Leptogenesis, mass hierarchies and low energy parameters

Werner Rodejohann*

Institut für Theoretische Physik, Universität Dortmund, Otto–Hahn–Str.4, 44221 Dortmund, Germany

Abstract

Leptogenesis in a left–right symmetric model is investigated for all possible neutrino mass hierarchies. The predictions of the model for low energy parameters as measured in neutrinoless double beta decay and in oscillation experiments are compared. The preferred values of the Majorana phases and limits on the smallest mass state are given. The main observation is that for the inverse hierarchy observable CP violation in oscillation experiments as well as a sizable signal in neutrinoless double beta decay can be expected. In a degenerate scheme, one Majorana phase is bounded to be around $\pi/2$ or $\pi$, and this ambiguity can easily be tested through neutrinoless double beta decay. The dependence of the baryon asymmetry on the different “Dirac” and “Majorana” phases is analyzed and a possibility to avoid the gravitino problem is discussed.

*E-mail: rodejoha@xena.physik.uni-dortmund.de
1 Introduction

The connection of leptogenesis [1] with low energy parameters has been investigated in a number of recent publications [2, 3, 4, 5]. Typically, a see–saw mechanism [6] connects the light neutrinos as indicated by oscillation experiments with the heavy Majorana neutrinos whose decay creates the observed baryon asymmetry of the universe. In [7] this very connection was analyzed within left–right symmetric models, for which a very simple connection between leptogenesis and neutrino oscillations was found [8]. This simple picture yields identical low and high energy sectors of the theory. Predictions of the scenario for $\langle m \rangle$, the effective Majorana mass of the electron neutrino, were given. Its predicted value lies around $10^{-3}$ eV, as expected in a normal hierarchical neutrino mass scheme. The allowed values of the $CP$ violating phases in the mixing matrix were investigated in [10].

In the present note we generalize these previous works by including also the inverse and degenerate mass hierarchies. In addition, we use a more appropriate fit [3] for the solution of the Boltzmann equations. Apart from the fact that in the inverse hierarchy $\langle m \rangle$ is now considerably larger than in the normal hierarchical scheme, the predictions for other low energy parameters differ, be it the preferred value of the leptonic Jarlskog parameter or the lower limit on the smallest mass eigenstate. The identical low and high energy sectors of the theory allow to show the resonance effect for degenerate neutrinos in a very simple manner. A two–fold ambiguity of the value of one Majorana phase is observed, which can easily be resolved through neutrinoless double beta decay. Finally, the gravitino problem is commented on and a possibility to avoid it in our scenario is discussed.

The paper is organized as follows: In Section 2 we shortly review the formalism of leptogenesis in left–right symmetric models. We then estimate the baryon asymmetry in Section 3 in both the normal and inverse hierarchy as well as for degenerate neutrinos. The full numerical results are given in Section 4 and we finally conclude in Section 5.

2 The model and leptogenesis

The gauge group of left–right symmetric models is $SU(2)_L \times SU(2)_R \times U(1)_{B–L}$ and the leptonic mass term reads:

$$\mathcal{L} = \bar{\psi}_{\alpha L} h_{\alpha \beta} \Phi \psi_{\beta R} + f_{\alpha \beta} \left[ \bar{\psi}_{\alpha L} \epsilon \bar{\Delta}_L \bar{\tau} \psi_{\beta L} + (L \rightarrow R) \right].$$

(1)

Here, $\psi_{\alpha L}$ ($\psi_{\alpha R}$) are the left– (right–)handed lepton doublets, $\Phi$ a Higgs bi–doublet and $\Delta_{L,R}$ are Higgs triplets. The Yukawa coupling matrices are denoted by $f$ and $h$. The presence of two Higgs triplets maintains the left–right symmetry and results in a type II see–saw mechanism. Symmetry breaking is achieved by receiving the following vacuum expectation values of the Higgses:

$$\langle \Phi \rangle = \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix} \quad \text{and} \quad \langle \Delta_{L,R} \rangle = \begin{pmatrix} 0 & 0 \\ 0 & v_{L,R} \end{pmatrix}.$$  

(2)

\footnote{See, e.g., [3] for the possibility of baryogenesis in left–right symmetric models.}
The light and heavy neutrino masses are then obtained by diagonalising
\[
\begin{pmatrix}
    m_L & \tilde{m}_D \\
    \tilde{m}_D^T & M_R
\end{pmatrix},
\]  
(3)
where \(m_L = f v_L\) and \(M_R = f v_R\) is a left–handed (right–handed) Majorana and \(\tilde{m}_D = h \kappa \simeq h v\) the Dirac mass matrix, which in this scenario is identical to the charged lepton matrix \(m_{lep}\). The weak scale is \(v = 174\) GeV. Diagonalization yields
\[
m_\nu = m_L - \tilde{m}_D M_R^{-1} \tilde{m}_D^T.
\]  
(4)
Since \(v_L v_R = \gamma v^2\) with \(\gamma = \mathcal{O}(1)\), it follows that \(M_R = \frac{v_R}{v_L} m_L \simeq \frac{v_R}{v_L} m_\nu\),

\[
i.e.,\ the\ low\ energy\ mass\ matrix\ is\ identical\ to\ the\ high\ energy\ matrix,\ thus\ the\ mass\ spectra\ are\ the\ same\ at\ both,\ the\ see–saw\ and\ the\ low\ energy\ scale.\ The\ matrix\ \(m_\nu\)\ is\ further\ diagonalized\ by
\[
U_L^T m_\nu U_L = \text{diag}(m_1, m_2, m_3).
\]  
(6)
\(U_L\) is therefore identical to the matrix that diagonalizes \(M_R\), whose eigenvalues are needed to compute the decay asymmetry. This asymmetry is caused by the interference of tree level with one–loop corrections for the decays of the heavy Majorana neutrinos, \(N_i \rightarrow \phi l^c\) and \(N_i \rightarrow \phi^\dagger l\):
\[
\varepsilon_i = \frac{\Gamma(N_i \rightarrow \phi l^c) - \Gamma(N_i \rightarrow \phi^\dagger l)}{\Gamma(N_i \rightarrow \phi l^c) + \Gamma(N_i \rightarrow \phi^\dagger l)}
= \frac{1}{8\pi v^2} \frac{1}{(m_D m_D)_{ii}} \sum_{j \neq i} \text{Im}(m_D^i m_D^j)^2 \left( f(M_j^2/M_i^2) + g(M_j^2/M_i^2) \right).
\]  
(7)
The contribution of the heavier neutrinos is washed out and only the asymmetry generated by the decay of the lightest one (i.e., \(M_1\) in normal hierarchies) survives. The Dirac mass matrix has been rotated by \(U_L\), thus changes to \(m_D = \tilde{m}_D U_L\). The function \(f\) stems from vertex \([1]\) and \(g\) from self–energy \([2, 3]\) contributions:
\[
f(x) = \sqrt{x} \left( 1 - (1 + x) \ln \left( \frac{1 + x}{x} \right) \right), \quad g(x) = \frac{\sqrt{x}}{1 - \frac{x}{1 + x}}.
\]  
(8)
For \(x \gg 1\) it holds \(f(x) + g(x) \simeq -\frac{3}{2\sqrt{x}}\). The resonance for degenerate neutrinos with \(x \simeq 1\) can be corrected to a order unity \(\varepsilon_i\) with a resummation formalism, which will be commented on below. Typically, one expects \(v_R = \mathcal{O}(10^{15})\) GeV and thus \(v_L\) of order of the light neutrino masses \(m_i \lesssim 0.05\) eV in hierarchical schemes.

The decay asymmetry \((7)\) is converted into a baryon asymmetry via\([4, 5]\)
\[
Y_B = c \frac{\kappa}{g^*} \varepsilon_1,
\]  
(9)
\(\text{The corresponding formula for supersymmetric scenarios is approximately the same, since both, } g^* \text{ and } \varepsilon_1, \text{ are enhanced by roughly a factor two.}\)
with $c = -28/51$, $g^* \simeq 110$ and $\kappa$ a dilution factor, which can be obtained through solving the Boltzmann equations [4]. There exists a convenient fit [3], which takes into account the suppression of $\kappa$ for large $M_1 \gtrsim 10^{14}$ GeV and small (large) $\tilde{m}_1 \lesssim 10^{-5}$ eV ($\tilde{m}_1 \gtrsim 10^{-2}$ eV), where $\tilde{m}_i$ is defined as
\[
\tilde{m}_i = \frac{(m_D^\dagger m_D)_ii}{M_i} = 3.0 \cdot 10^{-5} \gamma \left( \frac{(m_D^\dagger m_D)_ii}{\text{GeV}^2} \right) \left( \frac{10^{-3} \text{eV}}{m_i} \right) \left( \frac{10^{15} \text{GeV}}{v_R} \right)^2 \text{eV}.
\] (10)

In addition, the heavy Majorana masses are given by
\[
M_i = 3.3 \cdot 10^{13} \frac{1}{\gamma} \left( \frac{m_i}{10^{-3} \text{eV}} \right) \left( \frac{v_R}{10^{15} \text{GeV}} \right)^2 \text{GeV}.
\] (11)

The typical numbers one expects are $\kappa \simeq 10^{-1} \ldots 10^{-3}$ and $|\epsilon_1| \simeq 10^{-7} \ldots 10^{-5}$, which lead to the experimentally observed value [15] $Y_B \simeq 10^{-11} \ldots 10^{-10}$. Since in our scenario $(m_D^\dagger m_D)_ii$ is of order GeV$^2$, we have for $m_1 = 10^{-3}$ eV and the “natural parameters” $v_R = 10^{15}$ GeV and $\gamma = 1$ the values $M_1 \simeq 10^{13}$ GeV and $\tilde{m}_1 \simeq 10^{-5}$ eV. This leads to values of the Yukawa couplings of $f \simeq m_\nu/v_L \simeq 0.1 \ldots 1$. We will discuss the case of larger and smaller values of $m_i$ later.

### 3 Estimating the baryon asymmetry

We use the following parametrisation of $U_L$:
\[
U_L = U_{\text{CKM}} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)})
\]
\[
= \begin{pmatrix}
c_1c_3 & s_1c_3 & s_3e^{-i\delta} \\
-s_1c_2 - c_1s_2s_3e^{i\delta} & c_1c_2 - s_1s_2s_3e^{i\delta} & s_2c_3 \\
s_1s_2 - c_1c_2s_3e^{i\delta} & -c_1s_2 - s_1c_2s_3e^{i\delta} & c_2c_3
\end{pmatrix} \text{diag}(1, e^{i\alpha}, e^{i(\beta+\delta)}),
\] (12)

where $c_i = \cos \theta_i$ and $s_i = \sin \theta_i$. The “Dirac phase” $\delta$ appears in terrestrial lepton flavor violating processes, whereas the “Majorana phases” $\alpha$ and $\beta$ show up in lepton number violation, e.g., in neutrinoless double beta decay ($0\nu\beta\beta$).

The values of $\theta_2$ and $\Delta m^2_\alpha$ are known to a good precision, corresponding to maximal mixing $\tan^2 \theta_2 \simeq 0.5 \ldots 2$ and $\Delta m^2_\alpha \simeq (1 \ldots 5) \cdot 10^{-3}$ eV$^2$ [8]. Regarding solar neutrinos, two solutions are presently favored, the Large Mixing Angle (LMA) solution with $\tan^2 \theta_1 \simeq 0.2 \ldots 0.8$ and $\Delta m^2_\odot \simeq (1 \ldots 20) \cdot 10^{-5}$ eV$^2$ and the low $\Delta m^2_\odot$ (LOW) solution with $\tan^2 \theta_1 \simeq 0.5 \ldots 1$ and $\Delta m^2_\odot \simeq (0.3 \ldots 5) \cdot 10^{-7}$ eV$^2$ [17]. Latest data strongly favors the LMA solution [19]. The third angle $\theta_3$ is bounded to be smaller than about 15 degrees [20]. Typical best-fit points are mostly connected with very small $\theta_3$ and are $\Delta m^2_\odot = 2.5 \cdot 10^{-3}$ eV$^2$, $\tan^2 \theta_2 = 1$, $\Delta m^2_\odot = 4.5 \cdot 10^{-5}$ eV$^2$ and $\tan^2 \theta_1 = 0.4$ for LMA or $\Delta m^2_\odot = 1 \cdot 10^{-7}$ eV$^2$ and $\tan^2 \theta_1 = 0.7$ for LOW.

In the normal mass scheme the eigenstates are ordered as $m_3 > m_2 > m_1$ and are given as
\[
m_3 = \sqrt{\Delta m^2_\odot + m_1^2} \quad \text{and} \quad m_2 = \sqrt{\Delta m^2_\odot + m_1^2}.
\] (13)
In a strong hierarchy it holds $m_3 \simeq \sqrt{\Delta m_A^2} \gg m_2 \simeq \sqrt{\Delta m_{\odot}^2} \gg m_1$. In the inverse scheme one has $m_1 > m_2 > m_3$ with

$$m_1 = \sqrt{\Delta m_A^2 + m_3^2} \quad \text{and} \quad m_2 = \sqrt{-\Delta m_{\odot}^2 + m_1^2}. \quad (14)$$

In case of hierarchical masses, $m_1 \simeq m_2 \simeq \sqrt{\Delta m_A^2} \gg m_3$. We will now estimate the leptogenesis parameters with the simplifications $\tilde{m}_D = \text{diag}(0, 0, m_\tau)$, $\theta_2 = \pi/4$ and keeping only the leading order in $s_3$. This has been shown to be an excellent approximation within this scenario [8].

### 3.1 Normal hierarchy

Using the above mentioned simplifications together with Eqs. (5,7,8) we find for the decay asymmetry

$$\varepsilon_1 = -\frac{m_\tau^2}{8 \pi v^2} \frac{1}{t_1 - 2 s_3 c_3} \left[ \frac{1}{1 + t_1^2} \left( t_1 s_{2\alpha} + 2 s_3 (t_1^2 s_{2\alpha+\delta} - s_{2\alpha-\delta}) \right) \frac{m_1}{\sqrt{\Delta m_{\odot}^2}} + \left( t_1 s_{2(\beta+\delta)} - 2 s_3 s_{2\beta} \right) \frac{m_1}{\sqrt{\Delta m_A^2}} \right], \quad (15)$$

with $s_{2\alpha+\delta} = \sin 2\alpha + \delta$ and so on. Setting $t_1^2 = 1$ one obtains the formulas from [8]. The parameter $\tilde{m}_1$ reads

$$\tilde{m}_1 = 4.8 \cdot 10^{-5} \gamma \left( \frac{10^{15} \text{GeV}}{v_R} \right)^2 \left( \frac{10^{-3} \text{eV}}{m_1} \right) \frac{t_1}{1 + t_1^2} (t_1 - 2 s_3 c_3) \text{ eV}. \quad (16)$$

For the best–fit values mentioned in the last section, the dilution factor is $\kappa \simeq 0.04$ for $m_1 = 10^{-4}$ eV and the decay asymmetry simplifies to

$$-\varepsilon_1 \simeq \begin{cases} 4.4 \cdot 10^{-7} \left( s_{2\alpha} + 0.2 s_{2(\beta+\delta)} \right) \left( \frac{m_1}{10^{-4} \text{eV}} \right) & \text{LMA} \vspace{1em} \\ 7.7 \cdot 10^{-7} s_{2\alpha} \left( \frac{m_1}{10^{-4} \text{eV}} \right) & \text{LOW} \end{cases}. \quad (17)$$

Therefore, for LMA (LOW) values of $m_1$ around between $10^{-5}$ to $10^{-3}$ eV ($10^{-6}$ to $10^{-4}$) eV are required in order to produce a sufficient $\varepsilon_1$. The smallness of $\Delta m_{\odot}^2$ for the LOW solution leads to an almost vanishing contribution of $\Delta m_A^2$ to $\varepsilon_1$ and a larger asymmetry. It is seen that for large values of $m_1$ the decay asymmetry increases. For $m_1 = 10^{-4}$ eV one finds that $Y_B \simeq 10^{-11} (s_{2\alpha} + 0.2 s_{2(\beta+\delta)})$ for LMA and $Y_B \simeq 2 \cdot 10^{-10} s_{2\alpha}$ for LOW. Therefore, values of $\alpha$ around $(2n + 1)\pi/4$ are required to produce a sufficient asymmetry, which will be confirmed below. Values of $m_1$ lower than $10^{-5}$ ($10^{-6}$) eV for the LMA (LOW) solution...
render $\varepsilon_1$ too small and represent a rough lower limit on the smallest neutrino mass $[3]$. For identical masses the ratio of $Y_B$ for the two solutions reads

$$\frac{Y_B^{\text{LOW}}}{Y_B^{\text{LMA}}} \bigg|_{s_3^2=0, t_1^2=1} \approx \frac{r_\odot}{1 + s_2(\beta+\delta)/s_2 \alpha r^{\text{LMA}}} \approx r_\odot,$$  \hfill (18)

with $r_\odot^2 = (\Delta m^2_\odot)^{\text{LMA}}/(\Delta m^2_\odot)^{\text{LOW}} \gg r^2_{\text{LMA}} = (\Delta m^2_\odot)^{\text{LMA}}/\Delta m^2_\odot \ll 1$. Thus, the baryon asymmetry for the LMA solution is smaller by the square root of the ratio of the solar mass scales.

### 3.2 Inverse hierarchy

In the inverse hierarchy the lightest neutrino is now $M_3$. There is no contribution from the solar scale $\Delta m^2_\odot$ to the decay asymmetry:

$$\varepsilon_3 = -\frac{m_2^2}{8 \pi \mu^2} \frac{m_3}{\Delta m^2_A} \frac{1}{1 + t_1^2} \left[ s_2(\alpha - \beta + \delta) - t_1 (s_1 s_2(\beta+\delta) - 4 s_3 s_3 c_\delta + 2 \beta - \alpha) \right],$$  \hfill (19)

We will comment below on the potential enhancement of the decay asymmetry due to the degenerate masses $M_1$ and $M_2$. For our best–fit values this simplifies to

$$- \varepsilon_3 \simeq \begin{cases} 5.9 \cdot 10^{-8} \left( s_2(\alpha - \beta + \delta) - 0.4 s_2(\beta+\delta) \right) \left( \frac{m_3}{10^{-3} \text{ eV}} \right) & \text{LMA} \\ 2.1 \cdot 10^{-8} \left( s_2(\alpha - \beta + \delta) - 0.7 s_2(\beta+\delta) \right) \left( \frac{m_3}{10^{-3} \text{ eV}} \right) & \text{LOW} \end{cases}$$  \hfill (20)

being one order of magnitude below the values for the normal hierarchy. A smaller range for $m_3$ than in the normal hierarchy is found, values around $10^{-3}$ to $10^{-2}$ eV are now required: we find for $\tilde{m}_3$ that

$$\tilde{m}_3 = \tilde{m}_1 \frac{1 + t_1^2}{t_1} \frac{1}{t_1 - 2 s_3 c_\delta},$$  \hfill (21)

which is of the same order of magnitude as $\tilde{m}_1$. Therefore, the lower limit on the smallest mass eigenstate, for which $\varepsilon_3$ becomes too small, is roughly $10^{-4}$ eV. For $m_3 = 10^{-3}$ eV, $\kappa$ is about 0.006 and for $t_1^2 = 1$ one finds that $Y_B \simeq 4 \cdot 10^{-11} c_\alpha s_\alpha - 2(\beta+\delta)$. Therefore, for large $t_1^2$ values of $\alpha \simeq n\pi$ are favored, as will be confirmed later on. If we assume identical smallest mass states and $\kappa$, the fraction of the baryon asymmetry in the two hierarchies is

$$\frac{Y_{B}^{\text{norm}}}{Y_{B}^{\text{inv}}} \bigg|_{s_3^2=0, t_1^2=1} \approx \frac{s_2 \tau + 2 s_2(\beta+\delta)}{2 s_3 c_\delta + 2 \beta - \alpha},$$  \hfill (22)

where $r^2 = \Delta m^2_A/\Delta m^2_\odot \gg 1$. Thus, for comparable phases and masses, the baryon asymmetry in the normal hierarchical scheme is larger by the square root of the ratio of the atmospheric and solar mass scales.
3.3 Degenerate neutrinos

The question of leptogenesis with degenerate neutrinos has been addressed in the past in different models [21]. As seen from Eqs. (10,11), it seems difficult to have degenerate neutrinos \( m_i \gtrsim 0.1 \text{ eV} \) in our scenario, because the resulting large \( M_i \) and small \( \tilde{m}_i \) lead to a strong suppression of \( \kappa \). It is however possible to decrease \( M_i \) by changing \( \gamma \) and \( v_R \), which would then increase \( \tilde{m}_i \propto 1/M_i \). As an example, consider \( m_i = 1 \text{ eV} \), which for our “natural parameters” leads to \( M_i \approx 10^{16} \text{ GeV} \) and \( \tilde{m}_i \approx 10^{-8} \text{ eV} \). If one now decreases \( M_i \) by choosing \( v_R = 10^{13} \text{ GeV} \) and \( \gamma = 10 \), then one has \( M_i \approx 10^{11} \text{ GeV} \) and \( \tilde{m}_i \approx 10^{-4} \text{ eV} \), which are again acceptable values. The Yukawa couplings turn out to remain basically unchanged, \( f \approx m_\nu/v_L \approx 0.1 \ldots 1 \). The usual fit of the wash–out parameter \( \kappa \) assumes hierarchical neutrinos. We will therefore only consider the decay asymmetry, which for degenerate neutrinos displays a resonance behavior. For \( M_i^2/M_f^2 \approx 1 \) the vertex part of the decay asymmetry is of order \( (8 \pi v^2)^{-1} \approx 10^{-6} \). The precise form of \( \varepsilon_i \) for the self–energy part reads [13]

\[
\varepsilon_i = \sum_{j \neq i} \frac{\text{Im}(m^\dagger_D m_D)_{ij}}{(m_D^\dagger m_D)_{ii} (m_D^\dagger m_D)_{jj}} \frac{\Delta M^2_{ij}}{(\Delta M^2_{ij})^2 + M^2_i \Gamma^2_j} \equiv \sum_{j \neq i} \phi_{ij} \mu_{ij},
\]

(23)

where we separated the two fractions and introduced the decay width

\[
\Gamma_i = \frac{(m_D^\dagger m_D)_{ii}}{8 \pi v^2} M_i.
\]

(24)

For \( s_3 = 0 \) we find that \( \phi_{12} = s_{2\alpha}(1 + t_1^2) \), \( \phi_{13} = s_{2(\beta+\delta)} \) and \( \phi_{23} = s_{2(\delta-\alpha-\beta)} \). Using (1) and introducing \( c_i = (m_D^\dagger m_D)_{ii}/(8 \pi v^2) \), which in our scenario is \( \mathcal{O}(10^{-6}) \) to an excellent approximation, one finds

\[
\mu_{ij} = \frac{\Delta m^2_{ij} m_i m_j c_j}{(\Delta m^2_{ij})^2 + m_i^2 m_j^2 c_j^2} \approx \frac{10^6 \Delta m^2_{ij}}{m_0^2 \left( \frac{\Delta m^2_{ij}}{m_0^2} \right)^2 + 1},
\]

(25)

where the degenerate mass \( m_i \approx m_j \equiv m_0 \) was introduced. If \( \Delta m^2 \) is the atmospheric scale, then \( \mu_{ij} \) is of order \( 10^{-6} \) \( (10^{-4}) \) for \( m_0 = 0.1 \) \( (1) \) eV. If the scale is \( (\Delta m^2_{\text{LMA}}) \approx 10^{-5} \text{ eV}^2 \), then \( \mu_{ij} \approx 10^{-4} \) \( (10^{-2}) \) for \( m_0 = 0.1 \) \( (1) \) eV. If \( (\Delta m^2_{\text{LOW}}) \approx 10^{-7} \text{ eV}^2 \), then \( \mu_{ij} \approx 10^{-1} \) independent of \( m_0 \). Therefore, the self–energy part is never larger than order one. In the normal hierarchy we find for \( m_0 = 0.1 \text{ eV} \) that

\[
10^6 |\sum \varepsilon_i| \approx \begin{cases} 
10^2 s_{2\alpha}(1 + t_1^2) + 2 c_i s_{2(\beta+\delta)-\alpha} & \text{LMA} \\
10^5 s_{2\alpha}(1 + t_1^2) + 2 c_i s_{2(\beta+\delta)-\alpha} & \text{LOW}
\end{cases}
\]

(26)

and for \( m_0 = 1 \text{ eV} \)

\[
10^6 |\sum \varepsilon_i| \approx \begin{cases} 
10^4 s_{2\alpha}(1 + t_1^2) + 2 \cdot 10^2 c_i s_{2(\beta+\delta)-\alpha} & \text{LMA} \\
10^5 s_{2\alpha}(1 + t_1^2) + 2 \cdot 10^2 c_i s_{2(\beta+\delta)-\alpha} & \text{LOW}
\end{cases}
\]

(27)
Note that $\sum \epsilon_i$ can not be of order one for the LMA solution. If the wash–out factor $\kappa$ is around $10^{-1} \ldots 10^{-3}$ as for the hierarchal scheme, the numbers in the right–hand side of the last two equations have to be smaller or of the order 1 or 0.1. The first term proportional to $s_{2\alpha}$ dominates and can be made small for $\alpha \simeq n\pi/2$, which either minimizes or maximizes the second term proportional to $c_\alpha$. This demands fine–tuning of $\alpha$ to a precision of $10^{-3}$ to $10^{-5}$. The phase has to be closer to $n\pi/2$ for the LOW solution and for larger $m_0$. The ambiguity in $\alpha$ can be tested in neutrinoless double beta decay since for $t_3^2 \simeq 0$ the effective mass reads

$$\langle m \rangle \simeq \frac{m_0}{1 + t_1^2} \sqrt{(1 + t_1^2)^2 - 4 t_1^2 s_\alpha^2} \simeq m_0 c_\alpha,$$

(28)

where the last approximation corresponds to $t_1^2 \simeq 1$. Thus, the cases $\langle m \rangle \ll m_0$ and $\langle m \rangle \simeq m_0$ correspond to $\alpha \simeq \pi/2$ and $\alpha \simeq \pi$, respectively. Forthcoming experiments can very well test values of $\langle m \rangle$ below or around 0.01 eV.

In the last section the potential enhancement of the asymmetry in the inverse hierarchy has been mentioned. With the formalism discussed in this section, one can calculate now the contribution of the vertex contribution to the decay asymmetry from the Majorana neutrinos $M_1$ and $M_2$. We find that

$$\left| \sum \epsilon_i \right|_{M_1-M_2} \simeq 10^6 s_{2\alpha} \frac{r^2}{10^{12} + r^4},$$

(29)

with $r^2$ defined after Eq. (22). For LMA, this number is of order $10^{-6} s_{2\alpha}$ and for the disfavored LOW solution about $10^4$ times this value. One would find again that $\alpha \simeq n\pi/2$, which again is testable in neutrinoless double beta decay: for $t_3^2 m_3 \simeq 0$ and $t_1^2 \simeq 1$ the effective mass reads

$$\langle m \rangle \simeq \sqrt{\Delta m_A^2} c_\alpha.$$

(30)

Thus, the cases $\langle m \rangle \ll \sqrt{\Delta m_A^2}$ and $\langle m \rangle \simeq \sqrt{\Delta m_A^2}$ correspond again to $\alpha \simeq \pi/2$ and $\alpha \simeq \pi$, respectively. Since the masses $M_1$ and $M_2$ are heavier then $M_3$, the asymmetry caused by them suffers an additional reduction, we can for the inverse hierarchy and the strongly favored LMA solution safely work with the conventional form of $\epsilon_3$ as in Eq. (13).

For very small masses of $m_i \lesssim 10^{-8}$ eV, which lead to large $\tilde{m}_i$ and small $M_i$, it is possible to overcome this problem by increasing $v_R$ and decreasing $\gamma$. However, since $m_\nu \simeq 10^{-3}$ eV, the resulting values of the Yukawa couplings $f$ become now too large and spoil the naturalness of the model. We will therefore not discuss this possibility.

4 Numerical Results

We will now analyze the predictions of our scenario for low energy observables. For $\tilde{m}_D$ we took a typical charged lepton mass matrix

$$\tilde{m}_D = \begin{pmatrix} 0 & \sqrt{m_e m_\mu} & 0 \\
\sqrt{m_e m_\mu} & m_\mu & \sqrt{m_\tau m_\mu} \\
0 & \sqrt{m_\tau m_\mu} & m_\tau \end{pmatrix},$$

(31)
where \( m_{e,\mu,\tau} \) are the masses of the electron, muon and tau lepton. Fig. 1 shows \( Y_B \) as a function of the smallest mass state for the best-fit values of the oscillation parameters and \( t_3^2 = 0.005 \). In the inverse hierarchy we took the LMA solution to avoid the resonant enhancement as discussed above. The preferred value of the smallest mass is larger in the inverse hierarchy and larger for the LMA solution. Indicated in the plot are typical experimental values of \( Y_B \approx (1.7 \ldots 8.1) \cdot 10^{-11} \). For the following plots we fixed the smallest mass states to \( 10^{-4} \) \((10^{-3}) \) eV for LOW (LMA and inverse hierarchy).

In Fig. 2 we display scatter plots of the two Majorana phases \( \alpha \) and \( \beta \) for both schemes and solutions. The points are obtained by producing random values of the oscillation parameters in the range given above and also varying the three phases between zero and \( 2\pi \). When a sufficient baryon asymmetry is produced, the point is kept. Due to the smallness of \( t_2^3 \), \( \beta \) is basically a free parameter. The second phase \( \alpha \) lies around \( \pi/4 \) or \( 5\pi/4 \) in the normal hierarchy\(^3\) and around 0, \( \pi \) or \( 2\pi \) in the inverse hierarchy. These values confirm our estimates in Section 3. In the normal hierarchy, \( \langle m \rangle \) is a function of \( (\alpha - \beta) \), to be precise:

\[
\langle m \rangle^2 \simeq \Delta m^2 s_1^4 + \Delta m^2 t_3^4 + 2 s_1^2 t_3^2 \sqrt{\Delta m^2 \Delta m^2} e_{2(\alpha - \beta)} .
\]  

(32)

For sizable \( \langle m \rangle \) and a sizable dependence of \( \langle m \rangle \) on the phases large values of \( t_3^2 \) are required. It turns out that \( \alpha = \pi/4 \) or \( 5\pi/4 \) yield similar results for \( \langle m \rangle \) and that for large \( t_3^2 \) values of \( \beta \approx 3\pi/4 \) lead to unobservable \( \langle m \rangle \). See [10] for details.

In the inverse hierarchy however, the preferred values of \( \alpha \) are of particular interest for \( 0\nu\beta\beta \). From Eq. (30) it is seen that for values of \( \alpha = n\pi \) there are no cancellations in \( \langle m \rangle \) and it holds \( \langle m \rangle \simeq \sqrt{\Delta m^2} \). This is shown in Fig. 3, where scatter plots of \( t_3^2 \) and \( \langle m \rangle \) are shown. Practically all points for the inverse hierarchy lie above 0.02 eV, a value observable by future experiments [22]. In the normal hierarchy, approximately half of the points lie above 0.002 eV, which is a very ambitious limit planned to be achieved by the GENIUS experiment [23]. From the figure it becomes also clear, that the inverse hierarchy prefers large values of \( t_3^2 \), since most of the points lie above 0.01.

Finally, in Fig. 4 we display \( \Delta m^2_{\odot} \) against the \( CP \) violating leptonic Jarlskog invariant \( J_{CP} \), which is defined as

\[
J_{CP} = \frac{1}{8} \sin 2\theta_1 \sin 2\theta_2 \sin 2\theta_3 \cos \theta_3 \sin \delta
\]  

(33)

and may be measured in next generation neutrino experiments [24]. Necessary conditions are that LMA is the solution of the solar neutrino problem and that \( \Delta m^2_{\odot} \) is not too small. It is seen from the plots that there is a slight preference for large \( \Delta m^2_{\odot} \) in the normal hierarchy and a very strong one in the inverse hierarchy. Also, due to the large \( t_3^2 \) values, \( J_{CP} \) is larger in the inverse hierarchy.

\(^3\)For fixed values of the oscillation parameters the values of the phases can be different for the respective solutions, see [10]. Note however, that the dependence on \( m_1 \) of \( \kappa \) has not been fully taken into account in that analysis.
It is an interesting question to ask how the baryon asymmetry depends on the two different kinds of phases, the Dirac phase $\delta$ and the Majorana phases $\alpha$ and $\beta$. In the minimal $SO(10)$ model analyzed in [4] it has been observed that a sufficient baryon asymmetry can be produced alone with one single Majorana phase, whereas the Dirac phase alone is not enough. Reference [5], analyzing the minimal supersymmetric see–saw model, observes that leptogenesis is insensitive to the Dirac phase. From Eq. (15) one notes that in the normal hierarchy and the LOW solution, there is hardly any dependence on $\beta$ and $\delta$ and for the LMA solution the dependence is suppressed. The situation is thus similar to the one in [4]. In the inverse scheme and the LMA solution however, the phases can contribute comparably to $Y_B$, as can be seen from Eq. (19). Therefore, the Dirac phase alone is sufficient to produce the required amount of baryon asymmetry. In case of the LOW solution and also for the degenerate scheme, the Majorana phase $\alpha$ is dominant and the other phases play no role. It would be interesting to investigate this behavior in other models. Naively one expects minor dependence of the baryon asymmetry on $\delta$, since it appears in most parameterizations with the small quantity $s_3$. In addition, since leptogenesis requires lepton number violation, the Majorana phases can be expected to play the major role.

A final comment concerns the gravitino problem of leptogenesis, which appears once one embeds a theory in a supergravity framework. Majorana neutrinos with masses considerably below $10^{10}$ GeV can potentially evade this problem. The resulting smallness of $\kappa$ is fixed by the resonant behavior of $\varepsilon_i$. For instance, $M_i \simeq 10^8 \ (10^4)$ GeV together with $m_i \simeq 1$ eV leads to $v_R \simeq 10^{11} \ (10^9)$ GeV, $v_L \simeq 10^3 \ (10^5)$ eV and $f \simeq 10^{-4} \ (10^{-6})$, when $m_{\nu} \simeq 0.1$ eV is assumed. The mass $\tilde{m}_i$ will then be of the order of $1 \ (10^4)$ eV. Extrapolating $\kappa$ for Majorana masses of order $10^8$ GeV leads to $\kappa \simeq 10^{-6}$, thus $\sum \varepsilon_i \simeq 10^{-3} \ldots 10^{-2}$. Therefore, extreme fine–tuning of $\alpha \simeq \pi/2$ is required, leading to very small $\langle m \rangle$. Lower values of $M_i$ as in TeV scale leptogenesis [25] with high values of $\tilde{m}_i$ lead to extremely small values of $\kappa$ and demand $\sum \varepsilon_i$ to be of order one, in conflict with the favored LMA solution.

5 Conclusions

Leptogenesis in left–right symmetric models is investigated for all possible neutrino mass schemes. Depending on the solar solution and the mass scheme, the preferred value of the lowest mass state differs. In addition, predictions of low energy observables, such as $\langle m \rangle$ and $J_{CP}$ differ. Especially in the inverse hierarchy large values of $t_3^2$ are required, which predict observable $CP$ violation in oscillation experiments. Furthermore, only little cancellation in neutrinoless double beta decay is predicted, which leads to $\langle m \rangle \simeq \sqrt{\Delta m^2_{31}}$. The degenerate scheme predicts $\alpha$ to be around $\pi/2$ or $\pi$, which can easily be tested in next generation $0\nu\beta\beta$ experiments. The condition $\alpha \simeq \pi/2$ holds for the case of Majorana neutrinos with masses not much below $10^{10}$ GeV, as required in order to evade the gravitino problem.
Acknowledgments

I thank Anjan Joshipura for helpful comments and careful reading of the manuscript. This work has been supported by the “Bundesministerium für Bildung, Wissenschaft, Forschung und Technologie”, Bonn under contract No. 05HT1PEA9.

References

[1] For recent reviews see, e.g. A. Pilaftis, Int. J. Mod. Phys. A 14, 1811 (1999); W. Buchmüller, M. Plümacher, Int. J. Mod. Phys. A 15, 5047 (2000).

[2] For recent works, see e.g. G. Branco et al., Nucl. Phys. B 617, 475 (2001); W. Buchmüller, D. Wyler, Phys. Lett. B 521, 291 (2001); M.S. Berger, K. Siyeon, Phys. Rev. D 65, 053019 (2002); M. Fujii, K. Hamaguchi, and T. Yanagida, Phys. Lett. B 538, 107 (2002) H.B. Nielsen, Y. Takanishi, hep-ph/0204027; D. Falcone, hep-ph/0204333; S. Davidson, A. Ibarra, hep-ph/0206304.

[3] M. Hirsch, S.F. King, Phys. Rev. D 64, 113005 (2001).

[4] G. Branco et al., hep-ph/0202030.

[5] J. Ellis, M. Raidal, hep-ph/0206174.

[6] M. Gell–Mann, P. Ramond, and R. Slansky in Supergravity, p. 315, edited by F. Nieuwenhuizen and D. Friedman, North Holland, Amsterdam, 1979; T. Yanagida, Proc. of the Workshop on Unified Theories and the Baryon Number of the Universe, edited by O. Sawada and A. Sugamoto, KEK, Japan 1979; R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44, 912 (1980).

[7] A.S. Joshipura, E.A. Paschos, hep-ph/9906498; A.S. Joshipura, E.A. Paschos, and W. Rodejohann, Nucl. Phys. B 611, 227 (2001).

[8] A.S. Joshipura, E.A. Paschos and W. Rodejohann, JHEP 08, 029 (2001).

[9] R.N. Mohapatra, X. Zhang, Phys. Rev. D 46, 5331 (1992).

[10] K.R.S. Balaji, W. Rodejohann, Phys. Rev. D 65, 093009 (2002).

[11] M. Fukugita, T. Yanagida, Phys. Lett. B 174, 45 (1986); M.A. Luty, Phys. Rev. D 45, 455 (1992); M. Flanz, E.A. Paschos, and U. Sarkar, Phys. Lett. B 345, 248 (1995); W. Buchmüller, M. Plümacher, Phys. Lett. B 389, 73 (1996).

[12] L. Covi, E. Roulet, and F. Vissani, Phys. Lett. B 384, 169 (1996); M. Flanz et al., Phys. Lett. B 389, 693 (1996); L. Covi, E. Roulet, Phys. Lett. B 399, 113 (1997); W. Buchmüller, M. Plümacher, Phys. Lett. B 431, 354 (1998).
[13] A. Pilaftsis, Phys. Rev. D 56, 5431 (1997).

[14] See W. Buchmüller, M. Plümer in [12]; M. Flanz, E.A. Paschos, Phys. Rev. D 58, 113009 (1998).

[15] D.E. Groom et al., Eur. Phys. J. C 15, 1 (2000).

[16] S.M. Bilenky, J. Hosek, and S.T. Petcov Phys. Lett. B 94, 495 (1980); J. Schechter, J.W.F. Valle, Phys. Rev. D 22, 2227 (1980); Phys. Rev. D 23, 1666 (1980).

[17] J.N. Bahcall, M.C. Gonzalez–Garcia, and C. Peña–Garay, hep-ph/0204314.

[18] T. Toshito for the SuperKamiokande collaboration, hep-ex/0105023.

[19] SNO collaboration, Q.R. Ahmad et al., Phys. Rev. Lett. 89, 011301 (2002) and Phys. Rev. Lett. 89, 011302 (2002).

[20] S.M. Bilenky, S.T. Petcov, and D. Nicolo, Phys. Lett. B 538, 77 (2002).

[21] E.J. Chun, S.K. Kang, Phys. Rev. D 63, 097902 (2001); M. Fujii, K. Hamaguchi, and T. Yanagida, Phys. Rev. D 65, 115012 (2002); M. Fukugita, and T. Yanagida, hep-ph/0203194; J. Ellis, M. Raidal, and T. Yanagida, hep-ph/0206306.

[22] S.R. Elliot, P. Vogel, hep-ph/0202264.

[23] H.V. Klapdor–Kleingrothaus et al., hep-ph/9910205.

[24] P. Huber, M. Lindner, and W. Winter, hep-ph/0204352.

[25] T. Hambye, Nucl. Phys. B 633, 171 (2002) and references therein.
Figure 1: The baryon asymmetry as a function of the smallest mass state in both hierarchies for different representative values of the Majorana phases, the best-fit points as mentioned in the text and \( t^2 = 0.005 \). The solid line is for the LOW solution, the dotted for LMA and the dashed-dotted for the inverse hierarchy.

Figure 2: Scatter plot of the Majorana phases \( \alpha \) and \( \beta \) for both hierarchies.
Figure 3: Scatter plot of $\tan^2 \theta_3$ and $\langle m \rangle$ for both hierarchies in the LMA solution.

Figure 4: Scatter plot of $J_{CP}$ and $\Delta m^2_{\odot}$ for both hierarchies in the LMA solution.