Wake behind an offshore wind farm observed with dual-Doppler radars

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Abstract. We present dual-Doppler measurements of the wake behind an offshore wind farm. The measured wind speed in the wake is compared with two wake models, one resolving the single wakes from individual turbines, the other describing the wind farm as a distributed area of increased roughness. We analyse a case with steady wind direction approximately aligned with the turbine rows. The radar measurements allow us to track the evolution of the wakes through the wind farm and further downstream. In the case studied here the wake region extends at least 17 km downstream of the wind farm. Both models are in good agreement with observations provided a coastal gradient is accounted for. The generality of this conclusion is tested by comparing modelled wind speeds with observations aggregated over multiple cases with a similar inflow wind direction.

1. Introduction
Offshore wind farms are now often constructed in clusters, mainly due to consenting limitations, but also to better utilize high wind resources and optimize logistics or as multi-phase development of mega-sites. Therefore, understanding the influence of inter-farm wakes in existing and planned wind farm clusters remain a significant challenge [1,2].

Investigations of wakes and validations of wake models have mostly been confined to wakes internal to a wind farm [3-6]. Much less is known about the recovery of wakes in the downstream zone of wind farms, where no further energy is extracted from the flow. Offshore, wake measurements behind wind farms have been made with met masts at downstream distances up to 6 km [7,8], and with airborne probes [9,10]. These in-situ measurements are confined either to the met mast location or along the flight path of the aircraft. Extremely long wakes have also been observed in the surface winds through SAR retrievals [10-13]. But it is hard to accurately extrapolate the wind speeds retrieved from SAR images to hub height and the SAR scenes are only infrequently available. Questions remain about the detailed structure and recovery of the wake behind a wind farm. How fast does the wind speed recover? Can the wind farm wake be predicted by commonly used wake models? Do variations in the ambient flow speed influence the rate of wake recover? The answers to these questions determine the scale and importance of inter-farm wake losses and are therefore important for the planning and design of future wind farms and clusters.

Here we present extensive dual-Doppler radar measurements of the wake behind an offshore wind farm. The two radars form the cornerstone of the BEACon R&D project and are deployed near the
Westernmost Rough offshore wind farm off the coast from Yorkshire, UK. We demonstrate that the radars are a promising remote sensing technology for measuring the wake region downstream of a wind farm due to the large range combined with high spatial and temporal resolution [14-16]. We show how the radars can map out the structure and extent of the wind farm wake, while at the same time providing measurements of the inflow and the surrounding flow field. The latter reveals a coastal gradient interacting with the wake behind the wind farm. We characterise the wind farm wake for different inflow conditions and assess how the wind farm wake adapts. In addition, we compare the measured wind farm wakes with theoretical predictions from both the Park model and the top-down wake model, where the wind farm is considered as an area of increased roughness.

We begin this paper with a description of the dual-Doppler measurement setup, followed in Section 3 by an outline of the Park and top-down wake models. Section 4 presents a case study consisting of measurements over a one-hour period with a steady wind direction aligned roughly with the turbine rows. In Section 5 we compare model outputs with the measured wind speed and examine the predictive power of the models. We conclude in Section 6 with a summary of the findings and a discussion of their relevance for wind farm planning and production estimates.

2. Dual-Doppler radar measurement campaign
The Westernmost Rough offshore wind farm consists of 35 Siemens Gamesa turbines, each with a rated power of 6 MW. The wind turbines have a hub height of 106 m and a rotor diameter \( (D) \) of 154 m. They are located between 8 and 14 km from the coastline (Figure 1).

The BEACon campaign was commenced in July 2016, lasting until the spring of 2018. From the coastline the BEACon radars scan a three-dimensional volume encompassing the wind farm and part of the surrounding area, providing comprehensive measurements of the flow field at high resolution. Specifically, the radars scan separate 60° azimuth sectors sampling every 0.352° with a beam width of 0.5°. This is equivalent to a transverse resolution of 0.45\( D \) and 0.80\( D \) at the nearest and farthest turbines, respectively. Along the beam the spatial resolution is 15 m. The azimuthal sector is scanned at 13 different elevation tilts from 0.2° to 1.4°. A complete three-dimensional volume covering all elevation tilts at ranges from 3 km to 32 km from the shore is collected in 64 seconds.

The radial wind speed data from the two radars undergo automated filtering for various artefacts such as second trip echoes and sea clutter before being combined into dual-Doppler horizontal wind speed and wind direction on an interpolated Cartesian grid spanning nine vertical levels from 50 m to 250 m above mean sea level with a horizontal resolution of 25 m in and near the wind farm and 50 m in the outer part of the domain. The Cartesian grid follows the curvature of the earth, and there is no evidence of refraction of the radar beams at the heights considered here. Hence, the measurement heights are consistent across the domain.

The uncertainty of the BEACon wind speed measurements has been quantified through calibration of the radial wind speed with a co-located scanning lidar coupled with a spatial uncertainty model for the dual-Doppler processed data. Details of this will be reported elsewhere. The resulting uncertainty of the one-hour averaged horizontal wind speed in the case study presented here is less than 0.2 m/s inside the wind farm and its wake and reaches a maximum of 0.8 m/s on the north/south edges of the dual-Doppler domain for the cases included here.

3. Wind farm wake models
We compare the BEACon measurements with two simple wake models, the top-down wind farm roughness model and the Park (N. O. Jensen) model, which describes wakes emanating from individual turbines and their superposition. In the top-down model the wind farm is considered an area of increased roughness due to the thrust of the turbines [17,18]. The momentum balance between
the approximately constant stress layer below the turbine rotors with the equilibrium logarithmic layer above the wind farm yields an expression for the effective wind farm roughness [19]

\[ z_{0,WF} = z_H \left( 1 + \frac{D}{2z_H} \right)^\beta \exp \left( -\kappa \left[ \frac{z_H}{z_0} \left( \frac{1 - D}{2z_H} \right)^\beta \right] \right)^{-1/2} \]

Here \( z_0 \) is the roughness in the upstream region before the wind farm, while \( z_H \) is the wind turbine hub height. The scaled thrust coefficient is given by \( c_t = \pi C_T(U_0)D^2/\beta \sigma_D \sigma_c \) with \( C_T(U_0) \) the turbine’s thrust coefficient at the inflow wind speed, \( \sigma_D \) and \( \sigma_c \) are the mean turbine spacings in the downstream and cross-stream directions, respectively. Finally, \( \beta = v^*/(1 + v^*) \) and \( v^* \approx 28 \sqrt{c_t} \) is a non-dimensional eddy viscosity accounting for the additional mixing in the wake. For the upstream roughness we use a value of 0.1 mm. For the case study presented below, where the inflow wind speed is 10.4 m/s and direction aligned with the turbine rows, the equivalent wind farm roughness is 1.5 m. In general, \( z_{0,WF} \) will depend on both wind speed and wind direction.

The decrease of the hub height wind speed through the wind farm and the wake recovery behind it is governed by the formation of internal boundary layers (IBL) at the entrance and exit of the wind farm, respectively. Following Meneveau [20] we calculate the IBL height from

\[ h_i(x) \approx h_i(x_i) + \left( \frac{x - x_i}{z_0,WF} \right)^{4/5} \]

where \( i = 1 \) (2) signifies the IBL forming inside (behind) the wind farm and \( x - x_i \) is the downstream distance from the roughness change at the first (last) turbine. The initial height of the IBL inside the wind farm is \( h_i(x_1) = z_H \), while \( h_5(x_2) = 0 \). Inside the wind farm the reduction of the hub height wind speed relative to the free stream is given by

\[ \frac{U(x)}{U_0} = \frac{\ln(h_1(x)/z_0) \ln(z_H/z_0,WF)}{\ln(h_1(x)/z_0,WF) \ln(z_H/z_0)} \]

Behind the wind farm the hub height wind speed is

\[ \frac{U(x)}{U_0} = \frac{\ln(h_1(x)/z_0) \ln(h_2(x)/z_0,WF)}{\ln(h_1(x)/z_0,WF) \ln(h_2(x)/z_0)} \]

The Park model [21,22] has been used in many different implementations. Here we use the original version of the model with a wake decay parameter of \( k = 0.04 \), for details see [23].

4. Wind farm wake measurements
An example of a wind farm wake measured with the BEACon radars is shown in Figure 1. The data were collected on January 1, 2017 from 03:30 UTC to 04:30 UTC. Only the dual-Doppler wind speed in the horizontal plane at hub height is plotted. The positions of the radars and the extent of the azimuthal scan sectors are also included in the figure. The plotted data have been averaged over one hour with south-westerly wind, nearly aligned with the turbine rows. During this hour, the wind direction was nearly constant, varying only by about 5° and the inflow wind speed 5D upstream of the first row of turbines was between 10.1 m/s and 10.7 m/s, Figure 2. The inflow wind speed and direction are calculated as the mean of the observations at hub height along a line perpendicular to the direction of the rows and spanning the width of the wind farm. This is marked with the dashed line in
front of the wind farm in Figure 1. For completeness Figure 2 also gives the inflow shear and veer profiles.

A wake region of lower wind speed is visible, stretching to the end of the domain 17 km downstream of the wind farm. Individual wind turbine wakes are discernible up to six km behind the wind farm. Further downstream they merge into a single, contiguous wind speed deficit area.

![Figure 1](image)

**Figure 1.** One-hour averaged dual-Doppler wind speed at hub height. The black dots represent the wind turbines. The radars are indicated by the black squares and their azimuthal scan sectors are shown by the thin black lines. The blue and red lines mark the transects along which the wind speed is plotted in Figure 3. They are aligned with turbine rows 1 and 4, respectively. The dashed lines show the locations of the cross-sections in Figure 4 and the upstream plane. The shaded areas are freestream bands (see the text).

In Figure 1 a blue and a red solid line are included. These are aligned with turbine rows 1 and 4, respectively. The wind speed along these lines is plotted in Figure 3 as shaded bands. The centre of these bands is the time-averaged wind speed, while their width represents the uncertainty (at the 68% level) of the mean value. This is given by $\sigma^2 = \sum_{i=1}^{N} \sigma_i^2 / N$, where the $\sigma_i$ are the uncertainties of the individual $N$ samples in the averaging interval, determined by the spatial uncertainty model, given the local, instantaneous dual-Doppler wind speed and direction.

Inside the wind farm, the wind speed along the two rows exhibits minima behind the turbine positions marked with the vertical dashed lines in Figure 3. Note that row 4 has a two-turbine gap in the middle, across which the wind speed recovers partially. At the end of this gap, starting five rotor diameters before the third turbine in row 4, the wind speed starts dropping. We speculate that this
marks the induction zone of that turbine. A smaller wind speed reduction is seen in front of the first turbine in rows 1 and 4, but not in front of any of the other interior turbines. This might be due to the interaction between the induction and the wake recovery. After the last turbines, the wind speed begins a steady recovery towards the freestream flow speed.

The wind speed across the flow direction, along the dashed lines behind the wind farm in Figure 1, is plotted in Figure 4. Only the Park model is included in these plots of the structure of the wind field across the flow, since the top-down model does not resolve individual wakes and does not give an estimate of the spreading of the wind farm wake. Comparing cross-sections at different downstream distances, the wind farm wake is seen to expand slightly as it propagates downstream.

Figure 2. Time histories of the inflow wind speed (a) and wind direction (b). Both are at hub height and averaged spatially across the flow direction 5 rotor diameters in front of the wind farm. The inflow profiles of wind speed (c) and wind direction (d) are also shown.

5. Comparison with model predictions
In this section we compare the predictions of the two studied wake models with the dual-Doppler wind speed data. We are interested in how the simulations match the evolution of the wind speed, to some degree inside the wind farm, but principally in the wake behind the wind farm.

We run the Park model with a timeseries of wind speed and wind direction inputs derived from the hub height BEACon measurements 5\(D\) upstream of the wind farm by averaging along the upstream dashed line in Figure 1. The resulting Park model predictions are then averaged temporally to allow direct comparison with the one-hour average of the observations. Note that contrary to the common use of the model, we calculate the predicted wind speed not just at the turbine positions, but also along the cross-stream lines in Figure 1 as well as the lines aligned with rows 1 and 4. When plotting the model results we use the wind speed averaged over an imaginary rotor at each point with the same dimensions as the turbines in the wind farm.
Figure 3. One-hour averaged wind speed at hub height along the blue and red lines in Figure 1 aligned with rows 1 and 4 in the layout, respectively, and the estimated free stream wind speed. The shaded bands represent the uncertainty of the one-hour mean wind speed. Also plotted are the predictions of the two models. The vertical dashed lines indicate the turbine positions. Note that there is a two-turbine gap in the middle of row 4. Distance is relative to the last turbine in the rows.

The top-down model is less sensitive to the exact wind direction, since it does not resolve individual turbine wakes. For simplicity we therefore neglect wind direction variation and assume a wind direction aligned with the turbine rows. We can then approximate the product of the mean turbine spacing in the streamwise and spanwise directions by the mean area per turbine in the array. Further, we run the top-down model with an inflow wind speed of 10.4 m/s, corresponding to the observed mean inflow wind speed.

The wind speed map in Figure 1 shows that the wind speed in the background flow is increasing downstream. We interpret this as a coastal gradient induced by the transition from high roughness land to smooth ocean surface at the coastal boundary. This complicates the analysis of the deficit recovery in the wind farm wake, since some of the observed increase in wind speed is due to the variation of the ambient flow. To compare the observations reasonably with model predictions of the wind farm wake we must account for the coastal gradient. There are two options: either model the effect on the downstream flow of the roughness transition at the coast or adjust the wake model outputs using observations of the gradient outside the wake region. To limit the modelling to wakes we have opted for the latter. To this end we define bands following the row directions both north-west and south-east of the wind farm. The bands are 250 m wide and are separated from the nearest row by 10D. They are represented by the shaded areas in Figure 1. At each downstream distance we estimate the freestream wind speed by averaging the BEACon measurements across both these bands. We further assume that the freestream wind speed can be interpolated linearly between the northern and the southern band. A similar approach has been applied in the analysis of wind farm wakes in SAR images [11]. We thus account for the variation in the freestream wind speed by:

- Modelling the wake wind speed with the inflow conditions upstream of the wind farm as inputs.
- Scaling the modelled wind speed by the ratio between the local freestream wind speed (interpolated linearly between the north and the south bands) and the upstream wind speed.
For the scenario in Figure 1 the freestream wind speed increases by 17.3% along the northern transect and 5.9% along the southern transect relative to the upstream region.

In Figure 2 the Park model follows the same pattern along rows 1 and 4: the modelled drop in wind speed behind each turbine is too sharp and deep, but the wake recovery agrees with the measurements, such that the wind speed at the next turbine position is in line with the data. The only exception is the first turbine in both rows, where the wind speed drop is too small, causing the model to overshoot the wind speed at the location of the second turbine. The model captures the recovery across the two-turbine gap in row 4. Similarly, the rate of recovery of the wake behind the wind farm in the Park model matches reasonably with the observations, when corrected for the variation in the freestream wind speed, although the model generally overestimates the wind speed behind the wind farm along the lines aligned with rows 1 and 4. We will explain this after examining the cross-stream flow structure.

In Figure 4 we compare the Park model with the measurements in the cross-stream direction. Note that the cross-stream plots are viewed from behind, so the flow direction is out of the plane. The cross-stream axis is centred on row 4. Hence positive (negative) cross-stream distances correspond to points following the dashed lines north (south) of the red line in Figure 1. Row 1 is located at a cross-stream distance of -2.9 km. The Park model predicts the correct wake width and the level of recovery of the wake up to 10 km behind the wind farm. Further downstream the model overestimates the wind speed recovery rate, as is also evident from the streamwise plots in Figure 3.

![Figure 4](image)

**Figure 4.** Plots of the cross-stream flow at different downstream distances. The locations of the cross-sections are indicated with the dashed lines in Figure 1. The shaded bands represent the uncertainty of the one-hour averaged dual-Doppler wind speed. The black lines are the Park model predictions averaged over the same period.

At downstream distances where the individual turbine wakes are still visible a shift between the modelled and observed wind speed minima can be seen. This shift increases with downstream distance. The modelled wind speed has minima given by the inflow wind direction relative to the location of the turbine rows, hence the displacement between model and observations indicates that the wind direction is gradually turning behind the wind farm. The observed wakes are shifting to negative cross-stream values, corresponding to a clock-wise deflection of the wind farm wake. This has also been predicted from fluid dynamics simulations [24]. The displacement between modelled
and observed wake peaks due to the turning wind direction is responsible for the overestimation by the Park model of the wind speed along the lines aligned with rows 1 and 4 as shown in Figure 3.

The top-down model on the other hand does not capture the steep decrease and gradual recovery of the wind speed at and behind the individual turbine positions in Figure 3. This is because the model assumes the wake loss is distributed evenly through the wind farm, which does not consist of localized turbines but a uniform area of elevated roughness. The modelled wind speed decreases gradually through the wind farm. At the location of the first turbine in both rows it is significantly higher than the dual-Doppler wind speed, but it agrees quite well with the observed wind speed at the remaining turbine positions. Except for the first km behind the wind farm, the top-down model captures the wake recovery very well.

Beyond 11 km behind the last turbine both models predict that the flow has essentially recovered to the freestream. However, the observations show that the deficit persists further downstream, even increasing slightly. This is likely a manifestation of the imperfect assumptions we had to make about the structure of the background flow.

So far, we have considered a single case study. In Figure 5 we aggregate dual-Doppler data and Park model simulations across many ten-minute periods where the wind direction is approximately aligned with the turbine rows. Cases were included if the inflow wind direction was within 7° degrees of the row direction. Just as in the cases described above, the model was run for each inflow wind speed and direction (averaged across the flow direction in front of the wind farm) and corrected for the estimated downstream variation in the freestream velocity. Subsequently, both the model results and the BEACon data in each inflow wind speed bin were combined to ensemble mean values. The individual ten-minute wind speeds as well as the ensemble mean values for a line aligned with row 4 are plotted in Figure 5.

**Figure 5.** Observed and Park model wind speeds along a line aligned with row 4. The panels correspond to different intervals of inflow wind speed ($U_0$). The individual ten-minute data are shown with the thin lines. The thick lines correspond to the ensemble mean values. The number $n$ of data points is indicated in each panel.

As in the case study, we find that the Park model predicts the wake behind the wind farm well. The influence of the decreasing thrust with increasing wind speed is evident in a reduction of the wind
speed deficit in and behind the wind farm as the inflow wind speed increases. There does not appear to be a clear correlation between inflow wind speed and the magnitude of the coastal gradient. Presumably, the wind speed variation after the coastal transition is governed by variables such as atmospheric stability and turbulence that we are not binning on. This leads to substantially different coastal gradient profiles for different cases. Together with differences in dual-Doppler data coverage this leads to large jumps in the ensemble means at distances of more than 10 km downstream of the wind farm.

6. Discussion and conclusions
We have presented dual-Doppler radar measurement of wakes within and behind an offshore wind farm. The wake can be tracked for 17 km behind the wind farm. This is the limiting range of the radars given the experimental setup. For production estimate calculations the influence of neighbouring wind farms therefore needs to be included at least to this distance, but probably further.

We have presented a case study and assessed how the wind farm wake adapts to different inflow conditions by presenting results aggregated over different upstream wind speeds. The volumetric measurements of the radars reveal both the inflow and the detailed structure of the wind farm wake. Distinct wakes from the individual turbine rows are visible six km downstream of the wind farm. Beyond that distance the wakes merge into a single wind farm wake.

A strong coastal gradient complicates the comparison of the dual-Doppler data with wake models, since the gradient is in the direction where the longest wind farm wake can be tracked with the radars. We account for the presence of this wind speed variation in the background flow by scaling the modelled wind speeds to a local freestream value (relative to the inflow upstream of the wind farm), assuming this freestream velocity can be determined from the flow on the sides of the wind farm. This leads to a reasonable agreement between the tested models and the data. However, the gradient may also play an active role in the wake recovery, impeding it or accelerating it. This is not considered by engineering wake models and could be the subject of future research.

We have compared two different engineering wake models with measurements. Both the top-down model and the Park model predict the correct wake recovery up to 10 km behind the wind farm. The Park model appears to overestimate the wind speed on lines aligned with the turbine rows, but this is due a shift in wind direction behind the wind farm, which the model does not consider. This shift causes the model wind speed minima and maxima to be shifted relative to those of the observations. When looking down a line this leads to an apparent overestimation by the model wind speed at distances, where a model wind speed peak is aligned with a wind speed minimum in the observations. Furthermore, the park model resolves the individual wakes and their merging into an extended wind farm wake region. In the case study both models overestimate the wind speed recovery in the far wake beyond 10 km downstream of the wind farm. This could be a consequence of the simplifying assumptions we have made about the gradient in the background flow. For the aggregated cases in Figure 5 the Park model sometimes overestimates and sometimes underestimates the wind speed in the far field. This shows that the wind speed overestimation in the far wake found in the case study is not systematic.

In addition, given the large distance over which we track the wind farm wake, it takes an air parcel about 45 minutes to advect from the front row of turbines to the end of the domain for an inflow wind speed of 10 m/s. For the case study presented here, this is not an issue, since the variation of wind speed and direction in the inflow is small. But in other cases, where the temporal variation is larger, this effect needs to be accounted for when comparing observations and models. Furthermore, some of the variations between the observed wind farm wakes in Figure 5 are due to environmental variables like atmospheric stability, which we do not have access to. The advection time delay and the influence
of environmental variables are not accounted for in the Park model and could be a source of some of the discrepancies between model and observations in Figure 5. This will be the subject of future work.

Some authors have proposed combining models of the two types tested here [25], representing the wind farm as an area of increased roughness and using IBL theory to calculate a reduced inflow wind speed at each turbine position. Subsequently, the wake losses are calculated using a single-wake model with the IBL-corrected wind speeds as inputs. Based on the results presented here, it appears that such an approach will tend to overestimate the wake losses, since the top-down model alone predicts wind speeds roughly in alignment with the observations at the turbine locations. Coupling it with the Park model (or another similar single-wake model) is therefore expected to lead to too conservative estimates of the wind speed in the wake.

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