Quantitative Risk Analysis of Tunnel Instability in Layered Rock Mass Considering the Spatial Variability of Elastic Modulus

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Abstract: Layered rock mass is a typical excavation disturbance sensitive rock mass, which is prone to the large deformation and collapse of soft rock mass under tunnel unloading. This in turn causes the construction safety risk and delays the project construction. As the spatial variability of rock mechanics parameters existed widely, which could make the deterministic calculation results cannot really reflect the safety reserve of the rock mass by supports. Therefore, considering the spatial variability of elastic modulus of layered rock mass for study the risk of tunnel instability both has the theoretical and practical significance. According to the transverse anisotropy characteristics of layered rock mass in Baokang tunnel, China, elastic wave velocities were determined by the acoustic test method at different broken surrounding rock mass. Meanwhile, the elastic modulus was converted into the elastic modulus based on theoretical formula, and a database of the elastic modulus was set up. Using the variograms model of geostatistics, the autocorrelation distances of elastic modulus along the strike and dip of rock strata were determined, respectively. Furthermore, a two-dimensional parameter random field which can characterize the spatial variability of elastic modulus of layered rock mass was constructed by the Karhunen-Loève (K-L) expansion method. Taking the standard value of tunnel allowable deformation as the evaluation index, the limit state equation of tunnel allowable deformation in the soft rock was established. Finally, considering the spatial variability of elastic modulus of layered rock mass, a quantitative risk analysis of surrounding rock instability was realized by the Monte Carlo method and numerical simulation method. The engineering example shows that, this quantitative risk analysis method can effectively evaluate the safety state of the surrounding rock mass, and it can provide a theoretical guidance and technical support for the tunnel construction. This work could provide a reference for other layered rock mass tunnel construction.

Key words: Large deformation of soft rock; Elastic modulus; Random field; Monte-Carlo method; Risk analysis

1. Introduction
Spatial variability is an inherent characteristic of geotechnical materials, which has an important
impact on the reliability of the calculation results of geotechnical problems. It is an inevitable trend to adopt a probability limit state design method based on reliability theory to replace the traditional fixed value design method for the theoretical development of geotechnical engineering design[1]. The quantitative risk analysis method considering the spatial variability of strength parameters is mainly applied to slope engineering[2][3][4][5]. The risk analysis method based on random field theory for the rock instability is also gradually applied in amount of underground engineering[6][7][8][9]. At present, the spatial variability of elastic modulus on the reliability of tunnel structure has gradually become a research hotspot [1][9][10]. However, the quantitative risk analysis of large deformation considering the spatial variability of elastic modulus has not been reported for layered soft rock tunnels.

For layered rock mass, its easy bending and slippage characteristics frequently led to large deformation and collapse of the surrounding rock mass, which seriously restricted the successful construction of the tunnel project[11][12][13]. Researches show that the parameters as elastic modulus of layered rock mass have significant variability along the strike and dip directions according to the transversely isotropic characteristic[14]. To simulate the spatial variability of elastic modulus of layered rock mass, this paper would construct a elastic modulus database and construct a random field by K-L series expansion method. Finally, a quantitative risk analysis will be achieved by combining Monte Carlo method and numerical simulation method.

2. Method of quantitative risk analysis for tunnel instability of layered rock mass

2.1. Database of the elastic modulus

The construction of a random field is based on a statistical analysis of a large number of parameters. However, for elastic modulus, it is difficult to carry out a large number of field tests to build a database. Fortunately, the elastic modulus can be switched from the longitudinal wave velocity which is easy to be tested by acoustic test method in a tunnel[15][16][17]. At present, the most widely used conversion formula is equation (1). This formula is determined by a total of 589 sets of data from more than 20 large and medium hydropower stations in China[16].

\[ E_2 = 0.0894 \exp (1.0445 V_s) \]  

According the coordinates and values of elastic modulus, a database could be built. Based on this, the mean and standard deviation of elastic modulus can be determined by statistics, and autocorrelation function which is along the dip and strike direction respectively can be determined by geostatistics[18].

2.2. Random field simulation of elastic modulus of layered rock mass

The random field theory is to represent the point and spatial characteristics of the parameter randomness through an autocorrelation functio[19]. In order to reflect the random field into the numerical calculation, the random field must be discretized. The existing random field discretization methods are mainly: local average method, covariance matrix decomposition method, K-L expansion method[20][21], etc. Compared with other methods, the K-L expansion method has the advantages of not relying on the grid shape of the discrete region and having higher computational accuracy and efficiency[22][23], so this method is selected to constructed a random field in this paper. K-L expansion provides a second-moment characterization of a random process in terms of deterministic orthogonal functions as follows[24][25]:

\[ H(x_1, x_2; \theta) = \mu + \sum_{i=1}^{n} \sigma \sqrt{\lambda_i} f_i (x_1, x_2) \xi_i (\theta) \]  

Where: \( \mu \) and \( \sigma \) are the mean and standard deviation of characteristic parameters of the normal random field, respectively; \( x_1, x_2 \) are the coordinates of any point in the random region; \( \theta \) are the external spatial coordinates; \( \lambda_i \) and \( f(\cdot) \) are the eigenvalues and eigenfunctions of the
autocorrelation function respectively; $\zeta(\theta)$ is the independent standard normal random variable.

For a separable two-dimensional autocorrelation function, the eigenvalues and eigenfunctions can be simplified as the product of each eigenvalue and eigenfunction of its one-dimensional autocorrelation function[26]. The elastic modulus determined along the strike and dip directions are as follows:

$$E(x_i,x_j;\theta)=\mu+\sum_{j=1}^{n}\lambda_j f_j^i(x_i,x_j)\sqrt{\lambda_j} f_j^j(x_i,x_j)\zeta(\theta)$$

Where: $\lambda_i$ and $f_i^j(\cdot)$ are eigenvalues and eigenfunctions along the dip direction, respectively; $\lambda_i'$ and $f_i'(\cdot)$ are eigenvalues and eigenfunctions along the dip direction, respectively.

2.3. Calculation of Tunnel failure probability by Monte Carlo Method

The Grid elements in the FLAC^3D grid model can be regrouped according to the order of generation of random field element, and the random elastic modulus could be assigned to their corresponding grids using element traversal method. In order to calculate the failure probability of tunnel instability, the limit state equation should be determined. According to the Uniform Standard for Structural Stability Design of Railway Engineering[27] and the Technical Regulations for Monitoring and Measuring of Railway Tunnels[28], the functional function of the allowable deformation of the initial support of the tunnel is established as follows:

$$g(x)=X_m-x$$

where: $X_m$ is the allowable deformation value; $x$ is the horizontal relative displacement calculated from numerical calculation with random elastic modulus.

Furthermore, the failure probability can be calculated by Monte Carlo method, which is the simplest, the highest recognition and the most widely used reliability method. The failure probability can be calculated as:

$$P_t=\frac{N_f}{N}\times100\%$$

Where: $N$ is total calculation number; $N_f$ is the number of situations that $g(x)$ is smaller than 0.

3. Engineering application

3.1. Engineering background

This paper takes Baokang Tunnel of Zhengwan High Speed Railway as the research background. The tunnel is located in Baokang County, Xiangyang City, Hubei Province, China, with a total length of 14574.2784m and a maximum buried depth of about 504m. When completed, it will become an important link between central China and western China. The tunnel for the passenger special line tunnel, the design speed of 350Km/h. The net width of the designed tunnel is 14.02m and the net height is 12.36m. The exit zone of Baokang Tunnel passes through silurian system shale and sandstone. The surrounding rock mass of Baokang tunnel exit is gray argillaceous shale. It has thin layers and soft texture (figure 1). The tendency of the rock layer ranges from SE 140° to E 180°, and the dip angle varies from 0° to 60°, with a maximum burial depth of 300 m.

The tunnel exit is excavated by the three-step seven-procedure excavation method, and the main initial support is anchors (the length is 4 m and spacing is 1 m × 1.2 m), shotcrete (the thickness is 28 cm and the strength is C25) and steel arch (the specification is I20b and the spacing is 1 m). During the construction process, the maximum displacement which is near 300 mm is located on the left arch. The big deformation led to the cracking of the initial support and the twisting of the steel arch frame (figure 2), which is seriously threatening the construction safety. In order to quantitatively evaluate the instability risk of the tunnel under this excavation and support, it is urgent to calculate the failure probability.
3.2. Spatial variability of elastic modulus
To establish a database of elastic modulus, several acoustic tests were performed after tunnel excavation on 10 holes of 10 sections in Baokang tunnel. The sampling was performed 0.2 m for every tested hole. The result shows that the loosening depth of the left surrounding rock mass is about 6 m (Figure 3). Since the elastic modulus in the loose zone is directly related to the deformation of surrounding rock mass, the longitudinal wave velocities of this part are converted into elastic modulus according to equation (1) to build the database. The probability density function (PDF) curve was fitting as shown in Figure 4, and the optimal PDF of the elastic modulus is lognormal distribution. The statistical results show that the mean and standard deviation of elastic modulus are 0.6004 GPa and 0.0554 GPa, respectively. The probability density function of elastic modulus is:

\[
f(x) = \frac{1}{x \times 0.092 \times \sqrt{2\pi}} e^{-\frac{(\ln x + 0.5144)^2}{2 \times 0.092^2}}
\]  

The autocorrelation function of elastic modulus along strike and dip is:

\[
\gamma_{E-x} = 1.1 \times [1 - \exp(-h^2 / 2.5)]
\]  

\[
\gamma_{E-y} = 1.03 \times [1 - \exp(-h^2 / 1.5)]
\]  

3.3. Random field building and coupling with grid model
According to the statistical analysis, the PDF of elastic modulus obeys a logarithmic Gaussian distribution with mean of 0.6004 GPa and SD of 0.0554 GPa. Taking the natural logarithm, the corresponding mean and SD are 0.5978 GPa and 0.0553 GPa, respectively. Since the autocorrelation functions are Gaussian functions, the eigenvalue and eigenfunction along the strike and dip direction can be solved using the wavelet-Galerkin technique[25], respectively. Meanwhile, \( \hat{\xi}(\hat{\theta}) \) can be determined by Latin hypercube sampling[29]. And the truncation number n is calculated by the expectation energy ratio factor method[30], the value of 25 was selected in this paper. The random field of elastic modulus can be constructed by substituting the above parameters into the equation (3). Figure 5 shows the 1st and 100th random distributions of elastic modulus. It can be seen that the
distributions of the generated random field are continuous, smooth, and stable.

![Figure 5](image)

**Figure 5.** Random distributions of elastic modulus at the 1st and 100th calculation time: (a) first calculation; (b) hundredth calculation.

According to the tunnel excavation sequence, a three-dimensional grid model (figure 6(a)) was constructed, the excavation sequence is shown in figure 6(b), in which the loose zone was set as a random parameter region (figure 6(c)). Material parameters are shown in Table 1. For simulating the layered rock mass in FLAC3D software, the Bilinear Strain-Hardening/Softening Ubiquitous-Joint Model was used[31], which considers deformation with transverse isotropy. In the simulation, the supporting forms included grouting rock bolt and steel arch, which were simulated by cable and shell construction unit, respectively.

![Figure 6](image)

**Figure 6.** Mesh model for numerical simulation: (a) three-dimensional grid model; (b) excavation sequence; (c) random parameter region.

| $E^a$/Gpa | $c_0^b$/MPa | $c_j^c$/MPa | $\phi_0^d$/° | $\phi_j^e$/° | $\psi^f$ | $\psi_j^g$ | $\sigma^h$/MPa | $\sigma_j^h$/MPa | $\nu^i$ |
|-----------|-------------|-------------|---------------|---------------|---------|---------|-------------|-------------|--------|
| original  | 20          | 5           | 4             | 33            | 30      | 12      | 11          | 5           | 2      |
| degrade   | 3.5         | 2           | 31            | 28            | 11      | 10      | 2           | 0.5         | 0.3    |

* $E$ is elastic modulus.
* $c_0$ and $\phi_0$ is cohesion and internal friction angle of the rock matrix, respectively.
* $c_j$ and $\phi_j$ is the cohesion and internal friction angle of the bedded plane, respectively.
* $\psi$ and $\psi_j$ are dilatancy angles of the rock matrix and the bedded plane, respectively.
* $\sigma$ and $\sigma_j$ are the tensile strength of the rock matrix and the bedded plane, respectively.
* $\nu$ is Poisson ratio.

For simulating the random parameter in the FLAC3D software, the finite difference grids are regrouped by the random parameter sequence. The random parameter sequence is determined by the eigenfunctions of the autocorrelation function along the strike and dip directions, respectively. Furthermore, each random parameter of one simulation can be assigned to the corresponding finite difference grids using an element traversal method in FLAC3D software. Finally, the spatial variability and randomness of elastic modulus can be reflected in the numerical simulations[14]. Figure 7 is the 1st parameter assignments of elastic modulus which is compared with the corresponding random parameter distribution. It can be seen that the assigned elastic modulus is all located in the random field, and the distribution and values are both consistent. The random field and grid model have an
ideal coupling effect.

3.4. Failure probability calculated by Monte Carlo Method

According to the suggestion of literature[32], the simulation number is selected 200 in this paper. For determining the failure probability by Monte Carlo method, the limit state equation (4) should be calculated. For example, the convergence displacement between left and right arch waist is about 240 mm (figure 8), which is bigger than the allowable convergence value 234 mm. Therefore, the limit state equation result of the first random parameter simulation is -6 mm that is negative. Because the side wall span is large and the allowable convergence value is also large (302 mm), the measuring point is set at the waist of the arch. Similarly, the total 200 results can be calculated and counted as shown in figure 9 and figure 10. Finally, the failure probability of the arch waist reaches 27.5%. Actually, the results of 31 displacement monitoring sections under this excavation and support form show that there are 9 times of the convergence displacement which is bigger than 234 mm. Therefore, the actual failure probability is 29%. That means this quantitative risk analysis method can reflect the deformation failure probability of Baokang tunnel, and be used to optimize the support parameters to reduce the risk of large deformation and instability.

In order to significantly reduce the large deformation and instability risk of the tunnel, a strengthening support measure was put forward. The main strengthening supports include anchors (spacing is 1 m × 1 m), shotcrete (the thickness is 29 cm), steel arch (the specification is I22a and the spacing is 0.6 m), and advance small conduit grouting on the left arch waist. According to the calculation results (figure 11) of large deformation and instability probability of the tunnel after reinforced support, the damage probability is reduced to 0 (figure 12). The actual displacement monitoring results of reinforced support tunnel show that the average convergence displacement is 200 mm and the maximum convergence displacement is 220 mm (figure 13), which is less than the allowable convergence value 234 mm. At the same time, the feedback from the construction site shows
that the surrounding rock mass of the tunnel did not have large deformation and failure disasters under the strengthened support.

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig11.png}
\caption{Probability density function of displacement value of arch waist after strengthening support.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig12.png}
\caption{Accumulative distribution function of arch waist displacement value after strengthening support.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.45\textwidth]{fig13.png}
\caption{Displacement monitoring after strengthening support}
\end{figure}

4. Conclusion

In this paper, a database of elastic modulus of the layered rock mass can be built by switching from the longitudinal wave velocities. Meanwhile, a random field can be constructed by K-L expansion method to represent the spatial variability of the elastic modulus. Finally, by Monte Carlo method and the numerical calculation method, the quantitative risk of the tunnel instabilities can be analyzed for initial support and strengthening support, respectively, and several conclusions are drawn from this study:

1) The spatial variability of the elastic modulus was studied by geostatistics method. The results show that the optimal PDF of the elastic modulus of the layered rock mass of Baokang tunnel is log-normal distribution, the autocorrelation function is a Gaussian function, and the autocorrelation distance along the strike and dip direction is 2.5 m and 1.5 m, respectively.

2) The random field based on K-L expansion method can simultaneously characterize the spatial variability of elastic modulus along the strike and dip of rock strata. Meanwhile, the example showed that the random field can be coupled with the mesh model well.

3) On the basis of the random field, the failure probability calculated by Monte Carlo method can effectively represent the tunnel instability risk under different supporting conditions. The quantitative risk analysis method in the paper can provide technical support for the optimization of excavation and support of similar layered rock mass tunnel engineering.

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Reference

[1] Li J and Chen J 2020 Comprehensive reliability analysis of deformation induced by shield tunneling considering spatial variability of soil parameters, Journal of Yangtze River Scientific Research Institute, 37(06), 127-33
[2] Jiang S, Yang J, Yao C and Huang J 2018. Quantitative risk assessment of slope failure considering spatial variability of soil properties, Engineering, 35(01), 136-47

[3] Liu X, Wang Y and Li D 2019 Slope failure modes at large deformation in spatially variable soils, Journal of Engineering Geology, 27(05), 1078-84

[4] Zheng D, Li D, Cao J and Fang G et al 2017 Effect of spatial variability on correlation between slope failure modes and system reliability of slope stability, Rock and Soil Mechanics, 38(02), 517-524+533

[5] Zha B, Pei H and Yang Q 2019 Gaussian process regression-based response surface method and reliability analysis of slopes, Chinese Journal of Geotechnical Engineering, 41(S1), 209-212

[6] Pan Q and D Dias 2017 Probabilistic evaluation of tunnel face stability in spatially random soils using sparse polynomial chaos expansion with global sensitivity analysis, Acta Geotechnica, 12(6), 1415-29

[7] Li D 2020. Determining deformation control index for xiamen metro tunnel, Journal of Yangtze River Scientific Research Institute, 37(04), 90-95

[8] Wang C, Zhu H, Xu Z and Li J 2018 Ground surface settlement of shield tunnels considering spatial variability of multiple geotechnical parameters, Chinese Journal of Geotechnical Engineering, 40(02), 270-77

[9] Zhang W, Wang Q, Liu H and Chen F 2021 Influence of rock mass spatial variability on probability of tunnel roof wedge failure, Rock and Soil Mechanics, 42(05), 1462-72

[10] Li J, Chen J, Cheng H and Hu Z 2018 Study on surrounding soil deformation induced by twin shield tunneling based on random field theory, Chinese Journal of Rock Mechanics and Engineering, 37(07), 1748-65

[11] Chen J, Chen L, Luo Y Wang C and Liu W 2021 Mechanism and control method of large deformation for large-span chlorite schist tunnel, Journal of Traffic and Transportation Engineering, 21(02), 93-106

[12] Guo X, Tan Z, Li L, Luo N and Wu Y 2017 Deformation and failure mechanism of layered soft rock tunnel under high stress, China Civil Engineering Journal, 50(02), 38-44

[13] Li L, Tan Z, Guo X, Yu Y and Luo N 2019 Research on large deformation of tunnels with small intervals in squeezing steeply dipping phylite strata, Chinese Journal of Rock Mechanics and Engineering, 38(02), 276-86

[14] Chen D, Xu D, Ren G, Jiang Q, Liu G, Wan L and Li N 2019 Simulation of cross-correlated non-gaussian random fields for layered rock mass mechanical parameters, Computers and Geotechnics, 112(AUG.), 104-19

[15] Li J, Du J, Zhang Y Zhou H and Fang Z 2019 Study on the method for determining deformation parameters of foundation rock mass in a nuclear power project, Chinese Journal of Rock Mechanics and Engineering, 38(01), 2988-96

[16] Li W, Huang Z and Tan X 2010. Research and application of correlation between elastic modulus and wave velocity of rock mass in hydroelectric project, Chinese Journal of Rock Mechanics and Engineering, 29(01), 2727-33

[17] Xu Q, Wang J, Hao W, Wei Y and Zhou B 2019 Correlation between elastic modulus and wave velocity of wudongde dam foundation rock mass and its application, Chinese Journal of Underground Space and Engineering, 15(05), 1434-41

[18] Xiahou Y, Zhang S, Tan H, Liu X and Wu Q 2019 Study of structural cross-constraint random field simulation method considering spatial variation structure of parameters, Rock and Soil Mechanics, 40(12), 4935-45+62

[19] Wen M, Zhang D, Fang H, Yu F and Liu Y 2021 Optimization analysis on spatial variability of tunnel surrounding rock parameters based on constrained random field theory, Modern Tunnelling Technology, 58(01), 99-108

[20] Yang W, Zheng J, Zhang R and Qiao Y 2021 Face stability analysis of shield tunnel considering variability of soil parameters and support pressure in clay, Journal of Civil and Environmental Engineering, doi:10.11835/j.issn.2096-6717.2021.053

[21] Tan X, Dong F, Fei S, Xiu, L, Hou X and Ma H 2020 Reliability analysis method based on KL expansion and its application, Chinese Journal of Geotechnical Engineering, 42(05), 808-16

[22] Yang J, Zhang D and Lu Z 2004 Stochastic analysis of saturated-unsaturated flow in heterogeneous media by combining karhunen-loeve expansion and perturbation method, Journal of Hydrology, 294(1-3), 18-38

[23] Yang Z, Cao Z, Li D and Fang G 2017 Effect of spatially variable friction coefficient of granular materials on its macro-mechanical behaviors using biaxial compression numerical simulation, Engineering Mechanics,34(05), 235-246

[24] Li D, Jiang S, Zhou C and Fang G 2013 Reliability analysis of slopes considering spatial variability of soil parameters using non-intrusive stochastic finite element method, Chinese Journal of Geotechnical Engineering, 35(08), 1413-22

[25] Phoon K, Huang S and Quek S 2002 Implementation of Karhunen–Loeve expansion for simulation using a wavelet-Galerkin scheme, Probabilistic Engineering Mechanics, 17(3), 293-303

[26] Huang S, Quek S and Phoon K 2001 Convergence study of the truncated karhunen-loeve expansion for simulation of stochastic processes, International Journal for Numerical Methods in Engineering, 52(9), 1029–43

[27] National Standards Compilation Group of People’s Republic of China 2019 GB 50216-2019. Unified design standard for reliability of railway engineering structures. China Planning Press, Beijing.

[28] National Standards Compilation Group of People’s Republic of China 2015 Q/CR 9218-2015. Technical Code for Monitoring Measurement of Railway Tunnel. China Planning Press, Beijing.

[29] Jiang S, Li D and Zhou C 2013 Non-intrusive stochastic finite element method for slope reliability analysis based on Latin hypercube sampling, Chinese Journal of Geotechnical Engineering, 35(S2),70-76

[30] Huang S, Liang B and Phoon K 2009 Geotechnical probabilistic analysis by collocation-based stochastic response surface method: An Excel add-in implementation, Georisk, 3(2): 75-86
[31] Itasca Consulting Group, Inc., 2005 Fast Language Analysis of continua in 3 dimensions, version 3.0 user’s manual. Itasca Consulting Group, Inc.

[32] Meng J, Qin Y, Xie L and Yu G 2019 Deformation analysis of shield tunnel considering spatial variability of soil elastic modulus. Water Power, 45(11), 34-39+83