Fano effect in Aharonov–Bohm ring with topologically superconducting bridge

V V Val’kov¹, M Yu Kagan²⁻³ and S V Aksenov¹

¹ Kirensky Institute of Physics, Federal Research Center KSC SB RAS, Akademgorodok street 50/38, 660036 Krasnoyarsk, Russia
² P.L. Kapitza Institute for Physical Problems RAS, Kosygin street 2, 119334 Moscow, Russia
³ National Research University Higher School of Economics, Myasnitskaya street 20, 101000 Moscow, Russia

E-mail: vvv@iph.krasn.ru, kagan@kapitza.ras.ru and asv86@iph.krasn.ru

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Abstract
Taking into account an inner structure of the arms of the Aharonov–Bohm ring (AB ring) we have analyzed the transport features related to the topological phase transition which is induced in a superconducting wire (SC wire) with strong spin–orbit interaction (SOI). The SC wire acts as a bridge connecting the arms. The in-plane magnetic-field dependence of linear-response conductance obtained using the nonequilibrium Green’s functions in the tight-binding approximation revealed the Breit–Wigner and Fano resonances (FRs) if the wire is in the nontrivial phase. The effect is explained by the presence of two interacting transport channels in the system. As a result, the FRs are attributed to bound states in continuum (BSCs). The BSC lifetime is determined by both hopping parameters between subsystems and the SC-wire properties. It is established that the FR width and position are extremely sensitive to the type of the lowest-energy excitation in the SC wire, the Majorana or Andreev bound state (MBS or ABS, respectively). Moreover, it is shown that in the specific case of the AB ring, the T-shape geometry, the FR disappears for the transport via the MBS and the conductance is equal to one quantum. It doubles in the local transport regime. On the contrary, in the ABS case the local conductance vanishes. The influence of the mean-field Coulomb interactions and diagonal disorder in the SC wire on the FR is investigated.

Keywords: topological superconductivity, Majorana bound state, Andreev bound state, Aharonov–Bohm ring

(Some figures may appear in colour only in the online journal)

1. Introduction
In the last decades the development of paradigm concerning the band-structure topological properties exhibits serious achievements [1–3]. Among all the variety of solid-state systems where the topological phases are found one-dimensional (1D) or quasi-1D topological superconductors (SCs) attract interest since they provide the basis for novel quantum-electronic bits [4]. In these structures, by analogy with topological insulators, the bulk SC gap coexists with edge state. It was originally shown in the Kitaev model [5] that this state can be defined by two unpaired Majorana fermions (MFs) or Majorana bound state (MBS) localized at the opposite ends of 1D wire. The spatial separation is a key feature allowing to propose MBS-based bits as building blocks of topological quantum computers, in turn, making them stable against local decoherence processes. Additionally, the MFs are non-Abelian anyons then the state of such a topological qubit can be changed by means of braiding operations [6, 7].

To induce the MBS in 1D wire the combination of spin–orbit interaction (SOI), magnetic field and SC pairing is necessary [8, 9]. Following this recipe many experiments were done to detect the MBS in a last few years. In particular, tunneling spectroscopy of SC-covered semiconducting wires with strong SOI (in what follows we call this hybrid structure a SC wire) was performed to observe a zero-bias peak (ZBP)
in conductance as the MBS fingerprint [10–12]. Due to the fact that Majorana particle and antiparticle are the same object the resonant Andreev reflection in the lead-wire contact has to occur guaranteeing the height of ZBP equal to $2G_0$ (where $G_0 = e^2/h$—conductance quantum) [13, 14] and persistent under the chemical potential, magnetic field and tunnel barrier variations [15–17].

Until recently, a stable quantization at $2G_0$ remained unattainable [10–12, 18–20] stimulating a wide discussion in the field about both the reasons of low ZBP and other issues resulting in the ZBP and mimicking the MBSs. The saturation of $2G_0$ ZBP was prevented by proximity-induced soft rather than hard SC gap. There were different mechanisms suggested to be responsible for the SC phase breaking, partly related to the technological imperfections: disorder in the wire, electron scattering in the wire as well as interactions with electrons and phonons in the leads [21, 22]. It was shown that disorder in multiband quasi-1D wire can generate not quantized ZBP due to the appearance of either edge [23] or bulk [24] quasiparticles. Similar conductance peculiarity is observed in the Kondo resonance channels involving this physics. In [47] the QD in the middle between the QDs was considered as one of the contacts [52, 53]. The contributions of local and crossed Andreev reflection processes resulting in the FR were additionally analyzed for the different AB-ring geometries [54].

Starting from the seminal study of Aharonov and Bohm [35] the influence of distant electromagnetic potential on quantum particle propagation has been utilized in numerous works to research coherent transport in mesoscopic structures (see e.g. [36–38]). The Aharonov–Bohm (AB) effect in different geometries is actively employed to investigate the MBS features as well. The classic one implies the SC wire inserted into one of the AB-ring arms. In particular, the topological phase transition is signalized by a disorder-independent period doubling of the AB conductance oscillations since the MBS allows to transfer a single quasiparticle (not a Cooper pair) [39]. Similarly, it causes the fractional Josephson effect (4$\pi$-periodic) [5, 40] and the appearance of $h/e$ harmonic of persistent current finite at zero AB flux with a sign that is determined by the fermion parity of Coulomb-blockaded SC wire [41]. The floating SC wire was also used to point out the differences in the AB interference pictures mediated by MBSs and ABSs [42]. The same problem was considered in [43] where the conductance and noise were calculated for the AB ring including only one MBS or ABS.

Non-standard geometries include MF-QD-MF connection with the loop-shaped SC wire. Such a scheme is proposed to realize the tunnel-braid topological qubit. Then the non-Abelian rotations within its degenerate ground-state manifold can be realized as single-electron tunneling in and out of the Coulomb-blockaded QD. The magnetic flux penetrating the loop center tunes the MF-dot-MF system to the desired degenerate energy point [44, 45]. In order to distinguish the topological $h/e$-periodicity from the same AB effect provided by the normal ring, the resembling system with the Coulomb-blockaded SC wire to control the ground-state parity, by analogy with [46], was studied in [47]. In [48] the QD in the centre of the system was changed to 1D lead (called a loop in the article) coupled to the normal contact in the central part and described by the tight-binding model with no Zeeman and SOI effects taken into account. It was shown that the nontrivial phase of the SC wire results in 4$\pi$ conductance oscillations (if the flux quantum is defined as $\phi_0 = h/2e$) due to the MBS-assisted nonlocal tunneling process while no AB oscillations were observed in the trivial phase.

One of the features of the AB-ring coherent transport is the Fano effect [49] that is a manifestation of the destructive interference. Several schemes were proposed to detect the MBSs involving this physics. In [50] the modification of the Fano resonance (FR) in conductance was studied for the AB ring consisting of two QDs and the Kitaev wire side-coupled to one of them. Shang et al placed the wire between the QDs [51]. The SC wire was also considered as one of the contacts [52, 53]. The contributions of local and crossed Andreev reflection processes resulting in the FR were additionally analyzed for the different AB-ring geometries [54].

Usually, both reference arm of the AB ring and the leads which couple the SC wire (or another structure, e.g. QD) with the contact(s) are skipped or treated phenomenologically. In other words, the normal contact(s) is directly connected with the SC wire or with each other. At the same time, the possibility to fabricate quantum networks based on hybrid superconductor-semiconductor nanowires, including AB rings, was recently demonstrated in [55]. Taking it into account, here we consider lengthy leads in both AB arms, top and bottom, bridged by the SC wire as it is displayed in figure 1. The ring is subjected to an in-plane magnetic field, $B_z$. It induces the topological phase transition in the wire and allows to obtain non-topological low-energy excitations in the leads. As a result, the interaction between the transport channels of the wire and leads gives rise to the FRs. Using the nonequilibrium Green’s function technique the influence of different factors, such as the AB phase, SC pairing etc, on the FRs is discussed. It is interesting to note that the analogous geometry was investigated in [51]. However, the authors did not treat the leads and SC wire microscopically. The tight-binding treatment of such a scheme was used by Rainis et al [56] to study the transport signatures of fractional fermions in the wire without focusing on its interaction with the leads’ low-energy modes.

This article is organized as follows: in section 2 we present the theoretical model of the system sketched in figure 1. In section 3 the theory of quantum transport based on the nonequilibrium Green’s function method and tight-binding approximation is discussed. In section 4 the results of numerical calculations are analyzed. It includes a general discussion of the observed features of conductance (section 4.1), a comparison of the FR properties for the transport via the MBS and ABS (section 4.2) and the effects of mean-field Coulomb
repulsion and diagonal disorder (section 4.3). We conclude in section 5 with a summary.

2. The model Hamiltonian

Here we study the wire with the strong Rashba SOI deposited on the surface of s-wave SC. The wire is located between four leads forming the AB-type device (see figure 1) with two half-frings. The Hamiltonian of the system depicted in figure 1 has the following form:

\[ H = H_L + H_R + H_I + H_W + H_{\text{Ct}}. \]  (1)

First and second terms in (1) characterize the left and right paramagnetic metal contacts,

\[ \hat{H}_I = \sum_{j \sigma} \left( \xi_{s} + \frac{eV}{2} \right) \sigma \xi_{s}^{T_j \sigma} \xi_{s}^{T_j \sigma}, \quad i = L, R, \]  (2)

where \( \sigma \xi_{s}^{T_j \sigma} \xi_{s}^{T_j \sigma} \) is an annihilation operator of the electron on the left in the ith contact. \( \xi_{s} \) is a chemical potential of the system; \( eV \) is an energy of the source-drain electric field. Hereinafter we suppose that a magnetic field, \( B = (0, -B_x, B_y) \), acts in the device area (leads + SC wire). The third term, \( \hat{H}_I = H_{LT} + H_{LB} + H_{RT} + H_{RB} \), describes the left-top and -bottom (LT and LB, respectively) as well as right-top and -bottom (RT and RB, respectively) leads which, firstly, connect in parallel the SC wire with the corresponding contacts; secondly, present the arms of AB ring. All the contributions in \( \hat{H}_I \) are supposed to be identical, e.g.

\[ \hat{H}_{LT} = \sum_{\sigma j = 1}^{N} \xi_{s} \xi_{s}^{T_j \sigma} + \frac{1}{2} \sum_{\sigma j = 1}^{N} d_{LTj}^{\dagger} d_{LTj+1, \sigma} + \text{h.c.}, \]  (3)

where \( d_{LTj}^{\dagger} \) is an annihilation operator of the electron on the left in the ith contact with spin \( \sigma \) and spin-dependent energy \( \xi_{s} = t + \sigma V_{s} - \mu \); \( V_{s} = -\frac{1}{2} \mu_{B} B_y B_z \) is a magnetic flux threading the left (right) halfring with an area \( S_{L(R)} \); \( \mu_{B} \) is a magnetic field.

Note that the Hamiltonian (6) represents the effective interaction operator which is used to develop quantum-transport theory in the system.

Figure 1. The superconducting wire embedded in the Aharonov–Bohm ring that is formed by the left top and bottom (LT and LB) as well as right top and bottom (RT and RB) leads. The device is between metallic contacts.

\[ H_{W} = \sum_{j = 1}^{N_{W}} \left[ \sum_{\sigma} \xi_{j} a_{j \sigma}^{\dagger} a_{j \sigma} + U n_{j \uparrow} n_{j \downarrow} \right] + \frac{1}{2} \sum_{j = 1}^{N_{W}-1} \left( t_{j}^{+} a_{j+1, \sigma}^{\dagger} a_{j \sigma}^{\dagger} a_{j \sigma} a_{j+1, \sigma} + \text{h.c.} \right) + V \sum_{\sigma} n_{j \sigma} n_{j+1, \sigma'}, \]  (4)

where \( a_{j \sigma} \) is an annihilation operator of the electron on the ith site of the wire with spin \( \sigma \) and energy \( \xi_{j} \); \( \Delta \) is a SC pairing potential; \( \alpha = \alpha_B / a \) is a spin–orbit coupling strength obtained via the wire’s Rashba parameter, \( \alpha_B \), and a lattice constant, \( a \); \( U \) is an intensity of the intra- and intersite Coulomb interactions, respectively; \( n_{j \sigma} = a_{j \sigma}^{\dagger} a_{j \sigma} \) is an operator of particle number in the SC wire. The term \( \hat{H}_{W} \) takes into account the coupling between the SC wire and leads,

\[ \hat{H}_{W} = \sum_{\sigma} \left[ \left( t_{LT \sigma} d_{LT\sigma}^{\dagger} + t_{RT \sigma} d_{RT\sigma}^{\dagger} \right) a_{1 \sigma}^{\dagger} \right] + \left( t_{LB \sigma} d_{LB\sigma}^{\dagger} + t_{RB \sigma} d_{RB\sigma}^{\dagger} \right) a_{N_{W} \sigma} + \text{h.c.}, \]  (5)

where \( t_{LT \sigma}, t_{RT \sigma}, t_{LB \sigma}, t_{RB \sigma} \) are overlap integrals between the edges of LT(RB) leads and SC wire, respectively.

The tunnel Hamiltonian \( \hat{H}_{\text{Ct}} \) is responsible for the coupling between the contacts and leads, which are simultaneously the arms of AB ring,

\[ \hat{H}_{\text{Ct}} = \sum_{\sigma, \sigma'} \left[ c_{LT \sigma}^{\dagger} \left( t_{LT \sigma} e^{-i \phi_{LT \sigma}} + t_{LT \sigma} e^{i \phi_{LT \sigma}} \right) d_{RTj \sigma} + c_{RT \sigma}^{\dagger} \left( t_{RT \sigma} e^{-i \phi_{RT \sigma}} + t_{RT \sigma} e^{i \phi_{RT \sigma}} \right) d_{LTj \sigma} \right] + \text{h.c.}, \]  (6)

where \( t_{LTj \sigma}, t_{RTj \sigma} \) are overlap integrals between the edges of LT(RB) leads and corresponding contacts; \( \phi_{LT \sigma}, \phi_{RT \sigma} = 2 \pi \frac{2 \delta_{LT \sigma}}{\delta_{LT \sigma}} \) is a magnetic flux threading the left (right) halfring with an area \( S_{LT (RT)} \); \( \delta_{LT (RT)} \) is a magnetic field.
3. General quantum-transport theory in the AB ring

Transport properties of the AB ring is calculated numerically using the nonequilibrium Green’s functions [57–59]. These functions have to be defined in the spin ⊗ Nambu space to take into account both the SC pairing and spin-flip processes in the wire subsystem [15, 60].

In order to describe how the Green’s function technique is combined with the tight-binding approximation in case of the AB ring we introduce the set of basis field operators for \( i \)th subsystem (the leads, SC wire or contacts) each characterized by its own set of quantum numbers \( m, \hat{v}_m^{i,n} = \left( f_{in}^\dagger \ f_{in} \ f_{mi}^\dagger \ f_{mi} \right)^T \), where \( f_{in} = \{ d_{i\sigma}, \ a_{i\sigma}, \ c_{d\sigma} \} \)—an annihilation operator of the electron with quantum number \( m \) and spin \( \sigma \) acting in the \( i \)th Hilbert subspace. Specifically, the total field operators of the multi-site leads and SC wire are

\[
\hat{\psi}_i = \left( \hat{\psi}_1 \ldots \hat{\psi}_N \right)^T, \quad i = \text{LT, LB, RT, RB}
\]

\[
\hat{\psi}_W = \left( \hat{\psi}_W^1 \ldots \hat{\psi}_WN_W \right)^T.
\]

As a result, the summands in \( H \) become

\[
\hat{H}_W = \frac{1}{2} \sum_i \hat{\psi}_i^\dagger \hat{v}_W \hat{\psi}_W + \text{const}, \quad \hat{H}_i = \frac{1}{2} \sum_i \hat{\psi}_i^\dagger \hat{h}_i \hat{\psi}_i,
\]

\[
\hat{H}_{W1} = \frac{1}{2} \sum_k \left[ \hat{\psi}_W^\dagger \left( \hat{T}_{LT} \hat{\psi}_{LT} + \hat{T}_{RT} \hat{\psi}_{RT} \right) \right. \\
+ \left. \hat{\psi}_W^\dagger \left( \hat{T}_{LB} \hat{\psi}_{LB} + \hat{T}_{RB} \hat{\psi}_{RB} \right) \right] + \text{h.c.},
\]

\[
\hat{H}_{C1} = \frac{1}{2} \sum_k \left[ \hat{\psi}_k^\dagger \left( \hat{T}_{LT} \hat{\psi}_{LT} + \hat{T}_{LB} \hat{\psi}_{LB} \right) \right. \\
+ \left. \hat{\psi}_k^\dagger \left( \hat{T}_{RT} \hat{\psi}_{RT} + \hat{T}_{RB} \hat{\psi}_{RB} \right) \right] + \text{h.c.},
\]

where \( \hat{h}_i \)—the \( 4N_W \times 4N_W \) matrix with the following non-zero blocks without the Coulomb interactions

\[
H_{ij} = \begin{pmatrix}
\xi_+ & \Delta & 0 & 0 \\
\Delta & -\xi_+ & 0 & 0 \\
0 & 0 & \xi_+ & -\Delta \\
0 & 0 & -\Delta & -\xi_+
\end{pmatrix},
\]

\[
H_{ij+1} = H_{i,j-1} = \frac{1}{2} \begin{pmatrix}
-t & 0 & -\alpha & 0 \\
0 & t & 0 & -\alpha \\
\alpha & 0 & -t & 0 \\
0 & \alpha & 0 & t
\end{pmatrix}.
\]

Notice that the corrections to these blocks in case of \( U, \ V \neq 0 \) will be discussed in section 4.3. The blocks of leads’ matrix \( \hat{h}_i \) can be obtained from (8) by putting \( \Delta = \alpha = 0 \).

The tunnel matrices responsible for the contact-lead couplings, \( \hat{T}_{LT, LB} \) and \( \hat{T}_{RT, RB} \), are obtained by performing a gauge transformation [61, 62].
where \( \Gamma_{\alpha\beta} = 2\pi t_{\alpha\beta}^2 \) if \( \alpha = \beta \) and \( \Gamma_{\alpha\beta} = 2\pi t_{\alpha\beta} t_{\beta\gamma} \) if \( \alpha \neq \beta \)—the coupling strengths between the \( \theta \)th leads and the same contact; \( \rho_{i} \)—the DOS of the \( \theta \)th contact. In the calculations below the contacts are treated in the wide-band limit. It means that \( \Gamma_{\alpha\beta} = \text{const} \).

The retarded and lesser Green’s functions can be obtained from the Dyson and Keldysh equations, respectively,

\[
G' = (\omega - \hat{H} - \hat{\Sigma})^{-1}, \quad G^\alpha = (G')^+, \quad \hat{G}^\alpha = \hat{G}' \hat{\Sigma}^\alpha \hat{G}^\nu.
\]

4. Results and discussion

In numerical calculations we suppose that \( S_L = S_R = S_\Delta = \frac{LW}{\pi} \sqrt{L^2 - \frac{t^2}{m^*}} \), where \( L_W = a (N_W - 1), L = a (N - 1), N_W = 30, N = 20 \). It is important to note that we use small \( N_W \) and \( N \) which considerably reduces tight-binding transport calculations. Consequently, in order to get the MBSS in such a short SC wire it is necessary to consider unphysically large lattice constant \( a = 50 \text{nm} \) leading to small hopping parameter, \( t = \frac{\hbar^2}{m^* \pi^2} \), where \( m^* = 0.015m_0 \) is an experimentally accessed effective mass in semiconducting nanowires [10].

The other parameters are \( k_B T = 0, \mu = 0, \Delta = 250 \mu\text{eV}, \alpha_B = 0.2 \text{eV} \), \( A, g = 50, B_y \sim 0.1 - 1 \text{T}, B_x \sim 0.001 - 0.01 \text{T} \). In the forthcoming paragraphs, all the energy parameters are measured in units of \( t \). The transport parameters satisfy an inequality, \( \Gamma_i, \tau_j \ll t \), where \( i = L, L_N, R, R_N, j = LT, LB, RT, RB \), that corresponds to weak (tunnel) coupling. In addition, the linear response regime is studied in the article.

4.1. Breit–Wigner and Fano resonances

We start with the situation when the Coulomb interactions in the SC wire are neglected \( (U = V = 0) \) and consider the symmetric AB ring, i.e. \( \Gamma_{L1} = \Gamma_{L_N} = \Gamma_{R1} = \Gamma_{R_N} = \Gamma = 0.01 \), \( t_{LT} = t_{LB} = t_{RT} = t_{RB} = t_1 = 0.1 \). Initially the AB phases in the halfrings are chosen to be the same, \( \Phi_L = \Phi_R = \Phi \). Hereinafter, we mostly concentrate on an in-plane Zeeman-energy dependence of the conductance since from experimental point of view it is easier to control the magnetic field in comparison with the gate voltage, SOI or SC pairing parameters. The transport in the system is defined by the one-particle excitations in the vicinity of the Fermi level. The corresponding energies of the separate lead (blue dotted curve) and SC wire (red solid and green dashed curves) as functions of the in-plane Zeeman energy are shown in figure 2(a).

The lowest energy of the lead, \( E_{11} \), periodically becomes equal to zero if \( V_y \lesssim 2 \). Simultaneously, the topologically nontrivial phase develops in the SC wire if \( \mu^2 + \Delta^2 < V_y^2 \lesssim (2t - \mu)^2 + \Delta^2 \) [8,9]. In this regime the lowest energy of wire excitation, \( E_{W1} \), is split from the higher one, \( E_{W2} \), and oscillates as it is clearly seen in figure 2(a).

![Figure 2. The magnetic-field dependence of the lowest energies of the single-electron excitations of the lead and SC wire (a) and the conductance of the AB ring (b). Parameters: \( U = V = 0, \alpha = 0.195, \Delta = 0.243, \Phi = 0 \).](image-url)
Taking it into account, one can suppose that a positive magnetic field results in primarily spin-polarized transport. Moreover, the hopping between two levels in the central QDs as a result of SOI gives rise to the conductance (b) of the sextuple-QD structure for the BSC (infinitely narrow peak) at $v_y = 0$, $t = 0.1$. The other parameters are the same as in figure 2.

Figure 3. (a) The magnetic-field dependence of the TDOS (a) and conductance (b) of the sextuple-QD structure for $t_0 = 0.1$. The other parameters are the same as in figure 2.

As it was shown in the works [63–65] presence of the degenerate levels ($E_{1,2}$) as well as bonding and antibonding states ($E_{3,4}$ and $E_{5,6}$) for closed system results in the emergence of bound states in continuum (BSCs) if system is open. This concept is illustrated in figure 3(a) where the total density of states (TDOS), TDOS ($\omega = 0$; $V_y = -\text{Tr} \left[ \text{Im} \left\{ \hat{G}^\dagger (\omega = 0; V_y) \right\} \right] / \pi$, as function of the in-plane magnetic-field energy is plotted. It is evident that for the degenerate states the wide maximum coincides with the BSC (infinitely narrow peak) at $v_y = 1$ (see red solid curve). Additionally, two pairs of the separated maximum and BSC corresponding to two pairs of bonding and antibonding states are located on either side of this point. All three wide maxima in the TDOS manifest themselves as the BWRs in the conductance (see red solid curve in figure 3(b)). In opposite, there are no resonant features associated with the BSCs in the transport properties.

If spin degrees of freedom are taken into account that the combination of SOI and SC pairing gives rise to nonzero coupling between contacts and BSCs with energies $E_{1,0}$. As a result their peaks in the TDOS become broadened and two FRs emerge at the same $v_y$ in the conductance (see blue dashed curves in figures 3(a) and (b)). Finally, following the ideas of [64] we can break the system symmetry, e.g. due to the unequal contact-lead couplings or, alternatively, introducing the AB phase. Then all the BSC peaks in the TDOS acquire the finite width and the conductance contains three FRs (see black dotted curves in figures 3(a) and (b)). The last result can be realized even in the spinless regime (not shown here). Next, the energy splitting of bonding and antibonding states is defined by $t_0$. In particular, if $t_0 = 1$ then $t_0 \gg t_1$ and we get the situation when there are two wide maxima and two BSCs are around $v_y = 1$. Additionally, one maximum (BSC) is distinctly settled to the left (right) of them.

Note that the coupling between the BSCs and contacts vanishes and, consequently, the FRs in figure 3(b) disappear if $t = \alpha = 0$ (equivalently, $t_0 = 0$ in the spinless case) and $\Phi = 0$. Taking it into account, one can suppose that a possible reason for the absence of FRs in [51] in the situation of directly grounded SC wire is no interaction between the MBSs or zero AB phase.

Recurring to the initial system we get the set of such features in the TDOS. However, it is essential that the BSCs obtain finite lifetime only if the wire lowest-energy excitation is close to zero, in other words the topologically nontrivial phase has to be induced (see figure 2(a)). The typical structure of conductance resonances in this regime is displayed in figure 4(a). It is modified in comparison with the above-described picture of the sextuple-QD conductance. In particular, one of the FRs of sextuple-QD structure transforms to the set of BWRs in case of the AB ring. As a result, the FRs close to the $E_1$ ($V_y$) minima are only left (one of them is shown in figure 4(a)). Meanwhile, the BWR positions are substantially defined by the $E_{1,0}$ ($V_y$) zeros. When $V_y$, $\Delta$, $\alpha$ are fixed the BWR- and FR widths depend on $E_{1,1}$ and $E_{1,1}$, respectively. The higher $E_{1,1}$, the narrower the corresponding resonance.

Since the increase of the AB-ring size does not change the system symmetry the degenerate states have to remain. As it was shown above the nonzero AB phase allows to unveil the existence of the corresponding BSC. In figure 4(b) the new FR with extremely narrow width emerges if $\Phi \neq 0$ and it coincides with the $E_{11}(V_y)$ zero.

Thus, the occurrence of various resonances can be qualitatively explained by the fact that different interacting channels can provide the contributions to transport. If it is the wire channel, i.e. $\mu = E_{1,1}$, than the BWR appears. If the lead channel, $\mu = E_{1,1}$, mainly participates in the transport the Fano-type interference takes place. It is worth to note that
similar transport signatures were observed in other nanosize systems containing two interacting channels [64, 66].

4.2. FR properties: MBS versus ABS

4.2.1. The FR width and position. One can note from figures 2 and 3 that the FRs appear for any nonzero SOI and SC pairing if the in-plane magnetic field satisfies the inequality $\mu^2 + \Delta^2 < V_y^2 < (2t - \mu)^2 + \Delta^2$. The last means that the FR is present irrespective of the type of SC-wire state with $E_{W1}$ since the spatial distribution of the corresponding probability density continuously changes from the edge- (MBS) to bulk (ABS) one as the gap in the SC-wire excitation spectrum decreases [67]. However, the FR properties significantly depend on the type of this state that can be governed by e.g. the in-plane magnetic field or SC-pairing potential. The last case is shown in figure 5. One can compare the FRs at $\Delta = 0.243$ and $\Delta = 0.097$ (see red dotted and blue dashed curves in figure 5(a)). The width dramatically decreases as the state with $E_{W1}$ becomes Andreev-type at $\Delta = 0.097$ (see the top and middle insets of figure 5(a)). As a result, the FR width is not changed as long as the overlap of the MF probability densities gathered at the wire edges is close to zero (see red dotted and blue dashed curves in figure 5(b)). Finally, the observed FRs collapse (the FR width becomes infinitely small) when $\Delta$ vanishes even though $E_{W1} \approx 0$ (see black solid curves in figures 5(a) and (b)).

Using the toy model of sextuple-QD structure we demonstrated that the FR positions are defined, in particular, by the hopping parameters. It is interesting that after the increase of number of sites in the system the impact of $t_1$ change is greatly determined by the spatial distribution of the SC-wire low-energy state. Despite the growth of the FR width, its position is 'frozen’ in case of MBS as $t_1$ grows (see figure 6(a)). Otherwise, when the state is substantially delocalized and transforms to the ABS the increase of $t_1$ gives rise to the FR displacement (see figure 6(b)).

Figure 4. (a) The BWRs and FR in the conductance of the AB ring when the SC wire is in the nontrivial phase. The BWR/FR position (the left y-axis) coincides with the minimum of the energy $E_{W1}/E_0$ (the right y-axis). (b) The influence of the AB phase. Inset: the appearance of new FR if $\Phi \neq 0$. The parameters are the same as in figure 2.

Figure 5. The SC-ordering dependence of the FR for $N_W = 30$ (a) and $N_W = 150$ (b). Insets: spatial distributions of the probability density of the wire state with $E_{W1}$ for different $\Delta$ and $V_y = 0.375$. The other parameters are the same as in figure 2.
4.2.2. T-shape geometry. The MBS and ABS can be also distinguished due to the modification of the transport scheme. Let us consider the T-shape geometry, i.e. $\Gamma_{LN} = \Gamma_{RN} = 0$. In this situation the FR is extremely sensitive to the MBS overlap, $\Gamma_{MBS} = \sum_{j=N_W/2}^{N_W/2+1} \left( |u_{j\sigma}|^2 + |v_{j\sigma}|^2 \right)$, (16)

where $u_{j\sigma}, v_{j\sigma}$ are the Bogoliubov-transformation coefficients corresponding to the energy $E_{W1}$, site $j$ and spin $\sigma$. This parameter is plotted in the inset of figure 7(a). The MBS overlap is reduced by two orders of magnitude if the wire length increases from $N_W = 30$ to $N_W = 100$. Simultaneously, the SC-wire low-energy state is modified from the ABS to MBS and the FR minimum placed around $V_y = 0.777$ rises, i.e. $G \neq 0$ (compare red dotted and blue dashed curves in figure 7(a)). Further elongation of the wire results in the two more orders decrease of $\Gamma_{MBS}$. As a result, the $G_0$-plateau emerges in the conductance (see magenta dash-dotted and black solid curves).

The reasons of the obtained behavior become clear if we note that the left and right bottom leads in such a T-shaped scheme play a role of side-coupled structures which effective interaction with the contacts is defined by the type of the state with $E_{W1}$. If it is the MBS than these leads are virtually separated from the contacts and the FR does not appear. However, if the state becomes Andreev-type or, in other words, bulk-like than the system is equivalent to the Fano–Anderson model, which describes the interaction between a discrete linear chain and single impurity [68], and the corresponding resonance emerges in the conductance.

Finally, the behavior of local transport via the MBS and ABS is completely different. To demonstrate it we introduce an asymmetry coefficient, $\beta$, such that $t_{RT} = \beta t_1$. It is remarkable that in the MBS case the height of the conductance plateau doubles if the transport regime evolves from nonlocal, $\beta = 1$, to local, $\beta = 0$ (see black solid and circled curves in figure 7(b)). In turn, the ABS-mediated conductance vanishes in the local transport regime (compare red dotted and blue dashed curves in figure 7(b)).

The observed features of transport via the MBS and ABS in the T-shape device can be described analytically. In order to
achieve this it is essential to notice that transport into the MBS implies tunnel coupling with only one MF. Meanwhile, in the Majorana representation tunneling into the ABS, $\alpha^\dagger |0\rangle$, brings into play both MFs since $\alpha^\dagger = (\eta_1 + i\eta_2)/2$, where $\eta_{1,2}$ are the MF operators.

Let us consider these cases separately and start with an effective low-energy Hamiltonian $\hat{H}_W = i\xi_0 \eta_1 \eta_2/2$ describing the interaction between MFs with intensity $\xi_0$. For the sake of simplicity we substitute the four multi-site leads for single-level QDs and consider spinless case, then

$$\hat{H}_{W1} = t_1 \left[ d_{1,\uparrow}^\dagger - d_{1,\downarrow} + \beta \left( d_{2,\uparrow}^\dagger - d_{2,\downarrow} \right) \right] \eta_1 + t_1 \left[ d_{3,\uparrow}^\dagger - d_{3,\downarrow} + d_{4,\uparrow}^\dagger - d_{4,\downarrow} \right] \eta_2,$$

$$\hat{H}_{C1} = \sum_k \left( t_{1C} e^{+}_{1k} d_1 + i t_{2C} e^{+}_{2k} d_2 + \text{h.c.} \right),$$

$$\hat{H}_{l} = \sum_{j=1}^{4} \xi_j d_{j}^\dagger d_{j}, \quad \hat{H}_b = \sum_k \left( \xi_k \pm \frac{eV}{2} \right) c_{k,\downarrow}^+ c_{k,\uparrow}, \quad i = L, R.$$

Applying (11) for the symmetrical system ($t_L = t_R$) leads to the following expression for the left-contact current:

$$I_L = \frac{eV^2}{4\pi} \int_{-\infty}^{+\infty} \mathrm{d}\omega \left( n_{R} - n_{L} \right) \left[ 2|F_{11e}|^2 + |G_{12e}|^2 + |G_{12h}|^2 \right],$$

where $n_{L,(R)} = n(\omega \pm eV/2)$; $F_{11e}(\omega)$, $G_{12e}(\omega)$, $G_{12h}(\omega)$ are the Fourier transforms of the anomalous and normal retarded Green’s functions,

$$F_{11e}(t - t') = -i\Theta(t - t') \left\langle \left( d_{1,\uparrow}^\dagger(t) d_{1,\downarrow}(t') \right) \right\rangle,$$

$$G_{12e}(t - t') = -i\Theta(t - t') \left\langle \left( d_{1,\downarrow}(t) d_{2,\uparrow}(t') \right) \right\rangle,$$

$$G_{12h}(t - t') = -i\Theta(t - t') \left\langle \left( d_{1,\uparrow}(t) d_{2,\downarrow}(t') \right) \right\rangle.$$

The term proportional to $F_{11e}$ in (15) describes the contribution from the local Andreev reflection. The last two correspond to the processes of direct charge transfer. Note that the contribution from the crossed Andreev reflection is absent since $\mu_{L,(R)} = \mu \pm eV/2$. The linear-response conductance from (18) is equal to

$$G_L = \frac{\Gamma^2}{2} \left[ 2|F_{11e}(0)|^2 + |G_{12e}(0)|^2 + |G_{12h}(0)|^2 \right].$$

Utilizing the equation-of-motion technique the sought Green’s functions are

$$F_{11e}^r = -\frac{2t_{1e}^2 z C_{zB}}{z \xi_1^2 z_B - \xi_0^2 \xi_1 C_{zF} C_B},$$

$$G_{12e}^r = \frac{2 t_{1e}^2 z C_{12e}}{z \xi_1^2 z_B - \xi_0^2 \xi_1 C_{zF} C_B},$$

$$G_{12e} = G_{12h} = -\beta F_{11e}^r = \frac{2 \beta t_{1e}^2 \left( z^2 - 8 z_1^2 \right)}{C_{1z} Z_1},$$

where $Z_1 = \left( \xi^2 - 8 z_1^2 \right) \left( z C_{1e} - 4 z_1^2 \left( 1 + \beta^2 \right) \right) - \xi_0^2 z C_{1e}$. Thus, for $\beta = 1$ the linear-response conductance becomes $G_L = G_0/2$ and is in agreement with [45]. However, if $\beta = 0$ than $G_L = G_0$. The same results occur if $\xi_1 = \xi_2 = \xi_3 = 0$ and $\xi_0 = 0$ (see figure 7). The conductance (19) tends to zero in general case when $\xi_1, \xi_0 \neq 0$.

Next, consider the second case, i.e. the transport via the isolated ABS. Hence, the low-energy wire Hamiltonian is $\hat{H}_W = \xi_0^e \alpha^\dagger \alpha$. As the ABS- initial second-quantization operators are related by means of the Bogoliubov transformation, $a_{1(Lu)} = u \omega \pm v \alpha^\dagger \alpha$, the QD-ABS tunnel Hamiltonian has a following form [43]

$$\hat{H}_{W1} = t_{1e} \left( d_{1,\uparrow}^\dagger + \beta d_{2,\uparrow}^\dagger + d_{3,\downarrow}^\dagger + d_{4,\downarrow}^\dagger \right) \alpha + t_{1h} \left( d_{1,\downarrow}(1 + \beta d_{2,\downarrow} + d_{3,\downarrow} + d_{4,\downarrow}) \alpha + \text{h.c.} \right),$$

where $t_{1e(h)}$ is an electron (hole) tunneling amplitude.

The solution of the equations of motion for the Green’s functions for $\xi_1 = \xi_2 = \xi_3 = 0$ gives

$$F_{11e}^r = -\frac{t_{1e}^2 k_{12e}}{Z_2},$$

$$G_{12e}^r = \frac{\beta z}{C_{1z} Z_1 C_{12e}} \left[ z C_{1e} \left( r_{1e}^2 C_{12e} + r_{1h}^2 C_{12h} \right) \right] - \left( r_{1e}^2 - r_{1h}^2 \right)^2 \left( z \left( 1 + \beta^2 \right) + 2 C_{1e} \right),$$

where $C_{e(h)} = z \mp \xi_1$, $Z_2 = z^2 C_{1e} C_{1h} + \left( r_{1e}^2 - r_{1h}^2 \right)^2 \left( z \left( 1 + \beta^2 \right) + 2 C_{1e} \right) - 2 z^2 C_{1e} \left( r_{1e}^2 + r_{1h}^2 \right) \left( z \left( 1 + \beta^2 \right) + 2 C_{1e} \right)$. It is clearly seen from (23) that the linear-response conductance vanishes both in nonlocal and local transport via the ABS. Note that the general formulae for the Green’s functions are rather cumbersome to present them explicitly. However, it is necessary to emphasize that in this situation $F_{11e}^r (0) = 0$ for $\beta = 0, 1$ whereas $G_{12e}^r (0) \neq 0$ for $\beta = 1$. Such a behavior qualitatively coincides with the numerics in figure 7.
Thus, the proposed AB ring allows to detect the topological phase transition due to the appearance of FRs. Additionally, analyzing the FR properties one can separate the edge low-energy excitations of the SC wire from the ones having the bulk-like spatial distribution. The last option means that the corrections of the nonzero elements and new terms. In particular, the last ones are responsible for the on-site spin-flip processes and intersite SC pairing.

The influence of the Coulomb interactions on the FR is displayed in figure 8. It is seen that the on-site correlations slightly shift the maximum of the FR but the antiresonance remains at the same place (compare red dotted and blue dashed curves). Simultaneously, the BWR indicating the pure tunneling via the MBS moves to the left because of the corresponding shift of the MBS zero energy when $U \neq 0$. If now $V \neq 0$ the BWR goes to the opposite direction (see black solid curve). But the FR is stuck at the same place and its characteristics are changed negligible. As it follows from the used parameters the discussed results are obtained in the regime of low carrier concentration in the wire. At the high carrier-concentration regime ($\mu > t + \sigma V + U (\delta_{j\sigma}' a_{j\sigma} + a_{j\sigma}' \delta_{j\sigma})$) the observed behavior is qualitatively similar.

The effect on the linear-response conductance of the AB ring similar to the Coulomb interactions occurs if the diagonal disorder is present in the SC wire, $\xi_j = \delta_{j\xi}, j = 1, ..., N_W$ (where $\delta_{j\xi}$ is an $j$th site random potential which takes values with an uniform distribution in the interval $[-1/2, 1/2]$). In figure 9 the influence of this issue is shown for two independent distributions of $\delta_{j\xi}$ (see blue dashed and black dash-dotted curves).

5. Conclusion

In this article we investigated the linear-response conductance of the AB ring focusing on the interplay between the internal energy structures of the AB arms (leads) and SC wire which bridges the arms. Employing the nonequilibrium Green’s functions we showed that the BWRs and FRs emerge when the SC wire is switched in the topologically nontrivial phase by the in-plane magnetic field. Using the sextuple-QD toy model we demonstrated that the FRs are related to the dependence of BSCs’ lifetime on hopping parameters as well as the SOI and SC pairing in the wire. Connection between the type of the lowest-energy state in the SC wire, the MBS or ABS, and the FR properties is exhibited. Additionally, it is shown that the spatial distribution of this state can be examined via the T-shaped transport scheme. Thus, taking into account the ongoing debates on the ways to distinguish between the MBS and ABS the obtained FR can be used as a corresponding tool (in favor of the MBS). Our results provide additional data for new experiments aiming to observe the MBS manifestation in coherent transport.

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ORCID iDs
S V Aksenov https://orcid.org/0000-0001-9774-3852

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