The spin-dependent nd scattering length - a proposed high-accuracy measurement

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The understanding of few-nucleon systems at low energies is essential, e.g. for accurate predictions of element abundances in big-bang and stellar fusion. Novel effective field theories, taking only nucleons, or nucleons and pions as explicit degrees of freedom, provide a systematic approach, permitting an estimate of theoretical uncertainties. Basic constants parameterising the short range physics are derived from only a handful of experimental values. The doublet neutron scattering length $a_2$ of the deuteron is particularly sensitive to a three-nucleon contact interaction, but experimentally known with only 6\% accuracy. It can be deduced from the two experimentally accessible parameters of the nd scattering length. We plan to measure the poorly known "incoherent" nd scattering length $a_i,d$ with $10^{-3}$ accuracy, using a Ramsey apparatus for pseudomagnetic precession with a cold polarised neutron beam at PSI. A polarised target containing both deuterons and protons will permit a measurement relative to the incoherent np scattering length, which is known experimentally with an accuracy of $2.4 \times 10^{-4}$.

PACS numbers: 21.45.+v, 25.40.Dn, 28.20.Cz, 25.10.+s

1. Introduction

In the past few years, a new strategy has been developed to describe nuclear forces at low energy. Chiral perturbation theory ($\chi$PT) is an effective field theory, which describes interactions of pions and between pions and nucleons (N). It leads to a systematic expansion of the scattering amplitude in powers of ratios of small momenta and low-energy input parameters like the pion mass over the breakdown scale of the theory. For the first time the accuracy of calculations can be estimated in a model-independent theory of nuclear interactions, providing reliable predictions of many important low-energy quantities. These are ground state properties of bound systems and processes involving external and exchange currents, as e.g. cross sections relevant for big-bang nucleosynthesis and stellar fusion. Also the determination of fundamental properties of the neutron from experiments on few-nucleon systems mandates a model-independent subtraction of nucleon binding and meson exchange effects.

Weinberg pointed out that chiral three-nucleon (3N) forces appear naturally in $\chi$PT. The most relevant processes are: a two-pion exchange, a 2N contact interaction with pion exchange, and a 3N contact interaction. The contact interactions parameterise the short-range physics. As in Fermi’s theory of weak interaction, they are characterised by effective couplings, called low-energy constants (LECs). They have to be fixed by measured data of two independent low-energy 3N observables. Very recently, a first complete analysis of nd scattering at next-to-next-to leading order has been performed with impressive results. All observables are expanded in powers
Figure 1. The strengths $H$ of the point-like three-body forces of the effective field theory in which pions are integrated out (top) must be determined from experimental three-body data. At momenta above $m_{\pi c}$, they are partially resolved as pion-exchange with known couplings (bottom), but the core-strengths $\tilde{H}$ still need the input from three-nucleon observables. Solid (dashed) lines denote nucleons (pions).

of momenta and the pion mass over the $\chi$PT-breakdown scale of about 800 MeV.

An even simpler approach to the nuclear few-body problem is an effective field theory without pionic degrees of freedom [6,7]. This theory starts out from point-like interactions between nucleons, which only have to respect the symmetries of QCD. Like $\chi$PT, it describes phenomena in a systematic way, but is applicable only at energies well below a breakdown scale set by the pion mass. Again, only two LECs characterising 3N forces are required to predict observables with an accuracy of less than 1 % in processes involving three nucleons.

However, this accuracy can only be achieved if the experimental inputs are known correspondingly well. The binding energy of the triton and the doublet nd scattering length $a_2$ are particularly well suited to determine the LECs for $\chi$PT and for the pion-free theory. First, there are no Coulomb effects to be considered. Second, $a_2$ is very sensitive to 3N forces: in the quartet channel, the incident neutron has its spins parallel to the one bound in the deuteron, so that the Pauli principle prohibits 3N forces to play any sizeable role at low momentum transfer. In contrast, the $s$-wave in the doublet channel allows for a momentum-independent 3N interaction. This turns out even necessary to achieve results which are not sensitive to physics at high energy scales, or respectively, at short distances beyond the range of applicability of the theory. While the triton binding energy is known with an accuracy of $5 \times 10^{-7}$, the experimental knowledge of the nd doublet scattering length is only 6 %.

2. Present situation and accuracy goal

The scattering length of a neutron with spin $s$ and a nucleus with spin $I$ is given by

$$a = \frac{I + 1}{2I + 1} a_+ + \frac{I}{2I + 1} a_- + \frac{2 (a_+ - a_-)}{2I + 1} s \cdot I,$$

$$= a_c + \frac{2a_i}{\sqrt{I(I+1)}} s \cdot I,$$  \hspace{1cm} (1)

where $a_+$ and $a_-$ denote the scattering lengths in the state with total spin $I + \frac{1}{2}$, respectively, $I - \frac{1}{2}$. Since one cannot prepare the latter state, it is not possible to measure $a_-$ directly. Experimentally accessible are the spin-independent, coherent scattering length $a_c$, and the factor $a_i$ which parameterises the spin-dependence (sometimes called "incoherent" scattering length). For the neutron-deuteron system, the doublet and quartet scattering lengths, $a_- = a_2$, respectively $a_+ = a_4$, are given as the linear combinations

$$a_2 = a_{c,d} - \sqrt{2} a_{i,d},$$

$$a_4 = a_{c,d} + \frac{1}{\sqrt{2}} a_{i,d}.$$  \hspace{1cm} (2)

The best experimental value of $a_2$ was obtained 30 years ago [8], using a combination of a measurement of the scattering cross section of the free deuteron

$$\sigma_{s,d} = 4\pi \left( |a_{c,d}|^2 + |a_{i,d}|^2 \right),$$

with a gravity refractometric measurement of the bound coherent nd scattering length $b_{c,d}$ (through
the relation of bound and free scattering lengths \( b \), respectively \( a \) by

\[
a = \frac{M}{M + m} b,
\]

with corresponding indices. \( M \) is the mass of the nucleus and \( m \) the neutron mass). The experimental values were \( \sigma_{n,d} = 3.390 \pm 0.012 \) barn and \( b_{c,d} = 6.672 \pm 0.007 \) fm, leading to

\[
a_2 = 0.65 \pm 0.04 \text{ fm}.
\]

Recently, a new measurement of \( b_{c,d} \) was performed at the NIST interferometer at Washington DC \[9\]. Including the result, \( b_{c,d} = 6.6649 \pm 0.0040 \) fm, the present world average is

\[
b_{c,d} = 6.6683 \pm 0.0030 \text{ fm}.
\]

However, this improvement does not significantly reduce the experimental uncertainty of \( a_2 \), since this is dominated by the insufficient knowledge of \( a_{i,d} \). On the other hand, the authors of ref.\[9\] argue that, because \( a_4 \) should have a very small dependence on 3N forces, \( a_{i,d} \) may be derived from the experimental value stated in eq.\[6\] and a theoretical value of \( a_4 \), using eq.\[2\] and eq.\[4\]. That way, they obtain a semi-experimental value \( a_2 = 0.645 \pm 0.003 \text{ (expt)} \pm 0.007 \text{ (theory) fm} \).

The goal of the present experiment is a direct measurement of \( a_{i,d} \), which does not rely on any nuclear few-body theoretical input. As a minimum aim we hope to achieve an accuracy of \( 10^{-3} \). Using eq.\[2\] together with eq.\[6\], this shall provide a new value of \( a_2 \) with an uncertainty of \( 0.004 \) fm.

3. Method

The spin-dependent scattering length induces a spin-dependence of the neutron refractive index. As a result, \( b_i \) can be determined directly with a polarised neutron beam passing through a polarised target, via detection of pseudomagnetic neutron precession around the axis of nuclear polarisation \[10,11\]. The pseudomagnetic precession angle is given by

\[
\varphi^* = 2 \lambda d \sum_k \frac{I_k}{I_k + 1} P_k N_k b_{i,k}.
\]

\( N_k \) is the number density, \( I_k \) the nuclear spin and \( P_k \) the nuclear polarisation, with the sum index \( k \) extending over the nuclear species with spin. \( \lambda \) is the de-Broglie wavelength of the neutrons, and \( d \) is the thickness of the sample. The angle \( \varphi^* \) can be measured with an accuracy of at least one degree, using the method described in \[12,13,14\] based on Ramsey’s well-known resonance technique with two separated oscillatory fields. The target is situated in the homogeneous magnetic field between the two high-frequency \( \frac{\pi}{2} \) coils.

The uncertainty of the deuteron polarisation \( P_d \) would impose a severe limitation in accuracy if the deuterons were the only nuclei with spin in the target. This difficulty can be considerably relaxed in a measurement of \( b_{i,d} \) relative to \( b_{i,p} \), of the proton, which is known with the high accuracy of \( 2.4 \times 10^{-4} \). Using a single target which contains both deuterons and protons at hydrogen sites, absolute polarisation measurements can be avoided \[15\]. Many materials are suitable to apply the method of dynamic nuclear polarisation (DNP), by which both isotopes can be polarised simultaneously under still rather convenient conditions \[16\].

The method combines several measurements described in the following. First, the sample is polarised via DNP. After freezing the nuclear polarisation one determines the pseudomagnetic precession angle. According to eq.\[7\] and including an additional, instrumental phase \( \varphi_0 \), it is given by

\[
\varphi_1 = \varphi^*_d + \varphi^*_p + \varphi_0
\]

with

\[
\varphi^*_d = \sqrt{2} \lambda d P_d N_d b_{i,d},
\]

\[
\varphi^*_p = \frac{2}{\sqrt{3}} \lambda d P_p N_p b_{i,p}.
\]

Further, using hf-saturation, one can selectively depolarise either the protons or the deuterons, without significantly affecting the polarisation of the other spin species. The subsequent cross relaxation between the two spin systems can be held sufficiently slow by keeping the temperature sufficiently low. Saturating, e.g. first the protons, one can then measure

\[
\varphi_2 = \varphi^*_d + \varphi_0.
\]
After a subsequent depolarisation of the deuterons one measures
\[ \phi_3 = \varphi_0. \]  
(11)
Combining the measured values and using eq. (11), we obtain
\[ b_{i,d} = \sqrt{\frac{2}{3}} \frac{\phi_2 - \phi_3}{\phi_1} \frac{P_p N_p}{P_d N_d} b_{i,p}. \]  
(12)

The method is completed with measurements of deuteron and proton NMR signals \( I_k \), taken as integral of the corresponding rf-absorption line before and after each measurement of a pseudo-magnetic precession angle:
\[ I_k = C_k P_k N_k. \]  
(13)

Apart from natural constants, \( C_k \) is given as the product of \( g_k^2 / I_k \) (with \( g_k \) denoting the \( g \)-factor) \[ \text{[17]} \] and a factor accounting for the sensitivity of the resonance circuit. The latter would be difficult to determine absolutely. However, since only the ratio of \( P_i N_k \) for the two spin species occurs in eq. (12), replacing this by the ratio of NMR signals via eq. (13), the common instrumental factor cancels out. This requires using the same, linear resonance circuit at the same frequency in the measurements of \( I_d \) and \( I_p \), which therefore have to be performed at different main magnetic fields to account for the different gyromagnetic ratios.
Combining the various measurements into ratios, we thus finally obtain
\[ b_{i,d} = \frac{1}{\sqrt{6}} \frac{g_d^2 \phi_2 - \phi_3}{g_p^2 \phi_1 - \phi_2} \frac{I_p}{I_d} b_{i,p}. \]  
(14)
It is the gist of this method that we donot suppose any exact knowledge of neither \( N_d \), \( N_p \), \( P_d \), \( P_p \), \( d \) or \( \lambda \), nor of any absolute calibration factors.

### 4. Some practical comments

The choice of the sample is governed by several factors. First, we consider the isotopic composition. Best sensitivity is attained for \( \varphi_d^* \approx \varphi_p^* \). DNP keeps the spin temperatures of protons and deuterons equal [18]. Using the Brillouin functions in the high-temperature limit,
\[ \frac{P_d}{P_p} \approx \frac{4 \gamma_d}{3 \gamma_p} \approx 0.2. \]  
(15)
From eq. (9),
\[ \frac{N_d}{N_p} \approx 5 \frac{P_p \varphi_d^*}{P_d \varphi_p^*}. \]  
(16)
Thus, \( \varphi_d^* \approx \varphi_p^* \) for \( N_d / N_p \approx 4 \%). On the other hand, to keep the systematic uncertainties induced by non-linearities of the NMR resonance circuit small, requires \( I_d \approx I_p \), which happens to be true for about the same ratio. Considering a typical material with number density \( 6.7 \times 10^{23} \) cm\(^{-3}\) of hydrogen sites and the measurements being done at \( \lambda = 0.4 \) nm, the corresponding pseudomagnetic precession angles are
\[ \varphi_d^* = 90 \text{ rad} \quad \text{and} \quad \varphi_p^* = 440 \text{ rad}. \]  
(17)
Since the Ramsey technique is sensitive to precession angles of at least one degree, a nuclear polarisation of a few percent is sufficient already for a target only 3 mm long.

Relaxation times are known to be strongly temperature-dependent. Using a magnetic field of 2.5 T and a cryostat providing \( T \approx 100 \) mK, the nuclear spin systems are frozen. Typical spin-lattice relaxation times of 500 hours and cross relaxation time of 100 hours, obtained for (CH\(_2\)OH)\(_2\) doped with Cr(V) paramagnetic centres (radicals), were published in ref. [13]. These are sufficiently long, since, for the given neutron intensity, a single run of the experiment will take much less than one hour. Operating with an amount of radicals below the optimum value to reach maximum polarisation (which is not required here), spin relaxation may be suppressed even further.

Apart from stability requirements, keeping low the amount of radicals is also of interest to suppress systematic errors which may be due to their local magnetic fields. Nuclear spins close to the paramagnetic electrons contribute to pseudomagnetic precession but usually stay undetected by NMR, since their resonance frequencies are strongly shifted with respect to the NMR line of the nuclei in the bulk. Two measures may be taken. First, the radical concentration should be kept as low as possible. Second, deuterated radicals should be used, in order to keep the few protons of the sample away from the radicals. Also for practical reasons, a good candidate of a target
material is a plastic deuterated to the required abundance and doped with deuterated nitroxyl radicals [19]. Note in addition that, using a novel stroboscopic technique of simultaneous small angle neutron scattering and NMR measurements, detailed information can be obtained about the polarisation state of protons close to radicals dissolved in a deuterated matrix [20]. That way one can determine a correction, if necessary. Note also, that the paramagnetic electrons are always polarised very close to 100%, and their fields cause a small magnetic neutron precession, which is included in the instrumental phase $\phi_0$.

The NMR detection scheme is a crucial part of the setup and deserves special consideration. An analysis of several possibilities is described in a separate publication [21]. Apart from its special role in the measurement, NMR can also be used to monitor the polarisation state of the spin systems at any stage of the experiment. Many systematic variations of parameters can be performed to check the independence of the measurement with respect to the conditions which have dropped out in the derivation of eq. (14). Different samples with different chemical composition, varied degree of deuteration and polarisation will be used. A systematic variation of target and beam parameters should be able to demonstrate the reliability of the measurement (this strategy was also adopted in a recent determination of the spin-dependent $n^3$He scattering length [22]).

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