Rung-Rung Current Correlations on a 2-Leg t-J Ladder

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Abstract

We report the results of numerical calculations of rung-rung current correlations on a 2-leg t-J ladder with $J/t = 0.35$ for dopings $x = 0.125$ and $x = 0.19$. We find that the amplitude of these correlations decays exponentially. We argue that this can be understood within a bosonization framework in terms of the pinned phase variables associated with a CISO phase with $d_{x^2-y^2}$-like power law pairing correlations.

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Variational flux-phase states are known to provide interesting low energy estimates for the ground state of the 2D Heisenberg antiferromagnet. In addition, recent calculations have found signatures of a staggered-flux phase in current vorticity correlations in lightly-doped $t$-$J$ clusters. In particular, Ivanov et. al. found evidence for a power law decay of the staggered current vorticity for a projected $d$-wave variational wave function on a $10 \times 10$ lattice and Leung has reported staggered vorticity correlations from the exact diagonalization of a 32-site $t$-$J$ cluster with two holes. Prompted by these results, we have carried out density matrix renormalization group (DMRG) calculations of the rung-rung current correlations on doped 2-leg $t$-$J$ ladders. As is known, near half-filling an isotropic $t$-$J$ ladder, with $J/t$ in the physical range appropriate to the cuprates, retains a spin gap and exhibits power law $d_{x^2-y^2}$-like pair field and $4k_F$-CDW correlations. Thus, it is of interest to explore whether there may also be staggered current correlations on the 2-leg ladder.

Indeed, as discussed below, the DMRG calculations show that there are oscillating rung-rung current correlations. However, we find that they are incommensurate at a finite doping, relatively weak, and decay exponentially on a scale of several lattice spacings. Complementing the numerical results, we discuss a bosonization treatment of the 2-leg ladder. Here, one finds that the rung current depends exponentially on various boson phase fields. We argue that in a C1SO phase which has a spin-gapped ground state with power law $d_{x^2-y^2}$-like pairing correlations, a phase variable appearing in the rung-current is dual to a pinned variable and that this leads to an exponential decay of the rung-rung current correlations, as seen numerically.

Here we will consider an isotropic 2-leg $t$-$J$ ladder described by the Hamiltonian

\begin{equation}
H = t \sum_{i,\lambda,\alpha} (c_{i+1\alpha}^\dagger c_{i\lambda} + h.c.) - t \sum_{i,\alpha} (c_{i\lambda}^\dagger c_{i1\alpha} + h.c.) - J \sum_{i,\lambda} (\vec{S}_{i\lambda} \cdot \vec{S}_{i+1\lambda} - \frac{n_{i\lambda}n_{i+1\lambda}}{4}) - J \sum_{i} (\vec{S}_{i1} \cdot \vec{S}_{i2} - \frac{n_{i1}n_{i2}}{4})
\end{equation}

Here $c_{i\lambda\alpha}$ destroys an electron on rung $i$ and leg $\lambda = 1, 2$ with spin $\alpha = \uparrow, \downarrow$ and $\vec{S}_{i\lambda} = c_{i\lambda}^\dagger \vec{\sigma}/2 c_{i\lambda}$. The Hilbert space excludes all states with doubly-occupied sites. The rung current operator for the $j^{th}$ rung can be written as

\begin{equation}
J_j = it \sum_{\alpha} (c_{j2\alpha}^\dagger c_{j1\alpha} - c_{j1\alpha}^\dagger c_{j2\alpha}) = it \sum_{\alpha} (\psi_{j\alpha\alpha}^\dagger \psi_{j\alpha\alpha} - \psi_{j\alpha\alpha}^\dagger \psi_{j\alpha\alpha})
\end{equation}
Here the second form is expressed in terms of the even (bonding) and odd (antibonding) operators
\[ \psi_{je/o\alpha} = c_{j2\alpha} \pm c_{j1\alpha} \sqrt{2} \tag{3} \]
with \( e \) and \( o \) corresponding to + and − respectively. We will study the ground state rung-rung correlation function
\[ c(|k - j|) = \langle J_k J_j \rangle \tag{4} \]

Using DMRG techniques we have calculated the rung-rung current correlation function, Eq. (1), for a 2 \times 32 ladder with \( J/t = 0.35 \) at fillings \( \langle n_i \rangle = 1 - x \) with \( x = 0.125 \) and 0.19. The DMRG results are for a ladder with open-end boundary conditions and \( c(\ell) \) is obtained by averaging-over sites separated by \( \ell = |k - j| \). The calculations were done with the finite system DMRG method, keeping a maximum of 800 states per block, on a 32 \times 2 open system. The discarded weight was typically about \( 3 \times 10^{-6} \) in the final sweep. The correlation measurements were checked by repeating some of the calculations keeping 1000 states; the differences were quite small and would not affect any conclusions. In order to reduce the effect of the open boundaries, results for many different points with the same separation were averaged over to obtain each data point shown. As shown in Fig. 1(a) for \( x = 0.125 \), the rung-rung current correlations exhibit an incommensurate oscillation with a small amplitude and decay exponentially. Fig. 1(b) shows a semi-log plot of \( |c(\ell)| \) versus \( \ell \) for \( J/t = 0.35 \) and \( x = 0.125 \) and Fig. 1(c) shows a similar plot for \( x = 0.19 \). The slope of the solid line in Fig. 1(b) is 1/2.8 corresponding to an exponential decay of \( c(\ell) \) with a correlation length \( \xi = 2.8 \) lattice spacings. Similarly, as seen in Fig. 1(c), for \( x = 0.19 \) we find an exponential decay of the rung-rung current correlations with \( \xi = 4 \) lattice spacings.

Further insight into the nature of rung-rung current correlations on a 2-leg \( t-J \) ladder is provided within a bosonization framework. Here we pass to a continuum limit along the length of the 2-leg ladder and introduce the usual left- and right-moving fields
\[ \psi_{\lambda\alpha}(x) = e^{-ik_{F\lambda}x}\psi_{L\lambda\alpha}(x) + e^{ik_{F\lambda}x}\psi_{R\lambda\alpha}(x) \tag{5} \]
with \( \lambda = e \) and \( o \) corresponding to the even (bonding) and odd (antibonding) bands respectively with \( k_{Fe} \) and \( k_{Fo} \) the fermi wave vectors for these bands.
Using this representation, the rung current operator, Eq. (2), becomes

\[ J(x) = J_o(x) + \left( e^{i(k_F e + k_F o)x} J_{2k_F}(x) + \text{h.c.} \right) \] (6)

with

\[ J_o(x) = \imath t \sum_\alpha \left[ e^{i(k_F e - k_F o)x} \left( \psi_{Le}^\dagger(x) \psi_{Lo}(x) - \psi_{Ro}(x) \psi_{Re}^\dagger(x) \right) + \text{h.c.} \right] \] (7)

and

\[ J_{2k_F} = \imath t \sum_\alpha \left( \psi_{Le}^\dagger(x) \psi_{Ro}(x) - \psi_{Lo}(x) \psi_{Re}^\dagger(x) \right) . \] (8)

In the usual way, we represent the left and right moving fermion fields by left and right moving boson fields:

\[ \psi_{L/R\lambda}(x) \sim e^{i\sqrt{4\pi} \phi_{L/R\lambda}} \] (9)

Then, introducing the dual canonical Bose fields,

\[ \phi_{\lambda\rho} = \phi_{R\lambda\alpha} + \phi_{L\lambda\alpha} \quad \theta_{\lambda\alpha} = \phi_{R\lambda\alpha} - \phi_{L\lambda\alpha} \] (10)

we have, for example, for the first term in Eq. (8)

\[ \psi_{Le}^\dagger(x) \psi_{Ro}(x) \sim e^{i\sqrt{4\pi} (-\phi_{\rho\alpha} + \theta_{\rho\alpha} + \phi_{\sigma\alpha} + \theta_{\sigma\alpha})} . \] (11)

In terms of the even and odd parity charge and spin Bose fields with \( \lambda = e \) or \( o \)

\[ \phi_{\lambda\rho} = \left( \phi_{\lambda\uparrow} + \phi_{\lambda\downarrow} \right) / \sqrt{2} \]
\[ \phi_{\lambda\sigma} = \left( \phi_{\lambda\uparrow} - \phi_{\lambda\downarrow} \right) / \sqrt{2} \] (12)

and their dual fields, we can write the up spin part of Eq. (11) as

\[ \psi_{Le\uparrow}^\dagger(x) \psi_{Ro\uparrow}(x) \sim e^{i\sqrt{4\pi} (-\phi_{\rho\uparrow} - \phi_{\sigma\uparrow} - \theta_{\rho\uparrow} - \theta_{\sigma\uparrow})} . \] (13)

Here,

\[ \phi_{\pm\rho} = \left( \phi_{\rho\pm} \pm \phi_{\sigma\pm} \right) / \sqrt{2} \] (14)

with similar relations for \( \phi_{\pm\sigma} \), \( \theta_{\pm\rho} \), and \( \theta_{\pm\sigma} \). Actually, this last transformation is not canonical when the even and odd bosons have different velocities. However, we will follow the
standard practice \[5,6\] of assuming that this velocity difference is irrelevant. In this way, the rung current operators, Eqs. (7) and (8) become

\[ J_0(x) \sim i \sum_{\delta_1, \delta_2 = \pm 1} \delta_1 e^{i\pi(k_F e - k_F o)x} e^{-i\sqrt{\pi}[\phi_{-\rho} + \delta_2 \phi_{-\sigma} - \delta_1 \theta_{-\rho} - \delta_1 \delta_2 \theta_{-\sigma}]} + h.c. \]

(15)

(where \( \delta_1 \) labels the left or right term and \( \delta_2 \) labels spin) and

\[ J_{2k_F}(x) \sim \sum_{\delta_1, \delta_2 = \pm 1} \delta_1 e^{i\sqrt{\pi}[-\delta_1 \phi_{-\rho} - \delta_1 \delta_2 \phi_{-\sigma} + \theta_{+\rho} + \delta_2 \theta_{+\sigma}]} \]

(16)

(where now \( \delta_1 \) labels the \( e - o \) or \( o - e \) terms and \( \delta_2 \) labels spin).

Renormalization group analysis \[5,6\], based on the weak coupling 2-leg Hubbard model and earlier numerical results \[7–9\] on \( t-J \) ladders suggest that, for a realistic parameter range away from half-filling, the \( t-J \) model is in a “C1SO” phase in which both spin bosons are gapped and one of the charge bosons is gapped. When a boson is gapped, either \( \langle \phi \rangle \neq 0 \) or \( \langle \theta \rangle \neq 0 \). These expectation values must be specified to completely specify the phase which the system is in. The expected phase has

\[ \langle \theta_{\pm \sigma} \rangle \neq 0 \quad \langle \phi_{-\rho} \rangle \neq 0 . \]

(17)

In this phase the uniform part of the pair correlation function has power-law decay and the \( 2k_F e + 2k_F o \) part of the density correlation function has power law decay (but not the \( 2k_F e \) or \( 2k_F o \) parts), in agreement with the numerical results.

We may replace factors like \( e^{i\sqrt{\pi}\phi_{-\rho}} \) by their expectation values, so that:

\[ J_{2k_F} \sim \sum_{\delta_1, \delta_2 = \pm} \delta_1 e^{i\sqrt{\pi}[-\delta_1 \delta_2 \phi_{-\sigma} + \theta_{+\rho}]} . \]

(18)

Correlation functions involving exponentials of \( \phi_{-\sigma} \) decay exponentially since \( \phi_{-\sigma} \) is conjugate to the pinned dual field \( \langle \theta_{-\sigma} \rangle \). Therefore, we expect that the term in the rung current correlation function which oscillates at \( 2k_F \) will decay exponentially. Similarly, since \( J_0 \) depends exponentially on \( \phi_{-\sigma} \) and \( \theta_{-\rho} \) and both of these are conjugate to pinned phases, these correlation function will also decay exponentially.

Based upon this bosonization analysis, we expect that the asymptotic form of the rung-rung current correlations will oscillate with an incommensurate wave vector \( (k_F o + k_F e) = \pi(1 - x) \). Thus, for \( x = 0.125 \) the bosonization result gives
\[ c(\ell) \sim e^{-\ell/\xi} \cos(7\pi\ell/8 + \phi) \] (19)

Using the DMRG results, \( \xi = 2.8 \) for \( x = 0.125 \), Fig. 2 shows a comparison of Eq. (19) with the DMRG data.

We note that other phases have been suggested for the 2-leg Hubbard model and extended Hubbard models \[3,10\] some of which could have power law decay for the \( 2k_F \) component of the rung current correlation function. However, power law decay could only occur in a phase in which none of the boson fields dual to the ones appearing in Eq. (16) are pinned. This would mean that each boson field in Eq. (16) is either itself pinned or else is gapless. A possible C1S0 phase with this property has:

\[ \langle \theta^+ \sigma \rangle \neq 0, \quad \langle \phi^- \sigma \rangle \neq 0, \quad \langle \phi^- \rho \rangle \neq 0. \] (20)

In fact, such a C1S0 phase was predicted by Fabrizio \[5\] in the Hubbard model for small \( t_\perp \) and a small range of \( U \) near 7, based on a perturbative renormalization group analysis. Thus, it is possible that such a phase might also occur in the 2-leg \( t-J \) model or some generalization of it, for some range of parameters. However, Eq. (20) is not a sufficient condition for power law decay. Not only must these fields have non-zero ground state expectation values, but also these values must be such that a cancellation of the power-law part of the correlation function between the various terms in Eq. (18) does not occur. By explicitly writing pairing operators in the \( \phi/\theta, \rho/\sigma, +/\pm \) basis, used above, it can be checked that all pair correlations (singlet and triplet) have exponential decay in such a putative phase. We emphasize again that, based upon the \( d_{x^2-y^2} \)-like power law pairing correlations and density power law correlations which are observed for the \( t-J \) ladder for physically relevant \( J/t \) and doping regimes of interest, it would appear that the ground state of this model is characterized by pinned phases given by Eq. (17). In this ground state the current-current correlations decay exponentially.

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FIG. 1. The current-current correlation function $c(l)$ on a $32 \times 2$ lattice calculated with DMRG, at dopings of $x = 0.125$ and $x = 0.19$, with $J/t = 0.35$. In (a), we show results on a linear scale for $x = 0.125$, and in (b), we show the same results on a semi-log scale. The solid line in (b) corresponds to a decay length of $\xi = 2.8$. In (c), we show semi-log results for $x = 0.19$. The solid line corresponds to a decay length of $\xi = 4$. 

FIGURES
FIG. 2. (a) The current-current correlation function $c(l)$ data from Fig. 1(a) is compared with the asymptotic form expected from bosonization. Here $A = 0.0016$, $\phi = -2.0$, and as in Fig. 1(b) the decay length is $\xi = 2.8$ lattice spacings.