Study of the opportunities of analytical determination of the origin-destination flows in roundabout intersections based on data provided by fixed observers

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Abstract. For the purposes of the analysis of road traffic at the level of the area surrounding a roundabout intersection there are times when the flows of Origin-Destination (O-D flows) of the respective intersection are of interest - when it is not possible to improve the level of service of the intersection by means of increasing its capacity, but only by diverting some traffic flows from the respective area. Currently, the determining of these O-D flows is based on video recordings of the intersection traffic, followed by laborious and lengthy analyses of the images obtained. But a prolific way, yet not explored in specialised literature, can be the identification of the analytic relations between O-D flow volumes and the volumes of the intersection traffic flows that can be measured by fixed observers (incoming flows, outgoing flows and conflict flows). As a result, this paper presents the research conducted to identify the possible relationships between these deterministic O-D flows and the flows that can be measured by fixed observers. It was demonstrated that, when performing measuring of the flow of vehicles that turn right with fixed observers, there will be analytical deterministic relationships for the 3-armed intersection with return flows, as well as for the 4-armed intersection without return flows. This means that, for 3-armed intersections with return flows and for 4-armed intersections without return flows, it is possible to fully know the O-D flows based on the flows measured by fixed observers (compulsorily involving right-turn flows), which is an absolutely exciting and unique achievement, as knowing these Origin-Destination flows will allow better regulation of the traffic in the area where the roundabout intersection is, in order to facilitate the upstream diversion of some of O-D flows passing through the roundabout intersection.

1. Stating the problem
One of the major problems that major cities in the country face at the moment is the phenomenon of traffic congestion, which occurs mainly during traffic peak hours. This phenomenon is due to a very low level of service of the road network in these periods of time, and the level of service expresses the way in which the road network capacity meets the needs of traffic. Obviously, improving the level of service can be achieved by increasing network capacity and/or reducing traffic demand in the areas where the phenomenon of road congestion occurs [Florea D, Cofaru C and Soica A 2000, Neagu E 2003].

The road network consists of roads and road intersections, but it is to be noted that, almost always, the phenomenon of congestion is determined only by the unsatisfactory service level of road...
intersections [Boroiu A A and Neagu E 2015]. This leads to a keen interest in improving the service level of road intersections in major cities.

In terms of road intersections capacity, it is interesting that, depending on the type, there can be two different approaches to determining their capacity:

- for intersections with access regulated by traffic lights, their capacity is determined only by the geometrical characteristics of the intersection development and by the traffic lights cycle, not being influenced at all by traffic characteristics (even if the traffic is inhomogeneous, through the equivalence of vehicles in standard vehicles, the structure of traffic is eliminated as a determining factor), because there is no conflict traffic between the vehicle flows (there may be conflicts, however, but only among a flow of vehicles that change direction in the intersection and a flow of pedestrians on the specially-arranged crossing);

- in case of the intersections where traffic is regulated by priority signs or by the priority-to-the-right rule, as well as in the case of roundabout intersections, the traffic capacity of the various arms of the intersection (and therefore for the whole intersection) depends on the size of the conflict volumes between the flows of vehicles, which are generally continually variable over time due to the continuous variation of the Origin-Destination flows (O-D flows) that pass through the intersection (defined by the route followed in the intersection).

In the major cities of Romania, the recent years have witnessed a phenomenon of proliferation of roundabout intersections, there being the conviction that this type of intersection brings a number of advantages to the specifics of urban traffic. As a result, there is constant concern to improve the service level of these intersections.

The methodology for determining the service level of roundabout intersections is presented in the standard CNADNR 2009 - Standard for the development of intersections at public roads (created by the specialized company Search Corporation in accordance with Highway Capacity Manual 2003 - Transportation Research Board, National Academies of Sciences, USA, a reference work, published in a later edition in 2010 and recently revised in 2015).

The main mathematical models that evaluate the ability of each arm of the intersection are based only on the volume of the conflict flow that may easily be recorded by a traffic operator (an operator for each arm of the intersection).

There are two alternative mathematical models that can be used for a roundabout intersection with a single lane on each runway: one model has only one variable, i.e. the volume of the conflict flow, and the second has two variables - the volume of the conflict flow and the volume of the outgoing flow on the arm - dimensions that can be determined equally simply, by one or two observers for each arm of the intersection.

So, in order to assess the analytical capacity of a roundabout intersection, it is sufficient to measure all circular flows (and, for one of the methods, all outgoing flows) - and this is absolutely feasible by using 1 or 2 traffic monitors placed on each arm of the intersection - and the verisimilitude of the analytical model can be verified by measuring the actual ingoing or outgoing flow when congestion occurs on the entry lanes of the intersection arms.

As the development of a junction (with the regulation of the route followed by each access lane – see figure 1, with delimitation of the circular lanes by means of continuous markings or lane dividers) is very important to increase its capacity [Kimber R M 1980], there emerges the need to know the O-D traffic flows at the intersection (arm of entry in the intersection and arm of exit from it).

But even more important is the fact that, if one knows these O-D flows, one can reconsider the regulation of traffic in the area where the roundabout intersection is, by diverting some of the flow from the vicinity of the intersection so as to reduce the load of the intersection in question and, as a result, to eliminate the potential for traffic congestion [Boroiu A 2003].
Figure 1. Preselection depending on the desired route to avoid lane changing on the annular path [Kimber R M 1980].

It is very difficult to count these “Origin-Destination” flows because this would mean registering the arm of entry and the arm of exit for each vehicle that enters the respective intersection, i.e., in fact, tracing the route of each vehicle. This is practically impossible to be done by traffic monitors, the only feasible solution being that of recording the traffic in the whole intersection and then identifying each vehicle. But this method requires the use of expensive equipment and, moreover, is very laborious and consumes a lot of time.

As a result, the following question arises: is it possible for an intersection with the traffic regulated in roundabout, based on measured values (with fixed observers) for ingoing flows, outgoing flows and circular flows - figure 2a, to determine the Origin - Destination flows - figure 2b?

Figure 2. The flows that can be measured with fixed observers (a) and the Origin – Destination flows (b) in a roundabout intersection [CNADNR 2009, Highway Capacity Manual 2010].

2. Research conducted
To find the answer to this question, the road circulation in the roundabout intersection was formalized by noting the volume of traffic flows (number of standard vehicles in a particular timeframe) as similar as possible to the notations in specialised literature (written in Romanian or in a foreign language), as follows:
• ingoing flows volume: \( V_{i,m} \);
• outgoing flows volume: \( V_{o,n} \);
• conflict (circular) flows volume: \( V_{c,m} \);
• Origin - Destination flows volume: \( V_{m,n} \).

In the above notations, the index \( m \) represents the sequential number of the ingoing arm (origin), \( m = 1, 2, 3, 4, \ldots \), and \( n \) is the sequential number of the outgoing arm (destination), \( n = 1, 2, 3, 4, \ldots \).

The first three categories of flows are the flows that will be measured, so they represent the given data for the previously stated problem, and the last category of flows represents the values to be determined, therefore they constitute the unknown data of the problem. As a result, this issue will be analysed to discover if it can be solved, starting with the simplest intersection.

2.1. Roundabout intersection with two arms

This intersection is actually a mere road arrangement with the purpose of slowing the traffic (vehicles must slow down when crossing the annular path) and possibly to allow the return of vehicles.

In this case, all O-D flows (\( V_{11}, V_{12}, V_{21} \) and \( V_{22} \)) are in fact flows that can be measured directly [Deliu M 2016], so the issue does not exist in this case.

2.2. Roundabout intersection with three arms

The case with the fewest variables is the one in which there are no flows back (return flows), a situation in which the following relations apply:

\[
\begin{align*}
V_{i1} &= V_{i2} + V_{i3} \\
V_{i2} &= V_{i3} + V_{i1} \\
V_{i3} &= V_{i1} + V_{i2} \\
V_{c1} &= V_{c2} + V_{c3} \\
V_{c2} &= V_{c3} + V_{c1} \\
V_{c3} &= V_{c1} + V_{c2}
\end{align*}
\]

(1)

It is noted that in this case there are 9 linear analytical relations and that the 3 O-D flows for the left turn result, actually, directly from the measurement:

\[
\begin{align*}
V_{c2} &= V_{c1} \\
V_{c3} &= V_{c2} \\
V_{c1} &= V_{c3}
\end{align*}
\]

(2)

The other three Origin-Destination flows (for the right turn) result quite easily as relations between the flows measured.

\[
\begin{align*}
V_{i1} &= V_{c1} - V_{c3} = V_{i3} - V_{c1} \\
V_{i2} &= V_{c2} - V_{c1} = V_{i1} - V_{c2} \\
V_{i3} &= V_{c3} - V_{c2} = V_{i2} - V_{c3}
\end{align*}
\]

(3)

In conclusion, it is sufficient to measure the 3 circular flows (which will be needed to calculate the level of service) and the 3 ingoing flows or three outgoing flows (preferably, the 3 outgoing flows because they will be useful when determining the intersection capacity and the alternative method specified above).
The most general case for the intersection under study is that in which possible return flows are taken account of on each of the three arms of the intersection:
- there are 9 Origin-Destination flows (the number of O-D flows is equal to the square of the number of branches: \(3^2 = 9\));
- there are \(3 \times 3 = 9\) flows measurable with fixed observers (on each of the 3 arms there are 3 measurable flows: 1 ingoing flow, 1 outgoing flow and one flow of conflict);

It appears that in this case the number of analytical relationships between measured and Origin-Destination flows equals the number of unknowns (Origin-Destination flows), so this is the case of a system of 9 linear equations with 9 unknowns:

\[
\begin{align*}
V_{i1} &= V_{12} + V_{13} + V_{11} \\
V_{i2} &= V_{23} + V_{21} + V_{22} \\
V_{i3} &= V_{31} + V_{32} + V_{33} \\
V_{e1} &= V_{21} + V_{31} + V_{11} \\
V_{e2} &= V_{32} + V_{12} + V_{22} \\
V_{e3} &= V_{13} + V_{23} + V_{33} \\
V_{c1} &= V_{32} + V_{22} + V_{33} \\
V_{c2} &= V_{13} + V_{11} + V_{33} \\
V_{c3} &= V_{21} + V_{11} + V_{22}
\end{align*}
\]

This system of equations can be presented as a matrix to analyse if it is determined and solvable (table 1).

**Table 1.** The matrix of road flows in a 3-armed roundabout intersection (including the three return flows).

| Known data | \(V_{1,2}\) | \(V_{1,3}\) | \(V_{1,1}\) | \(V_{2,3}\) | \(V_{2,2}\) | \(V_{2,1}\) | \(V_{3,3}\) | \(V_{3,2}\) | \(V_{3,1}\) | \(V_{m,n}\) |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \(V_{i,m}\) | \(V_{i,1}\)  | 1            | 1            | 1            | 0            | 0            | 0            | 0            | 0            | 0            |
|            | \(V_{i,2}\)  | 0            | 0            | 0            | 1            | 1            | 1            | 0            | 0            | 0            |
|            | \(V_{i,3}\)  | 0            | 0            | 0            | 0            | 0            | 1            | 1            | 1            | 1            |
| \(V_{e,m}\) | \(V_{e,1}\)  | 0            | 0            | 1            | 0            | 1            | 0            | 1            | 0            | 0            |
|            | \(V_{e,2}\)  | 1            | 0            | 0            | 0            | 0            | 1            | 0            | 1            | 0            |
|            | \(V_{e,3}\)  | 0            | 1            | 0            | 1            | 0            | 0            | 0            | 0            | 1            |
| \(V_{c,m}\) | \(V_{c,1}\)  | 0            | 0            | 0            | 0            | 0            | 1            | 0            | 0            | 1            |
|            | \(V_{c,2}\)  | 0            | 1            | 1            | 0            | 0            | 0            | 0            | 0            | 1            |
|            | \(V_{c,3}\)  | 0            | 0            | 1            | 0            | 1            | 0            | 0            | 0            | 0            |

The Microsoft Excel program was used (Matlab can also be used) and it was found that the determinant of the matrix \(M_{9,9}\) (9 rows and 9 columns) is zero, so the system is undetermined. Continuing the analysis, it was found that the matrix rank is 6, so the only solvable case is that of the intersection with three arms without return flows, but that had already been solved before, even by a simpler method – the reduction method. Nevertheless, the research can be continued in the situation in which the right-turn flows are deemed to be measured (the only O-D flows which, however, can be measured by one fixed observer), so that the issue would be resumed with these flows as “known data”. There follows a system of equations whose associated matrix is presented in table 2 (\(V_{id}\) represents the volume of traffic on the arm that turns to the right).
Table 2. The matrix of road flows in a 3-armed roundabout intersection (including the three return flows) considering the known right-turn flows.

| Known data | $V_{1,3}$ | $V_{1,1}$ | $V_{2,1}$ | $V_{2,2}$ | $V_{3,2}$ | $V_{3,3}$ |
|------------|-----------|-----------|-----------|-----------|-----------|-----------|
| $V_{i,1}$- $V_{1d}$ | 1 | 1 | 0 | 0 | 0 | 0 |
| $V_{i,2}$- $V_{2d}$ | 0 | 0 | 1 | 1 | 0 | 0 |
| $V_{i,3}$- $V_{3d}$ | 0 | 0 | 0 | 0 | 1 | 1 |
| $V_{e,1}$- $V_{3d}$ | 0 | 1 | 1 | 0 | 0 | 0 |
| $V_{e,2}$- $V_{1d}$ | 0 | 0 | 0 | 1 | 1 | 0 |
| $V_{e,3}$- $V_{2d}$ | 1 | 0 | 0 | 0 | 0 | 1 |
| $V_{e,1}$ | 0 | 0 | 0 | 1 | 1 | 1 |
| $V_{e,2}$ | 1 | 1 | 0 | 0 | 0 | 1 |
| $V_{e,3}$ | 0 | 1 | 1 | 1 | 0 | 0 |

Using the Microsoft Excel program, it is noted that this matrix with 9 equations and 6 unknowns (M$_{9,6}$) has the rank 6, therefore the system is solvable: if the 3 right-turn flows are also measured, all the other O-D flows can be determined analytically (the 3 left-turn flows and the 3 flows of return).

2.3. Roundabout intersection with four arms

It is reasonable to consider at first the simplest scenario possible, when no flows of return on any arm are taken into account, and only if it is found that in this case the number of relations is higher than the number of unknowns (O-D flows) will the decision be taken to analyse more complex scenarios (with one or more flows of return). Thus, for the intersection with 4 arms, the relations between flows (no return flows are deemed to exist) will be:

$$V_{1} = V_{12} + V_{13} + V_{14}$$
$$V_{2} = V_{23} + V_{24} + V_{21}$$
$$V_{3} = V_{34} + V_{31} + V_{32}$$
$$V_{4} = V_{41} + V_{42} + V_{43}$$
$$V_{e1} = V_{21} + V_{31} + V_{41}$$
$$V_{e2} = V_{32} + V_{42} + V_{12}$$
$$V_{e3} = V_{43} + V_{13} + V_{23}$$
$$V_{e4} = V_{14} + V_{24} + V_{34}$$
$$V_{c1} = V_{32} + V_{42} + V_{43}$$
$$V_{c2} = V_{43} + V_{13} + V_{14}$$
$$V_{c3} = V_{14} + V_{24} + V_{21}$$
$$V_{c4} = V_{21} + V_{31} + V_{32}$$

(5)

It is noted that there are 12 linear analytical relations which comprise the 12 Origin-Destination flows (12 variables), so that this is the case of a linear system of 12 equations with 12 unknowns. Obviously, it is not appropriate to analyse a more complex scenario, not even with a return flow, as the number of unknowns would exceed the number of linear equations and, therefore, the problem would be unresolvable analytically. So, this is a linear system of 12 equations with 12 unknowns (12 1st-
degree equations that are polynomials with all variables in the 1st power), which can be submitted as a matrix to analyse if it is determined and if it is solvable (table 3).

**Table 3.** The matrix of road flows in a 4-armed roundabout intersection (without return flows).

| Known data | Unknown data, $V_{m,n}$ |
|------------|-------------------------|
| $V_{i,m}$  | $V_{i,1}$ $V_{i,3}$ $V_{i,4}$ $V_{2,3}$ $V_{2,1}$ $V_{3,4}$ $V_{3,1}$ $V_{3,2}$ $V_{4,1}$ $V_{4,2}$ $V_{4,3}$ |
| $V_{e,n}$  | $V_{e,1}$ $V_{e,2}$ $V_{e,3}$ $V_{e,4}$ |
| $V_{c,m}$  | $V_{c,1}$ $V_{c,2}$ $V_{c,3}$ $V_{c,4}$ |

Using the Microsoft Excel program, it was found that the matrix determinant is zero, so the system is not determined, that is it has no single solution. It was found that the rank of the matrix is 8, that is, only eight are main equations, which means that only 8 O-D flows can be determined analytically (depending on the measured flows and the remaining 4 O-D flows, which occur as parameters).

So, in the case of the roundabout intersection with 4 arms, the 12 Origin-Destination flows cannot be determined analytically depending on the 12 flows that can be measured with fixed observers.

However, since it was found that the matrix rank is 8, i.e. there are 8 main equations, which means that 8 O-D flows can be determined analytically, there is a continuation of the exploration that presents interest: if the flows of vehicles that turn right - $V_{12}$, $V_{23}$, $V_{34}$, $V_{41}$ – are considered to be parameters (it being, however, possible to measure these ones with the help of fixed observers), can the other $4 \times 2 = 8$ O-D flows be determined analytically (flow of vehicles going forward and flows of vehicles turning left)?

The known data will be those 12 in the $V_{12,12}$ matrix, plus the 4 flows of vehicles that turn right, i.e. 16 known data. Thus, the relations between the flows (introducing in the left member of the equation the flows that will be considered parameters - $V_{12}$, $V_{23}$, $V_{34}$, $V_{41}$) will be:

\[
\begin{align*}
V_{11} - V_{12} & = V_{13} + V_{14} \\
V_{22} - V_{23} & = V_{24} + V_{21} \\
V_{33} - V_{34} & = V_{31} + V_{32} \\
V_{44} - V_{41} & = V_{42} + V_{43} \\
V_{e,1} - V_{e,4} & = V_{e,1} - V_{e,4} \\
V_{e,2} - V_{e,3} & = V_{e,2} - V_{e,3} \\
V_{e,3} - V_{e,4} & = V_{e,3} - V_{e,4} \\
V_{c,1} - V_{c,4} & = V_{c,1} - V_{c,4}
\end{align*}
\]  

(6)
Noting in the left member of the first 8 equations (known data, calculated as difference between 2 measured flows) with \( V_{id} \) the flows on the arm \( i \) that turn to the right, the matrix of the system of 12 equations with 8 unknowns (and 12 known data, resulted from the measurement of 16 road flows - \( V_{j,i} \), \( V_{j,e} \), \( V_{j,c} \) and \( V_{j,d} \)) will be that presented in table 4.

**Table 4.** The matrix of the 8 road flows on the direction forward and on the direction to the left for a 4-armed roundabout intersection (with no return flows).

| Known data | \( V_{1,3} \) | \( V_{2,4} \) | \( V_{2,1} \) | \( V_{3,1} \) | \( V_{3,2} \) | \( V_{4,2} \) | \( V_{4,3} \) |
|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| \( V_{11}-V_{1d} \) | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| \( V_{12}-V_{2d} \) | 0 | 0 | 1 | 1 | 0 | 0 | 0 |
| \( V_{13}-V_{3d} \) | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| \( V_{14}-V_{4d} \) | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| \( V_{c1}-V_{4d} \) | 0 | 0 | 0 | 1 | 1 | 0 | 0 |
| \( V_{c2}-V_{1d} \) | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| \( V_{c3}-V_{2d} \) | 1 | 0 | 0 | 0 | 0 | 0 | 1 |
| \( V_{c4}-V_{3d} \) | 0 | 1 | 1 | 0 | 0 | 0 | 0 |
| \( V_{c4}-V_{4d} \) | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| \( V_{c2}-V_{c1} \) | 1 | 1 | 0 | 0 | 0 | 0 | 1 |
| \( V_{c3}-V_{c2} \) | 0 | 1 | 1 | 1 | 0 | 0 | 0 |
| \( V_{c4}-V_{c3} \) | 0 | 0 | 0 | 1 | 1 | 1 | 0 |

Using the Microsoft Excel program, it was found that the rank of the matrix \( M_{12,8} \) is eight, that there are 8 main equations, which means that the 8 O-D unknown flows can be determined analytically.

It is found that the last 4 equations (which require knowledge of the 4 volumes of conflict) and the first 4 equations (which require knowledge of the 4 ingoing and the 4 right-turn volumes) or the equations 5-8 (which require knowledge of 4 outgoing volumes and the 4 right-turn volumes) can be retained as principal equations.

*It follows that the 4 flows of forward running and the 4 flows of left turn can be determined analytically if there are measurements of the 4 volumes of conflict, the 4 volumes of right turn and other four flows: either the 4 ingoing flows, or the 4 outgoing flows.*

### 3. Conclusions
A broad theoretical research was developed, in an original way, to discover whether it is possible to determine analytically the Origin-Destination flows in the roundabout intersections based on traffic measurements performed by fixed observers.

We found that, in the case of the roundabout intersections with two arms (which, actually, represent circular arrangements on a road way, for the purposes of slowing the traffic and enabling the return of vehicles), analytical solving is not required.

In the case of the roundabout intersections with 3 arms, it is necessary to determine the flows of conflict and the ingoing or outgoing flows on the arms to get the 6 Origin-Destination flows. Instead, the scenario with return flows proved to be unsolvable based on flows measured with fixed observers. One single possibility exists in this case: measuring with fixed observers the flow of vehicles that turn right (this is possible, however), a situation in which the 6 Origin-Destination flows of vehicles can be determined for the vehicles that turn left and which turn back in the intersection.

The exploration of the theoretical research continued with the case of the intersection with 4 arms, where it was demonstrated that it is not possible to determine the 12 Origin-Destination flows on the
basis of the 12 flows measured by fixed observers, but it was found that, if the flows of vehicles that
turn right are considered to be parameters (which can be measured, yet, by fixed observers), the new
system of 12 equations with eight unknowns (the Origin-Destination flows of vehicles that go forward
or the Origin-Destination flows of vehicles that turn left) is solvable, with a unique solution for the 8
unknown O-D flows.

This means that, in the case of the intersections with 3 arms and flows of return and in that of the
intersections with four arms without flows of return it is possible to fully know the O-D flows based
on measurements performed by fixed observers (involving compulsorily the right-turn flows), which is
an absolutely interesting and unique achievement, since knowing these Origin-Destination flows will
allow the reconsideration of traffic regulation in the area of the roundabout junction, in order to make
the traffic smooth in the area and eliminate the possibilities of road congestion occurrence.

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