Ballistic persistent currents in disordered metallic rings: Origin of puzzling experimental values

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Typical persistent current ($I_{\text{typ}}$) in a normal metal ring with disorder due to random grain boundaries and rough edges is calculated microscopically. If disorder is due to the rough edges, a ballistic current $I_{\text{typ}} \approx e v_F L$ is found in spite of the diffusive resistance ($\propto L/l$), where $v_F$ is the Fermi velocity, $l$ is the mean free path, and $L \gg l$ is the ring length. This ballistic current has a simple interpretation: It is due to a single electron that moves (almost) in parallel with the rough edges and thus hits them rarely. Our finding agrees with a puzzling experimental result $I_{\text{typ}} \approx e v_F L$, reported by Chandrasekhar et al. [Phys. Rev. Lett. 67, 3578 (1991)] for metal rings of length $L \approx 1000$. If disorder is due to the grain boundaries, our results agree with theoretical results $I_{\text{typ}} \approx (e v_F L)/(l/L)$ that holds for the white-noise-like disorder and has been observed in recent experiments.

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A mesoscopic resistive metal ring pierced by magnetic flux (Φ) supports a persistent current [1,3]. At zero temperature, the ring supports the persistent current $I = \sum_{\Phi E_f < E_F} I_j$, where $I_j(\Phi) = -dE_j(\Phi)/d\Phi$ is the single-electron current carried by electron with eigen-energy $E_f(\Phi)$, and $E_F$ is the Fermi level [1]. Function $I(\Phi)$ is periodic with period $\Phi_0 = h/e$, which provides a clear-cut experimental sign of the persistent current [1,3]. If the ring is ballistic and possesses one conducting channel, the sum $\sum_j I_j$ changes its sign when a new occupied state $j$ is added. Due to the sign cancellation mainly the electron at the Fermi level contributes to the sum, and the amplitude of the current is $I_0 = e v_F L$, where $v_F$ is the Fermi velocity and $L$ is the ring circumference. If the ring is disordered, the size and sign of the current fluctuate from sample to sample due to the disorder fluctuations. It is then reasonable to assess a typical current in a single sample as $I_{\text{typ}} = \langle I^2 \rangle^{1/2}$, where $\langle \ldots \rangle$ means the ensemble average.

The number of the conducting channels ($N_c$) in disordered metal rings is typically $\gg 1$ and the rings obey the diffusive limit, $l \ll L \ll \xi$, where $l$ is the electron mean free path and $\xi \approx N_c l$ is the localization length. To estimate $I_{\text{typ}}$, one can assume that mainly the electron at the Fermi level contributes to the sum $\sum_j I_j$. Since $L \gg l$, the electron is expected to move around the ring by diffusion. Its transit time is $\tau_D = L^2/D$, where $D = v_F l/d$ is the diffusion coefficient and $d$ is the dimensionality of the sample. So $I_{\text{typ}} \approx e/\tau_D = (1/d)(e v_F L)/(l/L)$. A similar result follows from the Green functions theory for non-interacting electrons [6], if disorder is modeled by a random potential $V(x)$ obeying the white-noise condition $\langle V(x)V(x') \rangle \propto \delta(x-x')$. One obtains [6]

$$I_{\text{typ}}^{\text{heor}} = 2 \times \langle 1.6/d \rangle (e v_F L)/(l/L), \quad l \ll L \ll \xi. \quad (1)$$

Here 2 is the spin factor, $d = 1, 2, 3$, and 1.6 is from Ref. [7].

The first observation of persistent current in a single metallic ring was reported [2] for three Au rings of size $L \sim 1000$. The measured currents were ten-to-hundred times larger than the result [1], they ranged from $\sim 0.1 e v_F L$ to $\sim e v_F L$. This huge discrepancy has not been explained yet (see the reviews [8,9]). Other Au rings showed [3] currents a few times larger than result [1], and recent measurements of individual Au rings [4] and Al rings [5] agreed with result [1] very well.

Why did the similar measurements of diffusive Au rings [2,4] show quite different results, $I_{\text{typ}} \approx e v_F L$ and $I_{\text{typ}} \approx (e v_F L)/(l/L)$? A puzzle [2] is why a multichannel disordered ring of length $L \gg l$ carries the current $\sim e v_F L$, typical for a one-channel ballistic ring? This Letter wants to answer both questions. There is disorder due to polycrystalline grains and rough edges [3] even in pure Au rings. Using a scattering-matrix method for non-interacting electrons [7,10], we studied typical persistent currents in Au rings with grains and rough edges without the white-noise approximation.

If the disorder is due to the grains, our results agree with the white-noise-related formula [1] and experiments [4,5]. However, if the disorder is due to the rough edges, we find the ballistic-like result $I_{\text{typ}} \approx e v_F L$ albeit the resistance is diffusive ($\propto L/l$) and $L \gg l$, like in the experiment [2]. This ballistic current is due to a single electron that moves (almost) in parallel with the rough edges and thus hits them rarely. Briefly, result $I_{\text{typ}} \approx e v_F L$ in a metal ring of length $L \gg l$ is as normal as the result $I_{\text{typ}} \approx (e v_F L)/(l/L)$. Which result is observed depends on the nature of disorder.

For simplicity, we study two-dimensional (2D) rings and mention the 3D effect at the end. We start with a conductance study. Consider a 2D wire [Fig. 1] described by Hamiltonian

$$H = -(\hbar^2/2m)(\partial_x^2 + \partial_y^2) + U(x, y) + V(x, y), \quad (2)$$

where $m$ is the electron effective mass, $U$ is the grain boundary potential, and $V$ is the potential due to the edges (see Fig. 1 and Refs. [7,10]). We connect the wire to two ideal leads - clean long wires of width $W$. In the leads, the wave function of the electron with energy $E$ possesses the usual form [12]

$$\varphi(x, y) = \sum_{n=1}^{N} \left[ A_n^+(x) + A_n^-(x) \right] \sin\left( \frac{2\pi n x}{W} \right), \quad x \leq 0$$

$$\varphi(x, y) = \sum_{n=1}^{N} \left[ B_n^+(x) + B_n^-(x) \right] \sin\left( \frac{2\pi n y}{W} \right), \quad x \geq L \quad (3)$$

where $N$ is the considered number of channels (ideally $N = \infty$), $A_n^\pm(x) \equiv a_n^\pm e^{\pm ik_n x}$, $B_n^\pm(x) \equiv b_n^\pm e^{\pm ik_n x}$, and $k_n(E)$ is the wave vector given by equation $\pm k_n^2 \omega_n^2 + \frac{\hbar^2}{2m} = E$. The vectors $A_n^\pm(0)$ and $B_n^\pm(L)$ with components $A_n^\pm(0)$ and $B_n^\pm(L)$, respectively, obey the matrix equation

$$\begin{pmatrix} A_n^-(0) \\ B_n^+(L) \end{pmatrix} = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix} \begin{pmatrix} A_n^+(0) \\ B_n^-(L) \end{pmatrix}, \quad S = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}. \quad (4)$$
the diffusive law was shown in a full line. Second, all 

\[ U \]

\[ v \]

\[ c \]

\[ N \]

\[ \delta \]

\[ \rho \]

\[ \gamma \]

\[ k \]

\[ k_0 \]

\[ k_F \]

\[ k_\parallel \]

\[ l \]

\[ m \]

\[ n \]

\[ N_c \]

\[ N_e \]

\[ N_{e\parallel} \]

\[ N_{e\perp} \]

\[ C \]

\[ F \]

\[ h \]

\[ m \]

\[ \pi \]

\[ N \]

\[ N_c \]

\[ N_e \]

\[ N_{e\parallel} \]

\[ N_{e\perp} \]

\[ C \]

\[ F \]

\[ h \]

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\[ \pi \]

\[ N \]

\[ N_c \]

\[ N_e \]

\[ N_{e\parallel} \]

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\[ C \]

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\[ N_{e\parallel} \]

\[ N_{e\perp} \]

\[ C \]

\[ F \]

\[ h \]

\[ m \]

\[ \pi \]
words, the rough edges scatter all electrons except for a small part of those that move (in classical terms) almost in parallel with the edges. This small part, composed mainly of electrons occupying channel \( n = 1 \), hits the edges rarely and thus moves almost ballistically \([14]\). Eventually mainly the electron circulating at the Fermi velocity contributes, as in a one-channel ballistic ring. Thus \( I_{typ} \simeq ev_F/L \) albeit \( L \gg l \).

Now we discuss the assumption that the ring states coincide for \( L \gg W \) with the stripe states obeying the conditions (5). This standard approach describes the ring by the stripe-related Hamiltonian (Eq\(^2\)) that ignores the ring curvature. Consider the clean ring. If the ring curvature is included in the Hamiltonian, it produces the centrifugal force which is \( \propto \nu^2 \) and which pushes the radial wave functions towards the outer edge of the ring. Consequently, the radial wave functions become localized at the outer edge (especially for large \( \nu \)) and strongly differ from the form \( \sin(\eta r) \) even for \( L \gg W \). This result is exact in the non-interacting model but fails to describe metallic rings, because the localization of the radial wave functions at the outer edge produces the internal field due to the electron-ion and Hartree-Fock interaction. This internal field, ignored in the non-interacting model, tends to balance the centrifugal force and to delocalize the radial waves throughout the ring cross section. Once the balance is achieved, the resulting radial wave functions have to be close to the stripe-related form \( \sin(\eta r) \). Just this is implicitly assumed in the standard approach that omits the Hamiltonian both the ring curvature and the internal field. In reality a small deviation from \( \sin(\eta r) \) remains and produces the residual internal field balancing the centrifugal force \([16]\).

Consider the standard approach in terms of the semiclassical paths. Since the \( x \) axis is bent along the ring, the \( x \)-component of any straight-line path in the stripe is bent to follow the ring curvature; this curvature-mediated orbital effect is in fact due to the internal field that balances the centrifugal force. The standard approach thus strongly differs from the semiclassical-path-based approach \([17]\) that includes the ring curvature exactly in the non-interacting model but ignores the internal field. As the internal field is ignored, the paths that govern the wave functions are the straight lines \([17]\) and the radial wave functions are pushed toward the outer edge (this is manifested by the straight-line paths that hit solely the outer edge \([17]\)). If \( L \gg W \), any straight-line path unavoidably hits the ring edges many times \([17]\), unlike our path that circulates almost in parallel with the edges (for further insight see \([14]\)).

We now verify our estimates of persistent currents by microscopic calculations that rely on the standard approach. Using Eqs. (3), we write equations (5) in the matrix form

\[
\begin{pmatrix}
A^-(0) \\
B^+(L)
\end{pmatrix} =
\begin{pmatrix}
0 & Q^{-1}(\Phi) \\
Q(\Phi) & 0
\end{pmatrix}
\begin{pmatrix}
A^+(0) \\
B^-(L)
\end{pmatrix},
\]

where \( Q \) is the \( N \times N \) matrix with terms \( Q_{\alpha\beta} = e^{i2\pi \Phi/\Phi_0} \delta_{\alpha\beta} \). Equations (6) and (3) hold together for discrete energies \( E = E_j(\Phi) \) which we find for a given ring numerically \([10]\). Then we find \( I = -\sum_{E_j \leq E_F} dE_j/d\Phi \) and \( I_{typ} \equiv \langle I^2 \rangle^{1/2} \), where \( \langle I^2 \rangle \) is averaged over a small energy window at \( E_F \) \([10,14]\).

![Figure 3](image)

**Figure 3:** Typical persistent current \( I_{typ} \) in a disordered Au ring versus \( L/l \). The ring parameters are, \( \Phi = -0.25\hbar/e, l \) has been obtained from the wire resistivity (Fig. 2). The arrows point the parameters studied further in Ref. [13]. Symbols are our data, full lines show the formula \( I_{typ}^{ theor} = 1.6(ev_F/L)(l/L) \).

Figure 3 shows our main results. In the rings with grain boundaries, \( I_{typ} \) agrees (at large \( L \)) with expected result \( I_{typ}^{ theor} = 1.6(ev_F/L)(l/L) \), like in the experiments \([4,5]\). However, in the rings with rough edges, \( I_{typ} \) is systematically (not regarding the data fluctuations) close to the ballistic one-channel value \( I_0 = ev_F/L \), albeit \( L \gg l, N_c \gg 1 \) and \( \langle \rho \rangle \propto L \). All this agrees with the puzzling experiment \([2]\).

In the work \([2]\) the persistent current \( I_0 \) was observed in the Au ring with \( L \approx 100l \) and \( W = 90nm \). Indeed, the figure \( 3b \) shows \( I_{typ} \approx I_0 \) also for \( L/l \approx 100 \) and \( W = 90nm \). The difference is that in the work \([2]\) \( l \approx W \) (\( l = 70nm \) for \( W = 90nm \)) while our values of \( l \) in Fig. 3b \( [\text{also see Fig. } 2\text{a in } [14]) \] are at least two to three times larger than \( W \); the edge roughness alone cannot produce \( l \approx W \). In reality the edge roughness coexists with other types of disorder. Reference \([2]\) did not specify disorder in the measured samples, but Web mentions in Ref. \([18]\) that the grains in the Au rings of work \([2]\) were much larger than \( 1.5 \text{nm} \) while our values of \( W \) are at least two to three times larger than \( W \); the edge roughness alone cannot produce \( l \approx W \).

We now verify our estimates of persistent currents by microscopic calculations that rely on the standard approach. Using Eqs. 3, we write equations 5 in the matrix form

\[
\begin{pmatrix}
A^-(0) \\
B^+(L)
\end{pmatrix} =
\begin{pmatrix}
0 & Q^{-1}(\Phi) \\
Q(\Phi) & 0
\end{pmatrix}
\begin{pmatrix}
A^+(0) \\
B^-(L)
\end{pmatrix},
\]

where \( Q \) is the \( N \times N \) matrix with terms \( Q_{\alpha\beta} = e^{i2\pi \Phi/\Phi_0} \delta_{\alpha\beta} \). Equations (6) and (3) hold together for discrete energies \( E = E_j(\Phi) \) which we find for a given ring numerically \([10]\). Then we find \( I = -\sum_{E_j \leq E_F} dE_j/d\Phi \) and \( I_{typ} \equiv \langle I^2 \rangle^{1/2} \), where \( \langle I^2 \rangle \) is averaged over a small energy window at \( E_F \) \([10,14]\).
Wire/ring with rough edges and bamboo-like grains (d₀ = 7.7W)

| W[nm] | N₀ | δW[Å] | δ | d₀/[nm] | R₀ | α₀ | l/[nm] | ∆H/ε₀ |
|-------|----|-------|---|----------|----|-----|--------|--------|
| 90    | 347| 5.0   | 87| 700      | 0  | 2   | 0.2    | 85.2   |
| 90    | 347| 5.0   | 87| 700      | 0  | 2π/16| 97.0   | 0.90   |
| 90    | 347| 5.0   | 87| 700      | 0  | 2π/8 | 100   | 0.89   |

4. Transport in Au wires and Au rings with rough edges and bamboo-like grains. The angle α specifying the orientation of the grain boundary is chosen at random from the interval (−α₀, α₀), where α₀ = 0 means the ideal bamboo shape with the boundary perpendicular to the wire [20]. The table shows all parameters and the resulting l and ξ. Figure (a) shows the mean resistance <ρ> as a function of L/l, figure (b) show the transmission (T_n) versus L/l for α₀ = 0. The open symbols in figure (c) show the typical current in the ring, I_{typ}/I₀, as a function of L/l for various α₀, the full symbols show the corresponding maximum currents. Figure (d) shows the I_{typ} data from figure (c) normalized by I_{typ} = 1.6(ε_F/L)(l/L) and plotted in dependence on L/l.

with rough edges only [c.f. the right panel of Fig. 2(b)]. A suppression of the transmission, caused by a combined effect of the rough edges and bamboo-like grains, is visible for all 347 channels. Consequently, l is suppressed as well and we have l ≈ W. Similarly, the typical currents [Figs. 4(c) and 4(d)] are suppressed with the pure edge-roughness case [Fig. 3(b)], but they still grossly exceed the law I_{typ} = 1.6(ε_F/L)(l/L). Figure 4(c) shows the maximum currents, because the work [2] in fact reported the current amplitudes rather than I_{typ}. These amplitudes were between ~1.1I₀ and ~1.3I₀ and roughly the same (full symbols).

In conclusion, figure 3 naturally explains why the experiment 2 shows the result I_{typ} ≈ ε_F/L and experiments 3, 4 confirm the result I_{typ} ≈ (ε_F/L)(l/L). It suggests that disorder in samples of works 3, 4, 5 was white-noise-like (most likely mainly due to the random grain boundaries), while the dominant disorder in [2] was the edge roughness.

A few remarks at the end. (i) The samples of work 2 were 3D, but the 3D effects would change our 2D results insignificantly [14]. (ii) The step-shaped-roughness model in figure 1(b) is universal; our results hold also for any model with a smoothly varying roughness [14]. (iii) Our results are robust against the change of N_c, δ, ∆x, l, and L for a broad range of values. Therefore, the absence of the exact information on the nature of disorder in measured samples [4, 5] is not crucial for our conclusions. Anyway, our values of δ and ∆x are realistic (c.f. figures 1 and 2 in Ref. [21]). Experiments that would determine I_{typ} and l in correlation with the parameters of disorder can be useful. (iv) Note [14], that the transmissions T_{n=1} ≈ 1 in wires with rough edges have nothing in common with the bimodal distribution 1/√(1 − T^2), which exists in any diffusive conductor [13] and diverges for T = 1. Transmissions T in the bimodal distribution are the eigen-values of the t+τ matrix, while the meaning of our T_n is different.

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This supplemental material consists of six sections. In section I we explain in detail why our transport results obtained for the roughness model in figure (b) hold universally also for any other roughness model. In section II, the standard (wave-function-based) description of the ring states is compared with the semiclassical-paths-based description. In section III we provide a further insight into our result and focus solely on the typical current. In section V we explain in detail why our 2D study gives the results that hold very well also for 3D samples. Finally, in section VI we stress that the transmission eigen-value for any roughness model which is specified by the RMS parameter $\delta$ and roughness-correlation length $\Delta x$. We expect that the obtained transport results will agree with the results presented in the main text, if one compares the dependencies on the parameter $L/\xi$, where $\xi$ is the localization length. This expectation is motivated by a few fundamental findings.

First, the statistical ensemble of the macroscopically-identical mesoscopic conductors with impurity disorder is known to exhibit the conductance distribution which is essentially the same (for a given value of parameter $L/\xi$) for any model of the impurity disorder model; the weaker the disorder the better the accord of the conductance distribution of various models. Second, it seems that a similar universality (the independence on the specific model of disorder) holds also when the impurity disorder is replaced by disorder due to the edge roughness. In particular, the conductance calculations in Ref. [13], performed for the same step-shaped-roughness model as our model in figure (b), gives a quite similar results as the conductance calculations in the paper [SM1], performed for the smoothly varying roughness with the Gaussian correlation function. To demonstrate this universality by means of the direct comparison, we have performed the conductance calculations for the smoothly-varying roughness with the Gaussian correlation function (the model of Ref. [SM1], and we have compared them with our results for the step-shaped roughness in figure (b).

In figure (b) we show a typical output of such comparative study for two Au wires with the same number of the conducting channels ($N_c = 34$), so that one can compare directly the individual channel transmission. It can be seen that the in-
individual transmissions are in a very good agreement, which illustrates the above mentioned universality; note also that the individual transmissions for both roughness models coincide albeit the values of the parameters \( \delta \) and \( \Delta x \) in the considered roughness models are (intentionally) not the same. In addition, the universality with respect to the choice of \( \delta \) and \( \Delta x \) within the same roughness model is obvious for all our data in the main text and especially from our paper [7]. The last but not least, our main result (the ballistic-like persistent current \( I_{up} \approx e v_F/L \) in figure 3(b)) is universal simply because of the absence of the sensitivity to the edge roughness.

Any user of the scattering-matrix technique can perform a similar universality demonstration for other types of the smooth roughness. The prize for the use of the smoothly varying roughness is a much longer computational time, which is crucial especially for the persistent current calculations. Due to this reason our study in the main text relies on the step-shaped roughness, however, our results are universal as mentioned above.

II. Comparison of the standard wave-function-based approach with the semiclassical-path-based approach

The persistent current calculations presented in the main text rely on the standard approach. The key assumption of the standard approach, justified in the main text, is that the ring states coincide for \( L \gg W \) with the stripe states obeying the conditions 5. Just this assumption allows to describe the ring states by the stripe-related Hamiltonian (Eq 2) that ignores the ring curvature. The main text also mentions that the standard approach strongly differs from the semiclassical-path-based approach [17] that incorporates the ring curvature rigorously (within the non-interacting model) but ignores the internal Hartree-Fock-interaction-mediated field balancing the centrifugal force. In this approach the semiclassical paths in the ring fundamentally differ from those in the stripe, or in other words, the ring states and stripe states do not coincide. We wish to give a few more comments on the semiclassical-path-based approach [17].

Consider first the clean ring. In spite of the fact that the radial wave functions are pushed towards the outer edge of the ring by the centrifugal force, the semiclassical-path-based approach by Jalabert et al. [17] gives correct result for the ballistic persistent current. The problem arises when one considers the rough edges (see Samokhin’s paper [17]). Since only the straight-line paths are allowed (neglecting the orbital effect due to the magnetic field), in the annular geometry with \( L \gg W \) any straight-line path unavoidably hits the ring edges many times when it makes one trip around the ring [17]. On the contrary, the stripe geometry allows also very long straight-line paths that are (almost) parallel with the stripe edges and therefore do not feel the edge roughness. Within the approach of Refs. [17], these very long ballistic paths are changed in the annular geometry on the paths that hit exclusively the outer edge of the ring and become scattered by the edge roughness. On the other hand, in our standard approach these paths remain parallel with the ring edges and carry the ballistic-like current. This fundamental difference is due to the fact that the standard approach implicitly incorporates the balance between the centrifugal force and internal Hartree-Fock field (see the main text), while in the semiclassical-path-based approach [17] the centrifugal force remains unbalanced.

We also wish to address another fundamental difference. Samokhin [17] assumed that the straight-line path that hits the ring edge is reflected diffusively no matter what is the incidence angle (the angle between the path and the edge). Specifically, the probability of the diffusive reflection from the edge is described by the Fuchs coefficient which is equal to unity for all incidence angles. It should be mentioned that a realistic probability of the diffusive reflection, derived by Soffer and Ziman [SM2] for a free wave impinging the surface with uncorrelated roughness, strongly depends on the incidence angle. In particular, it is equal to unity for perpendicular incidence but approaches zero for small incidence angles. Samokhin [17] found in the ring with rough edges the diffusive persistent current. However, he would certainly not find such diffusive current, if the Fuchs coefficient is replaced by the angle-dependent probability of the diffusive reflection due to Ziman and Soffer [SM2]: Owing to the (almost) specular reflections at small angles, his result would most likely become more similar to our ballistic prediction. In our paper the correct angle dependence of the edge roughness scattering is included microscopically in the scattering matrix method.

Indeed, the tendency to a specular reflection at small angles is manifested by the channel transmission \( T_n \). Let us look at the right panel of figure 2b in detail. Semiclassically, the channel number \( n \) corresponds to the angle between the semiclassical trajectory and edge, and \( n = 1 \) corresponds to the smallest nonzero semiclassical angle allowed by the quantum confinement. Consider, say \( L \simeq 0.25 \xi \simeq 120 l \). In case of the diffusive reflection, for \( L/l = 120 \) one should observe
The current most ballistically. The resulting current, a single-channel ballistic ring.

...small part, composed mainly of electrons occupying channel contributions from other electrons tend to cancel like in a true (classically speaking) almost in parallel with the edges. This is due the electron that circulates with Fermi velocity, because the ring with rough edges supports the ballistic-like current for the currents with rough edges depends on the number of channels (\(N\)). Now we demonstrate that this is indeed the case.

FIG. S 3: Typical persistent current \(I_{typ}\) in the ring with rough edges as a function of the total number of channels (\(N\)) considered in the simulation. The same parameters and symbols are used as in figure 3b, the considered ring lengths are shown as \(L/l\).

Since \(\langle T_1 \rangle \sim 1\), one could naively think that the value \(I_{typ} \sim I_0\) will survive also if one chooses \(N\) as small as \(N = 1\). Figure 3 shows that this is not the case. For instance, in the ring with \(N_c = 347\) and \(L/l = 120\) the current approaches zero just for \(N \rightarrow 1\). This is easy to understand: Once the channel \(n = 1\) cannot communicate with other channels, the transmission \(\langle T_1 \rangle \sim 1\) tends to be suppressed to zero by Anderson localization, present in any sufficiently long 1D disordered system. Communication with a few other channels is needed to restore \(\langle T_1 \rangle \sim 1\) and to obtain \(I_{typ} \sim I_0\).

To provide further insight, figure 3 shows the sample-specific currents in two selected rings from figure 3 (bold arrows) and in a clean ring. Figure 3(a) shows the dependence \(I_j\) versus \(E_j\), figure 3(b) shows the total current \(I = \sum_{E_j < E_F} I_j\) versus \(E_F\). Evidently, the ring with rough edges exhibits remarkably larger currents than the ring with grain boundaries, albeit both rings are of the same size and posses the same value of \(l\).

Figures 3(c) and 3(d) focus on a small energy window below the Au Fermi level. One can see that \(I_j\) in the ring with rough edges exhibits sharp peaks with the sign alternating and oscillating with period \(\Delta E = 2n\hbar v_F / L\). This period is twice the inter-level distance in the ballistic single-channel ring, which suggests that the peaks are due to the quasi-ballistic channel \(n = 1\). [We recall that \(\langle T_1 \rangle \sim 1\) also for \(L/l \gg 1\), as is shown in the right panel of figure 2b.)] However, the height of the peaks is affected also by other channels, because, as discussed above, channel 1 cannot keep \(\langle T_1 \rangle \sim 1\) without communicating with a few other channels.

In figure 3(d) one can see that in the ring with rough edges also the total current \(I(E_F)\) oscillates with period \(\Delta E\). The amplitudes of the total current are close to \(I_0\), and therefore the typical currents of size \(\sim I_0\) appear in figure 3b.

In fact, already the data for the clean ring show \(I(E_F)\) oscillating with period \(\Delta E\). However, the amplitude of \(I\) is \(\sim \sqrt{N_c}\) and the amplitude of \(I_0\) is \(2I_0\), where the factor of 2 is due to the spin. Evidently, the rough edges reduce \(I\) from \(\sim \sqrt{N_c}\) to \(\sim I_0\), but they do not change the oscillation period set by the clean ring. Note that also the ring with grain boundaries exhibits the oscillating persistent current. These oscillations are chaotic and correlated with correlation length \(\sim (l/L)\Delta E\), predicted for the white-noise-like disorder.
dashed line. It can be seen that a reliable estimate of $I_{\text{typ}}$ in the ring with grain boundaries requires $N \gtrsim N_c$, while for the ring with rough edges one only needs $N \sim 10$ no matter how large $N_c$ is. This is due to the effective number $N_c^{\text{eff}} \sim 10$, as has already been explained in the beginning of this section.

IV. The problem of the mean persistent current

In our present work we have focused on the typical current and we have not discussed the mean current. The sign and amplitude of the mean current, measured in the pioneer experiment by Levy et al. [1] is another puzzling problem in the field. As can be seen from our description of the scattering matrix method, we can in principle provide also numerical data for the average persistent current. There are however a few serious reasons why our present manuscript is not focused on the average current.

First, the problem of the average current has been addressed by Bary-Soroker et al. [SM3] who attempted to explain it within the interacting electron model. However, these authors did not address the problem of the typical current, and the experiments [4, 5] showed, that the typical current is most likely not affected by electron-electron interaction and should be tractable within the non-interacting model. Just these reasons lead us to focus on the problem of the giant typical current and to solve it within the single-electron model.

Second, a complete scattering-matrix study of the amplitude and sign of the average current would require (perhaps) ten to hundred times more computational time than the study of the typical current, presented in our present manuscript. We have therefore decided to focus on the problem of the typical current, and already this problem was computationally cost.

V. On the robustness of our 2D results against the 3D effects

All our transport data in the main text were obtained within the 2D model depicted in figure 1 while the experimental samples of reference [2] were three-dimensional. Here we want to point out in detail that the extension of our 2D study to 3D (replacement of the rough edges by rough side-walls) would not change our major results remarkably. The effect of 3D on our 2D results can be estimated easy without any explicit calculation as follows.

In our 2D wire (1(b)) only the edge roughness scattering is considered, while in the 3D wire of reference [2] the roughness scattering is in general due to the wire edges (side-walls) and due to the top and bottom surfaces in addition. In spite of this difference it is evident that the 3D sample still preserves the key feature of our 2D model: The electrons which occupy the ground 1D channel (now the channel with quantum numbers $n_y = 1$ and $n_z = 1$, where $z$ is the vertical direction) still move almost in parallel with the sample edges and sample surfaces, and therefore avoid the roughness scattering quite similarly as in the 2D case. Due to this key feature, the almost
perfect transmission of the Fermi electron in the ground 1D channel, found for the 2D wire (Fig.2), has to persist also in the 3D wire. Owing to this ballistic Fermi electron, the 3D ring has to carry the persistent current $I_{typ} \simeq e v_F / L$ also for $L/l >> 1$, just like the 2D rings in figure[3(b)].

Furthermore, one can easily see that the roughness-mediated scattering in 3D does not modifies the mean free path $l$ in comparison with 2D remarkably. As already mentioned, in the 3D wires one should consider also the roughness-mediated scattering from the top surface and bottom surface. However, in real normal-metal 3D wires the roughness amplitude (RMS) of the top and bottom surfaces is usually of the order of one lattice constant ($\sim 0.5$ nm; see e.g. the paper [SM4]), which is one order of magnitude less than the roughness amplitude at the edges (RMS $\sim 5$ nm - 10 nm; see the experiment of [21], and our manuscript). Since the roughness-limited mean free path is proportional to the square of the RMS (see e.g. our paper [2] or references cited therein), the effect of the top and bottom surfaces on the mean free path is two orders of magnitude weaker than the effect of the edges. In other words, it is very likely, that in the 3D wires of reference [2] the roughness-mediated scattering is mainly due to the wire edges, while the reflections from the almost smooth top/bottom surface are (almost) specular and thus do not affect the electron transport. The case when the roughness of the top and bottom surfaces is comparable with the edge roughness is experimentally unlike: The roughness of the edges is due to the limitations of the electron lithography and lift-off, due to the roughness of the resist side walls, etc. (see [21]), while the origin of the surface roughness is quite different. However, even in this unlike case the rough surfaces would marginally affect the quasi-ballistic channel $n_y = 1$ and $n_z = 1$ and they would only reduce the edge-roughness-limited mean free path by a factor of $\sim 2$, as can be estimated from the Matthiessen rule.

Finally, unlike our 2D wire in figure[1(b), the edges of the 3D wire are in fact the side walls and the edge roughness at such side walls in general scatters the electrons also into the vertical (z) direction, in addition to the in-plane scattering considered in our 2D model. This may decrease the roughness-limited mean free path say by a few tens of percent, but cannot affect the fundamental feature (the ballistic-like motion in the ground 1D channel) responsible for the persistent current $I_{typ} \simeq e v_F / L$ also for $L/l >> 1$.

In principle, our scattering-matrix approach allows to produce also the numerical data for the 3D samples (we already did so for the wires with the grain boundaries in reference[10]), albeit not for the samples as large as those used in the experiment[2]. However, the computational time would be quite huge and we do not expect any new features in comparison with our 2D results.

VI. Feature $T_{n=1} \simeq 1$ in the wire with rough edges and $T = 1$ as a general feature of any diffusive wire: Two different things

We recall that the ring with the rough edges exhibits the persistent current $I_{typ} \simeq e v_F / L$ at $L \gg l$ (figure[3(b)], because the constituting rough wire exhibits the transmission $T_{n=1} \simeq 1$ (right panel of figure[2(b)]. We want to stress that the transmission $T_{n=1} \simeq 1$ in the wire with rough edges has nothing in common with the well-known bimodal distribution $1/\sqrt{(1-T)T^2}$, which exist in any diffusive conductor [13] and diverges for $T = 1$. Note that the transmissions $T$ in the bimodal distribution are the eigen-values of the $t+t$ matrix [13], while we speak about $T_n = \sum_{m=1}^{NC} |t_{n,m}|^2$, which are the diagonal elements of the $t+t$ matrix. In other words, the channels corresponding to the eigen-values $T$ in the distribution $1/\sqrt{(1-T)T^2}$ are the eigen-states of the $t+t$ matrix, while the channels corresponding to our diagonal elements $T_n$ are the usual plane-wave states. To avoid misunderstanding by readers who are accustomed to the latter definition, we add a few more remarks.

The bimodal distribution $1/\sqrt{(1-T)T^2}$ as a general property of any diffusive conductor with white-noise-like disorder [13] coexists with the diffusive persistent current $I_{typ} \simeq (e v_F / L)(l/L)$ in the corresponding disordered ring [6]. In other words, the eigenvalues $T = 1$ in the bimodal distribution do not cause any ballistic persistent current. The reason why the current is diffusive in spite of $T = 1$, is most likely that the eigenvalue $T = 1$ does not necessarily mean the ballistic motion (a well known example is the perfect transmission in case of resonant tunneling). The situation becomes fundamentally different for disorder due to the rough edges. In this case the eigen-values $T_n$ still follow the bimodal distribution $1/\sqrt{(1-T)T^2}$, however, this has nothing in common with the ballistic-like persistent current found by us. The ballistic-like current is due to the appearance of the diagonal element $T_{n=1} \simeq 1$: Namely, any wire from the statistical ensemble of wires with rough edges exhibits the diagonal element $T_{n=1} \simeq 1$ independently on the choice of the Fermi energy and wire length. These features are the attributes of ballistic electron motion within channel $n = 1$. Indeed, it is easy to check in our simulation, that the electron plane wave entering the wire in the channel $n = 1$ remains (almost) unscattered between any two successive scatterers inside the disordered region. As a result, the ring made of such wire supports the persistent current with typical size dominated by the ballistic channel $n = 1$, that is, $I_{typ} \simeq e v_F / L$. In summary, the reason for appearance of $I_{typ} \simeq e v_F / L$ is the ballistic behavior of the diagonal element $T_{n=1}$; the fact that the bimodal distribution gives eigenvalues $T = 1$ is irrelevant.

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