Fluctuations of the largest fragment size in percolation and multifragmentation

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Abstract

Aladin data on fragmentation of $^{197}$Au projectiles are remarkably well reproduced by a bond percolation model. A critical behavior is identified on the basis of fluctuations of the largest fragment size.

The present work is motivated by percolation studies [1] which demonstrated that a cumulant analysis of the largest fragment size distributions is a valuable tool in searching for a phase transition (critical behavior) in fragmenting systems. This method is applicable even to very small systems and therefore is well suited for applications to nuclear multifragmentation.

The simulations were performed with a three-dimensional bond percolation model on simple cubic lattices with free boundary conditions to account for the surface presence. Given a bond probability value $p$ (control parameter) and the total number of sites $Z_0$ (system size), the probability distribution $P(Z_{max})$ of the largest cluster size $Z_{max}$ is determined from a large sample of events. The statistical measures as the mean, variance, skewness and kurtosis contain the most significant information about the distribution. Of particular interest are the following cumulant ratios

\begin{align}
K_2 & \equiv \frac{\mu_2}{\langle Z_{max}\rangle^2} = \kappa_2/\kappa_1^2 \\
K_3 & \equiv \frac{\mu_3}{\mu_2^{3/2}} = \kappa_3/\kappa_2^{3/2} \\
K_4 & \equiv \frac{\mu_4}{\mu_2^2} - 3 = \kappa_4/\kappa_2^2,
\end{align}

(1)
where $\langle Z_{\text{max}} \rangle$ denotes the mean value, $\mu_i = \langle (Z_{\text{max}} - \langle Z_{\text{max}} \rangle)^i \rangle$ is the $i$th central moment, and $\kappa_i$ is the $i$th cumulant of $P(Z_{\text{max}})$. $K_2$ is the variance normalized to the squared mean, $K_3$ is the skewness which indicates the distribution asymmetry, and $K_4$ is the kurtosis excess measuring the degree of peakedness with respect to the normal distribution. In the transition region, these quantities obey with good accuracy finite-size scaling relations even for very small systems with open boundaries. This allows to identify universal (independent of the system size) features of $K_i$ at the percolation transition. The transition in finite systems (the pseudocritical point) is associated with the broadest and most symmetric $P(Z_{\text{max}})$ distribution, which is indicated by $K_3 = 0$ and the minimum value of $K_4$ of about $-1$. This criticality signal is approximately preserved when events are sorted by measurable variables correlated with the control parameter (e.g. $Z_{\text{bound}}$) [1].

![Figure 1: The cumulant ratios $K_i$ of the $P(Z_{\text{max}})$ distribution as a function of $Z_{\text{bound}}$. Bond percolation calculations versus the experimental data.](image)

The present work compares percolation predictions with the Aladin S114 data on fragmentation of projectile spectators in $^{197}$Au + Au, In, Cu collisions at the incident energies of 600-1000 AMeV. Details of the experiment
and general characteristics of the data were presented in [2].

Figure 1 examines the cumulants ratios $K_i$ of the largest fragment size distribution $P(Z_{\text{max}})$. The percolation results are plotted in the left diagrams as a function of $Z_{\text{bound}}$ normalized to the system size $Z_0$ for three different system sizes that span over a range expected in the transition region. Percolation events were generated for the bond probabilities uniformly distributed in the interval $[0,1]$, and then sorted according to $Z_{\text{bound}}$. As one can see, the pseudocritical point is located at $Z_{\text{bound}}/Z_0 = 0.84$ independently of the system size. The experimental results are shown in the right diagrams for the Au + Au systems at 600 and 1000 AMeV and for the summed data sets (all targets and energies). Here, $K_i$ are plotted as a function of $Z_{\text{bound}}$. The percolation and experimental patterns of the cumulants are very similar. In particular, the percolation pseudocritical point is well reflected in the data at $Z_{\text{bound}} = 54$. Based on this comparison, the (mean) system size at $Z_{\text{bound}} = 54$ can be estimated as $Z_0 = Z_{\text{bound}}/0.84 \approx 64$.

Figure 2: Percolation predictions versus the experimental data at $Z_{\text{bound}} = 54$: (a) the mean fragment multiplicity as a function of the fragment size (the largest fragment excluded), (b) the mean fragment size as a function of the fragment rank, (c) the multiplicity distribution of fragments with $Z > 2$, (d) the distribution of the second largest fragment charge.

Once the system size is established, we can examine other observables related to fragment charge partitions. In Fig. 2 we compare percolation results with the data at the pseudocritical point $Z_{\text{bound}} = 54$. Panel (a)
shows the fragment size distribution. The model well describes the data over four orders of magnitude. As expected for the percolation pseudocritical point, the distribution follows in some range the asymptotic power-law dependence shown by the dotted line [1]. Panel (b) shows the Zipf-type plot, i.e. the mean size of the largest, second largest,... r-largest fragments plotted against their rank r. The percolation model very well reproduces not only mean values but also event-to-event fluctuations. For example, the next panels show the multiplicity distribution of fragments with Z > 2 and the distribution of the second largest fragment charge.

Similar comparisons performed at other Z_{bound} values in the range from 36 to 66 have also shown an almost perfect resemblance between percolation predictions and the data. The system sizes determined in the analysis are in a good agreement with the experimental estimates [3].

In conclusions, fluctuations of the largest fragment charge observed in the 197 Au spectator fragmentations show the percolation pattern. In analogy to percolation, the pseudocritical point is identified at Z_{bound} = 54 which corresponds to the He-Li isotope temperature corrected for secondary decays of 5.2 ± 0.4 MeV [4]. Detailed comparisons have demonstrated that the experimental fragment charge partitions are remarkably well reproduced by the bond percolation model with no free parameters and no corrections for secondary decays in a wide range of Z_{bound}. In the context of the lattice gas model which is equivalent to bond percolation at the normal density, the success of percolation suggests that clusters are formed at the dense medium and isolated fragments are cold, in line with the "Little big bang" scenario of multifragmentation [5].

This work has been supported by the Polish Ministry of Science and Higher Education grant N202 160 32/4308 (2007-2009).

References

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