Nonperturbative Spontaneous Symmetry Breaking

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Abstract

A nonperturbative approach for spontaneous symmetry breaking is proposed. It is based on some properties of interacting field operators. As the consequences an additional terms like to $m^2A^2$ appears in the initial Lagrangian.

1 Introduction

One of the most astonishing results of quantum field theory is spontaneous symmetry breaking. One can say that it is the situation when something arises from the quantization. This means that on the quantum level one has something that was not present on the classical level. Coleman and Weinberg [1] write: “... higher-order effects may qualitatively change the character of a physical theory ...”. The main goal of the Coleman-Weinberg mechanism is to derive a Higgs potential from more fundamental principles, with as few arbitrary parameters as possible. In this mechanism the Higgs potential is induced by radiative corrections, rather than being inserted by hand. In this approach one can summarize over higher-loop graphs to induce an effective potential, which may then produce spontaneous symmetry breaking. Of course, it can be only in the theories with interactions (where we have these higher-loop graphs) : i.e. a nonlinearity (in Lagrangian) can lead to the interesting consequences for quantized theory. It is not very surprisingly because on the classical level we have the same : very simple behavior of a classical linear theory can be changed on the very complicated and surprising behavior of a classical nonlinear theory.

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For example, in nonlinear theories we have monopoles, instantons, black holes, strange attractors and so on. On the quantum level we can expect once more amazing stuffs if we add some nonlinear terms in Lagrangian. Probably one of such manifestations of a nonlinearity is confinement in the QCD.

The problem here is that we do not have detailed techniques for the nonperturbative calculations. Even on the perturbative level we do not sure that the result after the sum over all Feynman diagrams will the same as after the sum over a finite number of Feynman diagrams. Nevertheless, according to perturbative calculations we know that they change an initial Lagrangian so that an extra potential term arises in the Lagrangian (Coleman-Weinberg mechanism).

In this paper we work out a nonperturbative method which can be applied for strongly interacting fields and to show that extra terms (or nonperturbative spontaneous symmetry breaking) appear in an initial Lagrangian if the product of field operators have some properties concerning to the rearrangement of the brackets in a nonlinear potential term.

2 Nonlinear term in non-Abelian gauge theories

Our basic attention here is devoted to the non-Abelian gauge field SU(2) (for the simplicity we will consider only this gauge group). The Lagrangian is

$$L = \frac{1}{4g^2} F^a_{\mu\nu} F^{a\mu\nu}$$

where $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g \epsilon^{abc} A^b_\mu A^c_\nu$ is the field strength and $A^a_\mu$ is the gauge potential; $g$ is the coupling constant; $\epsilon^{abc}$ are the structural constants of the gauge group SU(2); $a = 1, 2, 3$. In the quantum case we have the operators $\hat{A}^a_\mu$ and $\hat{F}^a_{\mu\nu} = \partial_\mu \hat{A}^a_\nu - \partial_\nu \hat{A}^a_\mu + g \epsilon^{abc} \hat{A}^b_\mu \hat{A}^c_\nu$. Let us underline that all operators considered here are the operators of interacting fields in contrast with the perturbative techniques where these operators describe non-interacting fields.

Let us consider the nonlinear part of the field strength operator $\hat{F}^a_{\mu\nu}$:

$$(\hat{F}^{nl})^a_{\mu\nu} = \epsilon^{abc} \hat{A}^b_\mu \hat{A}^c_\nu.$$  

At first we assume that this product do not have any singularity as it is the product of interacting fields. Physically it means that there are situations in interacting field theories where these singularities do not occur (e.g. for flux tubes in Abelian or non-Abelian theory quantities like the “electric” field inside the tube, $\langle E^a_\mu \rangle < \infty$, and energy density $\varepsilon(x) = \langle (E^a_\mu)^2 \rangle < \infty$ are nonsingular). Here we take as an assumption that such singularities do not occur.

Thus we have such nonlinear term

$$\left( (\hat{F}^{nl})^a_{\mu\nu} \right) \left( (\hat{F}^{nl})^{a\mu\nu} \right) = \epsilon^{abc} \epsilon^{ade} \left( \hat{A}^b_\mu \hat{A}^c_\nu \right) \left( \hat{A}^{d\mu} \hat{A}^{e\nu} \right) =$$

$$(\delta^{bd} \delta^{ce} - \delta^{be} \delta^{cd}) \left( \hat{A}^b_\mu \hat{A}^c_\nu \right) \left( \hat{A}^{d\mu} \hat{A}^{e\nu} \right) =$$

$$\left( \hat{A}^b_\mu \hat{A}^c_\nu \right) \left( \hat{A}^{b\mu} \hat{A}^{c\nu} \right) - \left( \hat{A}^b_\mu \hat{A}^c_\nu \right) \left( \hat{A}^{c\mu} \hat{A}^{b\nu} \right).$$  

2
Our main assumption is that this nonlinear expression has some properties that allows us to say that

\[
\left( \hat{A}_a^a \hat{A}_b^b \right) \left( \hat{A}_a^{\beta \mu} \hat{A}_b^{\alpha \nu} \right) - \left( \left( \hat{A}_a^a \hat{A}_b^b \right) \hat{A}^{\beta \mu} \right) \hat{A}^{\alpha \nu} \neq 0
\]  

(3)

and

\[
\left( \hat{A}_a^a \hat{A}_b^b \right) \left( \hat{A}_b^{\alpha \nu} \hat{A}_a^{\beta \mu} \right) - \left( \left( \hat{A}_a^a \hat{A}_b^b \right) \hat{A}^{\alpha \nu} \right) \hat{A}^{\beta \mu} \neq 0
\]  

(4)

and these differences are not connected with the commutator \([\hat{A}_a^a, \hat{A}_b^b]\). The second term in Eq. (3) after some manipulations with brackets and permutations of \(A_a^a\) can be led to \((\hat{A}_a^a \hat{A}^{\alpha \nu})(\hat{A}_b^b \hat{A}^{\beta \mu})\). In this case this equation tell us that the square of nonlinear part \((\tilde{F}_{nl})_{\mu \nu}^a\) of the field strength is not equal to the square of the vector square \(\hat{A}^4 = (\hat{A}_a^a \hat{A}^{\alpha \nu})^2\), i.e. the r.h.s. of Eq. (3) is nonzero. In some sense it is like to Cooper pairing in the superconductivity. The initial potential term for electrons is

\[
\left( \hat{\psi}_\beta \right) \left( \hat{\psi}_\alpha \right)
\]

and after some manipulations we have

\[
\left( \hat{\psi}_\alpha \hat{\psi}_\beta \right) \left( \hat{\psi}_\beta \hat{\psi}_\alpha \right)
\]

where every pair of brackets \((\cdots)(\cdots)\) has an independent physical meaning: each pair describes annihilation and creation of Cooper pair. We note that these pairs were not present in the initial Lagrangian. Another words, at first we had electrons and then Cooper pairs.

The differences (3) (4) are connected only with the nonlinearity of the term \((\tilde{F}_{nl})^2\). In order to calculate this difference we will consider at first a very simple example.

3 Simple example

Let \(\hat{a}\) and \(\hat{b}\) are operators which have a nonassociative property

\[
\left( \hat{a} \hat{b} \right) \left( \hat{a} \hat{b} \right) - \left( \left( \hat{a} \hat{b} \right) \hat{a} \right) \hat{b} \neq 0.
\]  

(5)

In order to calculate this commutations relationship we compare its with the ordinary commutators in a linear field theory

\[
\hat{\phi}(x) \hat{\phi}(y) - \hat{\phi}(y) \hat{\phi}(x) = -i\hbar D(x - y)
\]  

(6)

where \(\hat{\phi}(x)\) is some field operator and \(D(x - y)\) is some function. We see that at the l.h.s. we have the production of two operators and at the r.h.s. the number of operators is \((2 - 2) = 0\).

This procedure we would like to apply for the expression (5). At the l.h.s. we have the production of four operators, consequently at the r.h.s. we should have the production of \((4 - 2) = 2\) operators. Thus, we have two possibilities

\[
\left( \hat{a} \hat{b} \right) \left( \hat{a} \hat{b} \right) - \left( \left( \hat{a} \hat{b} \right) \hat{a} \right) \hat{b} = \lambda \begin{cases} 0, & \text{if } \hat{b} = \hat{a}; \\ \lambda_1 \hat{a}^2 + \lambda_2 \hat{b}^2, & \text{if } \hat{b} \neq \hat{a}. \end{cases}
\]  

(7)
The next simple example is for the vectors $A_{\mu}$. Let us determine the following relation

$$
(\hat{A}_{\mu}\hat{A}_{\nu}) - \left( (\hat{A}_{\mu}\hat{A}_{\nu}) \hat{A}^{\nu} \right) \hat{A}^{\nu} = \lambda \left\{ \begin{array}{ll}
0, & \text{if } \mu = \nu \\
\hat{A}_{\nu} \hat{A}^{\nu}, & \text{if } \mu \neq \nu.
\end{array} \right.
$$

(8)

Here and up to end we will write $\sum$ if there is the sum, consequently in this equation (8) we have not any sum over repeating indices. Now we summarize over $\nu$

$$
\sum_{\nu=0}^{3} (\hat{A}_{\mu}\hat{A}_{\nu}) (\hat{A}^{\mu}\hat{A}^{\nu}) - \sum_{\nu=0}^{3} \left( (\hat{A}_{\mu}\hat{A}_{\nu}) \hat{A}^{\nu} \right) \hat{A}^{\nu} = \lambda \sum_{\nu \neq \mu} \hat{A}_{\nu} \hat{A}^{\nu} = \lambda \left( \sum_{\nu=0}^{3} \hat{A}_{\nu} \hat{A}^{\nu} - \hat{A}_{\mu} \hat{A}^{\mu} \right).
$$

(9)

After the sum over $\mu$ we have

$$
\sum_{\mu,\nu=0}^{3} (\hat{A}_{\mu}\hat{A}_{\nu}) (\hat{A}^{\mu}\hat{A}^{\nu}) - \sum_{\nu=0}^{3} \left( (\hat{A}_{\mu}\hat{A}_{\nu}) \hat{A}^{\nu} \right) \hat{A}^{\nu} = 3\lambda \sum_{\nu=0}^{3} \hat{A}_{\nu} \hat{A}^{\nu}
$$

(10)

We would like to underline again that the differences (8) (11) are not connected with the commutator $[\hat{a}, \hat{b}]$ which can be zero. This is only a manifestation of the nonlinearity of the corresponding expression $(\hat{a}\hat{b})(\hat{a}\hat{b})$.

### 4 Non-Abelian case

Let us come back to more realistic case: non-Abelian gauge theories. We suppose that for the first term of the r.h.s. of Eq. (2) we have such equation

$$
(\hat{A}_{\mu}^{a}\hat{A}_{\nu}^{b}) (\hat{A}^{a\mu}\hat{A}^{b\nu}) - \left( (\hat{A}_{\mu}^{a}\hat{A}_{\nu}^{b}) \hat{A}^{a\mu} \right) \hat{A}^{b\nu} = \\
\lambda \left\{ \begin{array}{ll}
0, & \text{if } b = a, \nu = \mu; \\
\hat{A}_{\nu}^{a} \hat{A}^{a\nu}, & \text{if } b = a, \nu \neq \mu; \\
\hat{A}_{\mu}^{b} \hat{A}^{b\mu}, & \text{if } b \neq a, \nu = \mu; \\
\hat{A}_{\nu}^{b} \hat{A}^{b\nu}, & \text{if } b \neq a, \nu \neq \mu.
\end{array} \right.
$$

(11)
Let us remind that here we have not the sum over repeating indices. Using the same calculations as in the previous section we have after summarizing over $b, \nu$

\[
\sum_{b,\nu} (\hat{A}_{\mu}^a \hat{A}_b^\nu) (\hat{A}^{a\mu} \hat{A}^{b\nu}) - \sum_{b,\nu} \left( (\hat{A}_{\mu}^a \hat{A}_b^\nu) \hat{A}^{a\mu} \right) \hat{A}^{b\nu} =
\]

\[
\lambda \left( \sum_{\nu \neq \mu} \hat{A}_\nu^a \hat{A}^{a\nu} + \sum_{b \neq a} \hat{A}_\nu^b \hat{A}^{b\nu} + \sum_{b \neq a, \nu \neq \mu} \hat{A}_\nu^b \hat{A}^{b\nu} \right) =
\]

\[
\lambda \left( \sum_{b,\nu} \hat{A}_\nu^b \hat{A}^{b\nu} - \hat{A}_\mu^a \hat{A}^{a\mu} \right).
\]

(12)

Finally, after summarizing over $a, \mu$ we have

\[
\sum_{a,\mu} \sum_{b,\nu} (\hat{A}_\mu^a \hat{A}_\nu^b) (\hat{A}^{a\mu} \hat{A}^{b\nu}) =
\]

\[
\sum_{a,\mu} \sum_{b,\nu} \left( (\hat{A}_\mu^a \hat{A}_\nu^b) \hat{A}^{a\mu} \hat{A}^{b\nu} + 11\lambda \sum_{b,\nu} \hat{A}_\nu^b \hat{A}^{b\nu} \right)
\]

(13)

For the next term $(\hat{A}_\mu^a \hat{A}_\nu^b)(\hat{A}^{a\mu} \hat{A}^{b\nu})$ in Eq. (3) we have some modifications. We have to calculate the following difference

\[
\sum_{a,\mu} \sum_{b,\nu} \left( (\hat{A}_\mu^a \hat{A}_\nu^b) \hat{A}^{a\mu} \hat{A}^{b\nu} \right) \hat{A}^{a\nu} =?
\]

(14)

At first we will consider the next cases with different relations between $b, a$ and $\nu, \mu$.

If $b = a$ and $\nu = \mu$

\[
\left( \hat{A}_\mu^a \hat{A}_\mu^a \right) (\hat{A}^{a\mu} \hat{A}^{a\mu}) - \left( \left( \hat{A}_\mu^a \hat{A}_\mu^a \right) \hat{A}^{a\mu} \right) \hat{A}^{a\mu} = 0.
\]

(15)

If $b = a$ and $\nu \neq \mu$

\[
\left( \hat{A}_\mu^a \hat{A}_\nu^a \right) (\hat{A}^{a\mu} \hat{A}^{a\nu}) - \left( \left( \hat{A}_\mu^a \hat{A}_\nu^a \right) \hat{A}^{a\nu} \right) \hat{A}^{a\nu} = \lambda \hat{A}_\mu^a \hat{A}^{a\nu}.
\]

(16)

If $b \neq a$ and $\nu = \mu$

\[
\left( \hat{A}_\mu^a \hat{A}_\mu^b \right) (\hat{A}^{b\mu} \hat{A}^{a\mu}) - \left( \left( \hat{A}_\mu^a \hat{A}_\mu^b \right) \hat{A}^{b\mu} \right) \hat{A}^{a\mu} = \lambda \hat{A}_\mu^b \hat{A}^{a\mu}.
\]

(17)

If $b \neq a$ and $\nu \neq \mu$

\[
\left( \hat{A}_\mu^a \hat{A}_\nu^b \right) (\hat{A}^{b\mu} \hat{A}^{a\nu}) - \left( \left( \hat{A}_\mu^a \hat{A}_\nu^b \right) \hat{A}^{b\mu} \right) \hat{A}^{a\nu} = 0.
\]

(18)

The relation (18) demands more careful discussion. In fact we assume here that the difference (14) is nonzero only if there is at least two identical factors (as in Eq's (15)–(17)). Let us consider the situation with two identical pairs
$$(\hat{A}_a^\mu \hat{A}_b^\nu)(\hat{A}^{a\mu}\hat{A}^{b\nu}).$$ After some permutations of brackets and factors we can derive $(\hat{A}_a^\mu \hat{A}_b^\nu)(\hat{A}^{a\mu}\hat{A}^{b\nu})$. If we average some quantum state $|Q\rangle$ we will have

$$\langle Q| (\hat{A}_a^\mu \hat{A}_b^\nu)(\hat{A}^{a\mu}\hat{A}^{b\nu}) |Q\rangle = \langle Q| (\hat{A}_a^\mu \hat{A}_b^\nu)(\hat{A}^{b\mu}\hat{A}^{b\nu}) |Q\rangle = \lambda \langle Q| (\hat{A}_a^\mu \hat{A}_b^\nu) |Q\rangle + (something)$$

(19)

here (something) is connected with commutators and permutations of brackets for three factors (for example, $(\hat{A}_a^\mu \hat{A}_b^\nu)\hat{A}^{a\mu} \rightarrow \hat{A}_a^\mu (\hat{A}_b^\nu\hat{A}^{a\mu})$ and so on). It means that the difference (19) can be nonzero only if a few factors are identical but if all factors are different then the first term on the r.h.s. of Eq. (19) is absent. Physically, one can say that the quantum correlation between four different components of the vector potential is the same for any combination of brackets (with an accuracy of the second term in Eq. (15)).

In view of Eq’s (13)- (17) the sum over $b, \nu$ in Eq. (14) gives us

$$\sum_{b,\nu} (\hat{A}_a^\mu \hat{A}_b^\nu)(\hat{A}^{b\mu}\hat{A}^{\nu}) - \sum_{b,\nu} (\hat{A}_a^\mu \hat{A}_b^\nu)(\hat{A}^{b\mu}\hat{A}^{\nu}) \hat{A}^{a\nu} =$$

$$\lambda \left( \sum_{\nu \neq \mu} \hat{A}_a^\nu \hat{A}^{a\nu} + \sum_{b \neq a} \hat{A}_b^\nu \hat{A}^{a\nu} \right) = \lambda \left( \sum_{\nu} \hat{A}_a^\nu \hat{A}^{a\nu} + \sum_{b} \hat{A}_b^\nu \hat{A}^{a\nu} - 2 \hat{A}_a^\mu \hat{A}^{a\mu} \right).$$

(20)

Now we can summarize over $a, \mu$ and Eq. (14) has the following form

$$\sum_{a,\mu} \sum_{b,\nu} (\hat{A}_a^\mu \hat{A}_b^\nu)(\hat{A}^{b\mu}\hat{A}^{\nu}) - \sum_{a,\mu} \sum_{b,\nu} (\hat{A}_a^\mu \hat{A}_b^\nu)(\hat{A}^{b\mu}\hat{A}^{\nu}) \hat{A}^{a\nu} =$$

$$\lambda \left( 4 \sum_{a,\mu} \hat{A}_a^\nu \hat{A}^{a\nu} + 3 \sum_{b,\mu} \hat{A}_b^\nu \hat{A}^{b\mu} - 2 \sum_{a,\mu} \hat{A}_a^\mu \hat{A}^{a\mu} \right) = 5 \lambda \sum_{a,\mu} \hat{A}_a^\mu \hat{A}^{a\mu}.$$

(21)

Consequently

$$\left( \left( \hat{F}_{\mu\nu} \right)_{\alpha}^{\mu} \right) = \left( \hat{F}_{\mu\nu} \right)_{\alpha}^{\mu} = \left( \hat{A}_a^\mu \hat{A}_b^\nu \right) \hat{A}^{a\mu} \hat{A}^{b\nu} - \left( \hat{A}_a^\mu \hat{A}_b^\nu \right) \hat{A}^{b\mu} \hat{A}^{a\nu} =$$

$$\left( \hat{A}_a^\mu \hat{A}_b^\nu \right) \hat{A}^{a\mu} \hat{A}^{b\nu} - \left( \hat{A}_a^\mu \hat{A}_b^\nu \right) \hat{A}^{b\mu} \hat{A}^{a\nu} + 6 \lambda \hat{A}_a^\mu \hat{A}^{a\mu}.$$

(22)

here we again restore the ordinary rule for the sum over repeating indices. It is evidently that our extra term $(6 \lambda \hat{A}_a^\mu \hat{A}^{a\mu})$ breaks the initial gauge symmetry of given Lagrangian (1), and consequently it is similar to Coleman-Weinberg symmetry breaking but on the nonperturbative level.

It is necessary to note that all these calculations (13)- (22) were done for the fields in one point $(x)$.  

6
5 Some explanations

Now we would like to explain why we need with equation (22). Let us assume that we have a quantum state $|\Phi\rangle$ and the action of the field operator $\hat{A}_\mu^a$ on this state is $|\Phi_1\rangle = \hat{A}_\mu^a|Q\rangle$. For the nonlinear term the problem is that we do not know the action of this term by the direct way

$$\left(\hat{A}_\mu^a \hat{A}_\nu^b\right) \left(\hat{A}^{a\mu} \hat{A}^{b\nu}\right)|Q\rangle =?$$

(23)

because we do not know the action of the operator $\hat{A}^{a\mu} \hat{A}^{b\nu}$ on the quantum state $|Q\rangle$. In order to determine the action (23) we need for the following transformations

$$\left(\hat{A}_\mu^a \hat{A}_\nu^b\right) |Q\rangle \rightarrow \left(\hat{A}_\mu^a \hat{A}_\nu^b\right) \hat{A}^{a\mu}|Q\rangle = \hat{A}_\mu^a \hat{A}_\nu^b |\Phi_2\rangle = \hat{A}_\mu^a |\Phi_3\rangle = |\Phi_4\rangle$$

(24)

where $|\Phi_1\rangle = \hat{A}_\mu^a|Q\rangle$ and so on for every $|\Phi_i\rangle$, $i = 2, 3, 4$.

6 Discussions and conclusions

In this paper we have shown that a nonlinear potential in quantum non-Abelian gauge theory can lead to the appearance of some extra terms in Lagrangian. In Ref. [2] it is shown that Ginzburg - Landau equation can be derived from a pure non-Abelian gauge theory. There was assumed that the quantization of such theory leads to the appearance of $m^2 A^2$-like term in the initial Lagrangian. In this paper we suggested a nonperturbative mechanism for this phenomenon.

A possibility that the operators of quantum fields with a nonlinear potential can have nonassociative properties was investigated in Ref. [3]. In this paper we continue this approach but there is one very essential distinction : we assume that the remainders in Eq’s (3) (4) are not a numerical function but is an operator which is the product of fields operators.

In Ref’s [4] it is shown that a Meissner-like effect in non-Abelian gauge theories arises. There was applied a nonperturbative quantization techniques based on the Heisenberg’s approach to a nonlinear spinor field. Heisenberg’s idea is to use field operator equations for receiving an infinite equations set for all Green’s functions. In this paper (remaining in the frame of Heisenberg’s approach) we show that this nonperturbative mechanism leads to spontaneously symmetry breaking.

Finally, we would like to say that probably nonperturbative quantum field theory with a strong interaction will have very unusual and unexpected properties in contrast with quantum field theory with a small coupling constant. This difference is similar with the difference between analytical and differentiable (but nonanalytical) functions. The first case is similar to a quantum field theory with a small coupling constant, where we can expand a function in
Taylor-series which for the quantum theory is Feynman diagrams. In the second case a function can not be expanded in any Taylor-series and respectively Feynman techniques can not be applied for such kind of quantum field theories. Probably, the QCD belong to the last case that leads to the fact that confinement can not be explained on the language of Feynman diagrams. In this case one can say \cite{5} that "fields are primary to particles".

Our opinion is that in this case we can use the Heisenberg’s nonperturbative quantization method \cite{6}. Something like this takes place in the classical case: the nonlinear classical theories have such nonperturbative phenomena as: self-organisation, monopoles, instantons, black holes, strange attractors and so on which are absent in linear theories and they can not be derived by the perturbative way.

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References

\cite{1} S. Coleman and E. Weinberg, Phys. Rev., D7, 1888 (1973).
\cite{2} V. Dzhunushaliev and D. Singleton, “Ginzburg - Landau equation from SU(2) gauge field theory”, in preparation.
\cite{3} V. Dzhunushaliev, Theor. Math. Phys., 100, 1082 (1994); V. Dzhunushaliev, “Nonperturbative operator quantization of strongly nonlinear fields” \cite{hep-th/0103172} V. Dzhunushaliev and D. Singleton, Nuovo Cimento, 117B, 137 (2002).
\cite{4} V. Dzhunushaliev and D. Singleton, Phys. Rev., D65, 125007 (2002).
\cite{5} J. Hansson, “A simple explanation of the non-appearnace of physical gluons ans quarks”, \cite{hep-ph/0208157}, to appear in Can. J. Phys.
\cite{6} W. Heisenberg, Introduction to the unified field theory of elementary particles, Max - Planck - Institut für Physik und Astrophysik, Interscience Publishers London, New York, Sydney, 1966; W. Heisenberg, Nachr. Akad. Wiss. Göttingen, N8, 111(1953); W. Heisenberg, Zs. Naturforsch., 9a, 292(1954); W. Heisenberg, F. Kortel und H. Mütter, Zs. Naturforsch., 10a, 425(1955); W. Heisenberg, Zs. für Phys., 144, 1(1956); P. Askali and W. Heisenberg, Zs. Naturforsch., 12a, 177(1957); W. Heisenberg, Nucl. Phys., 4, 532(1957); W. Heisenberg, Rev. Mod. Phys., 29, 269(1957).