On a fusion chain reaction via suprathermal ions in high-density H–\(^{11}\)B plasma

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Abstract

The \(^{11}\)B(p,3\(\alpha\)) fusion reaction is particularly attractive for energy production purposes because of its aneutronic character and the absence of radioactive species among reactants and products. Its exploitation in the thermonuclear regime, however, appears to be prohibitive due to the low reactivity of the H–\(^{11}\)B fuel at temperatures up to 100 keV. A fusion chain sustained by elastic collisions between the \(\alpha\) particles and fuel ions, this way scattered to suprathermal energies, has been proposed as a possible route to overcome this limitation. Based on a simple model, this work investigates the reproduction process in an infinite, non-degenerate H–\(^{11}\)B plasma, in a wide range of densities and temperatures which are of interest for laser-driven experiments \((10^{24} \lesssim n_e \lesssim 10^{28} \text{ cm}^{-3}, T_e \lesssim 100 \text{ keV}, T_i \sim 1 \text{ keV})\). In particular, cross section data for the \(\alpha–p\) scattering which include the nuclear interaction have been used. The multiplication factor, \(k_\infty\), increases markedly with electron temperature and less significantly with plasma density. However, even at the highest temperature and density considered, and despite a more than twofold increase by the inclusion of the nuclear scattering, \(k_\infty\) turns out to be of the order of \(10^{-2}\) only. In general, values of \(k_\infty\) very close to 1 are needed in a confined scheme to enhance the suprathermal-to-thermonuclear energy yield by factors of up to \(10^3\) or \(10^4\).

Keywords: proton–boron fusion, H–\(^{11}\)B fuel, advanced fusion fuels, fusion chain reaction, avalanche p–\(^{11}\)B fusion, p–\(\alpha\) scattering, aneutronic fusion

1. Introduction

The advanced fusion fuel (Dawson 1981) based on a H–\(^{11}\)B mixture would exploit the reaction

\[ ^{11}\text{B} + p \rightarrow 3\alpha + 8.7 \text{ MeV} \]

which is particularly attractive as it involves only abundant and stable isotopes in the reactants, and no neutron in the reaction products. The reaction cross section shows a main resonance at 612 keV in the centre-of-mass (CM) system (figure 1(a)), with a maximum value of 1.4 b and a width of 300 keV (Becker et al 1987, Nevins and Swain 2000, Sikora and Weller 2016). A second resonance of potential interest for fusion applications appears at lower energy (148 keV) and peaks to about 0.1 b, though it is only 5.3 keV wide. Summed over the three reaction channels—the low branching-ratio \(^{12}\)C\(^*\) direct breakup and the sequential decays via \(^4\)Be\(^*\) or \(^8\)Be\(^*\)—the energy spectrum of the generated \(\alpha\) particles is a continuum which extends up to about 6.7 MeV in the lab, for a 675 keV incoming proton. The spectrum is strongly peaked around 4 MeV (Stave et al 2011).

Neutron production in the fusion plasma is actually expected by the side reactions \(^{11}\)B(\(\alpha\),n)\(^{14}\)N and \(^{11}\)B(p,n)\(^{11}\)C, though at a tiny rate (Kernbichler et al 1985). A major drawback of
H-11B fuel compared to conventional deuterium-tritium (DT) fuel is instead its much lower reactivity (figure 1(b)). This issue poses tremendous challenges to the exploitation of p-11B fusion for energy production purposes, under both magnetic and inertial confinement schemes (Moreau 1977, Shmatov 2019). Nevertheless, the fact that three charged, massive, energetic particles are produced in the reaction, suggests that the fusion yield could effectively be enhanced by a non-thermal effect induced by those particles, which is the elastic scattering of fuel ions to energies corresponding to the highest values of the fusion cross section (Brueckner and Brysk 1973, Moreau 1977). While thermalising, some of the ions in these showers can undergo further fusion, eventually setting a chain reaction up. As a matter of fact, the fusion-born α’s can easily be stopped in the plasma, especially at very high densities. At those densities, moreover, the α’s tend to lose energy mostly to plasma ions rather than to electrons. This happens when the electrons’ Fermi velocity (or their thermal velocity) becomes comparable to the α-particle velocity while the ion thermal velocity remains substantially lower (Gryzinski 1958, Levush and Cuperman 1982, Son and Fisch 2004).

A fusion reaction progressing exclusively via nuclear reactions has also been proposed in H-11B fuel enriched with 10B (Belyaev et al 2015a, 2015b). It would be based on the high-energy protons produced in the reaction 11B(α,p)14C (Q = 0.8 MeV). The competing reaction 11B(α,n)14N, however, would remove α particles from the chain and should be complemented with the α-bearing reaction 10B(n,α)7Li. Notwithstanding this, it has finally been shown that at least for electron temperatures up to 100 keV, only a tiny fraction of α particles would be capable to react with 11B because of their loss of energy in the fuel, thus preventing the development of the chain (Shmatov 2016a).

In the present work, we focus on the suprathermal chain reaction. In an infinite, homogeneous H-11B plasma, an α particle emitted at a certain energy Eα,0 in a primary fusion event is characterised by the multiplication (or reproduction) factor kα(Eα,0), which is the average number of secondary α-particles generated via suprathermal processes during its slowing down. The corresponding average number of fusion events is just (1/3)kα, as three α-particles are emitted per event. By integrating over the spectrum of the primary α’s, φ(Eα,0), a multiplication factor in terms of fusion events, k∞, can be estimated through the relation

\[ k_\infty = \int k_\alpha(E_{\alpha,0}) \varphi(E_{\alpha,0}) \, dE_{\alpha,0} \]  

where φ is normalised to unity (note that the abovementioned factor of 1/3 has been absorbed in the normalisation of φ). Accordingly, each fusion event gives rise to new generations of events along a geometric progression with common ratio k∞. The number of events adds up, on average, to \(1 + k_\infty + k_\infty^2 + \ldots + k_\infty^l + \ldots\), where l is the number of generations. Along these lines, it is not difficult to show (see appendix A) that upon a thermonuclear rate per unit volume \(R\), the time evolution of the cumulative number of fusion events per unit volume, \(n_1(t)\), is given by

\[ n_1(t) = \frac{R \tau_g}{1 - k_\infty} \left[ \frac{t}{\tau_g} + k_\infty \ln k_\infty \left(1 - k_\infty^{\prime}/\tau_g\right)\right] \]  

where \(\tau_g\) is the average period between two consecutive generations, and the initial condition \(n_1(0) = 0\) has been assumed. Depending on whether (a) \(k_\infty < 1\), (b) \(k_\infty = 1\), or (c) \(k_\infty > 1\), \(n_1\) can respectively

(a) increase linearly with \(t\), asymptotically to \(Rt/(1 - k_\infty)\), for \(t \gg \tau_g\),

(b) increase quadratically with \(t\), as it reduces to \(R(1 + t^2/2\tau_g^2)\), or

(c) diverge exponentially, with the growth rate \(\ln k_\infty/\tau_g\).

It goes without saying that the capability to achieve a chain reaction with multiplicity \(k_\infty \gtrsim 1\) would play a significant—if not indispensable—role in the possible exploitation of H-11B.
fusion as an energy source, especially under recently proposed laser-driven schemes for plasma generation and confinement (Hora et al 2015, 2016, 2017).

Early computations by Weaver et al (1973), however, estimated the increase of the reaction rate due to non-thermal effects to vary only between 5% and 15% in the density range 10^{15}–10^{26} cm^{-3} and temperature range 150–350 keV (it is unclear whether the electron or ion density is reported, and the boron-to-hydrogen ion concentration is not given). Subsequent calculations by Moreau (1976, 1977) returned multiplication factors of the order of 10^{-2} in a plasma with 100 ≲ T_e ≲ 300 keV, T_i = 0, and Coulomb logarithm lnΛ = 5. The boron-to-hydrogen ion concentration used is unclear (perhaps 8.9%), the optimum for thermonuclear ignition; moreover, only the Coulomb interaction was taken into account in the α-ion scattering, neglecting the nuclear (hadronic) one, which can instead turn out to be predominant, as we shall see. Recently, Hora et al (2015, 2016) and Eliezer et al (2016) have claimed evidence of the chain reaction in experiments at the Prague Asterix Laser System, Czech Republic, where an unprecedented fusion yield (4 × 10^8 α’s per laser pulse) had been achieved by irradiation of an H-enriched, B-doped Si target (Picciotto et al 2014, Margarone et al 2015). On the contrary, Shmatov (2016b) and Belloni et al (2018) have refuted this possibility on the basis of stopping power and α–p collision rate arguments. Lately, Eliezer and Martinez-Val (2020) have proposed the concept of a possible H–B fusion reactor where the stopping power problem is circumvented by the application of an external electric field.

As it is evident, a systematic study of the chain reaction multiplicity in high-density H–B plasma has been missing so far. Studies of this kind exist for DT fuel (e.g. Brueckner and Brysk 1973, Peres and Shvarts 1975, Aley et al 1978, Kumar et al 1986) and will briefly be outlined in section 4. Prompted by recent theoretical and experimental advances in laser-driven p–B fusion (Hora et al 2017, Giuffrida et al 2020), the advent of ultra-high-power (tens PW) laser systems as well as the growing interest of the scientific community in the potentiality of H–B fuel for energy production (Rostoker et al 1997, Belyaev et al 2005, Cowen 2013, Labaune et al 2013), this work reports an analysis of the multiplication process in a wide range of plasma densities and temperatures which are of interest for current and future laser-based experiments. In particular, cross section data for the α–p scattering which include the nuclear interaction have been used. The aspiration is that results and conclusions can help inform the choice of parameters and the development of techniques in future experiments, with a view to maximising multiplication effects.

2. Theory

We assume a two-temperature (T_e, T_i), H–B plasma with n_j ions per unit volume (the subscript j stands for p or B); the density ratio n_B/n_p is denoted by γ. Indicating by E_{0,j} the energy of an ion just after the scattering by an α particle, we will consider as suprathermal those secondary ions for which \( E_{j,0} \gg T_e \). In this limit, the number of ions of the species j scattered into the energy interval \( (E_{j,0} - \alpha E_{j,0} + dE_{j,0}) \) through the path length dx of the α particle is given by

\[
d^2 N_j = n_j \sigma_j(E_{\alpha}, E_{j,0}) dE_{j,0} dx
\]

where \( \sigma_j \) is the differential scattering cross section. The spectral distribution of these ions through the entire path length of the α particle is then

\[
dN_j/dE_{j,0} (E_{j,0}; E_{\alpha,0}) = n_j \int_{E_{\alpha,0}}^{E_{j,0}} \sigma_j(E_{\alpha}, E) \left( \frac{dE_{\alpha}}{dx} \right)^{-1} dE_{\alpha}
\]

where \( dE_{\alpha}/dx \) is the stopping power of the α particle (taken as a positive quantity). For pure Coulomb scattering, \( \sigma_j \) is given by the well-known Rutherford cross section, \( \sigma_R \), which reads, in terms of \( E_{j,0} \), as

\[
\sigma_R(E_{\alpha}, E_{j,0}) = \frac{\pi (z_\alpha z_j e^2)^2}{E_{\alpha} E_{j,0}^2} \frac{m_\alpha H}{m_j} (E_{j,0}^{\text{max}} - E_{j,0})
\]

where the \( z \)'s are particle electric charges in units of the elementary charge e, the \( m \)'s are particle masses, \( H \) is the Heaviside step function, and the endpoint energy \( E_{j,0}^{\text{max}} \) is given, from basic kinematics, by

\[
E_{j,0}^{\text{max}} = \eta_{ij} E_{\alpha}
\]

with \( \eta_{ij} = 4 m_\alpha m_j / (m_\alpha + m_j)^2 \).

Denoting by \( k_{\alpha j} \) the contribution to \( k_{\alpha} \) of the suprathermal population of the species \( j \), it is straightforward to see that for an α particle of energy \( E_{\alpha,0} \), the spectrum of \( k_{\alpha j} \) over \( E_{j,0} \) is linked to the spectrum of \( N_j \) by

\[
\frac{d k_{\alpha j}}{dE_{j,0}} (E_{j,0}; E_{\alpha,0}) = 3 P_j(E_{j,0}) \frac{dN_j}{dE_{j,0}} (E_{j,0}; E_{\alpha,0})
\]

where \( P_j \) is the fusion probability of the ion throughout its thermalisation, and the factor of 3 is the number of α’s per fusion event. \( k_{\alpha j}(E_{\alpha,0}) \) is calculated by numerical integration of equation (7) over \( E_{j,0} \). In turn, \( k_{\alpha} \) is calculated by summing over the contribution of each ion species, i.e.

\[
k_{\alpha} (E_{\alpha,0}) = k_{\alpha p} (E_{\alpha,0}) + k_{\alpha B} (E_{\alpha,0}).
\]

Finally, \( k_\infty \) is calculated from \( k_{\alpha} (E_{\alpha,0}) \) through equation (1). By the same way, it is also meaningful to calculate the spectrum \( d k_{\alpha p}/dE_{j,0} \) from \( d k_{\alpha j}/dE_{j,0} \).

Concerning the fusion probability in equation (7), it is easy to show (e.g. Giuffrida et al 2020) that in the cold-ion approximation for the target species, and for \( P_j \ll 1 \) (which is our case, see section 3), the following relation holds

\[
P_{p(B)} (E_{p(B),0}) = n_{p(B)} \int_0^{E_{p(B),0}} \sigma_{f} (E_{\text{CM}}) \left( \frac{dE_{p(B)}}{dx} \right)^{-1} dE_{p(B)}
\]
where \( \frac{dE_j}{dx} \) is the stopping power, \( \sigma_f \) is the fusion cross section, and \( E_{\text{CM}} \) is the CM energy of the \( p-^{11}\text{B} \) system, i.e.

\[
E_{\text{CM}} = \frac{m_B(j)}{m_p + m_B} E_p(0),
\]

(10)

Calculations of the abovementioned quantities have been performed in conditions relevant to laser-driven fusion plasmas and are reviewed in section 3. In computations, we have adopted the following input data or models and considerations.

2.1. \( \alpha \) spectrum, \( \varphi(E_{\alpha,0}) \)

For the sake of simplicity, we have used the crude two-group approximation according to which, on average, one \( \alpha \) particle is emitted at energy \( E_{\alpha,1} = 1 \text{ MeV} \) and the other two at \( E_{\alpha,II} = 4 \text{ MeV} \) (Stave et al 2011). Consequently, \( \varphi(E_{\alpha,0}) \) in equation (1) is given by

\[
\varphi(E_{\alpha,0}) = \frac{1}{3} \left[ \delta(E_{\alpha,0} - E_{\alpha,1}) + 2 \delta(E_{\alpha,0} - E_{\alpha,II}) \right]
\]

(11)

where \( \delta \) is the Dirac delta function.

2.2. Fusion cross section, \( \sigma_f \)

The analytic approximation of Nevins and Swain (2000) has been used below 3.5 MeV, and an interpolation of TENDL evaluated data (Koning et al 2019) at higher energies (figure 1(a)).

2.3. Elastic scattering cross section, \( \sigma_e \)

For the \( \alpha-p \) scattering, evaluated angular cross section data have been retrieved from SigmaCalc (Gurbich 2016), interpolated, and finally converted into the form of \( \sigma_e \) as a function of the scattered energy. A comparison with \( \sigma_{qe} \) is displayed in figure 2, which shows that the nuclear contribution dramatically increases the cross section, within a factor of 3 for \( E_{\alpha} \lesssim 2 \text{ MeV} \), up to a factor of 10 around 4 MeV and at high values of \( E_{p,0} \), and by hundreds of times at progressively higher values of \( E_{\alpha} \) and \( E_{p,0} \). Nevertheless, for \( E_{\alpha} > 4 \text{ MeV} \) and \( E_{p,0} < 1 \text{ MeV} \), a wide area exists where \( \sigma_e/\sigma_R < 1 \), which is an effect of the interference between the Coulomb and nuclear scattering amplitudes (Perkins and Cullen 1981). For the \( \alpha-^{11}\text{B} \) scattering, the Rutherford cross section has been used as the higher Coulomb barrier makes the nuclear contribution negligible at the energies concerned.

2.4. Stopping powers, \( \frac{dE_e}{dx} \) and \( \frac{dE_p}{dx} \)

The Spitzer–Sivukhin model (Spitzer 1956, Sivukhin 1966) has been used, in the form of a multicomponent (electrons and ion species), two-temperature \( H-^{11}\text{B} \) plasma detailed in Levush and Cuperman (1982). Earlier, this model had also been used by Moreau (1976, 1977) for slowdown calculations in high-density \( H-^{11}\text{B} \) plasma. With a view to the subsequent discussion, it is worth recalling the form of the electronic stopping power in the expressions of \( dE_e/dx \) and \( dE_p/dx \), i.e.

\[
\frac{dE_{qe}}{dx} = n_e \ln \Lambda_{qe} (n_e, T_e) g_{qe} (E_q, T_e)
\]

(12)

where the subscript \( q \) stands for \( \alpha, p \) or B, and the functions \( \Lambda_{qe} \) and \( g_{qe} \) are given by Sivukhin (1966). In particular,

\[
\Lambda_{qe} = 1.632 \times 10^{14} \frac{(T_e [\text{keV}])^{3/2}}{z_qz_e (n_e [\text{cm}^{-3}])^{1/2}}.
\]

(13)

Formulas analogous to equation (12) hold for the \( q-p \) and \( q-^{11}\text{B} \) components of the stopping power.

2.5. Plasma electron density, \( n_e \)

With reference to equations (4) and (9), it is useful to write the ion densities appearing therein in terms of the electron density, \( n_e \), and \( \gamma \), i.e.

\[
n_p = n_e / (z_p + z_B \gamma)
\]

(14)

\[
n_B = n_e \gamma / (z_p + z_B \gamma).
\]

(15)

Orders of magnitudes between \( 10^{24} \) and \( 10^{28} \text{ cm}^{-3} \) have been considered for \( n_e \). As a term of reference, for amorphous boron in STP conditions, \( n_B = 1.3 \times 10^{23} \text{ cm}^{-3} \), hence \( n_e = 6.5 \times 10^{23} \text{ cm}^{-3} \).

2.6. Boron-to-hydrogen ion concentration, \( \gamma \)

If one considers, for the sake of simplicity, only the electronic stopping power in the expressions of \( dE_e/dx \) and \( dE_p/dx \) (equation (12)), it is straightforward to note that the apparent linear dependence on \( n_e \) in equations (4) and (9) actually cancels out, leaving factors which depend on \( \gamma \) according to equation (14). This implies that equation (7) and derived quantities depend on \( \gamma \) through the overall factor \( \gamma/(z_p + z_B \gamma)^3 \), which has a maximum at \( \gamma = 0.2 \) for \( z_p = 1 \) and \( z_B = 5 \). In reality, the dependence of the multiplication factors on \( \gamma \) is obviously much more complicated because of
the dependence on $n_i$ of the ion–ion components of the stopping powers. The optimum $\gamma$ has to be calculated numerically and depends, moreover, on the specific set of parameters entering the equations. Its value and the corresponding maximum values of $k_{\alpha}$ and $k_{\infty}$, however, are in general not dramatically affected, as shown in section 3.

### 2.7 Temperatures, $T_e$ and $T_i$

Minimising $dE_\alpha/dx$ and $dE_j/dx$ in equations (4) and (9), respectively, requires high values of both $T_e$ and $T_i$ in a classic (Maxwell–Boltzmann) plasma. As a precondition, one wants to deal with a fully ionised plasma in order to reduce the electronic component of the stopping power (Giuffrida et al. 2020); accordingly, $T_e$ should be much higher than the ionisation energy of $\text{B}^{4+}$, which is 0.34 keV for the isolated ion (Lide 2000) and less in high-density matter (More 1993). Even when $T_e$ is higher than the $\text{B}^{4+}$ ionisation energy, electrons can still be Fermi-degenerate at the high densities considered here. The Fermi energy $E_F$, which scales as $n_e^{2/3}$, has been plotted in figure 3 for reference. In view of further reducing the electronic stopping power, we have verified that in our density domain it is convenient to work at low degeneracy compared to the fully degenerate case (Son and Fisch 2004, Giuffrida et al. 2020). Accordingly, at a given $n_e$, we have chosen to work with a classic plasma at $T_e/E_F > 5$, a condition which in our case ensures both full ionisation and low degeneracy.

At a given $T_e$, the lower $T_i$ the more effective is the suprathermal energy transfer. Indeed, in the limit $T_i = 0$, all the scattered ions are obviously suprathermal; moreover, from basic kinematics, the collisional energy transfer from the $\alpha$ particle to the ion occurs as long as the velocity of the former is higher than that of the latter. Also in the light of the recent experiment of Giuffrida et al. (2020) on a shocked hydrogen-rich boron nitride target, where $n_e \sim 10^{24}$ cm$^{-3}$ and $T_i \sim T_e \sim 1$ keV have been estimated, we have chosen to set $T_i = 1$ keV, which is a good compromise between the needs to reduce the ion–ion component of the stopping power on the one hand, and to increase the $\alpha$-to-ion energy partition and ensure a suprathermal spectrum as wide as possible ($T_e \ll E_{\alpha,j}^{2/3}$) on the other hand. As a matter of fact, the contribution to $k_{\alpha\beta}$ of protons with $E_{p,0} < 10$ keV is absolutely negligible; see section 3. Incidentally, in a low-$T_i$ scheme the thermonuclear burn will be very modest and will just be used to seed the chain reaction, which is expected to provide most of the energy output. The first fusion events can also be initiated by injecting high-energy protons into the fuel or heating a small region of the fuel by a laser pulse.

The temperature requirements for the most effective induction of suprathermal effects appear to be in opposition to the condition $T_e < T_i$ (or $T_e \ll T_i$) usually pursued to achieve confined fusion. In this regime, indeed, bremsstrahlung losses are reduced and, under magnetic confinement, the requirements for containing plasma pressure are relaxed. For instance, Son and Fisch (2004) have shown that a ‘convenient’ ignition regime for $\text{H}^{-11}$B fuel can be achieved at $T_i = 200$ keV, $T_e = 27$ keV, and $n_e = 6.7 \times 10^{28}$ cm$^{-3}$ (with $\gamma = 0.3$). Apart from the fact these parameters are impracticable today, plasma regimes where $T_e > T_i$ are much more easily achievable in current laser-driven experiments, especially when hot electrons are generated in the shocked or inertially compressed target. Besides merely enhancing the fusion yield, the suprathermal chain reaction could drive plasmas of this kind towards ignition, by increasing $T_i$ quickly. In a plasma already ignited, the chain reaction would have a less critical impact, even though it would obviously contribute to increase the fusion gain.

On the basis of all the abovementioned considerations, the choice of parameters $10^{21} \leq n_e \leq 10^{28}$ cm$^{-3}$, $T_i = 1$ keV, max $|T_i, 5E_F(n_e)| \leq T_e \leq 100$ keV (figure 3) has appeared as the most urgent to investigate in this study.

### 3. Results

The $^{11}$B-ion contribution to $k_{\alpha}$ in equation (8) turns out to be in any case much smaller than that of the proton, by a factor of at least 100, as we have verified by direct computation for values of $E_{\alpha,0}$ up to 10 MeV. Accordingly, explicit results for suprathermal $^{11}$B ions will not be reported in the following. The physical reasons for the negligible role of $^{11}$B recoils in the multiplication process will be outlined in section 4. In the present section, focus is made on the results of calculations for $P_{f}(E_{p,0}), \text{d}N_{f}/\text{d}E_{p,0}, k_{\alpha\beta}/\text{d}E_{p,0}, \text{d}k_{\alpha\beta}/\text{d}E_{p,0}, k_{\alpha\beta}(E_{\alpha,0}),$ and $k_{\infty}$.

The fusion probability of equation (9) has been plotted in figure 4 as a function of proton energy for representative values of $n_e$ and $T_e$, with $T_i = 1$ keV and $\gamma = 0.2$. The shape of the curves resembles the main features of $\sigma_f$. The curves, in particular, exhibit marked knees at energies around the 148 and 612 keV cross section resonances. Furthermore, the fusion probability becomes negligibly small below 100 keV. Within our domain of parameters, values of $P_{f}$ of the order of $10^{-3}$ are attained at proton energies between approximately 400 and 500 keV. The fusion probability is enhanced more
the nuclear interaction. Such ‘oscillation’ in this proton energy effectively by increasing $T_e$ rather than $n_e$ (the curves at $10^{25}$ and $10^{26}$ cm$^{-3}$ are almost indistinguishable); furthermore, the $T_e$-driven enhancement is amplified by proton energy. While the dependence on $T_e$ is entirely due to the stopping power in equation (9), the slight dependence on $n_e$ results from the quasi cancellation of the density in the product between $n_B$ and $(dE_p/dx)^{-1}$, as discussed in section 2.6. Considering only the electron component of $dE_p/dx$—equation (12)—this residual dependence is of the form $1/\ln n_0(T_e)/n_e$, where $n_0$ is determined by comparison with equation (13). It is worth noticing, however, that given $E_{p,0}$, if $n_e$ is increased keeping $T_e$ fixed, $P_p$ remains quasi constant as long as the plasma does not become degenerate, i.e. up to values of $n_e$ such that $E_p(n_e) \sim T_e$ (figure 3). In a fully degenerate plasma, indeed, equation (12) no longer holds, and the electronic stopping power becomes almost independent of the density and proportional to the ion velocity (Son and Fisch 2004). Accordingly, the density cancellation no longer occurs in equation (9), and $P_p(B)$ increases linearly with $n_e$. This happens when $E_p > T_e$, which implies $n_e > (2m_e T_e)^{3/2}/\hbar^3/3\pi^2$.

The spectral analysis of scattered protons and multiplica-
tion factors is provided in figure 5. In panel (a), $dN_p/dE_{p,0}$ has been plotted according to equation (4), for the two $\alpha$-particle reference energies $E_{\alpha,1}$ and $E_{\alpha,II}$ (viz 1 and 4 MeV, respectively) and the elastic cross sections $\sigma_s$ and $\sigma_R$. First, one notes that the spectrum endpoint energies, as given by equation (6), are quite high, approaching 600 keV in the case of $E_{\alpha,1}$ and overcoming 2 MeV in the case of $E_{\alpha,II}$. Secondly, at the given values of $E_{\alpha,0}$, the trend of the curves corresponding to $\sigma_s$ and $\sigma_R$ is similar. Nevertheless, at values of $E_{p,0}$ of the order of 10 keV and beyond, one notes that the yield of scattered protons is significantly higher when the nuclear interaction is taken into account. At intermediate energies, of the order of 100 keV, the difference reduces significantly. At energies of the order of 1 MeV, the scattered yield is again enhanced by the nuclear interaction. Such ‘oscillation’ in this proton energy range is basically due to the interference between the scattering amplitudes of the nuclear and Coulomb potentials, as mentioned in section 2.3. In panel (b), $dk_{\alpha,p}/dE_{p,0}$ has been plotted according to equation (7), for the two reference values of $E_{\alpha,0}$ and the same set of parameters indicated in panel (a). The distribution $dk_{\alpha,\infty,p}/dE_{p,0}$ resulting from the $\alpha$ spectrum of equation (11) is also shown. Despite the increasing yield of scattered protons at low $E_{p,0}$, it is immediate to recognise that the contribution to $k_{\alpha,p}$ is negligible when $E_p < 100$ keV is negligible. This is obviously due to the extremely low fusion probability at those energies. One can also notice that the contribution to $k_{\alpha,\infty,p}$ of the low-energy $\alpha$ (blue curve) is rather limited.

A detailed analysis of the behaviour of $k_{\alpha,p}(E_{\alpha,0})$ with $T_e$ and $n_e$ is shown in figure 6. In panel (a), curves have been generated for three different values of $T_e$ (10, 50 and 100 keV), keeping $n_e$ fixed at $10^{26}$ cm$^{-3}$. As a term of comparison, a curve based only on the Rutherford $\alpha$–p scattering has been calculated at $T_e = 50$ keV. In panel (b), $n_e$ has been varied

![Figure 4](image-url) Proton fusion probability as a function of the incident energy, for different representative values of $n_e$ and $T_e$. The values of $T_e$ (in keV) are indicated next to the curves.

![Figure 5](image-url) Spectral analysis of scattered protons (a) and multiplication factors (b) for $\alpha$ particles of initial energies $E_{\alpha,1}$ and $E_{\alpha,II}$. The values of $n_e$, $T_e$, $T_i$ and $\gamma$ indicated in panel (a) have been used for calculations. Curves in panel (a) are given for both the scattering cross sections $\sigma_s$ and $\sigma_R$. 

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**Figure 4.** Proton fusion probability as a function of the incident energy, for different representative values of $n_e$ and $T_e$. The values of $T_e$ (in keV) are indicated next to the curves.

**Figure 5.** Spectral analysis of scattered protons (a) and multiplication factors (b) for $\alpha$ particles of initial energies $E_{\alpha,1}$ and $E_{\alpha,II}$. The values of $n_e$, $T_e$, $T_i$ and $\gamma$ indicated in panel (a) have been used for calculations. Curves in panel (a) are given for both the scattering cross sections $\sigma_s$ and $\sigma_R$. 

by 1-decade steps from $10^{22}$ to $10^{28}$ cm$^{-3}$ while keeping $T_e$ fixed at 100 keV. In all cases, the curves quickly drop below $E_{\alpha,0} \simeq 2$ MeV. Above 4 MeV, their shape is approximately linear in the semi-log plot, meaning an exponential increase with $E_{\alpha,0}$. A fit with a function of the form $k_{\alpha,p} \propto \exp(aE_{\alpha,0})$ on the curves of panel (b) returns $a \simeq 0.45$ MeV$^{-1}$ for $4 \leq E_{\alpha,0} \leq 10$ MeV, meaning that $k_{\alpha,p}$ increases by a factor of about 2.5 each time $E_{\alpha,0}$ increases by 2 MeV.

At a given $E_{\alpha,0}$, $k_{\alpha,p}$ increases with both $T_e$ and $n_e$, as expected from stopping power considerations. The slope of the straight portion of the curves in figure 6 slightly increases with $T_e$, whereas it is unaffected by variations of $n_e$. In the latter case, the spacing between adjacent curves increases with regularity upon tenfold increments of $n_e$. However, the logarithmic sensitivity of $k_{\alpha,p}$ on $n_e$ is very limited; for instance, $\partial \ln k_{\alpha,p} / \partial \ln n_e < 0.12$ at $T_e = 100$ keV, whereas for $n_e = 10^{26}$ cm$^{-3}$, $\partial \ln k_{\alpha,p} / \partial \ln T_e$ approximately ranges from 2.3 at $T_e = 10$ keV to 1.8 at $T_e = 100$ keV. We also remark the effect of the use of $\sigma_t$ instead of $\sigma_R$ in the $\alpha$–p scattering. In figure 6(a), the gap of the respective curves at $T_e = 50$ keV progressively increases with $E_{\alpha,0}$, resulting in a value of $k_{\alpha,p}$ which e.g. at $E_{\alpha,0} = 10$ MeV, is about 8 times higher when the nuclear interaction is taken into account. Despite this and the abovementioned favourable features, even at the highest values of $E_{\alpha,0}$, $n_e$ and $T_e$ considered in this work, $k_{\alpha,p}$ remains significantly lower than 1.

At $n_e = 10^{28}$ cm$^{-3}$ and $T_e = 100$ keV, one extrapolates $k_{\alpha,p} = 1$ for $E_{\alpha,0} \approx 13.6$ MeV, a value which could be achieved by using high-energy protons to trigger the fusion reaction (in a fast-ignition kind of approach) or accelerating the fusion-born $\alpha$’s, e.g. in a laser-induced electric field (Giuffrida et al 2020). It is not obvious, however, how $E_{\alpha,0}$ could be kept so high for more than one generation. Nevertheless, it can be of some comfort to notice that the ranges of the 13.6 MeV $\alpha$-particle (6.3 g cm$^{-2}$) and its knock-on proton (15.4 g cm$^{-2}$ for $E_{\alpha,0} = 8.7$ MeV), still fall well below the expected areal density ($\rho$) of an H–11B compressed pellet at ignition, $\rho \approx 50$ g cm$^{-2}$ (Duderstadt and Moses 1982). At $n_e = 10^{28}$ cm$^{-3}$ and $\gamma = 0.2$, this value translates into $r \approx 18.5 \mu$m as $\rho = 2.7 \times 10^4$ g cm$^{-3}$.

Finally, the dependence on $\gamma$ of $k_{\alpha,p}(E_{\alpha,0})$, $k_{\alpha,p}(E_{\alpha,II})$ and the resulting $k_{\infty}$ is shown in figure 7, in the range $0 \leq \gamma \leq 1$ and for the most multiplication-effective (viz the highest) values of $n_e$ and $T_e$ explored in this work. As it is obvious, the curves vanish for $\gamma \to 0$ (too few 11B ions for the fusion reaction to occur) and decrease smoothly to 0 for $\gamma > 1$ (too few protons available for scattering). In between, a maximum occurs at values of $\gamma$ which depend on $E_{\alpha,0}$ and are however not far from 0.2, the value argued in section 2.6 and used in the calculations above. More importantly, the differences in $k_{\alpha,II}(E_{\alpha,0})$, $k_{\alpha,II}(E_{\alpha,II})$ and $k_{\infty}$ between the case $\gamma = 0.2$ and the respective optimal $\gamma$’s are limited to the order of 10%. We remark that in the conditions of figure 7, the peak value for our estimate of $k_{\infty}$ is only of the order of $10^{-2}$.
4. Discussion

As anticipated, it is instructive to remark the physical reasons behind the very modest impact of suprathermal $^{11}\text{B}$ ions on the multiplication process. It is already evident from equation (10) for the $p-^{11}\text{B}$ CM energy that the fusion probability of suprathermal $^{11}\text{B}$ ions is much smaller than that of protons, at the same particle energy. Indeed, $E_{CM}$ is suppressed by a factor $m_p/m_B$, resulting in very low values of $\sigma_f$ in equation (9). Moreover, at the same particle energy, $dE_B/d\tau$ is larger than $dE_p/d\tau$, which still depresses the integrand in equation (9). On the opposite, at given $E_{0,0}$ and ion energy, $dN_{\text{thr}}/dE_{\text{thr},0}$ tends to be larger than $dN_p/dE_{p,0}$, by a factor $m_p/m_B$ $(z_B/z_p)^3 = 2.3$ when $\sigma_N = \sigma_f$ is assumed in equation (4). Also, the $^{11}\text{B}$ suprathermal spectrum is slightly wider than the proton one, since $E_{\text{thr},0}^{\text{max}}/E_{\text{p},0}^{\text{max}} = \eta_{\text{th}}/\eta_{\text{p}} = 1.2$. Nevertheless, the net result from equation (7) is such that the $^{11}\text{B}$ contribution to $k_\infty$ in equation (8) is in any case much smaller than the proton one.

Though the findings of section 3 prevent the possibility of achieving the chain reaction in realistic conditions, it is of utmost importance to study how and how much a weak multiplication regime, i.e. when $k_\infty < 1$ (and especially $k_\infty \ll 1$), can enhance the pure thermonuclear burn. In this respect, the ratio of the energy per unit volume produced during the confinement time $\tau_c$, $E$, to the energy stemming from the sole thermonuclear burn, $E_{\text{thr}}$, is just $n_l(\tau_c)/R\tau_c$, where $n_l(t)$ is given by equation (2). We prefer to study this ratio in the form of the fractional increment $I = (E - E_{\text{thr}})/E_{\text{thr}}$, which is equivalent to the suprathermal-to-thermonuclear energy yield, $\xi_{\text{st}}/\xi_{\text{thr}}$, since the suprathermal energy component, $\xi_{\text{st}}$, is obviously $E - E_{\text{thr}}$.

Explicitly,

$$I(k_\infty) = \frac{\xi_{\text{st}}}{\xi_{\text{thr}}}(k_\infty) = \frac{k_\infty}{1 - k_\infty}(1 + \frac{\tau_\alpha}{\tau_c} \frac{1 - k_\infty^{\alpha}/k_\infty}{\ln k_\infty})$$

(16)

where $\tau_\alpha$ is the spectrum-averaged thermalisation time of the $\alpha$'s, and we have used the fact that $\tau_\alpha \approx \tau_c$. Indeed, if one estimates $\tau_\alpha$ as the average extinction time of the $\alpha$-induced recoil shower, then $\tau_\alpha^2 \approx \tau^2 + \tau_c^2$, where $\tau_\alpha$ is the spectrum-averaged thermalisation time of the secondary protons; $\tau_c$ is obviously longer than the average thermalisation time of B ions, but much shorter than $\tau_\alpha$.

One immediately notes that $I$ depends on the ratio $\tau_c/\tau_\alpha$ as a parameter. At the plasma densities considered here and for $T_c \sim 5E_F$, $\tau_\alpha$ generally turns out to be of the order of 1 ps or lower. With $\tau_c \sim 1$ ns—typically assumed in inertial confinement or other laser-driven schemes (Hora et al 2017)—$\tau_c/\tau_\alpha$ can then reach the order of $10^2$ or $10^3$. Notice that for a self-sustaining chain reaction (i.e. $k_\infty \geq 1$), $\tau_c/\tau_\alpha$ represents the maximum possible number of $\alpha$-particle generations within the time $\tau_c$.

Plots of $I$ as a function of $k_\infty$ are shown in figure 8, for several orders of magnitude of $\tau_c/\tau_\alpha$ and $k_\infty < 1$. In the limit $\tau_c/\tau_\alpha \to \infty$, equation (16) yields the asymptotic behaviour $I \sim k_\infty/(1 - k_\infty)$, which, being independent of $\tau_c/\tau_\alpha$, explains the convergence of the curves observed at high values of the parameter. For $k_\infty < 1$, one recognises the approximate scaling $I \sim k_\infty$ in figure 8. This means that in the plasma conditions investigated in this work, the burn enhancement due to the multiplication is of the order of 1% at most.

In the limit $k_\infty \to 1$, equation (16) yields $I \to (1/2)\tau_c/\tau_\alpha$. This opens the possibility of very large increments in the energy output (and consequently, high fusion gains); however, at high $\tau_c/\tau_\alpha$, $I$ rises steeply when $k_\infty \to 1$, so that $k_\infty$ has to lie very close to 1 to allow increments of the order of $\tau_c/\tau_\alpha$, being approached (e.g. $I \approx 0.37\tau_c/\tau_\alpha$ when $1 - k_\infty = \tau_c/\tau_\alpha$ for $\tau_c/\tau_\alpha \approx 1$).

To summarise, our parametric analysis has shown that $k_\infty$ increases markedly with $T_c$ and less significantly with $n_\alpha$, with the optimum $\gamma$ lying between 0.2 and 0.4. The achievable fusion energy is further enhanced by the ratio $\tau_c/\tau_\alpha$. In the weak chain, however, the enhancement is quite limited moving from $\tau_c/\tau_\alpha \sim 10$ to $\tau_c/\tau_\alpha \sim 100$ while $k_\infty \lesssim 0.5$, and negligible beyond $\tau_c/\tau_\alpha \sim 100$ while $k_\infty \lesssim 0.9$. We note, moreover, that there is a trade-off between the requirements for raising $T_c$ up on the one hand, and keeping $\tau_c/\tau_\alpha$ sufficiently large on the other hand, since $\tau_c$ also increases with $T_c$ (on the contrary, $\tau_\alpha$ decreases with $n_\alpha$ as $1/n_\alpha \ln \Lambda$). For typical confinement times, however, values of $\tau_c/\tau_\alpha$ larger than at least 10 appear to be always ensured.

We conclude this section by making contact with previous representative findings for DT fuel. Peres and Shvarts (1975) have shown that a chain reaction via elastic recoils can proceed in a cold infinite DT plasma at densities above $8.4 \times 10^{22}$ ions cm$^{-3}$. The optimum isotopic ratio is $n_p/n_T = 0.72$. In the analysis, they have also considered recoil-induced DD and TT fusion reactions and the scattering by their products. The main contributor to the chain reaction turns out to be the DT-born 14.1 MeV neutron, while the 3.5 MeV $\alpha$ particle contributes only a few percent. Indeed, if the neutron is disregarded, the medium is not critical even at $10^{23}$ ions cm$^{-3}$, the highest density considered by the authors. This observation is of interest for inertial confinement experiments, where the neutron can easily escape the
compressed pellet. For a finite-temperature, infinite plasma, however, Afek et al (1978) have found lower critical densities; for instance, \(1 \times 10^{22} \text{ ions cm}^{-3}\) at \(T_e = T_i = 14 \text{ keV}\) and \(n_0/\tau_T = 0.64\). For the finite-temperature, finite-size case, Kumar et al (1986) have estimated an upper bound of 0.5 for the suprathermal fusion probability associated to the DT neutron in a pellet compressed to \(\rho r\) of a few g cm\(^{-2}\), at a density of \(6.0 \times 10^{25} \text{ ions cm}^{-3}\) (roughly 1000 times the solid density), \(T_e = T_i = 40 \text{ keV}\), and \(n_0/\tau_T = 1\). We conclude that the suprathermal contribution to the fusion yield is substantially lower in H–\(^{11}\)B fuel compared to DT fuel, in similar plasma conditions and for cases of practical interest.

5. Conclusion

We have investigated the possibility of a fusion chain reaction via \(\alpha\)-recoiled ions in high-density, non-degenerate H–\(^{11}\)B plasma, under conditions which are of interest for laser-driven experiments (\(10^{24} \lesssim n_e \lesssim 10^{28} \text{ cm}^{-3}\), \(T_e \lesssim 100 \text{ keV}\), \(T_i \sim 1 \text{ keV}\)). On the basis of a simple model, the multiplication factor for individual fusion events \((k_\infty)\) has been estimated in terms of the energy-dependent multiplication factor for individual \(\alpha\) particles \((k_\alpha)\), by averaging over the \(\alpha\) emission spectrum. A spectral analysis of the suprathermal proton population and of the multiplication factors is also reported.

We have found that the contribution of suprathermal \(^{11}\)B ions to \(k_\alpha\) and \(k_\infty\) is of the order of 1% only. In the case of the scattered proton, the complete elastic cross section, accounting also for the nuclear interaction, must be used in calculations. For instance, the value of \(k_\alpha\) for the most probable \(\alpha\) emission energy (about 4 MeV) turns out to be more than twice that found for a pure Coulomb scattering. The spectral analysis shows that only protons with recoil energies higher than or comparable to 100 keV contribute to the multiplication factors. This important limitation is essentially due to the drop of the fusion probability at lower energies.

The parametric analysis shows that \(k_\alpha\) increases with both \(T_e\) and \(n_e\), though it is much more sensitive to \(T_e\). The optimum \(\gamma\) lies between 0.2 and 0.4. In general, \(k_\alpha\) quickly drops below \(E_{\alpha,0} \approx 2 \text{ MeV}\), while it increases nearly exponentially above 4 MeV, up to at least 10 MeV, the highest \(\alpha\) energy considered in this work. Even at the highest values of \(n_e\) and \(T_e\) considered (\(10^{28} \text{ cm}^{-3}\) and 100 keV, respectively), \(k_\alpha\) (hence, \(k_\infty\)) remain significantly lower than 1: \(k_\alpha = 0.2\) for \(E_{\alpha,0} = 10 \text{ MeV}\), and \(k_\infty \approx 0.01\) over the reference fusion spectrum. While \(k_\infty \lesssim 0.3\), the fractional increment in the energy output relative to the thermonuclear burn scales linearly with \(k_\infty\), being practically insensitive to the parameter \(\tau_e/\tau_\alpha\) and remaining, therefore, quite limited. On the contrary, for \(k_\infty \rightarrow 1\), the burn enhancement approaches the order of magnitude of \(\tau_e/\tau_\alpha\), which can easily be made as large as \(10^3\) or \(10^4\) in experiments.

Increasing \(k_\infty\) above the order of \(10^{-2}\), however, appears problematic in realistic laser-driven plasma conditions, meaning those presently achievable or likely to be achieved in the near future. One notes, moreover, that \(k_\infty\) in H–\(^{11}\)B fuel is substantially lower than in DT fuel, ceteris paribus. Novel ideas are needed in order to exploit the full potential of suprathermal multiplication in H–\(^{11}\)B fuel. For instance, ion upscattering (by more than one \(\alpha\) particle) and the interaction between suprathermals from different \(\alpha\)-tracks should be investigated at high thermal reaction rates. Also, several authors (Belyaev et al 2005, Picciotto et al 2014, Giuffrida et al 2020) have reported \(\alpha\) spectra shifted towards higher energies (up to 10 MeV) in laser-driven p–\(^{11}\)B fusion experiments. Giuffrida et al have ascribed this phenomenon to the acceleration of the fusion-born \(\alpha\)’s by the same laser-induced electric field which accelerated the protons to MeV energies in those experiments. With a proper choice of target characteristics and laser parameters, such an effect could represent an excellent means to exploit the nearly exponential increase of \(k_\alpha\) at high \(\alpha\) energy.

Data availability statement

The data that support the findings of this study are available upon reasonable request from the author.

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Appendix A.

Equation (2) is derived in the following. With the notation and upon the assumptions of section 1, a thermal fusion event occurring at the time \(t = t’\) will result, on average, in a geometric progression of suprathermal fusion events which sum \(S_l\) is given by

\[ S_l = \sum_{i=0}^{l} k_\infty \]  

(A.1)

after \(l\) generations. \(S_l\) also accounts for the initial event and can be expressed in closed form as

\[ S_l = \frac{1 - k_\infty^{l+1}}{1 - k_\infty} \]  

(A.2)

Note that \(k_\infty = 1\) is a removable singularity in equation (A.2) and \(S_l\) is obviously continuous as a function of \(k_\infty\), yielding \(l + 1\) at \(k_\infty = 1\). The average number of generations after a period \(t - t’\) is obviously \((t - t’)/\tau_e\). The function corresponding to \(S_l\) in the domain of time, \(S(t - t’)\), is the Green function of the system, and it is easily obtained by substituting the
discrete variable $l$ with the continuous quantity $(t - t')/\tau_g$ in the expression on right-hand side of equation \((A.2)\).

The number of thermal fusion events per unit volume in the interval $(t', t' + dt')$ is $R dt'$, with the thermonuclear specific rate given by the usual relation

\[
R = n_p n_B < \sigma v > \quad \text{(A.3)}
\]

where $n_p$ and $n_B$ are the densities of H and $^{11}$B ions in the fuel, respectively, and $< \sigma v >$ is the fuel reactivity. After a period $t - t'$, the contribution of these events to the cumulative number of fusion events (both thermal and suprathermal) per unit volume is then

\[
dn_l = RS (t - t') dt'. \quad \text{(A.4)}
\]

Equation \((2)\) for $n_l(t)$ is finally obtained by (elementary) integration of equation \((A.4)\) over $t'$, from $t' = 0$ up to the generic time $t$. Note that the quantities $R$, $k_{\infty}$, and $\tau_g$ have been considered as independent of time during fuel burning.

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