Revealing the Secrets of Chinese Ivory Puzzle Balls: Quantifying the Crafting Process using X-ray Computed Tomography

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Introduction
Chinese ivory puzzle balls are known for their beauty, their finesse and their ability to arouse the viewer’s curiosity. An ivory puzzle ball consists of several freely rotating concentric spheres, called ‘layers’, into which ornate decorative openwork patterns are carved. In most cases, fourteen conical ‘peepholes’ are pierced through each layer, revealing glimpses of the internal structure. Often, one or more peepholes on the outermost layer are covered with a decorated ‘cap’.

In this article we examine two Chinese ivory puzzle balls – one from the Rijksmuseum’s collection (fig. 1, the Rijksmuseum ball) and one from the National Palace Museum, Taipei (fig. 2, the NPM ball). The balls differ in size, number of layers, and decorative design. The Rijksmuseum ball has nine layers. Its outer layer measures 8.3 cm in diameter and is decorated with a pattern of floral scrolls. Eleven of the fourteen peepholes are covered with floral caps. A chain is fastened to one of the caps. The ball is dated to roughly the 1770-80 period and its provenance can be traced to its first owner, Jean Theodore Royer (1737-1807), whose collection was formed around this time. It is one of very few examples with provenance which establishes that it arrived in Europe in the eighteenth century through global trade.

The NPM ball has twenty-three layers. It is 13.2 cm in diameter and its outer layer is carved with a pattern of cloud-and-dragon motifs. The ball is dated to around 1840-1900, when it entered the Chinese imperial collection as a local tribute from Canton to the court. The inner layers of both balls are carved with different openwork patterns. Just glimpses of these patterns can be perceived at any one time, as they can only be seen through the peepholes of an outer layer. Rotating an inner layer...
relative to the peepholes in the surrounding layers reveals the complete pattern.

Chinese ivory puzzle balls are fabricated by turning on a lathe; drilling and carving tools are set in the lathe and used to shape the work. Cantonese turners in the eighteenth and nineteenth centuries used very basic lathes and a small set of simple drilling and carving tools. (figs. 3, 4a, b). We will discuss the crafting process and crafting tools in more detail in the second part of the article.

The crafting process and tools prompt several art-historical questions. For example, did eighteenth-century artisans follow a fixed procedure when crafting balls? Which tools were used and how were they employed? How have these tools evolved over time? Is there a relationship between the number of layers in a ball and the accuracy required of the tools and fabrication process?

We address these questions in this article, proposing a non-invasive computational method for analyzing the crafting process. Our approach employs Computed Tomography (CT) scanning techniques, which use a series of X-ray pictures to generate a digital 3D computer model of the ivory ball. The advantage of CT is that it is non-destructive, and high precision three-dimensional images of the ball can be obtained. The morphological properties of a ball are measured from its digital representation and compared with the properties of a mathematical description of a ball. In this way, we are able to show how the crafting process of a ball can be quantified and assessed.2

A Brief History of the ‘Devil’s Work’ Spheres
The magnificent craft of carved Chinese ivory puzzle balls tends to fascinate viewers. The Chinese have described them as the ‘devil’s work’ (guigong 鬼工 or 鬼功).3 Making them
requires considerable practice, and a high degree of skill and patience. It has been said that the art and technique of crafting ivory puzzle balls has passed down through family workshops in Canton from generation to generation from the Qianlong period (r. 1736-1795) to the present day. Through the generations, every leading master has pursued the same goal – to create the most-layered ivory puzzle ball.4

This art form has been little researched until recent decades. Historically the balls were regarded as either luxury export goods (the Rijksmuseum ball) or local tribute items (the NPM ball).5 In 2007, Shih Ching-Fei’s breakthrough research showed that these objects are not merely trinkets, or emblems of China made for the Western market, but can support a fully fledged re-examination of both the general relationship between Europe and the Qing court, and the complex relationship between Canton craftsmen, the imperial workshops and the introduction of European crafts.6 Her further research investigated the influence of the European lathe on the Qing court workshops. She showed that related art works, namely turned and carved Chinese ivory spheres and similar objects, contain evidence that reveals an understanding of artistic and technological exchanges between the East and the West in the eighteenth century. The introduction of the lathe stimulated ivory craft not only in the imperial workshops but also in Canton.7

Although the exact date is not known, but certainly by 1720, the European rose engine lathe was introduced to the Chinese imperial court workshops. The Chinese court artisans used the European rose engine lathe to create turned ivory works (figs. 5a, b) during the Yongzheng period (r. 1722-35).8 In the Qing imperial collection there are seventeenth-century European turned hundred-layered goblets in wood and also turned ivory works (fig. 6).9 The final decoration of the seventeenth-
century European turned ivory Box Decorated with Concentric Ball has features in common with the Chinese ivory puzzle ball. But the operation of the European lathe in the Chinese court seemed to be a short-lived fashion that ended around 1736. The archives of the imperial workshops reveal that in 1777 Emperor Qianlong asked the western missionaries and the court turners of the western lathe (Xiyang xunchuang zhi ren 西洋鏇床之人) to check whether the western rose engine lathes were still in working order and able to turn patterns. It was discovered that the lathes had not been used for a long time and some parts were missing, so he ordered that patterns should be carved instead. Based on an account by John Thomson (1837-1921) in the late nineteenth century, there was no evidence to show that Cantonese ivory artisans actually used European equipment. There are gaps that remain to be filled for us to understand the technical exchanges in ivory turning between Europe and China. The impact of the European rose engine lathes on the ivory works in the Qing court and Canton workshops requires further research.

The earliest Chinese record of an ivory puzzle ball, under the heading of ‘devil’s work spheres’ appears in Cao Zhao’s 曹昭 book Essential Criteria of Antiquities (Gegu yaolun 格古要論, 1388). Cao’s text gives an example of an early ivory puzzle ball with three layers that appeared in the fourteenth century: ‘I have seen a hollow concentric ivory ball, which had two layers inside turned [by a lathe], both layers can revolve. It is called a “devil’s work sphere”. I was told that it was made for the Palace of the Song dynasty.’ Cao assumed it was made by an artisan who served in the court of the Song dynasty (960-1127), but this cannot be verified. Cao explicitly notes that the ball was made using a lathe. The term ‘devil’s work sphere’ was also used by Gao Lian 高濂 (1573-1620), an art connoisseur, in his book Eight Discourses on the Art of Living (Zunsheng bajian 遵生八箋, 1591), in which he describes a twelve-layered ‘devil’s work sphere’ carved from stone, but how it was made is unknown: ‘Ancient people like to carve this type of stone [Qingtian stone] into circular spheres which were similar to “devil’s work spheres”. I have seen one with twelve layers that from the outermost layer to the innermost one was diminishing in size and each layer could revolve. The innermost sphere was as tiny as a green bean. How it was made I do not know. It was “true devil’s work”.’ We can only imagine what it looked like since there is no extant artefact matching Cao’s or Gao’s description.

Gao Shiqi 高士奇 (1645-1704) mentioned a nine-layered European ivory ‘devil’s work sphere’ with ‘very many’ holes and a cube at its centre that he encountered in the Qing imperial court: ‘[I] entered the imperial court and saw an ivory sphere with a hundred holes. The sphere was made with nine layers, while others with seven or five layers. Gently pushing [the layers] with a gold hairpin, they rotate smoothly, and each layer is the same.’ What Gao Shiqi described seems to
match the appearance of the European turned ivory works (fig. 6) in the Qing imperial collection. This is the earliest reference to European ivory puzzle balls in the Chinese imperial court. The term ‘a hundred holes’ that Gao described can be rhetorical and refer to ‘many’.

The Palace Museum, Beijing, has another ivory ball in its collection that contains fifty holes on its outer surface and has at least five layers (fig. 7), its appearance seems to match Gao’s description. This ball is dated between Yongzheng (r. 1722-35) and Qianlong (r. 1736-95) reigns, but the reason for the dating is not given. Could this ball be dated to the Kangxi’s reign (r. 1661-1722) and considered as the prototype of the current Chinese ivory puzzle balls? Where it was made, whether in Europe or in China, in the imperial workshop or in Canton, or somewhere else, remains a mystery and requires further research.

Chinese ivory puzzle balls saw the peak of their fabrication around the mid-eighteenth century. Almost all the extant balls were made in Canton from the mid-eighteenth century onwards. The balls in the Rijksmuseum and National Palace Museum collections both belong to this category. The art and technique of crafting ivory puzzle balls in Canton has been passed on from generation to generation. Since the eighteenth century, ivory carvers in Canton dedicated themselves to mastering this art. The creation of puzzle balls showcased the ability and experience of the ivory carvers.

Wang Qishu (1728-99) recorded a thirteen-layered ivory ‘devil’s work sphere’ that he encountered in the Liulichang antique market in Beijing in his book, published in 1792. Wang’s account suggests that the ivory ‘devil’s work spheres’ were already circulating on the common art market in China in the late eighteenth century. Although Wang did not mention its origin, it is most likely the ivory puzzle ball from Canton.

**The Crafting Process**

The crafting process of Chinese ivory puzzle balls was described in various nineteenth-century travel accounts. For example, the Scottish photographer John Thomson wrote in his book *Through China with a Camera*: ‘The rough piece of solid ivory is first cut into a ball; it is then fixed into a primitive-looking lathe and turned with a sharp tool in various positions until it becomes perfectly round. It is then set again in the lathe and drilled with the requisite number of holes all round. After this one hole is centred, a tool bent at the end is passed in, and with this a groove is produced near the heart of the sphere; another hole is then centred, and after that another, the same operation being carried out with all the holes until all the grooves meet and a small ball drops into the centre. In this way all the balls, one within the other, are ultimately released. The next operation is carving the innermost ball; this is accomplished by means of long drills and other delicate tools, and in the same way all the rest of the balls are carved in succession, the carving gradually becoming more easy and elaborate until the outside ball is reached, and this is then finished with a delicate beauty that resembles the finer sorts of lace.‘

The western turning literature also describes the crafting process of Chinese ivory puzzle balls in detail, stating that the procedure consisted of six steps (fig. 8):

1. Select and saw off a suitable section of ivory
2. Turn the selected section into a sphere
3. Drill fourteen peepholes
4. Separate the layers by turning through each peephole
5. Create openwork patterns on each layer
6. Polish the outer surface
Crafting tools

Lathes were used to turn ivory puzzle balls. Two parts of the lathe are relevant to this discussion: the chuck and the tool post. The chuck is used to fasten the work into the lathe. Chucks can vary in size and shape. The chuck is rotated in the headstock and clamps the work. For example, a hollow hemispherical chuck is attached to the lathe, the ivory puzzle ball is then clamped into the recess in the chuck. In this case, the diameter of the chuck will be the same as the ball being worked. The tool post is used to support the carving tools as they act on the work being rotated in the chuck. Tool posts can have different functions and can be replaced during the crafting process, depending on the task.

According to the description in John Thomson’s account in 1899, very simple equipment was still being used by Chinese ivory puzzle ball artisans. The Chinese lathe was foot powered. The carving and drilling tools were also rather simple. Only a small number of hemispherical shaped wooden chucks were available. Depending on the size of the tusk, an appropriately sized chuck was chosen. The tool post consisted of only a curved wooden block. When needed, the artisans would mark the wooden block with chalk in order to align marked points with respect to the centre of rotation of the lathe. Marks were also placed on carving and drilling tools to determine the depth from the surface of the ball to an internal point. The artisans would use their hands to hold the tools on the tool post when performing the turning operation (figs. 3, 4a, b, 9, 10).

In contrast, European turners used more sophisticated lathes, chucks and tools. European lathes were able to control turning speed accurately. The tool post might be mounted on a measuring slide that allows the artisan to measure the depth of the carving tool with respect to the ball’s surface (Chinese artisans used chalk marks on the tool for depth measurements). In theory, the European equipment was more flexible and easier to use. Critically, a higher degree of accuracy could be achieved. Holtzapffel and Springett give detailed step-by-step procedures of the crafting process using European equipment. The procedures are straightforward but require advanced tool posts to support the tools during the drilling and separation steps (see fig. 8, steps 3 and 4).
Analysis and Discussion of the Manufacturing Process Modelling and Quantifying the Crafting Process

Turning an ivory puzzle ball requires knowledge of the overall geometry of the ball. The drawings in figs. 11a, b were used by artisans as a geometric model of an ivory ball with fourteen peepholes. The drawing in fig. 11a shows a cube, in which the centre of the cube corresponds to the centre of the ivory ball. Fourteen lines are traced from the centre of the cube through the eight corners and through the midpoints of the six faces. The fourteen lines define the centre lines of the fourteen peepholes. The drawing on the right shows a cross-section of a four-layered ball as it would appear on the central plane of a lathe if the ball is viewed through any of the six face peepholes. Each peephole has an aperture, which defines how large the peephole will be.

We use the model to quantify step 3 (making peepholes), step 4 (separating the layers) and step 5 (creating open-work patterns) of the crafting process. We define a ‘well crafted’ ball as one in which:

1. The orientation of a peephole in the crafted ball is the same as the orientation of the corresponding line defined in the model. If the orientation of the crafted peephole does not match the orientation line in the model, then the artisan did not mark the starting position of the peephole accurately, or the angle of attack of the tool used by the artisan to drill the peephole was not accurate.

2. The diameter of each peephole should be the same. If the diameters of the peepholes vary, the artisan did not guide the drilling tool very accurately when making the peephole.

3. The thickness of each layer and the distances between layers should be consistent. If the distance between the layers varies, then the artisan did not position the separation tools at the correct depth in each peephole.

Figs. 11a, b
Geometric models of an ivory ball with fourteen peepholes.

a) Chinese drawing of geometric solid model: a cube and six square pyramids. Fourteen vertices are defined by the eight corners of the cube and the six apices of the pyramids.
From Mei Wending. Complete Works on Calendar and Mathematics (Lisuan quanshu, 1723), vol. 4, Sikuquanshu edition, 1781.
b) A cross section of a four-layer ball as it would appear at the centre plane of the lathe.
From Holtzapffel and Holtzapffel 1881 (note 17), p. 427.
4. A geometric pattern carved on a layer consists of an arrangement of smaller elements. Many element types exist; e.g. a square element, a triangle element, a star shaped element. We define an element as a composition of small (curved) line segments. For example: a triangular shaped element is a composition of three-line segments. An ellipse element is a composition of two curved-line segments. The artisan creates an element by ‘punching’ a small carving knife into the ivory. The size of the knife will correspond to the length of a line segment. In this way, a triangular element can be made by three punches with a straight knife. An ellipse is made by two punches with a curved knife. Comparing the sizes of the elements on a layer provides an indication of the how accurately the artisans placed the carving knife when crafting an element. If the sizes of the elements vary, then the artisan did not position the carving knife very accurately.

Measurements
We now define three measurements that allow us to compute the crafting quality from the 3D CT data set.
Figs. 12a-c define three measurements:

a) Peephole Measurement (crafting process step 3): The diameter of a peephole is computed by fitting a cone onto the peephole data. The opening angle of the cone is used to compute the diameter of each hole (blue and red lines of left panel).

b) Layer Measurement (crafting process step 4): The thickness of each layer and the distance between each layer is computed from the 3D data (red and green lines in middle panel).

c) Pattern Measurement (crafting process step 5): The size of each element on each layer is measured (one circular element, four triangle elements, four elliptical elements).
Results

3D X-ray Imaging
An experimental CT scanner setup was used to scan and generate 3D data volumes of the ivory balls. Figs. 13a, b show two different views of the nine-layered Rijksmuseum ball. On the left (fig. 13a) is a grayscale cross section of the 3D data set. On the right (fig. 13b) is a 3D rendering of the nine layers.

Image processing techniques are used to segment layers in the 3D volume and to generate 3D surface meshes of each layer. Figs. 12a-c are computer renderings of the meshes of each layer. Models are scaled so that the detail of each layer can be seen more easily.

Figs. 14a, b are the starting point for the measurements. The images show the central cross section of the Rijksmuseum ball (fig. 14a) and the npm ball (fig. 14b). The images are generated by aligning the polygonal data towards a common centre point, and clipping away all polygonal data that does not belong to the central plane. We now present the measurements of the Rijksmuseum and npm ivory balls.

Peephole Measurements
Fig. 15 plots the aperture angle of each of the fourteen peepholes. The average angle of the Rijksmuseum ball (blue plot)
is 21.5° with a standard deviation of 0.9°. For the NPM ball (red plot) the average aperture angle is 23° with a standard deviation of 0.1°. The plot shows that the aperture angles of the Rijksmuseum ball are irregular, whereas the aperture angles of the NPM ball are more regular. The standard deviation for the NPM ball is significantly lower than for the Rijksmuseum ball.

A possible explanation is that the supporting tools used to craft the NPM ball were more accurate than the ones used to make the nine-layered ball. For the nine-layered ball, the variation is high, and it may well be that the artisan guided the peephole drilling tool by hand. For the twenty-three-layered ball, it is likely that the artisan used more sophisticated tool fixtures.

### Layer Measurements

Fig. 16a, b plot the layer thickness and separation for the two balls. For the Rijksmuseum ball (in blue), the average layer thickness and separation is 5.3 mm and 3.2 mm, with a standard deviation of 0.3 mm and 0.9 mm. For the NPM ball (in red), the average layer thickness and separation is 3 mm and 0.9 mm, with a standard deviation of 0.9 mm and 0.1 mm.

The standard deviation of the distance between layers of the Rijksmuseum ball is substantially larger than the NPM ball. For the NPM ball, the standard deviation shows that the distance between layers is the same for all layers. This implies that the artisan has placed the separation tool at the correct depth for each peephole. For the Rijksmuseum ball, the separation between the layers is larger but, more importantly, the standard deviation is almost ten times larger. This implies that the artisan had more difficulty in placing the separation tool at the correct depth.

One explanation is that, in the NPM case, the artisan had tools to measure depth accurately. In the Rijksmuseum case, it may well be that the artisan placed marks on the separation tool and used these marks for all peepholes.

### Pattern Measurements

Fig. 17 shows the patterns for both balls. The pattern on the Rijksmuseum ball (right row) has two elements; a square element and an almond shaped element. There are thirty-four square elements and seventy almond elements in the region displayed. Four punches are required for each element, resulting in 416 punches for the region.

Visual inspection will reveal defects in the patterns or in the elements. Defects in patterns occur if an element is misplaced or even missing. An example of a missing square element can be seen in the right row. Defects in patterns occur when the artisan mispunches an element. Examples of mispunching a T-shaped element can be seen in the left row.
Fig. 17
Patterns on layer two for the NPM ball (left) and Rijksmuseum ball (right). Images from top to bottom: a 3D rendering of the hemisphere of layer two; a small region of interest; punched elements; element classification.
Tool marks
Figs. 18a, b show the effects of placing the separation tool at different depths. The images show an outer (fig. 18a) and inner (fig. 18b) view of layer six of the Rijksmuseum ball. The tool marks left behind by the separation tool can be seen as deep grooves on the inner surface (indicated by the red arrow). In addition, small triangular shaped wedges can be seen around the central peephole (green arrow). Such triangles are the result of placing the separation tool at different depths in the surrounding peepholes.

Discussion and Conclusion
We used a non-destructive method to measure the morphological properties of Chinese ivory balls. We developed an assessment of how accurately the artisan performed the fabrication process by comparing the measurements with a governing mathematical model. Two ivory balls were measured; a nine-layered ball from the Rijksmuseum made in the eighteenth century and a twenty-three-layered ball the made in the nineteenth century from the National Palace Museum in Taipei.

Our results show a substantial difference in the accuracy of the crafting process between the two ivory balls. For the Rijksmuseum ball, the results show that the accuracy of approximately 0.9 mm in placing separation tool is at the required depth. The aperture of each peephole varies. It is conceivable that an expert ivory artisan can achieve this accuracy with basic tools. However, for the NPM ball, an accuracy of approximately 0.1 mm was achieved and the aperture of the peepholes was constant. It is not likely that even a very skilled artisan could achieve this accuracy using only handheld tools. A more likely explanation would be that different, more accurate, tools were used when crafting the NPM ball.

From an art-historical perspective, there may be an explanation for the measured differences. It is likely that the accuracy of the turning tools used by the Cantonese ivory artisans had improved during the eighteenth century. As Shih Ching-Fei pointed out, the European lathe and turning technologies were introduced in Canton during the eighteenth century. At the end of the nineteenth century, Holtzapfel documented how Chinese ivory balls were crafted using European tools. It is, however, unlikely that Cantonese artisans used the tools doc-

Figs. 18a, b
Tool marks on the inner layers of the Rijksmuseum ball. Deep circular grooves caused by the separation tool can be seen. Also, small triangular shaped wedges are visible, caused by placing the separation tool at different depths.
Chinese ivory puzzle balls are known for their beauty, finesse and their ability to intrigue viewers. From the eighteenth century until recently, they have been crafted by turning, using a simple lathe and a set of drilling and carving tools developed in the eighteenth century. The craft of Chinese ivory puzzle balls has been described as the ‘devil’s work’, as it requires a great deal of proficiency, accuracy and patience.

This study presents a novel method for quantifying the crafting process of Chinese ivory puzzle balls. The method is based on measuring the morphological properties of ivory balls in three-dimensional images obtained using X-ray Computed Tomography (CT) scanning techniques. The accuracy of the crafting process is obtained by comparing the measured properties with an underlying mathematical model of the ball. We apply the proposed method to ivory balls from the Rijksmuseum in Amsterdam and the National Palace Museum in Taipei. The results show substantial differences in the accuracy of the crafting process.

From an art-historical perspective, the results show that the accuracy of the crafting process evolved during the eighteenth century. They also suggest that the ivory balls we have analyzed have been crafted with different types of turning tools.

For future work we plan to scan and apply the proposed methods to a large collection of ivory balls. We will develop novel data processing techniques that will allow us to detect trends in the crafting process of Chinese ivory puzzle balls.

**NOTES**

1. For the overview of Royer’s collection see Jan van Campen, *De Haagse jurist Jean Theodore Royer (1737-1807) en zijn verzameling Chinese voorwerpen*, Hilversum 2000.

2. Two different laboratory X-ray CT scanner setups were used to scan and generate 3D data volumes of the ivory balls. The Rijksmuseum ball was scanned at the Flex-Ray laboratory at CWI, the national research institute for mathematics and computer science in the Netherlands. The NPM ball was scanned at the imaging facility of the National Palace Museum. The Astra toolkit was used to do the 3D reconstruction of the acquired data sets. In-house image processing tools, developed at CWI, were used for the segmentation of the layers, and for data analysis. The Visualization Toolkit (VTK) was used to draw the surface meshes.

3. At times, the work is also called ‘immortal’s work’ (xiangong 仙工). For the discussion of the terms of ‘devil’s work’ and ‘immortal’s work’ see Chi Jo-Hsin 倪若昕, ‘Cong “guigong” dao “xiangong”: Qingdai nanpái yadiao gongyi gaishu 從「鬼工」到「仙工」：清代南派牙雕工藝概述 [From ‘devil’s work’ to ‘immortal’s work’: on the craft of ivory carving of the southern school in the
revealing the secrets of chinese ivory puzzle balls

4 The Weng family is famous for this craft. The first generation can be traced back to Weng Wuzhang 翁五章 in the Qianlong period. He made an eleven-layered ivory puzzle ball for the Qing court. His son, Weng Tong 翁彤 made twenty-layered balls. His grandson, Weng Zhao 翁昭 made twenty-five-layered balls in 1915. The fourth generation, Weng Rongbiao 翁榮標 made a thirty-five-layered ball in 1957 and a forty-five-layered ball in 1979. The current fifth generation master, Weng Yaoxiang 翁耀祥 created a fifty-seven-layered ball in 2003 which is by far the most-layered ball. See Huang Danxiao 黃丹曉, ‘Xianzaishi, “miji”: Guangdong xiangyaqiu suojian zhi xueju jikan' [The technical details and inheritance of Canton ivory ball carving], in Zijincheng [Forbidden city] 203 (2011), pp. 72-85.

5 For balls as export art, see Craig Clunas (ed.), Chinese Export Art and Design, London 1987, pp. 96-104; as tribute, see Yang Boda 杨伯達, Guangdong gongpin 清代廣東貢品, London 1987, p. 96; as tribute, see Percival David, Chinese Connoisseurship: The Ko Ku Yao Lun, the Essential Criteria of Antiquities, London 1971, p. 133; for authors; in Percival David's translation, he omitted the fact that the inner layers of the ball were turned on a lathe, cf. Sir Victor Sassoon Collection, Sir Victor Sassoon Collection: the Camera Orientalis, London 1971, p. 133; for Cao's original text see ibid. p. 310.

6 Shih Ching-Fei, ‘Xiangyaqiu suojian zhi tongxie xijie yu chuancheng xiangyaqiu diaoke jiyi xijie yu chuancheng’ [The technical details and inheritance of Canton ivory ball carving], in Zijincheng [Forbidden city] 203 (2011), pp. 152-73.

7 Shih Ching-Fei, ‘Yeshi bolaipin: Qinggong zhong de huashi xuanchuang 也只是舶來品：清宮中的花式鏇床’ [Another item from over the sea: ornamental lathes at the Qing court], in Meishushi yanjiu jikan 美術史研究集刊, 32 (2012), pp. 171-238.

8 Shih Ching-Fei 2007 (note 6), pp. 87-138; Shih Ching-Fei 2012 (note 7), pp. 171-238; Shih Ching-Fei, ‘Unknown Transcultural Objects: Turned Ivory Works by the European Rose Engine Lathe in the Eighteenth-Century Qing Court’, in Anna Grasskamp, Monica Janea (eds.), EurAsian Matters: China, Europe, and the Transcultural Objects, 1600-1800, Heidelberg 2018, pp. 57-76.

9 Shih Ching-Fei, ‘A Hundred-Layered Goblet from the Western Ocean’, in Orientations 46:4 (2015), pp. 60-64; Liu Yue 劉岳, ‘Cong yijian Qinggong yiliu de xiangya qiwu suoqi 從一件清宮遺留的象牙器物說起’ [Talking from an ivory object left behind by the Qing palace], in Zijincheng 203 (2011), pp. 40-56.

10 ‘Guangmuzuo 廣木作 (carpentry workshop), the twelfth month of the forty-fourth year of Qianlong’s reign (1777),’ in Chinese University of Hong Kong (ed.), Qinggong neiwu zaobanchu dangan tonghui 清宮內務府造辦處檔案總匯, Beijing 2005, vol. 39, p. 708.

11 John Thomson, Through China with a Camera, London 1899, p. 70.

12 ‘昔有象牙國毬一個，中直通一孔，內車二重，皆可轉動，故謂之鬼功毬。或云宋內院中作者。’ Translation provided by the authors; in Percival David's translation, he omitted the fact that the inner layers of the ball were turned on a lathe, cf. Sir Percival David, Chinese Connoisseurship: The Ko Ku Yao Lun, the Essential Criteria of Antiquities, London 1971, p. 133; for Cao’s original text see ibid. p. 310.

13 ‘嘗有象牙國毬一個，中直通一孔，內車二重，皆可轉動，故謂之鬼功毬。或云宋內院中作者。’ Translation provided by the authors; in Percival David's translation, he omitted the fact that the inner layers of the ball were turned on a lathe, cf. Sir Percival David, Chinese Connoisseurship: The Ko Ku Yao Lun, the Essential Criteria of Antiquities, London 1971, p. 133; for Cao’s original text see ibid. p. 310.

14 ‘昔有象牙圓毬一個，中直通一孔，內車二重，皆可轉動，故謂之鬼功毬。或云宋內院中作者。’ Cited in Shih Ching-Fei 2016 (note 3), p. 18.

15 Chi Jo-Hsin 2007 (note 3), p. 59.
John Thomson 1899, p. 70.

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Holtzapffel, Turning and Mechanical Manipulation Intended as a Work of General Reference and Practical Instruction on the Lathe, and the Various Mechanical Pursuits Followed by Amateurs. Vol. iv: The Principle and Practice of Hand or Simple Turning, London 1881, pp. 426-45;
David Springett, Woodturning Wizardry, East Petersbury 1993, pp. 143-55.

Holtzapffel and Holtzapffel 1881 and Springett 1993 (note 17).

For the Chinese geometric solid model see Wang Chung-Chi 王崇齊, ‘Shuzi de mimi: Xiangyaqiu de jihe zhixu 數字的祕密：象牙球的幾何秩序 [Secrets behind numerals: geometric order of the ivory ball]’, in Zijincheng 203 (2011), pp. 68-71; for the European geometric solid model see Wang Ching-Ling 王靜靈, ‘Yuzhou yu shijie de zhuzai: Deleisidun suocang Sakesen wanghou de duomianti xiangya taoqiu zhizuo 宇宙與世界的主宰：德累斯頓所藏薩克森王侯的多面體象牙套球製作 [Domination of the cosmos and the world: production of polyhedral ivory balls collected by Princes of Saxony in Dresden]’, in Zijincheng 203 (2011), pp. 57-67. We also appreciate opinions provided by Professor Shih Ching-Fei of National Taiwan University through personal correspondences and discussion.

Holtzapffel and Holtzapffel 1881 (note 17), p. 427.

Sophia Coban et al., ‘Explorative Imaging and Its Implementation at the Flex-ray Laboratory’, MDPI Journal of Imaging, 6:4 (2020). In note 2 we provide additional information on the CT scanner and the settings used to scan the ivory balls.

Robert van Liere et al., ‘Imaging Ancient Chinese Ivory Puzzle Balls: Deducing the Make Process’, in Proceedings of the 6th International Workshop on Image Processing for Art Investigation (IPAI), Ghent 2018, pp. 48-50.

For reference, compare images in figs. 14a, b with Holtzapffel’s diagram in fig. 11b.

The plots disregard the outer most and inner most layer, since these are always much thicker than the other layers.

Shih Ching-Fei 2007 (note 6), 2012 (note 7) and 2018 (note 8).

Holtzapffel and Holtzapffel 1881 (note 17), p. 429.

> Fig. 19
Nine layers of the Rijksmuseum ball. Images are scaled to fit in panel.
The images presented in this appendix are intended to give an impression of the beauty and finesse of ancient Chinese ivory puzzle balls. It is clear from these images that crafting an ivory ball required a great deal of proficiency and patience.

Figs. 19 and 20a-d are visualizations generated from a high-resolution 3D CT volume of the nine-layered ball from the Rijksmuseum ball. Ambient occlusion, an advanced computer graphics rendering technique, is used so that even the finest details in the carvings can be perceived clearly.
Figs. 20a-d
Zooming into details of the outer layer of the Rijksmuseum ball.
REVEALING THE SECRETS OF CHINESE IVORY PUZZLE BALLS