Beyond CCT: The spectral index system as a tool for the objective, quantitative characterization of lamps

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Abstract

Correlated color temperature (CCT) is a semi-quantitative system that roughly describes the spectra of lamps. This parameter gives the temperature (measured in kelvins) of the black body that would show the hue more similar to that of the light emitted by the lamp. Modern lamps for indoor and outdoor lighting display many spectral energy distributions, most of them extremely different to those of black bodies, what makes CCT to be far from a perfect descriptor from the physical point of view. The spectral index system presented in this work provides an accurate, objective, quantitative procedure to characterize the spectral properties of lamps, with just a few numbers. The system is an adaptation to lighting technology of the classical procedures of multi-band astronomical photometry with wide and intermediate-band filters. We describe the basic concepts and we apply the system to a representative set of lamps of many kinds. The results lead to interesting, sometimes surprising conclusions. The spectral index system is extremely easy to implement from the spectral data that are routinely measured at laboratories. Thus, including this kind of computations in the standard protocols for the certification of lamps will be really straightforward, and will enrich the technical description of lighting devices.

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1. Introduction

Observational astronomy progressed for centuries having black bodies as its almost only matter of study: the stars. And for millennia, the only detector system in astronomy was the human eye, unaided or aided by optical devices, what defined the sensitivity curve of human sight as the only spectral band effectively available for the study of the universe.

The end of the xixth century brought the photographic revolution and, with it, a different sensitivity curve that covered a slightly different spectral region, biased towards bluer wavelengths. Even though photographic emulsions are less sensitive than the eye, this new technology allowed the study of much fainter celestial objects, thanks to the possibility to accumulate light during very long exposure times.

Approximately at the same time, spectroscopic techniques led to the discovery of non-thermal emitters in astrophysical contexts: emission nebulae whose light is made up mainly from narrow lines of ionized atoms such as hydrogen, oxygen or sulfur.

More and more non-thermal astrophysical sources have been discovered since then. Also, technological progress opened the whole electromagnetic
Figure 1: Normalized transmission curves for the five filters of the Johnson-Cousins system used in astronomical photometry. More information in Sect. 2.1 and Table 2. See Bessell (1990).

spectrum to astrophysics, and many different bands have been defined, even inside the optical window (that roughly covers from the near-UV to the near-IR). One of the most used photometric systems in observational astronomy is the so called Johnson-Cousins, based on a set of five filters (shown in Fig. 1 and that will be commented later in Sect. 2.1 and Table 2), but many others exist.

Interestingly enough, there exists a strong analogy between the evolution of observational astronomy and that of lighting engineering, that we have outlined in Table 1. Also this field began with black bodies as the only working matter (combustion of solids or gas, and incandescent lamps), and only one sensitivity curve was considered at the beginning: The photopic (day-time) sensitivity curve of the human eye. Somewhat later, the scotopic
Table 1: A comparison of observational astronomy and lighting technology evolution.

|                  | Observational astronomy                          | Lighting technology                      |
|------------------|--------------------------------------------------|------------------------------------------|
| Original spectra | Black bodies (stars)                             | Black bodies (incandescent lamps)        |
| Original band    | Human eye sensitivity curve                      | Human eye sensitivity curve (photopic)   |
| First additional band | Photography (biased towards blue)            | Scotopic sensitivity curve (biased towards blue) |
| Additional spectra | Non-thermal emitters: emission nebulae; synchrotron radiation; etc. Red-shifted spectra. | Non-thermal emitters: discharge lamps; light emitting diodes. |
| Later additional bands | Many bands inside and outside the visible spectrum. For instance, Johnson-Cousins photometric system $UBVRI$. | Bands linked to visual and non-visual pigments: $LC, MC, R, Z, CS$. Non-human sensitivity curves. |

(darkness-adapted) sensitivity curve was added, and it was found to be much more sensitive, and biased towards the blue. Later on, new light sources have appeared, that are not thermal emitters, such as discharge lamps and light-emitting diodes (LEDs).

Huge advancements have happened in recent times, in the research of photo-sensitive pigments in humans and in other species both animal and vegetal, what implied characterizing many spectral sensitivity curves that complement the traditional ones. In this context of non-thermal emitters and multiplicity of spectral bands, analog to the evolution experienced in astronomy, it arises the need to re-think those concepts used in lighting engineering that are based upon the properties of human vision and of thermal light sources.

Correlated Color Temperature (CCT) is a way to link human perception of the hue of lamps to the thermodynamic temperature of black bodies.
The current official definition can be found at CIE (2004). For non-black-bodies, CCT lacks rigorous physical meaning and it provides just a perceptual indication of the hue of the light. Significant differences in the perceived hue are admitted, even for sources having the same CCT. In a multi-band (even non-human-band) and non-thermal context, CCT loses most of its meaning, even in spite the efforts to bring this parameter to the limit of maximum numerical accuracy as in Changjun et al. (2016). We have to ask ourselves whether better methods do exist, to characterize the spectral properties of lamps. From the qualitative and unsatisfactory, one-number CCT descriptor, to the heavy power of giving the whole spectrum in high resolution as suggested by Lucas et al. (2014), there has to be some middle point that allows us to work with just a few numbers, with univocal and clear physical meanings, that should even make it possible to perform meaningful calculations (something completely out of place with CCT).

We explore the promising prospects that arise from the adaptation of some of the techniques developed in multi-band astronomical photometry, to lighting engineering. In particular we will work on the so-called color index system, that we propose to translate into a format suited to the description of lamps under the name of spectral index system: Converging solutions for two fields of study that have followed parallel trajectories during the last two centuries.

2. The spectral index system

Fortunately, the two worlds that converge into this scheme, astronomical photometry and lighting engineering, follow traditions fully compatible in what refers to the conventions used to describe spectral energy distributions and filters, what allows a soft join that may benefit both fields.

2.1. Basic concepts: spectrum and filter

Let us start commenting the basic ingredients needed to compute spectral indices: The spectral energy distribution of the light source, and the transmission curves of filters, equivalent in many senses to the spectral sensitivity curves of photopigments. Both communities prefer working in terms of wavelength (λ) rather than frequency (ν), and the standard way of characterizing filters and sensitivity curves is normalizing them in such a way that the maximum is set to unity. In this paper, all wavelengths are measured in
nanometers (nm). Spectra, also called spectral energy distributions (SED), are noted as $E(\lambda)$. Filters (or sensitivity curves) are noted as $F(\lambda)$.

Table 2 displays some descriptors for a set of filters of interest. Among these filters, there are the five of the astronomical photometric system Johnson-Cousins (see Bessell (1990) for more details). To avoid confusions that may arise from the coincidence of symbols, in this work we will label Johnson-Cousins filters as $U_J$, $B_J$, $V_J$, $R_C$, $I_C$. Table 2 includes, also, the photopic $V$ and scotopic $V'$ sensitivity curves of human vision (see Vos (1978) for $V$, and Wyszecki & Stiles (1982) or Schiergerling (2004) for $V'$), and the spectral sensitivity curves of five human photosensitive pigments given by CIE (2015): cyanopic ($SC$), melanopic ($Z$), rhodopic ($R$), chloropic ($MC$) and erythropic ($LC$), mentioned in Sect. 2.3. The filter descriptors are effective wavelength $\lambda_{\text{eff}}$ and filter width $\Delta \lambda$, defined as follows:

$$\lambda_{\text{eff}} = \frac{\int_0^\infty \lambda F(\lambda) d\lambda}{\int_0^\infty F(\lambda) d\lambda}; \quad \Delta \lambda = \frac{1}{F_{\text{max}}} \int_0^\infty F(\lambda) d\lambda$$

(1)

Where $F_{\text{max}}$ stands for the maximum value of the filter curve $F(\lambda)$, that normally will be equal to unity. Note that, in general, $\lambda_{\text{eff}}$ is not equal to the wavelength at which the curve $F(\lambda)$ reaches its maximum, although the two values should be very similar for filter functions symmetric in shape.

As we will show later, the units of $E(\lambda)$ are not particularly relevant, as long as they are expressed in terms of physical energy (not photon counts), and the function is well calibrated, not affected by instrumental biases nor other spectral filtering. These conditions rule for the material routinely produced in lighting engineering for lamp certification purposes. The spectrophotograph output at the laboratory may be expressed in terms of W/nm, $\mu$W/(cm$^2$ nm) at some standard distance, W/(m$^2$ sr nm), etc. As said, any of these will work perfectly when fed into our formalism.

From the filtered spectrum, $F(\lambda)E(\lambda)$, through integration, we get the integrated flux, $\Phi_{E,F}$:

$$\Phi_{E,F} = \int_0^\infty F(\lambda)E(\lambda) d\lambda$$

(2)

A particular case of filter is set by the absence of any filter at all or, in other words, $F(\lambda) = 1 \forall \lambda$. We refer to this non-filter as the bolometric filter, and it leads to the bolometric flux:
Table 2: Effective wavelengths and filter widths (see definitions in Eq. 1) for several sensitivity curves: Those of the astronomical photometric system Johnson-Cousins, photopic and scotopic curves $V$ and $V'$, and several human photopigments.

| Filter: | Johnson-Cousins | | | | |
|---------|-----------------|------------|-------------|-------------|-------------|
|         | $U_J$ | $B_J$ | $V_J$ | $R_C$ | $I_C$ |
| $\lambda_{\text{eff}}$ (nm): | 365.3 | 438.2 | 552.4 | 645.2 | 885.9 |
| $\Delta \lambda$ (nm): | 53.1 | 98.0 | 104.8 | 129.9 | 300.6 |

| Filter: | Human vision curves |
|---------|---------------------|
|         | $V'$ | $V$ |
| $\lambda_{\text{eff}}$ (nm): | 502.4 | 559.4 |
| $\Delta \lambda$ (nm): | 97.1 | 107.4 |

| Filter: | Photopigments | | | | |
|---------|--------------|------------|-------------|-------------|-------------|
|         | $SC$ | $Z$ | $R$ | $MC$ | $LC$ |
| $\lambda_{\text{eff}}$ (nm): | 452.8 | 496.5 | 512.8 | 542.5 | 566.9 |
| $\Delta \lambda$ (nm): | 51.4 | 83.1 | 96.9 | 110.8 | 118.6 |

$$\Phi_{E,\text{bol}} = \int_0^{\infty} E(\lambda) d\lambda$$  \hspace{1cm} (3)

2.2. Definition of the spectral index

The relative integrated flux among two filters $F_1$ and $F_2$, for a given spectrum $E$, is given by the quotient of integrated fluxes:

$$Q_{1,2}(E) = \frac{\Phi_{E,F1}}{\Phi_{E,F2}} = \frac{\int_0^{\infty} F_1(\lambda) E(\lambda) d\lambda}{\int_0^{\infty} F_2(\lambda) E(\lambda) d\lambda}$$  \hspace{1cm} (4)

The self-normalization implicit in Eq. 4 makes it evident that the specific units in which $E(\lambda)$ is expressed are not relevant.

Finally, the spectral index of spectrum $E$ for the pair of filters $F_1$ and $F_2$ is defined this way:

$$C_{1,2}(E) = -2.5 \log_{10} Q_{1,2}(E) = -2.5 \log_{10} \frac{\Phi_{E,F1}}{\Phi_{E,F2}} = -2.5 \log_{10} \int_0^{\infty} F_1(\lambda) E(\lambda) d\lambda$$  \hspace{1cm} (5)
\[+2.5 \log_{10} \int_0^\infty F_2(\lambda)E(\lambda)d\lambda\]

The quantity \(-2.5 \log_{10} \Phi_{E,F_i}; i = 1, 2\), that appears in Eq. 5, is named *instrumental magnitude of E in filter F_i* and it may be also represented as \(m_{F_i}(E)\). Thus, we can say that a spectral index is a difference of instrumental magnitudes:

\[C_{1,2}(E) = m_{F_1}(E) - m_{F_2}(E) \tag{6}\]

Magnitudes as a system to evaluate the apparent brightness of stars have been in use in astronomy for more than two thousand years, in a tradition that can be traced back at least to the time of Hipparchus of Nicaea (around year 150 B.C., see for instance Ch. 4 in North (1995)). This system is rooted into the peculiarities of human vision, what justifies its logarithmic nature, the instrumental magnitude being simply \(-2.5\) times the decimal logarithm of the integrated flux. It is very important to realize that with the negative sign introduced in the definition of instrumental magnitude, a larger integrated flux implies a lower numerical value for the associated instrumental magnitude. Number \(2.5\) fixes the scale in such a way that a difference of 5 magnitudes implies a factor 100 in integrated flux, and it was implicitly introduced by Hipparchus when he established that the fainter stars seen with the naked eye have magnitude equal to six, while the brighter ones have magnitude equal to one. There is a second and unexpected link of the magnitude scale with human vision: As we will see later (Sect. 3.3), the sensitivity contrast of human vision among photopic and scotopic conditions amounts almost exactly to one magnitude.

For the bolometric filter (i.e., in absence of filter) we get the bolometric instrumental magnitude, that is a measure of the total emission of the lamp across the whole spectrum:

\[m_{\text{bol}}(E) = -2.5 \log_{10} \int_0^\infty E(\lambda)d\lambda \tag{7}\]

An important convention, linked to the definition expressed in Eq. 5, is the need to sort the two filters, \(F_1\) and \(F_2\), in such a way that the first one is always the bluest of the pair. This convention leads to spectral indices with larger numerical values for redder sources, and smaller (even negative) values for bluer spectra. We will refer to this convention as the *bluer first* rule.
The election of decimal logarithms, the negative sign and the 2.5 coefficient may seem arbitrary, but they are directly drawn from the metrological system already in use in astronomical photometry, that already is widely spread to measure the brightness of astronomical sources of radiation like the stars, but also to describe natural and artificial sky brightness. The effects of artificial light at night on sky brightness is commonly measured in the stellar magnitude scale, and it leads in a very natural way to the evaluation of sky color in the same system (see for instance Sánchez de Miguel et al. (2017)). Thus, using the same methodology for the description of lamp spectra and its effects, will establish an interesting bridge between lighting engineering on the one side, and astronomy and the study of artificial light at night on the other side.

When the relative flux (Eq. 4) is of interest on itself, it can be retrieved from the corresponding spectral index in a straightforward way, inverting the definition (Eq. 5):

$$Q_{1,2}(E) = \frac{\Phi_{E,F1}}{\Phi_{E,F2}} = 10^{-C_{1,2}(E)/2.5}$$

(8)

2.3. Sensitivity curves used

The simple formalism sketched in Sect. 2.2 is absolutely general. In order to apply it we need, of course, some specific spectrum $E(\lambda)$ but, obviously, two spectral bands have to be selected and defined. Let us consider which filters or spectral sensitivity curves may be of interest for lighting technology. No doubt, the number of such bands may be very high. Among them we have to include the standard sensitivity curves of human vision, photopic $V(\lambda)$ and scotopic $V'(\lambda)$, described in Fig. 2 and in Table 2. The similarities between the astronomical Johnson $V_J$ band and the photopic $V$ function are very obvious and they are not casual, since the astronomical filter was specifically designed to have a match as good as possible with the sensitivity of human sight.

CIE (2015) defines the spectral sensitivity curves of five human photosensitive pigments, together with a common pre-receptoral transmittance curve that evaluates the spectral absorption of the tissues placed in front of the receptors themselves. From the table in Annex A of CIE (2015) we take five sensitivity curves named cyanopic ($SC$), melanopic ($Z$), rhodopic ($R$), chloropic ($MC$) and erythropic ($LC$). Applying to all of them the associated pre-receptoral transmittance curve $\tau(\lambda)$, and later normalizing the resulting
Figure 2: Normalized human eye standard sensitivity curves under photopic (\(V\), solid line) and scotopic (\(V'\), dashed) conditions. See Vos (1978), Wyszecki & Stiles (1982) and Schiergerling (2004).
Figure 3: Normalized sensitivity curves for the five human photo-receptors, after applying the pre-receptoral transmittance function. See Annex A to CIE (2015).
Figure 4: Action curves named blue-hazard function ($B$, dashed line) and aphakic hazard function ($A$, solid line), from ICNIRP (2013). The descriptors of $B$ function (Eqs. 1) are $\lambda_{\text{eff}} = 446.2$ nm, $\Delta \lambda = 69.0$ nm, not too far from the astronomical Johnson $B_J$ band (Table 2, Fig. 1). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
curves to max = 1, we get the sensitivity curves described in Table 2 and Fig. 3.

The International Commission on Non-Ionizing Radiation Protection defines in [ICNIRP (2013)] the blue light hazard function ($B$) and the aphakic hazard function ($A$) (see Fig. 4). For the reasons described in the source publication, and commented in Sect. 4.1 in the discussion of Fig. 10 curve $A$ cannot be normalized to max = 1, what has to be kept in mind when interpreting results involving this filter.

Several interesting, and very simple bandpasses can be defined as step functions, specially filters transparent to only short wavelengths. We label such filters as $L_{\lambda_0}$, where $\lambda_0$ is such that:

$$L_{\lambda_0} = \begin{cases} 1 & \forall \lambda \leq \lambda_0 \\ 0 & \text{otherwise} \end{cases} \quad (9)$$

We will refer to such filters as lowpass-$\lambda$, because we are working in the wavelength space and those filters block high values of $\lambda$, leaving low values unaltered. Of course, the same physical filter (maybe made from glass) would be described as a highpass if we were working in the frequency space.

In a similar way, highpass-$\lambda$ filters may be defined as:

$$H_{\lambda_0} = \begin{cases} 0 & \forall \lambda \leq \lambda_0 \\ 1 & \text{otherwise} \end{cases} \quad (10)$$

The product of a highpass by a lowpass specifies a window filter. Thus, filter $H_x \times L_y$ would be transparent to wavelengths between $x$ and $y$ nm.

Lowpass-$\lambda$, highpass-$\lambda$ and window filters are important because there are already several regulations and recommendations establishing spectral restrictions on lamps, on the basis of the quantity of radiation emitted below, above or between certain specific wavelengths. As an example, Table 3 (that will be commented in Sect. 3) summarizes some of these already existing specifications, translating them into the language of spectral indices.

### 3. Some useful specific indices

Now we get closer to the specific application of the formalism. In order to do that, in this section we review several pairs of filters that lead to spectral indices meaningful for the description of lamp spectra. First we consider filter pairs that include the bolometric filter to derive bolometric indices.
In a second step we review indices implying lowpass-$\lambda$ filters. Finally, we describe some generic indices made up from filter pairs of any kind.

### 3.1. Bolometric indices

We talk about bolometric indices when dealing with spectral indices that include the bolometric filter as one of the two sensitivity curves involved in the calculation (Eq. 5). The bluer first convention linked to the definition of spectral indices (Sect. 2.2) requires that the first filter of the pair has to be the bluer one. The bolometric filter, being described by a constant function equal to unity for all values of $\lambda$, is in fact an infinitely red filter. Thus, it is always the redder of any pair and has to be introduced in the formulae as $F_2$. Representing the first filter as $F$, whatever it is, and the bolometric one (the second) as bol, we have the definition of bolometric index of spectrum $E$ for filter $F$:

$$C_{F,bol}(E) = m_F(E) - m_{bol}(E)$$  \hspace{1cm} (11)

It is evident that under these conditions we will always have more radiation in the second filter (the bolometric one means absence of any filter), what will make $m_{bol}(E)$ smaller (i.e., 'brighter', due to the negative sign in the definition of instrumental magnitudes). As a conclusion, all bolometric indices will always be positive. They provide a way to evaluate the portion of energy emitted by a lamp in the band covered by filter $F$, compared to the total emission over the whole spectrum.

It is time to start applying the formalism to specific lamps. Let us take as first examples the spectra displayed in Fig. 5 corresponding to a PC amber LED lamp, a metal halide lamp, and a standard incandescent bulb. If we take the photopic sensitivity curve $V$ as $F_1$, the corresponding bolometric index will measure the amount of useful radiation (from a photopic point of view) emitted by the lamps, compared to their total emission. We get the results:

|          | $E$   | $C_{V,bol}(E)$ | $Q_{V,bol}(E)$ |
|----------|-------|----------------|----------------|
| PC amber | 0.721 | 0.515          |
| Metal halide | 0.819 | 0.470          |
| Incandescent | 2.096 | 0.145          |

These figures indicate a higher visual efficacy of the PC amber lamp, if the photopic emissions are compared to the total amount of radiation. In
Figure 5: Spectra of three lamps used as examples for the application of the spectral index system. A PC amber lamp (number 39 in the ancillary database Galadi-Enríquez (2017)), a discharge metal halide lamp (number 25) and a classical incandescent bulb (number 29).
Table 3: Some existing regulations whose requirements may be expressed in terms of spectral indices.

| Location                                      | Reference               | Requirements                                                                 |
|-----------------------------------------------|-------------------------|------------------------------------------------------------------------------|
| Andalusia (Spain)                             | BOJA (2010)             | In general: $L_{525} - \text{bol} > 0.753$                                  |
|                                               | Article 13.a            | At protected areas:                                                          |
|                                               |                         | $L_{440} - \text{bol} > 2.060$ for non-LED                                  |
|                                               |                         | $L_{500} - \text{bol} > 2.060$ for LED                                       |
| Canary Islands (Spain)                        | IAC (2017)              | IAC amber LED definition:                                                    |
|                                               | Section G.8             | $L_{500} - \text{bol} > 4.560$                                              |
|                                               |                         | IAC warm LED definition:                                                     |
|                                               |                         | $L_{500} - V > 1.505$                                                       |
|                                               |                         | IAC super-warm LED definition:                                               |
|                                               |                         | $L_{500} - V > 2.060$                                                       |
| Antofagasta, Atacama and Coquimbo (Chile)     | DORCh (2013)            | Simultaneously required:                                                     |
|                                               | Article 7               | $H_{300} \times L_{379} - H_{380} \times L_{780} > 2.060$                  |
|                                               |                         | $H_{380} \times L_{499} - H_{380} \times L_{780} > 2.060$                  |
|                                               |                         | $H_{380} \times L_{780} - H_{781} \times L_{1000} < -0.753$                |

In fact, the visible light emitted by the PC amber lamp is larger than 50% of the total, because its index is lower than $0.753 = -2.5 \log_{10}(0.5)$. The lower photopic efficacy corresponds in this case to the incandescent bulb: its very red (high value) index is due to its large amount of infra-red emission, and it has to be taken into account that the experimental spectrum used is truncated at certain infra-red wavelength, so the true value for its $C_{V,\text{bol}}$ index should be even larger, meaning in this case an even lower light efficacy.

The maximum efficacy in the bolometric index for an arbitrary filter would correspond to a lamp emitting monochromatic light at the wavelength in which the first filter curve reaches unity, and in this case the bolometric index would be equal to zero. The closer the bolometric index gets to zero, the higher the lamp efficacy for the filter used to perform the computations.

An interesting case is that posed by low- and highpass-\(\lambda\) bolometric indices. There are already some regulations and recommendations on lamp spectra that establish a certain limit for the fraction of the total radiation
emitted below or over a specific wavelength, or inside certain intervals. Table 3 shows several examples. Specifically the lowpass – bolometric criteria is included in the Andalusian regulation (BOJA (2010)), and in the Instituto de Astrofísica de Canarias (IAC) amber LED definition from IAC (2017). For instance, in one case it is required that the total emission below 500 nm has to be less than 15% of the total (Andalusian requirement for LEDs at protected areas). That means that the $L_{500} - \text{bol}$ index has to be larger than 2.060. In the case of the same three previous lamps, we get:

|          | $E$  | $C_{L500,\text{bol}}(E)$ | $Q_{L500,\text{bol}}(E)$ |
|----------|------|--------------------------|--------------------------|
| PC amber | 5.662| 0.005                    |                          |
| Metal halide | 1.652| 0.218                    |                          |
| Incandescent | 3.618| 0.036                    |                          |

In this case, the PC amber lamp would widely fulfill the requirement, but this specific metal halide lamp would not. It has to be noted that also the incandescent lamp taken as example fulfills this limit. This PC amber lamp qualifies, too, as "IAC amber LED", because its index is larger than 4.560 (see Table 3).

However, bolometric criteria can be criticized, because they measure efficacy comparing a certain band to the total amount of emission, including even non-visible wavelengths. For instance, the above-mentioned criterium based on a lowpass-$\lambda$ bolometric index, very clearly favours lamps with strong infra-red emissions. If going beyond a certain value of $C_{L500,\text{bol}}$ is required, this can be achieved not only by reducing the amount of light at the blue side, but also by increasing the wasteful infra-red radiation at the red side. This is the main reason why incandescent lamps display such large values for indices of this kind, as can be seen in the data accompanying this article, Galadí-Enríquez (2017).

3.2. Generic indices for pass-$\lambda$ filters

That caveat can be easily overcome just setting the right, non-bolometric band, as second filter for the calculations. An obvious enough option would be using the photopic curve $V$ as a reference. This curve has its effective wavelength around $\lambda = 560$ nm. The central wavelength of any lowpass-$\lambda$ filter $L_x$ is placed exactly at $x/2$. Given that normally the aim is to limit the amount of emission in blue bands, most often $x/2$ will be smaller than 560 nm, and the step filter will be the first (bluer first), and the photopic
curve will act as the second (redder) band for spectral index calculation. The resulting index $L_x - V$ would describe the quantity of energy emitted in the blue, below $\lambda = x$ nm, compared to the amount of photopically efficient light, what seems a fair and meaningful comparison. The recent update of the regulations at Canary Islands (see IAC (2017)) includes several specifications that may be easily translated into our formalism using exactly this index. They appear in Table 3, and they refer to the definitions of ”IAC warm LED” and ”IAC super-warm LED”, that require the index $L_{500} - V$ to reach at least the values 1.505 (warm) or 2.060 (super-warm). Going back to the same three lamps that we are using as an example, for $L_{500} - V$ we get the results:

| $E$       | $C_{L_{500},V}(E)$ | $Q_{L_{500},V}(E)$ |
|-----------|--------------------|--------------------|
| PC amber  | 4.491              | 0.016              |
| Metal halide | 0.833            | 0.464              |
| Incandescent | 1.522            | 0.246              |

Not surprisingly, the PC amber LED overruns both criteria, but also the incandescent lamp would fit in the ”warm” box (although these IAC definitions are intended only for LEDs). Now we get a significantly bluer (lower) value of the index for the incandescent bulb, compared to the bolometric result, because for index $L_{500} - V$ all infra-red emissions are kept out of the calculation. Something similar, to a lesser extent, happens to the metal halide lamp.

3.3. Totally generic indices

The flexibility of the spectral index system arises from its totally generic character, allowing to select any pair of spectral bands of interest.

As an illustration of the general scheme, let us translate into the spectral index formalism a classical photometric parameter, the so-called ”scotopic to photopic ratio”, or $S/P$. Traditionally, this ratio is computed from the lamp spectrum $E(\lambda)$, filtered trough the two standard sensitivity curves of human vision displayed in Fig. 3. The scotopic $V'(\lambda)$ and photopic $V(\lambda)$ functions are normalized to maximum equal to unity, and they can be introduced into Eq. 5 as $F_1$ and $F_2$ (bluer first) to compute the ratio $Q_{V',V}(E) = \frac{\Phi_{E,V'}}{\Phi_{E,V}}$ and, from there, the scotopic – photopic spectral index $C_{V',V}(E)$ for the lamp. Applying this to the same three examples, we get:
In this scheme, a null value would mean "same energy in both bands". The first lamp (PC amber) has an index value close to 2, meaning that the blue filter (scotopic) is receiving approximately 15% of the energy that goes through the red (photopic) one. The value for the incandescent lamp is close to the 50% energy ratio (photopically efficient energy doubles the amount of scotopically efficient flux).

But normally the S/P ratio is computed not from the normalized $V'$ and $V$ functions, but from the scaled versions of these functions, including the well-known scaling factors 1700 lm/W for the scotopic curve, and 683 lm/W for the photopic:

$$K'(\lambda) = 1700 \, V'(\lambda); \quad K(\lambda) = 683 \, V(\lambda)$$  \hspace{1cm} (12)

This way we move from the energy domain into the human perceptual domain, going further from the physical input and closer to the action exerted by this input. The logarithmic nature of the spectral index allows a straightforward transformation from the normalized index $C_{V',V}(E)$ to the scaled index $C_{K',K}(E)$ just applying an additive zero point equal to $-2.5 \log_{10}(1700/683)$. We find the happy coincidence that this zero point is almost exactly equal to $-1$, in fact it is $-0.990$, allowing the immediate conversion from the normalized scotopic – photopic index to its scaled version:

| $E$        | $C_{K',K}(E)$ | $S/P$ |
|------------|---------------|-------|
| PC amber   | 1.042         | 0.383 |
| Metal halide | -0.469       | 1.540 |
| Incandescent | -0.194       | 1.196 |

In the scaled version $C_{K',K}(E)$, a null value would mean "same efficacy (or action) in both bands". The figures indicate that the PC amber LED exerts an action some 2.5 times more intense photopically than scotopically, but the contrary happens with the other two lamps. The incandescent one is closer to the equilibrium of actions (null value), while the metal halide lamp displays a clearly stronger scotopic action. The classical S/P ratio can be derived from the scaled index through Eq. 8.
Figure 6: The scotopic – photopic normalised \((V' - V)\) spectral index (a measure of the \(S/P\) ratio) as a function of CCT, for the set of lamps discussed in Sect. 4. The correlation is poor, implying that CCT is not a good indicator for \(S/P\) considerations. The solid line marks the location of black bodies.
CCT is a poor proxy of the $S/P$ ratio, as can be seen in Fig. 6, that shows the normalized index $C_{V',V}$ as a function of CCT, for the set of lamps discussed in Sect. 4.

Indices related to the $S/P$ ratio may be of specific interest in lighting, but they are not specially useful to classify lamps according to their content of blue light, because the two filters $V'$ and $V$ are very close to each other, and they show a significant overlap. Studies on the environmental and health effects of artificial light at night place focus on the limitation of blue light and, for this purpose, at some point, some small set of standard filter pairs should arise from a consensus among the scientific and technical community interested in lamp characterization. Without any intention of making a firm proposal in this sense, and with the only aim of illustrating the system, we will consider the spectral index defined by means of the melanopic curve $Z$ (Fig. 3, Table 2) and the photopic $V$ function. The melanopic curve, with effective wavelength around $\lambda = 495$ nm, is related to the pigment active in the intrinsically photosensitive retinal ganglion cells, and it is linked to the regulation of the human circadian system, a matter of special interest for studies on artificial light at night and chronodisruption. The resulting $Z-V$ index may act as a measure of the input to the ganglion cells per each unit of photopically useful light. Often, redder (larger) values of this index will be preferred, meaning a smaller amount of potentially disruptive light (from a circadian point of view) per lumen.

The ancillary data in Galadí-Enríquez (2017) give this index for a large set of lamps. Here we show the values that are obtained for the three examples that we have been using in previous sections:

| $E$          | $C_{Z,V}(E)$ | $C_{Z,V}(E)$ |
|--------------|--------------|--------------|
| PC amber     | 2.937        | 0.067        |
| Metal halide | 0.787        | 0.484        |
| Incandescent | 1.082        | 0.369        |

Finally, let us comment that the Chilean regulation for their northern astronomical regions, DORCh (2013), specifies limits both on the amount of blue, and on the amount of red emission, compared to the quantity of light emitted in the central part of the visible interval. We express these criteria as conditions on indices built from window filters, in Table 3.
4. Application to a lamp sample

We illustrate the formalism deriving a set of selected spectral indices for a sample of more than sixty lamp spectra. The contents of the database are described in detail in Appendix A and they are available at Galadí-Enríquez (2017). In this section we discuss some conclusions that can be drawn from those results, and we compare our spectral index proposal with several similar ideas found in the literature.

4.1. Discussion

The results that can be found in the ancillary data set, Galadí-Enríquez (2017), lead to several conclusions.

The $Z - V$ melanopic – photopic index provides some insight into the blueness of light sources, in relation to the amount of useful light emitted. Fig. 7 displays the relation between this index and CCT for the whole sample of light sources studied. The solid line represents the black body locus. We see that, in general, all artificial lamps are redder than black bodies with the same CCT (spectral indices are larger than those of black bodies), a tendency that is specially strong towards low CCT values. The LED lamp with negative $Z - V$ index (source number 60) is a very special device used for signaling, not for lighting purposes. We clearly see a general correspondence between CCT and $Z - V$, but it is evident that this relation shows a significant spread, a spread that intensifies, too, in the low CCT area.

The correlation with CCT is worse for those sources whose spectra are more different to those of black bodies. Fig. 8 shows this for discharge lamps of two different technologies: fluorescent (low pressure mercury) and metal halide. Let us underline that in the second case, the lamp with highest CCT (number 27) is the reddest for this filter pair, contrary to what may be expected if CCT would be a good spectral descriptor.

Zooming into the low CCT area (Fig. 9), we can see that the spread of the relation makes CCT almost meaningless as a descriptor of blueness for this pair of filters, for CCT values larger than 2500 K, approximately. Over an interval that covers a span of some 2000 K, the spectral index $Z - V$ varies from 0.7 to 1.0, only 0.3 magnitudes, and not always in a monotonic way. LEDs clustering around CCT = 3000 K show a similar variation in $Z - V$, from 0.9 to 1.2 magnitudes.

The region of the extremely low CCT lamps (below 2500 K, the so-called ‘warm-light emitters’) shows the largest spread. Non-standard LEDs clearly
Figure 7: The melanopic – photopic ($Z - V$) spectral index (a measure of the circadian input per lumen) as a function of CCT.
Figure 8: The melanopic – photopic \((Z - V)\) spectral index as a function of CCT for fluorescent and metal halide lamps. The lack of correlation is specially outstanding for these technologies.
Figure 9: The melanopic – photopic ($Z - V$) spectral index as a function of CCT for sources below CCT = 4500 K. Inspecting this graph illustrates what kind of mistakes may be possible when classifying lamps according to CCT as if it was a measure of the amount of blue light in the spectra, even when dealing with only one technology. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)
Figure 10: The blue hazard – photopic \((B - V)\) spectral index as a function of CCT. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

...demonstrate their potential as light emitters with blue-light content levels even lower than those of the traditional sodium discharge lamps. However, let us keep in mind that such devices are currently seldom used actually for real lighting, where much bluer LEDs are normally used, in the CCT interval over 2500 K, usually with much bluer \(Z - V\) values, around or below \(Z - V = 1\).

Index \(B - V\) (blue-hazard – photopic) shows a better correlation with CCT, but still with a significant scatter. Redder light sources (CCT < 2000 K) show an extreme deviation from theoretical black bodies (Fig. 10). Several lamps are slightly bluer than the corresponding black bodies for this filter pair. In this index, non-standard LEDs are even redder than low-pressure sodium (pure amber LED, PC amber LED, Philips 1800 K LED).
As commented in Sect. 2.3, attention should be paid to the results derived for the aphakic–photopic index, due to the non-standard character of the aphakic $A$ curve, displayed in Fig. 4. The $A$ curve is an adaptation of the blue-hazard $B$ curve, to take into account the increased transparency of eye tissues to blue light in very young persons. It cannot be normalized to maximum unity because it has to follow the profile of curve $B$ at the red side. For this reason, aphakic indices are always much bluer (smaller) than the equivalents built from the standard blue-hazard function. From the pair of indices given in the ancillary data set, $B - V$ and $A - V$, a third one can be very easily deduced as the difference: $A - B$. This index will always be negative, and it provides a measure of how the potential risk of the blue light from a lamp may be increased for very young subjects.

We have already commented the similarity among the astronomical filter $V_J$ and the photopic function $V$. The blue-hazard $B$ curve is not too far from the Johnson-Cousins $B_J$ filter. As a result, there exists a significant correlation between the values of the indices $B - V$ and $B_J - V_J$, but it is not good enough to forget the differences. So, care has to be taken not to confuse the perceptual functions $B$ and $V$ with their Johnson-Cousins astronomical close relatives, named $B_J$ and $V_J$ in this paper, but labelled exactly with the same symbols $B$ and $V$ in the astronomical literature.

Index $L_{500} - V$ has some chances to become a kind of standard to classify lamps according to their amount of blue emission, if we want to evaluate this per unit of photopically efficient light (let us say, per lumen). Regulations already in use in Canary Islands [IAC (2017), Table 3] rely on this kind of criteria. As we see in Fig. 11, CCT would be a bad descriptor for the evaluation of the content of blue light, specially in the interval of the most frequent CCTs, from 2000 to 4000 K. All lamps analyzed are redder than the black bodies with the same CCT.

4.2. Comparison with alternative formalisms

The specific features of lamp spectra characterization by means of the spectral index system can be enumerated as follows:

1. The system is quantitative, providing the numerical and physical meaning that CCT lacks.
2. Flexibility and general character, free election of filter pairs according to the needs of each field of study.
Figure 11: The $L_{500}$ - photopic ($L_{500} - V$) spectral index as a function of CCT.
3. Unit-independent. Any index can be computed from any spectrum, expressed in the units you want. The mandatory comparison of two bands extracted from the same spectrum eliminates any worry about units. We may say that the index is self-normalized.

4. As a consequence of the previous point, no reference source has to be defined or used. Each spectrum acts as its own calibrator.

5. Standard data already obtained at labs for lamp characterization are perfect for spectral index computation. The calculations are even simpler than those routinely performed for CCT determination.

6. The logarithmic nature allows lamp comparison through simple additions and subtractions, as well as the inclusion of scale factors in the form of additive zero points.

7. The formalism is fully compatible with that already in use in astronomy, specially in the field of studies on artificial sky brightness and light pollution.

A spectral index specifies an elemental physical output from lamps, at a very low level. For instance, when using pigment sensitivity curves as those showed in Fig. 3, the resulting indices would describe the direct physical-chemical input on the corresponding sensitive cells. The possible relation of this elemental input to more complex physiological effects falls out of the scope of the formalism. We may, again, pose a comparison with astrophysics, where stellar spectra are described very often by means of the Johnson index $B_J - V_J$ (do not confuse with our blue hazard – photopic index defined above). That index constitutes an elementary description of the star, and establishing any further relation between this index and more complex quantities, such as effective temperature, falls on the side of the applications of the measurement, not on the side of performing the measurement itself.

Of course, the idea of describing lamp spectra performing computations from the integration on several bands has been present in lighting engineering from the beginning, and definitions such as that of luminous efficacy or $S/P$ ratio are already based on concepts of this kind. In recent years, with the increasing need of going multi-band, several proposals have circulated, pointing to ideas quite close to the spectral index formalism described in this paper. We will briefly comment a representative subset of them.

ˇZukauskas et al. (2012) study the way to optimize solid-state lamps for photobiologically friendly lighting. This leads them to consider quotients of integrated fluxes defined in a fashion similar to what we state in Eq. 2.
The main differences are that they apply scaling factors (in units of lm/W), and that their second filter is always the bolometric one, i.e., they normalize their 'luminous efficacy of radiation' estimators (equations 1, 2 and 3 of their paper) according to the total integrated flux emitted by the lamps. Their 'circadian efficacy of radiation' estimator is very similar to our quotient of integrated fluxes $\Phi_Z(E)/\Phi_{bol}$. The quotients of such estimators expressed in their equation 6 would be equivalent to a difference of our spectral indices (their dimensional multiplicative factors would transform into just an additive, spectrum-independent, zero point, by the way). Later on in the paper, the authors introduce these elementary quantitative estimators inside a non trivial model, to derive figures of merit for their scientific purposes, a process that may have been done using spectral indices too.

Aubé et al. (2013) developed a system that received even a denomination similar to ours. They define quotients of integrated fluxes very close to our $\Phi_1(E)/\Phi_V(E)$; note that this time the second filter is always the photopic one, in what they call 'constant lumen normalization' (equation 3 in their paper). After defining three filters of interest to be used as $F_1$, they compute their final 'indices' relative to the values of a standard illuminant (specifically the ICE standard illuminant D65). In the language of our system, this would translate into applying the corresponding spectral index of the standard illuminant as a subtractive zero point, due to the logarithmic nature of our proposal. For instance, in our scheme, $C_{Z,V}(D65) = 0.186$ and, thus, the same index for any other spectrum may be referred to the D65 scheme just subtracting this value: then, $C_{Z,V}(E) - C_{Z,V}(D65) = 0$ would mean 'the same Z/V ratio as the standard illuminant', while in our simpler scheme (not relative to any standard illuminant) $C_{Z,V}(E) = 0$ has a meaning closer to the physical reality: 'same energy in both filters'.

The sound work by Aubé et al. (2013) is not totally general. It is doubtful whether it is really necessary to rely on a standard illuminant as reference. The non-logarithmic character places the work further from astronomical tradition (what may be more relevant for their 'star light index') and, while turning easier the determination of their 'indices', it makes somewhat more cumbersome their later management in practical use. Their specific selection of filters is just one among many other possible, but maybe they are too complicated, mixing simple physical inputs with non-trivial considerations about effects and actions that, in our opinion, should be left for later stages in the interpretation. We pursue the computation of simple numbers as close as possible to the true, native and neat properties of the spectra.
Finally, Escofet & Bará (2015) delve into the complexities of circadian inputs going back to the elementary concept of integrated filtered flux of this work (Eq. 2), and of Zukauskas et al. (2012), to later combine several filters (or ‘weighting functions’) in a shape that may have been formulated, too, in our language of spectral indices. In our opinion, Escofet & Bará (2015) offer an interesting example of clean separation of inputs and actions or effects, with an approach at only one step from using a completely general formalism for the multiplicity of filters used by them. These authors deal with those filters on a one-by-one basis, handling a complex network of scaling factors and standards that we avoid in our spectral index system, seeking maximum simplicity and homogeneity.

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Appendix A. Lamp sample: contents of the database

The spectral index formalism has been applied to a database with more than sixty spectra kindly provided by the sources mentioned in the acknowledgments section, for the only purposes of this work. We have also used a set of lamp spectra from the public open repository Light Spectral Power
Several black body spectra have been generated by us in a straightforward way. The results are available at Galadí-Enríquez (2017), and they cover different lighting technologies, including experimental spectra obtained with different spectrometers. For each available spectrum we compute the following spectral indices:

- **Photopic – bolometric,** $V_{\text{bol}}$. A measure of the luminous efficacy of lamps, showing the fraction of spectral energy emitted inside the photopic band. For this index, bluer lamps (with lower values of the spectral index) would be more efficient from a lighting point of view.

- **Lowpass-λ 500 nm – photopic,** $L_{500} - V$. Comparing the amount of energy emitted below 500 nm with that efficient for lighting purposes, in photopic conditions. A good measure of the amount of blue light compared to lighting efficacy. If the aim is to reduce the amount of blue light, then larger values of this index are preferred.

- **Melanopic – photopic,** $Z - V$. It compares the amount of light active on the circadian receptors with the intensity efficient for lighting in photopic conditions. Again, larger values are better, in the sense that they indicate a lower input on the intrinsically sensitive retinal ganglion cells.

- **Lowpass-λ 500 nm – bolometric,** $L_{500} - \text{bol}$. Reflects the ratio between blue light and the total amount of energy radiated by the lamp.

- **Blue hazard – photopic,** $B - V$, and **aphakic – photopic,** $A - V$, according to the curves specified by ICNIRP (2013).

The results are shown for all lamps in the ancillary data attached to this article. For each index, the value $C_{1,2}(E)$ according to definition 5 is given but, also, for a better understanding of the system for people not well acquainted with the logarithmic scale, the relative integrated flux $Q_{1,2}(E) \ (\text{Eq. 4})$ is provided. A last column contains the value of the correlated color temperature (CCT) of the lamps, in kelvins, directly drawn from the sources that provided the spectra.

A graphic annex displays the spectra for 61 of the 69 light sources.

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