Energy-Aware JPEG Image Compression: A Multi-Objective Approach

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Abstract

Customer satisfaction is crucially affected by energy consumption in mobile devices. One of the most energy-consuming parts of an application is images. While different images with different quality consume different amounts of energy, there are no straightforward methods to calculate the energy consumption of an operation in a typical image. This paper, first, investigates that there is a correlation between energy consumption and image quality as well as image file size. Therefore, these two can be considered as a proxy for energy consumption. Then, we propose a multi-objective strategy to enhance image quality and reduce image file size based on the quantisation tables in JPEG image compression. To this end, we have used two general multi-objective metaheuristic approaches: scalarisation and Pareto-based. Scalarisation methods find a single optimal solution based on combining different objectives, while Pareto-based techniques aim to achieve a set of solutions. In this paper, we embed our strategy into five scalarisation algorithms, including energy-aware multi-objective genetic algorithm (EnMOGA), energy-aware multi-objective particle swarm optimisation (EnMOPSO), energy-aware multi-objective differential evolution (EnMODE), energy-aware multi-objective evolutionary strategy (EnMOES), and energy-aware multi-objective pattern search (EnMOPS). Also, two Pareto-based methods, including a non-dominated sorting genetic algorithm (NSGA-II) and a reference-point-based NSGA-II (NSGA-III) are used for the embedding scheme, and two Pareto-based algorithms, EnNSGAII and EnNSGAIII, are presented. Experimental studies show that the performance of the baseline algorithm is improved by embedding the proposed strategy into metaheuristic algorithms. In particular, EnMOGA, EnMOPS, and EnNSGA-II can perform competitively, among others. Furthermore, we statistically verify the proposed algorithm’s effectiveness based on the Wilcoxon-signed rank test. Finally, a sensitivity analysis of the parameters is provided. The source code for reproducing the results is available in: https://github.com/SeyedJalaleddinMousavirad/MultiobjectiveJPEGImageCompression

Keywords: JPEG image compression, energy, NSGA-II, genetic algorithm, metaheuristic
1 Introduction

Mobile devices such as smartphones and tablets are ubiquitous and receiving much attention for their energy efficiency since customer satisfaction relies heavily on battery uptime. In addition, battery uptime plays a crucial role for developers since anomalous draining usually warrants negative app store ratings [27].

Several studies [10, 23, 33, 35] have concentrated on documenting energy-aware programming trends in the context of Android, the leading mobile ecosystem, and finding better alternatives. But images have not been seriously discussed, while they are one of the most important components of mobile software, particularly in games.

JPEG (Joint Photographic Experts Group) format is the most commonly used method of compression for digital images, and is based on the Discrete Cosine Transform (DCT) [2]. The process of JPEG compression is started with representation of the original uncompressed image in YCbCr colour space, where Y, Cb and Cr indicate luminance, blue and red chrominance components, respectively; and each component is handled independently. We shall simply use the luminance component, Y, for the sake of simplicity, while for other components, the process is the same. The image’s component Y is divided into $8 \times 8$ blocks, each of which is separately modified. Before using the DCT, the $8 \times 8$ blocks are zero-shifted by deducting 128 from the element values. Then, each modified block is quantised. The primary mechanism for compression, quantisation, also results in information loss due to the representation of the DCT coefficients. Each block may be effectively entropy encoded after quantisation [7], with no information being lost in the process.

The quantisation table (QT) plays a crucial role in the JPEG image compression. Annex K variant [19], the most important variant of JPEG implementation, employs two quantisation tables, called luminance quantisation table (LQT) and chrominance quantisation table (CQT). The main responsibility of these two is to quantise the DCT coefficient blocks of luminance and chrominance elements, respectively. The process of finding proper values for both quantisation tables is a challenging and difficult task since each image needs its own table, although most implementations use a conventional value for the tables.

To design the best quantisation table, meta-heuristic algorithms (MA) such as genetic algorithm (GA) [47] and particle swarm optimisation (PSO) [40] can be used. MAs are iterative, stochastic, and problem-independent algorithms that solve an optimisation problem by using a number of operators to guide the search process. Also, they can provide a close to optimal solution, but they are not able to guarantee a global optimum solution.

In one of the first efforts to use MA for the construction of the JPEG QTs, [9] proposed a GA algorithm to find the quantisation table so that the chromosome is an array of size 64, while the objective function is the mean square error between the original image and the compressed image. In another work, [29] incorporated GA to design a JPEG image quantisation table to compress iris images in iris recognition systems. [4] proposed a knowledge-based GA to find the quantisation table. To this end, image characteristics and knowledge about image compression are integrated into the GA algorithm. Differential evolution (DE)
is used to design the quantisation table in another paper [32], and they showed that DE could outperform canonical GA. Another study [30] proposes a knowledge-based DE to improve the performance of DE. From the literature, we can also observe some other MA algorithms for designing QTs in JPEG, such as simulated annealing (SA) [18, 25], DE algorithm [46], particle swarm optimisation (PSO) [18], firework algorithm [43], and firefly algorithm [44]. Also, some researchers try to combine different MAs to improve the performance of designing QTs. For instance, [38] proposed a combination of FA and teaching-learning-based optimisation to select the QT. Also, reducing time complexity is considered in a few papers. For example, [31] employs a surrogate model-based DE algorithm to reduce computation time for optimising the QT.

While images are one of the primary sources of energy consumption in smartphones, it is challenging to measure the amount of energy consumed for a specific operation in a typical image. Most of the current methods in the literature can measure the power of a battery or, at best, for a particular application [16]. To tackle this, we used the energy profiler of Android Studio and Plot Digitiser software, manually and not in an automatic way, to verify that image quality and file size play a crucial role in the energy consumption of an application. In other words, smaller file sizes and lower image quality consume less energy. A developer has two main goals in selecting an image: 1) they tend to select an image with high quality, and 2) they prefer to choose an image with smaller file size. As a result, there are two conflicting criteria for a mobile developer. Since an operation’s energy for an image cannot be calculated as a straightforward process, file size and quality can act as a proxy for energy consumption.

These two criteria, file size and image quality, are two conflicting objectives. Therefore, multi-objective metaheuristic optimisation (MOMO) algorithms can be used to tackle this issue. MOMO addresses optimising a problem based on two or more conflicting objective. There are two general approaches for solving a multi-objective problem, namely, scalarisation and Pareto-based approaches [22]. Scalarisation approaches solve a multi-objective problem by converting it into a single-objective problem, while Pareto-based approaches generate a set of optimal solutions.

This paper proposes an energy-aware JPEG Image compression strategy. The main characteristics of this paper are as follows:

1. We investigate, based on an energy profiler, that there is a high correlation between energy consumption and image quality. Such a condition is also valid for image file size.

2. We propose a multi-objective strategy for handling both image file size and image quality.

3. The multi-objective strategy is embedded into five scalarisation methods, including genetic algorithm (GA), differential evolution (DE), particle swarm optimisation (PSO), evolutionary strategy (ES), and pattern search (PS). Therefore, five scalarisation-based multi-objective techniques for JPEG image compression are introduced, namely, EnMOGA, EnMODE, EnMOPSO, EnMOES, and EnMOPS.
4. We also embed the proposed strategy into two well-known Pareto-based approaches, the non-dominated sorting genetic algorithm (NSGA-II) [13] and reference-point based non-dominated sorting genetic algorithm (NSGA-III) [12]. As a result, two Pareto-based techniques are introduced here, namely, EnNSGAII and EnNSGAIII.

5. We provided an extensive set of experiments for validating the algorithms.

This paper is organised as follows. Section 2 explains briefly some challenges in the paper, while Section 3 introduces background knowledge. Section 4 explains the metaheuristic algorithms used in the paper. The proposed algorithms are introduced in Section 5 whereas we provide a set of extensive experiments in Section 6. Finally, the paper is concluded.

2 Key Challenge

One of the main challenges in calculating the energy consumption of an application is figuring out how to do it. Some research uses hardware devices for this purpose [34], which is hard to set up. They calculate the energy consumption of the battery and not an android application. Some others try to estimate the energy profile of an android application, which is not straightforward to do as well [16].

This section investigates the effect of image file size and image quality on energy consumption. To this end, we used an energy profiler in the Android Studio software [21] and a plot digitiser [3] to estimate energy consumption. Plot digitisers are tools to convert a specific curve to digitised numbers. To this end, first, the digitiser should be calibrated for the curve (here between 0 and 2000). In other words, we should specify the minimum and maximum values on the y-axis. In this case, the minimum and maximum values are not critical since we only need a comparison between the results (and not an exact number for energy consumption).

To design the experiment, we compress an image at different levels (90%, 70%, 50%, 30% and 10%), and the energy consumption (EC) for each image is calculated. To this end, an image loading program is written, located in a loop with 1000000 iterations and then, the energy profile is achieved for this during five independent runs (Figure 1). Then, the energy profiler yielded is fed to the plot digitiser to convert it to digit numbers, and for each run, the total energy consumption is estimated. Finally, the average over five runs is obtained as the EC measure. The size and quality (based on PSNR) are also reported for each image. The results can be seen in Table 1. From the table, we can conclude that:

1. By decreasing the image compression level, the EC is decreased as well.

2. By decreasing the image compression level, as expected, the file size is decreased as well.

3. By decreasing the image compression level, as expected, the image quality deteriorated.
Figure 1: Energy profiler for an image loading program in 5 independent runs.

Table 1: Effect of image size and image quality on the energy consumption.

| Level | EC      | File Size(Mb) | PSNR   |
|-------|---------|---------------|--------|
| Original | 2769.52 | 3.07          | inf    |
| 90     | 2734.73 | 1.90          | 38.6913|
| 70     | 2682.73 | 1.50          | 37.5774|
| 50     | 2638.61 | 0.98          | 35.1707|
| 30     | 2566.47 | 0.66          | 33.0362|
| 10     | 2511.69 | 0.27          | 28.3434|

4. By decreasing the image size, the image quality is also reduced.

All in all, reducing the compression level reduces the file size, image quality, and energy consumption, while the developer tends to use higher image quality and smaller file size. Therefore, these two objectives, image quality and image size, are in conflict.

The correlation between image size and image quality (based on the information available in Table 1) is demonstrated in Table 2 to show this contradiction. This table clearly verifies a conflict between higher image quality and smaller file size since the correlation is a high positive number, close to 1.

In conclusion, file size and image quality can be considered proxies of energy consumption. While developers tend towards smaller file sizes and higher image quality, these two objectives conflict with each other since higher image quality will increase the file size and energy consumption. As a result, it is necessary to strike a balance between image quality and file size.

Table 2: Correlation between EC criterion and file size with other.

| Correlation | File size | PSNR     |
|-------------|-----------|----------|
| EC          | 0.9433    | 0.9754   |
| File size   | 1         | 0.9615   |
3 Preliminaries

3.1 Multi-objective Optimisation

Multi-objective optimisation (MO) is the process of finding the minimum or maximum of two conflicting objective functions. Without loss of generality, a multi-objective optimisation problem (MOP), formally, can be stated as a minimisation problem as

\[
\text{Minimise } F(x) = (f_1, f_2, \ldots, f_M)
\]

subject to \( x \in \Omega \) (1)

where \( \Omega \) is the decision space, and \( F : \Omega \rightarrow \mathbb{R}^M \) is the objective function in which \( M \) is the number of different real-valued objective functions, and \( \mathbb{R}^M \) shows the objective space.

There are two general metaheuristic approaches for tackling multi-objective optimisation, scalarisation and Pareto-based approaches [22]. Scalarisation approaches solve a multi-objective problem by converting it into a single-objective problem, while Pareto-based techniques find a set of optimal solutions.

Scalarisation approaches incorporate multi-objective functions into one single scalar objective function as

\[
G(x) = w_1 f_1(x) + w_2 f_2(x) + \ldots + w_M f_M(x)
\] (2)

The real-valued positive weights, \( w_i, i = 1, 2, \ldots, M \), indicate the performance priority. A larger weight shows that the corresponding objective function has a higher priority than the objective function with a smaller weight. When the priority of objective functions is not clear in advance, one of the most common methods is to use Equal Weights [22], in which the weights are given by

\[
w_i = \frac{1}{M}
\] (3)

where \( i = 1, 2, \ldots, M \).

After scalarisation, all single-solution-based metaheuristic algorithms such as GA [47], PSO [26], and DE [41] can be used to find the optimal solution.

There is usually no single solution that can simultaneously minimise all the objective functions since the objectives are inherently competing. To tackle this, a set of optimal solutions, called Pareto optimal solutions, can be defined, with corresponding localisation in the objective space called the Pareto front.

In single-objective optimisation, the superiority of one solution over another can be easily obtained by comparing the objective functions, while the quality of a solution can be achieved by the concept of dominance in multi-objective optimisation.

**Definition 1** (Pareto dominance). A solution \( x_1 \) dominates another solution \( x_2 \) (denoted by \( x_1 \prec x_2 \)) if and only if:
1. \( \forall i \in \{1, 2, \ldots, M\} : f_i(x_1) \leq f_i(x_2) \), where \( M \) is the number of objective functions. In other words, in all objective functions, solution \( x_1 \) should not be worse than \( x_2 \).

2. \( \exists j \in \{1, 2, \ldots, M\} : f_i(x_2) < f_i(x_1) \); meaning that solution \( x_1 \) is strictly superior to solution \( x_2 \) in at least one objective function.

**Definition 2** (Non-dominated solution). A solution \( x_1 \) is called Pareto optimal solution or non-dominated solution if it is not dominated by other solutions in the whole search space. It can be mathematically defined as

\[
\nexists x_2 \in X : x_2 \preceq x_1
\]

Figure 2 indicates non-dominated solutions among other solutions in a bi-objective minimisation problem. \( f_1 \) and \( f_2 \) are two conflicting objectives which should be minimised simultaneously. From the figure, \( x_3 \) has a lower value than \( x_1 \) in both objective functions. Therefore, we can say that \( x_1 \) is dominated by \( x_3 \). In other words, \( x_3 \) is a non-dominated solution. In addition, \( x_2, x_4, x_5 \) and \( x_6 \) are also non-dominated solutions since there is no other solution that dominates them in both objective functions.

**Definition 3** (Pareto front). The set of all non-dominated solutions is called Pareto optimal set (PS), which is stated as

\[
PS = \{ u \in X | \nexists v \in X, u < v \}
\]

The Pareto front (PF) corresponds to the Pareto optimal set in the objective space, and is denoted as

\[
PF = \{ F(x) | x \in PS \}
\]
### 3.2 The JPEG Image Compression

Figure 3 shows the main components of JPEG image compression. The encoder is responsible for converting the original image into the JPEG compression variant of the original image, while the reverse task is carried out by the decoder. In the following, we explain the main components in more detail.

#### 3.2.1 DCT and IDCT Components

The source image is first divided into $8 \times 8$ blocks. Then, the values of the blocks are shifted from $[0, 2^p - 1]$ to $[-2^{p-1}, 2^{p-1} - 1]$, in which $p$ is the number of bits per pixel (in the baseline JPEG compression, $p = 8$). Each block of $8 \times 8$ pixels can be seen as a vector with a size of $64 \times 1$, which should be fed into the Discrete Cosine Transform (DCT) component. The DCT block decomposes the input signal into 64 basis-signal amplitudes, called DCT coefficients. Mathematically, the DCT can be expressed as

$$F(u, v) = \frac{1}{4} c_u c_v \left\{ \sum_{x=0}^{7} \sum_{y=0}^{7} f(x, y) \cos \left( \frac{(2x + 1)u\pi}{16} \right) \cos \left( \frac{(2y + 1)v\pi}{16} \right) \right\}$$

(7)

where

$$c_r = \begin{cases} \frac{1}{\sqrt{2}} & r = 0 \\ 1 & r > 0 \end{cases}$$

(8)

The coefficient corresponding to $u, v = 0$ is known as the DC coefficient, while the remaining 63 coefficients are the AC coefficients.

Inverse DCT (IDCT) is the reverse of DCT component to reconstruct the original image, which is stated as

$$F(x, y) = \frac{1}{4} c_x c_y \left\{ \sum_{u=0}^{7} \sum_{v=0}^{7} f(u, v) \cos \left( \frac{(2x + 1)u\pi}{16} \right) \cos \left( \frac{(2y + 1)v\pi}{16} \right) \right\}$$

(9)

In the absence of the quantisation step, the original 64-point signal is precisely restored.
3.2.2 The Quantisation and Dequantisation Components

The quantisation step works based on a 64-element quantisation table, which should be known in advance. Each table entry defines the step size of the quantiser for its related DCT coefficient and belongs to [1,255]. Quantisation aims to achieve compression while maintaining image quality by removing information that is not visually important.

The quantisation component is defined as

$$L(u, v) = \text{round}\left(\frac{F(u, v)}{Q(u, v)}\right),$$  \hspace{1cm} (10)

where $L(u, v)$ are the quantised DCT coefficients, $F(u, v)$ are the DCT coefficients, $Q(u, v)$ indicates the corresponding element of the quantisation table, and $\text{round}(x)$ is the closest integer number to $x$. It is worth mentioning that the larger the value of $Q(u, v)$, the larger the information loss.

The de-quantisation component of the decoder reverses the quantisation process to recreate a rough estimate of $F(u, v)$ from $L(u, v)$ as

$$\hat{F}(u, v) = L(u, v) \times Q(u, v),$$  \hspace{1cm} (11)

This step plays a crucial role in the process of JPEG compression since the quantisation table generates a loss of information. Thus, it is necessary to establish the quantisation table to strike a compromise between compression effectiveness and reconstructed image quality.

3.2.3 Symbol Coding

The 63 AC coefficients of the 8×8 block are handled independently from the DC coefficient after quantisation. The Differential Pulse Code Modulation (DPCM) is used to encode the DC coefficient as

$$\text{DIFF}_i = DC_i - DC_{i-1}$$  \hspace{1cm} (12)

where $DC_i$ and $DC_{i-1}$ are the DC coefficients for the current 8×8 block and the prior 8×8 block, respectively.

In order to format the quantised 63 AC coefficients for entropy coding, a zigzag scan [39] can be used. After the zigzag scan, the AC coefficients show diminishing variances and rising spatial frequencies.

3.2.4 Entropy Coding

After the quantisation process, there are often a few nonzero and several zero-valued DCT coefficients. Entropy coding’s goal is to compress the quantised DCT coefficients by making use of their statistical properties. The baseline technique used by JPEG is the Huffman coding, which employs two DC and two AC Huffman tables for the luminance and chrominance DCT coefficients, respectively [39].
4 Algorithms

Due to the introduction of a vast and varied range of metaheuristic techniques in the literature, it is evident that we cannot analyse all of them. Also, the main focus of this paper is not benchmarking all algorithms but introducing a general strategy for multi-objective JPEG image compression. Therefore, for our study, we have chosen a variety of state-of-the-art algorithms. In the following, we briefly outline the selected algorithms, while the cited publications are referred to for further details.

4.1 Scalarisation Methods

- Genetic algorithm (GA) [47] is the oldest metaheuristic algorithm, and includes three significant operators: selection, crossover, and mutation. The selection operator is responsible for selecting candidate solutions who contribute to the next generation’s population. The information from the parents is integrated into the crossover operator, while random modifications are made to one or more components of a potential solution in the mutation operator. Based on the ”survival of the fittest” premise, solutions are transferred from one iteration to the next.

- Differential evolution (DE) [41] is another metaheuristic algorithm including three main operators, mutation, crossover, and selection. Mutation creates candidate solutions based on the differences among candidate solutions as

  \[ v_i = x_{r1} + S F(x_{r2} - x_{r3}), \]

  where \( S F \) signifies a scaling factor, and \( x_{r1}, x_{r2}, \) and \( x_{r3} \) are three distinct randomly selected candidate solutions from the current population, and \( v_i \) is called a mutant vector. Crossover is responsible for integrating the mutant vector with a target vector selected from the current population. Eventually, a candidate solution is selected by a selection operator depending on its quality.

- Particle swarm optimisation [40] is a swarm-based optimisation approach, and its updating scheme is based on the best position found for each candidate solution and a global best position. The velocity vector of a particle is updated as

  \[ v_{t+1} = \omega v_t + c_1 r_1 (p_t - x_t) + c_2 r_2 (g_t - x_t), \]

  where \( t \) shows the current iteration, \( x_t \) is the current position, \( r_1 \) and \( r_2 \) are random numbers generated from a uniform distribution in the range of \([0, 1]\), \( p_t \) is the personal best position, and \( g_t \) indicates the global best position. Then, a candidate solution is updated as

  \[ x_{t+1} = x_t + v_{t+1}, \]

- Evolutionary strategy [48] is a metaheuristic algorithm where each offspring is generated based on a
Gaussian random number as

\[ x_{\text{new}} = x_{\text{old}} + N(0, \sigma^2), \]  

(16)

where \( N(0, \sigma^2) \) is a Gaussian random number with mean 0 and variance \( \sigma^2 \). Then, competition should be done for each individual and finally, the best individuals transfer to the next generation.

- Pattern search \[^{24}\] is a simple yet effective optimisation algorithm that, in an iterative manner, combines exploratory and pattern moves to find the best solution to a problem. The exploratory move tries one direction, and if that doesn’t work, it tries the other. In particular, it generates a new solution as

\[ x^+ = x + \rho, \]  

(17)

where \( x^+ \) is the new solution based on the current solution \( x \), and \( \rho \) is called the step size or exploratory radius. If this move can not improve the current solution, it attempts another direction

\[ x^+ = x - \rho, \]  

(18)

If the moves in all directions fail, then the radius is halved.

4.2 Pareto-based Techniques

4.2.1 Non-dominated Sorting Genetic Algorithm II

The Non-Dominated Sorting Genetic Algorithm (NSGA-II) \[^{13}\] is one of the state-of-the-art approaches for Pareto-based multi-objective optimisation. NSGA-II is based on four basic operators, including, Non-Dominated Sorting, Elite Preserving Operator, Crowding Distance, and Selection Operator, which are described below in more detail.

**Non-Dominated Sorting:** The notion of Pareto dominance is used in this process to sort the population members. In the first step, the non-dominated members of the initial population are assigned to the first rank. These top-ranked individuals are subsequently put in the first front and eliminated from the current population. The remaining population members are then subjected to the non-dominated sorting technique. The remaining population’s non-dominated individuals are given the second rank and positioned in the second front. This procedure continues until all population members are distributed across various fronts in accordance with their rankings, as seen in Figure \[^{4}\].

**Elite Preserving Operator:** Elite solutions are maintained by being immediately passed on to the next generation as part of an elite preservation strategy. In other words, the non-dominated solutions identified in each generation transfer to the next generations until some solutions dominate them.

**Crowding Distance:** The crowding distance determines the density of solutions around a specific solution. It is the average distance between two solutions along each of the objectives on each side of the
solution. When two solutions with varying crowding distances are compared, the solution with the greater crowding distance is assumed to be present in a less crowded area. The crowded distance of the $i$-th solution is computed based on the average side-length of the cuboid (Figure 5). Mathematically, the crowding distance is defined as

$$CD_i = \sum_{j=1}^{k} \frac{f_{i+1}^j - f_{i-1}^j}{f_{max}^j - f_{min}^j},$$

(19)

where $f_{i}^j$ shows the $j$-th value of an objective function for the $i$-th solution, $f_{max}^j$ and $f_{min}^j$ signify the maximum and minimum values of $j$-th objective function among the current population, and $k$ is the number of objective functions.

**Selection Operator:** A crowded tournament selection operator is used to choose the population for the next generation. This operator selects the population based on the rank of the population members and the crowding distances between them. The following rules apply when choosing one of two population members to represent the next generation: 1) If the two population members are of different ranks, the higher rank one is chosen; and 2) If the two population members are of the same rank, the member with the greater crowding distance is chosen.

**Procedure:** The NSGA-II algorithm starts by creating an initial population $P_t$ of size $N$. Following crossover and mutation operations on the population, $P_t$, a new population, $Q_t$, is produced. Then, the non-dominated sorting operation is carried out on the new population, $R_t$, created by combining the populations $P_t$ and $Q_t$. The $R_t$ population members are then divided into several fronts based on their non-dominance levels.

The next step is to choose $N$ candidate solutions from $R_t$ in order to produce $P_{t+1}$. If the size of the first front is greater than or equal to $N$, only $N$ members are chosen from the least crowded area of the first front to create $P_{t+1}$. The members of the first front are directly moved to the next generation if the size of
the first front is more than $N$, and the remaining members are taken from the second front’s least crowded area and added to $P_{t+1}$. The process is repeated for the subsequent fronts until the size of $P_{t+1}$ equals $N$, if the size of $P_{t+1}$ is still less than $N$. Following the same process, the populations $P_{t+2}$, $P_{t+3}$, $P_{t+4}$,... for subsequent generations are created until the stopping criterion is not met. Figure 6 shows the NSGAII procedure visually.

4.3 Reference-point Based Non-dominated Sorting Genetic Algorithm

The basic framework of Reference-point Based Non-dominated Sorting Genetic Algorithm (NSGA-III) [12] is similar to NSGA-II, but with significant modifications to its selection process. Unlike NSGA-II, NSGA-III adaptively updates several widely used reference points, which aids in maintaining diversity among
As previously mentioned, the NSGA-III employs a pre-defined set of reference points to guarantee diversity in the solutions produced. The standard NSGA-III algorithm benefits from Das and Dennis’s approach which assigns points to a normalised hyper-plane. The total number of reference points ($H$) in an $M$ objective problem by $P$ division can be calculated as

$$H = \binom{M + P - 1}{P}$$

For instance, in a problem with three objectives ($M = 3$), the reference points are made on a triangle whose apex is at $(1, 0, 0)$, $(0, 1, 0)$, and $(0,0,1)$. For each objective axis, four divisions ($P = 3$) will result, and therefore a total of 10 reference points (Figure 7).

NSGA-II utilises the crowding distance to pick the remaining members after non-dominated sorting, while with NSGA-III, the reference points are used to select the remaining members. To do this, the range of objective values and reference points are first normalised to be the same. Afterwards, the orthogonal distance between each reference line and a member of $P_t$ is calculated. The reference point with the shortest orthogonal distance is then used to associate the member with.

The number of individuals who are connected to each reference point, known as the niche count for each reference point, is then computed for further analysis. Then, the reference point with the lowest niche point is found and the member from the last front that is associated with it should be included in the final population. It is important to note that a reference point need not have any population members linked

![Figure 7: 3D plot of the 10 reference points with $p = 3$.](image-url)
with it and may have one or more related population members. For the $j$-th reference point, the number of population members that are associated with each reference point is counted and denoted as niche count ($\rho_j$).

NSGA-III employs a niche-preserving operator as follows. First, a reference point set, $J_{\text{min}} = j : \text{argmin}_j \rho_j$ is defined, including a minimum $\rho_j$. When there are several of these reference points, one ($j^* \in J_{\text{min}}$) is selected at random. If $\rho_{j^*} = 0$ (meaning that there is no associated member to the reference point $j$), two scenarios can happen. First, the reference point $j$ is already connected to one or more of the members in front of $F_1$. In this case, the one closest to the reference line perpendicularly is added to $P_{t+1}$. Second, the front $F_1$ does not have any members linked to the reference point. In this case, the reference point is not taken into account anymore for the current generation.

In the case of $\rho_j \geq 1$ (indicating that one member associated with the reference point exists in $P_t/F_L$), a randomly selected number from front $F_L$ that is associated with the reference point $\rho_j$ is added to $P_{t+1}$. After updating the niche counts, the process is repeated $K$ times in total to bring $P_{t+1}$’s population size to $N$.

## 5 Proposed Methods

This paper proposes a general strategy for multi-objective optimisation of JPEG implementation. As a result, it can be used with any optimisation algorithm. To this end, we embed our proposed scheme into five scalarisation algorithms, GA, DE, PSO, ES, and PS, and two Pareto-front-based algorithms, NSGA-II and NSGA-III. First, we define the solution representation and objective function in the following. Then, by embedding the proposed strategy within seven backbone optimisation algorithms, we obtain seven new algorithms, namely EnMOGA, EnMODE, EnMOPSO, EnMOES, EnMOPS, EnNSGAII, and EnNSGAIII, respectively.

### 5.1 Solution Representation

Our proposed algorithm aims to find multi-objective optimal quantisation tables, including luminance quantisation table (LQT) and chrominance quantisation table (CQT). To this end, each 8-by-8 quantisation table is reshaped to a 1-by-64 vector, and then both are concatenated. Therefore, the representation proposed in this paper is a vector of dimension 128 as

$$x = [LQT_{1,1}, ..., LQT_{8,8}, ..., CQT_{1,1}, ...CQT_{8,8}]$$  \hspace{1cm} (21)

where $LQT_{i,j}$ and $CQT_{i,j}$ show the corresponding element in the location $(i, j)$ in the LQT matrix and the CQT matrices. In other words, the first 64 entries are positive integer numbers in $[0, 2^p - 1]$ (where $p$ is the number of bits indicating a pixel, in our case $p = 8$) for the LQT table, while the remaining elements are reserved for the CQT table.
5.2 Objective Functions

This paper introduces two main objective functions, file size and image quality. One is to be minimised (file size) and the other to be maximised (image quality). To this end, first, the JPEG image should be achieved using the corresponding candidate solution for a typical image. The first objective function is file size defined as

\[ F_{\text{obj}} = \frac{FS_{\text{JPEG}}}{FS_{\text{org}}} \]  

(22)

where \( FS_{\text{JPEG}} \) is the file size for image after JPEG compression process, and \( FS_{\text{org}} \) is the image size for the original image. Lower \( F_{\text{obj}} \) shows a higher capability in the compression process.

The second objective function is Peak Signal to Noise (PSNR), as one of the most common measures for assessing image quality, which is computed as

\[ PSNR = 20 \log_{10}(\frac{255}{\text{RMS}E}) \]  

(23)

\[ \text{RMS}E = \sqrt{\sum_{i=1}^{M} \sum_{j=1}^{N} (I(i,j) - \hat{I}(i,j))^2} \]  

(24)

where \( M \) and \( N \) are the image dimensions, and \( I \) and \( \hat{I} \) are the original and the compressed images. A higher PNSR value indicates better performance.

Scalarisation approaches integrate the multi-objective functions into one objective function. Therefore, the objective function for the scalarisation methods is expressed as

\[ F(x) = w_1 F_{\text{obj}} + \frac{w_2}{PSNR} \]  

(25)

where \( w_1 \) and \( w_2 \) are two used-defined parameters, indicating the importance of each objective function.

Pareto-based approaches can work on our two objective functions independently, so there is no need to combine two objective functions. As a result, the two objective functions for Pareto-based approaches are \( F_{\text{obj}} \) and \( \frac{1}{PSNR} \).

5.3 Embedding Within Scalarisation Approaches

5.3.1 EnMOGA Algorithm

The EnMOGA begins with forming a random initial population from a uniform distribution. Over various generations, new populations are created by applying crossover, mutation, and selection operators. The pseudo-code of the EnMOGA algorithm is given in Algorithm[1] while the components are briefly explained below.
Selection: We use tournament selection, which promotes quicker convergence. In tournament selection, the top candidate solutions are chosen from a random subset of the population for each tournament. The size of the tournament is determined by the number of participants in each tournament.

Crossover: We use Simulated Binary Crossover (SBX) \cite{14} for the crossover operator. A binary notation can express real values, and then a point crossover can be performed. By using a probability distribution model of the binary crossover, SBX replicated this process. SBX benefits from two leading parameters, including the probability of a crossover and the distribution index ($\eta$).

Mutation: Polynomial Mutation \cite{14} is used in this paper, which follows the same probability distribution as the SBX operator in the parent's vicinity. It also has the same parameters as the SBX operator.

\begin{algorithm}
\caption{EnGAMO algorithm in the form of pseudo-code.}
1: // $L/U$: lower/upper bound; $N_{pop}$: population size; $N_{var}$: number of variables; $NFE_{max}$: maximum number of function evaluations; prob: the probability of the crossover, $\eta$: the distribution index
2: Initialise population of $N_{pop}$ candidate solutions using the representation introduced in Section 5.1
3: Calculate objective function values (OFV) of all candidate solutions (Section 5.2)
4: $x^*$ = best candidate solution in the initial population
5: $NFE = N_{pop}$
6: iter = 0
7: while $NFE <= NFE_{max}$ do
8: iter = iter + 1
9: Perform Tournament selection (Section 5.3.1).
10: Perform SBX crossover (Section 5.3.1).
11: Perform Polynomial mutation (Section 5.3.1).
12: Calculate objective function values of all new candidate solutions (Section 5.2).
13: Replace the old population by the new one.
14: $x^+$ = best candidate solution in the current population
15: if OFV of $x^+ <$ OFV of $x^*$ then
16: $x^* = x^+$
17: end if
18: $NFE = NFE + N_{pop}$
19: end while
\end{algorithm}

5.3.2 EnMODE Algorithm

Since EnMODE is a population-based metaheuristic, it is started with a random initial population. It has three main operators, including, mutation, crossover, and selection. For them, we used the standard operators, described in Section 4.1. Also, EnMODE benefits from Dither \cite{17}, a deterministic scheme of randomisation of the scale factor $SF$ (introduced in Section 4.1). Dither proposes selecting $SF$ from the interval $[0.5, 1.0]$ randomly for each generation. The pseudo-code of EnMODE is given in Algorithm 2.

5.3.3 EnMOPSO Algorithm

The EnMOPSO is based on the PSO algorithm. The updating strategy used here is similar to the standard PSO algorithm introduced in Section 4.1. Standard PSO uses two parameters, $c_1$ and $c_2$. Here, both parameters are updated based on the way proposed in \cite{49}. To this end, PSO has been placed in 4 states, including
Algorithm 2 EnMODE algorithm in the form of pseudo-code.

1: // L/U: lower/upper bound; \(N_{pop}\): population size; \(N_{var}\): number of variables; \(NFE_{\text{max}}\): maximum number of function evaluations; \(CR\): crossover rate.
2: Initialise population of \(N_{pop}\) candidate solutions using the representation introduced in Section 5.1.
3: Calculate objective function values of all candidate solutions (Section 5.2).
4: \(x^*\) = best candidate solution in the initial population
5: \(NFE = N_{pop}\)
6: \(\text{iter} = 0\)
7: while \(NFE < = NFE_{\text{max}}\) do
8: \(\text{iter} = \text{iter} + 1\)
9: Perform Dither operation (Section 5.3.2).
10: Perform Mutation operator (Section 4.1).
11: Perform Crossover operator (Section 4.1).
12: Calculate objective function values of all new candidate solutions (Section 5.2).
13: Perform Selection operator (Section 4.1).
14: \(x^*\) = best candidate solution in the current population
15: end while

convergence, exploitation, exploration, and jumping out. In each state, one of the following operations should be performed.

1. Increasing \(c_1\) and decreasing \(c_2\) in an exploration state,
2. Increasing \(c_1\) slightly and decreasing \(c_2\) slightly in an exploitation State,
3. Increasing \(c_1\) slightly and increasing \(c_2\) slightly in a convergence state,
4. Decreasing \(c_1\) and increasing \(c_2\) in a jumping out state.

The evolutionary states estimation process is as follows.

1. Calculate the mean distance of each particle \((d_i)\) with all other particles as

\[
d_i = \frac{1}{N-1} \sum_{j=1,j\neq i}^{N} \sqrt{\sum_{k=1}^{D} (x_{ki} - x_{kj})^2}
\]

where \(N\) and \(D\) are population size and the number of dimensions, respectively.

2. Calculate the evolutionary factor as

\[
ef = \frac{d_g - d_{\text{min}}}{d_{\text{max}} - d_{\text{min}}}
\]

where \(d_g\) means the distance for the global best position, and \(d_{\text{max}}\) and \(d_{\text{min}}\) are the maximum and minimum distances, respectively.

3. Classify \(ef\) into one of four sets (based on the rules introduced in [49]), which represents the states of exploration, exploitation, convergence, and jumping out.

In addition to updating \(c_1\) and \(c_2\), \(\omega\) also updates based on a Sigmoid function as

\[
\omega(ef) = \frac{1}{1 + 1.5e^{-2.6ef}}
\]
Algorithm 3 EnMOPSO algorithm in the form of Pseudo-code.

1: // L/U: lower/upper bound; \( N_{\text{pop}} \): population size; \( N_{\text{var}} \): number of variables; \( NFE_{\text{max}} \): maximum number of function evaluations; \( G \): maximum number of iterations.
2: \( g = 1 \)
3: \( NFE = N_{\text{pop}} \)
4: \( \text{iter} = 0 \)
5: Initialise population of \( N_{\text{pop}} \) candidate solutions using the representation introduced in Section 5.1.
6: Calculate objective function values of all candidate solutions (Section 5.2).
7: Initialise \( G_{\text{best}} \) as a candidate solution with the minimum value of the population.
8: Initialise \( P_{\text{best}} \) to its initial position for each candidate solution.
9: while \( NFE < NFE_{\text{max}} \) do
10: \( \text{iter} = \text{iter} + 1 \)
11: Estimate the evolutionary states of the algorithm and calculate evolutionary factor using Eq. 27.
12: Select one of 4 states, including convergence, exploitation, exploration, and jumping out to update the parameters.
13: Update \( \omega \) using Eq. 28.
14: Calculate the particle’s velocity according to Eq. 14.
15: Update particle’s position according to Eq. 15.
16: Calculate objective function values of all new candidate solutions (Section 5.2).
17: Update \( G_{\text{best}} \) and \( P_{\text{best}} \).
18: \( x^* = \) best candidate solution in the current population.
19: Update \( NFE \)
20: end while

Algorithm 4 EnMOES algorithm in form of Pseudo-code.

1: // L/U: lower/upper bound; \( N_{\text{pop}} \): number of bids; \( N_{\text{var}} \): number of variables; \( NFE_{\text{max}} \): maximum number of function evaluations; \( \sigma \): variance value for Gaussian distribution.
2: \( g = 1 \)
3: \( NFE = N_{\text{pop}} \)
4: \( \text{iter} = 0 \)
5: Initialise population of \( N_{\text{pop}} \) candidate solutions using the representation introduced in Section 5.1.
6: Calculate objective function values of all candidate solutions (Section 5.2).
7: \( x^* = \) best candidate solution in the initial population.
8: while \( NFE < NFE_{\text{max}} \) do
9: \( \text{iter} = \text{iter} + 1 \)
10: Select parents in a random manner.
11: Generate offspring using Eq. 16.
12: Calculate objective function values of all offspring (Section 5.2).
13: Select the best candidate solutions among the combination of offspring and parents.
14: \( x^* = \) best candidate solution in the current population.
15: end while

The EnMOPSO algorithm in the form of Pseudo-code is given in Algorithm 3.

5.3.4 EnMOES Algorithm

EnMOES algorithm is inspired by evolutionary strategy [48], and includes two leading operators, namely mutation and selection. Mutation operator is performed using Eq. 16 while selection is based on objective function ranking. The EnMOES algorithm in the form of pseudo-code is given in Algorithm 4.
Algorithm 5: EnMOPS algorithm in the form of Pseudo-code.

1: // L/U: lower/upper bound; Nvar: number of variables; NFE\textsubscript{max}: maximum number of function evaluations; ρ: step size
2: Generate a randomly candidate solution (x) using the representation introduced in Section 5.1
3: Calculate objective function values (OFV) of the candidate solution (Section 5.2).
4: NFE = 1
5: iter = 0
6: while NFE \leq NFE\textsubscript{max} do
7: iter = iter + 1
8: Generate one trial solution (x\textsuperscript{+}) using Eq. 17
9: Calculate objective function values of the new trial solution (Section 5.2).
10: if x\textsuperscript{+} is better than the current solution then
11: replace the current solution by x\textsuperscript{+}
12: else
13: Generate one trial solution (x\textsuperscript{+}) using Eq. 18
14: Calculate objective function values of the new trial solution (Section 5.2).
15: if x\textsuperscript{+} is better than the current solution then
16: replace the current solution by x\textsuperscript{+}
17: end if
18: end if
19: if x\textsuperscript{+} is worse than the current solution then
20: ρ ← ρ/2
21: end if
22: end while

5.3.5 EnMOPS Algorithm

EnMOPS works based on pattern search; therefore, it tries to find the optimal point by comparing, at each iteration, its value with a finite set of trial points. The Pseudo-code of EnPS-MO algorithm is given in Algorithm 5.

5.4 Embedding within Pareto-based Techniques

This subsection presents how to embed the proposed multi-objective JPEG image compression strategy into two well-known Pareto-based techniques, NSGA-II and NSGAIII.

5.4.1 EnNSGAII Algorithm

EnNSGAII algorithm is a Pareto-based technique and generates a set of solutions instead of a single solution. EnNSGAII employs the NGSA-II algorithm for the optimisation process introduced in Section 4.2.1. We have used the same operators for EnNSGAII algorithm including non-dominated sorting, elite preserving operator, crowding distance, and selection operator. For the evolutionary step, we have used similar operators to the ones used in EnMOGA. In other words, we have used tournament selection, SBX crossover, and Polynomial mutation. Algorithm 6 presents the pseudo-code for the EnNSGAII algorithm.
Algorithm 6 EnNSGAII algorithm in the form of Pseudo-code.

1: // \(L/U\): lower/upper bound; \(N_{\text{pop}}\): number of bids; \(N_{\text{var}}\): number of variables; \(NFE_{\text{max}}\): maximum number of function evaluations; \(\text{prob}\): the probability of a crossover, \(\eta\): the distribution index
2: 
3: Initialise population of \(N_{\text{pop}}\) candidate solution using the representation introduced in Section 5.1.
4: Calculate objective function values of all candidate solutions (Section 5.2).
5: Assign (level) rank based on Pareto sorting
6: \(NFE = N_{\text{pop}}\)
7: \(\text{iter} = 0\)
8: while \(NFE < NFE_{\text{max}}\) do
9: \(\text{iter} = \text{iter} + 1\)
10: Perform Tournament selection (Section 5.3.1).
11: Perform SBX mutation (Section 5.3.1).
12: Perform Polynomial mutation (Section 5.3.1).
13: Calculate objective function values of all new candidate solutions (Section 5.2).
14: \(R_t \leftarrow \text{Combine parent and offspring population}\)
15: Assign (level) rank based on Pareto sorting
16: Generate sets of non-dominated solutions
17: Add solutions to next generation starting from the first front to \(N_{\text{pop}}\) individuals.
18: Determine crowding distance
19: Select points on the lower front with high crowding distance
20: Update \(N\text{EF}\)
21: end while

5.4.2 EnNSGAIII Algorithm

EnNSGAIII is similar to EnNSGAII except that it employs reference directions rather than crowding distance. Therefore, all operators we used for EnNSGAIII are similar to EnNSGAII. In other words, EnNSGAIII employs non-dominated sorting, elite preserving operator, tournament selection, SBX operator, and Polynomial mutation for the optimisation process. The EnNSGAIII algorithm in the form of pseudo-code is given in Algorithm 7.

6 Experimental Results

To demonstrate the superiority of our proposed strategy, an extensive set of experiments is provided. To this end, we have used 7 popular benchmark images in image compression, including, Airplane, Barbara, Lena, Mandrill, Peppers, Tiffany, and Sailboat, as well as 6 images suggested in [37] for image quantisation benchmarking, including, Snowman, Beach, Cathedrals beach, Dessert, Headbands, and Landscape. Figure 8 shows the benchmark images.

Our proposed strategy is embedded in five scalarisation and two Pareto-based methods. All algorithms are run 30 times independently to provide a fair comparison, and their statistical results, including average and standard deviation, are presented. The population size and the number of function evaluations for all algorithms are set to 50 and 1000, respectively. For other parameters, we used the default parameters that can be seen in Table 3.

All algorithms are implemented in Python and with the Pymoo framework [6], an open-source frame-
Algorithm 7 EnNSGAIII algorithm in the form of Pseudo-code.

1: // L/U: lower/upper bound; \( N_{pop} \): number of bids; \( N_{var} \): number of variables; \( N_{FE_{\text{max}}} \): maximum number of function evaluations; \( prob \): the probability of a crossover, \( \eta \): the distribution index
2: 
3: Initialise population of \( N_{pop} \) candidate solution using the representation introduced in Section 5.1.
4: Calculate objective function values of all candidate solutions (Section 5.2).
5: Assign (level) rank based on Pareto sorting
6: \( N_{FE} = N_{pop} \)
7: \( iter = 0 \)
8: while \( N_{FE} <= N_{FE_{\text{max}}} \) do
9: \( iter = iter + 1 \)
10: Perform Tournament selection (Section 5.3.1).
11: Perform SBX mutation (Section 5.3.1).
12: Perform Polynomial mutation (Section 5.3.1).
13: Calculate objective function values of all new candidate solutions (Section 5.2).
14: \( R \leftarrow \) Combine parent and offspring population
15: Assign (level) rank based on Pareto sorting
16: Generate sets of non-dominated solutions
17: Add solutions to next generation starting from the first front to \( N_{pop} \) individuals.
18: Normalise objective function and create reference set
19: Assign each member to a reference point
20: Compute niche count of each reference point
21: add new members to the new population based on the niche count
22: Update \( N_{FE} \)
23: end while

Table 3: Parameter settings.

| Algorithm    | Parameter          | Value |
|--------------|--------------------|-------|
| EnMOGA       | \( Prob \) for crossover | 0.9   |
|              | \( \eta \) for crossover | 20    |
|              | \( Prob \) for mutation | 0.3   |
|              | \( \eta \) for mutation | 20    |
| EnMOPSO      | -                  |       |
| EnMODE       | CR                 | 0.2   |
| EnMOES       | -                  |       |
| EnMOPS       | \( \rho \)          | 0.5   |
| EnNSGA-II    | \( Prob \) for crossover | 0.9   |
|              | \( \eta \) for crossover | 20    |
|              | \( Prob \) for mutation | 0.3   |
|              | \( \eta \) for mutation | 20    |
| EnNSGA-III   | \( Prob \) for crossover | 0.9   |
|              | \( \eta \) for crossover | 20    |
|              | \( Prob \) for mutation | 0.3   |
|              | \( \eta \) for mutation | 20    |

work including state-of-the-art single-and multi-objective algorithms as well as features related to multi-objective optimisation such as visualisation, introduced in 2020.

6.1 Results of Scalarisation Approaches

This section aims to find answers to two central questions, as follows:

- Is the proposed strategy able to provide higher quality results than the baseline algorithm? (Here, the baseline algorithm means the standard JPEG compression method).
• Among the embedded scalarisation methods, which algorithm has been able to provide a better solution?

To this end, Table 4 compares the results of the baseline algorithm with other algorithms in terms of mean and standard deviation of the objective function yielded by each algorithm. All algorithms except the baseline algorithm are run 30 times since the baseline algorithm is deterministic. Therefore, we do not provide any standard deviation for the baseline algorithm. Also, the rank of each algorithm per image is indicated from the smallest mean to the highest mean in Table 4. In the last row of the table, the average rank of each algorithm and subsequently, the overall ranks are reported as well.
From Table 4 and by a comparison between the baseline algorithm with others, we can observe that the baseline algorithm in all cases and in comparison to all algorithms achieves the worst results. For instance, for the Airplane image, the objective function for the baseline algorithm is 1.7839, while for others, it is between 1.4185 and 1.4700, indicating a significant improvement in the proposed strategy. Therefore, in short, we can say that our strategy, independent of the embedding algorithm, can provide competitive results compared to the baseline algorithm.

As mentioned, we employed five scalarisation methods. Here, we compare the results of scalarisation methods together. The results can be seen in Table 4. From the table, we can observe that EnMOGA can achieve the first rank in 6 out of 13 images and the second rank in 7 out of 13 images. Also, EnMOPS is placed in the first rank with seven cases, and in the second rank with five images. EnMOES achieved the fifth or worst rank among the embedding algorithms in all cases. Therefore, from the last row of the table, we can say that EnMOGA and EnMOPS provide the best average rank and, subsequently, overall rank, while EnMOES provides the highest average and overall ranks.

Due to the non-deterministic behaviour of metaheuristic algorithms, non-parametric statistical is obligatory. In this case, the alternative hypothesis $H_1$ denotes a statistically significant difference between the algorithms, while the null hypothesis $H_0$ states that there is no statistical difference between the two algorithms. The null hypothesis is the initial statistical assertion, and the alternative hypothesis would be accepted if the null hypothesis were to be shown to be false. To this end, we carried out Wilcoxon signed rank test [15] at 5% significance level based on the mean objective function value to compare the results statistically.

Table 5 shows that EnMOGA and EnMOPS perform statistically superior to other algorithms since both win in 4 cases. Also, EnMOPS and EnMOGA are statistically the same. The overall following best working algorithm is EnMOPSO (3 wins, 2 losses). Among the scalarisation algorithms, EnMOES performs worst (1 win, 4 losses). Again, the baseline algorithm (0 win, 5 losses) fails against all proposed scalarisation methods.

We also investigated the convergence curves for all algorithms. Figure 9 shows plots of objective function values against the number of function evaluations on all images and for a single random run. It can be seen that EnMOPS and EnMOGA have faster convergence compared to other algorithms, while EnMOES suffers from low-speed convergence.

### 6.2 Results of Pareto-based Approaches

The evaluation method of the Pareto-based approaches differs from the scalarisation approaches because the Pareto-based approaches result in several solutions while the scalarisation methods produce only one solution. There are several measures to validate the results of Pareto-based approaches, such as Generational Distance (GD) [45] and Inverted Generational Distance (IGD) [8], while here, we cannot use these because
Table 4: A comparison between different scalarisation approaches and the baseline algorithm in terms of objective function.

| Images     | Baseline | EnMOGA | EnMOPSO | EnMODE | EnMOES | EnMOPS |
|------------|----------|--------|---------|--------|--------|--------|
| Airplane   | Mean     | 1.7839 | 1.4227  | 1.4370 | 1.4596 | 1.4700 | 1.4185 |
|            | Std.     |        | 0.0055  | 0.0064 | 0.0051 | 0.0034 | 0.0159 |
|            | rank     | 6      | 2       | 3      | 4      | 5      | 1      |
| Barbara    | Mean     | 1.7647 | 1.3571  | 1.3688 | 1.4052 | 1.4224 | 1.3731 |
|            | Std.     |        | 0.0076  | 0.0079 | 0.0079 | 0.0060 | 0.0265 |
|            | rank     | 6      | 1       | 2      | 4      | 5      | 3      |
| Lena       | Mean     | 1.8552 | 1.4312  | 1.4475 | 1.4665 | 1.4794 | 1.4337 |
|            | Std.     |        | 0.0062  | 0.0072 | 0.0054 | 0.0029 | 0.0205 |
|            | rank     | 6      | 1       | 3      | 4      | 5      | 2      |
| Mandrill   | Mean     | 1.8961 | 1.6998  | 1.6329 | 1.6666 | 1.6767 | 1.5703 |
|            | Std.     |        | 0.0073  | 0.0084 | 0.0041 | 0.0049 | 0.0196 |
|            | rank     | 6      | 2       | 3      | 4      | 5      | 1      |
| Peppers    | Mean     | 1.8604 | 1.5168  | 1.5313 | 1.5409 | 1.5507 | 1.5004 |
|            | Std.     |        | 0.0046  | 0.0048 | 0.0044 | 0.0036 | 0.0120 |
|            | rank     | 6      | 2       | 3      | 4      | 5      | 1      |
| Sailboat   | Mean     | 1.8458 | 1.5670  | 1.5839 | 1.6015 | 1.6103 | 1.5379 |
|            | Std.     |        | 0.0048  | 0.0063 | 0.0033 | 0.0027 | 0.0150 |
|            | rank     | 6      | 2       | 3      | 4      | 5      | 1      |
| Snowman    | Mean     | 1.8239 | 1.4041  | 1.4150 | 1.4467 | 1.4611 | 1.4120 |
|            | Std.     |        | 0.0071  | 0.0075 | 0.0080 | 0.0054 | 0.0189 |
|            | rank     | 6      | 1       | 3      | 4      | 5      | 2      |
| Tiffany    | Mean     | 1.8027 | 1.5109  | 1.5147 | 1.5229 | 1.5282 | 1.4867 |
|            | Std.     |        | 0.0018  | 0.0024 | 0.0026 | 0.0023 | 0.0093 |
|            | rank     | 6      | 2       | 3      | 4      | 5      | 1      |
| Beach      | Mean     | 1.8047 | 1.4847  | 1.4956 | 1.5282 | 1.5386 | 1.4723 |
|            | Std.     |        | 0.0061  | 0.0081 | 0.0058 | 0.0043 | 0.0186 |
|            | rank     | 6      | 2       | 3      | 4      | 5      | 1      |
| Cathedrals beach | Mean | 1.7551 | 1.3648  | 1.3748 | 1.4013 | 1.4110 | 1.3647 |
|            | Std.     |        | 0.0076  | 0.0080 | 0.0056 | 0.0056 | 0.0175 |
|            | rank     | 6      | 2       | 3      | 4      | 5      | 1      |
| Dessert    | Mean     | 1.8195 | 1.3760  | 1.3859 | 1.4113 | 1.4220 | 1.3804 |
|            | Std.     |        | 0.0067  | 0.0066 | 0.0076 | 0.0053 | 0.0180 |
|            | rank     | 6      | 1       | 3      | 4      | 5      | 2      |
| Headbands  | Mean     | 1.8027 | 1.3879  | 1.3971 | 1.4201 | 1.4269 | 1.3901 |
|            | Std.     |        | 0.0047  | 0.0058 | 0.0042 | 0.0037 | 0.0141 |
|            | rank     | 6      | 1       | 3      | 4      | 5      | 2      |
| Landscape  | Mean     | 1.8456 | 1.4272  | 1.4386 | 1.4731 | 1.4940 | 1.4303 |
|            | Std.     |        | 0.0066  | 0.0081 | 0.0087 | 0.0058 | 0.0203 |
|            | rank     | 6      | 1       | 3      | 4      | 5      | 2      |
| Average rank | 6      | 1.54   | 2.92    | 4.00   | 5.00   | 1.54   |
| Overall rank | 6      | 1.5    | 3       | 4      | 5      | 1.5    |

Table 5: Results of Wilcoxon signed rank test based on mean objective function value. ‡, †, and ≈ indicate that the algorithm in the corresponding row is statistically better than, worse than, or similar to the algorithm in the corresponding column. The last column summarises the algorithms' total wins (w), ties (t), and losses (l).

| Images     | Baseline | EnMOGA | EnMOPSO | EnMODE | EnMOES | EnMOPS | w/t/l |
|------------|----------|--------|---------|--------|--------|--------|------|
| Airplane   | ‡ † † † † | † † † † † | 0/0/5   | 4/1/0  | 3/0/2  | 2/0/3  | 1/0/4 |
| Barbara    | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Lena       | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Mandrill   | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Peppers    | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Sailboat   | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Snowman    | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Tiffany    | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Beach      | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Cathedrals beach | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Dessert    | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Headbands  | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |
| Landscape  | ‡ † † † † | ‡ † † † | 4/1/0   | 3/0/2  | 2/0/3  | 1/0/4  |      |

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Figure 9: Convergence curves for scalarisation approaches.
they require the true Pareto front, which is not available in this problem. Therefore, we used an alternative measure, called hyper-volume (HV) \[28\], which does not require a true Pareto-front to validate the results. The HV measure is regarded as a fair measure among other criteria \[5\] so that HV can take into consideration both closeness to the optimal solution and being well-distributed along the whole Pareto front. HV measure determines the area/volume that, in relation to a reference point, is dominated by the given set of solutions. A higher value of HV measure in a minimisation problem shows a better quality of the solution.

This paper integrated the proposed strategy into two Pareto-based algorithms, including NSGA-II and NSGA-III. The results based on the HV measure are given in Table \[6\]. The table shows that EnNSGAI\(\text{I}^I\) outperforms EnNSGAI\(\text{I}II\) in 10 out of 13 cases, while it fails in 3 cases. From the last row of the table, we can observe that the average rank of EnNSGAI\(\text{I}^I\) is lower than EnNSGAI\(\text{I}II\). In other words, EnNSGAI\(\text{I}^I\) overcomes EnNSGAI\(\text{I}II\).

To perform a deeper analysis, we also conducted a Wilcoxon signed rank test on the results. The achieved \(p\)-value is 0.0574, which means that there is a statistical difference between the two algorithms only at a 10\% significance level.

Finally, to have a more comprehensive view of the generated Pareto fronts, we plot the Pareto front for the algorithms in Figure \[11\]. It can be seen that EnNSGAI\(\text{I}^I\) provides more points in the Pareto front compared to EnNSGAI\(\text{I}II\).

Since there are two conflicting objective functions, we can not plot convergence curves in terms of objective functions. Therefore, in the next experiment, we indicated the convergence curves in terms of the HV measure rather than objective functions. Figure \[10\] shows the convergence curves for our two proposed algorithms. It is clear that, in most cases, EnNSGAI\(\text{I}^I\) provides a faster convergence rate.

### 6.3 Comparison between Scalarisation and Pareto-based Methods

Generally speaking, a comparison between scalarisation and Pareto-based methods is not possible since scalarisation methods only generate one solution based on a given set of weights, while Pareto-based methods generate a set of solutions. To tackle this problem, we select one solution from a Pareto-based method as

\[
PF_{\text{selected}} = \min_{i=1}^{N_{PF}} (w_1 f_i^1(x) + w_2 f_i^2(x) + \ldots + w_M f_i^M(x))
\]

(29)

where \(N_{PF}\) is the number of solutions in the generated Pareto front, \(w_1, \ldots, w_M\) are the corresponding weights in the scalarisation method, and \(f_i^1, \ldots, f_i^M\) are the objective function values for the \(i\)-th solution in the Pareto front.

Table \[7\] shows the results. It can be seen that EnNSGAI\(\text{I}^I\) can not work better than two others when we select only one solution, as expected since the Pareto-based approaches focus on a set of solutions and not only one solution. Despite the performance of the scalarisation method compared to the Pareto-based
Table 6: A comparison between EnNSGAI and EnNSGAII in terms of the HV measure.

| Images          | EnNSGAII | EnNSGAII |
|-----------------|----------|----------|
|                 | Mean     | Std.     |
| Airplane        | 7.0803   | 0.1805   |
|                 | 7.0920   | 0.2133   |
|                 | 2        | 1        |
| Barbara         | 7.1164   | 0.1717   |
|                 | 7.0369   | 0.2482   |
|                 | 1        | 2        |
| Lena            | 7.1206   | 0.2085   |
|                 | 7.1436   | 0.1077   |
|                 | 2        | 1        |
| Mandrill        | 7.0618   | 0.0483   |
|                 | 7.0585   | 0.0646   |
|                 | 1        | 2        |
| Peppers         | 7.2614   | 0.0634   |
|                 | 6.6218   | 0.5518   |
|                 | 1        | 2        |
| Sailboat        | 7.0653   | 0.1971   |
|                 | 7.0424   | 0.0986   |
|                 | 1        | 2        |
| Snowman         | 7.1900   | 0.0599   |
|                 | 7.0897   | 0.0727   |
|                 | 1        | 2        |
| Tiffany         | 6.9587   | 0.4292   |
|                 | 7.1036   | 0.1055   |
|                 | 2        | 1        |
| Beach           | 7.078745 | 0.03692  |
|                 | 6.960397 | 0.1994   |
|                 | 1        | 2        |
| Cathedrals beach| 7.1510   | 0.1207   |
|                 | 7.1149   | 0.0571   |
|                 | 1        | 2        |
| Dessert         | 7.1242   | 0.2396   |
|                 | 7.0103   | 0.3402   |
|                 | 1        | 2        |
| Heatbands       | 7.2309   | 0.0521   |
|                 | 7.1754   | 0.1084   |
|                 | 1        | 2        |
| Landscape       | 7.1799   | 0.0765   |
|                 | 7.1417   | 0.0837   |
|                 | 1        | 2        |
| Average rank    | 1.23     | 1.77     |
| Overall rank    | 1        | 2        |
Figure 10: Convergence curves for Pareto-based approaches in terms of the HV measure.
Figure 11: A comparison of the Pareto front of the proposed methods for each image.
algorithm, we cannot say that the Pareto-based algorithm did not work well because the output of the Pareto-based algorithm is a set of solutions with different weights, while the scalarisation method does not have such an ability. Also, from this experiment, it is worthwhile to mention that if we know the weights of each objective, scalarisation methods are preferable.

Table 7: A comparison between the scalarisation and Pareto-based methods. The values signify the objective function value defined for scalarisation method and $PF_{selected}$ for EnNSGAII. The best result for a given image is boldfaced.

| Images               | EnMOGA | EnMOPS | EnNSGAII |
|----------------------|--------|--------|----------|
| Airplane             | 1.4227 | 1.4185 | 1.4705   |
| Barbara              | 1.3571 | 1.3731 | 1.4048   |
| Lena                 | 1.4312 | 1.4337 | 1.4720   |
| Mandrill             | 1.6098 | 1.5703 | 1.6695   |
| Peppers              | 1.5168 | 1.5004 | 1.5462   |
| Sailboat             | 1.5670 | 1.5379 | 1.6076   |
| Snowman              | 1.4041 | 1.4120 | 1.5212   |
| Tiffany              | 1.5109 | 1.4867 | 1.4207   |
| Beach                | 1.4847 | 1.4723 | 1.3987   |
| Cathedrals beach     | 1.3648 | 1.3647 | 1.4155   |
| Dessert              | 1.3760 | 1.3804 | 1.4459   |
| Headbands            | 1.3879 | 1.3901 | 1.5244   |
| Landscape            | 1.4272 | 1.4303 | 1.4648   |

6.4 Sensitivity Analysis

The sensitivity analysis of the suggested algorithm’s control parameters is examined below. To serve as representatives, we chose two images, namely Airplane and Barbara. We selected EnMOGA as a scalarisation method, and EnNSGAII as a Pareto-based method, which show better performance, for our experiments.

6.4.1 Sensitivity to Population Size

Population size is one of the most critical parameters in metaheuristic algorithms. Metaheuristic algorithms with a large population size usually provide better results than small population size since a large population size supports higher diversity for the population, leading to higher exploration ability due to the recombination of its diverse members [20][36]. Nevertheless, sometimes it is more effective to use a small population size. The term micro-algorithm, $\mu$-algorithm, refers to a metaheuristic algorithm with a small population size [20].

This section aims to investigate the effect of population size on performance. To this end, the population size is set to 5, 10, 20, 30, 50, 100, and 200, while the number of function evaluations is fixed for all algorithms. In other words, for smaller population size, the number of iterations is higher than for larger population size. Figure 12a shows the objective function value achieved by different population sizes and for the EnMOGA algorithm. Both images show an upward trend; in other words, a larger population size leads to a higher objective function value. It means that lower population size is preferable.
The same experiment is performed by EnNSGAII, and the results are given in Figure 12. For the Airplane image, by increasing the population size from 5 to 20, the HV value is also increased, while there is a downward trend by increasing the population size from 20 to 200. For the Barbara image, the conditions are a bit different, and the trend is downward for all population sizes.

In short, it can be said that smaller population sizes can lead to better results for the scalarisation approach, whereas the Pareto-based approach works better with larger population sizes.

![Graphs showing objective function values for different population sizes.](image)

**Figure 12:** The objective function values obtained with different population sizes.

### 6.4.2 Sensitivity to $\text{prob}$ and $\eta$ in the Crossover Operator

Our crossover operator depends on two parameters, called $\text{prob}$ and $\eta$. To study the sensitivity of $\text{prob}$ and $\eta$, 18 combinations of $\text{prob}$ and $\eta$ are assessed ($\text{prob} = 0.5, 0.7, 0.9$ and $\eta = 2, 5, 10, 20, 30, 40$). All other parameters are fixed. Figures 13 indicates the objective function value with different $\text{prob}$ and $\eta$ combinations for the Airplane and Barbara images. For the Airplane image, we can see that the objective
function value for \( \text{prob} = 0.5 \) is higher than the other two for all \( \eta \) values. There is a fluctuation in comparison between \( \text{prob} = 0.7 \) and \( \text{prob} = 0.9 \); meaning that for \( \eta = 2, 5, 20 \), \( \text{prob} = 0.7 \) outperforms \( \text{prob} = 0.9 \), while for other \( \eta \) values, \( \text{prob} = 0.9 \) provides better results than \( \text{prob} = 0.7 \).

This experiment is also conducted for the EnNSGAI1 algorithm. The results are given in Figure 14. From Figure 14, it is clear that \( \text{prob} = 0.9 \) provides better results with more stability in all cases. By increasing the \( \eta \) values for \( \text{prob} = 0.5 \) and 0.7, the HV values also are improved. The similar results can be seen in Figure 14 for the Barbara image. In most cases, the \( \text{prob} = 0.9 \) outperforms other \( \text{prob} \) values, followed by \( \text{prob} = 0.7 \).

![Airplane image](image1)

![Barbara image](image2)

Figure 13: The objective function values of different \( \text{prob} \) and \( \eta \) values in the crossover operator of EnMOGA

### 6.4.3 Sensitivity to \( \text{prob} \) and \( \eta \) in the Mutation Operator

There are also two parameters, \( \text{prob} \) and \( \eta \), in the mutation operator. We investigated the effect of 18 combinations (\( \text{prob} = 0.1, 0.3, 0.5 \) and \( \eta = 2, 5, 10, 20, 30, 40 \)). The results of EnMOGA are given in
Figure 14: The HV values of different \textit{prob} and \textit{\eta} values in the crossover operator of EnNSGAII
For the Airplane image and $prob = 0.5$, $\eta$ values have a downward trend; in other words, by increasing the $\eta$ values, performance is also improved. For $\eta = 0.3$, there is a fluctuation, while for $\eta = 0.1$, the results are more stable. From Figure 15 we can observe that the EnMOGA is sensitive to these parameters. In particular, $\eta = 20$ and $prob = 0.3$ provided the best results, whereas the worst results are achieved by $\eta = 40$ and $prob = 0.5$.

Similar results for EnNSGAI in Figure 16 indicate that, again, these parameters can affect the performance. Figure 16 investigates that $\eta = 20$ can provide the highest HV values for most cases. Also, there is an upward trend from $eta = 2$ to $eta = 20$, while a downward trend can be seen from $eta = 20$ to $eta = 40$. The similar trends can also be observed in Figure 16 for Barbara image.

![Graph](image1.png)

Figure 15: The objective function values of $prob$ and $\eta$ in the mutation operator of EnMOGA
Figure 16: The HV values of $\text{prob}$ and $\eta$ in the mutation operator of EnNSGAII
7 Conclusions

This paper proposes an energy-aware multi-objective strategy to find the best values of quantisation tables in JPEG image compression. To this end, first, we investigated whether there is a high correlation between two main properties of images, image quality and file size, and energy consumption. As a result, these two can be considered a proxy for energy consumption. Then, we defined two conflicting objective functions, including image quality and file size, while a vector-based representation was used as the candidate solution. In the next step, we embedded the proposed strategy into seven metaheuristic algorithms. Five of them are among scalarisation methods, including energy-aware multi-objective genetic algorithm (EnMOGA), energy-aware multi-objective particle swarm optimisation (EnMOPSO), energy-aware multi-objective differential evolution (EnMODE), energy-aware multi-objective evolutionary strategy (EnMOES), and energy-aware multi-objective pattern search (EnMOPS), while two others are selected among Pareto-based approaches, including energy-aware non-dominated sorting genetic algorithm (EnNSGA-II) and energy-aware reference-based NSGA-II (EnNSGA-III). Our extensive results indicated that all algorithms could outperform the baseline. In particular, EnMOGA, EnMOPS, and EnNSGA-II offered better results.

Despite the effectiveness of the proposed strategy, this work can be extended in the future with the following hints.

1. This paper employed some well-established metaheuristic algorithms for the embedding process, while it can be improved by embedding the strategy into more recent algorithms such as L-SHADE [42].

2. This paper ignored decomposition-based approaches (DBA) for finding the conflicting objectives, while the literature shows that DBAs have an excellent capability for multiobjective optimisation. Therefore, DBAs can be used for this problem in the future.

3. The goal of this paper was to find the optimal points for quantisation tables. It will likely to provide better results by adding other parameters to the current representation, such as the quality factor.

4. This paper employed the default parameter of algorithms for the first stage of the embedding process, and we have not focused on the parameter settings for all algorithms. The optimal parameters also can be archived by a self-adaptation approach.

5. This paper only employed two objective functions, while this research can be extended to the many-objective optimisation problem in the quantisation table generation.
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