Magnetotransport in Cuprates: a Test of the Spin Fluctuation Model

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We report on a simple calculation of the magnetotransport in cuprate superconductors, based on the nearly antiferromagnetic Fermi liquid (spin fluctuation) model. We find that the model explains all important features seen experimentally: the violation of Köhler’s rule, the close relationship between the Hall angle and the magnetoresistance, the temperature dependence of the first high field correction to MR and the doping dependence of the low field MR data. In addition, the estimated values of \( \omega_c \tau \), calculated using parameters obtained from the NMR measurements, yield values in close agreement with those found experimentally for overdoped and optimally doped cuprates.

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Magnetotransport measurements of the Hall effect \[1\] and the magnetoresistance (MR) \[3\] in the normal state of cuprates \[4\], reveal the full peculiarity of these remarkable systems and pose a major challenge to candidate theories of high temperature superconductivity. Harris et al \[3\] find that for modest magnetic fields, \( B \), the relative shift in the longitudinal resistivity, \( \Delta \rho_{xx}/\rho_{xx} \), is proportional to \( \Theta_H^2 \), where \( \Theta_H \), the Hall angle, \( \equiv \rho_{xy}/\rho_{xx} \); this implies that Köhler’s rule is violated. Tyler et al \[4\], who study the relative magnetoresistance in overdoped cuprates placed in strong fields, \( B \sim 40 - 60 \) T, find a departure from the modest field, \( B^2 \) behavior,

\[
\frac{\Delta \rho_{xx}}{\rho_{xx}} = \frac{\alpha B^2}{1 + \beta^2 B^2} \tag{1}
\]

a departure which occurs at higher threshold fields for optimally doped or underdoped than for the overdoped cuprates. In addition, Tyler et al \[4\] find that at a given temperature and field, \( \Delta \rho_{xx}/\rho_{xx} \) is little changed as one goes from optimally doped to overdoped samples. A further challenge is explaining the specific values obtained in Ref. 3 for the parameter, \( \omega_c \tau \), where \( \omega_c \) is the cyclotron frequency and \( \tau \) is the effective relaxation rate.

Below we show that the spin fluctuation model of the superconducting cuprates, in which the system is described as a nearly antiferromagnetic Fermi liquid, provides a natural explanation of all the above results. In this model planar quasiparticles interact through an effective magnetic interaction which is proportional to the dynamical spin susceptibility, which is generally taken to be \[3\]

\[
V_{\text{eff}}^{\text{NAFL}}(q, \omega) = g^2 \chi_{q}(\omega) = \frac{g^2 \chi_Q}{1 + \xi^2(q - Q)^2} - i\frac{\Delta}{\xi^2}. \tag{2}
\]

Here \( \chi_Q = \alpha_{\text{AF}} \xi^2 \), with \( \alpha_{\text{AF}} \) constant, \( \xi \) is the AF correlation length, \( \omega_{sf} \) is the frequency of the low energy relaxational mode characteristic of a system which is nearly antiferromagnetic (AF), and \( g \) is the coupling constant. Because the interaction is strongly peaked near the commensurate AF wavevector, \( Q = (\pi, \pi) \), it produces a pronounced anisotropy in quasiparticle behavior near the Fermi surface. Hot quasiparticles, located near those parts of the FS which can be connected by \( Q \), are so strongly scattered into one another that their lifetimes and other properties are dramatically different from the behavior characteristic of a normal Landau Fermi liquid, while cold quasiparticles, which are typically located near the diagonals, \( |k_x| = |k_y| \), are scattered rather weakly and display near Landau Fermi liquid behavior. Detailed calculations, reported in Ref. 3, show that by taking this anisotropy into account one can deduce the quite different behavior of hot and cold quasiparticles from single crystal measurements of \( \rho_{xx} \) and \( \rho_{xy} \); one finds in this way that \( \tau_{\text{hot}} \ll \tau_{\text{cold}} \). These calculations also made it possible to explain quantitatively the doping and temperature dependence of both the longitudinal conductivity and the Hall conductivity, while Schmalian et al \[6\] find that the spin fluctuation model can explain the weak pseudogap behavior found in ARPES experiments.

We begin our consideration of MR phenomena with the Zener-Jones solution of the Boltzmann equation \[4\], which can be used to obtain the conductivity tensor to arbitrary order in \( B \):

\[
\sigma_{\mu\nu} = 2e^2 \sum_k v^\mu_k (1 + D)^{-1} \left\{ v^\nu_k \tau_k \left( \frac{\partial f_0}{\partial \epsilon_k} \right) \right\} \tag{3}
\]

where \( D = e v \times B \cdot \nabla k \) is a differential operator, \( v^\nu_k \equiv \nabla \epsilon_k \) is the Fermi velocity and \( \tau_k \) the quasiparticle lifetime at a point \( k \) on the FS. One can expand \( (1 + D)^{-1} \) in terms of the applied field: assuming \( B \) perpendicular to the \( x - y \) plane, and switching from the sum to integrals over the energy \( \epsilon \) and the tangential component of the momentum \( k \) one finally obtains:

\[
\sigma_{\mu\nu}^{(n)} = e^2 \int_{\text{FS}} \frac{dk}{\nu_l} v^\mu_k (-D)^n (v^\nu_k \tau_k) \tag{4}
\]

where \( D = eB v \tau_k (\partial/\partial k) \).
Consider now applied fields low enough that \( \omega_c \tau_k \ll 1 \). In this limit \( \Delta \rho_{xx}/\rho_{xx} \approx -\Delta \sigma_{xx}/\sigma_{xx} - \Theta_H^2 \). Because of symmetry considerations, the linear in \( B \) term in Eq. (1) vanishes for \( \sigma_{xx} \) and it is immediately clear that the low field MR is proportional to \( B^2 \), in agreement with experiment and as in normal metals [4,3,3]. In this limit, the above formula for \( \Delta \sigma_{xx} \) yields

\[
\Delta \sigma_{xx} \approx -e^2 \int_{\text{FS}} \frac{dk}{(2\pi)^2} \omega_c^2 \tau_k v_k \propto -\omega_c^3 \tau \tag{5}
\]

where we have introduced the mean free path, \( \ell_k = \tau_k v_k \), and neglected terms which contain \( \partial \tau_k / \partial k \), whose contribution turns out to be small.

Clearly, \( \Delta \sigma_{xx} \) and hence the MR are governed primarily by cold quasiparticles whose relaxation times are the largest. To take this anisotropy into account, we use the parametrization of Ref. [6], appropriate to a close-to-circular FS, in which the mean free path of a point near the FS can be characterized by an angle \( \theta \):

\[
\ell_{\text{cold}}^{-1} = \ell_{\text{cold}}^{-1} \frac{1 + a \cos 4\theta}{1 - a} \tag{6}
\]

where \( a = (1 - r)/(1 + r) \) is the anisotropy parameter \( r \equiv \ell_{\text{hot}}/\ell_{\text{cold}} \). We find, after some algebra, that

\[
- \frac{\Delta \sigma_{xx}}{\sigma_{xx}} \approx - \frac{\omega_c^2 \ell_{\text{cold}}^2}{8\hbar \nu_f} (3 + 2r + 3r^2). \tag{7}
\]

In the large anisotropy limit, making use of the results for \( \rho_{xx} \) and \( \cot \Theta_H \) (see Ref. [6]):

\[
\cot \Theta_H \approx \frac{2}{\omega_c \tau_{\text{cold}}} \tag{8a}
\]

\[
\rho_{xx} \approx \frac{m_{\text{cold}}}{n e^2 \tau_{\text{cold}} \sqrt{T}} \tag{8b}
\]

this yields the experimental result of Harris et al [2],

\[
- \frac{\Delta \sigma_{xx}}{\sigma_{xx}} \approx - \frac{3}{8} \omega_c^2 \tau_{\text{cold}}^2 \approx - \frac{3}{2} \Theta_H^2 \tag{9}
\]

and

\[
- \frac{\Delta \rho_{xx}}{\rho_{xx}} \approx - \frac{1}{8} \omega_c^2 \tau_{\text{cold}}^2 \approx \frac{1}{2} \Theta_H^2. \tag{10}
\]

Thus, although hot quasiparticles contribute to both \( \sigma_{xx} \) and \( \Delta \sigma_{xx} \), the ratio, \( \Delta \sigma_{xx}/\sigma_{xx} \), is determined entirely by the cold quasiparticles, in accord with the \( r \to 0 \) limit of Eq. (7). Since \( \cot \Theta_H \propto T^2 \) in optimally doped cuprates [2], Eq. (10) explains why Harris et al observed \( \Delta \rho_{xx}/\rho_{xx} \approx -T^{-4} \) in \( \text{YBa}_2\text{Cu}_3\text{O}_7 \) [2]. Moreover, since \( \sigma_{xx} \) involves both cold and hot quasiparticles, plotting \( \Delta \rho_{xx}/\rho_{xx} \) as a function of \( B^2/\rho_{xx}^2 \) will not produce a universal curve, independent of temperature, and hence in the spin-fluctuation model of cuprates Köhler’s rule is violated, again in agreement with experiment [3].

Having established the important role played by the cold quasiparticles, we consider next the magnitude of \( \tau_{\text{cold}} \). We use the expression derived in Ref. [6]:

\[
\frac{h}{\tau_{\text{cold}}} = \frac{h}{\tau_1} + \frac{\gamma k_B T}{T_0 + T}, \tag{11}
\]

where \( \gamma = \alpha g^2/8\hbar \nu_f(Dk) \), \( T_0 = \omega_d \xi_0^2(Dk)^2/\pi \), and we introduce the constant scattering rate, \( 1/\tau_1 \), independent of momentum and temperature, to model the influence of static disorder. We take the parameters which determine \( \tau_{\text{cold}} \) from our earlier calculations of the resistivity and Hall angle for \( \text{Tl} \, 2201 \) (see Refs. [6,9,10]): thus we assume that the spin fluctuations are in the mean field regime, with \( \omega_d \xi_0^2 = \text{const} = 50 \text{ meV} \), take \( \alpha = 15 \text{ states/eV} \), \( \hbar \nu_f = 0.5 \text{ eV} \), \( Dk \sim 1 \text{ inverse lattice spacings} \), \( g = 0.5 \) eV and \( m_c \approx 2m_e \), and so find \( \gamma = 0.93 \). In addition, on assuming that the role played by static disorder does not vary appreciably with doping, we estimate (see below) \( h/\tau_1 = 0.9 \text{ meV} \) from the resistivity measurements on an overdoped sample. Our calculated results [using Eqs. (11) and (12)] for \( \Delta \rho_{xx}/\rho_{xx} \) are compared with experiment in Fig. 1. As may be seen, quantitative agreement with the experimental results of Ref. [3] is found. We remark that the approach we followed is self-consistent, because for this choice of parameters \( \omega_c \tau_{\text{cold}} = 0.5 \) at \( T \sim 200 \text{ K} \) and \( B = 60 \text{ T} \).

We consider next the strong field behavior of the MR, i.e., what happens when for cold quasiparticles the weak field condition, \( \omega_c \tau_{\text{cold}} \ll 1 \) is no longer satisfied. For the above choice of parameters, it is clear from Eq. (11) that as the temperature is lowered to \( T \sim 50 \text{ K} \) this condition is violated. To study this we return to Eqs. (1) and (4) and consider the contributions of order \( B^2 \). A straightforward, but lengthy calculation shows that in the limit of large relaxation rate anisotropy, \( r \ll 1 \), one finds

\[
\beta^2 B^2 \approx \frac{5}{16} \omega_c \tau_{\text{cold}}. \tag{12}
\]

Of course, just as in the case of the parameter \( \alpha \), the exact value of \( \beta \) depends, through \( \tau_{\text{cold}} \), on details of band structure, although we do not expect it to change dramatically as one moves from optimally doped to overdoped samples within the same compound. We emphasize that our estimate of \( \beta \) is not the same as that obtained in the high field limit, where \( \omega_c \tau_k \gg 1 \) for all \( k \). In this limit, \( \beta^2 \propto m^*/\tau^{-1} \), and since the average scattering rate \( \tau \) is dominated by the hot quasiparticles [3], \( \beta \) would have a very different temperature dependence than that given by Eq. (12). Experiment shows that the value of \( \omega_c \tau \), is comparatively low (e.g., in \( \text{Tl} \, 2201 \) at \( T = 40 \text{ K} \) and \( B = 60 \text{ T} \) \( \omega_c \tau \sim 0.9 \) [3]) and therefore our estimate of \( \beta \) is a more reasonable one.

Before comparing our strong field result to experiment it is important to take into account the role of static disorder (the term \( 1/\tau_1 \) in Eq. (11)) and changes in the spin...
fluctuation and band parameters as one goes from the optimally doped to the overdoped material. Using the experimental result for \( \rho_{xx} \)\(^{3}\), which shows a well-defined residual resistivity, \( \rho_0 \), and comparing \( \rho_{xx}(T) - \rho_0 \) with \( \rho_{xx}(T) \), assuming that the temperature dependence of \( \rho_{xx} \) comes only from the spin fluctuations, we estimate \( h/\tau_1 = 0.9 \) meV, the value used above for the optimally doped case. This estimate does not depend on the carrier density and effective mass. ARPES measurements show that for large doping levels the observed FS agrees with Luttinger’s theorem, so that one expects a somewhat larger value of \( \Delta k \) in the overdoped material, while the overall coupling constant may be slightly reduced\(^{3}\). These changes lead to a reduced value of \( \gamma \) (of Eq. (11)) as the doping level increases. Nevertheless, since the relative MR is governed by the cold quasiparticles, which are not strongly scattered by the spin fluctuations, we expect this change to be slight and we take \( \gamma_{\text{over}} \sim 0.9\gamma_{\text{opt}} \).

Figure 2 compares our calculated result with experiment\(^\text{[3]}\): we have made use of Eq. (1) and assumed that the temperature dependence of \( \alpha \) and \( \beta \) comes directly from the temperature dependence of \( \tau_k \) [see Eqs. (11) and (12)], calculated using the above input parameters. Again, reasonable quantitative agreement is found at all temperatures.

In arriving at the above results, we have assumed that \( \gamma \) and \( T_0 \) and hence \( \tau_{\text{cold}} \) do not vary appreciably as one goes from optimally doped to overdoped systems. This means that at optimal doping and higher doping levels, the low field relative MR, \( \Delta \rho_{xx}/\rho_{xx} \propto \omega_c^2 \tau_{\text{cold}}^2 \) must be weakly doping dependent as well. In the inset of Fig. 1 we demonstrate that this is the case for Tl 2201: the results are within 20\% of each other at all fields and temperatures even though the superconducting transition temperatures are vastly different (\( T_c = 30 \) and 80 K respectively). Moreover, the temperature dependence of the low field MR in the two samples is essentially the same, even though the resistivity is qualitatively different. The slightly larger value of the relative MR in the overdoped sample can be attributed to the role of the hot spots, which are better defined in overdoped materials, and hence can contribute to transport more than they do in the optimally doped ones, as discussed below.

The onset of high field behavior is also seen in the Hall effect\(^\text{[3]}\): while the low field Hall resistivity is given by \( \rho_{xy} = R_H B \), where \( R_H \) is the Hall constant, higher order terms lead to considerable deviation of \( \rho_{xy} \) from linear in \( B \) behavior\(^\text{[3]}\). On using the same approximations as for the MR, we have found that, in Tl 2201 at \( T = 40 \) K and \( B = 60 \) T, the relative departure of \( \rho_{xy} \) from linearity, \( 1 - R_H \rho_{xy} = 0.3 \) in reasonable agreement with the experimental value of \( \sim 0.25 \).

The quantitative agreement with experiment we find is remarkably good, given the at first sight crude approximations we have made. For example, in Eq. (11) we have neglected terms which involve derivatives of \( \tau_k \) with respect to the momentum component parallel to the FS, while our analysis assumes that the FS in Tl 2201 is close to circular. There are several reasons why these assumptions are reasonable. First, in a NAFL the momentum dependence of the relaxation rates can typically be factorized from their temperature dependence (see Ref.\(^\text{[3]}\)). Therefore the inclusion of the terms involving these derivatives would only lead to somewhat different momentum weights for the different temperature dependencies seen in \( \tau_k \), but not to an overall changed behavior. Second, the hot spots are typically not special symmetry points on the FS. Hence contribution from the terms with derivatives is substantially reduced by geometric factors. Still, inclusion of the remaining terms could, in principle, lead to rather different values of the parameter \( \alpha \propto \Delta \rho_{xx}/\rho_{xx} \omega_c^2 \). For example, if the FS is close to rectangular, the value of \( \alpha \) can be much larger than that quoted here.

We consider next the role played by the hot quasiparticles, whose relaxation rate is given by\(^\text{[3]}\):

\[
\frac{1}{\tau_k} = \frac{\alpha^2 T \xi}{4\eta} \left[ 1 - \left( 1 + \frac{\pi T}{\omega_{sf}} \right)^{-\frac{1}{2}} \right].
\]

In Ref.\(^\text{[6]}\) we have shown that only in the limit \( T \gg \omega_{sf}/\pi \), is the behavior of the hot quasiparticles anomalous compared to the Fermi liquid behavior, leading to \( r \to 0 \). The overdoped materials tend to have larger values of \( \omega_{sf} \) (see Ref.\(^\text{[1]}\)), which, in turn, depends only moderately on temperature. Both experiment and a comparison of Eqs. (11) and (13) shows that for overdoped systems at low temperatures (\( T \sim 50 \) K) one no longer has \( r \ll 1 \), so that higher order (in \( r \)) terms must be taken into account. It is easy to show that these terms lead to a reduction of \( \alpha \) and in the limit \( r \to 1 \) (no anisotropy) yield a vanishing MR, as expected for a uniform scattering rate\(^\text{[3]}\). Therefore, only a full numerical calculation can confirm the quantitative agreement with experiment seen here\(^\text{[2]}\).

In conclusion, we have performed a simple analytical calculation of the MR in cuprate superconductors, based on the spin fluctuation (NAFL) model of cuprates. Quantitatively, the experimentally measured MR appears to be in reasonable agreement with the magnetic measurements in cuprates, indicating the close connection between the magnetic properties and the transport and therefore the importance of spin fluctuations for the normal state of cuprates. We find that the NAFL model gives a consistent description of all experimental data to date. It naturally explains a close relationship between the orbital MR and the Hall effect results\(^\text{[3]}\). It also predicts that the experimentally observed deviations from the low field behavior of the MR is due to onset, rather than the truly high field behavior.

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FIG. 1. The calculated relative (orbital) magnetoresistance in an optimally doped Ti 2201 compound, is compared with experiment (Ref. [3]): the symbols (lines), offset by 0.004 for clarity, correspond to the measured (calculated) result at (top to bottom) $T = 140$, 200 and 260K. The input parameters have been obtained from NMR measurements (see Refs. [6,9,10]) and are given in the text. Inset: the experimentally measured results of Tyler et al [3] for the low field orbital MR in the overdoped (solid lines) and optimally doped (symbols) Ti 2201 material at $T = 120$ (top) and $T = 180$ K (bottom). Note that the numerical values of the relative MR differ by less than 20%, in agreement with Eq. (9).

FIG. 2. The calculated relative magnetoresistance in an overdoped Ti 2201 compound, is compared with experiment (Ref. [3]): the results correspond to (top to bottom) $T = 40$, 50, 60, 80, 100, 120 and 150K. The theoretical curve (solid lines) is given by Eq. (1) where the coefficients $\alpha$ and $\beta$ have been determined from the spin fluctuation model, using parameters (see text) obtained from NMR measurements in the optimally doped system (Refs. [6,9,10]). Note the onset of high field effects at lower temperatures.