New phenomena in laser-assisted leptonic decays of the negatively charged boson $W^-$

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Abstract

The majority of studies and experiments performed at electron-positron colliders over the last two decades have focused on studying $W$ and $Z$ weak force-carrying bosons and accurately measuring all their properties, not only because they play an important role in establishing Standard Model theory and providing an accurate test of its predictions of particle interactions, but also because they are a unique tool for probing manifestations of the new physics beyond the standard model. Therefore, it would be particularly important to discuss some of the new phenomena and changes that can arise in these bosons when their decay occurs under an external electromagnetic field. In a recent paper, we investigated the laser effect on the final products of $Z$ boson decay and found that laser had an unprecedented effect on branching ratios. In this work and within the standard Glashow-Weinberg-Salam model of electroweak interactions, we study theoretically the leptonic decay of the $W^-$-boson ($W^- \rightarrow \ell^- \bar{\nu}_\ell$) in the presence of a circularly polarized electromagnetic field and we examine the laser effect, in terms of its field strength and frequency, on the leptonic decay rate and the phenomenon of multiphoton processes. The calculations are carried out using the exact relativistic wave functions of charged particles in an electromagnetic field. It was found that the laser significantly contributed to reducing the probability of $W^-$-boson decay. We show that the laser-assisted decay rate is equal to the laser-free one only when the famous Kroll-Watson sum rule is fulfilled. The notable effect of the laser on the leptonic decay rate was reasonably interpreted by the well-known quantum Zeno effect or by the opening of channels other than leptonic ones to decay. This work will pave the way for an upcoming one to study the hadronic decay of the $W^-$-boson and then explore the laser effect on its lifetime and branching ratios.

Keywords: electroweak interaction, laser-assisted processes, decay rate.

1. Introduction

The tremendous and rapid progress that laser technology has made since its invention in the 1960s has enabled a new field of theoretical and experimental studies to explore the interactions of electromagnetic (EM) fields with matter at high intensities [1]. This important development of laser technology has played an important role in the production of high-power laser systems with a maximum intensity of $10^{22}$W/cm$^2$ [2,3] and researchers currently expect that higher intensities can be
achieved in the near future [4–7]. Today, these high-power laser systems have become indispensable
equipment and are used in laboratories all over the world, thanks to which studies of the basic
processes of quantum electrodynamics (QED) [8–13] and electroweak theory [14,15] in the presence of a
strong laser field and realistic in experiments have continued to attract significantly the attention of
physicists. Through all this, scientists are seeking to know and understand how elementary particles
behave and how their properties change when inserted in an EM field. The theory describing the
fundamental interactions of these particles is called the Standard Model (SM) and it is a gauge the-
ory, which describes strong, weak and electromagnetic interactions by exchanging the corresponding
spin-1 gauge fields: eight massless gluons and one massless photon, respectively, for the strong and
electromagnetic interactions, and three massive bosons, $W^\pm$ and $Z^0$, for the weak interaction. It is
one of the most successful achievements of modern physics in that it provides a very elegant theoreti-
cal framework capable of describing known experimental facts in particle physics with great precision.
The discovery of the $W$ [16,17] and $Z$ [18,19] bosons in 1983 by the UA1 and UA2 collaborations at
the CERN’s $p\bar{p}$ collider provided direct confirmation of the unification of the weak and electromagnetic
interactions in a common framework that describes them together. This common framework
is the so-called electroweak theory predicted by physicists Sheldon Lee Glashow [20], Steven Wein-
berg [21] and Abdus Salam [22] in the late 1960s, in which they explain that the electromagnetic and
weak forces, which have long been considered as separate entities, are in fact manifestations of
the same fundamental interaction. In recognition of their role in the discovery of the $W$ and $Z$
particles, the CERN physicist Carlo Rubbia [23] and engineer Simon van der Meer [24] were awarded
the Nobel Prize in physics in 1984. Detectable $W$ particles can be produced in proton-antiproton
collision experiments by processes such as $ud$ or $\bar{u}d \to W$ followed by subsequent leptonic $(W \to \ell \nu)$
or hadronic decay $(W \to q\bar{q})$, where $\ell = e, \mu, \tau$ and $q$ or $q'$ represent one of the quarks $u, d, c, s, o b except t since top quark is heavier than the $W$ boson. Experiments at the Large Electron-
Positron Collider (LEP) and the Stanford Linear Collider (SLC) refine the measurements of the
$W$-boson properties (mass, total decay rate and cross sections of their production). At the same
time, the properties of $W$ bosons are studied in the Tevatron proton-antiproton accelerator. All
these experiments have given more precise measurements of the $W$-boson mass and its total decay
rate, which are now known to be $M_W = 80.379 \pm 0.012$ GeV and $\Gamma_W = 2.085 \pm 0.042$ GeV [25].
The accuracy obtained in these experiments makes it possible to test the predictions of the SM at
the level of radiative corrections. This work has, as an objective, to study the leptonic decay of
the negatively charged boson $W^-$ in the presence of a circularly polarized EM field. As mentioned
above, the $W^-$-boson can decay into a lepton and antineutrino $W^- \to \ell^- \bar{\nu}_\ell$ (leptonic decay) or into
a pair of quarks $W^- \to q\bar{q}'$ (hadronic decay). Here, we will only consider the first channel (leptonic
channel), where we will try to realize the effect of the EM field strength and frequency on the decay
rate. In 2004, the author Kurilin studied the leptonic decays of the $W^-$-boson in a strong crossed
EM Field, where he calculated the expression of the decay rate and studied its changes in terms of
the external-field-strength parameter $\chi = e M_W^2 \sqrt{-\langle F_{\mu\nu} F^{\mu\nu} \rangle}$, where $F_{\mu\nu}$ is the EM tensor [26].
Kurilin’s approach is completely different from the approach followed here. In his study, Kurilin
restricts his consideration to the case of a so-called crossed field, which is a superposition of constant
electric and magnetic fields whose strength vectors are equal and orthogonal. The wave functions of
the charged particles in this case have a simple expression in terms of the EM tensor $F_{\mu\nu}$, and this
makes the related mathematical calculations less complicated. Some of the transformations in our
calculations have been simplified by the introduction of the ordinary Bessel functions, whose order
is interpreted as the number of exchanged photons, while Kurilin calculations have been performed
with the help of special mathematical functions generally termed Airy functions [27]. Despite this
difference in the method and the nature of the EM field, we have compared our results obtained here
with the results obtained in his research and give some explanations for both results obtained. There is also an earlier paper who did the same study in brief giving the expression of the decay rate in the presence of a circularly polarized EM field \[28\]. An overview of the interaction of these bosons with the EM field and how to write the expression for the wave functions describing them can be found in refs. \[29, 30\]. Although \(W\) bosons are best known for their role in radioactive decay, their decay may provide the opportunity to discover supersymmetric particles and probe new physics beyond the SM \[31, 32\]. The present work is part of a series of studies we have conducted to know and understand the effect of the EM field on experimentally measurable quantities during electroweak decay processes (decay rate, lifetime, branching ratio). So far, we have calculated both the pion \[33\] and \(Z\)-boson \[34\] decays in the presence of an EM field in an attempt to contribute to enriching the debate on this topic. In this same context, we try to study the decay of the \(W^-\)-boson in two stages. First, we will study its leptonic decay and thus the effect of the EM field on the leptonic decay rate. Secondly, in a forthcoming research, we will study the hadronic decay and collect the results obtained with the present results in order to be able to study the effect of the laser on both the lifetime and the branching ratio for each channel as two important points that must be treated. It is therefore this work that will pave the way for the next one, and by that time we will have studied this decay and have sufficiently covered all its aspects. The rest of the paper is arranged as follows. In section 2., we will try to formulate, in detail, the theoretical expression of the decay rate, then in section 3., we will present the results obtained and highlight the extent of the laser influence on the decay rate. The last section 4. is devoted to summarizing the conclusions that we have reached. We only mention here that we used, during our calculations, known natural units \(c = \hbar = 1\).

### 2. Theoretical formulation of the decay rate in a laser field

We begin by considering the process of electroweak decay of a boson vector \(W^-\), with four-momentum \(p\) and mass \(M_W\), into a lepton \(\ell^-\) and corresponding antineutrino \(\bar{\nu}_\ell\) in the field of a plane EM wave

\[
W^- (p) \rightarrow \ell^- (p_1) + \bar{\nu}_\ell (p_2), \quad (\ell = e, \mu, \tau)
\] (1)

where the arguments are our labels for the four-momenta. The corresponding Feynman diagram is shown in figure 1. The laser field, which is assumed to be circularly polarized monochromatic, is described by the following classical four-potential

\[
A^\mu (\phi) = a_1^\mu \cos (\phi) + a_2^\mu \sin (\phi),
\] (2)

![Figure 1: Lowest order Feynman diagram of the leptonic \(W^-\)-boson decay.](image-url)
depending on a single variable, \( \phi = (k.x) \), the phase of the laser field. \( k = (\omega, \mathbf{k}) \) is the wave four-vector \( (k^2 = 0) \) and \( \omega \) is the laser frequency. The four-amplitudes \( a_{11} = |a|((0, 1, 0, 0) \) and \( a_2 = |a|((0, 0, 1, 0) \) are equal in magnitude and orthogonal, which implies \( (a_1, a_2) = 0 \) and \( a_1^2 = a_2^2 = -|a|^2 = -(E_0/\omega)^2 \) where \( E_0 \) is the amplitude of the electric field. We suppose that the four-potential satisfies the Lorentz gauge condition, \( k_\mu A^\mu = 0 \), which means \( (k.a_1) = (k.a_2) = 0 \), indicating that the wave vector \( \mathbf{k} \) is chosen to be along the z-axis.

The lowest-order scattering S-matrix element for the laser-assisted leptonic \( W^- \) decay reads [35]

\[
S_{fi}(W^- \rightarrow \ell^- \bar{\nu}_\ell) = \frac{i g}{2\sqrt{2}} \int d^4x \bar{\psi}_\ell(x) \gamma^\mu (1 - \gamma_5) \psi_\ell(x) W^\mu_\mu(x),
\]

where \( g \) is the electroweak coupling constant. \( \psi_\ell(x) \) is the wave function of the relativistic lepton \( \ell^- \) in an EM field given by the relativistic Dirac-Volkov functions normalized to the volume \( V \) [36]

\[
\psi_\ell(x) = \left[ 1 + \frac{e \bar{k}.A}{2(k.p_1)} \right] \frac{u(p_1, s_1)}{\sqrt{2Q_1 V}} \times e^{iS(q_1, x)},
\]

where regarding the sign one should note that \( e = -|e| \) is the charge of the electron, and

\[
S(q_1, x) = -q_1.x - \frac{e(a_1.p_1)}{k.p_1} \sin(\phi) + \frac{e(a_2.p_1)}{k.p_1} \cos(\phi).
\]

\( u(p_1, s_1) \) represents the Dirac bispinors for the free lepton with momentum \( p_1 \) and spin \( s_1 \) satisfying \( \sum_{s_1} u(p_1, s_1) \bar{u}(p_1, s_1) = 1 \) where \( m_\ell \) is the rest mass of the lepton. The 4-vector \( q_1 = (Q_1, q_1) \) is the quasi-momentum that the lepton acquires in the presence of the EM field

\[
q_1 = p_1 - \frac{e^2 a^2}{2(k.p_1)}k, \quad q_1^2 = m_\ell^2 - e^2a^2,
\]

where \( m_\ell^* \) is an effective mass of the lepton inside the EM field.

For the laser-dressed \( W^- \) (1-spin particle), its wave function can be written in the form [29]

\[
W^-_\mu(x) = \left[ g_{\mu\nu} - \frac{e}{(k.p)}(k_\mu A_\nu - k_\nu A_\mu) - \frac{e^2}{2(k.p)^2}A^2 k_\mu k_\nu \right] \varepsilon_\mu(p, \lambda) \frac{\varepsilon^*_{\lambda}(p, \lambda)}{\sqrt{2q_0 V}} \times e^{iS(q, x)},
\]

where \( g_{\mu\nu} = \text{diag}(1, -1, -1, -1) \) is the metric tensor of Minkowski space, \( \varepsilon_\mu(p, \lambda) \) is the \( W^- \)-boson polarization four-vector such that the summation over all three directions of polarization \( \lambda \) yields \( \sum_{\lambda=1}^3 \varepsilon_\mu(p, \lambda) \varepsilon^*_{\lambda}(p, \lambda) = -g_{\mu\nu} + p_\mu p_\nu/M_W^2 \), and

\[
S(q, x) = -q.x + \frac{e(a_1.p)}{k.p} \sin(\phi) - \frac{e(a_2.p)}{k.p} \cos(\phi),
\]

with the 4-vector \( q = p - \left[ e^2a^2/2(k.p) \right] k \) \( (q^2 = M_W^2) \) and \( M_W^* = \sqrt{M_W^2 - e^2a^2} \) are, respectively, the quasi-momentum and the effective mass that the boson \( W^- \) acquires inside the EM field. The outgoing antineutrino \( \bar{\nu}_\ell \) is treated as massless particle with four-momentum \( p_2 \) and spin \( s_2 \). According to the Feynman rules, it is represented by an incoming wave function with negative four-momentum as follows [35]:

\[
\psi_{\bar{\nu}_\ell}(x) = \frac{v(p_2, s_2)}{\sqrt{2E_2 V}} e^{ip_2.x},
\]
where \( E_2 = p_2^0 = |\mathbf{p}_2| \) and \( v(p_2, s_2) \) is the Dirac spinor satisfying \( \sum_{s_2} v(p_2, s_2) \overline{\nu}(p_2, s_2) = \hat{\mathbf{p}}_2 \). Substituting the lepton, antineutrino and \( W^- \)-boson wave functions (4), (7) and (9) into expression (3) for the S-matrix element, we obtain

\[
S_{fi}(W^- \to \ell^- \bar{\nu}_\ell) = \frac{i g}{2 \sqrt{2} \sqrt{S(E_2 Q_1 p_0)}} \int d^4 x \overline{\nu}(p_1, s_1) \left[ \left[ 1 + C(p_1) A \right] \gamma^\mu (1 - \gamma_5) \right.
\]

\[
\times \left[ g_{\mu \nu} - C(p) (k_\mu A_\nu - k_\nu A_\mu) - \frac{C(p)^2}{2} a^2 k_\mu k_\nu \right] \nu(p_2, s_2)
\]

\[
\times \varepsilon_\mu(p, \lambda) e^{ip_2 x} e^{i(S(q,x) - S(q_1,x))},
\]

where \( C(p_1) = e/(2(k_1 p_1)) \) and \( C(p) = e/(k_2 p) \). Now, let us transform the exponential term \( e^{i(S(q,x) - S(q_1,x))} \) by introducing the following parameter

\[
z = \sqrt{\alpha_1^2 + \alpha_2^2} \quad \text{with} \quad \alpha_1 = -e \left( \frac{a_1 p_1}{k_2 p_1} \right) \quad \alpha_2 = -e \left( \frac{a_2 p_2}{k_2 p_1} \right),
\]

this yields

\[
e^{i(S(q,x) - S(q_1,x))} = e^{i(q_1 - q)x} e^{-iz \sin(\phi - \phi_0)},
\]

with \( \phi_0 = \arctan(\alpha_2/\alpha_1) \). Therefore, the S-matrix element becomes

\[
S_{fi}(W^- \to \ell^- \bar{\nu}_\ell) = \frac{i g}{2 \sqrt{2} \sqrt{S(E_2 Q_1 p_0)}} \int d^4 x \overline{\nu}(p_1, s_1) \left[ C_0 + C_1 \cos(\phi) + C_2 \sin(\phi) \right]
\]

\[
\times v(p_2, s_2) \varepsilon_\mu(p, \lambda) e^{ip_2 x} e^{-iz \sin(\phi - \phi_0)},
\]

where the three quantities \( C_0, C_1, \) and \( C_2 \) are expressed as follows:

\[
C_0 = \gamma_\nu (1 - \gamma_5) - \frac{C(p)^2}{2} a^2 k_\mu k_\nu \gamma^\mu (1 - \gamma_5),
\]

\[
C_1 = C(p_1) \phi_1 \bar{k} \gamma_\nu (1 - \gamma_5) - C(p) \gamma^\mu (1 - \gamma_5) (k_\mu a_{1\nu} - k_\nu a_{1\mu}) - \frac{C(p)^2}{2} C(p_1) a^2 k_\mu k_\nu
\]

\[
\times \phi_1 \bar{k} \gamma^\mu (1 - \gamma_5),
\]

\[
C_2 = C(p_1) \phi_2 \bar{k} \gamma_\nu (1 - \gamma_5) - C(p) \gamma^\mu (1 - \gamma_5) (k_\mu a_{2\nu} - k_\nu a_{2\mu}) - \frac{C(p)^2}{2} C(p_1) a^2 k_\mu k_\nu
\]

\[
\times \phi_2 \bar{k} \gamma^\mu (1 - \gamma_5).
\]

The linear combination of the three different quantities in equation (13) can be transformed by the well-known identities involving ordinary Bessel functions \( J_s(z) \) [37]:

\[
\begin{bmatrix}
1 \\
\cos(\phi) \\
\sin(\phi)
\end{bmatrix} \times e^{-iz \sin(\phi - \phi_0)} = \sum_{s=-\infty}^{+\infty} \begin{bmatrix} B_s(z) \\ B_{1s}(z) \\ B_{2s}(z) \end{bmatrix} e^{-is\phi},
\]

where

\[
\begin{bmatrix} B_s(z) \\ B_{1s}(z) \\ B_{2s}(z) \end{bmatrix} = \begin{bmatrix} J_s(z)e^{i\phi_0} \\ (J_{s+1}(z)e^{i(\phi_0 + \phi_0/2)}/2 + J_{s-1}(z)e^{i(\phi_0 - \phi_0)/2})/2i \\ (J_{s+1}(z)e^{i(\phi_0 + \phi_0/2)}/2i - J_{s-1}(z)e^{i(\phi_0 - \phi_0)/2})/2i \end{bmatrix},
\]

(16)
where \( z \) is the argument of the Bessel functions defined in equation (11) and \( s \), their order, can be interpreted as the number of exchanged photons. Using these transformations in equation (13) and integrating over \( d^4x \), the matrix element \( S_{fi} \) becomes

\[
S_{fi}(W^- \rightarrow e^- \bar{\nu}_e) = \frac{ig}{2\sqrt{2}/\sqrt{8E_2}Q_1p_0\sqrt{v}} \sum_{s = -\infty}^{\infty} \mathcal{M}^s_{fi}(2\pi)^4 \delta^4(p_2 + q_1 - q - sk),
\]

where the quantity \( \mathcal{M}^s_{fi} \) is defined by

\[
\mathcal{M}^s_{fi} = \bar{u}(p_1, s_1)\Lambda[v(p_2, s_2)\varepsilon_{\mu}(p, \lambda)],
\]

where

\[
\Lambda_s = C_0B_s(z) + C_1B_{s1}(z) + C_2B_{s2}(z).
\]

Here, we call the quantity \( \mathcal{M}^s_{fi} \) "the spinorial part" since it is the only part of the matrix element which depends on spin. Multiplying the squared S-matrix element by the density of final states, summing over spins of leptons and antineutrinos, averaging over the polarization of the incoming boson, and finally dividing by the time \( T \) yields the decay rate

\[
\Gamma(W^- \rightarrow e^- \bar{\nu}_e) = \sum_{s = -\infty}^{\infty} \Gamma^s(W^- \rightarrow e^- \bar{\nu}_e),
\]

where \( \Gamma^s \), called the photon-number-resolved decay rate, is defined by

\[
\Gamma^s(W^- \rightarrow e^- \bar{\nu}_e) = \frac{g^2}{64q_0} \int \frac{d^3q_1}{(2\pi)^3Q_1} \int \frac{d^3p_2}{(2\pi)^3E_2} (2\pi)^4 \delta^4(p_2 + q_1 - q - sk)|\mathcal{M}^s_{fi}|^2,
\]

where we have used \([(2\pi)^4 \delta^4(p_2 + q_1 - q - sk)]^2 = VT(2\pi)^4 \delta^4(p_2 + q_1 - q - sk)\), and

\[
|\mathcal{M}^s_{fi}|^2 = \frac{1}{3} \sum_{\lambda} \sum_{s_1, s_2} |\mathcal{M}^s_{fi}|^2 = \frac{1}{3} \sum_{\lambda} \sum_{s_1, s_2} |\bar{u}(p_1, s_1)\Lambda[v(p_2, s_2)\varepsilon_{\mu}(p, \lambda)|]^2.
\]

Performing the integration over \( d^3p_2 \) and using \( \delta^4(p_2 + q_1 - q - sk) = \delta^3(p_2 + q_1 - q - sk)\delta(E_2 + Q_1 - Q - s\omega) \), the photon-number-resolved decay rate \( \Gamma^s \) becomes

\[
\Gamma^s(W^- \rightarrow e^- \bar{\nu}_e) = \frac{g^2}{64(2\pi)^2p_0} \int \frac{d^3q_1}{Q_1E_2} \delta(E_2 + Q_1 - Q - s\omega)|\mathcal{M}^s_{fi}|^2,
\]

with \( p_2 + q_1 - q - sk = 0 \). We choose the \( W^- \)-boson rest frame in which \( Q = q_0 = M_W^* \) and \( q = 0 \), then \( p_2 = sk - q_1 \). Hence, using \( d^3q_1 = |q_1|^2 d|q_1| d\Omega_1 \), we obtain

\[
\Gamma^s(W^- \rightarrow e^- \bar{\nu}_e) = \frac{g^2}{64(2\pi)^2p_0} \int \frac{|q_1|^2 d|q_1| d\Omega_1}{Q_1E_2} \delta\left(\sqrt{s\omega}^2 + |q_1|^2 - 2s\omega|q_1|\cos(\theta) + \sqrt{|q_1|^2 + m_{\ell}^2 - M_W^* - s\omega}\right)|\mathcal{M}^s_{fi}|^2.
\]

The remaining integral over \( d|q_1| \) can be solved by using the familiar formula [35]:

\[
\int dx f(x) \delta(g(x)) = \left. \frac{f(x)}{|g'(x)|} \right|_{g(x) = 0}.
\]
Thus we get

$$\Gamma^s(W^- \to \ell^- \nu_{\ell}) = \frac{g^2}{64(2\pi)^3p_0} \int \frac{|q_1|^2|d\Omega_{\ell}}{Q_1E_2|g'(|q_1|)|} |\bar{M}^s_{fi}|^2.$$ \hspace{1cm} (26)

where $g^2 = G_F M_W^2/\sqrt{2}$, with $G_F = (1.166 \pm 0.000 \, 02) \times 10^{-11} \text{ MeV}^{-2}$ is the Fermi coupling constant, and

$$g'(|q_1|) = \frac{|q_1| - s\omega \cos(\theta)}{\sqrt{(s\omega)^2 + |q_1|^2 - 2s\omega|q_1| \cos(\theta)}} + \frac{|q_1|}{\sqrt{|q_1|^2 + m_\ell^2}}.$$ \hspace{1cm} (27)

Using the fact that $d\Omega_{\ell} = \sin(\theta)d\theta d\varphi$ and $\int d\varphi = 2\pi$, the equation (26) can be written as follows:

$$\Gamma^s(W^- \to \ell^- \nu_{\ell}) = \frac{g^2}{128\pi p_0} \int |q_1|^2|\sin(\theta)d\theta|\frac{|d\Omega_{\ell}}{Q_1E_2|g'(|q_1|)|} |\bar{M}^s_{fi}|^2,$$ \hspace{1cm} (28)

where the integral over $d\theta$ is performed with the help of numerical integration.

The term $|\bar{M}^s_{fi}|^2$ can be calculated by converting the sums over the spins into traces as follows:

$$|\bar{M}^s_{fi}|^2 = \frac{1}{3} \left( -g_{\mu\nu} + \frac{p_{\mu}p_{\nu}}{M_W^2} \right) \text{Tr}\left[ (\not{\gamma}_1 + m_\ell)\gamma_5 \not{\phi}_1 \not{\Lambda}_s \not{\phi}_2 \not{\Lambda}_s \right],$$ \hspace{1cm} (29)

where

$$\not{\Lambda}_s = \gamma^0 \Lambda^\dagger_s \gamma^0,$$

$$= C_0 B_s^*(z) + C_1 B_s^*(z) + C_2 B_2^*(z),$$ \hspace{1cm} (30)

and

$$C_0 = \gamma^0 C^\dagger_0 \gamma^0 = \gamma_\mu(1 - \gamma_5) - \frac{C(p)^2}{2} a^2 k_\mu k_\nu \gamma^\nu(1 - \gamma_5),$$

$$C_1 = \gamma^0 C^\dagger_1 \gamma^0 = C(p_1)\gamma_\mu(1 - \gamma_5)\not{\phi}_1 - C(p)\gamma_\nu(1 - \gamma_5)(k_\mu a_{1\nu} - k_\nu a_{1\mu}) - \frac{C(p)^2}{2} C(p_1) a^2 k_\mu k_\nu \gamma^\nu(1 - \gamma_5)\not{\phi}_1,$$ \hspace{1cm} (31)

$$C_2 = \gamma^0 C^\dagger_2 \gamma^0 = C(p_1)\gamma_\mu(1 - \gamma_5)\not{\phi}_2 - C(p)\gamma_\nu(1 - \gamma_5)(k_\mu a_{2\nu} - k_\nu a_{2\mu}) - \frac{C(p)^2}{2} C(p_1) a^2 k_\mu k_\nu \gamma^\nu(1 - \gamma_5)\not{\phi}_2.$$

The trace calculation is performed with the help of FEYNCALC [38–40]. The detailed and explicit expression obtained for the averaged squared "spinorial part" $|\bar{M}^s_{fi}|^2$ is given by

$$|\bar{M}^s_{fi}|^2 = \frac{1}{3} \left[ A J_s^2(z) + B J_{s+1}^2(z) + C J_{s-1}^2(z) + D J_s(z) J_{s+1}(z) + E J_s(z) J_{s-1}(z) + F J_{s+1}(z) J_{s-1}(z) \right],$$ \hspace{1cm} (32)

where the six coefficients $A$, $B$, $C$, $D$, $E$ and $F$ are explicitly expressed by

$$A = \frac{4}{(k.p)^2 M_W^2} \left[ a^4 e^4(k.p_1)(k.p_2) + 2a^2 e^2(2(k.p_1)(k.p_2)M_W^2 + (k.p)^2(p_1.p_2) - (k.p)(p_1.p_2)(p.p_2) + (k.p)(p_1)(p.p_2) \right],$$ \hspace{1cm} (33)
\[ B = \frac{-e^2}{2(k.p)(k.p_1)^2 M_W^2} [4(k.p_1)(k.p_1)^2 - (a_1.p_1)(a_1.p_2)(k.p) + a^2((k.p)(p_1.p_2) \\
+ (k.p_2)(M_W^2 - (p.p_1)) + 2(k.p)(p.p_2) + (k.p_1)(p.p_2)) - a^2(k.p_1) \\
\times (a^2 e^2((k.p)(k.p_2)p_0 - (k.p_1)(k.p_2)p_0 - 2(k.p)^2 E_2) + 2(k.p)((k.p_1)p_0(p_1.p_2) \\
+ p_1^0(2(k.p_2)M_W^2 + 4(k.p)(p.p_2) + (k.p_1)(p.p_2) + 2(k.p)(k.p_1)s)) \omega + a^4 e^2 \\
\times (2(k.p)(k.p_2)M_W^2 + (k.p_1)^2(p_1.p_2) + (k.p)(4(k.p) + (k.p_1))(p.p_2))^2 \\
+ |a| |q_1|(k.p_1)(2|a|(k.p)(2((k.p_2)M_W^2 + 2(k.p)(p.p_2)) \omega + (k.p_1)(-(k.p_2)p_0 \\
+ 2(k.p)(p_1^0 + E_2) + (p.p_2) \omega)) \cos(\theta) + (a_1.p_1)(k.p_1) \omega(-2(k.p)p_0 + a^2 e^2 \omega) \\
\times \sin(\theta)], \] (34)

\[ C = \frac{e^2}{2(k.p)(k.p_1)^2 M_W^2} [-4(k.p)(k.p_1)^2 - (a_1.p_1)(a_1.p_2)(k.p) + a^2((k.p)(p_1.p_2) \\
+ (k.p_2)(M_W^2 - (p.p_1)) + 2(k.p)(p.p_2) + (k.p_1)(p.p_2)) - a^2(k.p_1)(a^2 e^2((k.p) \\
\times (k.p_2)p_0 - (k.p_1)(k.p_2)p_0 - 2(k.p)^2 E_2) + 2(k.p)((k.p_1)p_0(p_1.p_2) + p_1^0 \\
\times (2(k.p_2)M_W^2 + 4(k.p)(p.p_2) + (k.p_1)(p.p_2) + 2(k.p)(k.p_1)s)) \omega + a^4 e^2(2(k.p) \\
\times (k.p_2)M_W^2 + (k.p_1)^2(p_1.p_2) + (k.p)(4(k.p) + (k.p_1))(p.p_2))^2 + |a| |q_1|(k.p_1) \\
\times (2|a|(k.p)(2((k.p_2)M_W^2 + 2(k.p)(p.p_2)) \omega + (k.p_1)(-(k.p_2)p_0 + 2(k.p)(p_1^0 \\
+ E_2) + (p.p_2) \omega)) \cos(\theta) + (a_1.p_1)(k.p_1) \omega(-2(k.p)p_0 + a^2 e^2 \omega) \sin(\theta)], \] (35)

\[ D = \frac{e}{(k.p)^2(k.p_1)^2 M_W^2} [2(k.p)(k.p_1)((-a^2 e^2(k.p_1)(2(a_1.p_2)(k.p) + (a_1.p_2)(k.p) \\
+ (a_1.p_1)(k.p_2)) + 2(k.p)(-(a_1.p_2)(k.p_1)M_W^2 + (a_1.p_1)(k.p_2)M_W^2 \\
+ (a_1.p_2)(k.p_1)(p.p_1) + (a_1.p_1)(2(k.p) + (k.p_1))(p.p_2)) + |a| |q_1| \omega \\
\times (a^4 e^4((k.p)(2(k.p) + (k.p_1)) - 2(k.p_1)(k.p_2)) \omega - 4(k.p)^2(k.p_1)p_0(2(p.p_2) \\
+ (k.p_1)s + M_W^2(p_1^0 + s \omega)) + 2a^2 e^2(k.p)((-(k.p_1)(k.p_1)p_0 + 2(p_1^0 + E_2) \\
+ (k.p)(M_W^2 + 2(k.p_1)s) \omega + (k.p_1)(2(k.p_2)p_0 + 2(p.p_2) \omega + (k.p_1)s \omega)) \sin(\theta)], \] (36)

\[ E = \frac{e}{(k.p)^2(k.p_1)^2 M_W^2} [2(k.p)(k.p_1)((-a^2 e^2(k.p_1)(2(a_1.p_2)(k.p) + (a_1.p_2)(k.p) \\
+ (a_1.p_1)(k.p_2)) + 2(k.p)(-(a_1.p_2)(k.p_1)M_W^2 + (a_1.p_1)(k.p_2)M_W^2 \\
+ (a_1.p_2)(k.p_1)(p.p_1) + (a_1.p_1)(2(k.p) + (k.p_1))(p.p_2)) + |a| |q_1| \omega \\
\times (-(a^4 e^4((k.p)(2(k.p) + (k.p_1)) - 2(k.p_1)(k.p_2)) \omega + 2a^2 e^2(k.p)((k.p_1) \\
\times (-2(k.p_2)p_0 + (k.p)(p_0 + 2(p_1^0 + E_2))) - ((k.p)M_W^2 + 2(k.p_1)(p.p_2) \\
+ (k.p_1)(2(k.p) + (k.p_1)s) \omega + 4(k.p)^2(k.p_1)p_0(2(p.p_2) + (k.p_1)s \\
+ M_W^2(p_1^0 + E_2 - s \omega)) \sin(\theta)], \] (37)

\[ F = \frac{4e^2(a_1.p_1)(a_1.p_2)(k.p)}{(k.p_1)M_W^2}, \] (38)
where the different scalar products are evaluated in the rest frame of $W^-$-boson, and

$$p_0 = M_{W^-}^* + \frac{e^2 a^2 \omega}{2(k.p)},$$

$$p_1^0 = Q_1 + \frac{e^2 a^2 \omega}{2(k.p_1)}.$$  \hspace{1cm} (39)

The expression of $|q_1|$ is obtained by solving numerically the equation $g(|q_1|) = 0$, which is a required condition to apply the familiar formula given in (25). We mention here that the result obtained in the trace calculation contains the various antisymmetric tensors, $\epsilon(a,b,c,d) = \epsilon_{\mu\nu\rho\sigma} a^\mu b^\nu c^\rho d^\sigma$ for all four-vectors $a,b,c,d$, which arise when four matrices $\gamma$ meet the fifth matrix $\gamma_5$ inside the trace ($\text{Tr}[\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\gamma^5] = -4i\epsilon_{\mu\nu\rho\sigma}$). Because it was calculated analytically and replaced in our computational program, it does not appear in the result listed above. We give here an example showing how we calculate these tensors using Einstein’s summation and the Grozin convention $\epsilon_{0123} = 1$,

$$\epsilon(a_1,a_2,p_1,p_2) = \epsilon_{\mu\nu\rho\sigma} a_1^\mu a_2^\nu p_1^\rho p_2^\sigma,$$

$$= |a|^2 \left[ \epsilon_{1203} p_1^0 p_2^3 + \epsilon_{1230} p_1^3 p_2^0 \right],$$

$$= |a|^2 \left[ p_1^0 (s \omega - |q_1| \cos(\theta)) - \left( |q_1| \cos(\theta) + \frac{e^2 a^2 \omega}{2(k.p_1)} \right) p_2^0 \right].$$  \hspace{1cm} (40)

So far, we have given a detailed theoretical calculation in order to obtain the expression of the leptonic decay rate of the $W^-\!$-boson in the presence of an EM field. The most known and important quantity that comes directly after calculating the decay rate is the lifetime, and since we only took into account the leptonic decay without the hadronic, we are not allowed to talk about the lifetime of the boson $W^-$, because this quantity requires the expression of the total decay rate that combines the leptonic and hadronic $W^-\!$ decays. This is in our perspectives and will be the subject of an upcoming research in which we aim to study the hadronic decay of the boson $W^-$ in the presence of an EM field, and then we combine the results obtained with the present leptonic results so that we can explore the effect of the laser on the lifetime as a very important point that must be addressed.

3. Results and discussion

This section is devoted to the presentation, analysis, and discussion of the numerical results obtained in relation to the leptonic decay rate of the $W^-\!$-boson in the presence of a circularly polarized EM field. Through the following, we will see the effect that the laser may have in terms of its parameters (both its field strength and frequency) on the leptonic decay rate. We chose to have the reference origin on the boson $W$ which is at rest before decay. Let us recall here that the geometry we have worked on inside is a general geometry whose spherical coordinates are $\theta$ and $\varphi$. The angle $\varphi$ was chosen to be zero ($\varphi = 0^\circ$) in all of the results obtained. All the results below have been obtained for the following input parameters:

$$M_W = 80.379 \text{ GeV}, \hspace{1cm} \text{(from [25])}$$

$$m_e = 0.511 \times 10^{-3} \text{ GeV}, \hspace{0.5cm} m_\mu = 0.1057 \text{ GeV}, \hspace{0.5cm} m_\tau = 1.777 \text{ GeV},$$

$$e = -8.5424546 \times 10^{-2} \hspace{1cm} \text{(charge of electron in natural units)}.$$  \hspace{1cm} (41)
Remember that we have utilized the following unit transformation: in System International (SI) units, the electric field strength $\mathcal{E}_0[\text{SI}] = 4329.0844 \; [\text{V/cm}]$ corresponds, in natural units (NU), to $\mathcal{E}_0[\text{NU}] = 1 \; [\text{eV}^2]$. The first thing we are familiarized to checking in such processes occurring within the EM field is to make sure that the decay rate in the presence of the laser field is exactly equal to its corresponding one in the absence of the laser when we take the zero field strength ($\mathcal{E}_0 = 0 \; \text{eV}$) and without the exchange of any photons ($s = 0$). In this case, the argument of the Bessel functions is equal to $z = 0$. Thus all the terms that contribute to the “spinorial part” $|\mathcal{M}_{fi}^s|^2$ given in (32) will be null except for the term multiplied by the coefficient $A$, since $J_0(0) = 1$ and $J_1(0) = J_{-1}(0) = 0$. The expression of $|\mathcal{M}_{fi}^s|^2$ becomes $|\mathcal{M}_{fi}^s|^2 = AJ^2_s(z)/3$, and if we take $a^2 = 0$ we get the trace expression of leptonic $W^-$ decay in the absence of the laser field [35]; that is

$$
|\mathcal{M}_{fi}^{\text{laser-free}}|^2 = \frac{8}{3} \left( (p_1 \cdot p_2) + \frac{2(p \cdot p_1)(p \cdot p_2)}{M_W^2} \right).
$$

(42)

For the other quantities that compose the decay rate in equation (28), we finds at $\mathcal{E}_0 = 0$ and $s = 0$ their expressions in the absence of the laser field.

Figures 2-5 show the multiphoton decay rate $d\Gamma^s/d\theta$ (28) for the process ($W^- \rightarrow e^-(\bar{\nu}_e)$ versus the net photon number $s$ transferred between the decaying system and the laser field. In order to show numerically the effect of field strength and frequency on the phenomenon of multiphoton energy transfer, we display the $s$-resolved decay rate at different field strengths and frequencies for the angle $\theta = 90^\circ$. To examine the effect of the laser field strength in the first step, we set, in figures 2, 3 and 4 the frequency to the value $h\omega = 1.17 \; \text{eV}$ and changed the field strength $\mathcal{E}_0$ so that it is equal to $\mathcal{E}_0 = 10^6 \; \text{V/cm}$ in figure 3, $\mathcal{E}_0 = 10^7 \; \text{V/cm}$ in figure 4 and $\mathcal{E}_0 = 10^8 \; \text{V/cm}$ in figure 2. Comparing these three figures with each other, we can see that as the laser field strength increases at a fixed frequency, the number of photons exchanged enhances and increases, and this is evident through the value of the cutoff number in each case. The cutoff numbers in figures 3, 4 and 2 are, respectively, 20, 160 and 1500 (for the absorption side) and $-20$, $-160$ and $-1500$ (for the emission side). It can also be observed from these figures that the magnitude of $d\Gamma^s/d\theta$ decreases as the field strength increases. This is the effect of laser field strength.

For the effect of laser frequency, we have exactly reversed the previous situation. In figures 2, 3 and 5, we fixed the field strength to the value $\mathcal{E}_0 = 10^6 \; \text{V/cm}$ and changed the laser frequency from $h\omega = 0.117 \; \text{eV}$ in figure 2 and $h\omega = 1.17 \; \text{eV}$ in figure 3 to the value $h\omega = 2 \; \text{eV}$ in figure 5. Through these figures, we can see that, unlike the field strength effect, when the laser frequency increases, the number of exchanged photons decreases, and this is evident by the decrease of the cutoff number from the value 1500 at frequency $h\omega = 0.117 \; \text{eV}$ [figure 2] to 10 at frequency $h\omega = 2 \; \text{eV}$ [figure 5] with a constant field strength equal to $\mathcal{E}_0 = 10^6 \; \text{V/cm}$. In relation to the magnitude of $d\Gamma^s/d\theta$, it is also noted that it increases with the increase of the laser frequency until reaching, for example, $d\Gamma^s/d\theta = 17 \times 10^6 \; \text{eV}$ at frequency $h\omega = 2 \; \text{eV}$ and $s = 4$ [see figure 5]. Based on the foregoing, it can be said that for increasing field strength and fixed frequency, processes involving large numbers of photons make significant contributions, in contrast to the case of high frequencies and fixed field strength where only low-photon exchanging processes are important. This is demonstrated by the formula of the argument $z = -e(a_1 \cdot p_1)/(k \cdot p_1) \propto \mathcal{E}_0/\omega^2$ given in (11), which is responsible for the effects of the laser, as it decreases with higher frequencies and vice versa, meaning that at lower frequencies a greater number of photons can be exchanged; as can be deduced from the behavior of the Bessel function $J_s(z)$ and clearly seen in figures 2-5. Thus, for stronger field strengths or lower frequencies, an increased effect on the leptonic decay rate can be expected. The heights of the different photon-number peaks are critically dependent on the values of the ordinary Bessel functions. In all the previous results presented in figures 2-5, the contributions of the different $s$-photon processes are cut off at two edges that are symmetric with respect to $s = 0$. 

It also appears that all the spectra are symmetric envelopes with respect to both the positive and negative edges, and this indicates that the photon absorption processes \((s > 0)\) are exactly equal to the photon emission processes \((s < 0)\). The cutoff number can be explained by the well-known properties of the Bessel function, which decreases considerably when its argument \(z\) (equation 11) is approximately equal to its order \(s\). This has already been referred to in refs. [41, 42]. In fact, our goal in drawing these envelopes in terms of the number of exchanged photons is to obtain the cutoff numbers at each specified field strength and frequency. The cutoff number is defined as a fixed number of photons from which the \(s\)-resolved decay rate \(d\Gamma^s/d\theta\) suddenly falls off \((d\Gamma^s/d\theta = 0)\). This number is very important to us because with it we make sure that the famous Kroll–Watson sum rule [43] is fulfilled. By the way, and in this context, we mention here that this sum rule in our case
can be formulated mathematically as follows:
\[
\sum_{s=-\text{cutoff}}^{+\text{cutoff}} \Gamma_s(W^- \rightarrow \ell^- \bar{\nu}_\ell) = \Gamma^{\text{laser-free}}(W^- \rightarrow \ell^- \bar{\nu}_\ell),
\]  
(43)
and is achieved when the decay rate in the presence of the laser field tends to approach the laser-free results by increasing the number of exchanged photons until they are exactly equal when we reach the cutoff number. In the next stage, we will try to make sure that this sum rule is respected. Let us take for example the case presented in figure 4 where the field strength \( \mathcal{E}_0 = 10^7 \text{ V/cm} \) and frequency \( \hbar \omega = 1.17 \text{ eV} \), the cutoff number in this case is equal, as shown in figure 4, to about +160 (for absorption) and −160 (for emission). In reality, this number of exchanged photons cannot be considered a cutoff number according to the aforementioned definition, because the decay rate \( d\Gamma_s/d\theta \) is not exactly equal to 0; but it is so small that our numerical program gives it a zero value when drawing the curves. By checking the values numerically, we found that the decay rate \( d\Gamma_s/d\theta \) is zero exactly at \( s \geq 450 \) (\( s \leq -450 \)). Therefore, the sum rule must be satisfied essentially at this number of exchanged photons. To prove this, we plotted the decay rate \( \Gamma(W^- \rightarrow e^- \bar{\nu}_e) \) (20), at \( \mathcal{E}_0 = 10^7 \text{ V/cm} \) and \( h\omega = 1.17 \text{ eV} \), summed over an increasingly different numbers of exchanged photons and compared it with the decay rate in the absence of the laser field. The curves obtained in connection with this are shown in figure 6, where the numbers of exchanged photons over which we have summed the leptonic decay rate are as follows: \(-N \leq s \leq +N\), where \( N = 30, 60, 120, 180, 200, 450. \) Through this figure, it is evident to us that with the increase in the number of photons exchanged between the decaying system and the laser field, the effect of the latter on the leptonic decay rate gradually decreases until it becomes completely absent when the number of exchanged photons is equal to the cutoff number (\( -450 \leq s \leq +450 \) in this case), and thus we obtain two identical curves. In other words, we say that the leptonic decay rate summed over a number of exchanged photons approaches slowly and tends, as the number of exchanged photons increases, towards its corresponding one in the absence of the laser until it is exactly equal to it at a specified number of exchanged photons, which is the cutoff number. This is exactly what we mean by the sum rule which is, in our case, respected and successfully achieved.

The idea of the sum rule was originally conceived when Kroll and Watson derived, in 1973, the differential cross-section formula for an elastic electron scattering process in the presence of a laser when neglecting the interaction of the atom (scattering potential) with the laser. Within the framework of the Kroll-Watson approximation, the information obtained from the laser-assisted leptonic decay rate regarding the decay system is the same as the information obtained from the decay rate without the laser field. Figure 7 shows the same thing as figure 6, except that in this case the laser field strength and frequency are, respectively, \( \mathcal{E}_0 = 10^6 \text{ V/cm} \) and \( h\omega = 2 \text{ eV} \), which are the same values used for each of them to draw the envelope in figure 5. As shown in figure 5, the cutoff number equals 10 for absorption (−10 for emission). But its real value, from which the s-resolved decay rate is zero, is 123. Therefore, we see that the sum rule in figure 7 is fulfilled exactly at this number of exchanged photons. These are two clear examples that are sufficient to make sure that the well-known sum rule is respected and therefore can be served and considered as a proof of the correctness of our theoretical calculations. In order to realize the size of the laser effect on the leptonic decay rate and make it more clear, it is convenient to introduce here a quantity denoted by \( R_{\text{with/without}} \) and defined as the ratio between the summed leptonic decay rate in the EM field given in equation (20) and its corresponding one in the absence of the laser field; that is,
\[
R_{\text{with/without}} = \frac{\Gamma^{\text{with laser}}(W^- \rightarrow e^- \bar{\nu}_e)}{\Gamma^{\text{without laser}}(W^- \rightarrow e^- \bar{\nu}_e)}.
\]  
(44)
Without laser

\[ 20 \leq s \leq 20 \]

\[ 123 \leq s \leq 123 \]

\[ 2 \leq s \leq 2 \]

\[ 5 \leq s \leq 5 \]

\[ 10 \leq s \leq 10 \]

\[ 60 \leq s \leq 60 \]

\[ 180 \leq s \leq 180 \]

\[ 200 \leq s \leq 200 \]

\[ 0 \leq \theta \leq 180 \text{ degree} \]

Leptonic decay rate \[ eV \]

Figure 6: The variation of the summed leptonic decay rate \( \Gamma (W^- \to e^- \bar{\nu}_e) \) \((20)\) (in units of \(10^6\)) with and without a laser as a function of the angle \( \theta \) for various numbers of photons exchanged. The spherical coordinate \( \varphi = 0^\circ \). The laser field strength and frequency are \( \mathcal{E}_0 = 10^7 \text{ V/cm} \) and \( \hbar \omega = 1.17 \text{ eV} \).

In figures 8 and 9, we aim to illustrate the effect of laser frequency on both the ratio \( R_{\text{with/without}} \) \((44)\) and the total leptonic decay rate, which is equal to the sum of the electron, muon and tau decay rates. To do this, we plotted their changes in terms of laser field strength for two different frequencies, CO\(_2\) laser frequency \( (\hbar \omega = 0.117 \text{ eV}) \) and Nd:YAG laser frequency \( (\hbar \omega = 1.17 \text{ eV}) \). Based on figure 8, it can be seen that the CO\(_2\) laser affects the \( R_{\text{with/without}} \) ratio only when the field strength is above the threshold of \( 10^3 \text{ V/cm} \), while the effect of the Nd:YAG laser on the ratio starts only when the field strength is above \( 10^5 \text{ V/cm} \); so the CO\(_2\) laser, whose frequency is lower than the Nd:YAG one, can affect the \( R_{\text{with/without}} \) ratio more than the Nd:YAG laser even at low field strengths. For example,
Figure 10: The same as in figure 8, but for different numbers of exchanged photons and fixed laser frequency \( h\omega = 1.17 \text{ eV} \).

Figure 11: The same as in figure 9, but for different numbers of exchanged photons and fixed laser frequency \( h\omega = 1.17 \text{ eV} \).

The \( R_{\text{with/without}} \) value at \( \mathcal{E}_0 = 10^5 \text{ V/cm} \) in the presence of the CO\(_2\) laser (\( R_{\text{with/without}} = 0.4 \)) does not attain in the presence of the Nd:YAG laser except when \( \mathcal{E}_0 = 10^7 \text{ V/cm} \). Thus, having a high-frequency laser effect requires the availability of very high field strengths. The same can be observed completely with regard to the effect of the laser frequency on the total leptonic decay rate shown in figure 9. It can be said that the effect of the laser decreases at high frequency or we say that the high-frequency laser has little effect compared to the low-frequency laser. In figures 10 and 11, we have drawn the same two quantities as in figures 8 and 9 at different numbers of photons exchanged, but now we have set the laser frequency to \( h\omega = 1.17 \text{ eV} \). This is exactly the same as we did when we wanted to check the sum rule. We will show here via figures 10 and 11 how the sum rule is also respected in both the \( R_{\text{with/without}} \) ratio and the total leptonic decay rate. We have summed over 100 and three cutoff numbers 170, 450 and 1500. We notice that at small field strengths all the curves, regardless of the number of exchanged photons, are coincident and have a constant value of 1 with respect to \( R_{\text{with/without}} \) in figure 10 and a value \( 6.81192 \times 10^8 \text{ eV} \) for the total leptonic decay rate in the absence of the laser in figure 11. Above the strength \( 10^5 \text{ V/cm} \), we notice that both quantities, for each specified number of exchanged photons, decrease nonlinearly with increasing field strength. It appears clearly to us that with the increase in the number of exchanged photons, the laser’s influence whether on the ratio or the total decay rate decreases until it becomes completely absent when we reach the cutoff \(-1500 \leq s \leq +1500 \) at which the sum rule is fulfilled, and thus we obtain a constant horizontal curve at the field strengths between \( 10 \) and \( 10^8 \text{ V/cm} \). The same thing we have recently examined and verified concerning the lifetime in the case of the pion and Z-boson decays [33, 34], and we will confirm it in a future study regarding the lifetime of the W-boson. The final result that we can deduce from the above is that the effect of the laser on the decay rate remains present as long as the number of exchanged photons that we sum over it does not reach the cutoff number at which the sum rule is fulfilled. For illustration purposes, similarly to the 2-dimensional plot, the contour-plots in figures 12 and 13 show more information on the overall variation and shape of the ratio \( R_{\text{with/without}} \) and total leptonic decay rate with respect to both laser field strength and frequency for an exchange of \(-20 \leq s \leq +20 \) photons. This type of graphics is only a kind of 3-dimensional representation, in which one studies a certain quantity and its behavior by changing simultaneously two variables on a 2-dimensional plane and using colors to represent the variations of the quantity. Apparently, these two figures 12 and 13 are divided into many colored regions separated by contours, where each contour separating two different regions corresponds to a value on the bar legend. Looking at these
Figure 12: The 3-dimensional contour-plots of the same quantity as in figure 8 as a function of the laser field strength $\mathcal{E}_0$ and frequency $\omega$ for $-20 \leq s \leq +20$ exchanged photons.

Figure 13: The 3-dimensional contour-plots of the same quantity as in figure 9 as a function of the laser field strength $\mathcal{E}_0$ and frequency $\omega$ for $-20 \leq s \leq +20$ exchanged photons.

two figures, we notice that they have the same feature to some extent, which indicates that the two quantities represented there change in the same way in terms of field strength and frequency. For the variation with respect to the laser frequency, we observe that the effect of the laser field on the two quantities decreases at high frequencies, while it becomes significant with increasing laser field strength at each specific frequency. This is completely in agreement with everything we said above.

The important question now is how can we explain the great effect that the EM field has at strong field strengths. From figure 10, it appears that the value reached by the ratio $R_{\text{with}/\text{without}}$, when the number of exchanged photons is $-100 \leq s \leq +100$ and at $\mathcal{E}_0 = 10^8$ V/cm, is $R_{\text{with}/\text{without}} = 0.07$. This means that, the laser-assisted leptonic decay rate is 93% lower than its corresponding one in the absence of the laser field. This is a significant effect of the laser on the decay rate. A computer calculation shows, when summing over $-10 \leq s \leq +10$, that the minimum value reached by the quantity $R_{\text{with}/\text{without}}$ is $R_{\text{min}}^{\text{with}/\text{without}} = 2.11603 \times 10^{-10}$ at field strength $\mathcal{E}_0 = 10^{10}$ V/cm (Schwinger limit), meaning that the quantity $R_{\text{with}/\text{without}}$ is almost zero at this field strength. This means that the decay rate in the presence of the laser is negligible compared to its equivalent one in the absence of the laser. Since the decay rate reflects, in its origin, the probability of occurrence of the decay, we conclude that the presence of the laser has considerably reduced the probability of occurrence of the leptonic $W^-$-boson decay if we do not exaggerate and say that it can suppress or prevent its decay. To our knowledge, there are two explanations for this behavior of the $W$ boson in the presence of the EM field. The first is that the laser has suppressed these leptonic channels to then completely allow other hadronic ones, whether they are allowed in the absence of laser, such as $W^- \rightarrow \bar{u}d$, $\bar{c}s$, or not, such as $W^- \rightarrow \bar{t}b$, especially since the $W$ boson gains in the EM field an excess of mass (effective mass) which increases with increasing field strength (e.g. $M^*_W = 186.825$ GeV at $\mathcal{E}_0 = 10^{16}$ V/cm and $\hbar\omega = 1.17$ eV). This first explanation can only be confirmed by studying the $W^-$-hadronic decay in the presence of the EM field. The second explanation is to attribute this behavior to the so-called quantum Zeno effect. The quantum Zeno effect can generally be defined by the inhibition of transitions between quantum states by repeated measurements of the state. This
effect has attracted a lot of attention over the past years [44–49]. In 1977, Misra and Sudarshan showed, based on quantum measurement theory, that an unstable particle would never decay when observed continuously [44]. They were the first to give the name to this effect. In our case, the process of the $W$ leptonic decay is observed by inserting it into an EM field. Thus the laser as an external field here plays the role of the measurement instrument. In addition, the lifetime of the $W$ boson will certainly, as a result, be extended and long [45]. In this context, we want to link our results and compare them with the results obtained by Kurilin in 2004 when he calculated the partial width of the $W$-boson leptonic decay in a strong EM crossed field [26]. He studies the changes of the decay width in terms of a characteristic parameter $\kappa = e M_W^{-3} \sqrt{- \langle F_{\mu\nu} q^\nu \rangle^2}$, where $F_{\mu\nu}$ is the EM tensor. He has revealed that the partial decay width is a non-monotonic function of the field-strength parameter $\kappa$. In superstrong fields ($\kappa \gg 1$), the partial width is greater than the corresponding one in a vacuum by a factor of a few tens, which means that the laser-assisted leptonic decay occurs faster than that in the absence of a laser. This result he reached does not constitute any contradiction with what we obtained. In contrast to the Zeno effect, there is an opposite effect to it called the quantum anti-Zeno effect, where the decay of some quantum systems from an initial state might be accelerated by frequent interrogations when the system remains in that state [50–52]. This effect was discovered by Kofman and Kurizki [53, 54]. However, these two results, despite the difference between them due to the different approach followed in each of them, can be widely accepted with reference to the quantum Zeno effect or the anti-Zeno effect, depending on whether the decay is suppressed or enhanced.

4. Conclusion

A detailed theoretical calculation of the leptonic $W^-$-boson decay ($W^- \rightarrow \ell^- \bar{\nu}_\ell$) in the presence of a circularly polarized electromagnetic field has been performed and an exact expression for the laser-assisted leptonic decay rate has been analytically derived. Our obtained numerical results show that the probability of decay to occur in the presence of a strong electromagnetic field is very small and almost non-existent, as if the laser at strong strengths and appropriate frequency is trying to stop the boson and delay its decay. This behavior can be considered as a result of the so-called quantum Zeno effect. As a consequence, the lifetime of the $W^-$-boson will also be affected by the laser and it will be long and extended, and it is not excluded that the branching ratios are also altered by the presence of the laser. In any case, this issue appears to be of particular interest and deserves a dedicated investigation which we will undertake in a forthcoming work to study the hadronic decay of the $W^-$-boson.

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