Hot QCD equations of state and RHIC

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We show how hot QCD equations of states can be adapted to make definite predictions for quark-gluon plasma at RHIC. We consider equations of state up to $O(g^5)$ and $O[g^6(ln(1/g)+\delta)]$. Our method involves the extraction of equilibrium distribution functions for gluons and quarks from these equations of state by capturing the the interaction effects entirely in the effective chemical potentials. We further utilize these distribution functions to study the screening length in hot QCD and dissociation phenomenon of heavy quarkonia states by combining this understanding with the semi-classical transport theory.

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I. INTRODUCTION

The hot and dense matter created at RHIC behaves like a near perfect fluid as inferred from recent experimental observations at RHIC[1]. This form of the matter lies in the strongly interacting domain of QCD. It is of the interest to check if the domain is genuinely non-perturbative, or could be understood in the form of higher order terms in perturbation theory. Methods based on weak perturbative techniques [2] have achieved partial success in interpreting the results from RHIC. The aim of the present article is to investigate more fully, the viability of pQCD EOS in understanding the RHIC results. We implement this programme by extracting quasi-free equilibrium distribution functions from the EOS, capturing all the interaction effects in the effective fugacities[3, 4]. This procedure allows one to study various observables for hot and dense matter at RHIC.

II. QUASI-PARTICLE REALIZATION OF HOT QCD EOS

We consider hot QCD EOS in pQCD up to $O(g^5)$[7](EOS1) and $O[g^6(ln(1/g)+\delta)]$[8](EOS2). The free parameter $\delta$ in EOS2 encodes the unknown perturbative and non-perturbative contributions of $O(g^6)$ and can be fixed by matching the EOS with lattice predictions on EOS[5]. The ansatz for the determination of equilibrium distribution functions involves retaining the ideal distribution forms, with the effective fugacities ($z_g = \exp(\mu_g)$ and $z_f = \exp(\mu_f)$). Note that for the mass less quarks (u and d) which we consider to constitute the bulk of the plasma, $z_{g/f} = 1$ ($\mu_{g/f} = 0$) if they were not interacting.

As the first step in our approach, we express the hot EOS in the form

$$P = P_g + P_q + \Delta P_g + \Delta P_f$$

We seek to absorb the contributions from all the non-ideal terms in the effective chemical potentials $\mu_g$ and $\mu_f$. Since the equations of states are expected to be valid at $T > 2T_c$, we treat the
dimensionless quantity $\tilde{\mu}_{g,f} \equiv \beta \mu_{g,f}$ perturbatively. This determination of $\tilde{\mu}_{g/f}$ needs to be done self consistently. Accordingly, we expand the the grand canonical partition functions ($Z_{g/f}$) for gluons and quarks as a Taylor series in $\tilde{\mu}_{g,f}$. We obtain

$$\log(Z_g) = \sum_{k=0}^{\infty} (\tilde{\mu}_g)^k \partial_k \log(Z_g)|_{\tilde{\mu}_g = 0}; \log(Z_q) = \sum_{k=0}^{\infty} (\tilde{\mu}_f)^k \partial_k \log(Z_f)|_{\tilde{\mu}_f = 0}. \quad (2)$$

We equate $\log(Z)$ with the pressure by the well known relation: $\log(Z) = P/\beta V$, and determine $\tilde{\mu}_{g/f}$ self-consistently order by order. We find that it is sufficient to determine them up to cubic order. Details may be found in [3].

III. THE SCREENING LENGTH

The Debye mass from equilibrium distribution function can be obtained from the expression [9]

$$M_{D_{g/f}}^2 = (g')^2 C_{g/f} \int \frac{d^3p}{2\pi^3} \partial_0 f_{g/f}^{eq}, \quad (3)$$

where $C_g = 2N_c$, $C_f = -2N_f$ and $g'$ is the effective coupling which appears in the transport theory. We fix $g'$ by comparing the temperature dependence of the screening length with the lattice results of Kazmarek and Zantow[10]. The behavior of the screening length for pure gauge theory and full QCD for EOS1 and EOS2 are shown in Fig.1. Interestingly, the behavior of the screening length with temperature qualitatively matches with the lattice QCD results (see Fig.2 of Ref.[10]). The agreement for EOS2 is slightly better as compared to EOS1 and in both the cases agreement becomes better at higher temperatures ($T \equiv 3T_c$ and higher).

IV. THE HEAVY QUARK POTENTIAL AND DISSOCIATION OF QUARKONIA

In this section, we address the screening of heavy quark potential by combining the quasi-particle model for hot QCD introduced in Section(I) to the semi-classical transport theory[6]. The response
function (chromo-electric permittivity) has recently been determined from the transport theory by employing the ideal distribution functions for gluons and quarks by Ravishankar and Ranjan [6]. We determine these response functions for realistic distribution functions obtained from pQCD. Finally, the response functions is then used to study the screening properties of QGP. We outline the method (discussed in detail in [3, 4, 6]) below.

Consider the Cornell potential:

$$\phi(r) = -\frac{\alpha r}{2} + \Lambda r,$$

where $\alpha$ and $\Lambda$ are phenomenological constants. The first term dominates at small distance while the linear causes confinement, dominating at large distances. The medium modifies the expression for the potential (in the Fourier space), $\hat{\phi}(k) \to \hat{\phi}(k)/\hat{\epsilon}(k)$, where the response is evaluated at $\omega = 0$. Accordingly, the potential undergoes modification $\hat{\phi}(k) \to \hat{\phi}_m(k)$, which can be written as

$$\hat{\phi}_m(k) = -\sqrt{2\pi} \alpha k^2 + \frac{\Lambda}{\sqrt{2\pi} k^2 + m_D^2},$$

where

$$m_D^2 = 8\pi Q^2 T^2 \left[ N_f f_2(z_q) + g_2(z_g) \right],$$

For a gluonic plasma, the above expression reduces to $m_{Dg}^2 = 16\pi Q^2 T^2 g_2(z_g)$, where $z_g = \exp(\tilde{\mu}_g)$ and $z_q = \exp(\tilde{\mu}_q)$ Note that, $m_D$ has determined by the permittivity via: $\epsilon = 1 + \frac{m_D^2}{k^2}$. The modified form of the potential in r-space is then given by:

$$\phi_s(r) = \left( \frac{2\Lambda}{m_D^2} - \alpha \right) \frac{\exp(-m_D r)}{r} - \frac{2\Lambda}{m_D^2} - \frac{2\Lambda}{m_D} - \alpha m_D.$$

It is clear from Eq.(6) that the role played by the Debye mass in QGP is rather different from its role in electrodynamic plasma. To see this, note that at large $T$, Eq.(6) reduces to

$$\phi_s(r) \sim -\frac{2\Lambda}{m_D^2} - \alpha m_D.$$

Ignoring the additive contribution, the energy of the $q\bar{q}$ in the ground state is simply given by

$$E_g = \frac{m_q \Lambda^2}{m_D},$$

where $m_q$ is the mass of heavy quark. The binding energy is, of course, temperature dependent and approaches zero as $T \to \infty$. At any finite temperature though, the quarks possess a thermal energy $E_{Th} \sim \frac{3}{2} T$, by equipartition theorem, leading to an ionization of the quarkonium when $E_{Th}$ matches the binding energy. This leads to the melting temperature $T_d$ of $J/\Psi$ and $\Upsilon$ listed in Table(I). It is noteworthy that the dissociation temperatures are all roughly in the range $T_D \approx (2 - 3)T_c$, which is higher than the temperatures achieved so far. Since the temperatures expected at LHC is $\sim 2T_c - 3T_c$, one may expect to test these predictions there. And moreover these estimates are consistent with the recent results from other theoretical works [11].

V. SUMMARY AND CONCLUSIONS

In conclusion, we have developed a quasi particle model for hot QCD to extract the equilibrium distribution functions for gluons and quarks from the hot QCD EOS. We have shown that the interaction effects can entirely be captured in the effective fugacities for the gluons and quarks.
TABLE I: The dissociation temperature \(T_D\) for various quarkonia states (in unit of \(T_c\)).

| Hot EOS | quarkonium | Pure QCD \(N_f = 2\) | Pure QCD \(N_f = 3\) |
|---------|------------|---------------------|---------------------|
| EOS1    | \(J/\Psi\) | 2.2                 | 2.62                |
|         | \(\Upsilon\) | 2.5                 | 3.14                |
| EOS2    | \(J/\Psi\) | 1.86                | 2.38                |
| \(\delta = 0.8\) | \(\Upsilon\) | 2.12                | 2.76                |
| EOS2    | \(J/\Psi\) | 1.95                | 2.45                |
| \(\delta = 1.0\) | \(\Upsilon\) | 2.2                 | 2.83                |
| EOS2    | \(J/\Psi\) | 2.03                | 2.52                |
| \(\delta = 1.2\) | \(\Upsilon\) | 2.28                | 2.9                 |

We utilized these distribution functions to study the screening length as a function of temperature. We have determined the dissociation temperatures for \(J/\Psi\) and \(\Upsilon\) by studying the medium modifications to heavy quark potential which enters in the form of chromo-electric permittivity. The results on dissociation temperatures are consistent with other recent theoretical works. Our approach can easily be generalized to the hot QCD EOS at finite quark-chemical potential. This approach can also be employed to study the thermodynamic and transport properties of hot and dense matter at RHIC.

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