MULTICOLOR PHOTOMETRY OF THE TYPE II CEPHEID PROTOTYPE W VIRGINIS

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Received 2007 July 31; accepted 2007 August 30

ABSTRACT

We present the results of recent long-term $BVR_{IC}$ photometric monitoring of the type II Cepheid prototype W Virginis. These new observations, made during the 2006 and 2007 observing season, represent the longest homogeneous, multicolor light curve of W Vir to date. The $BVR_{IC}$ light and color curves show conclusively that W Vir exhibits modest but detectable cycle-to-cycle variations, the cause of which appears to be multiperiodicity rather than nonlinearity. We combined our $I$-band data with the five available years of ASAS-3 $I$-band photometry to obtain a 6.5 yr light curve that we then analyzed to obtain the pulsation spectrum of W Vir. We find a best-fit period $P_0 = 17.27134$ days; along with this period and the integer-ratio harmonics $P_0/2$ through $P_0/5$ inclusive, we clearly detect two additional periods, $P_1$ and $P_{low}$, that are close to but not exactly $2P_0/3$ and $2P_0$, respectively. The former, $P_1 = 11.52562$ days, we interpret to be the first overtone mode; the latter, $P_{low} = 34.59760$ days, is close to the beat period of $(P_1^{-1} - P_0^{-1})^{-1}$, as well as to the value of $2P_0$. We interpret the previously reported but thus far unconfirmed descriptions of alternating minima as manifestations of this multiperiodicity. Finally, we use the period derived from the $I$-band light curve to define a new ephemeris: $HJD_{max} = 2,452,758,172 + 17.27134E$. We compiled an $(O - C)$ diagram spanning 75 yr from 1932 to 2007 using a variety of published photometric data and visual observations from the American Association of Variable Star Observers and derived a period-change term for the ephemeris equal to $-9.9 \times 10^{-7}E^2$, indicating a period decrease.

Key word: Cepheids

1. INTRODUCTION

W Virginis (AAVSO 1320−02: R.A. = 13h26m1.993s, decl. = $-03^\circ 22^\prime 43.42^\prime\prime$ [J2000.0]) is the class prototype of the type II Cepheid variables. It has a period of about 17.3 days and is relatively bright, with maxima reaching $V \sim 9.5$. Like many bright variables, it has become less frequently monitored and studied in recent years. While several robotic telescopes monitor this and other objects as part of their all-sky campaigns, very little recent monitoring by visual observers, short-term observational work was done on this star other than long-term observational campaigns, and this is where the American Association of Variable Star Observers (AAVSO) and its members and partner observatories represent a remarkable new opportunity for the astronomical community to study neglected, interesting objects.

The AAVSO was asked to provide photometric support for high-resolution spectroscopic observations of W Vir (G. Wallerstein et al. 2008, in preparation) throughout 2006, and we initiated an observing campaign that included both amateur and professional observers, performing visual and instrumental photometry. These data are being used in the interpretation of the resulting spectroscopy, but they also represent a substantial new data set for this historic star. In particular, nightly $BVR_{IC}$ observations with the Sonoita Research Observatory (SRO) 14 inch (0.35 m) telescope in southern Arizona. An SBIG STL-1001E CCD camera was used with Johnson-Cousins $BVR_{IC}$ filters. The pixel scale is $1.25^\prime\prime$ pixel$^{-1}$, yielding a field of view of $21^\prime \times 21^\prime$. Each image was dark-subtracted and flat-fielded using standard techniques. All stars in each image were extracted using routines from DAOPHOT (Stetson 1987). Aperture photometry was performed, typically using apertures whose diameter was equal to 4 times the full width at half-maximum of the star profiles.

In this paper, we present new, multicolor photometry for W Vir taken during 2006 and 2007. We present continuous and folded light and color curves, describe the quality and statistical properties of the Sonoita data, and present a comprehensive analysis of this important new data set. In §2 we describe in full the newly acquired observations, as well as the archival data used in our analyses. In §3 we describe our analyses including derivation of the full variability spectrum, determine a new ephemeris for this star, and present the results of our $I$-band and multicolor analyses. In §4 we discuss the physical consequences of our findings with particular emphasis on the unambiguous detection of the first overtone mode and recommend future avenues for observational and theoretical research on W Vir and on the type II Cepheids in general.

2. OBSERVATIONS

Despite the astrophysical importance of W Vir, little concerted observational work was done on this star other than long-term monitoring by visual observers, short-term $UBV$ photometric campaigns covering a few cycles, or monitoring by automated systems (e.g., Hipparcos and ASAS-3). Because of this, the novel observations conducted here represent a major increase in the observational record for W Vir, and we discuss these observations in some detail.

2.1. Sonoita Research Observatory Observations

W Vir has been observed over two seasons using the SRO 0.35 m telescope, located in southern Arizona. An SBIG STL-1001E CCD camera was used with Johnson-Cousins $BVR_{IC}$ filters. The pixel scale is $1.25^\prime\prime$ pixel$^{-1}$, yielding a field of view of $21^\prime \times 21^\prime$. In this paper, we present new, multicolor photometry for W Vir taken during 2006 and 2007. We present continuous and folded light and color curves, describe the quality and statistical properties of the Sonoita data, and present a comprehensive analysis of this important new data set. In §2 we describe in full the newly acquired observations, as well as the archival data used in our analyses. In §3 we describe our analyses including derivation of the full variability spectrum, determine a new ephemeris for this star, and present the results of our $I$-band and multicolor analyses. In §4 we discuss the physical consequences of our findings with particular emphasis on the unambiguous detection of the first overtone mode and recommend future avenues for observational and theoretical research on W Vir and on the type II Cepheids in general.
Since W Vir is located at Galactic coordinates $l = 319.5659^\circ$, $b = +58.3713^\circ$, no stars of comparable magnitude lie within the field of view. We therefore offset the field to the west to acquire a handful of fainter, but usable, comparison stars. A total of six stars, ranging from $V = 11.0$ to 13.6, form an ensemble. Using the inhomogeneous ensemble techniques similar to Honeycutt (1992) we perform differential photometry of W Vir using these six stars, resulting in an average per-observation error of 5 mmag. The Celestron C14 optical telescope assembly has strong vignetting across the field. While flat-fielding removes essentially all of this vignetting, a small residual offset may remain. We positioned the field exactly the same on all visits to ensure that any residual will just cause a systematic offset from the standard system.

All stars in the 21$' \times 21$ field have been calibrated by the SRO telescope on 42 photometric nights. On each night, Landolt (1983, 1992) standards were observed to obtain transformation coefficients, along with a single A0 star that was followed from meridian to large air mass. These observations resulted in mean errors of about 5 mmag for the comparison stars. The stars used are found on the AA VSO Web site. Astrometry of these comparison stars was determined using the UCAC2 reference catalog (Zacharias et al. 2004) along with the SLALIB astronomy utility software package (Wallace 1994). Coordinates are given in decimal degrees and are valid for epoch 2006.77. Errors are standard deviations of the mean.

### 2.2. Other Observational Data

Other major sources of data include ASAS (Pojmanski 2002), Hipparcos photometry (van Leeuwen et al. 1997) transformed to $V$ using the calibration of Harmanec (1998), and the AAVSO International Database. We also made use of previously published observations of W Vir collected and generously provided by L. Berdnikov (2002, private communication); these data include a number of $UBV$ and $BV$ data sets published in Abt (1954; observed by Whitford and Code at Mount Wilson), Eggen et al. (1957), and Arp (1957). All but the ASAS-3 data were used solely in the compilation of our $(O - C)$ diagram, although we note that the early $UBV$ data of Arp (1957) were also used in the transformation of the Hipparcos data to $V$.

### 2.3. V-Band Light Curve

We merged 6.5 yr of $V$-band observations from SRO and from the archives of the ASAS-3 project (Pojmanski 2002) to form a long-term light curve and perform a detailed time-series analysis. We limited this light curve to these two data sources because (1) both are well sampled, relatively homogeneous, and fully calibrated on a standard photometric system, and (2) the rate of period change in W Vir is significant enough to introduce smearing of the signal over much longer timescales. The 6.5 yr long ASAS-3 + SRO $V$-band light curve of W Vir is shown in Figure 1. Aside from the large annual observing gaps due to solar conjunction, the $V$-band light curve of W Vir is nearly continuous with at least a few (and often many) points obtained per cycle and several cycles obtained during each observing season.

### 3. Analysis

We have three primary goals in performing this work. First, we want to use a homogeneous, photometrically calibrated data set to determine the pulsation spectrum of W Vir over a long interval and define a new ephemeris. Second, we want to study the long-term evolution of the light curve of W Vir, and determine whether the changes are due to multiple periods, secular period changes, nonlinear behavior, or some combination of these. Third, we want to use the shorter set of multicolor photometry to study the multiwavelength behavior of W Vir to gain more physical insight into the pulsations. To study the periodicities and period evolution of W Vir, we used both Fourier and $(O - C)$ analyses on the primary $V$-band data set, on the new SRO $BVRC_{I1}$ data set, and on data from the AAVSO International Database. We discuss each of these in turn below.

#### 3.1. Fourier Analysis of a 6.5 yr $V$-Band Light Curve

First, we performed a thorough Fourier analysis of the 6.5 yr $V$-band light curve, composed of ASAS-3 $V$ magnitudes and $V$ magnitudes obtained with SRO. We chose not to include transformed Hipparcos magnitudes because of the large gap between the end of Hipparcos observations and the start of ASAS-3 observations, and because the period changes significantly enough over the 17 yr span of the longer data set that it affects the Fourier analysis. Our analysis was performed using a newly written

### Table 1

| R.A. (J2000.0) | Decl. (J2000.0) | $V$ | $(B - V)$ | $(V - R_C)$ | $(R_C - I_C)$ | $(V - I_C)$ | err$(V)$ | err$(B - V)$ | err$(V - R_C)$ | err$(R_C - I_C)$ | err$(V - I_C)$ |
|---------------|---------------|-----|-----------|-------------|--------------|-------------|---------|-------------|----------------|----------------|----------------|
| 201.395994    | -3.296183     | 13.051 | 0.811    | 0.436       | 0.387        | 0.820       | 0.006   | 0.008       | 0.005          | 0.004          | 0.005          |
| 201.292630    | -3.312171     | 13.564 | 0.970    | 0.568       | 0.544        | 1.112       | 0.005   | 0.009       | 0.005          | 0.005          | 0.005          |
| 201.244004    | -3.399964     | 12.174 | 0.581    | 0.351       | 0.342        | 0.715       | 0.006   | 0.009       | 0.005          | 0.005          | 0.007          |
| 201.264303    | -3.546543     | 11.043 | 1.110    | 0.569       | 0.494        | 1.058       | 0.005   | 0.008       | 0.004          | 0.004          | 0.005          |
| 201.264437    | -3.528959     | 13.328 | 0.972    | 0.560       | 0.467        | 1.020       | 0.005   | 0.007       | 0.004          | 0.004          | 0.005          |
| 201.351088    | -3.528159     | 12.517 | 0.564    | 0.323       | 0.294        | 0.615       | 0.005   | 0.008       | 0.004          | 0.004          | 0.005          |

Note.—Units of right ascension and declination are decimal degrees.
The implementation of the Fourier “clean” algorithm outlined in Roberts et al. (1987), for which the FORTRAN 90 source code is available on request. For these calculations, we tested all frequencies between 0 and 0.8 cycles day$^{-1}$ with a resolution $\Delta f = 6.6 \times 10^{-6}$ cycles day$^{-1}$. We preceded the computation of the final, high-resolution spectrum with a low-resolution scan (at approximately the minimum frequency separation imposed by the span of the data set) to determine the correct mean value of the data from the zero-frequency amplitude. We heavily oversampled the final spectrum in the frequency domain by a factor of 64, and used a gain of 0.05 per clean iteration to make the cleaning process as numerically stable as possible; larger gain values and lower frequency resolution produced nearly identical frequencies at the cost of slightly higher noise, while the cost in oversampling is as numerically stable as possible; larger gain values and lower frequency resolution produced nearly identical frequencies at the cost of slightly higher noise, while the cost in oversampling is simply increased computation time. The final, cleaned spectrum of the 6.5 yr $V$-band data set is shown in Figure 2. There are three groups of frequencies of interest: the primary pulsation frequency, $f_0$, and its integer-multiple harmonics, $2f_0$ to $5f_0$; a secondary frequency, $f_1$; and a low-frequency peak, $f_{\text{low}}$, that appears to be a beat frequency of $f_{\text{low}} \sim f_1 - f_0$. The frequencies, amplitudes, and phases of these frequencies are given in Table 2.

The spectrum is dominated by the primary pulsation frequency of $f_0 = 5.788939 \times 10^{-2}$ cycles day$^{-1}$ ($P_0 = 17.27134$ days) and its integer multiples. The higher order harmonics have significant amplitude, and as a result the light curve is non-sinusoidal. We compared the $V$-band Fourier component parameters $\phi_{j1} = \phi_j - j\phi_1$ and $R_{j1} = a_j/a_1$ of these integer multiple frequencies with previously published values (Morgan 2003) and find that our values are reasonably consistent with prior observations. We note that our values were derived directly from the cleaned Fourier spectrum; these values are often derived via sine-wave fitting and are done to lower order, and so may not be strictly comparable. A comparison of our derived parameters and those published elsewhere is given in Table 3. Our derived values of $R_{j1}$, $\phi_{j1}$ are consistent with previously published values obtained from other observations, with the only minor exception being $R_{31}$, which is lower by a factor of 2 in our data. As a whole, this shows that the underlying pulsations driven at $P_0$ are stable throughout the recent observational record, indicating that the underlying pulsational waveform is unchanged over recent years. This is good evidence for W Vir being a stable pulsator despite it being a long-period Cepheid. Like the other long-period type II Cepheids (and unlike the BL Her stars and classical Cepheids), there is no clear trend in $R_{j1}$, $\phi_{j1}$ with period. However, the relatively large value of $R_{21} \sim 0.2$ is quite similar to those of other high-amplitude pulsators, while the value of $\phi_{21} \sim 5.9\pi$ (very close to $2\pi$) results in a nearly symmetric (if non-sinusoidal) light curve unlike the sawtooth shapes of shorter period Cepheids, where $\phi_{21} \sim 4.0$.

### 3.2. Updated Ephemeris and $(O - C)$ Analysis

The $V$-band data set is of sufficient length and photometric quality that we are able to compute an accurate, current ephemeris for W Vir. We can then compute an $(O - C)$ curve and establish a rate of period change based on these archival data. To determine the current ephemeris, we created a synthetic light curve using the frequencies, amplitudes, and phases for $f_0$ through $5f_0$ given in Table 2, excluding $f_1$ and $f_{\text{low}}$. After ensuring that this curve was a good fit to the $V$-band light curve, we chose the best-observed maximum (HJD 2,452,758.172) nearest the temporal center of the data set (HJD 2,453,078.271) as the zero point of our ephemeris:

$$HJD_{\text{max}} = 2,452,758.172 + 17.27134E.$$  

We then used this ephemeris to measure $(O - C)$ for all available data for W Vir. The available data includes visual observations from the AAVSO International Database spanning JD 2,419,500 to the present, along with previously published photometry of various kinds collected by L. Berdnikov and collaborators (L. Berdnikov 2002, private communication). When the data were sufficient to allow fitting, individual cycles were fit by eye using a synthetic mean curve placed at the closest predicted time of maximum. We adjusted both the magnitudes and times of

| Frequency (cycles day$^{-1}$) | Freq. Error ($\times 10^{-8}$) | Amplitude ($F$) | Amp. Error ($F$) | Phase (rad) | Phase Error (rad) | ID |
|------------------------------|-------------------------------|----------------|-----------------|-------------|-----------------|----|
| 0.05789939                   | 2                             | 0.590          | 0.001           | -0.716      | 0.001           | $f_0$ |
| 0.11579878                   | 12                            | 0.119          | 0.001           | -1.820      | 0.006           | $2f_0$ |
| 0.17369817                   | 43                            | 0.033          | 0.001           | 1.052       | 0.021           | $3f_0$ |
| 0.23159755                   | 27                            | 0.052          | 0.001           | -1.227      | 0.016           | $4f_0$ |
| 0.28949694                   | 157                           | 0.099          | 0.001           | 2.543       | 0.059           | $5f_0$ |
| 0.02889049                   | 32                            | 0.044          | 0.001           | 1.414       | 0.035           | $f_{\text{low}}$ |
| 0.08567634                   | 62                            | 0.023          | 0.001           | 2.791       | 0.054           | $f_1$ |

Notes.—The Fourier amplitudes and phases of the harmonics $2f_0$ through $5f_0$ are given at the exact frequency, rather than at the measured peak, because background noise shifts the location of the peak slightly from the true value. Phases are relative to HJD = 2,453.078.271, the time center of the ASAS-3 + SRO data set.
the observed data points to obtain a best fit to the synthetic cycle; a magnitude adjustment was often required due both to the difference in response between visual estimates and the $I'$ band and to cycle-to-cycle variations not included in the synthetic mean curve. For cases where there was no objective, unambiguous way to improve the fit with either a magnitude or a temporal offset, we left one or both equal to zero; for this reason, many of the $(O - C)$ measures are “0.0,” skewing the $(O - C)$ diagram close to the zero point of the ephemeris. For the visual data, we used individual observers when possible to minimize scatter between the different subjective responses by different observers, but the $(O - C)$ values obtained from visual data were marginal in general due to incomplete coverage of pulsation cycles. Although AAVSO data prior to 1932 exist, they are generally not of sufficient quantity or homogeneity to allow unambiguous cycle fitting. Fits to instrumental photometry were generally better, although they too suffer from incomplete coverage of cycles and are also affected by the intrinsic cycle-to-cycle variations. We initially estimated errors on the times of maxima to be around 0.5 days, and the resulting reduced $\chi^2$ values of the fits suggest the errors are likely around 0.35 to 0.4 days.

The resulting $(O - C)$ diagram for W Vir is shown in Figure 3, with first- and second-order polynomial fits superimposed. There is clearly a significant trend in the $(O - C)$ curve, and the period-change term is identical to within error bars for fits involving only a second-order term and a quadratic (first- and second-order term) fit. The reduced $\chi^2$ value for the pure second-order fit is the same as that for the quadratic fit. A fit involving only a first-order term (which measures only an error in the period, rather a period change) has a worse $\chi^2$ value than either of the other two fits. Because of this, we are confident that we have detected the period change of W Vir over the span of data used in this paper. The resulting ephemeris with a second-order correction is

$$\text{HJD}_{\text{max}} = 2,452,758.172 + 17.27134E - 9.9 \times 10^{-7}E^2.$$  (2)

Our result from the last 75 yr of data contradicts the inconclusive results found by Percy & Hoss (2000), who found essentially no period change when analyzing archival times of maximum and minimum dating to the 19th century. We have not included data prior to 1932 in our analysis, but we note the existence of both sparse AAVSO observations and published photometry along with archived times of maximum and minimum, and plan to reanalyze these data for a future paper.

### 3.3. $BVR_C$-$I_C$ Light Curves

A major part of this project was to observe W Vir in $BVR_C$-$I_C$ simultaneously to better understand the underlying physical behavior of the pulsations and provide full photometric coverage for the spectroscopy performed by G. Wallerstein et al. Very little multicolor work has been done on W Vir since the early photometric studies performed in the 1950s. Some multicolor amateur data exist, with the most notable being the $B$ photometry of M. Bonnardeau taken during the spring and summer of 2005 and the AAVSO data submitted as part of our support for the Wallerstein campaign. The observations we obtained with SRO represent the largest multicolor data set obtained for W Vir in its history. The full $BVR_C$-$I_C$ light curve taken at SRO is shown in Figure 4, and the $BVR_C$-$I_C$ light curve folded on a period of

![Fig. 3.—An $(O - C)$ diagram of W Vir using the ephemeris given in eq. (1). There is substantial variation in $(O - C)$ with each cycle, but the overall trend is parabolic, which indicates a secular period change. The reasons for the cycle-to-cycle scatter are (1) that the secondary pulsation frequencies affect the time of maximum and the goodness of the fit, and (2) that the sparse data for most of the cycles prior to the ASAS-3 data make it very difficult to fit a mean curve reliably. The approximate error in $(O - C)$ measures is 0.3 days.](image1)

![Fig. 4.—The $BVR_C$-$I_C$ light curve. The following magnitude offsets were applied to the $B$, $R_C$, and $I_C$ data for clarity: $B (+0.75)$, $R_C (−0.75)$, and $I_C (−1.5)$.](image2)
17.27134 days is shown in Figure 5. We note that these light curves include all data through the end of the 2007 spring observing season (JD 2,454,299), which were obtained and reduced after the bulk of the analysis for data through JD 2,454,255 was completed for this paper. The additional 40 days of data do not change the main results given in the prior sections of this work and are included to provide more complete phase coverage of the 2007 season.

3.3.1. Color Dependencies of the Pulsation Behavior

Pulsation amplitude is highest in $B$ and weakens progressively through $V$, $R_C$, and $I_C$. The $B$-band light curve has a sharp peak followed by a short plateau and decline, while the other three bands rapidly rise to a plateau and remain constant for approximately half a cycle. Maximum light occurs immediately following the rapid rise at $B$ and $V$, while $R_C$ remains essentially flat, and $I_C$ has a slowly rising plateau with peak approximately 0.2 cycles after $B$ and $V$. We determined the Fourier amplitude ratios $R_{j1} = a_j/a_1$ and phase differences $\phi_{j1} = \phi_j - \phi_1$ for $j = 2, 4$ for each of the $BVR_CI_C$ light curves. The amplitude ratios are shown in Figure 6, and the phase differences are shown in Figure 7. The amplitude ratios are largest in $B$ and smallest in $I_C$ for all harmonics, indicating less sinusoidal light variation at bluer wavelengths. The value of $\phi_{31}$ is close to $2\pi$ in all four bands, which results in a much more symmetric light curve, and it asymptotically approaches $2\pi$ at redder wavelengths. The values of $\phi_{31}$ and $\phi_{41}$ would produce larger asymmetries, but the harmonics have lower amplitude and thus a smaller effect on the overall light curve. Generally, the amplitude ratios and phase differences indicate that the light curves become increasingly symmetrical (although not purely sinusoidal) at redder wavelengths.

The $(B-V)$ color spans a much larger range (+0.4 at maximum to +1.1 at minimum) than do $(V-R_C)$ or $(R_C-I_C)$ because the temperature range is large ($\sim$5000–7000 K) and the spectral peak at $T_{max}$ lies within the $B$ band. The color change is very rapid during rising light, going from reddest to bluest within 0.2 cycles; the steepness of both the light curve and the color curve suggest a strong shock, and this is consistent with past spectroscopic studies (see Lèbre & Gillet 1992 and references therein).

3.3.2. Cycle-to-Cycle Variations

The folded light curve in Figure 5 shows much larger scatter (>0.1 mag) than the photometric errors (generally <0.01 mag), indicating that there are cycle-to-cycle variations. This is expected given the amplitudes of $f_{low}$ and $f_1$; folding on twice the fundamental period decreases the scatter considerably, although it does not remove it entirely, particularly for the 2007 season. Color-color plots of $(B-V)$ versus $(V-I_C)$ for single-season data folded on twice the fundamental period are shown in Figure 8. The color-color plot clearly shows that the deeper of the two minima (minimum 2) has a significantly redder $(B-V)$ color than...
minimum 1, indicating a cooler temperature. The fact that this difference between the two minima persists throughout the season indicates that the pulsations are being modulated with a period approximately twice that of the main period.

3.3.3. \( R_C \)-Band Irregularity

The \( R_C \)-band light curve shows much larger scatter than \( B, V \), or \( I_C \) when folded, indicating that there is a transient, irregular spectral feature occurring within the \( R_C \) band. We have no means of uncovering details of this feature with broadband photometry, but the work of Lébre & Gillet (1992) hinted strongly that it may be variable postshock emission lines in an extended atmosphere. Lébre & Gillet (1992) found very large variations in \( H_O \) flux over the course of the pulsation cycle, as well as irregularity in the evolution of the line-profile variations. The flux variation of the \( H_O \) emission alone is about 5%: the width of the line is about 0.5% of the width of the \( R_C \) band, and the flux varies between 0.5 and 6 times the continuum, which could generate variations at the level of about 0.1 mag. But what is striking is that there are large variations at a given phase from cycle to cycle, which indicates that the shock features are not occurring at the same time as they propagate into the extended atmosphere and do not have the same strength. This suggests that the response of the extended atmosphere is irregular, despite the regularity of the photospheric motions. High-resolution spectroscopy over several cycles may provide more insight into this behavior.

4. DISCUSSION

There are several important results of our work. We conclusively show that there are cycle-to-cycle variations in the light curve of \( W \) Vir, but that these variations appear to be due to the presence of two pulsation modes and not to inherent instabilities in the pulsation. The fact that these modes are well defined in the Fourier spectrum suggests that the pulsation behavior is largely linear in nature. Prewhitening of the \( V \)-band data with the frequency set in Table 2 does not fully remove the signal, which suggests that a second process is at work, either a secular period change or low-level nonlinearity. The period of \( W \) Vir is definitely changing, as was shown by the \((O-C)\) diagram in Figure 3, and the fact that the prewhitening is clearly worse at the temporal extremes of the light curve than at the temporal center suggests that a secular period change may explain long-term changes without requiring nonlinear behavior.

The secondary pulsation frequencies \( f_{low} \) and \( f_1 \) are real and are not aliases of the main frequency. The color differences between alternating minima seen in the 2006 \( BVR_CI_C \) data must have a physical origin, and a beat frequency of \( f_{low} = f_1 - f_0 \sim f_0/2 \) is the easiest interpretation. The fact that the pattern of alternating minima did not repeat as clearly during the 2007 observing season is caused by the low-frequency peak \( f_{low} \) not being exactly \( f_0/2 \). The two secondary spectral peaks could be a manifestation of period doubling, as was described, e.g., in Pollard et al. (2000). However, two facts suggest a purely modal explanation for \( W \) Vir rather than a fundamental change in pulsation behavior. First, the two peaks in question are not exact rational fractions of the main frequency to within the frequency precision of either peak. Any nonlinear process that “doubles” the primary period without making it exactly twice the period while keeping it relatively stable over time would be difficult to explain. Second, the frequency ratio \( f_0/f_1 \sim 0.667 \) is consistent with the likely frequency ratio of the radial fundamental and first overtone modes of Population II Cepheids having periods longer than 10 days. (See Buchler & Szabó 2007 for recent models.)

We propose that the cycle-to-cycle variations of \( W \) Vir are best explained in terms of combinations of linear pulsation modes, and that the strength of these variations is due to the near-resonant ratio of the fundamental and first overtone frequencies. We note that because this particular frequency ratio should only occur in well-evolved, long-period Cepheids (possibly those on their last crossing), this stage may be the last waypoint in the evolution of \( W \) Vir stars before they move up the asymptotic giant branch into the RV Tauri phase. For the general population of long-period type II Cepheids that show strong cycle-to-cycle variability, their apparent irregularity may be due to overtone modes instead of nonlinear pulsation behavior.

The link between the \( W \) Virginis stars and the RV Tauri stars has been discussed previously (see, e.g., Wallerstein 2002), and observational work on the Large Magellanic Cloud \( W \) Virginis and RV Tauri sample (Pollard et al. 2000) shows a progression in the physical and pulsational characteristics of these two classes. The physical distinction between the two in this scenario is that the \( W \) Virginis stars are slightly less evolved members of the same population whose lower luminosity-to-mass \((L/M)\) ratios keep them within the linear pulsation regime; the RV Tauri stars, having larger \( L/M \) ratios, are in the nonlinear regime. Interestingly, although RV Tauri stars are clearly nonlinear in nature, the work of Buchler et al. (1996) showed that their seemingly chaotic behavior is confined to a low-dimensional phase space, and they interpret this as being due to the interaction of as few as two pulsation modes—exactly what we see with \( W \) Vir.

The picture that emerges is that double-mode pulsation or multiperiodicity becomes increasingly likely at longer periods and larger \( L/M \) ratios among Population II giants, and that both the type II Cepheids of long period and the RV Tauri stars exhibit this behavior. The phenomenological difference between the two is then simply that the RV Tauri stars have reached the point where nonlinearities overwhelm the pulsational dynamics of the envelope, leading to irregularity in the light curve. We suggest three future studies that could confirm this picture. First, evolution and nonadiabatic, linear pulsation models of \( W \) Vir stars having periods longer than 10 days are crucial to studying the physical link between the two classes, and we strongly encourage the Cepheid modeling community to expand their model grids to these longer periods and higher luminosities into the RV Tauri region. Second, a long-term photometric study of other long-period type II Cepheids should be undertaken to search for linear multiperiodicities. The existence of many all-sky photometric surveys such as ASAS (Pojmanski 2002) greatly facilitates this, and such an analysis is currently underway (P. Wils & S. Otero 2007, private communication). Finally, an analysis of the chaotic dynamics of a larger sample of RV Tauri stars (and the type II Cepheids) should be done, as was performed on R Scuti (Buchler et al. 1996) and AC Her (Kollath et al. 1998). Such studies could easily be performed on both large-scale photometric survey databases and on existing archives of long-term visual observations.

5. CONCLUSIONS

Despite being the class prototype of a cosmologically important class of variable stars, our understanding of \( W \) Virginis and other type II Cepheids is incomplete. Until now, little concentrated observational work has been done on this object other than with single-cycle photometry or sparse coverage over a single observing season with photometry and spectroscopy. The use of an automated telescope with a well-planned observing queue made it possible to follow \( W \) Vir regularly for nearly two years. This highlights the power of small, automated telescopes for performing long-term studies of neglected stars, and also the fact
that there remains work still to be done even on bright, “well-studied” objects.

Our work has clarified some long-standing questions about the pulsation behavior and confirmed the existence of double-mode pulsations and the resulting cycle variations in this star. Despite the existence of a second pulsation period, the pulsational signature of the dominant mode is very stable over time, suggesting that W Vir at least does not show incipient instabilities like those seen in some long-period type II Cepheids. An interesting question then becomes where is the boundary between the parameter spaces of the type II Cepheids and the RV Tauri stars, and at what point do the linear pulsations of Cepheids give way to the nonlinear behavior shown in the RV Tauri stars? Linear, nonadiabatic pulsation models of both type II Cepheids and RV Tauri stars clearly need to be generated using modern opacities to map out the parameter space of mass, luminosity and temperature to determine fundamental and overtone mode periods and their nonadiabatic growth rates; hydrodynamic models will be key to understanding where the transition between linearity and nonlinearity lies, and we encourage the computation of both by the theoretical modeling community.

We wish to thank J. Gross, D. Terrell, and W. Cooney for their permission to use the Sonota Research Observatory for this project. As always, we acknowledge with gratitude the observations made by members of the AAVSO’s worldwide observing community over the last century, without whom this work would not be possible. We also thank L. Berdnikov for supplying us with the compiled published photometry in 2002, and we thank the anonymous referee whose comments helped to clarify and improve the paper. This research has made use of NASA’s Astrophysics Data System and the SIMBAD database, operated at CDS, Strasbourg, France.

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