Calculation of the optimal power indicators for the engine of a small spacecraft

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Abstract. The use of small space satellites is perspective for the development of sciences, development of new technologies, conducting small engineering tests, and tracking global earth processes. In this article the main dependencies between various parameters of a small space satellite, which operates on outboard air, have been determined. It make possible to carry out its most optimal design. An assessment of the possibility of extrapolation in the calculations of a small number of thermalized particles to volumes close to the real number is carried out. The influence of changing the parameters of the air intake of the installation on the power of its engine and the friction force acting on the installation has been investigated. As a conclusion restrictions on engine parameters are given. They were reduced to a dependence on the diameter of the air intake.

1. Introduction
Small spacecraft are used for testing and demonstrating technologies: systems and components of rocket and space technology, conducting research in the field of solar and space physics, planetology, astronomy and astrophysics, earth sciences, and space biology. They carry out the study of the boundaries of the earth’s atmosphere, daily and global observations of earth processes that change during the day, the study of the physicochemical properties of the surfaces of planets or their atmospheres, the study of the survival and adaptation of organisms to outer space (to microgravity and to high levels of radiation).

The propulsion system for small spacecraft, which operates on the outboard air, collects fractions of the residual atmosphere and uses them as a working fluid for the electric rocket engine to compensate for the braking force acting on the spacecraft. The system theoretically allows the satellite to stay in orbit for an unlimited amount of time without a reserve of working fluid.
Calculations for a propulsion system were made in the article. System is capable to maintain the orbit of a small spacecraft in a circular orbit of the Earth with a height of 200 km, with a lifetime of at least 5 years. The main task of the device is to conduct radar reconnaissance.

1.1. Assumptions and baseline data
A typical installation scheme for outboard air consists of an air intake, a collector, and an electric rocket engine. Accepted assumptions:

- Incoming fractions enter the air intake without hindrance. Then we can consider a problem in which a given flow rate $m$ falls into the volume of the collector $N$. This flow rate at a known altitude will depend only on the inlet diameter of the air intake.
- The flow of incoming fractions does not interact with the sidewalls of the initial path.
- The reverse flow and the flow of fractions to the engine are thermalized.
- There are no collisions of fractions with each other.

The return current from the air intake is represented by three components:

- Thermalized flow, driven by a pressure gradient.
- Fractions reflected from the walls of the air intake.
- Fractions that have reversed their direction as a result of collisions with other fractions.

Obviously, the greater the degree of pressure increase in the air intake, the greater the contribution of the first and third components.

The return current from the air intake is determined by the pressure difference created in it. In the calculations, we will neglect the effect of the incoming particle flow, which reduces the magnitude of the reverse current. In this case, we can use the usual equations of vacuum technology to estimate the flow.

Consider the initial design data, which were taken from the NRLMSIS-E model and reduced to the daily average:

| Title                              | Numerical value          |
|------------------------------------|--------------------------|
| Inlet fractions temperature        | 835K                     |
| Temperature of fractions in the collector | 300K                   |
| Average molar mass                 | 20.56 g / mol            |
| Density of fractions in orbit      | $2.68 \times 10^{-10}$ kg / m$^3$ |
| Pressure                           | $9 \times 10^{-5}$ Pa    |

At given pressures $p = 10^{-2} .. 10^{-5}$ Pa, a free-molecular flow regime is realized.

2. Main part

2.1. Air intake length
The efficiency of a mesh air intake is defined as the ratio of the number of fractions entering it to the number of fractions leaving the engine. To preserve the validity of the assumption of the motion of incoming fractions without collisions, the length of the air inlet $L$ must be less than the free path of the
fractions. In this case, the design pressure will be taken as the average between the inlet and the pressure in the manifold $p_{\text{in}} = 0.001$ Pa, then the average free path will be $\bar{\lambda} = 10$ m.

So, $L$ should not exceed 10 m. Since with an increase in the length of the air inlet, its conductivity decreases and, accordingly, the reverse current, it is worth taking the maximum possible length of the air inlet.

To increase the efficiency of the air intake, we propose the use of a mesh throughout the entire length of the inlet line. The minimum grid step is set, on the one hand, by technological conditions, and on the other, by optical transparency and the absence of particle collisions with the wall. After several iterations, the author found the following parameters that satisfy all boundary parameters: the minimum grid spacing $a = 11$ cm, the length of the air intake $L = 3$ m. In this case, the average number of particle collisions with the wall is also two. The mean free path along the inlet path will be 10 m.

2.2. Selecting the diameter of the air intake

An important parameter of the air intake, which determines the operation of the entire system, is its diameter. There are two restrictions on its size. The minimum air intake diameter is set by the air mass flow rate, at which a helicon discharge can be ignited at an acceptable level of input power. The maximum diameter is limited by the electrical power, which is allocated for the needs of the electric rocket engine. The larger the diameter of the air intake, the greater the force of aerodynamic drag and, accordingly, the greater the required engine thrust and power consumption.

On the other hand, it is important to choose the diameter of the discharge chamber corresponding to the diameter of the inlet. As shown earlier, the larger this diameter, the smaller the diameter of the air inlet can be. It is also important that the cross-section of the discharge chamber is significantly less than the diameter of the air intake. The larger the size of the discharge chamber, the more fractions fly through the system.

According to the author's analysis, the maximum diameter at which a helicon discharge can be ignited at an acceptable power level (up to 1 kW at a flow rate of no more than 1.5 mg / s) was 100 mm.

2.3. Numerical modeling of a collector

A collector is provided for thermalization of fractions entering the system. In this collector fractions repeatedly collide with the walls and lose their energy. However, it is not possible to estimate its size analytically. For this purpose, as well as to check a number of assumptions of the model, numerical simulations were carried out.

It is necessary to ensure that the numerical model corresponds to real conditions. This correspondence is determined by three key parameters: the concentration of fractions, the interaction of fractions with the wall, and the interaction of fractions with each other.

Within the framework of numerical calculations, a limitation is imposed on the concentration of fractions, since a large number of fractions requires huge volumes of computer RAM. In our case, for 32 GB of RAM, the upper limit for the number of modeled fractions became $N = 5 \times 10^5$, a similar number of fractions was used by other authors. At the same time, the concentration of fractions in the studied orbit is $N = 8 \times 10^{15}$. On the other hand, the calculation of the flight path of a large number of fractions is associated with time constraints. So, it took 4 days to calculate 1 ms for 10,000 fractions, which makes it impossible for this investigation to obtain a large amount of data.

In this regard, the first modeling task was to estimate the error in modeling a much smaller number of fractions.
At the initial moment of time, the entire volume of the object under study contained 1000 randomly located fractions, they moved in the same direction as the axis of the air intake at a speed of 7.8 km/s.

At the same time a “cylinder” appears in front of the investigated object, containing the investigated number of fractions. All the fractions that appeared at a given speed move from the cylinder to the air intake. The cylinder is replenished with fractions at that moment, when the last particle leaves its limits. This allows you to simulate a continuous stream of fractions.

In this model, the fractions do not interact with each other, since their free path lies in the range from 1 m to 380 m, which is much larger than the characteristic dimensions of the air intake. This means that the fractions mainly interact with the walls of the air intake.

2.4. Influence of the number of fractions on the model
To assess the effect of particle concentration on the calculation results, a calculation was performed with a different number of incident fractions: 500, 750, 1000, 2000, 5000, 10000.

In this problem, the estimated parameter is the number of fractions entering the hole on the back of the collector per unit time. This hole acts as a motor. Below (Figure 2) is a graph of the dependence of the number of fractions in time for different numbers of incident fractions.

The initial burst of particle flood is due to the exist of initial fractions in the air intake. It can be seen that the particle flood reaches the “shelf” in about 1 μs.

After analyzing the obtained result, we can see that the flood grows linearly with an increase in the number of fractions. This allows us to conclude that in the absence of a grid, an increase or decrease in the number of fractions does not cause additional effects. This means that the inlet volume and the meshless inlet in general can be modeled using fewer fractions and extrapolated to a larger number.

2.5. Particle scattering
All equations of the vacuum technique are obtained with the condition of diffuse collision of fractions with the wall. The validity of this condition has been confirmed by numerous experiments in the case of a thermalized flow.

However, in our case, the flow is thermalized only in the collector. Due to the high speed, a significant number of fractions can also mirror reflect from the wall of the air intake or from the walls of the mesh. Roughness also determines the magnitude of this effect. The smoother the surface, the more fractions are mirrored.

Since the influence of this effect is difficult to evaluate analytically, a numerical experiment was carried out. In this experiment was estimated the effect of the fraction of diffusely scattered fractions from specularly reflected ones. This ratio was taken to be 0 (diffuse scattering), 0.1, 0.5, 0.9, 1 (specular reflection).
Figure 2. Particle flow into the engine with different numbers of incident fractions

Figure 3. Particle flood into the engine at different values of the reflection coefficient

Figure 3 shows the change in the particle flood at different values of the coefficient. It can be seen that in the range of 0.5-1 its effect is insignificant, but in the range from 0 to 0.5 there is a sharp jump. It can be concluded that the effect of specular reflection of fractions insignificantly affects the particle flood, even with a significant number of specularly reflected fractions. For all future calculations the specular reflectance is assumed to be 0.8.

2.6. Optimal geometry
An important task solved by a numerical experiment is to find the maximum compression ratio in the reservoir.

Figure 4. Particle flow into the engine with different collector diameters.
At the maximum compression ratio the flow of fractions into the engine will also be maximum, since other parameters of the inlet channel do not change.

In the developed model was investigated a case with a constant reservoir length and variable diameter. The semidiameters were investigated: 11 cm (equal to the pipe diameter), 15 cm, 20 cm, 40 cm, 70 cm, 100 cm. It can be seen from the figure 4 that there is an optimal ratio of the collector diameter to the air intake diameter of 1.2.

2.7. Dependence of the characteristics of the installation on the parameters of the air intake

After calculating a remote control operating only on outboard air, it is necessary to assess the dependence of characteristics on the parameters of the installed confuser (air intake).

The coefficient of the collection efficiency of molecules (capture coefficient) can be defined as the ratio of fractions emitted through the nozzle of the propulsion system to the number of fractions that have passed through the confuser:

\[
\eta_c = \frac{N_r}{N_i}
\]

From the number of fractions it is possible to count the specific mass flow rate. Consequently the mass of fractions entering the confuser per unit time is:

\[
\dot{m}_i = \rho \cdot V \cdot S_1
\]

where \(S_1\) is the area of the inlet section of the confuser. So we can get:

\[
\eta_c = \frac{\dot{m}_r}{\rho \cdot V \cdot S_1}
\]

The engine thrust \(F = \dot{m}_r \cdot g \cdot I\), so a relation between specific impulse and particle capture coefficient:

\[
\eta_c \cdot g \cdot I = \frac{1}{2} \cdot V \cdot C_d
\]

The capture ratio \(\eta_c\) is determined by the geometric parameters of the air intake, which were described earlier.

![Figure 5. Dependence of the unit specific impulse on the efficiency of the air intake at different values of the shape factor.](image)
From the above, it is possible to obtain a dependence that makes it possible to assess the possibility of using various types of engines in outboard air systems to compensate for the friction force acting on the spacecraft.

In this work it was also shown that the ratio of thrust to friction force is constant. From this it can be concluded, that if any engine is capable of providing the required specific impulse on an oxygen-nitrogen mixture, then it is possible to find an orbit in which the thrust force for it will be equal to the friction force - there will be an orbit in which it will be able to compensate the frictional force.

In this calculation, however, the increase in the midsection due to the collector was not taken into account. Previously, the optimal ratio of the collector diameter to the diameter of the inlet path was obtained.

2.8. Calculation of aerodynamic drag and engine power
In the simplest case, the friction force acting on the spacecraft can be calculated as:

$$ F_d = \frac{1}{2} \rho \cdot V^2 \cdot S \cdot C_d $$

where $C_d$ is the shape factor, depends on the spacecraft shape and flight altitude.

For heights of 200-300 km, it lies within 2.1-2.7, $S$ is the spacecraft cross-sectional area, $V$ is the orbital velocity, $\rho$ is the density of the atmosphere.

In this case, the assumption of the continuity of the medium is used. However, the friction force calculated by this method is in good agreement with the existing data on satellite flights at low altitudes.
More complex models, in which the friction force is calculated when considering the fractions bombarding the spacecraft, gives an underestimated result. The results of calculating the friction force are shown in figure 7.

Knowing the price of engine thrust and taking into account the midsection of the collector, you can go to the required power:

![Figure 7. Calculation model type.](image)

3. Conclusion

From the analysis, restrictions on the parameters of the engine follow, which were reduced to a dependence on the diameter of the air intake:

- Required specific impulse: as the diameter of the air inlet increases, the efficiency of collecting fractions decreases. Accordingly, the larger the diameter of the air inlet, the greater the particle velocity must be to compensate for the friction force.
- Flow limitation: to ensure a minimum operating flow of 0.1 mg / s, the air intake diameter must be at least 0.28 m.
- Power limitation: with an increase in the midsection, the aerodynamic drag of the spacecraft increases, and the required thrust to maintain the orbit correspondingly increases. Knowing the energy price of thrust, you can get the electric power of the engine.

References

[1] Longmier B W et al. 2011 VX-200 magnetoplasma thruster performance results exceeding fifty-percent thruster efficiency J. Propul. Power 27(4) 915-20

[2] Stenzel R L 1976 Whistler wave propagation in a large magnetoplasma The Physics of Fluids 19(6) 857-64

[3] Boswell R W et al. 1997 Helicons—the early years IEEE Transactions on Plasma Science 25(6) 1229-44

[4] Hole M J and Simpson S W 1997 Performance of a vacuum arc centrifuge with a nonuniform magnetic field Physics of Plasmas 4(10) 3493-500

[5] Babashov N N and Chetvernin M Y 2021 Methods for improving accuracy in measuring deviations from roundness and cylindricity IOP Conference Series: Materials Science and Engineering 1047 012032

[6] Chen F F and Boswell R W 1997 Helicons—the past decade IEEE Transactions on Plasma Science 25(6) 1245-57

[7] Takahashi K et al. 2010 Characterization of the temperature of free electrons diffusing from a magnetically expandig current-free double layer plasma Journal of Physics D: Applied Physics
43(16) 162001-1-4

[8] World Wind Energy Association report for 2019 Retrieved from: https://wwindea.org/

[9] Masson-Delmotte V et al. 2018 Summary for Policymakers. In: Global Warming of 1.5°C. An IPCC Special Report on the impacts of global warming of 1.5°C above pre-industrial levels and related global greenhouse gas emission pathways, in the context of strengthening the global response to the threat of climate change, sustainable development, and efforts to eradicate poverty (Geneva, Switzerland: World Meteorological Organization) p 32

[10] Pan X and Kraines S 2001 Environmental Input-Output Models for Life-Cycle Analysis Environmental and Resource Economics 20 61-72

[11] Barbashov N N and Abdullina L R 2020 Selection of effective criteria for determining the volume of measurements Journal of Physics: Conf. Ser. 1515 052029