Integration of Fuzzy AHP-VIKOR Methods in Multi Criteria Decision Making: Literature Review

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Abstract. The rapid development of the times directly demands technology, culture and social development as well. The development of this era also gives us many problems that must be resolved. One of the problems that must be faced is the problem of making decisions. There are many methods for solving decision-making problems, one of which is Multiple-Criteria Decision Making (MCDM). MCDM is a decision-making method to determine the best alternative from a number of alternatives based on certain criteria. This journal will discuss one of the MCDM methods, namely Fuzzy AHP-TOPSIS. Basically, AHP breaks up a complex and unstructured situation into its component parts. Then arrange these parts or variables in a hierarchical arrangement and give numerical values to subjective considerations about the relative importance of each variable. The fuzzy set theory used to represent uncertainty, obscurity, inaccuracy, lack of information, and partial truth. In the VIKOR method, a ranking is performed on the weight that has been obtained in the FAHP method by comparing ratings from a series of alternatives. The VIKOR method calculate the ratio positive and negative ideal solution. The VIKOR method propose a compromise solution with profit rate. This method overcomes the drawbacks of other MCDM methods.

1. Introduction
Decision making is a result of the process of determining choices among several alternatives. The problem of decision making is one of the problems that are often encountered in daily life. Decision making problems occur in various scopes. In the problem of decision making there are several methods that can be done, one of which is Multiple-Criteria Decision Making (MCDM). MCDM is used to choose the best choice from several alternatives stem from several predetermined criteria. The criteria in question can be in the shape of measurements, standards or rules used in decision creation [1].

Management Decision System was the term that first revealed in 1971 as the concept of a decision support system by Michael Scoot Morton. Then many companies, research and education institute began to conduct research and anything related to support decision support systems, so that the resulting production can be concluded that this system aimed at assisting decision making in utilizing particular data and models to find an answer to various unarranged problems in a computer-based system [2].

As a decision support method, Analytic Hierarchy Process (AHP) is developed to complete problems by solving solutions problem, parse and finally arrange it into a hierarchical format. AHP is
an organized multicriteria procedure for sorting out and examining complex choices dependent on numerous models [12]. To obtain priority criteria, this method uses comparison criteria pairs with a measurement scale that is has been determined. The main source of the Analytic Hierarchy Process method is the perception that was gathered from experts or experts, which means we cannot deny existence of subjectivity in creating decision. There is also presence of inconsistency limits in this method if we take into account data validity. However, the accuracy of data and results which is obtained will be impacted by sufficient uncertainty and doubt much in the assessment. The Fuzzy Analytic Hierarchy Process method is a theory that is further developed based on this.

The Analytic Hierarchy Process (AHP) method developed with fuzzy logic theory is the Fuzzy Analytic Hierarchy Process, especially triangular fuzzy. Almost the same as the AHP method regarding problem solving steps with the Fuzzy AHP method. It's just that to get priority, the Fuzzy AHP method changes the AHP scale to a fuzzy triangle scale before finally, with an extensive analysis, further analysis is carried out on the modified data.

2. Theoretical Background

2.1. Multi Criteria Decision Making (MCDM)

As one way of making decisions, Multi Criteria Decision Making (MCDM) determines the best alternative based on certain criteria from a number of alternatives. Standards, measures or rules used in decision making are usually become the criteria. Multi Attribute Decision Making (MADM) and Multi Objective Decision Making (MODM) are, based on these decisions, the division of MCDM types.

MODM is used to solve problems in continuous space (such as in mathematical programming), whereas MADM is usually used to assess or select a number of alternatives [5].

2.2. Analytic Hierarchy Process (AHP)

AHP was developed by Thomas L. Saaty as a decision support model. Describing a complex multi-factor or multi-criteria problem into a hierarchy becomes the working principle in this decision support model. Hierarchy, according to Saaty, is defined as a representation of a complex problem. A multi-level structure where the first level is the goal, followed by the level of factors, criteria, sub-criteria and so on to the last level as an alternative to the problem description. The subjective and quantitative elements of dynamic cycles for all intents and purposes can be handled by AHP quickly and methodically [13]. A complex problem can be broken down hierarchically, into groups that will appear more structured and systematic because the problem is organized into a hierarchical form [6].

The stages of decision making in the AHP method are basically as follows:

- Define the issue and decide the ideal arrangement
- Create a progressive structure that begins with general targets, trailed by rules and elective decisions to be positioned.
- Form a pairwise examination lattice that outlines the overall commitment or impact of every component to every one of the goals or models level above it. Correlations are settled on dependent on the decision or judgment of the leader by evaluating the degree of significance of a component contrasted with different components.
- Normalize information is by isolating the estimation of every component in the combined grid with the all-out estimation of every section.
- Calculate the eigenvector worth and test its consistency, in the event that it isn’t reliable, at that point information recovery (inclination) should be rehashed. The eigenvector esteem being referred to is the most extreme eigenvector esteem got utilizing Matlab or physically.
- Repeat stages 3, 4, and 5 for all degrees of the chain of command.
• Calculates the eigenvector of each matched examination lattice. The eigenvector esteem is the heaviness of every component. This progression is to combine decisions in organizing components at the most reduced degree of the pecking order to the accomplishment of goals.

• Test the consistency of the progressive system. In the event that it doesn't meet with CR <0, 100 then the appraisal must be rehashed.

In resolving AHP issues, there are several principles that need to be understood including [7]:

• Decomposition

Decomposition needs to be done, namely breaking down the entire problem into its elements after the problem is defined first. If you want to get an accurate result, it is impossible for further solutions to be carried out so that some action can be obtained from the problem because the solution is also carried out on the elements. The analysis process is called a hierarchy for this reason,

![Figure 1. Hierarchy](image)

• Comparative Judgement

These standard methods making decisions about the overall significance of two components at a specific level according to the level above it. This evaluation is the center of AHP, on the grounds that it will influence the need of the components. The aftereffects of this appraisal will be set as a lattice called a pairwise examination framework. In setting up the size of interests utilizing benchmarks that can be found in Table 3.

**Table 1. Basic comparison criteria**

| Intensity Importance | Definition                                                                 |
|----------------------|-----------------------------------------------------------------------------|
| 1                    | The two components are similarly significant                               |
| 3                    | One component is somewhat more significant than the other                   |
| 4                    | One component is a higher priority than different components                |
| 7                    | One component is obviously more significant than another component         |
| 9                    | One supreme component is a higher priority than different components       |
| 2,4,6,8              | The qualities between the two contemplations are near one another            |

• Synthesis of Priority

Search for the eigenvector value to get local priority from each pairwise comparison matrix. Because to get global priority, synthesis must be carried out between local priorities in the pairwise comparison
matrix that exists at each level. Priority setting is the order of elements through the synthesis procedure according to relative importance.

- **Logical Consistency**

Consistency has two implications, first is that comparable items can be assembled by consistency and significance. The subsequent significance is identified with the degree of connection between objects dependent on specific models. The consistency pointer is estimated through the Consistency Index (CI) which is detailed:

\[
Z_{\text{max}} = \frac{\sum_{i=1}^{n} \text{Vector Consistency}}{n} \\
\text{CI} = \frac{Z_{\text{max}} - n}{n-1}
\]

Information:
- \(n\) = Number of items compared
- \(Z_{\text{max}}\) = Average value calculated earlier

Random consistency index can be seen in Table 2.

| Table 2. Random consistency index |
|----------------------------------|
| N  | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| RI  | 0.58| 0.90| 1.12| 1.24| 1.32| 1.41| 1.45| 1.51|

So, the degree of inconsistency for pair comparison in the decision criteria matrix in the previous example is calculated by the ratio of CI to RI:

\[
\text{CR} = \frac{\text{CI}}{\text{Random Consistency Index}}
\]

Information:
- CR = Consistency Ratio
- CI = Consistency Index
- RI = Random Index

In general, the degree of consistency is satisfactory if: \(\frac{\text{CI}}{\text{RI}} < 0.10\)

### 2.3. Fuzzy Association Theory

To represent uncertainty, ambiguity, inaccuracy, lack of information, and partial truth, fuzzy set theory is the mathematical framework used. Lack of information, in solving problems, is often found in various fields of life. The discussion of vagueness began in 1937, when a philosopher named Max Black expressed his opinion about obscurity. Black defines a proportion of obscurity as a proportion where the probable status of the proportion is not clearly defined. For example, to state that someone belongs to the young category, the statement “young” can give a different interpretation than by each individual, and we cannot give a certain age to say someone is young or not young [5].

There are several reasons why people use fuzzy logic, including:

- The idea of Fuzzy Logic is straightforward. The numerical idea fundamental Fuzzy Reasoning is exceptionally basic and straightforward.
- Fuzzy Logic is entirely adaptable.
- Fuzzy Logic has a capacity to bear wrong information.
- Fuzzy Logic can demonstrate nonlinear capacities that are exceptionally mind-boggling.
- Fuzzy Logic can manufacture and apply the encounters of specialists straightforwardly without experiencing a preparation cycle.
- Fuzzy Logic can work with traditional control procedures.
- Fluffy Logic depends on common language.

2.3.1. Membership Function. Membership function is a curve that shows the mapping of data input points into the membership value (degree of membership) which has an interval between 0 to 1. One way that can be used to obtain membership values is through the function approach [8].

2.3.2. Triangular Fuzzy Number. Triangular fuzzy number is a combination of two lines (Linear). The graph of the triangular membership function is depicted in the form of a triangular curve as seen in Figure 2[9].

Fuzzy set theory that helps in measuring using language or linguistics related to human subjective judgments is the definition of triangular fuzzy numbers. Pairwise comparisons depicted with ratio scales associated with fuzzy scales are the essence of fuzzy AHP. Fuzzy triangular numbers are symbolized and the following membership function provisions for 5 scale linguistic variables.

| AHP scale | Fuzzy Number | Invers Value of Fuzzy Number | Definition |
|-----------|--------------|-------------------------------|------------|
| 1         | (1,1,1)      | (1, 1, 1)                     | Equally important |
| 2         | (1,2,3)      | (1/3,1/2,1)                   | Scale between the same and a little more important |
| 3         | (2,3,4)      | (1/4,1/3,1/2)                 | Low dominance |
| 4         | (3,4,5)      | (1/5,1/4,1/3)                 | Scale between low dominance and high dominance |
| 5         | (4,5,6)      | (1/6,1/5,1/4)                 | High dominance |
| 6         | (5,6,7)      | (1/7,1/6,1/5)                 | Scale between high dominance and very high dominance |
| 7         | (6,7,8)      | (1/8,1/7,1/6)                 | Very high dominance |
| 8         | (7,8,9)      | (1/9,1/8,1/7)                 | Scale between very high dominance and absolute dominance |
| 9         | (8,9,9)      | (1/9,1/9,1/8)                 | Absolute dominance |

2.4. VIKOR
The VIKOR technique is a relevant MCDM strategy started by Opricovic [6]. It is improved as a multi standards dynamic strategy. VIKOR is utilized to take care of discrete choice issues with clashing standards which can help the leaders to enhance complex frameworks to get a last arrangement. This trade off positioning strategy centers around a lot of limited options that are assessed by every model capacity. The positioning could be performed by contrasting the proportion of closeness with the ideal options [10].

Figure 2. Triangular fuzzy number
The following stages in the VIKOR method:

- Compute normalized quantities by using Eq.
  Assume \( m \) alternatives and \( n \) criteria.
  With \( i=1,2,3,...,m; \text{ and } j=1,2,3,...n \)
  \[
  F_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}
  \tag{4}
  
- Determine the best (\( F_j^+ \)) and the worst (\( F_j^- \)) quantities in each area
  If we assume the \( j \)th function represents a benefit then \( F_j^+ = \max F_{ij} \) (or setting an aspired level) and \( F_j^- = \min F_{ij} \) (or setting a tolerable level).
  Alternatively, if we assume the \( j \)th function represents a cost/risk, then \( F_j^+ = \min F_{ij} \) (or setting an aspired level) and \( F_j^- = \max F_{ij} \) (or setting a tolerable level).

- Determine weights of the criteria
  The weights of the criteria should be computed to express the relative importance.

- Compute the values \( S_i \) and \( R_i \); \( i = 1,2,...,m \) by the equations
  \[
  S_i = \frac{\sum_{j=1}^{n} W_j (F_j - F_{ij})}{(F_j - F_{ij})} 
  \tag{5}
  \]
  \[
  R_i = \max \left[ w_j (F_j - F_{ij}) / (F_j - F_{ij}) \right] 
  \tag{6}
  
  Where \( w_j \) are the criteria’s weights, expressing their relative importance.
  \( S_i \) and \( R_i \) are respectively the \( L_1,i \) and \( L_{\infty},i \) in the \( L_p \)-metric used in the compromise programming method.

- Compute the values \( Q_i \); \( i = 1,2,...,m \) by the Eq.
  \[
  Q_i = v \left[ \frac{S_i - S'}{S - S'} \right] + (1 - v) \left[ \frac{R_i - R'}{R - R'} \right] 
  \tag{7}
  
  Where, \( S=\max S_i, \ S'=\min S_i, \ R=\max R_i, \ R'=\min R_i \)
  \( v \) is introduced as the weight of the strategy of “the majority of Criteria” (or “the maximum group utility”) and usually \( v = 0.5 \)

- Rank the alternatives, sorting by the values \( S_i, R_i, \text{ and } Q_i \), in decreasing order.

- Investigate as a compromise solution the alternative \( A' \), which is ranked the best Alternative according to the measure \( Q \) (minimum) if the following two conditions are satisfied:
  Condition 1. Acceptable advantage:
  \[
  Q(A'') - Q(A') \geq \frac{1}{m-1} 
  \tag{8}
  
  Where \( A'' \) is the alternative with second position in the ranking list by \( Q \); \( m \) is the number of alternatives.

  Condition 2. Acceptable stability in decision making: Alternative \( A' \), must also be the best ranked by \( S \) or/and \( R \). This compromise solution is stable within a decision-making process, which could be “voting by majority rule” (when \( v > 0.5 \) is needed), or “by consensus” \( v \approx 0.5 \), or “with veto” \( v < 0.5 \). Here, \( v \) is the weight of the decision-making strategy “the majority of criteria” (or “the maximum group utility”)
  If one of the conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:
  Alternatives \( A' \), and \( A'' \) if only condition 2 is not satisfied, or;
  Alternatives \( A'; A'',...,A^{(M)} \) if condition 1 is not satisfied; \( A^{(M)} \) is determined by the relation
  \[
  Q(A^{(m)}) - Q(A') < \frac{1}{m-1} 
  \tag{9}
  
  for maximum \( m \) (the positions of these alternatives are “in closeness”).
3. Research Methodology
The method used in this paper is a literature review study. The writing of this literature review is based on international and national journals. The collected journals are journals that discuss the integration of Fuzzy AHP and VIKOR. The journals are collected through the Google Scholar and Science Direct websites.

4. Result

4.1. Fuzzy AHP
The watched qualities in true issues are frequently uncertain or obscure. Loose or unclear information might be the aftereffect of unquantifiable, inadequate, and non-realistic data. They are frequently communicated with limited stretches, ordinal (position request) information or Fuzzy numbers.

To successfully deal with emotional recognitions and inaccuracy, Fuzzy numbers are coordinated with AHP, permitting the proper articulation of etymological assessment (Calabrese et al., 2016). Fluffy numbers are additionally used to manage vulnerabilities influencing abstract inclinations in evaluating true dynamic issues.

In spite of the comfort of AHP in dealing with both quantitative and subjective rules of MCDM issues dependent on leaders’ decisions, FAHP can decrease or even kill the Fuzziness; dubiousness existing in numerous dynamic issues may add to the loose decisions of chiefs in customary AHP approaches. The field of AHP has quickly developed. As found in this book, as of late, numerous analysts have planned FAHP models in numerous applications to manage circumstances where a portion of the information are uncertain or dubious. The FAHP strategy is along these lines fit to tackling dynamic issues concerning abstract assessments and is at present among the most generally utilized MCDM strategies in the fields of business, the board, assembling, industry and government.

The main FAHP technique was proposed by Van Laarhoven and Pedrycz utilizing Triangular Fuzzy Numbers (TFNs) in the pairwise correlation grid. Afterward, numerous different strategies were proposed, utilizing different sorts of Fuzzy numbers, for example, the Trapezoidal enrollment work or the chime molded/Gaussian participation work. In later years, FAHP has generally been applied in the territories of choice and assessment with noteworthy of writing on consolidating/incorporating FAHP with different instruments, especially with the TOPSIS, QFD, Delphi and ANP [4].

4.2. FAHP-VIKOR
The thought behind VIKOR is to choose the alternative that best accomplishes a harmony between two conditions: to be as close as conceivable to the positive-ideal arrangement. The positive-ideal arrangement speaks to the virtual most ideal choice that would have been made by choosing the best presentation for each boundary among real proposition; the negative-ideal arrangement speaks to the virtual most exceedingly awful alternative that would have been created by choosing the most exceedingly terrible exhibition for each boundary among the real recommendations.

VIKOR additionally can be utilized to rank choices in understanding to unique variable that estimates both their closeness to the virtual most ideal choice and their farness to the virtual most noticeably terrible alternative [11].

By utilizing the FAHP and VIKOR techniques, dynamic issues can be settled and as well as can be expected be gotten from tackling these issues. Choice of the best elective utilizing the VIKOR technique makes it simple to pick an appropriate answer for be applied to the issue

5. Discussion
This literature study shows that the AHP is a choice help model that will portray a complex multifaceted or multi-rules issue into a progression. To adequately deal with abstract observations and inaccuracy, Fuzzy numbers are incorporated with AHP, permitting the proper articulation of etymological assessment. The thought behind TOPSIS is to choose the alternative that best accomplishes a harmony between two conditions: to be as close as conceivable to the positive-ideal
arrangement. The positive-ideal arrangement speaks to the virtual most ideal choice that would have been made by choosing the best presentation for each boundary among genuine recommendations; the negative-ideal arrangement speaks to the virtual most exceedingly terrible choice that would have been made by choosing the most exceedingly terrible exhibition for each boundary among the real proposition.

In applying AHP and VIKOR technique there is additionally shortcoming in both strategies. The shortcoming of the AHP technique is the reliance of the AHP model on its primary data sources. This primary information is as a specialist's recognition so it includes subjectivity and AHP technique is just a numerical strategy without factual testing so there is no certainty cutoff of the accuracy of the model framed [9].

The soft spot for the VIKOR strategy is at the weighting stage, the weighting cycle is just parted with by the predominant/chief without a weighting consistency check like the AHP technique.

6. Conclusion

The AHP method is a support model that will depict a complex multifaceted or multicriteria in progressive system. The incorporated Fuzzy in Analytical Hierarchy Process technique to deal with emotional discernments and impreciseness appeared in Kamran Rezaie [9] diary which examines the utilization of the Fuzzy AHP and VIKOR strategies assessing execution of Iranian Cement Firms. The thought behind VIKOR is to choose the choice that best accomplishes a harmony between numerous conditions: to be as close as conceivable to the positive-ideal arrangement. The positive-ideal arrangement speaks to the virtual most ideal alternative that would have been formed by choosing the best exhibition for each boundary among genuine proposition; the negative-ideal arrangement speaks to the virtual most noticeably awful choice that would have been made by choosing the most noticeably terrible presentan for each boundary among the real recommendations.

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