Some Types of Mappings in Bitopological Spaces

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Abstract:

This work introduces some concepts in bitopological spaces, which are nm-joω-converges to a subset, nm-joω-directed toward a set, nm-joω-closed mappings, nm-joω-rigid set, and nm-joω-continuous mappings. The mainline idea in this paper is nm-joω-perfect mappings in bitopological spaces such that n = 1, 2 and m = 1, 2 n ≠ m. Characterizations concerning these concepts and several theorems are studied, where j = θ, δ, α, pre, b, β.

Key words: Filter base, nm-joω-converges, nm-joω-closed mappings, j-ω-rigid a set, nm-joω-perfect mappings.

Introduction and Preliminaries:

In 1963 Kelly J. C. (1) introduced the definition, a set G with two topologies σ_1 and σ_2 is said to be bitopological space and denoted by (G, σ_1, σ_2) and a subset K ⊆ G. The closure and interior of K in (G, σ_i) is denoted by σ_i-cl(K) and σ_i-int(K), where i = 1, 2. A topological space (G, σ) and a point g in G is said to be condensation point of K ⊆ G if every open neighborhood S in σ with g ∈ S, the set K ∩ S is uncountable (2). In 1982 the ω-closed set was first exhibited by H. Z. Heib in (3) defined it as a subset K ⊆ G is called ω-closed if it incorporates each its condensation points, and the ω-open set is the complement of the ω-closed set and the ω-closed of the set K ⊆ G denoted by ω-cl(K). The ω-interior of the set K ⊆ G is defined as the union of all ω-open sets content in K and is denoted by intω(K). In (4) a point g ∈ G is said to be ω-cluster points of K ⊆ G if clω(S) ∩ K ≠ φ for each open set S of G contained g. Also in (4) the set of each ω-cluster points of K is called the ω-closure of K and is denoted by ω-cl(K). A subset K ⊆ G is called ω-closed (4) if K = ω-cl(K). The complement of ω-closed set is said to be ω-open. A point g ∈ G is said to be ω-cluster points of K ⊆ G if clω(S) ∩ K ≠ φ for each ω-open set S of G containing g. The set of each ω-cluster points of K is called the ω-closure of K and is denoted by ω-cl(K). A subset K ⊆ G is called ω-closed (4) if K = ω-cl(K). The complement of ω-closed set is said to be ω-open. A subset K ⊆ G is said to be ω-closed (5) if K = ω-cl(K) = {g ∈ G : int(ω-cl(S)) ∩ K ≠ φ, S ∈ τ and g ∈ S}. The complement of ω-closed is called ω-open set, and K is ω-ω-closed if K = ω-clω(K) = {g ∈ G : intω-cl(S)) ∩ K ≠ φ, S ∈ τ and g ∈ S}. For other notions or notations not defined here, R. Engling (6) should be followed closely. Several characterizations of ω-closed sets were provided in (4, 5, 8, 9, and 10). Some of the results in (11), (12), (13), (14) and (15) will be built.

Definition 1. (1) A nonempty family ℑ of nonempty subsets of G is called filter base if M_1, M_2 ∈ ℑ then M_3 ⊆ M_1 ∩ M_2 for some M_1 ∈ ℑ.

The filter generated by a filter base ℑ consists of all supersets of elements of ℑ. An open filter base on a space G is a filter base with open members.

The set N_γ of all neighborhoods (nbds) of g ∈ G is a filter on G, and any nb base at g is a filter base for N_γ. This filter called the nbd filter at g.

Definition 2. (1) Let ℑ and φ be filter bases on G. Then φ is called finer than ℑ (written as ℑ < φ) if for all M ∈ ℑ, there is G ∈ φ. G ⊆ M also, that ℑ meets φ if M ∩ G ≠ φ for all M ∈ ℑ also, G ∈ φ.

Notice, ℑ → g iff N_γ < ℑ.

Definition 3. (7) A subset K of a space G is called:
Theorem 1. Let $G$ be an $nm_j$-open condensation point of a filter base $\mathcal{F}$ on $G$, then every $\sigma_j$-open nbd $S$ of $g$, the $j$-open closure of $S$ contains a member of $\mathcal{F}$ and thus contains a member of any filter base $\mathcal{F}^*$ minutes than $\mathcal{F}$, so that $\mathcal{F}^* nm_j \to g$.

Proof: ($\Rightarrow$) Let $g$ be an $nm_j$-open condensation point of a filter base $\mathcal{F}$ on $G$, then every $\sigma_j$-open nbd $S$ of $g$, the $j$-open closure of $S$ contains a member of $\mathcal{F}$ and thus contains a member of any filter base $\mathcal{F}^*$ minutes than $\mathcal{F}$, so that $\mathcal{F}^* nm_j \to g$.

($\Leftarrow$) Assume that $g$ is not an $nm_j$-open condensation point of a filter base $\mathcal{F}$ on $G$, then there existent an $\sigma_j$-open nbd $S$ of $g$, such that $j$-open closure of $S$ contains no member of $\mathcal{F}$, denote by $\mathcal{F}^*$ the family of sets $M^* = M \cap (G - (cl_j^ω(S)))$ for $M \in \mathcal{F}$, then the sets $M^*$ are nonempty. And $\mathcal{F}^*$ is a filter base and indeed is minute than $\mathcal{F}$, since $M_1^* = M_1 \cap (G - cl_j^ω(S))$ and $M_2^* = M_2 \cap (G - cl_j^ω(S))$, so there is an $M_3 \subseteq M_1 \cap M_2$ and this lead to:

$M_3^* = M_3 \cap (G - (cl_j^ω(S))) \subseteq M_1 \cap M_2 \subseteq (G - (cl_j^ω(S)))

By construction $\mathcal{F}^*$ not $nm_j$-convergent to $g$. This contradiction, and thus $g$ is an $nm_j$-open condensation point of a filter base $\mathcal{F}$ on $G$.

Definition 7. A filter base $\mathcal{F}$ on a bitopological space $(G, \sigma_1, \sigma_2)$ is said to be $nm_j$-open directed toward a set $K \subseteq G$ (written as $3nm_{j-\omega}$-dir-tow $\to K$) if for each filter base $\varphi$ finer $\mathcal{F}$ has an $nm_j$-open condensation point in $K$. i.e. $(nm_j$-open cod $\varphi) \cap

K \neq \varphi$. $3nm_{j-\omega}$-dir-tow $\to g$ used to mean $3nm_{j-\omega}$-dir-tow $\to \{g\}$, where $g \in G$, and $j = \theta, \delta, \alpha, \pre, b, \beta$.

Theorem 2. Let $\mathcal{F}$ be a filter base on a bitopological space $(G, \sigma_1, \sigma_2)$ and point $g \in G$, then $\mathcal{F}$ $nm_j \to g$ if and only if $3nm_{j-\omega}$-dir-tow $\to g$, where $j = \theta, \delta, \alpha, \pre, b, \beta$.

Proof: ($\Rightarrow$) Clear.

($\Leftarrow$) Assume that $\mathcal{F}$ is not an $nm_j$-open convergence to $g$, there exists an $\sigma_j$-open nbd $S$ of $g$, such that $M \subseteq cl_j^ω(S)$, for all $M \in \mathcal{F}$. Then $\varphi = \{(M \cap (G - (\sigma_j - cl_j^ω(S))) : M \subseteq \mathcal{F}\}$ is a filter base on $G$ finer than $\mathcal{F}$, and consequently $g \notin nm_j$-open cod $\varphi$. So $\mathcal{F}$ cannot be $nm_j$-open directed towards $g$.

Definition 8. A mapping $\lambda : (G, \sigma_1, \sigma_2) \to (H, \zeta_1, \zeta_2)$ is said to be $nm_j$-perfect if for every filter base $\mathcal{F}$ on $\lambda(G)$, $nm_j$-open directed towards some subset $L$ of $\lambda(G)$, the filter base $\lambda^{-1}(\mathcal{F})$ is $nm_j$-open directed towards $\lambda^{-1}(L)$ in $G$, where $j = \theta, \delta, \alpha, \pre, b, \beta$.
Theorem 4. Let \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) be a mapping. Then the following are equivalent:

(a) \( \lambda \) is \( nm-j-\omega \)-perfect.

(b) For every filter base \( \mathcal{F} \) on \( \lambda(G) \), which is \( nm-j-\omega \)-convergent to a point \( h \in H \), \( \lambda(\mathcal{F}) \) is \( nm-j-\omega \)-filter base on \( G \).

(c) For any filter base \( \mathcal{F} \) on \( G \), \( nm-j-\omega \)-cod \( \lambda(\mathcal{F}) \subset \mathcal{F} \), where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \). 

Proof: (a) \( \Rightarrow \) (b) Proof by Theorem (2).

(b) \( \Rightarrow \) (c) Let \( \mathcal{F} \) be \( nm-j-\omega \)-filter base on \( G \). Then \( \lambda(\mathcal{F}) \) is \( nm-j-\omega \)-filter base on \( \lambda(G) \).

(c) \( \Rightarrow \) (a) Suppose \( \mathcal{F} \) be a filter base on \( \lambda(G) \), it is \( nm-j-\omega \)-directed towards some subset \( L \) of \( \lambda(G) \). Let \( \mathcal{F} \) be a filter base on \( G \) finer than \( \lambda^{-1}(\mathcal{F}) \). Then, \( \lambda(\mathcal{F}) \) is \( nm-j-\omega \)-filter base on \( \lambda(G) \).

Definition 9. A mapping \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) is said to be \( nm-j-\omega \)-closed if the image of every \( nm-j-\omega \)-closed set in \( G \) is \( nm-j-\omega \)-closed in \( H \), where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).

Theorem 6. The \( nm-j-\omega \)-perfect mapping \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) is \( nm-j-\omega \)-closed, where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).

Proof: Follow from Theorem (5) and Theorem (3) (a) \( \Rightarrow \) (c) taking \( \mathcal{F} = \{K\} \).

Definition 10. A subset \( K \) of bitopological space \( (G, \sigma_1, \sigma_2) \) is said to be \( nm-Supra-\omega \)-rigid (written as \( nm-j-\omega \)-rigid ) in \( G \) if for every filter base \( \mathcal{F} \) on \( G \) with \( (nm-j-\omega \)-cod \( \mathcal{F} \) \) \( \cap K = \phi \), there is \( S \in \sigma_1 \) and \( M \in \mathcal{F} \), such that \( K \subset S \) and \( cl^\omega(S) \cap M = \phi \), or equivalent, if for every filter base \( \mathcal{F} \) on \( G \) whenever, 

\[
K \cap (nm-j-\omega \text{-cod } \mathcal{F}) = \phi,
\]
then for some \( M \in \mathcal{F} \), 

\[
K \cap (nm-j-\omega \text{-cl } \mathcal{F}) = \phi,
\]
then \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).

Theorem 7. If a mapping \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) is \( nm-j-\omega \)-closed such that for every \( h \in H \), \( \lambda^{-1}(h) \) is \( nm-j-\omega \)-rigid in \( G \), then \( \lambda \) is \( nm-j-\omega \)-perfect, where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).

Proof: Assume \( \mathcal{F} \) is a filter base on \( \lambda(G) \) such that \( \lambda^{-1}(\mathcal{F}) \) is \( nm-j-\omega \)-closed in \( H \), for some \( h \in H \). If \( \mathcal{F} \) is a filter base on \( G \) finer than the filter base on \( \lambda^{-1}(\mathcal{F}) \). Thus, \( \lambda(\mathcal{F}) \) is a filter base on \( \lambda(G) \), finer than \( \mathcal{F} \). Since \( \lambda^{-1}(\mathcal{F}) \) is \( nm-j-\omega \)-filter base on \( \lambda(G) \), \( \lambda(\mathcal{F}) \) is \( nm-j-\omega \)-filter base on \( \lambda(G) \).

Definition 11. A mapping \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) is said to be \( nm-Supra-\omega \)-continuous (written as \( nm-j-\omega \)-continuous) if for any \( \zeta_1 \)-open nbd \( T \) of \( \lambda(g) \), there exists a \( \sigma_1 \)-open nbd \( S \) of \( g \), \( \lambda(cl^\omega(S)) \subset cl^\omega(T) \), where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).

Definition 12. A mapping \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) is said to be weakly \( nm-j-\omega \)-continuous if for any \( \zeta_1 \)-open nbd \( T \) of \( \lambda(g) \), there exists a \( \sigma_1 \)-open nbd \( S \) of \( g \) such that \( \lambda(S) \subset cl^\omega(T) \), where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).

Definition 13. A mapping \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) is said to be strongly \( nm-j-\omega \)-continuous if for any \( \zeta_1 \)-open nbd \( T \) of \( \lambda(g) \), there exists a \( \sigma_1 \)-open nbd \( S \) of \( g \), \( \lambda(cl^\omega(S)) \subset cl^\omega(T) \), where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).

Definition 14. A mapping \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) is said to be super \( nm-j-\omega \)-continuous if for any \( \zeta_1 \)-open nbd \( T \) of \( \lambda(g) \), there exists a \( \sigma_1 \)-open nbd \( S \) of \( g \), \( \lambda(int^\omega_{\zeta_1}(cl^\omega(T))) \subset cl^\omega(T) \), for \( n, m \) = 1 and 2 such that \( n \neq m \), where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).

Definition 15. A mapping \( \lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2) \) is said to be almost \( nm-j-\omega \)-continuous if for any \( \zeta_1 \)-open nbd \( T \) of \( \lambda(g) \), there exists a \( \sigma_1 \)-open nbd \( S \) of \( g \), \( \lambda(S) \subset cl^\omega(T) \), for \( n, m \) = 1 and 2 such that \( n \neq m \), where \( j = \theta, \delta, \alpha \), pre, \( b, \beta \).
1 and 2 such that \((n \neq m)\), where \(j = \theta, \delta, \alpha, \text{pre}, b, \beta\).

The relation between weakly and strongly \(nm\)-\(j\)-\(o\)-continuous mappings are given by the following:

![Table 1. The relation between weakly and strongly \(nm\)-\(j\)-\(o\)-continuous mappings, where \(j = \theta, \delta, \alpha, \text{pre}, b, \beta\).]

| Strongly \(nm\)-\(j\)-\(o\)-continuous mapping | \(\Rightarrow\) | \(nm\)-\(j\)-\(o\)-perfect mapping |
|-----------------------------------------------|-------------|----------------------------------|
| Weakly \(nm\)-\(j\)-\(o\)-continuous mapping | \(\Rightarrow\) | \(nm\)-\(j\)-\(o\)-rigid mapping |

In the higher figure the converses not be true such that the demonstrated by the following examples:

**Example 1.** Let \(A\) be the upper half of the plane and \(B\) be the x-axis. Let \(G = A \cup B\). If \(r_{\text{dis}}\) be the half disc topology on \(G\) and \(\tau\) be the relative topology that \(G\) inherits by virtue of being a subspace of \(\mathbb{R}^2\). The identity mapping \(\lambda : (G, \tau) \rightarrow (G, r_{\text{dis}})\). Then, \(\lambda\) is weakly \(nm\)-\(j\)-\(o\)-continuous mapping but it is not \(nm\)-\(j\)-\(o\)-continuous mapping.

**Example 2.** Let \(\lambda : (G, \sigma, \sigma) \rightarrow (G, \zeta, \zeta)\) be a mapping such that \(G = \{u, v, w\}\), and \(\sigma = \{G, \phi, \sigma_2 = \{G, \phi, \{u, v\}\}\). Such that \(\lambda(u) = \lambda(v) = \lambda(w) = u\). Then \(\lambda\) is almost \(nm\)-\(j\)-\(o\)-continuous mapping but it is not \(nm\)-\(j\)-\(o\)-continuous mapping.

**Example 3.** Let \(\lambda : (\mathbb{R}, \tau) \rightarrow (\mathbb{R}, \tau)\) be a mapping. Define by \(\lambda(g) = g\), and let \((\mathbb{R}, \tau)\) where \(\tau\) is the topology with basis members are of the form \((a, b)\) and \((a, b) - N\) such that \(N = \{1/n; n \in \mathbb{Z}^+\}\). Then \((\mathbb{R}, \tau)\) is Hausdorff but is not \(o\)-regular. Then \(\lambda\) is \(nm\)-\(j\)-\(o\)-continuous mapping but it is not strongly \(nm\)-\(j\)-\(o\)-continuous mapping.

**Example 4.** Let \(\lambda : (G, \sigma_1, \sigma_2) \rightarrow (G, \sigma_1, \sigma_2)\) be identity mapping, such that \(G = \{u, v, w\}\) and \(\sigma_1 = \{G, \phi, \{u, v\}\}\). Then \(\lambda\) is super \(nm\)-\(j\)-\(o\)-continuous mapping but it is not strongly \(nm\)-\(j\)-\(o\)-continuous mapping.

**Theorem 8.** If an \(nm\)-\(j\)-\(o\)-continuous mapping \(\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta, \zeta)\) is \(nm\)-\(j\)-\(o\)-perfect, then:

(a) \(\lambda\) is \(nm\)-\(j\)-\(o\)-closed.

(b) For every \(h \in H\), \(\lambda^{-1}(h)\) is \(nm\)-\(j\)-\(o\)-rigid in \(G\), where \(j = \theta, \delta, \alpha, \text{pre}, b, \beta\).

**Proof:** (a) By Theorem (6) \(\lambda\) an \(nm\)-\(j\)-\(o\)-perfect mapping is \(nm\)-\(j\)-\(o\)-closed.

(b) To prove \(\lambda^{-1}(h)\) is \(nm\)-\(j\)-\(o\)-rigid, let \(h \in H\), and assume that \(\mathcal{I}\) be a filter base on \(G\) such that \((nm\)-\(j\)-\(o\)-cod \(\mathcal{I}\) \(\cap\) \(\lambda^{-1}(h) = \emptyset\). Then \(h \notin \lambda(\text{nm\)-\(j\)-\(o\)-cod \(\mathcal{I}\))\), since \(\lambda\) is \(nm\)-\(j\)-\(o\)-perfect, by Theorem (3 (a) \(\Rightarrow\) (c)). Then, \(h \notin (nm\)-\(j\)-\(o\)-cod \(\lambda(\mathcal{I}))\), so there exists an \(M \in \mathcal{I}\) such that \(h \notin \text{nm\)-\(o\)-cl}_{\mu}(\lambda(M))\), yond exists an \(\zeta_m\)-open nbd \(T\) of \(h\) also, \(\zeta_m\)-\(cl\)\(^{\mu}\)(\(T\) \(\cap\) \(\lambda(M) = \emptyset\), since \(\lambda\) is \(nm\)-\(j\)-\(o\)-continuous, for every \(g \in \lambda^{-1}(h)\), then \(\sigma_o\)-open nbd \(S\) of \(g\) such that \(\lambda(\text{cl}\^{\mu}(S)) \subset \zeta_m\)-\(cl\)\(^{\mu}\)(\(T\) \(\subset\) \(H\)-\(\lambda(M)\)). Then \(\lambda(\text{cl}\^{\mu}(S)) \cap \lambda(M) = \emptyset\), so that \(\text{cl}\^{\mu}(S) \cap M = \emptyset\), then \(g \notin \text{nm\)-\(o\)-cl}\^{\mu}\)(\(M\)), for every \(g \in \lambda^{-1}(h)\), then \(\lambda^{-1}(h) \cap (\text{nm\)-\(o\)-cl}\^{\mu}\)(\(M\)) = \emptyset\), so \(\lambda^{-1}(h)\) is \(nm\)-\(j\)-\(o\)-rigid in \(G\), where \(j = \theta, \delta, \alpha, \text{pre}, b, \beta\).

**Corollary 1.** An \(nm\)-\(j\)-\(o\)-continuous mapping \(\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2)\) is \(nm\)-\(j\)-\(o\)-perfect if \(\lambda\) is \(nm\)-\(j\)-\(o\)-closed and for every \(h \in H\), \(\lambda^{-1}(h)\) is \(nm\)-\(j\)-\(o\)-rigid in \(G\), where \(j = \theta, \delta, \alpha, \text{pre}, b, \beta\).

The results show that thereupon the higher theorem remainders aught if \(nm\)-\(j\)-\(o\)-closeness of \(\lambda\) is replaced by a strongly enfeeble condition which will be called as a weak \(nm\)-\(j\)-\(o\)-closeness and strong \(nm\)-\(j\)-\(o\)-closeness of \(\lambda\). Thus, these will be predefined as follows:

**Definition 16.** A mapping \(\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2)\) is called weakly \(nm\)-\(j\)-\(o\)-closed if for every \(h \in \lambda(G)\), and each \(\sigma_o\)-open set \(S\) containing \(\lambda^{-1}(h)\) in \(G\), there exists a \(\zeta_m\)-open nbd \(T\) of \(h\), \(\lambda^{-1}(\zeta_m\)-\(cl\)\(^{\mu}\)(\(T\) ) \(\subset\) \(\text{cl}\^{\mu}\)(\(S\)), for \(n, m = 1\) and 2 such that \((n \neq m)\), where \(j = \theta, \delta, \alpha, \text{pre}, b, \beta\).

**Definition 17.** A mapping \(\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2)\) is said to be strongly \(nm\)-\(j\)-\(o\)-closed if for each \(h \in \lambda(G)\), and each \(\sigma_o\)-open set \(S\) containing \(\lambda^{-1}(h)\) in \(G\), there exists a \(\zeta_m\)-open nbd \(T\) of \(h\), \(\lambda^{-1}(\zeta_m\)-\(cl\)\(^{\mu}\)(\(T\) ) \(\subset\) \(S\)), for \(n, m = 1\) and 2 such that \((n \neq m)\), where \(j = \theta, \delta, \alpha, \text{pre}, b, \beta\).

The relation between weakly and strongly \(nm\)-\(j\)-\(o\)-closed mappings are given by the following figure:

![Figure 2. The relation between weakly and strongly \(nm\)-\(j\)-\(o\)-continuous mappings, where \(j = \theta, \delta, \alpha, \text{pre}, b, \beta\).]

152
Theorem 9. An $nm-j$-$\omega$-closed mapping $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ is weakly nm-$j$-$\omega$-closed, where $j = \theta, \delta, \alpha, \beta$, pre, b, $\beta$.

Proof: Assume that $h \in \lambda(G)$ also, let $S$ be a $\sigma$-open set containing $\lambda^{-1}(h)$ in $G$, by Theorem (5) and $\lambda$ is $nm$-$j$-$\omega$-closed mapping, then $nm\text{-}cl_{G}^{j}(\lambda(G - cl_{j}^{\omega}(S)) \subset \lambda([\sigma \text{-} cl_{G}^{j}(G - cl_{j}^{\omega}(S)). Since h \notin \lambda([\sigma \text{-} cl_{G}^{j}(G - cl_{j}^{\omega}(S)), and h \notin nm\text{-}cl_{G}^{j}\lambda(G - cl_{j}^{\omega}(S)). Thus, there exists an $\zeta_{n}$-open nbd $T$ of $h$ in $H$, $\zeta_{n}\text{-}cl_{G}^{j}(T) \cap \lambda(G - cl_{j}^{\omega}(S)) = \Phi$, then $\lambda^{-1}(\zeta_{n}\text{-}cl_{G}^{j}(T)) \cap \lambda(G - cl_{j}^{\omega}(S)) = \Phi$, i.e., $\lambda^{-1}(\zeta_{n}\text{-}cl_{G}^{j}(T)) \subset cl_{j}^{\omega}(S)$, then $\lambda$ is weakly nm-$j$-$\omega$-closed.

The inversion of the Theorem (9) is not be right, it will be shown by next example:

Example 5. Let $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ be a constant mapping and $\sigma$, $\zeta$ and $\zeta_{2}$ be any topology, then $\lambda$ is weakly nm-$j$-$\omega$-closed for $n = m = 1$ and $2$ such that $(n \neq m)$, let $G = H = \Phi$. If $\zeta_{2}$ or $\zeta_{2}$ is discrete topology on $H$, then $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ given by $\lambda(g) = 0$, for every $g \in G$, is neither $j$-$\omega$-closed nor $2j$-$\omega$-closed, regardless of the topologies $\sigma$, $\sigma$ also, $\zeta$ (or $\zeta$), where $j = \theta, \delta, \alpha, \beta$.

Theorem 10. An strongly nm-$j$-$\omega$-closed mapping $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ is nm-$j$-$\omega$-closed, where $j = \theta, \delta, \alpha, \beta$.

Theorem 11. If an nm-$j$-$\omega$-continuous mapping $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ is nm-$j$-$\omega$-perfect, then:
(a) $\lambda$ is strongly nm-$j$-$\omega$-closed.
(b) for every $h \in H$, $\lambda^{-1}(h)$ is nm-$j$-$\omega$-rigid in $G$, where $j = \theta, \delta, \alpha, \beta$.

Theorem 12. Let $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ be nm-$j$-$\omega$-continuous mapping. Then $\lambda$ is nm-$j$-$\omega$-perfect, if:
(a) $\lambda$ is weakly nm-$j$-$\omega$-closed.
(b) for every $h \in H$, $\lambda^{-1}(h)$ is nm-$j$-$\omega$-rigid in $G$, where $j = \theta, \delta, \alpha, \beta$.

Proof: Assume that $\lambda$ is nm-$j$-$\omega$-continuous mapping then satisfying the condition for (a) and (b). To show that $\lambda$ is nm-$j$-$\omega$-perfect, Theorem (7) show that $\lambda$ is nm-$j$-$\omega$-closed, let $h \in nm\text{-}cl_{G}^{j}\lambda(K)$, for some non- null subset $K$ of $G$. However $h \notin \lambda(nm\text{-}cl_{G}^{j}(K))$, so $\mathcal{L} = \{K\}$ is a filter base on $G$, also $\lambda^{-1}(h) = \Phi$, by nm-$j$-$\omega$-rigidity of $\lambda^{-1}(h)$. There is $\sigma$-open set $S$ containing $\lambda^{-1}(h)$ such that $cl_{j}^{\omega}(S) \cap K = \Phi$, and by a mapping $\lambda$ is weakly nm-$j$-$\omega$-closed, there exists an $\zeta_{n}$-open nbd $T$ of $h$, such that $\lambda^{-1}(\zeta_{n}\text{-}cl_{j}^{\omega}(T)) \subset cl_{j}^{\omega}(S)$. Then $\lambda^{-1}(\zeta_{n}\text{-}cl_{j}^{\omega}(T)) \cap K = \Phi$, i.e. $\lambda^{-1}(\zeta_{n}\text{-}cl_{j}^{\omega}(T)) \cap \lambda(K) = \Phi$, this is impossible because that $h \in nm\text{-}cl_{G}^{j}\lambda(K)$. So $h \in \lambda(\text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)). Then $\lambda$ is nm-$j$-$\omega$-closed.

Study on some Types of j-$\omega$-perfect Mappings in Bitopological Spaces

In this section, nm-$j$-$\omega$-perfect mappings are given and used the definitions of characterizations theorems for an nm-$j$-$\omega$-continuous mapping and weakly nm-$j$-$\omega$-continuous mapping and strongly nm-$j$-$\omega$-continuous mapping and super nm-$j$-$\omega$-continuous mapping and almost nm-$j$-$\omega$-continuous mapping are indicated to this end, and $n, m = 1, 2$ where $j = \theta, \delta, \alpha, \beta$.

Theorem 13. A mapping $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ is nm-$j$-$\omega$-continuous if $\lambda(\text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)) \subset \text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)$, for $n, m = 1$ and $2$ such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, \beta$.

Proof: (\Rightarrow) Assume that $h \in \text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)$ and $T$ is $\zeta_{n}$ open nbd of $\lambda(g)$. Because of $\lambda$ is nm-$j$-$\omega$-continuous, there exists a $\sigma$-open nbd $S$ of $g$ such that $\lambda(\text{cl}_{j}^{\omega}(S)) \subset \text{cl}_{j}^{\omega}(T)$. Since, $\text{cl}_{j}^{\omega}(S) \cap K \neq \Phi$, then $\zeta_{n}$-cl$_{j}^{\omega}(T) \cap \lambda(K) \neq \Phi$. Thus, $\lambda(g) \in \text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)$. This shows that $\lambda(\text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)) \subset \text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)$ for $n, m = 1$ and $2$ such that $(n \neq m)$.

\hfill (\Leftarrow) Clear.

Theorem 14. A mapping $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ is weakly nm-$j$-$\omega$-continuous if $\lambda(\text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)) \subset \text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)$, for $n, m = 1$ and $2$ such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, \beta$.

Theorem 15. A mapping $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ is strongly nm-$j$-$\omega$-continuous if $\lambda(\text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)) \subset \text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)$, for $n, m = 1$ and $2$ such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, \beta$.

Theorem 16. A mapping $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ is super nm-$j$-$\omega$-continuous if $\lambda(\text{nm-$j$-$\omega$-int-cl}_{j}^{\omega}(K)) \subset \text{nm-$j$-$\omega$-int-cl}_{j}^{\omega}(K)$, for $n, m = 1$ and $2$ such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, \beta$.

Theorem 17. A mapping $\lambda: (G, \sigma, j) \rightarrow (H, \zeta, j)$ is almost nm-$j$-$\omega$-continuous if $\lambda(\text{nm-$j$-$\omega$-cl}_{j}^{\omega}(K)) \subset \text{nm-$j$-$\omega$-int-cl}_{j}^{\omega}(K)$, for $n, m = 1$ and $2$ such that $(n \neq m)$, and for every $K \subset G$, where $j = \theta, \delta, \alpha, \beta$.
Theorem 18. A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2)$ be $nm$-$j$-$\omega$-continuous and $nm$-$j$-$\omega$-perfect, Then $\lambda^{-1}$ preserves $nm$-$j$-$\omega$-rigidity, where $j = \theta, \delta, \alpha$, pre, $b, \beta$.

Proof: Assume that $L$ be an $nm$-$j$-$\omega$-rigid set in $H$ and suppose $\mathcal{Z}$ be a filter base on $G$, then $\lambda^{-1}(L) \cap (nm$-$j$-$\omega$-cod $\mathcal{Z}) = \phi$, since $\lambda$ is $nm$-$j$-$\omega$-perfect and $L \cap \lambda(nm$-$j$-$\omega$-cod $\mathcal{Z}) = \phi$. By Theorem (3) (a) $\Rightarrow$ (c) then $L \cap (nm$-$j$-$\omega$-cod$\lambda(\mathcal{Z})) = \phi$, now $L$ being an $nm$-$j$-$\omega$-rigid set in $H$, there exists an $M \in \mathcal{Z}$ such that $L \cap (nm$-$j$-$\omega$-$cl^\omega(M)) = \phi$, since $\lambda$ is $nm$-$j$-$\omega$-continuous, by Theorem (14) it follows that $L \cap \lambda(nm$-$j$-$\omega$-$cl^\omega(M)) = \phi$. Then $\lambda^{-1}(L) \cap (nm$-$j$-$\omega$-$cl^\omega(M)) = \phi$. This proves that $\lambda^{-1}(L)$ is $nm$-$j$-$\omega$-rigid.

Definition 18. A subset $K$ of a bitopological space $(G, \sigma_1, \sigma_2)$ is said to be $nm$-$j$-$\omega$-set in $H$ if for every $\sigma_n$-open cover $\mathcal{K}$ of $K$, there is a finite sub collection $L$ of $\mathcal{K}$ such that $K \subset \cup \{ cl^\omega_j(S); L \in L \}$, where $j = \theta, \delta, \alpha$, pre, $b, \beta$.

Theorem 19. Let $(G, \sigma_1, \sigma_2)$ be a bitopological space, and a subset $K$ of space for every filter base $\mathcal{Z}$ on $K$ such that $(nm$-$j$-$\omega$-cod $\mathcal{Z}) \cap K \neq \phi$, is an $nm$-$j$-$\omega$-set, where $j = \theta, \delta, \alpha$, pre, $b, \beta$.

Proof: Let $\mathcal{K}$ be an $\sigma_n$-open cover of $K$, $\sigma_n$-$j$-$\omega$-closed of union of any finite subcollection of $\mathcal{K}$ is not cover $K$. So $\mathcal{Z} = \{K / cl^\omega_j(\cup \epsilon(S)); L \in L \}$ is finite subcollection of $\mathcal{K}_j$ is a filter base on $K$ and $(nm$-$j$-$\omega$-cod $\mathcal{Z}) \cap K = \phi$, this contradiction yield that $K$ is an $nm$-$j$-$\omega$-set.

Theorem 20. If $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2)$ is $nm$-$j$-$\omega$-perfect, and $L \subset H$ is $nm$-$j$-$\omega$-set in $H$, then $\lambda^{-1}(L)$ is an $nm$-$j$-$\omega$-set in $G$, for $n, m \in \mathbb{N}$ and such that $(n \neq m)$, and where $j = \theta, \delta, \alpha$, pre, $b, \beta$.

Proof: Assume that $\mathcal{Z}$ is a filter base on $\lambda^{-1}(L)$, then $\lambda(\mathcal{Z})$ is a filter base on $L$. Because $L$ is an $nm$-$j$-$\omega$-set in $H$, such that $L \cap nm$-$j$-$\omega$-cod $\lambda(\mathcal{Z}) \neq \phi$, by Theorem (12). By Theorem (3) (a) $\Rightarrow$ (c), $L \cap \lambda(nm$-$j$-$\omega$-cod $\mathcal{Z}) \neq \phi$, so $\lambda^{-1}(L) \cap nm$-$j$-$\omega$-cod $\lambda(\mathcal{Z}) \neq \phi$. Therefore by Theorem (12), $\lambda^{-1}(L)$ is an $nm$-$j$-$\omega$-set in $G$.

The inversion of the Theorem (20) is not right, as shown by the example following:

Example 6. Let $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2)$ be an identity mapping and $\sigma_1$, $\sigma_2$ be the cofinite and discrete topologies respectively on $G$, and $\zeta_1$, $\zeta_2$ respectively denote the indiscrete and usual topologies on $H$ such that $G = H = \mathbb{R}$, then every subset of either of $(G, \sigma_1, \sigma_2)$ and $(H, \zeta_1, \zeta_2)$ is a $12$-$j$-$\omega$-set. Now, any nonvoid finite set $K \subset G$ is $12$-$j$-$\omega$-closed in $G$, but $\lambda(K)$ (i.e $K$) is not $12$-$j$-$\omega$-closed in $H$, (in fact, the only $12$-$j$-$\omega$-closed subset of $H$ are $H$ and $\phi$), where $j = \theta, \delta, \alpha$, pre, $b, \beta$.

The Theorem (20) and the above Example (6) allude the definition of a strictly weaker transcription of $nm$-$j$-$\omega$- perfect mapping as given below.

Definition 19. A mapping $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2)$ is said to almost $nm$-$j$-$\omega$-perfect if for every $nm$-$j$-$\omega$-set $K$ in $H$, $\lambda^{-1}(K)$ is $nm$-$j$-$\omega$-set in $G$, where $j = \theta, \delta, \alpha$, pre, $b, \beta$.

By analogy to Theorem (20), amnlest condition for a mapping to be almost $nm$-$j$-$\omega$-perfect, is prove as follows.

Theorem 21. Let $\lambda : (G, \sigma_1, \sigma_2) \rightarrow (H, \zeta_1, \zeta_2)$ be any mapping such that

(a) $\lambda^{-1}(h)$ is $nm$-$j$-$\omega$-rigid in $G$, such that for every $h \in H$

(b) $\lambda$ is weakly $nm$-$j$-$\omega$- closed.

Then $\lambda$ is almost $nm$-$j$-$\omega$-perfect, where $j = \theta, \delta, \alpha$, pre, $b, \beta$.

Proof: Assume that $L$ be an $nm$-$j$-$\omega$-set in $H$ and let that $\mathcal{Z}$ be a filter base on $\lambda^{-1}(L)$, then $\lambda(\mathcal{Z})$ is a filter base on $L$. Also, by Theorem (20), $(nm$-$j$-$\omega$-cod $\mathcal{Z}) \cap L \neq \phi$, let $h \in [(nm$-$j$-$\omega$-cod $\mathcal{Z})] \cap L$. Assume that $\mathcal{Z}$ has no $nm$-$j$-$\omega$- condensation point in $\lambda^{-1}(L)$, then $(nm$-$j$-$\omega$-cod $\mathcal{Z}) \cap \lambda^{-1}(h) \neq \phi$. Because of $\lambda^{-1}(h)$ is $nm$-$j$-$\omega$-rigid in $G$, there exists an $M \in \mathcal{Z}$ and a $\sigma_n$-open $S$ containing $\lambda^{-1}(h)$, such that $M \cap \sigma_n$-$cl^\omega_j(S) = \phi$.

By $\lambda$ is weakly $nm$-$j$-$\omega$- closed, then there is a $\zeta_n$ - open nbd $T$ of $h$, $\lambda^{-1}(\zeta_n - cl^\omega_j(T)) \subset \sigma_n$-$cl^\omega_j(S)$.

Therefore which implies that $\lambda^{-1}(\zeta_n - cl^\omega_j(T)) \cap M = \phi$, i.e., $\zeta_n - cl^\omega_j(T) \cap \lambda(M) = \phi$, which is a contradiction. Therefore by Theorem (20), $\lambda^{-1}(L)$ is an $nm$-$j$-$\omega$-set in $G$. So $\lambda$ is almost $nm$-$j$-$\omega$-perfect.

Conclusion.

The main purpose of the present work is the starting point for some application of pairwise supra-$\omega$-perfect mappings of abstract topological structures in filter base by using bitopological spaces. Definitions of characterizations theorems are used for an $nm$-$j$-$\omega$-continuous mapping and weakly $nm$-$j$-$\omega$-continuous mapping and strongly $nm$-$j$-$\omega$-continuous mapping and super $nm$-$j$-$\omega$-continuous mapping and almost $nm$-$j$-$\omega$-continuous mapping.
بعض انواع التطبيقات في الفضاءات التبولوجية الثنائية

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الخلاصة:
قدنا بعض المفاهيم في الفضاءات التبولوجية الثنائية وهي الاقتراب من المجموعة الجزئية من النمط، الاتجاه $\omega j$، $\omega j$، $\omega j$، $\omega j$، والخط الذي يربط القسم série من النمط. التطبيقات المستمرة من النمط $\omega j$، والخط الرئيسي لهذا البحث هو التكاملات التامة من النمط $\omega j$، والخط الذي يربط القسم série من النمط. التكاملات التامة من النمط $\omega j$، والخط الرئيسي هذا البحث هو التكاملات التامة من النمط $\omega j$، والخط الرئيسي هذا البحث هو التكاملات التامة من النمط $\omega j$، والخط الرئيسي هذا البحث هو التكاملات التامة من النمط $\omega j$، والخط الرئيسي هذا البحث هو التكاملات التامة من النمط $\omega j$.

المفتاحيات: المراحل الأساسية، التقارب من النمط $\omega j$، التكاملات التامة من النمط $\omega j$، $\omega j$، $\omega j$، $\omega j$، $\omega j$.