Ellipticity analysis of the BOOMERanG CMB maps

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Abstract. The properties of the Cosmic Microwave Background (CMB) maps carry valuable cosmological information. Here we report the results of the analysis of ellipticity of the hot and cold CMB anisotropy spots in the BOOMERanG 150 GHz map. We carried out this analysis for the map obtained by summing independent measurement channels (signal plus noise map) and for a comparison map (noise only map) obtained by differencing the same channels. The anisotropy areas (spots) have been identified for both maps for various temperature thresholds. The orientation (obliquity) of the spots is random for both maps. We computed the mean elongation of spots obtained from the maps at a given temperature threshold using a simple estimator. We found that for the sum map there is a region of temperature thresholds where the average elongation is not dependent on the threshold. Its value is \( \sim 2.3 \) for cold areas and \( \sim 2.2 \) for hot areas. The bias of the estimator is \( \sim +0.4 \). The presence of noise also biases the ellipticity by \( \sim +0.3 \). These biases have not been subtracted in the results quoted above. The threshold independent and random obliquity behaviour in the sum map is stable against pointing reconstruction accuracy and noise level of the data, thus confirming that these are actual properties of the dataset. In the sum map the anisotropy areas are elongated more homogeneously than in the difference map. Analogous elongation properties of CMB anisotropies had been detected for COBE-DMR 4 year data.

Key words. Cosmic Microwave Background

1. Introduction

The properties of the Cosmic Microwave Background (CMB) radiation continue to be a key window to our understanding of the early evolution and the present structure of the Universe. Recent experiments provided a detection of acoustic peaks in the angular power spectrum of the CMB, which is an important characteristic of the conditions at the last scattering epoch, and hence constrains a number of cosmological parameters (see e.g. de Bernardis et al. 2000, Netterfield et al. 2001, Lee et al. 2002, Leitch et al. 2001, Mason et al. 2002, Scott et al. 2002). Below we report the results of the analysis of the ellipticity of the anisotropies in the sky maps from BOOMERanG. At a given temperature threshold, spots with temperature larger (lower) than the threshold are identified. There are several definitions of the ellipticity of one spot, depending on the detailed procedure used to measure it. Loosely speaking, ellipticity is the ratio between the major and the minor semi-axes of the ellipse best fitting the contour of the spot. Ellipticity is expected in the CMB maps, as a result of the physical effects occurring before recombination, which induce correlations in the image of the CMB. This paper, however, is in the spirit of a model independent analysis of the data, so we will not attempt to compare to and constrain different models of the CMB anisotropy. Various descriptors have been proposed and already used for the study of CMB maps. The main aim was to check the
2. Complexity of CMB maps

The theory of algorithmic information provides tools to study the CMB maps and extract information on the general properties of the underlying dynamical systems without specification of cosmological models. Such a descriptor is the Kolmogorov complexity of the anisotropy areas (spots) (Gurzadyan 1993) which is the amount of information required to determine uniquely a given object. The conditional complexity \( K(x | y) = \min \{ l(p) \} \) is related to the amount of information on the object \( x \) with respect to the object \( y \) (Kolmogorov 1983, Chaitin 1987)

\[
I(y : x) = K(x) - K(x | y),
\]

and is the minimal length \( l(p) \) of the binary coded program required to describe the object \( x \) when the one for \( y \) is known. The complexity is related to Kolmogorov–Sinai (KS) entropy \( h \) via the relation

\[
K_u(t) - K_u(t_0) = \log_2 2^{h(t)}(t - t_0) = h(t)(t - t_0), \quad (1)
\]

quantifying the loss of information \( \Delta I \) at the evolution of CMB pattern from the initial state \( t_0 \) up to \( t \), i.e. from the last scattering epoch up to the observer. The ellipticity of anisotropy areas at each temperature threshold is the simplest descriptor of the complexity of the areas. The ellipticity \( \epsilon \) is defined via the divergence of the null geodesics in (3+1)-space

\[
\epsilon = \frac{L(t)}{L(t_0)}, \quad (2)
\]

where

\[
L(t) = L(t_0) \frac{a(t)}{a(t_0)} \exp(hs), \quad (3)
\]

\( a(t) \) is the scale factor of the Universe, \( s \) is the affine parameter of the geodesics and KS-entropy is the sum of the positive Lyapunov exponents (Gurzadyan and Kocharyan 1992): for more properties of the geodesics see Anosov 1967, Lockhart et al. 1982, Gurzadyan and Kocharyan 1994. The geodesic mixing is a statistical effect arising due to the exponential mixing, i.e. exponential decay of time correlation functions of the freely propagating photon beams at \( k = -1 \), is independent on the conditions on the last scattering surface, and is distinguished by the threshold independence and the randomness of the obliquities of the elongated areas. The ellipticity due to geodesics mixing has to vanish at precisely flat \( k = 0 \) and positively curved, \( k = +1 \), spaces.

3. Data

BOOMERanG is a millimetric telescope with bolometric detectors on a balloon borne platform (Piacentini et al. 2002, Crill et al. 2002). It was flown in 1998/99 and produced wide (4% of the sky), high resolution (\( \sim 10' \)) maps of the microwave sky (90 to 410 GHz) (de Bernardis et al. 2000). Two observation modes were used to map this sky patch: sky scans at a speed of \( 1^\circ /s \) and sky scans at \( 2^\circ /s \). This allows us to perform powerful tests for systematics (see Netterfield et al. 2001). In fact, at \( 2^\circ /s \), the sky temperature distribution produces signals in the detection chain at frequencies which are doubled with respect to observations at \( 1^\circ /s \), while instrument related effects, like 1/f noise, microphonic lines, and time-domain response remain at the same frequency. Comparing the maps obtained in the two observation modes is thus very effective in detecting instrumental artifacts. In the BOOMERanG maps the CMB structure is resolved with high signal to noise ratio, and hundreds of degree-scale hot and cold areas are evident. The rms temperature fluctuation of these areas is \( \sim 80 \mu K \). The detected fluctuations are spectrally consistent with the

\footnote{We avoid the often used terms 'open' and 'closed' Universe, since the geometry does not define the topology, e.g. the flat, \( k = 0 \) Universe can be not only \( R^3 \) but also \( S^1 \times R^2 \), \( T^3 \), \( R^3 \times T^2 \), etc., and similarly the negatively curved \( k = -1 \) Universe can be both 'open' and 'closed'.}
Masi et al. (2001) have shown that contamination from local foregrounds is negligible in the maps at 90, 150 and 240 GHz, and that the 410 GHz channel is a good monitor for dust emission. In our study we used two maps at 150 GHz. These maps have been obtained from the time ordered data using an iterative procedure (Natoli et al. 2001), which properly takes into account the system noise and produces a maximum likelihood map. The largest structures (scales larger than $10^\circ$) are removed in this procedure, to avoid the dominating effects of instrument drifts and 1/f noise. The two input maps, A and B, included 33111 pixels, each of $\sim 7$ arcmin in linear size, in a high Galactic latitude region covering about 1 % of the sky, with coordinates $RA > 70^\circ$, $-55^\circ < dec < -35^\circ$ and $b < -20^\circ$ (de Bernardis et al. 2000). The first map (A) has been obtained from the data of the B150A detector, while the second map (B) was obtained by averaging the maps from detectors B150A1 and B150A2. In this way we obtained two maps with similar noise per pixel. The (CMB) signal to noise ratio per pixel is of the order of 1 for our 7’ pixels. The sum (A+B) and difference (A-B) maps from all scans ($1^\circ$/s and $2^\circ$/s) are shown in fig.1 and fig.2 respectively. There are three AGN with significant flux in the maps (double circles in fig.1 and fig.2). This has been taken into account in the analysis.

4. Analysis

We studied the excursion sets in the BOOMERanG maps by means of a specially developed software (Gurzadyan and Kashin 2002). This enabled, in an interactive way, to change the input parameters, like the threshold level and the minimum and maximum number of pixels forming an anisotropy area to be included in the analysis, and allowed the visualization of all the intermediate steps of the analysis.

4.1. Algorithm

To define the excursion sets (hereafter ‘areas’) in the maps, a matrix of temperature data of the pixels with equal and higher than the given temperature threshold (lower, for negative thresholds) has been formed and the contours of those areas have been studied. The original maps follow the Healpix pixelization scheme (Gorski et al. 1998). We have reprojected the data using both Cartesian and curvilinear coordinates. We found that for our purpose the two are equivalent, and we used the Cartesian coordinates for simplicity. The Cartesian coordinates of pixels X, Y have been determined from the mean distance of the centers of the pixels. This procedure enabled us to check the sensitivity of the results with respect to the coordinate system and the cell size, for different total numbers of cells. In particular, the oversampled 1 arcmin cell (a 1692 x 1296 matrix)
appears to lead to more accurate results than the original 7.5 arcmin cell matrix. The procedure of the definition of the centers of areas, their semi-axes and their obliquity and ellipticity was as follows: • 1. Center of the area. We define the coordinates of the center as \( y_c = (y_2 - y_1)/2 \), \( x_c = (x_2 - x_1)/2 \), where \( y_1 \) and \( y_2 \) are the highest and lowest \( y \) coordinates of the pixels of the area (and analogously for \( x \) coordinates). • 2. Semi-major axis \( d_{\text{max}} \). It is estimated as the segment connecting \((x_c, y_c)\) and the center of the farthest pixel of the area. The inclination angle of the segment is the obliquity, and is measured counter-clock-wise from the positive \( x \) semi-axis (parallel to RA). • 3. Semi-minor axis \( d_{\text{min}} \). On either sides of the major axis we find the pixels having the maximum distance between the pixel center and the major axis. We take the average of the two distances as the length of the semi-minor axis. • 4. The ellipticity is computed as \( \epsilon = d_{\text{max}}/d_{\text{min}} \). For each temperature threshold the mean ellipticity of the anisotropy areas was estimated for both the A+B and A-B maps, along with the angular distribution of the obliquities. Our algorithm to estimate ellipticity is simpler and faster than the one used in Barreiro et al. 2001 but, in the presence of noise, is biased, as we show below. The level of biasing, however, is reasonably small, and completely acceptable for our purpose.

4.2. Simulations

Since the use of ellipticity is relatively new in the CMB literature, we carried out numerical simulations in order to show the performance of our estimator and the expected behaviour for the map of the CMB. In order to validate our algorithm we produced simulated maps, with circular, symmetric gaussians, well separated in the sky, with FWHM \( \sim 30 \) arcmin. We used the same pixelization used for the BOOMERanG data. The average ellipticity we measure is between 1.3 (lower thresholds) and 1.4 (higher thresholds). The deviation from the expected unit value is due to pixelization and to the algorithm used to define ellipticity. The result gets closer to 1 for larger FWHM. If we use gaussians with two axis (minor 30 arcmin, major 45 arcmin), and we keep them well separated, we get an ellipticity of \( \sim 1.8 \), probably due to the bias we have seen before. Finally, we moved randomly the symmetric gaussians, so that some of them merged and produced elliptical spots. In this case the mean ellipticity is a function of threshold, as expected. We conclude that our estimator of ellipticity is biased at a level of \( \lesssim +0.4 \). In fig.3 we illustrate the expected ellipticity behaviour of intrinsic CMB ellipticity in the context of the currently popular adiabatic inflationary model. Many realization of CMB maps of the sky region observed by BOOMERanG have been simulated, starting from the best fit angular power spectrum measured by BOOMERanG. For different temperature thresholds \( T_i \), the anisotropy spots hotter than \( T_i \) have been identified, and their ellipticities have been computed. In the left part of fig.3 the two lines define the 68% confidence intervals for the map-averaged ellipticity derived from these simulations. A map-averaged ellipticity \( \sim 2 \) is expected, basically independent of the temperature threshold. In the right part of fig.3 we simulate the presence of instrumental noise in the measurements of ellipticity, and show how to extract the CMB ellipticity signal from noisy sky maps. We have added realistic noise and filtering, at the same level present in the BOOMERanG 150 GHz channels, and analyzed the map-average ellipticity of simulations obtained summing (continuous lines) and differencing (dashed lines) maps from two independent measurement channels. The presence of noise increases the average ellipticity in the maps by \( \sim 0.3 \). In the difference maps, where only noise is present, the scatter of map-averaged ellipticities is higher.

Fig. 3. Average ellipticity of intrinsic CMB anisotropy (left) obtained from simulations of CMB maps with the best fit power spectrum measured by BOOMERanG. The two lines define the 68% confidence interval for the map-averaged ellipticity measured from the simulations. In the right part of the figure we add realistic noise and filtering, at the same level present in the BOOMERanG 150 GHz channels, and analyze the map-average ellipticity of simulations obtained summing (continuous lines) and differencing (dashed lines) maps from two independent measurement channels. The presence of noise increases the average ellipticity in the maps by \( \sim 0.3 \). In the difference maps, where only noise is present, the scatter of map-averaged ellipticities is higher.

4.3. Application to the BOOMERanG data

Our software enabled to explicitly follow the evolution of anisotropy areas with respect the temperature threshold and the role of each area in the final results. As an example, Table 1 contains the data for the hot and cold anisotropy areas at thresholds \( \pm 500 \mu K \). In fig.4 we plot a histogram of obliquity of the spots detected in the sum map A+B, at a temperature threshold of \( +400 \mu K \). Only
Table 1a.
Threshold: $-500 \mu K$, \hspace{1cm} A + B

| Area No | Coordinates, degree | Number of pixels | Ellipticity |
|---------|----------------------|------------------|------------|
| C1      | 243.9 -40.4          | 5                | 4.72       |
| C2      | 241.5 -39.7          | 3                | 1.42       |
| C3      | 241.1 -39.5          | 4                | 1.96       |
| C4      | 241.6 -39.4          | 3                | 2.43       |
| C5      | 239.4 -38.5          | 3                | 1.61       |
| C6      | 239.2 -38.2          | 4                | 4.33       |
| C7      | 253.1 -34.2          | 12               | 1.73       |
| C8      | 250.5 -33.9          | 6                | 2.20       |
| C9      | 255.0 -33.0          | 12               | 2.21       |
| C10     | 253.9 -31.7          | 8                | 1.69       |
| C11     | 242.0 -31.6          | 6                | 2.49       |
| C12     | 240.1 -29.2          | 4                | 3.17       |
| C13     | 260.2 -27.9          | 6                | 2.24       |
| C14     | 260.9 -26.9          | 4                | 2.39       |
| C15     | 260.5 -26.7          | 7                | 2.02       |
| C16     | 261.0 -24.0          | 3                | 1.28       |
| C17     | 243.4 -23.6          | 4                | 2.18       |
| C18     | 246.4 -21.9          | 3                | 2.48       |
| C19     | 247.0 -21.7          | 4                | 1.95       |

Note to table 1b: area H7 includes a known AGN. Other two AGNs are present in the map we have analyzed, but they fill less than 3 pixels and were discarded.

Table 1b.
Threshold: $+500 \mu K$, \hspace{1cm} A + B

| Area No | Coordinates, degree | Number of pixels | Ellipticity |
|---------|----------------------|------------------|------------|
| H1      | 258.9 -40.1          | 5                | 2.10       |
| H2      | 261.5 -40.0          | 3                | 1.60       |
| H3      | 253.0 -38.8          | 4                | 1.86       |
| H4      | 261.9 -37.9          | 5                | 2.52       |
| H5      | 246.3 -36.6          | 4                | 2.25       |
| H6      | 256.7 -32.9          | 3                | 2.84       |
| H7      | 240.7 -32.7          | 3                | 2.39       |
| H8      | 250.1 -31.1          | 5                | 1.68       |
| H9      | 251.9 -30.0          | 3                | 1.45       |
| H10     | 243.3 -28.2          | 5                | 1.89       |
| H11     | 243.3 -27.8          | 4                | 2.39       |
| H12     | 252.1 -27.4          | 3                | 1.57       |
| H13     | 240.6 -26.8          | 3                | 1.53       |
| H14     | 256.7 -26.4          | 4                | 1.71       |
| H15     | 243.9 -26.0          | 6                | 2.27       |
| H16     | 243.9 -25.5          | 3                | 2.41       |
| H17     | 254.9 -24.8          | 6                | 1.62       |
| H18     | 247.6 -24.2          | 7                | 1.61       |
| H19     | 244.2 -21.6          | 3                | 2.16       |

areas formed by 5 pixels or more have been considered, because the obliquity error due to pixelization is very large for areas with less pixels. The obliquities of these 25 areas are random. The $\chi^2$ for a uniform distribution is 6.13 with 5 DOF. The same typical behaviour has been found at different thresholds. Only when areas with relatively small number of pixels are included, there is a certain domination of alignment on 45 and 135 degrees, which disappears when such areas are abandoned, thus indicating the role of the rectangular shapes of the pixels themselves. This test is a strong indication that the origin for the detected ellipticity cannot be instrumental. Effects related to detectors time constants or to the scan strategy should be strongly anisotropic, as our scans are all within $\pm 12^\circ$ from the DEC=constant lines. In Fig.5 we plot the map-averaged ellipticity versus the temperature threshold for both the A+B map (signal plus noise) and the A-B map (noise only). In fig.6 we plot the same for a previous data release with higher noise (S/N of the order of 0.7 vs 1.0) and lower accuracy pointing reconstruction (4.5’ rms vs 2.5’ rms). These data have been considered in order to investigate if the results are robust against variations of the noise level and of the pointing accuracy. The use of perturbed data is a standard and powerful technique in the framework of theory of dynamical systems (Arnold 1989). We use it in order to check at once the effect of perturbations like inaccuracy in pointing, timestream filtering, detector noise, which are different for the two data releases we compare. Only the areas containing 3 to 200 pixels have been used, since areas with 1 and 2 pixels introduce biases related to the shape of the pixels. The mean elongation has been computed only for thresholds containing at least two areas. Including areas with more than 200 pixels is not informative, since only a few of such structures are present, and their topological properties are expected to be different from those of smaller areas. The lower boundary (in absolute value) of the temperature threshold interval of interest was determined as the level where most of the areas already have shapes for which an ellipticity can be assigned. The upper boundary marks the threshold where the number of areas present...
noise level enabled to compare some properties of A+B and A-B, to reveal the role of the noise. We proceeded as follows. We defined ‘equivalent’ sets of areas for the A+B and A-B maps, respectively, by selecting the two temperature threshold intervals with the same number of areas contributing to our measurement of ellipticity. For example, for positive thresholds, the interval [450,625] in the A+B map must be compared to the interval [300,400] in the A-B map, since both intervals include from 5 to over 50 areas. This scheme defines threshold intervals [-300, -450] µK and [300, 400] µK for A-B and [-475, -625] µK and [450, 625] µK for A+B, as seen in Fig. 5. We compute the standard deviation of the ellipticity in those threshold intervals, as \( \sigma^2 = \sum (\epsilon - \overline{\epsilon})^2 / (N-1) \), where \( N \) is the number of thresholds in the interval. We find that the peak to peak scatter, for negative thresholds, is over 5\( \sigma \) for A-B, while is 3\( \sigma \) for A+B, with the overwhelming majority of points inside the 2\( \sigma \) interval. Another feature not evident from Figure 5 is the scatter in ellipticities at a given threshold. In the first case (A+B) the distribution of areas ellipticities for a given threshold is in general narrower than in the second case (A-B) especially at high thresholds. For example, \( \delta\epsilon = \epsilon_{\text{max}} - \epsilon_{\text{min}} \) equals 1.3 for the threshold corresponding to 6 areas in A+B, (-600 µK), while it is 2.8 for the threshold corresponding to 5 areas in A-B (-400 µK). In other words, at high thresholds the areas in A+B possess more homogeneous ellipticities with respect to the areas in A-B. It is remarkable that in their other properties the A+B and A-B areas show no qualitative difference. Namely, they show very similar properties in: (A) the variation of the number of spots vs threshold, (B) the dependence of the mean area sizes, in pixel numbers per area, on the threshold (which is very similar within the "equivalent" intervals), (C) the ellipticity dependence on the area size. It thus appears that the two sets have identical properties except for the scatter of ellipticity over the thresholds. This behavior of ellipticity is robust against the variation of the parameters. For example, the change of the minimal number of the pixels per area from 3 to 5, 7, 10 does not change the properties of elongation in the A+B data. The grand-average ellipticities for the final maps are reported in Table 3.

### 4.4. Systematics

Possible systematic effect could be expected for very small areas due to the shape of the Healpix pixels. However, we find that this effect, for such small spots, is smeared by the intrinsic inaccuracy of the definition of the two semi-axes. We also checked the effect of including data closer to the Galactic plane (down to \( b < -13^\circ \)). We found that the flat ellipticity vs threshold behaviour was strongly distorted by the presence of low galactic latitude data. We repeated the analysis above separately for maps obtained from 1°/s scans only and for maps obtained from 2°/s only, and obtained consistent results. The difference map (1°/s map - 2°/s map) features a larger scatter of the map-averaged
ellipticity vs threshold diagram, as expected for noise only. This test strongly excludes an instrumental origin of the ellipticity of the areas.

5. Conclusions

The analysis of the BOOMERanG CMB sky map reveals threshold independent elongation of the temperature anisotropy in the meaningful interval of temperature thresholds. Selecting areas with three and more pixels, within temperature thresholds for [375, 575] µK (hot areas), we obtained a mean ellipticity $2.22 \pm 0.05$; in the range [-400,-600] µK (cold areas) we obtained a mean ellipticity 2.3 ± 0.06. The quoted errors are statistical only. The bias deriving from pixelization and algorithm is of the order of +0.3, while the bias due to the noise is also $\sim$ 0.3. The ellipticities quoted above have not been corrected due to the noise have not been subtracted.

Note to table 3: The quoted errors are statistical only. A bias of $\lesssim 0.3$ due to the algorithm, and a similar bias due to the noise have not been subtracted.

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sample simulated map with B98 power spectrum

average ellipticity vs. \( \Delta T \) (\( \mu K \))

- **A+B**
- **A - B**

- Sample data points for different values of \( \Delta T \) (\( \mu K \)) are shown in the graph.
