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STUDY OF \( \eta \pi^- \) PRODUCTION BY PIONS IN THE COULOMB FIELD

D.V. Amelin, E.B. Berdnikov, S.I. Bityukov,
G.V. Borisov, R.I. Dzhelyadin, V.A. Dorofeev,
A.V. Ekimov, Yu.P. Gouz\(^1\), A.K. Konoplyannikov,
A.N. Karyukhin, I.A. Kachaev, Yu.A. Khokhlov,
V.F. Konstantinov, S.V. Kopikov, M.E. Kostrikov,
V.V. Kostyukhin, S.A. Likhoded, V.D. Matveyev,
A.P. Ostankov, D.I. Ryabchikov, O.V. Solovianov,
A.A. Solodkov, E.A. Starchenko, N.K. Vishnevsky,\(^1\)
E.V. Vlassov, A.M. Zaitsev\(^2\)
IHEP, Protvino, Russia
G.G. Sekhniaidze, E.G. Tskhadadze,
IPh, Tbilisi, Georgia

Abstract
The production of \( \eta \pi^- \) system at low \( M_{\eta\pi^-} \) by the \( \pi^- \) beam in the Coulomb field of Be nuclei was studied. The cross section of the reaction \( \pi^- Be \rightarrow \eta \pi^- Be \) was compared to the experimental data on the decay \( \eta \rightarrow \pi^+\pi^-\gamma \) and to the theoretical predictions.

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\(^1\)E-mails: gouz@mx.ihep.su, Iouri.Gouz@cern.ch
\(^2\)E-mails: zaitsev@mx.ihep.su, Alexandre.Zaitsev@cern.ch

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Introduction.

We present here the study of the $\eta\pi^-$ system production at low mass ($M_{\eta\pi^-} < 1.18$ GeV/$c^2$) in the Coulomb field of $Be$ nuclei

$$\pi^- Be \rightarrow \eta\pi^- Be. \quad (1)$$

The subject of our study is the process

$$\pi^- \gamma \rightarrow \eta\pi^-. \quad (2)$$

Its amplitude at low $M_{\eta\pi^-}$ can be expressed in the following form:

$$M_{\eta\pi\pi\gamma} = -i\epsilon_{\mu\nu\rho\sigma} A^\mu p_\nu p_\pi p_\sigma F_{\eta\pi\pi\gamma}(s, t, u) \quad (3)$$

(notations are given in Fig.1, $s = (p_\pi + p_\eta)^2$, $t = (p_\eta - p_\pi)^2$, $u = (p_\pi - p_\eta)^2$).

In the chiral limit the amplitude of the process (2) is determined by the box anomaly, and $F_{\eta\pi\pi\gamma}(0, 0, 0)$ can be expressed as follows \[1, 2\]

$$F_{\eta\pi\pi\gamma}(0, 0, 0) = \frac{e}{4\pi^2 f_\pi^3} \left( \frac{f_\pi}{f_8} \cos\theta_p \right) - \frac{f_\pi}{f_0} \sqrt{\frac{2}{3}} \sin\theta_p. \quad (4)$$

Here $f_{\pi,0,8}$ are the pion, singlet and octet decay constants, $\theta_p$ is the singlet-octet mixing angle for pseudoscalars.

The value of $F_{\eta\pi\pi\gamma}$ at the chiral limit, as well as its dependence on the kinematical variables, are of interest for the theoretical analysis.

A model predicting the dependence of $F_{\eta\pi\pi\gamma}(s, t, u)$ on the kinematical variables was described in \[2\]; following this model, the authors performed the fit of experimental data on radiative decays of light pseudoscalar mesons

$$\eta, \eta' \rightarrow \pi^+\pi^-\gamma, \quad (5)$$

$$\eta, \eta' \rightarrow \gamma\gamma \quad (6)$$

and determined $f_0$, $f_8$ and $\theta_p$. For $F_{\eta\pi\pi\gamma}$ this fit yields

$$F_{\eta\pi\pi\gamma}(0, 0, 0) = 6.5 \pm 0.3 \text{ GeV}^{-3}. \quad (7)$$

The decays (3) proceed at positive $t = M^2_{\pi\pi}$. Study of the reaction (2) gives information on $F_{\eta\pi\pi\gamma}(s, t, u)$ in different region of kinematical variables, at negative $t$. Here we compare the experimentally measured cross section of the reaction (1) at low $M_{\eta\pi^-}$ with predictions of \[2\].

1 Main features of the $\eta\pi^-$ system production in the Coulomb field of nuclei

The cross section of the $\eta\pi^-$ system production in the Coulomb field of nuclei is given by the expression \[3\]

$$\frac{d\sigma}{dsdq^2} = \frac{Z^2 \alpha}{\pi} \frac{|q^2 - q^2_{\text{min}}|}{q^4} \frac{1}{s - m^2_{\pi}} \frac{p_\pi^2 p_i}{48\pi} F_{\eta\pi\pi\gamma}^2 G^2(q^2). \quad (8)$$
Here $Z$ is the charge of nucleus, $G(q^2)$ is its electromagnetic form-factor, $p_i$ and $p_\eta$ are absolute values of momenta of incident pion and outgoing $\eta$-meson in the c.m. system of produced $\eta$ and $\pi^-$. 

The differential cross section of this reaction is strongly peaking at low $q^2$. This feature facilitates the event selection; from the other hand, it leads to the fast drop of the cross section at higher masses of produced $\eta\pi^-$ due to the cut on $|q^2_{\min}| \sim \left(s - m^2_\pi\right)^2 / 4E^2_{\text{beam}}$. 

In case of $\eta\pi^-$ production at low mass in the Coulomb field, the $P$-wave is predominantly produced\(^1\) ($S$-wave corresponds to $0-0$ transition and is suppressed $\propto q^2 / m^2_\rho$, and higher waves are suppressed by the barrier factor $p^2$). However higher waves appear in the amplitude (5) due to the dependence of $F_{\eta\pi\pi\gamma}$ on kinematical variables, and their fraction increases with $M_{\eta\pi^-}$. 

The background for the process of production in Coulomb field is the process of hadronic production of the $\eta\pi^-$ system. In the hadronic production of $\eta\pi^-$ at high energies the amplitudes with positive exchange naturality with projection of 1 onto the Gottfried-Jackson axis are dominating (4). At low $|q^2|$ this is mainly diffractive processes, which proceed coherently on nuclei. They have characteristic $q^2$ distribution in form

$$\frac{d\sigma}{dq^2} \sim |q^2 - q^2_{\min}| e^{-b|q^2|}$$

with slope parameter $b \simeq 50$ GeV$^{-2}$ and distribution on azimuthal angle in Gottfried-Jackson frame (Treiman-Yang angle)

$$\frac{dN}{d\varphi_{\text{TY}}} \sim \sin^2 \varphi_{\text{TY}}.$$

The $\eta\pi^-$ can also be produced in processes with negative exchange naturality (e.g. by $b_1$-trajectory). Such exchanges, however, are suppressed in scattering on nuclei at high energies, which can be illustrated by the absence of signal from $a_0(980)$-meson in the $\eta\pi^-$ effective mass spectrum (4). At low $M_{\eta\pi^-}$ for the contribution of such exchanges one can expect zero projection onto the Gottfried-Jackson axis (uniform distribution on Treiman-Yang angle) and broad $q^2$-distribution of type

$$\frac{d\sigma}{dq^2} \sim e^{-b|q^2|}.$$ 

with slope parameter $b \simeq 7$ GeV$^{-2}$. As the contribution of negative naturality exchanges is expected to be small, such approximate description is sufficient for our studies.

Apart from hadronic production of $\eta\pi^-$, there exists instrumental background, which consists of events with different final state, where one or more slow particles were not detected. For this background one can expect broad $q^2$-distribution, which can also be parametrized in form $d\sigma/dq^2 \sim e^{-b|q^2|}$.

2 The VES setup

The experiment is performed at the VES setup (IHEP, Protvino) with $\pi^-$ beam at the momentum of 37 GeV/c. The setup is a large aperture magnetic spectrometer which

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\(^1\) Note that the $P$-wave in the $\eta\pi^-$ system has an exotic set of quantum numbers, $I^G J^P = 1^- 1^-$.  

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includes a system of proportional and drift chambers and a lead-glass electromagnetic calorimeter. The target was beryllium ($l = 4$ cm). The trigger conditions required presence of two or more charged particles in the forward hemisphere and absence of hard charged particles in the backward hemisphere. The process under study (1) was detected with $\eta$-meson decaying into $\pi^+\pi^-\pi^0$. In the region of $M_{\eta\pi^-} = 1$ GeV, the setup resolution on the effective mass of $\eta\pi^-$ is $\sigma(M_{\eta\pi^-}) \simeq 15$ MeV, and on the transverse momentum $\sigma(p_t) \simeq 17$ MeV.

3 Event selection, results

The main event selection criteria were the following:

- the event contains three tracks of charged particles with total charge of -1 and two photons in the final state;
- the total energy of the final state lies within the interval of $36 < E_{\text{tot}} < 39$ GeV;
- the charged particles are not identified as electrons;
- the effective mass of two photons lies within the interval of the $\pi^0$ mass: $105 < m_{\gamma\gamma} < 165$ MeV;
- the effective mass of one of $\pi^+\pi^-\pi^0$ subsystems lies in the interval of the $\eta$-meson mass: $531 < m_{\pi^+\pi^-\pi^0} < 567$ MeV. The constraint $m_{\gamma\gamma} = m_{\pi^0}$ was applied for the calculation of $m_{\pi^+\pi^-\pi^0}$.

Fig. 2 shows $m_{\gamma\gamma}$ (a) and $m_{\pi^+\pi^-\pi^0}$ (b) spectra. Signals from $\pi^0$ and $\eta$ mesons are clearly seen over smooth background.

Fig. 3 shows the $M_{\eta\pi^-}$ spectrum. It is dominated by $a_2(1320)$-meson signal. The arrow shows the cut used for the selection of events near threshold: $M_{\eta\pi^-} < 1.18$ GeV. The background for this spectrum was evaluated and subtracted using the events from control intervals on $m_{\pi^+\pi^-\pi^0}$ around the $\eta$-meson peak, with $503 < m_{\pi^+\pi^-\pi^0} < 521$ MeV or $577 < m_{\pi^+\pi^-\pi^0} < 595$ MeV. For all other spectra the method of bin filtering was used, i.e. the number of events in each bin was determined as the number of $\eta$-mesons obtained by fitting of the $m_{\pi^+\pi^-\pi^0}$ spectrum for this bin.

Fig. 4 shows the $|q^2 - q_{\text{min}}^2|$ distribution for selected events. It has characteristic Coulomb peak at low $|q^2 - q_{\text{min}}^2|$, as well as a broad structure corresponding to the hadronic production of $\eta\pi^-$. The result of the fit by superposition of $q^2$-distributions for Coulomb (8) and hadronic (9) production of $\eta\pi^-$, convoluted with experimental resolution of the setup, and instrumental background, is shown by dashed line in Fig. 4.

The parameters of the instrumental background were determined by the analysis of distributions on total energy and Treiman-Yang angle for the events with $0.01 < |q^2 - q_{\text{min}}^2| < 0.1$ GeV$^2$ and $M_{\eta\pi^-} < 1.18$ GeV/c$^2$ (Fig. 5a,b; in the first case the aforementioned selection on total energy was not applied). In the total energy distribution one can see the peak corresponding to fully reconstructed events. The background to the left of the peak, with $34 < E_{\text{tot}} < 35.5$ GeV, has uniform distribution on Treiman-Yang angle and $q^2$ distribution with slope parameter $b \simeq 7$ GeV$^{-2}$. The number of background events was
determined by fitting of the distribution on Treiman-Yang angle by superposition of (10) and constant. It is worth noticing that similar distributions on $\varphi_{TY}$ and $|q^2|$ are expected also for $\eta\pi^-$ production in isospin exchanges with negative naturality, which means that the number of background events obtained by fitting of Treiman-Yang angle distribution contains also possible contribution from such exchanges.

Free parameters of the fit to the $q^2$-distribution (Fig. 4) were number of events of Coulomb production of $\eta\pi^-$ (8) and number of events of its hadronic production. The interference of Coulomb and hadronic amplitudes was taken to be negligible, because the former is real, while the latter at high energies is almost imaginary. For the slope parameter $b$ in (9) the value of $b = 50 \text{ GeV}^{-2}$ was used, which corresponds to the diffractive production on Be nuclei. Varying this value by $\pm 10\%$ does not change the result. The number of events of instrumental background was taken to be equal to that found from fitting the distribution on Treiman-Yang angle. The effect of uncertainty in the number of background events on the error in determination of free parameters was taken into account by varying the number of background events within its standard deviation.

The number of events in the Coulomb peak, in the range of $|q^2 - q^2_{min}| < 0.09 \text{ GeV}^2$, is

$$N_{coul} = 109 \pm 23.$$  \hspace{1cm} (12)

In order to find the cross section, we used the results of [5], where the differential cross section of the reaction

$$\pi^- \text{Be} \rightarrow \pi^+\pi^-\pi^- \text{Be}$$  \hspace{1cm} (13)

was measured at 40 GeV, and also the results of the VES experiment on reactions

$$\pi^- \text{Be} \rightarrow \pi^+\pi^-\pi^- \text{Be}$$
$$\pi^- \text{Be} \rightarrow a_2^- (\pi^+\pi^-\pi^-) \text{Be}$$
$$\pi^- \text{Be} \rightarrow a_2^- (\eta(\pi^+\pi^-\pi^0)\pi^-) \text{Be}.$$  

These data allow us to determine the cross section:

$$\sigma_{coul} = 145 \pm 34 \text{ nb}.$$  \hspace{1cm} (14)

When calculating the error, we took into account the error in measurements of [5] (5%) and the systematic error (10%) which corresponds to the uncertainty in the calculation of the efficiency of the VES setup.

Besides the process under study (2), the Coulomb production of $a_2^-$-meson decaying into $\eta\pi^-$ also contributes to this cross section. Assuming positive interference between amplitude (3) and amplitude of Coulomb production of $\eta\pi^-$ via $a_2^-$-meson, one can calculate the expected cross section of Coulomb production in the same range of $M_{\eta\pi^-}$ and $q^2$, using the experimentally measured [6] width of the decay $a_2^- (1320) \rightarrow \pi^-\gamma$ and parametrization of the dependence $F_{\eta\pi\pi\gamma}$ on $s, t, u$ from [2]:

$$\sigma^{th}_{\eta\pi} = 119 \pm 13 \text{ nb},$$  \hspace{1cm} (15)

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2 Authors express their thanks to V.V. Ezhela and Yu.I. Ivanshin for the help in the extraction of necessary data from [6].
(the quoted error corresponds to the errors of experimental measurements of widths of $a_2(1320) \rightarrow \pi\gamma$ and $\eta \rightarrow \pi^+\pi^-\gamma$ decays). This value is in good agreement with the experimentally measured cross section [14] and confirms strong dependence of $F_{\eta\pi\pi\gamma}$ on $t$.

For the description of the dependence of $F_{\eta\pi\pi\gamma}$ on kinematical variables in various theoretical models [1, 2] the two substantial parameters are used — the value of $F_{\eta\pi\pi\gamma}$ in the chiral limit and the mass of vector meson in the $\pi\pi$ channel ($\rho$-meson). Using the cross section measured in our experiment (14) and the width of the decay $\eta \rightarrow \pi^+\pi^-\gamma$, we calculated $F_{\eta\pi\pi\gamma}(0, 0, 0)$ and the effective mass of vector meson $\tilde{m}_\rho$ within the framework of the model [2]. We obtained the values

$$F_{\eta\pi\pi\gamma}(0, 0, 0) = 6.9 \pm 0.7 \text{ GeV}^{-3}, \quad \tilde{m}_\rho = 900 \pm 120 \text{ MeV}/c^2 \quad (16)$$

which agree respectively with [7] and the $\rho$-meson mass from [6] within one standard deviation.

**Conclusion**

The cross section of the $\eta\pi^-$ production at low mass in the Coulomb field of beryllium nuclei at the beam momentum $p_{\text{beam}} = 37 \text{ GeV}/c, M_{\eta\pi^-} < 1.18 \text{ GeV}/c^2$ and $|q^2| < 0.09 \text{ GeV}^2$ was measured:

$$\sigma_{\text{coul}} = 145 \pm 34 \text{ nb}. \quad (17)$$

This value is in good agreement with theoretical predictions [2, 6].

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Figure 1: Reaction (1)
Figure 2: Effective mass spectra of $\gamma\gamma$ (a) and $\pi^+\pi^-\pi^0$ (b). The cuts used for the event selection are shown by arrows.
Figure 3: The effective mass spectrum of \( \eta \pi^- \). The arrow shows the cut used for the event selection (\( M_{\eta \pi^-} < 1.18 \text{ GeV}/c^2 \)).
Figure 4: The $|q^2 - q_{\text{min}}^2|$ distribution for events with $m_{\eta\pi^-} < 1.18$ GeV/$c^2$. The dashed line is the result of the fit (see text); the dotted line shows the contribution of hadronic production of $\eta\pi^-$ and background.
Figure 5: Distributions on total energy (a) and Treiman-Yang angle (b) for the $\eta\pi^-$ events with $M_{\eta\pi^-} < 1.18$ GeV, $0.01 < |q^2 - q_{\min}^2| < 0.1$ GeV$^2$. 