Brillouin phonon cooling by electro-optic feedback

Kien Phan Huy, Adrien Godet, Thibaut Sylvestre, and Jean-Charles Beugnot
Institut FEMTO-ST UMR 6174, Université Bourgogne Franche-Comté, CNRS, Besançon, France
(Dated: April 16, 2018)

We theoretically investigate the cooling of a propagating phonon through Brillouin scattering. To that end, we propose to introduce an external viscous force using Brillouin scattering and an electro-optic feedback. Short delays feedback show an efficient control of the Brillouin linewidth whereas long delays can induce Fano-like resonances.

I. INTRODUCTION

The study of microscopic mechanical devices and their interactions with light have attracted much interest recently [1,2]. Among the most fascinating phenomena is the cooling of an oscillator [3,5]. Optomechanical interactions wether they find their origin in radiation pressure or electrostriction usually involve the inelastic scattering of a pump laser that produces Stokes and Anti-Stokes waves as in Brillouin scattering. When Stokes light is produced, it is downshifted in frequency from the value of the oscillator resonant frequency. As the scattered light transfers energy to the oscillator, a heating process occurs. On the opposite, Anti-Stokes light generation corresponds to a process where photons gain energy and are blueshifted. It is thus a cooling process. In order to favour one process over the other, an optical cavity can be added to have the pump and anti-Stokes waves optically resonants whereas the Stokes process is off or slightly off resonance. As a result, the detuning with respect to the cavity resonances will lead to heating or cooling [6,7]. Another method is to introduce an electro-optic feedback [8,9]. In this case, the mechanical oscillation amplitude is optically measured and light is used to act as a supplementary feedback force on the mechanical device. If the supplementary force is proportional to the mechanical oscillation amplitude, the system behave as an opto-electronic oscillator [10,11]. However, If we send light at the Anti-Stokes frequency with the correct amplitude and phase relationship, it will interact with the pump to produce an electrostrictive force that will act as a viscous force on the phonon and affect the Brillouin linewidth as highlighted by the pink area. This method of phonon lifetime management may find several applications such as filtering or Brillouin lasers where the Brillouin linewidth is used as an asset [15,17] .

II. PRINCIPLE AND METHODOLOGY

First we shall consider the equations that rules Brillouin scattering as developped by Y. Pennec et al. [2]. The equations for pump $E^{(1)}$ and Stokes $E^{(2)}$ fields write

\[
\frac{\partial}{\partial t} \vec{E}^{(2)}_i - \frac{\omega_2}{c^2} \epsilon_r E^{(2)}_i = -\frac{(\omega_1 - \Omega)^2}{c^2} \chi_{ijkl} E^{(1)}_j u^*_k \tag{1}
\]

\[
\frac{\partial}{\partial t} \vec{E}^{(1)}_i - \frac{\omega_1}{c^2} \epsilon_r E^{(1)}_i = -\frac{(\omega_2 + \Omega)^2}{c^2} \chi_{ijkl} E^{(2)}_j u^*_k \tag{2}
\]

FIG. 1. Principles of Brillouin scattering and feedback cooling

The paper is organized as follow. First we give the theoretical background of our method and detail one possible experimental implementation. Then we will investigate both the cooling-heating process with a simple model and finally present the behaviour of the setup for long delays with the prediction of Fano-like resonance observation.

* Email:kphanhuy@univ-fcomte.fr
where $\omega_n$ are the respective optical pulsation, $c$ is the speed of light, $\varepsilon_r$ is the relative dielectric permittivity, $\chi_{ijkl}$ is the electrostriction tensor component, $\Omega$ is the frequency detuning between pump and signal and $u_k$ is the $k$ component of the displacement field. The comma followed by an index refers to the derivative with respect to the index coordinate. The acoustic equation reads

$$\rho \ddot{u}_i - (c_{ijkl} u_{k,l})_j + \rho \Gamma_B \dot{u}_i = T^{es}_{ij},$$

$$T^{es}_{ij} = -\varepsilon_0 \chi_{ijkl} E^{(1)}_k E^{(2)*}_l,$$

where $\rho$ is the material density, $c_{ijkl}$ is the elastic tensor, $T^{es}_{ij}$ is the electrostrictive force and $\varepsilon_0$ the vacuum dielectric permittivity and $\Gamma_B$ the viscous damping factor. The dot denotes the derivation with respect to time. Note that the propagation terms are usually neglected since the phonon propagates very slowly in comparison to light. The principle of our methods consist in adding a third field $E^{(3)}$ that will produce a secondary electrostrictive force $T^{fbk}$. If this force is made proportional to $\frac{\partial u}{\partial t}$ it can eventually translate on the left-hand side of the equation and act as an additional viscous force. This proportionality can be achieved by introducing a feedback in the setup. The setup is shown in Fig. 2.

![FIG. 2. Setup. C: Optical circulator; OMW: optical microwire; FBG1,2 Fiber Bragg grating; ISO: optical isolator; mod: EO phase modulator; BPF: RF-bandpass filter; PS: RF-Phase shifter; Amp: RF-amplifier.](image)

It is very similar to a Brillouin optoelectronic oscillator with few major differences nonetheless. A highly coherent laser light is first split in three path. LO and BRI paths are usan local oscillator and Brillouin pump path. The Brillouin pump (green arrows) goes through a circulator C and joint the optical microwire (OMW) where it experiences Brillouin backscattering. The back scattered light (red arrows) comes back to the circulator C and is routed to a fiber Bragg grating to isolate the Stokes light. This light is then mixed with the LO light to produce an optical beat measured by a photodiode. In a silica OMW of diameters inferior to 2 $\mu$m, boundary conditions are no more negligible and acoustic waves propagates as modes. As a result, all the components of the displacement field $(u_{nk})_{n\in\{x,y,z\}}$ are coherent and proportional to a wave function $U e^{-i(\Omega t - k z)}$ of amplitude $U$, pulsation $\Omega$ and wavevector $k$ that must satisfies the acoustic mode dispersion equation. As the same arguments already applies to the optical mode, in the nondepleted pump approximation, Eq. (1) becomes

$$A^{(2)}(z_0 - \Delta z) = \frac{\omega_1 - \Omega}{i 2 k_B c^2} \chi A^{(1)}(z_0) U^*(z_0) \Delta z.$$  

where $A^{(2)}$ is the amplitude of the Stokes wave function $A^{(2)} e^{-i(\omega_2 - k z)}$, $z_0$ is the coordinate of the OMW end, $\Delta z$ is the OMW length, $k_B$ is Stokes light wavevector, $\chi$ is the overall summed contribution of all the tensorial components of the electrostrictive interaction between the pump mode of amplitude $A^{(1)}$ and the acoustic mode of amplitude $U$. As a result the optical beat resulting from the mixing the LO and the Stokes waves is proportional to

$$|A^{(2)}(z_0 - \Delta z) + A^{(1)}|^2 = I^{(2)} + I^{(1)} + \frac{\omega_1 - \Omega}{j 2 k_B c^2} \chi \Delta z \times I^{(1)}(U + c.c.),$$

where $I^{(2)} = A^{(2)} A^{(2)*}$ and $I^{(1)} = A^{(1)} A^{(1)*}$ are respectively Stokes and pump intensity. One can readily see that the optical beat is proportional to the phonon wave function. This information can thus be used to produce a feedback on the Brillouin scattering process as described in the following. The RF signal from the photodiode propagates along the pink line in Fig. 2. The signal conditioning is performed thanks to a bandpass filter, a phase shifter and a RF amplifier before it feeds an electro-optic phase modulator (EOPM). The EOPM then modulates the laser light that followed the third path (FBK) to produce the feedback. To insure that the Anti-Stokes light is proportional to the phonon amplitude $U$, the modulator must be driven with low to moderate RF power (i.e. driving voltage $V \ll 2.4 V_z$). In this case the amplitude of the first harmonic amplitude would be $J_0 \left( \frac{V}{V_z} \right) \approx \frac{V}{2 V_z} \propto U$. The desired feedback modulated light thus leaves the modulator, goes through an isolator and a filter FBG2 that isolates the modulation sideband that corresponds to the Anti-Stokes frequency. Note that this is where our setup fundamentally differs from previous Brillouin optoelectronic oscillator schemes. In those schemes, the Stokes sideband is used to stimulate the Brillouin scattering and close the loop of the oscillator. Here since we use the Anti-Stokes frequency to perform cooling, our goal is not to close a loop and add enough gain to have sustained oscillations or observe nonlinear cavity dynamic. In our case, the Stokes signal never reaches the photodiode since it is strongly attenuated by the filter FBG1 before the photodiode and the gain on the Anti-Stokes feedback is kept low. This should prevent unwanted oscillations. However a last condition has to be fulfilled so that the added process act as a viscous force. If we add the Anti-Stokes process to Eq. (5) we obtain

$$\rho \ddot{u}_i - (c_{ijkl} u_{k,l})_j + \rho \Gamma_B \dot{u}_i = T^{es}_{ij} + T^{fbk}_{ij},$$

$$T^{fbk}_{ij} = -\varepsilon_0 \chi_{ijkl} E^{(1)}_k E^{(3)*}_l.$$
where $E_{(3)l}^{(1)}$ is the $l$ component of the amplitude of the feedback Anti-Stokes light. Thanks to the measurement of $U$ and the control of the amplitude and phase of the RF signal, if the feedback delay is short enough to be negligible, we can alter $E_{(3)l}^{(1)}$ and make the feedback electrostrictive force equal to $T_{(3)l}^{bfk} = i\Omega \rho_0 u_i$. Note that for an OMW, the Brillouin that is typically few tens of MHz large for a 9 GHz center frequency. As a consequence, multiplying by $i\Omega$ is similar to a time derivation. The phonon equation then writes

$$\rho \ddot{u}_i - (c_{ijkl}u_{k,l})_{,j} + \rho (\Gamma_B + \kappa) \dot{u}_i = T_{(3)l}^{ec}, \quad (9)$$

Following the calculation detailed in Boyd [18], we find in continuous-wave regime that the Stokes intensity then writes

$$\frac{\partial I_2}{\partial z} = -g_0 \frac{(\Gamma_B + \kappa)^2}{(\Omega_B - \Omega)^2 + (\Gamma_B + \kappa)^2} I_1 I_2,$$

where $g_0$ is the Brillouin gain [18]. In Fig. 3, we plot the normalized Brillouin gain for feedback neglected delay and various values of $\kappa$. One can see that for higher values of $\kappa$ the damping increases thanks to the extra viscous force and enlarge the Brillouin resonance similarly to Cohadon et al. experiment results [9]. Note that to obtain this result it is important that the feedback delay is shorter than the photon lifetime which in the case of Brillouin in silica is 10 ns. In the following paragraph we will investigate the system response if the delay is longer.

III. TEMPORAL RESPONSE WITH MODERATE DELAY

First, we remind that since the acoustic wave behave as a mode, each component of the displacement field $u_i$ are proportional the acoustic wave function $U(t) = U e^{-i\Omega t}$, an Eq. (7) can be rewritten in the simpler form of a damped oscillator equation when the propagation is neglected,

$$\ddot{U} + \Omega_B^2 U + \Gamma_B \dot{U} + \kappa U(t - \tau) = E_{(3)l}^{(1)}$$

where $\tau$ is the feedback delay and $E_{(3)l}^{(1)}$ is the electrostrictive force resulting from the pump-signal interaction. The dot denotes the derivation with respect to time. This equation is a constant delay differential equation that can be solved numerically [19, 20]. Here we resolve the free oscillation regime for $E_{(3)l}^{(1)} = 0$ and initial conditions $U(0 < t) = 1$. In Fig. 4, we show the resolution for a 9 GHz resonance and different feedback value when the delay is negligible ($\tau \ll \frac{1}{\Gamma_B}$). We have also decreased the life time to 0.3 ns to improve lisible. In Fig. 4(a), with no feedback involved, the lifetime is as expected 0.3 ns. As we apply a viscous force with $\kappa = \Gamma_B$, the acoustic lifetime reduces by a factor of two as shown by Fig. 4(b). Finally, Fig. 4(c) shows that if we switch the feedback sign, we can compensate the damping as shown for $\kappa = -0.8 \times \Gamma_B$. The same numerical resolution can be performed for moderate delay like $\tau = 1$ ns which is about three times the acoustic lifetime. In this case, Fig. 5(a-b), shows a very different behavior. The feedback revive the acoustic wave with a period corresponding to the loop. Since the overall loop gain is lower than unity, the successive replicas have less energy. However comparing Fig. 5(a) and Fig. 5(b) shows that the replicas arrive in phase with the original acoustic oscillation, the oscillator progressively accumulate some energy until it reaches sustainable oscillation. This is what one would expect from a standard electro-optic oscillator. On the opposite, when the replica arrives out of phase as shown by Fig. 5(b) at 1 ns, we then observe a sequence of further damped oscillations. As this behavior depends on phase, it should have a signature in the frequency domain that will be investigated in the next paragraph.

IV. “FANO” SPECTRAL RESONANCE WITH LONG DELAY

Taking into account the delay $\tau$, Eq. (10) rewrites

$$\frac{dI_1}{dz} = \frac{\omega^2 \gamma_c^2}{8\pi n c \rho_0} I_2 I_1^* 2\text{Re} \left\{ \frac{i}{\Omega_B^2 - \Omega^2 - i\Omega (\Gamma_B + \kappa)} \right\}$$

where $k$ is the acoustic wavevector, $\gamma_c$ the electrostrictive constant, $n$ the refractive index, $\rho_0$ the mean density of...
the medium. The right-hand side of the equation gives the Brillouin spectrum and can be computed for different delays. In the Fig. 5 we show computed spectra for delay $\tau = 20$ ns in linear (a) and dB scale (b) for a feedback strength $\kappa = \Gamma_B$ in blue. Comparing with the non feedback Brillouin spectrum in green, we can see the spectrum almost splits in two peaks showing the signature of a modulated signal. In the temporal domain, one thus expect replicas of the Brillouin signal to be successively reinserted via the feedback loop at this modulation rate similarly as the one shown previously in Fig. 5(a). Since the right-hand side Eq. (11) involves the real part of the Lorentzian expression, it reaches a maximum value when the denominator finds its lowest value. When $\Omega$ is taken close to $\Omega_B = \kappa e^{i\Omega \tau}$, this denominator is minimum. It is illustrated by Fig. 6(b) that shows in log scale the presence of Fabry-Perot-like fringes spaced by a free spectral range $2 \pi$ corresponding to a cavity formed by the loop. When $\Omega$ is taken close to $\Omega_B$, one can see that the frequencies that minimize the denominator are degenerated around the Brillouin frequency thanks to the complex nature of $\kappa e^{i\Omega \tau}$. The imaginary part of this term then adds to $\Omega_B^2$ in the equation leading to small resonances on both sides of the Brillouin frequency $\Omega_B$. When the delay is increased just enough to induce a $\pi$-phase shift, the spectrum flattens as seen in Fig. 6(c). Since a phase-shift in time domain results in a translation in frequency domain, the Brillouin resonance is now aligned with a Fabry-Perot dark fringe. However as shown in Fig. 6(d), the $\Omega_B^2 - \Omega^2$ dominate and no dip is found. Note that the spectrum widening finds its echo in the time domain where the replicas are alternatively in-phase and out of phase every loop time leading to a loss of coherence. Finally, Fig. 6(e) shows the most interesting behaviour as the delay is again increased of $\frac{\pi}{2}$. Here we observe a Fano-like resonance. Fano resonance occur when a background continuum of states is coupled to a resonant discrete state. Usually the spectrum due to the background states vary slowly enabling the resonant discrete state to induce a sharp asymmetric peak in the spectrum [21, 22]. Here the Brillouin resonance is the discrete state and the cavity formed by the feedback loop gives a set of states. However in our case the Brillouin resonance is large and play the role of the slow varying spectrum whereas the feedback loop states are sharp and resonant. It finds explanation as the losses from the feedback loop can be compensated by the RF amplifier. As illustrated by Fig. 6(f), the result is nonetheless very similar to a Fano resonance as a a sharp and asymmetric resonance emerge from the spectrum at the Fabry-Perot resonance position. Such a narrow linewidth resonance could be used in an optical tunable Brillouin filter or a Brillouin laser where the feedback length is not a problem.

V. CONCLUSION

In conclusion, we have theoretically investigated the control of the Brillouin linewidth by electro-optic feedback. Contrary to previous experiments where the combination of Brillouin scattering and optical-feedback was
intended to build an optoelectronic oscillator, we use the optical feedback to take control of the Brillouin linewidth by introducing an external viscous force. For delay shorter than the coherence time, the viscous force can damp or narrow the Brillouin linewidth depending on its sign and value. For longer delay, a behaviour similar to optoelectronic oscillator is predicted. However we also numerically observed the presence of Fano-like resonances when the feedback loop induces cavity states that are slightly detuned from the Brillouin linewidth. This could find various applications in the optical filtering and laser development.

VI. ACKNOWLEDGEMENTS

This work was supported by the FUNFILM project (ANR-16-CE36-0010-03), the OASIS project (ANR-14-CE36-0005-01), the support from the Région de Franche-Comté and the LABEX ACTION program (ANR-11-LABX-0001-01).

[1] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity Optomechanics (Germany: Springer) (2014)
[2] Y. Pennec, V. Laude, N. Papanikolaou, B. Djafari-Rouhani, M. Oudich, J. S. El, J. C. Beugnot, J. M. Escalante, and A. Martinez, Nanophotonics 3, 413 (2014).
[3] F. Marquardt, J. P. Chen, A. A. Clerk, and S. M. Girvin, Phys. Rev. Lett. 99, 093902 (2007).
[4] I. Wilson-Rae, N. Nooshi, W. Zwerger, and T. J. Kippenberg, Phys. Rev. Lett. 99, 093901 (2007).
[5] G. Bahl, M. Tomes, F. Marquardt, and T. Carmon, Nat Phys 8, 203 (2012).
[6] O. Arcizet, P.-F. Cohadon, T. Briant, M. Pinard, and A. Heidmann, Nature 444, 71 (2006).
[7] J. Chan, T. P. M. Alegre, A. H. Safavi-Naeini, J. T. Hill, A. Krause, S. Grblacher, M. Aspelmeyer, and O. Painter, Nature 478, 89 (2011).
[8] S. Mancini, D. Vitali, and P. Tombesi, Phys. Rev. Lett. 80, 688 (1998).
[9] P. F. Cohadon, A. Heidmann, and M. Pinard, Phys. Rev. Lett. 83, 3174 (1999).
[10] R. G. Harrison, J. S. Uppal, A. Johnstone, and J. V. Moloney, Phys. Rev. Lett. 65, 167 (1990).
[11] D. Yu, W. Lu, and R. G. Harrison, Phys. Rev. A 51, 669 (1995).
[12] X. S. Yao, Opt. Lett., OL 22, 1329 (1997).
[13] X. S. Yao, Brillouin opto-electronic oscillators, US Patent No. 5 917, 179A (1999).
[14] M. Merklein, B. Stiller, I. V. Kabakova, U. S. Mutugala, K. Vu, S. J. Madden, B. J. Eggleton, and R. Slavk, Opt. Lett., OL 41, 4633 (2016).
[15] K. O. Hill, B. S. Kawasaki, and D. C. Johnson, Applied Physics Letters 28, 608 (1976).
[16] T. Tanemura, Y. Takushima, and K. Kikuchi, Opt. Lett., OL 27, 1552 (2002).
[17] J. C. Yong, L. Thévenaz, and B. Y. Kim, Journal of Lightwave Technology 21, 546 (2003).
[18] R. W. Boyd, Nonlinear Optics, Third Edition, 3 edition (Academic Press, Burlington, MA, 2008).
[19] K. W. Neves, ACM Trans. Math. Softw. 1, 357 (1975).
[20] L. F. Shampine and S. Thompson, Applied Numerical Mathematics 37, 441 (2001).
[21] Y. S. Joe, A. M. Satanin, and C. S. Kim, Phys. Scr. 74, 259 (2006).
[22] U. Fano, Phys. Rev. 124, 1866 (1961).