Neutrino-induced Collapse of Bare Strange Stars
Via TeV-scale Black Hole Seeding

Peter Gorham, John Learned, and Nikolai Lehtinen

Department of Physics and Astronomy,
University of Hawaii at Manoa, 2505 Correa Rd., Honolulu, HI, 96822

Abstract

There is increasing observational evidence for the existence of strange stars: ultra-compact objects whose interior consists entirely of deconfined quark matter. If confirmed, their existence places constraints on the rate of formation of microscopic black holes in models which invoke a TeV-scale Planck mass. In such models, black holes can form with $\sim$ TeV masses through nuclear interactions of particles with PeV and greater energies. Once formed, these black hole states are unstable to Hawking radiation, and rapidly decay. However, if such a black hole forms in the interior of a strange star, the density is high enough that the decay may be counterbalanced by accretion, and the black hole can grow, leading to subsequent catastrophic collapse of the star. A guaranteed source of ultra-high energy particles is provided by the cosmogenic Greisen neutrinos, as well as by ultra-high energy cosmic rays, and the implied lifetimes for strange stars are extremely short, contrary to observations. The observed lifetimes of strange star candidates thus effectively exclude Planck mass scales of less than $\sim$ 2 TeV with comparable black hole masses, for up to 2 extra dimensions. Seeding of strange star collapse in scenarios with a larger number of extra-dimensions or with higher mass black holes remains a possibility, and may provide another channel for the origin of gamma-ray bursts.
To address the large disparity between the four-dimensional Planck mass \( M_4 \sim 10^{19} \text{ GeV} \) for gravitational symmetry-breaking and the corresponding electro-weak scale \( \sim 100 \text{ GeV} \), it has been proposed that space may contain compact extra dimensions with scales up to \( \sim 0.1 \text{ mm} \) \([1, 2]\). In such models, the fundamental \( n \)-dimensional Planck scale \( M_D \) (with \( n = D - 4 \) extra dimensions) could appear at energies of \( O(\text{TeV}) \). At this fundamental scale, gravitational interactions between particles can become strong enough to produce a black hole (BH) with a \( \sim \text{TeV mass} \) \([3, 4]\). At these masses, the black holes decay very rapidly due to Hawking radiation, unless there are sufficient further nuclear interactions within their lifetimes to offset their evaporation, requiring matter at near nuclear densities. Such material occurs naturally only in the cores of ultra-compact stars.

In the case of a neutron star, the density rises from \( \sim 1 \text{ gm cm}^{-2} \) to near-nuclear density \([3, 4]\) over the first 50-200 m. Any incoming particle with sufficient energy to produce a black hole will interact well before the high density region. The black hole thus formed initially encounters matter at only the crustal density, of order \( 10^{-6} \) to \( 10^{-4} \text{ fm}^{-3} \) \( (1 \text{ fm}^{-3} = 1.67 \times 10^{15} \text{ gm cm}^{-3}) \). This density is inadequate to stabilize the black hole against decay. However, if stars exist with interior densities that approach nuclear density very close to the surface, evaporation of the black hole may be offset by accretion of the surrounding matter. With sufficient density, runaway growth of the black hole will lead to catastrophic collapse of the star.

Theoretical predictions for the existence of strange stars arose from the hypothesis that \( u, d, s \) quark matter may be the final ground state of the strong interaction \([8]\), and thus the final stable state prior to gravitational collapse. Recently, a number of studies have led to several proposed candidates for such stars, among them the X-ray pulsar Her X-1 \([9]\), the bulge sources 4U 1728-34 \([11]\) and 4U 1820-30 \([12]\), the recently discovered ms pulsar SAX J1808.4-3658 \([10]\), the X-ray pulsar GRO J1744-28 \([13]\), and the isolated neutron star RX J185635-375 \([14, 15]\). Although further refinements of neutron star models may eventually fit some of these candidates, the observations favor masses and radii which are presently inconsistent with any neutron star model.

A distinguishing feature of such stars is that their mass-radius relation behaves in a manner opposite to that of classical neutron stars: whereas a neutron star’s radius decreases with
increasing mass, a strange star, by virtue of its uniform maximal nuclear density, must increase its radius with increasing mass \[16\]. It has been proposed that neutron stars may in fact be “seeded” with quark matter to convert them to strange stars \[17\]. In this letter, we consider a more radical seeding process: that of conversion of a quark star to a black hole by a TeV-mass black hole seed.

Formation of TeV-scale black holes is qualitatively a result of high energy collisions in which two particles pass within a Schwarzschild radius \(r_s\) of each other with a center of mass energy of \(E_{cm} = \sqrt{s}\) where for a nucleon at rest \(s = 2m_NE_i\) with \(m_N \simeq 1\) GeV and \(E_i\) the lab-frame energy of the incident particle. The cross section for this process is predicted to be approximately geometric with \(\sigma_{BH} \simeq \pi r_s^2\). The implied energy threshold for production on nucleon targets at rest is \(E_{i,thr} = 5 \times 10^{14}\) eV \((M_{BH} / 1\text{ TeV})^2\). It is evident that this threshold is at present out of reach of any fixed target accelerator. However, the Large Hadron Collider (LHC) has a goal of \(\sqrt{s} = 14\) TeV, corresponding to \(E \simeq 10^{17}\) eV for interactions with a fixed target. Thus production of black holes for \(M_{BH} \sim M_D\) up to several TeV may be within its reach \[4, 18, 19\].

This possibility has generated considerable recent interest.

There are several cosmic sources of particles of energies above 0.1 EeV which could play a role in black hole production. Two sources of particular interest are the high energy cosmic rays, and the related cosmogenic neutrino flux \[20\], which arises as a result of the integrated interactions of the highest energy cosmic rays (with energies above \(\sim 3 \times 10^{19}\) eV) with the cosmic microwave background radiation throughout the universe, the so-called Greisen-Zatsepin-Kuzmin (GZK) process \[21\]. The GZK neutrinos have been identified already as a likely source of black-hole production in existing neutrino detectors, and useful limits on the black-hole production cross section have already been derived from air shower data \[23, 24\].

We stress here in passing that the flux of GZK neutrinos is a generic result of the GZK cutoff, and is predicted in varying quantities for all standard models of ultra-high energy cosmic-ray propagation, as well as for many models in which the GZK cutoff is violated. The only assumptions necessary for the the existence of a GZK neutrino flux are that (a) the local universe is not greatly different from any other cosmic locale in its energy density of \(\geq 3 \times 10^{19}\) eV cosmic rays; and (b) that photopion production is well-behaved at the center-of-momentum-
frame energy ($\sim 1$ GeV) needed for the GZK process. Since cross sections are well-known at GeV energies, this latter condition reduces to a requirement on the accuracy of the Lorentz transformation for $\gamma \simeq 10^{11}$.

The crucial factor in determining the likelihood of black hole production by a given energetic particle is the black-hole production cross-section. Estimates are based on geometric arguments for interactions partons are based on the $D + n$-dimensional Schwarzshild radius [18]:

$$
\sigma_{BH} \simeq \pi r_s^2 = \frac{1}{M_D^2} \left[ \frac{M_{BH}}{M_D} \left( \frac{8 \Gamma(\frac{n+3}{2})}{n + 2} \right) \right]^{\frac{2}{n+1}}.
$$

Extensions of this argument for high energy neutrino interactions [26] gives cross sections of order $10^{-32}$ to $10^{-30}$ cm$^2$ for neutrinos from 0.1-1 EeV [22, 23, 25, 26]. Evaluation of production cross sections for $\sim$ few TeV black holes in $p\bar{p}$-interactions at the LHC indicate cross sections in the range of $10^{-34}$ to $10^{-32}$ cm$^2$ for $n$ in the range of 4 to 6 [18]. Black hole production in hadron collisions may thus be comparable to top quark production at the LHC.

There is still considerable debate over whether there is an exponential suppression of these values, leading to cross sections 1-3 orders of magnitude smaller [27]. In either case, it is evident that the fraction of cosmic ray interactions which can produce black holes will be at most the ratio of the black-hole cross section to the total hadronic interaction cross section, thus $\simeq 10^{-10}$ to $10^{-7}$ per interaction.

For ultra-high energy neutrinos, however, the black hole production cross section can exceed the standard model deep-inelastic hadronic cross section by 1-2 orders of magnitude [22, 23, 25, 26], making black-hole production the dominant process for neutrinos in many cases. Even for the exponentially-suppressed cases, the fraction of EeV interactions that can produce black holes is no less than about 10%. Thus, although the flux of GZK neutrinos is, for most models, less than that of the cosmic rays at these energies, the high probability for black-hole production in this scenario means that the GZK neutrino flux will dominate the rate.

If bare strange stars exist, their density profile is expected to be a nearly constant $10^{15}$ gm cm$^{-3}$ ($\sim 1$ fm$^{-3}$) up to within 1 fm of the surface [28]. Under these conditions, the neutrino interaction length is $L_\nu = (\rho_s \sigma_{tot} N_A)^{-1} \simeq 10^{-8}$ cm ($10^4$ pb/$\sigma_{tot}$)(1 fm$^{-3}$/\rho_s) where $\sigma_{tot} = \sigma_{BH} + \sigma_{sm}$ is the sum of the black hole] and standard model neutrino cross sections. The
interaction thus evidently takes place within a few atomic diameters of the surface.

The rest frame lifetime for the black hole due to evaporation via Hawking radiation is estimated to be \[ \tau \sim \frac{1}{M_D} \left( \frac{M_{BH}}{M_D} \right)^{(3+n)/(1+n)} . \] (2)

For \( M_D \approx M_{BH} \approx 1 \text{ TeV} \), the lifetime is \( \sim 10^{-27} \text{ s} \). Evaporation in the black hole rest frame is thus extremely rapid if there is nothing to prevent it.

The Lorentz factor of the black hole \( \gamma = E_{\nu}/M_{BH} \) has a significant effect on the black hole evaporation process \[29\]. The black hole is created within the highly degenerate Fermi gas of the interior of the strange star. For young strange stars this consists of a photon component with effective temperature of order 10-25 MeV, and a quark component with Fermi temperature \( T_F \approx 0.4 - 1 \text{ GeV} \), with an upper limit set by the \( c \)-quark mass at 1.15-1.35 GeV.

The Lorentz boost modifies the angular distribution of the thermal bath temperature by the factor \( T'(\theta) = T_{\text{bath}}[\gamma(1 + \beta \cos \theta)]^{-1} \). Using the Stefan-Boltzmann law (which is appropriate here since the boosted effective temperature is much higher than the fermion masses), this factor can be integrated over all directions to yield the effective temperature of the bath as observed in the black hole frame \[30\]:

\[ T_{\text{eff}} = T_{\text{bath}} \sqrt{\gamma} \left( 1 + \frac{\beta^2}{3} \right)^{\frac{1}{4}} . \] (3)

The bath temperature is determined by the temperatures of each of the partial pressure components in the thermal bath, but is dominated by the Fermi pressure of the quark-gluon plasma.

The Hawking temperature of the black hole is given by

\[ T_H = M_D \left( \frac{M_D}{M_{BH}} \frac{n + 2}{8 \Gamma(\frac{n+2}{2})} \right)^{\frac{1}{n+1}} \left( \frac{n + 1}{4\sqrt{\pi}} \right) , \] (4)

and falls typically in the range of \( T_H = 100 - 500 \text{ GeV} \) for \( M_D \approx 1 - 3 \text{ TeV} \) and similar black hole masses. To prevent immediate evaporation of the black hole, we require \( T_{\text{eff}} \geq T_H \) which gives a condition on the initial neutrino energy: \( E_{\nu,\text{thr}} \approx M_{BH}(3T_H/4T_{\text{bath}})^2 \). For \( T_{\text{bath}} = T_F, M_D \approx M_{BH} \leq 5 \text{ TeV}, n = 1, 2, 3, 4 \), this requirement yields \( E_{\nu,\text{thr}} = (1.5 \times 10^{19}, 5 \times 10^{19}, 10^{20}, 1.5 \times 10^{20}) \text{ ev} \). Black holes created by neutrinos of energy greater than this will be initially stabilized by the apparent temperature of the bath in their own rest frame.
FIG. 1: Black hole mass & mean free path evolution vs. number of collisions in the interior of a strange star for $M_{\text{BH, initial}} = 1 \text{ TeV}$, $E_\nu = 10^{20} \text{ eV}$ (the black hole creation is the first interaction). The largest fractional mass increase and drop in the mean free path happens at the first subsequent collision.

The mass evolution of the initially relativistic black hole through accretion can in principle also be treated by a quasi-thermodynamic approach. Kinematically, however, the first interaction of the boosted black hole with a quark is the most significant, since the black hole absorbs all of the mass energy of this particle, including the large portion of its own boost that the particle has in the center of momentum frame. Thus we treat the fermion accretion problem as collisions with a black hole with a capture cross section given by the classical value $\sigma_c = \frac{27}{4} \pi r_s^2$ which accounts for the impact parameter of geodesics into the hole [23, 31].

We have treated the black hole- parton accretion process both numerically and semi-analytically, and we find that, as long as the black hole effective lifetime is long enough for it to interact with a single parton with reasonable probability, the large amount of mass-energy absorbed in this first interaction leads invariably to the black hole eventually coming to rest with a mass equal to the energy of the particle which created it: $M_{\text{BH, final}} \simeq E_\nu$. For $E_\nu = 10^{20} \text{ eV}$,
\( n = 2 \), and initial \( M_{\text{BH}} \sim M_D \sim 1 \text{ TeV} \), this requires of order \( 10^{11} \) collisions with massive quarks in the star, over a total distance of order 0.1 mm. A plot of the first 50 collisions of the black hole mass evolution for this case is shown in Fig. 1.

At this stage the black hole is approximately at rest with a much reduced surface temperature due to its increased mass. If \( T_F \geq T_H \) at this stage the black hole can continue to grow; if not, evaporation will proceed again. This condition can be solved for the minimum incoming neutrino energy required for runaway black hole growth:

\[
E_{\nu, \text{min}} = \frac{M_D^{n+2}}{T_{\text{bath}}^{n+1}} \frac{n + 2}{8 \Gamma \left( \frac{n+3}{2} \right)} \frac{1}{\left( \frac{n+1}{4\sqrt{\pi}} \right)^{1/(n+1)}}.
\] (5)

For \( n = 1, 2, 3, 4 \), and \( M_{\text{BH}} \sim M_D \sim 1 \text{ TeV} \), and \( T_{\text{bath}} = 1 \text{ GeV} \), \( E_{\nu, \text{min}} = (2.5 \times 10^{16}, 2.8 \times 10^{19}, 3 \times 10^{22}, 4 \times 10^{25}) \text{ eV} \), which shows that there is a region of the \( n \leq 2 \) extra-dimension parameter space that is probed by the expected GZK neutrino flux.

We can place a lower limit on combinations of \((M_D, T_F)\) for \( n \leq 2 \) based on the known (at least) several year lifetimes \( \tau_s \) of existing strange star candidates. We invert equation (5) for \( E_{\nu}^{\text{max}} = 10^{20.5} \text{ eV} \), where GZK neutrinos are expected to have an integral flux comparable to the measured cosmic ray flux at this energy, \( F_{\nu}(E_\nu \geq 10^{20.5}) \simeq 0.01 \text{ km}^{-2} \text{ yr}^{-1} \text{ over} 4\pi \text{ sr} \). Using this flux, we probe values of \( M_D \) up to several TeV \((n = 2)\) or several tens of TeV \((n = 1)\). For all values of the neutrino-black hole production cross section (including exponential suppression) we find that the implied stellar lifetimes are at most several years, contrary to observations of the strange star candidates noted previously.

In Fig. 2, we plot the implied limits on \((M_D, T_F)\) based on probable strange star lifetimes greater than these predictions. There is no real dependence on \( M_{\text{BH}} \) in our analysis, since we find that, as long as the initial black hole mass is kinematically allowed, it rapidly acquires all the initial mass-energy of the incoming neutrino. Thus our results apply to black hole masses up to \( \sim 80 \text{ TeV} \). Limits are plotted as a function of \( T_F \) the effective thermal bath temperature of the stellar interior.

Although the standard spin-down lifetimes that can be estimated for isolated radio pulsars are not available for strange star candidates, their probable lifetimes are at least \( 10^6 \text{ yr} \), based on the fact that several of the candidates are in evolved binary systems. Thus the lower limit for
FIG. 2: Limits on black hole production and the gravity scale $M_D$ based on bare strange star lifetimes. The limit on $M_D$ is plotted as a function of the assumed Fermi temperature of the strange star interior $T_F$. Values of $350 \text{ MeV} \leq T_F \leq 1000 \text{ MeV}$ fall in the acceptable regions of strange star equations of state, with $T_F \sim 450 \text{ MeV}$ typical.

$M_D$ based on GZK neutrino fluxes is constrained only by the lack of fluxes at higher neutrino energies.

Other limits from astrophysical considerations are in fact more stringent that our limits, but depend on arguments that require analysis of stellar energy loss due to Kaluza-Klein (KK) gravitons [32, 33], or limits on the decay products of these particles [34]. Such KK modes may either be strongly suppressed [33], or the decays may proceed through invisible channels [36]. The present results do not depend in any way on the KK emission or decay processes. The black hole formation process is dependent only on geometric and kinematic arguments regarding the extra dimensions, and further work on understanding the potential stabilization and growth requirements for these black holes may still extend the application or limits to higher values of $n$. 
We have focused our discussion on the apparent limits that obtain for $M_D$ given that strange star candidates appear to be relatively stable. However, it is interesting to speculate on whether seeding of such stars with black holes formed via interactions of ultra-high energy cosmic rays and neutrinos could account to the observed rate of Gamma-ray burst (GRB) events, since stellar collapse to a black hole is one mechanism that appears capable of producing the observed energy of GRBs events. In fact, in the absence of other constraints on $M_D$, it is clearly possible to create almost any desired rate for stellar collapse of bare strange stars, since one can almost arbitrarily reduce the cross section and GZK neutrino fluxes by raising the minimum value for $M_D$ or $M_{BH}$. The addition of a thin layer of crust material allows for even further tuning, and thus it appears that black hole seeding of strange stars as a possible channel for stellar collapse is difficult to rule out at present.

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