Coherent state operators in loop quantum gravity

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Introduction

Idea: Coherent states can be used to construct operators corresponding to classical functions. [Klauder 1970’s→]; [Bergeron, Gazeau, arXiv:1308.2348]
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Ingredients:

- A complete set of coherent states $|q, p\rangle$ labeled by coordinates $q$ and momenta $p$

$$\mathbb{1} = \int d\mu(q, p) |q, p\rangle\langle q, p|$$

- A function $f(q, p)$ on the classical phase space
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\[ f(q, p) \rightarrow \hat{A}_f = \int d\mu(q, p) f(q, p) |q, p\rangle \langle q, p| \]
Part 1:

Coherent states on $SU(2)$
Coherent states on $SU(2)$: Construction of the states

Coherent states in QM:

$$\delta(x - x_0) = \int dk \ e^{-ik(x-x_0)}$$
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- Insert $e^{\sigma \nabla^2/2}$

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  $$\psi(x_0, p_0)(x) \sim e^{ip_0 x} e^{-(x-x_0)^2 / 2\sigma}$$
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1. $\delta_{g_0}(g) = \sum_j d_j \text{Tr} \, D^{(j)}(g_0 g^{-1})$
2. Insert $e^{t \nabla^2 / 2}$
3. $\sum_j d_j \, e^{-tj(j+1)/2} \, \text{Tr} \, D^{(j)}(g_0 g^{-1})$

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Coherent state operators in LQG

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\[
\sum_j d_j e^{-tj(j+1)/2} \operatorname{Tr} D^{(j)}(g_0 g^{-1})
\]

- Replace $g_0 \in SU(2)$ with $h \in SL(2, \mathbb{C})$

\[
\sum_j d_j e^{-tj(j+1)/2} \operatorname{Tr} D^{(j)}(hg^{-1})
\]
Coherent states on $SU(2)$

\[ \psi_h(g) = \sum_j d_j e^{-t_j(j+1)/2} \text{Tr} \, D(j)(hg^{-1}) \quad d_j = 2j + 1 \quad h \in SL(2, \mathbb{C}) \]

[Thiemann and Winkler, arXiv:hep-th/0005233, 0005234, 0005235 and 0005237]
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$$\psi_h(g) = \sum_j d_j e^{-t_j(j+1)/2} \text{Tr} D^{(j)}(hg^{-1})$$

$$d_j = 2j + 1$$

$$h \in SL(2, \mathbb{C})$$

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For an interpretation in terms of classical variables, decompose $h$ as

$$h = g_0 e^{p_0 \cdot \sigma / 2} = e^{p'_0 \cdot \sigma / 2} g_0$$

where $g_0 \in SU(2)$, and $(p'_0 \cdot \sigma) = g_0 (p_0 \cdot \sigma) g_0^{-1}$.
Relation to classical variables

It is natural to identify \( g_0 \) as the holonomy of the Ashtekar–Barbero connection along an edge \( e \):

\[
h_e = \mathcal{P} \exp \left( \int_e A \right)
\]
Relation to classical variables

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$$h_e = \mathcal{P} \exp \left( \int_e A \right)$$

Two conjugate variables can be associated to the edge by using the parallel transported flux variable:

$$E[p](S) = \int_S d^2\sigma \; n_a(\sigma) \; h_p \rightarrow \sigma \; E^a(\sigma) h_{\sigma \leftarrow p}$$

Choosing $p = \{s(e), t(e)\}$, one obtains two variables, which are related by

$$E[t(e)] = h_e E[s(e)] h_e^{-1}$$
Semiclassical properties

\[ \langle g | g_0, p_0 \rangle = \sum_j d_j e^{-t(j+1)/2} \text{Tr} D^{(j)} \left( g_0 e^{p_0 \cdot \sigma/2} g^{-1} \right) \]

\[ \langle jmn | g_0, p_0 \rangle = \sqrt{d_j} e^{-t(j+1)/2} D^{(j)}_{mn} \left( g_0 e^{p_0 \cdot \sigma/2} \right) \]

The state \( |g_0, p_0 \rangle \) is peaked on its labels in the following sense:
Semiclassical properties

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- \( \rho(g) = |\langle g | g_0, p_0 \rangle|^2 \) has a maximum at \( g = g_0 \);
  the peak is sharp when \( t \) is small.
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- \( \rho(j, m, n) = |\langle jmn | g_0, p_0 \rangle|^2 \) has a maximum at \( j \simeq |p_0|/t \);
  the peak is sharp when \( t \) is large.
Further properties

- The states $|g, p\rangle$ provide an overcomplete basis on the Hilbert space $L_2(SU(2), d\mu_{\text{Haar}})$:

$$1 = \int d\mu(g, p) |g, p\rangle \langle g, p|$$

where the measure has the form $d\mu(g, p) = d\mu_{\text{Haar}}(g) d\nu(p)$. 
Further properties

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where the measure has the form $d\mu(g, p) = d\mu_{\text{Haar}}(g) d\nu(p)$.

- Under a local $SU(2)$ gauge transformation $a(x)$, the states transform as

$$|g, p\rangle \rightarrow |a^{-1}(t)ga(s), R^{-1}(a(s))p\rangle$$

$$|g, p'\rangle \rightarrow |a^{-1}(t)ga(s), R^{-1}(a(t))p'\rangle$$

where $R(a)$ is the $\mathbb{R}^3$ rotation matrix associated with $a \in SU(2)$. 
Part 2:
Coherent state operators in LQG
A coherent state operator associated to a single link of a spin network will have the form

\[ \hat{A}_f = \int d\mu(g, p) f(g, p) |g, p\rangle \langle g, p| \]
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More generally,

$$\hat{A}_f = \int d\mu(g_1, p_1) \cdots d\mu(g_N, p_N) f(\{g\}, \{p\}) \times |g_1, p_1 \otimes \cdots \otimes g_N, p_N\rangle \langle g_1, p_1 \otimes \cdots \otimes g_N, p_N|$$
A coherent state operator associated to a single link of a spin network will have the form

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More generally,

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\[ \times |g_1, p_1 \otimes \cdots \otimes g_N, p_N\rangle\langle g_1, p_1 \otimes \cdots \otimes g_N, p_N| \]

Properties:

- \( \hat{A}_f \) is gauge invariant, if \( f(\{g\}, \{p\}) \) is invariant under the corresponding transformation of \( \{g\} \) and \( \{p\} \).
- If \( f(g, p) > 0 \) almost everywhere, then all eigenvalues of \( \hat{A}_f \) are strictly positive.
Part 3:
Examples
Holonomy operator

To construct the coherent state operator corresponding to the holonomy, choose
\( f(g, p) = D_{mn}^{(j)}(g) \):

\[
\hat{A}_{D_{mn}^{(j)}} = \int d\mu(g, p) \, D_{mn}^{(j)}(g) \, |g, p\rangle \langle g, p|
\]
Holonomy operator

To construct the coherent state operator corresponding to the holonomy, choose \( f(g, p) = D^{(j)}_{mn}(g) \):

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\hat{A}_{D^{(j)}_{mn}} = \int d\mu(g, p) D^{(j)}_{mn}(g) |g, p\rangle \langle g, p|
\]

The matrix elements of this operator between two spin network states \((g|jmn) = \sqrt{d_j D^{(j)}_{mn}(g)}\) are

\[
\langle j_1 m_1 n_1 | \hat{A}_{D^{(j)}_{mn}} | j_2 m_2 n_2 \rangle = C_t(j_1, j_2, j) \sqrt{\frac{d_{j_1}}{d_{j_2}}} C_{jj_1j_2} C_{nn_1n_2}
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$$= C_t(j_1, j_2, j) \langle j_1 m_1 n_1 | \hat{D}^{(j)}_{mn} | j_2 m_2 n_2 \rangle$$

where

$$C_t(j_1, j_2, j) \to 1 \quad \text{when} \quad t \to 0$$
The choice $f(g, p) = p^i$ ($h = g e^{p \cdot \sigma / 2}$) gives the coherent state operator of the left-invariant vector field:

$$\hat{A}_{p^i} = -i \int d\mu(g, p) \frac{p^i}{t} |g, p\rangle \langle g, p|$$

The scaling by $1/t$ gives a good large $j$ limit of the operator.
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The action of the operator on a spin network state is given by

\[
\hat{A}_{p^i} |jmn\rangle = E_t(j) \hat{L}^i |jmn\rangle
\]

where \( \hat{L}^i \) is the standard left-invariant vector field \( \left( L^i \psi(g) = \frac{d}{d\epsilon} \psi(ge^{-i\epsilon \sigma^i/2}) \bigg|_{\epsilon=0} \right) \).
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and

\[
E_t(j) \simeq 1 + \mathcal{O} \left( \frac{1}{j} \right) \quad \text{when} \quad j \gg 1
\]
To get the right-invariant vector field, use the variable $p' \ (h = e^{p' \cdot \sigma / 2g})$:

$$\hat{A}_{(p')}^i = i \int d\mu(g, p') \frac{(p')^i}{t} |g, p'\rangle \langle g, p'|$$
Right-invariant vector field

To get the right-invariant vector field, use the variable $p'$ ($h = e^{p' \cdot \sigma / 2g}$) :

$$\hat{A}(p')_i = i \int d\mu(g, p') \frac{(p')^i}{t} |g, p'\rangle \langle g, p'|$$

The action on a spin network is again

$$\hat{A}(p')_i |jmn\rangle = E_t(j) \hat{R}^i |jmn\rangle$$
Algebra of holonomies and fluxes

To study the structure of the quantization of holonomies and fluxes by coherent states, we compare $[\hat{A}_{D_{mn}}^{(i)}, \hat{A}_{p^i}]$ with the commutator of the corresponding canonical operators,

$$[\hat{D}_{mn}^{(j)}, \hat{L}^i] = \frac{i}{2} D_{\mu n}^{(j)}(\sigma^i) \hat{D}_{m \mu}^{(j)}$$
To study the structure of the quantization of holonomies and fluxes by coherent states, we compare \([\hat{A}_{D_{mn}}, \hat{A}_{p^i}]\) with the commutator of the corresponding canonical operators,

\[
[\hat{D}^{(j)}_{mn}, \hat{L}^i] = \frac{i}{2} D^{(j)}_{\mu n}(\sigma^i) \hat{D}^{(j)}_{m\mu}
\]

The result is

\[
\langle j_1 m_1 n_1 | [\hat{A}_{D_{mn}}, \hat{A}_{p^i}] | j_2 m_2 n_2 \rangle = \langle j_1 m_1 n_1 | \frac{i}{2} D^{(j)}_{\mu n}(\sigma^i) \hat{A}^{(j)}_{D_{mn}} | j_2 m_2 n_2 \rangle
\]

\[
+ \frac{i}{2} (E_t(j_2) - 1) D^{(j_2)}_{n_2 \mu}(\sigma^i) \langle j_1 m_1 n_1 | \hat{A}^{(j)}_{D_{mn}} | j_2 m_2 \mu \rangle - \frac{i}{2} (E_t(j_1) - 1) D^{(j_1)}_{\mu n_1}(\sigma^i) \langle j_1 m_1 \mu | \hat{A}^{(j)}_{D_{mn}} | j_2 m_2 n_2 \rangle
\]

where \((E_t(j) - 1) \to 0\) in the limit \(j \to \infty\).
Area operator

Classically, \( \text{Area}(S) = \sqrt{E_i(S)E_i(S)} \). Accordingly, the coherent state area operator is

\[
\hat{A}|p| = \int d\mu(g,p) \frac{|p|}{t} |g,p\rangle \langle g,p| \]
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Spin networks are eigenstates of this operator,

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$$\hat{A}_{|p|} |jmn\rangle = \alpha(j) |jmn\rangle$$

However, the eigenvalue is modified:

$$\alpha(j) = \left( j + \frac{1}{2} + \frac{1}{t(2j + 1)} \right) \text{erf} \left[ \sqrt{t} \left( j + \frac{1}{2} \right) \right] + \frac{1}{\sqrt{\pi t}} e^{-t(j+1/2)^2}$$
Area operator: Eigenvalues

For large $j$, the spectrum approaches that of the canonical area operator. The lowest eigenvalue $\alpha(0)$ is positive, even in the $t \to \infty$ limit.
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Angle operator

For a pair of links belonging to a spin network node, we define the angle operator

$$\hat{A}_{\theta(p_1, p_2)} = \int d\mu(g_1, p_1; g_2, p_2) \cos^{-1}\left(\frac{p_1 \cdot p_2}{|p_1||p_2|}\right) \langle g_1 p_1; g_2 p_2 \mid g_1 p_1; g_2 p_2 \rangle$$
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It is diagonal on (suitably coupled) spin networks:

$$\hat{A}_{\theta(p_1, p_2)} |j_1 \ y \ j_2 \ y \ k \rangle = \theta(j_1, j_2, k) |j_1 \ y \ j_2 \ y \ k \rangle$$
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It is diagonal on (suitably coupled) spin networks:

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$$\theta(j_1, j_2, k) = \int d\nu(p_1, p_2) \cos^{-1}\left(\frac{p_1 \cdot p_2}{|p_1||p_2|}\right) \begin{pmatrix} j_1 & j_2 & k \\ m_1 & m_2 & \mu \end{pmatrix} D^{(j_1)}_{m_1 n_1}(e^{p_1 \cdot \sigma}) D^{(j_2)}_{m_2 n_2}(e^{p_2 \cdot \sigma}) \begin{pmatrix} j_1 & j_2 & k \\ n_1 & n_2 & \mu \end{pmatrix}$$
Angle operator: Eigenvalues

\[ \theta_{\text{canonical}}(j, j, j) = \frac{2\pi}{3} \]
Angle operator: Eigenvalues

\[ \theta_{\text{canonical}}(j, j, j) = \frac{2\pi}{3} \]
The operator

\[ \hat{A}_{V_n} = \int d\mu(g_1, p_1) \cdots d\mu(g_n, p_n) V_n \left( \frac{p_1}{t}, \ldots, \frac{p_n}{t} \right) |g_1 p_1; \ldots; g_n p_n\rangle \langle g_1 p_1; \ldots; g_n p_n| \]

describes the volume associated to a node of a spin network.
Volume operator

The operator

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describes the volume associated to a node of a spin network.

$V_n(p_1, \ldots, p_N)$ is the volume spanned by the vectors $\{p_i\}$

- $V_3(p_1, p_2, p_3) = \frac{\sqrt{2}}{3} \sqrt{|p_1 \cdot (p_2 \times p_3)|}$
- $V_4(p_1, p_2, p_3, p_4)$: See [Haggard, arXiv:1211.7311]
The operator

\[ \hat{A}_V = \int d\mu(g_1, p_1) \cdots d\mu(g_n, p_n) V_n \left( \frac{p_1}{t}, \ldots, \frac{p_n}{t} \right) |g_1 p_1; \ldots; g_n p_n \rangle \langle g_1 p_1; \ldots; g_n p_n | \]

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- \[ V_4(p_1, p_2, p_3, p_4): \text{See [Haggard, arXiv:1211.7311]} \]

A similar volume operator is constructed in [Bianchi, Donà, Speziale, arXiv:1009.3402]
Volume operator: Action on spin networks

\[ \hat{A}_{V_3} \begin{array}{c} j_1 \\ j_3 \end{array} \begin{array}{c} j_2 \\ j_3 \end{array} \rangle = \nu(j_1, j_2, j_3) \begin{array}{c} j_1 \\ j_3 \end{array} \begin{array}{c} j_2 \\ j_3 \end{array} \]
Volume operator: Action on spin networks

\[ \hat{A}_{V_3} | j_1 \quad j_3 \quad j_2 \rangle = v(j_1, j_2, j_3) | j_1 \quad j_3 \quad j_2 \rangle \]

\[ v(j_1, j_2, j_3) = e^{-t(j_1(j_1+1)+j_2(j_2+1)+j_3(j_3+1))} \int d\nu(p_1, p_2, p_3) \left| \frac{p_1}{t} \cdot \left( \frac{p_2}{t} \times \frac{p_3}{t} \right) \right| \]

\[ \times \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} D_{m_1 n_1}^{(j_1)}(e^{p_1 \cdot \sigma}) D_{m_2 n_2}^{(j_2)}(e^{p_2 \cdot \sigma}) D_{m_3 n_3}^{(j_3)}(e^{p_3 \cdot \sigma}) \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix} \]
Volume operator: Action on spin networks

\[ \hat{A}_{V_3} | j_1 j_2 j_3 \rangle = v(j_1, j_2, j_3) | j_1 j_2 j_3 \rangle \]

\[ v(j_1, j_2, j_3) = e^{-t(j_1(j_1+1)+j_2(j_2+1)+j_3(j_3+1))} \int d\nu(p_1, p_2, p_3) \sqrt{|p_1/t \cdot (p_2/t \times p_3/t)|} \times \begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} D_{m_1 n_1}^{(j_1)} (e^{p_1 \cdot \sigma}) D_{m_2 n_2}^{(j_2)} (e^{p_2 \cdot \sigma}) D_{m_3 n_3}^{(j_3)} (e^{p_3 \cdot \sigma}) \begin{pmatrix} j_1 & j_2 & j_3 \\ n_1 & n_2 & n_3 \end{pmatrix} \]

\[ \hat{A}_{V_4} | j_1 j_2 j_3 j_4 \rangle = \int d\nu(p_1, p_2, p_3, p_4) V(p_1, p_2, p_3, p_4) F_{kl}(p_1, p_2, p_3, p_4) \]

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- Which coherent states are ideal for this construction?
- How to improve the geometric operators?
- What is the proper way to treat gauge invariance?
Thank You!