Deconfinement transition for nonzero baryon density in the Field Correlator Method.

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Abstract

Deconfinement phase transition due to disappearance of confining colorelectric field correlators is described using nonperturbative equation of state. The resulting transition temperature $T_c(\mu)$ at any chemical potential $\mu$ is expressed in terms of the change of gluonic condensate $\Delta G_2$ and absolute value of Polyakov loop $L_{fund}(T_c)$, known from lattice and analytic data, and is in good agreement with lattice data for $\Delta G_2 \approx 0.0035 \text{ GeV}^4$. E.g. $T_c(0) = 0.27; 0.19; 0.17 \text{ GeV}$ for $n_f = 0, 2, 3$ respectively.

1 Introduction

Phase transition at nonzero $\mu$ and dynamics of quark gluon plasma (qgp) is now of great interest because of impressive results of heavy ion experiments, see [1] for a recent review and references. The topic calls for a nonperturbative (NP) treatment of QCD degrees of freedom at nonzero $T, \mu$, which is especially important for not very large $T, \mu$. Below we are using the NP approach based on Field Correlator Method (FCM) [2], which was applied to nonzero $T$ in [3, 4].

The main advantage of FCM is a natural explanation and treatment of dynamics of confinement, as well as the deconfinement transition [3, 4], in terms of Color Electric (CE) $D^E(x), D^E_1(x)$ and Color Magnetic (CM) Gaussian (quadratic in $F_{\mu\nu}$) correlators $D^H(x), D^H_1(x)$.
The correlators \( D^E(x) \) and \( D^H(x) \) ensure confinement in the planes \((4,i)\) and \((i,k)\) respectively, \(i,k=1,2,3\) so that standard string tension \( \sigma^E \equiv \sigma = \frac{1}{2} \int D^E(x) d^2x \), and spatial string tension \( \sigma^H \equiv \sigma_s = \frac{1}{2} \int D^H(x) d^2x \). Correlators \( D^E_1, D^H_1 \) contain perturbative series, and \( D^E_1 \) plays an important role in that it contributes to the modulus of the Polyakov line; at \( T \geq T_c \) one has \([5]\)

\[
L^{(V)}_{\text{fund}} = \left| \frac{1}{N_c} \langle tr P \exp i g \int_0^\beta A_4 dz_4 \rangle \right| = \exp \left( -\frac{V_1(\infty, T)}{2T} \right) \tag{1}
\]

with the static \( Q\bar{Q} \) potential

\[
V_1(r, T) = \int_0^\beta d\nu (1 - \nu T) \int_0^\beta \xi d\xi D^E_1(\sqrt{\xi^2 + \nu^2}). \tag{2}
\]

Deconfinement phase transition in this language is the disappearance of \( D^E(x) \) (and \( \sigma^E \)) at \( T \geq T_c \), while \( L^{(V)}_{\text{fund}} \) and \( L^{(V)}_{\text{adj}} = (L^{(V)}_{\text{fund}})^{9/4} \) are nonzero then \([4]\). However the disappearance of \( D^E(x) = \frac{1}{N_c} \langle tr E_i(x) \Phi(x,y) E_i(y) \rangle \) implies vanishing of a part of vacuum energy density,

\[
\varepsilon_{\text{vac}} \equiv \frac{\beta(\alpha_s)}{16 \alpha_s} \langle (F_{\mu\nu}^a)^2 \rangle = -\left( \frac{11 - \frac{2}{3} n_f}{32} \right) G_2, \quad D^E(0) + D_1^E(0) + D^H(0) + D_1^H(0) = \frac{\pi^2}{9} G_2
\]

and \( G_2 \) is the gluon condensate \([6]\). At \( T = 0 \), \( D^E = D^H, D_1^E = D_1^H \) and for \( T > 0 \) both \( D^H, D_1^H \) do not change till \( T \approx 2T_c \), while \( D^E \) disappears at \( T \geq T_c \)[7] in agreement with the deconfinement mechanism suggested in \([4]\).

Particle data \([7]\) and analytic study \([8]\) imply that \( D_{1,0}^{(E,H)}(x) \approx 0.2 D_{1,0}^{(E,H)}(x) \), therefore one expects that \( \Delta G_2 = G_2(T < T_c) - G_2(T > T_c) \approx \frac{1}{2} G_2(T < T_c) \approx \frac{1}{2} G_2^{stl} \), where \( G_2^{stl} \approx 0.012 \text{ GeV}^4 \) \([6]\) (see \([9]\) for a recent gauge-string duality treatment of \( G_2^{stl} \) yielding \( G_2^{stl} \approx (0.01 \pm 0.002) \text{ GeV}^4 \)).

This \( \Delta G_2 \) taken as the change of free energy (pressure) across the phase boundary will be our basic element in finding the phase transition curve \( T_c(\mu) \) below.

\footnote{The subscript \( V \) in \( L^{(V)}_{\text{fund}} \) is to distinguish from \( L^{(F)}_{\text{fund}} \) calculated on the lattice with singlet free energy \( F_{Q\bar{Q}}^1(\infty, T) \) replacing \( V_1(\infty, T) \) in \([11]\). It is clear that \( L^{(F)} \) contains all bound states in addition to the ground state (selfenergy) \( V_1(\infty, T) \), hence \( F_{Q\bar{Q}}^1 \leq V_1 \). We shall ignore the difference in the first approximation in what follows and write \( L_{\text{fund}} \).}
To this end we introduce in the next section the NP EoS of qgp derived recently in [10], and express $T_c(\mu)$ in terms of $\Delta G_2$ and $L_{fund}(T_c)$.

Taking for the latter the lattice or analytic value, one obtains a set of curves $T_c(\mu)$ for $n_f=0,2,3$ depending on the only parameter $\Delta G_2$. These resulting curves and their end points $T_c(0),\mu_c(0)$ are discussed in conclusion.

2. In the NP approach to the qgp in [10] one introduces in the first approximation the interaction of single quarks and gluons with the vacuum, which is called the Single Line Approximation (SLA), leaving pair and triple, etc... correlations to the next steps. As a result one obtains in SLA the pressure $P_{SLA}^{q}$ of quarks (and antiquarks) and $P_{SLA}^{g}$ of gluons which are expressed through $L_{fund}$, namely [10].

$$p_q \equiv \frac{P_{SLA}^{q}}{T^4} = \frac{4N_c n_f}{\pi^2} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^4} L_{fund}^n \varphi_q^{(n)}(\mu) e^{-\mu n T}$$

(4)

$$p_{gl} \equiv \frac{P_{SLA}^{gl}}{T^4} = \frac{2(N_c^2 - 1)}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^4} L_{adj}^n$$

(5)

with

$$\varphi_q^{(n)}(T) = \frac{n^2 m_q^2}{2T^2} K_2 \left( \frac{m_q T}{T} \right) \approx 1 - \frac{1}{4} \left( \frac{nm_q}{T} \right)^2 + ...$$

(6)

In (4), (5) it was assumed that $T \lesssim 1 \lambda \equiv 1$ GeV, where $\lambda$ is the vacuum correlation length, e.g. $D_1^{(E)}(x) \sim e^{-|x|/\lambda}$, hence powers of $L_i^n$, see [10] for details.

With few percent accuracy one can replace the sum in (5) by the first term, $n = 1$, and this form will be used below for $p_{gl}$, while for $p_q$ this replacement is not valid for large $\mu T$, and one can use instead the form equivalent to (4),

$$p_q = \frac{n_f}{\pi^2} \left[ \Phi_\nu \left( \frac{\mu - \frac{V_1}{2}}{T} \right) + \Phi_\nu \left( -\frac{\mu + \frac{V_1}{2}}{T} \right) \right]$$

(7)

where $\nu = m_q/T$ and

$$\Phi_\nu(a) = \int_0^\infty \frac{z^4 dz}{\sqrt{z^2 + \nu^2}} \frac{1}{(e^{\sqrt{z^2 + \nu^2} - a} + 1)}.$$

(8)

Eqs. (7), (5) define $p_q, p_{gl}$ for all $T, \mu$ and $m_q$, which is the current (pole) quark mass at the scale of the order of $T$. 

Using (4)-(8) we can define the pressure $P_I$ in the confined phase, and $P_{II}$ in the deconfined phase, taking into account that vacuum energy density in two phases $\varepsilon_{\text{vac}}$ and $\varepsilon_{\text{dec}}$ respectively contributes to the free energy, and hence $|\varepsilon_{\text{vac}}|, |\varepsilon_{\text{dec}}|$ to the pressure. One has

$$P_I = |\varepsilon_{\text{vac}}| + P_{\text{hadron}}, \quad P_{II} = |\varepsilon_{\text{dec}}| + (p_{gl} + p_q)T^4. \tag{9}$$

Here $P_{\text{hadron}}$ is the pressure of the hadron gas at $T \leq T_c$. From $P_I(T_c) = P_{II}(T_c)$ one obtains $T_c$, neglecting $P_{\text{hadron}}$ in the first approximation

$$T_c(\mu) = \left( \frac{(11 - \frac{2}{3}n_f)\Delta G_2}{32(p_{gl} + p_q)} \right)^{1/4}. \tag{10}$$

In (10) enter only two parameters; $\Delta G_2$ and $L_{\text{fund}}(T_c) = \exp\left(-\frac{\kappa}{T_c}\right)$, $\kappa \equiv \frac{1}{2}V_1(\infty, T_c) \approx \frac{1}{2}F_{QQ}(\infty, T_c)$.

The latter can be found in 3 different ways: 1) from the direct lattice measurements [11] of $P_{1QQ} \approx 0.5$ GeV; 2) from analytic calculation of $D^E_1$ in [8], which yields $V_1(\infty, T < T_c) \approx \frac{6\alpha_s(M_0)^{1/3}}{M_0} \approx 0.5$ GeV with $M_0 \approx (0.8 - 1)$ GeV lowest gluelump mass [12]; 3) from lattice calculations of $D^E_1$ at $T > T_c$, [7, 13], which according to [2] yields $V_1(\infty, T_c) \approx 0.4 \div 0.6$ GeV. Therefore one can fix $V_1(\infty, T_c) = 0.50(5)$ GeV ($\kappa = 0.25$ GeV) and this value is independent of $n_f$ [11]. As a result $T_c(\mu)$ is a function of only $\Delta G_2$ and for each value of $\Delta G_2$ one finds a set of curves for $n_f = 2, 3, ...$. We choose $\Delta G_2 \approx \frac{1}{2}G_{\Delta}^2$ and in Fig.1 the curves computed numerically from (10) for $n_f = 2, 3$ are shown for $\Delta G_2 \approx 0.00341$ GeV$^4$ and zero quark pole masses.

The end points $T_c(0)$ and $\mu_c(0)$ can be found analytically e.g. for $T_c(\mu)$ with $\sim 5\%$ accuracy one has (expanding (10) in $\frac{p_{gl}}{p_q}$)

$$T_c(\mu) = T_c(0) \left( 1 - C \frac{9\mu^2}{T_c^2(0)} \right), \quad C = 0.0110(3), \tag{11}$$

with

$$T_c(0) \approx \frac{1}{2}T(0) \left( 1 + \sqrt{1 + \frac{\kappa}{T(0)}} \right), \quad T(0) = \left( \frac{(11 - \frac{2}{3}n_f)\pi^2\Delta G_2}{384n_f} \right)^{1/4}. \tag{12}$$

For $\Delta G_2 = 0.00341$ GeV$^4$ one obtains $T_c(0) = (0.27; 0.19; 0.17)$ GeV for $n_f = 0, 2, 3$ respectively, which agrees well with numerous lattice data, see
Figure 1: The phase transition curve $T_c(\mu)$ from Eq. (10) (in GeV) as a function of quark chemical potential $\mu$ (in GeV) for $n_f = 2$ (upper curve) and $n_f = 3$ (lower curve) and $\Delta G_2 = 0.0034$ GeV$^4$.

[14] for reviews. The value of $C = 0.011$ is inside the scattered set of lattice values [14].

Another end point, $\mu_c(0)$ can be found from the asymptotics of Eq. (8), $\Phi_0(a \to \infty) = \frac{a^4}{4} + \frac{\pi^2}{2} a^2 + ...$, which yields (for $m_q = 0$) $\mu_c(0) = \kappa + (48)^{1/4} T(0)$ and for the same $\Delta G_2$ as above one gets $\mu_c(0) = (0.63; 0.58)$ GeV for $n_f = 2, 3$.

One can check, that the derivative in $T$, $\frac{d\mu_c(T)}{dT}$ vanishes at $T = 0$.

3. The phase curve $T_c(\mu)$ in Fig.1 is in reasonable agreement with lattice data at least for $\mu \lesssim 0.25$ GeV, see [15] for review and references. Two important points are to be discussed here: 1) order of transition and possible critical point 2) approximations and assumptions of the present work.

1). The vacuum transition of our approach is evidently of the first order at least in the leading (SLA) approximation used for (10), and does not contain any critical points. This is in agreement with lattice $n_f = 0$ data, but the lattice results for $n_f = 2, 3$ depend on masses, discretization and are not fully conclusive. The softening of transition for $n_f > 0$ in our approach is explained by the increasing role of $P_{\text{hadron}}(T)$ for $n_f > 0$ near $T_c$, which suppresses the specific heat and makes the curve $P(T)$ more smooth. The chiral transition in our approach is caused by the deconfinement, since both
\langle \bar{q}q \rangle \text{ and } f_\pi \text{ are expressed via } D^E(x) \text{ [16] and vanish together with it, in agreement with lattice data, see e.g. [14]. The Polyakov loop is a good (approximate) order parameter for } n_f = 0(n_f > 0) \text{ since at } T < T_c \text{ it is expressed via } D^E(x) \text{ and vanishes (strongly decreases) (see Eq. (6) of [5]).}

2) In our derivation of (10)-(12) it was assumed a) that the only important part of qgp dynamics is the interaction with the NP vacuum –SLA; b) it is assumed that vacuum fields do not depend on } T, \mu \text{ in the phase diagram, except at the phase boundary where the shift } \Delta G_2 \text{ occurs; in particular neither } \Delta G_2 \text{ nor } L_{fund}(T_c) \text{ depend on } \mu . \text{ The latter point is partly supported by lattice data [17]. In general this picture of rigid vacuum is based on the notion of the dilaton scale } m_d \text{ of vacuum fields, which can be associated with the } 0^{++} \text{ glueball mass around 1.5 GeV and therefore for all external parameters (like } \mu \text{ or } T \text{) much less than } m_d \text{ vacuum fields are fixed.}

Another argument in favor of rigid vacuum is that all dependence on } \mu \text{ and } n_f \text{ does not appear in the lowest order of } 1/N_c \text{ expansion, since it comes from the quark loops. (Note that nevertheless } T_c(0) \text{ differs strongly for } n_f = 0 \text{ and } n_f = 2, 3; \text{ even through } \Delta G_2 \text{ was kept fixed, and this successful prediction of } T_c(0) = 0.27 \text{ GeV and 0.19 GeV respectively can be considered as another support of our picture). Several things were not taken into account. Quark masses are included trivially via Eqs.(6,7) and this can be checked } vs \text{ lattice data. Phase transition near } \mu_c(0) \text{ can be complicated due to strong } qq \text{ and } qqq \text{ interaction, which is not taken into account above and will be discussed elsewhere (see also [10]). therefore the possibility of color superconductivity is not commented here.}

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