Topology of Equivalent Unconstrained Systems in QCD

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Abstract

We consider the derivation of equivalent unconstrained systems for QCD given in the class of functions of nontrivial topological gauge transformations. We show that the unconstrained QCD obtained by resolving the Gauss law constraint contains a monopole, a zero mode of the Gauss law, and a rising potential, which can explain the phenomena of confinement and hadronization as well as spontaneous chiral symmetry breaking and the $\eta-\eta'$-mass difference.

(Key-words: QCD, Gauss law, topology, monopole, zero mode, hadronization, confinement, U(1)-problem)

1 Introduction

In this contribution I discuss the construction of gauge-invariant physical variables in QCD being adequate for the low energy nonperturbative region. The validity of the standard Faddeev-Popov (FP) integral \[1, 2\] for arbitrary gauges in non-Abelian theory has been proved by Faddeev \[3\] only for scattering amplitudes of degrees of freedom which are not observable in QCD. This proof is therefore not adequate for description of the phenomena of confinement and hadronization. The FP integral has been obtained from the Feynman path integral by construction of the non-Abelian equivalent unconstrained system in the class of functions of standard perturbation theory.

In the present paper we discuss the derivation of equivalent unconstrained systems for QCD in the class of functions of topologically nontrivial transformations, in order to demonstrate that the physical phenomena of hadronization and confinement in QCD can be explained by the explicit solutions of the non-Abelian constraints in this class of functions.

Before we consider our main idea for QCD, we illustrate it on the example of the equivalent unconstrained system derived for QED by Heisenberg and Pauli \[4\], and Dirac \[5\].

2 Equivalent Unconstrained Systems in QED

Let us consider the action for the Abelian theory

$$W = \int d^4x \left\{ -\frac{1}{4}G_{\mu\nu}^2 - A^{\mu}J^{\nu} \right\}. \quad (1)$$
This action contains superfluous (nonphysical) degrees of freedom and is not compatible with the simplest variational methods developed in the framework of the Newtonian mechanics.

To describe the gauge-invariant dynamics, Heisenberg and Pauli [4], and Dirac [5] have considered the equivalent system obtained by resolving the Gauss law constraint. Recall that all peculiarities of the application of classical variational methods, including the initial value problem, the spatial boundary conditions, the time evolution, the classification of constraints, and the equations of motion in the Hamiltonian approach, are defined only in a definite frame of reference distinguished by the time axis.

The Gauss law constraint in the frame of reference with an axis of time $l^{(0)}_{\mu} = (1, 0, 0, 0)$ is the equation for the time component of a gauge field

$$\frac{\delta W}{\delta A_0} = 0 \Rightarrow \partial_j^2 A_0 = \partial_k A_k + J^*_0.$$  \hspace{1cm} (2)

As it was shown in Refs. [4, 5] (see also [6]) the substitution of the solution of this constraint into the initial action (1) leads to the equivalent unconstrained system

$$W^{\ast}_{l_0}(A^*, J^*) = \int d^4x \left\{ -\frac{1}{2} (\partial_\mu A^*_k)^2 - A^*_i J^*_i - \frac{1}{2} J^*_0 \left( \frac{1}{\partial_j^2} J^*_0 \right) \right\}.$$  \hspace{1cm} (3)

By this way, in QED, one could obtain the electrostatic Coulomb interaction of sources $J^*_0$ in the monopole class of functions ($f(\vec{x}) = O(1/r), |\vec{x}| = r \to \infty$) and two transversal photons constructed by Dirac [5] as "dressed" variables $A^*$ in the explicit form

$$ieA^*_k = U(A)(ieA_k + \partial_k)U(A)^{-1}, \quad U(A) = \exp[ie \frac{1}{\partial_j^2} \partial_k A_k].$$  \hspace{1cm} (4)

The equivalent unconstrained system can be quantized by the Feynman path integral in the form

$$Z_F[l^{(0)}, J^*] = \int d^2A^* \exp \left\{ iW^{*}_{l_0}(A^*, J^*) \right\}.$$  \hspace{1cm} (5)

This path integral depends on the axis of time $l^{(0)}_{\mu} = (1, 0, 0, 0)$.

The relativistic covariance of the equivalent unconstrained system in QED has been predicted by von Neumann [4] and proven by Zumino [7] on the level of the algebra of generators of the Poincare group. In particular, a moving relativistic atom in QED is described by the usual boost procedure for the wave function, which corresponds to a change of the time axis $l^{(0)} \to l$, i.e., motion of the Coulomb potential [8] itself

$$W_C = \int d^4x d^4y \frac{1}{2} J^*_l(x) V_C(z^\perp) J^*_l(y) \delta(l \cdot z) \quad (\partial^2_k V_C(\vec{x}) = -\delta^3(\vec{x})), $$  \hspace{1cm} (6)

where $J^*_l = l\mu J^*_\mu$, $z^\perp = z_\mu - l_\mu (z \cdot l)$, $z_\mu = (x - y)_\mu$. A time axis is chosen to be parallel to the total momentum of a considered state. In particular, for bound states this choice means that the coordinate of the potential coincides with the space of the relative coordinates of the bound state wave function in the accordance with the Yukawa-Markov principle [9] and the Eddington concept of simultaneity (“yesterday’s electron and today’s proton do not make an atom”) [10]. In this case, we get the relativistic covariant dispersion law and invariant mass spectrum. The relativistic generalization of the Coulomb potential is not only the change of the form of the potential, but also the change of a direction of its motion in four-dimensional
space to lie along the total momentum of the bound state. The relativistic covariant unitary perturbation theory in terms of such the relativistic instantaneous bound states has been constructed in [8]. In this perturbation theory, each instantaneous bound state in QED has a proper equivalent unconstrained system. The manifold of frames corresponds to the manifold of ”equivalent unconstrained systems”. The relativistic invariance means that a complete set of physical states for any equivalent system coincides with the one for another equivalent system (this treatment belongs to Schwinger, see [11]).

This treatment of the relativistic invariance and covariance is confused with the naive understanding of the relativistic invariance as independence on the time-axis of the functional integral.

One supposes that the dependence on the frame \( l^0 \) can be removed by the transition from the Feynman integral (5) to perturbation theory in any relativistic-invariant gauge \( f(A) = 0 \) with the FP determinant

\[
Z_{FP}[J] = \int d^4A \delta[f(A)] \Delta_{FP} \exp \left\{ iW[A, J] \right\}.
\]

This transition is well-known as a ”change of gauge”, and it is fulfilled in two steps

I) the change of the variables \( A^* \), and

II) the change of the physical sources \( J^* \) of the type of

\[
A^*_k(A)J^*_k = U(A) \left( A_k - \frac{i}{e} \partial_k \right) U^{-1}(A)J^*_k \Rightarrow A_kJ^k.
\]

At the first step, all electrostatic monopole physical phenomena are concentrated in the Dirac gauge factor \( U(A) \) that accompanies the physical sources \( J^* \).

At the second step, changing the sources (8) we lose the Dirac factor together with the whole class of electrostatic phenomena including the Coulomb-like instantaneous bound state formed by the electrostatic interaction.

Really, the FP perturbation theory in the relativistic gauge contains only photon propagators with the light-cone singularities forming the Wick-Cutkosky bound states with the spectrum differing \(^1\) from the observed one which corresponds to the instantaneous Coulomb interaction. Thus, the restoration of the “explicit” relativistic form of the equivalent unconstrained system \( l^0 \) by the transition to a relativistic gauge loses all electrostatic ”monopole physics” with the Coulomb bound states.

3 Unconstrained QCD

3.1 Topological degeneration and class of functions

We consider the non-Abelian \( SU_c(3) \) theory with the action functional

\[
W = \int d^4x \left\{ \frac{1}{2} \left( G_{0i}^a \right)^2 - B_i^a \right\} + \bar{\psi}i\gamma^\mu \left( \partial_\mu + A_\mu \right) - m \psi \right\},
\]

where \( \psi \) and \( \bar{\psi} \) are the fermionic quark fields. We use the conventional notation for the non-Abelian electric and magnetic fields

\[
G_{0i}^a = \partial_0 A_i^a - D_i^{ab}(A) A_i^b, \quad B_i^a = \epsilon_{ijk} \left( \partial_j A_k^a + \frac{g}{2} f^{abc} A_j^b A_k^c \right).
\]

\(^1\)The author thanks W. Kummer who pointed out that in Ref. [12] the difference between the Coulomb atom and the Wick-Cutkosky bound states in QED has been demonstrated.
as well as the covariant derivative \( D_i^{ab}(A) := \delta^{ab}\partial_i + g f^{acb} A_c^i \).

The action (3) is invariant with respect to gauge transformations \( u(t, \bar{x}) \)

\[
\hat{A}_i^u := u(t, \bar{x}) \left( A_i + \partial_i \right) u^{-1}(t, \bar{x}), \quad \psi^u := u(t, \bar{x})\psi,
\]

(11)

where \( \hat{A}_\mu = g \lambda^a \lambda^a_\mu \).

It is well-known [2] that the initial data of all fields are degenerated with respect to the stationary gauge transformations \( u(t, \bar{x}) = v(\bar{x}) \). The group of these transformations represents the group of three-dimensional paths lying on the three-dimensional space of the \( SU_c(3) \)-manifold with the homotopy group \( \pi(3) (SU_c(3)) = Z \). The whole group of stationary gauge transformations is split into topological classes marked by the integer number \( n \) (the degree of the map) which counts how many times a three-dimensional path turns around the \( SU(3) \)-manifold when the coordinate \( x_i \) runs over the space where it is defined. The stationary transformations \( v^n(\bar{x}) \) with \( n = 0 \) are called the small ones; and those with \( n \neq 0 \)

\[
\hat{A}_i^{(n)} := v^n(\bar{x}) \hat{A}_i(\bar{x}) v^n(\bar{x})^{-1} + L_i^n, \quad L_i^n = v^n(\bar{x}) \partial_i v^n(\bar{x})^{-1},
\]

(12)

the large ones.

The degree of a map

\[
N[n] = -\frac{1}{24\pi^2} \int d^3 x \epsilon^{ijk} Tr[L_i^n L_j^n L_k^n] = n .
\]

(13)

as the condition for normalization means that the large transformations are given in the class of functions with the spatial asymptotics \( O(1/r) \). Such a function \( L_i^n \) (12) is given by

\[
v^n(\bar{x}) = \exp(n\hat{\Phi}_0(\bar{x})), \quad \hat{\Phi}_0 = -i\pi \frac{\lambda^a_A x^a}{r} f_0(r),
\]

(14)

where the antisymmetric \( SU(3) \) matrices are denoted by

\[
\lambda^1_A := \lambda^2, \quad \lambda^2_A := \lambda^5, \quad \lambda^3_A := \lambda^7,
\]

and \( r = |\bar{x}| \). The function \( f_0(r) \) satisfies the boundary conditions

\[
f_0(0) = 0, \quad f_0(\infty) = 1,
\]

(15)

so that the functions \( L_i^n \) disappear at spatial infinity \( \sim O(1/r) \). The functions \( L_i^n \) belong to monopole-type class of functions. It is evident that the transformed physical fields \( A_i \) in (12) should be given in the same class of functions \( \hat{\Phi}_i(\bar{x}) = O(1/r) \)

\[
A_i^c(t, \bar{x}) = \Phi_i^c(\bar{x}) + \bar{A}_i^c(t, \bar{x}),
\]

(16)

where \( \bar{A}_i \) is a weak perturbative part with the asymptotics at the spatial infinity

\[
\bar{A}_i(t, \bar{x})|_{\text{asymptotics}} = O\left( \frac{1}{r^{l+\epsilon}} \right) \quad (l > 1).
\]

(17)

We restrict ourselves to ordinary perturbation theory around a static monopole \( \Phi_i(\bar{x}) \), and use, as an example, the Wu-Yang monopole [13, 14]

\[
\Phi_i^{WY} = -i \frac{\lambda^a_A}{2} \epsilon_{iak} \frac{x^k}{r^2} f_1^{WY}, \quad f_1^{WY} = 1
\]

(18)
which is a solution of classical equations everywhere besides the origin of coordinates. To remove a singularity at the origin of coordinates and regularize its energy, the Wu-Yang monopole is changed by the Bogomol’nyi-Prasad-Sommerfield (BPS) monopole \(^{15}\)

\[
f^{WY}_i \Rightarrow f^{BPS}_i = \left[ 1 - \frac{r}{\epsilon \sinh(r/\epsilon)} \right], \quad \int d^3x [B^{a}_i(\Phi_k)]^2 = \frac{4\pi}{g^2\epsilon}, \tag{19}\]

to take the limit of zero size \(\epsilon \to 0\) at the end of the calculation of spectra and matrix elements.

The statement of the problem is to construct an equivalent unconstrained system for the non-Abelian fields in this monopole-type class of functions.

### 3.2 The Gauss Law Constraint

An equivalent unconstrained system is obtained by resolving the non-Abelian Gauss law constraint

\[
\frac{\delta W}{\delta A_0} = 0 \quad \Rightarrow \quad (D^2(A))^{ac} A_0^c = D_i^{ac}(A) \partial_0 A_i^c + j_a^c, \tag{20}\]

where \(j_a^c = g\bar{\psi} \frac{\lambda_a}{2} \gamma_\mu \psi\) is the quark current.

In lowest order of the perturbation theory this constraint takes the form

\[
(D_j^2(\Phi))^{ac} A_0^c = D_i^{ac}(\Phi) \partial_0 A_i^c, \tag{21}\]

In the considered case of BPS-monopole \(^{13}\), there is the zero mode of the covariant Laplace operator in the monopole field

\[
(D^2)_{ab}^{\text{BPS}}(\Phi^{\text{BPS}}_k(\Phi^{\text{BPS}}_0)(\vec{x})) = 0. \tag{22}\]

The nontrivial solution of this equation is well-known \(^{13}\); it is given by equation \(^{14}\) where

\[
f^{\text{BPS}}_0 = \left[ \frac{1}{\tanh(r/\epsilon)} - \frac{\epsilon}{r} \right] \tag{23}\]

has the boundary conditions \(^{15}\) of a phase of the topological transformations \(^{14}\).

This zero mode is a perturbative form of a zero mode of the Gauss law constraint \(^{20}\) \(^{6}\) \(^{16}\) \(^{17}\) as the solution \(Z^a\) of the homogeneous equation \(^{24}\)

\[
(D^2(A))^{ab} Z^b = 0, \tag{24}\]

with the asymptotics at the space infinity

\[
\hat{Z}(t, \vec{x})|_{\text{asymptotics}} = \hat{N}(t) \hat{\Phi}_0(\vec{x}), \tag{25}\]

where \(\hat{N}(t)\) is the global variable of an excitation of the gluon system as a whole. From the mathematical point of view, the zero mode means that the general solution of the inhomogeneous equation \(^{20}\) for the time-like component \(A_0\) is a sum of the homogeneous equation \(^{24}\) and a particular solution \(\hat{A}_0\) of the inhomogeneous one \(^{27}\): \(A_0 = Z^a + \hat{A}_0\).

The zero mode of the Gauss constraint and the topological variable \(N(t)\) allow us to remove the topological degeneration of all fields by the non-Abelian generalization of the Dirac dressed variables \(^{1}\)

\[
0 = U_Z(\hat{Z} + \partial_0)U_Z^{-1}, \quad \hat{A}_i^* = U_Z(\hat{A}_i^t + \partial_t)U_Z^{-1}, \quad \psi^* = U_Z \psi^t, \tag{26}\]
where the spatial asymptotics of $U_Z$ is

$$U_Z = T \exp\left[ \int_0^t dt' \hat{Z}(t', \vec{x}) \right]_{\text{asymptotics}} = \exp[N(t)\hat{\Phi}_0(\vec{x})] = U^{(N)}_{as},$$

(27)

and $A^I = \Phi + \bar{A}, \psi^I$ are the degeneration free variables. In this case, the topological degeneration of all color fields converts into the degeneration of only one global topological variable $N(t)$ with respect to a shift of this variable on integers: $(N \Rightarrow N + n, n = \pm 1, \pm 2, ...)$.

One can check $[18]$ that the Pontryagin index for the Dirac variables $(26)$ with the asymptotics $(17), (25), (27)$ is determined by only the difference of the final and initial values of the topological variable

$$\nu[A^*] = \frac{g^2}{16\pi^2} \int_{t_{in}}^{t_{out}} dt \int d^3 x G_{\mu\nu}^a G^{a\mu\nu} = N(t_{out}) - N(t_{in}).$$

(28)

Thus, we can identify the global variable $N(t)$ with the winding number degree of freedom in the Minkowski space. This degree of freedom plays the role of the Goldstone mode that removes the topological degeneration of initial data.

As we shall see below, the vacuum wave function of the topological free motion in terms of the Pontryagin index $(28)$ takes the form of a plane wave $\exp(iP_N \nu[A^*])$. The well-known instanton wave function $[19]$ appears for nonphysical values of the topological momentum $P_N = \pm i8\pi^2/g^2$. This fact points out that instantons are permanently tunnelling and correspond to quantum nonphysical solutions with the zero energy in Euclidean spacetime of the type of nonphysical ones for an oscillator $(\hat{p}^2 + q^2)\psi_0 = 0$.

### 3.3 Physical Consequences

The dynamics of physical variables including the topological one is determined by the constraint-shell action of an equivalent unconstrained system (EUS) as a sum of the zero mode part, and the monopole and perturbative ones

$$W^{(0)}_{\text{l}} = W_{\text{Gauss–shell}} = W_Z[N] + W_{\text{mon}}[\Phi_I] + W_{\text{loc}}[\bar{A}].$$

(29)

The action for an equivalent unconstrained system $(29)$ in the gauge $(35)$ with a monopole and a zero mode has been obtained in the paper $[18]$ following the paper $[3]$. This action contains the dynamics of the topological variable in the form of a free rotator

$$W_Z = \int dt \frac{\dot{N}^2 I}{2} ; \quad I = \int_V d^3 x (D_i^{ac}(\Phi_k)\Phi_0^c)^2 = \frac{4\pi}{g^2}(2\pi)^2 \epsilon,$$

(30)

where $\epsilon$ is a size of the BPS monopole considered as a parameter of the infrared regularization which disappears in the infinite volume limit. The dependence of $\epsilon$ on volume can be chosen as $\epsilon \sim V^{-1}$, so that the density of energy was finite.

The perturbation theory in the sector of local excitations $W_{\text{loc}}[\bar{A}]$ is based on the Green function as the inverse differential operator of the Gauss law

$$[D^2(\Phi)]^{ac} V_{cb}(x, y) = -\delta^3(x - y)\delta^{ab}$$

(31)

$^2$ The author is grateful to V.N. Gribov for the discussion of the problem of instantons during a visit in Budapest, May 1996.
which is the non-Abelian generalization of the Coulomb potential. As it has been shown in [18], the non-Abelian Green function (31) in the field of the Wu-Yang monopole is the sum of a Coulomb-type potential and a rising one. This means that the instantaneous quark-quark interaction leads to spontaneous chiral symmetry breaking [8, 20], goldstone mesonic bound states [8], glueballs [20, 21], and the Gribov modification of the asymptotic freedom formula [21]. If we choose a time-axis $l(0)$ along the total momentum of bound states [8] (this choice is compatible with the experience of QED in the description of instantaneous bound states), we get the bilocal generalization of the chiral Lagrangian-type mesonic interactions [8]. In this case, the U(1) anomalous interaction of $\eta_0$-meson with the topological variable [18] lead to additional mass of this isoscalar meson.

All these results can be described by the Feynman path integral for the obtained unconstrained system in the class of functions of the topological transformations (see [18])

$$Z_F[l(0), J^{a*}] = \int DN(t) \int \prod_{c=1}^{c=8} [d^2 A^{c*} d^2 E^{c*}]$$

$$\times \exp \left\{ iW_{l(0)}^*[A^*, E^*] + i \int d^4x [J^c_{\mu} A^{c*}_{\mu}] \right\} , \quad (32)$$

where $J^{c*}$ are physical sources.

The nonperturbative phase factors of the topological degeneration can lead to a complete destructive interference of color amplitudes [6, 17, 22] due to averaging over all parameters of the degenerations, in particular

$$<1|\psi^*|0> = <1|\psi^I|0> \lim_{L \to \infty} \frac{1}{2L} \sum_{n=-L}^{n=L} U^{(n)}_{as}(x) = 0 . \quad (34)$$

This mechanism of confinement due to the interference of phase factors (revealed by the explicit resolving the Gauss law constraint [17]) disappears after the change of ”physical” sources $A^*J^* \Rightarrow AJ$ that is called the transition to another gauge. In the lowest order of perturbation theory the Gauss law constraint for the degeneration free variables is compatible with only one gauge. It is the Coulomb-type gauge in the monopole field

$$D_{k}^{ac}(\Phi)\tilde{A}_{k}^{c} = 0 . \quad (35)$$

The change of variables $A^*$ of the type of (4) with the non-Abelian Dirac factor

$$U(A) = U_{Z} \exp \left\{ \frac{1}{D^2(\Phi)} D_{j}(\Phi)\hat{A}_{j} \right\} \quad (36)$$

and the change of the Dirac sources $J^*$ can remove all monopole physics, including confinement and hadronization, like similar changes (4), (8) in QED (to get a relativistic form of the Feynman path integral) remove all electrostatic phenomena in the relativistic gauges.
The transition to another gauge faces the problem of zero of the FP determinant $detD^2(\Phi)$ (i.e. the Gribov ambiguity $\mathcal{P}$ of the gauge (35)). It is the zero mode of the second class constraint. The considered example (32) shows that the Gribov ambiguity (being simultaneously the zero mode of the first class constraint) cannot be removed by the change of gauge as the zero mode is the inexorable consequence of internal dynamics, like the Coulomb field in QED. Both the zero mode, in QCD, and the Coulomb field, in QED, have nontrivial physical consequences discussed above, which can be lost by the standard gauge-fixing scheme.

4 Conclusion

There are “admissible” gauges for which the operations of varying and constraining commute [3, 5, 24]. This commutativity allows us to construct an equivalent unconstrained system (compatible with the Feynman path integral) directly in terms of gauge-invariant variables by using the substitution of a solution of the Gauss law constraint into the initial singular action.

In this paper we reproduced the Faddeev derivation of the non-Abelian unconstrained system in another class of functions. This class of functions is unambiguously defined by the normalization of nontrivial topological gauge transformations and contains a monopole and a zero mode of the Gauss law constraint.

We have shown that ”equivalent unconstrained systems” in QCD in the class of functions of nontrivial topological transformations contains a monopole and a zero mode of the Gauss law constraint. The monopole forms the rising potential of hadronization of color quarks and gluons. And the zero mode forms the phase factors of the topological degeneration and additional mass of the $\eta_0$-meson.

There is only one gauge for which the operations of varying and constraining commute. It is not Coulomb one, but its the covariant generalization in the presence of a monopole.

If we pass to another gauges on the level of the FP integral in relativistic gauges, all these monopole phenomena are lost.

Recall that Faddeev proved the equivalence of the Feynman integral to the Faddeev-Popov integral in an arbitrary gauge for the scattering amplitudes [2] only, i.e. when all particle-like excitations of the fields are on their mass-shell. However, for the cases of bound states in QED and QCD and other collective phenomena where these fields are off their mass-shell this equivalence has not been proved and might not exist.

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