Definition of service angle of android’s robot hand by method of small movements of gripper’s axis synthesis by speed vector

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Abstract. The paper presents a generalized method for determining the service solid angle based on the assigned gripper axis orientation with a stationary grip center. Motion synthesis in this work is carried out in the vector of velocities. As an example, a solid angle of the android robot arm is determined, this angle being formed by the longitudinal axis of a gripper. The nature of the method is based on the study of sets of configuration positions, defining the end point positions of the unit radius sphere sweep, which specifies the service solid angle. From this the spherical curve specifying the shape of the desired solid angle was determined. The results of the research can be used in the development of control systems of autonomous android robots.

1. Introduction
Modeling manipulation objects using robots requires clarifying dexterity and natural properties of actuation mechanisms at initial stages [1, 2]. One of the tasks at this stage is the determination of the service angle. The solution to this problem on a virtual level is possible by modeling small movements of manipulators’ mechanisms in the vector of velocities of the output link providing the specified fixed position of the center of the output link. In the paper [3] the method for determining the angle of service using the vector equation of the equivalent mechanism closure is introduced. However, this method is difficult to use in view of a complex structure of kinematic chains of manipulator mechanisms, specified different limit values of generalized coordinates and motional redundancy being equal to two or above. In the paper [4] the way of defining the solid angle by studying instantaneous states of mechanisms [5] is proposed. However, the axis movements synthesis of a gripper in different directions with a stationary grip center was not examined in the paper mentioned above. Only particular positions of the gripper axis corresponding to the allowable values of the vector of generalized velocities for a specified configuration were determined. In the work [6] a generalized method of determining the service angle based on the motion synthesis of the gripper axis clockwise and counterclockwise with a stationary grip center for planar manipulator mechanisms is proposed. It should be noted that the method for determining the service angle proposed in this work can be used only for planar manipulator mechanisms. In the paper [7] a method of determining the service angle based on Grashof criterion is proposed, however, this method can be used only for planar manipulator as previous one.

2. Task description
This research is a continuation of the geometric study related to the determination of the solid service angle for the android robot arm using motion synthesis of the gripper axis in various directions with the stationary grip center. The essence of the proposed method is based on finding such
configuration in which the service angle in the specified sector of unit radius sphere sweep increases [4]. This configuration is obtained by the implementation of discrete values of the vectors of generalized velocities $Q$ in the tolerance range. The tolerance range of the vector $Q$ of the android arm mechanism for different robot arm configurations was analysed in the work [8]. Geometric analysis of the service angle mapping allows estimating such natural properties of manipulators as manipulator dexterity and manoeuvrability for specified limiting values of generalized coordinates at different points of the configuration space.

Let the spatial seven link mechanism of the android arm AR600E (Fig. 1a) be assigned. The figure shows the position of the system of coordinates $O_{x_0,y_0,z_0}$ related to the characteristic point of the hand, the movement of which is carried out along a predetermined path.

![Figure 1. Definition of the parameters specifying the position of the point L in the sphere sweep: a - a kinematic diagram of the android arm mechanism and angle position $\alpha$ and $\beta$ in the inertial coordinate system; b - the position of the point L in the sphere sweep.](image)

The point $O_{12} = O_n$ defines the center of the output link which should not deviate from the assigned point of the predetermined path by the value $\delta$ referred to as the positioning accuracy. When conducting tests, the parameter value $\delta = 1$ mm was taken. The points $O_1$–$O_{12}$ specify the coordinate origins used in the assignment of the kinematic chain model. The linear system of equations defines the dependence of the gripper velocity vectors and generalized velocities $V$ and $Q$ of the manipulator mechanism [5]. Implementation of vectors $Q$ from the tolerance range [8] provides a new position for the kinematic chain and gripper axis. The new position of the kinematic chain defined by the values of generalized coordinates is understood under implementation. These coordinates are obtained by adding the generalized coordinates of the old configuration and calculated components values of the vector $Q$ according to the equations (under the assumption $Aq_i \approx \dot{q}_i$):

$$q_i = q_i + \dot{q}_i,$$

$$\dot{q}_i = q_i + \ddot{q}_i$$

where $\dot{q}_i$ are the components of the $Q$.

3. Theoretical Basis
With the aim of a graphic representation of the solid service angle for the spatial six-linked mechanism of the arm (Fig. 1a) let us draw a conditional sweep for the surface sphere $F$ of the unit radius. Between the points of that conditional sweep and points of the sphere $F$ there is an unambiguous correlation. Any point $D$ of the sphere surface $F$ of the radius $O_n D$ specifies a single point $L$ on this sweep defined by the polar coordinates $\alpha_p$ and $\beta_p$ (Fig. 1b). The segment $O_n D$ equals to unity. The positions of the point $L$ in the sweep of the unit radius sphere belonging to different sweep sectors will
correspond to the set of particular positions of the gripper axis $O_nz_n$ when implementing the vector values $Q$ and $\delta = 1$ mm. Let us assign the sweep sector with the angles $\beta_p^{\text{min}}$ and $\beta_p^{\text{max}}$. In this sector the angle $\beta_p$ varies in the interval $\beta_p^{\text{min}} \leq \beta_p \leq \beta_p^{\text{max}}$. The point $L$ in the sphere sweep is determined by the parameters $\alpha_p = \alpha$ and $\beta_p = \beta$. The parameter $\alpha_p$ in this sweep specifies the distance from the center of the circle $O_p$ to the point $L$ of the sweep and is determined by the angle between the original position of the longitudinal axis of the gripper $z_n$ and position of the axis $z_n'$ produced by implementing the vector $Q$ (Fig. 1a). The unit director vectors of these axes define the vectors $W_2$ and $W_2'$ (Fig. 2).

**Figure 2.** Definition of angles $\alpha$ and $\beta$ in the inertial coordinate system $O_0$ specifying the position of the point $L$ in the sphere sweep

The angle $\beta_p$ defines the direction while finding the point $L$ in the conditional sweep by means of the polar coordinates (Fig. 1b). This angle is determined by the angle $\beta$ between the vector projection $W_6$ of the gripper axis on the plane $\Pi \perp O_nz_n$ and the vector $W_1$ specifying the axis orientation $x_n$ (Fig. 1a and Fig. 2) ($W_1 \subset \Pi$). The vector $W_6$ is determined by the vector projection $W_5$ on the plane $\Pi$. The vector $W_5 = W_3 - W_3'$. To determine the point $O_{11}'$ (Fig. 2) we use the vector $W_3' = W_3 + W_7$. In addition the vector $W_7$ is determined by the vector projection $W_8$ in the direction of the vector $W_9$. The position of the vector $W_9$ is defined by the points $O_{11}'$ and $O_{11}''$ and vectors $W_8$ and $W_9$, where $W_9 = W_4 - W_3$, $W_8 = W_4 - W_3'$. The point $O_{11}''$ is the projection of the point $O_{11}'$ on the plane $\Pi$. Angle cosines $\alpha$ and $\beta$ are calculated by scalar products of vectors:

$$cos \alpha = \frac{W_2 \cdot W_2'}{|W_2| \cdot |W_2'|};$$

$$cos \beta = \frac{W_1 \cdot W_6}{|W_1| \cdot |W_6|}.$$  \hfill (2)

Let us construct the set of configurations and particular axis positions of the gripper, the angle value $\alpha$ for them being increased. Concurrently, the movement of the gripper axis will occur in a given sweep sector. The following configuration is constructed, if the value of the angle $\alpha$ is greater than the value of the angle calculated under the previous iteration. It should be noted that in this research the assumption $O_{12} \approx O_{12}'$ is adopted, where the points $O_{11}'$ and $O_{12}'$ are defined by instantaneous states implementation. The reference position of the android robot arm configuration defining the position of the point $O_{12}$ corresponds to the values of generalized coordinates $q_j (25^\circ, 20^\circ, -65^\circ, 25^\circ, -65^\circ)$. The maximum value of the angle $\alpha_{\text{max}}$ is determined at each step from the values $\alpha$ and $\alpha'$ (where $\alpha$ and $\alpha'$ are the values of the angle at the $k$-th and $k+1$ iterations).
(max α, α') → α^{max}.

After the configuration k+1 having been defined which and satisfies conditions:

\[ a_p^{k+1} > a_p^{max}, \]
\[ \beta_p^{min} > \beta > \beta_p^{max} \]  

the following configuration k+2 is found, with the list of generalized coordinates \( q_p = q_i \) being saved. At this time the configuration k+1 satisfying conditions (2) is adopted as the originally specified configuration. Specifying different values for the angles \( \beta_p^{min} \) and \( \beta_p^{max} \) allows performing the synthesis of the gripper axis movements while finding the point \( O_{12} \) inside the radius sphere of 1 mm and given sweep center.

The algorithm for computing positions of points \( L \) in the sphere sweep is shown in Fig. 3.

![Diagram of the calculation algorithm for the position of the end points (L) defining the spherical curve \( l_p \) in the sphere sweep](image)

**Figure 3.** Diagram of the calculation algorithm for the position of the end points \( (L) \) defining the spherical curve \( l_p \) in the sphere sweep

The following schematic designations are adopted: 1 is the input of values \( q_i(q_1, q_2, ..., q_5), q_i^{max}, q_i^{min}, a_p^{min} = 0, V, \beta_p^{min}, \beta_p^{max}, \delta_{min} = 1 \) mm; 2 is \( q_i = q_p \); 3 is the calculation of matrixes \( M_{0,0} \) defining the position of links and values of the vectors \( W_1 - W_4 \); 4 is the calculation of matrixes \( M_{0,k} \) by implementing the vector \( Q \) and determination of vector values \( W_1', W_2', W_3', W_4' - W_5' \); 5 is the linearization error calculation \( \delta_p \); 6 is \( \delta_p < \delta_{min} \); 7 is \( q_i^{min} < q_i < q_i^{max} \); 8 is the calculation of \( a_p \) and \( \beta \); 9 is the calculation of \( a_p^{max} \) (3); 10 is \( a_p > a_p^{max} \); 11 is \( q_i^{p} = q_i \); 12 is the variation of parameters \( k_i \) specifying the value of the vector \( Q \) with the motional redundancy [8]: \( k_i = k_i + 1 \); 13 is \( k_i > k_i^{max} \); 14 is the calculation of the vector \( Q, q_i = q_i + \Delta q_i \); 15 is the output positions of the end point \( L \) belonging to the spherical curve \( l_p \) and sector sphere sweep \( \beta_p^{min} \) and \( \beta_p^{max} \); 16 is \( \beta_p^{max} > 360° \); 17 is the assignment of the next sweep sector \( \beta_{p1}^{min} \) and \( \beta_{p1}^{max} \), 18 is the end.

4. Experimental Results

Fig. 4ab shows the results of studies related to the determination of the projection angle \( \alpha \) formed between the extreme positions of gripper axes on three planes of projections and determination of the path for the point displacement \( L \) in the conditional sphere sweep. For this motion synthesis of the gripper axis, the point \( L \) moves in the specified sweep sector. The sweep sector is specified by the angles of 90° and 120°. The limit values of the generalized coordinates to determine a spherical curve gripper axis, the point \( L \) moves in the specified sweep sector. The sweep sector is specified by the path for the point displacement \( L \) between the extreme positions of gripper axes on three planes of projections and determination of the...

...ki characterizes the manoeuvrability of a manipulator. The projection images \( D_m \) of the region \( D_n \) give only a certain geometric representation of the specified manipulator manoeuvrability in the determined sector of the sphere sweep under the assigned values \( q_i^{max} \) and \( q_i^{min} \).
Figure 4. Projection of the angle $\alpha$ and position of the point $L$ in the sphere sweep under the motion synthesis in one of the sweep sectors for the configuration $q_1 = 25^\circ$, $q_2 = 20^\circ$, $q_3 = -65^\circ$, $q_4 = 25^\circ$, $q_5 = -65^\circ$ when: a) $-75^\circ \leq q_i \leq 75^\circ$; b) $-120^\circ \leq q_i \leq 120^\circ$

Fig. 4ab represents particular positions of the gripper axis under the specified module of the vector $|V(V_x, V_y, V_z)| = 1 \text{ mm} / t$. The vector $V$ is in parallel to the axis $x$ of the fixed system of coordinates. Fig. 4ab shows the particular positions of the point $L$ within the defined sector. The end points $L$, being at a maximum distance from the point $O_p$ of the sweep center, assign the position of the spherical curve $l_p$. The area of the closed loop specified by this curve determines the solid service angle of the android robot arm.

5. Discussion
The findings of the study allow one to draw the following conclusions:
1 – The form and position of the spherical curve $l_p$ largely depend on the position of the android arm mechanism;
2 – When calculating the vector $Q$, maximum values of the parameters $k_i^{max}$ should be used. This significantly reduces the computational time for finding the position of a spherical curve.

6. Summary and Conclusion
The proposed algorithm for computing the projections of the solid service angle and the spherical curve allows one to determine a specified angle with a certain assumption. This algorithm is characterized by a generalized versatility and can be used for manipulators with arbitrary structure of the kinematic chains, an arbitrary degree of motional redundancy, and specified different limit values of generalized coordinates. The developed method for reorientation of the OL can be used in intelligent control systems of autonomously functioning robots, with the aim of changing the position of configuration in relation to specified forbidden regions.

7. References
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