Abstract

The Operator Product Expansion, in conjunction with the power counting of non-relativistic field theory, is used to examine the end-point region of the radiative decay of heavy quark bound states with $^3S_1$ quantum numbers, $Q\bar{Q} \to \gamma + X$. We identify an infinite class of operators that determine the shape of the photonic end-point spectrum. These operators can be resummed to form an octet structure function which parameterizes the energy of the dynamical gluon content of the leading octet Fock state component of the quarkonium. This color-octet contribution is important when the photon spectrum is examined with a resolution given by $\Delta E_\gamma \sim m_Qv^2$, where $v^2$ is the relative quark velocity squared. The formalism used makes explicit the shift of the end-point from its partonic to its hadronic value.
The direct photon spectrum in $\Upsilon(1S)$ decay has been studied experimentally\cite{1} and compared with predictions based upon the color-singlet model\cite{2, 3, 4, 5}. These predictions were refined by the inclusion of hadronization effects\cite{5} which one would expect to be important in the end-point region. In this letter we discuss an effect which is formally larger near the end-point which has been heretofore neglected. This effect stems from higher Fock states in the heavy quark bound state. Using an operator product expansion (OPE) in conjunction with non-relativistic QCD (NRQCD)\cite{7}, we show that the end-point spectrum receives a leading order contribution from a color octet structure function. Furthermore, the formalism shows explicitly the shifting of the maximal photon energy from its partonic to its hadronic value. Thus, studying the endpoint region of the spectrum becomes a useful laboratory for studying non-perturbative QCD effects.

Direct photons in quarkonia decay arise from the electromagnetic coupling of both the heavy and light quarks. The spectrum stemming from the couplings to the light quarks has been previously studied in\cite{4} using the leading logarithmic approximation. This fragmentation contribution to the photon energy spectrum is suppressed relative to that coming from the coupling to heavy quarks, especially near the photonic endpoint where the hadronic invariant mass is highly restricted. In this letter, we will focus on the spectrum due to the coupling to the heavy quarks.

The photonic decay of quarkonia has the merit that the decay rate contains an external variable, namely the photon energy, which allows one to analytically continue to the unphysical regime where the OPE is trustworthy. This is tantamount to the statement that we may smear over a range of hadronic invariant mass, resulting in an average which is dual to the parton model\cite{6}. This is in contrast to the total hadronic decay calculations which assume duality on a point by point basis.

For the case at hand, we begin by considering the imaginary part of the time ordered product of two electromagnetic currents

$$T = i \int d^4xe^{-iq\cdot x}\langle \Upsilon \mid T(J^\mu(x)J_\mu(0)) \mid \Upsilon \rangle, \quad (1)$$

where $J^\mu = \bar{b}\gamma_\mu b$ and the normalization convention for upsilon states is $\langle \Upsilon(p) \mid \Upsilon(p') \rangle = (2\pi)^3\delta^3(p' - p)$. In eq.(1) and hereafter, spin averaging of the $\Upsilon$ matrix element is understood.
At fixed, $q^2 = 0$, $T$ has two cuts which span the entire real $q_0$ axis (we work in the rest frame of the Υ) in the complex $q_0$ plane. There is a cut resulting from the physical process $\Upsilon \to \gamma + X$ in the region $-m_{\Upsilon}/2 \leq q_0 \leq m_{\Upsilon}/2$ (neglecting the pion mass) and another from the process $\gamma + \Upsilon \to X$ for which the final state, $X$, has invariant mass larger than $m_{\Upsilon}$. This overlap obstructs the usual dispersion relation analysis since the discontinuity across the cut in the physical region is polluted by the other process. However, this obstruction is a red herring. To see this, we note that the $\Upsilon \to \gamma + X$ decay rate can be extracted from the time ordered product

$$T' = i \int d^4x e^{-iq \cdot x} \langle \Upsilon_{12} | T(J^\mu_{12} J_{\mu}(0)) | \Upsilon_{34} \rangle,$$

(2)

where $J^\mu_{ij} = \bar{b}_i \gamma^\mu b_j$ and $b_i$ are differing degenerate b-type quark species. $T'$ has two cuts, however they are now separated as shown in Figure 1. To get the $\Upsilon \to \gamma + X$ rate we simply perform the contour integral over a contour which only picks up a contribution from the part we are interested in. We may then relate the decay rate to the imaginary part of the $T'$ via

$$\frac{d\Gamma}{dE_\gamma} = \frac{2e_b^2 \alpha}{\pi} E_\gamma \text{Im} T',$$

(3)

where $e_b = -1/3$ is the electric charge of the $b$-quark and $\alpha$ is the fine structure constant.

We now may perform an OPE on the product of currents and calculate the Wilson coefficients using perturbative QCD. The OPE is valid in the unphysical region which is related to the physical process via the contour shown in Figure 1. The contour approaches the cut at one point, and thus the level of rigor here is not at the same level as in deep inelastic scattering but should be considered at the same level as the predictions for semi-leptonic heavy meson decay [8, 9]. Performing the contour integral over the photon energy corresponds to the smearing mentioned above. We may choose different weightings for the contour to extract information regarding the spectrum. Furthermore, as will be seen below, the convergence of the OPE will be dictated by the size of the smearing region.

For a given photon energy $E_\gamma$, the final hadronic invariant mass is given by $m_X^2 = m_{\Upsilon}^2 (1 - 2E_\gamma/m_{\Upsilon})$. Consequently, the end-point region of the photon energy spectrum corresponds to low invariant mass hadronic final states. In this region, the photon energy spectrum eq.(3) must be smeared over a range $\Delta E_\gamma$. We will see that the OPE gives us quantitative
information regarding the necessary size of the smearing region. Indeed, the convergence of the OPE is predicated upon the prudent choice of smearing functions.

The leading order diagrams come from those shown in Figure 2b. This decay mode is dominated by the color-singlet Fock state. NRQCD velocity scaling rules place this contribution to the decay rate of order, $\alpha\alpha_s^2v^3/\pi^2$. Away from the end-point this is the leading piece. However, near the end point there will also be a contribution stemming from those diagrams shown in Figure 2a which are dominated by the Fock state where the quarks are in a relative octet state. This color-octet process yields contributions to the decay rate of order $\alpha\alpha_s v^6/\pi$ and $\alpha\alpha_s v^7/\pi$ for the $^1S_0$ and $^3P_J$ intermediate states, respectively [7]. If we use $v^2 \simeq \alpha_s/\pi$, then the contributions are down by a factors of $v$ and $v^2$ relative to the color-singlet process. However, this naive counting does not hold for the end-point region of the photon energy.
spectrum because the color octet contribution is highly singular there. Indeed, as will be shown below, after smearing over a region of photon energies, \( \Delta E_\gamma \sim m_b v^2 \), the octet process is of the same order as the singlet in the endpoint region.

The leading order octet contribution to \( T' \) is given by

\[
T'_8 = 4g^2s(\gamma_{12} \left[ \bar{b}_1 \gamma^\alpha \left( \frac{\not{p} + iD - \not{q} + m_b}{(p + iD - q)^2 - m_b^2} T^A \gamma_\nu b_2 \right) \right]
\]

\[
\frac{g^{\mu\nu}}{(2p - q + iD)^2 + i\epsilon} \left[ \bar{b}_3 T^A \gamma_\mu \left( \frac{\not{p} + iD - \not{q} + m_b}{(p + iD - q)^2 - m_b^2} \gamma_\alpha b_4 \right) \right] \mid \Upsilon_{34},
\]

where each of the heavy quarks carries momentum \( p = (m_b, \vec{0}) \) and \( D \) denotes a covariant derivative. \( T'_8 \) receives contributions from \( ^1S_0 \) and \( ^3P_J \) intermediate states so we write

\[
T'_8 = T'_8(^3P_J) + T'_8(^1S_0).
\]

We now match onto the NRQCD by expanding the fields \( b \) in terms of non-relativistic fields \( \psi \) and \( \chi \) for the particle and anti-particle respectively. In making the transition to the effective theory, we keep only the Dirac structures relevant to the decay at hand. The relation between \( b \)-quark fields in full QCD and NRQCD produces additional factors of \( D \).

After imposing spin symmetry we find that the contribution to the imaginary part of \( T' \) from the octets Fock state are given by

\[
\text{Im} T'_8(^1S_0) = g^2s C_{^1S_0} \int dk_8^+ O(^1S_0) f(k_8^+) O(^1S_0) \delta(m_b - E_\gamma + k_8^+ / 2),
\]

\[
\text{Im} T'_8(^3P_J) = g^2s C_{^3P_J} \int dk_8^+ O(^3P_J) f(k_8^+) O(^3P_J) \delta(m_b - E_\gamma + k_8^+ / 2),
\]

where we have defined

\[
f(k_8^+) = \langle \Upsilon \mid \left[ \psi^\dagger \sigma_i \frac{i}{2} \not{D}_j T^A \chi \right] \delta(k_8^+ - n \cdot iD) \left[ \chi^\dagger \sigma_i \frac{i}{2} \not{D}_j T^A \psi \right] \mid \Upsilon \rangle / O(^3P_J),
\]

\[
O(^3P_J) = \langle \Upsilon \mid \left[ \psi^\dagger T^A \chi \right] \left[ \chi^\dagger T^A \psi \right] \mid \Upsilon \rangle,
\]

\[
f(k_8^+) = \langle \Upsilon \mid \left[ \psi^\dagger T^A \chi \right] \delta(k_8^+ - n \cdot iD) \left[ \chi^\dagger T^A \psi \right] \mid \Upsilon \rangle / O(^1S_0),
\]

\[
O(^1S_0) = \langle \Upsilon \mid \left[ \psi^\dagger T^A \chi \right] \left[ \chi^\dagger T^A \psi \right] \mid \Upsilon \rangle,
\]
and the light-like four-vector $n = (2p-q)/m_b$ is taken to be, $n = (1, 0, 0, 1)$. The four-vector, $k_b^+$, is the light-cone momentum of the dynamical gluon in the octet Fock state. Furthermore, we have kept only the leading twist operators, which is to say we have dropped terms of order $(k_b^+)^2$ in the delta function. The constants $C_{3P_J}$ and $C_{1S_0}$ have the values

$$C_{3P_J} = \frac{7\pi}{18m_b^5}, \quad C_{1S_0} = \frac{\pi}{2m_b^3}. \quad (12)$$

The functions $f(k_b^+)^{3P_J}$ and $f(k_b^+)^{1S_0}$ are the normalized probabilities that the dynamical gluon in the octet Fock state has light-cone momentum $k_b^+$. A strong analogy may be made with the deep-inelastic structure functions by studying the Fourier transform of the $f(k_b^+)$'s. In the gauge, $n \cdot A = 0$, the Fourier transform introduces a factor of $\exp(tn \cdot \partial)$. This translates the combination of quark fields $[\chi^\dagger \psi]$ from the origin to the spacetime point $tn$ giving, for example in the case of $3P_J$,

$$\hat{f}(t)^{3P_J} = \langle \Upsilon | \left[ \psi^\dagger(0)\sigma_i \frac{i}{2} \overset{\leftrightarrow}{D}_j T^A \chi(0) \right] \frac{P\exp[-ig_s \int_0^t dt' n \cdot A(t')]}{O(3P_J)} \left[ \chi^\dagger(tn)\sigma_i \frac{i}{2} \overset{\leftrightarrow}{D}_j T^A \psi(tn) \right] | \Upsilon \rangle. \quad (13)$$

In eq.(13) a path-ordered exponential has been inserted to restore gauge invariance.

The analysis of the color singlet contribution is simplified by the fact that in this case we may use the vacuum saturation approximation, which is a well controlled expansion in $v^2$ for this case [7]. The net result of summing the leading twist corrections is simply to shift the maximal photon energy from its partonic to hadronic value. To see this, we note that the imaginary part of Figure 2a can be written as

$$ImT'_1(3S_1) = g_s^4G(E_\gamma) \sum_{n=0}^\infty \frac{(-1)^n}{n!} \frac{\partial^n}{\partial(2E_\gamma)^n} \theta(2m_b - 2E_\gamma) \langle \Upsilon | \left[ \psi^\dagger \sigma_i \chi \right] (n \cdot i\partial)^n \left[ \chi^\dagger \sigma_i \psi \right] | \Upsilon \rangle, \quad (14)$$

where the leading order Wilson coefficient, $G(E_\gamma)$, is a smooth function of $E_\gamma$, that does not vanish at the end-point. After factorizing this matrix element only the time derivatives contribute (for $\Upsilon$ states at rest) giving factors of the binding energy [10]. Consequently eq.(14) becomes

$$ImT'_1(3S_1) = g_s^4G(E_\gamma) \theta(m_\Upsilon - 2E_\gamma) \langle \Upsilon | \left[ \psi^\dagger \sigma_i \chi \right] | 0 \rangle \langle 0 | \left[ \chi^\dagger \sigma_i \psi \right] | \Upsilon \rangle, \quad (15)$$
Figure 2: 2a) Leading order diagram for the octet contribution. 2b) Leading order Diagram for the singlet contribution.

showing that the whole effect of the leading twist contributions for the color singlet part of the amplitude is to shift the end-point from its partonic value, \( m_b \), to the physical one \( m_\Upsilon/2 \).

We are now in a position to consider the relative sizes of \( T_1'(3S_1) \), \( T_8'(3P_J) \) and \( T_8'(1S_0) \) in the end-point region. If the smearing region is large enough, then we would expect that the singular nature of the octet contribution will not compensate for naive \( v \) suppression of the octet state contributions. Too small of a smearing region will lead to a breakdown of the OPE, as the subleading twist terms will begin to dominate the octet contribution. More quantitatively, we may expand the octet contribution into an infinite sum of leading twist matrix elements

\[
ImT_8'(3P_J) = C_{3P_J} g_s^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \langle \Upsilon | \left[ \psi^\dagger \sigma_i \frac{i}{2} \overset{\leftrightarrow}{D}_j T^A \chi \right] iD_{\mu_1}...iD_{\mu_n} \left[ \chi^\dagger \sigma_i \frac{i}{2} \overset{\leftrightarrow}{D}_j T^A \psi \right] | \Upsilon \rangle (16)
\]
\[ ImT_s^{(1)S_0} = C_1 S_0 g_s^2 \sum_{n=0}^{\infty} \frac{(-1)^n}{n!} \langle \Upsilon \mid [\bar{\psi}^A T^A \chi] iD_{\mu_1} \ldots iD_{\mu_n} [\chi^A T^A \bar{\psi}] \mid \Upsilon \rangle \]
\[ n^{\mu_1} \ldots n^{\mu_n} \delta^{(n)} (2E_\gamma - 2m_b), \]

where \( \delta^{(n)} \) denotes \( n \) derivatives with respect to \( 2E_\gamma \) acting on the delta function. Eqs.(16,17) are similar to the expression found in the analysis of the leptonic end-point spectrum in \( B \) decays \[1\] \[2\]. An operator with \( n \) factors of the covariant derivative scales as \[7\] \( v^{(6,7)+2n} \), as each derivative contributes a factor of \( v^2 \) to the scaling. If we smear, for example, using a Gaussian of width \( \Delta E_\gamma \) then for, \( \Delta E_\gamma << m_b v^2 \), the terms in the sum grow with \( n \), and the subleading-twist terms which have been dropped cannot be neglected. On the other hand, for \( \Delta E_\gamma >> m_b v^2 \), the singlet contribution will dominate over the octet. Thus, the finest energy resolution with which we can examine the photon endpoint spectrum, without introducing yet more higher twist structure functions, is of order \( m_b v^2 \). Note that this corresponds to smearing over a range of hadronic masses \( \Delta m_X \sim m_b v \) which for large \( m_b \) is much greater than the QCD scale. For the smearing width \( \Delta E_\gamma \sim m_b v^2 \) the octet \( 3P_J \) contribution is of the same order as the color-singlet since the naive octet suppression of \( v^2 \) is compensated by the fact that the singlet contribution starts with a theta function, whereas the octet starts with a delta function. The formally leading contribution \[1\] however, comes from the octet \( 1S_0 \) contribution which is enhanced by a power of \( 1/v \). Therefore, if we wish to resolve the photon energy spectrum within \( m_b v^2 \sim 500 \text{ MeV} \) of the end-point, we must know the value of the structure functions \( f(k^+_8)_{1S_0} \) and \( f(k^+_8)_{3P_J} \), which can be perhaps measured by the lattice or extracted from another process.

Let us now consider the size of other possible large effects near the end-point. As mentioned in the introduction there is another nonperturbative effect near the end point having to do with the hadronization of the partons. The gluonic partons are not really massless, but are actually finite invariant mass objects. This effect was addressed in ref. \[3\], where the author used a parton-shower Monte Carlo approximation to determine the effects of hadronization on the photon spectrum of the color-singlet contribution. Hadronization cuts off the photon distribution at the end point. This effect is important for hadronic masses,

\footnote{See erratum in ref. \[3\].}
of order the QCD scale and should be subdominant to those of the octet contribution. The success of hadronization models in describing the endpoint spectrum may indicate that the color octet matrix elements are smaller than expected on the basis of dimensional analysis.

In the end-point region there are also large perturbative corrections stemming from terms of the form \( \ln(1 - x) \) where \( x = E_\gamma / m_b \). These terms result from the fact that near the endpoint gluon radiation is suppressed ruining the delicate cancelation between real and virtual graphs. A resummation of these logarithms is expected to suppress the singlet spectrum near the end point. The calculations in ref. \[1\] indicate that this resummation does indeed suppress the color-singlet rate near the end-point and are important when \( E_\gamma > 0.4m_T \). There will be similar large logarithms in the Wilson coefficient for the octet contribution discussed above, resumming these logarithms should again suppress the end point region. A calculation of these effects is necessary to undertake any phenomenological investigation of the end-point region.

The results of this paper are applicable to both \( c\bar{c} \) and \( b\bar{b} \) quarkonia states. However, we feel that the charm quark mass is probably not large enough for our methods to be trustworthy. We also note that effects similar to those discussed here are important for fragmentation into quarkonia near \( z = 1 \) as well as direct production in some kinematic circumstances\[14\].

Acknowledgements

While we this paper was being written we became aware of \[13\] which deals with issues similar to those here for the case of \( \eta_Q \) decay. I.Z.R is supported by grant no. DOE-FG03-90ER40546. M.B.W. is supported by grant no. DE-FG03-92-ER40701.

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