Influence of the Multilayer Nature of Reinforcement of Open Thin-Walled Cylindrical Carbon Shells on Their Natural Vibrations

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Abstract. In the near future, mega-long-span structures will be required for creating stable microclimate over large areas, regardless of the natural conditions, for comfortable living of people. Thin-walled shell structures made of carbon materials with high strength properties are best suited for this purpose. The problem with thin-walled shells resides in vibrations caused by the action of external forces, which can lead to structural failure. The paper presents and experimental study of influence of the multilayer nature of reinforcement of open thin-walled cylindrical carbon shells on their natural vibrations. Experimental data have been compared to the theoretical calculations of open thin-walled cylindrical shell vibrations taking into account the frequency spectrum splitting. It has been found that with the increase in the number of carbon fabric reinforcement layers, vibration frequency deviation in open thin-walled cylindrical carbon shells increases at the initial stage as compared to the analytic model.

1. Introduction

Global climate change on Earth necessitates development of protection against the impact of the environment for human settlements, manufacturing facilities, and entire cities. This requires engineering of mega-long-span structures. Cylindrical and dome-shaped thin-walled shell structures are best suited for this purpose. These shells will be exposed to various loads: wind and snow loads, varying atmospheric pressure, seismic and temperature loads, loads from human-life-support engineering systems. For this reason, lightweight and strong structural materials should be used for making these shells. Carbon materials, which are three or more times stronger than metal, work best for this purpose. However, there is a problem of the presence of natural vibrations in open thin-walled cylindrical shells,
and cyclical or quasi-static external action will cause forced vibrations. Coalescence of the natural and forced vibration frequencies can result in unacceptable amplitudes of the structure vibrations which will cause failure of these shells [1, 2]. As of today, no analysis of the structure oscillatory behavior is performed when designing building structures, due to their relatively small size as well as absence of the relevant procedures and analytic models. Therefore, it is necessary to study various aspects of the structure oscillatory processes and, in particular, influence of the multilayer nature of reinforcement of open thin-walled cylindrical carbon shells on their natural vibrations.

2. Materials and method
Scaled-down models of open thin-walled cylindrical carbon shells were made at Komsomolsk-na-Amure State University, in order to experimentally study the influence of the multilayer nature of reinforcement of open thin-walled cylindrical carbon shells on their natural vibrations. Shell models with single-layer reinforcement were made of carbon fabric 2/2 12K-1000-600. Two-layer reinforcement was made up of one layer of carbon fabric 2/2 12K-1000-200 and one layer of fabric 2/2 12K-1000-400. Three layers of carbon fabric 2/2 12K-1000-200 were used for three-layer reinforcement. The following dimensions were taken for the open thin-walled cylindrical shell models: $H=R=200$ mm, $L=400$ mm, $h=1.2$ mm. An impulse hammer AU03 (force $P=0.2$ N) was used to cause vibrations. Amplitude and frequency of the shell vibrations were measured by means of a high speed vibrometer HSV2000 with the accuracy of 1 micron (amplitude) within the range of 1 Hz–50 kHz (frequency), Fig. 1.

![Figure 1. Test Set-Up for an Open Thin-Walled Cylindrical Shell.](image1)

![Figure 2. Piece of an Open Thin-Walled Cylindrical Carbon Shell with Three-Layer Reinforcement.](image2)
Theoretical calculation of vibration frequency for an open cylindrical shell was carried out using a motion equation [3, 4]

\[ w_0(y) = f_0 \sin(\beta y + \varphi_0); \quad \beta = \frac{n}{R}, \]

where \( f_0 \) is the amplitude of initial deviations; \( \varphi_0 \) is the initial angle of reference.

Considering that the initial imperfection leads to coupling of bending and radial vibrations of the shell [5, 6], the deflection of the latter will be represented as follows:

\[ w(x, y, t) = \sum_{m,n} [f_{1m,n}(t) \sin \beta_n y + f_{2m,n}(t) \cos \beta_n y + f_{3m,n}(t) \sin \alpha_m x]; \]

\[ \beta_n = \frac{n}{R}; \quad \alpha_m = \frac{m\pi}{l}. \]

Coordinates \( f_{1m,n} \) and \( f_{2m,n} \) in equation (2) correspond to bending vibrations of an open shell [7, 8], while coordinate \( f_{3m,n} \) – to radial ones.

Using this, we obtain the stress function \( \Phi(x, y, t) \):

\[ \Phi = E \left( \sum_{m,n} \left[ \Phi_{0m,n} \sin \beta_n y + \Phi_{1m,n} \cos \beta_n y + \Phi_{2m,n} \sin(\beta_0 + \beta_n)y + \Phi_{3m,n} \sin(\beta_0 - \beta_n)y \right. \right. 
\]
\[ + \left. \left. \Phi_{4m,n} \cos(\beta_0 + \beta_n)y + \Phi_{5m,n} \cos(\beta_0 - \beta_n)y + \Phi_{6m,n} \cos \beta_0 y + \Phi_{7m,n} \right] \sin \alpha_m x \right. 
\]
\[ + \frac{\Phi_{01} x^2}{2} + \Phi_{02} xy + \frac{\Phi_{03} y^2}{2} \right). \]

The first eight coefficients used in (3) are as follows:

\[ \Phi_{0m,n} = \frac{\alpha_m^2}{R(\alpha_m^2 + \beta_n^2)^2} f_{1m,n}; \quad \Phi_{1m,n} = \frac{\alpha_m^2}{R(\alpha_m^2 + \beta_n^2)^2} f_{2m,n}; \]
\[ \Phi_{2m,n} = -\frac{\alpha_m^2 \beta_n^2}{2(\alpha_m^2 + (\beta_0 + \beta_n)^2)^2} f_{1m,n}; \quad \Phi_{3m,n} = -\frac{\alpha_m^2 \beta_n^2}{2(\alpha_m^2 + (\beta_0 - \beta_n)^2)^2} f_{1m,n}; \]
\[ \Phi_{4m,n} = -\frac{\alpha_m^2 \beta_n^2}{2(\alpha_m^2 + (\beta_0 + \beta_n)^2)^2} f_{2m,n}; \quad \Phi_{5m,n} = -\frac{\alpha_m^2 \beta_n^2}{2(\alpha_m^2 + (\beta_0 - \beta_n)^2)^2} f_{2m,n}; \]
\[ \Phi_{6m,n} = -\frac{\alpha_m^2 \beta_n^2}{(\alpha_m^2 + \beta_n^2)^2} f_{3m,n}; \quad \Phi_{7m,n} = \frac{1}{\alpha_m^2 R} f_{3m,n}. \]

Having satisfied the boundary conditions [9, 10]:

\[ \frac{h}{b \pi R} \int_0^{b \pi R} \frac{\partial^2 \Phi}{\partial y^2} dy = -\frac{h}{b \pi R} \int_0^{b \pi R} \frac{\partial^2 \Phi}{\partial x \partial y} dy = 0 \text{ при } x = 0, x = l. \]

we obtain that \( \Phi_{01} = \Phi_{02} = \Phi_{03} = 0. \)

Orthogonalization of a linearized equation of motion is written as:

\[ \frac{1}{E} \nabla^4 \Phi = -\frac{1}{2} \left[ L(w_0 + w, w_0 + w) - L(w_0, w_0) \right] - \frac{1}{R} \frac{\partial^2 w}{\partial x^2}; \]

\[ \nabla^4 \Phi = \frac{1}{E} \left( \frac{\partial^4 \Phi}{\partial x^4} + \frac{\partial^4 \Phi}{\partial y^4} - \frac{1}{R^2} \frac{\partial^4 \Phi}{\partial x \partial y^2} \right) \to \frac{1}{E} \left( \frac{\partial^4 \Phi}{\partial x^4} + \frac{\partial^4 \Phi}{\partial y^4} - \frac{1}{R^2} \frac{\partial^4 \Phi}{\partial x \partial y^2} \right) \to \frac{1}{E} \left( \frac{\partial^4 \Phi}{\partial x^4} + \frac{\partial^4 \Phi}{\partial y^4} \right). \]
\[
\frac{D}{h} \nabla^4 w = L(\Phi, w_0 + w) \frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2} + \frac{q}{h} \frac{\partial^2 w}{\partial t^2}
\]

where \( \nabla^4 \) is the biharmonic Laplacian operator, and gives a system of coupled dynamic equations for the dimensionless coordinates \( \alpha_{imn} = \frac{f_{mn}}{h} \) (\( i = 1, 2, 3 \)).

Squared dimensionless natural frequencies of bending and radial vibrations of an ideal shell are, respectively, as follows:

\[
\omega_{mn}^2 = \frac{\varepsilon_n(1 + \theta_{mn}^2)^2}{12(1 - \mu^2)} + \frac{\theta_{mn}^4}{(1 + \theta_{mn}^2)^2}; \quad \mu^2 = 1 + \frac{\varepsilon_n \theta_{mn}^4}{12(1 - \mu^2)}; \quad \varepsilon_n = \left( \frac{n^2 h}{R} \right)^2;
\]

\[
\theta_{mn} = \frac{m\pi R}{nl}.
\]

Structure of equations (5) indicates that \( w_0(y) \) can lead to coupled bending-and-radial vibrations of the shell. Bending modes of shell vibrations with different wave number \( n \) (non-conjugate bending modes) also get coupled. No coupling of conjugate bending modes is observed at \( w_0(y) \) taking on form (1); however, splitting of the shell frequency spectrum does take place [11, 12].

By introducing normal coordinates, it can be shown that in order to determine the modes of predominantly bending vibrations of the shell with \( w_0(y) \) taking on form (5), it is necessary to "sum up" the modes of bending vibrations of an ideal open shell only in the circumferential coordinate \( y \). Thus, the deflection of the under consideration can be approximated by the following expression:

\[
w(x, y, t) = \sin \alpha_m x; \quad \sum_n \left[ f_{1m,n}(t) \sin \beta_n y + f_{2m,n}(t) \cos \beta_n y + f_{3m,n}(t) \right];
\]

In certain special cases with initial imperfection, when the number of waves in the circumferential direction is the same as in the longitudinal one, i.e., \( n = n_0 \) [13–15], the system of dynamic equations (5) takes on the following form

\[
\bar{a}_{1m,n} \left( c_{1m,n} + \alpha_0^2 c_{0m,n} \right) a_{3m,n} + a_0 c_{4m,n} a_{1m,2n} + \alpha_0^2 c_{7m,n} a_{1m,3n} = 0;
\]

\[
\bar{a}_{2m,n} + \left( d_{1m,n} + \alpha_0^2 d_{0m,n} \right) a_{2m,n} + a_0 d_{4m,n} a_{2m,2n} + \alpha_0^2 d_{7m,n} a_{2m,3n} + d_{8m,n} a_0 a_{3m,n} = 0;
\]

\[
\bar{a}_{3m,n} + g_{1m,n} a_{3m,n} + g_{2m,n} a_0 a_{2m,n} = 0.
\]

If only one term of the series is retained in expansion (7),

\[
w(x, y, t) = \left[ f_{1m,n}(t) \sin \beta_n y + f_{2m,n}(t) \cos \beta_n y + f_{3m,n}(t) \right] \sin \alpha_m x;
\]

we obtain the following (the subscripts \( m \) and \( \ell \) or \( n \) for \( \varepsilon, \theta, \omega, \rho \) are omitted) instead of (8):

\[
\bar{a}_{1m,n} + \left[ 1 + \frac{\varepsilon \theta^4}{4\omega^2(4 + \theta^2)^2} \right] a_{1m,n} = 0;
\]

\[
\bar{a}_{2m,n} + \left[ 1 + \frac{\varepsilon}{4\omega^2} \right] \frac{\theta^4}{2(4 + \theta^2)^2} a_{2m,n} - \frac{\varepsilon \theta^4}{2\omega^2} \left[ 1 + \frac{\theta^4}{(1 + \theta^2)^2} \right] a_0 a_{3m,n} = 0;
\]

\[
\bar{a}_{3m,n} + \left[ \frac{\varepsilon \theta^4}{2(1 + \theta^2)^2} \right] a_0 a_{2m,n} = 0.
\]

We seek a periodic solution of system (10) as follows:
\[ a_{1m,n}(\tau) = a_{1m,n} \cos \Omega_0 \tau; \quad \Omega_0 = \frac{\omega_0}{\omega}; \quad i = 1,2,3. \]

We deduce three dimensionless natural frequencies from the frequency equation: \( \Omega_{01} < 1 \leq \Omega_{02} \ll \Omega_{03} \). The first frequency corresponds predominantly bending vibrations of an imperfect shell, the second one — to purely bending vibrations, and the third one — to predominantly radial vibrations. The square of the second natural frequency, at which no radial vibrations are observed, can easily be obtained from the first equation of system (10):

\[
\Omega_{02}^2 = 1 + \frac{\varepsilon \theta^4}{4 \omega^2 (4 + \theta^2)^2} a_0^2
\]

or

\[
\omega_{02}^2 = \omega^2 + \frac{\varepsilon \theta^4}{4 (4 + \theta^2)^2} a_0^2
\]

Examination of formula (12) shows that the second frequency of the new solution is equal to the fundamental frequency of the conventional solution: \( \Omega_{02} = \Omega_{01k} \). Since the shell is open, we calculate the bending frequency of the spectrum for a shell with the parameters \( l/R = 2; R/h = 166 \) at \( m = 1, n = 10, \mu = 0.3 \), in order to compare it with the experimental data.

3. Results

Figure 3 shows the results of the theoretical calculations carried out according to the proposed mathematical model for calculating the vibration frequency for open thin-walled cylindrical shells, plotted against half-waves (graph 4). The results of the theoretical calculation of vibration frequency for an open thin-walled cylindrical shell are in satisfactory agreement with the results of testing of an open thin-walled cylindrical shell with single-layer reinforcement, the deviation equals 7–8%.

![Figure 3](image.png)

**Figure 3.** Relationship Between the Vibration Frequency and the Number of Half-Waves for Open Thin-Walled Cylindrical Carbon Shells with: 4 — Three-Layer Reinforcement; 3 — Two-Layer Reinforcement; 2 — Single-Layer Reinforcement; 1 — Graph of the Theoretical Calculation of Vibration Frequency of the Shell.

In case of an open thin-walled cylindrical carbon shell with two-layer reinforcement, the initial deviation of the vibration frequency from the analytic model amounts to 20%. In case of an open thin-
walled cylindrical carbon shell with three-layer reinforcement, the deviation of the vibration frequency from the analytic model increases to 45%.

4. Conclusion
The initial frequency of the shell vibrations increases with the increase in the number of carbon fabric reinforcement layers, while the amplitude decreases. An exponent taking into account the presence of multiple reinforcement layers should be introduced in the mathematical model for calculation of the vibration frequency for an open cylindrical shell.

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