Three-dimensional correlation of speckles in deep Fresnel region: extracting the roughness exponent of random surface

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Abstract
We provide a detailed analysis of the three-dimensional cross correlation properties of speckle patterns in the deep Fresnel region scattered by random surface. Basing on Kirchhoff approximation, we first derive the quantitative relationship between the longitudinal correlation function and the fractal characteristics of random surface. A novel measuring method is proposed for extracting the roughness exponent of random surface from the longitudinal correlation function of speckles. In experiment, the speckle patterns deviating different distance from the object plane are acquired by a microscopic imaging system. The longitudinal correlation function of the speckles descends with the increase of deviating distance in the form of negative power law. By calculating the Fourier transform of the longitudinal correlation function, we obtain the roughness exponent of random surface by making linear fitting in the large spectrum region. The extracted results are in good agreement with those measured by an atomic force microscopic, which indicate our method has a satisfying accuracy.

1. Introduction

Research on the rough surfaces is a long-standing problem that has attracted great interest in many scientific and technological fields such as the growth fronts [1, 2], the dynamic fractures [3], and so on. Due to the advantages of non-contact and non-destruction, scattering technique has become a powerful measuring method and has been widely applied to the surfaces ranging from atomic to macroscopic scales by using electromagnetic radiations of different bands of wavelengths [4–7]. The speckle measuring method is essentially based on the dependence of the statistical properties of the speckles on the parameters describing the surface morphology. A three-dimensional (3D) speckle field is produced when a coherent laser light illuminates an optical rough surface. By studying the space-time cross correlation function in the observation plane, the statistical properties of the speckles can be usually examined. In addition to its fundamental relevance, the 3D correlation characteristics of the speckles are important for several fields of speckle metrology, such as speckle holography [8], speckle interferometry [9] and speckle photography [10]. More recently, it also turned out to be a relevant issue in novel optical imaging techniques such as dynamic speckle illumination microscopy [11] and thermal ghost imaging [12]. In recent years, the study on the 3D spatial coherence characteristics of the speckles has gradually attracted much attention of many scholars and many relevant literatures have been reported in succession [13–16]. However, these literatures have mainly concentrated on the detailed analysis of the 3D spatial coherence properties of speckle fields and the discussions on the 3D average speckle sizes. Furthermore, these studies discuss on a general analysis of the coherence properties with unspecific assumptions about the random diffuser that originates the speckles. As far as we know, there are few papers to discuss the relationship between the 3D correlation properties of speckles and the surface topography or characterize the morphological features of the random surfaces by using the 3D correlation properties of speckles.

In this paper, we study the 3D cross correlation properties of speckle patterns in the deep Fresnel region scattered by random surface. Based on the Kirchhoff approximation of the scattering theories, we derive the expression of the longitudinal correlation function of the speckle fields. For the first time, we find the
quantitative relationship between the longitudinal correlation function and the morphological features of random surfaces, which was absent in previous literature on the 3D correlation properties of speckles. Furthermore, we propose a novel measuring method for extracting the roughness exponent of random surfaces from the longitudinal correlation function. In experiment, we setup a microscopic imaging system which can collect a wider range of the spatial spectrum components scattered from the random surface, and the speckle patterns in the deep Fresnel region are received by a CCD. While moving slightly the observation plane along optical axis, we obtain a series of corresponding speckle patterns and then calculate the Fourier transform of the longitudinal correlation function of the speckles. By making linear fitting in the large spectrum region, we can obtain the roughness exponent \( \alpha \) of random surfaces. The extracted results by our method agree well with those by an atomic force microscope (AFM), which indicates that our measuring method has a satisfying accuracy.

2. Theory

We consider the geometry schematically illustrated in figure 1, when a laser beam illuminates a rough scattering surface, the speckled light is produced due to the random superposition of the scattered light waves. The scattered light propagates to the observation plane \((x, y)\) and can be collected by a microscope objective (MO) with high numerical aperture. The observation plane is placed in the deep Fresnel region, which refers to the diffraction region from a few wavelengths near the surface to the traditional Fresnel region. The distance \( z \) from the observation plane to the random surface is adjustable. When \( z \) is much smaller than the illuminating beam diameter, the scattered waves will spread and a large number of scattering components on the random surface contribute to the light field at an observation point, and the resultant light field on the observation plane obeys a complex Gaussian random process. For a random surface, we have studied the two-dimensional transverse autocorrelation of the speckles in the deep Fresnel region with the diffraction theory of Kirchhoff approximation [17].

Now we need to extend the research on the correlation characteristics of the speckles in the deep Fresnel region to a 3D case. When the incident light wave with unity amplitude illuminates the random surface on the object plane, the light field at position \( r \) in the observation plane is given by using the Kirchhoff approximation [18]:

\[
A(r) = \iiint \exp[i2\pi(n-1)h(x_0, y_0)/\lambda] \times \frac{\exp(ikr)}{r} \cos \theta dx_0 dy_0,
\]

where \( h(x_0, y_0) \) is the surface height distribution of the random surface, \( \cos \theta \) is the inclination factor. The 3D cross correlation of the speckle fields in the different observation planes can be characterized as [19]:

\[
I_A(r_1, r_2) = \langle A(x_1, y_1, z_1) A^*(x_2, y_2, z_2) \rangle,
\]

in which \( A(x_1, y_1, z) \) and \( A(x_2, y_2, z) \) are the electric fields at two points \((x_1, y_1, z_1)\) and \((x_2, y_2, z_2)\) on the observation plane when the surface sample is on the object plane and deviates from the object plane with \( \Delta z \), respectively. Here \( \langle \ldots \rangle \) indicates an ensemble average and the star denotes complex conjugation. In the condition that the speckle field \( A(x, y, z) \) is a complex Gaussian process, the correlation function of the speckle intensity is given by [20]
\[ J_l(r_1, r_2) = \langle I \rangle^2 + |J_A(r_1, r_2)|^2, \]

where \( \langle I \rangle = \langle I(r_1) \rangle = \langle I(r_2) \rangle \) is the average intensity of the speckle field. Substituting equations (1)–(2) and calculating based on the Taylor series expansion, we derive the 3D cross correlation function of the speckles:

\[
J_A(r_1, r_2) = \frac{\lambda}{2^2} \int \int \exp \left\{ -[k(n-1)]^2 [w^2 - R_h(\Delta x_0, \Delta y_0)] \right\} \times \exp \{2\pi [k_\parallel (\Delta x_0 - \Delta x) + k_\perp (\Delta y_0 - \Delta y)] \} \times \exp \{2\pi \lambda \Delta z (k_\parallel^2 + k_\perp^2) d\Delta x_0 d\Delta y_0 dk_\parallel dk_\perp, \]

where

\[ \Delta x = x_2 - x_1, \quad \Delta y = y_2 - y_1, \quad \Delta x_0 = x_{02} - x_{01}, \quad \Delta y_0 = y_{02} - y_{01}, \quad \text{and} \quad k_\parallel = \sin \theta \cos \gamma / \lambda, \]

\[ k_\perp = \sin \theta \sin \gamma / \lambda \]

are the components of wave vector \( \mathbf{k} \) on the object plane, respectively, \( \theta \) and \( \gamma \) are the azimuths angles of \( \mathbf{k} \).

When the surface sample does not deviate from the object plane, i.e. \( \Delta z = 0 \), the 3D cross correlation function described in equation (4) will be simplified to the 2D autocorrelation function expressed as equation (9) in our previous research [17]. In this paper, we mainly study on the 3D cross correlation properties of speckles and its relationship with the morphological features of random surface. On the observation plane, the size of speckle grains far away from the optical axis is quite different from that of the on-axis grains, as mentioned in [15]. However, in the paraxial region, the size difference of speckle grains is very small. Therefore, we only consider the case of \( \Delta x = 0, \Delta y = 0 \) for reasons of mathematical simplicity, and then the 3D cross correlation function \( J_A(r_1, r_2) \) will be simplified to contain only one variable of \( \Delta z \). The integration of \( \Delta x_0 \) and \( \Delta y_0 \) in equation (4) is the duplex Fourier transform of \( \exp \left\{ -[k(n-1)]^2 [w^2 - R_h(\Delta x_0, \Delta y_0)] \right\} \), which is actually the scattered intensity profile:

\[
S(k_\parallel) = \int \exp \left\{ -[k(n-1)]^2 [w^2 - R_h(\rho)] \right\} \cdot \exp (i k_\parallel \cdot \rho) \, d^2 \rho, \]

where \( k_\parallel = 2\pi (k_\parallel i + k_\perp j) \) is the parallel vector of \( \mathbf{k}, R_h(\rho) \) is the height autocorrelation function of the random surface. This scattered intensity profile is the function of wave vector component \( k_\parallel \), and it will spread with the propagation distance \( z \) when it is observed in the real space coordinate \((x, y)\). When \( z \) is small, which is the case we will perform the experiment, the scattered intensity profile is smaller than the conventional aperture function of screen size or beam illumination size, so it plays the role of aperture function and determines the cross correlation function of the speckle fields. With the increase of the propagation distance, the spread area of scattered intensity profile will become much larger than the conventional aperture of beam illumination, and the cross correlation function will be determined by the conventional aperture. The [21–23] have demonstrated that the ground glass screens can describe self-affine fractal random surface every well and its height autocorrelation function is expressed as:

\[
R_h(\rho) = \langle h(r_2) h(r_0 + \rho) \rangle = w^2 \exp \left\{ -\rho^2 / (\xi^2 \alpha) \right\}, \]

where \( w \) is the root mean square deviation roughness, \( \xi \) is the lateral correlation length and usually regarded as the average size of the surface grains at which the autocorrelation function of surface height decreases to \( e^{-1} \), \( \alpha \) is roughness exponent and describes the local jaggedness of the surface, i.e. the smaller value of \( \alpha \), the greater local jaggedness.

The problem of calculating \( J_A(r_1, r_2) \) is thus reduced to that of calculating the scattered intensity profile \( S(k_\parallel) \). Substituting the equation (6) into the equation (5), we can obtain the scattered intensity profile \( S(k_\parallel) \) in principle. However, it is generally difficult to obtain the analytic expression of the equation (5). In most cases, the relationship between \( S(k_\parallel) \) and the random surface parameters can be established by using some approximate conditions [22–24]. It is known that the fractal characteristic of the random surface is mainly reflected in the region \( \rho \ll \xi \) in the height–height correlation function, which represents the high frequency component in spatial frequency domain. Thus, the fractal characteristic determines the asymptotic behavior of \( S(k_\parallel) \) in the high spatial frequency range \( (k_\parallel \gg 1/\xi) \). Based on the Fourier–Bessel transform and Taylor series expansion in the case of \( \rho \ll \xi \) with neglecting the higher-order terms, the analytic expression of \( S(k_\parallel) \) in the high frequency range can be written as:

\[
S(k_\parallel) |_{k_\parallel \gg 1/\xi} = 2\pi \int_{D_\parallel \rho} \{ 1 + [2\pi (n-1)/\lambda]^2 w^2 \times (\rho/\xi^{2\alpha}) J_0(k_\parallel \rho) \} \rho \, d\rho, \]

where \( D_\parallel \) is the small integral region at \( \rho \ll \xi \). The Fourier transformation of the constant term 1 in the right-hand integrand function is a center-point term, which has no effect on the behavior of \( S(k_\parallel) \) in the high frequency range, so it can be omitted. Therefore, the above equation can be further expressed as:

\[
S(k_\parallel) |_{k_\parallel \gg 1/2\pi \xi} = M [2\pi (n-1)/\lambda]^2 w^2 \xi^{-2\alpha} k_\parallel^{-2\alpha - 2}, \]

(8)
in which $k_i = k_\parallel / 2\pi$, $M = (2\pi)^{-1 - 2\alpha} \int_{D_i} J_0(t) t^{2\alpha + 1} dt$ is an integral constant which is independent of the surface characteristic, $t = 2\pi pk_i$, and $D_i$ is the integral region at $t \ll 1$. The above equation indicates that the scattered intensity profile $S(k_i)$ in the high frequency range decreased with the parallel component of the wave vector $k_i$ according to the negative power law and the power value is $l = 2\alpha + 2$, which is agreed with the result that mentioned in our previous research [22]. Substituting equation (8) into equation (4), we finally obtain the expression of the longitudinal correlation function in the paraxial region as following:

$$J_A(\Delta z) = N \int k_i^{-\alpha - 1} \exp(i2\pi \Delta zk_i) dk_i,$$

in which $N = \pi M [2\pi w(n - 1)^2(2\xi^2/\lambda)^{-\alpha}, k_i = \lambda(k_p^2 + k_q^2)/2$. The analytic solution of above integration equation is generally difficult to be obtained. It is well understood that the fractal characteristic is related to the local fluctuations of the random surface. From equations (8) and (9), we can see that the longitudinal correlation function is the Fourier transform of the scattered intensity profile, and the fractal structure corresponds to the segment of scattered intensity profile at large scattering angles. When $\alpha$ takes smaller values, the fractal characteristics become more obvious with large local fluctuations, and the scattered intensity profile decays more slowly, and vice versa. Since the power spectrum is defined as the Fourier transform of the correlation function [25], at larger spectrum region, it is the contribution of $J_A(\Delta z)$ at smaller $\Delta z$. It can be seen that in the large spectrum region, the power spectrum $FT\{J_A(\Delta z)\}$ decreases in the form of negative power law with the power value $m = \alpha + 1$. Moreover, the tangent slope of the power spectrum in the log–log scale is related to the fractal characteristics of random surface in the short–range region, which reveals the quantitative relationship between the longitudinal correlation function $J_A(\Delta z)$ and the roughness exponent $\alpha$ of the random surface. Therefore, we can first obtain the Fourier transform of $J_A(\Delta z)$ by numerically calculating based on the experimental data and equation (3), and then extract the roughness exponent $\alpha$ of the random surface by making linear fitting in the log–log scale.

3. Experiment

The ground glass screens are made by grinding a holographic plate with different sizes of silicon carbide powders and they are labeled as sample No. 1, No. 2 and No. 3, respectively. We use an atomic force microscope (AFM, PARK, AutoProbe CP) to measure the samples. The images of surface morphology are shown in figure 2 with scanned area 80 $\mu$m × 80 $\mu$m. Although the morphologies of the three samples are different, the common characteristics are large-scale height fluctuations. There are many large valleys and mounds on the sample surfaces, and a lot of smaller grains and fluctuations with different size scales are on these valleys and mounds, which indicate the random surface has obvious fractal characteristic in the short–range region [21]. From these AFM images, the height data $h(x_0, y_0)$ mentioned in equation (1) in the scanned area of each sample can be obtained. We calculate the height autocorrelation function and fit it with that of random self-affine fractal surface model defined in equation (6), respectively, and then we get the surface parameters of the three samples as shown in table 1.

We detect the speckle patterns in deep Fresnel region with a microscopic imaging system as depicted in figure 3. A laser beam with the wavelength of 532 nm is used as the light source, and is spatially filtered and expanded with a spatial filter. Lens L1 and L2 are used to collimate laser beam and adjust the image magnification, respectively. The distance from the random surface to the MO is about 1 cm, which is much smaller than the diameter of the incident beam (6 cm). Therefore, the aperture contributing to an observation point is within the area of the illumination beam. A MO with high numerical aperture (Nikon, Dry, 100×, N. A.0.9, WD 1 mm) is used to collect a wider range of the spatial spectrum components scattered from the random surface. Here WD represents the working distance of MO, and it is regarded as the front plane of the MO to the observation plane, which is the plane 1 cm behind the random surface. The measuring accuracy of surface
characteristic can be improved by using this MO due to the higher spatial spectrum components corresponding to the resolution for local tiny structure in the surface morphology. The surface sample is mounted on a 3D piezo nanometer stage (PI E516) for accurate positioning. A CCD (Roper, Cascade 1k) is used for receiving the speckle patterns. We first use a white light source to illuminate the sample for finding the accurate object plane. Due to the extremely short coherence length of the white light and the short-focus of MO, the image will become blurry when the sample is slightly deviated from the object plane. By slowly moving the surface sample along the optical axis, we can find a clear imaging position, which is the object plane of surface sample. Then we use laser beam instead of white light source to illuminate the sample for acquiring the speckle patterns. In order to study the longitudinal correlation properties of the speckle patterns in the deep Fresnel region, so we move slightly the surface sample away from the object plane along the optical axis, and the maximum deviating distance is 30 μm. Meanwhile, CCD records a series of corresponding speckle patterns in different deviating distances and the imaging range of CCD for collecting the speckle field is 34 × 34 μm². In our experimental setup, the magnification is 233.6. To ensure the accuracy, we measure the speckle fields at each deviating distance in five different transversal positions of each surface sample. Then, we average the light intensities in these five different positions at each deviating distance, and then the average value of longitudinal correlation at each deviating distance can be obtained.

### 4. Results and discussion

With the above optical system, we obtain the speckle patterns with different deviating distances in the deep Fresnel region. Figure 4(a) gives the speckle pattern on the object plane for surface sample No. 3, and figures 4(b)–(d) give the patterns deviating from the object plane 2 μm, 5 μm and 10 μm, respectively. It is shown that unlike the smooth local intensity in the traditional far field speckles, the speckle patterns have no obvious specific size and contain many scattered grains with different sizes from small to large scales due to the incomplete scattering in the deep Fresnel region. Inside the large-sized grains, there are many small-sized grains

| Table 1. The parameters of three surface samples, respectively. |
|-----------------------------|-----------------------------|-----------------------------|
| w (μm) | ξ (μm) | α |
| No. 1 | 0.384 ± 0.0028 | 3.572 ± 0.069 | 0.8627 ± 0.0058 |
| No. 2 | 0.432 ± 0.0021 | 4.537 ± 0.0829 | 0.7648 ± 0.00125 |
| No. 3 | 0.653 ± 0.0156 | 5.793 ± 0.056 | 0.68746 ± 0.0062 |

**Figure 3.** Diagram of the optical setup for collecting the speckle patterns in deep Fresnel region.

**Figure 4.** (a)–(d) Speckle patterns on the object plane and deviating from the object plane. 2 μm, 5 μm and 10 μm for surface sample No. 3, respectively.
which are arranged densely in somewhat order and even distribution. These phenomena indicate that the speckle patterns in the deep Fresnel region have distinct fractal characteristics. Moreover, we can see that there is only very slight difference between the speckle patterns in figures 4(a) and (b), which indicates that the longitudinal correlation coefficient of the speckle patterns is very large when the surface sample is close to the object plane. With the increase of the deviating distance, the speckle pattern gradually differs from that of the object plane. When the deviating distance is 10 µm, the speckle pattern in figure 4(d) is obviously different from the pattern in figure 4(a). For the other two samples, there are the same phenomenon, i.e. the difference between the speckle patterns become gradually obvious when the observation plane is far away from the object plane. It can be seen that the longitudinal correlation of speckles on different observation surfaces will decrease with the increase of the deviating distance.

These qualitative characteristics can be quantitatively described by the longitudinal correlation function of the speckle fields. Through numerical calculation of the experimental data, we can obtain the change curves of the longitudinal correlation function $J_d(\Delta z)$ with the deviating distance $\Delta z$ for the samples No. 1, No. 2 and No. 3 as shown in figure 5, respectively. It can be seen that the curves are divided into two parts by the division points at about $\Delta z = 2.5$ µm. In the region $\Delta z < 2.5$ µm, the curves descend quickly with the increase of the deviating distance, which is mainly caused by the fractal structure in the short-range region of random surfaces. With the increase of the deviating distance, the curves then decrease slowly. When $\Delta z > 10$ µm, the curves descend very slowly and finally tend to somewhat straight horizontal lines. This means that the longitudinal correlation function approach a limit value at the large deviating distance, and this limit value decreases with the increase of surface roughness. We can see that $J_d(\Delta z)$ decays approximately with the increase of $\Delta z$ in the form of negative power law for all of the three samples. These properties of the longitudinal correlation function are related to the scattering properties of the surface topography. We can imitate the definition of the lateral correlation length $\xi$ to define the longitudinal correlation length $\xi_l$ of speckle grains. $\xi_l$ may be defined as the deviating distance at which $J_d(\Delta z)$ decreases to $e^{-1}$ of the maximum value, and the longitudinal correlation lengths are 4.221 µm, 4.624 µm and 5.477 µm for No. 1–No. 3, respectively. It is obvious that the longitudinal correlation length of speckle patterns will rise with the increase of surface roughness, which indicates that the rougher random surface the larger longitudinal length size of speckle grains. This is in accordance with the above theoretical analysis, i.e. the longitudinal correlation function of speckles in the deep Fresnel region is exactly determined by the scattered profile and is the reflection of the surface topography.

In order to obtain the quantitative relationship between the fractal features of random surfaces and the longitudinal correlation of speckles, we first numerically calculate the Fourier transform of $J_d(\Delta z)$ basing on experimental data and equation (3), and then make linear fitting in the large spectrum region to acquire the roughness exponent of the random surface based on equation (9). The curves remarked by black square dots in figure 6 represent the Fourier transform of $J_d(\Delta z)$ produced by the samples No. 1, No. 2 and No. 3, respectively, and the red lines show the corresponding linear fittings in the large spectrum region. The fitting results are $\alpha_1 = 0.846 \pm 0.005 \pm 0.005$, $\alpha_2 = 0.767 \pm 0.003 \pm 0.003$ and $\alpha_3 = 0.681 \pm 0.003 \pm 0.003$, which are very consistent with the results measured by AFM. This shows that the method of extracting surface parameter from the longitudinal correlation function of the speckle patterns is feasible and has satisfactory accuracy.

Moreover, we also have calculated the probability density function of the speckle intensity, and the probability density curves generally approach the negative exponential decay, which indicates that the speckle fields could be taken as circular Gaussian speckles in our experiment.
5. Conclusion

In this paper, we discuss in detail the 3D cross correlation characteristics of speckle patterns scattered by random surface in the deep Fresnel region. We find that the 3D cross correlation function is exactly determined by the scattered profile and is related to the surface topography. Moreover, we first derive the quantitative relationship between the fractal characteristics of random surface and the longitudinal correlation function of speckles. In experiment, the speckle patterns deviating different distance from the object plane are measured with a microscopic imaging system. The longitudinal correlation function descends approximately with the increase of deviating distance in the form of negative power law, and the longitudinal correlation length of speckles rises with the increase of surface roughness. We calculate the Fourier transform of the longitudinal correlation function of speckles and obtain the roughness exponent $\alpha$ of random surface by making linear fitting in the large spectrum region. Comparison of the results with those measured by AFM indicates that our method has a satisfying accuracy.

In this method, we measured many speckle patterns with different deviating distances and calculated the corresponding longitudinal correlation between the points on these speckle patterns. This method contains richer data information and can better represent the surface ensemble average, so from a statistical point of view, this method should have higher precision compared with that of the 2D autocorrelation measurement. Moreover, the greater significance of this paper is that we first discover the quantitative relationship between the longitudinal correlation and the morphological features of random surfaces, which was absent in previous literature on the 3D correlation properties of speckles, so we believe that our work can be useful to further understand the characteristics of speckle fields in the deep Fresnel region.

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