SHOCK EMERGENCE IN SUPERNOVAE: LIMITING CASES AND ACCURATE APPROXIMATIONS

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ABSTRACT

We examine the dynamics of accelerating normal shocks in stratified planar atmospheres, providing accurate fitting formulae for the scaling index relating shock velocity to the initial density and for the post-shock acceleration factor as functions of the polytropic and adiabatic indices which parameterize the problem. In the limit of a uniform initial atmosphere, there are analytical formulae for these quantities. In the opposite limit of a very steep density gradient, the solutions match the outcome of shock acceleration in exponential atmospheres.

Key words: shock waves – supernovae: general

1. INTRODUCTION

Shock emergence at the surface of an exploding star is an important moment in the life of a supernova. Shock and post-shock acceleration in the outer stellar envelope, and the breakout of post-shock radiation from a thin layer beneath the photosphere, can have a number of significant consequences. The escaping flash of radiation gives an energetic precursor which can signal the supernova’s existence (Klein & Chevalier 1978) and carries physical information about the explosion (Matzner & McKee 1999; Calzavara & Matzner 2004; Nakar & Sari 2012, 2010; Sapir et al. 2011; Katz et al. 2010; Suzuki & Shigeyama 2010; Piro et al. 2010); traveling outward, it can ionize a circumstellar nebula like the one surrounding SN 1987A (Lundqvist & Fransson 1996) and produce an infrared light echo as it encounters dust (Dwek & Arendt 2008). Shock emergence launches the fastest ejecta, the first to host the supernova photosphere (Chevalier 1992) and the first to interact with circumstellar and interstellar matter, producing a synchrotron-emitting shell (Fransson et al. 1996). If they meet a companion star or dense circumstellar disk, then an additional X-ray signal can be produced (Metzger 2010; Kasen 2010).

In particularly compact and energetic explosions, the shock can become relativistic before emerging, and relativistic ejecta can create X-ray and γ-ray transients in their circumstellar collisions (Matzner & McKee 1999; Tan et al. 2001), and may produce light elements through spallation (Fields et al. 2002; Nakamura & Shigeyama 2004) and, potentially, ultra-high-energy cosmic rays (Wang et al. 2007; Budnik et al. 2008).

Potentially observable shock breakouts accompany several other types of astrophysical events, including the type Ia explosions (Piro et al. 2010) and accretion-induced collapses (Fryer et al. 1999; Tan et al. 2001) of white dwarfs, tidal disruptions of stars, jet and cocoon emergence in long-duration gamma-ray bursts, and (albeit in a less energy-conserving manner) superbubbles in galactic disks.

Underlying all these phenomena are the hydrodynamics of shock acceleration in the outer layers of a star, and anchoring these dynamics is the asymptotic problem of flow behind a normal, adiabatic shock accelerating through a planar medium which varies as a power law with depth. As Matzner & McKee (1999) first demonstrated, this asymptotic planar solution can be combined with the dynamics of a spherical, self-similar blastwave into an accurate approximate model for shock propagation and post-shock flow in a spherical explosion. This, in turn, can be used to predict the amount and upper speed limit of the fastest ejecta and properties of the breakout flash (Matzner & McKee 1999; Calzavara & Matzner 2004), transition to relativistic flow and aspects of the circumstellar interaction (Tan et al. 2001), and many other breakout-related phenomena. With advances in the theory of photon-mediated shocks and emission around the time of breakout (e.g., Katz et al. 2010, 2012; Nakar & Sari 2010; Sapir et al. 2011), of the ultra-relativistic self-similar problem (Perna & Vietri 2002; Nakayama & Shigeyama 2005; Pan & Sari 2006; Kikuchi & Shigeyama 2007), and of the interaction of relativistic ejecta with a stellar wind (Nakamura & Shigeyama 2006), among others, there are ample opportunities for these approximate global models to be improved and extended.

To advance this larger project, we focus here on the planar, adiabatic, non-relativistic problem of an accelerating normal shock. Our goal is to provide flexible yet highly accurate approximations for the most important flow quantities, the shock acceleration index and the post-shock acceleration factor, as functions of the adiabatic and polytropic indices (γ and γρ, respectively) which parameterize the problem. A secondary goal is to demonstrate that although the flow quantities must typically be found as eigenvalues of the dynamical problem, they adhere to well-understood limiting forms in several asymptotic cases.

The self-similar problem with a power-law atmosphere below vacuum was posed by Gandel’Man & Frank-Kamenetskii (1956) and solved in its Eulerian form by Sakurai (1960). We shall use Sakurai’s eigenvalue method to identify the shock acceleration index, but for the post-shock flow we employ the Lagrangian approach by Matzner & McKee (1999). This has the dual advantage that it continuously describes both the pre-breakout and post-breakout flow in a single function, and that it naturally connects each fluid element’s state at the shock front with those in the final state.

2. PROBLEM, METHOD, AND SOLUTIONS

Our problem involves one-dimensional flow with an altitude x relative to the stellar surface. The initial density distribution of cold matter is ρ0(x) ∝ (−x)n for x < 0 and ρ0(0) = 0 for x > 0. Here, n is the polytrope parameter, which is related to γρ by the hydrostatic relation with constant gravity g∗: if P = 0 at x = 0, then P(x) = g∗(−x)ρ(x)(n + 1) ∝ ρ(x)γρ with γρ = 1 + 1/n.

A strong adiabatic shock wave accelerates down this density gradient, reaching x = 0 at t = 0 with infinite velocity; neglecting radiative effects, this is the point of breakout.
For $t > 0$ the shock disappears and matter expands into the region of positive $x$. In the limit that all additional physical effects—curvature, gravity, and temperature of the star, finite depth, non-simultaneity of breakout, relativity, shock thickness, etc.—are negligible, the flow is self-similar. The shock velocity accelerates according to $v_s = \dot{x}_s(t) \propto (-\dot{x})^{-1} \propto \rho^{-\beta}$, where $\beta = \lambda/n$. The fluid motion is a universal function of self-similar variables like $x/\chi(t)/t$ or $x(m, t)/\chi_0(m)$, in which each fluid element (labeled by its mass coordinate $m$) accelerates from a post-shock velocity toward its terminal velocity, which is a unique multiple $v_f(m)/v_s(m)$ of the shock velocity which crossed that element. Our task is to find $\beta$ and $v_f/v_s$ as functions of $\gamma$ and $\gamma'$.  

2.1. Shock Acceleration Parameter and Its Limits

To find the shock acceleration index $\lambda$, or equivalently the velocity-density index $\beta$, we follow Sakurai (1960). In the limit that all additional physical effects—curvature, gravity, and temperature of the star, finite depth, non-simultaneity of breakout, relativity, shock thickness, etc.—are negligible, the flow is self-similar. The shock velocity accelerates according to $v_s = \dot{x}_s(t) \propto (-\dot{x})^{-1} \propto \rho^{-\beta}$, where $\beta = \lambda/n$. The fluid motion is a universal function of self-similar variables like $x/\chi(t)/t$ or $x(m, t)/\chi_0(m)$, in which each fluid element (labeled by its mass coordinate $m$) accelerates from a post-shock velocity toward its terminal velocity, which is a unique multiple $v_f(m)/v_s(m)$ of the shock velocity which crossed that element. Our task is to find $\beta$ and $v_f/v_s$ as functions of $\gamma$ and $\gamma'$.  

To find the shock acceleration index $\lambda$, or equivalently the velocity-density index $\beta$, we follow Sakurai (1960). Sakurai writes the conservation equations for mass, energy, and entropy in Eulerian form, introduces the self-similar ansatz, and arrives at a single, first-order differential equation for the spatial structure of the post-shock flow prior to breakout. This equation must pass smoothly from the conditions immediately behind the shock front, through a critical point at the sonic point of the flow; this is only possible for a unique value of $\lambda$ or $\beta$, which we identify by a shooting method. We present the solution space $\beta(n, \gamma)$ spanning $\log_{10} n = -6, \ldots, 6$ and $\log_{10}(\gamma - 1) = -5, \ldots, 6$ in Table 1. For the entire parameter space, the simple functional fit

$$
\beta = \left[ A + \left( \frac{B \gamma}{\gamma - 1} \right)^C \right]^{-1}
$$  

is quite accurate: the root-mean-square (rms) error relative to the values in Table 1 is 1.5%, and the error, which is concentrated at high $n$ and low $\gamma$, is at most 4.0%. High accuracy is necessary, because $\beta$ is the exponent of a number which becomes large around breakout. For instance, the energy in relativistic ejecta scale as $E_{\text{rel}} \propto [E_{\text{in}}/(M_\text{ej}c^2)]^{\gamma/(2\beta)}$, where $E_{\text{in}}$ is the explosion energy and $M_\text{ej}$ is the total ejected mass; in the model for SN 1998bw discussed by Tan et al. (2001), an error of $\beta$ leads to an error in $E_{\text{rel}}$ which is nine times greater. Higher accuracy can be obtained by interpolating our table, and the differential equations yield solutions to numerical accuracy. Several limiting forms of our fit to $\beta(n, \gamma)$ are readily apparent.  

In the limit $n \to 0$ of an effectively uniform stellar envelope, $\beta$ and its fit in Equation (1) reproduce the approximate expression derived by Whitham (1958 and 1974, simplifying and improving upon results by Chisnell 1955, 1957, and Chester 1960):

$$
\beta \to \left[ 2 + \left( \frac{2 \gamma}{\gamma - 1} \right)^{1/2} \right]^{-1} \cdot (n \to 0).
$$

Whitham arrived at this form by reasoning that quantities just behind the shock front should evolve similarly to those found along a forward-traveling sound wave, for which there is an exact equation. In general, this is only a good approximation, because the shock moves more slowly than these forward characteristics and because conditions vary from one characteristic to another. In the limit $n \to 0$, however, there is no variation among characteristics, and Whitham’s approximation becomes exact.  

The isothermal limit $\gamma \to 1$ is characterized by

$$
\beta \to 0 \cdot (\gamma \to 1).
$$

In this limit, a strong shock is infinitely compressive and governed by the conservation of momentum. The shock velocity is constant because the atmosphere above the shock front has negligible mass relative to the shell of post-shock material. We note that Whitham’s approximation is exact in this limit as well, because the shock no longer outruns forward characteristics.  

In the limit $n \to \infty$, the stellar structure is isothermal ($\gamma_p \to 1$) and transitions from power-law to exponential in its depth dependence. It is reassuring, therefore, that Equation (1) gives

$$
\beta \to \left[ 1.78 + \left( 4.321 \gamma \right)^{0.460} \right]^{-1} \cdot (n \to \infty)
$$

in this limit. As Hayes (1968) notes, this limit coincides with the case of an exponential atmosphere ($\gamma_p = 1$); we reproduce his solutions and those of Raizer (1964). Equation (1) demonstrates that the shock acceleration index is indistinguishable from the exponential case for all $n \lesssim 10^3$—or in practical terms, any time that the distance to the surface is very far when measured relative to the density scale height.  

In the $\gamma \to \infty$ limit of incompressible flow, $\beta$ takes the definite form $[A(n) + B(n)^{\gamma/(2\beta)}]^{-1}$. We know of no physical explanation for this result.  

We end this section by noting that for the specific case $\gamma = 3/2$ and $n = 5$, we find $\beta = 1/5$ and $\lambda = 1$ (at least to within a part in $10^3$, while the fit of Equation (1) gives $\lambda = 1.01$). This is unlikely to be a coincidence, although we have not identified any simplification in the dynamical equations for this case.  

3. POST-SHOCK FLOW AND ASYMPTOTIC FREE EXPANSION

After each parcel of gas has been swept into motion by the shock, it continues to accelerate until its internal energy is spent and it has reached the terminal velocity $v_f(m)$. To describe this, we employ the Lagrangian method of Matzner & McKee (1999). This naturally provides quantities like $v_f(m)/v_s(m)$, and continues smoothly through the point of breakout; however, it does not yield eigenvalues like $\beta$ as readily as Sakurai’s method. In order to correct a couple typos in Matzner & McKee’s Appendix (which do not affect their results), we write out the equation. The Lagrangian self-similar time and space coordinates are $\eta = t/t_0(m)$ and $\xi = x/x_0(m)$, where $t_0(m)$ is the time at which the shock crosses $x_0(m)$. For a given fluid element both $\eta$ and $S$ decline from unity to $-\infty$ as a fluid element accelerates outward; shock breakout (neglecting radiative effects) is at $\eta = 0$, and the element exits the boundaries of the progenitor somewhat later, when $S = 0$. As Matzner & McKee discuss, the pressure and density distributions and the resulting acceleration can be computed from these variables, and the equating resulting fluid
acceleration to $\dot{x}(m) = S''(\eta) x_0(t_0) t_0^2 m^2$ yields

\[
(1 + \lambda)^2 S''(\eta) = \frac{2}{\gamma + 1} \left( \frac{\gamma - 1}{\gamma + 1} \right)^{\gamma} \times \left\{ \frac{n - 2\lambda}{\Sigma(\eta)^{\gamma + 1}} - \frac{\lambda S'(\eta) (\lambda + 1) S''(\eta)}{\Sigma(\eta)^{\gamma + 1}} \right\},
\]

where $\Sigma(\eta) = S(\eta) - (\lambda + 1) S'(\eta)$. Note that relative to Matzner & McKee's Equation (A2) and its preceding discussion, all instances of $\lambda - 1$ have been corrected to $\lambda + 1$. We integrate Equation (2) from the post-shock conditions $S(1) = 1$, $S'(1) = 2/[(\lambda + 1) (\gamma + 1)]$ to a very large negative value of $\eta$ (typically $-\eta > 10^{20}$).

### 3.1. The Acceleration Factor and Its Limits

We use the asymptotic form $|S_f - S(\eta)| \propto (-\eta)^{(-\gamma - 1)}$, valid for $\gamma > 1$ (where $S_f' = S(\eta \to \infty)$, to deduce

\[
v_f(m)/v_m(m) = (\lambda + 1) S_f'/S(1). \]

We present this in Table 2; the approximation

\[
v_f(m)/v_m(m) \approx \gamma + 1 \left[ \left( \frac{2\gamma}{\gamma - 1} \right)^D + E \right],
\]

where

\[
D = 1 - \left( \frac{1.832}{(1.083/n)^{0.828} + (1.5/n)^{0.883}} \right) + \frac{1.655}{0.803},
\]

\[
E = 1 - \left( \frac{\sqrt{2.39/n}}{1.18 + 1} \right)
\]

is reasonably accurate: over the values in Table 2, the rms error is 0.6% and the maximum error is 1.8%. We do not have solutions for very small values of $\gamma - 1$, however, so we cannot check accuracy in that regime.

The form of Equation (3) gives some insight into the nature of post-shock acceleration, because the initial factor $(\gamma + 1)$ expresses the immediate post-shock velocity in units of $v_\gamma$, so the
factor \( (2γ/(γ−1))^2 + E \) captures the subsequent acceleration from the post-shock state to the final state of free expansion. A clear limit of Equation (3) is that in which the initial density distribution becomes uniform:

\[
\frac{v_f(m)}{v_i(m)} \rightarrow \frac{2}{\gamma + 1} \left[ \left( \frac{2γ}{γ−1} \right)^{1/2} + 1 \right] \cdot (n \rightarrow 0).
\]

This limit is a consequence of the isentropic nature of the post-shock flow, which ensures that the Riemann invariant \( v + 2cs/(γ−1) \) is conserved along outward-traveling sound fronts from the post-shock state to the freely expanding state; here, \( cs \) is the adiabatic sound speed.

In the opposite limit, corresponding to an exponential atmosphere,

\[
\frac{v_f(m)}{v_i(m)} \rightarrow \frac{2}{\gamma + 1} \left[ \left( \frac{2γ}{γ−1} \right)^{1/3} + \frac{1}{5} \right] \cdot (n \rightarrow \infty)
\]

to within 3%. Hayes (1968) and Grover & Hardy (1966) apply an Eulerian approach and do not provide this ratio. Raizer (1964) does compute post-shock acceleration in his Lagrangian treatment: Zel’dovich & Raizer (1967) report that he finds \( v + 2cs/(γ−1) \) is conserved along outward-traveling sound fronts from the post-shock state to the freely expanding state; here, \( cs \) is the adiabatic sound speed. In the opposite limit, corresponding to an exponential atmosphere,

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for a spherical explosion of energy \( E_{in} \) traveling through a mass distribution described by radius \( r \), density \( ρ_0 \), and enclosed ejecta mass \( m_{ej} \). This form matches the properties of an interior blastwave (an energy-conserving flow in which \( v_f(r)^2m_{ej}(r) \rightarrow E_{in} \) with those of planar shock acceleration (a second-type similarity solution with eigenvalue indices). Matzner & McKee (1999) propose the shock velocity approximation

\[
v_s = \Lambda \left( \frac{E_{in}}{m_{ej}} \right)^{1/2} \left( \frac{m_{ej}}{\rho_0^2r^2} \right) \beta \tag{4}
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\]
Table 1 and Equation (1) should allow for further improvements in which \( v_s \) better responds to the local conditions.\(^1\)

Second, for any hydrostatic envelope in which radiation pressure is initially negligible, there exists a range of shock strengths which are both strong and yet not dominated by radiation pressure in the post-shock state. These might develop in the steepening of finite-amplitude sound pulses (Wyman et al. 2004) or in the explosion launched by the impact of a comet or asteroid on the atmosphere (Chevalier & Sarazin 1994). Insofar as the post-shock gas is described by a characteristic value of \( \gamma \) different from \( 4/3 \), our results indicate how shock emergence is changed.

Third, any investigation of the phenomena surrounding shock acceleration and shock emergence (e.g., shock instability; Luo & Chevalier 1994) must use the ideal planar solution as its reference state. Understanding this solution’s parameter dependence and limits therefore adds insight into the phenomenon under study.

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\(^1\) It may also be possible to incorporate Waxman & Shvarts’s (1993) second-type similarity solutions for spherical blastwaves in steep density distributions (\( d \ln \rho / d \ln r < -3 \)), although the rapid variation of \( d \ln \rho / d \ln r \) in subsurface regions of stars poses a difficulty.