Light-front gauge propagator
reexamined

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November 5, 2018

Abstract

Gauge fields are special in the sense that they are invariant under
gauge transformations and "ipso facto" they lead to problems when we
try quantizing them straightforwardly. To circumvent this problem we
need to specify a gauge condition to fix the gauge so that the fields that
are connected by gauge invariance are not overcounted in the process of
quantization. The usual way we do this in the light-front is through the
introduction of a Lagrange multiplier, \((n \cdot A)^2\), where \(n_\mu\) is the external
light-like vector, i.e., \(n^2 = 0\), and \(A_\mu\) is the vector potential. This leads
to the usual light-front propagator with all the ensuing characteristics
such as the prominent \((k \cdot n)^{-1}\) pole which has been the subject of much
research. However, it has been for long recognized that this procedure
is incomplete in that there remains a residual gauge freedom still to be
fixed by some "ad hoc" prescription, and this is normally worked out to
remedy some unwieldy aspect that emerges along the way. In this work
we propose a new Lagrange multiplier for the light-front gauge that leads
to the correctly defined propagator with no residual gauge freedom left.
This is accomplished via \((n \cdot A)(\partial \cdot A)\) term in the Lagrangian density. This
leads to a well-defined and exact though Lorentz non invariant propagator.

1 Introduction

The history of the light-front gauge goes as far back as 1949 with the pioneering work of P.A.M.Dirac [1], where the front-form of relativistic dynamics was introduced as a well-defined possibility for describing relativistic fields. Since its début into quantum field theory it has known days of both glory and oblivion for varied reasons. On the one hand it seemed a solid grounded and more convenient approach to studying quantum fields, e.g., the only setting where a proof of the finiteness of the \(N = 4\) supersymmetric Yang-Mills theory could be carried out successfully was in the light-cone gauge (a facet of its glory) [2]. But on the other hand, manifest Lorentz covariance is lost and non-local terms sneak into the renormalization program (the other side of the coin that charges us with a price to pay).
One of the reasons why the light-front form has lured many into this field of research is due to the fact that its propagator structure seemed simple enough to deserve their special attention. However, its manifest apparent simplicity hide many complexities not envisaged at first glance nor understood without much hard work. For example, one of the, say, “ugly” aspects of the ensuing propagator is the emergence of the mistakenly so-called “unphysical” pole which in any physical processes of interest leads to Feynman integrals bearing these singularities. We say mistakenly because as it became understood later, it is in fact very much physical in that without a proper treatment of such a pole, one violates basic physical principles such as causality [3].

On the other hand, for the brighter side of it, the light-front gauge seemed advantageous in quantum field theory because it allowed the possibility of decoupling the ghost fields in the non-Abelian theories, since it is an axial type gauge, as shown by J. Frenkel [4], a property that can simplify Ward-Takahashi identities [5] and problems involving operator mixing or diagram summation [6].

Looking through the light-front literature we soon realize that there is a simple and standard gauge vector potential field propagator in which appears two terms [7], namely,

$$G^{\mu\nu}_{ab}(k) = -i\delta^{ab} \left\{ g^{\mu\nu} - \frac{k^{\mu}n^{\nu} + k^{\nu}n^{\mu}}{k \cdot n} \right\},$$  

(1)

where $a, b$ labels non-Abelian gauge group indices.

We see that the propagator (1) has one strictly covariant factor proportional to the space-time metric $g^{\mu\nu}$ and also the characteristic light-front factor proportional to $(k^{\mu}n^{\nu} + k^{\nu}n^{\mu})(k \cdot n)^{-1}$. For the majority of computations, be they in quantum field theory or in nuclear physics (Bethe-Salpeter, etc.) make use of this propagator. Some people have recognized the presence of a third term proportional to $(k^{2}n^{\mu}n^{\nu})(k \cdot n)^{-2}$ [8], i.e.,

$$G^{\mu\nu}_{ab}(k) = -i\delta^{ab} \left\{ g^{\mu\nu} - \frac{k^{\mu}n^{\nu} + k^{\nu}n^{\mu}}{k \cdot n} + \frac{k^{2}n^{\mu}n^{\nu}}{(k \cdot n)^{2}} \right\},$$  

(2)

but this third term has always been consistently dropped in the actual calculations on the grounds that it has been claimed long ago that such “contact terms” have no physical significance because they do not propagate any information. After all, from its inception, the paradigm has always been that gauge terms such as $k^{\mu}n^{\nu} + k^{\nu}n^{\mu}$ and $k^{2}n^{\mu}n^{\nu}$ must not contribute to any physical process because of current conservation. If that be the case, then we must squarely face the vexing question: Why one would drop only the “contact terms” in the calculations on the grounds that they do not have physical significance because propagates no information? However, more recently, it has been shown [9] that this is not the case. These “contact terms” do have physical significance being carriers of relevant information.

Our contribution in this paper is to show that the condition $n \cdot A = 0$ ($n^{2} = 0$) is necessary but not sufficient to define the light-front gauge. It leads to the standard form of the light-front propagator (1) which lacks the relevant
contact term of (2). The necessary and sufficient condition to uniquely define the light-front gauge is given by \( n \cdot A = \partial \cdot A = 0 \) so that the corresponding Lagrange multiplier to be added to the Lagrangian density is proportional to \((n \cdot A)(\partial \cdot A)\) instead of the usual \((n \cdot A)^2\). Note that the condition \( \partial \cdot A = 0 \) in the light-cone variables defines exactly (for \( n \cdot A = A^+ = 0 \)) the constraint

\[
A^- = \frac{\partial^+ A^\perp}{\partial^+} \Rightarrow \frac{k^+ A^\perp}{k^+}.
\]

This constraint, together with \( A^+ = 0 \), once substituted into the Lagrangian density yields the so-called two-component formalism in the light-front, where one is left with only physical degrees of freedom, and Ward-Takahashi identities and multiplicative renormalizability of pure Yang-Mills field theory is verified [10]. Thus, if we start off by correctly defining the gauge condition in the light-front form, the problems related to residual gauge freedom, zero modes and ..... are completely finessed.

2 Light-Front Dynamics: Definition

According to Dirac [1] it is “...the three-dimensional surface in space-time formed by a plane wave front advancing with the velocity of light. Such a surface will be called front for brevity.”. An example of a light-front is given by the equation \( x^+ = x^0 + x^3 \).

A dynamical system is characterized by ten fundamental quantities: energy, momentum, angular momentum, and boost. In the conventional Hamiltonian form of dynamics one works with dynamical variables referring to physical conditions at some instant of time, the simplest instant being given by \( x^0 = 0 \). Dirac found that other forms of relativistic dynamics variables refer to physical conditions on a front \( x^+ = 0 \). The resulting dynamics is called light-front dynamics, which Dirac called front-form for brevity.

A perusal into the specific literature will soon help us to discover that many different names are used to describe this form of dynamics and the corresponding gauge, such as light-front field theory, field theory in the infinite momentum frame, null plane field theory and light-cone field theory. We prefer the word light-front since the quantization surface is a light-front (tangential to the light cone).

The variables \( x^+ = x^0 + x^3 \) and \( x^- = x^0 - x^3 \) are called light-front “time” and longitudinal space variables respectively. Transverse variables are \( x^\perp = (x^1, x^2) \). We call the reader’s attention to the fact that there are many different conventions used in the literature. Here, we follow the conventions, notations and some useful relations employed in [7].

By analogy with the light-front space-time variables, we define the longitudinal momentum \( k^+ = k^0 + k^3 \) and light-front “energy” \( k^- = k^0 - k^3 \).

For a free massive particle, the on-shell condition \( k^2 = m^2 \) leads to \( k^+ \geq 0 \)
and the dispersion relation
\[ k^- = \frac{(k^\perp)^2 + m^2}{k^+}. \] (3)

This dispersion relation (3) is quite remarkable for the following reasons:

1. Even though we have a relativistic dispersion relation, there is no square root factor.
2. The dependence of the energy \( k^- \) on the transverse momentum \( k^\perp \) is just like in the nonrelativistic relation.
3. For \( k^+ \) positive (negative), \( k^- \) is positive (negative). This fact has several interesting consequences.
4. The dependence of energy on \( k^\perp \) and \( k^+ \) is multiplicative and large energy can result from large \( k^\perp \) and/or small \( k^+ \). This simple observation has drastic consequences for renormalization aspects [11, 4].

3 Massless vector field propagator

The Lagrangian density for the vector gauge field (for simplicity we consider an Abelian case) is given by
\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu A^\mu)^2, \] (4)
where the characteristic Lagrange multiplier proportional to \((\partial \cdot A)^2\) is the so-called gauge-breaking term, which defines a physical configuration space, the space of orbits, from projecting out the gauge fields onto this space.

The equations of motion in the light-front variables are
\[ \partial^+ \left[ \frac{1}{2} \partial^+ A^- + \frac{1}{2} \partial^- A^+ - \partial^\perp A^\perp \right] - (\partial^+ \partial^- - \partial^\perp^2) A^+ = 0 \] (5)
\[ \partial^i \left[ \frac{1}{2} \partial^+ A^- + \frac{1}{2} \partial^- A^+ - \partial^\perp A^\perp \right] - (\partial^+ \partial^- - \partial^\perp^2) A^i = 0 \] (6)
\[ \partial^- \left[ \frac{1}{2} \partial^+ A^- + \frac{1}{2} \partial^- A^+ - \partial^\perp A^\perp \right] - (\partial^+ \partial^- - \partial^\perp^2) A^- = 0 \] (7)

The usual procedure in the light-front milieu has been to make a gauge choice by taking \[ A^+ = 0. \] (8)

This gauge choice is known as infinite-momentum gauge, null-plane gauge, light-cone gauge and light-front gauge. From [13], we have
\[ \partial^+ A^- = 2\partial^+ A^\perp + F(x^+, x^\perp) \] (9)
Thus \( A^- \) is not a dynamical variable. Choosing \( F \) to be zero, the dynamical variables \( A^i \) obey the massless Klein-Gordon equation.
Since the dynamical variables $A^\mu$ obey massless Klein-Gordon linear equation, the general solution will be given by the superposition of plane waves:

$$A^\mu(x) = \int \frac{dk^+ d^2k_\perp}{2k^+(2\pi)^3} \sum_{\alpha=1,2} \delta_{j_\alpha} \left[ a_\alpha(k)e^{-ikx} + a_\alpha^\dagger(k)e^{ikx} \right].$$

(10)

The operator $a_\alpha(k)$ and $a_\alpha^\dagger(k)$ are annihilation and creation operators for photons. They satisfy the commutation relations

$$[a_\alpha(k), a_\beta^\dagger(k')] = 2(2\pi)^3 k^+ \delta_{\alpha\beta} \delta^3(k-k'),$$

$$[a_\alpha(k), a_\beta(k')] = 0, \quad [a_\alpha^\dagger(k), a_\beta^\dagger(k')] = 0.$$  

(11)

The equal $x^+$ commutation relation for the transverse components of the gauge field is

$$[A^\mu(x), A^\nu(y)]_{x^+=y^+} = -\frac{i}{4} \delta_{\mu\nu} \epsilon(x^- - y^-) \delta^2(x^\perp - y^\perp),$$

(12)

where the indices $j, l$ label transverse components of the field.

Taking into consideration the commutators among the field operators as derived above, we may write the momentum space expansions of the free field operator. Introducing the polarization vectors

$$\epsilon_\alpha^\mu(k) = \frac{1}{k^+} (0, 2k^1, k^+, 0), \quad \epsilon_2^\mu(k) = \frac{1}{k^+} (0, 2k^2, 0, k^+),$$

(13)

we can write

$$A^\mu(x) = \int \frac{dk^+ d^2k_\perp}{2k^+(2\pi)^3} \sum_{\alpha} \epsilon_\alpha^\mu(k) \left[ a_\alpha(k)e^{-ikx} + a_\alpha^\dagger(k)e^{ikx} \right].$$

(14)

We obtain the Lorentz condition (note that this is a posteriori condition)

$$\partial \cdot A = 0.$$  

(15)

Introducing the four-vector $n = (1,0,0,-1)$ we have the relation

$$\sum_{\alpha=1,2} \epsilon_\alpha^\mu(k) \epsilon_\alpha^\nu(k) = -g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{k^+} - \frac{n^\mu n^\nu}{(k^+)^2}.$$  

(16)

Let $G^{\mu\nu} = iS^{\mu\nu}$ denote the massless vector field propagator in the light-front theory. We have

$$S^{\mu\nu}(x-y) = -i \langle 0 | T^+ A^\mu(x) A^\nu(y) | 0 \rangle,$$

$$G^{\mu\nu}(x-y) = \theta(x^+ - y^+) \langle 0 | A^\mu(x) A^\nu(y) | 0 \rangle + \theta(y^+ - x^+) \langle 0 | A^\nu(y) A^\mu(x) | 0 \rangle.$$  

(17)

Using the expansion (14) we have

$$G^{\mu\nu}(x-y) = \int \frac{dk^+ d^2k_\perp}{2k^+(2\pi)^3} \frac{e^{-ik(x-y)}}{k^2 + i\varepsilon} \left[ -g^{\mu\nu} + \frac{n^\mu k^\nu + k^\mu n^\nu}{k^+} - \frac{n^\mu n^\nu}{(k^+)^2} \right].$$

(18)
4 Propagator with gauge fixing \((n \cdot A)(\partial \cdot A) = 0\)

In this (and in the subsequent appendices) instead of going through the canonical procedure of determining the propagator as done in the previous section, we shall adopt a more head-on, classical procedure by looking for the inverse operator corresponding to the differential operator sandwiched between the vector potentials in the Lagrangian density.

The relevant gauge fixing term that enters in the Lagrangian density we define as

\[(n \cdot A)(\partial \cdot A) = 0, \quad (18)\]

yielding for the Abelian gauge field Lagrangian density:

\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha} (2n_\mu A^\mu \partial_\nu A^\nu) = \mathcal{L}_E + \mathcal{L}_{GF} \quad (19)
\]

where the gauge fixing term is conveniently written so as to symmetrize the indices \(\mu\) and \(\nu\). By partial integration and considering that terms which bear a total derivative don’t contribute and that surface terms vanish since \(\lim_{x \to \infty} A_\mu(x) = 0\), we have

\[
\mathcal{L}_E = \frac{1}{2} A^\mu \left( \Box g_{\mu\nu} - \partial_\mu \partial_\nu \right) A^\nu \quad (20)
\]

and

\[
\mathcal{L}_{GF} = -\frac{1}{\alpha} (n \cdot A)(\partial \cdot A) = -\frac{1}{2\alpha} A^\mu (n_\mu \partial_\nu + n_\nu \partial_\mu) A^\nu \quad (21)
\]

so that

\[
\mathcal{L} = \frac{1}{2} A^\mu \left( \Box g_{\mu\nu} - \partial_\mu \partial_\nu - \frac{1}{\alpha} (n_\mu \partial_\nu + n_\nu \partial_\mu) \right) A^\nu \quad (22)
\]

To find the gauge field propagator we need to find the inverse of the operator between parenthesis in \(22\). That differential operator in momentum space is given by:

\[
O_{\mu\nu}(k) = -k^2 g_{\mu\nu} + k_\mu k_\nu + \frac{1}{\alpha} (n_\mu k_\nu + n_\nu k_\mu) \quad (23)
\]

so that the propagator of the field, which we call \(G^{\mu\nu}(k)\), must satisfy the following equation:

\[
O_{\mu\nu} G^{\nu\lambda}(k) = \delta^\lambda_\mu \quad (24)
\]

\(G^{\nu\lambda}(k)\) can now be constructed from the most general tensor structure that can be defined, i.e., all the possible linear combinations of the tensor elements that compose it (the most general form includes the light-like vector \(m_\mu\) dual to the \(n_\mu\) – but for our present purpose it is in fact indifferent):

\[
G^{\mu\nu}(k) = g^{\mu\nu} A + k^\mu k^\nu B + k^\mu n^\nu C + n^\mu k^\nu D + k^\mu m^\nu E + m^\mu k^\nu F + n^\mu n^\nu G + m^\mu m^\nu H + n^\mu m^\nu I + m^\mu n^\nu J \quad (25)
\]
Since (23) does not contain any $m_\mu$ factors it is straightforward to conclude that $E = F = H = I = J = 0$. Then, we have

$$A = -(k^2)^{-1}$$

(26)

$$A + (k \cdot n)C + \lambda (k \cdot n)B + \lambda n^2 C = 0$$

(27)

$$-k^2 C + \lambda A + \lambda k^2 B + \lambda (k \cdot n) C = 0$$

(28)

$$(k \cdot n) G + \lambda A + \lambda (k \cdot n) D + \lambda n^2 G = 0$$

(29)

$$-k^2 G + \lambda k^2 D + \lambda (k \cdot n) G = 0$$

(30)

where $\lambda \equiv \alpha^{-1}$.

From (30) we have

$$G = \frac{\lambda k^2}{(k^2 - \lambda k \cdot n)} D,$$

which inserted into (29) yields

$$D = \frac{(k^2 - \lambda k \cdot n)}{[\lambda (k \cdot n)^2 - 2k^2 k \cdot n - \lambda k^2 n^2]} A,$$

$$D = \frac{(\alpha k^2 - k \cdot n)}{[(k \cdot n)^2 - 2\alpha k^2 k \cdot n - k^2 n^2]} A,$$

(31)

so that substituting $D$ back in (4) gives

$$G = \frac{\lambda k^2}{[\lambda (k \cdot n)^2 - 2k^2 k \cdot n - \lambda k^2 n^2]} A,$$

$$G = \frac{k^2}{(k \cdot n)^2 - 2\alpha k^2 k \cdot n - k^2 n^2} A.$$  

(32)

From (31) and (28)

$$\begin{cases} A + \lambda (k \cdot n) B + (k \cdot n + \lambda n^2) C = 0 \\ \lambda A + \lambda k^2 B + (-k + \lambda k \cdot n) C = 0 \end{cases},$$

obtaining for the system

$$C = \frac{(\alpha k^2 - k \cdot n)}{[(k \cdot n)^2 - 2\alpha k^2 k \cdot n - k^2 n^2]} A = D$$

and

$$B = \frac{(\alpha^2 k^2 + n^2)}{[(k \cdot n)^2 - 2\alpha k^2 k \cdot n - k^2 n^2]} A.$$
In the light-front $n^2 = 0$ and taking the limit $\alpha \to 0$, we have

\[
\begin{align*}
B &= 0, \\
C &= -\frac{A}{k \cdot n}, \\
D &= -\frac{A}{k \cdot n}, \\
G &= \frac{k^2}{(k \cdot n)^2} A.
\end{align*}
\]

Then, it is a matter of straightforward algebraic manipulation to get the relevant propagator in the light-front gauge, namely,

\[
\begin{align*}
G^{\mu \nu}(k) &= -\frac{1}{k^2} \left\{ g^{\mu \nu} - \frac{k^{\mu} n^\nu + n^{\mu} k^\nu}{k \cdot n} + \frac{n^{\mu} n^\nu}{(k \cdot n)^2} k^2 \right\},
\end{align*}
\]

which has the outstanding third term commonly referred to as contact term.

## 5 Conclusions

We have constructed a Lagrange multiplier in the light-front that completely fixes the gauge choice so that no unphysical degrees of freedom are left. In other words, no residual gauge remains to be dealt with. Moreover this allows us to get the correct propagator including the important contact term. As have been proved, this term is of capital importance in the renormalization of (Bethe-Salpeter?) ...

The configuration space wherein the gauge potential $A_\mu$ is defined have by the gauge symmetry many equivalent points for which we can draw an imaginary line linking them. These constitute the gauge potential orbits. Gauge fixing therefore means to select a particular orbit. The light-front condition $n \cdot A = 0$ defines a hypersurface in the configuration space which cuts the orbits of the gauge potentials. This surface is not enough to completely fix the gauge. We also need the hypersurface $\partial \cdot A = 0$. The intersect between the two hypersurfaces defines a clear cut line and a preferred direction in the configuration space. The two together then completely fixes the gauge with no residual gauge freedom left.

ACKNOWLEDGEMENTS: A.T.Suzuki is partially supported by CNPq under process 303848/2002-2 and J.H.O.Sales is supported by FAPESP under process 00/09018-0

## A Propagator with gauge fixing $\partial \cdot A = 0$

The gauge fixing term known as Lorentz condition

\[
\partial \cdot A = 0,
\]
yields for the Abelian gauge field Lagrangian density:

\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha} (\partial_{\mu} A^{\mu})^2 = \mathcal{L}_E + \mathcal{L}_{GF} \]  

(36)

By partial integration and considering that terms which bear a total derivative don’t contribute and that surface terms vanish since \( \lim_{x \to \infty} A^{\mu}(x) = 0 \), we have

\[ \mathcal{L}_E = \frac{1}{2} A^{\mu} (\Box g_{\mu\nu} - \partial_{\mu} \partial_{\nu}) A^{\nu} \]  

(37)

and

\[ \mathcal{L}_{GF} = -\frac{1}{2\alpha} \partial_{\mu} A^{\mu} \partial_{\nu} A^{\nu} = \frac{1}{2\alpha} A^{\mu} \partial_{\mu} \partial_{\nu} A^{\nu} \]  

(38)

so that

\[ \mathcal{L} = \frac{1}{2} A^{\mu} \left( \Box g_{\mu\nu} - \partial_{\mu} \partial_{\nu} + \frac{1}{\alpha} \partial_{\mu} \partial_{\nu} \right) A^{\nu} \]  

(39)

To find the gauge field propagator we need to find the inverse of the operator between parenthesis in (39). That differential operator in momentum space is given by:

\[ O_{\mu\nu} = -k^2 g_{\mu\nu} + k_{\mu} k_{\nu} - \frac{1}{\alpha} k_{\mu} k_{\nu} \]  

(40)

so that the propagator of the field, which we call \( G^{\mu\nu}(k) \), must satisfy the following equation:

\[ O_{\mu\nu} G^{\mu\lambda}(k) = \delta_{\lambda}^{\mu} \]  

(41)

\( G^{\mu\lambda}(k) \) can now be constructed from the most general tensor structure that can be defined, i.e., all the possible linear combinations of the tensor elements that composes it:

\[ G^{\mu\lambda}(k) = [A g^{\nu\lambda} + B k^{\nu} k^{\lambda} + C n^{\nu} n^{\lambda} + D k^{\nu} n^{\lambda} + E k^{\lambda} n^{\nu}] \]  

(42)

where \( A, B, C, D \) and \( E \) are coefficients that must be determined in such a way as to satisfy (41). Of course, it is immediately clear that since (22) does not contain any external light-like vector \( n_{\mu} \), the coefficients \( C = D = E = 0 \) straightaway. So,

\[ G^{\mu\nu}(k) = -\frac{1}{k^2} \left\{ g^{\mu\nu} - (1 - \alpha) \frac{k^{\mu} k^{\nu}}{k^2} \right\} \]  

(43)

Of course, this is the usual covariant Lorentz gauge, which for \( \alpha = 1 \) is known as Feynman gauge and for \( \alpha = 0 \) as Landau gauge.

B Propagator with gauge fixing \( n \cdot A = 0 \)

The axial type gauge fixing is accomplished through the condition

\[ n_{\mu} A^{\mu} = 0 \]  

(44)
so that we can write the Lagrangian density as

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2\alpha} (n_\mu A^\mu)^2 = \mathcal{L}_E + \mathcal{L}_G$$  \hspace{1cm} (45)$$

In a similar way as before, we have:

$$\mathcal{L}_E = \frac{1}{2} A^\mu (\Box g_{\mu\nu} - \partial_\mu \partial_\nu) A^\nu$$  \hspace{1cm} (46)$$

and

$$\mathcal{L}_G = -\frac{1}{2\alpha} n_\mu A^\mu n_\nu A^\nu = -\frac{1}{2\alpha} A^\mu n_\mu n_\nu A^\nu.$$  \hspace{1cm} (47)$$

Therefore

$$\mathcal{L} = \frac{1}{2} A^\mu \left( \Box g_{\mu\nu} - \partial_\mu \partial_\nu - \frac{1}{\alpha} n_\mu n_\nu \right) A^\nu.$$  \hspace{1cm} (48)$$

In momentum space the relevant differential operator that needs to be inverted is given by

$$O_{\mu\nu} = -k^2 g_{\mu\nu} + k_\mu k_\nu - \frac{1}{\alpha} n_\mu n_\nu,$$  \hspace{1cm} (49)$$

so that, the general tensorial structure given in (42) that must satisfy (41) yields

$$G^{\mu\nu}(k) = -\frac{1}{k^2} \left\{ g^{\mu\nu} - \frac{k^\mu k^\nu}{(k \cdot n)^2} (n^2 - \alpha k^2) - \frac{k^\mu n^\nu + n^\mu k^\nu}{k \cdot n} \right\}.$$  \hspace{1cm} (50)$$

$$G^{\mu\nu}(k) = -\frac{1}{k^2} \left\{ g^{\mu\nu} - \frac{k^\mu k^\nu}{(k \cdot n)^2} (n^2 - \alpha k^2) - \frac{k^\mu n^\nu + n^\mu k^\nu}{k \cdot n} \right\}.$$  \hspace{1cm} (51)$$

Taking the limit $\alpha \to 0$ and using the light-like vector $n_\mu$ for which $n^2 = 0$ we have finally

$$G^{\mu\nu}(k) = -\frac{1}{k^2} \left[ g^{\mu\nu} - \frac{(k^\mu n^\nu + n^\mu k^\nu)}{(k \cdot n)} \right],$$  \hspace{1cm} (52)$$

which is the standard two-term light-front propagator so commonly found in the literature.

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