Determining the Energy Barrier for Decay out of Superdeformed Bands

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An asymptotically exact quantum mechanical calculation of the matrix elements for tunneling through an asymmetric barrier is combined with the two-state statistical model for decay out of superdeformed and normal-deformed wells for several nuclei in the 190 and 150 mass regions. The spin-dependence of the barrier leading to sudden decay out is shown to be consistent with the decrease of a centrifugal barrier with decreasing angular momentum. Values of the barrier frequency in the two mass regions are predicted.

I. INTRODUCTION

Since their first experimental observation \(^1\), superdeformed (SD) nuclear states, with their strong ellipsoidal deformation and special set of shell closures, have offered a tantalizing and unique window into subatomic physics. Their rapid decay-out, in particular, has been the subject of great interest (e.g., Refs. [2–15]). In the standard theoretical approach \(^2\), this process is modeled by a two-well potential function of deformation: Here, the nucleus is a single quantum mechanical particle, which tunnels between the two wells, and can escape the system via electromagnetically induced decay from either. Because the barrier between the SD and normal-deformed (ND) wells is a direct consequence of nucleon–nucleon interactions, an understanding of its shape for various nuclei and angular momenta would be of considerable importance to the study of nuclear structure. Thus, a common objective of theoretical studies is to bridge the gap between measured experimental data, such as lifetimes and nuclear spins, and the shape of this barrier. In this Letter, we show that a rapid decrease in barrier height with decreasing nuclear spin explains the SD decay mechanism.

It was previously shown \(^10\) that an elegant, two-state model \(^11\) of SD decay-out is sufficient to give an excellent picture of the system’s time-evolution. One of the principal advantages of such a straightforward technique was the extraction from experiment of such important quantities as the tunneling matrix element \(V\) and the spreading width for tunneling through the barrier \(\Gamma\). The purpose of the present Letter is to move beyond these phenomenological quantities, and extract the height of the barrier itself as a function of nucleus and nuclear spin, as shown in Fig. 1. Previous approaches \(^14\) used a semiclassical treatment that did not allow for an accurate computation of the tunneling rate prefactor. As we shall see, this can introduce a potential error of several orders of magnitude in the estimation of the tunneling width. In this paper, we go beyond previous treatments by computing the tunneling rate in a systematic and controlled fashion using a functional integral approach \(^17\).

II. PATH-INTEGRAL APPROACH TO TUNNELING

Absent additional information about the nuclear shell potential, the smoothest potential describing both the tunnel barrier and the SD well (which lies far above the ND yrast line at decay out) is a cubic polynomial:

\[
U(x) = M\omega^2 x^2/2 - \lambda x^3,
\]

where \(x\) is a coordinate describing the quadrupole deformation of the nucleus \((x = 0\) corresponds to the bottom of the SD well, while the ND well occurs for \(x > x_B = M\omega^2/3\lambda\)), \(\omega\) is the oscillator frequency of the SD well, and \(M\) is the inertia of the quadrupole vibrational mode. With a suitable choice of the parameters \(M\omega^2\) and \(\lambda\), \(U(x)\) provides a maximum entropy
(least-biased) fit to the unknown nuclear potential barrier. Note that for this simple potential, the barrier frequency \( \omega_B \equiv \sqrt{|U''(x_B)|/M} = \omega \).

The use of Euclidean complex-time path integrals over the tunneling coordinate provides a systematic approach for determining quantum tunneling rates at arbitrary temperature and dissipation [17], as an asymptotic expansion in \( \hbar \). This method allows calculation not only of the leading-order exponential dependence of the tunneling rate on potential parameters, but also the more computationally difficult subdominant asymptotics (i.e., the prefactor term). For tunneling through the potential barrier \( \Pi \) at zero temperature and friction [18–21], the mean-square tunneling matrix element out of the SD yrast state is found to be:

\[
\langle V^2 \rangle = \hbar \omega D_N \left( \frac{54 W}{\pi^2 \hbar \omega} \right)^{1/2} \exp\left( -\frac{36 W}{5 \hbar \omega} \right), \tag{2}
\]

where \( W \equiv U(x_B) = M^3 \omega^6/54\lambda^2 \) is the barrier height (as measured from the bottom of the SD well), and \( D_N \) is the mean level spacing in the ND band at the energy of the decaying SD state. The action \( S \) to tunnel out of the SD state through the barrier is proportional to the argument of the exponential function in Eq. (2):

\[
-\frac{2S}{\hbar} = -\frac{36 W}{5 \hbar \omega}, \tag{3}
\]

where the factor of two is due to the power of \( V \).

To make contact with experiment, the tunneling matrix element may be estimated using the two-state model of SD decay [5, 10, 15], which assumes the decay-out process is dominated by coupling of each SD state with its nearest-lying energy level in the ND band. The branching ratios \( F_N \) and \( F_S = 1 - F_N \) for decay out and intraband decay, respectively, are determined by three rates [10]:

\[
F_N = \frac{\Gamma_N \Gamma^1/(\Gamma_N + \Gamma^1)}{\Gamma_S + \Gamma_N \Gamma^1/(\Gamma_N + \Gamma^1)}, \tag{4}
\]

where \( \Gamma_S/\hbar \) and \( \Gamma_N/\hbar \) are the electromagnetic decay rates of the SD and ND states, respectively, and \( \Gamma^1/\hbar = \frac{2\hbar^2}{(\Delta^2 + \Gamma^2)} \) is the nucleus’ net tunneling rate through the barrier, with \( \Gamma = \frac{1}{3} (\Gamma_S + \Gamma_N) \), and \( \Delta \) the energy difference between the SD and ND states. Given the experimentally determined branching ratios and the electromagnetic widths, \( \Gamma^1 \) is known [15]:

\[
\Gamma^1 = \Gamma_S / \left( \frac{F_S}{F_N} - \frac{\Gamma_S}{\Gamma_N} \right). \tag{5}
\]

The tunneling matrix element \( V \) may then be determined statistically [10, 15], assuming the SD and ND levels are uncorrelated, and that the ND levels obey the Wigner surmise. The mean-square tunneling matrix element is found to be [15]

\[
\langle V^2 \rangle = D_N^2 \Gamma^1 / 6\pi \Gamma, \tag{6}
\]

where a numerically negligible correction, whose relative size is \( \mathcal{O}(\Gamma/D_N)^2 \), has been omitted.

Eqs. (4) and (5) may be combined to yield an expression for the tunneling width in terms of the properties of the nuclear potential barrier:

\[
\Gamma^1 = \frac{18 \hbar \omega \hat{\Gamma}}{D_N} \left( \frac{6W}{\pi \hbar \omega} \right)^{1/2} \exp\left( -\frac{36 W}{5 \hbar \omega} \right). \tag{7}
\]

Note that this result for the net tunneling width, which includes tunneling and electromagnetic decay on an equal footing, differs by a factor of \( 3\Gamma^1/D_N \) from the bare tunneling width into an infinitely broadened, fully continuum ND spectrum. From the values of \( \Gamma_S, \Gamma_N, \) and \( D_N \) listed in Table I, one sees that usage of such a bare ND-continuum result [5, 22] could result in an error of several orders of magnitude.

The energy barriers (in units of the barrier frequency) obtained by solving Eq. (7) for all SD decays for which the four parameters, \( F_N, \Gamma_S, \Gamma_N, \) and \( D_N \), are known are listed in Table I (see also Fig. I). Also listed is the tunneling action \( S = 3.6W/\hbar \omega \), which is a characteristic measure of the opaqueness of the barrier [5, 22]. Note that \( W/\hbar \omega \) depends only weakly (logarithmically) on the barrier frequency \( \omega \). In the literature, the value \( \hbar \omega = 0.6 \text{MeV} \) has been used [5, 22], but we shall determine \( \omega \) self-consistently in Section III.

We note that, of the four parameters in Eq. (7), only \( F_S \) and \( \Gamma_S \) are directly measured experimentally; typically, these are known to within a few percent. \( \Gamma_N \) and \( D_N \) must be calculated theoretically, with models fit to experimental data. The uncertainties in \( \Gamma_N \) and \( D_N \) could thus be appreciable, perhaps as large as a factor of two or more. A better determination of these quantities is a worthy goal for future studies of SD nuclei, but goes well beyond the scope of the present Letter.

For almost all decay-out sequences, we find that the barrier height decreases with decreasing angular momentum. One exception is the odd-spin \(^{194}\text{Hg} \) sequence, for which the two highest-spin calculated barriers are so close that statistical fluctuations about the mean-square matrix elements of Eqs. (2) and (6) are sufficient to reverse the trend. This could occur, for example, due to an accidental near-degeneracy of the SD and ND states in \(^{194}\text{Hg}(15) \), which would lead to a larger than expected branching ratio \( F_N \). The other exception is the first parameter set for \(^{194}\text{Pb}(10) \). The primary difference between the first and second parameter sets for \(^{194}\text{Pb} \) is Ref. [12]’s revised treatment of the pairing gap; it is thus seen that this consideration may play an important role in understanding the spin dependence of the SD decay-out barrier [33].

### III. CENTRIFUGAL TUNNEL BARRIER

Finally, we address whether the decrease in the tunnel barrier with decreasing spin (cf. Fig. I and Table I) is consistent with the centrifugal barrier of a spinning nucleus.
If the superdeformed nucleus and the saddle configuration at the top of the energy barrier are described as rigid rotors with moments of inertia $I_S$ and $I_B$, respectively, then the barrier height $W(I)$, as a function of the angular momentum quantum number $I$, is simply the $I = 0$ barrier $W(0)$, plus the rigid-rotor rotational increase in the energy of the barrier configuration, minus the rotational increase of the bottom of the SD well (from which $W(I)$ is measured), i.e.,

$$W(I) = W(0) + \frac{\hbar^2 I(I+1)}{2} \left( \frac{1}{I_B} - \frac{1}{I_S} \right). \quad (8)$$

Although the rigid-rotor model is a simplification, nevertheless the decrease in the barrier height between successive SD states can be rigorously expressed in terms of the kinetic moments of inertia:

$$W(I) - W(I-2) = \hbar^2 (2I-1) \left( \frac{1}{I_B^{(1)}} - \frac{1}{I_S^{(1)}} \right). \quad (9)$$

The kinetic moments of inertia $I_S^{(1)}$ of several SD yrast states in the 150 and 190 mass regions have been measured. For $^{152}$Dy, $I_S^{(1)} = 8592^2$/MeV and the aspect ratio $\eta \equiv b/a = 2.0$, with $a$ and $b$ the smaller and larger radii of the nucleus, respectively. For $^{192}$Hg, $I_S^{(1)} = 9092^2$/MeV and the aspect ratio $\eta = 1.65$. The moment of inertia of the barrier configuration $I_B^{(1)}$ is not measured, but must be determined theoretically. This can be done by applying the Strutinsky shell correction method to the cranking model. However, to account for pairing, we employ a phenomenological two-fluid model in which only the region outside the largest possible central sphere contributes to the moment of inertia. Within this two-fluid model, we find that the moment of inertia is

$$I^{(1)} = m_n r_0^2 \left( \frac{A}{\eta} \right)^{5/3} \frac{\eta^5 + \eta - 2}{5}, \quad (10)$$

where the nucleus has been taken as a prolate ellipsoid of

| nucleus(I) | $F_N$ | $\Gamma_S$ | $\Gamma_N$ | $D_N$ | $\Gamma^I$ | $\hbar \omega$ | $W/h\omega$ | $S/h$ | Refs. |
|-----------|-------|-----------|-----------|------|-----------|-------------|------------|------|-------|
| $^{192}$Hg(12) | 0.26 | 0.128 | 0.613 | 135. | 0.049 | 0.24 | 1.8 | 6.5 | [13, 24] |
| $^{192}$Hg(10) | 0.92 | 0.050 | 0.733 | 89. | 0.37 | 0.24 | 1.3 | 4.7 | [13, 24] |
| $^{192}$Pb(16) | <0.01 | 0.487 | 0.192 | 1.362. | <0.0050 | 0.24 | >1.8 | >6.5 | [8, 12] |
| $^{192}$Pb(14) | 0.02 | 0.266 | 0.201 | 1.258. | 0.0056 | 0.24 | 1.7 | 6.2 | [8, 12] |
| $^{192}$Pb(12) | 0.34 | 0.132 | 0.200 | 1.272. | 0.10 | 0.24 | 1.3 | 4.6 | [8, 12] |
| $^{192}$Pb(10) | 0.88 | 0.048 | 0.188 | 1.410. | 1.9 | 0.24 | 0.76 | 2.7 | [8, 12] |
| $^{192}$Pb(8) | >0.75 | 0.016 | 0.169 | 1.681. | >0.067 | 0.24 | <1.2 | <4.3 | [8, 12] |
| $^{194}$Hg(12) | 0.42 | 0.097 | 4.8 | 16.3 | 0.071 | 0.24 | 2.3 | 8.4 | [25, 26, 27, 28] |
| $^{194}$Hg(10) | >0.91 | 0.039 | 4.1 | 26.2 | >0.44 | 0.24 | <2.0 | <7.1 | [25, 26, 27, 28] |
| $^{194}$Hg(12) | 0.40 | 0.108 | 21. | 344. | 0.072 | 0.24 | 2.1 | 7.6 | [29] |
| $^{194}$Hg(10) | 0.97 | 0.046 | 20. | 493. | 1.6 | 0.24 | 1.6 | 5.8 | [29] |
| $^{194}$Hg(12) | 0.40 | 0.086 | 1.345 | 19. | 0.060 | 0.24 | 2.2 | 7.8 | [13, 27] |
| $^{194}$Hg(10) | >0.95 | 0.033 | 1.487 | 14. | ≥1.1 | 0.24 | <1.8 | <6.5 | [13, 27] |
| $^{194}$Hg(15) | 0.10 | 0.230 | 4.0 | 26.5 | 0.026 | 0.24 | 2.4 | 8.6 | [27, 28] |
| $^{194}$Hg(13) | 0.16 | 0.110 | 4.5 | 19.9 | 0.021 | 0.24 | 2.5 | 8.9 | [27, 28] |
| $^{194}$Hg(11) | >0.93 | 0.048 | 6.4 | 7.2 | >0.71 | 0.24 | <2.2 | <7.8 | [27, 28] |
| $^{194}$Pb(10) | 0.10 | 0.045 | 0.08 | 21.700. | 0.0053 | 0.24 | 1.1 | 4.1 | [28, 30, 31, 32] |
| $^{194}$Pb(8) | 0.38 | 0.014 | 0.50 | 2.200. | 0.0087 | 0.24 | 1.6 | 5.8 | [28, 30, 31, 32] |
| $^{194}$Pb(6) | >0.91 | 0.003 | 0.65 | 1.400. | >0.032 | 0.24 | <1.5 | <5.5 | [28, 30, 31, 32] |
| $^{194}$Pb(12) | <0.01 | 0.125 | 0.476 | 236. | <0.0013 | 0.24 | >2.2 | >8.0 | [12, 28] |
| $^{194}$Pb(10) | 0.10 | 0.045 | 0.470 | 244. | 0.0051 | 0.24 | 2.0 | 7.2 | [12, 28] |
| $^{194}$Pb(8) | 0.35 | 0.014 | 0.445 | 273. | 0.0077 | 0.24 | 1.9 | 6.9 | [12, 28] |
| $^{194}$Pb(6) | >0.96 | 0.003 | 0.405 | 333. | >0.088 | 0.24 | <1.5 | <5.4 | [12, 28] |
| $^{152}$Dy(28) | 0.40 | 0.10 | 17. | 220. | 11. | 0.56 | 1.6 | 5.8 | [29] |
| $^{152}$Dy(26) | 0.81 | 0.70 | 17. | 194. | 140. | 0.56 | 1.2 | 4.4 | [29] |
revolution with aspect ratio $\eta$ and atomic mass number $A$, $m_n$ is the mass of a nucleon, and $r_0 = 1.27$fm. With these parameters, the measured kinetic moments of inertia of $^{152}$Dy and $^{192}$Hg at decay-out are reproduced to within 1 To leading order in the quadrupole deformation parameter $\varepsilon$ (see Ref. [38], Eq. (10) gives

$$
\mathcal{T}^{(1)} \approx \frac{4\varepsilon}{5} A^{2/3} m_n r_0^2. 
$$

(11)

For $^{152}$Dy, the barrier occurs at an aspect ratio of $\eta = 1.7$ [38], so that $\mathcal{T}^{(1)}_S/\mathcal{T}^{(1)}_B = 1.3$ and $\Delta W = W(28) − W(26) = 0.21$MeV. Assuming a constant barrier frequency, and comparing to the results from Table I ($\Delta W/h \omega = 0.37$), implies a barrier frequency $h \omega = 0.56$MeV.

For $^{192}$Hg, the barrier is estimated to occur at an aspect ratio of $\eta \approx 1.4$ [34], so that $\mathcal{T}^{(1)}_S/\mathcal{T}^{(1)}_B = 1.5$ and $\Delta W = W(12) − W(10) = 0.12$MeV. Assuming a constant barrier frequency, and comparing to the results from Table II ($\Delta W/h \omega = 0.51$), implies a barrier frequency $h \omega = 0.24$MeV. Because the logarithmic dependence of $W$ on $\omega$ almost completely cancels out in such a calculation, the differences $\Delta W/\omega$ and $\Delta S$ are nearly independent of the choice of $\omega$.

IV. CONCLUSIONS

In conclusion, we have determined the barrier height $W$ and tunneling action $S$ for decay-out of a superdeformed band by combining an asymptotically exact quantum tunneling calculation with a two-state dynamical model. The Table presents our numerical results for all superdeformed decays for which sufficient experimental data are known. We find that the tunnel barrier decreases significantly with decreasing spin during the decay-out process. The spin-dependence of the barrier is explained quantitatively in terms of the variation of the centrifugal barrier within a two-fluid model of nuclear rotation, which in turn allows us to self-consistently predict the tunnel barrier’s curvature.

The results presented in this Letter complete the chain of reasoning needed to connect the intriguing phenomenology of the decay-out process in superdeformed nuclei with an understanding of the underlying nuclear structure. Our results indicate that the rapidity and universality of the decay-out profiles can be explained straightforwardly within our two-state dynamical model by the decrease of the centrifugal barrier between the super-deformed and normal-deformed energy wells with decreasing spin.

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