Friction Domination with Superconducting Strings

Konstantinos Dimopoulos *

and

Anne–Christine Davis †

Department of Applied Mathematics and Theoretical Physics,
University of Cambridge, Silver Street,
Cambridge, CB3 9EW, U.K.

September 5, 2021

Abstract

We investigate the evolution of a superconducting string network with arbitrary, constant string current in the friction dominated regime. In the absence of an external magnetic field the network always reaches a scaling solution. However, for string current stronger than a critical value, it is different than the usual, horizon-scaling of the non-superconducting string case. In this case the friction domination era never ends. Whilst the superconducting string network can be much denser than usually assumed, it can never dominate the universe energy density. It can, however, influence the cosmic microwave background radiation and the formation of large scale structure. When embedded in a primordial magnetic field of sufficient strength, the network never reaches scaling and, thus, eventually dominates the universe evolution.
1. Introduction

The microphysics of cosmic strings has received considerable attention. In particular, Witten [1] showed that cosmic strings become superconducting as a result of boson condensates or fermion zero modes in the string core. Such strings are capable of carrying a sizeable current, with the maximum current being about $10^{20} A$ for a grand unified scale string. Inevitably, such currents have cosmological and astrophysical [2] consequences. The consequences for emission of synchrotron radiation [3] and for high energy $\gamma$-rays [4][5][6] have been explored.

Unlike non-conducting strings, loops of superconducting string can be stabilised from collapse by the angular momentum of the current carriers, forming vortons [7]. If vortons are sufficiently stable then consistency with standard cosmology puts severe constraints on theories giving rise to such strings. However, such constraints have been derived assuming that the evolution of a network of superconducting strings is similar to that of ordinary strings. Early studies using both analytic [8] and numerical techniques [9][10][11] showed that the string evolution was indeed similar to that of ordinary cosmic strings. However, these studies neglected the very early times when the string is interacting strongly with the surrounding plasma. During this friction dominated period the string correlation length grows until is catches up with the horizon and a scaling solution sets in.

Since the interaction of particles with a superconducting string [3][12] is very different from that of an ordinary string [13], there is every reason to expect that the friction dominated period could be vastly different. Thus, it is important to investigate this and ascertain whether or not the superconducting string network really does reach a scaling solution. Even if the network does reach a scaling solution it could still be very different from that of the non-conducting case.

We first discuss the interaction of the plasma particles on the string and then show the evolution of the curvature radius assuming a constant string current. We show that, in the absence of a primordial magnetic field, there is a critical current above which the friction dominated period never ends. Instead the string reaches a so-called plasma scaling solution, where the density of strings is considerably greater than the usual horizon scaling. The cosmological consequences of this are investigated. When the string is embedded in a magnetic field of strength greater than a critical value, a string dominated universe results. Finally, we discuss our assumption that the string current is constant. Our conclusions are summarised in the final section. In what follows, unless stated otherwise, we use natural units ($\hbar = c = 1$).

2. Friction on superconducting cosmic strings

Friction on a cosmic string is caused by particle interaction with the string as it moves through the plasma. The friction force per unit length is,

$$f \sim \rho \sigma v \bar{v}$$  \hspace{1cm} (1)

where $\rho$ is the energy density of the plasma, $\sigma$ is the interaction cross-section, $v$ is the velocity of the string segment and $\bar{v} \simeq \Delta p/m$ is given by,
\[ \bar{v} \equiv \max\{v, v_{th}\} \quad (2) \]

where \( \Delta p \) is the particle’s momentum change, \( v_{th} \sim \sqrt{T/m} \) is the thermal velocity of the plasma particles, \( m^2 \simeq m_0^2 + \alpha T^2 \) is the mass of a plasma particle with rest mass \( m_0 \), \( \alpha = g^2/4\pi \) with \( g \) the gauge coupling) and \( T \) is the plasma temperature. Also, \( m_P = 1.22 \times 10^{19}\text{GeV} \) is the Planck mass.

Curves and wiggles on the strings tend to untangle due to string tension, which results in oscillations of the curved string segments on scales smaller than the causal horizon and larger than their curvature radii. Friction dissipates the energy of these oscillations and leads to their gradual damping. Thus, the strings become smooth on larger and larger scales with their curvature radius growing accordingly. The characteristic damping timescale is 

\[ t_d \equiv \frac{\mu v^2}{\dot{\mathcal{E}}} \sim \frac{\mu v}{f} \sim \frac{\mu}{\rho \sigma \bar{v}} \quad (3) \]

where \( \mu \) is the string mass per unit length and \( \dot{\mathcal{E}} \sim fv \), is the energy loss per unit time per unit length. The above timescale corresponds to a length-scale called the friction length \[15\], \[16\].

Initially the friction length is smaller than the curvature radius. In this case Kibble \[14\] has shown that the growth of the string curvature radius is,

\[ \frac{dR}{dt} \sim t_d \quad (4) \]

If the growth of the friction length is faster than that of \( R \), then at some point it will catch up with the curvature radius and \( (4) \) will cease to be valid. This occurs when the curvature radius reaches the horizon \[14\] and the strings become smooth on horizon scales. From then on \( R \sim t \), the friction domination era ends and the string network satisfies the well–known horizon–scaling solution. Thus, in order to see whether a network of superconducting cosmic strings ever reaches a scaling solution one has to study the evolution of the string curvature radius. For this, a closer look to the superconducting string system is required.

The current of a charged–current carrying superconducting string creates a magnetic field around the string core. This field is very strong near the string and does not allow the plasma particles to approach. As the charged particles encounter the magnetic field their motion is diverted in such a way that they create a surface current of opposite orientation than the string current \[3\], which screens the magnetic field. Thus, while the string moves in the plasma it is shielded by a magnetocylinder, which contains the magnetic field and does not allow the plasma to penetrate.

The flow of the plasma around the string creates a shock front, which is the border of the magnetocylinder \[3\], whose distance from the string is determined by the pressure
balance between the magnetic field and the plasma. The string magnetic field is given by the Biot-Savart law,

\[ B_s(r) \simeq \frac{2J}{r} \]  

(5)

where \( J \) is the string current and \( r \) is the distance from the string. All through this paper, unless stated otherwise, we will assume that the string current \( J \) is constant. The validity of this assumption will be discussed later but here we can just mention that it is primarily based on conservation laws due to topological index theorems [1]. The magnitude of the current will be treated as a free parameter and can have any value up to \( J_{\text{max}} \sim e\sqrt{\mu} \), where \( e \) is the charge of the current carriers. We also assume, for simplicity, that the current switches on at the phase transition that creates the string network.

The pressure balance, \( B_s^2(r_s) \sim \rho \bar{v}^2 \) suggests that the dimensions of the magnetocylinder are of order,

\[ r_s \sim \frac{J}{\sqrt{\rho \bar{v}}} \]  

(6)

The magnetocylinder cross-section is not circular of course [18] but (6) is a good estimate of the minimum distance of the shock front from the string [3]. We can use, therefore, \( r_s \) as the superconducting, charged-current carrying string cross-section. From (6) it is evident that the cross-section increases with the string current, in agreement with [12].

The above cross-section \( r_s \) has to be compared with the usual cross-section for a cosmic string,

\[ \sigma_{cs} \sim k^{-1} \]  

(7)

where \( k \sim (m\bar{v})^{-1} \) is the momentum of the incident particle in the frame of the string [13].

3. Evolution of the curvature radius

The evolution of the string curvature radius determines whether the string network reaches a scaling solution or overcloses the universe instead.

As suggested by numerical simulations [10], the intercommuting process of superconducting strings is very similar to the non-superconducting string case, i.e. the probability of producing string loops is of order unity. High intercommutation probability ensures the scaling of the network in the absence of friction. Moreover, since the superconducting network dissipates its energy more efficiently, due to additional radiation emission [8], the intercommutation probability can be much lower than the limit required for non-superconducting strings without undermining the scaling solution.

Assuming an intercommutation probability of order unity suggests that the network density evolves according to,

---

1For temperatures higher than the electroweak energy scale the electroweak symmetry is unbroken and the “magnetic” field of the string is actually due to the hypercharge generator. However, as shown in [17], its magnitude still follows, \( B \sim J/r \) as in (5).
where $a$ is the scale factor of the universe ($a \propto t^{1/2}$ for the radiation era).

The above implies that the typical inter-string distance grows as $\dot{R} \sim v$. From (8) it can be inferred that the network would satisfy a scaling solution, $\rho_s/\rho = const.$, provided $R \propto t$ for the radiation era. If $R$ manages to grow up to horizon size then scaling is ensured since $R \sim t$. In this case the inter-string distance is the horizon size and the string velocity is $v \sim 1$. We will refer to this scaling solution as horizon-scaling.

From (1) and (7) we find that the friction force for non-superconducting strings is,

$$f_{ns} \sim \frac{\rho v}{m} \quad (9)$$

Similarly, from (1) and (6) we obtain for the plasma friction force,

$$f_{pl} \sim J v \sqrt{\rho} \quad (10)$$

Note that the friction forces are not affected by the transition from the radiation to the matter era, since $\rho \sim (m_P/t)^2$ at all times.

Inserting the above into (3) one finds,

$$t_{ns}^d \sim \frac{\mu m}{\rho} \quad (11)$$

and

$$t_{pl}^d \sim \frac{\mu}{J \sqrt{\rho}} \quad (12)$$

From the above it can be inferred that the temperature dependence of the damping time is changed only due to the variation of the particle mass. This has an effect only when the plasma friction force is subdominant. In this case, using (1) we obtain,

$$R_{ns} \sim \frac{\eta}{m_P^3} t_5^{5/4} \quad (13)$$

and

$$R_{ns}' \sim \left[ \frac{\eta}{\sqrt{m_0 m_P}} \right] t_m^{-1/2} t_3^{3/2} \quad (14)$$

where (13) and (14) correspond to temperatures higher and lower than $T(t_m) \sim m_0$ respectively (with $m_0 \sim 1 GeV$) and we have used that, $\mu \sim \eta^2$ with $\eta$ being the scale of the symmetry breaking that produced the string network.

Similarly, in the case of plasma friction domination, for all temperatures, we have,

\footnote{The prime when used denotes the low temperature case.}
\[ R_{pl} \sim \frac{\eta}{\sqrt{J m_p}} t \]  

The above suggest that, if the plasma friction is subdominant, the curvature radius grows more rapidly than the horizon and will reach the horizon size at,

\[ t_s \sim \frac{m_P^3}{\eta^4} \quad (16) \]

or at,

\[ t'_s \sim \frac{m_P^2}{\eta^2 m_0} \sim \left[ \frac{\eta}{\sqrt{m_0 m_P}} \right]^2 t_s \quad (17) \]

for high and low temperatures respectively. Comparing with \( t_m \) we find,

\[ \eta > \sqrt{m_0 m_P} \iff t_s > t'_s > t_m \quad (18) \]

Thus, for \( \eta > \sqrt{m_0 m_P} \sim 10^9 \text{GeV} \) the network would reach horizon scaling before \( t_m \).

If, on the other hand, plasma friction dominates, then \( R \) follows (15) and the curvature radius will always remain in constant proportion to the horizon size. Also, the strings of the network assume a constant, terminal velocity of order,

\[ v_T \sim \dot{R} \sim \frac{\eta}{\sqrt{J m_p}} \quad (19) \]

Both the friction forces (9) and (10) are decreasing with time. However, the plasma friction force declines less rapidly, for all temperatures. Thus, there is always a time when plasma friction will come to dominate. By comparing the forces, for high and low temperatures respectively, we find,

\[ t_c \sim \frac{m_P}{J^2} \quad (20) \]

and

\[ t'_c \sim \frac{m_P}{m_0 J} \sim \left( \frac{J}{m_0} \right) t_c \quad (21) \]

However, the evolution of the network will be affected by this only if \( t_c \leq t_s \) (or \( t'_c \leq t'_s \)). In the opposite case \( f_{ns} \) remains dominant until the curvature radius grows up to horizon size and the horizon–scaling solution begins. Comparing the two critical times gives the critical string current,

\[ J_c \equiv \frac{\eta^2}{m_P} \sim \eta \sqrt{G \mu} \quad (22) \]

for all temperatures, where \( G \) is Newton’s gravity constant \((G = m_P^2)\). Note that, for high temperatures, \( T_c \equiv T(t_c) \sim J \) and \( J_c \sim T_s \equiv T(t_s) \).
If the string current is smaller than $J_c$ then the evolution of the curvature radius follows the usual pattern described in the literature \[19\]. If, however, $J > J_c$, the curvature radius grows more rapidly than the horizon until $t_c$ (or $t'_c$). After that, it follows \[13\]. Since, $R_{pl} \propto t$, the network evolves again in a self similar way but with typical inter-string distance smaller than the usual horizon–scaling by a factor of the magnitude of the terminal velocity. We will call this scaling solution \textit{plasma–scaling}.

From the above we also find,

$$J \gtrsim m_0 \Leftrightarrow t_c \lesssim t'_c \lesssim t_m$$

(23)

Thus, if $J > J_c$ and $J > m_0$ the network reaches plasma–scaling before $t_m$. Summing up, the above suggest that:

- For $J < J_c$, the network always reaches horizon–scaling at $t = \min\{t_s, t'_s\}$.
- For $J \geq J_c$, the network always reaches plasma–scaling at $t = \min\{t_c, t'_c\}$.

At this point we should point out that, for currents weaker than $J \sim m_0 \sim 1 \text{ GeV}$, it can be shown that the magnetocylinder system is not impenetrable to plasma particles and, thus, for $J_c < J < m_0$, the above analysis is not entirely reliable.\footnote{From \[22\] it follows that this regime can only be realized when, $\eta \leq \sqrt{m_0 m_P}$.}

4. The plasma–scaling solution

Let us, now, estimate the string network’s energy density during plasma–scaling. In the case of horizon–scaling ($J \leq J_c$) the energy density of the network of open strings can be easily found to be,

$$\frac{\rho_s}{\rho} \sim \left(\frac{\eta}{m_P}\right)^2 \sim G\mu$$

(24)

The above result is the usual estimate for a non-superconducting string network energy density \[13\].

Now, if friction continues to dominate ($J > J_c$) then the network reaches plasma–scaling and the energy density of the open string network is larger. Indeed, using $\rho_s \sim \mu/R_{pl}^2$ we find,

$$\frac{\rho_s}{\rho} \sim \frac{J}{m_P} \ll 1$$

(25)

The above shows that, \textit{the current carrying string network can never dominate the universe energy density}. For overcritical currents the network density and the string terminal velocity are,
\begin{align}
J_c & \leq J \leq J_{\text{max}} \\
1 & \geq v_T \geq \sqrt{\frac{\eta}{m_p}} \\
G\mu & \sim \left(\frac{\eta}{m_p}\right)^2 \leq \frac{\rho_s}{\rho} \leq \frac{\eta}{m_p} \sim \sqrt{G\mu}
\end{align} \tag{26}

From the above it can be inferred that the larger the current the slower the strings move. Thus, the inter-string distance and the curvature radius are smaller. As a result, although the number of intercommutations per unit time per unit volume \(\sim v/R\) \cite{20} is not reduced, the loops produced by the network are smaller and, therefore, less string length is lost. This results into a larger open string energy density. However, even the maximum \(\rho_s\) cannot dominate the overall energy density of the universe.

Still, the existence of an overdense string network could have other observable consequences. Indeed, were such a network to seed the large scale structure, it would produce structure of a smaller correlation. However, it would \textit{not} create density inhomogeneities that may be incompatible with the galaxy formation scenario. This can be seen as follows.

Setting \(\delta \rho \equiv \rho_s\), the fractional density fluctuation of the string network at horizon crossing is,

\begin{equation}
\left(\frac{\delta \rho}{\rho}\right)_s \sim \frac{G\mu}{v_T^2} \geq G\mu \tag{27}
\end{equation}

However, the overdensities generated by the strings are smaller. Indeed, while moving through the plasma, the open strings generate wakes of overdense matter due to the conical string metric \cite{21}. The overdensity inside a newly formed wake is of the order of \(\delta \rho_W/\rho \simeq 1\), its length is \(l_W \sim v_T t\) and its thickness is \(d_W \simeq v_T t \tan(\Delta/2) \sim (G\mu)v_T t\), where \(\Delta \simeq 8\pi G\mu\) is the deficit angle of the string metric. The linear mass overdensity of a wake \(\delta \mu_W = \delta \rho_W d_W l_W \sim \rho(G\mu)(v_T t)^2\). Thus, since \(R_{pl} \sim v_T t\), the total overdensity due to open string wakes is,

\begin{equation}
\left(\frac{\delta \rho}{\rho}\right)_w \simeq \frac{1}{\rho} \frac{\delta \mu_W}{R_{pl}^2} \sim G\mu \tag{28}
\end{equation}

The above shows that the overdensities generated by the string network are independent of the string velocity. This is so, because, although a denser network will create more wakes per horizon volume the length of such wakes will be shorter corresponding to filaments of small linear mass density. Thus, superconducting strings in grand unified theories (GUTs) could still account for the large scale structure observed even if they carry substantial currents.

Similar results are obtained for the temperature anisotropies in the microwave sky. The later are produced due to the boost of radiation from the string deficit angle. The anisotropy generated by a single string is given by,
\begin{equation}
\frac{\Delta T}{T}_s \sim (G\mu)v_T \leq G\mu \tag{29}
\end{equation}

However, the overall anisotropy includes contributions of all the strings that have crossed the line of sight until the present time. An analytical model to calculate the rms anisotropy from a string network is described in [22], where it is shown that, \((\Delta T/T)_\text{rms} \simeq (G\mu)v\sqrt{M}\), where \(M \simeq (H^{-1}/R)^2\) is the number of open strings inside a horizon volume, with \(H^{-1}\) being the Hubble radius. Thus, the rms temperature anisotropy in the case of a plasma scaling string network is,

\begin{equation}
(\Delta T/T)_\text{rms} \simeq (G\mu)vT \frac{H^{-1}}{R_{pl}} \sim G\mu \tag{30}
\end{equation}

The above shows that the rms anisotropy is again independent of the string velocity. Thus, GUT superconducting strings could generate the observed anisotropy regardless of their current. The only effect that a denser network would have on the pattern of temperature fluctuations would be to hide its non-Gaussian profile in smaller angular scales than the usually estimated 1° [23], since the inter-string distance would be smaller than the horizon size at decoupling.

Superconducting strings could add to the temperature anisotropies by emitting radiation themselves, mostly due to the decay of loops or small scale structure. However, as shown in [3], the radiational contribution of the open string network is of minor importance. The later could have an effect regarding only ultra high energy radiation [4] and could be a \(\gamma\)-ray source.

The plasma-scaling solution for the open string network is a direct consequence of assuming that intercommuting produces loops with efficiency of order unity. This scaling solution, however, is much different from the usual horizon scaling of non-superconducting strings, since the network is denser with slower moving strings. Note that, in this case, the friction force never becomes negligible, i.e. the friction domination era never ends.

In the above calculations we have implicitly assumed that the overall energy density is not dominated by the loops produced by the network. This is not a trivial assumption since current carrying string loops can avoid total collapse by forming stable vortons which could have lethal consequences to the universe evolution [4]. However, vorton production is beyond the scope of this paper.

5. In a primordial magnetic field

It would be interesting to embed the whole network in a primordial magnetic field and observe how its evolution is going to be (if at all) affected. It is obvious that, in principle, the current carrying strings do interact with an external magnetic field, since, at a distance, they appear not too different from current carrying wires. Thus, the magnetic force per unit length on the stings would be,

\begin{equation}
f_B \sim JB \sin \theta
\end{equation}
where $B$ is the magnitude of the external field and $\theta$ is the angle between the magnetic field lines and the string segment. Depending on $\theta$ the above force can accelerate or decelerate the string.

The magnetic force can be compared with $f_{ns}$ and $f_{pl}$ in order to determine under which conditions it will be the dominating one. For simplicity, we will consider the high temperature case only.

If $f_{ns}$ is the dominant friction force then by comparing (9) and (31) (with $\sin \theta = 1$) it can be found that the magnetic force will dominate at,

$$t^B_c \sim \frac{\eta^2}{(B_0 J)^4 m_P}$$

where $B_0$ is the magnitude of the magnetic field at network formation.

The magnetic force will never dominate provided the network reaches horizon-scaling before $t^B_c$. The condition for this is,

$$B_0 < \left( \frac{J_c}{J} \right) \eta^2$$

If $f_{pl}$ is the dominant friction force, then by comparing (10) with (31) we find that the magnetic force is subdominant if,

$$B_0 < \sqrt{J_c J} \eta^2$$

In order for the magnetic field energy density $\rho_B \sim B^2/8\pi$ not to dominate the overall energy density of the universe, we require, $B_0 \leq \eta^2$. In view of (33) and (34) this constraint implies that, $J < J_c$ ensures that the magnetic field will never dominate the forces acting on the strings. Thus, a weak current $J \leq J_c$ suggests that the string network will follow the standard non-superconducting string network evolution regardless of the existence of a primordial magnetic field. On the other hand, in the case of a strong current the magnetic field can influence the network evolution provided it is stronger than the constraint (34).

If the magnetic force was dominant then the network would evolve according to its action on the strings. In this case however, the curvature radius would not grow larger than the coherence scale of the magnetic field $R_B$. Indeed, since the string tension is dominated by the magnetic force, there is no driving force to “straighten” the strings over larger scales. The incoherence of the magnetic field would twist the strings and curve them over scales of order $R_B$. Also, any loops with dimensions larger than $R_B$, will not contract. So, over these scales, no string length is lost and, effectively, there is no loop production. Thus, $R \propto a$ and we have string domination [24]. Although the above have been calculated in the high temperature case only we expect similar results in low temperatures.

The ratio of energy densities is,

$$\frac{\rho_s}{\rho} \sim \left( \frac{\eta t}{R_B m_P} \right)^2$$

Thus, string domination could be avoided only if,
\[ R_B > \frac{\eta}{m_P} R_H \]  

(36)

where \( R_H \sim t \) is the horizon size. For current values \( R_B \sim 1 \text{kpc} \) and \( R_H \sim 10^4 \text{Mpc} \) we find that we can avoid string domination if \( \eta < 10^{11} \text{GeV} \).

The above is dependent on the assumption that the string current \( J \) remains constant. However, if the external magnetic field is strong enough, it would affect the string current.\(^4\) Still, even by taking into account the back-reaction of the external field on the string current, the qualitative picture is not significantly changed.

6. The string current assumptions

In the above we have assumed that the current switches on at the time of formation of the string network and remains constant during the subsequent network evolution. In this paragraph we take a closer look at these assumptions.

i) Current conservation

Witten has suggested that the superconducting strings will most probably carry a strong DC current, which would persist due to topological index theorems \(^1\). In general, current conservation is a direct outcome of the field equations for a straight and infinite superconducting string (see for example \(^25\)). However, for a realistic string there are some delicate points that have to be clarified for our assumption to be justified.

One point is the fact that Brownian contraction gradually decreases the string length \( L \) between two distant fixed points on the string. Since, \( L \sim \frac{d^2}{R} \) \(^19\) where \( d \) is the distance between the points, this process suggests that, \( \frac{\dot{J}}{J} \sim \frac{\dot{R}}{R} \). However, this effect is counteracted by the smoothing of the current flow.

The string current has its own correlation length \( l \). The orientation of the current on larger scales follows a one-dimensional random walk pattern. Thus, between two points with string length distance \( L > l \), the average string current is \( J_{rms} \sim J/\sqrt{(L/l)} \), where \( J \) is the coherent current inside a correlated string segment. Current conservation, suggests that the overall current \( J_{rms} \) remains constant if the string length is unaltered. Therefore, the local current has to diminish with time as \( l \) grows.\(^5\)

Putting together the two effects described above we see that the Brownian contraction would in fact increase \( J_{rms} \) with time and this may be enough to hold the local current \( J \) more or less constant, although its coherence length would grow. Note that in our treatment of the string system we were dealing, in fact with the local, coherent current on the string. The balance between the two effects depends delicately on the growth rate of the current coherence length, which is as yet unknown. A reasonable guess would be

\(^4\)One could argue that, since the current grows as \( \dot{J} \sim vB \) \(^1\), it will eventually reach its maximum value, \( J_{max} \sim \eta \) and from then on remain constant.

\(^5\)The growth of the current coherence length is due to the algebraic addition of the current at the interface between two initially uncorrelated domains, as have been shown numerically in \(^10\).
that the correlation length grows with the speed of light, since, inside the string, the charge carriers are massless.

Balancing the above effects suggests that, \( J_{rms}L \sim \sqrt{Lt} J \), where \( J_{rms}/J_{rms} \sim \dot{R}/R \). Thus, the time dependence of the current is,

\[
\frac{\dot{J}}{J} = \frac{1}{t} - \frac{\dot{R}}{R}
\]  

(37)

The above suggests that the current is indeed constant at a scaling solution, when \( R \propto t \). Thus, although the current will be time dependent during \( fn_s \) domination our results are unaffected since, in this period, the existence of a current does not influence the evolution of the network.

By means of (37), the initial critical current for the network can be found with the use of (16) and (22) as,

\[
J^0_c \sim J_c \sqrt{\frac{m_P}{\eta}}
\]

(38)

for both low and high temperatures.

However, apart from the above, the current built-up inside the strings could be affected by a number of other effects. Most of these effects concern the small scale structure of the strings, which may introduce an AC component to the string current \[11\][26][27]. However, the AC currents are likely to be suppressed by radiational back-reaction \[8\] and particle production in cusps \[5\][9].

The evolution of the current magnitude on the open strings is still unclear. If the current was not kept constant, then its variation could destabilise the delicate balance of the plasma–scaling solution. In the case that the string current decreased with time, \( f_{pl} \) would become less effective, the curvature radius would grow faster than in (13) and, thus, it would eventually catch up with the horizon resulting into horizon scaling. If the opposite was true and the current increased with time, then plasma friction would become larger and the curvature radius would not be able to follow the growth of the horizon at a constant ratio. Instead the strings would become slower and the network denser. However, the growth of \( J \) would have to end when it reached \( J_{max} \). Then, the network would assume the plasma–scaling solution at its maximum density case. In all cases, though, we still manage to avoid domination of the universe energy density from the open string network.

**ii) Initiation of the current**

Although we have assumed that the current switches on at network formation, in general, this could occur at a later stage. However, even if the network evolves initially without the presence of a current, the final picture is not severely altered.

Indeed, were this the case, the network would initially evolve according to the standard cosmic string evolution scenario. Thus, the curvature radius would grow as in (13) until it reached horizon scaling. If the time \( t_s \), when current switched on, was later than this then the network evolution would not be affected. If, on the other hand, it switched
on before $t_*$ then the evolution would be identical to the one described earlier provided $t_s \leq t_c$. If, however, $t_* > t_s > t_c$, then the string network would remain comovingly frozen due to excessive plasma friction, until the length-scale given by (15) reached the size of the network curvature radius. From then on the evolution would continue again as described earlier.

Switching on the current in later times could have an effect on the magnitude of $J$, since the later cannot be larger than the energy scale at the time. Indeed, suppose that the current switched on at the temperature, $T_s \sim \zeta \eta$, where $\zeta \leq 1$. Then, over a string segment of dimensions of the order of the curvature radius $R(t_s)$, the maximum average current at the current initiation time $t_s$ would be, $(J_{rms})_{max} \sim \zeta \eta / \sqrt{n}$, where $n \sim R(t_s)/(\zeta \eta)^{-1}$. Since $R$, until $t_s$, would evolve according to (13) the current would be,

$$(J_{rms})_{max} \sim \zeta^{7/4} \eta \left( \frac{\eta}{m_P} \right)^{1/4}$$

Assuming that the coherence length of the current grows with light-speed, after some time the current in the segment considered, will become coherent with magnitude $J \sim J_{rms}$. Thus, (29) will be the maximum possible string current. Comparing with $J_c$ of (22) we find that, in order for the current to affect the string network evolution $\zeta$ has to be greater than,

$$\zeta \equiv \left( \frac{\eta}{m_P} \right)^{3/7}$$

For GUT strings $\zeta_c \sim 10^{-2}$. Thus, the evolution of GUT strings that become current carrying at the electroweak transition [28] ($\zeta \sim 10^{-14}$) cannot be affected by their current.

### 7. Discussion and conclusions

In conclusion, we have investigated the evolution of a charged-current carrying, open string network. We have shown that, in the absence of a primordial magnetic field, the network, in general, reaches a scaling solution. This ensures that the network does not dominate the energy density of the universe. If the network is embedded in a strong enough magnetic field, then it is possible that it will never reach a scaling solution and will dominate the energy density of the universe.

We have also demonstrated that, in all cases considered, the existence of a current on the strings will have an effect only if the current is larger than a critical value $J_c$, given in (22). We have found a similar critical value for the possible influence of a primordial magnetic field. It is interesting that $J_c$ is also the critical current with respect to radiation emission from the string, over which electromagnetic radiation dominates gravitational radiation [29].

For string current stronger than $J_c$ we have shown that friction never ends and the scaling solution is very different than the standard cosmic string horizon-scaling. The curvature radius and the inter-string distance follow the horizon growth in constant proportion, but they could be much smaller than the horizon size. As a consequence, the string network would be a lot denser. Inside a horizon volume the strings would be more
curved and twisted and would move much slower. Thus, the loops produced by the network are smaller, although the intercommuting rates are unaffected. Therefore, even though the network is denser, it loses less mass by loop production and this is why so much of the string length is kept in the open strings. We called this scaling solution plasma-scaling.

If the network reached plasma-scaling then there are a number of consequences that may have observational importance. First of all, production of smaller loops could relax the vorton constraints [7]. Also, a denser network would generate large scale structure with much different features than the one produced by a horizon scaling network. Indeed, the slow moving strings would create filaments instead of thin wakes, whose separating distances could be much smaller than the horizon. Also the imprint of the strings on the microwave sky would be Gaussian in smaller angular scales than the horizon scale at decoupling. However, neither the magnitude of the overall density perturbations or the rms temperature fluctuations will be affected. Therefore, GUT superconducting strings can still satisfy observations even if they carry a substantial current.

The plasma scaling solution could also have important astrophysical effects. Indeed, a denser superconducting string network would result in substantial generation of high energy radiation [2][4][5][6]. Agreement with observations by adjusting accordingly the parameters of the model could provide information on the underlying theory.

The evolution of the open string network described in our paper is expected to be modified in the matter era due to streaming velocities developed by the plasma during the gravitational collapse of the protogalaxies. Such streaming velocities will tangle the strings since, if friction is dominant, the later are more or less “glued” to the plasma [30]. The situation resembles the case of a dominant primordial magnetic field. The network curvature radius and inter-string distance would follow the scale of the plasma streaming and this could lead to string domination.

In our treatment we have made a number of assumptions. Since, in order to explore the curvature radius evolution, we were primarily interested on larger scales, we chose to ignore the small scale structure of the strings and its consequences (AC currents, string linear energy density and tension renormalisation) since it is likely to be substantially suppressed by radiation back-reaction and particle production. We have also assumed that the string magnetocylinder is impenetrable and free of plasma. It can be shown that this assumption is valid for $J \geq m_0 \sim 1 GeV$.

One fundamental assumption made concerns current conservation. We have argued that the local string DC current would remain more or less constant. However, more work is required here, particularly on the evolution of the current’s coherence length and the current’s AC component. However, although a variable current may influence our results, it would never lead to string domination. Finally, we have not considered the spring/vorton problem, which is beyond the scope of this paper.

This work was partly supported by PPARC, the E.U. under the HCM program (CHRX-CT94-0423), the Isaac Newton Fund (Trinity College, Cambridge), and the Greek State Scholarships Foundation (I.K.Y.). We would like to thank B. Carter, P. Peter and N. Turok for discussions. Finally, we thank Observatoire de Paris (Meudon) for hospitality.
References

[1] E. Witten, Nucl. Phys. B 249 (1985) 557.

[2] E.M. Chudnovsky and A. Vilenkin, Phys. Rev. Lett. 61 (1988) 1043; R. Plaga
   Phys. Rev. D 47 (1993) 3635.

[3] E.M. Chudnovsky, G.B. Field, D.N. Spergel and A. Vilenkin, Phys. Rev. D 34
   (1986) 944.

[4] C.T. Hill, D.N. Schramm and T.P. Walker, Phys. Rev. D 36 (1987) 1007.

[5] B. Pacyński, Ap. J. 335 (1988) 525.

[6] R. Plaga, Ap. J. 424 (1994) L9.

[7] R.L. Davis and E.P.S. Shellard, Nucl. Phys. B 323 (1989) 209; R. Brandenberger,
   B. Carter, A.C. Davis and M. Trodden, Phys. Rev. D 54 (1996) 6059;

[8] M.N. Butler, R.A. Malaney and M. B. Mijic, Phys. Rev. D 43 (1991) 2535.

[9] P. Amsterdamski, Phys. Rev. D 39 (1989) 1524.

[10] P. Laguna and R.A. Matzner, Phys. Rev. D 41 (1990) 1751.

[11] D.N. Spergel, W.H. Press and R.J. Scherrer, Phys. Rev. D 39 (1989) 379; Nature
     334 (1988) 682.

[12] W.B. Perkins, Nucl. Phys. B 364 (1991) 451.

[13] A.E. Everett, Phys. Rev. D 24 (1981) 858; R. Brandenberger, A.C. Davis and
     A. Matheson, Nucl. Phys. B 307 (1988) 909; W. Perkins, L. Perivolaropoulos,
     A.C. Davis, R. Brandenberger and A. Matheson, Nucl. Phys. B 353 (1991) 237.

[14] T.W.B. Kibble, J. Phys. A 9 (1976) 1387; T.W.B. Kibble, Acta Phys. Pol. B 13
     (1982) 723; T.W.B. Kibble, Phys. Rep. 67 (1980) 183.

[15] M.B. Hindmarsh and T.W.B. Kibble, Rep. Prog. Phys. 58 (1995) 477.

[16] C.J.A.P. Martins and E.P.S. Shellard, Phys. Rev. D 54 (1996) 2535.

[17] J. Ambjørn and P. Olesen, Int. J. Mod. Phys. A 23 (1990) 4525; W.B. Perkins
     and A.C. Davis, Nucl. Phys. B 406 (1993) 377.

[18] J. Hurley, Phys. Fluids 4 (1961) 109.

[19] A. Vilenkin and E.P.S. Shellard, Cosmic Strings and other Topological Defects,
    Cambridge Monographs on Mathematical Physics, CUP, Cambridge 1994.
[20] T. Vachaspati and A. Vilenkin, Phys. Rev. D 30 (1984) 2036.

[21] J. Silk and A. Vilenkin Phys. Rev. Lett. 53 (1984) 1700; T. Vachaspati, Phys. Rev. Lett. 57 (1986) 1655; T. Hara and S. Miyoshi, Prog. Theor. Phys. 77 (1987) 1152.

[22] L. Perivolaropoulos, Phys. Lett. B 298 (1993) 305.

[23] B. Allen, R. Caldwell, E.P.S. Shellard, A. Stebbins and S. Veeraghavan, Phys. Rev. Lett. 77 (1996) 3061; D. Coulson, P. Ferreira, P. Graham and N. Turok, Nature 368 (1994) 27; L. Perivolaropoulos, Phys. Rev. D 48 (1993) 1530; M.N.R.A.S. 267 (1993) 529; (1997) CRETE-97-14 [astro-ph/9704011].

[24] T.W.B. Kibble, Phys. Rev. D 33 (1986) 328.

[25] A. Vilenkin and T. Vachaspati, Phys. Rev. Lett. 58 (1987) 1041.

[26] M. Aryal, A. Vilenkin and T. Vachaspati, Phys. Lett. B 194 (1987) 25.

[27] D.N. Spergel, T. Piran and J. Goodman, Nucl. Phys. B 291 (1987) 847.

[28] A.C. Davis and W.B. Perkins, Phys. Lett. B 390 (1997) 107.

[29] E. Copeland, D. Haws, M. Hindmarsh and N. Turok, Nucl. Phys. B 306 (1988) 908.

[30] E. Chudnovsky and A. Vilenkin, Phys. Rev. Lett. 61 (1988) 1043.