ERRATUM TO ‘MAXIMAL SUBALGEBRAS OF CARTAN TYPE IN THE EXCEPTIONAL LIE ALGEBRAS’

There is an error in the statement of [HS16, Lemma 2.8]. The first subalgebra $W \cong W_1 \subseteq H = H(2; \Phi(\tau))^{(1)}$, whose basis is given in the lemma is not a $p$-subalgebra. This is because, for the first element in the basis—the element representing $\partial$ in $W_1$—namely, $(1 - X^{(p-1)}Y^{(p-1)})\partial_X \in H$, one has the identity

$$
\left( (1 - X^{(p-1)}Y^{(p-1)})\partial_X \right)^{[p]} = -Y^{(p-1)}\partial_X,
$$

which is not 0, and not an element of $W$. In fact it is the element ‘$\Theta$’ as in [FSW14, (5.6)] and we shall refer to it as such in this erratum.

Now, one has that the $p$-closure, $W_p = W \oplus \langle \Theta \rangle$ and one has $\Theta[p] = 0$. We check that this does not affect the remainder of the paper.

**Observation 0.1.** Suppose $H$ and $W$ are restricted Lie algebras, with $W$ a subalgebra of $H$ such that the $p$-closure $W_p \cong W \oplus n$ where $n$ is a subspace of $H$ consisting of $p$-nilpotent elements which commute with $H$. Then for any restricted representation $V$ of $H$, we have that the restriction of $V$ to $W$ has restricted composition factors.

**Proof.** Let $U$ be a simple $W$-submodule of $V$. As $V$ is restricted, $n$ acts nilpotently on $V$, hence by Schur’s lemma, $n$ acts trivially on $U$. Thus the image of $W$ in $\mathfrak{gl}(U)$ is restricted. This is to say that $U$ is a restricted representation for $W$. The general result follows by induction after factoring out $U$. \qed

From the observation it follows that the statement of Lemma 2.9 can remain the same. Each representation under consideration is restricted for the minimal $p$-envelope $Z$ of $H$. The calculations in the proof for the case of the adjoint representation find vectors killed by the action of $\partial$ and their $X\partial$ weights, where $\partial$ and $X\partial$ represent the usual elements in $W$. By Observation 0.1 any simple submodule is restricted, hence must contain such an vector and the $X\partial$ weight on it determines the isomorphism type of the simple restricted submodule, thus the conclusion in this case can remain the same.

The arguments finding the restriction to $W$ of Verma modules $M(r)$ with $r \geq 2$ can be improved. Since we are only interested in the composition factors of $M(r)|W$ we may, by Observation 0.1 assume in the ensuing calculations that the element representing $\partial$, that is $x = (1 - X^{(p-1)}Y^{(p-1)})\partial$, acts such that $x^p = 0$, since it will do so on any simple submodule. In particular it does no harm to assume that $x$ kills $x^{p-1}y^b \otimes v_i$, for all $0 \leq a \leq p - 1, \ i \in \{-r, -r + 2, \ldots, r\}$. Thus $M(r)$ can be graded with $x^ay^b \otimes v_i$ in grade $2a$ and one has that $x(M(r)(2a)) \subseteq M(r)(2a + 2)$. As also each $x^ay^b \otimes v_i$ is a weight vector for $h = X\partial_X - Y\partial_Y$ representing $X\partial$ in $W$, with weight $i - a + b$, we have that $M(r)$ as a graded $\langle x, h \rangle$ module is identical to its analogue in the context of the proof of Lemma 2.6. Then Lemma 2.1 and Proposition 2.2 guarantee that the composition factors as a $W$-module are the same as those given in 2.6, (which concur with those in Lemma 2.9).

2010 Mathematics Subject Classification. 17B45.
Now let us recall the main strategy to prove that there are no Hamiltonian-type subalgebras of $g$, if $g$ is exceptional in good characteristic. The lists of all possible composition factors of $g|W$ are given in Tables 3 and 5. It is straightforward to compare these with those coming from Lemmas 2.6 and 2.9 and to see that this is incompatible with any $W_1$ subalgebra of any Hamiltonian Lie algebra.

References

[FSW14] Jörg Feldvoss, Salvatore Siciliano, and Thomas Weigel, *Restricted lie algebras with maximal 0-pim*.

[HS16] Sebastian Herpel and David I. Stewart, *Maximal subalgebras of Cartan type in the exceptional Lie algebras*, Selecta Math. (to appear) (2016).