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Robust operation of a GaAs tunable barrier electron pump

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Abstract

We demonstrate the robust operation of a gallium arsenide tunable-barrier single-electron pump operating with 1 part-per-million accuracy at a temperature of 1.3 K and a pumping frequency of 500 MHz. The accuracy of current quantisation is investigated as a function of multiple control parameters, and robust plateaus are seen as a function of three control gate voltages and RF drive power. The electron capture is found to be in the decay-cascade, rather than the thermally-broadened regime. The observation of robust plateaus at an elevated temperature which does not require expensive refrigeration is an important step towards validating tunable-barrier pumps as practical current standards.

Keywords: single electron pumps, primary electrical metrology, current standards

(Some figures may appear in colour only in the online journal)

1. Introduction

The controlled transport of single electrons in mesoscopic devices has attracted much attention as a conceptually simple primary standard of electric current [1]. Very precise control of electrons has been achieved using chains of mesoscopic normal metal islands [2], but limited to slow pumping rates ≲ 10 MHz due to the fixed RC time-constant of the junctions between the islands. At the present time, the most practically useful combination of accuracy and high electron pumping rate has been achieved using electrostatically gated semiconductor quantum dots (QDs) operated as non-adiabatic tunable-barrier pumps [3] in the low-temperature decay cascade regime [4]. Using state-of-the-art current measurement techniques [5, 6], there have been several reports of pumped current accurate at the part-per-million (ppm) level or better, at pump repetition rates in the range 0.5 GHz ≲ f ≲ 1 GHz, generating current 80 pA ≲ I_P = e f ≲ 160 pA [5, 7–11], where e is the elementary charge. These studies were performed on a variety of device architectures: etch-defined [5, 8, 10] and gate-defined [7] QDs in GaAs heterostructures, and silicon nano-wire MOSFETs [9]. While very promising for the metrological application of electron pumps, most of these studies were performed on carefully tuned devices. The required robustness of the current against changes in the pump control parameters has only recently begun to be investigated with high precision [11, 12], and only in one type of etch-defined pump.

In this study, we broaden the study of robustness, and investigate the gate-defined tunable barrier pump [7, 13]. Most significantly for the application of pumps as practical current standards, we perform our measurements at ≈ 1.3 K, the temperature of pumped helium-4. This is in contrast to previous robustness studies [11, 12, 14] which were carried out at dilution refrigerator temperatures. Using a rigorous statistical approach to evaluate the plateau extension and flatness, we find robust plateaus in all the tuning parameters we investigated, flat to within the ≈ 2 × 10⁻⁶ relative statistical uncertainty of each data point. Long measurements with the device in an optimally-tuned condition gave a current equal to e f within a relative total uncertainty of 8.6 × 10⁻⁷. We also
show that despite the elevated temperature, the pump was operating in the decay-cascade regime and not the thermally-broadened regime predicted [15] and observed [16] at higher temperatures. Furthermore, the device was affected by a significant amount of charge noise. The robust performance of the pump under these non-ideal conditions is encouraging evidence that the semiconductor electron pump can fulfill a role as a practical current standard.

This paper is structured as follows: section 2 describes the characterization and measurement technique. Section 3 presents the main experimental results in which we show that the pump current displays flat plateaus over a wide range of several tuning parameters. In section 4 we analyze the statistical fluctuations of the current on the plateaus, and show that there is no indication of structure on the plateaus within the measurement uncertainty. Finally in section 5 we show that the pump is operating in the decay-cascade regime, and not in the thermal-equilibrium regime, even at the elevated temperature.

2. Characterisation

The pump used in this study (see SEM image in figure 1(a)) was realised in a 2-dimensional electron gas (2-DEG) in a GaAs–AlGaAs heterostructure with metallic surface gates. The sample was fabricated using techniques described previously [7, 13], and measured at a temperature of ~1.3 K. DC voltages $V_{G1} - V_{G6}$ defined a quantum dot in the region between the gates, and a sinusoidal AC voltage at $f = 500$ MHz was added to gate 1 using a room-temperature bias-T, to pump electrons from the source to the drain. The AC source had an output power $P_{RF}$, calibrated for a $50 \Omega$ load, and the total attenuation of the $50 \Omega$ co-axial line between the source and the device was $\approx 4$ dB. A magnetic field $B = 13.5$ T was applied perpendicular to the plane of the sample [17–19]. The pump current $I_P$ was measured in two modes; normal-accuracy and high-accuracy. In normal-accuracy mode, used for rapid characterization, the current was amplified by a room-temperature high-accuracy transimpedance amplifier, with an uncertainty in the gain characterization, the current was amplified by a room-temperature high-accuracy transimpedance amplifier, with an uncertainty in the gain characterization, the current was amplified by a room-temperature high-accuracy transimpedance amplifier, with an uncertainty in the gain characterization, the current was amplified by a room-temperature high-accuracy transimpedance amplifier, with an uncertainty in the gain characterization, the current was amplified by a room-temperature high-accuracy transimpedance amplifier, with an uncertainty in the gain characterization, the current was amplified by a room-temperature high-accuracy transimpedance amplifier, with an uncertainty in the gain characterization, the current was amplified by a room-temperature high-accuracy transimpedance amplifier, with an uncertainty in the gain characterization. For high-accuracy measurements, $I_P$ was compared with a reference current derived from applying a voltage across a calibrated 1 $\Omega$ standard resistor [5, 7, 9]. In this mode the amplifier measures the small difference between the pump and reference currents, and provided this difference is made less than 0.05% of $I_P$, by tuning the reference current, the calibration uncertainty of the amplifier contributes less than $1 \times 10^{-7}$ to the total relative uncertainty. We are chiefly interested in the deviation of $I_P$ from its expected quantised value $ef$, so we define the dimensionless normalised deviation, $\Delta I_P \equiv (I_P - ef)/ef$. Likewise, all uncertainties in $\Delta I_P$ will be expressed as relative uncertainties in dimensionless units. The RF modulation of the entrance gate, and the reference current source are turned on and off synchronously with a cycle time of 40 s to eliminate instrumental offsets. The on-off cycle is repeated $n_{cyc}$ times. To reject linear drift in the offset current, our data analysis routine calculates $\Delta I_P$ using the data from the ‘off’ part of the cycle and half of the data from the two adjacent ‘on’ parts, thus generating $n_{cyc} = 1$ statistically independent values of $\Delta I_P$ with standard deviation $\sigma^4$. These values are then averaged to yield a mean $\Delta I_P$ with statistical uncertainty $U_{ST} = \sigma/\sqrt{n_{cyc}} - 1$ (all uncertainties reported in this paper are 1 sigma standard uncertainties). The relative systematic uncertainty in $\Delta I_P$ is dominated by the calibration uncertainty of the standard resistor, $U_{IG} = 8 \times 10^{-7}$, with an additional small contribution due to the voltage measurement $U_V^2 < 2 \times 10^{-7}$, so that the total uncertainty $U_T = \sqrt{U_{ST}^2 + U_{IG}^2 + U_V^2}$.

Figure 1(b) shows the derivative $dI_P/dV_{G2}$ as a function of $V_{G1}$ and $V_{G2}$, obtained from a normal-accuracy measurement, following an iterative tuning procedure to find the optimum settings for the DC gate voltages: $(V_{G1}, V_{G2}, V_{G3}, V_{G4}, V_{G5}, V_{G6}) = (-0.96, -0.7, 0.39, -0.78, 0.53, -1)$ V.

4 see supplementary figure S1 (stacks.iop.org/MET/54/299/mmedia).
and $P_{RF} = 5.2$ dBm. During the tuning procedure, plots of $I_p(V_{G1}, V_{G2})$ similar to figure 1(b) were obtained first while systematically stepping $V_{G3}$ and $V_{G5}$ with the aim of maximising the width of the 1e1f plateau. At minimum, a $4 \times 4$ matrix of $(V_{G3}, V_{G5})$ values were investigated. Having found the optimal values of $V_{G3}$ and $V_{G5}$, the procedure was repeated stepping $V_{G4}$ and $V_{G6}$. Note that the relatively large negative values of the voltages applied to the lower finger gates in figure 1(a), combined with the positive voltage applied to the plunger gate $V_{G3}$, has the effect of shifting the QD position above the axis of symmetry defined by the trench gate $V_{G5}$. The approximate location of the QD is indicated by a dashed red circle in figure 1(a) [13].

The data of figure 1(b) was taken as a series of $V_{G2}$ scans at fixed $V_{G1}$, with $V_{G1}$ incremented between scans. This plot, known as the ‘pump map’, shows clearly the regions of zero derivative, where the current is invariant in the two control voltages [20, 21]. The mis-alignment of regions of maximum derivative in successive scans visible in this data also shows that the device operation is affected by a random telegraph signal (RTS) well known to affect this type of 2-DEG structure [22, 23] and already observed in another sample [7] with a similar design to the one in this study. Despite the noise, a broad region can be identified on the one-electron plateau where the derivative is zero within the resolution of the data. In the next section, we use high-accuracy measurements to investigate the robustness of current quantization on the one-electron plateau.

3. High-accuracy plateau measurements

We made a total of 6 high-accuracy measurement scans as a function of the control parameters $V_{G1}, V_{G2}, V_{G3}$ and $P_{RF}$, denoted S1–S6, as well as normal-accuracy measurements over a wider range of each scanned parameter. We also made a further 4 measurements with the pump tuning parameters fixed to the optimal values and $n_{2e2} = 750, 1400, 900$ and $983$, denoted F1–F4. The six scans and four fixed-parameter measurements were made over a period of 14 d. In figure 2 we present data from four of the scans, with each set of high-accuracy data plotted (filled circles) on logarithmic (figures 2(a)–(d)) axes, with normal-accuracy data (open circles) also shown on the logarithmic plots. Each high-accuracy data point in the data of figure 2 is averaged from 70 on-off cycles. The error bars indicate the statistical uncertainty $U_{GT} \sim 2 \times 10^{-6}$, which for these relatively short averaging times is the largest component of the total uncertainty; $U_I \sim U_{GT}$. The normal-accuracy data has sufficient accuracy and signal-to-noise ratio to resolve relative deviations of $\Delta I_p$ from $ef$ as small as $10^{-4}$, and the logarithmic plot is a useful way to visualize the data during the iterative gate tuning procedure. In each scan plotted in figure 2, the fixed parameters were set to the optimum values noted in section 2. Two additional scans were performed, S1 and S2 (not shown in figure 2), with one fixed parameter slightly offset from the optimum: S1 was a $V_{G2}$ scan, with $V_{G1} = -0.975$ V, and S2 was a $V_{G1}$ scan with $V_{G2} = -0.695$ V.

The effect of RTS noise can be seen in the normal-accuracy data, particularly for scan S3, where individual RTS switching events are indicated by gray arrows in figure 2(a). Nevertheless, for each scan, the high-accuracy data exhibits a plateau where $\Delta I_p$ appears invariant in the control parameter within the uncertainty of the individual data points. Scans S3 and S4 can immediately be compared with similar data measured using an etch-defined pump [11], and we note that the plateaus in our gate-defined pump are approximately twice as wide in both entrance gate ($V_{G1}$)
and exit gate ($V_{G2}$) as those in the etch-defined pump. This may reflect a higher charging energy of the gate-defined pump, but it could also be an artifact of different lever arms (gate voltage to QD energy conversion factors) resulting from the very different geometries of the two types of device. Comparing scans S4 and S5 (figures 2(f) and (g)) the effect of the different lever arms of $V_{G2}$ and $V_{G3}$ on the QD level is clear: both of these gates control the depth of the QD, so $I_P$ has a similar functional dependence on either gate, but because $V_{G3}$ is coupled much more strongly to the QD than $V_{G2}$, the plateau occupies a smaller range of gate voltage.

To evaluate the plateau extension more quantitatively, two methods were used. Firstly (the ‘exponential fit method’), we fitted the high-accuracy data $I_P(x)$ to a sum of two exponential functions [15] 

$$I_{fit} = I_0 + e^{-\alpha_1(x-x_1)} + e^{\alpha_2(x-x_2)}$$  

where $\alpha_1$, $\alpha_2$, $x_1$, $x_2$, $\delta_1$ are fitting parameters. The parameter $\delta_1$ is the best-fit offset of the plateau from $ef$. We include it because we do not assume a priori that the plateau is exactly quantised. For runs S1-S6, we found $0.23 \times 10^{-6} \leq \delta_1 \leq 1.33 \times 10^{-6}$.

For runs S3 and S6, only the second exponential term was used for the fit because the data had no clear deviation from the plateau on the low-x axis side. We defined the plateau width as the range of the control parameter for which $|I_{fit}/ef| - 1 - \delta_1 \leq \delta_{fit}$, with $\delta_{fit} = 10^{-7}$. This choice of $\delta_{fit}$ reflects the lower limit to the statistical uncertainty achievable for realistic measurement times of order 1 d. Other studies [8, 11] used the same method to define the plateau, but without including the offset $\delta_1$, and with $\delta_{fit} = 10^{-8}$. The fits are shown in the lower panels of figure 2 as solid lines $^3$, and the resulting selections of data points (number of points $= N_{exp}$) are enclosed by a solid box. The standard deviation of the $N_{exp}$ data points in each scan is denoted $\sigma(\Delta I_P)$, and the statistical uncertainty of $\Delta I_P$ averaged over these points on the plateau is $U_{ST, pla} = \sigma(\Delta I_P)/\sqrt{N_{exp}}$. The scatter of the data points inside the boxes appears to be consistent with their individual uncertainties, but we will address this point more quantitatively in section 4.

Secondly, a purely empirical criterion was used, based on linear fits to sections of the high-accuracy data (the ‘linear fit method’). This method does not make any assumptions about the functional form of the data. For each scan, we found the largest number $N_{lin}$ of consecutive data points for which $|S| < U_{SLOPE}$, where $S$ is slope of a linear fit to the $N_{lin}$ points, and $U_{SLOPE}$ is the uncertainty in the slope $^6$. The resulting data ranges are enclosed by dashed boxes in the lower panels of figure 2, and the relevant parameters are shown in table 1. As with the exponential fit method, the statistical uncertainty of the averaged points is given by $U_{ST, pla} = \sigma(\Delta I_P)/\sqrt{N_{lin}}$. The linear fit method allows us to assign a numerical value to the plateau flatness given by $U_{SLOPE}$ multiplied by the plateau width. The flatness is comparable to the uncertainty of the data points from which it is derived, because the scatter of the data points determines the uncertainty in the linear regression. The flatness therefore is roughly between $1 \times 10^{-6}$ and $2 \times 10^{-6}$ for all the scans irrespective of the plateau width in the scanned units. For example, scans S4 and S5 have plateau widths in gate voltage units differing by roughly a factor 3 due to the different lever arms of $V_{G2}$ and $V_{G3}$ as noted above, but the flatness for both the plateaus is $\sim 2 \times 10^{-6}$. To evaluate the flatness with $10^{-7}$ uncertainty using the linear fit method would require long averaging times, but we note that this is the only unambiguous method of proving that a plateau is flat. The exponential fit method, on the other hand, allows the plateau extension to be estimated based on a much shorter measurement, under the strong assumption that the fitting function (in this case, an exponential) captures all of the physics relevant to the pump accuracy at the target level of uncertainty.

For all the scans, $N_{exp} < N_{lin}$ which is to be expected since we chose $\delta_{fit} \ll U_{ST}$, the exponential fit method estimates the plateau extension to be smaller than the linear fit method, because the latter is only constrained by $U_{ST} \sim 2 \times 10^{-6}$. For scan S3, scatter of some of the data points strongly constrained the range of points which satisfied the linear fit criterion. As can be seen from table 1, a similar scan, S2, exhibited a plateau in $V_{G1}$ more than twice as wide in gate voltage. The question of whether the scatter in run S3 is excessively large is addressed in section 4. Regarding the $P_{RF}$ scan S6, there are some indications in figure 2(h) that an exponential function does not adequately describe the increase of the current for $P_{RF} > 5.7$ dBm, and we speculate that rectification [24] or heating may play a role in the breakdown of quantised pumping at large gate drive amplitudes.

The current averaged over the plateaus, with the plateaus defined using both the exponential (closed triangles) and

$^6$ An estimate of the uncertainty in the slope is given by the square root of the second diagonal element in the $2 \times 2$ covariance matrix output by the fitting algorithm. See standard texts on regression analysis, for example Draper and Smith, *Applied Regression Analysis* (second edition), Wiley (1981), pp 82–85.

### Table 1. Fit and slope parameters.

| Scan number | Scanned variable | $n_{exp}$ | $N_{exp}$ | $N_{lin}$ | Plateau width | Slope $S \times 10^{-6}$ | $U_{SLOPE} \times 10^{-6}$ | Flatness $\times 10^{-6}$ |
|-------------|------------------|----------|----------|----------|--------------|--------------------------|--------------------------|--------------------------|
| S1          | $V_{G2}$         | 25       | 15       | 21       | 40 mV        | 74 V$^{-1}$              | 81 V$^{-1}$              | 3.24                     |
| S2          | $V_{G1}$         | 70       | 6        | 11       | 100 mV       | 19 V$^{-1}$              | 23.7 V$^{-1}$            | 2.37                     |
| S3          | $V_{G2}$         | 70       | 4        | 5        | 40 mV        | 18 V$^{-1}$              | 53 V$^{-1}$              | 2.12                     |
| S4          | $V_{G2}$         | 70       | 6        | 11       | 30 mV        | 24 V$^{-1}$              | 59 V$^{-1}$              | 1.77                     |
| S5          | $V_{G3}$         | 70       | 11       | 12       | 8.25 mV      | 233 V$^{-1}$             | 261 V$^{-1}$             | 2.15                     |
| S6          | $P_{RF}$         | 70       | 10       | 17       | 2.5 dBm      | −0.22 dBm$^{-1}$         | 0.51 dBm$^{-1}$          | 1.27                     |
The un-correlated uncertainty thus allows the different measurements of $\Delta I_P$ to be compared with each other without the additional uncertainty associated with linking to the SI unit system. For example, the two fixed-point runs with the lowest uncertainty, F3 and F4, are consistent within their combined uncertainty of $7.8 \times 10^{-7}$. If the plateau is defined using the exponential fit method, the average current is consistent within $\epsilon_f$ within the uncertainties, and furthermore there are no major inconsistencies between the data points when only the un-correlated uncertainty is considered. Averaging all the data from the four fixed-point runs (a total of 4033 cycles lasting 47h) reduced $U_{ST}$ such that $U_{T} \sim U_{IG}$ and yielded a best estimate of the pump current: $\Delta I_P = 0.28 \pm 0.86 \times 10^{-6}$. This is marginally more accurate than the previous best electron pump measurement using the current measurement system at NPL [9], although it falls short of the record low uncertainty of $1.6 \times 10^{-7}$ recently reported [11] using a measurement system based on a new type of ultra-stable current preamplifier known as an ‘ULCA’ [6]. Future efforts will aim to reduce $U_{IG}$ to around $2 \times 10^{-7}$ as well as implementing an ULCA-based measurement system at NPL. It is interesting to note that the accumulated precision measurements and associated theoretical fit lines [5, 7–9, 11], suggest that a tunable-barrier electron pump operated at an optimal working point is accurate at the $1 \times 10^{-7}$ level. With this premise, we could hypothetically consider the pump as a primary current standard, and the data of runs F1–F4 as constituting a calibration of the reference resistor with total uncertainty $\sim 3 \times 10^{-7}$, almost a factor 3 lower than the $U_{IG}$ presently achievable at NPL. However, we believe such a step would be premature, and that the robustness of these pumps requires further extensive investigation before a consensus can be reached on the required set of conditions for operation at a given accuracy level.

4. Statistical evaluation of plateau current

The data points on the plateaus in figure 2 show some scatter, and we now evaluate whether this scatter is consistent with statistical scatter about a stationary mean or whether it is a sign of structure on the plateau, or possibly drift in the pump current or the measurement system. We note that recent developments in the metrology of small currents [6, 25] have focused attention on the stability of high-value thick-film standard resistors, principally those of 100 MΩ value. The 1 GΩ standard resistor used in the reference current source is also a thick-film design, and may suffer from short-term instability at the sub-ppm level. However, the uncertainties in the data of figures 2 and 3 are too large for this to have a significant effect on the scatter of the data points. We focus on the more conservative (narrower) plateaus defined using the exponential fit method. For each scan, the mean of the $N_{exp}$ measurements of $\Delta I_P$ to have a given standard deviation, assuming that $\Delta I_P$ is normally distributed with standard deviation ($U_{ST}$). (c): Allan deviation of pumped current as a function of the number of on-off cycles, calculated from 3 of the runs at fixed operating point. The Allan deviation for run F1 (not shown) exhibited similar behavior. The gray dashed line shows the expected $1/\sqrt{T}$ dependence for frequency-independent Johnson–Nyquist noise in the reference resistor.

linear (open circles) fit methods, is plotted in figure 3(a) for runs S1–S6. Error bars show the un-correlated uncertainty $\sqrt{U_{ST,plat}^2 + U_N^2}$. The current measured in runs F1–F4 with the pump at fixed operating point is also plotted on the same graph (closed circles), with error bars indicating $\sqrt{U_{ST}^2 + U_N^2}$. The un-correlated uncertainty does not include $U_{IG}$, which is shown as a grey box centred on $\Delta I_P = 0$. The resistor was calibrated before and after the measurement campaign and its value was assumed constant during the campaign based on its long-term drift rate of $\sim 0.01(\mu \Omega/\text{s}) \times 10^{-7}$ [5]. In contrast, the voltage measurement was calibrated before and after each run. The un-correlated uncertainty thus allows the different measurements of $\Delta I_P$ to be compared with each other without the additional uncertainty associated with linking to the SI unit system. For example, the two fixed-point runs with the lowest uncertainty, F3 and F4, are consistent within their combined uncertainty of $7.8 \times 10^{-7}$. If the plateau is defined using the exponential fit method, the average current is consistent within $\epsilon_f$ within the uncertainties, and furthermore there are no major inconsistencies between the data points when only the un-correlated uncertainty is considered. Averaging all the data from the four fixed-point runs (a total of 4033 cycles lasting 47h) reduced $U_{ST}$ such that $U_{T} \sim U_{IG}$ and yielded a best estimate of the pump current: $\Delta I_P = 0.28 \pm 0.86 \times 10^{-6}$. This is marginally more accurate than the previous best electron pump measurement using the current measurement system at NPL [9], although it falls short of the record low uncertainty of $1.6 \times 10^{-7}$ recently reported [11] using a measurement system based on a new type of ultra-stable current preamplifier known as an ‘ULCA’ [6]. Future efforts will aim to reduce $U_{IG}$ to around $2 \times 10^{-7}$ as well as implementing an ULCA-based measurement system at NPL. It is interesting to note that the accumulated precision measurements and associated theoretical fit lines [5, 7–9, 11], suggest that a tunable-barrier electron pump operated at an optimal working point is accurate at the $1 \times 10^{-7}$ level. With this premise, we could hypothetically consider the pump as a primary current standard, and the data of runs F1–F4 as constituting a calibration of the reference resistor with total uncertainty $\sim 3 \times 10^{-7}$, almost a factor 3 lower than the $U_{IG}$ presently achievable at NPL. However, we believe such a step would be premature, and that the robustness of these pumps requires further extensive investigation before a consensus can be reached on the required set of conditions for operation at a given accuracy level.

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4. Statistical evaluation of plateau current

The data points on the plateaus in figure 2 show some scatter, and we now evaluate whether this scatter is consistent with statistical scatter about a stationary mean or whether it is a sign of structure on the plateau, or possibly drift in the pump current or the measurement system. We note that recent developments in the metrology of small currents [6, 25] have focused attention on the stability of high-value thick-film standard resistors, principally those of 100 MΩ value. The 1 GΩ standard resistor used in the reference current source is also a thick-film design, and may suffer from short-term instability at the sub-ppm level. However, the uncertainties in the data of figures 2 and 3 are too large for this to have a significant effect on the scatter of the data points. We focus on the more conservative (narrower) plateaus defined using the exponential fit method. For each scan, the mean of the $N_{exp}$ measurements of $\Delta I_P$ to have a given standard deviation, assuming that $\Delta I_P$ is normally distributed with standard deviation ($U_{ST}$). (c): Allan deviation of pumped current as a function of the number of on-off cycles, calculated from 3 of the runs at fixed operating point. The Allan deviation for run F1 (not shown) exhibited similar behavior. The gray dashed line shows the expected $1/\sqrt{T}$ dependence for frequency-independent Johnson–Nyquist noise in the reference resistor.
the plateau was drifting on the time-scale of the scan, or if the plateau was not flat, we expect \( \sigma(\Delta I_p) > \langle U_{ST} \rangle \). To assign a statistical significance to the ratio \( \sigma(\Delta I_p)/\langle U_{ST} \rangle \), we used a numerical simulation to assign a 68% confidence interval to the distribution of \( \sigma(\Delta I_p) \) expected for \( N_{\text{exp}} \) normally-distributed measurements with standard deviation \( \langle U_{ST} \rangle \)\(^3\). This is plotted as upper and lower horizontal bars in figure 3(b). The fixed-parameter runs F1–F4 were evaluated in the same way as the scans, by dividing the data into blocks of 70 cycles and analyzing each block separately. Over the whole data set, there is no statistically significant deviation of the ratio \( \sigma(\Delta I_p)/\langle U_{ST} \rangle \) from 1. One particular run, S3, appeared to have anomalously large scatter, visible in figure 2(e) and already discussed in section 3. This scatter is apparent in figure 3(b), in the relatively large ratio of \( \sigma(\Delta I_p)/\langle U_{ST} \rangle \). However, \( \sigma(\Delta I_p) \) is still just within the 68% confidence interval, clarifying that the data at different \( V_{G3} \) values cannot be distinguished from data drawn from the same distribution. Overall, we conclude from this analysis that the scatter of the data points on the plateaus is consistent with statistical fluctuations about a stationary mean.

This conclusion is supported by the Allan deviation of the current measured from runs F1–F4, all of which exhibited similar behavior. The Allan deviation plots for runs F2–F4 are shown in figure 3(c). They show no significant deviation from the expected \( \sqrt{t} \) behavior for frequency-independent noise out to the longest averaging times probed by the Allan deviation analysis \([26]\), roughly one quarter of the total measurement time, or \( \sim 3 \)h. For comparison, the dashed line shows the expected Allan deviation of frequency independent Johnson–Nyquist noise in the 1 GΩ resistor, \((4.2 \text{ fA} \sqrt{\text{Hz}}^{-1})/\sqrt{2\pi}\), where \( \tau = 40 \) s is the time for one on-off cycle. The Allan deviation of the pump current is increased above this theoretical level due to three inefficiencies in the duty cycle which reduce the effective averaging time: the on-off cycle means the pump current is only measured for half the time, auto zero in the readout voltmeters halves the measurement time again, and rejection of data points at the start of each half-cycle, to eliminate transient effects, further reduces the duty cycle. The latter two of these effects need to be optimized in future experiments to yield a lower overall statistical uncertainty \([11]\).

5. Pumping regime and noise broadening

The relatively high temperature of these measurements compared to previous high-precision studies motivated us to consider the mechanism of charge capture by the pump. At low temperatures, this occurs by a cascade of one-way tunneling events whereby electrons tunnel back to the source electrode as the QD is progressively isolated from the source \([4, 27]\). The experimental signature of the decay cascade is a characteristic double-exponential shape to the pump current as a function of the QD depth-tuning parameter. This tuning parameter can be the ‘exit gate’ voltage in simple two-gate pumps \([5, 8, 14]\), or a global top gate voltage \([9, 27]\), and

\[ F4 \]

\[ \text{Decay cascade} \]

\[ \text{Thermal} \]

\[ \chi^2 = 3.0 \times 10^{-4} \]

\[ \delta_2 = 20 \]

\[ \delta_1 = 20 \]

\[ B \]

\[ V_n \text{(mV)} \]

\[ F \]

\[ \text{Fit parameter} \]

\[ \text{Chi-Square} \]

\[ \text{Thermal} \]

\[ \text{Decay cascade} \]

\[ \text{Thermal} \]

\[ \chi^2 \]

\[ \delta_2 \]

\[ \delta_1 \]

\[ B \]

\[ V_n \text{(mV)} \]

\[ F \]

\[ \text{Fit parameter} \]

\[ \chi^2 \]

\[ \text{Thermal} \]

\[ \text{Decay cascade} \]

\[ \chi^2 \]

\[ \delta_2 \]

\[ \delta_1 \]

\[ B \]

\[ V_n \text{(mV)} \]

\[ F \]

\[ \text{Fit parameter} \]

\[ \chi^2 \]

\[ \text{Thermal} \]

\[ \text{Decay cascade} \]

\[ \chi^2 \]

in this work its role can be fulfilled by either \( V_{G2} \) or \( V_{G3} \). At higher temperatures, experimental \([16]\) and theoretical \([15, 28]\) work has indicated a cross-over to a thermal regime, in which back-tunneling is accompanied by forward tunneling into the QD from the source. This results in a symmetric shape to the current as a function of QD depth tuning parameter, reflecting the Fermi distribution of electrons in the leads. The cross-over to the thermal regime has been predicted to occur for \( 10 \times k_B T \gtrsim \Delta_{\text{ph}} \)\([15]\). Here, \( \Delta_{\text{ph}} \) is defined as the change in energy of the QD level when the entrance barrier transmission changes by a factor of Euler’s number \( \sim 2.718 \ldots \) and it thus quantifies the device-specific cross coupling between the modulated entrance barrier, and the QD energy level \([28]\). We crudely estimate \( \Delta_{\text{ph}} \approx 1 \text{ meV} = 8.9 \times k_B T \) for our device, based on the slope of representative conductance pinch-off data and typical lever arm factors between a gate voltage and QD energy level. From this estimate we expect the device to be between the two regimes, and we next examine experimental data to clarify the capture mechanism.

In figure 4(a), we plot the normalized pump current as a function of \( V_{G3} \), which functions as a QD depth-tuning gate.
A RTS is visible in the transition between the plateaus, where the pump current is a sensitive probe of changes in the electrostatic potential. On the plateau, the current is insensitive to the state of the RTS. For the data of figure 4(a), in the transition region between $I_P = 0$ and $I_P = e f$, the charge state causing the RTS noise appears to be in one state for the majority of the data points (filled points), and the points affected by a switch to the other state (open points) were excluded from fitting. The data is fitted to the decay cascade model [4]:

$$I_P = \frac{e f}{\chi} \sum_{n=1}^{2} \exp\left(-\exp\left(-aV_G + \Delta_m\right)\right)$$

over the full range (solid line), with reduced $\chi^2 = 2.6 \times 10^{-5}$, yielding the fit parameter $\delta_2 \equiv (\Delta_2 - \Delta_1) = 15.2$, and a thermal equilibrium (Fermi function) model [16, 28]:

$$I_P = \frac{1}{1 + e^{\frac{4(V_G - V_{th})}{\chi}}}$$

in the range $0.32 \leq V_{G3} \leq 0.38$ (dotted line) with reduced $\chi^2 = 3.5 \times 10^{-4}$. Close inspection of the fit lines shows that the decay-cascade model gives a better fit, and the thermal equilibrium model fails to reproduce the asymmetric plateau shape, with a sharp riser from $I_P = 0$ and a more gradual approach to $I_P = e f$. The reduced $\chi^2$ for the decay-cascade fit is more than a factor 10 smaller than the thermal equilibrium fit, suggesting that the pump is operating in the decay cascade regime. A similar conclusion was drawn by fitting the data of figure 4(a) with reduced $\chi^2$ values obtained from fitting the data of figure 4(a) with that calculated from noise broadening, we conclude that the experimentally measured data is not consistent with more than a few mV of noise broadening, and the pump is indeed operating in the decay cascade regime at our experimental temperature.

6. Conclusions

In conclusion, pumping in a GaAs tunable-barrier electron pump is robust against changes in the gate control parameters, and the RF drive amplitude, at the part-per-million level at a temperature of 1.3 K. The presence of two-level fluctuators did not affect the accuracy of the pump current. Compared to previous studies, this relaxes the experimental conditions required to observe quantised pumping at the part-per-million accuracy level, which is a promising step towards adoption of quantised charge pumps as current standards.

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