Students’ mathematics representation ability from picture form to equation

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Abstract. Conceptual thought is the basic thought in mathematics learning. Process of learning mathematics involves a representation interpretation from a representation form to others. The aim of this research was to analysis the representation process from picture form to mathematics equation. Moreover, this research was descriptive qualitative research. The data were gotten by using test and interview towards 34 students with heterogeneous ability in Surakarta. There were 4 stages in this research, they were unpacking the source, preliminary coordination, constructing the target and determining equivalency. This process involved some other representations such as symbol, schema, verbal, and numeric. Furthermore, the research result showed that the students still had not been able to do the representation process from picture form to equation well. Interpretation process started from picture to equation through other representation mediators like symbols, words and so on. Every student had various ways to do his or her representation. The problem about students’ low mathematics representation could be finished by using a learning medium-based RME. Besides, the suggestions for further research is to do deeper research related to students’ characteristics and to do the opposite research that is an analysis of the presentation process from the equation to picture form.

1. Introduction

For the advancement of a nation, education is an important thing. When education quality develops and is qualified, it can be said that the nation develops. One of the ways to develop education is by doing a learning activity. In Indonesia, mathematics learning is one of the priorities. However, that is the opposite of achievement. Based on a survey from Program for International Student Assessment (PISA), it is informed that students’ mathematics ability was still low, so Indonesia was in rate 62 among 70 countries taking part [1]. The students’ lowest national examination mean score in Surakarta was mathematics. Based on the result of observation and from the information gotten from the teacher at school, it is shown that students’ mathematics representation was still low. Seeing this lowness and the importance of mathematics in daily life, mastering mathematics side is a must [2]. There are some mathematics abilities, and one of them is mathematics representation ability which must be had by the students. Moreover, the purpose of learning mathematics is to develop some abilities such as: (1) mathematics communication, (2) mathematics logic, (3) mathematics solving, (4) mathematics connection, and (5) mathematics representation [3].

Representation is defined as a configuration of character, picture, concrete object and so on which can symbolize or represent another thing [4]. There are many experts dividing mathematics representation into some types such as graphic, picture, diagram, verbal, manipulative form, and symbol.
Furthermore, representation also can be seen as the way or form in which it explains, symbolizes, and relates to mathematics concept.

In learning mathematics, the use of mathematics representation enables students to concrete some concepts with the purpose of decreasing the students’ learning difficulty, so mathematics will be more interactive, interesting and can facilitate the students in connecting their cognition ability in mathematics representation[8].

In education, mathematics representation is divided into 5 parts: representation of real-world object, concrete, arithmetic symbol, verbal, and a picture or graphics [9]. Various researches about representation have been conducted such as the one related to symbol or number interpretation in mathematics [10], verbal representation [11], and others. Nevertheless, there have not been the studies about representation from a picture to mathematics equation representation.

Therefore, based on the problems related to the lowness of the students’ mathematics achievement, mathematics representation and inexistence of the study specified on mathematics representation process from picture to mathematics equation, the researcher was interested in conducting the research which focused on the students’ ability in expressing their mathematics ideas or thoughts into picture, diagram, or graphic which was interpreted into mathematics equation or expression.

2. Method
This research was descriptive qualitative research [12]. The method used was by analysing the students’ mathematics representation ability from a picture or graphics to mathematics equation form. The number of the subject was 34 students of grade eight at senior high school in Surakarta with heterogeneous ability. The subject sampling was by using purposive sampling technique. In addition, the data source was from an interview and test [13]. Mathematics representation data were conveyed through an exercise sheet in essay form. Furthermore, the topic was about Pythagoras. Generally, an interpretation activity passed 4 stages, they were: unpacking the source, preliminary coordination, constructing the target and determining equivalency [14].

3. Result and discussion
In this research, the students were given item in the form of essay with Pythagoras material with mathematics problem picture form. After doing testing towards the subject, the data were obtained that most students still had low mathematics representation ability. It could be seen from many students who not pass the score target as many as 94.1% with meant 52.451. It indicates that the students still faced difficulty in learning mathematics especially about mathematics representation. After that, the students’ difficulties were analysed by taking some students’ answer samples as follows.

Subject 1

Figure 1. The answer of subject 1.
The first activity done by S1 was unpacking the source. In this activity, S1 did it by paying attention to every keyword well. It could be seen when S1 explained by giving sign ‘?’ at the questioned variable. Here is the answer of S1 related to the process of unpacking the source.

\[ P: \text{From the problem in the question, what information do you catch?} \]
\[ S1: \text{Picture in the item is triangle picture with } p \text{ equals to 3 and } q \text{ equals to 9.} \]
\[ P: \text{What easiness do you get more?} \]
\[ S1: \text{Aha! What is looked for is a value.} \]
\[ P: \text{In your opinion, is the information in the item enough or not to answer the question?} \]
\[ S1: \text{Enough, Sir.} \]

The interview result shows that S1 could know the points of information in the questions given. It indicates that S1 did unpacking the source well enough.

When S1 understood well the matter given, then he related it a process of preliminary coordination. In this process, S1 tried to find the appropriate formula by assuming that the matter could be solved with Pythagoras formula, but he had the wrong assumption that in using it. He thought that value in the item could be solved with \( a^2 = q^2 - p^2 \) formula without looking for the value of the triangle direction. Here is the answer to S1 related to the process of preliminary coordination.

\[ P: \text{I see from your answer you used Pythagoras formula to answer the question given, did you?} \]
\[ S1: \text{Yes, Sir. I thought that formula was the solution to do the item.} \]
\[ P: \text{In your opinion, from the available data, how is the implementation in Pythagoras formula?} \]
\[ S1: \text{In my opinion, it was just used the available value in the item, so it could be found with } a^2 = q^2 - p^2 \text{ formula.} \]

The interview result showed that S1 chose the formula correctly to finish the matter, but there happened incorrect understanding related to the implementation of the formula. It caused the incorrectness of the result gotten. It indicates that S1 did preliminary coordination incorrectly.

The next step was constructing the target. In this process, S1 applied some thoughts into problems given in order to get a solution and answer. S1 inputted value 3 into variable \( p \) and value 9 into variable \( q \) and it was gotten a value 3\( \sqrt{8} \). Then, because in preliminary coordination S1 did a mistake, in this step S1 got incorrect result although in formula application of counting had been correct. Here is the answer to S1 related to constructing the target.

\[ P: \text{When you had known the appropriate formula from the problem, what did you do then?} \]
\[ S1: \text{The next step was I inputted the available value into Pythagoras formula to find a value which value was 9 and } p \text{ was 3. Then I counted and got the result } 3\sqrt{8}. \]

The interview result showed that S1 did construct the target well, but because the previous step was incorrect, the process result was incorrect too.

The last step was determining equivalency. In this stage, S1 checked again the steps that he had done before. S1 counted again the answer that was gotten whether there was a mistake or not. However, he did not do it well. He did not find the mistake he did. Here is the answer to S1 related to the process of determining equivalency.

\[ P: \text{Are you sure that your answer has been already correct?} \]
\[ S1: \text{I am sure, Sir.} \]
\[ P: \text{Have you checked again your answer?} \]
\[ S1: \text{Yes, I have.} \]
\[ P: \text{Please, explain how you checked again your answer!} \]
\[ S1: \text{I recounted my answer with } a^2 = 9^2 - 3^2 \text{ formula and the result was still } 3\sqrt{8}. \]
The interview result shows that S1 did not do determining equivalency process well. He only checked the measurement from the formula he believed. Nonetheless, the formula itself was incorrect. Based on the process done, S1 still had not been able to represent a mathematics matter into picture form which was changed to mathematics equation form well. It is shown that S1 did some mistakes in the process which caused an incorrect result.

**Figure 2.** The answer of subject 2.

The first activity done by S2 was **unpacking the source**. In this activity, S2 did it by paying attention to every keyword well. It can be seen when S2 explained with the way in giving variable R. From this activity, S2 also did symbol representation. Besides, S2 also drew again a triangle to finish the matter he found. Here is the answer of S2 related to the unpacking the sourcing process.

\[ P: \text{From the problem in the question, what information did you catch?} \]

\[ S2: \text{The point was I was asked to find a value in which } q \text{ equals to 9 and } p \text{ equals to 3, and the model is a triangle.} \]

\[ P: \text{In your opinion, is the information the item enough or not to answer the question?} \]

\[ S2: \text{Yes, enough.} \]

The interview result shows that S2 had been already known the points of information in the questions given. It indicates that S2 did unpacking the source activity well. S2 had understood about the matter given. Then, he related it to the **preliminary coordination** process. In this process, S2 tried to look for the appropriate formula by assuming that the matter could be solved with Pythagoras and triangle area formula. Here is the answer of S2 related to preliminary coordination process.

\[ P: \text{I see from your answer you drew 2 triangles, did you? Why did you do that?} \]

\[ S2: \text{Yes, Sir. I drew 2 triangle in which the first was to find the base value with Pythagoras, and the second one was to find a value with a triangle area formula.} \]

The interview result shows that S2 chose the formula well to solve the matter given. The next step was **constructing the target**. In this process, S2 applied his thought into the matter given in order to get the solution and answer. In this process, firstly S2 tried to find the triangle base with Pythagoras formula. He also added new variable R as the triangle base variable. The formula of Pythagoras that S2 used was \[ R^2 = 9^2 - 3^2 \], so it was gotten that \( R \)-value was \( 3\sqrt{8} \). In this step, S2 did the process well. However, in the step of finding a value by using triangle area the formula, S2 did mistake in counting part. The way had to be done by S2 is as follows.
\begin{align*}
\text{Area of } \Delta &= \frac{1}{2} \times \text{base} \times \text{height} \\
\frac{1}{2} \times 3 \times 9 &= \frac{1}{2} \times 3\sqrt{8} \times a \\
a &= \frac{3 \times 9}{3\sqrt{8}}
\end{align*}

It caused the final result which was incorrect. Here is the interview result from S2 related to constructing the target process.

\begin{quote}
P: When you had known the appropriate formula from the matter, what did you do then? \\
S2: What I did was inputting the available value into Pythagoras formula to find the triangle base signed by variable R, in which value \( q = 9 \), \( p = 3 \) and it was gotten \( R = 3\sqrt{8} \). Then I looked for value with triangle area formula and it was gotten \( a = 9\sqrt{10} \).
\end{quote}

\begin{quote}
P: Are you sure that triangle is the formula you used correctly in finding a value? \\
S2: If triangle area formula I have understood, Sir. However, actually I am rather confused in applying it which is to be the base and the height because there were 2 base values, those were \( R \) and could be 3 also.
\end{quote}

The interview result shows that S2 did a mistake in constructing the target process. The student was still confused in applying the concept of triangle formula in explaining stage.

In the process of determining an equivalency, S2 rechecked the steps he had done. He recounted the answer he got whether the mistake happened or not. However, in this stage, S2 did not do it well. He did not find the mistake he did. Here is the answer of S2 related to the determining equivalency process.

\begin{quote}
P: Are you sure with your answer? \\
S2: InsyaaAllah (God Will).
\end{quote}

\begin{quote}
P: Have you rechecked your answer? \\
S2: Already.
\end{quote}

\begin{quote}
P: Please, explain how you recheck your answer? \\
S2: I rechecked by paying attention to the item again and the formula I used whether had been correct or not and the counting, I checked again.
\end{quote}

The interview result shows that S2 did not do the determining equivalency well. Although he had checked again the overall, he did not find his own mistake in the counting process.

Based on the process done, S2 had not been able to represent a mathematics matter into picture form which was changed to mathematics equation well. It is shown that S2 did some mistakes in the process which caused an incorrect result.

Subject 3

![Figure 3](image)

**Figure 3.** The answer of subject 3.
In the process of unpacking the source, S3 did better than S1 and S2. It could be seen when the S3 was able to examine each point of the question and also its analysis in describing the answers in a complex manner without redrawing the questions, the activity had explained that S3 had mastered the problem well. Here are the answers from S3 regarding the process of source unpacking process.

**P:** From the problems in the question, what information do you get?

**S3:** The image on the question is an image of a right triangle where the length of the side is \( p = 3 \) and \( q = 9 \). From the problem, it is told to find the value of \( a \), but there are several steps before looking for the score of \( a \).

**P:** Do you think the information that given is enough for you in answering the question?

**S3:** It is really enough.

The result of the interview indicates that the S3 could recognize the points of information on the questions moreover S3 was able to describe it by convincing the steps which would be done. This shows that S3 did the process of unpacking the source well.

The next process is preliminary coordination. In this process, S3 tried to find the right formula and step by assuming that the problem can be solved by the formula of Pythagoras and the formula of the area of a triangle by doing several other steps. The following are the answers of S3 related to the preliminary coordination process.

**P:** You immediately wrote the formula without needing to draw it? Why did you do it?

**S3:** Right, Mr., because I already have an idea of what the solution will be. So I immediately go to the core of the answer.

**P:** If you have already understood the solution, please try to tell it.

**S3:** First, I look for a triangular base, I say "c", and then I look for the score of with a formula in finding the area of a triangle.

The interview result shows that S3 was able to choose several solutions well and also had an idea of what the final result would be.

The next step is constructing the target, in this process, S3 applied several completion formulas that have been believed by S3 to get a score. S3 started by finding the base symbolized of (c) using the Pythagoras formula and then looking for the score of \( a \) with the triangle area formula. Unfortunately, S3 did not simplify the base value (c) efficiently, which is 9.48. It should be \( 3\sqrt{8} \) to get an easier calculation process in the next step. However, this is not a mistake however that the efficiency of processing time was not too good. Here is an interview excerpt of S3 regarding the process of constructing the target:

**P:** When you have known the exact formula for the problem, what will you do next?

**S3:** I input the existing score into the Pythagoras formula to find the base of the triangle marked with the \( R \) variable, where the score \( q = a = 9, p = b = 3 \) and obtained \( c = 9.48 \). Then I found the score of \( a \) with the broad formula of the triangle and obtained \( a = 2.84 \).

**P:** You are looking for the \( c \) score until you get a score without root. Do you have trouble getting that score?

**S3:** I got a score of 9.48 from other paper scraps. \( \sqrt{90} \) is not a specific type of root that without being explained we will know it and I am worried if there are still roots, it will make me difficult for the next process. But on the contrary, looking for the rootless score is more difficult.

The results of the interview show that S3 did not do errors in the process, but S3 found the difficulties to find the score without roots. In calculations, S3 could answer the problem very well.

In the process of determining equivalency, S3 re-examined the steps that have been carried out. S3 corrected the answers that have been obtained whether an error occurred or not. In this process S3 did it well, as a result, the correct answer is obtained.
P: Are you sure with your answer?
S3: Yes sir.
P: Have you reviewed your answer?
S2: Yes, I have
P: Please explain how did you check your answer?
S3: I checked it by looking at the problem and the important points on the question, also checked the formulas, calculations, and steps that I did whether it was correct or not.

The results of the interview show that S3 did the process of determining equivalency very well. Based on the process that has been done, S3 had been able to represent a mathematical problem in the form of images that are converted into mathematical equations. This is indicated by S3 in answering the problems given by formulas, calculations and, good steps.

From several Subject analysis 1-3, it can be concluded that the ability of students' mathematical representation is still relatively low. Students still have difficulty in representing a mathematical problem in the form of images to form of mathematical equations. This is suitable with research that states that "students' mathematical representation is still relatively low" [15].

Based on translation activities, the subject has differences in each translation process. The translation process has different complexities depending on its activity [9]. This is in line with the opinions of Rahmawati D, Purwanto, Subanji, and Erri Hidayanto who argue that "Intermediate representation used by each subject varies, so the level of complexity of each translation process depends on the activity carried out" [16].

It can be seen that it is very important for the teacher to know the activities of students’ mathematical translation. In this case, the image representation to represent mathematical equations. Teachers who have a capability in the mastery of understanding representational abilities, teachers are expected to be able to improve students' abilities in the translation process will be better. Problems related to the low mathematical representation of students can be dealt by using an RME-based learning media in the learning process, or other media that has the characteristics in using context data, building student knowledge, using mathematical processes, and the existence of integrated interactivity and learning [17].

4. Conclusion
Based on this study, it was concluded that students had not been able to make the translation process from image representation to the same equation. The translation process stage is divided into four stages: unpacking the source, preliminary coordination, constructing the target and determining equivalency. The translation process starts from the image to the equation through other intermediary representations such as symbol representation, words and, others. Each student has a variety of ways of doing his representation because the level of difficulty of each student's translation process varies depending on the activity that he/she is doing. Problems related to the low mathematical representation of students can be dealt with using an RME-based learning media in the learning process. This research is only limited to the representation of images that are converted to the equations. The reverse process, the equation for the image is not visible.

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