Infrared non-perturbative QCD running coupling from Bogolubov approach

Boris A. Arbuzov

Skobeltsyn Institute of Nuclear Physics of MSU,
119992 Moscow, RF
E-mail: arbuzov@theory.sinp.msu.ru

We apply Bogolubov approach to QCD to demonstrate a spontaneous generation of three-gluon gauge invariant effective interaction which contributes significantly at the infrared region. The contribution of this interaction to $\alpha_s(p^2)$ leads to description of its behaviour in correspondence to phenomenology and lattice calculations without introduction of any additional parameter.

PACS: 11.30.Rd, 12.38.Lg, 12.39.-x, 12.40.Yx

In works [1, 2] with the use of method [3] inspired by N.N. Bogolubov approach [4, 5] a spontaneous generation of the effective interaction of the Nambu – Jona-Lasinio type is demonstrated. The interaction contains no additional parameters but the QCD ones. Results of calculation of hadron low-energy parameters: $m_\pi$, $f_\pi$, $m_\sigma$, $\Gamma_\sigma$, $<\bar{q}q>$ are quite consistent. In the present letter we apply the approach for calculation of infrared behaviour of the QCD running coupling constant.

We start with QCD Lagrangian with three light quarks ($u$, $d$ and $s$) with number of colours $N = 3$

$$L = \sum_{k=1}^{3} \left( \frac{1}{2} \left( \bar{\psi}_k \gamma_\mu \partial_\mu \psi_k - \partial_\mu \bar{\psi}_k \gamma_\mu \psi_k \right) - m_k \bar{\psi}_k \psi_k + g_s \bar{\psi}_k \gamma_\mu t^a A_\mu^a \psi_k \right) - \frac{1}{4} \left( F^a_{\mu\nu} F^{a\mu\nu} \right);$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f_{abc} A^b_\mu A^c_\nu.$$ (1)

where we use the standard QCD notations. In what follows we shall consider $m_k$ to be small enough and set them to zero. In accordance to the Bogolubov approach, application of which to such problems being described in details in work [3], we look for a non-trivial solution of a compensation equation, which is formulated on the basis of the Bogolubov procedure add – subtract. Namely let us write down the initial expression (1) in the following form

$$L = L_0 + L_{int};$$
\[
L_0 = \frac{1}{2} \left( \bar{\psi} \gamma_\mu \partial_\mu \psi - \partial_\mu \bar{\psi} \gamma_\mu \psi \right) - \frac{1}{4} F_0^{a \mu \nu} F_0^{a \mu \nu} + \frac{G}{3!} \cdot f_{abc} F_\mu^a F_\nu^b F_\rho^c ;
\]

\[
L_{int} = g_s \bar{\psi} \gamma^\mu A^\mu \psi - \frac{1}{4} \left( F_\mu^a F_\nu^a - F_0^{a \mu \nu} F_0^{a \mu \nu} \right) - \frac{G}{3!} \cdot f_{abc} F_\mu^a F_\nu^b F_\rho^c .
\]

Here \( \psi \) is the light quarks triplet, colour summation is performed inside of each fermion bilinear combination, \( F_0^{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \), and notation \( \frac{G}{3!} \cdot f_{abc} F_\mu^a F_\nu^b F_\rho^c \) means corresponding non-local vertex in the momentum space

\[
(2\pi)^4 G \cdot f_{abc} (g_\mu \nu (q_\rho p_\sigma - p_\rho q_\sigma) + g_\nu \rho (k_\mu p_\nu - q_\mu p_\nu) + g_\mu \rho (p_\nu q_\rho - k_\nu p_\rho) + q_\mu k_\nu p_\rho - k_\mu p_\nu q_\rho) \cdot F(p, q, k) \delta(p + q + k) + ...;
\]

where \( F(p, q, k) \) is a form-factor and \( p, \mu; q, \nu; k, \rho \) are respectfully incoming momenta and Lorentz indices of gluons and we mean also that there are present four-gluon, five-gluon and six-gluon vertices according to expression for \( F_\mu^a \).

Let us consider expression (2) as the new free Lagrangian \( L_0 \), whereas expression (3) as the new interaction Lagrangian \( L_{int} \). Then compensation conditions (see again [3]) will consist in demand of full connected tree-gluon vertices, following from Lagrangian \( L_0 \), to be zero. This demand gives a non-linear equation for form-factor \( F \).

These equations according to terminology of works [4, 5] are called compensation equations. In a study of these equations it is always evident the existence of a perturbative trivial solution (in our case \( G = 0 \)), but, in general, a non-perturbative non-trivial solution may also exist. Just the quest of a non-trivial solution inspires the main interest in such problems. One can not succeed in finding an exact non-trivial solution in a realistic theory, therefore the goal of a study is a quest of an adequate approach, the first non-perturbative approximation of which describes the main features of the problem. Improvement of a precision of results is to be achieved by corrections to the initial first approximation.

Thus our task is to formulate the first approximation. Here the experience acquired in the course of performing works [1, 2, 3] could be helpful. Now in view of obtaining the first approximation we would make the following assumptions.

1) In compensation equation we restrict ourselves by terms with loop numbers 0, 1.
2) In expressions thus obtained we perform a procedure of linearizing, which leads to linear integral equations. It means that in loop terms only one vertex contains the form-factor, being defined above, while other vertices are considered to be point-like. In diagram form equation for form-factor \( F \) is presented in Fig.1. Here four-leg vertex correspond to interaction of four gluons due to our effective three-field interaction. In our approximation we take here point-like vertex with interaction constant proportional to \( gG \).
3) We integrate by angular variables of the 4-dimensional Euclidean space. The necessary rules are presented in papers [1, 3].

At first let us present the expression for four-gluon vertex

\[
V(p, m, \lambda; q, n, \sigma; k, r, \tau, l, s, \pi) = gG \left( f^{amm} f^{arr} (U(k, l; \sigma, \tau, \pi, \lambda) - U(k, l; \lambda, \tau, \pi, \sigma) -
\right.
\]
-U(l, k; σ, π, τ, λ) + U(l, k; λ, π, τ, σ) + U(p, q; π, λ, σ, τ) − U(p, q; τ, λ, σ, π) −
- U(q, p; π, σ, λ, τ) + U(q, p; τ, σ, λ, π) − f_{\text{arn}} f_{\text{ams}}(U(p, l; σ, π, τ)) −
- U(l, p; σ, π, λ, τ) − U(l, p; τ, σ, λ, π) + U(l, p; τ, σ, π, λ) + U(k, q; π, τ, σ, λ) −
- U(q, k; π, σ, τ, λ) − U(k, q; λ, τ, σ, π) + U(q, k; λ, σ, τ, π) +
+ f_{\text{arn}} f_{\text{ams}}(U(k, p; σ, λ, τ, π) − U(p, k; σ, λ, π, τ) + U(p, k; π, λ, τ, σ) −
- U(k, p; π, λ, τ, σ) − U(l, q; λ, π, τ, σ) + U(l, q; τ, π, σ, λ) − U(q, l; τ, σ, π, λ) +
+ U(q, l; λ, σ, π, τ))
\end{equation} 

\begin{equation}
U(k, l; σ, π, λ, τ, π, λ) = k_a l_\sigma g_{πλ} - k_a l_\lambda g_{πτ} + k_π l_\lambda g_{στ} - (kl)g_{στ}g_{πλ}.
\end{equation}

Here a triad $p, m, λ$ etc means correspondingly momentum, colour index, Lorentz index of the gluon.

Let us formulate compensation equations in this approximation. For free Lagrangian $L_0$ full connected three-gluon vertices are to vanish. One can succeed in obtaining analytic solutions for the following set of momentum variables (see Fig. 1): left-hand legs have momenta $p$ and $-p$, and a right-hand leg has zero momenta. However in our approximation we need form-factor $F$ also for non-zero values of this momentum. We look for a solution with the following simple dependence on all three variables

\begin{equation}
F(p_1, p_2, p_3) = F\left(\frac{p_1^2 + p_2^2 + p_3^2}{2}\right);
\end{equation}

Really, expression (6) is symmetric and it return to $F(x)$ for $p_3 = 0, p_1^2 = p_2^2 = x$. We consider the representation (6) to be the first approximation and we plan to take into account the corresponding correction in the forthcoming study.

Now following the rules being stated above we obtain the following equation for form-factor $F(x)$

\begin{equation}
F(x) = -\frac{G^2 N}{64 \pi^2} \left( \int_0^Y F(y) y dy - \frac{1}{12 x^2} \int_0^x F(y) y^3 dy + \frac{1}{6 x} \int_0^x F(y) y^2 dy + \right)
+ \frac{x}{6} \int_x^Y F(y) dy - \frac{x^2}{12} \int_x^Y \frac{F(y)}{y} dy + \frac{G g N}{16 \pi^2} \int_0^Y F(y) dy +
+ \frac{G g N}{24 \pi^2} \left( \int_{3x/16}^{x/4} \frac{(3x - 16y)^2(3x - 8y)}{x^2(x - 8y)} F(y) dy + \int_{x/4}^Y \frac{(5x - 24y)}{(x - 8y)} F(y) dy \right).
\end{equation}

Her $x = p^2$ and $y = q^2$, where $q$ is an integration momentum. We introduce here an effective cut-off $Y$, which limited an infrared region where our non-perturbative effects act and consider the equation at interval $[0, Y]$ under condition

\begin{equation}
F(Y) = 0.
\end{equation}
We shall solve equation (7) by iterations. That is we expand the last line of (7) in powers of $x$ and take at first only constant term. Thus we have

$$F_0(x) = -\frac{G^2 N}{64 \pi^2} \left( \int_0^Y F_0(y) \, y \, dy - \frac{1}{12} x^2 \int_x^x F_0(y) \, y^3 \, dy + \frac{1}{6} x \int_0^x F_0(y) \, y^2 \, dy + \frac{x}{6} \int_0^Y F_0(y) \, dy - \frac{x^2}{12} \int_x^Y \frac{F_0(y)}{y} \, dy \right) + \frac{3 G \, g \, N}{16 \pi^2} \int_0^Y F_0(y) \, dy. \quad (9)$$

Expression (9) provide an equation of the type which were studied in papers [1, 2, 3], where the way of obtaining solutions of equations analogous to (9) are described. Following this way we have the unique solution of equation (9)

$$F_0(z) = \frac{1}{2} \left( -\frac{G^2 N}{64 \pi^2} \int_0^Y F(y) \, y \, dy + \frac{3 G \, g \, N}{16 \pi^2} \int_0^Y F(y) \, dy \right) \quad (10)$$

$$G_{15}^{31} \left( z | 0^{0}_{1,1/2,0,-1/2,-1} \right) = \frac{1}{2} z - G_{04}^{30} \left( z | 1/2, -1, -1/2 \right); \quad z = \frac{G^2 N \, x^2}{1024 \pi^2}. \quad (11)$$

Constants $C_1, C_2$ are defined by the following boundary conditions

$$\left[ 2 z \frac{d^3 F_0(z)}{dz^3} + 9 z \frac{d^2 F_0(z)}{dz^2} + \frac{d F_0(z)}{dz} \right]_{z = z_0} = 0;$$

$$\left[ 2 z \frac{d^2 F_0(z)}{dz^2} + 5 z \frac{d F_0(z)}{dz} + F_0(z) \right]_{z = z_0} = 1; \quad z_0 = \frac{G^2 N \, Y^2}{1024 \pi^2}. \quad (12)$$

Here as well as in the previous papers we obtain the solution in terms of Meijer G-functions [6]. Conditions (8, 11) defines set of parameters

$$z_0 = \infty; \quad C_1 = 0; \quad C_2 = 0. \quad (12)$$

However with these parameters the first integral in (9) diverges and we have no consistent solution. In view of this we consider the next approximation. We subtooite solution (10) into (7) and calculate the term proportional to $\sqrt{z}$. Now we have

$$F(z) = 1 + \frac{g \sqrt{z}}{4 \sqrt{3} \pi} \left( \ln(z) - 4 \gamma - 11 \ln(2) + \frac{7}{12} \right) +$$

$$+ \frac{2}{3} z \int_0^z F(t) \, dt \, t dt - \frac{4}{3 \sqrt{z}} \int_0^z F(t) \, \sqrt{t} \, dt -$$

$$- \frac{4 \sqrt{z}}{3} \int_z^{z_0} F(t) \, \frac{dt}{\sqrt{t}} + \frac{2}{3} \int_z^{z_0} F(t) \, \frac{dt}{t} \quad (13)$$

We look for solution of (13) in the form

$$F(z) = \frac{1}{2} G_{15}^{31} \left( z | 0^{0}_{1,1/2,0,-1/2,-1} \right) - \frac{g \sqrt{3}}{16 \pi} G_{15}^{31} \left( z | 1/2, -1, 1/2, -1/2, -1 \right)$$

$$+ C_1 G_{04}^{10} \left( z | 1/2, 1, -1/2, -1 \right) + C_2 G_{04}^{10} \left( z | 1, 1/2, -1, -1/2, -1 \right); \quad (14)$$
Boundary conditions also change to become in explicit form

\[
\frac{1}{2} G_{04}^{30}(z | 1/2, 0, -1/2, 0) - \frac{g \sqrt{3}}{16 \pi} \left( \frac{1}{2} G_{04}^{30}(z | 1/2, 0, 0, -3/2) + G_{15}^{31}(z | 1/2, 0, 0, -3/2) \right) + C_1 G_{04}^{10}(z | 0, 1/2, 0, -1/2) + C_2 G_{04}^{10}(z | 1/2, 0, 0, -1/2) - \frac{1}{2 \sqrt{z}} - \frac{g \sqrt{3}}{4 \pi} \left( \ln(z) + \frac{9}{4} - 4 \gamma - 11 \ln(2) \right) = 0; \quad z = z_0; \tag{15}
\]

Condition \(F(0) = 1\) leads to the following relation

\[
1 + \frac{G^2 N}{64 \pi^2} \int_0^Y F_0(y) \, dy = \frac{3 G g N}{16 \pi^2} \int_0^Y F_0(y) \, dy; \tag{16}
\]

which in explicit form reads

\[
1 + 8 \left( \frac{1}{2} G_{26}^{32}(z | 1, 1/2, 1/2, 0, 0) - \frac{g \sqrt{3}}{16 \pi} G_{26}^{32}(z | 1/2, 1/2, 1/2, 0, 0) \right) + C_1 G_{15}^{11}(z | 1/2, 1/2, 1/2, 0, 0) + C_2 G_{15}^{11}(z | 1/2, 1/2, 1/2, 0, 0) - \frac{3 \sqrt{3} g}{\pi} \left( \frac{1}{2} G_{26}^{32}(z | 1/2, 1/2, 1/2, 0, 0) - \frac{g \sqrt{3}}{16 \pi} G_{26}^{32}(z | 1/2, 1/2, 1/2, 0, 0) \right) + C_1 G_{15}^{11}(z | 1/2, 1/2, 1/2, 0, 0) + C_2 G_{04}^{10}(z | 3/2, 3/2, 0, 0, -1/2) = 0; \quad z = z_0. \tag{17}
\]

We have also condition

\[
F(z_0) = 0; \quad z_0 = 1.915838; \quad C_1 = 0.06172743; \quad C_2 = -0.1640803. \tag{19}
\]

that means smooth transition from the non-trivial solution to trivial one \(G = 0\). The conditions define values of parameters

\[
g(z_0) = 2.55779; \quad z_0 = 1.915838; \quad C_1 = 0.06172743; \quad C_2 = -0.1640803. \tag{20}
\]
We would draw attention to the fixed value of parameter $z_0$. The solution exists only for this value (12) and it plays the role of eigenvalue. As a matter of fact from the beginning the existence of such eigenvalue is by no means evident.

We consider the neglected terms of equation (7) as a perturbation to be taken into account in forthcoming studies.

We use Schwinger-Dyson equation for gluon polarization operator to obtain a contribution of additional effective vertex to the running QCD coupling constant $\alpha_s$. The corresponding diagram is presented at Fig.2. Due to this vertex being gauge invariant, there is no contribution of ghost fields. So the contribution under discussion reads

$$\Delta \Pi_{\mu\nu}(x) = \frac{g G N}{2 (2 \pi)^4} \int_0^\Gamma_0^{\mu\rho\sigma}(p, -q - \frac{p}{2}, q - \frac{p}{2}) \Gamma^{eff}_{\nu\rho\sigma}(-p, q + \frac{p}{2}, -q + \frac{p}{2}) F(q^2 + \frac{3\pi^2}{4}) dq \frac{(q^2 + p^2/4)^2 - (pq)^2}{(q^2 + p^2/4)^2}.$$  \hspace{1cm} (21)

Using expression (4) after angular integrations we have

$$\Delta \Pi_{\mu\nu}(x) = (g_{\mu\nu} p^2 - p_{\mu} p_{\nu}) \Pi(x); \quad x = p^2; \quad y' = q^2 + \frac{3x}{4};$$

$$\Pi(x) = -\frac{g G N}{32 \pi^2} \left( \frac{1}{x^2} \int_{3x/4}^\infty \frac{F(y')dy'}{y' - x/2} \left( 16 y'^3 - 48 y'^2 \frac{x}{x^2} + 45 y - \frac{27}{2} x \right) + \int_x^Y \frac{F(y')dy'}{y' - x/2} \left( -3y' + \frac{5}{2} x \right) \right).$$  \hspace{1cm} (22)

Here coupling $g$ corresponds to $g(Y)$. Integrals of Meijer functions depending on $y'$ multiplied by any power of $y'$ can be evaluated analytically. In view of this we expand the denominator in (22) in series in powers of $x/2$. Substituting solution (11) into integrals we obtain explicit expressions for each term of expansion of expression (22). Calculations show that results with different numbers of expansion terms begin practically coincide starting from three-term expansion. In what follows we present results with four terms.

So we have modified one-loop expression for $\alpha_s(p^2)$

$$\alpha_s(x) = \frac{4 \pi \alpha_s(p_0^2)}{4 \pi + b_0 \alpha_s(p_0^2) \ln(x/\Lambda^2) + 4 \pi \Pi(x)}; \quad x = p^2; \quad b_0 = 11 - \frac{2 N_f}{3}. \hspace{1cm} (23)$$

It is remarkable that function $\Pi(x)$ at $x = Y$ ($z = z_0$) turns to be very small, almost zero. We just expect this quantity to be zero exactly. So this property of the approximated polarization operator indicates the consistency of the procedure being used. Namely we normalize $\alpha_s(p^2)$ at point $p_0$, which correspond to our cut-off $Y$. Coupling constant $g$ entering in expressions (11) and (22) is just corresponding to this normalization point. Performing the well-known transformations in expression (23) we have for $u < u_0$

$$\alpha_s(u) = \frac{4 \pi}{b_0} \left( \ln(u) + \frac{8 \sqrt{N} \pi}{b_0 g} \right)^{-1}; \quad u = \frac{x}{\Lambda_{QCD}^2}; \quad u_0 = \frac{Y}{\Lambda_{QCD}^2} = 14.61328; \hspace{1cm}$$
\[
I = - \int_{\frac{z}{\sqrt{16}}}^{z} \frac{F_0(t)dt}{\sqrt{t(2\sqrt{t} - \sqrt{z})}} \left[ \frac{16}{z} t^{3/2} - \frac{48}{\sqrt{z}} t + \frac{45}{2} \sqrt{t} - \frac{27}{2} \sqrt{z} \right] + \\
+ \int_{z}^{z_{0}} \frac{F_0(t)dt}{\sqrt{t(2\sqrt{t} - \sqrt{z})}} \left[ -3\sqrt{t} + \frac{5}{2} \sqrt{z} \right] ; \quad z = \frac{G^2 N x^2}{1024 \pi^2} ; \quad b_0 = 9 .
\]

For \( u > u_0 \) we use the perturbative one-loop expression

\[
\alpha_s(u) = \frac{4\pi}{b_0 \ln(u)} .
\]

Two expressions (25), (24) give equal values \( \alpha_0 \) for \( \alpha_s \) at point \( z_0 \), while value \( \alpha_s(u_0) \) (20) is defined by equation (16). The self-consistent result for expressions (24, 25) with account of previous relations is unique and reads as follows

\[
z_0 = 1.915838 ; \quad u_0 = 14.61328 ; \quad \alpha_0 = 0.5206188 ; \\
C_1 = 0.06172743 ; \quad C_2 = -0.1640803 .
\]

Behavior of \( \alpha_s \) (24) with \( u = Q/\Lambda_{QCD} \) is presented at Fig. 3 for \( \Lambda_{QCD} = 0.2 GeV \) and \( 0.05 GeV < Q < 1 GeV \). The behaviour with maximum at \( Q \simeq 0.6 GeV \) and maximal value \( \alpha_s^{max} \simeq 0.55 \) agrees to calculations in work [7]. Qualitatively the result also corresponds to lattice calculations in work [8] (see also discussion in paper [9]). Note, that we begin plot at Fig.3 starting from \( p = 0.05 GeV \), because \( \alpha_s(u) \) has a pole at very small \( u \), which is analogous to the well-known perturbative pole at \( u = 1 \). Now this pole is shifted to the far infrared region. One may deal with it using the method proposed in work [10] and subtract from (24) the following term

\[
\frac{4\pi}{b_0 D (u - u_{00})} ; \quad u_{00} = 0.005769 ; \quad D = 170.1594 .
\]

This procedure practically does not change the result presented at Fig.3 in the denoted interval of \( Q \). For comparison we present the modified \( \alpha_s(Q) \) for interval \( 0.01 GeV < Q < 1 GeV \) at Fig 4. Value of \( \alpha_s(Q) \) at zero reads \( \alpha_s(0) = 1.4205 \).

All values are now expressed in terms of \( \Lambda_{QCD} \). Emphasize, that there is no additional parameters to describe the non-perturbative infrared region.

From definition of variables \( z, u \) and \( x \) we obtain value of new coupling constant \( G \)

\[
G = \frac{5.497571}{\Lambda_{QCD}^2} .
\]

We calculate non-perturbative vacuum average of the third power in gluon field, which is immediately connected with our results. We have

\[
< g^3 f_{abc} F_{\mu\nu}^{a} F_{\nu\rho}^{b} F_{\rho\mu}^{c} > = \frac{g^3 G}{(2\pi)^8} \frac{96 \pi^4}{I_1 I_2} = 110.77 \Lambda_{QCD}^6 .
\]
\begin{equation}
I_1 = \int_0^4 \left( 1 - \frac{3y}{4} \right) dy - \int_0^1 \frac{(1 - y)^2}{(1 - y/2)} dy - \int_1^4 \frac{(1 - y)^2(1 - 3y/4)}{y(1 - y/2)} dy = \nonumber \\
= 0.278756; \quad I_2 = \int_0^{Y} x^3 dx F(x) = 0.18062 \frac{(32 \pi)^4}{2 G^4 N^2}.
\end{equation}

Value (29) seems reasonable. However the accuracy of phenomenological and lattice definitions of this value is not yet satisfactory.

To conclude we would state, that method [3] being applied to non-perturbative \( \alpha_s \) proves its efficiency even in the first approximation, which is considered here. Bearing in mind also results of works [1, 2] on application of the approach to low-energy hadron physics we would express a hope, that in this way we could obtain the adequate tool to deal non-perturbative effects in QCD and, maybe, in other problems.

The mean non-perturbative value for \( \alpha_s \) turns to be here around 0.5. For this value results of works [1, 2] seem to be quite consistent.

The author express his gratitude to D.V. Shirkov for valuable discussions.

References

[1] B.A. Arbuzov, Phys. Atom. Nucl., 69, 1588 (2006); Yad. Fiz., 69, 1621 (2006).

[2] B.A.Arbuzov, M.K. Volkov and I.V. Zaitsev, Int. Journ. Mod. Phys. A, 21, 5721 (2006).

[3] B.A. Arbuzov, Theor. Math. Phys., 140, 1205 (2004); Teor. Mat. Fiz., 140, 367 (2004).

[4] N.N. Bogolubov. Physica Suppl., 26, 1 (1960).

[5] N.N. Bogolubov, Quasi-averages in problems of statistical mechanics. Preprint JINR D-781, (Dubna: JINR, 1961).

[6] H. Bateman and A. Erdélyi, Higher transcendental functions. V. 1 (New York, Toronto, London: McGraw-Hill, 1953).

[7] M. Baldicchi, A.V. Nesterenko, G.M. Prosperi et al., arXiv, 0705.1695 [hep-ph].

[8] E.-M Ilgenfritz, M. Müller-Preussker, A. Sternbeck and A. Schiller, arXiv, hep-lat/0601027.

[9] D.V. Shirkov, Eur. Phys. J. C, 22, 331 (2001).

[10] D.V. Shirkov and I.L. Solovtsov, Phys. Rev. Lett., 79, 1209 (1997).
Figure captions

Fig.1. Diagram representation of the compensation equation. Black spot corresponds to tree-gluon vertex with a form-factor. Simple point corresponds to a point-like vertex. Incoming momenta are denoted by the corresponding external lines.

Fig.2. Loop contribution to polarization operator.

Fig.3. Behaviour of $\alpha_s(Q)$, for $0.05\, GeV < Q < 1\, GeV$, $\Lambda_{QCD} = 0.2\, GeV$.

Fig.4. Behaviour of modified $\alpha_s(Q)$, for $0.01\, GeV < Q < 1\, GeV$, $\Lambda_{QCD} = 0.2\, GeV$. 
\[ p - p_0 + p - p_0 + p - p_0 + p - p_0 = 0 \]

Fig. 1.
Fig. 2.
\[ \alpha_s(Q) \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure3.png}
\caption{Figure 1: \[ \alpha_s(Q) \] vs. \( Q \) in GeV}
\end{figure}

Fig. 3
Fig. 4