Relativistic Ring-Diagram Nuclear Matter Calculations

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Abstract

A relativistic extension of the particle-particle hole-hole ring-diagram many-body formalism is developed by using the Dirac equation for single-particle motion in the medium. Applying this new formalism, calculations are performed for nuclear matter. The results show that the saturation density is improved and the equation of state becomes softer as compared to corresponding Dirac-Brueckner-Hartree-Fock calculations. Using the Bonn A potential, nuclear matter is predicted...
to saturate at an energy per nucleon of –15.30 MeV and a density equivalent to a Fermi momentum of 1.38 fm$^{-1}$, in excellent agreement with empirical information. The compression modulus is 152 MeV at the saturation point.

1 Introduction

The microscopic explanation of the properties of nuclear matter is one of the most fundamental traditional issues in theoretical nuclear physics. Being an infinite and homogeneous nucleon assembly, nuclear matter not only serves as an ideal system to test many-body theories but also provides constraints on the off-shell part of the nucleon-nucleon (NN) interaction which can not be determined by studying two-nucleon scattering data and the properties of the deuteron.

For many years, conventional Brueckner theory and variational methods have been widely used in the study of nuclear matter. However, there appears to be an intrinsic problem associated with these approaches. All calculations using these methods, no matter which potential is applied, fail to reproduce the empirical saturation properties of nuclear matter (energy per nucleon
$E/A \approx -16$ MeV and saturation density $\rho_0 \approx 0.17 \text{ fm}^{-3}$ [1, 2, 3]. To improve conventional nuclear matter predictions, two types of additional effects are needed: a general increase of the binding energy and a strongly density-dependent repulsive contribution.

In the early 1980’s, the Brooklyn group [4] motivated and worked out a relativistic extension of Brueckner theory, which was further developed by Horowitz and Serot [5], by Brockmann and Machleidt [6, 7], and by ter Haar and Malfliet [8]. The basic idea of these so-called Dirac-Brueckner approaches is the use of the Dirac equation for the single particle motion in nuclear matter. Due to the density dependence of the effective nucleon mass in the medium, the Dirac-spinor wave functions representing the nucleons become density dependent, and so does the nucleon-nucleon (NN) interaction. As a consequence of this, the attractive $\sigma$-boson exchange is suppressed causing a repulsive effect, which strongly increases with density. This effect allows to explain the empirical saturation density of nuclear matter. The correct binding energy of nuclear matter is obtained if for the single particle (s.p.) potential a continuous choice is applied and if a weak tensor-force NN interaction is used; both these factors increase the binding energy [3, 7].

So far, the Dirac-Brueckner approach has been applied only in the lowest
order in the reaction matrix, $K$ [Dirac-Brueckner-Hartree-Fock (DBHF)]. For non-relativistic Brueckner theory it is known that contributions beyond the lowest order in $K$ are not necessarily small. Therefore, it is most desirable to also investigate for the Dirac-Brueckner approach contributions of higher orders. It is the purpose of this paper to start such an investigation. As a first class of diagrams beyond past efforts, we will consider the ring-diagram series displayed in Fig. 1. In the non-relativistic many-body frame work, these diagrams have been investigated by the Stony Brook group [9, 10, 11, 12, 13, 14].

In the Stony Brook ring approach, hole-hole (hh) propagations in intermediate states are taken into account. These long-range correlations are ignored in standard Brueckner theory for nuclear matter. Short-range correlations [particle-particle (pp) propagations] are included in the ring formalism, as they are in Brueckner theory. In the non-relativistic calculations, it is found that the hole-hole propagations provide an effect that is repulsive and strongly density-dependent [15, 16]. Thus, nuclear matter saturation is improved. Moreover, the nuclear matter equation-of-state becomes softer, i. e., the compression modulus is reduced.

As a matter of fact, both the relativistic effects as obtained in DBHF and
the hh ring contributions shift the saturation density towards more realistic values. Yet, the predicted saturation densities are still above the empirical value. In the non-relativistic ring approach, one gets typically $k_F \gtrsim 1.42 \text{ fm}^{-1}$, and in DBHF calculations, $k_F \gtrsim 1.40 \text{ fm}^{-1}$ at the saturation point. Thus, there is still room for improvement. Therefore, it will be interesting to see what result the coherent contributions will yield. In other words, we like to investigate what role the ring-diagram effect will play in a relativistic framework, and whether in this approach the saturation density will be lowered even more than in the DBHF calculations. By doing so, we can at least show partial corrections to the DBHF results from certain diagrams of higher order in the reaction matrix.

In Section 2 we will explain the Dirac approach and its application to the ring formalism. Results are presented and discussed in Section 3. Finally, Section 4 contains a summary and some conclusions.

2 The Relativistic Ring-Diagram Approach

As mentioned, the essential point of the relativistic many-body approach is the use of the Dirac equation to describe the single-particle motion in
the nuclear medium (notation as in the textbook by J.D. Bjorken and S.D. Drell [17])

\[(\not{p} - M - \Sigma)\tilde{u}(p, s) = 0\] (1)

with the relativistic self-energy operator

\[\Sigma = U_s + \gamma_0 U_v\] (2)

where \(U_s\) is an attractive scalar and \(U_v\) the time-like component of a repulsive vector field. These two fields are strongly density-dependent. The momentum dependence is weak and will therefore be neglected.

The solution of Eq.(1) is

\[\tilde{u}(p, s) = \sqrt{\tilde{M}^2 + p^2} \begin{pmatrix} 1 \\ \sigma_p \\ \frac{\sigma_p}{\tilde{E}_p + \tilde{M}} \end{pmatrix} \chi_s\] (3)

with

\[\tilde{M} = M_N + U_s\quad\text{and}\quad \tilde{E}_p = \sqrt{\tilde{M}^2 + p^2},\] (4)

and \(\chi_s\) a Pauli spinor. The normalization is \(\tilde{u}^\dagger \tilde{u} = 1\).

In the approximation of an instant NN interaction, we can use two-time Green functions to define the particle-particle hole-hole propagations in the
relativistic framework. Neglecting negative-energy intermediate states (i.e. anti-nucleon contributions), the free two-particle propagator, in the basis defined by the self-consistent Dirac s.p. wavefunctions, can be written as

$$\tilde{F}_{\alpha_1\beta_1\gamma_1\delta_1}^{n_{phh}}(12,\omega) = \left\{ \frac{n_{1}n_{2}}{\omega - (\epsilon_{1} + \epsilon_{2}) - i0} - \frac{n_{1}n_{2}}{\omega - (\epsilon_{1} + \epsilon_{2}) + i0} \right\}(\Lambda_{1})_{\alpha_{1}\gamma_{1}}(\Lambda_{2})_{\beta_{1}\delta_{1}}$$

where $$\epsilon_{i} = \tilde{E}_{i} + U_{i}$$ is the relativistic s.p. energy, and

$$\Lambda_{i} = \frac{\tilde{p}_{i} + \tilde{M}}{2M} = \frac{\tilde{E}_{i}\gamma_{0} - p_{i}\gamma + \tilde{M}}{2M}$$

is a positive-energy nucleon projection operator.

With this propagator, the series of ppgh ring diagrams shown in Fig. 1 can be summed up similarly to the non-relativistic case \([10, 11]\). Summing up this series of infinite ring diagrams, the ground-state energy shift of nuclear matter is given by

$$\Delta E_{0}^{ppgh} = \frac{-1}{2\pi i} \int_{-\infty}^{\infty} d\omega e^{-i\omega t} Tr_{P}\left\{ \tilde{F}\tilde{K} + \frac{1}{2}(\tilde{F}\tilde{K})^{2} + \frac{1}{3}(\tilde{F}\tilde{K})^{3} + \ldots \right\}$$

where $$\tilde{K}$$ stands for a relativistic model-space Dirac-Brueckner reaction matrix (see below). $$Tr_{P}$$ represents a trace operation within the model-space $$(P)$$, i.e a sum over all states with $$|p| \leq k_{M}$$. Here $$k_{M}$$ is the momentum space boundary of the model-space. In this study, we use for $$k_{M}$$ the same value as in our previous work\([10, 11]\), namely $$k_{M} = 3.2 \text{ fm}^{-1}.$$
To calculate the relativistic reaction matrix $\tilde{K}$, we employ the relativistic three-dimensional Thompson equation [18] in nuclear matter [7, 19]

$$\tilde{K}_{ijkl}(\omega) = \frac{1}{2} \tilde{V}_{ijkl} + \frac{1}{2} \sum_{mn} \tilde{V}_{ijmn} \frac{Q_M(m, n)}{\omega - (\epsilon_m + \epsilon_n) + i0} \tilde{K}_{mnkl}(\omega)$$

with

$$Q_M(m, n) = \begin{cases} 
1, & \text{min}\left(|p_m|, |p_n|\right) > k_F \text{ and } \max\left(|p_m|, |p_n|\right) > k_M \\
0, & \text{otherwise.}
\end{cases}$$

The choice of the model-space reaction matrix can avoid the double counting problem in considering the particle-particle propagations in the ring-diagram series. [10, 11]

By using the pphh Green function and its RPA equation, the ground-state energy shift is derived from Eq.(7)

$$\Delta E_{0}^{\text{pphh}} = \int_{0}^{1} d\lambda \sum_{m} \sum_{ijkl \in P} Y_{m}(ij, \lambda) Y_{m}^{*}(kl, \lambda) \tilde{K}_{ijkl}[E_{m}(\lambda)],$$

where the transition amplitude $Y_{m}$ and eigenvalue $E_{m}(\lambda)$ are solutions of a pphh RPA-type secular equation

$$\sum_{ef}[(\epsilon_{i} + \epsilon_{j}) \delta_{ij,ef} + (\bar{n}_i \bar{n}_j - n_i n_j) \lambda \tilde{K}_{ijef}(\omega)] Y_{m}(ef, \lambda)$$

$$= \mu_{m}(\omega, \lambda) Y_{m}(ij, \lambda)$$

$$\Rightarrow$$

$$\sum_{ef}[(\epsilon_{i} + \epsilon_{j}) \delta_{ij,ef} + (\bar{n}_i \bar{n}_j - n_i n_j) \lambda \tilde{K}_{ijef}(\omega)] Y_{m}(ef, \lambda) = \mu_{m}(\omega, \lambda) Y_{m}(ij, \lambda)$$

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with the self-consistent condition

\[ \omega = \mu_m(\omega, \lambda) \equiv E_m(\lambda) \]  

(12)

and the requirement: \( \sum_{ef}(\bar{n}_e \bar{n}_f - n_e n_f)|Y_m(ef, \lambda)|^2 < 0 \). In the above \( n_i \) is the occupation number which is equal to 1 if \( p_i < k_F \) and 0 otherwise. \( \bar{n}_i \) is defined as \( 1 - n_i \). It should be noted that in practice we have actually include both two-body (the reaction matrix \( \tilde{K} \)) and one-body (self-energy insertion) terms in solving Eq. (11) \[10\], and have used the following normalization condition \[20\]

\[ \sum_{ef}(\bar{n}_e \bar{n}_f - n_e n_f)|Y_m(ef, \lambda)|^2 = \frac{-1}{1 - \frac{\partial \mu_m(\omega, \lambda)}{\partial \omega}}. \]  

(13)

As shown later, this brings about the dependence of the ring effect on the s.p. potential as well as on the nuclear matter density.

Formally speaking, the expressions presented here are rather similar to those used in the non-relativistic case. The difference is in the use of relativistic s.p. energies for \( (\epsilon_i) \) and in the reaction matrix elements, which are calculated using the medium-dependent Dirac-spinor s.p. wavefunctions Eq. (1).

We apply two types of the s.p. spectra, namely a model-space one and a
continuous one. They are defined by

\[ E_p = \begin{cases} 
\sqrt{(M_N + U_s)^2 + \mathbf{p}^2 + U_v}, & \text{if } |\mathbf{p}| \leq k_M \\
\sqrt{M_N^2 + \mathbf{p}^2}, & \text{otherwise}
\end{cases} \tag{14} \]

where \( k_M \) is finite for the model space spectrum (as mentioned, we choose \( k_M = 3.2 \text{ fm}^{-1} \)) while \( k_M \) is infinite for the continuous choice. For each \( k_F, \)

\( U_s \) and \( U_v \) are determined from

\[ \tilde{u}(\mathbf{p}, s)\gamma_0^\dagger[\gamma_0 U_s + U_v]\tilde{u}(\mathbf{p}, s) = \frac{\tilde{M}}{E_p} U_s + U_v \]

\[ = 2 \sum_{h<k_F} <ph|\tilde{K}(E_p + E_h)|ph>, \quad p \leq k_M, \tag{15} \]

where \( <ph|\tilde{K}(\omega)|ph> \) is the relativistic model-space reaction matrix as obtained from the solution of Eq. (8). For the continuous choice, \( Q_M \) in Eq.(8) is replaced by the common Pauli projection operator (i.e. \( k_M = k_F \)) and a principal value integration is employed to avoid any singularities.

In our calculations, the first step is to determine for each density the fields \( U_s \) and \( U_v \). This is achieved by solving Eqs. (8) and (15) selfconsistently. The resulting values are listed in Table 1 for both the model-space and the continuous s.p. spectrum. In the next step, the model-space relativistic reaction matrix is calculated using these self-consistent values and applied in the ring-diagram formalism.
3 Results and Discussion

We have calculated the energy per nucleon in nuclear matter for various densities applying the relativistic ring method outlined in the previous section. For the NN potential we use the relativistic meson-exchange interaction of the Bonn group [21], which applies the pseudovector coupling for the $\pi NN$ vertex. This potential has been used in the Dirac-Brueckner calculations of Ref. [3, 7]. The results of our relativistic ring calculations are listed in Table 1 and plotted in Fig. 2. In the figure, the solid line (with no label) shows the relativistic ring result. For comparison we also show the non-relativistic ring calculation using the same potential (solid line with label ‘nr’). Moreover, relativistic (DBHF) and non-relativistic Brueckner-Hartree-Fock results are also shown, again, using the same potential (dashed curves).

Though, the difference between the two relativistic calculations is obviously small (solid and dashed curves with no label), the effect of the infinite summation of the pphh ring diagrams can still be seen in the shift of the minimum of the curve towards a smaller density. Furthermore, the curvature of the relativistic ring curve is considerably smaller than the one of the DBHF curve, indicating a softer equation of state for the relativistic ring case.
As a matter of fact, the shift shows the existence of an additional density-dependent repulsive effects in the relativistic ring calculations as compared to the relativistic DBHF calculations. This feature can be seen more clearly in Table 2 where we present a \(\lambda\)-dependence of the average potential energy as defined by

\[
\Delta E_{0}^{pphh} = \int_{0}^{1} d\lambda \Delta E_{0}^{pphh}(\lambda). \tag{16}
\]

As already discussed in previous papers\[10, 11\], the \(\lambda\)-dependence characterizes the relative importance of the higher-order ring diagram contributions. Thus, the difference between \(\Delta E_{0}^{pphh}(\lambda = 1)\) and \(\Delta E_{0}^{pphh}(\lambda = 0)\) is a direct measure of this contribution. In Table 2, we list \(\Delta E_{0}^{pphh}\) for two \(\lambda\)'s and their density dependence for the partial-waves \(^3S_1\) plus \(^3D_1\). It is clearly seen that the difference \(\Delta E_{0}^{pphh}(\lambda = 0.887) - \Delta E_{0}^{pphh}(\lambda = 0.113)\) becomes less attractive with increasing density. This means a repulsive effect that increases with the density, and therefore shifts the saturation point.

One can see clearly from Fig. 2 that this ring effect is dramatically reduced in the relativistic calculations. (Compare the differences between solid and dashed curves for both the labeled and the unlabeled case.) This may have something to do with the different momentum dependence of the relativistic
and non-relativistic s.p. spectra. In the relativistic case, the energy depends approximately linearly on the momentum for large momentum. In the non-relativistic case, this relation is quadratic for all momenta. As a result, the \( \omega \)–dependence of the non-relativistic eigenvalues \( \mu_m(\omega) \) of Eq.(11) would be stronger than that of the relativistic ones. As seen from Eq.(10), the effect from the infinite summation of the ring diagrams depends sensitively on the normalization of the RPA amplitudes \( Y_m \) which are normalised using Eq.(13). Therefore, the stronger the \( \omega \)–dependence of \( \mu_m \), the smaller the \( Y_m \) (note \( d\mu_m/d\omega < 0 \)), and the larger the ring diagram repulsive effect.

For a closer comparison, we list in Table 3 the saturation properties of nuclear matter as predicted by the two different relativistic approaches. Particularly noteworthy is that in the relativistic ring approach the compression modulus comes out much smaller than in Dirac-Brueckner calculations. This implies that the equation of state is predicted softer by the ring approach.

Note that for the relativistic ring-diagram calculations we employ a model space s.p. spectrum while in DBHF we use a continuous choice. Thus, part of the observed differences could be attributed to the difference in the choice of the s.p. potential. To clarify this point, we have also performed a relativistic ring calculation with a continuous s.p. spectrum (cf. Table 1). It turns out
that the results are, indeed, very similar to the s.p. model space calculation.

We note that in the non-relativistic case\cite{15}, the differences between calculations employing different s.p. potentials are, in general, larger than in our present relativistic calculations. This fact can also be attributed to the different momentum dependence of the relativistic and non-relativistic s.p. spectra, as we mentioned earlier.

Comparing the two solid curves in Fig. 2, demonstrates the relativistic effect in the ring formalism. It is qualitatively of the same kind as in Brueckner-Hartree-Fock (compare the two dashed curves), but smaller. This is the consequence of the interplay of the relativistic and ring effects. The comparison also tells us that the relativistic effect is larger and more strongly density-dependent than the ring effect.

Finally, we wish to point out that in contrast to non-relativistic ring-diagram calculations, our present calculation is in favor of the NN interaction with the weaker tensor force. It appears that our results show support to previous DBHF calculations in the following two aspects: (1) We need to have a realistic NN interaction, which has a weak tensor force component. At present, it seems that the Bonn potential is a good candidate. This observation is also consistent with recent studies in finite nuclei.\cite{3, 19, 22} (2)
Without the consideration of anti-nucleon contribution, the infinite summation of the pphh ring diagrams is convergent and its correction to the DBHF is small. Therefore the DBHF approach may be a sufficient approximation for most purposes.

4 Summary and Conclusions

In this note, we have extended our ring-diagram formalism to incorporate relativistic effects by substituting the non-relativistic Schroedinger s.p. wavefunctions by relativistic Dirac-spinor s.p. wavefunctions. Our results show that in this relativistic framework, the general feature of the non-relativistic ring-diagram results are preserved. Thus, the ring effect shifts the saturation point to even lower saturation densities as compared to Dirac-Brueckner calculations, in perfect agreement with empirical information.

Furthermore, the equation of state comes out softer in the relativistic ring approach as compared to Dirac-Brueckner-Hartree-Fock calculations. Consequently, the compression modulus is predicted smaller by the relativistic ring formalism.

As a by-product of our investigation, we find that the dependence of
the nuclear matter energy on the choice of the s.p. spectrum is weaker in relativistic calculations. Mainly because of this, the relativistic pphh ring corrections to the DBHF nuclear matter calculations have turned out to be relatively small.

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Table 1. Energy per nucleon, E/A, and single-particle potential parameters, $U_s$ and $U_v$, (in units of MeV) for various densities as obtained in the present relativistic ring-diagram calculations and in the DBHF approach of Ref. [7]. In the case of ring calculations, two types of s.p. potentials are applied, namely the model-space and the continuous choices. In the DBHF calculations of Ref. [7] only the continuous choice was used. For the definition of $U_s$ and $U_v$ see Eqs. (2) and (15).

| $k_F$ ($\text{fm}^{-1}$) | Relativistic Rings | | DBHF | |
|-------------------------|-------------------|------------------|------------------|------------------|
|                         | Model-Space Choice | Continuous Choice | Continuous Choice | Continuous Choice |
|                         | E/A   | $U_s$   | $U_v$   | E/A   | $U_s$   | $U_v$   | E/A   | $U_s$   | $U_v$   |
| 1.20                    | -13.97 | -305.6 | 252.4 | -13.75 | -288.8 | 222.0 | -13.44 | -288.8 | 222.0 |
| 1.30                    | -15.00 | -344.9 | 283.5 | -14.66 | -331.6 | 255.0 | -14.86 | -331.6 | 255.0 |
| 1.40                    | -15.29 | -386.5 | 318.1 | -14.80 | -374.9 | 289.8 | -15.59 | -374.9 | 289.8 |
| 1.50                    | -14.62 | -431.0 | 361.3 | -13.83 | -416.3 | 325.7 | -14.88 | -416.3 | 325.7 |
| 1.60                    | -12.35 | -474.5 | 406.9 | -11.18 | -459.6 | 368.6 | -11.96 | -459.6 | 368.6 |
| 1.70                    | -6.77  | -505.5 | 442.2 | -5.64  | -497.2 | 412.8 | -5.88  | -497.2 | 412.8 |
| 1.80                    | +1.86  | -536.5 | 493.4 | +3.60  | -530.4 | 461.6 | +4.44  | -530.4 | 461.6 |
| 1.90                    | +13.92 | -549.5 | 518.5 | +15.11 | -554.8 | 512.0 | +19.72 | -554.8 | 512.0 |
| 2.00                    | +27.10 | -555.9 | 547.7 | +27.55 | -572.4 | 567.5 | +41.62 | -572.4 | 567.5 |
| 2.10                    | +35.91 | -558.1 | 611.9 | +36.88 | -590.2 | 640.3 | +71.20 | -590.2 | 640.3 |
Table 2. The $\lambda$-dependence of the average potential energy $\Delta E_{hh}^{pp}(\lambda)$ (in MeV) as defined in Eq. (16) in partial-waves $^3S_1+^3D_1$ for various densities.

| $k_F$ (fm$^{-1}$) | $\Delta E_{hh}^{pp}(\lambda)/A$ | $\lambda = 0.113$ | $\lambda = 0.887$ |
|------------------|-------------------------------|-------------------|-------------------|
| 1.20             | -11.30                        |                   | -20.87            |
| 1.30             | -12.52                        |                   | -21.32            |
| 1.40             | -13.41                        |                   | -21.41            |
| 1.50             | -13.94                        |                   | -21.13            |
| 1.60             | -14.07                        |                   | -20.20            |
| 1.70             | -13.66                        |                   | -18.68            |
Table 3. Nuclear matter saturation parameters as obtained in a Dirac-Brueckner-Hartree-Fock (DBHF) and a relativistic pphh ring-diagram (RR) calculation. Given are the energy per nucleon, $E/A$, the Fermi momentum, $k^0_F$, and the compression modulus, $\kappa$, at the saturation point.

|                | DBHF | RR  |
|----------------|------|-----|
| $E/A$ [MeV]    | -15.60 | -15.30 |
| $k^0_F$ [fm$^{-1}$] | 1.41 | 1.38 |
| $\kappa$ [MeV] | 296 | 152 |
**FIGURE CAPTIONS**

**Figure 1.** Particle-particle hole-hole ring-diagram series considered in this study. Each hatched box represents a relativistic model-space reaction matrix as defined by Eq.(8). Indices \(i, j, k, l, m, n\) denote model-space single-particle states.

**Figure 2.** Energy per nucleon in nuclear matter *versus* density expressed in terms of the Fermi momentum \(k_F\). Results from ring-diagram calculations are shown by solid curves, while Brueckner-Hartree-Fock predictions are represented by dashed lines. For both approaches, relativistic (no label) and non-relativistic (label ‘nr’) results are displayed. The shaded box covers empirical information on nuclear matter saturation \((E/A = -16 \pm 1 \text{ MeV}, k_F = 1.35 \pm 0.05 \text{ fm}^{-1})\).