Extraordinary Hall effect in hybrid ferromagnetic/superconductor (F/S) bilayer

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Extraordinary Hall effect (EHE) in bilayer F/S(N) was investigated theoretically. The conductivity tensor $\sigma_{\alpha\beta}$ is calculated in the Kubo formalism with Green functions found as the solutions of the Gorkov equations. We considered diffuse transport in the ferromagnetic layer, taking into account as a main mechanism of electron resistivity s-d scattering. In this model Gorkov equations for s-electrons in the ferromagnetic layer remain linear and are solved easily. It is shown that Hall field $E_i^H$ for both F/S and F/N contacts are step-functions of the coordinate perpendicular to the planes of the layers and have zero value in S(N) layer. The Andreev reflection increases the value of Hall constant $R_a$ for F/S case. The value of the Hall constant is $R_i^F/(S) = R_i^{bulk}((\sigma^+ + \sigma^-)/4\sigma^\uparrow \sigma^\downarrow)$, where $\sigma^\uparrow$ and $\sigma^\downarrow$ are conductivities of electrons with up and down spins, and $R_i^H$ is the Hall constant in the bulk ferromagnetic metal. In fact, $R_i^F/(S)$ coincides with EHE constant of the bilayer of two ferromagnetic metals with equal thickness and opposite directions of their magnetizations. So we can make a conclusion, that the ideal interface between ferromagnetic metal and superconductor may be considered like a mirror with inversion in spin space.

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The perpendicular spin-dependent transport in hybrid ferromagnetic/superconductor (F/S) and F1/F2/S structures has been previously investigated in Ref. [8]. It was shown that the Andreev reflection [4] at F/S interface causes a mixing of up and down spin channels and simultaneously a large spin accumulation on F/S interface arises. The total influence of both effects increases the resistance of the system at $T = 0$ comparing to its value when the superconductor is in the normal state. In the case of F1/F2/S these effects strongly influence the giant magnetoresistance (GMR) of the F1/F2 bilayer. For certain values of the parameters, the GMR could be even completely suppressed [4, 5].

Another spin-dependent transport effect, which has not been considered so far in F/S sandwiches, is the extraordinary Hall effect (EHE) [3]. The current $j$ is assumed to flow perpendicular to the interface, the magnetization $\mathbf{M}$ in the F-layer is in plane, and the Hall electrical field $E_i^H$ arises in plane in the direction perpendicular to $\mathbf{M}$. In such geometry, the Andreev reflection is expected to play a more complicated role than on the current-perpendicular-to-plane (CPP) GMR because EHE combines current-in-plane (CIP) and CPP features.

It is the purpose of this letter to investigate theoretically the EHE in F/S(N) bilayers in the diffuse regime in both situations where the non-magnetic layer is in its superconducting or normal states. In the present model, we assume that conductivities for spin-up and spin-down channels are different and the contribution of F/S interface resistance to the total resistance is small. This is the situation for which the CPP-GMR for F1/F2/S is destroyed by the Andreev reflection and spin accumulation [3].

I. GENERAL DEFINITIONS

For geometry used in the model, the following relation proposed by Smith and Sears [1] can be written

$$E_i^H = j_z (R_0 H_y + R_s M_y),$$

(1)

$R_0$ is the normal Hall effect coefficient whereas $R_s$ represents the EHE coefficient. $H_y$ and $M_y$ are respectively the external applied field and magnetization. The bias voltage is applied between the planes with coordinates $z = -a$ and $z = b$, and $z = 0$ is the position of interface.

As it was shown in [3, 4], extraordinary Hall effect has two different origins — skew-scattering and side-jump. As a first approach to EHE in these structures, we will investigate only the first one, taking into account elastic scattering on impurities. The approach that we use for calculation of the Hall coefficient for layered system has been described in [5]. Following the same method, we calculate the components of the conductivity tensor using Kubo formula [4].

$$\sigma_{\alpha\beta} = \frac{\hbar e^2}{\pi \Omega} \text{Tr} \left[ v_\alpha (G^+ - G^-)^\dagger v_\beta (G^+ - G^-) \right]$$

(2)

which includes simple "bubble" diagonal conductivity and vertex corrections for off-diagonal components, which are responsible for the extraordinary Hall effect. In (2) $v_\alpha$, $v_\beta$ are velocity components, indices $\alpha$, $\beta$ take the values $x$, $y$, $z$, $G^{+(-)}$ are retarded and advanced Green functions, $\Omega$ is the volume of the system.

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For the system under consideration, which is homogeneous in x\(y\)-plane and inhomogeneous in z direction, it is convenient to use \((\vec{k}, z)\)-representation, where \(\vec{k}\) is the in-plane electron momentum. The Green function \(G\) is defined by equations:

\[
G = G^0 + G^0 H^{so} G^0 \\
G^0 = G_{\text{eff}} + G_{\text{eff}} T G_{\text{eff}}
\]

(3)

where \(G^0\) is the Green function of the system in the absence of spin-orbit interaction, \(G_{\text{eff}}\) is the effective Green function, diagonal on in-plane vector \(k\), calculated in the coherent potential approximation (CPA) \([4]\). \(T\) is a scattering matrix and \(H^{so}\) is the spin-orbit interaction. To adopt for the ferromagnetic layer the model of totally disordered binary alloy \(A_xB_{1-c}\) (\(c\) is a concentration of the alloy's component \(A\)) we can write the expressions for \(T\)-matrix in singe-site approximation and for matrix elements of \(H^{so}\) in explicit form:

\[
T_{\sigma\sigma'}^{\pm}(z) = \frac{1}{N} \sum_n \epsilon_{\sigma'}(\vec{k} - \vec{r}_n) \times
\]

\[
\times \frac{\delta_{\sigma\sigma'} \nu(n, z) - \Sigma^{\sigma^+}}{1 - (\delta_{\sigma\sigma'} \nu(n, z) - \Sigma^{\sigma^+})} G^{\sigma^+}(z, z) = \frac{1}{N} \sum_n t_{\sigma\sigma'}^{\pm}(n, z)
\]

(4)

\[
H_{\sigma\sigma'}^{so}(z) = \frac{1}{N} \sum_n \epsilon_{\sigma'}(\vec{k} - \vec{r}_n) \times
\]

\[
\nu(n, z) i \lambda M_g [\vec{k} \times \vec{k}]_{y} = \frac{1}{N} \sum_n H_{\sigma\sigma'}^{so}(n, z)
\]

(5)

In \([3]\) and \([4]\) \(\delta_{\sigma}^\sigma = \varepsilon_{A}^\sigma - \varepsilon_{B}^\sigma\) is scattering parameter, \(\varepsilon_{A}^\sigma\) and \(\varepsilon_{B}^\sigma\) are the band centers for \(A\) and \(B\) components depending on spin \(\sigma\), \(\nu(n, z)\) is projection operator:

\[
\nu(n, z) = a_B(n, z)c - a_A(n, z)(1-c)
\]

(6)

\[
a_{\alpha}(n, z) = \begin{cases} 1, & \text{if the site } (\vec{r}_n, z) \text{ is occupied by } \alpha \text{ atom} \\ 0, & \text{in the opposite case} \end{cases}
\]

(7)

\(\lambda = \lambda_A - \lambda_B\) is parameter of spin-orbit scattering which is non-zero only in the ferromagnetic layer. \(\Sigma^{\sigma^+}\) is the coherent potential which is the solution of self-consistent equation \(<T^{\sigma^+}(z)> = 0\), where \(<\ldots>\) means averaging on impurities distribution, \(G^{\sigma^+}(z, z) = \frac{1}{N} \sum_{\sigma'} G^{\sigma^+}_{\sigma\sigma'}(z, z)\).

In \([3]\) the z-component of electron momentum vector \(\vec{k}\) in \([k \times \vec{k}]_y\) is the antisymmetric gradient operator: \(k_z = i(\nabla_z - \nabla_z').\)

It is important to note that from definitions \([3]\) and \([4]\) the first non-zero contribution into the vertex correction of formula \([3]\) linear on \(H^{so}\) is that containing \(<t_{\sigma\sigma'}^{\pm}(n, z) H_{\sigma\sigma'}^{so}(n, z')> \sim \delta_{mn} \delta(z - z')\).

In the adopted geometry the system of equations for Hall fields can be written as follows:

\[
j_{x}(z) = \int \sigma_{xx}(z, z') E_{x}^H(z') dz' + \\
+ \int \int \sigma_{xz}(z, z', z'') E_{x}(z'') dz' dz'' = 0 \quad (8a)
\]

\[
j_{z}(z) = \int \sigma_{zz}(z, z') E_{z}(z') dz' + \\
+ \int \int \sigma_{xz}(z, z', z'') E_{z}^H(z'') dz' dz'' \quad (8b)
\]

We consider \(H^{so} / \delta\) like a small parameter of the theory. So \(\sigma_{xz} \ll \sigma_{zz}\) and the second term in equation \((8b)\) can be omitted. The off-diagonal component of conductivity has a three-point character. The additional coordinate \(z''\) represents the scattering plane.

### II. MODEL

We consider a bilayer of the type F/S, where F is ferromagnetic layer, S is a superconducting layer. A simple two band (spin up and down) free electron model is adopted for this calculation. The Hamiltonian of the system is therefore written as:

\[
H = H_F + H_S
\]

(9a)

\[
H_F = \sum_{\sigma=\pm(+),-(-)} \int \left[ \left( \frac{\vec{p}^2}{2m} - \varepsilon_F + \text{sign}(\sigma) \varepsilon_{ex} \right) \times \psi_{\sigma}^* \psi_{\sigma} \right] \left( \psi_{\sigma}^d \psi_{\sigma} + h.c. \right) d^3r
\]

(9b)

\[
H_S = \int \sum_{\sigma} \left[ \left( \frac{\vec{p}^2}{2m} - \varepsilon_F \right) \psi_{\sigma}^* \psi_{\sigma} \right] + \left( \Delta(r) \psi_{\sigma}^d \psi_{\sigma}^* + h.c. \right) \right] d^3r
\]

(9c)

where \(\varepsilon_{ex} = \frac{p_{F_d}^2}{2m} - \frac{p_{F_d}^2}{2m}\) is the exchange energy, \(\varepsilon_F\), \(p_{F_d}\) are respectively the Fermi energy and momentum. The second term in \((9c)\) describes the scattering of quasi free s-electrons into almost localized d-states. In bulk ferromagnetic metals d-states may give contribute to the current \([11]\). However in the present situation, we consider that there are no d-states in the superconductor. Therefore d-electrons are completely reflected on F/S interface and do not contribute to the current. On other respects, s-d scattering in ferromagnetic dirty d-metal alloys remains the most important mechanism of s-electrons scattering \([11]\). In the case under consideration, we take it into account and consider that the random s-d scattering potential \(\gamma_{sd}\) is much smaller than \(\varepsilon_F\). We further calculate the mean free path in the Born approximation. \(\Delta\)
is the order parameter in the superconductor. Now the system of Gorkov equations $\hat{\text{G}}_{\text{G}}$ for the normal $G_{\text{G}}$ and anomalous $F_{\text{G}}$ Green functions can be written:

$$\left[ \frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial z^2} - \kappa^2 \right) + e_F + \varepsilon_{\text{ex}} - \gamma^2_G \Gamma_{dd}(z, z) \right] G_{zz}^{\uparrow\uparrow}(z, z') + \Delta F_{ss}^{\uparrow\uparrow}(z, z') = \delta(z - z') \quad (10a)$$

$$\Delta^* G_{zz}^{\uparrow\uparrow}(z, z') - \left[ \frac{\hbar^2}{2m} \left( \frac{\partial^2}{\partial z^2} - \kappa^2 \right) + e_F - \varepsilon_{\text{ex}} + \gamma^2_G \Gamma_{dd}(z, z) \right] F_{ss}^{\uparrow\uparrow}(z, z') = 0 \quad (10b)$$

We omitted index "eff" for the brevity. The terms $\varepsilon_{\text{ex}}$ and $\gamma^2_G \Gamma_{dd}$ are different from zero and $\Delta = 0$ if $z$ belongs to F-layer and vice versa in S-layers. The system (10) is written for spin $\uparrow$. For spin $\downarrow$, $\varepsilon_{\text{ex}}$ has to be changed to $-\varepsilon_{\text{ex}}$ and $\Gamma_{dd}^{\uparrow\downarrow}$ to $\Gamma_{dd}^{\downarrow\uparrow}$. The main difference between system (10) and usualy employed equation for F/S structures (see for example [13]) is that we took into account s-d scattering. Moreover, we considered this s-d scattering as the main mechanism determining the mean free path of s-electrons [11]. The function $G_{dd}^{\uparrow\downarrow}(z, z)$ may be considered as a constant in z-space if it is averaged over the short wave length $(\hbar/p_F)$ oscillations and the system of equation (10) may be solved analytically. Further, we set $\varepsilon_{\text{ex}} = 0$ for s-electron. The explicit expression for Green functions are:

$$G_{11}^{\uparrow\uparrow}(\infty < z, z' < 0) = \frac{e^{ik_1|z - z'|}}{2i k_1} \quad (11a)$$

$$F_{11}^{\uparrow\uparrow}(\infty < z, z' < 0) = \frac{e^{ik_1 z'}e^{-ik_1 z}}{k_1 + k_s^2} \quad (11b)$$

$$G_{12}^{\uparrow\downarrow}(\infty < z < 0 < z' < \infty) = \frac{e^{-ik_1 z'}e^{ik_1 z}}{i(k_1 + k_s)} \quad (11c)$$

$$F_{12}^{\uparrow\downarrow}(\infty < z < 0 < z' < \infty) = \frac{e^{ik_1 z}e^{-ik_1 z'}}{k_s + k_2} \quad (11d)$$

$$G_{22}^{\uparrow\uparrow}(0 < z, z' < \infty) = \frac{1}{4ik_s} \left[ e^{ik_s z}e^{-ik_s z'} + e^{ik_s z}e^{ik_s z'} \right] + \frac{1}{4ik_s} \left[ e^{ik_s z}e^{-ik_s z'} - e^{-ik_s z}e^{ik_s z'} \right] \quad (11e)$$

$$F_{22}^{\uparrow\uparrow}(0 < z, z' < \infty) = \frac{1}{4ik_s} \left[ e^{ik_s z}e^{-ik_s z'} + e^{ik_s z}e^{ik_s z'} \right] - \frac{1}{4ik_s} \left[ e^{ik_s z}e^{-ik_s z'} - e^{-ik_s z}e^{ik_s z'} \right] \quad (11f)$$

where $k_{1,2} = \sqrt{k_F^2 - \kappa^2 + i\Delta^2}$, $c_{1,2}$ and $d_{1,2}$ are mean free paths for up and down spins.

Perpendicular transport in bilayer F/S was investigated in [3] so we remind here only the result: $E_1^{\uparrow}l_1 = E_1^{\downarrow}l_2$, $E_1^{\uparrow}$ and $E_1^{\downarrow}$ are effective electrical fields acting on the carriers with spin up, down, $E_1^{\uparrow} = a(l_1 + l_2)/b$, $E_1^{\downarrow} = a(l_1 + l_2)/b$, $j_z \sim E_1^{\uparrow} l_1$. Off-diagonal components of conductivity in [2] as well as diagonal one has two contributions – normal and anomalous. For example, one has the form:

$$[\sigma_{zz}^{\text{norm}}(z, z', z'')]_{x} \sim L \left( e_{zz} G_{xx}^{zz}(z, z') \right)$

$$\left( T^+(z'') + H^{so}(z'') \right)_{x} \left( e_{zz} G_{xx}^{zz}(z', z') \right)$$

$$\left( T^-(z'') + H^{so}(z'') \right)_{x} \left( e_{zz} G_{xx}^{zz}(z', z') \right) \quad (12)$$

where $L$ means linear on $H^{so}$ part of this expression. Since $T_{so}^{zz} = T_{so}^{zz}$ and $H_{so}^{zz} = -H_{so}^{zz}$ term (12) is proportional to $(2iH^{so}(z) \text{Im} T^+(z))$ as well as all other contributions to $\sigma_{zz}$. In the Born approximation, we can write down for disordered binary system:

$$\left< 2iH^{so}(z) \text{Im} T^+(z) \right> = -\lambda M_g c(1 - c)(1 - 2c) (\delta^2)^2 \times$$

$$\times \text{Im} G^{zz}(z, z)[k \times \bar{k}]_y \quad (13)$$

from which it is easy to see that this term is proportional to $(1 - 2c)/l_\sigma$.

The unknown Hall field has to be found as a solution of the integral equation [30]. To solve it we take as a probe function for $E^H(z)$ the step function, taking value $E^{H\uparrow\downarrow}(z)$ inside the ferromagnetic layer and $E^{H\uparrow\downarrow}(z)$ in the superconductor for each direction of the electron’s spin $\uparrow, \downarrow$. In this case the equation (30) may be rewritten as a system of two equations:

$$E_1^{H \uparrow} l_1 \int \frac{\chi^3}{k_F^2} \left( 1 - e^{-2d_1(z + a)} - e^{2d_1 a} \right) dz +$$

$$+ 1/2 E_1^{H \downarrow} l_2 \int \frac{\chi^3}{k_F^2} e^{2d_2 a} (1 - e^{-2d_2 a}) dz +$$

$$+ 1/2 (E_1^{H \uparrow} + E_1^{H \downarrow}) \int \frac{\chi^3}{k_F^2} e^{2d_2 a} (1 - e^{-2d_2 a}) dz =$$

$$= R^\text{bulk} E_1^{H \uparrow} l_1 M_g \left( \frac{\sigma^+ + \sigma^-}{4} \right) \int \frac{\chi^3}{k_F^2} (1 -$$

$$- e^{-2d_1(z + a)} - e^{2d_1 z} e^{-2d_2 a}) dz; \quad a < z < 0 \quad (14a)$$
where $R_{s}^{\text{bulk}}$ is the Hall coefficient and $\sigma^{\uparrow\downarrow}$ are conductivities of up and down spin channels for the bulk ferromagnet. For spin down we have to change $\uparrow\rightarrow \downarrow$; $1 \rightarrow 2$.

Solving the system of equations (14a), (14b) we found that indeed the solution for the Hall field $E_{s}^{H}(z)$ in the form of the step-like function satisfies the system (14a), (14b) and consequently the integral equation 8d for any $z$. The results of the Hall fields are: $E_{s}^{H \uparrow} = E_{s}^{H \downarrow} = 0$, $E_{1}^{H \uparrow} = E_{1}^{H \downarrow} + 1$, $E_{1}^{H \uparrow} + E_{1}^{H \downarrow} = R_{s}^{\text{bulk}} E_{1}^{H \uparrow} M_{y} \frac{\sigma^{\uparrow} + \sigma^{\downarrow}}{2}$. So the extraordinary Hall coefficient for F/S bilayer is:

$$R_{s}^{F/S} = \frac{(\sigma^{\uparrow} + \sigma^{\downarrow})^{2}}{4\sigma^{\uparrow}\sigma^{\downarrow}} R_{s}^{\text{bulk}}$$  \hspace{1cm} (15)

III. F/N BILAYER

Now we recalculate the Hall coefficient when the nonmagnetic layer is in the normal state. In this case, all Green functions are diagonal in the spin space:

$$G_{11}^{\uparrow \downarrow}(-\infty < z, z' < 0) = \frac{e^{i k_{3} z - z'}}{2 i k_{1}}$$  \hspace{1cm} (16a)

$$G_{12}^{\uparrow \downarrow}(-\infty < z < 0 < z' < \infty) = \frac{e^{-i k_{3} z + k_{3} z'}}{i(k_{1} + k_{3})}$$  \hspace{1cm} (16b)

$$G_{22}^{\uparrow \downarrow}(0 < z, z' < \infty) = \frac{e^{i k_{3} z - z'}}{2 i k_{3}}$$  \hspace{1cm} (16c)

where $k_{3} = \sqrt{k_{F}^{2} - x^{2} + \frac{1}{2}k_{F}^{2}}$; $c_{3} + i d_{3}$, $l_{3}$ is the mean free path in the normal metal.

The following expressions are then obtained:

$$E_{1}^{\uparrow} = E_{1}^{\uparrow} a_{3} + b_{1} + \frac{4}{3}i l_{3}$$

$$E_{1}^{\uparrow} = 2(a_{3} + b_{1} + \frac{4}{3}i l_{3})$$  \hspace{1cm} (17a)

$$E_{1}^{\downarrow} = V l_{3}$$  \hspace{1cm} (17b)

The system of equations for the Hall fields assumed to be step-functions can be written in the form:

$$E_{1}^{H \uparrow} l_{1} \int \frac{x^{3}}{k_{F}^{3} c} \frac{1 - e^{-2d_{a}(z+a)}}{2} dx + \int \frac{x^{3}}{k_{F}^{3} c} \frac{1 - e^{-2d_{a}(z-b)}}{2} dx =$$

$$+ \frac{1}{2}E_{2}^{H \downarrow} l_{3} \int \frac{x^{3}}{k_{F}^{3} c} e^{2d_{b}z} (1 - e^{-2d_{b}b}) dx =$$

$$= R_{s}^{\text{bulk}} E_{1}^{H \uparrow} M_{y} \frac{\sigma^{\uparrow} + \sigma^{\downarrow}}{4} \int \frac{x^{3}}{k_{F}^{3} c} \times$$

$$\times (1 - \frac{e^{-2d_{a}(z+a)}}{2} - \frac{e^{2d_{a}(z-b)}}{2}) dx; -a < z < b \hspace{1cm} (18a)$$

$$1/2E_{2}^{H \uparrow} l_{1} \int \frac{x^{3}}{k_{F}^{3} c} e^{-2d_{a}z} (1 - e^{-2d_{a}a}) dx +$$

$$+ E_{2}^{H \downarrow} l_{3} \int \frac{x^{3}}{k_{F}^{3} c} (1 - \frac{e^{-2d_{b}z}}{2} - \frac{e^{2d_{b}b}}{2}) dx =$$

$$= R_{s}^{\text{bulk}} E_{1}^{H \uparrow} M_{y} \frac{\sigma^{\uparrow} + \sigma^{\downarrow}}{4} \int \frac{x^{3}}{k_{F}^{3} c} \times$$

$$\times (1 - \frac{e^{-2d_{a}a}}{2} - \frac{e^{2d_{a}a}}{2}) dx; -a < z < b \hspace{1cm} (18b)$$

For spin down we have to change $\uparrow\rightarrow \downarrow$; $1 \rightarrow 2$. Solution of this system gives us $E_{2}^{H \uparrow} = E_{2}^{H \downarrow} = 0$,

$$E_{1}^{H \uparrow} = \frac{V l_{3}(\sigma^{\uparrow} + \sigma^{\downarrow})M_{y}}{4(a_{3} + b_{1} + \frac{4}{3}i l_{3})} R_{s}^{\text{bulk}}$$  \hspace{1cm} (19a)

$$E_{1}^{H \downarrow} = \frac{V l_{3}(\sigma^{\uparrow} + \sigma^{\downarrow})M_{y}}{4(a_{3} + b_{1} + \frac{4}{3}i l_{3})} R_{s}^{\text{bulk}}$$  \hspace{1cm} (19b)

where $V$ is the total voltage drop across the F/N bilayer, and Hall coefficient is:

$$R_{s}^{F/N} = \frac{(l_{1} + l_{2}) 2a_{3} + b_{1} + \frac{4}{3}i l_{3}}{2} \frac{1}{a_{3}(l_{1} + l_{2}) + 2b_{1} l_{2} + \frac{4}{3}i l_{3}} R_{s}^{\text{bulk}} \hspace{1cm} (20)$$

With the same method it is easy to show that the Hall constant for a spin-valve bilayer in antiparallel magnetic configuration $F^{\uparrow}/F^{\downarrow}$ is equal to $R_{s}^{F_{1}/F_{2}} = \frac{(\sigma^{\uparrow} + \sigma^{\downarrow})^{2}}{4\sigma^{\uparrow}\sigma^{\downarrow}} R_{s}^{\text{bulk}}$.

IV. CONCLUSIONS

As can be see from (15) and (20), the relative change of the Hall coefficient in the presence of superconducting contact is equal:

$$\frac{R_{s}^{F/S} - R_{s}^{\text{bulk}}}{R_{s}^{\text{bulk}}} = \frac{(\sigma^{\uparrow} - \sigma^{\downarrow})^{2}}{4\sigma^{\uparrow}\sigma^{\downarrow}}$$

and

$$R_{s}^{F/S} - R_{s}^{F/N} \sim (\sigma^{\uparrow} - \sigma^{\downarrow})^{2}$$
Simultaneously it is shown that resistivity $\rho^{F/S}$ of F/S bilayer is equal $\rho^{F/S} = \frac{\rho_1 + \rho_2}{2}$, where $\rho_1$ and $\rho_2$ are resistivities of up and down spin channels, this resistivity as well as $R^{F/S}$ coincides correspondingly with resistivity and the Hall constant of the bilayer of two ferromagnetic metals with opposite direction of magnetizations. Therefore, we conclude that an ideal interface between a ferromagnetic metal and a superconductor can be considered like a quantum mirror with inversion in spin-space. Of course, the roughness of the interface may spoil the mirror image. In addition we considered the case where the spin-diffusion length was much larger than the thickness of the ferromagnetic layer. The influence of spin-flip processes on Hall effect will be considered in a forth coming paper.

Experimental investigation of EHE in the situation, close to one described in the letter, may be done if to use as a ferromagnetic layer the alloy CuNi with relatively small exchange splitting and high resistance. The thickness $t_{\text{CuNi}}$ of CuNi layer has to be in the interval $l_{\text{el}} \ll t_{\text{CuNi}} \ll l_{\text{sd}}$, where $l_{\text{sd}}$ is the spin-diffusion length and $l_{\text{el}}$ is the longest mean free path of the electron in bulk CuNi.

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