Structures of ground and excited states in C isotopes

Y Kanada-En’yo\textsuperscript{1}, F Kobayashi\textsuperscript{1} and T Suhara\textsuperscript{2}

\textsuperscript{1}Department of Physics, Kyoto University, Kyoto 606-8502, Japan
\textsuperscript{2}Institute of Physics, University of Tsukuba, Tsukuba 305-8571, Japan

Abstract. Structures of ground and excited states of C isotopes are theoretically investigated with the method of antisymmetrized molecular dynamics. For $^{14}$C, it is suggested that a linear-chain $3\alpha$ structure can be stabilized by the excess neutrons and may construct a rotational band near the $^{10}$Be+$\alpha$ threshold energy. In $^{11}$C and its mirror nucleus $^{11}$B, a cluster gas state and a linear-like cluster state are suggested in excited states. Possible assignment of the cluster states in $^{11}$B with the experimental energy levels are discussed. Deformations of proton and neutron densities of the ground bands in neutron-rich C are also discussed.

1. Introduction

Cluster aspect of unstable nuclei is one of the hot topics in physics of unstable nuclei. For instance, cluster structures of Be isotopes have been intensively investigated in theoretical and experimental works\cite{1}. It was revealed that a variety of cluster states exist in neutron-rich Be owing to a 2$\alpha$ core and surrounding excess neutrons. In addition to such two-center cluster structures suggested in Be, three-center cluster structures in C isotopes have been attracting a great interest. Rich cluster phenomena are expected in neutron-rich C due to the 3$\alpha$ core formation and valence neutrons.

Carbon nuclei are the fascinating nuclei that show coexistence of two kinds of features, shell-model and cluster aspects. The typical example is $^{12}$C which shows the coexistence of the $p_{3/2}$ sub-shell and the 3$\alpha$ cluster features. The ground state is considered to be admixture of the closed $p_{3/2}$ configuration and the 3$\alpha$ core structure. In the excited states near and above the 3$\alpha$ threshold energy, spatially well-developed 3$\alpha$ cluster structures are seen. In particular, a 3$\alpha$ cluster gas state suggested in the $^{12}$C(0\textsuperscript{+}$^+_2$) is one of the recent hot topics and it is often discussed in relation with alpha condensation \cite{2}. Moreover, searching for a linear-chain structure of 3$\alpha$ clusters has been a long standing problem.

Another interesting problem is the structure change of the ground band in C isotopes with increase of the neutron number. In the systematic study of ground states of C isotopes, it was found that neutron structure rapidly changes depending on the neutron number \cite{3}. In contrast to the drastic change of the neutron part, proton structure is rather stable in C because of the shell effect at $Z = 6$. As a result, a neutron-skin structure may develop in C near the neutron drip line. Another interesting feature in the ground bands of unstable C nuclei concerns deformation of proton and neutron densities. The decoupling, i.e., the difference between proton and neutron shapes was suggested in $^{10}$C, $^{16}$C, and $^{18}$C. The recently measured $E2$ transition strengths for the $2^+_1 \rightarrow 0^+_1$ in $^{10}$C\cite{4}, $^{16}$C\cite{5, 6}, $^{18}$C\cite{6, 7}, and $^{20}$C\cite{8} are useful information to clarify deformation properties of the ground bands in C isotopes. For instance, the quenching
of $E2$ transition strengths in $^{10}$C and $^{16}$C can be described by the decoupling of proton and neutron shapes.

Those phenomena tell us that cluster is one of the essential features in unstable nuclei as well as stable nuclei. Various kinds of cluster structure emerge depending on the excitation energy and also depending on the number and kind of core clusters as well as on the number of excess neutrons. In low-lying states, clusters are tightly bound in general and cluster features are characterized by the cluster correlation or the cluster formation. On the other hand, in highly excited states, one may often see remarkable cluster structures where clusters develop spatially. In the case of neutron-rich nuclei, excess neutrons play an important role in cluster structures.

In this paper, structures of ground and excited states of C isotopes are theoretically investigated with the method of antisymmetrized molecular dynamics. Three-center cluster structures in excited states of $^{14}$C and $^{11}$B are studied. Ground state structure of C isotopes is also investigated. The quenching and enhancement of $E2$ transition strength are discussed in relation with proton and neutron shape difference.

2. Formulation

In the framework of antisymmetrized molecular dynamics (AMD) [9], an $A$-nucleon wave function is given by a Slater determinant of Gaussian wave packets:

\[ \Phi_{\text{AMD}}(Z) = \frac{1}{\sqrt{A!}} A\{\varphi_1, \varphi_2, \ldots, \varphi_A\}, \]  

where the $i$th single-particle wave function is written as,

\[ \varphi_i = \phi_{X_i} \chi_i \tau_i, \]  

\[ \phi_{X_i}(r_j) \propto \exp\{-\nu(r_j - X_i)^2\}, \]  

\[ \chi_i = \left( \frac{1}{2} + \xi_i \right) \chi_+ + \left( \frac{1}{2} - \xi_i \right) \chi_. \]  

Here $\phi_{X_i}$ and $\chi_i$ are spatial and spin functions, and $\tau_i$ is the isospin function which is fixed to be up(proton) or down(neutron). We take an optimized value of the width parameter $\nu$ for each nucleus. Accordingly, an AMD wave function is expressed by a set of variational parameters, $Z \equiv \{X_1, X_2, \ldots, X_A, \xi_1, \xi_2, \ldots, \xi_A\}$, which indicate Gaussian centers and spin orientations for all nucleons. Those parameters are determined by the energy variation. The AMD wave function is quite similar to a wave function of fermionic molecular dynamics (FMD)[10, 11] where more generalized wave functions are used.

To study nuclear structure the energy variation and the spin and parity projections are performed using AMD wave functions. In the simple version of AMD, the variation is performed after the parity projection and then the spin projection is done after the variation. The variation after spin-parity projections is performed in the AMD+VAP calculation. For $^{14}$C and $^{11}$B, the AMD+GCM method with the $\beta$-$\gamma$ constraint [12] is also applied. More details of the present calculations are described in Refs. [13, 14, 15, 16]. General formulation of the AMD method is explained in Refs. [3, 17].

In the AMD wave function, all single nucleons are treated independently with localized Gaussians. Although any clusters are not assumed $a$ priori, multi-cluster structures can be described by grouping of single-nucleon Gaussian wave packets in the spatial configuration. On the other hand, if all Gaussian centers gather around a certain position, the AMD wave function becomes equivalent to a harmonic oscillator shell-model wave function around the position due to the effect of antisymmetrization. Thus, the AMD model space can describe both cluster and mean-field features with assembling and disassembling of Gaussian wave packets. If a system
A variety of developed 3\(^\alpha\) neutron skin structure may enhance in neutron-rich C near the drip line. Moreover, the shape is almost unchanged in spite of the rapid change of the neutron structure. Consequently, the 12\(^\alpha\) is rather stable in neutron-rich C against the neutron number increasing. Namely, 3\(^\alpha\)-cluster again in 12\(^\alpha\)

With the increase of the neutron number the cluster structure weakens in 10\(^\alpha\) results suggest that the ground state structure rapidly changes as the neutron number increases. The structures of Be, B, and C isotopes were investigated with the AMD method [3, 13]. The deformation and with the linear-chain structure in the energy region above the 10\(^\alpha\)K result, a appears in a higher 0\(^+\) state a few MeV above the 0\(^+\) state. This suggestion is supported by the recent experimental observation of 0\(^+\) states around 10 MeV excitation energy by Itoh et al. In 11C and its mirror nucleus 11B, similar cluster structures consisting of two \(\alpha\) and \(^3\)He(triton) clusters were suggested [14, 16]. That is, a compact triangle structure, a dilute cluster gas-like state, and a linear-chain structure are theoretically suggested. In particular, the developed 2\(\alpha\) + t cluster structure of the 11B(3/2\(^+\)) shows analogy with the 3\(\alpha\) structure of 12C(0\(^+\)). In the experimental side, candidates of rotational band members for the excited band on the top of the 3/2\(^+\) state in 11B were suggested by Yamaguchi et al. [22].

As mentioned before, the straight-line chain structure of three \(\alpha\) clusters may not be stable in 12C. In neutron-rich C isotopes, we will see further rich cluster phenomena in excited states because of three-center cluster structure with excess neutrons. Let us consider structure of 14C. By adding two neutrons to the 12C system, we can expect developed cluster structures consisting of 3 \(\alpha\) clusters with two excess neutrons in excited states of 14C. Ground and excited states of 14C are calculated with a method of \(\beta\)-\(\gamma\) constraint AMD in combination with the GCM[15]. A variety of developed 3\(\alpha\)-cluster core structures are suggested in excited states of 14C. It was found that the 3\(\alpha\) linear-chain structure can be stabilized by additional neutrons in 14C. As a result, a K\(^\pi\) = 0\(^+\) rotational band was suggested to be constructed from the elongate shape with the linear-chain structure in the energy region above the 10\(^{\scriptsize{Be}}\)+\(\alpha\) threshold energy.

4. Deformation and \(B(E2)\) of the ground bands in C

The structures of Be, B, and C isotopes were investigated with the AMD method [3, 13]. The results suggest that the ground state structure rapidly changes as the neutron number increases in each isotope chain. In Be isotopes, the well-developed 2\(\alpha\)-cluster structure is known in 8Be. With the increase of the neutron number the cluster structure weakens in 10\(^{\scriptsize{Be}}\) and enhances again in 12\(^{\scriptsize{Be}}\). In contrast to the rapid change of the cluster structure in Be, cluster core structure is rather stable in neutron-rich C against the neutron number increasing. Namely, 3\(\alpha\)-cluster structure in 12C weakens in 14C. With further increase of the neutron number, proton structure is almost unchanged in spite of the rapid change of the neutron structure. Consequently, the neutron skin structure may enhance in neutron-rich C near the drip line. Moreover, the shape difference between proton and neutron densities were suggested in 16C having prolate neutron and oblate proton shapes.

Let us discuss more details of the 16C structure. 16C consists of six protons and ten neutrons. Six proton systems tend to deform oblately, while ten neutrons favor a prolate deformation. Usually, proton and neutron deformations are expected to be consistent with each other because of the proton-neutron interaction. However, the results suggest that 16C is a unique nuclei where proton and neutron structures can decouple to each other. As a result of the decoupling, the neutron excitation is dominant in the 2\(^1\) state. Since the system is deformed, the excitation
energy of the $2^+_1$ state should be small. Nevertheless, proton contribution in the $2^+$ excitation is expected to be minor, and hence, the $E2$ transition strength for $2^+ \rightarrow 0^+$ may be suppressed. This suggestion is supported by the experimental measurements of the $E2$ strength and inelastic scattering cross sections [6, 13, 23, 24]. In contrast to the decoupling proton and neutron deformations in $^{16}\text{C}$, $^{20}\text{C}$ has an oblate neutron deformation which is consistent with the oblate proton deformation. It means that $^{16}\text{C}$ is the proton-neutron decoupling system while $^{20}\text{C}$ is the consistent system. Such the structure change may affect the $E2$ transition strength.

Recently, $E2$ strengths for $^{18}\text{C}$ and $^{20}\text{C}$ as well as that for $^{16}\text{C}$ were experimentally observed by life time measurements [5, 6, 7, 8]. They are useful data to clarify the ground band properties especially concerning proton structure. Figure 1 shows the $B(E2)$ values in C isotopes. It is experimentally known that the $2^+$ excitation energies in $^{16}\text{C}$, $^{18}\text{C}$, and $^{20}\text{C}$ are relatively small compared with those in lighter C isotopes. It suggests that these are deformed nuclei. The experimental $B(E2)$ value is small for $^{16}\text{C}$ indicating that the proton contribution is small in the spin 2 rotation. On the other hand, the recently measured $B(E2)$ value for $^{20}\text{C}$ is larger than that for $^{18}\text{C}$. The AMD and AMD+VAP calculations reproduce the systematics of $E2$ strengths in C isotopes. The quenching of $B(E2)$ for $^{16}\text{C}$ is described by the minor proton contribution because of the decoupling of proton and neutron shapes, while the enhancement of $B(E2)$ for $^{20}\text{C}$ can be understood by the consistency of proton and neutron deformations in $^{20}\text{C}$.

Figure 1. $E2$ transition strengths of C isotopes. The experimental data of $E2$ transition strengths are those in the compilation [25] and those measured by Ong et al. [6], Wiedeking et al. [5], Petri et al. [8], Voss et al. [7], McCutchan et al. [4]. The theoretical values are results calculated with AMD+VAP using MV1-case1($m = 0.62$)+LS($u_{ls} = 2600$ MeV) force and those with AMD using MV1-case3($m = 0.576$)+LS($u_{ls} = 900$ MeV) force. The shell model calculations are taken from Ref. [26].

5. Summary
Structures of ground and excited states of C isotopes were discussed based on theoretical calculations with the antisymmetrized molecular dynamics method. Possible cluster states were suggested in excited states of $^{14}\text{C}$, $^{11}\text{C}$ and $^{11}\text{B}$. In the study of ground states of C isotopes, the structure change with the increase of neutron number was suggested. The quenching and
the enhancement of $E2$ transition strengths in $^{16}C$ and $^{20}C$ were discussed in relation with decoupling and coupling between proton and neutron deformations, respectively.

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