Regularities of operation cartridge filters

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Abstract. The process of de-ironizing natural water by filtering through small-sized cartridge ceramic filters is considered. The process of filtration of the suspension through the filter wall, assuming that the filter wall and the sediment is not compressible. Considering, thus, the flow rate of filtrate is proportional to the pressure gradient of the square of the filter and inversely proportional to the resistance. In contrast to the works of V. A. Zhuzhikov, here the slurry is fed inside the cartridge filter. The results of theoretical and experimental studies are presented. Mathematical equations of filtration process are obtained when source water is supplied inside cartridge.

1. Introduction
Water de-gelation is an important point in water treatment technology. At present, there are many methods known to degenerate natural waters [1-7]. In this case, theoretical studies of water deselection by cartridge filters with a solid filter base are given. Cartridge filter is cylinder with cartridge of height $H$, inner diameter $D$, thickness of walls $\delta$, averaged diameter of pores $d$.

The main characteristics when considering the filtration process are: filter cycle duration, filtration rate and other factors [8,9].

As is known, the duration of the filter cycle is determined either by the quality of the filtrate (if impurities slip into the filtrate, the filter is stopped for washing) or by loss of head (the filter is encircled to such an extent that filtration does not occur) or by exhaustion of the volume provided to the precipitate.

2. Research problem statement and theoretical part
V.A. Zhuzhikov [10,11] offered the formula for definition of limit time of filtering for cartridge filters. He considered process of filtering of suspension through a filtering partition with formation of a deposit on the cartridge filter, having assumed that the filtering partition and a deposit are not squeezed. At the same time suspension moved outside of the filtering element. For time of $t_e$ during which the layer of a deposit accrues from the external radius of the filtering $R_{f,p,e}$ to $R_{s,e}$ deposit radius at external supply of suspension constant pressure difference $\Delta P$, it received expression:

$$t_e = \frac{R_{f,p,e}^2}{4x_p \Delta P} \left[ (y-a)(r_0 + r_{f,p}) \ln a + r_0 y \ln \frac{y}{a} \right]$$

(1)
where $x_0$ - the ratio of precipitate to filtrate volumes in the original slurry; $R_0$ and $r_{f.p.}$ - specific resistances of sediment and filter partition; $R_{f.p.e.}$, $R_{f.p.i.}$ - external and internal radiuses of a filtering partition; $a = (R_{f.p.e.} / R_{f.p.i.})^2$, $y = (R_{s.e.} / R_{f.p.i.})^2$.

![Figure 1](image-url)  
**Figure 1** Schematic section of filter cartridge with sediment layer: 1 - filter partition, 2 - precipitate at external water supply, 3 - precipitate at internal water supply.

Consider the filtration process under the same assumptions when feeding the suspension inside the cartridge filter [12]. Considering, like V.A. Zhujikov, the filtrate flow rate proportional to the pressure gradient of the filter area and inversely proportional to the resistance, we can record for an infinitely thin layer of filter baffle with a radius of $R_f$, located within the limits between the inner $R_{f.p.i.}$ and the outer $R_{f.p.e}$ radii:

$$
\frac{dV}{dt} = \frac{2\pi LR_{f.p.}}{r_{f.p.}} \frac{dP}{dR_{f.p.}}
$$

(2)

And for an infinitely thin layer of sediment with a radius $R_s$, located within the limits between the inner $R_{s.i.}$ and external $R_{s.e}$ radii:

$$
\frac{dV}{dt} = \frac{2\pi LR_s}{r_0} \frac{dP}{dR_s}
$$

(3)

By integrating equation (2) by the variable $R_{f.p.}$, ranging from $R_{f.p.i.}$ to $R_{f.p.e}$, and from $P_i$ to $P_e$, we will receive:

$$
P_e - P_i = \frac{r_{f.p.}}{2\pi \cdot L} \frac{dV}{dt} \ln \frac{R_{f.p.e}}{R_{f.p.i}}
$$

(4)

where through $P_i$ and $P_e$ the pressures are indicated on the inner and outer sides of the filter partition, respectively.

By integrating equation (3) on the variable $R_s$ within the range of $R_{s.i.}$ to $R_{s.e} = R_{f.p.i}$ and from $P_0$ to $P_l$, get:

$$
P_o - P_{l.p} = \frac{r_i}{2\pi \cdot L} \frac{dV}{dt} \ln \frac{R_{l.p}}{R_{f.p.e}}
$$

(5)

where $P_o$ - pressure on the inner surface of the sediment layer.

Subtracting from (5) (4) yields for pressure difference.
\[ \Delta P = P_0 - P_e = \frac{1}{2\pi \cdot L} \int_0^t \frac{dV}{dt} \left( r_0 \ln \frac{R_{s,i}}{R_{f,p,i}} - r \ln \frac{R_{f,p,e}}{R_{f,p,i}} \right) \]  

(6)

During filtration, the value of the internal radius of the deposit \( R_{s,i} \) is a variable value decreasing from \( R_{s,e} = R_{f,p,i} \).

Consider the differential relationship between the amount of \( dV \) filtrate produced and the thickness of the formed sediment layer \( dR_{s,i} \) radius.

\[ dV = \frac{2\pi \cdot R_{s,i} \cdot dR_{s,i}}{x_0} \]  

(7)

Substituting (7) in (6) and integrating the obtained differential equation within the range from 0 to \( t_i \) and from \( R_{s,e} = R_{f,p,i} \) to \( R_{s,i} \), for the time \( t_i \) during which the sediment layer increases from the inner radius of the filtering partition \( R_{f,p,i} \) to the inner radius of the \( R_{s,i} \) precipitate.

\[ t_i = \frac{R_{f,p,i}^2}{4x_0 \Delta P} \left[ (1 - z) \cdot \left( r \ln a + r_0 \right) + r_0 \cdot z \cdot \ln z \right] \]  

(8)

where \( z = R_{s,i}^2/R_{f,p,i}^2 \), in particular, at \( z = 0 \) (the whole inner surface of the filter is filled with sediment):

\[ t'_{lim} = \frac{R_{f,p,i}^2}{4x_0 \Delta P} \left( r \ln a + r_0 \right) \]  

(9)

The formula (9) defines the time of filtration before washing when the suspension is fed into the cartridge filter.

We will require equality of sediment volumes at external and internal suspension supply (in the limit case): \( y = 1 + a \). By dividing (1) by (9), we get:

\[ \frac{t'_{lim}^e}{t'_{lim}^i} = \frac{-r_0 + r \ln a + r_0 \left( 1 + a \right) \cdot \ln \left( 1 + \frac{1}{a} \right)}{r_0 + r \ln a} \]  

(10)

At a thickness tending to zero from (10) follows a thin filtering partition \( (a \rightarrow 1) \):

\[ \frac{t'_{lim}^e}{t'_{lim}^i} = 2\ln 2 - 1 = 0.386 \]  

(11)

Thus, the productivity of the cartridge filter at the external feed of the slurry is more than 2.5 times higher than at the internal feed at the same pressure difference.

Due to the fact that in the practice of cartridge filters the thickness of the filter element partition can be considerably higher than the thickness of the sediment layer formed on it (partition) [13], productivity of cartridge filters at internal supply of suspension to the filter element will naturally be less than productivity of such filter at external supply of suspension. However, this reduction in productivity does not correspond to expression (10), but rather depends on the overall dimensions of the cartridge.

It should be noted that in all known applications of cartridge filters the liquid flow was directed towards the center, i.e. radially inward [14,15]. In this case, the slurry experiences an increase in velocity as it passes through the medium, so that any suspended particles not retained on the outer layers are all less and less likely to be retained internally.

When the suspension is fed internally, on the contrary, as the slurry passes through the medium, the speed of the slurry is reduced and, consequently, the probability of the slurry being held is increased.
Thus, feeding the suspension inside the filter cartridge, slightly reducing the productivity of the cartridge filters, significantly improves the process parameters of water degilation and therefore can be advantageous when using cartridge filters as small water treatment plants for water degilation [16,17].

3. Research results and suggestions

In view of the above, studies were carried out using an experimental plant consisting of four cartridge filters.

The feed water after the pre-aeration was supplied to the inside of the ceramic porous cartridge, the filter surface of which was defined as $S_f = \pi D H$. One important characteristic of cartridge filters is the surface porosity $\varepsilon$ - the ratio of the pore area in a given section to the area of the entire section. Design characteristics of cartridge filters are given in Table 1.

| Filter number | Overall dimensions, mm | Filter surface, $S_f$, $m^2$ | Pore size, $d$, mm | Surface porosity, $\varepsilon$ |
|---------------|------------------------|-----------------------------|------------------|------------------|
| F1            | 206 56 9 0,0387 80 0,0024 |
| F2            | 220 40 15 0,0289 100 0,0019 |
| F3            | 105 50 3 0,0184 50 0,0038 |
| F4            | 80 29 7 0,0078 40 0,0028 |

Purification of water from iron compounds was carried out as it passed through filter pores. Through certain periods of $t$, the speed of filtering of $V_f$ and pressure difference $\Delta P$ was measured. The results of the experiments are summarized in Table 2 and Table 3.

| Time interval from the beginning of filtering, h | the Speed of filtering, $V_f$, m/h | Differential pressure, $\Delta P$, kPa |
|-----------------------------------------------|----------------------------------|-------------------------------------|
| F1                                           | F2 F3 F4                         | F1 F2 F3 F4                         |
| 0                                            | 2,45 2,45 2,60 0,66               | 14 19 9 11                          |
| 24                                           | 2,58 2,56 2,57 0,67               | 24 26 15 18                         |
| 48                                           | 2,51 2,49 2,60 0,67               | 35 37 22 32                         |
| 72                                           | 2,45 2,45 2,57 0,67               | 46 48 32 44                         |
| 82                                           | 2,39 2,40 2,54 0,66               | 56 57 40 48                         |

| Time interval from the beginning of filtering, h | Iron concentration in starting water, $C_{st}$, mg/dm$^3$ | Iron concentration in filtrate, $C$, mg/dm$^3$ |
|-----------------------------------------------|-----------------|-----------------|
| F1                                           | F2 F3 F4        | F1 F2 F3 F4     |
| 0                                            | 5,73 5,05 5,11 5,11 | 4,48 2,73 3,88 1,38 |
| 10                                           | 5,69 4,32 4,48 4,46 | 4,46 2,33 3,88 1,38 |
| 24                                           | 5,63 3,72 3,66 3,88 | 3,65 1,01 3,88 1,38 |
| 34                                           | 5,72 3,37 3,48 3,48 | 3,65 1,01 3,88 1,38 |
| 48                                           | 5,65 3,08 3,15 3,23 | 3,23 0,85 3,23 0,85 |
| 58                                           | 5,57 2,62 2,67 2,68 | 2,68 0,61 2,68 0,61 |
| 72                                           | 5,72 2,37 2,43 2,38 | 2,38 0,53 2,38 0,53 |
| 82                                           | 5,81 2,32 2,38 2,42 | 2,42 0,41 2,42 0,41 |
Processing of results of an experiment showed that at almost constant speed of filtering of $V_f$, differential pressure $\Delta P$ linearly increases in time:

$$\Delta P = \Delta P_0 + b_f t,$$

(12)

And the dependence of the iron concentration in the filtrate on the filtration time is well approximated by the equation

$$\ln \frac{C}{C_{mix}} = a + bt,$$

(13)

Mathematical processing of experimental data by least squares method allowed to obtain values of parameters included in equations (12) and (13). Their numerical values are shown in Table 4.

**Table 4.** Constants that determine the pressure drop and iron concentration in the filtrate versus time.

| Filter number | Constant value of equations (12) - (13) |     |     |     |
|---------------|----------------------------------------|-----|-----|-----|
|               | $\Delta P_0$, Pa                      | $b_f$, Pa/s | $a$ | $b$, 1/s |
| $\Phi 1$      | 12800                                  | 0,136 | 0,1676 | 2,7 $\cdot 10^6$ |
| $\Phi 2$      | 16900                                  | 0,125 | 0,1566 | 2,6 $\cdot 10^6$ |
| $\Phi 3$      | 7100                                   | 0,1   | 0,1325 | 2,7 $\cdot 10^6$ |
| $\Phi 4$      | 9300                                   | 0,131 | 0,525  | 7,7 $\cdot 10^6$ |

From physical representations clearly [18,19], that resistance $\Delta P$ to a stream in the cartridge filter consists of resistance of the filtering partition $\Delta P_0$ and resistance of a layer of a deposit. The resistance of the filter partition is subject to Stokes’ law:

$$\Delta P_0 = \frac{32 \cdot \mu \cdot \delta \cdot V_f}{\varepsilon \cdot d^2},$$

(14)

where $\mu$ - the dynamic viscosity of water.

The resistance of the sediment layer is proportional to its thickness at time $t$:

$$b_p \cdot t = k \cdot \frac{1}{S_f \cdot \rho_e} \int_0^t (C_m - C) \cdot S_f \cdot V_f \cdot dt \approx k V_f \cdot C_m \left(1 - e^{-a t}\right) \cdot t,$$

(15)

Hence for the proportionality factor $k$ we get:

$$k = \frac{b_p}{V_f \cdot C_m \left(1 - e^{-a t}\right)},$$

(16)

$K$ coefficient for all filters was identical $k = 240$ Pa/m $\cdot$ dm$^3$/mg.

The most important factor on which the constants $a$, $b$ of kinetic equation (13) depend is the filtration rate $V_f$. The second significant factor determining these constants is the ratio $\chi$ of the amount of adhesion surface on which the iron particles adhere to the volume of water entering the surface per unit time. The adhesive surface is the total surface of the inner channels of pores of diameter $d$ and length $\delta$ (wall thickness) [20].

$$S_{ad} = N \cdot \pi \cdot d \cdot \delta,$$

(17)

where $N = \frac{4 \cdot \varepsilon \cdot S_f}{\pi \cdot d^2}$ is the number of pores of diameter $d$ on the surface $S_f$.

Thus
\[ \chi = \frac{S_{\text{m}}}{V_f \cdot d} = \frac{4 \varepsilon \cdot \delta}{V_f \cdot d} \]  \hspace{1cm} (18)

Taking this into account, assuming the dependencies \( a, b \) to \( V_f \) and \( \chi \) linear and using the least squares method to find the coefficients from the experimental data, we obtain:

\[ a = -0.0085 + 125.52V_f + 4.82 \cdot 10^{-10} \cdot \chi \] \hspace{1cm} (19)

\[ b = 1.3712 \cdot 10^{-6} + 6.289 \cdot 10^{-4} \cdot V_f + 5.852 \cdot 10^{-10} \cdot \chi \] \hspace{1cm} (20)

In these equations, the velocity \( V_f \) is measured in m/s and \( \chi \) in s/m.

The above equations of the mathematical model of operation of the cartridge filter can be used to select filters with specified technological characteristics.

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