Quadrupole Interaction of Non-diffracting Beams and Their Influence on Atoms

Saud Al-Awfi1,* and Smail Bougouffa2,†

1Department of Physics, Faculty of Science, Taibah University, P.O.Box 30002, Madinah, Saudi Arabia
2Physics Department, College of Science, Al Imam Mohammad ibn Saud
Islamic University (IMSIU), P.O. Box 90950, Riyadh 11623, Saudi Arabia.

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In general the quadrupole interactions in atoms are assumed very weak and then can be ignored in the treatment of light-atom interaction. Recently it has show that the quadrupole interactions can be improved significantly as the atom interacts at near resonance with Laguerre-Gaussian mode. In this paper, we illustrate that the other kind of optical vertex can be also lead to a considerable enhancement of quadrupole interaction when the atom interacts with optical modes at near resonance. The calculations are performed on an interesting situation with Cs atom, where the process concerning dipole-forbidden, while quadrupole-allowed transitions with some specific values for atomic and optical mode parameters. Thus this appreciably improvement on quadrupole interaction can lead to new characteristics of atom-light interaction, which can have some constructive implications in experiment.

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I. INTRODUCTION

Since the emergence of atom optics during last decades, both theoretical and experimental investigations have been focused on the active-dipole interaction. This center of attention is reasonable and is mainly due to the control of this kind of interaction on the outcome situation between the atom and light field [1].

Furthermore, most of the models in quantum optics consider only the electric dipole interaction between systems and the electromagnetic field. In particular, the interaction of nearly resonant laser beams with atoms, which can be formulated as two-level system where the laser light requires dipole active transitions [2, 3]. In the context of dipole approximation, the studies of the aspects of diffraction of atoms in a laser field have guided to a several useful applications, including laser cooling, Bose-Einstein condensation, ultra cold atoms, atom lasers, simulation of condensed matter systems, production and study of strongly correlated systems, and production of ultra cold molecules [4–6].

The useful experimental progressions on the photon processes concerning the interaction of atoms and molecules with lasers, suggest deep theoretical investigations on atom-light interaction. In addition, recent advances in the optical measurement techniques on quadruple transitions have been realized and employed [7–9]. Then, some appropriate theoretical examinations that are concerned with the exploration of the effects of further nature of interactions have been suggested [10–12]. Some such studies are concerned with the exploration of the effects of magnetic-dipole interaction, in the context of the magnetic mirrors applications [13–15], while others are dealt with the electric-quadrupole aspects in the framework of the motion of the atoms in a laser field [7–12].

On the other hand, optical vortices can be considered as one of the most attractive branch that has emerged in atom optics. In the beginning their realization requires the cylindrical optical cavities [16]. However, with the ability of the experimental generation of Laguerre-Gaussian beams, the optical vortices can easily be created in space regions [17, 18]. More advances in the vortex laser beams that are characterized by a quantized orbital angular momentum, are established experimentally and determined through the surfaces optical vortices [19], which lead to an actual presence of optical tweezers.

In fact, the two main types of light that have considered in this area were Laguerre-Gaussian (LG) and Bessel (B) beams. Both beams are illustrated by the property of orbital angular momentum for all light modes greater than the fundamental mode [20, 21], which in both beams it is only able to produce the optical lattices [22]. This major characteristic is the main feature of Hermite-Gaussian (HG) light, which leads us to investigate Hermite-Gaussian light in this study beside the light types possessing orbital angular momentum. However, the feature of orbital angular momentum that possesses the two first types of beams makes them of particular significance in the case of the torque that robustly influences the rotational atomic motion.

The major goal of this paper is to extend the explorations on quadruple interactions in order to exploit the optical forces that are responsible on the motion and trapping small objects, neutral atoms, ions and molecules to well defined space regions. We will illustrate that the inclusion of the quadrupole interactions for some specified cases with forbidden dipole transitions can lead to interesting aspects. In particular, some recent investigations have pointed out the possibility of observing quadrupole interaction and even enhancing its magnitude. Moreover, the realization of optical lattices and vortices of atoms in space, in particular, requires the use of special types of light that are described as possessing non-diffracting beams. These kinds of beams can actually be produced in the laboratories at different orders.

Finally, the investigation of the quadrupole interaction effects for these three beams independently, gives an objective comparison between various aspects that can be generated with the different beams. Nevertheless, we can determine the most appropriate situation in the studies of laser light action on atoms.

The paper is organized as follows. In section II, the quadrupole interaction formalism is presented for atom-field interactions. Section III considers the mechanical motion of two level atom in the quadrupole interactions and the corresponding optical forces are identified. In section III, we are concerned with different kind of optical beams and the expression of Rabi-frequency is obtained in the quadrupole interaction. The spatial distribution of Rabi frequency is presented with typical parameters. Section V provides comments and conclusions.

II. QUADRUPOLE INTERACTION

The goal is to investigate the spatial distribution of the quadrupole Rabi frequency and its magnitude comparative to the case with different light modes. In addition, the effects of the optical vortices on this spatial distribution will be discussed. Thus, the consequences of this quadrupole Rabi frequency on the atom dynamics due to the forces and torque acting on the atom in the field of the optical vortex will be explored.

We consider here a two-level atom interacting with an optical vortex propagating along the z axis with an axial wave vector $k$. The interaction Hamiltonian in a multipolar series about the center of mass $\mathbf{R}$ can be read as

$$\hat{H}_{\text{int}} = \hat{H}_{dp} + \hat{H}_{qp} + \ldots,$$

(1)
where
\[
\hat{H}_{dp} = -\mathbf{\mu} \mathbf{E}(\mathbf{R}),
\] (2)
which represents the electric dipole interaction between system and the EM field, where \(\mathbf{\mu} = q \mathbf{r}\) is the dipole moment and \(\mathbf{E}(\mathbf{R})\) is the EM field. The quadrupole term is explicitly defined as
\[
\hat{H}_{qp} = -\frac{1}{2} \sum_{ij} \Omega_{ij} \frac{\partial E_i}{\partial r_j},
\] (3)
which represents the interaction between the quadrupole moments \(Q\) and the gradient of the electric field that is a function of the center-of-mass coordinate \(\mathbf{R}\) and \(r\) are the components of the internal position vector \(\mathbf{r}\). In order to simplify the problem, without lose of generality, we consider the electric field polarized along the \(x\) direction, which yields the following form of the quadrupole interaction Hamiltonian
\[
\hat{H}_{qp} = \frac{1}{2} \left( \hat{Q}_{xx} \frac{\partial E_x}{\partial x} + \hat{Q}_{xy} \frac{\partial E_x}{\partial y} + \hat{Q}_{xz} \frac{\partial E_x}{\partial z} \right),
\] (4)
where the elements of the quadrupole tensor operator for two-level system can be read as
\[
\hat{Q}_{ij} = Q_{ij} (\pi + \pi^\dagger),
\] (5)
where \(Q_{ij} = \langle i | \hat{Q}_{ij} | j \rangle\) are the quadrupole matrix element and \(\pi (\pi^\dagger)\) are the atomic level lowering and raising operators.

We assume that the EM field is linearly polarized along the \(x\) direction such that its quantized electric field as a function of the center-of-mass in cylindrical coordinates \(\mathbf{R} = (r, \phi, Z)\) can be read as
\[
\mathbf{E}(\mathbf{R}) = i u_k(r) \hat{a}_k e^{i \theta_k(\mathbf{R})} + H.c.
\] (6)
where \(u_k(r)\) and \(\theta_k(Z)\) are the amplitude and the phase of the EM field of the optical modes \(\{k\}\). Here \(\{k\}\) represents a group of indices that characterize the optical modes. \(\hat{a}_k\) is the annihilation operator for the field mode and \(H.c.\) refers to Hermitian conjugate. By substituting this form for the EM field in Eq. (4), we get the following form for the quadrupole interaction Hamiltonian
\[
\hat{H}_{qp} = \hbar \hat{a}_k \Omega_{qp}^{(k)}(\mathbf{R}) e^{i \theta_k(\mathbf{R})} + H.c.
\] (7)
where \(\Omega_{qp}^{(k)}(\mathbf{R})\) is the complex Rabi frequency.

### III. OPTICAL FORCES

Here, we link the previous formalism to the concept of mechanical force that acts by the EM field on the atom. Using the density matrix techniques, we can show that the steady state of total average quadrupole force \(\langle F_{k}^{opt} \rangle\) on the moving atom with velocity \(\mathbf{V} = \dot{\mathbf{R}}\) is given by
\[
\langle F_{k}^{opt}(\mathbf{R}, \mathbf{V}) \rangle = \langle F_{k}^{sp(\text{on})(\mathbf{R}, \mathbf{V})} \rangle + \langle F_{k}^{Q}(\mathbf{R}, \mathbf{V}) \rangle
\] (8)
where
\[
\langle F_{k}^{sp(\text{on})} \rangle = 2\hbar \Gamma_{Q} \left| \Omega_{k}^{Q}(\mathbf{R}) \right|^2 \left( \frac{\nabla \theta_k(\mathbf{R})}{\Delta_k^2(\mathbf{R}, \mathbf{V}) + 2 \left| \Omega_{k}^{Q}(\mathbf{R}) \right|^2 + \Gamma_{Q}^2} \right),
\] (9)
is the spontaneous force that represents the force due to the quadrupole absorption and re-emission of light by the atom and
\[
\langle F_{k}^{Q} \rangle = -2\hbar \nabla \left| \Omega_{k}^{Q}(\mathbf{R}) \right|^2 \left( \frac{\Delta_k(\mathbf{R}, \mathbf{V})}{\Delta_k^2(\mathbf{R}, \mathbf{V}) + 2 \left| \Omega_{k}^{Q}(\mathbf{R}) \right|^2 + \Gamma_{Q}^2} \right),
\] (10)
is the quadrupole force that arises from the nonconformity of the field distribution. Thus, it is responsible for confining the atom to maximal intensity regions of the field. \(\nabla \theta_k(\mathbf{R})\) stands for the gradient of the phase \(\theta_k(\mathbf{R})\). \(\Gamma_{Q}\) is the decay rate for the
quadrupole spontaneous emission and $\Delta_k(R, V)$ is the dynamic detuning which is a function of both the position and the velocity vectors of the atom

$$\Delta_k(R, V) = \Delta_0 - V \cdot \nabla \theta_k(R),$$

where $\Delta_0 = \omega - \omega_0$ is the static detuning, with $\hbar \omega_0$ is the atomic separation energy and $\omega$ is the light frequency. The second term in Eq. (11) $\delta = -V \cdot \nabla \theta_k(R)$ represents the Doppler shift due to the excited mode. The quadrupole force is derivable from a quadrupole potential $\langle F^Q_k \rangle = -\nabla \langle U^Q_k(R) \rangle$ with $U^Q_k(R)$ having the well known form

$$\langle U^Q_k(R) \rangle = -\frac{\hbar \Delta_0}{2} \ln \left( 1 + \frac{2 |\Omega^Q_k(R)|^2}{\Delta_0^2 + \Gamma^2_Q} \right).$$

The conventional derivation of average quadrupole force corresponding to Eq. (12) makes use of density matrix methods leading to the average forces given by equation (8) emerging in the steady state. For red detuned light $\Delta_0 < 0$, the quadrupole potential exhibits a minimum in the high intensity region of the beam tuned below resonance where $\Delta_0 < 0$. For blue detuned $\Delta_0 > 0$, we have trapping in the low-intensity (dark) regions of the field. On the other hand, in many experiment situations the large detuning is considered ($\Delta_0 \gg |\Omega^Q |$) and ($\Delta_0 \gg \Gamma_Q$), then the quadrupole potential can be approximate by

$$\langle U^Q_k(R) \rangle \approx -\frac{\hbar}{\Delta_0} |\Omega^Q_k(R)|^2.$$  

Thus, from the previous brief formalism we can see that the modulus squared Rabi frequency plays a crucial role to explore the dynamics atom relative to the field of mode $k$.

**IV. OPTICAL VORTEXES**

In the following, we will illustrate the previous concepts with different kind of the optical vortices and investigate the quadrupole spatial distribution of Rabi frequency with the most important optical vortices that can be experimentally established and have large applications.

**A. Laguerre Gaussian Modes**

Recently, it has shown that the weak optical quadrupole interaction in atoms can be improved considerably while the atom interacts at near resonance with an optical vortex. In addition, the Laguerre-Gaussian mode is investigated in the case of the quadrupole approximation and with an appropriate chose of the winding number $l$ of the vortex, the process involving the dipole-forbidden, but quadrupole-allowed, transitions in atoms can be realized [12]. Indeed, in the cylindrical coordinates, the laguerre-Gaussian mode is characterized by its amplitude and phase, which are given by

$$u_{klp}(r) = u^l_p(r) = E_{k00} \sqrt{\frac{p^l}{(l!)^2}} \left( \frac{r \sqrt{2}}{\omega_0} \right)^{|l|} L^{|l|}_p \left( \frac{2r^2}{\omega_0^2} \right) e^{-r^2/\omega_0^2},$$

where $L^{|l|}_p$ is the Laguerre polynomial of degree $p$ and $\omega_0$ is the radius at waist beat at $Z = 0$, $E_{k00}$ is the constant amplitude of the plane electromagnetic wave, and

$$\Theta_{klp} = kZ + l\phi,$$

is the phase of the optical mode. We have assumed here that the Laguerre-Gaussian mode has a long Rayleigh range and ignore all mode curvature effects. On the other hand, the optical forces and trapping potentials due to the kind of vortex field were extensively explored [12, 16, 18]. With Laguerre-Gaussian mode, we get for the complex Rabi frequency $\Omega^Q_{klp}$

$$\Omega^Q_{klp}(R) = \left( u^l_p(r)/\hbar \right) \left( \alpha \mathcal{Q}_{xx} + \beta \mathcal{Q}_{yy} + ik \mathcal{Q}_{zz} \right).$$
units of scaling factor of the Rabi frequency by \( \Omega \). We perform the numerical calculations with an interesting case of the Cs atom, which is recently illustrated for its quadrupole transition, which must be considered to improve the experimental investigations. [12] and pointed out that these results can provide a significant mechanical effects on atoms categorized by a quadrupole-allowed transition, which must be considered to improve the experimental investigations.

\[
\alpha = \left( \frac{|\ell| X}{r^2} - \frac{2X}{w_0^2} \frac{iY}{r^2} + \frac{1}{L_p^{(\ell)}} \frac{\partial L_p^{(\ell)}}{\partial X} \right),
\]

(17)

\[
\beta = \left( \frac{|\ell| Y}{r^2} - \frac{2Y}{w_0^2} + \frac{iX}{r^2} + \frac{1}{L_p^{(\ell)}} \frac{\partial L_p^{(\ell)}}{\partial Y} \right),
\]

(18)

In order to illustrate the effect of the quadrupole effects with Laguerre-Gaussian (LG) mode, we limit our exploration to case that is recently considered in [12]. We consider the case of an LG doughnut mode of winding number \( \ell \) and \( p = 0 \). In this case, the derivatives in \( \alpha \) and \( \beta \) in Eqs. (17,18) are null, as \( L_p^{(\ell)} \) is a constant for all \( \ell \). In addition, we also suppose that the atom is constrained to move in the \( X - Y \) plane and the quadrupole transition is such that \( Q_{xx} = Q_{zz} = 0 \). Within these assumptions the Rabi frequency Eq. (16) can be read as:

\[
\Omega_{\ell,0}^Q(R) = \left( u_0^{(\ell)}(r) / \hbar \right) \dot{Q}_{xx} \left( \frac{|\ell| X}{r^2} - \frac{2X}{w_0^2} \frac{iY}{r^2} \right).
\]

(19)

We perform the numerical calculations with an interesting case of the Cs atom, which is recently illustrated for its quadrupole transition \((6^2S_{1/2} \rightarrow 5^2D_{5/2})\) with an appropriate values of the essential parameters \( \omega_0 = \lambda / 2 \), \( \lambda = 675 \text{nm} \), \( Q_{xx} = 100 \text{a.u.} \), \( \Gamma_Q = 7.8 \times 10^8 \text{s}^{-1} \), \( \Delta_0 = 10^3 \Gamma_Q \) and for the intensity \( I = \epsilon_0 c E_{\epsilon 00}^2 / 2 = 10^9 \text{Wm}^{-2} \). In consequence, it is suitable to label the scaling factor of the Rabi frequency by \( \Omega_0 = \frac{\hbar}{\epsilon_0 c} \left( \Gamma_Q / \epsilon_0 \right)^{1/2} \), \( \hat{Q}_{xx} = 136 \Gamma_Q \). In Fig.1, we present the spatial distribution of \( |\Omega_{\ell,0}^Q|^2 \) in units of \( \Omega_0 \) for doughnut vortex of winding numbers \( |\ell| = 1 \) and \( |\ell| = 10 \).

FIG. 1. (Color online) The spatial distribution of the modulus of squared relative Rabi frequency \( |\Omega_{\ell,0}^Q|^2 \) for an atom in a (LG) doughnut mode. (a) in the case \( \ell = 1 \) and \( p = 0 \), (b) for the case \( \ell = 100 \) and \( p = 1 \). (c) The projection in the \( X - Y \) plane of the case \( \ell = 100 \) and \( p = 1 \). It is clear from Fig 1(a) that the presence of a maximum variation at the vortex core is a maintained only of the case \( \ell = 1 \). While Fig.1(b) illustrates the analogous variation of the relative Rabi frequency for the large winding number \( l = 100 \). From the Experiment point view, winding numbers as large as \( l = 300 \) can be accomplished, as shown recently [23]. On the other hand, the shape of the relative quadrupole Rabi frequency in the LG mode shows augmentation of both the trapping potential and the dissipative force in the procedures characterized by quadrupole transitions. These important consequences are recently explored [12] and pointed out that these results can provide a significant mechanical effects on atoms categorized by a quadrupole-allowed transition, which must be considered to improve the experimental investigations.

B. Bessel Modes

In this section, we follow same previous procedure to explore the quadrupole interaction of non-diffracting Bessel beam, which is essentially characterized at general position \( R = (r, \varphi, Z) \) in cylindrical polar coordinates, by its phase \( \theta_{km} \) and the complex Rabi frequency \( \Omega_{km}^Q \) that are given as [22]

\[
\theta_{km}(\varphi, Z) = kr + m\varphi,
\]

(20)
\[ \Omega_{km}(r, Z) = \left( \frac{g_m(r)}{\hbar} \right) \left[ \hat{\mathcal{Q}}_{xx} \eta + \hat{\mathcal{Q}}_{xy} \mu + \hat{\mathcal{Q}}_{xz} \sigma \right], \]  

where \( \mathbf{k} \) is the beam axial wave vector and \( m \) is the quantum number. The parameters \( \eta, \mu \) and \( \sigma \) are given respectively as

\[
\eta(r) = \left( \frac{1}{J_m} \partial J_m}{\partial r} \frac{imY}{r^2} \right),
\]

\[
\mu(r) = \left( \frac{1}{J_m} \partial J_m}{\partial r} \frac{imX}{r^2} \right),
\]

\[
\sigma(r, Z) = \left( \frac{2m+1}{2Z} - \frac{2Z}{Z_{\text{max}}} + ik_Z \right),
\]

The Bessel amplitude function \( g_m(r) \) can be read as

\[
g_m(r) = \sqrt{\frac{8\pi^2 k_0^2 w_0^2 l}{\varepsilon_0 c}} \left( \frac{Z}{Z_{\text{max}}} \right)^{m+1/2} \exp \left( -\frac{2Z^2}{Z_{\text{max}}^2} \right) J_m(k_{\perp} r),
\]

with \( J_m \) denoting the \( m \)th-order Bessel function of the first kind, \( r^2 = X^2 + Y^2, k_1^2 + k_2^2 = k_0^2 \) and \( k_0 = (2\pi/\lambda) \) being the wave number in free space while \( I \) is the beam intensity. In addition, \( k_{\perp} = k_0 \sin \alpha \) and \( k_Z = k_0 \cos \alpha \) are the transversal and longitudinal components of the wave vector respectively, while \( \alpha \) is the opening angle of the cone on which the wave traverses. In addition, \( w_0 \) is the input Gaussian beam’s waist, and \( Z_{\text{max}} \) is the typical ring spacing [24]. We point out here that the central spot of the zero-order Bessel mode (denoted by \( J_0 \)) is always bright (a central maximum) whereas that of all higher-order Bessel modes (denoted by \( J_m, m \) being an integer and \( \geq 1 \)) are always dark on the axis and are surrounded by concentric rings whose peak intensities decrease as \( r^{-1} \) [25]. Additionally, only the higher-order Bessel modes with \( m \geq 1 \) have an azimuthal phase dependence, \( \epsilon^{\text{imp}} \), on the mode axis, and, therefore, have a non-diffracting dark core. This property is directly related to the orbital angular momentum carried by the light modes, which is an addition to any spin angular momentum associated with their polar wave polarization. On the other hand, the factors \( \hat{\mathcal{Q}}_{ij} \) are the elements of the quadrupole tensor operator defined in Eq.(5).

For the numerical calculations, it is instructive therefore to consider the previous important case of the Cs atom that is recently exemplify for its quadrupole transition (\( 6^2S_{1/2} \rightarrow 5^2D_{3/2} \)) with a suitable values of the crucial parameters, which are illustrated previously. We shall also assume that the atom is constrained to move in the \( XY \) plane and the quadrupole transition is such that \( Q_{xy} = Q_{xz} = 0 \).

Here , it is appropriate to label the scaling factor of the Rabi frequency by \( \Omega_0 = \frac{\Omega_{0L}}{\hbar k_0} \sqrt{\frac{8\pi^2 k_0^2 w_0^2 l}{\varepsilon_0 c}} \). In Fig.2, we point out the spatial distribution of \( |\Omega_{km}|^2 \) in units of \( \Omega_0 \) for high-order vortex beam \( m = 0, m = 1 \) and \( m = 10 \).

We consider the rotational features including a rotational shift due to the azimuthal dependence of the field structure which arises in the interaction of the atom with any Bessel mode of order \( m > 0 \). Lastly,

C. Hermite-Gaussian Modes

In the active-quadrupole interaction, the essential features of a Hermite-Gaussian beam, at a general position \( R = (X, Y, Z) \), are the phase \( \theta_{km}(R) \) and the complex Rabi frequency \( \Omega_{km}^0(R) \). These are written as [1]

\[
\theta_{km}(Z) = (n + m + 1) \tan^{-1} (Z/Z_k) + kZ,
\]

and

\[
\Omega_{km}^0(R) = \xi_{000} \cdot \frac{C_{km}}{2\hbar} \cdot C_{km} \cdot F_{km}(R) \left\{ \alpha \hat{\mathcal{Q}}_{xx} + \beta \hat{\mathcal{Q}}_{xy} + \gamma \hat{\mathcal{Q}}_{xz} \right\},
\]

respectively, where \( \mathbf{k} \) is the axial wave vector of the beam mode, where the factor \( \xi_{000} \) is the amplitude for a corresponding plane wave of intensity \( I \) propagating in the dielectric medium of refractive index \( \eta \):

\[
\xi_{000} = \sqrt{2I/\eta^2 \varepsilon_0 c}
\]

and \( C_{km} \) is the normalization constant of the Hermite-Gaussian function and is given by

\[
C_{km} = \left[ 2/(2^n + m! \pi) \right]^{1/2},
\]
FIG. 2. (Color online) The spatial distribution of the modulus of squared relative Rabi frequency \( |\Omega_Q^{p}/\Omega_0| \) for an atom in a Bessel mode for \( Z = Z_{\text{max}} \). (a) in the case azimuthal mode \( m = 0 \), (b) for the case \( m = 1 \). (c) for the case \( m = 10 \). (d) The projection in the \( X - Y \) plane of the case \( m = 10 \).

for the case \( n \geq m \), while \( F_{knm}(R) \) can be read as

\[
F_{knm}(R) = \frac{w_0}{w(Z)} \exp \left[-ik \frac{(X^2 + Y^2)}{2R(Z)} \right] \times \exp \left[-\frac{(X^2 + Y^2)}{w^2(Z)} \right] \times H_n \left( \frac{\sqrt{2}X}{w(Z)} \right) \times H_m \left( \frac{\sqrt{2}Y}{w(Z)} \right).
\]  

(30)

Here \( R(Z) = (Z_R^2 + Z^2)/Z \) is the radius of curvature of the mode's wavefront with \( Z_R \) the Rayleigh range; \( w(Z) \) is the radius at which the Hermite-Gaussian mode amplitude and intensity drop to 1/e and 1/e^2 of their axial values, respectively,

\[
w^2(Z) = (Z_R^2 + Z^2)/kZ_R.
\]  

(31)

The position \( Z = 0 \), referred to as the Hermite-Gaussian mode waist, corresponds to the waist size \( w_0 \) of the Hermite-Gaussian mode, such that: \( w_0^2 = 2Z_R/k \). The special functions \( H_n(\cdot) \) is the Hermite polynomial of order \( n \). For this reason, Hermite-Gaussian modes are typically designated \( TEM_{mn} \) where \( m \) and \( n \) are the polynomial indices in the \( X \) and \( Y \) directions. Here
(X,Y) are transverse coordinates in the Cartesian coordinate systems. On the other hand, \( \alpha, \beta \) and \( \gamma \) are given respectively by

\[
\begin{align*}
\alpha &= \frac{1}{H_n} \frac{\partial H_n}{\partial X} - \frac{ik}{R(Z)} \frac{X}{w^2(Z)} - \frac{2X}{w^2(Z)} \\
\beta &= \frac{1}{H_m} \frac{\partial H_m}{\partial Y} - \frac{ik}{R(Z)} \frac{Y}{w^2(Z)} - \frac{2Y}{w^2(Z)} \\
\gamma &= \frac{1}{H_n} \frac{\partial H_n}{\partial Z} + \frac{1}{H_m} \frac{\partial H_m}{\partial Z} + \frac{i(n+m+1)}{k w^2(Z)} + \frac{(X^2+Y^2)}{R(Z)} \left[ \frac{ik}{R(Z)} - \frac{ik}{2Z} + \frac{2}{w^2(Z)} \right]
\end{align*}
\]

Assuming that the atom is constrained to move in the XY plane and the quadrupole transition is such that \( Q_{xy} = Q_{zc} = 0 \). Under this condition the electric-quadrupole interaction Eq.(27) takes the following form:

\[
\Omega_{\text{Qnm}}^0(R) = \frac{\xi_{100}}{2\hbar} \cdot C_{nm} \cdot F_{\text{kmn}}(R) \left\{ \tilde{Q}_{xx} \left( \frac{1}{H_n} \frac{\partial H_n}{\partial X} - \frac{ik}{R(Z)} \frac{X}{w^2(Z)} - \frac{2X}{w^2(Z)} \right) \right\}.
\]

In the following we will consider the same previous case for the Cs atom with the specific values of the essential parameters. In this case, we label the scaling factor of the Rabi frequency by \( \Omega_0 = \frac{Q_{xx}}{\eta_{00}} \sqrt{2I/\eta^2 \varepsilon_0 c} \). In Fig.3, we show the spatial distribution of \( |\Omega_{\text{Qnm}}^0|^2 \) for an atom in a Hermite Gaussian mode. (a) in the case \( n = 1 \) and \( m = 0 \), (b) for the case \( n = 1 \) and \( m = 1 \). (c) For the case \( n = 2 \) and \( m = 0 \).

![Spatial distribution of the modulus of squared relative Rabi frequency](image)

**FIG. 3.** (Color online) The spatial distribution of the modulus of squared relative Rabi frequency \( |\Omega_{\text{Qnm}}^0/\Omega_0|^2 \) for an atom in a Hermite Gaussian mode. (a) in the case \( n = 1 \) and \( m = 0 \), (b) for the case \( n = 1 \) and \( m = 1 \). (c) For the case \( n = 2 \) and \( m = 0 \).

**V. CONCLUSIONS**

In this work, we have presented the derivations of the quadrupole-active transition, acting on a two-level atom moving in three different types of non-diffracting beams. We have shown that the value of the Rabi frequency and hence the quadrupole trapping potential is sufficient enough to exploit in optical manipulation.

From our previous experience in the active-dipole interaction studies we know that this result can be enhanced by several techniques such as a combination of strong field gradients and high field intensity or using the mutual coupling of two co-propagating and counter-propagating beams [26, 27]. The mutual technique, in particular assists to avoid some undesirable effects of single beam interaction such as destabilizing dissipative force as well as it doubles the value of quadrupole force. In addition, it can also be supported significantly as demonstrated in some recent studies via the production of the plasmonic modes near a surfaces [19–22]. All these techniques could be generally described and used with ordinary light or non-ordinary.

However, the three types of light that we have worked on here can be enhancing by unique technique, which requires only higher order beams. Such kinds of beams have already produced in laboratories [28–33]. This technique, in particular, stands behind the importance of such types of light in various modern atomic applications. The remarkable concern that can be clearly seen from these results is that quadrupole interactions, which are usually ignored in atom optics, can be taken in consideration for some crucial cases, in particular the high order beams can be produced experimentally. Some applications can be envisaged.
at this stage such as the manipulation of adsorbed atoms and molecules held on surfaces by van der Waal force which received recently more attention in atom optics under the effects of active-dipole interaction. The corresponding results somewhat of quadrupole are expected. The research work along these lines is now in progress and the results will hopefully be reported in due course.

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