Is the monocentric urban economic model still empirically relevant? Assessing urban economic predictions in 192 cities on five continents

Charlotte Liotta\textsuperscript{a,b,*}, Vincent Viguié\textsuperscript{a}, Quentin Lepetit\textsuperscript{a}

\textsuperscript{a}CIRED (Ecole des Ponts ParisTech), Site du Jardin Tropical, 45bis, Av de la Belle Gabrielle, F-94736 Nogent-sur-Marne, France

\textsuperscript{b}TU Berlin, Straße des 17. Juni 135, D-10623 Berlin, Germany

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Abstract

Despite a large body of work that developed over more than 60 years, and numerous applications in theoretical papers, the empirical knowledge accumulated on the monocentric urban model and its extensions remains limited. Using a unique dataset gathering spatially explicit data on rents, population densities, housing sizes, and transport times in neighborhoods inside 192 cities on all continents, we investigate on a systematic basis the empirical relevance of the key stylized facts predicted by this model. Some of these predictions appear extremely robust: cities are more spread out when they are richer, more populated, and when transportation or agricultural land is less costly, and 95\% of the cities of our sample exhibit the predicted negative density gradient from the city center to suburbs. Rent variations inside cities are also significantly explained by transport times in most of the cities (159 cities). However, housing production (and population densities) seem significantly impacted by rents in only slightly more than half of the cities (106 cities). Nevertheless, high levels of informality, strong regulations and planning, specific amenities (e.g. coastal amenities) are, as expected by the theory, main factors leading to the discrepancies. Overall, several decades after its creation, the standard urban model seems to still capture surprisingly well the inner structure of many cities across the world, both in developed and in developing countries.

Keywords: Urbanisation, Standard Urban Model, Urban Spatial Structure, Cross-country Comparisons

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\*Corresponding author

Email address: liotta@centre-cired.fr (Charlotte Liotta)
1. Introduction

Understanding urbanization patterns and modeling urban dynamics have been subjects of research for at least a century (Duranton and Puga, 2015). These questions have been recently brought into a renewed spotlight in the context of the environmental crises, as urban sprawl is an important driver of both climate change (IPCC, 2014) and biodiversity losses (IPBES, 2019; Seto et al., 2012).

A well-established framework to work on these issues is the Standard Urban Model (SUM). It has been widely used in theoretical and applied papers, and numerous extensions have been derived. However, the empirical knowledge on the monocentric urban model, and its actual relevance, remains limited (see for instance Duranton and Puga (2015) for a review of these points). Taking benefit of the large quantity of data on worldwide cities that begins to become available, we assess here empirically the main assumptions and conclusions of this model on a broad cross-section of cities.

The SUM, initially developed by Alonso (1964), Muth (1969), and Mills (1967), describes the relationship between land use, land value and transportation costs in a city. More precisely, households trade-off between housing sizes and transportation costs to the CBD. In the center of the city, households pay high rents but bear low transportation costs, resulting in high housing supply by private developers, low dwelling sizes, and high densities. In the periphery, households pay low rents but bear high transportation costs. The housing supply is low, and households live in large dwellings.

The SUM, in its most standard form (Fujita, 1989), is actually the combination of two distinct mechanisms. The first enables to determine housing prices ("the real estate pricing model"). Potential tenants compete for dwellings, and landlords choose the highest bidder. This leads to the definition of "bid-rents", and to the idea that the higher the transportation costs, the lower the rents and/or real estate prices. To this mechanism is often added a central hypothesis: the city is monocentric, meaning that all jobs are located in the city center. The second mechanism enables to determine housing production, and population density ("the housing production model"). Developers build the buildings in the city, and either rent or sell the dwellings they make. With standard production functions, the higher the bid-rents, the higher is the housing supply.

Combining these mechanisms and the monocentric hypothesis, key results can be derived. Wheaton (1974) especially showed that, under the assumptions of the monocentric model with monetary commuting costs, large populations lead to large urbanized areas, high incomes lead to large urbanized areas, high transportation costs lead to small urban areas, and high agricultural land prices lead to small urban areas.

As highlighted by the review by Duranton and Puga (2015), a large number
of studies have checked empirically either some predictions of the SUM or part of its principles, but, mainly due to data availability, the analyses remain far from exhaustive. A first strand of literature uses spatially explicit data on population densities to assess whether cities exhibit the negative density gradient expected from the theory. This was actually begun even before the development of the SUM, following the seminal work by Clark (1951), which computed density gradients for 19 cities in 9 countries, and showed that they were close to exponential functions. Many papers followed, and recent examples include Bertaud and Malpezzi (2014), who compare population density gradients across 57 cities of different regions and levels of development, and Lemoy and Caruso (2017) who study 300 European cities. Both confirm the negative exponential curve for population densities for most of the cities they study. These papers use distance to the city center as a rough proxy for commuting costs. The recent paper by Qiang et al. (2020) on density profiles of US cities, is, to our knowledge, the only study going further and using actual transportation times as a proxy for transportation generalized costs.

Several papers also checked the aggregated predictions of the SUM on the size of urbanized areas, i.e. assessed whether the urbanized surface of a city is larger when population and income are larger and when agricultural land prices and transportation costs are lower. Recent examples include Paulsen (2012), which shows that the main predictions of the SUM are valid in the United States, with population and income determining urban areas. Oueslati et al. (2015) does a similar exercise for 282 European cities, Schmidt et al. (2020) for 92 cities in Germany and Jedwab et al. (2021) for 1010 cities in developing countries.

If a lot of studies empirically validate the predictions of the SUM in terms of urbanized areas and density gradients, the underlying mechanisms of the model, i.e. the fact that, in a city, locations with higher transportation costs exhibit lower bid-rents and that locations with lower rents exhibit lower housing supply, were however only assessed in a few studies. To our knowledge, the validity of the "the housing production model" was only studied in one paper (McMillen, 2006) using data on Chicago. The "real estate pricing model" was examined in more papers, several dating back from the 1970s (see for instance Yinger (1979), McDonald and Bowman (1979), see also McMillen (2010) and Duranton and Puga (2015) for reviews) and more recently by Ahlfeldt (2011). However, only a handful of cities were examined by these papers.

In this paper, we propose to assess in systematic way the ability of the SUM to represent cities across the world, using detailed data on 192 cities across all continent. Section 2 presents the data. Section 3, 4, 5 and 6 successively present our empirical assessments of the mechanisms and conclusions of the SUM, and section 7 concludes.
2. Data

Data availability on cities has dramatically increased in the past years, and many databases comparing cities have been made available by research teams and administrations (see for instance UN (2018), Lemoine-Rodriguez et al. (2020) or Nangini et al. (2019)). However, these databases generally contain aggregated information, such as the total population, the average income, the Gini coefficient etc. for each city as a whole. Detailed, spatialized information on the structure of the cities (data per neighborhood, or data on each pixel of spatial grids) are still rare, but are required to assess a model as the SUM. This is the type of data that we gathered in this study.

Using web-scrapping, we gathered spatially explicit gridded data on density, rents, dwelling sizes, land-use and transportation, inside 192 urban areas worldwide (Figure 1). The cities were chosen to get a wide geographical coverage, and represent different cultural and historical backgrounds, but data availability was the main selection criterion. The cities selected are medium to large cities (more than 300 000 inhabitants), and represent together more than 800 million people, i.e. 19% of the global urban population, or 34% of the people living in cities of more than 300 000 inhabitants.

![Cities studied](image)

Figure 1: Map of the cities

Each city was divided in a georeferenced grid of 1 km\(^2\) resolution, encompassing the whole urban area. We then aimed at getting values for each of the grid cells. To do this, we combined data from different sources (see Annex A.1 for all the information). For land-use, we used the European Space Agency (ESA) land cover data, available worldwide at a 300m spatial resolution. We reclassified
the data as “urbanizable” or “non urbanizable” areas (Annex B). For density, we used the GHSL data of the European Commission. Data on real estate, rents and dwelling sizes were most difficult to get, as no unified database exist. We obtained them through the web-scraping of real estate websites. The websites were carefully selected following four criteria: the website must have nation-wide coverage to ensure consistent results in a given country, it must geolocalize the dwellings, have values for both rent or sale prices and dwelling size, and read in local language and propose prices in local currency to limit real estate ads targeting expatriates. Data were aggregated at the grid cell level by taking the mean rents per square meter.¹

We estimated transport distances and durations from each grid cell to the city center using Google Maps and Baidu Maps APIs (Application Programming Interfaces). Different definitions of city centers exist in the urban economics literature. Most rely on job density data, which are unfortunately not available consistently in the cities of the database. We, therefore, defined city centers by a compromise between five qualitative criteria: the geographical center of the data, the historical center of the cities, the location of the public transports hub, the official central business district, and the city hall location. We collected, when available, both driving and public transport data, at typical afternoon rush hours. It was not possible to collect transport data from each grid cell, so we collected data from 10% of all cells, and then interpolated them using the INTERPP function from the R package AKIMA (see figure A.8 in appendix).

We finally also collected aggregated indicators at the city level, including for instance urbanized area of the city, total population, or income per capita. They are described in Annex A.3. Most are common except a qualitative "monocentricity city index" that we designed, and a "planned city index", equal to 1 when the city is planned and equal to 0 otherwise (and based on Bertaud and Malpezzi (2014) ).

In Annex A.2, we discuss the validity and the potential biases of the data that we collected through web-scrapping. We compared them to external databases, and we ran robustness checks on the different results if this paper (see Annex F).

3. Asssessing SUM predictions on urbanized areas surface

We first begin by assessing whether the aggregated predictions of the SUM on the size of urbanized areas seem valid in our sample of cities. More precisely, we aim at understanding to which extent urbanized areas increase with population and income and decrease with agricultural land prices and transportation costs.

To do this, we linearly regress urbanized areas on a range of city characteristics, including population, income, land prices, and monocentricity index. In addition,²

¹We assess the robustness of real estate data to different aggregation methods or outliers’ treatment in Annex F.1.
we build two commuting cost variables: "fuel cost", computed as the fuel efficiency multiplied by the fuel price and which measures the fuel price for 100km, and "commuting speed", which is the density-weighted average speed to go to the city center at night rush hour using the fastest transportation mode (car or public transport). We also present the result of a double-log specification.

### Table 1: Regression on urbanized areas

| Dependent variable: | Urbanized area surface | ln(urbanized area surface) |
|---------------------|------------------------|----------------------------|
| population          | 0.000***               | (0.000)                    |
| In(population)      | 0.856***               | (0.039)                    |
| income              | 0.014***               | (0.004)                    |
| In(income)          | 0.409***               | (0.037)                    |
| land price          | -169.932***            | (34.984)                   |
| In(land prices)     | -0.218***              | (0.037)                    |
| fuel cost           | -13199.329***          | (3740.833)                 |
| In(fuel cost)       | -0.160                 | (0.109)                    |
| commuting speed     | 39.789***              | (7.215)                    |
| In(commuting speed) | 0.448***               | (0.103)                    |
| monocentricity      | 201.458*               | (107.085)                  |
| constant            | -700.851**             | (350.325)                  |

| Observations        | 192                    | 192                        |
| $R^2$               | 0.736                  | 0.859                      |
| Adjusted $R^2$      | 0.728                  | 0.854                      |
| Residual Std. Error | 557.879(df = 185)      | 0.404(df = 185)            |
| F Statistic         | 16.073*** (df = 6.0; 185.0) | 240.218*** (df = 6.0; 185.0) |

*Note:* *p<0.1; **p<0.05; ***p<0.01

Table 1 shows the results. The adjusted $R^2$ are 0.73 for the main specification and 0.85 for the double log specification, meaning that city characteristics explain
a high share of the variance in urbanized areas among the cities of the sample. In addition, all the coefficients of the explanatory variables are significant with the expected sign, except fuel cost which is not significant in the double-log specification.

Our results are in line with existing studies, and confirm the empirical relevance of the predictions of the SUM on the size of cities. First, higher populations significantly increase urban areas, as it is expected (and as it was empirically assessed, for instance, in Paulsen, 2012; Oueslati et al., 2015; Schmidt et al., 2020; Spivey, 2008). We also find an elasticity of urbanized area with respect to population which is lower than one, as predicted by the SUM. The value that we find (0.86) is in line with Spivey (2008), which finds an elasticity of 0.91, and slightly higher than Paulsen (2012), which finds elasticities between 0.56 and 0.64.

Second, higher incomes also significantly increase urban areas, in line with the predictions of the SUM and of Paulsen (2012), Oueslati et al. (2015), and Schmidt et al. (2020), but in contradiction with Spivey (2008). Spivey (2008), working on US cities, find a negative and significant impact of incomes on urban areas, which she explains by the fact that the increase in incomes increases the demand for more affordable housing farther from the city center, but also raises the opportunity cost of time, the second effect outweighing the first one. Disaggregating our analysis by continent (table D.7 in appendix Annex D), we also find that the impact of incomes on urbanized areas is negative in the United States.

Third, as predicted by the SUM, higher agricultural land prices decrease urbanized areas. This result confirms those of Oueslati et al. (2015) and Paulsen (2012), whereas Spivey (2008) and Schmidt et al. (2020) found mixed or non-robust results.

Finally, both commuting costs variables are significant and have the expected sign in specification 1: higher fuel costs lead to smaller urban areas and higher commuting speeds lead to larger urban areas. Fuel cost appears however as less robust, as it is not significant in specification 2. Even if this result is in line with the predictions of the SUM, it is a noticeable result given that existing studies do not find robust evidence. Existing studies have had great difficulty finding consistent, available, and appropriate measures of transportation costs. Spivey (2008) uses the fraction of households owning a car, which is not robust in her main specification, and the Texas Transportation Institute’s “time travel index” and “congestion cost index,” with which she finds mixed evidence depending on the sample of cities. Paulsen (2012) does not include any commuting cost variables: the author considered the census average travel time to work, but finally dropped this variable. As no Europe-wide data on transport costs exist, Oueslati et al. (2015) rely on highway density data from the Eurostat regional dataset as a proxy for transport costs, with the assumption that highway investment reduces the time and the costs of commuting. They find a positive coefficient as expected. Finally, Schmidt et al. (2020) use commuting time as a proxy for the opportunity cost of
transportation and average diesel prices per liter as a monetary cost measure, but find non-robust results or even not the expected sign. Our proxies are close to those of Schmidt et al. (2020), but lead to significant and robust results for commuting speed, in line with the SUM.

The monocentricity index is positive and significant at a 10% level, in line with Spivey (2008), who finds that more sub-centers are associated with a smaller land area in the United States, which she interprets as the fact that sub-centers are more likely to develop in dense areas, or that increased density associated with sub-centers mitigates sprawl. However, our results contradict Schmidt et al. (2020), who finds no evidence on the impact of polycentricity on urban areas.

Figure D.12 in appendix Annex D compares urbanized areas data and predictions based on the double-log specification. We can see that the area of the large North American and Asian cities is overestimated by the model, and that among smaller cities, the size of European cities is slightly overestimated whereas the size of African cities is slightly underestimated.

4. Assessing SUM predictions on density gradients

Let us now examine a second prediction of the SUM model: the negative density gradient from the city center to the suburbs. More precisely, we aim at assessing whether, within cities, population density decreases when transportation costs increase.

4.1. Empirical strategy

We run the following regression on the cities of the database to estimate the impact of transportation costs on densities (this equation can be easily derived from the SUM, see C.8 in Annex C):

$$\ln(n_i) = \alpha_1 + \beta_1 \ln(Y - T_i) + \beta_2 \ln(L_i) - \beta_3 \ln(q_i) + \varepsilon_i$$  (1)

As in Qiang et al. (2020), we use commuting times and distances from Google Maps’ data. We are going two steps further by considering two transportation modes and by accounting for both opportunity and monetary costs. Transportation costs are computed with the following method:

- we assume that households choose the cheapest transportation mode between private car and public transport;
- for each transportation mode, the opportunity cost of time is derived from Google Maps’ data on transportation times, assuming that the opportunity cost of time is valued by the individuals at the hourly wage rate;
• the monetary cost of private cars comes from the distance of the commute from Google Maps, the efficiency of cars from the IEA (2019) and the fuel price from the World Bank;

• the monetary cost of public transport, assumed as being a fixed cost by commute (and thus not varying with the distance), is obtained from various sources.

We also run two robustness checks on the commuting cost variable. First, we use the commuting time $t_i = \min(t_{driving}, t_{public\_transport})$ instead of the income net of transportation cost $Y - T_i$. Second, we consider the euclidean distance to the city center $d_i$ instead of the income net of transportation cost $Y - T_i$ as in Bertaud and Malpezzi (2014) or Lemoy and Caruso (2017). Results are displayed in Annex F.2.

4.2. Results

Regression 1 lead to an overall good fit (table 2), with a median $R^2$ of 0.36 and a maximum $R^2$ of 0.73. The $R^2$ we estimate are not as high as the $R^2$ from Bertaud and Malpezzi (2014), which have a median of 0.800. This might come from the fact that we do not aggregate our density data per 1km annulus but use spatially explicit data per grid cell\(^2\). Using euclidean distance instead of commuting costs slightly improves the fit for the cities for which it is low but worsens it for the cities for which it is good. An example of cities for which using transportation costs improves the fit compared to euclidean distances can be found in ?? (figure ??).

In addition, for 179 cities, our parameter of interest (the impact of the income net of transportation costs on density $\beta_1$) is significant at the 5% level and positive as expected (table 3). The estimate of $\beta_1$ is not significant at the 5% level in 10 cities, and significantly negative in only two cities: Brasilia, a planned city created ex-nihilo in 1960, and Singapore, a highly dense and planned city (figure E.13 in Annex E). This result is consistent with Bertaud and Malpezzi (2014), which finds negative density gradients in all the cities of their dataset except Brasilia, Cape Town, Moscow and Seoul. Moscow and Brasilia are centrally planned cities, and Cape Town developed under apartheid. Even if the Korean economy has been more market driven, housing and land development have been heavily regulated in Seoul, which is characterized by Bertaud and Malpezzi (2014) as an "extraordinary regulatory environment" with a large greenbelt, a large public developer with substantial market power and a lot of public financing and subsidized.

We can use density gradients as a measure of the compactness of the city. As in the previous section, we expect the cities to be more spread out when the city is richer, more populated, when transportation is cheaper and when agricultural land is cheaper. As Bertaud and Malpezzi (2014), we find that density gradients flatten

\(^2\)Aggregating our data per 1km annulus lead to a median $R^2$ of 0.84 (see Annex F.3)
with income, with city population, and with falling transportation costs (table E.8 in appendix Annex E). Concerning commuting cost, only the commuting speed seems to be robustly impacting density gradients. More monocentric cities also flatten density gradients. This result confirms the results from the previous section.

5. Assessing the monocentric real estate pricing model

Let us now assess the real estate pricing model. Analyzing the housing demand by households, the real estate pricing model predicts that, within cities, bid-rents increase when transportation costs decrease. From Annex C, assuming standard Cobb-Douglas production and utility function, we can derive the following relation between rents (denoted \( R_i \)) and transportation costs.

\[
\ln(R_i) = \ln(R_0) - \frac{1}{\beta} \ln(Y) + \frac{1}{\beta} \ln(Y - T_i) \tag{2}
\]

We therefore run the following regression on the cities of the database.

\[
\ln(R_i) = \alpha_2 + \beta_4 \ln(Y - T_i) + \epsilon_i \tag{3}
\]

Regression 3 has a varying fit, with R2 from 0.00 to 0.59 (table 2). It is useful to note that the regression does not include control variables such as the proportion
Real estate pricing model
Impact of income net of transportation costs on rents

- Negative Gradient
- Not Significant
- Positive Gradient

Figure 2: Empirical assessment of the real estate pricing model: impact of income net of transportation costs on rents (equation 3).

of land that is suitable for housing construction or dwelling size, as in equation 1, but only measures the impact of transportation costs on rents.

For a large majority of the cities of the sample, the parameter of interest $\beta_4$ is
positive as expected and significant at the 5% level (table 3). It is not significant at the 5% level in 30 cities, and significantly negative in only 2 cities, Riga and Toluca. These results show that the real estate pricing model seems to hold in a large number of cities. Figure 2 shows that there is no clear geographical pattern in the cities for which parameter $\beta_4$ is significantly negative or not significant.

To understand why equation 3 leads to positive and significant parameter estimates in some cities, and to negative and non-significant parameter estimates in others, we regress the dummy indicating whether the estimate of $\beta_4$ is significant at the 5% level on a range of city characteristics (tab. 4).

The real estate pricing equation (equation 3) is more likely to lead to significant parameter estimates when the city is larger in terms of population, controlling for data quality (table 4), which is probably due to the fact that we have more observations and thus more explanatory power in larger cities. The real estate equation is also less likely to lead to significant parameter estimates when there is a large share of informal housing, as rents are extremely low or even nonexistent in informal housing areas no matter their locations. Coastal amenities also lead to non significant parameter estimates: indeed, rents in coastal areas might be very high in some cities due to coastal amenities even if they are far from the city center.

It is not possible, in our framework, to disentangle whether the model do not capture reality because a city is not monocentric or because the tradeoff between rent and transport is not valid. In Table 4, the monocentric index is surprisingly not significant, but this may be due to the definition of our index (see Annex A.1), which is not perfect (Schmidt et al. (2020) reached a similar conclusion, with a different indicator for polycentricity, in their empirical assessment of the SUM). As there are only big cities in our sample, and as such cities tend to be more polycentric than smaller ones (Ahlfeldt and Wendland, 2012a), it is remarkable that the monocentric real estate pricing model is coherent with the data of more than 80% of the cities. This may reflect the dominant role that the "historical" city centers keeps playing even when city grow and become progressively more polycentric (Ahlfeldt and Wendland, 2012a).

6. Assessing the housing production model

In this section, we finally aim at validating the housing production model. The housing production model predicts that, within cities, housing supply, and thus density, increase when bid-rents increase, resulting from the maximization of the profit of private developers. From Annex C, assuming standard Cobb-Douglas production and utility function, we can derive the following relation between population density and rents.
Table 4: Second-stage regression (demand side). We regress the dummy indicating whether the estimate of $\beta_4$ is significant at the 5% level on a range of city characteristics, including population, income, share of informal housing, monocentricity, Gini index, indicators of the quality of real estate data in our database (real estate data market cover and real estate data spatial cover, see Annex A.3), and on dummies indicating whether the city is a planned city and whether the city is a coastal city.

| Dependent variable: | $\beta_4$ not significant |
|---------------------|---------------------------|
| gini index          | 0.002                     |
|                     | (0.004)                   |
| informal housing    | 0.006**                   |
|                     | (0.003)                   |
| planned city        | 0.011                     |
|                     | (0.055)                   |
| monocentricity      | 0.022                     |
|                     | (0.068)                   |
| coastal city        | 0.129**                   |
|                     | (0.056)                   |
| population          | -0.000***                 |
|                     | (0.000)                   |
| income              | -0.000                    |
|                     | (0.000)                   |
| market data cover   | 0.000                     |
|                     | (0.000)                   |
| spatial data cover  | -0.618*                   |
|                     | (0.316)                   |
| constant            | 0.103                     |
|                     | (0.213)                   |

Observations 190  
$R^2$ 0.186  
Adjusted $R^2$ 0.145  
Residual Std. Error 0.338 (df = 180)  
F Statistic 3.849*** (df = 9.0; 180.0)

Note: *p<0.1; **p<0.05; ***p<0.01
\[
\ln(n_i) = \frac{1}{1-b} \ln(A) + \frac{b}{1-b} \ln(b) + \frac{b}{1-b} \ln(R_i) + \ln(L_i) - \ln(q_i)
\]

(4)

We therefore run the following regression on the cities of the database.

\[
\ln(n_i) = \alpha_3 + \beta_5 \ln(R_i) + \beta_6 \ln(L_i) - \beta_7 \ln(q_i) + \epsilon_i
\]

(5)

Regression 5 has a varying fit, with R2 from 0.005 to 0.85 (table 2). In terms of R2, the housing production equation seems to be more explanatory power than the real estate pricing equation, but looking at parameter estimates leads to different conclusions. Indeed, parameter \( \beta_5 \) is positive as expected and significant at the 5% level in only 106 cities out of 192. It is significantly negative in 15 cities, and it is not significant in 70 cities.

Figure 3 shows that the cities with a negative parameter estimate are mainly located in South Africa, Brazil, United States and United Kingdom. The cities for which the parameter is not statistically significant are mainly located in South America and Africa.

Qualitatively, looking at the cities for which the estimate of parameter \( \beta_5 \) is negative or not significant allows making some hypotheses on the city characteristics that threaten the validity of the SUM. Some of these cities are planned cities; it is for instance the case of Brasilia and Singapore, which exhibits negative density gradients. Others have a large share of informal housing: it is the case of Cape Town and Johannesburg, in South Africa. For instance, in Cape Town, informal settlements exhibit high densities but very low rents, contrary to the predictions of the housing production model. Some cities for which the estimates of \( \beta_5 \) are not significant or negative are coastal cities. For example, in San Diego, coastal amenities and tourism explain that the rents are high in the periphery, even if housing is spread out, contrary to the predictions of the SUM. Finally, other explanations are economic recession explaining that rents are not as high as expected in the city center (Birmingham) or polycentrism (Nottingham).

We try to verify these assumptions by running a multinomial logit regression on the dummy indicating whether the estimate of \( \beta_5 \) is positive, negative or non significant at the 5% level on a range of city characteristics, as in the previous section. We show that the housing production equation (equation 5) is more likely to lead to significant parameter estimates when the city is richer and larger in terms of population, and less likely to lead to significant parameter estimate when there is a large share of informal housing and when the city is a coastal city, controlling for real estate data quality (table 5). Real estate data spatial cover also seems to play a key role.
Figure 3: Empirical assessment on the housing production model: impact of rents on population density (equation 5).
### Second-stage regression (housing production model)

| Dependent variable: | $\beta_5$ negative | $\beta_5$ not significant |
|---------------------|--------------------|--------------------------|
| gini index          | 0.1759**           | 0.0340                   |
|                     | 0.078              | 0.030                    |
| population          | -5.901e-08         | -1.183e-07***            |
|                     | 8.03e-08           | 4.39e-08                 |
| income              | 1.142e-06          | -2.354e-05*              |
|                     | 1.83e-05           | 1.36e-05                 |
| monocentricity      | 0.6093             | 0.1292                   |
|                     | 0.939              | 0.459                    |
| constant             | -10.7329***        | -1.0499                  |
|                     | 3.533              | 1.630                    |
| informal housing    | -0.0473            | 0.0488**                 |
|                     | 0.067              | 0.023                    |
| spatial data cover  | 7.2976*            | -8.2646**                |
|                     | 4.151              | 3.509                    |
| market data cover   | -0.0001            | -1.15e-06                |
|                     | 0.000              | 1.83e-06                 |
| planned city        | 0.1476             | -0.1583                  |
|                     | 0.675              | 0.352                    |
| coastal city        | -0.3465            | 1.5579***                |
|                     | 0.647              | 0.413                    |
| Observations        | 192                |                          |
| Pseudo $R^2$        | 0.2469             |                          |
| Log-Likelihood      | -129.41            |                          |
| LL-Null             | -171.84            |                          |

**Note:** *p<0.1; **p<0.05; ***p<0.01

Table 5: Second-stage regression (housing production model)
7. Discussion and conclusion

More and more data now become available, but comparing cities worldwide still remains a challenging task because of the difficulty to gather data about the internal structure of cities. First, if we have been able to get data about city on each continent, we only managed to study a handful of African cities, mainly because of real estate data availability.

Second, we could only study the most simple version of the SUM, because key data were missing to enable to study, on a systematic way, more advanced versions. Gathering transport data to secondary job centers, for instance, to study polycentric models, would have taken too much time to be realistically doable in the context of our project. Similarly, we could not find global data sources on job density or on local income level per neighborhood, preventing to assess models such as Ahlfeldt et al. (2015) or Tsivanidis (2019). We guess, however, that the development of crowdsourced geospatial data could dramatically change this.

Third, the quality of the rent data that we use is difficult to fully validate. In Annex F, we do a series of robustness checks, in which we redo the analyses of sections 4, 5 and 6 by slightly changing the way we treat our gross data on rents (change in the outlier exclusion method etc.). They leave our results are broadly unchanged. Other data that we use, e.g. transport data from Google Maps, are difficult to assess, as they are provided by closed algorithms.

A few important ideas can be derived from our results. First, even if the monocentric SUM could be called a simplistic and old approach, originally developed to represent mainly US cities, it works surprisingly well to capture the structure of cities across the world. As already proven in the literature, its conclusions on urbanized surface and on density profile seem coherent with a large majority of the cities in our sample.

What is surprising, is that these conclusions of the model seem much more validated by the data than the internal mechanisms of the model (real estate pricing model and housing production model). One reason is the inertia of cities evolutions: as it was highlighted by McMillen (2006), "the primary problem is the static nature of the model. [...] densities reflect the past, whereas land values reflect expectations about the future". Densities and built environment are evolving slowly (as are transport times, to some extent), whereas rents and land prices can change quickly. It is therefore logical that relating transport to built environment and transport to rents would yield better results than relating rents to built environment. It is actually possible to note that the housing production model seems to work well in fast-growing cities in India and in China (Figure 3), where built environment evolutions are occurring at a speed close to real estate prices changes. Getting panel data on cities, and studying changes over times, would be required to study these questions in depth.
Still, this should not overshadow the fact that the internal mechanisms of the model seem coherent with data in more than half of the cities. It is also important to note that the factors which, in theory, limit the relevance of the SUM (high income inequalities in cities, strong locational amenities, polycentrism etc.) were indeed found to be important factors explaining why the model fails in practice to capture some city structures. In conclusion, the monocentric SUM remains a useful analytical tool, coherent with many city structures across the world. We hope that, with more data available, more precise knowledge on the domain of validity of its mechanisms and of its extensions will become possible in the near future.

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Annex A. Data

Annex A.1. Gridded data sources

Grid analysis. For each city, we designed a georeferenced grid of 1 km2 resolution, encompassing the whole urban area (Fig. A.4). This grid defines the resolution of our data: in each of the pixels of this grid, we tried to collect a representative value for land-cover, population density and housing price and size.

Figure A.4: Example of analysis grids.

Land cover data. We used the European Space Agency land cover data, available worldwide at a 300m spatial resolution on an annual basis from 1992 to 2015, and kept 2015 data.

Population density. We used the GHSL data of the European Commission, available for 1975, 1990, 2000 and 2015 at a 250m resolution, and kept 2015 data (Fig. A.5).\(^3\)

Rents and dwelling sizes. Real estate data on rents and dwelling sizes have been obtained from the webscraping of real estate websites from 2017 to 2020 (Fig. A.6). The real estate websites have been selected following four criteria:

- the website must have a nation-wide coverage to ensure consistent results in a given country,
it must geolocalize the dwellings,

- it must have values for both rent or sale prices and dwelling size, and

- it has to read in local language and to propose prices in local currency to limit real estate ads targeting expatriates.

Data were aggregated at the grid cell level by taking the mean rents per square meter.

Transportation. Transport distances and durations were collected using Google Maps and Baidu Maps APIs (Application Programming Interfaces). As in this project we are using monocentric models, identifying city centers was required. Different definitions have been used in the urban economics literature to define city centers (fig. A.7). Most rely on job density data (McDonald, 1987; Giuliano and Small, 1991; McMillen, 2001; McMillen and Smith, 2003; Redfearn, 2007; Ahlfeldt and Wendland, 2012a), which are unfortunately not available on a consistent basis in the cities that we study here. We therefore defined city centers here by a compromise between five qualitative criteria: the geographical center of the data, the historical center of the cities, the location of public transports hubs, the official central business district, and the city hall location.

Transport data collection was conducted from the centers defined above to each grid cell, and at typical afternoon rush hours (fig. A.8 and A.9). We collected, when available, both driving and public transport data.
It was not possible to collect transport data from each grid cell, so we collected data from 10% of all cells, and then interpolated them using `interp` function from R package akima.⁴

Annex A.2. Technical Validation of the gridded data, and potential biases

Let us examine now what we can say about the validity of the data that we collected.

Population densities. Population densities data are extracted from GHSL database (European Commission Global Human Settlement Project). This database is not perfect, but is generally considered a good representation of population densities across the world (see for instance Chowdhury et al. (2018); Florczyk et al. (2019); Liu et al. (2020) or the website of GHSL project⁵).

Transport data. The quality of Google map and Baidu maps data is more difficult to assess, as these websites use closed algorithms, based on users travel data to assess car travel (Wang, 2007; Barth, 2009). It should be expected that the quality of these data is a function of the number of users in each city, and should therefore be higher in developed country cities and in large population cities (Kreindler, 2016; Anderson, 2017). The quality of travel data for public transport depends

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⁴https://cran.r-project.org/web/packages/akima/index.html
⁵https://ghsl.jrc.ec.europa.eu/data.php
Figure A.7: Map of a sample of defined city centres.

Figure A.8: Driving times maps of a sample of cities.
on whether transport authorities have shared their data with Google or Baidu (this is indicated in our database). Another source of error comes from the number of datapoints that we use to measure transport times, and from the interpolation process. This source of error is also difficult to assess. We tried to mitigate it by using grid points close to each other near the center of the city, and further apart from each other when moving away from the center.

*Rents and real estate prices.* The quality of real estate prices, rents and dwelling size data differs from one city to another. A first source of mistake comes from the number of data points that we used to compute our averages per grid cell. If this number is small, idiosyncratic characteristics of the dwellings may not be averaged out, and may prevent to get actually representative estimates.

A second potential source of mistake is the systemic bias coming from our datasources. The websites that we scrapped present ads, which may not necessarily reflect the actual rents or prices, if margin of negociations exist, for instance. They also may be biased and present only dwellings which are not representative of the actual dwelling stock. This may especially be the case if, in a city, online ad websites are not the main way of buying or renting a dwelling. To mitigate these risks we tried to make sure that the websites we scrapped are actually used by locals (see section *Annex A.1*).

To assess these two risks, we checked the validity of our collected rents and real estate prices against 4 external databases providing averages per city. We used two crowd-sourced websites aiming at describing the cost of life across the world
for expatriates: Numbeo\textsuperscript{6} and Expatistan\textsuperscript{7} which give an estimation of rent and property prices around the world. We also used two databases built for real estate investors: UBS "Prices and Earnings" database (UBS, 2019), and data from CBRE, an expert in real estate and services based in London, whose data were obtained from various local sources (CBRE Residential, 2019). When averaged over the cities, or over the inner core and the outer core of the cities, our data broadly agree with these data (Fig. A.10a, A.10b, A.11a, A.11b).

\textsuperscript{6}https://www.numbeo.com/cost-of-living/
\textsuperscript{7}https://www.expatistan.com/cost-of-living
Comparison with Numbeo
Expressed in $  (R^2=0.665  74% datapoints are similar)
- Difference < 33%
- Difference > 33%

(a) For 74% of cities, the difference is lower than 33%.

Comparison with Expatistan
Expressed in $  (R^2=0.604  70% datapoints are similar)
- Difference < 33%
- Difference > 33%

(b) For 70% of cities, the difference is lower than 33%.

Figure A.10: Comparison of our database with Numbeo and Expatistan data for 2019.
(a) For 73% of cities, the difference is lower than 33% (PPA).

(b) For 52% of cities, the difference is lower than 33% (official exchange rate).

Figure A.11: Comparison of our database with Global Living (CBRE) and UBS data for 2019.
Annex A.3. Aggregated data and indicators

*Population and urban surface.* Population corresponds to the sum of the densities over the grid; similarly, urbanized area corresponds to the sum of the urbanized area data over the grid. We are aware that this definition of city limits is likely to overestimate population and urbanized area, but it allows the extent of "urban area" to be identical for the two variables and to be independent of city administrative boundaries, which vary a lot with countries.

*Income.* We use GDP per capita data at the country level from the World Bank.

*Agricultural rent.* We use agricultural rents data from the FAO, dividing agricultural GDP by the total agricultural area in the country.

*Monocentricity index.* We measure monocentricity using a qualitative index that shows to which extent the geographical center, the historical center, the public transport hub, the official CBD and the town hall are at the same place. The higher is this index, the more monocentric is the city.

*Transport price.* We also used World bank data on gasoline prices and International Energy Agency (2012) data on fuel efficiency to compute the cost of travels by car.

*Rent data quality indicator.* We computed two variables to assess rent data quality: the first one ("Real estate data market cover") corresponds to the average number of real estate ads that have been scrapped per grid cell in the city, and the second one ("real estate data spatial cover") corresponds to the ratio of grid cells for which have a density data for which we also have data on rents.

*Additional indicators.* We use informal housing data and Gini indices from the World Bank. Based on Bertaud and Malpezzi (2014), we designed a dummy equal to 1 when the city is planned and equal to 0 otherwise. For the urban areas that are not in Bertaud’s dataset, we set this dummy based on our judgment.

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8 It is equal to (population / number of ads).

9 It is equal to (number of pixels with real estate data / number of pixels with population > 0).
## Annex B. Land cover classification

| ESA CCI land cover category | Reclassification |
|-----------------------------|-------------------|
| 10 Cropland, rainfed        | Urbanizable       |
| 11 Herbaceous cover         | Urbanizable       |
| 12 Tree or shrub cover      | Non urbanizable   |
| 20 Cropland, irrigated or post-flooding | Urbanizable |
| 30 Mosaic cropland (>50%) / natural vegetation (<50%) | Urbanizable |
| 40 Mosaic natural vegetation (>50%) / cropland (<50%) | Urbanizable |
| 50 Tree cover, broadleaved, evergreen, closed to open (>15%) | Non urbanizable |
| 60 Tree cover, broadleaved, deciduous, closed to open (>15%) | Non urbanizable |
| 61 Tree cover, broadleaved, deciduous, closed (>40%) | Non urbanizable |
| 62 Tree cover, broadleaved, deciduous, open (15–40%) | Non urbanizable |
| 70 Tree cover, needleleaved, evergreen, closed to open (>15%) | Non urbanizable |
| 71 Tree cover, needleleaved, evergreen, closed (>40%) | Non urbanizable |
| 72 Tree cover, needleleaved, evergreen, open (15–40%) | Non urbanizable |
| 80 Tree cover, needleleaved, deciduous, closed to open (>15%) | Non urbanizable |
| 81 Tree cover, needleleaved, deciduous, closed (>40%) | Non urbanizable |
| 82 Tree cover, needleleaved, deciduous, open (15–40%) | Non urbanizable |
| 90 Tree cover, mixed leaf type (broadleaved and needleleaved) | Non urbanizable |
| 100 Mosaic tree and shrub (>50%) / herbaceous cover (<50%) | Non urbanizable |
| 110 Mosaic herbaceous cover (>50%) / tree and shrub (<50%) | Urbanizable |
| 120 Shrubland               | Non urbanizable   |
| 121 Evergreen shrubland     | Urbanizable       |
| 122 Deciduous shrubland     | Urbanizable       |
| 130 Grassland              | Urbanizable       |
| 140 Lichens and mosses      | Urbanizable       |
| 150 Sparse vegetation (tree, shrub, herbaceous cover) (<15%) | Urbanizable |
| 160 Tree cover, flooded, fresh or brakish water | Non urbanizable |
| 170 Tree cover, flooded, saline water | Non urbanizable |
| 180 Shrub or herbaceous cover, flooded, fresh/saline/brakish water | Non urbanizable |
| 190 Urban areas             | Urbanizable       |
| 200 Bare areas              | Urbanizable       |
| 201 Consolidated bare areas | Urbanizable       |
| 202 Unconsolidated bare areas | Urbanizable |
| 210 Water bodies            | Non urbanizable   |
| 220 Permanent snow and ice  | Non urbanizable   |

Table B.6: Reclassification of the ESA CCI land cover categories into "urbanizable" and "non urbanizable"
Annex C. The Standard Urban Model

The SUM, in its simplest form, can be summarized as follows (Fujita, 1989).

Real estate pricing model. In a city composed of I discrete locations indexed i, households seek to maximize their Cobb-Douglas utility:

$$\max \ U(z, q) = z^\alpha q^\beta$$  \hspace{1cm} \text{s.t.}  \hspace{1cm} z + qR + T \leq Y \quad (C.1)$$

with q the size of the dwellings, Y the income (assumed as being the same for every household in the city), R the rent, T the transportation cost to the city center (where we assume for simplicity that all jobs are located), and z the consumption of a composite good.

From equation (C.1), we find that:

$$q_i = \frac{\beta(Y - T_i)}{R_i} \quad (C.2)$$

$$z_i = \frac{\alpha(Y - T_i)}{u} \quad (C.2)$$

Writing u the uniform utility at equilibrium, and denoting $R_0$ the rent in the city center, we find:

$$R_i = R_0 \left(1 - \frac{T_i}{Y}\right)^{1/\beta} \quad \text{with} \quad R_0 = \frac{\alpha\beta Y^{1/\beta}}{u} \quad (C.3)$$

Housing production model. Absentee private developers produce housing from capital K and land L with the following Cobb-Douglas production function:

$$H(K, L) = AL^aK^b \quad (C.4)$$

With k = K/L (capital intensity per land surface) and h = H/L (housing density, i.e. number of m2 build per m2 on the ground), equation (C.4) can be rewritten:

$$h(k) = Ak^b \quad (C.5)$$

Developers seek to maximize their profit per land surface $\pi = Rh(k) - \rho k$ considering a capital cost (interest rate, for instance) $\rho$. Thus, housing supply writes:

$$H_i = A^{1/a}(bR_i/\rho)^{b/a}L_i \quad (C.6)$$

and population density $n_i$ writes:

$$n_i = H_i/q_i = A^{1/a}(bR_i/\rho)^{b/a}L_i/q_i \quad (C.7)$$

with $L_i$ accounting for natural constraints at location i.

The negative density gradient is a straightforward conclusion of the SUM. Combining eq. (C.3) and (C.7) we can derive a relation between density and transportation costs:

$$\ln(n_i) = \frac{1}{a} \ln(A) + \frac{b}{a} \ln(bR_i/\rho) - \frac{b}{a\beta} \ln(Y) + \ln(L_i) - \ln(q_i) + \frac{b}{a\beta} \ln(Y - T_i) \quad (C.8)$$
## Annex D. Supplementary results - SUM and urbanized area

**Dependent variable: Urbanized area**

|                  | North America | Europe | Oceania | South America | Asia | Africa |
|------------------|---------------|--------|---------|---------------|------|--------|
| population       | 0.000***      | 0.000*** | 0.000*** | 0.000***      | 0.000*** | 0.000*** |
|                  | (0.000)       | (0.000) | (0.000) | (0.000)       | (0.000) | (0.000) |
| income           | -0.142**      | 0.000   | 0.023*** | 0.014*        | -0.000 | -0.235 |
|                  | (0.066)       | (0.001) | (0.007) | (0.008)       | (0.000) | (0.172) |
| land prices      | -0.000**      | 10.073  | -50.321*** | -4003.310*** | -0.055 | -29926.989** |
|                  | (0.000)       | (118.130) | (6.490) | (1546.811)    | (0.093) | (14123.061) |
| commuting speed  | 30.937        | 6.826** | 37.199*** | 10.166*       | 0.029*** | 3.225 |
|                  | (24.724)      | (2.820) | (13.249) | (5.224)       | (0.111) | (8.824) |
| monocentricity   | 2783.493***   | 86.989* | -504.607*** | 12.893        | -0.001 | -169.377* |
|                  | (1359.344)    | (49.487) | (65.084) | (66.785)      | (0.171) | (95.088) |
| constant         | -0.000**      | -261.634** | -168.202*** | -156.615      | 4.755*** | 2164.355 |
|                  | (0.000)       | (128.258) | (21.695) | (196.909)     | (0.401) | (1372.491) |

| Observations     | 11            | 97     | 8       | 34            | 32    | 10     |
| Adjusted $R^2$   | 0.894         | 0.881  | 0.981   | 0.921         | 0.714 | 0.983  |

**Note:**

* $p<0.1$; ** $p<0.05$; *** $p<0.01$

**Table D.7: Regression on urbanized areas - by continent**

![Figure D.12: Difference between real and simulated urbanized areas - based on the double-log specification of table 1](image-url)
Annex E. Supplementary results - SUM and density gradients

| Dependent variable: |  |  |
|--------------------|---|---|
|                    | density gradient | ln(density gradient) |
| population         | -0.000***         | 0.000                  |
| ln(population)     | -0.245***         | 0.052                 |
| income             | 0.000             | 0.000                  |
| ln(income)         | 0.095             | 0.079                 |
| land prices        | 4.253***          | 0.633                 |
| ln(land prices)    | 0.185***          | 0.046                 |
| fuel cost          | -32.006           | 28.785                |
| ln(fuel cost)      | -0.397***         | 0.166                 |
| commuting speed    | -0.214***         | 0.054                 |
| ln(commuting speed) | -0.462***         | 0.146                |
| monocentricity     | 0.408             | 0.068                 |
| (1.002)            | (0.105)           |                       |
| constant           | 18.277***         | 5.632***              |
| (3.567)            | (1.137)           |                       |

Observations 179 179

$R^2$ 0.172 0.365

Adjusted $R^2$ 0.144 0.343

Residual Std. Error 7.108(df = 172) 0.525(df = 172)

F Statistic 13.090*** (df = 6.0; 172.0) 19.638*** (df = 6.0; 172.0)

Note: *p <0.1; **p <0.05; ***p <0.01

Table E.8: Second-stage regression of density gradients on city characteristics
Population density profile

Impact of income net of transportation costs on population density

- Negative Gradient
- Not Significant
- Positive Gradient

Figure E.13: Empirical assessment of the impact of income net of transportation costs on population density (equation 1).
Annex F. Robustness checks on real estate data

Annex F.1. Robustness checks on real estate data

In this study, our main robustness concerns are about the quality of real estate data. We run 7 robustness checks:

- Robustness check 1: aggregating rents data by taking the median instead of the mean in each grid cell.
- Robustness check 2: aggregating rents data by regressing rents on dwelling sizes in each grid cell.
- Robustness check 3: excluding outliers using the boxplot method.
- Robustness check 4: excluding outliers using the percentile method.
- Robustness check 5: excluding outliers using the hampel method.
- Robustness check 6: excluding the grid cells for which we have less than 4 data points.
- Robustness check 7: excluding the grid cells for which we have less than 10 data points.
| 191 obs. | Regression 1 | Regression 3 | Regression 5 |
|----------|--------------|--------------|--------------|
| min.     | 0.029        | 0.000        | 0.008        |
| 10th     | 0.191        | 0.009        | 0.063        |
| 25th     | 0.252        | 0.048        | 0.113        |
| 50th     | 0.362        | 0.141        | 0.205        |
| 75th     | 0.477        | 0.269        | 0.319        |
| 90th     | 0.585        | 0.396        | 0.446        |
| max.     | 0.734        | 0.586        | 0.848        |

(a)

| 191 obs. | Regression 1 ($\beta_1$) | Regression 3 ($\beta_4$) | Regression 5 ($\beta_5$) |
|----------|---------------------------|---------------------------|---------------------------|
| Positive and significant (5%) | 179 | 157 | 100 |
| Negative and significant (5%)   | 2  | 2  | 17  |
| Non significant (5%)             | 10 | 32 | 74  |

(b)

Figure F.14: Robustness check 1 - aggregating rents data by taking the median instead of the mean. a) percentiles of the R2 of the regressions. b) main parameters of the regressions.
| 191 obs. | Regression 1 ($\beta_1$) | Regression 3 ($\beta_3$) | Regression 5 ($\beta_5$) |
|----------|--------------------------|--------------------------|--------------------------|
| min.     | 0.029                    | 0.000                    | 0.007                    |
| 10th     | 0.191                    | 0.009                    | 0.069                    |
| 25th     | 0.252                    | 0.043                    | 0.113                    |
| 50th     | 0.362                    | 0.123                    | 0.198                    |
| 75th     | 0.477                    | 0.236                    | 0.311                    |
| 90th     | 0.585                    | 0.392                    | 0.425                    |
| max.     | 0.734                    | 0.585                    | 0.817                    |

(a)

| 191 obs. | Regression 1 ($\beta_1$) | Regression 3 ($\beta_3$) | Regression 5 ($\beta_5$) |
|----------|--------------------------|--------------------------|--------------------------|
| Positive and significant (5%) | 179 | 151 | 89 |
| Negative and significant (5%) | 2 | 5 | 21 |
| Non significant (5%) | 10 | 35 | 81 |

(b)

Figure F.15: Robustness check 2 - aggregating rents data by regressing rents on dwelling sizes in each grid cell. a) percentiles of the R2 of the regressions. b) main parameters of the regressions.
### (a)

| 191 obs. | Regression 1 | Regression 3 | Regression 5 |
|----------|--------------|--------------|--------------|
| min.     | 0.023        | 0.000        | 0.014        |
| 10th     | 0.183        | 0.012        | 0.058        |
| 25th     | 0.233        | 0.064        | 0.118        |
| 50th     | 0.349        | 0.179        | 0.193        |
| 75th     | 0.462        | 0.320        | 0.316        |
| 90th     | 0.567        | 0.432        | 0.437        |
| max.     | 0.738        | 0.659        | 0.772        |

### (b)

| 191 obs. | Regression 1 ($\beta_1$) | Regression 3 ($\beta_4$) | Regression 5 ($\beta_5$) |
|----------|---------------------------|---------------------------|---------------------------|
| Positive and significant (5%) | 179 | 159 | 112 |
| Negative and significant (5%) | 2 | 2 | 15 |
| Non significant (5%) | 10 | 30 | 64 |

Figure F.16: Robustness check 3 - excluding outliers using the boxplot method. a) percentiles of the $R^2$ of the regressions. b) main parameters of the regressions.
Figure F.17: Robustness check 4 - excluding outliers using the percentile method.

(a) percentiles of the R2 of the regressions.  
(b) main parameters of the regressions.
| 191 obs. | Regression 1 | Regression 3 | Regression 5 |
|----------|--------------|--------------|--------------|
| min.     | 0.023        | 0.000        | 0.014        |
| 10th     | 0.189        | 0.012        | 0.054        |
| 25th     | 0.234        | 0.060        | 0.112        |
| 50th     | 0.342        | 0.180        | 0.194        |
| 75th     | 0.462        | 0.319        | 0.328        |
| 90th     | 0.569        | 0.429        | 0.429        |
| max.     | 0.750        | 0.656        | 0.772        |

(a)

| 191 obs. | Regression 1 (β<sub>1</sub>) | Regression 3 (β<sub>4</sub>) | Regression 5 (β<sub>5</sub>) |
|----------|-------------------------------|------------------------------|-------------------------------|
| Positive and significant (5%) | 179                           | 156                          | 110                          |
| Negative and significant (5%)  | 2                             | 2                            | 17                           |
| Non significant (5%)           | 10                            | 33                           | 64                           |

(b)

Figure F.18: Robustness check 5 - excluding outliers using the hampel method. a) percentiles of the R2 of the regressions. b) main parameters of the regressions.
| 185 obs. | Regression 1 | Regression 3 | Regression 5 |
|----------|--------------|--------------|--------------|
| min.     | 0.063        | 0.000        | 0.007        |
| 10th     | 0.192        | 0.006        | 0.091        |
| 25th     | 0.293        | 0.086        | 0.173        |
| 50th     | 0.409        | 0.202        | 0.276        |
| 75th     | 0.556        | 0.397        | 0.417        |
| 90th     | 0.684        | 0.524        | 0.572        |
| max.     | 1.000        | 1.000        | 1.000        |

(a)

| 185 obs. | Regression 1 (β₁) | Regression 3 (β₄) | Regression 5 (β₅) |
|----------|--------------------|--------------------|--------------------|
| Positive and significant (5%) | 161 | 139 | 86 |
| Negative and significant (5%) | 3 | 2 | 14 |
| Non significant (5%) | 21 | 45 | 85 |

(b)

Figure F.19: Robustness check 6 - excluding the grid cells for which we have less than 4 data points. a) percentiles of the R2 of the regressions. b) main parameters of the regressions.
| 172 obs. | Regression 1 (β₁) | Regression 3 (β₄) | Regression 5 (β₅) |
|----------|-------------------|-------------------|-------------------|
| min.     | 0.115             | 0.000             | 0.006             |
| 10th     | 0.202             | 0.017             | 0.085             |
| 25th     | 0.315             | 0.074             | 0.164             |
| 50th     | 0.438             | 0.231             | 0.330             |
| 75th     | 0.659             | 0.459             | 0.496             |
| 90th     | 0.833             | 0.604             | 0.718             |
| max.     | 1.000             | 0.953             | 1.000             |

(a)

| 172 obs. | Regression 1 (β₁) | Regression 3 (β₄) | Regression 5 (β₅) |
|----------|-------------------|-------------------|-------------------|
| Positive and significant (5%) | 132 | 116 | 61 |
| Negative and significant (5%) | 4 | 3 | 15 |
| Non significant (5%) | 36 | 56 | 96 |

(b)

Figure F.20: Robustness check 7 - excluding the grid cells for which we have less than 10 data points. a) percentiles of the R2 of the regressions. b) main parameters of the regressions.
Annex F.2. Robustness checks on the commuting cost variable

| 191 obs. | Regression 1 | Regression 3 | Regression 5 |
|----------|--------------|--------------|--------------|
| min.     | 0.029        | 0.000        | 0.014        |
| 10th     | 0.166        | 0.016        | 0.058        |
| 25th     | 0.228        | 0.062        | 0.119        |
| 50th     | 0.338        | 0.167        | 0.193        |
| 75th     | 0.432        | 0.288        | 0.316        |
| 90th     | 0.547        | 0.407        | 0.437        |
| max.     | 0.728        | 0.626        | 0.772        |

(a) Percentiles of the R2 of the regressions.

| 191 obs. | Regression 1 ($\beta_1$) | Regression 3 ($\beta_4$) | Regression 5 ($\beta_5$) |
|----------|--------------------------|--------------------------|--------------------------|
| Positive and significant (5%) | 2                        | 0                        | 112                      |
| Negative and significant (5%)  | 175                      | 161                      | 15                       |
| Non significant (5%)            | 14                       | 30                       | 64                       |

(b) Main parameters of the regressions.

Figure F.21: Robustness check - using commuting time (minimum between cars and public transport) instead of income net of transportation costs.
| 191 obs. | Regression 1 | Regression 3 | Regression 5 |
|----------|--------------|--------------|--------------|
| min.     | 0.040        | 0.000        | 0.014        |
| 10th     | 0.207        | 0.018        | 0.061        |
| 25th     | 0.271        | 0.069        | 0.130        |
| 50th     | 0.365        | 0.190        | 0.206        |
| 75th     | 0.486        | 0.329        | 0.330        |
| 90th     | 0.573        | 0.460        | 0.454        |
| max.     | 0.716        | 0.631        | 0.772        |

(a)

| 191 obs. | Regression 1 ($\beta_1$) | Regression 3 ($\beta_4$) | Regression 5 ($\beta_5$) |
|----------|-------------------------|-------------------------|-------------------------|
| Positive and significant (5%) | 2           | 1                | 113                      |
| Negative and significant (5%) | 182         | 161              | 15                       |
| Non significant (5%)          | 7            | 29               | 63                       |

(b)

Figure F.22: Robustness check - using euclidian distance instead of income net of transportation costs. a) percentiles of the R2 of the regressions. b) main parameters of the regressions.

Annex F.3. Robustness checks on scale sensitivity
| 191 obs. | Regression 1 | Regression 3 | Regression 5 |
|----------|--------------|--------------|--------------|
| min.     | 0.293        | 0.000        | 0.062        |
| 10th     | 0.618        | 0.066        | 0.440        |
| 25th     | 0.738        | 0.195        | 0.590        |
| 50th     | 0.840        | 0.384        | 0.751        |
| 75th     | 0.914        | 0.552        | 0.845        |
| 90th     | 0.953        | 0.700        | 0.914        |
| max.     | 0.993        | 0.956        | 0.997        |

(a)

| 191 obs. | Regression 1 ($\beta_1$) | Regression 3 ($\beta_4$) | Regression 5 ($\beta_5$) |
|----------|--------------------------|--------------------------|--------------------------|
| Positive and significant (5%) | 158                      | 155                      | 117                      |
| Negative and significant (5%)   | 1                        | 1                        | 2                        |
| Non significant (5%)            | 31                       | 35                       | 71                       |

(b)

Figure F.23: Robustness check - aggregating data by 1 km annulus. a) percentiles of the R2 of the regressions. b) main parameters of the regressions.