Wave function for two-neutron halo states

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Abstract. In this work we investigate the theoretical aspects associated with the few-body universal properties of the weakly-bound two neutron halo in light nuclei. We emphasize that our study of wave functions can be used as inputs in calculations of reaction processes with light exotic nuclei. Starting from the Faddeev decomposition of the wave function for 3-body systems interacting through zero range forces, we calculate two-neutron halo states for large two-body scattering lengths coming closer to what is know today as Efimov’s Physics. We report our findings for the wave function for a three-body $n-n-C$ system, where we consider two neutrons as a halo around a compact core $C$. We focus our study on halo structure of Borromean systems like $^{11}$Li, $^{14}$Be, and $^{22}$C.

1. Introduction
Exotic neutron-rich light nuclei can be explored theoretically by considering few-body degrees of freedom. It forms a few-body system when it has a core nucleus surrounded by a halo of orbiting neutrons in large excess which are called halo nuclei or exotic nucleus. The halo is defined by neutrons which have a high probability to be found far away from the core. The core can be described by a more stable isotope of the element, which is surrounded by the halo neutrons. The small separation energy of the halo neutrons allows them to be distributed over a spatial region extending beyond the region occupied by the core, building a diffuse cloud that contains the neutrons, forming a halo. The concept of a short-range interaction between the particles and its implications are useful in understanding the quantum few-body physics of the halo [1]. The curious large size of the halo has motivated several studies, both experimental [2, 3] and theoretical [4], devoted to clarify the new aspects of the structure, stability and reaction of these nuclei and also the astrophysical implications.

In general, these light three-body halo nuclei systems have unusual properties in respect to the nuclear size: the radius of the neutron halo is much greater than the radius of the core, and it is assumed that the core has no structure [5, 6, 7]. We discuss here a situation which allows the use of concepts coming from short-range interactions. The two-neutron halo bound state has its properties dominated by the Efimov’s effect [8, 9, 10], which is associated with the Thomas collapse [11] that corresponds to infinite value of three body binding energy in the limit of a zero-range potential. The Efimov effect [12] corresponds to the appearance in the three-boson spectrum S-wave of states with binding energy geometrically separated and condensing at zero energy, when the scattering length is let to infinity and the dimer is bound at the scattering threshold. As the scattering is large with respect to the interaction range, the limit of infinite scattering length with respect to the interaction range, is physically the same as the range...
decreasing towards zero with a finite scattering length. This relates the Thomas collapse and the Efimov effect (for more discussions see [4]).

Here we will describe a three-body halo-nuclei with a neutron-neutron-core structure (we use the notation $n - n - C$). Examples of weakly-bound three-body systems are $^{11}$Li, $^{14}$Be, $^{20}$C, and $^{22}$C, where the universal aspects and Efimov physics can be explored [13]. In figure 1 (adapted from ref. [13]), we schematically represent the large sizes of the two neutron halo in $^{11}$Li and $^{22}$C. The two neutron binding energy in the $^{11}$Li isotope is the limit of the pairing energy at the nuclear surface. Coming closer to the neutron drip line the behavior of the separation energy of the neutrons is expected to decrease, but the interesting aspect of this nucleus is that while in $^{10}$Li the neutron is unbound and escapes, the two valence neutrons are bound to $^{11}$Li, an effect due to energy pairing. Thus, $^{11}$Li is divided into three parts, core plus two neutrons. Cores of this type, like $^{9}$Li which does not capture an extra neutron, but binds two are called Borromean. The heraldic symbol of the Renaissance Borromeo family represented by the three intertwined rings, such that if one ring is removed the others are too. Another important consequence of the large neutron halo is that it can be easily excited in a vibration mode against the core known as pigmy resonances (see e. g.[14]).

These isotopes along the neutron and proton driplines are important for understanding the limits of the nuclear stability. The study of nuclear structure and reactions of unstable weakly bound halo nuclei presents a challenge to standard methods in nuclear physics as the degrees of freedom of the neutron or proton halo should be taken into account explicitly.

In our work we model a class of halo nuclei described as a three-body system, composed by two neutrons and a core, where the pairwise interactions is modelled by Dirac $\delta-$potentials. The model is illustrated by computing a hypothetical highly excited Efimov state of the neutron halo. The present study brings to the nuclear context the previous work of Yamashita and co-workers [15] performed for cold atoms. The use of the contact interaction to study halo physics is justified by the large size of the halo compared with the range of the interaction. This simple model retains the essential physics of the weakly bound and large two-neutron halo systems. Furthermore, such structures are close to the Efimov limit and as we have discussed are associated to the collapse of the three-body state in the limit of a zero-range interaction. Due to the Thomas collapse, the low-energy properties of the three-body system are well defined if one three-body short range information is given. In addition the low-energy two-body physical informations should be supplied to the system. The ultraviolet divergences from the short range singularity coming from the contact potential can be regularised and renormalized [16].

2. The Wave Function

In this section we work out the wave function of the two-neutron halo, which is our goal. We use the notation $n$ for neutron and $C$ for core, but our approach is applicable to any three-particle system, which interacts via s-wave short-range interactions, where two of the particles are identical. The neutrons are supposed to be in a spin singlet state.

2.1. The Formalism

In a three-body system there are three two-body subsystems and pairwise potentials. We start by writing the Schroedinger equation for the three-body system as,

\[ (E - H_0)\psi = V\psi, \]

where $\psi$ is the bound state wave function. The potential $V$ is the sum of three-body potential and $H_0$ is the free hamiltonian, which should be written in terms of the Jacobi relative momenta. For a zero-range potential the three-body wave function for a system composed by two neutrons
and a core, can be written in terms of the spectator functions in the basis $|\vec{q}_c\vec{p}_c\rangle$, where $q_c$ is the relative momentum of the center of mass of the neutron in respect to the core, and $p_c$ is the relative momentum of the two neutrons. The two-neutron halo wave function is:

$$\langle \vec{q}_c\vec{p}_c | \Psi \rangle = f_{nn}(q_c) + f_{nc}(p_c - \frac{\vec{q}_c}{2}) + f_{nc}(p_c + \frac{\vec{q}_c}{2})$$

where the functions $f_{nn}$ and $f_{nc}$ are the spectator functions computed in the next section. The free hamiltonian is

$$H_0 = \frac{p_c^2}{2\mu_{nn}} + \frac{q_c^2}{2\mu_{nn,c}}$$

and the reduced masses are given by,

$$\mu_{nn,c} = \frac{2m_c m_n}{2m_n + m_c}, \quad \mu_{nc,n} = \frac{m_n(m_n + m_c)}{2m_n + m_c}, \quad \mu_{nn} = \frac{m_n}{2}, \quad \mu_{nc} = \frac{m_c m_n}{m_c + m_n}.$$ 

We can treat the $n-n-C$ wave function (2) in configuration space, and thus can get spatial distribution of the halo neutrons. As a matter of fact, the neutrons in the halo have a large probability to be found in the classically forbidden region where the wave-function is an eigenstate of the free Hamiltonian (1), and therefore the configuration space representation of the wave function (2) is written as,

$$\Psi(\vec{r}_n, \vec{r}_n') = \int d\vec{q} e^{-\kappa_{nn}|\vec{R}_{nn}|} e^{i\vec{q}.\vec{R}_c} f_{nn}(\vec{q}) + \int d\vec{q} e^{-\kappa_{nc}|\vec{R}_{nc}|} e^{i\vec{q}.\vec{R}_n} f_{nc}(\vec{q}) + \ldots,$$

where $\kappa_{nn} = \sqrt{2\mu_{nn} \left(S_{2n} + \frac{q_c^2}{2\mu_{nn,c}}\right)}$, $\kappa_{nc} = \sqrt{2\mu_{nc} \left(S_{2n} + \frac{q_c^2}{2\mu_{nc,n}}\right)}$ and $\vec{r}_n, \vec{r}_{n'}$ are neutron positions with respect to the center of mass as depicted in figure 2. $(S_{2n})$ is two-neutron separation energy of $^{11}\text{Li}$, which in the nuclear scale is fairly small with a value of $S_{2n} = 369.15(65)$ keV. There are another contribution to the wave function obtained from the last term by exchanging $n$ with $n'$ that is represented by dots in equation (4). $|\vec{R}_{nn}|$ and $|\vec{R}_{nc}|$ are relative distances for the $n-n$ and $n-core$ respectively. The relative position of the neutron to the centre of mass of the $n-C$ system is $R_n$. The relative positions are illustrated in fig.2.
2.2. Subtracted Faddeev Equations - Excited Bound States

The subtracted s-wave coupled integral equations for the spectator functions, $f$ of the $n - n - C$ for the zero-range interaction, generalizes the Skorniakov and Ter-Martirosyan [18], making them regularised, and for the case of $^{11}\text{Li}$ with two neutrons and a core, are written as (see e.g. [4]):

$$f_{nn}(q) = 2\tau_{nn}(q, E_3) \int_0^\infty dq' \frac{q'}{q} G_1(q, q', E_3) f_{nc}(q'),$$

$$f_{nc}(q) = \tau_{nc}(q, E_3) \int_0^\infty dq' \left[ \frac{q'}{q} G_1(q, q', E_3) f_{nn}(q') + A G_2(q, q', E_3) f_{nc}(q') \right],$$

where the absolute value of the three-body binding energy is $E_3$ and,

$$\tau_{nn}(q, E_3) = \frac{1}{\pi} \left[ \sqrt{E_3 + \frac{A + 2}{4A} q^2 + \sqrt{E_{nc}}} \right]^{-1},$$

$$\tau_{nc}(q, E_3) = \frac{1}{\pi} \left( \frac{A + 1}{2A} \right)^{3/2} \left[ \sqrt{E_3 + \frac{A + 2}{4A} q^2 + \sqrt{E_{nc}}} \right]^{-1},$$

$$G_1(q, q', E_3) = \log \frac{2A(E_3 + q^2 + q'q) + q^2(A + 1)}{2A(E_3 + q^2 - q'q) + q^2(A + 1)},$$

$$G_2(q, q', E_3) = \log \frac{2A(E_3 + q^2 + q'q) + q^2(A + 1)}{2A(E_3 + q^2 - q'q) + q^2(A + 1)} - \log \frac{2A(q + q^2 + q'q) + q^2(A + 1)}{2A(q + q^2 - q'q) + q^2(A + 1)},$$

we assume $\hbar = 1$ and $A = m_c/m_n$ defines the mass ratio and $m_n$ is the neutron mass. For the subsystems $n - n$ and $n - C$ we have the two-body binding energies as $E_{nn}$ and $E_{nc}$.

The numerical solution of the set of subtracted coupled equations (5) for the spectator function $f_{nn}(q)$ of the $n - n - C$ system in momentum space for a highly excited state is compared to the respective asymptotic formula in fig.3. The asymptotic form for large momentum is given by

$$f_{nn}(q)|_{q \rightarrow \infty} \rightarrow C_{nn} q^2 \sin(s \log(q/q_0)).$$

For more details about this form, as well as how to calculate parameter $s$, see reference [15]. The spectator function from the numerical solutions and the asymptotic one should coincide in the limit of an infinitely high excited state. But in practice both should be compared in the
Figure 3. Spectator function $f_{nn}(q)$ of a highly excited state for subsystem energies $E_{nn} = E_{nC} = 0$ and $A=9$, with $E_3 = 1.478 \times 10^{-12} E_0$, solution of the coupled equations (5) (blue line), compared with the asymptotic form (11) (red line). We have introduced $E_0 = \hbar^2 \mu^2 / m_n$ and we work in units where $\hbar = m_n = \mu = 1$.

window $k_0 << q << \mu$, where $k_0 = \sqrt{E_3}$, where the momentum is larger than the characteristic momentum of the state, $K_0$, and smaller than the subtraction scale $\mu$. We use $q$ to denote momentum and the boundary condition $f(q_0) = 0$ is chosen to match the numerical solution of couple subtracted equation, as we see in fig. 3.

3. Conclusions and Outlook
Weakly bound systems were addressed in the context of unstable neutron-rich nuclei. In the regime of a zero-range interaction, few scales determine the low-energy properties of two-neutron halo nuclei. The low-energy properties of such systems are dominated by the Efimov physics, which is universal and model independent. The integral equations for the bound state of three-body systems are easily solved numerically. After the regularization and renormalization of the model is done by fixing the three-body binding energy, by a convenient choice of the subtraction scale. We study one example of a highly excited Efimov state and the spectator function compared to the asymptotic form. The next step is to obtain the full wave function of the s-wave two neutron halo state in configuration space and study the one and two-body neutron densities.

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5. References
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