A Thermo-elasto-plastic Evaluation of Stress State of Mars

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Abstract. There is recently an increasing interest to explore the ways to find out the characteristics of other planets and possibility of exploiting their mineral resources. The images sent to Earth from Mars exploration rovers showed that rocks and geological and tectonic structures are strikingly similar to those in Earth. In this study, the stress state of Mars is investigated using a spherical symmetric body concept together with the consideration of a thermo-elasto-plastic behaviour of the Earth's constituting materials on the basis of the previous investigation of the stress state of Earth by Aydan [1,2,3]. Fundamentally four different situations, namely, fluid, elastic, elasto-plastic and thermo-elasto-plastic have been considered and the computational results are presented and their implications are discussed.

1. Introduction

Mankind is now exploring the ways to find out the characteristics of other planets and possibility of exploiting their mineral resources. One of most impressive images from the Apollo Program of NASA to the author was the man standing next to a fractured lunar rock mass. The images from recent Mars exploration rovers showed the striking similarities between rocks on Earth and those of Mars, which motivated the author to bring together some of his thoughts about the aspects of rock mechanics and rock engineering in Mars and to compare them with those of the earth [4,5,6,7].

Although the environmental conditions on Mars and other planets are different from those on Earth, the principles governing mechanical and engineering aspects of rocks on other planets should be quite similar to those developed for rocks of Earth. Therefore, the next generations of our discipline would definitely see its extension to the rocks of other planets. The establishment of the Commission on Planetary Rock Mechanics by the International Society for Rock Mechanics and Rock Engineering (ISRM) is one of the first steps for such a goal.

Aydan [1] investigated the stress state of Earth using different concepts and these studies are summarized in the publication in Aydan [3] together with other methods of stress measurement and inferences from geologic and tectonic features. Early investigations by Aydan (1993) clearly showed that Earth itself cannot be an elastic body as commonly assumed in geophysical models. On the basis of these earlier findings, Aydan [2] investigated the stress state of the Earth using a spherical symmetric body concept together with the consideration of a thermo-elasto-plastic behaviour of the Earth's constituting materials. In this study, the same computational method and constitutive models have been used to investigate the possible stress state in Mars. Fundamentally the material constituting Mars are assumed to behave according to four different constitutive models, namely, fluid, elastic, elasto-plastic and thermo-elasto-plastic with some knowledge from those of Earth and computations have been carried out. These computational results are presented and their implications are discussed.
2. Main Characteristics and Interior Structure of Mars

The radius of Mars is 3389 km, which almost the half of the radius of Earth and its gravitational acceleration at ground surface is 0.377g of Earth [8]. The mean density of Mars is about 3.9335 g/cm³ on the basis of the law of the universal gravitation law of Newton. Mars has a very thin atmosphere consisting of carbon dioxides, nitrogen, argon gases and it is a dusty, cold, desert environment. The rotation axis is tilted by 25 degrees with respect to the plane orbit around Sun and the year of Mars lasts about 687 Earth days and 669.6 solar days (abbreviated as sols). Furthermore, its rotation about its axis takes about 24.6 hours.

Many models for the inner structure and temperature distribution of Mars have been proposed by various researchers [9,10,11]. Particularly, Sohl and Spohn [9] utilized two meteorites came from Mars for estimating the internal structure of the Mars. All models are based on the concepts developed for Earth. Nevertheless, they are fundamentally estimations as there is no seismic network to explore Mars yet. NASA has deployed a three-component seismometer through Viking program in 1976 [12,13,14]. The second seismometer was installed as a part of INSIGHT instrument, which has been equipped with many sensors, on Mars in 2018. Viking seismometer recorded numerous vibrations during its operation. The InSight seismometer recorded the first marsquake in April, 2019. Nevertheless, the models for the inner structure of Mars still need some validation.

The crustal thickness of Mars is estimated to be 10-50 km. The mantle is divided mainly into two layers, which may be called upper and lower mantles. The core and mantle interface are estimated to be at a depth of 2000 km with a 150 km transition zone. Mars is estimated to be chemically composed of a silicate crust, Fe-Mg silicates mantle and metallic Fe core (figure 1). The average density is about 3.933 g/cm³. The gravitational acceleration is 0.377g. Figure 1 shows a plot of the estimated density, wave velocities and gravitational acceleration of Mars.

3. Rocks of Mars

Surface geology of Mars mapped by USGS and Scott and Carr [15] and MOLA data of NASA. The surface geology fundamentally involves volcanics, basalt, breccia, sedimentary deposits. Tectonics of Mars is not much active as compared to that of Earth. Nevertheless, very large scale Vallis rift zone, Tharsis volcanic chain, depression zones, fracture zones, faults, folding, metamorphism, discordant sedimentation and columnar jointing are found in Mars.

It is very likely that the rocks of Mars would be quite close to those of Earth. Therefore, the classification of rocks should be igneous, metamorphic and sedimentary on the basis of the images from many Mars explorers of NASA [13,14]. The images and drilling operations performed by Mars explorers (Opportunity, Sprit, Curiosity) on some selected rocks and rock blocks indicated that rocks are quite similar to those of the earth. Although it still needs some clarifications whether rocks seen in Earth fully exist on Mars, the sedimentary rocks (such as conglomerate, sandstone, siltstone, mudstone, sulphate...
deposits), metamorphic rocks such as phyllite, shale and serpentine and extrusive rocks (basalt, andesite-like (silicate bearing basalt), breccia) of Mars clearly similar to those of Earth. Furthermore, flow planes, bedding planes and schistosity planes, which are intrinsic to each rock class, are distinctly observed. In addition, some discordant sedimentation of sandstone layers was also distinctly observed.

4. Possible elasto-plastic constitutive models for rocks subjected to high pressure and temperature regimes

Many experiments on rocks under high pressure indicated that rocks exhibit elastic-strain hardening behavior. For example, Hirth and Tullis [16] reported the results of triaxial experiments on quartz aggregates under very high confining pressures, which is almost 6 times its uniaxial compressive strength. There are many yield criteria for the initiation and evolution of plastic behaviour of rocks. Figure 2 shows comparison of various yield criteria and the best fit to the experimental results is obtained for Aydan’s criterion [2] as seen in figure 2. Up to a confining pressure of 1000 MPa, the estimation by Mohr-Coulomb criterion is better than that by Hoek-Brown criterion [17]. Very high discrepancy is observed among the estimations by Hoek-Brown criterion and experimental results.

![Figure 2](image)

Figure 2. Comparisons of yield criteria for experimental results on quartz (from Aydan et al. [18], Aydan [4]).

As it is well-known the strength of rock decreases with increase in temperature. In geomechanics, there is almost no yield (failure) criterion incorporating effect of temperature on yield (failure) properties of rocks although there are some experimental researches (Hirth and Tullis [16]). Aydan’s criterion is the only criterion known incorporating the temperature and it was used to study the stress state of the earth (Aydan [2]). Figure 3 shows the experimental results for three different values of ambient temperature reported by Hirth and Tullis [16] while figure 4 shows the reduction of strength with temperature for a given confining pressure of 1.17-1.2 GPa.

Aydan’s criterion is the only yield criterion, which takes into account the effect of temperature on strength of rocks. Aydan’s yield function for thermo-plasticity yielding of rock is given by

$$\sigma_1 = \sigma_3 + [S_\infty - (S_\infty - \sigma_\epsilon)e^{-b_1\sigma_1}]e^{-b_2T}$$

(1)
Where $S_u$ is ultimate deviatoric strength while coefficients $b_1, b_2$ are empirical constants, $\sigma_1, \sigma_2, T$ are maximum, minimum principal stresses and temperature, respectively. Aydan’s yield (failure) criterion is applied to experimental results shown in figure 3 and results are shown in figure 4.

As rocks of Mars show close similarity to those of Earth, the constitutive laws and experimental results can be utilized, provided that environmental conditions are similar. In this study, the yield criterion, which is a function of principal stresses and temperature shown in figure 4 will be utilized for evaluating the stress state of Mars.

5. Thermo-elasto-plastic finite element method for analysing the stress state

As assumed by Aydan [2] to study the stress state of Earth, Mars is also assumed to be a spherical symmetric body so that the governing equilibrium equation is given by

$$\frac{\partial \sigma_r}{\partial r} + 2 \frac{\sigma_r - \sigma_0}{r} = \rho g$$  \hspace{1cm} (2)
where $\rho$ and $g$ are density and gravitational acceleration which may vary with depth. The constitutive law for elastic behaviour is written as:

$$
\begin{bmatrix}
\sigma_r \\
\sigma_\theta \\
\sigma_z
\end{bmatrix} = 
\begin{bmatrix}
\lambda + 2\mu & 2\lambda & 2\lambda \\
\lambda & 2(\lambda + \mu) & 2\lambda \\
\lambda & 2\lambda & \lambda + 2\mu
\end{bmatrix}
\begin{bmatrix}
\varepsilon_r \\
\varepsilon_\theta \\
\varepsilon_z
\end{bmatrix}
$$

or $\{\sigma\} = [D]\{\varepsilon\}$ (3)

It should be noted that Lame’s constants in the above equation can vary with depth. Using the general procedure of finite element discretization and taking variations on $\partial u$, with the use of Gauss divergence theorem and the following conditions

$$
\partial u \text{ on } \Gamma_u \text{ and } \hat{t} \text{ on } \Gamma_t
$$

the weak form of Eq. (2) takes the following form:

$$
\int_\Omega (\sigma, \partial \vec{e}_r + \sigma_\theta \partial \vec{e}_\theta) \, d\Omega = \int_\Omega \rho g \delta \vec{u} \, d\Omega
$$

Where $d\Omega = 4\pi r^2 \, dr$

Let us assume that the displacement in a given element is approximated by the following expression:

$$
u = [N]\{U\}
$$

where $[N]$ is shape function and $\{U\}$ is nodal displacement vector. Using the above approximate form and the constitutive law (3), the following finite element form is obtained for a typical finite element

$$
[K]\{U\} = \{F\}
$$

where

$$
[K] = \int_{\Omega_\delta} [B]^T [D] [B] \, d\Omega; \quad \{F\} = \int_{\Omega_\delta} \rho g [N]^T \, d\Omega
$$

If a linear type shape function is chosen as given below:

In analyses, it was assumed that the stress-strain response of rocks constituting the earth exhibit an elastic-perfectly plastic behaviour. The elastic constants of rocks were computed from the distributions of density, $p$ and $s$ wave velocities shown in figure 1. The initial stiffness technique was chosen as the iteration technique to deal with the plastic behaviour of rocks in finite element analyses (Owen and Hinton [19]). The temperature distribution of the earth was input as known by using a distribution proposed by Zharkov and Gudova [20].

### 6. Computational Results and Discussions

A series of case studies given below were carried:

**CASE 1:** Mars was in liquid state (hydrostatic)

**CASE 2:** the crust and mantle were elastic solids and the core was in liquid state (elastic)

**CASE 3:** the crust and mantle were elasto-plastic solids and the core was in liquid state (isothermic: elasto-plastic)

**CASE 4:** the crust and mantle were thermo-elasto-plastic solids and the core was in liquid state (non-isothermic)

Parameters $b_1, b_2, S_i, S_o$ of the yield criterion were chosen as $0.5 \ \text{GPa}^{-1}$, $0.0033 \ \text{°C}^{-1}$, 0 MPa, 3 GPa, respectively. Computed radial and tangential stresses are shown in figure 5. As noted from figure
5, liquid state and elastic provide some bounds for radial and tangential stresses. As expected, radial and tangential stresses have the same values and distributions is hydro-static. If the mantle and crust of Mars are assumed to be behaving elastically together with the assumption of core is in liquid state, the tangential stress in the mantle and crust is higher than the radial stress. As the mantle and crust support induced stress due to gravity, the radial and tangential stresses are smaller and show a hydrostatic distribution. It is interesting to note that the value of the tangential stress is about 3.3 GPa. This value is almost the half of the tangential stress for Earth behaving elastically as reported by Aydan [1,2]. As it is well-known, the uniaxial compressive strength of the strongest rock in Earth is about 0.6 MPa. It is natural to expect that the mantle and crust of Mars should exhibit an elasto-plastic behaviour under high tangential stress distribution.

Figure 5. Computed radial and tangential stresses for Mars for different constitutive models.

If mantle and crust is assumed to behave in elasto-plastic manner, the high tangential stress for elastic behaviour is greatly reduced and the whole mantle and crust become plastic. As a result, the radial and tangential stress in the core were increased. If the temperature distribution estimated by Zharkov and Gudova [20] is utilized and the yield strength of materials constituting the mantle and crust of Mars obeys to the yield criterion of Aydan [2] with chosen material properties, the tangential stress in the mantle and crust is further reduced while the radial stress is increased. In addition, the hydrostatic radial and tangential stresses in the core are increased. In other words, the consideration of the thermo-elasto plastic behaviour of the mantle and the crust of the earth further decreased the deviatoric stresses in the mantle and in the crust. These results fundamentally similar to those estimated for the Earth by Aydan [1-3].

Figures 6 and 7 show the distributions of radial and tangential stresses and the ratio of tangential stress to radial stress in the lithosphere for each case, respectively. As stated previously, the elasto-plastic analyses showed that the whole crust and mantle became plastic. Large tangential stresses seen in CASE 2 were dissipated when the plastic behaviour of the crust and mantle was considered. The radial stress (vertical stress) almost coincided with each other up to a depth of 30 km from the surface. They are almost equal to the vertical stress calculated from \( \rho g H \). The lateral stress coefficient, which has a value of infinity at the ground surface for CASE 2, also decreased in magnitude when the plastic behaviour
was considered. Lateral stress coefficient is roughly 2.34 for thermo-elastic behaviour for a crust with uniaxial compressive strength.

**Figure 6.** Computed radial and tangential stresses in the Martian lithosphere.

**Figure 7.** The distribution of lateral stress coefficient in the Martian lithosphere.

### 7. Conclusions
The rock mechanics aspects of Mars are quite similar to those of the Earth. The differences result from gravitational acceleration, climatic conditions (temperature, humidity, winds), thickness of atmosphere and none or limited amount of ground water or other fluids. The knowledge on the behaviour of rocks,
discontinuities and rock masses acquired on the Earth can be adopted in Mars with the consideration of the differences resulting from gravitation acceleration, climatic conditions and fluids in rock masses. On the basis of knowledge gained from rocks of Earth, the stress state of Mars is studied using a thermo-elastic-plastic behaviour constitutive model and spherical symmetric finite element method four different conditions. Although these results are preliminary, they are expected to provide some bases for future studies. The computational results may clearly explain why mountains are higher in Mars compared with those on Earth and why tectonics is less pronounced in Mars.

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