Leptonic color models from $Z_8$ orbifolded AdS/CFT

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Abstract

We study orbifold compactifications of the type $IIB$ superstring on $AdS_5 \times S^5/\Gamma$, where $\Gamma$ is the abelian group $Z_8$, which can lead to non-SUSY three and four family models based on quartification. In particular, we focus on two models, one fully quartified model and one a model with two trinification families and one quartification family, which reduces to the standard model with a minimal leptonic color sector.

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I. INTRODUCTION

Orbifold compactifications of the type $IIB$ superstring on $AdS_5 \times S^5$ (for a review see [3]) lead to gauge theories with $SU^n(N)$ gauge groups when the orbifolding group is abelian and of order $n$. Trinification models [4, 5] with gauge group $SU(3)_L \times SU(3)_C \times SU(3)_R$ and quartification [6, 7, 8, 9, 10, 11] models, where the gauge group is extended to $SU(3)_L \times SU(3)_C \times SU(3)_R$ are of this class. Thus a natural question to ask is whether one can derive models with the appropriate fermion content to allow for three families of quarks and leptons, and the appropriate scalar content to permit gauge symmetry breaking to the standard model and ultimately to $SU(3)_C \times U_{EM}(1)$. In [12, 13] two of us carried out a global search for $\Gamma = Z_n$ trinification models with three or more families. Here we will concentrate on phenomenologically interesting quartification models. These models contain a leptonic color sector to realize a manifest quark-lepton symmetry [14, 15, 16] and must contain at least three normal families to be phenomenologically viable, plus they contain the new fermions needed to symmetrize the quark and lepton particle content at high energies. We will consider both models with the full quartification (all families are quartification families),

$$3(3\bar{3}11) + (13\bar{3}1) + (113\bar{3}) + (\bar{3}113)$$

(1)

and hybrid models where two families are trinification families and the third is a quartification family,

$$2[(3\bar{3}1) + (13\bar{3}) + (1\bar{3}1)] + (3\bar{3}11) + (13\bar{3}1) + (113\bar{3}) + (\bar{3}113).$$

(2)

(Potentially, the family splitting could also be one trinification plus two quartification families.) The plan of the paper is as follows. We first review generic trinification and quartification models. Next we review the rules for generating models based on orbifold compactifications of the type $IIB$ superstring. We then restrict ourselves to $\Gamma = Z_8$ where we find the first phenomenologically interesting quartification and trinification–quartification hybrid models. We next study a specific semi-realistic four family quartification model and then an even more promising trinification–quartification hybrid model. We end with a discussion and summary of our main results.
II. REVIEW OF TRINIFICATION AND QUARTIFICATION MODELS

Trinification models \cite{4} are based on the gauge group $SU(3)_C \times SU(3)_L \times SU(3)_R$ where the electric charge operator is

$$Q = I_{3L} - \frac{Y_L}{2} + I_{3R} - \frac{Y_R}{2} = I_{3L} + \frac{Y}{2}. \quad (3)$$

In terms of $SU(3)_L \times SU(3)_R$, the leptons are

$$\ell \sim \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix}, \quad (4)$$

where $I_{3L} = (1/2, -1/2, 0)$ and $Y_L = (1/3, 1/3, -2/3)$ for the rows, and $I_{3R} = (-1/2, 1/2, 0)$ and $Y_R = (-1/3, -1/3, 2/3)$ for the columns and the exotic fermion $h(h^c)$, $E(E^c)$, and $N, N^c, S$ have charges $\mp 1/3, \mp 1, 0$ respectively. The quarks can also be arranged in matrix form

$$q \sim \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix}, \quad q^c \sim \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix}, \quad (5)$$

where $I_{3L} = (-1/2, 1/2, 0)$, $Y_L = (-1/3, -1/3, 2/3)$ for the columns in $q$ and $I_{3R} = (1/2, -1/2, 0)$, $Y_L = (1/3, 1/3, -2/3)$ for the rows in $q^c$. More compactly

$$l \sim (1, 3, \bar{3}), \quad (6)$$

$$q \sim (3, \bar{3}, 1), \quad q^c \sim (\bar{3}, 1, 3). \quad (7)$$

Vacuum expectation values (VEVs) for two scalar multiplets $\phi_a \sim (1, 3, \bar{3}) \quad (a = 1, 2)$ provide appropriate fermion masses and mixings. Trinification models can be nicely unified into $E_6$ but are not symmetric between their quark and lepton content.

Quartification models \cite{6, 7}, where the gauge group is extended to $SU(3)_L \times SU(3)_C \times SU(3)_R$, have quark-lepton symmetry, where

$$l \sim (3, \bar{3}, 1, 1), \quad l^c \sim (\bar{3}, 1, 1, 3), \quad (8)$$

$$q \sim (1, 3, \bar{3}, 1), \quad q^c \sim (1, 1, 3, \bar{3}). \quad (9)$$

The electric charge operator becomes

$$Q = I_{3L} - \frac{Y_L}{2} + I_{3R} - \frac{Y_R}{2} - \frac{Y_t}{2}. \quad (10)$$
Here $Y_l$ takes the same values as $Y_L$ or $Y_R$ depending on whether it is part of a triplet or antitriplet. The matrix representations of $l$ and $l^c$ are

$$l \sim \begin{pmatrix} x_1 & x_2 & \nu \\ y_1 & y_2 & e \\ z_1 & z_2 & N \end{pmatrix}, \quad l^c \sim \begin{pmatrix} x_1^c & y_1^c & z_1^c \\ x_2^c & y_2^c & z_2^c \\ \nu^c & e^c & N^c \end{pmatrix}. \quad (11)$$

Here the columns of $l$ have $Y_l = (-1/3, -1/3, 2/3)$ and the rows have $I_{3L} = (1/2, -1/2, 0)$, $Y_L = (1/3, 1/3, -2/3)$. The rows of $l^c$ have $Y_l = (1/3, 1/3, -2/3)$, and the columns have $I_{3R} = (-1/2, 1/2, 0)$, $Y_L = (-1/3, -1/3, 2/3)$.

Of the new particles $N$ and $N^c$ are neutral. The exotic $SU(2)_l$ doublet leptons $(x, y, z)$ have charges $(1/2, -1/2, 1/2)$ and $(x^c, y^c, z^c)$ have charges $(-1/2, 1/2, -1/2)$, respectively. Because of their half integral charges, the $SU(2)_l$ doublets have been dubbed “hemions”.

Symmetry breaking of quartification models to the standard model can be somewhat involved so we will delay a discussion until we get to specific models derived from AdS/CFT, but we note here that the fermions all fall into bifundamental representations of the gauge group, and these are naturally arranged into a moose or quiver diagram [17, 18], see Fig. 1.

### III. RULES FOR $AdS_5 \times S^5/Z_n$ MODEL BUILDING

In this paper we study conformal field theory models originating from the large $N$ expansion of the AdS/CFT correspondence. We choose $N = 3$ and as a consequence, the gauge group of the corresponding CFT model derived from an abelian orbifold is given by the product group $SU^n(3)$. The $\mathcal{N} = 4$ supersymmetry of $AdS_5 \times S^5$ is broken upon orbifolding $S^5 \rightarrow S^5/\Gamma$ where $\Gamma$ is a finite group embedded in the isometry $SU(4) \sim O(6)$ of $S^5$. Here we concentrate on the case $\Gamma = Z_n$ with $n = 8$. The choice of $Z_8$ is not arbitrary, in

![Quiver diagrams of $[SU(3)]^3$ trinification and $[SU(3)]^4$ quartification.](image-url)
fact a systematic search of \( Z_n \) orbifold models reveals that \( Z_8 \) is the minimal choice on which to base a phenomenological quartification model in which all product groups have the same coupling strength. The 4 of the \( SU(4) \) isometry must be neither real nor pseudoreal for chiral fermions to be present in the resulting quiver gauge theory. The number of surviving supersymmetries is \( N = 2, 1, 0 \) for \( \Gamma \) embedded nontrivially in \( SU(2), SU(3), \) or \( SU(4) \) respectively. For \( \alpha = \exp(2\pi i/n) \) the embedding is fixed by a choice \( 4 = (\alpha^A_1, \alpha^A_2, \alpha^A_3, \alpha^A_4) \).

We denote such a model as \( M_{A_1A_2A_3A_4} \) and define it as partition or double partition model, if \( A_1 + A_2 + A_3 + A_4 = n \) or \( 2n \), respectively (see [12, 13] for notational details). Partition or double partition models are particular attractive, as the construction of viable string theory non-partition models may not be possible [19]. For \( N = 0 \), the case of interest here, all four \( A_i \) need to be nontrivial. This determines the fermion content of the theory: fermions reside in the bilinear representations of the 1st and \( A_i \)-th product group and its cyclic permutations. Consider next the 6 of \( SU(4) \) which is the antisymmetric part in \( 4 \times 4 \). Consistency [19] requires a real embedding of the 6 which can be written in the form \( 6 = (A_1 + A_4, A_2 + A_4, A_3 + A_4, A_1 + A_2, A_2 + A_3, A_3 + A_1) \). This in turn determines the scalar content of the theory.

**IV. LEPTONIC COLOR MODELS FROM ADS/CFT**

It has been shown [13] that the viable \( Z_8 \) orbifolds of AdS/CFT include five partition models and one double partition model. In this work we study the symmetry breaking to the quartification group

\[
SU(3)_l \times SU(3)_L \times SU(3)_C \times SU(3)_R, \tag{12}
\]

where each factor is the diagonal subgroup of two of the original \( SU(3)^8 \) factor groups. This procedure yields quartification models in which the individual factor groups initially couple with the same strengths.

In general there exist 24 different symmetry breaking patterns of this type. A systematic search yielded only two viable models. One, a \( M_{1133} \) model, is a semi-realistic, four family quartification model and the other is an \( M_{1456} \) model, which leads to a more phenomenologically interesting hybrid fermion spectrum with two trinification families and one quartification family.
A. Quark-lepton quartification from AdS/CFT

The $SU(3)^8$ fermion spectrum of $M_{1133}$ is given by

$$2[(3\overline{3}111111) + (311\overline{3}1111)]_F + \text{cyclic permutations} \quad (13)$$

and the scalar spectrum is given by

$$[(31\overline{3}11111) + 4(3111\overline{3}111) + (31111\overline{3}1) + h.c.] + \text{cyclic permutations}. \quad (14)$$

We can break $SU(3)^8$ to $SU(3)^4$ by assigning VEVs to scalars in the representations $(31\overline{3}11111)$, $(131\overline{3}1111)$, $(1111\overline{3}111)$ and $(11111\overline{3}1)$. Omitting vectorlike fermions, singlets and octets the chiral $SU(3)^4$ fermions are

$$4[(3\overline{3}11) + (13\overline{3}1) + (113\overline{3}) + (\overline{3}11\overline{3})]_F \quad (15)$$

and the scalars are

$$10[(3\overline{3}1) + (13\overline{3}) + h.c.]_S + 2[(8111) + (1811) + (1181) + (1118)]_S. \quad (16)$$

Note that the chiral fermion content of this model is precisely four quartification families.

To proceed toward the standard model we first label the four remaining $SU(3)$’s as $SU(3)_l \times SU(3)_L \times SU(3)_C \times SU(3)_R$. With an octet $(1811)$ who’s VEV is proportional to $\lambda_8$ we can break $SU(3)_L$ to $SU(2)_L \times U_L(1)$. A second $(1811)$ with VEV proportional to $\lambda_1$ allows us to break $SU(2)_L$ completely but leave $U_L(1)$ unbroken. Likewise, VEVs for two octets of type $(1118)$ allows us to break $SU(3)_R$ down to $U_R(1)$. Next a $(13\overline{3}1) + h.c.$ can be used to break $U_L(1) \times U_R(1)$ to the diagonal subgroup $U_D(1)$. To achieve the final symmetry of the quartification we still need to break $SU(3)_l$. First a $\lambda_8$ type octet $(8111)$ VEV gives $SU(2)_l \times U_l(1)$. The $SU(2)_l$ needs to remain unbroken, but $U_D(1) \times U_l(1)$ is required to break to a linear combination that is the weak hypercharge. There are no remaining scalar representations that can do this, but it is possible that a leptonic color condensate forms that has both $U_D(1)$ and $U_l(1)$ charge, reducing the symmetry to the desired linear combination. However, since the $SU(2)_l$ scale $\Lambda_{\text{CDD}}$ is much below the color confinement scale determined by $\Lambda_{\text{QCD}},$ the formation of such a condensate provides a proof of principle rather that a viable phenomenology. We must proceed to the the $M_{1456}$ model if that is what we desire.
B. Minimal leptonic color from AdS/CFT

We now consider the more realistic double partition model $M_{1456}$: the particle spectrum of the unbroken $SU(3)^8$ theory at the string scale is given by

$$
(3\bar{3}111111)_F + (311\bar{1}3111)_F + (3111\bar{1}131)_F + (31111\bar{1}31)_F + c.h. + \text{cyclic permutations}
$$

fermion states and scalars in the

$$
(3\bar{3}111111)_S + (3\bar{1}311111)_S + (31\bar{1}31111)_S + (3111\bar{1}311)_S + (31111\bar{1}31)_S + (311111\bar{1}3)_S
$$

+ cyclic permutations

representations. $SU(3)^8$ is broken down to $SU(3)^4$ by assigning VEVs to $(3\bar{3}111111)_S$, $(1131\bar{1}3111)_S$, $(1113\bar{1}111)_S$ and $(11113111)_S$ which leaves chiral fermions now in the representations

$$
2[13\bar{3}1] + (11\bar{3}3) + (1\bar{3}13)]_F + [(3\bar{3}11) + (13\bar{3}1) + (1\bar{3}13) + (3\bar{1}13)]_F;
$$

and scalars in the representations

$$
3[(3\bar{3}11) + (13\bar{3}1) + (1\bar{3}13) + (3\bar{1}13) + h.c.]_S + 4[(3\bar{1}31) + (13\bar{1}3) + h.c.]_S
$$

+ $2[(8111) + (1811) + (1181) + (1118)]_S.
$$

We assume the two light families are the trinification families and the heavy quarks are in the color (anti-)triplet quartification multiplets. The third family leptons plus exotic matter can be obtained from the leptonic color (anti-)triplets.

We could give a VEV to a $(3\bar{3}11)$ and immediately break $SU(3)_L \times SU(3)_L$ to a new $SU(3)'_L$ which would yield a three family trinification model. We choose not to do this, as these models have already been extensively explored. Instead, in analogy with \[7\], and with the labeling $SU(3)_L \times SU(3)_L \times SU(3)_C \times SU(3)_R$, VEVs are given to two (1313) representations to generate realistic quark masses and non-zero mixing angles, and to $(3\bar{3}11)$ and to $(31\bar{1}3)$, by which $SU(3)_L$ is broken down to $SU(2)_L$. This way the scalar content given in (20) results in three Higgs doublets from the $(3\bar{3}11)$ representations and another 12 Higgs doublets from the $(131\bar{1}3)$ representations, which will be important for the phenomenology of the model.

Alternatively, the octets of type $(1811)_S$ and $(1118)_S$ are again sufficient to break $SU(3)_L \times SU(3)_R$ to $SU(2)_L \times U(1)_Y$. At this stage $SU(3)_L$ remains unbroken. It is interesting to note that at the $SU(3)_L \times SU(3)_L \times SU(3)_C \times SU(3)_R$ level, the $\beta$ function
for $SU(3)_l$ is the most negative. Ignoring the scalars for the moment, the fermionic term in $\beta_l$ is $4/3 N_l$ where $N_l = 3$ is the number of $SU(3)_l$ triplet Dirac fermions, while for $SU(3)_L \times SU(3)_C \times SU(3)_R$ we find fermionic terms with $N_L = 9$, $N_C = 9$, and $N_R = 9$. Consequently, $SU(3)_l$ is asymptotically free and its coupling constant $\alpha_l$ becomes of order one far above $\Lambda_{QCD}$.

V. PHENOMENOLOGICAL CONSEQUENCES

In the following we sketch some phenomenological aspects of the minimal leptonic color model. We follow here the discussion of the original quartification model in [7], but stress several interesting new aspects due to the two incomplete quartification families and the remnant particles from the $SU(3)^8 \rightarrow SU(3)^4$ symmetry breaking.

Gauge coupling unification: The renormalization-group evolution of the gauge couplings in leading order is given by

$$\frac{1}{\alpha_i(\mu)} - \frac{1}{\alpha_i(\mu')} = \frac{b_i}{2\pi} \ln \left(\frac{\mu'}{\mu}\right),$$

where $b_n$ are the one-loop beta-function coefficients,

$$b_3 = -11 + \frac{4}{3} N_g,$$

$$b_2 = -\frac{22}{3} + 2 N_q + \frac{4}{3} N_t + \frac{1}{6} N_H,$$

$$b_1 = \frac{13}{9} N_q + \frac{4}{3} N_t + \frac{1}{12} N_H.$$  

Here $N_g = 3$ is the number of generations and $N_q = 1$, $N_t = 2$ accounts for the number of quartification and trinification families. The running includes the contributions of the exotic weak-scale $SU(2)_l$ doublet “hemions” $[(x, y)$ is an $SU(2)_L$ doublet with $Y = 0$; $x^c$ and $y^c$ are $SU(2)_L$ singlets with $Y = \mp 1$] and $N_H$ Higgs doublets with $Y = \pm 1$. The initial values of the gauge couplings are

$$\alpha_3(M_Z) = 0.117,$$

$$\alpha_2(M_Z) = (\sqrt{2}/\pi)G_FM_W^2 = 0.034,$$

$$\alpha_1(M_Z) = \alpha_2(M_Z) \left(\tan^2\theta_W/\tan^2\theta_W(M_{GUT})\right) = 0.0181,$$

where $\sin^2\theta_W(M_{GUT}) = \sum I_{3L}^2/\sum Q^2 = 9/16$ (the sum running over all fermion representations) is determined by the embedding of $U(1)_Y$ in $[SU(3)]^4$. There are a total of 12
standard (uncolored) doublets of $SU(2)_L$. There are also 9 doublets with color and 9 more with leptocolor. All the standard doublets will be able to grow a mass when we have broken to the SM gauge group (or to SM gauge group × leptocolor), but we could keep them light by fine tuning, or by keeping the $U(1)$’s from $SU(3)_L$ and $SU(3)_R$ unbroken. This is similar to the symmetry breaking $E_6 \to SU(3)^3 \to SU(3) \times SU(2) \times U(1) \times U(1)' \times U(1)''$. As long as we keep all three $U(1)$’s unbroken, all components of the 27 of $E_6$ remain massless. The evolution of the couplings from the weak scale up to very high scales is shown in Fig. 2 using $N_H = 4$. The gauge couplings unify around $10^{13}$ GeV, at a somewhat higher energy scale as compared to \[7\]. An interesting consequence would be the chance to discover multiple Higgs doublets at the LHC.

For the coupling of the unbroken $SU(2)_l$ group the one-loop beta-function coefficient is given by

$$b_{2l} = -\frac{22}{3} + \frac{4}{3} N_q.$$  \hspace{1cm} (28)

The trinification families don’t contribute as they are singlets under $SU(2)_l$. This implies $\alpha_{2l}^{-1}(M_Z) \simeq 13$, which is between the weak and the strong couplings and will yield a similar phenomenology as in \[7\], albeit with a higher scale, where the leptonic color interaction becomes non-perturbative, somewhat below an MeV.

Hemion masses: As in \[7\], quartification scale hemion masses are forbidden by the $Z_8$ orbifold symmetry. TeV scale hemion masses could be generated by adding non-renormalizable operators that are suppressed by the Planck scale.

Electroweak precision data: Generally one should worry about electroweak precision data in view of the variety of new particles introduced by the model. However, the singlet and vectorlike symmetry breaking products will not affect these processes, and the hemions are vector-like under the SM gauge group, thereby their contribution would be suppressed by the hemion masses. The same argument applies to the Higgs sector. An important issue could be the discussion of potentially excessive flavor changing neutral currents, but this is beyond the scope of this work and will be studied elsewhere.

Proton decay: As usual in product group unifications scenarios, proton decay will not be mediated by gauge bosons. However, proton decay could be induced via couplings to the extended scalar sector given in (20), see [21].

Neutrino masses: The symmetry breaking chain discussed in section IV B provides several singlet fermions, e.g. from the $(3\bar{3}111111)$ representation after breaking the first two $SU(3)$
FIG. 2: Gauge coupling unification in the minimal leptonic color model, assuming two trinification and one quartification families of fermions, and 4 Higgs doublets. The couplings unify around $10^{13}$ GeV.

groups down to the diagonal subgroup by assigning a VEV to the scalar representation $(3\bar{3}111111)$. This makes the string inspired model superior to the simple quartification model, as a seesaw mechanism

$$m_\nu \sim \frac{m_{\nu\nu}^2}{M_{33111111}}$$

(29)
can be implemented to generate light neutrino masses without adding right-handed neutrinos by hand to the theory.

*Stickballs* are the glueballs of the leptonic color SU(2). These particles could act as a cold dark matter candidate.

VI. DISCUSSION

We have shown that it is possible to find quartification models based on orbifold compactifications of the type $IIB$ superstring on $AdS_5 \times S^5/Z_n$. These models have fermions in only bifundamental representations and can have a sufficient number of scalar fields to allow spontaneous symmetry breaking to the standard model. The first two models of this type arise at $n = 8$. The first fully quartified model is somewhat less than realistic since the final stage of symmetry breaking relies on condensates to provide a proof of principle rather
than a viable phenomenology. The other model does have sufficient number of scalar fields to allow the complete spontaneous symmetry breaking to the standard model. It has the interesting additional feature that one family is fully quartified, while the other two families are of the trinification type. This suggests the possibility of a natural family hierarchy in this hybrid model which could lead to interesting phenomenology, including a rich particle spectrum within reach of the LHC.

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