Matter effects in oscillations of neutrinos traveling short distances in matter

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Abstract

It is well known that when the distance $t$ traveled by neutrinos in matter is short, matter effects in oscillations of neutrino flavour are small and decrease with decreasing $t$ more rapidly than the oscillations effects themselves. We discuss the reason for this and demonstrate that under certain circumstances this statement is no longer correct. In particular, we show that if neutrinos propagate significant distances in vacuum before entering matter (or after exiting it), matter effects in short-$t$ neutrino oscillations can be significantly enhanced. Implications for oscillations of solar and atmospheric neutrinos with nearly horizontal trajectories inside the earth and for accelerator experiments are considered. We also comment on neutrino oscillations in matter due to flavour changing neutral currents.

Pacs numbers: 14.60.+Pq, 26.65.+t

Keywords: neutrino oscillations, matter effects

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1 Introduction

It is well known that when the distance \( t \) traveled by neutrinos in matter is short, the probabilities of oscillations of neutrino flavour in matter reduce to those in vacuum, i.e. with decreasing \( t \) matter effects on neutrino oscillations die out more rapidly than the oscillations effects themselves. In the present Letter we show that under certain conditions this is no longer true and matter effects in short-\( t \) experiments can be quite significant or even dominate the oscillation probability. In particular, we show that if neutrinos propagate significant distances in vacuum before entering matter (or after exiting it) matter effects in short-\( t \) neutrino oscillations can be strongly enhanced. We also discuss oscillations of neutrino mass eigenstates in matter, which are relevant for oscillations of solar and supernova neutrinos inside the earth. We show that these oscillations can have sizeable probabilities even when neutrino pathlengths in matter are relatively short.

2 Two-flavour neutrino evolution in matter

Consider the evolution equation for two-flavour neutrino oscillations in matter in the weak eigenstate basis [1, 2]:

\[
i \frac{\partial}{\partial t} \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} = \begin{pmatrix} -A(t) & B \\ B & A(t) \end{pmatrix} \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix} \equiv H(t) \begin{pmatrix} \nu_a \\ \nu_b \end{pmatrix}
\]

(1)

Here \( \nu_{a,b}(t) \) are the probability amplitudes of finding neutrinos of the corresponding flavor \( a, b \) at a time \( t \) (in particular, one of these two species can be a sterile neutrino \( \nu_s \)). The parameters \( A \) and \( B \) are

\[
A(t) = \cos 2\theta_0 \delta - V(t), \quad B = \sin 2\theta_0 \delta,
\]

(2)

where

\[
\delta = \frac{\Delta m^2}{4E}, \quad V(t) = \frac{G_F}{\sqrt{2}} N(t).
\]

(3)

Here \( G_F \) is the Fermi constant, \( E \) is neutrino energy, \( \Delta m^2 = m_2^2 - m_1^2 \), where \( m_{1,2} \) are the neutrino mass eigenvalues, and \( \theta_0 \) is the mixing angle in vacuum. The effective density \( N(t) \) depends on the type of the neutrinos taking part in the oscillations. For transitions between antineutrinos one should substitute \(-N\) for \( N \) in eq. (2). In a matter of constant density the probability of \( \nu_a \leftrightarrow \nu_b \) oscillations takes a very simple form

\[
P(\nu_a \rightarrow \nu_b; t) = \sin^2 2\theta \sin^2 \omega t,
\]

(4)

where \( \theta \) and \( 2\omega \) are the mixing angle in matter and the energy splitting between the matter eigenstates respectively:

\[
\sin 2\theta = \frac{B}{\omega} = \sin 2\theta_0 \delta, \quad \omega = \sqrt{A^2 + B^2}.
\]

(5)
It is now easy to see that in the short baseline limit ($\omega t \ll 1$) the oscillation probability

$$P(\nu_a \rightarrow \nu_b; t) \simeq \sin^2 2\theta (\omega t)^2 = \sin^2 2\theta_0 (\delta \cdot t)^2 = \sin^2 2\theta_0 \left( \frac{\Delta m^2}{4E} t \right)^2,$$

which is just the short-baseline limit of the oscillation probability in vacuum [3].

This raises a number of questions:

- Why do the matter effects on oscillations die out with decreasing $t$ more rapidly than the oscillation effects themselves?
- Is this true also in a matter of non-constant density?
- Is this true in the case of oscillations between more than two neutrino species?
- Are there any conceivable situations when the matter effects on oscillations of neutrinos traveling short distances in matter can be as large as the oscillation effects themselves?

The answer to the first of these questions will help us to find the answers to the rest of them.

3 Short-pathlength neutrino oscillations in matter

Consider the two-flavour evolution equation (1) in the short baseline (small oscillation phase) limit. In this limit the oscillation effects are known to be small, so one can solve the evolution equation in perturbation theory. Assume that the initial neutrino state is $\nu_a$. Then in the leading order in perturbation theory the amplitude $\nu_b(t) = -iBt$, which immediately yields (6). Thus, the fact that the matter effects on neutrino flavour oscillations disappear in the short baseline limit is the consequence of the two facts: (1) the oscillations effects are small and the perturbation theory applies; (2) since the initial state is a flavour eigenstate, in the leading order the transition amplitude is only determined by the off-diagonal terms in the effective Hamiltonian in (1), whereas the matter effects enter only through the diagonal terms.

From the above the answers to the other questions that we asked immediately follow: The functional $t$ dependence of matter density is irrelevant, i.e. our conclusion holds irrespective of whether the matter density is constant or not; it is also true in the case of oscillations between more than two species because, in the weak eigenstate basis, matter density enters only into the diagonal terms of the effective Hamiltonian for any number of flavours; the above conclusions need not be correct if the initial neutrino state is not a weak eigenstate, i.e. not a neutrino of a definite flavour.

Let us now consider the situation when the initial neutrino state entering the matter is a coherent superposition of the flavour eigenstates $\nu_a$ and $\nu_b$: $|\nu_i\rangle = |\nu(0)\rangle = a_1|\nu_a\rangle + a_2|\nu_b\rangle$, $|a_1|^2 + |a_2|^2 = 1$. In what follows, unless otherwise is specified, we shall always understand by neutrino pathlength the distance traveled by neutrinos in matter. It is straightforward
to find the transition probabilities to the second order in perturbation theory:

\[
P(\nu_i \rightarrow \nu_a; t) \simeq |a_1|^2 - (|a_1|^2 - |a_2|^2)(B_2)^2 - 2\text{Im}(a_2^*a_1)(B_2) - 4\text{Re}(a_2^*a_1)B \int_0^t dt_1 \int_0^{t_1} dt_2 A(t_2),
\]

\[
P(\nu_i \rightarrow \nu_b; t) \simeq |a_2|^2 + (|a_1|^2 - |a_2|^2)(B_2)^2 + 2\text{Im}(a_2^*a_1)(B_2) + 4\text{Re}(a_2^*a_1)B \int_0^t dt_1 \int_0^{t_1} dt_2 A(t_2),
\]

(7)

(8)

For our further discussion it is useful to write down the simplified versions of these expressions relevant for neutrino oscillations in a matter of constant density:

\[
P(\nu_i \rightarrow \nu_a; t) \simeq |a_1|^2 - (|a_1|^2 - |a_2|^2)(B_2)^2 - 2\text{Im}(a_2^*a_1)(B_2) - 2\text{Re}(a_2^*a_1)(AB_2^2),
\]

\[
P(\nu_i \rightarrow \nu_b; t) \simeq |a_2|^2 + (|a_1|^2 - |a_2|^2)(B_2)^2 + 2\text{Im}(a_2^*a_1)(B_2) + 2\text{Re}(a_2^*a_1)(AB_2^2).
\]

(9)

(10)

Matter effects enter through the parameters \(A\) in the last terms on the r.h.s. of eqs. (7)-(10). Notice that the leading order matter-induced contribution is \(\sim V\delta \cdot t^2\), whereas in the case when the initial state is a pure flavour eigenstate the leading order matter effect is \(\sim (V\delta \cdot t)^2\) (assuming \(V \geq |\delta|\)). Thus, matter effects in short-pathlength neutrino oscillations can be strongly enhanced when the initial state is not a pure flavour eigenstate.

As can be seen from (7)-(10), the oscillation effects themselves get enhanced in this case provided that \(\text{Im}(a_2^*a_1) \neq 0\), the leading contribution to the transition probability being now \(\sim \delta \cdot t\) rather than \(\sim (\delta \cdot t)^2\).

4 Two-media neutrino oscillations

How can one create an initial neutrino state which is not a pure flavour eigenstate? One possibility is to let neutrinos oscillate in a different medium before entering the medium of interest. For example, if neutrinos propagate in vacuum before they enter the matter, the initial state arriving at the vacuum-matter border is no longer a flavour eigenstate but rather a coherent superposition of the flavour eigenstates.

Assume that neutrinos are initially produced in the flavour eigenstate \(\nu_a\), propagate a distance \(t_1\) in vacuum and then the resulting state propagates a distance \(t\) in a matter of constant density. The oscillation phase acquired in vacuum is \(\phi_1 = \delta \cdot t_1\). Let us denote \(\sin \phi_1 = s_1, \cos \phi_1 = c_1\). Then the state \(|\nu_i\rangle\) entering the matter is characterized by \(a_1 = c_1 + i \cos 2\theta_0 s_1, a_2 = -i \sin 2\theta_0 s_1\). In the limit of short-pathlength oscillations in matter one finds from (7) and (10)

\[
P(\nu_i \rightarrow \nu_b; t) \simeq \sin^2 2\theta_0 \left\{ s_1^2 + (c_1^2 - s_1^2)(\delta \cdot t)^2 + 2s_1c_1(\delta \cdot t) + 2s_1^2 \cos 2\theta_0(V\delta \cdot t^2) \right\}
\]

(11)

\footnote{It is also important that the term \(V\delta \cdot t^2\) is sensitive to the sign of \(\Delta m^2\), whereas \((V\delta \cdot t)^2\) is not.}

\footnote{These probabilities can also be directly obtained (as a small \(\phi_2\) limit) from the general expression for the evolution matrix for neutrino oscillations in two layers of different constant densities derived in (4).}
and $P(\nu_i \to \nu_a; t) = 1 - P(\nu_i \to \nu_b; t)$. Notice that the l.h.s. of eq. (11) can also be understood as $P(\nu_a \to \nu_b; t_1 + t)$.

Eq. (11) has a simple physical interpretation. In the limit $t \to 0$ one has $P(\nu_i \to \nu_b; t) = \sin^2 2\theta_0 s_1^2$ which corresponds to propagation only in vacuum; the second and third terms in the curly brackets are due to the increase of the oscillation phase during the time interval $t$, neglecting the matter effects. These terms come from the expansion of $\sin^2(\phi_1 + \delta \cdot t)$ in small $\delta \cdot t$. The fourth term is the leading order matter contribution. Notice that we do not assume the smallness of the phase $\phi_1$, and in fact a sizeable enhancement of matter effects is only possible when it is not small.

Several comments are in order.

(i) The leading-order matter contribution vanishes in the case of maximal mixing in vacuum, $\theta_0 = \pi/4$. This may be useful for studying deviations of lepton mixing from the maximal one.

(ii) The first three terms in the curly brackets in eq. (11) are even in $\delta$ while the last, matter-induced, term is odd in it and so is sensitive to the sign of $\Delta m^2$. Thus, enhanced matter effects can facilitate studying the type of the neutrino mass hierarchy.

(iii) Although the absolute magnitude of matter effects, of course, strongly depends on $\theta_0$, their relative size is only mildly $\theta_0$ dependent provided that the vacuum mixing angle is not too close to $\pi/4$. This follows from the fact that $\sin^2 2\theta_0$ is the common factor in (11).

(iv) If the phase $\phi_1 = \delta \cdot t_1$ is not very close to $\pi/2$, the transition probability itself (i.e. neglecting matter effects) is also considerably enhanced as it now contains an $\sim \delta \cdot t$ term in addition to the $\sim (\delta \cdot t)^2$ ones. The relative size of the matter effects in this case is

\[
\cos 2\theta_0 \tan \phi_1 (Vt),
\]

while for $\phi_1 \approx \pi/2$ it is

\[
\cos 2\theta_0 (V/\delta).
\]

Thus for generic values $s_1, c_1 \sim 1$ the contribution of the matter effects to the transition probability, though strongly enhanced compared to the case of pure flavour eigenstate entering the matter, is relatively small. It may, however, still be noticeable as the short pathlength approximation applies even for $Vt$ as large as $\sim 1/3$ (the corrections are of the order $(Vt)^2$). When $\phi_1 \approx \pi/2$, matter effects give an important contribution to the transition probability over the time period $t$; they dominate in the limit $V \gg \delta$. This fact can be used for experimental searches of matter effects in short-pathlength neutrino experiments.

(v) The estimates (12) and (13) apply to the situations when the probability of finding $\nu_b$ at the vacuum-matter border is either experimentally known (two-detector experiments), or can be reliably estimated theoretically. If this is not the case, one has to compare the matter-induced contributions to $P(\nu_i \to \nu_b)$ with the probability of finding $\nu_b$ in the final

\[3\]More precisely, it is sensitive to the sign of $\cos 2\theta_0 \Delta m^2$ as only the sign of this quantity has a physical meaning. We adopt the convention $\cos 2\theta_0 \geq 0$ and allow for both positive and negative signs of $\Delta m^2$.}
state itself rather than with the increase of this probability due to neutrino propagation in matter. The relative contribution of matter effects is then

\[ \cos 2\theta_0 (V\delta \cdot t^2). \]  

(14)

For \( Vt \sim \delta \cdot t \sim 1/3 \) it can be about 10%.

(vi) Eq. (11) is also valid when neutrinos first propagate a distance \( t \) in matter and then a distance \( t_1 \) in vacuum. This is a consequence of the fact that two-flavour neutrino oscillations in matter are invariant under time reversal even if the matter density profile is not \( T \) invariant. Indeed, let the evolution matrix for eq. (1) be \( U(t_2, t_1) \), so that \(|\nu(t_2)\rangle = U(t_2, t_1)|\nu(t_1)\rangle\). The time-reversed evolution matrix is \( U(t_1, t_2) = U(t_2, t_1)^{-1} = U(t_2, t_1)^\dagger \). Since for any unitary \( 2 \times 2 \) matrix \(|U_{21}| = |U_{12}|\), the probability of neutrino flavour oscillations is \( T \) invariant. Notice that this is not in general true in the case of oscillations between \( n > 2 \) neutrino species.

Consider now a few numerical examples.

Atmospheric neutrinos coming to a detector from below the horizon propagate first in the air (which for the purposes of neutrino oscillations can be considered as vacuum) and then in the matter of the earth. The distances that neutrinos travel in the atmosphere \( t_1 \) and in the earth \( t \) are given by

\[ t_1 = -R|\cos \Theta| + \sqrt{(R + h)^2 - R^2 \sin^2 \Theta}, \quad t = -2R \cos \Theta, \]  

(15)

where \( R = 6371 \) km is the radius of the earth, \( \Theta \) is the zenith angle of the neutrino trajectory, and \( h \approx 15 \) km is the average height at which the neutrinos are produced in the atmosphere. We are interested in the regime \( t_1 \gg t \) which corresponds to nearly horizontal neutrino trajectories (zenith angles only slightly exceeding \( \pi/2 \)). In this case one can have sizeable phases \( \phi_1 \) whereas neutrino oscillations in the earth are in the short pathlength regime. This corresponds to \(|\cos \Theta| \approx 0.01\). Atmospheric neutrinos with nearly horizontal trajectories pass through the earth’s crust where the density is nearly constant and equal to about 2.8 \( g/cm^3 \) and the electron number fraction \( Y_e \approx 0.49 \) (the same is also true for short pathlength accelerator neutrino experiments). For oscillations between active neutrinos \( \nu_\mu \leftrightarrow \nu_e \) or \( \nu_e \leftrightarrow \nu_\tau \) this gives \( V \approx 5.17 \times 10^{-14} \) eV; for active-sterile neutrino oscillations \( V \) is a factor of two smaller.

Consider, e.g., atmospheric \( \nu_\mu \leftrightarrow \nu_e \) (or \( \nu_e \leftrightarrow \nu_\tau \)) oscillations for the neutrino trajectory with the zenith angle \( \Theta \approx 1.581 \) (\( \cos \Theta = -0.01 \)). From eq. (15) one finds \( t_1 \approx 378 \) km, \( t \approx 127 \) km, which gives \( Vt \approx 3.3 \times 10^{-2} \). For \( \Delta m^2 = 3.2 \times 10^{-3} \) eV\(^2 \), which is the current best-fit value of the Super-Kamiokande atmospheric neutrino data [4], and \( E = 1 \) GeV, one has \( \delta = 8 \times 10^{-13} \) eV, \( \phi_1 \approx 1.535, \delta \cdot t \approx 0.517 \). The relative matter contribution to the total transition probability is 4.3% and that to the increase of the probability due to neutrino propagation in the earth is 18%. This is a large effect, taking into account that the distance neutrinos travel inside the earth is only 127 km. If neutrinos did not oscillate in the atmosphere before entering the earth, this contribution would have been only about 0.5%.
It should be noted, however, that the short-pathlength regime is valid only in a narrow range of zenith angles. In addition, the effective mixing angle for $\nu_e \leftrightarrow \nu_x$ oscillations is known to be small [6].

Oscillations of $\nu_\mu$ into sterile neutrinos are disfavoured as the dominant channel of the atmospheric neutrino oscillations [7], but allowed as a subdominant channel with a weight that can be as large as about 50% [8]. For this channel, relative matter effects are a factor of two smaller than they are for the $\nu_\mu \leftrightarrow \nu_e$ or $\nu_e \leftrightarrow \nu_\tau$ channels, but their absolute value can be significantly larger because the corresponding mixing angle can be quite large.

Consider now the situation when neutrinos first propagate in matter and then in vacuum. This could, e.g., be realized in accelerator neutrino experiments with a detector placed on an earth’s satellite, which is certainly a rather remote possibility. Let us assume for definiteness that the distance that neutrinos propagate in the earth $t = 730$ km (the baseline of CERN – Gran Sasso and Fermilab – Soudan mine experiments). This corresponds to $\cos \Theta = -5.7344 \times 10^{-2}$, $Vt \simeq 0.191$ (for oscillations between active neutrinos). Assuming that the height of the satellite’s orbit is 750 km, the distance that neutrinos propagate after exiting the earth $t_1 \simeq 2837$ km. Let us again take $\Delta m^2 = 3.2 \times 10^{-3}$ eV$^2$. For $E = 8$ GeV, which is a typical energy of the accelerator neutrino experiments, one has $\delta = 10^{-13}$ eV, $\phi_1 \simeq 1.438$, $\delta \cdot t \simeq 0.37$. The relative matter contribution to the total transition probability is 13%, and that to the transition probability acquired due to neutrino propagation in the earth is a factor 1.3. This means that matter effects dominate the transition inside the earth in this case. If the detector is placed on the surface of the earth, i.e. $t_1 = 0$, matter effects constitute only about 3.5% of the oscillation probability for neutrinos that have traversed the earth, i.e. are almost a factor of 37 smaller. Thus, as paradoxical as it looks, the earth’s matter effects on the oscillation probability become stronger when neutrinos are detected farther from the surface of the earth.

Can one achieve a significant enhancement of the matter effects in the accelerator experiments by having very long decay tunnels? Unfortunately, this does not seem to be possible since a sizeable enhancement is only achieved when the distance traveled by neutrinos in vacuum is much larger than their pathlength in matter, and the latter should be at least a few hundred km.

5 Day-night effect in solar neutrino experiments

Another example of an initial state which is not a flavour eigenstate is given by the earth matter effects on solar neutrinos coming to a detector during night in the case of the MSW [1, 2] solutions of the solar neutrino problem. In this case the neutrino state arriving at the earth is an incoherent superposition of the mass eigenstates $\nu_1$ and $\nu_2$ (see, e.g., [3] for a recent discussion). The probability of finding a $\nu_e$ at the detector depends on the probability of $\nu_2 \rightarrow \nu_e$ oscillations inside the earth $P_{2e}$. Since in this case the initial state is a mass eigenstate $\nu_2$, one has $a_1 = \sin \theta_0$, $a_2 = \cos \theta_0$. In the short pathlength limit, in the case of
matter of constant density, one finds from (14)
\[ P_{2e} - (P_{2e})_{\text{init}} = P_{2e} - \sin^2 \theta_0 = \sin^2 \theta_0 (V \delta \cdot t^2). \]  
(16)

This expression vanishes in the \( V \to 0 \) limit because mass eigenstates do not oscillate in vacuum. However, it is nontrivial that it is of the order \( V \delta \cdot t^2 \) rather than \((V \delta \cdot t^2)^2\); because of this the day-night effect in solar neutrino experiments can be sizeable and must not be neglected even when the neutrino pathlengths inside the earth are relatively short. Since \( a_1 \) and \( a_2 \) are both real, there is no \( \sim \delta \cdot t \) contribution to the transition probability.

Another interesting point to notice is that the r.h.s. of (16) is \( \propto 1/E \). For sizeable pathlengths, the day-night effect increases with neutrino energy when \( \delta \gg V \) and decreases when \( \delta \ll V \) (see, e.g., fig. 4 in [10]). From (16) it follows that for short pathlengths it always decreases with \( E \), irrespective of the relative magnitudes of \( V \) and \( \delta \).

Let us consider a few numerical examples. For \( V t \sim \delta t \sim 1/3 \), which corresponds to the neutrino pathlength inside the earth \( t \simeq 1270 \) km and \( \delta \simeq V \simeq 5.2 \times 10^{-14} \) eV, and assuming \( \sin^2 \theta_0 \simeq 1 \), the matter-induced oscillation probability (16) is of the order of 10%. For 7Be solar neutrinos (\( E = 0.862 \) MeV) the above value of \( \delta \) corresponds to \( \Delta m^2 \simeq 1.8 \times 10^{-7} \) eV\(^2\), which is in the range of the MSW-LOW solution of the solar neutrino problem (for a recent analysis of the solar neutrino data see [11]). For the same distance \( t \simeq 1270 \) km, matter effects on the probability of neutrino flavor oscillations would constitute only about 1%, i.e. an order of magnitude smaller. For typical parameters of the MSW-LMA solution and \( E \simeq 10 \) MeV (typical energy of the 8B solar neutrinos) eq. (16) yields the probability \( \sim 1\% \) for the pathlengths as short as about 100 km, which has to be compared with the value \( 10^{-4} \) for the earth matter effect on neutrino flavour oscillations over the distance of 100 km.

Similar considerations apply to oscillations of supernova neutrinos inside the earth since those neutrinos also arrive at the earth as mass eigenstates. If the next supernova explosion occurs at such a time that its neutrinos come to a terrestrial detector passing through a short distance inside the earth, the earth’s matter effects on their oscillations may still be quite strong and should be taken into account [4].

Mass eigenstate neutrinos are also produced in neutral current reactions, i.e. in decays of real or virtual \( Z^0 \) bosons; it appears, however, technically rather difficult to produce sizeable beams of neutrinos born in neutral current reactions.

6 Neutrino oscillations due to FCNC

Neutrinos can oscillate in matter even if their masses are zero or negligible, provided that they have flavour changing neutral current (FCNC) interactions [1]. The evolution of the

\footnote{The importance of matter effects on oscillations of supernova neutrinos inside the earth was recently emphasized in [12]; however, the short pathlength limit was not discussed in these papers.}
system is described by eq. (1) with $A$ and $B$ being now both $t$ dependent:

$$A(t) = \frac{G_F}{\sqrt{2}} \epsilon' N(t), \quad B(t) = \sqrt{2} G_F \epsilon N(t), \quad (17)$$

Here $\epsilon$ and $\epsilon'$ are the FCNC parameters. In the short-baseline approximation, the transition probability $P(\nu_a \to \nu_b; t) \simeq B^2 t^2$. In contrast to the case of the ordinary neutrino oscillations in which the off-diagonal element $B$ of the effective Hamiltonian is independent of matter density $N(t)$, in the case of oscillations due to FCNC it is proportional to $N(t)$. Therefore the matter effect on the oscillation probability (which in this case coincides with the probability itself) is of the order $(\epsilon t)^2$ rather than $(V\delta \cdot t^2)^2$. This explains why in this case, unlike in the case of the ordinary neutrino flavour oscillations, matter effects can be quite sizeable even for as short baselines as that of the K2K experiment, $t = 250$ km (this fact was previously pointed out in [13, 14]). The expression $P(\nu_a \to \nu_b; t) \simeq B^2 t^2$ also explains why in the short baseline limit matter effects do not depend on $\epsilon'$ in the leading order [13].

## 7 Summary and conclusion

We discussed matter effects in short-pathlength neutrino oscillations and found out the reason why these effects generally decrease with decreasing pathlength more rapidly than the oscillation effects themselves. This happens because, in the leading order in perturbation theory, the transition amplitudes are determined by the off-diagonal terms of the effective Hamiltonian which are matter independent. This, however, is not true if the initial neutrino state is not a flavour eigenstate, in which case the diagonal terms of the effective Hamiltonian also contribute, or if the oscillations are due to flavour changing neutral currents, when the off-diagonal terms depend on matter density.

Initial states which are not flavour eigenstates can be obtained if neutrinos propagate significant distances in vacuum before entering matter (or after exiting it). In this case matter effects in short-pathlength neutrino oscillations can be strongly enhanced. We discussed implications of this observation for atmospheric neutrinos with nearly horizontal trajectories and for short pathlength accelerator experiments with the detector placed at large distances from the point where neutrinos exit the earth.

One also deals with initial neutrino states which are not flavour eigenstates when considering the earth matter effects on solar neutrinos in the case of the MSW solutions of the solar neutrino problem, or oscillations of the supernova neutrinos inside the earth. In these cases the initial states are mass eigenstate neutrinos. Matter effects for short neutrino pathlengths inside the earth are of the order $V\delta \cdot t^2$ rather than $(V\delta \cdot t^2)^2$ which would be expected if the initial state were flavour eigenstate.

In conclusion, we have shown that matter effects in oscillations of neutrinos traveling short distances in matter can be strongly enhanced when the initial neutrino state entering
the matter is not a flavour eigenstate, or when neutrinos are detected at significant distances from the point where they exit the matter. We believe that this is an interesting observation, even though the conditions for such an enhancement are generally difficult to realize. A notable exception is provided by solar and supernova neutrinos for which these conditions are realized as they arrive at the earth in mass rather than in flavour eigenstates.

The author is grateful to H. Minakata for asking a question during the NOW2000 workshop which led to the present study, and to A. Yu. Smirnov for useful discussions. This work was supported by Fundação para a Ciência e a Tecnologia through the grant JNICT-CERN/P/FIS/15184/99 and by the TMR network grant ERBFMRX-CT960090 of the European Union.

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