The signal calibration from a Sagnac polarized standing wave interferometer for displacement measurement

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Abstract. A Sagnac polarized standing wave interferometer (SPSWI) is extensively developed. It exploits two polarized standing wave signals formed in two orthogonal planes of oscillation to measure the displacement. However, two equal amplitude signals, which are complicated control, are sensitively required for displacement data analysis. Thus, the simple calibration process is necessary to accomplish this situation. In this paper, we present a methodology for calibrating signals from SPSWI by using Python programming to remove the constant signals, normalize oscillating signals and eliminate non-orthogonal error based on technique proposed by Fan. The relative displacement is derived from these calibrated signals.

1. Introduction
Interferometer is an important topic in engineering science, and industry. There are many techniques that can measure the displacement precisely, such as Michelson interferometer [1] and Sagnac interferometer [2]. All these interferometers are based on detection of an interference signal at the end of the setup. Moreover, there is another kind of interferometer that can be applied for displacement measurement. It is called the standing wave interferometer which exploits the standing wave signal to sense the displacement. The transparent thin photodiode detectors, therefore, are needed to insert into the travelling path of light to measure that signals. However, these optoelectronic devices must be required precisely controlled manufacturing process. In 2018, instead of using thin photodetectors, Lee et al. [3] proposed a polarized standing wave interferometer with the single-layer SiO₂ nano-sphere plate for scattering the intensity distribution of the standing wave and applied it to displacement measurement.

We propose the novel polarized wave interferometer based on Sagnac setup and exploits the diffracted light from a commercial grating to sense the intensity distribution of the two polarized standing wave and applied it to displacement measurement. Therefore, it is necessary to have an analysis of the displacement data from Sagnac polarized standing wave interferometer (SPSWI). In this paper, we applied the method of Fan [4] by using Python code to eliminate errors due to non-orthogonalization.

2. Method
Sagnac interference is named after the French physicist Georges Sagnac [5]. Normally, this phenomenon is found in interferometry caused by rotation. Sagnac interferometer is in a ring shape. A beam of light is split and are made to follow the same path but in opposite directions. If the polarizing beam splitter (PBS) is used instead of nonpolarizing beam splitter, the planes of oscillation of two beams are orthogonal. We take the advantage of this setup and apply it as standing wave interferometer.
The set up are shown in figure 1, the light from 632 nm He-Ne laser reflects the mirror, M₁ and M₂ and pass through a PBS. The light is separated into two beams depending on polarization: the vertical polarization (arrow) travels in clockwise direction, and the horizontal polarization travels in counter clockwise direction. Both polarizations reflect the mirror M₅ attached at the diaphragm of the speaker and travel back on the same path forming the standing wave of each polarization. The signals can be detected by inserting a grating into the setup to diffract light onto the photodetectors. The polarizers P₁ and P₂ were inserted to select H-polarized intensity for detector PD₁ and V-polarized intensity for detector PD₂.

![Figure 1. Schematic of a Sagnac polarized standing wave interferometer.](image)

The two signals of the interferometer can be theoretically analysed in term of the intensities. The two polarized standing wave intensities can be expressed as

\[ I_V = I_{V0} [1 - \cos (2k(L₀ + Δz))] \]

and

\[ I_H = I_{H0} [1 - \cos (2k(L₀ + ΔL + Δz) + φ)] \]

where \( L₀ + Δz \) is the distance from the grating to the mirror M₄ to the mirror M₅, \( L₀ + ΔL + Δz \) is the distance from the grating to the mirror M₁ to the mirror M₅, \( ΔL \) is the path difference between the path of horizontal and vertical polarization, \( Δz \) is the displacement of the mirror M₅, and \( φ \) denotes the non-orthogonal error (NOE) which results from the polarization mixing of the interference beams.

We can tune the position of the mirror M₃ or M₄ to change the value of \( ΔL \) such that \( I_V \) and \( I_H \) have phase difference of \( π/2 \). Thus, the intensities \( I_V \) and \( I_H \) are in the form of

\[ I_V = I_{V0} [1 - \cos (2k(L₀ + Δz))] \] (1)

and

\[ I_H = I_{H0} [1 - \sin (2k(L₀ + Δz) + φ)] \] (2)

respectively. The equation (1) and (2) explain the light intensity with DC part, \( I_{V0} \) and \( I_{H0} \), and AC part, which are phase quadrature. Then we can define

\[ φ = 2kΔz \] (3)

as the phase change from the displacement of the mirror M₅. Consequently, the displacement can be obtained from this phase shift in the equation (3). However, there is a phase error \( φ \) (NOE) resulting in periodic nonlinear error (PNE) in the displacement measurement.

According to the orthogonalization method [6], the signals become

\[ I'_V = \sqrt{2+2} \sin φ \cos (2k(L₀ + Δz) - δ) \]

(4)
and

\[ I_H = \sqrt{2+2\sin^2 \phi \sin (2k(L_0 + \Delta z) - \delta)} \]  

where \( \delta = \tan^{-1}(\cos \phi / (1 + \sin \phi)) \). Since the orthogonalized signals have different amplitude, we normalized the signal again. The calibrated signals become \( I_{VC} = \cos(2k(L_0 + \Delta z) - \delta) \) and \( I_{HC} = \sin(2k(L_0 + \Delta z) - \delta) \). The displacement of the movable mirror \( M_5 \) can be calculated as

\[ \Delta z = (1/2k) \tan^{-1}(I_{HC}/I_{VC}) \]  

3. Experiment and results

The speaker is driven by a sine wave signal from a function generator at frequency 300 Hz. When data from oscilloscope are collected, the two intensities of horizontal and vertical polarization are imported into the program written in Python language to eliminate the NOE and determine the displacement measurement with working principles according to figure 2.

\[ \text{Start} \]

- Import H- and V-polarized intensities data from the oscilloscope.
- Calculate DC level and AC amplitude of each intensity data.
- Calibrated DC term to zero and normalize AC.
- Eliminate NOE by orthogonalizing the signals.
- Normalize the signals again.
- Determine displacement by \( \Delta z = (1/2k) \tan^{-1}(I_{HC}/I_{VC}) \)
- Plot Lissajous from the calibrated signals.

\[ \text{End} \]

**Figure 2.** Flow chart of the calibrated-signal program.
The experimental results are shown in figure 3, which is raw data collected by an oscilloscope. The DC and AC signals of both intensities are obviously not equal. Figure 3(c) shows their Lissajous of the uncalibrated signal. Figure 3(b) is a calibrated interference signals after processing by our program and figure 3(d) is a Lissajous patterns of calibrated interference signals. These calibrated intensities are now removed the constant signals, normalize oscillating signals, and eliminate non-orthogonal error. Then the displacement is determined by using equation (6). Figure 4(a) is the displacement of the mirror M5 calculated from calibrated and uncalibrated signals as a function of time. The displacement oscillates between -4.00 to +6.00 µm. The different of the displacement is shown in figure 4(b). It shows that the nonlinear error of the displacement is a result of the NOE and the error is about 0.70 – 0.86 µm.
4. Conclusion
In this paper, we focus on the analysing process of the signals generated from SPSWI by using Python. The signals are shifted and normalized after processing to eliminate NOE. The analysing process is based on the orthogonalization technique by Fan et al. The displacement of the mirror can be determined in the scale of micrometre. The displacement of the mirror at various time in the range of 10 µm is obtained. The nonlinear error of the displacement is a result of the NOE and the error is in the range of 0.70-0.86 µm.

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