Installing of cosmological constant

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Abstract. An artificial scale of observable cosmological constant is dynamically related to its natural bare value due to an installation of relevant coherent state for the inflationary field in a finite volume of early Universe, because of exponential suppression of probability to find the state with zero number of quanta. Homogeneous quantum fluctuations of the field actually put hard constraints on the total amount of inflation.
1 Introduction

Since the Universe inflation [1–7] started with a final volume \( V_R \), a homogeneous inflationary field \( \phi(t) \) had got nonzero quantum fluctuations of both its value \( \delta \phi(t) \) and rate \( \dot{\delta} \dot{\phi}(t) \). Then the inflaton energy-momentum tensor \( T^\nu_\mu \) was averaged over a quantum state

\[
|\alpha\rangle = |0\rangle \langle 0| + |q\rangle \langle q| \alpha\rangle,
\]

where \(|0\rangle\) is the vacuum with the fluctuations relevant to the volume \( V_R \), while \(|q\rangle\) represents the sum over states with nonzero number of \( \phi \) quanta, so that the probability of vacuum \( w_{\text{vac}} = |\langle 0|\alpha\rangle|^2 \) in this state of inflation installation

\[
w_{\text{vac}} \sim e^{-\ell},
\]

with \( \ell \) being the average number of quanta in the final volume \( V_R < \infty \), hence, \( \ell < \infty \) and

\[
\langle \alpha|T^\nu_\mu|\alpha\rangle = w_{\text{vac}} \langle 0|T^\nu_\mu|0\rangle + (1-w_{\text{vac}}) \langle q|T^\nu_\mu|q\rangle + \sqrt{w_{\text{vac}}(1-w_{\text{vac}})} \left\{ \langle 0|T^\nu_\mu|q\rangle + \langle q|T^\nu_\mu|0\rangle \right\}. \tag{1.1}
\]

Here

\[
\langle 0|T^\nu_\mu|0\rangle = \rho^\text{bare}_\Lambda \delta^\nu_\mu
\]

is the bare cosmological term with the vacuum energy \( \rho^\text{bare}_\Lambda \) [8, 9], for instance,

\[
\rho^\text{bare}_\Lambda = \frac{1}{2} \langle 0|\dot{\phi}^2|0\rangle + \frac{m^2}{2} \langle 0|\phi^2|0\rangle = \frac{1}{2} \left( \delta \dot{\phi}_c^2 + m^2 \delta \phi_c^2 \right) \tag{1.2}
\]

in the simplest model of inflaton with a mass \( m \). Thus, the observed cosmological constant gets an artificial value

\[
\rho_\Lambda \sim e^{-\ell} \rho^\text{bare}_\Lambda \ll \rho^\text{bare}_\Lambda \quad \text{at} \; \ell \gg 1.
\]

In (1.1) the second term represents the dynamical contribution of inflaton quanta, while the third term corresponds to the cosmological creation (and annihilation) of those quanta from (and to) the vacuum due to the source of gravitational interaction, i.e. due to the energy-momentum tensor.

The bare cosmological constant can be evaluated in the following way:

It is natural to set the quantum fluctuations of kinetic and potential energies to be comparable for the inflationary field in the final volume \( V_R \) at the inflation start, hence,
\( \delta \dot{\phi} \sim m \delta \phi \). For inflaton potential \( V(\phi) \) the slow roll regime of field evolution at Hubble rate \( H \) gives

\[
\dot{\phi} \sim \frac{\partial V}{\partial \phi} \frac{1}{H} \sim \tilde{m}^2 \phi \frac{\tilde{m}_{\text{Pl}}}{m\phi} \sim m \tilde{m}_{\text{Pl}},
\]

where \( \tilde{m}_{\text{Pl}}^2 = (8\pi G)^{-1} \) is the reduced Planck mass squared. The slow rolling can be installed only if the rate \( \dot{\phi} \) begins to exceed its fluctuation \( \delta \dot{\phi} \), i.e. at \( \dot{\phi} \sim \delta \dot{\phi} \). Therefore,

\[
\rho_{\Lambda}^{\text{bare}} \sim \delta \dot{\phi}^2 \sim \dot{\phi}^2 \sim m^2 \tilde{m}_{\text{Pl}}^2.
\]

Empirically, \( m \sim 10^{13} \text{ GeV} [10] \), so the value of \( \rho_{\Lambda}^{\text{bare}} \sim (10^{15} \text{ GeV})^4 \) that is quite natural for high energy particle physics.

In this paper we consider the role of quantum fluctuations for the homogeneous inflationary field with the potential

\[
V = \frac{1}{2} m^2 \phi^2,
\]

in order to derive the condition of \( \delta \dot{\phi}^2 = \dot{\phi}^2 \) for the inflation start. We also argue for the coherent state of inflaton at the moment of inflation installation. For the number of inflationary field quanta in the primary volume \( V_R \) of inflation start the estimate \( \ell \sim 250 \) is obtained.

## 2 Problem treatment

As it was discovered by measuring magnitudes of brightness for standard candles of type Ia supernovas versus its red shifts [11–15], the accelerated expansion of Universe can be explained by the cosmological constant (see [8]), which was invented by Albert Einstein for effects of anti-gravity. So, the cosmological term determines the vacuum energy density that is given by fourth degree of scale about \( 10^{-3} \text{ eV} \), corresponding to 1 eV per cubed millimeter. At this scale the laws of physics are well studied and not related to any global properties of cosmos, hence, the observed value of cosmological constant does not match to the known dynamics. It is the essence of cosmological constant problem reviewed in [9] by S. Weinberg. The conflict with the dynamics can mean that the observed value of cosmological constant is artificial, i.e. reducible from a true initial quantity characteristic for the particle interactions and vacuum structure. In the quantum theory inherently involving the uncertainty principle, a field cannot be posed in a minimum of its potential without any motion, so that, by Zel’ dovich [16], the vacuum energy density is related to a non-zero energy for zero number of quanta of all physical fields, zero-point modes, while a scale of such the energy should be determined by relevant properties of forces in the nature, i.e., for instance, by a characteristic scale of grand unification of gauge interactions about \( 10^{15} \text{ GeV} \) [17, 18]. The extremely small scale of observed cosmological constant represents itself the problem.

However, since the measured value of vacuum energy density appears as a reflection of true initial quantity, we have to suppose that this reflection has been formed during interactions at the same times, when the bare cosmological constant itself could be essential, i.e. at energy densities of early Universe. These times refer to a rapid expansion of Universe, which is called the inflation [3–7] that is supported by certain real facts in its favor. So, the observed magnitude and spectrum of anisotropy in the cosmic microwave background radiation [19–22] can be explained dynamically, instead of introducing an occasional set of very specific initial data of evolution.
In the framework of model for the inflation of early Universe, the anisotropy is caused by the spatial inhomogeneity of quantum fluctuations of scalar field called inflaton $\phi$, as calculated in the method of secondary quantization in vicinity of classical solution for the field equation\footnote{The dot over a symbol denotes the derivative with respect to time.} with a potential $V(\phi)$

$$\ddot{\phi} + 3H \dot{\phi} + \frac{\partial V}{\partial \phi} = 0 \quad (2.1)$$

in the Universe expanding homogeneously and isotropically with the scale factor $a(t)$ in the metrics of Friedmann–Robertson-Walker–Lemtare

$$ds^2 = dt^2 - a^2(t) dr^2, \quad (2.2)$$

with the Hubble parameter $H = \dot{a}/a$, if the inflaton potential rather slowly changes near its stable minimum. For instance, in the simplest case of generic re-normalizable potential

$$V(\phi) = \frac{m^2}{8v^2} (\phi^2 - v^2)^2, \quad (2.3)$$

with two parameters: mass $m$ and vacuum expectation value $v$, the data prefer for the scenario of “new inflation” \cite{7}, when the field evolves from initial values in the region of potential plateau $V(0) = \frac{m^2}{8} v^2$ down to the minimum at $\phi = v$ (the “hilltop” scenario), so that we find empirically \cite{10}

$$m \approx (1.52 \pm 0.22) \cdot 10^{13} \ \text{GeV}, \quad (2.4)$$

$$2.5 \ m_{\text{Pl}} < v < 54 \ m_{\text{Pl}}, \quad (2.5)$$

where the Planck mass $m_{\text{Pl}}$ (in units $c = \hbar = 1$) is determined by the Newton gravitational constant $G$ via the relation $m_{\text{Pl}} = 1/\sqrt{G} = 1.22 \cdot 10^{19} \ \text{GeV}$. During the inflation the Hubble parameter $H$ in the field equation of (2.1) takes the role analogous to the coefficient of friction in the ordinary mechanics of point-like body, so that the regime of “slow roll” is established for the field tending to the minimum of potential. In this regime we can neglect the acceleration of $\ddot{\phi}$ in (2.1), while $H$ drifts in accordance to the general relativity with the metrics of (2.2):

$$\dot{H} = -4\pi G \dot{\phi}^2, \quad (2.6)$$

taking into account for the Friedmann equation in the case of matter saturated by the scalar field, which gives the energy density\footnote{All of other contributions to the energy density rapidly decline as powers of scale factor growing during the expansion.} $\rho$,

$$H^2 = \frac{8\pi G}{3} \rho = \frac{8\pi G}{3} \left(\frac{1}{2} \dot{\phi}^2 + V(\phi)\right). \quad (2.7)$$

When $H$ is decreasing during the evolution, it crosses a minimal critical value $H_{\text{min}}$ depending on the potential model. At lower densities $H < H_{\text{min}}$ the field goes out of the slow rolling and starts to rapidly oscillate in vicinity of potential minimum, with a damping. The quanta of inflaton vibrations are transformed to quanta of matter field, that causes the Universe reheating with the substance creation. The matter density repeats the profile of spatial inhomogeneity of inflaton \cite{3–6}. 


Like for any observable quantity, we need to quantize the global homogeneous field of inflaton, which is considered as the classical field in the description of early Universe, up to the introduction of small spatially inhomogeneous corrections in the framework of secondary quantization. This quantization of homogeneous field is necessary in order to clarify the role of nonzero dispersions of the field and its rate of change onto the run of Universe evolution. In the present article we consider the averaged quantum equations of inflaton evolution in the presence of fluctuations. For the sake of simplicity and clarity of emphasizing physical effects we study the model with potential

$$V = \frac{1}{2} m^2 \phi^2.$$  \hfill (1.3)

Because of fluctuations, the regime of inflation becomes possible in a restricted region of field energy density only if the Hubble parameter is less than a maximal critical value $H_{\text{max}}$, we find.

### 2.1 Quantization and coherent states

In a comoving volume $V_C$, setting the real physical volume $V_R = a^3 V_C$, the action of homogeneous scalar inflationary field in the metrics of (2.2)

$$S = V_C \int dt \, a^3(t) \left( \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} m^2 \phi^2 \right)$$  \hfill (2.8)

determines the canonical momentum of field

$$\hat{p}_\phi = V_R \dot{\phi},$$  \hfill (2.9)

so that the Hamiltonian

$$\mathcal{H}_\phi = \frac{\hat{p}_\phi^2}{2 V_R} + \frac{1}{2} V_R m^2 \phi^2$$  \hfill (2.10)

corresponds to the harmonic oscillator with mass $V_R$ and frequency $m$, hence, in the oscillatory units of the field $\phi^{(0)}$ and canonically conjugated momentum $p^{(0)}_\phi$

$$p^{(0)}_\phi = \sqrt{V_R m}, \quad \phi^{(0)} p^{(0)}_\phi = 1,$$  \hfill (2.11)

the operators of creation and annihilation of field quanta, $\hat{\alpha}^\dagger$ and $\hat{\alpha}$, are defined as

$$\hat{\alpha} = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi^{(0)} & i \hat{p}_\phi^{(0)} \\ \phi^{(0)} & p^{(0)}_\phi \end{pmatrix}. \hfill (2.12)$$

In the case of quadratic potential (1.3), authors of [23] have offered the description of inflationary dynamics in the method of parametric attractor in the scaled phase plane of coordinate–momentum, that is equivalent to the regime of slow rolling in the region of its applicability\(^3\). In this approach the phase trajectory of inflaton in terms of scaling variables\(^4\)

$$x = \sqrt{\frac{4 \pi}{3} \frac{\dot{\phi}}{m_{\text{Pl}} H}}, \quad y = \sqrt{\frac{4 \pi}{3} \frac{m \phi}{m_{\text{Pl}} H}},$$  \hfill (2.13)

\(^3\)The generalization of approach of parametric attractor to the case of potential (2.3) has been performed in [24].

\(^4\)The Friedmann equation of (2.7) takes the form of $x^2 + y^2 = 1$. 

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has got stable fixed points, i.e. solutions of equations \( \dot{x} = \dot{y} = 0 \), which positions depend on the Hubble parameter that causes a slow driftage of field in the phase plane during the inflation. In this way, the consideration on the phase plane allows us to exactly find the moment of inflation end due to the constraint on the existence of stable fixed points, that determines \( H_{\text{min}} = \frac{2}{3} m \) as shown in paper [23].

In terms of scaling variables of (2.13) the operator of quantum annihilation is reduced to the form of

\[
\hat{a} = \sqrt{\ell} (\hat{y} + i\hat{x}),
\]

where the number of quanta

\[
\ell = \frac{3}{8\pi m} H^2 m_{\text{Pl}}^2 V_R
\]
determines the energy spectrum in the volume \( V_R \)

\[
E = m \left( \ell + \frac{1}{2} \right),
\]

and the commutator

\[
[\hat{y}, \hat{x}] = \frac{i}{2\ell},
\]

setting the uncertainty relation

\[
\delta y^2 \delta x^2 \geq \frac{1}{16\ell^2}.
\]

The energy density of zero-point modes can be easily calculated in terms of field fluctuations in the vacuum \( \delta \dot{\phi}_v^2 \) and \( \delta \phi_v^2 \),

\[
\langle 0 | \hat{T}_0^0 | 0 \rangle = \frac{1}{2} \langle 0 | (\dot{\phi}_v^2 + m^2 \phi_v^2) | 0 \rangle = \frac{1}{2} (\delta \dot{\phi}_v^2 + m^2 \delta \phi_v^2),
\]

or in notations of scaling variables

\[
\langle 0 | \hat{T}_0^0 | 0 \rangle = \frac{3}{8\pi} H^2 m_{\text{Pl}}^2 \langle 0 | (x^2 + y^2) | 0 \rangle = \frac{1}{2\ell} \rho,
\]

where we have used the Friedmann equation and elementary identities for the vacuum

\[
\langle 0 | \hat{x}^2 | 0 \rangle = \delta x_v^2 = \frac{1}{4\ell}, \quad \langle 0 | \hat{y}^2 | 0 \rangle = \delta y_v^2 = \frac{1}{4\ell}.
\]

As we should expect, the energy density of zero-point modes in (2.20) does not depend on time, if we consider the number of quanta in a given physical volume \( V_R \), so that \( \ell \sim H^2 \sim \rho \) in accordance to (2.15).

Note that the energy density of vacuum is composed of contributions by not only zero-point modes of homogeneous field of inflaton but also by zero-point modes of secondary quantized inhomogeneous field, too, as well as by zero-point modes of all physical fields. Then, components of averaged energy-momentum tensor of vacuum fields diverge and one has to regularize infinities. If for the regularization one uses a method conserving the isotropy of space-time, for instance, a cut off four-momentum in four-dimensional Euclidean space, then the contribution of zero-point modes certainly results in the energy-momentum tensor proportional to the metrics, as it should be in the case of cosmological constant. Otherwise, if one uses a regularization conserving only the three-dimensional spatial isotropy, for instance, a cut off momentum of zero-point modes, then the renormalization of temporal and spatial
components of energy-momentum tensor has to be considered independently, i.e. it should involve two arbitrary constants. In the last case, the requirement of space-time isotropy allows us to relate these two constants of renormalization so that the vacuum tensor of energy-momentum becomes proportional to the metrics. Therefore, we get the same result as it has been obtained in the case of isotropic four-dimensional regularization (see also discussions in [25]). In our approach the finite density of vacuum energy is determined as the energy density calculated in the framework of canonical quantization of homogeneous inflaton field:

\[ \rho_{\Lambda}^{\text{bare}} = \langle 0 | \hat{T}_{00} | 0 \rangle. \tag{2.21} \]

At the inflation start, we suggest that the field and its rate of change fluctuate eventually, i.e. there is no correlation of its average values

\[ K = \langle xy + yx \rangle - 2 \langle x \rangle \langle y \rangle = 0. \]

In quantum mechanics, this situation can be realized for states minimizing the uncertainty relation for two operators, i.e. for generic coherent states being eigen-vectors for the operator

\[ \hat{\beta} = \frac{1}{2} \left( \frac{\hat{y}}{\delta y} + i \frac{\hat{x}}{\delta x} \right), \]

so that \( \hat{\beta} | \beta \rangle = \beta | \beta \rangle \) and \( K \equiv 0 \) for such the states. Thus, we conclude that the initial state of Universe before the inflation has corresponded to the generic coherent state with a maximal probability.

It is natural to put fluctuations of kinetic and potential energies for the inflaton to be comparable to each other at the inflation start or installation, hence, with a high probability \( \delta x_{\text{ins}}^2 \sim \delta y_{\text{ins}}^2 \). Below we will consider quantum equations of evolution and see that the field evolves slowly, while the evolution of its rate is very rapid. Therefore, a time, when fluctuations of scaling variables were exactly equal to each other \( \delta x^2 = \delta y^2 \) in practice coincides with the inflation start, when the fluctuations were comparable. Thus, with a high accuracy of leading approximation we can put \( \delta x_{\text{ins}}^2 = \delta y_{\text{ins}}^2 \).

Under the condition of equal fluctuations \( \delta x = \delta y \) the operator \( \hat{\beta} \) transforms into the operator annihilating the oscillator quanta \( \hat{\alpha} \), while the generic coherent state tends to the coherent state of oscillator \( | \alpha \rangle \).

In the oscillatory coherent state with a complex parameter of average values \( \alpha = \sqrt{\ell} (y + ix) \) we get \( \hat{\alpha} | \alpha \rangle = \alpha | \alpha \rangle \), and the fluctuations of scaling variables on the phase plane are determined by the average amount of quanta

\[ \delta x^2 = \delta y^2 = \frac{1}{4 \ell}, \tag{2.22} \]

exactly the same as in the vacuum. Therefore, the initial state of inflaton has got the vacuum fluctuations, that agrees with the representation about the inflation appearance from the state, which contains nothing or almost nothing except the virtual field and its fluctuations, of course.

The probability versus the number of quanta \( k \) is Poisson’s distribution with the average value \( \ell \):

\[ w_k = e^{-\ell} \frac{\ell^k}{k!}, \tag{2.23} \]
so that the probability of zero number of field quanta, i.e. the probability of vacuum, is equal to

\[ w_{\text{vac}} = e^{-\ell}. \]  

Note that the energy can be expressed in the form

\[ E = \rho_{\text{ins}} V_R + \rho_{\text{bare}}^\Lambda V_R, \]

where \( \rho_{\text{bare}}^\Lambda \) is the bare density of zero-point modes, \( \rho_{\text{ins}} \) is the energy density of inflaton, corresponding to the installation of inflation in the coherent oscillatory state of inflaton. Comparing with (2.16), we find

\[ \rho_{\text{bare}}^\Lambda = \frac{1}{2\ell} \rho_{\text{ins}}. \]  

2.2 Evolution of coherent state

Considering the quantized field, we have to take into account for that the scale factor is the classical quantity. Therefore, the equation of evolution for the Hubble parameter should be written in average, i.e.

\[ \dot{H} = -4\pi G \langle \dot{\phi}^2 \rangle. \]  

Introducing \( z = m/H \) and the derivative with respect to e-folding of scale factor \( N = \ln a/a_{\text{ins}} \),

\[ f' \equiv \frac{df}{dN} = \frac{\dot{f}}{N} = \frac{\dot{f}}{H}, \]

where \( a_{\text{ins}} \) is the scale factor at the inflation start, we get the equation of driftage

\[ z' = 3\langle \dot{z}^2 \rangle z. \]  

For the scaling variables on the phase plane we find

\[ \langle \dot{\hat{x}}^2 \rangle' = -6\langle \dot{\hat{x}}^2 \rangle + 6\langle \dot{\hat{x}}^2 \rangle^2 - 2z\langle \dot{\hat{x}} \rangle \langle \dot{\hat{y}} \rangle, \]

\[ \langle \dot{\hat{y}}^2 \rangle' = 6\langle \dot{\hat{x}}^2 \rangle \langle \dot{\hat{y}}^2 \rangle + 2z\langle \dot{\hat{x}} \rangle \langle \dot{\hat{y}} \rangle, \]

wherein we has taken into account for the coherent states,

\[ \langle \hat{x} \hat{y} + \hat{y} \hat{x} \rangle = 2\langle \hat{x} \rangle \langle \hat{y} \rangle. \]

It is easy to get \( \langle \dot{\hat{x}}^2 + \dot{\hat{y}}^2 \rangle' \equiv 0 \) in accordance to the Friedmann equation in average.

The inflation corresponds to stable fixed points of system (2.28), (2.29), when \( \langle \dot{\hat{x}}^2 \rangle' = \langle \dot{\hat{y}}^2 \rangle' = 0 \), which is possible only if

\[ z^2 \geq z_{\text{min}}^2 = 36 \delta x^2 (1 - \delta x^2) \approx \frac{9}{\ell}, \]

i.e.

\[ H^2 \leq \frac{\ell}{9} m^2 = H_{\text{max}}^2, \]

that can be easily derived by expressing \( z \) from the conditions of fixed points and considering this real parameter as the function of averaged values of scaling variables.

Thus, the canonical quantization of homogeneous inflaton field allows the existence of inflation regime in the coherent state, but the quantum fluctuations restrict the region of
inflation development under (2.31). This is caused by that the average rate of field change \( \langle \dot{x} \rangle \), corresponding to the fixed point in classics, can be essentially less than the quantum fluctuations of this rate. At \( z = z_{\text{min}} \) one get the equality \( \langle \dot{x}^2 \rangle = \delta x^2 \), while at \( z^2 \gg 36 \delta x^2 \) the classical limit is reached.

The maximal value of Hubble constant in (2.31) refers to the inflation installation in the oscillatory coherent state of inflationary field. It straightforwardly means that

\[
\rho_{\Lambda}^{\text{bare}} = \frac{1}{48\pi} \frac{m^2}{M_{\text{Pl}}^2},
\]

Then, the known estimate of inflaton mass in (2.4) determines the energy scale of bare density of vacuum energy \( \Lambda \approx 4 \cdot 10^{15} \text{ GeV} \). Such the scale is quite natural for the particle physics in models of Grand Unification \([17, 18]\). In this way, the contribution of bare cosmological constant in the energy density during the inflation is suppressed as \( 1/\ell \ll 1 \), so that it is inessential at that times.

If we put the maximal accessible value of Hubble parameter to be determined by the plateau height in the scenario of “new inflation”, i.e.

\[
H_{\text{max}}^2 = \frac{\pi}{3m_{\text{Pl}}^2} m^2 v^2,
\]

then

\[
\ell = \frac{3\pi v^2}{m_{\text{Pl}}^2}.
\]

### 2.3 Evolution of fluctuations

At nonzero correlations of scaling variables on the phase plane, equations for the field fluctuations take the form

\[
(\delta y^2)' = 6\langle \dot{x}^2 \rangle \delta y^2 + z(\langle xy + yx \rangle - 2\langle x \rangle \langle y \rangle),
\]

\[
(\delta x^2)' = -6\langle \dot{y}^2 \rangle \delta x^2 - z(\langle xy + yx \rangle - 2\langle x \rangle \langle y \rangle).
\]

Accounting for the evolution law of Hubble parameter, we deduce that for the coherent states the fluctuations of field and potential energy slowly grow with the decrease of Hubble constant as

\[
\delta y^2 = \delta y_{\text{ins}}^2 \frac{H_{\text{ins}}^2}{H^2},
\]

while the fluctuations of field rate and kinetic energy dominantly depend on the volume \( V_R = V_{R,\text{ins}} a^3 / a_{\text{ins}}^3 \), so that

\[
\delta x^2 = \delta x_{\text{ins}}^2 \frac{H_{\text{ins}}^2}{H^2} \frac{V_{R,\text{ins}}^2}{V_R^2}.
\]

Since the evolution of kinetic energy fluctuations is exponential with respect to the amount of e-foldings \( N \sim \ln a \), while the evolution of field fluctuations is the driftage, the moment of fluctuations equality \( N_* \), when \( \delta x^2 = \delta y^2 \), a little bit differs from the moment of inflation start \( N_{\text{ins}} \), i.e. \( |N_{\text{ins}} - N_*| \ll N_{\text{tot}} \). Therefore, the installation of oscillatory coherent state of inflationary field to the leading approximation practically coincides with the start of inflation. This fact is enough for our estimates. However, we have to remember, that further evolution of inflaton state hardly distorts the wave package in comparison with the moment, when this package was the oscillatory coherent state.
The total amount of e-folding during the existence of stable fixed point

\[ N_{\text{tot}} = \int_{z_{\text{min}}}^{z_{\text{max}}} \frac{dz}{3 \langle \dot{x}^2 \rangle z} \]

can be easily calculated in the limit \( z_{\text{min}} \ll z_{\text{max}} \), since \( \langle x^2 \rangle = \delta x^2 + \langle x \rangle^2 \) almost instantaneously reaches the classical limit \( \langle x \rangle^2 \approx z^2/9 \) because of rapid decline of kinetic energy fluctuations. Thus, for the potential under consideration in the leading approximation we deduce

\[ N_{\text{tot}} = \frac{1}{6} \ell. \]  

(2.36)

So, the field fluctuations restrict the total amount of inflation\(^5\).

### 3 Estimates

Empirically

\[ \rho_\Lambda = \frac{3}{8\pi} H_0^2 m_{\text{Pl}}^2 \Omega_\Lambda, \]

(3.1)

while in accordance with the data of Planck collaboration [21] the modern value of Hubble constant is equal to \( H_0 = 67.3 \pm 1.2 \text{ km sec}^{-1}\text{Mpc}^{-1} \), and the fraction of cosmological constant in the energy balance of flat Universe equals \( \Omega_\Lambda = 0.685 \pm 0.018 \pm 0.018 \).

On the other hand, the bare value of vacuum energy density in (2.32) results in the estimate

\[ \ell = \ln \frac{\rho_{\Lambda, \text{bare}}}{\rho_\Lambda} = \ln \frac{1}{18} \frac{m^2}{H_0^2 \Omega_\Lambda} \approx 250 \gg 1. \]

(3.2)

According to (2.33) the vacuum expectation value of inflaton equals

\[ v \approx 5.2 m_{\text{Pl}}. \]

(3.3)

In this way, we have to note that a further refinement of estimates needs to make a thorough numerical analysis in more realistic models, say, with potential (2.3), in comparison with the modern data [21] (see reviews in [27, 28]).

Nevertheless, basic physical effects considered in this paper are model independent: the observed contribution of vacuum energy density is suppressed by the factor of \( \exp\{-\ell\} \) in comparison with the initial bare cosmological constant, where \( \ell \) is the average number of quanta in the oscillatory coherent state of inflationary field, and the inflation dynamics is feasible only in the restricted region of energy density because of quantum fluctuations of the field. An application of particular model allows us to make comparison with empirical values and to pose constraints of confidence for a given model of inflation\(^6\).

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\(^5\)Empirical constraints on \( N_{\text{tot}} \) were studied in [26].

\(^6\)In particular, a primary spectrum of spatial inhomogeneity in the energy density for the simplest model of quadratic potential (1.3) disagrees to the measured angular anisotropy of cosmic microwave background radiation at the level of confidence with two standard deviations [21]. In addition, the total amount of inflationary e-folding \( N_{\text{tot}} \) in (2.36) is tensely small.
4 Conclusion

Our result cardinally changes the point of view onto the problem of cosmological constant [9, 10, 25, 29–38]: now we understand how the observed artificial scale of vacuum energy density about $10^{-3}$ eV is deduced from quite the reasonable cosmological constant with scale of the order of $10^{15}$ GeV because of the mechanism determined by the installation of initial coherent state for the inflationary field. Unfortunately, the main parameter of consideration is still empirical, though it gets the consistent value.

However, we stress that the simplest models of inflationary potential are marginally in conflict with recent Planck data, while the Starobinsky’s model [1, 2] is in perfect agreement with those data. Moreover, one has found the class of models with a nonminimal gravitational interaction of inflaton, that tends to the fiducial model by Starobinsky in the strong coupling limit [39]. We expect that the offered mechanism of cosmological constant installation can be successfully incorporated in that class of models, while the averaged number of inflaton quanta in the primary volume could be naturally fixed or predicted. Thus, the account for the homogeneous quantum fluctuations of inflationary field at the inflation start makes the inflation model for the early Universe to be more realistic. So, probably, developing the approach offered in the present paper, we will be able to prove that this actuality is dynamically inevitable.

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