Abstract—Surface waves are demonstrated theoretically to propagate along periodically corrugated surfaces made of non-conductive lossless materials with positive permittivity. An analytic derivation of the surface wave dispersion relation is presented in the case of the deeply subwavelength period of a corrugated structure using impedance boundary conditions (IBCs) at the interfaces. Thus obtained dispersion relation is verified numerically and limitations as well as the physicality of IBCs when modeling surface waves are discussed. Finally, suitable materials, potential experimental realization, and sensing applications of surface waves in the terahertz spectral range are discussed.

Index Terms—Corrugated surfaces, high-\(k\) dielectrics, surface waves, terahertz.

I. INTRODUCTION

THE strong spatial confinement of surface waves near interfaces allows for greatly enhanced performance of many practical devices in high-resolution sensing, spectroscopy, and subwavelength imaging [1], [2], [3], [4], [5], [6], [7], [8], [9]. Particularly, surface plasmon-polaritons propagation at the interface between a metal and an analyte revolutionized the field of bio-sensing in the visible spectral range [9], [10], [11]. Recently, terahertz (THz) spectral range emerged as the next frontier for non-destructive imaging and biomedical applications [12], [13], [14], [15], [16], [17], [18], [19]. Because of the relative transparency of dry dielectrics to THz radiation and the existence of spectral fingerprints for many biological molecules, THz sensing became an emerging research field that could also benefit from surface wave sensing modality. However, in the long wavelength spectral range, excitation and application of surface waves have been problematic due to the THz wave’s weak confinement (strong presence in the cladding) and high losses [20], [21]. This is because in the THz spectral range, the imaginary part of the dielectric constant of metal dominates and it can be several orders of magnitude larger than the real part unlike in the visible and near-IR spectral ranges, where the real part of the dielectric constant of metal is negative and much larger (in its absolute value) than the imaginary part. This entailed a search for new polaritonic materials capable of supporting THz surface waves with strong localization at the interface [22], [23], [24], [25], [26], [27], [28], [29].

One interesting alternative to surface plasmon-polaritons at long wavelengths is surface phonon-polaritons, which are excited on surfaces of some polar materials, typically in the near vicinity of material optical resonances. Such surface waves are a result of coupling between phonon and electromagnetic fields, with enabling materials abundant in the mid-IR and THz spectral ranges [30], [31] in the form of semiconductors and ceramics. Near resonances, optical properties of surface phonon-polaritons in the THz spectral range can be similar to those of a surface plasmon-polariton in the visible. This is because a polaritonic material can have a negative real part of the dielectric constant that is much larger than its imaginary part, which is similar to the case of metals in the visible/IR.

Recently, materials with subwavelength structuring were shown to offer an alternative to the traditional homogeneous materials for the excitation of surface waves as their interfacial optical properties are highly designable [32], [33], [34]. Specifically, in the THz spectral range, a large body of work now exists on structured interfaces that are capable of supporting surface states. Particularly, structured metallic surfaces featuring a periodic sequence of narrow air-filled grooves of various geometries such as rectangular [35], [36], [37], [38] and V-shape [39], [40], along with corrugated metallic wires [41], [42], and helically grooved wires [43] were shown to support surface waves called spoof surface plasmons that feature strong field localization in the vicinity of metal, similar to the classical plasmon-polaritons from the visible spectral range. Additionally, corrugated surfaces made of semiconductors can also support surface waves [44], [45]. In practice, however, due to high material losses of metals in the THz spectral range, and due to tight modal confinement in the narrow metallic grooves, propagation losses of such surface waves tend to be high. One way to reduce the loss of a surface wave is to push its fields into the low-loss air cladding by either using shallow grooves [46], metallic wedges [47], or by using corrugated metallic stripes [48], [49], [50], [51]. In those cases, however, surface wave confinement near the corrugated interface is decreased. Surface waves can also be excited on structured surfaces of ideal metals with an infinite negative dielectric constant (Perfect Electric Conductor) [52], [53], [54].

More generally, a detailed analysis shows that to support a guided wave at the interface between a lossless dielectric and a generalized material, it is only necessary for such a material

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to be lossy (have a non-zero imaginary part of the dielectric constant), while the sign of a real part of the dielectric constant is irrelevant [21], [55]. This suggests that surface modes could also be supported at the interface between two dielectrics with one of them featuring either absorption or scattering (radiation) losses. Excitation and characterization of such waves for the case of all-dielectric interfaces, however, proved to be not an easy task. The problem stems from the realization that reliable experimental characterization of such waves normally requires their relatively strong localization at the interfaces (transverse modal sizes of several tens to hundreds of wavelengths at most) to prevent significant interaction with the environment. Moreover, it is also desirable for the surface wave propagation distances to be much longer than the wave transverse size, as in this case surface wave resembles a guided mode that can be excited and probed much like a regular guided mode of a waveguide. Unfortunately, for many real materials and practical operation conditions, these surface waves are either extremely extended into the cladding and thus easily lost to the environment, or they feature very high transmission losses that result in the surface wave propagation length comparable to the wave transverse size. For these reasons, such surface waves are often difficult to excite and classify as true guided modes rather than some kind of local geometrical resonances.

In this article, we propose for the first time to our knowledge a rigorous semi-analytical derivation of the dispersion relation of a surface wave propagating on structured non-conductive surfaces made of loss-less dielectrics featuring real positive permittivities. While waves on structured non-conductive surfaces have been reported earlier (see [56], [57], [58]), the authors did not explicitly consider the geometric periodicity of the interface, while only considering periodically modulated material properties (such as complex permittivity) of a planar interface. We note that, in general, it is not trivial to equate the effect of geometrical variations at interfaces with variations of the flat surface effective material parameters as field discontinuities in high refractive index contrast systems are notoriously difficult to describe using perturbative or effective medium approximations (see [59], [60] for a detailed discussion). Moreover, in some cases, the planar surface material modulation models [56], [57], [58] and rigorous treatment of geometrical variations presented in our work give fundamentally different results. For example, the surface modulation models predict lossless surface modes when using lossless dielectric materials. In contrast, when using rigorous treatment of surface geometrical variations, the modes on corrugated dielectric interfaces are always lossy. Also, the dispersion relations derived in our work successfully describe both slow and fast waves as corrugation geometry is explicitly taken into account. For the same reason, the semi-analytic dispersion relation derived using IBC boundary conditions matches very well the dispersion relations of exact numerical solutions, which can speed up greatly design of realistic corrugated surfaces for THz applications. Finally, the semi-analytical theory presented in our work is elegant and universal as it describes the dispersion relation of surface waves propagating on corrugated interfaces between a dielectric and any other material that could have either positive or negative or complex dielectric constant. Finally, we show that a lossless surface wave exists in the limiting case of a dielectric with infinite positive permittivity, which is conceptually similar to the case of “spoof” waves on a corrugated PEC surface, which is a novel result. Particularly, we show that by using common high permittivity materials and judicious choice of the surface geometry one can realize strong confinement of surface waves near the interface, as well as propagation distances that greatly exceed the wave transverse size. Moreover, under certain conditions, surface waves can also propagate along structured surfaces made of a material with infinite positive dielectric constant (Perfect Isolator). More precisely, we study corrugated surfaces featuring 1-D periodic arrays of grooves filled with a lossless low-permittivity dielectric of the upper cladding. Such grooves are separated by notches made of high-permittivity dielectric of the lower cladding. Dispersion relations of surface waves are obtained semi-analytically in the long wavelength limit by solving Maxwell equations in the vicinity of a structured surface following a general approach detailed in [36] and [61]. One key difference in our derivation compared to the ones presented in the abovementioned references is that we use impedance boundary conditions (IBCs) at the interface with a structured interface, as well as positive values of the material dielectric constants [62]. This also entails an important and unique for this problem discussion about the effect of radiation leakage from the grooves into the notches, potential dissipation mechanisms of such leaked radiation inside the notches, as well as the necessity for suppression of radiation-induced coupling between adjacent grooves. Semi-analytical results with IBC were then confirmed with numerical simulations using COMSOL Multiphysics Software with IBC, and very good correspondence was observed. Finally, a full simulation using COMSOL using two cladding materials and a structured interface confirmed that our semi-analytical theory indeed describes well surface waves on non-conductive surfaces except for resonant frequencies at which structural resonances are excited inside the high dielectric permittivity notches. In principle, such resonances can be removed by various techniques as described further in the article. Although our derivations are not wavelength specific, without the loss of generality we present examples in the THz spectral range (0.1–1 THz) in view of future experimental work on the optical characterization of such waves.

II. ANALYTICAL DERIVATION OF THE DISPERSION RELATION

Consider a periodically corrugated surface made of a material with real and positive permittivity $\varepsilon_m > 0$ adjacent to a lossless dielectric filler of permittivity $\varepsilon_0 > 0$ as shown in Fig. 1. This periodic structure has a period $d$ and contains grooves of width $a$ and height $h$. In what follows we call notches the rectangular regions of length $d - a$ between grooves that are filled with material $\varepsilon_m$. Both materials are assumed to be non-magnetic. The dispersion relation of a surface wave supported by the interface can be found by solving the Maxwell equations in the semi-infinite upper zone II ($z > 0$) and grooves in the lower zone I ($z < 0$), while
respecting the continuity conditions of the fields at $z = 0$. Additionally, the IBC is used at the interfaces with the high dielectric material (zone III) in the form of (1) [63] where $\eta_s$ is the surface impedance and $s$ is the unit normal vector to the interface. This simplest form of IBC can be derived by considering a planewave incident from the material with lower dielectric permittivity onto a planar interface with material with higher dielectric permittivity. The incident E&M fields satisfy (1) at the interface, assuming that only an outgoing wave is present on the side of a higher permittivity dielectric

$$\eta_s s \times \mathbf{H} + s \times (s \times \mathbf{E}) = 0. \quad (1)$$

This boundary condition is attractive in its simplicity, but if it is to be universal and of practical utility, it is necessary that the surface impedance $\eta_s$ be independent of the incident field characteristics, such as polarization and angle of the incident plane wave. Following [63] one can demonstrate that if the two materials occupying the half-spaces are homogeneous and isotropic, and if the incident wave comes from material $\varepsilon_0$, while for the second material $|\varepsilon_m| \gg \varepsilon_0$, then surface impedance has the following simple form $\eta_s = 1/\sqrt{\varepsilon_m}$ (CGI units) for any polarization and incidence angle of the incoming wave. In the case of finite-size structures, such as sequence of grooves shown in Fig. 1(a), for the IBC to be valid at the interfaces with a filler dielectric, one has to demand that inside the notches only waves outgoing from the interfaces (into the notches) are excited, while no reflections from other surfaces reach those interfaces. Physically, this requires that waves propagating within the notches are either absorbed within the notch length, or that they are efficiently scattered within a notch and into the semi-infinite zone III, or that their coherence is somehow destroyed over the propagation distance within the notches via structured absorption of destructive interference as illustrated in Fig. 1(b)–(e). Note that when treating plasmon waves on corrugated metallic surfaces, the validity of IBC is easy to ascertain as long as the notch width is much larger than the metal skin depth. In this case, a wave transmitted into metal from the filler dielectric will decay exponentially fast from the interface into the notch and will have no presence on other surfaces. In contrast, in the case of surface waves propagating on corrugated surfaces of lossless dielectrics, for IBC to be valid, it is not sufficient to demand that $\varepsilon_m \gg \varepsilon_0$, in addition, one has to ensure that experimental system includes one of the energy dissipation mechanisms described earlier that would prevent interaction between different interfaces of the same notch.

We now present a brief derivation of the dispersion relation of surface waves propagating on corrugated interfaces of dielectrics. First, we start with zone I which defines a planar waveguide filled with low permittivity $\varepsilon_0$ material, and that supports modes propagating along the $OZ$ direction. The modes are confined between the walls of high permittivity $\varepsilon_m$ material. One end of the waveguide is opened into zone II, while the other one is bound by a high permittivity material. Assuming the validity of the IBC, one can solve the Maxwell equations and find the dispersion relation of the groove waveguide even modes with the propagation constant $\beta_z^{I}$

$$k_x^{I} \tan(k_y^{I} a/2) = -\frac{k_0 \varepsilon_0}{\sqrt{\varepsilon_m}} \quad (2)$$

with $\beta_z^{I} = \pm \sqrt{\varepsilon_0 k_0^2 - (k_x^{I})^2}$, where for a forward propagating wave (along the $OZ$ axis) $\text{Re} (\beta_z^{I}) > 0, \text{Im} (\beta_z^{I}) > 0$. Then, adopting an approach from the theory of spoo plasmons [64], in the limit of $dk_0\sqrt{\varepsilon_0} \ll 1$ (subwavelength corrugation), an analytic expression for the propagation constant $\beta_z^{SW}$ of a surface wave propagating on the corrugated interface along the $OZ$ axis can be found by solving Maxwell equations in zone II, while matching the fields with those of zone I and respecting the IBC along $z = 0$ on the notch surfaces

$$\left(\frac{\varepsilon_0 k_0}{\sqrt{\varepsilon_m}} + k_z^{SW, II} \right) \sin \left(\frac{\beta_z^{SW, II}}{2} \right) \sin \left(\frac{\beta_z^{SW, II}}{2} \right) = \frac{i \beta_z^{I} \sqrt{\varepsilon_m} \beta_z^{I}}{\sqrt{\varepsilon_m} \beta_z^{I} - i \varepsilon_0 k_0 \tan (\beta_z^{I} h)} + \frac{\varepsilon_0 k_0}{\sqrt{\varepsilon_m}} \quad (3)$$

where $k_z^{SW, II} = \pm \sqrt{\varepsilon_m k_0^2 - (\beta_z^{SW})^2}$ is the transverse (z-component) of the wave vector of a surface wave in the filler dielectric (zone II), while $k_z^{SW, II} = \pm \sqrt{\varepsilon_m k_0^2 - (\beta_z^{SW})^2}$ is the transverse (z-component) of the wave vector of a surface wave in the high permittivity dielectric (zone III) (see more details of derivation in the Appendix). Although the derivations presented above assume a non-conductive material with a purely real and positive value of $\varepsilon_m$, they also remain valid in case of lossy materials as long as $|\varepsilon_m| \gg \varepsilon_0$. Numerical methods such as the Newton–Raphson iteration method can then be used to solve (2) and (3) together. In addition, in the most general case, the two dispersion relations (2) and (3) are universal functions of several non-dimensional variables namely $h k_0 \sqrt{\varepsilon_0}, a k_0 \sqrt{\varepsilon_0}, b k_0 \sqrt{\varepsilon_0}$, and $\sqrt{\varepsilon_0}/\sqrt{\varepsilon_m}$. Indeed, defining surface wave effective refractive index as $n_{eff} = \beta_z^{SW}/k_0$ and that of a groove waveguide as $n_{eff} = \beta_z^{I}/k_0$, 

![Fig. 1. Schematic of the corrugated interface and validity of the IBC.](image-url)
as follows from the explicit forms of (2) and (3)
\[
\frac{n^{I}_{\text{eff}}}{\sqrt{\varepsilon_0}} = f_{n^{I}_{\text{eff}}} \left( a k_0 \sqrt{\varepsilon_0}, \sqrt{\varepsilon_0} \right) \tag{4}
\]
\[
\frac{n^{II}_{\text{eff}}}{\sqrt{\varepsilon_0}} \simeq f_{n^{II}_{\text{eff}}} \left( h k_0 \sqrt{\varepsilon_0}, a k_0 \sqrt{\varepsilon_0}, d k_0 \sqrt{\varepsilon_0}, \sqrt{\varepsilon_0} / \varepsilon_m \right). \tag{5}
\]
We will use (4) and (5) later in the article to evaluate the surface wave cut-off frequencies and their dispersion in a perfect isolator. We note that for a surface wave to be properly defined, one

wave must satisfy several physical conditions

\[
\text{Re}(\beta^I_{x}) > 0, \quad \text{Im}(\beta^I_{x}) > 0 \tag{6}
\]
as all the materials in the system have real and positive values of the dielectric constants and no coherent back reflection is expected for corrugation with a deeply subwavelength period. At the same time, a properly defined physical solution requires that electromagnetic energy be localized near the corrugated surface while decaying away from the interface in both zones II and III. Thus, the signs of \( k^\text{SW,II}_x \) and \( k^\text{SW,III}_x \) in (2) and (3) must be chosen so that

\[
\text{Im}(k^\text{SW,II}_x) > 0, \quad \text{Im}(k^\text{SW,III}_x) < 0. \tag{7}
\]
We generally find that in the case of a forward traveling surface wave with dispersion relation respecting (6) and (7), and similar to the case of a classic Brewster wave [21], the wave phase front in zone II is traveling toward the interface, while in zone III it travels away from the interface

\[
\text{Re}(k^\text{SW,II}_x) < 0, \quad \text{Re}(k^\text{SW,III}_x) < 0. \tag{8}
\]
This is also the reason why Brewster waves are closely related to the phenomenon of a Brewster angle [65]. Also, the modal propagation distance \( L_x \) as well as modal extent into the filler dielectric \( L^I_{II} \) can be defined as distances over which the fields decay by a factor \( e \) in the corresponding directions (+X, +Z)

\[
L_x = 1/\text{Im}(\beta^I_{x}) \tag{9}
\]
\[
L^I_{II} = 1/\text{Im}(k^\text{SW,II}_x). \tag{10}
\]
We note that for a surface wave to be properly defined, one must require that its transverse sizes be much smaller than the propagation distance \( L^I_{II} \ll L_x \).

A. Case of Infinite Positive Permittivity \( \varepsilon_m = +\infty \)

In this section, a case of a corrugated structure made of a perfect isolator (\( \varepsilon_m = +\infty \)) will be considered to find the surface wave cut-off frequencies and their dispersion in this simple limit. By considering \( \varepsilon_m = +\infty \) one can greatly simplify the analytic dispersion relations (2), (3) as follows:

\[
\beta^I_{x} = \pm \sqrt{\varepsilon_0 k_0^2 - \left( \frac{2\pi l}{a} \right)^2}, \quad l = 0, 1, 2 \ldots \tag{11}
\]
\[
k^\text{SW,II}_x \sin(\beta^I_{x} \frac{d}{2}) = i \beta^I_{x} \tan(\beta^I_{x} h). \tag{12}
\]
These expressions allow finding frequency regions of surface wave existence. Thus, a new branch of the surface wave dispersion relation will be pulled from the radiation continuum (lower cut-off frequency) when \( k^\text{SW,II}_x = 0 \rightarrow \beta^I_{x} h = \pi \cdot l_c \), while the same branch will be terminated at higher frequencies (upper cut-off frequency) when the surface wave propagation constant becomes infinite when \( \tan(\beta^I_{x} h) = \infty \rightarrow \beta^I_{x} h = \pi / 2 \cdot l_c \). From (11) we then find the following expression for the frequency regions of existence of the surface waves:

\[
v_c \in \frac{c}{\sqrt{\varepsilon_0}} \left( \sqrt{\left( \frac{l}{a} \right)^2 + \left( \frac{l_c}{2h} \right)^2} \right)^2 + \sqrt{\left( \frac{l}{2h} \right)^2 + \left( \frac{l_c + 0.5}{2h} \right)^2} \tag{13}
\]

While for the fundamental branch \( (l = 0; l_c = 0) \) it translates into \( v_c \in (0, c/(4h\sqrt{\varepsilon_0})) \).

Expression (12) can be further simplified in the regime of subwavelength corrugation \( |\beta^I_{x}| d \ll 1 \), where surface wave dispersion relation and its penetration depth into the filler dielectric \( L^I_{II} \) can be written as

\[
\beta^I_{x} = \sqrt{\varepsilon_0 k_0^2 + \left[ \frac{\beta^I_{x} \tan(\beta^I_{x} h)}{d} \right]^2} \tag{14}
\]
\[
L^I_{II} = \frac{1}{\beta^I_{x} \tan(\beta^I_{x} h)} \tag{15}
\]
From these expressions, it follows that the surface wave confinement in the filler dielectric can become subwavelength \( L^I_{II} / k_0 < 1 \) when certain conditions on the groove depth are satisfied. Thus, for the fundamental mode of a groove \( \beta^I_{x} = \sqrt{\varepsilon_0 k_0} \), condition of subwavelength confinement at the surface vicinity in the filler dielectric becomes

\[
\tan(\sqrt{\varepsilon_0 k_0} h) > \left( \frac{d}{a} \right) \cdot \frac{1}{\sqrt{\varepsilon_0}} \tag{16}
\]

Signifying that subwavelength confinement is observed mostly in the vicinity of an upper cut-off frequency. In Fig. 2(a) (red dashed curve), we present a typical dispersion relation of a surface wave using analytical dispersion relation (14) for \( d = 1.5 a \) while assuming that a subwavelength corrugation \( |\beta^I_{x}| d \ll 1 \) holds for all the parameters and all frequencies.

A subwavelength corrugation period \( d = 45 \mu m < \lambda_0 / 5 \) along with the depth of the grooves \( h = 75 \mu m \) are chosen so that the upper cut-off frequency of the fundamental branch is at \( v_c = 1 \text{ THz} \ (\lambda_0 = 300 \mu m) \), while we suppose that filler dielectric is air \( \varepsilon_0 = 1 \). In black curve, we present numerical solution of (11), (12) solved with the Newton–Raphson iteration method for the same structure. The gray region represents the regime where the condition of subwavelength corrugation does not hold. The surface wave penetration depths \( L^I_{II} / \lambda \) into the filler dielectric as calculated using the dispersion relation in the long wavelength limit (14) (solid curve), and numerically solved one (12) (dashed curve) are presented in Fig. 2(b). The subwavelength localization of the surface wave near the interface is possible near the upper cut-off frequency. Finally, we note that when approaching the upper cut-off frequency, solutions of (11), and (12) might also give high values of the surface wave propagation constant \( |\beta^I_{x}| d \gtrsim 1 \), which
eventually results in breaking down of the long wavelength approximation that was used to derive equations (3), (12). Thus, both expressions (12) and (14) are expected to give invalid solutions in the gray area of Fig. 2. Alternatively, when the subwavelength corrugation condition is respected, both dispersion relations (12) and (14) give similar results and can be used to represent the dispersion relation of a surface wave in the limit of $\varepsilon_m \to \infty$. Also, we note that when tighter confinement of a surface wave near surface is desired, one must resort to smaller periods $d$, that will allow operation closer to the upper cut-off frequency.

**B. Case of Finite Positive Permittivity $\varepsilon_m$**

Governing equations for the dispersion relation of a surface wave with finite permittivity $\varepsilon_m$ as given by (2) and (3) can be simplified in various limits. First of all, we note that in the case of subwavelength corrugation $ak_0 \ll 1$, (2) can be solved analytically via linearization as $|k_y^1|a/2 \ll 1$ for any choice of $a$, and $k_0$ (indeed, linearization of (2) only requires that $ak_0 \ll 2/s_0^1/\varepsilon_0/m$. Furthermore, assuming a long wavelength limit $|\beta^1_z|/d \ll 1$ we get an analytic expression for the effective permittivity of a lossy fundamental surface wave

$$\beta^1_z = \sqrt{\varepsilon_0 k_0^2 + i \frac{2k_0 \varepsilon_0}{a \varepsilon_m}}$$  \hspace{1cm} (17)

Note that in this regime, the two dispersion relations are only functions of a few non-dimensional parameters namely $h k_0/\sqrt{\varepsilon_0}$, $a/d$, $\sqrt{\varepsilon_0}/\sqrt{\varepsilon_m}$ and $1/ak_0/\sqrt{\varepsilon_m}$ such as

$$n_{eff}^{SW} = \frac{n^1_{eff}}{\sqrt{\varepsilon_e}} \left( \frac{1}{ak_0/\sqrt{\varepsilon_m}} \right)$$  \hspace{1cm} (19)

$$n_{eff}^{SW} \simeq f_n^{SW} \left( h k_0/\sqrt{\varepsilon_e}, a/ \sqrt{\varepsilon_m}, \sqrt{\varepsilon_0}/\sqrt{\varepsilon_m}, 1/ak_0/\sqrt{\varepsilon_m} \right).$$  \hspace{1cm} (20)

Understanding validity condition $|\beta^1_z h| \ll 1$ for (18) is somewhat complicated as in the vicinity of the upper cut-off frequency $\beta^1_z h \sim \pi/2$, the values of $\tan(\beta^1_z h)$ become large and the surface wave effective refractive index can also become large $(n_{eff}^{SW} \sim \sqrt{\varepsilon_0})$ both in its real and imaginary parts. Thus, for relation (18) to be valid for any choice of parameters, $a$, $d$, and $h$ in the vicinity of a cut-off frequency $\beta^1_z h \sim \pi/2$, one must require that $ak_0 \ll 1/\sqrt{\varepsilon_m}$. This requirement of extremely subwavelength grooves can lead to significant losses of surface waves due to inefficient confinement, and as a consequence, high radiation losses in such grooves are as follows from (17). In practice, to minimize radiation loss in the grooves one would rather choose $ak_0 \gg 1/\sqrt{\varepsilon_m}$, therefore (18) is not expected to be valid in the immediate vicinity of the upper cut-off frequency $\beta^1_z h \sim \pi/2$, while it will be generally valid somewhat away from the cut-off frequency. Thus, assuming $ak_0 \gg 1/\sqrt{\varepsilon_m}$ and $\sqrt{\varepsilon_0}/\sqrt{\varepsilon_m} \ll \tan(h k_0/\sqrt{\varepsilon_0}) \ll \sqrt{\varepsilon_m}/\sqrt{\varepsilon_e}$, (17) and (18) can be further simplified

$$\beta^1_z \simeq \sqrt{\varepsilon_0 k_0^2 + i \frac{\varepsilon_0}{a \varepsilon_m} \sqrt{\varepsilon_e}}$$  \hspace{1cm} (21)

$$n_{eff}^{SW} \simeq \varepsilon_0 + \frac{a \beta^1_z}{d k_0} \tan(\beta^1_z h)^2.$$  \hspace{1cm} (22)

Furthermore, in the case of a relatively shallow corrugation $\sqrt{\varepsilon_0}/\sqrt{\varepsilon_m} \ll \sqrt{\varepsilon_0} k_0 h \ll 1$, this can be further simplified to give a relatively simple expression for the surface wave refractive index. It can be also verified that in this limit surface wave is well defined in the sense that its penetration depth into zone II is much smaller than its propagation length

$$n_{eff}^{SW} \simeq \sqrt{\varepsilon_0} + \frac{\varepsilon_0^{3/2}}{2} \left( \frac{a k_0 h}{d k_0} \right)^2 + 2 \left[ \frac{a k_0 h}{d k_0} \right]^2 \varepsilon_0^{3/2} \frac{\varepsilon_0}{k_0a \sqrt{\varepsilon_m}}. \hspace{1cm} (23)$$

**C. Spatial Localization and Travel Distance of Surface Waves**

In Fig. 3, we study in more detail the choice of design parameters and their effect on the propagation distance and localization of surface waves in the semi-infinite cladding region (zone II). Our main question is whether surface waves are well defined. We consider that a surface wave is well defined when its propagation distance is much larger than
the wave extent into a lower permittivity cladding $L_z^{II} < L_x$ as defined in (9) and (10). In this study, we resort to the numerical solution of the full formulations (2), and (3) for the dispersion relation of the fundamental surface wave. Particularly, from analysis of (17) to (20) derived under the assumption of subwavelength corrugation ($\beta^SW d < 1$) it follows that a critical design parameter defining surface wave loss is $1/ak_0\sqrt{\varepsilon_m}$ as it also defines radiation losses of a single groove waveguide. Next, without the loss of generality, we fix $d = 1.5a$ and $v = 1$ Hz, and assume that the filler dielectric is air $\varepsilon_0 = 1$. We then study dependence of the normalized surface propagation distance $L_x k_0/\sqrt{\varepsilon_0}$, as well as the wave penetration into the filler dielectric $L_z^{II} k_0/\sqrt{\varepsilon_0}$ as a universal function of the corrugation depth parameter $h k_0/\sqrt{\varepsilon_0}$ and the groove waveguide loss parameter $1/ak_0\sqrt{\varepsilon_m}$ (see Fig. 3). Note that in the limit of low losses $1/ak_0\sqrt{\varepsilon_m} \ll 1$, we expect the maximal confinement near the upper cut-off frequency defined by $h k_0/\sqrt{\varepsilon_0} = \pi/2$. We also present results for several values of the high permittivity dielectric $\varepsilon_m = 100, 1000$, and 10,000. Additionally, in Fig. 3 in gray, we identify the regions where the subwavelength corrugation condition $\beta^SW d < 1$ does not hold, and, therefore, the found dependencies are not universal (or might have limited validity) when using the nondimensional parameters $1/ak_0\sqrt{\varepsilon_m}$, $h k_0/\sqrt{\varepsilon_0}$ for fixed values of $a/d$ and $(\varepsilon_0/\varepsilon_m)$ as predicted by (19), (20). Black regions in Fig. 3 define parameter space, where either numerical solution is not found or physical conditions (6), (7) for the existence of a forward propagating surface wave are not respected. Finally, dashed curves in Fig. 3 border the regions where the modal confinement in the filler dielectric becomes subwavelength $L_z^{II} = \lambda/\sqrt{\varepsilon_0}$ or where the surface wave becomes well defined $L_z^{II}/L_x = 1$. Thus, a design parameter space that would, for example, result in a well-defined surface wave with subwavelength extent into the filler dielectric will be between the two dashed lines shown in Fig. 3 (see 2nd and 3rd rows).

From the presented data we conclude that subwavelength confinement of a surface wave can be achieved near the upper cut-off frequency (larger values of $h k_0/\sqrt{\varepsilon_0}$) and propagation distance $L_x$ can become smaller than the modal transverse size $L_z^{II}$, thus rendering waves ill-defined. However, by somewhat reducing the groove depth $h k_0/\sqrt{\varepsilon_0}$ (while still operating outside of the gray region) allows the propagation distance $L_x$ to increase while retaining strong modal confinement near the interface. Finally, increasing the permittivity $\varepsilon_m$, allows to reduce the propagation loss of a surface mode, extend its propagation distance $L_x$, as well as to increase the validity region of the subwavelength corrugation condition $\beta^SW d < 1$ (resulting in smaller gray areas in Fig. 3). Validity of the subwavelength corrugation condition can be also extended by choosing a deeply subwavelength period $d$.

### III. Physical Meaning of the Surface Waves on Non-Conductive Interfaces

In the previous sections, a dispersion relation for the surface wave propagating along a corrugated non-conductive interface was derived and studied by using field expansions in terms of planewaves in various regions of a structure, while enforcing the field continuity and boundary conditions. While mathematically sound, the physical meaning of the surface waves was somewhat obscured. To demystify the nature of surface waves on the corrugated non-conductive interfaces, a simple effective medium model for the interfacial layer can be used. Assuming optically thin features of the corrugated surface $a \ll \lambda, (d - a) \varepsilon_m \ll \lambda$, as well as a period which is much smaller than the corrugation height $d \ll h$, the corrugated interface can be approximated as a uniform anisotropic layer of thickness $h$. The effective permittivity of the corrugated layer can be described using a diagonal tensor $(\varepsilon^x_c, \varepsilon^y_c, \varepsilon^z_c)$, where

\[
\varepsilon^x_c = d \left( \frac{a}{\varepsilon_0} + \frac{d - a}{\varepsilon_m} \right)^{-1} \quad (24)
\]

\[
\varepsilon^y_c = \varepsilon^z_c = \frac{a}{d} \varepsilon_0 + \frac{d - a}{d} \varepsilon_m. \quad (25)
\]

The interfacial layer is sandwiched between two homogeneous isotropic non-conductive materials with $\varepsilon_0 > 0, \varepsilon_m > 0$, while the permittivity constants satisfy $\varepsilon_0 < \varepsilon^x_c < \varepsilon^y_c = \varepsilon^z_c = \varepsilon_m$ as illustrated in Fig. 4. As we will show in the following, the interfacial layer supports a leaky mode that is guided along the interface, while showing evanescent decay into the top cladding and radiation leakage into the bottom cladding. This guided leaky mode is the surface mode analogous to what is described in the previous sections.

Indeed, following [66], a guided mode supported by the anisotropic interfacial layer can be found using a Transfer Matrix theory. Particularly, assuming only outgoing waves...
Fig. 4. Schematic of a corrugated coating, where the second layer can be approximated as a homogeneous layer with an effective permittivity $\varepsilon_c$ when the structure is operating in subwavelength regime.

Fig. 5. Typical dispersion relation of surface wave supported by the corrugated surface. (a) Complex dispersion relation of a fundamental surface wave as found by solving (2), (3) using Newton–Raphson iteration method (solid curves) are compared to those found by COMSOL (dashed curves). (b) Propagation distance $L_x$ (blue, cyan) and penetration depth into air $L_{II}^z$ (red, magenta) in units of $\lambda$. (c) Distribution of the electric and magnetic field amplitudes in air at 0.40 and 0.80 THz found using COMSOL. Here, $d = 50 \, \mu m$, $a = 35 \, \mu m$, $h = 75 \, \mu m$, $\varepsilon_0 = 1$, and $\varepsilon_m = 100$.

where $B \cdot \exp(i\beta x - i k_0^z z)$ in the top cladding, and $A \cdot \exp(i\beta x + i k_0^z z)$ in the bottom claddings, where

$$k_{0,m}^z = \sqrt{\varepsilon_{0,m} k_0^2 - \beta^2}$$

(26)

then, the amplitudes of the two outgoing waves are related by a $2 \times 2$ transfer matrix $T$ (see in the Appendix for details):

$$\begin{bmatrix} A \\ 0 \end{bmatrix} = T \begin{bmatrix} 0 \\ B \end{bmatrix}.$$ 

(27)

Solving (27) allows finding a complex dispersion relation of the interfacial leaky mode as detailed in the Appendix. In particular, in the case of $\varepsilon_m \to \infty$ we find that $\varepsilon_c^x \approx d/(a \varepsilon_0)$, $\varepsilon_c^z = \varepsilon_c^x \approx (d - a)/(d \varepsilon_m) \to \infty$, $k_c^z \approx \sqrt{\varepsilon_c} k_0$, which allows one to find dispersion relation of the guided mode in a simple analytical form

$$\varepsilon_{SW}^{\text{eff}} = \frac{\beta^2}{k_0^2} \leq \varepsilon_0 \left(1 + \frac{a}{d} \tan^2 \left(\sqrt{\frac{d}{a}} \varepsilon_0 k_0 h\right)\right).$$

(28)

Interestingly, the obtained dispersion relation (28) is somewhat different from the one found earlier (14). The difference stems from different approximations made in the derivation of these dispersion relations. Particularly, to approximate the corrugated layer as a uniform anisotropic dielectric it is important that all the corrugation features are optically thin $a \ll \lambda/\varepsilon_0$, $(d - a) \ll \lambda/\varepsilon_m$. Thus, in the case of a perfect isolator $\varepsilon_m \to \infty$, its thickness in the corrugation has to
be vanishingly small \((d - a) \to 0\). This is in contract with the assumptions made in the previous sections when deriving dispersion relation (14). There, we only assume a subwavelength corrugation period \(d \ll \lambda/\varepsilon_0\), while we put no limitation on the optical thickness of a perfect isolator. Instead, we assume that whatever radiation enters the perfect isolator layer, it is incoherently scattered or absorbed within the isolator.

IV. ANALYTICAL VersUS NUMERICAL RESULTS USING IBC

In this section, we study in more detail the dispersion relation of surface waves on corrugated surfaces as a function of its various parameters. The studied geometry is presented in Fig. 1(a), with all the boundaries between the filler dielectric and the high dielectric constant material being of the IBC type. The results of semi-analytical calculations (2) and (3) are verified using a finite element COMSOL mode solver with Floquet boundary conditions along the modal propagation direction (see more details in the Appendix). Fig. 5 compares dispersion relations of the fundamental surface wave supported by the corrugated surface made of materials with purely real and positive permittivity as obtained by solving equations (2), (3) using the Newton–Raphson iteration method (solid), with that computed using COMSOL solver (dashed). Here, we use \(d = 50 \, \mu m\), \(a = 35 \, \mu m\), and \(h = 75 \, \mu m\), and frequencies in the (0.1–1 THz) range. For the materials, we use \(\varepsilon_0 = 1\) and \(\varepsilon_m = 100\), which corresponds respectively to the relative dielectric permittivity of air and TiO\(_2\) [67]. The upper cut-off frequency for the fundamental surface mode is given by (13) where \(l_c = 0\), \(l = 0\), and it equals to 0.95 THz. Real and imaginary parts of the corresponding dispersion relation are shown in Fig. 5(a), while the corresponding modal propagation distance and transverse size are shown in Fig. 5(b). Note that near the upper cut-off frequency, modal transverse size decreases dramatically and quickly becomes subwavelength [see Fig. 5(c)]. However, when operating too close to the upper cut-off frequency, there is a significant increase in the radiation loss into zone III, thus leading to much shorter propagation distances. As seen from Fig. 5, there exists an optimal spectral region where transverse modal confinement is subwavelength, while propagation distance is larger than the wave transverse size.

Next, we study the effects of the groove width to period ratio \(a/d\), and groove height \(h\) on the surface wave dispersion, as well as the validity of the semi-analytical formulation (2), (3) when describing such waves. In Fig. 6, we present modal dispersion relation when varying groove width \(a\) for a fixed period \(d = 50 \, \mu m\) and groove height \(h = 75 \, \mu m\). From these results, we observe that increasing groove width \(a\) somewhat increases the upper cut-off frequency of the fundamental surface wave, while it significantly increases the modal losses. For high enough values of \(a\), dispersion relation of a surface wave crosses the light line of the filler dielectric and extends further toward higher values of the propagation constant, which leads to stronger modal confinement at the interface. To understand
Fig. 7. Propagation constant of surface wave for various groove height $h$. (a) Real part and (b) imaginary part of the propagation constant of the fundamental surface wave for three structures of increasing groove height $h = 50 \mu m$, $h = 75 \mu m$, and $h = 100 \mu m$. (c) Propagation distance $L_x$ and (d) penetration depth into air $L_{II}$ in units of $\lambda$. Results are found by solving (2), (3) using Newton–Raphson iteration method (solid curves) are compared to those found by COMSOL (dashed curves). The circles presented in (c) and (d) indicate the limit where the surface wave is well-defined $L_x/L_{II}^z = 1$. The period $d$ is $50 \mu m$, the groove width $a$ is $25 \mu m$ and the permittivity constants are $\varepsilon_0 = 1$ and $\varepsilon_m = 100$.

this, we note that larger groove sizes lead to a stronger modal presence in the grooves, as well as stronger radiation losses at the bottom of the planar waveguides (zone I) that radiate into zone III through their openings of size $a$.

Finally, in Fig. 7, we study the effect of the groove height on the modal dispersion relation, where $d = 50 \mu m$, and $a = 25 \mu m$. Here, as expected, we observe that deeper grooves result in lower cut-off frequencies and higher propagation losses. To appreciate the effect of the groove height on the dispersion relation, in Fig. 8, we also present the results at a fixed operational frequency of $1$ THz, and fixed period $d = 50 \mu m$ and groove width $a = 25 \mu m$, while varying the groove height $h$. From this data, we see that choosing shallow grooves (small $h$) results in longer propagation distances of a surface wave, as well as its stronger presence in the filler dielectric. Higher losses for deeper grooves are easy to understand by noting that radiation in a surface wave is partially guided along the leaky waveguide (zone I), so longer waveguides naturally result in higher (almost exponentially with $h$) losses. By increasing the groove height one can, in principle, retain acceptable modal propagation distances, while also increasing modal confinement to within several wavelengths at the structured surface.

V. VALIDITY OF THE IBCS AND LIMITATIONS

Up to now, the IBCs have been used to demonstrate the possibility of propagation of surface waves along corrugated structures made of non-conductive materials. Physically, IBC limits the field presence only to the low dielectric permittivity filler region, while neglecting the effects of field penetration into the high dielectric permittivity material. In reality, the electromagnetic fields localized at the corrugated interface can penetrate into structured high dielectric permittivity material and interfere within the notches. Although the electric field will be vanishingly small compared to the magnetic field in the limit $\varepsilon_m \rightarrow +\infty$, this alone does not prevent coupling of the fields in adjacent grooves via a non-zero magnetic field in the notches. Therefore, even in the limit of infinitely high $\varepsilon_m$, one still has to rely on one of the several decoherence mechanisms such as scattering, absorption, or radiation within the notches (see Fig. 1), so that the formulation using IBC remains valid. In practice, however, we often observe that numerical simulation of a complete system that includes high dielectric permittivity material indeed match those with IBC. This happens because the interaction between adjacent notches is suppressed via a combination of destructive interferences of the electromagnetic fields inside the notches, as well as strong radiation loss into the semi-infinite high permittivity dielectric from the narrow notches. For example, in Fig. 9, we compare dispersion relations of surface waves as computed using either a semi-analytical formulation (2), (3) with IBC, or using COMSOL with all the zones I–III explicitly present in simulations (zone III is terminated with the Perfectly Matched Layer absorbing boundary domain, for more information see in the Appendix). As seen from the figure, at most frequencies there is a good comparison between the two dispersion relations except for certain frequencies where resonance is excited inside the notches as seen in Fig. 9(c). We, therefore, conclude
Fig. 8. Propagation constant and modal sizes as a function of the groove depth $h$ at 1 THz operation frequency. (a) Real and imaginary parts of the fundamental surface wave propagation constant as a function of the groove depth. (b) Propagation distance $L_x$ and penetration depth into air $L^p_z$ in units of $\lambda$. Results are found by solving (2), (3) using Newton–Raphson iteration method (solid curves) are compared to those found by COMSOL (dashed curves). Here, $a = d/2$, $d = 50 \, \mu m$ and the permittivity constants are $\varepsilon_0 = 1$ and $\varepsilon_m = 100$.

Fig. 9. Dispersion relation of surface waves computed using full geometry of the corrugated surface (zones I, II, and III, without IBC) terminated with a perfectly matched layer. (a) Real (blue, cyan) and imaginary (reg, magenta) parts of the surface wave propagation constant as found using COMSOL simulation of a complete geometry (solid), and as computed equations (2), (3) (dotted). (b) Propagation distance (blue, cyan) and penetration depth into air (red, magenta) in units of $\lambda$. (c) Distribution of the electric and magnetic fields at various frequencies 0.50 (no resonance in the notch), 0.75, 1.00 (resonance in the notch), 1.05, 1.25, and 1.50 THz. Here, $d = 50 \, \mu m$, $a = 35 \, \mu m$, $h = 25 \, \mu m$, $\varepsilon_0 = 1$ and $\varepsilon_m = 100$.

that our semi-analytical formulations (2) and (3) derived using IBC, can indeed describe the dispersion relation of surface waves propagating on non-conductive corrugated surfaces. This semi-analytical model, however, is limited when adjacent grooves are strongly coupled via resonant field excitation inside of the high permittivity notches. One way to ensure the validity of the IBC approximation is to use strongly scattering or strongly absorbing high-permittivity material in
the notches to avoid field coupling in the adjacent grooves. Then, for a given notch width, one can test the limits of validity of the IBC approximation by comparing the results of the numerical simulations with IBC to those with high-loss material in the notches. For example, in Fig. 10, we fix the operational frequency at 1 THz and present the surface wave propagation constant as a function of the imaginary part of the refractive index of the high permittivity material while using the structure described in Fig. 9. The operational frequency was chosen to correspond to a resonant frequency at which a strong resonance is excited in the notches of a structure (assuming loss-less materials), thus inducing a strong coupling between the adjacent grooves, and invalidating the analytical derivations (2), (3). From Fig. 10, we see that for lower-loss materials in the notches (low values of the material RI imaginary part), results using IBC can deviate significantly from those using a full structure. However, when increasing losses of the notch material, results using IBC match well those obtained using the full structure calculations, as fields in the notches become strongly attenuated at higher values of the material losses. Thus, the semi-analytical formulation (based on the IBC approximation) is valid for subwavelength structures as long as coupling between adjacent grooves through the notches is somehow suppressed.

Similarly, for a given high-permittivity material, it is important to test the limits with respect to the corrugation period being sufficiently subwavelength for the approximation (2) and (3) to hold. To this end, in Fig. 11, the propagation constant of a surface wave is presented as a function of the normalized corrugation period $d/\lambda$, as calculated by using either an analytical model or the full structure simulations. For too small of a period $d/\lambda < 0.1$, one observes significant deviations between the analytic formulation and full structure simulations due to limitations of the IBC approximation (thin notches). At too large of a period $d/\lambda > 0.4$, surface modes become affected by the Bragg diffraction on the periodic structure. Finally, for the intermediate periods $0.1 < d/\lambda < 0.4$, they are small enough to disregard diffraction on the corrugated surface, while it is large enough to absorb the fields in the notches and render the IBC approximation valid. In this regime, analytical formulations (2) and (3) agree well with the results of full simulations.

VI. CONCLUSION

In this article, we demonstrate a new type of surface wave that propagates along periodically corrugated surfaces made of non-conducting materials featuring high positive permittivity. The existence of such waves is due to the possibility of the electromagnetic energy confinement in the planar groove waveguides limited by the high dielectric permittivity walls. Unlike the case of surface waves on structured metallic surfaces where electromagnetic energy is exponentially fast decreasing into metal, in our case, significant radiation presence is possible in the high dielectric permittivity material. Thus, careful consideration of radiative effects is important for the proper design of surface waves on non-conducting surfaces. An analytic derivation of the surface wave dispersion
relation is presented in the case of the deeply subwavelength period of a corrugated structure using IBCs at the interfaces. Thus, the obtained dispersion relation is then verified numerically using COMSOL Multiphysics software on a full system comprising a filler dielectric, a structured high permittivity dielectric, as well as radiation absorbing boundary conditions, and an overall good agreement between semi-analytical theory and numerical simulations is obtained. Finally, we confirmed that there exists a region of design space where surface wave on a non-conducting interface can be tightly confined at such an interface with its transverse size comparable to or even smaller than the wavelength of light in the filler dielectric. Additionally, such surface waves can feature propagation distances much longer than their transverse size, thus making such surface waves like the regular guided modes for practical purposes. As surface waves require high dielectric permittivity materials for their experimental realization, we believe that longer wavelengths spectral ranges (THz, microwave, etc.) are the most suitable for the demonstration of such waves due to the abundance of dielectrics with relative permittivity higher than 100.

APPENDIX

A. Spatial Localization and Travel Distance of Surface Waves

In the grooves (zone I, in Fig. 1), we have two waves propagating along the z-direction with propagation constants $\beta^I_z$ and $-\beta^I_z$. The wave propagating toward the positive $z$-direction come from the reflection at the bottom of the groove at $z = -h$. Also, the structure in zone I is similar to a waveguide made of two parallel planar materials of permittivity $\varepsilon_m$ separated by a dielectric of permittivity $\varepsilon_0$. Thus, for a TM mode inside the groove waveguide, one can easily solve the Maxwell equations and find the following magnetic and electric components [66]:

\[
\begin{align*}
H_x^+ &= A^+ \cos(k_{x}^I x) e^{i\beta^I_z z} \\
E_x^+ &= -i A^+ \frac{k_{x}^I}{\varepsilon_0 k_0} \sin(k_{x}^I x) e^{i\beta^I_z z} \\
E_x^- &= A^+ \frac{\beta^I_z}{\varepsilon_0 k_0} \cos(k_{x}^I x) e^{i\beta^I_z z}
\end{align*}
\]

where $A^+$ is a constant, and $k_{x}^I$ and $\beta^I_z$ are related as:

\[
k_{x}^I = \sqrt{\varepsilon_0 k_0^2 - (\beta^I_z)^2}.
\]

Using the impedance boundary conditions (33) with the expressions (29) and (30), we get:

\[
k_{x}^I \tan(k_{x}^I a/2) = -i \frac{k_0 \varepsilon_0}{\sqrt{\varepsilon_m}} \quad (36)
\]

where equation (36) with the relation (32) can be simplified as follows when the argument $k_{x}^I a/2$ is small:

\[
(k_{x}^I)^2 \simeq -i \frac{\varepsilon_0 k_0}{\sqrt{\varepsilon_m}} \quad (37)
\]

\[
\Rightarrow \beta^I_z = \sqrt{\varepsilon_0 k_0^2 - (k_{x}^I)^2} \simeq \sqrt{\varepsilon_0 k_0} + i \frac{\varepsilon_0}{\sqrt{\varepsilon_m}} \quad (38)
\]

A numerical method such as the Newton–Raphson iteration method can also be used to find the expression of $\beta^I_z$ from equations (32) and (36). The total electric and magnetic components in the first zone can then be written as the combination of the forward and backward propagating waves along the $z$-direction:

\[
\begin{align*}
H_y^I &= \cos(k_{x}^I x) \left[ A^+ e^{i\beta^I_z z} + A^- e^{-i\beta^I_z z} \right] \\
E_z^I &= -i A^+ \frac{k_{x}^I}{\varepsilon_0 k_0} \sin(k_{x}^I x) \left[ A^+ e^{i\beta^I_z z} + A^- e^{-i\beta^I_z z} \right] \\
E_x^I &= \frac{\beta^I_z}{\varepsilon_0 k_0} \cos(k_{x}^I x) \left[ A^+ e^{i\beta^I_z z} - A^- e^{-i\beta^I_z z} \right]
\end{align*}
\]

where $A^+$ and $A^-$ correspond to a wave with the propagation constant $\beta^I_z$. The relation between the coefficients $A^+$ and $A^-$ can be found using IBC at $z = -h$ and equations (39) and (41):

\[
A^+ = A^- e^{2i\beta^I_z h} \frac{\varepsilon_0 k_0^2 - \varepsilon_m k_0^2}{\varepsilon_0 k_0^2 + \varepsilon_m k_0^2}.
\]

Because of the impedance boundary condition and radiative loss at the waveguide boundaries, the amplitude of the wave propagating toward the positive $z$ direction is smaller than the wave propagating toward the bottom of the cavity. Also, the groove depth $h$ controls the phase difference between these two waves. These observations suggest that the groove depth will have a significant impact on the complex dispersion relation of the surface wave supported by the corrugated surface.

Next, the magnetic and electric components of the wave propagating along the $x$-axis in the region above the surface ($z > 0$) are given by:

\[
\begin{align*}
H_y^{II} &= \sum_n B_n e^{i\beta^{II}_x x} e^{ik^{II}_z z} \\
E_x^{II} &= \sum_n B_n \frac{k_{x}^{II,n}}{\varepsilon_0 k_0} e^{i\beta^{II}_x x} e^{ik^{II}_z z} \\
E_z^{II} &= -\sum_n B_n \frac{\beta^{II}_x}{\varepsilon_0 k_0} e^{i\beta^{II}_x x} e^{ik^{II}_z z}
\end{align*}
\]

\[
(k_{x}^{II,n})^2 = \varepsilon_0 k_0^2 - (\beta^{II}_x)^2; \quad \beta^{II}_x = \beta^{SW}_x + G_n,
\]

where $G_n = 2\pi n/d$ comes from the Bloch periodicity. At $z = 0$, we must match the components from zone I and II above the grooves $x \in (-a/2, a/2)$ and respect the impedance boundary condition otherwise for $x \in (-d/2, a/2) \cap (a/2, d/2)$. For the first case above the grooves, we have:

\[
\begin{align*}
H_y^{II} \big|_{z=0} &= H_y^I \big|_{z=0} \\
E_x^{II} \big|_{z=0} &= E_x^I \big|_{z=0}
\end{align*}
\]
While for the impedance boundary condition at the interface
at \( x \in (-d/2, -a/2) \cap (a/2, d/2) \):
\[
H_{I}^{II}|_{z=0} = -\sqrt{\varepsilon_m} e^{i k_I^{II}x}.
\] (49)

Here, to find the expression of the effective refractive
index of the surface waves, one must solve the previous
equations and conditions. To begin, a first relation between
the coefficients \( B_n \), \( A^+ \) and \( A^- \) can be found in relations (48) and (49):
\[
E_{x}^{II}|_{z=0} = \sum B_n e^{i k_I^{II}x} = \frac{A \beta_I^n}{\varepsilon_0 k_0} \cos(k_I^{II}x), \quad x \in \left(-\frac{a}{2}, \frac{a}{2}\right)
\]
\[+ \frac{1}{\sqrt{\varepsilon_m}} \sum B_n e^{i k_I^{II}x}, \quad x \in \left(-\frac{d}{2}, -\frac{a}{2}\right) \cap \left(\frac{a}{2}, \frac{d}{2}\right)
\] (50)

where \( A = A^+ - A^- \). In the limit of \( d \ll \lambda \), only \( n = 0 \) term
in (50) is of importance, which results in the following:
\[
E_{x}^{II}|_{z=0} = B_0 e^{i k_I^{SW}x} = \frac{A \beta_I^0}{\varepsilon_0 k_0} \cos(k_I^{SW}x), \quad x \in \left(-\frac{a}{2}, \frac{a}{2}\right)
\]
\[+ \frac{1}{\sqrt{\varepsilon_m}} B_0 e^{i k_I^{SW}x}, \quad x \in \left(-\frac{d}{2}, -\frac{a}{2}\right) \cap \left(\frac{a}{2}, \frac{d}{2}\right)
\] (51)

and after integrating (51) over \( (-\frac{d}{2}, \frac{d}{2}) \) we get:
\[
B_0 = \frac{A \beta_I^0}{\varepsilon_0 k_0} \sin(k_I^{SW}d/2) + \frac{1}{\sqrt{\varepsilon_m}} \sin(k_I^{SW}d/2).
\] (52)

A third relation between \( A^+, A^- \), and \( B_0 \) can be found with
the continuity condition of the magnetic component at \( z = 0 \)
represented by equation (47):
\[
\sum B_n e^{i k_I^{II}x} = (A^+ + A^-) \cos(k_I^{II}x).
\] (53)

In the limit of \( d \ll \lambda \), only \( n = 0 \) is of importance. So,
after integrating (53) over \( (-\frac{d}{2}, \frac{d}{2}) \) we get:
\[
B_0 e^{i k_I^{SW}x} = (A^+ + A^-) \cos(k_I^{SW}x),
\] (54)

\[
B_0 = (A^+ + A^-) \frac{k_I^{SW} \sin(k_I^{SW}d/2)}{k_I^{SW}}.
\] (55)

Using equations (42), (52), and (55), \( \beta_I^{SW} \) can now be found:
\[
\left(\frac{\varepsilon_0 k_0}{\sqrt{\varepsilon_m}} + k_I^{SW,II}\right) \sin(k_I^{SW}d/2) = \beta_I^{SW} \left[ \sin(k_I^{SW}d/2) + \frac{\varepsilon_0 k_0}{\sqrt{\varepsilon_m}} \right] + \frac{\varepsilon_0 k_0}{\sqrt{\varepsilon_m}} \sin(k_I^{SW}d/2) + \frac{\varepsilon_0 k_0}{\sqrt{\varepsilon_m}} \sin(k_I^{SW}d/2)
\] (56)

where \( k_I^{SW,II} = \sqrt{\varepsilon_0 k_0^2 - (\beta_I^{SW})^2} \) and \( \beta_I^{SW} = k_0 n_i^{SW,eff} \). So far, we
found analytic expressions for the dispersion relations of
waves propagating in the grooves (36) and on the corrugated
surface (56). One can then use a numerical method such as
the Newton–Raphson iteration method to solve for \( \beta_I^{SW}, \) and \( \beta_I^{SW} \). Further
simplifications can be applied under specific conditions.
Thus, the dispersion relation can be greatly simplified
in the ideal case where the corrugated structure is made
of material with infinite positive permittivity \( \varepsilon_m \rightarrow +\infty \):
\[
\beta_I^{SW} = \sqrt{\varepsilon_0 k_0^2 + \left(\beta_I^{SW} \tan(\beta_I^{SW}h) \sin(k_I^{SW}d/2) \right)^2}.
\] (57)

Furthermore, for a subwavelength corrugation \( \beta_I^{SW}d \ll 1 \):
\[
\beta_I^{SW} = \sqrt{\varepsilon_0 k_0^2 + \left(\beta_I^{SW} \tan(\beta_I^{SW}h) \sin(k_I^{SW}d/2) \right)^2}.
\] (58)

**B. Wave Propagating in Corrugated Coating**

Here, we derive the dispersion relation of leaky waves propa-
gating along the layer interfaces in a three-layer system. The
individual layers are characterized by permittivity constants \( \varepsilon_0, \varepsilon_c, \) and \( \varepsilon_m \), where the middle layer is anisotropic and
described by a diagonal permittivity tensor \( \varepsilon_c = (\varepsilon_c^x, \varepsilon_c^y, \varepsilon_c^z) \). In our
derivations, we use a standard transfer matrix theory for TM
polarized waves adapted to treat anisotropic materials having
two of its symmetry axis aligned with the planar interfaces.
Assuming only outgoing plane waves \( B \cdot \exp(i \beta x - ik_0^z z) \) in
the top cladding, and \( A \cdot \exp(i \beta x + ik_0^z z) \) in the bottom claddings,
then the amplitudes of the two outgoing waves are related by
a \( 2 \times 2 \) transfer matrix \( T \):
\[
\left[ \begin{array}{c}
A \\
0
\end{array} \right] = \left[ \begin{array}{cc}
T & 0 \\
0 & B
\end{array} \right], \quad T = M_{c,m} M_{0,c}
\] (59)

\[
M_{c,m} = \frac{1}{2} \left[ \begin{array}{cc}
(1 + q)e^{i \phi} & (1 - q)e^{-i \phi} \\
(1 - q)e^{i \phi} & (1 + q)e^{-i \phi}
\end{array} \right],
\] (60)

\[
M_{0,c} = \frac{1}{2} \left[ \begin{array}{cc}
1 + p & 1 - p \\
1 - p & 1 + p
\end{array} \right]
\] (61)

where
\[
p = \frac{\varepsilon_c^x k_0^x}{\varepsilon_0 k_0^x}, \quad q = \frac{\varepsilon_m k_0^z}{\varepsilon_c^x k_0^x}, \quad \phi = k_0^z h
\] (62)

and the transverse wave vectors in the isotropic and
anisotropic layers are given by:
\[
k_0^x = \sqrt{\varepsilon_0 k_0^2 - \beta^2}
\] (63)

\[
k_0^z = \sqrt{\varepsilon_0 k_0^2 - (\varepsilon_c^x / \varepsilon_c^z) \beta^2}.
\] (64)

One can then readily solve the matrix equation (61) to find
the equation for the leaky mode dispersion relation:
\[
1 + pq = i \tan \phi.
\] (65)

In the case where \( |\varepsilon_m| \rightarrow \infty \), (65) can be solved analytically:
\[
\varepsilon_{eff} \simeq \varepsilon_0 \left(1 + \frac{a}{d} \tan^2 \left( k_0^z d / \varepsilon_0 h \right) \right).
\] (66)

**C. Details of COMSOL Simulations**

To validate the analytic dispersion relation given by equations
(2)–(3) along with the study of the validity of the IBC,
the finite element software COMSOL Multiphysics 5.5 has
been used to numerically solve the electromagnetic surface
waves supported by the corrugated structure. All the
simulations were performed in the frequency domain, where
one period of the structure is modeled using the Floquet
periodic conditions at \( x = \pm d/2 \). This condition within
the COMSOL’s eigenfrequency study allows finding the complex
eigenfrequency \( \nu_{num}(\beta_d) \) that corresponds to a given real propagation constant \( \beta_d \). For the structure using IBC, a simple
domain made of the filler dielectric of dielectric permittivity
\( \varepsilon_0 \) representing the combination of the regions I and II is
delimited by the periodic conditions (region I) and IBC inside
the groove (region II). The region above \( z = 0 \), which extends
up to multiple times the operational wavelengths, is terminated with a perfect matched layer (PML) domain and scattering boundary condition at the outer boundary. For the full structure (without IBC), all the three zones illustrated in Fig. 1 along with the high dielectric permittivity material $\epsilon_m$ in zone III and below $z = -h$ is included into simulation cell. In this case, the computational cell is terminated both from the top and bottom with the PML domains and scattering boundary conditions.

The eigenfrequency search method was used to numerically find the complex dispersion relation $\nu_{\text{num}}(\beta_s) = \nu_1(\beta_s) + i\nu_{\text{num}}(\beta_s)$ of the surface waves, where $\beta_s$ is real and specified in the Floquet boundary conditions. In practical applications, however, one is rather interested in the complex propagation constant of the mode $\beta_{SW}^{\nu}(\nu) = \beta_{SW}^{\nu}(\nu) + i\beta_{SW}^{W}(\nu)$ for the pure real frequencies $\nu$. Thus, for practical applications, and to compare the semi-analytic dispersion relation given by (2)–(3) along with the numerical ones, one has to transform the dispersion relation in terms of a complex frequency $\nu_{\text{num}}(\beta_s)$ given by COMSOL into dispersion relations in terms of a complex propagation constant $\beta_{SW}^{\nu}(\nu)$. Assuming that the eigenfrequency $\nu_{\text{num}}(\beta_s)$ can be computed for any complex propagation constant $\beta_s$, we then look for $\beta_s$ that result in the purely real eigenfrequencies:

$$\nu_{\text{num}}(\beta_s^r, \beta_s^i) = 0.$$  \hspace{1cm} (67)

Multiple methods can be used to solve (67). One could start with a purely real propagation constant $\beta_{s,0} = (\beta_s^r, 0)$ that would result in the complex eigenfrequency $\nu_{\text{num}}(\beta_{s,0})$, and then use an iterative method (ex. method of secants) to find the corresponding imaginary part of the propagation constant $\beta_s^i$ so that the complex $\beta_s = (\beta_s^r, \beta_s^i)$ results in the purely real eigenfrequency (67). Particularly, for the n$^\text{th}$ iteration, one uses:

$$\beta_{s,n+1}^i = \beta_{s,n}^i - \frac{\nu_{\text{num}}(\beta_{s,n}^r, \beta_{s,n}^i) \cdot \nu_{\text{num}}(\beta_{s,n}^r - \beta_{s,n-1}^r, \beta_{s,n-1}^i)}{\nu_{\text{num}}(\beta_{s,n}^r, \beta_{s,n}^i) - \nu_{\text{num}}(\beta_{s,n}^r - \beta_{s,n-1}^r, \beta_{s,n-1}^i)}.$$  \hspace{1cm} (68)

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