Suppression of high-order multipole moments in a resonant periodic dipole chain

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Abstract. In this work we study the properties of optical states in transverse subdiffractional dipolar chains of periodically spaced scatterers. We demonstrate that in this system there are states with low values of emission rates, and we explain this as a reduction of different multipolar contributions up to high values of orbital angular momentum. We also show that the emission rate for these states decreases as \( \sim N^{-6.76} \) with the number of scatterers.

1. Introduction

One-dimensional systems have been attracting attention of researchers for many years already and many theoretical studies were dedicated to them starting from excitons in 1D [1], metallic [3] and dielectric [5] nanoparticle arrays and cold atomic chains [4]. The reason for this is that one-dimensional systems present an attractive object for studies from purely fundamental point of view. Moreover, they can find a potential application as a subwavelength waveguide to transmit optical signals in nanophotonics and plasmonics, or as a new protocol for single photon storage in quantum information processing [6].

In this work we show that it is possible to obtain a collective state of coupled dipoles with very large lifetime. We also explain the observed a substantially decreased emission rate by employing the multipole analysis and demonstrate that the effect appears due to simultaneous reduction of multipole contributions of high orders.

2. Theoretical framework: a toy model of coupled resonant dipolar scatterers

We begin by considering a periodic 1D array of electric dipole scatterers (Fig. 1, a), which are located on the \( y \) coordinate axis so that the axis origin is situated in the middle of a chain. The dipole moments are restricted to be aligned along the \( z \) axis. Different dipole scatterers are coupled through the electric dipole-dipole interaction, which can be described in the following way:

\[
d^{(n)} = \alpha(\omega)4\pi k^2 \sum_{l \neq n} G_{0,zz}(r_n, r_l, \omega) d^{(l)},
\]

where \( d^{(n)} \) is the dipole moment of the \( n^{th} \) scatterer, \( \alpha(\omega) \) is the polarizability, \( k = \omega/c \) and \( G_0(r, r', \omega) \) is the vacuum Green’s tensor. For the sake of simplicity we are going to replace \( \omega \) with a constant frequency \( \omega_0 \).
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scatterer we can set $\alpha$ with the value $\omega$ the number of scatterers $N$ emission rate for several values of $j$ a very small emission rate. c) Demonstrates the contribution of multipole moment $y$ versus the period of a chain $\Delta y$. Note the red circle marking a particular point of interest with a very small emission rate. The emission rate given by Re[$\lambda_k \alpha_0 k_0$] with $\lambda_k$ being the eigenvalue of a state $k$ with the smallest value of $\langle f^{(k)}_{i,i+1} \rangle$ plotted versus the period of a chain $\Delta y$. d) The minimal value of Re[$\lambda_k \alpha_0 k_0$] versus the number of scatterers $N$ (blue circles) and a linear fit in the double log plot (red solid line). A good matching with the linear fit says that emission rate scales as $\sim N^{-6.76}$.

Figure 1. a) Scheme of the system under consideration (top) for $N = 4$ dipole particles, arrows on each scatterer represent the orientation of dipoles moments in a state under consideration. Resonant electric dipolar response of a single scatterer (bottom). b) The emission rate given by $\alpha$ on each scatterer represent the orientation of dipoles moments in a state under consideration. c) Scheme of the system under consideration (top) for $N = 4$ dipole particles, arrows labeled by the system. For spherical multipoles in the case of a single point electric dipole the collective modes of the system. Now it is possible to perform a multipole expansion for field radiated by the system. For spherical multipoles in the case of a single point electric dipole labeled $n$ we have:

$$b^{(k)}_{j,m} = \frac{4\pi k^3}{\sqrt{j(j+1)}} \left( -\frac{im}{\sin \theta_n} j_j(kr_n) Y_{j,-m}(\theta_n, \phi_n) d^{(k,n)}_\theta - j_j(kr_n) \frac{\partial Y_{j,-m}(\theta, \phi_n)}{\partial \theta} \bigg|_{\theta=\theta_n} d^{(k,n)}_\phi \right);$$

$$d^{(k,n)}_\theta = \frac{4\pi k^3}{\sqrt{j(j+1)}} \frac{1}{kr_n} j_j(kr_n)(j+1) Y_{j,-m}(\theta_n, \phi_n) d^{(k,n)}_r + \frac{\partial(r j_j(kr))}{\partial r} \bigg|_{r=r_n} d^{(k,n)}_\theta + \frac{im}{\sin \theta_n} \frac{\partial(r j_j(kr))}{\partial r} \bigg|_{r=r_n} Y_{j,-m}(\theta_n, \phi_n) d^{(k,n)}_\phi,$$

where $Y_{j,m}(\theta, \phi)$, $j_j(kr)$ are the spherical harmonics and spherical bessel functions, respectively, $r_n, \theta_n, \phi_n$ are the coordinates of an $n$th dipole in spherical coordinate system. Index $k$ labels the eigenstate, while $n$ specifies a particular dipole in a chain. With (3) we can construct the expansion for a dipolar chain mode by simply performing the summation over different particles: $a^{(k)}_{j,m} = \sum_{n=1}^{N} a^{(k,n)}_{j,m}$. The coefficients $a^{(k)}_{j,m}, b^{(k)}_{j,m}$ express the field radiated by the system and observed at a point $r$ which is further away from the coordinate system center than any

with the value $\omega_0$ corresponding to the resonance of $\alpha(\omega)$. In this case for a lossless non-magnetic scatterer we can set $\alpha(\omega_0) = \alpha_0 = i\frac{3}{2k_0^3}$ leading to the following system:

$$M(\omega_0) \cdot \textbf{d} = 0, \quad M_{nn}(\omega_0) = -i\frac{2k_0^3}{3}, \quad M_{ln} = -4\pi k_0^2 G_{0,zz}(r_1, r_n, \omega_0)$$

(2)

For $N$ scatterers there are $N$ eigenvalues $\lambda^{(k)}$ and eigenvectors $d^{(k)}$ of (2), which gives us the collective modes of the system. Now it is possible to perform a multipole expansion for field irradiated by the system. For spherical multipoles in the case of a single point electric dipole labeled $n$ we have:
of dipoles (|\mathbf{r}| > |\mathbf{r}_n|): \textbf{E}_\omega^{(k)}(\mathbf{r}) = \sum_n \sum_{j=1}^{+\infty} \sum_{m=-j}^{+j} \left( a_{j,m}^{(k,n)} N_{j,m}(\mathbf{r}_n) + b_{j,m}^{(k,n)} M_{j,m}(\mathbf{r}_n) \right) \text{ with } N_{j,m} \text{ and } M_{j,m} \text{ being proportional to the usual spherical Mie harmonics (TM and TE waves, respectively).}

The system of \( N \) dipoles described by (1) has \( N \) eigenstates, in order to characterize them we can introduce the nearest-neighbor correlation function [2]: \( \langle f^{(k)}_{i,i+1} \rangle = \frac{1}{N-1} \sum_{i=1}^{N} \cos \left[ \arg(d^{(k,i+1)} c_{i+1}) - \arg(d^{(k,i)}) \right] \). It can be thought of as a function characterizing the degree of codirectionality of neighboring dipoles on average, and this function is always between \(-1\) and \(+1\).

3. Results
Among all the \( N \) eigenstates of the system let us focus on one with the smallest value of \( \langle f^{(k)}_{i,i+1} \rangle \), which means that for any neighboring pair of scatterers their dipole moments are out-of-phase. We can plot the value of Re \( \lambda k \alpha_0 k_0 \) which has a meaning of the emission rate of this state, versus system period \( \Delta y \) (see. Fig. 1, a). One can see a set of local minima out of which we are interested in the one with the smallest possible value of the emission rate, which is marked by a red circle. Simultaneously we can plot the contributions of the spherical multipoles for different \( j \) (Fig. 1, b and c). As can be seen from the figures, multipoles with different values of orbital angular momenta \( j \) exhibit a strong simultaneous reduction in this region. The emission rate for this state also demonstrates a rapid decrease with the number of scatterers following the \( \sim N^{-6.76} \) dependency (Fig. 1, d).

4. Conclusions
In this work we studied the transverse optical modes of a periodic 1D chain of dipole scatterers with a very low emission rate. We demonstrated that there exist such states that for specific system periods demonstrate a significant reduction in multipole coefficients with high orbital angular momentum values \( j \). We also showed that for these states the emission rate decreases as \( \sim N^{-6.76} \) with the number of particles.

These findings might find applications in the fields of nanophotonics serving as a subwavelength waveguides for optical excitations. It might also be important in the fields of quantum optics and light-matter interactions where such states with prolonged lifetimes are of importance for modern quantum technologies, e.g. quantum memory.

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