Research of the transfer relationship of the plastic region of the curved bridge based on the auxiliary system method

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Abstract. Based on the member bars of one end fixed and one end free, it is a new method to study the relationship between the two ends of the curved beam by using the principle of virtual work and the balance equation. By analyzing the plastic hinge and the plastic region model of the curved beam, the transfer relation of the plastic hinge and the plastic region model is deduced by using the transfer matrix method of auxiliary system. Compared with the traditional initial parameter method, the transfer matrix method is more accurate and the calculation program is simpler. The results show that the transfer matrix method is correct and effective, and it can simulate the failure process of elastoplastic better and can be used for the initial design and final check-up of the curved bridge structures.

1. Introduction
Curvilinear girder bridge structure is widely used, but due to the influence of curvature, the calculation of internal force and deformation of curvilinear beam is more complex, with the continuous development of curvilinear bridge, the elastoplastic analysis of curvilinear girder bridge is particularly important.

The basic idea of transfer matrix method is to transform the mechanical analysis problem of a whole structure into the mechanical analysis problem of “docking” and “transferring” of several units or substructures [1]. The auxiliary system method is an accurate method to solve the field matrix with the aid of the auxiliary system, which is the bar with one end fixed and one end free. The auxiliary system method was used to solve the spatial behavior transfer field matrix of curved beam [2], and then the elastic-plastic model of curved beam was analyzed, and the transfer relation of the model of plastic hinge was deduced, and on this basis, the transfer relation of the model of plastic domain of curved beam was deduced.

2. Analysis principle of bending elastic-plastic hinge model
After the element enters the elastic-plastic range, plastic hinge appears at the rod end, and an equivalent spring is set at both ends of the rod to reflect the elasto-plastic deformation characteristics of the rod end [3-7].

With the single-component model(Fig.1), the elasto-plastic deformation of the bar is reflected by the equivalent springs at both ends, and the stiffness can be determined by the arbitrary restoring force
model. If the rotation Angle of the rod end is set as $\theta$, the total rotation Angle $\theta'$ can be divided into elastic rotation Angle $\theta'$ and plastic rotation Angle $\theta''$, which can be written as formula (1):

$$\begin{align*}
\theta' &= \theta' + \alpha' \\
\theta'' &= \theta'' + \alpha''
\end{align*}$$

(1)

When $\alpha'$ and $\alpha''$ are equal to 0, the bar is in the state of elastic deformation.

Figure 1. The rotation corner diagram of single component model

In order to describe the plastic deformation, the three-line model $M$ with restoring force between moment $\theta$ at the rod end and Angle $M - \theta$ at the rod end in figure 2(a) can be adopted, and the model $M$ with moment $\alpha$ at the rod end and plastic Angle $M - \alpha$ separated from figure 2(a) can be adopted in figure 2(b).

According to formula (1), from figure 2 (a) can be obtained that:

$$M = M + \left( \theta' + \alpha - \frac{M}{k_i} \right) p_i k_i = M + \left( \frac{M}{k_i} + \alpha - \frac{M}{k_i} \right) p_i k_i$$

(2)

from figure 2 (b) can be obtained that:

$$M = M + \alpha f_i k_i$$

(3)

Among them, $k_i = 2EI/R\beta$ (bending stiffness).

If formula (2) and formula (3) are equal, the following equation can be obtained:

$$f_i = \frac{p_i}{1 - p_i} \quad \left( f_i' = \frac{p_i'}{1 - p_i} \right)$$

(4)

For example $p_i = 1$, is the elastic state, corresponding $f_i \to \infty$; For example $p_i = 0$, is the plastic flow state, corresponding to $f_i = 0$.

The state variables of the left section are denoted $S' = [N_i' \ Q_i' \ Q_i' M_i' M_i' \ u_i' \ v_i' \ \omega_i' \ \theta_i' \ \theta_i']^T$.

The state variables of the right section are denoted a $S' = [N_i' \ Q_i' \ Q_i' M_i' M_i' \ u_i' \ v_i' \ \omega_i' \ \theta_i' \ \theta_i']^T$. $D$ represents the lattice of bending plastic hinge.

The above formula can be written as a matrix:

$$S' = DS'$$

(5)

Among them,
3. Analysis of plastic domain model of curved beam

Under the real stress state, is the length of the plastic area, rather than a section (plastic hinge model without length), plastic domain can be said to "chain plastic hinge", namely the combination of multiple plastic hinge and curved beam section (as figure 3), so the length of the plastic domain can be said is very good, also be able to get plastic deformation, the structure of the area, make the elastic-plastic analysis of the structure more reasonable.

Figure 3. The schematic diagram of the plastic region of the curved beams

In formula (7) of formula \( I_p \) stipulated in China’s detailed rules for seismic design of highway and bridge [9] (JT G/T 02-01-2008), the calculation result is smaller.

\[
I_p = \left\{ \begin{array}{l}
0.08t + 0.022d, f_y \geq 0.044d, f_y \\
2 \frac{b}{3} 
\end{array} \right. 
\]

In the formula, \( b \) —— The short side dimensions of the rectangular section or the diameter of the circular section; \( l_p \) —— Component length.

Under the action of earthquake, the bridge structure is often cracked or destroyed in a certain section. The length of the plastic domain can be determined according to the size of the failure region. The plastic domain model of "chain plastic hinge" can calculate the deformation of the linear bridge structure in the plastic domain of any length.

4. The plastic domain transfer relation of curved beam

According to Figure 3, the plastic domain model of curved beam and the transmission matrix of curved beam and the dot matrix of plastic hinge, the transfer matrix of the bending plastic domain of curved beam can be derived.

\[
D = \begin{bmatrix} [D_1] & [D_2] & \vdots & [D_n] \end{bmatrix} \quad E_0 \quad \theta_0 \quad [D] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -l/f_i, k_i & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} 
\]

Figure 3. The schematic diagram of the plastic region of the curved beams

In figure 3, \( l_p \) represents the length of the plastic domain [8]. The plastic domain is represented as a set of \( n_p \) plastic hinges and \( n \) elastic segments with \( l_p/n \) lengths between plastic hinges.

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\[
\begin{bmatrix} \cos \phi & -\sin \phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\sin \phi & \cos \phi & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & R(\cos \phi - 1) & \cos \phi & -\sin \phi & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cos \phi & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix} 
\]

\[
T = \begin{bmatrix} \sin \phi \\
-\cos \phi \\
0 \\
0 \\
0 \cos \phi \\
0 \\
0 \\
0 \\
0 \\
1 \\
0 \\
0 \\
\end{bmatrix}
\]

In the equation,
\[ \phi = \frac{l_p}{nR}; \quad t_{11} = \frac{R'(\phi - \sin \phi)}{E_I}; \quad t_{12} = \frac{R'(1 - \cos \phi)}{E_I}; \]
\[ t'_{11} = \frac{R'(\phi \cos \phi + \sin \phi)}{2E_I} + R'(3\sin \phi \phi \cos \phi - 2\phi) + \mu R'(\sin \phi \phi \cos \phi); \quad t'_{12} = \frac{R'\phi \cos \phi + \sin \phi}{2nGA}; \]
\[ t'_{13} = \frac{R'(2 - 2\cos \phi \phi \sin \phi)}{2E_I} - \frac{\mu_l \sin \phi}{2nGA}; \quad t'_{21} = \frac{l_p \sin \phi - \mu_l \sin \phi}{2nEA}; \quad t'_{22} = \frac{l_p \sin \phi}{2nEA}; \quad t'_{23} = \frac{R'(2\cos \phi \phi \sin \phi - 2\phi)}{2E_I} + \mu_l \sin \phi; \]
\[ t'_{31} = \frac{R'(\sin \phi \phi \cos \phi)}{2EA} + \frac{R'(\sin \phi \phi \cos \phi)}{2GA}; \quad t'_{32} = \frac{R'(\sin \phi \phi \cos \phi)}{2EA}; \quad t'_{33} = \frac{R'\phi \cos \phi + \sin \phi}{2GA}; \]
\[ t'_{41} = \frac{R'(1 - \cos \phi \phi \sin \phi)}{2E_I} \quad \mu R'(\sin \phi \phi \cos \phi); \quad \mu = \frac{l_p}{nG}; \quad \mu = \frac{l_p}{nE_I}; \]
\[ t'_{42} = \frac{R'(2 - 2\cos \phi \phi \sin \phi)}{2E_I} - \frac{\mu_l \sin \phi}{2nEI}; \quad t'_{43} = \frac{R'(2 - 2\cos \phi \phi \sin \phi)}{2E_I} + \mu_l \sin \phi; \]
\[ t'_{44} = \frac{R'\phi \cos \phi + \sin \phi}{2GA}; \quad t'_{45} = \frac{R'(\phi \cos \phi - 2)}{2GA}; \quad t'_{46} = \frac{R'(\phi \cos \phi - 2)}{2GA}; \]

5. Example Calculation

The catastrophe model of the earthquake damage to the bridge is divided into three stages: elastic working stage, elastoplastic working stage and cataclastic fault-failure stage. In this paper, it is assumed that the crushing of concrete and the yielding of steel reinforcement occur simultaneously in the design. According to the code, the stress value of concrete entering the stage of fracture and failure is \( \sigma = 35\) MPa, while the stress value entering the stage of elastoplastic work is 0.4\( \sigma \).

A single span simply supported curved beam is subjected to vertical uniformly distributed load and concentrated load in the middle of the span. Its calculation diagram is shown in figure 4. Considering the shear deformation of the structure, the basic parameters of the curved beam are shown in table 1.

![Figure 4. The calculation diagram of the curved beam](image)

Table 1. The basic parameters of the curved beam

| R   | A   | E   | G   | I_3 | I_2 | I_1 | I_0 | \( \mu \) | q  | p   |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| m   | m^3 | MPa | MPa | m^4 | m^4 | m^4 | m^4 | kN·m^3 | kN  |
| 200 | 11.01 | 3 x 10^3 | 0.42 | 29.25 | 82.14 | 41.72 | 25.61 | 1.2 | 130 | 100 |

Matlab calculation software was used for programming. The maximum bending moment of the structure in this example appears in the mid-span section, and the structure is in the elastic-plastic working stage after reaching the ultimate bending. Figure 5 is the elastic-plastic analysis result of curved beam bridge calculated by the plastic hinge model.
According to the fracture failure of the structure, the length of the plastic domain is assumed to be $l/5$ (where $l$ represents the total length of the bridge), and Figure 6 shows the elastic-plastic analysis results of curved girder Bridges calculated by using the plastic domain model.

6. Conclusion
In this paper, the elastic-plastic model of curved beam is analyzed, and the transfer matrix of the bending plastic hinge model is derived by using the auxiliary system method, and the transfer matrix of the plastic domain model of curved beam is deduced. Matlab program was used to analyze the stress and deformation of single-span curved beam under load, indicating that the plastic domain model is more consistent with the failure characteristics of practical components than the plastic hinge model. The results of this paper can provide theory and method for the structural analysis of curved beam.

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