On the Signature of Short Distance Scale in the Cosmic Microwave Background

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Abstract

We discuss the signature of the scale of short distance physics in the Cosmic Microwave Background. In addition to effects which depend on the ratio of Hubble scale $H$ during inflation to the energy scale $M$ of the short distance physics, there can be effects which depend on $\dot{\phi}^2/M^4$ where $\phi$ is the classical background of the inflaton field. Therefore, the imprints of short distance physics on the spectrum of Cosmic Microwave Background anisotropies generically involve a double expansion. We present some examples of a single scalar field with higher order kinetic terms coupled to Einstein gravity, and illustrate that the effects of short distance physics on the Cosmic Microwave Background can be substantial even for $H \ll M$, and generically involve corrections that are not simply powers of $H/M$. The size of such effects can depend on the short distance scale non-analytically even though the action is local.

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I. INTRODUCTION

One of the highlights of inflationary cosmology [1,2] is the idea that astrophysical observables may be the results of physics at the microscopic scale. Since the pioneering work of [3], it has been known that quantum field fluctuations in the early universe are stretched by the enormous expansion of inflation to scales of astrophysical size, providing a first principle mechanism for density perturbations. In an inflationary universe, the temperature anisotropies of the Cosmic Microwave Background (CMB) and even the formation of galaxies are the results of quantum fluctuations writ large.

Most inflationary models have far more expansion than the 60 e-foldings required to solve the cosmological problems of standard cosmology. Consequently, astrophysical scales in the present universe may correspond to Planckian or sub-Planckian scales at the onset of inflation. Therefore, there is a hope that inflation may provide a kind of Planck scale microscope, magnifying short distance physics to observably large scale, allowing us to probe trans-Planckian or stringy physics from precision cosmological measurements [4–10].

More recently, this exciting prospect was revisited in Ref. [11], where it was claimed that a general argument assuming only low energy locality implies that the effects of short distance physics on the CMB are of the order of $(H/M)^{2n}$ where $n \geq 1$ is an integer, $H$ is the Hubble scale during inflation, and $M$ is the mass scale of short distance physics. This result seems to imply that stringy effects are far too small to be observed. However, it was pointed out in [12] that the argument of [11] assumes the universe is in the local vacuum state at horizon crossing, and hence the result of [11] is not in conflict with the seemingly different estimates of [7,8,13].

The purpose of this paper is to point out yet another way that the scale of short distance physics can affect the spectrum of CMB anisotropies. The effects of short distance physics (e.g., stringy $\alpha'$ corrections) on the low energy effective action of a scalar field generically include both higher derivative terms as well as higher order terms in the first derivative. Therefore, the effects of short distance physics on the CMB anisotropies is a double expansion in both $(H/M)^2$ and $\dot{\phi}^2/M^4$ where $\phi$ is the classical background of the scalar field. Contrary to the assertions of [11], the imprint of the short distance scale on the CMB generically involve corrections that are not simply powers of $(H/M)^2$. We find that the effects of short distance physics can be substantial even though $H << M$, and can in fact depend on the mass scale of short distance physics non-analytically even though the action is local.

The effects of higher derivative terms on the CMB anisotropies have been studied in [4–8,11]. Here we focus on the effects of the higher order terms in the first derivative. Let us consider the most general local action for a scalar field coupled to Einstein gravity, which involves at most first derivatives of the scalar field:

$$S = -\frac{1}{16\pi G} \int d^4x \sqrt{g} R + \int d^4x \sqrt{g} p(X, \phi)$$

(1)

where $g$ is the determinant of the metric, $R$ is the Ricci scalar and

$\delta \phi$ obeys a linear equation, and so the action contains at most terms quadratic in $\delta \phi$. Therefore, the remaining factors in these higher order terms are evaluated on the classical background.
\[ X = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi . \] (2)

The cosmological perturbations of this class of models have been studied in [14]. In this paper, we apply their results to some special cases to illustrate that the short distance effects on the Cosmic Microwave Background may not be captured simply by an expansion in \((H/M)^2\), and can depend on the short distance energy scale \(M\) in a non-analytic way even though the action which defines the model involves only local interactions.

The Lagrangian of the scalar field is denoted by \(p(X, \phi)\) since it plays the role of pressure [15], whereas the energy density is given by

\[ \varepsilon = 2X p_{,X} - p \] (3)

where \(p_{,X}\) denotes the derivative of the Lagrangian with respect to \(X\).

We consider the function \(p(X, \phi)\) of the following form:

\[ p(X, \phi) = F(X) - V(\phi) \] (4)

and hence,

\[ \varepsilon = 2X F_{,X} - F + V . \] (5)

The function \(F(X)\) includes the usual kinetic term \(X\) as well as higher order contributions. Therefore,

\[ F(X) = X + \alpha X f \left( \frac{X}{M^4} \right) \] (6)

where \(\alpha\) is a dimensionless number, \(f\) is a function of \(X/M^4\), and \(M\) is the mass scale associated with the short distance physics. If \(\alpha = 0\), we have the usual kinetic term. If \(\alpha \neq 0\), the kinetic term is not minimal and the higher order kinetic terms depend on \(M\).

The rationale behind Eq. (4), augmented by Eq. (6), is that it is a plausible prescription for encoding the physics of a small distance scale, \(M\), in a way that depends rather simply on fields and first derivatives of the fields, and can preserve causality if appropriate conditions on \(F(X)\) are satisfied. Dimensional analysis alone would allow other choices, such as

\[ p(X, \phi) = X G_K \left( \frac{X}{\phi^2 M^2} \right) - V(\phi) G_V \left( \frac{X}{\phi^2 M^2} \right) ; \] (7)

in some inflationary scenarios, a theory based on Eq. (7) could result in corrections to conventional inflation that can be expressed as a power series in \(H^2/M^2\) for small \(H/M\), as envisioned in [11]. For example, in the chaotic inflationary scenario developed in § III, the dimensionless combination \(X/\phi^2 M^2 \approx H^2/8M^2L^2\) during slow roll, where \(L\) is the number of e-folds remaining to the end of inflation. However, we shall see that the simpler prescription based on Eqs. (4) and (6) can lead to more complicated behavior.
II. GENERAL RESULTS FOR SLOW-ROLL INFLATION

A. Background Equations

We consider the background to be an expanding Friedmann universe:

\[ ds^2 = dt^2 - a^2(t)\delta_{ij}dx^i dx^j . \]  

The equations of motion for the background variables \( a(t) \) and \( \phi(t) \) are:

\[ H^2 = \frac{8\pi G}{3} \varepsilon , \]
\[ \dot{\varepsilon} = -3H (\varepsilon + p) . \]  

In the slow-roll approximation,

\[ |2XF_{,X} - F| << V \]
\[ |(2XF_{,X} - F)_{,X} \dot{\phi}| << \frac{\partial V}{\partial \phi} , \]

the background equations become

\[ H^2 \approx \frac{8\pi G}{3} V \]
\[ 3H \dot{\phi} F_{,X} \approx -\frac{\partial V}{\partial \phi} . \]  

B. Fluctuation Spectra

The power spectrum for the scalar fluctuations is given by [14]

\[ P_k^S = \frac{16}{9} \frac{G^2 \varepsilon^2}{c_s (\varepsilon + p)|_{c_s k = aH}} \]  

where the quantities on the right-handed side are evaluated at sound horizon crossing, \( i.e., aH = c_s k \). Here, \( c_s \) is the “speed of sound” for the scalar perturbations:

\[ c_s^2 = \frac{\varepsilon + p}{2X\varepsilon,X} = \frac{F_{,X}}{F_{,X} + 2XF_{,XX}} . \]  

Causality requires that \( c_s \leq 1 \) which implies that \( 2XF_{,XX}/F_{,X} \geq 0 \).

In the slow-roll approximation

\[ P_k^S \approx \frac{16}{9} \frac{G^2 V^2}{c_s \phi^2 F_{,X} |_{c_s k = aH}} \]  

where the quantities are evaluated at “sound horizon” crossing, but, on the other hand, the spectrum of tensor fluctuations is given by the usual expression.
\[ P_k^T = \frac{128}{3} G^2 \varepsilon \bigg|_{k=aH} \approx \frac{128}{3} G^2 V \bigg|_{k=aH} = \frac{16}{\pi} G H^2 \bigg|_{k=aH} , \]  

where the quantities are evaluated at the usual particle horizon crossing, \( k = aH \). This is not exactly the same time as the sound horizon crossing for the scalar perturbations but as we shall see, in the specific model that we consider, \( c_s \sim 1 \), so the two horizon crossing times differ by relatively small amounts.

Note that the short distance effects are contained in the function \( F(X) \) (and its derivatives) evaluated at horizon crossing. Therefore the short distance effects on the perturbation spectra should depend on \( \dot{\phi}^2 / M^4 \).

### III. EXAMPLES

#### 1. Chaotic Inflation Model

Let us consider the potential in chaotic inflation [16]:

\[ V(\phi) = \frac{1}{2} m^2 \phi^2 . \]  

The background equations are especially simple:

\[ H \approx \sqrt{\frac{4\pi G}{3}} m \phi \]

\[ F_{,X} \dot{\phi} \approx -\frac{m}{\sqrt{12\pi G}} . \]  

Hence, \( \dot{\phi} \) and \( F_{,X} \) are independent of time. We can express \( \phi \) and \( H \) as functions of the number of e-foldings remaining to the end of inflation, \( i.e., L = \ln(a_{end}/a) \):

\[ \phi^2 = \frac{L}{2\pi G F_{,X}} \]

\[ H^2 = \frac{2m^2 L}{3 F_{,X}} . \]  

Therefore, the scalar and tensor perturbations are

\[ P_k^S = \frac{4G m^2 L_S^2}{3\pi} \left[ \frac{(F_{,X} + 2XF_{XX})^{1/2}}{F_{,X}^{3/2}} \right] \]

\[ P_k^T = \frac{32 G m^2 L_T}{3\pi} \frac{F_{,X}}{F_{,X}} . \]  

The usual results are recovered if we take \( F(X) = X \). Since \( L_s - L_T \approx \frac{1}{2} \ln c_s^{-1} \), and \( c_s^2 \geq \left[ 1 + 2(n-1) \right]^{-1} \) if \( F(X) \) is a polynomial of index \( n \), the fractional difference \( (L_s - L_T)/L_T \ll 1 \) for large \( L_T \), so we neglect \( L_s - L_T \) below. From the equation of motion:

\[ F_{,X} \sqrt{2X} = -\frac{m}{\sqrt{12\pi G}} \]  

5
We see that the solution of $X$ is, in general, not analytic in $M$.

For example, consider

$$F(X) = X + \frac{\alpha}{M^4} X^2,$$

where $M$ is the mass scale characteristic of short distance physics, and we introduce a parameter $\alpha$ which we may absorb into the definition of $M$, but we carry along anyway to keep track of nonlinear terms in $F(X)$. (If such terms are present, we take $\alpha = 1$ below; $\alpha = 0$ means these terms are absent altogether.) With this $F(X)$, we find

$$P^S_k = \frac{4Gm^2L^2}{3\pi} \left[ \frac{(1 + 6\alpha X/M^4)^{1/2}}{(1 + 2\alpha X/M^4)^{3/2}} \right]$$

$$P^T_k = \frac{32}{3\pi} \frac{Gm^2L}{(1 + 2\alpha X/M^4)}$$

where $X$ is given by the solution of the cubic equation

$$\left( 1 + \frac{2\alpha}{M^4} X \right) \sqrt{2X} = \frac{m}{\sqrt{12\pi G}}.$$  

If $\alpha X/M^4 \ll 1$, then $X$ is independent of $M$ to lowest order, and so, to the same order, the power spectra are also independent of $M$. There are higher order corrections amounting to an expansion in $X/M^4$; in this sense, the power spectra depend on $M$ analytically at small $X/M^4$. However, if $\alpha X/M^4 \gg 1$, then $X \approx m^2/3 M^{4/3}$, and therefore $P^S_{kT} \sim Gm^2M^4/X \sim G^{4/3}m^{4/3}M^{4/3} = (M/M_{PL})^{8/3}(m/M)^{4/3}$, where $M_{PL} = G^{-1/2}$ is the Planck mass. Thus, the fluctuation amplitude depends on a fractional power of the mass scale associated with the short distance physics. Below we shall see that if we want $m \sim M$ (or perhaps $m \lesssim M$ but not $m \ll M$), then we will be forced to the large $X/M^4$ limit. In this case, it will be possible to have small $H/M$, but we shall also see that $m \sim M \ll M_{PL}$.

The spectral index for the tensor fluctuations is given by [14]:

$$n_T = -3\frac{(\varepsilon + p)}{\varepsilon} \approx -6XF \frac{X}{V} = -\frac{1}{L}$$

Apart from the small change in $L$, the spectral index is not very different from the usual case. Furthermore, since $c_s$ is of order 1, the “consistency condition” is only altered mildly:

$$\frac{P^T_k}{P^S_k} = -8c_sn_T$$

for the quadratic $F(X)$ adopted above, $c_s \geq 1/\sqrt{3}$.

So far, we have merely assumed that it is possible for the short distance physics to alter the kinetic energy of the scalar field enough to change both the expansion rate during inflation, and the fluctuations that result. We have seen that the main effect would be that the dependence of the fluctuation amplitude on mass scales of the theory is changed, and the relative amplitudes of the scalar and tensor fluctuations are altered by factors which, as long as $c_s$ is not too small, are of order unity in general. In the specific model
we have considered, the relative amplitude of the scalar and tensor fluctuations changes by a factor of at most $\sqrt{3} \approx 1.73$ relative to its value for $\alpha = 0$, a subtle effect that may be discernible observationally, particularly once polarization measurements become available for CMB fluctuations. This change is model dependent, though, and alternative $F(X)$ could yield different results, since $c_s$ could be smaller for large $X$. For example, $c_s \to M^4/2X$ for large $X/M^4$ if $F(X) = M^4(e^{X/M^4} - 1)$. In this case, the ”consistency condition” is violated significantly and may give rise to an observable effect.

An important question we have not asked is whether the large $X$ limit can be attained in any realistic inflation model. There are three different issues to be addressed here:

- Is the large $X/M^4$ limit consistent within the context of slow roll inflation?

- Under what circumstances do we expect large $X/M^4$ to arise? Are there any constraints on the mass scales of the theory implicit in the large $X/M^4$ limit?

- Is the large $X/M^4$ limit consistent with the idea of “integrating out” massive modes in that the length scales probed by inflation are larger than $M^{-1}$ even though $X/M^4$ is large?

To examine these issues, let us consider the chaotic inflation model more quantitatively.

First, let us check that the slow roll condition is not violated in the limit of large $X/M^4$. With the help of Eqs. (17) and (18) we see that $|\dot{\phi}|/H\phi \approx (2L)^{-1}$, so that the scalar field evolves slowly compared with the expansion rate of the Universe for all $L \gtrsim 1$, just as in conventional chaotic inflation. (Notice that this particular condition does not depend on $X/M^4$ at all.) From Eq. (10) we also see that the energy density of the Universe is dominated by $V(\phi)$ until

$$2XF_{,X} - F(X) \simeq V = \frac{m^2 L}{4\pi GF_{,X}} ;$$

where $V$ has been evaluated using the slow roll solution. This implies an end to the slow roll phase when

$$L_{\text{end}} \sim \frac{1}{3} \left(1 - \frac{F(X)}{2XF_{,X}}\right)$$

which implies $L_{\text{end}} \sim 1$ whether or not $X/M^4$ is large. Finally, in this particular cosmological model, $\ddot{\phi} = 0$ during the slow roll phase, so we do not have to consider additional constraints from the smallness of $\dot{\phi}$ as long as the other conditions underlying the slow roll approximation are satisfied. Consequently, we see that the conditions for slow roll to be valid are hardly affected by the magnitude of $X/M^4$.

To assess the conditions under which large $X/M^4$ are likely to arise, we need to find the mass scales implied in this regime. We shall see that large $X/M^4$ is required if $m \sim M$, but not otherwise.

To make quantitative estimates, we use the fact that the amplitude of scalar perturbations is fixed by observations of the CMB to be $P^S_k \sim 10^{-10}$ on length scales for which $L \approx 60$. If we define $u^2 = 2X/M^4$, then we find
\[
\frac{m}{M_{Pl}} = \frac{(3\pi P_s k^3)^{1/2}}{2L} \left[ \frac{(1 + u^2)^{3/4}}{(1 + 3u^2)^{1/4}} \right] \approx 2.6 \times 10^{-7} \left( 10^{10} P_k^S \right)^{1/2} \left( \frac{60}{L} \right) \left[ \frac{(1 + u^2)^{3/4}}{(1 + 3u^2)^{1/4}} \right]; \tag{28}
\]

for large values of \( u \) this becomes

\[
\frac{m}{M_{Pl}} \approx 1.9 \times 10^{-7} \left( 10^{10} P_k^S \right)^{1/2} \left( \frac{60}{L} \right) u. \tag{29}
\]

Thus, we see that \( m \propto u \) at large \( u \).

The value of the other mass scale associated with new physics can be found by combining Eq. (28) with Eq. (20); the result is

\[
\frac{M}{M_{Pl}} = \frac{(P_k^S)^{1/4}}{2\sqrt{L} u^{1/2}[(1 + u^2)(1 + 3u^2)]^{1/8}} \approx 2.0 \times 10^{-4} \left( 10^{10} P_k^S \right)^{1/4} \left( \frac{60}{L} \right)^{1/2} u^{1/2}[(1 + u^2)(1 + 3u^2)]^{1/8}; \tag{30}
\]

for large values of \( u \),

\[
\frac{M}{M_{Pl}} \approx 1.8 \times 10^{-4} \left( 10^{10} P_k^S \right)^{1/4} \left( \frac{60}{L} \right)^{1/2} u^{-1}. \tag{31}
\]

Thus, we see that \( M \propto u^{-1} \) at large \( u \).

The ratio of the two mass scales is

\[
\frac{m}{M} = \frac{\sqrt{3\pi} (P_k^S)^{1/4}}{\sqrt{L}} \left[ \frac{(1 + u^2)^{7/8} u^{1/2}}{(1 + 3u^2)^{1/8}} \right] \approx 1.3 \times 10^{-3} \left( 10^{10} P_k^S \right)^{1/4} \left( \frac{60}{L} \right)^{1/2} \left[ \frac{(1 + u^2)^{7/8} u^{1/2}}{(1 + 3u^2)^{1/8}} \right]. \tag{32}
\]

Thus we see that if the mass scales are comparable, we must have large values of \( u \). In the large \( u \) limit, we would have

\[
\frac{m}{M} \approx 1.1 \times 10^{-3} \left( 10^{10} P_k^S \right)^{1/4} \left( \frac{60}{L} \right)^{1/2} u^2, \tag{33}
\]

and we can solve this to find

\[
u \approx 30 \left( 10^{10} P_k^S \right)^{-1/8} \left( \frac{60}{L} \right)^{-1/4} \left( \frac{m}{M} \right)^{1/2}. \tag{34}\]

We can rewrite the two masses in terms of \( m/M \), assuming large values of \( u \):

\[
\frac{m}{M_{Pl}} \approx 5.9 \times 10^{-6} \left( 10^{10} P_k^S \right)^{3/8} \left( \frac{60}{L} \right)^{3/4} \left( \frac{m}{M} \right)^{1/2}, \tag{35}
\]

and \( M = m/(m/M) \). When \( m \) and \( M \) are comparable, the large \( u = \sqrt{2X/M^4} \) limit must apply, and the relevant mass scale is well below \( M_{Pl} \) i.e about \( 7 \times 10^{13} \) GeV.

If we insist that \( M \sim M_{Pl} \), no inconsistencies arise, but we must then require that \( u \ll 1 \), and therefore that \( m \ll M \). In this case, we find that \( m/M_{Pl} \approx 2.56 \times 10^{-7} \left( 10^{10} P_k^S \right)^{1/2} (60/L) \), or a mass scale \( m \approx 3 \times 10^{12} \) GeV, about a factor of 20 smaller than
is found if $m \sim M$. A theory with $M \sim M_{Pl} \gg m$ is consistent, and yields the conventional chaotic inflation picture, modulo correction terms that may be expressed as an expansion in $X/M^4 \simeq m^2 M_{Pl}^2/24\pi M^4 \simeq H^2 M_{Pl}^2/16\pi M^4 L$, which is $\sim H^2/M^2$ for $M \sim M_{Pl}$, as was suggested in [11]. However, not only is this behavior not required, but it is also strongly violated if $m \sim M$. Moreover, even if $X/M^4 \ll 1$ and $m \ll M$, the expansion parameter $X/M^4$ generally involves two dimensionless ratios, $H/M$ and $M/M_{Pl}$, not just $H/M$, and also depends on the number of e-folds remaining in inflation, $L$.

Finally, let us consider the third consistency issue, namely, whether the length and time scales involved during inflation are all long compared with $M^{-1}$, the fundamental scale of the theory. A truly complete treatment of this question is beyond the intended scope of this paper, but a necessary condition must be that $H/M$ be small. This condition must be satisfied in order to justify using an effective theory, such as Eq. (1), for computing the small scale fluctuations that arise from quantum effects during slow roll. If the inequality $H/M < 1$ is satisfied, the use of an effective theory for computing the evolution of the smooth background is justified automatically, since, in slow roll, $|\dot{\phi}/H\phi| \ll 1$, so that $|\dot{\phi}/M\phi| \ll H/M$. In the limit of large $X/M^4$, we find that

$$
\frac{H}{M} \approx 0.21(10^{10} P_k)^{1/8} \left(\frac{L}{60}\right)^{1/4} \left(\frac{m}{M}\right)^{1/2}.
$$

(36)

If $m \lesssim M$, then the inequality $H < M$ ought to be satisfied, but not very strongly. (We note that, by contrast, the limit $H/M_{Pl} \ll 1$ is satisfied easily irrespective of the value of $X/M^4$.) One may ask if we can trust an effective theory such as Eq. (1) when $X/M^4$ is large, since the higher order corrections to $F(X)$ may be important. However, our purpose here is to establish that there can be substantial effects even when the length and time scales involved during inflation are large compared with $M^{-1}$. Thus, we expect that a treatment of early Universe cosmology based on an effective theory such as Eq. (1) ought to be justified, but a more careful treatment than we have attempted here is needed to establish this point rigorously.

2. Other Potentials

Although we do not consider other inflation potentials in detail here, we should examine whether the behavior we have found is specific to inflation in the chaotic inflation potential $V(\phi) = \frac{1}{2} m^2 \phi^2$. To investigate this, let us ask whether conventional inflation would lead to large values of $X/M^4$ for other potentials, taking account of constraints on the scalar fluctuation amplitude. Let us consider instead potentials that are very flat functions of $\phi$, so that $V = V_0[1 + f(\phi/\phi_0)] \simeq V_0$ throughout slow roll inflation. Then, from the validity of the slow roll approximation, we know that $\dot{\phi}^2/M^4 \ll V_0/M^4$; if $V_0 \lesssim M^4$, then the small $X/M^4$ limit is guaranteed. In this case, we do expect the corrections to conventional inflation to be small, but they still need not be expressible simply in terms of $H/M$. For example, if $V(\phi) = V_0(1 - e^{-\phi/\phi_0})$, we find that

$$
\frac{X}{M^4} = \frac{H^2 M_{Pl}^4}{128\pi^2 \phi_0^2 M^4 (1 + L/8\pi G \phi_0^2)^2},
$$

(37)
and if \( V = V_0[1 - (\sigma + 1)^{-1}(\phi/\phi_0)^{\sigma + 1}] \), we find that

\[
\frac{X}{M^4} = \frac{H^2 M_{Pl}^4}{128\pi^2 \phi_0^2 M^4 [1 + (\sigma - 1)L/8\pi G \phi_0^2]^{\sigma+1}},
\]

both of which are proportional to \( H^2 \), but differ from \( H^2/M^2 \) in general. Irrespective of the precise form of \( V(\phi) \), Eq. (38) implies that in conventional (i.e. small \( X/M^4 \)) slow roll cosmology

\[
\frac{X}{M^4} = \frac{H^4}{8\pi^2 M^2 P^S_k}.
\]

Thus, in models where \( V = V_0[1 + f(\phi/\phi_0)] \), the slow roll approximation and the assumption of small \( X/M^4 \) are only valid simultaneously provided

\[
\frac{V_0}{M^2} \lesssim \frac{3M_{Pl}^2 P^S_k (P^S_k)^{1/2}}{2\sqrt{2}} \approx 1.1 \times 10^{-5} M_{Pl}^2 (10^{10} P^S_k)^{1/2}. \tag{40}
\]

For \( V_0 = \epsilon M^4 \), with \( \epsilon \lesssim 1 \), this condition implies \( M \lesssim 0.003 \epsilon^{-1/2} M_{Pl} \), or a characteristic mass scale \( M \lesssim 4 \times 10^{16} \epsilon^{-1/2} \) GeV. In other words, although small \( X/M^4 \) is more or less guaranteed by the slow roll assumption provided \( V_0 \lesssim M^4 \), the observational constraint imposed by the magnitude of \( P^S_k \) restricts the values of \( M \) for which the approximations in conventional slow roll cosmology are all consistent.

The situation is a bit trickier for models in which \( V(\phi) \) is a powerlaw in \( \phi \), \( V(\phi) = m^{4-n} \phi^n/n \), but with \( n \neq 2 \). (We also must consider \( n = 4 \) separately.) In this case, conventional inflation yields

\[
\dot{\phi} = -\sqrt{\frac{nm^{4-n}}{24\pi G}} \phi^{n/2-1} = -M_{Pl}^{n/2} m^{2-n/2} \left( \frac{\phi}{M_{Pl}} \right)^{n/2-1} \sqrt{\frac{n}{24\pi}}, \tag{41}
\]

and scalar fluctuations of amplitude

\[
P^S_k = \frac{128\pi}{3n^3} \left( \frac{\phi}{M_{Pl}} \right)^{n+2} \left( \frac{M_{Pl}}{m} \right)^{n-4}; \tag{42}
\]

the slow roll approximation requires

\[
\left| \frac{\dot{\phi}}{H\phi} \right| = \frac{n M_{Pl}^2}{8\pi \phi^2} \ll 1, \tag{43}
\]

so that \( \phi/M_{Pl} \) must be \( \gtrsim 1 \) in general. In fact, the slow roll solution for \( \phi \) is

\[
\frac{\phi}{M_{Pl}} = \sqrt{\frac{nL}{4\pi}}. \tag{44}
\]

From Eq. (42) we see that

\[
\frac{\phi}{M_{Pl}} = \left( \frac{3n^3 P^S_k}{128\pi} \right)^{1/\nu} \left( \frac{m}{M_{Pl}} \right)^{\nu+4}/\nu-2; \tag{45}
\]

for

\[
\nu = \frac{2n-4}{n+2}.
\]
since \( P^S_k \sim 10^{-10} \), the first factor is small, so the second must be large. Thus, for \( n > 4 \), slow roll requires large \( m/M_{Pl} \) and for \( n < 4 \), it requires small \( m/M_{Pl} \). If we regard the value of \( P^S_k \) to be fixed observationally, then we can solve Eq. (42) to find

\[
\frac{M_{Pl}}{m} = \left( \frac{3n^3 P^S_k}{128\pi} \right)^{\frac{1}{n-4}} \left( \frac{\phi}{M_{Pl}} \right)^{\frac{n+2}{n-4}} \left( \frac{4\pi}{nL} \right)^{\frac{n+2}{n-4}},
\]

using Eq. (44); combining with Eq. (41) implies

\[
\frac{X}{M^4} = \frac{n}{48\pi} \left( \frac{m}{M} \right)^4 \left( \frac{3n^3 P^S_k}{128\pi} \right)^{\frac{n}{n-4}} \left( \frac{nL}{4\pi} \right)^{\frac{4(n-1)}{n-4}}.
\]

Eq. (47) shows that unless \( m/M \ll 1 \), \( X/M^4 \gg 1 \) in conventional cosmology for all \( n < 4 \), and that unless \( m/M \gg 1 \), \( X/M^4 \ll 1 \) for all \( n > 4 \). For \( n = 4 \), we take the potential to be \( V(\phi) = \lambda\phi^4/4 \). In this case, Eq. (44) continues to hold true, and we find

\[
\dot{\phi} = -\sqrt{\frac{\lambda}{6\pi}} M_{Pl} \phi = -\sqrt{\frac{\lambda L}{6\pi^2}} M_{Pl}^2
\]

\[
P^S_k = \frac{2\lambda L^3}{3\pi^2}.
\]

Eqs. (48) may be combined to find

\[
\frac{X}{M^4} = \frac{P^S_k}{8L^2} \left( \frac{M_{Pl}}{M} \right)^4 \approx 3.5 \times 10^{-15} \left( 10^{10} P^S_k \right) \left( \frac{M_{Pl}}{M} \right)^4
\]

in this case, \( X/M^4 > 1 \) if \( M_{Pl}/M > 10^4 \) or so, and vice-versa.

Thus we see, overall, that for potentials that are very flat, we do not expect significant signatures of large mass scales in models with \( F(X) \neq X \), but for simple polynomial potentials \( \propto \phi^n \), such effects may be present, but are only likely for \( n < 4 \). The principal effect would be to change the dependence of the scalar and tensor fluctuation amplitudes on the parameters of the theory. This in turn would alter estimates of the energy scales associated with inflation based on \( P^S_k \) and \( P^T_k \), just as we found in § III 1 for chaotic inflation. Moreover, the ratio \( P^T_k/P^S_k \) is affected, but in a way that depends critically on the form of \( F(X) \), via \( c_s \) (see Eqs. (13) and (25)). Therefore, potentially, some information about the scale of short distance physics \( M \) may be encoded in the CMB fluctuations. Even though the determination of \( M \) from observations would be rather model-dependent, it may be possible, at least in principle, to probe short distance physics from cosmological measurements.

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