Cosmological tests of modified gravity: constraints on $F(R)$ theories from the galaxy clustering ratio

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The clustering ratio $\eta$, a large-scale structure observable originally devised to constrain the shape of the power spectrum of matter density fluctuations, is shown to provide a sensitive and model independent probe of the nature of gravity in the cosmological regime. We apply this analysis to $F(R)$ theories of gravity using the luminous red galaxy sample extracted from the Sloan Digital Sky Survey. We find that the absolute amplitude of deviations from GR, $f_{R_0}$, is constrained to be smaller than $3 \times 10^{-6}$ at the $1\sigma$ confidence level. This bound, improving by an order of magnitude on current constraints, makes cosmological probes of gravity competitive with Solar system tests.

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Perplexing observations, such as the accelerated expansion of the universe (the dark energy phenomenon \cite{1, 2}), the rotation curves of galaxies or the gravitational lensing from large clusters of galaxies (the dark matter phenomenon \cite{3, 4}), seem to point towards new phenomena beyond the physics already tested in the laboratory or in the Solar System. In particular, they may be associated with departures from Einstein’s theory of gravity \cite{5}.

Powerful tests on Solar System \cite{6} and astrophysical \cite{7, 8} scales, however, seem to confirm GR predictions and impose stringent limits on modified gravity models. As a consequence, realistic alternative theories should incorporate nonlinear mechanisms that ensure convergence to General Relativity on small scales and high-density environments. In particular, the fifth force which emerges as a generic prediction of modified gravity models, obtained by adding a single scalar degree of freedom $\nu$ to Einstein’s equations, can be effectively screened either by the Vainshtein \cite{9}, the Damour-Polyakov \cite{10}, or the chameleon \cite{11, 12} mechanisms. Models with the chameleon property converge to GR on cosmological scales too. As a consequence, intermediate, mildly non linear scales ($\sim 10 h^{-1}\text{Mpc}$) appear as a unique window of opportunity for detecting possible deviations from GR in a large class of models.

Modified gravity models must, in first approximation, reproduce the smooth background expansion history of the standard model of cosmology, the Λ-Cold Dark Matter (Λ-CDM) paradigm. To distinguish and falsify various competing gravitational proposals it is thus necessary to analyze characteristic observables of the perturbed sector of the cosmological model. Indeed, to lowest order in cosmological perturbations, non-standard gravitational scenarios effectively result in a time- and scale-dependent modification of Newton’s constant, that is, in a distortion of the dynamical and the statistical properties of characteristic clustering quantities such as the power spectrum \cite{13, 14} and the growing mode $D_+(k, t)$ [and its logarithmic derivative, the growth rate $f(k, t)$] of linear matter perturbations \cite{15, 16}.

Several observational shortcomings affect cosmological probes of modified gravity such as cosmic shear in weak lensing maps \cite{17, 19}, redshift-space distortions \cite{20}, and galaxy clustering \cite{21, 22}. The growth rate of dark matter $f$, for example, cannot be estimated from data without picking a particular model, or at least a parameterization, for gravity \cite{23}. Moreover, although the most generic extensions of GR predict a scale dependent growth rate, devising a method able to measure $f(k, z)$ at different scales is a formidable observational task \cite{24, 25}. Additionally, in virtually all the analyses it is assumed that the bias is the same for both modified and standard gravity models. This is a non-trivial ansatz, since not only the growth of structures is expected to be different in modified theories of gravity, but the gravitational potential, which plays a key role in the formation of galaxies, and hence in determining their biasing properties, changes. Finally, most of these probes rely on a precise, but nonetheless challenging measurement of the mean galaxy density on large cosmic scales \cite{26}.

Here we show how to address most of these issues via a new gravitational probe, the clustering ratio $\eta$ \cite{27} (hereafter BM14).\cite{28} Due to its peculiar definition, this cosmological observable naturally accounts for possible scale-dependent growth rates of matter fluctuations. In particular, on linear scales the $\eta$ amplitude is constant as a function of time in general smooth dark energy ($w$-CDM) models but acquires a characteristic time dependence for modified gravity models.

The galaxy clustering ratio $\eta = \xi_{g,x}(r)/\sigma_{g,x}^2$ is defined as the ratio of the correlation function to the variance of the galaxy over-density field $\delta_g$ smoothed on a scale $x$. 

\[\xi_{g,x}(r) = \frac{1}{2} \int_0^\infty k^2 P(k) \frac{\tilde{C}(k, r)}{C(k)} dk,\]

where $P(k)$ is the matter power spectrum, $C(k)$ is the one-point correlation function, $\tilde{C}(k, r)$ is the two-point correlation function of $g$ galaxies and $x$ the scale on which $\delta_x$ is smoothed.
To simplify the analysis, we consider \( \eta \) as a function of the smoothing scale for a fixed ratio \( n \) between the correlation \( (r) \) and the smoothing \( (x) \) scale. Its amplitude, on quasi-linear scales, is

\[
\eta(n, x, t) = \frac{\int_0^\infty dk k^2 P(k, t) W(kx)^2 \sin(knx) knx}{\int_0^\infty dk k^2 P(k, t) W(kx)^2}, \tag{1}
\]

where \( W(y) = 3[\sin(y) - y \cos(y)] / y^3 \) is the Fourier transform of the unit top-hat (the specific filtering scheme adopted by BM14 to smooth data) and \( P(k, t) \) is the matter power spectrum.

This second order statistic provides a measure of the power ratio at the characteristic scales \( r = nx \) and \( x \), as its amplitude is roughly \( \eta(n, x) \sim D_2^\Delta(1/nx, z, \Delta^2_{L0}(1/nx)/D_2^\Delta(1/x, z)) \Delta^2_{L0}(1/x) \), where \( \Delta^2_{L0}(k) = k^3 P_{L0}(k)/2\pi^2 \) is the initial linear power per logarithmic interval of \( k \) and \( D_2^\Delta(k, z) \) the linear growing mode (which can depend on scale in modified-gravity scenarios). It allows to extract information about the shape of the real-space mass power spectrum without the need of reconstructing the galaxy power spectrum in Fourier space (nor the correlation function of galaxies in configuration space), one-point statistics such as count in cells being all that is needed for its estimation. On top of simplicity and accuracy, the distinctiveness of \( \eta \) is that on large scales, i.e. distances \( x \) and \( r \) that are in the quasi-linear regime, and assuming standard gravity, i.e. a growing mode which does not depend on scales, its amplitude is not only independent from local galaxy biasing (however non linear and scale-dependent in real space), but also from redshift space distortions, linear growth rate of structures, cosmic time, and normalization of the matter power spectrum.

Our approach follows from the observation, developed in this paper, that at least one of these characteristic predictions breaks down if modified gravity is responsible for the large scale distribution of matter. Specifically, if a scale dependent growing mode \( D_+ \) is considered, \( \eta \) is not a universal number anymore. What is also crucial for our discussion, is that the estimation of a characteristic observable of the perturbation sector, such as the clustering ratio \( \eta \), only requires the knowledge of the expansion rate of the universe, \( i.e. \) prior information about the smooth sector of the theory only. In other words, the method does not presuppose the premise to be tested, \( i.e. \) the knowledge of a specific gravitational theory. A prescription for converting redshifts into distances is the only ingredient needed for estimating \( \eta \) from redshift surveys data. Since a large class of interesting modified gravity models predict distance-redshift relations which are indistinguishable from that of the \( \Lambda \)-CDM models, the \( \eta \) observable is such that we do not need to re-estimate it in each of the distinct gravitational scenarios we are testing. From this remark follows the central argument of this paper: instead of assuming a gravitational model and using \( \eta \) to fix the expansion rate of the universe in that model, as done by BM14, we here assume the expansion rate known from independent observations (specifically the Planck results \[30\]) and use \( \eta \) to distinguish different gravitational models, specifically theories where the Einstein-Hilbert action is supplemented by a general function \( F \) of the Ricci scalar.

As template for the \( F(R) \) gravity models, we choose the bi-parametric form \[31-33\]

\[
F(R) = -2\alpha c^2 - f_{R_{0}} e^{R c^2 R_{0}^2} \tag{2b}
\]

where \( f_{R_{0}} \) is a normalization factor and \( N > 0 \). This Lagrangian corresponds to the large-curvature regime of the model proposed in \[33\], which is consistent with Solar-System and Milky-Way constraints thanks to the chameleon mechanism, for \( |f_{R_{0}}| \lesssim 7 \times 10^{-7} \). In the following we will specialize our analysis to the cases \( N = 1 \) and \( 2 \). The background dynamics agree with the reference \( \Lambda \)-CDM scenario. The growth rate of density fluctuations, however, is slightly modified. At the linear level, this follows from the fact that the Newtonian potential \( \Psi_S \), or Newton’s constant, are effectively multiplied, in Fourier space, by a scale dependent factor \( 1 + \epsilon(k, t) \) where \( \epsilon(k, t) = \frac{k^2}{\Delta_{SCD-M}^2 + k^2} \), and \( m^{-2} = 3\alpha^2 F_{RR} = -3(n + 1) f_{R_{0}} e^{2 R_{0} c^2 R_{0}^2} \). On large scales, \( k \ll a \), \( \alpha \) is recovered, whereas on small scales, within this linear approximation, Newton’s constant is larger by a factor 4/3. Stronger gravity implies that structure formation is favored and ultimately results in a matter power spectrum amplitude which is larger than that characterizing the \( \Lambda \)-CDM model. For smaller scales and high densities nonlinear effects are no longer negligible and the chameleon mechanism ensures convergence to GR. As \( |f_{R_{0}}| \) goes to zero, \( m^2 \) goes to infinity and General Relativity is recovered, hence the \( \Lambda \)-CDM scenario, on all cosmological scales (hereafter, we consider as reference \( \Lambda \)-CDM, the spatially-flat six-parameter model shown in column 1 (Best fit) of Table 2 by \[30\]).

The amplitude of the clustering ratio expected in \( F(R) \) gravity is computed using the formalism described in \[15\]. This combines one-loop perturbation theory [that includes nonlinear effects beyond the \( \epsilon(k, t) \) factor, such as new quadratic and cubic vertices in the Euler equation generated by the \( F(R) \) theory] and a halo model [which takes into account the nonlinear impact of the \( F(R) \) theory on the halo mass function through the analysis of the modified spherical collapse]. This approach provides a realistic estimate of the matter density power spectrum, from large scales to small scales, that is automatically consistent with one-loop perturbation theory and agrees with numerical simulations up to the highest available wave number, \( k \lesssim 3 h\text{Mpc}^{-1} \) at \( z = 0 \) \[15\].

We estimate the clustering ratio of the luminous red galaxy (LRG) sample extracted from the SDSS data release 7 \[35\] as well as from the data release 10 \[36\]. The first catalogue \( (s1) \) covers the redshift interval \( 0.15 < z < 0.43 \), has a contiguous sky area of \( 120 \times 45 \text{deg}^2 \), and comprises 62,652 LRG. The second sample \( (s2) \), extracted from the SDSS DR10 after removing all the objects in common with \( s1 \), extends over a deeper interval \( 0.3 < z < 0.67 \) but shallower (and not contiguous)
field of view \( \sim 3000 \text{ deg}^2 \). The galaxy clustering ratio is estimated, assuming the redshift-distance conversion of the reference Λ-CDM model, as detailed in [29]. Error bars are derived from a 30 block-jackknife resampling of the s1/s2 data, excluding each time, a sky area of \( 12 \times 14 \deg^2/(10 \times 10 \deg^2) \). Results for scales \( 10 \lesssim x \lesssim 25h^{-1}\text{Mpc} \) and correlation indices \( n = 2, 3, 4 \) are shown in FIG. 1. Note that the lower limit on \( x \) ensures the independence from bias and non-linear random motions of galaxies, while the upper limit on \( n \) is set because measurements are progressively noisier when the correlation length \( r = nx \) increases.

A generic yet distinctive feature of the matter power spectrum in \( F(R) \) theories is an excess of power on weakly nonlinear scales, \( 0.1 \lesssim k \lesssim 10h\text{Mpc}^{-1} \), with respect to the Λ-CDM case. On the scales considered here, \( x > 10h^{-1}\text{Mpc} \), we thus expect these theories to predict a smaller clustering ratio \( \eta(n, x) \) since \( \eta \) measures the ratio of the power at the characteristic scales \( nx \) and \( x \). This is effectively what is shown by FIG. 1 which illustrates the scale dependence of \( \eta \) in both the reference Λ-CDM and \( F(R) \) scenarios. Note that the relative deviation between \( F(R) \) and Λ-CDM predictions is constant, at least over the range of scales displayed, while its amplitude grows with the correlation index \( n \). The uncertainty in the data, however, grows even faster, that is the signal-to-noise ratio decreases as a function of \( n \) thereby reducing the discriminatory power of the diagnostic at high \( n \).

In FIG. 2 the value of the clustering ratio estimated at the three different redshifts \( z = 0.29, 0.42, \) and \( 0.60 \) is shown (for the typical quasi-linear scale \( x = 16h^{-1}\text{Mpc} \)) and compared against theoretical predictions. While in a Λ-CDM cosmology the amplitude of \( \eta \) is expected to be almost constant in time, in modified-gravity scenarios, such as \( F(R) \) theories, the scale dependence of the effective Newton’s constant eventually results in a substantial redshift dependence of the predictions for the amplitude of \( \eta \). In particular the discrepancy between the \( F(R) \) and the Λ-CDM predictions amplifies with time as they are indistinguishable at early cosmic epochs. A detection of a redshift dependence of the clustering ratio \( \eta(n, x, z) \) on these quasilinear scales would be a strong and unequivocal signature of deviations from the Λ-CDM scenario.

We quantify the confidence level with which current data reject an \( F(R) \) gravitational scenario by means of the standard \( \chi^2 \) statistic. To this end, we consider \( F(R) \) models with exponents \( N = 1 \) or \( N = 2 \) and we assume that the background field value \( |f_{R0}| \) is the only free fitting parameter. Because the signals at different scales \( x \) are correlated, we only analyse the galaxy field filtered on the scale \( x = 16h^{-1}\text{Mpc} \), a trade-off between the precision of measurements (worsening as \( x \) increases) and of theory, i.e. of Eq. 11 (worsening as \( x \) decreases). We make separate analyses for the correlation indices \( n = 2 \) and 3, hereafter called respectively reference analysis and control analysis. Note that the covariance matrix is diagonal, since the \( \eta \) measurements in the three different redshift bins can be considered as independent estimates. The resulting 1D Log-likelihood profiles are shown in FIG. 3.

The most immediate conclusion drawn from the anal-
also shows a statistic more extreme than 2\(\sigma\) computed as the probability of having a \(\chi^2\) statistic in rejecting the more discriminatory hypothesis that the reference Λ-CDM does not provide a satisfactory description of clustering data is ruled out with a significance level of 25% (for \(n = 2\)) and 82% (for \(n = 3\)) computed as the probability of having a \(\chi^2\) statistic more extreme than 2.77 and 0.39 respectively. This result is at odds with results based on the analysis of the growth rate data which seems to favor models predicting slightly less growth of structures than the reference Λ-CDM model. FIG. 3 also shows that the smaller the index \(n\) the more discriminatory the \(\eta\) statistic in rejecting \(F(R)\) scenarios is, essentially because of the smaller error bars (see FIG. 2).

Interestingly, while the reference analysis \((n = 2)\) provides stronger constraints, \(|f_{R_0}| \lesssim 3.2 \times 10^{-6}(/5.6 \times 10^{-6})\) to the 1\(\sigma\) precision level in \(F(R)\) models with the exponent \(N = 1(/2)\) and \(|f_{R_0}| \lesssim 9.9 \times 10^{-6}(/1.9 \times 10^{-5})\) at the 2\(\sigma\) level, the control analysis \((n = 3)\), being run on different correlation scales, allows us to check the unbiasedness of our conclusions. We also remark that \(F(R)\) models with higher exponent \(N\) are progressively less constrained since, when compared to the \(N=1\) models, they display a faster convergence to the Λ-CDM model at high redshift.

These results should be compared to the bounds obtained from other observables. On cosmological scales, the best bound is \(f_{R_0} \lesssim 1.4 \times 10^{-5}\) from the measurement of the galaxy power spectrum by the wiggleZ experiment \(59\) on scales larger than 30hMpc\(^{-1}\). The clustering ratio, being able to delve into the quasi-linear part of the power spectrum where deviations from GR are larger than on linear scales, allows one to get more stringent constraints. On smaller scales of a few kpc’s, strong gravitational lensing effects of galaxies place a bound \(f_{R_0} \lesssim 2 \times 10^{-6}\) \(40\), which is stronger than the one from the linear power spectrum and of the same order as the one obtained using the clustering ratio. The absence of disruption of the dynamics of satellite galaxies of the Milky Way implies that the latter must be screened, implying a loose bound of \(f_{R_0} \lesssim 7 \times 10^{-7}\). Effects on distance indicators in dwarf galaxies \(7\), and the comparison between the gas and stellar dynamics in these galaxies \(8\) imply that \(f_{R_0} \lesssim 5 \times 10^{-7}\) and \(f_{R_0} \lesssim 10^{-7}\) respectively. Finally, the most severe constraint in the Solar System comes from the test of the strong equivalence principle by the Lunar Ranging experiment \(6\) at the 10\(^{-13}\) level, which results in a competitive bound \(f_{R_0} \lesssim 10^{-6}\) for \(N = 1\) and irrelevant ones for greater values of \(N\). All in all, we find that the clustering ratio provides a method to test the properties of modified gravity as sharp as Solar System experiments such as Lunar Ranging or strong lensing observations, and better than current observations of the growth of cosmological structures on linear scales. Only dwarf galaxies where the chameleon effects are enhanced between screened and unscreened objects are more discriminatory.

Looking into the future, we have used the Horizon mock surveys \(41\) to extrapolate some forecasts for the errors on \(\eta\) achievable by Euclid, a future redshift survey with characteristics similar to SDSS, but covering larger and deeper space volumes. These computations, which assumes the precise knowledge of the background cosmology, show that such a mission will be able to push the statistical error on measurements of \(\eta\) at \(z \sim 1(/1.5)\) below 0.9%/(1.1%) (we assume \(n = 2\) and \(z = 16h^{-1}\)Mpc). We find that, although the clustering ratio in \(F(R)\) scenarios significantly differs from that expected in the Λ-CDM model only at late epochs, when cosmic acceleration kicks in, high redshift Euclid measurements are expected to lower the 2\(\sigma\) bound on \(f_{R_0}\) by roughly a factor of 4. Therefore, even in the near future, cosmological constraints on \(F(R)\) gravity are not expected to improve on astrophysical bounds. This result is specific to models with the chameleon property. The analysis of alternative screening mechanisms like K-mouflage \(42\), where large objects such as galaxy clusters are not screened, will certainly make Euclid-like data more discriminatory.

In summary, we have assumed that Planck measurements provide an accurate mapping of redshifts into distances, i.e. a precise description of the smooth expansion...
rate history of the universe, and we have demonstrated that the reference $\Lambda$-CDM model fairly describes the linear clustering properties of SDSS galaxies in the redshift range $0.15 < z < 0.67$, that is Einstein’s General Relativity satisfactorily describes also the perturbed dynamics of the late universe. In particular, $F(R)$ models having the same expansion rate as the reference $\Lambda$-CDM model are excluded by current cosmological data at $1\sigma$ if $f_{R_0} > 3.2 \times 10^{-6}$.

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