Weight Calculation of GSM-R Network Quality of Service Evaluation Indexes Based on Analytic Hierarchy Process and Entropy Weight Method

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Abstract. In order to assign an objective and comprehensive weight to each index for evaluating the GSM-R network quality of service, this paper used the combination of subjective and objective weight value to overcome the singularity of determining the weight from one aspect. The entropy weight method was used to determine the objective weight, which was assigned to each index by the entropy calculation. The subjective weight was calculated on the basis of the improved analytic hierarchy process (AHP) and the scale construction method was used to construct the judgment matrix, which reduced the computational complexity and redundancy. In addition, three kinds of calculation methods were proposed for combining multiple experts’ opinions. Combining the subjective and objective weight vectors obtained by different combination methods, the results of multiple integrated weight vectors were obtained. Then the average value was calculated for each index. By comparing the correlation coefficient between each integrated weight vector and the average weight vector, a better multiple combining calculation method was determined to calculate the weight of each index in the evaluation system. This paper combined the actual test data of ZX section of XL high-speed railway and the experts’ questionnaires to obtain the weight assignment of GSM-R network quality of service indexes. It can be seen from the results that the voice quality, switching success ratio and average handover delay account for a large proportion, which is in line with reality. The judgment method of weight proposed in this paper is objective and feasible.

1. Introduction
GSM-R is an international wireless communication standard for railway communication and applications. The operation quality of GSM-R network is directly related to the safety of railway transportation. The development of a GSM-R operation quality evaluation system can improve network maintenance efficiency and operation quality [1], and the weight calculation of indexes is a key issue in constructing this evaluation system.

Reference [2] analyzes the statistical data of GSM-R network quality of service standards and indexes, and studies which indexes need to be weighted if the GSM-R network quality of service is evaluated. Reference [3] uses the AHP and fuzzy evaluation method to analyze the problem and calculates the index weight. But the above two methods are only considered from the subjective evaluation of experts, the calculation results are highly subjective. In order to reduce the influence of
subjective factors of experts, reference [4] adopts a calculation method combining subjective weighting and objective weighting, using an improved analytic hierarchy process to calculate subjective weights, and entropy weight method to calculate objective weights. However, when calculating the subjective weights, only the opinions of one not multiple experts are used, and the results obtained are more one-sided. Reference [5] provides a mathematical method for multiple combinations of the opinions of many experts. Its expert opinions are weighted by calculating the correlation coefficient of the experts’ subjective weight matrixes. Reference [6] compares the index scale in the analytic hierarchy process with the 1-9 scale, 9/9-9/1 scale, and 10/10-18/2 scale from the three aspects of the average weight deviation, the maximum weight deviation, and the consistency index. Therefore, in different practical applications using the analytic hierarchy process, these four scales can be used for calculation and comparison. Based on the ideas of [5] and [6], this paper proposes three calculation methods to gather expert opinions. When calculating the subjective weight to construct the judgment matrix, four scales are used to calculate the weight respectively.

In the analytic hierarchy process, the judgment matrix is established based on the experts’ human judgments, and the comparison and assignment of the relative importance of the two factors by the expert completely depends on the expert experience, so the subjectivity and uncertainty are relatively strong. In addition, the process of calculating the weight by the judgment matrix requires a consistency test in AHP. If the consistency test fails, the judgment matrix needs to be rebuilt. For a model with multiple evaluation indexes, the amount of calculation is too large. In order to overcome the shortcomings of the single weighting method, this paper uses a combination of subjective qualitative calculation and objective quantitative calculation to determine the assignment of weight values.

According to the characteristics of many evaluation indexes in the GSM-R model, this paper adopts the scale construction method to construct the judgment matrix, in order to avoid the large amount calculation of using the analytic hierarchy process.

For reducing the influence of individual expert’s subjective factors, this paper adopts the following methods. Firstly, three different methods are used to gather the opinions of the experts group. Secondly, the calculation result of the entropy weight method is used as the objective weight. The subjective weights are combined with the objective weights to obtain the comprehensive weight. Finally, the average value is calculated for each index in the multiple comprehensive weight vectors. And the correlation coefficient between the comprehensive weight vector and the average weight vector is compared to determine the best multiple combination calculation method. The final weight obtained under this method is used as the weight value assignment result of various indexes of the GSM-R network quality of service.

2. Determination of Subjective Weight

2.1. Basic Principles of AHP
AHP is a decision-making method for multi-scheme or multi-objective proposed by American operations researcher T. L. Satty. Use the AHP to logicalize and hierarchize the influencing factors in the problem to be decided, establish a hierarchical structure model, and then qualitatively analyze the different factors in the same level. And through quantitative calculations, the degree of influence of lower-level factors on upper-level problems can be obtained. The sum of the weights of the lower-level factors is equal to the weight of the upper-level factors to which it belongs.

2.2. Build a Hierarchical Structure Model
The hierarchical structure model can be roughly divided into three layers: target layer, criterion layer, and scheme layer. Among them, the criterion layer can be further subdivided into a first-level index layer and a second-level index layer. This paper divides the GSM-R network quality of service criterion layer into three layers, its hierarchical model is shown in Figure 1.
Comprehensive evaluation indexes of GSM-R network quality of service

2.3. Constructing the Judgment Matrix
The judgment matrix $W=[m_{ij}]$ constructed by the scale construction method for n indexes satisfies the following properties:

1) $m_{ij}=1/m_{ji}$

2) When $i=j$, $m_{ij}=m_{ji}=1$

3) $m_{ij}=m_{iz}m_{zj}$ (i,j,z=1,2,⋯,n)

where $m_{ij}$ is the scale value of the i-th index compared with the j-th index.

$W=[m_{ij}]$ is an n-order matrix of the following form:

$$
W = \begin{bmatrix}
1 & m_{12} & m_{13} & m_{14} & \cdots & m_{1n} \\
1/m_{12} & 1 & m_{23} & m_{24} & \cdots & m_{2n} \\
1/m_{13} & 1/m_{23} & 1 & m_{34} & \cdots & m_{3n} \\
1/m_{14} & 1/m_{24} & 1/m_{34} & 1 & \cdots & m_{4n} \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
1/m_{1n} & 1/m_{2n} & 1/m_{3n} & 1/m_{4n} & \cdots & 1
\end{bmatrix}
$$

This paper uses four scales to calculate separately, and the value and meaning of each scale are shown in Table 1.

Figure 1. GSM-R network quality of service hierarchy model.
Table 1. Value and Meaning of Scales.

| Index scale          | 1-9 scale | 9/9-9/1 scale | 10/10-18/2 scale |
|----------------------|-----------|---------------|------------------|
| Very important       | 2.08      | 5             | 1.8              | 2.333             |
| Slightly important   | 1.277     | 3             | 1.286            | 1.5               |
| Equally important    | 1         | 1             | 1                | 1                 |
| Unimportant          | 1/1.277   | 1/3           | 1/1.286          | 1/1.5             |
| Very slight          | 2.08      | 1/5           | 1/1.8            | 1/2.333           |

2.4. Single-level Weight Vector Calculation

According to the proof in reference [7], any two rows in the matrix are proportional and satisfy \( m_{ij}=1/m_{ji} \), which is a positive and negative interaction matrix and consistent. So \( W \) is a completely consistent positive and negative interaction matrix, it is not necessary to perform consistency check. Calculate the eigenvector of matrix \( W=[m_{ij}] \), the value contained in the eigenvector of matrix \( W \) is the weight of the corresponding \( n \) indexes [8]. The steps of eigenvector calculation are as follows:

1) Calculate the product of the elements of each row in the matrix \( W \):
\[
\bar{a}_i = \prod_{j=1}^{n} m_{ij}, \quad j=1,2,\ldots,n
\]  
(2)

2) Divide the product of each row to the power of \( n \):
\[
\bar{b}_i = \frac{1}{n} \bar{a}_i
\]  
(3)

3) Normalize \( \bar{b}_i \), namely:
\[
b_i = \frac{\bar{b}_i}{\sum_{i=1}^{n} \bar{b}_i}
\]  
(4)

4) Construct eigenvector:
\[
B'=[b_1,b_2,\ldots,b_n]^T
\]  
(5)

2.5. Expert Multiple Combination for Weight Calculation

Select the results of \( z \) experts’ questionnaires, and each questionnaire contains the expert's subjective judgment on the comparison of the importance of \( n \) indexes. Using the AHP for the opinions of \( z \) experts, the \( z \) group weight vectors can be obtained, that is, the subjective judgment result of the weighting of \( n \) indexes by \( z \) experts.

It is necessary to gather the opinions of \( z \) experts, and calculate a set of weight vectors \([Q_1,Q_2,\ldots,Q_n]\) as the calculation result of subjective weight values. This paper gives three comprehensive methods.

- Method 1: For each index, find the average value of this indicator’s weight in the \( z \) group weight vector, namely:
\[
q_{ij} = \frac{\sum_{i=1}^{z} p_{ij}}{z}, \quad j=1,2,\ldots,n
\]  
(6)

where \( p_{ij} \) is the weight of the \( j \)-th index in the weight vector evaluated by the \( i \)-th expert.

- Method 2: Assign different weights to the weight vector of each expert based on the working time of the expert. Record the working time of the \( i \)-th expert as \( a_i \), then the weight of each index:
\[
q_{2j} = \sum_{i=1}^{z} \frac{a_i}{\sum_{i=1}^{n} a_i} p_{ij}, \quad j=1,2,\ldots,n
\]  
(7)
Method 3: Assign weights according to the difference between the average value of each index weight obtained in Method 1. The calculation process is as follows:

1) Calculate the average value of the distance between the weight vector and the average weight vector of the i-th expert, namely:

\[ e_i = \frac{\sum_{j=1}^{n} (g_{ij} - q_1)^2}{n} \quad (8) \]

2) Calculate the expert multiple combination weight of each index:

\[ q_{3j} = \frac{\sum_{i=1}^{z} e_i z_i}{\sum_{i=1}^{z} p_{ij}} \quad j=1,2,\ldots,n \quad (9) \]

3. Entropy Weight Method to Calculate Objective Weight

3.1. Basic Principles

In information theory, entropy is a measure of uncertainty. The greater the uncertainty, the greater the entropy and the greater the amount of information contained. According to this characteristic, we can judge the randomness and disorder of an event by calculating the entropy. The greater the discrete degree of the index, the greater the impact of the index on the comprehensive evaluation, and the greater the weight of the index. Therefore, we use the data detected by objective monitoring to calculate the entropy of each index, and assign a weight to each index by the size of the entropy [9].

3.2. Steps of Calculating the Weight by Entropy Weight Method

1) For the selected y samples and n indexes, let \( x_{ij} \) \((i=1,2,\ldots,y;j=1,2,\ldots,n)\) be the value of the j-th index in the i-th sample.

2) Since the units and evaluation standards of each index are different, it is necessary to convert the absolute value into a relative value for normalizing the index.

Normalized calculation formula of positive index:

\[ x_{ij}' = \frac{x_{ij} - \min\{x_{1j},\ldots,x_{nj}\}}{\max\{x_{1j},\ldots,x_{nj}\} - \min\{x_{1j},\ldots,x_{nj}\}} \quad (10) \]

Normalized calculation formula of negative index:

\[ x_{ij}' = \frac{\max\{x_{1j},\ldots,x_{nj}\} - x_{ij}}{\max\{x_{1j},\ldots,x_{nj}\} - \min\{x_{1j},\ldots,x_{nj}\}} \quad (11) \]

where \( \max\{x_{1j},\ldots,x_{nj}\} \) represents the maximum value of all numbers from \( x_{1j} \) to \( x_{nj} \), and \( \min\{x_{1j},\ldots,x_{nj}\} \) represents the minimum value of all numbers from \( x_{1j} \) to \( x_{nj} \).

Calculate the proportion of the i-th sample in the j-th index:

\[ T_{ij} = \frac{x_{ij}'}{\sum_{i=1}^{y} x_{ij}'} \quad (12) \]

3) Calculate the entropy of the j-th index [10]:

\[ u_j = k \sum_{i=1}^{y} T_{ij} \ln(T_{ij}) \quad j=1,2,\ldots,n \quad (13) \]

where \( k = \frac{1}{\ln(y)} > 0 \), satisfying \( u_j > 0 \).

4) Calculate information entropy redundancy:

\[ d_j = 1 - u_j \quad j=1,2,\ldots,n \quad (14) \]

5) Calculate the weight of each index:
\[ W_j = \frac{d_j}{\sum_{j=1}^{n} d_j}, \quad j=1,2,\ldots,n \]  

4. Determination of Comprehensive Weight

The subjective weight is combined with the objective weight to obtain a comprehensive weight vector \( X \). In the process of integrating, different weight occupancy coefficients are assigned to the subjective value and the objective value. Assign the weight occupancy coefficient to the subjective weight as \( f \), and the objective weight as \( (1-f) \), then the comprehensive weight of each index is:

\[ X_i = fQ_i + (1-f)w_i \quad (i=1,2,\ldots,n) \]

5. Evaluation of Experts Opinion Multiple Combination

In the process of subjective weight calculation, in order to judge which scale and experts opinion combination method is better, this paper calculates an average value for each index in the comprehensive weight vectors to form the average weight vector \( \bar{Y} \). And we calculate the correlation coefficient between each comprehensive weight vector and the average weight vector respectively. The correlation coefficient is a statistical indicator used to study the degree of correlation between variables. The value of the correlation coefficient \( r \) ranges from -1 to 1. The closer \( r \) is to 1, the greater the linear correlation between the two variables and the higher the degree of fitting. When the correlation coefficient between a certain comprehensive weight vector and the average weight vector is larger, the fitting degree between the two is higher. This comprehensive weight vector is considered to be a more objective and reasonable result, so the one with high fitting degree is selected as the combination method. The formula of correlation coefficient is:

\[ r = \frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2 \sum_{i=1}^{n}(Y_i-\bar{Y})^2}} \]

where \( X_i \) represents the weight of the \( i \)-th index in the weight vector \( X \), \( \bar{X} \) represents the average value of all the index weight values in the weight vector \( X \); \( y_i \) represents the weight of the \( i \)-th index in the weight vector \( Y \), \( \bar{Y} \) represents the average value of all the index weight values in the weight vector \( Y \).

6. Instance Analysis of GSM-R Network Quality of Service

6.1. Determination of Subjective Weight

For the hierarchical structure model, we obtain the comparison results of the importance between two indexes in each layer through questionnaires. This paper selects questionnaires from 10 experts, and compares the GSM-R network quality of service indexes in accordance with the hierarchical structure model in Figure 1. Through the questionnaires of 10 experts, the comparison results of the importance between two indexes can be obtained. Then the analytic hierarchy process is used respectively to obtain the weight vector containing 12 indexes. The indexes are arranged in the following order: switching success ratio, average handover delay, probability of establishment failure, probability of connection establishment time <5s (10s), average transmission delay, probability of transmission interference time <0.8s, probability of transmission interference time <1.0s, probability of transmission error-free time >7s, probability of connection establishment failure, average connection establishment delay, voice quality (uplink), voice quality (downlink).

Record the index weight vector obtained by the \( i \)-th expert as \( y_i \), and establish the matrix \( K \):

\[ K = [y_1, y_2, y_3, y_4, y_5, y_6, y_7, y_8, y_9, y_{10}]^T \]

Based on the four different scales in Table 1, four matrices \( K1, K2, K3, K4 \) can be obtained. According to the index scale, the matrix \( K1 \) is obtained:
According to the 1-9 scale, the matrix K2 is obtained

\[
K_2 = \begin{bmatrix}
0.2826 & 0.2826 & \ldots & 0.1406 & 0.0360 & 0.0459 & 0.0587 & 0.0587 & 0.0651 & 0.0360 & 0.0632 & 0.0749 & 0.0587 & 0.0459 & 0.0360 & 0.1406 & 0.1406 \\
0.1069 & 0.1069 & 0.0833 & 0.0652 & 0.0436 & 0.0436 & 0.0436 & 0.0436 & 0.0436 & 0.2110 & 0.1653 \\
0.1089 & 0.1089 & 0.0613 & 0.0365 & 0.0466 & 0.0466 & 0.0466 & 0.0969 & 0.0969 & 0.1448 & 0.1448 \\
0.1576 & 0.2013 & 0.0233 & 0.0485 & 0.0351 & 0.0351 & 0.0351 & 0.0351 & 0.0351 & 0.1793 & 0.1793 \\
0.1312 & 0.1312 & 0.0801 & 0.0627 & 0.0419 & 0.0419 & 0.0419 & 0.0419 & 0.0419 & 0.1718 & 0.1718 \\
0.1715 & 0.0824 & 0.1014 & 0.0487 & 0.0190 & 0.0190 & 0.0504 & 0.0242 & 0.0242 & 0.2202 & 0.2202 \\
0.1494 & 0.1494 & 0.0714 & 0.0253 & 0.0323 & 0.0323 & 0.0527 & 0.0413 & 0.0413 & 0.1666 & 0.1666 \\
0.0850 & 0.0409 & 0.0354 & 0.0354 & 0.0319 & 0.0664 & 0.0664 & 0.0664 & 0.0520 & 0.0520 & 0.2057 & 0.2626 \\
0.0982 & 0.0982 & 0.0655 & 0.0655 & 0.0307 & 0.0307 & 0.0392 & 0.0500 & 0.0500 & 0.0500 & 0.2110 & 0.2110 \\
0.0864 & 0.0528 & 0.0393 & 0.0308 & 0.0328 & 0.0418 & 0.0328 & 0.0418 & 0.0534 & 0.0418 & 0.2826 & 0.2826
\end{bmatrix}
\]

According to the 9/9-9/1 scale, the matrix K3 is obtained

\[
K_3 = \begin{bmatrix}
0.1544 & 0.1200 & 0.0835 & 0.0649 & 0.0362 & 0.0651 & 0.0506 & 0.0506 & 0.0394 & 0.0306 & 0.1523 & 0.1523 \\
0.1064 & 0.1064 & 0.0833 & 0.0648 & 0.0440 & 0.0440 & 0.0440 & 0.0440 & 0.0440 & 0.2111 & 0.1641 \\
0.1136 & 0.1136 & 0.0636 & 0.0636 & 0.0347 & 0.0447 & 0.0447 & 0.0447 & 0.0804 & 0.0804 & 0.1581 & 0.1581 \\
0.1502 & 0.1931 & 0.0316 & 0.0568 & 0.0334 & 0.0334 & 0.0334 & 0.0334 & 0.0334 & 0.1840 & 0.1840 \\
0.1305 & 0.1305 & 0.0794 & 0.0618 & 0.0420 & 0.0420 & 0.0420 & 0.0420 & 0.0420 & 0.1730 & 0.1730 \\
0.1692 & 0.0940 & 0.0969 & 0.0539 & 0.0201 & 0.0201 & 0.0201 & 0.0466 & 0.0259 & 0.0259 & 0.2137 & 0.2137 \\
0.1442 & 0.1442 & 0.0682 & 0.0682 & 0.0241 & 0.0310 & 0.0310 & 0.0513 & 0.0399 & 0.0399 & 0.1789 & 0.1789 \\
0.0975 & 0.0542 & 0.0425 & 0.0425 & 0.0324 & 0.0584 & 0.0584 & 0.0584 & 0.0454 & 0.0454 & 0.2034 & 0.2616 \\
0.0976 & 0.0976 & 0.0650 & 0.0650 & 0.0306 & 0.0306 & 0.0394 & 0.0507 & 0.0507 & 0.0507 & 0.2110 & 0.2110 \\
0.0826 & 0.0642 & 0.0479 & 0.0373 & 0.0294 & 0.0294 & 0.0378 & 0.0486 & 0.0378 & 0.2737 & 0.2737
\end{bmatrix}
\]

According to the 10/10-18/2 scale, the matrix K4 is obtained
6.2. Calculating the Weight Vector of Expert Multiple Combination

Among the 10 selected experts, the working time of experts in railway communication field is shown in Table 2.

| Working time(year) | Expert 1 | Expert 2 | Expert 3 | Expert 4 | Expert 5 | Expert 6 | Expert 7 | Expert 8 | Expert 9 | Expert 10 |
|--------------------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 10                 | 30       | 24       | 40       | 26       | 8        | 15       | 25       | 20       | 13       |

The subjective weight vector calculated for each expert in each row of the matrix $K_1,K_2,K_3,K_4$ is combined according to the three methods in section 1.5, and the results are shown in Table 3, Table 4, and Table 5.

Table 3. Method 1 Expert Multiple Combination Weight Vector.

| Index scale          | 0.1224 | 0.1088 | 0.0642 | 0.0553 | 0.0333 | 0.0432 | 0.0415 | 0.0487 | 0.0484 | 0.0463 | 0.1934 | 0.1945 |
|----------------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1-9 scale            | 0.1315 | 0.1061 | 0.0366 | 0.0177 | 0.0318 | 0.0606 | 0.0504 | 0.0825 | 0.0779 | 0.0730 | 0.1590 | 0.1727 |
| 9/9-9/1 scale        | 0.1246 | 0.1118 | 0.0662 | 0.0579 | 0.0327 | 0.0407 | 0.0393 | 0.0459 | 0.0449 | 0.0430 | 0.1959 | 0.1971 |
| 10/10-18/2 scale     | 0.1257 | 0.1093 | 0.0596 | 0.0471 | 0.0328 | 0.0458 | 0.0431 | 0.0542 | 0.0528 | 0.0497 | 0.1887 | 0.1913 |
Table 4. Method 2 Expert Multiple Combination Weight Vector.

| Index scale | 0.1213 | 0.1186 | 0.0587 | 0.0554 | 0.0351 | 0.0432 | 0.0427 | 0.0469 | 0.0493 | 0.0481 | 0.1903 | 0.1905 |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1-9 scale   | 0.1070 | 0.1332 | 0.0294 | 0.0173 | 0.0381 | 0.0603 | 0.0569 | 0.0797 | 0.0839 | 0.0811 | 0.1498 | 0.1633 |
| 9/9-9/1 scale | 0.1225 | 0.1199 | 0.0615 | 0.0583 | 0.0345 | 0.0410 | 0.0407 | 0.0447 | 0.0458 | 0.0447 | 0.1932 | 0.1934 |
| 10/10-18/2 scale | 0.1207 | 0.1215 | 0.0537 | 0.0473 | 0.0355 | 0.0461 | 0.0453 | 0.0523 | 0.0544 | 0.0526 | 0.1846 | 0.1860 |

Table 5. Method 3 Expert Multiple Combination Weight Vector.

| Index scale | 0.1225 | 0.1089 | 0.0643 | 0.0553 | 0.0333 | 0.0432 | 0.0415 | 0.0487 | 0.0484 | 0.0463 | 0.1932 | 0.1943 |
|-------------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|--------|
| 1-9 scale   | 0.1320 | 0.1056 | 0.0370 | 0.0179 | 0.0325 | 0.0609 | 0.0511 | 0.0840 | 0.0784 | 0.0739 | 0.1567 | 0.1699 |
| 9/9-9/1 scale | 0.1246 | 0.1118 | 0.0663 | 0.0579 | 0.0327 | 0.0407 | 0.0393 | 0.0459 | 0.0449 | 0.0430 | 0.1958 | 0.1969 |
| 10/10-18/2 scale | 0.1258 | 0.1094 | 0.0597 | 0.0472 | 0.0328 | 0.0458 | 0.0431 | 0.0543 | 0.0528 | 0.0497 | 0.1885 | 0.1909 |

6.3. Calculation of Objective Weight by Entropy Weight Method
This paper investigated the detection data of 12 indexes on the ZX section of the XL high-speed railway in September 2018, and extracted ten sets of data, as shown in Table 6.

Table 6. Detection and Monitoring Results of Indexes.

| Switching success ratio | Average handover delay (s) | Probability of connection establishment failure | Probability of transmission time <5s (10s) | Average transmission delay (s) | Probability of transmission interference time <0.8s | Probability of transmission interference time <1s | Probability of transmission error-free time >7s | Probability of connection establishment failure | Averag e connection establishment delay (s) | Percentage of voice quality less than or equal to level 4 (uplink) | Percentera ge of voice quality less than or equal to level 4 (downlink) |
|-------------------------|---------------------------|-----------------------------------------------|---------------------------------------------|---------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|---------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| Group 1                 | 1                         | 216                                           | 0                                           | 1                                           | 375                                           | 1                                             | 1                                             | 1                                           | 0                                             | 4287                                           | 0.9995                                         | 1                                             |
| Group 2                 | 1                         | 220                                           | 0                                           | 0.9921                                      | 395                                           | 0.9716                                        | 0.983                                         | 0.9602                                      | 0.0043                                       | 4635                                           | 0.9995                                         | 0.9988                                        |
| Group 3                 | 1                         | 221                                           | 0                                           | 1                                           | 373                                           | 1                                             | 1                                             | 0.9942                                      | 0                                             | 4269                                           | 1                                             | 0.9997                                        |
| Group 4                 | 0.985                     | 221                                           | 0                                           | 0.9868                                      | 395                                           | 1                                             | 1                                             | 0.9932                                      | 0                                             | 3954                                           | 0.9996                                         | 0.999                                         |
| Group 5                 | 1                         | 220                                           | 0                                           | 1                                           | 365                                           | 0.9959                                        | 1                                             | 0.9959                                      | 0                                             | 4387                                           | 0.9996                                         | 1                                             |
| Group 6                 | 1                         | 221                                           | 0                                           | 1                                           | 386                                           | 1                                             | 1                                             | 1                                           | 0.0041                                       | 4208                                           | 0.9996                                         | 0.9999                                        |
| Group 7                 | 1                         | 219                                           | 0.0017                                      | 0.9983                                      | 371                                           | 0.9907                                        | 0.9953                                        | 0.9954                                      | 0                                             | 4345                                           | 0.9988                                         | 0.9992                                        |
| Group 8                 | 1                         | 218                                           | 0                                           | 1                                           | 386                                           | 0.9701                                        | 0.9701                                        | 0.9701                                      | 0                                             | 4439                                           | 0.9975                                         | 1                                             |
| Group 9                 | 1                         | 221                                           | 0.0026                                      | 0.9921                                      | 365                                           | 1                                             | 1                                             | 0.9832                                      | 0.0025                                       | 4232                                           | 0.9994                                         | 0.9997                                        |
| Group 10                | 1                         | 220                                           | 0                                           | 0.9845                                      | 387                                           | 0.9877                                        | 0.9939                                        | 0.9908                                      | 0                                             | 4681                                           | 0.9996                                         | 0.9985                                        |

According to formula (10)-(15), the weight vector calculated by the entropy weight method is obtained:
The comprehensive weight is calculated by combing the subjective weight vector $Q$ and the objective weight vector $W$, and the assigned weight are both 0.5. The results of calculating the comprehensive weight vector according to formula (16) are shown in Table 7, Table 8, and Table 9.

### Table 7. Method 1 the Comprehensive Weight Vector.

| Index scale   | $W$                                      |
|---------------|------------------------------------------|
| 1-9 scale     | $\begin{bmatrix} 0.0567 & 0.3388 & 0.0385 & 0.0484 & 0.0487 & 0.0403 & 0.0235 & 0.0332 & 0.0477 & 0.0467 & 0.1059 & 0.1718 \end{bmatrix}$ |
| 9/9-9/1 scale | $\begin{bmatrix} 0.0605 & 0.3283 & 0.0218 & 0.0154 & 0.0462 & 0.0561 & 0.0283 & 0.0558 & 0.0762 & 0.0731 & 0.0866 & 0.1516 \end{bmatrix}$ |
| 10/10-18/2 scale | $\begin{bmatrix} 0.0576 & 0.3466 & 0.0395 & 0.0504 & 0.0476 & 0.0378 & 0.0221 & 0.0311 & 0.0440 & 0.0432 & 0.1068 & 0.1733 \end{bmatrix}$ |

### Table 8. Method 2 the Comprehensive Weight Vector.

| Index scale   | $W$                                      |
|---------------|------------------------------------------|
| 1-9 scale     | $\begin{bmatrix} 0.0541 & 0.3596 & 0.0343 & 0.0472 & 0.0500 & 0.0392 & 0.0235 & 0.0311 & 0.0473 & 0.0473 & 0.1016 & 0.1640 \end{bmatrix}$ |
| 9/9-9/1 scale | $\begin{bmatrix} 0.0456 & 0.3819 & 0.0162 & 0.0139 & 0.0513 & 0.0517 & 0.0296 & 0.0500 & 0.0761 & 0.0753 & 0.0755 & 0.1328 \end{bmatrix}$ |
| 10/10-18/2 scale | $\begin{bmatrix} 0.0552 & 0.3634 & 0.0359 & 0.0497 & 0.0492 & 0.0372 & 0.0224 & 0.0297 & 0.0440 & 0.0439 & 0.1031 & 0.1664 \end{bmatrix}$ |

### Table 9. Method 3 the Comprehensive Weight Vector.

| Index scale   | $W$                                      |
|---------------|------------------------------------------|
| 1-9 scale     | $\begin{bmatrix} 0.0567 & 0.3390 & 0.0386 & 0.0484 & 0.0487 & 0.0402 & 0.0235 & 0.0332 & 0.0477 & 0.0467 & 0.1058 & 0.1716 \end{bmatrix}$ |
| 9/9-9/1 scale | $\begin{bmatrix} 0.0608 & 0.3269 & 0.0221 & 0.0156 & 0.0473 & 0.0564 & 0.0287 & 0.0569 & 0.0768 & 0.0741 & 0.0853 & 0.1492 \end{bmatrix}$ |
| 10/10-18/2 scale | $\begin{bmatrix} 0.0575 & 0.3466 & 0.0396 & 0.0504 & 0.0476 & 0.0378 & 0.0221 & 0.0311 & 0.0440 & 0.0433 & 0.1068 & 0.1732 \end{bmatrix}$ |

6.5. Evaluation of Experts Opinion Multiple Combination
The weight vector obtained by calculating the average value of each index in multiple comprehensive weight vectors:

\[
Y = \begin{bmatrix} 0.0563 & 0.3480 & 0.0324 & 0.0385 & 0.0485 & 0.0436 \\ 0.0248 & 0.0384 & 0.0549 & 0.0537 & 0.0984 & 0.1625 \end{bmatrix}
\]

Compare the correlation coefficient between each comprehensive weight vector and the average comprehensive weight vector. The results are shown in Table 10.
It can be seen from Table 10 that the maximum correlation coefficient 0.9994 is obtained when using method 2 on a scale of 10/10-18/2. At this time, the final result of the weight value of each index is shown in Table 11.

Based on experts’ questionnaires and the actual detection data of the ZX section of the XL high-speed railway in September 2018, the comprehensive weight value is calculated using the method in this paper. In the results, the weight of the three indexes: voice quality, switching success ratio, and average handover delay account for a large proportion, which is in line with the result of the weight assigned to the index when the GSM-R network quality of service is actually evaluated.

| Table 10. Comparison Results of Correlation Coefficients. |
|----------------------------------------------------------|
| Index scale | 1-9 scale | 9/9-9/1 scale | 10/10-18/2 scale |
| Method 1 | 0.9973 | 0.9872 | 0.9963 | 0.9991 |
| Method 2 | 0.9987 | 0.9801 | 0.9978 | 0.9994 |
| Method 3 | 0.9973 | 0.9863 | 0.9963 | 0.9992 |

| Table 11. The Final Calculation Result of the Weight of Each Index (10/10-18/2 scale, method 2 expert opinion multiple combination). |
|--------------------------------------------------------|
| Indexes | Weight |
| switching success ratio | 0.0539 |
| average handover delay | 0.3652 |
| probability of establishment failure | 0.0311 |
| probability of connection establishment time <5s (10s) | 0.0399 |
| average transmission delay | 0.0501 |
| probability of transmission interference time <0.8s | 0.0415 |
| probability of transmission interference time <1.0s | 0.0247 |
| probability of transmission error-free time >7s | 0.0344 |
| probability of connection establishment failure | 0.0517 |
| average connection establishment delay | 0.0512 |
| voice quality (uplink) | 0.0976 |
| voice quality (downlink) | 0.1586 |

7. Conclusion
In the process of GSM-R network quality of service assessment, the calculation of the weight of each index is particularly important. This paper combines subjective weights with objective weights, and makes a slight improvement on the basis of the analytic hierarchy process to make the assignment...
process easier and the results close to actual objective experience. According to the research results, it can be seen that in the analytic hierarchy process improved by the scale construction method, the 10/10-18/2 scale can get more reasonable results. In order to reduce the subjective problem of a single expert opinion, on the basis of the analytic hierarchy model in this article and sample data, the weight is assigned to each expert opinion according to the expert's working time, so as to calculate the weight of the multiple combination of experts. It can be seen from the final calculation results of the weights of various indexes that voice quality, switching success ratio and average handover delay account for a larger proportion in the GSM-R network quality of service evaluation.

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