Layered semi-quantum secure communication protocols

Rajni Bala, Sooryansh Asthana and V. Ravishankar

*Department of Physics, IIT Delhi, Hauz Khas, New Delhi, 110016, Delhi, India.

*Corresponding author(s). E-mail(s):
Rajni.Bala@physics.iitd.ac.in;
Contributing authors: sooryansh.asthana@physics.iitd.ac.in;
vravi@physics.iitd.ac.in;

Abstract
In this paper, we harness the potential offered by multidimensional states in secure communication with only one quantum participant. We propose four protocols for–(i) layered semi–quantum key distribution (LSQKD), (ii) layered semi–quantum secret sharing (LSQSS), (iii) integrated layered semi–quantum key distribution and secret sharing (ILSQKD+SS), and, (iv) integrated layered semi–quantum secure direct communication and key distribution (ILSQSDC+KD) to share secret information in an arbitrary layered network. All the four protocols allow for simultaneous distribution of secure information in all the layers of a network, thanks to multidimensional states.

Keywords: quantum network, semi–quantum cryptography, layered quantum cryptography, quantum secret sharing, quantum secure direct communication

1 Introduction
In recent times, there is an upsurge of interest in the study of quantum communication protocols [1–11]. Almost all the endeavours in this realm have been driven by applications of distinctive features of quantum mechanics to perform certain tasks. In particular, one of the key applications– quantum key distribution (QKD)– owes its security to the no–cloning theorem [12]. The first seminal
Layered semi–quantum communication protocols

A protocol for QKD was put forth by Bennett and Brassard [1]. Thereafter, it has been a vibrant area of research [2, 5, 11–13]. In addition to QKD, many other communication protocols such as quantum secret sharing (QSS), quantum secure direct communication (QSDC), quantum key agreement, etc. have also been advanced [3, 16–24].

Along the sidelines, there is one question underlying all the quantum algorithms: what are the minimal quantum resources needed for a particular task? In the context of QKD, which is of primal importance in this paper, this question was asked in 2007 by Boyer et al. This question has culminated in the proposal of a semi–QKD (SQKD) protocol [4]. In this protocol, only one of the two parties had access to quantum resources. Subsequently, SQKD has been generalised to both– higher dimensional quantum systems and higher number of parties (see [17] and references therein). The versatility of semi–quantum communication protocols is reflected in many proposals including semi–QSS (SQSS), semi–QSDC (SQSDC), semi–quantum key agreement, etc. [6, 9, 18, 25–27].

Moving on to more realistic scenarios, recently, study of distribution of quantum information over a network is a thriving field [28–33]. In fact, distribution of quantum information over a quantum network generates a large number of novel scenarios. For example, all the nodes of a network may not have equal preference; one node might belong to more than one layer, or, there may be a need for collaboration among different nodes for retrieval of information. This constitutes a rich area of research [19, 34–37]. In fact, both QKD and QSS protocols to distribute secret information in a network have been proposed by employing layered entangled states [38, 39]. Such states have already been experimentally realised in the orbital angular momentum degree of freedom [40, 41]. In these protocols, all the participants are assumed to have access to quantum resources. Nevertheless, in a realistic situation, some of the participants may be constrained to perform measurements only in one of the bases and so are termed as ‘classical’, following [4]. The participants, who are endowed with the capabilities of preparing a quantum state and measuring it in more than one basis, are termed as ‘quantum participants’.

These observations prompt us to ask the following question: is secure distribution of information in a layered network possible with classical participants? In this paper, we show that the answer to this question is in the affirmative and is embedded in the integration of semi-quantum communication protocols and layered quantum communication protocols. Essentially, these next–generation semi–quantum communication protocols constitute the thrust of this paper.

In this paper, we have proposed four protocols for distribution of secret information in a quantum network having both–honest and dishonest participants. The details of these protocols and nature of participants are presented in the table (1). Dishonest participants require a collaboration for retrieval of information. The last protocol is unique in that it is the integration of both SQKD and SQSDC protocols. Thanks to multidimensional states, secure information can be distributed in all the layers of a network in one go.
Table 1 Proposed protocols, nature of participants and requirement of collaboration

| S. No. | Protocol                                                                 | State employed               | Nature of participants | Requirement of collaboration |
|--------|---------------------------------------------------------------------------|------------------------------|------------------------|----------------------------|
| 1(a)   | Layered semi–quantum key distribution (LSQKD) – Protocol I                | two-qubit Bell and a three-qubit GHZ state | Honest                 | No                         |
| 1(b)   | LSQKD–Protocol II                                                        | entangled multiqubit         | Honest                 | No                         |
| 2      | Layered–SQSS (LSQSS)                                                     | separable multiqubit         | Dishonest              | Yes                        |
| 3      | Integrated layered semi–quantum key distribution and secret sharing (ILSQKD+SS) | entangled multiqubit         | Honest and dishonest   | only among dishonest participants |
| 4      | Integrated layered semi–quantum secure direct communication and key distribution (ILSQSDC+KD) | entangled multiqubit         | Honest                 | No                         |

The plan of the paper is as follows: in section (2), for the sake of completeness, layered entanglement is discussed. The central results of the paper are contained in sections (3-8). Section (3) presents the terminology that will be used throughout the paper. Section (4) proposes two LSQKD protocols, reflecting the importance of multiqubit layered entangled states. In section (5), a LSQSS protocol for a five–party network is proposed, followed by a discussion of its sifted key rate and confidentiality of the shared key. In section (6), an ILSQKD+SS protocol is proposed for a network of four participants, followed by a discussion of secrecy of the key in different layers. Section (7) proposes an ILSQSDC+KD protocol, that allows for direct communication in one layer whereas a key is distributed in the another layer, all at once, for a three–party network. Section (8) discusses generalisations of these protocols to share information in an arbitrary layered network. In section (9), security of the proposed protocols against a class of eavesdropping attacks is discussed. Section (10) concludes the paper with closing remarks.

2 Preliminaries

In this section, for the purpose of an uncluttered discussion, we briefly discuss layered entanglement [40] in pure states, which is a basic rudiment of all the protocols proposed in the paper, barring LSQSS. For a detailed discussion, please refer to [40, 42].

Layered entanglement: Multipartite higher–dimensional entangled states have a rich structure of entanglement. In these states, different subsystems
Layered semi–quantum communication protocols

may belong to Hilbert spaces of different dimensions. This variation in the
dimensionalities of subsystems owes to asymmetric nature of entanglement
amongst different parties. This feature is termed as ‘layered entanglement’. As
an example, consider the following tripartite entangled state,

\[ |\Psi\rangle \equiv \frac{1}{2}(|000\rangle + |111\rangle + |220\rangle + |331\rangle). \]  

The Schmidt ranks for the first, second and the third subsystems are 4, 4, 2 which may be arranged as a vector, (4, 4, 2), known as Schm"{i}dt vector of the state \(|\Psi\rangle\). Note that the Schmidt rank for each subsystem in a pure layered
entangled state is same as the local dimensionality of the subsystem.

3 General operations

In this section, we describe general operations to be used throughout the paper.

1. Quantum participant: In each protocol, there is only one quantum par-
ticipant, named Alice. She can prepare any state and perform measurements
in any basis. Alice has a dual role to play. In some protocols, she acts as
a participant of network itself, while in the others, she acts as an external
server who distributes information.

2. Classical participant: All the classical participants are named Bob_1, Bob_2, ··· and so on. Being classical, they can perform measurements only
in the computational basis.

3. Following [4], classical participants can perform only two operations– (i)
CTRL, and (ii) Reflect.

- CTRL operation: This corresponds to a classical participant performing
a measurement in the computational basis, noting down the result and
sending the post–measurement state back to Alice.

- Reflect operation: In the Reflect operation, a classical participant simply
resends the state to Alice.

Eavesdropping check: To check for the presence of an eavesdropper, Alice
performs a projective measurement, \(\Pi_\eta \equiv |\eta\rangle \langle \eta|\), where the state \(|\eta\rangle\) is
employed in the protocol to share secret information. In the rounds, in which
none of the participants perform CTRL operation, the state \(|\eta\rangle\) remains
unchanged. However, if Eve tampers with the state, in some of rounds, it
leads to a change in the state \(|\eta\rangle\). This change gets reflected in the projec-
tive measurement performed by Alice. Thus, the presence of eavesdropper is
detected.

We now proceed to present various protocols for distributing secret
information over a network.
4 Layered semi–quantum key distribution (LSQKD)

In this section, we propose two LSQKD protocols to distribute keys in two layers among three participants. The first layer consists of two participants Alice and Bob$_1$. The second layer consists of all the three participants, viz., Alice, Bob$_1$, and Bob$_2$. The two protocols differ in the resource states which are used to distribute keys, in the following way:

1. **Protocol I**: Employs two maximally entangled states—a two–qubit Bell state and a three–qubit GHZ state.
2. **Protocol II**: Employs a single three–qudit layered entangled state.

4.1 Protocol I

The first protocol is essentially equivalent to two standard SQKD protocols running in parallel to distribute two keys in a network.

Let the two states, 

\[ |\psi_1\rangle \equiv \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle), \quad |\psi_2\rangle \equiv \frac{1}{\sqrt{2}} (|000\rangle + |111\rangle), \tag{2} \]

be shared in the first and the second layer respectively. The steps of the protocols are as follows:

1. Alice prepares the states $|\psi_1\rangle$ and $|\psi_2\rangle$ and sends their subsystems to the respective participants.
2. The recipients perform either CTRL or Reflect operations randomly.
3. Alice, upon receiving the states, does one of the following two things:
   (a) She performs measurements in the computational basis, or,
   (b) She performs a projective measurement along $|\psi_1\rangle$ and $|\psi_2\rangle$ on the states in the first and the second layer respectively.
4. After a sufficient number of rounds, Bob$_1$ and Bob$_2$ reveal the rounds in which they have performed CTRL operations and Alice reveals the rounds in which she has performed measurements in the computational basis.
5. Alice analyses the data of those rounds in which none of the Bobs have performed CTRL operations to check for the presence of an eavesdropper.
6. The outcomes of those rounds, in which all the Bobs of a given layer have performed CTRL operations and Alice has measured in the computational basis, constitute a key in the given layer.

In this way, two keys are shared in the two layers. The sifted key rates for sharing keys in both the layers are 1 bit each. In what follows, we provide a protocol that distributes keys in the same structure but with only a single multidimensional state.
4.2 Protocol II

In this protocol, we show that the same task can be implemented more efficiently by employing a single multidimensional layered entangled state.

Let the state,

\[ |\Psi\rangle \equiv \frac{1}{2}(|000\rangle + |111\rangle + |220\rangle + |331\rangle) \],

be shared among Alice, Bob$_1$, and Bob$_2$. The first two subsystems are four-level systems and the third subsystem is a two-level system. The rationale underlying this choice of state has been explained in Appendix (A).

The steps of the protocol are as follows:

1. Alice prepares the state $|\Psi\rangle$ and sends the second and the third subsystem to Bob$_1$ and Bob$_2$ respectively.
2. Bob$_1$ and Bob$_2$, upon receiving the states, may perform either CTRL or Reflect operations randomly.
3. Alice, upon receiving the subsystems from both the Bobs, either performs measurements on the three subsystems in the computational basis or performs a projective measurement of $\Pi_\Psi \equiv |\Psi\rangle \langle \Psi|$.
4. This completes one round. This process is repeated for many rounds.
5. After a sufficient number of rounds, Bob$_1$ and Bob$_2$ reveal the rounds in which they have performed CTRL operation and Alice reveals the rounds in which she has performed measurements in the computational basis.
6. Alice analyses the data of those rounds in which none of the Bobs have performed CTRL operation to check for the presence of an eavesdropper.
7. The rounds, in which all the Bobs of a layer have performed CTRL operation and Alice has measured in the computational basis, constitute a key in the respective layer.

Fig. 1 Schematic diagram of (a) the first and (b) the second LSQKD protocol. The two rectangles represent the two layers in which keys are shared. A double-arrowed line between Alice and a Bob symbolises two and fro movement of states from Alice to the respective Bob. The difference in the two protocols is in the resource states employed to implement the task.
A schematic diagram, depicting the difference between the two protocols, is shown in figure (1).

4.2.1 Key generation rule
To obtain a key symbol in each layer, all the participants employ binary representations of their measurement outcomes. Let the outcomes of Alice, Bob$_1$, Bob$_2$ be $a$, $b_1$, $b_2$ respectively. In the binary representation$^1$, these are expressed as:

$$a = 2a^{(1)} + a^{(0)}, \quad b_1 = 2b_1^{(1)} + b_1^{(0)}.$$  \hspace{0.5cm} (3)

The symbols $a^{(1)}$, $b_1^{(1)}$ constitute a key in the first layer whereas the symbols $a^{(0)}$, $b_1^{(0)}$, $b_2$, in the second layer.

4.2.2 Confidentiality of the key
There are two key symbols in the first layer, corresponding to each key symbol in the second layer, occurring with an equal probability. Due to this probabilistic nature, Bob$_2$ cannot infer the key symbol in the first layer and hence, the two keys remain confidential.

We illustrate it through an example. Consider a case in which Bob$_2$ has obtained the key symbol 0, i.e., $b_2^{(0)} = 0$. In such a case, $a, b_1 \in \{0, 2\}$.

We conclude this section with the following remark. The employment of layered entangled states and semi-quantum secure communication protocols are not exclusive to QKD. In fact, the proposals of [18, 22, 25–27, 39] testify the generality of the secure communication with classical Bobs. Layered secure semi-quantum communication protocols, however, remain much less explored. In what follows, we make a foray into these protocols one-by-one.

5 Layered semi-quantum secret sharing (LSQSS)
LSQKD protocols, proposed in the last section, assume that all the participants are honest. Now we turn to the situation when the participants are individually dishonest but are obliged to behave honestly when they collaborate. In such a situation, a secret should be generated only if all the participants collaborate [22]. In this section, we propose LSQSS protocol to share a secret in three layers of a network having five participants.

Suppose that Alice wants to distribute three secrets in three layers of a network of five participants, viz., Bob$_1$, Bob$_2$, Bob$_3$, Bob$_4$, and Bob$_5$. The first layer has two participants—Bob$_1$, and Bob$_2$. The second layer has three participants—Bob$_3$, Bob$_4$, and Bob$_5$. The third layer comprises of all the five participants. The secrets that is shared in one of the layers should remain confidential from the participants not belonging to the same layer. Let the

$^1$The outcome $b_2$ of Bob$_2$ is already in binary form.
Layered semi–quantum communication protocols

resource state,

\[ |\Phi\rangle \equiv \frac{1}{2} (|0\rangle + |1\rangle + |2\rangle + |3\rangle) \otimes 5, \]  

be used to implement the task. The rationale for this choice of state has been given in appendix (B). The steps of the protocol are as follows:

1. Alice prepares the state \( |\Phi\rangle \) and sends the \( i^{th} \) subsystem to Bob\(_i\), \( i \in \{1, \ldots , 5\} \).
2. Each Bob, on receiving the state, performs CTRL or Reflect operation randomly with an equal probability.
3. Alice, upon receiving the state from each Bob, either performs a measurement on each subsystem in the computational basis or performs a projective measurement \( \Pi_{\Phi} \equiv |\Phi\rangle \langle \Phi| \).
4. After a sufficient number of rounds, each Bob reveals the rounds in which CTRL operations are performed. Alice reveals the rounds in which she has measured in the computational basis.
5. Alice analyses the data of those rounds in which none of the Bobs have performed CTRL operation to check for the presence of an eavesdropper.
6. The rounds, in which all the Bobs of a layer have performed CTRL operation and Alice has measured in the computational basis, constitute a key in the respective layer.
7. A secret is generated when all the Bobs in a given layer have performed CTRL operations, which they can retrieve collaboratively.

A schematic diagram of the protocol is given in figure (2).
5.1 Secret generation rule

As in the key generation rule of the LSQKD, each participant converts his measurement outcome in binary representation. Let outcomes of Bob_1, Bob_2, Bob_3, Bob_4, and Bob_5 be \( b_1, b_2, b_3, b_4, \) and \( b_5 \) respectively. In the binary representation, these are expressed as:

\[
  b_i \equiv 2^{b_i(1)} + 2^{b_i(0)}, \quad i \in \{1, \cdots, 5\}.
\]

A secret symbol in the first and the second layer with participants Bob_1, Bob_2, and Bob_3, Bob_4, Bob_5 is defined as \( s_1 \equiv b_1^{(1)} \oplus b_2^{(1)} \) and \( s_2 \equiv b_3^{(1)} \oplus b_4^{(1)} \oplus b_5^{(1)} \) respectively, where the symbol \( \oplus \) represents the operation of addition modulo 2. Similarly, a secret in third layer consisting of all the five participants is defined as, \( s_3 \equiv b_1^{(0)} \oplus b_2^{(0)} \oplus b_3^{(0)} \oplus b_4^{(0)} \oplus b_5^{(0)}. \)

5.2 Confidentiality of the secrets in different layers

In the protocol, for each outcome of a Bob, there are four different possible outcomes of the rest of the Bobs. In addition, a secret in a layer is generated by employing a particular bit of binary representation of the outcome. Since none of the Bobs can infer outcomes of the rest, the corresponding bits also remain unknown. In this manner, secrets shared in different layers remain confidential.

6 Integrated layered semi–quantum key distribution and secret sharing (ILSQKD+SS)

LSQKD and LSQSS protocols are designed for the two limiting cases—in which all the participants are honest and dishonest respectively. A generic situation corresponds to only some of the participants being dishonest. In such a case, the requirement of collaboration for generation of a key is limited only to dishonest participants.

The employment of layered multidimensional entangled states provides us with an avenue to integrate layered SQKD and SQSS in a single protocol. Thanks to the experimental advances in this area [37, 41], we believe that these next-generation integrated protocols for communication in quantum networks may also be implemented. In what follows, we propose a protocol for a network of four participants. Two out of the four participants viz., Bob_1, and Bob_2 are dishonest and the remaining two—Bob_3 and Bob_4—are honest. Suppose that Alice wants to share a secret between Bob_1 and Bob_2 and share a key among all the four participants. To implement this task, Alice prepares the following state:
Layered semi–quantum communication protocols

\[ |\chi\rangle = \frac{1}{4} \left\{ \left( |00\rangle + |02\rangle + |20\rangle + |22\rangle + |11\rangle + |13\rangle + |31\rangle + |33\rangle \right) |00\rangle \\
+ \left( |01\rangle + |10\rangle + |03\rangle + |30\rangle + |12\rangle + |21\rangle + |23\rangle + |32\rangle \right) |11\rangle \right\}. \quad (6) \]

The rationale for this choice of state has been given in Appendix (C). The steps of the protocol are as follows:

1. Alice prepares the state \( |\chi\rangle \) and sends the \( i \)th subsystem to Bob\(_i\) \((i \in \{1, 2, 3, 4\})\).
2. Each Bob performs CTRL or Reflect operations randomly, but with an equal probability.
3. Alice, after receiving the subsystems, either performs measurements on each subsystem in the computational basis or performs a projective measurement, \( \Pi_{\chi} = |\chi\rangle \langle \chi| \).
4. After a sufficient number of rounds, each Bob reveals the rounds in which CTRL operations are performed. Alice reveals the rounds in which she has performed measurement in the computational basis.
5. Alice analyses the data of those rounds, in which none of the Bobs have performed CTRL operations, to check for the presence of an eavesdropper.
6. The rounds, in which all the Bobs of a given layer perform CTRL operations and Alice measures in the computational basis, generate a key for the same layer.

A schematic diagram of the protocol is given in figure (3).
6.1 Key generation rule

To generate a key, all the Bobs rewrite their measurement outcomes in the binary representation. Let the outcomes of Bob_1, Bob_2, Bob_3, and Bob_4 be \( b_1 \), \( b_2 \), \( b_3 \), and \( b_4 \) respectively. In the binary representation, these are expressed as:

\[
b_1 = 2b_1^{(1)} + b_1^{(0)}, \quad b_2 = 2b_2^{(1)} + b_2^{(0)}. \tag{7}
\]

A secret in the first layer is defined as: \( s_1 \equiv b_1^{(1)} \oplus b_2^{(1)} \). Since Bob_3 and Bob_4 are honest participants, their outcomes \( b_3, b_4 \) correspond to the key symbols in the second layer. On the other hand, Bob_1, and Bob_2 have to collaborate with each other to obtain the key symbol which is defined as: \( k \equiv b_1^{(0)} \oplus b_2^{(0)} = b_3 = b_4 \).

6.2 Confidentiality of keys

The key shared between Bob_1 and Bob_2 is completely confidential from Bob_3 and Bob_4. This is because the latter cannot obtain any information about the measurement outcomes of the former and consequently, of the bits employed to generate a secret.

7 Integrated layered semi-quantum secure direct communication and key distribution (ILSQSDC+KD)

The last three protocols are sensitive to the behaviour (honest/dishonest) of the participants. However, a situation may be envisaged in which all the participants are honest, but the kind of communication required in different layers varies. As an example, it may be required to have SQSDC in one set of layers and SQKD in another set of layers. Such tasks require integration of SQKD and SQSDC protocols, which can be done, thanks to the multidimensional entangled states.

A SQSDC protocol may be implemented in two stages\[9\]: the first stage is used to check for the presence of an eavesdropper and to generate a correlation between the participants. The correlation so generated is utilised to have secure and direct communication, in the second stage. In contrast, SQKD protocol is implemented in a single stage.

Likewise, the integration of the SQSDC and SQKD protocols will have two stages. The first stage establishes correlation among the participants of the layer requiring direct communication. In addition, a key will also be generated among the participants of other layers. In the second stage, employing the correlation generated in the first stage, direct communication takes place in the respective layer.

\[2\]The outcomes \( b_3, b_4 \) of Bob_3, and Bob_4 are already in the binary form.
Consider, for example, a network of three participants \textit{viz.}, Alice, Bob\textsubscript{1}, and Bob\textsubscript{2}. Direct communication is required in the first layer consisting of Alice and Bob\textsubscript{1}, and a key is to be distributed in the second layer consisting of all the three participants. The same state as in LSQKD may be employed by Alice to implement this task. This is due to the fact that in both the protocols (SQKD and SQSDC), all the participants are assumed to be honest. The two protocols differ only in implementation, i.e., some additional steps are required in SQSDC in contrast to SQKD. Thus, Alice prepares the state,

\[
|\Psi\rangle = \frac{1}{2} \left( |000\rangle + |111\rangle + |220\rangle + |331\rangle \right),
\]

which is the same as the one employed in LSQKD protocol given in section (4.2). The steps in the first stage of the protocol are same as that of LSQKD. We recapitulate the same for completeness. In contrast, the second stage is unique to this protocol and we describe in detail the same as follows:

**First stage:**

1. Alice prepares the state $|\Psi\rangle$ and sends the second and the third subsystem to Bob\textsubscript{1} and Bob\textsubscript{2} respectively.
2. Bob\textsubscript{1} and Bob\textsubscript{2}, upon receiving the states, perform CTRL or Reflect operations randomly.
3. Alice, upon receiving the states from each participant, performs measurement on each subsystem either in the computational basis or performs a projective measurement, $\Pi_{\Psi} = |\Psi\rangle \langle \Psi|$. This completes one round.
4. After a sufficient number of rounds, Bob\textsubscript{1} and Bob\textsubscript{2} reveal the rounds in which they have performed CTRL operations. Alice reveals the rounds in which she has performed measurement in the computational basis.
5. Alice analyses the rounds, in which none of the Bobs have performed CTRL operations.
6. The rounds, in which both the Bobs have performed CTRL operation and Alice has measured in the computational basis, constitute a key in the second layer.

A key in the second layer is generated in the same way as is done in the LSQKD protocol proposed in section (4.2).

**Second stage:**

1. Alice prepares a sequence of those rounds\textsuperscript{3}, in which her measurement outcomes are perfectly correlated with those of Bob\textsubscript{1}.
2. For this sequence, she prepares a state $|\beta\rangle$ afresh same as the post-measurement state of Bob\textsubscript{1} for the round, and so $\beta \in \{0, 1, 2, 3\}$.
3. Alice employs two operations to encode her message on the prepared state—(i) identity operation ($1$) to encode 0, (ii) the transformation $X = |1\rangle \langle 0| + |2\rangle \langle 1| + |3\rangle \langle 2| + |0\rangle \langle 3|$ to encode 1. That is to say, she sends the state $|\beta\rangle$

\textsuperscript{3}These are the rounds, in which she has measured in the computational basis and Bob\textsubscript{1} has performed a CTRL operation.
or $X |\beta\rangle$ whenever she wants to send 0 or 1 as the message bit respectively to Bob$_1$.

4. Bob$_1$ performs CTRL operation on each state.

5. Since Bob$_1$ knows his outcomes of CTRL operations in the first stage, he can learn whether Alice has performed the transformation $X$ or not.

6. In this way, he decodes the message sent by Alice. A schematic diagram of the protocol is given in figure (4).

![Schematic diagram of ILSQSDC+KD](image)

**Fig. 4** Schematic diagram of ILSQSDC+KD, i.e. integration of SQSDC and SQKD protocol. A secure direct communication is required between Alice and Bob$_1$ and a key is being shared among all the three parties, i.e., among Alice, Bob$_1$, and Bob$_2$.

Though a key has already been shared in the first stage of the protocol, the direct communication in the first layer completes in the second stage. Therefore, in the second stage, it is sufficient for Alice to communicate only with Bob$_1$ and that is why she sends a four-dimensional state to Bob$_1$.

**Security in SQSDC:** Though there is no randomness in the second stage of the protocol, yet Eve cannot obtain any information about the message. This owes to the information encoding scheme. Information is encoded in the transformations that Alice may apply on the state to be sent to Bob$_1$ in the second stage of the protocol. To retrieve this information, one needs to have access to the outcomes of Bob$_1$’s measurement in both the stages. Eve cannot access this information unless she obtains information from the first stage. However, if she had done so, she would be detected and the protocol would be aborted. Thus, the intended participants can communicate securely.

**Confidentiality in the communication:** The communication between Alice and Bob$_1$ is completely confidential from Bob$_2$ because—(i) Bob$_2$ cannot obtain any information about the outcomes of Alice and Bob$_1$ by his measurement result, (ii) Bob$_2$ has no access to communication taking place in the second stage. Thus, the message being shared in the first layer is completely confidential from Bob$_2$. 
8 Generalisation of protocols to an arbitrary layered structure

In the preceding sections, various communication protocols have been proposed to share secure information in a particular layered structure of a network. The generalisation of these protocols to an arbitrary structure of a network is of primordial importance, so we undertake this task in this section. The steps to implement a generalised protocol in a given structure are the same as given in the preceding sections for illustrative protocols. The main task in the generalisation of the protocols is to find a multidimensional state having the requisite layered structure that may be employed to implement the task.

In order to identify the requisite state (a multidimensional state) for any of the four protocols, we start with a set of reference states involving multiqubit systems. Each reference state implements the task of sharing information in a particular layer of the network. This reference state is shared among all the participants in a given layer. This indicates that a participant has the same number of qubits as the number of layers to which he belongs. That is to say, if a participant \( u_j \) belongs to \( \ell_j \) layers, he has \( \ell_j \) qubits. Equipped with this knowledge, we now provide a procedure to identify the requisite state for all the four protocols one by one.

Note that the states employed to implement LSQKD and LQKD are the same. The only difference between the two is that the former has some classical participants. The procedure to identify the same has been given in [38]. In this section, we recapitulate the procedure to identify a state for LSQKD protocol, for the sake of completeness. In addition, we also provide a procedure to identify requisite states to implement LSQSS, ILSQKD+SS and ILSQSDC+KD protocols for an arbitrary structure in a given network.

Consider a network, labelled by the symbol \( \kappa \), in which secret information is to be distributed. The network, \( \kappa \), is characterised by the set \( \{n, k, \ell_j\} \), where \( n \) represents the number of participants, and \( k \) represents the total number of layers in \( \kappa \). The symbol \( \ell_j \) represents the number of layers to which the participant \( u_j \) belongs. Since each layer has exactly one key, the same label \( i \in \{1, \cdots, k\} \) is employed for both.

The general steps of the procedure are described as follows:

1. we start with the reference multiqubit state that distributes secure information in a layer.
2. A state that distributes information in the network \( \kappa \) is the tensor product of all the reference states, each sharing an information in a given layer.
3. All the qubits belonging to the same participants in this state are clubbed together.
4. Employing the isomorphism between the Hilbert spaces of identical dimensions, the basis states of Hilbert spaces of tensor product of \( \ell_j \) qubits maps to a \( d_j = 2^{\ell_j} \) dimensional single qudit. In short, each participant is in possession of a \( 2^{\ell_j} \) dimensional qudit state.
5. After employing the mapping, one obtains the requisite multidimensional state that distributes secure information simultaneously in all the $k$ layers of a network, $\kappa$.

We now employ this procedure to identify states for each of the four protocols.

### 8.1 LSQKD

Let, the reference state, employed to share a key in the $i^{th}$ layer be a GHZ state\(^4\) given as:

$$|\psi_i\rangle \equiv \frac{1}{\sqrt{2}} \left( \bigotimes_j |0\rangle_{u_j} + \bigotimes_j |1\rangle_{u_j} \right),$$

(8)

where $u_j$ are the participants belonging to the $i^{th}$ layer. The GHZ state is chosen to share a key because it provides a perfect correlation in the computational basis among all the participants. Since keys are to be shared in all the $k$ layers, a state that shares keys in a structure $\kappa$ is a tensor product of the GHZ states, each GHZ state sharing a key in a particular layer. That is, the state $|\Psi_\kappa\rangle$ can be written as:

$$|\Psi_\kappa\rangle \equiv \bigotimes_{i=1}^{k} |\psi_i\rangle.$$  

(9)

Given the state, we, next, club the qubits belonging to each participant together. Since there exists an isomorphism between the Hilbert spaces of identical dimensions, the following bijective mapping,

$$|m_1m_2\cdots m_l\rangle \leftrightarrow \left| \sum_{r=1}^{l_j} 2^{l_j-r} m_r \right\rangle \equiv |m\rangle, \quad m_r \in \{0,1\},$$

(10)

exists between the basis states of Hilbert spaces of tensor product of $l_j$ qubits and that of $d_j = 2^{l_j}$ dimensional Hilbert space of a single qudit. Employing this mapping, states of $\ell_j$ qubits, belonging to the participant $u_j$, can be replaced with an equivalent $2^{\ell_j}$-dimensional qudit state. Thus, after employing the mapping of equation (10), the state $|\Psi_\kappa\rangle$ maps to an $n$-partite higher-dimensional state having layered entanglement. This is the state which is employed to share keys simultaneously in all the layers of a structure $\kappa$. Note that each participant $u_j$ in a network is in possession of $2^{\ell_j}$-dimensional qudit state where $\ell_j$ is the number of layers to which he belongs.

For further clarification, we have illustrated this procedure in the appendix (A) for a network of three participants in which keys are to be shared in the two layers.

\(^4\)If a layer consists of only two participants, the GHZ state reduces to a Bell’s state.
Similarly, the procedures can be laid down to identify the requisite $n$–partite states that can be employed to implement rest of the three protocols, viz., LSQSS, ILSQKD+SS, and ILSQSDC+KD for any arbitrary structure $\kappa$. In what follows, we describe them one-by-one.

8.2 LSQSS

A LSQSS protocol differs from a LSQKD in that it requires a collaboration among all the participants of a given layer to generate a key. This constraint of collaboration is crucial in the choice of a state that shares a secret in a particular layer.

Let, a reference multiqubit state,

$$|\phi_i\rangle \equiv \bigotimes_j \frac{1}{\sqrt{2}} (|0\rangle_{u_j} + |1\rangle_{u_j}),$$

be employed to share a secret in the $i^{th}$ layer of the structure $\kappa$. The symbol $u_j$ ranges over the participants belonging to the $i^{th}$ layer. The state is so chosen that none of the participants, based on his outcome, can infer the outcomes of the rest of the participants. Thus, all the participants have to collaborate with one another to share a secret.

Since secrets are to be shared in all the $k$ layers of the network, a state that shares secrets in a structure $\kappa$ is a tensor product of all such states, $\{|\phi_i\rangle\}$. That is, the state $|\Phi_\kappa\rangle$ can be written as:

$$|\Phi_\kappa\rangle \equiv \bigotimes_{i=1}^k |\phi_i\rangle.$$  

As before, we next club the qubits belonging to each participant together. For each participant $u_j$, states of the respective $\ell_j$–qubits can be written as the equivalent states of a $2^{\ell_j}$–dimensional qudit by employing the mapping given in equation (10).

In this way, the state $|\Phi_\kappa\rangle$ maps to an $n$–partite higher–dimensional state having a requisite structure that simultaneously shares secrets in a structure $\kappa$. Similar to LSQKD, each participant $u_j$ in a network is in the possession of $2^{\ell_j}$–dimensional qudit state where $\ell_j$ is the number of layers to which he belongs. An illustration of the procedure to identify a state that implements the task of secret sharing in the three layers of a network having five participants is given in the appendix (B).

8.3 ILSQKD+SS

An ILSQKD+SS protocol is an integration of both LSQKD and LSQSS protocols. In this protocol, only some of the participants are dishonest, and so collaboration is required only among these participants. In this protocol, three
kinds of layers are possible—(i) a layer consisting of only the honest participants, (ii) a layer consisting of only the dishonest participants, and (iii) a layer in which some of the participants are dishonest and the rest are honest. The first and the second kind of layers are equivalent to key sharing and secret sharing, whereas the third one requires sharing of a key such that collaboration is required only among the dishonest participants.

We identify a reference state for each of the three layers as follows:

1. For each layer \( p \in \kappa \) in which all the participants are honest, the same reference state \( |\psi_p\rangle \), as employed in LSQKD can be used, i.e.,

\[
|\psi_p\rangle \equiv \frac{1}{\sqrt{2}} \left( \bigotimes_j |0\rangle_{u_j} + \bigotimes_j |1\rangle_{u_j} \right),
\]

where \( j \) ranges over all the honest participants belonging to the layer ‘\( p \)’.

2. For each layer \( q \in \kappa \) in which all the participants are dishonest, the same state, as employed in LSQSS can be used, i.e., the state \( |\phi_q\rangle \) is used for secret sharing,

\[
|\phi_q\rangle \equiv \bigotimes_t \frac{1}{\sqrt{2}} \left( |0\rangle_{u_t} + |1\rangle_{u_t} \right),
\]

where \( t \) ranges over all the dishonest participants belonging to the layer ‘\( q \)’.

3. For each layer \( r \in \kappa \) in which some of the participants are dishonest, the collaboration is required among the dishonest participants. The condition of collaboration leads to the following choice of a resource state,

\[
|\alpha_r\rangle \equiv \frac{1}{\sqrt{2}} \left\{ \left( \bigotimes_t |+\rangle_{u_t} + \bigotimes_t |\rangle_{u_t} \right) \bigotimes_j |0\rangle_{u_j} \\
+ \left( \bigotimes_t |+\rangle_{u_t} - \bigotimes_t |\rangle_{u_t} \right) \bigotimes_j |1\rangle_{u_j} \right\},
\]

where,

\[
|\pm\rangle_{u_t} = \frac{1}{\sqrt{2}} \left( |0\rangle_{u_t} \pm |1\rangle_{u_t} \right),
\]

and the index \( t \) and \( j \) range over the dishonest and honest participants in the layer ‘\( r \)’ respectively. The state \( |\alpha_r\rangle \) is so chosen that none of the dishonest participants can infer the outcomes of each other as well as that of honest participants. Collaboration among all the dishonest participants is required to infer outcomes of honest participants, and hence the key.
Therefore, the combined state for the structure $\kappa$ is,

$$|\chi_\kappa\rangle \equiv \bigotimes_p |\psi_p\rangle \bigotimes_q |\phi_q\rangle \bigotimes_r |\alpha_r\rangle,$$

where the symbols $p$ and $q$ ranges over all the layers consisting of only honest and dishonest participants respectively, and the symbol $r$ ranges over all the layers in which some of the participants are dishonest.

As before, by clubbing the qubits belonging to same participants together and employing the mapping given in equation (10), the state $|\chi_\kappa\rangle$ maps to an $n$-partite state having the requisite layered structure, where $n$ is the number of participants. This state shares secrets and keys in various layers of a structure $\kappa$ in a network simultaneously. For better appreciation, we have provided an illustration that implements the task of sharing a secret in a layer with two participants and sharing a key in another layer with all the four participants in the appendix (C).

Thus, following the procedure, a state can be identified that distributes secrets in some layers and keys in the rest of the layers.

### 8.4 ILSQSDC+KD

This protocol shares keys in some of the layers while direct communication takes place in the rest of the layers. Since all the participants in the protocol are honest, this task can be implemented with the same state as employed for LSQKD.

The only difference comes in the second stage, in which it suffices to communicate only with those participants who participate in direct communication. Alice prepares the post-measurement states of the respective participants, and resends these states after performing appropriate transformations.

In this way, by employing the above prescriptions, a state for any arbitrary key structure in a network for any of the four protocols can be identified.

### 8.5 Key generation rule

To obtain keys/secrets, each participant rewrites his outcomes in the binary representation. Let $b_i$ be the outcomes of the participant Bob$_i$ of a network. Then, in binary representation$^5$, these are expressed as:

$$b_i = \sum_{m=0}^{l_j-1} 2^m b_i^{(m)}.$$  \hspace{1cm} (13)

$^5$Each participant writes binary representation of his outcome upto the places which is equal to the number of layers he belongs to.
The symbols $b_i^{(m)}$ are used to generate keys/secrets in $(k - m)$th layer, where $k$ is the total number of layers in the network\textsuperscript{6}.

**Honest participants:** For all the honest participants, belonging to, say $p$th layer, the symbols $b_i^{(k-p)}$ correspond to the key symbols.

**Dishonest participants:** For all the dishonest participants, belonging to the same $p$th layer, the key symbol corresponds to $\oplus_t b_t^{(k-p)}$, where $t$ ranges over all the dishonest participants in the $p$th layer and the symbol $\oplus$ represents the operation of addition modulo 2.

\section{Security against various attacks}

In this section, we consider security of shared secret information against a class of attacks.

Semi–quantum protocols are two–way communication protocols\textsuperscript{7}. For the first time, a state travels from Alice to the respective Bob and for the second time, it travels back from the same Bob to Alice. Thus, Eve has two opportunities to interact with the state.

\subsection{Measurement attacks}

In the measurement attack, Eve performs a measurement on the state and keeps it with herself. However, if she does so, she will be detected in the basis reconciliation step, when the respective participant will report that he has not received any state for specific rounds.

\subsection{Intercept–resend attack}

In intercept–resend attack, Eve intercepts the travelling state, performs a measurement on it and sends the post–measurement state to the intended recipient. If Eve does so, she gets detected, when Alice analyses the data. This is because in addition to measurements in the computational basis, Alice also performs a projective measurement, $\Pi_{\eta}$, where $|\eta\rangle$ is the resource state employed to implements the task. For the rounds, in which none of the Bobs have performed CTRL operation, Eve’s measurement changes the statistics corresponding to the projective measurement, and hence, she gets detected.

\subsection{Entangle–and–Measure attack}

In the entangle–and–measure attack, Eve may couple her ancillary systems with each subsystem and may try to obtain information about the outcomes of classical participants by performing a measurement on her system. However,

\textsuperscript{6}If Alice is also the part of, say $\ell_j$ layers, she also writes her outcome in the binary representation, $a = \sum_{m=0}^{\ell_j-1} 2^m a^{(m)}$. Then, the symbols $a^{(m)}$ are used to generate keys/secrets in $(k - m)$th layer.

\textsuperscript{7}With the sole exception of SQSDC protocols in which a state travels three times. However, when the state travels for third time, even if Eve intercepts, she may not obtain any information. It is due to the choice of encoding scheme.
if she does so, she gets detected when Alice analyses the data of the rounds in which none of the Bobs have performed CTRL operation. This is due to the fact that Eve’s tampering changes the statistics of projective measurement, \( \Pi_{\eta} \), and so, she gets detected.

In this way, different interventions of Eve will get detected and hence, the proposed protocols are secure against such attacks. A more general security analysis for the proposed protocols constitutes a separate study and will be taken up elsewhere.

10 Conclusion

In summary, this work explores the potential offered by multidimensional states in semi–quantum communication protocols over a network. Layered entanglement has been employed in all the protocols, barring LSQSS which can be implemented with multidimensional product states.

Integration of communication protocols constitutes a rich structure and opens up avenues for further study of quantum communication. This work opens up possibilities of integration of various protocols, being quantum or semi–quantum, as per the requirement of a network.

Appendix A Equivalence of tripartite layered entangled state with tensor product of EPR and three–party GHZ state

In section (4), we have proposed two protocols for distribution of keys in the two layers of a three–party network. The first protocol uses two maximally entangled states, an EPR state and a three–party GHZ state. The second protocol uses a tripartite state having a layered entanglement.

The states, employed in the two protocols, implement the task in the same layered structure due to mathematical equivalence between the states. To show this, we start with the reference states (these are the same states as used in the first LSQKD protocol.),

\[
|\psi_1\rangle = \frac{1}{\sqrt{2}}(|0_{A'}0_{B'_1}\rangle + |1_{A'}1_{B'_1}\rangle), \quad |\psi_2\rangle = \frac{1}{\sqrt{2}}(|0_A0_{B_1}0_{B_2}\rangle + |1_A1_{B_1}1_{B_2}\rangle),
\]

(A1)

where the subscripts represent the participants to which qubits belong. The primes in the subscripts of the state \( |\psi_1\rangle \) are used to differentiate the subsystems from that of the state \( |\psi_2\rangle \).

The combined state that distributes keys in the two layers is,

\[
|\beta\rangle = |\psi_1\rangle |\psi_2\rangle = \frac{1}{2}(|0_{A'}0_{B'_1}\rangle + |1_{A'}1_{B'_1}\rangle) (|0_A0_{B_1}0_{B_2}\rangle + |1_A1_{B_1}1_{B_2}\rangle)
\]
Layered semi–quantum communication protocols

\[ \frac{1}{2} \left( |0_A^10_B^10_A0_B^2 \rangle + |0_A^10_B^11_A1_B^10_B^2 \rangle + |1_A^11_B^10_A0_B^2 \rangle + |1_A^11_B^11_A1_B^10_B^2 \rangle \right) \]

\[ \frac{1}{2} \left( |0_A^10_B^10_A0_B^2 \rangle + |0_A^11_A0_B^11_B^10_B^2 \rangle + |1_A^11_A0_B^10_B^2 \rangle + |1_A^11_A1_B^11_B^10_B^2 \rangle \right) . \]  

Equation (A2)

We employ the mapping defined in equation (10) of section (8), which for two qubits system translates to,

\[ |00 \rangle \rightarrow |0 \rangle , \quad |01 \rangle \rightarrow |1 \rangle , \quad |10 \rangle \rightarrow |2 \rangle , \quad |11 \rangle \rightarrow |3 \rangle . \]  

Equation (A3)

Under this mapping, the multiqubit state \(|\Psi\rangle\) gets mapped to the following equivalent multiqudit state,

\[ |\beta \rangle \rightarrow \frac{1}{2} (|000 \rangle + |111 \rangle + |220 \rangle + |331 \rangle) , \]  

Equation (A4)

which is the same as the state \(|\Psi\rangle\). Equation (A4) shows the mathematical equivalence between the states employed in the two LSQKD protocols.

Appendix B Identification of the requisite state for LSQSS

To identify the requisite state that shares secrets in all the three layers in a network of five participants, we start with three states, each sharing a secret in the respective layer. The three states are:

\[ |\phi_1 \rangle = \bigotimes_{i=1,2} \frac{1}{\sqrt{2}} (|0 \rangle + |1 \rangle)_{B_i} , \quad |\phi_2 \rangle = \bigotimes_{i=3} \frac{1}{\sqrt{2}} (|0 \rangle + |1 \rangle)_{B_i} , \]

\[ |\phi_3 \rangle = \bigotimes_{i=1}^5 \frac{1}{\sqrt{2}} (|0 \rangle + |1 \rangle)_{B'_i} , \]  

Equation (B5)

where the subscripts represent the participant to which qubits belong. The primes in the subscripts of the state \(|\phi_3\rangle\) are used to differentiate its subsystems from that of the states \(|\phi_1\rangle\) and \(|\phi_2\rangle\). The combined state that distributes secrets in the three layers is,

\[ |\gamma \rangle \equiv |\phi_1 \rangle |\phi_2 \rangle |\phi_3 \rangle \]

\[ = \bigotimes_{i=1,2} \frac{1}{\sqrt{2}} (|0 \rangle + |1 \rangle)_{B_i} \bigotimes_{i=3} \frac{1}{\sqrt{2}} (|0 \rangle + |1 \rangle)_{B_i} \bigotimes_{i=1}^5 \frac{1}{\sqrt{2}} (|0 \rangle + |1 \rangle)_{B'_i} \]

Clubbing the subsystems belonging to same participants together, and employing the mapping \(|00 \rangle \rightarrow |0 \rangle , \quad |01 \rangle \rightarrow |1 \rangle , \quad |10 \rangle \rightarrow |2 \rangle , \quad |11 \rangle \rightarrow |3 \rangle\), the state
Layered semi–quantum communication protocols

$|\gamma\rangle$ maps to,

$$
|\gamma\rangle \rightarrow \bigotimes_{i=1}^{5} \frac{1}{2}(|0\rangle + |1\rangle + |2\rangle + |3\rangle)_{B_i}
$$

$$
= \left( \frac{1}{2}(|0\rangle + |1\rangle + |2\rangle + |3\rangle) \right) \otimes^{5}, \quad (B6)
$$

which is same as the state $|\Phi\rangle$ in equation (4).

Appendix C Identification of the requisite state for ILSQKD+SS

In section (6), a protocol has been proposed that distributes a secret and a key in two layers of a network having four participants. The four participants are named Bob$_1$, Bob$_2$, Bob$_3$, and Bob$_4$. The first layer consists of two dishonest participants named Bob$_1$, and Bob$_2$. The second layer consists of all the four participants. To identify a state that implements the task, we start with reference states,

$$
|\phi_1\rangle \equiv |++\rangle_{B'_1B'_2} = \frac{1}{2}(|00\rangle + |01\rangle + |10\rangle + |11\rangle)_{B'_1B'_2},
$$

$$
|\alpha\rangle \equiv \frac{1}{2}\left\{ \left( |00\rangle + |11\rangle \right) |00\rangle + \left( |01\rangle + |10\rangle \right) |11\rangle \right\}_{B_1B_2B_3B_4}
$$

Thus, a state that implements the task is,

$$
|\zeta\rangle \equiv |\phi_1\rangle|\alpha\rangle
$$

$$
= \frac{1}{4}\left( |00\rangle + |01\rangle + |10\rangle + |11\rangle \right)_{B'_1B'_2} \left( |00\rangle + |11\rangle \right) |00\rangle + \left( |01\rangle + |10\rangle \right) |11\rangle \right\}_{B_1B_2B_3B_4}, \quad (C7)
$$

Clubbing the subsystem belonging to the same participant and employing the mapping, $|00\rangle \rightarrow |0\rangle$, $|01\rangle \rightarrow |1\rangle$, $|10\rangle \rightarrow |2\rangle$, $|11\rangle \rightarrow |3\rangle$, the state $|\zeta\rangle$ maps to,

$$
|\zeta\rangle \rightarrow \frac{1}{4} \left\{ \left( |00\rangle + |02\rangle + |20\rangle + |22\rangle + |11\rangle + |13\rangle + |31\rangle + |33\rangle \right) |00\rangle 
+ \left( |01\rangle + |10\rangle + |03\rangle + |30\rangle + |12\rangle + |21\rangle + |23\rangle + |32\rangle \right) |11\rangle \right\}, \quad (C8)
$$

which is the same state as $|\chi\rangle$. 
References

[1] Bennett, C., Brassard, G.: Quantum cryptography: public key distribution and coin tossing. Proc. IEEE Int. Conf. on Comp. Sys. Signal Process (ICCSSP), 175 (1984). https://doi.org/10.1103/RevModPhys.38.447

[2] Ekert, A.K.: Quantum cryptography based on bell’s theorem. Phys. Rev. Lett. 67, 661–663 (1991). https://doi.org/10.1103/PhysRevLett.67.661

[3] Long, G.-l., Deng, F.-g., Wang, C., Li, X.-h., Wen, K., Wang, W.-y.: Quantum secure direct communication and deterministic secure quantum communication. Frontiers of Physics in China 2(3), 251–272 (2007)

[4] Boyer, M., Kenigsberg, D., Mor, T.: Quantum key distribution with classical bob. Phys. Rev. Lett. 99, 140501 (2007). https://doi.org/10.1103/PhysRevLett.99.140501

[5] Gisin, N., Ribordy, G., Tittel, W., Zbinden, H.: Quantum cryptography. Reviews of modern physics 74(1), 145 (2002)

[6] Shukla, C., Thapliyal, K., Pathak, A.: Semi-quantum communication: protocols for key agreement, controlled secure direct communication and dialogue. Quantum Information Processing 16(12), 1–19 (2017)

[7] Long, G.-L.: Quantum secure direct communication: principles, current status, perspectives. In: 2017 IEEE 85th Vehicular Technology Conference (VTC Spring), pp. 1–5 (2017).

[8] Gao, T.: Controlled and secure direct communication using ghz state and teleportation. Zeitschrift für Naturforschung A 59(9), 597–601 (2004)

[9] Zhang, M.-H., Li, H.-F., Xia, Z.-Q., Feng, X.-Y., Peng, J.-Y.: Semiquantum secure direct communication using epr pairs. Quantum Information Processing 16(5), 117 (2017)

[10] Bechmann-Pasquinucci, H., Tittel, W.: Quantum cryptography using larger alphabets. Phys. Rev. A 61, 062308 (2000)

[11] Bala, R., Asthana, S., Ravishankar, V.: Contextuality-based quantum conferencing. Quantum Information Processing 20(10), 352 (2021). https://doi.org/10.1007/s11128-021-03286-8

[12] Wootters, W.K., Zurek, W.H.: A single quantum cannot be cloned. Nature 299(5886), 802–803 (1982)

[13] Singh, J., Bharti, K., Arvind: Quantum key distribution protocol based on contextuality monogamy. Phys. Rev. A 95, 062333 (2017). https://doi.org/10.1103/PhysRevA.95.062333
Layered semi–quantum communication protocols

[14] Pirandola, S.: Quantum discord as a resource for quantum cryptography. Scientific reports 4(1), 1–5 (2014)

[15] Vazirani, U., Vidick, T.: Fully device-independent quantum key distribution. Phys. Rev. Lett. 113, 140501 (2014). https://doi.org/10.1103/PhysRevLett.113.140501

[16] Pirandola, S., Andersen, U.L., Banchi, L., Berta, M., Bunandar, D., Colbeck, R., Englund, D., Gehring, T., Lupo, C., Ottaviani, C., et al.: Advances in quantum cryptography. Advances in Optics and Photonics 12(4), 1012–1236 (2020)

[17] Iqbal, H., Krawec, W.O.: Semi-quantum cryptography. Quantum Information Processing 19(3), 1–52 (2020)

[18] Rong, Z., Qiu, D., Mateus, P., Zou, X.: Mediated semi-quantum secure direct communication. Quantum Information Processing 20(2), 1–13 (2021)

[19] Shukla, C., Malpani, P., Thapliyal, K.: Hierarchical quantum network using hybrid entanglement. Quantum Information Processing 20(3), 1–19 (2021)

[20] Hillery, M., Bužek, V., Berthiaumne, A.: Quantum secret sharing. Physical Review A 59(3), 1829 (1999)

[21] Chong, S.-K., Hwang, T.: Quantum key agreement protocol based on bb84. Optics Communications 283(6), 1192–1195 (2010)

[22] Sun, Y., Wen, Q.-y., Gao, F., Chen, X.-b., Zhu, F.-c.: Multiparty quantum secret sharing based on bell measurement. Optics communications 282(17), 3647–3651 (2009)

[23] Li, C., Ye, C., Tian, Y., Chen, X.-B., Li, J.: Cluster-state-based quantum secret sharing for users with different abilities. Quantum Information Processing 20(12), 1–14 (2021)

[24] Proietti, M., Ho, J., Grasselli, F., Barrow, P., Malik, M., Fedrizzi, A.: Experimental quantum conference key agreement. Science Advances 7(23), 0395 (2021)

[25] Li, L., Qiu, D., Mateus, P.: Quantum secret sharing with classical bobs. Journal of Physics A: Mathematical and Theoretical 46(4), 045304 (2013)

[26] Li, H.-H., Gong, L.-H., Zhou, N.-R.: New semi-quantum key agreement protocol based on high-dimensional single-particle states. Chinese Physics B 29(11), 110304 (2020)
[27] Yan, L., Zhang, S., Chang, Y., Sheng, Z., Sun, Y.: Semi-quantum key agreement and private comparison protocols using bell states. International Journal of Theoretical Physics 58(11), 3852–3862 (2019)

[28] Simon, C.: Towards a global quantum network. Nature Photonics 11(11), 678–680 (2017)

[29] Cavalcanti, D., Skrzypczyk, P., Aguilar, G., Nery, R., Ribeiro, P.S., Walborn, S.: Detection of entanglement in asymmetric quantum networks and multipartite quantum steering. Nature communications 6(1), 1–6 (2015)

[30] Ghosh, S., Kar, G., Roy, A., Sarkar, D., Sen, U.: Entanglement teleportation through ghz-class states. New Journal of Physics 4(1), 48 (2002)

[31] Sun, Q.-C., Mao, Y.-L., Chen, S.-J., Zhang, W., Jiang, Y.-F., Zhang, Y.-B., Zhang, W.-J., Miki, S., Yamashita, T., Terai, H., et al.: Quantum teleportation with independent sources and prior entanglement distribution over a network. Nature Photonics 10(10), 671–675 (2016)

[32] Lee, J., Lee, S., Kim, J., Oh, S.D.: Entanglement swapping secures multiparty quantum communication. Physical Review A 70(3), 032305 (2004)

[33] Huang, Y.-B., Li, S.-S., Nie, Y.-Y.: Controlled dense coding between multi-parties. International Journal of Theoretical Physics 48(1), 95–100 (2009)

[34] Ma, S., Wang, N.: Hierarchical remote preparation of an arbitrary two-qubit state with multiparty. Quantum Information Processing 20(8), 1–29 (2021)

[35] Qi, Z., Li, Y., Huang, Y., Feng, J., Zheng, Y., Chen, X.: A 15-user quantum secure direct communication network. Light: Science & Applications 10(1), 1–8 (2021)

[36] Epping, M., Kampermann, H., Bruß, D., et al.: Multi-partite entanglement can speed up quantum key distribution in networks. New Journal of Physics 19(9), 093012 (2017)

[37] Liu, H., Wang, W., Wei, K., Fang, X.-T., Li, L., Liu, N.-L., Liang, H., Zhang, S.-J., Zhang, W., Li, H., et al.: Experimental demonstration of high-rate measurement-device-independent quantum key distribution over asymmetric channels. Physical review letters 122(16), 160501 (2019)

[38] Pivoluska, M., Huber, M., Malik, M.: Layered quantum key distribution. Physical Review A 97(3), 032312 (2018)
Layered semi–quantum communication protocols

[39] Qin, H., Tang, W.K., Tso, R.: Hierarchical quantum secret sharing based on special high-dimensional entangled state. IEEE Journal of Selected Topics in Quantum Electronics 26(3), 1–6 (2020)

[40] Malik, M., Erhard, M., Huber, M., Krenn, M., Fickler, R., Zeilinger, A.: Multi-photon entanglement in high dimensions. Nature Photonics 10(4), 248–252 (2016)

[41] Hu, X.-M., Xing, W.-B., Zhang, C., Liu, B.-H., Pivoluska, M., Huber, M., Huang, Y.-F., Li, C.-F., Guo, G.-C.: Experimental creation of multi-photon high-dimensional layered quantum states. npj Quantum Information 6(1), 1–5 (2020)

[42] Huber, M., De Vicente, J.I.: Structure of multidimensional entanglement in multipartite systems. Physical review letters 110(3), 030501 (2013)