Resonant wave–filament interactions as a loss mechanism for HHFW heating and current drive

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Abstract

Perkins et al (2012 Phys. Rev. Lett. 109 045001) reported unexpected power losses during high harmonic fast wave (HHFW) heating and current drive in the National Spherical Torus Experiment (NSTX). Recently, Tierens et al (2020 Phys. Plasmas 27 010702) proposed that these losses may be attributable to surface waves on field-aligned plasma filaments, which carry power along the filaments, to be lost at the endpoints where the filaments intersect the limiters. In this work, we show that there is indeed a resonant loss mechanism associated with the excitation of these surface waves, and derive an analytic expression for the power lost to surface wave modes at each filament.

Keywords: wave-filament, HHFW, mode conversion

(Some figures may appear in colour only in the online journal)

1. Introduction

Reportedly [1, 2], during high harmonic fast wave (HHFW) heating and current drive in the edge plasma of the National Spherical Torus Experiment (NSTX), a substantial fraction of the radiofrequency power fails to reach the core plasma. Instead, it flows along field lines and hits the divertors. Over the years, many explanations for these unexpected losses have been put forward, involving fast waves (FW) [3], slow waves (SW) [4, 5], sheath rectification [6, 7] and parametric decay [8].

Recently, we proposed a mechanism that can explain why such losses are observed on NSTX but not elsewhere [9] (i.e. mainly in conditions with HHFW and small parallel wavenumber ($k_\parallel$)). Knowing that small (non-ELM) filaments are plentiful in the NSTX SOL [10, 11], we used an analytic solution for wave scattering at plasma filaments [12], which shows that surface waves on such filaments can be resonantly excited and carry power along the filaments and thus along the field lines. This is more common in the HHFW heating scenarios used in NSTX than in generic ion cyclotron range of frequencies (ICRF) heating scenarios. We hypothesized that the power would be lost due to some endpoint loss mechanism (e.g. sheath rectification, imperfect reflection, surface resistivity) where the filament finally intersects a divertor.

In this work, we quantify the amount of power that is redirected to the surface waves in an idealized case (infinite magnetic field lines) where the end point mechanisms are not present. We follow an approach that should be generic to any resonant surface-wave-like oscillations. The main steps are as follows.

(a) Derive the resonance condition for surface wave excitation on a given filament. Specifically, find the parallel wavenumber $k_\parallel$ at which a resonance occurs for prescribed
filament properties (density, radius). Several resonant $k_\parallel$ values may exist.

(b) Introduce a causality parameter $\nu$, formally similar to a collision frequency, as explained in [13].

c) Calculate the total power dissipated in the filament from step (a), with $\nu > 0$.

d) The resulting integrand has Lorentzian terms with peaks at the resonant $k_\parallel$, growing higher and narrower, but remaining integrable, as $\nu$ decreases.

e) Take the limit $\nu \to 0^+$. If the filament is long enough, we show that the asymptotic dissipated power remains finite, independent of $\nu$. We claim that the result is fairly independent of the physical dissipation mechanism described phenomenologically by $\nu$ (appendix A).

(f) Average the dissipated power over the filament type. Filaments are inherently random structures in terms of density/radius, with each a specific probability to be present at a given time, and a specific resonant $k_\parallel$ value that matches a specific part of the launched $k_\parallel$ spectrum for the incident waves. This work is organized as follows: in section 2 we analytically perform step (a) for one specific branch of resonant modes relevant in the HHFW regime. In section 3 we derive the power redirected due to this resonant surface wave excitation (steps (b)–(e)). Section 4 contains a discussion of the statistical nature of the filaments and the average power converted per filament (step (f)). An example is given in section 5, and the conclusion is given in section 6.

2. Derivation of the resonance condition

We consider a wave scattering problem much like that discussed in [9, 12]: a cylindrical filament with radius $r_f$ and constant density $n_f$, in a background plasma with constant density $n_0$. The filament is along the confining magnetic field, which is along $z$, parallel to $B$. The perpendicular direction (subscript $\perp$) is perpendicular to $B$. We use a cylindrical coordinate system $(r, \theta, z)$ or $(r, z)$ centered on the filament. In [9], we identified multiple kinds of conditions under which surface waves exist on the filament. Here, we try to find an approximate analytical expression for the resonance condition for the ones we expect to be physically relevant in the HHFW regime, the ones which occur when $\frac{n_f}{m_i}$ is not much greater than 1.

We start by writing the dielectric tensor in the HHFW limit ($\Omega_f \ll \omega \ll \omega_i \ll \omega_i^*$)

$$\epsilon = \begin{bmatrix} \epsilon_\perp & -i\epsilon_r & 0 \\ i\epsilon_r & \epsilon_\perp & 0 \\ 0 & 0 & \epsilon_\parallel \end{bmatrix} = \epsilon^H$$

$$\epsilon^H = \begin{bmatrix} -\frac{\omega_i^2}{\omega^2} & -i\frac{\omega_i^2}{\omega^2} & 0 \\ -i\frac{\omega_i^2}{\omega^2} & -\frac{\omega_i^2}{\omega^2} & 0 \\ 0 & 0 & -\frac{\omega_i^2}{\omega^2} \end{bmatrix}$$

where $\omega_i = \sqrt{\frac{m_i q_i}{e}}$, $\omega_e = \sqrt{\frac{m_e q_e}{e}}$, $\Omega_i = \frac{e B}{m_i}$, $\omega = 2\pi f$, with $f$ the launched wave frequency, and $q_i$ and $m_i$ the ion charge and mass. For simplicity, we assume a single-ion plasma. In the range of plasma parameters of interest for this work, the slow wave is evanescent, and its perpendicular wavevector $k_\perp$ is purely imaginary, with imaginary part $\kappa$

$$\kappa^2 = k_\perp^2 \frac{\epsilon_\parallel}{\epsilon_\perp} \approx k_\parallel^2 \frac{m_i}{m_e}$$

which is conveniently independent of the density, in the HHFW limit. Throughout this work, we will assume without loss of generality that $k_\parallel$ and $\kappa$ are positive. Following [14], we introduce a spectral representation of the scattered SW field, in the filament and in the background

$$E_\parallel = \int A_t(k_\parallel) \hat{E}_t(r, \theta, k_\parallel) \exp(ik_\parallel z) dk_\parallel$$

$$E_\perp = \int A_b(k_\parallel) \hat{E}_b(r, \theta, k_\parallel) \exp(ik_\parallel z) dk_\parallel$$

with the amplitudes $A_t(k_\parallel), A_b(k_\parallel)$ to be determined. We will see that the poles of $A_t(k_\parallel)$ and $A_b(k_\parallel)$ are the resonances for which we are looking. Next, we write $\hat{E}$ as the gradient of a potential [14]

$$\hat{E}_t \exp(ik_\parallel z) = \sum_m -\nabla \left( I_m(kr) \exp(im\theta + ik_\parallel z) \right)$$

$$\hat{E}_b \exp(ik_\parallel z) = \sum_m -\nabla \left( K_m(kr) \exp(im\theta + ik_\parallel z) \right)$$

where $I_m, K_m$ are Bessel functions. This approximation requires that $\epsilon_\perp$ is negligible w.r.t. $n_f^{-1}$, which fails for $k_\parallel \approx 0$. In practice, ICRF antennas avoid exciting very low $k_\parallel$ modes, especially when operating in dipole, which is common. As will be clear soon, only one azimuthal mode can be resonant, $m = \pm 1$ depending on the type of filament. Therefore, to simplify the notation in the following we omit the sum over $m$. For the fast wave, since $k_\perp, r_f << 1$, the perpendicular electric field of the incident FW is nearly constant across the radial scale of the filament. Mathematically, this is equivalent to approximating $E_\perp$ by $\nabla \perp \cdot r$

$$E_\perp \approx \int -\nabla \perp \cdot (E_\perp, r) \exp(ik_\parallel z) dk_\parallel$$

where $E_{\perp, r}(k_\parallel)$ is the spectrum of $E_\perp$ of the incident FW launched by the antenna. Thus, the perpendicular component of the solution of the wave equation can be approximated by the perpendicular gradient of the following potential

$$\Phi = \int \left( E_{\perp, r}(k_\parallel) \cdot r \right)$$

$$+ \left\{ \begin{array}{ll} A_t(k_\parallel) I_m(kr) \exp(im\theta) & r < r_f \\
A_b(k_\parallel) K_m(kr) \exp(im\theta) & r \geq r_f \end{array} \right.$$
In the spirit of the Born approximation [15], we omit the contribution from the scattered fast wave since it is negligible to the lowest order. Although our approach should remain valid in the case where the scattered fast wave is evanescent, this approximation is especially well-justified in the case where it is propagative: the surface wave cannot radiate energy away from the surface, and thus cannot contain a significant contribution of a propagating fast wave. As a consequence of this approximation, we will see the surface waves on the filament gain energy from the incident fast wave, but we will not see the fast wave lose energy—an approximation that is only really valid in a perturbative regime where at most a small fraction of the incident power is lost to the surface waves. Within these approximations, of the six boundary conditions on the interface [12], only two are not trivially satisfied: the continuity of $\Phi (E_f)$ from the fast wave is continuous by assumption since we neglect the scattered fast wave, and the continuity of $E_\parallel$ from the scattered slow wave follows from the continuity of $\Phi$ and the continuity of $D_n = \epsilon_{\perp} E_\omega - i \epsilon_\times E_\phi$ at the filament surface. These two boundary conditions suffice to constrain the unknown coefficients $A_t$ and $A_b$. The first gives

$$A_t(k_\parallel) I_m(\kappa r_\parallel) = A_b(k_\parallel) K_m(\kappa r_\parallel).$$

(10)

For the continuity of $D_n$, we consider a single $k_\parallel$ mode, where we may assume without loss of generality that $E_{\perp,i}$ points along the $x$ axis, and $|E_{\perp,i}| = |E_{\perp,f}|$. Then

$$\frac{E_r}{\exp(k_\parallel z)} = -E_{\perp,i} \cos(\theta) - \kappa \left\{ \frac{A_t' I_m(\kappa r) \exp(i m \theta)}{A_b K_m(\kappa r) \exp(i m \theta)} \right\} \quad r < r_1$$

$$\frac{E_\theta}{\exp(k_\parallel z)} = E_{\perp,i} \sin(\theta) - \frac{\imath}{r} \left\{ \frac{A_t I_m(\kappa r) \exp(i m \theta)}{A_b K_m(\kappa r) \exp(i m \theta)} \right\} \quad r > r_1.$$  

(11)

The matching condition for $D_n$ is

$$\epsilon_{\perp,f} E_{r,f} - \imath \epsilon_\times f E_{\theta,f} = \epsilon_{\perp,b} E_{r,b} - \imath \epsilon_\times b E_{\theta,b}$$

and writing it all out:

$$\epsilon_{\perp,f} \left( -E_{\perp,i} \cos(\theta) - \kappa A_t' I_m(\kappa r) \exp(i m \theta) \right)$$

$$- \imath \epsilon_\times f \left( E_{\perp,i} \sin(\theta) - \frac{\imath}{r} A_t I_m(\kappa r) \exp(i m \theta) \right)$$

$$= \epsilon_{\perp,b} \left( -E_{\perp,i} \cos(\theta) - \kappa A_b K_m(\kappa r) \exp(i m \theta) \right)$$

$$- \imath \epsilon_\times b \left( E_{\perp,i} \sin(\theta) - \frac{\imath}{r} A_b K_m(\kappa r) \exp(i m \theta) \right).$$

(12)

It is clear that only $m = \pm 1$ modes can be excited by the incident FW. Projecting (14) onto $m = \pm 1$ using $\cos(\theta) \to \frac{1}{2} \exp(i m \theta)$, $\sin(\theta) \to \frac{\imath}{2} \exp(i m \theta)$,

$$\epsilon_{\perp,f} \left( -E_{\perp,i} r_1 - \kappa A_t' I_m \right) - \imath \epsilon_\times f \left( \frac{m E_{\perp,i} r_1 - \imath A_t I_m}{2i} \right)$$

$$= \epsilon_{\perp,b} \left( -E_{\perp,i} r_1 - \kappa A_b K_m' \right) - \imath \epsilon_\times b \left( \frac{m E_{\perp,i} r_1 - \imath A_b K_m}{2i} \right)$$

(15)

where $\xi = \kappa r_1$. The argument of the Bessel functions and their derivatives is from now on assumed to be $\xi$ when not explicitly given. Collecting terms in the unknown amplitudes $A_t, A_b$, (15) becomes

$$A_t G_t - A_b G_b = S$$

(16)

where

$$G_t = \epsilon_{\perp,f} A_t' I_m + \epsilon_\times f m I_m$$

$$G_b = \epsilon_{\perp,b} A_b K_m' + \epsilon_\times b m K_m$$

(17)

$$S = \frac{\epsilon_{\perp,b} - \epsilon_{\perp,f} + m \epsilon_\times b - m \epsilon_\times f}{2}.$$  

(18)

The solution which obeys both continuity conditions, (10) and (16), is

$$A_t = \frac{SK_m}{K_m G_t - G_b K_m},$$

$$A_b = \frac{SI_m}{K_m G_t - G_b K_m}.$$  

(19)

In the HHFW limit, $\epsilon_{\perp}$ and $\epsilon_\parallel$ are both proportional to the density. Normalizing everything to the background density (i.e. unless otherwise noted, $\omega_i$ and $\omega_\perp$ are calculated from the background density $n_b$), and using $R \equiv \frac{n}{n_b}$ and $\varphi \equiv \frac{\varphi}{\Omega_i}$ ($\varphi \gg 1$ in HHFW),

$$G_t = \frac{\omega^2_i}{\omega} R (\varphi m I_m - \xi I_m')$$

$$G_b = \frac{\omega^2_i}{\omega} (\varphi m K_m - \xi K_m')$$

(20)

$$S = \frac{E_{\perp,b} r_1}{2} (1 - R) \frac{\omega^2_i}{\omega^2} (m \varphi - 1).$$

(21)

The resonance condition $K_m G_t = G_b I_m$, where $A_t$ and $A_b$ have a pole, becomes

$$R = \frac{n_i}{n_b} = \frac{\varphi m - \xi K_m'}{\varphi m - \xi K_m'}.$$  

(22)

In figure 1 we see $\frac{\varphi}{\Omega_i} > 0$ and $\frac{\varphi}{\Omega_i} < 0$. To determine the forward or backward nature of these waves, we can solve the resonance condition for $\varphi$

$$\varphi = \frac{\omega}{\Omega_i} = \frac{R \xi I_m' - \xi K_m'}{m R - 1}.$$  

(23)

Figure 2 shows (25) for various values of $\varphi$. From both theory [16, 17] and experiment [11], we know that filaments with $\frac{n}{n_b}$ not much greater than 1 are common, but filaments with much higher $\frac{n}{n_b}$ are rare. It becomes clear that this mechanism is relevant for HHFW ($\varphi \gg 1$) but not for traditional minority and
We now introduce the causality parameter $\nu$ (so-called 'hole filaments', where $r_0 < 1$). For filaments whose density is below that of the background $\rho_\infty$, we have previously determined the locations of these resonances within an exact electromagnetic full-wave solution [9]. In figure 3, we see that (25) is indeed a good approximation to the resonances found in that exact full-wave solution, which justifies our approximations. Numerical calculations in [18, 19] and theoretical arguments in [20] also confirm that such resonances exist even for nonidealized filaments.

2.1. Blob filaments ($R > 1$)

$\xi, \varphi, R$ and $\ell^\prime_r$ are all positive while $\kappa^\prime_r$ is negative (figure 1). Thus, for filaments whose density exceeds that of the background (so-called ‘blob filaments’, where $R > 1$), (25) predicts that resonances can only exist at $m = 1, R > \frac{\varphi_1}{\varphi + 1} > 1$, and $\xi < \xi_\infty$, where $\xi_\infty$ is the positive root of $-\varphi I_1(\xi) - \xi I'_1(\xi)$. From (26), $\partial_\varphi \omega > 0$ for $k || > 0$, so the surface waves resonantly excited on blob filaments are forward in the parallel direction.

2.2. Hole filaments ($R < 1$)

For filaments whose density is below that of the background (so-called ‘hole filaments’, where $R < 1$), (25) predicts that resonances can only exist at $m = -1, R < \frac{\varphi_1}{\varphi + 1} < 1$, and $\xi < \xi_0$, where $\xi_0$ is the positive root of $-\varphi K_{-1}(\xi) - \xi K'_{-1}(\xi)$. From (26), $\partial_\varphi \omega > 0$ for $k || > 0$, so the surface waves resonantly excited on hole filaments are forward in the parallel direction.

3. Derivation of the filament losses

We now introduce the causality parameter $\nu$, formally similar to a collision frequency, in the dielectric tensor

$$
e = \epsilon^H + i\epsilon^A$$

(27)

The case where $\nu$ plays the role of a physical collision frequency, and the electrons and ions have different collision frequencies, will be treated in appendix A. The introduction of $\nu$ modifies $G_t$ and $G_b$

$$
G_t = \frac{\omega^2}{\omega^2 R} \left(\varphi m I_m - \xi I'_m + \frac{i\nu}{\omega} \xi I'_m\right)
$$

(29)

$$
G_b = \frac{\omega^2}{\omega^2} \left(\varphi m K_m - \xi K'_m + \frac{i\nu}{\omega} \xi K'_m\right).
$$

(30)

The power loss density $P$, in SI units, is [22]

$$
P = \frac{\omega}{2} E^* \epsilon_0 \epsilon^A E.
$$

(31)

The total power absorbed within $r < \rho$ is

$$
P_t = P_t + P_b
$$

(32)

$$
P_t = \int_{-\infty}^{\infty} \int_{-\varphi}^{\varphi} \int_{0}^{r_f} P r dr d\varphi dz.
$$

(33)

$$
P_b = \int_{-\infty}^{\infty} \int_{-\varphi}^{\varphi} \int_{r_f}^{\rho} P r dr d\varphi dz.
$$

(34)

We want to calculate the total power in the $\nu \to 0^+$ limit

$$
P_0 = \lim_{\nu \to 0^+} \lim_{\rho \to \infty} P_t.
$$

(35)

Taking the limit $\rho \to \infty$ is a mathematical convenience, in practice the integrands vanish within a few SW decay lengths away from the filament surface.

It will be useful to reparametrize (4) and (5) in terms of the dimensionless quantity $\xi = k || r_1 \sqrt{m_e/m_i}$. Then

$$
P = \frac{\omega}{2 r_1^2 \sqrt{m_i/m_e}} \left(\int A_r^2 \xi_1^2 \exp \left(\frac{-i\xi_1 z}{r_1 \sqrt{m_i/m_e}}\right) d\xi_1\right)
$$

$$
\times \epsilon_0 \epsilon^A \left(\int A_i \xi_1 \exp \left(\frac{i\xi_2 z}{r_1 \sqrt{m_i/m_e}}\right) d\xi_2\right)
$$

$$
= \frac{\omega}{2 r_1^2 \sqrt{m_i/m_e}} \int A_r^2 \xi_1^2 \epsilon_0 \epsilon^A A_i \xi_1 \exp \left(\frac{i(\xi_2 - \xi_1) z}{r_1 \sqrt{m_i/m_e}}\right) d\xi_1 d\xi_2.
$$

(36)

Let us now integrate over $z$ (i.e. along $\mathbf{B}$, along the filament). The length of the filament is much larger than its radius. We approximate the finite filament by taking the integral from $-\infty$ to $\infty$. In reality, the filaments have finite length, and the ones...
Figure 2. $R \equiv \frac{n_i}{n_b}$ given by (25), for blob filaments ($R > 1, m = 1$, left) and for hole filaments ($R < 1, m = -1$, right). We show $k_\parallel$ on the upper x axis, with $\xi = k_\parallel r_1 \sqrt{m_i/m_e}$ assuming deuterium plasma and $r_1 = 1$ cm.

Figure 3. $R \equiv \frac{n_i}{n_b}$ given by (25) as white dashed lines, overlaid on the resonances found in [9] within a full electromagnetic solution for wave scattering at a filament. There is a second set of resonances at the bottom of the figure. These resonances arise when the SW can propagate inside the filament but not in the background [14], but they are not relevant for the rest of this work. The side plots give the wavelengths of the FW and SW, black for propagative, red for evanescent.

outside of the last closed flux surface end where they intersect the divertor. See appendix B for further discussion on this point. Making use of Parseval’s identity,

$$\int_{-\infty}^{\infty} Pdz = \frac{2\pi^2 e_0 R^2}{\omega^2 r_1 \sqrt{m_i/m_e}} \int |A_1|^2 E_1^* e^{i\xi} e^{i\omega_1 t} d\xi.$$  

Integrating also over the azimuthal angle, and inserting (28) (recall $\omega_i, \omega_e$ are background quantities, hence the factor $R$),

$$\int_{-\pi}^{\pi} \int_{-\infty}^{\infty} Pdz d\theta = \frac{2\pi^2 e_0 R^2}{\omega^2 r_1 \sqrt{m_i/m_e}} \int |A_1|^2 E_1^* \times \begin{bmatrix} \omega_i^2 & 0 & 0 \\ 0 & \omega_e^2 & 0 \\ 0 & 0 & \omega_e^2 \end{bmatrix} \xi d\xi$$

Let the resonance be at $\xi_R$, i.e. $\xi = \xi_R$ obeys (25). We linearize the denominator of (20) around the resonance,

$$K_m G_i - G_k I_m = -a(\xi - \xi_R) + ib\nu$$

$$b = \frac{\omega_i^2}{\omega_e^2} \beta = \frac{\omega_i^2}{\omega_e^2} \left( R K_m \xi t_m' - K_m' \xi t_m \right) \bigg|_{\xi \to \xi_R}$$

$$a = \frac{\omega_i^2}{\omega_e^2} \alpha = \frac{\omega_i^2}{\omega_e^2} \left( (1 - R) \xi m K_m t_m + R K_m' \xi t_m' - K_m' \xi t_m \right) \bigg|_{\xi \to \xi_R}$$

where the prime is the derivative w.r.t $\xi$ at constant $R$. The dimensionless quantities $\alpha$ and $\beta$ defined in (40) and (41) are shown in figures 4 and 5. We will soon see that the power $P_0$ scales with $\frac{1}{\xi^2}$. From these figures, we see that for blob filaments at constant $k_{\parallel}$, $\alpha$ and $\beta$ decrease with increasing $\xi_R$, so the power increases with increasing $\xi_R$. Thus, resonant interactions not only become rarer at lower $\xi_R$, they also lose less power.

At this point, we note that $\nu |A_1|^2$ behaves, in a neighbourhood of the resonance/pole, like a Lorentzian. As we approach the $\nu \to 0^+$ limit,

$$\nu |A_1|^2 \to |SK_m|^2 \frac{\nu^3}{\omega^2 (\xi - \xi_R)^2 + \nu^2 \omega^2}.$$  

The factor $\frac{\nu^3}{\omega (\xi - \xi_R) + \nu^2}$ becomes higher but more narrowly peaked around $\xi = \xi_R$ as $\nu$ approaches 0, such that its integral remains finite even in the $\nu \to 0^+$ limit. We find this integral by complex contour integration. The poles are at $\xi = \xi_R \pm \frac{i\nu}{2}$.
and the corresponding residues, \( \pm \frac{1}{2ab} \), do not depend on \( \nu \). Thus

\[
\int_{-\infty}^{\infty} \frac{\nu}{\alpha^2(\xi - \xi_R)^2 + \beta^2\nu^2} d\xi = \frac{\pi}{ab}.
\]

(43)

We have the following ‘Dirac delta-like’ behaviour

\[
\lim_{\nu \to 0^+} \int_{-\infty}^{\infty} \nu |A_f|^2 f(\xi) d\xi = \frac{\pi}{ab} |S_{Km}(\xi_R)|^2 f(\xi_R).
\]

(44)

\[
\lim_{\nu \to 0^+} \int_{-\infty}^{\infty} \nu |A_b|^2 f(\xi) d\xi = \frac{\pi}{ab} |S_{Im}(\xi_R)|^2 f(\xi_R).
\]

(45)

Thus, in the filament

\[
\lim_{\nu \to 0^+} \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} P d\xi d\theta = \frac{2\pi^2 \epsilon_0 |S_{Km}(\xi_R)|^2}{\omega^2 r_{ab} \sqrt{m_i/m_e}} R |Km(\xi_R)|^2
\]

\[
\times \begin{bmatrix} \omega_f^2 & 0 & 0 \\ 0 & \omega_f^2 & 0 \\ 0 & 0 & \omega_f^2 \end{bmatrix} \mathcal{E}_f
\]

(46)

Similarly, in the background

\[
\lim_{\nu \to 0^+} \int_{-\pi}^{\pi} \int_{-\infty}^{\infty} P d\xi d\theta = \frac{2\pi^2 \epsilon_0 |S_{Im}(\xi_R)|^2}{\omega^2 r_{ab} \sqrt{m_i/m_e}} \mathcal{E}_b^*
\]

\[
\times \begin{bmatrix} \omega_i^2 & 0 & 0 \\ 0 & \omega_i^2 & 0 \\ 0 & 0 & \omega_i^2 \end{bmatrix} \mathcal{E}_b.
\]

(47)

\[
P_0, \text{ the total power converted to the surface wave mode, is then simply}
\]

\[
P_0 = \frac{2\pi^2 \epsilon_0 \omega_i^2 |S|^2}{\omega^2 r_{ab} \sqrt{m_i/m_e}} R |Km(\xi_R)|^2
\]

\[
\times \int_{-\pi}^{\pi} \left( |\mathcal{E}_{f,r}|^2 + |\mathcal{E}_{f,\theta}|^2 + \frac{m_i}{m_e} |\mathcal{E}_{f,z}|^2 \right) r dr
\]

\[
+ \frac{2\pi^2 \epsilon_0 \omega_f^2 |S|^2}{\omega^2 r_{ab} \sqrt{m_i/m_e}} R |Km(\xi_R)|^2
\]

\[
\times \int_{-\pi}^{\pi} \left( |\mathcal{E}_{b,r}|^2 + |\mathcal{E}_{b,\theta}|^2 + \frac{m_i}{m_e} |\mathcal{E}_{b,z}|^2 \right) r dr.
\]

(48)
From (6) and (7) we have

$$|\mathcal{E}_i|^2 = \kappa^2 \begin{cases} |I_m^i(kr)|^2 & r < r_t \\ |K_m^i(kr)|^2 & r \geq r_t \end{cases}$$

(49)

$$|\mathcal{E}_d|^2 = \frac{1}{r^2} \begin{cases} |I_m(kr)|^2 & r < r_t \\ |K_m(kr)|^2 & r \geq r_t \end{cases}$$

(50)

$$|\mathcal{E}_c|^2 = \kappa^2 \begin{cases} |I_m(kr)|^2 & r < r_t \\ |K_m(kr)|^2 & r \geq r_t \end{cases}$$

(51)

Thus,

$$P_0 = \frac{2\pi^3\epsilon\omega^2}{\omega^2 r_t ab \sqrt{m_0^2}} (R\Psi_t(\xi_R) + \Psi_b(\xi_R))$$

(52)

where

$$\Psi_t(\xi_R) \equiv |K_m(\xi_R)|^2 \times \int_0^{\xi_R} \left( |I_m(\rho)|^2 + \frac{1}{\rho^2} |K_m(\rho)|^2 + |K_m(\rho)|^2 \right) \rho d\rho$$

(53)

$$\Psi_b(\xi_R) \equiv |I_m(\xi_R)|^2 \times \int_{\xi_R}^\infty \left( |K_m(\rho)|^2 + \frac{1}{\rho^2} |K_m(\rho)|^2 + |K_m(\rho)|^2 \right) \rho d\rho.$$  

Inserting \( S \) from (24) and \( \alpha \) and \( \beta \) from (40) and (41) into (52):

$$P_0 = \frac{\pi^3\epsilon\omega^2 r_t |E_{\perp,|}|^2 (1 - R) (m\varphi - 1)|^2}{2\omega\alpha\beta \sqrt{m_0^2}} \times (R\Psi_t(\xi_R) + \Psi_b(\xi_R)).$$

(55)

Recall that \(|E_{\perp,|}|^2\) is the power spectrum of the perpendicular component of the incident FW electric field, in units of \( V^2 \). Thus, \( P_0 \) has units of power. There is also a resonance at \(-\xi_R\), and the total power converted by the filament is the sum of \( P_0 \) over both resonances. If \(|E_{\perp,|}|^2\) is a symmetric power spectrum, this amounts to doubling \( P_0 \).

4. Filament statistics

Since all the RF calculations in this work are performed within linear electrodynamics, the powers dissipated by two filament modes with different resonant \( k_{||} \) values are additive. Thus, we can calculate the individual power \( P_0 \) for isolated filaments and then perform a statistical average \( \langle P_0 \rangle \) of the individual powers when \( \sim 10^2 \) filaments \([11]\) are present simultaneously in front of the antenna. The most important averaging is over \( R = \frac{\xi}{\xi_0} \), the statistics of which have been studied theoretically by \([16, 17]\), and empirically by \([10, 11]\). Given some probability density function \( P(R) \), a corresponding probability distribution over the resonant \( \xi_R \), \( P(\xi_R) \), can be readily derived using (25)

$$P(\xi_R) = P(R(\xi_R)) \frac{\partial R(\xi_R)}{\partial \xi_R}.$$  

(56)

Thus,

$$\int \mu_0 P(\xi_R) d\xi_R.$$  

(57)

Equation (57) is a bilinear operator from the power spectrum and the probability density to the expected power lost per filament \( \langle |k_0| \Psi_0|^2, P(\xi_R) \rangle \rightarrow \langle P_0 \rangle \).

Garcia et al \([16, 17]\) predict that \( \frac{n-\langle n \rangle}{\sigma_n} \), where \( n \) is the fluctuating density, \( \langle n \rangle \) is the time-averaged background density, and \( \sigma_n \) is the standard deviation of the fluctuating density, obeys a Gamma distribution

$$P \left( \frac{n-\langle n \rangle}{\sigma_n} \right) = \frac{\gamma^{1/2}}{\Gamma(\gamma)} \left( \gamma + \frac{1}{2} \frac{n-\langle n \rangle}{\sigma_n} \right)^{-\gamma} \exp \left( \frac{\gamma}{2} \frac{n-\langle n \rangle}{\sigma_n} - \gamma \right).$$  

(58)

The parameter \( \gamma \) is the so-called ‘intermittency parameter’ \( \gamma = \frac{\langle n \rangle^2}{\sigma_n} \). From all this, we can derive the cumulative distribution function

$$P \left( \frac{n-\langle n \rangle}{\sigma_n} > x \right) = P \left( \frac{n-\langle n \rangle}{\sigma_n} > \frac{x}{\sigma_n} \right) = P \left( \frac{n}{\langle n \rangle} > \frac{x}{\langle n \rangle} \right) = P \left( \frac{n}{\langle n \rangle} > \gamma \frac{x}{\langle x-1 \rangle} \right) = \int_{\gamma \frac{x}{\langle x-1 \rangle}}^{\infty} P \left( \frac{n}{\langle n \rangle} \right) d\frac{n}{\langle n \rangle}.$$  

(59)

Let \( t = \gamma + \frac{1}{2} \frac{n-\langle n \rangle}{\sigma_n} = \frac{d}{\gamma} \frac{n-\langle n \rangle}{\sigma_n} = \gamma \frac{x}{\langle x-1 \rangle} \rightarrow t = \gamma x \)

$$P \left( \frac{n}{\langle n \rangle} > x \right) = \frac{\Gamma(\gamma, \gamma x)}{\Gamma(\gamma)}$$  

(60)

Identifying \( \frac{d}{\gamma} \) with \( \frac{\sigma_n}{\langle n \rangle} \), the probability distribution function is

$$P \left( \frac{n}{\langle n \rangle} = \frac{d}{\gamma} \right) = \frac{\Gamma(\gamma, \gamma x)}{\Gamma(\gamma)} \left( \frac{n}{\langle n \rangle} \right)^{\gamma}.$$  

(61)

The probability that a given filament is a blob filament \( \langle R \rangle > 1 \) is

$$P_{\text{blob}} = \int_{1}^{\infty} P \left( R \right) dR = \frac{\Gamma(\gamma, \gamma)}{\Gamma(\gamma)}.$$  

(62)
The curves with amplitude $A_i$ vertical component of the incident FW be a simple dipole of we must assume a power spectrum. Let the spectrum of the general negligible, we consider some NSTX-like values. First, and $r$ normal distribution, with a positive correlation between

$$P_{\text{hole}} = \int_0^1 P(R) dR = 1 - \frac{\Gamma(\gamma, \gamma)}{\Gamma(\gamma)} = 1 - P_{\text{blob}}. \quad (63)$$

The probability that a given filament is a hole filament ($R < 1$) is

$$P_{\text{resonance}} = \int_0^{\phi_1} P(R) dR + \int_{\phi_1}^{\phi_2} P(R) dR$$

$$= \frac{\Gamma(\gamma, \gamma, \phi_2 - \phi_1)}{\Gamma(\gamma)} - \frac{\Gamma(\gamma, \gamma, \phi_1 - \phi_2)}{\Gamma(\gamma)} + 1$$

$$= 1 - \frac{4\exp(-\gamma)}{\nu_1^2} + O\left(\frac{1}{\nu_2^2}\right). \quad (64)$$

For completeness, we note that very recently, Biswas et al [23] argued that $P(R, r)$ should be given by a bivariate skew-normal distribution, with a positive correlation between $R$ and $r$.

5. Example

To show that the power involved in this mechanism is not in general negligible, we consider some NSTX-like values. First, we must assume a power spectrum. Let the spectrum of the vertical component of the incident FW be a simple dipole of amplitude $A_i$, peaking at $k_{||,0} = \pm k_{||,0}$

$$E_y = A_i \exp\left(-\frac{1}{2} \left(\frac{k_{||,0}}{k_{||,0}}\right)^2\right) k_{||,0}. \quad (65)$$

The power spectrum we need is that of the perpendicular component of the incident fast wave. Within the HHFW limit

$$|E_{\perp,1}|^2 = |E_{\perp,1}|^2 = |E_y|^2 + |E_{\perp,0}|^2 = |E_{\perp,0}|^2 \left(1 + \frac{\varphi}{\omega - \frac{\varphi}{\omega_i}} + 1\right). \quad (66)$$

6. Conclusion

We have discussed a resonant loss mechanism for high harmonic fast waves in tokamak edge plasmas. This loss mechanism is associated with the resonant excitation of surface waves on naturally occurring plasma filaments.

We have derived this loss mechanism for the specific case of resonant surface waves in HHFW-heated plasmas, where the resonant surface wave excitation is approximately described by the result of section 2. The derivation of this loss mechanism does not depend on details of the resonance condition. Analogous loss mechanisms likely exist for any resonantly excited surface wave. If resonant wave–filament interactions occur for heating systems other than HHFW, such as lower hybrid [25], it is likely that similar loss mechanisms are relevant there as well.

Throughout this paper, we interpret $P_0$ as the power lost from the incident fast wave, and transferred to the surface wave along the filament. In this interpretation, it is a transfer of power from one wave mode to another, which is seen as a loss from the point of view of using the incident wave for heating or current drive, but physically the power is not dissipated, it ends up in the other mode. The ultimate fate of this mode-converted power cannot be described by our model (see appendix B). It likely involves loss mechanisms at the filament endpoints.
Finally, from the estimate we provide for NSTX-like parameters, we find that in a HHFW heating or current drive scenario, this mechanism can easily redirect several kW of power per filament. Further work will focus on the detailed application of this theory to the case of the HHFW edge losses observed in NSTX.

Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Appendix A. Independent electron and ion collision frequencies

If we interpret \( \nu \) as a physical collision frequency, we may want to use a different collision frequency for ions and electrons. Let the ion frequency be \( \nu \) and the electron frequency \( \nu_e \), usually with \( \nu_e \gg \nu \), from momentum conservation considerations \( \frac{\nu_e}{\nu} \approx \frac{m_i}{m_e} \). Then, we replace (28) by

\[
e^A = \frac{\nu}{\nu_e} \begin{bmatrix} \frac{\nu}{\nu_e} & 0 & 0 \\ 0 & \frac{\nu_e}{\nu} & 0 \\ 0 & 0 & \frac{\nu}{\nu_e} \end{bmatrix}.
\] (A.1)

We take the collisionless limit keeping the ratio \( \frac{\nu_e}{\nu} \) constant. This has two main consequences. First, it changes (54) to

\[
\Psi_e(\xi_R) = \left| K_m(\xi_R) \right|^2 \left( |I_m(\rho)|^2 + \frac{1}{\rho^2} |I_m(\rho)|^2 + \frac{\nu_e}{\nu} |I_m(\rho)|^2 \right) d\rho
\] (A.2)

\[
\Psi_i(\xi_R) = \left| I_m(\xi_R) \right|^2 \left( |K_m(\rho)|^2 + \frac{1}{\rho^2} |K_m(\rho)|^2 + \frac{\nu_e}{\nu} |K_m(\rho)|^2 \right) d\rho.
\] (A.3)

Note that \( \Psi_e, \Psi_i \) are now first degree polynomials in \( \frac{\nu}{\nu_e} \).

Second, it modifies \( \xi \)

\[
\xi = k \eta \sqrt{\frac{m_i}{m_e}} \left( 1 + \frac{1 - (\nu_e/\nu)}{2\omega} \right)
\] (A.4)

which affects the linearisation (39)

\[
D = K_m G_1 - G_0 I_m
\] (A.5)

\[
D = \frac{\partial D}{\partial \xi} \frac{\partial \xi}{\partial \nu} + \frac{\partial D}{\partial \xi} \frac{\partial \xi}{\partial \nu} \nu + \cdots
\] (A.6)

\[
= -a(\xi - \xi_R) + \left( bi - a \frac{\xi}{\nu} \right) \nu + \cdots
\] (A.7)

\[
= -a(\xi - \xi_R) + i \left( b - a \xi_k \frac{1 - (\nu_e/\nu)}{2\omega} \right) \nu + \cdots
\] (A.8)

so \( a \) remains as it was in (41), but \( b \) gets an extra term, it becomes a first degree polynomial in \( \frac{\nu_e}{\nu} \). Thus, both the numerator and denominator in \( P_0 \) (52) are now first degree polynomials in \( \frac{\nu_e}{\nu} \). Remarkably, the polynomial in the numerator is proportional to the polynomial in the denominator: the dependence on \( \frac{\nu_e}{\nu} \) cancels out. \( P_0 \) (52) does not depend on \( \frac{\nu_e}{\nu} \). We prove the case of blob filaments (\( m = 1 \)). To show that the ratio \( \frac{\nu_e}{\nu} \) is independent of \( \frac{\nu_e}{\nu} \), we need to show that \( b/a = d/c \). We apply this observation to the numerator polynomial \( P_N \), obtained by working out the integrals (A.2) and (A.3)

\[
P_N = R K_1^2 \left( \xi_R^2 I_2^2 - (\xi_R^2 + 2) I_1^2 \right) - I_1^2 \left( \xi_R^2 K_0^2 - (\xi_R^2 + 2) K_1^2 \right)
\]

\[
+ \left( \frac{\nu_e}{\nu} \xi_R \left( R K_1^2 (I_1^2 - I_0 I_2) - I_1^2 (K_1^2 - K_0 K_2) \right) \right)
\] (A.9)

and the denominator polynomial \( P_D \) which follows from (A.8)

\[
P_D = \beta - \frac{\alpha \xi_R}{2} + \frac{\nu_e}{\nu} \frac{\alpha \xi_R}{2}.
\] (A.10)

Making use of Bessel function identities, and eliminating \( \phi \) using (26), we can simplify the expression for \( \alpha \) to

\[
\alpha I_1 K_1 = \xi_R \left( R K_1^2 (I_2^2 - I_0 I_2) - I_1^2 (K_1^2 - K_0 K_2) \right)
\] (A.11)

and the constant term in (A.9) to

\[
P_N = \xi_R^2 R K_1^2 \left( - \frac{I_0}{2} + \frac{I_1^2 - I_0 I_2}{2} + \frac{I_2}{2} \right)
\]

\[
- \xi_R^2 \beta I_1 \left( \frac{K_1^2}{2} - \frac{K_0^2 + K_2^2}{2} - K_0 K_2 \right) + \frac{\nu_e}{\nu} \cdots
\] (A.12)

The condition for independence of \( \frac{\nu_e}{\nu} \), the ratio of the constant coefficient over the linear coefficient for the numerator polynomial, must equal that of the denominator polynomial, becomes

\[
\frac{\beta}{\alpha \xi_R} - 1 = -1 - \frac{1}{2} \frac{R K_1^2 \left( - \frac{I_0}{2} + \frac{I_1^2 - I_0 I_2}{2} + \frac{I_2}{2} \right)}{R K_1^2 \left( I_2^2 - I_0 I_2 \right) - I_1^2 \left( K_1^2 - K_0 K_2 \right)}
\] (A.13)
\[
\frac{4 (- I_1 K_1' + K_1 I_1[R])}{\alpha} = \frac{R K_1^2 (- F_0 + F_2') - F_1' (- K_0^2 + K_2^2)}{R K_2^2 (F_1' - I_2 I_2) - F_1' (K_1^2 - K_0 K_2)}.
\]

Insert (A.11)

\[
\frac{4 I_1 K_1 (- I_1 K_1' + K_1 I_1[R])}{\xi_R} = - R K_1^2 (- F_0 + F_2') + F_1' (- K_0^2 + K_2^2).
\]

Working out the derivatives on the lhs and \((- F_0^2 + F_2') = (I_2 + I_0) (I_2 - I_0) = - (I_1 + I_0) \frac{2 I_1}{\xi_R}\) and \((- K_0^2 + K_2^2) = (K_2 + K_0)(K_2 - K_0) = \frac{2 K_2}{\xi_R}\) on the rhs,

\[
F_1'(K_0 + K_2) K_1 + R K_1^2 (I_0 + I_2) I_1 = F_1' (K_2 + K_0) K_1
+ R K_2^2 (I_2 + I_0) I_1
\]

which is clearly true.

**Appendix B. Finite filaments**

Our model is only valid in a strict sense for infinite magnetic field lines, where the parallel integration (37) can be performed from \(-\infty\) to \(+\infty\). Yet one would like to use this model for interpreting spurious interactions in the SOL of tokamaks, where open magnetic field lines connect to the material walls. In the presence of dissipation (\(\nu > 0\)), the nearly-resonant wave-filament modes have a small yet finite spectral width in the \(k_m\) domain. From the full width at half maximum of the Lorentzian (42)

\[
\Delta k_m = \frac{\beta}{\alpha R \sqrt{m_0/m_e} \nu}.
\]

Our model is an asymptotic theory valid for \(\nu \ll \text{small enough}\), such that \(\Delta k_m\) is far smaller than any other spatial scale-length in the problem (this is the condition for the ‘Dirac delta-like’ relations (44) and (45) to hold). Spatially, the nearly-resonant wave-filament modes should have a large yet finite parallel extent \(\Delta z \approx \frac{1}{\Delta k_m}\). This is the length scale over which the SW fields have non-negligible amplitude, as well as the length scale over which the mode-converted RF power is fully dissipated in the plasma (filament) volume. It is reasonable to think that our approach remains valid for bounded domains if the parallel extent of the nearly-resonant modes is smaller than the typical connection length \(L_{\parallel}\) of open magnetic field lines. This leads to the criterion

\[
\Delta k_m L_{\parallel} > 1
\]

which defines a lower bound for the parameter \(\nu\). It is possible that in a more complete treatment of this problem, a small contribution of the scattered propagative fast wave (which we neglected in this work) will provide a natural lower bound to the losses. If (B.2) does not hold, if \(\Delta k_m L_{\parallel} < 1\), we expect that not all the estimated mode-converted power can be dissipated in the finite filament in a single pass. Non-negligible RF fields can then reach the field line endpoints. There, they may be partially reflected back to the plasma, partially absorbed by lossy metallic walls, and/or excite sheath oscillations and enhance heat loads to the wall via sheath rectification. However, our model does not capture what happens at these endpoints and is less reliable in assessing the lost power in this regime.

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