Diamagnetism versus paramagnetism in charged spin-1 Bose gases

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Received 26 September 2010, in final form 26 November 2010
Published 16 December 2010
Online at stacks.iop.org/JPhysCM/23/026003

Abstract
It has been suggested that either the diamagnetism or paramagnetism of Bose gases, due to the charge or spin degrees of freedom respectively, appears solely to be extraordinarily strong. We investigate the magnetic properties of charged spin-1 Bose gases in an external magnetic field, focusing on the competition between the diamagnetism and paramagnetism, using the Lande-factor \( g \) of particles to evaluate the strength of the paramagnetic effect. We propose that a gas with \( g < \frac{1}{\sqrt{8}} \) exhibits diamagnetism at all temperatures, while a gas with \( g > \frac{1}{2} \) always exhibits paramagnetism. Moreover, a gas with the Lande-factor in between shows a shift from paramagnetism to diamagnetism as the temperature decreases. The paramagnetic and diamagnetic contributions to the total magnetization density are also calculated in order to demonstrate some details of the competition.

1. Introduction

Bose gas plays a significant role in understanding a series of exotic quantum phenomena, including superfluidity and superconductivity. It is well known that the ideal Bose gas exhibits Bose–Einstein condensation (BEC) below a critical temperature. In 1938, London first connected BEC to the \( \lambda \) transition in helium [1]. In 1946, Ogg proposed that BEC of bosonic electron pairs might result in superconductivity [2]. Furthermore, Schafroth [3] and Blatt and Butler [4] showed that an ideal gas of charged bosons exhibits the essential equilibrium features of a superconductor. Although the Bardeen–Cooper–Schrieffer (BCS) theory [5] revealed that electrons in a superconductor form Cooper pairs, as opposed to real-space pairs, the Schafroth–Blatt–Butler theory is helpful to the understanding of superconductivity and more importantly has stimulated research interest in charged Bose gases.

The charged Bose gas (CBG) is solely of academic interest in its own right. In particular, it exhibits nontrivial magnetic properties. For example, the condensation phenomenon in CBGs is strongly affected by the external magnetic field, owing to the quantization of the orbital motion of charged particles in a magnetic field [3, 6–12]. Schafroth clearly indicated that an arbitrarily small value of the magnetic field introduces qualitative changes: BEC no longer occurs [3]. May extended this idea to a \( d \)-dimensional CBG and found that it can condense only for \( d > 4 \) [7]. This point was reexamined by other researchers based on different methods [10, 11]. Meanwhile, the orbital motion results in extremely large Landau diamagnetism in CBGs. It was pointed out that the three-dimensional CBG displays a Meissner effect at low temperatures [3, 12, 13]. More recently, Alexandrov quantitatively accounted for the enhanced normal-state diamagnetism of superconducting cuprates using the CBG model [14].

On the other hand, neutral bosons with spin (spinor bosons) have also been studied theoretically [15]. The realization of spinor Bose condensate in optically trapped alkali atoms [16] stimulated new research interest. The constituent atoms, such as \( ^8\text{Rb} \), \( ^{23}\text{Na} \), and \( ^7\text{Li} \) (hyperfine) spin degrees of freedom and thus a magnetic moment. Their spin degrees of freedom become active in purely optical traps and thus investigation of their magnetic properties becomes possible. Yamada [15], and Simkin and Cohen [17] calculated the magnetization of neutral spinor bosons in a magnetic field and found that once BEC takes place, the magnetization remains finite even if \( H = 0 \), as if the system was magnetized spontaneously. The zero-field susceptibility tends to diverge as the temperature goes down to the BEC temperature. Moreover, rigorous proofs presented by Eisenberg and Lieb...
show that the magnetization and zero-field susceptibility at finite temperatures exceed that of a pure paramagnet [18]. All the results indicate that the neutral spinor bosons take on extraordinary paramagnetic effects in a magnetic field.

Bearing in mind the two contrary features of the Bose gas, one immediate question which must be raised is: which kind of magnetism will manifest itself if the bosons possess both the charge and spin degrees of freedom? Analogously, this question has been answered for charged Fermi gases, e.g., the electron gas, where both the paramagnetism and diamagnetism are relatively weak. For the electron gas, the diamagnetic part of the zero-field susceptibility is one-third (in absolute value) of the paramagnetic part, so altogether the gas is paramagnetic. For charged spinor Bose gas, since both the paramagnetism and diamagnetism are extremely large, their competition is a more interesting problem.

In this paper, we study the competition between the paramagnetism and diamagnetism of a charged ideal spin-1 Bose gas in an external magnetic field. In section 2, a model consisting of both the Landau diamagnetic effect and Pauli paramagnetic effect is proposed. The total magnetization density as well as its paramagnetic and diamagnetic parts are calculated respectively. Section 3 presents a detailed discussion of the obtained results. In section 4, a brief summary is given.

2. The model

The orbital motion of a charged boson with charge $q$ and mass $m^*$ in a constant magnetic field $B$ is quantized into the Landau levels

$$\epsilon_{jk} = \left(\frac{1}{2} + j\right)\hbar\omega + \frac{h^2 k^2}{2m^*},$$

where $j = 0, 1, 2, \ldots$ labels different Landau levels and $\omega = qB/(mc)$ is the gyromagnetic frequency. Since $\omega \propto B$, $\omega$ can be used to indicate the magnitude of the magnetic field in the following discussions. We assume the magnetic field is in the $z$ direction. Each Landau level is degenerate with degeneracy equal to

$$D_L = \frac{qBL_iL_y}{2\pi\hbar c}.$$  

Here we suppose the gas is in a box with $L_i \to \infty$, where $L_i$ is the length of the box in the $i$th direction. For a spin-1 boson, the Zeeman energy levels split in the magnetic field due to the intrinsic magnetic moment associated with the spin degree of freedom,

$$\epsilon_\sigma = -\frac{\hbar q}{m^*c}gB = -g\sigma\hbar\omega,$$

where $\sigma$ refers to the spin-$z$ index of Zeeman state ($F = 1, m_F = \sigma$) ($\sigma = +1, 0, -1$) and $g$ is the Lande-factor. The quantization of the orbital motion and the Zeeman effect give rise to the Landau diamagnetism and Pauli paramagnetism, respectively.

We consider an assembly of $N$ bosons, whose effective Hamiltonian reads

$$\hat{H} = -\mu N = D_L \sum_{j,k,\sigma}(\epsilon_{jk} + \frac{h^2 k^2}{2m^*} - \mu)n_{j,k,\sigma},$$

where $\mu$ is the chemical potential. The charged spinor bosons have been discussed theoretically in the context of relativistic pair creation [19]. However, the magnetism of charged spinor Bose gases is less studied and of major interest in the present work.

The grand thermodynamic potential is formally expressed as

$$\Omega_{T \ne 0} = -\frac{1}{\beta}\ln \text{Tr}e^{-\beta(\hat{H} - \mu N)}$$

$$= \frac{1}{\beta}D_L \sum_{j,k,\sigma} \ln[1 - e^{-\beta(\epsilon_{jk} + \frac{h^2 k^2}{2m^*} - \mu)}].$$

where $\beta = (k_B T)^{-1}$. Converting the sum over $k_z$ to a continuum integral, we get

$$\Omega_{T \ne 0} = \frac{\omega m^*V}{2\pi^2\hbar^2} \sum_{j=0}^{\infty} \sum_{\sigma} \int dk_z$$

$$\times \ln[1 - e^{-\beta((\epsilon_{j\sigma} + \frac{h^2 k_z^2}{2m^*} - \mu))}],$$

where $V$ is the volume of the system. Then using the Taylor expansion and performing the integral over $k_z$, we have

$$\Omega_{T \ne 0} = \frac{\omega V}{\hbar^2} \left(\frac{m^*}{2\pi^2\beta}\right)^{3/2} \sum_{j=1}^{\infty} \sum_{\sigma} \frac{l^{-2 - j/2}}{1 - e^{-\beta\mu - \frac{l^2}{2\hbar^2}}}. (7)$$

This treatment has been used by Standen and Toms [11] to deal with scalar Bose gases. Similarly, we introduce some compact notation for the class of sums,

$$\Sigma^\alpha_\sigma[\alpha, \delta] = \sum_{j=1}^{\infty} \sum_{\sigma} \frac{\mu^*e^{-j/2-2\sigma - \mu + \delta}}{(1 - e^{-\beta\mu})^{\alpha}},$$

where $x = \beta\hbar\omega$. With this notation we may rewrite equation (7) as

$$\Omega_{T \ne 0} = -\frac{\omega V}{\hbar^2} \left(\frac{m^*}{2\pi^2\beta}\right)^{3/2} \Sigma^\alpha_\sigma[-D, 0],$$

where $D = 3$. Then the density of particles $n = N/V$ can be derived from the thermodynamic potential,

$$n = \frac{1}{V} \left(\frac{\partial \Omega}{\partial \mu}\right)_{T,V}$$

$$= x \left(\frac{m^*}{2\pi^2\beta\hbar}\right)^{3/2} \Sigma^\alpha_\sigma[2 - D, 0].$$

The magnetization density $M$ is written as

$$M_{T \ne 0} = -\frac{1}{V} \left(\frac{\partial \Omega}{\partial B}\right)_{T,V} = \frac{\hbar q}{m^*c} \left(\frac{m^*}{2\pi^2\beta\hbar}\right)^{3/2}$$

$$\times \sum_{\sigma} \Sigma^\alpha_\sigma[-D, 0] + x(g\sigma - \delta)$$

$$\times \Sigma^\alpha_\sigma[2 - D, 0] - x \Sigma^\alpha_\sigma[2 - D, 1].$$

It is convenient to introduce some dimensionless parameters, such as $M' = m^*cM/(\hbar q)$, $\tilde{\omega} = \omega/(k_BT)$, $T = T/T^*$ and $x' = \tilde{\omega}/t$, to re-express equations (10) and (11),

$$M_{T \ne 0} = \frac{1}{2^{1/2}} \sum_{\sigma} \Sigma^{a\sigma}_\sigma[-D, 0] + x'(g\sigma - \delta)$$

$$\times \Sigma^{a\sigma}_\sigma[2 - D, 0] - x' \Sigma^{a\sigma}_\sigma[2 - D, 1].$$
and 3, respectively. Inset: the solid line, dashed line, and dotted line correspond to the scalar boson model which exhibits strong diamagnetism as a function of $g$ for fixed magnetic fields at $t = 0.1$. Here the solid line, dashed line, and dotted line correspond to $\bar{\omega} = 0.05, 0.3,$ and 3, respectively. Inset: $m$ as a function of $g$.

Here the characteristic temperature $T^*$ is given by $k_B T^* = 2\pi \hbar^2 n^{2/3}/m^*$. The Bose–Einstein condensation temperature of spin-1 Bose gas with density $n$ is just defined as $k_B T_c = 2\pi \hbar^2 n^{2/3}/[m^* [3(3/2)^{2/3}]] \approx k_B T^*/3.945$. The dimensionless notation $\bar{\Sigma}_\alpha' [\alpha, \delta]$ should be

$$\bar{\Sigma}_\alpha' [\alpha, \delta] = \sum_{\sigma = \pm} \frac{\mu'_{\alpha} \bar{\omega}^{-1/2}(1/2-\pi \sigma - \mu'(\bar{\omega}+b)}}{\left(1 - e^{-i\pi \sigma}\right)}$$

where $\mu' = \mu/(k_B T^*)$. The two variables $\mu'$ and $\bar{M}$ are determined by equations (12) and (13). As shown on the right side of equation (12), in the low temperature limit, $t^{1/2}$ tends to zero but $t^{1/2} \bar{\Sigma}_\alpha'$ [2 – $D, 0]$ keeps finite, so $\mu'$ has a subtle dependence on the temperature $t$. This makes it more difficult to solve the equation numerically at low temperatures. The results of our theory are more credible at high temperatures and a similar situation has been dealt with by Standen and Toms [20].

3. Results and discussions

In our model, both the charge and spin degrees of freedom are taken into account, which are described by the Landau energy term $e_{ik}^\lambda$ and the Zeeman energy term $e_{ik}^\sigma$ in the Hamiltonian (4), respectively. In the case that the Lande-factor $g$ tends to zero, the model degenerates into a charged scalar boson model which exhibits strong diamagnetism as already intensively discussed [7–12]. As $g$ becomes larger, the paramagnetic effect is strengthened.

We calculate the dimensionless magnetization density $\bar{M}$ as a function of $g$, as shown in figure 1(a). $\bar{M}$ is negative in the small $g$ region, which means that the diamagnetism dominates. The absolute value of $\bar{M}$ is larger in the stronger field $\bar{\omega}$. For each given value of $\bar{\omega}$, $\bar{M}$ increases monotonically with $g$. $\bar{M}$ changes its sign from negative to positive at a critical value of $g$, noted as $g_c$ hereinafter, reflecting that the paramagnetism becomes dominant as $g$ increases. Note that the slope of the $\bar{M}$ curve is dependent on $g$. When $g$ is near to zero, $\bar{M}$ increases slowly but the slope rises quickly with $g$. This means that the interplay between diamagnetism and paramagnetism is complex and nonlinear. However, in the strong paramagnetic region, $\bar{M}$ grows almost linearly with $g$.

Figure 1(b) plots the paramagnetic contribution to $\bar{M}$ (named as the paramagnetization density), $\bar{M}_p = g m$, with an inset showing $m = n_+ - n_-$, and figure 1(c) shows the diamagnetic contribution to $\bar{M}$ (named as the diamagnetization density), $\bar{M}_d = \bar{M} - \bar{M}_p$. The increasing tendency of $\bar{M}_p$ is similar to that of $\bar{M}$. It is noteworthy that the diamagnetization density is not suppressed, but enhanced as $g$ becomes larger. Comparing figures 1(a)–(c), it can be seen that the increase in $\bar{M}$ comes from the paramagnetic effect. In the small $g$ region, both $\bar{M}_p$ and $\bar{M}_d$ are strengthened nonlinearly with increasing $g$. As shown in the inset of figure 1(b), $m$ grows very quickly. Nevertheless, both $m$ and $\bar{M}_d$ flatten out in the large $g$ region. So the slope of the $\bar{M}$ curve is mainly due to the paramagnetization.

According to the discussions above, the critical value of the Lande-factor, $g_c$, is an important parameter to describe the competition between the diamagnetism and paramagnetism. $g_c$ is a function of the temperature $t$ and the magnetic field $\bar{\omega}$. Figure 2 shows $g_c$ as a function of $1/t$ in different magnetic fields $\bar{\omega} = 10, 3, 0.5, 0.3, 0.1$ and 0.05. Obviously, $g_c$ varies monotonically with the temperature $t$, while its dependence on the field $\bar{\omega}$ is not simple. For example, in the low temperature region, $g_c$ decreases with decreasing $\bar{\omega}$ at a given temperature as $\bar{\omega}$ is still larger than 0.3, then it rises up as $\bar{\omega}$ goes down further from $\bar{\omega} \approx 0.3$. 

![Figure 1](image1.png)  
Figure 1. (a) The total magnetization density ($\bar{M}$), (b) the paramagnetization density ($\bar{M}_p$), and (c) the diamagnetization density ($\bar{M}_d$) as a function of $g$ for fixed magnetic fields at $t = 0.1$. Here the solid line, dashed line, and dotted line correspond to $\bar{\omega} = 0.05, 0.3$, and 3, respectively. Inset: $m$ as a function of $g$.

![Figure 2](image2.png)  
Figure 2. Plots of the critical value of Lande-factor, $g_c$, as a function of $1/t$ for fixed values of $\bar{\omega}$. The field is chosen as $\bar{\omega} = 10$ (dash–dot–dotted line), 3 (dotted line), 0.5 (short dotted line), 0.3 (dashed line), 0.1 (short dashed line), and 0.05 (solid line).
As already mentioned, the results of our theory are more credible at high temperatures. In the high temperature limit, \( g_c \) seems universal with respect to different choices of magnetic field, \( g_c|_{t \to \infty} \approx 0.35356 \). For a given magnetic field, \( g_c \) increases as the temperature falls down. This suggests that the diamagnetic region is larger at low temperatures than at high temperatures. Although the exact value of \( g_c \) cannot be obtained at very low temperatures, its variation trend can be estimated from figure 2. It seems that \( g_c \) ranges from 0.475 to 0.50 in various magnetic fields.

It is useful to reexamine the high temperature behaviors of \( g_c \) by generalizing the above calculations to a spin-1 Boltzmann gas, since the Bose–Einstein statistics reduce to Maxwell–Boltzmann statistics, respectively, with fixed \( c \). The value of \( g_c \) for a Boltzmann gas is reasonably equal to that of a Bose gas in the high temperature limit. Figure 3 plots the numerical solutions of equation (19) and compares with the Bose gas. An interesting point is that the low-temperature-limit value of \( g_c \) is also consistent with that of a Bose gas at low temperature but in a high magnetic field, although the Maxwell–Boltzmann statistics are just valid in the high temperature region. The stronger the field, the better the accordance.

The value of \( g_c \) can be derived from equation (19) exactly in two limit cases: \( g_c|_{t \to \infty} = 1/\sqrt{8} \approx 0.35355 \) and \( g_c|_{0} = 1/2 \). The value of \( g_c \) for a Boltzmann gas is in sequence as 10 (dash–dot–dotted line), 3 (dotted line), 0.3 (dashed line), and 0.05 (solid line). (a) corresponds to \( g = 0.35 \), (b) corresponds to \( g = 0.45 \). (c) corresponds to \( g = 0.5 \).

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The \( g_{c}/\text{e}^{t} \) curves in figures 2 and 3 mark the boundary between the diamagnetic and paramagnetic regions. The gas exhibits diamagnetism at all temperatures and in all magnetic fields when \( g < g_{c}|_{t \to \infty} \approx 0.35355 \), while always exhibiting paramagnetism when \( g \geq 0.5 \). The magnetic properties seem more complicated in the intermediate region, \( g_{c}|_{t \to \infty} < g < 0.5 \). Figures 4(a)–(c) illustrate the dimensionless magnetization density \( M \) for the three different cases.

Figure 4(a) denotes the case of \( g = 0.35 \lesssim g_{c}|_{t \to \infty} \). The temperature-dependence of \( M \) is very similar to that of charged scalar Bose gases [11], as if the paramagnetic effect associated with the spin degree of freedom is thoroughly hidden. The diamagnetism is even stronger at lower temperatures. As the external field tends to be weak, a sharp bend appears gradually on the curve, which is located at the point corresponding
to the BEC temperature in zero field [11]. In our model, the BEC temperature for a spin-1 gas is \((1/\omega_c) \approx 3.945\). Figure 4(c) shows \(\bar{M}\) for a paramagnetic case to the contrary when \(g = 0.5\). \(\bar{M}\) is always positive in the field at all temperatures. An interesting phenomenon is that the \(\bar{M}-1/t\) curve shows up a peak in this case. The decline in \(\bar{M}\) at low temperatures is attributed to the diamagnetic effect. When weakening the magnetic field, the peak is lowered and moves to low temperatures. Figure 4(b) depicts the case with \(g\) in the intermediate region, which looks quite similar to figure 4(c). The key difference is that \(\bar{M}\) can change its sign from positive to negative as the temperature decreases, indicating that the system undergoes a shift from paramagnetism to diamagnetism.

Figures 4 demonstrate the total magnetic performance of the charged spin-1 Bose gas. We now turn to examine the underlying paramagnetic and diamagnetic effects for each corresponding case. As shown in figures 5, both the paramagnetization density and the diamagnetization density become more stronger at lower temperatures. As the external field is reduced, the strengthening of \(M_p\) and \(M_d\) becomes fast near the temperature close to the BEC point in zero field. As \(g\) grows, \(M_d\) is only slightly strengthened but \(M_p\) increases significantly and thus it can go beyond \(M_d\). \(M_p\) thoroughly exceeds \(M_d\) when \(g\) rises to 0.5. If \(g\) increases further, the paramagnetic effect can be so strong as to cover up the diamagnetic effect completely. Then the total magnetization density \(\bar{M}\) becomes monotonously increasing with lowering temperature and finally reaches a plateau, instead of a peak, at low temperatures.

4. Summary

This paper has revealed the interplay between paramagnetism and diamagnetism of the ideal charged spin-1 Bose gas. The Lande-factor \(g\) was introduced to describe the strength of paramagnetic effect caused by the spin degree of freedom. The gas exhibits a shift from diamagnetism to paramagnetism as \(g\) increases. The critical value of \(g\), \(g_c\), is determined by evaluating the dimensionless magnetization density \(\bar{M}\). Our results show that \(g_c\) increases monotonically as \(t\) decreases. In the high temperature limit, \(g_c\) goes to a universal value in all different magnetic fields, \(g_c|_{t \to \infty} = 1/\sqrt{8}\). At low temperatures, our results indicate that \(g_c\) ranges from 0.475 to 0.50 as the magnetic field varies. Therefore, a gas with \(g < 1/\sqrt{8}\) exhibits diamagnetism at all temperatures, but a gas with \(g > 1/2\) always exhibits paramagnetism. In cases where \(1/\sqrt{8} < g < 1/2\), the Bose gas undergoes a shift from paramagnetism to diamagnetism as the temperature decreases.

In order to depict some details of the competition between paramagnetism and diamagnetism, the paramagnetic and diamagnetic contributions to the total magnetization density have also been calculated. There is no doubt that the paramagnetism is enhanced with increasing \(g\). Surprisingly, the diamagnetism is not suppressed, but slightly strengthened. This implies that the competition between paramagnetism and diamagnetism is nontrivial. When \(g\) is fixed, both the paramagnetism and diamagnetism become stronger as \(t\) decreases. As in the scalar case, there is no Bose–Einstein condensation when the magnetic field is present, no matter how small it is. However, evidence of the condensation can be seen in the magnetization density as the magnetic field is reduced.

Acknowledgments

This work was supported by the Fok Ying Tung Education Foundation of China (No. 101008), the Key Project of the Chinese Ministry of Education (No. 109011), the National Natural Science Foundation of China (Grant No. 11004006), and the Fundamental Research Funds for the Central Universities of China.

References

[1] London F 1938 Phys. Rev. 54 947
[2] Ogg R A Jr 1946 Phys. Rev. 69 243
[3] Schafroth M R 1955 Phys. Rev. 100 463
[4] Blatt J M and Butler S T 1955 Phys. Rev. 100 476
[5] Bardeen J, Cooper L N and Schrieffer J R 1957 Phys. Rev. 108 1175
[6] Osborne M F M 1949 Phys. Rev. 76 400
[7] May R M 1965 J. Math. Phys. 6 1462
[8] Arias T A and Joannopoulos J D 1989 Phys. Rev. B 39 4071
[9] Alexandrov A S 1993 Phys. Rev. B 48 10571
[10] Daicic J, Frankel N E and Kowalenko V 1993 Phys. Rev. Lett. 71 1779
    Daicic J, Frankel N E, Gailis R M and Kowalenko V 1994 Phys. Rep. 237 63
    Daicic J and Frankel N E 1996 Phys. Rev. D 53 5745
[11] Toms D J 1994 Phys. Rev. B 50 3120
    Toms D J 1995 Phys. Rev. D 51 1886

Staden G B and Toms D J 1999 Phys. Rev. E 60 5275
[12] Rojas H P 1996 Phys. Lett. B 379 148
[13] Kirsten K and Toms D J 1997 Phys. Rev. D 55 7797
[14] Alexandrov A S 2006 Phys. Rev. Lett. 96 147003
[15] Yamada K 1982 Prog. Theor. Phys. 67 443
    Carfano D'Auria A, De Cesare L and Rabuffo I 1996 Physica A 225 363
[16] Stamper-Kurn D M, Andrews M R, Chikkatur A P, Inouye S, Miesner H-J, Stenger J and Ketterle W 1998 Phys. Rev. Lett. 80 2027
    Stenger J, Inouye S, Stamper-Kurn D M, Miesner H-J, Chikkatur A P and Ketterle W 1998 Nature 396 345
[17] Simkin M V and Cohen E G D 1999 Phys. Rev. A 59 1528
[18] Eisenberg E and Lieb E H 2002 Phys. Rev. Lett. 89 220403
[19] Daicic J and Frankel N E 1995 Phys. Rev. D 52 7174
[20] Staden G B and Toms D J 1997 arXiv:cond-mat/9712141