Perturbative QCD for $B_s \to a_1(1260)(b_1(1235))P(V)$ Decays

Zhi-Qing Zhang *

Department of Physics, Henan University of Technology, Zhengzhou, Henan 450052, P.R.China

(Dated: May 5, 2014)

Abstract

Within the framework of perturbative QCD approach, we study the charmless two-body decays $B_s \to a_1(1260)(b_1(1235))P(V)$ ($P, V$ represent the light pseudo-scalar and vector mesons, respectively.). Using the decays constants and the light-cone distribution amplitudes for these mesons derived from the QCD sum rule method, we find the following results: (a) The decays $\bar{B}_s^0 \to a_1^0 K^+(K^{*+})$ have the contributions from the factorization emission diagrams with a large Wilson coefficient $C_2 + C_1/3$ (order of 1), so they have the largest branching ratios and arrive at $10^{-5}$ order. While for the decays $\bar{B}_s^0 \to a_1^0 K^0(K^{*0})$, the Wilson coefficient is $C_1 + C_2/3$ in tree level and color suppressed, so their branching ratios are small and fall in the order of $10^{-7} \sim 10^{-8}$. For the decays $\bar{B}_s^0 \to b_1 K(K^*)$, all of their branching ratios are of order few times $10^{-6}$. (b) For the pure annihilation type decays $\bar{B}_s^0 \to a_1(b_1)\rho$ except the decays $\bar{B}_s^0 \to a_1 \pi$ having large branching ratios of order few times $10^{-6}$, the most other decays have the branching ratios of $10^{-7}$ order. The branching ratios of the decays $\bar{B}_s^0 \to a_1^0(b_1^0)\omega$ are the smallest and fall in the order of $10^{-8} \sim 10^{-9}$. (c) The branching ratios and the direct CP-asymmetries of decays $\bar{B}_s^0 \to a_1^0(b_1^0)\eta^{(')}$ are very sensitive to take different Gegenbauer moments for $\eta^{(')}$. (d) Except for the decays $\bar{B}_s^0 \to a_1^0 K^{*0}, a_1^0 \omega, b_1^0 \omega$, the longitudinal polarization fractions of other $\bar{B}_s^0 \to a_1(b_1)V$ decays are very large and more than 90%. (e) Compared with decays $\bar{B}_s^0 \to a_1(b_1)P$, most of $\bar{B}_s^0 \to a_1(b_1)V$ decays have smaller direct CP asymmetries.

PACS numbers: 13.25.Hw, 12.38.Bx, 14.40.Nd

* Electronic address: zhangzhiqing@haut.edu.cn
I. INTRODUCTION

In general, the mesons are classified in $J^{PC}$ multiplets. There are two types of orbitally excited axial-vector mesons, namely $1^{++}$ and $1^{-+}$. The former includes $a_1(1260), f_1(1285), f_1(1420)$ and $K_{1A}$, which compose the $^3P_1$-nonet, and the latter includes $b_1(1235), h_1(1170), h_1(1380)$ and $K_{1B}$, which compose the $^1P_1$-nonet. There is an important character for these axial-vector mesons except $a_1(1260)$ and $b_1(1235)$, that is each different flavor state can mix with one another, which comes from the other nonet meson or the same nonet one.

$B^0 \to a_1^+(1260)\pi^\pm$ are the first decay modes with an axial-vector in the final state observed by BarBar and Belle [1–3]. Measuring their time-dependent CP asymmetries can provide the information of Cabibbo-Kobayshi-Maskawa (CKM) weak phase $\alpha$. After these measurements, many other charmless decays $B \to AP, AV$ ($P, V$ stand for the light pseudo-scalar and vector mesons) have also been reported by experiments [4–10].

On the theoretical side, many methods are employed to research these decays, such as the naive factorization approach [11, 12], the generalized factorization approach [13], the QCD factorization approach [14, 15], the PQCD approach [17]. Though the factorization approach holds only approximately and its predictions are at odds with experiments for some decays, many factorization approaches can explain the data in many cases. So these results predicted by the different factorization approaches are useful to investigate production mechanism of axial vectors in $B$ meson decays, extract the information of Cabibbo-Kobayshi-Maskawa (CKM) weak phase, probe the structures of axial vectors, even calculate the relative strong phase between tree and penguin diagrams. To our knowledge there is still lacking the study of charmless decays $B_s \to AP, AV$ both in experiments and theories. Our aim is to fill in this gap and provide a ready reference to the forthcoming experiments to compare their data with the predictions in the PQCD approach. In view of the fact that $a_1(1260)$ and $b_1(1235)$ can not mix with each other because of the opposite $C$-parities and they do not also mix with other mesons, we would like to study the decays $B_s \to a_1(1260)P(V), b_1(1235)P(V)$ in detail.

In the following, $a_1(1260)$ and $b_1(1235)$ are denoted as $a_1$ and $b_1$ in some places for convenience. The layout of this paper is as follows. In Sec. II decay constants and light-cone distribution amplitudes of the relevant mesons are introduced. In Sec. III we then analyze these decay channels using the PQCD approach. The numerical results and the discussions are given in Sec. IV. The conclusions are presented in the final part.

II. DECAY CONSTANTS AND DISTRIBUTION AMPLITUDES

For the wave function of the heavy $B_s$ meson, we take

$$\Phi_{B_s}(x, b) = \frac{1}{\sqrt{2N_c}}(p_{B_s} + m_{B_s})\gamma_5\phi_{B_s}(x, b). \tag{1}$$

Here only the contribution of Lorentz structure $\phi_{B_s}(x, b)$ is taken into account, since the contribution of the second Lorentz structure $\bar{\phi}_{B_s}$ is numerically small [17] and has been neglected. For the distribution amplitude $\phi_{B_s}(x, b)$ in Eq.(1), we adopt the following
model:

\[ \phi_{B_1}(x, b) = N_{B_1} x^2(1-x)^2 \exp \left[ -\frac{M_{B_1}^2 x^2}{2\omega_b^2} - \frac{1}{2} (\omega_b b)^2 \right], \]

where \( \omega_b \) is a free parameter, we take \( \omega_b = 0.5 \pm 0.05 \text{ GeV} \) in numerical calculations, and \( N_{B_1} = 63.671 \) is the normalization factor for \( \omega_b = 0.4 \).

The wave functions for the pseudo-scalar (P) mesons \( K, \pi \) are given as

\[ \Phi_P(P, x, \zeta) \equiv \frac{1}{\sqrt{2N_C}} \gamma_5 \left[ P \phi^A(x) + m_0 \phi^P(x) + \zeta m_0 (\not{\phi} - v \cdot n) \phi^T(x) \right], \]

where the parameter \( \zeta \) is either +1 or −1 depending on the assignment of the momentum fraction \( x \). The chiral scale parameter \( m_0 \) is defined as \( m_0 = \frac{M^2}{m_{q_1} + m_{q_2}} \). The distribution amplitudes are expanded as:

\[ \phi^A_{K,\pi}(x) = \frac{3 f_{K,\pi}}{\sqrt{6}} x(1-x) \left[ 1 + a_{1(K,\pi)} C_1^{3/2}(t) + a_{2(K,\pi)} C_2^{3/2}(t) \right], \]

\[ \phi^P_{K}(x) = \frac{3 f_K}{2\sqrt{6}} \left[ 1 + 0.43 \frac{C_2^{1/2}(t)}{t}; \phi^P_{\pi}(x) = \frac{3 f_\pi}{2\sqrt{6}} \left[ 1 + 0.24 \frac{C_2^{1/2}(t)}{t} \right], \]

\[ \phi^T_{K}(x) = \frac{-f_K}{2\sqrt{6}} \left[ C_1^{1/2}(t) + 0.35 C_3^{1/2}(t) \right]; \phi^T_{\pi}(x) = \frac{-f_\pi}{2\sqrt{6}} \left[ C_1^{1/2}(t) + 0.55 C_3^{1/2}(t) \right], \]

with Gegenbauer polynomials defined as:

\[ C_1^{3/2}(t) = 3t, C_2^{3/2}(t) = 1.5(5t^2 - 1), \]

\[ C_1^{1/2}(t) = t, C_2^{1/2}(t) = 0.5(3t^2 - 1), C_3^{1/2}(t) = 0.5(5t^2 - 3). \]

As for the distribution amplitudes of the pseudo-scalar mesons \( \eta \) and \( \eta' \), we use the quark flavor basis mixing mechanism proposed by Refs.\[18\] and take the same formulae and parameter values as those in Ref.\[19\].

For the vector mesons, their distribution amplitudes are defined as

\[ \langle V(P, \epsilon_L^*)|\bar{q}_2\beta(z)q_1\alpha(0)|0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx \ e^{ixp_\mu}[m_V \psi_L^* \phi_V(x) + \psi_L^* P \phi_V(x) + m_V \phi_V^s(x)]_{\alpha\beta}, \]

\[ \langle V(P, \epsilon_T^*)|\bar{q}_2\beta(z)q_1\alpha(0)|0 \rangle = \frac{1}{\sqrt{2N_c}} \int_0^1 dx \ e^{ixp_\mu} \left[ m_V \psi_T^* \phi_V^s(x) + \psi_T^* P \phi_V^s(x) + m_V i \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon^{\nu\rho}_{\alpha\beta} n^\sigma \phi_V^s(x) \right], \]

where \( n(v) \) is the unit vector having the same (opposite) direction with the moving of the vector meson and \( x \) is the momentum fraction of \( q_2 \) quark. The distribution amplitudes of the axial-vectors have the same format as those of the vectors except the factor \( i \gamma_5 \) from the left hand:

\[ \langle A(P, \epsilon_L^*)|\bar{q}_2\beta(z)q_1\alpha(0)|0 \rangle = \frac{i \gamma_5}{\sqrt{2N_c}} \int_0^1 dx \ e^{ixp_\mu}[m_A \psi_L^* \phi_A(x) + \psi_L^* P \phi_A(x) + m_A \phi_A^s(x)]_{\alpha\beta}, \]

\[ \langle A(P, \epsilon_T^*)|\bar{q}_2\beta(z)q_1\alpha(0)|0 \rangle = \frac{i \gamma_5}{\sqrt{2N_c}} \int_0^1 dx \ e^{ixp_\mu} \left[ m_A \psi_T^* \phi_A^s(x) + \psi_T^* P \phi_A^s(x) + m_A i \epsilon^{\mu\nu\rho\sigma} \gamma_5 \gamma^\mu \epsilon^{\nu\rho}_{\alpha\beta} n^\sigma \phi_A^s(x) \right]. \]
TABLE I: Decay constants and Gegenbauer moments for each meson (in MeV). The values are taken at $\mu = 1$ GeV.

| $f_{K^-}$ | $f_{K^+}^T$ | $f_{\phi}$ | $f_{\phi}^T$ |
|----------|-------------|------------|-------------|
| $209 \pm 2$ | $165 \pm 9$ | $231 \pm 4$ | $186 \pm 9$ |
| $f_K$ | $f_\pi$ | $f_{a_1}$ | $f_{b_1}^T$ |
| $160$ | $130$ | $238 \pm 10$ | $-180 \pm 8$ |
| $f_\rho$ | $f_\rho^T$ | $f_\omega$ | $f_\omega^T$ |
| $209 \pm 2$ | $165 \pm 9$ | $195 \pm 3$ | $151 \pm 9$ |
| $a_{1K}$ | $a_{1\pi}$ | $a_{2K}$ | $a_{2\pi}$ |
| $0.17$ | $0$ | $0.2$ | $0.44$ |
| $a_1^V(K^*)$ | $a_1^V(K^*)$ | $a_2^V(K^*)$ | $a_2^V(K^*)$ |
| $0.03 \pm 0.02$ | $0.04 \pm 0.03$ | $0.11 \pm 0.09$ | $0.10 \pm 0.08$ |
| $a_2^V(\rho, \omega)$ | $a_2^V(\rho, \omega)$ | $a_2^V(\phi)$ | $a_2^V(\phi)$ |
| $0.15 \pm 0.07$ | $0.14 \pm 0.06$ | $0.18 \pm 0.08$ | $0.14 \pm 0.07$ |
| $a_2^V(a_1(1260))$ | $a_2^V(a_1(1260))$ | $a_2^V(b_1(1235))$ | $a_2^V(b_1(1235))$ |
| $-0.02 \pm 0.02$ | $-1.04 \pm 0.34$ | $-1.95 \pm 0.35$ | $0.03 \pm 0.19$ |

As for the upper twist-2 and twist-3 distribution functions of the final state mesons, $\phi_{V(A)}$, $\phi_{V(A)}^T$, $\phi_{V(A)}^\perp$, $\phi_{V(A)}^\parallel$, $\phi_{V(A)}^\perp$, and $\phi_{V(A)}^\parallel$ can be calculated by using the light-cone QCD sum rule. We list the distribution functions of the vector ($V$) mesons, namely $\rho(\omega, \phi)$, as follows

$$
\left\{
\begin{array}{ll}
\phi_V(x) &= \frac{f_V}{2\sqrt{2}N_c} \phi_{\parallel}(x), \phi_V^T(x) = \frac{f_V^T}{2\sqrt{2}N_c} \phi_{\perp}(x), \\
\phi_{V}^\perp(x) &= \frac{f_{\perp}}{2\sqrt{2}N_c} h_{(t)}(x), \phi_{V}^\parallel(x) = \frac{f_{\parallel}}{2\sqrt{4N_c}} N_c \frac{d}{dx} h_{(s)}(x), \\
\phi_{V}^T(x) &= \frac{f_{V}^T}{2\sqrt{2}N_c} g_{(t)}(x), \phi_{V}^\perp(x) = \frac{f_{\perp}^T}{2\sqrt{4N_c}} N_c \frac{d}{dx} g_{(s)}(x).
\end{array}
\right.
$$

(11)

The axial-vector ($A$) mesons, here $a_1$ and $b_1$, can be obtained by replacing each $\phi_V$ with $\phi_A$, by replacing $f_V^T(f_V)$ with $f$ in Eq. (11). Here we use $f$ to present both longitudinally and transversely polarized mesons $a_1(b_1)$ by assuming $f_{a_1}^T = f_{a_1} = f$ for $a_1$ and $f_{b_1} = f_{b_1}^T = f$ for $b_1$. In Eq. (11), the twist-2 distribution functions are in the first line and can be expanded as

$$
\phi_{\parallel, \perp} = 6x(1-x) \left[ 1 + a_2^{\parallel, \perp} \frac{3}{2} (5t^2 - 1) \right], \quad \text{for } V \text{ mesons};
$$

(12)

$$
\phi_{\parallel, \perp} = 6x(1-x) \left[ a_0^{\parallel, \perp} + 3a_1^{\parallel, \perp} t + a_2^{\parallel, \perp} \frac{3}{2} (5t^2 - 1) \right], \quad \text{for } A \text{ mesons},
$$

(13)

where the zeroth Gegenbauer moments $a_0^A(a_1) = a_0^\parallel(b_1) = 0$ and $a_0^A(a_1) = a_0^\perp(b_1) = 1$.

As for twist-3 LCDAs, we use the asymptotic forms for $V$ mesons:

$$
\begin{align*}
\begin{align*}
&h_{(t)}^\parallel(x) = 3t^2, h_{(s)}^\parallel(x) = 6x(1-x), \\
g_{(t)}^\perp(x) = 6x(1-x), g_{(s)}^\perp(x) = \frac{3}{4}(1 + t^2).
\end{align*}
\end{align*}
$$

(14)
And we use the following forms for $A$ mesons:

\[ h_{\parallel}^{(t)}(x) = 3a_{\parallel}^0t^2 + \frac{3}{2}a_{\parallel}^1t(3t^2 - 1), \]
\[ h_{\parallel}^{(a)}(x) = 6x(1 - x)(a_{\parallel}^0 + a_{\parallel}^1t), \]
\[ g_{\perp}^{(a)}(x) = 6x(1 - x)(a_{\parallel}^0 + a_{\parallel}^1t), \]
\[ g_{\perp}^{(v)}(x) = \frac{3}{4}a_{\parallel}^0(1 + t^2) + \frac{3}{2}a_{\parallel}^1t^3. \]  

(15)

In Eqs. (4)-(8) and Eqs. (12)-(15), the function $t = 2x - 1$. The decays constants and the Gegenbauer moments $a_{\parallel,\perp}^n$ for each meson are quoted the numerical results [20–25] and listed in Table I.

III. THE PERTURBATIVE QCD CALCULATION

The PQCD approach is an effective theory to handle hadronic $B_s$ decays. Because it takes into account the transverse momentum of the valence quarks in the hadrons, one will encounter double logarithm divergences when the soft and the collinear momenta overlap. Fortunately, these large double logarithm can be re-summed into the Sudakov factor [26]. There are also another type of double logarithms which arise from the loop corrections to the weak decay vertex. These double logarithms can also be re-summed and resulted in the threshold factor [27]. This factor decreases faster than any other power of the momentum fraction in the threshold region, which removes the endpoint singularity. This factor is often parameterized into a simple form which is independent on channels, twists and flavors [28]. Certainly, when the higher order diagrams only suffer from soft or collinear infrared divergence, it is ease to cure by using the eikonal approximation [29]. Controlling these kinds of divergences reasonably makes the PQCD approach more self-consistent.

Here we take the decay $\bar{B}_s^0 \rightarrow a_1^0K^0$ as an example, whose all of the single hard gluon exchange diagrams are shown in Figure 1. These diagrams contain all of the leading order contributions to the decay $\bar{B}_s^0 \rightarrow a_1^0K^0$ in the PQCD approach. Diagrams 1(a) and 1(b) are called factorizable emission diagrams, where $a_1(1250)$ is at emitted position, the corresponding amplitude is presented as $F_{aK}$. In the PQCD approach, the form factor can be extracted from this amplitude. If $a_1(1250)$ is replaced with $b_1(1235)$, the amplitude contributed by the $(V-A)(V\pm A)$ operators would be zero due to the vanishing decay constant $f_{b_1}$. Diagrams 1(c) and 1(d) are called nonfactorizable emission diagrams, the corresponding amplitude is represented as $M_{aK}$. For the decay $\bar{B}_s^0 \rightarrow a_1^0K^0$, $F_{ak}$ included the color suppressed Wilson coefficient $C_2/3 + C_1$ gives the dominated contributions, while for the decay $\bar{B}_s^0 \rightarrow b_1^0K^0$, $M_{ak}$ included the color allowed Wilson coefficient $C_2$ gives the dominated contribution. Diagrams 1(e), 1(f) and 1(g), 1(h) are called nonfactorizable and factorizable annihilation diagrams, respectively, and the corresponding amplitudes are written as $M_{ak}$ and $F_{ak}$. For the decays $\bar{B}_s^0 \rightarrow a_1(b_1)\pi$, only these annihilation type amplitudes can contribute to the final results. If the meson $K^0$ is replaced with a vector meson $K^*(\rho, \omega, \phi)$, the amplitudes will become complicated, for both longitudinal and transverse polarizations can contribute to the decay width. So we can get three kinds of polarization amplitudes $M_{L}$ (longitudinal) and $M_{N,T}$ (transverse) by calculating these diagrams. Because of the aforementioned distribution amplitudes of the axial-vectors having the same format as those of the vectors except a factor, so the formulas of here
FIG. 1: Diagrams contributing to the decay $\bar{B}_s^0 \rightarrow a_1^0 K^0$.

considered $\bar{B}_s^0 \rightarrow a_1(b_1)V$ decays can be obtained from the ones of $\bar{B}_s^0 \rightarrow VV$ decays by some replacements.

IV. NUMERICAL RESULTS AND DISCUSSIONS

We use the following input parameters in the numerical calculations [30, 31]:

\[
\begin{align*}
    f_{B_s} & = 230 MeV, \quad M_{B_s} = 5.37 GeV, \quad M_W = 80.41 GeV, \\
    \tau_{B_s} & = 1.472 \times 10^{-12} s, \quad \alpha = 91.0^\circ, \quad \gamma = 67.2^\circ, \\
    |V_{td}| & = 8.58 \times 10^{-3}, \quad |V_{ts}| = 0.03996, \quad |V_{tb}| = 0.999, \\
    |V_{ud}| & = 0.97425, \quad |V_{us}| = 0.22539, \quad |V_{ub}| = 3.54 \times 10^{-3}.
\end{align*}
\]

In the $B_s$-rest frame, the decay rates of $\bar{B}_s \rightarrow a_1(b_1)V$, where $V$ represents $K^\ast, \rho, \omega, \phi$, can be written as

\[
\Gamma = \frac{G_F^2 (1 - r_{a_1(b_1)}^2)}{32\pi M_B} \sum_{\sigma=L,N,T} M^\sigma_{a_1(b_1)} M^\sigma, \tag{20}
\]

where $M^\sigma$ is the total decay amplitude of each considered decay. The subscript $\sigma$ is the helicity states of the two final mesons with one longitudinal component and two
transverse ones. The decay amplitude can be decomposed into three scalar amplitudes $a, b, c$ according to

$$\mathcal{M} = \epsilon_{2\alpha}^{*}(\sigma)\epsilon_{3\beta}^{*}(\sigma) \left[ a g^{\mu\nu} + \frac{b}{M_{2}M_{3}} P_{B}^{\mu} P_{B}^{\nu} + i \frac{c}{M_{2}M_{3}} \epsilon^{\mu\alpha\beta} P_{2\alpha} P_{3\beta} \right]$$

$$= \mathcal{M}_{L} + \mathcal{M}_{N} \epsilon_{2}(\sigma = T) \cdot \epsilon_{3}^{*}(\sigma = T) + i \frac{\mathcal{M}_{T}}{M_{B}^{2}} \epsilon^{\alpha\beta\gamma} \epsilon_{2\alpha}^{*}(\sigma) \epsilon_{3\beta}^{*}(\sigma) P_{2\gamma} P_{3\rho},$$

(21)

where $M_{2}$ and $M_{3}$ are the masses of the two final mesons $a_{1}(b_{1})$ and $K^{*}(\rho, \omega, \phi)$, respectively. The amplitudes $\mathcal{M}_{L}, \mathcal{M}_{N}, \mathcal{M}_{T}$ can be expressed as

$$\mathcal{M}_{L} = a \epsilon_{2}^{*}(L) \cdot \epsilon_{3}^{*}(L) + \frac{b}{M_{2}M_{3}} \epsilon_{2}^{*}(L) \cdot P_{3} \epsilon_{3}^{*}(L) \cdot P_{2},$$

$$\mathcal{M}_{N} = a, \quad \mathcal{M}_{T} = \frac{M_{B}^{2}}{M_{2}M_{3}} c.$$  

(22)

We can use the amplitudes with different Lorentz structures to define the helicity amplitudes, one longitudinal amplitudes $H_{0}$ and two transverse amplitudes $H_{\pm}$:

$$H_{0} = M_{B}^{2} \mathcal{M}_{L}, \quad H_{\pm} = M_{B}^{2} \mathcal{M}_{N} \mp M_{2}M_{3} \sqrt{r^{2} - 1} \mathcal{M}_{T},$$

(23)

where the ratio $r = P_{2} \cdot P_{3}/(M_{2}M_{3})$. After the helicity summation, we can get the relation

$$\sum_{\sigma = L, N, T} \mathcal{M}_{\sigma}^{*} \mathcal{M}_{\sigma} = |\mathcal{M}_{L}|^{2} + 2 (|\mathcal{M}_{N}|^{2} + |\mathcal{M}_{T}|^{2}) = |H_{0}|^{2} + |H_{+}|^{2} + |H_{-}|^{2}.$$  

(24)

The matrix elements $M_{j}$ of the operators in the weak Hamiltonian can be calculated by using PQCD approach, which are written as

$$M_{j} = V_{ub} V_{ud(s)}^{*} T_{j} - V_{tb} V_{td(s)}^{*} P_{j}$$

$$= V_{ub} V_{ud(s)}^{*} T_{j} (1 + z_{j} e^{i(\alpha(\gamma) + \delta_{j})}),$$

(25)

where $j = L, N, T$ and $\alpha$ and $\gamma$ are the Cabibbo-Kobayashi-Maskawa weak phase angles, defined via $\alpha = arg[-V_{ub}/V_{ud}]$ and $\gamma = arg[-V_{tb}/V_{td}]$, respectively. $\delta_{j}$ is the relative strong phase between the tree and the penguin amplitudes, which are denoted as "$T_{j}$" and "$P_{j}$", respectively. The term $z_{j}$ describes the ratio of penguin to tree contributions and is defined as

$$z_{j} = \left| \frac{V_{tb} V_{td(s)}^{*}}{V_{ub} V_{ud(s)}} \right| \left| \frac{P_{j}}{T_{j}} \right|.$$  

(26)

In the same way, it is easy to write decay amplitude $\overline{\mathcal{M}}_{j}$ for the corresponding conjugated decay mode:

$$\overline{\mathcal{M}}_{j} = V_{ub}^{*} V_{ud(s)} T_{j} - V_{tb}^{*} V_{td(s)} P_{j}$$

$$= V_{ub}^{*} V_{ud(s)} T_{j} (1 + z_{j} e^{i(-\alpha(\gamma) + \delta_{j})}),$$

(27)
So the CP-averaged branching ratio for each considered decay is defined as

\[ B = \left( |\mathcal{M}_j|^2 + |\overline{\mathcal{M}}_j|^2 \right)/2 = |V_{ub}V_{ub}^*|^2 \left[ T^2_L(1 + 2z_L \cos \alpha(\gamma) \cos \delta_L + z_L^2) \right. \]

\[ + 2 \sum_{j=N,T} T^2_j(1 + 2z_j \cos \alpha(\gamma) \cos \delta_j + z_j^2) \right]. \quad (28) \]

Like the decays $\bar{B}^0_s \to VV$, there are also 3 types of helicity amplitudes, so corresponding to 3 types of $z_j$ and $\delta_j$, respectively. Compared with the decays $\bar{B}^0_s \to a_1(b_1) V$, the calculation formula for the branching ratios of other considered decay modes $\bar{B}^0_s \to a_1(b_1) P$ are simpler, for only the longitudinal polarized component of the axial-vector combining with the distribution amplitudes of the pseudo-scalar meson can contribute to the final branching ratio.

Using the input parameters and the wave functions as specified in this section and Sec.II, it is easy to get the branching ratios for the considered decays which are listed in Table II, where the first two errors are the $B_s$ wave function shape parameter $\omega_b = 0.5 \pm 0.05$ GeV and the $B_s$ meson decay constant $f_{B_s} = 0.23 \pm 0.02$ GeV, respectively. The third error is induced by the hard scale-dependent varying from $\Lambda_Q^{(5)} = 0.25 \pm 0.05$ GeV. The last error is from threshold resummation parameter $c$, varying from 0.3 to 0.4. The dominant topologies contributing to these decays are also indicated through the symbols $T$(tree), $P$(penguin), $P_{EW}$(electroweak penguins), $C$(color-suppressed tree) and $ann$ (annihilation).

**A. $\bar{B}^0_s \to a_1(b_1) K(K^*)$**

The decays $\bar{B}^0_s \to a_1^-(b_1^+) K^+(K^{*+})$ have the contributions from the factorization emission diagrams with a large Wilson coefficient $C_2 + C_1/3$ (order of 1), so they have the largest branching ratios and arrive at $10^{-5}$ order. While for the decays $\bar{B}^0_s \to a_1^0 K^{0}(K^{*0})$, the Wilson coefficient is $C_1 + C_2/3$ in tree level and color suppressed, so their branching ratios are small and fall in the order of $10^{-7}$ $\sim$ $10^{-8}$. Although the decay $\bar{B}^0_s \to a_1^0 K^0$ is tree dominated, the contributions from tree operators between the factorization and nonfactorization emission diagrams cancel each other mostly, which induces its tree amplitudes to have a very small real part. It does not happen in the channel $\bar{B}^0_s \to a_1^0 K^{*0}$. At the same time, there exist three polarization states for the final mesons and the transverse polarizations are about 30%. So the decay mode $a_1^0 K^{*0}$ has a larger branching ratio compared with the mode $a_1^0 K^0$. For the decay $\bar{B}^0_s \to b_1^0 K^0$, the amplitude of the nonfactorization emission diagrams $M_{ek}^{T}$ ($T$ denotes the contribution from tree operators) including the large Wilson coefficient $C_2$ receives a larger value, which is about 5 times the decay $a_1^0 K^0$. Furthermore, because of the vanishing decay constant $f_{b_1}$, the amplitude $F_{ek}$ becomes zero for the decay $b_1^0 K^0$, while which has large value but the opposite sign with amplitude $M_{ek}$ for the decay $a_1^0 K^0$. So one can find that there is much larger contribution from the tree operator for the decay $b_1^0 K^0$ than that for the decay $a_1^0 K^0$. The decay $\bar{B}^0_s \to b_1^0 K^{*0}$ has large branching ratio, which is also because of the large contribution from the nonfactorizable emission diagrams.
TABLE II: Branching ratios (in units of $10^{-6}$) for the decays $\bar{B}_s^0 \to a_1(b_1)\bar{K}(\pi, \eta, \eta')$ and $\bar{B}_s^0 \to a_1(b_1)K^* (\rho, \omega, \phi)$. In our results, the errors for these entries correspond to the uncertainties from the $B_s$ meson wave function shape parameter $\omega_B$, the $B_s$ meson decay constant $f_{B_s}$, the QCD scale $\Lambda_{QCD}^{(5)}$ and the threshold resummation parameter $c$, respectively.

| Class | $\overline{B}_s^0 \to a_1^0 K^0$ | Br($10^{-6}$) | Class | $\overline{B}_s^0 \to a_1^0 K^{*0}$ | Br($10^{-6}$) |
|-------|---------------------------------|----------------|-------|---------------------------------|----------------|
| $\bar{B}_s^0 \to a_1^{-1} K^+$ | 21.4$^{+8.1}_{-5.5}$ | $0.081^{+0.016+0.005+0.033}{_{-0.010-0.005-0.032}}$ | $\bar{B}_s^0 \to a_1^{-1} \bar{K}^+$ | 29.4$^{+10.3+0.1+0.6}{_{-7.2-1.1-0.8}}$ |
| $\bar{B}_s^0 \to a_1^{-1} \pi^+$ | ann | $0.012^{+0.004+0.000+0.003}{_{-0.008-0.001-0.003}}$ | $\bar{B}_s^0 \to a_1^{-1} \rho^+$ | ann | $0.0049^{+0.0003+0.0003+0.0005}{_{-0.0003-0.0004-0.0002}}$ |
| $\bar{B}_s^0 \to a_1^{+1} \pi^-$ | ann | $0.030^{+0.009+0.000+0.010}{_{-0.008-0.001-0.003}}$ | $\bar{B}_s^0 \to a_1^{+1} \omega$ | ann | $0.33^{+0.13+0.00+0.02}{_{-0.08-0.00-0.03}}$ |
| $\bar{B}_s^0 \to a_1^{+1} K^0$ | T | $2.9^{+0.5+0.1+0.1+0.1}{_{-0.4-0.0-0.3-0}}$ | $\bar{B}_s^0 \to a_1^{+1} K^{*0}$ | T | $3.5^{+0.6+0.1+0.6+0.1}{_{-0.6-0.0-0.3-0}}$ |
| $\bar{B}_s^0 \to b_1^{-1} K^+$ | C | $1.3^{+0.1+0.0+0.1+0.0}{_{-0.2-0.1-0.3-0}}$ | $\bar{B}_s^0 \to b_1^{-1} \bar{K}^+$ | C | $2.0^{+0.2+0.1+0.2+0.3}{_{-0.2-0.1-0.3-0}}$ |
| $\bar{B}_s^0 \to b_1^{-1} \pi^+$ | ann | $0.079^{+0.013+0.001+0.006}{_{-0.013-0.000-0.004}}$ | $\bar{B}_s^0 \to b_1^{-1} \rho^+$ | ann | $0.88^{+0.06+0.01+0.19+0.05}{_{-0.08-0.02-0.18-0.05}}$ |
| $\bar{B}_s^0 \to b_1^{-1} \pi^-$ | ann | $0.17^{+0.02+0.002+0.002}{_{-0.02-0.000-0.000}}$ | $\bar{B}_s^0 \to b_1^{-1} \omega$ | ann | $1.1^{+0.1+0.0+0.2}{_{-0.1-0.0-0.3}}$ |
| $\bar{B}_s^0 \to b_1^{01} \pi^0$ | ann | $0.085^{+0.025+0.000+0.002}{_{-0.017-0.000-0.003}}$ | $\bar{B}_s^0 \to b_1^{01} \rho^0$ | ann | $0.95^{+0.04+0.01+0.25+0.03}{_{-0.06-0.01-0.24-0.03}}$ |
| $\bar{B}_s^0 \to b_1^{01} \eta$ | $P_{EW}$ | $0.13^{+0.05+0.001+0.001}{_{-0.05-0.000-0.000}}$ | $\bar{B}_s^0 \to b_1^{01} \omega$ | $P_{EW}$ | $0.011^{+0.001+0.000+0.002}{_{-0.001-0.000-0.000}}$ |
| $\bar{B}_s^0 \to b_1^{01} \phi$ | $P_{EW}$ | $0.32^{+0.09+0.002+0.002}{_{-0.04-0.000-0.000}}$ | $\bar{B}_s^0 \to b_1^{01} \phi$ | $P_{EW}$ | $0.21^{+0.04+0.000+0.003}{_{-0.03-0.000-0.000}}$ |

B. $\bar{B}_s^0 \to a_1(b_1)\pi(\rho, \omega)$

These channels belong to the annihilation type decays, contributed by the $W$–annihilation and $W$–exchange diagrams. The decays $\bar{B}_s^0 \to a_1(b_1)\pi(\rho)$ are sensitive to the wave functions of the final states. If the final mesons are $\pi$ and $a_1$, the branching ratios can arrive at $10^{-6}$ order, while for the $\pi$ and $b_1$ final states, the branching ratios become $10^{-7}$ order even smaller. In a word, $\mathcal{B}(\bar{B}_s^0 \to a_1\pi) > \mathcal{B}(\bar{B}_s^0 \to b_1\pi)$. The condition is contrary for the decay modes $a_1(b_1)\rho$. The branching ratios of decays $\bar{B}_s^0 \to \rho^+a_1^0, \rho a_1^0, \rho^-a_1^0$ are very near each other. There exists the similar case with the decays $\bar{B}_s^0 \to \rho^+\pi^-, \rho^0\pi^0, \rho^-\pi^+$, whose branching ratios are predicted as $(2.2, 2.3, 2.4) \times 10^{-7}$ [32], respectively. We also show the Cabibbo-Kobayashi-Maskawa angle $\gamma$ dependence of the branching ratios of decays $\bar{B}_s^0 \to a_1(b_1)\pi(\rho)$ in Fig.2. It is easy to see that the branching ratio for the decay with two neutral mesons in the final state lies between those of other two decays in most range of $0 < \gamma < 180^\circ$.

As for the other two annihilation type decays $\bar{B}_s^0 \to a_1^{0}\omega, b_1^{0}\omega$, whose branching ratios are in the order of $10^{-8} \sim 10^{-9}$. It is easy to see that this kind decay is sensitive to the quark structure of the final mesons. Compared with the decays $\bar{B}_s^0 \to a_1^{0}(b_1^{0})\rho^0$, the difference is mainly from the signs of $\bar{d}\bar{d}$ component in the mesons $\omega$ and $\rho^0$, which induces different interference effects between the amplitudes from the penguin operators: constructive for the decays $a_1^{0}(b_1^{0})\rho^0$, destructive for the decays $a_1^{0}(b_1^{0})\omega$. From our calculations, we find that the penguin amplitude for the decay $a_1^{0}(b_1^{0})\rho^0$ is about 20.4(48.2) times
FIG. 2: The dependence of the branching ratios on the Cabibbo-Kobayashi-Maskawa angle $\gamma$. In these panels, the solid lines are for the decays $\bar{B}_s^0 \to a_1^0(b_1^0)\pi^0, a_1^0(b_1^0)\rho^0$, dotted lines for $\bar{B}_s^0 \to a_1^-(b_1^-)\pi^+, a_1^-(b_1^-)\rho^+$, dashed lines for $\bar{B}_s^0 \to a_1^+(b_1^+)\pi^-, a_1^+(b_1^+)\rho^-$. of that for the decay $a_1^0(b_1^0)\omega$.

C. $\bar{B}_s^0 \to a_1(b_1)\eta^{(s)}$

The main contributions to these four decays are from the electro-weak penguin operators. Although the contributions from the tree operators have a prominent increase for the decays $\bar{B}_s^0 \to b_1^0\eta^{(s)}$ compared with those for the decays $\bar{B}_s^0 \to a_1^0\eta^{(s)}$. The former are about 5(7) times larger than the later. For the tree operator contributions are the Cabibbo-Kobayashi-Maskawa suppressed by a factor 50, so the increased tree operator contributions for the decays $\bar{B}_s^0 \to b_1^0\eta^{(s)}$ bring a slight increase to the branching ratios.

We also checked the sensitivity to the values on the Gegenbauer moments for all the considered decays. If one takes smaller Gegenbauer moments, such as $a_1^K = 0.05 \pm 0.02$ \cite{33}, $0.10 \pm 0.12$ \cite{34}, $a_2^{\pi K} = 0.115$ \cite{35}, the branching ratios have a few percent change for most of decays $\bar{B}_s^0 \to a_1(b_1)\pi(K)$, more than 10 percent change for only very few
channels. So we considered that the uncertainties caused by the Gegenbauer moments are small and can be neglected. But it is not the case for the decays $B_s^0 \to a_1(b_1)\eta^{(')}$. If one takes the newer Gegenbauer moments as given in Ref.\cite{35}:

$$a_2^s = 0.115, a_4^s = -0.015,$$

(29)

The branching ratios will have a prominent change,

$$\mathcal{B}(B_s^0 \to a_1^0\eta) = (0.97^{+0.33+0.01+0.01+0.23}_{-0.34-0.02-0.17-0.23}) \times 10^{-7},$$

(30)

$$\mathcal{B}(B_s^0 \to a_1^0\eta') = (2.1^{+0.7+0.0+0.4+0.8}_{-0.5-0.0-0.2-0.8}) \times 10^{-7},$$

(31)

$$\mathcal{B}(B_s^0 \to b_1^0\eta) = (0.21^{+0.04+0.03+0.05+0.02}_{-0.05-0.05-0.05-0.11}) \times 10^{-7},$$

(32)

$$\mathcal{B}(B_s^0 \to b_1^0\eta') = (0.75^{+0.04+0.04+0.16+0.06}_{-0.17-0.16-0.16-0.35}) \times 10^{-7},$$

(33)

where the errors come from the $B_s$ meson wave function shape parameter $\omega_B = 0.5 \pm 0.05$ GeV, the $B_s$ meson decay constant $f_{B_s} = 0.23 \pm 0.02$ GeV, the QCD scale $\Lambda^{(5)}_{QCD} = 0.25 \pm 0.05$ GeV and threshold resummation parameter $\xi$ varying from 0.3 to 0.4, respectively. Especially for the decays $B_s^0 \to a_1^0\eta^{(')}$, their branching ratios are sensitive to Gegenbauer moments and increase to $7 \sim 8$ times by using the newer Gegenbauer moments. Certainly, the increases of the branching ratios for decays $B_s^0 \to b_1^0\eta^{(')}$ are not so large. It is need to clarify which Gegenbauer moments are more reasonable.

D. $B_s^0 \to a_1(b_1)\phi$

These two decays are dominated by the electro-weak(EW) penguin operators. Though their branching ratios are small, these two decays are interesting to invest the effect from the electro-weak penguins, where there might exits new physics \cite{36}. The presence of a new physics contribution from EW can enhance the branching ratios of the decays $B_s^0 \to \pi(\rho)\phi$, which are used to improve the $B \to \pi K$ ”puzzle” \cite{37}. If here considered two decays have such effect, it is deserve more research attention.

V. POLARIZATION FRACTIONS OF THE DECAYS $B_s^0 \to a_1(b_1)V$

For the decays $B_s^0 \to a_1(b_1)V$, another equivalent set of helicity amplitudes are often used, that is

$$A_0 = -M_B^2 M_L,$$

$$A_{\parallel} = \sqrt{2} M_B^2 M_N,$$

$$A_{\perp} = M_2 M_3 \sqrt{2(r^2-1)} M_T.$$  (34)

Using this set of helicity amplitudes, we can define three polarization fractions $f_{0,\parallel,\perp}$:

$$f_{0,\parallel,\perp} = \frac{|A_0,\parallel,\perp|^2}{|A_0|^2 + |A_{\parallel}|^2 + |A_{\perp}|^2}.$$  (35)

The formalism of the wave function has great influence to the polarization fractions for some decays. In Ref.\cite{38}, the author suggested that taking the asymptotic models for the
TABLE III: Longitudinal polarization fraction ($f_L$) and two transverse polarization fractions ($f_{||}, f_{\perp}$) for the decays $B^0_s \to a_1(b_1) V$. In our results, the uncertainties of $f_L, f_{||}, f_{\perp}$ come from the $B_s$ meson wave function shape parameter $\omega_b$, the $B_s$ meson decay constant $f_{B_s}$, the QCD scale $\Lambda_{QCD}^{(5)}$ and threshold resummation parameter $c$, respectively.

| $B^0_s \to a_1^0 K^{*0}$ | $B^0_s \to b_1^0 K^{*0}$ | $B^0_s \to b_1^0 K^{*+}$ | $B^0_s \to b_1^0 K^{*0}$ |
|--------------------------|--------------------------|--------------------------|--------------------------|
| $f_L(\%)$ | $f_{||}(\%)$ | $f_{\perp}(\%)$ | $f_L(\%)$ |
| $68.9_{-6.4}^{+6.1}$$f_{||}(9.3_9^{+4.1}_{4.2})$ | $15.1_{-3.0}^{+3.1}$$f_{||}(9.3_9^{+4.1}_{4.2})$ | $16.0_{-3.1}^{+3.4}$$f_{||}(9.3_9^{+4.1}_{4.2})$ |
| $f_{||}(9.3_9^{+4.1}_{4.2})$ | $f_{\perp}(9.3_9^{+4.1}_{4.2})$ | $f_{\perp}(9.3_9^{+4.1}_{4.2})$ | $f_{\perp}(9.3_9^{+4.1}_{4.2})$ |

$K^*$ meson distribution amplitudes instead of its traditional formalism leads to a smaller $B \to K^*$ form factor ($A_0 \sim 0.3$). The smaller form factor responds to the smaller longitudinal polarization fraction. Another result is that the strengthened penguin annihilation and nonfactorizable contributions further bring it down. In the decays $B^0_s \to a_1(b_1)K^*$, we also take the asymptotic models for the $K^*$ meson wave functions and only find the decay mode $a_1^0K^{*0}$ with smaller longitudinal polarization fraction about 70%. If we neglect penguin annihilation contribution in the decay $B^0_s \to a_1^0K^{*0}$, and find that the branching ratio changes from $6.9 \times 10^{-7}$ to $5.5 \times 10^{-7}$, while the longitudinal polarization receives a larger increase and arrives at 93.1%. If we neglect nonfactorizable contribution, both the branching ratio and the polarization fractions will become much smaller. Compared with $B^0_s \to a_1^0K^{*0}$ and $B^0_s \to b_1^0K^{*0}$ decays, we argue that the polarization fractions are also connected with the symmetric properties of $a_1$ and $b_1$ distribution amplitudes, which might have a sensitive effect in the penguin annihilation contribution. If one neglects penguin annihilation contribution in the decay $B^0_s \to b_1^0K^{*0}$, the longitude fraction can amount to 95.4% and the branching ratio decreases by 30%. In a word, the contributions from the penguin annihilation diagrams are very sensitive to the final polarization fractions for some decays.

In Table III we list the longitudinal polarization fraction ($f_L$) and the transverse polarization fractions ($f_{||}, f_{\perp}$) for the decays $B^0_s \to a_1(b_1) V$, where the errors come from the $B_s$ meson wave function shape parameter $\omega_b = 0.5 \pm 0.05$ GeV, the $B_s$ meson decay constant $f_{B_s} = 0.23 \pm 0.02$ GeV, the QCD scale $\Lambda_{QCD}^{(5)} = 0.25 \pm 0.05$ GeV and the threshold
TABLE IV: Direct CP-violating asymmetries (in units of %) for the decays $\bar{B}_s^0 \to a_1(b_1)K(\pi, \eta, \eta')$ and $\bar{B}_s^0 \to a_1(b_1)K^*(\rho, \omega, \phi)$ (except $\bar{B}_s^0 \to a_1^0K^{*0}, a_1^0\omega, b_1^0\omega$). In our results, the errors for these entries correspond to the uncertainties from $\omega_b, f_{B_s}$, the QCD scale $\Lambda_{QCD}^{(5)}$ and the threshold resummation parameter $c$, respectively.

| Class | $\text{Br}(10^{-6})$ | Class | $\text{Br}(10^{-6})$ |
|-------|-----------------------|-------|-----------------------|
| $\bar{B}_s^0 \to a_1^0K^0$ | $C$ | $-66.1^{+10.4+3.4+4.2+3.7+3.3}_{-6.5-3.3-5.1-3.7-6.2}$ | $\bar{B}_s^0 \to b_1^0K^0$ | $T$ |
| $\bar{B}_s^0 \to a_1^0K^+$ | $T$ | $-9.7^{+1.4+0.8+0.4+0.7}_{-1.0-0.9-0.2-0.7}$ | $\bar{B}_s^0 \to b_1^0K^+$ | $C$ |
| $\bar{B}_s^0 \to a_1^-\pi^+$ | ann | $20.5^{+1.3+0.3+0.4+0.2}_{-1.3-0.0-0.6-0.2}$ | $\bar{B}_s^0 \to b_1^-\pi^+$ | ann |
| $\bar{B}_s^0 \to a_1^+\pi^−$ | ann | $3.2^{+0.3+0.0+0.6+0.2}_{-0.3-0.1-0.8-0.2}$ | $\bar{B}_s^0 \to b_1^+\pi^−$ | ann |
| $\bar{B}_s^0 \to a_1^0\eta$ | ann | $14.0^{+1.1+0.1+0.2+0.0}_{-1.2-0.0-0.0-0.0}$ | $\bar{B}_s^0 \to b_1^0\eta$ | ann |
| $\bar{B}_s^0 \to a_1^0\eta_P$ | $P_{EW}$ | $-31.3^{+2.8+2.2+2.5+2.1}_{-2.0-1.3-0.4-2.1}$ | $\bar{B}_s^0 \to b_1^0\eta_P$ | $P_{EW}$ |
| $\bar{B}_s^0 \to a_1^0\phi$ | $P_{EW}$ | $-10.2^{+1.4+1.2+2.3+2.1}_{-0.0-1.3-0.4-2.1}$ | $\bar{B}_s^0 \to b_1^0\phi$ | $P_{EW}$ |
| $\bar{B}_s^0 \to a_1^0\eta_P$ | $P_{EW}$ | $-1.9^{+1.4+0.0+1.0+1.0}_{-0.7-0.3-0.4-0.9}$ | $\bar{B}_s^0 \to b_1^0\eta_P$ | $P_{EW}$ |

resummation parameter $c$ varying from 0.3 to 0.4, respectively. Except the decays $\bar{B}_s^0 \to a_1^0K^{*0}, a_1^0\omega, b_1^0\omega$, the longitudinal polarization fractions of other $\bar{B}_s^0 \to a_1(b_1)V$ decays are very large and more than 90%.

VI. DIRECT CP ASYMMETRY

Now we turn to the evaluations of the CP-violating asymmetries in PQCD approach. In view that most of $\bar{B}_s^0 \to a_1(b_1)V$ decays have small transverse polarization fractions and only about few percent. So we can neglect them in our calculations and the expression for the direct CP-violating asymmetries of the decays $\bar{B}_s^0 \to a_1(b_1)V$ (except $\bar{B}_s^0 \to a_1^0K^{*0}, a_1^0\omega, b_1^0\omega$) become simple, which can be got by using Eq. (25) and Eq. (27):

$$A_{CP}^{\text{dir}} = \frac{\mathcal{M}^2 - |\mathcal{M}|^2}{|\mathcal{M}|^2 + |\mathcal{M}|^2} = \frac{2z_L \sin \alpha(\gamma) \sin \delta_L}{(1 + 2z_L \cos \alpha(\gamma) \cos \delta_L + z_L^2)}.$$

(36)

The direct CP-violating asymmetries for the decays $\bar{B}_s^0 \to a_1(b_1)P$ have similar expression. Using the input parameters and the wave functions as specified in this section and Sec. III one can calculate the PQCD predictions (in units of $10^{-2}$) for the direct CP-violating asymmetries of the considered decays, which are listed in Table IV where the errors induced by the uncertainties of $\omega_b = 0.5 \pm 0.05$ GeV, $f_{B_s} = 0.23 \pm 0.02$ GeV, $\Lambda_{QCD}^{(5)} = 0.25 \pm 0.05$ GeV and the threshold resummation parameter $c$ varying from 0.3 to 0.4, respectively. We find the following points:
FIG. 3: The dependence of the direct CP-violating asymmetries on the Cabibbo-Kobayashi-Maskawa angle $\gamma$. In these panels, the solid lines are for the decays $\bar{B}_s^0 \to a_1^0(b_1^0)\pi^0, a_1^0(b_1^0)\rho^0$, dotted lines for $\bar{B}_s^0 \to a_1^-\pi^+, a_1^-\rho^+$, dashed lines for $\bar{B}_s^0 \to a_1^+\pi^-, a_1^+\rho^-$.  

- Like the decay $\bar{B}_s^0 \to \pi^0K^0$, whose direct CP-asymmetry is more than 40% predicted by several methods \cite{32, 39, 40}, the decays $\bar{B}_s^0 \to a_1^0(b_1^0)K^0$ also have large direct CP-asymmetries. Unlike the channel $B_s^0 \to b_1^-K^+$, the decay $B_s^0 \to a_1^-K^+$ has a smaller direct CP-asymmetry. It is because that though there are near penguin amplitudes in these two decays, the tree amplitude of the latter is about 3 times as large as that of the former, and the sine values of their strong phases are close to each other. The direct CP-asymmetries in the decays $\bar{B}_s^0 \to a_1(b_1)K^+$ are small.

- The direct CP-asymmetries of the decays $\bar{B}_s^0 \to b_1^0\eta^{(')}$ are sensitive to taking different Gegenbauer moments for $\eta^{(')}$. If we take the newer Gegenbauer moments given in Eq. (29), their direct CP-asymmetries will change not only in magnitudes but also in signs.

- The decays $B_s^0 \to a_1(b_1)\rho$ except the channel $B_s^0 \to b_1^-\rho^+$ have smaller direct CP-violating asymmetries compared with the decays $B_s^0 \to a_1^-\pi$. The direct CP-
violating asymmetry for the decay $\bar{B}_s^0 \to b^- \rho^+$ is very sensitive to the tree operator contribution from the nonfactorization annihilation diagrams: if we neglect such contribution, its branching ratio can increase 14%, while the direct CP-violating asymmetry becomes only 1.3%. In Fig.3, we show the dependence of the direct CP-violating asymmetries for the decays $\bar{B}_s^0 \to a_1(b_1)\pi(\rho)$ on the Cabibbo-Kobayashi-Maskawa angle $\gamma$.

- There only exist factorization and nonfactorization emission diagrams for the decays $\bar{B}_s^0 \to a_1(b_1)\phi$. The direct CP-violating asymmetries in these two decays are small, because the interactions between tree and penguin contributions are small. From our calculations, we find the ratios of penguin to tree amplitudes for decays $\bar{B}_s^0 \to a_1\phi$ and $\bar{B}_s^0 \to b_1\phi$ are about 0.06 and 0.004, respectively. The strong phases penguin and tree amplitudes are only 0.15 and 0.026 rad, respectively.

- Compared with decays $\bar{B}_s^0 \to a_1(b_1)P$, most of $\bar{B}_s^0 \to a_1(b_1)V$ decays have smaller direct CP-violating asymmetries.

VII. CONCLUSION

In this paper, by using the decay constants and the light-cone distribution amplitudes derived from QCD sum-rule method, we research the decays $\bar{B}_s^0 \to a_1(b_1)P, a_1(b_1)V$ in PQCD approach and find that

- The decays $\bar{B}_s^0 \to a_1^-K^+(K^{*-})$ have the contributions from the factorization emission diagrams with a large Wilson coefficient $C_2 + C_1/3$ (order of 1), so they have the largest branching ratios and arrive at $10^{-5}$ order. While for the decays $\bar{B}_s^0 \to a_0^0K^0(K^{*-})$, the Wilson coefficient is $C_1 + C_2/3$ in tree level and color suppressed, so their branching ratios are small and fall in the order of $10^{-7} \sim 10^{-8}$. For the decays $\bar{B}_s^0 \to b_1K(K^*)$, all of their branching ratios are of order few times $10^{-6}$.

- For the pure annihilation type decays $\bar{B}_s^0 \to a_1(b_1)\rho$ except the decays $\bar{B}_s^0 \to a_1\pi$ having large branching ratios of order few times $10^{-6}$, the most of them have the branching ratios of $10^{-7}$ order. The branching ratios of the decays $\bar{B}_s^0 \to a_1^0(b_1^0)\omega$ are the smallest and fall in the order of $10^{-8} \sim 10^{-9}$.

- The branching ratios and the direct CP-asymmetries of decays $\bar{B}_s^0 \to a_1^0(b_1^0)\eta^{(*)}$ are very sensitive to take different Gegenbauer moments for $\eta^{(*)}$.

- Except for the decays $\bar{B}_s^0 \to a_1^0K^{*0}, a_1^0\omega, b_1^0\omega$, the longitudinal polarization fractions of other $\bar{B}_s^0 \to a_1(b_1)V$ decays are very large and more than 90%.

- Compared with decays $\bar{B}_s^0 \to a_1(b_1)P$, most of $\bar{B}_s^0 \to a_1(b_1)V$ decays have smaller direct CP-violating asymmetries.
Acknowledgment

This work is partly supported by the National Natural Science Foundation of China under Grant No. 11147004, and by Foundation of Henan University of Technology under Grant No. 2009BS038.

[1] B. Aubert, et al., [BABAR Collaboration], Phys. Lett. B 97, 051802 (2006).
[2] K. Abe, et al., [Belle Collaboration], arXiv:0706.3279 [hep-ex].
[3] B. Aubert, et al., [BABAR Collaboration], Phys. Lett. B 98, 181803 (2007).
[4] B. Aubert, et al., [BABAR Collaboration], Phys. Lett. B 100, 051803 (2008).
[5] F. Blanc, invited talk presented at Moriond QCD, La Thuile, Italy, March 17-24, 2007.
[6] J. P. Burke, talk presented at International Europhysics Conference on High Energy Physics, Manchester, England, July 19-25, 2007.
[7] K. Abe, et al., [Belle Collaboration], Phys. Rev. Lett. 87, 161601 (2001).
[8] B. Aubert, et al., [BABAR Collaboration], arXiv:hep-ex/0207085 (2002).
[9] B. Aubert, et al., [BABAR Collaboration], Phys. Rev. D 74, 031104 (2006), arXiv:hep-ex/0605024.
[10] B. Aubert, et al., [BABAR Collaboration], Phys. Rev. D 80, 051101 (2009), arXiv:hep-ex/0907.3485v1.
[11] V. Laporta, G. Nardulli, and T. N. Pham, Phys. Rev. D 74, 054035 (2006).
[12] G. Calderón, J.H. Muñoz and C.E. Vera, Phys. Rev. D 76, 094019 (2007).
[13] C. H. Chen, C. Q. Geng, Y. K. Hsiao, and Z. T. Wei, Phys. Rev. D 72, 054011 (2005).
[14] H. Y. Cheng, K. C. Yang, Phys. Rev. D 76, 114020 (2007).
[15] H. Y. Cheng, K. C. Yang, Phys. Rev. D 78, 094001 (2008).
[16] W. Wang, R. H. Li, C. D. Lu, Phys. Rev. D 78, 074009 (2008).
[17] C.D. Lu and M.Z. Yang, Eur. Phys. J. C 28, 515 (2003).
[18] T. Feldmann, P. Kroll and B. Stech, Phys. Rev. D 58, 114006 (1998); Phys. Lett. B 449, 339 (1999).
[19] Z.J. Xiao, Z.Q. Zhang, X. Liu, L.B. Guo, Phys. Rev. D 78, 114001 (2008).
[20] C. Amsler, et al., [Particle Data Group], Phys. Lett. B 667, 1 (2008).
[21] P. Ball, G. W. Jones and R. Zwicky, Phys. Rev. D 75, 054004 (2007).
[22] P. Ball and R. Zwicky, JHEP 0604, 046 (2006).
[23] P. Ball and G. W. Jones, JHEP 0703, 069 (2007).
[24] K. C. Yang, JHEP 0510, 108 (2005).
[25] K. C. Yang, Nucl. Phys. B 776, 187 (2007).
[26] H. n. Li and B. Tseng, Phys. Rev. D 57, 443 (1998).
[27] H. n. Li, Phys. Rev. D 66, 094010 (2002).
[28] H. n. Li and K. Ukai, Phys. Lett. B 555, 197 (2003).
[29] H. n. Li and H. L. Yu, Phys. Rev. D 53, 2480 (1996).
[30] K. Nakamura, et al., [Particle Data Group], J. Phys. G37, 481 (2010).
[31] CKMfitter Group, [http://ckmfitter.in2p3.fr]
[32] A. Ali, et al., Phys. Rev. D 76, 074018 (2007).
[33] A. Khojamirian, Th. Mannel, M. Melcher, Phys. Rev. D 70, 094002 (2004).
[34] V.M. Braun and A. Lenz, Phys. Rev. D 70, 074020 (2004).
[35] P. Ball and R. Zwicky, Phys. Rev. D 71, 014015 (2005).
[36] A. J. Buras, et al., Nucl. Phys. B 697, 133 (2004).
[37] L. Hofer, D. Scherer, L. Vernazza, Acta Phys. Polon. B3:227-233 (2010).
[38] H. n. Li, Phys. Lett. B 622, 63 (2005).
[39] M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003).
[40] A. Williamson and J. Zupan, Phys. Rev. D 74 014003 (2006); Erratum-ibid. D74, 039901 (2006).