An ETA-PDE Method for the Fragility Analysis of Concrete Dam

Qiang Xu1,2, Shutong Xu2, Jianyun Chen1,2, Jing Li1,2 and Qibin Jia2

1 State Key Laboratory of Coastal and Offshore Engineering, Dalian University of Technology, Dalian, China
2 Faculty of Infrastructure Engineering, Dalian University of Technology, Dalian, China
E-mail: xushutong@mail.dlut.edu.cn

Abstract. A nonlinear damage ETA-PDE method is proposed for the fragility analysis of concrete dam. The endurance time analysis (ETA) method and the generalized probability density evolution (PDE) method are constructed by optimization and the method of lines (MOL) algorithm especially for the randomness of ground motion. Then, by integrating the ETA method and generalized PDE method, a nonlinear damage ETA-PDE method is established. To verify the applicability and reliability of the proposed ETA-PDE method, by using the tensile damage model, the nonlinear dynamic calculation and the fragility analysis are carried out for the Koyna concrete gravity dam. The results show the proposed PDE method can capture details of the probability density distribution for the fragility indicators (such as the plastic dissipation energy, the damage dissipation energy, the damage volume and so on). The proposed ETA-PDE method has special advantages on the calculating scale and solving probability density. Thus, it is suitable for practical engineering applications.

1. Introduction
In the early study, plenty of the fragility analysis methods for concrete structures have been proposed. Vamvatsikos et al. [1] expounded the principle and the key steps of IDA method, discussing the association with conventional static pushover analysis. Baker [2] investigated different structural performance analysis methods, including IDA and Multiple stripe analysis (MSA) which had produced higher precision than IDA method in the aspect of fragility. Han and Chopra [3] put forward an MPA-based method that was aimed to estimate structural seismic capacity for the limit states of different indexes. Many researchers [4] used the Monte Carlo simulation (MCS) method and IDA method to analyze the uncertain parameters of structures under seismic loads. For the problem of large calculations of these methods, in recent years, Endurance time analysis (ETA) method, which is first proposed by Estekanchi [5] has less amount of calculation to make the fragility analysis of concrete structures. One-time calculation of the ETA is equivalent to a group of ground motions with different peak accelerations. Pang et al. [6] used the PDE method to analyze the probability density distribution of the fragility indicators of earth-rockfill dam under stochastic earthquake excitation. Hariri-Ardebili et al. [7] applied ETA method to dams and used the indicators such as displacement and stress to evaluate structural seismic ability, subsequently, the crack propagation of dams under endurance time acceleration functions (ETAFs) have been predicted. ETA method has acceptable accuracy and can predict the behavior of dams. For the problem of the probability density distribution of these methods, Li and Chen [8] developed the probability density evolution (PDE) method to gain the probability
density distribution of the fragility indicators in dynamic analysis.

In this paper, to take advantage of the ETA method and the PDE method, a nonlinear damage ETA-PDE method is established by coupling the ETA method and the PDE method. To verify the applicability and reliability of the proposed ETA-PDE method, the fragility analysis is carried out for the Koyna gravity dam.

2. Seismic Fragility ETA-PDE Method for Dam

2.1. Seismic Fragility Analysis

Seismic fragility curves express the probability of structural damage when structures are subjected to given seismic intensity measure (IM). They can be developed by many methods. These methods evaluate the fragility of structures by nonlinear dynamic time history analysis under several groups of ground motions with increasing peak accelerations. Seismic fragility analysis contains two parts, which are obtaining samples and calculating seismic fragility probability. In this study, the ETA method is used to obtain samples of damage indices of dam subjected to given increased seismic IM. And the PDE method is used to estimate the seismic fragility probability of dam.

2.2. Endurance Time Analysis (ETA) Method

In ETA method, an endurance time of acceleration history (ETAH) is used to replace a series of seismic histories with different acceleration peaks. As shown in figure 1, the ETA method uses different interval times to satisfy the linear growth of design response spectrum with the factor of $t/t_{\text{target}}$. The relation of the target acceleration spectrum $S_{\text{at}}(T,t)$, the target displacement response spectrum $S_{\text{at}}(T,t)$ and the codified design acceleration spectrum $S_{\text{ad}}(T)$ is

$$S_{\text{at}}(T,t) = \frac{t}{t_{\text{target}}} S_{\text{ad}}(T), \quad S_{\text{at}}(T,t) = \frac{t}{t_{\text{target}}} S_{\text{ad}}(T) \times \frac{T^2}{4\pi^2}$$  (1)

where $t_{\text{target}}$ is the target duration of ETAH, $T$ is the period of free vibration, $S_{\text{ad}}(T)$ is the design maximum acceleration spectrum, $S_{\text{at}}(T,t)$ and $S_{\text{at}}(T,t)$ are the target acceleration spectrum and the displacement spectrum of interval times of ETAH from $0s$ to $t$.

And ETA method optimizes the ETAH $\alpha$ by the unconstrained optimization formula in time domain as follows:

$$\text{Minimize} \ F(\alpha) = \int_0^{t_{\text{end}}} \int_0^{t_{\text{max}}} \left[ \{S_{\text{ad}}(T,t) - S_{\text{at}}(T,t)\}^2 + \alpha [S_{\text{ad}}(T,t) - S_{\text{at}}(T,t)]^2 \right] dt dT$$  (2)

where $\alpha$ is the weighted parameter. In this paper, the displacement spectrum is not considered so $\alpha=0$. Figure 1 shows the incremental relationship of response spectrum with ETAH in ETA method.

2.3. Probability Density Evolution (PDE) Method

The dynamic equation of structure subjected to earthquake motion is

$$M\ddot{X} + C\dot{X} + KX = -MI\ddot{X}_g$$  (3)

where $M$ and $C$ are the mass matrices and damping matrices respectively; $\ddot{X}, \dot{X}$ and $X$ are the acceleration, velocity, and displacement vectors, respectively; $\ddot{X}_g$ is the earthquake ground motion; $I$ is the load position vector.

Because of the randomness $\theta$ of $\ddot{X}_g$, the responses $Z$ (such as damage volume, the plastic dissipation energy, and the damage dissipation energy) are also random. According to the principle of probability conservation, the PDE equation is

$$\frac{\partial p_{Z,\theta}(Z,\theta,t)}{\partial t} + \dot{Z}(\theta,t) \frac{\partial p_{Z,\theta}(Z,\theta,t)}{\partial Z} = 0$$  (4)
where $p_{Z,\theta}(Z,\theta,t)$ is the conditional probability density function of $Z(t)$ and $\hat{X}_Z(\theta)$. For the randomness $\theta$ of the earthquake ground motion, we can generate several $n$ ETAHs. Thus, the probability of each ETAH is $1/n$. We use the method of lines (MOL) algorithm to solve the PDE equation.

2.4. The ETA-PDE Method

Figure 2 shows the overall process ETA-PDE method for concrete structure. The main idea of ETA-PDE method is that using the ETA method to obtain samples of damage indices of dam subjected to given increased seismic IM and using the PDE method estimate the seismic fragility probability of dam.

Figure 1. The incremental relationship of response spectrum with ETAH.  
Figure 2. The proposed procedure for ETA-PDE method for dam.

3. Seismic Fragility Analysis of Konya Concrete Gravity Dam Based on ETA-PDE Method

3.1. Nonlinear Damage Constitutive for Concrete

The damage behavior of concrete material can be depicted using concrete plastic-damage constitutive model proposed by Lee and Fenves [9], the stress-strain relationship of uniaxial tensile load is shown in figure 3.

3.2. The Koyna Dam Model

The ETA-PDE method is used to make a seismic fragility analysis for the Konya concrete gravity dam. The thickness of model is 1m and the two-dimensional plane strain model for the Koyna concrete gravity dam is performed. As shown in figure 4, the dam’s length and height are 70m and 103m, respectively. The upstream water level is 91.7 m and the hydrodynamic pressure is calculated by the generalized Westergaard method. For dam concrete material, the density, the Young’s modulus, and the Poisson’s ratio are 2643 kg/m³, 31 GPa, and 0.2, respectively. The upstream water level is 91.7 m and the hydrodynamic pressure is calculated by the generalized Westergaard method. For dam concrete material, the density, the Young’s modulus, and the Poisson’s ratio are 2643 kg/m³, 31 GPa, and 0.2, respectively. The Rayleigh damping ratio is 0.05. For the dynamic parameters in figure 3, the tensile strength $f_t$ of the concrete used in the analysis is 2.9 MPa. The fracture energy $G_f$ is assumed to be 200 N/m based on experience. The limiting tensile strain $\varepsilon_t$ is set at 0.00037, the characteristic length $l_c$ is 0.5 m. The damping of concrete dam system is assumed to be of Rayleigh type. The first sixth natural frequencies and periods are listed in table 1.

The 20 groups of ETAHs are generated by ETA method as shown in figure 5. The maximum horizontal peak ground acceleration (PGA) is 0.6g, and the maximum vertical PGA is 0.4g. Figure 6 shows that the response spectrums of ETAHs fit the target acceleration spectrums well. And the interval times of ETAHs 0-5s, 0-10s, 0-15s and 0-20s are corresponding to horizontal PGAs 0.15g, 0.3g, 0.45g, and 0.6g, respectively.
Figure 3. Concrete constitutive behavior.  

Figure 4. The model of the Koyna gravity dam.

Table 1. Dynamic characteristics of Koyna gravity dam.

| Mode | Frequency (Hz) | Mode | Frequency (Hz) | Mode | Frequency (Hz) |
|------|----------------|------|----------------|------|----------------|
| 1    | 3.01           | 3    | 10.85          | 5    | 24.05          |
| 2    | 7.99           | 4    | 15.75          | 6    | 24.07          |

Figure 5. ETAHs for the Koyna dam model corresponding to (a) Horizontal direction; (b) Vertical direction.

Figure 6. ETAH response spectrum in different interval times corresponding to (a) Horizontal direction; (b) Vertical direction.

3.3. Dynamic Results of the Koyna Dam

Figure 7 shows that the damage profiles of the Koyna dam in 3 typical times of 1 group of ETAH. The results show that the main damage region of the dam is at the downstream slope. Thus, it suggests that the Koyna gravity dam damage degree is related to the damage degree and damage region at the downstream slope. Thus, the ‘medium’ damage is defined that upstream and downstream of dam is
penetrated in which the damage factor is more than 0.0. When the damage factor is more than 0.7, it generally believed that the macro cracks have occurred, and the damage of structure is very serious at this time. Therefore, the ‘serious’ damage is regarded that upstream and downstream of dam is penetrated in which the damage factor is more than 0.7. Moreover, the damage volume is regarded that the volume of dam in which the damage factor is more than 0.0. The reference damage volume is regarded that the volume of dam in which the damage factor is more than 0.7. Figure 8 shows that the growth trend of the failure probability for the plastic dissipation energy and the damage dissipation energy is greater than that of the damage volume and the reference damage volume. By comparing the NDS method with PDE method, figure 8 shows that the results of the failure probability by two methods are similar. However, the results based on NDS method are conservative and overestimate the failure probability. Figure 9 shows that the plastic dissipation energy, the damage dissipation energy, the damage volume, and the reference damage volume have similar exponential evolutionary trends. It suggests that these fragility indicators have some equivalence. Meanwhile, figure 9 shows that the discreteness of plastic dissipation energy is least while that of damage volume is greatest. Figure 9 also shows that the discreteness of damage dissipation energy and the reference damage volume is relatively large.

Figure 7. Damage distribution in typical times (PGAs) of an ETAH for the Koyna dam corresponding to (a) 6 s (0.18 g); (b) 11.35 s (0.3405 g); (c) 20 s (0.6 g).

Figure 8. The failure probability of the different fragility indicators for the Koyna dam corresponding to (a) Medium damage; (b) Severe damage.
3.4. Seismic Fragility Analysis of the Koyna Dam

In the ideal state, with the increase of the number of samples, the results of the probability density evolution process will reach the real probability density distributions. To illustrate the validity of the proposed ETA-PDE method, 20 groups of the different fragility indicators under ETAHs are calculated. We use the Monte Carlo (MC) method as a reference. And then we give the probability density function (PDF) distribution by the PDE method, the normal distribution supposition (NDS) and the MC method, respectively.

Figures 10 and 11 show that the probability density distribution by the PDE method is closer to the reference by the MC method than that by NDS. It suggests that the normal distribution does not necessarily apply to all probability distributions of the fragility indicators. There is widespread partial peaks phenomenon of the probability density distribution for the fragility indicators, which cannot be described by NDS. It suggests that the probability density distribution by NDS has some limitations. The NDS can only reflect the variation and the peak value of probability density curve. In contrast, the PDE method can reflect the variation, the peak value and the deviation of the probability density distribution. And the proposed PDE method is sensitive to the samples, which indicates the proposed PDE method can capture details of the probability density distribution for the fragility indicators.

![Figure 9](image)

**Figure 9.** The time history curve of the different fragility indicators under ETAH for the Koyna dam corresponding to (a) The plastic dissipation energy; (b) The damage dissipation energy; (c) The damage volume; (d) The reference damage volume.
Figure 10. The probability density evolutionary process of the different fragility indicators for the Koyna dam corresponding to (a) The plastic dissipation energy; (b) The damage dissipation energy; (c) The damage volume; (d) The reference damage volume.

Figure 11. The PDE process of the plastic dissipation energy, damage dissipation energy, damage volume and reference damage volume by PDE method, NDS and MC method for the Koyna dam.
4. Conclusion
The nonlinear damage ETA-PDE method for concrete structure is proposed. The applicability of the proposed ETA-PDE method is demonstrated and several conclusions and recommendations are proposed accordingly:

(1) The results suggest that the normal distribution does not necessarily apply to all probability distributions of the fragility indicators. There is a widespread partial peaks phenomenon of the probability density distribution for the fragility indicators, which cannot be described by NDS. It suggests that the probability density distribution by NDS has some limitations. The NDS can only reflect the variation and the peak value of probability density curve. In contrast, the PDE method can reflect the variation, the peak value and the deviation of the probability density distribution. And the proposed PDE method is sensitive to the samples, which indicates the proposed PDE method can capture details of the probability density distribution for the fragility indicators.

(2) The fragility indicators such as the plastic dissipation energy, the damage dissipation energy, the damage volume and the reference damage volume all have similar exponential evolutionary trends. Due to the small number of samples, no exponential formulas are fitting for the fragility indicators because of stringency. The results suggest that these fragility indicators also have some equivalence. The upper and lower confidence intervals of the fragility indicators are often asymmetrical.

Acknowledgements
The paper is supported by the National Key R & D Program of China (2017YFC0404903), the Funds of the Innovation Support Plan for Researcher of Dalian (No. 2016RQ015).

References
[1] Vamvatsikos D and Cornell C A 2002 Incremental dynamic analysis Earthq. Eng. Struct. D 31 (3) 491-514.
[2] Baker J W 2015 Efficient analytical fragility function fitting using dynamic structural analysis Earthquake Spectra 31 (1) 579-599.
[3] Han S W and Chopra A K 2006 Approximate incremental dynamic analysis using the modal pushover analysis procedure Earthq. Eng. Struct. D 35(15) pp 1853-1873.
[4] Asgarian B, Sadrinezhad A and Alanjari P 2010 Seismic performance evaluation of steel moment resisting frames through incremental dynamic analysis J. Constr. Steel Res. 66 (2) 178-190.
[5] Estekanchi H E, Valamanesh V and Vafai A 2007 Application of endurance time method in linear seismic analysis Engineering Structures 29 (10) 2551-2562.
[6] Pang R, Xu B, Kong X and Zou D 2018 Seismic fragility for high CFRDs based on deformation and damage index through incremental dynamic analysis Soil Dynamics and Earthquake Engineering 104 432-436.
[7] Hariri-Ardebili M A, Mirzabozorg H and Kianoush R 2012 A study on nonlinear behavior and seismic damage assessment of concrete arch dam-reservoir-foundation system using endurance time analysis Iran University of Science & Technology 2 (4) 573-606.
[8] Li J and Chen J 2005 Dynamic response and reliability analysis of structures with uncertain parameters Int. J. Numer. Meth. Eng. 62 (2) 289-315.
[9] Lee J and Fenves G L 1998 A plastic-damage concrete model for earthquake analysis of dams Earthq. Eng. Struct. D 27 (9) 937-956.