Comparison of RBFs Interpolation for Load Transfer on Wing Structure

Pan Cheng1,*, Yang Zhao1, Yuchen Sun1
1Shanghai Aircraft Design & Research Institute, Shanghai, P.R. China

*Corresponding author: chengpan@comac.cc

Abstract. Load transfer is of importance in simulation of aeroelasticity by coupling with CFD and FEM. Different RBFs (Radial Basis Function) with variant parameters are tested in load transfer and their results are compared in this paper. Result comparison shows that different RBFs with variant parameter will lead to different results. More approximate load on structure can be gained by MQB and TPS functions. The compact support RBFs should be used with wisely chosen support radius to gain approximate results.

Keywords: Aerodynamics, Aeroelasticity, Load transfer, RBF.

1. Introduction
With subsonic civil transports operating in the transonic regime, it is becoming important to determine the effects of static aeroelasticity caused by coupling between aerodynamic loads and elastic forces. Since aeroelasticity contributes significantly to the design of these aircraft, there is a strong need in the aerospace industry to predict these fluid-structure interactions computationally.

To analyze static aeroelastic problem under transonic condition, fluid structure interaction should be applied with Computational Fluid Dynamic (CFD) analysis tools coupling with Finite Element Analysis (FEA) analysis tools. The CFD and FEA code have been well developed by which the nonlinear behavior of aerodynamic and structure can be predicted. Since the source codes are not always available, the coupling process will run CFD and FE codes by turn and transfer loads and motion between aerodynamic mesh and finite element model[1][2]. The loads transfer is of important since the transferred loads on structure should precisely reflect the loads on aerodynamic mesh.

The loads transfer has been implemented with two different mesh strategies. The first strategy focuses on the relative position of aerodynamic mesh cells and structure grid so that geometry relation information need to be found out in transfer[3][4]. This class of method is very accurate in some specific problem, however it is difficult to be used automatically or to be applied on non-match grid. The second strategy is based on energy conservative law and the topological relation between aerodynamic mesh and structure grid is unnecessary. Therefore it is easy to automatize this method[5][6]. Nonetheless the loads transfer might be difficult to control, which means the energy is conservative meanwhile the local distribution could be non-match.

The radial basis function (RBF) interpolation is a well-established tool for conservative interpolation and variant basis function to be chosen in fluid-structure-interaction. The object of this research is to find the best way to do the RBF interpolation in the point of view of mechanics, by comparing variant
basis functions and parameters for static aeroelastic problems of civil transporter wing in the transonic regime.

2. The principal of load transfer

Since the field of aerodynamics and structure are discreted by different mesh due to different physical characteristics of fluid and structure, their boundary do not match each other in most cases. In another word, they do not share same discreted points on boundaries so that the discreted field of structure with lesser grid points could differ significantly from the smoother aerodynamic surface mesh with much more points. The structure model can be built as a finite element model in Lagrange system meanwhile the fluid can be built as finite volume and surface mesh in Euler system. The interpolation should convert the external aerodynamic loads (pressure) $f_j$ to the internal structure stress forces(stress), and then transfer the displacement of structure $\delta u_j$ caused by loads to the displacement of fluid boundary mesh $\delta u_f$.

Since the works performed by aerodynamic forces and structure stress should keep equivalent during coupling meanwhile the aerodynamic force counterbalance with structure stress, the energy conservation law is shown as

$$\delta w = \delta u_f^T \cdot f_j = \delta u^T \cdot \bar{f}_j = \delta u^T \cdot f_s$$  (1)

A couple matrix $H$ could be introduced to connect the displacement of fluid and structure with approximated linear relation

$$\delta u_f = H \cdot \delta u_j$$  (2)

And the linear relation between aerodynamic forces and structure forces.

$$f_s = H^T \cdot f_j$$  (3)

If the coupling matrix $H$ can be constructed, the matrix can be used for transferring displacement from structure to fluid and its transpose for transferring aerodynamic forces to structure forces.[3]

2.1. Radial basis function interpolation

Radial Basis Function is a widely used multivariable interpolation tool for scatter data. Besides the fluid structure interaction, the RBF also plays an important role in engineering area, such as image processing, CAD, CAE[7][8][9]. The application of RBF in fluid-structure-interaction will be discussed in this part.

Generally, the multivariable interpolation can be described as following. A sample of multivariable scatter data is assumed to be given as a set of points $X = \{x_{1L}, x_2\} \subseteq R^d$ and a set of values on these points given as $u_{1L}, u_2$ respectively. To construct a function that can generate values $u_{1L}, u_2$ only from point evaluation, an interpolation function $s_{u,X} (x)$ can be expressed as

$$s_{u,X} (x) = \sum_{j=1}^{N} \alpha_j \phi (\| x - x_j \|) + p(x)$$  (4)

The interpolation basis function $\phi (\| \cdot \|)$ is RBF only in respect of Euclidean distance and $p(x)$ is polynomial with number d of variables. These two parts to be determined should satisfy following conditions [10]

(1) The value of interpolation function at point $x_j$ equals to known value $u_j$:

$$s_{u,X} (x_j) = u_j, \quad 1 \leq j \leq N$$  (5)

(2) For any polynomial $q(x)$ with lower or equal order than that of $p(x)$:
The interplant will be unique if the basis function is conditionally positive definite. The linear polynomial \( p(x) = w_0 + w_1x + w_2y + w_3z \) could be applied if the basis function is conditionally definite positive with order lower than two. For fluid structure integration, the linear polynomial is adaptable and Euclidean space is limited as 3-dimension.

For the fluid structure problem, the \( N_j \) fluid mesh points \( X'_j = (x'_j, y'_j, z'_j) \), \( 1 \leq j \leq N_j \) with forces and pressure on them need to be interpolated as \( N_j \) forces on structure points \( X_j = (x_j, y_j, z_j) \), \( 1 \leq j \leq N_s \). Also, the displacement of fluid points need \( u'_j \) to be recovered from structure points displacement \( u_j \). The structure points could be setup as the set of \( X \) in the multivariable interpolation and displacement as the function value to be evaluated. The coefficients of the constructed interpolation function could be solved by linear equations as following

\[
\begin{bmatrix}
0 \\
{u'_j}
\end{bmatrix} = \begin{bmatrix}
0 & P^T & M \\
P & M & \alpha
\end{bmatrix}
\]

(7)

\( u'_j \) is the displacement of known structure point. \( w \) is the vector composed of coefficients \( (w_0, w_1, w_2, w_3)^T \) in \( p(x) \). \( a \) is the vector of coefficients of interpolating basis functions, expressed as \( (a_1, K_a, a_{N_s}) \). \( M \) is a sub matrix with size and the elements of it are radial basis functions \( f_\| = f(||x'_j - x'_j||) \). \( P \) is a \( N_s \times 4 \) matrix with rows expressed as \((1, x'_j, y'_j, z'_j)\). The coefficients \([w a]^T\) of basis functions could be solved by linear equations (7) to determine the interpolating function \( s_{u, x}(x) \) and further more to gain the displacement of fluid points \( u'_j = s_{u, x}(x'_j) \).

The transforming matrix in equation (2) then could be expressed as \( H = A_{fi} \cdot C_{ss}^{-1} \). \( C_{ss} \) is a matrix consist of coefficients of equation (6) and \( A_{fi} \) can be expressed as following matrix

\[
A_{fi} = \begin{bmatrix}
1 & x'_1 & y'_1 & z'_1 & \phi_{1,1}^{fi} & L & \phi_{1,N_s}^{fi} \\
1 & x'_2 & y'_2 & z'_2 & \phi_{2,1}^{fi} & L & \phi_{2,N_s}^{fi} \\
M & M & M & M & O & M & \phi_{M,N_s}^{fi} \\
1 & x'_{N_s} & y'_{N_s} & z'_{N_s} & \phi_{N_s,1}^{fi} & L & \phi_{N_s,N_s}^{fi}
\end{bmatrix}
\]

(8)

in which \( \phi_{ij}^{fi} = \phi(||x'_j - x'_i||) \). \( C_{ss}^{-1} \) represents the determination of interpolating coefficients, and \( A_{fi} \) represents the basis function of interpolation. The matrix \( H \) will be the load transferring matrix.

### 2.2. Choice of radial basis functions

RBFs could be categorized as compact[10][11][12] and globally supported functions. Globally supported RBFs will lead to values not equal to zero in whole domain. The common RBFs are listed in table 1. The MQB method uses a parameter \( a \) to control the shape of the basis function. Larger \( a \) leads a more flat shape while smaller \( a \) makes the function more sharp. Generally the value of \( a \) is chosen in range of \( 10^{-3} \): \( 10^{-3} \) [13]. The effect caused by different value of \( a \) will be investigated in this paper.
Table 1

| No. | Name | $f(\|x\|)$ |
|-----|------|-------------|
| 1   | TPS  | $\|x\|^2\log(\|x\|^2)$ |
| 2   | Gauss | $e^{\|x\|^2}$ |
| 3   | MQB  | $\sqrt{a^2+\|x\|^2}$ |
| 4   | QB   | $1+\|x\|^2$ |

The compactly supported RBFs characterized by

$$\phi(\|x\|) = \begin{cases} f(\|x\|) & 0 \leq x \leq 1, \\ 0 & x>1, \end{cases} \quad (9)$$

The value of RBFs can be compulsorily set to non-zero inside a circle or sphere with radius of $r$ and to zero outside, by defining $f_r(x) = f(x/r)$. The region that a point affects can be controlled by set $r$ to variant value, which means the local characteristic will be easily controlled if the compactly supported RBFs are used. Common compactly supported RBFs are listed in table 2[14].

Table 2

| No. | Name | $f(x)$, $(x = \|x\|/r)$ |
|-----|------|-------------------------|
| 1   | CPC0 | $(1-\xi)^2$ |
| 2   | CPC2 | $(1-\xi)^2(4\xi+1)$ |
| 3   | CPC4 | $(1-\xi)^6(353\xi^2+6\xi+1)$ |
| 4   | CPC6 | $(1-\xi)^8(32\xi^3+25\xi^2+8\xi+1)$ |

Theoretically the compact support radius should be set to different value on different interpolating points, but it could lead to singular matrix. So uniform radius will be chosen for all points in one case[15]. The radius also need to be chosen carefully to cover enough points locally while make no effect on points that far away. The influence of radius will be investigated in this paper.

3. Test cases and results

3.1. Compare between different radial basis function interpolation

A typical transport wing is adopted as fluid model. The aerodynamic pressure will be calculated from CFD, shown in Fig 1(A). The structure model simply consists of beams, shown in Fig 1(B). It can be described as a stiffness axis and beams and ribs attached on it.

For a wing and its structure, the bend and twist deformation, which produced by bending and twisting torque relative to stiffness axis, will significantly affect the aerodynamic performance. So the consistency of torque offered by fluid pressure and torque interpolated from pressure, which will be exerted on structure, is considered as an important criterion to evaluate the interpolating capabilities of different RBFs. All RBFs in table 1 and CPC0 and CPC2 are chosen to be tested.

Three values are chosen for the shape control parameter of MQB, $a=1e-3$, $a=1e-4$, $a=1e-5$. The radius for compactly supported RBFs also varies in three values. The first two are maximum and minimum distance between any two mesh points, say R=18.0 and R=2.0. The third is a value between max and min, say R=9.0. The field for interpolation will also be cut into pieces and interpolating in split area, by which the interpolating region will be narrowed thus the inconsistence could be limited.
The interpolated twisting torque about to be exerted on structure and original torque produced by fluid pressure are shown in Fig 2. The original torque distribution can be integrated from the fluid mesh and keep relative high precision because of dense CFD mesh. The results calculated by RBFs with varying parameters are shown in sub figure A-C. Figure D-F in fig 2 show the results using same RBFs and parameters on spitted region. Globally supported RBFs are only applied in A, D. The results from varying shape control parameters are shown in B, E and varying support radius shown in C, F. The results show that TPS and MQB function with varying parameters have the capability to interpolate torque distribution that well match with original torque. QB and Gauss show similar trend to the original torque but not approximately match. The RBFs with compactly support do not show well results that expected. The distribution of torque interpolated by compactly supported RBFs looks like scatter data spread around the original distribution. The results interpolated on spitted regions show better agreement. If we differentiate them carefully on spitted regions, the compactly supported RBFs will get better results when radius set to 9 between max and min. For MQB function, the shape control parameters in suggested range do not apparently affect the results.

The interpolated bending torque distributions are shown in Fig.4A, comparing with those of original torque. All the globally supported RBFs show good approximation of interpolation, while the compactly supported RBFs act better when medium value is set for supporting radius. Especially, the CPC0 function shows bad approximation if the supporting radius set to a minimum value. The RBFs with compactly support do not show well approximation. The interpolated twisting torque about to be exerted on structure and original torque produced by fluid pressure are shown in Fig 2. The original torque distribution can be integrated from the fluid mesh and keep relative high precision because of dense CFD mesh. The results calculated by RBFs with varying parameters are shown in sub figure A-C. Figure D-F in fig 2 show the results using same RBFs and parameters on spitted region. Globally supported RBFs are only applied in A, D. The results from varying shape control parameters are shown in B, E and varying support radius shown in C, F. The results show that TPS and MQB function with varying parameters have the capability to interpolate torque distribution that well match with original torque. QB and Gauss show similar trend to the original torque but not approximately match. The RBFs with compactly support do not show well results that expected. The distribution of torque interpolated by compactly supported RBFs looks like scatter data spread around the original distribution. The results interpolated on spitted regions show better agreement. If we differentiate them carefully on spitted regions, the compactly supported RBFs will get better results when radius set to 9 between max and min. For MQB function, the shape control parameters in suggested range do not apparently affect the results.

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3.2. Splitting zones to improve the loads matching

The results on splitting region shows better accordance between loads distribution before and after interpolation. Splitting region could be cut by any kind of plane sets. In this section, the different cut plane set will be studied to help find a relation between cut plane interval and matching degree. Because the typical wing box structure consists of a serial of ribs section. Several planes along with the structure ribs will be setup and some of them will be picked to remain as a cut plane. So the cut plane interval is defined as how many ribs are removed. The cut plane will cut both structure model and aerodynamic model and make the structure grids and CFD mesh points into a few group. In each group, load will calculated sectional and then interpolated on to according mesh points. If one CFD cell is cut by planes, it will be split into two groups. The pressure distribution on the cell will be separated into two parts and classified to two groups.

Figure 3. Rib position and cut planes.

As shown in Fig 3, the cut planes will setup along ribs. Two different ribs interval number will be test to setup cut plane set, 2 ribs and 4 ribs. The results of TPS and Gauss radius basis function interpolation will be put together for comparing.

Figure 4. Twist and bending moment transfer results by splitting zones.

The results show that splitting by cut planes will obviously improve the matching level before and after load transfer. Although the TPS and Gauss radius basis function interpolation results almost doesn’t match at each rib, the twist moment distribution interpolated in splitting way quickly converge to aerodynamic twist moment distribution. Different number of cut planes slightly affect the matching level but not big enough to affect the loads transfer in FSI problem.
4. Conclusions
In this paper, several interpolation methods for loads transfer by different RBFs are compared. The comparison indicated that RBFs are very accurate tools to transfer loads from fluid mesh (pressure) to structure points (forces) by interpolation if the parameters are chosen properly. The TPS and MQB act best in interpolating, and other RBFs could be limited in small split regions to gain better results. Supporting radius need to be considered carefully if compactly supported RBFs are used. The medium value between maximum and minimum distance will gain better results while the two border of the range will lead totally obviously inconsistent between interpolated and original distributions of torques. Splitting aerodynamic mesh and stricture grids by planes will obvious improve matching level before and after interpolation. The zone separated by different planes interval do not affect the twist and bending moment distribution results. The RBF interpolation could be practically valued in engineering and provide reference for coupling between CFD and FE, which is widely applied in fluid structure interaction simulation.

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Acknowledgments
This work was not financially supported by any fund.