An investigation into the nonstationary characteristics of separation-bubble formation on a smooth circular cylinder in the critical transition range

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ABSTRACT

To investigate the characteristics of the separation-bubble formation of flow over a smooth circular cylinder in the critical transition range, a sliding-window (SW) method and a peak-valley (PV) method were proposed to identify the intermittent jumps, named the characteristic events, in the real-time pressure signal obtained on both sides of the cylinder model. By evaluating the counts of the qualified events of the SW and PV methods, the PV method was found less sensitive to the small-scale disturbances in the pressure signal, therefore was adopted for later analysis. With the PV method, the characteristic events were identified from the pressure signal and categorized into two types: Type-1 is referred to the events of pressure descending and Type-2 is referred to the events of pressure ascending. Subsequently, the count per minute of the characteristic events was determined for describing the intermittency of the separation-bubble formation, and the time scale of each of the characteristic event was regarded as the time length of the separation-bubble formation. The count per minute of the characteristic events appeared to be the highest in the transitional regime. While the time scales of the characteristic events were varying with Reynolds number, the weighting-averaged normalized time scales in the transitional regime of the three cases studied were found comparable to the normalized time scale of the lift jump noted in the literature. Physically, the characteristic events found in the pressure signals in this study can be attributed to the three-dimensional aspect of separation-bubble formation.

KEYWORDS: flow over circular cylinder, critical regime, nonstationary characteristics, time scale

1. INTRODUCTION

The investigations on the characteristics of the flow passing a circular cylinder have been continued for decades because of the complicated flow phenomena sensitive to the Reynolds number [1, 2]. For instance, the drag coefficient of the circular cylinder, which is frequently discussed in aerodynamics, varies obviously with the Reynolds number [1–5]. A drastic decrease of the drag coefficient occurs while the Reynolds number is around the order of $10^5$ [1–6], which is called the critical transition range in this study. Roshko [5] indicated that the critical phenomenon is involved with transition of boundary layer and flow separation on the surface of a circular cylinder. Referring to the literature [2, 4, 7–11], a series of flow states during the critical transition range have been identified, which are named the precritical, one-bubble and two-bubble states. In the precritical state for Reynolds number around $2 \times 10^5$, a discrete transition from the subcritical state could be identified that the periodicity of vortex shedding was degraded due to the unsteadiness of formation of small separation bubbles on the two sides of the circular cylinder, which was accompanied with higher base pressure measured [11]. As the Reynolds number increased to the one-bubble state, the asymmetric flow state with steady lift [4, 8–10] and the asymmetric distribution of the pressure on the surface [7, 12] would be extraordinary. This asymmetric phenomenon is caused by a notable separation bubble, which occurs on either side of a circular cylinder. According to the literature [4–6, 8, 10], the separation bubble is featured with the laminar flow separation, subsequently transition to turbulent flow in the separated shear layer, and then reattachment to the surface. The asymmetric flow pattern with one-sided separation bubble on the circular cylinder is sensitive to the Reynolds number [8, 13]. As a follow-on of the one-bubble state, the two-bubble state occurs as the Reynolds number further increased around $4 \times 10^5$ [2, 4], for which the flow gets symmetric because of the separation bubbles of comparable sizes formed on both sides of the circular cylinder with a significantly low drag and higher base pressure coefficient [4, 7, 12–14]. Overall speaking, in the critical transition range not only the distinguished transitions of the flow states take place within a small range of Reynolds number, but also the flow phenomena are highly nonstationary and three-dimensional due to coupling of the processes of flow separation and laminar–turbulent transition [8, 9, 11, 14–18]. Bearman [4] and Farell and Blessman [7] investigated the nonstationary phenomenon of separation-bubble bursting by the hot-wire measurement in the wake of the circular cylinder. Miau et al. [14] and Lin et al. [18] examined...
the intermittent presence of the separation bubble by the pressure measurements on the surface of the circular cylinder. Cadot et al. [16] investigated the discrete flow states based on the histograms of the real-time pressure coefficients obtained at different Reynolds numbers. Desai et al. [17] looked into the characteristics of the real-time pressure fluctuations at a section on the circular cylinder using a proper orthogonal decomposition method at high subcritical Reynolds numbers.

The present authors were intrigued by the nonstationarity of the flow characteristics in the critical transition range, which remains a challenge for further exploration. According to Bendat and Piersol [19], a random data process is called nonstationary if its mean value and the correlation of the ensemble are varying with time. In addition, the nonstationary process can be briefly categorized into three kinds: time-varying mean value, time-varying mean square value and time-varying frequency [19]. To resolve the characteristics of the nonstationary data, various methods have been reported in the literature, for instance, the short-time Fourier spectral analysis, wavelet transform [20], Hilbert–Huang transform (HHT) [21] and piecewise linear representation (PLR) [22]. The first three methods, especially HHT, are useful for analyzing the nonstationary data characterized by the time-varying frequency [11], but not suitable for the cases with intermittent jumps in the signal. On the other hand, the methods of PLR including a number of algorithms, for instance, the linear regression with the sliding-window (SW) method or peak–valley (PV) determination, are suitable to pick up the intermittent jumps in a time-series signal trace [22]. The algorithms employed in this study are capable of identifying the jump-like fluctuations, which are caused by the intermittent formation of the separation bubble on the circular cylinder.

This study is intended to describe the nonstationarity of the flow phenomena observed in the critical transition range with the quantities defined. Given the fact that the flow phenomena learned from the literature as well as the experimental data reported in this paper are highly nonstationary, it is impossible to reduce the meaningful quantities based on the time-averaging algorithms, which are mainly for the data assumed to be stationary. Instead, the approach made in this study was to develop the techniques pertaining to extracting the information regarding the intermittent presence of separation bubble on the circular cylinder. The methods of analysis proposed in this study could help to better understand the nonstationary characteristics of pressure or force fluctuations experienced by a structural body in the critical regime, which could be crucial to the considerations of structural loads in practical applications, for instance, the gust loads on aircraft [23].

2. EXPERIMENTAL METHOD

2.1 The wind tunnel

The experiments were conducted in the ABRI (Architecture and Building Research Institute) environmental wind tunnel located in the Kuei-ren Campus of National Cheng Kung University. The wind tunnel is a closed-loop type driven by an axial fan with the rated power of 450 kW [24]. The size of the main test section, where the present experiments were conducted, is 4 m in width, 2.6 m in height and 36 m in length. The maximum wind speed in the main test section can reach 36 m/s and the turbulence intensity in the free stream is less than 0.5% according to Kao [24]. The boundary layer thickness at the location of the circular cylinder model is about 60 mm measured on the floor of the main test section at the free-stream velocity of 20 m/s [24].

2.2 The circular cylinder model

The circular cylinder employed in this study was made of stainless steel, 0.32 m in diameter denoted as $D$ and 2.6 m in height denoted as $L$, which is shown in Fig. 1. Referring to [14], the relative surface roughness of the circular cylinder is $3.88 \times 10^{-5}$, as normalized by the diameter of the circular cylinder. The circular cylinder spanned the ceiling and the floor of the test section so that the blockage ratio is 8%. The center of the circular cylinder model was located 2.9 m from the inlet of the main test section.

2.3 Pressure taps

Pressure measurements were made on the circular cylinder with four pressure taps located in the middle section of the model. See Fig. 2 for the locations of the pressure taps at $0^\circ$, $90^\circ$, $-90^\circ$ and $180^\circ$, where $\theta = 0^\circ$ denotes the frontal stagnation point with regard to the incoming flow. Each of the pressure taps was connected to a diaphragm-type pressure transducer Validyne DP-103, whose response frequency was about 50 Hz, installed inside the model. In the experiment, the pressure signals obtained
at the four pressure taps were sampled simultaneously at a rate of 1000 Hz for a sampling time of 120 s.

In this study, the Reynolds number $R_{ref}$ is based on $D$ and the characteristic velocity $V_{ref}$, which was deduced from a reference pressure, $p_{ref}$, defined in Eq. (1):

\[ p_{ref} = p_0 - p_{static}, \]
\[ V_{ref} = \sqrt{\frac{2p_{ref}}{\rho}}, \]
\[ Re_{ref} = \frac{V_{ref}D}{v}. \]  

(1)

In Eq. (1), $p_0$ denotes the pressure measured at $\theta = 0^\circ$ and $p_{static}$ denotes the static pressure in the free stream that was obtained from a Pitot tube situated immediately downstream from the inlet of the test section, which was introduced from the top wall.

Table 1 provides the information concerning the three experiments made for this study, which are named Cases 14, 16-1 and 17-1, respectively, signifying the time periods of the experiments made. The dimensions of the pressure tubes between the pressure taps and pressure transducers are also given in the table. According to Bergh and Tijdeman [25], the dynamic response and the phase lag of pressure oscillations through the transmission tube could be estimated by the dimensions of the tube and the volume in the transducer. Given that in the Reynolds number range of concern the characteristic frequency of vortex shedding would be about 10 Hz, it was estimated that the amplitude attenuation due to the dynamic response was negligible and the phase lag was less than $5^\circ$, regardless of the fact that the inner diameters of the pressure tubes of the three cases were not the same. Moreover, the characteristic timescale of the intermittent pressure variations of concern in this study was twice or longer than that of vortex shedding. Thus, for the present purpose the pressure signal distortions due to the pressure tube system can be neglected.

### Table 1 The information about the experiments.

| Code number | Date of experiment | Inner diameter (m) | Length (m) |
|-------------|--------------------|--------------------|------------|
| Case 14     | 24–26 September 2014| 0.004              | 0.6        |
| Case 16-1   | 8–13 September 2016 | 0.002              | 0.6        |
| Case 17-1   | 21–25 August 2017  | 0.002              | 0.6        |

The definition of the time-mean pressure coefficient is given in Eq. (2), where $p_0$ represents the time-averaged pressure measured at $90^\circ$, $90^\circ$ or $180^\circ$:

\[ C_{p,\theta} = \frac{p_\theta - p_{static}}{p_{ref}}. \]  

(2)

Moreover, the definition of the time-mean base pressure coefficient $C_{pb}$ is given in Eq. (3), where $p_{180^\circ}$ denotes the time-mean pressure at $\theta = 180^\circ$:

\[ C_{pb} = \frac{p_{180^\circ} - p_{static}}{p_{ref}}. \]  

(3)

The definition of the pressure fluctuation coefficient $C_{p,\theta,rms}$ is given in Eq. (4), where $p_{\theta,rms}$ denotes the root-mean-square value of the pressure fluctuations:

\[ C_{p,\theta,rms} = \frac{p_{\theta,rms}}{p_{ref}}. \]  

(4)

### 3.2 Linear regression with the SW method

The linear regression with the SW method is one of the algorithms in the PLR methods [22] to simplify a time-series sampled record into a series of lines. In this study, the SW method is used to identify the intermittent ramps in the pressure signals obtained from the surface of the circular cylinder. The procedures of the SW method include linear regression with the sliding window, grouping and sifting, which are demonstrated in Fig. 3. Step 1 is to choose the window size and the step size for obtaining a regression relation in each window. The window size has to be large enough that the noise due to the small disturbances can be averaged out; on the other hand, it has to be smaller than the timescale of the ramp to be determined. Thus, the window size of the sliding window chosen in this study was 50 data points and the step size was 1 data point. After the linear regression, a first-order polynomial in a sliding window would be qualified and kept if the coefficient of determination, $R^2$, is greater than 0.98. In Step 2, the qualified ramps would be integrated into a characteristic event. However, in order to distinguish different types of events, namely the ramps could be in the descending or ascending trend, they were separated into different groups. In Step 3, a characteristic event would be qualified through a sifting process based on the criteria explained below.

According to Achenbach’s work [13], the pressure coefficient would be lower than $-2$ if a separation bubble is formed. Thus, in the present work if the pressure coefficient in a ramp event lower than $-2$ it signifies the formation of a separation bubble. This threshold condition is included in the criteria shown in Table 2. In the table, Type 1 refers to the events of significant pressure decrement, which is attributed to the appearance of a separation bubble, and Type 2 refers to the events of significant pressure increment, which is attributed to the disappearance of a separation bubble. After the sifting process, the qualified events of the two types are collected separately for characterizing the intermittent formation of the separation bubbles. Furthermore, a timescale $\tau_F$ associated with each of the qualified events was determined to represent the temporal characteristics of the pressure change. $\tau_F$ can be normalized by the characteristic length of the circular cylinder model, $D$ and the free-stream velocity $V_{ref}$, which is shown in Eq. (5):

\[ \text{normalized timescale} = \frac{D}{V_{ref} \tau_F}. \]  

(5)
Figure 3 Procedures of the SW method.

Table 2 Criteria of the types of events.

| Event type | Cp in the event | Description               |
|------------|-----------------|---------------------------|
| Type 1     | max($C_p$) > -1.5, min($C_p$) < -2, $C_p$ is decreasing | First kind of pressure change |
| Type 2     | max($C_p$) > -1.5, min($C_p$) < -2, $C_p$ is increasing | Second kind of pressure change |

3.3 Verification of the SW method

To verify the SW method, two characteristic events in a sampled time trace of pressure coefficient obtained from Case 14 are highlighted in Fig. 4 for discussion. As noted, the characteristic events can be distinguished if their slopes are monotonically increasing or decreasing; however, those with inflection points present in the increment or decrement process could not be identified. For instance, this method could only identify the characteristic event during $t = 87.5 - 89$ s without any small-scale fluctuations, but failed to identify the event during $t = 105 - 106$ s featuring more complicated variations in the pressure trace.

3.4 PV determination

The PV determination is another method in the PLR methods [22] to identify the characteristic points in a time-series trace. Namely, the local maximum (peak) and the local minimum (valley) are identified as the characteristic points. In this study, a PV method is used to identify the intermittent events of the bubble formation in the pressure signals. The procedures of the method are explained in Fig. 5, which include PV determination, grouping and sifting. In Step 1 of the method, the characteristic peaks and valleys in a real-time pressure coefficient trace were identified. The neighboring peak and valley that differed by less than 0.03 in value would be ignored. In Step 2, if the peak and valley points differed by more than 0.3, they would be connected as a single ramp. However, there might be some disturbances between ramps. Thus, an extra condition was imposed: if the amplitude of a disturbance was smaller than 0.12, then the disturbance would be ignored. Subsequently, two neighboring ramps of the same type would be grouped into one ramp and so on until interrupted by a significant ramp of the other type. In Step 3, the characteristic events were examined by the criteria given...
in Table 2. Similar to the previous SW method, not only the characteristic events in the pressure-coefficient signals were collected, but also the timescale $\tau_F$ of each qualified characteristic event was determined and normalized according to Eq. (5).

### 3.5 Verification of the PV method

A sampled pressure coefficient trace with a number of significant jumps in Case 14 was selected for the verification of the PV method proposed. The result is shown in Fig. 6. Since the characteristic events are composed of the peaks and valleys, the ramps containing inflection points can be identified conditionally, like the characteristic event identified at $t \sim 22$ s in Fig. 6. As seen, not only the monotonical variations in the signal trace can be identified, but also those of more sophisticated variations in the trace, like $t \sim 105.5$ s in Fig. 6, are qualified by the PV method. Thus, the PV method developed is capable of identifying more characteristic events containing small-scale disturbances than the SW method mentioned earlier.

### 3.6 Comparison of the SW and PV methods based on the outcome results

The characteristic events in a time-series pressure coefficient trace can be determined by either the SW method or PV method, but the results reduced from two methods are quite different. In Figs 7 and 8, the characteristic events determined by two methods for two sampled traces, which correspond to the Reynolds numbers featuring different characteristic behaviors, are compared. As mentioned above, the SW method could only identify the characteristic events without any inflection point. On the other hand, the PV method conditionally worked for the inflection points existing in the characteristic events. In both figures, it is seen that the number of characteristic events identified by the PV method is substantially more than that identified by the SW method.

A comparison of the number of characteristic events identified by the SW and PV methods for all the data obtained in the three experiments over the critical transition range is made in Fig. 9, in which the distributions depicting the counts of the characteristic events versus the Reynolds number are given. Ideally speaking, a decrement of the pressure would be always followed by an increment; therefore, the number of Type-1 events would have been the same as that of Type-2 events. Nevertheless, in reality the small-scale disturbances present in the real-time traces might interrupt the identification of a characteristic event by either the SW method or PV method. Hence, the total numbers of Type-1 and Type-2 events shown in Fig. 9 are actually not equal using either method. Specifically speaking, the differences between the numbers of Type-1 and Type-2 events identified by the SW method appear to be more severe than those identified by the PV method. This finding suggests that the PV method is superior to the SW method in tolerating the presence of the inflection points in a qualified event. Therefore, in the following, only the results obtained by the PV method will be presented for discussion.

### 4 RESULTS AND DISCUSSION

#### 4.1 General characteristics of the pressure signals measured in the critical transition range

A comparison of the base pressure coefficients obtained from Cases 14, 16-1 and 17-1 with respect to the Reynolds numbers over the critical transition range, together with the data obtained from the literature \cite{4, 7, 14, 26}, is made in Fig. 10. Although the base pressure coefficients of the three cases at Reynolds numbers around $2 \times 10^5$ appear to be somewhat different from the data reported in the literature, the trend of a dramatic drop in the range of $Re \sim 2 - 4 \times 10^5$ is in good agreement with those reported in the literature.
Variations of the time-averaged pressure coefficients obtained at $\theta = \pm 90^\circ$ and $180^\circ$ with respect to the Reynolds numbers in the critical transition range for the three experiments studied are presented in Figs 11–13 for discussion. As seen, they are further distinguished into five regimes, which are indicated by different colors and further explained in Table 3. Overall speaking, the Reynolds number ranges corresponding to the five regimes in the three cases vary because the critical phenomenon of concern is very sensitive to the experimental conditions, including at least the free-stream disturbance, the temperature variations and the surface roughness of the model. Even with great care in the experiment, minute changes in the experimental conditions could lead to noticeable differences in the results obtained.

Notable features are described below.

In Case 14, the transition from the one-bubble to two-bubble state, numbered regime (4) in Fig. 11, can be clearly identified. Nevertheless, it is not distinguishable in Fig. 12 for Case 16-1 and Fig. 13 for Case 17-1, which strongly show that this is a transitional state, very unstable and easily bypassed to the one-bubble or two-bubble regime.

In all the three figures, the time-averaged pressure coefficients show remarkable variations in regime (2) named for the transition from the precritical to one-bubble state. To illustrate this characteristic behavior, Fig. 14 presents the distributions of the root-mean-square values of the pressure fluctuation coefficients, $C_{p,\text{rms}}$, with respect to the Reynolds number for all the three cases studied. Remarkably high $C_{p,\text{rms}}$ in regime (2) for all the three cases are noted, which strongly show the nonstationary characteristics of the flows. Physically, this is attributed to the intermittent formation of the separation bubble on the circular cylinder. Desai et al. [17] also pointed out the transitional flow states between subcritical, steady one-bubble and steady two-bubble regimes that were identified from the variations of the sectional drag coefficient, where the drag coefficient was deduced from the pressure distribution around a section of the circular cylinder.

In viewing that the flow behaviors in the critical transition range can be highly unsteady and nonstationary, it is not meaningful to further carry out data analysis based on the conventional time-averaging methods, for instance, the Fourier analysis or the auto- or cross-correlation analysis.

4.2 Counts of the characteristic events versus Reynolds number

In the following, the real-time pressure signals obtained at $\theta = \pm 90^\circ$ were examined for the intermittent formation of the separation bubble. First of all, the numbers of data sets made in the three experiments over the critical transition range are shown in Fig. 15 for reference. Note that each of the data sets contained

![Figure 5 Procedures of the PV method.](image-url)
120,000 data points, which were sampled over 120 s at the rate of 1 kHz.

In data analysis, the number of characteristic events found in each of the data sets by the PV method was documented. For illustration, Fig. 16a shows the counts of the characteristic events over the critical transition range found in Case 14. Further, in order to determine the count of events per minute, the numbers of characteristic events in Fig. 16a were divided by the time in minutes of the data sets in the corresponding Reynolds number range; the distribution of data sets over the Reynolds number range studied is provided in Fig. 16b. As a result, a quantity called the count of the events per minute was deduced, which is given in
Figure 9 Comparison of the counts of the characteristic events obtained by the SW and PV methods at (a) $\theta = +90^\circ$ and (b) $\theta = -90^\circ$.

Fig. 10 Variations of the base pressure coefficient in the critical transition range including a comparison with other works.

Fig. 11 Variations of the pressure coefficients at $\theta = \pm 90^\circ$ and $180^\circ$ with respect to the Reynolds number in the critical transition range, Case 14.

Subsequently, comparisons for this quantity corresponding to the two types of characteristic events of all the three cases can be seen in Fig. 17. In the figure, a common trend noted is that the numbers of counts per minute corresponding to the Type-1 and Type-2 events appear to be the greatest in regime (2), the transition from the precritical to one-bubble state, followed by regime (1), the precritical state. Further discussion on the results in Fig. 17 is provided below.

In regime (1), while the time-mean pressure coefficient values show small deviations from the subcritical level, the real-time data indicate large-amplitude fluctuations intermittently. As a result, a significant number of counts of the characteristic events can be seen in Fig. 17. Physically speaking, the real-time pressure fluctuations with significant intermittency show that temporarily formation of separation bubbles is prevailing. In this case, the count per minute of the Type-1 events is close to that of Type-2 events. Here, it is also worth mentioning that the intermittency considered in this study is different from the term describing the flow characteristics in the turbulent boundary layers [27–29], for which the intermittency factor was deduced from the root-mean-square values of the velocity fluctuations within a time duration.

In regime (2), the real-time pressure signals measured at $\theta = \pm 90^\circ$ showed pronounced fluctuations. Clearly, this is a strong indication of unstable formation of the separation bubble. In this regime, not only the time-mean pressure coefficients at $\theta = \pm 90^\circ$ showed remarkable scattering as seen in Figs 11–13, but also the counts per minute of the characteristic events appear to vary significantly among the three cases shown in Fig. 17.

It is worth mentioning that in Case 14 regime (4), the transition from the one-bubble to two-bubble state, was identified.
While the counts per minute of the characteristic events at $\theta = +90^\circ$ appear to be very low, the counts per minute at $\theta = -90^\circ$ are rather remarkable, which shows that unsteady formation of separation bubbles was prevailing on this side.

Generally speaking, the counts per minute of the characteristic events among the three cases differ widely. This is attributed to the fact that in the critical transition range flow over the surface of the circular cylinder could be sensitive to all the factors associated with the experiment.

### 4.3 Normalized timescales of the characteristic events versus Reynolds number

The timescale of each of the characteristic event could be normalized according to Eq. (5). A histogram depicting the distribution of the normalized timescales at $Re = 3.41 \times 10^5$ of Case 14 for which the real-time pressure signals were obtained at $\theta = +90^\circ$ is presented in Fig. 18 as a sample for illustration.

Moreover, the histograms of the normalized timescales for all the Reynolds numbers studied in Case 14 are assembled in Fig. 19a and b for a three-dimensional view and a top view, respectively.

Using the same method, the histograms of the normalized timescales deduced from the three cases studied are presented in Figs 20–22 for discussion. Note that in each of the figures, the results of Type 1 and Type 2 corresponding to the real-time pressure signals obtained at $\theta = +90^\circ$ and $\theta = -90^\circ$ are sorted into four plots. Overall speaking, the histograms deduced from the pressure measurements at $\theta = \pm 90^\circ$, the locations on the two shoulders of the circular cylinder, are not consistent. This is attributed to strong asymmetry of the flow in regime (2), which can also be learned from the time-mean pressure data in Figs 11–13 and the root-mean-square values of pressure fluctuations in Fig. 14. This characteristic of asymmetry was also pointed out in the literature [16, 17].

For Case 14 in Fig. 20, the histograms of the normalized timescale of Type 1 and Type 2 events deduced at $\theta = +90^\circ$ and $\theta = -90^\circ$ appear to spread widely. In detail, in Fig. 20a the histograms of the normalized timescale at $\theta = +90^\circ$ mostly fall in the range from 0.03 to 0.15 in regimes (1) and (2). The appearances of the histograms of Type 1 and Type 2 events are rather different. In Fig. 20b, the histograms of $\theta = -90^\circ$ show a similar feature that most of the normalized timescales fall in the range of 0.03 to 0.15.

For Case 16-1, Fig. 21 indicates that the histograms are seen in a range of Reynolds number narrower than those in Case 14, which are mainly in regime (2). Moreover, the range of Reynolds number seen in Fig. 21a for $\theta = +90^\circ$ in which the characteristic events could be identified is even narrower than that in Fig. 21b for $\theta = -90^\circ$.

For Case 17-1, Fig. 22 indicates that the histograms are seen in both regimes (1) and (2). The normalized timescale appears to be limited to lower values compared to those in Cases 14 and 16-1. Essentially, the maximum value in each of the plots is limited below 0.1, in particular the response of Type-2 events identified from both sides of the circular cylinder.

Referring to Fig 18 for the probability histogram of the normalized timescale at a fixed Reynolds number, one can further find the maximum, minimum and averaged values from the plot. Moreover, according to Eq. (5), the maximum value of the normalized timescale represents the shortest timescale of the characteristic events, and vice versa. Along with the observations, the averaged, maximum and minimum of the normalized timescales deduced from the pressure real-time signals at $\theta = +90^\circ$ and $\theta = -90^\circ$ for all the three cases are shown in Figs 23 and 24 for comparison. Particular attention is paid to the data in regime (2) that the averaged values for Case 14 appear as the highest among the three cases, whereas those for Case 17-1 are the lowest. It is also noted that the minimum values of the normalized timescale of the characteristic events at $\theta = \pm 90^\circ$ are almost unchanged in regimes (1) and (2). This finding leads to a postulation that the lowest normalized timescale of the characteristic events might be independent of the Reynolds number. Further discussion in this regard is provided below.
models were actually employed for the experiments made in a pressurized wind tunnel. Schewe [9] was able to determine the normalized timescale corresponding to the lift jumps for the two different cylinder models at Reynolds number around $3.2 \times 10^5$. For reference, the normalized timescales deduced from Schewe’s experiments [9] for the two circular cylinder models are listed in Table 4. Schewe [9] mentioned that the timescale of the lift jump could be considered as a time length required for the transition of the separation bubble to spread along the whole circular cylinder. It is shown in the last column in Table 4 that if the timescales characterizing the lift jumps were normalized by the total lengths of the two circular cylinders, respectively, the nondimensional values would be about 0.3.

Inspired by Schewe’s [9] argument, an alternate definition of the normalized timescale based on the spanwise dimension, namely $L/V_{ref}\tau_F$, is adopted here. First of all, referring to the data reported by Schewe [9], the equivalent value in terms of $D/V_{ref}\tau_F$ for the present circular cylinder would be about 0.0369 as the aspect ratio of the present model $L/D = 8.1$. The level of this value is marked in Figs 23 and 24 for reference. Interestingly noted is that the reference value is comparable to the averaged values of the normalized timescales with respect to the three regimes.

Table 3 Characteristics of flow in each regime in the critical transition range.

| Number | Regime                              | Instruction                                                                 |
|--------|-------------------------------------|------------------------------------------------------------------------------|
| (1)    | Precritical regime                  | It is determined by the signal where the percentage of vortex-shedding frequency is lower than 80% [11]. |
| (2)    | Transition from the precritical to one-bubble regime | The scattering of the time-averaged pressure coefficients at $\theta = \pm 90^\circ$ is significant. |
| (3)    | Steady one-bubble regime            | The scattering of the 2-min averaged pressure coefficients at $\theta = \pm 90^\circ$ is small, but the difference between them is significant. |
| (4)    | Transition from the one-bubble to two-bubble regime | It is between the steady one-bubble and two-bubble regimes. The scattering of the time-averaged pressure coefficients at $\theta = \pm 90^\circ$ is significant. |
| (5)    | Steady two-bubble regime            | The variations and difference of the time-averaged pressure coefficients at $\theta = \pm 90^\circ$ are small compared to the one-bubble regime. |
Table 4 Results reported by Schewe [9].

| Model     | Re       | D (m) | L/D | Blockage ratio (%) | D/V_ref \( \tau_F \) | L/V_ref \( \tau_F \) |
|-----------|----------|-------|-----|-------------------|----------------------|----------------------|
| Model 1   | \( \sim 3.2 \times 10^5 \) | 0.06  | 10  | 10                | 0.029                | 0.29                 |
| Model 2   | \( \sim 3.2 \times 10^5 \) | 0.034 | 18  | 5.7               | 0.017                | 0.306                |

Table 5 Weighting-averaged normalized timescales for the three cases in regime (2), the transition from the precritical to one-bubble state, \( \theta = +90^\circ \).

| Case 14 | Case 16-1 | Case 17-1 |
|---------|-----------|-----------|
| Type 1  | Type 2    | Type 1    | Type 2    | Model 1 [9] | Model 2 [9] |
| Weighting-averaged \( L/V_{\text{ref}} \tau_F \) | 0.581 | 0.667 | 0.384 | 0.386 | 0.253 | 0.165 | 0.29 | 0.306 |

Table 6 Weighting-averaged normalized timescales for the three cases in regime (2), the transition from the precritical to one-bubble state, \( \theta = -90^\circ \).

| Case 14 | Case 16-1 | Case 17-1 |
|---------|-----------|-----------|
| Type 1  | Type 2    | Type 1    | Type 2    | Model 1 [9] | Model 2 [9] |
| Weighting-averaged \( L/V_{\text{ref}} \tau_F \) | 0.563 | 0.471 | 0.331 | 0.278 | 0.170 | 0.134 | 0.29 | 0.306 |

Figure 15 A distribution of the data sets obtained from the three experiments.

cases. Specifically speaking, the averaged normalized timescales of Case 14 are higher than the reference value, the values in Case 16-1 are close to the reference value, but those in Case 17-1 are lower.

For further comparison, a weighting-averaged timescale is proposed here, based on the timescales of the characteristic events and the corresponding counts obtained in regime (2). Normalized by the spanwise length of the circular cylinder according to the definition of \( L/V_{\text{ref}} \tau_F \), the results of the weighting-averaged count per minute of the characteristic events in regime (2) are presented in Tables 5 and 6 for all the three cases at \( \theta = +90^\circ \) and \( -90^\circ \), respectively. Both tables indicate that in Case 14 the weighting-averaged normalized timescales of the characteristic events are almost two times higher than the reference value of 0.3 [9]. In Case 16-1, the weighting-averaged normalized timescales are close to the reference value. In Case 17-1, the weighting-averaged normalized timescales are somewhat lower than the reference value. Overall speaking, the weighting-averaged normalized timescales of the characteristic events found in the three cases of the experiments range between 2 and 0.5 times the reference value of 0.3 reported in [9]. Therefore, it is reasonable to say that the present findings deduced from the real-time pressure measurements on the circular cylinder support the viewpoint of Schewe [9]. Namely, the weighting-averaged timescale associated with the characteristic events deduced from the real-time pressure data can be regarded as a timescale associated with the propagation of bubble formation on the circular cylinder along the spanwise direction. Physical evidence concerning the three-dimensional characteristics of the separation bubble in the critical transition range can also be found in our previous studies [14, 18].

5. CONCLUSION

In this study, the SW and PV methods were proposed to identify the characteristic events in the real-time pressure signal obtained at \( \theta = \pm 90^\circ \), due to the intermittent formation of the separation bubble on the circular cylinder in the critical transition range. Specifically, the events of pressure decrement are referred to as Type 1, and the events of pressure increment are referred to as Type 2. By evaluating the outcome results
of the SW and PV methods, the PV method was found less sensitive to small-scale fluctuations embedded in the real-time pressure signals, and therefore was favored for the present data analysis.

The counts per minute of the two-type characteristic events deduced from the real-time pressure signals were determined to describe the intermittency of the occurrence of the separation-bubble formation. As found, the characteristic events occur mostly in the transition from the precritical to one-bubble state, named regime (2) in this study.

From the probability histograms of the normalized timescales of the two-type characteristic events corresponding to the
three cases of experiments, it is realized that the timescales of the characteristic events vary widely. On the other hand, the weighting-averaged normalized timescales of the characteristic events, based on the spanwise length of the circular cylinder, for all the three cases are found comparable to the characteristic timescale corresponding to the lift jump reported in the literature [9]. This finding gives a strong support to a physical viewpoint that the characteristic events identified in the

Figure 18 Probability histogram of the normalized timescales of the characteristic events reduced from the real-time pressure signals obtained at $\theta = +90^\circ$, $Re = 3.41 \times 10^5$, for Case 14.

Figure 19 Variation of the probability histogram of normalized timescale versus Reynolds number for Case 14: (a) a three-dimensional view and (b) a top view.
Figure 20 Variations of normalized timescale histogram versus Reynolds number at (a) $\theta = +90^\circ$ and (b) $\theta = -90^\circ$, for Case 14.

Figure 21 Variations of normalized timescale histogram versus Reynolds number at (a) $\theta = +90^\circ$ and (b) $\theta = -90^\circ$, for Case 16-1.
real-time pressure signals in the present study are associated with the three-dimensional aspect of bubble formation on the circular cylinder.

The present study found that the normalized timescales associated with the intermittent pressure fluctuations, in terms of $L/V_{ref} \tau_F$, fall in the range of 0.1–0.6. Nevertheless, more data would be needed in the future to further verify this finding. Meanwhile, more advanced data analysis techniques, which could have artificial intelligence or state-of-the-art schemes, incorporated to identify the characteristic events under the
non-stationary flow conditions, would be desirable in the future.

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