Effect of double local quenches on the Loschmidt echo and entanglement entropy of a one-dimensional quantum system

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Abstract. We study the effect of two simultaneous local quenches on the evolution of the Loschmidt echo (LE) and entanglement entropy (EE) of a one-dimensional transverse Ising model. In this work, one of the local quenches involves the connection of two spin-1/2 chains at a certain time and the other corresponds to a sudden change in the magnitude of the transverse field at a given site in one of the spin chains. We numerically calculate the dynamics associated with the LE and the EE as a result of such double quenches, and discuss the various timescales involved in this problem using the picture of quasiparticles (QPs) generated as a result of such quenches. We perform a detailed analysis of the probability of QPs produced at the two sites and the nature of the QPs in various phases, and obtain interesting results. More specifically, we find partial reflection of these QPs at the defect center or the site of \( h \)-quench, resulting in new timescales which have never been reported before.

Keywords: entanglement in extended quantum systems (theory), quantum quenches, spin chains, ladders and planes (theory), quantum phase transitions (theory)
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1. Introduction

Recently, quantum information theoretic measures such as decoherence [1–3], entanglement and fidelity [4–7] have become the subject of immense interest. Close to the quantum critical point (QCP) of a quantum many-body system, these quantities show peculiar behaviors and hence can be regarded as a tool for detecting the QCP [4]. Decoherence that signifies the loss of coherence of the system when it interacts with the environment is an important observable for quantum computation.

In this context the Loschmidt echo (LE), which quantifies the decoherence, is studied extensively [8–14]. The LE is defined as the square of the overlap of the two wave functions \( |\psi(t)\rangle \) and \( |\psi_i(t)\rangle \) evolving with two different Hamiltonians \( H \) and \( H_0 \), respectively, i.e.

\[
\mathcal{L}(t) = |\langle \psi(t)|\psi_i(t)\rangle|^2.
\]

Initially, both states are prepared in the ground state \( H_0 \), given by \( |\psi_i\rangle \). The LE provides information about how small perturbations during an evolution can result in the decoherence of the state of the system. For example, the Hamiltonian \( H \) can be generated from the system Hamiltonian \( H_0 \) by coupling a qubit to \( H_0 \) globally or locally at a
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site. In this case, it can be shown that the LE in fact measures the decoherence of the qubit as a result of coupling to the system Hamiltonian [8]. The more a qubit decoheres, the greater is its chance of losing the stored information, making LEs an important quantity for information processing and storage. On the other hand, LEs can also be used to detect the presence of a QCP by showing a sharp dip in the Hamiltonian at the QCP when \( H \) and \( H_0 \) are close to each other. The quantity LE was later used by Heyl et al [15] to define a new concept called dynamical phase transitions, which uses the resemblance between the canonical partition function of an equilibrium system and the amplitude of the LE, \( G(t) \), defined as \( G(t) = \langle \psi | e^{-iHt} | \psi \rangle \).

At the same time, entanglement in quantum many-body systems, which is the measure of quantum correlations between the two subsystems of the main system, has also become a topic of intensive research interest across the last few years [16–33]. One of the quantities used to measure the entanglement between the two subsystems is the von Neumann entropy [34–37]. Consider a bipartite system divided into two subsystems \( A \) and \( B \) of length \( L_A \) and \( L_B \) with a total length \( L = L_A + L_B \). If the whole system is in a quantum pure state \( | \psi \rangle \) with density matrix \( \rho = | \psi \rangle \langle \psi | \), then the von Neumann entropy \( S \) of subsystem \( A \) with reduced density matrix \( \rho_A = \text{Tr}_B(\rho) \) is defined as

\[
S = -\text{Tr}(\rho_A \ln \rho_A). \tag{2}
\]

The entanglement entropy (EE), or \( S \), increases with increasing quantum correlations (entanglement) between the two subsystems. The EE exhibits distinct scaling relations at and close to a QCP with the shortest length scale of the system, i.e. \( S \sim \ln L_A \) near the critical point whereas it scales as \( S \sim \ln \xi \) when away from critical point [18, 20, 34].

With our understanding of the behavior of the EE in an equilibrium system getting better, a considerable amount of focus is also given to the EE in systems which are out of equilibrium [38–46]. The experimental demonstration of such non-equilibrium dynamics using optical lattices [47] also contributes to the sudden upsurge in studies related to decoherence and entanglement in out of equilibrium systems. One of the ways of generating such non-equilibrium dynamics is a sudden quench. A sudden quench in the system can be performed locally or globally. In a global quench, a parameter of the Hamiltonian is changed suddenly at all sites resulting in non-equilibrium dynamics. In this process, the EE generally shows a linear increase in time \( t \) up to some time \( t_0 \) [38]. The local quench is defined as a local change of a parameter of the Hamiltonian. For example, the EE between two critical subsystems \( A \) and \( B \) of a homogeneous one-dimensional periodic chain which are disconnected for \( t < 0 \) and connected at \( t = 0 \) increases as \( S \sim \frac{2c}{3} \ln t \) for \( t \ll L/v_{\text{max}} \) [39, 40, 45], provided \( t \gg a/v \) where \( a \) is the lattice spacing, \( v \) is the velocity of the excitations at the critical point and \( v_{\text{max}} \) is its maximum value; the factor 2 in the prefactor of the expression of \( S \) is not present for open boundary conditions. Such studies are important in the context of information propagation through a quantum many-body system.

In this paper, we consider two independent transverse field Ising spin chains in the ferromagnetic or critical phase. For the times \( t < 0 \), the two spin chains, which are in their respective ground states, are disconnected and are later joined at \( t = 0 \) (\( J \)-quenching). Simultaneously, we change the magnitude of the transverse field at a single site of one of the chains (\( h \)-quenching). This results in a non-equilibrium evolution of the state of the system. Such a double quench physically corresponds to the
removal of a weak bond and the addition of some magnetic impurity in the system, which causes a local increase in the strength of the magnetic field or the magnetization at that site. While such local defects, corresponding to removal of a weak bond, can appear in x-ray absorption problems [39, 48], it is difficult to control their occurrence such that both quenches occur at the same time. On the other hand, due to the limitation of the numerical technique that we employ here, it is important that the two quenches happen simultaneously. Nevertheless, this problem involves studying the scattering of waves (quasiparticles) at the defect center (site of h-quench), as we shall show later, and hence is a step towards extending the quantum mechanical scattering problem to a many-body system. In short, we investigate the effect of local increases in the magnetic field which may act as scattering centers for the wave-like quasiparticles (QPs) produced after the local J-quench.

Our aim here is to understand this non-trivial evolution using the picture of QPs generated by the above mentioned local quenches. In the process of understanding the dynamics of the QPs produced at the two sites, we observe the following salient features of the dynamics in the quenches studied: (i) for the specific type of quenches studied in this paper, QPs are produced at the site of an h-quench with a smaller probability than they are produced at the site of a J-quench. If both the quenches are of same order, the QP production probability is also of same order; (ii) A partial/full reflection of QPs occurs at the site of the h-quench; (iii) We observe the particle-like nature of QPs when away from the critical point but an extended nature in the critical region [49]. To the best of our knowledge, this has never been reported anywhere else in quantum information theoretic measures. We find some interesting timescales in double quenches which can be successfully explained using the reflection picture of the QPs generated.

The organization of the paper is as follows: we discuss the model studied in this paper and briefly mention the numerical techniques in section 2.1 followed by a discussion of the semiclassical theory of QP generation in section 2.2. In section 3, we present the results of the LE and the EE in the critical region for various geometries, and we present the results for the ferromagnetic region in section 4. A comparison between the two quenches studied in this paper is made in section 5, using the semiclassical picture. We conclude the paper with our main results in section 6. We have also added two appendices outlining the numerical techniques used.

2. Model

2.1. Exact diagonalization

The Hamiltonian we consider here is that of a 1D Ising chain in a transverse field given by

\[ H = -\sum_{n}(J_n\sigma_n^x\sigma_{n+1}^x + h_n\sigma_n^z), \]  

where \( h_n \) and \( J_n \) are the site-dependent transverse magnetic fields and cooperative interactions, respectively, and \( \sigma_n^x \) and \( \sigma_n^z \) are standard Pauli matrices at the lattice site \( n \). For the homogeneous case (\( h_n = h \) and \( J_n = J \)), the model in equation (3) has a QCP at \( J = h \).
that separates the ferromagnetic and quantum paramagnetic phases. Using Jordan–
Wigner transformations followed by a Fourier transformation for a homogeneous and
periodic chain, the energy spectrum obtained for the Hamiltonian \( (3) \) is \([50, 51]\)
\[
\Lambda_q = \pm 2J \sqrt{(h + \cos q)^2 + \sin^2 q},
\]
where \( q \) is the momentum which takes discrete values given by \( q = 2\pi m/L \) with
\( m = 0 \cdots L - 1 \) for a finite system of length \( L \).

On the other hand, such homogeneous systems are very rare in nature. At the very
least one finds some local defects, no matter how pure the material is. The general method
adopted in order to study systems which are not homogeneous, which we use in this paper,
is outlined below. Following Jordan–Wigner transformation, the Hamiltonian in equation
\((3)\) can be described by a quadratic form in terms of spinless fermions \( c_i \) and \( c_i^\dagger \) \([50]\)
\[
H = \sum_{i,j} \left[ c_i^\dagger A_{i,j} c_j + \frac{1}{2}(c_i^\dagger B_{i,j} c_j^\dagger + \text{h.c.}) \right].
\]
where, \( A \) is a symmetric matrix due to hermiticity of \( H \) and \( B \) is an antisymmetric
matrix which follows from the anticommutation rules of \( c_i \) operators. The elements of
the matrices thus obtained are:
\[
A_{i,j} = -(J\delta_{i,j+1} + J\delta_{i,j-1}) - 2h\delta_{i,j},
B_{i,j} = -(J\delta_{i,j+1} - J\delta_{i,j-1}).
\]
The above Hamiltonian can be diagonalized in terms of the normal mode spinless Fermi
operators \( \eta_k \) given by the relation \([50]\),
\[
\eta_k = \sum_i (g_k(i) c_i + h_k(i) c_i^\dagger),
\]
where \( g_k(i) \) and \( h_k(i) \) are real numbers. In terms of these operators, the Hamiltonian
takes the diagonal form
\[
H = \sum_k \Lambda_k \left( \eta_k^\dagger \eta_k - \frac{1}{2} \right),
\]
with \( \Lambda_k \) being the energy of different fermionic modes with index \( k \). These \( \Lambda_k \)s are given
by the solutions of the eigenvalue equations,
\[
(A - B)(A + B)\Phi_k = \Lambda_k^2 \Phi_k
(A + B)(A - B)\Psi_k = \Lambda_k^2 \Psi_k.
\]
It can be shown that the elements of the eigenvectors are related to the \( g \) and \( h \)
matrices used to diagonalize the Hamiltonian as follows: \( \Phi_k(i) = g_k(i) + h_k(i) \) and
\( \Psi_k(i) = g_k(i) - h_k(i) \). We calculate the LE and the EE using \( \Phi, \Psi, g \) and \( h \), as discussed
in appendices A and B.

### 2.2. Semiclassical theory of quasiparticles

When a system at zero temperature is taken away from its ground state by applying
some perturbation, the state of the system undergoes a non-equilibrium evolution with

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respect to the final Hamiltonian. The initial state, which now is an excited state, is a source of QPs corresponding to the final Hamiltonian. Recently, such non-equilibrium dynamics have been studied using a semiclassical picture of the QPs generated for global [49, 52] and local quenches [53], and excellent agreement between the numerical results and the semiclassical theory has been obtained. We now briefly describe this theory, which will be used to explain the various timescales observed in our numerical calculations. For global quenches from $h = 0$ to a very small $h$ value it can be shown that these QPs are wave packets of low-lying excitations, as discussed in detail in [49]. Due to the conservation of momentum, QPs of a given momentum are always produced in pairs, with their group velocity $v_g(k) = |\frac{\partial \epsilon_k}{\partial k}|$ being equal and opposite to each other. As discussed in [49], these QPs in the small $h$ limit can be considered as classical particles (sharply defined QPs) which, when they cross a site, simply flip the spin at that site. Though this picture is discussed for a very specific quench (a small quench), it has been verified for stronger quenches and for quenches in the paramagnetic phase with slight modifications. It is also argued that these QPs are no longer point particles but are extended objects as the critical point is approached due to the large correlation length. In the following sections, we shall try to explain our numerical results, at least qualitatively, with this point-like picture of QPs for spin chains in the ferromagnetic and in the critical region.

3. Loschmidt echo and entanglement entropy for a critical chain

We first study double quenches for a critical chain where some work has already been done in [40, 45, 53] for local $J$-quenches. As discussed before, we consider the simultaneous application of two types of local perturbations to the system and study the time evolution of the LE and von Neumann EE as a result of such quenches. Initially the spin chain is prepared in the ground state of $H = H_1 + H_2$ where $H_1$ and $H_2$ are the Hamiltonians of the two decoupled homogeneous Ising chains of length $L_1$ and $L_2$, respectively, with open boundary conditions ($J_{L_1} = J_{L_2} = 0$). Two simultaneous quenches are performed at $t = 0$, namely (i) the two spin-$1/2$ chains are suddenly connected together resulting in a chain of total length $L = L_1 + L_2$ and (ii) the transverse field at a particular site $L'$ belonging to either the chain 1 or 2 is changed from $h$ to $h + \delta$. The system then evolves with the final Hamiltonian

$$H_f = H_1 + H_2 + H_{12}' - \delta \sigma^z_{L'},$$

(10)

where $H_{12}'$ defines the connection between the two spin chains of length $L_1$ and $L_2$ and is of the form $J \sigma^x_{L_1} \sigma^x_{L_1+1}$. At the same time, the term $-\delta \sigma^z_{L'}$ in the Hamiltonian corresponds to the $h$-quenching which changes the magnitude of the transverse field at site $L'$ from $h$ to $h + \delta$. We incorporate these quenches numerically by considering a single spin chain of total length $L = (L_1 + L_2)$ with the first $L_1$ spins forming system 1 and the remaining ones forming system 2. They are disconnected for $t < 0$, which can be incorporated in the numerics by putting $J_{L_1} = 0$ in the initial Hamiltonian. At $t = 0$ it is increased to $J$, which is also the interaction strength at all the other sites. For all of our calculations, we have set $J = 1$. The details of the numerical calculations for the LE and EE are outlined in the appendix, see also [10, 42].

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As we switch on the two local perturbations discussed above, there is a local increase in the energy of the system at the site of the local perturbations [40, 53]. These sites then become the source of QP production. Henceforth, we shall refer to the QPs created due to the $h$-quenching at $L'$ as QP$^1$, and the corresponding left and right moving QPs as QP$^1_L$ and QP$^1_R$, respectively. Similarly, the left and right moving QPs created at the $L_1$ site of $J$-quenching will be referred to as QP$^2_L$ and QP$^2_R$, respectively. Below, we discuss various timescales in the double quench problem and present our results for the evolution of the LE and the EE for various cases, in the light of QP propagation.

### 3.1. Timescales

The evolution of LE and EE following double quenches can be explained by defining the few timescales present due to the propagation of the generated QPs at the site of local quenches. In general, for the readability of the paper, we shall assume $L' < L_1 < L_A$ ($L_A$ is explained later in the context of the EE) throughout, but the case with $L' > L_1$ is also presented and discussed in the captions of various figures. These timescales are determined by the fastest moving QPs, with maximal group velocity $v_{\text{max}} = \max_k v_g(k)$, and are given by $t_1 = (2L')/v_{\text{max}}$ (time taken for QP$^1_L$ to come back), $t_2 = 2(L - L')/v_{\text{max}}$ (time taken for QP$^1_R$ to come back), $t_3 = (2L_1)/v_{\text{max}}$ (time taken for QP$^2_L$ to come back) and $t_4 = (2L_2)/v_{\text{max}}$ (time taken for QP$^2_R$ to come back). Other than the obvious timescales mentioned above, two more timescales are observed numerically. The first is $t'$, given by $2|L_1 - L'|/v_{\text{max}}$, which appears to be the time taken by QP$^2_L$ (for $L_1 > L'$) to reach $L'$ and get partially reflected at $L'$ where it finds a change in potential from $h$ to $h + \delta$. The second timescale is the same as $t_2$ in magnitude, but is due to QP$^2$ and is given by $t'' = 2(L - L')/v_{\text{max}}$, which is the time taken by the QP$^1_L$ (QP$^1_R$) to get reflected at $L'$ (right-hand boundary) and come back to $L_1$ after getting fully (partially) reflected at the right-hand boundary ($L'$). At time period $T = 2L/v_{\text{max}}$, all of the QPs return to their origin if there is no partial reflection. It should be noted that $v_{\text{max}}$ for the homogeneous transverse Ising model with elements as defined in equation (6) is 2 at both the critical point and in the paramagnetic phase. We shall use the same value of $v_{\text{max}}$ in our case since the numerically obtained value of $v_{\text{max}}$ by differentiating the eigenvalues is also close to 2.

### 3.2. Evolution of the Loschmidt echo and the entanglement entropy

We now present the numerical results showing the effect of local quenching of the spin chain on the evolution of LE and EE. Let us first study the evolution of the LE. In figure 1, we present the evolution of the LE for different values of $\delta$ with $L_1 = L_2$ and $L' = L/3$. As the strength of the $h$-quench increases ($\delta$ increases), the LE shows a faster decay, as expected. On the other hand, the effect of $L'$ on the timescales is shown in figure 2. For $J$-quenching alone [45], the LE shows three timescales given by $t_3$, $t_4$ and the time period $T$, as discussed in section 3.1 (see the $L' = 0$ plot of figure 2). Since the LE is the overlap of two wave functions which had complete overlap initially, it starts decreasing from one at $t = 0$ until one of the QPs reaches the boundary. Intuitively, the return of the reflected QP will undo the effect of its dynamics, resulting in an increase in the LE. Thus, one expects to see peaks at $t_3$ and $t_4$ when both of the pair of QPs

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Figure 1. The LE as a function of time for different values of $\delta$ with the $J$-quench at $L_1 = L/2$ and the $h$-quench at site $L' = L/3$. The transverse field at $L'$ is changed from 1 to $1 + \delta$. For the $J$ quenching alone (i.e. $\delta = 0$), the LE has a peak at $t_3 = L/v_{\text{max}} = 150$ and $T = 2t_3$, where $v_{\text{max}} = 2$ and $L = 300$. By applying two local perturbations simultaneously at time $t = 0$, we observe a small peak at $t' = 50$ and a comparatively stronger peak at $t'' = 200$. We also note small fluctuations near $t_1 = 100$ which are more clearly seen for $\delta = 1$ curve.

return to their origin. When the transverse field term at $L'$ is changed from $h$ to $h + \delta$ together with $J$-quenching, we expect to see the following additional timescales, in parallel with the discussion in the previous section: $t_1 = (2L')/v_{\text{max}}$, $t' = 2(L_1 - L')/v_{\text{max}}$, $t_2 = t'' = 2(L - L'/v_{\text{max}}}$. All of the above mentioned timescales can be clearly seen in figure 2 (main), except for $t_1$. We shall try to set out the argument for this later. If we simply have $h$-quench from 1 to 2, we find that the decay in the LE is very small compared to the $J$-quench, but we do observe $t_1$ and $t_2$ as shown in inset of figure 2.

We now focus on the EE as a function of time for the above scenario. In this case, another parameter is the size $L_A$ of subsystem $A$ of which we calculate the EE for the remaining system of size $L - L_A$. Let us firstly consider the simplest case where $L_1 = L_A$, i.e. the location of the $J$-quenching also determines the size $L_A$ of subsystem $A$. Interestingly, the bipartite EE of two critical transverse Ising chains can also detect the response of $h$-quenching. We see that for $J$-quenching alone or for the single quench, the EE shows perfect periodic oscillations with a sharp dip at $t_3 = 2L_1/v_{\text{max}}$ and an increase at $t_4$ (see figure 3), also discussed in [45] using conformal field theory. This can also be explained using the QP picture. A pair of QPs will increase the entanglement between subsystem $A$ and the rest if one of them is in subsystem $A$ and the other is in $B$. The QP pairs are generated at $L_1 = L_A$ and travel in opposite directions, resulting in an immediate increase in $S(t)$ for $t > 0$. This is not the case when $L_1 \neq L_A$ and this will be discussed separately. As one of the QPs reaches the boundary and gets reflected, $S$ starts decreasing and eventually shows a sharp dip at $t_3$ when $\text{QP}_1^2$ enters subsystem $B$ so that both the QPs belonging to a pair are in $B$. The decrease continues until time $t_4$ when $\text{QP}_2^2$ enters subsystem $A$, so that the QPs are in different subsystems resulting in an increase after $t_4$. At $T = 2L/v_{\text{max}}(\approx L)$, both the QPs arrive at the starting point and $S$ dips once again, after which the pattern repeats. If we now perform local $h$-quenching at a general site $L'$ of the spin chain, we observe that the evolution of the EE in double quenches follows the single quench case but is accompanied by deviations at certain
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Figure 2. Main: the LE as a function of time for double quenches with $L_1 = 100$, $L_2 = 100$ and $L = 300$ and different $L'$. The first peak of the LE occurs at time $t' = 2(L_1 - L')v_{\text{max}}$ ($t' = 25$ and $50$ for $L' = L/4$ and $L/6$, respectively). The other timescales are $t_3 = 100$, $t_4 = 200$ and $t_3'$. We note that $t'' = 225$ and $250$ for $L' = 75$ and $50$, respectively. Inset: $h$-quench only, at $L/3$ with $L = 300$. Timescales $t_1 = 100$ and $t_2 = 200$ are both detected in this case. Note that the decay in this case is small in magnitude.

Figure 3. Time evolution of the EE with $L_1 = L_2 = 100$ and different $L'$. For $L' = L/4$, the deviation from the single $J$-quench case appears at $t'/2 = 12.5$ whereas at $t' = 25$, $\text{QP}_{L}^2$ enters subsystem $B$ after getting reflected at $L'$ where its partner $\text{QP}_{R}^2$ is already present. The decrease in the EE continues until $\text{QP}_{R}^2$ enters subsystem $A$ at $t = L_2 = 200$. We also see a dip at $t = t'' = 225$ where $\text{QP}_{L}^2$ and $\text{QP}_{R}^2$ exchange systems. One can make a similar argument for the evolution of the EE when $L' = 3L/4$. The deviation from the single quench case begins at $t = 62.5$. In this case, $\text{QP}_{L}^2$ enters subsystem $B$ at $t = 100$ which causes a sharp decrease at this time. On the other hand, $\text{QP}_{R}^2$ enters subsystem $A$ at $t = 125$ resulting in an increase in the EE as its counterpart is still in subsystem $B$. The natural increase at $t = 200$, which is only there for $J$-quenching, can also be observed. This might be due to the fact that any reflection at $L'$ is not perfect and there is a possibility of getting a transmitted component of the QP wave, also discussed in sections 4 and 6. The timescales $t'' = 225$ and $T = 300$ are also present.
times which can once again be explained using the QP picture. We observe that the double quench case follows the single quench case \((L' = 0.0)\) until \(t = t'/2\), after which there is a sudden deviation or increase from the single quench case. This is because one of the QPs \((\text{QP}^1_L)\) produced at \(L'\) (which is not present in single quench case) enters subsystem \(B\) at this time whereas the other QP of the same pair remains in subsystem \(A\). This results in an extra increase in \(S\). On the other hand, at \(t = t'\), the \(\text{QP}^2_L\) gets back to \(L_1 = L_A\) after reflection at \(L'\) where we find a sharp decrease in \(S\) as both \(\text{QP}^2_L\) and \(\text{QP}^2_R\) are now in subsystem \(B\). The EE keeps on decreasing (after a slight increase) until \(\text{QP}^2_R\) enters subsystem \(A\) at \(t = 2(L - L_1)/v_{\text{max}}\) as a result of getting reflected from the right-hand boundary, after which we observe a sharp increase in the EE. One more observed timescale corresponds to \(t'' = 2(L - L_1)/v_{\text{max}}\) which was also observed in the LE. At this time, the \(\text{QP}^2_R\) and \(\text{QP}^2_L\) return back to \(L_A\) and exchange systems, as also discussed earlier with reference to the LE. All of the timescales mentioned above are shown in figure 3. We have found that this is also the case for the EE—as the strength of the \(h\)-quench is increased, the deviation (or increase) from the \(J\)-quench alone also increases.

The main difference between the analysis of the LE and the EE is that in the case of the LE we are interested in timescales at which the QPs produced come back to their origin. In the case of the EE, we are interested in the timescales in which one of the QPs belonging to a pair crosses one system and goes in to the other system. If this crossover results in both the QPs being in the same system then the EE decreases, otherwise it increases. It should also be noted that the initial increase in the EE in presence of both of the quenches would be similar to the \(J\)-quench alone. The deviation due to the \(h\)-quench only begins at \(t'/2\).

In conclusion, we find that for the quenches studied here, the dominant effects are due to \(\text{QP}^2\) at times \(t', t'', t_3\) and \(t_4\). We also note that the effect at \(t''\) is stronger, which may be due to the fact that at this time three different QPs return to their origin after reflection at various points, as discussed below: (i) \(\text{QP}^1_R\) after reflection from the right-hand boundary, (ii) \(\text{QP}^2_R\) after reflection from the right-hand boundary and a second reflection at \(L'\) causing it to return to \(L_1\) (iii) \(\text{QP}^2_L\) after reflection at \(L'\) and a second reflection at the right-hand boundary resulting in its return to \(L_1\). Finally, a peak is also observed at \(t = L\) which is the return time of all of the fastest QPs back to their origin when there is no reflection at \(L'\), thus giving us a hint that there may also be a transmitted component of the QP. We shall comment on this further after discussing the results in the ferromagnetic phase.

It should be noted that the presence of timescale \(t_1\) due to \(\text{QP}^1\) is almost negligible in these figures, though we do observe some perturbation at this time. On the other hand, it is too early to disregard the presence of \(\text{QP}^1\) as its effect is very clearly observed in the evolution of the EE at \(t'/2\). We shall try to set out the argument about the absence of \(t_1\) scale in the evolution of the LE in section 5.

We now consider the most general situation with \(L_1 \neq L_2 \neq L_A\) and calculate the time evolution of the EE. Here, \(J\)-quenching is again performed at \(L_1\), but the subsystem is assumed to be of length \(L_A = L/2\), and different from \(L_1\). The EE as a function of time after the double quenches is shown in figure 4. Let us define \(t(= L_A - L_1)\) as the distance between the right-hand side of the subsystem \(A\) and the site of the \(J\)-quenching. As
in other cases, we explicitly discuss the case with \( L' < L_1 < L_A \) below, and try to present some other examples through the figures. For \( \delta = 0 \), the EE remains at a constant value for small \( t \) and starts increasing at \( t = l/v_{\text{max}} \), when QP\(_L\) hits the boundary of the two subsystems and enters subsystem \( B \) [39]. Note the contrast between \( L_1 = L_A \) where \( \dot{S}(t) \) increases immediately and \( L_1 \neq L_A \) where \( \dot{S}(t) \) is constant initially. The EE shows a sharp decrease at \( t = (2L_1 + l)/v_{\text{max}} \) when QP\(_L\) enters subsystem \( B \) after getting reflected from the left-hand boundary. Similarly to the previous case (i.e. for \( L_1 \neq L_2 \) and \( L_1 = L_A \)), there is a ‘decay region’ between the time range \([2L_1 + l/v_{\text{max}}, (L_2 + L_B)/v_{\text{max}}]\) where the EE decays very slowly, after which there is an increase as the QPs are now in different subsystems. In this case, since the site for \( J \)-quenching does not coincide with the subsystem size, there are many additional timescales (as discussed below), the appearance of which puts the picture of travelling QPs on stronger footing. Let us now come back to the double quenches. Following the double quenches, the EE more or less follows the single quench case. The first deviation resulting in an increase in the EE (similar to the one discussed before) occurs at \( t = (L_A - L')/v_{\text{max}} \), when QP\(_L\) enters the subsystem \( B \). By definition, timescale \( t' \) is the time taken by QP\(_L\) to get reflected at \( L' \) and come back to subsystem \( A \), which in this particular case is given by \( t' = (2(L_1 - L') + l)/v_{\text{max}} \). On the other hand, timescale \( t'' \), defined as the time taken by QP\(_L\) (or QP\(_R\)) to undergo double reflection at \( L' \) and one of the boundaries, gets divided into two scales. This is because the distance travelled by QP\(_L\) is smaller than that travelled by QP\(_R\), which was not the case in our previous discussions where \( L_1 \neq L_A \). Let us define these two timescales as \( t'' = (2(L_1 - L') + l + 2L_B)/2 \) (for QP\(_L\)) and \( t'' = t'' + 2l/v_{\text{max}} \) (for QP\(_R\)). All of these timescales are clearly shown in figure 4. We would like to point out here that the basic physics related to tracking the QPs remains the same when we change the position of \( L' \), but the formula for these timescales may have to be changed, as can be seen in the \( L' = 3L/4 \) case discussed in figure 4. Also, the above discussion will be correct if \( (L_A - L')/2 < L_1 - L' \), i.e. QP\(_R\) reaches \( L_A \) before QP\(_L\). In the opposite case too, one needs to simply to apply the same ideas to get the right picture of the dynamics. It should also be mentioned that we observe some extra timescales, some of which can be explained and are discussed in the caption of figure 4.

4. Entanglement entropy for a ferromagnetic chain

In this section, we briefly discuss the evolution of EE when the total spin chain is in the ferromagnetic phase. Here, we consider the local quenching of asymmetric spin chains \((L_1 = L_2)\) with \( L_1 = L_A \). We concentrate on two different cases in order to calculate the EE after single or double quenches: one where the total spin chain is deep inside the ferromagnetic phase (see figure 5(a)) and the other where the spin chain is close to the critical point (see figure 5(b)). Let us first consider the spin chain with \( h = 0.5 \) at all sites. Figure 5(a) shows the time evolution of the EE after a single quench at \( L_1 \) (\( J \)-quench) and also after the double quenches, namely, an \( h \)-quench at \( L' \) and a \( J \)-quench at \( L_1 \). For the single quench \((L' = 0)\), the EE detects \( t_3 \) and \( t_4 \) successfully.

\[ \text{doi:10.1088/1742-5468/2016/04/043107} \]
Note the difference between the critical and ferromagnetic region for times up to \( t_3 \).

In the critical case, the EE increases at \( t = 0 \) followed by a decrease which starts at around \( t_3/2 \), when the QP\(_L^2\) gets reflected from the boundary, though the decrease is sharper at \( t_3 \). On the other hand, in the ferromagnetic region, we see a sudden increase in the EE followed by an almost constant EE region up to \( t_3 \) (no decrease at \( t_3/2 \)), after which it decreases suddenly. This hints at the fact that in the ferromagnetic region QPs are more point-like particles, and hence their location can be known precisely. In the critical case, however, these QPs are extended wave packets, as also mentioned in [52], and hence the reflection at the boundary is also felt at \( L_1 \). Similarly to the critical case (see section 3.2), the EE decays between \( t_3 \) to \( t_4 \). In this time range the fastest moving QPs do not contribute to the EE. Let us now discuss the time evolution of the EE after double quenches. One can clearly observe the timescales \( t'/2 \) and \( t' \) from figure 5. Interestingly, the EE starts decreasing after \( t' \) and this continues up to \( t_4 \). The sharp increase in the EE at \( t_4 \) is due to the fact that at \( t_4 \), QP\(_R^2\) enters subsystem \( A \) whereas QP\(_L^2\) is still in subsystem \( B \). It should be mentioned that in the ferromagnetic case \( t'' \) is not clearly visible, which once again can be explained by the point-like nature of QPs in the ferromagnetic region. For \( t < t'' \), QP\(_L^2\) is in subsystem \( B \) and QP\(_R^2\) is in subsystem
A. They exchange systems at $t''$, thus contributing to the entropy equally for $t < t''$ and $t > t''$. On the other hand, the critical case distinguishes between QPs approaching $L_1$ and moving away from $L_1$ due to the finite extent of the QP. Figure 5(b) presents the evolution of the EE when $h = 0.99$ and hints at a more extended QP picture, similar to the critical phase showing signatures of partial reflection at $L'$.

5. Comparison between QP$^1$ and QP$^2$

In this section we try to explain why the effect of QP$^2$ is stronger than that of QP$^1$. These QPs, which are locally produced at the site of perturbation, have energies given by the eigenvalues of the final Hamiltonian. Let $k$ identify the QP $\eta_k$ having eigenenergy $\Lambda_k$. QPs $\eta_k$ are produced at the perturbation site with probability $f_k$. Numerically, one can obtain $f_k$ by calculating the expectation value

$$f_k = \langle \psi_i | \eta_k^\dagger \eta_k | \psi_i \rangle,$$

which is proportional to the number of QPs $\eta_k$ corresponding to the final $H$ present in the initial state $|\psi_i\rangle$. This expression can be written in terms of $\Phi_k, \Psi_k$ of the final Hamiltonian and the matrix $G^i$ (see appendix B) with respect to the initial Hamiltonian. A comparison of $f_k$ for the $h$-quench alone ($h$ changed from 1 to 2 at $L'$), $J$-quench alone (0 to 1 at $L_1$) and both quenches together is shown in figure 6. Clearly, the QP creation probability is an order of magnitude higher in case of the $J$-quench alone when compared to the $h$-quench. This hints at the fact that the $J$-quench is the main source of QP production and hence our numerical results are dominated by the dynamics of QP$^2$ for the particular quench that we have studied. To explore this point further, we...
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compared the $f_k$ for $J$-quench from 0 to 1 at a given site with the $h$-quench from 0 to 1. We find that for such quenches, of similar magnitude, $f_k$ is of the same order and hence the effect of QP$^1$ and QP$^2$ should be equally felt. In figure 7, we plot the LE for the double quench case, where $J$ is switched from 0 to 1 and $h$ from 0 to 2 so that a site of defect is present in the final Hamiltonian. The evolution clearly shows the strong presence of QP$^1$ along with QP$^2$. Thus, we conclude that it is the strength of the quench which determines the QP creation probability, and thereby the evolution of various quantities.

6. Conclusion and discussion

In this paper, we studied the effect of two simultaneous local quenches in an otherwise uniform transverse Ising chain of length $L$ with open boundary conditions. Initially, the system is prepared in the ground state of the transverse Ising chain having a uniform transverse field $h$ and the interaction strength set to unity at all sites except $J_L$, and $J_{L'}$, where it is zero. The first quench corresponds to a sudden increase of $J_{L'}$ from 0 to 1, and the second quench involves the sudden change in the transverse field from $h$ to $h + \delta$ at site $L'$. We argue that the sites of the two local quenches are the source of QP production as there is a local increase in energy due to the quenches. These QPs are wave packets of the low-lying excitations of the final Hamiltonian. As discussed in [49, 52], the QPs are localized in the ferromagnetic region and behave more like classical particles, whereas they are extended objects/wave packets as the critical point is approached. Our numerical results on double quenches establish this fact very clearly, which has never been done before to the best of our knowledge.
We numerically studied the evolution of the LE and EE after the double quenches and explained the evolution using the QP picture. The envelope of the curve is dictated by the fastest moving QPs. We showed through examples that most of the timescales can be explained using the propagation of QPs. The most interesting phenomena that is observed numerically is the partial or full reflection of the QPs at \( L' \), the site of the \( h \)-quench. Only if we include such a phenomena can we explain certain numerically observed timescales. As discussed in detail in the paper, for the quenches studied here the most relevant timescales are \( t_1 \), \( t_2 \), \( t_3 \) and \( t'' \) (for \( L_1 = L_4 \)) which are all due to the QP\(^2 \) pair, or the pair produced at the \( J \)-quenching site. The presence of the other set of QPs, namely QP\(^1 \), is clearly seen in the evolution of the EE. We have shown that the probability of QPs produced due to \( h \)-quench is roughly an order of magnitude smaller than the \( J \)-quench, which could be the reason for the stronger effect of QP\(^2 \) in the evolution of the LE and the EE. On the other hand, the above statement is quench specific. If both quenches are of similar magnitude we find that both types of QP are equally strong, showing peaks or dips in the LE and the EE.

The double quenches deep inside the ferromagnetic phase can be very nicely described by the point-like QPs where all the timescales are sharply observed. We have contrasted this ferromagnetic case with the double quenches in the critical phase and proposed reasons for the differences.

Our main aim in this paper was to study the dynamical evolution of the LE and the EE after double quenches and to see if one can explain the behavior, at least qualitatively, using the propagation of QPs. We have demonstrated here that this is indeed possible. Although we cannot propose a general formula for all of the timescales involved, as it depends on which QP arrives at the subsystem first which in turn depends on the location of \( L' \), \( L_1 \) and \( L_A \), the basic idea gives us the right picture. We have also checked this for other cases which are not presented in this paper. The QP picture does explain many, if not all, features of the dynamical evolution of the LE and the EE that occur in double quenches studied here.
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Appendix A. Loschmidt echo for a general quadratic fermionic system

Here we discuss the method for evaluating the time evolution of the LE in real space for a general quadratic fermionic system [10]. We rewrite the Hamiltonian in equation (5) in the following form

\[ H = \frac{1}{2} C^\dagger \mathcal{H} C, \]  

(A.1)

where \( C^\dagger = (c_1^\dagger, \ldots, c_L^\dagger, c_1, \ldots, c_L) \) and \( \mathcal{H} = \sigma^x \otimes A + i\sigma^y \otimes B \).

The LE, defined in equation (1), can be evaluated in this case by the following relation [10, 54]

\[ \mathcal{L}(t) = |\langle \psi_i | e^{-iH_f t} | \psi_i \rangle| = |\det(1 - R + Re^{-iH_f t})|, \]  

(A.2)

where \( H_f \) is the final Hamiltonian after double quenches. \( R \) is the \( 2L \times 2L \) correlation matrix whose elements are two-point correlation functions of fermionic operators \( R_{kl} = \langle \psi_i | c_k^\dagger c_l | \psi_i \rangle \), where \( | \psi_i \rangle \) is the ground state of the initial Hamiltonian \( H_i \). It can be shown that the matrix \( R \) can be written in terms of \( g \) and \( h \) matrices (see [10] and discussion around equation (7)).

Appendix B. Time-dependent entanglement entropy for a general quadratic fermionic system

For EE evolution [55], let us define two Clifford operators which are also related to the Majorana fermion operators \( a_{2i-1} \) and \( a_{2i} \) in the following way,

\[ \mathcal{A}_i = c_i^\dagger + c_i = a_{2i-1} \quad \text{and} \quad \mathcal{B}_i = c_i^\dagger - c_i = ia_{2i}. \]  

(B.1)

The operators \( \mathcal{A} \) and \( \mathcal{B} \) can be written in terms of the free-fermion operators \( \eta_k \)

\[ \mathcal{A}_i = \sum_{k=1}^L \Phi_k(i) (\eta_k^\dagger + \eta_k), \quad \mathcal{B}_i = \sum_{k=1}^L \Phi_k(i) (\eta_k^\dagger - \eta_k). \]  

(B.2)

The time evolution of these operators is obtained from the time dependence of the fermionic operators, i.e. \( \eta_k(t) = e^{-i\Lambda_k t} \eta_k \), where \( \eta_k \) and \( \Lambda_k \) are QPs and eigenenergies corresponding to the final Hamiltonian \( H_f \). This is given by...
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\[ A_i(t) = \sum_{j=1}^{L} \{ \langle A_i A_j \rangle L + \langle A_i B_j \rangle L \}, \]

\[ B_i(t) = \sum_{j=1}^{L} \{ \langle B_i A_j \rangle L + \langle B_i B_j \rangle L \}, \]

where the time-dependent contractions are

\[ \langle A_i A_j \rangle L = \sum_{k=1}^{L} \cos(\Lambda_k t)\Phi_k(i)\Phi_k(j), \]

\[ \langle A_i B_j \rangle L = \langle B_j A_i \rangle L = \sum_{k=1}^{L} \sin(\Lambda_k t)\Phi_k(i)\Phi_k(j), \]

\[ \langle B_i B_j \rangle L = \sum_{k=1}^{L} \cos(\Lambda_k t)\Phi_k(i)\Phi_k(j). \]

The total system is divided into two subsystems \( A \) and \( B \) of lengths \( L_A \) and \( L_B \), respectively. In order to evaluate the EE between these two subsystems after the double quench, we have to calculate the reduced density matrix \( \rho_A \) of subsystem \( A \). This can be reconstructed from the \( 2L_A \times 2L_A \) correlation matrix of the Majorana operators

\[ \langle \psi | a_m(t) a_n(t) | \psi \rangle = \delta_{m,n} + \imath (\Gamma_A)_{mn}, \]

where \( m, n = 1, 2, 3, \ldots, 2L_A \) and \( | \psi \rangle \) is the initial ground state. The matrix \( \Gamma_A \) is an antisymmetric matrix which can be brought into the block-diagonal form by an orthogonal matrix say, \( V \). Therefore, the eigenvalues of \( \Gamma_A \) are purely imaginary and of the form \( \pm \nu_l \) with \( l = 1, 2, \ldots, L_A \). This can be used in order to write the reduced density matrix as a direct product of \( L_A \) uncorrelated modes \( \rho_A = \bigotimes_{l=1}^{L_A} \varrho_l \), where each \( \varrho_l \) has eigenvalues \((1 \pm \nu_l)/2\). Thus the bipartite EE for \( \rho_A \) is the sum of entropies of \( L_A \) uncorrelated modes given by

\[ S_L(L_A) = -\sum_{l=1}^{L_A} \left( \frac{1 + \nu_l}{2} \ln \frac{1 + \nu_l}{2} + \frac{1 - \nu_l}{2} \ln \frac{1 - \nu_l}{2} \right). \]

The time-dependent expectation values in equation (B.5) are calculated using equation (B.3) as

\[ \langle a_{2l}(t) a_{2m}(t) \rangle = -\langle B_l(t) B_m(t) \rangle, \]

\[ \langle a_{2l-1}(t) a_{2m-1}(t) \rangle = -\langle A_l(t) A_m(t) \rangle, \]

\[ \langle a_{2l}(t) a_{2m-1}(t) \rangle = -\langle B_l(t) A_m(t) \rangle, \]

\[ \langle a_{2l-1}(t) a_{2m}(t) \rangle = -\langle A_l(t) B_m(t) \rangle. \]

Thus we get the time-dependent correlation matrix \( \Gamma_A \) and for each time we can obtain the EE from equation (B.6). The elements of the matrix \( \Gamma_A \) are given by

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\[ \Gamma^{A}_{2l-1, 2m-1} = -i \sum_{k_1, k_2} G^{i}_{k_1k_2} \langle A_{l}B_{k_2} \rangle \langle A_{m}A_{k_1} \rangle + i \sum_{k_1, k_2} G^{i}_{k_2k_1} \langle A_{l}A_{k_1} \rangle \langle A_{m}B_{k_2} \rangle \]

\[ \Gamma^{A}_{2l-1, 2m} = \sum_{k_1, k_2} G^{i}_{k_1k_2} \langle A_{l}A_{k_2} \rangle \langle B_{m}B_{k_1} \rangle - \sum_{k_1, k_2} G^{i}_{k_2k_1} \langle A_{l}B_{k_1} \rangle \langle B_{m}A_{k_2} \rangle \]

\[ \Gamma^{A}_{2l, 2m-1} = -\sum_{k_1, k_2} G^{i}_{k_1k_2} \langle B_{l}B_{k_2} \rangle \langle A_{m}A_{k_1} \rangle + \sum_{k_1, k_2} G^{i}_{k_2k_1} \langle B_{l}A_{k_1} \rangle \langle A_{m}B_{k_2} \rangle \]

\[ \Gamma^{A}_{2l, 2m} = -i \sum_{k_1, k_2} G^{i}_{k_1k_2} \langle B_{l}B_{k_2} \rangle \langle B_{m}B_{k_1} \rangle + i \sum_{k_1, k_2} G^{i}_{k_1k_2} \langle B_{l}B_{k_2} \rangle \langle B_{m}A_{k_1} \rangle. \]  

where \( G^{i}_{k_1k_2} = -\sum_{k} \Psi^{i}_{k}(k_1)\Psi^{i}_{k}(k_2) \) is the equilibrium correlation function which is calculated with the initial Hamiltonian \( H_{i} \).

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