Parameterized FEM for estimation of product unit assembly parameters

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Abstract. The article analyzes the existing calculation methods for assembly dimensional chains. The analysis revealed factors that are not considered in the methodology as a result, thus reducing reliability of the calculations. One of such factors is link interconnection for some dimensional chains, which must be considered while calculating. The parametrized finite-element model (FEM) of an aviation engine rotor is proposed considering: link interconnection of dimensional chains; geometrical deviations of part surfaces; axial forces from bolt connection; volume deformations. The developed FEM has been used in theoretical and experimental studies of geometric accuracy of the rotor assembly parameters. The research results for interrelationship of end face and radial run-out values of controlled rotor surfaces from geometrical errors of its parts are presented.

1. Introduction
Civil and military aircraft engines are subject to high requirements for geometric accuracy. This is due to the fact that it has a significant impact on the main quality indicators and technical characteristics of the aircraft engine [1,2]. The main measure to assess geometric accuracy of the assembled engine in the technological process is the control operation for assembly parameters (end face and radial run-outs, gaps, tension and linear dimensions).

The accuracy of the controlled assembly parameters is affected by the following factors. The first factor is the errors in the manufacture of the parts from which the unit or product [3-6] is assembled. The second factor is the errors that occur during assembling the unit or product. In order to estimate the influence of the given factors, the analysis task for dimensional chains is solved. To ensure the assemblability of the product by assigning tolerances on the geometric parameters of the parts, the synthesis problem is solved [7-9].

2. The calculation methods for dimensional chains of the products in mechanical engineering
The most common methods for calculating dimensional chains are the maximum-minimum method (max-min) and the probability method [10,11]. The first method is sometimes referred to as the method of full interchangeability, and the second one is referred to as the method of incomplete interchangeability. Existing methods and models for calculating assembly dimensional chains have the following drawbacks. The first one is an inaccurate calculation for the closing link accuracy, which is due to the lack of establishing the relationship for the parts in the product. When calculating dimensional chains using the maximum-minimum method, the nominal representation of part surfaces is used. In
reality, mating surfaces of the parts have defects (dimensional discrepancy, form and location deviation). The second drawback is the lack of consideration for influence of interconnected assembly dimensional chains, which are present in the design of scientific products.

Let's consider interrelation of dimensional chains on a compressor rotor assembly unit of an aviation engine. Figure 1 shows the assembly dimensional chain scheme of the considered rotor. The presented scheme contains two closing links $Z_1$ and $Z_2$, which have common component links $A_1(B_6)$, $A_2(B_5)$ and $A_3(B_1)$. The master link of the assembly dimensional chain can be a gap, tension, linear or angular size, accuracy of which is specified in the specifications for manufacture and assembly of the product. Master links $Z_1$ and $Z_2$ are the gap and tension. Bolt connection on diameter $D_1$ and $D_2$ forms a power circuit which provides rotor stiffness and connects the parts to the assembly unit. During assembly of the considered rotor, its discs are tightened by axial forces of bolted connections on corresponding diameters $D_1$ and $D_2$, at the same time at diameter $D_3$, there is a tension or gap, which leads to deformation of the discs of 10 and 11 stages. Arising deformations of the disks lead to appearance of end face $T_i$ and radial $R_i$ run-outs of the considered rotor. It is known that disc deformations are complex nonlinear. Unknown is the nature of the control surface run-out dependencies of the considered rotor on the values of geometric errors of its part surfaces. Using classical theory of dimensional bonds to calculate the considered dimensional chain of the considered rotor leads to large errors of its assembly parameter estimation.

Interconnection of dimensional chains while assembling the considered rotor can be considered through developing a parametrized finite-element model. Implementing such a model is possible by establishing interconnection between CAD and CAE systems. Pattern model creation considering geometrical errors is done in a CAD system. Assembly process of the considered rotor is simulated in a CAE system.

A sufficient number of studies have been conducted in recent decades on the representation of admission, analysis and synthesis. The review papers [12-19] present the main and perspective methods for representation and analysis of tolerances in three dimensions and their comparison. The following methods are presented: tolerance map (Tolerance-Map T-Map), matrix model, single model Jacobian-Torsor, direct linearization method (DLM), GapSpace model. Their benefits and drawbacks are considered.

3. Parameterized FEM for assessment of assembly parameters
The paper presents a parametrized FEM for assessment of aircraft engine rotor assembly parameters, which is implemented through interaction of software systems presented in the form of a flow chart in figure 2.
Before using the parametrized FEM, preparatory operations are performed. In "Excel" program system, the user sets the values of the initial data for creating parametric surfaces according to the preset variables in "NX" system. Parametric end face and radial surfaces in "NX" system are defined by corresponding systems of equations. End surfaces are described by a system of equations:

$$\begin{align}
  x_t &= R_i \cdot \cos(360 \cdot t_i) \\
  y_t &= R_i \cdot \sin(360 \cdot t_i) \\
  z_t &= H_i + A_i \cdot \sin(360 \cdot t_i \cdot k_i)
\end{align}$$

being $x_{i,j}$, $y_{i,j}$, $z_{i,j}$ — functions describing the spline along axis $OX$, $OY$, $OZ$ respectively; $R_{i,j}$ — surface cross-section radius (mm); $t_{i,j}$ — constant curve functions describing the deviations; $H_{i,j}$ — distance to the surface or between surfaces (mm); $A_{i,j}$ — amplitude of the spline curve deviation (mm); $k_{i,j}$ — curve oscillation phase; $z_{i,j}$ — surface displacement along axis $ZO$ (mm).

Radial surfaces are described by a system of equations:

$$\begin{align}
  x_t &= (R_j + A_j \cdot \sin(360 \cdot t_j \cdot k_j)) \cdot \cos(360 \cdot t_j) \\
  y_t &= (R_j + A_j \cdot \sin(360 \cdot t_j \cdot k_j)) \cdot \sin(360 \cdot t_j) \\
  z_t &= H_j + z_j
\end{align}$$

The following operations are performed in "ANSYS Workbench" program system: loading and binding the parametric model, setting the material properties and boundary conditions, setting the postprocessor to save the model in "stl" format [20]. MATLAB software system is used to automate the processes of calculating FEM through interaction of the used software systems. Let’s consider the operation stages of the parameterized FEM. At the first stage, the FEM calculation process control program is launched in MATLAB environment. The control script reads the values of the parameters defined in Excel and saves them to the clipboard.

In the second stage, "ANSYS Workbench" is launched. The control script starts the project, previously created in "ANSYS Workbench" environment. Then the command is transmitted, and the executed operations are controlled via APDL programming language.

In the third stage, the parameters of the linked CAD model are checked for compliance with the loaded ones in the first stage. If the result is satisfactory, the parameterized values of the three-dimensional solid model are replaced. "ANSYS Workbench" and "Siemens NX" work in pairs by default. After starting reconfigurations, "ANSYS Workbench" communicates and runs "Siemens NX", which in turn performs the model reconfiguration. As a result of finite-element modeling, we obtain a 3D solid-state model with actual parameters. After issuing the APDL code indicating that the operation is successful, the process continues.

In the fourth stage, the gap of the mating part surfaces is eliminated. The calculation is carried out according to the following formula:

$$\varepsilon = -\delta - 5 \cdot 10^{-5}$$
being $\varepsilon$ – tension value, mm; $\delta$ – distance between mating surfaces, mm. The program launches the "Static Structural" module, which updates and reads information about the inter-contact gap. While using a control code, the program calculates the gap size and moving direction of the parts, for their further pairing. Thus, the result of the fourth stage is to obtain the inter-contact tension with the value of no more than 0.05 µm.

During the fifth stage, the program, using the APDL environment, sends the command for calculation. This step ends with saving the deformed facet model in "stl" format. The above stages are repeated for the remaining experiments.

The final stage is the calculation of assembly parameters. The program gradually loads into the clipboard the specified coordinates of points from faceted bodies forming a surface that determines the specified assembly parameters (end face and radial run-outs, run-out flatness, conicity, etc.) [21].

To find the end face run-out for each model point, the distance to the plane is determined by the point on the rotation axis and its guide vector. The specified distance is determined by the formula:

$$ R_i = -\frac{(\vec{x} - \vec{p}, \vec{v})}{(\vec{v}, \vec{v})}, $$

(4)

being $R_i$ – the distance from the $i$ point of the calculated surface to the target plane; $\vec{x}$ – radius of the point vector on the part rotation axis; $\vec{p}$ – radius of the calculated surface point; $\vec{v}$ – guiding vector of the rotation axis.

After all distances are found, the end face run-out is calculated using the formula:

$$ \Delta_{RF} = \max_i R_i - \min_j R_j. $$

(5)

The calculation of the radial run-out is done in two stages. At the first stage is the maximum distance from the rotation axis of the calculated surface. The distance from the point to the rotation axis is calculated by the formula:

$$ R = -\frac{\left\| \vec{v} \cdot (\vec{p} - \vec{x}) \right\|}{\|\vec{v}\|}, $$

(6)

The second stage searches for the point closest to the rotation axis. The specified coordinate is searched for on all borders of triangular areas of the calculated surface. The radial run-out is calculated using a formula:

$$ \Delta_{RR} = \Delta_{RFmax} - \Delta_{RFmin}. $$

(7)

At the end of the stage, the MATLAB program saves the values of assembly parameters in the "xlsx" format and finishes the work.

4. Experimental and theoretical research for the geometric accuracy influence of parts on the controlled assembly parameters

During statistical data gathering, the deviation values of the form and an arrangement of compressor rotor part surfaces of an aircraft engine have been defined. Parts have been measured on a coordinate measuring machine. In the course of studies, it has been found that the end face and radial contact surfaces of rotor parts have a harmonic deflection form with two maximum and minimum values. During the statistical data gathering, it has been determined that the displacement of the part contact surfaces is insignificant.

Let’s consider using the parametrized FEM on a compressor rotor assembly example of an aviation engine which structure includes the parts shown in figure 3.
In the course of theoretical study, the scattering fields of controlled parameters from the run-out value of the mating surfaces are researched. CAD models with surfaces repeating geometrical errors of measured parts have been used for modeling. In order to determine the interrelation between run-outs and geometric errors of the parts, the experiment has been divided into 3 stages. During the first stage, the deviations of the end mating surfaces of the parts have been preset. All models have been assigned the same surface form deviation value, but its mutual angular position has been set by random values. In order to determine the scattering field values for each deviation value, seven experiments have been performed. The second stage of the theoretical experiment for radial surfaces has been conducted in a similar way. In the third stage, the end face and radial surfaces have been changed. As a result, the theoretical results presented in Table 1 have been obtained.

Table 1. Theoretical research for interrelation of the scattering field for run-outs with the surface deviation value.

| preset deviation of surfaces, mm | surface run-out scattering fields, mm | specified surface deviation, mm | surface run-out scattering fields, mm |
|---------------------------------|----------------------------------------|----------------------------------|----------------------------------------|
| radial                          | end face                               | radial, E                        | end face, F                            | end face, E                        | radial, F |
| 0                               | 0.005                                  | 0.021                            | 0.002                                  | 0.005                                | 0.0075    | 0.013    | 0.000    |
| 0                               | 0.0075                                 | 0.028                            | 0.002                                  | 0.0075                                | 0.0075    | 0.012    | 0.002    |
| 0                               | 0.01                                   | 0.032                            | 0.003                                  | 0.01                                  | 0.0075    | 0.013    | 0.001    |
| 0                               | 0.025                                  | 0.062                            | 0.013                                  | 0.0075                                | 0.01      | 0.039    | 0.004    |
| 0                               | 0.05                                   | 0.081                            | 0.018                                  | 0.01                                  | 0.01      | 0.029    | 0.001    |
| 0                               | 0.075                                  | 0.099                            | 0.023                                  | 0.025                                | 0.01      | 0.045    | 0.005    |
| 0                               | 0.1                                    | 0.092                            | 0.023                                  | 0.01                                  | 0.025    | 0.057    | 0.006    |
| 0.005                           | 0                                      | 0.001                            | 0.000                                  | 0.025                                | 0.025    | 0.048    | 0.005    |
| 0.0075                          | 0                                      | 0.001                            | 0.001                                  | 0.05                                  | 0.025    | 0.037    | 0.004    |
| 0.01                            | 0                                      | 0.004                            | 0.001                                  | 0.025                                | 0.05      | 0.050    | 0.004    |
| 0.025                           | 0                                      | 0.010                            | 0.003                                  | 0.05                                  | 0.05      | 0.065    | 0.005    |
| 0.05                            | 0                                      | 0.024                            | 0.005                                  | 0.05                                  | 0.075    | 0.085    | 0.015    |
| 0.075                           | 0                                      | 0.040                            | 0.007                                  | 0.075                                | 0.075    | 0.086    | 0.017    |
| 0.1                             | 0                                      | 0.054                            | 0.006                                  | 0.1                                  | 0.075    | 0.130    | 0.019    |
| 0.005                           | 0.005                                  | 0.010                            | 0.002                                  | 0.075                                | 0.1      | 0.132    | 0.025    |
| 0.0075                          | 0.005                                  | 0.015                            | 0.003                                  | 0.1                                  | 0.1      | 0.152    | 0.019    |

The dependencies shown in figure 4 are based on Table 1.
Figure 4. Dependence of the scattering field for run-outs on the preset surface deviation value: a) end face run-outs; b) radial run-outs.

In order to definite the character of the scattering field distribution for run-outs on all compressor assembly of an aviation engine, experiments with the preset deviations of end face and radial surfaces with value equal 0.01 mm have been simulated. Run-outs have been determined by end faces: A, C, E, G, I, K, M, O, Q, S, U, W and radial surfaces: B, D, F, H, J, L, N, P, R, T, V, X. Following the achieved values of run-outs, the scattering fields, which are presented in Table 2, have been obtained.

Table 2. The scattering fields for experiments with a preset end face and radial surface deviation of 0.01 mm.

| Diameter | \( \omega_{1f} \), mm | \( \omega_{2f} \), mm | \( \omega_{3f} \), mm | \( \omega_{1R} \), mm | \( \omega_{2R} \), mm | \( \omega_{3R} \), mm |
|----------|---------------------|---------------------|---------------------|---------------------|---------------------|---------------------|
| I        | 0.012               | 0.005               | 0.005               | 0.006               | 0.015               | 0.003               |
| II       | 0.022               | 0.026               | 0.018               | 0.017               | 0.001               | 0.001               |
| III      | 0.029               | 0.026               | 0.011               | 0.009               | 0.004               | 0.004               |

Having analyzed Table 3, we can conclude that as the diameter of the rotor control surface increases, the size of the end run-out scattering field increases \( \omega_{f} \). In case of radial run-outs of \( \omega_{R} \) rotor control surfaces the situation is reversed.

5. Conclusion
Experimental research of the form and arrangement of compressor rotor part mating surfaces of an aircraft engine has shown that in the sum of geometrical deviations, the greatest importance is made by the form deviations. For example, for flat surfaces of rotor parts, the deviation from the flatness of the end faces in their aggregate deviation from the nominal value is more than 65%. In this regard, it can be assumed that the values of assembly parameters at the considered production conditions are most influenced by the deviations of the surface form, represented by normalized parameters: deviations from run-out flatness and non-roundness. The work includes researching the influence of run-out flatness and non-roundness of the surfaces on the values of rotor assembly parameters, represented by end face and radial run-outs on control surfaces. The topology for form deviation of the mating end and radial surfaces which is characteristic more than in 80% of cases is revealed. It is revealed that end and radial surfaces of rotor parts have harmonic form deviation with two maximum and minimum values. Theoretical research of the compressor rotor assembly process allowed to reveal the scattering fields of run-outs of its control surfaces depending on the preset values of run-out flatness and roundness of the parts. Dependence of scattering field values of run-outs on the deviation preset values of end face and radial surfaces have linear character. The deviation value from roundness of the radial surfaces has a relatively
small impact on the scattering fields of radial run-outs of $\omega_R$ rotor control surfaces. Surface run-out scattering fields $\omega_F$ of the rotor surfaces are more dependent on the run-out flatness of the part end surfaces. As the diameter of the rotor control surface increases, the value of the end run-out scattering field increases $\omega_F$. In case of radial run-outs $\omega_R$ of rotor control surfaces the situation is reversed.

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