Buoyant convective transport of nanofluids in a non-uniformly heated annulus

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Abstract. This paper reports the influence of non-uniform thermal conditions on buoyancy-driven convection of water based nanofluids in a cylindrical annulus. Annular geometry is formed by two upright co-axial cylinders. In this analysis, two different non-uniform temperature profiles are applied at bottom boundary, while the side boundaries are kept at lower temperature and top boundary is taken as thermally insulated. For the first case, the bottom boundary is sinusoidally heated, while linear thermal profile is applied in the second case. The annular gap is filled with water based nanofluids with copper nanoparticle. Using ADI based finite difference technique, the model equations are solved for vast range of parametric values. Numerical simulation results reveal the bi-cellular flow pattern for both non-uniform thermal conditions at all range of Rayleigh numbers. Further, the heat transport rates are highly sensitive to non-uniform conditions supplied at the bottom wall. The results of this analysis could be utilized for applications involving non-uniform thermal conditions in an annular geometry.

1. Introduction
Buoyant convection in an upright annulus with the differently heated cylindrical side walls and thermally insulated horizontal walls is a widely used standard physical configuration representing many industrial and technological applications. As a result, several theoretical and experimental investigations of natural convection in this geometry has been carried out. The pioneering attempt on numerical study of buoyant convective flows in a vertical annular geometry is by Davis and Thomas [1]. Later, Kumar and Kalam [2] made a detailed convective heat transport analysis in this geometry and discussed the discrepancy exists in the earlier study [1]. Venkatachalappa et al. [3] discussed the influence of rotation of cylindrical walls of the annulus on convective flows and the associated thermal transport processes. Sankar and co-workers [4, 5, 6, 7, 8] investigated the impacts of size and position of single and double thermal source positioned in the inner wall of the annulus and found the optimum size and location to enhance the thermal transport. In an open-ended double-passage porous annuli, Girish et al. [9] analyzed the impacts of a baffle on developing convective flow and thermal characteristics. Sankar et al. [10] made an attempt to explore the baffle effects on buoyant convection in an annulus and found the length and location of baffle make prominent changes in flow pattern and thermal transport. The impacts of partial heating and cooling on natural convection in a square geometry is estimated by Sankar et al. [11].

Nanofluids is a new type of thermal transport fluids, designed through mixing nanometer-sized solid particles in conventional fluids like water and ethylene glycol. Study of nanofluids becoming a fast upcoming multidisciplinary arena for nanoscientists, nanotechnologists, and
thermal engineers. As a result, these traditional cooling methods may not provide the solution to the cooling requirement of electronic industries. Since Choi [12] innovated the novel concept of nanofluids, the research has attracted remarkable interest from many investigators. Putra et al. [13] made a detailed analysis of different nanofluids by considering different models of physical properties of fluid as well as nano-particles. Khanafer et al. [14] demonstrated the enhancement of convection by utilizing nanofluids in a rectangular geometry. Some of prominent studies on thermal enhancement using nanofluids in finite geometries are due to Basak and Chamkha [15] and Sabour et al. [16]. Buoyant convection of different nanofluids in upright annular space has also been investigated for uniform as well as discrete thermal conditions [17, 18, 19]. Das et al. [20] and Haddad et al. [21] presented detailed review of thermal enhancement due to nanofluids in various geometries under different thermal conditions. Oztop [22] predicted the impacts of discrete heating on buoyant convection of nanofluids in square geometry. Abu-Nada [23] performed numerical simulations to account for variation in thermal conductivity and viscosity in convection of nanofluids. For many thermal design of equipments, the geometrical shape need not be regular. In such situations, a prior knowledge of flow and thermal properties in such enclosures provides vital information to the design engineers [24, 25]. From the above detailed literature survey, it is found that the effect of axially varying thermal profiles on convective nanofluids flow in an annular geometry has not been studied in the existing literature. Therefore, in the present analysis, we made an attempt to investigate two different types of axially varying thermal conditions on convection of nanofluids in an upright annular geometry.

2. Mathematical formulation

The present study focuses on buoyant convection of Cu-water nanofluid filled in an annular space with adiabatic top wall, isothermally cold side boundaries and spacial-varying temperature at lower wall as shown in Fig. 1. We considered two different types of thermal conditions on the lower boundary; sinusoidal thermal profile (Case-I) and linear temperature (Case-II). We presumed that the nanofluid to be incompressible, the flow to be laminar and axisymmetric. Also, base fluid (water) and Cu-water nanofluid are taken to be in thermal equilibrium and the thermophysical properties of the nanofluids remain invariant except density change which is taken through the Boussinesq approximation through body force term of momentum equation. Under these assumptions, the governing equations can be written in vorticity-stream function form as follows:

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial R} + W \frac{\partial T}{\partial Z} = \frac{\alpha_{nf}}{\alpha_f} \nabla^2 T, \quad (1)
\]

\[
\frac{\partial \zeta}{\partial t} + U \frac{\partial \zeta}{\partial R} + W \frac{\partial \zeta}{\partial Z} - \frac{U \zeta}{R} = \frac{\mu_{nf}}{\rho_{nf} \alpha_f} \left[ \nabla^2 \zeta - \frac{\zeta}{R^2} \right] - \frac{(\rho \beta)_{nf}}{\rho_{nf} \beta_f} Ra Pr \frac{\partial T}{\partial R}, \quad (2)
\]

\[
\zeta = \frac{1}{R} \left[ \frac{\partial^2 \psi}{\partial R^2} - \frac{1}{R} \frac{\partial \psi}{\partial R} + \frac{\partial^2 \psi}{\partial Z^2} \right], \quad (3)
\]

\[
U = \frac{1}{R} \frac{\partial \psi}{\partial Z}, \quad W = -\frac{1}{R} \frac{\partial \psi}{\partial R}. \quad (4)
\]

Here, \(\nabla^2 = \frac{\partial^2}{\partial R^2} + \frac{1}{R} \frac{\partial}{\partial R} + \frac{\partial^2}{\partial Z^2}\)

The following non-dimensional variables are adopted in this analysis:

\[
R = \frac{(r, z)}{D}, \quad U = \frac{u D}{\alpha_f}, \quad W = \frac{w D}{\alpha_f}, \quad t = \frac{t^*}{(D^2/\alpha_f)}, \quad T = \frac{(\theta - \theta_c)}{(\theta_h - \theta_c)}, \quad P = \frac{p D^2}{\rho_{nf} \alpha_f},
\]

\[
\zeta = \frac{\zeta D^2}{\alpha_f}, \quad \psi = \frac{\psi D}{\alpha_f}, \quad Ra = \frac{g \beta \Delta T D^3}{\nu \alpha}, \quad Pr = \frac{\nu}{\alpha}.
\]
Figure 1. Physical configuration, coordinate system and boundary conditions

Dimensionless conditions along the boundary are given by:

At $t = 0$, 

$$ U = W = T = 0, \quad \psi = \zeta = 0 $$

At $t > 0$, 

$$ T = 0 \quad \text{at inner and outer cylinders} $$

$$ \frac{\partial T}{\partial Z} = 0 \quad \text{at upper wall} $$

$$ T = \begin{cases} 
\sin(\pi R), & \text{case(I) at the lower wall} \\
1 - R, & \text{case(II) at the lower wall} 
\end{cases} $$

The nanofluid properties are estimated using the below relationships combining the base-fluid and nanoparticles properties:

$$ \rho_{nf} = (1 - \phi)\rho_f + \phi\rho_s \quad (5) $$

$$ \mu_{nf} = \frac{\mu_f}{(1 - \phi)^{2.5}} \quad (6) $$

$$ (\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_s \quad (7) $$

$$ k_{nf} = \frac{k_s - 2k_f - 2\phi(k_f - k_s)}{(k_s - 2k_f) + \phi(k_f - k_s)} \quad (8) $$

$$ \alpha_{nf} = \frac{k_{nf}}{(\rho C_p)_{nf}} \quad (9) $$

$$ (\rho C_p)_{nf} = (1 - \phi)(\rho C_p)_f + \phi(\rho C_p)_s \quad (10) $$

In the above, the subscripts $f$ and $s$ respectively denote the fluid and nanoparticles. The thermophysical characteristics of water (base fluid) and nanoparticle are provided in Table 1.
The overall thermal transport rates from bottom \((\overline{Nu_B})\) and side \((\overline{Nu_S})\) walls are estimated from the following global Nusselt numbers

\[
\overline{Nu_B} = \int_0^1 Nu_BdR \quad \text{and} \quad \overline{Nu_S} = \int_0^1 Nu_SdZ,
\]

where \(Nu_B = \pm \frac{k_{nf}}{k_f} \frac{\partial T}{\partial Z}\) and \(Nu_S = \pm \frac{k_{nf}}{k_f} \frac{\partial T}{\partial R}\).

3. Solution procedure and validation

An implicit time-splitting method (ADI and SLOR methods) is applied to convert the model equations to a linear system of algebraic equations. The reduced tri-diagonal system of equations is inverted through the efficient Thomas algorithm. All spacial derivatives are approximated using suitable second-order finite difference approximations. The details of method can be found in our earlier studies \([7, 8, 9, 10, 11]\) and hence not repeated here for brevity. Present numerical simulations are verified with the predictions of Basak et al. \([15]\) in Fig. 2 and found fairly good agreement.

![Comparison of isotherms (top) and streamlines (bottom) of pure water (dotted line) and Cu-water nanofluid (solid line) with \(\phi = 0.1\) and \(Ra = 10^5\) for sinusoidal heating case. Present study (left) and Basak et al. \([15]\) (right).]
Figure 3. Effect of nanoparticle volume fraction ($\phi = 0.1$) and $Ra$ on streamlines and isotherms for case-I at $Ra = 10^4$ (top), $Ra = 10^5$ (center) and $Ra = 10^6$ (bottom). Solid line is for water and dotted line is for nanofluid.

are varied in the ranges of $10^3 \leq Ra \leq 10^6$, $\phi = 0.0 - 0.2$ and the radius ratio is fixed at $\lambda = 2$. Figure 3 displays the effect of Rayleigh number on streamlines and isothermal contours at a constant magnitude of $\phi = 0.1$ for sinusoidal heating of bottom wall. For all values of $Ra$, bi-cellular flow is generated in the annulus and isotherms reveal the nature of sinusoidal thermal variation. The size of eddy near inner wall reduces while that of outer wall eddy increases with Rayleigh number. The variation of isothermal structure with $Ra$ reveals the dominance of convective strength. The effect of linear thermal variation of bottom wall on flow and thermal contours is depicted in Fig. 4 for three magnitudes of $Ra$. In this type of non-uniform heating too, the streamline and isothermal pattern reveals the bi-cellular structure similar to sinusoidal heating of bottom wall.
Figure 4. Effect of nanoparticle volume fraction ($\phi = 0.1$) and $Ra$ on streamlines and isotherms for case-II at $Ra = 10^4$ (top), $Ra = 10^5$ (center) and $Ra = 10^6$ (bottom). Solid line is for water and dotted line is for nanofluid.

The global thermal transport rates from the bottom and side walls are shown in Fig. 5 for different Rayleigh numbers and fixed value of $\phi$. Heat dissipation from the bottom wall steadily increases with Rayleigh number, however, the $Nu_S$ increases up to $Ra = 10^5$ and decreases at $10^5$. Interestingly, among the two different non-uniform heatings considered along the bottom wall, linear heating of bottom wall produces higher thermal transport rates as compared to sinusoidal heating.

Figure 6 presents the global thermal transport rates from the lower and vertical walls for different nanoparticle volume fractions for a fixed value of Rayleigh number. It is detected
Figure 5. Effect of $Ra$ on the global Nusselt number at bottom wall (top) and side wall (bottom) for $\phi = 0.1$

that total heat transport rates can be enhanced by increasing percentage of nanoparticle. Also, sinusoidal thermal profile along the lower boundary produces less heat transport as compared to linear heating.

5. Conclusions
In this paper, buoyant convective heat transport of water-Cu nanofluids in an annular region bounded by two upright, co-axial cylinders is numerically investigated. From the extensive numerical simulations, it has been detected that the bi-cellular convective flow is observed in
Figure 6. Effect of $\phi$ on the global Nusselt number at side wall (top) and bottom wall (bottom) for $Ra = 10^5$

the annulus for both non-uniform heating. Further, the overall heat transport rate could be improved either by augmenting the nanoparticle volume fraction or imposing linear heating along the bottom wall.

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