Analysis of equipment utilization at random order arrivals and random output

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Abstract. The aim of this work is to study equipment utilization of an enterprise. To examine the random order arrivals and random output, we use the supply and demand model, in which demand is equipment utilization and supply is product output. There are no restrictions on the type of value, deterministic or random, and on the form of the distribution function of demand and/or supply. The initial data are integer random supply and demand with arbitrary distribution functions. The output parameters of the model are supported and unsupported demand, and unused supply. These parameters are also integer random variables for which the model allows obtaining their functional characteristics. We show that the conventional deterministic approach to calculating the quantity of equipment as a ratio of the average product demand to the average machine capacity leads to both equipment downtime and consumer unsupported demand. We solve this problem for the enterprise manufacturing temperature sensors. When two pieces of equipment are used to produce a certain type of sensors under study, the average unsupported demand is about 12% of the average monthly order and varies from 17 to 112 (non-produced) sensors. Only five months a year the order is fully executed within a month; the equipment is fully utilized for 7–8 months a year. The model allows calculating not only the average values of parameters, but also their distribution functions.

1. Introduction

The problem of planning and optimizing equipment utilization is one of the most important among a wide range of tasks of modern industry. This problem can be easily solved with constant (scheduled) equipment utilization and scheduled deterministic output. However, in the market economy, production orders are dispatched unevenly, and therefore, the priority is to take into account random fluctuations in machine workload. The ultimate goal of solving this problem is to ensure the smooth production flow, the violation of which can lead to delivery delay, inefficient use of fixed assets and working capital [1].

There is a variety in approaches to accounting for the random nature of both product order and output. In [2], a formula for the load factor was obtained and linear and parabolic approximations of the output volume were investigated. The estimation of output as a random variable was considered in [3].

The evolutionary-genetic approach to optimization problems with random loading of machines was used, for example, in [4–7] and the branch-and-bound method was applied in [8]. The equipment
utilization model developed in [9] used the Petri nets, and in [10], the authors dealt with a two-stage calculation algorithm using databases and linear optimization methods. In [11], the random load factor was described by the diffusion process. In [12], the authors considered the problem of equipment utilization for cases of fuzzy objective function and fuzzy restrictions. An individual study was performed in the case when the parameters of the problem were random variables with a normal distribution law. In [13], a uniformly distributed demand model sensitive to the sell price was considered. Random output was studied in [14].

2. The model of supply and demand
To analyze random order arrivals and random output, we used the model of supply and demand [15], in which demand was equipment utilization and supply was product output. Unlike the papers [3, 12, 13], the model imposed no restrictions on the type of value, deterministic or random, and on the form of the distribution function of demand and/or supply.

The initial data are integer random demand $N$ and supply $Z$, with arbitrary distribution functions $A(n)=P(N<n)$ and $C(n)=P(N\geq n)$, respectively. We denoted the corresponding density functions for these and other functions by lowercase letters, as generally accepted.

The model allows to calculate the following indicators: supported demand $N_s$ and, equal to it, used supply $Z_s$, unsupported demand $N_u$, and unused supply $Z_u$.

Functional characteristics of the required parameters were calculated as follows: the distribution function of supported demand $N_s$ and equal used supply $Z_s$:

$$H(n) = P(N_s = Z_s < n) = 1 - \left[1 - A(n)\right] \cdot \left[1 - C(n)\right]$$

(1)

the distribution function of unsupported demand $N_u$:

$$Q(n) = P[N_u < n] = \sum_{i=0}^{\infty} a(i) \cdot \left[1 - C(i - 1 - n)\right]$$

(2)

the distribution function of unused supply $Z_u$:

$$G(n) = P[Z_u < n] = \sum_{i=0}^{\infty} c(i) \cdot \left[1 - A(i - 1 - n)\right]$$

(3)

3. Results and discussion
We solved this problem for the enterprise manufacturing temperature sensors. The special service of the enterprise regularly collected orders for sensors manufactured, forming a production program for a month at least. For a certain type of sensors, there was a sample of orders for 2 years (table 1, data slightly changed).

| Month number | Demand, pcs | Month number | Demand, pcs | Month number | Demand, pcs | Month number | Demand, pcs |
|--------------|-------------|--------------|-------------|--------------|-------------|--------------|-------------|
| 1            | 150         | 7            | 160         | 13           | 250         | 19           | 230         |
| 2            | 220         | 8            | 225         | 14           | 165         | 20           | 145         |
| 3            | 150         | 9            | 185         | 15           | 220         | 21           | 270         |
| 4            | 100         | 10           | 185         | 16           | 170         | 22           | 305         |
| 5            | 95          | 11           | 190         | 17           | 310         | 23           | 155         |
| 6            | 115         | 12           | 195         | 18           | 170         | 24           | 310         |

Temperature sensors were manufactured using special expensive equipment. Due to the different number of working days in a month and some subjective and objective reasons, a unit of equipment produced a different number of sensors per month. Table 2 presents the distribution of the number of products manufactured per unit of equipment.
The results of calculating the reference parameters of the model are as follows: the average number of orders for the sensors under consideration (average demand) was 194.6 pieces. A unit of equipment can produce 99 sensors per month on the average (average supply magnitude). Therefore, according to the average values, we need two units of equipment to complete a monthly order. In this case, an average of 198 sensors can be manufactured. The distribution density of the number of products made on two machines can be calculated by the formula [16]:

\[ c_2^*(n) = c(n) = \sum_{j=0}^{n} c_1(n - j)c_1(j), \]  

(4)

where \( c_1(i) \) is the distribution density of the number of sensors produced on one machine.

Within the problem under consideration, the characteristics of the demand and supply system have the following meaning. Unsupported demand shows how many devices will not be manufactured on available equipment. The values of the distribution function \( Q(n) \) of unsupported demand, calculated by (2), are given in table 3.

**Table 3.** The values of the distribution function \( Q(n) \)

| \( n \) | 0  | ... | 17  | ... | 21  | ... | 56  | ... | 110 | 111 | 112  | ... | 212 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| \( Q(n) \) | 0.623 | ... | 0.626 | ... | 0.663 | ... | 0.832 | ... | 0.928 | 0.954 | 0.980 | ... | 1.0 |

Comments on the table:

- There will be no unsupported demand \( (N_0 = 0) \) or, in other words, demand will be supported in 62.3% of months; that is, with the current demand for sensors, two units of equipment can fully fulfill the order only seven months out of twelve.
- In eight months of the year, unsupported demand will not exceed 21 sensors.
- In ten months of the year, unsupported demand will not exceed 56 sensors.
- The minimum number of non-produced sensors is 17, the maximum is 112. The individual values of the unsupported demand distribution density \( q(n) \) are small. Thus, 111 and 112 sensors will not be produced with a probability of 0.026.

Average unsupported demand is 23 sensors, which is about 12% of the average monthly order. That is, on average, every month consumers will not receive 23 sensors of the type under consideration.

Unused supply has the meaning of equipment downtime, expressed in the number of sensors that could have been manufactured. According to the distribution function calculated by (3), there will be no downtime of an equipment unit in 37.8% of months \( (G(0) = 0.378) \). The maximum downtime will be equal to production time of 112 sensors \( (G(112) = 1.0) \), and the probability of this event will be 0.1% \( (g(112) = 0.001) \). Average unused supply \( \overline{z}_0 = 26.3 \) sensors.

The process of manufacturing sensors includes universal operations for all versions without readjustment of equipment and special operations that are performed for specific versions. Therefore, in the absence of orders for some versions of sensors, the corresponding specialized equipment stays idle.

We investigate how the existing universal equipment will be loaded when manufacturing sensors of all types (Table 4).

Average demand was \( \bar{n} = 1583.3 \) sensors. Distribution of the number of the produced sensors on all universal equipment gives \( \bar{z} = 1499.9 \), i.e., average supply is approximately equal to average demand on the enterprise production. This ratio corresponds to the generally accepted approach to calculating the quantity of equipment.
Table 4. Monthly data on orders for all types of sensors

| Month number | Demand, pcs | Month number | Demand, pcs | Month number | Demand, pcs | Month number | Demand, pcs |
|--------------|-------------|--------------|-------------|--------------|-------------|--------------|-------------|
| 1            | 1230        | 7            | 2000        | 13           | 1000        | 19           | 1800        |
| 2            | 990         | 8            | 1900        | 14           | 1500        | 20           | 1100        |
| 3            | 2020        | 9            | 1400        | 15           | 1300        | 21           | 1400        |
| 4            | 2000        | 10           | 1500        | 16           | 1750        | 22           | 1700        |
| 5            | 1570        | 11           | 1710        | 17           | 1300        | 23           | 1680        |
| 6            | 1310        | 12           | 1950        | 18           | 2000        | 24           | 1800        |

Average unsupported demand was $\bar{n} = 186.3$ sensors, that is, the customers will not receive an average of 186.3 sensors per month. The values of the distribution function $Q(n)$ and density $q(n)$ calculated by (2) show that:

- demand will be supported in 44.4% of months; therefore, ensuring the equality of the supply and demand averages results in the order being fully completed within a month for only five months a year;
- unsupported demand of up to 73 sensors is expected within six months a year;
- in any month, there will be no unsupported demand in the amount of 523 sensors or more.

Average unused supply was $\bar{z} = 102.9$ sensors. This means an average monthly downtime of all universal equipment during the manufacturing time of 102.9 sensors.

The values of the distribution function $G(n)$ and density $g(n)$ calculated by (3) show that:

- there will be no equipment downtime in 61.3% of months ($G(0) = 0.613$), i.e., within 7—8 months a year;
- downtime equal to the manufacturing time of one sensor will be 1% of months a year ($g(1) = 0.010$), two sensors – 0.2% of months ($g(2) = 0.002$), etc.;
- the maximum downtime will be equal to the manufacturing time of 512 sensors ($G(512) = 1.0$);
- the probability of this event will be 0.1% ($g(512) = 0.001$).

The enterprise management directive is that the generated order (monthly plan) must be completed in full. This implies at least two fundamentally different ways of solving the problem.

First, the opportunity of acquiring universal and, especially, special equipment should be explored. It can be the single-type equipment, and in this case the main indicator will be the payback period. It can be adaptive equipment since the enterprise produces more than 10 types of sensors. To organize rational operation of such equipment, it is necessary to make management decisions based on the use of methods that allow calculating the scheduler of equipment utilization taking into account the readjustment time for the manufacture of products of different types. And this, in turn, requires the availability of specialists skilled in the above methods and the corresponding software for implementing optimal methods of equipment utilization.

The second direction to solve the arisen problem is the organization of double shift work, which will also require certain costs.

4. Conclusion

The model under consideration allows obtaining a real picture of equipment utilization and downtime at the enterprise with a random demand for its products. In particular, it is shown that, with the generally accepted deterministic approach to the calculation of the quantity of equipment as the ratio of the average product demand to the average machine capacity, there is both downtime and unsupported consumer demand. For these cases, the model allows the calculation of not only the average parameters, but also their distribution.

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