Kaon photoproduction on the nucleon: Contributions of kaon-hyperon final states to the magnetic moment of the nucleon

S. Sumowidagdo and T. Mart
Jurusan Fisika, FMIPA, Universitas Indonesia, Depok 16424, Indonesia

Abstract

By using the Gerasimov-Drell-Hearn (GDH) sum rule and an isobaric model of kaon photoproduction, we calculate contributions of kaon-hyperon final states to the magnetic moment of the proton and the neutron. We find that the contributions are small. The approximation of $\sigma_{TT'}$ by $\sigma_T$ clearly overestimates the value of the GDH integral. We find a smaller upper bound for the contributions of kaon-hyperon final states to the proton’s anomalous magnetic moment in kaon photoproduction, and a positive contribution for the square of the neutron’s magnetic moment.

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The internal structure of the nucleon is still an interesting topic of investigations nowadays. The existence of this structure is responsible for the ground state properties of the nucleon, such as hadronic and electromagnetic form factors and the anomalous magnetic moment. At higher energies this finite internal structure yields a series of resonances in the mass region of $1 - 2$ GeV. It was then found that the nucleon’s ground state properties and the nucleon’s resonance spectra are not all independent phenomena; they are related by a number of sum rules [1].

One of these sum rules is the Gerasimov-Drell-Hearn (GDH) sum rule, which connects the nucleon’s magnetic moments and the helicity structures in the resonance region. Although the GDH sum rule was proposed more than 30 years ago, no direct experiment had been performed to investigate whether or not the sum rule converges. However, with the advent of the new high-intensity and continuous-electron-beam accelerator, accurate measurements of the contribution to the GDH integral from individual final states are made possible.

Previously, Hammer, Drechsel, and Mart (HDM) suggested that by using the Gerasimov-Drell-Hearn sum rule it is possible to estimate strange contributions to the magnetic moments of the proton [2]. They used experimental data and an isobaric model for the photoproduction of $\eta$, $\phi$, as well as $K$ mesons, in order to estimate the transversely unpolarized total cross section $\sigma_T$ and, therefore, to calculate the upper bounds of strange contributions to the anomalous magnetic moment of the proton. It is the purpose of this Brief Report to update the contributions of kaon-hyperon final states, by means of the latest isobaric model which fits all available experimental data, including the recent data from SAPHIR [3].

The GDH sum rule [4] (for a review see Ref. [1]) relates the anomalous magnetic moment of the nucleon $\kappa_N$ to the difference of its polarized total photoabsorption cross section

$$\kappa_N^2 = \frac{m_N^2}{8\pi^2\alpha} \int_0^\infty \frac{d\nu}{\nu} [\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)],$$

where $\sigma_{3/2}$ and $\sigma_{1/2}$ denote the cross sections for the possible combinations of spins of the nucleon and photon (i.e., $\sigma_{3/2}$ for total spin = $3/2$ and $\sigma_{1/2}$ for total spin = $1/2$), $\alpha$ is the fine structure constant, $\nu$ is the photon energy in the laboratory frame, and $m_N$ the mass of the nucleon. The derivation of GDH sum rule is based on general principles: Lorentz and gauge invariance, crossing symmetry, causality, and unitarity. The only assumption in deriving Eq. (1) is that the scattering amplitude goes to zero for the limit $|\nu| \to \infty$, thus there is no subtraction hypothesis [5].

In photoproduction processes, however, the spin-dependent cross section is related to the total cross sections by

$$\sigma_T = \frac{\sigma_{3/2} + \sigma_{1/2}}{2},$$

$$\sigma_{TT'} = \frac{\sigma_{3/2} - \sigma_{1/2}}{2}. \quad (3)$$

The first cross section can be measured using unpolarized real photons while the second can be measured with longitudinally polarized electrons and polarized nucleon targets or hyperon recoils. Experimentally, the latter must be done using electroproduction, i.e. virtual photons. Nevertheless, the momentum transfer of the electrons ($Q^2$) can be minimized close to the photon point. Numerically, $\sigma_{TT'}$ can be calculated by using photoproduction, since Eq.
(4) needs $Q^2 = 0$ and $\sigma_{TT'} = \sigma_{TT'}(F_1, F_2, F_3, F_4)$, where the $F_i$’s are the CGLN amplitudes for real photons (3).

Unlike the calculation in the previous paper, here we use both

$$\kappa^2_N = \frac{m_N^2}{\pi^2 \alpha} \int_0^{\nu_{\text{max}}} \frac{d\nu}{\nu} \sigma_{TT'}$$

and

$$\kappa^2_N \lesssim \frac{m_N^2}{\pi^2 \alpha} \int_0^{\nu_{\text{max}}} \frac{d\nu}{\nu} \sigma_T$$

where the GDH Integral is already saturated at $\nu_{\text{max}} \approx 2$ GeV (10), in order to measure the deviations of the approximation made by the previous work from the expected values. This was not done in the previous work since experimental data for kaon photoproduction were very scarce at that time, especially for the $\gamma p \rightarrow K^0 \Sigma^+$ channel, thus predictions of $\sigma_{TT'}$ were somewhat unreliable.

We use the latest and modern elementary operator (7), which was guided by recent coupled-channel results (8) and includes the newest data (3). The model consists of a tree-level amplitude that reproduces all available $K^+ \Lambda$, $K^+ \Sigma^0$ and $K^0 \Sigma^+$ photoproduction observables and thus provides an effective parametrization of these processes. The background terms contain the standard $s$-, $u$-, and $t$-channel contributions along with a contact term that was required to restore gauge invariance after hadronic form factors had been introduced (3). This model includes the three nucleon resonances that have been found in the coupled-channels approach to decay into the $K\Lambda$ channel, the $S_{11}(1650)$, $P_{11}(1710)$, and $P_{13}(1720)$. For $K\Sigma$ production further contributions from the $S_{31}(1900)$ and $P_{31}(1910)$ $\Delta$ resonances were added.

In Fig. 1 we show the total cross sections $\sigma_T$ and $-\sigma_{TT'}$ as a function of the photon laboratory energy $\nu$ for the six isospin channels in kaon photoproduction. Since there are no experimental data for productions on the neutron, we consider the three right panels in Fig. 1 as predictions. Obviously, the model can remarkably reproduce the experimental data for the productions on the proton. In the former calculation, contribution from the $\gamma p \rightarrow K^0 \Sigma^+$ channel could not properly be calculated since previous elementary models mostly overpredict $K^0 \Sigma^+$ total cross section by a factor of up to 100 (11). With the new SAPHIR data available in three isospin channels, the elementary model becomes more reliable to explain kaon photoproduction on the proton and to predict the production on the neutron.

The elementary model predicts negative sign for $\sigma_{TT'}$ (note that we have plotted $-\sigma_{TT'}$), except for the $K^0 \Lambda$ channel, where it produces a negative sign for the GDH integral of the neutron, thus yielding positive values for $\kappa^2$ of the neutron, albeit $\gamma n \rightarrow K^+ \Sigma^-$ and $\gamma n \rightarrow K^0 \Sigma^0$ channels show a different behavior.

In Table I we list the numerical values obtained both by Eqs. (1) and (3), using a cutoff energy where we found the elementary model is still reliable. It is found that the result is not sensitive to the cutoff energy $\nu_{\text{max}}$ around 2 GeV, i.e. there is no significant change in the integral in the energy interval $1.8 - 2.2$ GeV, especially in the case of photoproduction on the proton where the cross sections show a convergence at higher energies. From Table I it is already obvious that replacing Eq. (1) by Eq. (3) would overestimates the value of the GDH Integral, especially since we know that $\sigma_T$ is positive definite, while $\sigma_{TT'}$ is not.
FIG. 1. Total cross sections $\sigma_T$ (solid lines) and $-\sigma_{TT'}$ (dotted lines) for the six isospin channels plotted as a function of photon laboratory energy $\nu$ (GeV). The elementary model is from Ref. [7], experimental data are taken from Ref. [3], and references therein. The elementary model fits not only total cross section data shown in this figure, but also differential cross section and polarization data (not shown). Total cross sections for the $n(\gamma,K^0)\Lambda$ channel are scaled with a factor of $\frac{1}{2}$. 
TABLE I. Numerical values for the contribution of kaon-hyperon final states to the square of proton’s and neutron’s anomalous magnetic moments $\kappa^2_N(K)$. Column (1) is obtained from Eq. (4), while column (2) is evaluated by using Eq. (5).

| Channel          | $\kappa_p^2(K)$ | $\kappa_n^2(K)$ |
|------------------|-----------------|-----------------|
| $\gamma p \rightarrow K^+\Lambda$ | −0.026 | 0.075 |
| $\gamma p \rightarrow K^+\Sigma^0$ | −0.024 | 0.030 |
| $\gamma p \rightarrow K^0\Sigma^+$ | −0.013 | 0.019 |
| Total            | −0.063 | 0.031 |

We find that our present calculation yields a slightly different result for $\gamma p \rightarrow K^+\Lambda$ channel, but not in the $\gamma p \rightarrow K^+\Sigma^0$ channel, where previous work seems to overestimate the present calculation.

Should the contributions add up coherently, our calculation would yield values of $\kappa_p^2(K) = −0.063$ and $\kappa_n^2(K) = 0.031$, or $|\kappa_p(K)|/\kappa_p \leq 0.14$ and $\kappa_n(K)/\kappa_n \leq 0.094$. This puts even smaller values for the upper bound of the magnitude of kaon-hyperon final states contributions to the proton’s magnetic moment, compared to the previous result of HDM, $\kappa_p^2(K) = −0.07$ [4]. An interesting feature is that our calculation yields a positive value for contributions to the $\kappa_n^2(K)$, therefore increases the calculated value of the GDH Integral for the neutron.

In conclusion, we have refined the calculation of kaon-hyperon final states contributions to the anomalous magnetic moment of the proton and predicted the contributions for the case of the neutron, based on the experimental data of kaon photoproduction and a modern isobaric model. Experimental data for $\sigma_T$ in neutron’s channels and $\sigma_{TT'}$ in all six isospin channels will strongly suppress the uncertainties in our calculation. Therefore, future experimental proposals in MAMI, ELSA, TJNAF, or GRAAL should address this topic as an important measurement in order to improve our understanding of the nucleon’s structure.

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