Design of Nonlinear Excitation Controller for Generator Based on Differential Algebra

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Abstract: With the appropriate transformation to the system model, this paper proposed the nonlinear control strategy for differential algebraic system based on the linear control law. Applying the method to single machine infinite-bus power system, which belongs to a nonlinear differential algebraic model, a nonlinear excitation control law has been designed. Simulation results show that the control law can enhance the stability of the system, also it can improve the regulative precision of generator terminal voltage, make each state trace for their given values rapidly. It manifests that the proposed method can effectively solve the design problem for nonlinear differential algebraic system and it is valuable for application.

1. Introduction

In practical engineering, the engineering model of complex system is generally described by nonlinear differential algebraic system equation. The nonlinear characteristics of this kind of system determine that the control design is difficult [1-5]. As what Wang Jie and Chen Chen (2001) stated, the method is applied to the generator excitation control system with differential algebraic model, but it is difficult to coordinate the dynamic and static performance of the system. [2] In Masahico Nambu’s paper (1996), the local structure theory of differential algebraic system is discussed by using differential geometry method, which is mainly based on the basic theory of differential equations on differential manifolds. [6] The input-output decoupling problem of affine nonlinear differential algebraic systems is discussed in Reich S’s paper [7], and the conditions for realizing input-output decoupling control by static feedback are given.

In order to solve the control problem of this kind of nonlinear system, Li Xiaocong (2004) discussed the selection of output function of nonlinear control system from the perspective of performance index of closed-loop control system, and found that the performance of control system was closely related to the selection of output function. On the basis of the previous research work, a multi index control method for differential algebraic nonlinear systems is proposed, which can solve the control problem of differential algebraic systems with multiple control objectives. In order to solve the problem of unknown parameters, with the help of Hartmann theorem [9], parameters can be configured according to the performance index of the system. Generator excitation control is an important means to improve the stability of power system and suppress low-frequency oscillation [10-11]. In the design case, a single machine infinite bus condensing power generation system is used, and a nonlinear excitation control law is designed.
The multi index control method based on differential algebra is proposed in this paper. The mathematical model of the generator is described by differential algebraic equations [12], which is different from the commonly used differential geometry method. It increases the algebraic constraint of generator operation, better reflects the actual operation state of the generator, and makes the control method more effective for the control of the generator. The multi index method is used to express the output, which is reflected in the nonlinear control. The variables of the system influence each other and restrict each other. It can realize the coordinated control of dynamic and static performance more effectively than using only one variable, which ensures the control precision and control effect of the system control target.

The simulation results show that the proposed nonlinear control law is effective for multi index differential algebraic systems with unknown parameters.

2. Design principle of nonlinear control for differential algebraic systems

2.1. Design of multi index nonlinear control
Consider the MIMO nonlinear differential algebraic system

\[
\begin{align*}
\dot{x} &= f(x, y) + \sum_{i=1}^{m} g_i(x, y)u_i, \\
p(x, y) &= 0, \\
zh &= h(x, y)
\end{align*}
\]

(1)

Where \(x\) is the n-dimensional state vector of the system, \(y\) is the m dimensional constraint vector; \(f(x, y) \in \mathbb{R}^n\), \(g_i(x, y) \in \mathbb{R}^n\), \(p(x, y) \in \mathbb{R}^m\) is a nonlinear vector field differentiable to both \(x\) and \(y\); \(zh = h(x, y)\) is a smooth vector field, \(u_i\) is an input.

Definition 1: the M derivative of each output \(h(x, y)\) to the vector field \(f(x, y)\) is

\[
M_i h(x, y) = E(h) f(x, y)
\]

Where \(E(h) = \frac{\partial h}{\partial x} - \frac{\partial h}{\partial y} \left[ \frac{\partial p}{\partial y} \right]^{-1} \frac{\partial p}{\partial x}\)

Definition 2: If the M derivative \(M_i M_j^{\sigma_1} h_j, \cdots, M_i M_j^{\sigma_m} h_j\) of each \(h_j(x, y)\) to the system (1) is zero and \(M_i M_j^{\sigma_m} h_j\) is not zero, then the relative order of \(h_j(x, y)\) to the system is \(r_j\). For MIMO systems, each output has a relative order \(r_j\), all relative orders of the system are a set \(r = (r_1, r_2, \cdots, r_m)\), and the total relative order of the system \(r = \sum_{r=1}^{m} r_j\).

When the total relative order \(r\) of the system is less than the system dimension \(n\), i.e. \(r < n\), there are \(n - r\) coordinate mappings \(\mu_j(x, y)(j = 1, 2, \cdots, n - r)\) satisfying

\[
M_i \mu_j(x, y) = 0 \quad (j = 1, \cdots, n - r).
\]

The nonlinear system (1) can be written in the following format under coordinate transformation

\[
\Phi(x, y) = [\mu_1, \mu_2, \cdots, \mu_r, \eta_1, \eta_2, \cdots, \eta_{n-r}]^T
\]
2. 2 Selection of output function

The key to the design of multi index nonlinear control is to select the output function correctly. Each output in the nonlinear system is affected by many other physical quantities at the same time in the process of system motion. Therefore, the linear combination of the output can make the dynamic and static performance of the controlled system better coordinated. The selected variable may be the sum of state variables One or more. [8] The bundle variables are expressed as follows:

\[ z_j = h_j(x, y) = \alpha x + \beta y \]  

(4)

The unknown parameters \( \alpha \) and \( \beta \) are taken as the coefficients of state variable \( x \) and constraint variable \( y \).

2.3 Design of multi index nonlinear control law

For MIMO nonlinear systems, each output has a set of corresponding linear systems and a nonlinear equation in system (3). Therefore, the system (3) can be divided into \( m \) groups of linear subsystems \( \zeta_{ji}(j = 1, 2, \ldots, r_i - 1; k = 1, 2, \ldots, m) \) and \( M \) nonlinear system equations \( M_j \eta_j(x, y) \) (5)

For the linear part, the control law of the system is obtained based on the linear control theory [13-14]

\[ v_j = -\sum_{j=1}^{n} k_{ij} \zeta_j \]  

(6)

For nonlinear systems, the control law is used

\[ u = B^{-1}(x, y)(-A(x, y) + v) \]  

(7)

In the formula,

\[ u = (u_1, u_2, \ldots, u_m) \quad v = (v_1, v_2, \ldots, v_m) \quad A(x, y) = \left[ \begin{array}{c} c A^T \\ \end{array} \right]_{m \times n}, B(x, y) = \left[ M_{gl} M_f^{-1} h_1 \right]_{m \times n} \]

Suppose \( (x_0, y_0) \) is the equilibrium point of the system, the corresponding initial \( \zeta \) value is \( \zeta_0 \), suppose \( \Delta f = f - f_0 \), then the multi index nonlinear control law of the system (1) can be obtained.

\[ u = u_0 - \frac{\Delta A(x, y) + k \Delta \zeta}{\left[ M_{gl} M_f^{-1} h_1 \right]_{m \times n}} \]  

(8)

in the formula,

\[ \Delta A = \left( M_{g1}^T h_1 \quad \cdots \quad M_{gn}^T h_1 \right)^T \]

\[ k \Delta \zeta = \left( \sum_{j=1}^{n} k_{i1} (\zeta_j - \zeta_{i0}) \quad \cdots \quad \sum_{j=1}^{n} k_{i} (\zeta_j - \zeta_{i0}) \right)^T \]

Analysis: from the above control law, when the system is not disturbed, the control law is always equal to \( u_0 \), and the system states will not change; when the system is disturbed, each state changes, and the disturbance rejection term is no longer zero.
3. Design of generator excitation control law based on differential algebraic model

3.1 Nonlinear model of condensing steam turbine

The nonlinear system of condensing steam turbine generator set connected to infinite bus power system is considered, and its differential algebraic model is as follows [15]:

\[
\begin{align*}
\dot{E}_q' &= -\frac{1}{T_d}(E_q' - u_f) \\
\dot{\delta} &= \omega - \omega_0 \\
\dot{\omega} &= \frac{1}{T_j}a_h(P_n - P_f) - \frac{D}{T_j}(\omega - \omega_0) \\
P_m &= -\frac{P_n}{T_{\Sigma}} + \frac{U_f}{T_{\Sigma}}
\end{align*}
\]  

(9a)

\[
\begin{align*}
V_f^2 &= \left(\frac{x_{qf}}{x_{ef}}V\sin\delta\right)^2 - \left(\frac{x_{qf}'}{x_{ef}}V\cos\delta + \frac{x_{ef}}{x_{eff}}E_q'\right)^2 = 0 \\
E_q + \frac{x_{qf}'}{x_{ef}}V\cos\delta - \frac{x_{ef}}{x_{eff}}E_q' &= 0 \\
P_e + \frac{V^2(x_{qf}' - x_{ef})}{2x_{ef}x_{df}}\sin 2\delta - \frac{E_q'V}{x_{df}}\sin\delta &= 0
\end{align*}
\]  

(9b)

Where E'q, δ, ω, Pm are state variables, which represent generator transient potential, generator power angle, generator rotor angular velocity and turbine input mechanical power respectively; Eq, Pe and Vf are constraint variables, which are no-load electromotive force, electromagnetic power and generator terminal voltage respectively, U_f, UT represents the excitation voltage of generator and control signal of steam valve opening; T'd0 is the excitation winding time constant when the generator stator is open circuit, Tj is the rotor inertia time constant of the generator set; D is the generator damping coefficient; ω0 is the synchronous angular speed of generator rotor; T_{\Sigma} is the equivalent time constant of steam turbine, xd is the direct axis reactance, xq is the quadrature axis reactance, x'd is the direct axis transient reactance, xe is the total equivalent reactance of voltage transformer and transmission line; x'd∑=x'd+xe; xq∑= xq+xe. V is the infinite bus voltage. The unit of δ is (rad), the unit of ω is (rad / s), the unit of time constant is (s), and the rest are unit values.

The mathematical model of condensing steam turbine described in equation (9) is composed of four state variables and three algebraic variables. The differential algebraic system as shown in equation (1) is written, and each quantity is as follows:

\[
p(x, y) = V_f^2 - \left(\frac{x_{qf}}{x_{ef}}V\sin\delta\right)^2 - \left(\frac{x_{qf}'}{x_{ef}}V\cos\delta + \frac{x_{ef}}{x_{eff}}E_q'\right)^2
\]

\[
+ \frac{V^2(x_{qf}' - x_{ef})}{2x_{ef}x_{df}}\sin 2\delta - \frac{E_q'V}{x_{df}}\sin\delta
\]  

(10)
\begin{align*}
0 &= p_1(x,y) = E_y' + \frac{x_{\alpha\Sigma} - x_{\beta\Sigma}}{x_{\alpha\Sigma}} V \cos \delta - \frac{x_{\beta\Sigma}}{x_{\alpha\Sigma}} E_y' \\
0 &= p_2(x,y) = V_y^2 \left( \frac{x_y}{x_{\alpha\Sigma}} V \sin \delta \right)^2 - \left( \frac{x_y'}{x_{\alpha\Sigma}} V \cos \delta + \frac{x_y}{x_{\alpha\Sigma}} E_y' \right)^2 \\
0 &= p_3(x,y) = P_e + \frac{V^2 (x'_{\alpha\Sigma} - x'_{\beta\Sigma})}{2x'_{\alpha\Sigma} x_{\alpha\Sigma}} \sin 2\delta - \frac{E V}{x_{\alpha\Sigma}} \sin \delta
\end{align*}

\[ x = \begin{bmatrix} E_y' & \delta & \omega & P_e \end{bmatrix}^T, y = \begin{bmatrix} E_y' & V_f & P_e \end{bmatrix}^T \]

\[ f = (f_1, f_2, f_3, f_4) \quad f_1 = -E_y/T_p, \quad f_2 = (\omega - \omega_0); \]

\[ f_3 = -[P + D(\omega - \omega_0)]/T_s, \quad f_4 = -P_e/T_p \]

\[ g_1 = [-1/T_p, 0, 0, 0]^T; \]

\[ g_2 = [0, 0, 1, 1/T_p]^T; \]

3.2 Design of multi index excitation controller

For the single machine infinite bus system described in equation (16), the expression of the output function is selected according to the form of equation (4)

\[ y_i = h_i(x, y) = \alpha_i x + \beta_i y (i, j = 1, 2, \ldots) \]

The selection of parameters \( \alpha \) and \( \beta \) is based on the following principles:

Firstly, in the design of the controller, the implementation of the controller becomes more complex because it is not measurable, so the observable measurement can only be taken as the output variable. \( E'q, \delta \) and \( Eq \) are all unmeasurable, so intervention control law should be avoided.

Second, the terminal voltage and generator speed determine the power quality, which can be selected as control variables.

Thirdly, when the mechanical power is disturbed, the unbalanced torque generated on the rotor destroys the stable operation of the generator. In order to accurately track the given value and suppress the electromagnetic oscillation, it can be selected as the control variable.

Finally, the output function is determined as follows:

\[ \begin{aligned}
    y_1 &= h_1(x, y) = \alpha_{13} \Delta \omega + \beta_{12} \Delta V_f \\
    y_2 &= h_2(x, y) = \alpha_{23} \Delta \omega + \beta_{23} \Delta P_e
\end{aligned} \]

Then,

\[ \frac{\partial p}{\partial y} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2V_f & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \frac{\partial p}{\partial x} = \begin{bmatrix} \rho_{11} & \rho_{12} & 0 & 0 \\ \rho_{21} & \rho_{22} & 0 & 0 \\ \rho_{31} & \rho_{32} & 0 & 0 \end{bmatrix} \]

where

\[ \rho_{11} = -x_{\alpha\Sigma}/x'_{\alpha\Sigma}; \quad \rho_{12} = (x'_{\alpha\Sigma} - x_{\alpha\Sigma}) V \sin \delta / x'_{\alpha\Sigma}; \]

\[ \rho_{21} = -2x_{\alpha\Sigma} \left( \frac{x_y'}{x'_{\alpha\Sigma}} \cos \delta + \frac{x_y}{x'_{\alpha\Sigma}} E_y' \right); \]
\[ \rho_{22} = \frac{\left( x' \right)^2 - \left( \frac{x}{x'} \right)^2}{V^2 \sin 2\delta + \frac{2x'x'E}{(x')^2}} \]

\[ \rho_{31} = -\frac{V \sin \delta}{x'}, \rho_{32} = V^2 (x' - x) \cos 2\delta(x') - VE \sin \delta(x'), \]

Therefore, according to the basic operations and properties of differential algebraic systems, the total relative order \( r_1 + r_2 = 3 \) of the output function \( h_1, h_2 \) is less than the system dimension \( n = 4 \).

Therefore, the mapping function \( \Phi(x, y) \) needs another mapping \( \eta_1(x, y) \), so that \( M_{\eta} \eta_1(x, y) = 0 \), it is easy to know that if \( \eta_1(x, y) = \delta \) is taken, the condition is satisfied.

So the mapping function is taken

\[ \Phi(x, y) = \begin{bmatrix} M_{\eta}^T h_1(x, y) & M_{\eta}^T h_2(x, y) & M_{\eta}^T h_3(x, y) & \eta_1(x, y) \end{bmatrix}^T \]

\[ = \begin{bmatrix} h_1(x, y) & M_{\eta}^T h_2(x, y) & h_2(x, y) & \delta \end{bmatrix}^T \]

The system (9) can be transformed into the following dynamic equation

\[ \Phi(x, y) = \begin{bmatrix} M_{\eta}^T h_1(x, y) & M_{\eta}^T h_2(x, y) & M_{\eta}^T h_3(x, y) & \eta_1(x, y) \end{bmatrix}^T \]

\[ = \begin{bmatrix} h_1(x, y) & M_{\eta}^T h_2(x, y) & h_2(x, y) & \delta \end{bmatrix}^T \]

In the above formula, the correlation is calculated as follows:

\[ E(h_1(x, y)) = \frac{\partial h_1}{\partial x} - \frac{\partial h_1}{\partial y} (\frac{\partial p}{\partial y}) \frac{\partial p}{\partial x} \]

\[ = (\frac{\beta_{32} \rho_{31}}{2V_f} - \frac{\beta_{31} \rho_{32}}{2V_f} \quad \delta \quad 0) \quad (17) \]

Then

\[ M_{\eta} h_1 = \frac{\beta_{32} \rho_{31}}{2V_f} f_1 - \frac{\beta_{31} \rho_{32}}{2V_f} f_2 + \bar{c}_{13} f_3 \quad (18a) \]

\[ M_{\eta} h_2 = -\beta_{31} \rho_{32} f_1 - \beta_{32} \rho_{31} f_2 + \bar{c}_{13} f_1 + \beta_{32} f_4 \]

\[ M_{\eta} M_{\eta} h = M_{\eta} h_1 = \frac{\beta_{31} \rho_{31}}{2V_f \tau_{d0}} \quad M_{\eta} M_{\eta} h = M_{\eta} h_2 = \frac{\beta_{32} \rho_{32}}{2V_f \tau_{d0}} \]

\[ M_{\eta} M_{\eta} h = M_{\eta} h_1 = \frac{\bar{c}_{13}}{\tau_{y2}} \quad M_{\eta} M_{\eta} h = M_{\eta} h_2 = \frac{\bar{c}_{23}}{\tau_{y2}} \]

\[ M_{\eta} h_1 = M_{\eta} h_1 = \bar{c}_{13} (f_1 - f_n) - \frac{1}{2V_f} \sum^{2}_{i=1} \beta_{31} \rho_{31} (f_i - f_n) \]

\[ M_{\eta} h_2 = M_{\eta} h_2 = \bar{c}_{13} (f_1 - f_n) + \beta_{31} (f_n - f_m) - \frac{2}{2V_f} \sum^{2}_{i=1} \beta_{31} \rho_{31} (f_i - f_n) \quad (18c) \]

By substituting the above equation into equation (8), the excitation control law of DASMINC is obtained as follows:
3.3 Numerical simulation and analysis

In this paper, the simulation experiment is carried out on a single machine infinite bus system. The system parameters are as follows: \( x_q = x_d = 2.12; x'd = 0.26; x_e = 0.24; D = 2; T_j = 4.06; T'd_0 = 5.8. \)

In order to verify the effectiveness of the proposed nonlinear control strategy, dynamic simulation and comparative analysis are carried out for the following two cases. The proposed multi index nonlinear control law based on differential algebraic system (solid line 1 in the figure) and linearization control (dotted line 2) are adopted.

At the beginning, the system was in stable state. At 0.4s, three-phase short circuit occurred at the high-voltage side of generator outlet transformer. After 0.15s, the fault was removed and reclosed successfully. The following figure shows the response curve of the system under the disturbance.

\[
\begin{align*}
\begin{pmatrix} u_1 \\ u_2 \end{pmatrix} &= u_q - \begin{pmatrix} \frac{M_1 h_1}{M_f h_2} + \frac{\alpha_{13} \Delta \omega + \beta_{12} \Delta V_e}{\alpha_{23} \Delta \omega + \beta_{21} \Delta P_e} \\ \frac{M_2 h_1}{M_2 h_2} \end{pmatrix} \\
\end{align*}
\]

(19)

**Analysis:** when large disturbance occurs in the system, this method can suppress the frequency oscillation of the system, improve the transient stability of the system, suppress the overshoot of the generator terminal voltage, and maintain the stability of the system. From the simulation results, it can be seen that the proposed multi index nonlinear excitation coordinated control law for differential algebraic systems can quickly transfer the generator power angle and bus voltage to the equilibrium state of the system after large disturbance, and has good performance in stabilizing the system voltage.

**Figure 1. System responses to a three-phase fault**

- (a) bus voltage response curve
- (b) power angle response curve
- (c) Valve opening control signal curve
- (d) electromagnetic power response curve

**4. Conclusion**

In this paper, a nonlinear control strategy for nonlinear differential algebraic systems is proposed. Based on the analysis of differential algebraic systems, a nonlinear control strategy for multiple control objectives is designed. The simulation results show that the control strategy adopted can effectively solve the control problem of nonlinear systems, make them track their given values quickly and accurately, and improve the stability of nonlinear systems. It is proved that this method can effectively solve the control problems of nonlinear systems considering the dynamic and static aspects.
performance of the system, the application range of differential algebraic system to nonlinear problems is extended.

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References
[1] Lu Qiang, Mei Shengwei, sun Yuanzhang. (2008) Nonlinear control of power system. Beijing: Tsinghua University Press. pp. 57-113.
[2] Wang Jie, Chen Chen. (2001) Nonlinear control of differential algebraic models in power systems. Acta electrical engineering Sinica, 21 (8): 16-19.
[3] Wang Jiang, Li Tao, Zeng Qiming, et al. (2005) Differential algebraic nonlinear control of permanent magnet synchronous motor based on observer. Chinese Journal of electrical engineering, 1,25 (2): 87-91.
[4] Wang J, Chen C. (2001) Exact linearization of nonlinear control of differential algebraic systems. In:Proceedings of International Conference on Info-Tech and Info-Net. Beijing, China, IEEE, 284-290.
[5] Fong K. Mak. (1992) Design of Nonlinear Generator Exciters Using Differential Geometric Control Theory. In: Proceeding of the 31st Conference on Decision and Control.tucson, Arizona, 1149-1153.
[6] Masahico Nambu, Yasuharu Ohsawa. (1996) Development of an Andvanced Power System stabilizer Using a Strict Linearization Approach. IEEE Transactions on Power Systems, Vol.11, No. 2. May: 813-818.
[7] Reich S. (1995) On the local qualitative behavior of differential algebraic equations. Circuits Systems Signal Process,14(4): 427-443.
[8] Li Xiaocong, CHENG Shijie,WEI Hua, etal. (2004) Important Affection of the Output Function in MIMO Nonlinear Control System Design (in Chinese). Proceedings of the CSEE,24(10):50-56.
[9] L.Baratchart, M.Chyba, J.B.Pomet. (1999) On the differentiability of feedback linearization and the Hartman-Grobman theorem for control systems. Proceedings of the 38th Conference on Decision & Control Phoenix, Arizona USA, December:1617-1622.
[10] Q.Lu, Y.Sun, Z..Xu, etal. (1996) Decentralized Nonlinear Optimal Excitation Control, IEEE Transaction on Power Systems, Vol. 11, No. 4, November :1958-1960.
[11] Liu Hui, LI Xiao-cong, Wei Hua. (2007) Nonlinear Excitation Control for Generator Unit Based on NCOHF(in Chinese). Proceedings of the CSEE,1,27(1):14-18.
[12] Wang Jie, Chen Chen, Wu Hua, et al. (2002) Design theory and method of parameter adaptive control for multi machine power systems. Acta electrical engineering Sinica, 22 (5): 5-9.
[13] Zheng Dazhong. (2005) Linear system theory. Tsinghua University Press, Beijing. pp. 180-209.
[14] Hu Shousong, Wang Zhiquan, et al. (2005) Optimal control theory and system. Science Press, Beijing. pp.197-199.
[15] Li Xiaoxiang, Cheng Shijie, Wei Hua, et al. (2003) A high performance nonlinear excitation control. Acta electrical engineering Sinica, 12,23 (12): 37-42.