Non–decoupling, triviality and the $\rho$ parameter

Kenichiro Aoki$^1$ and Santiago Peris$^2$

$^1$Department of Physics  
University of California at Los Angeles  
Los Angeles, CA 90024–1547

$^2$Department of Physics  
The Ohio State University  
Columbus, OH 43210–1106

Abstract

The dependence of the $\rho$ parameter on the mass of the Higgs scalar and the top quark is computed non–perturbatively using the $1/N_c$ expansion in the standard model. We find an explicit expression for the $\rho$ parameter that requires the presence of a physical cutoff. This should come as no surprise since the theory is presumably trivial. By taking this cutoff into account, we find that the $\rho$ parameter can take values only within a limited range and has finite ambiguities that are suppressed by inverse powers of the cutoff scale, the so called “scaling–violations”. We find that large deviations from the perturbative results are possible, but only when the cutoff effects are also large.
1. Introduction

In the standard model\[1\], all particles acquire masses through their interaction with the vacuum. In perturbation theory, these masses are proportional to the corresponding coupling constant times the vacuum expectation value of the scalar field. For a fixed vacuum expectation value, the masses are larger only if the coupling constants are stronger. In this situation, the decoupling theorem does not apply\[2\], and there exists the interesting possibility of having physical effects at low energy that do not vanish as a particle becomes heavier, called non–decoupling effects. The most celebrated example of such an effect is the so called $\rho$ parameter\[3]\[4], which not only does not vanish, but grows with the mass of the top quark or the Higgs boson. The $\rho$ parameter measures the relative strength of the neutral current to the charged current interactions at low energy, as measured, for instance, in neutrino scattering experiments.

The top quark and the Higgs scalar are the only particles in the standard model that are yet to be found. At one loop, the contribution to the $\rho$ parameter from the top grows quadratically with its mass as

$$\rho_{\text{top}} \approx 1 + \frac{3}{(4\pi)^2} \sqrt{2} G_F m_t^2$$

where $m_t$ is the top mass and $G_F$ is the Fermi constant. However in the case of the Higgs, the contribution has only a logarithmic dependence on its mass in accordance with the screening theorem\[5] and reads

$$\rho_{\text{Higgs}} \approx 1 - \frac{3}{4} \frac{g'^2}{(4\pi)^2} \log\left(\frac{M_H^2}{M_W^2}\right)$$

where $g'$ is the U(1)$_Y$ weak hypercharge gauge coupling, $M_H$ is the Higgs mass and $M_W$ is the $W$ mass.

As $m_t$ and $M_H$ grow, so do their respective contributions to $\rho$. Since $\rho$ is measured to be unity to roughly a percent, this in principle can place bounds on $m_t$ and $M_H$. In practice, it is only the top contribution that grows fast enough to violate the experimental constraint within the range of applicability of perturbation theory, under which (1.1) was derived. This gives the ubiquitous bound of $m_t \lesssim 200 GeV$\[6] quoted in the literature. The Higgs contribution grows extremely slowly and by the time it reaches the experimental limit, the Higgs mass is far too large to be in the perturbative regime, so that (1.2) does not give any useful bound on $M_H$. The fact that $\rho$ grows with $m_t$ and $M_H$ is a manifestation of non–decoupling.

An a priori unrelated feature of the standard model is the fact that there is evidence that the non–gauge sector of the theory is “trivial”. This, very succinctly, means that the theory has a built–in cutoff and the Higgs\[7]\[8]\[9]\[10] and the top\[11]\[12]\[13] masses have theoretical upper bounds set by requiring the internal consistency of the theory.

In this paper we would like to study these non–decoupling effects taking the $\rho$ parameter as an example, and address the question of what happens to the top and the Higgs contributions to the $\rho$ parameter when the couplings are so large that perturbation theory is not valid. Do they still grow with the mass of the particle, or do they saturate somehow?
How does the presence of a physical cutoff affect these contributions? It should become clear that our results are applicable to more general non–decoupling effects than just the contributions to $\rho$.

To try to answer these questions we shall use the $1/N_F$ expansion where $N_F$ is the number of components of the scalar field. (In the standard model $N_F$ is two.) The standard model needs to be generalized accordingly as we shall explain below; the scalar sector is none other than the classic $O(2N_F)$ model of \cite{7,10} and the fermion sector was generalized in \cite{12}.

The $1/N_F$ expansion entails a systematic truncation of the Schwinger–Dyson equations and is non–perturbative in the coupling constant. It resums an infinite set of diagrams. We shall see that a consequence of this resummation is a pole in the loop momentum that develops in the expression for the $\rho$ parameter. Because of this pole, were we to integrate the loop momentum from zero to infinity — as we usually do — the expression would diverge. This pole necessitates the presence of a built–in cutoff, only up to which the integration is to be performed. We identify this momentum scale with the cutoff scale obtained from the triviality arguments. This further clarifies the crucial role played by the cutoff in the theory, bringing up an intriguing connection between non–decoupling and triviality.

Of course, once a cutoff is present there will be effects that depend on the fine details of the actual cutoff procedure, often referred to as “scaling violations”. These effects are completely negligible for all practical purposes within the perturbative regime, where the cutoff is very large, since they are suppressed by inverse powers of the cutoff. In the non–perturbative regime, however, the cutoff scale is not much higher than the physical momentum scale under consideration and these scaling violations are not suppressed at all. It is only in this situation that we find large deviations from the perturbative results.

Also, the $1/N_F$ expansion for the $\rho$ parameter includes graphs that intrinsically have arbitrary number of loops in the resummation. Since the two loop computations have been performed in the standard model both for the Higgs\cite{14} and for the top\cite{15}, our results provide a non–trivial comparison of the results of the $1/N_F$ expansion against those of perturbation theory. We find that our simple expressions for the $\rho$ parameter contain the essential features of the perturbative results. Furthermore, by comparing the perturbative results to those derived using the $1/N_F$ expansion, the region where perturbation theory is valid becomes clear.

There are already calculations of the $\rho$ parameter utilizing a $1/N$–type expansion in the literature for the contributions of the Higgs\cite{16}, and the top quark\cite{17}. However, the role of the physical cutoff in radiative corrections with perturbative non–decoupling is, to the best of our knowledge, a novel feature of the present work. The presence of a physical cutoff was apparently not considered in \cite{13}; and in \cite{17}, $\rho$ was computed only to leading order in $1/N_{\text{color}}$ wherein the cutoff does not appear.

Admittedly, $N_F=2$ is not $N_F=\infty$. However, we think that our results should be reliable, at least qualitatively. The existence of a physical cutoff in the theory is the most crucial ingredient of our calculation and this is a quite firm result that has been established both using the lattice\cite{8,13} and large–$N$ methods\cite{7,10,11,12}. The need to cut off the loop momenta and the appearance of scaling violations are natural consequences of having
2. The dynamics of the Nambu–Goldstone bosons and the $\rho$ parameter

At very low energies the standard model can be described by an effective Lagrangian of the form

$$-L_{JJ} = \frac{g^2}{8M_W^2} J^+_\mu J^-_\mu + \frac{1}{2} \frac{g^2}{8M_Z^2 \cos^2 \theta_W} J^0_\mu J^0_\mu$$

$$= \frac{G_F}{\sqrt{2}} \left( J^+_\mu J^-_\mu + \frac{1}{2} \rho J^0_\mu J^0_\mu \right).$$

(2.1)

Here $J^\pm \equiv J^1 \pm iJ^2$, $J_0 \equiv J^3 - \sin^2 \theta_W J_{em}$ where $J_i$, $(i = 1, 2, 3)$ and $J_{em}$ denote the isospin and electromagnetic currents respectively. $\sin^2 \theta_W$ is the electroweak mixing angle and $G_F$ is the Fermi constant. (Here and below, we use the spacelike metric $(-1, +1, +1, +1).)$ Clearly from (2.1) one finds the following expression for the $\rho$ parameter

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} \bigg|_{\text{zero momentum}}. \quad (2.2)$$

At tree level, $\rho$ equals unity due to a global symmetry, the so–called custodial SU(2) symmetry under which $W^\pm$ and $Z/\cos \theta_W$ transform like a triplet [18]. Beyond the tree level one finds

$$\rho \approx 1 - \frac{\Pi_W}{M_W^2} + \frac{\Pi_Z}{M_Z^2} \quad (2.3)$$

where $\Pi_{W,Z}$ are the vacuum polarization functions of the $W, Z$ bosons, and the approximation $\Pi_{W,Z}/M_{W,Z}^2 \ll 1$ has been used.

As discussed in the introduction, we would like to calculate in the $1/N_F$ expansion how $\rho$ is affected by a heavy top and a heavy Higgs. Therefore, the non–perturbative effects one wants to compute are caused by the Yukawa coupling, $y$, and the quartic scalar self-coupling, $\lambda$, and have nothing to do with the gauge sector of the theory which only plays the rather passive role of transmitting these effects into the $\rho$ parameter where we can measure them. In fact, the SU(2)$_L$ and U(1)$_Y$ gauge couplings, $g$ and $g'$, are small and ordinary perturbation theory in these couplings should be an excellent approximation.

In the $1/N_F$ expansion, one always has the arbitrariness of how to generalize the initial Lagrangian for arbitrary $N_F$. The above discussion suggests that the obvious generalization to a SU($N_F)_L$ (or Sp($N_F)_L$ as in Ref. [16]) gauge theory is an unavoidable complication in this case. Our approach, following [19], will be to define the $\rho$ parameter in terms of the Nambu–Goldstone fields, making contact with what is actually measured in the standard model by first taking the limit $N_F \to 2$ and then gauging the SU(2)$_L$ group at the very end. In this way the SU(2)$_L$ gauge bosons act solely as external fields.

Let us briefly summarize then how one can compute the $\rho$ parameter in terms of the Nambu–Goldstone bosons. The relevant part of the Lagrangian is the kinetic term for these bosons and, when radiative corrections are taken into account, reads

$$-L_\chi = Z_{\chi^+} \left| \partial_\mu \chi^+ - \frac{g v}{2} W^+_\mu \right|^2 + \frac{1}{2} Z_{\chi^0} \left( \partial_\mu \chi^0 - \frac{g v}{2 \cos \theta_W} Z_\mu \right)^2 + \text{other terms} \quad (2.4)$$
where the above combination is determined by the SU(2)\(_L\) × U(1)\(_Y\) gauge invariance. The vacuum expectation value of the Higgs field \(v\) in general includes the shifts due to quantum corrections and is known to be about 246 GeV in the standard model. The “effective” masses for the \(W\) and \(Z\) are

\[
-\mathcal{L}_{\text{mass}} = Z_{\chi^+} \left( \frac{gv}{2} \right)^2 W_{\mu}^+ W_{\mu}^- + \frac{1}{2} Z_{\chi^0} \left( \frac{gv}{2 \cos \theta_W} \right)^2 Z_{\mu}^2 .
\]  

These are the masses that appear in (2.1) and therefore one can express the \(\rho\) parameter as

\[
\rho = \frac{Z_{\chi^+}}{Z_{\chi^0}} \bigg|_{\text{zero momentum}} \simeq 1 + (Z_{\chi^+} - Z_{\chi^0}) \quad \text{for} \quad (Z_{\chi^+,0} - 1) \ll 1 \quad (2.6)
\]

in terms of the wave function renormalization constants of the Nambu–Goldstone fields defined at zero momentum. We would like to emphasize that the first equality in Eq. (2.6) is exact up to contributions \(\mathcal{O}(g^2, g'^2)\). We would also like to point out here the advantage of the approach using Nambu–Goldstone bosons in the case of a possible future lattice calculation of \(\rho\), doing away with the non–Abelian gauge bosons altogether. To leading order in the SU(2)\(_L\) gauge coupling \(g\), one does not need to gauge SU(2)\(_L\) at all.\footnote{We cannot do without the U(1)\(_Y\) gauge coupling \(g'\), since this is responsible for the custodial symmetry breaking in the case of the Higgs.}

Of course, if one wished to do so, one could also obtain \(\rho\) by calculating the vacuum polarization of the gauge bosons as in Eq. (2.3) since, after all, both calculations are related by gauge invariance. We shall also do so where appropriate and, by comparing with the calculation with the Nambu–Goldstone bosons, we shall see some interesting effects in the non-perturbative regime due to the presence of the physical cutoff.

3. The Higgs contribution to the \(\rho\) parameter

3.1. The Higgs sector in the large–\(N_F\) limit

We first summarize the aspects of the Higgs sector in the large–\(N_F\) limit necessary for our purposes, following [7][10]. The Lagrangian for the scalar sector of the standard model with no gauge coupling is

\[
-\mathcal{L}_\phi = \partial_\mu \phi \dagger \partial^\mu \phi + \lambda (\phi \dagger \phi - v^2/2)^2 .
\]  

Here, \(\phi\) is in a \(N_F\) dimensional irreducible representation of SU(\(N_F\))\(_L\). Without any loss of generality, the scalar field develops a vacuum expectation value \(\langle \phi \rangle = (v/\sqrt{2} \ 0 \ 0 \ldots 0)^T\). This vacuum expectation value breaks the symmetry of the model from O(2\(N_F\)) to O(2\(N_F\) – 1). This O(2\(N_F\) – 1) unbroken symmetry rotates all the Nambu–Goldstone bosons among themselves. Therefore it plays the role of the custodial symmetry in that \(\rho\), as defined for instance in Eq. (2.3), is necessarily unity. We define \(\chi^0\) and \(\chi^-\) as
\[ \phi = ((v + H + i\chi^0) / \sqrt{2}, i\chi^-, \ldots)^T \] where \( \phi \) is a vector with \( N_F \) components. The spectrum consists of one massive real scalar and \( 2N_F - 1 \) Nambu–Goldstone bosons. The mass of the scalar, \( H \), is \( \sqrt{2\lambda v} \) at tree level.

The large–\( N_F \) limit is taken by keeping \( \lambda N_F \) and \( v^2 / N_F \) fixed while taking \( N_F \) to infinity. The quantum corrections that contribute to leading order in the large–\( N_F \) limit are the massless Nambu–Goldstone bubble graphs. The vacuum expectation value is not corrected to this order. The bare coupling constant, \( \lambda_{\text{bare}} \), needs to be renormalized and we define the renormalized coupling, \( \lambda(s_0) \), in the following manner

\[
\lambda(s_0) = \frac{\lambda_{\text{bare}}}{1 - \lambda_{\text{bare}} N_F / (8\pi^2) \ln s_0 / s_{\text{bare}}^F}.
\]

(3.2)

Here, \( s_{\text{bare}}^F \) denotes a regulator dependent quantity whose explicit expression is not needed in the subsequent analysis. The renormalized coupling constant, as defined above, has the meaning of the strength of scattering at the momentum (squared) scale \( s_0 \). The renormalization scale, \( s_0 \), is arbitrary. Another definition of the renormalized coupling, used in most lattice computations is through the physical Higgs mass in units of the vacuum expectation value. In this case, the relation between the bare and the renormalized coupling is not as explicit as the definition we use (3.2).

The propagators for the Nambu–Goldstone bosons are not modified to leading order in \( 1/N_F \). However, the propagator of the Higgs field, \( H \), receives the bubble contributions as in fig. 1 and may be expressed in terms of the renormalized parameters as

\[
D_H(p^2) = \left[ p^2 + \Sigma_H(p^2) \right]^{-1}, \quad \text{where} \quad \Sigma_H(p^2) \equiv \frac{2\lambda(s_0)\mu^2}{1 - \lambda(s_0) N_F / (8\pi^2) \ln p^2 / s_0}.
\]

(3.3)

Notice that \( \Sigma_H(p^2) \), and as a consequence \( D_H(p^2) \), are \( s_0 \)-independent. The quantum effects, roughly speaking, change the mass into a momentum dependent mass. The physical mass and width of the Higgs particle are determined by the location of the pole of the above propagator in the complex plane as \( p^2 = -(M_H - i\Gamma_H / 2)^2 \).

At the momentum scale \( s^F_{\text{triv}} \equiv s_0 \exp(8\pi^2 / (\lambda(s_0) N_F)) \), cross sections diverge. We identify this scale as the physical cutoff scale in the theory, usually called the “triviality scale”. This cutoff scale is a physical quantity that is independent of the renormalization scheme chosen. We note that the cutoff scale has a typical non–perturbative dependence on the coupling. As the renormalized coupling constant increases, at some point the model becomes physically unacceptable for at least two reasons; the cross sections at the scale of the mass of the Higgs diverge and the cutoff scale of the theory becomes as small as the mass. This leads to a bound on the Higgs mass of about 1 TeV [7][8][9]. The mass, width and the triviality scale are plotted against the renormalized coupling in fig. 2. We also plot \( 0.1 \sqrt{s_{\text{triv}}^F} \) for convenience. In the plot, the renormalization scale \( s_0 \) was chosen to be \( |p^2| = M_H^2 + \Gamma_H^2 / 4 \) where \( p^2 \) is the location of the pole of the propagator (3.3) in the complex plane. In this case, the cutoff scale \( s_{\text{triv}}^F \) becomes equal to the mass scale \( s_0 = M_H^2 + \Gamma_H^2 / 4 \) and \( M_H \) becomes equal to \( \Gamma_H / 2 \) when \( \lambda(s_0) \) is infinity.

3.2. The corrections to the \( \rho \) parameter

When the O(2\( N_F - 1 \)) custodial symmetry is unbroken, \( \rho = 1 \). The interactions in the scalar potential of the standard model do not break the custodial symmetry. The weak
hypercharge, however, does break this custodial symmetry and therefore one should expect corrections to the \( \rho \) parameter due to the scalars to be of order \( g' \), where \( g' \) is the \( U(1)_Y \) weak hypercharge gauge coupling. Therefore, we need to gauge \( U(1)_Y \) to compute the \( \rho \) parameter. The Lagrangian is

\[
-\mathcal{L}_{\phi B} = \frac{1}{4} B^2_{\mu\nu} + \frac{1}{2a} (\partial_{\mu} B_{\mu})^2 + |D_{\mu} \phi|^2 + \lambda (\phi^\dagger \phi - v^2/2)^2 \tag{3.4}
\]

where

\[
B_{\mu\nu} \equiv \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu}, \quad D_{\mu} \phi \equiv (\partial_{\mu} + ig'2B_{\mu})\phi. \tag{3.5}
\]

We let the gauge parameter \( a \) go to zero and enforce the transverse polarization, \( \partial \cdot B = 0 \), which eliminates the mixing term between \( \chi^0 \) and \( B_{\mu} \). In this gauge, the gauge boson \( B \) has mass \( M_B \equiv g'v/2 \) and the Nambu–Goldstone bosons are massless.

The leading order corrections to the Nambu–Goldstone propagators arise from the graphs in fig. [3] which we call \( \Pi^H_{\chi^0} \). As usual, one may check that the class of graphs in fig. \( 3 \) that contribute to the mass of the Nambu–Goldstone bosons cancel. Also there are contributions to the Nambu–Goldstone propagators of \( O(1/N_F) \) that do not depend on the \( U(1)_Y \) coupling \( g' \) and hence do not contribute to the \( \rho \) parameter at this order. The Nambu–Goldstone propagators become

\[
D_{\chi^+}(p^2) = \left( p^2 - \Pi^H_{\chi^+}(p^2) \right)^{-1}, \quad D_{\chi^0}(p^2) = \left( p^2 - \Pi^H_{\chi^0}(p^2) \right)^{-1} \tag{3.6}
\]

so that

\[
Z_{\chi^+}(p^2) = 1 - \Pi^H_{\chi^+}(p^2), \quad Z_{\chi^0}(p^2) = 1 - \Pi^H_{\chi^0}(p^2) \tag{3.7}
\]

where the prime denotes the derivative with respect to \( p^2 \).

A simple computation secures the following expressions for \( \Pi^H_{\chi^+}, \Pi^H_{\chi^0} \).

\[
\Pi^H_{\chi^+}(p^2) = g'^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2p^2 - (k \cdot p)^2}{((k+p)^2 + M_B^2)(k+p)^2k^2} \tag{3.8}
\]

\[
\Pi^H_{\chi^0}(p^2) = g'^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2p^2 - (k \cdot p)^2}{((k+p)^2 + M_B^2)(k+p)^2} D_H(k^2). \tag{3.8}
\]

\( D_H \) is the Higgs propagator in the large–\( N_F \) limit in (3.3). Here and below, the integrals have been continued to Euclidean space. These expressions are logarithmically divergent in the ultraviolet and they can be regularized, for instance, by the dimensional regularization.

Using (2.6), we derive the \( \rho \) parameter to leading order in the \( U(1)_Y \) gauge coupling as

\[
\rho - 1 \bigg|_{\text{Higgs}} = \frac{Z_{\chi^+(0)}}{Z_{\chi^0}(0)} - 1 = \Pi^H_{\chi^0}(0) - \Pi^H_{\chi^+}(0) + O(g'^2, g'^4, g'^2/N_F) \tag{3.9}
\]

\[
= \frac{3}{4g'^2} \int_{k^2 < \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{\Sigma_H(k^2)}{k^2(k^2 + M_B^2)(k^2 + \Sigma_H(k^2))}. \tag{3.9}
\]

The \( \rho \) parameter is expressed in terms of renormalized parameters and is independent of the renormalization scale \( s_0 \), as it should be. A crucial comment is in order; the above
integral is integrable both at \( k^2 = 0 \) and \( k^2 = \infty \). However it is not integrable in the region in between. There is a pole in the region \( s_{\text{triv}}^H < k^2 < \infty \). The reason for this disease can be traced back directly to the existence of a pole in the expression (3.3) for the Higgs self-energy, which in turn is a direct consequence of the pole in Eq. (3.2), i.e. of the Landau pole. As was discussed above, this region above the triviality scale is not physically consistent and we shall cutoff the integral at \( \Lambda^2 \), below the triviality scale \( s_{\text{triv}}^H \).

It is important to note that it is not only physically reasonable to cut off the integral at \( \Lambda^2 \), below the triviality scale \( s_{\text{triv}}^H \). This region above the triviality scale is not physically consistent and we shall cutoff the integral at \( \Lambda^2 \), below the triviality scale \( s_{\text{triv}}^H \).

The need to impose the cutoff does not appear to any order in perturbation theory.

Since the cutoff is now at a well defined finite momentum scale, the physical parameters develop explicit ambiguities that have to do with the details of the cut-off procedure which are somewhat arbitrary. These ambiguities are of order \( \mathcal{O}(p^2/\Lambda^2) \) or \( (M_H^2/\Lambda^2) \), where \( p^2 \) is the momentum scale of the process under consideration. These suppressions are essentially guaranteed by the renormalizability of the theory. These ambiguities correspond to the “scaling violations” in the corresponding lattice theory.

The \( \rho \) parameter may also be obtained computing the pieces proportional to \( \delta_{\mu\nu} \) in the self-energy of the \( W, Z \) gauge bosons in fig. 5, \( \Pi_W^H, \Pi_Z^H \). (We use the unitary gauge.) We obtain

\[
\frac{\Pi_W^H(0)}{M_{W,\text{tree}}^2} = g^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^2 + M_{W,\text{tree}}^2} D_H(k^2), \quad \frac{\Pi_Z^H(0)}{M_{Z,\text{tree}}^2} = g^2 \int \frac{d^4k}{(2\pi)^4} \frac{k^2}{k^2 + M_{Z,\text{tree}}^2} D_H(k^2).
\]

(3.10)

Here, we defined \( M_{W,\text{tree}} \equiv g v/2 \), \( M_{Z,\text{tree}} \equiv g v/(2 \cos \theta_W) \). The additional seagull contributions of fig. 5 cancel exactly in their contributions to the \( \rho \) parameter. Then up to Higgs independent contributions, one finds

\[
\rho - 1 \bigg|_{\text{Higgs}} = \left. \frac{\Pi_Z^H}{M_{Z,\text{tree}}^2} \right|_{\text{Higgs}} - \left. \frac{\Pi_W^H}{M_{W,\text{tree}}^2} \right|_{\text{Higgs}} = -3 g^2 \int_{k^2 < \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{\Sigma_H(k^2)}{(k^2 + M_{W,\text{tree}}^2)(k^2 + M_{Z,\text{tree}}^2)(k^2 + \Sigma_H(k^2))}.
\]

(3.11)

This agrees with the expression obtained using the scalars in (3.9) up to terms that do not depend on the Higgs propagator and terms that are of order \( \mathcal{O}(M_W^2/M_H^2, M_Z^2/M_H^2) \). The limit of this expression for large \( M_H \), which is what we are interested in, is the same as that of Eq. (3.9).

It is instructive to compare the results with the perturbative results. We may expand the expression for the \( \rho \) parameter obtained using the \( 1/N_F \) expansion in (3.9) in powers of \( \lambda(M_{H,\text{pert}}^2) \) as

\[
\rho - 1 \bigg|_{\text{Higgs}} = -\left( \frac{g}{4\pi} \right)^2 \left[ \frac{3}{4} \ln \frac{M_{H,\text{pert}}^2}{M_B^2} + \left( \frac{G_F M_{H,\text{pert}}^2}{16\pi} \right)^2 + \mathcal{O}(\lambda^3(M_{H,\text{pert}}^2)) \right].
\]

(3.12)
where we introduced $M_{H,pert}^2 \equiv 2\lambda(M_{H,pert}^2)v^2$. $M_{H,pert}^2$ is the mass of the Higgs particle defined as the pole of the propagator to one loop, which is sufficient for the comparison with the two loop result. We have expanded up to three loop order and kept only contributions that grow with the mass of the Higgs. We may compare this to the expression up to two loops given in [14]

$$\rho - 1\bigg|_{Higgs,2\text{-loop}} = -\left(\frac{g'}{4\pi}\right)^2 \left[\frac{3}{4} \ln \frac{M_{H,pert}^2}{M_W^2} - 5.37 \times 10^{-3} G_F M_{H,pert}^2 + \mathcal{O}(\lambda^2)\right].$$

(3.13)

When comparing these two results, we need to keep in mind that by definition, what we call “the Higgs contribution to the $\rho$ parameter” is well defined only up to Higgs mass independent terms of order $g^{'2}$. The one–loop term of course agrees, but the order $\lambda$ term seen in the two–loop result (3.13) is not present to this order in the $1/N_F$ expansion (3.12).

In the perturbative result of [14], it was remarked that the coefficient for this term has a strong cancellation and this is perhaps the reason why it is not seen in our computation. The results of the $1/N_F$ expansion are in agreement, as it should be, with the screening theorem [3].

We plot the $\rho$ parameter obtained in the $1/N_F$ expansion against the coupling $\lambda(s_0)$ and the Higgs mass in fig. 7 and fig. 8, respectively. For illustrative purposes, we use two values for the cutoff, $\Lambda/\sqrt{s_{H,\text{triv}}} = 0.5$ and $\Lambda/\sqrt{s_{H,\text{triv}}} = 0.1$. For comparison, the one and two loop results are also shown. We chose the same convention in fig. 7 for the renormalization scale $s_0 = M_H^2 + \Gamma_H^2/4$ as in fig. 2.

The $1/N_F$ result for the $\rho$ parameter agrees very well with the perturbative results when the coupling constant is small and deviates from them for larger larger values of the coupling. At some point, the cutoff scale becomes as small as the momentum scale of the mass, $M_H^2 + \Gamma_H^2/4$, at which point the results cease to make sense and the $\rho$ parameter is plotted only up to this coupling. The deviations from the perturbative results and the scaling violations are clearly visible in the plot of $\rho - 1$ against the coupling in fig. 7. However, from the plot against the mass in fig. 8, we see that the large deviations from the perturbative results and the scaling violations occur only in the region where the mass is almost at its triviality bound. The maximum of the mass of the Higgs in the $1/N_F$ expansion is $3.5v$. (See fig. 2.) Typical numbers we obtain are the following: The relative difference between $\rho - 1$ computed with the cutoffs for the integral $\Lambda/\sqrt{s_{H,\text{triv}}} = 0.1, 0.5$ is 10% at $M_H/v = 3.3$. At this point, $\lambda(s_0) = 6.9$, $\sqrt{s_{H,\text{triv}}}/v = 60$, $\Gamma_H/M_H = 0.59$ and the discrepancy from the two–loop result is few in $10^{-4}$ which is of the order of the terms not enhanced with respect to the Higgs mass which we did not compute. When the results computed for the two cutoffs differ by 1%, $M_H/v = 2.9$, $\lambda(s_0) = 4.6$, $\sqrt{s_{H,\text{triv}}}/v = 220$ and $\Gamma_H/M_H = 0.44$. We notice that there is a small region in which the $1/N_F$ result differs from the perturbative result while the cutoff effects are small. It will be interesting to see what happens when the experimental precision reaches the level needed to see the cutoff effects.

$1/N_F$ is not that small in the standard model and the rough trend of the corrections may be deduced by comparing the $1/N_F$ results to the one loop results as was discussed in [20]. The expression for the renormalized coupling constant, $\lambda(s_0)$, should be contrasted
to the one obtained by integrating the one–loop renormalization group equation, in which the only difference is that the factor $N_F$ in Eq. (3.2) is replaced by $(N_F + 4)$. At the same time, in the contribution of the Nambu–Goldstone bosons to the Higgs propagator (3.3), the factor $N_F$ should be replaced by $(N_F - 1/2)$ at one loop. The effect of this discrepancy between the two factors of $N_F$ may be summarized by saying that for a given Higgs mass, the cutoff is smaller than the estimate given by the large–$N_F$ limit. The most significant net effect is to make the physically acceptable region smaller, and it does not change the results qualitatively.

Our results seem to be in agreement with [14], where cutoff effects were not discussed. However, it is difficult to make a detailed comparison. In the region of interest, when the $\rho$ parameter differs from the perturbative results substantially, the cutoff effects are important and at some point the cutoff becomes as small as the Higgs mass.

4. The top contribution to the $\rho$ parameter

4.1. The quark–Higgs sector in the large–$N_F$ limit

The classical Lagrangian for the quark–Higgs sector when a quark is massive is

$$-\mathcal{L}_{\phi q} = -\mathcal{L}_\phi + \overline{q}_L \partial q_L + \overline{t}_R \partial t_R + y (\overline{q}_L \phi t_R + \overline{t}_R \phi^\dagger q_L)$$

(4.1)

$q_L$ and $t_R$ are irreducible representations of $SU(N_F)_L$ of dimensions $N_F$ and 1, respectively. The model has a $SU(N_F)_L \times U(1)_L \times U(1)_R$ symmetry under the following transformations

$$q_L \mapsto U_L e^{i\alpha_L} q_L, \quad t_R \mapsto e^{i\alpha_R} t_R, \quad \phi \mapsto U_L e^{i(\alpha_L - \alpha_R)} \phi, \quad U_L \in SU(N_F)_L, \alpha_L, \alpha_R \in \mathbb{R}$$

(4.2)

The vacuum expectation of $\phi$ breaks the symmetry to $SU(N_F - 1)_L \times U(1)_L + R$ and gives mass to the $t$–fermion. We rewrite the $q_L$–field for convenience as $q_L \equiv (t_L, b_1^L, b_2^L, \ldots, b_{N_F-1}^L)^T$. The spectrum consists of one massive Dirac fermion, $t \equiv t_L + t_R$, and $N_F - 1$ Weyl fermions $b_i^L$. Classically, the mass of $t$ is $yv/\sqrt{2}$. When the Yukawa coupling $y$ vanishes, one recovers the $O(2N_F) \to O(2N_F - 1)$ symmetry breaking pattern of the pure scalar theory. The Yukawa coupling breaks the custodial symmetry $O(2N_F - 1)$ and will yield corrections to the $\rho$ parameter.

The large–$N_F$ limit is taken by keeping $y^2 N_F, v^2 / N_F$ fixed as we take $N_F$ to infinity. In the large–$N_F$ limit, the only one–particle irreducible graphs that contribute are the corrections to the $t$–propagator in fig. 9. The full propagator for the $t$–field, in terms of the bare quantities, is

$$S_t(p) = \left\{ i \sqrt{A_{R,bare}(p^2)P_R + P_L} + y_{bare} v \right\}^{-1}, \quad A_{R,bare}(p^2) \equiv 1 - \frac{y_{bare}^2 N_F}{2(4\pi)^2} \ln \frac{p^2}{s_{bare}^i}$$

(4.3)

Here, $P_L, P_R$ are projection operators onto the left, right–handed fields and $s_{bare}^i$ denotes a regulator dependent quantity whose explicit expression we shall not need. In particular, there are no corrections to this order to the scalar sector of the model so that the two
sectors may be solved independently. The coupling constant $y_{bare}$ is renormalized similarly to the scalar self-coupling (cf. (3.2))

$$y^2(s_0) = \frac{y_{bare}^2}{1 - y_{bare}^2 N_F / (32\pi^2) \ln s_0 / s_{bare}^t}$$

(4.4)

where the renormalization scale $s_0$ is arbitrary. The renormalized coupling corresponds to the scattering strength at the momentum (squared) scale $s_0$.

This coupling diverges at the scale $s_{triv}^t = s_0 \exp\{32\pi^2 / (y^2(s_0) N_F)\}$ and we identify this scale $s_{triv}^t$ as the physical cutoff scale in the theory. The mass, width and the cutoff scale may be determined numerically and is identical to the Higgs case in fig. [3] provided we make the replacement $\lambda(s_0) \mapsto y^2(s_0) / 4$. (In the plot, the renormalization scale $s_0$ should be understood to be the norm of the pole of the propagator in (1.3) to agree with the conventions of the Higgs case.) For details on the solution of this model, we refer to [12]. As in the Higgs case, there is a bound on the top mass of around 1 TeV in the $1/N_F$ expansion [12]. When the custodial symmetry is not broken, the mass bound was established in [11]. Also, a considerable amount of numerical work exists in the lattice approach that supports this point of view [13], although the results should be viewed as still being preliminary.

4.2. The corrections to the $\rho$ parameter

The contribution of the top to the $\rho$ parameter is qualitatively different from that of the Higgs. The Yukawa coupling of the fermions breaks the custodial symmetry so that the correction to the $\rho$ parameter is not proportional to the gauge coupling. Furthermore, no analog of the screening theorem exists so that the correction may become an order of magnitude larger than the case of the Higgs. The leading order corrections to the Nambu–Goldstone propagator are of order $O(1/N_F)$ and arise from the graphs in fig. [10]. These graphs may be computed simply and are expressed using the renormalized parameters as

$$\Pi_{\chi^+}^t(p^2) = 2 y^2(s_0) N_c \int \frac{d^4k}{(2\pi)^4} \frac{k(k + p)}{(k + p)^2} \left[ A_R(k^2) k^2 + \hat{m}_t^2 \right]$$

$$\Pi_{\chi^0}^t(p^2) = 2 y^2(s_0) N_c \int \frac{d^4k}{(2\pi)^4} \left[ A_R((k + p)^2) k(k + p) + \hat{m}_t^2 \right] \left[ A_R((k + p)^2)(k + p)^2 + \hat{m}_t^2 \right]$$

where $N_c$ is the number of colors, $\hat{m}_t \equiv y^2(s_0) \bar{v}^2 / 2$ and $A_R(s) \equiv 1 - y^2(s_0) N_F / (32\pi^2) \ln s / s_0$.

The reader is cautioned that $\hat{m}_t$ is related, but not equal to, the mass of the top. At the same order, there is a tadpole graph for the Higgs field $H$ as in fig. [11] which leads to a shift in its vacuum expectation value. The shift in the vacuum expectation value removes the mass term from the Nambu–Goldstone fields. (See, for instance, [21].) Since this effect is momentum independent, it does not at all affect our calculation of the momentum dependent pieces below.

Using (1.3), the expression $Z_{\chi^+} = 1 - \Pi_{\chi^+}^t(0), Z_{\chi^0} = 1 - \Pi_{\chi^0}^t(0)$ and the equation (2.4) for the $\rho$ parameter, we derive

$$\rho - 1 \bigg|_{top} = y^2(s_0) N_c \int_{k^2 \ll \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{\hat{m}_t^2 \left( k^2 y^2(s_0) N_F / (32\pi^2) + \hat{m}_t^2 \right)}{k^2 (A_R(k^2) k^2 + \hat{m}_t^2)^3}.$$  

(4.6)
As in the Higgs case, the integral needs to be cut off at a scale \( \Lambda^2 \lesssim s^t_{\text{triv}} \). Then the above expression is well defined.

Just as in the Higgs case, we may compute the \( \rho \) parameter directly from the two point functions of the \( W \) and \( Z \) gauge bosons. The custodial symmetry breaking effects come both from the Yukawa coupling and the \( U(1)_Y \) weak hypercharge coupling. Since we are only interested in the leading effect which is not suppressed by the gauge coupling, we may set \( g' \) to zero in the calculation. This simplifies the computation and the two self energy graphs in fig. 12 yield the following expression for the \( \rho \) parameter

\[
\rho - 1 \bigg|_{\text{top}} = \frac{N_c}{v^2} \int_{k^2 < \Lambda^2} \frac{d^4k}{(2\pi)^4} \frac{\hat{m}^4_t}{k^2 (A_R(k^2)k^2 + \hat{m}^2_t)^2} . \tag{4.7}
\]

Unlike the Higgs case, this expression differs from the one obtained using the Nambu–Goldstone bosons in (4.6). Notice that, in general, one should not expect these two expressions to agree since our naive implementation of the cutoff \( \Lambda \) with a step function breaks gauge invariance explicitly. We do not know how to improve on this within the present non–perturbative context. If we use integration by parts and ignore the surface terms, the two expressions agree. Since the surface terms are non–vanishing only because of the existence of the finite cutoff, both forms are the same to all orders within perturbation theory. Once the cutoff effects are considered, they agree only up to terms of order \( \mathcal{O}(m_t^2/\Lambda^2) \). When the two formulas disagree by an appreciable amount, we are in the region where the scaling violations also are substantial.

We expand the expression for the \( \rho \) parameter obtained using the \( 1/N_F \) expansion in (4.6) or (4.7) in the coupling constant up to three loop order as

\[
\rho - 1 \bigg|_{\text{top}} = N_c \left[ x + N_F x^2 + N_F^2 \left( \frac{1}{2} + \frac{\pi^2}{6} \right) x^3 + \mathcal{O}(x^4) \right] , \quad \text{where} \quad x \equiv \frac{G_F m^2_{t,pert}}{8\sqrt{2}\pi^2} . \tag{4.8}
\]

with \( m^2_{t,pert} \equiv y^2(m^2_{t,pert})v^2/2 \) analogously to the Higgs case, and compare with the result up to two loops [15]

\[
\rho - 1 \bigg|_{\text{top},2\text{-loop}} = N_c \left[ x + (N_c + (19 - 2\pi^2))x^2 + \mathcal{O}(x^3) \right] . \tag{4.9}
\]

The two expressions are qualitatively in agreement. The order \( x \) term agrees and the coefficient for the order \( x^2 \) term has the same sign while the magnitude differs by 13%.

The \( \rho \) parameter may be computed numerically using the above expressions (4.7) (“NG”) or (4.8) (“gauge”) and is compared to the one and two loop results in fig. 13 and fig. 14. In the plot of \( \rho - 1 \) against the coupling in fig. 13, the renormalization scale \( s_0 \) was chosen to be the norm of the pole of the \( t \) propagator (4.3), analogously to the Higgs case. The \( \rho \) parameter obtained using the \( 1/N_F \) expansion is plotted only in the region \( m_t^2 + \Gamma^2_t/4 < \Lambda^2 \). We use two values for the cutoff, \( \Lambda/\sqrt{s^t_{\text{triv}}} = 0.5 \) and \( \Lambda/\sqrt{s^t_{\text{triv}}} = 0.1 \). Even though the contribution to the \( \rho \) parameter is an order of magnitude larger than that of the Higgs, the gross features are similar. When the coupling constant is small, all methods of computation agree, as they should. The effect of using different cutoffs, the deviation from the two loop results and the discrepancy between the calculation using the
scalars and the gauge bosons are clearly visible in the region where the coupling constant is large. The scaling violations are appreciable only when the mass of the top is also close to its maximal value in which case the triviality scale is also not much larger. There is a small region around $m_t/v \sim 2.3$ where the cutoff effects are small and the deviation from the two loop result is substantial. The $1/N_F$ expansion perhaps captures the essence of higher loop effects and it would be interesting to compare with the three loop result if such is computed. The interest in this point, however, is only theoretical, since the $\rho$ parameter is already measured to be one within a percent.

As in the Higgs case, we list some typical numbers: When the relative discrepancy between the $\Lambda/\sqrt{s_{\text{triv}}} = 0.1, 0.5$ is $10\%$, $m_t/v = 2.9$. At this point, $y^2(s_0) = 20$, $\sqrt{s_{\text{triv}}}/v = 160$ and $\Gamma_t/m_t = 0.42$. The calculation using the scalars and the gauge bosons differ by $4\%$ and the $1/N_F$ result differs from the perturbative results by $30\%$. When the relative difference between the results for the two cutoff scales $\Lambda/\sqrt{s_{\text{triv}}} = 0.1, 0.5$ is $1\%$, $m_t/v = 2.6$, $y^2(s_0) = 14$, $\sqrt{s_{\text{triv}}}/v = 670$ and $\Gamma_t/m_t = 0.30$. The scalar and the gauge calculations differ by $0.5\%$, and both differ from the two loop result by $9\%$ at this point.

5. Discussion

There is evidence that the standard model without gauge couplings needs to be defined with a physical cutoff at a finite momentum scale. This is clearly displayed in the $1/N_F$ expansion of the standard model. In this work, we computed the contribution of the Higgs and the top to the $\rho$ parameter in the standard model using the $1/N_F$ expansion. The $\rho$ parameter is not finite unless the cutoff is imposed also on the intermediate physical states. We could treat the two contributions separately since the corresponding couplings do not influence each other in the large–$N_F$ limit, apart from the fact that the cutoff of the combined theory is the lower of the two cutoffs. In particular, the contribution to the $\rho$ parameter from the Higgs and the top may just be summed.

The existence of the physical cutoff at a finite scale leads to a qualitatively different behavior from perturbation theory, in the region of strong coupling. For one thing, it leads to ambiguities in the physical predictions of the theory. These ambiguities are of the order of the mass scale over the cutoff scale. But more interestingly, the growth of the non–decoupling effect with respect to the particle mass that one observes in the perturbative regime is saturated. Heuristically, one may understand this fact by looking, for instance, at the expression for the $\rho$ parameter derived from the vacuum polarization of the gauge bosons (4.7). Making the crude approximation of letting $A_R(k^2) \rightarrow 1$, we may perform the integral explicitly to obtain

$$\rho - 1 \bigg|_{\text{top}} \approx \frac{N_c}{2(4\pi)^2} y^2(s_0) \frac{\Lambda^2}{\Lambda^2 + y^2(s_0)v^2/2}.$$  \hspace{1cm} (5.1)

In the perturbative regime, for $y(s_0)$ very small and consequently $\Lambda$ extremely large, one finds the well known growth with the Yukawa coupling, $\delta \rho \sim y^2$. However, for $y(s_0)$ very large, one notices that $y(s_0)v$ can overcome $\Lambda$ (which actually tends to a constant value for large $y(s_0)$) so that in the non–perturbative regime one finds instead that $\delta \rho \sim \Lambda^2/v^2$, i.e.
it is $y(s_0)$–independent. Of course, the latter result is cutoff dependent in a most obvious way so that cutoff effects will be of order one in this regime. This is indeed what we see, for instance, in fig. [3] and fig. [4]. The same approximation applied to $\delta \rho$ obtained from the Nambu–Goldstone boson propagators (4.6) agrees with the previous $\delta \rho$ for $y \ll 1$ while it is a factor of two larger when $y^2(s_0)v^2 \gg \Lambda^2$, again in good agreement with the plots. Similar arguments applied to the Higgs case yield

$$\rho - 1 \bigg|_{Higgs} \approx -\frac{3}{4} \left( \frac{g'}{4\pi} \right)^2 \left[ \ln \frac{8\lambda(s_0)}{g'^2} - \ln \frac{\Lambda^2 + 2\lambda(s_0)v^2}{\Lambda^2} \right]. \quad (5.2)$$

We again see that for $\lambda(s_0)v^2 \gg \Lambda^2$ the result is $\lambda(s_0)$–independent but cutoff dependent, while for $\lambda(s_0) \ll 1$ the result reduces to the one loop expression.

It should perhaps be emphasized that substantial scaling violations can occur for non–decoupling effects even at low energy since they can behave as the mass over the cutoff. This is unlike the case of decoupling effects where scaling violations are of the order of the momentum scale over the cutoff and therefore vanish at low energies.

The non–decoupling effects may become very different than the perturbative results if the only requirement is that the mass of the particle be smaller than the cutoff scale. We may further require that the cutoff effects be small so that the cutoff scale is much larger than the mass scale. The smaller we demand the cutoff effects to be, the deeper we are pushed into the perturbative regime. These properties were established by analyzing the $\rho$ parameter in detail. However, we believe that the qualitative features are shared by non–decoupling effects in trivial theories in general.

In the present work, we employed the characterization of the $\rho$ parameter entirely in terms of the Nambu–Goldstone bosons. This is conceptually simpler and, we believe, it will prove very useful in the event of a future lattice calculation.

One of our goals was to determine whether the effects of a heavy particle grow unboundedly with the mass of the particle as in perturbation theory or, as we have found, are tamed by non–perturbative effects. This question is of paramount importance for the fundamental question of defining the standard model on the lattice and its associated problem of non–decoupling effects due to the fermion doublers. Non–decoupling effects due to fermion doublers in the lattice formulation of the standard model have recently been discussed in [22].

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2 For example, in the case of the $\rho$ parameter, the relevant energy scale is effectively zero.
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Figure Captions

Fig. 1. The leading order one–particle irreducible quantum contributions in the $1/N_F$ expansion to the Higgs propagator.

Fig. 2. The mass, width, and the triviality scale in units of the vacuum expectation value, $v$, vs. the renormalized coupling $\lambda(s_0)$. $y^2/4$ in the label for the $x$–axis refers to the Yukawa coupling which appears in the next section.

Fig. 3. The leading order contributions to the Nambu–Goldstone propagators. The blob denotes the full propagator.

Fig. 4. The seagull and Higgs tadpole graph contributions to the propagators of Nambu–Goldstone bosons that cancel.

Fig. 5. Leading order self energy contributions to the gauge bosons due to the Higgs.

Fig. 6. The seagull graph contributions for the gauge bosons due to the Higgs.

Fig. 7. The Higgs contribution to the $\rho$ parameter in the $1/N_F$ expansion plotted against the coupling, $\lambda(M_H^2 + \Gamma^2/4)$. “Cutoff” denotes the factor $\Lambda/\sqrt{s_{\text{triv}}}$. 

Fig. 8. The Higgs contribution to the $\rho$ parameter in the $1/N_F$ expansion plotted against the mass in units of $v$. “Max. mass” denotes the maximum mass of Higgs as seen from the spectrum fig. 2.

Fig. 9. Leading order one–particle irreducible contributions to the $t$ propagator in the large–$N_F$ limit.

Fig. 10. The leading order top corrections to the Nambu–Goldstone propagators. The blobs denote the full propagators.

Fig. 11. The tadpole contribution due to the top for $H$ of order $1/N_F$.

Fig. 12. Self energy graphs for the gauge bosons due to the top.

Fig. 13. Contribution of the top to the $\rho$ parameter plotted against the Yukawa coupling $y^2(s_0)$ in comparison with the 1,2–loop results. Cutoff denotes the factor $\Lambda/\sqrt{s_{\text{triv}}}$. “NG” and “gauge” stands for the calculation using the Nambu–Goldstone bosons and the gauge bosons respectively.

Fig. 14. Contribution of the top to the $\rho$ parameter plotted against the mass in units of $v$ in comparison with the 1,2–loop results. “Max. mass” denotes the maximum mass of top as seen from the spectrum fig. 2.