Extrapolation of Three-dimensional Magnetic Field Structure in Flare-productive Active Regions with Different Initial Conditions

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Abstract

Nonlinear force-free field (NLFFF) modeling has been extensively used as a tool to infer three-dimensional (3D) magnetic field structure. In this study, the dependency of the NLFFF calculation with respect to the initial guess of the 3D magnetic field is investigated. While major parts of the previous studies used the potential field as the initial guess in NLFFF modeling, we adopt linear force-free fields with different constant force-free alpha as the initial guesses. This method enables us to investigate the uniqueness of the magnetic field obtained by the NLFFF extrapolation with respect to the initial guess. The dependence of the initial conditions on NLFFF extrapolation is smaller in the strong magnetic field region. Therefore, the magnetic field at lower heights (<10 Mm) tends to be less affected by the initial conditions (correlation coefficient C > 0.9 with different initial conditions); although, the Lorentz force is concentrated at lower heights.

Unified Astronomy Thesaurus concepts: Active sun (18); Solar active regions (1974); Solar active region velocity fields (1976)

1. Introduction

Solar active regions contain strong magnetic fields and sometimes produce explosive events, such as solar flares and coronal mass ejections (CMEs) by releasing magnetic energy. The magnetic reconnection releases non-potential magnetic energy, and it heats and accelerates the plasmas. One of the most important open questions for solar flares and CMEs is: what are the conditions for the onset of solar flares and CMEs? Inferring a correct 3D magnetic field is a crucial task to answer this question.

The magnetic field in the solar atmosphere is usually measured through spectropolarimetric observations with slit-based or filter-based instruments. State-of-the-art spacecraft, such as Hinode (Kosugi et al. 2007) and Solar Dynamics Observatory (SDO; Pesnell et al. 2012), have enabled us to observe the magnetic field in the photosphere with high spatial resolution and high polarimetric accuracy. While the photospheric magnetic field can be inferred with high accuracy through spectropolarimetric observations, it is difficult to observe the magnetic field in the corona. The coronal lines have low brightness, are optically thin, and suffer from large Doppler broadening.

Force-free field modeling is one of the alternative methods for inferring the 3D magnetic field in the solar corona. The main concept of force-free field modeling is to extrapolate the coronal magnetic field from the spatial map of the magnetic field in the photosphere based on two assumptions (Wiegelmann & Sakurai 2012). The first assumption is the mechanical equilibrium of the plasmas in the solar corona. The second assumption is the domination of the Lorentz force in the solar corona. In the solar corona, the plasma $\beta = 8\pi n p / B^2$, which is the ratio between the plasma pressure and the magnetic pressure, is thought to be sufficiently small, $\beta \ll 1$ (Gary 2001). The above two assumptions lead to the condition that the Lorentz force vanishes in the solar corona,

$$j \times B = 0,$$

where $j$ is the current density. The current density follows the Ampère’s law,

$$\nabla \times B = \frac{4\pi}{c} j.$$

Equation (1) can be written as

$$\nabla \times B = \alpha(r) B,$$

where $\alpha$ is the force-free parameter. Generally, $\alpha$ is not a constant in space, and such a magnetic field distribution is called a nonlinear force-free field (NLFFF).

There exist two main approaches to solve NLFFF equations. The first approach is to solve the NLFFF equations with the boundary conditions, which is a mathematically well-posed problem. An example of this approach was proposed by Grad & Rubin (1958) and so is often called the Grad–Rubin method. In this approach, the distribution of $\alpha$ in only one polarity and the vertical magnetic field are prescribed to the bottom boundary. One weakness of this approach is that the vector magnetic field on the bottom boundary is not consistent with the observed photospheric magnetic field. The second approach is to find the closest force-free equilibrium field matching the observed vector magnetic field in the photosphere, which is prescribed to the bottom boundary. While this approach keeps the bottom boundary consistent with the observed magnetic vector in the photosphere, this approach is an ill-posed problem, and there is no proof of unique and stable solutions.

The examples of this approach are the optimization methods (Wheatland et al. 2000; Wiegelmann & Neukirch 2006) and the magnetohydrodynamics (MHD) relaxation methods (Chodura & Schlueter 1981; Mikic & McClymont 1994; Inoue et al. 2014).
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The NLFFF applications to the solar observations have some uncertainties, e.g., observational errors and numerical methods. The question is, in a practical way, how large are the uncertainties of NLFFF modeling? In other words, is there a possibility that completely different NLFFF results could be obtained based on the same bottom boundary? Schrijver et al. (2006) applied six NLFFF algorithms to analytical force-free field solutions and compared these solutions with each other. The best method reproduced the total energy in the magnetic field with a 2% error. Metcalf et al. (2008) also compared NLFFF methods by using a solar-like reference model. They applied NLFFF methods to both forced (not force-free) and force-free bottom boundaries. They showed that while the NLFFF with the force-free bottom boundary reproduced a helical flux bundle, the NLFFF with the forced bottom boundary did not reproduce the reference model well. De Rosa et al. (2006) applied various kinds of NLFFF algorithms with the force-free bottom boundary reproduced a helical force-free bottom boundary. They show that while the NLFFF reproduce the reference model well. De Rosa et al. (2006) applied six NLFFF algorithms to analytical force-free field solutions and compared these solutions with each other. The best method reproduced the total energy in the magnetic field with a 2% error. Metcalf et al. (2008) also compared NLFFF methods by using a solar-like reference model. They applied NLFFF methods to both forced (not force-free) and force-free bottom boundaries. They showed that while the NLFFF with the force-free bottom boundary reproduced a helical flux bundle, the NLFFF with the forced bottom boundary did not reproduce the reference model well. De Rosa et al. (2009) applied various kinds of NLFFF algorithms (Grad–Rubin, optimization, and MHD relaxation) to the photospheric magnetic field observed with Hinode and investigated the degrees of difference between the solutions from different algorithms. They found that the field lines from the NLFFF do not agree with the EUV images, and the average misalignment is 20–40 degrees. Thalmann et al. (2013) investigated the effect of the observational instrument on the solution of NLFFF. They used a photospheric magnetic field observed with different instruments, i.e., the Hinode/SOT SP and SDO/Helioseismic and Magnetic Imager (HMI; Schou et al. 2012). They concluded that the relative estimates such as normalized magnetic energy and the overall structure of the magnetic fields might be reliable; although, there were remarkable deviations in the absolute value between the instruments. DeRosa et al. (2015) investigated the influence of spatial resolution on NLFFF with three kinds of methods similar to De Rosa et al. (2009). They showed that the free energy tends to be higher with increasing resolutions, and magnetic helicity values vary significantly among different resolutions.

The previous studies mentioned above show how different NLFFF results are produced. The questions remains whether completely different solutions are produced or not based on the same bottom boundary. In other words, how unique is the solution when focusing on one method? If completely different solutions exist, how can we obtain the result most consistent with the coronal imaging observations? Although many previous studies applied NLFFF to solar active regions and showed qualitative agreement between NLFFF and coronal images (Régnier & Amari 2004; Inoue et al. 2012; Sun et al. 2012; Valori et al. 2012; Jiang et al. 2014; Wang et al. 2014; Kawabata et al. 2017; Muhamad et al. 2018), there still exist some regions where the NLFFF produces quite different field lines from the coronal images (De Rosa et al. 2009). Therefore, the problems we have to tackle are (1) to reveal whether completely different solutions with the same bottom boundary can exist or not in practical calculations and (2) what we should do in the case that the modeled field lines do not agree with the coronal images.

To investigate whether completely different solutions with the same bottom boundary exist or not, we evaluate NLFFF extrapolation with different initial conditions. As is often the case with nonlinear inverse problems, the different initial guesses may often produce completely different converged solutions. Therefore, the uniqueness of the NLFFF calculation can be studied by giving different initial conditions.

The NLFFF extrapolation is usually performed as follows:

1. Set the 3D initial condition by using the photospheric vertical magnetic field.
2. Give the information of the horizontal magnetic field to the bottom boundary.
3. Perform some relaxation process.

In process (1), almost all previous studies use the potential field as an initial condition. We perform the NLFFF calculation not only with the potential field but also with the linear force-free field as an initial condition. Several force-free alpha values are given in the linear force-free case. Comparisons among the NLFFF results with different initial conditions will provide some insights into the uniqueness of the NLFFF extrapolation. As described above, previous studies show that different results are obtained depending on the calculation methods used in the NLFFF modeling (Schrijver et al. 2006; Metcalf et al. 2008; Schrijver et al. 2008; De Rosa et al. 2009). One of the causes of the difference between the Grad–Rubin method and the other two methods (optimization and MHD relaxation methods) is the treatment of the bottom boundary. Because the former method relaxes the 3D magnetic field so that the force-free alpha along the magnetic field line is constant, the horizontal component of the magnetic field at the bottom boundary is different from the observational magnetic field in the photosphere. On the other hand, the latter methods keep the bottom boundary as the observational magnetic field vector in the photosphere. We give priority to keeping the magnetic field measured in the photosphere at the bottom boundary. Therefore, we focus on one method, the MHD relaxation method, which was developed by Inoue et al. (2014).

This paper is organized as follows: We describe the observations and data reduction in Section 2. The method of NLFFF extrapolation is described in Section 3. We present and discuss the results in Sections 4 and 5, respectively. We summarize our results in Section 6.

2. Observations

As targets to analyze, we chose two active regions, NOAA 11692 and NOAA 11967 to investigate how significantly different coronal magnetic field structures are derived depending on initial conditions. The examined active regions are two extreme examples; one has a rather simple bi-pole magnetic distribution at the photosphere, and the other shows a complicated field distribution at the photosphere. The photospheric vertical magnetic field distributions are shown in the left upper and lower panels of Figure 1. While the former is composed of bi-pole magnetic fields and shows a weak twist in the photosphere, the latter has multipole magnetic fields and shows a strong twist. The global alpha is one of the index values for expressing the degree of twist in the active regions. Tiwari et al. (2009a, 2009b) defined global alpha \( \alpha_g \) as

\[
\alpha_g = \sum \frac{(\partial B_y - \partial B_x)}{\sum B_z^2} \cdot (\partial B_z - \partial B_x) B_z, \tag{4}
\]

which can be calculated from the photospheric vector magnetic field, \( B_x \), \( B_y \), and \( B_z \), where the z-axis is defined as pointing in the direction vertical to the solar surface. The global alpha of NOAA 11692 is \( \alpha_g = -1.0 \times 10^{-8} \text{ m}^{-1} \), and NOAA 11967 has \( \alpha_g = -5.0 \times 10^{-8} \text{ m}^{-1} \). Absolute values of \( \alpha_g \) in various
active regions usually vary from 0 to $5 \times 10^{-8}$ m$^{-1}$ (Pevtsov et al. 1995, 1997). Therefore, the two active regions we analyzed are contrasting examples in terms of the twist.

2.1. Observations at NOAA 11692

NOAA active region 11692 was a typical active region with a round leading sunspot consisting of an umbra and penumbra. Several opposite polarity magnetic fields are broadly distributed at the following area. This active region produced 14 C-class flares and two M-class flares between 2013 March 12 and 2013 March 22. The vector magnetic field map in the upper left panel of Figure 1 was observed with Hinode/SOT SP and SDO/HMI. The field of view (FOV) of the SP observation is shown by the green box in Figure 1. To increase the narrow FOV of SP when extrapolating the coronal magnetic field by NLFFF modeling, we used the data obtained by the HMI. The gray scale shows the vertical components of the magnetic field to the solar surface, and the green arrows show the horizontal magnetic field, which are derived by the inversion of the spectro-polarimetric data described in Section 2.3. The leading spot has a negative polarity, and a positive polarity is dominant at the following region. The negative spot has a counterclockwise horizontal magnetic field, while the strong horizontal magnetic field cannot be identified in the positive region that follows. The SP performs spectropolarimetric observations with two magnetically sensitive Fe I lines at 6301.5 Å and 6302.5 Å with a spectral sampling of 21.5 mÅ per pixel and scanned this region between 03:00 and 03:35 UT on 2013 March 15. Although the SP provides the polarimetric information based on the slit observations with the high spectral sampling of 21.549 mÅ pix$^{-1}$, the slit observations limit the FOV. NOAA 11692 was located around the disk center ($-161^\prime$, $257^\prime$) at the time of the SP scanning. The HMI measures polarization based on the 6 narrow bands (bandwidth: 76 mÅ $+/-$ 10 mÅ) observations around Fe I 6173 Å line. The HMI has the advantage of regularly observing the full disk of the Sun with a spatial sampling of 0.5 pix$^{-1}$. We also used the HMI data obtained at 03:11 UT on 2013 March 15.

To evaluate the validity in the result from the NLFFF extrapolations, we utilized the soft X-ray image observed with the X-ray telescope (XRT; Golub et al. 2007) on board Hinode, as shown in the upper right panel of Figure 1. The image was observed with Be-thin filter at 03:57 UT on 2013 March 15 with an FOV of 395″ × 395″. The pixel sampling is 1″. A sigmoidal structure can be clearly identified in the X-ray image.

2.2. Observations at NOAA 11967

NOAA active region 11967 was a flare-productive active region, which produced 10 M-class and 38 C-class flares from...
2014 January 31 to 2014 February 9. In the same way as the data for NOAA 11692, we combined a magnetic field map of SP and HMI. We used one of the SP scanned maps obtained at 07:50–08:45 UT on 2014 February 3, as shown by the green box in the lower left panel of Figure 1. The gray scale shows the vertical components of the magnetic field to the solar surface, and the green arrows show the horizontal magnetic field, which are derived by the inversion of the spectropolarimetric data described in Section 2.3. The map has an effective pixel size of 0′′.3 with an FOV of 280′′ × 130′′. NOAA 11967 was located at almost the disk center (−100′′, −100′′) at the time of the SP scanning. We used the HMI data obtained at 07:47 UT. NOAA 11967 was mainly composed of four magnetic polarities (P1, N1, P2, and N2 in Figure 1). While P1 shows round-shape structure, N1, N2, and P2 show elongated structures. At the polarity inversion line (PIL) between N1 and P2, the sheared horizontal magnetic fields are well visible.

We also used X-ray images obtained with XRT with the B-thin filter at 07:12 UT on 2014 February 3 with an FOV of 512′′ × 512′′, as shown in the lower right panel of Figure 1. Compared to NOAA 11692, NOAA 11967 had a complex structure of coronal loops, because this active region has multiple locations of both polarities. The sheared magnetic field lines between N1 and P2 also can be seen in the X-ray image.

### 2.3. Data Reduction

For the calibration of the Stokes profiles obtained with Hinode/SOT SP, we used the Solarsoft routine SP_PREP (Lites & Ichimoto 2013) and applied a Milne–Eddington atmosphere (ME) model in order to derive the physical parameters by a nonlinear least-squares fitting using the code based on MELANIE (Socas-Navarro 2001). SP_PREP corrects the wrap-around of the Stokes I, dark and flat, instrumental polarization, spectral line curvature, thermal drift, and orbital Doppler shift. When we derive the magnetic field azimuth, there is well-known ambiguity called 180 degree ambiguity in the LOS reference frame (Landi Degl’I Innocenti & Landolfi 2004). The 180° ambiguity in the transverse magnetic field direction was solved with the minimum energy ambiguity resolution method (Metcalf 1994; Leka et al. 2009).

For the data of HMI, the vector magnetic field data products were provided by the HMI team, which is called Space-weather HMI Active Region Patches (SHARP; Bobra et al. 2014). The HMI data was used to expand the FOV of the 4 × 4 binned data of SP (1′′/2/pix) for NOAA 11692 and the 2 × 2 binned data of SP (0′′6/pix) for NOAA 11967. The binning was performed in order to reduce the calculation time of the NLFFF extrapolation. The spatial resolution affects the magnetic energy and free energy of the NLFFF models (DeRosa et al. 2015), because the binning process changes the magnetic energy and free energy at the bottom boundary. In this study, we focus on the dependence of the initial condition on the results of the NLFFF based on the same bottom boundary. Therefore, we perform the NLFFF modeling based on the bottom boundary with a single binning factor for each active region (1′′/2/pix for NOAA 11692 and 0′′6/pix for NOAA 11967).

### 3. MHD Relaxation Method and Numerical Settings

The nonlinear force-free field extrapolation is performed by the MHD relaxation method (Inoue et al. 2014; Inoue 2016), which uses the following equations:

\[ \frac{∂v}{∂t} = -(v \cdot \nabla)v + \frac{1}{ρ} j \times B + ν \nabla^2 v, \]

\[ \frac{∂B}{∂t} = \nabla \times (v \times B - η j) - \nabla φ, \]

\[ j = \nabla \times B, \]

\[ \frac{∂φ}{∂t} + c^2 φ ∇ \cdot B = -\frac{c^2}{η} φ, \]

where \( ρ \) is the pseudo density, which is assumed to be equal to \( |B| \) to ease the relaxation by equalizing the Alfvén speed in space, \( φ \) is the convenient potential for \( ∇ \cdot B \) cleaning, and \( ν \) is the viscosity, which is set to a constant (1.0 × 10^{-3}). The length, magnetic field, velocity, and time were normalized by \( L_0 = 157 \) Mm for NOAA 11967 and \( L_0 = 314 \) Mm for NOAA 11692 and \( B_0 = 4000 \) G, \( V_0 = 0.06 \) (\( 4πρ_0 \))^{1/2}, and \( τ_0 = L_0/V_0 \), where \( V_0 \) is the Alfvén velocity. Equations (5)–(8) are the equation of motion, the induction equation, Ampère’s law, and \( ∇ \cdot B \) cleaning introduced by Dedner et al. (2002), respectively. The parameters \( c^2 \) and \( η \) are the advection and diffusion coefficients, respectively, and are fixed at 0.1 and 0.04. The nondimensional resistivity \( η \) is given by

\[ η = η_0 + η_1 \frac{|j \times B| |v|^2}{|B|^2}, \]

where \( η_0 \) and \( η_1 \) are fixed at 5.0 × 10^{-5} and 1.0 × 10^{-3} in nondimensional units. The second term is introduced to accelerate the relaxation to the force-free state.

The velocity field at each grid was adjusted at each time step below in order to avoid becoming large in value. When the value of \( ν^* \) becomes larger than the value of \( ν_{max} \),

\[ ν \rightarrow ν_{max}, \]

where \( ν^* = |ν|/|ν_{max}| \) and \( ν_{max} = 0.1 \).

In previous NLFFF calculations, the potential field has been used as an initial guess for the 3D magnetic field structure. We chose the linear force-free field as an initial condition, including a potential field, which satisfies \( ∇ \times B = α_0 B \), where \( α_0 \) is a constant. We examined five different initial conditions \( α_0 = [0, ±1.2, ±2.3] \times 10^{-8} \) m^{-1} for NOAA 11692 and 12 initial conditions \( α_0 = [0, ±0.70, ±1.2, ±2.3, ±4.6, ±7.0, −12] \times 10^{-8} \) m^{-1} for NOAA 11967. The values of \( α_0 \) are chosen by the pixel size. The \( α_0 \) values of [0, ±0.70, ±1.2, ±2.3, ±4.6, ±7.0, −12] \times 10^{-8} m^{-1} correspond to [0, ±0.003, ±0.005, ±0.01, ±0.02, ±0.03, −0.05] pix^{-1} for NOAA 11967. Because the magnetic field of NOAA 11967 is less affected by the initial conditions than that of NOAA 11692, we have more initial conditions for NOAA 11697.

We used the equations of Alissandrakis (1981) for the calculations of the linear force-free field, and the resulting initial conditions are shown in Figures 2 and 3. Although the case \( α_0 = +12 \times 10^{-8} \) m^{-1} for NOAA 11967 was also calculated, the calculation did not converge. Therefore, we do not include it in the results in this paper. The numerical domain
is set to \((0, 0, 0) < (x, y, z) < (1.0, 0.7, 0.7)\) resolved by \(360 \times 252 \times 252\) nodes for NOAA11692 and \((0, 0, 0) < (x, y, z) < (1.5, 1.0, 0.5)\) resolved by \(540 \times 360 \times 180\) nodes for NOAA 11967. In order to set the same top boundary for all calculations, the initial conditions above \(z = 0.417\) are set to the potential field.

The magnetic field at the top was fixed for the initial state (potential field), and the normal component of the magnetic field on the bottom boundary was also fixed. The side boundary was periodic. We varied the transverse component on the bottom boundary \(B_{BC}\) as follows:

\[
B_{BC} = \gamma B_{obs} + (1 - \gamma) B_{initial},
\]

\(\gamma\) being the volume filling factor, \(B_{obs}\) the observed horizontal component, \(B_{initial}\) the initial horizontal component, and \(\gamma \leq 1\). The volume filling factor is either 0 or 1, the former being the case of an empty bottom boundary, and the latter the case of a perfectly filled bottom boundary. The constant \(\gamma\) is set to 0.5, and the horizontal component of the field on the bottom boundary is the average of the observed field and the initial field. The observed field is shown in Figure 2.

**Figure 2.** The morphology of the magnetic field lines (green solid lines) in NOAA 11692 at the initial condition for the NLFFF extrapolation. The background grayscale image shows the soft X-ray images observed with Hinode/XRT.
where $B_{\text{obs}}$ and $B_{\text{initial}}$ are the transverse components of the observational and initial bottom boundaries, respectively. We increased $\gamma = \gamma + d\gamma$ when $\int |j \times B|^2 dV$ dropped below a critical value. In this study, we set $d\gamma = 0.1$. When $\gamma$ becomes equal to 1, $B_{BC}$ is consistent with the observed field. The number of calculation steps was set to 24,000 steps for NOAA 11692 and 25,000 steps for NOAA 11967.

Spatial derivatives are calculated by the second-order central differences, and temporal derivatives are integrated by the Runge–Kutta–Gill method to a fourth-order accuracy.

4. Results

4.1. Properties of the Active Regions

In this Section, the property of two active regions in the photosphere are summarized. Figure 4 shows the histograms of the horizontal magnetic field (left panel) and vertical magnetic field (right panel) for NOAA 11692 (blue solid line) and NOAA 11967 (red solid line). For both the horizontal and vertical magnetic fields, NOAA 11967 has larger frequencies at larger magnetic fields. While 6.3% of the horizontal magnetic field in the FOV is larger than 1000 G for NOAA 11967, 0.49% is larger than 1000 G for NOAA 11692. Regarding the vertical magnetic field, the ratios of $B_z > 1000$G are 7.5% and 0.47% for NOAA 11967 and NOAA 11692, respectively. The unsigned vertical magnetic flux of NOAA 11967 is $8.4 \times 10^{22} \text{Mx}$, which is 2.3 times larger than that of NOAA 11692, $3.7 \times 10^{22} \text{Mx}$.

One issue with performing NLFFF extrapolations from the photosphere is that this layer is considered to contain Lorentz forces (e.g., Gary 2001). Some previous studies have investigated force-freeness in the active regions in the photosphere based on the necessary condition of the force-free approximation shown by Low (1985). The Lorentz force can be
The values of force-freeness of the active regions from Equations (13)–(15) are listed in Table 1. The values of \(|F_x|/F_p\), \(|F_y|/F_p\), and \(|F_z|/F_p\) of NOAA 11967 are sufficiently small, i.e., the active region satisfies the necessary condition of the force-free field. On the other hand, the absolute value of the force-freeness of NOAA 11692 is slightly larger than 0.1 in \(|F_x|/F_p\) and \(|F_y|/F_p\). However, this value is not so large and not far from 0.1 compared to other active regions reported in previous studies (Metcalf et al. 1995; Moon et al. 2002; Liu et al. 2013). Therefore, the two active regions NOAA 11692 and NOAA 11967 are comparatively appropriate regions to apply to the assumption of force-free modeling.

4.2. Morphology of Field Lines from NLFFF

Figure 5 shows the magnetic field lines (green solid lines) in NOAA 11692 as a result of the NLFFF extrapolation with five different initial conditions. The magnetic field lines are chosen randomly around the region where the sigmoidal structure is identified. Background grayscale images are X-ray images observed with the XRT. As can be clearly seen, the 3D morphology of the magnetic field lines strongly depends on the initial conditions. In other words, the morphology of the magnetic field lines from the NLFFF extrapolation is not far from that of the initial conditions. When we use the potential field as an initial condition (\(\alpha_0 = 0\) case), which is usually chosen, the field lines are potential-like and the sigmoidal structure cannot be correctly reproduced. On the other hand, when we choose appropriate initial conditions (e.g., \(\alpha_0 = -2.3 \times 10^{-8}\) m\(^{-1}\) in Figure 5), many magnetic field lines from the NLFFF extrapolation are almost parallel to the direction of the sigmoidal structure in the X-ray image. The value \(\alpha_0 = -2.3 \times 10^{-8}\) m\(^{-1}\) is larger than the global alpha estimated from the photospheric magnetic field, \(\alpha_g = -1.0 \times 10^{-8}\) m\(^{-1}\). Figure 6 shows a side view of the magnetic field lines. Only the cases of \(\alpha_0 = -2.3 \times 10^{-8}\), 0, and \(2.3 \times 10^{-8}\) m\(^{-1}\) are shown. The heights of the field lines are at around 100–200 Mm. Not only are the long loop lines with loop tops located more than 100 Mm different among different solutions, but the short field lines with loop tops located below 100 Mm are also different among different solutions.

Figure 7 shows the magnetic field lines (green solid lines) in NOAA 11967 as a result of the NLFFF extrapolation with 12 different initial conditions. Compared to NOAA 11692, the results are less dependent on the initial condition. When we use large \(|\alpha_0| > 7.0 \times 10^{-8}\) m\(^{-1}\) as an initial condition, the result shows a slightly different morphology. The result of \(\alpha_0 = -1.2 \times 10^{-8}\) m\(^{-1}\) does not seem to converge to a reasonable solution. As shown in Figure 8, the height of the field lines is around 30 Mm.

| Table 1: Force-freeness of the Active Regions Derived from Equations (13)–(15) |
|----------------------------------|------------------|------------------|
| NOAA 11692                       | NOAA 11967       |
| \(|F_x|/F_p\)                     | 0.053            | 0.029            |
| \(|F_y|/F_p\)                     | 0.128            | 0.0089           |
| \(|F_z|/F_p\)                     | 0.17             | 0.089            |

Figure 4. The histograms of the horizontal magnetic field (left panel) and vertical magnetic field (right panel) for NOAA 11692 (blue solid line) and NOAA 11967 (red solid line).
4.3. Total Magnetic Energy, Total Free Energy, and Extrapolation Metrics

To evaluate the difference due to the initial condition quantitatively, we focus on the total magnetic energy, total free energy, and extrapolation metrics, as shown in Table 2 for NOAA 11692 and Table 3 for NOAA 11967. The first column shows the constant force-free alpha $\alpha_0$, which is used for the initial condition of the NLFFF extrapolation. The second, third, and fourth columns show the magnetic energy of the initial condition $E_{\text{init}}$, the magnetic energy of the NLFFF $E$, and the free magnetic energy $E_{\text{free}}$. The third and fourth columns are normalized by the magnetic energy of the potential field $E_{\text{pot}}$.

Figure 5. The morphology of the magnetic field lines (green solid lines) in NOAA 11692 as a result of the NLFFF extrapolation. The background grayscale image shows the soft X-ray images observed with Hinode/XRT.
Figure 6. A side view of the magnetic field lines in NOAA 11692, obtained by the NLFFF extrapolation with three initial conditions. The background grayscale image projected onto the bottom surface shows the soft X-ray image observed with Hinode/XRT.

Figure 7. The morphology of the magnetic field lines (green solid lines) in NOAA 11967 as a result of the NLFFF calculation. The background grayscale image shows the soft X-ray image observed with Hinode/XRT.
Figure 8. A side view of the magnetic field lines in NOAA 11967, obtained by the NLFFF extrapolation with three initial conditions. The background grayscale image projected onto the bottom surface shows the soft X-ray image observed with Hinode/XRT.

| Initial $\alpha_0$ (10$^{-8}$ m$^{-1}$) | $E_{\text{init}}$ (10$^{32}$ erg) | $E/E_{\text{pot}}$ | $E_{\text{free}}/E_{\text{pot}}$ | $\langle \text{CW sin $\theta$} \rangle$ (10$^{-5}$) | $\langle |j| \rangle$ |
|---------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $-$2.3                                | 8.12            | 1.14            | 0.14            | 0.38            | 5.58            |
| $-$1.2                                | 6.46            | 1.06            | 0.06            | 0.54            | 7.36            |
| 0 (potential)                         | 6.27            | 1.05            | 0.05            | 0.58            | 7.90            |
| 1.2                                   | 6.54            | 1.06            | 0.06            | 0.57            | 8.09            |
| 2.3                                   | 8.39            | 1.14            | 0.14            | 0.39            | 6.35            |

Note. First column: the constant force-free alpha $\alpha_0$ used for the initial condition of the NLFFF extrapolation. Second column: the magnetic energy of the initial condition $E_{\text{init}}$. Third column: the magnetic energy of the NLFFF $E$. Fourth column: the free magnetic energy $E_{\text{free}}$. Fifth column: the mean sine of the angle $\theta$ between $j$ and $\mathbf{B}$ weighted by $j$, which represents force-freeness of the NLFFF extrapolation. Sixth column: the fractional flux ratio, which represents the divergence-freeness of the NLFFF extrapolation.

For NOAA 11692, when we choose the potential field as an initial condition, the resulting magnetic energy of the NLFFF becomes almost potential. When the magnetic energy of the initial condition is larger, the resulting magnetic energy of the NLFFF becomes larger. However, the magnitude of the difference of $E/E_{\text{pot}}$ among the different initial conditions is not very large compared to that of $E_{\text{init}}$. The ratios of the maximum total magnetic energy and free energy ($\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$) to the minimum ones ($\alpha_0 = 0$ m$^{-1}$) are 1.08 and 2.64, respectively. NOAA 11697 has a larger magnetic energy and free energy than those of NOAA 11692, as shown in Tables 2 and 3. As mentioned in Section 4.2, the result from $\alpha_0 = -12 \times 10^{-8}$ m$^{-1}$ does not seem to be physically reasonable. Therefore, we focus on the results except for $\alpha_0 = -12 \times 10^{-8}$ m$^{-1}$ for NOAA 11697. Even when we choose the potential field as an initial condition, the resulting NLFFF has $E_{\text{free}}/E_{\text{pot}} = 0.14$. This notable result for NOAA 11697 is in spite of the large initial constant $\alpha_0$ such that $|\alpha_0| > 2.3 \times 10^{-8}$ m$^{-1}$, and the resulting magnetic field energy and free energy do not show large differences among the different initial conditions. The ratios of the maximum total magnetic energy and free energy ($\alpha_0 = -7.0 \times 10^{-8}$ m$^{-1}$) to the minimum ones (0 m$^{-1}$) are 1.03 and 1.28, respectively.

| Initial $\alpha_0$ (10$^{-8}$ m$^{-1}$) | $E_{\text{init}}$ (10$^{33}$ erg) | $E/E_{\text{pot}}$ | $E_{\text{free}}/E_{\text{pot}}$ | $\langle \text{CW sin $\theta$} \rangle$ (10$^{-5}$) | $\langle |j| \rangle$ |
|---------------------------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $-$12                                 | 20.0            | 1.31            | 0.31            | 0.19            | 8.26            |
| $-$7                                  | 7.17            | 1.18            | 0.18            | 0.23            | 7.60            |
| $-$4.7                                | 4.41            | 1.16            | 0.16            | 0.25            | 5.82            |
| $-$2.3                                | 3.20            | 1.14            | 0.14            | 0.25            | 7.42            |
| $-$1.2                                | 3.05            | 1.14            | 0.14            | 0.26            | 5.66            |
| $-$0.7                                | 3.03            | 1.14            | 0.14            | 0.27            | 5.16            |
| 0 (potential)                         | 3.02            | 1.14            | 0.14            | 0.27            | 4.50            |
| 0.7                                   | 3.03            | 1.14            | 0.14            | 0.27            | 4.07            |
| 1.2                                   | 3.06            | 1.14            | 0.14            | 0.27            | 3.97            |
| 2.3                                   | 3.22            | 1.14            | 0.14            | 0.27            | 5.07            |
| 4.7                                   | 4.38            | 1.15            | 0.15            | 0.26            | 5.89            |
| 7.0                                   | 7.19            | 1.17            | 0.17            | 0.24            | 6.16            |

Note. The definition of each column is the same as those in Table 3.

The fifth column shows $\langle \text{CW sin $\theta$} \rangle$, the mean sine of the angle $\theta$ between $j$ and $\mathbf{B}$ weighted by $j$, which is defined as follows,

$$\langle \text{CW sin $\theta$} \rangle = \frac{\sum |j| \text{ sin $\theta$}}{\sum |j|}. \quad (17)$$

The metric is based on the property of the force-free field that the electric current is parallel to the magnetic field. Therefore, $\langle \text{CW sin $\theta$} \rangle$ represents how force-free an obtained NLFFF solution is. When the solution is close to the force-free state, $\langle \text{CW sin $\theta$} \rangle$ is close to zero. For NOAA 11692, $\langle \text{CW sin $\theta$} \rangle$ has the smallest value when $\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$, while it has the largest value when $\alpha_0 = 0$ m$^{-1}$. For NOAA 11967, the magnitude of $\langle \text{CW sin $\theta$} \rangle$ is smaller than that of NOAA 11692. Similar to NOAA 11692, $\langle \text{CW sin $\theta$} \rangle$ tends to have a smaller value when $|\alpha_0|$ becomes larger. The magnetic field tends to be force-free, where the field strength is large in our NLFFF modeling, as shown in the analysis of the Appendix. This means that an increase in the number of pixels with strong magnetic fields leads to smaller values of $\langle \text{CW sin $\theta$} \rangle$. When the absolute initial alpha is large, the number of pixels with strong magnetic fields is also large.
The sixth column shows the fractional flux ratio \( |f_i| \)

\[
f_i = \frac{\int_{\Delta S} \mathbf{B} \cdot dS}{\int_{\Delta S} |\mathbf{B}| dS} = \frac{(\nabla \times \mathbf{B}) \times (\Delta x)^3}{|\mathbf{B}| \times 6(\Delta x)^2} = \frac{(\nabla \times \mathbf{B}) \Delta x}{6|\mathbf{B}|},
\]

where \( \Delta S \) and \( \Delta x \) are the small discrete surface and the grid spacing. From one of the Maxwell equations, the divergence of the magnetic field must vanish in the calculation box. \(|f_i|\) represents the degree of divergence-freeness of an obtained NLFFF solution. When \( \nabla \cdot \mathbf{B} \) very nearly vanishes in the results of the NLFFF extrapolation, \(|f_i|\) approaches zero. For NOAA 11692, \(|f_i|\) has the smallest value, when the initial condition is \( \alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1} \). As shown in Section 4.2, the field lines of the NLFFF from \( \alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1} \) are almost parallel to the direction of the sigmoidal structure in the X-ray image. On the other hand, for NOAA 11967, while the morphologies of the field lines are not so different among the NLFFF results, each solution has different \(|f_i|\) values. \(|f_i|\) has the smallest value when the initial condition is \( \alpha_0 = 1.2 \times 10^{-8} \text{ m}^{-1} \), and it tends to have a large value when \( |\alpha_0| \) is large. One of the reasons for this result is the setting of the initial conditions. In order to set the same top boundary for all calculations, the initial conditions above \( z = 0.417 \) are set to the potential field. If the absolute initial alpha is large, there is a strong discontinuity in the initial conditions. This might be the reason for the large \( \nabla \cdot \mathbf{B} \). The effect of nonzero \( \nabla \cdot \mathbf{B} \) on the energy can be estimated by deriving the non-solenoidal magnetic field proposed by Valori et al. (2013). Although we do not analyze the non-solenoidal magnetic field in this paper, the effect of nonzero \( \nabla \cdot \mathbf{B} \) on energy is a few percent if \(|f_i| < 10^{-5}\) from the results of Valori et al. (2013).

### 4.4. Comparison of Solutions at Each Height

Figure 9 shows the global alpha derived at each height from the NLFFF results. The global alpha was calculated by Equation (4). Different colors shows the results from different initial conditions. For both NOAA 11692 and NOAA 11967, the global alpha shows larger deviations among the calculations at higher layers than at lower layers. For NOAA 11692, the global alpha is \(-1.0 \times 10^{-8} \text{ m}^{-1}\) at the photospheric height, and depending on the initial condition, the global alpha diverges to positive and negative values as the height increases. For NOAA 11967, the global alpha is \(-5.0 \times 10^{-8} \text{ m}^{-1}\) at the photospheric height. Independent of the initial conditions, the global alpha increases as the height increases, and beyond 20 Mm, the global alpha starts to deviate depending on the initial conditions.

The spatial distributions of the vector magnetic fields in NOAA 11692 at 2.6 Mm and 26 Mm height are shown in Figures 10 and 11. The grayscale color shows the vertical magnetic field, and green arrows show the horizontal magnetic field. The horizontal magnetic field in the negative spot is less dependent on the initial condition at 2.6 Mm. In the region between the positive and negative polarities, however, the horizontal magnetic field is a bit different between the results from different initial conditions. This tendency can be clearly seen at the higher height, as shown in Figure 11. The horizontal magnetic field around the polarity inversion line in the case of \( \alpha_0 = 2.3 \times 10^{-8} \text{ m}^{-1} \) is completely different from that in the case of \( \alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1} \). On the other hand, the vertical magnetic field distribution shows the simple two-polarity configuration and that they are similar to each other. Figure 12 shows the number density distribution in the vector magnetic field with different initial conditions for NOAA 11692 at heights of 2.6 Mm and 26 Mm. The result with \( \alpha_0 = 0 \text{ m}^{-1} \) is compared to that with \( \alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1} \). As seen in the difference between Figures 10 and 11, the dispersion is larger in 26 Mm compared to in 2.6 Mm. While the values of correlation coefficient \( C \) are 0.96 and 0.95 for \( B_x \) and \( B_y \) at 2.6 Mm, respectively, those at 26 Mm are 0.82 and 0.60, respectively. The dispersion also can be seen at 2.6 Mm in the regions with small
magnetic fields (∼100G). In the number density distribution of $B_y$, some pixels have large $B_y$ in $\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$, while some have small $B_y$ in $\alpha_0 = 0$ m$^{-1}$. This distribution reflects the horizontal magnetic field distribution around the polarity inversion line, as described above.

The spatial distributions of the vector magnetic field in NOAA 11967 at 2.6 Mm and 26 Mm height are shown in Figures 13 and 14. The grayscale color shows the vertical magnetic field, and the green arrows show the horizontal magnetic field. The spatial distribution of the vector magnetic field is quite similar at 2.6 Mm among NLFFF results from different initial conditions. At 26 Mm, there exist differences not only in the horizontal magnetic field but also in the vertical magnetic field. Figures 15, 16, and 17 show the number density

![Image of diagrams showing spatial distributions of vector magnetic fields]

**Figure 10.** The spatial distributions of the vector magnetic field for each solution in NOAA 11692 at 2.6 Mm height. The grayscale color shows the vertical magnetic field, and the green arrows show the horizontal magnetic field. The length of the red arrow shows the field strength of 1000 G.
distribution in the vector magnetic field of NOAA 11967 with different initial conditions at heights of 2.6 Mm and 26 Mm. The result of $\alpha_0 = 0 \text{ m}^{-1}$ is compared to the results of $\alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1}$, $\alpha_0 = -4.7 \times 10^{-8} \text{ m}^{-1}$, and $\alpha_0 = -7.0 \times 10^{-8} \text{ m}^{-1}$, respectively. For all comparisons, the vector magnetic field shows a strong correlation at the 2.6 Mm height.

Figure 11. Same as Figure 10 but at 26 Mm. The length of the red arrow shows the field strength of 200 G.
even though the initial conditions are quite different. These coefficients of NOAA 11967 are larger than those of NOAA 11692. Similar to NOAA 11692, the small deviation can be seen in the weak magnetic field (<200 G) region. At 26 Mm, the number density distributions do not show a strong correlation, suggesting that the NLFFF results are affected by the initial conditions; although, the correlation coefficients are larger than those of NOAA 11692. The correlation coefficients become small for the comparison between the potential and the large absolute value of the initial alpha. However, the strong magnetic field (>200 G) shows good correlation even at 26 Mm height.

4.5. Convergence in Each Calculation

The volume integral of the Lorentz force must vanish in the force-free condition. However, it does not strictly vanish in numerical NLFFF computations. A nonzero Lorentz force may be caused by the non-force-freeness at the bottom boundary and the inconsistency of setting the top boundary and side boundary. For the reasons mentioned above, the Lorentz force often appears near the boundary of the calculation box. Therefore, we usually stop the relaxation process when the volume integral of the Lorentz force converges to a certain value. Figure 18 shows the evolution of the volume integral of the Lorentz force as a function of time step. The Lorentz force is normalized by the values described in Section 3. Each color of the lines shows the initial absolute value of the constant force-free alpha. The solid lines correspond to an initial negative force-free alpha, whereas the dotted lines correspond to a positive one. The Lorentz force increases between 10 and 100 step numbers because the electric current is transported from the bottom boundary according to Equation (11). For both active regions, the convergence speed becomes faster for negative values in comparison with the opposite value in the same absolute $\alpha_0$. For NOAA 11692, the volume integral of the Lorentz force converges to a similar value for all initial conditions. For NOAA 11967, when the absolute value of the initial constant alpha has a smaller value, e.g., the black, yellow, purple, orange, and blue lines in Figure 18, the volume integral of the Lorentz force tends to be smaller at the same time step compared to the large initial force-free alpha, e.g., the green and red lines. The converged values of the Lorentz force are higher than the initial values. This is because we fixed the bottom boundary to the non-force-free (observed) photospheric magnetic field during the late stage of the MHD relaxation. As a result, the Lorentz force is mainly concentrated at lower heights, as shown in the Appendix. On the other hand, we use a force-free (potential or LFF) bottom boundary for the initial conditions, and the total Lorentz force is not so large in the initial stage.

Figure 12. All panels show the number density distribution in the vector magnetic field for different initial conditions at 2.6 Mm (upper panels) and 26 Mm (lower panels) height. Comparison is shown between $\alpha_0 = 0 \text{ m}^{-1}$ and $\alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1}$ in NOAA 11692.
5. Discussions

We calculated NLFFF with five and 12 different initial conditions for NOAA 11692 and 11967, respectively. A summary of our results is as follows:

1. According to the comparison with the soft X-ray image, the NLFFF shows better correspondence in the simple active region NOAA 11692, when the initial constant force-free alpha has the same sign as the global alpha calculated from the photospheric magnetic field, as shown in Figure 5. On the other hand, in the complex multipole active region NOAA 11967, the results of the NLFFF extrapolations are less dependent on the initial condition, as shown in Figure 7.

2. The total magnetic energy of the NLFFF extrapolation does not strongly depend on the initial conditions, as shown in Tables 2 and 3. The dependence of the free energy is larger compared to the total magnetic energy.

3. The solution of NLFFF in the region where the strong magnetic field exists, e.g., magnetic fields at lower...
heights, tends to be less affected by the initial conditions, as shown in Figure 9. On the other hand, the region in the weak magnetic field around the polarity inversion line tends to be affected by the initial conditions.

4. Except for the case $\alpha_0 = -12 \times 10^{-8} \text{ m}^{-1}$ in NOAA 11967, the NLFFF extrapolation is considered to be converged, as shown in Figure 18. The increase of the calculation steps may not affect the results in this study.

From Figure 9, the magnetic field in the lower height region tends to be less affected by the initial conditions, while the magnetic field at the higher region is strongly affected by the initial conditions. There is a possibility that this result is caused by the convergence problem of the NLFFF modeling, but it is ruled out by the analysis in Appendix, which analyzes the Lorentz forces as a function of height and iteration step number.

Our results suggest that NLFFF models calculated using relaxation methods, including the method used in this study, permit different results (depending on the initialization field used) in the upper regions of the computational domain. In other words, there exist several local minima for the force-free equilibrium in the higher regions. Because the initial condition is force-free everywhere except at the bottom boundary, a
larger degree of freedom is allowed at the higher regions compared to the lower regions. For example, in our MHD relaxation method, the information of the bottom boundary is transported by the propagation of the pseudo-Alfvén wave as mentioned above. The region that is far from the boundary tends to be less affected by the pseudo-Alfvén wave and remains almost unchanged from the initial equilibrium state (potential-like or LFFF-like). Therefore, the NLFFF extrapolation cannot reproduce the magnetic field of the active regions well, e.g., having twisted structures in the corona, but has a potential-like photospheric magnetic field. Note that this possibility does not prove that the degree of freedom of the NLFFF solution is mathematically larger at higher regions because a certain amount of the Lorentz force remains in our calculation. To solve this problem, we suggest that additional observational limitations should be given to the current NLFFF models such that the magnetic field at higher regions can converge to the NLFFF result, which is consistent with X-ray and/or EUV imaging observations.

We compared the NLFFF results with coronal loops observed with Hinode/XRT. For NOAA 11692, the 3D magnetic field configuration matches well and appears to be more consistent with the X-ray observation when the absolute value of the initial force-free alpha is smaller. On the other hand, NOAA 11697 shows strong initial condition dependence in Figure 5. The clear difference between NOAA 11697 and 11692 is the complexity of the photospheric magnetic field. Our results suggest that the NLFFF result of NOAA 11697 is less affected by the initial conditions than that of NOAA 11692. Because we analyze only two active regions, we cannot determine the cause of this result. Therefore, we discuss candidates of the cause of the difference in terms of the properties of the two active regions. There are five possibilities that could explain this cause, presented as follows:

1. NOAA 11697 has more magnetic flux, $8.4 \times 10^{22}$ Mx, than that of NOAA 11692, $3.7 \times 10^{22}$ Mx. This suggests that there are more strong magnetic field regions above the photosphere.
2. Unbalanced magnetic flux at the photospheric height may also be the cause. In the NLFFF modeling, magnetic flux unbalance at the bottom boundary may produce inconsistent results and may be related to the initial condition dependence. The net vertical magnetic flux normalized by the total magnetic flux is $-0.03$ for NOAA 11692 and 0.17 for NOAA 11697. This shows that the magnitude of the unbalance of the vertical magnetic flux is larger in NOAA 11697 than in NOAA 11692. Because the NLFFF results of NOAA 11697 are less affected by the initial conditions, the flux unbalance in this study may not affect the initial condition dependence.

Figure 15. All panels show the number density distribution in the vector magnetic field from different initial conditions at 2.6 km (upper panels) and 26 Mm (lower panels) height. Comparison is shown between $\alpha_0 = 0 \text{ m}^{-1}$ and $\alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1}$ in NOAA 11697.
3. The force-freeness at the photospheric height is different between the active regions. Although force-freeness of both active regions is relatively small, that of NOAA 11967 \((F_0 = 0.089)\) is smaller than that of NOAA 11692 \((F_0 = 0.17)\). The large value of the force-freeness may affect the dependence of the initial conditions on the NLFFF result.

4. The length scale of the field lines may affect the results. While the height of the loop top in NOAA 11692 is around 100 Mm, as shown in Figure 6, that in NOAA 11967 is around 30 Mm, as shown in Figure 8. This result means that the height of the coronal loops identified in the soft X-ray images are different between NOAA 11692 and 11967. Our result shows that the magnetic field in the higher regions tends to be affected more strongly by the initial conditions than that in the lower regions. Therefore, in comparison with the soft X-ray image in NOAA 11692, we compared the field lines, which tend to be affected by the initial conditions. However, only the difference of the length scale of the coronal loops cannot explain the difference of the initial condition dependency. As shown in Figures 12 and 15, NOAA 11692 is more dependent on the initial conditions than NOAA 11967 even at the same height. This result indicates that only the difference of the length scale would not be the cause of the difference of the dependence between the two active regions.

5. There is a difference of the horizontal field distribution between the two active regions. As shown in Figure 1, a strong horizontal magnetic field occupies a large part of the FOV of NOAA 11967 for all four polarities. On the other hand, in NOAA 11692, while the horizontal magnetic field can be seen in the negative sunspot, there is no strong horizontal magnetic field in the positive polarity. As shown in Figure 4, while 6.3% of the horizontal magnetic field in the FOV is larger than 1000 G for NOAA 11967, 0.49% is larger than 1000 G for NOAA 11692. This result indicates that in relaxation process, it is difficult to give sufficient electric currents (non-potential magnetic field) to the upper atmosphere in the case of NOAA 11692 because there are fewer currents and forces in the photosphere. As a result, there is a strong dependency on the initial conditions in the NLFFF modeling of NOAA 11692. In the NOAA 11967 case, the active region has a strong horizontal field, which gives strong perturbations to the upper atmosphere in the relaxation process. This may be why the NLFFF modeling of NOAA 11967 converged to a consistent solution with different initial conditions.

Figure 16. All panels show the number density distribution in the vector magnetic field from different initial conditions at 2.6 km (upper panels) and 26 Mm (lower panels) height. Comparison is shown between \(\alpha_0 = 0 \text{ m}^{-1}\) and \(\alpha_0 = -4.7 \times 10^{-8} \text{ m}^{-1}\) in NOAA 11967.
As shown in Tables 2 and 3, the total magnetic energy does not strongly depend on the initial conditions. On the other hand, the free energy shows larger differences among each solution than the total magnetic energy. Since the free energy is defined as the deviation between the magnetic energy and potential magnetic energy, the small difference of the total magnetic energy becomes a large difference in free energy. This ratio is smaller than those in previous studies that focused on other dependences, such as method dependence (De Rosa et al. 2009, total energy: 1.9 between Reg$^+$ and Am1$^-$), instrument dependence between Hinode and SDO (Thalmann et al. 2013, total energy: ∼2.2, free energy: ∼2.5), and spatial resolution dependence (DeRosa et al. 2015, total energy: ∼1.4, free energy: ∼2 in magnetofrictional method). This means that the initial condition dependence of the total magnetic energy and free energy in our MHD relaxation method is comparatively small. The uniqueness of total energy and free energy can be explained by using our results. Since the magnetic energy and free energy concentrate at lower heights, they become less dependent on the initial conditions.

Our study shows that the NLFFF results determined from relaxation methods may be strongly affected by the initial conditions at higher levels of the computational domain. We therefore recommend checking whether the field lines from the NLFFF models are consistent with the coronal images. Our results also indicate that the sigmoidal structure can be well reproduced by changing initial conditions even when the photospheric magnetic field has less twist. Regarding NOAA 11692, our NLFFF modeling produces more consistent results with X-ray observations when we use an initial value of $\alpha_0 = -2.3 \times 10^{-8} \text{m}^{-1}$. This value has the same sign of the global alpha in the photosphere ($\alpha_g = -1.0 \times 10^{-8} \text{m}^{-1}$).

Although the global alpha does not strictly correspond to the global magnetic twist when the magnetic field is not force-free, the sign of the global alpha gives the sign of the global shear signed angle (Tiwari et al. 2009b). Therefore, our results offer an important suggestion that the photospheric global alpha can be used for a rough estimation of the initial conditions for better NLFFF modeling.

### 6. Summary

A summary of our findings is as follows:

1. The solution of NLFFF at regions where the strong magnetic field exists, e.g., magnetic field at lower heights (<10 Mm), tends to be less affected by the initial conditions; although, the Lorentz force is concentrated at lower heights.

Figure 17. All panels show the number density distribution in vector magnetic field from different initial conditions at 2.6 km (upper) and 26 Mm (lower) height. Comparison is shown between $\alpha_0 = 0 \text{m}^{-1}$ and $\alpha_0 = -7.0 \times 10^{-8} \text{m}^{-1}$ in NOAA 11967.
The total magnetic energy of the NLFFF extrapolation does not strongly depend on the initial conditions.

The NLFFF extrapolation of the complex active region NOAA 11967 is less dependent on the initial conditions compared to that of NOAA 11692.

We proposed the problem of whether completely different solutions with the same bottom boundary exist or not. We conclude that we obtain completely different 3D NLFFF structures from the different initial conditions with the same bottom boundary. However, the initial condition dependence is small (correlation coefficient $C > 0.9$) where the magnetic field is strong, e.g., at lower heights ($<10$ Mm). We also reveal that the 10–100 times larger Lorentz force, which is normalized by the square of the magnetic field strength, remains at lower heights ($<10$ Mm) than that at higher regions ($>10$ Mm). The magnitude of the dependence is also different between the two active regions.

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Appendix

Analysis of Lorentz Force

Although the volume integral of the Lorentz force appears to converge in Figure 18, there is a possibility that the magnetic field has not yet converged at the higher region because the Lorentz force may be mostly concentrated at lower heights in the calculation box. We investigate the height distribution of the Lorentz force. Panels (a) and (c) in Figure A1 show the height distribution of the Lorentz force for NOAA 11692, which is averaged at each height for $\alpha_0 = 0$ m$^{-1}$ and $\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$, respectively. This is the result at 24,000 steps. As expected, the Lorentz force is mainly concentrated at lower heights for both solutions. Panels (b) and (d) in Figure A1 show the Lorentz force, which is normalized by the square of the magnetic field strength and is averaged at each height for (a) $\alpha_0 = 0$ m$^{-1}$ and (b) $\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$. For both $\alpha_0 = 0$ m$^{-1}$ and $\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$, the normalized Lorentz force is also mainly concentrated at lower heights. Below 10 Mm, the normalized Lorentz force is of the order of 10–100, while above 10 Mm, the normalized Lorentz force is of the order of 1–10. Panels (e) and (f) show the difference of the height distribution of the Lorentz force between panels (a) and (c), and panels (b) and (d), respectively. Red asterisks show panels (a) > (c) or (b) > (d), while blue asterisks show panels (c) > (a) or (d) > (b). The difference of the Lorentz force is around $10^{-4}$. The Lorentz force of $\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$ is larger above 20 Mm. At most of the height, the normalized Lorentz force of $\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$ is smaller than that of $\alpha_0 = 0$ m$^{-1}$. Although the difference of the normalized Lorentz force is larger at lower heights, the ratio to the normalized Lorentz force is smaller compared to the higher region. At lower heights, the difference of the normalized Lorentz force is of the order of 0.1–1, while the normalized Lorentz force is of the order of 10–100. The ratio of the difference is 1% at lower heights. On the other hand, at higher regions, the difference of the normalized Lorentz force is of the order of 1. The ratio of the difference is 10% at the higher region. Panels (g) and (f) show the height distribution of the absolute value of the force-free alpha, which is averaged at each height for $\alpha_0 = 0$ m$^{-1}$ and $\alpha_0 = -2.3 \times 10^{-8}$ m$^{-1}$, respectively. Note that the force-free alpha shows a similar distribution to that of the Lorentz force. At lower heights, the
force-free alpha is of the order of $10^{-100}$, while at higher regions, the force-free alpha is of the order $10^{-10}$. The force-free alpha is smaller than the normalized Lorentz force at lower heights, while it is larger at higher regions. This result indicates that the normalized Lorentz force is sufficiently small at higher regions, while at lower heights, there exists a significant normalized Lorentz force.

Panels (a) and (c) of Figure A2 show the Lorentz force and the normalized Lorentz force distribution at 2600 km height for NOAA 11692 for $\alpha_0 = 0 \ m^{-1}$ (left panels) and $\alpha_0 = -2.3 \times 10^{-8} \ m^{-1}$ (right panels). Panels (b) and (d) show the Lorentz force and the normalized Lorentz force distribution at 26 Mm height for NOAA 11692 for $\alpha_0 = 0 \ m^{-1}$ (left panels) and $\alpha_0 = -2.3 \times 10^{-8} \ m^{-1}$ (right panels). At 2600 km height, the strong Lorentz force is concentrated around the negative sunspot because the strong magnetic field is concentrated. The normalized Lorentz force is small in strong magnetic field regions, such as negative spot umbras, while the normalized Lorentz force is large in the spot penumbra and weak magnetic field region. At 2600 km height, there are few differences between the distribution of $\alpha_0 = 0 \ m^{-1}$ and $\alpha_0 = -2.3 \times 10^{-8} \ m^{-1}$. At 26 Mm height, the distributions of the Lorentz force and the normalized Lorentz force are similar. This indicates that the magnetic
field is more uniformly distributed at higher regions. The Lorentz force is concentrated in the polarity inversion line. At 26 Mm, the Lorentz force shows a slightly different distribution between $\alpha_0 = 0 \text{ m}^{-1}$ and $\alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1}$. The NLFFF of $\alpha_0 = 0 \text{ m}^{-1}$ has a stronger Lorentz force in the polarity inversion line compared to that of $\alpha_0 = 0 \text{ m}^{-1}$.

Panels (a) and (b) of Figure A3 show the height distribution of the Lorentz force and the normalized Lorentz force for NOAA 11692, which is averaged at each height. These are the results of $\alpha_0 = 0 \text{ m}^{-1}$. The colors show each time step at 20 (black), 100 (yellow), 500 (red), 900 (green), 24,000 (blue), and 60,000 (orange) steps, respectively. Panels (c) and (d) show the temporal variations of the height distribution of the Lorentz force and the normalized Lorentz force. The colors show the time interval between 20 and 100 step (yellow), 100 and 500 step (red), 500 and 900 step (green), 900 and 24,000 step (blue), and 24,000 and 60,000 step (orange), respectively. The solid lines show the increase, while the dashed lines show the decrease. Both the Lorentz force and the normalized Lorentz force show similar behaviors. As can clearly be seen, the Lorentz force and the normalized Lorentz force are transported from lower heights to higher regions with increasing the calculation step. Between the step of 24,000 (blue) and 60,000 (orange), the Lorentz force and the normalized Lorentz force show little change, which means that the magnetic field does not change significantly during these steps.

Figure A4 shows the temporal evolution of the normalized Lorentz force distribution at 2600 km for NOAA 11692. This is the result of the normalized Lorentz force of $\alpha_0 = 0 \text{ m}^{-1}$. Each
Figure A3. Panels (a) and (b): the solid lines show the height distribution of the Lorentz force and the normalized Lorentz force for NOAA 11692, which is averaged at each height, respectively. This is the result of $\alpha_0 = 0 \text{ m}^{-1}$. The colors show each time step at 20 (black), 100 (yellow), 500 (red), 900 (green), 24,000 (blue), and 60,000 (orange) step, respectively. Panels (c) and (d): the temporal variations of the height distribution of the Lorentz force and the normalized Lorentz force. The colors show the time interval between 20 and 100 step (yellow), 100 and 500 step (red), 500 and 900 step (green), 900 and 24,000 step (blue), and 24,000 and 60,000 step (orange), respectively. The solid lines show the increase, while the dashed lines show the decrease.

Figure A4. The temporal evolution of the normalized Lorentz force distribution at 2600 km for NOAA 11692. This is the normalized Lorentz force of $\alpha_0 = 0 \text{ m}^{-1}$. Each panel shows each time step at panels (a) 20 step, (b) 100 step, (c) 500 step, (d) 900 step, (e) 24,000 step, and (f) 60,000 step, respectively.
panel shows each time step at panels (a) 20 step, (b) 100 step, (c) 500 step, (d) 900 step, (e) 24,000 step, and (f) 60,000 step, respectively. The normalized Lorentz force at 2600 km increases between 20 and 500 steps, while the distribution of the Lorentz force does not change significantly after 500 steps. This result indicates that the magnetic field at lower heights does not change significantly from early calculation steps; although, the normalized Lorentz force remains to a certain amount.

Figure A5 shows the temporal evolution of the normalized Lorentz force distribution at 26 Mm for NOAA 11692. Each panel shows each time step at panels (a) 20 step, (b) 100 step, (c) 500 step, (d) 900 step, (e) 24,000 step, and (f) 60,000 step, respectively. At the time step of 20 step and 100 step, there is little normalized Lorentz force because the normalized Lorentz force has not been transported from the bottom boundary as seen in Figure A3. After 500 steps, the normalized Lorentz force increases at 26 Mm, and the distribution of the normalized Lorentz force does not change significantly after 24,000 step.

Figures A6, A7, A8, A9, and A10 show the results of the analysis of the Lorentz force for NOAA 11967 in the same way as for NOAA 11692. The results are similar to those of NOAA 11692.

From the above results, there are two possible reasons that could explain the stronger dependence of the initial condition on the NLFFF modeling at higher regions. The first possible reason is that the information of the bottom boundary has not reached at the higher regions yet. In the MHD relaxation method, the NLFFF is achieved by the propagation of the disturbance as pseudo-Alfvén wave, which is produced by the artificial change of the bottom boundary according to Equation (11). Because we assume \( \rho = |B| \) in the MHD calculation, the Alfvén velocity is \( B/\sqrt{4\pi \rho} \sim \sqrt{B} \). At higher regions and weak magnetic field region, there is a possibility that the pseudo-Alfvén wave has not reached yet. This possibility is rejected by the result in Figures A3 and A8, which show that the Lorentz force is transported to the higher region.

The second possibility is the difference of the convergence speed among each height. We used the resistivity defined as Equation (9), which becomes large when the Lorentz force or the velocity becomes large. Because the Lorentz force and the velocity tend to become large at lower heights, the resistivity tends to be larger in the lower regions than in the higher regions. The large resistivity allows the magnetic field to be relaxed faster at lower heights. For the second possibility, it is unlikely that the magnetic field at the higher regions converges to one unique solution by increasing iteration steps. Regarding the convergence of the NLFFF results, from the result in Figures A5 and A10, the distribution of the Lorentz force does not significantly change after the 24,000 step for NOAA 111692 and the 25,000 step for NOAA 11967. This result indicates that the current scheme cannot reduce the Lorentz force at higher regions any more.
We have to note that although the magnetic field at lower heights is less affected by the initial conditions, a certain amount of the Lorentz force remains, as shown in Figures A1 and A6. The main reasons for the remaining Lorentz force in the calculation box may be due to the non-force-freeness of the photospheric magnetic field at the bottom boundary. Since the photospheric magnetic field is not ideally force-free, the nonzero Lorentz force will be produced in the current NLFFF scheme. Therefore, strictly speaking, we do not obtain ideal force-free solution in the current NLFFF scheme. We will investigate how this non-force-freeness at the bottom boundary affects the accuracy of the NLFFF modeling by comparing the NLFFF and chromospheric observations (Y. Kawabata et al. 2020, in preparation). This problem should be solved by using more force-free bottom boundary, such as the chromospheric magnetic field.

Figure A6. Panels (a) and (c): the height distribution of the Lorentz force for NOAA 11967, which is averaged at each height for $\alpha_0 = 0 \text{ m}^{-1}$ and $\alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1}$. This is the result at 25,000 step. Panels (b) and (d): the height distribution of the normalized Lorentz force. Panels (e) and (f): the difference of the height distribution of the Lorentz force and the normalized Lorentz force between panels (a) and (c), and panels (b) and (d), respectively. Red asterisks show panels (a) > (c) or (b) > (d), while blue asterisks show panels (c) > (a) or (d) > (b).
Figure A7. Panels (a) and (c): the Lorentz force and the normalized Lorentz force distribution at 2600 km height for NOAA 11967 for $\alpha_0 = 0 \text{ m}^{-1}$ (left) and $\alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1}$ (right), respectively. Panels (b) and (d): the Lorentz force and the normalized Lorentz force distribution at 26 Mm height for NOAA 11967 for $\alpha_0 = 0 \text{ m}^{-1}$ (left) and $\alpha_0 = -2.3 \times 10^{-8} \text{ m}^{-1}$ (right), respectively.
Figure A8. Panels (a) and (b): the solid lines show the height distribution of the Lorentz force and the normalized Lorentz force for NOAA 11967, which is averaged at each height. This is the result of $\alpha_0 = 0 \text{ m}^{-1}$. The colors show each time step at 20 (black), 100 (yellow), 500 (red), 900 (green), 25,000 (blue), and 45,000 (orange) step, respectively. Panels (c) and (d): the temporal variation of the height distribution of the Lorentz force and the normalized Lorentz force. The colors show the time interval between 20 and 100 step (yellow), 100 and 500 step (red), 500 and 900 step (green), 900 and 25,000 step (blue), and 45,000 step (orange), respectively. The solid lines show the increase, while the dashed lines show the decrease.

Figure A9. The temporal evolution of the normalized Lorentz force distribution at 2600 km for NOAA 11967. This is the result of $\alpha_0 = 0 \text{ m}^{-1}$. Each panel shows each time step at panels (a) 20 step, (b) 100 step, (c) 500 step, (d) 900 step, (e) 25,000 step, and (f) 45,000 step, respectively.
Figure A10. The temporal evolution of the normalized Lorentz force distribution at 26 Mm for NOAA 11967. This is the result of $\alpha_0 = 0 \text{ m}^{-1}$. Each panel shows each time step at panels (a) 20 step, (b) 100 step, (c) 500 step, (d) 900 step, (e) 25,000 step, and (f) 45,000 step, respectively.

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