Ground-State Scalar $\bar{q}q$ Nonet: $SU(3)$ Mass Splittings and Strong, Electromagnetic, and Weak Decay Rates

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By comparing $SU(3)$-breaking scales of linear mass formulae, it is shown that the lowest vector and scalar mesons all have a $\bar{q}q$ configuration, while the ground-state octet and decuplet baryons are $qqq$. Also, the quark-level linear $\sigma$ model is employed to predict similar $\bar{q}q$ and $qqq$ states. Furthermore, the approximate mass degeneracy of the scalar $a_0(985)$ and $f_0(980)$ mesons is demonstrated to be accidental. Finally, it is shown that various strong, electromagnetic, and weak mesonic decay rates are successfully explained within the framework of the quark-level linear $\sigma$ model.

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I. INTRODUCTION

In the quark model, one usually assumes that pseudoscalar ($P$), and vector ($V$) mesons are $\bar{q}q$, whereas octet ($O$) and decuplet ($D$) baryons are $qqq$ states. However, it is now argued [1] that the light scalar ($S$) mesons are non-$\bar{q}q$ candidates, in view of their low masses and broad widths. In this paper, we shall show that the ground-state meson nonets $P$, $S$, and $V$ are all $\bar{q}q$, hence including the light scalars, while the lowest $O$ and $D$ baryons are $qqq$ states.

In Sec. 2, $SU(3)$ mass splittings for loosely bound $V$ and $S$ states are shown to have symmetry-breaking scales of 13% and 18%, respectively, using linear mass formulae. We apply the latter formulae to $qqq$ $O$ and $D$ states in Sec. 3, leading to $SU(3)$-breaking scales of 13% and 12%, respectively. Then in Sec. 4, we employ the quark-level linear $\sigma$ model ($\sigma$M) to predict similar $\bar{q}q$ and $qqq$ states as in Secs. 2 and 3. Next in Sec. 5, we study the $S$ $\bar{q}q$ states and argue why the $V$ states have slightly higher masses, on the basis of the nonrelativistic quark model. Moreover, the approximate mass degeneracy of the $S$ $a_0(985)$ and $f_0(980)$ mesons is shown to be just accidental. Finally, in Secs. 6, 7, and 8 we successfully determine, in an $\sigma$M framework, mesonic decay rates for strong, electromagnetic, and weak processes, respectively. In Sec. 9 we summarize our results and draw some conclusions.

II. MASS SPLITTINGS FOR $U(3) \times U(3)$ $V$ AND $S \bar{q}q$ MESONS

Although meson masses are expected to appear quadratically in model Lagrangians, while they must appear so for $P$ states [2], for $V$ and $S$ states a Taylor-series linear form for $SU(3)$ mass splittings is also possible. Thus, consider a Hamiltonian density $H = H(\lambda_0) + H_{ss}(\lambda_8)$ using Gell-Mann matrices. Then the vector-meson-nonet
masses $m_V = \sqrt{2/3} \ m_V^0 - d_{\delta t_i} \ \delta m_V$ are

$$m_{\rho, \omega} = \sqrt{\frac{2}{3}} \ m_V^0 - \frac{1}{\sqrt{3}} \ \delta m_V \approx 776 \text{ MeV},$$

$$m_{K^*} = \sqrt{\frac{2}{3}} \ m_V^0 + \frac{1}{2\sqrt{3}} \ \delta m_V \approx 894 \text{ MeV}, \quad (1)$$

$$m_\phi = \sqrt{\frac{2}{3}} \ m_V^0 + \frac{2}{\sqrt{3}} \ \delta m_V \approx 1020 \text{ MeV},$$

with $\phi \approx s\bar{s}$. Measured vector masses [1] suggest average mass splittings

$$m_V^0 \approx 1048 \text{ MeV}, \quad \delta m_V \approx 141 \text{ MeV}, \quad (2)$$

giving an $SU(3)$-breaking scale of $\delta m_V/m_V^0 \approx 13\%$.

Such considerations can be repeated for axial-vector mesons as well, even though it is now hard to draw any decisive conclusions, also in view of the experimental situation. This is why regarding these mesons we limit ourselves to the following observations. In the case of axial-vector $a_1$ states, we assume the $f_1(1420)$ is mostly $s\bar{s}$, because the PDG [1] reports $f_1(1285) \rightarrow K^0 K \pi$, $K^* K$ as dominant, while $f_1(1285)$ are almost absent. Thus, $f_1(1285)$ is mostly $\bar{n}n$, like the nonstrange $a_1(1260)$ (with $a_1 \rightarrow \sigma \pi$ seen, but $a_1 \rightarrow f_0(980)\pi$ not seen, because $f_0(980)$ is mostly $s\bar{s}$).

Also the scalar masses (not incompatible with Ref. [1]) predicted from the $L\sigma M$ discussed in Sec. 4 obey the mass-splitting pattern (for the chiral limit (CL) in $SU(2)$ and $SU(3)$, see Refs. [3] and [4], respectively)

$$m_{\sigma_n} = \sqrt{\frac{2}{3}} \ m_S^0 - \frac{1}{\sqrt{3}} \ \delta m_S \quad \begin{cases} \text{CL} & \Rightarrow 2 \hat{m}_{CL} \ = \ 650 \text{ MeV,} \\ \sigma \bar{\sigma} \quad \text{CL} & \Rightarrow 922 \text{ MeV,} \quad (3) \\ \sigma \bar{\sigma} \quad \text{CL} & \Rightarrow 940 \text{ MeV.} \end{cases}$$

Here, $m_{\sigma_n}(650)$ is near the PDG average [1] $m_{f_0(600)}$. $m_{\kappa}(780)$ is near the E791 value [5] 797 ± 19 MeV, and $m_{\sigma_s}(940)$ is near the PDG value $m_{f_0(980)}$, which is thus mostly $s\bar{s}$. The masses from Eqs. (3) then give the CL average mass splittings

$$m_S^0 \quad \begin{cases} \text{CL} & \Rightarrow 922 \text{ MeV,} \\ \delta m_S \quad \begin{cases} \text{CL} & \Rightarrow 167 \text{ MeV,} \\ \delta m_S/m_S \quad \begin{cases} \text{CL} & \Rightarrow 18 \% . \end{cases} \end{cases} \end{cases} \quad (4)$$

The fact that the $\bar{q}q$ scalars have an $SU(3)$-breaking CL scale of 18%, larger than the 13% scale of $V$ ground states, further suggests that, whereas the $V$ are $\bar{q}q$ loosely bound states, the $\bar{q}q$ $S$ states (with quarks touching in the NJL scheme [6]) are “barely” elementary-particle partners of the tightly bound $P$ states (discussed in Sec. 4).

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1 Recall that $d_0_{ij} = \sqrt{2/3} \ \delta_{ij}, \ d_{n\bar{n}} = 1/\sqrt{3}, \ d_{s\bar{s}} = d_{K\bar{K}} = -1/(2\sqrt{3}), \text{ and } d_{s\bar{s}} = -2/\sqrt{3}$.
III. LOOSELY BOUND QQQ BARYONS

In this same Taylor-series spirit, the octet ($O$) baryon $SU(3)$ mass splitting $m_O = m_O^0 - \delta m_O (d_{ss}^8 + f_{ss} f^{8i})$, with $d_{ss} + f_{ss} = 1$, predicts (the index $ss$ means semistrong)

$$m_N = m_O^0 - \frac{\delta m_O}{2\sqrt{3}} (-d_{ss} + 3f_{ss}) \approx 939 \text{ MeV},$$
$$m_{\Lambda} = m_O^0 + \frac{\delta m_O}{\sqrt{3}} d_{ss} \approx 1116 \text{ MeV},$$
$$m_{\Sigma} = m_O^0 - \frac{\delta m_O}{\sqrt{3}} d_{ss} \approx 1193 \text{ MeV},$$
$$m_{\Xi} = m_O^0 + \frac{\delta m_O}{2\sqrt{3}} (d_{ss} + 3f_{ss}) \approx 1318 \text{ MeV}.$$ (5)

The $(d/f)_{ss}$ ratio can be found from Eqs. (5) as

$$\left( \frac{d}{f} \right)_{ss} = \frac{3}{2} \frac{m_{\Sigma} - m_{\Lambda}}{m_{\Xi} - m_N} \approx -0.305 \ , \ d_{ss} \approx -0.44 \ , \ f_{ss} \approx 1.44.$$ (6)

Thus, Eqs. (5) predict the average mass splittings

$$m_O^0 \approx 1151 \text{ MeV} \ , \ \delta m_O \approx 150 \text{ MeV} \ , \ \frac{\delta m_O}{m_O^0} \approx 13\%.$$ (7)

The $SU(3) D$ baryon masses $m_D = m_O^0 + \delta m_D$ have $m_D^0$, weighted by wave functions

$$\Psi_{(abc)}^{(abc)} \Psi_{(abc)} = \Delta \Delta + \Sigma \Sigma^* + \Xi \Xi^* + \Omega \Omega,$$ (8)

and $\delta m_D$ is weighted by

$$3 \Psi_{(ab3)}^{(ab3)} \Psi_{(ab3)} = \Sigma \Sigma^* + 2 \Xi \Xi^* + 3 \Omega \Omega.$$ (9)

Then the $SU(3) D$ masses are predicted (in MeV) to be

$$m_{\Delta} = m_O^0 \approx 1232,$$
$$m_{\Sigma^*} = m_O^0 + \delta m_D \approx 1385 \ , \ \text{with} \ \delta m_D \approx 153,$$
$$m_{\Xi^*} = m_O^0 + 2\delta m_D \approx 1533 \ , \ \text{with} \ \delta m_D \approx 151,$$
$$m_{\Omega} = m_O^0 + 3\delta m_D \approx 1672 \ , \ \text{with} \ \delta m_D \approx 147.$$ (10)

This corresponds to average mass splittings

$$m_D^0 \approx 1232 \text{ MeV} \ , \ \delta m_D \approx 150 \text{ MeV} \ , \ \frac{\delta m_D}{m_D^0} \approx 12\%.$$ (11)

It is interesting that both loosely bound $qqq O$ and $D$ symmetry-breaking scales of about 150 MeV are near the $\bar{q}q V, S$ mean mass-splitting scale of $\delta m = 141 \text{ MeV}, 167 \text{ MeV}$. However, the CL $SU(3)$-breaking scale of 18% for scalars is about 50% greater than the 12–13% scales of $V, D$ states. This suggests that $V, O, D \bar{q}q$ or $qq$ states are all loosely bound, in contrast with the elementary-particle $\bar{q}q S$ and, of course, the $P$ states (see above). In fact, the latter Nambu–Goldstone $P$ states are massless in the CL $p^2 = m_N^2 = 0, p^2 = m_K^2 = 0$, as the tightly bound measured $[1] \ pi^-$ and $K^+$ charge radii indicate [7].

IV. CONSTITUENT QUARKS AND THE QUARK-LEVEL LσM

Formulating the $P$ and $S \bar{q}q$ states as elementary chiral partners [8], the Lagrangian density of the $SU(2)$ quark-level linear $\sigma$ model (LσM) has, after the spontaneous-symmetry-breaking shift, the interacting part [9] (for $f_\pi = (92.42 \pm 0.27) \text{ MeV} \approx 93 \text{ MeV}$)

$$L_{\text{int}} = \bar{g} \bar{\psi}(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) \psi + g' \sigma (\pi^2 + \pi^2) - \frac{\lambda}{4} (\pi^2 + \pi^2)^2 - f_\pi g \bar{\psi} \psi,$$ (12)
with tree-order CL couplings related as \((f^{CL}_\pi \to 90 \text{ MeV})\)

\[
g = \frac{m_q}{f_\pi}, \quad g' = \frac{m^2_q}{2f_\pi} = \lambda f_\pi.
\]  

(13)

The \(SU(2)\) and \(SU(3)\) chiral Goldberger–Treiman relations (GTRs) are

\[
f_\pi g = \hat{m} = \frac{1}{2}(m_u + m_d), \quad f_K g = \frac{1}{2}(m_s + \hat{m}).
\]  

(14)

Since \(f_K/f_\pi \approx 1.22\) \cite{1}, the constituent-quark-mass ratio from Eq. (14) becomes

\[
m_\sigma = 2m_q, \quad g = \frac{2\pi}{\sqrt{N_c}}, \quad \text{for } N_c = 3.
\]  

(16)

Here, the first equation is the NJL relation \cite{6}, now true for the \(L\sigma M\) as well. The second equation in Eqs. (16) was first found via the \(Z = 0\) compositeness relation \cite{10}, separating the elementary \(\pi\) and \(\sigma\) particles from the bound states \(\rho\), \(\omega\), and \(a_1\).

We first estimate the nonstrange and strange constituent quark masses from the GTRs (14), together with the \(L\sigma M\) loop-order result (16):\(^2\)

\[
\hat{m} \approx g f_\pi \approx \frac{2\pi}{\sqrt{3}} (93 \text{ MeV}) \approx 337 \text{ MeV} \quad \overset{\text{CL}}{\rightarrow} \quad 325 \text{ MeV}, \\
m_s = \left(\frac{m_s}{\hat{m}}\right) \hat{m} \approx 1.44 \hat{m} \approx 485 \text{ MeV} \quad \overset{\text{CL}}{\rightarrow} \quad 470 \text{ MeV}.
\]  

(17)

These quark-mass scales in turn confirm the mass-splitting scales found in Secs. 2, 3:

\[
\delta m_V \approx \delta m_S \approx \delta m_O \approx (485 - 337) \text{ MeV} = 148 \text{ MeV} \quad \overset{\text{CL}}{\rightarrow} \quad (470 - 325) \text{ MeV} = 145 \text{ MeV},
\]  

(18)

near 141, 167, 150, 150 MeV, respectively. Also the \(SU(3)\) non-vanishing masses are predicted as

\[
m^0_V \approx \sqrt{\frac{3}{2}} (m_s + \hat{m}) \approx 1007 \text{ MeV}, \\
m^0_O = m^0_D \approx m_s + 2\hat{m} \approx 1160 \text{ MeV},
\]  

(19)

near the 1048, 1151, and 1232 MeV \(m^0\) masses in Secs. 2, 3.

To verify that the pion and kaon are tightly bound \(\bar{q}q\) mesons, we compute the \(\pi^+\) and \(K^+\) charge radii as \cite{7} \(r_\pi = \frac{1}{\hat{m}}\) \(\text{CL} = 0.61 \text{ fm}\) and \(r_K = \frac{2}{(m_s + \hat{m})\text{CL}} = 0.50 \text{ fm}\), near data \cite{1} \(0.672 \pm 0.008 \text{ fm}\) and \(0.560 \pm 0.031 \text{ fm}\), respectively. Likewise, to verify that the proton is a \(qqq\) touching pyramid, we estimate the proton charge radius as \(R_p = (1 + \sin30^\circ) r_\pi \approx 0.9 \text{ fm}\), near data \cite{1} \(0.870 \pm 0.008 \text{ fm}\).

\section*{V. S Scalars and Accidental Degeneracies}

We begin with the non-CL NJL-\(L\sigma M\) scalar masses \(m_{\sigma_n} = 2\hat{m} = 674 \text{ MeV}, m_\kappa = 2\sqrt{\hat{m}m_s} = 809 \text{ MeV},\) and \(m_{\sigma_s} = 2m_s = 970 \text{ MeV}.\)

\(^{2}\) The resulting quark masses are well in agreement with the values obtained on the basis of the magnetic moments of the respective baryons (see e.g. Ref. \cite{11}). The proton magnetic moment \(\mu_p \simeq 2.7928\) e.g. yields \(\hat{m} = m_p/\mu_p = 336 \text{ MeV}.\)
An almost degenerate case in the nonrelativistic quark model (NRQM) is [12], in the context of QCD,\(^3\)

\[ m_S \approx m_V + \frac{2\alpha_s}{m_{\text{dyn}}^2} \left( \frac{\vec{L} \cdot \vec{S}}{r^3} \right) = 780 \text{ MeV} - 140 \text{ MeV} = 640 \text{ MeV} , \tag{20} \]

where the ground-state vector mesons have \( L = 0 \) and so no spin-orbit contribution to the mass. This corresponds to \( m_{\pi(650)} \approx m_{\omega(782)} - 140 \text{ MeV} = 642 \text{ MeV} \). Equivalently, invoking the \( I = 1/2 \) CGC of 1/2, one predicts via the NRQM \( m_{\kappa(800)} \approx m_{\phi(892)} - 70 \text{ MeV} = 822 \text{ MeV} \). Or invoking instead the \( s\bar{s} \) CGC of 1/3, one gets \( m_{\sigma(970)} \approx m_{\phi(1020)} - 47 \text{ MeV} = 973 \text{ MeV} \). In a similar way we obtain also \( m_{a_0(985)} = m_{\rho(770)} + (3/2) 140 \text{ MeV} = 980 \text{ MeV} \).

As an alternative way to examine the latter, in the case of the elementary-particle \( \mathcal{P} \) and \( S \) states, one should invoke the infinite-momentum-frame (IMF, see Appendix) scalar-pseudoscalar \( SU(3) \) equal-splitting laws (ESLs), reading [14]

\[ m_{\sigma}^2 - m_{\pi}^2 \approx m_{\kappa}^2 - m_{K^*}^2 \approx m_{a_0} - m_{\text{avg}} \approx 0.40 \text{ GeV}^2 , \tag{21} \]

where \( m_{\text{avg}} \) is the average \( \eta, \eta' \) mass 753 MeV. These ESLs hold for the non-CL NJL-L\(\mathcal{M}\) scalar mass values. Using the ESLs (21) to predict the \( a_0 \) mass, one finds

\[ m_{a_0} = \sqrt{0.40 \text{ GeV}^2 + m_{\text{avg}}^2} \approx 983.4 \text{ MeV} , \tag{22} \]

very close to the PDG value 984.7 \( \pm \) 1.2 MeV. Thus, the nearness of the \( a_0(985) \) and \( f_0(980) \) masses, the latter scalar being mostly \( s\bar{s} \) and so near the vector \( s\bar{s} \phi(1020) \) (see above), is indeed an accidental degeneracy. Note that a similar (approximate) degeneracy is found in the dynamical unitarized quark-meson model of Ref. [15], where the same \( \bar{q}q \) assignments are employed as here.

This ground-state scalar \( ^1 \text{S}_0 \) nonet \([\sigma(650), \kappa(800), f_0(980), a_0(985)]\) is about 500–700 MeV below the \( ^1 \text{P}_1 \) nonet \([f_0(1370), K^*_0(1430), f_0(1500), a_0(1450)]\), just as the ground-state \( ^1 \text{S}_0 \)–\( ^1 \text{P}_1 \) vector nonet \([\rho(770), \omega(782), K^*(892), \phi(1020)]\) is about 600–800 MeV below the \( ^1 \text{S}_0 \) nonet \([\rho(1450), \omega(1420), K^*(1680), \phi(1680)]\).

VI. STRONG-INTERACTION SCALAR-MESON DECAY RATES

Given the above scalar-meson nonet \([\sigma(650), \kappa(800), f_0(980), a_0(985)]\), compatible with present data and also with the \( SU(3) \) mass splittings in Secs. 2, 3, 5 and the quark-level \( \mathcal{L} \) in Sec. 4, we now predict \( \mathcal{L} \) decay rates based on the \( SU(3) \) Lagrangian density \( \mathcal{L}_{\mathcal{L}}^{\mathcal{L}} = g_{\sigma\pi\pi} d_{ijk} S_i S_j P_k \), with \( \mathcal{L} \) coupling \( g_{\sigma\pi\pi} = (m_{\sigma}^2 - m_{\pi}^2)/(2 f_{\pi}) \approx 2.18 \text{ GeV} \), where \( f_{\pi} = (92.42 \pm 0.27) \text{ MeV} \) and \( m_{\pi} \approx 650 \text{ MeV} \) (the latter stems from the CL \( m_q \approx 325 \text{ MeV} \) [3]). Thus, the \( \sigma \rightarrow 2 \pi \) decay rate, for \( p_{cm} = 294 \text{ MeV} \) and \( \phi_{a} = \pm 18 \text{°} \), becomes

\[ \Gamma_{\sigma\pi\pi} = \frac{p_{cm}}{8 \pi m_{\sigma}^2} \left( \frac{3}{2} \right) \left[ 2 g_{\sigma\pi\pi} \cos \phi_{a} \right]^2 \approx 714 \text{ MeV} . \tag{23} \]

Here the factor of 2 is due to Bose statistics (see e.g. Ref. [17]), and this broad width \( \Gamma_{\sigma} \approx m_{\sigma} \) is expected from data [18] and from phenomenology [19].

Next, the \( a_0(985) \rightarrow \eta\pi \) width for \( p_{cm} = 321 \text{ MeV} \) is

\[ \Gamma_{a_0\eta\pi} = \frac{p_{cm}}{8 \pi m_{a_0}^2} \left[ 2 g_{\sigma\pi\pi} \cos \phi_{ps} \right]^2 \approx 138 \text{ MeV} , \tag{24} \]

where \( \phi_{ps} \approx 42 \text{°} \) is in the quark nonstrange(\( \bar{n}n \))-strange(\( s\bar{s} \)) basis [20]. This predicted \( \mathcal{L} \) width is not incompatible with the high-statistics decay rate [21] \( \Gamma_{a_0\eta\pi} = (95 \pm 14) \text{ MeV} \).

Furthermore, the \( \kappa \rightarrow K\pi \) decay rate, for \( p_{cm} = 218 \text{ MeV} \) and \( m_{\kappa} \approx 800 \text{ MeV} \), is

\[ \Gamma_{\kappa K\pi} = \frac{p_{cm}}{8 \pi m_{\kappa}^2} \left( \frac{3}{4} \right) \left[ 2 g_{\sigma\pi\pi} \right]^2 \approx 193 \text{ MeV} , \tag{25} \]

\( ^3 \) Note that we follow Ref. [12], and use \( \alpha_s(m_{\sigma}^2) \approx \pi/4 \) (see also Ref. [13]), \( \vec{L} \cdot \vec{S} = -2, m_{\text{dyn}} = 315 \text{ MeV} \), while \( \langle r^{-3} \rangle = 4 \beta^3/(3 \sqrt{\pi}) \) is obtained employing harmonic-oscillator wave functions with \( \beta \approx 180 \text{ MeV} \).

\( ^4 \) For convenience, we use here the same value of the mixing angle \( \phi_{a} \) as in Ref. [16], i.e., \( \phi_{a} = \pm 18 \text{°} \).
which is of the same order as the E791 data [5]
\[ \Gamma_{K^0\pi^-}^{E791} = (410 \pm 43 \pm 87) \text{ MeV}, \quad m_\kappa = (797 \pm 19 \pm 43) \text{ MeV}, \]  
and especially the very recent data of the BES collaboration [22]
\[ \Gamma_{K^0\pi^-}^{\text{BES}} = (220^{+225}_{-169} \pm 97) \text{ MeV}, \quad m_\kappa = (771^{+164}_{-221} \pm 55) \text{ MeV}. \]  
Lastly, we estimate (see e.g. Ref. [16]) the \( f_{2} \) and especially the very recent data of the BES collaboration [22] for mixing angle \( \pm 18^\circ \) in the quark basis [23], for \( p_{\text{cm}} = 470 \text{ MeV} \):
\[ \Gamma_{f_0\pi^\pm} = \frac{p_{\text{cm}}}{8\pi m_{f_0}^2} \left( \frac{3}{2} \right) [ 2 g_{\sigma\pi\pi} \sin \phi_\sigma ]^2 \approx 53 \text{ MeV}, \]  
not too distant from the recent E791 measurement [5]
\[ \Gamma_{f_0\pi^\pm}^{E791} = (44 \pm 2 \pm 2 \text{ MeV}, \quad m_{f_0} = (977 \pm 3 \pm 2) \text{ MeV}. \]  

VII. ELECTROMAGNETIC MESON DECAY RATES INVOLVING \( \bar{Q}Q \) SCALARS

Next we study the five electromagnetic meson decays \( \sigma \rightarrow 2\gamma, a_0 \rightarrow 2\gamma, f_0 \rightarrow 2\gamma, \phi \rightarrow f_0\gamma, \) and \( \phi \rightarrow a_0\gamma \). Again assuming \( m_\sigma \approx 650 \text{ MeV} \) (because \( \hat{m} \approx 325 \text{ MeV} \approx m_N/3 \) in the CL, so that the NJL-LooM scalar mass is \( m_\sigma = 2\hat{m} \approx 650 \text{ MeV} \)), the quark-loop amplitude magnitude is, for \( f_\pi = (92.42 \pm 0.27) \text{ MeV} \) [20] (see e.g. Eq. (11a) in Ref. [24], and the considerations in Ref. [16])
\[ |M(\sigma \rightarrow 2\gamma)| = \frac{5}{3} \frac{\alpha}{\pi f_\pi} + \frac{1}{3} \frac{\alpha}{\pi f_\pi} \approx 5.0 \times 10^{-2} \text{ GeV}^{-1}. \]  
Here, the first term is due to the nonstrange quark triangle, while the second term stems from the charged-kaon and -pion triangle graphs. This result (30) is compatible with the data estimate [25]
\[ \Gamma_{\sigma 2\gamma} = \frac{m_\sigma^2}{64 \pi} |M(\sigma \rightarrow 2\gamma)|^2 = (3.8 \pm 1.5) \text{ keV}, \]  
or (for \( m_\sigma \approx 650 \text{ MeV} \))
\[ |M(\sigma \rightarrow 2\gamma)| \approx (5.3 \pm 1.0) \times 10^{-2} \text{ GeV}^{-1}. \]  

Now we examine \( a_0(985) \rightarrow 2\gamma \). A nonstrange-quark triangle loop predicts the gauge-invariant induced amplitude magnitude [26] (for \( m_{a_0} \approx (984.7 \pm 1.2) \text{ MeV} \))
\[ |M(a_0 \rightarrow 2\gamma)|_{\text{quark-loop}} = \left| 2 \xi \left[ 2 + (1 - 4\xi) I(\xi) \right] \frac{\alpha}{\pi f_\pi} \right| \]
\[ = \left| 2.03 \pm 0.07 + i (1.89 \pm 0.03) \right| \times 10^{-2} \text{ GeV}^{-1} \]
\[ = (2.78 \pm 0.06) \times 10^{-2} \text{ GeV}^{-1}, \]  
for \( \xi = \hat{m}^2/m_{a_0}^2 \approx 0.109 \pm 0.004 < 1/4 \) in the CL, with (see e.g. Ref. [16])
\[ I(\xi) = \int_0^1 dy \int_0^1 dx \frac{y}{\xi - xy(1-y)} \]
\[ \xi^{<1/4} \frac{\pi^2}{2} - 2 \ln^2 \left[ \frac{1}{\sqrt{4\xi}} + \sqrt{\frac{1}{4\xi} - 1} \right] + 2\pi i \ln \left[ \frac{1}{\sqrt{4\xi}} + \sqrt{\frac{1}{4\xi} - 1} \right] \]
\[ = 3.03 \pm 0.08 + i (6.13 \pm 0.13). \]  
However, adding to Eq. (33) the charged-kaon-loop amplitude [26] \( 0.97 \times 10^{-2} \text{ GeV}^{-1} \) (as required by the LoM), which has the opposite sign as compared to the fermionic quark-loop amplitude, in turn predicts [27]
\[ |M(a_0 \rightarrow 2\gamma)| \approx |M(a_0 \rightarrow 2\gamma)|_{\text{quark-loop}} + M(a_0 \rightarrow 2\gamma)_{\text{kaon-loop}}| \]
\[ = |1.07 \pm 0.44 + i (1.89 \pm 0.03)| \times 10^{-2} \text{ GeV}^{-1} \]
\[ = (2.17 \pm 0.22) \times 10^{-2} \text{ GeV}^{-1}. \]
The latter result is too large as compared to data, assuming $a_0 \rightarrow \eta \pi$ is dominant [1]:

$$\Gamma_{a_0} = \frac{m_{a_0}^3}{64 \pi} |M(a_0 \rightarrow 2\gamma)|^2 = (0.24 \pm 0.08) \text{ keV},$$

or

$$|M(a_0 \rightarrow 2\gamma)| = (0.7 \pm 0.2) \times 10^{-2} \text{ GeV}^{-1}. \quad (37)$$

However, upon disregarding the imaginary part of the quark-loop amplitude, which is reasonable in view of quark confinement, we come much closer to the data, as

$$\text{Re} \left[ M(a_0 \rightarrow 2\gamma)_{\text{quark-loop}} + M(a_0 \rightarrow 2\gamma)_{\text{kaon-loop}} \right] = (1.07 \pm 0.44) \times 10^{-2} \text{ GeV}^{-1}. \quad (38)$$

Next we study $f_0 \rightarrow 2\gamma$. Assuming for the moment that $f_0(980)$ is purely $\bar{s}s$, the strange-quark loop gives, for $N_c = 3$ [28] (see also Ref. [16])

$$|M(f_0 \rightarrow 2\gamma)|_{\text{quark-loop}} = \frac{\alpha N_c g_{f_0 SS}}{9 \pi m_s} \simeq 8.19 \times 10^{-3} \text{ GeV}^{-1}, \quad (39)$$

taking the $\Lambda\sigma\pi$ value $m_s = 485$ MeV from Eq. (17), with the $\Lambda\sigma\pi$ coupling $g_{f_0 SS} = 2\pi\sqrt{2}/3 \approx 5.13$. In fact, Eq. (39) is surprisingly near the observed amplitude [1]

$$\Gamma_{f_0} = \frac{m_{f_0}^3}{64 \pi} |M(f_0 \rightarrow 2\gamma)|^2 = (0.39 \pm 0.12) \text{ keV}, \quad (40)$$

or (with $m_{f_0} \simeq (980 \pm 10)$ MeV)

$$|M(f_0 \rightarrow 2\gamma)| = (9.1 \pm 1.5) \times 10^{-3} \text{ GeV}^{-1}. \quad (41)$$

Nevertheless, a more detailed analysis based on kaon and pion loops, and allowing a small $\bar{n}n$ admixture in the $f_0(980)$, essentially confirms this nice result [16].

Let us now analyse the decay $\phi(1020) \rightarrow f_0(980)\gamma$. Since the $\phi(1020)$ is known to be dominantly $\bar{s}s$, just as we assume the $f_0(980)$ to be, the $s$-quark loop gives (with $g_\phi = 13.43$ from $\Gamma_{\phi ee}$ and $e = \sqrt{4\pi\alpha} = 0.30282\ldots$)

$$|M(\phi \rightarrow f_0 \gamma)|_{\text{quark-loop}} = \frac{2 g_\phi e g_{f_0 SS}}{4 \pi^2 m_s} \cos \phi_s \simeq 2.07 \text{ GeV}^{-1}. \quad (42)$$

However, the charged-kaon loop is known to give the rate [29]

$$\Gamma_{\phi f_0 \gamma} \mid_{\text{kaon-loop}} = 8.59 \times 10^{-4} \text{ MeV}, \quad (43)$$

or

$$|M(\phi \rightarrow f_0 \gamma)|_{\text{kaon-loop}} = 0.75 \text{ GeV}^{-1}. \quad (44)$$

Subtracting this kaon-loop amplitude (44) from the quark-loop amplitude (42) predicts in turn

$$|M(\phi \rightarrow f_0 \gamma)| \approx 2.07 \text{ GeV}^{-1} - 0.75 \text{ GeV}^{-1} = 1.32 \text{ GeV}^{-1}, \quad (45)$$

near the recent KLOE data [30], for $p_{cm} \simeq (38.69 \pm 9.62)$ MeV,

$$\Gamma_{\phi f_0 \gamma} \mid_{\text{KLOE}} = \frac{p_{cm}^2}{12\pi} |M(\phi \rightarrow f_0 \gamma)|^2 \approx (19 \pm 1) \times 10^{-4} \text{ MeV}, \quad (46)$$

or

$$|M(\phi \rightarrow f_0 \gamma)| \approx (1.11 \pm 0.42) \text{ GeV}^{-1}, \quad (47)$$

as the branching rate for $\phi \rightarrow f_0 \gamma$ is $(4.47 \pm 0.21) \times 10^{-4}$.

Lastly we note that the KLOE observed branching ratio (BR) is [31]

$$\text{BR}(\phi \rightarrow f_0 \gamma/a_0\gamma) = 6.1 \pm 0.6. \quad (48)$$

Because we know that $\phi$ is dominantly $\bar{s}s$, this BR Eq. (48) being much greater than unity strongly suggests that $a_0(985)$ is mostly $\bar{n}n$ and $f_0(980)$ is mostly $\bar{s}s$. The latter assumption we have continually made throughout this paper, while it had been a conclusion of Ref. [16] (see also Ref. [32]).
VIII. W-EMISSION WEAK DECAY RATES

In this section we study the five weak decays $K^+ \to \pi^0\pi^+$, $D^+ \to \overline{K}^0\pi^+$, $D^+ \to \sigma\pi^+$, $D^+ \to \pi^0\pi^+$, and $D_s \to f_0(980)\pi^+$, via tree-level $W$-emission graphs. Recalling from Refs. [16] and [32], the amplitudes due to $W$-emission are,

$$|M(K^+ \to \pi^0\pi^+)| = \frac{G_F |V_{ud}| |V_{us}|}{2\sqrt{2}} f_\pi (m_{K^+}^2 - m_{\pi^+}^2)$$

$$= (1.837 \pm 0.020) \cdot 10^{-8} \text{ GeV} ,$$

near data [1] $(1.832 \pm 0.007) \cdot 10^{-8} \text{ GeV}$,

$$|M(D^+ \to \overline{K}^0\pi^+)| = \frac{G_F |V_{ud}| |V_{us}|}{2} f_\pi (m_{D^+}^2 - m_{\overline{K}^0}^2)$$

$$= (177 \pm 27) \cdot 10^{-8} \text{ GeV} ,$$

near data [1] $(136 \pm 6) \cdot 10^{-8} \text{ GeV}$, and

$$|M(D_s^+ \to f_0\pi^+)| = \frac{G_F |V_{ud}| |V_{cs}|}{2} f_\pi (m_{D_s^+}^2 - m_{f_0}^2)$$

$$= (159 \pm 25) \cdot 10^{-8} \text{ GeV} ,$$

near data [1] $(178 \pm 40) \cdot 10^{-8} \text{ GeV}$. In the latter case we have assumed that $f_0(980)$ is all $s\bar{s}$.

Now we also consider $D \to \pi^0\pi^+$ and $D \to \sigma\pi^+$ (with $m_\sigma = 650$ MeV), again in this $W$-emission scheme, predicting

$$|M(D^+ \to \pi^0\pi^+)| = \frac{G_F |V_{ud}| |V_{cd}|}{2\sqrt{2}} f_\pi (m_{D^+}^2 - m_{\pi^+}^2)$$

$$= (28.9 \pm 2.1) \cdot 10^{-8} \text{ GeV} ,$$

near data [1] $(38.6 \pm 5.4) \cdot 10^{-8} \text{ GeV}$ (also see Ref. [33], with $p_{cm} = 925$ MeV), and

$$|M(D^+ \to \sigma\pi^+)| = \frac{G_F |V_{ud}| |V_{cd}|}{2\sqrt{2}} f_\pi (m_{D^+}^2 - m_\sigma^2)$$

$$\simeq 25.5 \cdot 10^{-8} \text{ GeV} ,$$

near recent data [1] $(37.6 \pm 4.5) \cdot 10^{-8} \text{ GeV}$. This latter amplitude follows from the decay rate (with $p_{cm} = 815$ MeV, $\tau = 1051 \times 10^{-15} \text{ s}$)

$$\Gamma_{D^+\sigma\pi^+} = \frac{p_{cm}}{8\pi m_{D^+}^2} |M(D^+ \to \sigma\pi^+)|^2$$

$$= \frac{\hbar}{2\pi \tau} (2.1 \pm 0.5) \times 10^{-3} = (1.32 \pm 0.31) \times 10^{-15} \text{ GeV} .$$

Not only are the above $D^+ \to \pi^0\pi^+$ and $D^+ \to \sigma\pi^+$ $W$-emission amplitudes near data, they are even of about the same magnitude. This is another example of the $\sigma$ and $\pi$ being chiral partners [8].

IX. SUMMARY AND CONCLUSIONS

Throughout this paper we have dealt with all ground-state mesons as $q\bar{q}$ nonets in the context of the LqM. In Sec. 2 we studied SU(3) mass splittings for $V$ and $S$ $q\bar{q}$ mesons, with $V$ loosely bound states, and $P$, $S$ tighter $q\bar{q}$

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5 We use here $G_F = 1.16639(1) \times 10^{-5} \text{ GeV}^{-2}$, $|V_{ud}| = 0.9735 \pm 0.0008$, $|V_{us}| = 0.2196 \pm 0.0023$, $|V_{cd}| = 0.2241 \pm 0.016$, $|V_{cs}| = 1.04 \pm 0.16$, $m_{D^+} = (1869.4 \pm 0.5) \text{ MeV}$, and $m_{f_0} = (1909.0 \pm 1.4) \text{ MeV}$.

6 At this point we should keep in mind that the uncertainty in $m_\sigma$ is of the order of $m_\sigma$!
elementary particles. In Sec. 3 we reviewed $qqq$ octet and decuplet baryons. In Sec. 4 we briefly summarized the quark-level $\Lambda\sigma$M theory, while in Sec. 5 we explained the accidental degeneracy of the $a_0(985)$ and $f_0(980)$ scalars. In Sec. 6 we computed a few strong scalar-meson decay widths, while in Sec. 7 we performed a similar analysis for some electromagnetic decays involving scalar mesons. Finally, in Sec. 8 we employed $W$-emission graphs to describe several hadronic weak-decay processes.

The usual field-theory picture is that meson masses should appear quadratically and baryon masses linearly in Lagrangian models based on the Klein–Gordon and Dirac equations. However, in Secs. 2 and 3 we studied both mesons and baryons in a $\text{SU}(3)$-symmetry Taylor-series sense. Instead, in Sec. 5 we studied symmetry breaking in the IMF, with $E = [p^2 + m^2]^{1/2} \approx p + m^2/2p + \ldots$. Here, between brackets, the 1 indicates the symmetry limit, and the quadratic mass term means that both meson and baryon masses are squared in the mass-breaking IMF for $\Delta S = 1$ ESLs. While the former mass-splitting approach (with linear masses) fits all $V$, $S$, $O$, and $D$ ground-state $SU(3)$-flavor multiplets, so does the latter (with quadratic masses) for the IMF-ESLs. Nevertheless, Nambu–Goldstone pseudoscalars $\mathcal{P}$ always involve quadratic masses. Both approaches suggest that all ground-state mesons ($P$, $S$, $V$) are $\bar{q}q$ states, while baryons ($O$, $D$) are $qqq$ states. This picture is manifest in the quark-level $\Lambda\sigma$M of Sec. 4. The accidental scalar degeneracy between the $\bar{s}s$ $f_0(980)$ and the $\bar{n}n$ $a_0(985)$ was explained in Sec. 5, via the IMF quadratic-mass ESLs — also compatible with mesons being $\bar{q}q$ and baryons $qqq$ states.

Concerning the mass splittings in general, we observed the remarkable feature that the real parts of masses of resonances in mesonic and baryonic ground-state multiplets nicely follow an $SU(3)$ splitting pattern, despite the enormous disparities in decay widths and thus in the imaginary parts. This may be understood in the unitarized picture of Ref. [15], in which both real and virtual decay channels contribute to the physical masses of e.g. the scalar mesons as dressed $\bar{q}q$ states. We also verified in Secs. 6, 7, and 8 that mesonic decay rates can be simply explained on the basis of the flavor and chiral symmetry underlying the quark-level $\Lambda\sigma$M. This is another indication that the lowest lying mesons are all $\bar{q}q$, while the considered baryons are $qqq$.

So far we have taken the mass and coupling parameters of the quark-level $\Lambda\sigma$M — in particular $m_{\sigma}$ — to be real numbers (“narrow-width approximation”). A recently developed formalism [34] may allow us to go beyond this approximation in the near future.

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**APPENDIX A: KINEMATIC INFINITE-MOMENTUM FRAME**

The infinite-momentum frame (IMF) has two virtues: (i) $E = [p^2 + m^2]^{1/2} \approx p + m^2/2p + \ldots$, for $p \to \infty$, requires squared masses when the leading term is eliminated, using $SU(3)$ formulae with coefficients $1 + 3 = 2 + 2$, as e.g. the Gell-Mann–Okubo linear mass formula $\Sigma + 3\Lambda = 2N^2$ valid to 3%; (ii) when $p \to \infty$, dynamical tadpole graphs are suppressed [35]. In fact, $\Sigma^2 + 3\Lambda^2 = 2N^2 + 2\Xi^2$ is also valid empirically to 3%. This squared $qqq$ baryon mass formula can be interpreted as a $\Delta S = 1$ ESL, which holds for both $O$ and $D$ baryons [14]:

\[
\sum \Lambda - N^2 \approx \Xi^2 - \Sigma \Lambda \approx \frac{1}{2} \left( \Xi^2 - N^2 \right) \approx 0.43 \text{ GeV}^2, \tag{A1}
\]

\[
\Sigma^2 - \Delta^2 \approx \Xi^2 - \Sigma^2 \approx \Omega^2 - \Xi^2 \approx \frac{1}{2} \left( \Omega^2 - \Sigma^2 \right) \approx 0.43 \text{ GeV}^2.
\]

However, the $\bar{q}q$ pseudoscalar and vector $\Delta S = 1$ ESLs have about one half this scale (also empirically valid to 3%), viz.

\[
m_K^2 - m_{\rho}^2 \approx m_{\phi}^2, \quad m_{\rho}^2 \approx m_{\sigma}^2 - m_{\rho}^2 \approx \frac{1}{2} \left( m_{\phi}^2 - m_{\rho}^2 \right) \approx 0.22 \text{ GeV}^2, \tag{A2}
\]

as roughly do the $\bar{q}q$ scalars found in Sec. 2, i.e.,

\[
m_{K_{\text{800}}}^2 - m_{\sigma_{(940)}}^2 \approx m_{\sigma_{(940)}}^2 - m_{\rho_{(800)}}^2 \approx 0.22 \ldots 0.24 \text{ GeV}^2. \tag{A3}
\]

This approximate factor of 2 between Eqs. (A1) and Eqs. (A2,A3) is because there are two $\Delta S = 1$ $qqq$ transitions, whereas there is only one $\Delta S = 1$ transition for $\bar{q}q$ configurations.
So if we take Eq. (A3) as physically meaningful, we may write

$$2m_k^2 \approx m_{\sigma(600)}^2 + m_{\sigma(980)}^2 \approx m_{\sigma(650)}^2 + m_{\sigma(940)}^2 \approx 1.31 \ldots 1.32 \text{ GeV}^2 \ ,$$

(A4)

yielding \( m_\kappa \approx 811 \text{ MeV} \) close to experiment, which again suggests these scalars are \( \bar{q}q \) states.

These IMF quadratic mass schemes, along with the non-CL NJL-L\( \kappa \) mass \( m_{\kappa(800)} = 2\sqrt{m_\kappa} = 809 \text{ MeV} \) or the averaged\(^7\) mass value of 800 MeV, again suggest (as do the empirical scales of Eqs. (A2) and (A3) vs. Eqs. (A1)) that all ground-state meson nonets are \( \bar{q}q \), whereas the baryon octet and decuplet are \( qqq \) states.

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