Disturbance: It’s a Feature, not a Bug

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Results of measurements give legitimacy to a physical theory: What if acquiring these results in the first place necessitates what the same theory considers to be an interaction? In this note, we assume that theories account for interactions so that they are empirically traceable, and that observations necessarily go with such an interaction with the observed system. We investigate consequences of this assumption: The unfolding language game, inspired by “quantum logic,” leads to a class of contextual and probabilistic theories. Disturbance becomes a means to render interactions, thus, also measurements, empirically tangible. The measurement “problem” arises in all such theories, not only quantum mechanics: It is a consequence of the need for empirical evidence of interactions.

I. INTRODUCTION

The infamous Wigner’s-friend experiment [1–3] serves to illustrate the measurement problem: If we imagine Wigner performing a measurement on his friend who measured another system, there are different—in fact, incommensurable—uses of the term “measurement:

(M1) If the friend’s “measurement” of a state in an equal superposition with respect to his measurement basis is regarded as an interaction between two systems—modelled by a physical evolution—, then it corresponds to a unitary on the joint system, yielding an entangled joint state;

(M2) if the “measurement” leads to an account of experience that serves as a normative judgement about the validity of a theory, we expect exclusively one of several possible outcomes.

Statements (M1) and (M2) are in conflict in two respects: On the one hand, there is a fundamental dualism due to the nature of language: Either we regard a measurement as an interaction captured by the formal language of the theory or as a meaningful account of experience [4]. On the other hand, the linearity of quantum mechanics cannot be reconciled with value-definiteness—the outcome being exclusively one of several possibilities.

The latter incommensurability is not so much a peculiarity—or defect—of quantum mechanics. Instead, we argue that it appears in theories that (a) account for interactions so that they are empirically significant, and (b) require that an observation necessarily goes with such an interaction. An observation is then itself empirically traceable. The two requirements above are combined in the interaction assumption:

(IntA) Interactions are empirically traceable. An observation necessitates such an interaction.

The incommensurability in the measurement problem can, therefore, be regarded as a consequence of the interaction assumption.

In order to determine the accounts of experience that are compatible quantum mechanics—or, conversely, the accounts that falsify quantum mechanics—, we have to connect statements (M1) and (M2). If they are incommensurable, then a bridge is needed: the Born rule. As such, the necessity of the Born rule is tightly connected to the measurement problem and can as well be traced back to the interaction assumption. We argue that both appear in a wider class of physical theories that satisfy the interaction assumption, not just quantum mechanics.

The structure of the article is illustrated in Figure 2:

Figure 1. There is a connection between the necessity of the Born rule and the incommensurability exposed in the measurement problem. They both arise from the incommensurability of statements (M1) and (M2). The two can be regarded as arising from the interaction assumption.

Figure 2. Outline: We first motivate the interaction assumption before discussing a concept of verifiable information facilitating the scientific paradigm of verification. We then investigate how the notion of verifiable information can be combined with the interaction assumption in a physical theory. We examine the characteristics of such theories and how to regard quantum mechanics as an instance of such a theory.
causality of our sensory perceptions. In Section III we develop the notion of verifiable information, a concept constitutive to natural sciences. We then investigate in Sections IV and V how the notion of verifiable information can be reconciled with empirically tangible interactions as required by \( \text{IntA} \). Here we employ, inspired by quantum logics, the calculus of orthocomplemented lattices (see Appendix C). In Appendix A how the resolution restriction emerges in theories with an interaction assumption. Finally, in Appendix B we discuss repercussions of the above for quantum mechanics.

II. MEANING, INTERACTION, SYSTEM

In this section, we contextualize the interaction assumption \( \text{IntA} \) see Figure 3.

In Section II A we discuss the need for external entities. The external is often regarded as the cause of experience, and as such endowed with ultimate explanatory authority. Even if one hesitates to enthrone external entities as sufficient reason behind sensory perceptions, the external plays a crucial role in establishing meaning and reference.

In Sections II B and II C we examine the notion of a “system”—the referent in physical theories: On the one hand, the scientific quest for empirical confirmation and reproducibility requires a context in which statements can only depend on that context. On the other hand, as discussed in Section II D if an observation is not a “spooky action at a distance,” the observer and the observed entity cannot be regarded as independent systems.

The interaction assumption reflects the intent to render the interactive access to something external a meaningful concept—i.e., subject to our experience.

A. The external cause of meaning

The intent of explaining our experience—including observations and measurements—by means of a reduction to an ultimate level goes with a complementary picture of cognition:

The cognitive process of perception is often conceived as follows: the world consists of things (such as material bodies or atoms) which exist in themselves; these things produce effects (phenomena); and we capture some of these effects by means of our sensory organs; that is to say, the things are the cause of our sensations. [6, §3.2]

There is something external that causes—at least in part—the perceived actuality. Conversely, the effective sensory perceptions allow us to refer to their external cause. Even if one is sceptical towards the existence of an immediate correspondence between the external referent and the reference \( \text{IntA} \), the externally caused sensory input can be considered necessary to establish meaning and reference. If, for instance, the problem of first words is resolved by trimming —by training strict behavioral patterns—the acquisition of language crucially relies on experience with an external cause. In this constructive perspective onto language, externally caused experience is necessary to establish meaning. Establishing meaning, thus, relies on adding something to language that is not inherent to it in the first place.

B. Separability

The scientific method relies essentially on empirically confirming statements—as we discuss in greater detail in Section III. We, therefore, require that there is a context in which there are statements with a clearly confined dependence. Einstein’s separability assumption—effectively a non-signalling assumption—ensures that we can meaningfully say something about an “entity” or “system,” independently of its environment, that is, independently of parts external to the system.

1 “It seems perfectly clear, at least since Wittgenstein and Sellars, that the ‘meaning’ of typographical inscription is not an extra ‘immaterial’ property they have, but just their place in a context of surrounding events in a language-game, in a form of life. This goes for brain-inscriptions as well.” [7, §1.2]

2 “We need to make a distinction between the claim that the world is out there and the claim that truth is out there. To say that the world is out there, that it is not our creation, is to say, with common sense, that most things in space and time are the effects of causes which do not include human mental states. To say that truth is not out there is simply to say that where there are no sentences there is no truth, that sentences are elements of human languages, and that human languages are human creations.” [6, §1]

3 “Man muss schon etwas wissen (oder können), um nach der Benennung fragen zu können. Aber was muss man wissen?” [6, §30]

4 “Das Lehren der Sprache ist hier [für das Kind] kein Erklären, sondern ein Abrichten.” [6, §5]

5 Quantum mechanics accounts for this by the partial trace being sufficient to derive all measurement results about the subsystem (see, e.g., [11]).
that which we conceive as existing (‘actual’) should somehow be localized in time and space. That is, the real in one part of space, A, should (in theory) somehow ‘exist’ independently of that which is thought of as real in another part of space, B. If a physical system stretches over the parts of space A and B, then what is present in B should somehow have an existence independent of what is present in A. What is actually present in B should thus not depend upon the type of measurement carried out in the part of space, A; it should also be independent of whether or not, after all, a measurement is made in A. Einstein combines the assumption of systems being separately describable with the observation of the “worldly origin” of our terms: Our choice of a “system” should not affect the ability to formulate independent statements about that system.

Whereas a theory with this flexibility as to what counts as a “system” allows for ample applications, it deprives the term of its sortal character and thus its ability to establish identity: If anything can be regarded as a system, nothing is essentially a system. The mere character trait of “constituting a system” does not provide the means to identify the system. To meaningfully say something about a system, however, requires us to identify and refer to that system. So, a theory that states what can be said about a system, and that comes with the flexibility regarding the choice of the system, necessarily relies on other linguistic means to establish the reference to that system. Furthermore, if a theory does not allow to identify its systems, it does also not allow to identify any potential thing-in-itself associated with systems (see also Section 11).

**D. No observation at a distance**

Despite the possibility of statements independent of the environment of a system, we have to allow for changes of a system that are the effect of an external cause. Regarding an observer as a system, we have to account for the external cause of his sensory perceptions as discussed in Section 11A. This does not imply that this description of the observer is exhaustive, and it does not imply the ability to conclude what is the observer’s account of experience. It merely requires an interaction as the necessary requirement for the external cause of his sensory perception. We, thus, restrict separability in the sense that a system is either independently describable—it is isolated—or it interacts with its environment—i.e., with other systems external to it. There is, there-

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7“A criterion is a way of recognizing something, or a feature by which we can recognize something. A criterion of identity is something by which we can recognize the correctness of a statement of identity. Questions of identity make sense only once we have specified what kind of thing is at issue. One cannot simply ask ‘Is this the same as that?’, but only ‘Is this the same S as that’, where ‘S is a sortal term like ‘table’ or ‘planet’. Accordingly, a criterion of identity is a feature which determines whether or not an object falling under ‘S’ that we encounter in one context or refer to in one way is the same as an object falling under ‘S’ that we encounter in another context or refer to in another way.” [16, §2.2]

8“Another possible interpretation of the slogan ‘no entity without identity’ [...] might run something like this: ‘There is nothing you can sensibly talk about without knowing, at least in principle, how it might be identified.’ I have nothing to say against this admirable maxim.” [17, §1]

9In [18], Mittelstaedt proposes a “quantum ontology.” The author rightly points out the inability to reidentify a “quantum object.” Thus, the “quantum ontology” cannot suffice to meaningful refer to either a system or a “quantum object.” Even if quantum mechanics is “nearer to the ‘truth’ than classical mechanics” [18, p.9], it cannot be the “whole truth.”

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6“Terms that have proven useful for the ordering of things attain easily such an authority over us so that we forget their worldly origin and we accept them as unalterable facts. They are, then, put down as ‘thinking-necessities,’ ‘a priori given,’ etc. The path of scientific progress is often made impassable for a long time by such misconceptions.” [13, p. 102, own translation]
fore, a dichotomy between systems that are independently describable versus systems that interact with their environment—with such an interaction, seemingly contradictory, being at the core of “saying something about a system.”

The interaction assumption \([\text{IntA}]\) demands that these interactions themselves have empirically detectable effects. An interaction is, therefore, not an abstract term beyond our experience but it becomes itself meaningful. If an observation is necessarily accompanied by an interaction, it must be empirically detectable as well. There is no “observation at a distance,” just as there is no action at a distance.

The importance of an interaction has been emphasized before. Bohr, in his reply to the EPR paper, emphasizes the role of interactions to refute the notion of a “[prediction] without without in any way disturbing a system”—the idea of innocently reading off measurement results:

"Indeed the finite interaction between object and measuring agencies conditioned by the very existence of the quantum of action entails—because of the impossibility of controlling the reaction of the object on the measuring instruments if these are to serve their purpose—the necessity of a final renunciation of the classical ideal of causality and a radical revision of our attitude towards the problem of physical reality."

In a similar vein, Popper in exposed an essential shortcoming of classical mechanics: If classical mechanics did account for the interactions that finally led to our experiences, it would be indeterministic. Instead, classical mechanics relies on other theories to account for causal connections in an observation: The electromagnetic interactions allowing to measure position and momentum have negligible disturbing effects.

### III. VERIFIABILITY

In Section [II A] we discuss the need for an equivalence in order to allow for a verifiability. Questions, their relation to information, and meaning is briefly addressed in Section [III B]. In Section [II C] we introduce a formal representation of questions as an orthocomplemented lattice. The lattice is then, in Sections [II D] and [II E] endowed with the equivalence relations as depicted in Figure 4.

#### A. Equivalent questions

If physics, in the spirit of Einstein’s quote in Section [II C] above, strives to make “statements about parts of the world,” then we require that statements can repeatedly be empirically confirmed—they are verifiable. Popper states in that the “scientifically meaningful physical effect” is characterized by being reproducible—regularly and by anybody who builds the experiment according to the instructions. This supposes that there exists the possibility to refer to or mean the “same experiment” for different points in space and time (“regularly”), and by different observers (“anybody”). We, therefore, assume:

\[
\text{(EQ) There is an equivalence between questions. If and only if an observation yields an equal answer to an equivalent question, then the second answer confirms the first.}
\]

The assumption does not imply that there is an a priori fixed meaning of “experiment built according to the instructions” captured in a particular privileged language.

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10 In [22], Aerts distinguishes between “theories of system” and “theories of measurements.” The former “predict” results of measurements. The latter account for disturbances by the measurement. Classical and quantum mechanics both belong to the former. Complementarity is then, according to Aerts, not a disturbance by a measurement, but the inability to predict with certainty. We neither follow the distinction regarding theories nor the author’s statement that “the aim of a physical theory of the physical system is to ‘predict’ the result of a certain test, and this prediction is done before the test is carried out and no matter whether the test will be carried out.” [22, p.2442] This characterization of the aim of physics requires an a priori meaning of a “certain test” independent from running that test, contrary to our belief that meaning is established within the context of a language game.

11 “Der wissenschaftlich belangvolle physikalische Effekt kann ja geradezu dadurch definiert werden, daß er sich regelmäßig und von jedem reproduzieren läßt, der die Versuchsanordnung nach Vorschrift aufbaut.” [23, §1.8]

12 We do not specify how this equivalence is established (see also last paragraph in Section [II A]).
If meaning and reference are abilities rather than occurrences \([8, \S 1]\), then the existence of such a privileged language is in doubt. Thus, the assumption \([EQ]\) of the possibility to ask semantically equivalent questions is to be read as: There is the possibility to establish identical meaning and thus equivalent questions for both different points in space and time, as well as different observers.

While we assume that there are equivalent questions, we do not assume that equivalent questions always have equal answers. Rather, we use in Section \([IV.A]\) that answers can depend on the context, i.e., they can be contextual.

### B. The meaning of “information”

Sections \([III.B\) D specify our use of the term “information” and build the foundation for the discussion in Sections \([IV]\ and \([VI]\)

a. Information. Say, an agent makes two observations about an entity—for instance: two consequent measurements in a Stern-Gerlach experiment, the same measurement in two different runs of a Stern-Gerlach experiment, or the measurement of momentum and position of a solid ball. These observations allow to answer questions such as, “Does the silver atom have spin-up or spin-down?” Each question-answer pair constitutes information. Importantly, neither the question nor the answer alone suffice. If one regards a string of zeros and ones stored on a hard disk as information—assuming already the context to meaningfully refer to “zeros and ones stored on a hard disk”—it is because one believes that they can answer binary questions, that is, questions with two possible answers. Zeros and ones constitute bits of information merely in light of corresponding questions. Just as there is no privileged notion of a “system,” there is no privileged notion of “information” that could be reified as the basic building blocks.

b. Distinguishability. We take a question to have multiple—necessarily more than one—mutually exclusive possible answers. The question is answered if one has selected one of these possibilities. While a theory singles out a specific answer, experience might suggest another—contradicting—one, and, thereby, falsify the theory \([23]\).

c. Meaning. A piece of information carries meaning only insofar as the respective question does. In light of the arguments by, e.g., Wittgenstein \([2, 21, 22]\), Quine \([26, 27]\), and Putnam \([8, 28]\), there are doubts whether a question can have an inherent semantics. Rorty summarizes that

the ‘meaning’ of typographical inscription is not an extra ‘immaterial’ property they have,

but just their place in a context of surrounding events in a language-game, in a form of life. This goes for brain-inscriptions as well. \([7, \S 1.2]\)

d. No a-priori states. The above-introduced notion of information does not rely on an a-priori assumption on the nature and number of “states in which a system can be” \([23, \S I]\). Quite oppositely, we aim to develop a notion of “state” in view of an idea of how we speak about things, and, therefore, we attempt to sail around an a-priori commitment to states of any sort.

### C. Formal description

To capture the above formally, we assume that questions in a given experimental context are represented by elements of a set \(Q\). Two questions \(Q_1, Q_2 \in Q\) are equivalent in the above sense (see Section \([III.A]\) if and only if \(Q_1 \sim Q_2\).

a. Binary questions. We now endow the set of questions \(Q\) with additional, admittedly rather ad hoc, structure. The first move in this language game \([6]\) is to assume that the questions in \(Q\) are binary, i.e., questions with two possible answers, \(t\) and \(f\). One may, as well, assume that questions have no less than three possible answers and, thereby, commit to ternary logic instead of binary logic in our case. \([14]\). In the binary case, questions in \(Q\) can be regarded as statements that are either true or false. A statement can imply another,

\[
(Q_1, t) \Rightarrow (Q_2, t)
\]

where “⇒” is the notion of implication in ordinary language. Note that the equivalence of questions is not equal to the bi-directional implication

\[
(Q_1, t) \Rightarrow (Q_2, t) \land (Q_1, t) \Leftrightarrow (Q_2, t) .
\]

If \((Q_1, t) \Rightarrow (Q_2, t)\) and an inquiry yields \((Q_1, t)\), then it follows that \((Q_2, t)\). If, then, one attempts to confirm \((Q_2, t)\), one inquires about a

\[
\equiv Q_2 \equiv Q_2 .
\]

Importantly, if we “ask the same question again” at a different space-time point, we are referring to two different questions that are not bi-directionally implied (as in \([1]\)), but \(\sim\)-equivalent questions. \([15]\) Depending on the context these might have different answers (see Section \([IV.A]\)).

b. Lattice of questions. Implication and negation of statements allow to regard the \(Q\) as a complemented lattice. The implication of statements, \((Q_1, t) \Rightarrow (Q_2, t)\), allows to define an order relation \(\preceq\) on \(Q\):

\[
Q_1 \preceq Q_2 \text{ if and only if } (Q_1, t) \Rightarrow (Q_2, t) .
\]

13 Initially, we aim for a qualitative rather than a quantitative characterization of “information.” In particular, when referring to “information” we do not mean mutual information.

14 The argument against deterministic theories in Section \([IV.A]\) relies crucially on binary logics.

15 No two questions that are actually inquired about are identical or mutually implied (as in \([1]\)). They may or may not be \(\sim\)-equivalent.
If, in addition, \((Q_2, t)\) does not imply \((Q_1, t)\), then we write \(Q_1 \prec Q_2\). The negation of a question \(Q\) has an answer \(t\) if and only if the answer to \(Q\) is \(f\).

\((-Q, t) \Leftrightarrow (Q, f)\).

We assume that there exists a question \(Q_i\) that is always answered with \(t\) — a tautological question. Similarly we assume that there is a question \(f\) that is always answered with \(f\) — the absurd question. Then, for all questions \(Q \in Q, Q_i \preceq Q\) and \(Q \preceq Q_o\). Assuming that the joins and meets in the partially ordered set \((Q, \preceq)\) are unique, the set of questions together with the implication and negation forms a complemented lattice (for a brief, visual summary of lattices, see Appendix [C]). It seems natural to assume the negation to be involutional and order reversing. This yields an orthocomplemented lattice.

If we “ask the same question twice,” we are referring to two \(\sim\)-equivalent questions that do, however, not imply each other in the sense of [II]. Within the lattice, the two questions have the trivial lower and greatest lower join, i.e., the absurd and the tautological question. In other words: The tautological question and the absurd one hold together the sublattices associated with inquiries at different times or in different runs. The equivalence relations defined above (see, e.g., Figure III) relate questions across these sublattices.

c. Deterministic theories. For now, we take a theory to be deterministic in the sense that it associates each question \(Q \in Q\) with an answer \(A \in \{t, f\}\). The pair \((Q, A)\) then constitutes information in the sense above. The association of an answer to a question may be contextual in the sense that the theory associates answers to any finite subset of questions \(Q \subset Q\) dependent on that subset. Conversely, a theory is called non-contextual if it associates answers to questions independent of other questions.

D. Identical systems

In the above-mentioned example of two subsequent measurements in one run of a Stern-Gerlach experiment, we assume that we refer to two measurements of the same silver atom—i.e., of the identical entity. Observations then yield attributes of one single entity. Thus, two questions may be equivalent insofar as they refer to the identical entity. This yields a second equivalence relation, \(\sim^*\), on \(Q\). While questions equivalent with respect to \(\sim\) may refer to the same kind of entity in different runs of the same experimental setup, questions that are equivalent with respect to \(\sim^*\) refer to the same entity within the one particular run.

The equivalence relation of \(\sim\)-equivalent questions referring to the identical system, i.e., the intersection of \(\sim\) and \(\sim^*\), is then denoted as \(\equiv\). In this sense, performing two measurements in the same basis does not constitute asking the same question but asking equivalent questions with respect to the relation \(\equiv\).

In summary, we have introduced two notions of equivalence of questions that reflect essentially asking the same question about the same (type of) entity, \(\sim\), and about the same (identical) entity, \(\equiv\). The latter corresponds to asking \(\sim\)-equivalent questions referring to the identical system, i.e., that are \(\sim^*\)-equivalent.

IV. INTERACTION ASSUMPTION FOR VERIFIABLE INFORMATION

We combine the notion of verifiable information established in Section III with the interaction assumption [IntA] motivated in Section II. First, in Section [IV.A], we contrast isolated systems against systems that interact with their environment. This leads us to turn to probabilistic and contextual theories. As discussed in Section [IV.B], a Born rule then becomes an essential part of the theory. In Section [IV.C], we examine connections to the BB84 key agreement protocol. Subsequently, in Section [IV.D], we consider interactions between different parts of an isolated system, and how they relate to the measurement problem.

A. Interactions and isolated systems

a. Isolated systems. We first characterize what it means for a system not to interact with its environment. Therefore, we add to Einstein’s separability restriction: A system \(S\) is isolated if and only if equivalent questions referring to that system yield same answers. In other words: Information inquired about a system remains valid in the sense that it can be reproduced or verified if and only if that system is isolated. Formally,
Figure 6. By transitivity of the equivalence relation \(\sim\) asking an equivalent question \(Q_2 \equiv Q_1\) does not disturb the system: The answer to another equivalent question \(Q_3 \equiv Q_1\)—short for \(Q_2 \sim Q_1 \wedge Q_1 \sim Q_2\)—is still the same, \(A_3 = A_1\). The same extends to any \(Q_2\) implied by \(Q_1, Q_1 \Leftarrow Q_1\).

Figure 7. Asking a non-equivalent question about a system disturbs the systems in the sense that equivalent questions about it, \(Q_1 \equiv Q_3\), yield different answers.

the above reads as

\[
S \text{ is isolated if and only if } \forall Q \text{ with } (Q, A) : \forall Q' : Q : (Q', A).
\]

For isolated systems, \(\equiv\)-equivalent questions (\(\sim\) and \(\sim^*\)-equivalent, see Section III D) obtain the same answers, i.e., they mutually imply each other (see Figure 5).

b. Transitivity. We take the relations \(\sim\) and \(\sim^*\) to be proper equivalence relations, in particular transitive. To ensure consistence with our notion of isolated systems, an interaction that reproduces valid information—that yields an answer to a previously asked equivalent question—leaves an isolated system undisturbed. An isolated system may interact with its environment to inquire about equivalent questions without being disturbed as depicted in Figure 6. This extends to inquiries about implied questions. We call a question \(Q_2\) implied by another question \(Q_1\) if there exists a question \(Q'_2\) such that

\[
Q'_2 \equiv Q_1 \land Q'_2 \preceq Q_2.
\]

In other words: By transitivity we cannot distinguish whether a system does not interact with its environment at all or whether it does interact to inquire implied information.

c. Empirically tangible interactions. The interaction assumption demands that we are able to distinguish whether a system is isolated or, on the contrary, whether it has been interacted with to inquire non-implied information. We extend the interaction assumption (\(\text{IntA}\)) slightly: Not only do we assume that there is an interaction corresponding to every inquiry about a system. For every interaction, there is also a corresponding question an observer equivalently could have inquired about.\(^{17}\)


Figure 8. In a deterministic theory, binary questions with an interaction assumption lead to a contradiction: As between each pair of the equivalent questions, \(Q_1 \equiv Q_3 \equiv Q_5\), there is a non-implied question, the answers should all differ albeit \(\neg A_1 = A_5\).

We, therefore, treat all interactions of a system with its environment as if an observer is inquiring about a question.

If, in the spirit of Einstein’s separability criterion, a previous interaction becomes empirically evident only from inquiries about that system, then there must be questions whose answer depends on these previous interactions. If we require the empirical evidence for an interaction to stem merely from the system under consideration, we are lead to establish an interaction as a violation of the criterion for isolated systems: An interaction has to disturb the system. We are, thus, left with a contextual theory in the sense of Paragraph III C.c.

To exemplify the above, let us consider successive inquiries about three question that refer to the same system, i.e., that are \(\sim^*\)-equivalent, as depicted in Figure 7. Say we inquired about the first question \(Q_1\) and obtained an answer \(A_1\). Any inquiry about a non-implied question \(Q_2\)—this implies that \(Q_2 \not\equiv Q_1\)—should alter the answer to questions \(Q_3 \equiv Q_1\). In a deterministic theory with an interaction assumption, the answer \(A_3\) is then \(\neg A_1\) (“not \(A_1\”). This however has problematic consequences for binary questions in \(Q\): Let us consider inquiries about \(\equiv\)-equivalent questions, \(Q_1, Q_3,\) and \(Q_5\). Between these, there are inquiries about non-implied questions, \(Q_2\) and \(Q_4\), as shown in Figure 8. Both, \(Q_2\) and \(Q_4\), disturb the answers to the other questions. Thus, inquiring about \(Q_5\) yields an answer

\[
A_5 = \neg A_3 = \neg (\neg A_1) = A_1.
\]

The pieces of redundant information, \((Q_1, A_1)\) and \((Q_5, A_5)\), let the system appear to be isolated—despite the inquiries of non-implied questions. Therefore, if one holds merely these two pieces of redundant information, the interaction assumption is violated. As noted already in Paragraph III C.a it is essential here to assume binary logics.

d. Probabilistic theories. The problem can be solved by turning to probabilistic theories with an adapted notion of isolated systems:\(^{18}\) A system is isolated if and only if equivalent questions regarding the same system yield same answers with certainty. That is, any set

\(^{17}\)In fact, requiring that there should not be any qualitative difference between interactions can be read this way.

\(^{18}\)We assume that the set of answers is a Borel set.
of \(\sim\)-equivalent questions can be asked again with the same result:

\[
S \text{ is isolated if and only if } \forall Q \subseteq Q : \forall Q \in \tilde{Q} \text{ with } (Q, A) : \forall Q' \equiv Q : ((Q', A) .
\]

Previously, in [3], a system is isolated if inquiring about equivalent questions about an identical system, in one single run of an experiment, yields the same answer. In [1], however, a system is isolated if for arbitrarily many runs of the experiment inquiring about \(\equiv\)-equivalent questions yields same answers—if in each run we can reassure ourselves by asking a \(\equiv\)-equivalent question about the identical system. The answers across different runs do not need to be the same—only within the same run.

e. In retrospect. The notion of an “equivalence of questions” has been fundamental for establishing a notion of “isolated systems.” The first should, however, not be regarded as logically prior to the latter—rather, the two have to be thought of as a mutually dependent, and to be made sense of together (see Figure 9). In order to establish what is to be considered an equivalent question, one usually relies on an equivalent use: Two questions are then equivalent if they yield same answers in the same context.

\section*{B. Born rule in a contextual theory}

The above considerations on the interaction assumption have left us with a contextual and probabilistic theory: For a given question, the theory yields a probability distribution over the answers, and this probability distribution depends on inquiries about other questions. Generally, such a theory assigns to a finite set of questions \(\overline{Q} \subseteq Q\) a joint probability distribution. We assume that the possible answers to any questions in \(\overline{Q}\) are contained in a set \(A\). Further, we denote with \(\mathbb{P}(Q)\) the set of finite ordered subsets of \(Q\). A probabilistic theory can then be regarded as a map that assigns to each ordered set \(\overline{Q} \subseteq \mathbb{P}(Q)\) a joint probability distribution,

\[
T : \overline{Q} \rightarrow \{(x_1, \ldots, x_n) \in A^n \rightarrow P(x_1, \ldots, x_n)\} . \tag{5}
\]

The theory is contextual if the map \(T\) does not derive from a map

\[
T' : Q \rightarrow \text{Prob}(A)
Q \rightarrow \{x \in A \rightarrow P(x)\} \tag{6}
\]

that assigns to each question independently a probability distribution.

a. Non-contextual theories. In the case of a theory of the form (6), the theory is equivalent to a relation \(\{(Q, x, P(x))\} \subset Q \times A \times [0, 1]\) that associates each question-answer pair with a probability weight. Such a theory might stem from a probability distribution over a Boolean lattice \(Q \times A\) (see Appendix C). Then, an interaction cannot “disturb” the measurement result, in the sense that it alters the probability distribution. The association of probability distributions is independent of other measurements. An interaction is merely empirically tangible insofar as it changes the probability distribution over the possible answers of questions for a given set of questions—just as the Born rule does. It can thus be regarded as a generalization of the Born rule. In the particular case of quantum mechanics, the states, according to Gleason’s theorem [30], correspond to probability distributions over an orthomodular, non-distributive lattice (see Appendix C).

As we have argued above: If a theory is to satisfy the interaction assumption, then this map cannot be reduced. Following the introductory comments Section [1] we expect a “measurement problem” that corresponds to the necessity of the Born rule. We return to this point in Section [V]

\section*{C. Generalized BB84}

Any theory satisfying the interaction assumption [IntA] allows to derive a key-exchange protocol as in [31]. If interactions can be traced empirically, then also the action of an eavesdropper provided that he cannot guess what is an equivalent question: Alice inquires a system about a randomly chosen question \(Q_1 \in Q\), and then sends the system to Bob who also chooses a random question \(Q_2 \in Q\) and inquires about it. If the two questions are equivalent, \(Q_1 \equiv Q_2\), then Alice and Bob obtain the same answer. For an eavesdropper to learn something about the obtained answers, he will have to inquire an equivalent question. If he does not know Alice’s and Bob’s question, he can merely guess a question. With non-zero probability, he will choose a non-implied question, and thus disturb the system. The disturbance reveals his interaction to Alice and Bob.
The above shows that the BB84 protocol relies crucially on the dichotomy between isolated systems and systems interacting with their environment: Either a system is isolated or it interacts with its environment. To learn something about the system, an interaction is necessary. If interactions leave traces, then one can tell whether someone could have learnt something about that system. Conversely, the BB84 protocol can be used to characterize what we mean by referring to “isolated systems” or to “interactions”: Instead of making interactions “empirically tangible,” we could equivalently have strived for making “an eavesdroppers actions detectable.”

V. INTERACTIONS WITHIN AN ISOLATED SYSTEM

So far, we have merely considered interactions of the environment (or of observers in the environment) with the system under consideration. We now turn to interactions within an isolated system, i.e., between different parts of a joint system. Two systems, $S_1$ and $S_2$, together can again be regarded as a system assuming the ability to refer to $S_1$ and $S_2$ suffices to refer to the corresponding combined system. The joint system consisting of $S_1$ and $S_2$ is denoted $S_1 \cup S_2$.

A. Equivalent questions revisited

How does the notion of an isolated system for a combined system relate to the notion of isolated systems for the subsystems? How do questions in $Q_{S_c}$ about the combined system $S_c = S_1 \cup S_2$ relate to questions about the subsystems, i.e., to elements in the Cartesian product $Q_{S_1} \times Q_{S_2}$? If any question that one can ask about $S_c$ can be separated into questions on the subsystem—i.e., if there is a one-to-one correspondence between the elements in $Q_{S_c}$ and in $Q_{S_1} \times Q_{S_2}$—, then a combined system is isolated if and only if the subsystems are isolated.

According to \((\text{IntA})\) there is no difference if an observer interacted with $S_1$ or another system $S_2$ did. Let us consider the scenario depicted in Figure 10. If one inquires about two equivalent questions $Q_1 \equiv Q_3$ about $S_1$—before and after an interaction with the other subsystem $S_2$—, then the answers must differ unless the interaction corresponds to inquiring about a question implied by $Q_1$. The same holds, vice versa, for $S_2$ with respect to inquiries about equivalent questions $Q_2 \equiv Q_4$. Thus, for a given interaction between $S_1$ and $S_2$, there are questions on the subsystem that detect the interaction. The combined system $S_c$, however, \textit{did not interact with its environment} between the pair of inquiries $(Q_1, Q_2)$ and $(Q_3, Q_4)$. The combined system should still be isolated. Therefore, if two subsystems interact,

1. it is not sufficient for the joint system to be isolated that the subsystems are isolated, and

2. the notion of equivalent questions for $S_c$ does not simply derive from the notion of equivalent questions for $S_1$ and $S_2$.

In particular, the pairs $(Q_1, Q_2)$ and $(Q_3, Q_4)$ are not equivalent questions for $S_c$ unless the interaction corresponds to implied questions for both $Q_1$ on $S_1$ and $Q_3$ on $S_2$.

B. The measurement problem

If we assume that there is no qualitative difference between a system interacting with another, and a system interacting with an observer inquiring about a question, then one is tempted to regard an observer just as a system. As one inquired about a question for any system,
one might inquire about the measurement result of an observer. This leads to a general Wigner’s-friend experiment: Let us assume that Wigner inquires a system $F$ about an initial question $Q_{iF}$ with three possible answers, $A_{iF} = \{0, 1, \Delta\}$, and a system $S$ about an initial question $Q_{iS}$ with two possible answers, $A_{iS} = \{0, 1\}$ as depicted in Figure 11. Wigner obtains an answer $A_{iF} = \Delta$ for his inquiry about $Q_{iF}$. The systems $F$ and $S$ then interact so that if Wigner initially obtains $(A_{iS}, \Delta)$, then he finally gets $A_{iS} = A_{iF} = A_{iS}$ upon inquiring about $Q_{iF} \equiv Q_{iF}$ and $Q_{iS} \equiv Q_{iS}$. As the answers change (deterministically), the pairs $(Q_{iS}, Q_{iF})$ and $(Q_{iS}', Q_{iF}')$ are not equivalent questions for the interacting joint system $S \cup F$:

$$(Q_{iS}, Q_{iF}) \not\sim (Q_{iS}', Q_{iF}') \equiv Q_{int}.$$ 

Wigner now changes the initial question $Q_{iS}$ to another, non-equivalent $Q_{iS}' \not\sim Q_{iS}$. Previously, the interaction corresponded for $S$ to the inquiry about a question implied by $Q_{iS}$. This is not the case anymore for $Q_{iS}'$. So the inquiry about an equivalent question $Q_{int}' \equiv (Q_{iS}', Q_{iF})$ disturbs both systems: Inquiring about $Q_{iF}' \equiv Q_{iF}^2$ before and after inquiring about $Q_{int}'$ yields with non-zero probability different results, respectively for $Q_{iS}^1 \equiv Q_{iS}'^1$. Conversely, inquiring about $Q_{int}'$ after $(Q_{iS}, Q_{iF})$ yields with non-zero probability an answer different from $(A_{iS}, A_{iF})$. If one regards the interaction as a measurement of Wigner’s friend, then one knows either that the joint system $S \cup F$ is isolated and attributed the right notion of equivalent questions corresponding to the interaction, or one knows that the friend got the right result. But inquiring about one of these invalidates the other. The two pieces of information—whether the friend interacted with $S$ and what he got as a result—are not simultaneously valid.

VI. CONCLUSION

If we require that there is “no observation at a distance,” and if we regard this requirement itself not exempt from our experience, then we are lead to assume that any interaction—including those that accompany our observations—leaves traces. Disturbance and complementarity in contextual, probabilistic theories with dichotomic notions of an isolated system and an interaction are then not a defect. Instead, they turn out to be a means to render interactions empirically tangible: An interaction of a system with its environment alters the context, and, thus, the probability distribution for other measurements performed on that system. The necessity of a Born rule, a non-trivial map from ordered sets of measurements to probability distributions, is then an immediate consequence.

If interactions within isolated joint systems are qualitatively no different from interactions of systems with an observer, as they are regarded necessary by the interaction assumption, then there emerges a measurement problem: In a Gedankenexperiment with encapsulated observers, à la Wigner’s friend, one can construct inquiries so that either one can inquire about whether the encapsulated observer interacted with the system she measured or what result the encapsulated observer obtained. Either of the inquiries invalidates previously obtained answers to the respective other. We cannot commensurate the idea of a measurement yielding a definite result with the idea of a measurement being an interaction.

VII. EPILOGUE

The antagonism [between the actual language and our demand for a crystal-clear logic] becomes unbearable; the demand is on the verge to become something empty. — We got onto the clear ice where there is no friction, where the conditions are, in a sense, ideal, but also where we cannot walk. We want to walk; so, we need the friction. Back onto the rough ground! [§ 107 (own translation)]

The picture of an external cause behind sensory perceptions without the necessity of traceable interactions may yield a crystal-clear logic—in the form of a non-contextual and deterministic theory. Then, however, we find ourselves on the clear, friction-less ice: How can we meaningfully speak of an external cause if there is no empirical evidence of an interaction?

If, on the contrary, an interaction that is regarded necessary for an observation leaves empirically tangible traces, one has the ground to meaningfully refer to an external cause. The measurement problem is here rather a characteristic of contextual, probabilistic theories than a defect of quantum mechanics. The question is not necessarily how to solve the measurement problem but how it can be made sense of.

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Appendix A: Resolution restriction

In [20] and [32], it is assumed that there is an upper bound on how much information one can have about a system. We examine how this resolution restriction emerges in theories satisfying the interaction assumption.

a. Refinement. An ordered family of questions, \( \{Q_i\}_i \), that refer to the same system, i.e., that are \( \sim \)-equivalent, is called a refinement, if any \( Q_i \) is implied by its successor \( Q_{i+1} \) as defined in Paragraph IV A b, i.e.,

\[
\forall i : \exists Q_i' : Q_i' \equiv Q_{i+1} \land Q_i' \succ Q_{i+1}.
\]

b. Reassurance. A refinement can be reassured in the following sense: If we inquire about the questions in its given order, then, at any time, we can reassure ourselves (i.e., inquire again about) previous questions, [21] without disturbing the system. If the system is isolated, we will then obtain the same answer as to the equivalent question inquired about previously.

c. Resolution restriction. If the lattice \( (Q, \preceq) \) is atomic, then any refinement is finite. Then there is only a finite number of questions that are not \( \equiv \)-equivalent and that can be inquired about without invalidating some of the questions. The resolution restriction, i.e., the assumption that there is a maximal number of question one can simultaneously know the answer to, can, thus, be regarded as a consequence of the interaction assumption together with the assumption that the lattice \( (Q, \preceq) \) is atomic.

Appendix B: The “state” of a quantum “system”

How does the above discussion change the perspective onto quantum mechanics? In the following, we examine some repercussions.

a. Privileged questions. We have argued in Section II that semantic intricacies taint the notion of an ultimate thing-in-itself, and, thus, also the notion of the state exhaustively characterizing such an independently existing thing-in-itself—forming a “state-in-itself.” The interaction assumption adds to the scepticism regarding a reification of the state symbols: Even if the state-in-itself existed, the interaction assumption would bar the epistemic access to it. After having asked a question \( Q \), one may ask refining questions. By the interaction assumption, however, there exists a question \( Q' \) that allows to detect the inquiry of \( Q \). Then, two pieces of information constituted from the inquiries about \( Q \) and \( Q' \) cannot be valid simultaneously. Which of the two pieces of information does then tell us something about the state-in-itself?

Bohmian mechanics qualifies a “position measurement” as the measurement that reveals the state-in-itself. But there seems little in the way to single out the “momentum measurement.” [22]

b. Observer independence. The formulation of the “realism assumption” in [33] comes with similar issues:

One [assumption] is that a system has a ‘real physical state’—not necessarily completely described by quantum theory, but objective and independent of the observer. The assumption only needs to hold for systems that are isolated, and not entangled with other systems. [32]

The notion of an isolated system differs here from the one above: The system-in-itself has a state-in-itself—indeed of an observer. This puts the state in an in-principle, demon perspective [23]: If one can merely say something about that state after experience and corresponding interactions, then a state independent of the observer remains a rather abstract concept.

c. Gleason’s theorem. Instead of regarding epistemically inaccessible symbols or answers to distinguished questions as the “state-in-itself,” we follow the direction of Gleason’s theorem [35]: If we choose to represent the \( \sim \)-equivalence classes of questions by projectors on a Hilbert space, then—if the dimension is greater than two—the probability distributions over these projectors are in a one-to-one correspondence with the density matrices on that Hilbert space. Therefore, the state symbols of quantum mechanics can be regarded as the probability distributions over the \( \sim \)-equivalence classes. The collapse is an initial choice of a probability distribution. It corresponds to the assumption that, without further knowledge, we take a system to be isolated and, therefore, expect to obtain the same answer to subsequent, \( \equiv \)-equivalent questions.

---

[21] Strictly speaking: we mean an inquiry about a \( \equiv \)-equivalent question.

[22] Instead of assuming that “everything is, in the end, a position measurement,” one may follow the argumentation in Section II or return to Bohr’s or Popper’s argument (see Section III) against a simple reading off of pointer positions: If a measurement goes necessarily with an interaction, then everything might be regarded as a “momentum measurement.” In the “reading-off-picture” the position measurement is fundamental, in an “interaction-picture” the momentum measurement can be regarded as fundamental.

[23] “One might say that all these difficulties arise from the fact that the story of the Laplacean demon is an attempt to eliminate the vague and dangerous phrase ‘in principle.’ For what it tries to explain is what we mean when we say that the future states of a system can be ‘in principle’ predicted on the basis of a knowledge of past or present states. ‘In principle’ means here something like ‘not in practice, because human knowledge is never sufficiently precise and complete.’ No wonder that, in attempting to explain what we mean by ‘in principle,’ Laplace was driven to a superhuman intelligence. But the Laplacean demon is unsatisfactory, we may say, just because infinitely precise and complete knowledge is also ‘in principle’ unattainable.” [21]
d. Infinitely many questions. Even though one may assume that there are merely finitely many answers—we restrict ourselves to binary questions above—, this does not imply restrictions on the number of \( \sim \)-equivalence classes of questions. If one does not assume a bound on the number of equivalence classes, then there is no lower bound on the distance between two equivalence classes. Thus, there is no upper bound on the number of probability distributions associated with the disturbance an inquiry about a question from one equivalence class causes to inquiries about questions from another. Therefore, the number of states must be equally unbounded. This yields a variant of Hardy’s theorem [34]. In [35] the continuity of the underlying mathematical structure is regarded as a strange aspect of quantum mechanics. 24

Hardy’s theorem together with results such as Holevo’s bound and the discreteness of errors, show that precisely the opposite is the case: it is instead the continuity of quantum physics that is so strange. How can it be that we have a continuum of quantum states that ostensibly behave discretely but we do not have, and cannot have, an underlying discrete structure? [35, §3.2]

The continuity merely appears strange if one elevates the quantum state to the thing-in-itself together with an assumption of epistemic transparency—the tenet of the immediate sensory accessibility of the thing-in-itself.

The conclusion that even the most primitive quantum system must contain an infinite amount of information [35, §3.2]

relies on an imprecise use of the term “information”. Even in an atomic lattice of questions with a resolution restrict resulting from the interaction assumption as argued in Appendix A there may be infinitely many \( \sim \)-equivalence classes of question. While there may be infinitely many non-equivalent questions, only a finite number of them might have simultaneously valid, i.e., reproducible, answers. The ability of asking arbitrarily many questions should not be confused with the ability of having reproducible answers to all these questions.

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24 As noted in [35], the step from infinite states to a continuous state space is shown in [36].
Appendix C: Overview for quantum logic

Figure 12 gives, first, a brief introduction to lattices (see, e.g., in [37]). The terms in the blue boxes form a sequence of narrowing definitions, starting from a general lattice (Box 1) towards a Boolean lattice (Box 5). The figure also summarizes some results in quantum logic: The theorems by Jauch and Piron [38], Kochen and Specker [39], and Gleason [40] limit the possibility of quantum mechanics being a non-contextual theory. In the contrary, we raise the question why a theory should be contextual, and, thus, not require the structure of a distributive lattice.

Figure 12. A brief summary of lattices and quantum logic.