Phonon spectral functions of photo-generated hot carrier plasmas: effects of carrier screening and plasmon–phonon coupling

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Abstract

We investigate spectral behavior of phonon spectral functions in an interacting multi-component hot carrier plasma. Spectral analysis of various phonon spectral functions is performed considering carrier–phonon channels of polar and nonpolar optical phonons, acoustic deformation-potential, and piezoelectric Coulomb couplings. Effects of phonon self-energy corrections are examined at finite temperature within a random phase approximation extended to include the effects of dynamic screening, plasmon–phonon coupling, and local-field corrections of the plasma species. We provide numerical data for the case of a photo-generated electron–hole plasma formed in a wurtzite GaN. Our result shows the clear significance of the multiplicity of the plasma species in the phonon spectral functions of a multi-component plasma giving rise to a variety of spectral behaviors of carrier–phonon coupled collective modes. A useful sum rule on the plasma-species-resolved dielectric functions is also found.

Keywords: multi-component plasma, phonon spectral function, plasmon–phonon coupling, phonon self-energy, electron–phonon coupling

(Some figures may appear in colour only in the online journal)
would support a great variety of spectral behaviors giving rise to novel features (absent in the scp) such as the coupling of individual plasmons to polar optical phonons. The mode coupling properties are expected to depend on the plasma species, because the dispersive behavior of self-sustaining oscillations varies for different plasma species [3].

Phonon spectral function of a solid state plasma is of fundamental importance in understanding the behavior of coupled plasmon–phonon modes in a solid and depends strongly on the nature of the screening and collective behavior of the carriers. The carrier–carrier interaction introduces dielectric screening and would weaken the carrier–phonon coupling. For example, Jain and co-workers proposed a coupling of LO phonons to quasiparticle excitations (QPE) and studied its effect on the hot electron energy loss rate at low temperature in a single-component electron plasma [15]. However, little is known about the nature of the low energy coupled phonon-QPE modes and the spectral behavior of the QPE modes. Since there are several polarization functions in the case of a mcp, a careful examination of the spectral behavior is required. The spectral behavior in a mcp is expected to be quite involved and different from that of a single-component one [16]. The effects of dynamic screening of plasma species on collisional broadening of the phonon spectral function need to be clarified in a systematic way in a mcp-phonon coupled system.

The purpose of this paper is to investigate various phonon spectral functions of a mcp in the \( \omega-q \) plane and analyze their behavior in detail to elucidate the effects of dynamic screening and various plasmon–phonon coupling at finite temperature. The effects of dynamic screening to the carrier–phonon interactions is included by extending the random phase approximation (rpa) to take into account local-field corrections (LFCs) of carriers, and carrier–phonon coupling channels of polar and nonpolar optical phonons are considered. We present numerical results applied to the case of electrons and holes optically generated in a solid focusing our study on the influence of dynamic screening and plasmon–phonon coupling on various phonon spectral functions. We briefly review the formulation of phonon spectral function of a mcp in section 2 and discuss results in section 3 for the cases of optically excited wurtzite GaN. The main conclusions of our study are in section 4. A brief account of some preliminary analysis on dielectric responses in a mcp has recently been reported [17]. The supplemental formulation and numerical results of linear responses of a mcp prerequisite for phonon spectral analysis are given in appendix B. We find a useful sum rule giving a simple relation between plasma-species resolved dielectric functions. Table 1 shows the physical parameters of a wurtzite GaN used in our numerical calculation of polarization and dielectric response functions for an idealized mcp. In GaN, the formation of hot electron–hole plasma is expected for carrier densities well beyond the Mott density of GaN \( (\approx 1.8-3.8 \times 10^{18} \text{ cm}^{-3}) \) [1, 27]. In a wurtzite GaN, the band extrema of conduction and valence bands are located at the center of the Brillouin zone, the former being of \( \Gamma_7 \) symmetry and the latter splitting into heavy-hole, light-hole, and split-off bands\(^4\). As one increases the degree of photo excitations, the number of plasma species can be tuned from a two-component (conduction electron-heavy hole) plasma at weak excitation and a three-component (conduction electron-heavy hole-light hole) one at strong excitation.

### 2. Phonon spectral functions

The phonon spectral function of a material describes the probability distribution of having phonons of wave number \( q \) and frequency \( \omega \). The bare phonon modes are modified in a solid due to carrier screening and phonon–plasmon coupling, and the poles of the dressed phonon propagator determine the renormalized phonon dispersion relations [28]. In the presence of plasmon–phonon coupling, multiple peak structure is expected in the phonon spectral function of the material. The LO phonon and the plasmon are coupled because each produces a longitudinal electric field interacting with the charge density of the other [6]. The mode coupling of LO phonons and plasmons introduces a pair of branches named \( L^+ (\omega, q) \) and \( L^- (\omega, q) \), the former (latter) representing high-frequency (low-frequency) coupled mode. Detailed analyses of the longitudinal coupled modes for a single-component solid state plasma have been reported extensively in the past [29, 30]\(^5,6\).

We consider a mcp consisting of electrons in the conduction band and various holes in the valence bands generated by optical excitations in an intrinsic semiconductor [1, 2]. We limit our consideration to a weakly interacting electron and holes at

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**Table 1.** Material parameters used for numerical results of a wurtzite GaN.

| Parameter                      | GaN   | References |
|--------------------------------|-------|------------|
| \( \epsilon_{\infty} \)       | 5.35  | [18]       |
| \( \epsilon_0 \)              | 9.7   | [19]       |
| \( \omega_{LO} \)             | 92    | [20]       |
| \( \omega_{TO} \)             | 66    | [20]       |
| \( m_e/m_0 \)                  | 0.22  | [21]       |
| \( m_{hh0}/m_0 \)             | 1.30  | [21]       |
| \( m_{lh0}/m_0 \)             | 0.3   | [21]       |
| \( \rho \)                     | 6.15  | [19]       |
| \( \epsilon_c \)              | 5     | [22–24]    |
| \( \epsilon_v \)              | 2     | [25]       |
| \( \rho \)                     | 10\(^8\) |           |
| \( s \)                        | 7.67 \times 10\(^3\) | [26] |

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\(^4\) Each band is assumed to have two-fold spin degeneracy with \( \Gamma_6^\pm \) (heavy), \( \Gamma_6^\pm \) (light), and \( \Gamma_7 \) (split-off) symmetries, respectively.

\(^5\) See, for example, Cochran et al [29]. The authors illustrated the dispersion relation of the coupled modes.

\(^6\) See, for example, Cohen [30]. The author discussed the behavior of the coupled modes neglecting the damping effect in the long wavelength limit.
high density region above the Mott density. In such a high density regime, the interaction energy would be much weaker than the kinetic energy and the exciton binding energy is suppressed to vanish breaking excitons into electrons and holes [31].

The effective phonon spectral function $A(q, \omega)$ of a solid is, in general, the sum of contributions from individual carrier–phonon coupling channel such that

$$A(q, \omega) = \sum_{j \nu} A_{j\nu}(q, \omega).$$

(1)

Here $A_{j\nu}(q, \omega)$ describes the probability distribution of having phonons (of $j$th type) dressed by plasma species $\nu$ and is given, in terms of retarded phonon propagator $D_{j\nu}(q, \omega)$ for the individual phonon mode, as [28]

$$A_{j\nu}(q, \omega) = -\frac{1}{\pi} i m D_{j\nu}(q, \omega),$$

(2)

where $q$ and $k$ denote vectors $\hat{q}$ and $\hat{k}$, respectively. In compound semiconductors, charge carriers couple to phonons via various channels such as couplings to LO and TO phonons, and also through acoustic deformation potential and piezooacoustic (AP) couplings. The phonon spectral behavior of a solid state plasma would depend on the carrier screening and scattering channels (distinguished by a parameter $j$) of various carrier–phonon interaction $H^{(j)}_{c\nu}$ via a specific coupling of carrier (c) and phonon (ph). The interaction of specific carrier–phonon coupling is represented by [32–34]

$$H^{(j)}_{c\nu} = \sum_{k\nu} M^{(j)}_{k\nu} \hat{n}_{k+q \nu} (\epsilon^{(j)}_{qk} + \epsilon^{(j)}_{qk}),$$

(3)

where $\hat{n}_{k+q \nu} (= \hat{c}^{\dagger}_{k+q \nu} \hat{c}_{k \nu})$ and $M^{(j)}_{k\nu} (= M^{(j)}_{k\nu})$ are the Fourier transform of the carrier density operator $\hat{n}(r)$ and the matrix element of carrier–phonon coupling for the phonon mode $\omega_{q}$ with distinct polarization, respectively. Here $\epsilon_{k\nu}$ ($\epsilon^{(j)}_{qk}$) and $d_{qk}^{(j)}$ ($d_{qk}^{(j)}$) are the ordinary annihilation (creation) operators of a carrier in the band $\nu$ and of a phonon with frequency $\omega_{q}$ [35], respectively. We consider the case that carriers (electrons and holes) and the lattice are weakly coupled so that carrier–phonon interaction $H^{(j)}_{c\nu}$ would be treated as a perturbation. (See appendix A for further discussion on various matrix elements $M^{(j)}_{k\nu}$, used in our study.)

The dressed phonon propagator $D_{j\nu}$ with collisional broadening is written, in general, as [32, 33]

$$D_{j\nu}(q, \omega) = \frac{2\omega_{q}}{\omega^2 - \omega^2_{q} - 2\omega_{q}|M_{j\nu}(\omega)|^2 \Pi_{q}(q, \omega)/\hbar},$$

(4)

where $\omega_{q}$ and $\Pi_{q}(q, \omega)$ are the bare (undoped crystal) phonon frequency of mode $j$ and the polarization propagator defined by equation (B.5), respectively, the behavior of the latter being discussed in detail in appendix B. The dielectric screening in many carrier systems gives rise to renormalized electron–phonon coupling and thus to a dressed phonon propagator [36]. This collisional broadening modifies the phonon dispersion relations along with the phonon spectral function. In equation (4), $\Pi_{q}(q, \omega)$ is the full retarded polarization propagator of each plasma component (distinguished by $\nu$) and $|M_{j\nu}(\omega)|^2 \Pi_{q}(q, \omega)/\hbar$ in the denominator represents the (complex) phonon self-energy correction $\gamma_{j\nu}(q, \omega)(\equiv \Delta_{j\nu} - i \Gamma_{j\nu}/2)$ via polarization function of plasma species $\nu$, which is known to introduce low energy quantum interference branch in the phonon spectral function [15]. The real and imaginary parts of the phonon self-energy, $\Delta_{j\nu}$ and $\Gamma_{j\nu}$, are given, respectively, by

$$\Delta_{j\nu}(q, \omega) = |M_{j\nu}(\omega)|^2 \Re \Pi_{q}(q, \omega)/\hbar$$

(5)

and

$$\Gamma_{j\nu}(q, \omega) = -2|M_{j\nu}(\omega)|^2 \Im \Pi_{q}(q, \omega)/\hbar.$$ 

(6)

The former describes the frequency renormalization correction due to the electronic screening of the long-ranged Coulomb fields associated with the phonons and the latter a measure of phonon lifetime $\tau_{q}$ or the width of the spectral function due to collisional broadening. Phonon self-energy and, hence, $\Delta_{j\nu}$ and $\Gamma_{j\nu}$ are also functions of $\omega$ and $q$, and $\Im \Pi_{q}(q, \omega) \leq 0$, in general, as illustrated in figures B2(c) and (d) below in appendix B. Ignoring the phonon renormalization, $D_{j\nu}$ reduces, with an infinitesimal positive $\eta$, to [28]

$$D_{j\nu}^{(0)}(q, \omega) = \frac{2\omega_{q}}{\omega^2 - \omega^2_{q} + i\eta},$$

(7)

giving rise to the well-known bare phonon spectral function $A_{j\nu}^{(0)}(q, \omega) = \delta(\omega - \omega_{q}) - \delta(\omega + \omega_{q})$.

In the presence of electron–phonon coupling, the phonon spectral function $A_{j\nu}(q, \omega)$ is now written as

$$A_{j\nu}(q, \omega) = \frac{\omega^2 - \omega^2_{q} - 2\omega_{q}\Delta_{j\nu}(q, \omega) + \omega_{q}\Gamma_{j\nu}(q, \omega)}{\omega^2 - \omega^2_{q} - 2\omega_{q}\Delta_{j\nu}(q, \omega)^2 + \omega_{q}\Gamma_{j\nu}(q, \omega)^2}.$$ 

(8)

We note that the structure of equation (8) is of Fano-like profile [6, 37]. The phonon self-energy $\gamma_{j\nu}(q, \omega)$ due to the dielectric screening represents the frequency shift and the broadening of the spectral line shape playing the role of the complex 'line shape parameter', as was discussed in detail in [6] and the references therein. In the present case, the electron–phonon interaction couples the quasi-discrete phonon level to the continuum of electronic excitations, and the quantum mechanical interference results manifesting an asymmetry of the symmetric bare phonon lines, and the line shape can be fitted in terms of the Fano–Breit–Wigner profile. The discrete phonon states are renormalized by its interaction with the continuum in the case under study. The line shape change can be monitored in the Raman study of the system varying exciting wavelength over a wide range of carrier concentrations, and the phonon self-energy corrections in the solid state plasma would be extracted from the line shape profile of the spectral function [6, 38].

The denominator of $A_{j\nu}(q, \omega)$ can be rewritten, in terms of phenomenological renormalized phonon frequency $\tilde{\omega}_{q}$ and phonon lifetime $\tau_{q}$, as $\omega^2 - \tilde{\omega}_{q}^2 - \frac{i}{2\tau_{q}}$, where $\tilde{\omega}_{q}$ satisfies a quadratic equation given by
(a) and (b): dressed phonon spectral functions at effective carrier temperatures 25 and 300 K. (c) and (d): cross-sectional views of species-resolved phonon spectral function $A_c(q, \omega)$ at four different values of wave number $q$. Contributions via electron–phonon coupling channels of polar (LO) and nonpolar (TO) optical phonons, and of piezoelectric (AP) and nonpolar acoustic deformation (AD) potentials are summed.

$$\omega^2 - \omega^2_j (1 + 2 \Delta_{jj}/\omega_j) \omega^2_{\nu} - \omega^2_\nu \Gamma^2_{\nu} = 0$$

with $\Gamma^{-1}_{\nu} = 2 \tau^{-1}_\nu \Pi_{j\nu}$. For each phonon mode $j$, the electron–phonon interaction introduces the phonon self-energy to change the bare phonon frequencies $\omega_j$ to the new frequencies $\omega^\pm_j$ with finite lifetime $\tau_j$. Equation (9) gives rise to, along with the renormalized primary mode close to $\omega^+_j(q, \omega) \approx \omega_j(1 + 2 \Delta_{jj}/\omega_j + 1/2)\Gamma^2_{\nu}$ for $2\Delta_{jj} > -\omega_j$, a secondary (low energy) mode $\omega^-_j(q, \omega) \approx \Gamma_{jj}/(1 + 1/2\Delta_{jj})$ for $2\Delta_{jj} < -\omega_j$ with negative self-energy correction $\Delta_{jj} < 0$. We find that the latter mode $\omega^-_j$ would be well-defined only with finite values of $\Gamma_{jj}(q, \omega)$ in the region of $\Re \Pi_{\nu}(q, \omega) < 0$ in the $\omega - q$ space. $\Gamma_{jj}(q, \omega)$ is finite only in the $\omega - q$ plane of finite $\Im \Pi_{\nu}(q, \omega)$, which occurs in the presence of single-particle excitations as shown in figure B2 in appendix B. Deep valley with negative $\Re \Pi_{\nu}(q, \omega)$ occurs in the (very) low energy region of high damping inside the single-particle excitation continuum on the $\omega - q$ plane. (See figures B2(a) and (b).)

**3. Results and discussion**

In order to illustrate numerical results for the spectral behaviors of the phonon spectral functions in an ideal mcp, use has been made of effective masses $m_\infty = 0.22 m_0$, $m_{hh} = 1.3 m_0$, and $m_{hh} = 0.30 m_0$ of a simplified wurtzite GaN with parabolic bands. The bare plasma frequencies for carrier concentration of $2 \times 10^{19} \text{ cm}^{-3}$ are $\omega_{p,e} \approx 153 \text{ meV} (= 1.68 \omega_{LO})$, $\omega_{p,hh} \approx 63 \text{ meV} (= 0.69 \omega_{LO})$, and $\omega_{p,lb} \approx 131 \text{ meV} (= 1.44 \omega_{LO})$ for scp plasmas of electrons, heavy holes, and light holes, respectively. In the present work, frequencies and wave numbers are scaled by the longitudinal bare phonon frequency $\omega_{LO}$ and the Thomas–Fermi screening wave number $q_{sc}$, respectively. We consider a simplified non dispersive model of Einstein for bare optical phonons with $\omega_{LO} = 92 \text{ meV}$ and $\omega_{TO} = 66 \text{ meV} (= 0.72 \omega_{LO})$ and a Debye-type model for bare acoustic phonons of $\omega_{jj} = sq$ in an undoped GaN. Here $s \approx 7.67 \times 10^3 \text{ m s}^{-1}$ is the mean speed of sound wave in the material. For a scp, $q_{sc}$ is given, in terms of particle number density $n$ and the chemical potential $\mu$, by [39] $q_{sc}^2 = 4 \pi n \mu/\rho$, and we have $q_{sc}^{(e)} = 9.2 \times 10^5 \text{ cm}^{-1}$ and $q_{sc}^{(hh)} = 2.2 \times 10^7 \text{ cm}^{-1}$ at 25 K and $q_{sc}^{(e)} = 8.6 \times 10^5 \text{ cm}^{-1}$.
and $q_{\text{cc}}^{\text{hh}} = 2.1 \times 10^{-7}$ cm$^{-1}$ at 300 K, respectively, for carrier concentration of $2 \times 10^{19}$ cm$^{-3}$. For a mcp, $q_{\text{cc}}$ is written as

$$q_{\text{cc}}^2 = \frac{4\pi e^2}{\epsilon_0 m_0} \sum_{\nu, \mu} \frac{n_\nu}{m_\nu},$$

where $n_\nu$ and $m_\mu$ are, respectively, the particle number density and the chemical potential of the band occupied by the plasma species $\nu$. For a 2cp consisting of conduction electrons and heavy holes with electron concentration of $2 \times 10^{19}$ cm$^{-3}$, we have $q_{\text{cc}} = 2.94 \times 10^{-7}$ cm$^{-1}$ at 25 K and $q_{\text{cc}} = 2.54 \times 10^{-7}$ cm$^{-1}$ at 300 K, respectively.

In figure 1 the effective phonon spectral function $A(q, \omega)$ of a 2cp is shown for conduction electron concentration of $2 \times 10^{19}$ cm$^{-3}$ at effective carrier temperatures 25 and 300 K, respectively. For optically generated plasmas of weak excitation having not too much carrier concentration, for example, $\sim 10^{19}$ cm$^{-3}$, one can have a 2cp consisting of conduction electrons and heavy holes. Boundaries of allowed single-particle excitations for electrons and holes are designated by a pair of dashed and dotted lines in panels (a) and (b). Species-resolved frequency dependences of $A(q, \omega)$ are displayed separately in panels (c) and (d) at 25 and 300 K for the purpose of spectral analysis. The cross-sectional view of $A(q, \omega)$ and $A_{\text{hh}}(q, \omega)$ are illustrated for four different values of $q$. In a multi-component solid state plasma, the phonon spectral function would reveal multi-component features of the plasma—many sets of phonon-plasmon coupled modes distinct from the case of the scp. Two sets of LO phonon–plasmon coupled modes ($\omega_L$) are shown in addition to the sharp TO modes of $\omega_{\text{TO}} = 0.72 \omega_{\text{LO}}$. At $n_\nu = 2 \times 10^{19}$ cm$^{-3}$, the bare plasmon frequencies are $\omega_{\text{p}, e} = 1.68 \omega_{\text{LO}}$ and $\omega_{\text{p}, hh} = 0.69 \omega_{\text{LO}}$ for the electrons and heavy holes, respectively, well separated from both optical phonon modes $L^{(+)}$ and $L^{(-)}$. In a 2cp, there occur both optic and acoustic plasmons, and thus one can expect not only the individual LO phonon-optic plasmon coupled spectral behaviors of lighter and heavier species as in a scp, but also the contribution from the acoustic plasmon modes [6, 14]. Our result shows that the multiple plasma species give substantial influence on the spectral behavior of the phonon spectral function. Couplings of conduction electrons and heavy holes to both optical and acoustical phonon spectral functions appear simultaneously in the case of 2cp. In panels (a) and (b), contributions to the spectral functions $A_{\text{AD}}(q, \omega)$ of the deformation-potential induced acoustic phonon and $A_{\text{AP}}(q, \omega)$ of piezoelectric Coulomb interaction are also illustrated showing sharp peaks around the low-frequency bare $\omega_{\text{c}}(q)$. For the case of piezoelectricity induced Coulomb interaction, $A_{\text{P}}(q, \omega)$ shows little but slightly enhanced collisional broadening compared to $A_{\text{AD}}(q, \omega)$ of acoustic deformation-potential mechanism.

The spectral intensity for the high-frequency plasmon-like $\omega^{(e)}(q)$ conduction electron–plasmon coupled modes shows strong dispersive behavior starting with $\omega_{\text{p}, e}(\sim 1.7 \omega_{\text{LO}})$ at $q = 0$ and then approaching the boundary of electronic single-particle excitation continuum. The spectral intensities for phonon-like coupled branches of $\omega^{(e)}(q)$ and $\omega_{\text{hh}}^{(e)}(q)$ are of very weak dispersion with peaks at frequencies close to $\omega_{\text{LO}}$ and $\omega_{\text{TO}}$, respectively. Both branches show very sharp peak structures and vanish for the wave numbers beyond $q = 1.5 q_{\text{cc}}$. For the low-frequency plasmon-like $\omega_{\text{hh}}^{(e)}(q)$ phonon-heavy hole plasma coupled modes, we understand that the spectral intensity of the mode is strongly suppressed because the low-frequency heavy-mass species plasma oscillation is so heavily screened by the light-mass species (conduction electrons) that the self-sustaining oscillations of hole plasmons are not permitted any more in this quasi-static region. We conjecture that additional little spectral strength with some broadening near the zero frequency peaked at around $q \sim 0.2 q_{\text{cc}}$ is the so-called QPE-like branch [10], which appears in the region of finite $Zm \Pi_{\nu, e}$ and $R_{\nu, e} \Pi_{\nu, e} < 0$ inside the continuum of single-particle excitations. We observe that individual dressed branches are well resolved with slight overlap as seen in the panels (c) and (d).

In figure 2, spectral behaviors of effective phonon spectral functions $A(q, \omega)$ are compared for the cases of 2cp having conduction electron concentration $5 \times 10^{19}$ cm$^{-3}$ (panels (a) and (c)) and of three-component plasma (3cp) having conduction electron concentration $5 \times 10^{19}$ cm$^{-3}$ (panels (b) and (d)) at effective carrier temperature 25 K. The contributions via electron–phonon coupling channels of polar LO and nonpolar TO phonons, and of piezoelectric and nonpolar acoustic deformation potentials are summed. Cross-sectional views of $A_{\text{LO}}^{(e)}(q, \omega)$ ($\nu = e, hh$) for $3 \times 10^{19}$ cm$^{-3}$ and $A_{\text{TO}}^{(e)}(q, \omega)$ ($\nu = e, hh, lh$) for $5 \times 10^{19}$ cm$^{-3}$ are illustrated in panels (c) and (d), respectively, at four different values of wave number $q$. In a mcp of increased carrier concentrations, the compressive (optical) plasmon-like coupled modes begin with higher frequencies (for example, $\omega \geq 3 \omega_{\text{LO}}$ for $3 \times 10^{19}$ cm$^{-3}$) at $q = 0$, and the corresponding branches of very weak strengths are seen at $\omega \sim 3.2 \omega_{\text{LO}}$ for $3 \times 10^{19}$ cm$^{-3}$ (panels (a) and (c)) and $\omega \sim 4.2 \omega_{\text{LO}}$ for $5 \times 10^{19}$ cm$^{-3}$ (panel (d)) at $q \approx 0.2 q_{\text{cc}}$. At 25 K, $q_{\text{cc}} = 3.15 \times 10^{7}$ cm$^{-1}$ (3.49 $10^{7}$ cm$^{-1}$) for an electron–hole plasma of conduction electron concentrations $3 \times 10^{19}$ cm$^{-3}$ (5 $10^{19}$ cm$^{-3}$). The spectral behavior shown in panels (a) and (c) for $3 \times 10^{19}$ cm$^{-3}$ is very similar to a weakly excited case of $2 \times 10^{19}$ cm$^{-3}$ shown in figures 1(a) and (c), except the fact that the branch $\omega^{(e)}(q)$ occurs at much increased frequency, for example, $\omega \sim 3.2 \omega_{\text{LO}}$ at $q \approx 0.2 q_{\text{cc}}$. For a photo-generated 3cp having conduction electrons of $5 \times 10^{19}$ cm$^{-3}$ at effective carrier temperature 25 K, the charge carriers of the plasma can be resolved into conduction electrons of concentration $5 \times 10^{19}$ cm$^{-3}$, heavy holes of concentration $4.968 \times 10^{19}$ cm$^{-3}$, and light holes of concentration $3.2 \times 10^{19}$ cm$^{-3}$. Species-resolved phonon spectral functions $A_{\text{LO}}(q, \omega)$ and $A_{\text{TO}}(q, \omega)$ of a 3cp ($\nu = e, hh$, and lh) with conduction electron concentration $5 \times 10^{19}$ cm$^{-3}$ are displayed in figures 3 and 4. Boundaries of each individual allowed single-particle excitations are designated by a pair of dashed lines in

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7 In this region, the dielectric screening is quasi-static, since the plasmon frequency of light-mass species is considerably greater than the frequency $\omega_{\text{hh}}^{(e)}(q)$ of the plasmon-like heavy-mass species plasmon–LO phonon coupled mode.
Figure 2. Spectral behaviors of effective phonon spectral functions $A(q, \omega)$ of a two-component plasma (2cp) having conduction electron concentration of $3 \times 10^{19}$ cm$^{-3}$ and of a three-component plasma (3cp) having conduction electron concentration of $5 \times 10^{19}$ cm$^{-3}$ at effective carrier temperature 25 K. (a) and (b): phonon spectral functions of $A(q, \omega)$. (c) and (d): cross-sectional views of species-resolved $A_{2\text{cp}}^\nu(q, \omega)$ ($\nu = \text{e, hh}$) and $A_{3\text{cp}}^\nu(q, \omega)$ ($\nu = \text{e, hh, lh}$) at four different values of $q$. Contributions via electron–phonon coupling channels of polar (LO) and nonpolar (TO) optical phonons, and of piezoelectric (AP) and nonpolar acoustic deformation (AD) potentials are added.

Figure 3. Species-resolved phonon spectral function $A_{\text{c}}(q, \omega)$ ($\nu = \text{e, hh, lh}$) of a three-component plasma formed by conduction electrons of concentration $5 \times 10^{19}$ cm$^{-3}$, heavy holes of concentration $4.968 \times 10^{19}$ cm$^{-3}$, and light holes of concentration $3.2 \times 10^{17}$ cm$^{-3}$ at effective carrier temperature 25 K. Contributions of (a) conduction electron plasmon–LO phonon coupling, (b) heavy hole plasmon–LO phonon coupling, and (c) light hole plasmon–LO phonon coupling.
each panel. It is seen that, with the present choice of sample parameters, the effects of collisional broadening through carrier screening are not significant on the nonpolar phonon spectral functions $A_{\text{TOe}}(q, \omega)$ and $A_{\text{TOlh}}(q, \omega)$, because of very low concentration of light hole carriers.

In figure 5, the frequency and wave-number dependences of phonon self-energy correction $\Gamma_{\text{LOe}}(q, \omega)$ in a two-component plasma via LO phonon-conduction electron coupling for concentration of $2 \times 10^{19}$ cm$^{-3}$ for each species. (a) and (b): real part of the self-energy correction $\Delta_{\text{LOe}}(q, \omega)$ at 25 K and 300 K, respectively. (c) and (d): imaginary parts of the self-energy correction $\Delta_{\text{LOe}}(q, \omega)$ at 25 K and 300 K, respectively.
Contributions of the light-mass species (conduction electrons) $\omega^{\Delta}_{LOe}$ and $\Gamma^{\Delta}_{LOe}$ are illustrated, respectively, in the $\omega$-$q$ space. The contours of $\omega^{\Delta}_{LOe}$ are indicated by thin solid lines in panels (a) and (b). The spectral behaviors of $\nu^{P}_{j}$ reveals a very similar structure as that of the dressed polarization function $\nu^{\Pi}_{R}$ of the plasma species, since $\Delta^{\nu}_{j}$ and $\Gamma^{\nu}_{j}$ are proportional to $\nu^{\Pi}_{R}$ and $\nu^{\Pi}_{I}$, respectively. (See figures B2(a) and (c) below in appendix B.) The sign of $\omega^{\Delta}_{LOe}$ is the same as that of $\nu^{\Pi}_{R}$. The real part of the self-energy correction $\Delta^{\nu}_{LOe}$ also changes the sign crossing the zeros of $\nu^{\Pi}_{R}$ with peaks just above and below the branches of high-frequency optic and low-frequency acoustic plasmon modes. On the other hand, $\Gamma^{\nu}_{j}$ is defined to be $\Delta^{\nu}_{j}$.

Figure 6. Spectral behavior of the plasmon–phonon coupled modes $\omega^{\pm}_{LOe}(q, \omega)$ in a photo-generated two-component plasma of conduction electron concentrations $2 \times 10^{19}$ cm$^{-3}$ at (a) 25 K and (b) 300 K.

Figure 7. Cross-sectional views of renormalized phonon modes $\omega^{\pm}_{LOe}(q, \omega)$ for electron concentrations of $2 \times 10^{19}$ cm$^{-3}$ (a) and (b): behavior at representative values of frequency $\omega$ at 25 K and 300 K, respectively. (c) and (d): behavior at representative values of wave number $q$ at 25 K and 300 K, respectively.
nominate and shows peak structure along the well-defined high-frequency compressive optic and low-frequency acoustic plasmon branches near the plasma cut-offs. We observe that the effect of plasma temperature on the phonon frequency renormalization (panels (a) and (b)) is moderate, but the broadening in the self-energy correction (panels (c) and (d)) is more pronounced at higher temperature as was also seen in the case of the spectral function $A(q, \omega)$ illustrated in figures (a) and (b). The phonon self-energy corrections $\tilde{P}_{LO}(q, \omega)$ of a photo-generated mcp for two different conduction electron concentrations $n_c = 3 \times 10^{19}$ cm$^{-3}$ and $5 \times 10^{19}$ cm$^{-3}$ at 25 K are compared in figure C1 below in appendix C. In the case of $3 \times 10^{19}$ cm$^{-3}$ (panels (a) and (c) of figure C1), the plasma consists of conduction electrons and heavy holes forming a 2cp and the spectral behavior of $\tilde{P}_{LO}(q, \omega)$ is very close to that of $2 \times 10^{19}$ cm$^{-3}$ shown in figure 5. Apparent difference is that the bare frequencies of both optic and acoustic plasmon modes are increased accordingly in a mcp as the carrier concentration increases. For a plasma of conduction electron concentration $5 \times 10^{19}$ cm$^{-3}$, the light hole band is also occupied becoming a 3cp of $n_h = 4.968 \times 10^{19}$ cm$^{-3}$ and $n_h = 3.2 \times 10^{19}$ cm$^{-3}$. Panels (b) and (d) of figure C1 illustrate the spectral behavior of $\Delta_{LO}$ and $\Gamma_{LO}$, respectively, of a 3cp with conduction electron concentration $n_c = 5 \times 10^{19}$ cm$^{-3}$. The effect of light hole carriers on $Re \Pi_{LO}$ and $Im \Pi_{LO}$, thus, to the LO phonon self-energy corrections $\Delta_{LO}$ and $\Gamma_{LO}$ are found to be negligible due to relatively too small concentration of light holes.

Spectral behaviors of the renormalized LO phonon frequencies $\tilde{\omega}_{LO}(q, \omega)$ and $\tilde{\omega}_{LO}(q, \omega)$ are shown in figure 6 for a 2cp of electron concentration $2 \times 10^{19}$ cm$^{-3}$ at 25 K and 300 K, respectively. The primary mode $\tilde{\omega}_{LO}(q, \omega)$ occurs in the domain of $2\Delta_{LO} > -\omega_{LO}$ while the mode $\tilde{\omega}_{LO}(q, \omega)$ in the domain of $2\Delta_{LO} < -\omega_{LO}$ only with finite values of $\Gamma_{LO}$ and negative $\Delta_{LO}$. The latter mode is expected to be observable in the case of large $\Gamma_{LO}$. In figure 7, renormalized phonon modes $\tilde{\omega}_{LO}(q, \omega)$ and $\tilde{\omega}_{LO}(q, \omega)$ shown in figure 6 are resolved for representative values of frequency $\omega$ (panel (a) and (b)) and of wave number $q$ (panel (c) and (d)), respectively. The frequency and wave number are scaled by the longitudinal bare phonon frequency $\omega_{LO}$ and Thomas–Fermi screening wave number $q_{sc}$, respectively. We note that $\tilde{\omega}_{LO}(q, \omega) \approx \omega_{LO}$ over most of the domain satisfying the condition $2\Delta_{LO}(q, \omega) > -\omega_{LO}$ in the $\omega$ plane because the regions of well-defined plasmonic collective modes becoming $\tilde{\omega}_{LO}(q, \omega) \gg \omega_{LO}$. Below the small opening gap (as indicated by white blank in panel (a)) of $2\Delta_{LO}(q, \omega) < -\omega_{LO}$, we observe the secondary mode $\tilde{\omega}_{LO}(q, \omega) \approx \omega_{LO}$ over the domain designated in the insets. The mode $\tilde{\omega}_{LO}(q, \omega)$ is expected in the region of $2\Delta_{LO}(q, \omega) < -\omega_{LO}$ with finite values of $\Gamma_{LO}(q, \omega)$, as shown in figures 5(c) and (d). Spectral behaviors of the renormalized phonon frequencies $\tilde{\omega}_{LO}(q, \omega)$ and $\tilde{\omega}_{LO}(q, \omega)$ for a 2cp of conduction electron concentrations $3 \times 10^{19}$ cm$^{-3}$ and a 3cp of $5 \times 10^{19}$ cm$^{-3}$ are compared in appendix C. (See figure C2 below.) The results illustrated in figures C2(a) and (b) are very similar to that of electron concentration $2 \times 10^{19}$ cm$^{-3}$ shown in figure 6(a) except the shifts in locations of the extrema along the frequency axis at a given effective carrier temperature. Cross-sectional views of the corresponding renormalized phonon modes $\tilde{\omega}_{LO}(q, \omega)$ and $\tilde{\omega}_{LO}(q, \omega)$ are given in figure C3 for representative values of frequency and wave number. At increased conduction electron concentration of $5 \times 10^{19}$ cm$^{-3}$, the frequencies of collective modes of plasmon–phonon coupling shift to higher values, but the presence of light hole carriers of minor concentration is not seen to give noticeable effects in the spectral behavior of the phonon renormalization.

4. Summary and conclusions

We have investigated spectral behavior of the phonon spectral functions of a mcp in an extended random phase approximation. The effects of dynamic screening, plasmon–phonon coupling, and exchange-correlations of the plasma species are examined. We have applied the formulation to the case of an electron–hole plasma of carrier concentrations beyond the Mott density generated optically in an ideal wurtzite GaN, and scattering channels of various carrier–phonon couplings are considered. Clear significance of the multiplicity of the plasma species is shown in the dielectric responses and phonon spectral behaviors of a mcp. From the comparative study of the responses and phonon spectral functions of a mcp with that of a scp, we find that dynamic screening and plasmon–phonon coupling are essential in understanding the spectral behavior of phonon spectral functions in a mcp.

By extending linear response calculation to a mcp, we have detailed the spectral behavior of the dressed polarization functions $\Pi_{LO}(\omega) (\nu = e, hh)$ and examined plasma species-resolved dielectric functions $\epsilon_{\mu \nu}$. A sum rule of $\sum_{\mu \nu}(\epsilon_{\mu \nu}^{-1} - \delta_{\mu \nu}) = 0$ is found and multi-component character of the plasmonic oscillations was investigated. From the zeros of the effective dielectric function $Re \epsilon_{eff}(q, \omega)$ of a 2cp, it is found that optic and acoustic plasmon branches are well separated from each other. (See figure B4.) Hubbard-like local-field corrections of carriers are found to shift both branches slightly to reduced plasmon frequencies for given values of wave number. In a two-component electron–hole plasma, the higher frequency electron plasmon mode is almost intact and well defined near the bare plasma frequency $\omega_{p,e}$. However, the lower frequency bare plasmon mode of heavier species (heavy holes) at $\omega_{p, hh}$ is heavily screened by the lighter mass species (conduction electrons) resulting in an acoustic branch positioned inside the continuum of electron excitations, the latter mode being subject to Landau damping by the light-mass species. $Im \epsilon_{eff}(q, \omega)$ reveals double peak structure in a 2cp each peak appearing inside the single-particle excitation continua of electrons and holes of the plasma, unlike the case of a scp. The effects of local-field corrections of the carriers are appreciable only in the region of low frequency and long wavelength.

Dressed phonon propagators are evaluated in the $\omega$-$q$ plane and we have demonstrated that the dielectric screening in many carrier system gives rise to renormalized electron–phonon
coupling modifying the phonon dispersion relations along with phonon spectral function. The phonon frequency renormalization $\Delta \omega_p(q, \omega)$ and the phonon spectral broadening $\Gamma_p(q, \omega)$ are analyzed in terms of $\Re \Pi_{\nu \nu}(q, \omega)$ and $\Im \Pi_{\nu \nu}(q, \omega)$, respectively, and it is found that the effect of plasma temperature on the phonon frequency renormalization is moderate, but the phonon broadening is more pronounced at higher temperature. We have demonstrated that the plasmon–LO phonon coupling gives a pair of branches $L^+\omega(q, \omega)$ and $L^{-}\omega(q, \omega)$ and, in a 2cp, two sets of LO phonon–plasmon coupled modes $(\omega_{L})$ are seen for each plasma species. The spectral intensity for the high-frequency plasmon-like coupled mode $\omega_{p,c}(q)$ shows strong dispersive behavior starting with $\omega_{p,c}$ at $q = 0$ and approaching the boundary of electronic single-particle excitation continuum. The spectral intensities for phonon-like coupled branches of $\omega_{c}^{(e)}(q)$ and $\omega_{c}^{(b)}(q)$ are of very weak dispersion with peaks at frequencies close to $\omega_{L,0}$ and $\omega_{TO}$. The low-frequency plasmon-like coupled mode $\omega_{p,c}^{(b)}(q)$ is strongly suppressed so that the self-sustaining oscillations of hole plasmons are not permitted any more in this quasi-static region. Moreover, we have observed an additional peak of little intensity at $q \sim 0.2 k_{\text{F}}$ near the zero frequency in the domain of finite $\Im \Pi_{\nu \nu}$ and $\Re \Pi_{\nu \nu} < 0$ inside the continuum of single-particle excitations. The origin for the little peak is conjectured to be the so-called QPE-like branch [10]. In a 3cp of conduction electron concentration $5 \times 10^{19}$ cm$^{-3}$, the effects of light hole carriers on $\Re \Pi_{\nu \nu}$ and $\Im \Pi_{\nu \nu}$ and, thus, to the LO phonon self-energy corrections $\Delta L_{0c}$ and $\Gamma_{L0}$ are found to be negligible due to relatively too small concentration of light holes. The spectral behaviors of the phonon self-energy correction reveals a very similar structure to that of the dressed polarization function $\Pi_{\nu \nu}(q, \omega)$ of the plasma species.

In conclusion, in a multi-component solid state plasma, multiple plasma species give substantial influences on the spectral behavior of the phonon spectral function, which is very distinct from that in a single component plasma commonly seen in doped semiconductors. The results of our computations can be utilized in investigations of energy relaxation behavior of optically generated solid state plasma and are suitable for experimental observation. The spectral behaviors demonstrated in the present work would be confirmed with various energy loss scattering measurements and hot carrier spectroscopies on photo-generated electron–hole plasmas in polar semiconductors. Dynamic spectral shifts and the line broadening would be tested using time-resolved light scattering and/or neutron scattering measurements. The incident angle-dependent/backscattering Raman measurements would reveal the spatial dispersive behavior in the spectral line shapes. The low frequency acoustic plasmon-like coupled modes at long wavelength may be tested by time-resolved Brillouin scattering or ultrasonic measurements.

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Appendix A. Matrix elements of carrier–phonon coupling

Here we briefly summarize matrix elements of carrier–phonon coupling channels employed in our discussion of spectral behavior of phonon spectral functions. Of various channels of carrier–phonon scattering, we consider four distinct carrier–phonon coupling channels would be important in compound semiconductors such as wurtzite GaN material [40]: couplings to polar longitudinal optical (LO) and nonpolar transverse optical (TO) phonons and couplings through acoustic deformation potential and piezoelectricity.

In polar crystals, the longitudinal modes due to long-ranged Fröhlich-type Coulombic interaction induces a long ranged electrical polarization field. For the case of carrier-polar optical phonon coupling, the squared coupling matrix element is given by [28]

$$|M_{q, \nu}^{\text{LO}}(q)|^2 = \frac{4\pi\hbar^2}{q^2} \frac{\omega_{\nu \nu}^2}{2m_e} \alpha,$$

(A.1)

where $\alpha$ is the dimensionless Fröhlich coupling constant $\alpha = e^2/\hbar m_e$ where $\hbar \omega_{\nu \nu}$ is the transition frequency between the states $\nu$ and $\nu'$. $e$ and $\epsilon_{\infty}$ are, respectively, the optical and static dielectric constants of the material.

In GaN, much enhanced carrier-polar phonon interaction is expected due to its higher ionicity giving, for example, the Fröhlich coupling constant $\alpha_{\text{GaN}} \sim 6\alpha_{\text{GaAs}}$ with $\omega_{\nu \nu}(\text{GaN}) \sim 3\omega_{\nu \nu}(\text{GaAs})$ [41]. For crystals lacking a center of symmetry, acoustic phonons also induce an electric polarization field giving rise to piezoelectricity through Coulombic interaction. The piezoelectricity is most commonly found in the wurtzite structure, and the matrix element $M_{q, \nu}^{\text{AP}}(q)$ for piezoelectric acoustic (AP) phonon coupling with frequency $\omega(q)$ and velocity $\mathbf{v}(q)$ is given by [32]

$$|M_{q, \nu}^{\text{AP}}(q)|^2 = \frac{2\hbar s(q)}{\rho_v q^2 \mathbf{v}(q)^2} \sum_{ijk} q_i e_{ijkl} \xi_2(\lambda, q) q_j^2,$$

(A.2)

Here $\rho_v$, $s(q)$, $\xi_2(\lambda, q)$, and $e_{ijkl}$ denote the average mass density, dielectric constant of the material, the unit vector of the acoustic lattice polarization $\lambda (= \text{TA or LA})$, and the piezoelectric coupling constants of a third rank tensor, respectively [32]. Because strain modifies local band structure of the material, the deformation potentials can also induce carrier-nonnolar TO and acoustical phonon couplings. The matrix element $M_{q, \nu}^{\text{TO}}(q)$ for nonpolar optical phonon coupling is given, in terms of short-ranged optical deformation potential, by [40]

$$|M_{q, \nu}^{\text{TO}}(q)|^2 = \frac{\hbar D^2}{2\rho_v \omega_{\nu \nu} V}.$$

(A.3)

Here $\mathbf{D}$ and $\mathbf{V}$ are the optical deformation potential constant and the volume of the sample, respectively. On the other hand,
the matrix element $M_{q,i}^{AD}(q)$ for acoustical deformation potential scattering is written as [40, 42]

$$|M_{q,i}^{AD}(q)|^2 = \frac{\hbar \mathcal{E}_c^2 q^2}{2\rho_i \omega_q} V_i^n.$$  \hspace{1cm} (A.4)

Here $\mathcal{E}_c$ is the deformation potential constants of the longitudinal acoustical waves for the carriers in the conduction and valence bands.

**Appendix B. Linear response calculation of a multi-component plasma**

Let us consider an electron–hole plasma, a two-component plasma (2cp), beyond the Mott density subject to a weak external potential field $\phi_{ext}(q, \omega)$ caused by some external test charge distribution $\rho_{ext}$. In such a high density regime, dynamic screening suppresses the exciton binding energy to vanish [31]. In the present work, we consider only the particle scatterings neglecting the excitonic contributions, the latter would be prominent in the opposite regime [43]. Electrons and holes respond differently to an external disturbance resulting in nonidentical polarization functions $\Pi_{i\nu}^{eq}(q, \omega)$ ($\nu = e$ or hh). The potential field $\phi_{ext}(q, \omega)$ gives rise to the corresponding potential energy $V_{ext}$ for a carrier of type $i$. Here $i$ denotes different species of the plasma and we limit our consideration to the simplest case of a nondegenerate valence band in order to simplify our discussion, i.e. $i = 1$ (2) for electrons (holes) of the plasma. We note that $V_{ext}^{(i)}(q, \omega) = e_i \phi_{ext}(q, \omega)$ for the carriers of electric charge $e_i = \mp e$, where $-e$ is the elementary charge of an electron.

### B.1. Self-consistent formulation

The external potential will cause changes in densities of each plasma species, and the charge density fluctuation $\delta \rho_i$ in the $i$th component is written as

$$\delta \rho_i(q, \omega) = \sum_j \Pi_{i\nu}^{eq}(q, \omega)V_{ext}^{(j)}(q, \omega),$$  \hspace{1cm} (B.1)

where $\{V_{ext}^{(1)}, V_{ext}^{(2)}\}$ are the carrier density probe and $\Pi_{i\nu}^{eq}(q, \omega)$ is the reducible polarization propagator—the Fourier transform of the density–density response function of a mcp. That is, Re $\Pi_{i\nu}(q, \omega)$ is a measure of the response of an electron liquid to a bare external disturbance. The imaginary part of the retarded polarization function, Im $\Pi_{i\nu}(q, \omega)$, is directly linked to the real part of the conductivity—a measure of dissipative processes, in which quanta of wave number $q$ and frequency $\omega$ are absorbed by the carriers in the plasma. Now, the density fluctuation $\delta \rho_i(q, \omega)$ introduces polarization field to the carriers in the plasma, and the effective (self-consistent) potential energy of a carrier in the $i$th plasma component is given by

$$V_{eq}^{(i)}(q, \omega) = V_{ext}^{(i)}(q, \omega) + \sum_j \psi_{ij}(q, \omega) \delta \rho_j(q, \omega).$$  \hspace{1cm} (B.2)

Here the second term on the right-hand side denotes the additional potential due to the polarization of the system, and $\psi_{ij}$ (or $\psi_{ji}$ in general) is the effective interaction between carriers of the $i$th and $j$th components including exchange and correlation effects. In general, $\psi_{ij}$ is nonlocal and needs be determined self-consistently because exact expression for $\psi_{ij}$ is not available. Within the local density functional scheme of Kohn and Sham [44, 45], $\psi_{ij}$ can be represented as $\psi_{ij} = v_{ij}[1 - G_0(q)]$ with $v_{ij} = \frac{4\pi e_i e_j}{\omega_q^2}q$ and generalized local-field corrections $G_i(q)$ as of Hubbard [46, 47]8). Here $\kappa$ is the background dielectric constant of the material. Our discussion on the exchange-correlation effect of carriers parallels that of Kukkonen and Overhauser [45] and Hedin and Lundqvist [48] but with extension to the case of mcp [49]. (For our convenience, we will minimize recalling the wave-vector and frequency dependencies of the quantities explicitly in each expression now on. For example, $\Pi_{i\nu}(q, \omega)$ will be written as $\Pi_{i\nu}$.)

The density fluctuation $\delta \rho_j$ can also be determined by treating the plasma as a noninteracting system responding to the self-consistent effective potential $V_{ext}^{(i)}$ such as

$$\delta \rho_j(q, \omega) \equiv \check{\Pi}(q, \omega)V_{ext}^{(i)}(q, \omega).$$  \hspace{1cm} (B.3)

Here $\check{\Pi}$ denotes the temperature-dependent proper polarization function of the multi-component many carrier system and describes the response of a plasma component of type $i$ to the effective potential $V_{ext}^{(i)}$, the sum of the external and polarization potentials. Hence $\check{\Pi}$ is a measure of the density fluctuation induced by a screened external charge $\rho_{ext}^{(i)}(= \rho_{ext}^{(i)}(\omega))$.

Combining equations (B.2) and (B.3) gives rise to coupled equations for $\delta \rho_i$ and $\delta \rho_j$. One can easily solve the equations for $\delta \rho_i$ and $\delta \rho_j$ in terms of $V_{ext}^{(i)}$ and $V_{ext}^{(j)}$ to write

$$\begin{pmatrix} \delta \rho_1 \\ \delta \rho_2 \end{pmatrix} = \begin{pmatrix} \Pi_{11} & \Pi_{12} \\ \Pi_{21} & \Pi_{22} \end{pmatrix} \begin{pmatrix} V_{ext}^{(1)} \\ V_{ext}^{(2)} \end{pmatrix},$$  \hspace{1cm} (B.4)

where individual components of the polarization propagators $\Pi_{i\nu}(q, \omega)$ are now given by

$$\Pi_{ii} = \check{\Pi}_i(1 - \check{\Pi}_i \psi_{ij})/\Delta$$  \hspace{1cm} (B.5)

for the intra-species (diagonal) parts ($i = 1$ or $2$) and

$$\Pi_{ij} = \check{\Pi}_i \check{\Pi}_j \psi_{ij}/\Delta$$  \hspace{1cm} (B.6)

for the inter-species (off-diagonal) parts ($i \neq j$). Here $\Delta$ is defined by $\Delta = 1 - \check{\Pi}_1 \psi_{11} - \check{\Pi}_2 \psi_{22} + \check{\Pi}_1 \check{\Pi}_2 (\psi_{12} \psi_{22} - \psi_{12} \psi_{21})$ and plays the role of the effective dielectric function in a mcp. (See equation (B.20) below.)

In the mean-field rpa, the proper polarization function $\check{\Pi}_i$ is replaced with the noninteracting expression commonly known as Lindhard polarizability $\Pi_{0i}(\equiv \Pi_{0i}^{eq})$ for simplicity of

\hspace{1cm}8 Hubbard [46]: an example of Hubbard’s local-field corrections is $G_i(q) = \frac{1}{\sqrt{q^2 + k_F^2 + \mathcal{E}_c^2 q^2}}$, where $k_F$ and $\mathcal{E}_c$ are the Fermi and Thomas–Fermi wave numbers, respectively, of the $i$th species of the multi-component plasma [28]. In this work, we have used this expression of $G_i(q)$ extended to include ladder diagrams in our evaluation of $\psi_{ij}$ and $\Delta(q, \omega)$.}
notation). It means that the effects of many-body correla-
tions are neglected to let the effective pair interaction $v_{ij}$
become the bare Coulomb interaction $v_{ij}(=v_{ji})=\mp v$
with $v=4\pi e^2/(\varepsilon a^2)$. The generalized noninteracting polarization function $\Pi_i^0$ is given by \[28, 50\]

$$\Pi_i^0(q, \omega) = 2 \sum_k \frac{f_{k+q,i}^{(0)} - f_{k,i}^{(0)}}{\varepsilon_{k+q,i} - \varepsilon_{k,i} - \hbar\omega - i\eta}, \quad (B.7)$$

where $f_{k,i}^{(0)}$ is the temperature dependent carrier distribution function. The Lindhard-type expression of equation (B.7) is analogous to the case of spin-resolved polarization functions in a multi-component spin system \[49\]. The intra- and inter-species components of the polarization propagator $\Pi_{ij}$ are written in the rpa, as

$$\Pi_{ij}^{\text{ii}} = \Pi_i^0(1 - v\Pi_j^0)/[1 - v(\Pi_i^0 + \Pi_j^0)] \quad (B.8)$$

and

$$\Pi_{ij}^{\text{vv}} = -v\Pi_i^0\Pi_j^0/[1 - v(\Pi_i^0 + \Pi_j^0)]. \quad (B.9)$$

We note that $\Delta^{\text{ii}} = 1 - v(\Pi_i^0 + \Pi_j^0)$ is the effective macroscopic dielectric function $\epsilon_{\text{eff}}^{\text{ii}}$ in a mcp. (See further discussion on $\epsilon_{\text{eff}}^{\text{ii}}$ below.) Further reducing to the case of single-component system such as in doped semiconductors, we have $\Pi_{ij}^{\text{ii}} = \Pi_i^0(1 - v\Pi_j^0)$, the well known expression of mean-field polarizability \[28, 51, 52\].

In figure B1 real and imaginary parts of $\Pi_i^0(q, \omega) (i = e)$ of a conduction electron plasma is illustrated in the $q-\omega$ plane for the carrier density of $2 \times 10^{19} \text{ cm}^{-3}$ at effective temperatures $T_{\text{eff}} = 25 \text{ K}$. Each inset illustrates the wave number dependence of $Re \Pi_i^0(q, \omega)$ and $Im \Pi_i^0(q, \omega)$, respectively, for representative frequencies at 25 K and 300 K. Real part of the noninteracting polarization function $Re \Pi_i^0(q, \omega)$ shows a broad valley structure within the single-particle excitation continuum at small frequency $q < q_{\text{dc}}$ and a peak structure of moderate width along the upper boundary of the continuum. The line of $Re \Pi_i^0(q, \omega) = 0$ lies in the continuum region of the single-particle excitations and, at small $q$, $Re \Pi_i^0(q, \omega)$ changes sign from negative to positive as $\omega$ increases sweeping across the continuum region. On the other hand, $Im \Pi_i^0(q, \omega)$ is finite and negative inside the continuum region of the single-particle excitations showing a sharp dipped structure along the zero line of $Re \Pi_i^0(q, \omega)$. Nonlocal dynamic behaviors of dressed $\Pi_\nu(q, \omega)$ (\(\nu = e \text{ or } h\hbar\)) are illustrated in figure B2 for a 2cp consisting of conduction electrons and heavy holes, each species of carrier concentration $2 \times 10^{19} \text{ cm}^{-3}$ at effective carrier temperature 25 K. Boundaries of a pair of single-particle excitation continua for electrons and heavy holes are indicated with steep dashed (for electrons) and slower dotted (for holes) lines, respectively, and branches of the optic and acoustical plasmon excitations are clearly distinguished in strong color intensities. The well defined optic modes dominated by the lighter species are intact to be seen clearly in $Re \Pi_i(q, \omega)$ and $Im \Pi_i(q, \omega)$ occurring well outside the single-particle excitation continuum at frequencies higher than that of the bare LO phonons. The longitudinal low-frequency (optic) bare modes of the heavier species are drastically screened by the lighter mass species to become acoustic \[6\]. Insets illustrate the frequency or wave-number dependences of $Re \Pi_\nu(q, \omega)$ for representative values of wave number or frequencies.

### B.2. Plasma-species-resolved dielectric functions

Since we now have all the components of the polarization propagator $\Pi_{ij}$, the self-consistent interaction of equation (B.2) can now be written, with $\delta_{ij}$’s given by equation (B.4), as

$$\left( \begin{array}{c} V^1_{\text{ext}} \\ V^2_{\text{ext}} \end{array} \right) = \frac{1}{\Delta} \left( \begin{array}{cc} \Pi^0_{ij} & \Pi_i^0 \psi_{ij1} - \Pi_i^0 \psi_{ij2} \\ \Pi_i^0 & 1 - \Pi_i^0 \psi_{ij1} \end{array} \right) \left( \begin{array}{c} V^1_{\text{ext}} \\ V^2_{\text{ext}} \end{array} \right)$$

\quad \equiv \left( \begin{array}{cc} \varepsilon_{ij1}^{-1} & \varepsilon_{ij2}^{-1} \\ \varepsilon_{ij1} & \varepsilon_{ij2} \end{array} \right) \left( \begin{array}{c} V^1_{\text{ext}} \\ V^2_{\text{ext}} \end{array} \right). \quad (B.10)$$

Here the $2 \times 2$ matrix multiplied to the column of external probe potentials $(V^1_{\text{ext}}, V^2_{\text{ext}})$ on the right hand side is just the inverse of ‘plasma-test charge’ dielectric tensor $\varepsilon$ in the $2 \times 2$ space of plasma species and each component is given by

$$\varepsilon_{ij} = \delta_{ij} + \sum_{\ell=1,2} \psi_{\ell ij} \Pi_{ij}. \quad (B.11)$$

(Similar description corresponding to the case of spin-polarized electrons was investigated earlier by one of us \[50\].) The intra-species ‘electron-test charge’ dielectric function $\varepsilon_{ii} = 1 - \Pi_i^0 \psi_{ii1} - \Pi_i^0 \psi_{ii2}(1 - \Pi_i^0 \psi_{ii2})$ reduces, in the rpa, to $\varepsilon_{ii}^{\text{rpa}} = 1 - \Pi_i^0(1 - \Pi_i^0)$ and to $\varepsilon_{ii}^{\text{rpa}} = 1 - \Pi_i^0$ in the case of single species electron liquid \[48\]. On the other hand, the inter-species component $\varepsilon_{ij} = \Delta(\psi_{ij1})^2$ reduces to $\varepsilon_{ij}^{\text{rpa}} = 1 - (1 - \Pi_i^0)(1 - \Pi_j^0)$ in the rpa and is undefined in the case of single-species electron system. We find that

$$\sum_{ij} (\varepsilon_{ij}^{-1} - \delta_{ij}) = [(\psi_{11} + \psi_{22})\Pi_i^0 + (\psi_{21} + \psi_{12})\Pi_j^0$$

$$-2(\psi_{12}\psi_{21} + \psi_{11}\psi_{22})\Pi_i^0\Pi_j^0]$ \quad (B.12)$$

and, hence, that $\sum_{ij} (\varepsilon_{ij}^{-1} - \delta_{ij}) = 0$ since $\psi_{ij} = -\psi_{ij}(i \neq j)$ in the rpa.

In response to the potential field $\phi_{\text{ext}}$ due to an external test charge $\rho_{\text{ext}}^i$ of electrical charge $e_i$ ($e_j$), another probing test charge would experience the ‘test charge-test charge’ interaction $V_{ij}^{\text{rpa}}(\rho_{\text{ext}}^i)$ written as

$$V_{ij}^i(q, \omega) = v_{ij}(\rho_{\text{ext}}^i + \delta_{ij}) + v_{ij}\delta_{ij}. \quad (B.13)$$

and, similarly, $V_{ij}^j(q, \omega)$ with the indices $i$ and $j$ interchanged in $V_{ji}^i$. In general, the dielectric function $\epsilon(q, \omega)$ of a material is defined, in terms of ‘test charge-test charge’ interaction, by

$V_{ij}^i(q, \omega) = V_{\text{ext}}(q, \omega)\epsilon(q, \omega) \quad (45)$,

and can be extended to a mcp as follows. Substituting the density fluctuations $\delta_{ij}$’s of equation (B.4) into equation (B.13), $V_{ij}^i$ and $V_{ij}^j$ are written, in terms of $V_{\text{ext}}^i$ and $V_{\text{ext}}^j$ as

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Figure B1. Real and imaginary parts of noninteracting polarization functions $\Pi_0^q(q, \omega)$ of conduction electron plasma for the carrier density of $2 \times 10^{19} \text{ cm}^{-3}$ at effective temperature $T_{\text{eff}} = 25$ K. Insets illustrate the wave-number dependence of $\Pi_0^q(q, \omega)$ for representative frequencies at 25 and 300 K. Pair of dashed lines denotes the region of allowed single-particle excitations in a wurtzite GaN.

Figure B2. Real and imaginary parts of dressed polarization functions, $\Pi_\nu^q(q, \omega)$, for conduction electrons ($\nu = e$) and heavy holes ($\nu = hh$) of concentration $2 \times 10^{19} \text{ cm}^{-3}$ and effective carrier temperature 25 K. Insets illustrate the frequency or wave-number dependences of $\Re \Pi_\nu^q(q, \omega)$ for representative values of wave number or frequencies. Pairs of dashed and dotted lines denote the allowed regions of single-particle excitations for conduction electrons and heavy holes, respectively, in a wurtzite GaN.

Figure B3. Dyson equations for the effective Coulomb interaction $V_{ij}$ between carriers $i$ and $j$ ($i = e$ or $h$) in a multi-component plasma.
Here $\varepsilon^{-1}_{ij}$'s are the components of the inverse dielectric tensor $\varepsilon^{-1}$ of a mcp and are given in terms of the polarization propagators. The intra- and inter-species components are expressed, respectively, as $\varepsilon^{-1}_{ii} = 1 + \nu(\Pi_{ij} - \Pi_{ji})$ and $\varepsilon^{-1}_{ij} = \nu(\Pi_{ij} - \Pi_{ji})$ and can be combined to be written, in general, as

$$\varepsilon^{-1}_{ij} = \delta_{ij} + (-1)^{\nu + 1} \sum_{\ell = 1, 2}^{\nu} \nu_{\ell} \Pi_{ij}. \quad (B.15)$$

By substituting $\Pi_{ij}$'s of equations (B.5) and (B.6) into equation (B.15), one can obtain, in a mcp, a sum rule written as

$$\sum_{ij} (\varepsilon^{-1}_{ij} - \delta_{ij}) = 0. \quad (B.16)$$

In the rpa, $V_{ii}^{\nu} = V_{ii}^{\nu}$ and, hence, $\tilde{\delta}_{ij}$ and $\epsilon_{ij}$ are identical. If we neglect inter-species correlations (i.e. $G_{ij} = G_{\delta_{ij}}$ like Hubbard’s local-field correction [46]9), we have

$$\epsilon_{\nu} = 1 - \frac{\Lambda_{\nu} \Pi_{ij}^{0}}{1 - \Lambda_{\nu} \Pi_{ij}^{0}} \quad (B.17)$$

and

$$\epsilon_{ij} = 1 - \frac{\Lambda_{\nu} \Pi_{ij}^{0}}{\Lambda_{\nu} \Pi_{ij}^{0}}. \quad (B.18)$$

Here the vertex function $\Lambda_{i}$ is defined by $\Lambda_{i} = [1 - (\psi_{\nu} - v)\Pi_{ij}^{0}]^{-1}$ generally known as $1/(1 + vG_{ii}^{\nu} \Pi_{ij}^{0})$ [28]. For the case of a single species system, one resumes $\epsilon_{\nu}^{-1} = 1 + \nu\Pi_{ij}^{0}/(1 - \psi_{\nu} \Pi_{ij}^{0})$ giving rise to $\epsilon_{\nu} = 1 - \Lambda_{\nu} \Pi_{ij}^{0}$. Since $\Lambda_{i} \rightarrow 1$ in the rpa, we have $\epsilon_{\nu}^{\text{rpa}} = 1 - (1 - \nu \Pi_{ij}^{0}/(v \Pi_{ij}^{0})$ and $\epsilon_{ij}^{\text{rpa}} = 1 - \nu \Pi_{ij}^{0}/(1 - \nu \Pi_{ij}^{0})$ further reducing to $\epsilon_{\nu}^{\text{rpa}} = 1 - \nu \Pi_{ij}^{0}$ in a single species electron gas.

**B.3. Effective dielectric constant and local-field corrections on it**

Let us introduce the macroscopic effective dielectric constant $\epsilon_{\text{eff}}$ of a mcp by equating $[\epsilon_{\text{eff}}^{-1}(q, \omega) - 1] \rho_{\text{ext}}$...
phenomenologically in the presence of the external test charge $\rho_{\text{ext}}$ to the net charge density induced in the plasma as follows [14]

$$\varepsilon_{\text{eff}}(q, \omega) = 1 + \frac{v[\Pi_1^0 + \Pi_2^0 - \Pi_1^1\Pi_2^1 \sum_{j=1,2} \psi_{ij}]}{\Delta} = (\varepsilon_{11}^{-1} - 1) + (\varepsilon_{22}^{-1} - 1) = -(\varepsilon_{11}^{-1} + \varepsilon_{21}^{-1}), \quad \text{(B.20)}$$

where the identity of equation (B.16) has been observed in writing the last equality. In the rpa, $\varepsilon_{\text{eff}}^{\text{rpa}} = 1 - v[\Pi_1^0 + \Pi_2^0]$ is the effective dielectric constant, which is the same as that suggested by Vashishta et al and others [47, 53]. However, we note that Vashishta et al introduced their effective dielectric constant $\varepsilon_{\text{eff}}$ differently. They defined their $\varepsilon_{\text{eff}}(q, \omega) - 1$ as $\sum_{ij}(\varepsilon_{ij}^{-1} - \delta_{ij})$, which vanishes according to the identity given by equation (B.16). Our description of $\varepsilon_{\text{eff}}$ is consistent with that of the Dyson equation approach. In a multi-component many carrier system (consisting of electrons and various holes in the present case), the effective (dressed) interactions $\tilde{V}_{ij}$ between carriers of types $i$ and $j$ are the solutions of Dyson equations. For example, in terms of full retarded (proper) polarization function $\tilde{\Pi}_i$ and bare Coulomb interaction $v_{ij}$, it is written as

$$\tilde{V}_{ij} = v_{ij} + \sum_{\ell=e,h} v_{\ell i}\tilde{\Pi}_i\tilde{V}_{\ell j}. \quad \text{(B.21)}$$

Here the bare Coulomb interaction $v_{ij}$ is $v_{ij}(\equiv \frac{4\pi e^2}{\omega})$ for $i = j$ or $-v_q$ for $i \neq j$. The internal Coulomb interactions of the carriers in the plasma are renormalized in exactly the same way as the external potential fields. Diagrammatic representation of equation (B.21) for dressed interactions $\tilde{V}_{ij}$ is given in figure B3.

Figure B5. Effective dielectric functions $\text{Im} \varepsilon_{\text{eff}}(q, \omega)$ of two-component plasma formed in a wurtzite GaN with conduction electron density of $2 \times 10^{19}$ cm$^{-3}$ at effective carrier temperature 25 K: (a) $\text{Im} \varepsilon_{\text{eff}}(q, \omega)$, (b) $\text{Im} \varepsilon_{\text{eff}}^{\text{rpa}}(q, \omega)$, (c) $\text{Im} [\varepsilon_{\text{eff}}(q, \omega) - \varepsilon_{\text{eff}}^{\text{rpa}}(q, \omega)]$, and (d) $\text{Im} \varepsilon_{\text{eff}}(q, \omega)$ and $\text{Im} \varepsilon_{\text{eff}}^{\text{rpa}}(q, \omega)$ for $\omega = 0.1\omega_{1LO}$. Pairs of dashed lines denote the corresponding boundaries of allowed single-particle excitation continua.
One can solve the coupled equations of equation (B.21) explicitly for \( \tilde{V}_{ee} \), \( \tilde{V}_{hh} \), and \( \tilde{V}_{eh} \) to write \( \tilde{v}/\epsilon_{ij} = \epsilon_{ij}^{\text{eff}} \) confirming \( \omega = -\sum \Pi(q_i) = \epsilon_{qi}^{\text{eff}} \).

In figures B4(a) and (b), dispersive behaviors of the real part of effective dielectric function \( \epsilon_{\text{eff}}(q, \omega) \) of equation (B.20) and \( \epsilon_{\text{eff}}^I(q, \omega) \) are shown, respectively, for a 2cp formed of conduction electrons and heavy holes each with conduction electron concentrations 3 \times 10^{19} \text{ cm}^{-3} and 5 \times 10^{19} \text{ cm}^{-3}.

**Figure C1.** Phonon self-energy corrections \( \Phi_{\text{LOe}}(q, \omega) \equiv \Delta_{\text{LOe}} - i\Pi_{\text{LOe}/2} \) for a two-component plasma and a three-component plasma at 25 K. (a) and (c): real and imaginary parts of the self-energy correction for a two-component plasma with conduction electron concentration 3 \times 10^{19} \text{ cm}^{-3}. (b) and (d): real and imaginary parts of the self-energy correction for a three-component plasma with conduction electron concentration 5 \times 10^{19} \text{ cm}^{-3}.

**Figure C2.** Spectral behavior of the renormalized phonon frequencies \( \tilde{\omega}_{\text{LOe}}(q, \omega) \) in a photo-generated plasma at 25 K of conduction electron concentrations (a) 3 \times 10^{19} \text{ cm}^{-3} and (b) 5 \times 10^{19} \text{ cm}^{-3}.
concentration $2 \times 10^{19}$ cm$^{-3}$ at effective carrier temperature 25 K. In panel (a), the zero value contours of $\varepsilon_R(q, \omega) = \varepsilon_{R,0}^{\text{eff}}$ are indicated with dark solid lines, each denoting the dispersion curves of high-frequency ‘optic’ and low-frequency ‘acoustic’ plasmon modes. Pairs of dashed and dotted lines indicate the boundaries of allowed single-particle excitations for electrons and heavy holes. In the region of long wavelength and high frequency, $\varepsilon_R(q, \omega) \approx 0$ allowing well-defined dissipationless self-sustaining collective oscillations. The difference of $\varepsilon_{R,0}^{\text{rpa}}(q, \omega)$ is illustrated in panel (c), and the wave-number dependences of $\varepsilon_{R,0}^{\text{rpa}}(q, \omega)$ are compared in panel (d) for $\omega = 0.1 \omega_{\text{LO}}$. In panel (b), the contour of $\varepsilon_{R,0}^{\text{rpa}}(q, \omega) = 0$ is indicated by green dotted line for comparison. In figure B4(a), we find that a pair of plasmon branches are observed in a 2cp, and that optic and acoustic branches are well separated within the rpa, each damped through single-particle excitations of electrons and holes, respectively. However, the local-field corrections of carriers modify the plasmon branches slightly reducing plasmon frequencies of both the optical and acoustical branches for given values of wave number $q$. We note that, in the 2cp of $n_e = 2 \times 10^{19}$ cm$^{-3}$, the higher frequency electron plasmon mode of bare frequency $\omega_{p,e} \approx 1.8 \omega_{\text{LO}}$ is almost intact and well defined but that the lower frequency plasmon mode of heavier species (heavy holes) at $\omega_{p,\text{hh}} \approx 0.69 \omega_{\text{LO}}$ is screened by the lighter conduction electrons giving rise to an acoustic branch, the latter mode being subject to Landau damping by the lighter species, since the branch is located well inside the electron excitation continuum [17]. The corresponding imaginary parts $\varepsilon_{I,0}^{\text{rpa}}(q, \omega)$ and $\varepsilon_{I,0}^{\text{rpa}}(q, \omega)$ are shown in figure B5. While $\varepsilon_{I,0}^{\text{rpa}}(q, \omega)$ has a single peaked structure in an $\omega - q$ plane for a scp [16], it reveals double peaked structure each appearing inside the single-particle excitation continua of electrons and holes in a 2cp, respectively, right below the upper boundaries of each continuum in the region of low frequency and long wavelength. The effects of local-field corrections are appreciable only in the region of low frequency and long wavelength as illustrated in panels (c) and (d) of figure B5.

Appendix C. Spectral behavior of phonon self-energy corrections and renormalized phonon modes of two-component and three-component plasmas

In a photo-generated electron–hole plasma, the number of plasma species can be modulated as a function of the carrier concentration $n_e$ in the conduction band. For higher values of $n_e$, both valence bands of heavy holes and light holes can

Figure C3. Wave-number and frequency dependences of renormalized phonon modes $\tilde{\omega}_{\text{LO}}^\pm(q, \omega)$ at 25 K. (a) and (b): behavior for representative values of frequency $\omega$ for electron concentrations of $3 \times 10^{19}$ cm$^{-3}$ and $5 \times 10^{19}$ cm$^{-3}$, respectively. (c) and (d): behavior for representative values of wave number $q$ for electron concentrations of $3 \times 10^{19}$ cm$^{-3}$ and $5 \times 10^{19}$ cm$^{-3}$, respectively.
be occupied giving rise to 3cp. For a plasma of conduction electron concentration $5 \times 10^{19}$ cm$^{-3}$, the light hole band is also occupied becoming a 3cp of $n_{hh} = 4.968 \times 10^{19}$ cm$^{-3}$ and $n_{he} = 3.2 \times 10^{17}$ cm$^{-3}$. Phonon self-energy corrections $\Phi_{LO}(q, \omega)$ of a photo-generated mc-p for two different conduction electron concentrations $n_c = 3 \times 10^{19}$ cm$^{-3}$ and $5 \times 10^{19}$ cm$^{-3}$ at 25 K are given in figure C1. In the case of $3 \times 10^{19}$ cm$^{-3}$ (panels (a) and (c) of figure C1), the plasma consists of conduction electrons and heavy holes forming a 2cp and the spectral behavior of $\Phi_{LO}(q, \omega)$ is very close to that of conduction electron concentration $2 \times 10^{19}$ cm$^{-3}$ shown in figure 5. Noticeable difference is that the bare frequencies of both optic and acoustic plasmon modes are increased accordingly in a mc-p as the carrier concentration is increased. Panels (b) and (d) illustrate the spectral behavior of $\Delta \omega_{p}$ and $\Gamma_{p}$, respectively, of a 3cp with $n_c = 5 \times 10^{19}$ cm$^{-3}$. The effect of light hole carriers on $Re \Phi_{LO}$ and $Im \Phi_{LO}$ and, thus, to the LO phonon self-energy corrections $\Delta \omega_{LO}$ and $\Gamma_{LO}$ are found to be negligible due to relatively too small concentration of light holes.

Spectral behaviors of renormalized phonon frequencies $\omega_{LO}(q, \omega)$ and $\omega_{LO}(q, \omega)$ are compared in figure C2 for a 2cp of $n_c = 3 \times 10^{19}$ cm$^{-3}$ and a 3cp of $n_c = 5 \times 10^{19}$ cm$^{-3}$, respectively. In panel (b), boundaries of the continuum of single-particle excitations for light holes are indicated by a pair of dash-dotted lines. The frequency and wave number are scaled by the longitudinal bare phonon frequency $\omega_{LO}$ and Thomas–Fermi screening wave number $q_{sc}$, respectively. The results illustrated in figures C2(a) and (b) are very similar to that of electron concentration $2 \times 10^{19}$ cm$^{-3}$ shown in figure 6(a), but the locations of the extrema are shifted along the frequency axis at a given effective carrier temperature. In figure C3, cross-sectional views of the renormalized phonon modes $\omega_{LO}(q, \omega)$ and $\omega_{LO}(q, \omega)$ shown in figure C2 are illustrated for representative values of frequency and wave number. In panels (a) and (b), the wave-number dependences of $\omega_{LO}(q, \omega)$ are shown for constant values of frequency $\omega$ at 25 K, while the frequency dependences at representative values of wavenumber $q$ are shown in panels (c) and (d) for electron concentrations of $3 \times 10^{19}$ cm$^{-3}$ and $5 \times 10^{19}$ cm$^{-3}$, respectively. For conduction electron concentration of $5 \times 10^{19}$ cm$^{-3}$, the frequencies of plasmon–phonon coupled modes shift to higher values, but the presence of light hole carriers of minor concentration is not seen to give observable effects in the spectral behavior of the phonon renormalization.

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