

\[3PF_2\] pairing in high-density neutron matter

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The onset of the \(3PF_2\) superfluid phase in high-density neutron matter is studied within the BCS framework with two and three body forces. Owing to the strong correlations the energy gap is so sizeably quenched as to demand to reconsider the role of superfluidity in neutron-star phenomena.

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I. INTRODUCTION

So far, the neutron stars (NS) have been considered as a rich laboratory of various superfluid phases of nuclear matter \([1-10]\). Recently the interest has been focussed on the NS interior, where both the vortex pinning responsible for the observed period glitches\([11]\) and the nucleon superfluidity responsible for the main cooling mechanisms\([12]\) are supposed to be located. In particular the cooling phase responsible for the main cooling mechanism in high density nuclear matter, that however all previous calculations neglect (see Ref.\([13]\) and references therein).

The density region, where the \(3PF_2\) component of the nuclear interaction is the most attractive, may extend to several times the saturation density of nuclear matter. In such a high-density regime, the short-range correlations are so strong that the momentum distribution around the Fermi level departs significantly from the typical profile of a degenerate Fermi system. This departure is measured by the so-called Z-factor \((0 < Z < 1)\)\([14]\). Since the deformation of the Fermi surface hinders particle transitions around the Fermi energy \(\epsilon_F\), the pairing gap is expected to get reduced in any case, even for the low density \(1S_0\) pairing channel (see, e.g.,\([15]\)). Concerning the high-density neutron-neutron pairing in the \(3PF_2\) channel, the calculations so far reported in the literature ignore the effect of the Z-factor, owing to the large uncertainty still existing in the magnitude of the nucleus-nucleon \((NN)\) interaction in the \(3PF_2\) channel, including both the two body force (2BF) and the three-body force (3BF) component. Other many-body effects, such as screening effect, have been neglected for the same reason.

In this note, we present a study of the Z-factor effect on the \(3PF_2\) pairing in pure neutron matter. In principle, we should consider asymmetric nuclear matter for application to NS core, but the small proton fraction is not relevant in this context, as discussed below. The deformation of the Fermi surface and the Z-factor are studied in the framework of the Brueckner theory with 2BF and 3BF\([10]\). The energy gap is then calculated within the generalized BCS theory\([17]\), including in the pairing interaction not only 2BF, but also 3BF. The latter, in fact, is dominant at high density and therefore it is expected to directly influence pairing gap in addition to the Z-factor.

II. FORMALISM AND RESULTS

A. Nucleon propagator in neutron matter

The neutron Green’s function is given by

\[ G^{-1}(p, \omega) = \omega - \frac{p^2}{2m} - \Sigma(p, \omega) + \epsilon_F, \]

where \(\epsilon_F\) denotes the Fermi energy and \(\Sigma(p, \omega)\) the self-energy. Expanding the latter in a series of powers of the quasiparticle energy around the Fermi surface, we obtain

\[ G^{-1}(p, \omega) \approx Z(p)^{-1}(\omega - \epsilon_p), \]

where the quasi-particle energy and the quasi-particle strength are

\[ \epsilon_p = \frac{p^2}{2m} + \Sigma(p, \epsilon_p) - \epsilon_F \]

\[ Z(p) = \left[ 1 - \frac{\partial \Sigma(p, \omega)}{\partial \omega} \right]^{-1}_{\omega=\epsilon_p}, \]

respectively. The quasiparticle strength \(Z(p)\) measures the deviation of a correlated Fermi system from the ideal degenerate Fermi gas. The occupation numbers \(n(p)\) and the \(Z(p)\) factors have been calculated in the framework of the Brueckner theory\([18, 19]\), with the inclusion of the three body forces the hole-line expansion can be extended up to high densities. The self-energy, truncated to the second order (see Ref.\([20]\) for details), provides us with a good reproduction of the empirical nuclear mean field\([21]\) and the optical-model potential\([22]\), so that we are quite confident that the next orders are irrelevant for the present calculation. We employ as 2BF the meson exchange Bonn B potential\([23]\), whose meson parameters are constrained by the fit with the experimental
phase shifts of NN scattering. The microscopic meson exchange 3BF is the one constructed by Li et al. [16]. It is consistent with the 2BF because it adopts the same meson parameters as Bonn B, so that there are no free parameters in the model.

The Z(p) factor is related to the depletion of the occupation number n(p) around the Fermi surface. According to the Migdal-Luttinger theorem [24], its value \( Z = Z(p_F)/(Z\text{-factor}) \) equals the discontinuity of the occupation number at the Fermi surface, i.e.,

\[
\lim_{\epsilon \to 0} [n(p_F^- - \epsilon) - n(p_F^+ + \epsilon)] = Z(p_F),
\]

where \( p_F \) is the Fermi momentum. In our approximation \( \Sigma(p, \omega) = \Sigma_1(p, \omega) + \Sigma_2(p, \omega) \), where \( \Sigma_1(p, \omega) \) determines the left limit and \( \Sigma_2(p, \omega) \) the right limit of the preceding equation for \( \epsilon \to 0 \).

In Fig. 1(a), we display the calculated occupation numbers vs. momentum at a given density (upper panel) and Z-factors vs. density (lower panel) in pure neutron matter. The effect of 3BF is shown in both panels. In the lower panel the calculations are reported for two approximations of the self-energy.

In Fig. 1(a), we display the calculated occupation probability in pure neutron matter at density \( \rho = 0.3 \text{ fm}^{-3} \). One easily observes the remarkable deviation from the ideal Fermi gas (solid line) due to the strong short-range correlations. As expected, the deviation is slightly enhanced by 3BF. In Fig. 1(b) the calculated Z-factor is displayed vs. density in the two different approximations for the self-energy, i.e., \( \Sigma \approx \Sigma_1 \) and \( \Sigma \approx \Sigma_1 + \Sigma_2 \), respectively. The calculation of \( Z_F \) from Eq.(3) requires a high numerical accuracy: increasing the accuracy the calculated \( Z_F \) gets lower and lower until the converging value is reached. It is noticed that without 3BF, the Z-factors decrease slowly as a function of density. Adding the contribution of \( \Sigma_2 \) leads to an overall reduction of the Z factor. The 3BF reduces further the Z-factor and its effect increases rapidly with density. As a consequence, including 3BF makes the decrease of the Z-factor as a function of density much more rapid than that obtained by adopting pure 2BF. Therefore 3BF induces a strong extra deviation from the ideal Fermi gas model.

**B. Gap equation in the \( ^3PF_2 \) channel**

The \( ^3PF_2 \) superfluidity in pure neutron matter has been investigated by using various theoretical approaches [25, 31] with 2BF, and later extended to microscopic 3BF forces by Zuo et al. [17]. In this case, the pairing gaps are determined by the two coupled equations:

\[
\begin{align*}
\left( \Delta_L(p) \right) & = -\frac{1}{\pi} \int_0^\infty dp' Z(p) Z(p') E_{p'} \\
& \times \left( V_{L,L'}(p, p') \begin{pmatrix} \Delta_L(p) \\ \Delta_L(p') \end{pmatrix} V_{L+2,L+2}(p, p') \right) \left( \Delta_L(p') \right),
\end{align*}
\]

where \( E_{p'} = (\epsilon_p - \mu)^2 + \Delta_p^2 \) and \( \Delta^2 = \Delta_L^2 + \Delta_{L+2}^2 \). \( V_{L,L'}(p, p') \) are the matrix elements of the realistic NN interaction in the coupled \( ^3PF_2 \) channel. In the gap equation, the Z-factors and the single-particle energy \( \epsilon_p \) are calculated from the Brueckner theory. As for the pairing interaction \( V_{L,L'}(p, p') \) in the \( ^3PF_2 \) coupled channel, we adopt the same 2BF and 3BF as in the Brueckner calculation. The 3BF cannot be neglected in the gap calculation because the \( ^3PF_2 \) pairing is expected to occur in the high density domain, where 3BF is quite sizeable specially in the \( ^3PF_2 \) channel.

The results are summarized in Fig.2. Neglecting the Z-factor effect (upper panel), the magnitude of the \( ^3PF_2 \) gap with the new interaction does not differ from previous calculations with AV18 potential [17]. On the other hand, the 3BF enhances the \( ^3PF_2 \) superfluidity significantly at higher densities. But, as shown in the lower panel of Fig.2, the introduction of the Z-factor effect finally quenches the pairing gaps to a value less 0.05 MeV, one order of magnitude smaller than the value with full interaction. The effect of the Z-factor appears to be extremely sizeable at high densities. It is worth noticing that this effect is opposite to the 3BF effect on the \( ^3PF_2 \) pairing in neutron matter: the former turns out to be much stronger than the latter. In conclusion, the departure of the system from the pure degenerate limit
drives the pairing attenuation at high density, regardless of whether the 3BF is included or not.

III. CONCLUSIONS

In conclusion, we have studied the anisotropic $^3PF_2$ pairing in pure neutron matter. The effects of the Fermi surface depletion (Z-factor) have been included in the calculation of the energy gap. In the pure degenerate limit, the $^3PF_2$ superfluid phase extends over a broad density range with a gap peak value of about 0.2 MeV without 3BF and 0.5 MeV with 3BF. The inclusion of the Z-factor leads to a rapid decrease of the gap magnitude by one order of magnitude: its peak value drops to less than 0.05 MeV and the superfluidity domain shrinks to $0.1 - 0.4$ fm$^{-3}$. In neutron stars the proton fraction in $\beta$-equilibrium with neutrons could in principle affect the $^3PF_2$ pairing, but in the non-vanishing gap range it is less than 15% of the total density. Recent calculations on the isospin dependence of the quasi-particle strength show that the enhancement of the neutron Z-factor is negligible for such small proton fraction \cite{13}. Even including the additional screening suppression \cite{14}, the conclusion remains that the departure from the Fermi gas limit is the main cause of pairing disappearance in high-density nuclear matter. This result makes doubtful the role of the $^3PF_2$ pairing in NS core.

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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2.png}
\caption{(Color Online)Effect of the Fermi surface depletion on $^3PF_2$ pairing gap in pure neutron matter. Notice the y-scale change from (a) to (b).}
\end{figure}

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