Evolution of angular momenta and energy of the Earth-Moon system

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Abstract

We have developed a model for the evolution of the Earth-Moon angular momenta, energy dissipation and tidal torque valid for the entire history of the Earth-Moon system. The model is supported by present observational data.

Key Words: Earth-Moon system: tidal torque, angular momentum, rotation, dissipation

1 Introduction

Very recently, we have developed a model to calculate the length of year, day, month and angular momentum of the Earth-Moon system in the past time (Arbab, 2003 b). By considering the effect of expansion of the universe on the Earth - Moon system, we were able to explain the geologic evolution of the Earth’s past rotation. We have shown that the perturbation affecting our Earth-Moon system can be modelled by an introducing an effective gravitational constant which embodies these perturbations, so that we keep Newton’s law of gravitation and Kepler’s third law invariant, and applicable to our system. We have found that the this cosmic evolution is manifested in several geologic phenomena (e.g. tides, which are the result of gravitational forces, etc). Gravitational forces in the Earth-Mon system cause tides, or bulge in the shape Earth and the Moon. It costs a lot of energy to deform the Earth, and this energy is lost through the internal friction of rock rubbing against rock within the Earth to raise the solid
body tides. Tides raised in the oceans by the Sun and Moon on Earth dissipate significant energy, and transfer angular momentum from the spin of the Earth to the orbit of the Moon. However, since much of the dissipation occurs in shallow marginal seas, the dissipation rate will not be constant. Glacial cycle changes in sea level and tectonic changes in coastal morphology will both influence the evolution of the lunar orbit.

Various theories of the Earth’s evolution have suggested that its moment of inertia has gradually changed. While Lyttleton finds that the Earth’s moment of inertia has decreased by 30% during the last 3 billion years, others predict that it was smaller in the past than at present. Changes in mass distribution can significantly change the rotational dynamics of the Earth-Moon system. Carey has suggested a doubling of the Earth’s moment of inertia (C) since the paleozoic. Any change in the moment of inertia will induce a change in the rate of the Earth’s rotation. The transfer of angular momentum from Earth to Moon resulted in an appreciable increase in the length of day and a decrease in the length of month. Since the Earth’s spinning rate is slowing down the Moon’s orbital angular momentum (L) has to change in order to keep the total angular momentum of the system constant. It is widely understood that an increase in the angular momentum of the Moon immediately implies an increase in the Moon-Earth distance. However, we have shown that is not always the case. In an expanding universe one can satisfy this if the gravitational constant is replaced with an effective constant involving a time coordinate that takes care of the universe expansion, keeping the normal Newton’s constant invariant (Arbab, 2003a).

With the present Apollo Moon’s retreat of 3.82 cm/y when extrapolated backward would mean the Moon can not exist as a stable body more than 1.5 billion years ago. The tidal force would have torn it apart. It is also remarked by some astronomers and geologists that the tidal force was smaller in the past than now. Slichter (1963) remarked that if ”for some unknown reason” the tidal torque was much less in the past than in the present (where ”present” means roughly the last 100 million years), this would solve the problem. But he could not supply the reason, and concluded his paper by saying that the time scale of the Earth-Moon system ”still presents a major problem”. Stacey (1977, p.103) concluded that ”...tidal friction was very much less in the remote past than we would deduce on the basis of present-day observations”.So far geologists have not found any geologic evidence of mega-tide to support a close approach of the Moon in the past. The tidal evolution is critical for all theories of the evolution of the Earth-Moon system. The schematic backward integrations reaches a narrow state in too short a time into the past, in contradiction with the lack of any geological evidence. Older fossils
show that tides existed as long as 2.8 billion years ago and that the month was 17 days long (Kaula and Harris, 1975). Our model however, gives a value of 19.6 days. Pannella (1972) suggests that there may have been 39 days per synodic month 2 billion years ago. However, our model says there are 37.6 days per synodic month.

2 Earth-Moon angular momenta, angular velocities and torque

Runcorn (1964) has shown that the value of the orbital angular momentum of the Moon at present \( L_0 \) to its value 370 million years ago (since Devonian)

\[
\frac{L_0}{L} = 1.016 \pm 0.003.
\]  

(1)

He then found that the lunar tidal torque acting on Earth since the Devonian to be \( 3.9 \times 10^{16} \) Nm. Conservation of the angular momentum of the Earth-Moon system implies

\[
L + S = L_{\text{tot}} = \text{constant},
\]

(2)

where \( S \) is the spin angular momentum of the Earth. At present one has

\[
\frac{L_0}{S_0} = 4.83
\]

(3)

Hereafter the subscript ‘0’ denotes present day quantity.

We have found earlier (Arbab, 2003b) that the

\[
L = L_0 \left( \frac{t_0 - t}{t_0} \right)^{0.44},
\]

(4)

where \( t \) is the time measured before present and \( t_0 = 11 \times 10^9 \) y (the present age of the universe\(^1\)). With \( L_0 = 2.878 \times 10^{34} \) kg m\(^2\) eq.(3) gives \( S_0 = 0.5958 \times 10^{34} \) kg m\(^2\) (see Touma and Wisdom, 1994) so that \( L_{\text{tot}} = 3.4738 \times 10^{34} \) kg m\(^2\).

Sonett \textit{et al.} (1996) have obtained a value of \( 2.727 \times 10^{34} \) J s for the orbital angular momentum of the Moon whereas our value is \( 2.774 \times 10^{34} \) J s.

The Moon mean orbital motion is given by (Arbab, 2003b)

\[
n = n_0 \left( \frac{t_0 - t}{t_0} \right)^{1.3}
\]

(5)

\(^1\)See Chaboyer, B. \textit{et al.}, 1998. \textit{Astrophys. J.}, 494, 96.
and the deceleration rate of the Earth’s spin

\[ \omega = \omega_0 \left( \frac{t_0}{t_0 - t} \right)^{2.6}. \]  

(6)

The above two equations yield the relation

\[ \omega n^2 = \omega_0 n_0^2, \]

(7)

which relates directly the Earth angular velocity to the Moon angular velocity. Thus if the Earth deceleration rate is known, the Moon acceleration rate can be found. The above equation yields

\[ \left( \frac{\dot{\omega}}{\omega} \right) = -2 \left( \frac{\dot{n}}{n} \right), \]

(8)

where the dot defines the derivative with respect to time. Equations (5) and (6) show that the length of day and month were respectively, 6 hours and 14 days (preset days) when the Earth-Moon system formed. Thus, the month lengthens to 29.5 days and the day to 24 hours!

From eqs. (2) and (3) the moment of inertia becomes

\[ C = \frac{L - L_{\text{tot}}}{\omega}, \]

(9)

where \( \omega_0 = 7.2921 \times 10^{-5} \text{ rad s}^{-1} \), \( n_0 = 2.66170 \times 10^{-6} \text{ rad s}^{-1} \). We remark that during the time between 1000-3000 million years ago the Earth has undergone a fast deceleration, and we attribute this to the emergence of abundant water on Earth. We have noticed that the moment of inertia has changed very slightly over the last 100 million years but has been doubled since the Earth formation. According to our model, the present increase in the moment of inertia is given by

\[ \left( \frac{dC}{dt} \right)_0 = 3.9 \times 10^{29} \text{ kg m}^2 \text{ cy}^{-1} \]

(10)

in comparison with a decreasing value obtained by Bursa (1987) of

\[ \left( \frac{dC}{dt} \right)_0 = -(4.54 \pm 0.49) \times 10^{29} \text{ kg m}^2 \text{ cy}^{-1}, \]

(11)

where \( C_0 = 8.17 \times 10^{37} \text{ kg m}^2 \). The values of \( L, C, S \) and \( \tau \) are tabulated in Table 1 for different geologic epochs. Bursa (1986) has evaluated the effects due to changes in the Earth’s polar moment of inertia due to the secular time variations in the second zonal harmonic:

\[ \frac{dC}{dt} = -\frac{2}{3} MR^2 \frac{dJ_2}{dt} \]

(12)
where $MR^2 = 2.4296 \times 10^{38}$ kg m$^2$ ($R =$ Earth’s radius, $M =$ Earth’s mass). Yoder et al. (1983) have estimated that

$$\left( \frac{dJ_2}{dt} \right)_0 = (-2.8 \pm 0.6) \times 10^{-9}$/cy (13)

from LAGEOS tracking data and attribute it to the effects of postglacial rebound. Our model, however, gives

$$\left( \frac{dJ_2}{dt} \right)_0 = -2.4 \times 10^{-9}$/cy (14)

showing a good agreement with observational data.

According to this model, the lunar tidal torque ($\tau$) is given by

$$\tau = \frac{dL}{dt} = -\frac{dS}{dt}, \quad (15)$$

or

$$\tau = \tau_0 \left( \frac{t_0}{t_0 - t} \right)^{0.56}, \quad (16)$$

where $\tau_0 = 3.64764 \times 10^{16}$ N m is the present tidal torque. The torque acting on Earth since the Devonian is thus $3.83 \times 10^{16}$ N m, which agrees with Runcorn (1964) result. It also agrees with that given from the measurements of the longitudes of the Sun and Moon (Murray, 1957). Kolenkiewicz et al. (1973) obtained a torque of $(4.8 \pm 0.8) \times 10^{16}$ N m. It is found recently by Sonett et al. (1996) that $(900 \pm 100)$ million years ago the torque was $9.6 \times 10^{16}$ N m estimated from a study of laminated tidal sediments. However, our model gives a value of $3.826 \times 10^{16}$ N m. In particular, the torque has decreased by 26 % since the Earth formed. Holmberg (1952) obtained a torque of $3.7 \times 10^{15}$ N m and Munk and MacDonald (1960) gave a torque of $3 \times 10^{15}$ N m. While the Earth’s spin has decreased by 50 %, the Moon’s orbital momentum has increased by 25 % since their formation (4.5 billion years ago). The slow decrease of the Earth’s spin rate is partly due to an increase in the Earth’s moment of inertia in addition to a decrease in the Earth’s angular velocity.

One must consider the possibility that small relative movements due either to tidal forces or to mechanical stresses on the surface of the Earth may have taken place between the different plates. The Earth’s moment of inertia is changed periodically by tides raised by the Sun and the Moon, which distort the shape of the Earth. The moment of inertia can also change due to changes in the Earth’s internal temperature. A decline in the Earth’s spin velocity would lead to an increase in the polar diameter and a decrease in the equatorial diameter. Any redistribution of mass in the Earth’s interior would
affect the spinning rate of the Earth. This mass redistribution is mainly due to tidal force that acing on Earth since its existence. It dictates the Earth to maintain its equilibrium condition and in turns the Earth exhibits some geological disturbances (e.g., volcanos, earthquakes, etc) in order to comply with the ever changing conditions. Therefore, the Earth must have been suffering a lot from the continuous geological disturbances in order to remain in equilibrium. If tidal forces were bigger in the past then its consequences on paleo-rocks must have been very prominent. High tides would impose a big energy dissipation on Earth in the past. However, Precambrian geology shows no global melting could have occurred during the last 2.7 billion years, and probably much earlier than that. We observe that the conservation of angular momentum of the Earth-Moon system is responsible for the increasing moment of inertia of the Earth. In this work we analyze the evolution of the Earth-Moon system in regard to angular momentum, energy and their dissipation. We came up with excellent agreement with observations. These results encourage us to look for other astronomical and geophysical application of this model.

3 Earth-Moon energy and tidal dissipation

The Earth’s rotational energy is given by

\[ E_1 = \frac{1}{2} C \omega^2, \quad (17) \]

and the Moon’s orbital energy is given by

\[ E_2 = -\frac{G m M}{2r}, \quad (18) \]

where \( r \), \( m \) and \( M \) are the Earth-Moon distance, Moon’s mass and Earth’s mass, respectively, and \( C \) is given by eq.(9).

The rate of total tidal energy \( (E = E_1 + E_2) \) dissipated in the Earth-Moon system is given by

\[ P = -\frac{dE}{dt}. \quad (19) \]

Recently, Sonett et al. (1996) have found that the orbital energy of the Moon 900 million years ago to be \(-4.242 \times 10^{28}\) J. This coincides with our model prediction, which is \(-4.124 \times 10^{28}\) J. MacDonald (1964) estimated an energy of \(1.5 \times 10^{31}\) J to be released on an overall slowing down of the rotation of the Earth from a 3 hour to 24 hour period. Monin et al. (1987) estimated that 4.0-3.2 billion years ago the Earth released a tidal energy of \(8.5 \times 10^{29}\) J, which was sufficient to melt the upper mantle to a depth of 350-400
km and which does not differ largely from our anticipated value of $7.8 \times 10^{29}$ J. Our model shows that the total energy lost by the Earth-Moon system over the past 4.5 billion years was $1.48 \times 10^{30}$ J. This enormous energy must have been distributed (utilized) somewhere by the Earth’s body or oceans. Such a problem can be better tackled by invoking geophysical and oceanographical treatment. We, however, remark that the MacDonald’s value above is two order of magnitude higher than observations might suggest.

Wunsch (2000) estimated that the total presently observed energy dissipation is about $3 \times 10^{12}$ W. Egbert and Ray (2000) reported from a TOPEX/Poseidon satellite that about $1 \times 10^{12}$ W (25-30% of the total tidal energy dissipation) occurs in the deep ocean. While the Earth has lost about 87% of its original rotational energy, the Moon lost only 21% of its original orbital energy. The Earth present rotational energy is $2.17 \times 10^{29}$ J which was $1.687 \times 10^{30}$ J some 4.5 billion years ago. Thus early in the Earth history the orbital source of power generation may have competed with radiogenic one. This rotational source of energy may have powered core convection and dynamo though they operate today at a smaller rate. They are still strong to be detected by present lunar laser techniques.

The astronomical estimate of the amount of tidal energy dissipated in the Earth-Moon system is given by (Lang, 1992; Lambeck, 1980)

$$P = -4 \times 10^{12} \text{ W},$$

(20)

to be compared with the one calculated from the present recession rate of 3.82 cm/y, deceleration rate, and with constant moment of inertia, which is

$$P = -2 \times 10^{12} \text{ W}.$$  

(21)

It is also suggested by Jeffreys (1976) that most reliable values indicate that $P = -2.9 \times 10^{12}$ W, and is twice the value he obtained previously; and compared with astronomical values $P = -2.7 \times 10^{12}$ W and $P = -0.6 \times 10^{12}$ W (Jeffreys, 1976). The analysis of Hendershott (1972) favored a value between 3 and $4 \times 10^{12}$ W. Munk and MacDonald obtained (1960) a value of $P = -3.2 \times 10^{12}$ W. Smith and Jungles (1970) obtain a 3 to $5 \times 10^{12}$ W from observational considerations of gravitationally determined tidal phase lag. According to our model, we obtain a very close value of

$$P = -2.98 \times 10^{12} \text{ W},$$

(22)

compared with the observed value in eq.(18). This coincides exactly with the result obtained by Kagan and Kivman (1995) for the $M_2$ ocean tide. The same number is also found by Pekeris and Accad (1969).
It is believed that oceans provide the predominant sink for the tidal energy: dissipation in the solid Earth and Moon can be at most about 10% of the total. Our model, however gives the total contribution of the energy dissipation without giving the details. (Stacey & Stacey, 1999) have found that the total tidal dissipation in 4.5 billion years to be about $2 \times 10^{30} \text{ J}$ in comparison with our model’s value of $1.47 \times 10^{30} \text{ J}$. This shows how close our results are compatible with present data. Fig.1 - Fig.4 show a graphical variation $\omega$, $L$, $\tau$ and $E$ as functions of time. Our model will benefit very from the future space and geophysical results.

As to date there is no complete account of the Earth history related to its rotation, this model would satisfactory fill the gap. We remark that it is the first simple model that relates the Earth-Moon system parameters to time directly and explicitly.

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Table 1: The Earth-Moon system parameters at different geologic times

| Time * | 600  | 900  | 1000 | 1300 | 1750 | 2170 | 3000 | 3500 | 4500 |
|--------|------|------|------|------|------|------|------|------|------|
| \( L \times 10^{34} \text{ kg m}^2 \) | 2.809 | 2.774 | 2.762 | 2.726 | 2.671 | 2.619 | 2.510 | 2.441 | 2.295 |
| \( I \times 10^{37} \text{ kg m}^2 \) | 7.875 | 7.684 | 7.614 | 7.390 | 7.013 | 6.624 | 5.777 | 5.232 | 4.115 |
| \( \tau \times 10^{16} \text{ N m} \) | 3.764 | 3.826 | 3.847 | 3.913 | 4.019 | 4.125 | 4.359 | 4.520 | 4.897 |
| \( \omega \times 10^{-5} \text{ rad s}^{-1} \) | 8.437 | 9.1004 | 9.342 | 10.112 | 11.442 | 12.911 | 16.689 | 19.738 | 28.365 |
| \( n \times 10^{-6} \text{ rad s}^{-1} \) | 2.47453 | 2.38213 | 2.35152 | 2.26023 | 2.12487 | 2.00031 | 1.75940 | 1.61781 | 1.34319 |
| \( E \times 10^{28} \text{ J} \) | 23.85 | 27.72 | 29.12 | 33.73 | 41.94 | 51.33 | 76.73 | 98.31 | 165.33 |

* Time: In million years before present,

\( S \)= Spin angular momentum,

\( L \)= Orbital angular momentum,

\( I \)= Moment of Inertia,

\( \tau \)= Torque.

\( E \)= Total Earth-Moon system energy
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