Yamabe solitons on 3-dimensional cosymplectic manifolds

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**Abstract:** In this paper, it has been proved that if a 3-dimensional cosymplectic manifold \(M^3\) admits a Yamabe soliton, then either \(M^3\) is locally flat or the potential field is a contact vector field. Some special potential vector fields of Yamabe solitons on 3-dimensional cosymplectic manifolds have been considered and some other results have been obtained. Also, for general \((2n + 1)\)-dimensional case, it will be shown that if an \(f -\) cosymplectic manifold \(M^{2n+1}\) admits a contact Yamabe soliton structure, then \(M^{2n+1}\) is a cosymplectic manifold. Finally, an example of Yamabe soliton on a 3-dimensional cosymplectic manifold is provided.

**Keywords:** contact geometry; cosymplectic manifold; yamabe soliton

1. Introduction

The notion of Yamabe flow was introduced by R. S. Hamilton in 1988 to study Yamabe’s conjecture stating that any metric is conformally related to a metric with constant scalar curvature. Also, Yamabe solitons serve as self-similar solutions of Yamabe flow. This notion is the subject of many types of research in the last decade. A Riemannian manifold \((M^n, g)\) is said to be a Yamabe soliton if there is a smooth vector field \(V\) and a constant \(\lambda\) such that

\[
\mathcal{L}_V g = 2(\lambda - S)g.
\]

where \(S\) is the scalar curvature, and \(\mathcal{L}\) stands for the Lie derivative operator. The soliton is shrinking, steady and expanding according to as \(\lambda < 0\), \(\lambda = 0\) and \(\lambda > 0\). If \(V = Vf\) for some real-valued

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**PUBLIC INTEREST STATEMENT**
The notion of Yamabe flow was introduced by R. S. Hamilton in 1988 in order to study Yamabe’s conjecture stating that any metric is conformally related to a metric with constant scalar curvature. In this research article, we discuss the geometry of Yamabe solitons on 3-dimensional cosymplectic manifolds. We prove that if a 3-dimensional cosymplectic manifold \(M^3\) admits a Yamabe soliton, then either \(M^3\) is locally flat or the potential field is a contact vector field. We consider some special potential vector fields of Yamabe solitons on 3-dimensional cosymplectic manifolds and obtain some other results. Also, for general \((2n + 1)\)-dimensional case, we will show that if an \(f -\) cosymplectic manifold \(M^{2n+1}\) admits a contact Yamabe soliton structure, then \(M^{2n+1}\) is a cosymplectic manifold.
smooth function $f$ on $M$, then it is called the gradient Yamabe soliton, and $f$ is called the potential function. In this case, the Equation 1) can be rewritten as follows:

$$\nabla^2 f = (\lambda - S)g.$$ 

On the other hand, the geometry of several kinds of almost contact metric manifolds is the subject of many kinds of research in the last decade. Especially, the notion of Ricci soliton as a generalization of the Einstein metric, on some classes of almost contact metric manifolds has been investigated by many authors (Ghosh, 2011; Sharma & Ghosh, 2011). For instance, it is proved that any 3-dimensional Sasakian manifold admitting a non-trivial Ricci soliton is homothetic to the standard Sasakian structure on a 3-dimensional Heisenberg group (Sharma & Ghosh, 2011). Cho proved that a Kenmotsu 3-manifold $M$ admitting a Ricci soliton with a transversal potential vector field (orthogonal to the Reeb vector field) is of constant sectional curvature $-1$. Also he has shown cosymplectic 3-manifold admitting a Ricci soliton with the Reeb potential vector field or a transversal vector field is of constant sectional curvature 0 (Cho, 2013). In (Ghosh, 2011), Ghosh proved that a 3-dimensional Kenmotsu manifold equipped with a Ricci soliton is necessarily of constant sectional curvature $-1$. Recently, one of the present authors in a joint work has studied Ricci $\rho$-soliton on 3-dimensional $\eta$-Einstein almost Kenmotsu manifolds (Azami and Fasihi-Ramandi, 2020). The results show that if an $\eta$-Einstein almost Kenmotsu manifold $M^3$ admits a $\rho$-Ricci soliton, then $M^3$ is a Kenmotsu manifold with constant sectional curvature $-1$ and the soliton is expanding with $\lambda = 2$. Ricci solitons on 3-dimensional cosymplectic manifolds have been studied by Wang (Wang, 2017), and a rigidity theorem on these types of manifolds is obtained.

In this paper, motivated by the recent work of Wang in (Wang, 2017), Yamabe solitons on 3-dimensional cosymplectic manifold $M^3$ have been considered. After preliminaries on contact and cosymplectic manifolds, it has been proved that if the cosymplectic manifold $M^3$ admits a Yamabe soliton, then either $M^3$ is locally flat or the potential field is a contact vector field. Some special potential vector fields of Yamabe solitons on 3-dimensional cosymplectic manifolds have been considered and some other results have been acquired. Also, it has been proved if an $f$-cosymplectic manifold $M^{2n+1}$ admits a contact Yamabe soliton, then $M^{2n+1}$ is a cosymplectic manifold. Finally, an example of Yamabe soliton on a 3-dimensional cosymplectic manifold will be given.

The paper is organized as follows: Section 2 concerns the cosymplectic manifolds, the structure of contact manifolds and some aspects have been described. In section 3, the main results will be given and then an example have been provided. Section 4 concerns the conclusions.

2. Cosymplectic manifolds

In this section, some basic definitions of contact manifolds, with emphasis on those aspects that will be needed in the next section, have been summarized. For more details, one can consult (Blair, 2010).

An almost contact structure on a $(2n + 1)$-dimensional smooth manifold $M$ is a triple $(\phi, \xi, \eta)$, where $\phi$ is a $(1, 1)$-type tensor field, $\xi$ is a global vector field and $\eta$ a 1-form, such that

$$\phi^2 = -\text{id} + \eta \otimes \xi, \quad \eta(\xi) = 1,$$

where, $\text{id}$ denotes the identity mapping, which implies that $\phi(\xi) = 0$, $\eta \circ \phi = 0$ and $\text{rank}(\phi) = 2n$. Generally, $\xi$ is called the characteristic vector field or the Reeb vector field.

A Riemannian metric $g$ on $M^{2n+1}$ is said to be compatible with the almost contact structure $(\phi, \xi, \eta)$ if for every $X, Y \in \mathfrak{X}(M)$, we have

$$g(\phi(X), \phi(Y)) = g(X, Y) - \eta(X)\eta(Y).$$
An almost contact structure endowed with a compatible Riemannian metric is said to be an almost contact metric structure. The fundamental 2-form $\Phi$ of an almost contact metric manifold $M^{2n+1}$ is defined by

$$\Phi(X, Y) = g(X, \phi(Y))$$

for any vector fields $X, Y$ on $M^{2n+1}$.

An almost contact metric manifold is defined as an almost cosymplectic manifold, such that $d\eta = 0$ and $d\phi = 0$. In particular, an almost cosymplectic manifold is said to be a cosymplectic manifold if $\nabla \phi = 0$. Moreover, on a cosymplectic manifold, we have the following relation:

$$\nabla \xi = 0 \quad (\iff \nabla \eta = 0),$$

which implies that $\xi$ is a Killing vector field. It follows directly that

$$R(\ldots)\xi = 0 \quad (\iff \text{Ric}(\xi) = 0),$$

where Ric stands for the Ricci operator. An almost contact structure is said to be almost $\alpha -$ Kenmotsu if $df = 0$ and $d\omega = 2\alpha \eta \wedge \omega$ for a non-zero constant $\alpha$. More generally, if the constant $\alpha$ is any real number, then an almost contact structure is said to be almost $\alpha -$ cosymplectic (Öztürk et al., 2010). Moreover, Aktan et al. (2014) generalized the real number $\alpha$ to a smooth function $f$ on $M$ and defined an almost $f -$ cosymplectic manifold, which is an almost contact metric manifold $(M, \phi, \xi, \eta, g)$ such that $d\omega = 2f \eta \wedge \omega$ and $d\eta = 0$ for a smooth function $f$ satisfying $df \wedge \eta = 0$. Also, if the almost $f -$ cosymplectic structure on $M$ is normal, we say that $M$ is an $f -$ cosymplectic manifold. Obviously, if $f$ is constant, then an $f -$ cosymplectic manifold is either cosymplectic under condition $f = 0$, or $\alpha -$ Kenmotsu ($\alpha = f \neq 0$). For a $(2n + 1)$ - dimensional $f -$ cosymplectic manifold the following identity is valid:

$$\nabla_X \xi = -f \phi^3 X \quad (4)$$

3. Main results

In this section, we present our main results and provide their proofs. The following Theorem is one of our main results in this paper.

**Theorem 3.1.** If a 3-dimensional cosymplectic manifold $M^3$ admits a Yamabe soliton, then either $M^3$ is locally flat, or the potential field is a contact vector field.

**Proof.** Let $(M^3, g, V, \lambda)$ is a Yamabe soliton on a cosymplectic manifold $M$ of dimension 3. Recall that the curvature tensor $R$ of any 3-dimensional Riemannian manifold can be written as

$$R(X, Y)Z = g(Y, Z)\text{Ric}X - g(X, Z)\text{Ric}Y + \text{Ric}(Y, Z)X$$

$$- \text{Ric}(X, Z)Y - \frac{S}{2} \{g(Y, Z)X - g(X, Z)Y\},$$

for any vector fields. If we replace both $Y$ and $Z$ by $\xi$ in the formula (3.5), using $\nabla \xi = 0$ yields

$$\text{Ric} = \frac{S}{2} \text{id} - \frac{S}{2} \eta \otimes \xi,$$

where $S$ denotes the scalar curvature. Therefore $M^3$ is an $\eta$-Einstein manifold. It follows directly from (3.6) that

$$\nabla_X \text{Ric} = \frac{1}{2} X(S)Y - \frac{1}{2} X(S)\eta(Y)\xi, \quad \forall X, Y, Z \in \mathfrak{X}(M).$$

If we contract $X$ in (3.7) and use the following well-known formula,
\[
\text{trace}(X \rightarrow (\nabla Y) \nabla X) = \frac{1}{2} Y(S).
\]

we obtain \(\xi(S)\nabla Y = 0\) for any vector field \(Y\) on \(M\), and this is equivalent to

\[
\xi(S) = 0.
\]

On the other hand, by (1.1) we obtain

\[
(\nabla Y)g)(Y, Z) = 2(\lambda - S)g(Y, Z).
\]

Taking the covariant differentiation from both sides of the above formula along an arbitrary vector field \(X\), we obtain the following equality for any vector fields \(Y\) and \(Z\) on \(M\).

\[
(V_X^\nabla)(Y, Z) = -2X(S)g(Y, Z).
\]

But we know the following formula from Yano (1970),

\[
(\nabla Y) - \nabla Y \nabla^\nabla Y - \nabla_{[Y, X]} g)(Y, Z) = -g((\nabla Y)(X, Y), Z) - g((\nabla Y)(X, Z), Y).
\]

Since \(V\) is the Levi-Civita connection of \(M\) we have \(Vg = 0\) and then the above formula becomes

\[
(V_X^\nabla)(Y, Z) = g((\nabla Y)(X, Y), Z) + g((\nabla Y)(X, Z), Y).
\]

One can easily check that the operator \((\nabla Y)\) is a symmetric tensor field of type \((1, 2)\), i.e.

\[
(\nabla Y)g)(X, Y) = (\nabla Y)g)(Y, X).
\]

In fact, this symmetry is a consequence of Jacobi identity in the Lie algebra of real smooth function on \(M\). Hence, a simple combinatorial argument shows that

\[
g((\nabla Y)(X, Y), Z) = \frac{1}{2}(V_X^\nabla)g)(Y, Z) + \frac{1}{2}(V_Y^\nabla)g)(Z, X) - \frac{1}{2}(V_Z^\nabla)g)(X, Y).
\]

Using (3.10) and (3.9) the following formula is obtained,

\[
(\nabla Y)(X, Y) = g(X, Y)V^\nabla Y - X(S)Y - Y(S)X.
\]

Taking the covariant differentiation of \((\nabla Y)(Y, Z)\) along an arbitrary vector field \(X\), we obtain

\[
(V_X^\nabla)(Y, Z) = g(Y, Z)\nabla^\nabla Y - g(Z, V_X^\nabla Y)Y - g(Y, V_X^\nabla Y)Z
\]

The following tensorial identity is well-known (see Yano, 1970),

\[
(\nabla Y)Z = (\nabla Y)Z - (\nabla Z)(\nabla Y)(X, Z).
\]

for any vector fields \(X\), \(Y\), and \(Z\). By a straightforward computation we obtain

\[
(\nabla Y)(X, Y)Z = g(Y, Z)\nabla^\nabla Y - g(Z, \nabla^\nabla Y)Y
\]

\[
- g(X, Z)\nabla^\nabla Y + g(Z, \nabla^\nabla Y)X
\]

\[
- g(Y, \nabla^\nabla Y)Z + g(X, \nabla^\nabla Y)Z
\]

for any vector fields \(X\), \(Y\), and \(Z\). Applying (2.2) and (3.8) we obtain \(\nabla^\nabla Y = 0\). Thus, using this and contracting the tensorial relation (3.11) over \(X\), then a straightforward computation shows

\[
(\nabla Y)\nabla Y)(X, Z) = g(Y, Z)\nabla^\nabla Y
\]

\[
\text{for any vector fields } X, Y, \text{ and } Z. \text{ Applying (2.2) and (3.8) we obtain }\nabla^\nabla Y = 0. \text{ Thus, using this and contracting the tensorial relation (3.11) over } X, \text{ then a straightforward computation shows}
\]

\[
(\nabla Y)\nabla Y)(X, Z) = g(Y, Z)\nabla^\nabla Y
\]

\[
\text{for any } Y, Z \in X(M). \text{ Moreover, keeping in mind that } M \text{ is an } \eta\text{-Einstein manifold, by (3.7) and a straightforward calculation we obtain that}
\]

\[
(\nabla Y)g(Y, Z) = \frac{1}{2} V(S)g(Y, Z) + \frac{5}{2} g(\nabla^\nabla Y, Z) + g(\nabla^\nabla Y, Y)
\]
\[
\frac{1}{2} V(S)\eta(Y)\eta(Z) - \frac{S}{2} \eta(\nabla_Y V)\eta(Z) - \frac{S}{2} \eta(\nabla_Z V)\eta(Y)
\]

for any vector fields \(Y, Z \in \mathcal{X}(M)\). Subtracting (3.12) from this equation gives the following equation,

\[
(2\Delta(S) - V(S) - S(\lambda - S))g(Y, Z) + V(S)\eta(Y)\eta(Z) + S\eta(\nabla_Y V)\eta(Z) + S\eta(\nabla_Z V)\eta(Y) = 0
\]

Putting \(Z = \xi\) and \(Y\) orthogonal to \(\xi\) into (3.13), gives \(Sg(\nabla_Y V, \xi) = 0\). This implies that either \(S = 0\) or \(g(\nabla_Y V, Y) = 0\). In the first case, because of (3.6) \(M^3\) is Einstein and according to (2.3), it is easily seen that \(M^3\) is locally flat. If \(g(\nabla_Y V, Y) = 0\), then putting \(f = \eta(\nabla_Y V)\) we have \(\nabla_Y V = f\xi\). On the other hand, from (1.1) we have

\[
g(\nabla_Y V, Y) + g(\nabla_Y V, X) = 2(\lambda - S)g(X, Y),
\]

for any vector field \(X, Y\). Putting, \(X = \xi\) and \(Y\) orthogonal to \(\xi\) into above equation gives \(\eta(\nabla_Y V) = 0\), which is equivalent to \(Y(\eta(V)) = 0\) for any vector field \(Y \in \{\xi\}^\perp\). Now, we can write

\[
(\nabla_Y \eta)(X) = \nabla_Y \eta(X) - \eta(\nabla_Y X - \nabla_X Y) = \eta(\nabla_X V) = \eta(\nabla_Y V) = f\eta(X)
\]

for any vector field \(X\), where we have applied the fact that \(\eta(V)\) is invariant along only \(Y \in \{\xi\}^\perp\). In this context, we say that the potential vector filed \(V\) is a contact vector field, that is, \(\nabla_Y V = f\eta\). This completes the proof.

The above theorem has been proved about Ricci soliton on 3- dimensional cosymplectic manifolds by Wang (2017). Now we get the following result:

**Corollary 3.2.** If a 3-dimensional cosymplectic manifold admits a Yamabe soliton with \(\xi\) the potential vector field or a unit transversal potential vector field \(\omega(\perp \xi)\) then the potential vector field is Killing.

**Proof.** Suppose that a 3-dimensional cosymplectic manifold \(M^3\) admits a Yamabe soliton with the potential vector field \(V = \xi\). It is obviously Killing vector field. When \(V\) is orthogonal to \(\xi\) and of unit length, from above theorem, we have \(g(\nabla_Y V, \xi) = 0\). On the other hand, putting \(X = \xi\) and \(Y = \xi\) into Equation 14) gives

\[
g(\nabla_Y V, \xi) = (\lambda - S)g(\xi, \xi)
\]

Consequently, \(\lambda - S = 0\), which completes the proof.

Note that the above corollary when a 3-dimensional cosymplectic manifold \(M^3\) admits a Ricci soliton, reduced to this fact that the soliton is steady (Cho, 2013). Yano has proved that if the holonomy group of a Riemannian \(n\)-manifold leaves a point invariant, then there exists a vector field \(V\) on \(M\) which satisfies

\[
\nabla_Y V = Y,
\]

for any vector \(Y\) tangent to \(M\) (Yano & Chen, 1971). The vector field \(X\) mentioned above is called concurrent vector field. We consider the concurrent vector field \(V\) and obtain the following corollary:

**Corollary 3.3.** If a 33-dimensional cosymplectic manifold admits a Yamabe soliton with concurrent potential vector field \(V\), then either the soliton is expanding with \(\lambda = 1\) or \(V\) is an infinitesimal contact transformation.
Proof. By (3.14) we can write
\[2g(X, Y) = 2(\lambda - S)g(X, Y),\]
for any vector field \(X\) and \(Y\). Hence, \(\lambda - S = 1\). If \(S = 0\), then \(\lambda = 1\) and consequently, the soliton is expanding. Otherwise,
\[(\mathcal{L}_Vf)(X) = \nabla_Vf(X) - \eta(\nabla_XV - \nabla_Xf) = \eta(\nabla_XV) = \eta(X),\]
for any vector field \(X\). In this context, we say that the potential vector field \(V\) is an infinitesimal contact transformation (see (Tanno, 1962)), that is, \(\mathcal{L}_Vf = \eta\).

Finally, we consider \(f\) - cosymplectic manifold \(M^{2n+1}\) and get the following theorem:

**Theorem 3.4.** If an \(f\) - cosymplectic manifold \(M^{2n+1}\) admits a contact Yamabe soliton, then \(M^{2n+1}\) is cosymplectic manifold.

**Proof.** In view of (2.4), we have
\[(\mathcal{L}_f g)(X, Y) = 2f[g(X, Y) - \eta(X)\eta(Y)]\]

Therefore it implies from the Yamabe soliton (1.1) with \(V = \xi\) that
\[(\lambda - S)g(X, Y) = fg(X, Y) - f\eta(X)\eta(Y),\]

therefore for any vector field \(X\) on \(M\), we have
\[\eta(X)\xi = f\eta(X)\xi.\]

By taking \(X = \xi\) and using (3.15), we get
\[\eta(\xi)\xi = f\eta(\xi)\xi,\]

thus \((S - \lambda)\xi = 0\). It follows that \(S = \lambda\). Consequently, \(\mathcal{L}_f g = 0\) and \(\xi\) is Killing vector field.

It is well known that any Euclidean space of odd dimension admits a cosymplectic structure. In the following example, we construct a gradient Yamabe soliton on 3-dimensional cosymplectic manifold (for more details see (Wei & Wylie, 2009): example 2.1).

**Example 3.5.** Let \((\mathbb{R}^3, g_0)\) be the Euclidean 3-space equipped with the usual flat metric \(g_0\). Set \(V = \nabla f\), where the potential function \(f = 2|x|^2\) for some non-zero constant \(\lambda\). One can easily check that \(\text{Hess} f = \lambda g_0\), that is \((\mathbb{R}^3, g_0, f, \lambda)\), is a gradient Yamabe soliton.

**4. Conclusions**
In this paper, we considered Yamabe solitons on 3-dimensional cosymplectic manifold \(M^3\). We proved that if a cosymplectic manifold \(M^3\) admits a Yamabe soliton structure, then either \(M^3\) is locally flat or the potential vector field is a contact vector field. Also, we considered Yamabe soliton structure on cosymplectic manifold with concurrent potential field and obtained some interesting results. Moreover, we proved that if on \(f\)-cosymplectic manifold \(M^{2n+1}\) admits a contact Yamabe soliton, then \(M^{2n+1}\) is cosymplectic manifold. In this paper, we concentrate on Yamabe solitons on 3-dimensional manifolds. Studying this structure for general dimension manifold could be the topic of another article.
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