New technique for a simultaneous estimation of the level density and radiative strength functions of dipole transitions at $E_{\text{ex}} \leq B_n - 0.5$ MeV
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Abstract

The new, model-independent method to estimate simultaneously the level densities excited in the $(n, \gamma)$ reaction and the radiative strength functions of dipole transitions is developed. The method can be applied for any nucleus and reaction followed by cascade $\gamma$-emission. It is just necessary to measure the intensities of two-step $\gamma$-cascades depopulating one or several high-excited states and determine the quanta ordering in the main portion of the observed cascades. The method provides a sufficiently narrow interval of most probable densities of levels with given $J^\pi$ and radiative strength functions of dipole transitions populating them.

1 Introduction

The observed parameters of the cascade $\gamma$-decay of the compound nucleus can be reproduced in the calculation if one determines (in the frameworks of some model) at least

1. the mean density $\rho$ of the excited states with given spin and parity $J^\pi$, and
2. the mean width $\Gamma_{\lambda i}$ of $\gamma$-transitions between the arbitrary states $\lambda$ and $i$.

The objects of primary interest are the total radiative width $\Gamma_{\gamma}$ of the compound nucleus (neutron resonance) and the spectrum of $\gamma$-emission. It may be, for example, the intensity $I_{\gamma\gamma}$ of the cascades of two successive $\gamma$-transitions between the compound state and given low-lying level via a great number of intermediate levels. The experimental data on $I_{\gamma\gamma}$ (as a function of the energy of their intermediate level) are obtained for over 30 nuclei from the mass region $114 \leq A \leq 200$ (see, e.g., [4]) with a precision of approximately 10%. The experimental values of $\Gamma_{\gamma}$ are known within the same accuracy. Unfortunately, such accuracy cannot be achieved in the calculation of these parameters for an arbitrary nucleus because there are no models that would predict $\rho$ and $\Gamma_{\lambda i}$ with the mentioned above precision.

This is seen from the data of Table 1 which represent the mean ratio between the experimental cascade intensities $I_{\gamma\gamma}^{\text{exp}}$ and those calculated $I_{\gamma\gamma}^{\text{cal}}$ using the known enough models of level density [2, 3] and radiative strength functions [4, 5].

| Models: | [2, 4] | [2, 5] | [3, 4] | [3, 5] |
|--------|--------|--------|--------|--------|
| R      | 2.2(2) | 2.7(2) | 1.5(1) | 1.7(1) |

Table 1: The ratio $R = I_{\gamma\gamma}^{\text{exp}} / I_{\gamma\gamma}^{\text{cal}}$ averaged over 30 nuclei.

Precise $\gamma$-decay parameters are, however, necessary for the calculation of the interaction cross-sections of neutrons with unstudied target nuclei and the understanding of the behavior of nuclear matter with increasing excitation energy. An analysis of the existing methods for the determination of the level density [3, 4] and radiative strength functions ($k$) [5]

$$k = \Gamma_{\lambda i} / (E_{\gamma}^3 \times A^{2/3} \times D_{\lambda})$$

in deformed nuclei, for example, shows that it is not possible to obtain sufficiently precise experimental level densities for certain intervals of their energies and quantum numbers as well as the
widths of the corresponding transitions. Analysis of contributions of different sources of systematical errors in determination of the level density from the evaporation spectra was performed by H.Vonach [9] mainly for light spherical nuclei. The total uncertainty evaluated by him amounts to about 20-30\%. It should be noted, however, that an accuracy in calculation of cascade intensities within the models [2, 3, 4, 5] in the lightest nuclei (from the range of the minimum of the neutron strength function) studied by us is also considerably better than that for deformed nuclei: the ratio \( R \) for \(^{114}Cd\) and \(^{124,125}Te\) varies in limits from 0.7 (\(^{124}Te\)) up to 1.4 (\(^{114}Cd\)).

Therefore, without developing new methods for the determination of nuclear parameters under discussion one cannot expect any progress in the modification of the existing theoretical models, first of all, for deformed nuclei. (In eq. \( \Gamma_{\lambda i} \) is the partial radiative width of \( \gamma \)-transition with the energy \( E_\gamma \), \( D_\lambda \) is the average level spacing of the decaying state and quantity \( A \) is the nucleon number).

A new and sufficiently perspective way to obtain such information for the entire energy interval below \( B_n \) seems to be the investigation [14, 11] of the two-step \( \gamma \)-cascades between the compound state \( \lambda \) and the given low-lying level \( f \) through all possible intermediate states \( i \). The algorithms [12, 13, 4] developed for the analysis of \( \gamma \)-\( \gamma \) coincidences registered by ordinary Ge detectors allow one to determine the intensity distribution of the cascades as a function of the energy of the cascade intermediate levels over the whole energy region up to \( E_{ex} \simeq B_n \) with an acceptable systematic error (which decreases as the efficiency of the \( \gamma \)-spectrometer increases).

The intensity \( i_{\gamma \gamma} \) of an individual cascade is

\[
i_{\gamma \gamma} = \frac{\Gamma_{\lambda i}}{\Gamma_\lambda} \times \frac{\Gamma_{i f}}{\Gamma_i},
\]

where \( \Gamma_{\lambda i} \) and \( \Gamma_{i f} \) are the partial widths of the transitions connecting the levels \( \lambda \rightarrow i \rightarrow f \), \( \Gamma_\lambda \) and \( \Gamma_i \) are the total widths of the decaying states \( \lambda \) and \( i \), respectively. The sum intensity \( I_{\gamma \gamma} \) of the cascades is related to an unknown number of intermediate levels \( n_{\lambda i} = \rho \times \Delta E \) and unknown widths of primary and secondary transitions via the equation

\[
I_{\gamma \gamma} = \sum_{\lambda, f} \sum_i \frac{\Gamma_{\lambda i} \Gamma_{i f}}{\Gamma_\lambda \Gamma_i} = \sum_{\lambda, f} \frac{\Gamma_{\lambda i}}{\Gamma_i} \left\{ \begin{array}{ll} n_{\lambda i} \frac{\Gamma_{i f}}{m_{\lambda i}} & \text{if } \Gamma_{\lambda i} > m_{\lambda i} \\ \Gamma_{i f} & \text{if } \Gamma_{\lambda i} < m_{\lambda i} \end{array} \right\}
\]

The summation is over a certain set of quantum numbers of intermediate, initial, and final states for the purpose of comparison with the experimental data. The thermal neutron capture cross-section for two possible spins of compound states are listed in [13], for example. The \( J^\pi \) values for the initial and final cascade levels are also known. The latter, however, is true if the energy \( E_f \) of the final state does not exceed \( \simeq 1 \text{ MeV} \). The optimal width of the interval \( \Delta E \) and the number \( N \) of such intervals in eq. (3) are determined by the statistics of \( \gamma \)-\( \gamma \) coincidences (as a square detector efficiency) and the necessity to obtain detailed energy dependence for \( I_{\gamma \gamma} \). The width of \( \Delta E \) does not exceed 0.5 MeV even in the case of a 10\% efficiency detector, however. The total radiative widths \( \Gamma_\lambda \) of the capturing states are also known from the corresponding experiments for all stable nuclei [15]. The mean partial widths \( \langle \Gamma_{\lambda i} \rangle \), \( \langle \Gamma_{i f} \rangle \) and the total numbers \( m_{\lambda i}, m_{i f} \) of levels excited by \( E1 \) and \( M1 \) transitions after the decay of the states \( \lambda \) and \( i \), respectively, to be found in the analysis are related to the total radiative widths as

\[
\Gamma_\lambda = \langle \Gamma_{\lambda i} \rangle \times m_{\lambda i}
\]
\[
\Gamma_i = \langle \Gamma_{i f} \rangle \times m_{i f}
\]

The contribution of higher multipolarities to eqs. (3) and (4) is smaller than the error of the determination of \( I_{\gamma \gamma} \). Equations (3) and (4) and their obvious combination

\[
\Gamma_\lambda \times I_{\gamma \gamma} = \sum_{J, \pi} \Gamma_{\lambda i} \times n_i \times (\Gamma_{i f} / \langle \Gamma_{i f} \rangle \times m_{i f})
\]
allow three ways of the estimation of the parameters of the cascade γ-decay using the experimental data on $I_{\gamma\gamma}$ and $\Gamma_{\lambda}$:

(a) the level density can be estimated from eq. (3) using model calculated partial radiative widths;

(b) the partial widths of cascade transitions can be estimated from eq. (5) using model calculated level densities with certain $J^\pi$;

(c) simultaneous estimation of the intervals of probable level densities and radiative strength functions which satisfy eqs. (3) and (4) in general.

It is clear that the level density and strength functions found according to variants (a) and (b) inevitably contain errors caused by the uncertainties of experimental and model values used as parameters of the analysis. However, the influence of these uncertainties on the final result is suppressed because of the correlation (determined by the used type of the functional relations (3) and (5)) between the experimental $\Gamma_{\lambda}^{\text{exp}}$, $I_{\gamma\gamma}^{\text{exp}}$ and the parameters under study $\rho$, $\Gamma$.

In accordance with the variant (a) the sufficiently narrow interval of probable $\rho$ was determined for almost 30 nuclei from the mass region $114 \leq A \leq 200$ for some set of possible models of γ-transition strength functions. An important conclusion made in [16] is that the best description of the level density in the interval from $\sim 0.5B_n$ to $B_n$ was achieved in the framework of the generalized model of the superfluid nucleus [3]. Besides, simple enough models [4, 5] of radiative strength functions cannot provide a correct description of the experiment and also need modification. An analysis by variant (b) was performed by us, as well. The main result is that there are no strength function models for $E1$ and $M1$ transitions in deformed nuclei which could reproduce the dependence $\Gamma_{\lambda} \times I_{\gamma\gamma}$ at primary transition energies $E_1 \leq 2 - 3$ MeV if the level density is set by the model of a non-interacting Fermi-gas. Therefore, the understanding and correct description of the γ-decay of the compound nucleus with a high level density require experimental determination of the level density and radiative strength functions over the entire excitation energy region.

Further investigations [17] have shown that the level density at excitations from 1-2 to 3-4 MeV in, first of all, deformed nuclei deviates strongly from the exponential energy dependence derived on the basis of the idea that the nucleus is a non-interacting Fermi-gas [2]. Moreover, it is not excluded that the level density in this energy interval can be almost constant or even decrease with increasing excitation energy. These confirm and complement the results obtained in [16].

2 Analysis

The variant (c) of analysis of the experimental intensities of two-step γ-cascades between the capturing state and several low-lying levels allowed us to suggest an original method for the solution (although partial) of this problem. It is based on an obvious circumstance that $N + 1$ equations (3) and (4) together with $6N$ conditions

$$\rho(\pi = +) > 0; \quad \rho(\pi = -) > 0$$
$$\Gamma(E1) > 0; \quad \Gamma(M1) > 0$$

(6)

(separately for primary and secondary transitions in the case of radiative widths) restrict some interval of possible level densities and partial radiative widths which provide a simultaneous reproduction of $\Gamma_{\lambda}^{\text{exp}}$ and $I_{\gamma\gamma}^{\text{exp}}$. This interval can be estimated using modern computers and the existing computational algorithms. Its width, however, cannot equal zero even at zero uncertainty of the experiment. It should be added that $I_{\gamma\gamma}$ in the form of eq. (3) is inversely proportional
(qualitatively) to the total number of states excited in the process under study and is proportional to the ratio of cascade transition widths to their mean values. Therefore, the method of analysis described below has a maximum sensitivity at minimum density of the excited states (unlike the methods [6, 7]).

As in the case of other reactions (followed by $\gamma$-emission) used for the determination of $\rho$, all values obtained experimentally in the $(n_{th}, \gamma)$ measurements are determined by the product $\Gamma_{\lambda i} \times \rho$. Hence, in the calculation deviation of one of the two parameters from its mean value is compensated by deviation of the other one with the corresponding magnitude and sign. This circumstance should be taken into account in data processing — a minimum or maximum value of the level density derived from the experimental data results, e.g., in a maximum or minimum value of the corresponding strength functions.

It should be noted that deviation of the calculated level density from the true value is completely compensated by deviation of strength functions when $\Gamma_{\lambda}$ is only calculated. In the case of the calculation of $I_{\gamma\gamma}$ the compensation is incomplete. This very circumstance allows one to select the intervals of $\rho$ and $\Gamma_{\lambda i}$ which provide the description of the $I_{\gamma\gamma}$ and $\Gamma_{\lambda}$ parameters with an acceptable uncertainty. This analysis can be performed by means of finding large enough sets of random values of $\rho$ and $\Gamma_{\lambda i}$ which reproduce completely the parameters $\Gamma^{exp}_{\lambda}$ and $I^{exp}_{\gamma\gamma}$ and belong to the intervals that contain true values. This means that most probable values of the level density and radiative strength functions of dipole $\gamma$-transitions and intervals of their uncertainties can be found by selection of pairs of random $\rho$ and $k$ which satisfy, in general, eqs. (3) and (4) or (3) and (5). This requires numerous repetitions of the procedure and statistical methods of analysis.

It is clear that the widths of the intervals of probable $\rho$ and $k$ satisfying eqs. (3) and (4) increase with increasing number of unknown parameters in the equations. According to experimental conditions, the summation in eqs. (3) and (5) as over all intermediate states of the cascades. Since the summed data included cascade transitions of different multipolarities, we could not obtain the strength functions of $E1$ and $M1$ transitions and the level density for different parities separately with a good precision. In practice, from a combination of eqs. (3) and (5) the sum of strength functions and the sum of level densities of both parities should be only derived and compared with model predictions. The corresponding summation reduces considerably the uncertainty of the observed result due to anti-correlation of elements.

Indeed, an analysis of the available data confirms that the dispersion of each set of $\rho(\pi = +)$, $\rho(\pi = -)$, $k(E1)$ and $k(M1)$ random values is too large to make any conclusions about independent correspondence of individual values to the model.

A sufficiently large $N$ and the nonlinearity of eqs. (3) and (4) stipulate the choice of the way to solve the system of equations and inequalities - the Monte Carlo method. The simplest iterative algorithm [16] was used for this aim: we set some initial values for $\Gamma(E1)$, $\Gamma(M1)$, $\rho(\pi = -)$, and $\rho(\pi = +)$ and then distort them by means of random functions. If these distortions decrease the parameters $\Delta = (I^{exp}_{\gamma\gamma} - I^{cal}_{\gamma\gamma})^2$ at this step of the iteration procedure, then the distorted values are used as initial parameters for the next iteration. Agreement between the experimental and calculated cascade intensities and the total radiative widths, respectively, is usually achieved after several thousand iterations. As a result we get two random ensembles of level densities and partial widths for every $N$ energy intervals. Examples of intermediate and final results of one of many variants of the calculation for two nuclei are shown in Figs. 1 and 2. It is obvious that such iterative process can be realized in an unlimited number of ways. We chose a sufficiently simple and effective way: the Gaussian curve is used as a distorting function for logarithms of $\rho$ and $f$

$$f(E) = A \times \exp\left(-\frac{(E - E_0)^2}{\sigma^2}\right)$$

(7)

Its parameters are independently chosen for the level density and strength functions from the
intervals \([-0.2;0.2]\), \([E_d; B_n]\) and \([0.3 \text{ MeV}; B_n]\) for \(A\), \(E_0\), and \(\sigma\), respectively using a standardized random value distributed uniformly in \([0;1]\). Here \(E_d\) is the maximum excitation energy of the known discrete level involved in the calculation. Numerous repetitions of the iterative calculation with different initial parameters (including obviously unreal values of \(\Gamma\) and \(\rho\)) for \(\sim 30\) nuclei from the mass region \(114 \leq A \leq 200\) show that this algorithm yields rather narrow intervals of the sum level density of both parities and of the sum partial widths of \(E1\) and \(M1\) transitions. The use of eq. (7) with mentioned parameters allows one to get a set of different, smooth enough functional dependences for both \(\rho\) and \(k\). In this case for the majority of the studied nuclei the values of level density are in good agreement with the number of the observed intermediate levels of the cascades resolved as the pairs of peaks. In some nuclei, however, the mean level density (which together with the mean strength functions provides reproduction of cascade intensities) is less than the number of intermediate levels observed below \(\approx 2\) MeV. The main portion of this discrepancy is removed in all cases if one foresees a possibility of additional local variation of \(k\) for high-energy transitions in the energy interval which, as a rule, does not exceed 0.1-0.2 MeV. One of the examples of this kind is shown in Fig. 1. The necessity to account this effect can be due to both insufficient averaging of the random partial widths of primary transitions and their possible dependence on the structure of the excited low-lying level. This can result, for instance, from concentration of the strength of the fragmented single-particle or phonon states.

3  Asymptotical uncertainty of the obtained parameters

The method suggested by us for estimation of \(\rho\) \(k\) cannot give unique value of these parameters at a given energy of the excitation or quantum energy. Therefore the question arises about the value of their uncertainty at different energies and degree of possible systematical deviations of the observed parameters from the modal values. The results of modelling for \(^{156}\text{Gd}\) and \(^{198}\text{Au}\) shown in Fig. 3 answer these questions. Intensity of cascades for these nuclei were calculated under assumption that the strength function \(k(E1)\) is described by model [4] and value of \(k(M1) = \text{const}\); level density exponentially increases with the energy or have some step-like structure. Below the excitation energy \(\approx 1 – 2\) MeV the calculation used experimental decay scheme. Consequently, the calculated intensity distribution of cascades in function of the primary transition energy has one or two maxima. (Other conditions of the calculation completely corresponded to the experiment).

Figure 3(b) shows that the model level density is reproduced practically without systematical error and the width of the interval of its probable values does not exceed 20-30%.

Discrepancy between the experimental and model sum \(k(E1) + k(M1)\) results from that the total radiative width calculated according model [4] does not correspond to the experimental value. Energy dependence of \(k(E1) + k(M1)\) is reproduce rather well – sharp changes in the first derivative with respect to the quantum energy is not observed (unlike some other nuclei studied by us).

So, one can summarize that the suggested method provides reliable enough estimation of the level density and radiative strength functions of dipole transitions.

4  Approach used in calculation

The insufficient experimental data on cascade \(\gamma\)-transitions (only cascades terminating at low-lying levels \((E_f < 1\) MeV) of nuclei were studied [4]) does not allow us to determine the level densities and gamma-widths without the following important assumption: the strength functions of transitions of a given multipolarity only depend on the transition energy and do not depend on the structure and energy of the corresponding excited states. Their nonequal values for \(\gamma\)-transitions of equal
energies but populating different levels is, in part, compensated by the circumstance that the left part of eq. (5) depends on absolute radiative strength function values of primary transitions and depend only on the ratio of strength functions in the case of secondary transitions. These decrease the effect of the discussed assumption on the $k(E_1) + k(M1)$ values but do not remove it completely.

There is no necessity in introduction of any hypotheses of spin dependence of level density differing from that predicted in models [4, 5].

5 Sources of errors in the determination of strength functions and level densities

The presence of the statistic and systematic errors in determination of $I_{\gamma\gamma}$, $\Gamma_\lambda$ and specific problems of extraction of level density and radiative strength functions cause noticeable uncertainties of the determined parameters. The influence of the different sources of errors on the obtained results manifest itself in a different degree.

1. Uncertainties of the measuring of terms in eqs. (3) and (5) result in errors of strength functions and level density. Owing to a linear relation between $\Gamma_\lambda$, $I_{\gamma\gamma}$ and $\Gamma_{\lambda i}$ in eq. (5), $\simeq 10\%$ errors of $\Gamma_\lambda$ and $I_{\gamma\gamma}$ achieved in the experiment cause rather a small error in the determination of $\Gamma_{\lambda i}$ and $\rho$ as compared to dispersion of the obtained data.

2. The more considerable source of uncertainty in the determination of the strength functions and $\rho$ is a systematic error of decomposition [13, 14] of the experimental spectra into two components corresponding to solely primary and solely secondary transitions. The analysis [17] showed that the error in $\Delta I_{\gamma\gamma}$ caused by this procedure does not usually exceed $\simeq 20\%$ for primary transition energy $E_1 < 3 - 4$ MeV. Intensities of cascades (histograms in Figs. 1, 2, 4-13) at these primary transition energies can be overestimated, as a maximum, by the above value. At the higher energies they can be decreased by the same value (the total intensity is preserved). In order to estimate the influence of $\Delta I_{\gamma\gamma}$ on the final results, the $I_{\gamma\gamma}$ values were varied within a level of 25%. These variations caused changes in $k(E_1) + k(M1)$ and $\rho$ which did not exceed the dispersion of the data plotted in Figs. 4-13.

3. The maximum uncertainty of level density and radiative strength functions results from the use of condition (6). It dominates at any possible precision in determination of $I_{\gamma\gamma}$ and $\Gamma_\lambda$.

The simplest way to estimate these errors at any $E_1$ and $E_{ex}$ is the following:

(a) taking into account that the probabilities of deviations with opposite sign of the random $\rho_i$ and $f_i$ values with respect to their mean values are equal and decrease as the absolute values of deviations increase; and

(b) assuming that mathematical expectations of the random ensembles of the $\rho_i$ and $k_i$ values satisfying eqs. (3)-(5) correspond to their real values

one can consider the mean-square deviations of the random values relative to their arithmetical means as the estimations of the errors. These errors can be attributed to level density and strength functions separately in spite of their strong anti-correlation. Just these uncertainties are shown for the radiative strength functions and level density plotted in Figs. 4-13.

On the whole we can summarize the situation as the following. At the presently achieved accuracy for experimental determination of $I_{\gamma\gamma}$ and $\Gamma_\lambda$, level densities and strength functions are derived from eqs. (3)-(5) with the mean total uncertainties of about 40-50% in the worst case. Asymptotic value of this uncertainty at zero statistic and systematic errors of the experiment is equal, in the average, for both $\rho$ and $k$ and cannot be less than $\simeq 20\%$.  

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4. There are two ways to decrease the errors of the level density and strength functions determined from eqs. (3)-(5):

(a) the increase of the volume of the experimental data on the cascade intensities;
(b) the reduction of the number of parameters in eqs. (3)-(5) owing to the use of additional information or introduction of some new assumptions.

In the first case the problem can be easily solved experimentally: the use of a Compton-suppressing spectrometer consisting of HPGe detectors with an efficiency of not less than 30-40% allows the selection from a mass of \( \gamma - \gamma \) coincidences of two-step cascades for a considerably larger number of their final levels than at present. From a combination of eq. (3) for the sum over all final levels of cascades and an individual final level \( f \) one can determine the ratio \( \Gamma_{i f}/<\Gamma_{i f}> \times m_{i f} \) for all possible values of \( i \) and \( f \), i.e., determine energy dependence of the experimental sum \( k(E1) + k(M1) \) for any possible secondary transitions, get rid of the only approach used in the analysis, and reduce the number of parameters in the analysis.

The data shown in Figs. 4-13 were obtained under assumption about a constancy of the ratio

\[
\Gamma_{\lambda i}/\Gamma_{i f} = \text{const}
\]

for the transitions with equal multipolarity and energy \( E_1 \) in all interval of the neutron binding energy.

The comparison of the total \( \gamma \)-spectra and population of low-lying levels calculated in this way with the available experimental data including spectroscopic information shows that even such assumption provides better accuracy in calculating the parameters of cascade \( \gamma \)-decay than the approach using the models. Unfortunately, we could not achieve complete correspondence between the estimated level density and available spectroscopic information. Nevertheless, the obtained values demonstrate certain correspondence of our level density with the numbers of the excited levels observed in the experiment. Some nuclei, however, demonstrate residual discrepancy (for example, \(^{170}Tm\), Fig. 1). This discrepancy can be attributed, partially, to both insufficient precision of assumption and inexactitude of the spectroscopic data. Their errors can be also considerably decreased using more efficient spectrometer of \( \gamma - \gamma \) coincidences than that used by authors.

On the whole, in spite of the uncertainties mentioned above one can conclude that at a given stage of the experimental investigation of the cascade \( \gamma \)-decay of compound states our method provides more reliable results than methods.

6 Main results of analysis

The type of relation between \( k \) and \( \rho \) on the one hand and between \( \Gamma_{\lambda} \) and \( I_{\gamma \gamma} \) on the other hand does not allow one to determine \( k \) and \( \rho \) unambiguously and independently. Some deviation of, for example, \( \rho \) from a real value is inevitably compensated by deviation of strength functions of the corresponding magnitude and sign. Nevertheless, the results obtained in the present analysis can be used for the verification of nuclear models and, if necessary, for the determination of the direction of the further development of these models. The main argument in favour of this statement is relatively weak dependence of the final results on the initial values of strength functions and \( \rho \) in the iterative process. As an example, Figs. 1 and 2 show the strength function and \( \rho \) values obtained for their unreal initial values: \( \rho(E_{ce}) = \rho(B_n) \), the strength functions decrease linearly as
the transition energy increases. Nevertheless, the final results of the iterative process quite agree with a general picture obtained for a large enough set of different real and unreal initial values of $k$ and $\rho$. This confirms the conclusion that the strength functions and level density obtained from the analysis can be considered as most probable.

The strength functions $k(E_1) + k(M1)$ and level densities $\rho$ obtained in the present analysis are plotted in Figs. 4-13. For every set of random $\rho$ at a given excitation energy $E_{ex}$ and $k(E_1) + k(M1)$ at a given primary transition energy $E_1 = B_n - E_{ex}$ there were determined both their mean values and probable dispersion using usual relationships of statistical mathematics. The results of the analysis are compared with predictions of the level density models [2, 3] and models of radiative widths [4, 5]. In the case of radiative strength functions a comparison is performed in the following manner: the $k(E1)$ values calculated according to the models [4] and [5] (upper and lower curves, respectively) are summed with $k(M1) = const$ which is normalized so that the ratio $\Gamma(M1)/\Gamma(E1)$ would be approximately equal to the experimental data at $E_\gamma \simeq B_n$.

A comparison of the results of the analysis with predictions of the models [2, 3, 4, 5] (often used by experimentalists) shows that:

1. the sums $k(E1)+k(M1)$ and $\rho$ are not monotonic functions of the energy and, probably, reflect the most common peculiarities of the structures of the states connected by the corresponding $\gamma$-transitions;

2. the energy dependence of $k(E1)+k(M1)$ differs strongly from predictions of the models [2, 3] in the case of even-even compound nuclei from the region of the $4s$-resonance of the neutron strength function, at least;

3. the $k(E1) + k(M1)$ functions increase from near-magic to deformed nuclei and from complicated highly-excited states to simpler low-lying levels which are populated by $\gamma$-transitions under consideration;

4. relative deviations of the obtained strength functions and level densities from the mean values are characterized by strong negative correlation. In the majority of nuclei the correlation coefficient changes from -0.6 to -1.0. This means that the strength functions and level densities are not independent variables in eqs. (3) and (5), which provides the possibility of their simultaneous determination;

5. the probable level density determined in the present analysis conforms to the picture obtained in previous experiments [14, 17]: up to the excitation energy 1-2 MeV, our data are not in contradiction with the exponential extrapolation of $\rho(E_{ex})$ predicted by the Fermi-gas back-shift model [2]. The energy dependence of the level density in the interval from 1-2 to some threshold value $E_b$ is considerably weaker than it follows from any existing level density model. Above $E_b \approx 3$ MeV for $N$-odd and $\approx 4$ MeV for $N$-even nuclei, the level density, most probably, corresponds better to the predictions of the generalized model of the superfluid nucleus in its simplest form [3].

This change in the behaviour of the level density in the vicinity of the excitation energy $E_b$ may signify a qualitative change in the nuclear properties. The observation [18] of the probable harmonicity of the excitation spectra of the intermediate levels of the most intense cascades in a large group of nuclei from the mass region $114 \leq A \leq 200$ allows an assumption that the nuclear properties at low energy are mainly determined by vibrational excitations (probably, a few phonons of rather high energy). A very quick exponential increase in the level density above $E_b$ says about the probable dominant influence of the inner, many-quasiparticle type of excitations of these states.
7 Discussion

The method suggested in present work allows model independent, simultaneous estimation of intervals of probable values of the level densities with given spins and summed strength functions of primary dipole transitions populating them. The method is effective in investigations of any stable nucleus. The main differences of this algorithm from the known methods of determination of level densities [6, 7, 8] and radiative strength functions [8] are the following:

1. Our method does not permit one to get the sole values of $\rho$ and $k$ for a given energy. But the width of the intervals of their probable magnitudes depends very weakly on the uncertainty in determination of $\Gamma_\lambda$ and $I_{\gamma\gamma}$ at the achieved precision of the experiment, at the one hand, and is narrow enough in order to get new information on nuclear matter, from the other hand.

2. The most correct and reliable data on the level density is derived from the evaporation spectra at the highest excitation energies; analysis of the cascade intensities provides similar data for the lowest energies. So, both methods mutually add each other.

3. Analysis of cascade intensities allows direct determination of the absolute level densities, evaporation spectra usually provide [6, 7] information on nuclear temperature.

4. Systematical uncertainties of both methods do not relate. Discrepancies in the independently determined level densities at some energies indicate to necessity, for example, to determine more precisely the barrier transmission factor for the evaporated particle or to take into account different energy dependence of $k$ of the primary and secondary transitions of the $\gamma$-cascades. Besides, they can testify to necessity to describe more correctly direct and pre-equilibrium processes in nuclear reactions for deformed nuclei or to define more precisely the nuclear excitation energy above which thermodynamical parameters of a nucleus are determined mainly by quasiparticle excitations.

5. Energy dependence of the data in Figs. 4-13 can be reproduced well enough in the framework of modern version of the generalized model of the superfluid nucleus [19] if the temperature of the phase transition is diminished up to the value $T'_{cr} \approx 0.7T_{cr}$, where

$$T_{cr} = \delta/1.76$$

is the temperature of the transition from the superfluid to normal phase of homogeneous Fermi-system [20]. But re-determination of the entropy and temperature predicted by model [19] should be done so that nuclear temperature below $T'_{cr}$ will not increase with decreasing excitation energy.

6. Additional and independent arguments in favor of reliability of step-like structure in level density are:

(a) combinatorical calculation [21] of density of the states with $K^\pi = 1/2+$ in $^{165}Dy$ below $B_n$, providing similar to Figs. 4-13 picture;

(b) analysis [22] of the experimental data from the reaction $^{165}Ho(p, n)^{165}Er$. This also demonstrates some step-like structure in the total level density at low excitations;

(c) precise analysis [23] of the neutron cross sections for actinides testify to necessity to take into account the influence of the pairing interaction on the level density for the wide interval of the neutron energies manifesting itself, in particular, as irregularities in the energy dependence of the level density.
7. It is obvious that the structures shown in Figs. 4-13 can be inherent not to the total level density with given $J^\pi$, but only to that part of them which are really excited in $(n, \gamma)$ reaction. Then, unlike the existing notions, this reaction is selective and structures of the excited states must be taken into account in any calculations of parameters of this reaction in the entire excitation energy region below $B_n$.

8 Conclusions

A new method is suggested for a simultaneous estimation of the probable level density populated by dipole primary transitions in the $(n_{th}, \gamma)$ reaction and the sum strength functions $k(E1) + k(M1)$ of these transitions. Unlike other methods used for the investigations of nuclear properties below the excitation energy 6-9 MeV, this method allows the estimation of $\rho$, radiative strength functions, and intervals of their probable variations without any model notions of the nucleus.

The method is universal – it can be used for any nucleus and reaction with $\gamma$-emission. The latter is possible if the excitation energy interval of high-lying states is narrow enough in order to use the sum coincidence technique. Besides, the most probable quanta ordering in the cascades must be determined for the main part of the observed cascade intensity. It should be noted, that in the case of a lack of the experimental values of the total radiative widths of decaying high-lying states the absolute radiative strength functions cannot be determined. In this case only relative energy dependence of the radiative strength functions can be obtained.

The most important (although preliminary and qualitative) physical result is that the level density below the neutron binding energy (first of all in deformed nuclei) cannot be reproduced to a precision achieved in the experiment without more precise than in [19] accounting for the co-existence and interaction of superfluid and usual phases of nuclear matter in this whole excitation energy interval.

The obtained results demonstrate very serious and obvious discrepancies with the existing ideas of the structure of the deformed nuclei. These data agree completely with an earlier obtained qualitative picture [18] of the studied process: considerable influence of vibrational excitations on the nuclear properties below the excitation energy $E_b$ and a transition to dominant influence of quasiparticle excitations above this energy.

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Fig. 1. The examples of $\rho$ (a) and $k$ (b) intermediate values and the corresponding distributions of cascade intensities (c) for the $^{170}\text{Tm}$ odd-odd nucleus in function of the primary transition energy $E_1$ or excitation energy $E_{\text{ex}}$. Letters next to the lines mean the number of iterations. Triangles show number of levels excited by the primary dipole transitions with the energy $E_1$ in the energy interval of 100 keV. The dashed curve (a), (b) represents model predictions, the histograms (c) represent the experimental cascade intensities with statistical errors.
Fig. 2. The same as in Fig. 1, for the $^{190}\text{Os}$ even-even nucleus.
Fig. 3. The intensity of two-step cascades (a) calculated using level density [2, 3] shown by solid lines in (b) and radiative strength [4, 5] functions - line 1 in (c) (line 2 in (c) represents predictions of model [5]). Points with error bars represent the interval of possible values of \( \rho \) (b) and \( k \) (c) providing acceptable precision in reproduction of cascade intensities shown in (a).
Fig. 4. Experimental cascade intensities $I_{\gamma\gamma}$ in 0.5 MeV energy bins with ordinary statistical errors for $^{114}Cd$ and $^{124}Te$ (histograms. Curves represent calculation performed like that shown in Fig. 3. Points with errors represent number of levels per 100 keV energy interval and sums $k(E1) + k(M1)$, respectively.
Fig. 5. The same as in Fig. 4 for $^{137}\text{Ba}$ and $^{138}\text{Ba}$. 
Fig. 6. The same as in Fig. 4 for $^{139}\text{Ba}$ and $^{146}\text{Nd}$. 
Fig. 7. The same as in Fig. 4 for $^{150}Sm$ and $^{156}Gd$. 
Fig. 8. The same as in Fig. 4 for $^{158}\text{Gd}$ and $^{160}\text{Tb}$. 
Fig. 9. The same as in Fig. 4 for $^{164}\text{Dy}$ and $^{168}\text{Er}$. 
Fig. 10. The same as in Fig. 4 for $^{170}\text{Tm}$ and $^{174}\text{Yb}$. 
Fig. 11. The same as in Fig. 4 for $^{181}$Hf and $^{183}$W.
Fig. 12. The same as in Fig. 4 for $^{192}$Ir and $^{196}$Pt.
Fig. 13. The same as in Fig. 4 for $^{198}$Au and $^{200}$Hg.