MODEL BASED IMAGE RECOVERY IN PHOTOACOUSTIC TOMOGRAPHY USING HYBRID NON-CONVEX REGULARIZATION

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ABSTRACT

We propose a novel model based image reconstruction method for photoacoustic tomography (PAT) involving a novel form of regularization, and a novel computational scheme for implementing the forward model without building large matrices. The regularization is constructed by combining second order derivatives and intensity into a non-convex form so as to exploit the properties of typical PAT images that we observe: in PAT images, high intensities and high second order derivatives are jointly sparse. The specific form of regularization proposed here is a modification of a form that was proposed for fluorescence image restoration. As non-convex regularized reconstruction requires large number of iterations, and the PAT forward model has to be applied in the iterations, we propose efficient computational structures for matrix-free implementation of the forward model. We evaluate the proposed method on both simulated and real measured data sets and compare with recent fast iterative shrinkage threshold algorithm (FISTA)-based reconstruction method.

1 Introduction

Photoacoustic tomography (PAT) [1][2][3][4][5][6][7] provides high resolution and high contrast images of deep tissues by imaging the spatial distribution of certain substances that can absorb near infra-red optical energy. Upon shining with a laser pulse, the substance under investigation absorbs the optical energy and undergo thermoelastic expansion; as a result, the spatial distribution of the concentration of the substance gets translated into the distribution of pressure-rise. This initial pressure rise travels outwards as ultrasound waves which are collected by ultrasound transducers placed at the boundary. From the acoustic pressure measured by the transducers as function of time, a PAT reconstruction method recovers an estimate of the initial pressure-rise by solving the associated inverse problem [8]. Achieving accuracy in image reconstruction is a fundamental challenge in PAT: the reconstructed images suffer from artifacts that depend on the measurement geometry mainly because the reconstruction problem is complex.

There are a class of methods known as analytical inversion methods, which obtain reconstruction (initial pressure rise) by some transformation on the measured data. Among such methods, filtered back projection (FBP) [9][10][11][12] and

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delay and sum method [15] are fast and memory efficient, but require a large number of measurements for good quality reconstructed images. This leads to increased scan time or expensive instrumentation setups. Time reversal methods [15] are the least demanding class of methods in the category of analytical methods, and can be used for any detection geometry [16], acoustic absorption and dispersion [17], and can accommodate the heterogeneities present in the medium [18]. However, in many applications of PAT, due to the geometrical limitations [19] or by choice to accelerate data acquisition [20], we may have to put restrictions on the spatial and/or temporal sampling of the photoacoustic (PA) signal. In such situations, these methods suffer from blurring and curved-line artifacts.

Model-based iterative reconstruction methods outperform the direct analytic reconstruction methods in the case of limitation in the size of measured data, and yield better quality in reconstruction [21][22][23][24]. These methods express the measured pressure samples as a linear transformation on the initial pressure rise. This transformation is implemented by means of matrix-vector multiplication with the vector representing the pixel values of the initial pressure, where the matrix is typically very large. This transformation goes into the data fidelity term, which is minimized along with a regularization functional to achieve reconstruction. The regularization represents a prior belief on the spatial characteristics of image to be recovered [25][26][27][28]. The minimization is achieved by means of iterations involving repeated application of the above-mentioned transformation and its adjoint leading to high computational burden and memory-burden. To reduce the memory requirements and computational complexity, several methods have been reported. For example, the memory overhead can be reduced by decreasing the number of measurements [29] or by calculating the matrix-vector products on-the-fly without explicitly storing the model matrix [30]. But since the same operations need to be performed multiple times, the computational time required is longer. A fast inversion can be achieved by simplifying the forward model with the assumption that the photoacoustic sources lie in a plane [31]. The inversion can be made faster by decoupling the inversion into smaller subproblems using a discrete wavelet packet decomposition [32]. But the usage of interpolation methods in calculating the model matrix leads to modeling errors as it is dependent on image resolution and hence regularization is needed to suppress the imaging artifacts. A more accurate model matrix calculation based on the direct discretization of Poisson-type integral showed better reconstruction accuracy in high-noise and low-SNR imaging conditions [33]. The accuracy of the model matrix is dependent on the number of sensors which makes the above method not useful in the limited data scenarios.

We develop a novel model based reconstruction method that yields significantly improved resolution in the reconstruction. Our contributions are threefold. First, we note that existing model-based methods in PAT use a generic regularization forms developed for general images. A regularization function that can cater to the specific nature of the photoacoustic images is not used until now. We first observe that PAT images has the following characteristics which is also observed in fluorescence images: in these images high intensities and high second order derivatives are jointly sparse. This property was exploited in [34] by constructing a regularization that combines the intensity and the second order derivatives. Here we modify this form such that it is more suitable for the current reconstruction problem. Second, we develop an efficient method for on-the-fly computation of matrix-vector products derived using a model that uses an exact time propagator to calculate the acoustic field [35][36]. By taking advantage of the filtering approach given in the proposed formulation, forward and adjoint matrix operations can be accelerated using filters with its complexity independent of the number of transducers. Finally, we construct a novel preconditioned gradient method for minimizing the cost function.

1.1 Forward model in photoacoustic tomography

The forward problem in PAT accounts for the calculation of the pressure fields in space and time \( p(\mathbf{r}, t) \) from a known photoacoustic source \( H(\mathbf{r}, t) \), which represents the light energy deposited in the medium per unit volume per unit time. The induced pressure waves \( p(\mathbf{r}, t) \) under the condition of thermal and stress confinements obey the following differential equation for an acoustically homogeneous medium [37]

\[
\left( \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right) p(\mathbf{r}, t) = -\Gamma \frac{d}{dt} H(\mathbf{r}, t),
\]  

where \( \Gamma \) is a dimensionless parameter called the Grüneneisen coefficient, that describes the conversion efficiency of heat to pressure and \( c \) is the speed of sound in the medium. By recognizing that the temporal duration of the laser pulse is shorter than the temporal resolution of the ultrasound detectors in most of the photoacoustic imaging applications, the PA source \( H(\mathbf{r}, t) \) may be approximated by \( H(\mathbf{r})\delta(t) \), where \( H(\mathbf{r})\delta(t) \) is the deposited energy per unit volume. Then the solution to (1) can be written as [8]

\[
p(\mathbf{r}, t) = \frac{\Gamma}{4\pi c} \frac{\partial}{\partial t} \int_{|\mathbf{r}'| = ct} H(\mathbf{r}') \frac{1}{|\mathbf{r} - \mathbf{r}'|} \, d\mathbf{r}',
\]  

where the initial pressure field \( p_0(\mathbf{r}) = p(\mathbf{r}, t = 0) \) can be written as

\[
p_0(\mathbf{r}) = \Gamma \, H(\mathbf{r}).
\]
The pressure distribution, \( p(\mathbf{r}, t) \), can also be expressed as [36, 35]

\[
p(\mathbf{r}, t) = \mathcal{F}^{-1}\left\{ \hat{P}_o(k) \cos(c_0 |k| t) \right\},
\]

where \( \hat{P}_o(k) \) is the Fourier transform of \( p_0(\mathbf{r}) \), with \( k \) denoting the 2D Fourier frequency.

### 1.2 Discrete forward model

The discrete representation of the forward problem is the basis of model-based reconstruction algorithms in PAT. Since the measured pressure field is linearly related to the photoacoustic source, the discretization of the forward problem may be written in a matrix form as [22]

\[
\mathbf{p}_m = \mathbf{H}\mathbf{p}_0,
\]

where \( \mathbf{p}_m \) is a \( LM \times 1 \) vector representing the total number of discrete pressure measurements from \( L \) transducers each taking \( M \) time-samples and \( \mathbf{H} \) is the model matrix of size \( LM \times N \) with \( N \) being the total number of pixels. The initial pressure distribution \( p_0(\mathbf{r}) \) is represented spatially using a 2D imaging grid having \( N_x \) and \( N_y \) grid points along \( x \) and \( y \) directions respectively and is denoted by a \( N \times 1 \) vector \( \mathbf{p}_0 \) where \( N = N_x \times N_y \). The model matrix can be calculated by discretizing the integral relation in (2) in the ideal case of homogeneous lossless medium and point detectors. Several methods to improve the accuracy of the model by incorporating transducer responses, heterogeneity in the medium and interpolation techniques for accurate discretization has been reported [25, 38, 31]. Including the measurement noise, the modified imaging model can be written as

\[
\mathbf{p}_m = \mathbf{H}\mathbf{p}_0 + \mathbf{\eta},
\]

where \( \mathbf{\eta} \) represents measurement noise, which is Gaussian.

### 1.3 Model based image reconstruction

The aim of the reconstruction task in PAT is to recover the initial pressure distribution \( \mathbf{p}_0 \) from the noisy transducer measurement data \( \mathbf{p}_m \). In the limited data case, the PA image reconstruction problem is ill-posed and hence constraints are imposed on the required solution in the form of a regularization. In this case, the image reconstruction problem can be treated as an optimization problem where the solution is obtained by minimizing a cost function. The reconstruction problem can be written as

\[
\mathbf{p}_0 = \underset{\mathbf{p}_0}{\text{arg min}} \ J(\mathbf{p}_0)
\]

where \( J(\mathbf{p}_0) \) is the cost function and is given by

\[
J(\mathbf{p}_0) = ||\mathbf{p}_m - \mathbf{H}\mathbf{p}_0||_2^2 + \lambda \ R(\mathbf{p}_0).
\]

Here \( R(\mathbf{p}_0) \) is the regularization functional and \( ||\cdot||_2 \) represents the \( L_2 \) norm. The regularization parameter \( \lambda \), controls the amount of roughness in the solution and fidelity to the measured data.

The regularization functional typically should be able to smooth the noise and hence contains derivative terms in their formulation. A quadratic and differentiable regularization functional called Tikhonov regularization has been used in limited data cases [8, 39] and is given by

\[
R(\mathbf{p}_0) = \sum_i ||\mathbf{D}_i\mathbf{p}_0||_2^2 = \sum_{r=1}^N \sum_i (\langle \mathbf{D}_{o,i}\mathbf{p}_0 \rangle_r)^2
\]

where \( \langle \cdot \rangle_r \) denotes the \( r \)th components of its vector argument, and \( \mathbf{D}_{o,i} \) represents the matrix of \( i^{th} \) derivative filter of order \( o \). For example, \( \mathbf{D}_{1,i} \), \( i = 1, 2 \), are the matrix equivalents of filtering by discrete filters that implement the operators \( \frac{\partial}{\partial x} \) and \( \frac{\partial}{\partial y} \). Further, \( \mathbf{D}_{2,i} \), \( i = 1, 2, 3 \), are the matrix equivalents of filtering by discrete filters that implement the operators \( \frac{\partial^2}{\partial x^2} \), \( \frac{\partial^2}{\partial y^2} \), \( \sqrt{2} \frac{\partial}{\partial x \partial y} \). The resulting minimization of the convex quadratic cost function yields a closed form solution given by

\[
\mathbf{p}_0 = [\mathbf{H}^T\mathbf{H} + \lambda \sum_i \mathbf{D}_{o,i}^T\mathbf{D}_{o,i}]^{-1}\mathbf{H}^T\mathbf{p}_m
\]

As it precludes any large derivative values, Tikhonov regularization tends to smooth edges in the reconstructed image along with the noise.
Widely used non-quadratic functional is the Total Variation (TV) \([40, 41, 42, 25]\) and it is superior to the quadratic functional in its ability to preserve edges and is robustness to noise. The discrete total variation is given by

\[
R_{TV}(p_0) = \sum_{r=1}^{N} \sqrt{\sum_{i} (D_{o,i} p_0)_r^2}
\]  

(11)

So far, only \(o = 1\) has been used, and this form favors images having piece wise constant regions. Often a differentiable yet convex approximation of the total variation is used which can be written as

\[
R_{TV}(p_0) = \sum_{r=1}^{N} \sqrt{\epsilon + \sum_{i} (D_{o,i} p_0)_r^2}
\]  

(12)

where \(\epsilon\) is a small positive number.

2 The proposed reconstruction method

2.1 Proposed Regularization functional

The photoacoustic images have high contrast due to the differential absorption of light in the near-infrared region by chromophores such as hemoglobin. Due to this, high values of initial pressure, \(p_0(r)\), are sparsely distributed. Further, regions of having high derivative values are also sparsely distributed. This effect has been observed in fluorescence images in the work presented \([34]\), where the regularization was modified by adding an intensity term to capture this effect. The combined point-wise cost goes into a logarithmic function and summed over all pixels. Here we replace the log by a fractional power, and write the proposed regularization as

\[
R_{h,1}(p_0) = \sum_{r=1}^{N} \left( \epsilon + \alpha (p_0)_r^2 + (1 - \alpha) \sum_{i} (D_{o,i} p_0)_r^2 \right)^{p},
\]  

(13)

where the weight \(\alpha\) controls the penalization for the intensity term. The advantage of this modification is that it allows an optimization strategy that can efficiently handle non-convex cost function, which will be demonstrated later. Here, we choose \(p < 0.5\) meaning that the resulting cost functional is non-convex. We also consider a variant of the above form, which is given below:

\[
R_{h,2}(p_0) = \alpha \sum_{r=1}^{N} \left( \epsilon + (p_0)_r^2 \right)^{p} + (1 - \alpha) \sum_{r=1}^{N} \left( \epsilon + \sum_{i} (D_{o,i} p_0)_r^2 \right)^{p}
\]  

(14)

2.2 The complete cost functional

The initial pressure distribution in PAT is proportional to the fluence distribution and absorption coefficient in the tissue and hence the recovered PAT images should contain only non-negative values in it. Therefore, a non-negativity constraint is imposed on the solution of the optimization problem. The modified optimization problem to be solved is given as

\[
\hat{p}_0 = \arg \min_{p_0 \geq 0} J(p_0)
\]  

(15)

where

\[
J(p_0) = \|p - Hp_0\|_2^2 + \lambda R_{h,i}(p_0) \quad i = 1 \text{ or } 2.
\]  

(16)

However, as the cost is non-convex, the above constrained optimization problem is challenging to solve. To handle this problem, we modify cost by adding quadratic penalty term to enforce positivity as given below

\[
J(p_0) = \|p - Hp_0\|_2^2 + \lambda R_{h,i}(p_0) + \lambda_p \|\mathcal{P}^-(p_0)\|_2^2
\]  

(17)

where

\[
(\mathcal{P}^-(p_0))_r = \begin{cases} 
0 & \text{if } (p_0)_r \geq 0 \\
(p_0)_r & \text{if } (p_0)_r < 0.
\end{cases}
\]  

(18)

This translates the constrained optimization problem into an unconstrained problem. Although this modified formulation does not completely eliminate negative values in the reconstruction, this formulation allows a sufficient trade-off between reconstruction quality and computational complexity.
2.3 Proposed Algorithm

We adapt the preconditioned gradient search for minimizing the cost function. To this end, we first need to write the expression of the gradient. For notational convenience, we use $x$ in the place of $p_0$. The gradient expression is given by

$$\nabla J(x) = A^{(x)} x - H^T p_m,$$

where

$$A^{(x)} = H^T H + \lambda \alpha W^{(x)} + \lambda \sum_i D_{o,i}^T W^{(x)} D_{o,i} + \lambda_p N^{(x)}. \tag{20}$$

Here $W^{(x)}$ is the diagonal matrix with diagonal elements given by $\{W^{(x)}\}_{ji} = w_j(x)$ with $w_j(\cdot)$ given by

$$w_j(x) = q \left( \epsilon + \alpha(x) j^2 + \sum_i (D_{o,i} x)^2 \right)^{q-1}. \tag{21}$$

Further, $N^{(x)}$ is a diagonal matrix with diagonal elements given by $\{N^{(x)}\}_{ji} = n((x)_j)$, where $n(x) = 0.5(1 - \text{sign}(x))$. The superscript '(x)' in the diagonal matrices signify their dependence on $x$, which makes the operation $A^{(x)} x$ a non-linear operation.

The preconditioned gradient search proceeds as follows: given a current estimate of the minimum, $x^{(k)}$, we update the estimate as $x^{(k+1)} = x^{(k)} - \beta_k \bar{p}(\nabla J(x^{(k)}))$, where $\bar{p}(\cdot)$ is the preconditioning, and $\beta_k$ is the step-size. To describe how the preconditioning is done, let $g^{(k)} = \nabla J(x^{(k)})$ and $\bar{g}^{(k)} = \bar{p}(\nabla J(x^{(k)}))$. Then $\bar{g}^{(k)}$ is determined by solving the linear system of equations $A^{(k)} \bar{g}^{(k)} = g^{(k)}$, where

$$A^{(k)} = H^T(H + \lambda \alpha W^{(k)} + \lambda \sum_i D_{o,i}^T W^{(k)} D_{o,i} + \lambda_p N^{(k)}). \tag{22}$$

Here $A^{(k)}$, $W^{(k)}$, and $N^{(k)}$ are simplified notations for $A^{(x^{(k)})}$, $W^{(x^{(k)})}$ and $N^{(x^{(k)})}$, where we have used only the iteration index for the superscript. The step size $\beta_k$ is chosen such that $J(x^{(k+1)}) < J(x^{(k)})$ by means of a back-tracking procedure. Specifically, starting with $\beta_k = 1$, the required $\beta_k$ is determined by series of checks on the condition $J(x^{(k+1)}) < J(x^{(k)})$ with iterated multiplication of $\beta_k$ with a factor $p \in (0, 1)$.

Since the regularization functional is non-convex, the result of the optimization method described above, might be dependent on the parameters such as the number of iterations in the Conjugate Gradient (CG) method used for computing $g^{(k)}$ from $\bar{g}^{(k)}$, and the multiplication factor used for line-search, $p$. To alleviate this problem, we introduce an outer loop in reconstruction method, where $q$ is varied from 0.5 to 0.25 in small steps. This the main advantage of replacing the logarithm used in the original formulation $\left[34\right]$ by the fractional power $p$. The overall algorithm is given in the Algorithm Panel.

2.4 Matrix-free implementation

Finding the gradient $g^{(k)}$ of the cost function and the search direction $\bar{g}^{(k)}$ using CG involves the repeated application of $H^T H$; this is computationally very expensive and needs large amount of memory. For example, generating $H$ for image size $512 \times 512$ corresponding to 128 transducers with each transducer taking 1024 samples results in $10^8$ elements in $H$, and thus requires 256 GB RAM $\left[31\right]$. In this work, we derive a formula for matrix free implementation of the proposed algorithm by doing on-the-fly computation of multiplication with $H^T H$ without explicitly constructing $H$.

We first note that $H$ is of the form $H = [H_1^T \ H_2^T \ \cdots \ H_L^T]^T$, where $H_i$ represents the operation of obtaining samples from $L$ transducers at $i$th time instant. Let $v$ be some vector that undergo multiplication by $H$. Let $v(x)$ be image obtained by putting elements of $v$ into image form. Then the operator that is the equivalent of multiplying $v$ with $H_i$ can be written by using the forward model of the equation $\left[4\right]$ as

$$H_i v = P \mathcal{F}^{-1} \{ F(A v) \cos(c_0 |k| t_i) \}, \tag{23}$$

where $P$ is the operation that represents retrieval of samples from transducer locations $\{r_s, s = 1, \ldots, L\}$ into vector form, and $A$ represents the operation of assembling the $N \times 1$ vector into an $N_x \times N_y$ image. Then, multiplication with $H_i^T$ can be expressed as

$$H_i^T u = A^o \mathcal{F}^{-1} \{ \cos(c_0 |k| t_i) F \{ P^o u \} \}, \tag{24}$$

where $A^o$ and $P^o$ are the adjoints of $A$ and $P$ respectively. $A^o$ represents scanning a $N_x \times N_y$ image into a vector of size $N \times 1 (N = N_x N_y)$, Further, assuming that the transducer locations $\{r_s, s = 1, \ldots, L\}$ are in a subset of image
Algorithm 1 Proposed algorithm for PA image reconstruction

Input: $x^{(1)}$, $p_m$, $\alpha$, $\lambda$, $\lambda_p$, $\epsilon$, $\rho$

Output: $y^{(q)}$

Initialization: $q \leftarrow 0.5$, $y^{(q)} \leftarrow x^{(1)}$

while $q \geq 0.25$ do
  $\beta_k \leftarrow 1$, $k \leftarrow 0$, $x^{(k)} \leftarrow y^{(q)}$
  while $\frac{\left\|x^{(k+1)} - x^{(k)}\right\|_2}{\left\|x^{(k)}\right\|_2} \geq 10^{-6}$ do
    $k \leftarrow k + 1$
    $W_{jj}^{(k)} \leftarrow q \left[ \epsilon + \alpha (x^{(k)})_j^2 + \sum_i \left( D_{a,i} x^{(k)} \right)_j q^{-1} \right]$
    $N_{jj}^{(k)} \leftarrow 0.5 \left[ 1 - \text{sign}((x^{(k)})_j) \right]$
    $A^{(k)} \leftarrow H^T H + \lambda \alpha W^{(k)} + \lambda \sum_i D_{a,i}^T W^{(k)} D_{a,i} + \lambda_p N^{(k)}$
    $g^{(k)} \leftarrow A^{(k)} x^{(k)} - H^T p_m$
    $\hat{g}^{(k)} \leftarrow \text{CG}(A^{(k)}, g^{(k)})$
    $\beta_k \leftarrow \text{LS}(x^{(k)}, \hat{g}^{(k)}, \rho)$
    $x^{(k+1)} \leftarrow x^{(k)} - \beta_k \hat{g}^{(k)}$
  end
  $y^{(q)} \leftarrow x^{(k)}$
  $q \leftarrow q - (0.25/10)$
end
return $y^{(q)}$

Algorithm 2 Line Search Algorithm: LS

Input: $x^{(k)}$, $\hat{g}^{(k)}$, $\rho$

Output: $\beta_k$

Initialization: $\beta_k \leftarrow 1$

$x^{(k+1)} \leftarrow x^{(k)} - \beta_k \hat{g}^{(k)}$

Evaluate cost $J(x^{(k)})$ and $J(x^{(k+1)})$ using (17)

while $J(x^{(k+1)}) > J(x^{(k)})$ do
  $\beta_k \leftarrow \rho \beta_k$ (Backtracking Line search)
  $x^{(k+1)} \leftarrow x^{(k)} - \beta_k \hat{g}^{(k)}$
  Evaluate cost $J(x^{(k)})$ and $J(x^{(k+1)})$ using (17)
end
return $\beta_k$

grid points, $\mathcal{P}^a$ becomes the operation of embedding an $L \times 1$ vector into an zero-image of size $N_x \times N_y$. Then, the operator-equivalent of $H_i^T H_i$ can be expressed as

$$H_i^T H_i v = A^a F^{-1} \left\{ \cos(c_0 |k| t_i) F \{ \mathcal{P}^a \mathcal{P} F^{-1} \{ F(A v) \cos(c_0 |k| t_i) \} \} \right\}$$

(25)

It can be shown that $\mathcal{P}^a \mathcal{P}$ becomes equivalent to multiplication by a binary images of 1s and 0s, with 1s corresponding to the transducer locations. Let $S(r)$ be this binary image.

$$H_i^T H_i v = A^a F^{-1} \left\{ \cos(c_0 |k| t_i) F \{ S(r) F^{-1} \{ F(A v) \cos(c_0 |k| t_i) \} \} \right\}$$

(26)
This results in the following form for $y = H^T H v$

$$y = H^T H v = \sum_{i=1}^{M} H_i^T H_i v \quad (27)$$

$$= A^o \sum_{i=1}^{M} F^{-1} \left\{ \cos(c_0 |k| t_i) F \left\{ S(r) F^{-1} \{ F(A v) \cos(c_0 |k| t_i) \} \right\} \right\} \quad (28)$$

In actual implementation, we do not build matrices. We used vector matrix notation in the description given above only for notational convenience. Every vector involved in the algorithm is kept in image form. Specifically, the input to the above operator will be image form and output will also be in image form. If $v(r)$ is the $N \times N$ image equivalent of the $N^2 \times 1$ vector $v$, and $y(r)$ is the image equivalent of the vector $y$ the above operator can be represented by

$$y(r) = \sum_{i=1}^{M} F^{-1} \left\{ \cos(c_0 |k| t_i) F \left\{ S(r) F^{-1} \{ F(v(r)) \cos(c_0 |k| t_i) \} \right\} \right\} \quad (29)$$

Figure 1: a) Schematic diagram of PA data acquisition geometry with ultrasound transducers (shown by red dots) around the imaging region of size 12.8 mm x 12.8 mm. The computational imaging grid size is 51.2 mm x 51.2 mm.

3 Reconstruction results

3.1 Reconstruction from simulated data

We use a blood vessel image of size 128 x 128 with designated physical size 12.8 mm x 12.8 mm as model for generating the synthetic data. The synthetic data is generated as per the geometry given in the Figure 1, where the dotted circle, whose radius is 12 mm, denotes the trajectory of possible locations for transducers. The required image (image to be reconstructed), is defined to be on larger grid of size 512 x 512 with the equivalent physical size of 51.2 mm x 51.2 mm. This is done to accommodate for the boundary effect caused by two Fourier based convolutions represented in (29). The forward data is generated using (4) and added with Gaussian noise to form simulated data having SNR levels of 20 dB, 30 dB and 40 dB. Values for $t$ in the model of (4) is chosen as $\{ \delta_i, i = 1, \ldots, M \}$ with $1/\delta_i = 100$ MHz and $M = 1600$. The number of transducers to mimic the limited data scenario in our experiments is taken as 64 and 128. The speed of the sound in the medium is assumed to be 1.5 mm/µs and we consider the medium to be homogeneous with no dispersion or absorption of sound. The parameters $\lambda$ and $q$ are determined using the model itself, as done in most methods that focus of the design of regularization. We choose a recent FISTA-based method [25] for comparison.
In the first experiment, we generated measurements corresponding to 128 transducers with SNR of 20 dB. Figure 2 compares the reconstructed results obtained from this measurement set. It is clear that the proposed method with both forms of regularization yields better reconstruction over FISTA-based method, although the result with second form of regularization gives slightly inferior result. This is reflected by the improved SSIM score (0.7622 and 0.7548 against 0.7052). The proposed method was also able to reconstruct the peak amplitude of the initial pressure distribution (1 Pa) more accurately than FISTA algorithm as evident from the Figures 2A, 2B and 2C, 2D.

![Figure 2](image.png)

Figure 2: Comparison of reconstructions obtained from simulated data with 128 transducers and 20 dB measurement noise. (A): Reference phantom model (the maximum initial Pressure rise is assumed to be 1 Pa); (B): Reconstruction obtained by proposed method with form I regularization (SSIM score: 0.7622); (C) Reconstruction from FISTA-based method with $\lambda$ chosen for best SSIM with SSIM score of 0.7052; (D): Reconstruction obtained by proposed method with form II regularization (SSIM score: 0.7548)

Figure 3 shows the scan line based intensity profile of reconstructions in Figure 2. The intensity profiles show that our method closely follows the ground truth compared to the FISTA based method which is also validated by superior SSIM scores.

In the second experiment, we consider three simulated data sets obtained from the same numerical phantom with the same number of transducers (128). However, here we consider three noise levels viz. 20 dB, 30 dB and 40 dB. The results are compared in the Figure 4. It is clear from the figure that the proposed method outperforms the FISTA-based method. Since the first form of proposed regularization outperforms the second, we present the reconstruction result only for the first form. Note that the SSIM scores of the reconstructions from data sets with input SNRs 20 dB, 30 dB, and 40 dB are 0.7622, 0.7636, and 0.7643 respectively, which indicates that the proposed method retrieves most of the quality that can be obtained from nearly noise-free data from the noisy data. On the other hand, the improvement in reconstruction yielded by FISTA-based method with respect to increase in the input SNR, is more gradual, as indicated by the SSIM scores 0.7052, 0.7370, and 0.7427 corresponding to input SNRs 20 dB, 30 dB, and 40 dB. Further, it also
Figure 3: Scan line based intensity profiles of reconstructed images from Fig. 2.

clear that the difference in performance between the proposed and FISTA is higher for lower input SNR. The third experiment is the repetition of the second experiment with one change: we used 64 transducers instead of 128. We observed similar pattern in comparison between the proposed method and the FISTA method as shown in the Figure 5. All the SSIM scores reported above are summarized in the Table 1.

| SNR (dB) | 64 transducers | 128 transducers |
|----------|----------------|-----------------|
|          | FISTA based    | Proposed        | FISTA based    | Proposed        |
| 20       | 0.6408         | 0.7588          | 0.7052         | 0.7622          |
| 30       | 0.7236         | 0.7623          | 0.7370         | 0.7636          |
| 40       | 0.7437         | 0.7626          | 0.7427         | 0.7643          |

Table 1: SSIM scores of reconstruction with varying input noise levels and different number of transducers

3.2 Reconstruction from real measured data

We use a triangular shaped physical phantom constructed using horse hair to generate real measured data. The details of the experimental set up can be found in Fig. 2 of Ref. [43]. A Q-switched Nd:YAG laser operating at 532 nm delivered laser pulses with 5 ns width at 10 Hz repetition rate onto the sample. One uncoated Plano-concave lens (LC1715, Thorlabs) and four right-angle uncoated prisms (PS911, Thorlabs) were utilized to provide a light fluence of 9 mJ/cm² (< 20 mJ/cm²: ANSI safety limit). The hair phantom having the side-length and diameter of 10 and 0.15 mm, respectively, was attached to the pipette tips adhered on acrylic slab. For recording the PA data, a 2.25 MHz flat ultrasound transducer (Olympus-NDT, V306-SU) of 13 mm diameter active area and 70% nominal bandwidth was rotated continuously for 360 deg around the sample. A pulse amplifier (Olympus-NDT, 5072PR) first amplified and filtered the acquired PA signals and then a data acquisition (DAQ) card (GaGe, compuscope 4227) recorded the signals using a sampling frequency of 25 MHz. A sync signal from laser was used for synchronization of data acquisition with laser illumination. The reconstructed PA imaging region has a size of 12.8 mm by 12.8 mm containing 128 by 128 pixels and data from 400 transducer positions is used to do the reconstruction. Since the actual values of initial pressure rise are unknown here, we have used following figure-of-merit as used in the reference [44] to compare different methods.

\[
FOM = 20 \times \log_{10} \left( \frac{S}{n} \right)
\]  

(30)
where $S$ is peak intensity value of the reconstructed image and $n$ is the standard deviation of the intensity.

Figure 6 compares reconstructed images for horse hair phantom data. For the proposed method, since we do not have the ground truth for evaluation, we present reconstruction results obtained by using both form of regularization. Figure 6A shows the image reconstructed using the first form of regularization term and Figure 6B shows the result obtained by using second form regularization. Figure 6C shows the corresponding result obtained from FISTA based method. The proposed methods were able to give sharp images while giving a 3dB improvement in the FOM values when compared with the FISTA based result as given in figure.

4 Conclusions

A novel model based algorithm to improve the quality of reconstructed PAT images in the limited data scenario was developed. The joint sparse nature of intensity and derivatives in PAT images was exploited to design a novel regularization scheme along with the prior on the non negativity of initial pressure distribution. We proposed two forms of regularization function to capture the structures in the image and being robust to noise. The resulting non convex cost function was minimized by a novel preconditioned gradient search algorithm to yield superior quality in the reconstruction. A new formula was derived for fast computation of on-the-fly vector matrix products using filters to reduce the computational burden of the algorithm. The algorithm was compared against FISTA based method of [25] for various noise levels (20, 30 and 40 dB) and with few number of transducers (64 and 128) to mimic the limited data scenario. We considered both real and simulated data sets for our experiments and our proposed method yielded superior reconstruction quality.
Figure 5: Reconstructed images from input datasets corresponding to 64 transducers for various noise levels. (A), (B), and (C): reconstructions obtained from the proposed method with first form of regularization corresponding to SNRs 20 dB, 30 dB, and 40 dB with SSIM scores 0.7588, 0.7623, and 0.7626 respectively. (D), (E), and (F): reconstructions obtained from the FISTA method corresponding to SNRs 20 dB, 30 dB, and 40 dB with SSIM scores 0.6408, 0.7236, and 0.7437.

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Figure 6: Reconstructed images from horse hair phantom data using 400 transducers. (A): reconstruction from proposed method with first form of regularization (FOM 47.42 dB). (B): reconstruction from proposed method with second form of regularization (FOM 47.02 dB); (C): reconstruction from FISTA-based method (FOM 44.21 dB).

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