Wideband Spectrum Sensing in Dynamic Spectrum Access Systems Using Bayesian Learning

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Abstract. The commercialization and growth of Cognitive radio technology demand a spectrum sensing system that reacts in real-time to smart decisions, unlike mobile standards, which do not have inherent features for assigning a vacant band to the secondary user when the primary user is not present. It is incapacitating because of standard provisions due to varying size, varying weight, varying power, and cost limitations. Often, it was found that some vacant areas in bands are unused by the primary user/group[4]. In this case, it will be more economical to assign those vacant spaces (bandwidth) to a secondary group. The algorithm developed has to sense and analyze its native environment to identify its vacant spectrum. Compressed sensing technique [5], [6] can provide higher bandwidth service, increased spectrum utilization.

Keywords: Wideband spectrum sensing, Compressed Sensing, Cognitive radio networks, Bayesian learning.

1. Introduction
Cognitive Radio is a technology that can be designed and programmed to use the foremost effective wireless channel in its neighboring areas to remove user interference, noise and congestion while operating dynamically [1]. A Wideband radio network accommodates a broad range of frequencies [2] or wavelengths [3]. Spectrum sensing refers to the assignment of identifying the frequency support of a given input signal. The commercialization and growth of cognitive Radio technology demand a spectrum sensing system that reacts in real-time to smart decisions, unlike mobile standards, which cannot have inherent features to assign a vacant band to the secondary user when the primary user is not present. It is incapacitating because of standard provisions due to varying size, varying weight, varying power, and cost limitations. Often, it was found that some vacant areas in bands are unused by the primary user/group[4]. In this case, it will be more economical to assign those vacant spaces (bandwidth) to a secondary group. The algorithm developed has to sense and analyze its native environment to identify its vacant spectrum. Compressed sensing technique [5], [6] can provide higher bandwidth service, increased spectrum utilization.
In the wideband spectrum sensing technique, the first step is to sample the wide band of interest. The difficulty here is to sample the spectrum at the Nyquist rate. The need to process the ADC samples at a high rate in a wideband is not affordable[7], which leads to an enormous amount of computation, power consumption, and complexity. The problem is, sampling rate to be reduced and to find a way to recover the signal. Compressed sampling turns it into an opportunity to recover the signal with fewer samples than the Nyquist sampling rate[10]. Sub Nyquist spectrum sensing techniques will sample the wideband spectrum at a sub-Nyquist sampling rate to use the block sparsity in the wideband spectrum; the spectrum[11] of interest will be transformed into the frequency domain DFT. Sub Nyquist wideband sensing technique assumes that an average sparsity over a multi-band in the entire spectrum[12].

In the proposed work, bands that exhibit similar occupancy patterns will be grouped into a block with an attempt to take advantage of heterogeneous wideband to inherit a block structure to design efficient sub-Nyquist or compressed spectrum sensing technique. The frequency band is partitioned into blocks, and these blocks are grouped based on varying sparsity levels. Block Sparse Bayesian Learning will be adapted to recover the support vector. The prior knowledge of intra-block correlation leads to a case with negligible noise. In contrast, the lack of information on block partition, i.e., with no intra-block correlation present, results in a case with noise.

1.1. Related Works
Compressed sensing technique is widely used in sub-Nyquist wideband spectrum sensing due to the humongous number of samples processed at high speed, leading to increased complexity and cost. Sub Nyquist sensing technique will use fewer samples than the Nyquist rate to recover the K-sparse wideband signal [13],[14]. Previous works mentioned here consider fixed sparsity in the wideband signal, i.e., several non-zero elements are fixed that is impractical. In real-time, the sparsity level will change after some duration; hence the research works uses a two-stage scheme that estimates the sparsity and performs measurements [15].

Key contributions:
- BSBL with correlation can find the occupied band in the wideband spectrum when the number of measurements is greater than 75.
- Mean Square Error (MSE) in BSBL method is less than 0.3 in the SNR range 0 to 5dB (irrespective of sparsity information).
- The rest of the paper is organized as follows: Section II gives the proposed system model. Section III discusses the recovery of the signal using Block sparse Bayesian learning (BSBL) with correlation and without correlation. Section IV gives numerical simulation and results. Finally, Section V presents the conclusions from this work.

2. System Model
Consider a wide band spectrum where the sub-bands are allocated to individual primary users (licensed users). Assume that within the spectrum of interest, different types of user applications can be accommodated. User applications have similar primary user occupancy pattern will be considered as a group. The underlying thread - the primary user is from different applications allocated in different portions of the spectrum- has a common band occupancy pattern. This paper uses an inherent common occupancy pattern between the different primary users with different applications for block-based occupancy over the spectrum of interest, grouping similar occupancy patterns of the primary user as shown in Figure 1.a and Figure1.b. Assume that the bands occupied will be accessed for a finite amount of time, and the average sparsity will not be changed before the system will sense the occupied bands.
The Fast-Fourier transform (FFT) of this signal is performed, after which the signal is down-sampled by a factor (pair of co-primes) [18]. Co-prime sampled signals will have two different sampling channels. Here, ADC1 and ADC2 change the analog input signal to digital form. L1T and L2T are the intervals considered to sample in the ADC’s L1T and L2T, have to be co-prime integers, and perform the downsampling sub-Nyquist level. ‘L.’ represents the LCM of L1 and L2, which means L is the product of L1 and L2, i.e., L=L1*L2. The average sampling rate is (L1+L2)/(LT). Co-prime sampling has resulted in being recognized as an efficient sampling technique.

Two ADCs help in the sampling of signals. 1/MT and 1/NT are the rates at which they function. 1/T = 2fmax is the basic universal Nyquist rate formula. The synchronous aspect of co-prime sampling in which L1=3 and L2=4. x1[n] and x2[m] are the sampled signals taken into the two Analog to Digital converters. The synchronicity means that x1[n] and x2[m] can be regarded as the Nyquist grid’s downsamples. A block consists of grouped Nyquist grids (quantity=L). L denotes the greatest common divisor of L1 and L2. Every block now consists of identical samples L taken from outputs of the converters. In every block, C1 and C2 represent the timestamps. These timestamps give us an idea of the indices of the output samples from the converters. The grouped sets C1 and C2 are shown below in equations (1) and (2).

$$C_1 = \{c_1^1 | c_1^1 = L_1 i : i = 0, 1, \ldots, (L_1 - 1)\}$$ (1)

$$C_2 = \{c_2^2 | c_2^2 = L_2 i : i = 0, 1, \ldots, (L_2 - 1)\}$$ (2)

M (M = L1 + L2 - 1) samples are not uniform per block. The indices associate with C = C1 U C2.

3. Signal Recovery

Consider the following linear system model equation:

$$y = \Phi x + \nu$$ (3)

where the K-block sparse signal x has the ‘K’ non-zero blocks [x1, \ldots, xd1, \ldots, xd_{g-1}, \ldots, xd_g]^T and \nu is AWGN noise.

y ∈ R^M×1 is a measurement vector consisting of an M compressed signal.

Φ ∈ R^{M×N} (M < N) is a measurement matrix.

x ∈ R^{N×1} is the sparse block signal that needs to be recovered from the compressed measurements. \nu is an unknown noise vector.

x is considered as a K-block sparse signal, which is a signal consisting of ‘K’ non-zero blocks in an N-dimensional space where K << N, [x1, x2, \ldots, xN] block-sparse signal.

Block sparsity refers to the fact that x can be viewed as a concatenation of blocks of equal size, assumed throughout the process to be of length ‘d’. In the block partition, d1, \ldots, d_g are not the same.
Some non-zero blocks are present in ‘g’ blocks. This prototype is often referred to as a block-sparse structure. The above model indicates that every block \( x_i \in \mathbb{R}^{d_i \times 1} \) satisfies a Gaussian distribution:

\[
p(x_i) \sim N_x(0, \gamma_i B_i)
\]

where \( i = 1, 2, 3, ..., g \), \( \gamma_i \) is the non-negative value, and \( \gamma_i (\forall i) \) always tends to 0.

### 3.1. BSBL Expectation-Maximization Known block partition algorithm

\( B_i \in R_{d_i \times d_i} \) is non-negative, not indefinite matrix, giving correlation sequence of the ith blocks.

Getting the prior of the input signal

\[
p(x) \sim N_x(0, \Sigma_0)
\]

\( \Sigma \) is a block-diagonal matrix. Every main block here equals \( y_i \beta_i \). \( p(v) \sim N_0(0, \lambda I) \), is presumed to be satisfied by the vector of noise signal. In this, \( \lambda \) is considered as non-negative value. Output posterior is given by

\[
p(x|y; \lambda, y_i, B_i) = N_x(\mu_x, \Sigma_x) \text{with } \mu_x = \Sigma_0 \Phi^T (\lambda I + \Phi \Sigma_0 \Phi^T)^{-1} y \text{ and } \Sigma_x = (\Sigma_0^{-1} + \lambda \Phi \Sigma_0 \Phi^T)^{-1}.
\]

Once the hyper-parameters \( \lambda, y_i, B_i \) are analysed, input vector can be estimated with the help of the hyper-parameters. The hyper parameters for the estimation are given as the following expressions (6a) - (6d):

\[
\mu_x = \Sigma_0 \Phi^T (\lambda I + \Phi \Sigma_0 \Phi^T)^{-1} y \\
\Sigma_x = \Sigma_0 - \Sigma_0 \Phi^T (\lambda I + \Phi \Sigma_0 \Phi^T)^{-1} \Phi \Sigma_0 \\
\lambda = \frac{1}{N} \sum_{i=1}^{M} \text{Tr}[B_i^{-1} (\Sigma_x + \mu_x \mu_x^T)] \\
\gamma_i = \frac{1}{d_i} \sum_{i=1}^{N} [\text{Tr} (B_i^{-1} (\Sigma_x + \mu_x \mu_x^T))]
\]

Upon the convergence of the process, the actual input signal is found using estimation of \( \mu_x \).

The constriction of \( B_i = B (\forall i) \) gives

\[
B = \left[ \sum_{i=1}^{g} \frac{\Sigma_x + \mu_x \mu_x^T}{\gamma_i} \right]^{-1}
\]

\( B \) is Toeplitz \((1, r, \cdots, r^{d-1})\)

\[
B = \begin{bmatrix}
1 & r & r^2 & \cdots & r^{d-1} \\
r & 1 & r & \cdots & r^{d-2} \\
\vdots & \vdots & \ddots & \ddots & \vdots \\
& & & 1 & r \\
& & & r & 1
\end{bmatrix}
\]

Where \( r = 0.99 \), a reasonable bound value for correlation. When the block size is equal and known, limit the blocks to have the same correlation structure. In this scenario, \( B \) is obtained from equation 8. This limiting of blocks to a specific correlation structure will improve the performance of the algorithm.

### 3.2. Unknown block partition-Algorithm

In this case, all the non-zero blocked groups have the same size \( h \). Because the blocks are randomly situated, they tend to overlap plausibly, resulting in non-identical bigger blocks. \( N - h + 1 \) possible group in the input signal that overlaps one another. The ending point of this block is the \((i + h - 1)\) element. In some of these blocks, the non-zero entries of the inputs are found. A multivariate Gaussian distribution is considered for the \(i\)th block, where \( \gamma_i B_i \) is the covariance matrix.

Prior of input signal:

\[
p(x) \sim N_x(0, \Sigma_0)
\]

Since blocks now are seemingly overlapping, \( \Sigma_0 \) now does not represent a block diagonal matrix. \( \Sigma_0 \) has the build where every \( \gamma_i B_i \) is found along the main diagonal of the matrix \( \Sigma_0 \); it also finds itself overlapping other \( \gamma_i B_i \). This prototype algorithm helps us develop an estimation method of hyper parameters \( \lambda, \gamma_i B_i (\forall i) \). Since \( \Sigma_0 \) is not a block-diagonal covariance matrix anymore, now, the covariance matrix is:
\[ \Sigma_0 = B \text{diag}(\gamma_1 B_1, \ldots, \gamma_p B_p) \in R^{p \times p} \] represents a block-diagonal matrix in the above equation and has its main diagonal blocks represented as \( \gamma_1 B_1, \ldots, \gamma_p B_p \). Note that now \( \gamma_i B_i \) does not overlap other \( \gamma_i B_i \).

The original model now becomes

\[ y = \sum_{i=1}^{p} \phi E_i z_i + v \triangleq Az + v, \] (12)

where \( A \) is represented as \([A_1, \ldots, A_p]\) and \( A_i \), \( \phi E_i \). The resulting model resembles a BSBL one. The algorithm equations (14) to (e).

\[ \mu \leftarrow \frac{\Sigma_0 AT (\lambda I + A \Sigma_0 AT)^{-1}}{\Sigma_0^{-1} \Sigma_0 AT (\lambda I + A \Sigma_0 AT)^{-1} A \Sigma_0^{-1}} \] (13a)

\[ \Sigma_z \leftarrow \frac{\Sigma_0^{-1} \Sigma_0 AT (\lambda I + A \Sigma_0 AT)^{-1} A \Sigma_0^{-1}}{\lambda} \] (13b)

\[ \gamma_i \leftarrow \frac{\text{Tr}[B^{-1}(\gamma_i z_i + \mu_i z_i^2)^T]}{h}, \forall i \] (13c)

\[ B \leftarrow \frac{1}{\sum_{i=1}^{p} \Sigma_z^2 + \mu_z^2 (\mu_z^2)^T}} \] (13d)

\[ \lambda \leftarrow \frac{\|y - A \mu_z\|^2 + \lambda \|\phi - \text{Tr}(\Sigma_z^{-1})\|^2}{M} \] (13e)

where \( \mu_z^2 \) is the corresponding \( i \)th block in \( \mu_z \), which is also the corresponding \( i \)th main diagonal block in \( \Sigma_z \), and restriction of \( B \) for more robust performance can be further implemented. Failure Rate is indicated as total trials * 100. Success rate = (Number of successful trials / Total trials)*100. In some cases defined by expression (15)

\[ MSE = \frac{||\hat{X} - X_{gen}||_2^2}{||X_{gen}||_2^2} \] (14)

Where \( \hat{X} \) is the estimate of source matrix \( X_{gen} \).

4. Numerical Results and Discussion

Secondary users will sense the vacant band’s wideband spectrum when block partition is unknown, \( \phi \) is off the scale \( 80 \times 256 \). The number of non-zero elemental constitutes within the input is set at 32. Non-zero blocks are randomly chosen, and the number of non-zero entries will be equal to 32. Therefore, every block has a completely random dimension, and the non-zero element location is completely random. To vividly observe the efficiency of the scenario, the intra-block correlation is zero. Mean square values are noted for different block sizes at SNR = 25dB.

As shown in Figure 2.b, the graph between several measurements \( M \) and success rate with \( N \) being constant (\( N=512 \)) with and without Intra-block correlation characteristics was plotted. Figure 2.a with includes gives much better success rates at a lower number of samples or measurements than the plot without correlation that reaches a success rate much later \( M>60 \).
Figure 2: a) Mean square error of various block size h. b) Graph between several measurements and success rate.

Figure 3: MSE comparison for various SNR of BSBL with correlation (proposed), LASSO, LASSO, and OMP.

Analyzed the algorithms for various measurement numbers 'M'. 10 out of 64 non-zero blocks will be randomly chosen. For values of $k = 1, \ldots, 10$, the elemental constituents of the $k$-th blocks sprouted due to Gaussian variance $N \sim (0, \Sigma_k)$ is developed with MATLAB code’s help. $\beta_k$ is the parameter that measures the intra-block correlation. In every iteration, it had been arbitrarily estimated between 0.9-0.99. The success rate is plotted in figure 2.b., where several measurements $M$ takes values between 60 and 110, varying the number of measurements. In figure 2.b, BSBL with correlation case performance is better than the BSBL without correlation. The success rate of BSBL with correlation and correlation for a different number of measurements has been tabulated as shown in Table 1. Figure 3 shows the better detection performance than the other methods even in the low SNR range assumed that the correlation information is available.

Table 1 shows the success rate, and it is recovered with correlation and without correlation. Figure 3 shows the success rate for different measurement vectors for both cases. It is evident to say BSBL with correlation improves the performance to detect the support vector accurately with a lesser number of measurements for the same SNR.
Table 1: Success Rate Of Bsbl With Correlation And Without Correlation For Various Number Of Measurements

| S.No. | Measurement | #Trials | Success Rate in % |
|-------|-------------|---------|-------------------|
|       |             |         | Average with Correlation | Average without correlation |
| 1     | 60          | 10      | 27.7              | 0                        |
|       |             | 30      | 27.7              | 0                        |
|       |             | 50      | 27.7              | 0                        |
|       |             | 10      | 72.4              | 39.1                     |
| 2     | 65          | 30      | 72.4              | 39.1                     |
|       |             | 50      | 72.4              | 39.1                     |
|       |             | 10      | 91                | 73.1                     |
| 3     | 70          | 30      | 91                | 73.1                     |
|       |             | 50      | 91                | 73.1                     |
|       |             | 10      | 99.3              | 81.7                     |
| 4     | 75          | 30      | 99.3              | 81.7                     |
|       |             | 50      | 99.3              | 81.7                     |
|       |             | 10      | 100               | 95.3                     |
| 5     | 80          | 30      | 100               | 95.3                     |
|       |             | 50      | 100               | 95.3                     |
|       |             | 10      | 100               | 100                      |
| 6     | 85          | 30      | 100               | 100                      |
|       |             | 50      | 100               | 100                      |
|       |             | 10      | 100               | 100                      |
| 7     | 90          | 30      | 100               | 100                      |
|       |             | 50      | 100               | 100                      |
|       |             | 10      | 100               | 100                      |
| 8     | 100         | 30      | 100               | 100                      |
|       |             | 50      | 100               | 100                      |

5. Conclusion
This paper presented a BSBL methodology to identify the unoccupied bands in the block-based structure for the heterogeneous networks' occupancy pattern. It is evident from the simulations that the proposed BSBL algorithm performs better than the state-of-art methods even at low range SNR; better recovery performance with intra-block correlation needs a lesser number of measurements with correlation case than without correlation. The proposed BSBL method's detection performance is better than the state-of-art methods like WLASSO, OMP, and LASSO.

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