, we obtain

\[ q_{,\omega} = -1 + \frac{3}{2} \left( \frac{\Omega_{\text{DM}}}{M^3 H^2} \right) \left( \frac{3}{2} \left( \frac{M^3 H^2}{\Omega_{\text{DM}}} \right)^{1/2} \right) \]

for OLECADE and

\[ q_{,\omega} = -1 + \frac{3}{2} \left( \frac{\Omega_{\text{DM}}}{M^3 H^2} \right) \left( \frac{3}{2} \left( \frac{M^3 H^2}{\Omega_{\text{DM}}} \right)^{1/2} \right) \]

for NLECADE model.

On the other hand, the effective state of parameter can be calculated as follow

\[ w^{\text{eff}} = w_D + \frac{Q}{3H \rho_D} = w_D + \frac{\delta \Omega_m}{3H \Omega_D} \]

where using Eqs. (16) and (28) we obtain

\[ w_{\omega}^{\text{new}} = -1 + \frac{3}{2} \left( \frac{3}{2} \left( \frac{M^3 H^2}{\Omega_{\text{DM}}} \right)^{1/2} \right) \frac{3}{2} \left( \frac{M^3 H^2}{\Omega_{\text{DM}}} \right)^{1/2} \]

and

\[ w_{\omega}^{\text{new}} = -1 + \frac{3}{2} \left( \frac{3}{2} \left( \frac{M^3 H^2}{\Omega_{\text{DM}}} \right)^{1/2} \right) \frac{3}{2} \left( \frac{M^3 H^2}{\Omega_{\text{DM}}} \right)^{1/2} \]

Summary and Discussion

Here, we investigated a logarithmic entropy corrected model of OADE and NADE and studied the correspondence between them and a chameleon-tachyon DE model, i.e., a tachyon scalar field which is coupled to matter Lagrangian. We reconstructed the tachyonic potential in each case and obtained its evolution with respect to time and scale factor parameter. On the other hand, we calculated the deceleration parameter and the effective equation of state parameter of both models to deal with the cosmological implications of our model.

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