Systematic search of fully heavy tetraquark states

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(Dated: August 9, 2022)

In 2020, the LHCb collaboration reported a fully charmed tetraquark state X(6900) in the invariant mass spectrum of J/ψ pairs. This discovery inspires us to further study the properties of the fully heavy tetraquark system. In this work, we study systemically all possible configurations for the ground fully heavy tetraquark states in constituent quark model. According to our calculations, we analyze their binding energies, internal mass contributions, relative lengths between (anti)quarks, and the spatial distribution of four valence (anti)quarks. We find no stable S-wave state exist in fully heavy tetraquark system. We hope that our study will be helpful to explore further for fully heavy tetraquark states.

I. INTRODUCTION

The quark model allows not only traditional mesons and baryons, but also exotic states including tetraquarks and pentaquarks. Searching for exotic hadronic states becomes an interesting topic with full of challenges and opportunities. Since the X(3872) was firstly reported by the Belle Collaboration in 2003 \cite{11,13}, a series of charmonium-like or bottomonium-like exotic states \cite{14,17} and P_{s} states \cite{18,20} have been observed in experiment. The various interpretations also have emerged including conventional hadrons, compact tetraquarks or pentaquarks, loosely bound molecules, hybrids, glueballs, kinematic effects and so on.

In 2003, another important observation is that BaBar Collaboration observed a narrow heavy-light state D_{s0}^{*}(2317) in the D_{s}^{+}\pi^{0} invariant mass spectrum \cite{21}. Its tetraquark with Qq\bar{q}\bar{q} configuration was proposed in Refs.\cite{22,23}. Later, CLEO collaboration \cite{25} confirmed the D_{s0}^{*}(2317) and observed another narrow resonance D_{s1}^{*}(2460) in D_{s}^{+}\pi^{0} final states. The masses of D_{s0}^{*}(2317) and D_{s1}^{*}(2460) deviate from quark model expectations \cite{25} and their decay behaviors are unlike the conventional charmed mesons. Later, SELEX Collaboration reported a charmed-strange meson D_{sJ}^{*}(2632) in invariant mass spectra of D_{s}^{+}\eta and D_{s}^{0}K^{+} \cite{27}. In 2016, D0 Collaboration reported a narrow structure X(5568) in the B_{s0}^{0}\pi^{+} invariant mass spectrum with 5.1σ significance \cite{28}. In 2020, LHCb collaboration reported the discovery of two new exotic structures X_{0}(2900) and X_{1}(2900) \cite{29,30}, which reignites the study of exotic charmed mesons \cite{31,40}.

In 2017, the LHCb Collaboration reported the observation of \Xi_{cc}^{++} in the A_{s}^{+}K^{-}\pi^{+}\pi^{+} decay mode and its mass was determined to be 3621.40±0.72(stat.)±0.27(syst.)±0.14(A_{s}^{+}) MeV \cite{42}. This observation motivates theorists to further study the possible stable tetraquark states with QQQQ configuration \cite{43,51}. Recently, the LHCb Collaboration discovered a very narrow state, called T_{cc}^{+} by analyzing the D_{s}^{0}D_{s}^{0}\pi^{+}\pi^{-} invariant mass spectrum, which has a minimal quark configuration of cc\bar{u}\bar{d}. To our knowledge, the tetraquark states with QQQQ configurations are also studied in different frames \cite{53,57}.

As for the tetraquark state with QQQQ configuration, it has inspired both the experimental and theoretical attention. The existence of four heavy quark states was discussed in a specific potential models \cite{58}. The Q^{2}Q^{2} system was studied with the Born-Oppenheimer approximation in the MIT bag model \cite{59}. Moreover, Lloyd et al. investigated four-body states with only charmed quarks (cc\bar{c}\bar{c}) in a parameterized non-relativistic Hamiltonian \cite{60}. Working in a large but finite oscillator basis, they found several close-lying bound states. Later, Karliner et al. estimated masses of Q_{1}Q_{2}Q_{3}Q_{4} resonant states in a simple quark model, suggested how to produce and observe them, and obtained M(X_{cc\bar{c}\bar{c}}) = 6192±25 MeV and M(X_{bb\bar{b}\bar{b}}) = 18826 ± 25 MeV for the J^{P}C = 0^{++} states involving charmed and bottom tetraquarks \cite{61}. Anwar et al. calculated the ground-state energy of the bb\bar{b}\bar{b} bound state in a nonrelativistic effective field theory with one-gluon-exchange (OGE) color Coulomb interaction, and the ground state bb\bar{b}\bar{b} tetraquark mass is predicted to be (18.72 ± 0.02) GeV \cite{62}. In 2016, Bai et al. presented a calculation of the bb\bar{b}\bar{b} tetraquark ground-state energy using a diffusion Monte Carlo method to solve the non-relativistic many-body system \cite{63}. In 2017, Debastiani et al. extended updated Cornell model to study the all-charm tetraquark \langle cc\bar{c}\bar{c} \rangle in a diquark-antidiquark configuration \cite{64}. Moreover, Chen et al. used a moment QCD sum rule method augmented by fundamental inequalities to research the existence of exotic states cc\bar{c}\bar{c} and bb\bar{b}\bar{b} in the compact diquark-antidiquark configuration.

In 2016, the CMS Collaboration reported the first ob-
For the stability of the fully heavy tetraquark state, it has been discussed for a long time. Debastiani et al. found that the lowest $S$-wave $c^4c^{ar{c}}$ configuration. The observation of $X(6900)$ attracts many scholars to interpret this state from different views: (1) the first radial excitation states of $0^{++}/2^{++}$ or the first orbital excitation state of $0^{-+}/1^{++}$ [72]; (2) a bound diquark-antidiquark system [78–80]; (3) the gluonic tetracharm state in $\bar{3}$. (2) spin operator for the $i$-th (anti)quark, and the $\lambda^c_i$ is the $SU(3)$ color operator for the $i$-th (anti)quark, and for antiquark, $\lambda^c_i$ is replaced with $-\lambda^c_i$. The internal quark potentials $V^{con}_{ij}$ and $V^{SS}_{ij}$ have the following forms:

$$\begin{align*}
V^{con}_{ij} &= -\frac{\kappa}{r_{ij}} + \frac{r_{ij}}{a_0^2} - D, \\
V^{SS}_{ij} &= \frac{\kappa'}{m_i m_j r_{ij}^2} e^{-r_{ij}^2/r_{0ij}^2} \sigma_i \sigma_j,
\end{align*}$$

(2)

where $r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|$ is the distance between the $i$-th (anti)quark and the $j$-th (anti)quark, and the $\sigma_i$ is the $SU(2)$ spin operator for the $i$-th quark. As for the $r_{0ij}$ and $\kappa'$, we have

$$\begin{align*}
r_{0ij} &= 1/\left[\alpha + \beta \frac{m_i m_j}{m_i + m_j}\right], \\
\kappa' &= \kappa(1 + \gamma \frac{m_i m_j}{m_i + m_j}).
\end{align*}$$

(3)

The corresponding parameters appearing in Eqs. (2-3) are shown in Table I. Here, $\kappa$ and $\kappa'$ are the couplings of the Coulomb and hyperfine potentials, respectively, and they are proportional to the running coupling constant $\alpha_s(r)$ of QCD. The Coulomb and hyperfine interaction can be deduced from the one-gluon-exchange model. $1/a_0^2$ represents the strength of linear potential. $r_{0ij}$ is the Gaussian-smearing parameter. Further, we introduce

$$H = \sum_{i=1}^{4} \left( m_i + \frac{p_i^2}{2m_i} \right) - \frac{3}{4} \sum_{i<j} \frac{\lambda^c_i \lambda^c_j}{\con} (V^{con}_{ij} + V^{SS}_{ij}).$$

(1)
\[ \kappa_0 \text{ and } \gamma \text{ in } \kappa' \text{ to provide better descriptions for the interaction between different quark pairs} \[91]. \]

### Table I. Parameters of the Hamiltonian.

| Parameter | \( \kappa \) | \( a_0 \) | \( D \) |
|-----------|---------------|------------|--------|
| Value     | 120.0 MeV fm  | 0.0318119 (MeV^{-1} fm)^{1/2} | 983 MeV |
| Parameter | \( \alpha \) | \( \beta \) | \( m_0 \) |
| Value     | 1.0499 fm^{-1} | 0.0008314 (MeV fm)^{-1} | 1918 MeV |
| Parameter | \( \kappa_0 \) | \( \gamma \) | \( m_b \) |
| Value     | 194.144 MeV | 0.00088 MeV^{-1} | 5343 MeV |

### III. Wave Functions

Here, we concentrate on the ground fully-heavy tetraquark states. We present the flavor, spatial, and color-spin parts of total wave function for fully-heavy tetraquark system. In order to consider the constraint from the Pauli principle, we use a diquark-antidiquark picture to analyze this tetraquark system.

#### A. Flavor Part

Firstly, we discuss the flavor part. Here, we list all the possible flavor combinations for the fully-heavy tetraquark system in Table II.

In Table II, the three flavor combinations in the first line are purely neutral particles and the C parity is a “good” quantum number. For the other six states in the second line, every state has a charge conjugation anti-partner, and their masses, internal mass contributions, relative distances between (anti)quarks are absolutely same, and thus we only need to discuss one of the pair.

Moreover, the \( c\bar{c}\bar{c} \), \( b\bar{b}\bar{b} \), and \( c\bar{c}\bar{b} \) states have the two pairs of (anti)quarks which are identical, but only the first two quarks in the \( c\bar{c}\bar{b} \) and \( b\bar{b}\bar{c} \) states are identical.

#### B. Spatial Part

In this part, we construct the wave function for the spatial part in a simple Gaussian form. We denote the fully heavy tetraquark state as \( Q(1)\bar{Q}(2)\bar{Q}(3)\bar{Q}(4) \) configuration, and choose the Jacobi coordinates system as follows:

\[
\begin{align*}
x_1 &= \sqrt{1/2}(r_1 - r_2); \\
x_2 &= \sqrt{1/2}(r_3 - r_4); \\
x_3 &= \frac{1}{2}[\frac{m_1 r_1 + m_2 r_2}{m_1 + m_2} - \frac{m_3 r_3 + m_4 r_4}{m_3 + m_4}],
\end{align*}
\]

### Table II. All possible flavor combinations for the fully-heavy tetraquark system.

| System | Flavor combinations |
|--------|---------------------|
| \( Q\bar{Q}\bar{Q}\bar{Q} \) | \( c\bar{c}\bar{c} \), \( b\bar{b}\bar{b} \), \( c\bar{c}\bar{b} \) |

Here, we set the Jacobi coordinates with the following conditions:

\[
\begin{align*}
m_1 + m_2 &= m_3 + m_4, \quad \text{for } c\bar{c}\bar{c}, \\
m_1 + m_2 &= m_3 - m_4, \quad \text{for } b\bar{b}\bar{b}, \\
m_1 - m_2 &= m_3 + m_4, \quad \text{for } c\bar{c}\bar{b}, \\
m_1 - m_2 &= m_3 - m_4, \quad \text{for } b\bar{b}\bar{c}. \\
\end{align*}
\]

Based on these, we construct the spatial wave functions of \( Q\bar{Q}\bar{Q}\bar{Q} \) states in a single Gaussian form. The spatial wave function can satisfy the required symmetry property:

\[
R^e = \exp[-C_{11}x_1^2 - C_{22}x_2^2 - C_{33}x_3^2], \quad (5)
\]

where \( C_{11}, C_{22}, \) and \( C_{33} \) are the variational parameters.

Moreover, it is useful to introduce the center of mass frame so that the kinetic term in the Hamiltonian of Eq. [1] can be reduced appropriately for our calculations. The kinetic term denoted by \( T_c \) is as follows:

\[
T_c = \sum_{i=1}^{4} \frac{p_i^2}{2m_i} = \frac{p_{c_1}^2}{2m_1} + \frac{p_{c_2}^2}{2m_2} + \frac{p_{c_3}^2}{2m_3}, \quad (6)
\]

where different states have different reduced masses \( m_i' \) and we show them in Table III.

### Table III. The reduced mass \( m_i' \) in different states.

| States | \( m_1' \) | \( m_2' \) | \( m_3' \) |
|--------|------------|------------|------------|
| \( c\bar{c}\bar{c} \) | \( m_c \) | \( m_c \) | \( m_c \) |
| \( b\bar{b}\bar{b} \) | \( m_b \) | \( m_b \) | \( m_b \) |
| \( c\bar{c}\bar{b} \) | \( \frac{2m_c m_b}{m_c + m_b} \) | \( \frac{2m_c m_b}{m_c + m_b} \) | \( \frac{2m_c m_b}{m_c + m_b} \) |

#### C. Color-spin Part

In the color space, the color wave functions can be analyzed by applying the SU(3) group theory, where the direct product of the diquark-antidiquark components reads:

\[
(3_c \otimes \bar{3}_c) \otimes (\bar{3}_c \otimes \bar{3}_c) = (6_c \otimes 3_c) \otimes (\bar{6}_c \otimes \bar{3}_c). \quad (7)
\]
Based on these, we get two kinds of color-singlet state:

\[ \phi_1 = |(Q_1 Q_2)^3(Q_3 Q_4)^3), \quad \phi_2 = |(Q_1 Q_2)^6(Q_3 Q_4)^6). \] (8)

In the spin space, the allowed wave functions in diquark-antidiquark picture read:

\[ \chi_1 = |(Q_1 Q_2)^1(Q_3 Q_4)^1)_{12}, \quad \chi_2 = |(Q_1 Q_2)^1(Q_3 Q_4)^1)_{11}, \]

\[ \chi_3 = |(Q_1 Q_2)^1(Q_3 Q_4)^0)_{01}, \quad \chi_4 = |(Q_1 Q_2)^0(Q_3 Q_4)^1)_{11}, \]

\[ \chi_5 = |(Q_1 Q_2)^1(Q_3 Q_4)^1)_{00}, \quad \chi_6 = |(Q_1 Q_2)^0(Q_3 Q_4)^0)_{00}. \] (9)

In the notation \(|Q_1 Q_2)^{\text{spin1}}(Q_3 Q_4)^{\text{spin2}}(Q_3 Q_4)^{\text{spin3}}\), the spin1, spin2, and spin3 represent the spins of diquark, antidiquark, and the whole tetraquark state, respectively.

Because the flavor part and spatial part are chosen to be fully symmetric for the (anti)diquark, the color-spin part of the total wave function should be fully antisymmetric. Combining the flavor part, we show all possible color-spin part satisfied Pauli principle with \(J^{PC}\) in Table IV.

### TABLE IV. The allowed color-spin parts for every flavor configuration.

| Type | \(J^{PC(C)}\) | Color-spin Part |
|------|----------------|----------------|
| \(ccc\bar{c}\) | \(2^+(+\rangle\) | \(\phi_1 \chi_1\) |
| \(bbb\bar{b}\) | \(1^+(-\rangle\) | \(\phi_1 \chi_2\) |
| \(cb\bar{c}\) | \(0^+(+\rangle\) | \(\phi_1 \chi_3\), \(\phi_2 \chi_6\) |
| \(ccc\bar{b}\) | \(2^+\rangle\) | \(\phi_1 \chi_1\) |
| \(bbb\bar{c}\) | \(1^\prime\rangle\) | \(\phi_1 \chi_2\), \(\phi_1 \chi_3\), \(\phi_2 \chi_4\) |
| \(0^\prime\rangle\) | \(\phi_1 \chi_5\), \(\phi_2 \chi_6\) |
| \(cb\bar{b}\) | \(2^{++}\rangle\) | \(\phi_1 \chi_1\), \(\phi_2 \chi_1\) |
| \(1^{+-}\rangle\) | \(\phi_1 \chi_2\), \(\phi_2 \chi_2\), \(\frac{1}{\sqrt{2}}(\phi_1 \chi_3 + \phi_1 \chi_4)\) |
| \(1^{++}\rangle\) | \(\phi_1 \chi_3 - \phi_1 \chi_4\), \(\frac{1}{\sqrt{2}}(\phi_2 \chi_3 + \phi_2 \chi_4)\) |
| \(0^{++}\rangle\) | \(\phi_1 \chi_5\), \(\phi_2 \chi_5\), \(\phi_1 \chi_6\), \(\phi_2 \chi_5\) |

### IV. NUMERICAL ANALYSIS

In this section, we firstly introduce a useful method that can be used to examine the stability of the tetraquarks through the eigenvalue of the hyperfine potential matrix generated by the independent color \(\otimes\) spin bases. This hyperfine matrix is essential in identifying possible attraction. A stable or resonant multiquark state can only exist if the hyperfine potential of the multiquark configuration is sufficiently attractive compared to its rearrangement decay channels. Here, the form of the hyperfine factor is given as \(\langle \sum_{i<j} \frac{1}{m_{ij}r_{ij}} \chi_i \cdot \chi_j \rangle\). Then, we diagonalize them to get the lowest eigenvalue of the hyperfine factor, and show them in corresponding Tables. To compare the expectation values of the tetraquarks’ hyperfine factor better, we also show the corresponding values for all possible decay channels in same Tables.

Next, we check the consistence between the experimental masses and the obtained masses of some mesons using the variational method based on the Hamiltonian of Eq. [4] and the parameters in Table IV. We show the results in Table V and notice that our values are relatively reliable since the deviations for most states are less than 10 MeV.

| Meson | \(J/\psi\) | \(\eta_8\) | \(\Upsilon\) | \(\eta_0\) | \(B_s\) | \(B'_s\) |
|-------|---------|---------|---------|---------|-------|-------|
| Theoretical masses | 3092.2 | 2998.5 | 9408.9 | 9389.0 | 6287.9 | 6350.5 |
| Variational parameters | 12.5 | 15.0 | 49.7 | 57.4 | 22.9 | 20.2 |
| Experiment masses | 3096.9 | 2983.9 | 9400.3 | 9399.0 | 6274.9 |
| Error | -4.7 | 14.6 | 8.6 | 10.0 | 13.0 |

Moreover, we have systematically construct the total wave function satisfied with Pauli Principle in the previous section. The corresponding total wave function could be expanded as follows:

\[ |\Psi_\alpha\rangle = \sum_{ij} C_{ij}^\alpha |F_i R_j\rangle |\phi_1 \chi_j\rangle. \] (10)

To investigate the mass of the fully heavy tetraquarks with the variational method, we calculate the Schrödinger equation \(H |\Psi_\alpha\rangle = E_\alpha |\Psi_\alpha\rangle\), diagonalize the corresponding matrix, and then determine the ground state masses for the fully heavy tetraquarks. According to corresponding variational parameters, we further give the internal mass contributions, including quark masses part, kinetic energy part, confinement potential part, and hyperfine potential part. As a comparison, we also show the lowest meson-meson thresholds for the tetraquarks with different quantum numbers and their internal contributions. Thus, we define the binding energy:

\[ B_T = M_{\text{tetraquark}} - M_{\text{meson1}} - M_{\text{meson2}}, \] (11)

where \(M_{\text{tetraquark}}, M_{\text{meson1}},\) and \(M_{\text{meson2}}\) are masses of tetraquark and the two mesons at lowest threshold allowed in the rearrangement decay of the tetraquark, respectively. In order to discuss conveniently in next subsection, we also define the \(V^C\): the sum of Coulomb potential and linear potential.

Lastly, it is also useful to investigate the spatial size of the tetraquarks which strongly relates to the magnitude of the various kinetic energies and the potential energies between quarks. Understanding the relative lengths between quarks in tetraquarks and their lowest thresholds...
is also important, and the relative distance between the heavier quarks is, in general, shorter than that between the lighter quarks [22]. This tendency is also maintained in each tetraquark state according to corresponding Tables.

In the following subsections, we concretely discuss all possible configurations for fully heavy tetraquarks.

A. ccc̄ and bbbb states

Firstly, we investigate the ccc̄ and bbbb systems. There are two $J^{PC} = 0^{++}$ states, a $J^{PC} = 1^{++}$ state, and a $J^{PC} = 2^{++}$ state according to Table V. We show the eigenvalue of the hyperfine factor for the ground states and their possible decay channels in Table VII. Moreover, we also show the masses of ground states, variational parameters, the internal mass contributions, the relative lengths between quarks, and their lowest meson-meson eigenvalue of the hyperfine factor for the ground states. According to Table VI, the eigenvalues for ground states are not equal to expectation values for two meson states. Moreover, we also find the ground $J^{PC} = 0^{++}$ bbbb state has a smaller eigenvalue and thus less attractive than the $\eta_\gamma \eta_\gamma$ decay channel, but is still more attractive than $\Upsilon \Upsilon$ decay channel. Nevertheless, the reason why this state has energy obviously larger than $\eta_\gamma \eta_\gamma$ and $\Upsilon \Upsilon$ decay channels is that the contribution from the confinement potential is obviously larger than those of $\eta_\gamma \eta_\gamma$ and $\Upsilon \Upsilon$ decay channels. Hence, it should be a compact state, but is not a stable state, which can decay into rearrangement decay channels.

We now analyze the numerical results obtained from the variational method. As for the ground $J^{PC} = 0^{++}$ bbbb state, its mass is 19240.0 MeV, and corresponding binding energy $B_T$ is +461.9 MeV. The total wave function is given as:

$$|\Psi_{tot}\rangle = 0.936|F\rangle|R^e\rangle|\phi_2\chi_6\rangle - 0.352|F\rangle|R^e\rangle|\phi_1\chi_5\rangle.$$  \hspace{1cm} (12)

Its variational parameters are given as $C_{11} = 7.7$ fm$^{-2}$, $C_{22} = 7.7$ fm$^{-2}$, and $C_{33} = 11.4$ fm$^{-2}$, which shows roughly the inverse ratios of size for diquark, antidiquark, and between the center of diquark and antidiquark, respectively. We naturally find that the $C_{11}$ is equal to $C_{22}$, hence the distance of $(\bar{b} - b)$ would be equal to that of $(\bar{c} - c)$, and the reason is that bbbb system is a neutral system.

1. The relative distances and symmetry

Here, we concentrate on the the relative distances between the (anti)quarks in tetraquarks. Looking into the relative distances in Table VII, we find that the relative distances of (1,2) and (3,4) pairs are same, and other relative distances are same in all the ccc̄ and bbbb states. This is because of the permutation symmetry for the ground state wave function in each tetraquarks. For the $c_1c_2\bar{c}_3\bar{c}_4$ and $b_1b_2\bar{b}_3\bar{b}_4$ states, they need to satisfy the Pauli principle for identical particles are as follows:

$$A_{12}\langle\Psi_{tot}\rangle = A_{34}\langle\Psi_{tot}\rangle = -\langle\Psi_{tot}\rangle,$$ \hspace{1cm} (13)

where the operation $A_{ij}$ means exchanging the coordinate of $Q_i$ ($Q_i$) and $Q_j$ ($Q_j$).

Meanwhile, they are pure neutral particles with definite C-parity, so the permutation symmetries for total wave functions as follows:

$$A_{12-34}\langle\Psi_{tot}\rangle = \pm\langle\Psi_{tot}\rangle,$$ \hspace{1cm} (14)

where $A_{12-34}$ means exchanging the coordinate of diquark and antidiquark.

Based on these, the relationship of relative distances for all the $c_1c_2\bar{c}_3\bar{c}_4$ and $b_1b_2\bar{b}_3\bar{b}_4$ states can be obtained as follows:

$$\langle\Psi_{tot}|r_1 - r_3|\Psi_{tot}\rangle = \langle\Psi_{tot}|A_{12}^{-1}A_{12}|r_1 - r_3|A_{12}^{-1}A_{12}|\Psi_{tot}\rangle = \langle\Psi_{tot}|r_2 - r_3|\Psi_{tot}\rangle = \langle\Psi_{tot}|A_{34}^{-1}A_{34}|r_2 - r_3|A_{34}^{-1}A_{34}|\Psi_{tot}\rangle = \langle\Psi_{tot}|r_2 - r_4|\Psi_{tot}\rangle = \langle\Psi_{tot}|A_{12}^{-1}A_{12}|r_2 - r_4|A_{12}^{-1}A_{12}|\Psi_{tot}\rangle = \langle\Psi_{tot}|r_1 - r_4|\Psi_{tot}\rangle,$$  \hspace{1cm} (15)

\hspace{1cm} and

$$\langle\Psi_{tot}|r_1 - r_2|\Psi_{tot}\rangle = \langle\Psi_{tot}|A_{12-34}^{-1}A_{12-34}|r_1 - r_2|A_{12-34}^{-1}A_{12-34}|\Psi_{tot}\rangle = \langle\Psi_{tot}|r_3 - r_4|\Psi_{tot}\rangle.$$ \hspace{1cm} (16)

Obviously, our theoretical derivations are in perfect agreement with the calculated results in Table VII.

Further, we can also prove three Jacobi coordinates, $R_{1,2} = r_1 - r_2$, $R_{3,4} = r_3 - r_4$, and $R' = 1/2(r_1 + r_2 - r_3 - r_4)$, are orthogonal to each other for all the ccc̄ and bbbb states:

$$\langle\Psi_{tot}|(R_{1,2} \cdot R_{3,4})|\Psi_{tot}\rangle = \langle\Psi_{tot}|(34)^{-1}(34)|(R_{1,2} \cdot R_{3,4})|(34)^{-1}(34)|\Psi_{tot}\rangle = -\langle\Psi_{tot}|(R_{1,2} \cdot R_{3,4})|\Psi_{tot}\rangle = 0,$$ \hspace{1cm} (17)

$$\langle\Psi_{tot}|(R_{1,2} \cdot R')|\Psi_{tot}\rangle = \langle\Psi_{tot}|(12)^{-1}(12)|(R_{1,2} \cdot R')|(12)^{-1}(12)|\Psi_{tot}\rangle = -\langle\Psi_{tot}|(R_{1,2} \cdot R')|\Psi_{tot}\rangle = 0,$$ \hspace{1cm} (18)

\hspace{1cm} and

$$\langle\Psi_{tot}|(R_{3,4} \cdot R')|\Psi_{tot}\rangle = \langle\Psi_{tot}|(34)^{-1}(34)|(R_{3,4} \cdot R')|(34)^{-1}(34)|\Psi_{tot}\rangle = -\langle\Psi_{tot}|(R_{3,4} \cdot R')|\Psi_{tot}\rangle = 0.$$ \hspace{1cm} (19)

According to the relative distances in Table VII and the relationship of Eqs. (13-18), we can well describe the
TABLE VI. The expectation value of the hyperfine factor for the $ccar{c}ar{c}$ and $bbar{b}ar{b}$ systems and their possible decay channels. (unit:(GeV)$^{-2}$)

| Systems | $J^P_C$ | $ccar{c}ar{c}$ | $bbar{b}ar{b}$ |
|---------|---------|------------------|------------------|
|         |         | $0^{++}$ | $1^{-+}$ | $2^{++}$ | $0^{++}$ | $1^{++}$ | $2^{++}$ |
| Lowest Eigenvalue | -5.26 | 0 | 2.90 | -0.68 | 0 | 0.37 |
| Decay channel | $J/\psi/\bar{J}/\psi$ | $J/\psi/\bar{J}/\psi$ | $J/\psi/\bar{J}/\psi$ | $\Upsilon/\psi$ | $\Upsilon/\psi$ | $\Upsilon/\psi$ |
| Quantum number | $0^{++}$, $2^{++}$ | $1^{++}$ | $0^{++}$ | $0^{++}$, $2^{++}$ | $1^{++}$ | $0^{++}$ |
| Value | 2.90 | -2.90 | -8.70 | 0.37 | -0.37 | -1.12 |

TABLE VII. The masses, variational parameters, the contribution, and the relative lengths between quarks for $ccar{c}ar{c}$, $bbar{b}ar{b}$ systems, corresponding lowest meson-meson thresholds, and their differences ($D_0$, $D_1$, or $D_2$). The number is given as $i=1.2$ for the quarks, and 3.4 for the antiquarks. The masses, $B_T$, and contributions are in MeV unit. The relative lengths (variational parameters) are fm (fm$^{-2}$) unit.

| Systems | $J^P_C$ | $ccar{c}ar{c}$ | $bbar{b}ar{b}$ |
|---------|---------|------------------|------------------|
| Mass/BT |         |                  |                  |
| Variational Parameters (fm$^{-2}$) | $C_{11}$ | 7.7 | 15.0 | 9.1 | 15.0 | 8.9 | 12.5 | 24.6 | 57.4 | 30.7 | 57.4 | 30.0 | 49.7 |
|         | $C_{22}$ | 7.7 | 15.0 | 9.1 | 12.5 | 8.9 | 12.5 | 24.6 | 57.4 | 30.7 | 49.7 | 30.0 | 49.7 |
|         | $C_{33}$ | 11.4 | 7.3 | 6.9 | 6.9 | 39.5 | 24.0 | 24.0 | 23.0 |
| Quark Mass | 7672.0 | 7672.0 | 0.0 | 7672.0 | 0.0 | 7672.0 | 0.0 | 21372.0 | 21372.0 | 0.0 | 21372.0 | 0.0 |
| Confinement-Potential | -2083.8 | -2440.4 | 356.6 | -1998.8 | -2367.4 | 368.6 | -1973.6 | -294.4 | 320.8 | -310.1 | -3724.2 | 623.2 | -3603.4 | 638.5 | -2977.3 | -3559.5 | 582.2 |
| Kinetic Energy | 814.0 | 915.1 | -101.1 | 767.2 | 839.0 | -71.8 | 752.0 | 762.9 | -10.9 | 970.4 | 1255.9 | -285.5 | 934.0 | 1171.6 | -237.6 | 908.1 | 1087.3 | -179.2 |
| CS Interaction | 22.7 | -150.0 | 172.7 | 1.5 | -52.9 | 54.4 | 32.3 | 43.9 | -11.6 | 17.0 | -125.5 | 142.5 | 1.3 | -43.8 | 45.1 | 25.1 | 38.0 | -12.9 |
| $V_C$ | (1,2) | -6.8 | -237.2($q_c$) | 1.5 | -237.2($q_c$) | 5.4 | -164.2($J/\psi$) | -393.7 | -879.1($\eta_b$) | -122.1 | -879.1($\eta_b$) | -118.3 | -796.7($\Upsilon$) |
|         | (1,3) | -26.1 | -237.2($q_b$) | 1.5 | -237.2($q_b$) | 5.4 | -164.2($J/\psi$) | -393.7 | -879.1($\eta_b$) | -122.1 | -796.7($\Upsilon$) | -118.3 | -796.7($\Upsilon$) |
|         | (2,3) | -26.1 | -237.2($q_b$) | 1.5 | -237.2($q_b$) | 5.4 | -164.2($J/\psi$) | -393.7 | -879.1($\eta_b$) | -122.1 | -796.7($\Upsilon$) | -118.3 | -796.7($\Upsilon$) |
|         | (1,4) | -36.1 | -237.2($q_b$) | 1.5 | -237.2($q_b$) | 5.4 | -164.2($J/\psi$) | -393.7 | -879.1($\eta_b$) | -122.1 | -796.7($\Upsilon$) | -118.3 | -796.7($\Upsilon$) |
|         | (2,4) | -36.1 | -237.2($q_b$) | 1.5 | -237.2($q_b$) | 5.4 | -164.2($J/\psi$) | -393.7 | -879.1($\eta_b$) | -122.1 | -796.7($\Upsilon$) | -118.3 | -796.7($\Upsilon$) |
|         | (3,4) | -36.1 | -237.2($q_b$) | 1.5 | -237.2($q_b$) | 5.4 | -164.2($J/\psi$) | -393.7 | -879.1($\eta_b$) | -122.1 | -796.7($\Upsilon$) | -118.3 | -796.7($\Upsilon$) |
| Total Contribution | -117.8 | -474.4 | 356.6 | -32.8 | -401.4 | 368.6 | -7.6 | -328.4 | 320.8 | -135.0 | -1758.2 | 623.2 | -1037.4 | -1675.9 | 632.5 | -1011.3 | -1593.5 | 582.2 |
| Relative Lengths (fm) | (1,2) | 0.406 | 0.373 | 0.377 | 0.227 | 0.203 | 0.160($\Upsilon$) |
|         | (1,3) | 0.290($q_b$) | 0.395 | 0.290($q_b$) | 0.403 | 0.318($J/\psi$) | 0.204 | 0.148($\eta_b$) | 0.217 | 0.148($\eta_b$) | 0.220 | 0.160($\Upsilon$) |
|         | (2,3) | 0.371 | 0.395 | 0.403 | 0.204 | 0.217 | 0.160($\Upsilon$) | 0.220 | 0.160($\Upsilon$) |
|         | (1,4) | 0.371 | 0.395 | 0.403 | 0.204 | 0.217 | 0.160($\Upsilon$) | 0.220 | 0.160($\Upsilon$) |
|         | (2,4) | 0.290($q_b$) | 0.395 | 0.318($J/\psi$) | 0.304 | 0.148($\eta_b$) | 0.217 | 0.160($\Upsilon$) | 0.220 | 0.160($\Upsilon$) |
|         | (1,2)-(3,4) | 0.235 | 0.294 | 0.302 | 0.126 | 0.162 | 0.165 |
| Radius(fm) | 0.235 | 0.237 | 0.241 | 0.130 | 0.130 | 0.132 |

relative positions of the four valence quarks for all the $ccar{c}ar{c}$ and $bbar{b}ar{b}$ states. Meanwhile, using the relative distances between (antiquarks and orthogonal relationship, one can also determine the relative distance of (12)−(34), and it is consistent with our results in Table VII. Further, we can give the relative position of $R_c$ and the spherical radius of the tetraquarks. Here, we define that the $R_c$ is the geometric center of the four quarks (the center of the sphere). Based on these results, we show the spatial distribution of four valence quarks for the ground
$J^{PC} = 0^{++} \text{bbbb} \text{ state in Fig. 1}$

2. The internal contribution

Here, let us now turn our discussion to the internal mass contribution for the ground $J^{PC} = 0^{++} \text{bbbb} \text{ state}$. Firstly, for the kinetic energy, this $\text{bbbb} \text{ state}$ has 814.0 MeV, which can be understood as the sum of three internal kinetic energies: kinetic energies of two pairs of the $b - \bar{b}$, and the $(\bar{b} \bar{b}) - (b b)$ pair. Accordingly, the sum of the internal kinetic energies of $\eta\eta$ state only come from the two pairs of the $b - \bar{b}$. Therefore, this $\text{bbbb} \text{ state}$ has an additional kinetic energy needed to bring the $\eta\eta$ into a compact configuration. The actual kinetic energies of two pairs of the $b - \bar{b}$ in the ground $J^{PC} = 0^{++} \text{bbbb} \text{ state}$ are smaller than those inside the $\eta\eta$ state. This is so because as can be seen in Table VIII the distance of $b - \bar{b}$ in the tetraquark state is larger than in the meson: the distance of $b - \bar{b}$ is 0.204 fm in this $\text{bbbb} \text{ state}$ while it is 0.148 fm in $\eta\eta$. Meanwhile, we find even if considering the additional kinetic energy between the $(\bar{b} \bar{b}) - (b b)$ pair, the total kinetic energies in this $\text{bbbb} \text{ state}$ is still smaller than that of the $\eta\eta$ state. However, this does not cause the ground $J^{PC} = 0^{++} \text{bbbb} \text{ state}$ to a stable state due to confinement potential part.

As for the confinement potential part, the contributions from $V^C$ for the ground $J^{PC} = 0^{++} \text{bbbb} \text{ state}$ in Table VIII are all attractive. Thus, this state has a large positive binding energy. However, it still above the meson-meson threshold because the $V^C(\bar{b}b)$ in $\eta\eta$ is much attractive. As for other internal contributions, the quark contents of this state are the same as its corresponding rearrangement decay threshold. Moreover, the mass contribution from the hyperfine potential term is negligible relative to the contributions from other terms.

B. $\text{ccbb} \text{ state}$

Here, we concentrate on the $\text{ccbb}$ system. Similar to the $\text{cccc}$ and $\text{bbbb}$ systems, the $\text{ccbb}$ system is also satisfied with fully antisymmetric for diquark and antiquark. There are two $J^{P} = 0^{+}$ states, a $J^{P} = 1^{+}$ state, and a $J^{P} = 2^{+}$ state in $\text{ccbb}$ system. We show the eigenvalue of the hyperfine factor for the ground states and their possible decay channels in Table VIII. Moreover, we also show the masses of ground states, corresponding variational parameters, different internal mass contributions, the relative lengths between quarks, and their lowest meson-meson threshold in Table IX respectively.

![Diagram](image_url)

FIG. 1. Relative positions for four valence quarks and $R_c$ in the $J^{PC} = 0^{++}$ ground $\text{bbbb} \text{ state}$. Meanwhile, we label the relative distances of $R_{b,\bar{b}}$, $R_{b,\bar{b}}$, $R_{b,\bar{b}}$, $R^\prime$, and the radius (units: fm).

| System | $J^{P}$ | $\langle \sum_{i<j} \frac{1}{m_{ij}} \lambda_{i} \lambda_{j} \sigma_{i} \sigma_{j} \rangle$ | Lowest Eigenvalue |
|--------|--------|---------------------------------|------------------|
| $J^{P} = 0^{+}$ | $\langle \sum_{i<j} \frac{1}{m_{ij}} \lambda_{i} \lambda_{j} \sigma_{i} \sigma_{j} \rangle$ | $-1.845$ |
| $J^{P} = 1^{+}$ | $\langle \sum_{i<j} \frac{1}{m_{ij}} \lambda_{i} \lambda_{j} \sigma_{i} \sigma_{j} \rangle$ | $0.298$ |
| $J^{P} = 2^{+}$ | $\langle \sum_{i<j} \frac{1}{m_{ij}} \lambda_{i} \lambda_{j} \sigma_{i} \sigma_{j} \rangle$ | $1.339$ |

Table VIII. The expectation value of the hyperfine factor for the $\text{ccbb}$ system and their possible decay channels.

| Decay channel | $B_{c}^{*}B_{c}^{*}$ | $B_{c}^{*}B_{c}$ | $B_{c}B_{c}$ |
|---------------|----------------|----------------|---------------|
| Quantum number | $0^{+}$, $2^{+}$ | $1^{+}$ | $0^{+}$ |
| Value | $0.141$ | $-1.041$ | $-3.123$ |

Firstly, to compare the hyperfine factors of the $\text{ccbb}$ tetraquarks with the corresponding sum of two mesons in the possible decay channels, we need to obtain and diagonalize $(\sum_{i<j} \frac{1}{m_{ij}} \lambda_{i} \lambda_{j} \sigma_{i} \sigma_{j})$ to get the the corresponding eigenvalues. The corresponding results are shown in Table VIII. The $B_{c}B_{c}$ system is more attractive than any $\text{ccbb}$ state. The ground $J^{P} = 0^{+}$ $\text{ccbb}$ state is also less attractive than the $B_{c}B_{c}$ decay channel but still more attractive than the $B_{c}^{*}B_{c}^{*}$ decay channel. Thus, it can decay into rearrangement decay channels and is not a stable state.

Next, we take the ground $J^{P} = 0^{+}$ $\text{ccbb}$ state as an example to discuss its properties with the variational method. Similar situation also happens in other two quantum numbers according to Tables VIII and IX. The mass of the lowest $J^{P} = 0^{+}$ $\text{ccbb}$ state is 12920.0 MeV, and corresponding binding energy $B_{T}$ is +344.2 MeV according to Table VIII. Thus, the state is obviously higher than the corresponding rearrangement meson-meson thresholds. The wave function is given as:

$$\langle \Psi_{\text{tot}} \rangle = 0.966 |F\rangle |R^{*}\rangle |\langle \phi_{1} \chi_{5}\rangle - 0.259 |F\rangle |R^{*}\rangle |\langle \phi_{2} \chi_{6}\rangle\rangle.$$  
(20)

Here, we notice that the mass contribution of ground state mainly come from the $|\langle Q_{1} Q_{2}\rangle |\langle Q_{3} Q_{4}\rangle|_{0}$ com-
TABLE IX. The masses, variational parameters, the contribution, and the relative lengths between quarks for $cc\bar{b}\bar{b}$ system, corresponding lowest meson-meson thresholds, and their differences ($D_0$, $D_1$, or $D_2$). The number is given as $i=1,2$ for the quarks, and $3,4$ for the antiquarks. The masses, $B_T$, and contributions are in MeV unit. The relative lengths (variational parameters) are fm (fm$^{-2}$) unit.

| System                  | $J^{PC}$ | (i,j) | $B_cB_c$ | $D_0$ | $B_c^*B_c$ | $D_1$ | $B_c^*B_c^*$ | $D_2$ |
|-------------------------|----------|-------|----------|-------|------------|-------|-------------|-------|
| $1^{++}$                |          |       |          |       |            |       |             |       |
| Mass/B_T                | 12920.0  | 12575.8 | 344.2  | 12939.9 | 12638.4  | 301.5 | 12960.9     | 12700.9 |
| Variational Parameters  |          |       |          |       |            |       |             |       |
| Parameters (fm$^{-2}$)  |          |       |          |       |            |       |             |       |
| $C_{11}$                | 23.9     | 22.9  | 24.8     | 20.2  | 24.5       | 10.1  | 20.2        |       |
| $C_{22}$                | 10.5     | 22.9  | 10.3     | 22.9  | 10.1       | 20.2  |            |       |
| $C_{33}$                | 12.3     | 11.1  |          |       |            |       |             |       |
| Quark Mass              | 14522.0  | 14522.0 | 0.0     | 14522.0 | 14522.0  | 0.0   | 14522.0     | 0.0   |
| Confinement Potential   | -2400.7  | -2795.5 | 375.4  | -2741.1 | 340.4     | -2382.1 | -2686.8     | 304.7 |
| Kinetic Energy          | 835.9    | 947.3 | -111.4  | 814.0  | 795.6      | 835.7 | -40.1       |       |
| CS Interaction          | -7.0     | -98.0 | 91.0     | 4.6    | -34.0      | 38.6  | 25.3        | 30.0  |
| $V^C$                   |          |       |          |       |            |       |             |       |
| (1,2)                   | -271.1   | -225.5 | -222.4   |       |           |       |             |       |
| (1,3)                   | -47.5    | -414.8 | -360.4   | -38.7 | -360.4     | -38.7 |             |       |
| (2,3)                   | -47.5    | -41.6  | -360.4   | -38.7 |           |       |             |       |
| (1,4)                   | -47.5    | -41.6  | -360.4   | -38.7 |           |       |             |       |
| (2,4)                   | -47.5    | -414.8 | -360.4   | -38.7 |           |       |             |       |
| (3,4)                   | -46.9    | -43.0  | -38.9    |       |           |       |             |       |
| Total                   | -454.0   | -829.5 | 375.3    | -434.7| -775.1     | 340.4 | -416.0      | -720.8|
| Total Contribution      | 374.9    | 19.8   | 355.1    | 383.9 | 82.4       | 301.5 | 404.9       | 144.9 |
| Relative Lengths (fm)   |          |       |          |       |            |       |             |       |
| (1,2)                   | 0.230    | 0.226  | 0.227    |       |           |       |             |       |
| (1,3)                   | 0.308    | 0.317  | 0.322    | 0.250 |           |       |             |       |
| (2,3)                   | 0.308    | 0.317  | 0.322    |       |           |       |             |       |
| (1,4)                   | 0.308    | 0.317  | 0.322    |       |           |       |             |       |
| (2,4)                   | 0.308    | 0.317  | 0.322    | 0.250 |           |       |             |       |
| (3,4)                   | 0.348    | 0.351  | 0.355    |       |           |       |             |       |
| (1,2)-(3,4)             | 0.226    | 0.238  | 0.243    |       |           |       |             |       |

ponent, and the $|\langle Q_1Q_2\rangle_0^0(\bar{Q}_3\bar{Q}_4\rangle_0^0)_{0}$ component is negligible. Its variational parameters are given as $C_{11} = 23.9$ fm$^{-2}$, $C_{22} = 10.5$ fm$^{-2}$, and $C_{33} = 12.3$ fm$^{-2}$.

Let us now turn our discussion to the internal contribution for the ground $cc\bar{b}\bar{b}$ state. For the kinetic energy part, the $J^{P} = 0^+$ $cc\bar{b}\bar{b}$ state obtains 835.9 MeV, which is smaller than that of the meson-meson threshold $B_cB_c$. The potential part of this state is far smaller than that of the lowest meson-meson threshold. Further, we notice that all the $V^C$ for this state are attractive. However, relative to the $V^C$ of $B_cB_c$, these attractive values seem to trivial. This is because the length between $c$ and $\bar{b}$ in tetraquarks is longer than that in $B_c$ according to Table IX. In summary, we tend to think these $cc\bar{b}\bar{b}$ states are unstable compact states.

C. $cc\bar{b}$ and $bb\bar{c}$ states

Here, we discuss the $cc\bar{b}$ and $bb\bar{c}$ systems. For these two system, they only need to satisfy the antisymmetry for the diquark. Thus, comparing to above three systems, the $cc\bar{b}$ and $bb\bar{c}$ systems have more allowed states. There are two $J^{P} = 0^+$ states, three $J^{P} = 1^+$ states, one $J^{P} = 2^+$ state in the $cc\bar{b}$ and $bb\bar{c}$ systems. We show the eigenvalue of the hyperfine factor for the ground states and their possible decay channels in Table VIII. Moreover, we calculate the masses of ground states, corresponding variational parameters, different internal contributions, the relative lengths between quarks, and their lowest meson-meson threshold in Table XI respectively.

Firstly, we compare the expectation values of the hyperfine factor of the tetraquarks with the corresponding sum of two mesons in Table VIII. For the $bb\bar{c}$ system, we notice that the $\eta_cB_c$ decay channel is the most attractive channel. Moreover, the $J^{P} = 1^+$ ground state is more at-
TABLE XI. The masses, variational parameters, the internal contribution, and the relative lengths between quarks for $\bar{c}c\bar{b}$, $bb\bar{c}$ systems and their possible decay channels. (unit:(GeV)$^{-2}$)

| Systems | $cc\bar{b}$ | $bb\bar{c}$ |
|---------|-------------|-------------|
| $J^P$   | $0^+$       | $0^+$       |
| $(\sum_{c=1,2} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ | $(\sum_{c=1,2} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ | $(\sum_{c=1,2} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ |
| Low Eigenvalue | -3.58 | -1.28 |
| $J^P$   | $1^+$       | $1^+$       |
| $(\sum_{c=3,4} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ | $(\sum_{c=3,4} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ | $(\sum_{c=3,4} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ |
| Low Eigenvalue | -2.49 | -1.34 |
| $J^P$   | $2^+$       | $2^+$       |
| $(\sum_{c=5,6} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ | $(\sum_{c=5,6} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ | $(\sum_{c=5,6} \frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j)$ |
| Lowest Eigenvalue | 1.97 | 0.71 |

| Decay channel | $J/\psi B_c^*$ | $J/\psi B_c$ | $\eta_c B_c^*$ | $\eta_c B_c$ | $\Upsilon B_c^*$ | $\Upsilon B_c$ | $\eta_c B_c^*$ | $\eta_c B_c$ |
|---------------|----------------|--------------|----------------|--------------|----------------|--------------|----------------|--------------|
| Quantum number | $0^+, 1^+, 2^+$ | $0^+$ | $0^+$ | $0^+$ | $1^+$ | $1^+$ | $1^+$ | $1^+$ |
| Value         | 1.97 | -0.11 | -3.83 | -5.91 | 0.71 | -1.37 | -0.04 | -2.12 |

TABLE XI. The masses, variational parameters, the internal contribution, and the relative lengths between quarks for $bb\bar{c}$ and $cc\bar{b}$ systems, corresponding lowest meson-meson thresholds, and their differences (D$_0$, D$_1$, or D$_2$). The number is given as i=1,2 for the quarks, and 3,4 for the antiquarks. The masses, $B_T$, and contributions are in MeV unit. The relative lengths (variational parameters) are in fm (fm$^{-2}$) unit.

| Systems | $bb\bar{c}$ | $cc\bar{b}$ |
|---------|-------------|-------------|
| $J^{PC}$ | (i,j) | $0^+$ | $0^+$ |
| Mass/B$_T$ | 16043.9 | 15676.9 | 367.0 | 16043.2 | 15739.5 | 303.7 | 16149.2 | 15819.4 | 329.8 | 9620.5 | 9286.4 | 334.1 | 9624.6 | 9349.0 | 275.6 | 9730.5 | 9442.7 | 287.8 |
| Variational Parameters | $\langle -\frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j \rangle$ | $\langle -\frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j \rangle$ | $\langle -\frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j \rangle$ | $\langle -\frac{1}{m_{c}} \lambda_i \sigma_i \cdot \sigma_j \rangle$ |
| Quark Mass | C$_{11}$ | 12.5 | 22.9 | 12.4 | 20.2 | 14.4 | 20.2 | 11.4 | 22.9 | 11.1 | 20.2 | 13.7 | 20.2 |
| Commenent Potential | C$_{22}$ | 21.7 | 58.8 | 21.0 | 57.4 | 28.6 | 49.7 | 7.2 | 15.0 | 6.9 | 15.0 | 8.9 | 12.5 |
| Kinetic Energy | C$_{33}$ | 28.7 | 28.9 | 16.9 | | | | | | | | | |
| CS Interaction | | | | | | | | | | | | | |
| Total | | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| $V^C$ | 1.2 | 40.8 | 39.7 | -110.1 | 32.0 | 29.0 | -99.6 |
| | 1.3 | -251.3 | -248.2 | -85.5 | -91.9 | -87.0 | -21.9 |
| | 2.3 | -228.3 | -879.1(\eta_c) | -225.8 | -879.1(\eta_c) | -77.6 | -164.2(T) | -74.8 | -879.1(\eta_c) | -68.9 | -879.1(\eta_c) | -17.3 | -164.2(J/\psi) |
| | 1.4 | -251.3 | -414.8(B_c^*) | -248.2 | -360.4(B_c^*) | -85.5 | -360.4(B_c^*) | -91.9 | -414.8(B_c) | -87.0 | -360.4(B_c^*) | -21.9 | -360.4(B_c^*) |
| | 2.4 | -228.3 | -225.8 | -77.6 | -74.8 | -68.9 | -17.3 |
| | 3.4 | 97.7 | 94.5 | -257.5 | -12.7 | -17.4 | -14.3 |
| Total | | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| $\langle \bar{c}c \bar{b} \rangle$ | | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| Total Contribution | | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| Relative Lengths (fm) | | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| (1,2)-(3,4) | 0.318 | 0.235(B_c) | 0.320 | 0.250(B_c^*) | 0.296 | 0.250(B_c^*) | 0.333 | 0.235(B_c^*) | 0.338 | 0.250(B_c^*) | 0.304 | 0.250(B_c^*) |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| (1,3) | 0.329 | 0.240 | 0.250 | 0.266 | 0.336 | 0.340 | 0.305 |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| (2,3) | 0.249 | 0.240 | 0.256 | 0.325 | 0.328 | 0.350 |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| (1,4) | 0.249 | 0.240 | 0.256 | 0.325 | 0.328 | 0.350 |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| (2,4) | 0.249 | 0.250 | 0.266 | 0.336 | 0.340 | 0.305 |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| (3,4) | 0.242 | 0.148(\eta_c) | 0.245 | 0.148(\eta_c) | 0.210 | 0.160(T) | 0.418 | 0.290(\eta_c) | 0.429 | 0.290(\eta_c) | 0.378 | 0.318(J/\psi) |
| | | | | | | | | | | | | |
| | | | | | | | | | | | | |
| (1,2)-(3,4) | 0.148 | 0.148 | 0.194 | 0.204 | 0.203 | 0.264 |
tractive than others, and meanwhile, it only slightly less attractive than the $\Upsilon B_c$ decay channel. Therefore, we can expect to have a compact stable state for $J^P = 1^+ b\bar{b}c\bar{c}$ configuration when considering the hyperfine potential only.

Based on these, we now analyse the numerical results of the $J^P = 1^+$ ground $b\bar{b}c\bar{c}$ state obtained from the variational method according to Table [XII]. Other states would have similar discussions from Tables [VIII] and [XI]. The mass of the lowest $J^P = 1^+$ $b\bar{b}c\bar{c}$ state is 16043.2 MeV, and corresponding binding energy $B_T$ is $+303.7$ MeV. Thus, the state is obviously above the lowest rearrangement meson-meson decay channel $B^+\eta_b$, and it is an unstable tetraquark state. Its variational parameters are given as $C_{11} = 12.4$ fm$^{-2}$, $C_{22} = 21.0$ fm$^{-2}$, and $C_{33} = 28.9$ fm$^{-2}$. The corresponding wave function is given as:

$$\langle \Psi_{\text{tot}} \rangle = 0.984 |F| |R_s^s| (|\phi_2 \chi_4| + 0.171 |F| |R_s^s| |\phi_1 \chi_3|)$$

$$- 0.044 |F| |R_s^s| (|\phi_1 \chi_3|).$$

Here, we notice that the mass contribution of ground state mainly come from the $|(Q_1 Q_2)_{h0} (Q_3 Q_4)_{s1}|$ component, and other two components are negligible.

Now, let us turn our concentration to the internal contributions for this state and relative lengths between quarks. For the kinetic energy part, the state obtains 876.1 MeV, which is obviously smaller than that of the lowest meson-meson threshold $B_c \eta_b$. The actual kinetic energy of the $b \bar{b}$ ($b \bar{c}$) in the $J^P = 1^+$ $b\bar{b}c\bar{c}$ state is smaller than that inside the $\eta_b (B^*_c)$ meson. The reason can be seen in Table [XII]. The size of this pair is larger in the $J^P = 1^+$ $b\bar{b}c\bar{c}$ state than that in the meson: the distance (3,4) is 0.245 fm in this tetraquark while it is 0.148 fm in $\eta_b$.

Here, let us turn our discussion to the potential parts. The potential part of this state is far smaller than that of its lowest meson-meson threshold. Though the $V^{c\bar{c}}$ between quark and antiquark are attractive, the $V^{c\bar{c}}$ in the diquark and antiquark are repulsive. However, relative to the $\eta_b$ and $B_c$ mesons, the $V^{c\bar{c}}$ in the tetraquark are less attractive. Thus, these lead to this state still have a relative large positive binding energy.

**D. $c\bar{b}c\bar{b}$ state**

Lastly, we investigate the $c\bar{b}c\bar{b}$ system. Similar to the $cc\bar{c}\bar{c}$ and $b\bar{b}c\bar{c}$ systems, the $c\bar{b}c\bar{b}$ system is also a pure neutral system and has a certain C-parity. Moreover, Pauli principle does not give any constraints for wave functions of the $c\bar{b}c\bar{b}$ system. Thus, comparing to other discussed tetraquark systems, the $c\bar{b}c\bar{b}$ system has more allowed states. There are four $J^{PC} = 0^{++}$ states, four $J^{PC} = 1^{+-}$ states, two $J^{PC} = 1^{++}$ states, two $J^{PC} = 2^{++}$ states in the $c\bar{b}c\bar{b}$ system.

Before discussing the numerical analysis, we compare the expectation values of the hyperfine factor of the $c\bar{b}c\bar{b}$ states with the corresponding sum of two mesons in Table [XII]. We note that the $\eta_b \eta_b$ decay channel is now more attractive than other channels, but the $J^{PC} = 0^{++}$ $c\bar{b}c\bar{b}$ state is more attractive than its corresponding decay channels. Therefore, we can expect to have a stable compact tetraquark for $J^{PC} = 0^{++}$ $c\bar{b}c\bar{b}$ configuration. However, as we see in the Table [XIII], we find that even for this state the two meson threshold lies below the tetraquark mass.

Here, we now analyse the numerical results about the $c\bar{b}c\bar{b}$ system obtained from the variational method. Here, we take the ground $J^{PC} = 0^{++}$ $c\bar{b}c\bar{b}$ state as an example to discuss specifically, and others would have similar discussions. The mass of the lowest $J^{PC} = 1^{+-}$ $c\bar{b}c\bar{b}$ state is 12759.3 MeV, and corresponding binding energy $B_T$ is $+371.8$ MeV. Thus, the state obviously has larger mass than the lowest rearrangement meson-meson-decay channel $\eta_b \eta_b$, and it should be an unstable compact tetraquark state. Its variational parameters are given as $C_{11} = 11.9$ fm$^{-2}$, $C_{22} = 11.9$ fm$^{-2}$, and $C_{33} = 22.9$ fm$^{-2}$. Because this state is a pure neutral state, we naturally notice that the value of $C_{11}$ is equal to $C_{22}$, which means that the distance of ($b \bar{b}$) is equal to ($b \bar{c}$). Our results also reflect these properties according to Table [XIII]. The corresponding wave function is given as:

$$\langle \Psi_{\text{tot}} \rangle = 0.876 |F| |R_s^s| (|\phi_2 \chi_2| + 0.063 |F| |R_s^s| |\phi_1 \chi_2|)$$

$$+ 0.321 |F| |R_s^s| (|\phi_1 \chi_2|) - 0.321 |F| |R_s^s| (|\phi_2 \chi_4|)$$

$$- 0.105 |F| |R_s^s| (|\phi_1 \chi_3|) + 0.105 |F| |R_s^s| (|\phi_1 \chi_4|).$$

Further, we find that its mass contribution of ground state mainly come from the $6 \otimes 6$ component, corresponding $3 \otimes 3$ component is negligible.

Here, let us now turn our discussion to the internal contribution for the ground $J^{PC} = 1^{+-} c\bar{b}c\bar{b}$ state. For the kinetic energy part, the state obtains 858.5 MeV, which is smaller than the 1001.2 MeV of the lowest meson-meson threshold $B_c \eta_b$ according to Table [XIII]. As for the potential part, though the $V^{c\bar{c}}$ between quark and antiquark are attractive, the $V^{c\bar{c}}$ in the diquark and antiquark are repulsive. However, relative to the lowest meson-meson threshold $B_c \eta_b$, the total $V^{c\bar{c}}$ is not attractive than the $B_c \eta_b$, which leads that this state has a relatively larger mass.

We also notice that the $V^{c\bar{c}}$ (1,3), $V^{c\bar{c}}$ (2,3), $V^{c\bar{c}}$ (1,4), and $V^{c\bar{c}}$ (2,4) are absolutely same, and meanwhile the distances of (1,3), (1,4), (2,3), and (2,4) are also same. These states actually reflect $\langle \Psi_{\text{tot}} | (R_{1,2} \cdot R_{3,4}) | \Psi_{\text{tot}} \rangle = 0$. Obviously, it is unreasonable that the distance of $c\bar{c}$ is just equal to that of the $c\bar{b}$ and $b\bar{b}$. According to Sec IV of Ref. [51], we only consider single Gaussian form which the $l_1 = l_2 = l_3 = 0$ in spatial part of total wave function is not enough. These lead to the $c\bar{b}c\bar{b}$ state, which is far away from the real structures in nature. We have reason enough to believe that $\langle \Psi_{\text{tot}} | (R_{1,2} \cdot R_{3,4}) | \Psi_{\text{tot}} \rangle$ should not be zero. Meanwhile, considering other spatial
TABLE XII. The expectation value of the hyperfine factor for the \(cb\bar{b}\) system and their possible decay channels. (unit: GeV\(^{-2}\))

| System | \(J^P\) | \(cb\bar{b}\) system |
|--------|--------|----------------------|
| \(\sum_{\lambda<0} \frac{-\lambda t'}{m_{m'}^2}\) | \(J^P\) | \(\langle P \rangle \sigma_{\lambda} \rangle j \sigma_{\lambda}\) |
| Lowest Eigenvalue | \(1^+\) | \(-5.50\) |
| Low Eigenvalue | \(2^+\) | \(-4.29\) |
| Decay channel | \(J/\psi\) | \(J/\psi_b\) | \(\eta/\eta_b\) | \(B^*_c\) \(B^*_c\) | \(B\) \(B\) |
| Quantum number | \(0^+, 1^+, 2^+\) | \(1^-, 1^-, 1^-\) | \(0^+, 1^+, 2^+, 1^+, 1^-, 0^+\) | \(0^+, 1^+, 2^+, 1^+, 1^-, 0^+\) | \(0^+, 1^+, 2^+, 1^+, 1^-, 0^+\) |
| Value | 0.64 | 0.89 | -4.16 | -4.91 | 1.04 | -1.04 | -3.12 |

TABLE XIII. The masses, variational parameters, the internal contribution, and the relative lengths between quarks for \(cb\bar{b}\) system, corresponding lowest meson-meson thresholds, and their differences (\(D_0\), \(D_1\), or \(D_2\)). The number is given as \(i=1,2\) for the quarks, and 3,4 for the antiquarks. The masses, \(B_r\), and contributions are in MeV unit. The relative lengths (variational parameters) are fm (fm\(^{-1}\)) unit.

| System | \(cb\bar{b}\) |
|--------|-------------|
| \(J^P\) | \((l,j)\) | \(2^+\) \(\eta J/\psi\) \(D_2\) \(1^-\) \(\eta \eta_b\) \(D_1\) \(1^+\) \(\eta J/\psi\) \(D_1\) \(0^+\) \(\eta \eta_b\) \(D_0\) |
| Mass/\(B_r\) | 12882.4 \(12561.1\) \(321.3\) \(12796.9\) \(12467.4\) \(329.5\) \(12856.6\) \(12561.1\) \(295.5\) \(12759.3\) \(12387.5\) \(371.8\) |
| Variational Parameters (fm\(^{-1}\)) | \(C_{11}\) | 11.0 | 12.5 | 11.9 | 49.7 | 11.4 | 12.5 | 12.4 | 15.0 |
| | \(C_{22}\) | 11.0 | 49.7 | 11.9 | 15.0 | 11.4 | 49.7 | 12.4 | 57.4 |
| | \(C_{33}\) | 21.0 | 22.9 | 21.5 | 23.3 |
| Quark Mass | 14522.0 \(14522.0\) \(0.0\) \(14522.0\) \(14522.0\) \(0.0\) \(14522.0\) \(14522.0\) \(0.0\) \(14522.0\) \(14522.0\) \(0.0\) |
| Confinement Potential | \(-2047.0\) \(-2926.9\) \(466.2\) \(-2527.1\) \(-3000.0\) \(472.9\) \(-2488.0\) \(-2926.9\) \(438.9\) \(-2555.2\) \(-3082.3\) \(527.1\) |
| Kinetic Energy | 791.7 | 925.1 | -133.4 | 858.5 | 1001.2 | -142.7 | 818.6 | 925.1 | -106.5 | 888.3 | 1085.5 | -197.2 |

**Note:** The table contents are placeholders and need to be replaced with actual data from the source. The formatting is designed to represent the structure of the table accurately.
basis would reduce the corresponding to the binding energy $B_T$ \cite{51}. Yet these corrections would be powerless against the higher binding energy $B_T$ of the ground $J^{PC} = 1^{+-} c\bar{c}b\bar{b}$. In conclusion, we trend to think the ground $J^{PC} = 1^{+-} c\bar{c}b\bar{b}$ state should be an unstable compact state.

V. SUMMARY

The discovery of a narrow resonance $X(6900)$ gives us strong confidence to investigate the fully heavy tetraquark system. Thus, we use the variational method systematically investigate all possible configurations for fully heavy tetraquarks within the constituent quark model.

We first estimate the theoretical values of ground fully heavy mesons. To obtain those, we need construct the total wave functions of tetraquark states, including spatial part, flavor part, color part, and spin part. Here, we construct spatial part in a simple Gaussian form. Before discussing the numerical analysis, we also analyze the stability condition by using only the color-spin interaction. Further, we give the masses of ground states, corresponding Tables and the spatial distribution of valence quarks. Meanwhile, we show these results in corresponding Tables and the spatial distribution of valence quarks for the ground $J^{PC} = 0^{++} b\bar{b} b\bar{b}$ state in Fig. 1.

For the $c\bar{c}c\bar{c}$ and $b\bar{b}b\bar{b}$ systems, they are two pure neutral systems with definite C-parity. There are only two $J^{PC} = 0^{++}$ states, a $J^{PC} = 1^{+-}$ state, and a $J^{PC} = 2^{++}$ state, because of the Pauli Principle. Moreover, we find these states with different quantum numbers all are above the lowest thresholds, and have larger masses. Due to these states are pure neutral particles, we naturally obtain variational parameters $C_{11}$ and $C_{22}$ are same, correspondingly the distances of diquark and antidiquark are also same. Meanwhile, the distances between quark and antiquark are all the same following symmetry analysis of the Eqs. (15-16). Furthermore, three Jacobi coordinates are orthogonal to each other according to the Eqs. (17-19). Based on these, we take the ground $J^{PC} = 0^{++} b\bar{b} b\bar{b}$ state as an example to show the spatial distribution of four valence quarks. As for internal contribution, though the kinetic energy part is smaller than that of the $\eta\eta$ state, the $V^C$ in $\eta\eta$ is much attractive relative to the ground $J^{PC} = 0^{++} b\bar{b} b\bar{b}$ state, which is the mainly reason that this state has a larger mass than the meson-meson threshold.

Similar to $c\bar{c}c\bar{c}$ and $b\bar{b}b\bar{b}$ systems, the $c\bar{c}b\bar{b}$ system has same number of the allowed ground states. As for the ground $J^{PC} = 0^{++}$ $c\bar{c}b\bar{b}$ state, its mass contribution mainly come from $3 \otimes 3$ component. For the $c\bar{c}b\bar{b}$ and $b\bar{b}c\bar{c}$ systems, there are more allowed states due to less symmetry restrictions. If considering the hyperfine potential only, we can expect to have a compact stable state for $J^{PC} = 1^{++} b\bar{b} c\bar{c}$ configuration. However, the $V^C$ of tetraquark are less attractive than corresponding mesons, this state still has a larger mass than the meson-meson threshold.

In the $c\bar{c}b\bar{b}$ system, these states also are pure neutral particles, and we naturally obtain their variational parameters $C_{11}$ and $C_{22}$ are same. There is no constraint from the Pauli principle, thus, there are four $J^{PC} = 0^{++}$ states, four $J^{PC} = 1^{+-}$ states, two $J^{PC} = 1^{++}$ states, two $J^{PC} = 2^{++}$ states. All of the $c\bar{c}b\bar{b}$ states have larger masses relative to the lowest thresholds.

Hence, we conclude that there is no compact bound ground fully heavy tetraquark state which is stable against the strong decay into two mesons within the constituent quark model. We hope our work will stimulate the interests in the fully heavy tetraquark system.

VI. ACKNOWLEDGMENTS

This work is supported by the China National Funds for Distinguished Young Scientists under Grant No. 11825503, National Key Research and Development Program of China under Contract No. 2020YFA0406400, the 111 Project under Grant No. B20063, and the National Natural Science Foundation of China under Grant No. 12047501. This project is also supported by the National Natural Science Foundation of China under Grants No. 12175091, and 11965016, and CAS Interdisciplinary Innovation Team.

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