Flux lattice melting and the onset of $H_{c2}$ fluctuations

Stephen W. Pierson$^{a,1}$ Oriol T. Valls$^b$

$^a$Department of Physics, Worcester Polytechnic Institute, Worcester, MA 01609-2280, USA
$^b$Physics Department and Minnesota Supercomputer Institute, University of Minnesota, Minneapolis, MN 55455, USA

Abstract
The flux lattice melting temperature in YBa$_2$Cu$_3$O$_{7-\delta}$ has been shown to be very close to that of the onset of fluctuations around $H_{c2}(T)$. Here, we present a theoretical argument in support of the idea that this occurs because the increased strength of the fluctuations as a function of magnetic field pushes away the first order flux lattice melting transition. The argument is based on hydrodynamic considerations (the Hansen-Verlet freezing criterion). It is not specific to high-temperature superconductors and can be generalized to other systems.

Keywords: Flux Lattice Melting; Hansen-Verlet freezing criterion; fluctuations

A prominent feature of the remarkably rich field-temperature ($H$-$T$) phase diagram in high-$T_c$ superconductors[1] (HTSC’s) is the flux lattice melting (FLM) temperature $T_M(H)$ line. Theories of $T_M(H)$ typically treat that phase transition as a stand-alone phenomenon, as in, for example, the earliest approach of examining the elastic moduli of the flux lattice to establish the Lindemann criterion for the melting temperature.[2]

In contrast, we have recently presented evidence[3] that $T_M(H)$ should be viewed as intimately connected to a nearby feature in the phase diagram: the superconducting/normal crossover line $H_{c2}(T)$. In Ref. [3], it was shown that the FLM temperature coincides with that of the onset of fluctuations around $H_{c2}$, as the temperature is increased. The essential idea, which is appealing in its simplicity, is best captured by considering the system as it is being cooled: as one does so, the vortices cannot freeze into a lattice until the $H_{c2}$ fluctuations have died down. When the strength of the fluctuations is estimated through the field-dependent Ginzburg criterion $G_i(H)$, the freezing temperature of the vortices is determined by,

$$G_i(H) = \frac{T_m(H) - T_c(H)}{T_c(H)} \propto H^{2/3},$$

where the constant of proportionality is presented in Refs. [3] and [4].

In this paper, we present a simple theoretical argument, based on very general hydrodynamic considerations and the Hansen-Verlet[5] (HV) freezing criterion (the freezing version of the better-known Lindemann melting rule), that explains the coincidence of the FLM with the edge of the $H_{c2}$ fluctuation region.

The evidence for the relation between the FLM and $H_{c2}$ fluctuation lines is twofold.[3] The fit of
the theoretical expressions for the specific heat of Tešanović and Andreev[6] to the YBa$_2$Cu$_3$O$_{7−δ}$ (YBCO) data of Schilling et al.[7] (see Fig. 1), establishes that the specific heat for temperatures above the FLM (where it has a spike) can be attributed to $H_{c2}$ fluctuations and that the spike coincides with the onset of those fluctuations. Secondly, the positions of the FLM specific heat spikes are determined by fluctuations around $H_{c2}$ as described by the formula of Herbut and Tešanović.[4] [The formula is essentially Eq. (1).]

Here we argue on very simple and general theoretical grounds that a first order freezing transition cannot occur until the fluctuations from a nearby second order transition have subsided. We use only two fundamental hydrodynamic relations and the Hansen-Verlet criterion. The HV criterion states that freezing occurs when, as one cools, the magnitude of the first finite wavevector peak in the static structure factor $S(k)$ reaches a certain value, typically ranging from 3 to 6. There are two hydrodynamic constraints[8]; the long wavelength limit of $S(k)$ is related to the average density $\rho$ and the compressibility $\chi$ by

$$\lim_{k \to 0} S(k) = k_B T \rho \chi,$$

while on the other hand, the integral of $S(k) - 1$ over all $k$ varies with $T$ only very slowly, through $\rho$.

Combining the two hydrodynamic results with the HV criterion, one immediately sees that the fluctuations arising from the higher temperature second order transition must have died down before the first order transition may occur. Just below the second order phase transition, where $\chi$ diverges, $S$ still has a large peak at small $k$. Because of the constraint on the integral over all $k$ of $S(k)$ the magnitude of the finite $k$ peaks must remain fairly small. Only as one moves further below the second order phase transition, and the zero $k$ peak shrinks, can the magnitude of the first peak increase to a value large enough to satisfy the Hansen-Verlet rule. Hence, freezing can occur only sufficiently far from the second order phase transition. Simple free energy models involving two coupled order parameters can be built to illustrate this conclusion.

Given the generality of this argument, it is clear that it applies to any system where a first order phase transition is near a second order one. Candidates include liquid crystals, heavy-fermion superconductors, materials with structural and ferroelectric transitions, and Langmuir monolayers.

We have shown that FLM in the $H-T$ phase diagram of HTSC’s cannot occur until the fluctuations from the $H_{c2}(T)$ line die down, evidence for which was given in Ref. [3]. Our theory is sufficiently general that it can be applied to other systems.

We thank I. Herbut and Z. Tešanović for discussions. SWP thanks the Petroleum Research Fund, administered by the ACS, for their support.

References

[1] G.Blatter, et al., Rev.Mod.Phys. 66 (1994) 1125.
[2] A. Houghton, R. A. Pelcovits, and A. Sudbø, Phys. Rev. B 40 (1989) 6763.
[3] S.W.Pierson, O.T.Valls, Phys.Rev.B 57, (1998) 8143.
[4] I.F.Herbut and Z.Tesanovic, Physica C 255 (1995) 324.
[5] J.P. Hansen and L. Verlet, Phys. Rev. 184 (1969) 151.
[6] Z.Tesanovic, A.V.Andreev, Phys.Rev.B 49 (1994) 4064.
[7] A.Schilling, et al., Phys.Rev.Lett. 78 (1997) 4833.
[8] J.P. Boon and S. Yip, Molecular Hydrodynamics, Dover, New York, (1980) pp 24-27.