Abstract

Compressing convolutional neural networks (CNNs) by pruning and distillation has received ever-increasing focus in the community. In particular, designing a class-discrimination based approach would be desired as it fits seamlessly with the CNNs training objective. In this paper, we propose class-discriminative compression (CDC), which injects class discrimination in both pruning and distillation to facilitate the CNNs training goal. We first study the effectiveness of a group of discriminant functions for channel pruning, where we include well-known single-variate binary-class statistics like Student’s T-Test in our study via an intuitive generalization. We then propose a novel layer-adaptive hierarchical pruning approach, where we use a coarse class discrimination scheme for early layers and a fine one for later layers. This method naturally accords with the fact that CNNs process coarse semantics in the early layers and extract fine concepts at the later. Moreover, we leverage discriminant component analysis (DCA) to distill knowledge of intermediate representations in a subspace with rich discriminative information, which enhances hidden layers’ linear separability and classification accuracy of the student. Combining pruning and distillation, CDC is evaluated on CIFAR and ILSVRC-2012, where we consistently outperform the state-of-the-art results.

1. Introduction

Convolutional neural networks (CNNs) have become a mainstream machine learning model for various computer vision tasks, such as image classification [51, 15], object detection [9, 47], and semantic segmentation [41, 49]. To gain better recognition performance, a popular approach is to grow deeper and wider models. However, such CNNs require a larger storage space and higher computational cost, making them unsuitable for edge devices like mobile phones and embedded sensors.

Many methods have been proposed for CNN compression. For example: weight quantization [2, 4], tensor low-rank factorization [23, 29], network pruning [14, 13, 61], and knowledge distillation [21, 48]. Among them all, a combination of channel pruning and knowledge distillation is the preferable method to learn smaller dense models, which can easily leverage Basic Linear Algebra Subprograms (BLAS) libraries [31].

While CNNs are fundamentally trained to differentiate objects from different classes, the study of discrimination based network compression is quite limited. Prior class-discriminative pruning works [44, 61, 37, 28, 55] lack effectiveness study for their pruning metrics, where they propose and evaluate their metrics singly without comparing to other well-known discriminant functions, like Maximum Mean Discrepancy [11]. Besides, these works ignore the hierarchical nature of CNNs’ semantic extraction and only use fine class discrimination for both early and later layers, which could be sub-optimal. For knowledge distillation, while the output distillation scheme [21, 57] normally incorporates class discrepancy knowledge, intermediate discriminative distillation has rarely been attempted.

To this end, we propose a novel approach to compress classification CNNs, dubbed class-discriminative compression (CDC), which leverages class discrepancy for channel pruning and knowledge distillation, fitting seamlessly with CNN class-discriminative training.
2. Related Work

Channel Pruning. Channel pruning is a promising approach to enhance network efficiency [56, 54, 19, 28, 16, 12, 36, 3]. Some works leverage norm statistics of weight parameters [31, 39, 19], feature maps reconstruction losses [20, 42], ranks of the feature maps [36], and self-learned importance coefficients [22, 3] to evaluate channel redundancy, without the use of discriminative information.

In line with our work, several discrimination-based pruning methods are proposed [44, 61, 37, 55, 28]. Molchanov et al. [44], Lin et al. [37], and Zhong et al. [55] use Taylor expansion to estimate the accuracy/entropy loss caused by removing a channel. They omit high order terms, making the evaluation less accurate, and require back propagation of class information from the outputs to early layers, where the inter-layer dependency could affect channel saliency measurement. On the contrary, we adopt closed-form metrics to directly evaluate channels’ activations discrepancy without approximation and back-propagation. Zhuang et al. [61] and Kung et al. [28] use an entropy loss and a closed-form discriminant function to directly evaluate channel activations discrepancy, but require time-consuming iterative optimization and heavy matrix operations. In contrast, our metric doesn’t need iterative steps and expensive matrix computation, which is more time efficient.

Moreover, these studies solely focus on a single discriminant loss/metric, whilst we provide an analysis on the effectiveness of a group of closed-form metrics. We also introduce a novel hierarchical pruning paradigm where the class discrimination is measured on different label granularities for early and later layers. Such adaption of label granularity in channel pruning fits seamlessly with the nature of coarse to fine semantic understanding in CNNs.

Knowledge Distillation. The idea of knowledge distillation is pioneered by Hinton et al. [21] to allow a student classifier to mimic the output of its teacher by a soft label entropy loss. Romero et al. [48] propose a popular hint-layer method for intermediate distillation (similar methodology adopted in [1, 32]), which uses gradient-learned weights to linearly transform student’s hidden layers to the same depth as the teacher’s and minimize their norm-based loss, while no classification information is distilled. Moreover, the full space of the layer could contain redundant knowledge for classification, making [48] less effective. In our work, we propose to distill classification information in the subspace of hidden layers by discriminant component analysis (DCA) [27], where we achieve better distillation results. There is also an attention transfer approach for intermediate distillation [58], yet again no supervised class information is distilled. To the best of our knowledge, only Li et al. [33] propose a discriminative intermediate distillation scheme by extracting local patterns in the feature maps. However, this method requires a significant amount of overhead to train the class-wise attention module, which could hinder its usage on large high-performing teacher networks, e.g., ResNet-50. On the contrary, our subspace distillation scheme is more scalable that allows us to use the full-size ResNet-50 as the teacher.

Class-Discriminative Analysis. Our work is also closely related to techniques for class-discriminative analysis, such as high-dimensional class-discriminative functions of Discriminant Information (DI) [28] and Maximum Mean Discrepancy (MMD) [11]. We also include a set of single-variate binary-class discriminant metrics, Student’s T-Test (Ttest) [30], Absolute SNR (AbsSNR) [10], Sym-
metric Divergence (SD) [43], and Fisher Discriminant Ratio (FDR) [46] in our study. These metrics are originally defined to measure the significance of two class’s difference for a univariate dataset and have been adopted for machine learning tasks. For example, SD is used to select discriminative individual features in bioinformatics feature vectors for dimension reduction and efficient classification [43]. However, no prior work has applied them under the context of channel pruning, and we take the first step to generalize them for high-dimensional multi-class channel scoring.

Discriminant component analysis (DCA) [27] also plays a key role in our work. It can be seen as a multi-class linear discriminant analysis (LDA) and a supervised version of principle component analysis (PCA). It finds components of a data matrix that can transform it into a subspace with the most class linear separability. We apply DCA to achieve class-discriminative distillation at intermediate layers.

3. Methodology

3.1. Discriminant Functions

To evaluate the class discrepancy of a channel, we formulate it as $\mathcal{M}(\mathbf{F}, \mathbf{Y})$, where $\mathcal{M}$ is the discriminant metric, $\mathbf{F}$ is the set of feature maps obtained at a channel, and $\mathbf{Y}$ is the label numbering scheme. Although discriminant metrics are all mathematically well-defined, their empirical effectiveness on channel pruning remains understudied. As shown in Fig. 2, the very first thing we want to know is which discriminant metric works best for channel pruning.

We study a group of closed-form metrics including MMD and DI, which can be applied on feature maps directly without back-propagation and iterative optimization. We also generalize four univariate binary-class metrics, T-test, AbsSNR, FDR, and SD, for channel pruning. We use SD [26] as an example to illustrate our generalization method. Let us denote an $n$-sample 2-class single-variate dataset as $\mathcal{D} = \{(x_i, b_i)\}_{i=1}^n$, where $b_i \in \{0, 1\}$ is the binary labeling scheme $\mathcal{B}$. Let $\mathcal{D}^+ = \{x_i \mid (x_i, b_i) \in \mathcal{D}, b_i = 1\}$ and $\mathcal{D}^- = \{x_i \mid (x_i, b_i) \in \mathcal{D}, b_i = 0\}$ denote two partitions of $\mathcal{D}$ based on $\mathcal{B}$. SD of $\mathcal{D}$ is defined as:

$$
\text{SD}(\mathcal{D}, \mathcal{B}) = \frac{1}{2} \left( \frac{\sigma_{D^+}^2}{\mu_{D^+}^2} + \frac{\sigma_{D^-}^2}{\mu_{D^-}^2} \right) + \frac{1}{2} \left( \frac{\mu_{D^+} - \mu_{D^-}}{\sigma_{D^+}^2 + \sigma_{D^-}^2} \right)^2 - 1
$$

(1)

where $\mu_{D^+}$ and $\sigma_{D^+}^2$ are the sample mean and variance of $\mathcal{D}^+$, and $\mu_{D^-}$ and $\sigma_{D^-}^2$ are the statistics for $\mathcal{D}^-$. For an $N$-sample $Y$-class dataset, we denote the feature maps of a CNN channel as $\mathbf{F} = \{(f_i, y_i)\}_{i=1}^N$, where $f_i \in \mathbb{R}^{W \times H}$ denotes the feature map of the $i$-th input image, $y_i \in [1:Y]$ is the $i$-th class label, and $W/H$ are the spatial sizes. We first partition $\mathbf{F}$ as $\mathbf{F}^c$ and $\mathbf{F}^c$ where $\mathbf{F}^c = \{f_i \mid (f_i, y_i) \in \mathbf{F}, y_i = c\}$ and $\mathbf{F}^- = \{f_i \mid (f_i, y_i) \in \mathbf{F}, y_i \neq c\}, \forall c \in [1:Y]$. By this partition, denoted as $\mathcal{B}_c$, we can find the statistics in Eqn. 9 in a two-class manner. Note each $f_i$ in $\mathbf{F}^c$ is a 2D feature map with $W \times H$ activations, and thus there are $|\mathbf{F}^c| \times W \times H$ activations in $\mathbf{F}^c$ in total. We then define two statistics operators $g_{\text{mean}}$ and $g_{\text{var}}$ on $\mathbf{F}^c$, which return the mean and variance over these $|\mathbf{F}^c| \times W \times H$ activations. We thus get $\mu_c = g_{\text{mean}}(\mathbf{F}^c)$ and $\sigma_c^2 = g_{\text{var}}(\mathbf{F}^c)$ for $\mathbf{F}^c$, and their counterparts $\mu_{-c}$ and $\sigma_{-c}^2$. The SD score for $\mathcal{B}_c$ is thus:

$$
\text{SD}(\mathbf{F}, \mathcal{B}_c) = \frac{1}{2} \left( \frac{\sigma_{\mu-c}^2}{\sigma_{\mu-c}^2} + \frac{\sigma_{\sigma-c}^2}{\sigma_{\sigma-c}^2} \right) + \frac{1}{2} \left( \frac{\mu_\mu - \mu_{\mu-c}}{\sigma_{\mu}^2 + \sigma_{\sigma-c}^2} \right)^2 - 1
$$

(2)

SD($\mathbf{F}, \mathcal{B}_c$) captures the discriminativeness of class $c$ relative to the other classes in $\mathbf{F}$. In general, we want to select channels that distinguish all classes well on average. Thus, under the $Y$-class setting, the generalized Symmetric Divergence (G-SD) of $\mathbf{F}$ is formally defined as:

$$
\text{G-SD}(\mathbf{F}, \mathbf{Y}) = \frac{1}{Y} \sum_{c=1}^{Y} \text{SD}(\mathbf{F}, \mathcal{B}_c)
$$

(3)

Such generalization method is applicable to other single-variate binary-class metrics, and incurs no expensive operations (e.g., matrix inversion, SVD), which makes the generalized metrics scalable to large networks and datasets.

3.2. Hierarchical Pruning

In our formulation $\mathcal{M}(\mathbf{F}, \mathbf{Y})$, $\mathbf{F}$ is determined by input images and network’s weights, while $\mathbf{Y}$ can be calibrated for different layers. After finding out the best metric $\mathcal{M}$, we investigate the settings of $\mathbf{Y}$ for more effective discriminative pruning. It is widely recognized that CNN learns coarse semantics (fruit, vehicle) in the early layers while extracting finer class (apple, truck) concepts in the later layers [59]. Inspired by the nature of hierarchical semantic separation in CNN, we propose to adapt the granularity of $\mathbf{Y}$ based on layer positions for class discrepancy measurement as shown in Fig. 3. Specifically, we define a watershed layer $l_{W \times S}$, where we evaluate the channel discrepancy by a coarse class labeling scheme $\mathcal{M}(\mathbf{F}, \mathcal{Y}_l)$ when $l \leq l_{W \times S}$, and use fine label pruning $\mathcal{M}(\mathbf{F}, \mathcal{Y}_l)$ when $l > l_{W \times S}$.

While most image datasets only provide fine class labels $\mathcal{Y}_l \in [1:F]$, the coarser labeling scheme $\mathcal{Y}_l \in [1:C]$ need to be derived on our own. To tackle this issue, we use a pre-trained CNN Net to group similar fine categories into the same coarse category, i.e., to learn a disjoint many-to-one mapping $Q : [1:F] \rightarrow [1:C]$. We investigate two methods.
Clustering on Class Centroids. We randomly sample a held-out set of images from the training set. We feed these images through $Net$ and get the last hidden activations. We calculate the activations’ class centroid for each fine label, denoted as $\mathcal{H} = \{h_1, h_2, \ldots, h_F\}$, and run a K-means clustering on $\mathcal{H}$ with $C$ clusters to get the mapping $Q$.

Clustering on Confusion Matrix. We feed the held-out images through $Net$ to get their predicted labels. Based on the predicted and true labels, we construct a confusion matrix $M \in \mathbb{R}^{F \times F}$, where $M_{i,j}$ denotes the number of images with true label $i$ but predicted as label $j$. Then we run a spectral clustering on $M$ with $C$ clusters which gives us $Q$.

3.3. Intermediate Class Discrepancy Distillation

After we obtain the pruned networks, we retrain them with a combined loss of cross entropy $L_{CE}$ and knowledge distillation to recover their accuracies. In particular, we propose to distill classification information at intermediate layers’ subspaces found by discriminant component analysis (DCA) [27] in Fig. 4. We experimentally compare the DCA-based distillation with the hint layer [48] in Sec. 4.3, and find that distillation at the subspace with rich class information results in better performance.

**DCA.** Let $D = \{(x_i, y_i)\}_{i=1}^{N}$ denote an N-sample Y-class dataset, the within-/between-class scatter matrix $S_W/S_B$:

$$
S_W = \sum_{y=1}^{Y} \sum_{j=1}^{N_y} (x_{y,j} - \bar{x}_y)(x_{y,j} - \bar{x}_y)^T \quad (4)
$$

$$
S_B = \sum_{y=1}^{Y} N_y(\bar{x}_y - \bar{x})(\bar{x}_y - \bar{x})^T \quad (5)
$$

where $N_y$, $x_{y,j}$ are the number of samples and the $j$-th sample of class $y$, $\bar{x}_y$ and $\bar{x}$ are the $y$-class centroid and overall data centroid. Note the center adjusted scatter matrix $\bar{S} = S_W + S_B$. The discriminant components of $D$ are:

$$
W^{DCA} = \arg\max_{W: W^T \bar{S} W = I} tr(W^T \bar{S} W) \quad (6)
$$

As LDA only finds one component that best separate two classes, DCA can be seen as a multi-class version of it where $S_W$ and $S_B$ incorporate information for all classes. Moreover, DCA’s objective is the same as PCA while it includes an extra within-class matrix $S_W$ in its constraint, making DCA a supervised version of PCA. As evidenced in the original paper [27], the DCA subspace is more linearly separable than the one found by PCA.

To learn DCA weights for intermediate layers, we feed the held-out set of images through the network and get its intermediate activation $A \in \mathbb{R}^{N \times C \times H \times W}$, which is reshaped as $A \in \mathbb{R}^{N \times D}$, $D = C \times H \times W$. With a labeling scheme $Y$, we apply Eqn. 6 to get its top $Y$ components, $W^{DCA} \in \mathbb{R}^{D \times Y}$, as $S_B$ has a rank of $Y$.

**Distillation.** We learn DCA for the teacher, $W^{DCA}_T$, at the start of training, and learn the student’s DCA, $W^{DCA}_S$, every $d$ epochs. These weights are not updated in the back-propagation, and the loss is constructed as:

$$
L_{KD}^{Inter} = \sum_{l=1}^{L} ||A_T(l)W^{DCA}_T(l) - A_S(l)W^{DCA}_S(l)||_1 \quad (7)
$$

where $A_T(l), W^{DCA}_T(l)$ denote the activation and DCA weights for layer $l$ of the teacher, and $A_S(l), W^{DCA}_S(l)$ are those for the student. Mathematically speaking, $L_{KD}^{Inter}$ imposes regularization on student’s hidden layers to push the transformed subspace as linearly separable as its teacher.

We adopt the output distillation loss $L_{Out}^{KD}$ in [21], and our training loss for the student is formally defined as:

$$
L = L_{CE} + \lambda L_{KD}^{Inter} + \gamma L_{Out}^{KD} \quad (8)
$$

where $\lambda, \gamma$ are the weights for distillation losses.

4. Experimental Results

4.1. Function Effectiveness Study

We conduct one-shot pruning tests with the pool of metrics in Sec. 3.1 for VGG-16 on CIFAR-10 and ResNet-38 on CIFAR-100, with results shown in Fig. 5. For each metric, we uniformly prune 10%, 20%, 30%, and 40% of the least discriminative channels (measured by fine labels) in each layer. These pruned models are fine-tuned by $L_{CE}$ only with the same training parameters. We include random pruning as the baseline.
The discriminant metrics outperform random pruning in nearly all ratios, with and without fine-tuning, clearly indicating their pruning effectiveness. We observe a leading metric, G-SD, which consistently achieves the best results. Without retraining, G-SD outperforms DI by 5.5% accuracy on CIFAR-10 with 40% of channels removed and has an 8% accuracy gain over MMD on CIFAR-100 with 30% of channels removed. With retraining, G-SD gains around 0.5% accuracy over MMD on CIFAR-100 when 30% of channels are removed. Based on such consistent winning results, we adopt G-SD in our CDC pipeline for channel pruning. We also visually demonstrate the advantage of G-SD over other state-of-the-art pruning methods in Fig. 8.

4.2. Hierarchical Pruning Study

To study the hierarchical pruning (HP) scheme, we use G-SD to uniformly remove 45% of the channels in each layer from a ResNet-164 on CIFAR-100 and retrain it by \( \mathcal{L}_{CE} \) only with results in Fig. 6. CIFAR-100’s ground truth coarse label can well serve as reference for the study.

Effectiveness. We first show the effectiveness of HP by comparing it with three other labeling schemes for class discrepancy measurement. All four pruning modes are: (1) All layer fine label \( Y_f \) pruning. (2) All layer coarse label \( Y_c \) pruning. (3) Front layer \( Y_f \) + rear layer \( Y_c \) pruning. (4) Front layer \( Y_c \) + rear layer \( Y_f \) pruning (HP). We use the ground truth coarse label for \( Y_c \), and set the watershed layer \( l_{WS} = 0.5L \), where \( L \) is the total number of layers in the network. As shown in Fig. 6a, HP (Mode 4) achieves the best accuracy among all schemes. This suggests that the channels’ class discrimination shall be measured based on its semantic granularity for better pruning performance.

Watershed Layer. We vary the placement of the watershed layer, parameterized by \( l_{WS} = \alpha L, \alpha \in (0, 1) \). As shown in Fig. 6b, we find that \( \alpha = 0.5 \) gives the best result, suggesting that the CNN is processing coarse class semantics in the first half of the layers, and extracting the finer concepts in the second half.

Class Hierarchies. While most image datasets don’t have ground truth coarse labels, we further evaluate proposed HP using the coarse labels learned by the clustering algorithms in Sec. 3.2, and set \( l_{WS} = 0.5L \). We set the number of learned coarse classes to be 20, which is the same as the ground truth scheme. As shown in Fig. 6c, the coarse class labels learned by spectral clustering on the accuracy confusion matrix could even outperform the ground truth scheme. This indicates that HP is generally effective, even without the ground truth coarse label. We include a study with multiple coarse levels in Supplementary Material.

Comparison to State of the Arts. We further compare our G-SD HP scheme with other state-of-the-art pruning methods in Tab. 1, where our method outperforms all of them. We achieve a 2.51% accuracy gain over LCCL [7] with 41.9% less FLOPs. Compared to DI [28], we achieve 1.66% higher accuracy and 5.2% less FLOPs. More comparison results on ILSVRC-2012 are in Supplementary Material.

4.3. Intermediate Distillation Study

We then combine \( \mathcal{L}_{CE} \) with different distillation losses to retrain HP-pruned ResNet-164 with results in Fig. 7. We investigate the following distillation modes which have similar computational budgets: (1) No distillation. (2) Only output distillation [21]. (3) Output + hint-layer intermediate distillation [48]. (4) Output + \( Y_f \) DCA intermediate distillation. (5) Output + \( Y_c \) DCA intermediate distillation.

In this study, we set \( \lambda = 10.0 \) and \( \gamma = 1.0 \) in Eqn. 8 and we only insert intermediate loss at the watershed layer for all intermediate distillation schemes. We include study on inserting losses at multiple intermediate layers in Supplementary Material. We find adding hint-layer distillation (Mode 3) does not improve over output only distillation (Mode 2). On the contrary, DCA-based distillation approaches.
(a) Investigation of Label Pruning Modes
(b) Choice of Watershed Layer
(c) Effects of Different Class Hierarchies

Figure 6: Exploring different settings for hierarchical pruning (HP) with ResNet-164 on CIFAR-100. (a) The front-layer $\mathcal{Y}_f$ + rear-layer $\mathcal{Y}_r$ HP scheme outperforms other label placement schemes for class-discriminative pruning. Notably, HP largely improves over the all-layer fine label pruning, which is used in other literatures. (b) Setting $\alpha = 0.5$, i.e., setting the center layer as the watershed layer gives the best performance for HP. (c) The coarse label learned by spectral clustering on the confusion matrix outperforms the ground truth, suggesting HP can be effective even without ground truth coarse labels.

Figure 7: Comparing different knowledge distillation methods. Adding hint-layer distillation [48] (Mode 3) doesn’t improve over output only distillation (Mode 2). On the contrary, DCA-based distillation (Mode 4-5) further improves the distillation performance, where using the coarse label for DCA learning (Mode 5) achieves the best result.

(Mode 4-5) improves the distillation quality, where using the coarse label $\mathcal{Y}_c$ to learn DCA weights (Mode 5) achieves the best results, which again emphasizes that we should adopt the class granularity of the layer for class discrepancy analysis. These results demonstrate the advantage of DCA-based intermediate distillation over the hint layer [48].

Combining HP with DCA-based distillation, the 2.7×-accelerated ResNet-164 derived by our CDC pipeline achieves an accuracy of 78.05% on CIFAR-100, further advancing the state-of-the-art compression results.

4.4. Comparing to State of the Arts

Benchmarks. Combining HP and DCA-based distillation, we evaluate CDC with VGGNet [51], ResNet [15], and MobileNet-V2 [50] on CIFAR-25 [25] and ILSVRC-2012 [5]. We report compressed models at different FLOPs by adding a letter suffix (e.g., CDC-A and CDC-B). We compare various state-of-the-art compression methods (including their baseline acc.), e.g., TAS [8], LeGR [3], DMCP [12], SSR-GR [53], and CC [34], where CDC outperforms all of them.

Training Settings. We use the Nesterov SGD optimizer [45] with a momentum of 0.9. The weight decay factor is set to be 0.0001. We use the standard data augmentation scheme [15] for all datasets. On CIFAR, we fine-tune 200 epochs with a minibatch size of 128. The learning rate

| Network Method | Test Acc. (%) | Acc. ↓ | FLOPs (M) Pruned (%) |
|----------------|--------------|-------|---------------------|
| ResNet 56      |              |       |                     |
| CDC            |              |       |                     |
| CDC-A          |              |       |                     |
| CDC-B          |              |       |                     |
| CDC-A          |              |       |                     |
| CDC-B          |              |       |                     |
| LeGR           |              |       |                     |
| Di [28]        |              |       |                     |
| CDC            |              |       |                     |

| Network Method | Test Acc. (%) | Acc. ↓ | FLOPs (M) Pruned (%) |
|----------------|--------------|-------|---------------------|
| ResNet 164     |              |       |                     |
| SLIM [39]      |              |       |                     |
| CDC            |              |       |                     |
| CDC-A          |              |       |                     |
| CDC-B          |              |       |                     |
| CDC-A          |              |       |                     |
| CDC-B          |              |       |                     |
| LeGR           |              |       |                     |
| Di [28]        |              |       |                     |
| CDC            |              |       |                     |
| CDC-A          |              |       |                     |
| CDC-B          |              |       |                     |

Table 2: CIFAR-10 Compression Results

| Network Method | Test Acc. (%) | Acc. ↓ | FLOPs (M) Pruned (%) |
|----------------|--------------|-------|---------------------|
| ResNet 56      |              |       |                     |
| SLIM [39]      |              |       |                     |
| CDC            |              |       |                     |
| CDC-A          |              |       |                     |
| CDC-B          |              |       |                     |
| CDC-A          |              |       |                     |
| CDC-B          |              |       |                     |
| LeGR           |              |       |                     |
| Di [28]        |              |       |                     |
| CDC            |              |       |                     |
| CDC-A          |              |       |                     |
| CDC-B          |              |       |                     |

Table 3: CIFAR-100 Compression Results
is initialized at 0.05 and multiplied by 0.13 at epoch 80 and 160. On ILSVRC-2012, we use a batch size of 128 to fine-tune 100 epochs. The learning rate is started at 0.025 with cosine decay learning rate schedule.

**CDC Settings.** We adopt the same one-shot uniform hierarchical pruning scheme\(^1\) as in Sec. 4.2. On CIFAR, all training data are used for G-SD channel scoring, while we randomly sample 10,000 training images for G-SD scoring on ILSVRC-2012. We learn 5/20 coarse classes for CIFAR-10/100 and 100 coarse classes for ILSVRC-2012 by spectral clustering on the confusion matrix. We set the watershed layers for all networks to be the middle layers. For distillation, we set \(\lambda = 10.0\) and \(\gamma = 1.0\) in Eqn. 8 and learn the DCA weights by coarse class with the intermediate loss only inserted at the watershed layer. The student DCA is updated at the 40% and 80% of the total epochs.

**CIFAR Results.** On CIFAR-10, our VGG-16 has a 3\times faster inference speedup with respect to GAL [38], while having 0.36% higher accuracy. Our CDC-A of ResNet-56 outperforms DCP [61] with 0.52% accuracy gain at the same FLOPs reduction ratio. Compared to LFPC [16] on ResNet-110, CDC-B achieves 94.08% accuracy, which is 1.01% higher with the same FLOPs. On CIFAR-100, our compressed VGG-19 has an accuracy gain of 0.48% compared to SLIM [39] with 22.4% less FLOPs. Compared to LFPC [16] and LeGR [3], CDC achieves an accuracy gain of 1.52% and 1.31%, respectively, while having 5% less FLOPs on ResNet-56. On ResNet-164, CDC outperforms DI [28] by 1.94% in accuracy with 5% less FLOPs.

**ILSVRC-2012 Results.** On ResNet-50, CDC-A increases top-1 and top-5 accuracies from the baseline by 0.04% and 0.16% with 44.3% FLOPs pruned. Compared to DMCP [12] and TAS [6], CDC-A has a 0.69% top-1 accuracy gain. Moreover, CDC-B achieves a top-1 accuracy of 76.35% with 53.5% FLOPs reduction, surpassing all prior methods. On ResNet-18, CDC-A achieves a 3.06% accuracy gain with a higher FLOPs reduction ratio compared to LCCL [7]. CDC-B demonstrates top-1 accuracy gains of 1.76% and 1.51% with respect to SFP [17] and DCP [61]. We finally evaluate CDC on MobileNet-V2, which is a much compact network suitable for mobile deployment. CDC-A achieves a top-1 accuracy of 71.97% with 56.3% FLOPs reduction, outperforming AMC [18], LeGR [3], and CC [34]. CDC-B advances DCP [61] by 5% and 1.02% top-1 accuracy, when 53.4% of FLOPs are pruned. CDC’s superior performance well indicates the effectiveness of discriminative compression.

### 5. Ablation Study

**G-SD.** In Fig. 8, we visualize channels at Res1,2 in ResNet-50 on ILSVRC-2012 to intuitively show the effectiveness of G-SD. In Col. 1-3, we observe that the channel

---

\(^1\)For pruned MobileNet-V2, we round the number of channels to its closest integer that is divisible by 8 in each layer, as suggested in [50].
Figure 8: Channel selection analysis. Col. 1: Input images. Col. 2-3: Channels with low and high G-SD values. The low G-SD channel generates responses that are indistinguishable to different classes, while the high one produces informative activations. Col. 4-8: Average responses of the top-10 channels selected by different metrics. From left to right, the metrics are: $\ell_1$-weight [31], batch-norm scaling factor [39], filter’s geometric median [19], DI [28], and G-SD. In general, the channels picked by G-SD preserves the most classification information.

Figure 9: Left: Results of different number of coarse classes for hierarchical pruning. Including an extra coarse class hierarchy improves network pruning performance, while using 20 coarse classes (same number of classes as ground truth) achieves the best performance. Right: Linear separability (measured by classification accuracy of linear SVM) of the intermediate layers trained by different intermediate distillation methods. The layer trained by coarse class DCA distillation shows the best linear separability.

Hierarchical Pruning. As the optimal number of clusters for spectral clustering is mainly based on different heuristics [60, 52], we conduct an empirical study on the number of coarse classes we should learn for $\mathcal{Y}_c$ in hierarchical pruning. We investigate it on CIFAR-100 and use $\mathcal{Y}_c$ of different number of coarse classes to hierarchically prune 56% FLOPs from a ResNet-56 and retrain it only by $\mathcal{L}_{CE}$, with the results shown in Fig. 9. We find that using the coarse class information from $\mathcal{Y}_c$ generally improves over the non-hierarchical pruning. Moreover, we find that learning a 20-class $\mathcal{Y}_c$ achieves the best result, which could be explained that the ground truth labeling of CIFAR-100 also has 20 coarse classes.

DCA-Based Distillation. We measure the linear separability of the intermediate layers by evaluating their classification accuracies on linear SVMs. We compare four different intermediate distillation schemes: (1) No distillation. (2) Hint layer [48]. (3) Fine class DCA. (4) Coarse class DCA. We train linear SVMs (SVM parameter $C$ searched on $[1e-5, 1e-4, ..., 1, 10]$) on the watershed layers of ResNet-164 (both full-size teacher and 63%-pruned students) on CIFAR-100 with results in Fig. 9. We find that the hint layer [48] (Mode 2) has only slight improvement on layer’s linear separability compared to no distillation (Mode 1). Moreover, the coarse class DCA scheme (Mode 4) best improves the linear separability of the student’s hidden layer.

6. Conclusion

In this paper, we propose class-discriminative compression (CDC) to learn efficient neural networks. While limited attempts has been made to leverage classification information for network compression, we design a unified framework for discriminative pruning and distillation, fitting seamlessly with the discriminative training objective. To better identify channels’ redundancy for class-discriminative pruning, we study the pruning effectiveness of a group of closed-form discriminant functions and propose a hierarchical pruning paradigm. Moreover, we make the first attempt to distill discriminative information in hidden layers’ subspace by discriminant component analysis. Combining the pruning and distillation approaches, CDC is evaluated on CIFAR and ILSVRC-2012, outperforming state-of-the-art methods by a clear margin.
References

[1] Guobin Chen, Wongun Choi, Xiang Yu, Tony Han, and Mannmohan Chandraker. Learning efficient object detection models with knowledge distillation. In Proceedings of the 31st International Conference on Neural Information Processing Systems, pages 742–751, 2017.

[2] Wenlin Chen, James Wilson, Stephen Tyree, Kilian Weinberger, and Yixin Chen. Compressing neural networks with the hashing trick. In International Conference on Machine Learning, pages 2285–2294, 2015.

[3] Ting-Wu Chin, Ruizhou Ding, Cha Zhang, and Danuta Marculescu. Towards efficient model compression via learned global ranking. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 1518–1528, 2020.

[4] Matthieu Courbariaux, Itay Hubara, Daniel Soudry, Ran El-Yaniv, and Yoshua Bengio. Binarized neural networks: Training deep neural networks with weights and activations constrained to +1 or -1. arXiv preprint arXiv:1602.02830, 2016.

[5] Jia Deng, Wei Dong, Richard Socher, Li-Jia Li, Kai Li, and Li Fei-Fei. Imagenet: A large-scale hierarchical image database. 2009.

[6] Xin Dong, Shangyu Chen, and Sinno Pan. Learning to prune deep neural networks via layer-wise optimal brain surgeon. In Advances in Neural Information Processing Systems, pages 4857–4867, 2017.

[7] Xuanyi Dong, Junshi Huang, Yi Yang, and Shuicheng Yan. More is less: A more complicated network with less inference complexity. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 5840–5848, 2017.

[8] Xuanyi Dong and Yi Yang. Network pruning via transformable architecture search. In Advances in Neural Information Processing Systems, pages 759–770, 2019.

[9] Ross Girshick. Fast r-cnn. In Proceedings of the IEEE international conference on computer vision, pages 1440–1448, 2015.

[10] Todd R Golub, Donna K Slonim, Pablo Tamayo, Christine Huard, Michelle Gaasenbeek, Jill P Mesirov, Hilary Coller, Mignon L Loh, James R Downing, Mark A Caligiuri, et al. Molecular classification of cancer: Class discovery and class prediction by gene expression monitoring. science, 286(5439):531–537, 1999.

[11] Arthur Gretton, Karsten M Borgwardt, Malte J Rasch, Bernhard Schölkopf, and Alexander Smola. A kernel two-sample test. Journal of Machine Learning Research, 13(Mar):723–773, 2012.

[12] Shaopeng Guo, Yujie Wang, Quanquan Li, and Junjie Yan. Dmcp: Differentiable markov channel pruning for neural networks. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 1539–1547, 2020.

[13] Song Han, Huizi Mao, and William J Dally. Deep compression: Compressing deep neural networks with pruning, trained quantization and huffman coding. arXiv preprint arXiv:1510.00149, 2015.

[14] Song Han, Jeff Pool, John Tran, and William Dally. Learning both weights and connections for efficient neural network. In Advances in neural information processing systems, 2015.

[15] Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 770–778, 2016.

[16] Yang He, Yuhang Ding, Ping Liu, Linchao Zhu, Huan-wang Zhang, and Yi Yang. Learning filter pruning criteria for deep convolutional neural networks acceleration. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 2009–2018, 2020.

[17] Yang He, Guoliang Kang, Xuanyi Dong, Yanwei Fu, and Yi Yang. Soft filter pruning for accelerating deep convolutional neural networks. arXiv preprint arXiv:1808.06866, 2018.

[18] Yihui He, Ji Lin, Zhijian Liu, Hanrui Wang, Li-Jia Li, and Song Han. Anc: Automl for model compression and acceleration on mobile devices. In Proceedings of the European Conference on Computer Vision (ECCV), pages 784–800, 2018.

[19] Yang He, Ping Liu, Ziwei Wang, Zhilan Hu, and Yi Yang. Filter pruning via geometric median for deep convolutional neural networks acceleration. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 4340–4349, 2019.

[20] Yihui He, Xiangyu Zhang, and Jian Sun. Channel pruning for accelerating very deep neural networks. In Proceedings of the IEEE International Conference on Computer Vision, pages 1389–1397, 2017.

[21] Geoffrey Hinton, Oriol Vinyals, and Jeff Dean. Distilling the knowledge in a neural network. arXiv preprint arXiv:1503.02531, 2015.

[22] Zehao Huang and Naiyan Wang. Data-driven sparse structure selection for deep neural networks. In Pro-
Alex Krizhevsky and Geoffrey Hinton. Learning multiple layers of features from tiny images. Technical report, Citeseer, 2009. 6

Hao Li, Asim Kadav, Igor Durdanovic, Hanan Samet, Erich L Lehmann and Joseph P Romano. Testing statistical hypotheses. 2014. 1

Alex Krizhevsky and Geoffrey Hinton. Learning multiple layers of features from tiny images. Technical report, Citeseer, 2009. 6

S.Y. Kung. Kernel methods and machine learning. Cambridge University Press, 2014. 3, 12

S.Y. Kung. Discriminant component analysis for privacy protection and visualization of big data. Multimedia Tools and Applications, 76(3):3999–4034, 2017. 2, 3, 4, 15

S.Y. Kung, Zejiang Hou, and Yuchao Liu. Methodical design and trimming of deep learning networks: Enhancing external bp learning with internal omnipresent-supervision training paradigm. In ICASSP 2019-2019 IEEE International Conference on Acoustics, Speech and Signal Processing (ICASSP), pages 8058–8062. IEEE, 2019. 1, 2, 5, 6, 7, 8, 12, 13, 16

Vadim Lebedev, Yaroslav Ganin, Maksim Rakhuba, Ivan Oseledets, and Victor Lempitsky. Speeding-up convolutional neural networks using fine-tuned cp-decomposition. arXiv preprint arXiv:1412.6553, 2014. 1

Erich L Lehmann and Joseph P Romano. Testing statistical hypotheses. Springer Science & Business Media, 2006. 2, 12

Hao Li, Asim Kadav, Igor Durdanovic, Hanan Samet, and Hans Peter Graf. Pruning filters for efficient convnets. arXiv preprint arXiv:1608.08710, 2016. 1, 2, 6, 8, 13, 16

Tianhong Li, Jianguo Li, Zhuang Liu, and Changshui Zhang. Few sample knowledge distillation for efficient network compression. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 14639–14647, 2020. 2

Xiaojie Li, Jianlong Wu, Hongyu Fang, Yue Liao, Fei Wang, and Chen Qian. Local correlation consistency for knowledge distillation. In European Conference on Computer Vision, pages 18–33. Springer, 2020. 2

Yuchao Li, Shaohui Lin, Jianzhuang Liu, Qixiang Ye, Mengdi Wang, Fei Chao, Fan Yang, Jincheng Ma, Qi Tian, and Rongrong Ji. Towards compact cnns via collaborative compression. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 6438–6447, 2021. 6, 7

Shaohui Lin, Rongrong Ji, Yuchao Li, Yongjian Wu, Feiyue Huang, and Baochang Zhang. Accelerating convolutional networks via global & dynamic filter pruning. In IJCAI, pages 2425–2432, 2018. 1, 2

Shaohui Lin, Rongrong Ji, Chenqian Yan, Baochang Zhang, Liujuan Cao, Qixiang Ye, Feiyue Huang, and David Doermann. Towards optimal structured cnn pruning via generative adversarial learning. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 2790–2799, 2019. 6, 7

Zhuang Liu, Jianguo Li, Zhiqiang Shen, Gao Huang, Shoumeng Yan, and Changshui Zhang. Learning efficient convolutional networks through network slimming. In Proceedings of the IEEE International Conference on Computer Vision, pages 2736–2744, 2017. 2, 5, 6, 7, 8, 13, 16

Zechun Liu, Haoyuan Mu, Xiangyu Zhang, Zichao Guo, Xin Yang, Kwang-Ting Cheng, and Jian Sun. Metapruning: Meta learning for automatic neural network channel pruning. In Proceedings of the IEEE/CVF International Conference on Computer Vision, pages 3296–3305, 2019. 7, 14

Jonathan Long, Evan Shelhamer, and Trevor Darrell. Fully convolutional networks for semantic segmentation. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 3431–3440, 2015. 1

Jian-Hao Luo, Jianxin Wu, and Weiyao Lin. Thinet: A filter level pruning method for deep neural network compression. In Proceedings of the IEEE international conference on computer vision, pages 5058–5066, 2017. 2

Man-Wai Mak and S.Y. Kung. A solution to the curse of dimensionality problem in pairwise scoring techniques. In International Conference on Neural Information Processing, pages 314–323. Springer, 2006. 3, 12
[44] P. Molchanov, S. Tyree, T. Karras, T. Aila, and J. Kautz. Pruning convolutional neural networks for resource efficient inference. In 5th International Conference on Learning Representations, ICLR 2017-Conference Track Proceedings, 2017. 1, 2

[45] Yurii E Nesterov. A method for solving the convex programming problem with convergence rate o(1/k^2). In Dokl. akad. nauk Sssr, volume 269, pages 543–547, 1983. 6, 13

[46] Paul Pavlidis, Jason Weston, Jinsong Cai, and William Noble Grundy. Gene functional classification from heterogeneous data. In Proceedings of the fifth annual international conference on Computational biology, pages 249–255, 2001. 3, 12

[47] Shaoqing Ren, Kaiming He, Ross Girshick, and Jian Sun. Faster R-CNN: Towards real-time object detection with region proposal networks. In Advances in Neural Information Processing Systems (NIPS), 2015. 1

[48] Adriana Romero, Nicolas Ballas, Samira Ebrahimi Kahou, Antoine Chassang, Carlo Gatta, and Yoshua Bengio. Fitnets: Hints for thin deep nets. arXiv preprint arXiv:1412.6550, 2014. 1, 2, 4, 5, 6, 8

[49] Olaf Ronneberger, Philipp Fischer, and Thomas Brox. U-net: Convolutional networks for biomedical image segmentation. In International Conference on Medical image computing and computer-assisted intervention, pages 234–241. Springer, 2015. 1

[50] Mark Sandler, Andrew Howard, Menglong Zhu, Andrey Zhmoginov, and Liang-Chieh Chen. Mobilenetv2: Inverted residuals and linear bottlenecks. In Proceedings of the IEEE conference on computer vision and pattern recognition, pages 4510–4520, 2018. 6, 7, 14

[51] Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition. arXiv preprint arXiv:1409.1556, 2014. 1, 6

[52] Ulrike Von Luxburg. A tutorial on spectral clustering. Statistics and computing, 17(4):395–416, 2007. 8

[53] Zi Wang, Chengcheng Li, and Xiangyang Wang. Convolutional neural network pruning with structural redundancy reduction. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 14913–14922, 2021. 6, 7

[54] Jianbo Ye, Xin Lu, Zhe Lin, and James Z Wang. Rethinking the smaller-norm-less-informative assumption in channel pruning of convolution layers. arXiv preprint arXiv:1802.00124, 2018. 2

[55] Zhonghui You, Kun Yan, Jinmian Ye, Meng Ma, and Ping Wang. Gate decorator: Global filter pruning method for accelerating deep convolutional neural networks. In Advances in Neural Information Processing Systems (NeurIPS), 2019. 1, 2, 7

[56] Ruichi Yu, Ang Li, Chun-Fu Chen, Jui-Hsin Lai, Vlad I Morariu, Xintong Han, Mingfei Gao, Ching-Yung Lin, and Larry S Davis. Nisp: Pruning networks using neuron importance score propagation. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 9194–9203, 2018. 2, 6

[57] Sukmin Yun, Jongjin Park, Kimin Lee, and Jin-woo Shin. Regularizing class-wise predictions via self-knowledge distillation. In Proceedings of the IEEE/CVF Conference on Computer Vision and Pattern Recognition, pages 13876–13885, 2020. 1

[58] Sergey Zagoruyko and Nikos Komodakis. Paying more attention to attention: Improving the performance of convolutional neural networks via attention transfer. arXiv preprint arXiv:1612.03928, 2016. 2

[59] Matthew D Zeiler and Rob Fergus. Visualizing and understanding convolutional networks. In European conference on computer vision, pages 818–833. Springer, 2014. 2, 3

[60] Pietro Zelnik-Manor, Lihiard Perona. Self-tuning spectral clustering. Advances in Neural Information Processing Systems, pages 1601–1608, 2004. 8

[61] Zhuangwei Zhuang, Mingkui Tan, Bohan Zhuang, Jing Liu, Yong Guo, Qingyao Wu, Junzhou Huang, and Jinhui Zhu. Discrimination-aware channel pruning for deep neural networks. In Advances in Neural Information Processing Systems, pages 883–894, 2018. 1, 2, 6, 7, 14
The supplementary material is organized as follows. In Sec. 7, we present the definitions of our group of discriminant functions and a mathematical analysis on our adopted metric, G-SD. The detailed settings of the experiments in our main paper are included in Sec. 8. We discuss additional ablation studies on G-SD, hierarchical pruning, and DCA-based distillation in Sec. 9. Additional visualizations of the channel selection analysis are provided in Sec. 10.

7. Discriminant Functions

7.1. Single-Variate Binary-Class Metrics

We adopt the same notation of the n-sample binary-class single-variate dataset as in the main paper. Our implementations of Symmetric Divergence (SD) [43], Absolute SNR (AbsSNR) [10], Fisher Discriminant Ratio (FDR) [46], and Student’s T-Test (Ttest) [30] are:

\[
\text{SD}(\mathcal{D}, \mathcal{B}) = \frac{1}{2} \left( \frac{\sigma^2_{\mathcal{D}^+} + \sigma^2_{\mathcal{D}^-}}{\sigma^2_{\mathcal{D}^+} + \sigma^2_{\mathcal{D}^-}} \right) + \frac{1}{2} \left( \frac{(\mu_{\mathcal{D}^+} - \mu_{\mathcal{D}^-})^2}{\sigma^2_{\mathcal{D}^+} + \sigma^2_{\mathcal{D}^-}} \right)^{-1}
\]

(9)

\[
\text{AbsSNR}(\mathcal{D}, \mathcal{B}) = \frac{|\mu_{\mathcal{D}^+} - \mu_{\mathcal{D}^-}|}{\sigma_{\mathcal{D}^+} + \sigma_{\mathcal{D}^-}}
\]

(10)

\[
\text{FDR}(\mathcal{D}, \mathcal{B}) = \frac{(\mu_{\mathcal{D}^+} - \mu_{\mathcal{D}^-})^2}{\sigma^2_{\mathcal{D}^+} + \sigma^2_{\mathcal{D}^-}}
\]

(11)

\[
\text{Ttest}(\mathcal{D}, \mathcal{B}) = \frac{|\mu_{\mathcal{D}^+} - \mu_{\mathcal{D}^-}|}{\sqrt{\frac{\sigma^2_{\mathcal{D}^+}}{|\mathcal{D}^+|} + \frac{\sigma^2_{\mathcal{D}^-}}{|\mathcal{D}^-|}}}
\]

(12)

where |\mathcal{D}^+| and |\mathcal{D}^-| denote the number of samples in the two classes of \mathcal{D}.

We observe that the computation of the three metrics (not appeared in the main paper), AbsSNR, FDR, and Ttest, mainly requires four statistics, \mu_{\mathcal{D}^+}, \sigma^2_{\mathcal{D}^+}, \mu_{\mathcal{D}^-}, and \sigma^2_{\mathcal{D}^-}, which are the same as SD. Therefore, we can adopt the same way explained in the main paper to generalize them for channel scoring; (1) Partition the feature maps of a channel in a one-versus-all manner based on their labels; (2) Apply specialized statistics extractors to obtain the metric score for each partition; (3) Aggregate the scores from all partitions to a single scalar. Via such way, we derive G-AbsSNR, G-FDR, and G-Ttest for high-dimensional multi-label channel scoring and pruning.

We also provide an analysis on the mathematical merits of SD as follows. The discriminant power of a scalar feature is quantified by the first two terms in SD. The first term measures the divergence of two classes symmetrically. It achieves a high score for the feature whose \sigma^2_{\mathcal{D}^+} or \sigma^2_{\mathcal{D}^-} is much smaller than the other, which indicates that one class of the feature has a more concentrated distribution, and thus the feature is more class distinguishable. The second term\(^2\) awards features that have large “Signal-to-Noise Ratio (SNR)” [24] where the centroid distance \((\mu_{\mathcal{D}^+} - \mu_{\mathcal{D}^-})^2\) can be viewed as “class discrimination signal” and the class variance sum \(\sigma^2_{\mathcal{D}^+} + \sigma^2_{\mathcal{D}^-}\) is regarded as “class variational noise”. Therefore, SD’s ability to find discriminant features is mathematically supported.

7.2. High-Dimensional Metrics

We present the definitions of Discriminant Information (DI) [28] and Maximum Mean Discrepancy (MMD) [11] here. We use the same notation of N-sample Y-class dataset in the main paper and regard \(f_i\) as a flattened 1D vector.

\[
\text{DI}(\mathcal{F}) = \text{tr}((\bar{\mathbf{S}} + \rho \mathbf{I})^{-1} \mathbf{S_B})
\]

(13)

where \(\rho\) is a small ridge factor to ensure invertibility. In practice, we set \(\rho = 0.0001\).

MMD. We define the rbf kernel as:

\[
k(x, y) = \exp\left\{-\frac{||x - y||^2}{2\sigma^2}\right\}
\]

(14)

The two-class MMD on a \((\mathcal{F}^c, \mathcal{F}^{-c})\) partition can thus be defined as:

\[
\text{MMD}_{\text{two}}(\mathcal{F}^c, \mathcal{F}^{-c}) = \frac{1}{|\mathcal{F}^c|^2} \sum_{f_i, f_j \in \mathcal{F}^c} k(f_i, f_j)
\]

\[
+ \frac{1}{|\mathcal{F}^{-c}|^2} \sum_{f_i, f_j \in \mathcal{F}^{-c}} k(f_i, f_j)
\]

\[
- \frac{2}{|\mathcal{F}^c| \times |\mathcal{F}^{-c}|} \sum_{f_i \in \mathcal{F}^c, f_j \in \mathcal{F}^{-c}} k(f_i, f_j)
\]

(15)

following the similar one-versus-all setting, the MMD score of a channel is given as:

\[
\text{MMD}(\mathcal{F}) = \frac{1}{Y} \sum_{c=1}^{Y} \text{MMD}_{\text{two}}(\mathcal{F}^c, \mathcal{F}^{-c})
\]

(16)

In practice, we set \(\sigma = 1\) for the rbf kernel.

8. Experimental Details

8.1. Training Parameters

For the experiments we conduct in Sec. 4.1-4.3 and Sec. 5 of the main paper, we use the following settings to retrain our pruned networks.

\(^2\)The expressions of SD’s second term vary a bit in [26] and [43]. In this work, we adopt the one in [26].
Figure 10: The CPU wall time for scoring all channels in VGG-16 layers by G-SD and DI on ILSVRC-2012. G-SD shows a significant advantage in computational efficiency.

We use the SGD optimizer with Nesterov momentum [45] to fine-tune VGG-16 and ResNet-38/56/164, where the momentum is set as 0.9. The fine-tuning process takes 200 epochs with a batch size of 128, and the weight decay is set to be 1e-3. We augment the training samples with a standard data augmentation scheme [15]. The learning rates of VGG-16 and ResNet-38/56/164 are started as 0.006 and 0.05, and multiplied by 0.28 and 0.14 at 40% and 80% of the total number of epochs. Our training codes are implemented in PyTorch.

8.2. CDC Parameters

To learn the coarse class hierarchy, we use scikit-learn [?] to implement the KMeans clustering algorithm as well as the spectral clustering algorithm. The confusion matrices and the last hidden-layer activations are obtained from the full size ResNet-110, ResNet-164, and ResNet-50 for CIFAR-10, CIFAR-100, and ILSVRC-2012. For our DCA-based distillation, we use Scipy [?] to solve Eqn. 6 for the DCA weights and use scikit-learn to train linear SVMs. For coarse class DCA distillation, we use the coarse class learned by spectral clustering. For the output knowledge distillation [21], we set the temperature parameter $T = 1$ for all experiments.

9. Additional Ablation Study

9.1. Time Complexity of G-SD.

We conduct an empirical study to investigate the time complexity of two discriminant functions, G-SD and DI, for channel scoring in Fig. 10. We pick 3,000 ILSVRC-2012 samples and compute their feature maps in the channels of a VGG-16. The maps are computed via an NVIDIA Tesla P100 GPU and the scorings of feature maps are executed on an Intel Xeon E5-2680 v4 CPU. We calculate the CPU wall time for scoring all the channels in each layer by the two discriminant functions (excluding the time to obtain the maps). We note that G-SD generally has much less time complexity than DI in the scoring process of each layer. Moreover, for layer conv1_1 whose feature maps are of size $224 \times 224$, G-SD shows a 4000× speedup over DI.

9.2. Quantitative Channel Selection

We demonstrate the advantage of G-SD over other state-of-the-art pruning criteria [31, 39, 19, 28] quantitatively in Fig. 11 with ResNet-110 on CIFAR-10. We use each metric to uniformly remove $r\%$ of the lowest-scored channels in all layers and include random scoring as the baseline, where $r \in \{5, 10, 15, 20, 25, 30, 35, 40\}$. We then evaluate these pruned models’ accuracies directly without retraining. We observe that G-SD wins against all other methods in nearly all FLOPs reduction ratios.

9.3. Learned Class Hierarchy

We further investigate the coarse hierarchy learned by spectral clustering on the confusion matrix. On CIFAR-10, we learn 5 coarse classes, with the fine class grouping as: (1) automobile, truck; (2) airplane, ship; (3) cat, dog; (4) deer, horse; (5) bird, frog. These coarse classes can be labeled as vehicle transportation, other transportation tools, small mammals, large mammals, and non-mammals. This result shows that our method can well learn the coarse class semantics.

On CIFAR-100, we show the confusion matrices of ResNet-164 in Fig. 12. We first show the confusion ma-
9.4. Hierarchical Pruning on ILSVRC-2012

We further compare our hierarchical pruning (HP) scheme with state-of-the-art pruning methods on ILSVRC-2012 in Tab. 5 and 6. On both ResNet-50 and MobileNet-V2, HP again demonstrates its superiority by achieving higher accuracy with less inference cost.

9.5. DCA-Based Distillation Weight

We investigate the knowledge distillation weight $\lambda$ for DCA-based intermediate distillation in Fig. 13. We use hierarchical pruning to prune 56% FLOPs from a ResNet-56 trained on CIFAR-100, and fine-tune it with output distillation weight $\gamma = 1.0$ while vary the intermediate distillation weight to be $\lambda \in \{0.1, 1, 10, 100\}$. We find that setting $\lambda = 10$ achieves the best result for our scheme.

9.6. PCA-Based Distillation

We investigate using PCA components to transform layer activations for subspace distillation. We keep all other settings (layer pruning ratio, weights learning frequency, inserted layers, etc.) to be the same and compare this scheme with our DCA-based distillation with pruned ResNet-56/164 (45% of channels uniformly removed from each layer) on CIFAR-100 in Fig. 14. We find that our DCA-based distillation generally improves over the PCA
Figure 14: Comparing PCA with DCA for subspace distillation with ResNet-56 and ResNet-164 on CIFAR-100. DCA outperforms PCA for the distillation quality on both networks.

Figure 15: Results of multi-hierarchy CDC with ResNet-56 and ResNet-164 on CIFAR-100. Including a coarse hierarchy for network compression generally works better than the non-hierarchical scheme, while increasing the number of hierarchies to 3 gives similar results as the 2-hierarchy scheme.

9.7. Multi-Hierarchy CDC

We study a 3-hierarchy compression scheme where we learn multiple class hierarchies and insert multiple distillation losses. We compare it with the 2-hierarchy scheme from the main paper and the non-hierarchical (only using the fine label) scheme in Fig. 15. In the 3-hierarchy scheme, we learn two coarse class labels, $Y_{c1}$ and $Y_{c2}$, where $Y_{c2}$ is derived by spectral clustering on the confusion matrix of $Y_{c1}$, and $Y_{c1}$ is the coarse hierarchy adopted in the 2-hierarchy scheme. We conduct experiments with ResNet-56/164 on CIFAR-100, where $Y_{c1}$ has 20 classes and $Y_{c2}$ has 4 classes. We set the watershed layers at the locations of 1/3 and 2/3 of the total layers. We use $Y_{c2}$, $Y_{c1}$, and $Y_f$ to evaluate the channel class discrepancy for the first 1/3, the 1/3 to the 2/3, and the last 1/3 of the layers, and we uniformly remove 45% of channels from each layer. Two DCA-based distillation losses are inserted at two watershed layers, where the early one use $Y_{c2}$ to learn weights and the later one use $Y_{c1}$. We find both 2-hierarchy and 3-hierarchy schemes achieve better performance than the non-hierarchical one, while using 3 hierarchies gives similar results as the 2-hierarchy scheme.

10. More Visualizations

We repeat the channel selection analysis in Sec. 5 of the main paper with additional images from different classes (not shown in the main paper) in Fig. 16. Visually speaking, we again observe that our G-SD selects better class-discriminative channels compared to other methods.
Figure 16: More visualizations of the channel selection analysis. **Col. 1:** Input images. **Col. 2-3:** Channels with low and high G-SD values. The low G-SD channel generates responses that are indistinguishable to different classes, while the high one produces informative activations. **Col. 4-8:** Average responses of the top-10 channels selected by different metrics. From left to right, the metrics are: $\ell_1$-norm [31], batch-norm scaling factor [39], filter’s geometric median [19], DI [28], and G-SD. In general, the channels picked by G-SD preserves the most classification information.