The Significance to Quantum Computing of the Classical Harmonic Nature of Energy Eigenstates

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Abstract
Since a pure quantum system is incapable of faithfully simulating the solutions of the Schrödinger equation that actually pertains to itself, it is proposed that quantum computing technology (as opposed to cryptographic technology) not be based on pure quantum systems such as qubits but instead on physical systems which by their nature faithfully simulate the solutions of Schrödinger equations. Every Schrödinger equation is within a unitary transformation of being a set of mutually independent classical simple harmonic oscillator equations. Thus classical simple harmonic oscillators, or “chobits”, are the mathematically fundamental building blocks for all Schrödinger equations. In addition, classical harmonic oscillators are, as a practical matter, far easier to deal with than any pure quantum system—e.g., their phases and absolute amplitudes are readily physically accessible, they have little predilection for environmental decoherence, and they abound as cavity electromagnetic standing-wave modes. We study in mathematical detail the use of chobits to compute discrete quantum Fourier transforms, including gates, chobit counts, and chobit operation counts. The results suggest that thirty chobits and under a thousand chobit phase operations could generate discrete quantum Fourier transforms of a billion terms. Chobits can be technologically realized as semiconductor dynatron-type electronic oscillator circuits, which ought to be amenable to very considerable miniaturization.

Introduction: the Schrödinger equation’s classical canonical character
The procedures of second quantization [1] foster awareness that a Schrödinger equation describes a classical dynamical system in canonical form. This basic but not intuitively expected fact is almost never pointed out, however, in expositions of quantum mechanics that do not treat second quantization. Therefore we now explicitly show that the complex-valued Schrödinger equation,

\[ i\hbar \dot{\psi} = H\psi, \]  

for an \( M \)-state quantum system is equivalent to a purely real-valued classical canonical equation system. To this end we define the following two purely real-valued vectors that each have \( M \) components,

\[ \begin{align*}
q & \overset{\text{def}}{=} (\hbar/2)^{\frac{1}{2}}(\psi + \psi^*) \\
\dot{p} & \overset{\text{def}}{=} -i(\hbar/2)^{\frac{1}{2}}(\psi - \psi^*)
\end{align*} \]  

which are such that,

\[ \psi = (q + ip)/(2\hbar)^{\frac{1}{2}}. \]

We also define the following two purely real-valued \( M \) by \( M \) matrices,

\[ \begin{align*}
H_S & \overset{\text{def}}{=} (H + H^*)/2 = (H + H^T)/2 \\
H_A & \overset{\text{def}}{=} -i(H - H^*)/2 = -i(H - H^T)/2
\end{align*} \]  

where the second equality in each of the two parts of Eq. (1d) follows from the fact that \( H \) is Hermitian, i.e., that \( H^* = H^T \). Therefore \( H_S \) is a symmetric matrix as well as being a real-valued one, and \( H_A \) is an antisymmetric matrix as well as being real-valued. The definitions of \( H_S \) and \( H_A \) also imply that,

\[ H = H_S + iH_A. \]

Putting Eqs. (1c) and (1e) into Eq. (1a), the Schrödinger equation, produces,

\[ -\dot{p} + i\dot{q} = ((H_S/\hbar)q - (H_A/\hbar)p) + i((H_S/\hbar)p + (H_A/\hbar)q), \]
which implies the two purely real-valued equations,
\[ \dot{q} = (H_S/h)p + (H_A/h)q \quad \text{and} \quad \dot{p} = -(H_S/h)q + (H_A/h)p. \]  

(1g)

It is also readily verified that Eq. (1g) together with the definitions given in Eqs. (1b) and (1d) implies Eq. (1a). Therefore Eq. (1g) is equivalent to Eq. (1a), the Schrödinger equation. Now if a classical Hamiltonian \( H_{cl}^{(H)}(q,p) \) exists such that the two equalities given by Eq. (1g) are the same as,
\[ \dot{q} = \nabla_p H_{cl}^{(H)}(q,p) \quad \text{and} \quad \dot{p} = -\nabla_q H_{cl}^{(H)}(q,p), \]

(1h)

then Eq. (1g) is a classical dynamical equation system in canonical form. It is in fact readily verified, using the facts that \( H_S \) is a real-valued symmetric matrix and \( H_A \) is a real-valued antisymmetric matrix, that the particular classical Hamiltonian,
\[ H_{cl}^{(H)}(q,p) \overset{\text{def}}{=} ([q,H_S q] + [p,H_S p] + 2[p,H_A q])/2h, \]

(1i)
does indeed fulfill the condition that Eq. (1h) is the same as Eq. (1g). Therefore the generic Schrödinger equation of Eq. (1a) is indeed equivalent to a classical dynamical equation system in canonical form.

Finally, if we use the antisymmetric nature of the matrix \( H_A \) to reexpress the classical Hamiltonian of Eq. (1ii) as,
\[ H_{cl}^{(H)}(q,p) = ([q,H_S q] + [p,H_S p] + [p,H_A q] - [q,H_A p])/2h, \]

(1j)

and then substitute the definitions given by Eqs. (1b) and (1d) into the right-hand side of Eq. (1i), there results, after a slightly tedious gathering and cancellation of terms,
\[ H_{cl}^{(H)}(q,p) = ([\psi,H^*\psi^*] + [\psi^*,H\psi])/2 = ([\psi^*,H\psi]), \]

(1k)

where the last equality follows from the fact that \( H^* = H^T \). Therefore the classical Hamiltonian for a Schrödinger equation is equal to the quantum expectation value of that equation’s Hamiltonian matrix, a result which is very familiar in the context of second quantization [1].

While we have so far been dealing with the generic Schrödinger equation, there is no real loss of generality from assuming that its Hermitian Hamiltonian matrix is diagonal, because that diagonalization can always in principle be achieved by a unitary transformation, which is invertible and even of course linear. We shall now see that the Schrödinger equation for an \( M \)-state quantum system describes, when its Hermitian Hamiltonian matrix is diagonal, nothing more than \( M \) mutually independent classical simple harmonic oscillators.

Schrödinger equation simulation by classical simple harmonic oscillators

When the \( M \) by \( M \) Hermitian Hamiltonian matrix \( H \) of the Schrödinger equation of Eq. (1a) is diagonal, then \( H = H^T = H^* \), and therefore from Eq. (1d) we will have that \( H_S = H \) and \( H_A = 0 \), which simplifies the classical dynamical equations of Eq. (1g) to,
\[ \dot{q} = (H/h)p \quad \text{and} \quad \dot{p} = -(H/h)q. \]

(2a)

Furthermore, the diagonal \( M \) by \( M \) Hermitian Hamiltonian matrix \( H \) will satisfy,
\[ H|m\rangle = E_m|m\rangle, \quad m = 0,1,\ldots,M-1, \]

(2b)

where \( |0\rangle,|1\rangle,\ldots,|M-1\rangle \) are the natural orthonormal basis vectors for the \( M \)-dimensional vector space which have a single sequentially selected component set equal to unity with the rest set equal to zero, and \( E_0,E_1,\ldots,E_{M-1} \) are the corresponding diagonal elements (i.e., energy eigenvalues) of \( H \). We can use this natural complete orthonormal basis set to decompose the vectors \( q \) and \( p \) into their components,
\[ q = \sum_{m=0}^{M-1} q_m|m\rangle \quad \text{and} \quad p = \sum_{m=0}^{M-1} p_m|m\rangle. \]

(2c)
Eq. (2c) in conjunction with both Eqs. (2a) and (2b) implies that,

\[ \dot{q}_m = \frac{E_m}{\hbar}p_m \quad \text{and} \quad \dot{p}_m = -\frac{E_m}{\hbar}q_m, \quad m = 0, 1, \ldots, M - 1, \]  

(2d)

which we recognize as the classical dynamical equations of \( M \) mutually independent classical simple harmonic oscillators. We can likewise decompose the complex-valued vector \( \psi \) into its complex-valued components,

\[ \psi = \sum_{m=0}^{M-1} \psi_m |m\rangle, \]  

(2e)

which in conjunction with Eq. (2b) and the Schrödinger equation given by Eq. (1a) implies that,

\[ \dot{\psi}_m = -i\frac{\hbar}{E_m}\psi_m, \quad m = 0, 1, \ldots, M - 1. \]  

(2f)

These \( M \) independent differential equations have the general solutions,

\[ \psi_m(t) = \psi_m(t_0) \exp(-iE_m(t - t_0)/\hbar), \quad m = 0, 1, \ldots, M - 1. \]  

(2g)

Eqs. (2g) and (2e) yield the general solution to the Schrödinger equation given by Eq. (1a), namely,

\[ \psi(t) = \sum_{m=0}^{M-1} \psi_m(t_0) \exp(-iE_m(t - t_0)/\hbar)|m\rangle. \]  

(2h)

Eq. (2h) is, in conjunction with Eq. (2b), readily shown to indeed satisfy the Schrödinger equation given by Eq. (1a). Furthermore, Eq. (2h) in conjunction with Eq. (1b) implies that,

\[ q(t) = (2\hbar)^{\frac{1}{2}} \sum_{m=0}^{M-1} |\psi_m(t_0)| \cos(\arg(\psi_m(t_0)) - E_m(t - t_0)/\hbar)|m\rangle, \]

\[ p(t) = (2\hbar)^{\frac{1}{2}} \sum_{m=0}^{M-1} |\psi_m(t_0)| \sin(\arg(\psi_m(t_0)) - E_m(t - t_0)/\hbar)|m\rangle, \]  

(2i)

which, in conjunction with Eq. (2b), is readily shown to indeed satisfy the classical dynamical equations given by Eq. (2a). From Eq. (2i), which precisely corresponds to Eq. (2c), we readily isolate the fully solved dynamics of the \( M \) mutually independent classical simple harmonic oscillators whose classical dynamical equations are given by Eq. (2d),

\[ q_m(t) = (2\hbar)^{\frac{1}{2}} |\psi_m(t_0)| \cos(\arg(\psi_m(t_0)) - E_m(t - t_0)/\hbar), \]

\[ p_m(t) = (2\hbar)^{\frac{1}{2}} |\psi_m(t_0)| \sin(\arg(\psi_m(t_0)) - E_m(t - t_0)/\hbar), \]  

(2j)

where \( m = 0, 1, \ldots, M - 1 \). The \( q_m(t) \) and \( p_m(t) \) of Eq. (2j) are readily shown to indeed satisfy the classical dynamical equations that are given by Eq. (2d). These \( M \) mutually independent classical simple harmonic oscillator solutions furthermore have exactly the required absolute amplitudes, namely \((2\hbar)^{\frac{1}{2}}|\psi_m(t_0)|\), and the required phases, namely \( \arg(\psi_m(t_0)) \), to precisely simulate the general solution \( \psi(t) \) given by Eq. (2h) to the Schrödinger equation for an \( M \)-state quantum system that is given by Eq. (1a). Indeed, combining Eq. (2j) with Eqs. (2c) and (1c) yields precisely Eq. (2h).

Thus we see that the solutions of Schrödinger equations for \( M \)-state quantum systems can always be faithfully simulated by \( M \) mutually independent classical simple harmonic oscillators. We therefore now dub the most basic classical dynamical entity for the faithful simulation of Schrödinger equations the “chobit”, and note that the “chobit” is, of course, a single classical simple harmonic oscillator—the prefix “cho” abbreviates “classical harmonic oscillator”. The chobit, given its amplitude and phase, represents a single complex number. A complex number of course comprises far more data than the integer modulo two represented by an ordinary bit.

It is very worthwhile to take notice at this point of the fact that the physical quantum system to which a given Schrödinger equation pertains, in principle cannot faithfully simulate the solutions of that Schrödinger equation because the quantum mechanics of a physical system is not in one-to-one correspondence with the solutions of the Schrödinger equation that pertains to it. For example, some of the information that is an inherent part of Schrödinger-equation solutions is systematically lost because of the probabilistic requirement that physical meaning be attached to only the absolute squares of inner products of state vectors—this makes the inherent overall phase of any state vector physically meaningless. Further information that is an inherent
part of Schrödinger-equation solutions is lost because of the probabilistic requirement that state vector norms be devoid of physical meaning. In fact, single-state quantum systems, which are described by the simplest Schrödinger equations that are possible (and are simulated by a single chobit), have all of their Schrödinger-equation solution information, namely both their single phase and their single absolute amplitude, made physically meaningless by these two probabilistic requirements. Consequently, single-state quantum systems do not physically exist at all. Physical two-state quantum systems, the celebrated qubits, whose Schrödinger equations require two chobits to simulate, i.e., two complex numbers, have a very significant fraction of that information made physically meaningless by these probabilistic requirements—this has the consequence that physical qubits can bear information equivalent to only two angles, one azimuthal and one polar. Thus a physical qubit falls drastically short of being able to faithfully simulate the solutions of the two-chobit Schrödinger equation which pertains to it.

Furthermore, even information that is borne by a quantum system can sometimes be so awkward to recover as to be poorly suited to computational applications. For example, the superposition of orthonormal basis states with equal absolute amplitudes but differing phases is annoyingly recalcitrant with regard to recovery of the relative phase information: the simple inner product of that superposition state with any one of the basis states results in the complete obliteration of the phase information by the probabilistically mandatory subsequent taking of the absolute square of such an inner product. For computational applications chobits thus absolutely shine by comparison with actual quantum systems such as physical qubits because the classical nature of chobits in principle makes all the information they bear readily physically accessible. (But by the very same token actual physical quantum systems offer superior potential for ingenious cryptography.)

The fact that chobit phases are in principle readily physically accessible is of particular relevance to carrying out the unitary discrete quantum Fourier transform [2] whose definition is,

$$U_F^{(M)} |m\rangle \overset{\text{def}}{=} M^{-\frac{1}{2}} \sum_{m'=0}^{M-1} e^{2\pi i m'M/M} |m'\rangle$$

where the integers $m'$ in Eq. (3a) satisfies $0 \leq m' \leq p^n - 1$, we can write the $p$-nary expansion of $m'$ as,

$$m' = \sum_{l=1}^{n} m'_l p^{n-l},$$

where the integers $m'_l$ satisfy $m'_l \in \{0, 1, \ldots, p - 1\}$ for $l = 1, 2, \ldots, n$. This expansion permits us to write the integer $m'$ as its $n$-digit $p$-nary representation $m' = m'_1 m'_2 \ldots m'_n$. This $n$-digit $p$-nary representation of $m'$ in turn permits us to express the basis state $|m'\rangle$ of the $p^n$-state system as an $n$-fold direct product of basis states of $p$-state systems,

$$|m'\rangle = |m'_1\rangle \otimes |m'_2\rangle \otimes \ldots \otimes |m'_n\rangle = \otimes_{l=1}^{n} |m'_l\rangle.$$
where the integers $m_l$ satisfy $m_l \in \{0, 1, \ldots, p - 1\}$ for $l = 1, 2, \ldots, n$. Thus, exactly as for the integer $m$ above, we can write the integer $m$ as its $n$-digit $p$-nary representation $m = m_1 m_2 \ldots m_n$, which in turn permits us to express the basis state $|m\rangle$ of the $p^n$-state system as an $n$-fold direct product of basis states of $p$-state systems,

$$|m\rangle = |m_1\rangle \otimes |m_2\rangle \otimes \ldots \otimes |m_n\rangle = \otimes_{l=1}^n |m_l\rangle.$$  

(3e)

We shall be using Eq. (3c) for the basis state $|m\rangle$ which appears on the left-hand side of Eq. (3a), but for the $m$ which appears in the exponent on the right-hand side of Eq. (3a) we shall in due course be using the rightmost summation representation given in Eq. (3d).

In light of Eq. (3b), we now replace the factors $m'/M = m'/p^n$ in the exponent on the right-hand side of Eq. (3a) by $\sum_{l=1}^n m_l p^{-i}$. In light of Eq. (3c) we also replace $|m\rangle$ on the right-hand side of Eq. (3a) by $\otimes_{l=1}^n |m_l\rangle$. To properly match these decompositions involving $m'$, we note that the following notational changes must also be carried out on the right-hand side of Eq. (3a),

$$M^{-\frac{1}{p^n}} \sum_{m' = 0}^{p^n - 1} \langle p^n \rangle^{-\frac{1}{p^n}} \sum_{m' = 0}^{p^n - 1} p^{-\frac{1}{p^n}} \sum_{m'_l = 0}^{p - 1} \ldots \sum_{m'_0 = 0}^{p - 1}.$$  

With these changes and a subsequent convenient rearrangement of factors, Eq. (3a) becomes,

$$U_F^{(p^n)} |m\rangle = \otimes_{l=1}^n \left[ p^{-\frac{1}{p^n}} \sum_{m'_l = 0}^{p - 1} e^{i2\pi m'_l p^{-1} m_l} |m'_l\rangle \right] \quad \text{where } m \in \{0, 1, \ldots, p^n - 1\}.  

(3f)

We now note from Eq. (3e) that $|m\rangle = \otimes_{l=1}^n |m_l\rangle$, where each $m_l \in \{0, 1, \ldots, p - 1\}$ for $l = 1, 2, \ldots, n$. We also note from the rightmost summation representation given in Eq. (3d) that $p^{-1} m = \sum_{r=1}^{n} m_{n+1-r} p^{-1-r-1}$, and that all the terms of this sum for which $r \geq l + 1$ have integer values. Therefore Eq. (3f) becomes,

$$U_F^{(p^n)} (\otimes_{l=1}^n |m_l\rangle) = \otimes_{l=1}^n \left[ p^{-\frac{1}{p^n}} \sum_{m'_l = 0}^{p - 1} e^{i2\pi m'_l \sum_{r=1}^{l} m_{n+1-r} p^{-1-r-1} |m'_l\rangle} \right],  

(3g)

which is more transparent when expressed in terms of ascending powers of $p^{-1}$,

$$U_F^{(p^n)} (\otimes_{l=1}^n |m_l\rangle) = \otimes_{l=1}^n \left[ p^{-\frac{1}{p^n}} \sum_{m'_l = 0}^{p - 1} e^{i2\pi m'_l \sum_{r=1}^{l} m_{n+1-r} p^{-1-r-1} |m'_l\rangle} \right].  

(3h)

By using “$p$-nary point” notation, Eq. (3h) can alternatively be written,

$$U_F^{(p^n)} (\otimes_{l=1}^n |m_l\rangle) = \otimes_{l=1}^n \left[ p^{-\frac{1}{p^n}} \sum_{m'_l = 0}^{p - 1} e^{i2\pi m'_l (0,m_{n+1-l},m_{n+2-l} \ldots m_n) |m'_l\rangle} \right].  

(3i)

Note that when $n = 1$ one recovers from Eq. (3h), (3i) or (3g) the original Eq. (3a) with $M = p$, $m = m_1$, and $m' = m'_1$.

The salient point of the discrete quantum Fourier transform $p$-nary direct-product representations of Eqs. (3h),(3i) or (3g) is that they feature only $n$ times $p - 1$ phases that require chobits, whereas the original discrete quantum Fourier transform representation of Eq. (3a) features $M - 1 = p^n - 1$ phases that require chobits. An interesting small exercise is to hold $p^n$, the total number of terms of the discrete quantum Fourier transform (which is related to the resolution achieved by that discrete Fourier transform), fixed while simultaneously attempting to minimize $n(p - 1)$, the number of chobits needed to accommodate the number of phases in the $p$-nary direct product representation of that discrete quantum Fourier transform. Since $n(p - 1) = \ln(p^n) / (p - 1) / \ln(p)$, we need to minimize $(p - 1) / \ln(p)$, a function of $p$ that increases monotonically when $p > 0$. Therefore the $p = 2$ binary base version of the direct-product representation of the discrete quantum Fourier transform that is presented in Ref. [2] minimizes the number of chobits needed for a given number of terms of that transform. With $p = 2$, $n$ chobits suffice for $2^n$ terms.

Ref. [2] shows how the particular $p = 2$ binary version of the direct-product representation of the discrete quantum Fourier transform given by Eq. (3i) or (3h) can be built up from interwoven repetitions of a few elementary unitary gates, namely operations that are each based on data stored in a single (binary) digit. The small number of unitary single binary-digit gates which are so utilized in Ref. [2] can be all be straightforwardly generalized into unitary single $p$-nary-digit gates. The $p$-nary generalization of the unitary “Hadamard gate” of Ref. [2] is an especially interesting example of such a gate generalization. Given a $p$-nary
the preceding generalized Hadamard gate. This changes Eq. (4b) to,

\[ p^{-\frac{1}{2}} \sum_{m_{n+1-1} = 0}^{p-1} e^{i2\pi m_{n+1-1} m_{1}/p} |m_{n+1-1} \rangle. \]  

(4a)

Comparing \(|m_{l}\rangle\) and the one-digit superposition state given by Eq. (4a) with the two sides of Eq. (3a), we see that the unitary “generalized Hadamard gate” is a microcosmic one-digit analog of the unitary discrete quantum Fourier transform itself.

We proceed now to build up Eq. (3h) in very close analogy with the detailed interwoven repetitions of gate applications presented in Ref. [2]. Starting with the \(n\)-digit direct-product state \(|m\rangle = |m_{1}\rangle \otimes \ldots \otimes |m_{n}\rangle = \bigotimes_{i=1}^{n} |m_{i}\rangle\) that appears on the left-hand side of Eq. (3h), we commence by applying the “generalized Hadamard gate” of Eq. (4a) to its leftmost digit \(|m_{1}\rangle\), which yields,

\[ p^{-\frac{1}{2}} \sum_{m_{n} = 0}^{p-1} e^{i2\pi m_{n} m_{1}/p} |m_{n} \rangle \otimes \bigotimes_{i=2}^{n} |m_{i}\rangle. \]  

(4b)

Note from Eq. (4b) that the effect of the generalized Hadamard gate on the leftmost digit \(|m_{1}\rangle\) has been to change it into a one-digit superposition state which is a one-digit analog of the discrete quantum Fourier transform itself, with the digit \(|m_{2}\rangle\) remaining in place to become the successor leftmost digit, i.e., the generalized Hadamard gate has, inter alia, effectively consumed the leftmost digit \(|m_{1}\rangle\), leaving \(|m_{2}\rangle\) behind as the leftmost available digit.

We next need to modify the one-digit superposition state that the generalized Hadamard gate has just created with the information that is stored in the remaining available digits \(|m_{2}\rangle, |m_{3}\rangle, \ldots, |m_{n}\rangle\), and we achieve that end with a cascading sequence of gates that are closely similar to each other. Those remaining available digits themselves, however, are in no way modified and therefore are not consumed by this gate cascade.

We begin the gate cascade by applying the generalized \(R_{2}\) gate [2] to the leftmost available digit \(|m_{2}\rangle\) in Eq. (4b) in order to insert that digit’s information into the one-digit superposition state that was created by the preceding generalized Hadamard gate. This changes Eq. (4b) to,

\[ p^{-\frac{1}{2}} \sum_{m_{n} = 0}^{p-1} e^{i2\pi m_{n} |m_{1}\rangle/p \otimes |m_{n} \rangle} \otimes \bigotimes_{i=2}^{n} |m_{i}\rangle. \]  

(4c)

We now continue with the rest of the generalized \(R_{2}\) gate [2] cascade by applying the similar generalized \(R_{3}\) gate [2] to the \(|m_{3}\rangle\) digit in order to insert that digit’s information into the one-digit superposition state that was created by the preceding generalized Hadamard gate, then applying the generalized \(R_{4}\) gate [2] to the \(|m_{4}\rangle\) digit in order to do the same with that digit’s information, and so forth, finally applying the generalized \(R_{n}\) gate [2] to the \(|m_{n}\rangle\) digit in order to do so with that digit’s information. The result of this particular entire generalized \(R_{k}\) gate [2] cascade applied to Eq. (4b) is,

\[ p^{-\frac{1}{2}} \sum_{m_{n} = 0}^{p-1} e^{i2\pi m_{n} \sum_{i=1}^{n} m_{i} p^{-i}} |m_{n} \rangle \otimes \bigotimes_{i=2}^{n} |m_{i}\rangle. \]  

(4d)

Having finished this particular generalized \(R_{k}\) gate [2] cascade, which has not consumed any of the digits that were available in Eq. (4b), we now apply the generalized Hadamard gate to the leftmost available digit in Eq. (4d), namely \(|m_{2}\rangle\), which consumes that digit and produces a second one-digit superposition state, so that Eq. (4d) becomes:

\[ p^{-\frac{1}{2}} \sum_{m_{n} = 0}^{p-1} e^{i2\pi m_{n} \sum_{i=1}^{n} m_{i} p^{-i}} |m_{n} \rangle \otimes \left[ p^{-\frac{1}{2}} \sum_{m_{n-1} = 0}^{p-1} e^{i2\pi m_{n-1} m_{2}/p} |m_{n-1} \rangle \right] \otimes \bigotimes_{i=3}^{n} |m_{i}\rangle. \]  

(4e)

Now we modify this second one-digit superposition state which has been created by the latest generalized Hadamard gate with another generalized \(R_{k}\) gate [2] cascade. These generalized \(R_{k}\) gate [2] cascades always begin with the application of the generalized \(R_{2}\) gate [2] to the leftmost available digit which here is, from Eq. (4e), \(|m_{3}\rangle\). This generalized \(R_{k}\) gate cascade continues from there, with the generalized \(R_{3}\) gate applied to \(|m_{4}\rangle\), the generalized \(R_{4}\) gate applied to \(|m_{5}\rangle\) and so forth, finally ending with the generalized \(R_{n-1}\) gate
applied to $|m_n\rangle$. The upshot of this generalized $R_k$ gate cascade, which modifies the one-digit superposition state that was created by the most recent generalized Hadamard gate, is to change Eq. (4e) to,

$$
\left[ p^{-\frac{1}{2}} \sum_{m'_n=0}^{p-1} e^{i2\pi m'_n \sum_{s=1}^{n} m_s p^{-s}} |m'_n\rangle \right] \otimes \left[ p^{-\frac{1}{2}} \sum_{m'_n=0}^{p-1} e^{i2\pi m'_n \sum_{s=1}^{n-1} m_{s+1} p^{-s}} |m'_{n-1}\rangle \right] \otimes (\otimes_{l=3}^{n} |m_l\rangle),
$$

which, of course, has not consumed any of the digits that were available in Eq. (4e).

We continue in this way with the application of the generalized Hadamard gate to the leftmost available digit (in Eq. (4f) that would be the digit $|m_3\rangle$), which consumes that digit, followed by a generalized $R_k$ gate cascade that begins with application of the generalized $R_2$ gate to the leftmost available digit and ends after application of a generalized $R_k$ gate to the rightmost available digit, which is $|m_n\rangle$, followed by the application of yet another generalized Hadamard gate to the leftmost available digit, which consumes that digit, etc. This procedure finally produces, after the very last available digit $|m_n\rangle$ is consumed by a generalized Hadamard gate,

$$
\otimes_{l=1}^{1 \leq n} \left[ p^{-\frac{1}{2}} \sum_{m'_l=0}^{p-1} e^{i2\pi m'_l \sum_{s=1}^{n} m_{n-s+1} p^{-s}} |m'_l\rangle \right],
$$

which has its direct product factors in the reverse order to those of Eq. (3h). As is explained in Ref. [2], swap operations, which involve additional gates, are then applied to turn the order of these reversed factors around, which produces Eq. (3h).

Exactly as in the straightforward gate census described in Ref. [2], we readily see that there are altogether $n(n+1)/2$ generalized $R_k$ and Hadamard gates, plus $n/2$ swap operations. Each generalized $R_k$ or generalized Hadamard gate requires $p-1$ changes in phases or initializations of phases. Therefore the total number of chobit operations required is of order $(n(p-1))^2$, i.e., it is of the order of the square of the total number $n(p-1)$ of chobits required. Again holding the total number of terms $p^n$ of the discrete quantum Fourier transform fixed while simultaneously attempting to minimize the total required number of chobit operations $(n(p-1))^2$, we note that $(n(p-1))^2 = (\ln(p^n))/(p-1)/\ln(p))^2$, and $[(p-1)/\ln(p)]^2$ also increases monotonically for $p > 0$. Therefore the $p = 2$ binary base version of the direct-product representation of the discrete quantum Fourier transform that is presented in Ref. [2] minimizes the number of chobit operations needed for a given number of terms of that transform. With $p = 2$, $n$ chobits and of order $n^2$ chobit operations are sufficient for $2^n$ terms.

It is interesting to note at this point that the direct-product representation of the discrete quantum Fourier transform that is given by Eq. (3h) or (3i) has, from Eq. (3f), the equivalent but more suggestive form,

$$
U_F^{(p^n)} |m\rangle = \otimes_{l=1}^{n} \left[ p^{-\frac{1}{2}} \sum_{m'_l=0}^{p-1} e^{i2\pi m'_l (m/p - [m/p]_{\text{GIP}}) p^{-l}} |m'_l\rangle \right] \mbox{ where } m \in \{0, 1, \ldots, p^n-1\},
$$

and where $[\ ]_{\text{GIP}}$ denotes the “greatest integer part” of the argument enclosed by its square brackets. Now instead of using the very complicated interwoven repetitions of gates set out in Eqs. (4) to build up the phases of the direct-product representation, which is the approach taken in Ref. [2], we can very simply develop, in $p$-nary multiple-precision floating point representation, the $n$ successive numbers $m/p, m/p^2, \ldots, m/p^n$ that Eq. (5) needs through $n$ successive divisions by $p$ which can be achieved by mere subtractions of unity from the $p$-nary floating-point exponent, and we can simultaneously very simply develop the accompanying $n$ successive $p$-nary multiple-precision fixed-point integers $[m/p]_{\text{GIP}}, [m/p^2]_{\text{GIP}}, \ldots, [m/p^n]_{\text{GIP}}$ that Eq. (5) needs through calculations of $[m/p^l]_{\text{GIP}}$ from $[m/p^{l-1}]_{\text{GIP}}$ that eliminate the latter’s least significant $p$-nary digit via a mere single-digit rightward shift. Since, as has been noted above, the optimum value of $p$ is 2, we are in fact here talking about merely the familiar binary multiple-precision floating-point and fixed-point representations. Moreover, since $2^{30} \approx 10^9$, the values of $n$ that are required in practice likely permit mere double-precision or even single-precision binary representation of the $n$ successive number pairs which are based on the integer $m$ that Eq. (5) needs.

The straightforward simplicity of the procedure just outlined contrasts sharply with the convoluted complexity of the standard application of quantum gates set out in Eqs. (4) and in Refs. [2] and [3]. Such simplicity has an interesting echo in earlier work on the discrete quantum Fourier transform by R. B. Griffiths.
and C. S. Niu [4], who pointed out that the conversion of intermediate results of quantum gate operations into classical signals that control subsequent quantum gate operations permits considerably fewer and simpler such gates. That moving away from the use of quantum hardware can in fact increase computational effectiveness is, of course, not fortuitous happenstance—an M-state physical quantum system is sufficiently ill-suited to computation that it cannot simulate the full solution space of the Schrödinger equation which actually pertains to itself. The inherently awkward, complex and convoluted characteristics of quantum hardware that ill suit it to straightforward computation obviously do provide, however, marvelously fertile ground for the development of cryptography.

Conclusion

Chobit-based computing automatically and naturally removes the awkward bottlenecks which are inherent to attempts to base computing technology on actual quantum systems such as physical qubits. Whereas the latter are by their very nature incapable of faithfully simulating the full solution space of the very Schrödinger equations which actually pertain to them, chobits, which themselves faithfully simulate the full set of one-state-system Schrödinger-equation solutions, are the fundamental building blocks for simulating all Schrödinger-equation solutions: M chobits suffice to faithfully simulate the full solution sets of Schrödinger equations which pertain to M-state quantum systems.

The inherent information content of the chobit is the complex number, which could hardly be more convenient for an immense variety of computing applications. In the starkest possible contrast, the probabilistic nature of quantum mechanics deprives the actual quantum counterpart of the chobit of even an iota of information content, so that such a one-state quantum system cannot physically exist. Even qubits, which when conceptualized within the Schrödinger-equation domain are simply two-chobit systems, are in quantum physical reality bereft of a considerable fraction of that information content: instead of two complex numbers the qubit’s information content is two angles, one azimuthal and the other polar, and as is well known, the probabilistic nature of quantum mechanics can make even that diminished information capacity of the qubit extremely awkward to access in practice—it is indeed the Byzantine nature of quantum information which gives it such great promise for cryptographic applications.

The chobit, which is a classical simple harmonic oscillator, in contrast should in principle present no issues at all with regard to access to the phase and absolute amplitude information it carries. Chobits, being classical systems, in addition do not suffer from the environmental decoherence issues that are a natural aspect of many ultra-microscopic pure quantum systems.

One possible technological realization of the chobit is a “dynatron-type” electronic oscillator circuit [5], in which a powered “negative-resistance” element is placed in parallel with a basic electronic oscillator circuit in order to cancel out that circuit’s innate electrical resistance, thus enabling it to indefinitely sustain simple harmonic current oscillation at its natural frequency. If this active “negative-resistance” element is an appropriate semiconductor device such as a tunnel diode [6] or Gunn diode [7], such a “dynatron-type” electronic oscillator circuit ought to be amenable to very considerable miniaturization. One can therefore envision such “chobit circuits” eventually being routinely incorporated into the designs of large-scale integrated circuits that are intended for applications in which chobits can be useful.

Gunn diodes, which can operate at higher power levels than tunnel diodes, are frequently used to provide active negative resistance for electromagnetic cavity oscillators [7]. The multitudinous electromagnetic standing-wave modes of such a cavity could in principle comprise a very large number of chobits, but it would be challenging to accurately detect and manipulate the phases and absolute amplitudes of such a large set of standing-wave modes. Nonlinear devices which mix external wave signals with internally generated reference frequencies to produce “beat” frequencies, a technique known in radio engineering parlance as “heterodyning”, could at least in principle be the key to carrying out such a task [8].

The above considerations concerning conceivably multitudinous chobits notwithstanding, is is worth noting that the beauty of the direct-product representation of the discrete quantum Fourier transform is that a modest number of chobits suffices to carry that transform out. For example, thirty chobits suffice for a discrete quantum Fourier transform of a billion terms, and forty chobits will do for a trillion terms.
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