The universe evolution as a possible mechanism of formation of galaxies and their clusters

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The Kepler problem is considered in a space with the Friedman–Lemaitre–Robertson–Walker metrics of the expanding universe. The covariant differential of the Friedmann coordinates (X=a(t)x) is considered as a possible mechanism of the formation of galaxies and clusters of galaxies. The cosmic evolution leads to decreasing energy of particles, causing free particles to be captured in bound states. In this approach the evolution of the universe plays the role usually inscribed to Cold Dark Matter.

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The description of a Newtonian motion of a galaxy in a gravitational field of mass of a cluster of galaxies is used for analysis of Cold Dark Matter in the modern cosmological researches\textsuperscript{[1,2,3,4]}. Here we face with the following contradiction: the Newtonian motion of a galaxy is described in the flat space-time \((ds^2) = (dt)^2 − \sum_i(dx^i)^2\); whereas the observational data are analysed in terms of the Friedman–Lemaitre–Robertson–Walker (FLRW) metrics

\[(ds^2) = (dt)^2 − \sum_i a^2(t)(dx^i)^2. \tag{1}\]

Therefore, it is worth to study the Newtonian motion of a particle in a gravitational field

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in the space-time with the FLRW metrics where observational coordinates of the expanding Universe are considered as

$$X^i = a(t)x^i, \quad dX^i = a(t)dx^i + x^i da(t), \quad (2)$$

and instead of the differential of the Euclidean space $dX^i$, we use the covariant differential of the FLRW space coordinates

$$a(t)dx^i = d[a(t)x^i] - x^i da(t) = dX^i - X^i \frac{da(t)}{a(t)}. \quad (3)$$

Just this problem is considered in this note to study a possible mechanism of the formation of galaxies and their clusters, taking into account that both the Newton motion and the cold dark matter problem should be formulated in the terms of the FLRW metrics (1).

One can check that the interval (1) in terms of these variables (2) becomes

$$(ds^2)^2 = (dt)^2 - \sum_i (dX^i - H(t)X^i dt)^2), \quad (4)$$

where $H(t) = \dot{a}(t)/a(t)$ is the Hubble parameter. In the space with the interval (1) and the covariant derivative $(\dot{X}^i - H(t)X^i)$ the Newton action takes the form

$$S_A = \int_{t_i}^{t_f} dt \left[ \sum_i \left( P_i(\dot{X}^i - H(t)X^i) - \frac{P_i^2}{2m_I} \right) + \frac{\alpha}{R} \right], \quad (5)$$

where $\alpha = M_0 m_I G$ is a constant of a Newtonian interaction of a galaxy with a mass $m_I$ in a gravitational field of a cluster of galaxies of mass $M_0$.

Let us consider a particle moving in a plane in the cylindrical coordinates

$$X^1 = R \cos \Theta, \quad X^2 = R \sin \Theta \quad (6)$$

described by the action (5) in terms of this coordinates

$$\int_{t_i}^{t_f} dt \left[ P_R(\dot{R} - H(t)R) + P_{\Theta} \dot{\Theta} - \frac{P_R^2}{2m_I} + \frac{P_{\Theta}^2}{2m_I} R^2 + \frac{\alpha}{R} \right]. \quad (7)$$

In this action $P_{\Theta} = J_I$ is the integral of motion. The total energy reads as

$$E_{\text{tot}}(t) = H(t)RP_R + E_N, \quad (8)$$

where

$$E_N = \frac{P_R^2}{2m_I} + \frac{J_I^2}{2m_I R^2} - \frac{\alpha}{R}.$$
is customary Newtonian energy of the system (7). The total energy (8) is not conserved, due to the expansion of the Universe, and it can be rewritten in the form

$$E_{\text{tot}}(t) = \frac{m_I [\dot{R}^2 - H^2(t)R^2]}{2} + \frac{J_I^2}{2m_I R^2} - \frac{\alpha}{R}.$$  \hfill (9)

One can see, that in the comparison with the Newtonian energy in the flat space where $H(t) = 0$, an additional term $H^2(t)R^2$ is appearing. It can be treated as a friction potential induced by metric (4). What is consequence of this friction?

To know the role of the cosmic evolution of universe in a motion of a particle in the expanding universe, we consider as an example a solution of the equation of motion

$$\ddot{R} - (\dot{H}(t) + H(t)^2)R - \frac{J_I^2}{m_I R^3} + \frac{\alpha}{m_I R^2} = 0.$$  \hfill (10)

Let a particle started from a point where its total energy is equal to zero $E_{\text{tot}}(t_I) \equiv 0$ with the initial data $(t = t_I, \ R = R_I, \ \dot{R} = 0)$.

For simplicity we restrict ourselves by the case of the rigid state $\Omega$

$$H(t) = \frac{H_I}{1 + 3H_I(t - t_I)},$$ \hfill (11)

where $H_I = H(t = t_I)$.

Solution of the equations for the case of $\Omega$ in units $y = R/R_I, \ x = H_I(t - t_I)$ is given in Fig. 1 for $\alpha/m_I R_I^3 H_I^2 = 1, \ J_I^2/m_I R_I^4 H_I^2 = 3$. This solution gives a remarkable fact: later when $H_I(t - t_I) \geq 10$ the energy of this particle becomes negative $E_{\text{tot}} = E_{\text{tot}}/(m_I H_I^2 R_I^2) = -0.0405$, and particle is bound.

This solution shows us that the cosmic evolution can form the Kepler bound states such as galaxies and their clusters, as the cosmic evolution decrease energy of fragments, urging free fragments to capture in bound states, and free galaxies, in clusters of galaxies. The lowering of energy which leads to bound state is the influence of friction appearing in eq. $\Omega$. Fig. 2 shows us a pathway of a particle in the cartesian coordinates, that starts at the moment $t_I$ from the point $(1,0)$ with the zero value of the total energy $E(t_I) = 0$.

It is worth reminding that the energy conservation law $\dot{E}_N(t) = 0$ in the conventional Newton theory in the flat space-time gives the link of the initial data $v_{I0}, \ R_I = R(t_I)$ at $H = 0$

$$v_{I0}(R_I) = \sqrt{\frac{\alpha}{m_I R_I}} \equiv \sqrt{\frac{r_g}{2R_I}},$$ \hfill (12)
Figure 1: At the upper panel the numerical solution of the equation (5), in dimensionless variables $y(x) = R/R_I$ and $x = H_I(t - t_I)$ with boundary conditions $y(x = 0) = 1$ and $y'(x = 0) = 0$ are displayed. The curve at lower panel demonstrates the evolution of the total energy given by (8).

Figure 2: Pathway of a motion of a particle in the cartesian coordinates $(X^1, X^2)$ which starts at point $(1, 0)$ with zero value of the total energy (8).

where $r_g = 2\alpha/m_I \simeq 3 \times 10^5 M$ cm is the gravitational radius of an object, and $M$ is the mass of an object expressed in the solar mass.

In the considered case of the nonzero Hubble velocity $H \neq 0$ (for week variation of the
Hubble parameter) the link of the initial data takes the form
\[ v_I(R_I) = \sqrt{\frac{r_g}{2R_I} + 2(H_I R_I)^2}. \]  
(13)

From (13) follows that the Newton theory is not valid for large radii
\[ R_I \geq R_{cr} = \left(\frac{r_g}{H_I^2}\right)^{1/3}. \]  
(14)

The present-day value of the Hubble parameter \( H_0^{-1} \approx 10^{28} \) cm gives the value of the critical radius
\[ R_{cr}[M] \approx 10^{20} M^{1/3} \text{cm}, \]  
(15)

where \( M \) is the mass of an object expressed in the solar mass. One can see that the critical radial distance (15) is very close to the size of galaxies as well as to galaxy groups and galaxy clusters when adequate masses of those structures are taken into account:
\[ R_{cr}[M \approx 10^9] \approx 10^{23} \text{cm} \approx 30 \text{kpc}, \]
\[ R_{cr}[M \approx 10^{12}] \approx 10^{24} \text{cm} \approx 0.3 \text{Mpc}, \]
\[ R_{cr}[M \approx 10^{15}] \approx 10^{25} \text{cm} \approx 3 \text{Mpc}, \]

and it even coincides with the size of the Coma \( R_{size} \approx 3 \times 10^{25} \) cm [1].

Let us define the size of a galaxy \( R_{size} \) so that \( \bar{v} = v_{10}(R_{size}) = v_I(R_{size}) \sqrt{2/(2 + \gamma)} \), where \( \gamma = (R_{size}/R_{cr})^3 \leq 1 \) (for the example, if \( R_{size} = R_{cr}/2 \), we have \( \gamma = 1/8 \) and \( \bar{v} = (r_g H_I)^{1/3} \approx 3 \times 10^8 M^{1/3} \) at \( H_I \approx H_0 \)).

Then the rotational curve of the circular velocity – radius relation (13) can be considered in terms of the ratio \( \xi = R/(2R_{size}) \)
\[ \frac{v_I}{\bar{v}} = \sqrt{\frac{1}{\xi} + (2\xi)^2 \gamma}. \]  
(16)

The dependence (16) of circular velocity \( v_I/\bar{v} \) from the radius is given on Figure 3, where the first curve on the hyper-surface at \( \gamma = 0 \) corresponds to the Newtonian case, and the curves at \( \gamma \neq 0 \) deviate from the Newtonian case. Their behaviour imitates the Cold Dark Matter halos beyond the validity region of the Newton approximation at \( R \geq R_{cr} \) [2, 3, 4].

So, the violation of the virial theorem observed in spiral galaxy rotation curves which is usually considered as an evidence of the existence of the Cold Dark Matter halos in galaxies,
Figure 3: The dependence of circular velocity $v_l/\bar{v}$ from the radius $\xi = R/(2R_{\text{size}})$ where $R_{\text{size}}$ is chosen so that a rotation curve $\bar{v}$ coincides with the Newton one $v_0$: $\bar{v} = v_0(R_{\text{size}}) = \sqrt{2/(2 + \gamma)}v_l(R_{\text{size}})$ at $\xi = 0.5$, and $\gamma = (R_{\text{size}}/R_{\text{cr}})^3 \leq 1$; for $\gamma = 1/8$, $R_{\text{size}} = R_{\text{cr}}/2$.

in fact can be interpreted as an evidence of the Hubble evolution. Moreover we have seen that this cosmic evolution can be considered as one of the mechanism of galaxies and galaxy clusters formation.

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