Polarization observables in $\pi$-photoproduction on the deuteron

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A general analysis of polarization observables for the $d(\gamma, \pi)NN$ reaction with polarized photons and/or oriented deuterons is presented. The unpolarized differential cross section, linear photon asymmetry, vector and tensor target asymmetries are predicted for forthcoming experiments.

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I. INTRODUCTION

Meson photo- and electroproduction on light nuclei is primarily motivated by the following possibilities: (i) study of the elementary neutron amplitude in the absence of a neutron target, (ii) investigation of medium effects, i.e., possible changes of the production operator in the presence of other nucleons, (iii) it provides an interesting means to study nuclear structure, and (iv) it gives information on pion production on off-shell nucleon, as well as on the very important $\Delta N$-interaction in a nuclear medium.

Polarization observables will give additional valuable information for checking the spin degrees of freedom of the elementary pion production amplitude of the neutron, provided, and this is very important, that one has under control all interfering interaction effects which prevent a simple extraction of this amplitude. As an illustration of these various aspects, incoherent single pion photoproduction on the deuteron in the $\Delta(1232)$-resonance region is investigated with special emphasis on polarization observables [1, 2]. The importance of this process derives from the fact that the deuteron, being the simplest nuclear system, plays a similar fundamental role in nuclear physics as the hydrogen atom plays in atomic physics.

II. POLARIZATION OBSERVABLES

The most general expression for all possible polarization observables in terms of the transition matrix elements is given by [1, 2, 3]

$$O = \sum_{\alpha \alpha'} \int_0^{\Omega_{\text{max}}} dq \int d\Omega_{\text{pNN}} \rho_s \mathcal{M}_{s'm',m'_d}^{(t'\mu')} \bar{\Omega}_{s'm'sm}^{\gamma} \mathcal{M}_{sm,m_d}^{(t\mu)} \rho_s^{d}_{m,m'} \rho_{s'm'd'}^{d} ,$$

where we have introduced as a shorthand for the quantum numbers $\alpha = (s, m, t, m_{\gamma})$ and $\alpha' = (s', m', t', m'_{\gamma})$. Furthermore, $m_{\gamma}$ denotes the photon polarization, $m_d$ the spin projection of the deuteron, $s$ and $m$ total spin and projection of the two outgoing nucleons, respectively, $t$ their total isospin, and $\mu$ the isospin projection of the pion. $\rho^{d}_{m,m'}$ and $\rho^{d}_{m'd'}$ denote the density matrices of initial photon polarization and deuteron orientation, respectively, $\bar{\Omega}_{s'm'sm}$ is an operator associated with the observable, which acts in the two-nucleon spin space. For details with respect to the density matrices we refer to [3]. The transition $\mathcal{M}$-matrix elements of the $d(\gamma, \pi)NN$ reaction as well as the phase space factor $\rho_s$ are given in [1].

As in our previous work [2], the unpolarized differential cross section is given by

$$\frac{d^3\sigma}{d\Omega_{\pi}dq} = \frac{1}{6} \sum_{\alpha} \int d\Omega_{\text{pNN}} \rho_s |\mathcal{M}_{smm',m_d}^{(t\mu)}|^2 .$$

The photon asymmetry for linearly polarized photons is given by [1]

$$\Sigma = \frac{2}{A} \text{Re} \sum_{smtm_d} \int_0^{\Omega_{\text{max}}} dq \int d\Omega_{\text{pNN}} \rho_s \mathcal{M}_{sm+1m_d}^{(t\mu)} \mathcal{M}_{sm-1m_d}^{*(t\mu)} ,$$

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with
\[ A = \sum_{\alpha} \int_{0}^{q_{\text{max}}} dq \int d\Omega_{\text{NN}} \rho_{s} |M_{s_{\text{smm}},m_{\gamma}}^{(\mu)}|^2. \]  
(4)

The vector target asymmetry \( T_{11} \) is given by
\[ T_{11} = \frac{\sqrt{6}}{A} \Re \sum_{s_{\text{smm}}} \int_{0}^{q_{\text{max}}} dq \int d\Omega_{\text{NN}} \rho_{s} [M_{s_{\text{smm}},-1}^{(\mu)} - M_{s_{\text{smm}},+1}^{(\mu)}] M_{s_{\text{smm}},0}^{(\mu)} \ast . \]  
(5)

The tensor target asymmetries are given by
\[ T_{20} = \frac{1}{\sqrt{2}A} \sum_{s_{\text{smm}}} \int_{0}^{q_{\text{max}}} dq \int d\Omega_{\text{NN}} \rho_{s} [M_{s_{\text{smm}},+1}^{(\mu)} + M_{s_{\text{smm}},-1}^{(\mu)} - 2 |M_{s_{\text{smm}},0}^{(\mu)}|^2], \]  
(6)
and
\[ T_{21} = \sqrt{6} A \Re \sum_{s_{\text{smm}}} \int_{0}^{q_{\text{max}}} dq \int d\Omega_{\text{NN}} \rho_{s} [M_{s_{\text{smm}},-1}^{(\mu)} - M_{s_{\text{smm}},+1}^{(\mu)}] M_{s_{\text{smm}},0}^{(\mu)} \ast . \]  
(7)
and
\[ T_{22} = \frac{2\sqrt{3}}{A} \Re \sum_{s_{\text{smm}}} \int_{0}^{q_{\text{max}}} dq \int d\Omega_{\text{NN}} \rho_{s} M_{s_{\text{smm}},-1}^{(\mu)} M_{s_{\text{smm}},+1}^{(\mu)} \ast . \]  
(8)

### III. RESULTS AND DISCUSSION

For the evaluation of the above mentioned observables we have chosen the laboratory frame of the deuteron. As coordinate system a right-handed one is taken with \( z \)-axis along the momentum \( \vec{k} \) of the incoming photon and \( y \)-axis along \( \vec{k} \times \vec{q} \), where \( \vec{q} \) is the pion momentum. As seen above, all observables are calculated by integrating over the pion momentum \( q \), the polar angle \( \theta_{\text{NN}} \) and the azimuthal angle \( \phi_{\text{NN}} \) of the relative momentum \( p_{\text{NN}} \) of the two outgoing nucleons. These integrations are carried out numerically. The number of integration points was being increased until the accuracy of calculated observable becomes good to 1%.

The contribution to the pion production amplitude is evaluated by taking a realistic \( NN \) potential model for the deuteron wave function. For our calculations we have used the wave function of the Paris potential [7], which is in excellent agreement with \( NN \) scattering data [6]. For the elementary pion photoproduction operator, we have taken the effective Lagrangian model of Schmidt et al. [7]. This model had been constructed to give a realistic description of the \( \Delta(1232) \)-resonance region. It is given in an arbitrary frame of reference and allows a well defined off-shell continuation as required for studying pion production on nuclei. Therefore, this model for the elementary photoproduction amplitude is quite satisfactory for our purpose, namely to incorporate it into the reaction on the deuteron.

Fig. 1 shows our results for the \( \pi \)-meson spectra [9] as a function of pion momentum \( q \). One sees that when \( q \) reaches its maximum, the absolute value of the relative momentum \( p_{\text{NN}} \) of the two outgoing nucleons vanishes, and thus a narrow peak is appears in the forward pion angles. In the lower part of Fig. 1 we see that the unpolarized differential cross section is small and the narrow peak which appears at forward angles is disappears. The same effect appears in the coherent process of charged pion photo- and electroproduction on the deuteron [8, 9], in deuteron electrodissintegration [10] as well as in \( \eta \)-photoproduction [11]. It is also clear that the maximum value of \( q \) (when \( p_{\text{NN}} \to 0 \)) decreases with increasing the pion angle. It changes from \( \sim 300 \text{ MeV} \) at forward angles to \( \sim 200 \text{ MeV} \) at backward ones. In principle, the experimental observation of this peak in the high \( \pi \)-momentum spectrum may serve as another evidence for the understanding of the \( \Delta(1232) \)-meson spectra.

Our results for the photon asymmetry \( \Sigma \) for linearly polarized photons [3] for all different charge states of the pion of \( d(\vec{q},\pi)\text{NN} \) are plotted in Fig. 2 at different photon lab-energies as a function of pion angle \( \theta_{\pi} \) in the laboratory frame. First of all, we see that the photon asymmetry has always a negative values at forward and backward pion angles. One notes qualitatively a similar behaviour for charged pion channels whereas a totally different behaviour is seen for the neutral pion channel.

For extreme forward and backward pion angles one sees, that the effect of Born contributions is relatively small in comparison to the results when \( \theta_{\pi} \) changes from \( 60^\circ \) to \( 120^\circ \). One notices also, that the contribution from Born
terms are much important in this region. In the energy range of the $\Delta(1232)$-resonance, one sees that the contribution from Born terms are important in the case of charged pion channels. For the neutral pion channel we see, that this contribution is very small at 330 MeV. For lower and higher energies, one sees again the sizeable effect from Born terms which arise mainly from the Kroll-Rudermann term. One sees also, that $\Sigma$ is sensitive to the energy of the incoming photon. Finally, we observe that the interference of the Born terms with the $\Delta(1232)$-resonance contribution causes considerable changes in the linear photon asymmetry. Experimental measurements will give us more valuable information on this asymmetry.

Our results for the vector target asymmetry $T_{11}$ are depicted in Fig. 3 for $\gamma \vec{d} \rightarrow \pi^- pp$ (left panels), $\pi^+ nn$ (middle panels), and $\pi^0 np$ (right panels), respectively. The asymmetry $T_{11}$ clearly differs in size between charged and neutral pion photoproduction channels, being even opposite in phase. For charged pion photoproduction reactions we see, that the vector target asymmetry has always a negative values which mainly come from the Born terms. A small positive contribution from the $\Delta$-resonance is found only at pion forward angles. At backward angles, the negative values for $T_{11}$ come from an interference of the Born terms with the $\Delta(1232)$-resonance contribution. For all energies one observes at forward angle the strongest effect of the Born terms.

With respect to the neutral pion photoproduction channel, we see that the vector target asymmetry is always positive. For energies below the $\Delta$-resonance, a very small negative value is found at extreme backward pion angles while a relatively large positive value at forward angles is found. We see also, that $T_{11}$ is sensitive to the Born terms. The same effect was found by Blaazer et al. [12] and Wilhelm and Arenhövel [13] for the coherent pion photoproduction reaction on the deuteron. The reason is that $T_{11}$ depends on the relative phase of the matrix elements as can be seen from (5). It would vanish for a constant overall phase of the $t$-matrix, a case which is approximately realized if only the $\Delta(1232)$-amplitude is considered. Finally, we notice that $T_{11}$ is vanishes at $\theta_\pi = 0$ and $\theta_\pi = \pi$ which is not the case for the linear photon asymmetry.

Let us discuss now the results of the tensor target asymmetries $T_{20}$, $T_{21}$, and $T_{22}$ as shown in Figs. 4 and 5 for $\gamma \vec{d} \rightarrow \pi^- pp, \pi^+ nn$, and $\pi^0 np$, respectively. We start from the tensor asymmetry $T_{20}$ which is plotted in the left panels. For $\gamma d \rightarrow \pi NN$ at forward and backward pion angles, the asymmetry $T_{20}$ allows one to draw specific conclusions about details of the reaction mechanism. In comparison to the results for photon and vector target asymmetries we found here, that the contribution from the Born terms is very small both for charged and neutral pion production channels. It is also noticeable, that for charged channels the asymmetry $T_{20}$ has a relatively large positive values at pion forward angles while a small negative ones at backward angles are found. For the neutral pion production channel we see, that $T_{20}$ has a negative values at forward angles and a positive ones at backward angles. Only for energies above the $\Delta$-resonance we note, that it has a small negative values at extreme backward angles.

The tensor target asymmetry $T_{21}$ of $\gamma \vec{d} \rightarrow \pi^- pp, \pi^+ nn$, and $\pi^0 np$ is plotted in the middle panels of Figs. 4 and 5 respectively. It is clear that $T_{21}$ differs in size between charged and neutral pion production channels. One notices, that for charged pion channels $T_{21}$ asymmetry is sensitive to the Born terms, in particular at forward pion angles. In the case of $\pi^0$ channel one sees, that the contribution of the Born terms is much less important at all energies. In comparison to the results for photon and vector target asymmetries we found also here, that the contribution from the Born terms is small both for charged and neutral pion production channels. It is also noticeable, that in the case of charged pion channels the asymmetry $T_{21}$ has a relatively large positive values at pion forward angles. For the neutral pion channel we see, that $T_{21}$ has a negative values at forward angles. Furthermore, as in the case of vector target asymmetry, we found that $T_{21}$ vanishes at $\theta_\pi = 0$ and $\theta_\pi = \pi$.

In the right panels of Figs. 4 and 5 we depict our results for the tensor target asymmetry $T_{22}$ for the reactions $\gamma \vec{d} \rightarrow \pi^- pp, \pi^+ nn$, and $\pi^0 np$, respectively. One readily notes the importance of Born terms at extreme forward pion angles. Like the results of the $T_{20}$ and $T_{21}$ asymmetries, the $T_{22}$ asymmetry is sensitive to the values of pion angle $\theta_\pi$. At $\theta_\pi = 60^\circ$ we see, that the Born terms are important for $\pi^0$ production channel while these terms are very important for charged pion channels at extreme forward angles. Moreover, we found that $T_{22}$ is also vanishes at $\theta_\pi = 0$ and $\theta_\pi = \pi$.

IV. CONCLUSION

I would like to conclude that the results presented here for polarization observables in the $d(\gamma, \pi)NN$ reaction in the $\Delta(1232)$-resonance region can be used as a basis for the simulation of the behaviour of polarization observables and for an optimal planning of new polarization experiments of this reaction. It would be very interesting to examine our predictions experimentally.
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FIG. 1: The $\pi$-meson spectra in the $d(\gamma, \pi)NN$ reaction as a function of pion momentum $q$ at a photon energy of 330 MeV for different values of pion angles $\theta_\pi$. The solid curves show the results of the full calculations while the dashed curves represent the results when only the $\Delta(1232)$-resonance is taken into account. The left, middle and right panels represent the results for $\gamma d \rightarrow \pi^- pp$, $\pi^+ nn$ and $\pi^0 np$, respectively.
FIG. 2: Linear photon asymmetry of $d(\gamma, \pi)NN$. Notation as in Fig. [1]
FIG. 3: Vector target asymmetry of $d(\gamma, \pi)NN$. Notation as in Fig. I.
FIG. 4: Tensor target asymmetries of $\vec{d}(\gamma, \pi^-)pp$. Notation as in Fig. 1.
FIG. 5: Tensor target asymmetries of $d(\gamma, \pi^+)nn$. Notation as in Fig. □.
FIG. 6: Tensor target asymmetries of $\bar{d}(\gamma, \pi^0)np$. Notation as in Fig. [4]