Accelerating D-branes

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Abstract

Higher derivative terms are computed in the one-loop effective action governing the interactions of D3-branes, in two ways: (1) in a formalism with $N = 2$ supersymmetry, and (2) in the standard background field formalism, with only on-shell supersymmetry. It is shown that these calculations only agree using tree-level equations of motion. The off-shell supersymmetric calculation exhibits acceleration terms that appear in terms with four derivatives. These may imply disagreement at two-loop order between supergravity and Yang-Mills descriptions of D-brane dynamics.

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1 Introduction

Witten[1] proposed that low-velocity, short-distance interactions of D-branes should be described by supersymmetric Yang-Mills(SYM) theories in various dimensions, corresponding to the dimensions of the world-volumes of the D-branes. The conjecture of Banks, Fischler, Shenker and Susskind[2] uses this description of D0-brane interactions, but interpreted in a remarkable manner as a non-perturbative definition of M-theory in an infinite momentum frame. In this context, the SYM description should hold for arbitrary dynamics and at arbitrary distances, with agreement with supergravity expected at long distances. Much evidence has been accumulated for the validity of this conjecture[3].

Some of this evidence relies on phase-shift calculations for straight-line trajectories[3], finding agreement even at two-loop order[7]. Given that finite-time calculations show new terms appearing even at one-loop order[8], it is of interest to consider the effects of curved trajectories.

We do not address Matrix theory[2] directly in this paper, though this work, combined with the work of Taylor[5], is our main motivation. We consider the dynamics of D3-branes as described by $N = 4$ 4d SYM theory. We calculate higher-derivative terms at one-loop order in two ways. The first calculation, presented in Sections 2 and 3, is done in a formalism that preserves $N = 2$ supersymmetry, and gives $v^4/r^4$ terms at one-loop order, but automatically accompanied by $v^2a/r^3$ and $a^2/r^2$ terms. The second calculation (Section 4), done in a background field formalism with only on-shell supersymmetry, exhibits acceleration terms of the form $v^2a^2/r^6 +$ higher orders. For recent work along the lines of this calculation, see [4]. The coefficient of the $v^4/r^4$ term agrees precisely with supergravity in both calculations. We discuss this comparison, and further inferences, in Section 5.

2 N=2 implications

According to Witten[1], the low-velocity short-distance dynamics of D3-branes should be described by $N = 4$ SYM theory. Off-shell $N = 4$ supersymmetry in four dimensions is not well-understood, so we consider what can be said about the effective action in a formalism with only $N = 2$ supersymmetry preserved off-shell.
The leading term in a momentum expansion of the $N = 2$ superspace effective action is the imaginary part of a chiral integral of a holomorphic function $\mathcal{F}(W)$ [10], where $W$ is the $N = 2$ gauge superfield strength, whose $N = 1$ superfield components are the $N = 1$ gauge superfield strength $W_\alpha$ and the chiral superfield $\phi$. The next term in the expansion is a full ($N = 2$) superspace integral of a real function $\mathcal{H}(W, \bar{W})$. This term was first studied in [11, 12]. The function $\mathcal{H}$ must be dimensionless so it is a function of dimensionless combinations of $W, \bar{W}$ and some scale $\Lambda$. Furthermore, since $\mathcal{F}$ saturates the R-anomaly, $\mathcal{H}$ must be $U(1)_R$ invariant. That fixes the form of $\mathcal{H}$ to be

$$\mathcal{H} = \mathcal{H}^0 + c \left( \ln \frac{W^2}{\Lambda^2} + g^0(W) \right) \left( \ln \frac{\bar{W}^2}{\Lambda^2} + \bar{g}^0(\bar{W}) \right). \tag{1}$$

The function $\mathcal{H}^0$ is a function of dimensionless ratios of $W$ and $\bar{W}$ and the perturbative one-loop contributions to it have been calculated [12, 13]. For a $U(1)$ theory one cannot write such a term.

In the case we are interested in, the theory with $N = 4$ supersymmetry and gauge symmetry $SU(2)$ spontaneously broken to $U(1)$, there is no scale and no $\mathcal{H}^0$, and hence the whole term can be written as

$$\mathcal{H} = c \ln \frac{W^2}{\Lambda^2} \ln \frac{\bar{W}^2}{\Lambda^2}. \tag{2}$$

Notice that there is no scale dependence since the terms dependent on the scale are purely holomorphic or anti holomorphic and thus are killed when integrating over the full $N = 2$ superspace measure. In [14] it was argued that since two or higher loop corrections and non-perturbative correction would introduce non-trivial scale and coupling constant dependence which would violate the symmetries of the $\mathcal{H}$-term, there can be no such corrections. This means that (2) is the full perturbative and non-perturbative form of this term. The only unknown is the constant $c$. It will be calculated in the next section.

There have been several partial checks of the claims in [14] in the literature. The instanton calculations in [15, 16] and the two loop calculation in [17] all seem to support the result.

To find the terms relevant to D3-brane scattering in the SYM description, we expand $\mathcal{H}$ in bosonic components assuming it has the form (2). The
relevant terms are (see also [18, 19])

\[ \int d^4x 4c \left( \frac{\vert \phi \vert^4}{\phi} - \frac{\ddot{\phi} \phi^2}{\phi \phi^2} - \frac{\dddot{\phi} \phi^2}{\phi \phi^2} + 2 \frac{\vert \phi \vert^2}{\phi^2} \right) \].

(3)

The complex field \( \phi \) is to be interpreted as the coordinates of the brane in two of the six transverse dimensions (the other two are in the scalar components of the hypermultiplet). We may therefore rewrite this formula as

\[ 4c v^4 + 2v^2 (\vec{r} \cdot \vec{a}) - 4 (\vec{r} \cdot \vec{v}) (\vec{a} \cdot \vec{v}) + 2r^2 a^2 \]

(4)

Note that this term contains accelerations, as well as the expected \( v \) dependence, and that this follows purely from \( N = 2 \) supersymmetry. The only unknown is the constant \( c \). \( c \) cannot be zero, since that would imply a vanishing potential at \( v^4/r^4 \) order.

Integrating the terms with acceleration by parts can change the coefficient of the \( v^4/r^4 \) term. However, this also leads to terms proportional to radial velocities, as well as additional acceleration terms. Such changes do not affect the equations of motion, obviously, so we have written the \( v^4 \) term with no radial velocity factors.

### 3 N=1 calculation

We now turn to the problem of calculating the unknown constant \( c \) from the previous section.

The method we use is adapted from that used in [12, 17]. The idea is to perform perturbative calculations in the bare non-abelian action around an expectation value for the chiral field \( \phi \). If one is interested in the effective action for the massless fields only, one can choose all external fields to lie in the the same (abelian) direction which simplifies the group theory substantially. Furthermore, to calculate \( c \) it is not enough to keep only \( \dot{\phi} \) fields external since for the theory we are interested in, there is no contribution to the \( N = 1 \) Kähler potential of \( \phi \) coming from \( \mathcal{H} \). Instead we focus on another term in the \( N = 1 \) expansion of \( \mathcal{H} \). Following [12], we find the \( N = 1 \) expansion

\[ S_{\mathcal{H}} = \int d^4x d^4\theta \left( \cdots + i\mathcal{H}_{AB} \bar{W}^{Ba} \nabla^a W^A + \cdots \right), \]

(5)
where $\mathcal{H}_{AB} = \frac{\partial^2 \mathcal{H}}{\partial \phi^A \partial \phi^B}$. Note that this is the only term in the $N = 1$ expansion of $\mathcal{H}$ that contains two field strengths and no derivatives on $\phi$.

By performing a calculation with two external vector fields at non-zero momentum and an arbitrary number of external $\phi$ fields at zero momentum we are able to read off the coefficient and we find the value

$$c = \frac{1}{4(4\pi)^2}.$$  \hspace{1cm} (6)

This can be checked by comparing to the four gauge boson one-loop scattering amplitude computed in 4d $N = 4$ SYM theory using the supersymmetric background field method\,\cite{20}. This result, eq. (6), disagrees with calculations performed in harmonic superspace\,\cite{21, 22} where the authors find $c = 0$. A detailed presentation of the calculation will appear in\,\cite{23}.

4 The Background Field Method

We wish now to calculate the effective action at one-loop order in the background field method, starting from the dimensional reduction of the ten-dimensional SYM action to four dimensions. The point of interest for our purposes is that we wish to compute terms with derivatives in the effective action. The easiest way of computing these is to take advantage of the fact that in background field gauge, gauge potentials can only occur in covariant derivatives. Thus, if we start from a background field configuration with constant gauge potential matrices, we can extract terms in the effective action with arbitrary numbers of covariant derivatives.

The effective action is given by the logarithm of a product of determinants at one-loop order. For the $N = 1$ 10d SYM theory, with fields that realize supersymmetry on-shell, the determinants of interest are all of the form $X + F_{mn}J^{mn}$, where $F_{mn}$ is the curvature of the gauge potential in the adjoint representation, $J^{mn}$ are the Lorentz generators in appropriate representations (vector, spinor or scalar), and

$$X \equiv -\partial^2 + i(\partial \cdot A + A \cdot \partial) + A \cdot A$$ \hspace{1cm} (7)

is a Lorentz scalar operator which is a matrix in the (enveloping algebra of the) adjoint representation. It is a trivial exercise to obtain from these expressions the dimensionally reduced effective action. All derivatives with
indices \(m, n \in \{4, \ldots, 9\}\) are set equal to zero. We will only need a configuration where \(F_{mn}\) is non-zero only for indices \(m, n\) ranging over four values, say, \(m, n \in \{0, 1, 4, 5\}\). Thus, we may as well use the standard ten-dimensional generators of the Lorentz algebra in the \(F_{mn}J^{mn}\) term. We have six scalars, one vector, two Dirac spinors, and one ghost determinant (which exactly cancels two scalar determinants at one-loop order) in four dimensions, or equivalently, one Majorana-Weyl fermion, one vector and one ghost determinant in ten dimensions. The effective action is

\[
\Gamma = \int \sum \frac{(-)^n}{n} \text{tr}_G \left( ((X^{-1}F)^n) \left[ \frac{1}{8} \text{tr}_F J^n - \frac{1}{2} \text{tr}_V J^n \right] \right)
\]

(8)

where \(\text{tr}_G\) denotes a trace over gauge indices, \(\text{tr}_F\) denotes a trace over Dirac indices, and \(\text{tr}_V\) denotes a trace over vector indices, and all traces are computed in ten dimensions. The Lorentz indices on the product \(J^n\) are contracted with the Lorentz indices on the \(F_{mn}\) matrices.

We see immediately that since

\[
\frac{1}{4} \text{tr}_F J^{mn} J^{pq} = \text{tr}_V J^{mn} J^{pq}
\]

(9)

and

\[
\frac{1}{4} \text{tr}_F J^{mn} J^{pq} J^{rs} = \text{tr}_V J^{mn} J^{pq} J^{rs}
\]

(10)

that if \([A_4, A_5] = 0, A_1 = 0\), there are no terms with fewer than four derivatives of \(A_4\) or \(A_5\). The term with \(J^4\) does not cancel between the fermion and vector traces. This is, of course, the well-known fact that there is no renormalization of the coupling constant in \(N = 4\) 4d SYM theory, and that the first term in the effective action has four derivatives. Additional derivatives arise in expanding \(X^{-1}\) assuming \(A_0\) small. Every term with an \(A_0\) must arise from a covariant derivative \(D_0\), and hence contains all necessary information regarding higher partial derivatives. Working out the normalization of the \((\partial_0 A_i)^4\) term explicitly, we find a coefficient \(1/(4\pi)^2\). We compare this with the term \(|\phi|^4/|\phi|^4\) in eq. 3 and we see that it corresponds to \(c = 1/(4\pi)^2\), which is precisely the value given in eq. 6. This value \(c = 1/(4\pi)^2\) can be checked by comparing to the 15/16 coefficient expected in a dimensional reduction to 1 space-time dimension. Ref.’s 21, 22 found the value \(c = 0\), which disagrees with both our eq. 6 and our background field calculation.
Even though the value $c = \frac{1}{4}(4\pi)^2$ agrees between the background field method, and the calculation presented in Sect.’s 2,3, the crucial point is that the effective action eq. 8 does not agree with eq. 3—we now turn to a detailed examination of this discrepancy, in the next section (Sect. 5).

5 Comparison to supergravity

We wish to compare the higher derivative terms calculated in these two different ways with the interpretation of the 4d SYM action as a description of the dynamics of a collection of three-branes in the BPS limit, due to Witten[1]. However, while Witten’s proposed description was in the limit of small separations, we will consider how this description matches up at large separation, following Douglas, Kabat, Pouliot and Shenker[6].

The spacetime metric produced by an extremal three-brane[9] is
\begin{equation}
\ ds^2 = f^{-1/2}(d-1^2 + dx^\mu dx^\nu) + f^{1/2} dx^i dx^j,
\end{equation}
where $\mu = 1,2,3$, are indices parallel to the three-brane, $i \in \{4, \ldots ,9\}$, are transverse indices, and $f(r^2 \equiv x^ix^i) \equiv 1+R^4/r^4$. The 4-form potential arising from the fact that the three-brane carries Ramond charge is related to $f$ by
\begin{equation}
(\partial C)_{\mu\nu\gamma\rho i} = \epsilon_{\mu\nu\gamma\rho} \partial_i f^{-1}, \quad (\partial C)_{ijklm} = \epsilon_{ijklmn} \partial_n f.
\end{equation}
Note that $R^4 = N/2\pi^2 T_3$, where $N$ is the Ramond charge of the three-brane, and $T_3$ is the tension of the three-brane.

The world-volume action of a test three-brane in the background of such an extremal three-brane is given by the sum of a geometric induced volume term and a term arising from the coupling to the background 4-form field. The volume term and the 4-form term appear in precisely the manner needed for a vanishing static potential for the test three-brane. In static gauge, the action, for motions in which the test three-brane stays parallel to the three-brane producing the background, is then
\begin{equation}
S_{\text{test}} = \int d^4x \left[ \frac{T_3}{2} v^2 + \frac{N}{16\pi^2} \frac{v^4}{r^4} + \ldots \right].
\end{equation}
We want to compare this world-volume action to the Yang-Mills theory description. From the effective actions computed in the first part of this paper,
it is trivial to see that, with \( A^i \equiv \phi^i \equiv g\sqrt{T_3}x^i \), \( i \in \{4, \ldots, 9\} \), we get precise agreement between SYM theory and supergravity for the background field calculation (eq. 8), but there are no terms in the supergravity test particle action that correspond to four derivative terms with fewer than four scalar fields that \textit{must} arise in the calculation with off-shell supersymmetry, eq. 4.

What is the physical meaning of this discrepancy? The \( S \)-matrix is, in general, related to the effective action evaluated at a stationary point (with prescribed asymptotics) of the effective action. The background-field calculation has only on-shell supersymmetry, hence the effective action computed from it possesses only supersymmetry using the \textit{tree-level} equations of motion. There is no reason why radiative corrections in effective actions in formalisms with or without off-shell supersymmetry should agree, except when using tree-level equations of motion. This is entirely consistent with the two calculations given above, since the tree-level equations of motion indeed set higher derivative terms to zero, and therefore lead to an agreement between the two calculations. However, since we are interested in the \( S \)-matrix, we do not want to use the tree-level equations of motion, but rather the loop-corrected equations of motion. Then, it would seem that the effective action computed in a manner preserving off-shell supersymmetry is the correct action to use for computing the \( S \)-matrix. Carrying this line of thought to its logical conclusion, it would seem that one must use an off-shell formulation that preserves \textit{all} of the supersymmetries to get definitive answers.

Another way to state the key point here is that the \( v^4/r^4 \) potential term appears only at one-loop order. Scattering solutions are therefore analogous to solitons in theories where symmetry breaking only occurs due to radiative corrections, and care is needed in applying perturbative recipes\[24\]. Effective actions with different numbers of off-shell supersymmetries need not agree for configurations that break some of the supersymmetry, such as those with non-zero velocities. For long distance phenomena, the equations of motion give

\[
a \sim g^2 \frac{v^4}{r^5}
\]

and hence

\[
\frac{v^2 a}{r^3} \sim g^2 \frac{v^6}{r^8},
\]

(14)
which is a two-loop effect. The agreement with supergravity at the lowest order is therefore unaffected. If there had been a tree level potential, this would not be the case.

Since the two-loop term, calculated in the background field formalism, has been shown to agree between supergravity and Matrix theory for D0-branes without acceleration,[7] it is of some interest to check if an off-shell calculation in the D0-brane case is consistent with this check. A naïve computation suggests that a correspondence between D0-brane and D3-brane terms of the following form

\[
\begin{align*}
\frac{1}{r^7} & \leftrightarrow \frac{1}{15\pi^2 r^4}, & \frac{1}{r^8} & \leftrightarrow \frac{1}{32\pi^3 r^3}, & \frac{1}{r^9} & \leftrightarrow \frac{1}{6\pi^2 r^2}.
\end{align*}
\]  

(16)

Thus, inclusion of a term of the form \(v^2a/r^6\) at one-loop order in the D0-brane calculation would lead to a contribution at two-loop order of the form \((105/4\pi)v^6/r^{14}\). This additional contribution would imply a disagreement between supergravity and D0-brane dynamics at two-loop order. A check of this term directly in D0-brane quantum mechanics is under way.

6 Acknowledgements

We are grateful to Diego Bellisai, Marc Grisaru, Igor Klebanov, Gilad Lifshytz, Yaron Oz, Martin Roček, Yuri Shirman, and Wati Taylor for useful discussions. This work was supported in part by NSF grant PHY96-00258.

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