Exact Path Integral Quantization of 2-D Dilaton Gravity

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We demonstrate that in the absence of ‘matter’ fields to all orders of perturbation theory and for all 2D dilaton theories the quantum effective action coincides with the classical one. This resolves the apparent contradiction between the well established results of Dirac quantization and perturbative (path-integral) approaches which seemed to yield non-trivial quantum corrections. For the Jackiw-Teitelboim (JT) model, our result is even extended to the situation when a matter field is present.

In recent years, stimulated by the ‘dilaton black hole’ numerous studies of quantized gravity in \(d = 2\) were performed using the second order formalism

\[
\mathcal{L}_{(1)} = \sqrt{-g} \left( -X \frac{R}{2} - \frac{U(X)}{2} (\nabla X)^2 + V(X) \right),
\]

(1)

A common feature of all these studies is that due to the particular structure of the theory the constraints can be solved exactly, yielding a finite dimensional phase space. This remarkable property raises hope that one will be able to get insight into the information paradox. However, in the presence of an additional matter field again an infinite number of modes must be quantized.

Using the path integral approach one-loop quantum corrections have been considered perturbatively yielding even for pure dilatonic gravity a highly non-trivial renormalization structure, clearly in contradiction to the results from the above approaches.

Here we close the gap between these two approaches by demonstrating that for there are no local quantum corrections for the path integral approach as well. Adding matter fields we are still able to quantize the JT-model exactly. Finally we comment on higher loop contributions for the general case.

1 Exact Path Integral Quantization – Matterless Case

Let our starting point be the first order action (\(X^+, X^-\) and \(X\) are auxiliary fields)

\[
\mathcal{L}_{(2)} = X^+ D e^- + X^- D e^+ + X d\omega + \epsilon(V(X) + X^+ X^- U(X)),
\]

(2)

where \(D e^a\), \(d\omega\) and \(\epsilon\) are the torsion, curvature and volume two form, respectively. The quantum equivalence of (2) to the second order form (1) was demonstrated in

We will be working in a ‘temporal’ gauge \(e^+_0 = \omega_0 = 0, \quad \epsilon_0 = 1\) which, by applying Permanent address: Department of Theoretical Physics, St. Petersburg University, 198904 St. Petersburg, Russia
the canonical BVF formalism, produces the determinant $F = (\det \partial_0)^2 \det(\partial_0 + X^+ U(X))$. The generating functional for the Green functions is

$$ W = \int (\mathcal{D}X)(\mathcal{D}X^+)(\mathcal{D}e^-_1)(\mathcal{D}e^+_1)(\mathcal{D}\omega_1) F \exp \left[ i \int x \mathcal{L}(2) + \mathcal{L}_s \right], \quad (3) $$

where $\mathcal{L}_s$ denotes the contribution of the sources ($j^\pm$, $J^\pm$, $J$) corresponding to the fields ($e^-_1, \omega_1, X^+, X$). Integrating over the zweibein components and the spin connection results in

$$ W = \int (\mathcal{D}X)(\mathcal{D}X^+)(\mathcal{D}X^-) \delta(1) \delta(2) \delta(3) F \exp \left[ i \int J^+ X^- + J^- X^+ + J X \right], \quad (4) $$

$$ \delta(1) = \delta \left( -\partial_0 X^+ + j^+ \right) \quad (5) $$

$$ \delta(2) = \delta \left( -(X^+ U(X) + \partial_0) X^- + j^- - V(X) \right) \quad (6) $$

The remaining integrations can be performed most conveniently in the order $X^+, X^-, X$ to yield

$$ Z = -i \ln W = \int \frac{1}{\partial_0 + U(X)} \frac{1}{\partial_0 - U(X)} j^+ \quad (8) $$

where $X$ has to be replaced by $X = \partial_0^{-1} j^+ + \partial_0^{-2} j$. Note that the determinant $F$ is precisely canceled by these last three integrations. Eq.(8) gives the exact non-perturbative generating functional for connected Green functions and it does not contain any divergences, because it clearly describes tree–graphs only. Hence no quantum effects remain.

In [6] it was demonstrated that by inserting the explicit expressions for the mean fields (e.g. $\bar{X}^\pm = \frac{\delta Z}{\delta J^\pm}$) into the effective action $\Gamma$ one obtains exactly the classical action (2) in our temporal gauge, up to surface terms. As expected this clearly re-demonstrates the absence of (local) quantum effects.

### 2 Exact Path Integral Quantization – JT-Model with Matter

Coupling a scalar field minimally $\dagger$ to the gravitational action leads to the well known Polyakov action $\mathcal{L}_P = \sqrt{-g}R \Box^{-1} R$. Clearly $\mathcal{L}_P$ is not linear in the zweibein anymore and will therefore, in general, destroy the procedure of section 1. However, for the JT model $\dagger\dagger$ ($U(X) = 0, V(X) = \text{const}X$), we are able to perform a complete integration of the gravitational and the matter action. Beginning with the $X^+$ and $X^-$ integration we arrive (starting from (3)) at

$$ W = \int (\mathcal{D}e^-_1)(\mathcal{D}e^+_1)(\mathcal{D}\omega_1) \delta(X^-) \delta(X^+) \delta(X^-) F \exp i \int j \omega_1 + j^+ e^-_1 + j^- e^+_1 + \mathcal{L}_s \quad (9) $$

The case of non-minimal coupling has been treated in [8].
\[
\delta(X) = \delta \left( \partial_0 \omega_1 + J - \alpha e^+_1 \right) \]  \hspace{1cm} (10)
\[
\delta(X^+) = \delta \left( \omega_1 + J^+ + \partial_0 e^-_1 \right) \]  \hspace{1cm} (11)
\[
\delta(X^-) = \delta \left( \partial_0 e^+_1 + J^+ \right) \] . \hspace{1cm} (12)

The remaining integrations can be done with the use of (10 to 12) during which the term \( F \) gets cancelled again. As a final result we arrive at \( Z = -i \ln W = \int j\omega_1 + j^+ e^-_1 + j^- e^+_1 + \mathcal{L}_P \) where the zweibein and connection have to be expressed as the solutions of (10 to 12). This gives the exact non-perturbative generating functional for connected Green functions, even in the presence of matter. In the absence of external matter sources therefore the quantum JT model with matter is locally equivalent to the classical JT model with Polyakov term, i.e. the 'semiclassical' approximation becomes exact.

3 Global Considerations and the General Case with Matter

All Dirac quantization approaches treat models whose kinetic dilaton was removed by a conformal transformation, which drastically change the global structure already at the classical level. It is therefore by no means clear how the quantum theory is affected. As we demonstrated here, local quantum effects are not affected by that transformations, however there are sources of conformal non-invariance which may change global effects.

Two-loop contributions from scalar matter for \( U(X) = 0 \) and arbitrary \( V(X) \) have been considered in 6. To this order, interestingly enough the effective action only experiences a renormalization of the dilaton potential, i.e.

\[
\Gamma = S_{cl}(e^-_1, \omega, X) + hS_P(e^-_1, e^+_1) + O(h^3) \]  \hspace{1cm} (13)

where \( V \rightarrow V - h^2\gamma V'' \), \( \gamma \) being a field independent constant.

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