Exploring the sources of p-mode frequency shifts in the CoRoT target HD 49933

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Abstract Oscillations of the solar-like star HD 49933 have been thoroughly observed by CoRoT. Two dozen frequency shifts, which are closely related to the change in magnetic activity, have been measured. To explore the effects of magnetic activity on frequency shifts, we calculate frequency shifts for the radial and $l = 1$ p-modes of HD 49933 with the general variational method, which evaluates the shifts using a spatial integral of the product of a kernel and some sources. The theoretical frequency shifts reproduce the observation well. The magnitudes and positions of the sources are determined according to a $\chi^2$ criterion. We predict the source that contributes to both the $l = 0$ and $l = 1$ modes is located $0.48 - 0.62$ Mm below the surface of the star. In addition, based on the assumption that $A_0$ is proportional to the change in the MgII activity index $\Delta i_{\text{MgII}}$, we obtain that the change in MgII index between the minimum and maximum of the cycle during the period of HD 49933 is about 0.665. The magnitude of the frequency shifts compared to the Sun already demonstrates that HD 49933 is much more active than the Sun, which is further confirmed in this paper. Furthermore, our calculation of the frequency shifts for $l = 1$ modes indicates the variation of turbulent velocity in the stellar convective zone may be an important source for the $l = 1$ shifts.

Key words: stars: individual (HD 49933) — stars: evolution — stars: oscillation — stars: modeling

1 INTRODUCTION

HD 49933, also known as HR 2530 and HIP 32851, is an F5V main sequence star with a surface rotation period of 3.5 d. It has a temperature ranging between $6450 \pm 75$ K (Kallinger et al. 2010) and $6780 \pm 130$ K (Bruntt et al. 2004), a $\log(L/L_{\odot})$ between $4.24 \pm 0.13$ (Bruntt et al. 2008) and 4.3 ± 0.2 (Bruntt et al. 2004) and a radius of $1.42 \pm 0.04 R_{\odot}$. Its metallicity, ranging from $-0.46 \pm 0.08$ (Bruntt et al. 2008) to $-0.30 \pm 0.11$ (Bruntt et al. 2004), is slightly metal poor compared to the Sun and Procyon. All these characteristics are summarized in Table 1. Its oscillation, with

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Table 1: Observational Data for HD 49933

| $T_{\text{eff}}$ (K) | $\log (L/L_\odot)$ | [Fe/H] | $R$ ($R_\odot$) | Reference |
|---------------------|-------------------|--------|----------------|------------|
| 6570 ± 60           | –0.44 ± 0.03      | Bruntt (2009) |
| 6450 ± 75           | –0.46 ± 0.08      | Kallinger et al. (2010) |
| 6780 ± 130          | 4.24 ± 0.13       | Bruntt et al. (2008) |
| 6735 ± 53           | 4.26 ± 0.08       | Gillon & Magain (2006) |
| 6780 ± 70           | 4.3 ± 0.2         | Bruntt et al. (2004) |
|                     | 1.42 ± 0.04*      | Bigot et al. (2011) |

* Data used by Liu et al. (2014).

Table 2: Observed Frequencies and Their Shifts for the Star HD 49933 (Salabert et al. 2011)

| Frequency ($\mu$Hz) | Frequency shift ($\mu$Hz) | Frequency ($\mu$Hz) | Frequency shift ($\mu$Hz) |
|---------------------|---------------------------|---------------------|---------------------------|
| l = 0               |                           | l = 1               |                           |
| 1544.69 ± 0.83      | 0.753 ± 1.007             | 1500.54 ± 0.94      | –0.336 ± 0.848            |
| 1631.10 ± 0.22      | 0.780 ± 1.123             | 1586.62 ± 0.61      | –0.111 ± 0.666            |
| 1714.49 ± 0.61      | –0.606 ± 2.108            | 1670.48 ± 0.81      | 1.139 ± 0.948             |
| 1799.75 ± 1.03      | 2.073 ± 1.508             | 1755.30 ± 0.78      | –0.496 ± 0.981            |
| 1884.82 ± 0.59      | 0.815 ± 2.050             | 1840.68 ± 0.79      | 1.657 ± 0.853             |
| 1972.73 ± 1.14      | 1.344 ± 1.158             | 1928.13 ± 1.48      | 0.233 ± 0.750             |
| 2057.82 ± 0.96      | 3.059 ± 1.674             | 2014.38 ± 0.93      | 0.717 ± 0.992             |
| 2147.10 ± 1.05      | 2.808 ± 0.863             | 2101.58 ± 1.67      | 1.091 ± 1.021             |
| 2236.46 ± 0.39      | 1.868 ± 1.658             | 2190.81 ± 2.32      | –0.578 ± 1.095            |
| 2322.10 ± 1.66      | 1.430 ± 2.193             | 2277.89 ± 1.29      | –3.338 ± 1.826            |
| 2408.56 ± 0.83      | 2.174 ± 0.989             | 2450.76 ± 3.34      | –0.336 ± 0.848            |
| 2495.76 ± 3.34      | 11.444 ± 2.812            | 2450.76 ± 3.34      | –0.336 ± 0.848            |

* Data kindly provided through private communication.

The first observation of Doppler velocity done by Mosser et al. (2005), was observed three times by CoRoT in recent years, but the last run is still being processed by the CoRoT team. The time span of the first observation was 60 days at the beginning of 2007 during the first CoRoT run (IRa01) while the second was 137 days in 2008 during the first CoRoT long duration run (LRa01). García et al. (2010) analyzed these two sets of data and discovered that the p-mode frequencies and amplitudes of HD 49933 varied with magnetic activity and showed a period of at least 120 d. Subsequently, Salabert et al. (2011) analyzed the second set of data by dividing the 137-day light curve into two subseries corresponding to periods of low- and high-stellar activity based on the work of García et al. (2010). They extracted 24 frequencies and their shifts for $l = 0$ and $l = 1$ modes using a local maximum likelihood fitting analysis, which had 12 $l = 0$ modes and 12 $l = 1$ modes. These are listed in Table 2 (Salabert et al., private communication).

We can find from Table 2 that, for $l = 0$ modes, most frequency shifts are located in the range of 1–3 µHz, quite high compared to the frequency shifts of the Sun and β Hyi. In addition, the frequency shifts of HD 49933 reach a maximal value of about 3 µHz around 2100 µHz. For frequencies larger than 2100 µHz, the variation in the p-mode frequency shifts indicates a downturn followed by an upturn for both $l = 0$ and $l = 1$ modes. Such a frequency dependence of the frequency shifts measured in HD 49933 is comparable with the one observed in the Sun (Salabert et al. 2004), suggesting the solar-like star HD 49933 could have a similar physical mechanism driving the frequency shifts as the ones taking place in the Sun, which is thought to arise from changes in the outer layers due to its magnetic activity (Salabert et al. 2011).

In dynamo modeling, frequency shifts are thought to arise from either changes in propagation speed near the surface due to a direct magnetic perturbation (Goldreich et al. 1991), or a slight decrease in the radial component of the turbulent velocity in the outer layers and the associated
Table 3 Parameters for the Sun and $\beta$ Hyi

| Star      | $A_0$   | Ref. | $\Delta_{\text{MgII}}$ | Ref. | Frequency shift (\(\mu\)Hz) | Ref. |
|-----------|---------|------|-------------------------|------|-----------------------------|------|
| Sun       | 0.3116  | [1]  | 0.0135                  | [1]  | most < 0.8                  | [1]  |
| $\beta$ Hyi | 0.33    | [1]  | 0.015                   | [1]  | 0.1 ± 0.4                   | [2]  |
| HD 49933  | 14.63*  |      | 0.665*                  |      | most 1–3                    | [3]  |

[1] Matcalfe et al. (2007); [2] Bedding et al. (2007); [3] Salabert et al. (2011). *Results in this paper.

In our previous work (Liu et al. 2014) we used the small frequency separation ratios $r_{01}$ and $r_{10}$ to constrain the evolution parameters of the stellar models and determined the size of the convective core and the extent of overshooting for HD 49933. In the present work, we will utilize the stellar models we have obtained in Liu et al. (2014), with the method developed by Metcalfe et al. (2007) to study the observed frequency shifts of HD 49933.

In Section 2 we outline the general variational method for modeling frequency shifts. Then we apply this method to study the shifts in radial and nonradial modes of HD 49933 in Section 3. Finally, we discuss our results and give conclusions in Section 4.

2 METHOD

2.1 General Formulations

In order to evaluate frequency shifts that are related to activity, we use a general variational expression given by Metcalfe et al. (2007),

$$\Delta \nu_{nlm} = \int \frac{d^3r}{2I_{nl} \nu_{nlm}} K_{nlm} S,$$

(1)

where

$$I_{nl} = \int_0^R \rho \left[ \xi_r^2 + \Lambda \xi_h^2 \right] r^2 dr = R^5 \bar{\rho} \tilde{I}_{nl},$$

(2)

is the mode inertia. $\Lambda = l(l + 1)$ and $\bar{\rho}$ is the stellar mean density. The dimensionless mode inertia $\tilde{I}_{nl}$ is defined as

$$\tilde{I}_{nl} = \int_0^1 \tilde{\rho} \left[ y^2 + \Lambda z^2 \right] x^4 dx,$$

(3)

where $x = \frac{r}{R}$, $\tilde{\rho} = \frac{\rho}{\bar{\rho}}$, $y = \frac{\xi_r}{\tilde{\rho}}$, and $z = \frac{\xi_h}{\tilde{\rho}}$ are corresponding dimensionless quantities. Since all the derived kernels have leading terms proportional to $|\text{div} \xi_{nlm}(r)|^2$, for simplicity Metcalfe et al. (2007) adopted the common kernel

$$K_{nlm} = |\text{div} \xi_{nlm}(r)|^2 = q_{nl}(D)|Y_{ml}^m|^2,$$

(4)
where $D$ denotes the depth below the photosphere and $Y^m_l$ is the spherical harmonic. It is easily verified that

$$q_j(D) = \left[ \frac{1}{r^2} \frac{\partial}{\partial r}(r^2 \xi_v(r)) - \frac{1}{r} \xi_h(r) \Lambda \right]^2.$$  

(5)

The following simple form of the source was assumed in Metcalfe et al. (2007)

$$S_k(D) = 1.5 \times 10^{-11} A_k \delta(D - D_{c,k}) \mu Hz^2,$$

(6)

where $A_k$ and $D_{c,k}$ are adjustable parameters representing the strength and position of the source, respectively, and will be determined by fitting the measured shift data. The numerical factor is arbitrary. Combining Equations (6), (4) and (1) leads to

$$\Delta \nu_{nlm} = R M l \sum_k A_k Q_{nl}(D_{c,k}) \kappa_{k,lm},$$

(7)

where

$$Q_{nl}(D_{c,k}) = 1.5 \times 10^{-11} \frac{q_j(D_{c,k})}{I_{nl} \nu_{nl}},$$

(8)

$$\kappa_{k,lm} = \int \int |Y^m_l|^2 P_{2k}(\mu) d\mu d\phi = P_{2k}^l(m) Z^l_k,$$

(9)

and

$$Z_{k,l} = (-1)^k \frac{(2k - 1)!!(2l + 1)!!(l - 1)!}{k!(2l + 2k + 1)!!(l - k)!}.$$  

(10)

In these equations, $R$ and $M$ are expressed in solar units, frequencies are expressed in $\mu Hz$ and $P_{2k}^l(m) = lP_{2k}(m/l)$ are orthogonal polynomials of order $2k$ (see Schou et al. 1994).

### 2.2 Frequency Shifts of $l = 0$ and $l = 1$ Modes

Given the values of $A_k$ and $D_{c,k}$, we can evaluate the change in the frequencies within individual multiples of low-degree modes using Equation (7). However, for radial modes ($l = 0$), both $k$ and $m$ only have single values that are zero, and thus only $A_0$ and $D_{c,0}$ are needed. Simple calculation shows $\kappa_{k,lm} = 1$ for $k = l = m = 0$. Then Equation (7) becomes

$$\Delta \nu_{n0} = \frac{R}{M} A_0 Q_{n0}(D_{c,0}).$$

(11)

The values of $A_0$ and $D_{c,0}$ can be determined by fitting the measured frequency shifts (see Table 2) for radial modes. Using the least squares fitting technique we have

$$A_0(D_{c,0}) = \frac{M}{R} \frac{\Sigma_n Q_{n0}(D_{c,0}) \Delta \nu_{n0}^{obs}}{\Sigma_n Q_{n0}^2(D_{c,0})/\sigma_{n0}^{obs}^2},$$

(12)

where $\Delta \nu_{n0}^{obs}$ are the measured $l = 0$ shifts and $\sigma_{n0}^{obs}$ are the measured uncertainties. For any given $D_{c,0}$, we can calculate $A_0$ through Equation (12) and $\Delta \nu_{n0}$ through Equation (11). By brute-force searching from the star’s interior to the surface, we obtain the best estimate of $D_{c,0}$ that minimizes $\chi^2$ (see Fig. 1)

$$\chi^2 = \sum_n \left( \frac{\Delta \nu_{n0} - \Delta \nu_{n0}^{obs}}{\sigma_{n0}^{obs}} \right)^2 = \sum_n \left[ \frac{R}{M} A_0(D_{c,0}) Q_{n0}(D_{c,0}) - \Delta \nu_{n0}^{obs} \right]^2.$$  

(13)
It should be noted that Equations (12) and (13) are different from equations (8) and (9) in Metcalfe et al. (2007). The latter are problematic, because they always lead to too small $A_0$ and too large a discrepancy between calculated and measured shifts.

If we have measurements of the individual mode frequencies within multiplets, we could use Equation (7) to directly calculate $A_k$ and $D_{c,k}$ for $k$ up to $l$. However, such measurements are difficult, and we only get the mean frequency shifts (averaged over multiplet components) for $l = 1$ modes as shown in Table 2. In order to calculate the mean frequency shifts, the following formula is proposed in Dziembowski (2007)

$$\Delta \nu_{nl} = \frac{2l + 1}{2} \frac{R}{M} \sum_{k=0}^{l} \left[ \sum_{m=-l}^{m=l} \left| Y_l^m(\theta_0, 0) \right|^2 \kappa_{k,lm} \right] A_k Q_{n1}(D_{c,k}) ,$$

(14)

where $\theta_0$ represents the inclination of the rotation axis to the line of sight. For $l = 1$ modes, there are two sources of perturbation that contribute to the frequency shifts, corresponding to $k = 0$ and $k = 1$ (see Eq. (6)), respectively. The above equation becomes

$$\Delta \nu_{nl} = \frac{3}{2} \frac{R}{M} \sum_{k=0}^{1} \left[ \sum_{m=-1}^{m=1} \left| Y_1^m(\theta_0, 0) \right|^2 \kappa_{1,lm} \right] A_k Q_{n1}(D_{c,k})$$

$$= \frac{9}{8\pi M} A_0 Q_{n1}(D_{c,0}) + \frac{9}{40\pi M} \left( 2 \cos^2 \theta_0 - \sin^2 \theta_0 \right) A_1 Q_{n1}(D_{c,1}).$$

(15)

Thus, four parameters, $A_0$, $A_1$, $D_{c,0}$ and $D_{c,1}$, in Equation (15) need to be determined. $A_0$ and $D_{c,0}$ can be determined through Equations (12) and (13), based on the frequency shift of radial modes, while $A_1$ and $D_{c,1}$ are calculated based on the frequency shift of $l = 1$ modes by minimizing the following $\chi^2$ criterion

$$\chi^2 = \sum_{n} \left( \frac{\Delta \nu_{nl} - \Delta \nu_{nl}^{obs}}{\sigma_{nl}^{obs}} \right)^2,$$

(16)

with steps analogous to those used in calculating $A_0$ and $D_{c,0}$.

3 NUMERICAL RESULTS

3.1 Radial Modes

In order to calculate the frequency shifts of HD 49933 using Equation (11), a proper stellar model is needed. To reproduce the observed characteristics of HD 49933, in Liu et al. (2014) we computed a grid of evolutionary tracks with the Yale Rotation Evolution Code (Pinsonneault et al. 1989; Guenther et al. 1992; Yang & Bi 2007). The initial parameter range of masses and heavy metal abundances are $1.08 - 1.34 M_\odot$ and $0.006 - 0.030$, respectively. Theoretical analysis has been carried out for the star. A total of fifty-four best-fitting models were identified out of hundreds of evolutionary tracks in Liu et al. (2014), among which parameters of 17 models are listed in Table 4. These 17 models can not only reproduce, like the other 37 models, the measured temperature, luminosity, and large frequency separation of the star, but also fit the variation pattern of the small frequency separations well in terms of frequencies.

The computational results of all 17 models are summarized in Table 5. Here, we take model 16 from Table 4 as representative of our analysis, since it has the smallest $\chi^2_0$ compared to other models. Among the 12 measured frequency shifts of radial modes, we rule out the last (and largest) shift and only fit the other 11 ones, because this shift is probably an outlier. Values of $\chi^2_0(r)$, defined by Equation (13), depend on the radius of the star. The variation of $\chi^2_0(r)$ with radius in the vicinity of the star’s surface is shown in Figure 1. Note that in this figure, the $\chi^2_0$ curve has two minima.
### Table 4: Evolutionary Models (Liu et al. 2014) for HD 49933

| Model | $M$ ($M_\odot$) | $(Z/X)_s$ | $T_{\text{eff}}$ (K) | $L/L_\odot$ | $R/R_\odot$ | Age (Gyr) |
|-------|----------------|-----------|----------------------|-------------|-------------|-----------|
| 1     | 1.26           | 0.0061    | 6603                 | 3.532       | 1.439       | 1.536     |
| 2     | 1.26           | 0.0073    | 6626                 | 3.597       | 1.441       | 1.639     |
| 3     | 1.28           | 0.0097    | 6602                 | 3.626       | 1.453       | 1.595     |
| 4     | 1.30           | 0.0092    | 6546                 | 3.475       | 1.454       | 1.277     |
| 5     | 1.30           | 0.0106    | 6664                 | 3.548       | 1.459       | 1.410     |
| 6     | 1.26           | 0.0059    | 6608                 | 3.540       | 1.452       | 1.580     |
| 7     | 1.26           | 0.0071    | 6632                 | 3.614       | 1.442       | 1.696     |
| 8     | 1.28           | 0.0069    | 6557                 | 3.467       | 1.445       | 1.402     |
| 9     | 1.28           | 0.0082    | 6583                 | 3.540       | 1.448       | 1.521     |
| 10    | 1.28           | 0.0094    | 6608                 | 3.614       | 1.452       | 1.646     |
| 11    | 1.28           | 0.0135    | 6537                 | 3.491       | 1.458       | 1.837     |
| 12    | 1.29           | 0.0132    | 6577                 | 3.595       | 1.462       | 1.763     |
| 13    | 1.30           | 0.0104    | 6568                 | 3.483       | 1.459       | 1.454     |
| 14    | 1.28           | 0.0133    | 6541                 | 3.499       | 1.458       | 1.867     |
| 15    | 1.29           | 0.0125    | 6569                 | 3.565       | 1.460       | 1.731     |
| 16    | 1.28           | 0.0131    | 6545                 | 3.508       | 1.458       | 1.902     |
| 17    | 1.29           | 0.0123    | 6573                 | 3.573       | 1.459       | 1.758     |

### Table 5: Computational Results of the Evolutionary Models in Table 4

| Model | $A_0$ | $D_{c,0}$ | $\Delta i_{\text{MgII}}^*$ | $\chi^2_0$ | $A_1$ | $D_{c,1}$ | $\chi^2_1$ |
|-------|-------|-----------|----------------------------|------------|-------|-----------|------------|
| 1     | 13.57 | 0.527     | 0.617                      | 2.734      | -3814 | 3.842     | 7.675      |
| 2     | 12.93 | 0.516     | 0.588                      | 2.699      | -3749 | 3.806     | 7.764      |
| 3     | 14.23 | 0.561     | 0.647                      | 2.700      | -4149 | 3.868     | 7.810      |
| 4     | 15.16 | 0.554     | 0.689                      | 2.748      | -4421 | 3.910     | 7.685      |
| 5     | 16.55 | 0.620     | 0.752                      | 2.735      | -4720 | 3.985     | 7.497      |
| 6     | 13.49 | 0.528     | 0.613                      | 2.742      | -3737 | 3.818     | 7.658      |
| 7     | 13.20 | 0.534     | 0.600                      | 2.702      | -3689 | 3.801     | 7.719      |
| 8     | 14.46 | 0.539     | 0.657                      | 2.761      | -4084 | 3.872     | 7.638      |
| 9     | 14.14 | 0.543     | 0.643                      | 2.723      | -4105 | 3.874     | 7.719      |
| 10    | 14.18 | 0.564     | 0.645                      | 2.707      | -4069 | 3.857     | 7.787      |
| 11    | 12.76 | 0.484     | 0.580                      | 2.644      | -4133 | 3.745     | 7.933      |
| 12    | 14.23 | 0.557     | 0.647                      | 2.643      | -4288 | 3.825     | 7.898      |
| 13    | 15.14 | 0.577     | 0.688                      | 2.736      | -4639 | 3.897     | 7.745      |
| 14    | 13.49 | 0.513     | 0.613                      | 2.641      | -4179 | 3.758     | 7.921      |
| 15    | 12.91 | 0.504     | 0.587                      | 2.672      | -4085 | 3.785     | 7.870      |
| 16    | 14.63 | 0.558     | 0.665                      | 2.639      | -4333 | 3.809     | 7.886      |
| 17    | 13.69 | 0.534     | 0.622                      | 2.672      | -4221 | 3.832     | 7.869      |

* $\Delta i_{\text{MgII}} = A_0/22$.

However, the position of the source is unlikely to be determined by the left minimum, since the right one is a global minimum and the value of $Q_{n0}$ at the left minimum is far lower than that at the right minimum. The best estimate of $D_{c,0}$, determined by the right minimum for $\chi^2_0$, is 0.558 Mm. The corresponding $A_0$, obtained by Equation (12), is 14.63. The value of $A_0$ is much bigger than that of the Sun and $\beta$ Hyi obtained by Metcalfe et al. (2007) (see Table 3). Considering that the measured frequency shifts for HD 49933 are quite large compared to those of the Sun and $\beta$ Hyi, this result is not surprising. Because the value of $A_0$ is directly related to variation in the magnetic field near the convective zone of the star, we confirm that the magnetic field of HD 49933 shows more variations through the period of its cycle than the Sun or $\beta$ Hyi.
Fig. 1 $\chi^2_0$ (dot-dashed line) and $\chi^2_1$ (solid line) vs. relative radius $r/R$ for model 16. The vertical lines mark the positions $D_{c,0}$ (right) and $D_{c,1}$ (left) of the source.

Fig. 2 Observed frequency shifts (dots) for the $l = 0$ mode of HD 49933 and theoretical results computed from Eq. (7) (triangles). The dashed lines correspond to weighted linear fits to the observed shifts, while the solid lines correspond to linear fits to the calculated shifts.

The frequency shifts calculated from Equation (11) for the radial modes of HD 49933, and the observed frequency shifts are shown in Figure 2. The corresponding least squares linear fits, for which the 1st to the 7th points and the 7th to the 11th points are fitted separately, are also drawn in the figure. A rising trend with increasing frequencies for the calculated shifts is evident, which is consistent with the results of previous work (Goldrich et al. 1991; Dziembowski & Goode 2004; Metcalfe et al. 2007). The measured shifts also grow with frequencies up to 2100 $\mu$Hz (Salabert et al. 2011), but the four shifts in the range of 2100 – 2400 $\mu$Hz seem to subsequently drop, which cannot be reproduced by the present model. However, due to the poor quality of observational data, this dropping pattern is questionable. Generally speaking, the calculations are in good agreement with the measurements. The value of $\chi^2_0$ is 2.639.
The last measured frequency shift, whose value is 11.44 µHz, is exceedingly large compared to other measured frequency shifts. We cannot get a good fitting with it. Such a bimodal pattern, with different behaviors in low and high frequencies, was also found in frequency shifts of the Sun, for which the fractional frequency shifts rapidly rise at low frequencies and precipitously decline at high frequencies (above \( \nu \approx 4 \) mHz). This behavior points to different locations of the sources producing the frequency shifts. Since Goldreich et al. (1991) ascribed the sudden decline in the solar frequency shifts at high frequencies to a rise in the solar chromospheric temperature, we naturally assume that it may be the decrease in the stellar chromospheric temperature that leads to the abrupt rise of frequency shift for HD 49933. The precipitous nature of the rise results from a chromospheric resonance that occurs at \( \nu \approx 2500 \) µHz.

Unlike \( \beta \) Hyi and the Sun, which have been studied in Metcalfe et al. (2007), the measured \( \text{MgII} \) index data for HD 49933 are not currently available. Metcalfe et al. (2007) have obtained a relationship of \( A_0 = 22\Delta i_{\text{MgII}} \) for the Sun. According to the assumption that the ratio between \( A_0 \) and \( \Delta i_{\text{MgII}} \) is invariant in different solar-like stars (Metcalfe et al. 2007), we can predict that the change in the \( \text{MgII} \) index between the minimum and maximum of the cycle during the period of HD 49933 should be about 0.665, a number yet to be validated by future observations.

### 3.2 Nonradial Modes

Benomar et al. (2009) measured the inclination angle of HD 49933 to be \( \theta_0 = 17^{\circ} \pm 9^{\circ} \). According to Equation (15), only \( A_1 \) depends on the value of \( \theta_0 \), but \( \theta_0 \) has no impact on \( D_{c,1} \) and the resulting \( \chi_1^2 \). So, we choose the central value of \( \theta_0 = 17^{\circ} \) and calculate \( A_1 \) and \( D_{c,1} \) by fitting the measured frequency shifts of \( l = 1 \) modes with the \( \chi^2 \) criterion, i.e. Equation (16). In this calculation, we disregard the last observed frequency shifts.

The \( \chi_1^2(r) \) curve in the vicinity of the star’s surface is plotted in Figure 1. Figure 3 illustrates the calculated frequency shifts for the \( l = 1 \) modes, together with the observed data. We can see that though the agreement between theoretical results and measured shifts for \( l = 1 \) modes (with \( \chi_1^2 = 7.886 \)) is not as good as that of the radial modes, the general trend of shifts with frequency is reproduced. The best estimate of \( D_{c,1} \) is 3.809 Mm (see Fig. 1), which means the position of the

![Fig. 3](image-url) The computed frequency shifts (triangles) and the measured shifts (dots) for \( l = 1 \) p-modes of HD 49933.
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$k = 1$ source is deeper below the stellar surface than that of the $k = 0$ source. The calculated $A_1$ is negative, with a value of $-4333$. This means the $k = 1$ source may represent a variation of turbulent velocity in the outer convective zone. It is commonly accepted that the magnetic field impedes convection, and so an increase in the magnetic field will reduce turbulent velocity. Dziembowski (2004) showed that a decrease in the turbulent pressure causes frequency to increase. Therefore, if we assume the variation of turbulent pressure in the stellar convective zone is a type of source, the corresponding $A_1$ should be negative.

4 DISCUSSION AND CONCLUSIONS

In this work, we reproduce the observed frequency shifts well for the radial ($l = 0$) and nonradial ($l = 1$) oscillation modes of a solar-like star HD 49933. Our results show that magnetic activity of HD 49933 may be more active than that of the Sun and β Hyi, and we predict the change in MgII activity index $\Delta i_{\text{MgII}}$ between the minimum and maximum of the stellar activity cycle for HD 49933 should be much larger than that observed in the Sun and β Hyi. Moreover, the position of the source that contributes to both $l = 0$ and $l = 1$ modes is limited to be in the range $0.48 \pm 0.62$ Mm below the stellar surface.

It is commonly assumed that magnetic fields impede convection, that is, decrease the convective velocity. Our calculation for the frequency shifts of $l = 1$ modes indicates that the decrease in turbulent velocity induced by the increasing magnetic field in the rising phase during the active period of HD 49933 may significantly contribute to the frequency shifts for $l = 1$. Based on mixing-length theory, a decrease in the convective velocity is associated with a decrease in temperature in the convective zone (Dziembowski & Goode 2005), which can be reflected by lower effective temperature of a star. Since perturbations to effective temperature and chromospheric temperature are both related to activity in the magnetic field, the relationship between variations in the two types of temperatures during the cycle of stellar activity is also an issue deserving further research.

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