The Constrained Exceptional Supersymmetric Standard Model

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Abstract

We propose and study a constrained version of the Exceptional Supersymmetric Standard Model (E\textsubscript{6}SSM), which we call the cE\textsubscript{6}SSM, based on a universal high energy scalar mass $m_0$, trilinear scalar coupling $A_0$ and gaugino mass $M_{1/2}$. We derive the Renormalisation Group (RG) Equations for the cE\textsubscript{6}SSM, including the extra $U(1)_N$ gauge factor and the low energy matter content involving three 27 representations of $E_6$. We perform a numerical RG analysis for the cE\textsubscript{6}SSM, imposing the usual low energy experimental constraints and successful Electro-Weak Symmetry Breaking (EWSB). Our analysis reveals that the sparticle spectrum of the cE\textsubscript{6}SSM involves a light gluino, two light neutralinos and a light chargino. Furthermore, although the squarks, sleptons and $Z'$ boson are typically heavy, the exotic quarks and squarks can also be relatively light. We finally specify a set of benchmark points which correspond to particle spectra, production modes and decay patterns peculiar to the cE\textsubscript{6}SSM, altogether leading to spectacular new physics signals at the Large Hadron Collider (LHC).
1. Introduction

Supersymmetry (SUSY) provides an attractive framework that allows one to link gravity with the other fundamental forces of nature. Indeed, it is well known that local SUSY (Supergravity) leads to a partial unification of the Electro-Weak (EW), strong and gravitational interactions [1]. At some high energy scale local SUSY in Supergravity (SUGRA) models can be spontaneously broken in a hidden sector. Then the low–energy limit of such a theory is described by a global SUSY Lagrangian plus a set of soft SUSY–breaking terms [2] which do not induce quadratic divergences, thus preserving the Supersymmetric solution to the hierarchy problem [3] (for a recent review see [4]). A set of soft SUSY–breaking terms involves gaugino masses \( M_a \), soft scalar masses \( m_i^2 \), plus bilinear (\( B_i \)) and trilinear (\( A_i \)) scalar couplings [5]. If the SUSY–breaking scale is within a few TeV then the \( SU(3)_C, SU(2)_W \) and \( U(1)_Y \) gauge couplings converge to a common value near the scale \( M_X \approx 2 - 3 \cdot 10^{16} \text{ GeV} \) [6], which allows one to embed SUSY extensions of the Standard Model (SM) into Grand Unified Theories (GUTs) [7]. The rational \( U(1)_Y \) charges, which are postulated \textit{ad hoc} in the SM, then appear in a natural way in the context of SUSY GUT models after the breakdown of the extended symmetry – such as \( SU(5) \), \( SO(10) \) or \( E_6 \) – at the scale \( M_X \).

However, the incorporation of the simplest SUSY extension of the SM — the Minimal Supersymmetric Standard Model (MSSM) — into SUGRA or SUSY GUT models leads to the \( \mu \)–problem [4]. The Superpotential of the MSSM contains one bilinear term \( \mu \hat{H}_d \hat{H}_u \) that can be present before SUSY is broken. One would naturally expect the parameter \( \mu \) to be either zero or of the order of the Planck scale. On the one hand, if \( \mu \approx M_{Pl} \) then the Higgs scalars acquire a huge positive contribution \( \sim \mu^2 \) to their squared masses and EW Symmetry Breaking (EWSB) does not occur. On the other hand, if \( \mu = 0 \) at some scale \( Q \) the mixing between Higgs doublets is not generated at any scale below \( Q \) due to non–renormalisation theorems [8] so that \( \langle H_d \rangle = 0 \) and down–type quarks and charged leptons remain massless. The correct pattern of EWSB requires \( \mu \) to be of the order of the SUSY–breaking (or EW) scale.

An elegant solution to the \( \mu \)–problem naturally arises in the framework of Superstring inspired \( E_6 \) models. Ten–dimensional heterotic Superstring theory based on \( E_8 \times E_8' \) [9] can play a role in the ultraviolet completion of the non–renormalisable SUGRA models. In the strong coupling regime of an \( E_8 \times E_8' \) heterotic string theory, which is described by eleven dimensional Supergravity (M–theory) [10], the string scale can be compatible with the unification scale \( M_X \) [11]. Compactification of the extra dimensions results in the breakdown of \( E_8 \) down to \( E_6 \) or one of its subgroups in the observable sector [12]. The remaining \( E_8' \) couples to the usual matter representations of the \( E_6 \) group only by virtue
of gravitational interactions and comprises a hidden sector that gives rise to spontaneous breakdown of local SUSY. At low energies the hidden sector decouples from the observable one. The only signal it produces is a set of soft SUSY–breaking terms characterised by the gravitino mass \( m_{3/2} \) scale\(^1\) which spoil the degeneracy between bosons and fermions within one Supermultiplet.

At the string scale, \( E_6 \) can be broken via the Hosotani mechanism \([14]\). The breakdown of the \( E_6 \) symmetry results in several models based on rank–5 or rank–6 gauge groups. Therefore Superstring inspired \( E_6 \) models may lead to low–energy gauge groups with one or two additional \( U(1)' \) factors in comparison to the SM. In particular, \( E_6 \) can be broken directly to the rank–6 subgroup \( SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_{\psi} \times U(1)_{\chi} \). Two anomaly-free \( U(1)_\psi \) and \( U(1)_\chi \) symmetries of the rank-6 model are defined by \([15]\): \( E_6 \to SO(10) \times U(1)_{\psi}, \ SO(10) \to SU(5) \times U(1)_{\chi} \). This rank–6 model can be reduced further to an effective rank–5 model with only one extra gauge symmetry \( U(1)' \) which is a linear combination of \( U(1)_{\chi} \) and \( U(1)_{\psi} \):

\[
U(1)' = U(1)_{\chi} \cos \theta + U(1)_{\psi} \sin \theta .
\]

If \( \theta \neq 0 \) or \( \pi \) the extra \( U(1)' \) gauge symmetry forbids an elementary \( \mu \) term but allows an interaction of the extra SM singlet Superfield \( \hat{S} \) with the Higgs Supermultiplets \( \hat{H}_d \) and \( \hat{H}_u \) in the Superpotential: \( \lambda \hat{S} \hat{H}_d \hat{H}_u \). After EWSB the scalar component of the SM singlet Superfield \( \hat{S} \) acquires a non-zero VEV breaking \( U(1)' \) and an effective \( \mu \)–term of the required size is automatically generated \([16]\). Thus in Superstring inspired \( E_6 \) models the \( \mu \)–problem is solved in a similar way to the Next–to–Minimal Supersymmetric Standard Model (NMSSM) \([17]\), but without the accompanying problems of singlet tadpoles or domain walls \([18]\).

\( E_6 \) inspired SUSY models with an extra \( U(1)' \) have been extensively studied \([15]\), \([19]\). In general the models predict extra exotic matter beyond the MSSM and NMSSM. The large couplings of exotic quarks (\( D, \bar{D} \)) to the SM singlet \( S \) of the form \( \kappa S(D \bar{D}) \) may induce radiative breakdown of the extra \( U(1)' \) symmetry \([20]\), \([21]\)–\([24]\). An important feature of \( E_6 \) inspired SUSY models is that the mass of the lightest Higgs particle can be substantially larger in these scenarios than in the MSSM and NMSSM \([24]\). Previously, the implications of \( E_6 \) inspired SUSY models with an additional \( U(1)' \) gauge symmetry have been studied for EWSB \([21]\)–\([24]\), neutrino physics \([25]\)–\([26]\), leptogenesis \([27]\)–\([28]\), EW baryogenesis \([29]\), muon anomalous magnetic moment \([30]\), electric dipole moment of electron \([31]\) and tau lepton \([32]\), lepton flavour violating processes like \( \mu \to e \gamma \) \([33]\) and CP-violation in the Higgs sector \([34]\). Such models have also been proposed as the

\(^{1}\)In the most general case a complete set of expressions for the soft SUSY–breaking parameters can be found in \([13]\).
solution to the tachyon problems of anomaly mediated SUSY breaking, via $U(1)'$ D-term contributions \cite{35}, and used in combination with a generation symmetry to construct a model explaining fermion mass hierarchy and mixing \cite{36}.

Recent publications have focused on a particular $E_6$ inspired SUSY model with an extra $U(1)_N$ gauge symmetry in which right handed neutrinos do not participate in the gauge interactions. This corresponds to $\theta = \arctan \sqrt{15}$. Only in this Exceptional Supersymmetric Standard Model ($E_6$SSM) \cite{37,38} right–handed neutrinos may be superheavy, shedding light on the origin of the mass hierarchy in the lepton sector and providing a mechanism for the generation of lepton and baryon asymmetry of the universe \cite{27,28}. Supersymmetric models with an additional $U(1)_N$ gauge symmetry in which right–handed neutrinos have zero charge have been studied in \cite{26} in the context of non–standard neutrino models with extra singlets, in \cite{39} from the point of view of $Z–Z'$ mixing, in \cite{23} and \cite{39,40} where the neutralino sector was explored, in \cite{23} where the RG flow of couplings was examined and in \cite{22,24} where EWSB was studied.

In a recent letter \cite{41} we presented predictions from a constrained version of the above $E_6$SSM, referred to as the $cE_6$SSM\cite{1}, in which the soft SUSY–breaking scalar masses, gaugino masses and the trilinear scalar couplings are each assumed to be universal at the scale $M_X$, i.e. $m_i^2(M_X) = m_0^2$, $M_i(M_X) = M_{1/2}$ and $A_i(M_X) = A_0$. We discussed scenarios of the $cE_6$SSM with the lowest values of $m_0$ and $M_{1/2}$ consistent with both EWSB and experimental constraints, leading to very light exotic quarks, inert Higgs/Higgsinos and $Z'$ masses. As such these represented scenarios which could be discovered early at the LHC using “first data”. Since the emphasis was on early discovery we did not explore the $cE_6$SSM parameter space thoroughly and did not present a set of benchmarks which represent all the qualitatively different spectra of TeV scale $cE_6$SSM scenarios. For brevity we also omitted the renormalisation group equations (RGEs) used in our analysis and did not provide full details of our mass spectra calculations.

In this paper we provide a comprehensive study of the parameter space of the $cE_6$SSM and the TeV scale predictions of the model. We present two–loop RGEs for the gauge and Yukawa couplings together with two–loop RGEs for the gaugino masses and trilinear scalar couplings as well as one–loop RGEs for the soft scalar masses, in order to calculate the values of all masses and couplings at the EW scale for each set of fundamental parameters at the GUT scale $M_X$. Two–loop corrections to the $\beta$–functions are important for the analysis of the particle spectrum because in $E_6$ inspired SUSY models the $\beta$–function of the $SU(3)$ gauge coupling and the gluino mass vanish in the one–loop approximation. We perform a numerical RG analysis for the $cE_6$SSM, imposing the usual low energy experimental constraints and enforcing successful EWSB. Our analysis reveals that there

\footnote{See also Ref. \cite{42} for a preliminary account.}
is a substantial part of the cE\(_6\)SSM parameter space where the correct breakdown of the
gauge symmetry can be achieved and all experimental constraints can be satisfied. We
then perform a scan of the parameter space of the cE\(_6\)SSM and specify a set of benchmark
points that highlight particular characteristics of the particle spectrum within the cE\(_6\)SSM
parameter space. A general feature of the benchmark spectra is a light sector of SUSY
particles consisting of a light gluino, two light neutralinos and a light chargino, resulting
from the relative smallness of the low energy gaugino masses \(M_i\) due to the stronger gauge
running. Although the squarks, sleptons and \(Z'\) boson are typically much heavier, the
exotic quarks and squarks can be also relatively light leading to spectacular new physics
signals at the LHC.

The paper is organised as follows. In the next section we introduce the E\(_6\)SSM and
define the cE\(_6\)SSM. In section 3 we discuss the breakdown of gauge symmetry in the
cE\(_6\)SSM. In section 4 we provide analytical expressions for the mass matrices and masses
of all new particles appearing in our model. In section 5 we study the RG flow of all masses
and couplings and summarise the results of our studies of the particle spectrum. Section
6 is reserved for our conclusions and outlook. Appendix A contains explicit expressions
for the one–loop corrections to the mass matrix of the CP–even Higgs bosons calculated
in the leading approximation. In Appendix B we specify the complete system of RGEs
that we use in our analysis.

2. From the E\(_6\)SSM to the cE\(_6\)SSM

The E\(_6\)SSM is based on the \(SU(3)_C \times SU(2)_W \times U(1)_Y \times U(1)_N\) gauge group which is a
subgroup of \(E_6\). The extra \(U(1)_N\) gauge symmetry is defined such that right–handed neutrinos carry zero charges. The E\(_6\)SSM can originate from an \(E_6\) GUT gauge group which
is broken at the GUT scale \(M_X\). In E\(_6\) theories the anomalies are cancelled automatically;
all models that are based on the \(E_6\) subgroups and contain complete representations of \(E_6\)
should be anomaly–free. Consequently, in order to make a Supersymmetric model with
an extra \(U(1)_N\) anomaly–free, one is forced to augment the minimal particle spectrum
by a number of exotics which, together with ordinary quarks and leptons, form complete
fundamental 27 representations of \(E_6\). Thus the particle content of the E\(_6\)SSM involves at
least three fundamental representations of \(E_6\) at low energies. These multiplets decompose
under the \(SU(5) \times U(1)_N\) subgroup of \(E_6\) as follows:

\[
27_i \rightarrow (10, 1)_i + (5^*, 2)_i + (5^*, -3)_i + (5, -2)_i + (1, 5)_i + (1, 0)_i .
\]  

(2)

The first and second quantities in brackets are the \(SU(5)\) representation and extra \(U(1)_N\)
charge respectively, while \(i\) is a family index that runs from 1 to 3. An ordinary SM family,
which contains the doublets of left–handed quarks $Q_i$ and leptons $L_i$, right-handed up– and down–quarks ($u^c_i$ and $d^c_i$) as well as right-handed charged leptons, is assigned to $(10, 1)_i + (5^*, 2)_i$. Right-handed neutrinos $N^c_i$ should be associated with the last term in Eq. (2), $(1, 0)_i$. The next-to-last term, $(1, 5)_i$, represents SM-singlet fields $S_i$, which carry non-zero $U(1)_N$ charges and therefore survive down to the EW scale. The pair of $SU(2)_W$–doublets ($H^d_i$ and $H^u_i$) that are contained in $(5^*, -3)_i$ and $(5, -2)_i$ have the quantum numbers of Higgs doublets. They form either Higgs or Inert Higgs $SU(2)_W$ multiplets. Other components of these $SU(5)$ multiplets form colour triplets of exotic quarks $D_i$ and $D^c_i$ with electric charges $-1/3$ and $+1/3$, respectively.

In addition to the complete $27_i$ multiplets the low energy matter content of the $E_6$SSM is supplemented by an $SU(2)_W$ doublet $H^\prime$ and anti-doublet $\bar{H}^\prime$ from the extra $27^\prime$ and $\bar{27}^\prime$, in order to preserve gauge coupling unification. These components of the $E_6$ fundamental representation originate from $(5^*, 2)$ of $27^\prime$ and $(5, -2)$ of $\bar{27}^\prime$ by construction. The analysis performed in [43] shows that the unification of gauge couplings in the $E_6$SSM can be achieved for any phenomenologically acceptable value of $\alpha_3(M_Z)$ consistent with the measured low energy central value, unlike in the MSSM which, ignoring the effects of high energy threshold corrections, requires significantly higher values of $\alpha_3(M_Z)$, well above the experimentally measured central value. The splitting of $27^\prime$ and $\bar{27}^\prime$ multiplets can be naturally achieved, for example, in the framework of orbifold GUTs [44].

| $Q$ | $u^c$ | $d^c$ | $L$ | $e^c$ | $N^c$ | $S$ | $H^d_2$ | $H^u_1$ | $D$ | $\bar{D}$ | $H^\prime$ | $\bar{H}^\prime$ |
|-----|-------|-------|-----|-------|-------|-----|---------|---------|-----|-------|---------|---------|
| $\sqrt{5/3}Q^Y_i$ | $1/3$ | $-2/3$ | $1/3$ | $-1/2$ | $1$ | $0$ | $1/2$ | $-1/2$ | $-1/3$ | $1/3$ | $-1/2$ |
| $\sqrt{40}Q^N_i$ | $1$ | $1$ | $2$ | $2$ | $1$ | $0$ | $5$ | $-2$ | $-3$ | $-2$ | $3$ | $2$ |

Table 1: The $U(1)_Y$ and $U(1)_N$ charges of matter fields in the $E_6$SSM, where $Q^N_i$ and $Q^Y_i$ are here defined with the correct $E_6$ normalisation factor required for the RG analysis.

The matter content of the $E_6$SSM with correctly normalized Abelian charges of all matter fields is summarised in Table 1. Because right–handed neutrinos $\hat{N}^c$ do not participate in gauge interactions they are expected to gain masses at some intermediate scale after the breakdown of $E_6$ [37], [45]. The remaining matter survives down to the EW scale near which the gauge group $U(1)_N$ is broken. Thus, in addition to a $Z'$ corresponding to the $U(1)_N$ symmetry, the $E_6$SSM involves extra matter beyond the MSSM with the quantum numbers of three $5 + 5^*$ representations of $SU(5)$ plus three $SU(5)$ singlets with $U(1)_N$ charges. The presence of a $Z'$ boson and exotic quarks predicted by the $E_6$SSM provides spectacular new physics signals at the LHC which were discussed in [37–38], [46].

*We use the terminology “Inert Higgs” to denote Higgs–like doublets that do not develop VEVs.
Since the right–handed neutrinos are heavy, the three known doublet neutrinos $\nu_e$, $\nu_\mu$ and $\nu_\tau$, acquire small Majorana masses via the see–saw mechanism. At the same time the heavy Majorana right-handed neutrinos may decay into final states with lepton number $L = \pm 1$, thereby creating a lepton asymmetry in the early universe. In the $E_6$SSM the Yukawa couplings of exotic particles are not constrained by neutrino oscillation data. As a result substantial values of the CP–asymmetries can be induced even for a relatively small mass of the lightest right–handed neutrino ($M_1 \sim 10^6$ GeV) so that successful thermal leptogenesis may be achieved without encountering a gravitino problem [28].

In general $E_6$ symmetry does not forbid lepton and baryon number violating operators that result in rapid proton decay. Moreover, exotic particles in $E_6$ inspired SUSY models give rise to new Yukawa interactions that induce unacceptably large non–diagonal flavour transitions. To suppress these effects in the $E_6$SSM an approximate $Z_2^H$ symmetry is imposed. Under this symmetry all superfields except one pair of $H_1$, and $H_2$, (say $H_d \equiv H_{13}$ and $H_u \equiv H_{23}$) and one SM-type singlet field ($S \equiv S_3$) are odd. The $Z_2^H$ symmetry reduces the structure of the Yukawa interactions to (see [37])

$$W_{E_6SSM} \rightarrow \lambda_i \hat{S}(\hat{H}_i^d \hat{H}_i^u) + \kappa_i \hat{S}(\hat{D}_i \hat{D}_i) + f_{\alpha \beta} \hat{S}_\alpha (\hat{H}_d \hat{H}_\beta^u) + \tilde{f}_{\alpha \beta} \hat{S}_\alpha (\hat{H}_\beta^d \hat{H}_u) + \frac{1}{2} M_{ij} \hat{N}_i^c \hat{N}_j^c + \mu' (\hat{H} \hat{H}^c) + h_{ij}^E (\hat{H}_d \hat{H}^\prime) \hat{e}_j^c + h_{ij}^N (\hat{H}_u \hat{H}^\prime) \hat{N}_j^c + W_{MSSM}(\mu = 0),$$

where $\alpha, \beta = 1, 2$ and $i, j = 1, 2, 3$. In Eq. (3) we choose the basis $H_1^d$, $H_1^u$, $D_i$ and $\hat{D}_i$, so that the Yukawa couplings of the singlet field $S$ have flavour diagonal structure. The $SU(2)_W$ doublets $\hat{H}_u$ and $\hat{H}_d$, that are even under the $Z_2^H$ symmetry, play the role of Higgs fields generating the masses of quarks and leptons after EWSB. The singlet field $S$ must also acquire a large VEV in order to induce sufficiently large masses for the exotic charged fermions and $Z'$ boson and avoid conflict with direct particle searches at present and past accelerators. This requires the Yukawa couplings $\lambda_i$ and $\kappa_i$ to be reasonably large. If $\lambda_i$ or $\kappa_i$ are large at the GUT scale they affect the evolution of the soft scalar mass $m_S^2$ of the singlet field $S$ rather strongly resulting in negative values of $m_S^2$ at low energies that triggers the breakdown of the $U(1)_N$ symmetry.

Because $H_u$, $H_d$ and $S$ generate masses of all quarks, leptons and exotic fermions, it is natural to assume that only these fields acquire non–zero VEVs. To guarantee this, a certain hierarchy between the Yukawa couplings must exist. Defining $\lambda \equiv \lambda_3$, we impose $\kappa_i \sim \lambda_3 \lesssim \lambda_{1,2} \gg f_{\alpha \beta}, \tilde{f}_{\alpha \beta}, h_{ij}^E, h_{ij}^N$. Although $f_{\alpha \beta}$ and $\tilde{f}_{\alpha \beta}$ are expected to be considerably smaller than $\lambda_i$ and $\kappa_i$, they cannot be negligibly small since the fermion components of the Superfields $\hat{S}_1$ and $\hat{S}_2$ would become extremely light. The induced masses of singlinos $\tilde{S}_1$ and $\tilde{S}_2$ should be as large as a few MeV, otherwise the extra states
could contribute to the universe expansion rate prior to nucleosynthesis, thereby changing nuclear abundances.

Although $Z^H_2$ eliminates any problem related with baryon number violation and non-diagonal flavour transitions it also forbids all Yukawa interactions that would allow the exotic quarks to decay. Since models with stable charged exotic particles are ruled out by different experiments [48] the $Z^H_2$ symmetry must be broken. But the breakdown of $Z^H_2$ should not give rise to the operators leading to rapid proton decay. There are two ways to overcome this problem: the Lagrangian must be invariant with respect to either a $Z^L_2$ symmetry, under which all Superfields except lepton ones are even (Model I), or a $Z^B_2$ discrete symmetry, which implies that exotic quark and lepton Superfields are odd whereas the others remain even (Model II). If the Lagrangian is invariant under the $Z^L_2$ symmetry transformations then the terms in the superpotential which permit exotic quarks to decay and are allowed by the $E_6$ symmetry can be written in the following form

$$W_1 = g^{Q}_{ijk} \hat{D}_i (\hat{Q}_j \hat{Q}_k) + g^{q}_{ijk} \hat{D}_i \hat{Q}_j \hat{Q}_k.$$  

(4)

that implies that exotic quarks are diquarks. If $Z^B_2$ is imposed then the following couplings are allowed:

$$W_2 = g^{E}_{ijk} \hat{D}_i \hat{Q}_j \hat{Q}_k + g^{D}_{ijk} (\hat{Q}_i \hat{L}_j) \hat{D}_k.$$  

(5)

In this case the baryon number conservation requires exotic quarks to be leptoquarks.

Since $Z^H_2$ violating operators lead to non–diagonal flavour interactions, the corresponding Yukawa couplings are expected to be small, and must preserve either the $Z^B_2$ or $Z^L_2$ symmetry to ensure proton stability. In order to guarantee that the contribution of new particles and interactions to $K^0 - \overline{K^0}$ oscillations and to the muon decay $\mu \rightarrow e^- e^+ e^-$ are suppressed in accordance with experimental limits, it is necessary to assume that the Yukawa couplings of exotic particles to ordinary quarks and leptons are less than $10^{-3} - 10^{-4}$. In this case, they do not affect the RG flow of other masses and couplings and can safely be ignored in our analysis of the particle spectrum.

The hierarchical structure of the Yukawa interactions allows one to simplify the Superpotential substantially. Integrating out heavy Majorana right–handed neutrinos and keeping only Yukawa interactions whose couplings are allowed to be of order unity we find

$$W_{E_6SSM} \simeq \lambda \hat{S}(\hat{H}_d \hat{H}_u) + \lambda_\alpha \hat{S}(\hat{H}_d^\alpha \hat{H}_u^\alpha) + \kappa_i \hat{S}(\hat{D}_i \hat{L}_1)$$

$$+ h_u(\hat{H}_u \hat{Q}) \hat{t}^c + h_b(\hat{H}_d \hat{Q}) \hat{b}^c + h_\tau(\hat{H}_d \hat{L}) \hat{\tau}^c + \mu'(\hat{H}' \overline{\hat{H}'}),$$

(6)

where the Superfields $\hat{L} = \hat{L}_3$, $\hat{Q} = \hat{Q}_3$, $\hat{t}^c = \hat{u}_3^c$, $\hat{b}^c = \hat{d}_3^c$ and $\hat{\tau}^c = \hat{e}_3^c$ belong to the third generation. The Superpotential [48] includes only one bilinear term which is solely
corresponding mass term is not suppressed by the $E_6$ symmetry and is not involved in the process of the EWSB. Therefore, the parameter $\mu'$ remains arbitrary. Gauge coupling unification requires $\mu'$ to be within 100 TeV \cite{43}. The simplified Superpotential (6) that we use in our analysis of the cE$_6$SSM contains seven new couplings compared to the MSSM with $\mu = 0$: the parameter $\mu'$ and six new Yukawa couplings $\lambda_i$ and $\kappa_i$.

The most general scalar potential of the E$_6$SSM that ensures soft SUSY–breaking can be presented as a sum

$$V = V_F + V_D + V_{soft},$$

where $V_F$ and $V_D$ are the contributions of $F$ and $D$ terms respectively, while $V_{soft}$ contains a set of soft SUSY–breaking couplings:

$$V_{soft} = m^2_{S_i} |S_i|^2 + m^2_{H^0_i} |H^0_i|^2 + m^2_{H^\pm_i} |H^\pm_i|^2 + m^2_{D_{\alpha_i}} |D_{\alpha_i}|^2 + m^2_{D_{\beta_i}} |\overline{D}_{\beta_i}|^2 + m^2_{Q_i} |Q_i|^2
+ m^2_{[S_i]} |u_i|^2 + m^2_{[\overline{D}_{\beta_i}]} |d_i|^2 + m^2_{[D_{\alpha_i}]} |L_i|^2 + m^2_{[\overline{D}_{\beta_i}]} |e_i|^2 + m^2_{H^\pm_i} |H^\pm_i|^2 + m^2_{H^0_i} |H^0_i|^2
+ \left[ B' \mu'(H^0 \overline{H}) + h.c. \right] + \left[ \lambda_i A_{\lambda_i} S(H^d_i H^\nu_i) + \kappa_i A_{\kappa_i} D_i \overline{D}_i \right]
+ h_t A_t (H_u Q) t^c + h_b A_b (H_d Q) b^c + h_\tau A_\tau (H_d L) \tau^c + h.c. \right] .$$

The soft breakdown of SUSY gives rise to many new couplings. The six additional Yukawa couplings are accompanied by six extra trilinear scalar couplings, $A_{\lambda_i}$ and $A_{\kappa_i}$ \cite{8}. Soft SUSY–breaking also induces the bilinear scalar coupling $B'$ that corresponds to the mass term $\mu' H'^0 \overline{H}'$ in the Superpotential (6). In addition, the scalar potential of the E$_6$SSM includes 15 extra soft scalar masses: six masses of exotic squarks $m_{\bar{D}_i}$ and $m_{\overline{D}_i}$, four masses of Inert Higgs fields $m_{H^d_i}$ and $m_{H^\pm_i}$, two soft scalar masses of $H'$ and $\overline{H}'$ and three masses of SM singlet scalar fields $m^2_{S_i}$. Due to the extra Yukawa couplings, the parameter $\mu'$ and the new trilinear scalar and bilinear scalar couplings (that can be complex), even the simplified version of the $Z^H_2$–symmetric E$_6$SSM considered here involves 43 new parameters in comparison to the MSSM with $\mu = 0$. Fourteen of them are phases, some of which (but not all) can be eliminated by an appropriate redefinition of the new fields. However, the number of fundamental parameters reduces drastically in the cE$_6$SSM, defined at the GUT scale $M_X$, where all gauge couplings coincide, i.e. $g_1(M_X) \simeq g_2(M_X) \simeq g_3(M_X) \simeq g'_1(M_X)$, while the off–diagonal gauge coupling $g_{11}(M_X)$ vanishes. Constrained SUSY models impose extra unification constraints on the soft SUSY–breaking parameters. In particular, all soft scalar masses are set to be equal to $m^0_0$ at the scale $M_X$. Gaugino masses $M_i(M_X)$ are equal to an overall gaugino mass $M_1/2$ at the GUT scale and all trilinear and bilinear scalar couplings coincide at this scale, i.e. $A_i(M_X) = A_0$ and $B_i(M_X) = B$. Thus the cE$_6$SSM is uniquely characterised by the set of Yukawa couplings...
\[ \lambda_i(M_X), \kappa_i(M_X), h_l(M_X), h_b(M_X) \text{ and } h_\tau(M_X), \]
the universal soft scalar mass \( m_0 \), the universal gaugino mass \( M_{1/2} \) and the universal trilinear scalar coupling \( A_0 \). The phases of the dimensionless couplings in the Superpotential are selected by appropriate field redefinitions and are chosen so that all the dimensionless couplings are real. In order to guarantee correct EWSB, \( m_0^2 \) has to be positive. To simplify our analysis we also assume that \( A_0 \) is real and \( M_{1/2} \) is positive — this then naturally leads to real VEVs of the Higgs fields.

The set of parameters mentioned above should be in principle supplemented by \( B' \) and \( \mu' \). However, since \( \mu' \) is not constrained by EWSB and the term \( \mu' \hat{H} \hat{H}^\dagger \) in the Superpotential is not suppressed by the \( E_6 \) symmetry, the parameter \( \mu' \) can be as large as 10 TeV. Therefore we assume that the scalar and fermion components of the Superfields \( \hat{H}' \) and \( \hat{H}' \) are very heavy so that they decouple from the rest of the particle spectrum. As a consequence the parameters \( B' \) and \( \mu' \), that determine the masses of the survival components of \( 27' \) and \( \overline{27}' \), are irrelevant for our analysis.

3. EWSB and \( Z-Z' \) mixing

As described in the previous section, the Higgs sector of the model involves two Higgs doublets \( H_u \) and \( H_d \), as well as the SM–singlet field \( S \). The corresponding Higgs effective potential can be written as,

\[
V = \lambda^2 |S|^2 (|H_d|^2 + |H_u|^2) + \lambda^2 |(H_d H_u)|^2 + \frac{g'_2}{8} \left( H_d^\dagger \sigma_a H_d + H_u^\dagger \sigma_a H_u \right)^2 \\
+ \frac{g^2}{8} \left( |H_d|^2 - |H_u|^2 \right)^2 + \frac{g'_2}{2} \left( \tilde{Q}_1 |H_d|^2 + \tilde{Q}_2 |H_u|^2 + \tilde{Q}_S |S|^2 \right)^2 \\
+ m_S^2 |S|^2 + m_1^2 |H_d|^2 + m_2^2 |H_u|^2 + \left[ \lambda A_s S(H_u H_d) + h.c. \right] + \Delta V , \tag{9}
\]

where \( g' = \sqrt{\frac{3}{5}} g_1 \) is the low energy (non-GUT normalised) gauge coupling and \( \tilde{Q}_1 \), \( \tilde{Q}_2 \) and \( \tilde{Q}_S \) are the effective \( U(1)_N \) charges of \( H_d \), \( H_u \) and \( S \) defined below. The first two terms in Eq. (9) correspond to \( F\)–term contributions while the subsequent three represent \( D\)–term contributions associated with \( SU(2)_W, U(1)_Y \) and \( U(1)_N \) gauge interactions. The term in Eq. (9) proportional to \( g_1^2 \) corresponds to the \( D\)–term contribution due to the extra \( U(1)_N \) interaction, which is not present in the MSSM or NMSSM. The value of \( g_1' \) at the EW scale can be determined by assuming gauge coupling unification.

The last term in Eq. (9) \( \Delta V \) represents the contribution of loop corrections to the Higgs effective potential. Here we take into account only the dominant contribution to \( \Delta V \) that comes from loop diagrams involving the top–quark and its Superpartners. In
the leading one–loop approximation we find

$$\Delta V = \frac{3}{32\pi^2} \left[ m_{t_1}^4 \left( \ln \frac{m_{t_1}^2}{Q^2} - \frac{3}{2} \right) + m_{t_2}^4 \left( \ln \frac{m_{t_2}^2}{Q^2} - \frac{3}{2} \right) - 2m_t^4 \left( \ln \frac{m_t^2}{Q^2} - \frac{3}{2} \right) \right]$$  \hspace{1cm} (10)$$

where \( m_t, m_{t_1}, m_{t_2} \) are the masses of the top quark and its Superpartners. The analytical expressions for \( m_{t_1} \) and \( m_{t_2} \) are specified in the next section.

At the physical minimum of the scalar potential \((9)\) the Higgs fields develop VEVs

$$\langle H_d \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} v_1 \\ 0 \end{array} \right), \quad \langle H_u \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_2 \end{array} \right), \quad \langle S \rangle = \frac{s}{\sqrt{2}}.$$  \hspace{1cm} (11)

The equations for the extrema of the Higgs boson potential are:

$$\frac{\partial V}{\partial s} = m_2^2 s - \frac{\lambda A_\lambda}{\sqrt{2}} v_1 v_2 + \frac{\lambda^2}{2} (v_1^2 + v_2^2) s$$
$$+ \frac{g_1^2}{2} \left( \tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_s s^2 \right) \tilde{Q}_s s + \frac{\partial \Delta V}{\partial s} = 0,$$  \hspace{1cm} (12)

$$\frac{\partial V}{\partial v_1} = m_1^2 v_1 - \frac{\lambda A_\lambda}{\sqrt{2}} s v_2 + \frac{\lambda^2}{2} (v_1^2 + s^2) v_1$$
$$+ \frac{g_1^2}{2} \left( \tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_s s^2 \right) \tilde{Q}_1 v_1 + \frac{\partial \Delta V}{\partial v_1} = 0,$$  \hspace{1cm} (13)

$$\frac{\partial V}{\partial v_2} = m_2^2 v_2 - \frac{\lambda A_\lambda}{\sqrt{2}} s v_1 + \frac{\lambda^2}{2} (v_1^2 + s^2) v_2$$
$$+ \frac{g_1^2}{2} \left( \tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_s s^2 \right) \tilde{Q}_2 v_2 + \frac{\partial \Delta V}{\partial v_2} = 0.$$  \hspace{1cm} (14)

where \( \tilde{g} = \sqrt{g_1^2 + g_2^2} \). Instead of \( v_1 \) and \( v_2 \), it is more convenient to use \( \tan \beta = v_2/v_1 \) and \( v = \sqrt{v_1^2 + v_2^2} = 246 \text{ GeV} \).

The VEVs of the Higgs fields \((11)\) induce masses for the gauge bosons and lead to \( Z–Z' \) mixing. In this context, note that the \( U(1)_Y \) and \( U(1)_N \) mix at low energies even before EWSB because gauge symmetries do not forbid a mixing term in the \( E_6 \text{SSM} \) Lagrangian,

$$\mathcal{L}_{\text{mix}}^{\text{kin}} = -\frac{\sin \chi}{2} F_{\mu \nu}^Y F_{\mu \nu}^N,$$  \hspace{1cm} (15)

where \( F_{\mu \nu}^Y \) and \( F_{\mu \nu}^N \) are field strengths for the \( U(1)_Y \) and \( U(1)_N \) gauge interactions. The parameter \( \sin \chi \) is expected to be equal to zero at the GUT scale. Nevertheless a small value of \( \sin \chi \) is generated at low energies due to loop effects. The mixing in the gauge kinetic part of the Lagrangian \((15)\) can be eliminated by means of a non–unitary transformation of the two \( U(1) \) gauge fields \([21, 49, 50]\). In this case all physical phenomena related to the gauge kinetic term mixing can be described by using effective \( U(1)_N \) charges

$$\tilde{Q}_i \equiv Q_i^N + Q_i^Y \delta,$$  \hspace{1cm} (16)
where $\delta = g_{11}/g_1'$

$$g_1 = g_Y, \quad g'_1 = g_N / \cos \chi, \quad g_{11} = -g_Y \tan \chi, \quad (17)$$

while all $U(1)_Y$ charges remain the same.

Initially the EWSB sector involves ten degrees of freedom. However, four of them are massless Goldstone modes which are eaten by the $W^\pm$, $Z$ and $Z'$ gauge bosons. The charged $W^\pm$ bosons gain masses via the interaction with the neutral components of the Higgs doublets in the same way as in the MSSM so that $M_{W'} = \frac{g_2}{2} v$. In contrast, neutral gauge bosons get mixed leading to the formation of two mass eigenstates $Z_1$ and $Z_2$. Letting $Z'$ be the gauge boson associated with $U(1)_N$ we get

$$Z_1 = Z \cos \alpha'_{Z2} + Z' \sin \alpha'_{Z2}, \quad Z_2 = -Z \sin \alpha'_{Z2} + Z' \cos \alpha'_{Z2}, \quad (18)$$

$$M_{Z_1, Z_2}^2 = \frac{1}{2} \left[ M_Z^2 + M_{Z'}^2 \mp \sqrt{(M_Z^2 - M_{Z'}^2)^2 + 4\Delta^4} \right],$$

where

$$M_Z^2 = \frac{g_2^2}{4} v^2, \quad \Delta^2 = \frac{g_1' g_2}{2} v^2 \left( \tilde{Q}_1^2 \cos^2 \beta - \tilde{Q}_2^2 \sin^2 \beta \right), \quad M_{Z'}^2 = g_1'^2 v^2 \left( \tilde{Q}_1^2 \cos^2 \beta + \tilde{Q}_2^2 \sin^2 \beta \right) + g_2'^2 \tilde{Q}_S^2 s^2, \quad (19)$$

$$\alpha'_{Z2} = \frac{1}{2} \arctan \left( \frac{2\Delta^2}{M_{Z_1}^2 - M_{Z_2}^2} \right).$$

Phenomenological constraints typically require the mixing angle $\alpha'_{Z2}$ to be less than $1 - 2 \times 10^{-3}$ and the mass of the extra neutral gauge boson to be heavier than 860 GeV. A suitable mass hierarchy and mixing between $Z$ and $Z'$ are maintained if the field $S$ acquires a large VEV $s \gtrsim 1.5 - 2$ TeV. Then the mass of the lightest neutral gauge boson $Z_1$ is very close to $M_Z$ whereas the mass of $Z_2$ is set by the VEV of the singlet field $M_{Z_2} \simeq M_{Z'} \approx g_1' \tilde{Q}_S s$.

### 4. Particle spectrum

#### 4.1 The squarks and sleptons

In Supersymmetric theories, each quark and lepton state with a specific chirality has a scalar Superpartner. In principle, all scalars with the same electric charge, R–parity and colour quantum numbers can mix with one another. This means that the mass eigenstates of the squarks and sleptons should be obtained by diagonalising three $6 \times 6$ squared–mass matrices for up–type squarks ($\tilde{u}_L, \tilde{c}_L, \tilde{t}_L, \tilde{u}_R, \tilde{c}_R, \tilde{t}_R$), down–type squarks ($\tilde{d}_L, \tilde{s}_L, \tilde{b}_L, \tilde{d}_R, \tilde{s}_R, \tilde{b}_R$) and charged leptons ($\tilde{e}_L, \tilde{\mu}_L, \tilde{\tau}_L, \tilde{e}_R, \tilde{\mu}_R, \tilde{\tau}_R$) and one $3 \times 3$ matrix for sneutrinos ($\tilde{\nu}_e, \tilde{\nu}_\mu, \tilde{\nu}_\tau$). However, since the first and second family quarks and leptons
have negligible Yukawa couplings the mixing angles of the corresponding squark and
electron states are very small so that their masses are set by the appropriate diagonal
elements. Thus one finds,

\[ m_{\tilde{d}_{L_{i}}}^2 \simeq m_{Q_{i}}^2 + \left( -\frac{1}{2} + \frac{1}{3} \sin^2 \theta_W \right) M_Z^2 \cos 2\beta + \Delta_Q, \]

\[ m_{\tilde{u}_{L_{i}}}^2 \simeq m_{Q_{i}}^2 + \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) M_Z^2 \cos 2\beta + \Delta_Q, \]

\[ m_{\tilde{u}_{R_{i}}}^2 \simeq m_{u_{c_{i}}}^2 + \frac{2}{3} M_Z^2 \sin^2 \theta_W \cos 2\beta + \Delta_{u_{c}}, \]

\[ m_{\tilde{d}_{R_{i}}}^2 \simeq m_{d_{c_{i}}}^2 - \frac{1}{3} M_Z^2 \sin^2 \theta_W \cos 2\beta + \Delta_{d_{c}}, \]

\[ m_{\tilde{e}_{L_{i}}}^2 \simeq m_{L_{i}}^2 + \left( -\frac{1}{2} + \sin^2 \theta_W \right) M_Z^2 \cos 2\beta + \Delta_{L}, \]

\[ m_{\tilde{\nu}_{i}}^2 \simeq m_{L_{i}}^2 + \frac{1}{2} M_Z^2 \cos 2\beta + \Delta_{L}, \]

\[ m_{\tilde{e}_{R_{i}}}^2 \simeq m_{e_{c_{i}}}^2 - M_Z^2 \sin^2 \theta_W \cos 2\beta + \Delta_{e_{c}}. \]

The first terms on the right–hand side of Eqs. (20)-(26) are soft scalar masses while all
other terms come from the \( SU(2)_W \), \( U(1)_Y \) and \( U(1)_N \) D–term quartic interactions in the
scalar potential (7) when the Higgs fields get VEVs. In particular,

\[ \Delta_{\phi} = \frac{g_Y^2}{2} \left( \tilde{Q}_1 v_1^2 + \tilde{Q}_2 v_2^2 + \tilde{Q}_S s^2 \right) \tilde{Q}_{\phi}, \]

are contributions of \( U(1)_N \) D–term to the masses of squarks and sleptons. In general the
terms in Eqs. (20)-(26) which are proportional to \( M_Z^2 \) or \( g_Y^2 v^2 \) are typically much smaller
than the soft scalar masses squared and \( g_Y^2 s^2 \). As a consequence the D–term contributions
to the squark and slepton masses are governed by \( \Delta_{\phi} \) which in the leading approximation
are given by

\[ \Delta_Q \simeq \Delta_{u_{c}} \simeq \Delta_{e_{c}} \simeq \frac{1}{10} M_Z^2, \]

\[ \Delta_{d_{c}} \simeq \Delta_{L} \simeq \frac{1}{5} M_Z^2. \]

We emphasise that the extra \( U(1)_N \) D–term gives positive contributions to the masses of
squarks and sleptons because the \( U(1)_N \) charges of the SM-singlet Superfield \( S \) and the
charges of quark and lepton Supermultiplets have the same sign.

Let us now consider the masses of squarks and sleptons of the third generation. In
contrast with the first two families the top quark Yukawa coupling is always large at
the EW scale resulting in substantial mixing between left–handed and right–handed top
squarks. Diagonalising the $2 \times 2$ top squark mass matrix it is easy to see that

$$m^2_{\tilde{t}_1,\tilde{t}_2} = \frac{1}{2} \left\{ m^2_{Q_3} + m^2_{u_3} + \frac{1}{2} M_Z^2 \cos 2\beta + \Delta_Q + \Delta_{u^c} + 2m_t^2 + \right.
\frac{1}{2} \left[ m^2_{Q_3} - m^2_{u_3} + \left[ \frac{1}{2} - \frac{4}{3} \sin^2 \theta_W \right] M_Z^2 \cos 2\beta + \Delta_Q - \Delta_{u^c} \right] + 4m_t^2 X_t^2 \right\},$$

(29)

where $X_t = A_t - \frac{\lambda_s}{\sqrt{2} \tan \beta}$ is a stop mixing parameter. The large value of $X_t$ induces a significant mixing in the stop sector which reduces the mass of the lightest top squark so that it may become one of the lightest eigenstates in the particle spectrum.

With increasing $\tan \beta$, the $b$–quark and $\tau$–lepton Yukawa couplings grow. At large values of $\tan \beta \gg 10$ the couplings $h_b$ and $h_\tau$ become comparable with the top quark Yukawa coupling at the EW scale. This leads to substantial mixing between left–handed and right–handed sbottoms as well as left–handed and right–handed staus. The eigenvalues of the corresponding $2 \times 2$ matrices are given by

$$m^2_{\tilde{b}_1,\tilde{b}_2} = \frac{1}{2} \left\{ m^2_{Q_3} + m^2_{d_3} - \frac{1}{2} M_Z^2 \cos 2\beta + \Delta_Q + \Delta_{d^c} + \right.
\frac{1}{2} \left[ m^2_{Q_3} - m^2_{d_3} + \left[ \frac{1}{2} + \frac{2}{3} \sin^2 \theta_W \right] M_Z^2 \cos 2\beta + \Delta_Q - \Delta_{d^c} \right] + 4m_b^2 X_b^2 \right\},$$

(30)

$$m^2_{\tilde{\tau}_1,\tilde{\tau}_2} = \frac{1}{2} \left\{ m^2_{L_3} + m^2_{e_3} - \frac{1}{2} M_Z^2 \cos 2\beta + \Delta_L + \Delta_{e^c} + \right.
\frac{1}{2} \left[ m^2_{L_3} - m^2_{e_3} + \left[ \frac{1}{2} + 2 \sin^2 \theta_W \right] M_Z^2 \cos 2\beta + \Delta_L - \Delta_{e^c} \right] + 4m_\tau^2 X_\tau^2 \right\},$$

(31)

where $X_b = A_b - \frac{\lambda_s}{\sqrt{2} \tan \beta}$ and $X_\tau = A_\tau - \frac{\lambda_s}{\sqrt{2} \tan \beta}$. From Eqs. (30)-(31) one can see that the magnitude and importance of mixing in the sbottom and stau sectors depend on $\tan \beta$. If $\tan \beta$ is not too large ($\lesssim 10$) the sbottoms and staus are not strongly effected by the mixing terms because $m_b$ and $m_\tau$ are small. In this case the mass eigenstates are very nearly the same as the gauged eigenstates $\tilde{b}_L, \tilde{b}_R, \tilde{\tau}_L$ and $\tilde{\tau}_R$. while their masses can be calculated using Eqs. (20)-(26). For larger values of $\tan \beta$, the mixing effects are non–negligible, and the lightest sbottom and stau mass eigenstates can be significantly lighter than their first and second family counterparts.
4.2 The gluino

The gluino is a colour octet fermion. Therefore, it can not mix with any other particle in SUSY models. Since the gluino is strongly interacting, its running mass $M_3$ changes rather quickly with the renormalisation scale $Q$. Consequently, for an accurate estimate of the gluino mass one should use the scale–independent mass $\tilde{M}_\tilde{g}$ at which the renormalised gluino propagator has a pole. Including one–loop corrections to the gluino propagator that arise from gluon/gluino and quark/squark loops one finds that the gluino’s pole mass is given in terms of the running mass in the DR scheme by

$$M_{\tilde{g}} = M_3(Q) \left[1 - \Delta_{\tilde{g}}(Q)\right]^{-1},$$

where,

$$\Delta_{\tilde{g}}(Q) = \frac{g_3^2(Q)}{16\pi^2} \left\{9 \ln \left(\frac{Q^2}{M_3^2}\right) + 15 - \sum_q \sum_{i=1}^2 B_1(M_3, m_{q'}, m_{\tilde{q}'}) - \sum_q \frac{m_q}{M_3} \sin(2\theta_q) \left[B_0(M_3, m_q, m_{\tilde{q}_1}) - B_0(M_3, m_q, m_{\tilde{q}_2})\right]\right\},$$

and,

$$B_1(p, m_1, m_2) = \frac{1}{2p^2} \left[A_0(m_2) - A_0(m_1) + (p^2 + m_1^2 - m_2^2)B_0(p, m_1, m_2)\right],$$
$$A_0(m) = m^2 \left[1 - \ln \frac{m^2}{Q^2}\right],$$
$$B_0(p, m_1, m_2) = -\ln \left(\frac{p^2}{Q^2}\right) - f_B(x_+) - f_B(x_-),$$

with,

$$f_B(x) = \ln(1 - x) - x \ln(1 - x^{-1}) - 1, \quad x_{\pm} = \frac{s \pm \sqrt{s^2 - 4p^2(m_1^2 - i\varepsilon)}}{2p^2},$$

and $s = p^2 - m_2^2 + m_1^2$. This expression for the gluino’s pole mass (32) automatically incorporates the one–loop renormalisation group resummaton. The first two terms in the right hand side of Eq. (33) correspond to the gluon/gluino one–loop contributions while other terms represent quark/squark one–loop corrections to the gluino mass. Indices $q'$ and $\tilde{q}'$ in Eq. (33) denote light quarks and their Superpartners. In the case of the light quarks we neglect the mixing between left–handed and right–handed squark states. The sum over $q$ in the bottom line of Eq. (33) includes only heavy quarks for which mixing effects parametrised via the mixing angle $\theta_q$ can not be ignored. The corrections specified above can be as large as 20%–30% because the gluino is strongly interacting, with a large group theory factor due to its colour, and because it couples to all of the squark–quark pairs.
4.3 The charginos and neutralinos

After EWSB, all Superpartners of the gauge and Higgs bosons acquire non–zero masses. Since the Supermultiplets of the $Z'$ boson and SM-singlet Higgs field $S$ are electromagnetically neutral they do not contribute any extra particles to the chargino spectrum. Consequently the chargino mass matrix and its eigenvalues remain the same as in the MSSM, namely

$$m_{χ^±}^2 = \frac{1}{2} \left[ M_2^2 + \mu_{\text{eff}}^2 + 2M_W^2 \right. \pm \sqrt{\left( M_2^2 + \mu_{\text{eff}}^2 + 2M_W^2 \right)^2 - 4(M_2\mu_{\text{eff}} - M_W^2 \sin 2\beta)^2} \right], \quad (37)$$

where $M_2$ is the $SU(2)$ gaugino mass and $\mu_{\text{eff}} = \frac{\lambda v}{\sqrt{2}}$. LEP searches for SUSY particles including data collected at $\sqrt{s}$ between 90 GeV and 209 GeV set a 95% CL lower limit on the chargino mass of about 100 GeV [53]. This lower bound constrains the parameter space of the $E_6$ SSM restricting the absolute values of the effective $\mu$–term and $M_2$ from below, i.e. $|M_2|, |\mu_{\text{eff}}| \geq 90 - 100$ GeV.

In the neutralino sector there are two extra neutralinos besides the four MSSM ones. One is an extra gaugino coming from the $Z'$ vector Supermultiplet. The other is an additional Higgsino $\tilde{S}$ (singlino). In the interaction basis $(\tilde{B}, \tilde{W}_3, \tilde{H}_1^0, \tilde{H}_2^0, \tilde{S}, \tilde{B}')$ the neutralino mass matrix takes a form

$$M_{\tilde{\chi}^0} = \begin{pmatrix}
M_1 & 0 & -\frac{1}{2}g'v_1 & \frac{1}{2}g'v_2 & 0 & 0 \\
0 & M_2 & \frac{1}{2}gv_1 & -\frac{1}{2}gv_2 & 0 & 0 \\
-\frac{1}{2}g'v_1 & \frac{1}{2}gv_1 & 0 & -\mu_{\text{eff}} & -\frac{\lambda v_2}{\sqrt{2}} & \tilde{Q}_1g'v_1 \\
\frac{1}{2}g'v_2 & -\frac{1}{2}gv_2 & -\mu_{\text{eff}} & 0 & -\frac{\lambda v_1}{\sqrt{2}} & \tilde{Q}_2g'v_2 \\
0 & 0 & -\frac{\lambda v_2}{\sqrt{2}} & -\frac{\lambda v_1}{\sqrt{2}} & 0 & \tilde{Q}_sg'v_2 \\
0 & 0 & \tilde{Q}_1g'v_1 & \tilde{Q}_2g'v_2 & \tilde{Q}_sg'v_2 & M_1'
\end{pmatrix}, \quad (38)$$

where $M_1$, $M_2$ and $M_1'$ are the soft gaugino masses for $\tilde{B}$, $\tilde{W}_3$ and $\tilde{B}'$ respectively. In Eq. (38) we neglect the Abelian gaugino mass mixing $M_{11}$ between $\tilde{B}$ and $\tilde{B}'$ that arises at low energies as a result of the kinetic term mixing even if there is no mixing in the initial values of the soft SUSY–breaking gaugino masses near the GUT or Planck scale [49]. The top–left $4 \times 4$ block of the mass matrix (38) contains the neutralino mass matrix of the MSSM where the parameter $\mu$ is replaced by $\mu_{\text{eff}}$. The lower right $2 \times 2$ submatrix represents the extra components of neutralinos. The neutralino sector in $E_6$ inspired SUSY models was studied recently in [23], [31–33], [39–40], [54–55].
As one can see from Eqs. (37)–(38) the masses of charginos and neutralinos depend on \( \lambda, s, \tan \beta, M_1, M_1', \) and \( M_2. \) In SUGRA models with uniform gaugino masses at the GUT scale, the RGE flow yields a relationship \( M_1' \approx M_1 \approx 0.5M_2. \) Due to stringent constraints on the mass of the \( Z' \) boson, the VEV of the SM singlet field \( S \) has to be large (\( s \gtrsim 2 \text{ TeV} \)). This implies that \( \tilde{Q}g'_1s \) and \( \mu_{\text{eff}} \) are much larger than other entries in the neutralino mass matrix (38). As a result the mass matrix (38) can be approximately diagonalised and the expressions for the chargino masses (37) can be substantially simplified. In this case one chargino and two neutralinos are almost degenerate with mass \( |\mu_{\text{eff}}|, \) i.e.

\[
|m_{\chi^\pm_1}| \approx |m_{\chi^0_3}| \approx |m_{\chi^0_4}| \approx |\mu_{\text{eff}}|.
\]

They are formed predominantly from the neutral and charged Superpartners of the Higgs bosons. Two other neutralinos are mixtures of the \( U(1)_N \) gaugino \( \tilde{B}' \) and singlino \( \tilde{S}. \) Their masses are closely approximated by

\[
|m_{\chi^0_{5,6}}| \approx \frac{1}{2} \left[ \sqrt{M_1'^2 + 4M_2'^2} \mp \sqrt{M_1'^2 + 4M_2'^2} \right].
\]

Since the masses of extra neutralino states are controlled by the \( Z' \) boson mass they tend to be heavy (\( \sim 1 \text{ TeV} \)) so that their direct observation is unlikely in the near future. The Superpartners of the \( SU(2) \) gauge bosons compose another chargino and neutralino whose masses are governed by \( |M_2|. \) Finally, the mass of the neutralino state that is predominantly bino, \( \tilde{B}, \) is set by \( |M_1|. \)

### 4.4 The exotic particles

In addition to the NMSSM-like particle content, the \( E_6 \) SSM involves exotic matter that forms three families of down–type quark Superfields \( (\tilde{D}_i \text{ and } D_i) \), two generations of Inert Higgs Supermultiplets \( (H_\alpha^d \text{ and } H_\alpha^u) \), two families of extra singlets \( S_\alpha \) and a vector–like doublet Superfield associated with the survival components of the extra 27' and \( \overline{27}' (H' \text{ and } \overline{H}') \) which manifest themselves in the Yukawa interactions (3) as fields with lepton number \( L = \pm 1. \) The masses of the fermion and scalar components of this vector–like lepton Supermultiplet are set by \( \mu' \) which is expected to be of the order of \( 10 \text{ TeV}. \) Therefore these exotic lepton fields are normally very heavy and decouple from the rest of the particle spectrum. The masses of the fermion components of the exotic quark and Inert Higgs Supermultiplets are determined by the VEV of the SM-singlet field \( S \) and by the Yukawa couplings \( \kappa_i \) and \( \lambda_\alpha. \) They are given by

\[
\mu_{\tilde{D}_i} = \frac{\kappa_i}{\sqrt{2}} s, \quad \mu_{\tilde{H}_\alpha} = \frac{\lambda_\alpha}{\sqrt{2}} s,
\]

where \( \mu_{\tilde{D}_i} \) are exotic quark masses, while \( \mu_{\tilde{H}_\alpha} \) are the masses of the Inert Higgsinos. The experiments at LEP, HERA and the Tevatron set stringent lower bounds on the masses
of exotic quarks and new charged particles, so the Yukawa couplings $\kappa_i$ and $\lambda_\alpha$ cannot be negligibly small.

Relatively large masses of exotic quarks give rise to a substantial mixing between the corresponding exotic squark states. Since we choose a field basis such that the Yukawa couplings of $D_i$ and $\bar{D}_i$ to $S$ are flavour diagonal, the calculation of the exotic squark masses reduces to the diagonalisation of three $2 \times 2$ matrices whose eigenvalues can be written as

$$M_{D_1, D_2}^2 = \frac{1}{2} \left\{ m_{D_i}^2 + m_{\bar{D}_i}^2 + 2\mu_{D_i}^2 + \Delta_D + \Delta_{\bar{D}} \right\}$$

$$\pm \sqrt{\left[ m_{D_i}^2 - m_{\bar{D}_i}^2 + \frac{2}{3} M_Z^2 \cos 2\beta \sin^2 \theta_W + \Delta_D - \Delta_{\bar{D}} \right]^2 + 4\mu_{D_i}^2 X_{D_i}^2} \right\},$$

where $X_{D_i} = A_{\kappa_i} - \frac{\lambda}{2\sqrt{2}s}v^2\sin 2\beta$ and $\Delta_\phi = \frac{g_1'^2}{2} \left( \tilde{Q}_1v_1^2 + \tilde{Q}_2v_2^2 + \tilde{Q}_SS^2 \right)\tilde{Q}_\phi$. Relatively heavy Inert Higgsinos also lead to significant mixing effects in the Inert Higgs boson sector. Once again, the flavour diagonal structure of the Yukawa couplings of $H_d^\alpha$ and $H_u^\alpha$ to the singlet field $S$, leads to mixing only between the Inert Higgs bosons from the same family. Diagonalising the appropriate $2 \times 2$ mass matrices one finds,

$$m_{H_{\alpha_1}, H_{\alpha_2}} = \frac{1}{2} \left\{ m_{H_d^\alpha}^2 + m_{H_u^\alpha}^2 + 2\mu_{H_{\alpha}}^2 + \Delta_{H_d} + \Delta_{H_u} \right\}$$

$$\pm \sqrt{\left[ m_{H_d^\alpha}^2 - m_{H_u^\alpha}^2 + M_Z^2 \cos 2\beta + \Delta_{H_d} - \Delta_{H_u} \right]^2 + 4\mu_{H_{\alpha}}^2 X_{H_{\alpha}}^2} \right\},$$

$$m_{H_{\alpha_1}^\pm, H_{\alpha_2}^\pm} = \frac{1}{2} \left\{ m_{H_d^\alpha}^2 + m_{H_u^\alpha}^2 + 2\mu_{H_{\alpha}}^2 + \Delta_{H_d} + \Delta_{H_u} \right\}$$

$$\pm \sqrt{\left[ m_{H_d^\alpha}^2 - m_{H_u^\alpha}^2 - M_Z^2 \cos 2\beta \cos 2\theta_W + \Delta_{H_d} - \Delta_{H_u} \right]^2 + 4\mu_{H_{\alpha}}^2 X_{H_{\alpha}}^2} \right\},$$

where $X_{H_{\alpha}} = A_{\lambda_{\alpha}} - \frac{\lambda}{2\sqrt{2}s}v^2\sin 2\beta$. The magnitude of the mixing in the exotic squark and Inert Higgs sectors is governed by the mixing parameters $X_{D_i}$ and $X_{H_{\alpha}}$ as well as by the Yukawa couplings $\kappa_i$ and $\lambda_\alpha$. If the Yukawa couplings that determine the mixing of the exotic scalar fields are large, the mixing effects can be so substantial that the corresponding lightest exotic squarks and/or Inert Higgs bosons may be among the lightest SUSY particles in the spectrum of the $E_6$SSM. Additionally, when $\kappa_i$ or $\lambda_i$ are relatively small the appropriate exotic quarks or Inert Higgsinos may be sufficiently light that they can be discovered at the LHC.
Since we neglect the couplings $f_{\alpha\beta}$ and $\tilde{f}_{\alpha\beta}$ in the Superpotential (3), the scalar components of the SM-singlet Superfields $S_\alpha$ do not mix with other scalar fields. Their masses are given by

$$M^2_{S_\alpha} = m^2_{S_\alpha} + \Delta_S,$$  \hspace{1cm} \text{(45)}

where $m^2_{S_\alpha}$ are soft scalar masses while $\Delta_S$ is a $U(1)_N$ D–term contribution. In the leading approximation, the $U(1)_N$ D–term contributions to the masses of the exotic scalars are set by $M^2_{Z'}$:

$$\Delta_D \simeq \Delta_{H_u} \simeq -\frac{1}{5} M^2_{Z'}, \hspace{1cm} \Delta_{\bar{D}} \simeq \Delta_{H_d} \simeq -\frac{3}{10} M^2_{Z'}, \hspace{1cm} \Delta_S \simeq \frac{1}{2} M^2_{Z'}.$$ \hspace{1cm} \text{(46)}

We emphasise that in contrast with the ordinary squarks and sleptons, the $U(1)_N$ D–term gives negative contributions to the masses of exotic squarks and Inert Higgs bosons because the $U(1)_N$ charge of the SM-singlet Superfield $S$ and the $U(1)_N$ charges of the exotic quarks and Inert Higgs Supermultiplets are opposite. The $U(1)_N$ D–term gives the largest contributions to the masses of the scalar components of the SM-singlet Superfields $S_\alpha$, making these fields rather heavy.

### 4.5 The Higgs bosons

Due to electric charge conservation the charged components of the Higgs doublets do not mix with neutral Higgs fields. They form a separate sector whose spectrum is described by a $2 \times 2$ mass matrix. Its determinant has zero value leading to the appearance of two Goldstone states which are absorbed into the longitudinal degrees of freedom of the $W^\pm$ gauge boson. Their orthogonal linear combination gains mass

$$m^2_{H^\pm} = \frac{\sqrt{2} \lambda A \lambda}{\sin 2\beta} v^2 - \frac{\lambda^2}{2} v^2 + \frac{g^2}{2} v^2 + \Delta_\pm,$$ \hspace{1cm} \text{(47)}

where $\Delta_\pm$ represents the contribution of loop corrections to the charged Higgs boson mass in the $E_6$SSM.

The imaginary parts of the neutral components of the Higgs doublets and imaginary part of the SM-singlet field $S$ compose the CP–odd Higgs sector of the model. This sector includes two Goldstone modes $G_0$, $G'$ which are swallowed by the $Z$ and $Z'$ bosons after EWSB, leaving only one physical CP–odd Higgs state $A$ which acquires mass

$$m^2_A = \frac{\sqrt{2} \lambda A \lambda}{\sin 2\varphi} v + \Delta_A, \hspace{1cm} \tan \varphi = \frac{v}{2s} \sin 2\beta,$$ \hspace{1cm} \text{(48)}

where $\Delta_A$ is the contribution of loop corrections.
The CP–even Higgs sector involves Re $H_d^0$, Re $H_u^0$ and Re $S$. In the field space basis $(h, H, N)$ rotated by an angle $\beta$ with respect to the initial one

$$Re \ H_d^0 = (h \cos \beta - H \sin \beta + v_1)/\sqrt{2},$$

$$Re \ H_u^0 = (h \sin \beta + H \cos \beta + v_2)/\sqrt{2},$$

$$Re \ S = (s + N)/\sqrt{2},$$ (49)

the mass matrix of the Higgs scalars takes the form [56]:

$$M^2 = \begin{pmatrix}
\frac{\partial^2 V}{\partial v^2} & \frac{1}{v} \frac{\partial^2 V}{\partial v \partial \beta} & \frac{1}{v} \frac{\partial^2 V}{\partial v \partial \bar{s}} \\
\frac{1}{v} \frac{\partial^2 V}{\partial v \partial \beta} & \frac{\partial^2 V}{\partial v^2} & \frac{1}{v} \frac{\partial^2 V}{\partial v \partial s} \\
\frac{1}{v} \frac{\partial^2 V}{\partial v \partial \bar{s}} & \frac{1}{v} \frac{\partial^2 V}{\partial v \partial s} & \frac{\partial^2 V}{\partial \bar{s}^2}
\end{pmatrix} = \begin{pmatrix}
M^2_{11} & M^2_{12} & M^2_{13} \\
M^2_{21} & M^2_{22} & M^2_{23} \\
M^2_{31} & M^2_{32} & M^2_{33}
\end{pmatrix}. \quad (50)

Taking second derivatives of the Higgs boson effective potential and substituting $m_1^2$, $m_2^2$, $m_3^2$ from the minimisation conditions (12)-(14) one obtains,

$$M^2_{11} = \frac{\lambda^2}{2} v^2 \sin^2 2\beta + \frac{g_2^2}{4} v^2 \cos^2 2\beta + g_1^2 v^2 (\tilde{Q}_1 \cos 2\beta + \tilde{Q}_2 \sin 2\beta)^2 + \Delta_1,$$

$$M^2_{12} = M^2_{21} = \left(\frac{\lambda^2}{4} - \frac{g_2^2}{8}\right) v^2 \sin 4\beta + \frac{g_1^2}{2} v^2 (\tilde{Q}_2 - \tilde{Q}_1) (\tilde{Q}_1 \cos 2\beta + \tilde{Q}_2 \sin 2\beta) \sin 2\beta + \Delta_2,$$

$$M^2_{22} = \frac{\sqrt{2} \lambda A_2}{\sin 2\beta} s + \left(\frac{g_2^2}{4} - \frac{\lambda^2}{2}\right) v^2 \sin^2 2\beta + \frac{g_1^2}{4} (\tilde{Q}_2 - \tilde{Q}_1)^2 v^2 \sin^2 2\beta + \Delta_2,$$

$$M^2_{23} = \frac{\lambda A_2}{\sqrt{2}} v \cos 2\beta + \frac{g_1^2}{2} (\tilde{Q}_2 - \tilde{Q}_1) \tilde{Q}_S v s \sin 2\beta + \Delta_2,$$

$$M^2_{32} = -\frac{\lambda A_2}{\sqrt{2}} v \sin 2\beta + \lambda^2 v s + g_1^2 (\tilde{Q}_1 \cos 2\beta + \tilde{Q}_2 \sin 2\beta) \tilde{Q}_S v s + \Delta_3,$$

$$M^2_{33} = \frac{\lambda A_2}{2 \sqrt{2} s} v^2 \sin 2\beta + g_1^2 \tilde{Q}_S^2 s^2 + \Delta_3. \quad (51)$$

In Eqs. (51) the $\Delta_{ij}$’s are loop corrections to the mass matrix of the CP–even Higgs bosons in the E6SSM. The explicit expressions for $\Delta_{ij}$, calculated in the leading one–loop approximation, are given in Appendix A.

When the SUSY–breaking scale $M_S$ and VEV of the singlet field are considerably larger than the EW scale, the mass matrix (50)–(51) has a hierarchical structure. Therefore the masses of the heaviest Higgs bosons are closely approximated by the diagonal entries $M^2_{22}$ and $M^2_{33}$ which are expected to be of the order of $M^2_S$ or even higher. All off–diagonal matrix elements are relatively small $\lesssim M_S M_Z$. As a result the mass of one CP–even Higgs boson (approximately given by $H$) is governed by $m_A$ while the mass of another one (predominantly the $N$ singlet field) is set by $M_{Z'}$. Since the minimal eigenvalue of
the mass matrix \((50)-(51)\) is always less than its smallest diagonal element at least one Higgs scalar in the CP–even sector (approximately \(h\)) remains light even when the SUSY–breaking scale tends to infinity, i.e. \(m^2_{h_1} \lesssim M^2_{11}\). In contrast with the MSSM, the lightest Higgs boson in the \(E_6\) SSM can be heavier than \(110 - 120\) GeV even at tree level. In the two–loop approximation the lightest Higgs boson mass does not exceed \(150 - 155\) GeV \cite{37}–\cite{38}. The Higgs sector in the \(E_6\) inspired SUSY models was studied recently in \cite{37}, \cite{55}, \cite{57}.

5. Constructing realistic \(cE_6\)SSM scenarios

5.1 RG flow of couplings in the \(cE_6\)SSM

Below the GUT scale, the RG flow causes the gauge couplings and the soft SUSY–breaking parameters to split from the universal values \(g_0\), \(m_0^2\), \(M_{1/2}\) and \(A_0\). This splitting is described by the RGEs of the model, presented in Appendix \[E\]. For the gauge and Yukawa couplings two–loop RGEs are given as well as two–loop RGEs for \(M_a(\mu)\) and \(A_i(\mu)\) and one–loop RGEs for \(m^2_i(\mu)\).

This complete set of \(E_6\)SSM RGEs can be separated into two parts. The first describes the evolution of gauge and Yukawa coupling constants and is a nonlinear set of equations even in the one–loop approximation. Therefore it is extremely difficult or even impossible to find either exact or approximate solutions of these equations. The remaining subset of RGEs describes the running of fundamental parameters which break SUSY in a soft way. If the renormalisation group flow of the gauge and Yukawa couplings is known, this part of the RGEs can be considered as a set of linear differential equations for the soft SUSY–breaking terms. To solve them, first one integrates the equations for the gaugino masses \(M_i\). In the one–loop approximation we find,

\[
M_i(t) = \frac{g^2_i(t)}{g_0^2} M_{1/2}, \quad M'_i(t) = \frac{g'^2_i(t)}{g_0^2} M_{1/2},
\]

where the index \(i\) runs from 1 to 3 and \(t = \ln \frac{Q}{M_X}\), with \(Q\) being the renormalisation scale at which Eq. \((52)\) holds true.

Next one integrates the one–loop RGEs for the trilinear scalar couplings \(A_i(t)\) which can be written as,

\[
\frac{dA_i(t)}{dt} = S_{ij}(t)A_j(t) + F_i(t).
\] The dependence of \(F_i\) on \(t\) comes from the gaugino masses appearing in the one–loop RGE of the trilinears. One then finds the solution of this system of linear differential
where we have introduced \( \Phi_{ij}(t) \), which is the solution of the homogeneous equation 
\[
d\Phi_{ij}(t)/dt = S_{ik}(t)\Phi_{kj}(t),
\]
with the boundary conditions \( \Phi_{ij}(0) = \delta_{ij} \). From the universality constraint and exploiting Eq. (52) to write \( F_i(t) \propto M_{1/2} \), the solution of the RGEs

for the trilinear scalar couplings takes the form

\[
A_i(t) = e_i(t)A_0 + f_i(t)M_{1/2}.
\]

(55)

The obtained solution Eq. (55) can be substituted into the right–hand sides of the RGEs for the soft scalar masses which may be presented in the following form,

\[
\frac{dm_i^2(t)}{dt} = \tilde{S}_{ij}(t)m_j^2(t) + \tilde{F}_i(t).
\]

(56)

Due to the scalar mass universality constraints and the fact that the functions \( \tilde{F}_i(t) \) contain terms which are proportional to \( A_0^2, A_0M_{1/2}, \) and \( M_{1/2}^2 \) the solution of the linear system of differential Eq. (56) reduces to,

\[
m_i^2(t) = a_i(t)m_0^2 + b_i(t)M_{1/2}^2 + c_i(t)A_0M_{1/2} + d_i(t)A_0^2.
\]

(57)

Analytic expressions for \( e_i(t), f_i(t), a_i(t), b_i(t), c_i(t), \) and \( d_i(t) \), which determine the evolution of \( A_i(t) \) and \( m_i^2(t) \), are unknown, since an exact analytic solution of the \( E_6 \) SSM RGEs is not available.

The sensitivity of these functions to the Yukawa and gauge couplings at \( M_X \) is again very strong. In particular it is important to reiterate that the one–loop \( \beta \)–function for the gauge coupling of strong interactions is zero. So the running of \( g_3 \) and \( M_3 \) is dictated solely by the two–loop contributions and these two–loop \( \beta \)–functions can change the RG flow substantially. In this study the two–loop \( \beta \)–functions for the gaugino masses and trilinear couplings were included. The solution of two–loop RGEs for the \( M_i(t) \) can be written as,

\[
M_i(t) = p_i(t)A_0 + q_i(t)M_{1/2}.
\]

(58)

One can see that in the two–loop approximation gaugino masses depend not only on the universal gaugino mass, \( M_{1/2} \), but also on the trilinear scalar coupling, \( A_0 \). The numerical calculations show that the dependence of \( M_i(t) \) on \( A_0 \) is rather weak, i.e. \( p_i(t_0) \ll 1 \). However the change in the co-efficient \( q_i(t) \) is substantial and at low–energies the gaugino masses change by 20–40%.

The general form of the solutions of RGEs for \( m_i^2(t) \) and \( A_i(t) \) remains intact after the inclusion of two–loop effects. At the same time some of the coefficient functions \( f_i(t) \),
\[ t = \ln\left[ \frac{Q^2}{M_X^2} \right] \]

Figure 1: Two-loop RG flow of gauge couplings within the $E_6$ SSM for $T_{\text{MSSM}} = T_{\text{ESSM}} = M_t = 175$ GeV (upper lines) and $T_{\text{MSSM}} = 250$ GeV, $T_{\text{ESSM}} = 1500$ GeV (lower lines). Here we fix $\tan \beta = 10$ and $\alpha_3(M_Z) = 0.118$.

| $T_{\text{MSSM}}$ (GeV) | 250 | 250 | 250 | 175 | 175 | 175 |
|--------------------------|-----|-----|-----|-----|-----|-----|
| $T_{\text{ESSM}}$ (GeV)  | 1500| 800 | 250 | 1500| 250 | 175 |
| $g_0^2$                  | 1.54| 1.60| 1.78| 1.61| 1.88| 1.95 |
| $M_X$ (GeV)              | $3.5 \cdot 10^{16}$ | $3.3 \cdot 10^{16}$ | $3.5 \cdot 10^{16}$ | $3.7 \cdot 10^{16}$ | $4 \cdot 10^{16}$ | $4 \cdot 10^{16}$ |

Table 2: The dependence of $g_0^2$ and $M_X$ on the threshold effects in the exceptional SUSY model. Here we fix $\tan \beta = 10$ and $\alpha_3(M_Z) = 0.118$.

$b_i(t)$ and $c_i(t)$ change significantly. The two-loop corrections to the $\beta$–functions have the strongest impact on the RG flow of the soft SUSY–breaking terms which are sensitive to strong interactions.

The RG flow of the gauge couplings, $g_i(t)$, is also quite sensitive to threshold effects. In Fig. 1 the running of $\alpha_i(t)$ is presented for two different sets of threshold scales, $T_{\text{MSSM}} = T_{\text{ESSM}} = 175$ GeV and $T_{\text{MSSM}} = 250$ GeV, $T_{\text{ESSM}} = 1500$ GeV. The threshold $T_{\text{MSSM}}$ is a common scale for the sparticles of ordinary matter, while $T_{\text{ESSM}}$ is a common mass scale for new exotic particles not present in the MSSM. The unified gauge coupling at $M_X$ changes from 1.24 to 1.4 between the two threshold choices. This result and also the value of $g_0^2$ for several other threshold choices, $T_{\text{MSSM}}$ and $T_{\text{ESSM}}$, are summarised in Tab. 2. Since soft SUSY–breaking terms depend very strongly on the values of the gauge couplings at the GUT scale, the uncertainty related to the choice of the threshold scales limits the accuracy of our calculations of the particle spectrum. The results of our numerical analysis presented in Tab. 2 and Fig. 1 indicate that it is unrealistic to expect an accuracy, in the calculation of the sparticle masses, better than 10%.

In our analysis thresholds are used only in the SUSY preserving sector where full two–loop RGE are employed and are neglected in the soft SUSY–breaking sector where only one–loop RGE are used for the scalar masses. The thresholds are chosen before the spectrum is determined and are therefore only an estimate. A more accurate analysis is left for a further study. We chose $T_{\text{MSSM}} = 600$ GeV and $T_{\text{ESSM}} = 3$ TeV to be the mass
5.2 Procedure of our analysis

To calculate the particle spectrum within the cE\(_6\)SSM one must find masses and couplings which are consistent with both the high scale universality constraints and the low scale EWSB constraints. To evolve between these two scales we use two–loop renormalisation group equations (RGEs), presented in Appendix B in a modified version of SOFTSUSY 2.0.5 [58]. The details of the procedure we followed are summarized below.

1. The gauge and Yukawa couplings are determined independently of the soft SUSY breaking mass parameters as follows:
   (i) We select values for \( s = \sqrt{2} \langle S \rangle \) and \( \tan \beta = v_2/v_1 \).
   (ii) We set the gauge couplings \( g_1, g_2 \) and \( g_3 \) equal to the experimentally measured values at \( M_Z \).
   (iii) We fix the low energy Yukawa couplings \( h_t, h_b, \) and \( h_\tau \) using the relations between the running masses of the fermions of the third generation and VEVs of the Higgs fields, i.e.
   \[
   m_t(M_t) = \frac{h_t(M_t)}{\sqrt{2}} v \sin \beta, \quad m_b(M_t) = \frac{h_b(M_t)}{\sqrt{2}} v \cos \beta, \quad m_\tau(M_t) = \frac{h_\tau(M_t)}{\sqrt{2}} v \cos \beta.
   \]
   (iv) The gauge and Yukawa couplings are then evolved up to the GUT scale \( M_X \). Using the beta functions for QED and QCD, the gauge couplings are evolved up to \( m_t \). Between \( m_t \) and \( T_{MSSM} \) we evolve the gauge and Yukawa couplings with SM RGEs and between \( T_{MSSM} \) and \( T_{ESSM} \) we employ the MSSM RGEs. At \( T_{ESSM} \) the values of E\(_6\)SSM gauge and Yukawa couplings, \( g_1, g_2, g_3, h_t, h_b \) and \( h_\tau \), form a low energy boundary condition for what follows. Initial low energy estimates of the new E\(_6\)SSM Yukawa couplings, \( \lambda_i \) and \( \kappa_i \) are also input here, and all SUSY preserving couplings are evolved up to the unification scale using the two–loop E\(_6\)SSM RGEs.
   (v) At the unification scale \( M_X \) we set \( g'_1 = g_0 \) and select values for \( \kappa_i(M_X) \) and \( \lambda_i(M_X) \), which are input parameters in our procedure. An iteration is then performed between \( M_X \) and the low energy scale to obtain the values of all the gauge and Yukawa couplings which are consistent with our input values for \( \kappa_i(M_X) \), \( \lambda_i(M_X) \), gauge coupling unification and our low scale boundary conditions, derived from experimental data.

2. Now that the values of the gauge and Yukawa couplings have been obtained, the coefficients \( e_i(t) \), \( f_i(t) \), \( a_i(t) \), \( b_i(t) \), \( c_i(t) \), \( d_i(t) \), \( p_i(t) \) and \( q_i(t) \), appearing in Eq. (55), Eq. (57) and Eq. (58), can be obtained for \( t = \ln[T_{ESSM}/M_X^2] \). Low energy soft mass parameters are then functions of the GUT scale values of \( A_0 \), \( M_{1/2} \) and \( m_0 \). These coefficients are determined numerically as follows:
(i) Set $A_0$ and $M_{1/2}$ to zero at $M_X$ while giving $m_0$ a non-zero value and run the full set of E6SSM parameters down to the low scale to yield the coefficients proportional to $a_i(t)$ in the expressions for each low energy scalar (mass)$^2, m_i^2$.

(ii) Repeat for $A_0$ and $M_{1/2}$ to obtain coefficients $b_i(t)$ and $d_i(t)$ for each $m_i^2$; coefficients $c_i(t)$ and $f_i(t)$ for each low energy trilinear soft mass $A_i$ and coefficients $p_i(t)$ and $q_i(t)$ for each low energy gaugino soft mass $M_i$.

(iii) The coefficients, $c_i(t)$, of the $A_0 M_{1/2}$ terms appearing in the semi-analytic expressions for each $m_i^2$ are then determined using non-zero values of both $A_0$ and $M_{1/2}$ at $M_X$, using the results in part (ii) to isolate this term.

3. The semi-analytic expressions for the soft masses from step 2 above provide the set of low energy constraints on the soft masses coming from our cE$_6$SSM universality conditions. These are then combined with the conditions for correct EWSB, appearing in Eqs. (12)-(14), at low energy and determine sets of $m_0, M_{1/2}$ and $A_0$ which are consistent with EWSB, as follows:

(i) Working with the tree–level potential $V_0$ (to start with) we impose the minimisation conditions $\frac{\partial V_0}{\partial s} = \frac{\partial V_0}{\partial v_1} = \frac{\partial V_0}{\partial v_2} = 0$. In the tree–level approximation each of the EWSB conditions are quadratic functions of $\lambda_3(\mu)$, where $\mu$ is the energy scale at which the EWSB conditions are imposed. Using the semi-analytic approach described above to replace the third generation soft Higgs and Singlet masses and $A_3$ reveals that each EWSB condition also has quadratic dependence on the soft unification scale parameters $m_0, M_{1/2}$ and $A_0$. With three constraints and three soft mass parameters, the equations can be reduced to two second order equations with respect to $A_0$ and $M_{1/2}$, or equivalently one quartic equation with respect to $A_0$. This equation is solved numerically, and the resulting value for $A_0$ is used to obtain $M_{1/2}$ and $m_0$. For fixed values of gauge couplings, Yukawas and VEVs (determined from choices of $\tan\beta$ and $s$ with $v$ known from experiment) there are four sets of soft masses $A_0, M_{1/2}$ and $m_0$, though some or all can in principle be complex. Here we restrict our consideration to the scenarios with real values of fundamental parameters which do not induce any CP–violating effects. Therefore our routine deals with between 0 and 4 sets of real solutions to the soft masses.

(ii) For each solution $m_0, M_{1/2}$ and $A_0$ the low energy stop soft mass parameters are determined and the one–loop Coleman-Weinberg Higgs effective potential $V_1$ is calculated. The new minimisation conditions for $V_1$ are then imposed, and new solutions for $m_0, M_{1/2}$ and $A_0$ are obtained.

(iii) The procedure in (ii) is then iterated until we find stable solutions. For some values of $\tan\beta, s$ and Yukawa couplings the solutions with real $A_0, M_{1/2}$ and $m_0$ do not exist. There is a substantial part of the parameter space where there are only two solutions with real values of fundamental parameters. However, there are also some regions of the
parameters where all four solutions of the non–linear algebraic equations are real.

Although correct EWSB is not guaranteed in the cE6SSM, remarkably, there are always solutions with real $A_0$, $M_{1/2}$ and $m_0$ for sufficiently large values of $\kappa_i$, which drive $m_S^2$ negative. This is easy to understand since the $\kappa_i$ couple the singlet to a large multiplicity of coloured fields, thereby efficiently driving its squared mass negative to trigger the breakdown of the gauge symmetry.

4. Using the obtained solutions we calculate the masses of all exotic and SUSY particles, using expressions given in section 4, for each set of fundamental parameters.

Finally, at the last stage of our analysis we vary Yukawa couplings, $\tan \beta$ and $s$ to establish the qualitative pattern of the particle spectrum within the cE6SSM. To avoid any conflict with present and former collider experiments as well as with recent cosmological observations we impose the set of constraints specified in the next section. We then demonstrate how these bounds restrict the allowed range of the parameter space in the cE6SSM by performing scans over our input parameters.

5.3 Experimental and Theoretical Constraints

The experimental constraints applied in our analysis are: $m_h \geq 114$ GeV, all sleptons and charginos are heavier than 100 GeV, all squarks and gluinos have masses above 300 GeV and the $Z'$ boson has a mass which is larger than 860 GeV [52]. We also impose the most conservative bound on the masses of exotic quarks and squarks that comes from the HERA experiments [59], by requiring that they are heavier than 300 GeV. Finally, we require that the Inert Higgs and Inert Higgsinos are heavier than 100 GeV to evade limits from LEP.

In addition to setting bounds from the non–observation of new particles in experiment, we impose some theoretical constraints. We require that the Lightest Supersymmetric Particle (LSP) should be a neutralino. We also restrict our consideration to the values of the Yukawa couplings $\lambda_i(M_X)$, $\kappa_i(M_X)$, $h_i(M_X)$, $h_b(M_X)$ and $h_\tau(M_X)$ less than 3 to ensure the applicability of perturbation theory up to the GUT scale.

In our exploration of the cE6SSM parameter space we looked at scenarios with a universal coupling between exotic coloured Superfields and the third generation singlet field $\hat{S}$, $\kappa_{1,2,3}(M_X) = \kappa(M_X)$ and fixed the Inert Higgs couplings $\lambda_{1,2}(M_X) = 0.1$. In fixing $\lambda_{1,2}$ like this we are deliberately pre-selecting for relatively light Inert Higgsinos. The third generation Yukawa $\lambda = \lambda_3$ was allowed to vary along with $\kappa$. Splitting $\lambda_3$ from $\lambda_{1,2}$ seems reasonable since $\lambda_3$ plays a very special role in E6SSM models in forming the effective $\mu$–term when $S$ develops a VEV.

The first results we found were for a very large singlet VEV, $s \approx 10 - 20$ TeV, and this leads to a very heavy particle spectrum where many of the new particles would be out of
reach of current collider experiments. This can be seen in Fig. 2 where the dependencies of the soft mass parameters $m_0$, $M_{1/2}$ and $A_0$ on $\lambda$ for $s = 20$ TeV and a particular value of $\kappa = 0.25$ are plotted. One can see that for each value of $\lambda$ there are two different values of each soft mass. This is because we find that for these points, of the four solutions to our quartic equation, two are complex, leaving only the two real solutions appearing in the plots. We find the existence of two real solutions and two complex solutions to be typical for the parameter space we have examined.

![Figure 2: cE6SSM solutions with $\tan \beta = 10$, $s = 20$ TeV and $\kappa_{1,2,3} = 0.25$, $\lambda_{1,2} = 0.1$ fixed showing the relationship between $\lambda$ and $m_0$ (top), $M_{1/2}$ (bottom left) and $A$ (bottom right). Points in green (light gray) satisfy all experimental constraints from LEP and Tevatron data, while points in black are ruled out.](image)

Notice also that the solutions presented above possess a certain symmetry. This is because there is an invariance under the transformation $A_0 \rightarrow -A_0$, $M_{1/2} \rightarrow -M_{1/2}$, $\lambda \rightarrow -\lambda$. However, we exploit this symmetry to adopt a convention whereby $M_{1/2} \geq 0$ is fixed, and therefore are only admitting physical solutions with $M_{1/2} \geq 0$, with the result that this symmetry is not apparent for our valid solutions shown in green (light gray).

After further study, we also discovered solutions that are allowed by all experimental
constraints and have a significantly lighter $s$ for a smaller range of $\lambda_3$ and our universal $\kappa$. This is illustrated in Fig. 3 where the soft mass dependencies on $\lambda$ for $s = 5$ TeV and $\kappa = 0.25$ (which is within this narrow range allowing $s$ to be relatively light). Since many particles in the cE$_6$SSM have their masses set by the singlet VEV it is of clear phenomenological interest to study the parameter space with low values of $s$.

![Figure 3: cE$_6$SSM solutions with $\tan \beta = 10$, $s = 5$ TeV and $\kappa_{1,2,3} = 0.25$, $\lambda_{1,2} = 0.1$ fixed showing the relationship between $\lambda$ and $m_0$ (top left and magnified top right) $M_{1/2}$ (bottom left) and $A$ (bottom right). Points in green (light gray) satisfy all experimental constraints from LEP and Tevatron data, while points in black are ruled out.](image)

To further explore this interesting region of the cE$_6$SSM parameter space, for different fixed values of $\tan \beta = 3, 10, 30$, we scan over $s$, $\kappa_i$ and $\lambda$. From these input parameters, the sets of soft mass parameters, $A_0$, $M_{1/2}$ and $m_0$ which are consistent with the correct breakdown of the EW symmetry are found.

We find that for fixed values of the Yukawas the soft mass parameters scale with $s$, while if $s$ and $\tan \beta$ are fixed, varying the Yukawas, $\lambda$ and $\kappa_i$ then produces a bounded region of allowed points.

The value of $s$ determines the location and extent of the bounded regions. As $s$
is increased the lowest values $m_0$ and $M_{1/2}$, consistent with experimental searches and EWSB requirements, increase. This is shown in Fig. 4 where the allowed regions for three different values of the singlet VEV, $s = 3$ TeV, 4 TeV and 5 TeV, are compared, with the allowed regions in red (dark grey), green (light grey), magenta (medium grey) respectively and the excluded regions in white. Note that these regions overlap since we are finding soft masses consistent with EWSB conditions that have a non-linear dependence on the VEVs and Yukaws.

![Figure 4: Physical solutions with $\tan \beta = 10$, $\lambda_{1,2} = 0.1$, $s = \{3, 4, 5\}$ TeV fixed and $\lambda \equiv \lambda_3$ and $\kappa \equiv \kappa_{1,2,3}$ varying, which pass experimental constraints from LEP and Tevatron data. On the left hand side of each allowed region the chargino mass is less than 100 GeV, while underneath the Inert Higgses are less than 100 GeV or becoming tachyonic. The region ruled out immediately to the right of the allowed points is due to $m_h < 114$ GeV.

Further scanning over $s$, leaving only $\tan \beta$ fixed, we find a lower limit on the ratio $m_0/M_{1/2}$ which is a weak function of the singlet VEV $s$. For example, consider Fig. 5 (top, left). The region to the left of the allowed space is ruled out by the lightest chargino mass, $m_{\tilde{\chi}_1^\pm} < 100$ GeV, while the lower right region is ruled out by Inert Higgs bosons with masses below experimental bounds or tachyonic. This boundary implies that for $\tan \beta = 10$, over the allowed ranges shown, $m_0/M_{1/2}$ varies from $\approx 1.4$ to $\approx 0.8$.

This boundary can be understood as follows. For fixed $m_0$, maximizing $M_{1/2}$ requires the singlet VEV $s$ to be increased, as well as varying the Yukawas, $\lambda$ and $\kappa$. However,
the squared masses of the Inert Higgs bosons receive a positive contribution from $m_0^2$ and a negative contribution from the auxiliary D–term which varies with $s^2$ (see Eqs. (43) and (44)). Due to this D–term contribution the mass of the lightest Inert Higgs boson decreases with $s$ and at some point falls below experimental limits, bounding $M_{1/2}$ from above. The larger $m_0$ is, the larger the negative contribution must be in order to drive the Inert Higgs mass below its lower limit. Further, if one assumes that $m_0 \sim s$ and $A_\lambda \sim M_{1/2}$ then EWSB conditions imply $s \sim M_{1/2} \tan \beta$. This suggests not only the observed limit on $m_0/M_{1/2}$ but also that it will be more severe for large $\tan \beta$ and shallower for low $\tan \beta$.

The allowed region for $\tan \beta = 30$ in Fig. 5 (top, right) has a similar shape but in this case $m_0/M_{1/2}$ varies from $\approx 1.9$ to $\approx 1.4$, so for this larger $\tan \beta = 30$ the limit on ratio $m_0/M_{1/2}$ is enhanced. For $\tan \beta = 3$ in Fig. 5 (bottom) the situation is somewhat different. The region to the left of the allowed parameter space is still ruled out by experimental limits on the chargino mass. However the lower-right region is ruled out, not by the Inert Higgs masses, but by a light Higgs which is lower than the LEP limit. This change has two
underlying reasons. Firstly the Inert Higgs bosons obtain positive contributions to their masses from $m_0$ (with a coefficient of $\approx 1$) and $M_{1/2}$, while, due to the auxiliary D–term contribution, the Inert Higgs masses decrease with $s$. Since decreasing $\tan \beta$ reduces the hierarchy between $s$ and $M_{1/2}$, this negative contribution to the mass of the Inert Higgs is smaller and does not decrease their mass as rapidly when $m_0$ is reduced. Secondly we observe that the lightest Higgs mass reduces with $\tan \beta$ as in the MSSM. At $\tan \beta = 3$ the maximal value of the mass of the lightest Higgs boson is rather close to the LEP bound. As a result the variations of parameters can result in the increase of the mixing in the CP–even Higgs sector, which provides a negative contribution to the lightest Higgs boson mass, so that it becomes lower than the LEP limit of 114 GeV.

5.4 Benchmark Scenarios

A remarkable feature of the cE$_6$SSM is that the low energy gluino mass parameter $M_3$ is driven to be smaller than $M_{1/2}$ by RG running. The reason for this is that the E$_6$SSM has a much larger (super)field content than the MSSM (three 27’s instead of three 16’s) so much so that at one–loop order the QCD beta function (accidentally) vanishes in the E$_6$SSM, and at two loops it loses asymptotic freedom (though the gauge couplings remain perturbative at high energy). This implies that the low energy gaugino masses are all less than $M_{1/2}$ in the cE$_6$SSM, being given as roughly $M_3 \sim 0.7 M_{1/2}$, $M_2 \sim 0.25 M_{1/2}$, $M_1 \sim 0.15 M_{1/2}$. These should be compared to the corresponding low energy values in the MSSM, $M_3 \sim 2.7 M_{1/2}$, $M_2 \sim 0.8 M_{1/2}$, $M_1 \sim 0.4 M_{1/2}$.

Thus, in the cE$_6$SSM, since the low energy gaugino masses $M_i$ are driven by RG running to be small, the lightest SUSY states will generally consist of a light gluino of mass $\sim M_3$, a light wino-like neutralino and chargino pair of mass $\sim M_2$, and a light bino-like neutralino of mass $\sim M_1$, which are typically all much lighter than the Higgsino masses of order $\mu = \lambda s/\sqrt{2}$, where $\lambda$ cannot be too small for correct EWSB. The remaining neutralinos are mainly a superposition of the $U(1)_N$ gaugino and singlet Higgsino. Their masses are governed by $M_{Z'}$. The mass of the $Z'$ is set by the singlet VEV, i.e. $M_{Z'} \approx g'_1 Q_s s$ ($g'_1 \approx g_1$) and therefore is also much heavier than gluino, lightest neutralino and chargino. The heaviest CP–even Higgs state is degenerate with the $Z'$ while another CP–even Higgs, CP–odd and charged Higgs bosons have almost the same masses which are relatively close to the masses of charged and neutral Higgsinos. Since $m_0$ tends to be larger than $M_{1/2}$ for each value of $s$ (as may be seen in Fig.4) the Superpartners of ordinary quarks and leptons are considerably heavier than the light gauginos as well. This is a general prediction of the cE$_6$SSM. Moreover as follows from benchmark 1 (Fig.4) all extra exotic particles in the cE$_6$SSM can be also relatively heavy so that the light sector of the sparticle spectrum includes only gluino, two light neutralinos and light chargino.
Figure 6: The particle mass spectra for cE$_6$SSM Benchmark Point 1, with tan$\beta$ = 10, $s = 4.0$ TeV, $M_{1/2} = 389$ GeV, $m_0 = 725$ GeV, $A = -1528$ GeV, $\lambda_{1,2}(M_X) = 2.6$, $\lambda_3(M_X) = -2.0$, $\lambda_3(\mu_S) = -0.259$, $\kappa_{1,2,3} = 2.5$, $\kappa_3(\mu_S) = 0.728$.

Nonetheless, even the pessimistic scenario described by benchmark 1 leads to the striking collider signature. Indeed, because gluino, two light neutralinos and light chargino have relatively small masses in the considered case the pair production of $\chi^0_2 \chi^0_2$, $\chi^\pm_1 \chi^\mp_1$ and $\tilde{g} \tilde{g}$ should be possible at the LHC.

With increasing VEV of the SM-singlet field the structure of the particle spectrum becomes more hierarchical. Due to the hierarchical spectrum the gluinos can be relatively narrow states because $\Gamma_{\tilde{g}} \propto M_{\tilde{g}}^5/m_{\tilde{q}}^4$. In particular their width can be comparable to that of $W^\pm$ and $Z$ bosons. They will decay through $\tilde{g} \rightarrow q\bar{q}^* \rightarrow q\bar{q} + E_T^{miss}$, so gluino pair production will result in an appreciable enhancement of the cross section for $pp \rightarrow q\bar{q}q\bar{q} + E_T^{miss} + X$, where $X$ refers, hereafter, to any number of light quark/gluon jets. The second lightest neutralino decays through $\chi^0_2 \rightarrow \chi^0_1 + l\bar{l}$ and so would produce an excess in $pp \rightarrow lll\bar{l} + E_T^{miss} + X$, which could be observed at the LHC.

Notice however that, while these are general predictions of the model, it is also possible that more exciting signatures could originate in the cE$_6$SSM. For example, when the Yukawa couplings $\kappa_i$ of the exotic fermions $D_i$ and $\overline{D}_i$ have a hierarchical structure, some of them can be relatively light so that their production cross section at the LHC can be comparable with the cross section of $t\bar{t}$ production [37]. In the E$_6$SSM the $D_i$ and $\overline{D}_i$ fermions are SUSY particles with negative $R$–parity so they must be pair produced and decay into quark–squark (if diquarks) or quark–slepton, squark–lepton (if leptoquarks).
Assuming that $D_i$ and $\overline{D}_i$ fermions couple most strongly to the third family (s)quarks and (s)leptons the presence of light exotic quarks in the particle spectrum can lead to a substantial enhancement of the cross section of either $pp \rightarrow t\bar{t}b\bar{b} + E_T^{\text{miss}} + X$ if exotic quarks are diquarks or $pp \rightarrow t\bar{t}\tau\bar{\tau} + E_T^{\text{miss}} + X$ and $pp \rightarrow b\bar{b} + E_T^{\text{miss}} + X$ if new quark states are leptoquarks. The scenarios with light exotic quarks, light stop and a TeV scale $Z'$, which have early discovery potential at the LHC, are considered in our companion paper [11].

In this work we concentrate on the various scenarios with universal $\kappa$ couplings which have distinctive phenomenology and could provide interesting novel signatures at the LHC. In Tab. 3 we specify a set of benchmark points, which demonstrate different patterns of the particle spectrum that can be obtained in the considered case. The first block of Tab. 3 shows the input parameters which define the benchmark points. These benchmarks cover three different values of $\tan \beta = 3, 10, 30$. We deliberately restricted ourselves here to $s = 3.4 - 5.5 \text{ TeV}$ and $(m_0, M_{1/2}) < (1100, 950) \text{ GeV}$ in order to get a relatively light particle spectrum that can be observed at the LHC. Since we focus on the solutions with $s = 3.4 - 5.5 \text{ TeV}$, the allowed range of the cE6SSM parameter space remains rather narrow and the lightest Higgs boson mass is always relatively close to the LEP limit of 114 GeV. Because we have taken the $\kappa_i$ to be universal at the GUT scale these couplings have to be large enough to trigger EWSB. Since the $\kappa_i$’s control the exotic coloured fermion masses, this implies that all the $D_i$ and $\overline{D}_i$ fermions are all very heavy in the considered cases. For benchmarks presented in Tab. 3 the exotic coloured fermions have masses in the range $1.2 - 2.2 \text{ TeV}$.

In all of the scans carried out in the previous section and for the most of benchmark scenarios we have chosen $\lambda_{1,2}(M_X) = 0.1$ so that $|\lambda_3(M_X)| \gg \lambda_{1,2}(M_X)$. Low values of $\lambda_{1,2}(M_X)$ result in relatively light Inert Higgsinos (see benchmarks 2–6) because their masses are proportional to the corresponding couplings. For benchmarks 2–6 the Inert Higgsinos are much lighter than squarks, sleptons and the exotic coloured fermions and have masses below $400 - 500 \text{ GeV}$. In contrast, the Inert Higgs bosons can be light or heavy depending on the free parameters.

Benchmark 2 (shown in Fig. 7) is a scenario with very light Inert Higgs bosons ($m_{H_{a,1}} = 182 \text{ GeV}$) and fairly light Inert Higgsinos ($\mu_{\tilde{H}} = 418 \text{ GeV}$). The presence of light Inert Higgs bosons in the particle spectrum is caused by the large mixing effects in the Inert Higgs sector. The negative contributions from the $U(1)_N$ D–term to the diagonal entries of the Inert Higgs mass matrices also reduce masses of the corresponding mass eigenstates. The light Inert Higgs bosons decay via the $Z^H$ violating terms $h^\nu_{iak} \hat{N}_i^c \hat{H}_a^L \hat{L}_k$, $h^U_{iak} \hat{u}_i \hat{H}_a^L \hat{Q}_k$, $h^D_{iak} \hat{d}_i \hat{H}_a^L \hat{Q}_k$ and $h^E_{iak} \hat{e}_i \hat{H}_a^L \hat{L}_k$, where the Inert Higgs Superfields are $SU(2)$ doublets with \( \hat{H}_a = (\hat{H}_a^u, \hat{H}_a^d) \) and \( \hat{H}_a^u = (\hat{H}_a^u, \hat{H}_a^d) \). These interactions are analogous
to the Yukawa interactions of the Higgs Superfields, $\hat{H}_u$ and $\hat{H}_d$. So the neutral Inert Higgs bosons decay predominantly into third generation fermion–anti-fermion pairs, like $H_{01} \rightarrow b\bar{b}$. The charged Inert Higgs bosons decays are also into fermion–anti-fermion pairs, but in this case it is the antiparticle of the fermions' EW partner e.g. $H_{-1} \rightarrow \tau\bar{\nu}_\tau$.

The Inert Higgs bosons may also be quite heavy, so that the only light exotic particles are the Inert Higgsinos. Benchmark 3 (Fig. 5) is an example of this, emphasising the need to search for both the Inert Higgsinos as well as the Inert Higgs bosons at future colliders.

The $Z_2^H$ symmetry violating couplings mentioned above also govern the decays of the Inert Higgsinos. The electromagnetically neutral Higgsinos predominantly decay into fermion anti-sfermion pairs (e.g. $\tilde{H}_0 \rightarrow t\tilde{t}^*$, $\tilde{H}_0 \rightarrow \tau\tilde{\tau}^*$). The charged Higgsino decays are similar, but in this case the sfermion is the Supersymmetric partner of the EW partner of the fermion, (e.g. $\tilde{H}^+_0 \rightarrow t\tilde{b}$, $\tilde{H}^-_0 \rightarrow \tau\tilde{\nu}$).

Unfortunately the production cross sections of the Inert Higgs bosons and Inert Higgsinos at the LHC will not be large because they do not participate in strong interactions. In this context it is more interesting to study scenarios with light coloured particles. Benchmark 4 represents such a scenario (spectra shown in Fig. 6). In this case the lightest exotic squarks have masses 312 GeV and can be efficiently produced at the LHC. Once again the presence of light exotic squarks in the particle spectrum is caused by the mixing effects in the exotic squark sector. The RGEs for the soft SUSY–breaking masses, $m_{\tilde{D}_i}^2$ and $m_{\tilde{D}_i}^2$, 

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Figure 7: Benchmark point 2, with $\tan \beta = 10$, $s = 4.4$ TeV, $M_{1/2} = 775$ GeV, $m_0 = 799$ GeV, $A = 919$ GeV, $\lambda(M_X) = -0.3698$, $\lambda(\mu_S) = -0.3736$, $\lambda_{1,2}(M_X) = 0.1$, $\kappa_{1,2,3}(M_X) = 0.1780$, $\kappa_{1,2,3}(\mu_S) = 0.4935$. 

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Figure 8: Benchmark point 3, with $\tan\beta = 10$, $s = 3.8\ TeV$, $M_{1/2} = 390\ GeV$, $m_0 = 998\ GeV$, $A = 768\ GeV$, $\lambda(M_X) = -0.3066$, $\lambda(\mu S) = -0.2845$, $\lambda_{1,2}(M_X) = 0.1$, $\kappa_{1,2,3}(M_X) = 0.2463$, $\kappa_{1,2,3}(\mu S) = 0.5935$.

are very similar with $\frac{d}{dt}(m_{D_i}^2 - m_{\tilde{D}_i}^2) = g_1^2 M_1^2$, resulting in comparatively small splitting between these soft masses. Therefore, although the diagonal entries of the exotic squark mass matrices acquire large contributions proportional to $s^2$ that come from the F–term quartic interactions in the scalar potential, mixing can be large even for moderate values of $A_0$, leading to a large mass splitting between the two scalar partners of the exotic coloured fermion. Recent, as yet unpublished, results from Tevatron searches for di-jet resonances \[60\] rule out scalar diquarks with a mass less than 630 GeV, however, scalar leptoquarks may be as light as 300 GeV since at hadron colliders they are pair produced through gluon fusion.

Scalar leptoquarks decay through $Z_2^H$ violating terms, $g_{ijk}^N \hat{N}_i^c \tilde{D}_j \hat{\tilde{D}}_k$, $g_{ijk}^E \hat{\tilde{D}}_j \hat{\tilde{D}}_k$ and $g_{ijk}^D(\hat{Q}_i \hat{\tilde{L}}_j)\hat{D}_k$. Thus in the cE$_6$SSM light scalar leptoquarks decay into quark–lepton final states. If the $Z_2^H$ symmetry is mostly broken by the operators involving quarks and leptons of the third generation each scalar leptoquark gives one top quark and one $\tau$–lepton in the final state. Since scalar leptoquarks can be pair produced through gluon fusion, light scalar leptoquarks should lead to an enhancement of $pp \to t\bar{t}\ell \bar{\ell} + X$ at the LHC \[46\]. Notice that SM production of $t\bar{t}\tau^+\tau^-$ is $(\alpha_W/\pi)^2$ suppressed in comparison

\[4\] Note that in this case positive contributions to the diagonal entries of the exotic squark mass matrices from the F–terms dominate over negative contributions that originate from $U(1)_N$ D–term quartic interactions in the scalar potential.
Figure 9: Benchmark point 4, with $\tan\beta = 30$, $s = 5.0\,\text{TeV}$, $M_{1/2} = 725\,\text{GeV}$, $m_0 = 1074\,\text{GeV}$, $A = 1726\,\text{GeV}$, $\lambda(M_X) = -0.3847$, $\lambda(\mu_S) = -0.3788$, $\lambda_{1,2}(M_X) = 0.1$, $\kappa_{1,2,3}(M_X) = 0.1579$, $\kappa_{1,2,3}(\mu_S) = 0.4559$.

Figure 10: Benchmark point 5, with $\tan\beta = 30$, $s = 3.4\,\text{TeV}$, $M_{1/2} = 361\,\text{GeV}$, $m_0 = 993\,\text{GeV}$, $A = 1121\,\text{GeV}$, $\lambda(M_X) = -0.33$, $\lambda(\mu_S) = -0.32$, $\lambda_{1,2}(M_X) = 0.1$, $\kappa_{1,2,3}(M_X) = 0.18$, $\kappa_{1,2,3}(\mu_S) = 0.51$.

to the light scalar leptoquark production cross section. Therefore light scalar leptoquark
should produce a strong signal with low SM background at the LHC.

Figure 11: Benchmark point 6, with $\tan \beta = 3$, $s = 5.5 \text{TeV}$, $M_{1/2} = 931 \text{GeV}$, $m_0 = 918 \text{GeV}$, $A = 751 \text{GeV}$, $\lambda(M_X) = -0.434$, $\lambda(\mu_S) = -0.375$, $\kappa_{1,2,3}(M_X) = 0.1$, $\kappa_{1,2,3}(\mu_S) = 0.23$, $\kappa_{1,2,3}(\mu_S) = 0.56$.

The decays of the lightest scalar diquarks are induced by the $Z_2^H$ symmetry violating symmetry operators $g^Q_{ijk} \tilde{D}_i (\tilde{Q}_j \tilde{Q}_k)$ and $g^d_{ijk} \tilde{D}_i (\tilde{d}_j \tilde{c}_k)$ in the Superpotential. This results in the decays of $\tilde{D}_{i1}$ into quark–quark final states. Assuming that exotic squarks couple most strongly to the third family quarks each $\tilde{D}_{i1}$ gives $t$ and $b$ quarks in the final state. It is worth to emphasise here that exotic squarks are particles with positive $R$–parity. Therefore they can decay without missing energy from the LSP. The production and decay of isosinglet charge -1/3 quark and its scalar partner were explored in [61].

Another very intriguing feature of the cE6SSM is the presence of a $U(1)_{N^Z}$ gauge boson. In our benchmark point 5 (Fig. 10) this $Z'$ is fairly light ($M_{Z'} = 1.285 \text{TeV}$) and within the reach of the LHC. Signatures for the $Z'$ have already been discussed in [37] and are to be explored further in a follow up study [62]. However, the $Z'$ mass can be significantly heavier, of order 2 TeV, as is shown in our final benchmark point 6 (Fig. 11). The spectrum for this point is rather heavy with even the lightest chargino being as heavy as $m_{\chi_1^\pm} = 262 \text{GeV}$ and the lightest neutralino having $m_{\chi_1^0} = 148 \text{GeV}$.

The full spectrum for each of the benchmark points is given in Tab. 3. The Higgs spectrum for all the benchmark points contains a very light SM–like CP–even Higgs boson $h_1$ with a mass close to the LEP limit of 114 GeV. Other Higgs states have masses in the range 600–2100 GeV making them difficult to discover. The benchmark points all
exhibit the characteristic SUSY spectrum described above containing a relatively light gluino, a light wino-like neutralino and chargino pair, and a light bino-like neutralino, with other sparticle masses being much heavier.
|                  | BM 1 | BM 2 | BM 3 | BM 4 | BM 5 | BM 6 |
|------------------|------|------|------|------|------|------|
| \tan\beta        | 10   | 10   | 10   | 30   | 30   | 3    |
| \lambda_3(M_X)   | -2.0 | -0.37| -0.31| -0.38| -0.33| -0.43|
| \lambda_{1,2}(M_X)| 2.6  | 0.1  | 0.1  | 0.1  | 0.1  | 0.1  |
| \kappa_{1,2,3}(M_X) | 2.5  | 0.18 | 0.25 | 0.16 | 0.18 | 0.23 |
| s[TeV]           | 4.4  | 3.8  | 5.0  | 3.4  | 5.5  |      |
| M_{1/2}[GeV]     | 389  | 775  | 390  | 725  | 361  | 931  |
| m_0 [GeV]        | 725  | 799  | 998  | 1074 | 933  | 918  |
| A[GeV]           | -1528| 919  | 768  | 1726 | 1121 | 751  |

|                  | BM 1 | BM 2 | BM 3 | BM 4 | BM 5 | BM 6 |
|------------------|------|------|------|------|------|------|
| m_{\tilde{D}_1}(1,2,3)[GeV] | 1948 | 821  | 1363 | 312  | 884  | 1567 |
| m_{\tilde{D}_2}(1,2,3)[GeV]   | 2200 | 2363 | 2077 | 2623 | 1860 | 2997 |
| \mu_D(3)[GeV]         | 2060 | 1535 | 1595 | 1612 | 1221 | 2187 |

|                  | BM 1 | BM 2 | BM 3 | BM 4 | BM 5 | BM 6 |
|------------------|------|------|------|------|------|------|
| | m_{\chi^0_2}| [GeV] | 1490 | 1603 | 1405 | 1832 | 1256 | 2006 |
| m_{h_3} \simeq M_{Z'}[GeV] | 1518 | 1664 | 1437 | 1890 | 1285 | 2079 |
| m_{\chi^0_1}| [GeV] | 1490 | 1603 | 1405 | 1832 | 1256 | 2006 |

|                  | BM 1 | BM 2 | BM 3 | BM 4 | BM 5 | BM 6 |
|------------------|------|------|------|------|------|------|
| m_S(1,2)[GeV]    | 1290 | 1446 | 1430 | 1732 | 1351 | 1763 |
| m_{H^+}(1,2)[GeV] | 1172 | 765  | 875  | 1117 | 966  | 714  |
| m_{H^0}(1,2)[GeV] | 903  | 182  | 694  | 220  | 689  | 121  |
| \mu_{\tilde{g}}(1,2)[GeV] | 1302 | 418  | 324  | 491  | 323  | 471  |

|                  | BM 1 | BM 2 | BM 3 | BM 4 | BM 5 | BM 6 |
|------------------|------|------|------|------|------|------|
| m_{u_1}(1,2)[GeV]| 1007 | 1398 | 1211 | 1557 | 1173 | 1666 |
| m_{d_1}(1,2)[GeV]| 1023 | 1446 | 1225 | 1595 | 1186 | 1724 |
| m_{u_2}(1,2)[GeV]| 1023 | 1446 | 1225 | 1595 | 1186 | 1724 |
| m_{d_2}(1,2)[GeV]| 1113 | 1488 | 1292 | 1664 | 1241 | 1785 |
| m_{e_2}(1,2)[GeV]| 1015 | 1176 | 1207 | 1427 | 1165 | 1409 |
| \mu_{\tilde{e}}(1,2)[GeV] | 873  | 992  | 1105 | 1254 | 1080 | 1173 |

|                  | BM 1 | BM 2 | BM 3 | BM 4 | BM 5 | BM 6 |
|------------------|------|------|------|------|------|------|
| m_{\tau_2}[GeV] | 1012 | 1172 | 1203 | 1363 | 1117 | 1409 |
| m_{\tau_1}[GeV] | 867  | 982  | 1095 | 1102 | 973  | 1172 |
| m_{b_1}[GeV]    | 1108 | 1473 | 1282 | 1491 | 1133 | 1784 |
| m_{b_2}[GeV]    | 907  | 1216 | 1036 | 1193 | 914  | 1472 |
| m_{\tilde{e}}[GeV] | 921  | 1259 | 1070 | 1248 | 964  | 1511 |
| m_{\tilde{\mu}}[GeV] | 777  | 853  | 787  | 837  | 694  | 1056 |

|                  | BM 1 | BM 2 | BM 3 | BM 4 | BM 5 | BM 6 |
|------------------|------|------|------|------|------|------|
| | m_{\chi^0_1}| [GeV] | 739  | 1168 | 771  | 1343 | 784  | 1463 |
| m_{h_1} \simeq M_A \simeq m_{H^\pm}[GeV] | 615  | 1145 | 963  | 998  | 748  | 1508 |
| m_{h_1}[GeV]    | 116  | 114  | 121  | 114  | 119  | 114  |

|                  | BM 1 | BM 2 | BM 3 | BM 4 | BM 5 | BM 6 |
|------------------|------|------|------|------|------|------|
| m_{\tilde{g}}[GeV] | 350  | 673  | 362  | 642  | 338  | 805  |
| | m_{\chi^0_1}| [GeV] | 106  | 217  | 110  | 206  | 102  | 262  |
| m_{\chi^0_1}| [GeV] | 59   | 122  | 62   | 116  | 58   | 148  |

Table 3: Particle spectra for our constrained $E_6$SSM benchmark points.
6. Conclusions

In this paper we have considered the constrained version of the Exceptional Supersymmetric Standard Model (E\textsubscript{6}SSM). The E\textsubscript{6}SSM is based on the $SU(3)\text\text{C} \times SU(2)\text{W} \times U(1)\text{Y} \times U(1)\text{N}$ gauge group, which can originate from the breakdown of the $E_6$ symmetry at high energies. In this $E_6$ inspired SUSY model the right-handed neutrino does not participate in gauge interactions, allowing it to be used for both the see–saw mechanism and leptogenesis. To ensure anomaly cancellation and gauge coupling unification, the particle content of the E\textsubscript{6}SSM includes three complete fundamental 27 representations of $E_6$ as well as the doublet $H'$ and anti-doublet $\bar{H}'$ from extra 27' and $\bar{27}'$ representations. Thus, in addition to a $Z'$ corresponding to the $U(1)_N$ symmetry, the E\textsubscript{6}SSM involves extra matter beyond the MSSM that form three families of new exotic charge 1/3 quarks and squarks, three generations of $SU(2)$ doublets of Inert Higgs bosons and Inert Higgsinos, as well as three SM-singlet bosons and their fermionic Superpartners, which carry $U(1)_N$ charges. The baryon number conservation requires exotic quarks and squarks to be either diquarks (E\textsubscript{6}SSM Model I) or leptoquarks (E\textsubscript{6}SSM Model II).

The extra $U(1)_N$ gauge symmetry forbids the term $\mu \hat{H}_d \hat{H}_u$ in the Superpotential. Nevertheless one of the SM-singlet bosons $S$ develops a VEV $\langle S \rangle = s/\sqrt{2}$, breaking the extra $U(1)_N$ symmetry and providing the effective $\mu$ term for the Higgs doublets, as well as masses for exotic quarks, Inert Higgsinos and $Z'$. In general, the $E_6$ inspired SUSY models involves lots of new Yukawa couplings in comparison to the SM and MSSM. Some of these new couplings give rise to unacceptably large non–diagonal flavour transitions, which have not been observed. To suppress flavour changing processes, we have imposed an approximate $Z_2^H$ symmetry under which only the Higgs Superfields $H_u, H_d$ and $S$ are even while all other Supermultiplets are odd. This discrete symmetry can only be an approximate one because it forbids all terms that allow the lightest exotic quarks to decay.

The number of new couplings is further reduced within the constrained E\textsubscript{6}SSM (cE\textsubscript{6}SSM). The cE\textsubscript{6}SSM demands that all soft scalar masses, gaugino masses and trilinear scalar couplings are universal at the GUT scale. We analysed the RG flow of the gauge and Yukawa couplings, as well as soft SUSY breaking terms, using two–loop RGEs for the gauge and Yukawa couplings together with two–loop RGEs for the gaugino masses and trilinear scalar couplings and one–loop RGEs for the soft scalar masses. Since the E\textsubscript{6}SSM has a much larger Superfield content than the MSSM, the RG flow of the gauge and Yukawa couplings and soft SUSY breaking terms is entirely different from the minimal SUSY model. For example, due to the presence of three families of exotic quarks and squarks, the QCD beta function vanishes at one loop and at two loops it loses asymptotic
freedom (though the gauge couplings remain perturbative at high energy). Thus, the $E_6$SSM gauge couplings are considerably larger at high energies than in the MSSM, and the RG flows of gaugino and soft scalar masses are entirely different. For the same values of $M_{1/2}$, the gaugino masses in the $cE_6$SSM are much smaller than in the $cMSSM$ at low energies. One remarkable feature is that the low energy gluino mass parameter $M_3$ is driven to be smaller than $M_{1/2}$ by RG running.

For each set of $\tan \beta$ and SUSY preserving couplings we established semi–analytic relations between the soft SUSY breaking terms at the SUSY breaking scale and their values at the GUT scale. Then we imposed EWSB constraints, which can be considered as a system of non–linear algebraic equations with respect to $A_0$, $m_0$ and $M_{1/2}$ and found the solutions of these equations for fixed $\tan \beta$, $s$ and Yukawa couplings. At the last stage of our analysis, we varied the Yukawa couplings, $\tan \beta$ and $s$ to establish the qualitative pattern of the particle spectrum. To avoid any conflict with present and former collider experiments, as well as recent cosmological observations, we imposed a set of experimental and theoretical constraints which restrict the allowed region of parameter space.

The results of our analysis indicate that $m_0$ tends to be considerably larger than $M_{1/2}$ in the allowed region. As a consequence, the Superpartners of ordinary quarks and leptons are significantly heavier than the gluino and lightest neutralino and chargino, which are predominantly gaugino. Some of the exotic squarks can also be relatively light due to large mixing effects induced by the corresponding Yukawa couplings and $A_0$. The substantial mixing and negative $U(1)_N$ D–term contributions can lead to the presence of light Inert Higgs bosons as well. The masses of exotic quarks and Inert Higgsinos which originate from complete 27 plets are controlled by the corresponding Yukawa couplings and can be relatively light if some of these couplings are small.

The mass terms of the right–handed neutrinos and survival components of $27'$ and $\overline{27}'$ are not forbidden by the gauge symmetry and therefore the scalar and fermion components of these Supermultiplets are expected to be rather heavy, so they decouple from the rest of the particle spectrum. The mass of the $Z'$ is set by the singlet VEV, i.e. $M_{Z'} \approx g'_1 Q_S s$ where $Q_S \approx 5/\sqrt{40}$ and $g'_1 \approx g_1$. As a result, the $Z'$ is considerably heavier than the gluino, lightest neutralino and chargino. The lightest neutralino, $\chi^0_1$, is essentially pure bino, while the second lightest neutralino $\chi^0_2$ and the lightest chargino $\chi^\pm_1$ are the degenerate components of the wino. The Higgsino states are degenerate and much heavier with the masses given by the effective $\mu$ term. The remaining neutralinos are mainly a superposition of the $U(1)_N$ gaugino and singlet Higgsino. Their masses are governed by $M_{Z'}$. The heaviest CP–even Higgs state is degenerate with the $Z'$ while another CP–even Higgs, CP–odd and charged Higgs bosons have almost the same masses and are considerably heavier than the lightest SUSY particles. For $s = 3 - 5$ TeV, the lightest
Higgs boson mass is rather close to the LEP limit of 114 GeV. In this work we specified a set of benchmark points that illustrate all the features of the particle spectrum discussed above.

Thus, throughout all cE6SSM regions of parameter space, there is a general prediction that the lightest sparticles always include the gluino $\tilde{g}$, two lightest neutralinos $\chi^0_1, \chi^0_2$, and the lightest chargino $\chi^\pm_1$, which are considerably lighter than all the sfermions of ordinary matter. The corresponding hierarchical structure of the particle spectrum is caused by the RG flow. As a consequence, at the LHC one should observe pair production of $\chi^0_2 \chi^0_2$, $\chi^0_2 \chi^\pm_1$, and $\tilde{g}\tilde{g}$. Due to the hierarchical spectrum, the gluinos can be relatively narrow states so their width can be comparable to that of $W^\pm$ and $Z$ bosons. Gluino pair production would result in an appreciable enhancement of the cross section for $pp \to q\bar{q}q\bar{q} + E_T^{\text{miss}} + X$.

Since the second lightest neutralino decays through $\chi^0_2 \to \chi^0_1 + l\bar{l}$, its pair production would produce an excess in $pp \to l\bar{l}l\bar{l} + E_T^{\text{miss}} + X$, which can be also observed at the LHC.

Other possible manifestations of the E6SSM at the LHC are related to the presence of a $Z'$ and exotic multiplets of matter. A TeV scale $Z'$ will provide an unmistakable signal that can be observed soon after the LHC starts. If exotic quarks are relatively light, their production cross sections can be comparable with the cross section of $t\bar{t}$ production. The lifetime and decays of light exotic quarks are determined by the $Z'^H$ violating Yukawa couplings. If $D_i$ and $\overline{D}_i$ couple most strongly to the third family of (s)quarks and (s)leptons, then light exotic quarks lead to a substantial enhancement of the cross section of either $pp \to t\bar{t}b\bar{b} + E_T^{\text{miss}} + X$ (if they are diquarks) or $pp \to t\bar{t}\tau\bar{\tau} + E_T^{\text{miss}} + X$ and $pp \to b\bar{b} + E_T^{\text{miss}} + X$ (if they are leptoquarks). When scalar exotic quarks are light, they can decay into quark–quark (if diquarks) or quark–lepton (if leptoquarks) without missing energy from the LSP. As a result, their pair production leads to the enhancement of the cross section of either $pp \to t\bar{t}b\bar{b} + X$ or $pp \to t\bar{t}\tau\bar{\tau} + X$. Since the SM production cross sections of $pp \to t\bar{t}b\bar{b} + X$ or $pp \to t\bar{t}\tau\bar{\tau} + X$ are suppressed by many orders of magnitude compared to the cross section for $t\bar{t}$ production, the light exotic quarks and squarks should produce a strong signal with low SM background at the LHC.

The production cross sections of the Inert Higgs bosons and Inert Higgsinos will be much smaller at the LHC than the exotic (s)quark one. Nevertheless, their detection might also be possible if the corresponding states are light. Assuming that Inert Higgs bosons and Inert Higgsinos couple most strongly to the third family (s)quarks and (s)leptons, the lightest Inert Higgs bosons decay predominantly into third generation fermion–anti-fermion pairs like $H^0_1 \to b\bar{b}$ and $H^-_1 \to \tau\bar{\nu}_\tau$, while Inert Higgsinos predominantly decay into third generation fermion-anti-sfermion pairs. At an ILC the production rates of the light exotic (s)quarks and Inert Higgs bosons (Higgsinos) can be comparable, allowing their simultaneous observation.
We have not considered the question of cosmological cold dark matter (CDM) relic abundance due to the neutralino LSP and so one may be concerned that a bino-like lightest neutralino mass of around 100 GeV might give too large a contribution to $\Omega_{CDM}$. Indeed a recent calculation of $\Omega_{CDM}$ in the USSM [63], which includes the effect of the MSSM states plus the extra $Z'$ and the active singlet $S$, together with their superpartners, indicates that for the benchmarks considered here that $\Omega_{CDM}$ would be too large. However the USSM does not include the effect of the extra inert Higgs and Higgsinos that are present in the E$_6$SSM. While we have considered the inert Higgsino masses given by $\mu_{R_\alpha} = \lambda_\alpha s/\sqrt{2}$, we have not considered the mass of the inert singlinos which are generated by mixing with the Higgs and inert Higgsinos, and are thus of order $f v^2/s$ where their masses are controlled by additional Yukawa couplings $f$ which we have not specified in our analysis. Since $s \gg v$ it is quite likely that the LSP neutralino in the cE$_6$SSM will be an inert singlino with a mass lighter than 100 GeV. This would imply that the state $\chi^0_1$ considered here is not cosmologically stable but would decay into lighter singlinos. The question of the calculation of the relic abundance of such an LSP singlino within the framework of the cE$_6$SSM is beyond the scope of this article and will be considered elsewhere. In summary, it is clear that one should not regard the benchmark points with $|m_{\chi^0_1}| \approx 100$ GeV as being excluded by $\Omega_{CDM}$.

The discovery of $Z'$ and new exotic particles predicted by the E$_6$SSM at future colliders will open a new era in elementary particle physics. It will represent a possible indirect signature of an underlying $E_6$ gauge structure at high energies and may provide a window into string theory.

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A One–loop corrections to the Higgs masses

Higgs masses are obtained by taking double derivatives of the effective potential with respect to the Higgs fields.

The tree–level Higgs masses for the CP–even Higgs sector were presented in section 4.5 Eq. (51). The expression for the one–loop contribution, $\Delta V^{(1)}$, to the effective potential also appears in Eq. (10) and the physical masses of the stops, appearing in this equation, are calculated in the tree–level approximation,

$$m^2_{\tilde{t}_1, \tilde{t}_2} = \frac{1}{2} \left\{ m^2_{Q_3} + m^2_{u_3} + \frac{1}{2} M^2_Z \cos 2\beta + \Delta Q + \Delta w + 2m^2_t \mp \sqrt{M^4_{QQ} + 4m^2_t X^2_t} \right\}, \quad (A.1)$$

where

$$\Delta Q = \frac{g_1'}{80} (-3v^2_1 - 2v^2_2 + 5s^2), \quad \Delta w = \frac{g_1'}{80} (-3v^2_1 - 2v^2_2 + 5s^2), \quad (A.2)$$

$$M^2_{QQ} = m^2_{Q_3} - m^2_{u_3} + \left[ \frac{1}{2} - \frac{4}{3} \sin^2\theta_W \right] M^2_Z \cos 2\beta + \Delta Q - \Delta U, \quad (A.3)$$

$$X_t = A_t - \frac{\lambda s}{\sqrt{2} \tan \beta}, \quad (A.4)$$

and for further convenience defining,

$$r_t \equiv M^4_{QQ} + 4m^2_t X^2_t \quad \text{and} \quad R_{QQ} \equiv M^2_{QQ}(g_2^2 - g_1^2), \quad (A.5)$$

$$\mu_{eff} \equiv \frac{\lambda s}{\sqrt{2}} \quad \text{and} \quad \bar{g} \equiv \sqrt{g_2^2 + \frac{3g_1^2}{5}} \quad (A.6)$$

Including only stop/top contributions we find,

$$\frac{\partial \Delta V}{\partial x} = \frac{3}{32\pi^2} \left[ 2a_0(m_{t_1}) \frac{\partial}{\partial x} m^2_{t_1} + 2a_0(m_{t_2}) \frac{\partial}{\partial x} m^2_{t_2} - 4a_0(m_t) \frac{\partial}{\partial x} m^2_t \right], \quad (A.7)$$

where

$$a_0(m) \equiv m^2 \left[ \ln \frac{m^2}{Q^2} - 1 \right]. \quad (A.8)$$

Now, defining

$$\Delta_x m_i \equiv a_0(m_i) \frac{\partial}{\partial x} m^2_t, \quad (A.9)$$

it follows that

$$\frac{\partial^2 \Delta V}{\partial y \partial x} = \frac{3}{32\pi^2} \left[ 2 \frac{\partial}{\partial y} \Delta_x m_{t_1} + 2 \frac{\partial}{\partial y} \Delta_x m_{t_2} - 4 \frac{\partial}{\partial y} \Delta_x m_t \right], \quad (A.10)$$

$$\frac{\partial}{\partial x} \Delta_x m = (\frac{\partial}{\partial x} m^2) \ln \frac{m^2}{Q^2} + a_0(m) \frac{\partial^2}{\partial x^2} m^2, \quad (A.11)$$

$$\frac{\partial}{\partial y} \Delta_x m = (\frac{\partial}{\partial y} m^2) \ln \frac{m^2}{Q^2} + a_0(m) \frac{\partial^2}{\partial y \partial x} m^2. \quad (A.12)$$
Here we present the corrections in the basis \((v_1, v_2, v_3 \equiv s)\), with \(\Delta'_{ij} = \frac{\partial^2}{\partial v_i \partial v_j} \Delta V\) such that \(\Delta'_{11} = \frac{\partial^2}{\partial v^2_1} \Delta V\) etc. The corrections, \(\Delta_{ij}\) appearing in Eq. (51) can be obtained from these using the relations,

\[
\begin{align*}
\Delta_{11} &= \cos^2 \beta \Delta'_{11} - 2 \sin \beta \cos \beta \Delta'_{12} + \sin^2 \beta \Delta'_{22} \quad (A.13) \\
\Delta_{22} &= \sin^2 \beta \Delta'_{11} - 2 \sin \beta \cos \beta \Delta'_{12} + \cos^2 \beta \Delta'_{22} \quad (A.14) \\
\Delta_{33} &= \Delta'_{33} \quad (A.15) \\
\Delta_{12} &= (\cos^2 \beta - \sin^2 \beta) \Delta'_{12} + \sin \beta \cos \beta (\Delta'_{22} - \Delta'_{11}) \quad (A.16) \\
\Delta_{31} &= \cos \beta \Delta'_{13} + \sin \beta \Delta'_{23} \quad (A.17) \\
\Delta_{32} &= \cos \beta \Delta'_{23} - \sin \beta \Delta'_{13} \quad (A.18)
\end{align*}
\]

\[
\begin{align*}
\Delta'_{11} &= \frac{3}{16\pi^2} \left\{ \left[ \left( \frac{g^2}{8} - \frac{3g_1^2}{40} \right)^2 v_1^2 + \frac{1}{r_t} \left( \frac{v_1}{8} R_{QQ} - 2m_t^2 X_t \frac{s\lambda}{\sqrt{2}v_2} \right)^2 \right] \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} \\
&\quad + \frac{v_1}{32} \left( \frac{g^2}{8} - \frac{3g_1^2}{40} \right)^{1/2} R_{QQ} \left( \frac{v_1}{8} R_{QQ} - 16m_t^2 X_t \frac{s\lambda}{\sqrt{2}v_2} \right) \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} \\
&\quad + \left( \frac{g^2}{8} - \frac{3g_1^2}{40} \right) (a_0(m_i_1) + a_0(m_i_2)) + \frac{1}{32} \left[ r_t^{1/2} (4R_{QQ} + (g_2^2 - g_1^2)v_1^2 \\
&\quad + 16y_t^2 s^2 \lambda^2 - (v_1 R_{QQ} - 16m_t^2 X_t \frac{s\lambda}{\sqrt{2}v_2} \right) \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} \right] (a_0(m_i_2) - a_0(m_i_1)) \right\} \quad (A.19) \\
\Delta'_{22} &= \frac{3}{16\pi^2} \left\{ \left[ \left( \frac{g_2^2}{8} - \frac{g_1^2}{20} \right)^2 + \frac{8X_t A_t y_t^2 - R_{QQ}}{64r_t} \right] \frac{1}{v_2^2} \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} \\
&\quad + \frac{v_2}{4\sqrt{r_t}} \left( \frac{g_2^2}{8} - \frac{g_1^2}{20} \right) (8y_t^2 X_t A_t - R_{QQ}) \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} \\
&\quad + \left( \frac{g_2^2}{8} - \frac{g_1^2}{20} \right) (a_0(m_i_1) + a_0(m_i_2)) + \frac{1}{\sqrt{r_t}} \left[ \frac{(g_2^2 - g_1^2)v_2^2}{32} - R_{QQ} \right] \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} \\
&\quad - \frac{8X_t A_t y_t^2 - R_{QQ}}{32r_t} \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} \right] (a_0(m_i_2) - a_0(m_i_1)) - 2y_t^2 v_2^2 \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} - 2y_t^2 a_0(m_t) \right\} \quad (A.20) \\
\Delta'_{33} &= \frac{3}{16\pi^2} \left\{ \left[ \frac{g_2^4 s^2}{64} + \frac{2m_t^4 X_t^2 \lambda^2}{r_t \tan^2 \beta} \right] \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} - \frac{g_1^2 m_t^2 X_t \mu_{eff}}{2\sqrt{r_t} \tan \beta} \ln \frac{m_i^2}{m_{i_2}^2} \frac{Q^4}{Q^4} \\
&\quad + \frac{g_1^2}{8} (a_0(m_i_1) + a_0(m_i_2)) + \frac{\lambda^2 m_t^2}{\sqrt{r_t} \tan^2 \beta} \left[ 1 - \frac{4X_t^2 m_t^2}{r_t} \right] (a_0(m_{i_2}) - a_0(m_{i_1})) \right\} \quad (A.21)
\end{align*}
\]
\begin{align*}
\Delta'_{12} &= \frac{3 y_t^2}{16\pi^2} \left\{ \left[ \left( \frac{g^2}{8} - \frac{3 g_1^2}{40} \right) + \left( \frac{R_{QQ}}{8} - y_t^2 X_t \mu_{\text{eff}} \tan \beta \right) \left( \frac{g^2}{8} \right) + \left( \frac{R_{QQ}}{8} - y_t^2 X_t A_t \right) \right] v_1 v_2 \ln \frac{m_{t_1}^2 m_{t_2}^2}{Q^4} \right. \\
&\quad \times \left[ \left( y_t^2 X_t A_t - \frac{R_{QQ}}{8} \right) v_1 v_2 \ln \frac{m_{t_1}^2 m_{t_2}^2}{Q^4} + \left[ \left( \frac{g^2}{8} - \frac{3 g_1^2}{40} \right) \left( \frac{R_{QQ}}{8} - y_t^2 X_t A_t - \frac{R_{QQ}}{8} \right) \right] v_1 v_2 \ln \frac{m_{t_2}^2}{m_{t_1}^2} \\
&\quad - \left[ \left( \frac{g_2}{8} - \frac{g_1^2}{32} \right) v_1 v_2 + A_t y_t^2 \mu_{\text{eff}} \right] + \left( \frac{R_{QQ}}{8} - y_t^2 X_t \mu_{\text{eff}} \tan \beta \right) (2 y_t^2 X_t A_t \right. \\
&\quad - \left. \frac{R_{QQ}}{4} \right) v_1 v_2 \left( a_0(m_{t_2}) - a_0(m_{t_1}) \right) \left( \sqrt{\frac{1}{r_t}} \right) \right\}
(A.22)
\end{align*}

\begin{align*}
\Delta'_{13} &= \frac{3 y_t^2}{16\pi^2} \left\{ \left[ \left( \frac{g^2}{8} - \frac{3 g_1^2}{40} \right) \frac{g_1^2}{8} s v_1 - \left( \frac{R_{QQ}}{8} - y_t^2 X_t \mu_{\text{eff}} v_2 \right) \frac{2 m_t^2 X_t \lambda}{r_t \sqrt{2 \tan \beta}} \right] \ln \frac{m_{t_1}^2 m_{t_2}^2}{Q^4} \right. \\
&\quad - \left. \left[ \left( \frac{g^2}{8} - \frac{3 g_1^2}{40} \right) \frac{2 v_1 m_t^2 X_t \lambda}{\sqrt{2} \tan \beta} \frac{g_1^2}{8} \ln \frac{m_{t_2}^2}{m_{t_1}^2} \right] v_1 v_2 R_{QQ} X_t \right. \\
&\quad + \left. \frac{y_t^2 v_1 \lambda \mu_{\text{eff}}}{\sqrt{2} r_t} \left[ 1 - \frac{X_t \tan \beta}{\mu_{\text{eff}}} - \frac{4 X_t^2 m_t^2}{r_t} + v_1 v_2 R_{QQ} X_t \right] \left( a_0(m_{t_2}) - a_0(m_{t_1}) \right) \right\}
(A.23)
\end{align*}

\begin{align*}
\Delta'_{23} &= \frac{3 y_t^2}{16\pi^2} \left\{ \left[ \frac{g_1^2}{8} \left( y_t^2 - \frac{g_2}{8} - \frac{g_1^2}{20} \right) s - \frac{2 m_t^2 X_t \lambda}{r_t \sqrt{2} \tan \beta} \left( y_t^2 X_t A_t - \frac{R_{QQ}}{8} \right) \right] v_2 \ln \frac{m_{t_1}^2 m_{t_2}^2}{Q^4} \right. \\
&\quad + \frac{r_t}{\sqrt{2} r_t} \left[ \frac{g_1^2}{8} \left( y_t^2 X_t A_t - \frac{R_{QQ}}{8} \right) - \frac{2 m_t^2 X_t \lambda}{\sqrt{2} \tan \beta} \left( y_t^2 - \frac{g_2}{8} - \frac{g_1^2}{20} \right) \right] v_2 \ln \frac{m_{t_2}^2}{m_{t_1}^2} \\
&\quad - \frac{y_t^2 v_1 A_t}{\sqrt{2} r_t} \left[ 1 - \frac{4 m_t^2 X_t^2}{r_t} + \frac{v_2^2 X_t R_{QQ}}{4 A_t r_t} \right] \left( a_0(m_{t_2}) - a_0(m_{t_1}) \right) \right\}
(A.24)
\end{align*}

These complicated expressions can be simplified by keeping only the dominant contributions. Neglecting those auxiliary D-term contributions to the stop masses which are proportional to \( v_1^2 \) and \( v_2^2 \) we obtain the following simpler expressions,

\begin{align*}
\Delta'_{11} &\approx \frac{3 y_t^2}{16\pi^2} \left\{ \frac{m_t^2 X_t^2 s^2 \lambda^2}{r_t} \ln \frac{m_{t_1}^2 m_{t_2}^2}{Q^4} + \frac{\mu_{\text{eff}}}{\sqrt{r_t}} \left[ 1 - \frac{4 m_t^2 X_t^2}{r_t} \right] \left( a_0(m_{t_2}) - a_0(m_{t_1}) \right) \right\}
(A.25)
\end{align*}

\begin{align*}
\Delta'_{22} &\approx \frac{3 y_t^2}{16\pi^2} \left\{ 2 m_t^2 \left( 1 + \frac{X_t^2 A_t^2}{r_t} \right) \ln \frac{m_{t_1}^2 m_{t_2}^2}{Q^4} + \frac{A_t^2}{\sqrt{r_t}} \left[ 1 - \frac{4 m_t^2 X_t^2}{r_t} \right] \left( a_0(m_{t_1}) - a_0(m_{t_2}) \right) \right. \\
&\quad + \left( a_0(m_{t_1}) + a_0(m_{t_2}) - 2 a_0(m_t) \right) + \frac{4 m_t^2 X_t A_t}{r_t} \ln \frac{m_{t_2}^2}{m_{t_1}^2} - 4 m_t^2 \ln \frac{m_{t_2}^2}{Q^2} \right\}
(A.26)
\end{align*}
\[ \Delta'_{33} \approx \frac{3}{16\pi^2} \left\{ \frac{g_1^2 s^2}{64} + \frac{2 m_i^4 X_i^2 \lambda^2}{r_t \tan^2 \beta} \ln \frac{m_{i_2}^2 m_{i_1}^2}{Q^4} - \frac{g_1^2 m_i^2 X_i \mu_{\text{eff}}}{2 \sqrt{r_t} \tan \beta} \ln \frac{m_{i_2}^2}{m_{i_1}^2} \right. \\
+ \left. \frac{g_1^2}{8} \left( a_0(m_{i_1}) + a_0(m_{i_2}) \right) + \frac{\lambda^2 m_i^2}{\sqrt{r_t} \tan^2 \beta} \left[ 1 - \frac{4X_i^2 m_i^2}{r_t} \right] (a_0(m_{i_2}) - a_0(m_{i_1})) \right\} \quad (A.27) \]

\[ \Delta'_{12} \approx \frac{3y_1^2}{16\pi^2} \left\{ -\frac{2 m_i^2 \mu_{\text{eff}} A_i X_i^2}{r_t} \ln \frac{m_{i_1}^2 m_{i_2}^2}{Q^4} - \frac{2 m_i^2 X_i \mu_{\text{eff}}}{\sqrt{r_t}} \ln \frac{m_{i_2}^2}{m_{i_1}^2} \right. \\
- \left. \frac{\mu_{\text{eff}} A_i}{\sqrt{r_t}} \left[ 1 - \frac{4m_i^2 X_i^2}{r_t} \right] (a_0(m_{i_2}) - a_0(m_{i_1})) \right\} \quad (A.28) \]

\[ \Delta'_{13} \approx \frac{3}{16\pi^2} \left\{ \frac{2 m_i^4 X_i^2 \lambda^2 s}{r_t \tan \beta v_2} \ln \frac{m_{i_1}^2 m_{i_2}^2}{Q^4} - \frac{g_1^2 s^2 m_i^2 X_i \lambda}{4 v_2 \sqrt{2 r_t}} \ln \frac{m_{i_2}^2}{m_{i_1}^2} \right. \\
+ \left. \frac{y_1^2 v_1 \lambda \mu_{\text{eff}}}{\sqrt{2 r_t}} \left[ 1 - \frac{X_i \tan \beta}{\mu_{\text{eff}}} - \frac{4X_i^2 m_i^2}{r_t} \right] (a_0(m_{i_2}) - a_0(m_{i_1})) \right\} \quad (A.29) \]

\[ \Delta'_{23} \approx \frac{3y_1^2}{16\pi^2} \left\{ \left( \frac{g_1^2 s v_2}{8} - \frac{2 m_i^2 X_i^2 \lambda v_2}{r_i \sqrt{2} \tan \beta} A_t \right) \ln \frac{m_{i_1}^2 m_{i_2}^2}{Q^4} + \frac{(g_1^2 A_i v_2 s - 8 \sqrt{2} m_i^2 \lambda v_1) X_i}{8 \sqrt{r_t}} \ln \frac{m_{i_2}^2}{m_{i_1}^2} \right. \\
- \left. \frac{\lambda v_1 A_i}{\sqrt{2 r_t}} \left[ 1 - \frac{4m_i^2 X_i^2}{r_t} \right] (a_0(m_{i_2}) - a_0(m_{i_1})) \right\} \quad (A.30) \]
The running of the gauge couplings from the GUT scale to the EW scale is determined by a set of RGEs. In our analysis, we use two-loop RGEs for the gauge and Yukawa couplings together with two-loop RGEs for the gaugino masses $M_a(\mu)$ and trilinear scalar couplings $A_i(\mu)$, as well as one-loop RGEs for the soft scalar masses $m_i^2(\mu)$. A simplified set of one-loop RG equations may be found in [23]. The two-loop RGEs can be derived using general results presented in [64].

In the E6SSM the RGEs for the gauge couplings can written,

$$\frac{dG}{dt} = G \times B, \quad \frac{dg_2}{dt} = \frac{\beta_2 g_2^3}{(4\pi)^2}, \quad \frac{dg_3}{dt} = \frac{\beta_3 g_3^3}{(4\pi)^2}, \quad (B.1)$$

where $t = \ln[Q/M_X]$, while $B$ and $G$ are $2 \times 2$ matrices describing the RG flow of the Abelian gauge couplings, which is affected by the kinetic term mixing,

$$G = \begin{pmatrix} g_1 & g_{11} \\ 0 & g'_1 \end{pmatrix}, \quad B = \begin{pmatrix} B_1 & B_{11} \\ 0 & B'_1 \end{pmatrix} = \frac{1}{(4\pi)^2} \left( \begin{array}{cc} \beta_1 g_1^2 & 2g_1 g_1' \beta_{11} + 2g_1 g_{11} \beta_1 \\ 0 & g_1'^2 \beta_1 + 2g_1' g_{11} \beta_1 + g_{11}' \beta_1 \end{array} \right). \quad (B.2)$$

In the one–loop approximation $\beta_{11} = \frac{\sqrt{6}}{3}$. The two–loop diagonal $\beta$–functions of the gauge couplings are given by

$$\begin{align*}
\beta_3 &= -9 + 3N_g + \frac{1}{16\pi^2} \left[ g_3^2(-54 + 34N_g) + 3N_g g_2^2 + N_g g_1^2 \\
&\quad + N_g g_{11}^2 - 4 h_t^2 - 4 h_b^2 - 2 \Sigma_{\kappa} \right], \\
\beta_2 &= -5 + 3N_g + \frac{1}{16\pi^2} \left[ 8N_g g_3^2 + (-17 + 21N_g) g_2^2 + \left( \frac{3}{5} + N_g \right) g_1^2 \\
&\quad + \left( \frac{2}{5} + N_g \right) g_{11}^2 - 6 h_t^2 - 6 h_b^2 - 2 h_\tau^2 - 2 \Sigma_{\lambda} \right], \\
\beta_1 &= \frac{3}{5} + 3N_g + \frac{1}{16\pi^2} \left[ 8N_g g_3^2 + \left( \frac{9}{5} + 3N_g \right) g_2^2 + \left( \frac{9}{25} + 3N_g \right) g_1^2 \\
&\quad + \left( \frac{6}{25} + N_g \right) g_{11}^2 - \frac{26}{5} h_t^2 - \frac{14}{5} h_b^2 - \frac{18}{5} h_\tau^2 - \frac{6}{5} \Sigma_{\lambda} - \frac{4}{5} \Sigma_{\kappa} \right], \\
\beta_1' &= \frac{2}{5} + 3N_g + \frac{1}{16\pi^2} \left[ 8N_g g_3^2 + \left( \frac{6}{5} + 3N_g \right) g_2^2 + \left( \frac{6}{25} + N_g \right) g_1^2 \\
&\quad + \left( \frac{4}{25} + 3N_g \right) g_{11}^2 - \frac{9}{5} h_t^2 - \frac{21}{5} h_b^2 - \frac{7}{5} h_\tau^2 - \frac{19}{5} \Sigma_{\lambda} - \frac{57}{10} \Sigma_{\kappa} \right], \\
\Sigma_{\lambda} &= \lambda_1^2 + \lambda_2^2 + \lambda_3^2, \quad \Sigma_{\kappa} = \kappa_1^2 + \kappa_2^2 + \kappa_3^2.
\end{align*}$$

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The Yukawa couplings appearing in the Superpotential of the cE6SSM obey the following system of two–loop RGEs:

\[
\frac{d\lambda_i}{dt} = \left(\frac{\lambda_i}{(4\pi)^2}\right) \left[ 2\lambda_i^2 + 2\Sigma_{\lambda} + 3\Sigma_{\kappa} + \left(3h_i^2 + 3h_b^2 + h_r^2\right) \delta_{i3} - 3g_2^2 - \frac{3}{5}g_1^2 - \frac{19}{10}g_1' ^2 + \frac{\beta_{\lambda_i}^{(2)}}{(4\pi)^2} \right],
\]

\[
\frac{dk_i}{dt} = \left(\frac{k_i}{(4\pi)^2}\right) \left[ 2k_i^2 + 2\Sigma_{\lambda} + 3\Sigma_{\kappa} - \frac{16}{3}g_3^2 - \frac{4}{15}g_1^2 - \frac{19}{10}g_1' ^2 + \frac{\beta_{k_i}^{(2)}}{(4\pi)^2} \right],
\]

\[
\frac{dh_t}{dt} = \left(\frac{h_t}{(4\pi)^2}\right) \left[ \lambda^2 + 6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2 - \frac{3}{10}g_1' ^2 + \frac{\beta_{h_t}^{(2)}}{(4\pi)^2} \right],
\]

\[
\frac{dh_b}{dt} = \left(\frac{h_b}{(4\pi)^2}\right) \left[ \lambda^2 + h_t^2 + 6h_b^2 + h_r^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2 - \frac{7}{10}g_1' ^2 + \frac{\beta_{h_b}^{(2)}}{(4\pi)^2} \right],
\]

\[
\frac{dh_r}{dt} = \left(\frac{h_r}{(4\pi)^2}\right) \left[ \lambda^2 + 3h_b^2 + 4h_r^2 - 3g_2^2 - \frac{9}{5}g_1^2 - \frac{7}{10}g_1' ^2 + \frac{\beta_{h_r}^{(2)}}{(4\pi)^2} \right],
\]

where the two–loop contributions to the corresponding \(\beta\)–functions are given by

\[
\beta_{\lambda_i}^{(2)} = -2\lambda_i^2 \left(\lambda_i^2 + 2\Sigma_{\lambda} + 3\Sigma_{\kappa}\right) - 4\Pi_{\lambda} - 6\Pi_{\kappa} - \lambda^2 \left(3h_i^2 + 3h_b^2 + h_r^2\right) (2 + \delta_{i3}) - \left[9h_i^4 + 9h_b^4 + 6h_t^2h_b^2 + 3h_r^4\right] \delta_{i3} + 16g_3^2\Sigma_{\kappa} + 6g_2^2\Sigma_{\lambda} + g_1^2 \left(\frac{4}{5}\Sigma_{\kappa} + \frac{6}{5}\Sigma_{\lambda}\right) + g_1' ^2 \left(\frac{5}{2}\lambda_i^2 - \frac{9}{5}\Sigma_{\kappa} - \frac{6}{5}\Sigma_{\lambda}\right) + 16g_3^2 \left(h_t^2 + h_b^2\right) + g_1^2 \left(\frac{4}{5}h_t^2 - \frac{2}{5}h_b^2 + \frac{6}{5}h_r^2\right) + g_1' ^2 \left(-\frac{3}{10}h_t^2 - \frac{1}{5}h_b^2 - \frac{1}{5}h_r^2\right) \delta_{i3} + 3g_2^2 \left(3N_g - \frac{7}{2}\right) + \frac{3}{5}g_1^4 \left(3N_g + \frac{9}{10}\right) + \frac{19}{10}g_1' ^4 \left(3N_g + \frac{27}{20}\right) + \frac{9}{5}g_2^2g_1^2 + \frac{39}{20}g_2^2g_1' ^2 + \frac{39}{100}g_1^2g_1' ^2,
\]

\[
\beta_{k_i}^{(2)} = -2k_i^2 \left(k_i^2 + 2\Sigma_{\lambda} + 3\Sigma_{\kappa}\right) - 4\Pi_{\lambda} - 6\Pi_{\kappa} - 2\lambda^2 \left(3h_i^2 + 3h_b^2 + h_r^2\right) + 16g_3^2\Sigma_{\kappa} + 6g_2^2\Sigma_{\lambda} + g_1^2 \left(\frac{4}{5}\Sigma_{\kappa} + \frac{6}{5}\Sigma_{\lambda}\right) + g_1' ^2 \left(\frac{5}{2}k_i^2 - \frac{9}{5}\Sigma_{\kappa} - \frac{6}{5}\Sigma_{\lambda}\right) + \frac{16}{3}g_3^2 \left(3N_g - \frac{19}{3}\right) + \frac{4}{15}g_1^4 \left(3N_g + \frac{11}{15}\right) + \frac{19}{10}g_1' ^4 \left(3N_g + \frac{27}{20}\right) + \frac{64}{45}g_2^2g_1^2 + \frac{52}{15}g_2^2g_1' ^2 + \frac{13}{75}g_1^2g_1' ^2,
\]
\[ \beta^{(2)}_{h_t} = -22h_t^4 - 5h_b^4 - 5h^2_t h_b^2 - h^2_t h_b^2 - \lambda^2 \left( \lambda^2 + 3h_t^2 ight) \\
+ 4h_b^2 + h^2_t + 2\Sigma_\lambda + 3\Sigma_\kappa \right) + 16g_3^2 h_t^2 + 6g_2^2 h^2_t + g_1^2 \left( \frac{6h_t^2}{5} + \frac{2}{5}h_b^2 \right) \\
+ g_1^2 \left( \frac{3}{2}h^2_t + \frac{3}{10}h_b^2 + \frac{3}{5}h_b^2 \right) + \frac{16}{3}g_3^4 \left( 3N_g - \frac{19}{3} \right) + 3g_4^2 \left( 3N_g - \frac{7}{2} \right) \\
+ \frac{13}{15}g_4^4 \left( 3N_g + \frac{31}{30} \right) + \frac{3}{10}g_1^4 \left( 3N_g + \frac{11}{20} \right) + 8g_3^2 g_2^2 + \frac{136}{45}g_3^2 g_1^2 \\
+ \frac{8}{15}g_3^2 g_1^2 + g_2^2 g_1^2 + \frac{3}{4}g_2^2 g_1^2 + \frac{53}{300}g_1^2 g_1^2, \\
\beta^{(2)}_{h_b} = -5h_t^4 - 22h_b^4 - 5h_t^2 h_b^2 - 3h_b^2 h_t^2 - 3h^4_t - \lambda^2 \left( \lambda^2 + 4h_t^2 ight) \\
+ 3h_b^2 + 2\Sigma_\lambda + 3\Sigma_\kappa \right) + 16g_3^2 h_b^2 + 6g_2^2 h^2_b + g_1^2 \left( \frac{4}{5}h_b^2 + \frac{2}{5}h_b^2 + \frac{6}{5}h^2_t \right) \\
+ g_1^2 \left( \lambda^2 + \frac{1}{5}h^2_t + h_b^2 - \frac{1}{5}h^2_t \right) + \frac{16}{3}g_3^4 \left( 3N_g - \frac{19}{3} \right) + 3g_4^2 \left( 3N_g - \frac{7}{2} \right) \\
+ \frac{7}{15}g_4^4 \left( 3N_g + \frac{5}{6} \right) + \frac{7}{10}g_1^4 \left( 3N_g + \frac{3}{4} \right) + 8g_3^2 g_2^2 + \frac{8}{9}g_3^2 g_1^2 + \frac{4}{3}g_3^2 g_1^2 \\
+ g_2^2 g_1^2 + \frac{3}{2}g_2^2 g_1^2 + \frac{49}{150}g_1^2 g_1^2, \\
\beta^{(2)}_{h_\tau} = -9h_b^4 - 3h_t^2 h_b^2 - 9h_b^2 h_t^2 - 10h^4_\tau - \lambda^2 \left( \lambda^2 + 3h_t^2 ight) \\
+ 3h^2_\tau + 2\Sigma_\lambda + 3\Sigma_\kappa \right) + 16g_3^2 h^2_\tau + 6g_2^2 h^2_\tau + g_1^2 \left( -\frac{2}{5}h^2_b + \frac{6}{5}h^2_\tau \right) \\
+ g_1^2 \left( \lambda^2 - \frac{1}{5}h^2_b + \frac{13}{10}h^2_\tau \right) + 3g_2^4 \left( 3N_g - \frac{7}{2} \right) + \frac{9}{5}g_1^4 \left( 3N_g + \frac{3}{2} \right) \\
+ \frac{7}{10}g_1^4 \left( 3N_g + \frac{3}{4} \right) + \frac{9}{5}g_2^2 g_1^2 + \frac{39}{20}g_2^2 g_1^2 + \frac{51}{100}g_1^2 g_1^2, \]

and

\[ \Pi_\lambda = \lambda^4_1 + \lambda^4_2 + \lambda^4_3, \quad \Pi_\kappa = \kappa^4_1 + \kappa^4_2 + \kappa^4_3. \]

Using the two–loop β–functions for the gauge and Yukawa couplings and a method proposed in [65], one can obtain the two–loop RGEs for the gaugino masses and trilinear scalar couplings:

\[ \frac{dM_3}{dt} = \frac{g_3^2}{16\pi^2} \left[ (-18 + 6N_g)M_3 + \frac{1}{16\pi^2} \left( -216 + 136N_g \right) g_3^2 M_3 + 6N_g g_2^2 (M_2 + M_3) \right] \\
+ 2N_g g_1^2 (M_1 + M_3) + 2N_g g_1^2 (M_1' + M_3) - 8h^2_t (A_t + M_3) - 8h^2_b (A_b + M_3) \\
- 4\Sigma_\kappa - 4\Sigma_\kappa M_3 \right], \]

\[ \frac{dM_2}{dt} = \frac{g_2^2}{16\pi^2} \left[ (-10 + 6N_g)M_2 + \frac{1}{16\pi^2} \left( 16N_g g_3^2 (M_3 + M_2) + ( -68 + 84N_g) g_2^2 M_2 \right) \right] \\
+ \left( \frac{6}{5} + 2N_g \right) g_1^2 (M_1 + M_2) + \left( \frac{4}{5} + 2N_g \right) g_1^2 (M_1' + M_2) - 12h^2_t (A_t + M_2) \\
- 12h^2_b (A_b + M_2) - 4h^2_\tau (A_\tau + M_2) - 4\Sigma_\lambda - 4\Sigma_\lambda M_2 \right]. \]
\[ \frac{dM_1}{dt} = \frac{g_1^2}{16\pi^2} \left[ \left( 6 \frac{4}{5} + 6N_g \right) M_1 + \frac{1}{16\pi^2} \left( 16N_g g_3^2 (M_3 + M_1) \right) \right. \\
\] 
\[ + \left( 12 \frac{4}{5} + 6N_g \right) g_2^2 (M_2 + M_1) + \left( 12 \frac{25}{25} + 2N_g \right) g_1^2 M_1 \right. \\
\[ + \left( 16 \frac{12}{25} + 12N_g \right) g_1^2 (M_1' + M_1) - \frac{52}{5} h_i^2 (A_t + M_1) - \frac{28}{5} h_i^2 (A_b + M_1) \left. \right] \] 
\[ - \frac{36}{5} h_i^2 (A_r + M_1) - \frac{12}{5} \Sigma A_{\lambda} - \frac{12}{5} \Sigma A_{\kappa} - \frac{8}{3} \Sigma A_{\lambda} - \frac{8}{3} \Sigma A_{\kappa} \right), \\
\] 
\[ \frac{dM_1'}{dt} = \frac{g_1^2}{16\pi^2} \left[ \left( 4 \frac{4}{5} + 6N_g \right) M_1' + \frac{1}{16\pi^2} \left( 16N_g g_3^2 (M_3 + M_1') \right) \right. \\
\[ + \left( 12 \frac{4}{5} + 6N_g \right) g_2^2 (M_2 + M_1') + \left( 12 \frac{25}{25} + 2N_g \right) g_1^2 (M_1 + M_1') \right. \\
\[ + \left( 16 \frac{12}{25} + 12N_g \right) g_1^2 (M_1' - \frac{18}{5} h_i^2 (A_t + M_1') - \frac{42}{5} h_i^2 (A_b + M_1') \left. \right] \] 
\[ - \frac{14}{5} h_i^2 (A_r + M_1') - \frac{38}{5} \Sigma A_{\lambda} - \frac{38}{5} \Sigma A_{\kappa'} - \frac{57}{5} \Sigma A_{\lambda} - \frac{57}{5} \Sigma A_{\kappa'} \right), \\
\] 
\[ \frac{dA_{\lambda}}{dt} = \frac{1}{(4\pi)^2} \left[ 4 \lambda^2 A_{\lambda} + 4 \Sigma A_{\lambda} + 6 \Sigma A_{\kappa} + (6 h_i^2 A_t + 6 h_i^2 A_b + 2 h_i^2 A_r) \delta_{\lambda 3} \right. \\
\[ - 6 g_2^2 M_2 - \frac{6}{5} g_1^2 M_1 - \frac{19}{5} g_1^2 M_1' + \frac{\beta_{A_{\lambda}}^{(2)}}{(4\pi)^2} \right], \\
\] 
\[ \frac{dA_{\kappa}}{dt} = \frac{1}{(4\pi)^2} \left[ 4 \kappa^2 A_{\kappa} + 4 \Sigma A_{\lambda} + 6 \Sigma A_{\kappa} - \frac{32}{3} g_3^2 M_3 - \frac{8}{15} g_1^2 M_1 \right. \\
\[ - \frac{19}{5} g_1^2 M_1' + \frac{\beta_{A_{\kappa}}^{(2)}}{(4\pi)^2} \right], \\
\] 
\[ \frac{dA_t}{dt} = \frac{1}{(4\pi)^2} \left[ 2 \lambda^2 A_{\lambda} + 12 h_i^2 A_t + 2 h_i^2 A_b - \frac{32}{3} g_3^2 M_3 - 6 g_2^2 M_2 \right. \\
\[ - \frac{26}{15} g_1^2 M_1 - \frac{3}{5} g_1^2 M_1' + \frac{\beta_{A_t}^{(2)}}{(4\pi)^2} \right], \\
\] 
\[ \frac{dA_b}{dt} = \frac{1}{(4\pi)^2} \left[ 2 \lambda^2 A_{\lambda} + 2 h_i^2 A_t + 12 h_i^2 A_b + 2 h_i^2 A_r - \frac{32}{3} g_3^2 M_3 - 6 g_2^2 M_2 \right. \\
\[ - \frac{14}{15} g_1^2 M_1 - \frac{7}{5} g_1^2 M_1' + \frac{\beta_{A_b}^{(2)}}{(4\pi)^2} \right], \\
\] 
\[ \frac{dA_r}{dt} = \frac{1}{(4\pi)^2} \left[ 2 \lambda^2 A_{\lambda} + 6 h_i^2 A_b + 8 h_i^2 A_r - 6 g_2^2 M_2 - \frac{18}{5} g_1^2 M_1 - \frac{7}{5} g_1^2 M_1' + \frac{\beta_{A_r}^{(2)}}{(4\pi)^2} \right], \\
\] 

where the two–loop contributions to the \( \beta \)–functions of trilinear scalar couplings are given
by

\[ \beta_{A_{\Lambda}}^{(2)} = -4\lambda_i^2 \left( \lambda_i^2 + 2\Sigma_\Lambda + 3\Sigma_\kappa \right) A_{\Lambda_i} - 4\lambda_i^2 \left( \lambda_i^2 A_{\Lambda_i} + 2\Sigma_\Lambda + 3\Sigma_\kappa \right) - 16\Pi_{A_{\Lambda}} - 24\Pi_{A_{\kappa}} \\
-2\lambda^2 \left( 3h_t^2 + 3h_b^2 + h_r^2 \right) (2 + \delta_{i3}) A_{\Lambda} - 2\lambda^2 \left( 3h_t^2 A_t + 3h_b^2 A_b + h_r^2 A_r \right) (2 + \delta_{i3}) \\
-12 \left[ 3h_t^4 A_t + 3h_b^4 A_b + h_r^2 h_b^2 (A_t + A_b) + h_r^4 A_r \right] \delta_{i3} + 32g_3^2 \left( \Sigma_\kappa M_3 + \Sigma_{A_{\kappa}} \right) \\
+12g_2^2 \left( \Sigma_\Lambda M_2 + \Sigma_{A_{\Lambda}} \right) + 2g_1^2 \left[ \left( 4 \frac{\Sigma_\kappa}{5} + \frac{6}{5} \Sigma_\lambda \right) M_1 + \frac{4}{5} \Sigma_{A_{\kappa}} + \frac{6}{5} \Sigma_{A_{\Lambda}} \right] \\
+2g_1^2 \left[ \left( \frac{5}{2} \lambda_i^2 - \frac{9}{5} \Sigma_\kappa - \frac{6}{5} \Sigma_\lambda \right) M_1 + \frac{5}{2} \lambda^2 A_{\Lambda_i} - \frac{9}{5} \Sigma_{A_{\kappa}} - \frac{6}{5} \Sigma_{A_{\Lambda}} \right] \\
+32g_3^2 \left( h_t^2 + h_b^2 \right) M_3 + h_t^2 A_t + h_b^2 A_b \left[ \delta_{i3} + 2g_1^2 \left[ \left( \frac{4}{5} h_t^2 - \frac{2}{5} h_b^2 \right) M_1 + \frac{4}{5} h_t^2 A_t \right. \\
- \frac{2}{5} h_b^2 A_b + \frac{6}{5} h_r^2 A_r \right] \delta_{i3} + g_1^2 \left[ \left( -\frac{3}{5} h_t^2 - \frac{2}{5} h_b^2 \right) M_1 - \frac{3}{5} h_t^2 A_t - \frac{2}{5} h_b^2 A_b \right. \\
- \frac{2}{5} h_r^2 A_r \right] \delta_{i3} + 12g_1^2 \left[ 3N_g - \frac{7}{2} \right] M_2 + \frac{12}{5} g_1^4 \left( 3N_g + \frac{9}{10} \right) M_1 + \frac{38}{5} g_1^4 \left( 3N_g + \frac{27}{20} \right) M_1' \\
+ \frac{18}{5} g_3^2 g_1^4 \left( M_2 + M_1 \right) + \frac{39}{10} g_2^2 g_1^2 \left( M_2 + M_1' \right) + \frac{39}{50} g_1^2 g_1^2 \left( M_1 + M_1' \right),\]

\[ \beta_{A_{\kappa}}^{(2)} = -4\kappa_i^2 \left( \kappa_i^2 + 2\Sigma_\Lambda + 3\Sigma_\kappa \right) A_{\kappa_i} - 4\kappa_i^2 \left( \kappa_i^2 A_{\kappa_i} + 2\Sigma_\Lambda + 3\Sigma_\kappa \right) - 16\Pi_{A_{\Lambda}} - 24\Pi_{A_{\kappa}} \\
-4\lambda^2 \left( 3h_t^2 + 3h_b^2 + h_r^2 \right) A_{\Lambda} - 4\lambda^2 \left( 3h_t^2 A_t + 3h_b^2 A_b + h_r^2 A_r \right) + 32g_3^2 \left( \Sigma_\kappa M_3 + \Sigma_{A_{\kappa}} \right) \\
+12g_2^2 \left( \Sigma_\Lambda M_2 + \Sigma_{A_{\Lambda}} \right) + 2g_1^2 \left[ \left( \frac{4}{5} \Sigma_\kappa + \frac{6}{5} \Sigma_\lambda \right) M_1 + \frac{4}{5} \Sigma_{A_{\kappa}} + \frac{6}{5} \Sigma_{A_{\Lambda}} \right] \\
+2g_1^2 \left[ \left( \frac{5}{2} \kappa_i^2 - \frac{9}{5} \Sigma_\kappa - \frac{6}{5} \Sigma_\lambda \right) M_1 + \frac{5}{2} \kappa_i^2 A_{\kappa_i} - \frac{9}{5} \Sigma_{A_{\kappa}} - \frac{6}{5} \Sigma_{A_{\Lambda}} \right] \\
+64g_3^4 \left( 3N_g - \frac{19}{3} \right) M_3 + \frac{16}{15} g_1^4 \left( 3N_g + \frac{11}{15} \right) M_1 + \frac{38}{5} g_1^4 \left( 3N_g + \frac{27}{20} \right) M_1' \\
+ \frac{128}{45} g_3^4 g_1^2 \left( M_3 + M_1 \right) + \frac{104}{15} g_3^2 g_1^2 \left( M_3 + M_1' \right) + \frac{26}{75} g_3^2 g_1^2 \left( M_1 + M_1' \right) ,\]

\[ \beta_{A_{\Lambda}}^{(2)} = -88h_t^4 A_t - 20h_b^4 A_b - 10h_r^2 h_b^2 (A_t + A_b) - 2h_r^2 h_r^2 (A_b + A_r) - 2\lambda^2 \left( 2\lambda^2 + 3h_t^2 \\
+ 4h_b^2 + h_r^2 + 2\Sigma_\Lambda + 3\Sigma_\kappa \right) A_{\Lambda} + 3h_t^2 A_t + 4h_b^2 A_b + h_r^2 A_r + 2\Sigma_{A_{\Lambda}} + 3\Sigma_{A_{\kappa}} \\
+32g_3^2 h_t^2 \left( M_3 + A_t \right) + 12g_2^2 h_t^2 \left( M_2 + A_t \right) + 2g_1^2 \left[ \left( \frac{6}{5} h_t^2 + \frac{2}{5} h_b^2 \right) M_1 + \frac{6}{5} h_t^2 A_t \right. \\
+ \frac{2}{5} h_b^2 A_b \right) + 2g_1^2 \left[ \left( \frac{3}{2} \lambda^2 + \frac{3}{5} h_t^2 + \frac{3}{5} h_b^2 \right) M_1 + \frac{3}{2} \lambda^2 A_{\Lambda} + \frac{3}{10} h_b^2 A_b \right. \\
+ \frac{64}{3} g_3^4 \left( 3N_g - \frac{19}{3} \right) M_3 + 12g_2^4 \left( 3N_g - \frac{7}{2} \right) M_2 + \frac{52}{15} g_1^4 \left( 3N_g + \frac{31}{30} \right) M_1 \\
+ \frac{6}{5} g_1^4 \left( 3N_g + \frac{11}{20} \right) M_1' + 16g_3^2 g_2^2 \left( M_3 + M_2 \right) + \frac{272}{45} g_3^2 g_1^2 \left( M_3 + M_1 \right) \\
+ \frac{16}{15} g_3^2 g_1^2 \left( M_3 + M_1' \right) + 2g_3^2 g_1^2 \left( M_2 + M_1 \right) + \frac{3}{2} g_2^2 g_1^2 \left( M_2 + M_1' \right) + \frac{53}{150} g_1^2 g_1^2 \left( M_1 + M_1' \right),\]
\[\beta^{(2)}_{\lambda_b} = -20 h_t^4 A_t - 88 h_b^4 A_b - 10 h_b^2 h_t^2 \left( A_t + A_b \right) - 6 h_t^2 h_b^2 \left( A_b + A_\tau \right) - 12 h_t^4 A_\tau - 2\lambda^2 \left[ \left( 2\lambda^2 + 4 h_t^2 + 3 h_b^2 + 2\Sigma_\lambda + 3\Sigma_\kappa \right) A_\lambda + 4 h_t^2 A_t + 3 h_b^2 A_b + 2\Sigma A_\lambda + 3\Sigma A_\kappa \right] + 32 g_3^2 h_b^2 \left( M_3 + A_b \right) + 12 g_3^2 h_t^2 \left( M_2 + A_\tau \right) + 4 g_1^2 \left[ \left( \frac{2}{5} h_t^2 + \frac{1}{5} h_b^2 + \frac{3}{5} h_t^2 \right) M_1 + \frac{2}{5} h_t^2 A_t + \frac{1}{5} h_b^2 A_b + \frac{3}{5} h_b^2 A_\tau \right] + 2 g_1^2 \left[ \left( \lambda^2 + \frac{1}{5} h_t^2 + h_b^2 - \frac{1}{5} h_t^2 \right) M'_1 + \lambda^2 A_\lambda \right] + \frac{1}{5} h_t^2 A_t + h_b^2 A_b - \frac{1}{5} h_b^2 A_\tau \right] + \frac{2}{3} g_3^4 \left( 3 N_9 - \frac{19}{3} \right) M_3 + 12 g_3^4 \left( 3 N_9 - \frac{7}{2} \right) M_2 + \frac{28}{15} g_1^4 \left( 3 N_9 + \frac{5}{6} \right) M_1 + \frac{14}{5} g_1^4 \left( 3 N_9 + \frac{3}{4} \right) M'_1 + 16 g_3^2 g_2^2 \left( M_3 + M_2 \right) + \frac{16}{9} g_3^2 g_1^2 \left( M_3 + M_1 \right) + \frac{8}{3} g_3^2 g_1^2 \left( M_3 + M'_1 \right) + 2 g_2^2 g_1^2 \left( M_2 + M_1 \right) + 3 g_2^2 g_1^2 \left( M_2 + M'_1 \right) + 4 g_1^2 \left( M_1 + M'_1 \right),
\]

\[\beta^{(2)}_{A_\tau} = -36 h_t^2 A_b - 6 h_t^2 h_b^2 \left( A_t + A_b \right) - 18 h_b^2 h_t^2 \left( A_b + A_\tau \right) - 40 h_t^4 A_\tau - 2\lambda^2 \left[ \left( 2\lambda^2 + 3 h_t^2 \right) \right] + 3 h_b^2 + 2\Sigma_\lambda + 3\Sigma_\kappa \right) A_\lambda + 3 h_b^2 A_b + 3 h_t^2 A_\tau + 2\Sigma A_\lambda + 3\Sigma A_\kappa \right] + 32 g_3^2 h_t^2 \left( M_3 + A_b \right) + 12 g_3^2 h_t^2 \left( M_2 + A_\tau \right) + 4 g_1^2 \left[ \left( \frac{1}{5} h_b^2 + \frac{3}{5} h_t^2 \right) M_1 - \frac{1}{5} h_t^2 A_b + \frac{3}{5} h_t^2 A_\tau \right] + 2 g_1^2 \left[ \left( \lambda^2 - \frac{1}{5} h_b^2 + \frac{13}{10} h_t^2 \right) M'_1 + \lambda^2 A_\lambda - \frac{1}{5} h_b^2 A_b + \frac{13}{10} h_t^2 A_\tau \right] + 12 g_2^2 \left( 3 N_9 + \frac{3}{4} \right) M_1 + \frac{36}{5} g_1^4 \left( 3 N_9 + \frac{3}{2} \right) M'_1 + \frac{14}{5} g_1^4 \left( 3 N_9 + \frac{3}{4} \right) M'_1 + \frac{18}{5} g_2^2 g_1^2 \left( M_2 + M_1 \right) + \frac{39}{10} g_2^2 g_1^2 \left( M_2 + M'_1 \right) + \frac{51}{50} g_1^2 g_1^2 \left( M_1 + M'_1 \right),
\]

whereas

\[
\Sigma_{A_\lambda} = \lambda_1^2 A_{\lambda_1} + \lambda_2^2 A_{\lambda_2} + \lambda_3^2 A_{\lambda_3}, \quad \Sigma_{A_\kappa} = \kappa_2 A_{\kappa_1} + \kappa_3 A_{\kappa_2} + \kappa_3 A_{\kappa_3},
\]

\[
\Pi_{A_\lambda} = \lambda_1^2 A_{\lambda_1} + \lambda_2^2 A_{\lambda_2} + \lambda_3^2 A_{\lambda_3}, \quad \Pi_{A_\kappa} = \kappa_1^2 A_{\kappa_1} + \kappa_2 A_{\kappa_2} + \kappa_3 A_{\kappa_3}.
\]

The one–loop RGEs for the soft scalar masses can be written as

\[
\frac{d m_{S_i}^2}{d t} = \frac{1}{(4\pi)^2} \left[ \sum_{j=1,3} 4 \lambda_j^2 \left( m_{H_j}^2 + m_{H_j}^4 + m_S^2 + A_{\lambda_j}^2 \right) \delta_{i3} \right.
\]

\[
+ \sum_{j=1,3} 6 \kappa_j^2 \left( m_S^2 + m_{D_j}^2 + m_{D_j}^4 + A_{\kappa_j}^2 \right) \delta_{i3} - 5 g_1^2 M_1^2 + \frac{g_1^2}{4} \Sigma_1',
\]

\[
\frac{d m_{H_i}^2}{d t} = \frac{1}{(4\pi)^2} \left[ 2 \lambda_i^2 \left( m_{H_i}^2 + m_{H_i}^4 + m_S^2 + A_{\lambda_i}^2 \right) + 6 h_t^2 \left( m_{H_i}^2 + m_Q^2 + m_{t_4}^2 + A_t^2 \right) \delta_{i3} \right.
\]

\[
- 6 g_2^2 M_2 - \frac{6}{5} g_1^2 M_1^2 - \frac{4}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 \Sigma_1 - \frac{g_1^2}{10} \Sigma_1',
\]

\[52\]
\[
\frac{dm^2_{H_d}}{dt} = \frac{1}{(4\pi)^2} \left[ 2\lambda_i^2 \left( m_{H_d}^2 + m_{H_d}^2 + m_5^2 + A_{L_i}^2 \right) + 6\alpha_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \right] + 6\alpha_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} \\
+ 2\alpha_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{9}{5} g_1^2 M_1^2 \delta_{i3} - \frac{3}{5} g_1^2 \Sigma_1 - \frac{3}{20} g_1^2 \Sigma_1',
\]

\[
\frac{dm^2_{Q_i}}{dt} = \frac{1}{(4\pi)^2} \left[ 2\lambda_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} + 2\alpha_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} \\
- \frac{32}{3} g^2_2 M_3^2 - 6g_2^2 M_2^2 - 2\frac{2}{15} g_1^2 M_1^2 - \frac{1}{5} g_1^2 M_1^2 + \frac{1}{5} g_1^2 \Sigma_1 + \frac{9}{20} \Sigma_1',
\]

\[
\frac{dm^2_{\phi_c}}{dt} = \frac{1}{(4\pi)^2} \left[ 4\lambda_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{1}{5} g_1^2 M_1^2 \delta_{i3} \\
- \frac{3}{5} g_1^2 \Sigma_1 + \frac{g_1^2}{20} \Sigma_1',
\]

\[
\frac{dm^2_{\phi_e}}{dt} = \frac{1}{(4\pi)^2} \left[ 4\lambda_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} - \frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 M_1^2 \delta_{i3} \\
- \frac{3}{5} g_1^2 \Sigma_1 + \frac{g_1^2}{10} \Sigma_1',
\]

\[
\frac{dm^2_{\phi_i}}{dt} = \frac{1}{(4\pi)^2} \left[ 2\lambda_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} - 6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{4}{5} g_1^2 M_1^2 \delta_{i3} \\
- \frac{3}{5} g_1^2 \Sigma_1 + \frac{g_1^2}{10} \Sigma_1',
\]

\[
\frac{dm^2_{\phi_{D_i}}}{dt} = \frac{1}{(4\pi)^2} \left[ 2\lambda_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} - \frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 M_1^2 \delta_{i3} \\
- \frac{2}{5} g_1^2 \Sigma_1 - \frac{g_1^2}{10} \Sigma_1',
\]

\[
\frac{dm^2_{\phi_{B_i}}}{dt} = \frac{1}{(4\pi)^2} \left[ 2\lambda_i^2 \left( m_{H_d}^2 + m_Q^2 + m_{\phi}^2 + A_{L_i}^2 \right) \delta_{i3} - \frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 M_1^2 \delta_{i3} \\
+ \frac{2}{5} g_1^2 \Sigma_1 - \frac{g_1^2}{10} \Sigma_1',
\]

\[
\frac{dm^2_{\phi'}}{dt} = \frac{1}{(4\pi)^2} \left[ -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{4}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 \Sigma_1 + \frac{g_1^2}{10} \Sigma_1' \right],
\]

\[
\frac{dm^2_{\phi'}}{dt} = \frac{1}{(4\pi)^2} \left[ -6g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{4}{5} g_1^2 M_1^2 + \frac{3}{5} g_1^2 \Sigma_1 - \frac{g_1^2}{10} \Sigma_1' \right],
\]

where

\[\Sigma_1 = \sum_{i=1}^{3} \left( m_Q^2 - 2m_{\phi_i}^2 + m_{\phi_i}^2 + m_{\phi_i}^2 - m_{\phi_i}^2 - m_{H_d}^2 + m_{H_d}^2 - m_{H_d}^2 \right) - m^2_{H_d} + m^2_{H_d},\]

\[\Sigma_1' = \sum_{i=1}^{3} \left( 6m_Q + 3m_{\phi_i}^2 + 6m_{\phi_i}^2 + m_{\phi_i}^2 + 4m_{L_i}^2 - 4m_{H_d}^2 + 5m_{H_d}^2 + 6m_{L_i}^2 - 9m_{B_i}^2 - 6m_{B_i}^2 \right) + 4m^2_{H_d} - 4m^2_{H_d}.\]
References

[1] H.P. Nilles, Phys. Rept. 110 (1984) 1; A.B. Lahanas, D.V. Nanopoulos, Phys. Rept. 145 (1987) 1.

[2] A. H. Chamseddine, R. L. Arnowitt and P. Nath, Phys. Rev. Lett. 49 (1982) 970; R. Barbieri, S. Ferrara, C. Savoy, Phys. Lett. B 119 (1982) 343; H. P. Nilles, M. Srednicki, D. Wyler, Phys. Lett. B 120 (1983) 345; L. Hall, J. Lykken, S. Weinberg, Phys. Rev. D 27 (1983) 2359; S. K. Soni, H. A. Weldon, Phys. Lett. B 126 (1983) 215. P. Nath, R. L. Arnowitt and A. H. Chamseddine, Nucl. Phys. B 227 (1983) 121.

[3] E. Witten, Nucl. Phys. B 188 (1981) 513; N. Sakai, Z. Phys. C 11 (1981) 153; S. Dimopoulos, H. Georgi, Nucl. Phys. B 193 (1981) 150; R. K. Kaul, P. Majumdar, Nucl. Phys. B 199 (1982) 36.

[4] D. J. H. Chung, L. L. Everett, G. L. Kane, S. F. King, J. Lykken, L. T. Wang, Phys. Rept. 407 (2005) 1.

[5] L. Girardello, M.T. Grisaru, Nucl. Phys. B 194 (1982) 65.

[6] J. Ellis, S. Kelley, D. V. Nanopoulos, Phys. Lett. B 249 (1990) 441; J. Ellis, S. Kelley, D. V. Nanopoulos, Phys. Lett. B 260 (1991) 131; U. Amaldi, W. de Boer, H. Furstenau, Phys. Lett. B 260 (1991) 447; P. Langacker, M. Luo, Phys. Rev. D 44 (1991) 817.

[7] H. Georgi, S. L. Glashow, Phys. Rev. Lett. 32 (1974) 438.

[8] A. Salam, J. Strathdee, Phys. Rev. D 11 (1975) 1521; M. T. Grisaru, W. Siegel, M. Rocek, Nucl. Phys. B 159 (1979) 429.

[9] M. B. Green, J. H. Schwarz, E. Witten, “Superstring Theory” (Cambridge University Press, 1987).

[10] P. Horava, E. Witten, Nucl. Phys. B 460 (1996) 506 and Nucl. Phys. B 475 (1996) 94.

[11] E. Witten, Nucl. Phys. B 471 (1996) 135; T. Banks, M. Dine, Nucl. Phys. B 479 (1996) 173; K. Choi, H. B. Kim, C. Muñoz, Phys. Rev. D 57 (1998) 7521.

[12] F. del Aguila, G. A. Blair, M. Daniel, G. G. Ross, Nucl. Phys. B 272 (1986) 413.

[13] V.S. Kaplunovsky, J. Louis, Phys. Lett. B 306 (1993) 269; A. Brignole, L.E. Ibañez, C. Muñoz, Nucl. Phys. B 422 (1994) 125 [Erratum-ibid. B 436 (1995) 747].
[14] Y. Hosotani, Phys. Lett. B 129 (1983) 193.

[15] J.L. Hewett, T.G. Rizzo, Phys. Rept. 183 (1989) 193; P. Langacker, arXiv:0801.1345 [hep-ph].

[16] P. Athron, S. F. King, D. J. Miller, S. Moretti, R. Nevzorov, J. Phys. Conf. Ser. 110 (2008) 072001.

[17] H. P. Nilles, M. Srednicki, D. Wyler, Phys. Lett. B 120 (1983) 346; J. M. Frere, D. R. T. Jones, S. Raby, Nucl. Phys. B 222 (1983) 11; J. P. Derendinger, C. A. Savoy, Nucl. Phys. B 237 (1984) 307; M. I. Vysotsky, K. A. Ter-Martirosian, Sov. Phys. JETP 63 (1986) 489; J. Ellis, J. F. Gunion, H. Haber, L. Roszkowski, F. Zwirner, Phys. Rev. D 39 (1989) 844; L. Durand, J. L. Lopez, Phys. Lett. B 217 (1989) 463; M. Drees, Int. J. Mod. Phys. A 4 (1989) 3635; C. Panagiotakopoulos, K. Tamvakis, Phys. Lett. B 446 (1999) 224 and Phys. Lett. B 469 (1999) 145; C. Panagiotakopoulos, A. Pilaftsis, Phys. Rev. D 63 (2001) 055003; A. Dedes, C. Hugonie, S. Moretti, K. Tamvakis, Phys. Rev. D 63 (2001) 055009.

[18] S. A. Abel, S. Sarkar, P. L. White, Nucl. Phys. B 454 (1995) 663.

[19] J. F. Gunion, H. E. Haber, G. L. Kane, S. Dawson, “The Higgs Hunter’s Guide” (Westview Press, 2000) [Erratum arXiv:hep-ph/9302272; P. Binetruy, S. Dawson, I. Hinchcliffe, M. Sher, Nucl. Phys. B 273 (1986) 501; J. R. Ellis, K. Enqvist, D. V. Nanopoulos, F. Zwirner, Mod. Phys. Lett. A 1 (1986) 57. L. E. Ibanez, J. Mas, Nucl. Phys. B 286 (1987) 107; J. F. Gunion, L. Roszkowski, H. E. Haber, Phys. Lett. B 189 (1987) 409; H. E. Haber, M. Sher, Phys. Rev. D 35 (1987) 2206; J. R. Ellis, D. V. Nanopoulos, S. T. Petcov, F. Zwirner, Nucl. Phys. B 283 (1987) 93; M. Drees, Phys. Rev. D 35 (1987) 2910; J. F. Gunion, L. Roszkowski, H. E. Haber, Phys. Lett. B 189 (1987) 409. H. Baer, D. Dicus, M. Drees, X. Tata, Phys. Rev. D 36 (1987) 1363; J. F. Gunion, L. Roszkowski, H. E. Haber, Phys. Rev. D 38 (1988) 105.

[20] M. Cvetič, P. Langacker, Phys. Rev. D 54 (1996) 3570; M. Cvetič, P. Langacker, Mod. Phys. Lett. A 11 (1996) 1247; M. Cvetič, D. Demir, J. R. Espinosa, L. L. Everett, P. Langacker, Phys. Rev. D 56 (1997) 2861 [Erratum-ibid. D 58 (1998) 119905].

[21] P. Langacker, J. Wang, Phys. Rev. D 58 (1998) 115010.

[22] D. Suematsu, Y. Yamagishi, Int. J. Mod. Phys. A 10 (1995) 4521.

[23] E. Keith, E. Ma, Phys. Rev. D 56 (1997) 7155.
[24] Y. Daikoku, D. Suematsu, Phys. Rev. D 62 (2000) 095006.

[25] J. H. Kang, P. Langacker, T. J. Li, Phys. Rev. D 71 (2005) 015012.

[26] E. Ma, Phys. Lett. B 380 (1996) 286.

[27] T. Hambye, E. Ma, M. Raidal, U. Sarkar, Phys. Lett. B 512 (2001) 373.

[28] S. F. King, R. Luo, D. J. Miller, R. Nevzorov, JHEP 0812 (2008) 042.

[29] E. Ma, M. Raidal, J. Phys. G 28 (2002) 95; J. Kang, P. Langacker, T.-J. Li, T. Liu, Phys. Rev. Lett. 94 (2005) 061801.

[30] J. A. Grifols, J. Sola, A. Mendez, Phys. Rev. Lett. 57 (1986) 2348; D. A. Morris, Phys. Rev. D 37 (1988) 2012.

[31] D. Suematsu, Mod. Phys. Lett. A 12 (1997) 1709.

[32] A. Gutierrez-Rodriguez, M. A. Hernandez-Ruiz and M. A. Perez, Int. J. Mod. Phys. A 22, 3493 (2007) [arXiv:hep-ph/0611235].

[33] D. Suematsu, Phys. Lett. B 416 (1998) 108.

[34] S. W. Ham, J. O. Im, E. J. Yoo and S. K. Oh, JHEP 0812, 017 (2008) [arXiv:0810.4194 [hep-ph]].

[35] M. Asano, T. Kikuchi and S. G. Kim, [arXiv:0807.5084 [hep-ph]].

[36] B. Stech and Z. Tavartkiladze, Phys. Rev. D 77, 076009 (2008) [arXiv:0802.0894 [hep-ph]].

[37] S. F. King, S. Moretti, R. Nevzorov, Phys. Rev. D 73 (2006) 035009.

[38] S. F. King, S. Moretti, R. Nevzorov, Phys. Lett. B 634 (2006) 278.

[39] D. Suematsu, Phys. Rev. D 57 (1998) 1738.

[40] E. Keith, E. Ma, Phys. Rev. D 54 (1996) 3587.

[41] P. Athron, S. F. King, D. J. Miller, S. Moretti, R. Nevzorov, [arXiv:0901.1192 [hep-ph]].

[42] P. Athron, S. F. King, D. J. Miller, S. Moretti, R. Nevzorov, [arXiv:0810.0617 [hep-ph]].

[43] S. F. King, S. Moretti, R. Nevzorov, Phys. Lett. B 650 (2007) 57.
[44] Y. Kawamura, Prog. Theor. Phys. 105 (2001) 999; G. Altarelli, F. Feruglio, Phys. Lett. B 511 (2001) 257; L. J. Hall, Y. Nomura, Phys. Rev. D 64 (2001) 055003; A. Hebecker, J. March-Russell, Nucl. Phys. B 613 (2001) 3; T. Asaka, W. Buchmuller, L. Covi, Phys. Lett. B 523 (2001) 199; L. J. Hall, Y. Nomura, T. Okui, D. R. Smith, Phys. Rev. D 65 (2002) 035008.

[45] R. Howl, S. F. King, JHEP 0805 (2008) 008.

[46] S. F. King, S. Moretti, R. Nevzorov, arXiv:hep-ph/0601269; S. Kraml et al. (eds.), Workshop on CP studies and non-standard Higgs physics, CERN–2006–009, hep-ph/0608079; S. F. King, S. Moretti, R. Nevzorov, AIP Conf. Proc. 881 (2007) 138; R. Howl, S. F. King, JHEP 0801 (2008) 030; S. F. King, S. Moretti, R. Nevzorov, In *Moscow 2006, ICHEP* 1125-1128.

[47] M. Yu. Khlopov, A. D. Linde, Phys. Lett. B 138 (1984) 265; J. R. Ellis, J E. Kim, D. V. Nanopoulos, Phys. Lett. B 145 (1984) 181.

[48] J. Rich, M. Spiro, J. Lloyd–Owen, Phys. Rept. 151 (1987) 239; P. F. Smith, Contemp. Phys. 29 (1988) 159; T. K. Hemmick et al., Phys. Rev. D 41 (1990) 2074.

[49] D. Suematsu, Phys. Rev. D 59 (1999) 055017.

[50] K. S. Babu, C. Kolda, J. March–Russell, Phys. Rev. D 54 (1996) 4635; T. G. Rizzo, Phys. Rev. D 59 (1999) 015020.

[51] P. Abreu et al. [DELPHI Collaboration], Phys. Lett. B 485 (2000) 45; R. Barate et al. [ALEPH Collaboration], Eur. Phys. J. C 12 (2000) 183. J. Erler, P. Langacker, S. Munir and E. R. Pena, arXiv:0906.2435 [hep-ph].

[52] J. F. Grivaz, arXiv:0809.0531 [hep-ex] and references therein.

[53] A. C. Kraan, hep-ex/0505002.

[54] T. Gherghetta, T. A. Kaeding, G. L. Kane, Phys. Rev. D 57 (1998) 3178; S. Hesselbach, F. Franke, H. Fraas, Eur. Phys. J. C 23 (2002) 149; V. Barger, P. Langacker, H. S. Lee, Phys. Lett. B 630 (2005) 85; S. Y. Choi, H. E. Haber, J. Kalinowski, P. M. Zerwas, Nucl. Phys. B 778 (2007) 85; V. Barger, P. Langacker, I. Lewis, M. McCaskey, G. Shaughnessy and B. Yencho, Phys. Rev. D 75 (2007) 115002.

[55] V. Barger, P. Langacker, G. Shaughnessy, New J. Phys. 9 (2007) 333.

[56] P. A. Kovalenko, R. B. Nevzorov, K. A. Ter-Martirosian, Phys. Atom. Nucl. 61 (1998) 812 [Yad. Fiz. 61 (1998) 898]; R. B. Nevzorov, M. A. Trusov, J. Exp. Theor. Phys.
[57] V. Barger, P. Langacker, H. S. Lee and G. Shaughnessy, Phys. Rev. D 73 (2006) 115010.

[58] B. C. Allanach, Comput. Phys. Commun. 143 (2002) 305.

[59] A. Aktas et al. [H1 Collaboration], Phys. Lett. B 629 (2005) 9.

[60] F. Abe et al. [CDF Collaboration], http://www-cdf.fnal.gov/physics/, see CDF Notes 9246.

[61] J. Kang, P. Langacker and B. D. Nelson, Phys. Rev. D 77 (2008) 035003.

[62] P. Athron, S. F. King, D. J. Miller, S. Moretti, R. Nevzorov, in preparation.

[63] J. Kalinowski, S. F. King, J. P. Roberts, JHEP 0901 (2009) 066.

[64] D. R. T. Jones, Phys. Rev. D 25 (1982) 581; M. E. Machacek, M. T. Vaughn, Nucl. Phys. B 222 (1983) 83; M. E. Machacek, M. T. Vaughn, Nucl. Phys. B 236 (1984) 221; S. P. Martin, M. T. Vaughn, Phys. Rev. D 50 (1994) 2282.

[65] D. I. Kazakov, Phys. Lett. B 449 (1999) 201.