Measurement of Branching Fractions and CP-Violating Charge Asymmetries for $B$ Meson Decays to $D^{(*)}D^{(*)}$, and Implications for the CKM Angle $\gamma$

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We present measurements of the branching fractions and charge asymmetries of $B$ decays to all $D^{(*)}\bar{D}^{(*)}$ modes. Using 232 million $B\bar{B}$ pairs recorded on the $\Upsilon(4S)$ resonance by the $\BABAR$ detector at the $e^+e^-$ asymmetric $B$ factory PEP-II at the Stanford Linear Accelerator Center, we measure the branching fractions

$$B(B^0 \rightarrow D^{*+}D^{*-}) = (8.1 \pm 0.6 \pm 1.0) \times 10^{-4},$$

$$B(B^0 \rightarrow D^{*\pm}D^{\mp}) = (5.7 \pm 0.7 \pm 0.7) \times 10^{-4},$$
where in each case the first uncertainty is statistical and the second systematic. We also determine the limits

$$\mathcal{B}(B^0 \to D^{(*)0} \bar{D}^{(*)0}) < 0.9 \times 10^{-4},$$
$$\mathcal{B}(B^0 \to D^{(*)+} \bar{D}^{(*)0}) < 2.9 \times 10^{-4},$$
$$\mathcal{B}(B^0 \to D^{(*)--} \bar{D}^{(*)0}) < 0.6 \times 10^{-4},$$

each at 90% confidence level. All decays above denote either member of a charge conjugate pair. We also determine the CP-violating charge asymmetries

$$\mathcal{A}(B^0 \to D^{(*)+} \bar{D}^{(*)-}) = 0.03 \pm 0.10 \pm 0.02,$$
$$\mathcal{A}(B^0 \to D^{(*)+} \bar{D}^{(*)0}) = -0.15 \pm 0.11 \pm 0.02,$$
$$\mathcal{A}(B^0 \to D^{(*)--} \bar{D}^{(*)0}) = -0.06 \pm 0.13 \pm 0.02,$$
$$\mathcal{A}(B^0 \to D^{(*)+} \bar{D}^{(*)-}) = 0.13 \pm 0.18 \pm 0.04,$$
$$\mathcal{A}(B^0 \to D^{(*)+} \bar{D}^{(*)0}) = -0.13 \pm 0.14 \pm 0.02.$$

Additionally, when we combine these results with information from time-dependent CP asymmetries in $B^0 \to D^{(*)+} D^{(*)-}$ decays and world-averaged branching fractions of $B$ decays to $D^{(*)+} \bar{D}^{(*)0}$ modes, we find the CKM phase $\gamma$ is favored to lie in the range $[0.07 - 2.77]$ radians (with a $+0$ or $+\pi$ radians ambiguity) at 68% confidence level.

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I. INTRODUCTION

We report on measurements of branching fractions of neutral and charged $B$-meson decays to the ten double-charm final states $D^{(*)} \bar{D}^{(*)}$. For the four charged $B$ decays to $D^{(*)} \bar{D}^{(*)}$ and for neutral $B$ decays to $D^{(*)+} D^{(*)-}$, we also measure the direct CP-violating time-integrated charge asymmetry

$$A_{CP} \equiv \frac{\Gamma_- - \Gamma_+}{\Gamma_- + \Gamma_+},$$

where in the case of the charged $B$ decays, the superscript on $\Gamma$ corresponds to the sign of the $B^{(*)}$ meson, and for $D^{(*)+} D^{(*)-}$, $\Gamma^+$ refers to $D^{(*)+} D^{(*)-}$ and $\Gamma^-$ to $D^{(*)+} D^{(*)-}$.

In the neutral $B \to D^{(*)+} D^{(*)-}$ decays, the interference of the dominant tree diagram (see Fig. 1a) with the neutral $B$ mixing diagram is sensitive to the Cabibbo-Kobayashi-Maskawa (CKM) phase $\beta \equiv \arg \left[-V_{td} V_{*_{cb}} / V_{td} V_{cb}^* \right]$, where $V$ is the CKM quark mixing matrix [1]. However, the theoretically uncertain contributions of penguin diagrams (Fig. 1b) with different weak phases are potentially significant and may shift both the observed CP asymmetries and the branching fractions by amounts that depend on the ratios of the penguin to tree contributions and their relative phases. A number of theoretical estimates exist for the resulting values of the branching fractions and CP asymmetries [2–6].

The penguin-tree interference in neutral and charged $B \to D^{(*)} \bar{D}^{(*)}$ decays can provide sensitivity to the angle $\gamma = \arg \left[-V_{td} V_{*_{cb}} / V_{td} V_{cb}^* \right]$. With additional information on the branching fractions of $B \to D^{(*)} \bar{D}^{(*)}$ decays, the weak phase may be extracted, assuming SU(3) flavor symmetry between $B \to D^{(*)} \bar{D}^{(*)}$ and $B \to D_s^{(*)} \bar{D}^{(*)}$. For this analysis, we assume that the breaking of SU(3) can be parametrized via the ratios of decay constants $f_{D^{(*)}} / f_{D^{(*)}}$, which are quantities that can be determined either with lattice QCD or from experimental measurements [9].

In addition to presenting measurements of the $B^0 \to D^{(*)+} D^{(*)-}$ and $B^+ \to D^{(*)+} \bar{D}^{(*)0}$ branching fractions, and the CP-violating charge asymmetries for the latter modes and for $B^0 \to D^{(*)+} D^{(*)-}$, we search for the color-suppressed decay modes $B^0 \to D^{(*)0} \bar{D}^{(*)0}$, which have not been previously measured, and determine limits on those branching fractions [10]. If observed, the decays $B^0 \to D^{(*)0} \bar{D}^{(*)0}$ would provide evidence of W-exchange or annihilation contributions (see Fig. 1c,d). In principle, these decays could also provide sensitivity to the CKM phase $\beta$ if sufficient data were available. By combining all of these results with information from time-dependent CP asymmetries in $B^0 \to D^{(*)+} D^{(*)-}$ decays and world-
averaged branching fractions of $B$ decays to $D_{s}^{(*)}\overline{D}^{(*)}$ modes, we determine the implications for $\gamma$ using the method of Refs. [7, 8].

II. DETECTOR AND DATA

The results presented in this paper are based on data collected with the BABAR detector [11] at the PEP-II asymmetric-energy $e^{+}e^{-}$ collider [12] located at the Stanford Linear Accelerator Center. The integrated luminosity is 210.5 fb$^{-1}$, corresponding to 231.7 million $B\overline{B}$ pairs, recorded at the $\Upsilon(4S)$ resonance (“on-peak”, at a center-of-mass (c.m.) energy $\sqrt{s} = 10.58$ GeV).

The asymmetric beam configuration in the laboratory frame provides a boost of $\beta\gamma = 0.56$ to the $\Upsilon(4S)$. Charged particles are detected and their momenta measured by the combination of a silicon vertex tracker (SVT), consisting of five layers of double-sided detectors, and a 40-layer central drift chamber (DCH), both operating in the 1.5-T magnetic field of a solenoid. For tracks with transverse momentum greater than 120 MeV/$c$, the DCH provides the primary charged track finding capability. The SVT provides complementary standalone track finding for tracks of lower momentum, allowing for reconstruction of charged tracks with transverse momentum $p_{T}$ as low as 60 MeV/$c$, with efficiencies in excess of 85%. This ability to reconstruct tracks with low $p_{T}$ efficiently is necessary for reconstruction of the slow charged pions from $D_{s}^{(*)} \rightarrow D^{(*)}\pi^{\pm}$ decays in $B \rightarrow D^{(*)}\overline{D}^{(*)}$ signal events. The transverse momentum resolution for the combined tracking system is $\sigma_{p_{T}}/p_{T} = 0.0013 p_{T} + 0.0045$, where $p_{T}$ is measured in GeV/$c$. Photons are detected and their energies measured by a CsI(Tl) electromagnetic calorimeter (EMC). The photon energy resolution is $\sigma \varepsilon / E = \left\{2.3 / (E(\text{GeV})^{1/4} \pm 1.4)\right\} \%$, and their angular resolution with respect to the interaction point is $\sigma_{\theta} = (4.2 \text{ mrad}) / \sqrt{E(\text{GeV})}$. The measured $\pi^{0}$ mass resolution for $\pi^{0}$’s with laboratory momentum in excess of 1 GeV/$c$ is approximately 6 MeV/$c^{2}$.

Charged-particle identification (PID) is provided by an internally reflecting ring-imaging Cherenkov light detector (DIRC) covering the central region, and the most probable energy loss ($dE/dx$) in the tracking devices. The Cherenkov angle resolution of the DIRC is measured to be 2.4 mrad, which provides over 5$\sigma$ separation between charged kaons and pions at momenta of less than 2 GeV/$c$. The $dE/dx$ resolution from the drift chamber is typically about 7.5$\sigma$ for pions. Additional information to identify and reject electrons and muons is provided by the EMC and detectors embedded between the steel plates of the magnetic flux return (IFR).

III. CANDIDATE RECONSTRUCTION AND $B$ MESON SELECTION

Given the high multiplicity of the final states studied, very high combinatorial background levels are expected. Selection criteria (described in Sec. III A–E) are designed to minimize the expected statistical error on the $B$ branching fractions (as described in Sec. III F). A GEANT4-based [13] Monte Carlo (MC) simulation of the material composition and the instrumentation response of the BABAR detector is used to optimize signal selection criteria and evaluate signal detection efficiency. We retain sufficient sidebands in the discriminating variables to characterize the background in subsequent fits.

A. Charged track and $K_{s}^{0}$ selection

Charged particle tracks are selected via pattern recognition algorithms using measurements from the SVT and DCH detectors. We additionally require all charged-particle tracks (except for those from $K_{s}^{0} \rightarrow \pi^{+}\pi^{-}$ decays) to originate within 10 cm along the beam axis.
candidates must have an invariant mass between 115 and $B$ when combined with other tracks or neutrals to form mass and energy resolution of the parent particles. This procedure improves the expected decay points, and their masses are constrained to originate from their decay mode, while keeping almost all of the signal decays.

D. $D$ and $D^*$ meson selection

We reconstruct $D^0$ mesons in the four decay modes $D^0 \to K^-\pi^+$, $D^0 \to K^-\pi^0$, $D^0 \to K^-\pi^-\pi^+$, and $D^0 \to K^0_S\pi^+\pi^-$, and $D^+$ mesons in the two decay modes $D^+ \to K^-\pi^+\pi^+$ and $D^+ \to K^0_S\pi^+$. We require $D^0$ and $D^+$ candidates to have reconstructed masses within $\pm 20$ MeV/$c^2$ of their nominal masses [14], except for $D^0 \to K^-\pi^+\pi^0$, for which we require $\pm 40$ MeV/$c^2$ due to the poorer resolution for modes containing $\pi^0$S. These criteria correspond to approximately 2.5$\sigma$ of the respective mass resolutions. The $D^0 \to K^-\pi^+\pi^+$ decays must also satisfy a criterion on the reconstructed invariant masses of the $K^-\pi^+$ and $K^-\pi^0$ pairs: the combination of reconstructed invariant masses must lie at a point in the $K^-\pi^+\pi^0$ Dalitz plot [16] for which the expected density normalized to the maximum density ("Dalitz weight") is at least 6%. Additionally, the daughters of $D^0$ and $D^+$ candidates must have a probability of originating from a common point in space greater than 0.1%, and are then constrained both to originate from that common spatial point and to have their respective nominal invariant masses.

Candidate $D^{*+}$ and $D^{*0}$ mesons are reconstructed in the decay modes $D^{*+} \to D^0\pi^+$, $D^{*+} \to D^+\pi^0$, $D^{*0} \to D^0\pi^+$, and $D^{*0} \to D^0\gamma$, using pairs of selected $D^0$, $D^+$, $\pi^0$, $\pi^+$, and $\gamma$ candidates. The $\pi^+$ from $D^{*+} \to D^0\pi^+$ decays is additionally required to have a c.m. momentum of less than 450 MeV/$c$. Candidate $\pi^0$ mesons from $D^{*+} \to D^+\pi^0$ and $D^{*0} \to D^0\pi^0$ are required to have c.m. momenta $p^*$ in the range $70 < p^* < 450$ MeV/$c$. Photons from $D^{*0} \to D^0\gamma$ decays are required to have energies in the laboratory frame greater than 100 MeV and c.m. energies less than 450 MeV. The $D^*$ daughter particles are constrained to originate from a common point in space. After this constraint is applied, the mass differences $\Delta m$ of the reconstructed masses of the $D^*$ and $D$ candidates are required to be within the ranges shown in Table I. As shown in Fig. 2, the excellent res-

| Mode           | Minimum $\Delta m$ (MeV/$c^2$) | Maximum $\Delta m$ (MeV/$c^2$) |
|----------------|---------------------------------|---------------------------------|
| $D^{*+} \to D^0\pi^+$ | 139.6                           | 151.3                           |
| $D^{*+} \to D^+\pi^0$   | 135.0                           | 146.3                           |
| $D^{*0} \to D^0\pi^0$   | 135.0                           | 149.3                           |
| $D^{*0} \to D^0\gamma$  | 100.0                           | 170.0                           |

TABLE I: Allowed $\Delta m(D^*-D)$ ranges for the four $D^*$ decay modes.
olution in \( \Delta m \) for signal candidates makes the \( \Delta m \) requirement a very powerful criterion to reject background (see next section), especially for decay modes containing a \( D^{*+} \to D^0 \pi^+ \).

E. Variables used for \( B \) meson selection

A \( B \)-meson candidate is constructed by combining two \( D^{(*)} \) candidates that have both passed the selection criteria described previously. The pairs of \( D^{(*)} \) candidates are constrained to originate from the same point in space.

We form a likelihood variable, \( \mathcal{L}_{\text{Mass}} \), that is defined by a product of Gaussian distributions for each \( D \) mass and \( D^* - D \) mass difference.

For example, in the decay \( B^0 \to D^{**} D^{*-} \), \( \mathcal{L}_{\text{Mass}} \) is the product of four terms: Gaussian distributions for each \( D \) mass and double Gaussian (i.e. the sum of two Gaussian distributions) terms for each \( \Delta m \) term (the \( D^* - D \) mass difference). Defining \( G(x; \mu, \sigma) \) as a normalized Gaussian distribution where \( x \) is the independent variable, \( \mu \) is the mean, and \( \sigma \) is the resolution, \( \mathcal{L}_{\text{Mass}} \) for \( B^0 \to D^{**} D^{*-} \) decays is defined as:

\[
\mathcal{L}_{\text{Mass}} = G(m_D; m_{PDG}, \sigma_{m_D}) \times G(m_{\bar{D}}; m_{PDG}, \sigma_{m_{\bar{D}}}) \times \\
\left[ f_{\text{core}}G(\Delta m_{D^{**}}; \Delta m_{PDG}, \sigma_{\Delta m_{\text{core}}}) + (1 - f_{\text{core}}) \frac{1}{E} G(\Delta m_{D^*}; \Delta m_{PDG}, \sigma_{\Delta m_{\text{tail}}}) \right] \times \\
\left[ f_{\text{core}}G(\Delta m_{D^{*-}}; \Delta m_{PDG}, \sigma_{\Delta m_{\text{core}}}) + (1 - f_{\text{core}}) \frac{1}{E} G(\Delta m_{D^{**}}; \Delta m_{PDG}, \sigma_{\Delta m_{\text{tail}}}) \right],
\]

(2)

where the subscript “PDG” refers to the nominal value [14], and all reconstructed masses and uncertainties are determined before mass constraints are applied. For \( \sigma_{m_D} \), we use errors calculated candidate-by-candidate. The parameter \( f_{\text{core}} \) is the ratio of the area of the core Gaussian to the total area of the double Gaussian distribution. This, along with \( \sigma_{\Delta m_{\text{core}}} \) and \( \sigma_{\Delta m_{\text{tail}}} \), is determined separately for each of the four \( D^* \) decay modes given above, using MC simulation of signal events that is calibrated to inclusive samples of the \( D^* \) decay modes in data. For each of the \( B \) decay modes, a higher value of \( \mathcal{L}_{\text{Mass}} \) tends to indicate a greater signal likelihood. The distributions of \( -\ln(\mathcal{L}_{\text{Mass}}) \) for the representative signal mode \( B^0 \to D^0 \bar{D}^0 \) and for the corresponding combinatorial background from generic \( B^0 \bar{B}^0 \), \( B^+ B^- \), \( \psi \), and \( (u \bar{s} + d \bar{d} + s\bar{s}) \) decays, are shown in Fig. 3a. We use \( \mathcal{L}_{\text{Mass}} \) in selecting signal candidates, as will be described in the upcoming section.

We also use the two variables for fully-reconstructed \( B \) meson selection at the \( \Upsilon(4S) \) energy: the beam-energy-substituted mass \( m_{ES} \equiv [(s/2 + \vec{p}_B \cdot \vec{p}_{\bar{B}})^2/E^2 - \vec{p}_B^2]^{1/2} \), where the initial total \( e^+ e^- \) four-momentum \( (E, \vec{p}_e) \) and the \( B \) momentum \( \vec{p}_B \) are defined in the laboratory frame; and \( \Delta E \equiv E_{\text{cm}} - \sqrt{s}/2 \) is the difference between the reconstructed \( B \) energy in the c.m. frame and its known value. The normalized distribution of \( \Delta E \) for the representative signal mode \( B^0 \to D^0 \bar{D}^0 \), and for the corresponding combinatorial background components, is shown in Fig. 3b.

In addition to \( \mathcal{L}_{\text{Mass}}, m_{ES}, \) and \( \Delta E, \) a Fisher discriminant \( \mathcal{F} \) [17] and a \( D \)-meson flight length variable \( L \) are used to help separate signal from background. The
Fisher discriminant assists in the suppression of background from continuum events by incorporating information from the topology of the event. The discriminant is formed from the momentum flow into nine polar angular intervals of 10° centered on the thrust axis of the $B$ candidate, the angle of the event thrust axis with respect to the beam axis ($\theta_T$), and the angle of the $B$ candidate

\begin{table}[h]
\centering
\begin{tabular}{cccc}
\hline
Mode & Expected $B$ & $F_{\text{min}}$ & $L_{\text{min}}$ \\
\hline
$B^0 \rightarrow D^{(*)} D^{(*)}$ & 1.0 \times 10^{-5} & 0.47 & -0.4 \\
$B^+ \rightarrow D^{(*)} D^{(*)}$ & 4.4 \times 10^{-4} & 0.53 & -1.3 \\
$B^0 \rightarrow D^{(*)} D^{(*)}$ & 3.0 \times 10^{-4} & 0.53 & 0.5 \\
\hline
\end{tabular}
\end{table}
momentum with respect to the beam axis \((\theta_B)\):
\[
F \equiv \sum_{i=1}^{11} \alpha_i x_i. 
\] (3)

The values \(x_i (i = 1, \ldots, 9)\) are the scalar sums of the momenta of all charged tracks and neutral showers in the polar angle interval \(i\), \(x_{10} = |\cos \theta_T|\), and \(x_{11} = |\cos \theta_B|\). The coefficients \(\alpha_i\) are determined from MC simulation to maximize the separation between signal and background [17]. The normalized distribution of \(F\) for the representative signal mode \(B^0 \rightarrow D^0 \bar{D}\), and for the corresponding background components, is shown in Fig. 3c.

The flight length variable \(L\) that we consider is defined as \((\ell_1 + \ell_2)/\sqrt{\sigma_x^2 + \sigma_y^2}\), with the decay lengths \(\ell_i\) of the two \(D\) mesons defined as
\[
\vec{x}_{D_i} = \vec{x}_B + (\ell_i \times \vec{p}_{D_i})
\] (4)
where \(\vec{x}_D\) and \(\vec{x}_B\) are the measured decay vertices of the \(D\) and \(B\), respectively, and \(\vec{p}_D\) is the momentum of a \(D\). The \(\sigma_i\) are the measured uncertainties on \(\ell_i\). This observable exploits the ability to distinguish the long \(D\) lifetime. Thus, background events have an \(L\) distribution centered around zero, while events with real \(D\) mesons have a distribution favoring positive values. The normalized distribution of \(L\) for the representative signal mode \(B^0 \rightarrow D^0 \bar{D}\), and for the corresponding background components, is shown in Fig. 3d.

F. Analysis optimization and signal selection

We combine information from the \(L_{\text{Mass}}, \Delta E, F,\) and \(L\) variables to select signal candidates in each decay mode. The fractional statistical uncertainty on a measured branching fraction is proportional to \(\sqrt{N^s + N^b}/N^s\), where \(N^s\) is the number of reconstructed signal events and \(N^b\) is the number of background events within the selected signal region for a mode. The values \(N^s\) and \(N^b\) are calculated, using detailed MC simulation of the signal decay modes as well as of \(B\bar{B}\) and continuum background decays, by observing the number of simulated \(B\) decay candidates that satisfy the selection criteria for \(-\ln(L_{\text{Mass}}), |\Delta E|, F,\) and \(L\). We choose criteria which minimize the expected \(\sqrt{N^s + N^b}/N^s\) for each mode. Note that to calculate the expected number of signal events \(N^s\), one must assume an expected branching fraction, as well as the ratios of \(B\bar{B}\) and continuum events using their relative cross-sections. These are given, along with the requirements on \(F\) and \(L\), in Table II.

For each possible combination of \(D_s^+, D_s^0, D^+,\) and \(D^0\) decay modes, we determine the combination of selection criteria on \(-\ln(L_{\text{Mass}})\) and \(|\Delta E|\) that minimizes the overall expected \(\sqrt{N^s + N^b}/N^s\) for each \(B\) decay mode (see Tables III, IV, and V). The selection criteria for \(F\) and \(L\) are chosen, however, only for each \(B\) decay mode and not separately for each \((s)\) mode combination. The restrictiveness of the kaon identification selection is also optimized separately for each charged and neutral \(D^{(*)}\) mode.

Between 1% and 34% of selected \(B \rightarrow D^{(*)} \bar{D}\) events have more than one reconstructed \(B\) candidate that passes all selection criteria in \(L_{\text{Mass}}, \Delta E, F,\) and \(L\), with the largest percentages occurring in the decay modes \(B^0 \rightarrow D^0 \bar{D}^0\) and \(B^0 \rightarrow D^0 \bar{D}^0\), and the smallest occurring in \(B^0 \rightarrow D^{(*)} D^{(*)}\) and \(B^0 \rightarrow D^{(*)} D^{(*)}\). In such events, we choose the reconstructed \(B\) with the largest value of \(L_{\text{Mass}}\) as the signal \(B\) candidate.

IV. EFFICIENCY AND CROSSFEED DETERMINATION

The efficiencies are determined using fits to \(m_{\text{ES}}\) distributions of signal MC events that pass all selection criteria in \(L_{\text{Mass}}, \Delta E, F,\) and \(L\). There is a small, but non-negligible probability that a signal \(B\) decay of mode \(i\) is reconstructed as a different signal decay mode \(j\). We refer to this as crossfeed. Thus, efficiencies can be represented as a matrix \(\epsilon_{ij}\), where each contributing generated event is weighted by the \(D\) and \(\bar{D}\) decay mode branching fractions. To determine the elements of \(\epsilon_{ij}\), we fit the \(m_{\text{ES}}\) distributions of signal MC events generated as \(B\) decay mode \(i\) and reconstructed as \(B\) decay mode \(j\). The distributions are modeled as the sum of signal and background probability distribution functions (PDFs), where the PDF for the signal is a Gaussian distribution centered around the \(B\) mass, and the PDF for background is an empirical function [18] of the form
\[
f(x) \propto x \sqrt{1 - x^2} \exp[-\kappa(1 - x^2)],
\] (5)
where we define \(x \equiv 2m_{\text{ES}}/\sqrt{s}\), and \(\kappa\) is a parameter determined by the fit. In \(B\bar{B}\) MC samples containing signal and background decays, we find that the \(m_{\text{ES}}\) distribution is well-described by adding a simple Gaussian function to the empirical shape in Eq. 5. We fit the \(m_{\text{ES}}\) distributions of signal MC events generated as \(B\) decay mode \(i\) and passing selection criteria in mode \(j\) to the above distribution by minimizing the \(\chi^2\) of each fit with respect to \(\epsilon_{ij}\) (the \(\kappa\) parameter for each mode \((i,j)\)), the number of signal events \(N^s_{ij}\), and the number of background events \(N^b_{ij}\). We determine the efficiencies \(\epsilon_{ij}\) as \(N^s_{ij}/N^b_{ij}\), where \(N^b_{ij}\) is the total number of signal MC events that were generated in mode \(i\). The diagonal elements of the \(\epsilon_{ij}\) matrix (i.e. the numbers typically denoted as “efficiencies”) are in the range \((0.2 - 1.5) \times 10^{-3}\). The main crossfeed source is misidentification between \(D^0\) and \(D^{(*)}\) candidates. The matrix \(\epsilon_{ij}\) and the uncertainties on the elements of this matrix are given in Table VI. Crossfeed between different \(D\) submodes (i.e. mode numbers 12–15 in Table III) is negligible.
### TABLE III: Key to mode numbers used in Tables IV and V below.

| Mode | # |
|------|---|
| $D^+ \rightarrow (K^- \pi^+)\pi^+$ | 1 |
| $D^+ \rightarrow (K^- \pi^0)\pi^+$ | 2 |
| $D^+ \rightarrow (K^- \pi^0 \pi^+ \pi^-)\pi^+$ | 3 |
| $D^+ \rightarrow (K_0^\mp \pi^-)\pi^+$ | 4 |
| $D^+ \rightarrow (K^- \pi^+ \pi^-)\pi^+ \pi^- | 5 |
| $D^+ \rightarrow (K^- \pi^0)\pi^0 | 6 |
| $D^+ \rightarrow (K^- \pi^0)\pi^0 | 7 |
| $D^+ \rightarrow (K^- \pi^0 \pi^-)\pi^0 | 8 |

Table IV: Optimized $- \ln(L_{\text{Mass}})$ selection criteria used for all $B \rightarrow D^{(*)} \overline{D}^{(*)}$ modes. Selected events in a given mode must have $- \ln(L_{\text{Mass}})$ less than the given value. The $D^{(*)}$ decay modes 1 – 15 are defined in Table III above. Elements with "—" above and on the diagonal are modes that are unused since, due to high backgrounds, they do not help to increase signal sensitivity.

| Mode | D$^+$ | D$^{*0}$ | D$^-$ | D$^0$ |
|------|------|--------|------|------|
| 1 | 13.0 | 12.0 | 17.3 | 19.8 | 10.5 | 14.6 | 17.5 | 9.2 | 8.2 | 8.4 | 7.8 | 8.8 |
| 2 | 10.6 | 11.0 | 18.3 | 9.5 | 11.5 | 9.8 | 10.7 | 8.7 | 8.4 | 7.8 | 8.8 | 8.8 |
| 3 | 11.7 | 11.0 | 9.8 | 11.7 | 9.6 | 10.4 | 9.0 | 8.8 | 9.3 | 9.4 | 9.0 | — |
| 4 | — | — | — | — | — | — | — | 9.6 | 15.1 | 9.2 | — | — |
| 5 | — | — | 8.2 | — | — | — | — | 6.6 | — | — | — | — |
| 6 | — | — | — | — | 7.6 | 9.9 | 7.6 | 6.7 | 7.2 | — | — | — |
| 7 | — | — | — | — | — | 7.5 | — | — | — | — | — | — |
| 8 | — | — | — | — | — | — | 9.2 | — | — | — | — | — |
| 9 | — | — | — | — | 5.8 | — | — | — | — | — | — | — |
| 10 | — | — | — | — | — | — | — | — | — | — | — | — |
| 11 | — | — | 6.0 | 7.3 | 5.8 | 6.5 | 6.2 | — | — | — | — | — |
| 12 | — | — | — | — | 5.2 | 6.8 | — | — | — | — | — | — |
| 13 | — | — | — | — | — | 6.2 | — | — | — | — | — | — |
| 14 | — | — | — | — | — | 6.9 | — | — | — | — | — | — |

Table V: Optimized $\Delta E$ selection criteria used for all $B \rightarrow D^{(*)} \overline{D}^{(*)}$ modes. Selected events in a given mode must have $|\Delta E|$ (in MeV) less than the given value. The $D^{(*)}$ decay modes 1 – 15 are defined in Table III above. Elements with "—" above and on the diagonal are modes that are unused since, due to high backgrounds, they do not help to increase signal sensitivity.

| Mode | D$^+$ | D$^{*0}$ | D$^-$ | D$^0$ |
|------|------|--------|------|------|
| 1 | 35.5 | 33.8 | 30.4 | 35.2 | 25.5 | 35.7 | 21.0 | 26.0 | — | 43.6 | 18.0 | 18.1 | 20.2 | 17.1 | 19.9 |
| 2 | 34.5 | 29.6 | 23.5 | 27.4 | 49.9 | 23.9 | 21.4 | — | 29.3 | 19.4 | 25.9 | — | 19.5 | — | — |
| 3 | 23.5 | 23.7 | 18.2 | 34.0 | 30.6 | 20.6 | 27.3 | 18.6 | 19.0 | 20.4 | 17.1 | — | — | — | — |
| 4 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 5 | — | — | — | — | — | — | 19.1 | 16.9 | 19.7 | — | — | — | — | — | — |
| 6 | — | — | — | — | — | — | 19.1 | 16.9 | 19.7 | — | — | — | — | — | — |
| 7 | 35.1 | 23.0 | 27.3 | 25.5 | — | — | — | — | — | 20.0 | — | — | — | — | — |
| 8 | — | — | — | — | — | — | — | 24.5 | — | — | — | — | — | — | — |
| 9 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 10 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 11 | — | — | 15.1 | 15.5 | 19.2 | 15.4 | 15.5 | — | — | 18.7 | 16.1 | — | — | 19.0 | — | 15.9 |
| 12 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 13 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 14 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
| 15 | — | — | — | — | — | — | — | — | — | — | — | — | — | — | — |
TABLE VI: Elements of the efficiency and crossfeed matrix $\epsilon_{ij}$, and their respective uncertainties, used to calculate the branching fractions and charge asymmetries, as described in the text. All values are in units of $10^{-4}$. Uncertainties on the last digit(s) are given in parentheses. Elements with “—” correspond to values that are zero (to three digits after the decimal point). The column corresponds to the generated mode and the row corresponds to the reconstructed mode.

| Mode       | $D^+D^{*-}$ | $D^+\bar{D}^*$ | $D^0D^*$ | $D^{**}D^*$ | $D^+D^{-}$ | $D^0\bar{D}^0$ | $D^+D^{-0}$ | $D^+\bar{D}^{0*}$ | $D^+\bar{D}^0$ | $D^+\bar{D}^{0*}$ |
|------------|-------------|----------------|----------|-------------|------------|----------------|------------|----------------|-------------|----------------|
| $D^+D^{*-}$| 14.24(6)    | 0.010(3)       | —         | —           | —          | 0.18(1)       | —          | —              | —           | —              |
| $D^+\bar{D}^*$| 0.020(3)   | 11.52(6)       | —         | —           | —          | 0.010(3)      | 0.040(3)   | 0.08(1)       | —           | —              |
| $D^0D^*$   | —           | —              | 9.51(8)   | —           | —          | —              | —          | —              | —           | —              |
| $D^{**}D^*$| 0.080(3)    | —              | 2.60(2)   | 0.030(3)    | —          | 0.42(1)       | 0.010(3)   | —              | —           | —              |
| $D^+D^{-}$ | —           | —              | 0.020(3)  | 3.40(2)     | —          | 0.010(3)      | 0.46(1)    | 0.010(3)      | —           | —              |
| $D^0\bar{D}^0$| —         | —              | —         | 0.010(3)    | 12.02(10)  | 0.010(3)      | 0.020(3)   | —              | —           | —              |
| $D^+D^{-0}$| 2.60(2)     | —              | 0.23(1)   | 0.010(3)    | —          | 7.52(4)       | 0.07(1)    | —              | —           | —              |
| $D^+\bar{D}^{0*}$| 0.040(3) | 0.06(2)       | —         | 0.11(5)     | —          | 0.03(2)       | 13.51(25)  | 0.040(3)      | —           | —              |
| $D^+\bar{D}^0$  | —          | 0.41(1)       | —         | 0.010(3)    | 0.010(3)   | —              | —          | 0.070(3)      | 3.70(3)    | —              |
| $D^+\bar{D}^{0*}$| —         | 0.020(3)      | 0.06(1)   | —           | —          | 0.050(3)      | —          | 0.010(3)      | 0.020(3)   | 14.93(9)     |

V. BRANCHING FRACTION RESULTS

In order to determine the number of signal events in each mode, one must not only account for background which is distributed according to combinatorial phase space, but also for background which can have a different distribution in $m_{ES}$. It is possible for a component of the background to have an $m_{ES}$ distribution with a PDF that is more similar to signal (i.e., a Gaussian distribution centered around the $B$ mass) than to a phase-space distribution. Such a component is known as “peaking” background and typically derives from background events that have the same or similar final state particles as the signal decay mode. For example, in $B^0 \to D^+D^-$, peaking background primarily comes from the decays $B^0 \to DKK$ or $B^0 \to D\pi X$, where $D \to K\pi\pi$ and $X$ is $K^0$, $\rho$, $a_1$ or $\omega$, and the light mesons ($KX$ or $\pi X$) form a $D \to K\pi\pi$ decay. The optimization procedure that was detailed in Sec. III F eliminates decay submodes that have a large enough amount of peaking (in addition to combinatorial) background to decrease, rather than increase, the sensitivity for a particular decay; the final selection was detailed in Tables II, IV, and V. We determine the amount of peaking background $P_i$ in each $B$ decay mode $i$ via fitting the $m_{ES}$ distributions of $B\bar{B}$ MC simulated events. We minimize the $\chi^2$ of each fit, allowing the variables $\kappa_i^p$ (representing the “ARGUS parameter” described earlier), the number of expected peaking background events in data $P_i$, and the number of phase-space background events $N_i^{MCbkg}$, to float. The fitted number of peaking background events $P_i$ is compatible with zero, within two standard deviations, for all modes $i$.

We then fit the actual data to determine the number of reconstructed signal events in each mode. We fit the $m_{ES}$ distributions of reconstructed $B$ decays that pass all selection criteria in each mode $i$ to a sum of a Gaussian distribution and a phase space distribution (Eq. 5), similar to the PDFs used for efficiency and peaking background fits described above. We minimize the $\chi^2$ of each data fit, allowing the parameter $\kappa_i$, the number of signal events in data $N_i^{sig}$, and the number of background events in data $N_i^{bkg}$, each to float. The $m_{ES}$ distributions and the results of the fits are shown in Fig. 4. The branching fractions $B_i$ are then determined via the equation

$$\sum_j \epsilon_{ij} B_j N_B = N_i^{sig} - P_i$$

(6)

where $N_B = N_B^{mc} = (231.7 \pm 2.6) \times 10^6$ is the total number of charged or neutral $B$ decays in the data sample, assuming equal production rates of charged and neutral $B$ pairs.

We determine the branching fractions as

$$B_i = \sum_j \epsilon_{ij}^{-1} (N_i^{sig} - P_j)/N_B,$$

(7)

where $\epsilon_{ij}^{-1}$ is the inverse of matrix $\epsilon_{ij}$) yields the branching fractions given in Table VII. Note that the measured branching fractions for the three modes $B^0 \to D^{(*)0}\bar{D}^{(*)0}$ are not significantly greater than zero. Thus, we have determined upper limits on the branching fractions for these modes. The 90% confidence level (C.L.) upper limits quoted in Table VII are determined using the Feldman-Cousins method [19] and include all systematic uncertainties detailed below. Since the branching fractions can be correlated through the use of Eq. 6, we also provide the covariance matrix, with all systematic uncertainties included, in Table VIII. The covariance matrix is obtained via the approximation given in [20].

VI. BRANCHING FRACTION SYSTEMATIC UNCERTAINTIES

Table IX shows the results of our evaluation of the systematic uncertainties on the branching fraction mea-
FIG. 4: Distributions of $m_{ES}$ for selected candidates in each $D^{(*)} \overline{D}^{(*)}$ mode. The error bars represent the statistical errors only. The solid lines represent the fits to the data, and the shaded areas the fitted background.
TABLE VII: Results of the fits for the ten signal decay modes: the number of events for fitted signal $N^{\text{sig}}$, the peaking background $P$, and the crossfeed $C$, the branching fractions $B$, 90% C.L. upper limits on branching fractions, previous measurements of branching fractions (for modes that have previous measurements), and charge asymmetries. The uncertainties are statistical. For the final branching fraction and charge asymmetry results, the systematic errors are also given.

| Mode       | $N^{\text{sig}}$ | $P$  | $C$  | $B$ (10$^{-4}$) | U.L. | Previous $B$ results (10$^{-4}$) | $A_{CP}$ |
|------------|-------------------|------|------|----------------|------|---------------------------------|----------|
| $B^0 \to D^+ D^-$   | 270±19            | -1±2 | 4±1  | 8.1±0.6±1.0    | 8.1±0.8±1.1 [21] | 8.3±1.6±1.2 [22] | 9.9±1.2±1.2 [23] |
| $B^0 \to D^\pm D^\mp$ | 156±17            | 1±3  | 2±1  | 5.7±0.7±0.7    | 8.8±1.0±1.3 [24] | 11.7±2.6±2.2 [25] | 6.7±2.0±1.1 [26] |
| $B^0 \to D^+ D^-$   | 63±9              | 1±2  | 0±0  | 2.8±0.4±0.5    | 1.91±0.5±0.30 [28] | < 270 [29] |
| $B^0 \to D^*0 \bar{D}^{*0}$ | 0±6              | -2±2 | 0±0  | 1.3±1.1±0.4 | 0.9 | < 270 [29] |
| $B^0 \to D^*0 \bar{D}^{*0}$ | 10±8              | -2±3 | 1±1  | 1.0±1.1±0.4 | 2.9 | < 270 [29] |
| $B^0 \to D^*0 \bar{D}^{*0}$ | -11±12            | -8±4 | 0±0  | -0.1±0.5±0.2 | 0.6 | < 270 [29] |
| $B^+ \to D^+ \bar{D}^{*0}$ | 185±20            | -5±4 | 34±4 | 8.1±1.2±1.2 | 10.5±2.8±2.0 [26] | < 110 [29] | -0.15±0.11±0.02 |
| $B^+ \to D^+ \bar{D}^{*0}$ | 115±16            | 1±4  | 3±1  | 3.6±0.5±0.4 | 4.57±0.7±0.56 [28] | < 130 [29,30] | -0.06±0.13±0.02 |
| $B^+ \to D^+ \bar{D}^{*0}$ | 63±11             | 3±3  | 9±2  | 6.3±1.4±1.0 | 4.57±0.7±0.56 [28] | < 130 [29,30] | 0.13±0.18±0.04 |
| $B^+ \to D^+ \bar{D}^{*0}$ | 129±20            | -2±5 | 1±1  | 3.8±0.6±0.5 | 4.83±0.78±0.58 [28] | < 67 [29] | -0.13±0.14±0.02 |

TABLE VIII: Covariances of $B \to D^{(*)} \bar{D}^{(*)}$ branching fractions (with all systematic uncertainties included), in units of 10$^{-8}$.

| Mode       | $D^{+}D^{+}$ | $D^{+}D^{+}$ | $D^{+}D^{-}$ | $D^{+}D^{0}$ | $D^{+}D^{0}$ | $D^{+}D^{0}$ | $D^{+}D^{0}$ | $D^{+}D^{0}$ | $D^{+}D^{0}$ |
|------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| $D^{+}D^{+}$ | 1.26         | 0.55         | 0.22         | -0.15        | -0.01        | 0.07         | -0.01        | 0.73         | 0.33         | 0.54         | 0.30         |
| $D^{+}D^{+}$ | 0.91         | 0.26         | -0.08        | 0.04         | 0.04         | 0.46         | 0.19         | 0.37         | 0.26         |              |              |
| $D^{+}D^{+}$ | 0.39         | -0.03        | 0.02         | 0.00         | 0.16         | 0.08         | 0.26         | 0.16         |              |              |              |
| $D^{+}D^{+}$ | 1.27         | -0.04        | 0.00         | -0.53        | -0.06        | -0.13        | -0.05        |              |              |              |              |
| $D^{+}D^{+}$ | 1.25         | 0.00         | 0.07         | -0.02        | 0.05         | 0.02         |              |              |              |              |              |
| $D^{+}D^{+}$ | 0.22         | -0.01        | 0.00         | -0.01        | 0.00         |              |              |              |              |              |              |
| $D^{+}D^{+}$ | 2.60         | 0.31         | 0.55         | 0.27         |              |              |              |              |              |              |              |
| $D^{+}D^{+}$ | 0.43         | 0.19         | 0.11         |              |              |              |              |              |              |              |              |
| $D^{+}D^{+}$ | 2.61         | 0.27         |              |              |              |              |              |              |              |              |              |

a. **Submode branching fractions** The central values and uncertainties on the branching fractions of the $D$ and $D^*$ mesons are propagated into the calculation of the branching fraction measurements. The world average measurements [14] are used.

b. **Charged track finding efficiency** From studies of absolute tracking efficiency, we assign a systematic uncertainty of 0.8% per charged track on the efficiency of finding tracks other than slow pions from charged $D^*$ decays and daughters of $K_s^0$ decays. For the slow pions, we assign a systematic uncertainty of 2.2% each, as determined from a separate efficiency study (using extrapolation of slow tracks found in the SVT into the DCH tracking detector and vice-versa). Track finding efficiency uncertainties are treated as 100% correlated among the tracks in a candidate. These uncertainties are weighted by the $D$ and $D^*$ branching fractions.

c. **$K_s^0$ reconstruction efficiency** From a study of the $K_s^0$ reconstruction efficiency (using an inclusive data sample of events containing one or more $K_s^0$, as well as corresponding MC samples), we assign a 2.5% systematic
TABLE IX: Estimates of branching fraction systematic uncertainties (as percentages of the absolute values of the branching fraction central values) for all $B$ modes, after propagating the errors through Eq. 6. The totals are the sums in quadrature of the uncertainties in each column. Note that the term “Dalitz weight” refers to the selection on the reconstructed invariant masses of the $K^-\pi^+$ and $K^-\pi^0$ pairs for $D^0 \to K^-\pi^+\pi^0$ decays that was described in Sec. III D.

| Mode       | $D^{++}D^{*-}$ | $D^{++}D^{*-}$ | $D^{+}D^{-}$ | $D^{00}D^{00}$ | $D^{00}D^{00}$ | $D^{0}D^{0}$ | $D^{0}D^{0}$ | $D^{+}D^{0}$ | $D^{+0}D^{00}$ | $D^{+0}D^{00}$ | $D^{+0}D^{00}$ | $D^{+}D^{0}$ |
|------------|----------------|----------------|--------------|----------------|----------------|--------------|--------------|--------------|----------------|----------------|----------------|--------------|
| $D^{+0}$ BFs | 1.4            | 0.7            | 0.0          | 0.9            | 0.1            | 0.0          | 0.7          | 0.7          | 0.0            | 0.0            | 0.0            | 0.0          |
| $D^{0}$ BFs     | 0.0            | 0.0            | 0.0          | 4.9            | 1.6            | 0.0          | 2.1          | 0.0          | 4.4            | 0.0            | 4.4            | 0.0          |
| $D^{0}$ BFs     | 5.0            | 2.7            | 0.0          | 7.4            | 3.7            | 5.7          | 5.2          | 4.5          | 3.3            | 2.7            | 3.3            | 2.7          |
| $D^{+}$ BFs     | 1.4            | 6.5            | 13.2         | 0.1            | 0.2            | 0.4          | 0.1          | 0.3          | 6.5            | 6.5            | 6.5            | 6.5          |
| Tracking efficiency | 7.9            | 6.5            | 4.8          | 7.9            | 0.0            | 0.9          | 0.9          | 0.9          | 3.8            | 4.4            | 4.4            | 4.4          |
| $K_2^0$ efficiency | 0.3            | 0.2            | 0.0          | 0.0            | 0.1            | 0.0          | 0.1          | 0.2          | 0.3            | 0.2            | 0.2            | 0.2          |
| Neutrals efficiency | 2.5            | 1.0            | 0.0          | 8.4            | 2.9            | 1.9          | 4.6          | 1.6          | 4.3            | 1.0            | 4.3            | 1.0          |
| Kaon identification | 4.6            | 4.7            | 5.0          | 7.3            | 4.9            | 5.4          | 5.0          | 4.6          | 4.6            | 4.7            | 4.7            | 4.7          |
| $\mathcal{L}_{\text{mass}}$ cut | 1.1            | 1.1            | 1.1          | 1.1            | 1.1            | 1.1          | 1.1          | 1.1          | 1.1            | 1.1            | 1.1            | 1.1          |
| $\mathcal{F}$ cut | 0.0            | 0.0            | 0.9          | 0.9            | 0.9            | 0.9          | 0.9          | 0.9          | 0.9            | 0.9            | 0.9            | 0.9          |
| $L$ cut         | 0.0            | 0.0            | 0.8          | 0.8            | 0.8            | 0.8          | 0.8          | 0.8          | 0.8            | 0.8            | 0.8            | 0.8          |
| $\Delta E$ cut | 1.1            | 1.1            | 1.1          | 1.1            | 1.1            | 1.1          | 1.1          | 1.1          | 1.1            | 1.1            | 1.1            | 1.1          |
| Dalitz weight cut | 1.0            | 0.5            | 0.0          | 1.4            | 0.2            | 1.0          | 1.0          | 0.8          | 0.7            | 0.5            | 0.7            | 0.5          |
| $P(\chi^2)$ cut | 3.8            | 3.8            | 3.8          | 3.8            | 3.8            | 3.8          | 3.8          | 3.8          | 3.8            | 3.8            | 3.8            | 3.8          |
| Fit model       | 1.8            | 3.6            | 3.1          | 5.4            | 6.7            | 44.6         | 4.9          | 2.8          | 7.0            | 3.6            | 3.6            | 3.6          |
| Spin alignment  | 1.0            | 0.0            | 0.0          | 6.1            | 0.0            | 0.1          | 4.1          | 0.0          | 0.0            | 0.0            | 0.0            | 0.0          |
| Peaking background | 0.9            | 2.0            | 2.9          | 24.5           | 32.3           | 144.6        | 3.1          | 3.4          | 4.9            | 4.0            | 4.0            | 4.0          |
| Crossfeed       | 0.4            | 0.6            | 0.8          | 1.9            | 1.1            | 1.6          | 0.6          | 0.4          | 1.0            | 0.6            | 1.0            | 0.6          |
| $N_{\mu\pi}$ | 1.1            | 1.1            | 1.1          | 1.1            | 1.1            | 1.1          | 1.1          | 1.1          | 1.1            | 1.1            | 1.1            | 1.1          |
| Total          | 12.0           | 12.3           | 16.1         | 31.0           | 34.2           | 151.7        | 13.6         | 11.0         | 14.8            | 11.9            | 11.9            | 11.9         |
ifying our use of $\sigma_{MC}$ in obtaining the data yield. We then find the weighted average of $\Delta \sigma_i$, which is given by $(0.11 \pm 0.08)$ MeV/$c^2$. As a conservative estimate, we repeat the data yield determinations by moving $\sigma$ up and down by 0.2 MeV/$c^2$, and take the average of the absolute values of the changes in each data yield as the systematic uncertainty of fixing $\sigma$ to the MC value for that $B$ mode. A combined fit of common modes in data is used to determine the nominal values for $\mu$ and for the endpoint of the $m_{ES}$ distribution $\sqrt{s}/2$. Hence, we move the parameters up and down by their fitted errors (0.2 MeV/$c^2$ for $\mu$ and 0.1 MeV/$c^2$ for $\sqrt{s}/2$) to obtain their corresponding systematic uncertainties. The quadratic sum of the three uncertainties from $\mu$, $\sigma$ and $\sqrt{s}/2$ gives the systematic uncertainty of the fit model for each $B$ mode.

h. Spin-alignment dependence The $B^0 \rightarrow D^{*+}D^{-}$, $B^0 \rightarrow D^{*0}\bar{D}^{0}$, and $B^+ \rightarrow \bar{D}^{*0}D^{++}$ decays are pseudoscalar $\rightarrow$ vector vector (VV) transitions described by three independent helicity amplitudes $A_0$, $A_\|$, and $A_\perp$ [31]. The lack of knowledge of the true helicity amplitudes in the $B \rightarrow VV$ final states contributes a systematic uncertainty to the efficiency. The dominant source of this effect originates from the $p_T$-dependent inefficiency in reconstructing the low-momentum “soft” pions in the $D^{*+}$ and $D^{*0}$ decays, and the fact that the three helicity amplitudes contribute very differently to the slow pion $p_T$ distributions. To estimate the size of this effect, MC samples are produced with a phase-space angular distribution model for the decay products. Each event is then weighted by the angular distribution for given input values of the helicity amplitudes and phase differences. The efficiency is then determined for a large number of amplitude sets and the observed distributions in efficiencies are used to estimate a systematic uncertainty. For a given iteration, a random number, based on a uniform PDF, is generated for each of the three parameters: $R_\perp$, $\alpha$, and $\eta$, where

$$R_\perp = \frac{|A_\perp|^2}{|A_\||^2 + |A_\||^2 + |A_\|^2}, \quad \alpha = \frac{|A_\|^2 - |A_\|^2}{|A_\| + |A_\|},$$

and $\eta$ is the strong phase difference between $A_\|$ and $A_\|. Since $R_\perp$ for $B^0 \rightarrow D^{*+}D^{-}$ has already been measured [36], a Gaussian PDF with mean and width fixed to the measured values is used instead for that mode. The events of the MC sample are weighted by the corresponding angular distribution and the efficiency is determined (after applying all selection cuts) by fitting the $m_{ES}$ distribution and dividing by the number of generated events. The procedure is repeated 1000 times for each $B \rightarrow VV$ sample. The relative spread in efficiencies (rms divided by the mean) is used to estimate the systematic uncertainty due to a lack of knowledge of the true amplitudes.

i. Peaking background and crossfeed The uncertainties on the peaking background vector $P_i$ and on the efficiency matrix $\epsilon_{ij}$ are dominated by the available MC statistics. The resulting uncertainties on each element of the vector and matrix are propagated through to the branching fraction results via the formalism of Eq. 6.

j. Number of $B\bar{B}$ The number of $B\bar{B}$ events in the full data sample, and the uncertainty on this number, are determined via a dedicated analysis of charged track multiplicity and event shape [15]. The uncertainty introduces a systematic uncertainty of 1.1% on each of the branching fractions.

VII. MEASUREMENT OF $CP$-VIOLATING CHARGE ASYMMETRIES

To obtain the charge asymmetries $A_{CP}$ (defined in Eq. 1), we perform unbinned extended maximum likelihood fits to the $m_{ES}$ distributions of the selected events in each of the four charged-$B$ decay modes $D^{*+}\bar{D}^{0}$, $D^{*+}\bar{D}^0$, $D^{*0}\bar{D}^0$, and $D^{*+}\bar{D}^+$, and their respective charge conjugates, and in the neutral-$B$ decay mode $D^{*+}D^+$, using Eq. 5 as the PDF for the combinatorial background for both charges in each pair. The free parameters of each of the five fits individually are: 1) the combinatorial background shape parameter $\kappa$, 2) the total number of signal events, 3) the total number of background events, and 4) the “raw” charge asymmetry $A$. Parameters 1 and 3 are considered (and thus constrained to be) the same for both charge states in each mode; this assumption is validated in MC simulation of the background as well as in control samples of $B^0 \rightarrow D^{*+}\rho^+$ and $B^0 \rightarrow D^{*+}a_1^0$ decays in data. The results of the fits are shown in Fig. 5. Two potentially biasing effects must be considered: there can be a asymmetry in the efficiencies for reconstructing positively- and negatively-charged tracks, and peaking background and crossfeed between the modes can cause a small difference between the measured (“raw”) asymmetry and the true asymmetry. The former of those two effects is discussed in Sec. VIII below. Regarding the latter, to obtain the charge asymmetries $A_{CP}$ from the “raw” asymmetries $A$, very small corrections for peaking background and crossfeed between modes must be made. Using the terminology of Eq. 6, and considering the branching fractions $B_i$ to be sums of a “+” mode (with a $B^0$ or $B^+$, containing a $\bar{b}$ quark, as the initial state) and a “−” mode (with a $\bar{B}^0$ or $B^−$, which contain a $b$ quark, as the initial state): $B_i = B_i^+ + B_i^−$, we have the two equations

$$\sum_j \epsilon_{ij} B_j^\pm N_B = N_i^{\text{sig}\pm} - P_i^\pm$$

for the “+” and “−” states respectively, which imply

$$B_i^- + B_i^+ = \sum_j \epsilon_{ij}^{-1} [(N_j^{\text{sig}−} - P_j^-) \pm (N_j^{\text{sig}+} - P_j^+)]/N_B.$$ 

As

$$A_{CP,i} = \frac{\Gamma_i^- - \Gamma_i^+}{\Gamma_i^- + \Gamma_i^+} = \frac{B_i^- - B_i^+}{B_i^- + B_i^+},$$

(11)
FIG. 5: Fitted distributions of $m_{ES}$ for the two conjugate states of each of the five relevant modes. The error bars represent the statistical errors only. The solid lines represent the fits to the data, and the shaded areas the fitted background. The raw asymmetries $A$ are the normalized differences in the amount of signal between the members of each conjugate pair.
we have
\[ A_{CP,i} = \frac{\sum_j \epsilon_{ij}^{-1}[(N_j^{\text{sig}^-} - P_j^-) - (N_j^{\text{sig}^+} + P_j^+)]}{\sum_j \epsilon_{ij}^{-1}[(N_j^{\text{sig}^-} - P_j^-) + (N_j^{\text{sig}^+} + P_j^+)]} \] (12)

Since \( N_j^{\text{sig}} = N_j^{\text{sig}^-} + N_j^{\text{sig}^+} \) and the “raw” asymmetry
in a mode \( A_j = \frac{N_j^{\text{sig}^+} - N_j^{\text{sig}^-}}{N_j^{\text{sig}^+} + N_j^{\text{sig}^-}} \), we have
\[ A_{CP,i} = \frac{\sum_j \epsilon_{ij}^{-1}[A_j N_j^{\text{sig}} - A_j P_j]}{\sum_j \epsilon_{ij}^{-1}[N_j^{\text{sig}} - P_j]} \] (13)

where \( A_j^P = \frac{P_j^- - P_j^+}{P_j^- + P_j^+} \) are the charge asymmetries of the
peaking backgrounds. The total yields \( N_j^{\text{sig}} \), peaking
backgrounds \( P_j \), and efficiency matrix \( \epsilon_{ij} \) are identical
to those used for reconstructing the fraction measurements
and are given in Tables VII and VI. The values \( A_j^P \) are
nominally set to 0 and are varied to obtain systematic uncertainties
due to the uncertainty on the charge asymmetry of the peaking background (see Sec. VIII). Thus,
Eq. 13 is used to determine the final \( A_{CP} \) values from
the measured asymmetries, in order to account for the small effects due to peaking background and crossfeed
between modes. The measured \( A_{CP} \) values are given in
Table VII. They are all consistent with zero, and their
errors are dominated by statistical uncertainty.

VIII. SYSTEMATIC UNCERTAINTIES ON
CHARGE ASYMMETRY MEASUREMENTS

Table X shows the results of our evaluation of the various
sources of systematic uncertainty that are important
for the \( A_{CP} \) measurements.

a. Slow \( \pi^\pm \) charge asymmetry A charge asymmetry
in the reconstruction efficiency of the low-transverse-momentum charged pions from \( D^\pm \rightarrow D^0 \pi^\pm \) decays can
cause a shift in \( A_{CP} \) by biasing the rates of positively charged vs. negatively charged decays for each mode.
We estimate this systematic uncertainty by using data
control samples of \( B^0 \rightarrow D^{*+} X^+ \) and \( B^0 \rightarrow D^{*+} X^- \)
decays, where \( X \) is either \( \pi, \rho \), or \( a_1 \), and determining if
there is an asymmetry in the number of \( D^{*+} \) vs. \( D^{*-} \)
reconstructed. There are two potential biases of this technique: 1) a charge asymmetry in tracks other than the slow charged pions, and 2) the presence of doubly-Cabibbo-suppressed \( B^0(\bar{B}^0) \rightarrow D^{*\pm} X^\pm \) decays which could potentially introduce a direct-CP-violating asymmetry between the two states in the control sample. Discussion of 1) is detailed in the paragraph below, and the rate of 2) has been determined in analyses such as Refs. [32] and [33] to be of order 0.1%, well below the sensitivity for this measurement. We combine the information from the control sample modes and determine an uncertainty of 0.5% for each \( A_{CP} \) measurement for modes with a charged slow pion.

b. Charge asymmetry from tracks other than slow \( \pi^\pm \)
Auxiliary track reconstruction studies place a stringent
bound on detector charge asymmetry effects at transverse
momenta above 200 MeV/c. Such tracking and PID sys-
tematic effects were studied in detail in the analysis of
\( B \rightarrow \phi K^* \) [34]. We assign a 0.2% systematic per charged
track, thus an overall systematic of 0.4% per mode (as
the positively charged and negatively charged decays for
each mode have, on balance, one positive vs. one nega-
tive track respectively). This systematic uncertainty is
added linearly to the slow pion charge asymmetry system-
tic due to potential correlation.

c. Amount of peaking background Peaking back-
ground can potentially bias \( A_{CP} \) measurements in two
ways: 1) a difference in the total amount of peaking back-
ground from the expected total amount can, to second
order, alter the measured asymmetry between the posi-
tively charged and negatively charged decays, 2) a more
direct way for peaking background to alter the measured
\( A_{CP} \) would be if the peaking background itself were to
have an asymmetry between the amount that is recon-
structed as positively charged and the amount recon-
structed as negative. 1) is discussed here; 2) is discussed
in the paragraph below. The systematic uncertainty due
to the uncertainty on the total amount of peaking back-
ground in the five decays is determined via the formalism
of Eq. 13. Namely, the uncertainty is given by
\[ \delta A_{CP,i} = \frac{(\sum_j \epsilon_{ij}^{-1} A_j N_j^{\text{sig}}) \times \sqrt{\sum_j (\epsilon_{ij}^{-1})^2 (\delta P_j)^2}}{(\sum_j \epsilon_{ij}^{-1} [N_j^{\text{sig}} - P_j])^2} \] (14)

where \( (\delta P)_j \) are the uncertainties on the amount of peaking
background (which are given, along with the other parameters in the equation, in Table VII).

d. \( A_{CP} \) of peaking background The systematic un-
certainty due to the \( A_{CP} \) of the peaking background is
also determined using the formalism of Eq. 13. Namely,
the uncertainty is given by
\[ \delta A_{CP,i} = \frac{(\sum_j \epsilon_{ij}^{-1} A_j N_j^{\text{sig}}) \times \sqrt{\sum_j (\epsilon_{ij}^{-1})^2 (\delta A^P)^2 P_j^2}}{(\sum_j \epsilon_{ij}^{-1} [N_j^{\text{sig}} - P_j]) (\sum_j \epsilon_{ij}^{-1} A_j N_j^{\text{sig}})} \] (15)

Investigation of the sources of the peaking background in
these modes motivates a conservative choice of 0.68 for
the \( (\delta A^P)_j \) values.

e. Amount of crossfeed The systematic error due
to uncertainties in the amount of crossfeed between
the modes is also determined via the formalism of Eq. 13.
Namely, the uncertainty is given by
The covariance between the elements of the inverse efficiency matrix is obtained using the method of Ref. [20]. The very small systematic uncertainty due to crossfeed is thus obtained using Eq. 16 and the amounts of crossfeed and their uncertainties that are given in Table VI. The very small systematic uncertainty due to crossfeed is obtained using the method of Ref. [20].

The uncertainties in $m_{ES}$ resolution and the beam energy $\sqrt{s}$ are determined by varying these parameters within their fitted $\pm 1 \sigma$ ranges and observing the resulting changes in $A_{CP}$. The uncertainty in the reconstructed $B$ mass can also have an impact on the fitted $m_{ES}$ distributions and thus on the fitted $A_{CP}$ values. Varying the $B$ mass between the fitted value and the $\pm 1 \sigma$ range of the nominal $B^0$ or $B^+$ invariant mass allows the determination of the resulting effect on the $A_{CP}$ values.

$f$. Uncertainty in $m_{ES}$ resolution, $B$ mass, and $\sqrt{s}$

The uncertainties in $m_{ES}$ resolution and the beam energy $\sqrt{s}$ are determined by varying these parameters within their fitted $\pm 1 \sigma$ ranges and observing the resulting changes in $A_{CP}$. The uncertainty in the reconstructed $B$ mass can also have an impact on the fitted $m_{ES}$ distributions and thus on the fitted $A_{CP}$ values. Varying the $B$ mass between the fitted value and the $\pm 1 \sigma$ range of the nominal $B^0$ or $B^+$ invariant mass allows the determination of the resulting effect on the $A_{CP}$ values.

$g$. Potential fit bias

Uncertainties in the potential biases of the $A_{CP}$ fits are determined by performing the fits on large samples of MC simulation of the signal decay modes and of $B \bar{B}$ and continuum background decays. All results are consistent with zero bias, and the uncertainties of the fitted asymmetries on the simulated data samples are conservatively assigned as systematic uncertainties from biases of the fits.

### IX. IMPLICATIONS FOR $\gamma$

Information on the weak phase $\gamma$ may be obtained by combining information from $B \rightarrow D^{(*)} \bar{D}^{(*)}$ and $B \rightarrow D^{(*)} \bar{D}^{(*)}$ branching fractions, along with $CP$ asymmetry measurements in $B \rightarrow D^{(*)} \bar{D}^{(*)}$, and using an SU(3) relation between the $D^{(*)} \bar{D}^{(*)}$ and $D_s^{(*)} \bar{D}^{(*)}$ decays [7, 8]. For this analysis, we assume that the breaking of SU(3) can be parametrized via the ratios of decay constants $f_{D_s^{(*)}}/f_{D^{(*)}}$, which are quantities that can be determined either with lattice QCD or from experimental measurements [9].

In this model, one obtains the relation (for $B^0 \rightarrow D^{(*)} \bar{D}^{(*)}$ and individual helicity states of $B^0 \rightarrow D^{(*)} \bar{D}^{(*)}$):

\[
\delta_0 A_{CP, i} = \left( \sum_{jk} A_j N_j^{*\text{sig}} \text{cov}(\epsilon_{ij}^{-1}, \epsilon_{ik}^{-1}) A_k N_k^{*\text{sig}} \right) \left( \sum_j \epsilon_{ij}^{-1}[N_j^{*\text{sig}} - P_j] \right) \times \left( \sum_j \epsilon_{ij}^{-1}[N_j^{*\text{sig}} - P_j] \right) /
\left( \sum_j \epsilon_{ij}^{-1}[N_j^{*\text{sig}} - P_j] \right)^2
\]

(16)

where

\[
B \equiv \frac{1}{2}(|A_D|^2 + |\bar{A_D}|^2) = A_{ct}^2 + A_{ut}^2 + 2A_{ct}A_{ut} \cos \delta \cos \gamma,
\]

(18)

\[
a_{dir} = \frac{1}{2}(|A_D|^2 - |\bar{A_D}|^2) = -2A_{ct}A_{ut} \sin \delta \sin \gamma,
\]

(19)

\[
a_{indir} = \Im[e^{-2i\beta}(A_D^* \bar{A_D})] = -A_{ct}^2 \sin 2\beta - 2A_{ct}A_{ut} \cos \delta \sin(2\beta + \gamma) - A_{ut}^2 \sin(2\beta + 2\gamma),
\]

(20)

and

\[
a_R^2 \equiv B^2 - a_{dir}^2 - a_{indir}^2.
\]

(21)
$A_D$ and $\bar{A}_D$ represent amplitudes of a given $B^0$ and $\bar{B}^0 \to D^{(*)+}D^{(*)-}$ decay respectively, $B$ represents the corresponding average branching fraction, and $a_{dir}$ and $a_{indir}$ represent the corresponding direct and indirect $CP$ asymmetries respectively. The phases $\beta$ and $\gamma$ are the CKM phases and $\delta$ is a strong phase difference. $\mathcal{A}_{ct} \equiv \langle |T + E - P_c - P_t - P_{EW}^c| V_{cb} V_{cd}^* \rangle$ and $\mathcal{A}_{at} \equiv \langle |P_u - P_{EW}^t V_{ub} V_{ud}^*| V_{cb} V_{cd}^* \rangle$ are the magnitudes of the combined $B \to D^{(*)\bar{D}^{(*)}}$ decay amplitudes containing $V_{ub} V_{ud}^*$ and $V_{cb} V_{cd}^*$ terms respectively, and the $T$, $P$, and $E$ terms are the trees, penguin, and the sum of exchange and annihilation amplitudes respectively [7]. One can directly measure the parameters $B$, $a_{dir}$, and $a_{indir}$ using information from $B \to D^{(*)\bar{D}^{(*)}}$ decays; the parameter $\mathcal{A}_{ct}$ using information from $B \to D_+^{(*)}\bar{D}^{(*)}$ decays; and the weak phase $\beta$ can be obtained from the measurements of $\sin 2\beta$ based on $B^0 \to c\bar{c}K_0^*$ decays [35] thus allowing for solution of $\gamma$ (up to two discrete ambiguities) via Eq. 17. As the vector-pseudoscalar modes $B^0 \to D^{*\pm}D^{*\mp}$ are not $CP$ eigenstates, a slightly more complicated analogue to Eq. 17 is needed for these modes [8]. Measurement of $\mathcal{A}_{CP}$ for $D^{*\pm}D^{*\mp}$ is also necessary to obtain information on $\gamma$ from the vector-pseudoscalar modes.

Using these relations, there are four variables besides $\beta$ for each $B \to D^{(*)\bar{D}^{(*)}}$ decay for which to solve: $\mathcal{A}_{ct}$, $\mathcal{A}_{at}$, $\delta$, and $\gamma$. The branching fraction and the direct and indirect $CP$ asymmetries of the $B \to D^{(*)\bar{D}^{(*)}}$ decay provide three measured quantities. The other measurement that can be used is the branching fraction of the corresponding $B \to D_+^{(*)}\bar{D}^{(*)}$ decay, by using the relation expressed in Eq. 22.

The values $a_{indir}$ can, of course, only be measured in the neutral $B \to D^{(*)}\bar{D}^{(*)}$ decays. However, the charged $B \to D^{(*)\bar{D}^{(*)}}$ decays can supplement the neutral decays by adding information on $B$ and $a_{dir}$, assuming only isospin symmetry between the charged and neutral modes. Thus, information from the charged $B$ decay modes can assist the $\gamma$ determination.

SU(3)-breaking effects can distort the relation between $D^{(*)}\bar{D}^{(*)}$ and $D_+^{(*)}\bar{D}^{(*)}$ decays as expressed in Eq. 17. However, the SU(3)-breaking can be parametrized by the ratio of decay constants $f_{D^{(*)}/f_{D^{(*)}}}$, such that the amplitude for $B \to D_+^{(*)}\bar{D}^{(*)}$ decays

$$\mathcal{A}_{ct} = f_{D^{(*)}/f_{D^{(*)}}} \times \mathcal{A}_{at} / \sin \theta_c \quad \text{(22)}$$

where $\theta_c$ is the Cabibbo angle [14] and the parentheses around the asterisks correspond to the $B \to D^{(*)\bar{D}^{(*)}}$ and $B \to D_+^{(*)}\bar{D}^{(*)}$ decays that are used. The theoretical uncertainty of this relation is determined to be 10% [7].

We thus use the information from the vector-vector (VV) decays $B^0 \to D^{*+}D^{-*}$ and $B^+ \to D^{*+}\bar{D}^{*-0}$ and pseudoscalar-pseudoscalar (PP) decays $B^0 \to D^+D^-$ and $B^+ \to D^+\bar{D}^{-0}$, as well as the vector-pseudoscalar (VP) decays $B^0 \to D^{*+}D^{-*}$, $B^+ \to D^{*+}\bar{D}^{-0}$, and $B^+ \to D^+\bar{D}^{-0}$, to form constraints on $\gamma$ using the method of Refs. [7, 8].

To use the VV decays, we must make the assumption that the strong phases for the 0 and $\| \helicity$ amplitudes are equal. The constraints from the VP decays require no such assumption. The assumption of equal 0 and $\| \helicity$ amplitudes is theoretically supported by a QCD factorization argument described in [8].

Then, using Eq. 17, we combine the $B^0 \to D^{*+}D^{-*}$ and $B^+ \to D^{*+}\bar{D}^{-0}$ branching fractions and $\mathcal{A}_{CP}$ information given above with measurements of the $B^0 \to D^{*+}D^{-*}$ and $B^+ \to D^{*+}\bar{D}^{-0}$ branching fractions [14], measurements of the $B^0 \to D^{*+}D^{-*}$ time-dependent $CP$ asymmetries [21, 36], and the world-average values of $\sin 2\beta$ [35] and $\sin \theta_c$ [14].

We use a fast parametrized MC method, described in Ref. [8], to determine the confidence intervals for $\gamma$. We consider 500 values for $\gamma$, evenly spaced between 0 and $2\pi$. For each value of $\gamma$ considered, we generate 25000 MC experiments, with inputs that are generated according to Gaussian distributions with widths equal to the experimental errors of each quantity. For each experiment, we generate random values of each of the experimental inputs according to Gaussian distributions, with means and sigmas according to the measured central value and total errors on each experimental quantity. We make the assumption that the ratio $f_{D^*/f_D}$ is equal to $f_{D^*/f_D} = 1.20 \pm 0.06 \pm 0.06$ [9], allowing for the additional 10% theoretical uncertainty [7]. We then calculate the resulting values of $\mathcal{A}_{ct}$, $a_{dir}$, $a_{indir}$, and $B$, given the generated random values (based on the experimental values). When the quantities $a_{dir}$, $a_{indir}$, and $B$, along with $\beta$ and the value of $\gamma$ that is being considered, are input into Eq. (17), we obtain a residual value for each experiment, equal to the difference of the left- and right-hand sides of the equation. Thus, using Eq. 17, the 25000 trials per value of $\gamma$ provide an ensemble of residual values that are used to create a likelihood for $\gamma$ to be at that value, given the experimental inputs. The likelihood, as a function of $\gamma$, can be obtained from $\chi^2(\gamma)$, where $\chi^2 = (\mu/\sigma)^2$, $\mu$ is the mean of the above ensemble of residual values, and $\sigma$ is the usual square root of the variance. The value of $\chi^2(\gamma)$ is then considered to represent a likelihood which is equal to that of a value $\chi$ standard deviations of a Gaussian distribution from the most likely value(s) of $\gamma$. We define the “exclusion level,” as a function of the value of $\gamma$, as follows: the value of $\gamma$ is excluded from a range at a given C.L. if the exclusion level in that range of $\gamma$ values is greater than the given C.L.

We now turn to the VP decays. The method using VP decays shares the advantage with PP decays that no assumptions on strong phases are required. The disadvantage is that, as we will see, the constraints from the VP modes are weak.

We combine the information given above on the $B^0 \to D^{*+}D^{-*}$, $B^+ \to D^{*+}\bar{D}^{-0}$, and $B^+ \to D^{*+}\bar{D}^{-0}$ branching fractions and $\mathcal{A}_{CP}$ information with measurements of the $B^0 \to D^{*+}D^{-*}$, $B^0 \to D^{*+}\bar{D}^{-0}$, and $B^+ \to D^{*+}\bar{D}^{-0}$ branching fractions [14], measurements of the $B^0 \to D^{*+}D^{-*}$ time-dependent $CP$ asymmetries [24, 37], and the world-average values of $\sin 2\beta$ [35].
and $\sin^2 \theta_L$ [14]. Similar to the MC $\gamma$ determination for the VV and PP modes, we generate random values of each of the experimental inputs according to Gaussian distributions, with means and sigmas according to the measured central value and total errors on each experimental quantity. We again obtain a confidence level distribution as a function of $\gamma$.

Finally, we can combine information from the VV, PP, and VP modes. The resulting measured exclusion level as a function of $\gamma$ from each of the three sets of modes, as well as from their combination, is shown in Fig. 6. From the combined fit, we see that $\gamma$ is favored to lie in the range $[0.07 - 2.77]$ radians (with a $+0$ or $+\pi$ radians ambiguity) at 68% confidence level. This corresponds to $[4.1^\circ - 158.6^\circ]$ ($+0^\circ$ or $180^\circ$).

These constraints are generally weaker than those found in Ref. [8] due to the fact that the measured $CP$ asymmetry in $B^0 \rightarrow D^{*+}\bar{D}^{*-}$ has moved closer to the world-average $\sin 2\beta$, with the newer $B^0 \rightarrow D^{*+}\bar{D}^{*-}$ measurements in Ref. [38]. The closer this $CP$ asymmetry is to $\sin 2\beta$, the weaker the resulting constraints are on $\gamma$, due to the fact that the closeness of the $CP$ asymmetry to $\sin 2\beta$ favors the dominance of the tree amplitude, rather than the penguin amplitude whose phase provides the sensitivity to $\gamma$. Although the constraints are not strong, they contribute to the growing amount of information available on $\gamma$ from various sources.

X. CONCLUSIONS

In summary, we have measured branching fractions, upper limits, and charge asymmetries for all $B$ meson decays to $D^{(*)}\bar{D}^{(*)}$. The results are shown in Table VII. This includes observation of the decay modes $B^0 \rightarrow D^+D^-$ and $B^+ \rightarrow D^{*+}\bar{D}^{*-}$, evidence for the decay modes $B^+ \rightarrow D^+\bar{D}^{*-}$ and $B^+ \rightarrow D^+\bar{D}^{0}$ at $3.8\sigma$ and $4.9\sigma$ levels respectively, constraints on $CP$-violating charge asymmetries in the four decay modes $B^+ \rightarrow D^{(*)+}\bar{D}^{(*)0}$, measurements of (and upper limits for) the decay modes $B^0 \rightarrow D^{*0}\bar{D}^0$ and $B^0 \rightarrow D^0\bar{D}^0$, and improved branching fractions, upper limits, and charge asymmetries in all other $B \rightarrow D^{(*)}\bar{D}^{(*)}$ modes. The results are consistent with theoretical expectation and (when available) previous measurements. When we combine information from time-dependent $CP$ asymmetries in $B^0 \rightarrow D^{(*)+}\bar{D}^{(*)-}$ decays [38, 39] and world-averaged branching fractions of $B$ decays to $D_{s}^{(*)}\bar{D}_{s}^{(*)}$ modes [14] using the technique proposed in Ref. [7] and implemented in Ref. [8], we find the CKM phase $\gamma$ is favored to lie in the range $[0.07 - 2.77]$ radians (with a $+0$ or $+\pi$ radians ambiguity) at 68% confidence level.

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FIG. 6: The measured exclusion level, as a function of \( \gamma \), from the combined information from vector-vector, vector-pseudoscalar, and pseudoscalar-pseudoscalar modes. The combined information implies that \( \gamma \) is favored to lie in the range \([0.07 - 2.77]\) radians (with a +0 or +\( \pi \) radians ambiguity) radians at 68% confidence level.