Spatial Phase-Shifting Interferential System on Polarization Interference and Grating Beam-Splitting: Phase-Shifting Error Testing

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Abstract. In spatial phase-shifting interferential system on polarization interference and grating beam-splitting, the azimuth errors of the polarizers are the central error sources of phase-shifting steps and have 1:1 effect for the steps. It applies two ways to determine the azimuth errors of the ploarizers: experimental testing and software calculating. The basic principle of experimental testing is that the polarizer has selectivity for the polarizing direction. CCD records the intensity changing and it determines the direction of light axis of each polarizer by the turn-point of the intensity. The way of software calculating includes two algorithms: iterative phase-shifting algorithm and Fourier-tranform method of phase-shift determination. It builds an experimental system to test the azimuth errors of the polarizers, and analyzes the calculating errors of two algorithms by simulative phase-shifting fringe patterns.

1. Introduction
The optical interferometry is an important way to realize the measurement with high precision, the introduction of phase-shifting technique improve greatly the measurement precision and automation level. The interferometers on temporal phase-shifting have been applied wide in research and manufacturing. In order to improve the dynamic characteristic of the interferometers, spatial phase-shifting techniques are researched and 4D technology Corporation has produced some modes’ interferometers on these kinds of techniques. In research and manufacturing, there are many demand for the spatial phase-shifting interferometers with good dynamic characteristic and high precision.

There are two ways to realize spatial phase-shifting: one is that the a few of CCDs are added to record the phase-shifting fringe patterns(usually 3 or 4 maps) in the skillfully designed interferometers, for examples, Smythe system [1], Bureket system [2], Haasteren system [3], Four-camera interferometer [4], IPSI system [5] and so on. These systems almost apply polarization principle to realize spatial phase-shifting. The second way is that the interferometer only uses one CCD as the registering element. That is to say that the phase-shifting fringe patterns(usually 3 or 4 maps) are imaged on the same CCD, for examples, Kujawinska’s ESPI system[6], Kranz system [7], real-time interferential system on polarization phase-shifting [8], PhaseCam interferometer [9], Pixelated interferometer [10] and so on. These systems mainly apply grating beam-splitting and polarization phase-shifting to realize spatial phase-shifting. And Pixelated interferometer applies special designed
pixlated phasemask as its phase-shifting elements and remains high spatial resolutions. The interferential systems on few CCDs is easier to obtain the optical elements with high precision, so in theory, it is easier to get high measurement precision. But the systems are usually with big scales, demanded more careful adjustment and high stability in testing. Correspondingly, the systems on single CCD are with simple and compact setups and better stability and easy to use in different situations. The disadvantages are the high demand for the beam-splitting elements and phase-shifting elements. Spatial phase-shifting techniques on single CCD have become important ones in actual applications.

In phase-shifting interferential system, the errors of the phase-shifting values between fringe patterns are the main factors to effect measurement precision. Among the few effect factors, the azimuth errors of the polarizers are the main ones. It introduces two ways to test this kind of errors: experimental testing and software calculating. It builds an experimental system to test the azimuth error and analyzes the calculating algorithms of the errors of phase-shifting steps by simulative data.

2. Spatial phase-shifting interferential system on polarization interference and grating beam-splitting

The sketch map of the experimental system is shown in figure 1.

In figure 1, it gets linearly polarized light through the polarizer and then is expanded and collimated. This ray is split into two polarized light with vertical directions of light axes. The beams returning from the object surface and the reference mirror project along the receiving road. The two linearly polarized lights become circularly polarized lights after passing through the quarter wave plate. And then they are through a 2D phase grating to be diffracted into four ties, which includes two +1 diffraction rays and two -1 diffraction rays with the same intensity and phase distributions in theory. The four ties of rays are projected through four polarizers, which stitch together and the axis directions with the one of the assigned polarizer are 0, 45, 90 and 135 degree. So, CCD can receives four fringe patterns at the same plane and time with phase-shifting values 0 degree, 90 degree, 180 degree and 270 degree corresponding to an assigned fringe pattern (usual the first pattern and its phase-shifting value is supposed as 0 degree).

The main parameters of the polarization interference element and beam-splitting elements are as follows:

(1) Polarizer group: It is composed of four polarizers with $10 \times 10$ mm aperture each. The axis directions with the the assigned polarizer are 0, 45, 90 and 135 degree. The are cut and then stitched together by a suitable way and precision. This work is completed by Shanghai laser research institute.
(2) Beam-splitting grating: It applies a 2D Ronchi phase grating as the beam-splitting elements. The Ronchi phase grating applied in the experimental system is designed by Shanghai optical and mechanical institute. Its parameters are: grating points: $2 \times 2$; basis material: grass; sculpture depth: 0.62um; spatial occupying ratio: 1/2; spitting ratio: $I_0/I_{+1} \approx 1.6\%$, in which $I_0$ is the intensity of the diffraction light of zero order. $I_{+1}$ is the +1 and -1 order.

In experimental system, a few factors will affect the final measurement results, including the retardation and azimuth errors of quarter-wave plate, azimuth errors of the polarizers, phase and intensity errors by the grating and lens and the pixel matching errors between the fringe patterns, and so on. Reference [11] did detail analysis for the effect introduced by these errors for the measurement in theory. Among these errors, the azimuth errors of the polarizers are a central factor for the error of phase-shifting steps of the fringe patterns. The paper introduces two ways to test the errors of the phase-shifting steps. One way is by testing the azimuth errors of the polarizers, which introduces the phase-shifting steps 1:1, by experimental method. The second way is calculate the phase-shifting step errors by suitable algorithms. This work can present a basis of the precision analysis and algorithms choosing afterward.

3. Experimental testing of the errors of the phase-shifting steps in spatial phase-shifting system

Figure 2 is the sketch map of the testing system of the azimuth errors of the polarizers. The principle is that when rotating the polarizer, the intensity of the passing light will changed periodically, from maximum to vanishing and then again. So by the intensity changing, it can determine the light axis of the polarizers (related to the maximum intensity). The angle difference between the light axes of the polarizers and the axis of the assigned polarizer is the actual phase-shifting steps of the recorded fringe pattern by CCD.

The testing procedure is:

(1) Build the experiment system, shown in Figure 2.

(2) Moving the polarizer group to make the ray passes through the polarizer group from its center. Then polarizer (the one adjacent to the laser) is rotated and the changing intensity of one light point passing through polarize 1 is recorded each 10 degree. Therefore there are 36 points are registered in whole rotating period. The intensity changing curve is drawn, shown in Figure 3. By this curve, the vanishing position, which is vertical to the axis position of the each polarizer, is determined roughly.

(3) Around the vanishing position, the intensity is recorded each 2 degree (this value can be set smaller according to the testing precision) and the intensity changing curve is drawn and fit. From the fitting curve, the turning point is determined and this correspond position is set as A. The experimental testing curve and fitting curve are shown in Figure 4 and 5.

![Figure 2](image-url)  
Figure 2. Sketch map of the system to test the azimuth of the polarizers.  

![Figure 3](image-url)  
Figure 3. Intensity change of polarizer 1. (position*10 degree)
(4) By A as the datum mark, A+45degree is the rough vanishing position for polarizer 2. Now it tests the light point passing through polarizer 2. The same procedure is done as (3). The vanishing position is set as B. The experimental testing curve and fitting curve are shown in Figure 6 and 7.

(5) The same procedure is repeated and the vanishing positions for polarizer 3 and 4 are determined, set as C and D. The experimental curves and fitting curves are omitted.

(6) Do (B-A),(C-A) and (D-A), then the azimuth of each polarizer can be got, shown in table 1.

![Image](image1)

**Figure 4.** Intensity changing of polarizer 1 (position*2 degree).

![Image](image2)

**Figure 5.** Fitting curve of Figure 4 (turning point:9.646, correspond to 161.34 degree) (position*2 degree).

![Image](image3)

**Figure 6.** Intensity changing of polarizer 2 (position*2 degree).

![Image](image4)

**Figure 7.** Fitting curve of Figure 6 (turning point:8.78, correspond to 205.78 degree) (position*2 degree).

| Table 1. Azimuths of polarizers relative to polarizer 1. |
|----------------------------------|
| polarizer | 1  | 2  | 3    | 4    |
| Theory value(degree) | 0  | 90 | 180  | 270  |
| Testing value(degree) | 0  | 88.87 | 179.17 | 271.40 |

4. Software calculating the errors of the phase-shifting steps in spatial phase-shifting system

The paper introduces two kinds of algorithms to calculate the actual phase-shifting steps: Kong-kim's algorithms [12] and Fourier-tranform algorithm [13].

4.1. Kong-kim phase-shifting algorithm [12]

Reference [14] puts forward an algorithm on an least square and iterative calculation to determine the phase map and phase-shifting steps by the phase-shifting patterns directly, which supposes that the phase-shifting steps of all points on the same pattern are equal. Kong-kim algorithm predigests this algorithm, which assumes that the background is unchanged during phase-shifting.
The paper applies Kong-kim algorithm to calculate the phase-shifting steps of simulative fringe patterns, in which some step errors are introduced. The results are shown in Table 2.

| Steps in theory (degree) | Calculating errors (degree) |
|--------------------------|----------------------------|
| Four steps               |                            |
| 0 89 179 271             | 0 0.0163 -0.0008 0.0166    |
| (Standard step is 90°)    |                            |
| 0 88 178 270             | 0 -0.9487 0.0345 -0.9833   |
| 0 88 181 269             | 0 -1.5208 0.0263 -1.4946   |
| 0 91 179 272             | 0 1.4946 -0.0263 1.5208    |
| Five steps               |                            |
| 0 62 119 190 245         | 0 0.0050 0.0051 0.0048 0.0078 |
| (Standard steps is 60°)   |                            |

It can be seen from the table that Kong-kim algorithm is suitable for the small errors of phase-shifting steps. If increasing the step number, its calculating precision will be reduced with big step errors. But considering the complex and the spatial resolution of the system, it is unpractical to add the pattern numbers. So, the better way is to get the polarizer group with less azimuth errors.

4.2. Fourier-transform algorithm [12]

The pattern is added suitable carrier by titling the reference mirrors or other ways. Then each fringe pattern can be processed by Fourier-transform algorithm to calculate the phase map. The average values of the phase maps of the phase-shifting patterns are the phase-shifting steps.

It simulates four carrier fringe patterns with phase-shifting values related to the first one are 88, 185, 276 degree. The phase maps by Fourier-transform algorithm are shown in Figure 8-10 (shown in looking-down mode). The steps are calculated by averaging the phase values of the points around the center (means including the points at the edges). The results are shown in Table 3.

| Phase-shifting steps | Second fringe | Third fringe | Fourth fringe |
|----------------------|---------------|--------------|---------------|
| Values in theory     | 88°           | 185°         | 276°          |
| Values calculated    | 88.00°        | 185.00°      | 276.01°       |

From the results, it can be seen the Fourier-transform algorithm has higher calculating precision with good restraining ability for the errors and the same calculating precision for any phase-shifting value.
5. Conclusion
The paper tests the azimuth errors of the polarizers in the polarizer group by simple experimental way and these errors will present the basis for choosing the calculating algorithm of the phase map and analyzing the system error. Then the paper introduces two algorithms to calculate the phase-shifting steps and does simulative analysis. The results show that Fourier-transform has higher calculating precision and has the same accuracy for all phase-shifting steps. But the fringe patterns in this algorithm demand suitable carrier. It can applies a smooth surface as the object to test the phase-shifting steps and these values are introduced the calculating algorithm of the phase map as the standard steps. Kong-kim algorithm can calculate the phase-shifting steps and the phase map at the same calculating. So it is a timely way. This algorithm has a high precision when the windages with the standard phase-shifting steps is small. But if the windages are big, the calculating errors will increase. So by the analysis, the paper thinks that it can apply experimental testing or Fourier-transform algorithm to get the phase-shifting steps and these values are introduce the typical algorithm by the least squares to calculate the phase map. Of course, the applied algorithm has to consider other error factors, such as the uneven intensity distribution of the fringe patterns.

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