Constraining the symmetry energy at subsaturation densities using isotope binding energy difference and neutron skin thickness

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We show that the neutron skin thickness $\Delta r_{np}$ of heavy nuclei is uniquely fixed by the symmetry energy difference $L(\rho)$ at a subsaturation cross density $\rho_0 \approx 0.11 \text{ fm}^{-3}$ rather than at saturation density $\rho_0$, while the binding energy difference $\Delta E$ between a heavy isotope pair is essentially determined by the magnitude of the symmetry energy $E_\text{sym}(\rho)$ at the same $\rho_c$. Furthermore, we find a value of $L(\rho_0)$ leads to a negative $E_\text{sym}(\rho_0)-L(\rho_0)$ correlation while a value of $E_\text{sym}(\rho_0)$ leads to a positive one. Using data on $\Delta r_{np}$ of Sn isotopes and $\Delta E$ of a number of heavy isotope pairs, we obtain simultaneously $E_\text{sym}(\rho_0) = 26.65 \pm 0.20$ MeV and $L(\rho_0) = 46.0 \pm 4.5$ MeV at 95% confidence level, whose extrapolation gives $E_\text{sym}(\rho_0) = 32.3 \pm 1.0$ MeV and $L(\rho_0) = 45.2 \pm 10.0$ MeV. The implication of these new constraints on the $\Delta r_{np}$ of $^{208}\text{Pb}$ and the core-crust transition density in neutron stars is discussed.

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I. INTRODUCTION

The determination of density dependence of the symmetry energy $E_{\text{sym}}(\rho)$, which characterizes the isospin dependent part of the equation of state (EOS) of asymmetric nuclear matter, is of fundamental importance due to its multifaceted roles in nuclear physics and astrophysics \cite{Typel2009,Typel2010} as well as some issues of new physics beyond the standard model \cite{Domingo2010,Engel2009}. Due to the particularity of nuclear saturation density $\rho_0$ ($\sim 0.16 \text{ fm}^{-3}$), a lot of works have been devoted to constraining quantitatively the magnitude and density slope of the symmetry energy at $\rho_0$, i.e., $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$, by analyzing terrestrial nuclear experiments and astrophysical observations. Although significant progress has been made during the last decade, large uncertainties on $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ still exist (See, e.g., Refs. \cite{Typel2010,Typel2009,Li2008,Typel2013}). For example, while the value of $E_{\text{sym}}(\rho_0)$ is determined to be around $30 \pm 4$ MeV, the extracted $L(\rho_0)$ varies drastically from about 20 to 115 MeV, depending on the observables and analysis methods. To better understand the model dependence of the constraints and reduce the uncertainties is thus of critical importance and remains a big challenge in the community. In this work, we show the isotope binding energy difference and neutron skin thickness of heavy nuclei can be used to stringently constrain the subsaturation density behavior of the symmetry energy.

The neutron skin thickness $\Delta r_{np} = \langle r_n^2 \rangle^{1/2} - \langle r_p^2 \rangle^{1/2}$ of heavy nuclei, i.e., the difference of the neutron and proton rms radii, has been shown to be a good probe of $E_{\text{sym}}(\rho)$ \cite{Typel2009,Typel2010}, and this provides a strong motivation for the Lead Radius Experiment (PREX) being performed at the Jefferson Laboratory to determine the $\langle r_n^2 \rangle^{1/2}$ of $^{208}\text{Pb}$ to about 1% accuracy by measuring the parity-violating electroweak asymmetry in the elastic scattering of polarized electrons from $^{208}\text{Pb}$ \cite{Typel2009,Typel2010}. Physically, the $\Delta r_{np}$ depends on the pressure of neutron-rich matter in nuclei which will balance against the pressure due to nuclear surface tension \cite{Typel2010}. Since the pressure of neutron-rich matter is essentially controlled by the density dependence of $E_{\text{sym}}(\rho)$ and the characteristic (average) density in finite nuclei is less than $\rho_0$ (See, e.g., Ref. \cite{Typel2009}), one expects that the $\Delta r_{np}$ should depend on the subsaturation density behaviors of the $E_{\text{sym}}(\rho)$ \cite{Typel2010}. Brown and Typel \cite{Typel2009} noted firstly that the $\Delta r_{np}$ of heavy nuclei from model calculations is linearly correlated with the pressure of pure neutron matter at a subsaturation density of 0.1 fm$^{-3}$. The linear correlation of the $\Delta r_{np}$ with both $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ has also been observed in mean-field calculations \cite{Typel2010,Typel2009} using many existing nuclear effective interactions.

Recently, a remarkable negative correlation between $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ has been obtained by analyzing existing data on $\Delta r_{np}$ of Sn isotopes \cite{Typel2009}, showing a striking contrast to other constraints that essentially give a positive $E_{\text{sym}}(\rho_0)-L(\rho_0)$ correlation (See, e.g., Refs. \cite{Typel2009,Typel2010}). A negative correlation between $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ has also been observed for a fixed value of $\Delta r_{np}$ in $^{208}\text{Pb}$ \cite{Typel2009}. It is thus of great interest to understand the physics behind this negative $E_{\text{sym}}(\rho_0)-L(\rho_0)$ correlation. Using a recently developed correlation analysis method \cite{Typel2009}, we show here that the $\Delta r_{np}$ of heavy nuclei is uniquely fixed by the $L(\rho)$ at a subsaturation cross density $\rho_0 \approx 0.11 \text{ fm}^{-3}$, which naturally leads to a negative $E_{\text{sym}}(\rho_0)-L(\rho_0)$ correlation. Furthermore, we demonstrate that the binding energy difference between a heavy isotope pair is essentially determined by the magnitude of $E_{\text{sym}}(\rho)$ at the same $\rho_c$. For the first time, we obtain simultaneously in the present work stringent constraints on both the magnitude and density slope of the $E_{\text{sym}}(\rho)$ at $\rho_c \approx 0.11 \text{ fm}^{-3}$.

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by analyzing data of the isotope binding energy difference for heavy nuclei and $\Delta r_{np}$ of Sn isotopes, which has important implications on the values of $E_{sym}(\rho_0)$ and $L(\rho_0)$, the $\Delta r_{np}$ of $^{208}\text{Pb}$, and the core-crust transition density $\rho_t$ of neutron stars.

II. MODEL AND METHOD

The EOS of asymmetric nuclear matter at baryon density $\rho$ and isospin asymmetry $\delta = (\rho_n - \rho_p) / (\rho_p + \rho_n)$, given by its binding energy per nucleon, can be expanded to 2nd-order in $\delta$ as

$$E(\rho, \delta) = E_0(\rho) + E_{sym}(\rho)\delta^2 + O(\delta^4), \quad (1)$$

where $E_0(\rho) = E(\rho, \delta = 0)$ is the binding energy per nucleon in symmetric nuclear matter, and the nuclear symmetry energy is expressed as

$$E_{sym}(\rho) = 1 \frac{\partial^2 E(\rho, \delta)}{\partial \delta^2}|_{\delta=0}.$$ \quad (2)

Around a reference density $\rho_r$, $E_{sym}(\rho)$ can be characterized by using the value of $E_{sym}(\rho_r)$ and the density slope parameter $L(\rho_r)$, i.e.,

$$E_{sym}(\rho) = E_{sym}(\rho_r) + L(\rho_r)\chi_0 + O(\chi_0^2), \quad (3)$$

with $\chi_0 = (\rho - \rho_r)^2/3\rho_r$.

In the present work, we use the Skyrme-Hartree-Fock (SHF) approach with the so-called standard form of Skyrme force (see, e.g., Ref. [30]) which includes 10 parameters, i.e., the 9 Skyrme force parameters $\sigma$, $\lambda_0 - \lambda_3$, $x_0 - x_3$, and the spin-orbit coupling constant $W_0$. This standard SHF approach has been shown to be very successful in describing the structure of finite nuclei, especially global properties such as binding energies and charge radii [31, 32]. Instead of using directly the 9 Skyrme force parameters, we can express them explicitly in terms of 9 macroscopic quantities, i.e., $\rho_0$, $E_0(\rho_0)$, the incompressibility $K_0$, the isoscalar effective mass $m_{s,0}^*$, the isovector effective mass $m_{v,0}^*$, $E_{sym}(\rho_r)$, $L(\rho_r)$, $G_S$, and $G_V$. The $G_S$ and $G_V$ are respectively the gradient and symmetry-gradient coefficients in the interaction part of the binding energies for finite nuclei defined as

$$E_{\text{grad}} = G_S(\nabla \rho)^2/(2\rho) - G_V [\nabla (\rho_n - \rho_p)]^2/(2\rho). \quad (4)$$

Then, by varying individually these macroscopic quantities within their known ranges, we can examine more transparently the correlation of properties of finite nuclei with each individual macroscopic quantity. Recently, this correlation analysis method has been successfully applied to study the neutron skin [28] and giant monopole resonance of finite nuclei [33], the higher order bulk characteristic parameters of asymmetric nuclear matter [34], and the relationship between the nuclear matter symmetry energy and the symmetry energy coefficient in the mass formula [35], where the reference density $\rho_r$ has been set to be $\rho_0$.

III. RESULTS AND DISCUSSIONS

To examine the correlation of the $\Delta r_{np}$ of heavy nuclei with each macroscopic quantity, especially on $E_{sym}(\rho_r)$ and $L(\rho_r)$, we show in Fig. 1 the $\Delta r_{np}$ of $^{208}\text{Pb}$ from SHF with the Skyrme force MSLO [28] by varying individually $L(\rho_r)$ (a), $G_V$ (b), $G_S$ (c), $E_0(\rho_0)$ (d), $E_{sym}(\rho_r)$ (e), $K_0$ (f), $m_{s,0}^*$ (g), $m_{v,0}^*$ (h), $\rho_0$ (i), and $W_0$ (j) for $\rho_r = 0.06$, 0.11, and 0.16 fm$^{-3}$. The $E_{sym}(\rho_r)$ is shifted by adding 15 and 7 MeV for $\rho_r = 0.06$ and 0.11 fm$^{-3}$, respectively.

![Graph showing the correlation of $\Delta r_{np}$ of $^{208}\text{Pb}$ from SHF with the Skyrme force MSLO, varying $L(\rho_r)$, $G_V$, $G_S$, $E_0(\rho_0)$, $E_{sym}(\rho_r)$, $K_0$, $m_{s,0}^*$, $m_{v,0}^*$, $\rho_0$, and $W_0$ for $\rho_r = 0.06$, 0.11, and 0.16 fm$^{-3}$, with $E_{sym}(\rho_r)$ shifted by adding 15 and 7 MeV for $\rho_r = 0.06$ and 0.11 fm$^{-3}$, respectively.]

In order to determine the magnitude of the symmetry energy at $\rho_c$, i.e., $E_{sym}(\rho_c)$, we propose here to use the difference of binding energy per nucleon between an isotope pair, denoted as $\Delta E$. Before detailed quantitative calculations, it is instructive to estimate the $\Delta E$ from the well-known semiempirical nuclear mass formula in which the binding energy per nucleon for a nucleus with $N$ neutrons and $Z$ protons ($A = N + Z$) can be approximated
by

\[ E(N, Z) = a_{\text{vol}} + a_{\text{surf}} A^{-1/3} + a_{\text{sym}}(A) \left( \frac{N - Z}{A} \right)^2 + a_{\text{Coul}} \frac{Z(Z - 1)}{A^{4/3}} + E_{\text{pair}}. \]

(5)

For heavy spherical even-even nuclei, the \( \Delta E \) can then be expressed approximately as

\[ \Delta E = E(N + \Delta N, Z) - E(N, Z) \approx a_{\text{sym}}(A) \frac{4Z(N - Z)}{A^2} \cdot \frac{\Delta N}{A} - a_{\text{Coul}} \frac{4Z(Z - 1)}{3A^{4/3}} \cdot \frac{\Delta N}{A} \cdot A \]

(6)

if we assume \( \Delta N \) is significantly less than \( A \) and \( N - Z \) and \( a_{\text{sym}}(A + \Delta N) \approx a_{\text{sym}}(A) \). Since the Coulomb term is relatively well-known and the contribution of the surface term to \( \Delta E \) is generally small compared to that of the symmetry energy term for heavy neutron-rich nuclei (empirically we have \( a_{\text{sym}}(A) \approx 25 \text{ MeV} \) and \( a_{\text{surf}} \approx 18 \text{ MeV} \)), thus the \( \Delta E \) essentially reflects the symmetry energy of finite nuclei, i.e., \( a_{\text{sym}}(A) \). For heavy nuclei, one has the empirical relation of \( a_{\text{sym}}(A) \approx E_{\text{sym}}(\rho_c) \frac{12}{52} \), and thus expects that the \( \Delta E \) for heavy isotope pairs should be a good probe of \( E_{\text{sym}}(\rho_c) \). To see more quantitative results, similarly as in Fig. 1 we show in Fig. 2 the \( \Delta E \) for isotope pair \( ^{212}\text{Pb} \) and \( ^{178}\text{Pb} \) from SHF with MSL0 by varying individually the 9 macroscopic quantities and \( W_0 \) for \( \rho_c = 0.06, 0.11 \) and \( 0.16 \text{ fm}^{-3} \). As expected, it is seen from Fig. 2 that, for \( \rho_c = 0.11 \text{ fm}^{-3} \), the \( \Delta E \) exhibits a very strong correlation with \( E_{\text{sym}}(\rho_c) \) while it displays relatively weak dependence on \( L(\rho_c) \), \( G_V \), \( G_S \), \( m^*_A \) and \( W_0 \), and essentially no dependence on other macroscopic quantities. For \( \rho_c = 0.06 \) and \( 0.16 \text{ fm}^{-3} \), the \( \Delta E \) also exhibits strong correlation with \( L(\rho_c) \). These results indicate that the \( \Delta E \) for heavy isotope pairs indeed provides a good probe of \( E_{\text{sym}}(\rho_c) \).

Note here that using other standard Skyrme forces instead of MSL0 or using other heavy nuclei such as Sn isotopes leads to similar results as shown in Fig. 1 and Fig. 2. We have also checked with a number of existing standard Skyrme forces and obtained the similar conclusion [30].

Experimentally, there are very rich and accurate data on ground state binding energy of finite nuclei. To constrain \( E_{\text{sym}}(\rho_c) \) from \( \Delta E \), here we select 19 heavy isotope pairs which are all spherical even-even nuclei, namely, \( ^{114,120,126,132}\text{Sn}, ^{192,196}\text{Pt}, ^{216,218}\text{Po}, ^{212,218}\text{Pb}, ^{208,178}\text{Pb}, ^{206,178}\text{Pb}, ^{136,138,140}\text{Te}, ^{132,134,136,138,140}\text{Sn}, ^{132,134,136,138,140}\text{Sn}, ^{46,48,50,52,54,56}\text{Cd}, ^{46,48,50,52,54,56}\text{Cd}, ^{46,48,50,52,54,56}\text{Cd}, ^{46,48,50,52,54,56}\text{Cd} \), and \( ^{46,48,50,52,54,56}\text{Cd} \). On the other hand, the \( \Delta \tau_{np} \) of heavy Sn isotopes has been systematically measured using various methods, and here we use the existing 21 data on \( \Delta \tau_{np} \) of Sn isotopes [37, 42] to constrain the \( L(\rho_c) \). Firstly, with all other parameters fixed at their default values in MSL0, we calculate the \( \chi^2 \) from the difference between theoretical and experimental \( \Delta E \) (\( \Delta \tau_{np} \)) with different \( E_{\text{sym}}(\rho_c) \) (\( L(\rho_c) \)), and the results are shown by dashed lines in Fig. 3. For the evaluation of \( \chi^2 \), since the experimental precision of the binding energy is much better than what one can expect from the mean-field description due to the model limitation, here we adopt the usual strategy (See, e.g., [32]), namely, to assign a theoretical error to the binding energy. In particular, when we evaluate the \( \chi^2 \), we use the experimental errors for \( \Delta \tau_{np} \), while assign a theoretical relative error of 23% for \( \Delta E \) so that the minimum value of \( \chi^2 \) is close to the number of data points to make the \( \chi^2 \)-analysis valid [13]. From the dashed lines in Fig. 3 obtained with MSL0, one can extract a value of \( E_{\text{sym}}(\rho_c) = 26.08 \pm 0.17 \text{ MeV} \) and \( L(\rho_c) = 47.3 \pm 4.5 \text{ MeV} \) at 95% confidence level.

The values of \( E_{\text{sym}}(\rho_c) = 26.08 \pm 0.17 \text{ MeV} \) and \( L(\rho_c) = 47.3 \pm 4.5 \text{ MeV} \) are obtained by assuming the
other macroscopic quantities are fixed at their default values in MSL0 when $E_{\text{sym}}(\rho_c)$ or $L(\rho_c)$ is varied, and thus the correlations of $\Delta E$ ($\Delta r_{np}$) with the other macroscopic quantities are totally neglected. To be more precise, one should consider the possible variations of the other macroscopic quantities due to the correlations introduced by fitting the theoretical predictions to some well-known experimental observables or empirical values. Taking this into account, for a fixed $E_{\text{sym}}(\rho_c)$ ($L(\rho_c)$), we optimize all the other parameters, instead of simply keeping them at their default values in MSL0, by minimizing the weighted sum of $\chi^2$ evaluated from the difference between SHF prediction and the experimental data for some selected observables using the simulated annealing method (See, e.g., Ref. [44]). In the optimization, we select the binding energy per nucleon $E_B$ and charge rms radii $r_c$ of 25 spherical even-even nuclei, i.e., $^{204,206,208,196,198,197,192,190,188,186,184,180}$Pb, $^{124,122,120,118,116,112}$Sn, $^{50,52,54}$Sn, $^{64,66,68,70}$Ni, and $^{50,52,54,44,46,47,48}$Ca. A theoretical error of 0.013 MeV is assigned to $E_B$ and 0.01 fm to $r_c$ so that the respective $\chi^2$ is roughly equal to the number of the corresponding data points. We further constrain the macroscopic parameters in the optimization by requiring (1) the neutron $3p_{3/2} - 3p_{1/2}$ energy level splitting in $^{208}$Pb should lie in the range of 0.8 − 1.0 MeV; (2) the pressure of symmetric nuclear matter should be consistent with constraints obtained from data flow in heavy ion collisions [45]; (3) the binding energy of pure neutron matter should be consistent with constraints obtained from the latest chiral effective field theory calculations with controlled uncertainties [46]; (4) the critical density $\rho_{cr}$, above which the nuclear matter becomes unstable by the stability conditions from Landau parameters, should be greater than $2\rho_0$; and (5) $m^{*}_{s,0}$ should be greater than $m^{*}_{c,0}$ and here we set $m^{*}_{s,0} - m^{*}_{c,0} = 0.1 m$ ($m$ is nucleon mass in vacuum) to be consistent with the extraction from global nucleon optical potentials constrained by world data on nucleon-nucleus and (p,n) charge-exchange reactions [47]. In the optimization, for a fixed $E_{\text{sym}}(\rho_c)$ ($L(\rho_c)$), the contribution of the 21 data on $\Delta r_{np}$ of Sn isotopes (the 19 $\Delta E$ data) is included in the weighted sum of $\chi^2$.

By using the optimized values for other macroscopic quantities at a fixed $E_{\text{sym}}(\rho_c)$ ($L(\rho_c)$), we show the $\chi^2$ evaluated from the difference between theoretical and experimental $\Delta E$ ($\Delta r_{np}$) as a function of $E_{\text{sym}}(\rho_c)$ ($L(\rho_c)$) by solid circles in Fig. 3. From the new relation between $\chi^2$ and $E_{\text{sym}}(\rho_c)$ ($L(\rho_c)$), one can extract a value of $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV and $L(\rho_c) = 46.0 \pm 4.5$ MeV at 95% confidence level. It is seen that the $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV obtained with optimization is larger than the $E_{\text{sym}}(\rho_c) = 26.08 \pm 0.17$ MeV extracted using MSL0 without optimization by about 0.6 MeV, implying the correlations of $\Delta E$ for heavy isotope pairs with $L(\rho_c)$, $G_V$, $G_S$, $m^{*}_{c,0}$, and $W_0$ observed in Fig. 2 play a certain role in the extraction of $E_{\text{sym}}(\rho_c)$. On the other hand, it is remarkable to see that the extracted value of $L(\rho_c) = 46.0 \pm 4.5$ MeV with optimization agrees well with $L(\rho_c) = 47.3 \pm 4.5$ MeV obtained using MSL0 without optimization, indicating the correlations of $\Delta r_{np}$ with heavy nuclei with other macroscopic quantities play minor roles in the extraction of $L(\rho_c)$, as shown in Fig. 4. Besides the $E_B$ and $r_c$ of the 25 spherical even-even nuclei as well as the constraints on the neutron $3p_{3/2} - 3p_{1/2}$ energy level splitting in $^{208}$Pb, the pressure of symmetric nuclear matter and the binding energy of pure neutron matter, the critical density $\rho_{cr}$, and $m^{*}_{s,0} - m^{*}_{c,0} = 0.1 m$, by further including simultaneously the 21 data on $\Delta r_{np}$ of Sn isotopes and the 19 data of $\Delta E$ in the optimization, we obtain a globally optimized parameter set of Skyrme force denoted as MSL1 as shown in Table I. Unlike the MSL0 interaction which is obtained by directly using their empirical values for the macroscopic parameters, the MSL1 interaction is obtained by fitting experimental data or constraints by optimization. The main difference between MSL0 and MSL1 is the latter predicts a significantly smaller (larger) value of $L(\rho_0) = 45.25$ MeV ($G_V = 68.74$ MeV·fm$^5$) compared to the $L(\rho_0) = 60$ MeV ($G_V = 5$ MeV·fm$^5$) from MSL0. As expected, the MSL1 nicely reproduces the optimized values of $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$ obtained from $\chi^2$ analysis with optimization for $\Delta E$ and $\Delta r_{np}$, respectively, as shown by solid circles in Fig. 4.

Since what we have directly constrained from $\Delta E$ and $\Delta r_{np}$ of Sn isotopes are $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV and $L(\rho_c) = 46.0 \pm 4.5$ MeV at $\rho_c = 0.11$ fm$^{-3}$, the extrapolation is necessary to obtain information on the symmetry energy at $\rho_0$ from $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$. Shown in Fig. 4 are contours in the $E_{\text{sym}}(\rho_c) - L(\rho_0)$ plane for $E_{\text{sym}}(\rho_c)$ and $L(\rho_c)$ from SHF calculation by assuming the other 7 macroscopic quantities $\rho_0$, $E_0(\rho_0)$, $K_0$, $m^{*}_{s,0}$, $m^{*}_{c,0}$, $G_S$, and $G_V$ are fixed at their default (optimized) values in MSL1. It is interesting to see that a fixed value of $L(\rho_c)$ leads to a negative $E_{\text{sym}}(\rho_c) - L(\rho_0)$ correlation, which was observed previously by directly constraining $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ from analyzing the $\Delta r_{np}$ data of Sn isotopes [48]. On the other hand, a fixed value of $E_{\text{sym}}(\rho_c)$ leads to a positive $E_{\text{sym}}(\rho_0) - L(\rho_0)$ correlation.
correlation as expected (See, e.g., Ref. [33]). Combining the constraints from $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV and $L(\rho_c) = 46.0 \pm 4.5$ MeV leads to a quite stringent constraint on both $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ as indicated by the shaded region in Fig. 4, giving $E_{\text{sym}}(\rho_0) = 32.3 \pm 1.0$ MeV and $L(\rho_0) = 45.2 \pm 10.0$ MeV, which is essentially consistent with other constraints extracted from terrestrial experiments and astrophysical observations as well as theoretical calculations with controlled uncertainties [8, 11, 46] but with much higher precision. In particular, our results are in surprisingly good agreement with the constraint of $E_{\text{sym}}(\rho_0) = 31.2-34.3$ MeV and $L(\rho_0) = 36-55$ MeV (at 95% confidence level) obtained recently from Bayesian analysis of currently available neutron star mass and radius measurements [48] as well as that of $E_{\text{sym}}(\rho_0) = 29.0-32.7$ MeV and $L(\rho_0) = 40.5-61.9$ MeV obtained recently from the experimental, theoretical and observational analyses [49]. Our results also agree with the constraint of $E_{\text{sym}}(\rho_0) = 32.0 \pm 1.8$ MeV and $L(\rho_0) = 43.125 \pm 15$ MeV obtained by analyzing pygmy dipole resonances (PDR) of $^{130,132}\text{Sn}$ [41] and that of $E_{\text{sym}}(\rho_0) = 32.3 \pm 1.3$ MeV and $L(\rho_0) = 64.8 \pm 15.7$ MeV from analyzing PDR of $^{68}\text{Ni}$ and $^{132}\text{Sn}$ [50]. Furthermore, our results are consistent with the constraint of $E_{\text{sym}}(\rho_0) = 32.5 \pm 0.5$ MeV and $L(\rho_0) = 70 \pm 15$ MeV obtained recently from a new and more accurate finite-range droplet model analysis of the nuclear mass of the 2003 Atomic Mass Evaluation [51] although the two constraints only have a small overlap for $L(\rho_0)$.

The core-crust transition density $\rho_t$ of neutron stars play crucial roles in neutron star properties [1] and it is strongly correlated with the $L(\rho_0)$ [52, 53]. We notice a similar strong correlation also exists between $\rho_t$ and $L(\rho_c)$. Using a dynamical approach (See, e.g., Ref. [12]), we obtain $\rho_t = 0.082 \pm 0.005$ fm$^{-3}$ from $L(\rho_c) = 46.0 \pm 4.5$ MeV, which agrees well with the empirical values [1]. Furthermore, we find the $L(\rho_c) = 46.0 \pm 4.5$ MeV leads to a quite strong constraint of $\Delta r_{np} = 0.170 \pm 0.016$ fm for $^{208}\text{Pb}$, which is in good agreement with the $\Delta r_{np} = 0.156^{+0.025}_{-0.021}$ fm obtained from the $^{208}\text{Pb}$ dipole polarizability [54] and within the experimental error bar also consistent with the $\Delta r_{np} = 0.33^{+0.16}_{-0.08}$ fm extracted recently from the PREX [23].

IV. SUMMARY AND OUTLOOK

In summary, we show that while the binding energy difference $\Delta E$ between a heavy isotope pair is essentially determined by the magnitude of the symmetry energy $E_{\text{sym}}(\rho)$ at a subsaturation cross density $\rho_c \approx 0.11$ fm$^{-3}$, the symmetry energy density slope $L(\rho)$ at the same $\rho_c$ fixes uniquely the neutron skin thickness $\Delta r_{np}$ of heavy nuclei. Our results demonstrate that the global properties (e.g., the neutron skin thickness and binding energy difference) of heavy nuclei can be effectively determined by the EOS of nuclear matter at a subsaturation cross density $\rho_c \approx 0.11$ fm$^{-3}$ rather than at $\rho_0$, which is nicely consistent with the recent finding in Ref. [27] where the giant monopole resonance of heavy nuclei has been shown to be constrained by the EOS of symmetric nuclear matter at $\rho_c \approx 0.11$ fm$^{-3}$ rather than at $\rho_0$.

Furthermore, we find a fixed value of $L(\rho_c)$ leads to a negative correlation between $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ at saturation density $\rho_0$ while a fixed value of $E_{\text{sym}}(\rho_c)$ leads to a positive $E_{\text{sym}}(\rho_0)$-$L(\rho_0)$ correlation. The existing data on $\Delta r_{np}$ of Sn isotopes and $\Delta E$ for a number of heavy isotope pairs put simultaneously stringent constraints on the magnitude and density slope of the $E_{\text{sym}}(\rho)$ at $\rho_c$, i.e., $E_{\text{sym}}(\rho_c) = 26.65 \pm 0.20$ MeV and $L(\rho_c) = 46.0 \pm 4.5$ MeV at 95% confidence level, whose extrapolation gives $E_{\text{sym}}(\rho_0) = 32.3 \pm 1.0$ MeV and $L(\rho_0) = 45.2 \pm 10.0$ MeV. The obtained $E_{\text{sym}}(\rho_0)$ and $L(\rho_0)$ are essentially consistent with other constraints extracted from analyzing terrestrial experiments and astrophysical observations as well as theoretical calculations with controlled uncertainties but with higher precision. The extracted $L(\rho_0)$ also leads to a strong constraint of $\Delta r_{np} = 0.170 \pm 0.016$ fm for $^{208}\text{Pb}$ and $\rho_t = 0.082 \pm 0.005$ fm$^{-3}$ for the core-crust transition density of neutron stars.

Our results in the present work are only based on the standard SHF energy density functional. It will be interesting to see how our results change if different energy-density functionals, e.g., the extended non-standard SHF energy density functional or relativistic mean field model, are used. These studies are in progress and will be reported elsewhere.
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