A not so brief commentary
on
cosmological entropy bounds

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Abstract

There has been, quite recently, a discussion on how holographic-inspired bounds might be used to encompass the present-day dark energy and early-universe inflation into a single paradigm. In the current treatment, we point out an inconsistency in the proposed framework and then provide a viable resolution. We also elaborate on some of the implications of this framework and further motivate the proposed holographic connection. The manuscript ends with a more speculative note on cosmic time as an emergent (holographically induced) construct.

I. BACKGROUND AND MOTIVATION

The holographic principle can reasonably be viewed as a semi-classical manifestation of quantum gravity that places a fundamental limit on informational-storage capacities [1]. To translate this notion of holography into a cosmological setting has been an ongoing concern of the theoretical community; beginning with the pioneering work of Fischler and Susskind [2] (also see, e.g., [3–7]). For the most part, the challenge has been to find a way of placing a meaningful and well-defined upper bound on the entropy (i.e., the potential for information) in various cosmological situations. Optimistically speaking, one would prefer a bound that limits the entropy on a spacelike hypersurface; in particular, the observable universe at a specified moment in time. Certainly, this issue has been addressed by many interesting strategies and proposals. Nonetheless, progress in this direction has been impeded by the ambiguous nature of both entropy and the holographic principle itself.

An especially pertinent question is what (if anything) holography can tell us about inflationary [8] fluctuations. This could be of interest at early times in the era, when inflation can be modeled as an effective (quantum) field theory within the confines of the horizon; as well as at later times, after many of the fluctuations have been classically frozen at super-horizon scales. As a for instance, it has been asked if holographic bounds can enable us to place an upper limit on the ultraviolet-cutoff scale of the effective field theory (for relevant work, see [9–13]). One might also inquire as to the viability of holographically implicating
the so-called dark energy [14]; that is, the exotic energy source that mimics a cosmological constant \(^1\) and is responsible for our current period of acceleration. In fact, it has become fashionable, as of late, to call upon holographic arguments for the purpose of rationalizing the (inexplicably) small magnitude of the dark energy (e.g., [15–19]).

Let us emphasize that early-universe inflation also represents a cosmic period of acceleration; hence, there appears to be some sort of relationship — if only a superficial one — between the inflationary potential and the dark energy. It is therefore quite natural, in the context of our discussion, to ask if this notion of a connection could be strengthened by appealing to the holographic paradigm. In this regard, let us point out a recent paper of interest by Myung [20], who discussed how a particular holographic framework may be used to encompass both of these cosmic events. To elaborate, \(^2\) this author primarily focused on a pair of holographic bounds; with these being originally proposed by E. Verlinde for a closed, radiation-dominated (Friedmann-Robertson-Walker) universe [21], and later generalized by Youm to incorporate flat and open topologies as well [22]. Guided by these antecedent treatments, the author [20] begins by distinguishing between a weakly gravitating and a strongly gravitating phase of the universe. Essentially, \(HR < 1\) corresponds to the former regime and \(HR > 1\), the latter; with \(R\) being the cosmological scale factor, \(H = \dot{R}/R\) being the Hubble parameter (assumed to be positive), and a dot representing a differentiation with respect to cosmic time. [Technically speaking, the product \(HR\) should be compared with \(\sqrt{2 - k}\), where \(k\) takes on a value of \(-1, 0\) and \(+1\) for an open, flat and closed universe respectively [22]. We will, however, consistently ignore numerical factors of the order unity throughout the paper. Some further remarks on current conventions: We have chosen to work with a four-dimensional universe, although anything said in this paper can readily be extended to higher-dimensional models. Also, all fundamental constants — except for the Planck mass, \(M_p = G^{-1/2}\) — will always be set to unity.]

After discriminating between weak and strong gravity, one may then utilize, as prescribed by [21,22], a respective pair of holographic bounds:

\[
S < S_B \sim RE \quad \text{if} \quad HR < 1 \quad (1)
\]

and

\[
S < S_H \sim \frac{VH}{G} \quad \text{if} \quad HR > 1 , \quad (2)
\]

where \(S\) and \(E\) are the universal entropy and energy respectively, and \(V \sim R^3\) is the volume. [Actually, Myung prefers to replace equation (1) with the following bound on the energy: \(E < R/G\), so that \(E\) is limited by the Schwarzschild radius of a universal-sized black hole. But consider that, in the case of “very weak” gravity or \(HR << 1\), it is commonly recognized that \(S < S_B\) is the strongest (viable) holographic constraint [23,1]; while near the

\(^1\)It is, of course, within the realm of possibility that the dark energy mimics a cosmological constant because it is a cosmological constant.

\(^2\)In the first two sections, our attention will mainly be directed at the framework used in [20]. Meanwhile, we will defer discussing the (proposed) holographic connection, itself, until Section III.
saturation point or $HR \sim 1$ (which is really the main focus of either [20] or our subsequent discussion), this energy bound and equation (1) are parametrically equivalent [21]. Given these observations, we can see no good reason for dispensing with the usual convention of expressing holographic bounds in terms of the entropy.]

Before we proceed onwards, some elaboration on these bounds would appear to be in order. First of all, $S_B$ essentially represents Bekenstein’s famed entropy bound [23], which is believed by many to be a necessary restriction on any weakly gravitating system (e.g., [24]),

Now let us consider near the saturation point, where the system should just be on the cusp of black hole formation ($E \sim R/G$). Here, it becomes evident that $S_B \to S_{BH} \sim R^2/G$ as $HR \to 1$; where $S_{BH}$ is readily identifiable as the Bekenstein–Hawking entropy [27,28] of a universal-sized black hole. It follows that $S < S_{BH}$ must be true for any weakly gravitating system, which is obviously an anticipated result. Meanwhile, $S_H$ can be interpreted as the amount of entropy that is stored in a volume of space $V$ when it has been filled-up with Hubble-sized black holes (since $V/H^{-3} \times H^{-2}/G \sim S_H$). Common-sense considerations suggest that this would likely be the largest allowable entropy in a strongly gravitating region, inasmuch as $H^{-1}$ is expected to be the maximally sized stable black hole under such conditions. In fact, the so-called Hubble bound of equation (2) has, like the Bekenstein bound, been proposed in literature [3] that predates Verlinde’s treatment.

II. CRITIQUE AND RESOLVE

To reiterate a pivotal point, the Verlinde–Youm holographic bounds were central to the discussion in [20] on inflationary perturbations, current-day dark energy and their possible connection. Although sympathetic to the general philosophy of [20], we feel there are two critical issues that still need to be addressed. Firstly, the Verlinde–Youm framework (and, hence, any resultant bound) is specific to a radiation-dominated universe. As explicitly shown in [29], for a universe with an arbitrary equation of state ($p = \omega \rho$),

the analogous bounds take on much different form when $\omega$ differs from its radiative value of $+1/3$. It should be clear that, for the the eras of particular interest (early inflation and the current acceleration), the universe cannot possibly be regarded as being radiation dominated. Secondly and much more aesthetically, it can perhaps be argued that a truly encompassing consequence of holography should be reducible to a single entropic bound. In this sense, we find the distinction between strongly and weakly gravitating regimes to be somewhat unsettling.

Actually, one can resolve the second issue (but not the first) by returning to the seminal paper by Verlinde [21]. There, it was argued that the bound $S_C < S_{BH}$ — with $S_C$ repre-

3The Bekenstein bound is not, however, without its fair share of controversy; e.g., [25,26].

4As per usual conventions, we assume perfect-fluid matter for which $p$ is the pressure, $\rho$ is the energy density, and $\omega$ is the equation-of-state parameter. (For future reference, $T$ will be the temperature.) Note that $\omega$ equals $+1/3$ for pure radiation, $0$ for dust, must be less than $-1/3$ for an accelerating universe, and saturates $-1$ for a true cosmological constant. The present-day observed value happens to be bounded by $\omega < -0.75$ [14].
senting an appropriately defined Casimir entropy — is universal in the sense of being equally applicable to regimes of strong and weak gravity. Which is to say, the Casimir entropy of the universe should never be able to exceed that of a universal-sized black hole. Let us be a bit more explicit about what is meant, in this particular context, by the Casimir entropy. To arrive at this quantity, one first identifies a Casimir energy $E_C$ with the violation of the thermodynamic Euler identity (leading to $E_C \sim k[E + pV - TS]$ [21,22]), and then relates $S_C$ to $E_C$ by way of the saturated Bekenstein bound (i.e., by fixing $S_C \equiv RE_C$ up to a neglected numerical factor). As might have been anticipated, the Casimir entropy then turns out to be a sub-extensive quantity [21], scaling rather like an area! It is, therefore, not difficult to see how the bound $S_C < S_{BH}$ could very well have universal applicability, at least for the case of a closed universe. Unfortunately, this definition does become problematic for a flat or open universe, insofar as $E_C \propto k$; meaning that $S_C$ would then be vanishing or negative (respectively). On the other hand, one only needs to follow Youm’s prescription [22] to obtain a Casimir entropy that is strictly positive for all relevant topologies. This generalization does, however, seem to be somewhat contrived.

As alluded to above, Verlinde’s bound on the Casimir entropy resolves our second point of contention but not our first. However, we can proverbially kill two birds with one stone by calling upon a closely related but yet distinct entropy bound. Here, we have in mind a bound that was originally proposed by Brustein et al [30] and can essentially be viewed as an incarnation of the (so-called) causal entropy bound [7]. The causal entropy bound can, in turn, be accurately perceived as a generalization of the Hubble bound. (More specifically, to obtain the causal bound from $S_H$, one replaces the heuristic length scale $H^{-1}$ with a covariantly and rigorously defined “causal-connection scale”.) To elaborate on the logistics, let us follow [30] and first define a sub-extensive entropy ($S_{SUB}$) in accordance with the relation

$$S = \sqrt{2S_BS_{SUB}} ;$$

which is to say, $S_{SUB} \sim S^2/ER$. Then, as documented in [30], the causal entropy bound can be used to verify the validity — under quite generic circumstances — of the following holographic bound:

$$S_{SUB} < S_{BH} \sim \frac{V}{GR} \quad \text{for any value of } HR .$$

There are a few points of interest regarding this “improved” form of sub-extensive entropy bound; namely:

(i) Comparing equation (3) with the Cardy–Verlinde formula or [21]

$$S = \sqrt{2S_BS_C - S_C^2}$$

(as applicable to a closed, radiation-dominated universe), one finds that $S_{SUB}$ and $S_C$ are in agreement up to terms of the order $S_C/S_B$. Nevertheless, $S_{SUB}$ must be viewed as the preferable choice on which to base an entropy bound, being a quantity that maintains its integrity irrespective of the equation of state, topology, etcetera.
(ii) It is also prudent to compare equation (3) with Youm’s generalization of the Cardy–Verlinde formula or [22]

\[ S = \sqrt{2S_B S_C - kS_C^2} \tag{6} \]

(as applicable to a radiation-dominated universe that is closed, open or flat, and recall that \( S_C \) is defined so as to be a strictly positive quantity). Interestingly, \( S_{SB} \) and \( S_C \) happen to agree perfectly for the \( k = 0 \) case of a flat universe. Yet, any bound that is based on \( S_C \) remains problematic, insofar as neither equation (5) nor (6) holds up when \( \omega \) differs from \( +1/3 \) [29]. That is, one cannot justifiably expect, a priori, that the Verlinde–Youm formalism will directly translate into a context of cosmological acceleration.

(iii) It can be show that, for a regime of weak gravity \( (HR < 1) \), the “improved” bound of equation (4) ensures the validity of the Bekenstein bound (1). To see that this is correct, let us begin by employing \( S_{SB} \sim S^2/ER \) \( [cf, \text{equation (3)}] \) to rewrite equation (4) as follows:

\[ S < \sqrt{VE \over G} \tag{7} \]

which is — not coincidentally — yet another incarnation of the causal entropy bound [7]. Now using the weak-gravity constraint, as well as the first Friedmann equation or \( H^2 \sim GE/V \), \(^5\) we have

\[ \sqrt{V} > R\sqrt{GE} \tag{8} \]

Combining these last two equations, we immediately obtain the Bekenstein bound (1).

(iv) Moreover, the improved bound of equation (4) guarantees the validity of the Hubble bound (2) when the system is strongly gravitating. For the purpose of verifying this outcome, let us again utilize the first Friedmann equation, along with the bound \( HR > 1 \), to obtain

\[ \sqrt{E} > R^{-1} \sqrt{V \over G} \tag{9} \]

Substituting this result into equation (7), we then find

\[ S < \frac{V}{GR} \tag{10} \]

But, since \( R^{-1} < H \) for a strongly gravitating system, the Hubble bound (2) necessarily follows.

\(^5\) Although we work with a flat universe for simplicity, the results of both (iii) and (iv) readily carry through for an open or closed universe, with \( H^2 + kR^{-2} \sim GE/V \). One can confirm this assertion by inserting all the numerical factors and then imposing the “technically correct” definition of weak (strong) gravity; that is, \( HR < \sqrt{2 - k} \ (HR > \sqrt{2 - k}) \) [22].
It is these last two points, in particular, that provide the (previously missing) justification for one to apply the holographic bounds (1,2) in an inflationary or dark-energy context. Which is to say, these equations can now be motivated by the “improved” holographic bound of equation (4), which has been argued to have universal validity [30]. Conversely, the same claim can *not* be made, in good faith, about the Verlinde–Youm framework, which cannot necessarily be extrapolated away from radiation-dominated systems. Moreover, by reducing the framework to just the single entropic bound (4), we are able to punctuate the argument [20] that temporally well-separated systems can have a close holographic connection.

### III. IMPLICATIONS AND SPECULATIONS

For the sake of closure, let us now elaborate on some of the holographic implications in the cosmological regimes of interest. We will consider, in turn, the field-theoretical (or sub-horizon) inflationary perturbations, the classically frozen (or super-horizon) inflationary perturbations, and the present-day dark energy. (For further, partially overlapping discussion, see [20].) We end with some summarizing and speculative comments.

#### A. Sub-Horizon Perturbations

Here, we would like to begin by assigning an entropy to the inflationary (perturbative) modes at sub-horizon scales or, equivalently, at early times in the era. For this purpose, one normally proceeds — in lieu of a definitive explanation for the “inflaton” — by adopting the pragmatic model of an effective (quantum) field theory. However, even within this simplified framework, it is still not entirely clear as to how the entropy should be quantified. This ambiguity is a reflection of the global picture; which is essentially that of a pure state and, consequently, one of vanishing entropy. It follows that any meaningful calculation of the local (sub-horizon) entropy will necessarily require a suitable process of coarse graining. (For recent discussion on these and related matters, see [11,12].)

Conveniently for us, Gasperini and Giovannini [33] have already accomplished the stated task; having related the entropy (per comoving mode) to the “squeezing” that is induced by the time-dependent (and, hence, particle-creating) background spacetime. When all is said and done, they were able to express this coarse-grained entropy in terms of the Hubble parameter, as well as an ultraviolet (energy) cutoff for the effective field theory. More explicitly [33],

![Image]

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6On this note, we should point out that Myung [20] did briefly discuss another Verlinde-inspired holographic bound. This being \( T > -\dot{H}/(2\pi H) \) when \( HR > 1 \) [21]. Unfortunately, we are yet unable to substantiate this bound by appealing to equation (4). Nevertheless, this lower temperature bound is actually just a restatement of one of the slow-roll conditions of inflation [31], so it almost certainly has validity in (at least) an inflationary context.

7Also, for a particularly rigorous treatment on holography and the dark energy, see [32].
\[ S_{cg} \sim \frac{\Lambda^2}{H^2}, \]  

(11)

where \( \Lambda \) represents the ultraviolet cutoff and we have, as per usual, ignored the numerical factors.

Now what about holography? As appropriate for a local observer during the early stages of inflation, we should restrict considerations to the region \( R < H^{-1} \); meaning, a regime of weak gravity. Hence, this field-theoretic (coarse-grained) entropy is required to satisfy the Bekenstein bound. So let us insert equation (11) into the bound (1) and then replace \( E \) via the Schwarzschild (weak-gravity) constraint \( E < R M_p^2 \); thus obtaining

\[ \frac{\Lambda}{H} < M_p R. \]  

(12)

It stands to reason that this bound should be closest to saturation when \( R \to H^{-1} \). To put it another way, \( H^{-1} \) — being the proper size of the horizon — can also be viewed as the implicit infrared (length) cutoff for the field theory. Hence, we can expectantly write

\[ \Lambda < M_p. \]  

(13)

Insofar as the Planck mass is the natural energy scale for quantum gravity, this restriction on \( \Lambda \) is reassuring but, all by itself, not particularly useful.

There is, however, an existent viewpoint that the ultraviolet and infrared cutoffs should not, in spite of appearances, be regarded as independent quantities. As first argued by Cohen et al [15], these cutoffs should adhere to

\[ \Lambda^4 L^3 \lesssim M_p^2 L. \]  

(14)

This relation follows, quite simply, from the notion that the maximal energy of the effective field theory (the left-hand side) is not large enough to collapse the region into a black hole. Hence, one obtains the (saturating) relation \( \Lambda^2 \sim M_p / L \); meaning that, for \( L \sim H^{-1} \),

\[ \Lambda \sim \sqrt{M_p H}. \]  

(15)

The essential point here is that this outcome can also be rationalized in the context of our current framework: Firstly, equation (13) tells us that a field-theoretic description is indeed justifiable throughout the entire inflationary phase. Hence, \( E \sim \Lambda^4 R^3 \) should provide an accurate description of the energetics, and so the Bekenstein bound (1) can just as legitimately be expressed as \( S_{cg} < \Lambda^4 R^4 \) or

\[ \frac{1}{H^2 R^4} < \Lambda^2. \]  

(16)

Applying \( H < R^{-1} \), as well as recalling equation (13), we obtain

\[ H < \Lambda < M_p. \]  

(17)

Now — inasmuch as \( H, M_p \) and \( \Lambda \) are the only available energy scales — one need only appeal to Occam’s Razor to see that equation (15) naturally follows.
Taken together, equations (13) and (15) paint the following picture: The ultraviolet-cutoff scale is effectively “born” at the Planck time (or, just as suitably, later) and then decreases, dynamically, at a rate that is synchronized with the cosmic expansion of the universe. 8 We will have more to say on this observation below.

B. Super-Horizon Perturbations

As inflation proceeds, the quasi-exponential expansion of the scale factor rapidly drives the perturbations outside of the slowly rolling 8 horizon. (Note that any given perturbative mode has a proper wavelength that increases linearly with $R$.) After exiting, these inflationary perturbations are effectively frozen; forming what may be regarded as a fluid of classical modes. Ideally, for current purposes, we would like to estimate the amount of entropy that has been carried out in this manner at late (inflationary) times. But, just like in the preceding case, a procedure of coarse graining is once again called for. One natural choice is to initially adopt the perspective of a local (sub-horizon) observer [12] and then trace over the degrees of freedom in the unobservable (exterior) region; thus leading to what is known as an entropy of entanglement [34]. One could, in principle, do the analogous calculation of tracing over the interior region and — inasmuch as the total state is pure — both subregions would necessarily be assigned the same amount of entropy. Hence, any measurement of the entanglement entropy by an interior observer should apply equally well to the frozen super-horizon perturbations. As it so happens, an explicit calculation of this nature reveals that [35]

$$S_{\text{ent}} \sim \frac{\Lambda^2}{H^2},$$

(18)

where $\Lambda$ is, once again, the ultraviolet cutoff for the interior field theory.

Since the super-horizon region dictates that $R > H^{-1}$ (i.e., strong gravity), we are now in a regime for which the Hubble entropy bound (2) is most appropriate. Let us, therefore, consider the ratio

$$\frac{S_{\text{ent}}}{S_H} \sim \frac{\Lambda^2}{M_p^2 H^3 R^3} < \frac{\Lambda^2}{M_p},$$

(19)

where the inequality follows from $RH > 1$. Hence, it is quite clear that the Hubble bound will always be satisfied; providing $\Lambda < M_p$, which is now known to be the case [cf, equation (13)]. This outcome can be viewed as further support for the notion that holography has, as of yet, no obvious utility as a means of constraining inflation [10–12].

C. Dark Energetics

Due to our current state of affairs — namely, an accelerating cosmological phase with $\omega < -0.75$ — there must be some rather exotic form of negative-pressure matter or “dark

8 It is perhaps useful to note that, for many relevant models, $H \sim 10^{-5} M_p$ at the onset of inflation; after which $H$ “rolls” slowly downwards until the inflationary era terminates [8].
“energy” that dominates the universe [14]. Let us assume, for the sake of argument, that we are not dealing with a true cosmological constant. It then stands to reason that the dark energetics can, just like the inflationary modes, be described by an effective field theory at sub-horizon (or weakly gravitating) scales. And so, by analogy with our previous discussion, there should also be an ultraviolet cutoff for this effective theory; namely \( \Lambda \sim \sqrt{M_p H_0} \).

Here, \( H_0 \) represents the current day Hubble parameter; roughly \( 10^{-60} \) in Planck units. As has been pointed out elsewhere (e.g., [15]), this line of reasoning provides a de facto solution of the “cosmological constant problem” [36]. 9 More to the point, equation (20) predicts a dark-energy density of

\[
\rho_{\Lambda} \sim \Lambda^4 \sim M_p^2 H_0^2 \sim 10^{-120} M_p^4 ,
\]

which does indeed comply with the observational evidence. Unfortunately, even if there is more than an element of truth to this deduction, it is still conspicuously lacking an underlying (dynamical) mechanism. On the other hand, M. Li has suggested that the dark energy could have a purely holographic origin and, by taking the infrared cutoff to be the future event horizon, has constructed a dynamical framework that is consistent with the current observations [18]. 10 This scheme, although enticing, does however appear rather ad hoc. Clearly, further input will be required to motivate this choice. (For a preliminary attempt, see [37].) Alternatively, it could be argued that the field-theory cutoff \( \Lambda \) is the “fundamental” dynamic entity (cf, the observation at the end of Subsection A), which then acts to fix the infrared cutoff by way of the equation (14). If this were the case, one would not want to identify, a priori, the infrared cutoff with any particular horizon; rather, the dynamics of the various horizons (Hubble, future, particle, etc.) would best be viewed as manifestations of a dynamically evolving \( \Lambda \). We will continue on with this line of reasoning at the very end of the paper.

D. Final Thoughts

Having scrutinized three distinct cosmological eras, we are now in a position to reflect upon Myung’s proposal [20] of an encompassing holographic connection. Hence, a few pertinent comments are in order: First of all, for both inflation and the current accelerating period, there appears to be a holographic energy content — by which we mean a non-conventional matter source for which the energetics can be described strictly in terms of

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9 The problem being, simply stated, why is the energy scale for the dark energy some 60 orders of magnitude smaller than the Planck mass?

10 Given this particular framework, some other logical choices for the infrared cutoff — such as the past event (or particle) horizon or the precise location of the Hubble horizon itself — turn out to be incompatible with the observed equation of state [17,18].

9
an ultraviolet and infrared cutoff. Moreover, it has been argued (both here and elsewhere; e.g., [15]) that the two cutoff scales will most likely be implicitly related. And so, this holographic picture really boils down to just a single cutoff parameter; say, \( \Lambda \). It is then the \((\text{holographically induced})\) dynamics of \( \Lambda \) that would enable the proposed connection between the two temporally distant eras. Another plausible example of a holographic-inspired connection follows from an inspection of equation (18). Here, we see that the super-horizon inflationary modes — which can be regarded as \emph{classically} frozen — also “know” about the holographic cutoffs; this, in spite of the fact that the cutoff parameters are intrinsic to a \emph{quantum} field theory which is confined to the horizon \emph{interior}. Finally, let us re-emphasize the particular significance of the current work: The underlying formalism has now been essentially reduced to just the single — and universally justifiable [30] — holographic bound of equation (4).

Let us finish the paper on a somewhat more speculative note. It is, first of all, useful to recall (and then reinterpret) our observation at the very end of Subsection A: Namely, the cutoff-energy scale appears to progressively decrease from the Planck mass — passing through a value of roughly \( 10^{-5} \) during inflation — to the present day value of \( 10^{-60} \). Such behavior, a trajectory of ever decreasing scale, is quite reminiscent of a renormalization-group flow [38].\(^1\) To fill out this analogy, we can regard \( \Lambda \) as the (holographic) \emph{c}-function, time as the scale factor, and the (perceived) evolution of the universe as the renormalization-group trajectory. (At least from the viewpoint of a local observer, for whom \( \Lambda \) takes on a definite meaning. A hypothetical global observer would likely have a much different interpretation, as could also be anticipated on the grounds of horizon complementarity [41].) This framework has the esoteric appeal of a truly holographic universe, with one of the spacetime dimensions — cosmic time — being an emergent (holographically induced) construct. Such an outcome is consistent with the notion that the fundamental quantum theory of gravity should ultimately have a background-independent meaning [42]. As for the viability of such speculations, presumably only time (and quantum gravity) will tell.

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\(^{11}\)Note that Strominger [39] and Balasubramanian \emph{et al} [40] were the first to argue, on the basis of holography, that the evolution of the universe could be interpreted as such a flow. However, their perspective, not to mention flow direction, differs from here.
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