The Minimal Supersymmetric Standard Model (MSSM) with $R$-Parity Violation

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Abstract

In this lectures, we give a review about the Minimal Supersymmetric Standard Model (MSSM) with $R$-Parity Violation because it provides an attractive way to generate neutrino masses, lepton mixing angles in accordandce to present neutrino data.

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1 Introduction

Although the Standard Model (SM) describes the observed properties of charged leptons and quarks it is not the ultimate theory. However, the necessity to go beyond it, from the experimental point of view, comes at the moment only from neutrino data [1].

On September of 2018 from 5-7, happpend in Paris, France, the Interntional Conference “History of the Neutrino (1930-2018)”, where we get the following intersting presentations:

- Birth of the neutrino, from Pauli to the Reines-Cowan experiment [3];
- Prehistory of neutrino oscillation [4];
- The Nature of the Neutrino (Dirac/Majorana) and Double Beta Decay with or without Neutrinos [5];
- The Mikheyev-Smirnov-Wolfenstein (MSW) Effect [6];
- Neutrino Mistakes: Wrong tracks and Hints, Hopes and Failures [7];
- Neutrinos and Particle Physics Models [8];
- Neutrino Masses and Mixing: A Little History for a Lot of Fun [9];

and many others interesting talks where it was presented in details that neutrinos are massive particle, they can oscilate and they can be Dirac particles [10] or Majorana fermions [11, 12]. A very nice review about the status of Neutrinos can be found at [13].

Supersymmetry (SUSY) or symmetry between bosons (particles with integer spin) and fermions (particles with half-integer spin) has been introduced in theoretical papers nearly 30 years ago independently by Golfand and Likhtman [14], Volkov and Akulov [15] and Wess and Zumino [16].
The supersymmetry algebra was introduced in \[14\]. There and in the Wess-Zumino article \[16\], the supersymmetry generator \(Q\) relates bosons with fermions in the usual sense. The Volkov-Akulov article, however, deals only with fermions. The supersymmetry generator acts in a non-linear way, turning a fermion field into a composite bosonic one made of two fermion fields \[15\]. This illustrates that the supersymmetric algebra by itself does not require superpartners – in contrast with what is commonly said or thought now. Since that time there appeared thousands of papers. The reason for this remarkable activity is the unique mathematical nature of supersymmetric theories, possible solution of various problems of the SM within its supersymmetric extensions as well as the opening perspective of unification of all interactions in the framework of a single theory \[18, 19, 20, 21, 22\]. About the first version of the Supersymmetric Standard Model see \[17\].

On this review we will present a short introduction for the Minimal Supersymmetric Standard Model with \(R\)-Parity violation. First we present the model at Sec.(2), then we present our results when the left handed stau-neutrino get vev, Sec.(4). After it we present in a short way the same results when all the left-handed sneutrinos get vev. Our conclusion are found in the last section.

2 Review of the MSSM with \(R\)-Parity Interactions.

The Minimal Supersymmetric Standard Model (MSSM), known as MSSM, is a good candidate to be the physics beyond the Standard Model, as presented in several books and review about this subject \[18, 19, 20, 21, 22\]. It is the supersymmetric extension of the SM that contains a minimal number of states and interactions. The model has the gauge symmetry defined as

\[
SU(3)_C \otimes SU(2)_L \otimes U(1)_Y
\]

extended by the supersymmetry to include the supersymmetric partners of the SM fields which have spins that differ by \(+1/2\) as required by the supersymmetric algebra. The MSSM with \(R\)-Parity conservation has 124 free parameters \[19\].

The partner of fermion masses and mixing constitutes one of the most important issues in modern physics, the only new fermion that does not mixing is the gluinos and the only new bosons that does not mix is the sneutrinos.

However in the MSSM there are interactions that volates Lepton or Baryon Number conservation \[23, 24\]. Some years ago it was proposed a model for the structure of the lepton mixing which account for the atmospheric and the solar anomalies. It is based on the simplest one-parameter extension of the MSSM with bi-linear \(R\)-Parity violation \[28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40\].

We will introduce the following chiral superfields

\[
\begin{align*}
\tilde{L}_{iL} & = \begin{pmatrix} \hat{\nu}_{iL} \\ \hat{l}_{iL} \end{pmatrix} \sim (1, 2, -1), & \hat{\nu}_{iR} \sim (1, 1, +2), \\
\hat{Q}_{iL} & = \begin{pmatrix} \hat{u}_{iL} \\ \hat{d}_{iL} \end{pmatrix} \sim \begin{pmatrix} 3, 2, 1 \\ 3, 1, 2 \end{pmatrix}, & \hat{\nu}_{iR} \sim \begin{pmatrix} 3, 1, -4 \\ 3, 1, -2 \end{pmatrix}, & \hat{d}_{iR} \sim \begin{pmatrix} 3, 1, 2 \\ 3, 1, 3 \end{pmatrix}, \\
\hat{H}_1 & = \begin{pmatrix} \hat{H}_1^0 \\ \hat{H}_1^\pm \end{pmatrix} \sim (1, 2, +1), & \hat{H}_2 = \begin{pmatrix} \hat{H}_2^0 \\ \hat{H}_2^- \end{pmatrix} \sim (1, 2, -1). 
\end{align*}
\]
It is possible to rotate the chiral superfields $\hat{H}_{2}, \hat{L}_{3L}$ to a new basis $\hat{H}'_{2}, \hat{L}'_{3L}$ by a linear transformation given by \[18, 19, 39\]
\[
\hat{H}'_{2} = \frac{1}{\mu^2 + \mu_3^2} (\mu \hat{H}_2 - \mu_3 \hat{L}_{3L}),
\]
\[
\hat{L}'_{3L} = \frac{1}{\mu^2 + \mu_3^2} (\mu \hat{H}_2 + \mu_3 \hat{L}_{3L}).
\]
(3)

The non-zero vacuum expectation value of this model are denoted as
\[
\langle H_1 \rangle \equiv \frac{1}{\sqrt{2}} [v_1 + \sigma_1^0 + \nu \varphi_1^0], \quad \langle H_2 \rangle \equiv \frac{1}{\sqrt{2}} [v_2 + \sigma_2^0 + \nu \varphi_2^0],
\]
\[
\langle \tilde{L}_i \rangle \equiv \frac{1}{\sqrt{2}} [v_i^L + \tilde{\nu}_i^R + \nu \tilde{\varphi}_i^I].
\]
(4)

The superpotential is given by
\[
W = W_2 + W_3, \quad (5)
\]
\[
W_2 = \mu \left( \hat{H}_1 \hat{H}_2 \right) - \sum_{i=1}^{3} \mu_i \left( \tilde{L}_i \hat{H}_2 \right),
\]
\[
W_3 = \sum_{i,j,k=1}^{3} \left[ f_{ij}^l (\hat{H}_1 \hat{L}_i) \hat{\tilde{c}}_{jR} + f_{ij}^d (\hat{H}_1 \hat{Q}_i) \hat{d}_{jR} + f_{ij}^u (\hat{H}_2 \hat{Q}_i) \hat{\tilde{c}}_{jR} + \lambda_{ijk} (\tilde{L}_i \tilde{\tilde{L}}_j) \hat{\tilde{c}}_{kR} + \chi'_{ijk} (\tilde{L}_i \hat{Q}_j) \hat{d}_{kR} \right],
\]
(6)

where
\[
\left( \hat{A}\hat{B} \right) \equiv \epsilon_{\alpha\beta} \hat{A}^\alpha \hat{B}^\beta
\]
(7)

and we introduce 12 new parameters in addition to the MSSM with $R$-Parity conservation model.

We can add the following soft supersymmetry breaking terms to the MSSM \[18, 19, 20\]
\[
\mathcal{L}^{\text{MSSM}}_{\text{Soft}} = \mathcal{L}^{\text{MSSM}}_{\text{SMT}} + \mathcal{L}^{\text{MSSM}}_{\text{GMT}} + \mathcal{L}^{\text{MSSM}}_{\text{INT}},
\]
(8)

where the scalar mass term $\mathcal{L}^{\text{SMT}}$ is given by the following relation
\[
\mathcal{L}^{\text{MSSM}}_{\text{SMT}} = - \sum_{i,j=1}^{3} \left[ \tilde{L}_{iL} (M^2_L)_{ij} \tilde{L}_{jL} + \tilde{L}_{iR} (M^2_R)_{ij} \tilde{L}_{jR} + M^2 H^1_1 H_1 + M^2 H^2_2 H_2 \right],
\]
(9)

The $3 \times 3$ matrices $M^2_L$ and $M^2_R$ are hermitian and $M^2_L$ and $M^2_R$ are real. The gaugino mass term is written as
\[
\mathcal{L}^{\text{MSSM}}_{\text{GMT}} = - \frac{1}{2} \left[ \left( M_3 \sum_{a=1}^{8} \chi_{aL}^C \chi_{aC}^l + M \sum_{i=1}^{3} \chi_i^l \chi_i^l + M' \lambda \lambda \right) + h_c \right].
\]
(10)

Here, $M_3, M$ and $M'$ are complex. Finally, there is an interaction term $\mathcal{L}^{\text{INT}}$ of the form
\[
\mathcal{L}^{\text{MSSM}}_{\text{INT}} = - B \mu (H_1 H_2) - \sum_{i=1}^{3} \left\{ B_i \mu_i (H_2 \tilde{L}_i) + \sum_{j,k=1}^{3} \left[ (H_1 \tilde{L}_{iL}) A^E_{ij} \tilde{c}_{jR} - \chi'_{ijk} (\tilde{L}_{iL} \hat{Q}_{jL}) \hat{d}_{kR} \right] \right\} + h_c.
\]
(11)

The parameters $B \mu, B_i \mu_i, A^E_{ij}$ and $\chi'_{ijk}$ are in general complex \[18, 19\].
3 Masses

Here we will present the masses of all particle of this model. The masses of gluinos are the same as presented at MSSM with R-Parity conservation, due this fact we will not present them here, for more detail see .

We first will present the results when $\tilde{L}_3 \neq 0$, at Sec.(4), then the results for the general case, when all the left-handed sneutrinos gain vev at Sec.(5).

4 Results with only $\tilde{L}_3 \neq 0$.

4.1 Bosons Masses

On the other hand, $\mathcal{L}_{Higgs}$ give mass to the gauge bosons, throught the following expression:

$$ (D_m H_1)^\dagger (D_m H_1) + (D_m H_2)^\dagger (D_m H_2) + \sum_{i=1}^{3} \left( D_m \tilde{L}_3 \right)^\dagger \left( D_m \tilde{L}_3 \right), $$

(12)

where $D_m$ is covariant derivates of the SM given by:

$$ D_m H_1 \equiv \partial_m H_1 + igT^i W^i m H_1 + ig' \left( \frac{Y_{H_1}}{2} \right) b Y H_1, $$

$$ D_m H_2 \equiv \partial_m H_2 + igT^i W^i m H_2 + ig' \left( \frac{Y_{H_2}}{2} \right) b Y H_2, $$

$$ D_m \tilde{L}_i \equiv \partial_m \tilde{L}_i + igT^i W^i m \tilde{L}_i + ig' \left( \frac{Y_{\tilde{L}_i}}{2} \right) b Y \tilde{L}_i. $$

(13)

Before calculate expression for the covariant derivatives it is important to remember

$$ \sum_{i=1}^{3} \sigma^{i} W^{i}_m = \begin{pmatrix} W^{3}_m & \sqrt{2} W^{+}_m \\ \sqrt{2} W^{-}_m & -W^{3}_m \end{pmatrix}, \quad \sum_{i=1}^{3} \sigma^{i*} W^{i}_m = \begin{pmatrix} W^{3*}_m & \sqrt{2} W^{-*}_m \\ \sqrt{2} W^{+*}_m & -W^{3*}_m \end{pmatrix}, $$

(14)

where the charged gauge bosons are defined as

$$ W^{\pm}_m = \frac{1}{\sqrt{2}} (V^{1}_m \mp iV^{2}_m). $$

(15)

Using those informations we can get

$$ D_m \langle H_1 \rangle = \frac{\nu_1}{2\sqrt{2}} \begin{pmatrix} g \sqrt{2} W^{*-}_m \\ -g W^{3}_m + g' W^{i}_m \end{pmatrix}, $$

$$ D_m \langle H_2 \rangle = \frac{\nu_2}{2\sqrt{2}} \begin{pmatrix} -g W^{3}_m + g' W^{i}_m \\ -g \sqrt{2} W^{+}_m \end{pmatrix}, $$

$$ D_m \langle \tilde{L}_3 \rangle = \frac{\nu_3}{2\sqrt{2}} \begin{pmatrix} -g W^{3}_m + g' W^{i}_m \\ -g \sqrt{2} W^{+}_m \end{pmatrix}, $$

(16)
from those equations, we can show

\[
(D^m\langle H_1\rangle)^\dagger (D_m\langle H_1\rangle) = \frac{v_1^2}{8} \left[ 2g^2 \bar{W}^+ W_m^+ + g^2 W^3 W_m^3 - gg' W^3 W_m^3 - gg' W'^3 W_m^3 \right. \\
+ \left. (g')^2 W'^3 W_m^3 \right],
\]

\[
(D^m\langle H_2\rangle)^\dagger (D_m\langle H_2\rangle) = \frac{v_2^2}{8} \left[ 2g^2 \bar{W}^- W_m^- + g^2 W^3 W_m^3 - gg' W^3 W_m^3 - gg' W'^3 W_m^3 \right. \\
+ \left. (g')^2 W'^3 W_m^3 \right],
\]

\[
(D^m\langle \tilde{L}_3\rangle)^\dagger (D_m\langle \tilde{L}_3\rangle) = \frac{(v_3^L)^2}{8} \left[ 2g^2 \bar{W}^- W_m^- + g^2 W^3 W_m^3 - gg' W^3 W_m^3 - gg' W'^3 W_m^3 \right. \\
+ \left. (g')^2 W'^3 W_m^3 \right].
\]

(17)

We get the following expression to the charged gauge bosons masses

\[
M^2_W = \frac{g^2}{4} (v_1^2 + v_2^2 + (v_3^L)^2).
\]

(18)

We can define the new angle \( \beta \) in the following way

\[
\tan \beta = \frac{v_2}{v_1},
\]

(19)

and we can define a new angle \( \theta \) in the following way

\[
\begin{align*}
v_1 &= v \cos \beta \sin \theta, \\
v_2 &= v \sin \beta \sin \theta, \\
v_3^L &= v \cos \theta.
\end{align*}
\]

(20)

This new angle \( \theta \) tends to \((\pi/2)\) rad in the limit of the MSSM when \( v_3^L \) go to zero.

The neutral massive gauge boson \((Z^0)\) get the following mass

\[
M^2_Z = \frac{g^2}{4 \cos^2 \theta_W} (v_1^2 + v_2^2 + (v_3^L)^2) = \frac{M^2_W}{\cos^2 \theta_W},
\]

(21)

where \( \theta_W \) is the Weinberg angle and it is defined as

\[
e = g \sin \theta_W = g' \cos \theta_W,
\]

(22)

and get a massless foton \( A_m \). The rotation in this case is

\[
\begin{pmatrix}
A_m \\
Z_m
\end{pmatrix} = \begin{pmatrix}
\sin \theta_W & \cos \theta_W \\
\cos \theta_W & -\sin \theta_W
\end{pmatrix} \begin{pmatrix}
V_3^m \\
V_m
\end{pmatrix},
\]

(23)

it is the exact expression we get in the SM. Therefore the neutral boson gauge sector is exact the same as in the SM.
4.2 Charginos Masses

The supersymmetric partners of the $W^\pm$, together with the usual charged leptons $l_{3L}, l_{3R}$ and the $H^\pm$ mix to mass eigenstates called charginos $\chi_i^\pm$ ($i = 1, 2, 3$) which are four–component Dirac fermions.

From our Lagrangian we get the following terms to get the mass matrix for charginos

$$M (\lambda^-) (\lambda^+) + \frac{g v_1}{\sqrt{2}} (\lambda^+) \tilde{H}^-_1 + \frac{g v_2}{\sqrt{2}} (\lambda^-) \tilde{H}^+_2 + \frac{g v_3^L}{\sqrt{2}} (\lambda^+) l_{3L} + \mu \tilde{H}_1 \tilde{H}^+_2$$

$$- \mu_3 l_{3L} \tilde{H}^+ - \frac{f^r v_1}{\sqrt{2}} l_{3L} l_{3R}^c + \frac{f^r v_3^L}{\sqrt{2}} \tilde{H}^-_1 l_{3R}^c + h\text{c}$$

(24)

where

$$\lambda^\pm = \frac{1}{\sqrt{2}} (\lambda^1 \mp i\lambda^2),$$

(25)

see definition of $W$ boson given at Eq.(15) and $f^r \equiv f^r_{33}$ and all the fermions fields are two-component Weyl-van der Waerden fermions [25, 26, 27].

In order to get the mass matrix for charginos, we start with the basis

$$\psi^- = \left( \lambda^- \tilde{H}^-_1, l_{3L}^- \right)^T, \quad \psi^+ = \left( \lambda^+, \tilde{H}^+_2, l_{3R}^c \right)^T.$$  

(26)

The mass terms of the lagrangian of the charged gaugino–higgsino system can then be written as

$$\mathcal{L}_m = \frac{1}{2} \left( \psi^{+T} Y^\pm \psi^- \right) + h\text{c}$$

(27)

where

$$Y^\pm = \begin{pmatrix} 0 & X^T \\ X & 0 \end{pmatrix},$$

(28)

therefore we can rewrite Eq.(27) in the following way

$$\mathcal{L}_m = \frac{1}{2} \left[ \psi^{+T} X \psi^- + \psi^- T X^T \psi^+ \right] + h\text{c}$$

(29)

taken into account Eq.(24), we get the following expression

$$X = \begin{pmatrix} M & \frac{g v_2}{\sqrt{2}} \\ \frac{g v_1}{\sqrt{2}} & \mu & -\frac{f^r v_3^L}{\sqrt{2}} \\ \frac{g v_2}{\sqrt{2}} & \frac{f^r v_3^L}{\sqrt{2}} & -\mu_3 \end{pmatrix}.$$  

(30)

The $3 \times 3$ matrix $X$ of Eq.(64) can be put into diagonal form in the following way [18, 19, 28, 30]

$$XX^T = X^T X = \begin{pmatrix} M^2 + \frac{g^2 v_2^2}{2} & \frac{g v_2}{\sqrt{2}} (M v_1 + \mu v_2) & \frac{g v_3^L}{\sqrt{2}} (M v_3^L + \mu_3 v_2) \\ \frac{g v_2}{\sqrt{2}} (M v_1 + \mu v_2) & \frac{1}{2} \left( g^2 v_1^2 + 2 \mu^2 + (f^r)^2 (v_3^L)^2 \right) & \frac{g v_3^L}{A} (M v_3^L + \mu_3 v_2) \\ \frac{g v_3^L}{\sqrt{2}} (M v_3^L + \mu_3 v_2) & \frac{g v_3^L}{A} (M v_3^L + \mu_3 v_2) & \frac{1}{2} \left( (f^r)^2 v_1^2 + g^2 (v_3^L)^2 + 2 \mu_3^2 \right) \end{pmatrix},$$

(31)

where

$$A = \frac{1}{2} \left( (f^r)^2 v_1 v_3^L + g^2 v_1 v_3^L - 2 \mu \mu_3 \right).$$

(32)
From Eq.\((31)\) it is easy to get the following analytical results

\[
det(XX^T) = det(X^T X) = \frac{(f_r)^2}{8} \left[ g^2 v_2 \left( v_1^2 - (v_3^L)^2 \right) - 2M (\mu v_1 + \mu_3 v_3^L) \right]^2, \tag{33}
\]

\[
Tr(XX^T) = Tr(X^T X) = \frac{1}{2} \left[ 2(M^2 + \mu^2 + \mu_3^2) + g^2(v_1^2 + v_2^2 + (v_3^L)^2) + (f_r)^2(v_1^2 + (v_3^L)^2) \right]. \tag{34}
\]

Therefore, the smallest eigenvalues of \(XX^T\) and \(X^T X\) with tau mass \(m_\tau\).

Using \(\tan \beta = 1\)\(^3\) and \(M_Z = 91.187\text{ GeV}, s_W^2 = 0.223\)\(^4\), \(\mu_3 = 0.9\text{ GeV}, v_3^L = 0\text{ GeV}\) (this value is consistent with that of Ref. [37, 40]), \(\mu = 150\text{ GeV}, M = 250\text{ GeV}\) we obtain from Eq.\((31)\) the masses 1.777 (in GeV) for the tau, and 124.3 and 275.7 GeV for the charginos.

### 4.3 Neutralinos Masses

In this review we choose the basis [\^?\]

\[
\psi_{\text{MSSMRPV}}^0 = (\ i\lambda' \ i\lambda^3 \ \tilde{H}_1^0 \ \tilde{H}_2^0 \ \nu_{3L})^T. \tag{35}
\]

From our Lagrangian we get the following terms to get the mass matrix for neutralinos

\[
\frac{M}{2} (i\lambda^3) (i\lambda^3) + \frac{M'}{2} (i\lambda') (i\lambda') + \frac{g v_1}{\sqrt{2}} (i\lambda^3) \tilde{H}_1^0 - \frac{g v_2}{\sqrt{2}} (i\lambda^3) \tilde{H}_2^0 + \frac{g v_3^L}{\sqrt{2}} (i\lambda^3) \nu_{3L}
\]

\[
- \frac{g v_1}{\sqrt{2}} (i\lambda') \tilde{H}_1^0 + \frac{g v_2}{\sqrt{2}} (i\lambda') \tilde{H}_2^0 + \frac{g v_3^L}{\sqrt{2}} (i\lambda') \nu_{3L} - \mu \tilde{H}_1^0 \tilde{H}_2^0 + \mu_3 \nu_{3L} \tilde{H}_2^0 + h.c \tag{36}
\]

The mass terms of the neutral gaugino–higgsino system can then be written as

\[
\mathcal{L}_m = \frac{1}{2} (\psi_{\text{MSSMRPV}}^0)^T Y_{\text{neutralino}}^0 \psi_{\text{MSSMRPV}}^0 + h.c \tag{37}
\]

with

\[
Y_{\text{neutralino}}^0 = \begin{pmatrix}
M' & 0 & -\frac{g v_1}{2} & \frac{g v_2}{2} & \frac{g v_3^L}{2} \\
0 & M & \frac{g v_1}{2} & -\frac{g v_2}{2} & \frac{g v_3^L}{2} \\
-\frac{g v_1}{2} & \frac{g v_1}{2} & 0 & -\mu & 0 \\
\frac{g v_2}{2} & -\frac{g v_2}{2} & -\mu & 0 & \mu_3 \\
\frac{g v_3^L}{2} & \frac{g v_3^L}{2} & 0 & \mu_3 & 0
\end{pmatrix}. \tag{38}
\]

We can get numerical results for Eq.\((38)\), using the values presented after Eq.\((34)\) together with \(M' = -200\text{ GeV}\), we obtain a massive neutrino and its mass is \(m_{\nu_3} = 0.051\text{ eV}\) and four heavy neutralinos with masses 269.30, -202.88, -100.55, 84.2 GeV.

This model break lepton number and therefore necessarily generate non-zero Majorana neutrino masses [18, 19, 31, 32]. At tree-level only one of the neutrinos pick up a mass by mixing with

\[^3\text{See Eq.}[19].\]

\[^4\theta_W\text{ is the weak mixing angle and see Eq.}[22].\]

\[^5\text{The atmospheric neutrino mass scaleai is } m^2 = 3 \cdot 10^{-3}\text{eV}^2.\]
Figure 1: One-loop contribution to neutrino masses, coming from the interaction in our Superpotential defined at Eq. (6). This figure was taken from [24].

neutralinos [33, 34], leaving the other two neutrinos massless [35]. The LSP in this case is the neutralino $\tilde{\chi}^0_1$ and some of two body decay are [18, 19, 28, 30]

$$\tilde{\chi}^0_1 \rightarrow \tau^\pm W^\mp, \quad \tilde{\chi}^0_1 \rightarrow \nu_\tau Z,$$

etc.

More realistic neutrino masses require radiative correction and it is shown at Fig. (1) and in this way the solar mass is explained and more details about it can be found at [18, 19, 24, 28, 36, 37, 38, 39, 40].

We can explain the mixing angle as discussed [30].

### 4.4 Higgs masses

The Higgs potential of our model has the following form

$$V_{MSSM+PV}^H = V_F + V_D + V_{soft}. \quad (40)$$

The first term at Eq. (40) is

$$V_F = \sum_i F_i^\dagger F_i, \quad (41)$$

where $l = H_{1,2}, \tilde{L}_3$ and $\tilde{\bar{c}}_3$; the $F$ terms are

$$F_{H_1}^\dagger = -\frac{\mu}{2} H_2 - \frac{\mu_3}{2} \tilde{L}_{3L},$$

$$F_{H_2}^\dagger = -\frac{\mu}{2} H_1 - \frac{f^\tau}{3} \tilde{L}_{3L} \tilde{\bar{c}}_3,$$

$$F_{L_{3L}}^\dagger = -\frac{\mu_3}{2} H_1 - \frac{f^\tau}{3} \tilde{L}_{3L} H_2,$$

$$F_{l_{3R}}^\dagger = -f^\tau \left( H_1 \tilde{L}_3 \right). \quad (42)$$
where we have defined \( f^\tau \) after Eq. (24) and we get

\[
V_F = \frac{\mu^2}{4} \left( |H_1|^2 + |H_2|^2 \right) + \frac{\mu_2^2}{4} \left( |\bar{L}_{3L}|^2 + |H_2|^2 \right) \\
+ \frac{(f^\tau)^2}{9} \left[ \left( |\bar{L}_{3L}|^2 + |H_1|^2 \right) |\bar{L}_{3R}|^2 + |H_1^\dagger \bar{L}_{3L}|^2 - |H_1|^2 |\bar{L}_{3L}|^2 \right] \\
+ \frac{\mu_H \mu_3}{4} \left( H_2^\dagger \bar{L}_{3L} + \bar{L}_{3L}^\dagger H_2 \right) + \frac{\mu_H f^\tau}{6} \left( H_1^\dagger \bar{L}_{3L} |\bar{L}_{3R}| + H_1^\dagger \bar{L}_{3L} |\bar{L}_{3R}^c| \right) \\
+ \frac{\mu_3 f^\tau}{6} \left( H_2^\dagger H_1 |\bar{L}_{3R}^c| + H_1^\dagger H_2 |\bar{L}_{3R}^c| \right),
\]

(43)

where \( |H_1|^2 \equiv H_1^\dagger H_1 \) as usual.

The soft term that contribute to the scalar potential is given by

\[
V_{soft} = -\mathcal{L}_{SMT} - \mathcal{L}_{Int}.
\]

(44)

Therefore

\[
V_{soft} = M_{H_1}^2 |H_1|^2 + M_{H_2}^2 |H_2|^2 + M_{\bar{L}_{3L}}^2 |\bar{L}_{3L}|^2 + M_{\bar{L}_{3R}}^2 |\bar{L}_{3R}|^2 + B_H (H_1 H_2) + B_3 \mu_3 \left( H_2 \bar{L}_{3L} \right) \\
+ A_{33}^E \left( H_1 \bar{L}_{3L} \right) |\bar{L}_{3R}| + h.c.
\]

(45)

The third term at Eq. (40) is

\[
V_D = \frac{1}{2} \left[ D^i D^i + (D')^2 \right],
\]

(46)

where \( i = 1, 2, 3 \). There is one \( D \)-term came from \( \mathcal{L}_{Scalar} \) and from superpotential for each of the four gauge groups

\[
SU(2)_L : \quad D^i = -\frac{g}{2} \left[ H_1^\dagger \sigma^i H_1 + H_2^\dagger \sigma^i H_2 + \bar{L}_{3L}^\dagger \sigma^i \bar{L}_{3L} \right],
\]

\[
U(1)_Y : \quad D' = -\frac{g'}{2} \left[ |H_1|^2 - |H_2|^2 - |\bar{L}_{3L}|^2 + 2|\bar{L}_{3R}|^2 \right].
\]

(47)

Using the following relation

\[
\sigma^i_{ab} \sigma^j_{cd} = 2 \delta_{ad} \delta_{bc} - \delta_{ab} \delta_{cd},
\]

(48)

we finally get

\[
V_D = \frac{g^2}{8} \left[ (|H_1|^2 - |H_2|^2)^2 + 4( |H_1^\dagger \bar{L}_{3L}|^2 + |H_2^\dagger \bar{L}_{3L}|^2 - 2(|H_1|^2 + |H_2|^2)|\bar{L}_{3L}|^2 + (|\bar{L}_{3L}|^2)^2 \right] \\
+ \frac{(g')^2}{8} \left[ |H_1|^2 - |H_2|^2 - |\bar{L}_{3L}|^2 + 2|\bar{L}_{3R}|^2 \right].
\]

(49)
where we have defined the following new parameters

\[ \begin{align*}
V_{MSSMRPV}^H &= m_{1h}^2 |H_1|^2 + (m_{2h}^2 + \mu_3^2) |H_2|^2 + (M_L^2 + \mu_3^2) |\tilde{L}_{3L}|^2 + M_t^2 |\tilde{t}_R|^2 + [B\mu (H_1 H_2) + B_3\mu_3 \left( H_2 \tilde{L}_{3L} \right) + A_3^E \left( H_1 \tilde{L}_{3L} \right) |\tilde{t}_R|^2 + h.c.] + \frac{(f^r)^2}{9} \left[ \left( |H_1|^2 + |\tilde{L}_{3L}|^2 \right) |\tilde{e}_R|^2 - |H_1|^2 |\tilde{L}_{3L}|^2 + |H_1^* \tilde{L}_{3L}|^2 \right] + \frac{\mu H \mu_3}{6} \left( H_1^* \tilde{L}_{3L} + \tilde{L}_{3L}^* \mu_3 \right) + \frac{\mu H f^r}{6} \left[ (H_2 \tilde{L}_{3L}) \tilde{e}_R^* + (H_2 \tilde{L}_{3L}) \tilde{e}_R' \right] + \frac{\mu_3 f^r}{6} \left[ (H_2^* H_1) \tilde{e}_R^* + (H_2^* H_1) \tilde{e}_R' \right] + \frac{g^2}{8} \left[ (|H_1|^2 - |H_2|^2)^2 + 4(|H_1^* H_2| + |H_1^* \tilde{L}_{3L}|^2 + |H_2^* \tilde{L}_{3L}|^2) - 2(|H_1|^2 + |H_2|^2) |\tilde{L}_{3L}|^2 + (|\tilde{L}_{3L}|^2)^2 \right] + \frac{(g')^2}{8} \left[ |H_1|^2 - |H_2|^2 - |\tilde{L}_{3L}|^2 + 2 |\tilde{e}_R|^2 \right],
\end{align*} \]

(50)

where we have defined the following new parameters

\[ \begin{align*}
m_{1h}^2 &= M_{H_1}^2 + \frac{\mu^2}{4}, \quad m_{2h}^2 = M_{H_2}^2 + \frac{\mu^2}{4}.
\end{align*} \]

(51)

From Eq. (50) it is easy to reproduce the minimum of our potential scalar and it is given by [18]

\[ \begin{align*}
V_{MSSMRPV}^{\text{min}} &= \frac{(g^2 + (g')^2)}{32} \left( v_1^2 - v_2^2 + (v_3^L)^2 \right)^2 + \frac{m_{1h}^2}{2} v_1^2 + \frac{m_{2h}^2}{2} v_2^2 - B\mu v_1 v_2 + B_3\mu_3 v_2 v_3^L + \frac{M_L^2}{2} (v_3^L)^2
\end{align*} \]

and the constraint equations are

\[ \begin{align*}
m_{1h}^2 v_1 - B\mu v_2 - \mu_3 v_3^L + \frac{(g^2 + (g')^2)}{8} v_1 (v_1^2 - v_2^2 + (v_3^L)^2) &= 0,
(m_{2h}^2 + \mu_3^2) v_2 - B\mu v_1 - B_3\mu_3 v_3^L + \frac{(g^2 + (g')^2)}{8} v_2 (v_1^2 - v_2^2 + (v_3^L)^2) &= 0,
(M_L^2 + \mu_3^2) v_3^L - \mu_3 v_1 v_2 + \frac{(g^2 + (g')^2)}{8} v_3^L (v_1^2 - v_2^2 + (v_3^L)^2) &= 0,
\end{align*} \]

(53)

those equations are in agreement as presented at [18].

In the bases

\[ \begin{align*}
\begin{pmatrix} H_1^+ & H_2^+ & \tilde{e}_R' \end{pmatrix},
\begin{pmatrix} H_1^- & H_2^- & \tilde{e}_R \end{pmatrix},
\end{align*} \]

(54)

the mass matrix is given by

\[ \begin{align*}
M_{H^\pm}^2 &= \begin{pmatrix} M_{H_1^+}^2 & M_{H_2^+}^2 & M_{\tilde{e}_R'}^2 \\ M_{H_1^-}^2 & M_{H_2^-}^2 & M_{\tilde{e}_R}^2 \end{pmatrix},
\end{align*} \]

(55)
where

\[
M^2_{hh} = \begin{pmatrix}
A & B\mu + \frac{g^2}{4}v_1v_2 \\
\frac{g^2}{4}v_1v_2 & C
\end{pmatrix},
\]

\[
M^2_{lh} = \begin{pmatrix}
D & E^* \\
E & F
\end{pmatrix},
\]

\[
M^2_{ll} = \begin{pmatrix}
G & H \\
I & J
\end{pmatrix},
\]

with

\[
A = B\mu \frac{v_2}{v_1} + \frac{g^2}{4}(v_2^2 - (v_3^L)^2) + \mu\mu_3 \frac{v_3^L}{v_1} + \frac{|f'^*|^2}{2}(v_3^L)^2,
\]

\[
C = B\mu \frac{v_1}{v_2} + \frac{g^2}{4}(v_1^2 + (v_3^L)^2) - \mu\mu_3 \frac{v_3^L}{v_2},
\]

\[
D = \frac{|f'^*|^2}{2}v_1^2 - \frac{g^2}{4}(v_1^2 - v_2^2) + \mu\mu_3 \frac{v_1}{v_3^L} - B_3\mu_3 \frac{v_2}{v_3^L} + M^2_L,
\]

\[
E = \frac{f'}{\sqrt{2}}\mu_3 v_2 \left( A^{E_3} + \mu^* \tan \beta \right),
\]

\[
F = M^2_l + \frac{|f'^*|^2}{2}(v_1^2 + (v_3^L)^2) - \frac{g^2}{4}(v_1^2 - v_2^2 + (v_3^L)^2),
\]

\[
G = -\mu\mu_3 - \frac{|f'|^2}{2}v_1v_3^L + \frac{g^2}{4}v_1v_3^L,
\]

\[
H = -\frac{1}{\sqrt{2}} \left( f'^* v_2 - f'^{**} A^{E_3} v_3^L \right),
\]

\[
I = -B_3\mu_3 + \frac{g^2}{4}v_2v_3^L,
\]

\[
J = -\frac{f'^*}{\sqrt{2}} (\mu v_3^L + \mu v_1),
\]

(56)

those equations are in agreement as presented at [18].

We can show the following analytical results

\[
det \left( M^2_{H^{\pm}} \right) = 0,
\]

\[
Tr \left( M^2_{H^{\pm}} \right) = \frac{1}{v_1v_2v_3^L} \left\{ v_1v_2v_3^L \left[ 4M^2_l + 2g^2v_2^L - (g')^2 \left( v_1^2 - v_2^2 + (v_3^L)^2 \right) \right] + 4B\mu v_3^L \left( v_1^2 + v_2^2 \right) 
- 4\mu_3 \left[ B_3v_1 \left( v_2^2 + (v_3^L)^2 \right) + \mu v_2 \left( v_1^2 + (v_3^L)^2 \right) \right] \right\} .
\]

(57)

The pseudoscalar squared mass matrix is given

\[
M^2_A = \begin{pmatrix}
B\mu \frac{v_2}{v_1} + \mu\mu_3 \frac{v_3^L}{v_1} & B\mu \\
B\mu \frac{v_2}{v_1} & B\mu \frac{v_2}{v_1} - B_3\mu_3 \frac{v_3^L}{v_2} - B_3\mu_3 \\
-\mu\mu_3 & -B_3\mu_3 & \mu_3 \frac{v_1}{v_3^L} - B_3\mu_3 \frac{v_2}{v_3^L}
\end{pmatrix},
\]

(58)

and we get

\[
det \left( M^2_A \right) = 0,
\]

\[
Tr \left( M^2_A \right) = \frac{1}{v_1v_2v_3^L} \left\{ 4B\mu v_3^L \left( v_1^2 + v_2^2 \right) - \mu_3 \left[ B_3v_1 \left( v_2^2 + (v_3^L)^2 \right) + \mu v_2 \left( v_1^2 + (v_3^L)^2 \right) \right] \right\} .
\]

(59)
those equations are in agreement as presented at [18].

We can also get the mass squared mass matrix for the scalar sector but we will not show it here.

5 Results with all $\tilde{L}_i \neq 0$.

5.1 Bosons Masses

In this case, we continue to using the Eq. (60) plus the following terms

$$\left( D_m \tilde{L}_1 \right)^\dagger \left( D_m \tilde{L}_1 \right) + \left( D_m \tilde{L}_2 \right)^\dagger \left( D_m \tilde{L}_2 \right),$$

(60)

same the same procedure presented at Sec.(4.1) we get the following expression to the charged gauge bosons masses

$$M_W^2 = \frac{g^2}{4} \left( v_1^2 + v_2^2 + (v_1^L)^2 + (v_2^L)^2 + (v_3^L)^2 \right).$$

(61)

then we can rewrite

$$v_1 = v \sin \theta_1 \sin \theta_2 \sin \theta_3 \sin \beta,$n

$$v_2 = v \sin \theta_1 \sin \theta_2 \sin \theta_3 \cos \beta,$n

$$v_3^L = v \sin \theta_1 \sin \theta_2 \cos \theta_3,$n

$$v_2^L = v \sin \theta_1 \cos \theta_2,$n

$$v_1^L = v \cos \theta_1,$n

(62)

and the neutral massive gauge boson ($Z^0$) get the following mass

$$M_Z^2 = \frac{g^2}{4 \cos^2 \theta_W} \left( v_1^2 + v_2^2 + (v_1^L)^2 + (v_2^L)^2 + (v_3^L)^2 \right).$$

(63)

5.2 Charginos Masses

In this case the $X$ non diagonal mass matrix for charginos, defined at Eq. (28), is given by

$$X = \begin{pmatrix} M_{\tilde{\chi}}^{MSSM} & M_{\tilde{\chi}L} \\ M_{HL} & M_{\tilde{\chi} \text{lepton}}^{Yukawa} \end{pmatrix},$$

(64)

where

$$M_{\tilde{\chi}}^{MSSM} = \begin{pmatrix} M_{\tilde{\chi}} & \frac{v_1}{\sqrt{2}} \\ \frac{v_2}{\sqrt{2}} & \mu_H \end{pmatrix}, \quad M_{\tilde{\chi} \text{lepton}}^{Yukawa} = \frac{v_1}{\sqrt{2}} \begin{pmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{pmatrix}.$$ 

(65)

The second matrix at Eq. (64) is

$$M_{\tilde{\chi}L} = \begin{pmatrix} 0 & 0 & 0 \\ A & B & C \end{pmatrix},$$

(66)
where we have defined

\[ A = -\frac{1}{\sqrt{2}} (f_{11} v_1^L + f_{21} v_2^L + f_{31} v_3^L), \]

\[ B = -\frac{1}{\sqrt{2}} (f_{12} v_1^L + f_{22} v_2^L + f_{32} v_3^L), \]

\[ C = -\frac{1}{\sqrt{2}} (f_{13} v_1^L + f_{23} v_2^L + f_{33} v_3^L), \]

while the last matrix at Eq.(64) has the following expression

\[ \mathcal{M}_{HL} = \begin{pmatrix} g v_1^L & -\mu_1 \\ g v_2^L & -\mu_2 \\ g v_3^L & -\mu_3 \end{pmatrix}, \]

It is easy to realize that, from Eqs.(66,68), chargino sector decouple from the usual lepton sector in the limit

\[ \mu_1 = \mu_2 = \mu_3 = 0, \quad v_1^L = v_2^L = v_3^L = 0. \]

Using the parameters given before Eq.(34), plus the following parameters

\[ f_{11} = 6.3 \cdot 10^{-6}, \quad f_{22} = 1.21 \cdot 10^{-3}, \quad f_{33} = 2.3 \cdot 10^{-2}, \quad f_{12} = f_{13} = f_{23} = f_{21} = f_{31} = f_{32} = 10^{-9} \]

we obtain from Eq.(64) the masses 0.0005, 0.105, 1.777 (in GeV) for the usual leptons, and 105.36 and 294.65 GeV for the charginos.

### 5.3 Neutralinos Masses

Using Eq.(37) we can write in this case the following mass matrix for neutralinos at non-diagonal expression

\[ Y_{\text{neutralino}}^{\text{MSSMRP}} = \begin{pmatrix} \mathcal{M}_{\tilde{\chi}^0} & m^T \\ m & 0 \end{pmatrix}. \]

where

\[ \mathcal{M}_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -\frac{g v_2}{2} \\ 0 & M_2 & -\frac{g v_3}{2} \\ -\frac{g v_2}{2} & -\frac{g v_3}{2} & -\mu_H \end{pmatrix} \]

\[ m = \begin{pmatrix} \frac{-g' v_1}{2} & \frac{g v_1}{2} & 0 & \mu_1 \\ \frac{-g' v_2}{2} & \frac{g v_2}{2} & 0 & \mu_2 \\ \frac{-g' v_3}{2} & \frac{g v_3}{2} & 0 & \mu_3 \end{pmatrix}. \]

Using the same parameters are given after Eq.(38), but only changing the values of \( \mu_3 \) for \( \mu_3 = 11.4 \), we obtain a massive neutrino and its mass is \( m_{\nu_3} = 0.051 \text{ eV} \), and four heavy neutralinos with masses 259.05, −208.42, −101.23, 100.65 GeV.
5.4 Higgs masses

The $F$ terms are

\[
F_{H_1}^+ = -\frac{\mu_1}{2}H_2 - \sum_{i=1}^{3} \frac{\mu_i}{2} \tilde{L}_{iL}, \\
F_{H_2}^+ = -\frac{\mu_1}{2}H_1 - \sum_{i,j=1}^{3} \frac{f_{ij}}{3} \tilde{L}_{iL} \tilde{L}_{jR}, \\
F_{L_{iL}}^+ = -\frac{\mu_i}{2}H_1 - \sum_{j=1}^{3} \frac{f_{ij}}{3} \tilde{L}_{jR}H_2, \\
F_{i_{cR}}^+ = -\sum_{j=1}^{3} (f_{ij})^2 \left( H_1 \tilde{L}_{jL} \right).
\]

The $D$-term are

\[
SU(2)_L : \quad D^i = -\frac{g}{2} \left[ H_1^i \sigma^i H_1 + H_2^i \sigma^i H_2 + \sum_{j=1}^{3} \tilde{L}_{jL}^i \sigma^j \tilde{L}_{jL} \right], \\
U(1)_Y : \quad D' = -\frac{g'}{2} \left[ |H_1|^2 - |H_2|^2 - \sum_{j=1}^{3} |\tilde{L}_{jL}|^2 + 2 \sum_{j=1}^{3} |\tilde{L}_{jR}|^2 \right].
\]

The soft term that contribute to the scalar potential is given by

\[
V_{soft} = M^2_{H_1} |H_1|^2 + M^2_{H_2} |H_2|^2 + \sum_{i,j=1}^{3} \tilde{L}_{iL} \left( M^2_{L_i} \right)_{ij} \tilde{L}_{jL} + \sum_{i,j=1}^{3} \tilde{L}_{iR} \left( M^2_{L_i} \right)_{ij} \tilde{L}_{jR} + B\mu (H_1 H_2) \\
+ \sum_{i=1}^{3} B_i \mu_i (H_2 \tilde{L}_{3L}) + \sum_{i,j=1}^{3} A_{ij}^E (H_1 \tilde{L}_{iL}) \tilde{L}_{jR} + h.c.
\]

Using Eqs.(72,73,74), we can reproduce at tree-level the scalar mass matrices presented at [28, 30].

6 Conclusions

In this article we have presented the MSSM with $R$-Parity violation. We hope this review can be useful to all the people wants to learn about Supersymmetry.

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