The isoscalar dipole response of heavy spherical nuclei in the low-energy region is studied by using a semiclassical model, based on the solution of the linearized Vlasov kinetic equation for finite Fermi systems. In this translation-invariant model, the excitations of the center of mass motion are exactly separated from the internal ones. The isoscalar dipole strength function displays three resonance structures in the energy region up to 15 MeV. Calculations of the velocity fields associated with resonance structures at centroid energies show the vortex (toroidal) nature of two overlapping resonances. The main toroidal resonance gives a qualitative description of the low-energy isoscalar dipole resonance, which is observed in heavy spherical nuclei. The origin of the lowest isoscalar dipole resonance structure is apparently related to dipole single-particle excitations. Its centroid energy is close to the minimum energy of the dipole single-particle spectrum, and taking into account the residual interaction leads only to an insignificant shift of the centroid energy towards lower energy. However, the inclusion of residual interaction noticeably enhances the velocity field associated with the lowest resonance, which indicates collective effects in this resonance structure.

Keywords: kinetic model, low-energy resonance structures, velocity field, toroidal resonances.

1. Introduction

The nuclear isoscalar dipole response reveals the low-energy resonance [1 - 4]. Theoretical studies of low-energy isoscalar dipole resonance had been carried out both within quantum approaches [5 - 15] and using semiclassical ones [16 - 20]. They showed that this resonance has a substantial vortex (toroidal) character. However, quantum calculations of the isoscalar dipole response show several toroidal resonance structures in the low-energy region (below 15 MeV) [5, 8, 10, 11]. The main resonance is interpreted as a nuclear toroidal mode. The semiclassical dipole strength function also has several resonance structures in the low-energy region [17, 20]. It is of interest to study the nature of these resonance structures in order to compare them with quantum studies.

In this paper, the velocity fields associated with low-energy collective isoscalar dipole excitations are studied within the translation-invariant kinetic model of small oscillations of finite Fermi systems [21]. This article is a continuation of [20], in which attention was focused on studying the nature of the main nuclear isoscalar dipole resonances (toroidal and compression modes). In Section 2, we briefly recall the formalism of the kinetic model of collective dipole excitations in nuclei. The internal response function and the velocity field associated with the isoscalar dipole excitations are considered. In Section 3, the low-energy resonance structures of the isoscalar dipole response function are discussed, and the results of numerical calculations of the velocity fields associated with the low-energy resonance structures of the dipole strength function are shown.

2. Formalism

The translation-invariant kinetic model of small oscillations of finite Fermi systems based on the direct solution of the Vlasov equation for a Fermi system with a moving surface is used to study the collective isoscalar dipole excitations of heavy nuclei [17, 19, 20]. In this model, a nucleus is treated as a gas of interacting fermions confined to a spherical cavity with a moving surface. Within our kinetic model, we can find the explicit expression for the fluctuation of the phase-space distribution function related to the collective isoscalar dipole excitations. By using this function, we can calculate the response function [19] as well as the local dynamical quantities, in particular, the velocity field [20].

Isoscalar dipole excitations in finite Fermi systems are an effect of the second order for the dipole moment (in the first order, they reduce to the center-of-mass motion). So, we consider the collective isoscalar dipole modes excited by a weak external field of the kind

\[ V(\vec{r}, t) = \beta \delta(t) Q^{(3)}(r) Y_{10}^{*}(\theta, \varphi), \]

(1)

where \( Q^{(3)}(r) = r^3 \) is the second-order dipole moment, \( \delta(t) \) is the Dirac delta-function in time, and \( \beta (\beta = \text{const} \alpha, \quad \alpha << 1) \) is a parameter that describes the external field strength. Within the kinetic model, assuming a simplified residual interaction of separable form

\[ v(\vec{r}, \vec{r'}) = k_1 \sum_M r'(Y_{1M}^{*}(\theta, \varphi) Y_{1M}(\theta', \varphi')), \]

(2)
the explicit solution for the fluctuation of the phase-space distribution function can be found. Since the external field (1) can also excite the center of mass, the problem arises of extracting spurious strength from the response function. Our translation-invariant model allows for a clear way to evaluating the intrinsic response function associated with the field (1). By looking at the response of the center of mass induced by the external field (1), we can get [20]

\[ \tilde{R}^{(13)}_{\text{cm}}(s) = \frac{3}{8\pi} \frac{AR^4}{e_\rho s^2}, \]  

(3)

where \( R \) is the equilibrium radius of the system, \( s = \omega R / v_F \) is a convenient dimensionless frequency (\( v_F \) is the Fermi velocity) and \( e_\rho \) is the Fermi energy. Since this response function has no poles for \( s \neq 0 \), it does not give spurious dissipation at positive \( s \). Then the internal response function that is related to the collective isoscalar dipole excitations can be determined as

\[ \tilde{R}_{ss}(s) = \tilde{R}_{33}(s) - \tilde{R}^{(13)}_{\text{cm}}(s), \]  

(4)

\[ \tilde{R}^0_{ss}(s) = \frac{9A}{16\pi} \frac{1}{e_\rho} \sum_{n=-\infty}^{\infty} \sum_{N=-1}^{1} \int_0^1 dx x^2 s_{N\delta}(x) Q^k_{\alpha\nu}(x) \mathcal{Q}^k_{\alpha\nu}(x) \left( s + i\epsilon - s_{N\delta}(x) \right) \]  

(6)

where \( \mathcal{Q}^k_{\alpha\nu}(x) \) are the classical limit of the quantum-mechanical radial matrix elements of the dipole operators and are given by

\[ \mathcal{Q}^k_{\alpha\nu}(x) = (-)^\delta R \frac{1}{s_{N\delta}(x)}, \]  

(8)

\[ Q^3_{\alpha\nu}(x) = 3R^2 \mathcal{Q}^1_{\alpha\nu}(x) \left( 1 + \frac{4}{3} N \sqrt{1-x^2} - \frac{2}{s_{N\delta}(x)} \right). \]  

(9)

The response functions (6) involve an infinite sum over \( n \), however, in practice, it is sufficient to include only a few terms around \( n = 0 \) in order to fulfill the energy-weighted sum rule with good accuracy.

The moving-surface contribution \( S_{33}(s) \) to the internal response function (3) can be evaluated explicitly as

\[ S_{33}(s) = \frac{1}{1-\kappa \kappa_{0}(s) \left\{ -\chi_1(s) \right\}} \frac{[\chi_0^{(s)}(s) - \chi_0^{(0)}(0) \kappa \kappa_{0}(s)]}{1 - \kappa \kappa_{0}(s) \left\{ -\chi_1(s) \right\}}, \]  

(10)

where the functions \( \chi_0^{(s)}(s), (k = 1,3) \), and \( \chi_1(s) \) describe the dynamical surface effects and are defined as in Ref. [17]:

\[ \chi_0^{(s)}(s) = \frac{9A}{8\pi} \sum_{n=-\infty}^{\infty} \sum_{N=-1}^{1} \int_0^1 dx x^2 s_{N\delta}(x) \left( \frac{-)^\delta \mathcal{Q}^{(k)}_{\alpha\nu}(x) \right) \left( s + i\epsilon - s_{N\delta}(x) \right) \]  

(11)

\[ \chi_1(s) = \frac{9A}{4\pi} e_\rho \sum_{n=-\infty}^{\infty} \sum_{N=-1}^{1} \int_0^1 dx x^2 \frac{1}{s + i\epsilon - s_{N\delta}(x)}, \]  

(12)
The poles of the internal response function (4) determine the frequencies of collective isoscalar dipole modes. Neglecting residual interaction (κ = 0) in Eqs. (5), (10), we obtain the internal response function (4) in the zeroth-order approximation. Due to a self-consistent coupling between the motion of nucleons and the moving surface, the collective isoscalar dipole excitations originate in our kinetic model already in the zeroth-order approximation.

To get the information about the origin of collective isoscalar dipole excitations, it is interesting to consider the velocity field associated with the dipole collective motion. This local dynamic quantity describes the spatial distribution of the average nucleon velocity during collective excitation and provides information on the nature of excitation. In our kinetic model, the time Fourier-transform of the velocity field is determined as

\[ \tilde{u}(\tilde{r}, \omega) = \frac{1}{m\rho_0} \int d\tilde{p} \tilde{\rho} \delta n(\tilde{r}, \tilde{p}, \omega), \]  

where \( \delta n(\tilde{r}, \tilde{p}, \omega) \) are the (Fourier transformed in time) fluctuations of the phase-space particle distribution induced by a weak external field (1). \( \rho_0 \) is the nuclear equilibrium density and \( m \) is the nucleon mass. Choosing the Z axis in the direction of the external field, we will consider the velocity field in the meridian plane XZ that usually exploited in the RPA calculations [5, 22]. In this representation, the radius-vector of the particle is \( \tilde{r} = (x, y, 0, z) \) or \( \tilde{r} = (r, \varphi = 0) \) in the spherical coordinates and the velocity field (13) can be written as

\[ \tilde{u}(r, \varphi = 0, \omega) = u_e(r, \varphi = 0, \omega) \hat{e}_x + u_z(r, \varphi = 0, \omega) \hat{e}_z, \]  

where \( u_e(r, \varphi, \omega) \) and \( u_z(r, \varphi, \omega) \) are the projections of the velocity field vector into the X and Z axes, respectively, and \( \hat{e}_x, \hat{e}_z \) are unit vectors directed along these axes. The expressions for the functions \( u_e(r, \varphi, \omega) \) and \( u_z(r, \varphi, \omega) \) can be written as, see Ref. [20],

\[ u_e(r, \varphi, \omega) = \frac{\sqrt{5}}{\sqrt{N}} Y_{11}(\theta, 0) u_{12}(r, \omega), \]  

\[ u_z(r, \varphi, \omega) = Y_{00}(\theta, 0) u_{10}(r, \omega) - \frac{\sqrt{5}}{\sqrt{N}} Y_{20}(\theta, 0) u_{12}(r, \omega). \]  

Here \( Y_{mn}(\theta, 0) \) are the spherical harmonics, while the radial functions \( u_{10}(r, \omega) \) and \( u_{12}(r, \omega) \) are defined as

\[ u_{10}(r, \omega) = -i \sqrt{\frac{2}{3}} \frac{1}{\rho_0} \int de \int dl l \sum_{N=1}^{l} \left\{ i e [\delta n^+_m(r, e, l, \omega) - \delta n^-_m(r, e, l, \omega)] + \frac{N}{2} \frac{l}{p(r, e, l, r)} [\delta n^+_m(r, e, l, \omega) + \delta n^-_m(r, e, l, \omega)] \right\}, \]  

\[ u_{12}(r, \omega) = -i \sqrt{\frac{2}{3}} \frac{1}{\rho_0} \int de \int dl l \sum_{N=1}^{l} \left\{ i e [\delta n^+_m(r, e, l, \omega) - \delta n^-_m(r, e, l, \omega)] + \frac{l}{p(r, e, l, r)} [\delta n^+_m(r, e, l, \omega) + \delta n^-_m(r, e, l, \omega)] \right\}, \]  

where \( e \) is the particle energy, \( l \) is the magnitude of its angular momentum and \( p(r, e, l) = \sqrt{2me - (l/r)^2} \) is the magnitude of the particle radial momentum. The fluctuations of the phase-space particle distribution functions \( \delta n^+_m(r, e, l, \omega) \) are the solutions of the linearized Vlasov kinetic equation for a finite system with a moving surface. The explicit expressions of these functions are given in Ref. [20]. It should be noted that the dipole external field (1) results in the dipole radial (and tangential) component of the velocity field (13), while the angular dependence of the x and z projections of the velocity field vector (15), (16) is determined by the quadrupole and monopole spherical harmonics. Indeed, using the relation \( \hat{e}_x = \hat{e}_z \sin \theta + \hat{e}_r \cos \theta \), where \( \hat{e}_r \) is a unit vector directed along the radius-vector, and (15), (16) we can get that \( \tilde{u}(r, \varphi = 0, \omega) \hat{e}_r = Y_{00}(\theta, 0) u_{10}(r, \omega) \).

Our semiclassical projections (15) and (16) of the velocity field vector are similar to the quantum ones, see e.g., [22]. They have the same angular dependence as the corresponding quantum projections, while the radial form factors are calculated using the RPA-type equations of motion.
3. Low-energy resonances

In Fig. 1 we display the low-energy part (up to 20 MeV) of the isoscalar dipole strength function (\( E = \hbar \omega \))

\[
S(E) = -\frac{1}{\pi} \text{Im} \bar{R}_0(E).
\]

(19)

Fig. 1. Isoscalar dipole strength function in the low-energy region taking into account the residual interaction between nucleons (solid curve) and in the zero-order approximation (dashed curve). The system contains \( A = 208 \) nucleons.

We study the isoscalar dipole response of a sample “nucleus” of \( A = 208 \) nucleons. Dipole response functions calculated for other values of \( A \), corresponding to other medium-heavy spherical nuclei, are qualitatively similar to the case shown in Fig. 1. The dashed curve is obtained from the internal response function (4) in the zero-order approximation (\( \kappa_1 = 0 \)), while the solid curve shows the internal response function (4) taking into account the residual interaction between nucleons. It can be seen from Fig. 1 that the strength function has three resonance structures already in the zero-order approximation. The inclusion of the residual interaction leads to an insignificant shift of the resonance structures towards low energies.

The strength parameter \( \kappa_1 \) of the isoscalar dipole interaction (2) can be related to nuclear incompressibility \( K_A \) by using the inverse energy-weighted sum rule [23, 17]. We determine the value of the monopole incompressibility \( K_A^{\text{mon}} = 160 \text{ MeV} \) by comparison with the giant monopole-resonance data in \(^{208}\text{Pb} \) within our kinetic model [19]. Then, by assuming \( K_A = K_A^{\text{mon}} \), we get the dipole strength parameter \( \kappa_1 = -7.5 \cdot 10^{-3} \text{ MeV/fm}^2 \). The numerical calculations were carried out using standard values of nuclear parameters: \( r_0 = 1.25 \text{ fm}, \ e_f = 30.94 \text{ MeV}, \text{ and } m = 936 \text{ MeV} \).

The resonances of the dipole strength function (19) are determined by the poles of the intrinsic dipole response function (4), which are given by the solutions of the equation [19]

\[
[-\chi_1(s)][1-\kappa_1 R_0^0(s)] + \kappa_1 (\chi_0^0(s)-\chi_0^0(0))^2 = 0.
\]

(20)

In this equation, the functions \( \chi_1(s) \) and \( \chi_0^0(s) \), which describe dynamic surface effects, see Eqs. (11), (12), and the single-particle response function \( R_0^0(s) \), see Eq. (6), are calculated as integrals over classical trajectories determined by dipole eigenfrequencies (7). In our model, the dynamic (moving) surface provides translational invariance in the same way as in the liquid drop model (LDM). However, our semiclassical approach is based on dynamics in phase-space and, thus, takes into account the deformation of the Fermi surface at nuclear excitation. The absence of the low-energy isoscalar dipole modes in the LDM gives reason to assume that, in our model, the formation of the low-energy resonances is essentially related to the dynamic deformation of the Fermi surface.

We calculate the velocity field (14) associated with the resonance structures of the dipole strength function at the centroid energies. It can be found that in the low-energy region only two branches of the dipole eigenfrequencies (7) at \( n = 0, N = 1 \) and \( n = 1, N = -1 \) contribute to the dipole response function (4) as well as to the dipole velocity field (14). Thus, the nature of the dipole velocity field is associated with the properties of the dipole eigenfrequencies in the low-energy region. In particular, the branch \( s_0(x) \) is associated with a purely angular motion of particles and therefore can be involved in the formation of vortex motion.

In Fig. 2 the velocity fields for the lowest resonance structure are shown in the zero-order approximation at the centroid energy 7.2 MeV (a) and with regard for the residual interaction at the centroid energy 7.1 MeV (b). It can be seen that the velocity fields for the lowest resonance structure do not show vortex motion. Its centroid energy is close to the minimum energy of the dipole single-particle spectrum (7) determined by \( s_{\text{min}} = s_0(x=1) = 1 \). Given that \( s = \omega R/v_r \), we obtain \( E_{\text{min}} = \hbar v_r / R \) and, using the parameters of our model, we can find that \( E_{\text{min}} \approx 7.2 \text{ MeV} \). Taking into account the residual interaction leads to an insignificant shift of the centroid energy to lower energy, see Fig. 1 (solid curve). Thus, this resonance structure is probably associated with single-particle dipole excitations. However, taking into account the residual interaction leads to a visible strengthening of the velocity field,
Fig. 2. The velocity fields in the XZ plane associated with the lowest resonance structure, see Fig. 1, in the zero-order approximation at the centroid energy of 7.2 MeV (a) and taking into account the residual interaction between the nucleons at the centroid energy of 7.1 MeV (b). The system contains A = 208 nucleons.

see Fig. 2, b. This behavior of the velocity field shows collective effects in the lowest resonance structure. This collectivity can be induced by the dynamic deformation of the Fermi surface. To clarify this point, it is necessary to study the properties of the momentum flux tensor associated with this resonance. Such a study can be carried out within our semiclassical approach; however, this will be left for future work.

On the other hand, the velocity fields associated with the overlying resonances of the strength function have the vortex (toroidal) character already in the zero-order approximation, see Figs. 3 and 4.

Fig. 3. Velocity fields in the XZ-plane associated with the second resonance structure, see Fig.1, in the zero-order approximation at the centroid energy 10.2 MeV (a) and taking into account the residual interaction between nucleons at the centroid energy 10.2 MeV (b). The system contains A = 208 nucleons.
In Fig. 3, the results of numerical calculations of the velocity fields associated with the second resonance of the strength function are shown in the zero-order approximation (a) and taking into account the residual interaction (b). It can be seen from Fig. 3 that the inclusion of the residual interaction leads to strengthening the vortex motion associated with this resonance.

Fig. 4, a shows the velocity field for the main toroidal resonance in the zero-order approximation at the centroid energy 12 MeV, while Fig. 4, b displays this velocity field taking into account the residual interaction at the centroid energy 11.5 MeV. This resonance reproduces the nuclear low-energy resonance observed in heavy nuclei [20].

4. Conclusions

The velocity fields associated with the low-energy resonance structures of the isoscalar dipole strength function have been studied within the kinetic model. In this model, taking into account the dynamical-surface degree of freedom, it is possible to obtain an exact treatment of the center of mass motion. It is found that our semiclassical model predicts two toroidal resonances in the energy region below 15 MeV. The main toroidal resonance (the centroid energy 11.5 MeV), see Fig. 4, b, reproduces the nuclear low-energy resonance. The neighboring resonance in the region of lower energies (the centroid energy 10.2 MeV), see Fig. 3, b, also has a vortex (toroidal) character. The results of our semiclassical model are in qualitative agreement with the previous results of the relevant random-phase-approximation (RPA) calculations [5, 10, 11]. The quantitative comparison of our semiclassical model and the quantum approaches is rather difficult due to the different nature of calculations. The centroid energy of the lowest resonance structure is close to the minimum energy of the dipole single-particle spectrum ($E_{\text{min}} = h v_r / R$). The velocity field for this resonance structure does not show vortex motion, however, taking into account the residual interaction leads to collectivity in this resonance, see Fig. 2, b, which may be due to the dynamic deformation of the Fermi surface.

Our semiclassical approach makes it possible to obtain additional information on the nature of collective isoscalar dipole excitations in heavy nuclei. In particular, it would be interesting to study the nature of the momentum flux associated with collective isoscalar dipole excitations. This study could clearly show the effect of the dynamic deformation of the Fermi surface on the formation of nuclear low-energy resonance.
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IСОСКАЛЯРНИЙ ДИПОЛЬНИЙ ВІДГУК ВАЖКИХ ЯДЕР

В ОБЛАСТІ НИЗЬКИХ ЕНЕРГІЙ У КІНЕТИЧНИЙ МОДЕЛІ

Ізоскалярний дипольний відгук важких сферичних ядер в області низьких енергій вивчається в напівлінійній моделі, що епітетована на явний розв’язок лініаризованого кінетичного рівняння Власова для скінченних фермі-систем. У цій транспліційно-інваріантній моделі рух центра має тільки відділяється від внутрішніх збуджень. Ізоскалярна дипольна сила функція має три резонансні структури в області енергій до 15 МeV. Розрахунки полів швидкостей, пов’язаних з резонансними структурами при енергіях центроїда, виявляють вигляд (торідальну) природу двох верхніх резонансів. Основний торідальний резонанс дає якісний опис низькоенергетичного ізоскалярного дипольного резонансу, що спостерігається у важких сферичних ядрах. Похідки функції залежної дипольної дипольної резонансної структури, очевидно, пов’язані з дипольними одночасниками збуженнями. Її енергія центроїда близька до мінімальної енергії дипольного одночастинкового спектра, і врахування залишкової взаємодії призводить лише до незначної зсуву енергії центроїда в бік нижчої енергії. Проте включення залишкової взаємодії помітно посилює поле швидкостей, пов’язане з найнижчим резонансом, що вказує на колективні ефекти в цій резонансній структурі.

Ключові слова: кінетична модель, резонансні структури в області низьких енергій, поле швидкостей, торідальні резонанси.
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ИЗОСКАЛЯРНЫЙ ДИПОЛЬНЫЙ ОТКЛИК ТЯЖЕЛЫХ ЯДЕР
В ОБЛАСТИ НИЗКИХ ЭНЕРГИЙ В КИНЕТИЧЕСКОЙ МОДЕЛИ

Изоскалярный дипольный отклик тяжелых сферических ядер в области низких энергий изучается в полуклассической модели, которая опирается на явное решение линеаризованного кинетического уравнения Власова для конечных ферми-систем. В этой трансляционно-инвариантной модели движение центра масс точно отделяется от внутренних возбуждений. Изоскалярная дипольная сила функция имеет три резонансные структуры в области энергий до 15 МэВ. Расчеты полей скоростей, связанных с резонансными структурами при энергиях центроида, показывают вихревую (тороидальную) природу двух вышележащих резонансов. Основной торoidalный резонанс дает качественное описание низкоэнергетического изоскалярного дипольного резонанса, который наблюдается в тяжелых сферических ядрах. Происхождение нижней изоскалярной дипольной резонансной структуры, очевидно, связано с дипольными одночастичными возбуждениями. Ее энергия центроида близка к минимальной энергии дипольного одночастичного спектра, и учет остаточного взаимодействия приводит только к незначительному смещению энергии центроида к более низкой энергии. Однако включение остаточного взаимодействия заметно усиливает поле скоростей, связанное с самым низким резонансом, что указывает на коллективные эффекты в этой резонансной структуре.

Ключевые слова: кинетическая модель, резонансные структуры в области низких энергий, поле скоростей, торoidalные резонансы.

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