METALLICITIES FOR DOUBLE-MODE RR LYRAE STARS IN THE LARGE MAGELLANIC CLOUD

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ABSTRACT

Metallicities for six double-mode RR Lyrae stars (RRd's) in the Large Magellanic Cloud have been estimated using the AS method. The derived [Fe/H] values are in the range [Fe/H] = −1.09 to −1.78 (or −0.95 to −1.58, adopting a different calibration of [Fe/H] versus AS). Two stars in our sample are at the very metal-rich limit of all RRd's for which metal abundance has been estimated, either by direct measure (for field objects) or on the basis of the hosting system (for objects in globular clusters or external galaxies). These metal abundances, coupled with mass determinations from pulsational models and the Petersen diagram, are used to compare the mass-metallicity distribution of field and cluster RR Lyrae variables. We find that field and cluster RRd's seem to follow the same mass-metallicity distribution, within the observational errors, strengthening the case for uniformity of properties between field and cluster variables. At odds to what is usually assumed, we find no significant difference in mass for RR Lyrae variables in globular clusters of different metallicity and Oosterhoff types, or there may even be a difference contrary to the commonly accepted one, depending on the metallicity scale adopted to derive the masses. This “unusual” result for the mass-metallicity relation is probably due, at least in part, to the inclusion of updated opacity tables in the computation of metal-dependent pulsation models.

Key words: Magellanic Clouds — stars: abundances — stars: oscillations — stars: variables: other — techniques: spectroscopic

1. INTRODUCTION

The well-known dichotomy between short and long distance scales derived from old, Population II stars has plagued astronomers for a long time and not even the impressive improvements in measuring distances due to the Hipparcos mission (see, e.g., Gratton et al. 1997) seem to provide a universally accepted solution. This is pointed out by the disagreement existing between results based on field or cluster stars. Methods founded on field stars, like statistical parallaxes or the Baade-Wesselink method applied to field RR Lyrae stars or direct analysis of field horizontal-branch (HB) stars (see Gratton 1998), seem to favor the short scale. Instead, cluster star-based distances, derived from main-sequence fitting to local subdwarfs (see, e.g., Gratton et al. 1997) or from pulsational properties of RR Lyrae stars in globular clusters (Sandage 1993) or from the calibration of the HB luminosity level using the Cepheid distance modulus of the Large Magellanic Cloud (LMC; Walker 1992), seem to support the long scale.

Following a calibration of the absolute magnitude of horizontal-branch, field metal-poor stars with good parallaxes from Hipparcos, Gratton (1998) suggested that a difference (at an ~0.1–0.2 mag level) might actually exist between the luminosity of HB stars in globular clusters (GCs) and HB stars in the field. This result appeared to be confirmed by the difference in the average magnitude for field and cluster RR Lyrae stars in the LMC measured by Alcock et al. (1996) and Walker (1992), respectively (but see Clementini et al. 2001, hereafter C01). Another indication in favor of this possible difference comes from the latest evolutionary models by Sweigart (1997), which include some extra mixing of He and other heavy elements (C, N, and O). As a matter of fact, there is observational evidence for extra mixing in cluster red giants but not among field stars (Gratton et al. 2000). On the other side, studies by Catelan (1998) and Carretta, Gratton, & Clementini (2000) put this suggestion to the test using the pulsational properties of a selected sample of field and cluster RR Lyrae stars. Carretta et al. (2000) show in a quantitative way that in our Galaxy field and cluster RR Lyrae stars cannot be distinguished in a $\log P_{\text{field-cluster}}$-[Fe/H] plane: this goes toward excluding the possibility of a luminosity difference. In fact, the pulsational period-mass-luminosity-T_eff relation (van Albada & Baker 1971) tells us that, at fixed T_eff, a difference in the period distribution (or its absence) indicates that there is (or there is not) a difference in the mass-luminosity ratio for the variables. The determination of star masses with the high precision required to settle the question is still one of the most difficult tasks in astrophysics. In this context, double-mode RR Lyrae stars are extremely powerful tools because their masses can be estimated from the ratio of the two pulsational periods using pulsational models hence independently from stellar evolution models. Unluckily, the $M$-[Fe/H] relation can currently be studied in our Galaxy only using double-mode pulsators in clusters, since only a handful of RRd's have been so far identified in the field of our Galaxy.

The field RR Lyrae stars in the LMC bar play a key role in this respect, since (1) projection effects are negligible and they can be considered at the same distance from us, and (2) plenty of double-pulsating variables have been identified in the field of the LMC by the MACHO experiment (Alcock et al. 1997, hereafter A97). Field RR Lyrae stars in the LMC with good [Fe/H] determinations may allow the derivation of both an accurate $L = f([\text{Fe}/\text{H}])$ (using single-mode...
pulsators) and a $M = f([\text{Fe/H}])$ relation, if any (using double-mode pulsators), within a homogeneous sample of field variables. The accuracy required to settle the question of the magnitude difference can be estimated from the pulsation equation of van Albada & Baker (1971), computed at fixed temperature and period. In order to appreciate a supposed difference of $\Delta \log L \sim 0.05$ (or 0.12 mag), we must detect a $\Delta \log M \sim 0.06$ (i.e., $\sim 0.12 M_\odot$) difference in mass between field and cluster variables. Cox (1991), using a fixed metallicity and the new OPAL (Rogers & Iglesias 1992) opacity tables, found masses of $0.65 M_\odot$ for RR Lyrae pulsators in Oosterhoff type I clusters (Oo I; Oosterhoff 1939) and masses of 0.75 up to even 0.85 $M_\odot$ in Oosterhoff type II clusters (Oo II). Since the approximate difference in metallicity between the two Oosterhoff-type clusters he studied is $0.5$–$0.7$ dex, we estimate that an accuracy of $\pm 0.2$ dex in $[\text{Fe/H}]$ is then required.

We started an observational program on the field RR Lyrae stars of the LMC to (1) derive their average apparent luminosity and (2) determine their metallicities (in particular for the RRd pulsators). The first part of the program, mostly dealing with new photometric data acquired by our team near the bar of the LMC, is described in C01 and Di Fabrizio et al. (2001). The main results are the very accurate determination of the mean apparent magnitude of the field RR Lyrae stars of the LMC and the derivation of an independent reddening estimate.

From the light curves so derived, we determined epochs to properly time the spectroscopic observations. In this paper, we present metallicities with an accuracy of $\Delta [\text{Fe/H}] \sim 0.2$ dex for six RRd's in the LMC. We used the $\Delta S$ index (Preston 1959), defined as $\Delta S = 10[\text{SpT}(H) - \text{SpT}(K)]$, where $\text{SpT}(H)$ and $\text{SpT}(K)$ are the spectral types measured at minimum light, [in units of tenths of a spectral type class], based on the hydrogen lines and on the Ca II K line, respectively. In § 2, we present the observational data, and in § 3 we describe the derivation of the $\Delta S$ values for the six stars. Section 4 is devoted to the determination of metallicities; in § 5, we derive masses of our targets from the Petersen diagram and discuss the mass-metallicity distribution. A summary and our conclusions are presented in § 6.

2. Observations and Reduction

The MACHO program discovered about 7900 RR Lyrae stars in the 22 40 arcmin$^2$ fields centered on the LMC bar (Alcock et al. 1996) and published coordinates and periods (but no epochs) for 73 RRd's in the LMC bar (A97; differential light curves can be found on the MACHO Web site). Since the $\Delta S$ method requires the acquisition of spectra when variables are at minimum light, new photometric observations were obtained in order to derive complete ephemerides for some of A97's double-mode pulsators. All observations were carried out in La Silla, Chile, in 1999 January. Four nights at the Danish 1.54 m telescope, one of which was of photometric quality, were dedicated to Johnson $V$ and $B$ photometry (see C01 for a complete description) and immediately reduced to produce light curves and derive ephemerides (epochs, in particular) for our targets. We tried to maximize the number of RRd's observable in a single Danish Faint Object Spectrograph and Camera pointing (field of view of 13.5 arcmin$^2$), and for the present program, we selected two positions (in MACHO fields No. 6 and 13) for a total of nine RRd's; we refer to them as "field A" ($\alpha_{2000} = 05^{h}22^{m}44^{s} \delta_{2000} = -70^034'15") and "field B" ($\alpha_{2000} = 05^{h}17^{m}28^{s} \delta_{2000} = -71^000'14")". Henceforth, RRd's in the two fields will be indicated by CA or CB followed by an ordering number derived from Table 1 of A97. Periods in A97 were used together with the epochs obtained from the new photometric data to properly schedule at minimum light the subsequent spectroscopic observations done on two nights only 10 days afterward. We did not use our own derived periods, since C01 sampling of the light curve of the double-mode pulsators (about 60 data points in four consecutive nights) does not allow the derivation of periods for the RRd's as accurately as in A97. We show in Figure 1 the light curve for one of the RRd's based on C01 data and phased using A97's first-overtone period, $P_1$. Finding charts for the six stars discussed here are presented in Figure 2, taken from C01 photometry; maps are 200 arcsec$^2$, and the position of each RRd is indicated by a cross (not in scale) right at the center of the fields.

The spectroscopic observations were carried out at the 3.6 m ESO telescope during the nights 1999 January 17–18. The ESO Faint Object Spectrograph and Camera 2 was used, mounting the CCD No. 40, a Loral 2K $\times$ 2K chip, binned 2 $\times$ 2, in combination with grism No. 7 (600 lines mm$^{-1}$, 3270–5240 Å) and a $1.5'$ wide slit, resulting in a $\sim 9$ Å resolution (or $R \approx 450$). Whenever possible, the slit was rotated in order to exclude nearby contaminating stars. Sky conditions were good enough for spectroscopic observations but not of photometric quality, and the seeing varied from about $1'$ to $1.9'$, with an average value around $1.5'$.

We observed close to minimum light seven of the nine RRd's within these fields, exploiting the previously described ephemerides, obtaining usable spectra for six of them (for one of the stars, the spectrum is heavily contaminated by the very close bright object falling into the slit, and we excluded it). Identifications and information on these six targets are provided in Table 1. Equatorial coordinates come from the transformation of pixel positions to right ascension and declination, based on about 100 stars individuated both in each of the two fields and on the Digitized...
Fig. 2.—Finding charts for the six program stars derived from C01 photometry (top, left to right: CA 02, 48, and 67; bottom, left to right: CB 45, 49, and 61). Each RR\(d\) is at the center, indicated by a cross (coordinates are found in Table 1); the fields shown cover 200 arcsec\(^2\), with north at the top and east to the left.

Sky Survey,\(^6\) they differ by only a few arcseconds from the ones published by A97. The \(\langle V \rangle\) values are intensity averages and should be taken as preliminary, since a new calibration of the photometric data is under way (see C01); epochs of maximum light are given in column (8). The fundamental and first-overtone periods, \(P_0\) and \(P_1\), are taken from A97. In columns (9) and (10), we give the Heliocentric Julian Date of the spectroscopic observations of our targets and the corresponding phases computed using A97’s first-overtone pulsation periods and the epochs in column (8). Given the faintness of our targets (\(V \approx 19.5\) at minimum light), exposure times ranged from 30 to 50 minutes in order to reach the maximum possible signal-to-noise ratio (S/N) and avoid phase blurring on the pulsation cycle. Typical S/Ns range from 10 to 30.

Thirteen nonvariable stars in the open cluster Collinder 140 (Cr 140; Claria & Rosenzweig 1978, hereafter CR78), observed with the same instrumental configuration, were chosen as spectral type standard stars, to apply the “classical” approach to the \(\Delta S\) method via spectral types. The observed stars are indicated in Table 2, together with their Johnson \(B, V,\) and Strömgren \(b - y\) photometry, and

\(\Delta S\) method.

\(^{6}\) The Digitized Sky Survey was produced at the Space Telescope Science Institute under US government grant NAG W-2166.

| Name  | MACHO ID | \(\alpha\) (J2000) | \(\delta\) (J2000) | \(\langle V \rangle\) (mag) | \(P_0\) (days) | \(P_1/P_0\) | Epoch \(-2,451,100\) (days) | HJD \(-2,451,100\) (days) | \(\varphi\) |
|-------|-----------|---------------------|---------------------|--------------------------|----------------|---------------|--------------------------|--------------------------|--------|
| CA 02 | 13.6691.4052 | 5 21 34.1 | -70 31 52.1 | 19.67 | 0.46087 | 0.74266 | 83.62317 | 97.79014 | 0.74 |
| CB 45 | 13.6080.591 | 5 18 16.6 | -70 55 17.6 | 18.91 | 0.48089 | 0.74394 | 83.81805 | 97.62984 | 0.72 |
| CA 48 | 66.6811.651 | 5 22 45.2 | -70 36 35.7 | 19.35 | 0.48336 | 0.74457 | 83.70869 | 97.74300 | 0.03 |
| CB 49 | 13.5836.525 | 5 16 27.0 | -71 02 32.8 | 19.19 | 0.48406 | 0.74453 | 85.67901 | 97.54534 | 0.51 |
| CB 61 | 13.5958.518 | 5 17 37.5 | -71 00 26.3 | 19.49 | 0.49861 | 0.74467 | 84.73568 | 97.58639 | 0.77 |
| CA 67 | 13.6810.428 | 5 22 35.9 | -70 38 28.4 | 19.14 | 0.51159 | 0.74555 | 86.70999 | 96.73102 | 0.59 |

\(^{6}\) Two spectra are available for CA 48. The one at HJD = 51,197.74300 was observed around maximum light, while that taken at HJD = 51,196.55699 (\(\varphi = 0.58\)) is of low S/N. However, the phase-corrected \(\Delta S\) we derive from the spectrum at early phase (\(\Delta S = 5.2\)) is in extremely good agreement with that derived from the low S/N spectrum (\(\Delta S = 5.3\); see col. [6] of Table 5).

\(^{6}\) Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds.

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TABLE 2

INFORMATION ON THE SPECTRAL TYPE OF STANDARD STARS OBSERVED IN Cr 140

| Number | $V$   | $B-V$  | $b-y$  | Spectral Type (CR78) | Spectral Type ($b-y$) | Spectral Type (adopted) |
|--------|-------|--------|--------|----------------------|-----------------------|-------------------------|
| 6      | 7.12  | -0.04  | ...    | A0                   | ...                   | A0                      |
| 8      | 7.49  | 0.02   | ...    | A2                   | ...                   | A2                      |
| 9      | 7.59  | -0.09  | -0.042 | A0                   | B6.3                  | B8                      |
| 15     | 8.59  | -0.06  | -0.020 | A0                   | B9.3                  | A0                      |
| 16     | 8.68  | 0.40   | ...    | F2                   | ...                   | F2                      |
| 17     | 8.70  | 0.70   | 0.467  | F5                   | G0.2                  | ...                     |
| 19     | 8.83  | -0.02  | -0.017 | A0                   | B9.7                  | A0                      |
| 22     | 8.97  | 0.42   | ...    | F2                   | ...                   | F2                      |
| 24     | 9.00  | 0.19   | ...    | A3                   | ...                   | A3                      |
| 26     | 9.15  | 0.20   | 0.111  | A2                   | A5.5                  | A4                      |
| 29     | 9.28  | 0.14   | ...    | A2                   | ...                   | A2                      |
| 32     | 9.34  | 0.02   | -0.005 | A0                   | A9.9                  | A0                      |
| 38     | 9.57  | 0.22   | 0.140  | A5                   | A7.5                  | A6                      |
| 42     | 9.71  | -0.07  | ...    | A0                   | ...                   | A0                      |

* This star was observed but not used, since it is not on the main sequence.

spectral types derived both from CR78 and from the Strömgren $b-y$ photometry (see the next section). The last column of the table gives the spectral type adopted in the present analysis from either CR78 values or the average of CR78 types with those derived from the $b-y$, when both estimates are available.

For calibration purposes (i.e., to check that our $\Delta S$ values are on the standard system), we took spectra at minimum light of six field $ab$-type RR Lyrae stars of known $\Delta S$, namely, IU Car, X Crt, WY Ant, AF Vel, U Lep, and TV Leo; phases were derived from published ephemerides and checked a posteriori by the spectral types (see note to § 3.1). We also acquired 10 spectra along the pulsation cycle (from phase 0.0 to 0.65) of one $c$-type RR Lyrae star (T Sex, also of known $\Delta S$) in order to derive phase corrections for the spectra not exactly taken at minimum light.

Spectroscopic data were reduced using IRAF and the standard procedure for long-slit spectra. Images were trimmed, bias-subtracted, and flat-fielded; spectra were traced, extracted, and wavelength-calibrated. No flux calibration was needed for our goals. We retained for further analysis only the wavelength range 3750–5210 Å, in which all lines of interest are located. The most delicate parts of the whole procedure were spectrum extraction and background subtraction, since the observed fields are very crowded, and the seeing conditions were not optimal. The risk of contamination from nearby objects is high; this happened, for instance, in the case of one of our targets. The background-subtracted spectra of the six program stars are shown in Figure 3.

3. $\Delta S$ DERIVATION

3.1. Measurements of $\Delta S$

Spectral types were derived for our targets by comparing measured pseudo–equivalent widths (EWs) of the H$\beta$, H$\gamma$, and Ca II K lines with those measured in the stars of Cr 140. The pseudo-EWs were computed by dividing the instrumental fluxes within a small spectral region $(f)$ centered on the selected feature, with the average of those in two comparison spectral regions $(c_1$ and $c_2)$ located on both sides of each feature and defining the local continuum (see Table 3). All spectra were shifted to zero velocity before measuring the EWs. The (rest-frame) wavelength bins used are given in Table 3.

The spectral types for stars in Cr 140 given by CR78 are quite rough estimates. To reduce this source of error, they...
TABLE 3

Spectral Regions Used to Compute the Pseudo-EWs

| Line     | Blue Continuum (Å) | Line Region (Å) | Red Continuum (Å) |
|----------|--------------------|-----------------|-------------------|
| Ca II K  | c1 = 3850–3870     | f = 3925–3950   | c2 = 3940–3960    |
| Hγ       | c1 = 4220–4280     | f = 4320–4360   | c2 = 4420–4480    |
| Hβ       | c1 = 4680–4780     | f = 4840–4890   | c2 = 4950–5050    |

were averaged, whenever possible, with the spectral types deduced from *ubwby* colors (Hauck & Mermilliod 1998), using the calibrations by Crawford (1975, 1978, 1979). We then constructed curves calibrating EWs against spectral types using 13 of the 14 stars observed in Cr 140 (see Table 2 for identification); star No. 17 was excluded because it is not on the main sequence. Finally, we derived the best spectral type for each program spectrum by entering the measured EWs for the program star and reading out the corresponding spectral type. The final adopted H spectral type is the average of the values obtained from Hγ and Hβ. Following Preston (1959), the observed ΔS values were simply the difference between the H and Ca II K spectral types in tenths of spectral types. Results for the program stars are shown in Table 5, where we give for each of our targets the H and Ca spectral types and the ΔS values derived applying both no phase corrections (as suggested by Kemper 1982) or using phase corrections deduced from T Sex (see § 3.2). The corresponding metallicities are also presented in Table 5 and will be discussed in § 4.

Our resolution is too coarse to separate the stellar and interstellar components of the Ca II lines, and we did not apply any correction for the contribution due to interstellar absorption, since its effect on the ΔS value, hence on [Fe/H], is negligible. Gratton, Tornambe, & Ortolani (1986), applying the ΔS method to the globular cluster ω Cen, employed a correction of 0.2 spectral subclasses in the sense of an earlier spectral type) to their ΔS values, based on a reddening of $E(B-V) = 0.11$ mag. In our case, the reddening is comparable (C01), and the resulting correction would imply a difference in the derived [Fe/H] of about 0.04 dex; this is negligible with respect to the total error associated with the method.

To estimate scale and random errors in our derived ΔS values, we have compared the values we derived for the six comparison *ab*-type RR Lyrae stars with those given in the literature (see Table 4 for individual values; the comparison is done between our values and the average of literature ΔS values for each star). If we apply the phase corrections suggested by Butler (1975), our ΔS values are, on average, smaller than the literature values by 0.2 ± 0.4 units of ΔS, with an rms of 1.2 units of ΔS type for individual stars. Since the accuracy of the literature values is of the order of 0.5 units of ΔS, we conclude that our ΔS are on the standard scale and have random errors of less than 1 unit of ΔS.

3.2. Phase Corrections

Before interpreting the observed ΔS in terms of metallicity, we have to apply a correction for the observed phase, since ΔS is known to vary during the pulsation cycle. The exact form of the phase correction to be applied for *d*-type RR Lyrae stars is not known. In their study of Galactic double pulsators in the field, Clement, Kinman, & Suntzeff (1991) argued that the best correction is the same used for *c*-type RR Lyrae stars, since in general the amplitude of the first harmonic is larger than the amplitude of the fundamental mode for double pulsators (this is also true for our program variables). Furthermore, the colors at minimum light of the *d*-type RR Lyrae stars are much closer to those of the *c*-type at minimum than to the *ab*-type ones. Unfortunately, the phase corrections to ΔS for the *c*-type variables are not very well defined. Kemper (1982) studied the variation of ΔS along the light curve of three *c*-type RR Lyrae stars (RU Psc, T Sex, and DH Peg). He concluded that phase corrections are small (of the same order of the measuring error, i.e., ± 1 unit in ΔS) and should not be applied when the H spectral type is later than A7–A8, while he recommended to reject ΔS estimates from spectra where the derived H spectral type is earlier. An alternative procedure is to apply phase corrections to the ΔS according to the phase variation of ΔS derived from a well-studied *c*-type template star. We have used to this purpose our observations of T Sex and derive (see Fig. 4)

$$\Delta S_{\text{corr}} = \Delta S_{\text{obs}} + 0.587[10.5 - \text{SpT(H)}]$$

where we give the correction simply as a linear function of the hydrogen spectral type, and according to Kemper 8

8 All phases in column (2) of Table 4 were inferred from the Sp(H)'s of our spectra and agree with those computed from literature echemeterides in all cases, except AF Vel and U Lep. These two stars have varying periods. Two echemeterides are available for AF Vel and the corresponding phases (0.58: Eggen 1994; 0.63: Lub 1979) are slightly later than we derive; we suggest that both the echemeterides may be no longer adequate. Four echemeterides are available for U Lep (0.60: Firmey, Derevyagin, & Lysova 1985; 0.67: Lub 1977; 0.69: Eggen 1994; and 0.80: Fernley, Skillen, & Burki 1983), and the phase we derive agrees with Eggen's prediction.

TABLE 4

Field *ab*-Type RR Lyrae Stars: Comparison between Literature and Newly Derived ΔS Values

| Star  | φ    | ΔS (literature) | Reference (literature) | SpT(H) | SpT(K) | ΔS (observed) | ΔS (corrected) |
|-------|------|----------------|-----------------------|--------|--------|---------------|----------------|
| IU Car | 0.80 | 9              | 1                     | F3.2   | A5.2   | 8.0           | 8.2            |
| X Crt | 0.77 | 10, 10.4       | 1, 2                  | F3.2   | A4.3   | 8.9           | 9.1            |
| WY Ant | 0.83 | 10, 10.4       | 1                     | F3.5   | A4.7   | 8.8           | 8.8            |
| AF Vel | 0.55 | 0.58–0.63      | 1                     | F0.9   | A5.2   | 5.7           | 7.2            |
| U Lep | 0.69 | 8, 9, 9.4, 9.22| 1, 3, 4, 5            | F2.6   | A4.3   | 8.3           | 8.8            |
| TV Leo | 0.69 | 10, 10, 9.9, 10.49, 10.8 | 1, 3, 4, 5, 6 | F1.7   | A3.1   | 8.6           | 9.7            |

REFERENCES—(1) Lub 1977; (2) Kinman & Carretta 1992; (3) Preston 1959; (4) Butler 1975; (5) Suntzeff, Kinman, & Kraft 1994; (6) Walker & Terndrup 1991.
(1982), we neglect the small difference existing between the ascending and descending part of the light curve. This seems a reasonable assumption for the c-type pulsators (but see Fig. 1 of Smith 1986) given the fairly symmetrical shape of their light curves, which all show slow climbs to maximum light, and given that spectra of RRc’s do not seem to show the hydrogen line emissions and doubling, which are the signature of shock waves propagating through the atmosphere, found in the spectra of the RRab’s during the rise to maximum light (Preston & Paczyński 1964; Chadid & Gillet 1998; Gillet & Crowe 1988).

Using the above relationship, we obtain for T Sex $\Delta S = 6.81 \pm 0.09$, with an rms scatter of 0.26 units of $\Delta S$. This value is to be compared with that determined using the procedure suggested by Kemper (1982), that is, $\Delta S = 6.28 \pm 0.17$ with an rms scatter of 0.50 units of $\Delta S$. In addition, for comparison the value given by Kemper (1982) for this star is $\Delta S = 6.1$ (6.3 if the star were analyzed as an ab-type variable). However, T Sex has the largest variation of $\Delta S$ with phase among the three stars considered by Kemper (1982), so this procedure may lead to somewhat too large of phase corrections (implying too large of $\Delta S$ values and too low of metallicities). In Table 5, we give the $\Delta S$ values of our targets obtained using both the Kemper procedure (i.e., with no phase corrections and only using spectra later than A7–A8) and the phase corrections derived from T Sex light curve. On average, $\Delta S$ values corrected according to the T Sex curve are larger by $1.2 \pm 0.2$ spectral subtypes. Our adopted values are simply the average of those determined using the two different procedures. We estimate that uncertainties related to the phase correction are half the average difference between these two estimates, i.e., about 0.6 spectral subtypes.

![Graph](image)

**Fig. 4.—** T Sex, variation of $SpT(H)$ and of the corresponding $\Delta S$ for the 10 spectra from which we have derived the phase corrections described in § 3.2.

### 4. Metallicities of the Program Stars

The $\Delta S$ values of our targets (both with and without phase corrections) were translated to metallicities using two different relations. The first one is given by Clementini et al. (1995, hereafter C95) and is based on their spectroscopic study at high resolution and high S/N of 10 field RRab’s, on data adapted from Butler (1975) and Butler & Deming (1979), and on globular clusters having literature metallicities derived from high-resolution spectra:

$$[\text{Fe/H}] = -0.194 \Delta S - 0.08 .$$

(1)

The relation has been derived for ab-type RR Lyrae stars; however, Kemper (1982) found that, on average, $\Delta S$ values derived for ab- and c-type variables in GCs agree with each other, so d-type RR Lyrae stars should also obey approximately the same $\Delta S$–[Fe/H] relation. Metallicities derived from this relation are quite similar to the Zinn & West (1984, hereafter ZW84) metallicity scale for GCs.

The second one is given by Gratton (1999, hereafter G99) and recalibrates the $\Delta S$ index using the new metallicity scale for GCs found by Carretta & Gratton (1997, hereafter CG97), which differs from ZW84’s, especially at intermediate metallicities, usually giving somewhat higher metallicities. The metallicity dependence on $\Delta S$ is in this case

$$[\text{Fe/H}] = -0.176 \Delta S - 0.03 ,$$

(2)

which is only marginally consistent with the relation in C95. In the following, we will be using metallicities derived from both relations:

**C95.—** $\Delta S$ values, both using no phase correction, as suggested by Kemper, and corrected using T Sex, are given in columns (4) and (5) of Table 5, and the adopted $\Delta S$ (the average of the two) is given in column (6). Columns (7) and (8) of Table 5 give the metallicities of the program stars computed using the $\Delta S$ estimates, according to both the

### Table 5

| STAR   | SpT(H) | SpT(K) | $\Delta S$ Kem | $\Delta S$ T Sex | $\Delta S$ Adopted | [Fe/H] Kem | [Fe/H] T Sex | [Fe/H] Adopted |
|--------|--------|--------|----------------|------------------|-------------------|------------|-------------|---------------|
| CA 02…| F1.6   | A3.6   | 8.0            | 9.1              | 8.6               | -1.63      | -1.85       | -1.74         |
| CB 45…| F1.9   | A4.2   | 7.5            | 8.4              | 8.0               | -1.54      | -1.71       | -1.62         |
| CA 48…| A7.2   | A5.7   | 1.5            | 5.2              | 5.2               | ...        | -1.09       | -1.09         |
| CB 49…| F2.2   | A7.3   | 4.9            | 5.7              | 5.3               | -1.03      | -1.19       | -1.11         |
| CB 61…| F0.9   | A4.3   | 6.6            | 8.1              | 7.4               | -1.36      | -1.65       | -1.50         |
| CA 67…| F0.5   | A2.6   | 7.9            | 9.7              | 8.8               | -1.61      | -1.96       | -1.78         |

**Clementini et al. (1995)**

| [Fe/H] Kem | [Fe/H] T Sex | [Fe/H] Adopted |
|------------|--------------|---------------|
| -1.63      | -1.85        | -1.74         |
| -1.54      | -1.71        | -1.62         |
| ...        | -1.09        | -1.09         |
| -1.03      | -1.19        | -1.11         |
| -1.36      | -1.65        | -1.50         |
| -1.61      | -1.96        | -1.78         |

**Gratton (1999)**

| [Fe/H] Kem | [Fe/H] T Sex | [Fe/H] Adopted |
|------------|--------------|---------------|
| -1.44      | -1.63        | -1.54         |
| -1.35      | -1.51        | -1.43         |
| ...        | -0.95        | -0.95         |
| -0.89      | -1.03        | -0.96         |
| -1.19      | -1.46        | -1.33         |
| -1.42      | -1.74        | -1.58         |
Kemper procedure and T Sex phase corrections, respectively, adopting the C95 relation. In the case of variable CA 48, we cannot give any metal abundance using Kemper procedure for the spectrum taken near maximum light, since the H spectral type at which this star was observed is too early; the lower S/N spectrum, instead, can be used to determine $\Delta S$ values with both methods. Both spectra give similar results, and in the following, we will be using the metallicity derived from the higher S/N spectrum. Column (9) contains the adopted metallicities on the C95 scale: they are simply the average of the determinations obtained with the two different procedures. We estimate that internal uncertainties in these metallicities are of about $\pm 0.2$ dex (from typical errors of $\pm 1$ in our $\Delta S$ values). Systematic errors are mainly due to uncertainties in the phase corrections ($\pm 0.1$ dex) and in the metal abundance calibrations. The latter are, however, small (likely $\pm 0.1$ dex) in the abundance range of interest for the program stars.

The average metal abundance of the program stars is $[\text{Fe/H}] = -1.46 \pm 0.09$ (five stars, rms = 0.21 dex) when the Kemper procedure is used. A somewhat lower average value of $[\text{Fe/H}] = -1.60 \pm 0.13$ (six stars, rms = 0.33 dex) is obtained when the phase corrections appropriate for T Sex are applied to the sample. Note that in the latter case we include in the average one more star (variable CA 48) that happens to be the most metal-rich of the sample. Finally, if we use the adopted $[\text{Fe/H}]$ shown in column (9), we obtain an average $[\text{Fe/H}] = -1.49 \pm 0.11$ (6 stars, rms = 0.28 dex). If we include the systematic errors present in our determinations, we conclude that, on the C95 scale, our sample of six $d$-type RR Lyrae stars in the bar of the LMC has an average $[\text{Fe/H}] = -1.5 \pm 0.2$.

This compares rather well with the results obtained by Alcock et al. (1996). They used a similar method (line strengths of Ca II K and Hδ, measured on medium-low-resolution spectra of quite low S/N) and applied it to 15 field LMC RR Lyrae stars. Alcock et al. do not expand much on the subject: they do not give RR Lyrae star types or individual values for metallicities. They only say that the most frequent $[\text{Fe/H}]$ value is about $-1.6$, with values in the range from $-2.4$ to $-0.8$ (see their Fig. 8), and that the estimated accuracy is $\pm 0.25$ dex.

G99.—Columns (10) and (11) of Table 5 contain the analogues of columns (7) and (8), but use the G99 relation, and column (12) shows the adopted $[\text{Fe/H}]$, a simple mean of the two above. In this case, the average values are as follows: $[\text{Fe/H}] = -1.29 \pm 0.07$ (five stars, rms = 0.16 dex), adopting Kemper procedure; $[\text{Fe/H}] = -1.41 \pm 0.12$ (six stars, rms = 0.30 dex), adopting phase corrections derived from T Sex; and $[\text{Fe/H}] = -1.32 \pm 0.10$ (six stars, rms = 0.25 dex), adopting the recommended values in column (12). Again including systematic errors, we then conclude that, on the G99 scale, our sample has a somewhat larger average metallicity of $[\text{Fe/H}] = -1.3 \pm 0.2$.

The value compares slightly worse to Alcock et al. (1996), but this is not surprising, since CG97 scale tends to produce higher metallicities at this intermediate metal abundance. In any case, 15 and six objects are too small samples to attach significance to a $\sim 0.3$ dex difference, well within the quoted uncertainties.

Note that we have two RRd's with $[\text{Fe/H}] > -1.5$ (C95) or $-1.3$ (G99), hence with metal abundances larger than the Oo I clusters M3 and IC 4499: these are the most metal-rich RRd's identified so far.

5. MASSES

5.1. Petersen Diagram

Masses for the double-mode pulsators can be evaluated from the ratio between the first-overtone ($P_1$) and fundamental ($P_0$) pulsation periods. Petersen (1973) introduced the use of what is now universally known as “the Petersen diagram,” in which the ratio $P_1/P_0$ is plotted versus the value of $P_0$ for each RRd. Pulsation models define loci of constant mass in this diagram, hence RRd masses can be determined by the position of the star in the Petersen diagram, interpolating or extrapolating between these models.

Figure 5 shows the position in the Petersen diagram of the six LMC RRd's studied in this paper (we have omitted from the figure the remaining 67 LMC RRd's in A97 sample for clarity, but our small sample is well representative of the general $P_1/P_0$ versus $P_0$ distribution of the LMC RRd's; see Fig. 6), together with the other RRd's found in our Galaxy (clusters and field) and in the Draco and Sculptor dwarf spheroidal galaxies. Data plotted in Figure 5 have been taken from Garcia-Melendo & Clement (1997) for NSV 09295; from Clementini et al. (2000) for CU Com; from Clement et al. (1991) and Clement, Ferance, & Simon (1993) for AQ Leo, RR VIII-10, and RR VIII-5; from Clement et al. (1993) for NGC 2419 (one object), and NGC 6426 (one object); from Corwin, Carney, & Allen (1999) for M3 (five objects); from Walker & Nemec (1996) for IC 4497 (17 RRd's); from Walker (1994) for M68 (12 RRd's); from Nemec (1985b) for M15 (14 RRd's); from Nemec (1985a) for Draco (10 RRd's); from Kaluzny et al. (1995) for Sculptor (one object); and from A97 for the LMC (73 RRd's).

This figure is the analog of Figure 2 in A97; they also plotted the latest pulsational models available in literature by Bono et al. (1996), henceforth BCCM96, computed for metallicity $Z = 0.0001$ and for the three masses 0.65, 0.75, and 0.80 $M_\odot$. BCCM96 computed nonlinear, nonlocal,
time-dependent pulsational models based on up-to-date opacities (Rogers & Iglesias 1992). These models, whose properties and physical and numerical assumptions are described in Bono & Stellingwerf (1994), BCCM96, and Bono et al. (1997a, 1997b), are claimed by these authors to reconcile values for masses and luminosities based on pulsation and stellar evolution theories.

However, as already noted by A97, several objects (about 40 of the 73 double-mode LMC RR Lyrae stars, among which our CA 02, and one of the RRd in M3) fall into the region of the Petersen diagram below the $0.65M_\odot$ BCCM96 model, and extrapolation of the BCCM96 models would produce rather small masses of about $0.55M_\odot$ for these objects. This would not be simple to explain for the stars in this region with measured metallicity: M3 is the prototype Oosterhoff I cluster, and CA 02 and the Galactic field star RR VIII-58 have similar abundances, intermediate between Oo I and Oo II clusters ([Fe/H] $\approx -1.7$ on the C95 scale or $-1.5$ on the G99 scale). A97 discusses the problem somewhat, also suggesting, but in the end discarding, the possibility of these stars having a larger metal content, hence a mass larger than obtained by the BCCM96 models.

5.2. Pulsation Models: Derivation of the Masses

Since we now have information on the metallicity of six LMC RRd’s, we have decided to test the dependence of masses derived from the Petersen diagram also on metallicity. In order to do this, we derive masses for all objects in Figure 5 for which the metallicity has been estimated, using the most appropriate models. Metallicities for all objects were adopted as follows: In the case of the LMC RRd’s, we used the $\Delta S$ values (this paper), as for three Milky Way (MW) field RRd’s (AQ Leo, RR VIII-10, RR VIII-58; Clement et al. 1991), and converted them to [Fe/H] using both the C95 and G99 relations. In the case of CU Com, we used the [Fe/H] value in Clementini et al. (2000), obtained from high-resolution spectroscopy (note that at these low metal abundances the ZW84 and CG97 scales coincide). For the dSph’s Draco and Sculptor, we took the literature values (Mateo 1998) on the ZW84 scale and also converted them to CG97 scale, according to the transformation relations provided by CG97 (see eq. [7] in that paper). For Galactic GCs, since $\Delta S$ values were not available for all of them, we used the [Fe/H] values available in the literature (ZW84 for all of them, CG97 direct measurements for M3, M15, M68, and the transformation to the CG97 scale for NGC 2419, NGC 6426, and IC 4499). Metallicities chosen in the above way should be on a homogenous base for field and cluster variables, since the metallicity derived from the C95 relation (eq. [1]) is close to the ZW84 scale, while that derived from the G99 relation (eq. [2]) is close to the CG97 one.

Since both mass and metallicity affect the predicted Petersen diagram, information about the metallicity is required for a proper evaluation of the “pulsational” mass. At the same time, for each derived stellar mass the luminosity level can be inferred on the basis of the fundamental period range. We note that the only pulsation models that populate the Petersen diagram are those located in the double-mode (“OR”) region of the predicted instability strip, that is, between the theoretical fundamental blue edge and the theoretical first-overtone red edge, for each assumed luminosity level.

For each mass, models with different luminosity levels were computed in order to cover the full range of evolutionary predictions (which are dependent on the adopted input physics) for the luminosity of horizontal-branch stars and also to take into account evolutionary effects (RR Lyrae stars evolving from their zero-age horizontal-branch location). As already noticed by BCCM96, nonlinear computations offer the opportunity to disentangle the mass and luminosity effects on the location in the Petersen diagram. As shown in Figure 5, once the mass and metallicity are fixed, the period values identify the luminosity level. However, we consider the luminosity derivation beyond the aims of the present paper.

Models with $Z = 0.0001, 0.0004, 0.0006$, and 0.0008 have been computed for different masses and luminosities, using the same treatment as in BCCM96. In particular, we have used the following models: (1) $Z = 0.0001$ at 0.85, 0.83, 0.80, 0.75, 0.65, and 0.63 $M_\odot$ and with three luminosity levels, $log L = 1.61, 1.72, and 1.81$; (2) $Z = 0.0004$ at 0.75, 0.70, 0.65, 0.64, 0.63, 0.62, and 0.60 $M_\odot$ and three luminosity levels, $log L = 1.61, 1.65, and 1.72$; (3) $Z = 0.0006$ at 0.80, 0.75, and 0.65 $M_\odot$ and two luminosities, $log L = 1.61 and 1.72$; and (4) $Z = 0.0008$ at 0.85 and 0.65 $M_\odot$ and three luminosities, $log L = 1.61, 1.72, and 1.81$. A few additional models at $Z = 0.0002, 0.001, and 0.002$ have also been computed but not extensively used to determine masses. The main parameters of the adopted models ($Z$, mass, luminosity, $P_0$, and $P_1$) are given in Table 6.

A subset (indicated by footnote “a” in Table 6) of the $Z = 0.0001, 0.0004, 0.0006$, and 0.0008 models is shown in Figure 5, where for each metallicity, the upper curve refers to the higher mass, and the luminosity levels increase from right to left; now the models completely cover the range observed for the RRd’s. An enlargement of the region of interest for our six LMC RRd’s is shown in Figure 6, where all the LMC RRd’s from A97 are also plotted.

We derived masses for our targets, as well as for all RRd’s for which a metal abundance estimate was available (either direct or of the hosting system), interpolating between the
set of pulsational models at the four indicated metallicities. For each object, we obtained a table with four mass values, one for each Z, and a fit with a parabola produced a quadratic relation mass versus Z. We then entered the empirical [Fe/H] value for the star (we do it for both metallicity scales used in this paper) and derived the mass with the associated error (based on the error on the fit and an assumed 0.2 dex error in [Fe/H]). In some cases, this procedure was slightly modified because the parabolic interpolation was not completely stable: for NGC 2419 and 6426, we also used the Z = 0.0002 models; for CU Com, the most metal-poor object of the sample, we found it more adequate to use only the models at Z = 0.0001; similarly for CA 48 and CB 61, we used only the models at Z = 0.0008; and for CB 49, only the models either at Z = 0.0006 (on the C95 scale) or at Z = 0.0008 (on the G99 scale). Results for masses and associated errors for the six LMC RRd’s, MW field RRd’s, and the two dSph’s are given in Table 7, while Table 8 presents results for galactic cluster RRd’s.

Note that mass values for M3 are the most uncertain because of the lower precision of the periods $P_0$ and $P_1$ given for its RRd’s. However, this could not be an explanation for the rather unusual masses found for the two Oo I clusters, since IC 4499 has exactly the same behavior but has periods as precise as those of the other GCs.

5.3. Mass-Metallicity Distribution

Figure 7 shows these results graphically; in (a), the [Fe/H]'s are on the C95 scale, and in (b), they are on the G99 metallicity scale. The almost flat distribution of masses

| $Z$          | $M/M_\odot$ | log $L/L_\odot$ | $P_0$ (days) | $P_1$ (days) | $Z$          | $M/M_\odot$ | log $L/L_\odot$ | $P_0$ (days) | $P_1$ (days) |
|--------------|--------------|-----------------|--------------|--------------|--------------|--------------|----------------|--------------|--------------|
| 0.0001 ...... | 0.85         | 1.81            | 0.5259       | 0.3928*      | 0.85         | 1.81            | 0.5531        | 0.4124*      |
|              | 0.85         | 1.81            | 0.5822       | 0.4334*      | 0.85         | 1.81            | 0.5341        | 0.3985      |
|              | 0.83         | 1.72            | 0.4498       | 0.3376      | 0.83         | 1.72             | 0.4976        | 0.3720        |
|              | 0.80         | 1.81            | 0.5459       | 0.4064       | 0.80         | 1.81             | 0.5742        | 0.4271*     |
|              | 0.80         | 1.72            | 0.5086       | 0.3796*      | 0.80         | 1.72             | 0.5358        | 0.3991*     |
|              | 0.75         | 1.81            | 0.5670       | 0.4214*      | 0.75         | 1.81             | 0.5964        | 0.4428*     |
|              | 0.75         | 1.81            | 0.6282       | 0.4656*      | 0.75         | 1.81             | 0.4775        | 0.3563*     |
|              | 0.75         | 1.72            | 0.5021       | 0.3741*      | 0.75         | 1.72             | 0.5284        | 0.3932*     |
|              | 0.75         | 1.72            | 0.5568       | 0.4136*      | 0.75         | 1.72             | 0.5535        | 0.4089*     |
|              | 0.65         | 1.61            | 0.4241       | 0.3164*      | 0.65         | 1.61             | 0.4460        | 0.3322*     |
|              | 0.65         | 1.72            | 0.4985       | 0.3706*      | 0.65         | 1.72             | 0.5242        | 0.3893*     |
|              | 0.65         | 1.72            | 0.5515       | 0.4089*      | 0.65         | 1.61             | 0.4357        | 0.3247     |
|              | 0.63         | 1.61            | 0.5389        | 0.3994      | 0.63         | 1.72             | 0.5389        | 0.3410     |
|              | 0.63         | 1.72            | 0.5671        | 0.4198      | 0.63         | 1.72             | 0.5389        | 0.3994     |
| 0.0002 ...... | 0.85         | 1.81            | 0.5272       | 0.3931*      | 0.85         | 1.81             | 0.5548        | 0.4128*     |
|              | 0.75         | 1.72            | 0.4816       | 0.3599*      | 0.75         | 1.72             | 0.5065        | 0.3769*     |
|              | 0.75         | 1.72            | 0.5331       | 0.3961*      | 0.75         | 1.81             | 0.5721        | 0.4244*     |
|              | 0.75         | 1.72            | 0.6019        | 0.4459*     | 0.75         | 1.81             | 0.6019        | 0.4459*     |
|              | 0.65         | 1.61            | 0.4278        | 0.3188*      | 0.65         | 1.72             | 0.4500        | 0.3348*     |
|              | 0.65         | 1.72            | 0.5290        | 0.3921*      | 0.65         | 1.72             | 0.5565        | 0.4120*     |
|              | 0.75         | 1.72            | 0.4833        | 0.3597*      | 0.75         | 1.72             | 0.5083        | 0.3778*     |
| 0.0004 ...... | 0.75         | 1.72            | 0.4810        | 0.3575     | 0.70         | 1.72             | 0.5317        | 0.3943     |
|              | 0.70         | 1.61            | 0.4092        | 0.3055      | 0.70         | 1.61             | 0.4527        | 0.3369     |

* Models plotted in Fig. 5.
with varying metallicities, particularly in Figure 7a, shows no indication of a mass-metallicity relation. In fact, mass and metallicity have an opposite effect on the position of an RRd in the Petersen diagram, leading to a sort of degeneracy. A mass-metallicity relation is only obtained using a fixed metallicity to compute the pulsational models, then using the derived masses in connection with the empirical abundances.

**TABLE 7**

| Star/Cluster | $\Delta S$ | [Fe/H] (C95) | [Fe/H] (G99) | Mass (C95) ($M_\odot$) | Mass (G99) ($M_\odot$) | Reference (for $\Delta S$) |
|--------------|------------|---------------|---------------|-------------------------|-------------------------|---------------------------|
| LMC--CA 02   | 8.6        | $-2.00 \pm 0.15$ | $-1.83 \pm 0.15$ | 0.758 $\pm$ 0.042 | 0.786 $\pm$ 0.054 | ... |
| LMC--CB 45   | 8.0        | $-2.83 \pm 0.19$ | $-2.38 \pm 0.20$ | 0.835 $\pm$ 0.120 | 0.835 $\pm$ 0.120 | ... |
| LMC--CA 48   | 5.2        | $-2.83 \pm 0.19$ | $-1.66 \pm 0.20$ | 0.708 $\pm$ 0.063 | 0.812 $\pm$ 0.143 | ... |
| LMC--CA 61   | 6.2        | $-0.11 \pm 0.20$ | $-0.23 \pm 0.20$ | 0.352 $\pm$ 0.120 | 0.835 $\pm$ 0.120 | ... |
| LMC--CA 67   | 8.8        | $-1.83 \pm 0.15$ | $-1.58 \pm 0.20$ | 0.765 $\pm$ 0.102 | 0.928 $\pm$ 0.196 | ... |
| MW--VII--10  | 9.2        | $-1.86 \pm 0.20$ | $-1.66 \pm 0.20$ | 0.708 $\pm$ 0.063 | 0.812 $\pm$ 0.143 | ... |
| MW--VIII--58 | 8.6        | $-1.75 \pm 0.20$ | $-1.55 \pm 0.20$ | 0.605 $\pm$ 0.075 | 0.722 $\pm$ 0.138 | ... |
| MW--VIII--10 | 8.6        | $-1.75 \pm 0.20$ | $-1.55 \pm 0.20$ | 0.605 $\pm$ 0.075 | 0.722 $\pm$ 0.138 | ... |
| MW--AQ Leo   | 8.9        | $-1.81 \pm 0.20$ | $-1.60 \pm 0.20$ | 0.831 $\pm$ 0.098 | 0.987 $\pm$ 0.199 | ... |
| Draco--VIII--116 | 8.7 | $-1.74 \pm 0.20$ | $-1.54 \pm 0.20$ | 0.588 $\pm$ 0.077 | 0.691 $\pm$ 0.128 | ... |
| Draco--VII--72 | ... | ... | ... | ... | ... | ... |
| Draco--VIII--58 | 8.6 | $-1.75 \pm 0.20$ | $-1.55 \pm 0.20$ | 0.605 $\pm$ 0.075 | 0.722 $\pm$ 0.138 | ... |
| Draco--VIII--112 | 9.2 | $-1.86 \pm 0.20$ | $-1.66 \pm 0.20$ | 0.708 $\pm$ 0.063 | 0.812 $\pm$ 0.143 | ... |
| Draco--VIII--112 | 9.2 | $-1.86 \pm 0.20$ | $-1.66 \pm 0.20$ | 0.708 $\pm$ 0.063 | 0.812 $\pm$ 0.143 | ... |
| Draco--VIII--112 | 9.2 | $-1.86 \pm 0.20$ | $-1.66 \pm 0.20$ | 0.708 $\pm$ 0.063 | 0.812 $\pm$ 0.143 | ... |
| Draco--VIII--112 | 9.2 | $-1.86 \pm 0.20$ | $-1.66 \pm 0.20$ | 0.708 $\pm$ 0.063 | 0.812 $\pm$ 0.143 | ... |
| Draco--VIII--112 | 9.2 | $-1.86 \pm 0.20$ | $-1.66 \pm 0.20$ | 0.708 $\pm$ 0.063 | 0.812 $\pm$ 0.143 | ... |

**NOTES.**—$\Delta S$ values, metal abundances on the two metallicity scales discussed in the text, and individual masses for field RRd's. Metallicities for Sculptor and Draco are for the whole systems not for individual RRd's, and they are given only once.

**REFERENCES.**—(1) This paper; (2) Clementini et al. 2000; (3) Clement et al. 1991; (4) Mendes De Oliveira & Smith 1990.

![Fig. 7](image-url). Mass-metallicity plots using (a) metallicities on the ZW84 scale for GCs and dSph's and the C95 relation for RR Lyrae stars; and (b) metallicities on the CG97 scale for GCs and dSph's and the G99 relation for RR Lyrae stars. See the discussion in § 5.3 for extensive comments on the high mass of the Oo I clusters. In both panels, the program stars are indicated by filled circles, field RRd's in our Galaxy by filled triangles, RRd's in GCs by open squares, and dSph's by filled squares. References for the original papers are given in § 5. Lines indicate the variation of mass at the RGB tip with metallicity, for ages of 14 Gyr (solid line), 12 Gyr (dotted line), 10 Gyr (dot-dashed line), 8 Gyr (long dashed line), and 6 Gyr (dot-long dashed line), as deduced from Bertelli et al. (1994) and Girardi et al. (1996). The arrows indicate the effect on mass determination of a 0.2 dex error in the assumed [Fe/H].
This is what was often done in the past (see, e.g., BCCM96 and Fig. 1 in Cox 1991), but it implies neglecting the effect of the metal abundance on the position in the Petersen diagram, hence an error on the derived mass for all objects whose metallicities differ from the metal abundance adopted in the pulsational model computations. Petersen (1991), on the basis of linear calculations, found that the difference in metal abundance between Oo I and Oo II clusters produces a very small mass variation, whereas Cox (1991) predicts that the mass of all GC RRd variables should be close to 0.8 $M_\odot$, when metallicity is taken into account and the OPAL opacity tables for the proper composition are used. Our nonlinear computations show that the metallicity effect on the Petersen diagram is strong, since it acts in different ways on the fundamental and first-overtone periods and heavily influences their ratio. This result is in agreement with the predictions by Kovacs, Buchler, & Marom (1991) and Cox (1995), who also found a noticeable dependence of the Petersen diagram on metallicity when the new OPAL opacities are used in pulsation computations. In particular, Cox (1995) performed computations with $Z = 0.0003$ and found a mass increase of Oo I RRd with respect to those derived from models at $Z = 0.0001$. However, he reproduced the IC 4999 RRd's
location with \( Z = 0.0003, M = 0.70 \pm 0.05 \) \( M_\odot \) models, whereas the metallicity used in our paper for IC 4999 RRd's is equal or larger than 0.0006, depending on the metallicity scale. This most likely justifies the larger masses we find for Oo I RRd's with respect to Cox (1995) results. In fact, as shown in Figure 6, our \( Z = 0.0002-0.0004, M = 0.75 M_\odot \) models fit the IC 4999 RRd's location too.

Further analysis on the metallicity dependence of pulsational models is required, since there is still the possibility that models could somehow overestimate the effect as a result of the adopted opacity tables (the most recent Livermore ones; Iglesias & Rogers 1996; see also the discussion by Cox 1991) or that, as suggested by Kovacs et al. (1992), the Petersen diagram depends not only on the heavy-element abundance, but also on the chemical mixture; however, this issue is quite beyond the scope of the present paper.

In Figure 7a, the globular clusters of both Oosterhoff groups have similar masses with very similar internal scatter, while in Figure 7b the two Oo I clusters, M3 and IC 4499, show a (unrealistically?) higher mass than the Oo II GCs, as a result of the different (higher) metallicity scale adopted. Indeed, some weak hints of a mass-metallicity relation with larger masses at higher metal abundances could be seen also among the Oo I field and cluster RRd's in Figure 7a. Whether this finding might imply an actual difference between GCs or is simply to impute to a too high sensitivity of the model-derived masses on metallicity, not well tuned yet, is an open question. The offset between the two metallicity scales is about 0.2 dex at the typical metallicity of the Oo I clusters; comparison between Figures 7a and 7b tells quite clearly that a systematic error of 0.2 dex in the metallicity of the Oo I clusters, through the effect on the RRd's masses derived from pulsational models, goes in the direction of producing a mass-metallicity relation (of any kind) that might actually not be there. The arrows in Figure 7 show graphically the variation of derived masses for a 0.2 dex increase in the assumed [Fe/H], comparable to the error associated with abundances obtained with the \( \Delta S \) method.

The derived masses are not in contrast to the values expected from stellar evolutionary models for an age of 10–14 Gyr and no extensive mass loss on the red giant branch (RGB). This is shown in Figure 7, where the lines represent the mass at the RGB tip for ages of 6, 8, 10, 12, and 14 Gyr; they are taken from the evolutionary models by Bertelli et al. (1994) for \( Z = 0.004, 0.001, \) and 0.0004 and by Girardi et al. (1996) for \( Z = 0.0001; \) however, any other choice of isochrone sets would produce similar results. Within the error bars (see Tables 7 and 8), all objects in Figure 7a are consistent with the plotted isochrone sets. Only the two most metal-rich RRd's of the LMC lie above the 10 Gyr isochrone; however, the large error bar associated with their mass estimates on one hand and/or the possibility for these field objects to be slightly younger than 10 Gyr might account for their position in the figure. Figure 7b is more difficult to explain, since RRd's in both M3 and IC 4499 lie well above the 10 Gyr isochrone, even taking into account error bars, and ages younger than 10 Gyr might be at the lower limit of the acceptable values for these GCs.

This could be due to uncertainty in the pulsational masses: models could have too high sensitivity to abundances at these relatively large metallicities, either through the total heavy-element abundance or the adopted chemical mixture. Another possibility is too large of an abundance assumed for these clusters (e.g., an overestimate of about 0.15 dex in CG97 computations for intermediate-metallicity clusters). This last possibility can be checked for M3, which has also been studied with high-resolution spectroscopy and fine abundance analysis by Kraft et al. (1992); they find that [Fe/H] = -1.47 ± 0.01, compared with [Fe/H] = -1.34 ± 0.02 in CG97. However, also adopting the Kraft et al. value, the pulsational masses are larger than for the Oo II clusters. Furthermore, work on high-resolution spectra of turnoff stars in NGC 6752, obtained with UV-Visual Echelle Spectrograph on the Very Large Telescope (Gratton et al. 2001), obtains the same iron abundance already derived for giants in CG97 for this intermediate-metallicity cluster ([Fe/H] = -1.42). Small overestimates may be present but do not seem to explain completely the effect noted in Figure 7b. Finally, we wish to note that, based on a recent study of M92 by King, Stephens, & Boesgaard (1998) and on spectra of turnoff stars in NGC 6397 (Gratton et al. 2001), it may be necessary to also reassess the low-metallicity end of the GC scale in the sense of lower abundances: this would go in the direction of increasing the metallicity difference between Oo I and Oo II clusters. Discussing the validity of metallicity scales is outside the scope of the present paper, and we prefer to derive masses using different ones, leaving to the reader the final choice about which one to believe more.

In any case, the major result here is that masses of Oo I and Oo II clusters do not significantly differ, or if they do, they differ in the sense of Oo I GCs having higher masses, contrary to what is usually accepted. This stems from the use of appropriate metallicities in the pulsational models. This result is summarized in Table 9, where we also compare the average mass values derived for the field and cluster variables of differing Oosterhoff types, adopting the two metallicity scales separately. The cut between Oosterhoff types was set at [Fe/H] = -1.7 and -1.5 in ZW84 and CG97, respectively, with the Oo II variables being at [Fe/H] ≤ -1.7 (-1.5).

The field RRd's have a larger scatter, in both metallicity and mass; however, most stars lie in the same region of the diagram occupied by the GC stars. The bulk of the field Galactic RRd's, as well as RRd's in Draco and Sculptor, concentrate in the low-metallicity region occupied by the Oo II clusters, while almost all the LMC RRd's in our sample are in the region of the Oo I clusters or extend further toward metallicity values that have no counterparts in our Galaxy or in the two dSph's, where RRd's had been found in the past. This confirms the C01 finding that pulsational properties of the RR Lyrae variables in their

| Metal Abundances | \( \langle M \rangle_{GC} \) | \( \langle M \rangle_{field} \) |
|------------------|----------------|----------------|
| ZW84:            |                |                |
| [Fe/H] ≤ -1.7... | 0.772 (σ = 0.032) | 0.744 (σ = 0.072) |
| [Fe/H] ≥ -1.7... | 0.835 (σ = 0.048) | 0.928 (σ = 0.155) |
| CG97:            |                |                |
| [Fe/H] ≤ -1.5... | 0.765 (σ = 0.031) | 0.814 (σ = 0.082) |
| [Fe/H] ≥ -1.5... | 1.127 (σ = 0.059) | 1.008 (σ = 0.081) |
observed regions of the LMC bar seem to follow the period-amplitude relation of Oo I clusters like M3.

Finally note that, no matter which metallicity scale is adopted, cluster and field stars of similar metallicity and/or Oosterhoff type do not show any systematic difference in the derived mass.

6. SUMMARY AND CONCLUSIONS

In order to investigate the possibility of a systematic difference in the mass and mass-metallicity distribution for RR Lyrae in GCs and in the general field, we have used the Preston ΔS method to derive metallicities for six RRd's (double-pulsating variables) in the bar of the LMC. We have then combined these values with literature data for four field Galactic RRd's, making up a total of 10 field RRd's, whose metallicity has been directly measured, and with data for RRd's in the Draco and Sculptor galaxies, for which the same metallicity as the host galaxy has been assumed. For these stars, pulsational masses were derived using an extension of the BCCM96 models to enlarged mass and metallicity ranges, purposefully computed for this paper. The same procedure has been applied to GCs RRd's, both of Oo I and Oo II types, similarly finding their masses.

We have then compared the position of the field and cluster RRd's in the mass-metallicity diagram: we find that, on average, the two samples follow the same mass-metallicity distribution.

Since field and cluster RR Lyrae stars also obey the same mass-luminosity-metallicity relation (Catelan 1998; Carretta et al. 2000), we conclude that they should also obey the same luminosity-metallicity relation and that there is no difference in luminosity between field and cluster RR Lyrae stars. This result implies that results found for field RR Lyrae stars can be safely used also to derive properties of cluster RR Lyrae stars, like, e.g., the absolute luminosity and the absolute luminosity-metallicity relation.

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