Constraints on kinematic models from the latest observational data

Jianbo Lu
Department of Physics, Liaoning Normal University, Dalian 116029, P. R. China

Lixin Xu
Korea Astronomy and Space Science Institute, Daejon 305-348, Korea and
School of Physics and Optoelectronic Technology, Dalian University of Technology, Dalian, 116024, P. R. China

Molin Liu
College of Physics and Electronic Engineering, Xinyang Normal University, Xinyang, 464000, P. R. China

Kinematical models are constrained by the latest observational data from geometry-distance measurements, which include 557 type Ia supernovae (SNIa) Union2 data and 15 observational Hubble data. Considering two parameterized deceleration parameter, the values of current deceleration parameter \( q_0 \), jerk parameter \( j_0 \) and transition redshift \( z_T \), are obtained. Furthermore, we show the departures for two parameterized kinematical models from \( \Lambda \) CDM model according to the evolutions of jerk parameter \( j(z) \). Also, it is shown that the constraint on jerk parameter \( j(z) \) is weak by the current geometrical observed data.

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1. Introduction

The recently cosmic observations [1][2][3] suggest that the expansion of present universe is speeding up. The accelerated expansion of the universe is usually attributed to the fact that dark energy (DE) is an exotic component with negative pressure, such as cosmological constant, quintessence [4], phantom [5], quintom [6], generalized Chaplygin gas [7], agegraphic dark energy [8], etc [9]. Also, the accelerating universe is related to the modification of the gravity theory at large scale, such as \( f(R) \) modified gravity theory [10] and higher dimensional theory [11], etc. These approaches correspond to the dynamics of the universe. For more information about dynamics of universe, please see review papers [12].

Another route is the kinematical approach, which holds true regardless of the underlying cosmic dynamics [13], i.e., it depends neither on the validity of any particular metric theory of gravity nor on the matter-energy content of the observed universe [14]. It is only related to the weaker assumption that space-time is homogeneous and isotropic so that the FRW metric is still valid [14]. Then the kinematic approach is also called cosmokinetics [15], cosmography [16], or Friedmannless cosmology [17]. For kinematic equations, there are Hubble parameter \( H = \frac{\dot{a}}{a} \), deceleration

*Electronic address: lvjianbo819@163.com
parameter \( q = -\ddot{a}/\dot{a}^2 \), and jerk parameter \( j = -\dddot{a}/(a\dot{a}^3) \). It can be seen that parameters \( H \), \( q \) and \( j \) are purely kinematical, since they are independent of any gravity theory, and all of them are only related to scale factor \( a \) (or redshift \( z \), since \( a = \frac{1}{1+z} \)).

The benefit of the kinematical analysis is that it has the fewer assumptions and a different set of models are explored for comparison with dynamical scenario. Since the origin of cosmic acceleration is unknown, the choice of parameterized kinematical model is essentially arbitrary. But inappropriate kinematical model could imply an unphysical universe at earlier time. For instance, for the model \( q(z) = q_0 + q_1 z \), it has \( q(z) > 1/2 \) at high redshift \(^{18}\), which is not consistent with the matter dominated universe\(^1\). The reason is simple that the model \( q(z) = q_0 + q_1 z \), a expansion of \( q(z) \) at low redshift, is not reliable at high redshift. As a complementarity to dynamical approach, in this paper we constrain two parameterized kinematical models by using the latest observational data: 557 type Ia supernovae (SNIa) Union2 dataset and 15 observational Hubble data.

2. The kinematical approach and models

The dimensionless Hubble and deceleration parameters are defined by the first and second derivative of scale factor

\[
H \equiv \frac{\dot{a}}{a} = -\frac{1}{1+z} \frac{dz}{dt},
\]

\[
q \equiv -\frac{1}{H^2} \left( \frac{\dot{a}}{a} \right)^2 = \frac{1}{2} \left(1 + z\right) \frac{\left[H(z)^2\right]'}{H(z)^2} - 1,
\]

where "dot" denotes the derivative with respect to cosmic time \( t \). \( H \) and \( q \) describe the rate of expansion and acceleration of universe. The relation between \( H \) and \( q \) is written as

\[
H = H_0 \exp\left(\int_0^z [1 + q(u)] d\ln(1 + u)\right),
\]

\( H_0 \) is Hubble constant. In this paper, subscript "0" denotes the current value of cosmological quantities.

Similar, the jerk parameter \( j \) is defined as the dimensionless third derivative of scale factor with respect to cosmic time

\[
j \equiv -\frac{1}{H^3} \left( \frac{\dot{a}}{a} \right)^3 = -\frac{1}{2} \left(1 + z\right)^2 \left[H(z)^2\right]'' \left[H(z)^2\right] - \left(1 + z\right) \left[H(z)^2\right]' \left[H(z)^2\right] + 1.
\]

The use of the cosmic jerk parameter provides a more natural parameter space for kinematical studies, and transitions between phases of different cosmic acceleration are more naturally described by models incorporating a cosmic jerk \(^{19}\). Especially, since for flat ΛCDM model it has a constant jerk with \( j(z) = -1 \), jerk parameter can provides us with a simple, convenient approach to search for departures from the cosmic concordance model, ΛCDM, just as deviations from \( w = -1 \) done in more standard dynamical analyses. The jerk parameter is related with the deceleration parameter by the following differential equation

\[
j = -[q + 2q^2 + (1 + z) \frac{dq}{dz}],
\]

\(^1\) For the matter dominated universe, it has \( q(z) = 1/2 \), then there must be \( q(z) \leq 1/2 \) for any kinematical or dynamical model.
where luminosity distance of distance modulus and can be given by the SNIa dataset; the expression of theoretical distance modulus related to the apparent magnitude of supernova at peak brightness can be minimized the quantity [24] universe up to high redshift for comparison with the present rate. Theoretical cosmic parameters are determined by paper, too.

From Eqs. (3) and (5), we can see that the expressions of Hubble parameter and jerk parameter can be given by deceleration parameter. Here we consider two parameterized deceleration parameters, \( q(z) = q_0 + \frac{q_1}{1 + z} \) (\( M_1 \)) [20] and \( q(z) = \frac{1}{2} + \frac{q_1}{1 + z} \) (\( M_2 \)) [21]. \( M_1 \) is one order expansion of scale factor \( a \) at present \( (a = 1) \), i.e., \( q(a) = q_0 + q_1 (1 - a) \). \( M_2 \) is an alternative parametrization to the three-epoch model [2]. For these two models, the expressions of Hubble parameter, deceleration parameter and jerk parameter are shown in Table I.

For other parameterized deceleration parameter appeared in Ref. [14], such as model \( q(z) = q_0 + q_1 z \) and \( q(z) = q_0 \) = constant, we will not discuss: since the former model is only interested at low redshift, not all the observed data from 557 SNIa Union2 dataset (redshift interval \( 0.015 \leq z \leq 1.4 \)) and 15 observational Hubble data (redshift interval \( 0 \leq z \leq 1.75 \)) can be used to constrain its evolution; though the latter one indicates an accelerating universe [14], it does not describe a transition of universe from decelerated expansion to accelerated expansion. Furthermore, for two-epoch model \( q = q_0, z \leq z_T; q = q_1, z > z_T \), the \( q(z) \) function is not smooth, so it will not appeared in the paper, too.

### 3. Data and analysis Methods

Since SNIa behave as excellent standard candles, they can be used to directly measure the expansion rate of the universe up to high redshift for comparison with the present rate. Theoretical cosmic parameters are determined by minimizing the quantity [24]

\[
\chi^2_{SNIa}(H_0, \theta) = \frac{\sum_{i=1}^{N} (\mu_{obs}(z_i) - \mu_{th}(z_i, H_0, \theta))^2}{\sigma_{obs,i}^2},
\]

where \( N = 557 \) for SNIa Union2 data; \( \sigma_{obs,i}^2 \) are errors due to flux uncertainties, intrinsic dispersion of SNIa absolute magnitude and peculiar velocity dispersion, respectively; \( \theta \) denotes the model parameters; \( \mu_{obs} \) is the observed values of distance modulus and can be given by the SNIa dataset; the expression of theoretical distance modulus \( \mu_{th} \) is related to the apparent magnitude of supernova at peak brightness \( m \) and the absolute magnitude \( M \),

\[
\mu_{th}(z_i) \equiv m_{th}(z_i) - M = 5 \log_{10}(D_L(z)) + \mu_0,
\]

where luminosity distance

\[
D_L(z) = H_0 d_L(z) = (1 + z) \int_0^z \frac{H_0 dz'}{H(z'; \theta)},
\]

\[\text{ TABLE I: The expressions of Hubble parameter } H(z), \text{ deceleration parameter } q(z) \text{ and jerk parameter } j(z) \text{ for two models.}\]

| \( H(z) \) | \( q(z) \) | \( j(z) \) |
|------------------|------------------|------------------|
| \( H_0(1+z)^{1+q_0+q_1} \exp(\frac{q_1}{1+z}) \) | \( q_0 + \frac{q_1}{1+z} \) | \( -q_0 - \frac{q_1}{1+z} - 2(q_0 + \frac{q_1}{1+z}) - (1+z)[\frac{q_1}{(1+z)^2} - \frac{q_0}{1+z}] \) |
| \( H_0(1+z)^2 \exp(\frac{2q_0+q_1+q_2}{1+z}) \) | \( \frac{1}{2} + \frac{q_1}{1+z} \) | \( -\frac{q_1}{1+z} - 2(\frac{q_1}{1+z} + \frac{q_2}{1+z})^2 - (1+z)[\frac{q_1}{(1+z)^2} - \frac{2(q_1+q_2)}{(1+z)^3}] \) |

\[\text{ The three-epoch model [22] is a alternative scenario for the model } q(z) = q_0 + q_1 z, \text{ but the problem is that this function } q(z) \text{ is not smooth [21].}\]
\[ \mu_0 = 5 \log_{10}(\frac{H_0^{-1}}{Mpc}) + 25 = 42.38 - 5 \log_{10} h. \]  

(9)

It should be noted that \( \mu_0 \) is independent of the data and the dataset, though it is a nuisance parameter. By expanding the \( \chi^2 \) of Eq. (6) relative to \( \mu_0 \), the minimization with respect to \( \mu_0 \) can be made trivially \[25\][26]

\[ \chi^2_{SNIa}(\theta) = A(\theta) - 2\mu_0 B(\theta) + \mu_0^2 C, \]

(10)

where

\[ A(\theta) = \sum_{i=1}^{N} \frac{[\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0, \theta)]^2}{\sigma_i^2}, \]

(11)

\[ B(\theta) = \sum_{i=1}^{N} \frac{\mu_{obs}(z_i) - \mu_{th}(z_i; \mu_0 = 0, \theta)}{\sigma_i^2}, \]

(12)

\[ C = \sum_{i=1}^{N} \frac{1}{\sigma_i^2}, \]

(13)

Evidently, Eq. (6) has a minimum for \( \mu_0 = B/C \) at

\[ \tilde{\chi}^2_{SNIa}(\theta) = A(\theta) - B(\theta)^2/C. \]

(14)

Since \( \chi^2_{SNIa,min} = \chi^2_{SNIa, min} \) and \( \tilde{\chi}^2_{SNIa} \) is independent of nuisance parameter \( \mu_0 \), here we utilize expression (14) to displace (6) for SNIa constraint. Alternatively, one can also perform a uniform marginalization over the nuisance parameter \( \mu_0 \) thus obtaining \[27\][29]

\[ \chi^2_{SNIa}(\theta) = A(\theta) - \frac{B(\theta)^2}{C} + \ln(\frac{C}{2\pi}). \]

(15)

Comparing Eq. (14) with Eq. (15), it can be seen that two \( \chi^2 \)'s are equivalent for using them to constrain cosmological models, since they are only different from a constant term \( \ln(\frac{C}{2\pi}) \), i.e. if one marginalize over all values of \( \mu_0 \), as in Ref. [30], that would just add a constant and would not change the constraint results.

Recently, Stern et al obtained the Hubble parameter \( H(z) \) at 12 different redshifts from the differential ages of passively evolving galaxies \[31\]. And in Ref. \[32\], authors obtained \( H(z = 0.24) = 79.69 \pm 2.32, H(z = 0.34) = 83.8 \pm 2.96, \) and \( H(z = 0.43) = 86.45 \pm 3.27 \) by taking the BAO scale as a standard ruler in the radial direction. Using these data we can constrain cosmological models by minimizing

\[ \chi^2_{Hub}(H_0, \theta) = \sum_{i=1}^{N} \frac{[H_{th}(z_i) - H_{obs}(H_0, \theta, z_i)]^2}{\sigma_{obs;i}^2}, \]

(16)

where \( H_{th} \) is the predicted value for the Hubble parameter, \( H_{obs} \) is the observed value, \( \sigma_{obs;i}^2 \) is the standard deviation measurement uncertainty. Here the nuisance parameter \( H_0 \) is marginalized in the following calculation with a Gaussian prior, \( H_0 = 74.2 \pm 3.6 \) km s\(^{-1}\) Mpc\(^{-1}\) \[32\].

The likelihood function is written as \( L \propto e^{-\chi^2/2} \), and the total \( \chi^2 \) equals,

\[ \chi^2_{total} = \chi^2_{SNIa} + \chi^2_{Hub}, \]

(17)
FIG. 1: The best-fit evolutions of $q(z)$ with 1σ confidence level constrained from 557 SNIa Union2 dataset and 15 observational Hubble data.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
model & $\chi^2_{\text{min}}$ & $\chi^2_{\text{min}}$/dof & $q_0$ (1σ) & $j_0$ (1σ) & $z_T$ (1σ) \\
\hline
$M_1$ & 554.335 (478.060) & 0.973(1.163) & -0.701$^{+0.089}_{-0.089}$ ($-0.653^{+0.092}_{-0.093}$) & -2.000$^{+0.443}_{-0.442}$ ($-1.823^{+0.456}_{-0.457}$) & 0.689$^{+0.227}_{-0.117}$ (0.674$^{+0.257}_{-0.124}$) \\
$M_2$ & 554.560 (478.334) & 0.973(1.167) & -0.749$^{+0.103}_{-0.103}$ ($-0.698^{+0.106}_{-0.106}$) & -2.619$^{+0.602}_{-0.602}$ ($-2.386^{+0.619}_{-0.620}$) & 0.687$^{+0.198}_{-0.121}$ (0.667$^{+0.215}_{-0.126}$) \\
$\Lambda$CDM & 557.008 (479.283) & 0.975(1.166) & -0.598$^{+0.028}_{-0.027}$ ($-0.575^{+0.030}_{-0.030}$) & -1 & 0.761$^{+0.055}_{-0.055}$ (0.716$^{+0.056}_{-0.056}$) \\
\hline
\end{tabular}
\caption{The values of the current deceleration parameter $q_0$, jerk parameter $j_0$, and transition redshift $z_T$ against the model, obtained by using 557 SNIa Union2 data and 15 observational Hubble data (the numerical results in brackets correspond to the constraints from 397 SNIa Constitution data and 15 observational Hubble data).}
\end{table}

FIG. 2: The best-fit evolutions of $j(z)$ with 1σ confidence level constrained from 557 SNIa Union2 dataset and 15 observational Hubble data.

where $\chi^2_{\text{SNIa}}$ and $\chi^2_{\text{Hub}}$ are the ones described in Eq. (14) and Eq. (16), respectively. It is easy to see that the matter density $\Omega_m$ are not contained explicitly in the $\chi^2_{\text{total}}$. Then, the constraint results may not depend on the dynamic variables $\Omega_m$, and gravitation theory.

The evolutions of two kinematical parameters, $q(z)$ and $j(z)$, are plotted in Fig. 1 and 2 for two models, respectively. According to these figures, constraint results of cosmic parameters are obtained and shown in table II with the constraints on model parameters shown in table III. Relative to the evolutions of deceleration parameters $q(z)$ it can be seen that the constraints on jerk parameters $j(z)$, defined by the third derivative with respect to scale factor $a$, are
The values of model parameters (1σ)

| model | The values of model parameters (1σ) |
|-------|-------------------------------------|
| $M_1$ | $q_0 = -0.701^{+0.089}_{-0.089}$ $(q_0 = -0.653^{+0.092}_{-0.093})$, $q_1 = 1.718^{+0.475}_{-0.504}$ $(q_1 = 1.622^{+0.493}_{-0.529})$ |
| $M_2$ | $q_1 = -0.253^{+0.653}_{-0.673}$ $(q_1 = -0.286^{+0.692}_{-0.714})$, $q_2 = -1.249^{+0.106}_{-0.158}$ $(q_2 = -1.198^{+0.165}_{-0.162})$ |
| ΛCDM | $\Omega_{\Lambda m} = 0.268^{+0.019}_{-0.018}$ $(\Omega_{\Lambda m} = 0.283^{+0.021}_{-0.021})$ |

TABLE III: The 1σ confidence level of model parameters for the models: $q(z) = q_0 + \frac{q_1}{1+z}$, $q(z) = \frac{q_1}{1+z}$, and ΛCDM model by using the 557 SNIa Union2 data and 15 observational Hubble data (the numerical results in brackets correspond to the constraints from 397 SNIa Constitution data and 15 observational Hubble data).

weaker for these two models from the distance-measurement data. Furthermore, in Fig. 1 as a contrast we also plot the evolution of deceleration parameter $q(z)$ for ΛCDM model by the current geometry-distance observed data (and for this model, jerk parameter $j(z) = -1$ almost). From table III it is shown that for the dynamical ΛCDM model, the constraint results tend to favor the bigger values of current deceleration parameter $q_0$ and transition redshift $z_T$ relative to the cases of two kinematical models.

4. Conclusions

In this paper, kinematic models are constrained by the latest observational data: 557 SNIa Union2 dataset and 15 observational Hubble data. Generally, the expansion rhythm of current universe $q_0$ and transitional time from decelerated expansion to accelerated expansion $z_T$, depends on the parameterized form of kinematical equations. Here we consider two parameterized deceleration parameter. The best fit values of $z_T$, $q_0$ and $j_0$ with 1σ confidence level are obtained. From table III we can see that the values of $z_T$ indicated by these two models much approach each other. From Fig. 1 and 2 it can be seen that for the two kinematical models, the constraints on jerk parameters $j(z)$ are weak by the current observed data. In addition, we also can see the deviation of jerk parameter from $j = -1$ according to the Fig. 2, with measuring the departures for kinematical models from ΛCDM model. Furthermore, considering Refs. [14][19][34], where most models indicate that current data favors $j_0 < -1$ case, which is consistent with our results. At last, for comparing the differences of constraint results on cosmic parameters between the different SNIa data, we also consider the case of displacing the 557 SNIa Union2 data[23] with 397 SNIa Constitution data[36] in the above combined constraints, and the latter constraint results are listed in brackets in table III. According to table III it seems that the constraint results favor a bigger value of current deceleration parameter $q_0$ and jerk parameter $j_0$, and a smaller value of transition redshift $z_T$.

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3 The SNIa Union2 data are obtained, by adding new datapoints (including the high redshift SNIa) to the SNIa Union[35] data, making a number of refinements to the Union analysis chain, refitting all light curves with the SALT2 fitter.

4 The 397 Constitution data are obtained by adding 90 SNIa from CfA3 sample to 307 SNIa Union sample[35]. CfA3 sample are all from the low-redshift SNIa, $z < 0.08$, and these 90 SNIa are calculated with using the same Union cuts.
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