1. INTRODUCTION

The motions of stars in the immediate vicinity of Sgr A* have been tracked since 1992 at the NTT/VLT and since 1995 at the Keck telescope (Eckart & Genzel 1996; Ghez et al. 1998). With the detection of accelerations (Ghez et al. 2000) and the determination of the first orbit (Schödel et al. 2002) these measurements provided firm evidence for the existence of a massive black hole (MBH) at the center of the Milky Way, coincident with the radio-source Sgr A*. The stars are used as test particles for the gravitational potential in which they move. Particularly important is the bright (m_K ≈ 14) star S2 (S0-2 in the Keck nomenclature) orbiting the MBH in 15.9 years on an ellipse with an apparent diameter of about 0.2". Together with radial velocity data, the astrometric data allow for a geometric determination of R_0, the distance to the Galactic Center (Salim & Gould 1999; Eisenhauer et al. 2003). The number of orbits has increased to more than 20 since then (Ghez et al. 2005; Eisenhauer et al. 2005; Gillessen et al. 2009), and in particular the full S2 orbit has recently been used for improved determinations of the mass and of R_0 (Ghez et al. 2008; Gillessen et al. 2009).

These two astrometric data sets are independent since they were obtained at different telescopes and were analyzed by different teams using different tools. On the other hand, the radial velocity data attached to the astrometric data sets largely overlap, mainly because this is technically straightforward. The radial velocities refer to the local standard of rest and hence the inclusion of other data into a given data set needs no special care. The situation is different for astrometric data, for which only an approximate realization of an absolute reference frame is currently possible (Reid et al. 2007). This means that the exact definition of coordinates is a matter of the respective data analysis, and simply merging two lists of astrometric positions will fail (see Figure 1).

The two groups (Ghez et al. 2008; Gillessen et al. 2009) come to very similar conclusions for the mass of Sgr A* and R_0. From Very Large Telescope (VLT) data Gillessen et al. (2009) derived
one should obtain a data set which combines the respective advantages.3

2. DATABASE

The data used here (Table 2) are shown in Figures 1 and 2. We omit a detailed discussion of the data, since it is well presented in the original works and would be beyond the scope of this Letter.

The astrometric data from the NTT/VLT we are using are identical to the data from Gillessen et al. (2009) up to 2008 August. Since then, one more epoch has been observed in 2008 and 11 more epochs in 2009. We treated these data in precisely the same way as described in Gillessen et al. (2009), where the somewhat cumbersome derivation of astrometric positions from the imaging data is described in detail. As done previously, we assigned lower weights to the VLT data from 2002 to account for the fact that the star might have been confused in that year. In total, the NTT/VLT data set contains 70 epochs from 1992 March to 2009 August. Furthermore, we added one more epoch (2009 March) to the measurements of reference stars by which the coordinate system is tied to the International Celestial Reference Frame.

In their Table 3 Ghez et al. (2008) presented the measured positions for all Keck epochs.4 We use the given 26 epochs, ranging between 1995 and 2008.

3. ORBITAL FITS

Figure 1 shows that the two astrometric data sets cannot be put together in a simple fashion. To account for the difference we make the following, simple assumption: the two data sets only differ in their definition of the coordinate system, namely the position of the origin and the zero velocity.5 This yields four parameters ($\Delta x, \Delta y, \Delta v_x, \Delta v_y$) by which the difference can

Table 1

| Epoch     | R.A. (mas) | Decl. (mas) | Source |
|-----------|------------|-------------|--------|
| 1992.224  | -6.4 ± 4.6 | 172.0 ± 4.7 | NTT    |
| 1994.314  | -28.5 ± 4.8| 179.0 ± 3.4 | NTT    |
| ...       | ...        | ...         | ...    |
| 2002.250  | -3.1 ± 4.0 | -6.6 ± 4.0  | VLT    |
| 2002.355  | 6.6 ± 2.7  | -7.6 ± 2.7  | VLT    |
| 1995.439  | -42.6 ± 1.0| 164.1 ± 1.0 | Keck   |
| 1996.485  | -53.0 ± 9.5| 155.4 ± 9.5 | Keck   |
| ...       | ...        | ...         | ...    |

Table 1: Orbital Data for S2 Used in This Work4

| Epoch     | $v_\text{LSR}$ (km s$^{-1}$) | Source |
|-----------|-----------------------------|--------|
| 2000.487  | 1199 ± 100                  | Keck   |
| 2002.418  | -495 ± 40                   | Keck   |
| 2003.353  | -1512 ± 49                  | VLT    |
| 2003.446  | -1428 ± 63                  | VLT    |
| ...       | ...                         | ...    |

Note.4 The astrometric data from the NTT and VLT are valid for our current definition of the reference system.

For the radial velocities, we use the combined database from Gillessen et al. (2009) and Ghez et al. (2008).5 Finally, we add one more VLT epoch from 2009, which brings the total number of radial velocity points to 36.

5 In Table 4 of the latter paper, two radial velocity measurements from the year 2002 are missing. They are shown in the figures and clearly have been used in the fit. Otherwise the numbers quoted in the text for the degrees of freedom would not match. The values can be found in Ghez et al. (2003) though.

6 A difference in rotation is very unlikely, since the images can be oriented very well by the means of stars across the field of view.
be described. Given that both data sets contain many more data points, it is feasible to solve for these four parameters during the fitting procedure. These four parameters come hence in addition to the six orbital elements and the parameters describing the gravitational potential ($M_{\text{MBH}}$, $R_0$, position, zero velocity, $v_z$).

We have implemented the additional four parameters in the code of Gillessen et al. (2009). For (most of) the orbital fits, we used the same assumptions as for the preferred fit in Gillessen et al. (2009), namely we imposed priors on the NTT/VLT coordinate system definition which reflect our best estimate of how well an absolute coordinate system was established:

\[
\begin{align*}
\mathcal{P}(x) &= 0 \pm 1.0 \text{ mas } \\
\mathcal{P}(y) &= 0 \pm 2.5 \text{ mas } \\
\mathcal{P}(v_x) &= 0 \pm 0.1 \text{ mas yr}^{-1} \\
\mathcal{P}(v_y) &= 0 \pm 0.1 \text{ mas yr}^{-1} \\
\mathcal{P}(v_z) &= 0 \pm 5 \text{ km s}^{-1}.
\end{align*}
\]

Using that the combined data set can be described very well by a Keplerian orbit (Figure 3). For the 211 degrees of freedom (96 astrometric epochs which count twice and 36 radial velocity points minus 17 parameters) we obtained $\chi^2 = 271$ for the fit shown in Figure 3, corresponding to a reduced $\chi^2$ of 1.28.

The fit result for the four additional parameters is

\[
\begin{align*}
\Delta x &= -3.7 \pm 0.6 \text{ mas} \\
\Delta y &= -4.1 \pm 0.6 \text{ mas} \\
\Delta v_x &= -0.68 \pm 0.11 \text{ mas yr}^{-1} \\
\Delta v_y &= 0.26 \pm 0.10 \text{ mas yr}^{-1}.
\end{align*}
\]

We conclude that these parameters are well-determined from the fit, which validates our approach for combining the two data sets. The positional difference ($\Delta x$, $\Delta y$) is somewhat larger than the actual mismatch between the two coordinate systems. This happens simply since ($\Delta x$, $\Delta y$) refer to the epoch 2005 May, the zero point in time of the NTT/VLT coordinate system, while the Keck data use 2004 July for that. Nevertheless, it is worth checking how well the numbers in Equations (4) compare to the expected accuracies of the coordinate systems. The uncertainty of the NTT/VLT coordinate system (Equations (3)) is smaller than the values in Equations (4). Ghez et al. (2003) concluded that their coordinate system is accurate to ($\Delta x$, $\Delta y$) = \pm (5.7, 5.7) mas in position and to ($\Delta v_x$, $\Delta v_y$) = \pm (0.6, 0.9) mas yr\(^{-1}\). This is consistent with Equations (4). The larger uncertainty of the Keck coordinate system is due to the shorter timeline used for determining the motions of the reference stars (roughly 2 years compared to 6 years for the NTT/VLT data).

The best-fitting orbital elements are

\[
\begin{align*}
a &= 0'.1246 \pm 0.0019, \\
\Omega &= 226^\circ 53 \pm 0.72, \\
e &= 0.8831 \pm 0.0034, \\
\omega &= 64^\circ 98 \pm 0.81, \\
i &= 134^\circ 87 \pm 0.78, \\
t_p &= 2002.3293 \pm 0.0066,
\end{align*}
\]

where the errors are rescaled such that the reduced $\chi^2$ is 1. The corresponding parameters describing the gravitational potential are presented in the first row of Table 2. We continued by making different choices for the priors, for the 2002 VLT data and for the model of the gravitational potential. These fits are also listed in Table 2, and the respective choices are indicated in each row.

Comparing the results with Table 4 of Gillessen et al. (2009) one notes that generally the statistical fit errors for $M_{\text{MBH}}$ and $R_0$ are 20%–30% smaller in the combined orbit fit. Also the errors on the position and two-dimensional velocity of the MBH have decreased moderately. Generally, the fitted position of the MBH has moved by 2.5 mas east and 1.5 mas north. This is (marginally) consistent with the accuracy by which the radio-source Sgr A* can be identified in the infrared images.

The comparison also shows that for the combined data set the various fits differ less from each other than for the NTT/VLT only data set. For example, including or not including the 2002 VLT data (comparing the first and the second row) changed $R_0$ in Gillessen et al. (2009) by 0.95 kpc, while in the combined data set the same change is only 0.62 kpc. Also the sixth row (a fit excluding the 2002 VLT data and neglecting priors) shows that the results are much closer in the combined set. Gillessen et al. (2009) reported $R_0 = 6.63 \pm 0.91$ kpc, while the combined data yield $R_0 = 7.73 \pm 0.57$ kpc. The range of values for $R_0$ is in the combined data set 1 kpc only, while for Gillessen et al. (2009) it exceeded 2 kpc.

Next, we turned to post-Newtonian orbit models. A relativistic orbit fit is shown in row 9 of Table 2, yielding a slight increase in $R_0$ compared to the Keplerian fit. This bias was already noted by Zucker et al. (2006). Assuming the relativistic $\beta^2$ effects for the radial velocity (Zucker et al. 2006) we then checked if the combined data allow us to constrain the relativistic periastron shift. For this purpose, we introduced a parameter $f$, which is 0 for a Keplerian model and 1 for the correct relativistic model. The fit yielded a very large error $\Delta f \approx 10$, and we conclude that even the combined data set is not yet sensitive to the relativistic precession induced by the Schwarzschild metric.

We also repeated the tests of Gillessen et al. (2009) for an extended mass component, using three different power-law slopes for the density profile: $\rho(r) \propto r^{-\alpha}$, $\alpha = 1.4$, 7/4, 2.1. The normalization of the density profile was a parameter, which we determined by the fit. We express the results for this parameter in Table 2 by giving $\eta$, the mass between peri and apocenter of the S2 orbit divided by the fitted mass of the MBH. The results in rows 9–12 are not directly comparable to the

![Figure 3. Result of the combined orbit fit for the star S2. Blue: NTT/VLT measurements. Red: Keck measurements. The black line shows the Keplerian fit (row 1 in Table 2).](image)
numbers in Gillessen et al. (2009) who imposed an additional prior on $R_0$ (obtained from the other S stars) for the fits with an extended mass component. The results from the combined data yield nevertheless very similar upper limits. At most, very few percent of the central mass can make up an extended mass component inside the S2 orbit.

Overall, the fits presented here show that the chosen scheme to combine the two independent data sets is valid and that consequently (more) smaller statistical errors are obtained. Still, the approach is not yet sufficient to detect post-Newtonian effects in the S2 orbit.

4. DISCUSSION

For more than 10 years, two groups have assembled independently astrometric data sets for the stars in the central arcsecond of the Milky Way. The data sets are truly independent. They were obtained at different telescopes with different instruments. The analysis chains used different tools (deconvolution in the NTT/VLT case, point-spread function fitting for the Keck data), and the definition of astrometric coordinates is implemented in different ways. While it was reassuring that Ghez et al. (2008) and Gillessen et al. (2009) concluded very similarly for $M_{\text{MBH}}$ and $R_0$ it was not clear that the agreement actually also holds for the underlying data. We were able to show that indeed the most simple assumption—namely that the two data sets only differ in the coordinate system definition—is sufficient to perfectly map the two data sets on top of each other.

We presented for the first time a combined orbit fit, which however only moderately improves the accuracy by which $M_{\text{MBH}}$ and $R_0$ can be derived from the S2 data:

$$R_0 = 8.34 \pm 0.27 |_{\text{stat}} \pm 0.5 |_{\text{sys}} \text{kpc}$$

$$M_{\text{MBH}} = 4.40 \pm 0.27 |_{\text{stat}} \pm 0.5 |_{\text{sys}} \times 10^6 M_{\odot}.$$  

(6)

For the sake of completeness, we cite also the updated numbers for a fit using not just the combined S2 data, but in addition S1, S8, S12, S13, and S14, the same stars Gillessen et al. (2009) had used:

$$R_0 = 8.28 \pm 0.15 |_{\text{stat}} \pm 0.29 |_{\text{sys}} \text{kpc}$$

$$M_{\text{MBH}} = 4.30 \pm 0.20 |_{\text{stat}} \pm 0.30 |_{\text{sys}} \times 10^6 M_{\odot}. \quad (7)$$

Besides the expected improvement in $R_0 |_{\text{stat}}$ from 0.17 kpc to 0.15 kpc, also the systematic errors have decreased mildly compared to the previous work due to the smaller influence of the S2 2002 data.

We conclude that the combination does not help much in overcoming current limitations. The true value of the two data sets actually is their independence, allowing for cross-checks and lending credibly to the results.

Further substantial improvements in measuring the gravitational potential from Sgr A* by means of stellar orbits probably will come from improved astrometry on existing data, from longer timelines with the existing instruments and finally from advances in instrumentation.

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