SOFT POMERON PHYSICS ON THE LATTICE

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We use lattice QCD to investigate some topics in strong interaction phenomenology which are usually related to soft pomeron exchange. These include hadronic cross-sections at high energies and diffractive scattering at HERA. Some numerical results are presented in the framework of the Landshoff-Nachtmann pomeron model, and strategies for further studies are discussed.

1 Introduction

It has been known for a long time that the behaviour of the total hadronic cross-section at high energies could be described in the framework of Regge theory. In particular, the rise of the cross section was attributed to the Regge trajectory with the quantum numbers of the vacuum: the Pomeron. Renewed interest in Pomeron models has been recently triggered by the analysis of diffractive scattering at HERA. In particular, the discovery of events with large rapidity gaps has been interpreted as evidence for Pomeron exchange.

Recovering the phenomenology of Pomeron exchange from first principles QCD is a major theoretical problem. The lattice formulation of QCD, together with the Monte Carlo method, provides an invaluable tool for nonperturbative, first principles calculations. The aim of the present note is to describe some attempts to use such a tool in order to test models of diffraction and pomeron exchange from the point of view of QCD. In the following we first recall some basic facts about lattice QCD, then we describe an application of the method to test the Landshoff-Nachtmann (LN) model, and finally we outline a general approach to the lattice investigation of diffractive physics.
The transition from the continuum formulation of QCD to the lattice one can be summarised as follows: first the theory is continued to imaginary times ($t \to it$) and 4-dimensional spacetime is mapped onto the sites of a (usually hypercubic) lattice. Then a gauge-invariant discretisation of the QCD Lagrangian is obtained after defining fermion fields $\psi(n), \bar{\psi}(n)$ at the lattice sites and $SU(3)$-valued parallel transport operators $U(n, n')$, connecting nearest-neighbour sites. The relation between the link variables $U$ and the continuum gauge potential $A$ is given by $U = \exp(iagA)$, where $a$ is the lattice spacing and $g$ is the bare coupling constant. In this process two fundamental length scales are introduced: the lattice spacing $a$, which acts as an ultraviolet cutoff, and the volume of the box $V = L^4$, which regulates the infrared behaviour. Thus the above procedure produces a gauge-invariant, nonperturbative regularisation of the (Euclidean) theory. Finally, path integrals yielding Green functions of quark, gluon and hadronic fields can be numerically computed by means of the Monte Carlo method.

In the continuum limit, the dependence of the bare coupling constant on the value of $a$ is dictated by the renormalisation group equations. Asymptotic freedom implies $g_0 \to 0$ when $a \to 0$. In practice, the continuum limit $a \to 0$ can be studied numerically by letting $g_0$ approach zero. In such a limit physical quantities will not depend on the value of the cutoff, while ultraviolet-divergent Green functions will, and suitable renormalisation conditions can be imposed. Because of computer power limitations, most lattice QCD studies, including the present one, are still performed in the quenched approximation, which amounts to neglecting quark loops. Unlike in the continuum formulation of the theory, gauge-invariant, physical quantities can be directly computed on the lattice without gauge fixing. However, for our purposes one also needs to compute gauge-dependent Green functions. In this case a consistent gauge-fixing procedure has to be implemented. For the studies discussed here the lattice version of the Landau gauge was used.

3 The Landshoff-Nachtmann Pomeron on the Lattice

In the LN model Pomeron exchange between quarks behaves like a $C = +1$ photon-exchange diagram, with amplitude

$$i\beta_0^2(\bar{u}\gamma_{\mu}u)(\bar{u}\gamma^\mu u).$$  (1)
\( \beta_0 \) represents the strength of the Pomeron coupling to quarks, and is related to the (non-perturbative) gluon propagator by

\[
\beta_0^2 = \frac{1}{36\pi^2} \int d^2 p \left[ g^2 D(p) \right]^2, \tag{2}
\]

where \( g \) is the gluon-quark coupling. \( \beta_0 \) can be determined from, for example, the total \( pp \) cross section. As a consequence, the model yields simple formulae for \( pp \) scattering, exclusive \( \rho \) production in deep inelastic scattering and the \( J/\Psi \)– nucleon total cross section, which all contain integrals in momentum space of \( g^2 D(p) \).

In order to extract predictions from the model, one needs an expression for \( g^2 D(p) \). Obviously, convergence of (2) requires that the infra-red pole of the gluon propagator be removed by some nonperturbative mechanism. By inserting in the LN model the gluon propagator as computed on the lattice, we aimed to perform a strong consistency test from the point of view of QCD. We performed calculations on two sets of hypercubic lattices, corresponding to different physical volumes and lattice spacings. In addition, we used data for the gluon propagator as evaluated by Marenzoni et al. Comparing results from different data sets is important in order to study possible lattice artifacts in our calculations. We extracted the scalar gluon self-energy \( D_{\text{lat}}(p) \) from our data and then fitted it to various analytical expressions. Then the curves corresponding to the best fits were inserted in the LN formulae. In this process, we assumed that in the continuum limit the propagator is multiplicatively renormalisable, as it is in perturbation theory. Also, we neglected the running of the QCD coupling, i.e. we made the approximation \( g(p) = g \). As the scale for the momenta in \( D_{\text{lat}}(p) \) was set from independent measurements, we only had one free parameter to fix in the expression \( g^2 D_{\text{lat}}(p) \). This is a multiplicative factor that corresponds to the product of a gluon wavefunction renormalisation constant times a numerical value for \( g^2 \). We call this parameter \( g_{\text{eff}}^2 \). It can be determined by using (2) as a nonperturbative renormalisation condition, i.e. by imposing that \( \beta_0 \) attains its phenomenological value of 2.0 GeV\(^{-1}\):

\[
\beta_0^2 = \frac{1}{36\pi^2} \int d^2 p \left[ g_{\text{eff}}^2 D_{\text{lat}}(p) \right]^2 = 4 \text{ GeV}^{-2} \tag{3}
\]

It is the combination \( g_{\text{eff}}^2 D_{\text{lat}}(p) \) that we insert in the formulae of the LN model. In using such formulae, we adopt the analysis procedure of Halzen and collaborators.
3.1 Proton-proton elastic scattering

The calculation of $\sigma^0_{\text{tot}}$ and $\frac{d\sigma^0}{dt}$, i.e. the energy-independent part of the total and elastic differential cross section for proton-proton scattering, provides a benchmark for the LN model of the Pomeron. The model has already been explored using nonperturbative propagators obtained from approximate solutions of Schwinger-Dyson equations. Single Pomeron exchange is expected to dominate the differential cross-section up to $-t \approx 0.5 \text{ GeV}^{-2}$. To fully describe the energy dependence, the intercept of the Pomeron trajectory is taken to be somewhat larger than 1. The measured total and elastic differential cross sections are usually parametrised as follows:

$$\sigma_{\text{tot}} = \left(\frac{s}{m_p^2}\right)^{0.08} \sigma^0_{\text{tot}}, \quad \frac{d\sigma}{dt} = \left(\frac{s}{m_p^2}\right)^{0.168} \frac{d\sigma^0}{dt}. \quad (4)$$

For small $t$, the elastic differential cross section behaves like $e^{Bt}$, and the model is characterised by two parameters, $\sigma^0_{\text{tot}}$ and $B$.

We computed $\sigma^0_{\text{tot}}$ and $B$ on each lattice using the lattice gluon propagator and the effective coupling $g_{\text{eff}}$. We obtained values of $\sigma^0_{\text{tot}}$ ranging from 18.12 mb to 19.85 mb and $B$ from 12.6 $\text{GeV}^{-2}$ to 13.6 $\text{GeV}^{-2}$, thus in very good agreement with each other, suggesting that both quantities are subject to only small discretisation and finite volume effects. They are also encouragingly close to the phenomenological values of $\sigma^0_{\text{tot}} \approx 22.7 \text{ mb}$ and $B \approx 11 \text{ GeV}^{-2}$. In Fig. 1 we show ISR data for the differential elastic cross section at $\sqrt{s} = 53 \text{ GeV}$ together with the lattice predictions, with the energy correction of Eq. (4).

3.2 $J/\psi$-Nucleon Scattering

This process provides a further important test of the LN model. Two phenomenological features emerge from our calculations. Firstly, the quark-counting rule is closely satisfied in the case of the hadrons composed of light quarks, as we get for the ratio $\sigma_{pp}/\sigma_{\pi p}$ values in the range $1.5 - 1.8$. Secondly, the Pomeron couples more weakly to mesons composed of heavier quarks. This can be seen in Fig. 2, where we show the meson-nucleon cross section as a function of the pole radius, together with a Regge fit to the energy-independent part of the $\pi^- p$ and $K^- p$ cross sections. However, we observe now sizeable differences between the results on the different lattices, reinforcing the need to repeat the calculation for a wider range of lattice parameters.
Figure 1: Data for the $pp$ elastic cross section at $\sqrt{s} = 53$ GeV together with the lattice predictions, corrected for the energy dependence, on the 16$^4$ lattices at $\beta = 6.2$ (solid) and $\beta = 6.0$ (dots), and on the 24$^3 \times 32$ lattice at $\beta = 6.0$ (dashes).
Figure 2: The meson-nucleon total cross section as a function of the radius used in the meson form factor, on the 16^4 lattices at $\beta = 6.2$ (solid) and $\beta = 6.0$ (dots), and on the 24^4 × 32 lattice at $\beta = 6.0$ (dashes). Also shown are the radii and total cross sections corresponding to the $\pi$, $K$ and $J/\psi$. 
4 Diffractive Physics: the Factorisation Hypothesis

In this section we summarise some attempts to develop a general approach for investigating diffractive scattering using lattice QCD. Our approach starts from assuming *factorisation*. To clarify this concept, consider diffractive scattering at HERA in the one-photon approximation:

\[ \gamma^*(q) + p(p) \rightarrow \tilde{p}(\tilde{p}) + X(p_X). \]  

Following Arens *et al.* we say that if the above process factorises, one can describe it in two steps: first the original proton emits a Pomeron, \( p(p) \rightarrow p(\tilde{p}) + IP(\Delta) \), then the Pomeron collides with the \( \gamma^* \) to produce the final hadronic state \( X \), \( \gamma^*(q) + IP(\Delta) \rightarrow X(p_X) \). In other words, the amplitude for \( (5) \) can be written in a factorizable form:

\[ A(\gamma^*p \rightarrow Xp) = A(\gamma^*IP \rightarrow X) \otimes (IP - \text{propagator}) \otimes (ppIP - \text{vertex}). \]  

Notice that the above assumption does not imply that we treat the Pomeron like a particle. Also, we emphasise that although the physics we discuss here is usually interpreted in terms of Pomeron exchange, we want to analyse it from a more general point of view, where we only assume factorisation and the emission of a colour singlet excitation. In this sense, our use of the word Pomeron in the present discussion is simply for illustrative purposes. On the lattice we want to study the second and third factors in the above formula, i.e. Pomeron propagation and the effective coupling of the Pomeron to a proton. One motivation for such a study is the investigation of the helicity structure of the Pomeron, as it has recently been argued that a nontrivial spin structure in the Pomeron-proton vertex may be related to quantities measurable at HERA.

Given some *ansatz* for a composite QCD operator \( O_{IP}(x) \) which creates the Pomeron from the QCD vacuum, the effective Pomeron-proton coupling takes the form of a QCD 3-point vertex function, which we can study in momentum space as a function of the momentum of the proton, \( p \), and the momentum of the Pomeron, \( \Delta \). Both because of technical limitations and because of theoretical prejudice, we will assume that \( O_{IP} \) only contains gluonic fields. Usually composite QCD operators are classified according to \( J^{PC} \), i.e. spin, parity and charge conjugation. Since the Pomeron is a Regge trajectory, all the information we have *a priori* is that \( O_{IP} \) should have a \( J^{++} \) structure. As we do not want to assign a value to \( J \), it makes sense to allow \( O_{IP} \) to have arbitrary Lorentz structure, i.e. to be the sum of a scalar, a vector, and tensors of arbitrary rank.

Numerical work is in progress, aiming to test different *ansätze* for \( O_{IP} \). We are currently focusing on two-gluon operators, and we have obtained encouraging results as far as the numerical feasibility of the project is concerned.
Acknowledgments

This work was supported by the United Kingdom Particle Physics and Astronomy Research Council (PPARC) under grant GR/J21347. CP and DGR acknowledge the support of PPARC through Advanced Fellowships held at the University of Liverpool and at the University of Edinburgh respectively.

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