Source-Level Bitwise Branching for Temporal Verification

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Abstract—There is increasing interest in applying verification tools to programs that have bitwise operations. SMT solvers, which serve as a foundation for these tools, have thus increased support for bitwise reasoning through bit-blasting and linear arithmetic approximations. Still, verification tools are limited on termination and LTL verification of bitvector programs.

In this work, we show that similar linear arithmetic approximation of bitvector operations can be done at the source level through transformations. Specifically, we introduce new paths that over-approximate bitwise operations with linear conditions/constraints, increasing branching but allowing us to better exploit the well-developed integer reasoning and interpolation of verification tools. We present two sets of rules, namely rewriting rules and weakening rules, that can be implemented as bitwise branching of program transformation, the branching path can facilitate verification tools widen verification tasks over bitvector programs. Our experiment shows this exploitation of integer reasoning and interpolation enables competitive termination verification of bitvector programs and leads to the first effective technique for LTL verification of bitvector programs.

I. INTRODUCTION

In many case, verification tools (e.g. Ultimate [1]) fail in termination and LTL verification tasks over bitvector programs, due to complex bitwise reasoning. In this work, we introduce two sets of program transforming rules namely bitwise branching, which transform a bitwise program to an over-approximated version by adding linear constraints, this enable verification tools verify bitvector programs in integer domain.

For example, while ranking functions synthesis is critical in proving termination and temporal properties of programs, it is challenging to synthesize ranking functions over the domain of bitvectors due to the complexity of bitvector logics [2]. Ultimate, a state-of-the-art verifier for LTL verification, over-approximates bitwise operators as uninterpreted functions, and returns Unknown results for the following bitvector programs:

| (1) Termination | (2) LTL $\phi = \Box (\Diamond (n < 0))$ |
|-----------------|---------------------------------|
| $a = \ast; \text{assume}(a > 0)$; while ($x > 0$) { $a = a - 1$; $x = x \& a$; } | while ($x > 0$) { $a = a - 1$; $x = x \& a$; } |

In this work, we show that the key benefits of bitwise branching arise when concerned with termination and LTL. Example (1) involves a simple loop, in which $a$ is decremented, but the loop condition is on variable $x$, whose value is a bitvector expression over $a$.

Critical to verifying termination of this program are (1) proving the invariant ($I$) $x > 0 \land a > 0$ within the body of the loop and (2) synthesizing a rank function. To prove the invariant, tools must show that it holds after a step of the loop’s transition relation $T = x > 0 \land a' = a - 1 \land x' = x \& a'$, which requires reasoning about the bitwise-$\&$ operation because if we simply treat the $\&$ and as an uninterpreted function, $I \land T \land x' > 0 \implies I'$.

The bitwise branching strategy we introduce in Sec. [II] helps the verifier infer these invariants (and later synthesize rank functions) by transforming the bitvector assignment to $x$ into linear constraint $x \geq 0$ and $a \geq 0$.

In this case, bitwise branching translates the loop in Example (1) as depicted in the gray box to the right. This transformation changes the transition relation of the loop body from $T$ (the original program) to $T'$:

$T' = x > 0 \land a' = a - 1 \land ((x \geq 0 \land a' \geq 0 \land x' \leq a'))$

$\lor \neg (x \geq 0 \land a' \geq 0 \land x' = x \& a')$)

Importantly, when $I$ holds, the else branch with the $\&$ is infeasible, and thus we can treat the $\&$ and as an uninterpreted function and yet still prove that $I \land T' \land x' > 0 \implies I'$. With the proof of $I$ a tool can then move to the next step and synthesizing a ranking function $R(x, a)$ that satisfies $I \land T' \implies R(x, a) \geq 0 \land R(x, a) > R(x', a')$, namely, $R(x, a) = a$.

Bitwise branching also enables LTL verification of bitvector programs. We also examine the bitwise behavior of programs such as Example (2) above, with the LTL property $\Box (\Diamond (n < 0))$. Our experiments show that with bitwise branching, our implementation can prove LTL property of this program in 8.04s.

II. BITWISE-BRANCHING

We build our bitwise-branching technique on the known strategy of transforming bitwise operations into integer approximations [3, 4] but explore a new direction: source-level transformations to introduce new conditional paths that
Approximate many (but not all) behaviors of a bitvector program. These new paths through the program have linear input conditions and linear output constraints and frequently cover all of the program’s behavior (with respect to the goal property), but otherwise fall back on the original bitvector behavior when none of the input conditions hold. We provide two sets of bitwise-branching rules:

1. **Rewriting rules** of the form $C \vdash_E e_{bv} \rightsquigarrow e_{int}$ in Fig. 1A. These rules are applied to bitwise arithmetic expressions $e_{bv}$ and specify a condition $C$ for which one can use integer approximate behavior $e_{int}$ of $e_{bv}$.

In other words, rewriting rule $C \vdash_E e_{bv} \rightsquigarrow e_{int}$ can be applied only when $C$ holds and a bitwise arithmetic expression $e$ in the program structurally matches its $e_{bv}$ with a substitution $\delta$. Then, $e$ will be transformed into a conditional approximation: $C \delta ? e_{int} \delta : e_{bv}$. Note that, although modulo-2 is computationally more expensive, it is often more amenable to integer reasoning strategies. For conciseness, we omitted variants that arise from commutative re-ordering of the rules (in both Figs. 1A and 1B).

2. **Weakening rules** of the form $C \vdash_S s_{bv} \rightsquigarrow s_{int}$ are in Fig. 1B. These rules are applied to relational condition expressions (e.g., from assumptions) and assignment statements $s_{bv}$, specifying an integer condition $C$ and over-approximation transition constraint $s_{int}$.

When the rule is applied to a statement (as opposed to a conditional), replacement $s_{int}$ can be implemented as $\text{assume}(s_{int})$. When a weakening rule $C \vdash_S s_{bv} \rightsquigarrow s_{int}$ is applied to an assignment $s$ with substitution $\delta$, the transformed statement is $\text{if } C \delta \text{ then } \text{assume}(s_{int} \delta) \text{ else } s_{bv}$. In addition, when $s_{bv}$ of a weakening rule can be matched to the condition $c$ in an $\text{assume}(c)$ of the original program via a substitution $\delta$, then the $\text{assume}(c)$ statement is transformed to $\text{if } C \delta \text{ then } \text{assume}(s_{int} \delta) \text{ else } \text{assume}(c)$.

### III. Experiments & Conclusion

We implemented bitwise branching via a translation algorithm, in a fork of Ultimate, which is the state-of-the-art LTL prover and the only mature LTL verifier that supports bitvector programs, we denote our branch UltimateBwB. To our knowledge, there are no available bitwise benchmarks with LTL properties so we create new benchmarks for this purpose:

(i) New hand-crafted benchmarks called LTLBit of 42 C programs with LTL properties, in which bitwise operations are heavily used in assignments, loop conditions, and branching conditions. There are 22 programs in which the provided LTL properties are satisfied (√) and 20 programs in which the LTL properties are violated (✗). (ii) Benchmarks adapted from the BitHacks programs [3], consisting of 26 programs with LTL properties (18 satisfied and 8 violated).

| Table I: LTL Verification over Bitvector programs results |
|---------------------------------------------------------|
| | (i) BitHacks | (ii) LTLBit |
| | Ultimate | w. BwB | Ultimate | w. BwB |
| ✔ | 3 | 10 | - | 21 |
| ✗ (Unknown) | 21 | 5 | 42 | - |
| T (TimeOut) | 1 | 1 | 1 | 1 |
| M (Out of Memory) | 1 | 3 | - | - |

The rules in Fig. 1A and Fig. 1B were developed empirically, from the reachability/termination/LTL benchmarks, especially, based on patterns found in decompiled binaries (enable us to verify properties over binaries). We then generalized these rules to expand coverage, proofs for each rule were done with Z3.

**References**

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