Synchronous Behavior for Memristive Synapse-Connected Chay Twin-Neuron Network and Hardware Implementation

Quan Xu, Xiao Tan, Dong Zhu, Mo Chen, Jie Zhou, and Huagan Wu

School of Information Science and Engineering, Changzhou University, Changzhou 213164, China

Correspondence should be addressed to Huagan Wu; wuhg@cczu.edu.cn

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Synchronous behavior can be responsible for the function or dysfunction of a neural network. To employ a memristor with threshold memductance as a bidirectional synapse, a memristive synapse-connected Chay twin-neuron network is constructed. This paper numerically presents the synchronous behavior for four representative firing activities in the memristive twin-neuron network by utilizing time-domain waveforms, synchronized transition states (STSs), and mean synchronization errors (MSEs). Indeed, the synchronous behaviors are truly related to the coupling strength and initial condition of the memristor. Besides, utilizing the powerful XC7Z020 FPGA, a digitally circuit-implemented electro-neuron and the memristive synapse-connected Chay twin-neuron network are made. Thereafter, the four representative firing activities and their STSs are experimentally captured to further confirm the numerical simulations.

1. Introduction

Biological system propagates and handles neural signal via abundant collective behaviors [1–5]. As one of the most outstanding collective behaviors in neuronal network, synchronous behavior is important for the realization of signal propagation, information handling, and other activities [6–8]. Synchronous behavior for neuron-coupled models abstracted from the biological system has attracted much interest due to their simplicity and without loss of generality [9–15]. Numerous investigations of synchronous behavior in gap junction-coupled Huber–Braun neurons [9], gap junction-coupled or noise-sustained FitzHugh–Nagumo neurons [10–12], feedback unidirectional-coupled Hindmarsh–Rose neurons [7, 13], and chemical synaptic-coupled Hindmarsh–Rose neurons [14] have been explored in theoretical and numerical levels. Besides, excessive synchronization can be achieved through stimulating neuron by external control signal, which can be employed to cure some disorder diseases of Alzheimer’s, epilepsy, Parkinson’s, and hand shaking [15–17].

Based on the law of electromagnetic induction theory, Lv et al. have come up with the idea that the memristor can be employed as a connected synapse to describe transmembrane current generated by the membrane potentials’ difference between two coupled neurons [18, 19]. When two coupled neurons own different initial conditions, i.e., different neuron membrane potentials, electromagnetic induction current can be generated to exchange the charged ions across the membrane. In this way, the neuron membrane potentials are altered and synchronization could be achieved [7]. The memristor is a novel circuit device with its memductance really depending on the input and with memory effect [20], which is regarded as a valid candidate for neuronal synapse [21]. Thus, it is interesting to find the emergence of synchronous behavior for memristor synapse-coupled two neurons having different membrane potentials between them [22–25]. The coexisting firing activities and synchronous behaviors associated to initial condition of the memristor in a memristive synapse-connected Morris–Lecar double neurons’ network are explored [22]. Besides, the coexisting firing activities and initial-associating bifurcation behaviors, as well as the extreme events are disclosed in two adjacent Hindmarsh–Rose neurons connected by flux-controlled memristor [26]. However, the memristor initial condition-associated synchronous behavior still needs further investigation.

Three-dimensional (3D) Chay neuron model is a more physiologically excitable three-variable neuron model with simple mathematical form and rich essential features of electrical activities [27]. In essence, the 3D Chay neuron
model has the feature of a fast-slow structure to represent the membrane potential and ionic events in excitable membranes as in [28]. The 3D Chay neuron model can give birth to abundant firing activities of periodic and chaotic bursting/spiking behaviors [29]. Beyond these investigations, by selecting four sets of the maximal conductances for the two ions currents, four representative firing activities are revealed. Besides, the synchronization for these four representative firing activities in coupled Chay neuron models are barely disclosed, which hinders the process of unveiling living biological activities.

The theoretical explorations and numerical simulations are classical methods to investigate neural dynamics, but hardware implementations and experimental measurements have become increasingly promising for diverse neuron-based engineering applications [34–43]. To date, on the benefit of real-time modification, easy software control, and adjustment, field programmable gate array (FPGA) has been employed to verify the dynamical behaviors of Hindmarsh–Rose neuron model [34, 35], Wilson neuron model [36], Morris–Lecar neuron model [37], and FitzHugh–Nagumo neuron model [38], and FitzHugh–Nagumo neuron model [39] in the hardware level. One of another remarkable benefit is that the FPGA-based experiments can assign the desired initial conditions in software accurately and easily, which is conducive to investigate initial condition-associated electrical activities in the hardware level [40]. To consider the virtue of FPGA-based realization method, there should be important significance to develop various digital electronic neurons for utilizing in hardware level synthesizers for large-scale neuromorphic circuits. Besides, the digitally circuit-implemented neuron network can effectively imitate the synchronous behavior in the network, which can promote the integrated circuit design and diverse neuron-based engineering applications [22]. To our best knowledge, there is no comprehensive exploration about the Chay neuron model in this valuable approach.

The arrangement of this paper is well designed as follows. The representative firing activities in a 3D Chay neuron model are briefly reviewed by employing numerical plots of phase trajectories and time-domain waveforms. Besides, a memristive synapse-connected Chay twin-neuron network is built in Section 2. Section 3 explores synchronous behavior in the memristive Chay twin-neuron network by utilizing time-domain waveforms, STSs, and MSEs. A FPGA-based electronic neuron and the memristive synapse-connected Chay twin-neuron network are realized and the hardware tests are executed to verify the numerically simulated ones in Section 4. Eventually, the conclusion is drawn.

2. Memristive Synapse-Connected Chay Twin-Neuron Network

2.1. Minireview for 3D Chay Neuron Model. The 3D Chay neuron model is an effective candidate in the numerical simulation for electrical activities in single biological neuron [27], which is described as

\[
\begin{align*}
\frac{dV}{dt} &= g_l m^{\alpha_m} h^{\alpha_h} (\sqrt{I} - V) + g_r n^{\alpha_n} (V - V_K) + g_k c \frac{C}{1 + C} (V_K - V) + g_L (V_L - V), \\
\frac{dn}{dt} &= \frac{(n_{\infty} - n)}{\tau_n}, \\
\frac{dC}{dt} &= \rho \left[ m^{\alpha_m} h^{\alpha_h} (V_C - V) - k_{\infty} C \right],
\end{align*}
\]

where the three dynamic variables \( V, n, \) and \( C \) correspond to the membrane potential, open probability of the voltage-sensitive potassium ions channel, and intracellular concentration of calcium ions, respectively. The model parameter in (1) is assigned and shown in Table 1. Besides, \( y_{\infty} = \alpha_y/\beta_y \) is unified to the expressions of \( m_{\infty}, h_{\infty}, \) and \( n_{\infty} \), in which \( y \) presents the alphabets \( m, h, \) and \( n \). Herein, the expression formats for \( \alpha_m, \beta_m, \alpha_n, \beta_n, \alpha_h, \beta_h, \) and \( \tau_n \) are
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On account of the fast-slow effect, the bursting and spiking behaviors are two representative firing activities generated in this neuron model [27]. By using MATLAB ODE45-based method with time step $10^{-3}$ s, fixing the initial conditions (0.1 mV, 0.1, 0.1 nmol/L), and employing the model parameters in Table 1, note that “MaxStep” and “RelTol” in MATLAB-based ODE45 algorithm should be assigned as $10^{-3}$ s and $10^{-4}$ s, respectively.

The firing activities related to the reversal potentials [30, 32], external stimulus [31, 33], and time kinetic constant [41] have been investigated in numerical survey. Differently, by selecting four sets of model parameters for the maximal conductance $g_1$ of mixed Na$^+$-Ca$^{2+}$ channel and maximal conductance $g_{K,V}$ of potassium ions channel, four representative types of firing activities including periodic and chaotic bursting and chaotic spiking behaviors are revealed by utilizing the trajectories in the $C$-$V$ plane and the time-domain waveforms of the membrane potential $V$ and intracellular concentration $C$ of calcium ions, as shown in Figure 1. Figures 1(a) and 1(d) exhibit two kinds of periodic bursting behaviors, Figure 1(b) demonstrates chaotic bursting behavior with disordered spikes, and Figure 1(c) illustrates conventional chaotic spiking behavior. The bifurcation mechanisms for the two kinds of periodic bursting behaviors are different. Those are periodic fold/fold bursting in Figure 1(a) and periodic fold/homoclinic bursting in Figure 1(d) [42, 43]. Figure 2(b) shows the chaotic fold/homoclinic bursting behaviors, and Figure 1(c) denotes the chaotic spiking activity. The synchronous behaviors in a memristive synapse-connected Chay twin-neuron network under the four sets of model parameters will be disclosed in the next sections, respectively.

2.2. Modeling for Memristive Synapse-Connected Twin-Neuron Network. Support two identical neurons are bidirectionally coupled by memristor synapse to represent the electromagnetic induction effect induced with the membrane potential differences between them. Herein, the memristor with threshold memductance $W(\varphi) = \tanh(\varphi)$ and the membrane potential difference $V_M = V_1 - V_2$ are employed [26], so the induced current $I_M$ and memristor synapse characteristic model can be described as

$$I_M = W(\varphi)V_M = \tanh(\varphi)(V_1 - V_2),$$

with two identical Chay neurons coupled by the threshold memristor, and a memristive synapse-connected Chay twin-neuron network can be described as

$$\frac{dV_1}{dt} = g_{m_{1_{1_{1}}}h_{1_{1_{1}}}}(V_1 - V_1) + g_{K,V}n_1(V_K - V_1) + g_{K,C}C_1(V_K - V_1) + g_L(V_L - V_1) + k\tanh(\varphi)(V_1 - V_2),$$

$$\frac{dn_1}{dt} = \left(n_{1_{1_{1}}} - n_1\right) / \tau_{1_{1_{1}}},$$

$$\frac{dC_1}{dt} = \rho\left[n_{1_{1_{1}}}h_{1_{1_{1}}}(V_C - V_1) - k_C C_1\right],$$

$$\frac{dV_2}{dt} = g_{m_{2_{2_{2}}}h_{2_{2_{2}}}}(V_2 - V_2) + g_{K,V}n_2^4(V_K - V_2) + g_{K,C}C_1(V_K - V_2) + g_L(V_L - V_2) - k\tanh(\varphi)(V_1 - V_2),$$

$$\frac{dn_2}{dt} = \left(n_{2_{2_{2}}} - n_2\right) / \tau_{2_{2_{2}}},$$

$$\frac{dC_2}{dt} = \rho\left[n_{2_{2_{2}}}h_{2_{2_{2}}}(V_C - V_2) - k_C C_2\right],$$

$$\frac{d\varphi}{dt} = V_1 - V_2,$$
Besides, the coupling strength $k$ connected item just inputs additive current leading to that the membrane potentials which can be regulated. Also, (4) explained the mechanism for memristive feedback control. Besides, $y_{m,\infty} = \alpha_y/\left(\alpha_y + \beta_y\right)$ can be unified to the explications of $m_{\infty, o}$, $h_{\infty, o}$, and $n_{\infty, o}$, in which $y$ stands for $m$, $h$, and $n$ and $i = 1, 2$. The explicit expressions for $\alpha_{im}$, $\beta_{im}$, $\alpha_{ih}$, $\beta_{ih}$, $\alpha_{in}$, $\beta_{in}$, and $\tau_{in}$ are

$$
\alpha_{im} = \frac{0.1 \left(V + V_i\right)}{1 - e^{-\left(V + V_i + 25/10\right)}} \quad \beta_{im} = 4e^{-\left(\left(V + 50\right)/18\right)} \\
\alpha_{ih} = 0.07e^{-\left(V + 50/20\right)} \quad \beta_{ih} = \frac{1}{1 + e^{-\left(V + 20/10\right)}} \\
\alpha_{in} = 0.01\left(20 + V_i\right) \quad \beta_{in} = 0.125e^{-\left(V + 30/80\right)} \\
\tau_{in} = \frac{1}{\left[\alpha_{in} + \beta_{in}\right]}
$$

where $V_1$ and $V_2$ are the membrane potential, $n_1$ and $n_2$ are probability of the voltage-sensitive $K^+$ channel for the two identical Chay neuron models, $C_1$ and $C_2$ are the intracellular concentration of calcium ions for them, respectively, and $k \tan h_p (V_1 - V_2)$ stands for the memristor synapse-coupled item with $k$ as positive to feature the coupling strength. In (4), the coupling strength $k$ can control the effect of memristor synapse. Actually, the memristor synapse-connected item just inputs additive current leading to that the membrane potentials which can be regulated. Also, (4) explained the mechanism for memristive feedback control. Besides, $y_{m,\infty} = \alpha_y/\left(\alpha_y + \beta_y\right)$ can be unified to the explications of $m_{\infty, o}$, $h_{\infty, o}$, and $n_{\infty, o}$, in which $y$ stands for $m$, $h$, and $n$ and $i = 1, 2$. The explicit expressions for $\alpha_{im}$, $\beta_{im}$, $\alpha_{ih}$, $\beta_{ih}$, $\alpha_{in}$, $\beta_{in}$, and $\tau_{in}$ are

$$
\alpha_{im} = \frac{0.1 \left(V + V_i\right)}{1 - e^{-\left(V + V_i + 25/10\right)}} \quad \beta_{im} = 4e^{-\left(\left(V + 50\right)/18\right)} \\
\alpha_{ih} = 0.07e^{-\left(V + 50/20\right)} \quad \beta_{ih} = \frac{1}{1 + e^{-\left(V + 20/10\right)}} \\
\alpha_{in} = 0.01\left(20 + V_i\right) \quad \beta_{in} = 0.125e^{-\left(V + 30/80\right)} \\
\tau_{in} = \frac{1}{\left[\alpha_{in} + \beta_{in}\right]}
$$
Thus, the synchronous behaviors for the memristive synapse-connected Chay twin-neuron network can be disclosed by system (4). Following on this, we mainly focus on exploring the memristor initial condition and coupling strength associated synchronous behaviors in the proposed memristive network.

3. Synchronous Behaviors for the Chay Twin-Neuron Network

The exploration of synchronous behaviors for the Chay twin-neuron network is performed by MATLAB numerical simulations. Herein, we mainly focus on the two coupled neurons with only difference initial membrane potentials between them. Thus, the Chay twin-neuron network is triggered by initial conditions (0.1 mV, 0, 0.1 nmol/L, 1 mV, 0, 0.1 nmol/L, 0) without loss of the generality, within which the memristor initial condition \( \varphi_0 \) is tunable.

The electrical activity is chaotic spiking under the model parameters in Table 1, for each neuron in the memristive synapse-connected Chay twin-neuron network. The synchronized transition states (STSs) are plotted at the right in Figures 2(a)–2(d). There exist no errors between the two membrane potentials with a line in the \( V_1 - V_2 \) plane, which means that the two neurons are in sync. Otherwise, the relatively large errors between the two membrane potentials indicate the two neurons are out of synchronization. When \( \varphi_0 = -2 \text{ mWb} \), the difference between the two membrane potentials become smaller with \( k = 1 \) and 3 increasingly and disappear with \( k = 4 \), as shown in Figures 2(a)–2(c). This manifests that the two coupled neurons are asymptotically synchronized with increasing \( k \). However, when \( k = 4 \) with increasing \( \varphi_0 \) as \(-2 \text{ mWb} \) and \(1 \text{ mWb} \), the difference between the two membrane potentials become larger, as shown in Figures 2(c) and 2(d). It is indicated that the two coupled neurons are loose their synchronization. Summarily, by increasing the coupling strength \( k \) or decreasing memristor initial condition \( \varphi_0 \), the synchronization can be realized and the difference between the two membrane potentials is becoming smaller in the memristive synapse-connected Chay twin-neuron network. The mechanism for this process is that the coupling memristor exchanges the magnetic flux. Thus, the induced current is the carrier to drive the two Chay neurons in sync. Otherwise, the two connected Chay neurons are loss synchronization under tiny electromagnetic induction outputs.

To fully explore the synchronous behavior associated to the coupling strength \( k \) and initial condition \( \varphi_0 \) of the memristor synapse simultaneously, mean synchronization error (MSE) \( E \) is employed and defined as

\[
E = \frac{1}{N} \sum_{j=1}^{N} \frac{\sqrt{[V_1(j) - V_2(j)]^2 + [n_1(j) - n_2(j)]^2 + [C_1(j) - C_2(j)]^2}}{\sqrt{V_1(j)^2 + V_2(j)^2 + n_1(j)^2 + n_2(j)^2 + C_1(j)^2 + C_2(j)^2}}
\]

where \( V_1(j), n_1(j), \) and \( C_i(j) (i = 1, 2) \) are the \( j \)th sampling values and \( N \) is the number of total samples. The normalized MSE of the Chay twin-neuron network is becoming zero for synchronous state and nonzero for out of synchronization. Herein, the time sequence interval (600 s and 700 s) and the time step 0.01 s are selected. Thus, the number of total samples is \( N = 10000 \). The normalized MSEs for different coupling strength \( k \) and initial condition \( \varphi_0 \) as well as different \( g_T \) and \( g_{K,V} \) are given in the \( \varphi_0 - k \) plane, as plotted.
in Figure 3. The regions padded by red indicate the two connected Chay neurons in sync with $E=0$, whereas the regions padded by other colors denote the two connected Chay neurons lose synchronization with $E > 0$. In general, as coupling strength $k$ becoming larger and more negative initial condition $\varphi_0$, the normalized MSEs $E$ drops near zero. Thus, the two connected Chay neurons become sync. Contrarily, the two connected Chay neurons are in asynchronous state with $k$ becoming smaller and small negative or positive $\varphi_0$.

The normalized MSEs is given in Figure 3. The results show that the synchronous behaviors for the memristive synapse-connected Chay twin-neuron network are really associated with the memristor synapse coupling strength and the initial condition. Such an initial condition related synchronous behavior has been rarely reported in previous literature [22].

4. FPGA-Based Hardware Implementation

It is more complex to physically realize the memristive synapse-connected Chay twin-neuron model by FPGA. Thus, only the memristive synapse-connected Chay twin-neuron model by the digital electronic platform is demonstrated with representativeness. For this aim, fourth-order Runge–Kutta algorithm is utilized to obtain the discrete-time form for model (4) and given as

$$(V_1)_N = (V_1)_{N-1} + \frac{1}{4} \left[ D_1k_{(V_1)_1} + D_2k_{(V_1)_2} + D_3k_{(V_1)_3} + D_4k_{(V_1)_4} \right],$$

$$(n_1)_N = (n_1)_{N-1} + \frac{1}{4} \left[ D_1k_{(n_1)_1} + D_2k_{(n_1)_2} + D_3k_{(n_1)_3} + D_4k_{(n_1)_4} \right],$$

$$(C_1)_N = (C_1)_{N-1} + \frac{1}{4} \left[ D_1k_{(C_1)_1} + D_2k_{(C_1)_2} + D_3k_{(C_1)_3} + D_4k_{(C_1)_4} \right],$$

$$(V_2)_N = (V_2)_{N-1} + \frac{1}{4} \left[ D_1k_{(V_2)_1} + D_2k_{(V_2)_2} + D_3k_{(V_2)_3} + D_4k_{(V_2)_4} \right],$$

$$(n_2)_N = (n_2)_{N-1} + \frac{1}{4} \left[ D_1k_{(n_2)_1} + D_2k_{(n_2)_2} + D_3k_{(n_2)_3} + D_4k_{(n_2)_4} \right],$$

$$(C_2)_N = (C_2)_{N-1} + \frac{1}{4} \left[ D_1k_{(C_2)_1} + D_2k_{(C_2)_2} + D_3k_{(C_2)_3} + D_4k_{(C_2)_4} \right],$$

$$\varphi_N = \varphi_{N-1} + \frac{1}{4} \left[ D_1k_{\varphi_1} + D_2k_{\varphi_2} + D_3k_{\varphi_3} + D_4k_{\varphi_4} \right].$$

(7)

where $i$ is the sampling interval, $N$ is the total number of iterations, and $D_q$ ($q=1, 2, 3, 4$) denotes the coefficient of variation. To achieve the unique expression of intermediate vector $k_{(V)_q}$, $k_{(n)_q}$, $k_{(C)_q}$, and $k_{\varphi_q}$ ($i=1, 2$ and $q=1, 2, 3, 4$), the assumption of $k_{(V)_0} = k_{(n)_0} = k_{(C)_0} = k_{\varphi_0} = 0$ is applied. Then, one yields

$$K_{(V)_q} = g_1m_N^2h_N \left[ V_1 - \left( (V_1)_{N-1} + \frac{I_k(V_1)_{q-1}}{D_q} \right) \right]$$

$$+ g_{K_n}m_N^4 \left[ (V_1)_{N-1} + \frac{I_k(V_1)_{q-1}}{D_q} \right]$$

$$+ \frac{g_{K_C}}{1 + (C_1)_{N-1}} \left[ V_C - \left( (V_1)_{N-1} + \frac{I_k(V_1)_{q-1}}{D_q} \right) \right]$$

$$+ \frac{g_L}{(C_1)_{N-1}} \left[ V_L - \left( (V_1)_{N-1} + \frac{I_k(V_1)_{q-1}}{D_q} \right) \right]$$

$$+ k \tan \varphi_{N-1} \left( (V_1)_{N-1} + \frac{I_k(V_1)_{q-1}}{D_q} - (V_2)_{N-1} \right),$$

$$K_{(n)_q} = \frac{d_N \left( (n_1)_{N-1} + \frac{I_k(n_1)_{q-1}D_4}{D_q} \right)}{\tau_N},$$

$$K_{(C)_q} = \rho \left[ m_N^2h_N (V_C - (V_1)_{N-1}) - K_C \left( (C_1)_{N-1} + \frac{I_k(C_1)_{q-1}}{D_q} \right) \right],$$

$$K_{(\varphi)_q} = \frac{d_N \left( (n_2)_{N-1} + \frac{I_k(n_2)_{q-1}D_4}{D_q} \right)}{\tau_N},$$

$$K_{(C)_q} = \rho \left[ m_N^2h_N (V_C - (V_2)_{N-1}) - K_C \left( (C_2)_{N-1} + \frac{I_k(C_2)_{q-1}}{D_q} \right) \right].$$

(8)

in which $q=1, 2, 3, 4,$ and
Figure 3: Normalized MSEs of the memristive synapse-connected Chay twin-neuron network in the $\phi_0 - k$ plane. (a) $g_I = 1250 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (b) $g_I = 1850 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (c) $g_I = 1925 \text{ mS/cm}^2$ and $g_{K,V} = 1700 \text{ mS/cm}^2$; (d) $g_I = 1800 \text{ mS/cm}^2$ and $g_{K,V} = 1650 \text{ mS/cm}^2$.

Figure 4: Hierarchical structure of the Verilog HDL program for the memristive synapse-connected Chay twin-neuron model.
Figure 5: RTL schematics of the discrete-time memristive synapse-connected Chay twin-neuron model executed on XC7Z020 FPGA.
To achieve the discrete-time memristive synapse-connected Chay twin-neuron model (7), we employ a low-cost yet powerful XC7Z020 FPGA to execute the model for the first time. The FPGA software program using Verilog language is coded, within which the number of iteration \( N = 150000 \), the sampled interval \( i = 0.001 \), and the coefficients of variation \( D_1 = D_4 = 1 \) and \( D_2 = D_3 = 2 \) are set up. The parameters \( g_n, g_KV, k, \) and \( \varphi_0 \), which impact the firing activities and synchronous behaviors, are changed by the software program to capture the four typical firing activities, time-domain waveforms, and STIs corresponding to the numerical simulations.

The hierarchical structure of the Verilog HDL program is illustrated in Figure 4. A main controller is set to reset the electro-neuron and to start the iteration form of initial conditions for \( V_n, n_0, C_n \), and \( \varphi \) while the power is on. The floating-point operation IPs are contained in the lowest inherent IP layer, which can be instanced to construct our own customized IP. In the customized IP layer, add-, subtract-, multiply-, divide-, and exponent-operators are instanced to construct each function of the right side of continuous-time model (4) (expressed as \( f_{V1}, f_{n1}, f_{C1}, f_{V2}, f_{n2}, f_{C2}, \) and \( f_\varphi \)). The intermediate variables \( m_{i10}, m_{i20}, m_{i20}, h_{100}, n_{100}, m_{200}, h_{200}, \) and \( n_{200} \) have to be calculated before the calculations of \( f_{V1}, f_{n1} \) and \( f_3 \) thus, a sublayer for intermediate variables is hired in this layer. In the iteration layer, the functions \( f_{V1}, f_{n1}, f_{C1}, f_{V2}, f_{C2}, \) and \( f_\varphi \) are transmitted by time multiplexer way to compute the intermediate vectors \( k_{(V1)}, k_{(n1)}, k_{(C1)}, \) and \( k_{eq} \) (marked as \( K_1, K_2, K_3, \) and \( K_4 \)) in (8), and then the \( (V1)_N, (n1)_N, (C1)_N, (V2)_N, (n2)_N, (C2)_N, \) and \( \varphi_N \) are updated. Finally, an output layer is employed to adjust the output signal for visualizing expeditiously. The RTL schematic for the digitally electronic twin-neuron network is generated, as shown in Figure 5.

Since the outputs of a FPGA are digital, they are fed to a two-channel 14-bit D/A converter (AD9767) combined with the peripheral circuit to convert the digital outputs into analog ones. Herein, an oscilloscope Agilent DSO-X 3012A is employed to display the output analog signals. Note that, all variables are in the single-precision floating-point during the computation process, so they must be converted into integers and enlarged to the range of \([-8192, 8191]\) to take full use of 14 bits of the DAC digital input ports. The time-domain waveforms of \( V_1 \) and \( V_2 \) and
synchronous transition states in the $V_1 - V_2$ plane for different coupling strength $k$ and initial condition $\phi_0$ in the Chay bi-neuron network are captured and displayed in Figure 6. The amplitudes of $V_1$ and $V_2$ on the oscilloscope are differed from that of MATLAB simulations in Figure 2, but they are proportional with each other due to the same enlargement. It is clearly shown that the proposed FPGA-based digital hardware can verify the synchronous behavior for the memristive synapse-connected Chay twin-neuron network. Note that the initial states in our digital circuit experiment are accurately assigned in the software [44]. This is very different from the randomly sensed way of acquiring initial states by repeatedly switching on and off the power supply in an analog circuit experiment [45]. Hence, the initial states are determined in the digital circuits but undetermined in the analog ones.

Besides, a FPGA-based digital hardware electronic neuron is simply realized to confirm the four representative firing activities in the 3D Chay neuron model. The trajectories in the $V_1 - V_2$ phase plane and time sequences of $V_1$ and $V_2$ are captured and displayed in Figure 7. The model parameters are selected the same as these utilized in Figure 1.

5. Conclusion

Four kinds of representative firing activities classified on the dependence of two maximal conductances in a 3D Chay neuron model are briefly reviewed. Then, a memristive synapse-connected Chay twin-neuron network is built, upon which synchronous behaviors are explored by utilizing time-domain waveforms, STSs, and MSEs. The numerical simulations demonstrated the success and effectiveness of employing the memristor synapse to achieve synchronization. It is found that, associating with the large coupling strength and more negative initial condition of the memristor, synchronous behaviors are achieved. An effective approach to implement the electronic neuron and the Chay twin-neuron network via FPGA are employed, from which the four kinds of representative firing activities of chaotic and periodic bursting/spiking behaviors, as well as the STSs are experimentally captured to confirm the correctness of the numerical ones. Synchronous behavior disclosed in neuronal network can well reveal the benefit for understanding the dynamical intricacy in the biological neurons and reflect the feasibility of diverse neuron-based applications.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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