Realistic theory of microscopic phenomena;  
a new solution of hidden-variable problem.

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Abstract

Einstein’s critique of the Copenhagen interpretation of quantum mechanics was based fully on logical and philosophical arguments, while this interpretation seemed to be strongly supported by von Neumann who argued that any local (or hidden-variable) theory is not admissible by the quantum-mechanical mathematical model. However, his statement was not true as shown convincingly by J. Bell in 1964; an unphysical assumption was involved in von Neumann’s approach. After modifying the corresponding axiomatic basis Bell derived some inequalities, which was believed to hold for any kind of local (hidden-variable) theories applied to EPR experiments in Bohm’s modification (coincidence measurement of spin orientations). These inequalities were then violated by experimental data, which was interpreted mostly as an important support of the Copenhagen interpretation. However, Einstein’s objections were not removed, and thus the question has remained: Do the assumptions involved in Bell’s approach cover really all possible hidden-variable theories or not? After the experience with von Neumann’s proof such a question has been fully justified. Doubts should concern the fact that some non fully reasoned mathematical operations (interchange of probability factors) was made used of in deriving Bell’s inequalities. These operations have been shown recently (http://xxx.lanl.gov/quant-ph/9808005/) to be allowed only if an important simplification in a general local theory has been introduced. Similar a-priori assumptions have been involved in other kinds of derivations of the given inequalities.

As to the quantum-mechanical mathematical model (representing the basis of all considerations) the central problem is to be seen in the definition of the Hilbert space in which all solutions of time-dependent solutions of Schroedinger equation have been represented. The actual physical properties of such solutions have been strongly modified if Hamiltonian eigenfunctions have been chosen as the vector basis of the given Hilbert space and an (even basic) physical meaning has been attributed to them. Having chosen another (suitably extended) vector basis it has been possible to propose a realistic theory of microscopic phenomena covering all experimental results that have been claimed to be in agreement with predictions of the standard quantum-mechanical model. It concerns not only all stationary characteristics of microscopic physical systems but also the EPR experiments even if they must be interpreted rather differently now as the internal space (geometrical) structure of measuring devices must be taken into account. In addition to spin orientations they are also impact parameters in collisions of photons with individual interaction centers that play the role of hidden variables from the point of view of the standard quantum mechanics. All previous logical problems have been removed and any paradoxical behavior need not be required more in the proposed extended model.

1. Introduction

The most physicists believe in Copenhagen interpretation of the quantum mechanics including paradoxical behavior of the microscopic world, even if it has not been shown until now how the laws of macroscopic world might be derived from such a basic microscopic
concept. Consequently, discussions concerning the famous controversy between Einstein and Bohr have intensively continued in the whole century and corresponding symposia have been held in regular intervals (see, e.g., ).

In all these discussions the standard quantum-mechanical mathematical model has been practically always assumed to represent the only possible description of microscopic physical phenomena. The discussions have concentrated practically to the question, which of the two different interpretations of such a model should be preferred:

(i) orthodox (Copenhagen) - all information about a physical system at a given instant \( t \) being contained in a \( \psi(x,t) \)-function derived by solving the corresponding time-dependent Schrödinger equation and represented by a vector in the Hilbert space spanned on Hamiltonian eigenfunctions;

(ii) ensemble (statistical) - only statistical characteristics being described by such a \( \psi(x,t) \)-function, which corresponds to Einstein’s opinion that quantum mechanics cannot be considered complete.

The latter interpretation requires, of course, the existence of some other parameters that would characterize a physical system at a given time in addition to stationary quantities defining the usual \( \psi(x) \)-function. That requires, however, to look for a mathematical model in which the corresponding so called hidden variables will be included. However, practically until now any actual serious attempt of proposing a corresponding extended model has not been done (or at least has not been successful).

Such an extended model and its physical characteristics and consequences will be described in the following. However, to make the problem more understandable it is necessary to start with the analysis of the contemporary quantum-mechanical model. In individual sections the following items will be treated:

(i) the source of paradoxical properties in the standard quantum-mechanical model will be discussed;

(ii) an extended model will be proposed and its properties will be shown;

(iii) the actual meaning of EPR experiments (and corresponding experimental results) will be discussed in the light of the new extended model;

(iv) some predictions of the extended model (differing from those of the standard quantum-mechanical model) will be mentioned;

(v) the way how the deterministic (semiclassical) behavior and the probabilistic one combine in the extended model will be discussed;

(vi) consequences concerning some physical concepts as well as general thinking of human society will be mentioned.

2. Quantum mechanics and the origin of paradoxical properties

It is possible to state that the following assumptions represent basic ingredients of the quantum-mechanical model:

(i) the behavior of any physical system is assumed to be described by a \( \psi(x,t) \)-function obtained as a solution of time-dependent Schrödinger equation

\[
i \frac{\partial}{\partial t} \psi(x_j; t) = H\psi(x_j; t) \tag{1}\]
where $H$ is the corresponding Hamiltonian and $x_j$ represent space coordinates of all involved mass objects;

(ii) individual functions $\phi_\tau(x_j) \equiv \psi(x_j;\tau)$ are represented by vectors in an Hilbert space; all physical characteristics being derived with the help of rules holding for operators and vectors in the given Hilbert space, if the vectors represent individual physical states and the operators physical quantities (see, e.g., Ref. [4]).

However, these two basic assumptions would not have to lead to any paradoxes (as will be shown later) if the structure of corresponding Hilbert space were not defined with the help of the additional assumption:

(iii) the vector basis of the given Hilbert space is formed by Hamiltonian eigenfunctions $\psi_E(x_j)$:

$$H\psi_E(x_j) = E\psi_E(x_j) ,$$

(2)

and a basic physical meaning is attributed to these eigenfunctions. In addition to, the superposition principle has been introduced and each unit vector of such a Hilbert space has been assumed to correspond to a possible state of a given physical system.

We will show now that a kind of discrepancy exists between assumption (iii) and assumption (i). Schrödinger equation (describing a physical system defined by corresponding Hamiltonian $H$) provides different solutions characterized by a set $\kappa$ of physical quantities being conserved during whole evolution and by initial conditions of changing quantities (i.e., before all by positions and momenta of individual matter objects). The values of all these quantities are defined as expectation values of corresponding operators. It is possible to derive time-dependent positions and momenta of all objects from individual solutions $\psi(x_j; t)$, i.e., as the expectation values of corresponding operators, which may be brought to the correspondence with solutions of Hamilton equations (describing the system consisting of $N$ matter objects with the help of the same Hamiltonian). Consequently, they might be represented in principle by corresponding trajectories in the standard ($6N$-dimensional) phase space; individual trajectories being mutually fully separated (i.e., without any common points). And it might be expected that evolution of a physical system (consisting of $N$ stable matter objects) will be represented by trajectories with similar properties also in Hilbert space chosen in a suitable way in harmony with assumption (ii).

However, the situation have changed drastically when assumption (iii) has been added to the first two assumptions, and especially, when Hamilton eigenfunctions have corresponded to basic physical states. Individual solutions of Schrödinger equation (1) may be then represented by sequences of unit vectors in such an Hilbert space or by trajectories crossing mutually; i.e., states belonging to differently chosen initial function $\psi(x_j;0)$ or to different values of $\kappa$ are often represented by the same vectors. In such a case a special measurement postulate had to be introduced enabling to derive predictions concerning experimental data. And one can say that practically all quantum-mechanical paradoxes follow from assumption (iii).

Representing the solutions of Schrödinger equation in an Hilbert space has provided, however, a suitable basis for analyzing microscopic phenomena when some processes must be described as probabilistic. And one must ask if it is possible to modify assumption (iii) to be in better agreement with the characteristics of Schrödinger equation. The possibility of using a more suitably defined (extended) Hilbert space was discussed, e.g., by Rosenbaum in
1969 (and subsequently by some other authors). However, the extension considered in [6] is to be denoted as too formal and too general; it did not reflect actual physical conditions. In the next section a kind of minimum extension reflecting the physical situation will be described and some physical consequences derived.

3. Extended Hilbert space

A suitable extension of the Hilbert space was proposed in principle independently by three groups of authors many years ago [7, 8, 9]. They were Lax and Phillips [7] in 1967 who defined such a mathematical model for the first time; they used it for the description of some acoustic and optical phenomena, which invoked impression that the model was suitable for being applied to some semiclassical problems only. Later the same Hilbert structure was derived by Alda et al. [8] in solving the problem of a purely exponential decay law of unstable particles. And finally, Newton [9] showed that it was possible to define regularly the time operator in the case of harmonic oscillator when a similar extended Hilbert structure was made use of, while it was not possible to do it in the standard Hilbert space as pointed to by Pauli in 1938.

We shall illustrate basic characteristics of the mentioned approach with the help of the physical system consisting of two zero-spin particles, which represents the smallest system exhibiting time evolution. Its behavior in the center-of-mass system may be described by Schrödinger equation (1) with Hamiltonian

$$H = \frac{p_j^2}{2m} + V(q_j); \quad (3)$$

where $m$ is the reduced mass of the particle pair, and the operators of relative coordinates $q_j$ and of momentum components $p_j$ of one particle (in the center-of-mass system) are assumed to fulfill the following commutation relations

$$[q_j, p_k] = i\delta_{jk}, \quad [p_j, p_k] = 0, \quad [q_j, q_k] = 0. \quad (4)$$

Introducing two other operators

$$Q = q_j^2, \quad R = \frac{1}{2}\{p_j, q_j\} \quad (5)$$

and assuming $V(q_j) = V(Q)$ (i.e., the mutual potential depends on the distance between particles) one can write further

$$i[H, q_k] = \frac{p_k}{m}, \quad i[H, p_k] = -2q_k \frac{dV(Q)}{dQ}, \quad (6)$$

$$i[H, Q] = \frac{2}{m}R, \quad i[H, R] = 2\{H - V - Q \frac{dV(Q)}{dQ}\}. \quad (7)$$

It is also possible to introduce angular-momentum operator fulfilling relations

$$M_{jk} = [p_j, q_k], \quad [M_{jk}, H] = 0; \quad (8)$$
and further operator
\[ M = M_{jk}M_{jk} \]  
fulfilling relations
\[ [M, M_{jk}] = [M, H] = 0. \]

Individual trajectories corresponding to different solutions of Schrödinger equation may be then characterized by expectation values of mutually commuting operators; i.e., by \( <H> \), \( <M> \), and, e.g., by \( <M_{12}> \), which may represent the mentioned \( \kappa \) set for a given particle pair. Different points on individual trajectories may be then distinguished with the help of expectation values of operator \( R \).

Assuming the particle pair to be in an unbound state (i.e., belonging to continuous energy spectrum of the Hamiltonian) it follows from Eq. (7) that the expectation value of \( R \) always increases for smooth repulsive potentials; i.e., when the function \( dV(Q)/dQ < 0 \). It may rise, of course, for attractive potentials, too, as far as the kinetic energy is sufficiently large; the expectation value of it increases in principle from \( -\infty \) to \( +\infty \). Negative values of \( <R> \) characterize incoming states of the particle pair and positive values outgoing states; minimum distance between both the particles corresponding to \( <R> = 0 \).

For the states belonging to the discrete part of Hamiltonian spectrum (particle pair being in a bound state) the operator \( R \) ceases to exhibit a monotone behavior; it changes periodically during the evolution. In such a case its expectation values are not sufficient to distinguish between all different physical states and some other operators must be introduced, as will be shown in the other part of this section (the case of harmonic oscillator).

In any case the standard Hilbert space (defined according to assumption (iii)) is not sufficient to characterize behavior of a particle pair in agreement with reality as all states belonging to different values of \( R \) are represented practically by one common vector. To distinguish all different states (e.g., pair particles at different distances) the mentioned extended Hilbert space must be made use of. As to the continuous Hamiltonian spectrum it is the Hilbert space introduced in Refs. [7, 8]. For bound states (corresponding to discrete spectrum) the structure of Hilbert space is a little different; see the example of harmonic oscillator (comp. also Ref. [9]).

**Continuous Hamiltonian spectrum**

We will discuss the special case of conformal potential
\[ V(Q) = \eta Q^{-1}. \]  
The corresponding Hamiltonian has continuous spectrum for all real values of \( \eta \), and only collision states exist. It holds then in such a case
\[ i[H, R] = 2H \]  
and the operator \( R \) exhibits evidently a constant increase for any (positive) energy, independently of \( \eta \) value. It is possible to define operator
\[ T = \frac{1}{4}\{H^{-1}, R\} \]
fulfilling the commutation relation:

\[ i[H, T] = 1. \]  

(14)

It means that it is possible to define the time operator as a function of operators \( q_j \) and \( p_j \). One-to-one correspondence exists between expectation values of \( R \) and \( T \); zero values for both these operators corresponding to the minimum distance between the two particles.

It holds for expectation values of \( T \) corresponding to instantaneous states \( \psi_\kappa(x_j; \tau) \) of a particle-pair system:

\[
< \psi_\kappa(x_j; \tau)|T|\psi_\kappa(x_j; \tau) > = \tau
\]

(15)

for any \( \kappa \) and \( \tau \). Introducing the evolution operator

\[
U(t) = e^{-iHt} \quad (t > 0)
\]

(16)

it holds also

\[
U(t) |\psi_\kappa(x_j; \tau) > = |\psi_\kappa(x_j; \tau + t) > ,
\]

(17)

which indicates that expectation values of the operator \( T \) defined by Eq. (13) may be hardly identified with the parameter of flowing time. They characterize instantaneous states in a special scale, i.e., with the help of time expressing the distance from the state \( \psi_\kappa(x_j; 0) \). The evolution operator moves the states always to higher values of \( \tau \), or from in-states to out-states (belonging to negative, resp. positive, expectation values of \( T \)).

As to the structure of the extended Hilbert space we have already mentioned that it was introduced by Lax and Phillips [7] and derived in Ref. [8] by requiring an exact exponential decay law to hold for unstable objects. A detailed mathematical description of such an Hilbert space may be found also in Ref. [10]. Taking a \( \psi(x_j) \) function (i.e., for a special value of \( t \)) from possible solutions of time-dependent Schrödinger equation one can easily see that the same function may belong to an incoming state as well as an outgoing one. To distinguish such two states it is necessary for the corresponding Hilbert space to consist of two mutually orthogonal subspaces (corresponding to incoming and outgoing states); the bases of each of them being formed by all functions \( \psi(x_j) \) derived by solving the time-dependent Schrödinger equation; i.e. by all functions \( \phi_{\tau,\kappa}(x_j) \equiv \psi_\kappa(x_j; \tau) \) belonging to all possible values of \( \kappa \) and \( \tau \). The basis of the whole Hilbert space must be then defined with the help of function pairs; i.e. by

\[
\Phi_{\kappa,\tau}(x_j) \equiv \left\{ \frac{1-\varepsilon(\tau)}{2}.\phi_{\tau,\kappa}(x_j), \frac{1+\varepsilon(\tau)}{2}.\phi_{\tau,\kappa}(x_j) \right\}
\]

(18)

where

\[
\varepsilon(\tau) = sign \tau
\]

(19)

and \( \kappa \) represents the set of expectation values \( < H >, < M > \) and \( < M_{12} > \).

The solutions of the time-dependent Schrödinger equation for the conformal potential may be found e.g. in Ref. [11]. We will limit here to the special case of \( \eta = 0 \); the general structure of the Hilbert space being fully conserved. One can write then in Eq. (18)

\[
\phi_{\tau,\kappa}(\vec{x}) = \int d\vec{k} \, g_\kappa(\vec{k}) \, e^{ik(\vec{x}-\vec{x}_i-\vec{k}\tau)/2m}
\]

(20)
where $\tilde{x}_i$ and $\tau_i$ represent values corresponding to an initial state and function $g_\kappa(\vec{k})$ fulfils the condition

$$
(2\pi)^{-3} \int d\vec{k} \mid g_\kappa(\vec{k}) \mid^2 = 1
$$

is an arbitrary function of vector $\vec{k}$, at least in principle; it must be chosen so as to correspond to values $\kappa$ in an initial three-dimensional state.

The states in the extended model are characterized also by expectation values of operator $M_{jk}$, which enables to establish immediately impact parameter for two-particle collision states. It may be defined as a position vector corresponding to the minimum distance between the particle pair during the evolution (i.e., in the state characterized by $< R > = < T > = 0$). The impact parameter value may be derived from $\kappa$ set and represents an indivisible part of characteristics belonging to the extended model. If two extended objects collide important changes of the physical system may occur in transitions from in- to out-states in dependence on impact parameter value, which is, however, the problem lying outside the scope of this paper. In the standard quantum-mechanics the impact parameter has been eliminated from the description by the $\psi(x)$-function and its value (or its probability distribution) has had to be added. In the extended model the statistical distribution of initial value $\tilde{x}_i$ is responsible for probabilistic characteristics of the quantum-mechanical measurements.

**Discrete Hamiltonian spectrum**

It is then possible to represent internal evolution of a bound particle pair in a correspondingly extended Hilbert space in a similar way. There are, of course, some differences against the preceding case as the expectation value of $R$ changes periodically and it is not possible to make use of it in defining individual states of a bound system. It is necessary to introduce more suitable operators.

We will demonstrate the corresponding approach on the example of harmonic oscillator when the Hamiltonian possesses a mere discrete spectrum; i.e., only bound states may exist. The potential between two particles may be written now as

$$
V(Q) = \frac{k}{2} Q .
$$

The attractive force aims always to the common center of mass and the behavior of a three-dimensional harmonic oscillator may be described as the product of three (or at least two) linear harmonic oscillators. In the following we will limit ourselves to the simple one-dimensional case.

Instantaneous states of linear oscillator may be described with the help of operators

$$
C = \sqrt{\frac{k}{2}} \left\{ H^{-1/2}, q \right\} ,
$$

$$
S = -\sqrt{\frac{1}{2m}} \left\{ H^{-1/2}, p \right\} ,
$$

which fulfill relations

$$
i [H, S] = \omega C ,
$$

7
\[ i[H, C] = -\omega S, \]  
\[ <C^2 + S^2> = 1, \]  
\[ <CS - SC> = 0 \]  
(26)  
(27)  
(28)

where
\[ \omega = \sqrt{\frac{k}{m}}. \]  
(29)

It is then possible to introduce the phase operator
\[ \Phi = \arccos C = \arcsin S \]  
(30)

and also the time operator
\[ T = \frac{1}{\omega} \Phi \]  
(31)

fulfilling Eq. (14).

The representation Hilbert space is now more complicated than in the preceding continuous case. While physically different states are distinguished with the help of expectation values of \( C \) and \( S \) the expectation values of \( \Phi \) and \( T \) may be equal to any real value as evolution operator (16) evokes their steady increase. Two mutually orthogonal subspaces should then correspond to any interval \((2n\pi, 2(n+1)\pi)\) of expectation values of phase operator \( \Phi \). The evolution operator moves the states from one subspace to another; physically important values \(<C>\) and \(<S>\) changing periodically. Details concerning this Hilbert space structure for harmonic oscillator will be given elsewhere.

As to the given structure it is possible to say that it may be denoted as a solution of the phase-operator problem opened already by Dirac in 1927 [12] and not yet satisfactorily solved (see, e.g., Lynch [13]). Very recently Ozawa [14] showed that the problem might be solved in the framework of an extended Hilbert space constructed by him to such a purpose. It might be interesting to compare the structures of the Hilbert space of Ozawa and that of ours.

**Semiclassical properties of the extended model**

The proposed extended model enables to describe the behavior of microscopic physical systems consisting of a fixed number of stable particles practically in a semiclassical and fully realistic way. The evolution is represented by a trajectory in a Hilbert space characterized by corresponding values of \( \kappa \). Consequently, any additional measurement postulate is not more needed. There is not any difference in the predictions of the stationary characteristics (being conserved during whole evolution) by the standard quantum mechanics and by the extended model.

Important difference concerns the description of dynamic processes as there is not more possible for states belonging to different stationary characteristics to combine (and to form new pure states by superpositions); any transitions between different \( \kappa \) values are not possible, either. They are also Hamiltonian eigenfunctions that do not belong to extended Hilbert space. The proposed extended model enables then to describe newly transition phenomena, i.e., inelastic collisions and decay processes (see Sec. 6). Also the EPR problem may be now
discussed from a quite new point of view.

4. EPR experiments and Bell’s inequalities

The original goal of the EPR Gedankenexperiment proposed by Einstein et al. [1] was to argue on logical grounds that quantum mechanics could not be considered complete, and consequently, that it was necessary to add some other characteristics (i.e., the so-called hidden variables) if any microscopic system is to be described fully and in a realistic way. However, the logical arguments were not sufficient for the then physical community. The most physicists believed in the standard mathematical model more than in a realistic interpretation. And Bohr’s arguments [2] were almost generally accepted at that time.

Nevertheless, the discussion concerning the controversy between Einstein and Bohr has continued during this century. It is not possible to repeat the whole story of this problem here. We shall start with the impact that came when J. Bell [15] derived his inequalities. It was hoped that it would be possible to solve the old controversy with the help of experimental results on their basis. A new search for feasible experiments of EPR type was initiated.

Experiments based on the coincidence measurements of two equally polarized photons passing through polarizers in opposite directions were proposed and also performed. If \( a_\alpha \) and \( b_\beta \) are probabilities of single photons passing through individual (opposite) polarizers at given settings \( \alpha \) and \( \beta \) then it should hold according to Bell in the realistic interpretation for the combination of any four coincidence probabilities:

\[
\alpha \beta + a_\alpha b_\beta + a_\alpha b_\beta' - a_\alpha' b_\beta' \leq 2.
\] (32)

The series of corresponding experiments started approximately in 1971 and were finished practically in 1982 (see Ref. [16]) with the following results:

(i) Bell’s inequalities (32) have been surely violated for specially chosen orientations of polarizer axes, for which according to quantum-mechanical predictions a value greater than two should be obtained;

(ii) the experimental results may be regarded as being in agreement with quantum-mechanical predictions.

These results seemed to prove definitely the quantum-mechanical concept of particle non-locality. And a series of physicists started to develop world picture based on such a concept. Discussions were initiated how to make use of the concept of non-locality in practical applications; see a series of articles published in Physics World [17] about the so-called teleportation, cryptography and so on.

However, all theoretical considerations concerning these problems have been based on some additional assumptions going often far beyond the standard quantum mechanics; sometimes even contradicting them; see Ref. [18]. And all experimental results being claimed as a support of these new ideas have been interpreted incorrectly as argued recently also by Klyshko [19]. Sometimes the given interpretation has been based on the reversal of logical implication. There is not any doubt, either, that the so-called interference phenomena (i.e., periodically changing results in all these experiments) are based on changing time of flight of individual photons between two macroscopic objects and should be interpreted on the same basis as Newton fringes as mentioned also in Ref. [18].
There are, however, also papers looking for a more realistic concept of the physical world. E.g., S. Goldstein [21] returned recently to Bell’s ideas starting from Bohm’s results, which made it possible to attribute a definite track and to go back to an ontological interpretation of the microscopic world. However, such a goal can be hardly reached if one tries to solve the problems in the framework of the standard quantum-mechanical mathematical model. The ontological tendencies are, of course, in full harmony with the just discussed extended model the principles of which were mentioned for the first time in Ref. [21] and more systematically explained in Ref. [22].

It is the violation of Bell’s inequalities by experimental data which still seems to represent an important argument in favor of the standard quantum-mechanical model and, consequently, against the extended model. However, it has been shown recently that these inequalities have been applied to experimental data in an inadequate way.

It has been always believed that Bell inequalities have been fully based on locality condition only. However, already in the first derivation Bell had to use an additional mathematical operation (interchange of some factors belonging to different coincidence combinations), which was regarded as fully acceptable. In fact this operation has represented a step by which the locality concept has been significantly limited. And similar limiting assumption has been used in all other kinds of deriving these inequalities, the problem having been analyzed in Ref. [23].

While the space orientation of photon spin has been taken always into account the additional assumptions (corresponding to used approaches) has allowed to respect neither an actual space orientation of the photon pair nor any internal microscopic structure of polarizers (or measuring devices). Then it is not possible to respect exact impact parameters of individual photons into the atom grids of polarizers at given settings (and their corresponding weight distributions); any averaging in coincidence experiments must be performed over given pairs (not over events in single polarizers). Bell inequalities cannot be derived when a full realistic (locality) concept is taken into account. And it is not more possible to argue that the results of EPR experiments contradict the locality of microparticles [24].

Then of course, one point more remains to be explained: the fact that EPR results may be taken as being in agreement with quantum-mechanical predictions. In this case it is necessary to start with explaining the actual essence of EPR experiments. Their result does not concern any direct predictions of quantum-mechanical model. The experiments consist in demonstrating that the results are the same in one-side arrangement as well as in the coincidence one:

\[
\begin{align*}
\beta & \quad \alpha \\
\o & \quad \rightarrow \\
\alpha & \quad \beta \\
\leftarrow & \quad \rightarrow \\
\o & \quad \rightarrow \\
\end{align*}
\]

I.e., in showing that the transfer of non-polarized light through two polarizers may be de-
scribed in both the cases with the help of the generalized Malus law ($\beta$ being put zero)

$$M(\alpha) = (1 - \varepsilon)\cos^2\alpha + \varepsilon$$  \hspace{1cm} (33)

where $\varepsilon$ is always a non-zero quantity. To obtain identical results in both the different arrangements follows immediately from the quantum mechanics for ideal polarizers (when $\varepsilon = 0$). And it is assumed (without any actual proof) that the same may be expected for real polarizers, too.

One can, however, show that the same predictions may be derived for both the arrangements in a very simple local (hidden-variable) theory. Assuming for simplicity (in the first approximation) that the change of photon polarization during its passage through a polarizer may be neglected, one can write in both the arrangements

$$M(\alpha) = \int d\lambda p_1(\lambda) p_2(\lambda - \alpha)$$  \hspace{1cm} (34)

where $p_2(\lambda)$ are transfer probabilities of the light (photons) through individual polarizers; $\lambda$ representing the deviation of the polarization direction from the axis of the first polarizer (putting again $\beta = 0$).

Consequently, in contradistinction to common opinion the results of EPR experiments with polarized photons cannot bring any decision concerning the preference between the standard quantum mechanics and the new extended realistic model. However, the decision between these two models may be given with the help of other experiments, e.g., of those concerning the light transfer through three polarizers.

5. Experiments with three polarizers

As shown in the preceding the EPR experiments can hardly bring any decision between the predictions of quantum-mechanics and those of a hidden variable theory. And we may ask, whether it is possible to find an experiment which could contribute to the solution of this question. Combining Eqs. (33) and (34) and putting $p_2(\lambda) = p(\lambda)$ it has been possible to determine the shape of the function $p(\lambda)$ when the generalized Malus law is to hold for a pair of polarizers. And having used this function in deriving angle dependence of light transfer through three polarizers the predictions differing significantly from quantum-mechanical ones have been obtained. The function $p(\lambda)$ differs rather significantly from $\cos^2(\lambda)$-function used by Belifante [24] in arguing that the predictions of a local theory are significantly different from the quantum-mechanical predictions.

It means that a detailed analysis based on experiments with three polarizers could bring the whole problem by an important step further; especially, where preliminary measurements (see Refs. [25, 26]) indicate that the results are rather far from any quantum-mechanical characteristics. Predictions corresponding approximately to experimental results may be obtained with the help of Müller matrices starting from the description of the polarized light proposed by Stokes. However, neither Müller matrices seem to be able to reproduce fully actual experimental results. Thus, the given experiments open new deeper questions, whether commonly used mathematical approaches are sufficient to characterize different degrees of polarization; the problem being discussed recently also by Movilla et al. [27].
6. Deterministic and probabilistic behaviors of physical systems and realistic model

Let us go now back to a physical system consisting of two non-bound particles (states corresponding to continuous Hamiltonian spectrum). As mentioned above the individual subspaces in the extended representation Hilbert space may be denoted as subspaces of incoming and outgoing states. And one can aver that any evolution of a physical system should be regarded as irreversible since the evolution operator \( \tau \) transforms always the states from negative \( \tau \) to positive \( \tau \) and never in the opposite direction (i.e., from “in” to ”out”). Even inside the individual subspaces the evolution goes always from lower values to greater values of \( \tau \) (see \([8, 10]\)).

Some more complicated behavior may occur around the value of \( \tau = 0 \). In the case of an elastic collision the evolution goes continuously from ”in”- to ”out”-states, being represented by Eq. (18). However, if the mutual impact parameter is sufficiently small then an inelastic collision of non-point objects may occur and the characteristics of the system may change substantially. In such a case the system may pass to an out-state belonging to a quite different type of particles. However, the following evolution goes again (in the corresponding out-subspace) along the trajectory characterized by expectation values being conserved during the whole evolution (i.e., by the set of given \( \kappa \) values). Some additional hidden parameters describing internal structures of colliding particles and their instantaneous values may play, of course, an important role in such transitions.

At the present we are forced to describe the transitions from a given in-space to a different out-space with the help of phenomenological probability functions derived from experimental data. In some cases probability values may be predicted on the basis of stripping theory (e.g., in the case of nuclear collisions). In the case of hadrons the extended model represents a new challenge to look for a corresponding more realistic model of internal characteristics of these matter objects and of their interactions; the most hitherto hadron collision theories being very far from a considered realistic concept.

It is possible to conclude that the deterministic as well as probabilistic behaviors are described in the framework of one mathematical model. The extended model enables to describe the deterministic evolution of semiclassical systems (i.e., when the numbers and kinds of objects do not change) in the framework of individual subspaces of the total Hilbert space. Probabilistic processes (e.g., inelastic collisions) may be then characterized with the help of transition probabilities from one subspace to another one.

The extended mathematical model opens the possibility to describe in a realistic way not only inelastic collisions but also spontaneous decay of unstable particles (see Refs. \([8, 10]\)). However, an unstable particle may be hardly represented by one vector in the Hilbert space but by a subspace being orthogonal to all other parts of the total Hilbert space. In the first approximation this subspace might be taken as \((n + 1)\)-dimensional when the given particle decays into \(n\) different channels. Such a structure of an unstable particle was considered already in Ref. \([28]\) (see also \([29]\)), in solving a kind of deviations from the simple Breit-Wigner formula in resonance scattering. The unstable (resonance) particle has been assumed to exhibit some random transitions between its internal structures corresponding to different decay channels before an actual decay has occurred.
7. Discussion and comments

The representation of physical characteristics with the help of Hilbert space spanned on Hamiltonian eigenfunctions caused that sets of different physical states were represented by one mathematical symbol, i.e., by one vector in the given Hilbert space. All quantum-mechanical paradoxes have followed from such a representation. The given mathematical framework has not been adequate to describe the richness of real physical structures. To cope at least partially with this problem two kinds of different physical states (pure and mixed states) have been introduced even if it was not practically possible to distinguish between them on experimental basis.

In the extended Hilbert space each actual physical state is represented by a vector \( \phi_{\tau,\kappa}(x_j) \) where \( \kappa \) represents the set of characteristics being conserved during the whole evolution. The meaning of parameter \( \tau \) may be illustrated, e.g., with the help of a two-particle system; \( \tau \) being the expectation value of the operator \( T \) defined by Eq. (13) or Eq. (31) in the two mentioned special cases. In the former case of conformal potential (and similarly for all non-bound particle pairs) \( \tau = 0 \) for the lowest possible mutual distance between both the particles during the evolution of a given physical system. Consequently, a numerical value of parameter \( \tau \) characterizes the instantaneous mutual distance of given particles (expressed in time units). During the evolution the change of \( \tau \) value is given by evolution operator (16); \( \tau \) increasing by \( t \).

Hamiltonian eigenfunctions do not belong to the extended Hilbert space and do not represent any physical states in the case of the extended model; the superposition principle cannot be applied to, either. Any superposition of vectors \( \phi_{\tau,\kappa}(x_j) \) corresponds always to a statistical mixture. And one should conclude that an actual shape of \( \psi(x_j) \) function has not any direct physical meaning. The physical meaning may be attributed to expectation values of corresponding operators, only. The same evolution may be then described with the help of differently chosen \( \psi(x_j) \) functions (comp. Eq. (20)), which may raise the question whether it is necessary to use the representation Hilbert space defined over the field of complex numbers in the extended model; even if the use of Hilbert space may be still very helpful.

As already stressed there is not any reason more to attribute the so much discussed quantum paradoxes to the microscopic world. The extended model seems to be based fully on particle picture of reality (including photons representing the quanta of light). In contradistinction to the standard quantum-mechanical model the extended model does not represent any closed physical theory; any theory of everything has not any place here. The model is open for exploring still deeper characteristics of the microworld. On the other side, it is not possible to state that a simple particle picture represents fundamental and definite features of matter reality; the question remaining open for further exploration.

There are many other new questions which have been opened now and which should be solved step by step. One of the central problems might concern the existence of time operator and its definition; time operator being a function of \( q_j \) and \( p_j \). One should ask whether the time is a basic quantity added to the space characteristics or a quantity derived from the behavior of matter objects in the usual three-dimensional space.

And finally, it is necessary to stress that one should respect again standard logical rules,
especially to take into account that any science or research cannot bring a decisive verification of our ideas or hypotheses. The scientific methods are based on the falsification approach, in principle. Since the only logical contradiction provides us with a reliable response to our questions we may know only the untruth with certainty, not the truth. It is not, of course, possible to denote as a truth what was already falsified in the past.

On the other side, the falsification approach does not entitle us to deny the existence of one truth about the world even if it must be perpetually looked for in the region of possibilities left by the certainly known untruth. There is not more possible, either, to argue on the basis of the physical science that many-valued logic should be applied to natural phenomena. There is not any reason to believe in the plurality of truth concerning the world even if many modern philosophers try to convince us about it, arguing often by the standard quantum-mechanical model and by its paradoxes.

8. Conclusion

Concluding I should like to mention an actual position being claimed for the extended model in the story of the physical science. It is possible to say that in addition to Planck’s discovery of the quantum structure of energy transfer and Rutherford’s experiments they were Einstein and Bohr who contributed mainly to the progress of the then physical research: Einstein by the prediction of the photon [30] and Bohr by formulating basic postulates concerning the atom orbit structure [31]. And one can also maintain that both these ideas belong still to fundamentals of the contemporary physics, being based fully on the realistic view of matter nature.

However, it is necessary to admit that in other papers of theirs both these physicists contributed fundamentally to that the realistic objective view was changed to a formal mathematical description of physical phenomena: Einstein by formulating the theory of special relativity [2] and Bohr by carrying out Copenhagen interpretation of the quantum-mechanical mathematical model [33]. Even if it is possible to say that Einstein personally returned to physical realism in papers concerning general relativity it is evident that the common thinking in the whole century has been fundamentally influenced by formal principles of special relativity and mainly of quantum mechanics.

As to the extended model it may be hardly regarded as a mere formal mathematical description of reality. It is rather possible to say that through this model the whole physical story may return to the physical views hold in the beginning of this century and characterized by the two formerly mentioned papers of Einstein and Bohr.

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