The Yaglom Scaling of the Third-order Structure Functions in the Inner Heliosphere Observed by Helios 1 and 2

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Abstract

The third-order scaling law, Yaglom law, of Elsässer fluctuations in the solar wind is believed to reflect the inertial range energy cascade of the MHD turbulence and provides an approach to evaluate the cascade rate. However, the occurrence ratio with the Yaglom scaling law, the fraction of the intervals where the Yaglom linear scaling is observed, is reported to be low (0.05–0.30) in the high-latitude solar wind observed by the Ulysses spacecraft. Whether the occurrence ratio could be higher in other conditions remains unknown. Here, we analyze the occurrence of the third-order scaling in the inner heliosphere with the first 100 days of observation of the Helios 1 and Helios 2 spacecraft. We investigate 162 intervals in the leading edges and 323 intervals in the trailing edges of the high-speed streams, respectively. All of these intervals have a time duration of 9 hr. We find that in the inner heliosphere the occurrence ratio is relatively high in the leading edges (0.58) and moderate in the trailing edges (0.45). Among the data intervals with the Yaglom scaling in the leading edges, 94.7% of intervals give positive rates, while in the trailing edges 78.6% give negative rates. The variations of the occurrence ratio with various turbulence parameters are shown. The cascade rate is found to be higher than the proton heating rate calculated from the data, which have third-order scaling. These new results raise several questions related to the nature and origin of the third-order scaling law and may initiate new studies on solar wind turbulence.

Unified Astronomy Thesaurus concepts: Solar wind (1534); Interplanetary turbulence (830); Magnetohydrodynamics (1964)

1. Introduction

Turbulence is inherently multiscale, serving as a channel for the energy cascade through nonlinear interactions (Verscharen et al. 2019) and playing an important role in the origin and evolution of the solar wind (Tu & Marsch 1995; Bruno & Carbore 2013). Tu et al. (1984) created a WKB-like turbulence model to explain the radial evolution of the magnetic power spectra, and Tu (1988) found that the cascade rate obtained from the WKB-like turbulence model accounted for the heating rate in the fast solar wind observed by the Helios spacecraft. Wu et al. (2020) proposed that the energy is injected into the inertial range by the low-frequency break sweeping mechanism and found the energy supply rate consistent with the observed heating rate in the slow solar wind. Wu et al. (2021a) verified this consistency in the fast solar wind. These studies build a comprehensive connection scenario between turbulence and heating: The energy is supplied by the energy reservoir in the energy-containing range, cascades into the inertial range from large scales to small scales, and ultimately dissipates into heat. The turbulent dynamics in the inertial scale remains one of the intricate questions about space plasma physics.

Solar wind turbulence is often described in the framework of magnetohydrodynamics (MHD). The von Kármán–Howarth equation (von Karman & Howarth 1938) describes the time evolution of the correlation functions in homogeneous hydrodynamic turbulence. The Kolmogorov four-fifths law for the third-order structure functions follows immediately from the von Kármán–Howarth equation. Politano & Pouquet (1998a, 1998b) extended the equation and derived the appropriate equations for incompressible isotropic MHD in terms of Elsässer variables, from which the Yaglom four-thirds scaling law for the mixed third-order structure functions follows under the assumption of incompressibility, stationarity, homogeneity, and isotropy. The von Kármán decay rate is used to evaluate the energy supply rate by the energy-containing eddies under the phenomenological similarity decay theory for the energy and similarity scale and is found to be comparable to the heating rate in the solar wind (Wu et al. 2022). The Yaglom scaling law, however, represents an exact relation in the inertial range for the MHD turbulence without a phenomenological approximation. The linear scaling of the mixed third-order structure functions within the timescale of 1–100 hr is observed by Ulysses in a significant fraction of the periods, which indicates the existence of a well-defined inertial energy cascade range in plasma turbulence (Sorriso-Valvo et al. 2007). The Yaglom scaling law is further verified in some samples of both the ecliptic (MacBride et al. 2008; Smith et al. 2009; Marino et al. 2011) and polar wind (Marino et al. 2012). The range of scales generally extends from a few minutes up to 1 day or more, which is larger than the usual range of scales where Kolmogorov’s relation has been observed, say up to about a few hours (Carbone 2012).

The Yaglom scaling law has been applied to solar wind at different locations measured by various spacecraft missions, including ACE/Wind/TEHIS at 1 au (Stawarz et al. 2009, 2010, 2011; Coburn et al. 2012, 2014; Hadid et al. 2017; Sorriso-Valvo et al. 2021), Ulysses at high latitude in the
fast solar wind (Sorriso-Valvo et al. 2007; Marino et al. 2008, 2012), PSP from 0.17 to 0.25 au (Bandyopadhyay et al. 2020; Hernández et al. 2021), and Cluster at 1 au using multispacecraft techniques (Osman et al. 2011). It was developed from efforts to extend the analysis to compressive fluctuations (Carbone et al. 2009a; Banerjee & Galtier 2013; Banerjee et al. 2016; Andrés and Sahraoui 2017; Andrés et al. 2021; Simon & Sahraoui 2021), to the anisotropic form (MacBride et al. 2008; Carbone et al. 2009b), and with velocity shear (Wan et al. 2009; Stawarz et al. 2011). The Yaglom scaling law provides an approach to estimate the energy cascade rate and opens a window to study the dependence of the energy cascade rate on varying solar wind conditions. Smith et al. (2009) found that the cascade rates vary with the cross-helicity and become negative when the cross-helicity is close to 1, which was expanded by Stawarz et al. (2010) to larger energy ranges. Stawarz et al. (2011) found that the negative cascade rates turn positive after taking the shear effect into account. Vásquez et al. (2018) argued that the negative cascade rates are organized by the inward pseudo-energy, which is generally a minor component of energy. They found that this inward pseudo-energy is small within the rarefactions that have decreasing wind speed. Stawarz et al. (2009) found that the energy cascade rate is consistent with the inferred proton heating rate under the assumption of a radial temperature profile (Vásquez et al. 2007) at 1 au. Marino and Stawarz (2008) found that the energy cascade rate contributes to 5%-100% of the heating rate in the outer heliosphere from 3 to 4 au.

The Yaglom scaling law is not present everywhere in the data sets (Sorriso-Valvo et al. 2007). Marino et al. (2012) analyzed the occurrence of the Yaglom scaling law in the high-latitude solar wind using the Ulysses measurements. They calculated the occurrence ratio as the fraction of 11 day intervals where the Yaglom scaling is observed (in their paper, the occurrence ratio is indicated by the “scaling ratio”) and found that the occurrence ratio is smaller at higher latitudes and closer to the Sun (about 10%) and increases considerably away from the poles and far from the Sun (approximately 30%). However, the occurrence of the Yaglom scaling and how the occurrence ratio varies with the solar wind conditions in the inner heliosphere remain unknown. This information is critical to interpreting the energy cascade process in the solar wind turbulence because the solar wind conditions may influence not only the cascade rate but also the occurrence ratio. The comparison of the energy cascade rate with the observed heating rate calculated directly from the radial profiles has not been done for the solar wind in the inner heliosphere before.

In this study, we utilize the Helios measurements in the inner heliosphere from 0.3 to 1 au and analyze the mixed third-order structure functions. We focus on the occurrence of the Yaglom scaling and present the occurrence ratio in the leading edge and trailing edge. The variations of the occurrence ratio with turbulent parameters are also shown. We also compare the energy cascade rate with the observed heating rate. This paper is organized as follows. In Section 2, we describe the Helios measurements and the calculation of the mixed third-order structure functions, the turbulent parameters, and the observed heating rate. In Section 3, we present typical behaviors of the third-order structure functions and show the occurrence ratio variations in different conditions. In Section 4, we draw our conclusions and discuss the questions related to the nature and origin of the Yaglom scaling law.

2. Data and Method

Here, we use the first 100 days of observations from the Helios 1 and Helios 2 primary missions. The magnetic data $B$ are measured by the fluxgate magnetometers (Musmann et al. 1975) with a maximum sampling rate of 8 vectors/s. The plasma data are measured by the 3D plasma analyzer (Rosenbauer et al. 1977; Marsch et al. 1982) utilizing quadrasperical electrostatic devices, which obtain a full three-dimensional proton velocity distribution every 40.5 s. The proton number density $n_p$ and proton bulk velocity $V$, as well as the perpendicular temperature $T_{\perp}$, can be evaluated from the distribution (Marsch et al. 1982; Yeo et al. 2010).

The mixed third-order structure functions are defined as

$$Y^\pm(l) = \langle l \cdot \Delta Z^\pm | \Delta Z^\pm \rangle^2,$$

where $\Delta Z^\pm(x, l) = Z^\pm(x + l) - Z^\pm(x)$ are the increments of the Elsässer variables $Z^\pm$ at position $x$ and spatial scale $l$ and $\langle \rangle$ denotes the spatial average. The Elsässer variables $Z^\pm$ are defined as

$$Z^\pm(x) = V(x) \pm V_A(x),$$

where $V_A = B / \sqrt{4\pi n_p m_p}$ is the magnetic field in Alfvén units and $m_p$ is the proton mass.

The Yaglom scaling law is an exact relation derived for the mixed third-order structure functions directly from the dynamical MHD equations (Politano & Pouquet 1998a, 1998b), similar to Yaglom’s four-thirds law for the scalar-field convection in hydrodynamic turbulence (Yaglom 1949). Under the assumption of homogeneity and isotropy, the Yaglom scaling law is obtained as

$$Y^\pm(l) = \frac{4}{3} \epsilon^\pm m^l,$$

where $\epsilon_m^\pm$ is the energy cascade rate for the respective Elsässer energies per unit mass in the inertial range. The total cascade rate $\epsilon_m = (\epsilon_m^+ + \epsilon_m^-)/2$. For single-spacecraft observations, the mixed third-order structure functions are calculated with varying temporal lags $\tau$. The solar wind velocity $V_{sw}$ is used to interpret temporal lags as spatial lags through the Taylor hypothesis (Taylor 1938), $l = -V_{sw}\tau$ and $l = V_{sw}/V_{sw}$. Therefore, the Yaglom scaling law for the total third-order structure function $Y = (Y^+ + Y^-)/2$ can be written as

$$Y(\tau) = \frac{4}{3} \epsilon_m V_{sw} \tau.$$

This relation is a fundamental relation for MHD turbulence describing the energy cascade and provides a possible approach to estimate the energy cascade rate $\epsilon_m$ in the solar wind turbulence.

We divide the data into 9 hr intervals and require that the data gap is less than 10%. We obtain 237 intervals from the Helios 1 observations and 248 intervals from the Helios 2 observations. We calculate $Y(\tau)$ for each interval, here $\tau = \delta$, $1\delta$, $2\delta$, $3\delta$, $300\delta$ and $\delta = 81$ s. We calculate the positive (negative) occurrence ratio as the fraction of intervals where a linear scaling and positive (negative) values of $Y(\tau)$ are found. We calculate the occurrence ratio for the intervals in the fast solar wind ($V_{sw} > 600$ km s$^{-1}$) and in the slow solar wind ($V_{sw} < 400$ km s$^{-1}$), respectively. We also separate the intervals into the leading-edge group with increasing wind speed and in the
trailing-edge group with decreasing wind speed. We calculate the occurrence ratio for the two groups, respectively. We analyze the variation of the positive (negative) occurrence ratio with the radial distance \( r \), the solar wind speed \( V_{sw} \), and those dimensionless parameters to describe the state of solar wind turbulence (Tu & Marsch 1995; Wang et al. 2020): the normalized cross-helicity \( \sigma_c \), the normalized residual energy \( \sigma_r \), the correlation coefficient \( C_{vb} \) between \( \delta v_A \) and \( \delta v \), and the Alfvén ratio \( r_A \). They are defined as

\[
\sigma_c = 2 \frac{\langle \delta v \cdot \delta v_A \rangle}{\langle \delta v^2 \rangle + \langle \delta v_A^2 \rangle},
\]

\[
\sigma_r = \frac{\langle \delta v^2 \rangle - \langle \delta v_A^2 \rangle}{\langle \delta v^2 \rangle + \langle \delta v_A^2 \rangle},
\]

\[
C_{vb} = \frac{\langle \delta v \cdot \delta v_A \rangle}{\sqrt{\langle \delta v^2 \rangle \langle \delta v_A^2 \rangle}},
\]

\[
r_A = \frac{\langle \delta v_B^2 \rangle}{\langle \delta v_A^2 \rangle},
\]

where \( \delta v_A \) is the magnetic field fluctuations in Alfvén units and \( \delta v \) is the velocity fluctuations. In the following, we use \( C_{vb}' \) instead of \( C_{vb} \) so that the positive (negative) sign of \( C_{vb}' \) corresponds to the outward (inward) sense of the fluctuations.

For those intervals with linear scaling, we perform a linear fit for \( Y \) in the range 800 s < \( r \) < 8000 s, and the obtained slope is used to calculate \( \epsilon_m \). The positive (negative) values of \( Y(\tau) \) lead to a positive (negative) cascade rate. The positive and negative cascade rates are associated with direct and inverse cascades, respectively, and we consider them two different groups of intervals with Yaglom scaling. We compare the cascade rate with the proton heating rate separately for each group. The proton heating rate per unit volume is estimated using the left-hand side of the steady Chew, Goldberger, and Low (CGL) equation for the perpendicular pressure with nonzero heat flux (Chew et al. 1956; Sharma et al. 2006; Wu et al. 2020), corresponding to the gradient of magnetic moments. Assuming \( T_{1,0} = T_{0,0} r^{\alpha_n} \), \( n_p = n_p r^{\alpha_n} \), and \( B = B_0 r^{\alpha_B} \), the empirical heating rate can be obtained as

\[
H(\tau) = \frac{V_{sw} n_p \rho \kappa}{1 \text{ au}} \frac{T_{1,0}}{(\alpha_T - \alpha_B) r^{\alpha_n + \alpha_B - 1}},
\]

where \( \kappa \) is the Boltzmann constant. We obtain the radial heating rate profiles using the fitted parameters \( (\alpha_0, \alpha_1) \) for the relationship \( \alpha = \alpha_0 r^{\alpha_1} \) for the intervals with a positive (negative) cascade rate, respectively, where \( \alpha \) represents \( n_p, B \), and \( T_{1,0} \). In order to compare with the heating rate, we transfer \( \epsilon_m \) per unit mass into \( \epsilon \) per unit volume with \( \epsilon = n_p m_p \epsilon_m \).

3. Results

Figure 1 illustrates four typical behaviors of the third-order structure functions \( Y(\tau) \) within 81 s < \( r \) < 24,300 s. In Figure 1(a), \( Y(\tau) \) presents a linear scaling, and the linear fit gives a positive cascade rate \( \epsilon = 3.95 \times 10^{-16} \text{ W m}^{-3} \). This behavior indicates a direct energy cascade from large scales to small scales, which is reasonable as a channel for the turbulent heating of the solar wind. In Figure 1(b), the linear scaling of \( Y(\tau) \) is very clear, and the values of \( Y(\tau) \) are all negative, which leads to a negative cascade rate, \( \epsilon = -1.80 \times 10^{-16} \text{ W m}^{-3} \). The negative cascade rate suggests an inverse energy cascade from small scales to large scales. However, the inverse cascade is not yet understood, raising questions, especially on what serves as the energy reservoir and where the energy goes. Figure 1(c) shows that the sign of \( Y(\tau) \) varies from negative to positive and from positive to negative, but with an overall linear scaling for \( \log |Y(\lg \tau)| \). The cascade rate cannot be determined if one considers the sign of the cascade rate to be meaningful. The above three behaviors of \( Y(\tau) \) have been reported in previous studies and are interpreted as evidence of the cascade in the inertial range of the solar wind turbulence. In Figure 1(d), \( Y(\tau) \) behaves differently. No linear scaling can be found within 81 s < \( r \) < 8000 s, and the values of \( Y(\tau) \) scatter. Within 8000 s < \( r \) < 24,300 s, \( Y(\tau) \) can be linearly fitted in log–log space but not in the linear space. The cascade rate cannot be determined for \( Y(\tau) \), which behaves like Figure 1(d). In the following, we focus on the analyses of the behaviors in Figures 1(a) and (b), which allow us to determine the energy cascade rate.

Table 1 presents the number of intervals and the occurrence ratios in different groups. From the first row, we show that 237 intervals measured by Helios 1 are analyzed, among which 60 have a linear scaling with a positive cascade rate and 57 have a linear scaling with a negative cascade rate. Thus, the positive occurrence ratio is 0.253 and the negative scaling rate is 0.241. The second row shows that the positive occurrence ratio is 0.242 and the negative scaling rate is 0.250, measured by Helios 2. Both the positive and the negative occurrence ratios are around 0.25. The third and fourth rows are for 166 intervals in the fast wind (\( V_{sw} > 600 \text{ km s}^{-1} \)) and 104 intervals in the slow wind (\( V_{sw} < 400 \text{ km s}^{-1} \)). Both the positive and the negative occurrence ratios (0.283 and 0.311) are higher in the fast wind than those (0.240 and 0.154) in the slow wind.

We divide all the intervals observed by Helios 1 and Helios 2 into the leading-edge group (162 intervals) and the trailing-edge group (323 intervals), and the results are shown in the last two rows of Table 1. In the leading edge, the positive occurrence ratio is 0.549, and the negative scaling rate is 0.031. Among these intervals with linear scaling, 94.7% have a positive cascade rate. In the trailing edge, the positive occurrence ratio is 0.096, and the negative scaling rate is 0.353. Among these intervals with linear scaling, 78.6% have a positive cascade rate. These results indicate that the Yaglom scaling is strongly affected by the velocity gradient, and the direct cascade tends to occur in the compression region while the inverse cascade tends to occur in the rarefaction. This can also be clearly seen from the time series of \( V_{sw} \) in Figure 2, where red represents the intervals with a positive cascade rate and blue represents the intervals with a negative cascade rate. It is obvious that the red is located mostly on the leading edges and the blue is located mostly on the trailing edges.

Figure 3 shows the variations of the positive occurrence ratio (top panels) and negative occurrence ratio (bottom panels). The positive occurrence ratio is around 0.2–0.3 and does not seem to vary with the radial distance and the solar wind speed, while it probably depends on the absolute value of \( \sigma_r \) and \( r_A \). The positive occurrence ratio decreases as \( |\sigma_r| \) increases. This trend is consistent with the results for high-latitude solar wind measured by Ulysses in Marino et al. (2012). For those intervals with \( 0 < |\sigma_r| < 0.33 \), it is 0.33 while for those with \( 0.67 < |\sigma_r| < 1 \), it is 0.20. The positive occurrence ratio increases from 0.18 to 0.44 as \( r_A \) increases from 0 < \( r_A < 0.5 \) to 1 < \( r_A < 1.5 \). The negative occurrence ratio, however, shows
variability with the radial distance and the solar wind speed. It changes from 0.15 in the slow wind to 0.31 in the fast wind. A distinct trend is presented where the negative occurrence ratios increase from 0.09 to 0.33 as $|\sigma_c|$ increases. It is 0.17 for those intervals with $0 < r_A < 0.5$ and larger for those intervals with larger $r_A$.

Figure 4 presents the distributions of the positive occurrence ratio (left) and negative occurrence ratio (right) in the $C_{\text{VB}} - \sigma_t$ intervals.
plane. We only reserve the pixel with more than 8 intervals for statistical purposes. We can see that the positive occurrence ratio is 0.24 in the upper-right pixel with the most highly Alfvénic intervals. The lower $C_{VB}$ corresponds to the higher positive occurrence ratio when $\sigma_r$ are the same, while the lower $\sigma_r$ corresponds to the lower positive occurrence ratio when $C_{VB}$ are the same. This can be interpreted as the positive scaling being the lowest when the intervals contain magnetic-velocity alignments structures (Wang et al. 2020; Wu et al. 2021b). As for the negative occurrence ratio, the variation is different. We can see that it is the highest in the upper-right pixel with the most highly Alfvénic intervals and it decreases as the $C_{VB}'$ decreases. It is still relatively large for those intervals with magnetic-velocity alignments structures.

The energy cascade rate is an estimation of the turbulent heating rate. In the left panel of Figure 5, we compare the

Figure 2. Top panel: the time series of $V_{sw}$ for the intervals used to calculate $Y$ measured in the first 100 days by Helios 1. The red lines indicate the intervals with a positive linear scaling of $Y$, similar to that in Figure 1(a), and the blue lines indicate the intervals with a negative linear scaling of $Y$, similar to that in Figure 1(b). The black lines indicate the intervals without a determined cascade rate. Bottom panel: the time series of $V_{sw}$ measured in the first 100 days by Helios 2 in the same format as the top panel.

Figure 3. Top: the variations of the positive occurrence ratio with radial distance (a1), solar wind speed (a2), $|\sigma_c|$ (a3), $r_A$ (a4) for positive linear third-order structure functions. The values of the occurrence ratio are also shown. Bottom: The variations of negative occurrence ratio with radial distance (b1), solar wind speed (b2), $|\sigma_c|$ (b3), and $r_A$ (b4) for negative linear third-order structure functions.
positive cascade rate with the observed proton heating rate of those solar winds with a positive linear scaling. Most positive cascade rates are above the heating rates at the same \( r \), whose fitted profile (red solid line) is about an order larger than the observed heating rate profile (blue solid line). The red dashed line adapted from Tu (1988) represents the cascade rate obtained from a WKB-like turbulence model (Tu 1988). In the right panel of Figure 5, we plot both the absolute value of negative cascade rates (black points) evaluated from the negative linear scaling of \( Y \) and the proton heating rate (blue solid line) calculated using the data intervals with a negative linear scaling of \( Y \). The black solid line is the linear fit of the negative cascade rates in log–log space with its standard error shown.

4. Conclusions and Discussion

In this study, we analyze the Yaglom scaling of the third-order structure functions in the inner heliosphere observed by Helios 1 and 2 for the first time. We present four typical behaviors of the third-order structure functions, two of them possessing good linearity in log–log space and the other two having scattered values. The two with linearity conform to the Yaglom scaling law and can be used to determine the energy cascade rates. We find that the occurrence ratio is 0.58 in the leading edge and 0.45 in the trailing edge. The occurrence ratio in the inner heliosphere is larger than that in the high-latitude solar wind in the outer heliosphere. We also find that 94.7% of the intervals with the Yaglom scaling have a positive cascade rate in the leading edge and 78.6% of the intervals with the Yaglom scaling have a negative cascade rate in the trailing edge. We show the occurrence of the Yaglom scaling under...
different solar wind conditions. We find that the occurrence ratio with the positive and negative Yaglom scaling have different dependencies on the turbulent parameters. The positive occurrence ratio decreases as $|\sigma_c|$ increases, and the negative occurrence ratio increases as $|\sigma_c|$ increases or the solar wind speed increases. In the $C_{\text{fl}}=|\sigma_c|$ plane, the distributions show that the positive occurrence ratio is the lowest for those intervals with magnetic-velocity alignment structures and the negative occurrence ratio is the highest for those intervals with high Alfvénity. We also provide the radial dependences of the positive cascade rates and the negative cascade rates from 0.3 to 1 au and compare them to the observed heating rate calculated using the same data intervals. We find that both the positive cascade rates and the absolute values of the negative cascade rates are higher than the observed heating rates, suggesting turbulence cascade could provide enough energy to proton heating.

Our results have raised several questions related to the nature and origin of the third-order scaling law. First, we find that the Yaglom scaling law tends to give a positive cascade rate in the leading-edge compression and a negative cascade rate in the trailing-edge rarefaction. We should note that the Yaglom scaling law is derived under the hypotheses of incompressibility, stationarity, homogeneity, and isotropy. These conditions might be violated in the leading-edge compression and trailing-edge rarefaction. However, occurrence ratios as high as 0.58 and 0.45 indicate that despite the difficulties in constructing a rigorous description for the solar wind turbulence, a simplified phenomenological conjecture of the energy cascade can be obtained. Compression and rarefaction could be the reason for the larger occurrence ratio here than in the high-latitude solar wind found by Marino et al. (2012) and for the larger cascade rate than that obtained from the theoretical turbulence model (Tu 1988). The Yaglom scaling law has been investigated by taking the break in compressibility, anisotropy, stationarity, and the homogeneity into account. How the occurrence ratio varies with these conditions and how these conditions affect the behavior of the third-order structure functions await future work.

Second, the negative scaling law leads to a negative cascade rate. It is understandable that a positive cascade rate, which represents a direct energy cascade rate from large scales to small scales, can be related to solar wind heating as the turbulent source. However, a negative cascade rate means an inverse cascade from small scales to large scales. Some studies proposed that the negative cascade rate is a result of intermittent dynamics (Coburn et al. 2014; Smith et al. 2018) and using the absolute value of the energy cascade rate to evaluate the heating rate becomes customary (Hadid et al. 2018; Bandyopadhyay et al. 2020). Here we propose the possibility that the inverse cascade is a phenomenon caused by the stretch in the rarefaction, similar to the inverse cascade in thick turbulent flows resulting from the suppression of vertical motions by strong planar vortex (Xia et al. 2011) and the inverse cascade in Jupiter’s atmosphere (Young & Read 2017). However, two critical questions arise: What serves as the energy source for the inverse cascade, and where does the energy go? This may initiate new studies on solar wind turbulence in the future.

Third, we find linearity for both positive and negative third-order structure functions within 800 s < $\tau$ < 8000 s. The scale 8000 s is approximately larger than or at least comparable to the commonly accepted inertial range scales in the solar wind whose position is determined by the phenomenological power spectrum. However, as Sorriso-Valvo et al. (2010) argued, a formal definition of the inertial range of a turbulence cascade requires the linear scaling of the third-order structure functions. On one hand, the reason why linearity exists at such large scales stimulates the reinspection of the traditional inertial range. On the other hand, how the third-order structure functions behave within smaller scales in the inner heliosphere requires further investigation, and PSP and Solar Orbiter measurements with higher time resolution provide good opportunities for answering this question.

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