Constraints on background torsion from birefringence of CMB polarization

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Abstract

We show that a non-minimal coupling of electromagnetism with background torsion can produce birefringence of the electromagnetic waves. This birefringence gives rise to a B-mode polarization of the CMB. From the bounds on B-mode from WMAP and BOOMERanG data, one can put limits on the background torsion at $\xi_1 T_1 = (-3.35 \pm 2.65) \times 10^{-22}$ GeV$^{-1}$. 

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1. INTRODUCTION

In general relativity, the connection is defined through the covariant derivatives,

\[ \nabla_\mu V_\nu = \partial_\mu V_\nu + \Gamma^\alpha_{\mu\nu} V_\alpha \]  

(1)

\( \Gamma^\alpha_{\mu\nu} \) is symmetric in lower indices and is known as the Levi-Civita connection. On the other hand, in Einstein-Cartan theory \([1]\), which is an extension of general relativity, \( \Gamma^\alpha_{\mu\nu} \) is asymmetric with its symmetric part \( \{\alpha_{\mu\nu}\} \), the Levi-Civita part, also known as the Christoffel symbol and the torsion part \( T^\alpha_{\mu\nu} \) which is antisymmetric in \( \mu \) and \( \nu \) as given by,

\[ \Gamma^\alpha_{\mu\nu} \equiv \{\alpha_{\mu\nu}\} + T^\alpha_{\mu\nu} \]  

(2)

and

\[ \{\alpha_{\mu\nu}\} = \frac{1}{2} g^{\alpha\lambda} \left( \partial_\mu g_{\lambda\nu} + \partial_\nu g_{\lambda\mu} - \partial_\lambda g_{\mu\nu} \right), \quad T^\alpha_{\mu\nu} = \frac{1}{2} \left( \Gamma^\alpha_{\mu\nu} - \Gamma^\alpha_{\nu\mu} \right). \]  

(3)

In this paper we shall consider the generalized connection restricting ourselves to metric theories such that we still have \( \nabla_\mu g_{\alpha\beta} = 0 \) (metric a covariant constant) and use the conventions \( \eta^{\mu\nu} = \text{diag}(1, -1, -1, -1) \), for the metric and \( \epsilon_{123} = 1, \epsilon^{123} = -1 \), for the Levi-Civita symbol. The mathematical structure of torsion theories has been elucidated in \([2]\).

Coupling torsion to electromagnetism in a gauge-invariant fashion was first carried out by Novello \([3]\), De Sabbata and Gasperini \([4]\) and Duncan, Kaloper and Olive \([5]\). They pointed out that if the dual of the torsion tensor, \( T_\mu = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} T_{\nu\alpha\beta} \) is the divergence of a scalar, \( T_\mu = \partial_\mu \phi \), then it can be coupled to electromagnetic interactions in a gauge invariant way by the interaction \( T_\mu A_\nu \tilde{F}^{\mu\nu} = \phi F_\mu\nu \tilde{F}^{\mu\nu} \). A propagating scalar give rise to long range forces which can be constrained by the observations of energy loss of Hulse-Taylor binary pulsar \([6]\). A cosmological scalar field which couples to electromagnetism causes birefringence of electromagnetic waves causing polarization of radio signals from distant galaxies as pointed out by Harari and Sikivie \([7]\). Carroll and Field \([18]\) put strong constraints on the presence of cosmological background from the polarization of radio galaxies. Hammond \([10]\) introduced an antisymmetric two-index torsion potential \( \psi_{\mu\nu} \), related to the torsion tensor by \( T_{\alpha\mu\nu} = \partial_{[\alpha} \psi_{\mu\nu]} \) and coupled it to electromagnetism through an interaction of the form \( F^{\mu\nu} \psi_{\mu\nu} \). This interaction is similar to the kinetic mixing between photons and para-photons studied by Masso and Redondo \([11, 12]\). If the scalar-torsion fields have heavier mass, then there are...
laboratory constraints on their coupling to electromagnetism in optical polarization of lasers in external magnetic field \cite{15, 16} as proposed in \cite{13, 14} and in long range non-Newtonian forces \cite{17}. Constraints on heavy propagating torsion which may be produced in accelerators or in supernovae have been surveyed in \cite{18, 19}.

In this paper we study non-minimal coupling of a cosmological background torsion field to electromagnetic field of the form \(\xi_1 T^{\alpha \lambda \rho \sigma} F_{\alpha \sigma} \partial_{\lambda} \tilde{F}^{\rho \sigma} + \xi_2 T^{\sigma \gamma \delta \rho} F_{\sigma \rho} \partial_{\gamma} F_{\delta \rho}\). We find that the first type of coupling gives rise to birefringence in the electromagnetic waves propagating through a background torsion field over cosmological distances. The \(\xi_1\) type of coupling is similar in structure to the form \(\frac{1}{E_{\mu} n^{\alpha} F_{\alpha \sigma} n \cdot \partial (n_\beta \tilde{F}^{\beta \sigma})\) proposed by Myers and Pospelov \cite{20} motivated by quantum gravity arguments. For second type of coupling, the transverse modes of electromagnetic waves propagates along null geodesics despite the presence of torsion coupling. Therefore, the \(\xi_2\) type of coupling has no effect on the wave propagation. These types of couplings are also similar in form to the phenomenological Lorentz violating couplings of electromagnetism studied in Kostelecky and Mewes \cite{21} who proposed that such couplings will mix the \(E\) and \(B\) mode CMB polarization due to birefringence. A detailed study of the birefringence arising from Myers-Pospelov interaction on the CMB polarization was done by Gubitosi et al. \cite{22}. By an analysis of the CMB polarization data from WMAP and BOOMERanG, Gubitosi et al. put a limit on the angle of rotation of the plane of polarization of CMB signal at \(\alpha = (-2.4 \pm 1.9)\)°. We use this result of the angle of rotation of CMB to put the limits on the non-minimal coupling of torsion with electromagnetism.

2. NON-MINIMAL TORSION COUPLING WITH ELECTRODYNAMICS

We start with the Lagrangian for the electromagnetic field on a manifold with torsion but no curvature. We consider two non-minimal couplings of torsion with electromagnetism described by,

\[
\mathcal{L} = -\frac{1}{4} F_{\mu \nu} F^{\mu \nu} + \xi_1 T^{\alpha \rho \sigma} F_{\alpha \sigma} \left(\partial_{\rho} \tilde{F}^{\sigma \rho}\right) + \xi_2 T^{\sigma \gamma \delta \rho} F_{\sigma \rho} \left(\partial_{\gamma} F_{\delta \rho}\right)
\]

(4)

We will consider the two coupling terms separately.
2.1. Case-I: when $\xi_1 \neq 0$ and $\xi_2 = 0$

Using the Euler-Lagrangian equation,

$$\partial_{\mu} \left( \frac{\partial L}{\partial (\partial_{\mu} A_{\nu})} \right) - \frac{\partial L}{\partial A_{\nu}} = 0$$

(5)

the equation of motion will be,

$$\partial_i E^i = \xi_1 T^{\alpha\lambda} \partial_{\alpha} \partial_{\lambda} B^i$$

(6)

$$\partial_0 E^j + \partial_i \epsilon^{0ijk} B_k = \xi_1 \partial_{\alpha} \partial_{\lambda} \left[ -T^{\alpha\lambda} B^j + T^{\alpha\lambda} \epsilon^{0ijk} E_k \right]$$

(7)

and from the Maxwell’s equation for $F_{\mu\nu}$ ,

$$\partial_k \epsilon^{0kjl} E_l - \partial_0 B^j = 0$$

(8)

$$\partial_j B^j = 0$$

(9)

Using the eq. (8), the eq. (7) can be written as,

$$- \partial_0 \partial_0 E^j - \partial^m \left( \partial_m E^j - \partial^j E_m \right) = -\xi_1 \partial_{\alpha} \partial_{\lambda} \left[ T^{\alpha\lambda} \epsilon^{0mjn} \partial_m E_n - T^{\alpha\lambda} \epsilon^{0ijk} \partial^0 E_k \right]$$

(10)

which is the equation of motion for the electric field. We consider the plane wave solution where, the electric field can be written as, $E_m(x) = E_m(k) e^{-ik^0x^0}$ and eq. (10) reduces to,

$$k^0 k_0 E^j + k^m \left( k_m E^j - k^j E_m \right) = -i R(k_\alpha) [T^{\alpha\lambda} \epsilon^{0mjn} k_m E_n + T^{\alpha\lambda} \epsilon^{0ijk} k^0 E_k]$$

(11)

Choosing Z-axis along the direction of propagation i.e. $k_\alpha = (\omega, 0, 0, p)$, the three equations of motion given by the eq. (11) take the form,

$$(\omega^2 - p^2) E^1 = 2i \xi_1 T_1 p^3 E^2$$

$$(\omega^2 - p^2) E^2 = -2i \xi_1 T_1 p^3 E^1$$

$$\omega^2 E^3 = -i \xi_1 (T_2 E^1 - T_3 E^2) p^3$$

(12)

where,

$$T_1 = T^{00} + T^{33}$$

$$T_2 = T^{00} + T^{03} + T^{30} + T^{33}$$

$$T_3 = T^{00} + T^{03} + T^{30} + T^{33}$$
The equation of motion for the transverse modes are,

\[
\begin{pmatrix}
(\omega^2 - p^2) & 2i \xi_1 T_1 p^3 \\
-2i \xi_1 T_1 p^3 & (\omega^2 - p^2)
\end{pmatrix}
\begin{pmatrix}
E^1 \\
E^2
\end{pmatrix} = 0
\] (13)

Therefore, the equations of motion for the right- and left- circularly polarized fields \(E_{\pm} = E_1 \hat{e}_x \pm i E_2 \hat{e}_y\) turn out to be,

\[
(\omega^2 - p^2) (E_1 \pm i E_2) = 2 \xi_1 p^3 T_1 (E_1 \pm i E_2) = 0
\] (14)

From the dispersion relation (13), \(\omega_{\pm}\) will be;

\[
\omega_{\pm} = p (1 \pm \xi_1 p T_1)
\] (15)

The rotation of the plane of polarization can be written in terms of the difference of the \(\omega_{\pm}\) as following,

\[
\alpha = (\omega_- - \omega_+) t
\]

\[
= 2 \xi_1 p^2 T_1 t
\] (16)

where \(t\) is the propagation time. We notice that only the combination, \(T_1 = T_{03}^0 - T_{33}^0\) enters the equation because of our choice of coordinates. Experimental constraints on \(\alpha\) have been put \((-2.4 \pm 1.9)^{\circ}\) from the observation of CMB polarization by WMAP and BOOMERanG. We will make use of these constraints to put bounds on the torsion coupling \(\xi_1 T_1\).

2.2. Case-II : when \(\xi_2 \neq 0\) and \(\xi_1 = 0\)

Putting \(\xi_1 = 0\) in the correction term in the lagrangian, the equations of motion will be,

\[
\partial_i E^i = \xi_2 T^{\alpha \lambda}_i \partial_\alpha \partial_\lambda E^i
\] (17)

\[
-\partial_0 E^j + \partial_j \epsilon^{0ijk} B_k = \xi_2 \partial_\alpha \partial_\lambda \left[ -T^{\alpha \lambda}_0 E^j + T^{\alpha \lambda}_i \epsilon^{0ijk} B_k \right]
\] (18)

Using again the Maxwell’s eq. (8), eq. (18) can be written as,

\[
- \partial^0 \partial_0 E^j - \partial^m (\partial_m E^j - \partial^j E_m) = -\xi_2 \partial_\alpha \partial_\lambda \left[ T^{\alpha \lambda}_0 \partial^0 E^j - T^{\alpha \lambda}_i (\partial^j E^i - \partial^i E^j) \right]
\] (19)
which is the equation of motion for the electric field. Considering again the plane wave solution where, the electric field can be written as, $E_m(x) = E_m(k) e^{-ik^0 x}$, eq. (19) reduces to,

$$k^0 k^0 E^j + k^m (k_m E^j - k^j E_m) = -i \xi_2 \kappa_\alpha \kappa_\lambda \left[ T^{\alpha \lambda} \delta \left( k^0 E^j - T^{\alpha \lambda} \right) (k^j E^i - k^i E^j) \right]$$  \hspace{1cm} (20)

With the choice of Z-axis along the direction of propagation i.e. $k_\alpha = (\omega, 0, 0, p)$, the three equations of motion given by the eq. (11) take the form,

$$\left( \omega^2 - p^2 \right) E^1 = 0$$

$$\left( \omega^2 - p^2 \right) E^2 = 0$$

$$\omega^2 E^3 = -i \xi_2 \left[ T_1 E^3 + (T_3 E^1 + T_2 E^2) \right] \rho^3$$  \hspace{1cm} (21)

Here, we can see that the transverse modes propagates along null geodesics despite the presence of torsion coupling. So, we conclude that a coupling of the form $\xi_2 T^{\sigma \gamma} \delta (\partial_\gamma F^{\delta \nu})$ has no effect on the wave propagation. Therefore, we can not put the limits on $\xi_2$ from the consideration of CMB polarization.

3. POLARIZATION OF COSMIC MICROWAVE BACKGROUND

Thomson scattering of CMB photons on the last scattering surface gives rise to linear polarization. This linear polarization can be expressed by the two Stokes parameters Q and U. The Boltzmann equation, which describes the time evolution of the polarization perturbation will be,

$$\dot{\Delta}_Q + i k \mu \Delta_Q = -\dot{\tau} \left[ \Delta_Q + \frac{1}{2} \left( 1 - P_2(\mu) \right) S_p \right] \hspace{1cm} (22)$$

$$\dot{\Delta}_U + i k \mu \Delta_U = -\dot{\tau} \Delta_U$$  \hspace{1cm} (23)

where

$$S_p = \Delta T_2 + \Delta Q_2 - \Delta Q_0 ,$$

$$\dot{\tau} = \frac{x_e n_e \sigma_T a}{a_0} .$$

and $x_e$ the ionization fraction, $n_e$ the electron number density, $\sigma_T$ the Thomson scattering cross section, and $a$ is the scale factor. If any physical mechanism, which causes the rotation
of the polarization plane, then the time evolution of the stokes parameters will be modified. Considering only the scalar perturbation, the modified Boltzmann equation will be,

\[ \dot{\Delta}_Q + i k \mu \Delta_Q = -\dot{\tau} \left[ \Delta_Q + \frac{1}{2} \left( 1 - P_2(\mu) \right) S_p \right] + 2w \Delta_U \]  \hspace{1cm} (24)

\[ \dot{\Delta}_U + i k \mu \Delta_U = -\dot{\tau} \Delta_U - 2w \Delta_Q \]  \hspace{1cm} (25)

and \( \omega \) is the rate of change of angle of the polarization due to new physics, which in this case is the background torsion.

Changing the variable as \( 23 \),

\[ \tilde{\Delta}_Q \equiv e^{ik\mu \eta - \tau} \Delta_Q, \]

\[ \tilde{\Delta}_U \equiv e^{ik\mu \eta - \tau} \Delta_U, \]

\[ \tilde{S}_p \equiv e^{ik\mu \eta - \tau} S_p. \]  \hspace{1cm} (26)

Eq. (24) and eq. (25) reduce to the form,

\[ \dot{\tilde{\Delta}}_Q = -\dot{\tau} \frac{1}{2} \left( 1 - P_2(\mu) \right) \tilde{S}_p + 2w \tilde{\Delta}_U \]  \hspace{1cm} (27)

\[ \dot{\tilde{\Delta}}_U = -2w \tilde{\Delta}_Q \]  \hspace{1cm} (28)

These two eqs. (27) and eqs. (28) can be combined as,

\[ \dot{\tilde{\Delta}}_Q \pm i \dot{\tilde{\Delta}}_U = -\dot{\tau} \frac{1}{2} \left( 1 - P_2(\mu) \right) \tilde{S}_p \pm 2wi \left( \tilde{\Delta}_Q \pm i\tilde{\Delta}_U \right) \]  \hspace{1cm} (29)

Now define,

\[ \tilde{\Delta}_Q \pm i\tilde{\Delta}_U = P(\eta) \]  \hspace{1cm} (30)

Eqs. (29) can be written as,

\[ \dot{P}(\eta) \pm 2\omega i P(\eta) = -\dot{\tau} \frac{1}{2} \left( 1 - P_2(\mu) \right) \tilde{S}_p \]

\[ = -\frac{3}{4} \tilde{\tau}(\eta) \left( 1 - \mu^2 \right) \tilde{S}_p \]  \hspace{1cm} (31)

Eq. (31) can be solved for \( P(\eta) \) to give

\[ P(\eta) = \left[ -\frac{3}{4} \left( 1 - \mu^2 \right) \int \tilde{\tau}(\eta) \tilde{S}_p d\eta e^{\pm 2i \int \omega(\eta) d\eta} \right] e^{\pm 2i \int \omega(\eta) d\eta} \]  \hspace{1cm} (32)
Therefore, $\Delta Q$ and $\Delta U$ will be,

$$
\Delta Q = \left[ -\frac{3}{4} (1 - \mu^2) \left( \int \tau(\eta) \tilde{S}_p e^{\pm 2i \int \omega(\eta) d\eta} d\eta \right) \cdot e^{i k \mu \eta - \tau} \right] \cos (2\alpha) \quad (33)
$$

$$
\Delta U = - \left[ -\frac{3}{4} (1 - \mu^2) \left( \int \tau(\eta) \tilde{S}_p e^{\pm 2i \int \omega(\eta) d\eta} d\eta \right) \cdot e^{i k \mu \eta - \tau} \right] \sin (2\alpha) \quad (34)
$$

where the angle of rotation $\alpha = \int \omega(\eta) d\eta$.

When $\alpha = 0$ i.e. $\omega = 0$, then

$$
\Delta Q|_{\omega=0} = \left[ -\frac{3}{4} (1 - \mu^2) \left( \int \tau(\eta) \tilde{S}_p e^{\pm 2i \int \omega(\eta) d\eta} d\eta \right) \cdot e^{i k \mu \eta - \tau} \right]
$$

$$
\Delta U|_{\omega=0} = 0
$$

If both scalar and tensor perturbation are present, there will be non zero $\Delta U$, even when $\alpha = 0$.

$$
\Delta Q = \Delta Q|_{\omega=0} \cos (2\alpha) + \Delta U|_{\omega=0} \sin (2\alpha) \quad (35)
$$

$$
\Delta U = - \Delta Q|_{\omega=0} \sin (2\alpha) + \Delta U|_{\omega=0} \cos (2\alpha) \quad (36)
$$

In general, these stokes parameter $Q$ and $U$ can be related to $E$ and $B$ modes of polarization as follows in [24],

$$
(Q \pm iU)(\eta, \vec{x}, \hat{n}) = \int \frac{d^3q}{(2\pi)^3} \sum_{l=2}^{2} \sum_{m=-l}^{l} \left( E^{(m)}_l \pm B^{(m)}_l \right) \pm 2 \gamma_{lm} \quad (37)
$$

where, $\gamma_{lm}(\vec{x}, \hat{n}) = (-i)^l \sqrt{\frac{4\pi}{2l+1}} [s Y_l^m(\hat{n})] \exp(i \vec{k} \cdot \vec{x})$.

Using the expression for the power spectra $C_{XY}^{X'Y'} \sim \int dk [k^2 \Delta_X \Delta_X]$ , $X,Y = T,E,B$ ; we can derive the correlations for $T, E$ and $B$ for CMB in terms of $\alpha$ as follows,

$$
C_{l}^{EE} = \tilde{C}_{l}^{EE} \cos^2(2\alpha) + \tilde{C}_{l}^{BB} \sin^2(2\alpha),
$$

$$
C_{l}^{BB} = \tilde{C}_{l}^{EE} \sin^2(2\alpha) + \tilde{C}_{l}^{BB} \cos^2(2\alpha),
$$

$$
C_{l}^{EB} = \frac{1}{2} \left( \tilde{C}_{l}^{EE} - \tilde{C}_{l}^{BB} \right) \sin(4\alpha),
$$

$$
C_{l}^{TE} = \tilde{C}_{l}^{TE} \cos(2\alpha),
$$

$$
C_{l}^{TB} = \tilde{C}_{l}^{TE} \sin(2\alpha). \quad (38)
$$

From these relations we see that an initial $E$-mode polarization generated by Thomson scattering in the last scattering surface can source a $B$-mode polarization during propagation through a torsion background.
Using the polarization data from WMAP and BOOMERanG, Gubitosi et al. [22] have put the limits on the rotation of the plane of polarization $\alpha = (-2.4 \pm 1.9)^\circ$. Using the relation (16), the values $H_o = 72 (Km/s)/Mpc$, $z = 1100$, $E_p = 1.22 \times 10^{19}$, $p = 100\text{GHz}$, $\Omega_m = 0.3$, $\Omega_\lambda = 0.7$ and taking the time of propagation as,

$$t = \frac{1}{H_o} \int_0^Z \frac{(1 + z)}{\sqrt{\Omega_m(1 + z)^3 + \Omega_\Lambda}} dz$$

we can convert this to limits on the background torsion as,

$$\xi_1 T_1 = (-3.35 \pm 2.65) \times 10^{-22} \text{GeV}^{-1}$$ (39)

The limits on $\xi_1 T_1$ is comparable to the limits on the Lorentz violating interaction of electromagnetism studied in [21].

4. DISCUSSIONS

We have studied non-minimal coupling of electromagnetism to torsion background of the form $\xi_1 T^{\alpha\beta} F_{\alpha\nu} (\partial_\lambda \tilde{F}^{\rho\nu})$ and $\xi_2 T^{\sigma\gamma} F_{\sigma\nu} (\partial_\gamma \tilde{F}^{\delta\nu})$. The first of these interaction is similar in structure to the non-minimal coupling $\frac{1}{E_p} n^\alpha F_{\alpha\sigma} n . \partial (n_\beta \tilde{F}^{\beta\sigma})$, studied by Myers-Pospelov [20], where $n^\alpha$ is some fixed lorentz violating vector. In our case, the role of $n^\alpha$ is played by the background torsion. The second interaction $\xi_2 T^{\sigma\gamma} F_{\sigma\nu} (\partial_\gamma \tilde{F}^{\delta\nu})$ has the interesting property that irrespective of the magnitude of $\xi_2 T$, the electromagnetic waves propagate along null geodesics and suffer no polarization and one can’t put the constraints on this coupling from the observation of the CMB.

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