Three-Dimensional Modeling of Multiphase Fluid Flow With Linear Temperature Gradients In Porous Media Using Lattice Boltzmann Method Rothman-Keller

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Abstract. In oil and gas production, only about 5% to 20% of hydrocarbons can be released through production wells with natural reservoir pressure. In the oil and gas industry, there is a method called enhanced oil recovery (EOR) to remove hydrocarbons trapped in rock pores. EOR is not easy and expensive, so fluid flow modeling and simulation are required to reduce the cost. In fluid modeling, there is a continuous approach using the Navier-Stokes (NS) equation. Not only the continuous approach, but fluid modeling also can be done by using Molecular Dynamics (MD) approach. Between these two methods, there is a method that bridging the advantages between the two. This method uses a macroscopic and microscopic approach (mesoscale) called Lattice Boltzmann Method (LBM). With this approach, the results obtained are quite accurate without the use of high computing devices. The LBM used in this research is the Rothman-Keller (RK) / color gradient method. This method assumes that the interaction force between two fluids is directly proportional to the two-fluid density ratio and has two collision operators, BGKW operator, and perturbation operator. This method is then subjected to pressure on the boundary region introduced by Zou-He. Then the temperature is given linearly and be used to calculate body force due to the temperature gradient. This study shows that the model that has been made is successfully simulating the flow of multiphase fluid in rock pores. In the model viscosity as a function of the temperature found that the average velocity of the fluid flow decreases when the temperature increase. In the temperature gradient model, when the temperature gradient increases, the average velocity, and saturation of the wetting fluid increases.

1. Introduction
In the world of oil and gas exploitation, only about 5% to 20% of hydrocarbons can come out through production wells with natural reservoir pressure[1]. The oil and gas industry there is a method called enhanced oil recovery (EOR) to lift hydrocarbons that trapped in rock pores. Various reservoir researches have been carried out for the last 15-20 years shows that wettability modification through variations of ion composition from water injected to prove that EOR water-based engineering is environmentally friendly because without the addition of external chemicals so that it is accepted by oil and gas companies and researchers[2].

EOR is not easy and requires a lot of money, so modeling and simulation are needed, one of modeling method is fluid modeling in the rock pore scale. The first method is continuous fluid modeling approach using the Navier-Stokes (NS) equation, but this method is difficult because NS equations are not linear, too complex to define boundary conditions and geometrical limitations[3]. In addition to using a continuous approach, fluid modeling can also be done using the Molecular Dynamics (MD) approach.
This method sees the fluid consist of small particles determined by the interaction between particles with the particle radius parameters in the molecular system. This Molecular Dynamics Method requires high-performance computing devices, due to a large number of particles that must be calculated, for example, 1 mole of water (18 grams) consisting of $6.23 \times 10^{23}$ particles, thus modeling on a larger scale such as a reservoir certainly takes a long time.

Among the two methods above there are methods that bridging the advantages between the two. This method uses a macroscopic and microscopic (mesoscale) approach called the Lattice Boltzmann Method (LBM). In contrast to the MD method which considers fluids to consist of particles, this method looks at the fluid as a group of particles which is expressed in a distribution function so that its behavior can be known. Of course, this can save computing resources so it can save costs. Inside the reservoir, there are various types of fluids such as water, oil, and gas. In this modeling, two-phase fluid fluids were used, oil and water. Multiphase LBM modeling was first introduced by Gunstensen[4]. This model uses red and blue colored fluids to distinguish two different fluids by separating phases resulting from repulsive interactions based on color gradients and color momentum[7].

In EOR conditions in the real world, the reservoir injected with fluid in one of the wells causes a pressure difference on both sides. In LBM modeling, this can be overcome by using the boundary conditions[9] to give the effect of the pressure difference. In addition, EOR is usually used with a fluid that has a high temperature that causes temperature differences at the edges. For this reason, a force is given due to temperature differences by using the entropy force.

2. Methods
In this study, multiphase fluid flow in the rock pores is simulated. At the beginning of the simulation, the rock pores are filled with non-wetting fluid, after that wetting fluid is invaded from the inlet area to push non-wetting fluid to the outlet area. There is three kinds of models used in this study, the first is an ordinary multiphase LBM-RK model. The second is multiphase LBM-RK with viscosity depends on temperature. And the third is LBM-RK with a temperature gradient.

2.1. Lattice Boltzmann Method
LBM is a method that is introduced to solve fluid flow problem using statistical mechinic. Fluid is assumed as a number of particles that satisfies Maxwell-Boltzmann statistic. LB equation is derived using BGK as the basic equation of this method

$$\frac{\partial f}{\partial t} + c \cdot \nabla f = \frac{1}{\tau} (f^{eq} - f) \quad (1)$$

Eq. 1 tell that $f_i$ depend on equilibrium and relaxation time $\tau$. Macroscopic value of fluid can be obtain from:

$$\rho_v(r, t) = \sum_{i=1}^{6} f_i c_i \quad (2)$$

The streaming process can be calculated using equation below (Latva-Kokko dan Rothman, 2005).

$$f_i^{+} (x, t) = f_i^{-} (x, t) + (\Omega_i^k)^{1} + (\Omega_i^k)^{2} \quad (3)$$

Where $(\Omega_i^k)^{1}$ is collision operator adopted from BGKW scheme[14]
And \( (\Omega^k)^2 \) is perturbation operator [22]

\[
(\Omega^k)^2 = \frac{A^k}{2} \left| \begin{bmatrix} w_i \left( \frac{e \cdot F}{|F|^2} \right) - B_i \end{bmatrix} \right|
\]  

(5)

\( A^k \) is a surface tension parameter for the fluid and \( f \) is the interaction force between two fluids.

For the equilibrium distribution function can be calculated using this formula below,

\[
f_{i}^{eq}(x,t) = \rho_k \left( C_i^k + w_i \left[ \frac{e_i \cdot u}{c_i^2} + \frac{(e_i \cdot u)^2}{2c_i^4} - \frac{(u)^2}{2c_i^4} \right] \right)
\]  

(6)

2.2. Viscosity as a function of the temperature model

The first model is viscosity as a function of temperature. The viscosity of water can be calculated using Arrhenius equation[16]

\[
\mu(T) = 2.414 \times 10^{-5} \times T^{247.8/140}
\]  

(7)

And then the oil viscosity can be calculated using exponential asymptotic function

\[
\ln \mu = \ln \mu_c + \frac{C}{T - T_c}
\]  

(8)

Where \( \mu_c \) is a pre-exponential value of viscosity and \( C \) is from empirical data.

2.3. Temperature gradient model

The first approach for gradient temperature force in this model uses entropy force[17]

\[
F = -S V T
\]  

(9)

\( S \) is entropy that we can calculate from[18]

\[
S = -N k_B \sum_i p_i \ln p_i
\]  

(10)

Where \( N \) is the number of particle and \( k_B \) is Boltzmann constant and \( p \) is the probability of the particle that we can calculate from,

\[
p_i = \frac{f^i}{f^1 + f^2}
\]  

(11)

\( f \) is fluid distribution and \( i \) index is indicating wetting fluid or non-wetting fluid.

2.4. Rock Sample
Rock model used in this simulation is created using a computer algorithm by putting spheres as rock granules with different size randomly in a box, this computer model created by Latief and Fauzi (2011). This rock model has dimension 30×65×30 voxels and 50% of porosity.

3. Results and Discussion
As mentioned earlier, this study uses three kinds of LBM-RK simulation models. From those models obtained data that will be discussed below

3.1. The ordinary LBM-RK model result
Figure 1 below shows that wetting fluid invade non-wetting fluid, wetting fluid colored with red, black is for rock matrix and empty space are non-wetting fluid. When timestep reaches 200, saturation of wetting fluid is 4%, we can see from the figure that just a few of red fluid in the pore. When the timestep reaches 1000, the saturation of wetting fluid is 18%, and after 5000 timesteps, red fluid is almost filled up all pore space with the saturation of wetting fluid 74%. Saturation of water reaches 99.9% after 7600 timesteps.

![Figure 1](image-url)
3.2. Viscosity as a function of temperature model result

The second simulation is viscosity as a function of temperature. In this simulation, the temperature is set to $T = 300 \, K$, $T = 350 \, K$, and $T = 400 \, K$ and then viscosity can be calculate using eq. (4) and eq. (5).

| $T$ (K) | $\mu_w$ | $\mu_{nw}$ | $\tau_w$ | $\tau_{nw}$ | $\mu_w/\mu_{nw}$ |
|-------|---------|------------|---------|------------|------------------|
| 300   | 0.008541| 0.152955   | 0.725622| 1.158866   | 0.055837         |
| 350   | 0.003654| 0.138100   | 0.710961| 1.114299   | 0.026457         |
| 400   | 0.002167| 0.121876   | 0.706500| 1.065627   | 0.017779         |

In the table above shows that for every addition of temperature, the viscosity and relaxation time will decrease. If we see the ratio between the viscosity of wetting fluid and non-wetting fluid, the value also decreases.

After that, the graph of the average velocity and wetting saturation for each temperature variation are made.

Figure 2. Average velocity for every variation of temperature.

Figure 2 shows the data comparison of the average velocity for every temperature variations. From the graph above shows that the velocity curve at $T = 300 \, K$ is higher than $T = 350 \, K$ and $T = 400 \, K$ seems to be the lowest value. This shows that when the temperature rises up, the average velocity decrease. In Table 1, it can be seen that when the temperature increases, the ratio of the wetting and non-wetting fluid viscosity ($\mu_w/\mu_{nw}$) decreases. Because the wetting fluid that push the non-wetting fluid has a lower viscosity, it makes harder for the fluid to flow so it can cause the velocity decrease.
Figure 3. Saturation of wetting fluid for every variation of temperature

Figure 3 is a visualization of wetting saturation against timestep for every temperature variation. In the graph above, the three wetting saturation curves of three different temperature show at the beginning of the saturation is almost the same, then at T = 300 K wetting fluid saturation increases faster than T = 350 K and T = 350 K Wetting fluid saturation increases faster than T = 400 K. But when the wetting fluid saturation is close to 1, the saturation curve of the three different temperature shows the same. As with the average velocity curve, this can be caused by a decrease in the ratio of wetting to non-wetting fluid ($\mu_w/\mu_{nw}$) when the temperature is increased so that the fluid at higher temperatures will be harder to flow.

3.3. Temperature gradient model result

In this section, the LBM algorithm for the 3-dimensional multiphase fluid flow model is given an external force driven by the temperature gradient. The calculation of force due to temperature uses entropy $S = 10^{-5}$. Simulation is carried out three times, first without temperature gradient, then temperature gradient variation $\nabla T = 1.5^\circ C / voxel$ and $\nabla T = 2.5^\circ C / voxel$. From the variation of temperature gradient then the result compared each other.

Figure 4. Average velocity for every variation of the temperature gradient
In Figure 4 above, the average data velocity is displayed and compared for every different temperature gradient variations. In the graph above the average velocity curve shows has a higher value when the temperature gradient is raised. It can be seen that the temperature gradient increase the average velocity of the fluid flow increase also, even though the value is insignificant.

![Graph showing average velocity curve with different temperature gradients.](image)

**Figure 5.** Saturation of wetting fluid for every variation of the temperature gradient

Furthermore, in Figure 5, the simulation results of wetting saturation compared to different temperature gradient variations. In the graph above, the three wetting saturation curves of three different temperature gradients increase when the temperature gradient is increased. As in the average velocity curve, it can be seen that the temperature gradient can accelerate fluid flow. It can be seen from the wetting fluid saturation curve comparison, even though the difference is insignificant.

4. **Conclusion**

LBM-RK has been able to model multiphase fluid flow in 3-dimensional space well. This model has met the standards as explained in the theory. The visualization shows that this method can be used to simulate fluid invasion in rock pores.

In the viscosity as a function of temperature, the average velocity decreases when the temperature is raised. Likewise, on the wetting fluid saturation graph, there is a decrease when the temperature is raised. This occurred because when the temperature is increased, the ratio between the wetting fluid and non-wetting fluid (\(\mu_W / \mu_{NW}\)) decreases, causing the wetting fluid to become harder to push the non-wetting fluid and makes the flow slow down.

In the temperature gradient model, the increasing of wetting fluid saturation and average fluid flow velocity is not significant, it can be caused by the small force due to the temperature gradient. The small force is influenced by the entropy value which has a very small value, which is around \(10^{-5}\), compared to the average velocity which has value around \(10^{-3}\) to \(10^{-2}\), causing the force due to the temperature gradient to have no significant effect.

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