Design of gain scheduled fuzzy PID controller for multi-link robot manipulators

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Abstract. Gain Scheduling (GS) controller is one of the most popular approaches to nonlinear control design. Commonly, Gain Scheduling (GS) controllers have better performance than robust ones. In this paper, the structure of a proposed gain scheduled PID fuzzy controller (GS-PIDF) designed and analyzed. The Fuzzy PID controller gains weighted through gain scheduling designed system to grantee both the stability and the best performance of a multi links robot manipulator system. The overall designed controller system is applied to two links robot manipulator system, which is subject to modelling using Lagrangian dynamics method and simulation via MatLab and Simulink package to minimize the vibration and acquire best performance.

1. Introduction

Gain scheduling is widely technique applied for nonlinear control strategy. It has been used in many applications. Historically, gain-scheduling method applied with flight control, gas turbine control, robotics and under actuated mechanical systems [1, 2, 4 and 8].

Most techniques of gain scheduling control design are based on finding approximation linearized system at a number of selected operating points. The control parameters were then calculated based on different criteria at each operating points [1, 2, 4, 6 and 9].

The control parameters were applied to the nonlinear system under conditions of slow performance, near to the operating point and with or without stability guarantee. Most old studies put several strategies to overcome the disadvantages of the gain scheduling method; e.g. working close to the operating point, slow change between operating points, freeze the system between the operating points and even global shut down the system if show unacceptable performance [1, 2, 4, 6 and 8].

In this paper, a suggested strategy of gain scheduling combined with PID fuzzy controller applied to nonlinear two links robot arm is designed. The whole operating range of the robot arm is divided into several local operating points. Linearized dynamic model of the robot arm is calculated at each operating point. The control parameters range at each operating point is calculated using Routh stability criteria. The control gains for all operating points are tabulated and interpolated in between the operating point to cover the whole operating range. The gain-scheduled system is designed to calculate and feed the control gains to the PID fuzzy control at any operating point to force the robot arm to follow the input trajectory to the system.

The paper is organized as follows. In section two, modelling of mechanical systems with equation of motion and linearized model are represented. Section three provides the design control strategy and
steps. In section four, simulated results of the gain scheduling PID fuzzy controller with two links robot arm are represented. In section five, the conclusions are summarized.

2. Modelling of Mechanical Systems

Generally, the mathematical models for multi links robot manipulators are highly nonlinear; the mathematical models of such systems usually consist of a set of linear or nonlinear differential equations derived by using some form of estimation and simulation. The dynamic models of multi links robot manipulators may depend not only on their structure but also on their internal characteristics [7 and 9].

2.1. Equation of Motion

The two link robot manipulator is a nonlinear time invariant system. The general form of the Lagrangian dynamics equation of the system is given by:

\[ M(x)\ddot{x} + c(x, \dot{x})\dot{x} + G(x) = L\dot{u}(t) \]  \hspace{1cm} (1)

\[ M(x)\ddot{x} + N(x) = L\dot{u}(t) \]  \hspace{1cm} (2)

Where, \( x = [\theta_1, \theta_2, \dot{\theta}_1, \dot{\theta}_2]^T \) the state variables of the system which taken to be the angle and angular velocity of each link, \( \dot{u}(t) \) is the applied torque to the links, \( M(x) \) include the inertia matrix and \( N(x) \) is a vector including Carols and Centrifugal \( c(x, \dot{x}) \), gravity forces \( G(x) \) and damping effect.

\[ \begin{bmatrix}
    a_{1} & a_{2}c_{12} & a_{3} \\
    a_{2}c_{12} & a_{3} \\
  \end{bmatrix}
  \begin{bmatrix}
    \dot{\theta}_1 \\
    \dot{\theta}_2 \\
  \end{bmatrix}
+ \begin{bmatrix}
    B_1 & a_2s_{12}\dot{\theta}_2 \\
    -a_2s_{12}\dot{\theta}_1 & B_2 \\
  \end{bmatrix}
  \begin{bmatrix}
    \dot{\theta}_1 \\
    \dot{\theta}_2 \\
  \end{bmatrix}
+ \begin{bmatrix}
    -a_4s_1 \\
    -a_5s_2 \\
  \end{bmatrix}
= \begin{bmatrix}
    T_1 \\
    T_2 \\
  \end{bmatrix} \]  \hspace{1cm} (3)

Where,

\[ s_1 = \sin (\theta_1) \]
\[ s_{12} = \sin (\theta_1 - \theta_2) \]
\[ c_{12} = \cos (\theta_1 - \theta_2) \]
\[ a_1 = (m_1 + m_2)l_1^2 \]
\[ a_2 = m_2l_1l_2 \]
\[ a_3 = m_2l_1^2 \]
\[ a_4 = (m_1 + m_2)gl_1 \]
\[ a_5 = m_2gl_2 \]  \hspace{1cm} (4)

\[ \theta_1 \]
\[ \theta_2 \]
\[ \text{Figure 1. Two link robot arm.} \]
2.2. Linearized Model of Two Links Robot Arm

In order to drive the linearized model of the system the mathematical equations of the system (3-4) approximated near to each operating points representing the whole range of required trajectory the robot will take.

Equation (5) represents the linear model of two-link robot arm:

\[
\begin{bmatrix}
  a_1 & 0 \\
  0 & a_3
\end{bmatrix}
\begin{bmatrix}
  \dot{\theta}_1 \\
  \dot{\theta}_2
\end{bmatrix}
+ \begin{bmatrix}
  B_1 & 0 \\
  0 & B_2
\end{bmatrix}
\begin{bmatrix}
  \theta_1 \\
  \theta_2
\end{bmatrix}
+ \begin{bmatrix}
  -a_4 c_1 \\
  -a_5 c_2
\end{bmatrix}
= \begin{bmatrix}
  T_1 \\
  T_2
\end{bmatrix}
\]  

(5)

Where;

\[ c_1 = \cos(\theta_1) \]
\[ c_2 = \cos(\theta_2) \]

Bi is the damping coefficient at each hinge.

T_i is the torque at each hinge.

2.3. Case Study

The physical parameters of the two links robot arm designed as shown in Table 1:

| Link       | 1st Link | 2nd Link |
|------------|----------|----------|
| The link length; \( l_i \) | \( l_1 = 0.5 \text{ [m]} \) | \( l_2 = 0.5 \text{ [m]} \) |
| Link mass; \( m_i \) | \( m_1 = 5 \text{ [kg]} \) | \( m_2 = 4.5 \text{ [kg]} \) |
| Damping coefficient at each hinge; \( B_i \) | \( B_1 = 20 \text{ [kgm}^2/\text{s}] \) | \( B_2 = 20 \text{ [kgm}^2/\text{s}] \) |

3. Controller Design

The gain scheduled PID fuzzy (GS-PIDF) controller system is designed according the suggested steps:

- The whole operating range of the robot arm is divided into small interval operating points, \( \theta_i = 0^\circ \rightarrow 90^\circ, \Delta \theta_i = 10^\circ \)
- The nonlinear time invariant mathematical model of the two links robot arm is linearized at the selected operating points.
- Each linearized model is applied to classical PID close loop control system.
- Using Routh stability Criteria for each operating point to calculate the range of each gain. Where, each link has:

\[
\begin{align*}
    k_{pi_{min}} &< k_{pi} < k_{pi_{max}} \\
    k_{ii_{min}} &< k_{ii} < k_{ii_{max}} \\
    k_{di_{min}} &< k_{di} < k_{di_{max}}
\end{align*}
\]  

(6)

Where;

\( i \) = number of the links.
\( k_{pi} \) = Proportional gain,
\( k_{ii} \) = Integral gain,
\( k_{di} \) = Derivative gain.

- The selected gain of each operating point is calculated as the norm of the maximum and minimum values.
- The selected gain is applied to the PID fuzzy controller system at each operating point to perform the whole GS-PIDF controller system [3 and 8].

The designed control system shown in the following block diagram, figure 2
Figure 2. The GS-PID Fuzzy Controller System of Two Links Robot Arm.

4. Simulation Results
The dynamic model of two links robot arm in Equation (3) is simulated using MatLab Simulink Package using the physical parameters given in table 1. The gains variation for each link at the selected operating points is shown in figure 3. The variation of P and I gains $k_{p1}$, $k_{p2}$, $k_{i1}$, $k_{i2}$ for both links are the same for each operating point in figure 3[a-b]. While, the D gains for each link $k_{d1}$, $k_{d2}$ show large deviation at each operating point in figure 3[c].

![Figure 3. The gains variation for each link at the selected operating points.](image)

In this study, for each link two fuzzy logic controller systems were designed; PI fuzzy logic controller has inputs: the position error (e), the integration of the error (Ie) and PD fuzzy controller has inputs: the position error (e), change in the error (de) and one output is the change of torque (du). Input and
output fuzzy members functions are symmetric and triangle member functions were used in membership functions [3, 5, and 6].

The performance of the GS-PIDF controller for unit step input is shown in figure 4. The maximum overshoot of [21% - 27%] at [0.5 – 1 sec] and zero steady state error.

Figure 4. The Performance of the GS-PIDF Controller for Unit Step Input.

The GS-PIDF controller is applied to the two links robot arm with different trajectories. The controller system successes to force the robot arm to flow the given trajectory with minimum error as shown in figure 5.
Figure 5. The Performance of the GS-PIDF Controller for Different Input Trajectories.

5. Conclusions
In this paper, brief acknowledge of the gain scheduled control systems was introduced. Then a suggested control strategy was designed based on linearized the nonlinear dynamic model of two links robot arm at selected operating points. The strategy is based on grantee the stability of the system at each operating point by calculating the PID gains range at each operating point using Routh stability criteria. Also, these grants the stability performance of the system in between any two operating points. The norm PID gains at each operating point were the applied to a PID fuzzy controller to force the robot arm to follow the selected trajectory. The simulation designed for the system used MatLab and Simulink package. The control strategy shows acceptable performance of the robot arm with different given trajectories.

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