Synthesis of multi-band filters based on multi-prototype transformation

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Abstract
In this article, a new synthesis method for generalized multi-band filters has been proposed. Different from the traditional frequency transformation method, which transforms a low-pass prototype into a multi-band filter, multiple low-pass prototypes have been transformed into a multi-band filter. This study introduces the idea of transforming more than two lowpass prototypes into a filter, called multi-prototype transformation. It divides the multi-band filter to be synthesized into different segments each of which corresponds to a low-pass prototype. After frequency transformation, these segments are combined together to get the final transfer function. The multi-band filter synthesized by this method can have different orders, bandwidths or return losses in different passbands. Prescribed transmission zeros can also be realized. The return loss adjustment, segment boundary estimation and equal-ripple realization of this method are further explained in theory. To verify this method, an asymmetric quad-band filter is synthesized as an example and a symmetric triple-band filter is synthesized, designed, fabricated and measured. The obtained results show good agreement with the simulation.

1 INTRODUCTION

Wireless communication has various applications, such as global positioning systems, bluetooth, wireless local-area networks and so on. In recent decades, there is a trend to integrate these applications into a single system [1], called the multi-band communication system. This system needs multi-band filters to incorporate the multiple passbands within a single filter structure. The main advantages of using multi-band filters are saving mass and volume and simplifying the circuit. To design multi-band filters, two methods are usually used: (1) using multi-mode resonators [2] and (2) filter synthesis method [3].

Many multi-mode resonators have been used to design multi-band filters. Pal et al. [4] made a review of different types of multi-mode resonators, including stepped-impedance resonators [5], stub-loaded resonators [6], ring-loaded resonators [7] and fractal-shape resonators [8], for the design of multi-band filters. In Ref. [9], a novel multi-mode square ring loaded resonator is proposed to design a tri-band balanced bandpass filter. Multi-band filters realized by multi-mode resonators can have a compact size, according to Ref. [10,11]. However, their design process is not general. Different multi-mode resonators have different design processes and a multi-mode resonator is not applicable for all filters.

The filter synthesis method has an important role in multi-band filter design [12]. It is more flexible and simple since it can be realized through the topologies and technologies commonly used in single-band filter design. Many research studies have been done on the synthesis of multi-band filters. The frequency transformation method has widespread applications. In Ref. [13–15], a dual-band filter was realized by a quadratic transformation equation. An asymmetrical dual-band bandpass filter was synthesized by modified Richard’s transformation in Ref. [16]. Using a cubic transformation equation, symmetric and asymmetric triple-band filters were synthesized as reported in Ref. [17]. In Ref. [18] this method was extended to the case of arbitrary number.
of bands at the cost of increasing transformation equation complexity. In 2019, Wu et al. [19] proposed a direct synthesis method for quad-band filters by frequency transformation. However, all these transformation methods face a generality problem. Because all the passbands are derived from the same lowpass prototype, they cannot deal with generalized multi-band filters which have different orders or return loss in different passbands. To solve the generality problem of multi-band filter synthesis, a dual-band filter synthesis method based on two lowpass prototypes was proposed in Ref. [20]. The dual-band filter synthesized by this method can have different orders, bandwidths, or return losses in different passbands and the filter prescribed transmission zeros can also be realized. Musonda et al. [21] proposed a generalized multi-band filter synthesis method based on the linear optimization method. It can also realize the filter prescribed transmission zeros. Multi-band filters designed by coupling matrix synthesis are usually realized by a signal model resonator. Thus, they are not so compact.

More recently, some research studies tried to combine the method of using multi-mode resonators and the method of filter synthesis. Ma et al. [22] proposed a synthesis method for multi-band filters. With this method, a tri-band microstrip filter was synthesized and designed through tri-mode resonators. Guo et al. [23] combined the single-band/dual-band synthesis method and the multi-mode resonator theory together. Then, a design method for multi-band filters in substrate integrated waveguide was proposed. Combining the filter synthesis method with multi-mode resonators can not only be more generalized but can also result in compact size.

In this article, in order to design generalized multi-band filters, we propose a new synthesis method which can realize assigned transmission zeros and assigned return losses in each passband. Different from the traditional transformation method, which transforms a lowpass prototype to a multi-band filter, we transform multiple lowpass prototypes to a multi-band filter, called multi-prototype transformation. The multi-band filter to be synthesized is divided into different segments, each of which corresponds to a lowpass prototype. The filter segment boundaries can be derived from the prescribed return loss in each passband. Combining all the poles and zeros of lowpass prototypes together, the multi-band filter frequency response can be obtained. The synthesis of dual-band filter reported in Ref. [20] is a special case of this method. Compared with the method described in Ref. [20] which can be used to synthesize only a dual-band filter, the method presented here can synthesize an arbitrary number of bands. Besides, this article also provides an estimation of segment boundaries according to the prescribed return losses and other deeper explanations and examples of this method. To verify this proposed method, a quad-band filter is synthesized as an example. Then a symmetric triple-band filter is designed and fabricated using open-loop resonators [24], the measured results match well with the design.

This article is organized as follows. Section 2 describes the design theory including the filter transformation equation, passbands combination and filter synthesis process. In Section 3, an example of quad-band filter synthesis is given. In Section 4, return loss adjustment, segment boundaries estimation and realization of equal-ripple are explained in theory. In Section 5, the synthesis, design and fabrication of a symmetric triple-band filter are discussed. Finally, a conclusion is given in Section 6.

2 | DESIGN THEORY

2.1 | Proposal

Figure 1 shows the filter frequency responses in three different frequency domains. Ω-domain is the normalized frequency domain of single band lowpass filter prototypes. In our synthesis method, there are several independent lowpass prototypes. Ω'-domain is an auxiliary normalized frequency domain of the multi-band filter prototype in which a coupling matrix is synthesized. It is in general unrealizable due to asymmetry with respect to the origin. ω-domain is the actual frequency domain where the multi-band filter operates. The filter response in this domain is realizable. There are two frequency transformations in our design of multi-band filters: transformation from Ω domain to Ω' domain and transformation from Ω' domain to ω domain.

Suppose the multi-band filter to be designed in the ω domain has \( m \) passbands \( (\omega_{Li}, \omega_{Ui}) \) \((i = 1, 2, \ldots, m)\) and predefined transmission zero \( \omega_{TZ} \). Each passband has its assigned return loss \( RL_i \) and order \( N_i \). So the total order of this multi-band filter is \( N = \sum_{i=1}^{m} N_i \). Through the transformation from \( \omega \) domain to \( \Omega' \) domain, the \( i \)th passband \( (P_{Li}, P_{Ui}) \) in the \( \Omega' \) domain can be obtained. In order to use different lowpass prototypes, \( \Omega' \) domain is divided into \( m \) segments \( (Z_{i-1,j}, Z_{i,j+1}) \) with segment...
boundary $Z_{i-1,j}$ located between $P_{Li}$ and $P_{Li}$ satisfying $(P_{Li}, P_{Uj}) \subset (Z_{i-1,j}, Z_{i+1,j})$. More exact $Z_{i-1,j}$ can be estimated according to the return loss $RL_i$ and is explained in Section 4.2. The frequency response in the $i$th segment $(Z_{i-1,j}, Z_{i+1,j})$ is obtained from the $i$th lowpass prototype by the transformation from $\Omega$ domain to $\Omega'$ domain. These segments are combined together to get the response in the whole frequency range. With the frequency response in the $\Omega'$ domain, the coupling matrix can be synthesized.

In the following, we will introduce the two transformation equations, the combination of segments, and the synthesis process.

### 2.2 | Transformation equation

#### 2.2.1 | From $\Omega'$ domain to $\Omega$ domain

The frequency transformation from $\Omega'$ domain to $\Omega$ domain for the $i$th passband can be expressed as follows:

$$S = \frac{S'}{a_i} + \frac{b_i}{S'-jZ_{i-1,j}} + \frac{b_i}{S'-jZ_{i+1,i}} (Z_{i-1,j} \leq S' \leq Z_{i+1,i})$$  \hspace{1cm} (1)

where $S = \Omega$ and $S' = \Omega'$. $a_i$ and $b_i$ are unknown variables. $-\infty$, $-1$, and $+\infty$ in the $\Omega$ domain are transformed into $Z_{i-1,j}$, $P_{Li}$, $P_{Uj}$ and $Z_{i+1,i}$ in the $\Omega'$ domain, respectively, as shown in Figure 2. Thus resulting in:

$$-j = \frac{jP_{Li}}{a_i} + \frac{b_i}{jP_{Li} - jZ_{i-1,j}} + \frac{b_i}{jP_{Li} - jZ_{i+1,i}}$$  \hspace{1cm} (2)

Solving Equation (2), $a_i$ and $b_i$ can be derived as:

### Figure 3

Frequency transformation from $\Omega'$ domain to $\omega$ domain

$$b_i = \frac{P_{Li}}{P_{Uj} - Z_{i-1,j}} + \frac{P_{Li}}{P_{Li} - Z_{i+1,i}} + \frac{P_{Uj}}{P_{Li} - Z_{i-1,j} - P_{Li} - Z_{i+1,i}}$$

$$a_i = \frac{1}{P_{Li} - Z_{i-1,j}} + \frac{1}{P_{Li} - Z_{i+1,i}}$$  \hspace{1cm} (3)

If $P_{Uj} = -P_{Li}$ and $Z_{i+1,i} = -Z_{i-1,j}$, the two equations in Equation (2) are the same. This means there is one equation and two unknown variables. For the $b_i$ in Equation (3), both the numerator and the denominator are equal to 0. In this case, we can set $b_i = 1$ and then obtain

$$b_i = 1$$

$$a_i = \frac{P_{Li}}{1 + b_i(P_{Li} - Z_{i-1,j} + P_{Li} - Z_{i+1,i})}$$  \hspace{1cm} (4)

Substituting Equations (3) or (4) into Equation (1), we can get the transformation equation from $\Omega$ domain to $\Omega'$ domain for the $i$th passband.

If there are only two passbands, then $Z_{01} = -\infty$ and $Z_{23} = +\infty$. The frequency transformation Equation (1) from $\Omega'$ domain to $\Omega$ domain can be simplified as

$$S = \frac{S'}{a_i} + \frac{b_i}{S'-jZ_{i-1,i}} (S' \leq Z_{i2})$$

$$S = \frac{S'}{a_2} + \frac{b_2}{S'-jZ_{i2}} (Z_{i2} \leq S')$$

It is equal to Equation (1) in Ref. [20]. The synthesis method reported in Ref. [20] is a specific case of this work.
It should be noted that different passbands have the same transformation expression (Equation (1)). But their segment boundary \((Z_{i-1,i}, Z_{i,i+1})\) and passband \((P_{L_i}, P_{U_i})\) are different from each other. Therefore, the \(a_i\) and \(b_i\) in Equation (1) for different passbands may have different values.

2.2.2 From \(\Omega'\) domain to \(\omega\) domain

The frequency transformation from \(\Omega'\) domain to \(\omega\) domain for multi-band filters similar to the traditional lowpass to bandpass transformation can be expressed as [3]:

\[
S' = \frac{s}{d_1} + \frac{d_2}{s}
\]  
(5)

where \(s = j\omega\). \(d_1\) and \(d_2\) are unknown variables that can be determined when applying the transformation from \(P_{L_i}\) to \(\omega_{L1}\) and \(P_{U_m}\) to \(\omega_{U_m}\), as shown in Figure 3. These transformations can be written as

\[
jP_{L_i} = \frac{j\omega_{L1}}{d_1} + \frac{d_2}{j\omega_{L1}}
\]

\[
jP_{U_m} = \frac{j\omega_{U_m}}{d_1} + \frac{d_2}{j\omega_{U_m}}
\]  
(6)

then we have:

\[
d_1 = \frac{\omega_{U_m}^2 - \omega_{L1}^2}{P_{L1}\omega_{U_m} - P_{L1}\omega_{L1}}
\]

\[
d_2 = \frac{\omega_{L1}\omega_{U_m}(P_{L1}\omega_{L1} - P_{L1}\omega_{U_m})}{\omega_{U_m}^2 - \omega_{L1}^2}
\]  
(7)

2.3 Passbands combination

In order to realize multi-band filters with prescribed transmission zeros \(\omega_{TZ_i}\) in the \(\omega\) domain, we need to find the transmission zeros \(\omega_{TZ_i}\) and reflection zeros \(\omega_{RZ_i}\) of the \(i\)th lowpass prototype in the \(\Omega\) domain as well as the transmission zeros \(\omega_{TZ_i}\) and reflection zeros \(\omega_{RZ_i}\) of the \(i\)th segment in the \(\omega'\) domain according to \(\omega_{TZ}\). Then combine the \(m\) segments in the \(\Omega'\) domain together to find the multi-band filter transfer function. The process is as follows:

1. \(\omega_{TZ}\) is firstly transformed into \(\Omega_{TZ}\) in the \(\Omega'\) domain based on Equation (5).
2. The elements of \(\Omega_{TZ}\) located in the \(i\)th segment \((Z_{i-1,i}, Z_{i,i+1})\) are classified as \(\Omega_{TZ_i}\).
3. Then \(\Omega_{TZ_i}\) is transformed into \(\Omega_{TZ_i}\) according to Equation (1).
4. With \(\Omega_{TZ_i}\), \(\omega_{RZ_i}\) can be solved according to Ref. [25].
5. Using Equation (1), we can transform \(\omega_{RZ_i}\) to \(\Omega'\) domain to get \(\Omega'_{RZ_i}\).

![FIGURE 4 Frequency response of the quad-band filter in the \(\Omega'\) domain. (a) before tuning \(TP\). (b) after tuning \(TP\).](image)

6. Combining all the \(\Omega_{TZ}(i = 1, 2, \ldots, m)\) and \(\Omega_{RZ}(i = 1, 2, \ldots, m)\) together and removing the infinite transmission zeros, the transmission and reflection zeros of the multi-band filter can be given as follows:

\[
\begin{align*}
\Omega_{TZ'} &= \Omega_{TZ_1}, \Omega_{TZ_2}, \ldots, \Omega_{TZ_m} \\
\Omega_{RZ'} &= \Omega_{RZ_1}, \Omega_{RZ_2}, \ldots, \Omega_{RZ_m}
\end{align*}
\]  
(8)

It should be noted that if the \(i\)th lowpass prototype has transmission zero at \(-\infty/\infty\), according to (1), \(Z_{i-1,i}/Z_{i,i+1}\) is also the member of \(\Omega_{TZ}\), \(\Omega_{TZ'}\) consists of two parts

\[
\Omega_{TZ'} = \Omega_{TZ}, \Omega_{TZ_{reg}}
\]  
(9)

One part is \(\Omega_{TZ}\), the assigned transmission zeros. The other part is \(\Omega_{TZ_{reg}}\), the transmission zeros introduced by the infinite transmission zeros of the lowpass prototypes. \(\Omega_{TZ_{reg}}\) consists of some or all elements of \(Z_{i-1,i}/Z_{i,i+1}\). The characteristic function of a multi-band filter can be written as a rational function

\[
C_N(S') = \prod_{j=1}^{N} \frac{(S' - \Omega_{RZ}'(j))}{\prod_{k=1}^{N} (S' - \Omega_{TZ}(k))},
\]  
(10)

where \(N_z\) is the number of finite transmission zeros. This characteristic function determines the frequency response of the filter. The transfer function can be written as

\[
T^2(S') = \frac{1}{1 + e^2 C_N(S')}
\]  
(11)
where \( \epsilon \) is defined as the first passband ripple constant of the multi-band filter and is related to the passband return loss \( RL_i \)

\[
\epsilon = \frac{1}{C_N(-j)\sqrt{10^{RL_i} - 1}} \quad (12)
\]

Combining Equations (8)–(12), the transfer function of multi-band filter with \( RL_i \) fixed is obtained.

### 2.4 Synthesis process

Figure 4 shows the flowchart of the proposed multi-band filter synthesis method. Two synthesis examples are given in Sections 3 and 5 to describe the method.

### 3 QUAD-BAND FILTER SYNTHESIS EXAMPLE

The proposed method can synthesize generalized multi-band filters. Therefore, both symmetric and asymmetric multi-band filters can be synthesized. In this section, synthesis of an asymmetric quad band filter is described as an example. In Section 5, synthesis and design of a symmetric triple-band filter is described.

A quad-band filter \( (m = 4) \) can have at most four lowpass prototypes. Suppose the four passbands in the \( \omega \) domain are specified as in Table 1. The \( \omega_L_i \) and \( \omega_U_i \) for the four passbands are optionally selected.

After transforming \( \omega_L_i \) and \( \omega_U_i \) into \( P_{L_i} = -1 \) and \( P_{U_i} = 1 \), respectively, we can get \( d_1 = 3.7 \times 10^9 \), \( d_2 = 8.5189 \times 10^9 \) in Equation (5). The segment boundaries finally used are \( Z_{i2} = -0.6500, Z_{23} = 0.2400, Z_{34} = 0.8400 \).

Table 2 lists the passbands in the \( \Omega' \) domain and unknown variables in Equation (1). Tables 3 and 4 list the transmission and reflection zeros obtained in the synthesis, respectively.

Combining \( TZ' \) \((i = 1, 2, 3, 4)\) together and removing infinite transmission zeros, we get:

\[
TZ' = j[-1.1521, -0.6500, -0.6500, -0.5360, 0.1481, 0.2400, 0.2400, 0.3170, 0.7673, 0.8400, 0.8400, 1.1845]
\]

Combining together, we get:

\[
RZ' = j[-0.9823, -0.8600, -0.7715, -0.4668, -0.3935, -0.1284, 0.0131, 0.3461, 0.3780, 0.4791, 0.5867, 0.9054, 0.9390, 0.9907]
\]

With these transmission and reflection zeros, the filter frequency response is obtained and shown in Figure 4a. We can see that the return losses in the first and second passband are not flat. This unevenness will lead to a large variation in group delay. Therefore, return losses are further flattened by adjusting three elements of \( RZ' \) using a trial-and-error method. The \( RZ' \) after tuning are:

\[
RZ' = j[-0.9823, -0.8600, -0.7715, -0.4668, -0.3610, -0.1180, 0.0131, 0.3461, 0.3780, 0.4791, 0.5867, 0.9054, 0.9390, 0.9907]
\]

With this new \( RZ' \), the coupling matrix can be obtained according to Ref. [26] and is listed in Table 5. This coupling matrix can be transformed by matrix similarity transformation [25] in order to design different filter topologies. The filter frequency response is shown in Figure 4b. It can be seen that the return loss is almost flat.

When \( Z_{i2} \) and \( Z_{34} \) are fixed, the filter return loss will change with \( Z_{23} \). We can see from Figure 5 that, as \( Z_{23} \) moves from left to right, \( RL_i \), on the right side of \( Z_{23} \), becomes smaller. Whereas \( RL_i \) on the left side of \( Z_{23} \), except \( RL_1 \), becomes larger. This phenomenon is described in Section 4.1. The derivation of segment boundaries \( Z_{i1}, Z_{23}, \) and \( Z_{34} \) from the given \( RL_i \) is explained in Section 4.2. The following is a simple segment boundaries deriving process.

To get the segment boundaries, first, choose an initial value \( Z_{i2} = -0.62, Z_{23} = 0.20, Z_{34} = 0.78 \). Then \( RZ' \) is derived as:

\[
RZ' = j[-0.9823, -0.8690, -0.7722, -0.4673, -0.3700, -0.1200, 0.0133, 0.3462, 0.3794, 0.4824, 0.5873, 0.9057, 0.9911]
\]

According to \( \Omega'_{TZ}, RL_i \), and (20), \( TZ'_{seg} = [-0.6090, -0.6090, 0.1907, 0.1907, 0.8158, 0.8158] \) and \( TZ'_{seg,U} = [-0.6489, -0.6489, 0.2676, 0.2676, 0.8397, 0.8397] \) can be obtained. So \( TZ'_{seg} = [-0.6290, -0.6290, 0.2292, 0.2292, 0.8278, 0.8278] \). But the filter return loss still has a slight deviation. With some adjustment, the final \( TZ'_{seg} \) is \([-0.6500, -0.6500, 0.2400, 0.2400, 0.8400, 0.8400] \) and \( Z_{i2} = -0.6500, Z_{23} = 0.2400, Z_{34} = 0.8400 \).

### 4 FURTHER THEORETICAL EXPLANATION

#### 4.1 Return loss adjustment

In the following, we will qualitatively analyse why \( RL_i \) will change with the segment boundaries.
TABLE 3 Transmission zeros in the \( \Omega' \) domain and \( \Omega \) domain

| \( i \) | \( TZ'_i \) | \( TZ_i \) |
|---|---|---|
| 1 | \([-j[0000, 1.1521, 0.6500] \) | \([-j1.4915, \pm \infty] \) |
| 2 | \([-j0.6500, -0.5360, 0.1481, 0.2400] \) | \([-j1.9894, 2.9840, \pm \infty] \) |
| 3 | \([-j0.2400, 0.1370, 0.7673, 0.8400] \) | \([-j1.6599, 3.1389, \pm \infty] \) |
| 4 | \([-j0.8400, 1.1845, \infty] \) | \([-j1.9920, \pm \infty] \) |

Table 4 Reflection zeros in the \( \omega \) domain and \( \Omega \) domain

| \( i \) | \( RZ_i \) | \( RZ'_i \) |
|---|---|---|
| 1 | \([-j0.9293, -0.2581, 0.8025] \) | \([-j0.9823, 0.8600, 0.7715] \) |
| 2 | \([-j0.9373, -0.4397, 0.3568, 0.9231] \) | \([-j0.4668, -0.3935, -0.1284, 0.0131] \) |
| 3 | \([-j0.9441, -0.4750, 0.3285, 0.9190] \) | \([-j0.3461, 0.3780, 0.4791, 0.586] \) |
| 4 | \([-j0.8214, 0.1800, 0.9106] \) | \([-j0.9054, 0.9390, 0.9907] \) |

Figure 6 and Figure 2 in Ref. [20] show that, as the segment boundary \( Z_{i-1,j} \) located between \( P_{Li-1} \) and \( P_{Li} \) moves to the right, all transmission zeros remain unchanged except for the one introduced by \( Z_{i-1,j} \) which moves to the right. Compared with this transmission zero, the change of \( RZ' \) is smaller. We neglect the change of \( RZ' \) and analyse the change of return loss approximately.

According to Equations (10) and (11), we can get:

$$
\begin{align*}
\varepsilon^2 C_N(S') & = \frac{C_N(S')}{(\varepsilon |C_N(j)| \sqrt{10^{0.1RL_i} - 1})^2} \\
& = \left( \frac{N}{j} (S' - RZ'(j)) \right)^2 \left( \frac{N}{j} (j - RZ'(j)) \right)^2 \left( 10^{0.1RL_i} - 1 \right)
\end{align*}
$$

(13)

Neglecting the change of \( RZ' \) we have:

$$
\varepsilon^2 C_N(S') = \left( \frac{N}{j} (j - RZ'(k)) \right)^2 C_i
$$

(14)

where \( C_i \) is a constant.

Suppose \( Z_{i-1,j} \) moves from A to B in Figure 6. The filter transmission zero also moves from A to B in Figure 6b. When \( S' \) is in the passband right to \( Z_{i-1,j} \), as shown in Figure 6a. We have \( |j - B| > |j - A| \) and \( |S' - B| < |S' - A| \), therefore:

$$
\frac{|j - B|}{|S' - B|} > \frac{|j - A|}{|S' - A|}
$$

(15)

Substituting Equation (15) into Equation (14), \( \varepsilon^2 C_N(S') \) becomes larger. Thus, \( \varepsilon^2 C_N(S') \) becomes smaller based on Equation (11). Therefore, \( RL_i \) becomes smaller.

When \( S' \) is in the passband left to \( Z_{i-1,j} \) and \( S' \neq -j \), as shown in Figure 7b. Because of \( |j - A| > |S' - A| \), we have:

$$
\frac{|j - B|}{|S' - B|} > \frac{|j - A|}{|S' - A|} + |B - A| < \frac{|j - A|}{|S' - A|}
$$

(16)

Substituting Equation (16) into Equation (14), \( \varepsilon^2 C_N(S') \) becomes smaller. As a result, \( RL_i \) becomes larger.

When \( S' = -j \), substitute it into Equation (14), \( \varepsilon^2 C_N(S') = C \). Hence, \( RL_1 \) does not change.

4.2 Estimation of segment boundary \( Z_{i-1,j} \)

In Section 4.1, how segment boundaries influence return losses is qualitatively explained. In this section, we will derive the segment boundaries from the given return losses.

In this multi-band filter synthesis, no matter what the segment boundaries \( Z_{i-1,j}(i = 1, 2, \ldots, m) \) are, the filter prescribed transmission zeros \( \Omega'_{TZ} \) are invariant. The variation of filter reflection zeros \( RZ' \) is small (this is demonstrated in Section 3) and approximately neglected. Based on this property, we can firstly initialize \( Z_{i-1,j} \). Then obtain the corresponding \( RZ' \). With \( \Omega'_{TZ}, RZ' \) and \( RL_i \), the exact \( Z_{i-1,j} \) can be estimated. The relationship between characteristic function and filter return loss is

$$
\varepsilon |C_N(S')| = \frac{1}{\sqrt{10^{0.1RL(S') - 1}}}
$$

(17)

Substituting Equation (10) and \( TZ' = [\Omega'_{TZ}, \Omega'_{TZ_{seg}}] \) Equation (9) into Equation (17), we get:

$$
\varepsilon \left| \frac{\prod_{j=1}^{N}(S' - RZ'(j))}{\prod_{k=1}^{N}(S' - \Omega'(k))} \right| = \frac{1}{\sqrt{10^{0.1RL(S') - 1}}}
$$

(18)
Move the invariant to the right hand side and we have

\[
\frac{1}{L} \prod_k (S' - T \bar{Z}_e(k)) = \sqrt{1(0^0)RL(S') - 1} \prod_{\mu} (S' - RZ'(j)) \prod_k (S' - \Omega_{TZ}(k))
\]

(19)

When \( S' = jP_L \) and \( S' = jP_U \), the required filter insertion loss should be \( RL_i \), so

\[
\frac{1}{L} \prod_k (jP_L - T \bar{Z}_{segL}(k)) = \sqrt{1(0^0)RL(-1) - 1} \prod_{\mu} (jP_L - RZ'(j)) \prod_k (jP_L - \Omega_{TZ}(k))
\]

\[
\frac{1}{L} \prod_k (jP_U - T \bar{Z}_{segU}(k)) = \sqrt{1(0^0)RL(-1) - 1} \prod_{\mu} (jP_U - RZ'(j)) \prod_k (jP_U - \Omega_{TZ}(k))
\]

(20)

where \( i = 1, 2, \ldots, m \). The first equation in Equation (20) determines \( L_L \) and \( T \bar{Z}_{segL} \). The second equation in Equation (20) determines \( L_U \) and \( T \bar{Z}_{segU} \). \( T \bar{Z}_{seg} \) can be chosen as the average of \( T \bar{Z}_{segL} \) and \( T \bar{Z}_{segU} \)

\[
T \bar{Z}_{seg} = \frac{T \bar{Z}_{segL} + T \bar{Z}_{segU}}{2}
\]

(21)
So $Z_{i-1,j}$ can be determined. If the filter return loss still dissatisfies the $RL_i$ requirement, we can run another iteration with the new $Z_{i-1,j}$ or adjust $Z_{i-1,j}$ slightly.

The transmission zero $TZ'$ consists of two parts, $\Omega^2TZ$ and $TZ'_{seg}$ as expressed in Equation (9). $\Omega^2TZ$ represents the assigned transmission zeros. $TZ'_{seg}$ represents the transmission zeros introduced by the infinite transmission zeros of the lowpass prototypes. It is return loss dependent. In fact, for a filter with $m$ passbands, it is required that at least $m - 1$ pairs of transmission zeros must be dependent if the characteristic function is constrained to a given passband specification [21]. This means that a satisfactory solution could not always be achieved. One of the possible scenarios is that the determined $TZ'_{seg}$ is located in one of the passbands. This may be the correct solution for Equation (21) mathematically, but it is not the desired solution for the filter requirement. This further implies that the assigned transmission zeros $\Omega^2TZ$ can only take certain positions that would force the dependent transmission zeros $TZ'_{seg}$ to lie outside any of the passbands.

4.3 | Realization of equal-ripple

As the multi-band filter is synthesized by combining each single passband, we should consider the influence from other passbands when realizing equal-ripple in one passband. If a filter has no infinite transmission zeros, as shown in Figure 7, the filter characteristic function will quickly reach an almost fixed value when the frequency is larger than the transmission zeros. Suppose there is a dual band filter in-domain with a characteristic function:

$$C_N(S') = \left| \frac{\prod_{j=1}^{N_1} (S - RZ_1(j)) \prod_{j=1}^{N_1} (S - RZ_2(j))}{\prod_{k}^{N_2} (S - TZ_1(k)) \prod_{k}^{N_2} (S - TZ_2(k))} \right|$$

If its first passband in the $\omega'$ domain has no infinite transmission zeros, and its second passband $(PL_2, PU_2)$ is placed at the frequency where the first filter characteristic function is almost constant, then

$$C_N(S') = \text{const} \quad \left| \frac{\prod_{j=1}^{N_1} (S - RZ_1(j))}{\prod_{k}^{N_2} (S - TZ_1(k))} \right|, \quad PL_2 \leq S' \leq PU_2$$ (23)

This means $C_N(S')$ is obtained by multiplying the characteristic function of the second passband by a constant, so the first passband only affects the return loss level of second passband but has no influence on the equal-ripple in the second passband. The validity of Equation (23) may require a certain distance between passbands and no infinite transmission zeros. Sometimes even if these conditions are not met, if the first characteristic function where the second passband is placed varies not too much, the influence on second passband will be very small and can be neglected. So the second passband needs no post processing. However, if the first characteristic function where the second passband is placed changes substantially, the second passband needs to be optimized. In a similar way, the equal-ripple in other passbands can be achieved.

5 | FILTER SYNTHESIS, DESIGN AND MEASUREMENT

The filter to be designed is a symmetric triple-band ($m = 3$) filter with passbands of 1.900–1.920, 1.985–2.015 and 2.080–2.100 GHz in the $\omega$ domain. The assigned transmission zeros are located at 1.8815, 1.9315, 2.0685 and 2.1185 GHz. Each passband is four orders and $RL_1 = RL_2 = 20$ dB, $RL_2 = 30$ dB. It should be noted that if the filter bandwidth is narrow enough, it can be approximated that the passbands’ bandwidth in the $\omega'$ domain are proportionate to the corresponding passbands bandwidth in the $\omega$ domain, which means $(P_{U_i} - P_{L_i-1})/(P_{U_i} - P_{L_i}) = (\omega_{U_{i-1}} - \omega_{L_{i-1}})/(\omega_{U_i} - \omega_{L_i})$. Therefore, $(P_{U_i} - P_{L_i})$ and $\Omega^2TZ$ in the $\omega'$ domain can be obtained directly. The segment boundaries are $Z_{01} = -\infty$, $Z_{12} = -Z_{23}$, $Z_{34} = +\infty$. $Z_{12}$ is estimated according to Equation (20) and its value is $-0.572$. With these boundaries, the variables $a_i$ and $b_i$ in Equation (1) can be derived. It is worth mentioning that when $i = 2$, $P_{L_2} = P_{U_2}$ and $Z_{i-1,j} = -Z_{j+1,i}, a_i$ and $b_i$ are derived from Equation (4) instead of Equation (3).

Table 6 lists the variables in this synthesis process.

With $TZ'_i$ and $RZ'_i$, $TZ'$ and $RZ'$ are obtained:

$$TZ' = j \left[ \pm 1.185, \pm 0.685, \pm 0.572, \pm 0.572, \pm 0.572 \right]$$

$$RZ' = j \left[ \pm 0.991, \pm 0.927, \pm 0.848, \pm 0.805, \pm 0.110, \pm 0.061 \right]$$ (24)

The filter frequency response derived from Equation (24) is shown in Figure 8a.
TABLE 6 Variables in the synthesis process for the triple-band filter

|     | $i = 1$       | $i = 2$       | $i = 3$       |
|-----|---------------|---------------|---------------|
| $P_{L_i} P_{L_{i2}}$ | $(-1.000, -0.800)$ | $(-0.150, 0.150)$ | $(0.800, 1.000)$ |
| $\Omega^T_{TZ}$ | $[\pm 1.185, \pm 0.685]$ | $[\pm 0.572, \pm 0.572]$ | $[+\infty, 1.185, 0.572, 0.685]$ |
| $a_i$ | 0.374 | 9.757 | 0.374 |
| $b_i$ | 0.715 | 1 | 0.715 |
| $TZ_i$ | $[-\infty, -1.185, -0.685, -0.572]$ | $[\pm 0.572, \pm 0.572]$ | $[+\infty, 1.185, 0.572, 0.685]$ |
| $T_i$ | $[-1.998, 5.000, \pm \infty]$ | $[\pm \infty, \pm \infty]$ | $[-5.000, 1.998, \pm \infty]$ |
| $RZ_i$ | $[-0.940, -0.460, 0.327, 0.917]$ | $[\pm 0.924, \pm 0.383]$ | $[-0.917, -0.327, 0.460, 0.940]$ |
| $RZ_i$ | $[-0.991, -0.927, -0.848, -0.805]$ | $[\pm 0.140, \pm 0.061]$ | $[0.991, 0.927, 0.848, 0.805]$ |

FIGURE 8 Frequency response of a filter with no infinite transmission zeros

We can also have another design choice of removing two pairs of the triplicate transmission zeros at $\pm 0.5720$ to reduce the number of cross coupling. After removal, the second band return loss will change. Then $Z_{12}$ and $Z_{23}$ should be re-estimated using Equation (21), thus resulting in $Z_{12} = Z_{23} = -0.428$. That is to say $TZ'$ is:

$$TZ' = j[\pm 1.185, \pm 0.685, \pm 0.428]$$  \hspace{1cm} (25)

Without the triplicate transmission zeros, the filter return loss will be uneven. Using simple optimization, we get

$$RZ' = j[\pm 0.993, \pm 0.943, \pm 0.864, \pm 0.808, \pm 1.39, \pm 0.058]$$  \hspace{1cm} (26)

Figure 8b shows the frequency response of the triple-band filter derived from Equations (25) and (26). We can see that in Figure 9b the lower stopband and upper stopband are much better than those displayed in Figure 9a. This is because for a filter with $N$ orders, there are totally $N$ transmission zeros. Figure 9a shows $N_z = 10$ finite transmission zeros and two infinite transmission zeros. Figure 9a shows $N_z = 6$ finite transmission zeros and six infinite transmission zeros. There are four more infinite transmission zeros in Figure 9b. So Figure 9b shows better lower and upper stopbands. The coupling matrix shown in Figure 9b corresponding to the filter topology in Figure 9a is solved and listed in Table 7.
TABLE 7 Coupling matrix of the triple-band filter in Figure 9b

|   |   |   |   |
|---|---|---|---|
| M(1, 2) = M(11, 12) | 0.679 | M(6, 7) | 0.074 |
| M(2, 3) = M(10, 11) | 0.650 | M(5, 8) | 0.005 |
| M(3, 4) = M(9, 10) | 0.327 | M(4, 9) | 0.212 |
| M(4, 5) = M(8, 9) | 0.583 | M(3, 10) | −0.043 |
| M(5, 6) = M(7, 8) | 0.629 | Rs = RL | 0.455 |

M(i, j) = M(j, i).

All the elements not listed equal to zero.

microstrip open-loop resonators are adopted. Open-loop resonators can provide both electric and magnetic couplings between two resonators, thus, can be used for realizing both positive and negative coupling.

A substrate with a thickness of 0.787 mm, a relative dielectric constant of 2.2 and a loss tangent of 0.0009 is used. The layout of the designed filter is shown in Figure 10b. The size of the resonator (a, w, g) is determined by the central frequency 2 GHz. The tapping position (g01) is determined by the external coupling coefficient (Rs) and the distances between two resonators (g12, ..., g310) are determined by inter-resonator coupling coefficients (M(i, j)). Magnetic coupling is defined as the positive coupling. Thus, the coupling structure between resonator 5 and 8 should be as shown Figure 10a according to M(5, 8) = 0.005. And the coupling structure M(4, 5), M(5, 6) will be the mixed coupling shown in Figure 11b which is well described in Ref. [28] Chapter 8. This coupling is electric coupling when gij is small, whereas it
becomes magnetic coupling when $g_{ij}$ is larger but is too weak to support the strong coupling $M(4, 5)$ and $M(5, 6)$. To solve this problem, we make an approximation and regard $M(5, 8)$ as $-0.000$. Therefore, the final coupling structure is shown in Figure 10b.

The gaps ($g_{12}, ..., g_{110}$) are directly decided from the curve of the coupling coefficient varying with the gap between two resonators. This curve is obtained by simulation as in Ref. [24]. No tuning is applied to the gaps or any other size parameters to get coupling coefficients with more accuracy. The discrepancies between the actual and desired coupling coefficients will cause performance degradation.

Figure 11 is the simulated S-parameters based on ADS momentum and the measured S-parameters. Compared with the filter synthesis frequency response shown in Figure 8b, the simulated insertion loss is larger and the return loss is smaller especially in the first and third band. There is a right shoulder collapse in the third band. The main reason for this performance degradation is the approximation of the coupling coefficient $M(5, 8)$. As shown in Figure 12, after changing $M(5, 8)$ from 0.005 to $-0.000$, the return losses of the coupling matrix in the first and third band deteriorate. Therefore, the simulated and measured S-parameters designed according to this coupling matrix will also deteriorate. Other potential sources of errors are frequency-dependent coupling coefficients, discrepancies between the actual and desired coupling coefficients, unwanted couplings and the simulation error of $R_S$.

The measured S-parameters have larger insertion loss than the simulated S-parameters. This may be because of the loss of SMA, fabrication error, simulation inaccuracy, measured inaccuracy and the simulation and measurement environment mismatch. The asymmetry of the simulated S-parameters can be ascribed to the approximation when calculating $(P_{L1}, P_{U1})$.

The proposed method in this article mainly focuses on multi-band filter synthesis. Although the measured results are not very good, the synthesized filter frequency response and the coupling matrix meet the filter performance requirement. If the synthesized coupling matrix can be exactly realized, the designed filter can achieve the desired performances.

Table 8 shows the comparison of the proposed synthesis method based on multi-prototype transformation with other synthesis methods. References [16–20] report frequency transformation-based synthesis methods. Reference [21] describes an optimization-based method. Compared with the other methods, our method can synthesize an arbitrary number of bands with different orders, return losses, and bandwidths in different passbands. The assigned transmission zeros can also be realized. For generalized multi-band filter synthesis, especially when the required return loss is different in each passband, the proposed method provides an alternative choice.

### 6 CONCLUSION

This article proposes a novel multi-band filter synthesis method. Unlike conventional transformation techniques which transform a lowpass prototype into a multi-band filter, the proposed method transforms multiple lowpass prototypes into a filter, called multi-prototype transformation. This method can synthesize multi-band filters with different orders, return losses and bandwidths in different passbands. Filter prescribed transmission zeros can also be realized. An asymmetric quad-band filter with different bandwidths and orders in different passbands is synthesized to validate the proposed synthesis method. The return loss adjustment, segment boundary estimation and equal-ripple realization of this method are explained. Finally, a symmetric triple-band microstrip filter is synthesized, designed and fabricated. This method can be used to design generalized multi-band filters with assigned transmission zeros and assigned return losses in each passband.

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