The effect of energy dissipation on the dynamic response of reinforced concrete structure

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Abstract. Solving problems associated with protecting buildings from progressive collapse and minimizing resources is becoming increasingly important. In many countries, ensuring such protection is set in accordance with the requirements of national regulatory documents and, therefore, researches aimed at developing effective ways to protect constructive systems from progressive collapse under special actions are relevant. In this regard, the present work presents studies of the influence of energy dissipation in constructive reinforced concrete elements during their sudden structural rearrangement caused by the removal of one of the supporting elements. The hysteretic dispersion accompanying the nonlinear load-unload cycle of the cross-section during crack formation is taken into account. The damping mechanism of reinforced concrete elements of the constructive system is described by the equivalent viscous damping model, based on the hypothesis of proportionality of the damping force and the oscillation velocity. Approbation of the calculated model is carried out in relation to the reinforced concrete frame structures, the physical models of which were tested under the considered loading regime. A comparison of the experimental and calculated parameters shows the effectiveness of the proposed model for assessing the dynamic response of reinforced concrete structures with cracks under the static-dynamic loading regime.

1. Introduction

The number of studies aimed at protecting buildings and structures from progressive collapse has recently grown worldwide. These studies are aimed at solving survivability problems of physically and structurally nonlinear systems [1-4], problems of determining the stress-strain state in constructive elements after removing one of the bearing elements in them [5-8], and formulation of the criteria for the bearing capacity of constructive systems for special actions [9-12], experimental determination of the parameters of the static-dynamic deformation of structures under such loading conditions [13-16], etc. This topic also includes solving problems of finding effective ways to protect buildings and structures that minimize the consumption of materials and the cost of such protection [17-20]. Studies on taking into account the dissipative properties of materials that increase the damping properties of structures [21-24] can also be attributed to this direction.

In this regard, taking into account the relevance of the noted problems, this work is devoted to the study of the influence of energy dissipation on the dynamic response of reinforced concrete structure that works with cracks under special action caused by its sudden structural rearrangement.
2. Methods

It is known that in reinforced concrete structure there are three main types of energy dissipation, namely: hysteretic, accompanying the load-unload cycle of such inelastic material as reinforced concrete, energy dissipation due to friction on the contact surface between concrete and reinforcement in the crack formation zone, and dispersion caused by the formation of sound waves. Below is a mechanism for the formation of the first two types of energy dissipation.

2.1. Hysteretic dispersion accompanying the concrete load-unload cycle

Consider the cross-section of a bending reinforced concrete element and stress-strain diagram of the compressed concrete zone before crack formation (Figure 1). During structural oscillations, a loading and unloading process occurs in the compressed concrete zone. Since concrete is a nonlinear material, irreversible deformation ($\varepsilon_{irr}$) will appear in it when loading concrete. This deformation leads to the attenuation of structural oscillations. The diagram “stresses-total relative deformations” for a one-time cycle of static-dynamic load-unload of concrete (static section $oa$ and dynamic $ab$) has the form shown in Figure 1b [13].

![Figure 1. Mechanism description scheme of hysteresis dispersion in reinforced concrete element: a - scheme of forces in cross-section of the element at $M < M_{cre}$; b- diagram "$\sigma_b - \varepsilon_b$" under static-dynamic loading of concrete ($\varepsilon_{re}$-reversible part of deformation; $\varepsilon_{irr}$ - irreversible part).](image)

The ascending branch $oa$ of the diagram reflects the stage of static loading to the operational load level. It can be described by a nonlinear function G.A. Geniev [25]:

$$\sigma_{oa} = E_0 \left(1 - \frac{\varepsilon}{2\varepsilon_{bu}}\right)$$

(1)

where $\varepsilon_{bu} = 2R_b / E_0$ is the ultimate deformation of concrete.

The straight-line $ab$ describes the process of dynamic additional loading of a reinforced concrete element. As shown by the results of experimental studies [13,14,26], it can be described by a linear function in value of the coordinates of point $a$ and the value of angle inclination of a straight line $ab$, which expresses the dynamic modulus of concrete deformation:

$$\sigma_{a-b} = E_d \left(\varepsilon - \varepsilon_{at}^d\right) + E_0 \left(1 - \frac{\varepsilon^d}{2\varepsilon_{bu}}\right)$$

(2)
The descending branch \(bd\) of the diagram describes the unloading stage. The area of the figure \(oabe\) is equal to the amount of potential energy \(W\) related to a unit volume of the body under static-dynamic loading when full deformation is achieved \(\varepsilon^d\):

\[
W = S_{oae} + S_{abe} = \int_0^{\varepsilon^d} \sigma_{ae} \, d\varepsilon + \int_{\varepsilon^d}^{\varepsilon^u} \sigma_{de} \, d\varepsilon
\]

or after integration and corresponding transformations:

\[
W = \frac{E_0}{2} \left(1 + \frac{2\varepsilon^u}{3\varepsilon_{bu}} - \frac{\varepsilon^d}{\varepsilon_{bu}}\right) + E_0 \left(\frac{\varepsilon^d}{2\varepsilon_{bu}} \left(\varepsilon^d - \varepsilon^u\right) + \frac{\varepsilon^u}{2\varepsilon_{bu}} \left(\varepsilon^d - \varepsilon^u\right)\right)
\]

The area of the figure \(bde\) is equal to the part of the potential energy spent on restoration of the body geometry during unloading \(W_{re}\) (corresponds to the reversible part of the deformation \(\varepsilon_{re}\)):

\[
W_{re} = S_{bde} = \frac{1}{2} \frac{\left(\sigma_{n-1}\right)^2}{E_0} = \frac{1}{2} \frac{1}{E_0} \left[E_d \left(\varepsilon^d_{n-1} - \varepsilon^u\right) + E_0 \left(1 - \frac{\varepsilon^u}{2\varepsilon_{bu}}\right) \varepsilon^u\right]^2
\]

The area of the figure \(oabd\) (hysteresis loop) is equal to the amount of potential energy \(\Delta W_h\) dissipated during deformation (corresponds to the irreversible part of the deformation \(\varepsilon_{irr}\)). Based on the adopted energy exchange model, the dissipation of accumulated potential energy during unloading will be:

\[
\Delta W_h = W - W_{re}
\]

Then the hysteretic energy absorption coefficient in concrete is determined from the expression:

\[
\psi_h = \frac{\Delta W_h}{W}
\]

or after the corresponding transformations we get:

\[
\psi_h = 1 - \frac{3 \left[2E_d \varepsilon_{bu} \left(\varepsilon^d_{n-1} - \varepsilon^u\right) + E_0 \varepsilon^u \left(2\varepsilon_{bu} - \varepsilon^u\right)\right]^2}{4E_0 E_{bu} \left[9E_0 \varepsilon^u \left(\varepsilon^u\right)^2 - 3E_0 \varepsilon^u \left(\varepsilon^d_{n-1} + \varepsilon_{bu}\right) + 6E_0 \varepsilon_{bu} \varepsilon^d_{n-1}\right] + 3E_0 E_{bu} \left(\varepsilon^d_{n-1} - \varepsilon^u\right)^2}
\]

For example, consider heavy concrete B40 with the characteristics: \(R_b = 40MPa\), \(E_0 = 36000MPa\), \(\varepsilon_{bu} = 2R_b / E_0 = 0.0022\). Table 1 shows the results of assessing the effect of the level of static loading \(\varepsilon^u / \varepsilon_{bu}\) and dynamic module \(E_d\) on the energy absorption coefficients in reinforced concrete according to the Formula (8):

From the analysis of the table, it follows that the hysteretic energy absorption coefficient in concrete under static-dynamic loading significantly depends on the level of static loading \(\varepsilon^u / \varepsilon_{bu}\), as well as the value of the dynamic deformation modulus of concrete. With an increasing ratio \(E_d / E\), the energy absorption coefficient decreases. With increasing levels of static loading \(\varepsilon^u / \varepsilon_{bu}\), the energy absorption coefficient increases. The maximum value of the energy absorption coefficient reaches at \(\varepsilon^u / \varepsilon_{bu} = 1\) and is 62.5%.

In this case, it is appropriate to note that the stress state of concrete varies according to the coordinates of the structure, so the values \(\Delta W_h, W_h\) must be calculated by integration over its entire volume.
Table 1. The energy absorption coefficient $\psi_h$ at various relative values of the level of static loading $\varepsilon^{u\text{st}}_n / \varepsilon_{\text{bu}}$ and dynamic modulus $E_d / E$.

| $\varepsilon^{u\text{st}}_n / \varepsilon_{\text{bu}}$ | $E / E_0$ | 1.1 | 1.2 | 1.3 | 1.4 |
|------------------------------------------------|----------|-----|-----|-----|-----|
| 0.2                                             | 0.8      | 0.121 | 0.044 | 0 | 0 |
| 0.4                                             | 0.6      | 0.326 | 0.277 | 0.227 | 0.177 |
| 0.6                                             | 0.4      | 0.488 | 0.466 | 0.442 | 0.419 |
| 0.8                                             | 0.2      | 0.590 | 0.584 | 0.579 | 0.573 |
| 1                                               | 0        | 0.625 | 0.625 | 0.625 | 0.625 |

Notes: $\varepsilon^{u\text{st}}_n$, $\varepsilon_{\text{bu}}$ - deformation under static loading and ultimate deformation of concrete; $E_0$, $E$ - the initial and current deformation modulus; $E_d$ - deformation modulus during dynamic loading.

2.2. Energy dissipation from friction between concrete and reinforcement in a crack

In reinforced concrete elements with cracks, damping is caused simultaneously by two types of energy dissipation mechanisms: hysteretic energy dissipation in concrete in a compressed zone, and dispersion due to friction between concrete and reinforcement along their contact surface (Figure 2).

When $M > M_{\text{cre}}$ in the stretched zone of the reinforced concrete structure, cracks form. The friction force between concrete and reinforcement during opening and closing (dynamic process) of crack is written as:

$$F_c = \mu N$$  \hspace{1cm} (9)

where: $\mu$ is the coefficient of friction, which depends only on the parameters of the contact materials, $N$ is the compressive force acting between the surfaces of the reinforcement and concrete.

The energy loss from friction during the first half-cycle of oscillations is equal to the work of the friction force at a distance $(a^{n-1,d}_{\text{cre}} - a^{n,st}_{\text{cre}})$ and accordingly will be:

$$\Delta W_c = F_c (a^{n-1,d}_{\text{cre}} - a^{n,st}_{\text{cre}}) = \mu N (a^{n-1,d}_{\text{cre}} - a^{n,st}_{\text{cre}}),$$  \hspace{1cm} (10)

where $a^{n-1,d}_{\text{cre}}$, $a^{n,st}_{\text{cre}}$ is the crack opening width in the dynamic and static sections of deformation, respectively.

From Formula (10), it follows that the energy loss from friction depends on the reinforcement ratio of the reinforced concrete element, the diameter, and profile of the reinforcement bar, as well as the presence of prestressing in it. Since cracks in reinforced concrete elements are discrete character, an exact calculation of the energy loss caused by friction is equal to the sum of the work of the friction forces on all cracks. The practical solution to this problem can be an approximate character.
It is known [22] that the energy loss due to sound dispersion is much less than in the two cases described; therefore, in the calculation we can neglect them. As a result, the equivalent dispersion of the reinforced concrete structure with cracks is the result of a simple summation of energy: the hysteretic component and the component from the friction in the cracks.

3. A calculated model describing the energy dissipation of the structure during the sudden column removal

The noted difficulties in the mathematical description of damping mechanisms for a reinforced concrete structure as a whole (differences in the dissipation energy over the entire volume of the structure due to heterogeneity of the stress state and the discrete nature of the location of cracks) in many studies, for example [22-24], a simplified model of an equivalent viscous damper based on the hypothesis of proportionality of the damping force of the oscillation velocity is used to describe the energy dissipation process (Figure 3). Equivalent viscous damping (external damper model) is the most common damping model due to the simplicity of its practical use.

![Figure 3](https://example.com/figure3.png)

**Figure 3.** To the simulation of energy dissipation of a structure with a sudden removal of the middle support: a - a physical model of a two-span beam structure; b- equivalent calculated model with a total stiffness $K$ and a viscous damping coefficient $\zeta$.

Then the equation of motion of the structure in the form of a two-span beam after suddenly turning off the middle support, taking into account equivalent viscous damping, following [27] is written in the form:

$$u(t) = \left( u_n \right)_0 \begin{cases} \frac{1}{t_r} \left[ t - \frac{2z^2}{t_r \omega_D} e^{-\omega_D t} \sin \omega_D t - \frac{2z}{t_r \omega_n} \left( 1 - e^{-\omega_n t} \cos \omega_n t \right) \right] & \text{if } 0 \leq t \leq t_r; \\ \frac{1}{t_r} \left[ 1 + \frac{2z}{t_r \omega_n} \left( e^{-\omega_n t} \cos \omega_n t - e^{-\omega_n (t-t_r)} \cos \omega_n (t-t_r) \right) - \frac{1}{t_r \omega_D} \left( e^{-\omega_D t} \sin \omega_D t - e^{-\omega_D (t-t_r)} \sin \omega_D (t-t_r) \right) \right] & \text{if } t > t_r, \end{cases} \quad (11)$$

where: $\omega_n = \omega \sqrt{1 - \zeta^2}$ is the natural frequency of the structure, taking into account energy dissipation; $\left( u_n \right)_0 = P_0 / (m \omega_n^2)$ - static deflection of the structure; $\zeta = c / (2m \omega_n)$ - equivalent viscous damping coefficient.

4. Analysis of the test results of reinforced concrete frame structures with special action

Two-span three-story reinforced concrete monolithic frames of three series were designed, manufactured, and tested, two specimens in each series [13-15,26].

The first series (Figure 4a) is a normally reinforced beams structure ($\xi \leq \xi_k$) with one A500 rod and 8 mm in diameter so that when the central column is turned off, failure occurs due to the yielding flow of longitudinal reinforcement.
The second series's frames are designed with reinforcement two rods A500 of 8 mm in diameter in the stretched zone and one rod of 8 mm in diameter in the compressed zone of cross-section (over-reinforced sections \( \xi > \xi_b \)). Reinforcement of the second series's frames is designed so that when turning off the central column of experimental structure, brittle failure of beams on compressed concrete from reaching ultimate deformations in compressed zone occurred and longitudinal reinforcement ruptured in the stretched zone on the support sections of beams near central column at the moment of the sudden removal of this rack.

The third series (Figure 4b) of experimental structures were designed with prestressed beams in the upper and lower zones of their cross-section from A500 class rebar with 8 mm in diameter [26].

Figure 4. The formwork and reinforcement scheme for the experimental structures of the first, second (a) and third (b) series: 1,2- lower and upper reinforcement of beam, 3- equipment modeling loss of column, 4- prestressed reinforcement.

The loading of all experimental frames was carried out in two stages. At the first stage, the structures were loaded with a static load in the form of two concentrated forces in each beam's span. At the second stage, the structures were loaded with dynamic action in the form of the sudden removal of the middle or extreme column. The crack formation, oscillation, and damping of experimental frames under the considered regime of static-dynamic loading had a number of features.

The failure of the frame structures of the first series after a special action was characterized by a sharp increase in the number and width of cracks in the stretched zone along the entire lower surface of the beams and cracks in the stretched support upper zones of the beams adjacent to the extreme columns of the frame.

In the structure of the second series's frame with over-reinforced beams, the scheme of formation and opening of cracks after removing the middle column was similar to that in the first series's frame. At the same time, comparing the overall picture of the cracks, we can see that in the over-reinforced structure of this series, a significantly smaller number of cracks distributed over the surface of the beams was observed, and the width of their opening after special action was two times smaller than in the first series's frame.

In the structure of the third series's frame with prestressed beams of the first floor, no cracks were formed during the static application of the load. After the application of special action, some cracks were formed with an insignificant opening.

Based on the obtained experimental data for all frames, the equivalent viscous damping coefficient was determined from the expression:
\[
\frac{u(t) - (u_0)}{u(t + T) - (u_0)} = e^{\zeta \omega_0 T}, 
\]

where \(u(t), u(t + T)\) respectively, are the experimental values of the deflections of the structure at time \(t\) and \(t + T\); \(\omega_0\) - natural frequency of the structure.

According to the experimental data, the following values of viscous damping coefficients were obtained. For the beams of the first series's frames \(\zeta = 0.075\), the second series's frames \(\zeta = 0.035\), respectively, for the third series's frames \(\zeta = 0.015\).

The energy absorption coefficient from hysteresis dispersion \(\psi_h\) calculated from theoretical dependencies, from the friction between concrete and rebar in cracks and calculated from these values viscous damping coefficient \(\zeta\) for experimental structures of the first, second and third series, respectively, was: 0.06, 0.02, 0.008. At the same time, a substructure of the frame system in the form of a two-span beam of the first floor above the removed middle support was considered as a constructive scheme (see Figure 3). The most stressed cross-sections of the beams were used when calculating the energy loss in the first approximation.

A comparison of experimental and calculated values of relative dynamic displacements of the beam substructure of the first-floor calculated from the given experimental and calculated values of the damping coefficient (Figure 5) showed a satisfactory agreement between them.

![Figure 5. Graph of changes in relative dynamic displacements overtime after a special action for the first-floor beams of frames of the first (a), second (b), and third (c) series: 1- experiment; 2- calculation.](image)

Comparing the above graphs shows that the maximum surge of displacement at the first half-wave is observed in the prestressed beams of the third series's frames, the viscous damping coefficient for which has a minimum value.

5. Conclusions

For reinforced concrete bending elements under regimes of static-dynamic loading, analytical dependencies are obtained for determining the hysteresis energy dissipation in the concrete of the compressed zone and energy dissipation due to friction between the concrete and the reinforcement on the contact surface in the stretched zone.

A calculated model of equivalent viscous damping is constructed using these dependencies to assess the effect of energy dissipation on the dynamic response of a reinforced concrete structure working with cracks under a special action caused by its sudden structural rearrangement.
Comparison of the results of the numerical analysis of static-dynamic deformation of models of reinforced concrete frames with the results of tests showed the effectiveness of the proposed calculated model for evaluating the dynamic response of such reinforced concrete structures working with cracks.

The analytical dependencies suggested in the article on account of damping properties of conventional and prestressed structures with consideration for cracking can be used in the design of protection of reinforced concrete frames of buildings from progressive collapse under special actions.

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