Bibliometrics for collaboration works

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Abstract

An important issue in bibliometrics is the weighting of co-authorship in the production of scientific collaborations, which are becoming the standard modality of research activity in many disciplines. The problem is especially relevant in the field of high-energy physics, where collaborations reach 3000 authors, but it can no longer be ignored also in other domains, like medicine or biology. We present theoretical and numerical arguments in favour of weighing the individual contributions as $1/N_{\text{aut}}^{\alpha}$, where $N_{\text{aut}}$ is the number of co-authors. When counting citations we suggest the exponent $\alpha \approx 1$, and when counting the number of papers we suggest $\alpha \approx 1/3 - 1/2$, with the former (latter) value more appropriate for larger (smaller) collaborations.

Introduction

In many research fields, scientific collaboration has become the standard way of operating, and moreover, due to the increasing complexity of the problems to be faced, the number of scientists with different competences involved in each single collaboration is increasing. In the extreme case of high energy physics numbers have already reached the four-digit level, but in many other domains, like medicine of biology, it is not unusual to find two-digit collaborations.

In the context of bibliometrics this hyper-authorship phenomenon pose a very important question, concerning the individual degree of property that must be assigned to the authors of a common scientific article and to the citations received by a paper. On one side, it is rather clear that attributing the full credit of a paper to each of the author is mystifying and tends to encourage fictitious collaborations, because of the obvious competitive advantage resulting from the much larger number of articles that a collaboration may produce in the same amount of time in comparison with an isolated author. Moreover, also the number of citations received is strongly correlated with the typical dimensions of the collaborations operating in a given field of research.

The choice of fractionally counting papers by attributing a $1/N_{\text{aut}}$ weight to each of the $N_{\text{aut}}$ co-authors of a paper would imply a strong penalty and would discourage collaborations, due to the obvious impossibility for $N_{\text{aut}}$ authors to produce $N_{\text{pap}}$ papers in the same time in which a single author may produce a single paper.
A reasonable compromise between these two extreme (and conceptually wrong) choices would therefore be quite welcome.

The bibliometric literature documents that collaboration papers tend to have higher impact than single-authored papers. Beaver (1986) studied physics, finding that co-authored research tends to be of higher quality than solo research. Bordons, García-Jover, and Barrigon (1993) studied Spanish publications in pharmacology and pharmacy finding that internationally co-authored documents have higher impact than the remaining collaborative documents or non-collaborative ones. Avkiran (2013) found that collaboration leads to articles of higher impact in finance, up to 4 collaborators. Gazni and Didegah (2011) found a significant positive correlation between the number of authors and the number of citations in Harvard publications. Hsu and Huang (2011) considered 90k articles in natural sciences, finding that the average number of citations scales as $N_{\text{aut}}^{1/3}$ (data extend up to about 10 co-authors), up to wide fluctuations. Lee and Bozeman (2005) found that the number of peer-reviewed journal papers is strongly and significantly associated with the number of collaborators, unlike the number of fractionally-counted papers. Katz and Hicks (1997) studied how the average number of citations per paper varies with different types of collaborations. See also the works of van Raan (1997), Sooryamoorthy (2009) and Birnholtz (2006).

We will better quantify and understand the bibliometric output of collaborations.

General theoretical arguments concerning scale-free systems suggest that the scientific productivity of collaborations and the corresponding frequency distribution of citations should show some, at least approximate, power law dependence on $N_{\text{aut}}$. Empirical evidence appears to support these arguments. Finding the most appropriate exponents for these scaling laws would offer the possibility of weighting the production of collaborations in the bibliometric estimate of the (quantitative) value of their results in such a way as to discourage adaptive and opportunistic behaviours while encouraging more appropriate practices in the indication of co-authorship.

In Section 2 we develop and present some theoretical arguments in favour of weighing the individual contributions to a single paper as $1/N_{\text{aut}}^\alpha$, where $\alpha \approx 1/3 - 1/2$, with the former (latter) value more appropriate for larger (smaller) collaborations. When counting overall citations we suggest the exponent $\alpha = 1$, corresponding to fractional counting. Fractional counting has been extensively discussed in the literature. For instance, it has been considered in the context of metrics and rankings by Aksnes, Schneider, and Gunnarsson (2012), Bouyssou and Marchant (2016), Carbone (2011), Egghe (2008), Hooydonk (1997), Leydesdorff and Bornmann (2010, 2011), Leydesdorff and Ophof (2010), Leydesdorff and Shin (2011), Rousseau (2014), Strumia and Torre (2019), and in the context of constructing research networks by Leydesdorff and Park (2017), Perianes-Rodríguez, Waltman, and van Eck (2016).

In Section 3 we analyze empirical data concerning a very large number of collaborations active in fundamental physics, where the range of available values of $N_{\text{aut}}$ allows for sufficiently convincing estimates of the exponents describing the dependence on $N_{\text{aut}}$ of the total number of papers and of the mean and total number of citations.

A theoretical approach

Scaling

The behaviour of collaborations with $N_{\text{aut}}$ authors can be viewed as scale-free phenomena for a wide range of values of $N_{\text{aut}}$. Any upper limit on $N_{\text{aut}}$ would be sufficiently large to exclude any sensible effect on the equilibrium distributions in the range of values we are interested to explore (3 to 4 orders of magnitude).

We therefore expect that the various indices $N_I$ that characterise bibliometric outputs of collaborations
are distributed at equilibrium following a power-law behaviour, which we parametrise as follows

$$\langle N_I \rangle = C_I N_{aut}^{p_I}$$  \hspace{1cm} (1)

where $C_I$ and the powers $p_I$ are constants. For example this applies to the number of papers $N_{pap} = C_{pap} N_{aut}^{p_{pap}}$ of papers produced (in a definite amount of time) by a scientific collaboration and to the average number of citations per paper $N_{cit} = C_{cit} N_{aut}^{p_{cit}}$. The total number of citations $N_{totcit}$ received by the papers of a collaboration then scales as

$$N_{totcit} = N_{cit} N_{pap} = C_{cit} C_{pap} N_{aut}^{p_{totcit}}, \hspace{1cm} p_{totcit} = p_{cit} + p_{pap}.$$  \hspace{1cm} (2)

**Scaling of the total number of citations**

Assuming a collective rational behaviour, and that on average the number of citations received by scientific papers may be considered as a reasonable proxy for their quality, we might expect that individual and collective choices would lead at equilibrium to

$$p_{totcit} \approx 1,$$  \hspace{1cm} (3)

namely that the total number of citations received by collaborations scales, on average, with the number of members. This expresses the fact that the (average) value of work made by $N_{aut}$ scientists should approximately be equal to $N_{aut}$ times the work made by a single scientist. A lower power $p_{totcit}$ would arise in the presence of gift authorships, namely of authors who sign papers without substantially contributing.

This means that the total number of citations is not a fair indicator for authors, as it grows with the number of co-authors. Similarly, the $h$-index introduced by Hirsch (2005), being on average proportional to square root of the number of citations, grows on average as the square root of the number of co-authors.

A bibliometric index which does not overestimate nor underestimate individual contributions to a collaboration is then the number of fractionally-counted citations $N_{fcit}$ received by each author. This means that a fraction $f_A$ of each paper is attributed to each co-author $A$ such that the fractions $f_A$ sum up to unity. When there is no information about who contributed mostly, we assume a common $f_A$ equal to the inverse of the number of authors. On average $N_{fcit}$ scales with power index $p_{fcit} = p_{totcit} - 1 \approx 0$, showing that $N_{fcit}$ is a scale-invariant quantity that (unlike the number of citations) cannot be arbitrarily inflated grouping authors.

**Scaling of total number of papers**

In order to implement these concepts into actual bibliometric indices for the total number of papers or for the average number of citations, we must offer arguments in favour of explicit values for the exponents $p_{pap}$ and $p_{cit}$. We present here a simple “theoretical” argument. As the goal of collaborations is achieving more than what single authors can achieve, we expect $p_{cit} > 0$. Assuming rational behaviour in the formation of collaborations one may expect that (at least for not-too-big groups), individual competence of partners be as far as possible complementary, and therefore “orthogonal” in some abstract $N$-dimensional “space of competencies”. We may therefore regard the qualitative output of a collaboration as the vector sum of $N$ orthogonal vectors.

The limit where all authors have “orthogonal” competencies corresponds to $N = N_{aut}$: then the length of such vector scales as $N_{aut}^{1/2}$. This corresponds to

$$p_{pap} = 1/2, \hspace{1cm} p_{cit} = 1/2.$$  \hspace{1cm} (4)
Sometimes, more than one collaborator is needed to fulfill a needed competency: realistic collaborations organise in $N_{\text{sub}} \leq N_{\text{aut}}$ sub-collaborations that work on “orthogonal” topics. We assume the number of sub-collaborations satisfies the scaling law
\begin{equation}
N_{\text{sub}} = N_{\text{aut}}^s \quad \text{with exponent } s \leq 1.
\end{equation}

Then, the average number of citations of each paper scales as the square root of the number of “orthogonal” competencies $N = N_{\text{sub}}$:
\begin{equation}
N_{\text{cit}} \propto \sqrt{N} \propto N_{\text{aut}}^{s/2}.
\end{equation}

Assuming again an optimal distribution of resources, $N_{\text{totcit}}$ is expected to scale as $N_{\text{aut}}$, and thereby the number of papers is expected to scale as
\begin{equation}
N_{\text{pap}} \propto N_{\text{aut}}^{1-s/2}.
\end{equation}

It is reasonable to assume that the number of papers scales as $N$, the number of topics about which the collaboration has competencies. This leads to $s = 1 - s/2$, solved by $s = 2/3$, and thereby to
\begin{equation}
p_{\text{pap}} = 2/3, \quad p_{\text{cit}} = 1/3.
\end{equation}

A weaker growth of the number of papers with $N$ leads to smaller $p_{\text{pap}}$.

**Data about collaborations in fundamental physics**

In this section we present bibliometric data in fundamental physics, that offer support for
\begin{equation}
p_{\text{pap}} \approx 0.5 - 0.6, \quad p_{\text{cit}} \approx 0.4 - 0.5, \quad p_{\text{totcit}} \approx 1, \quad p_{\text{fcit}} \approx 0.
\end{equation}

We use the INSPiRE\textsuperscript{2} database that gives a picture of fundamental physics world-wide from $\sim 1970$ to 2019: 1.3 millions of scientific papers, 32 millions of references, 70 thousands of identified authors. Fundamental

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\textsuperscript{2}High-Energy Physics Literature INSPiRE Database (https://inspirehep.net/).
physics contains large collaborations, up to 3000 authors, and is thereby a good sample to study how the bibliometric outputs of collaborations scale with the number of collaborators. We will show data for two different kinds of collaborations:

1. **Official collaborations.** We consider the 5965 (mostly experimental) collaborations listed in the INSPIRE database. Each collaboration produced a certain number \( N_{\text{pap}} \) of papers, roughly written with the same group of \( N_{\text{aut}} \) authors. The left panel of fig. 1 gives some demographic information. We will show scatter plots where each collaboration is plotted as a dot, with main collaborations indicated by their name. Furthermore, we show the mean (median) as a red (magenta) curve. A blue dotted line highlights the scaling with the number of authors.

2. **Occasional collaborations.** Many more multi-authored papers have been written by collaborations that form for one or few papers. To study them we proceed as follows. For each author in the INSPIRE database we compute the average number of authors of his/her papers, \( \langle N_{\text{aut}} \rangle \geq 1 \), as well as his/her bibliometric indices (number of papers, of citations, etc). In view of the large number of authors we avoid showing scatter plots: we only show averages. Furthermore, results are shown separately within the main topics of fundamental physics: experiment, theory, astro/cosmo. The first category includes all papers in the hep-ex (high-energy experiments) and nucl-ex (nuclear experiments) category of arXiv. The latter category includes papers in astro-ph, which contains astrophysics and cosmology. Theoretical papers are those appeared in hep-ph (high-energy phenomenology), hep-th (high-energy theory), hep-lat (lattice), nucl-th (nuclear theory), gr-qc (general relativity and quantum cosmology).

**Scaling of the number of papers**

Figure 2 shows that the number of papers produced by official (left panel) or occasional (right panel) collaborations scales with the number of authors as

\[
N_{\text{pap}} \propto N_{\text{aut}}^{0.5 - 0.6}.
\]
In the right panel, theoretical papers with many authors fall below the scaling. These are rare outliers: almost all papers in theoretical categories have few authors. Theoretical papers with many authors mostly are collections of separate contributions grouped together, rather than big collaborations.

**Scaling of the mean number of citations**

Figure 3 shows that the mean number of citations received by papers written by an official collaboration (left panel) or by an author (right panel) roughly scales with the average number of co-authors as

\[ \frac{N_{\text{cit}}}{N_{\text{pap}}} \propto N_{\text{aut}}^{0.4-0.5}. \]  

(11)

This result is in agreement with Hsu and Huang (2011), which (in a much smaller sample) found a power \( \approx \frac{1}{3} \) up to \( N_{\text{aut}} \sim 10 \).

**Scaling of the total number of citations**

Figure 4 shows that the total number of citations received by an official collaboration (left panel) or author (right panel) grows roughly linearly with the average number of co-authors:

\[ N_{\text{cit}} \propto N_{\text{aut}}^1. \]  

(12)

This is expected by combining the two previous scalings: the total number of citations of a collaboration can be decomposed as the product of the number of papers written by the collaboration, times the average number of citations received per paper: these factors roughly scale as \( N_{\text{aut}}^{0.5-0.6} \times N_{\text{aut}}^{0.4-0.5} \).

Figure 5 shows that the total number of fractionally-counted citations \( N_{\text{fcit}} = \sum_p N_{\text{pcit}}/N_{\text{paut}} \) received by papers \( p \) written by an official collaboration (left panel) or author (middle panel) is roughly independent of the average number of co-authors. A similar result holds for a related quantity, “individual citations”, defined as fractionally counted citations divided by the number of references of the citing papers:

\[ N_{\text{fcit}}, N_{\text{icit}} \propto N_{\text{aut}}^0. \]  

(13)

This means that fractionally-counted citations or individual citations neither reward nor penalise working in big collaborations, while citations or the \( h \) index reward authors who prefer working in big collaborations.
We studied the bibliometric output of collaborations.

In section we presented theoretical arguments: various bibliometric indices that quantify the output of collaborations (total number of papers, of citations, etc) are expected to scale as a power of the number of authors. We presented expected values of the scaling exponents based on conservation of work, collective rational behaviour, and on plausibility argument about combining “orthogonal” competencies.

In section we computed the mean and median bibliometric output of official and occasional collaborations in fundamental physics, finding that averages satisfy scaling behaviours with power indices in rough agreement with the expectations of section .

So far we studied the means of distributions. To conclude, we extend the discussion to the full distributions, or at least to their variances. The right panel of fig. 1 shows the distributions of individual citations received by all papers in our database, splitting them according to their number of authors. We again see that papers with more authors are more cited. We also see that distributions have large variabilities: the distributions of individual citations are approximatively log-normal with log-scale means that scale as $\langle N_{\text{cit}} \rangle \propto N_{\text{aut}}^{0.5}$ and with log-scale widths that remain approximatively constant. This behaviour is obtained from our initial theoretical considerations adding one extra assumption: that collaborations tend to equalise the total amount of skill within each competence, such that the distribution in $N_{\text{cit}}$ of a collaboration is simply obtained rescaling the distribution of single-author papers. The bibliometric output of collaborations formed as random groups of authors would instead show larger variabilities.

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Figure 5. Total number of fractionally-counted citations versus number of collaborators.

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