STANDARD AND GENERALIZED NEWTONIAN GRAVITIES AS "GAUGE" THEORIES OF THE EXTENDED GALILEI GROUP - II: DYNAMICAL THREE-SPACE THEORIES

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Abstract

In a preceding paper we developed a reformulation of Newtonian gravitation as a \textit{gauge} theory of the extended Galilei group. In the present one we derive two true generalizations of Newton’s theory (a \textit{ten-fields} and an \textit{eleven-fields} theory), in terms of an explicit Lagrangian realization of the \textit{absolute time} dynamics of a Riemannian three-space. They turn out to be \textit{gauge invariant} theories of the extended Galilei group in the same sense in which general relativity is said to be a \textit{gauge} theory of the Poincaré group. The \textit{ten-fields} theory provides a dynamical realization of some of the so-called “Newtonian space-time structures” which have been geometrically classified by Künzle and Kuchař. The \textit{eleven-fields} theory involves a \textit{dilaton-like} scalar potential in addition to Newton’s potential and, like general relativity, has a three-metric with \textit{two} dynamical degrees of freedom. It is interesting to find that, within the linear approximation, such degrees of freedom show \textit{graviton-like} features: they satisfy a wave equation and propagate with a velocity related to the scalar Newtonian potential.
1 Introduction

The present paper must be seen, from the technical point of view, as a natural development of a previous one \[1\], in which Newtonian gravitation has been reformulated as a gauge theory of the extended Galilei group. The role of the Galilei group has thereby been transformed from that of a symmetry group to that of a covariance group, in the sense of Anderson \[2\]. Yet, from a substantial point of view, this paper contains true generalizations of Newton’s theory: we propose here two new gravitational theories which provide non-relativistic (Galilean) Lagrangian descriptions of the evolution of a Riemannian dynamical three-space in absolute time.

The recasting of Newton’s theory has been obtained in \[1\] in the form of a three-dimensional Galilean (absolute time respecting) generally-covariant Action principle. Thus our three-dimensional formulation differs substantially from the well-known four-dimensional reformulations of Newton’s theory in geometrical terms by Élie Cartan \[3\], Havas \[4\], Anderson \[2\], Trautman \[5\], Künzle \[6\] and Kuchař \[7\]. All these reformulations describe the Newtonian inertial-gravitational structure in terms of an affine connection compatible with the temporal flow $t_\mu$ and a rank-three spatial metric $h^{\mu\nu}$. While the curvature of the four-dimensional affine connection is different from zero because of the presence of matter, the Newtonian flatness of the absolute three-space is guaranteed by the further requirement that Poisson’s equation be satisfied, in the covariant form $R_{\mu\nu} = G\rho(z)t_\mu t_\nu$, where $R_{\mu\nu}$ is the Ricci tensor of the affine connection and $\rho(z)$ is the matter density. In this way, the four-dimensional description is dynamical, while the three-dimensional one is not.

It is clear that, in order to achieve the main scope we are interested in, it necessary first of all to get rid, in some way, of the flatness condition of the absolute three-space metric $g_{ij}$, which is expressed by the above covariant Poisson equation in the four-dimensional formulations and by the explicit vanishing of the three-dimensional Ricci tensor ($R_{ij} = 0$) in our three-dimensional formulation. Now, in \[1\] we have obtained essentially the following results: (1) By exploiting the gauge methodology originally applied by Utiyama \[8\] to the Lorentz group within the field theoretic framework, all the inertial-gravitational fields which
can be coupled to a non-relativistic mass-point have been characterized. (2) A suitable non-
relativistic limiting procedure (for $c^2 \to \infty$) from the four-dimensional level has then been
utilized. Precisely, the limiting procedure has been applied to the Einstein-Hilbert-De Witt
action for the gravitational field plus a matter action corresponding to a single mass point,
under the assumption of the existence of a global 3+1 splitting of the total Action, and of a
suitable parametrization of the 4-metric tensor in terms of powers of $c^2$. Once the expansion
in powers of $1/c^2$ has been explicitly calculated, we have made the Ansatz of identifying
the basic Galilean Action $A$ with the zero-th order term of the expansion itself. 3) The
resulting Action turned out to be dependent on 27 fields beside the degrees of freedom
of the mass-point. Eleven among these fields are gauge fields of the Galilei group, having
definite inertial-gravitational properties, while the remaining 16 fields are not coupled to
matter and play the role of auxiliary fields that guarantee the general covariance of the
theory.

In force of these results, unlike the case of the four-dimensional framework which does
not lend itself to any easy generalization, a way out of the above constraint appears natu-
rally in our formulation and is suggested by the very structure of the total Galilean Action
$A$. Indeed, it is natural to try to eliminate all the auxiliary fields that are not coupled to
matter. This result is obtained through the new Ansatz constituted by putting equal to
zero, by hand, as tensor equations, all the auxiliary fields. As a matter of fact, in this way
we define a new variational problem which turns out to provide consistently the follow-
ing results: (a) The theory contains only fields coupled to matter. (b) The theory is still
gauge-invariant (properly quasi-invariant) under the local Galilei group. (c) The condition
of Newtonian flatness ($R_{\mu\nu} = 4\pi G \rho(z) t_\mu t_\nu$) no longer appears and Riemannian three-spaces
with non-zero curvature are allowed.

It turns out that, within the theory obtained in this way, the eleventh gauge field $\Theta(t)$,
originally generated by the central extension of the Galilei group, has no dynamical meaning
and can be reabsorbed in the definition of the absolute time, so that the theory has ten
effective fields. Then, the constraint analysis shows that the three-metric $g_{ij}$ possesses
three dynamical degrees of freedom. The structure of the constraints chains of this theory
is rather involved, and its analysis has not been carried through completely in the present paper.

A more interesting theory, with eleven field, is obtained by allowing the \( \Theta \) field to depend on the space variables \( z \), besides the time \( t \). This field, which appears to be a classical analogue of the dilaton field, gives rise to a quite different constraint structure, which is more like that of general relativity. It turns out, in fact, that the three-metric \( g_{ij} \) has now two dynamical degrees of freedom, while all the other fields are constrained either by gauge conditions corresponding to first-class constraints (as the inertial force vector \( A_k \)) or by second-class pairs of constraints (as the scalar gravitational potential and the dilaton field). From this point of view, it can be said that this theory shares, so to speak, an intermediate status between pure Newtonian theory in which there is only the gravitational "force" associated to the scalar potential \( \varphi \), and general relativity in which there is no "force" and the whole dynamical description is provided by the 4-metric. It is interesting to find that within a linear approximation of the eleven-fields theory, the dynamical degrees of freedom of the spatial 3-metric show a graviton-like nature. Indeed, they satisfy an hyperbolic wave equation and propagate with a velocity related to the square root of the zero\(^{th}\)-order weak-field approximation of the scalar Newtonian potential. In this way the latter plays the additional role of cosmological background. This result seems quite remarkable from both a conceptual and a historical point of view. It is worth recalling that Einstein, in his first attempts towards a relativistic theory of gravitation, introduced a variable speed of light playing the role of the gravitational potential (see, for example Norton [9]).

Section 2 is dedicated to the presentation and discussion of the generalized Newtonian gravitational theories as gauge-invariant theories of the Galilei group: the ten-fields-theory (Section 3), and the eleven-field-theory (Section 4). Many calculations and special results are reported in three Appendixes.
2 Generalized Newtonian Gravities: Galilei gauge-invariant theories for some Special Newtonian Manifolds

In a previous paper [1] we have shown that it is possible to implement the standard Newtonian gravity as a covariant field theory. The fundamental fields of this theory are a 3-dimensional Euclidean metric $g_{ij}$, an inertial-gravitational vector field $A_i$, a scalar field $A$ which plays the role of a generalized Newton’s potential, and the time-reparametrization field $\Theta$. It was shown, moreover, that these fields are the gauge fields associated to the reinterpretation of the Galilei group as a localized group and that they can be exploited to define the Special Newtonian space-time structure on which the four dimensional Cartan’s reformulations of Newtonian Gravity [3] is based. The price that we had to pay for the above result was the introduction of 16 “auxiliary” fields, say $\alpha_0$, $\alpha_i$, $\gamma_{ij}$, $\beta_{ij}$, that do not have any physical role besides that of allowing a generally covariant formulation. Moreover, they do not couple to matter (a mass-point), and correspond to non-propagating degrees of freedom in the standard Newtonian theory. Our reformulation has no physical degree of freedom and, of course, flat metric.

We want now to search for a possible true generalization of Newton theory which, essentially, has to allow for a non-flat metric and a possible dynamical evolution of it in the spirit of general relativity (though, of course, in absolute time). As we shall see, contrary to what seems to be a widespread opinion, this is in fact realizable. Since we have, so to speak, to reduce the set of conditions which force the flatness and absolute nature of Newton’s space, a natural way for this generalization is already inscribed in the structure of the 27 fields Newtonian theory, because we have here the liberty to try to constrain or even eliminate some or all of the auxiliary fields without modifying the variety of fields physically interacting with matter. After all, only the original eleven fields are directly connected with the gaugeization of the Galilei group and, furthermore, only these latter are correlated to the geometric space-time-Newtonian structures. As a matter of fact, we will derive theories which remain gauge-invariant theories of the Galilei group in a peculiar way.
We will adopt here the simplest choice: we will put equal to zero, by hand, as tensor equations, all the auxiliary fields $\alpha_0, \alpha_i, \gamma_{ij}, \beta_{ij}$. It should be clear that, in this way, we are not dealing with, say, a subsector of the old variational problem, but we are in fact constituting an entirely different variational problem. In fact, as we shall see, we will obtain a variational principle for the description of the dynamics of some special Newtonian Manifolds \[6\] (ten-fields theory). A different theory can be obtained by allowing the field $\Theta$ to be a function of $t$ and $z$. This new theory (eleven-field theory) describes an additional dilaton-like degree of freedom. Both these theories describe fields coupled to matter. It is not clear, however, at the present level of analysis whether the formal results are completely consistent from a distributional point of view (singularity on the world-line like in the relativistic case of particles plus fields). Let us remark that, in absence of matter, if we allow $\alpha_0$ to be different from zero, the resulting theory is a subsector of the ten-fields one. Probably, in order to fit a formally consistent theory in presence of matter one has to add extra couplings of $\alpha_0$ to matter which cannot be obtained by a limiting procedure from general relativity. Yet, even if we have not carried out a complete analysis of all the possibilities, it is most likely that the only formally consistent theories without extra coupling to matter are the ten and eleven-fields theories just mentioned.
If we keep only the fields that explicitly interact with the mass-point, i.e., if we set
\( \alpha_0 = \alpha_i = \gamma_{ij} = \beta_{ij} = 0 \), in an arbitrary reference frame, the \( c^{-2} \) expansion of the total action of [1] can be rewritten as:

\[
\tilde{S} = S_F + S_M = \\
\frac{c^3}{16\pi G} \int dtd^3z \sqrt{\tilde{g}} N \left[ R + \frac{1}{N^2} g^{ik} g^{jl} (B_{ij} B_{kl} - B_{ik} B_{jl}) \right] \\
-mc \int d\lambda \sqrt{-g_{\mu\nu} x^\mu x^\nu} \\
= c^4 \left[ \frac{1}{16\pi G} \int dtd^3z \sqrt{g} \Theta R \right] \\
+ c^2 \left[ \frac{1}{16\pi G} \int dtd^3z \sqrt{g} \left[ -\frac{A}{\Theta} R + \Theta g^{ik} g^{jl} (B_{ij} B_{kl} - B_{ik} B_{jl}) \right] - m \int d\lambda \Theta t' \right] \quad (2.1) \\
+ \frac{1}{16\pi G} \int dtd^3z \sqrt{g} \left[ -\frac{A^2}{2\Theta^3} R + \frac{A}{\Theta^3} g^{ik} g^{jl} (B_{ij} B_{kl} - B_{ik} B_{jl}) \right] \\
+ m \int d\lambda \frac{m}{\Theta t'} \left[ \frac{1}{2} g_{ij} (x'^i + g^{ik} A_k t') (x'^j + g^{jl} A_l t') + A t' \right] \\
+ O(1/c^2) . 
\]

Now, the zeroth order term can be written:

\[
\tilde{S} = \tilde{S}_F + \tilde{S}_M \\
= \frac{1}{16\pi G} \int dtd^3z \sqrt{g} \left[ -\frac{A^2}{2\Theta^3} R + \frac{A}{\Theta^3} g^{ik} g^{jl} (B_{ij} B_{kl} - B_{ik} B_{jl}) \right] \\
+ m \int dtd^3z \frac{m}{\Theta} \left[ \frac{1}{2} g_{ij} (\dot{x}^i + g^{ik} A_k) (\dot{x}^j + g^{jl} A_l) + A \right] \delta[z - x(t)] . \quad (2.2)
\]

As in the previous paper [1] we will make the Ansatz that the total action for the generalized Galilean field theory with a mass-point be the zeroth order expression (2.2). The meaning of the symbols here is the following: \( g_{ij} \) is a three-dimensional metric (with signature 3), \( A_i \) is an inertial-gravitational vector field, \( A \) is a gravitational scalar field, and \( \Theta \) is the time-reparametrization field. \( R \) denotes the three-dimensional scalar curvature associated to the unique symmetric covariant derivative \( \nabla_i \) compatible with \( g_{ij} \), and \( B_{ij} \) is given by:

\[
B_{ij} = \frac{1}{2} \left[ \nabla_i A_j + \nabla_j A_i - \frac{\partial g_{ij}}{\partial t} \right] \quad (2.3)
\]
The Euler-Lagrange equations for the mass-point and the fields result:

\[
\begin{align*}
\mathcal{L}_A & \equiv \frac{1}{16\pi G} \sqrt{g} \left[ -AR + g^{ik} g^{jl} (B_{ij} B_{kl} - B_{ik} B_{jl}) \right] + \frac{m}{\Theta} \delta^3 [z - \mathbf{x}(t)] \equiv 0 \\
\mathcal{L}_{A_i} & \equiv \frac{1}{8\pi G} \Theta^3 \left\{ \partial_j \left[ -\sqrt{g} A [g^{ik} g^{jl} - g^{ij} g^{kl}] B_{kl} \right] \\
& + \left[ -\sqrt{g} A [g^{rk} g^{sl} - g^{rs} g^{kl}] B_{kl} \right] \Gamma^i_{rs} \right\} + \frac{m}{\Theta} \delta^3 [z - \mathbf{x}(t)] \left[ \dot{x}^i + g^{ij} A_j \right] \equiv 0 \\
\mathcal{L}_\Theta & \equiv \int d^3z \left\{ \frac{3}{16\pi G} \sqrt{g} \left[ \frac{A^2}{2\Theta^4} R - \frac{A}{\Theta^3} g^{ik} g^{jl} (B_{ij} B_{kl} - B_{ik} B_{jl}) \right] \\
& - \frac{m}{\Theta^4} \left[ \frac{1}{2} g_{ij} (\dot{x}^i + g^{ik} A_k) (\dot{x}^j + g^{jl} A_l) \right] \delta^3 [z - \mathbf{x}(t)] \right\} \equiv 0 \\
\mathcal{L}_{g_{ij}} & \equiv \frac{1}{16\pi G} \left\{ \sqrt{g} (g^{ir} g^{js} - g^{ij} g^{rs}) \nabla_r \nabla_s \left[ \frac{A^2}{\Theta^3} \right] + \sqrt{\frac{g^2}{A^2}} \left[ R^{ij} - \frac{1}{2} g^{ij} R \right] \right\} (2.4) \\
& + \frac{2\sqrt{g} A}{\Theta^3} [B^{ir} B^{js} g_{rs} - B^{ij} \text{Tr} B] \\
& + \frac{d}{dt} \left[ \frac{\sqrt{g}}{\Theta^3} (g^{ir} g^{js} - g^{ij} g^{rs}) B_{rs} \right] \right\} + \frac{m}{2\Theta} (\dot{x}^i + g^{ik} A_k) (\dot{x}^j + g^{jl} A_l) \delta^3 [z - \mathbf{x}(t)] \equiv 0 \\
\mathcal{L}_{x^i} & = \dot{x}^i + \Gamma^i_{kl} \dot{x}^k \dot{x}^l \\
& + \frac{\dot{\Theta}}{\Theta} \left[ \dot{x}^i + g^{ij} A_j \right] + g^{ij} \frac{\partial g_{jl}}{\partial t} \dot{x}^l \\
& - g^{ij} \left[ \frac{\partial A_0}{\partial x^j} - \frac{\partial A_j}{\partial t} \right] - g^{ij} \left[ \frac{\partial A_l}{\partial x^j} - \frac{\partial A_j}{\partial x^l} \right] \dot{x}^l \equiv 0 \,.
\end{align*}
\]

Let us see that, in this way, as in the case of the standard Newtonian theory developed in the previous paper [1], we have constructed a theory which is invariant under the local Galilei group. In that paper it was shown that the localized Galilei group operations for the mass-point coordinates and for the fields are naturally defined by the following infinitesimal transformations:

\[
\begin{align*}
\delta t(\lambda) & = -\varepsilon(t(\lambda)) \\
\delta x^i(\lambda) & = \varepsilon^i(x, t) - c_{ij} \omega^j(x, t) x^k - t v^i(x, t) \\
& \equiv \eta^i(x, t) \\
\end{align*}
\] (2.5)
\[
\begin{align*}
\delta \Theta &= \dot{\epsilon}(t)\Theta(t) \\
\delta g_{ij} &= -\frac{\partial \tilde{\eta}^k(x,t)}{\partial x^i} g_{kj} - \frac{\partial \tilde{\eta}^k(x,t)}{\partial x^j} g_{kj} \\
\delta A_0 &= 2\dot{\varepsilon} A_0 - A_i \frac{\partial \tilde{\eta}^i}{\partial t} - \Theta \frac{\partial}{\partial x^i} \left[ g_{ij} \dot{v}^j x^j \right] \\
\delta A_i &= \dot{\varepsilon} A_i - A_j \frac{\partial \tilde{\eta}^j}{\partial x^i} - g_{ij} \frac{\partial \tilde{\eta}^j}{\partial t} - \Theta \frac{\partial}{\partial x^i} \left[ g_{ij} \dot{v}^j x^j \right].
\end{align*}
\]

(2.6)

where \( c_{jk}^i = \epsilon_{ijk} \) are the usual structure constant of the O(3) rotation group, \( \varepsilon \) is the parameter of the infinitesimal time-translation, \( \varepsilon^i \) are the parameters of the infinitesimal space-translations, \( \omega^i \) are the parameters of the infinitesimal space rotations, and the \( v^i \) are those of the infinitesimal Galilei boosts.

In fact, if we now adopt these transformation rules, the variation of the total action under the transformations of the mass-point coordinates and of the gauge fields (2.5,2.6) results:

\[
\begin{align*}
\delta \tilde{S} &= \int dtdz \left\{ \dot{\xi} \hat{L} + \frac{1}{16\pi G} \frac{\sqrt{g}}{\Theta^2} (-AR + \Gamma) + m\delta^3 [z - x(t)] \frac{\partial F}{\partial t} - A_r g^{rs} \frac{\partial F}{\partial z^s} \\
&\quad - \frac{1}{8\pi G} \left[ \frac{\partial}{\partial z^j} \left( \frac{\sqrt{g}}{\Theta^2} [B^{ij} - (\text{Tr} B) g^{ij}] \right) + \sqrt{g} A \frac{\partial}{\partial z^j} \left( \frac{\sqrt{g}}{\Theta^2} [B^{ij} - (\text{Tr} B) g^{ij}] \Gamma_{rs}^{ij} \right) \right] \\
&\quad - m\delta^3 [z - x(t)] \left( \dot{\xi}^i + g^{ij} A_j \right) \frac{\partial F}{\partial z^i} + \frac{1}{8\pi G} \frac{\partial}{\partial z^i} \left( \frac{\sqrt{g}}{\Theta^2} [B^{ij} - (\text{Tr} B) g^{ij}] \frac{\partial F}{\partial z^j} \right) \right\} \\
&= \int dtdz \left\{ \dot{\xi} \hat{L} + \frac{\Theta \mathcal{E}_A}{\Theta^2} \left( \frac{\partial F}{\partial t} - A_r g^{rs} \frac{\partial F}{\partial z^s} \right) + \frac{\partial}{\partial z^i} \left( \frac{\sqrt{g}}{\Theta^2} [B^{ij} - (\text{Tr} B) g^{ij}] \frac{\partial F}{\partial z^j} \right) \right\}.
\end{align*}
\]

(2.7)

Therefore we have found the important result that the total action is quasi-invariant under the transformations (2.5,2.6) in force of the equations of motion. We can thereby conclude that the theory has a local Galilei invariance modulo the equations of motion. Let us remark that this peculiarity is precisely what it should be expected in the case of a variational principle corresponding to a singular Lagrangian.
3 A ten-fields theory

It is easy to show that, as in the first variational problem of the preceding paper (27-fields theory), the field $\Theta(t)$ has no real dynamical content also in the variational problem corresponding to the action \((2.2)\), since its effect amounts only to a redefinition of the evolution parameter $t$ in the expression $T(t) = \int_0^t d\tau \Theta(\tau)$. Indeed, by redefining the fields $A_0$ and $A_i$ as in the standard Newtonian case of \([1]\):

\[
\begin{align*}
\tilde{A}_0 &\equiv \frac{A_0}{\Theta^2} ; \quad \tilde{A} \equiv \frac{A}{\Theta^2} \\
\tilde{A}_i &\equiv \frac{A_i}{\Theta} \\
\tilde{B}_{ij} &\equiv \frac{B_{ij}}{\Theta} = \frac{1}{2} \left[ \nabla_i \tilde{A}_j + \nabla_j \tilde{A}_i - \frac{\partial g_{ij}}{\partial T} \right],
\end{align*}
\]

(3.1)

the total action \((2.2)\) can be re-written as:

\[
\begin{align*}
\tilde{S} &\equiv \int dT L[T] \\
&= \frac{1}{16\pi G} \int dT d^3z \sqrt{g} \left[ -\frac{\tilde{A}^2}{2} R + \tilde{A} g^{ik} g^{jl} (\tilde{B}_{ij} \tilde{B}_{kl} - \tilde{B}_{ik} \tilde{B}_{jl}) \right] \\
&\quad + m \int dT d^3z \left[ \frac{1}{2} g_{ij} \frac{d\tilde{x}^i}{dT} + g^{ik} \tilde{A}_k \left( \frac{d\tilde{x}^j}{dT} + \frac{\partial \tilde{g}_{ij}}{\partial T} \right) \right] \delta^3[z - \mathbf{x}(T)] ,
\end{align*}
\]

(3.2)

i.e., in a form independent of $\Theta(t)$. Correspondingly, the Euler-Lagrange equations are just the Eq.(2.4) without $\Theta$.

We shall deal now with the constraint analysis within the Hamiltonian formalism. The canonical momenta $[\dot{f} = \frac{\partial f}{\partial \dot{f}}]$ are defined by:

\[
\begin{align*}
p_k &\equiv \frac{\partial L[T]}{\partial \dot{x}^k} = m [g_{ki}\dot{x}^i + \tilde{A}_k] \\
\pi^i &\equiv \frac{\delta L}{\delta \dot{A}_i} = 0 \\
\pi_A &\equiv \frac{\delta L}{\delta \dot{\tilde{A}}} = 0 \\
\pi^{rs} &\equiv \frac{\delta L[T]}{\delta \dot{g}_{ij}} = -\sqrt{g} \tilde{A} \left( g^{rk} g^{si} - g^{rs} g^{kl} \right) \tilde{B}_{kl} .
\end{align*}
\]

(3.3)

Therefore, since the Lagrangian is independent of the velocities $\dot{\tilde{A}}$ and $\dot{\tilde{A}}_i$, we have, first of all, the primary constraints

\[
\pi^i \simeq 0 \quad , \quad \pi_A \simeq 0 .
\]

(3.4)
The Dirac Hamiltonian is given by:
\[ \begin{align*}
H_c &= \dot{x}^k p_k + \int d^3z \left[ \pi^i \dot{A}_i + \pi_A \dot{\lambda}_A + \pi^{ij} \dot{g}_{ij} \right] - L[T] \\
H_d &= \int d^3z \left[ \frac{\dot{A}^2}{2mG} \mathcal{H}_I + \frac{16\pi G}{\Lambda} \mathcal{H}_E + \left[ \frac{1}{2m} g^{ij} p_i p_j - m \dot{A} \right] \delta^3(z - x(T)) \right] \\
&\quad + \int d^3z \left[ \dot{A}_i g^{ij} \phi_j + \pi^i \lambda_i + \pi_A \lambda^A \right],
\end{align*} \]
where for further convenience we have introduced the following notation 3:
\[ \begin{align*}
\mathcal{H}_I &= \sqrt{g} R \\
\mathcal{H}_E &= \frac{1}{\sqrt{g}} [g_{ik} g_{jl} - \frac{1}{2} g_{ij} g_{kl}] \pi^{ij} \pi^{kl} \\
\phi_i &= 2g_{ij} \nabla_k \pi^{jk} + p_i \delta^3(z - x(T)) .
\end{align*} \]

For future use, we list here the relevant algebraic relations involving the above quantities:
\[ \begin{align*}
\{ \phi_i(z, T), \phi_j(z', T) \} &= -\left[ \phi_j(z, T) \frac{\partial}{\partial z^i} + \phi_i(z', T) \frac{\partial}{\partial z^j} \right] \delta^3(z - z') \\
\{ \phi_i(z, T), \mathcal{H}_I(z', T) \} &= -\mathcal{H}_I(z', T) \frac{\partial}{\partial z^i} \delta^3(z - z') \\
\{ \phi_i(z, T), \mathcal{H}_E(z', T) \} &= -\mathcal{H}_E(z', T) \frac{\partial}{\partial z^i} \delta^3(z - z') \\
\{ \mathcal{H}_I(z, T), \mathcal{H}_E(z', T) \} &= -2\pi rs \left[ R_{rs} - \frac{1}{4} g_{rs} R \right] \delta^3(z - z') \\
&\quad + 2\pi rs(z', T) \left[ \frac{\partial^2}{\partial z^r \partial z^s} + \Gamma^{k}_{rs} (z', T) \frac{\partial}{\partial z^k} \right] \delta^3(z - z') ,
\end{align*} \]
where \( \nabla \) is the covariant derivation with respect to the metric \( g_{ij} \).

Notice that the \( \phi_i \)'s are the canonical generators of the coordinate transformations (diffeomorphism group) of the three-space with fixed absolute time since we have
\[ \int d^3z' \{ g_{ij}(z, T), \phi_k(z', T) \} \xi^k(z', T) = -g_{ijk} \xi^k - g_{ik} \frac{\partial \xi^k}{\partial z^j} - g_{kj} \frac{\partial \xi^k}{\partial z^i} \]
\[ \int d^3z' \{ \pi^r s(z, T), \phi_k(z', T) \} \xi^k(z', T) = -\pi^r s \xi^k - \pi^{rs} \xi^k + \pi_{rk} \frac{\partial \xi^s}{\partial z^k} + \pi_{ks} \frac{\partial \xi^r}{\partial z^k} , \]
a fact which is well-known from the canonical 3+1 formulation of general relativity 10.

We apply now the Dirac-Bergman procedure. By imposing time-conservation of the primary constraints, we get:
\[ \begin{align*}
\phi_i(z, T) &= -g_{ij}(z, T) \dot{\pi}^j(z, T) \equiv -g_{ij}(z, T) \{ \pi^i(z, T), H_d \} \simeq 0 \\
\chi_A(z, T) &\equiv \pi_A(z, T) = \{ \pi_A(z, T), H_d \} \\
&= \frac{16\pi G}{\Lambda^2} \mathcal{H}_I - \frac{\dot{A}}{16\pi G} \mathcal{H}_I + m \delta^3(z - x(T)) \simeq 0 ,
\end{align*} \]

2Note that, while the expressions of \( \phi, \mathcal{H}_I \) and \( \mathcal{H}_E \) are identical to the corresponding quantities introduced in the usual Hamiltonian formalism of general relativity, the ADM super-Hamiltonian is instead \( \mathcal{H}_\perp = \mathcal{H}_I - \mathcal{H}_E \).
which must be imposed as secondary constraints. In turn, time-conservation of these latter gives:

\[
\psi_i(z, T) \equiv \dot{\phi}_i(z, T) = \{\phi_i(z, T), H_d\} \simeq 0
\]

\[
\psi_A(z, T) \equiv \dot{\chi}_A(z, T) = \{\chi_A(z, T), H_d\}
\]

\[
= 3\pi^r s \left[ R_{rs} - \frac{1}{4} g_{rs} \bar{R} \right] - 6\pi^r s \frac{1}{A^2} \partial_r \bar{A} \partial_s \bar{A} + \frac{16\pi G}{m A^2 \sqrt{g}} \pi^r s \left[ p_r p_s - \frac{1}{2} g_{rs} g^{lm} p_l p_m \right] \delta^3 \left[ z - \mathbf{x}(T) \right] - 2\lambda A \left[ \frac{1}{32\pi G} \mathcal{H}_I + \frac{16\pi G}{A^3} \mathcal{H}_E \right] \simeq 0.
\]

(3.10)

While we have \(\psi^i \simeq 0\) as a consequence of the primary and secondary constraints, the \(\psi_A\) must be put equal to zero as a further condition, whose nature must be discussed in detail.

This condition becomes an equation for the multiplier \(\lambda_A\), and the constraints chain stops consequently, if the quantity

\[
\bar{\chi} \equiv \frac{1}{32\pi G} \mathcal{H}_I + \frac{16\pi G}{A^3} \mathcal{H}_E
\]

is not identically zero. For simplicity, we will discuss only two particular cases. Actually, a complete treatment of the problem of the degrees of freedom of the theory would involve a thorough analysis of the various possible independent constraint sectors corresponding to different classes of initial conditions [11].

a) The simplest constraint structure obtains for a sector in which \(\bar{\chi} \neq 0\) everywhere. It is clear that not all the allowed initial conditions are compatible with this restriction. In this case we have:

1) 3 chains of first class constraints \(\pi^i \simeq 0, \phi_i \simeq 0\), so that \(\bar{A}_i\) and 3 components of \(g_{ij}\) are gauge variables to be fixed with some gauge fixing, for instance \(\bar{A}_i \equiv 0\) (Kuchař’s [7] non-rotating observer);

2) 1 chain containing a pair of second class constraints \(\pi_A \simeq 0, \chi_A \simeq 0\), so that \(\bar{A}\) is determined by \(\chi_A \simeq 0\) (see eqs.3.3): note that this equation for the potential hidden inside \(\bar{A}\) is not Poisson-like. Actually, as we know, it cannot be so, in the present conditions since the three-space is non-flat in general. Its form is precisely the following:

\[
2\sqrt{g} \bar{A} \Delta \bar{A} + 2\sqrt{g} g^{ij} \nabla_i \bar{A} \nabla_j \bar{A} = +4\pi m G \bar{A} \delta^3 \left[ z - \mathbf{x}(T) \right].
\]

(3.12)
It is interesting to note that, unlike the standard Poisson equation, Eq. (3.12) allows for the solution $A \equiv 0$, corresponding to the strong equation: $\pi^{rs} = 0$. This fact could have some interest in connection with the needs of a non-relativistic cosmology (see for example Rindler and Friedrichs [12]).

In this sector there are 3 physical degrees of freedom in $g_{ij}$ (one more than in the general relativistic case). The structure of the constraints is illustrated in Fig.1.

b) Let us remark that in the sector $\bar{\chi} \simeq 0$, one has proliferation [11] of constraints, i.e. $\psi_A \simeq 0$ is replaced by $\bar{\chi} \simeq 0$ and $\psi'_A = \psi_A |_{\chi=0} \simeq 0$. It is extremely difficult to analyze it and we are not sure that it is self-consistent. Anyway, where it consistent in absence of matter, it would imply the vanishing of both $\mathcal{H}_I$ and $\mathcal{H}_E$, as a consequence of $\chi_A \simeq 0$, $\bar{\chi} \simeq 0$: recall that the vanishing of both $\mathcal{H}_I$ and $\mathcal{H}_E$ is a particular solution of the ADM superHamiltonian constraints also in the general relativistic case (see footnote 2). Furthermore one should discuss here too the problem of the central charge $c^2\mathcal{M} + c^4\mathcal{N}$, mentioned at the end of Section 6 of the previous work.
3.1 An eleven-fields theory

Let us now generalize the field $\Theta(t)$ to an expression depending on time and on the space coordinates: $\Theta = \Theta(z, t)$. This generalization is not quite natural from a Galilean point of view since it introduces dilaton-like degrees of freedom into a Newtonian framework. In this case, of course, the field $\Theta$ cannot be reabsorbed and the total action (2.2) can be written as:

$$\tilde{S} = \int dt L[t]$$

$$= \frac{1}{16\pi G} \int dt d^3z \sqrt{g} \left[ -\frac{A^2}{2\Theta^2} R + \frac{A}{\Theta^3} g^{ik} g^{jl}(B_{ij}B_{kl} - B_{ik}B_{jl}) \right]$$

$$+ \int dt d^3z \frac{m}{\Theta} \left[ \frac{1}{2} g_{ij}(\dot{x}^i + g^{ik}A_k)(\dot{x}^j + g^{jl}A_l) + A \right] \delta^3[z - \mathbf{x}(t)].$$

(3.1)

Here again, we shall deal with the problem of investigating the true dynamical degrees of freedom of the theory by means of a constraint analysis within the Hamiltonian formalism. In this case too is profitable to adopt $\tilde{A} = A/\Theta^2$ as a dynamical variable. The Action then becomes:

$$\tilde{S} = \int dt L[t]$$

$$= \frac{1}{16\pi G} \int dt d^3z \sqrt{g} \left[ -\frac{\tilde{A}^2}{2\Theta^2} R + \frac{\tilde{A}}{\Theta} g^{ik} g^{jl}(B_{ij}B_{kl} - B_{ik}B_{jl}) \right]$$

$$+ \int dt d^3z \frac{m}{\Theta} \left[ \frac{1}{2} g_{ij}(\dot{x}^i + g^{ik}A_k)(\dot{x}^j + g^{jl}A_l) + \Theta^2 \tilde{A} \right] \delta^3[z - \mathbf{x}(t)].$$

(3.2)

The Euler-Lagrange equation of this Action are reported in appendix A.

Footnote: Allowing this generalization is tantamount to perform a conformal transformation on the original four-dimensional metric. In this way, the opening-out of the light-cones taking place through the limiting procedure no longer occur uniformly across the four-dimensional manifold.
The canonical momenta, \( \dot{f} = \frac{\partial f}{\partial \dot{x}} \) result:

\[
\begin{align*}
\pi^k &\equiv \frac{\partial L}{\partial \dot{x}^k} = m \frac{\Theta}{\Theta} [g_{k,i} \dot{x}^i + A_k] \\
\dot{\pi} &\equiv \frac{\delta L[t]}{\delta \dot{A}_i} = 0 \\
\pi_{\Theta} &\equiv \frac{\delta L[t]}{\delta \dot{\Theta}} = 0 \\
\pi_A &\equiv \frac{\delta L[t]}{\delta \dot{A}} = 0 \\
\pi^{rs} &\equiv \frac{\delta L[t]}{\delta \dot{g}_{ij}} = -\frac{\sqrt{g} \tilde{A}}{16\pi G \Theta} (g^{rk} g^{sl} - g^{rs} g^{kl}) B_{kl} .
\end{align*}
\]

Therefore, since the Lagrangian is independent of the velocities \( \dot{A}_i, \dot{\Theta} \) and \( \dot{\tilde{A}} \), we have first of all the primary constraints

\[
\begin{align*}
\pi^i &\approx 0 \\
\pi_{\Theta} &\approx 0 \\
\pi_A &\approx 0 .
\end{align*}
\]

The Dirac Hamiltonian is given by:

\[
\begin{align*}
H_c &\equiv \dot{z}^k p_k + \int d^3 z \left[ \pi^i \dot{A}_i + \pi_{\Theta} \dot{\Theta} + \pi^{ij} \dot{g}_{ij} \right] - L[t] \\
H_d &\equiv \int d^3 \Theta \left[ \frac{\tilde{A}^2}{32\pi G} \mathcal{H}_I + \frac{16\pi G}{A} \mathcal{H}_E + \left[ \frac{1}{2m} g^{ij} p_i p_j - m \tilde{A} \right] \delta^3 [z - x(t)] \right] \\
&\quad + \int d^3 z \left[ -A_i g^{ij} \phi_j + \pi^i \lambda_i + \pi_A \lambda^A + \pi_{\Theta} \lambda^\Theta \right] ,
\end{align*}
\]

where notation and algebraic properties of the quantities involved are the same of eqs. (3.6) and (3.7).

We apply now the Dirac-Bergmann procedure. By imposing time-conservation of the primary constraints, we get:

\[
\begin{align*}
\phi_i(z,t) &\equiv -g_{ij}(z,t) \dot{\pi}^j(z,t) = -g_{ij}(z,t) \{ \pi^j(z,t), H_d \} \\
&= 2 g_{ij} \nabla_k \pi^{jk} + p_i \delta^3 [z - x(t)] \approx 0 \\
\chi_\Theta(z,t) &\equiv \dot{\pi}_\Theta(z,t) = \{ \pi_\Theta(z,t), H_d \} \\
&= - \frac{\tilde{A}^2}{32\pi G} \mathcal{H}_I - \frac{16\pi G}{\tilde{A}} \mathcal{H}_E - \left[ \frac{1}{2m} g^{ij} p_i p_j - m \tilde{A} \right] \delta^3 [z - x(t)] \approx 0 \\
\chi_A(z,t) &\equiv \dot{\pi}_A(z,t) = \{ \pi_A(z,t), H_d \} \\
&= \Theta \left\{ \frac{16\pi G}{\tilde{A}^2} \mathcal{H}_E - \frac{\tilde{A}}{16\pi G} \mathcal{H}_I + m \delta^3 [z - x(t)] \right\} \approx 0 ,
\end{align*}
\]
which must be imposed as secondary constraints. In turn, time-conservation of these latter, implies that the following weak equations be satisfied on the constraint hypersurface:

\[
\begin{align*}
\hat{\phi}_i(z, t) &= \{\phi_i(z, t), H_d\} \\
&= -\chi_A \tilde{A}_i - \chi_\Theta \tilde{\Theta}_i + \phi_i g^{ik} A_{k,i} + A_i \partial_k [g^{ik} \phi_k] \simeq 0 \\
\hat{\chi}_\Theta(z, t) &= \{\chi_\Theta(z, t), H_d\} \\
&\simeq 6\pi^r \partial_r \tilde{A}_s \Theta + 3\pi^r \Theta \nabla_r \nabla_s \tilde{A} + \tilde{A} \partial_i [\nabla_i \pi^r] \\
&- 2\tilde{A} \Theta \nabla_i \pi^r \left[ \frac{\Theta_k}{\Theta} - \frac{A_k}{\tilde{A}} \right] \\
&+ \frac{pg_{jk}}{m} H_M \Theta_{,k} + \partial_k \left[ \frac{pg_{jk}}{m} - H_M \Theta \right] \simeq 0 ,
\end{align*}
\]

(3.7)

where we have defined \( H_M(z) = (\frac{1}{2m}g^{ij}p_ip_j - m\tilde{A})\delta^3[z - x(t)]. \)

Now, since \( \psi_i \equiv \dot{\phi}_i \simeq 0 \) already holds as a consequence of the primary and secondary constraints, the condition that the remaining weak equations be satisfied amounts to imposing the following tertiary constraints:

\[
\begin{align*}
\psi_\Theta(z, t) &\equiv \dot{\chi}_\Theta \simeq 0 \\
\psi_A(z, t) &\equiv \dot{\chi}_A \simeq 0 .
\end{align*}
\]

(3.8)

Let us note that the second of eqs. (3.7) determines the Dirac multipliers \( \lambda_A \) on the particle world-line. Again, by imposing time-conservation of the constraints (3.8), we finally obtain the quaternary constraints \( \xi_A(z, t) \) and \( \xi_\Theta(z, t) \) whose explicit form is reported in Appendix B (Eqs. B.1 and B.2).

Eqs. (B.1) and (B.2) allow in principle to solve for the multipliers \( \lambda_\Theta \) and \( \lambda_A \) and to close the constraints chains. Since eqs. (B.1) and (B.2) are partial differential equations for \( \lambda_\Theta \) and \( \lambda_A \) this is a non-trivial problem which could be possibly connected with the presence of residual gauge degrees of freedoms. Looking at the algebraic relations existing among all
the constraints, we see that the only first-class ones are:

\[
\left\{ \begin{array}{l}
\tilde{\phi}_i = \phi_i - \pi_A \tilde{A}_i, - \pi_\Theta \Theta_i + \pi^k A_i + \pi^k (A_{i,k} - A_{k,i}) \approx 0 \\
\pi^i \approx 0
\end{array} \right. \tag{3.9}
\]

The whole constraints chains are summarized in Fig. 2, while the complete constraints algebra is given in Appendix C.

While the Gauss constraints \( \phi_i \approx 0 \) correspond to the pure gauge nature of three degrees of freedom of \( g_{ij} \) (that have to be fixed by three coordinate conditions), the constraints \( \pi^i \approx 0 \) correspond in turn to the gauge nature of the fields \( \tilde{A}_i \).

Although a complete discussion of the role of the second class constraints in restricting the number of degrees of freedom could be carried out only in connection to the properties of the solutions of Eq.\([B.1, B.2]\) for the Dirac multipliers, it is reasonable to expect six generally second class constraints, as shown in Fig.2: they fix the fields \( \Theta(z,t), \tilde{A}(z,t) \) and one of the remaining degrees of freedom of \( g_{ij}(z,t) \) as functionals of all the other fields \([13]\). Therefore we are left with two degrees of freedom of the three-metric, as in general relativity. It is interesting to see that these degrees of freedom are "graviton-like", and that their propagation properties can be explicitly exhibited by means of a linear approximation,
in a region far from matter.

In order to show this, we choose a gauge fixing chain of the gauge variables associated to the first-class constraints in such a way that the first ones are just the "harmonic coordinate-conditions". Precisely, following [14], we start adding to the first class secondary constraints \( \phi_i \simeq 0 \), the gauge fixing constraints

\[
\Omega_k \equiv g_{kl} \Gamma^l_{rs} g^{rs} \simeq 0 .
\]  

(3.10)

Then, the condition

\[
M_k \equiv \dot{\Omega}_k \simeq 0 ,
\]  

(3.11)

provides the gauge-fixing for the first class primary constraints \( \pi^i \simeq 0 \). The fields \( A_i \) are determined by these equations. The time-derivatives of the constraints (3.11)

\[
\dot{M}_k \simeq 0 ,
\]  

(3.12)

fix in turn the multipliers \( \lambda_i \). Clearly, the local Galilei invariance is thereby broken.

The explicit calculations of the linear approximation will be worked out by starting with a weak-field approximation for the fields, based on the following Ansatz for the zeroth-order terms:

\[
\begin{aligned}
\left\{ 
\begin{array}{l}
g_{ij}(z, t) = \delta_{ij} + \epsilon h_{ij}(z, t) + O(\epsilon^2) \\
\Theta(z, t) = K_\Theta + \epsilon \Theta^{[1]}(z, t) + O(\epsilon^2) \\
\tilde{A}(z, t) = -K_A + \epsilon \tilde{A}^{[1]}(z, t) + O(\epsilon^2) \\
A_i(z, t) = 0 + \epsilon A_i^{[1]}(z, t) + O(\epsilon^2)
\end{array}
\right.
\end{aligned}
\]  

(3.13)

where \( K_A \) and \( K_\Theta \) are positive real constants. Moreover, the tensor \( h_{ij} \) will be decomposed in the usual transverse-traceless form:

\[
h_{ij} = h^{TT}_{ij} + \frac{1}{2} \left( \delta_{ij} h^T - (\Delta^{-1} h^T)_{,ij} \right) + h^L_{ij} + h^L_{ji} .
\]  

(3.14)

Given (3.13), we obtain the following weak-field expressions for the non-vanishing canonical momenta

\[
\begin{aligned}
\pi^{ij} &= \epsilon \frac{1}{32\pi G K_\Theta} [\delta^{ir} \delta^{js} - \delta^{ij} \delta^{rs}] [A^{[1]}_{r,s} + A^{[1]}_{s,r} - \dot{h}_{rs}] + O(\epsilon^2) \\
\pi_\Theta &= 0 \\
\pi_A &= 0 \\
\pi_i &= 0 .
\end{aligned}
\]  

(3.15)
From equations (3.13-3.14), we get the following expansion for the constraints:

\[
\chi_A \simeq - \frac{K_\Theta K_A}{16 \pi G} \epsilon \nabla h^T + O(\epsilon^2)
\]

\[
\chi_\Theta \simeq \frac{K_A^2}{16 \pi G} \epsilon \nabla h^T + O(\epsilon^2)
\]

\[
\psi_A \simeq 0 + O(\epsilon)
\]

\[
\psi_\Theta \simeq 0 + O(\epsilon)
\]

\[
\Omega_k \simeq \epsilon \left[ \frac{1}{2} \partial_k h^T + \nabla h^T_L \right] + O(\epsilon^2)
\]

\[
\phi_k \simeq \epsilon \frac{K_A}{16 \pi G K_\Theta} \left[ \partial_k h^T + \nabla \left( A_{k,r}^1 - A_{r,k}^1 - h^L_{k,r} + h^L_{r,k} \right) \right] + O(\epsilon^2)
\]

\[
M_k \simeq \epsilon M_k^1 \left[ K_A, K_\Theta; A_{k}^1, h^L_k, h^T \right] + O(\epsilon^2)
\]

The vanishing of the terms of order \( \epsilon \) determines the quantities \( h^T, h^L_k, A_{k}^1 \) while leaves the quantities \( h_{ij}^{TT}, \Theta^1, \tilde{A}^1 \) undetermined.

Now, let us write the multipliers \( \lambda^\Theta, \lambda^A, \lambda_i \) as power series in \( \epsilon \):

\[
\lambda^\Theta(z, t) = \lambda^0_{\Theta}(z, t) + \epsilon \lambda^1_{\Theta}(z, t) + O(\epsilon^2)
\]

\[
\lambda^A(z, t) = \lambda^0_{A}(z, t) + \epsilon \lambda^1_{A}(z, t) + O(\epsilon^2)
\]

\[
\lambda_i(z, t) = \lambda^0_i(z, t) + \epsilon \lambda^1_i(z, t) + O(\epsilon^2)
\]

(3.16)

The equations (3.11,B.1,B.1) for the Dirac multipliers become:

\[
\xi^A = 0 + \epsilon \frac{3K_A}{32 \pi G} \left[ \delta^{ir} \delta^{js} - \delta^{ij} \delta^{rs} \right] [A_{r,s}^1 + A_{s,r}^1 - \dot{h}_{rs}] \partial_r \partial_s \lambda^0_{\Theta} + O(\epsilon^2) \simeq 0
\]

\[
\xi^\Theta = 0 + \epsilon \frac{3K_A}{32 \pi G} \left[ \delta^{ir} \delta^{js} - \delta^{ij} \delta^{rs} \right] [A_{r,s}^1 + A_{s,r}^1 - \dot{h}_{rs}] \partial_r \partial_s \lambda^0_{A} + O(\epsilon^2) \simeq 0
\]

\[
\dot{M}_k = - \Delta \lambda^0_k + \epsilon \left( \frac{1}{2} K_\Theta^2 \Delta \dot{A}_k^1 - \frac{1}{2} K_\Theta K_A \Delta \Theta^1_k - \frac{1}{8} K_\Theta K_A^2 \Delta h_{k}^{TT} - \Delta \lambda_k^1 \right) + O(\epsilon^2) \simeq 0
\]

(3.17)

From these equations, it is seen that the Ansatz (3.13) is indeed consistent since Eqs.(3.16) admit the solutions \( \lambda^0_{\Theta}(z, t) = 0, \lambda^0_{A}(z, t) = 0, \lambda^0_i(z, t) = 0 \) at the zeroth order in \( \epsilon \).

Then, the Hamilton equations of motion for \( \pi^{ij} \) result:

\[
\dot{\pi}^{ij} = \{ \pi^{ij}, H_d \}
\]

\[
= \epsilon \left[ \delta^{ir} \delta^{js} - \delta^{ij} \delta^{rs} \right] \left[ - K_\Theta K_A^2 \frac{1}{2} \Delta h_{rs}^{TT} + K_\Theta K_A^2 \frac{1}{4} \Delta h_{rs}^{TT} \right. \]

\[
- K_A^2 \partial_r \partial_s \Theta^1 + 2 K_A K_\Theta \partial_r \partial_s \tilde{A}^1_k + O(\epsilon^2)
\]

(3.18)
On the other hand, from time differentiation of eqs. (3.15), one also gets
\[ \dot{\pi}^{ij} = \frac{K_A}{16\pi G K_\Theta} \left[ \lambda^{[1]}_{r,s} + \lambda^{[1]}_{s,r} - \frac{d^2}{dt^2} \left[ h^{TT}_{ij} + \frac{1}{2} \left( \delta_{ij} h^T - \Delta^{-1} h^T_{ii} \right) + h^L_{i,j} + h^L_{j,i} \right] \right]. \]  

(3.19)

Finally, by confronting (3.18) and (3.19) (collecting the expression 
\[ -\frac{K_A^2 K_{rs}}{32\pi G} \left[ \delta^{ir} \delta^{js} - \delta^{ij} \delta^{rs} \right] \]), inserting in (3.19) the expression for \( \lambda^{[1]} \) which comes out from the last equation (3.17) at the first order, and separating out the transverse traceless, the trace and longitudinal parts of the resulting equations, respectively, it follows:
\[
\begin{cases}
\frac{d^2}{dt^2} h^T = 0 + O(\epsilon) \\
\frac{d^2}{dt^2} h^L_i = 0 + O(\epsilon) \\
\frac{2}{K_A^2 K_\Theta} \frac{d^2}{dt^2} h^{TT}_{ij} = \nabla h^{TT}_{ij} + O(\epsilon).
\end{cases}
\]

(3.20)

Note that the first two equations (3.20) are compatible with the constraints for \( h^T, h^L_i \). The third one is the wave-equation for the ”graviton-like” degrees of freedom of the three-metrics which, consequently, propagate with a velocity given by:
\[ V = K_\Theta \sqrt{\frac{K_A}{2}} = \Theta^{[0]} \sqrt{\frac{\tilde{A}^{[0]}}{2}} = \sqrt{-\frac{A_0^{[0]}}{2}}. \]

(3.21)

It is seen that, under the above conditions, the potential \( A_0 \), at the lowest order, assumes, as it were, the role of a cosmological background. At this order, instead, \( \Theta^{[1]} \) and \( \tilde{A}^{[1]} \) remain undetermined. This result seems to us quite remarkable from both a conceptual and a historical point of view. It is worth recalling that Einstein, in his first attempts towards a relativistic theory of gravitation, introduced a variable speed of light playing the role of the gravitational potential (see Ref. [3]).

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Appendix A: The Euler-Lagrange equations of the eleven fields theory.

The Euler-Lagrange equations of the eleven fields theory result:

\[
\begin{aligned}
\mathcal{E}_{L_A} &= \frac{1}{16\pi G} \frac{\sqrt{g}}{\Theta^3} \left[ -AR + g^{ik}g^{jl}(B_{ij}B_{kl} - B_{ik}B_{jl}) \right] + \frac{m}{\Theta} \delta^3[z - x(t)] = 0 \\
\mathcal{E}_{L_{A_i}} &= \frac{1}{8\pi G} \left\{ \frac{1}{\Theta^3} \sqrt{g} A \left[ g^{ik}g^{jl} - g^{ij}g^{kl} \right] B_{kl} \right\}_{\theta = 0} + \frac{m}{\Theta} \delta^3[z - x(t)] \left[ \dot{x}^i + g^{ij}A_j \right] = 0 \\
\mathcal{E}_{L_\Theta} &= \frac{3}{16\pi G} \sqrt{g} \left\{ \frac{A^2}{2\Theta^4} R - \frac{A}{\Theta^4} g^{ik}g^{jl}(B_{ij}B_{kl} - B_{ik}B_{jl}) \right\} \\
&\quad - \frac{m}{\Theta^2} \left[ \frac{1}{2} g_{ij} (\dot{x}^i + g^{ik}A_k)(\dot{x}^j + g^{jl}A_l) + A \right] \delta^3[z - x(t)] = 0 \\
\mathcal{E}_{L_{g_{ij}}} &= \frac{1}{16\pi G} \left\{ \sqrt{g} (g^{ir}g^{js} - g^{ij}g^{rs}) \nabla_s \nabla_r \left[ \frac{A^2}{\Theta^3} \right] + \sqrt{g} \frac{A^2}{2\Theta^3} \left[R^{ij} - \frac{1}{2} g^{ij} R \right] \right\} + \frac{2\sqrt{g} A}{\Theta^3} \left[B^{ir}B^{js}g_{rs} - B^{ij}TrB \right] \\
&\quad + \frac{d}{dt} \left[ \sqrt{g} A \left( g^{ir}g^{js} - g^{ij}g^{rs} \right) B_{rs} \right] \\
&\quad + \frac{m}{2\Theta} (\dot{x}^i + g^{ik}A_k)(\dot{x}^j + g^{jl}A_l) \delta^3[z - x(t)] = 0 \\
\mathcal{E}_{x^i} &= \ddot{x}^i + \Gamma^i_{kl} \dot{x}^k \dot{x}^l \\
&\quad + \frac{\Theta}{\Theta} \left[ \dot{x}^i + g^{ij}A_j \right] + g^{ij} \frac{\partial g_{jl}}{\partial t} \dot{x}^l \\
&\quad + g^{ij} \left[ \frac{\partial A_0}{\partial x^i} - \frac{\partial A_j}{\partial t} \right] - g^{ij} \left[ \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i} \right] \dot{x}^l = 0 
\end{aligned}
\]

(A.1)
Appendix B: Quaternary constraints of the eleven fields theory.

The quaternary constraint of the eleven fields theory are given by:

\[ \xi_\Theta(z, t) \equiv \dot{\psi}_\Theta(z, t) = \{\psi_\Theta(z, t), H_d\} \]

\[ = \mathcal{F}^{rs} \left[ 6 \tilde{A}_r \Theta, s + 3 \Theta \nabla_r \nabla_s \tilde{A} \right] + 3 \Theta L^A \\
+ 3 \pi^{rs} \Theta \nabla_r \nabla_s \tilde{\lambda}^A + 6 \pi^{rs} \Theta \tilde{\lambda}^A, s + 4 \nabla_l \pi^{kl} \Theta \tilde{\lambda}^A, k \\
- \frac{\partial}{\partial z^k} \left[ \Theta(z) p_l g^{lk}(z) \delta^3(z - x(t)) \right] \tilde{\lambda}^A \\
+ 3 \pi^{rs} \nabla_r \nabla_s \tilde{\lambda}^A + 6 \pi^{rs} \tilde{A}_r \tilde{\lambda}^A, s \\
- 2 \nabla_l \pi^{kl} \tilde{\lambda}^2 \frac{\partial}{\partial z^k} \left[ \frac{\tilde{\lambda}^A}{A} \right] + \frac{\partial}{\partial z^k} \left[ \frac{1}{m} g^{lk} p_l \mathcal{H}_M(z) \delta^3(z - x(t)) \right] \tilde{\lambda}^A \\
+ \frac{2}{m} g^{lk} p_l \mathcal{H}_M(z) \delta^3(z - x(t)) \tilde{\lambda}^A \tilde{\lambda}_k \simeq 0 , \\
\xi_A(z, t) \equiv \dot{\psi}_A(z, t) = \{\psi_A(z, t), H_d\} \]

\[ = \mathcal{F}^{rs} \left[ 3 \Theta (R_{rs} - \frac{1}{4} g_{rs} R) - \frac{3 \Theta^2}{A^2} \tilde{A}_r \tilde{A}_s - 3 \Theta \nabla_r \nabla_s \Theta \right] \\
+ \mathcal{F}^{rs} \frac{16 \pi G \Theta^2}{m \sqrt{g} A^2} [ p_r p_s - \frac{1}{2} g_{rs} g^{ij} p_i p_j ] \delta^3(z - x(t)) \\
+ 3 G_{rs} \Theta \left[ \pi^{rs} - \frac{1}{4} \pi g^{rs} \right] - 3 \Theta L^\Theta \\
+ \frac{16 \pi G \Theta^2}{A} \frac{3}{2} R_{ij} \left[ - g^{ij} \mathcal{H}_E + \frac{1}{\sqrt{g}} (\pi \pi^{ij} - \frac{1}{2} g^{ij} \pi^2) \right] \\
- \frac{3 \pi G \Theta^2}{m \sqrt{g} A^3} \left[ \pi^{rs} p_r p_s - \frac{1}{2} \pi g^{rs} p_r p_s \right] \delta^3(z - x(t)) \tilde{\lambda}^A \\
- \frac{(16 \pi G)^2 \Theta^3}{m \sqrt{g} A^4} \left[ - \frac{1}{\sqrt{g}} \left( \pi^{rs} + \frac{1}{2} \pi^2 g^{rs} \right) + \mathcal{H}_E g^{rs} \right] p_r p_s \delta^3(z - x(t)) \tilde{\lambda}^A
\[ + \frac{2}{A^2(\pi G)} \delta^i_j [z - x(t)] (\lambda^A)^2 \]

\[ + \frac{2}{A^2(\pi G)} \mathcal{H}_M(\pi G) \delta^i_j [z - x(t)] A_i(\pi G) g^{ij}(\pi G) \lambda^A_j \]

\[-6\pi^r_s \Theta A^2 \tilde{A}_r \tilde{A}_s \lambda^\Theta + 3\pi^r_s (\pi G) \Theta (\pi G) \nabla_r \nabla_s \lambda^\Theta \]

\[ + \frac{16\pi G \Theta}{m \sqrt{g} A^2} \left[ \pi^r_s p_r p_s - \frac{1}{2} \pi g^{rs} p_r p_s \right] \delta^i_j [z - x(t)] \lambda^\Theta + \psi \lambda^\Theta \]

\[ + 2\Theta(z) \nabla_i \pi^{kl}(z) \left[ 2 \frac{\tilde{A}_k(z)}{\lambda(z)} \lambda^\Theta - \tilde{\lambda}^\Theta_k \right] \]

\[-\frac{2}{A^2} \mathcal{H}_M \delta^i_j [z - x(t)] \lambda^A \lambda^\Theta \simeq 0 , \]

where \( \pi = g_{ij} \pi^{ij} \), \( \lambda^A \equiv \lambda^A - A_i g^{ij} \tilde{A}_j \), \( \lambda^\Theta \equiv \lambda^A - A_i g^{ij} \Theta j \) and we have defined:

\[ F^{rs}(z, t) \equiv \int d^3 z' \{ \pi^{rs}(z, t), -\Theta(z', t) \chi_\Theta(z', t) \} \]

\[ = \frac{\sqrt{g}}{16\pi G} \left[ \Theta A^2 [R^{rs} - \frac{1}{2} g^{rs} R] - [g^{ri} g^{sj} - g^{rs} g^{ij}] \nabla_i \nabla_j [\Theta A^2] \right] \]

\[ + \frac{\Theta}{2m} g^{rs} p_r p_j \delta^i_j [z - x(T)] \]

\[ G_{rs}(z, t) \equiv \int d^3 z' \{ R_{rs}(z, t), -\Theta(z', t) \chi_\Theta(z', t) \} \]

\[ = (\delta^{in} g^{jn} + \delta^{in} g^{jm} - \delta^{in} g^{mj} - \delta^{mn} g^{ij}) \nabla_i \nabla_j \left[ \frac{16\pi G \Theta}{\sqrt{g} A} (\pi_{mn} - \frac{1}{2} g_{mn} \pi) \right] \]

\[-\frac{1}{4} g^{km} (\delta^{ij} R^l_{mks} + \delta^{ij} R^l_{mkr} - \delta^{ij} R^l_{ksm} - \delta^{ij} R^l_{ksr} + 4 \delta^{ij} R^l_{rsm}) \]

\[ \left[ \frac{16\pi G \Theta}{\sqrt{g} A} (\pi_{ij} - \frac{1}{2} g_{ij} \pi) \right] \]

\[ L^A(z, t) \equiv \int d^3 z' \pi^{rs}(z, t) \{ \nabla_r \nabla_s \tilde{A}(z, t), -\Theta(z', t) \chi_\Theta(z', t) \} \]

\[ = \frac{16\pi G \Theta \tilde{A}_k}{A} \left[ \Theta \frac{\tilde{A}_k}{A} - \frac{\tilde{A}_k}{A} \right] \left[ \frac{2}{\sqrt{g}} (\pi^{kl} \pi^{lj} g_{ij} - \frac{1}{2} \pi^{kl} \pi) - g^{kl} \mathcal{H}_E \right] \]

\[ + \frac{16\pi G \Theta \tilde{A}_k}{A} \left[ 2g_{lj} \pi^{ij} \nabla_i [\pi^{kl} - \frac{1}{2} g^{kl} \pi] - [\pi_{ab} - \frac{1}{2} g_{ab} \pi] g^{kj} \nabla_i \pi^{ab} \right] \]
\[ L^\Theta(z, t) \equiv \int d^3 z' \pi^{rs}(z, t) \{ \nabla_s \Theta(z, t), -\Theta(z', t) \chi_\Theta(z', t) \} \]

\[
= \frac{16\pi G}{A} \left[ \frac{\Theta_i}{\Theta} - \frac{\tilde{A}_k}{A} \right] \left[ \frac{2}{\sqrt{g}} (\pi^{ki} \pi^{lj} g_{ij} - \frac{1}{2} \pi^{kl} \pi) - g^{ik} \mathcal{H}_E \right] + \frac{16\pi G}{A} \Theta \Theta_k \left[ 2g_{ij} \pi^{ij} \nabla_i [\pi^{kl} - \frac{1}{2} g^{kl} \pi] - [\pi_{ab} - \frac{1}{2} g_{ab} \pi] g^{ki} \nabla_i \pi_{ab} \right]
\]

and \( \delta^{rs}_{ij} \) is the symmetrized expression \( \frac{1}{2} (\delta^i_j \delta^s_r + \delta^s_i \delta^r_j) \).

**Appendix C: Constraints algebra of the eleven-fields theory**

We summarize here the relevant part of the constraints algebra of the eleven fields theory:

\[ \{ \chi_A(z, t), \phi_i(z', t) \} = +\tilde{A}_i(z', t) \{ \chi_A(z, t), \pi_A(z', t) \} \]

\[ +\Theta_i(z', t) \{ \chi_A(z, t), \pi_\Theta(z', t) \} \]

\[ -\chi_A(z', t) \partial_i \delta^3[z - z'] \]

\[ \{ \chi_\Theta(z, t), \phi_i(z', t) \} = +\tilde{A}_i(z', t) \{ \chi_\Theta(z, t), \pi_A(z', t) \} \]

\[ +\Theta_i(z', t) \{ \chi_\Theta(z, t), \pi_\Theta(z', t) \} \]

\[ -\chi_\Theta(z', t) \partial_i \delta^3[z - z'] \]

\[ \{ \chi_A(z, t), \pi_A(z', t) \} = \frac{2}{A^2} \left[ \chi_\Theta \left( \frac{1}{2m} \delta^3[z - x(T)] \right) \delta^3[z - z'] \right] \]

\[ \{ \chi_\Theta(z, t), \pi_A(z', t) \} = \frac{1}{\Theta} \chi_A \delta^3[z - z'] \]

\[ \{ \chi_A(z, t), \pi_\Theta(z', t) \} = \frac{1}{\Theta} \chi_A \delta^3[z - z'] \]

\[ \{ \chi_\Theta(z, t), \pi_\Theta(z', t) \} = 0 \]

\[ \{ \psi_A(z, t), \phi_i(z', t) \} = +\tilde{A}_i(z', t) \{ \psi_A(z, t), \pi_A(z', t) \} \]
\[\begin{align*}
\{\psi_\Theta(z, t), \phi_i(z', t)\} &= +\Phi_{i}(z', t)\{\psi_{\Theta}(z, t), \pi_{\Theta}(z', t)\} \\
-\psi_A(z', t)\partial_t\delta^q[z - z']
\end{align*}\]

\[\begin{align*}
\{\psi_\Theta(z, t), \phi_i(z', t)\} &= +\tilde{A}_i(z', t)\{\psi_{\Theta}(z, t), \pi_A(z', t)\} \\
+\Phi_{i}(z', t)\{\psi_{\Theta}(z, t), \pi_{\Theta}(z', t)\} \\
-\psi_\Theta(z', t)\partial_t\delta^q[z - z']
\end{align*}\]

\[\begin{align*}
\{\chi_A(z, t), \chi_A(z', t)\} &= -2\frac{\Theta^2(z)}{A(z)} \left[ \nabla_i \pi^{kl}(z) - 3\tilde{A}_i(z)\pi^{kl}(z) \right] \partial_k \delta^q[z - z'] \\
-2\frac{\Theta^2(z')}{A(z')} \left[ \nabla_i \pi^{kl}(z') - 3\tilde{A}_i(z')\pi^{kl}(z') \right] \partial_k \delta^q[z - z']
\end{align*}\]

\[\begin{align*}
\{\chi_A(z, t), \chi_\Theta(z', t)\} &= -3\Theta \left[ \pi^{ij} R_{ij} - \frac{1}{4} \pi R \right] \delta^q[z - z'] + 6\pi^{ij} \frac{\Theta}{A^2} \tilde{A}_i \tilde{A}_j \delta^q[z - z'] \\
+3\Theta(z, t)\pi^{ij}(z, t) \left[ \partial_i \partial_j \delta^q[z - z'] - \Gamma_{ij}^k(z, t) \partial_k \delta^q[z - z'] \right] \\
-\frac{16\pi G}{m} \frac{\Theta}{\sqrt{g}A^2} \left[ \pi^{ij} - \frac{1}{2} g^{ij} \pi \right] \pi_i \pi_j \delta^q[z - z'] \\
-\Theta(z') \left[ 2\nabla_i \pi^{kl}(z') + p_i g^{k}(z') \delta^q[z' - x(t)] \delta^q[z - z'] \right] \\
-\Theta_{i}(z, t) \left[ 2\pi_{ij}(z, t) + p_j g^{ij}(z, t) \delta^q[z - x(t)] \right] \delta^q[z - z'] \\
\end{align*}\]

\[\begin{align*}
\{\chi_\Theta(z, t), \chi_\Theta(z', t)\} &= \left[ \tilde{A}(z) \nabla_i \pi^{kl}(z) - 3\tilde{A}_i(z)\pi^{kl}(z) \right] \partial_k \delta^q[z - z'] \\
+\left[ \tilde{A}(z') \nabla_i \pi^{kl}(z') - 3\tilde{A}_i(z')\pi^{kl}(z') \right] \partial_k \delta^q[z - z'] \\
-\left[ \frac{pg^{rk}(z, t)}{m} H_M (z, t) + \frac{pg^{rk}(z', t)}{m} H_M (z', t) \right] \partial_k \delta^q[z - z'] \\
\end{align*}\]

\[\begin{align*}
\{\psi_A(z, t), \pi_A(z', t)\} &= -12\pi^{kl} \frac{\Theta^2}{A^2} \tilde{A}_k \partial_t \delta^q[z - z']
\end{align*}\]
\[+12\pi^{kl}\frac{\Theta^2}{A^3}A_{k}A_{l}\delta^{q}z - z']
\]
\[-2\frac{\Theta^2}{A}\nabla_{l}\pi^{kl}\partial_{k}\delta^{q}z - z']
\]
\[+2\frac{\Theta^2}{A^2}\nabla_{l}\pi^{kl}\tilde{A}_{k}\delta^{q}z - z']
\]
\[-\frac{32\pi G\Theta^2}{m\sqrt{g}A^3}\left[\pi^{rs}p_{r}p_{s} - \frac{1}{2}\pi g^{rs}p_{r}p_{s}\right]\delta^{q}z - x(t)\delta^{q}z - z']
\]
\[+\frac{2}{A^2(z)}m\delta^{q}z - x(t)\left[\lambda A(z) - A_{i}A_{j}(z)\tilde{A}_{i}(z)\right]\delta^{q}z - z']
\]
\[+\frac{2}{A^2(z)}\mathcal{H}_{M}(z)\delta^{q}z - x(t)A_{i}(z)g^{ij}(z)\partial_{j}\delta^{q}z - z']
\]
\[\{\psi_{A}(z, t), \pi_{\Theta}(z', t)\} = \left\{3\pi^{rs}\left[R_{rs} - \frac{1}{4}g_{rs}R\right] - 3\pi^{rs}\nabla_{r}\nabla_{s}\Theta\right\}\delta^{q}z - z']
\]
\[-12\pi^{rs}\frac{\Theta}{A^2}A_{r}\tilde{A}_{s}\delta^{q}z - z']
\]
\[+3\pi^{rs}(z)\Theta(z)\partial_{r}\partial_{s}\delta^{q}z - z']
\]
\[-3\pi^{rs}(z)\Theta(z)\Gamma^{k}_{rs}(z)\partial_{k}\delta^{q}z - z']
\]
\[-\frac{32\pi G\Theta}{m\sqrt{g}A^2}\left[\pi^{rs}p_{r}p_{s} - \frac{1}{2}\pi g^{rs}p_{r}p_{s}\right]\delta^{q}z - x(t)\delta^{q}z - z']
\]
\[+2\Theta(z)\nabla_{l}\pi^{kl}(z)\left[2\frac{\tilde{A}_{k}(z)}{A(z)} - \frac{\Theta_{k}(z)}{\Theta(z)}\right]\delta^{q}z - z']
\]
\[+\Theta(z)\left[2\nabla_{l}\pi^{kl}(z) - p_{l}g^{lk}(z)\delta^{q}z - x(t)\right]\delta^{q}z - z']
\]
\[+\left[2\nabla_{l}\pi^{kl}(z) + p_{l}g^{lk}(z)\delta^{q}z - x(t)\right]\partial_{k}\Theta(z)\delta^{q}z - z']
\]
\[\{\psi_{\Theta}(z, t), \pi_{A}(z', t)\} = 3\pi^{rs}(z)\Theta(z)\left[\partial_{r}\partial_{s}\delta^{q}z - z' - \Gamma^{k}_{rs}\partial_{k}\delta^{q}z - z']\right]
\]
\[+6\pi^{rs}(z)\partial_{r}\Theta(z)\partial_{s}\delta^{q}z - z']
\]
\[+\Theta(z)\left[2\nabla_{l}\pi^{kl}(z) - p_{l}g^{lk}(z)\delta^{q}z - x(t)\right]\partial_{k}\delta^{q}z - z']
\]
\[-\frac{\partial}{\partial z^{k}}\left[\Theta(z)p_{k}g^{lk}(z)\delta^{q}z - x(t)\right]\delta^{q}z - z']
\]
\[-\left[2\nabla_{l}\pi^{kl}(z) + p_{l}g^{lk}(z)\delta^{q}z - x(t)\right]\partial_{k}\Theta(z)\delta^{q}z - z']
\]
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\[ \{ \psi_\Theta(z, t), \pi_\Theta(z', t) \} = 3 \pi^x(z) \nabla_r \nabla_s \tilde{A}(z) \delta^y[z - z'] + 6 \pi^x(z) \partial_r \tilde{A}(z) \partial_s \delta^y[z - z'] \]

\[ + 2 \nabla_l \pi^{kl}(z) \partial_k \tilde{A}(z) \delta^y[z - x(t)] - 2 \nabla_l \pi^{kl}(z) \tilde{A}(z) \partial_k \delta^y[z - x(t)] \]

\[ + \frac{\partial}{\partial z^k} \left[ \frac{1}{m} g^{lk} p_l \mathcal{H}_M(z) \delta^y[z - x(t)] \right] \delta^y[z - z'] \]

\[ + \frac{2}{m} g^{lk} p_l \mathcal{H}_M(z) \delta^y[z - x(t)] \partial_k \delta^y[z - z'] \]

\[ - \left[ 2 \nabla_l \pi^{kl}(z) + p_l g^{lk} (z) \delta^y[z - x(t)] \right] \partial_k \Theta(z) \delta^y[z - z'] \]

References

[1] R. De Pietri, L. Lusanna and M. Pauri: “Standard and Generalized Newtonian Gravity as “Gauge” Theories of the Extended Galilei Group - I. Standard Theory.” submitted for publication to Classical and Quantum Gravity.

[2] J. L. Anderson: Principles of Relativity Physics, Academic Press, London 1967.

[3] É. Cartan: “Sur les variétés à connexion affine et la théorie de la relativité generalisée (suite)”, Ann. École Norm. Sup. 40,(1923),325-412

[4] P. Havas: “Four-Dimensional Formulations of Newtonian Mechanics and their Relation to the Special and the General Theory of Relativity”, Rev. Mod. Phys. 36,(1964),938.

[5] A. Trautman, ”Theories of Space, Time and Gravitation” in Lectures on General Relativity, S. Deser and K.W. Ford, eds., Prentice-hall, Englewood Cliffs, 1965.

[6] H.P. Künzle: “Galilei and Lorentz Structures on Space-Time: Comparison of the Corresponding Geometry and Physics”, Ann. Inst. Henry Poincaré, 42,(1972),337.

[7] K. Kuchař: “Gravitation, Geometry, and Non-Relativistic Quantum Theory”, Phys. Rev. 22D,6,(1980),1285.
[8] R. Utiyama: “Invariant Theoretical Interpretation of Interactions”, Phys. Rev. 101,(1956),1597.

[9] A. Einstein: “On the Relativity Principle and the Conclusions Drawn From It”, Jahrbuch der Radioaktivität und Elektronik 4(1907),411-462, reprint in The collecting Papers of Albert Einstein, vol 2: The Swiss years: writings, 1900-1909, English transl. Anna Beck, Princeton University Press 1989; see also J. D. Norton: “General Covariance and the Foundations of General Relativity: Eight Decades of Dispute”, Rep. Prog. Phys. 56(1993),791-858.

[10] B.S. De Witt: “Quantum Theory of Gravity. I. The Canonical Theory”, Phys. Rev. 160,5,(1967),1113.

[11] L. Lusanna: “The Second Noether Theorem as the Basis of the Theory of Singular Lagrangians and Hamiltonians Constraints”, Riv. Nuovo Cimento 14,3,(1991),1.

[12] C: W.Rindler: Essential Relativity: Special, General and Cosmological, Revised Second Edition, Springer-Verlag, 1977, Chap. 9.

[13] M. Chaichian, D. Louis Martinez and L. Lusanna: “Dirac’s Constrained System: The Classification of Second-Class Constraints”, preprint Helsinki HU-TFT-93-5.

[14] R. Sugano, Y. Kagraoka and T. Kimura: “Gauge Transformations and Gauge-Fixing Condition in Constraint System”, Int. J. of Mod. Phys. A 7,(1992),61.