Determination of $\gamma$ and $\alpha$ from non-leptonic B decays with SU(3) flavour symmetry

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We describe in detail the method we have used to determine the CKM angles $\gamma$, $\alpha$ and $\beta$ using flavour symmetries between non-leptonic B decays. This method is valid in the context of the SM but also in presence of New Physics not affecting the amplitudes.

1 Introduction

B factories are opening a new exciting period in the precision Flavour Physics [1]. A set of very interesting non-leptonic B decays: $B \to \pi K$ and $B \to \pi\pi$ are now accessible at the $e^+e^-$ B factories. These modes together with the CP-asymmetry of $B_d \to J/\Psi K_S$ will allow us to determine the CKM angles, $\gamma$, $\alpha$ and $\beta$.

In this talk we try to answer the question of how precise we can get to determine the CKM angles using as inputs experimental data and symmetries, and trying to minimize as much as possible hadronic uncertainties from QCD. Since data seems to indicate that penguin diagrams play a fundamental role, any method should include their contribution[2][3][4]. We shall discuss, here, the method we have used in[2][3][5] to determine the CKM angles. This method is based on flavour symmetries between non-leptonic B decays. Another very interesting approach to non-leptonic B decays in the literature tries to predict directly from QCD some of the hadronic parameters, like, for instance, QCD Factorization[5] and PQCD[7].

We focus, here, on the recently measured CP-violating $B_d \to \pi\pi$ observables. We construct the method, step by step, with emphasis on its advantages and how to improve it when data from hadronic machines[3] will be available. We follow the notation of[3][4].

2 Description of the Method

We start by writing down the most general parametrization in the SM of the amplitude corresponding to $B_d^0 \to \pi^+\pi^-$, using the Wolfenstein parametrization[9][3]:

$$A(B_d^0 \to \pi^+\pi^-) = A_u^{(d)} (A_{CC}^{\pi} + A_{pen}^{\pi}) + A_c^{(d)} A_{pen}^{\pi} + A_s^{(d)} A_{pen}^{\pi} = C (e^{i\gamma} - d e^{i\theta})$$

This amplitude includes current-current contributions and QCD and EW penguin diagrams. All the hadronic information is collected in:

$$d e^{i\theta} = \frac{1}{R_b} \left( \frac{A_{pen}^{\pi}}{A_{CC}^{\pi} + A_{pen}^{\pi}} \right), \quad C = \lambda^2 (1 - \lambda^2) / R_b (A_{CC}^{\pi} + A_{pen}^{\pi})$$

with $A = |V_{cb}|^2 / 12$, $R_b = (1 - \lambda^2 / 2) |V_{ub} / V_{cb}|$ and $A_{pen}^{\pi} \equiv A_{pen}^{\pi} - A_{pen}^\ast$. We can construct, using this amplitude, the direct and mixing induced CP-asymmetries of $B_d \to \pi^+\pi^-$:

$$\mathcal{A}_{CP}^{dir} = - \frac{2 d \sin \gamma}{1 - 2 d \cos \gamma}$$

$$\mathcal{A}_{CP}^{mix} = \frac{\sin(\phi_d + 2\gamma) - 2 d \sin \cos \phi_d + \gamma + d^2 \sin \phi_d}{1 - 2 d \cos \gamma}$$

Here, the counting of parameters shows that we have two hadronic parameters $d$ and $\theta$ and $d$ are two weak parameters: weak mixing phase $\phi_d$ and $\gamma$, but only two observables.

However, we know that there is a closely related process $B_s \to K\bar{K}$, where a similar description can be used. A general amplitude parametrization[9][3] in the SM is:

$$A(B_s^0 \to K^+K^-) = \left( \frac{\lambda}{1 - \lambda^2 / 2} \right) C \left[ e^{i\gamma} + \frac{1 - \lambda^2}{\lambda^2} d e^{i\theta} \right]$$

It contains also two hadronic parameters $d'$ and $\theta'$, with the same functional dependence of penguin diagrams as in Eq. (1), with the only difference that the quarks $d$ and $s$ are interchanged in the external legs of penguins.

The corresponding CP asymmetries of $B_s \to K^+K^-$ are:

$$\mathcal{A}_{CP}^{dir} = 2 d' \sin \theta' \sin \gamma$$

$$\mathcal{A}_{CP}^{mix} = \frac{\sin(\phi_s + 2\gamma) + 2 d' \sin \cos \phi_s + \gamma + d'^2 \sin \phi_s}{1 + 2 d' \cos \gamma}$$

They also depend on two hadronic parameters: $d' \equiv d' / \epsilon$ with $\epsilon \equiv \lambda^2 / (1 - \lambda^2) \sim 0.05$ and $\theta'$ and two weak parameters: $\phi_s$ (negligibly small in SM) and the CKM angle $\gamma$.

Finally, if we combine both processes and their parameters using the U-spin symmetry[9][2][3], that implies:

$$d e^{i\theta} = d' e^{i\theta'}$$
we will have four observables and five parameters (out of the initial seven): $\gamma, \phi_d, d, \theta$ and $\phi_s$. Moreover, $\phi_s$ will be determined from $A_{CP}(B_s \rightarrow J/\psi \phi)$.

Last but not least, we can test the U-spin symmetry breaking in two different ways:

a) One can define U-spin breaking parameters: $\xi = d'/d$ and $\Delta \theta = \theta' - \theta$ and test the sensitivity of the results to these parameters.

b) Once the data from $B_s \rightarrow KK$ will be available and $A_{CP}(B_s \rightarrow J/\psi \phi)$ measured ($\phi_s$), we will be able to reduce to three the number of parameters: $\gamma, d$ and $\theta$, since ($\phi_d$ is taken from $B_d \rightarrow J/\psi K_s^0$), so we can test $\xi$ or $\Delta \theta$.

Looking a bit more in detail one realizes that $d$ is not a fully free parameter, we can constrain, and indeed substitute it introducing a new observable called $H$ (see \[10, 3\]):

$$H \propto \frac{\text{BR}(B_d \rightarrow \pi^+\pi^-)}{\text{BR}(B_s \rightarrow K^+K^-)}$$

that in the U-spin limit depends only on $\cos \theta \cos \gamma$ and $d$. Although data on $B_s \rightarrow KK$ is still available, we can already now apply the method using the data from B-factories, using the observation that $B_d \rightarrow \pi^+K^-$ and $B_s \rightarrow K^+K^-$ differ in their spectator quarks, meaning:

$$A_{CP}^\text{dir}(B_s \rightarrow K^+K^-) \approx A_{CP}^\text{dir}(B_d \rightarrow \pi^+K^-) \quad \text{BR}(B_s \rightarrow K^+K^-) \approx \text{BR}(B_d \rightarrow \pi^+K^-) \frac{\tau_B}{\tau_B}$$

This relation requires that the ’exchange’ and ’penguin annihilation’ contributions to $B_s \rightarrow KK$ absent in $B_d \rightarrow \pi^+K^-$ play a minor role [11]. But in case they would be enhanced we can also control them through data on $B_s \rightarrow \pi^+\pi^-$. This allows us to determine now $H$ yielding:

$$H \approx \frac{1}{\epsilon} \left( \frac{f_K}{f_{\tau}} \right)^2 \left[ \frac{\text{BR}(B_d \rightarrow \pi^+\pi^-)}{\text{BR}(B_d \rightarrow \pi^+K^-)} \right] = 7.5 \pm 0.9$$

and use it to write $d$ in terms of $d = f(H, \theta, \gamma; \xi, \Delta \theta)$ [3].

3 Exploring the allowed region in $B_d \rightarrow \pi\pi$ to the SM and beyond

The starting point is the general expression [3]:

$$A_{CP}^\text{dir}(B_d \rightarrow \pi^+\pi^-) = \frac{\sqrt{4p^2 - (u + v p^2)^2} \sin \gamma}{(1 - u \cos \gamma) + (1 - v \cos \gamma)p^2}$$

(2)

where $u, v, p$ are functions of four observable quantities $\mathcal{R}_{CP}^\text{max}, H, \phi_\approx$ obtained from $A_{CP}(B_d \rightarrow J/\psi K_s)$ and CKM-angle $\gamma$ (see [3] for details). They also depend on the two U-spin breaking parameters: $\xi$ and $\Delta \theta$. We start the analysis in the U-spin limit ($\xi = 1, \Delta \theta = 0$) and we explore

![Figure 1](https://example.com/figure1.png)

**Figure 1.** $A_{CP}^\text{dir}(B_d \rightarrow \pi\pi)$ as a function of $\gamma$ for $A_{CP}^\text{dir}(B_d \rightarrow \pi\pi) \in [0, 1]$ and $H = 7.5$. The curves correspond to fixed values for $A_{CP}^\text{mix}(B_d \rightarrow \pi\pi)$. a) corresponds to the solution $\phi_d = 47^\circ$ and b) $\phi_d = 133^\circ$. Horizontal band correspond to the experimental value for $A_{CP}^\text{dir}$ while the internal grey-shaded region corresponds to the experimental value for $A_{CP}^\text{mix}(B_d \rightarrow \pi\pi)$.

in Sec.3.2 the sensitivity of the results to deviations from this limit. An interesting remark is the symmetry [3] that Eq. (2) exhibits:

$$\phi_d \rightarrow 180^\circ - \phi_d \quad \gamma \rightarrow 180^\circ - \gamma$$

(3)

The present world average $\sin \phi_d = 0.734 \pm 0.054$ gives rise to two possible solutions: $\phi_d = (47^\circ \pm 5^\circ) \vee (133^\circ \pm 5^\circ)$. The first solution has positive $\cos \phi_d$ and the second negative $\cos \phi_d$. Our approach allow us to explore both. These two solutions together with the symmetry of Eq. (3) will have important consequences as we will see in a moment.

3.1 Determination of $\gamma$

The experimental situation is still uncertain and the present naive average is [12] (including PDG enlarged errors):

$$A_{CP}^\text{dir}(B_d \rightarrow \pi^+\pi^-) = -0.51 \pm 0.19 (0.23)$$

$$A_{CP}^\text{mix}(B_d \rightarrow \pi^+\pi^-) = +0.49 \pm 0.27 (0.61)$$

(4)

Taking Eq. (2) and varying $A_{CP}^\text{mix}(B_d \rightarrow \pi^+\pi^-)$ in all the positive range, with $H = 7.5$ for each solution of $\phi_d$ we find [3]:

![Figure 2](https://example.com/figure2.png)

**Figure 2.** The present world average $\sin \phi_d = 0.734 \pm 0.054$ gives rise to two possible solutions: $\phi_d = (47^\circ \pm 5^\circ)$ and $\phi_d = 133^\circ$. Horizontal band correspond to the experimental value for $A_{CP}^\text{dir}$ while the internal grey-shaded region corresponds to the experimental value for $A_{CP}^\text{mix}(B_d \rightarrow \pi\pi)$.

The present world average $\sin \phi_d = 0.734 \pm 0.054$ gives rise to two possible solutions: $\phi_d = (47^\circ \pm 5^\circ)$ and $\phi_d = 133^\circ$. Horizontal band correspond to the experimental value for $A_{CP}^\text{dir}$ while the internal grey-shaded region corresponds to the experimental value for $A_{CP}^\text{mix}(B_d \rightarrow \pi\pi)$.
Figure 2. Sensitivity to H

![Figure 2](image1)

Figure 3. Sensitivity to ξ

![Figure 3](image2)

3.2 Sensitivity to H and ξ, Δθ

Here we examine the sensitivity of CKM-angle γ to the variation of the different hadronic parameters.

- H: Fig. 2 shows the change in the prediction for γ when H is varied between 6.6 to 8.4, for φ_d = 47°. The region shown corresponds to the restriction of $\mathcal{R}^{\text{mix}}_{\text{CP}}$ inside the experimental range Eq. (4). The error induced in the determination of γ is only of ±2°. For the second solution φ_d = 133° exactly the same conclusion can be drawn. One can enlarge the range of H as it was done in[11] to take into account the uncertainty associated to the spectator-quark hypothesis used to determine H, and the error is still under control. Notice that with the future data on $B_s \rightarrow KK$, this hypothesis will not be needed.

- U-spin breaking parameter ξ. This is the most important source of uncertainty. However, as can be seen in Fig. 3, the error induced in the determination of γ is ±5° even if we allow for a very large ±20% U-spin breaking.

- U-spin breaking parameter Δθ. The effect on the determination of γ for values of Δθ up to 40° is completely negligible.

Other studies on the use and evaluation of U-spin can be found in[13].

3.3 Determination of α and β in the SM and in presence of New Physics only in the mixing

So far so good for γ, next question is how to determine α and β. Here we will also allow for Generic New Physics affecting the $B_d^0$-$\overline{B_d^0}$ mixing, but not to the $\Delta(B,S) = 1$ decay amplitudes. In order to do so, we will use three inputs[5,13]:

- $R_b \equiv |V_{ub}V_{cb}^*/V_{ub}V_{ub}^*|$ obtained from exclusive/inclusive transitions mediated by $b \rightarrow u\tau\bar{\nu}_\tau$ and $b \rightarrow c\ell\bar{\nu}_\ell$. Two important remarks are: a) It is not expected that New Physics can affect significantly this quantity, b) already from $R_b^{\text{max}} = 0.46$ we can extract a maximum possible value for β: $|\beta^{\text{max}}| = 27°$.

- γ obtained as discussed in Sec.3.1.

- φ_d from $\mathcal{A}^{\text{mix}}_{\text{CP}}(B_d \rightarrow J/\psi K_S)$ is used as an input for the CP asymmetries of $B_d \rightarrow \pi\pi$, but NOT to determine β, since we assume that New Physics could be present. For the same reason also $\Delta M_d$ and $\Delta M_s/\Delta M_d$ are not used as inputs.

Using these inputs we obtain two possible determinations for α and β, corresponding to two different scenarios.
with same errors associated to $\xi$ as in Scenario A. It is interesting to notice that this second solution gives a better agreement with data for certain very rare decays like $K^+ \to \pi^+ \nu \bar{\nu}$ than the SM solution.

In conclusion the method described allow us to determine the CKM angles using flavour symmetries with data from non-leptonic B decays and $R_b$. The method is valid for the SM and in presence of New Physics not affecting the amplitudes. Finally, the method provides self-consistency checks to control the impact of hypothesis and ways to eliminate some of them (spectator-quark hypothesis) when data from hadronic machines will be available.

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