Field dependence of the superconducting gap in YPd$_2$Sn: A $\mu$SR and NMR study

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Abstract. We have performed muon spin rotation/relaxation and $^{119}$Sn nuclear magnetic resonance (NMR) measurements to study the vortex state of polycrystalline samples of YPd$_2$Sn ($T_c = 5.4$ K), over a wide range of applied magnetic fields up to $B_{c2}(T)$. Measurements in the vortex state provide the temperature dependence of the effective magnetic penetration depth $\lambda(T)$ and the field dependence of the superconducting gap $\Delta(0)$. The results are consistent with a very dirty $s$-wave BCS superconductor with $\lambda(0) = 212(1)$ nm, a gap $\Delta(0) = 0.85(3)$ meV, and a Ginzburg-Landau coherence length $\xi_{GL}(0) \approx 23$ nm. The $\mu$SR data in a broad range of applied fields are well reproduced by taking into account a field-related reduction of the effective superconducting gap. Interestingly, the ratio $2\Delta(0)/(k_B T_c)$ appears to a good approximation to be field-independent, with a value at low field of 3.85(9), implying a field dependence of the gap $\Delta(T = 0, B) = \Delta(T = 0, B = 0) \sqrt{1 - \frac{B}{B_{c2}(0)}}$. We discuss the significance of this result.

The compound YPd$_2$Sn is a so-called full Heusler compound with general formula $AT_2M$ ($M$ main group element, $T$ transition element, and $A$ either a rare earth or another transition element). Out of many hundreds of Heusler compounds only less than 30 (with Pd, Ni, or Au at the $T$ site) are superconducting at ambient pressure. YPd$_2$Sn is the one with the highest superconducting transition temperature, $T_c = 5.4$ K.

Besides superconductivity Heusler materials [1] display a large variety of interesting electronic properties such as different types of magnetic order (ferro-, antiferro-, and ferrimagnetism), heavy fermion behavior or half metallic ferromagnetism [2]. Some materials of this class may also exhibit properties typical of a topological insulator [3]. Their electronic tunability and multifunctionality make them attractive candidates for spintronics applications.

Recent transport and thermodynamic studies have identified strong electron-phonon coupling as the most important factor leading to superconductivity in these families. However, in contrast to the simple BCS theory, no conventional dependence of $T_c$ as a function of the BCS parameters, such as $N(0)$ or the Debye temperature, was found [4].

Unlike conventional BCS superconductors, several Pd-based compounds, such as YbPd$_2$Sn [5] and ErPd$_2$Sn [6], display co-existing superconductivity and long-range antiferromagnetic order. A clear and complete understanding of the origin of superconductivity, magnetism, and especially of their coexistence or interplay in the full Heusler compounds, is still missing.
In YPd$_2$Sn, earlier µSR data also showed an anomaly of possible magnetic origin setting in at $T_c$. For a non-magnetic superconductor such as YPd$_2$Sn, the large variation of $T_c$’s reported in the literature is also striking. We have therefore characterized the magnetic and superconducting properties of YPd$_2$Sn at microscopic level also to clarify the role of Pd, which due to its large Stoner factor is near to ferromagnetism. The detailed results of these investigations are reported in [7].

Here we focus on investigations of the vortex state and we discuss the possible field dependence of the energy gap in a conventional BCS superconductor. The low $B_{c2}(0) = 0.57$ T of this material allowed to study the $B - T$ phase diagram in the vortex state over a large range of fields up to the phase boundary and to investigate how the BCS gap is affected by a large magnetic field. We analyzed the µSR data by using a sum of Gaussian functions. This has the advantage to be model independent and provides also the best fit to the data over the entire range of applied fields [8]. After subtraction of the background contribution, the time spectra were fitted with the following expression

$$A(t) = e^{-\sigma_i^2t^2/2} \sum_{i=1}^{N} A_i e^{-\sigma_i^2t^2/2} \cos (\gamma_i B_i t + \varphi).$$  (1)

Here $\varphi$ is the initial muon spin phase, while $A_i$, $\sigma_i$, and $B_i$ are the amplitude, relaxation rate and first moment of the internal field of the $i$-th Gaussian component, respectively. $\sigma_n$ is the small contribution to the field distribution arising from the nuclear moments, independent of temperature and determined well above $T_c$.

The standard deviation of the multi-Gaussian internal field distribution is then given by:

$$\langle \Delta B^2 \rangle = \frac{\sigma_s^2}{\gamma_{\mu}^2} = \sum_{i=1}^{N} \frac{A_i}{A_1 + \ldots + A_N} \left[ \left( \frac{\sigma_i}{\gamma_{\mu}} \right)^2 + \langle B_i \rangle^2 \right],$$  (2)

with

$$\langle B \rangle = \sum_{i=1}^{N} \frac{A_i B_i}{A_1 + A_2 + \ldots + A_N}. $$  (3)

In the fit the number of components $N$ was increased until the $\chi^2$ of the fit did not change significantly. Typically, $N$ is between 1 (high-field data) and 3 (low-field data).

The ability of µSR to measure the field distribution and its moments in the vortex state is a powerful tool to determine characteristic length scales of superconductors such as the magnetic penetration depth and the coherence length, and electronic properties such as the superconducting gap and its symmetry. The ideally periodic flux line lattice (FLL) of known symmetry has been computed from various theories, see [9, 10, 11, 12] and references therein. The field distribution of the vortex lattice contains detailed information about the superconducting state. Recently, for instance, the diffraction of Cooper pairs on the vortex lattice of a very clean superconductor predicted by the Delrieu treatment of Gorkov theory has been detected in Nb crystals by a careful analysis of the TF spectra [12, 13].

The London and Ginzburg-Landau theories have the practical advantage to provide relatively simple expressions for quantities such as the field variance $\langle \Delta B^2 \rangle = \frac{\sigma_s^2}{\gamma_{\mu}^2}$ and are therefore widely used to analyse the µSR data. However, the original London model neglects the vortex core, corresponding to the case of an extreme type-II superconductor $\kappa = \frac{1}{\xi} \gg 1$ and small applied fields $B$ or, equivalently, small $b \equiv \langle B \rangle / B_{c2}$, $\langle B \rangle$ average local field, which is generally close to the applied field; in our experiment equal to it within 1%. On the other hand the Ginzburg-Landau theory, which takes into account the vortex core, is actually valid only close to $T_c$. 

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Figure 1. Temperature dependence of the muon spin depolarization rate in the vortex state of YPd$_2$Sn, measured after cooling below $T_c$ in different applied fields. Also plotted are the NMR linewidths measured at 0.463 T and converted into equivalent $\mu$SR relaxation rates. The fit lines were obtained using the procedure explained in the text.

In the range $0.13/\kappa^2 \ll b \ll 1$ the London model gives the well known field-independent $\langle \Delta B^2 \rangle = 0.00371 \frac{\Phi_0^2}{\lambda^4}$ [14], or equivalently $\sigma_{sc,L}(T)[\text{\mu s}^{-1}] = \frac{1.0728 \times 10^5}{\chi^2(T)[\text{nm}^4]}$, where the subscript $L$ indicates the London limit.

If the applied field is not small with respect to $B_{c2}$, the standard deviation becomes field dependent, because of the effect of the finite size of the vortex cores and of the reduced intervortex distance. The effect of the finite size of the vortex cores and the expected field dependence of the field variance have been taken into account in the London model by introducing a Gaussian cutoff in the Fourier coefficients of the local field [9, 11]. Ginzburg-Landau theory provides a natural description of the vortex cores of the FLL [10].

One finds that the standard deviation in an applied field is a function of $b$ and can be written as

$$\sigma_s(T, b) = \sigma_{s,L} f(b).$$

(4)

Analytical expressions for $f(b)$ have been computed from the London model with cutoff (LC) [9] and from a numerical solution of the Ginzburg-Landau equation (GL) [10]. Ref. [11] also presented an approximate analytical solution and numerical calculations of this function. In the first two cases $f(b)$ simplifies to:

$$f_{LC}(b) = 0.45175(1 - b)\sqrt{1 + 3.9(1 - b^2)}$$

(5)

$$f_{GL}(b) = 0.45249(1 - b)\left[1 + 1.21(1 - \sqrt{b})^3\right]$$

(6)

where the coefficient have been chosen so that for $b = 0$, which formally corresponds to the London limit, we obtain $\sigma_{s,L}$. Both expressions have been computed for a wide range of fields in the mixed state, the first for $0 < b < 1$ and the second for $0.25/\kappa^{1.3} \ll b \ll 1$ for $\kappa \geq 5$.

Eq. 4 has been used to analyze data in several cases over the full temperature range, where a decrease in the width of the internal field distribution when increasing the field toward $B_{c2}$ has
been observed (see for instance [15]). Generally, for each temperature the $\sigma_s$ values obtained at various $b$ are fitted with Eq. 5 or 6 by leaving the parameters $\lambda$ and $B_{c2}$ free and finally obtaining from the global temperature data $B_{c2}(T)$ and $\lambda(T)$. This procedure assumes no field dependence of $\lambda$. This is true and justified at $T = 0$ for a conventional superconductor with a single isotropic gap, to which the calculations of Ref. [9, 10] are related, but not at $T > 0$; i.e., the temperature dependence of $\lambda$ cannot be considered as field independent. This follows from the fact that the magnetic penetration depth diverges at the critical temperature, which in turn depends on the applied field $T_c(B)$. Since the phase transition from the normal state to the vortex state is a second-order transition, this implicitly introduces for all $T > 0$ a field dependence of $\lambda$.

The temperature dependence of the magnetic penetration depth is a measure of the superfluid density $n_s \propto 1/\lambda^2$. Within the BCS approach it is determined by the temperature dependence of the BCS gap. To take the above described field effects on the magnetic penetration depth into account we introduce a field-dependent effective superconducting gap $\Delta(0, B)$. Physically, this can be understood by considering that fields of not negligible magnitude with respect to $B_{c2}$ act as pair breakers, smear out the sharp edges of the spectroscopic gap, and lead to an effective reduction of $\Delta(0)$.

Following Ref. [16] we parametrize the BCS gap as

$$\Delta(T, B) = \Delta(0, B) \tanh \left[ \frac{\pi k_B T_c(B)}{\Delta(0, B)} \sqrt{\frac{T_c(B)}{T} - 1} \right].$$  

(7)

Our set of data of $\sigma_s(T, b)$ can be well fitted using the expression for a dirty superconductor with a single $s$-wave gap [17] (see Fig. 1)

$$\frac{n_s(T)}{n_s(0)} = \frac{\lambda^2(0)}{\lambda^2(T)} = \frac{\Delta(T, B)}{\Delta(0, B)} \tanh \frac{\Delta(T, B)}{2k_B T}.$$  

(8)

The dirty character is confirmed by our independent determination of the normal and the superconducting state parameters via transport and magnetization measurements [7], where
we find that, while remaining a good metal with \( k_F \ell \approx 60 \), YPd\(_2\)Sn has a very low value of \( \ell/\xi_0 = 0.008 \) (\( \xi_0 \), BCS coherence length). Fig. 1 also shows the NMR line width measured at 0.463 T scaled to the muon data by taking into account the different gyromagnetic ratios (\( \gamma_{\text{Sn}}/\gamma_\mu = 0.1178 \)). It is interesting to note that the magnitude of the field broadening obtained by the two techniques agrees fairly well. However, the NMR data seem to be affected by additional broadening effects at low temperatures, probably due to the proximity to the phase boundary. This reflects the limitations of NMR with respect to \( \mu \)SR when studying the vortex state of superconductors with low \( B_c2 \) values.

From the fit we obtain \( T_c(B) \) and \( \Delta(0, B) \). The field dependence of the critical temperature obtained in such a way is in good agreement with the \( B_c2(T) \) curve obtained from magnetoresistivity and ac magnetization measurements and is well reproduced by the prediction of the Werthamer-Helfand-Hohenberg (WHH) expression \([7]\). This agreement supports our analysis. Fig. 2 shows the field dependence of the ratio \( 2\Delta(0)/(k_B T_c) \). Interestingly, it appears to a good approximation to be field-independent, with an average value of 3.85(9). From the usual approximation \( B_c2(T) = B_c2(0)[1 - (T/T_c)^2] \), one obtains \( T_c(B) = T_c(0)\sqrt{1 - B/B_c2(0)} \). Therefore, a constant gap-to-\( T_c \) ratio implies a field dependence of the gap \( \Delta(0, B) = \Delta(0, 0)\sqrt{1 - B/B_c2(0)} \).

Assuming a field independent gap-to-\( T_c \) ratio, the field and temperature dependence of the gap can be described by the compact formula

\[
\Delta(T, B) = \Delta(0, 0)\sqrt{1 - \frac{B}{B_c2(0)}} \tanh \left[ R \sqrt{\frac{T_c(B)}{T} - 1} \right].
\]  

In our specific case \( R = 1.6152 \), and for an ideal (weak limit) BCS superconductor, \( R = 2\pi/3.5278 = 1.781 \). In the microscopic BCS theory for superconductivity the external magnetic field is treated as a weak perturbation. Therefore, e.g., the solution of the Meissner effect is a low-field limit solution and the gap has no field dependence. In Ginzburg-Landau theory the spatially averaged Ginzburg-Landau order parameter has a field dependence \( \langle |\Psi_{\text{GL}}(\vec{r}, B)|^2 \rangle = \Psi_{\text{GL}}^2[1 - \frac{B}{B_c2(0)}] \). Since the Gorkov gap (or pair) potential \( \Delta_G(\vec{r}) \propto \Psi_{\text{GL}}(\vec{r}) \), it is interesting to note that, although the BCS (energy) gap is not necessarily the same as the Gorkov gap potential (see e.g., gapless superconductivity), a field reduction of the BCS gap following \( \sqrt{1 - \frac{B}{B_c2(0)}} \) is obtained if we identify the BCS gap with the spatially averaged Gorkov gap potential \( \Delta(T = 0, B)^2 = \langle |\Delta_G(\vec{r})|^2 \rangle \propto \langle |\Psi_{\text{GL}}(\vec{r}, B)|^2 \rangle = \Psi_{\text{GL}}^2[1 - \frac{B}{B_c2(0)}] \).

In conclusion, we find that the \( \mu \)SR data of YPd\(_2\)Sn are consistently reproduced in a broad range of applied fields with a field-dependent isotropic superconducting gap, which decreases with field with a similar functional dependence as the critical temperature. The ratio \( 2\Delta(0)/(k_B T_c) = 3.85(9) \) is consistent with the presence of an important electron-phonon coupling in this compound. The effective magnetic penetration depth is \( \lambda(0) = 212(1) \text{ nm} \). Assuming a field independent gap-to-\( T_c \) ratio a simple parametrization of the gap can be used to take into account the reduction of the superfluid density (for \( T \neq 0 \)) of a conventional s-wave BCS superconductor in a magnetic field.

References

[1] Heusler F 1903 Verh. Dtsch. Phys. Ges. 5, 219
[2] Graf T et al. 2011 Prog. Solid State Chem. 39 1
[3] Butch N P et al. 2011 Phys. Rev. B 84 220504(R)
[4] Klimczuk T et al. 2012 Phys. Rev. B 85 174505
[5] Kierstead H A et al. 1985 Phys. Rev. B 32 135
[6] Shelton R N et al. 1986 Phys. Rev. B 34 199
[7] Saadaoui H et al. 2013 Phys. Rev. B 88 094518
[8] Maisuradze A et al. 2009 J. Phys.: Condens. Matter 21 075701
[9] Brandt E H 1988 Phys. Rev. B 37 2349
[10] Brandt E H 2003 Phys. Rev. B 68 054506
[11] Yaouanc A, Dalmas de Réotier P and Brandt E H 1997 Phys. Rev. B 55 11107
[12] Maisuradze A and Yaouanc A 2013 Phys. Rev. B 87 134508
[13] Maisuradze A et al. 2013 Phys. Rev. B 88 140509(R)
[14] Barford W and Gunn J M F 1988 Physica C 156 515
[15] Shermadini Z et al. 2010 Phys. Rev. B 82 144527
[16] Gross F et al. 1986 Z. Phys. B 64 175
[17] Tinkham M 2004 Introduction to Superconductivity 2nd ed. (New York: Dover Publications)