Dark Matter Admixed Neutron Star Properties in the Light of X-Ray Pulse Profile Observations

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Abstract

The distribution of the dark matter (DM) in DM-admixed neutron stars (DANSs) is supposed to result in either a dense dark core or an extended dark halo, subject to the DM fraction of the DANS ($f_{DM}$) and the DM properties, such as the mass ($m_\chi$) and the strength of the self-interaction ($\gamma$). In this paper, we perform an in-depth analysis of the formation criterion for dark cores/dark halos, and point out that the relative distribution of these two components is essentially determined by the ratio of the central enthalpy of the DM component to that of the baryonic matter component inside the DANSs. For the critical case where the radii of the DM and the baryonic matter are the same, we further derive an analytical formula to describe the dependence of $f_{DM}^{\text{crit}}$ on $m_\chi$ and $\gamma$ for a given DANS mass. The relative distribution of the two components in DANSs can lead to different observational effects. We here focus on the modification of the pulsar pulse profile, due to the extra light-bending effect in the case of a dark halo existence, and conduct the first investigation into the dark halo effects on the pulse profile. We find that the peak flux deviation is strongly dependent on the ratio of the halo mass to the radius of the DM component. Last, we perform Bayesian parameter estimation on the DM particle properties, based on the recent X-ray observations of PSR J0030+0451 and PSR J0740+6620 by the Neutron Star Interior Composition Explorer.

Unified Astronomy Thesaurus concepts: Dark matter (353); Neutron stars (1108); Pulsars (1306)

1. Introduction

Neutron stars (NSs) could act as astrophysical laboratories for testing the possible effects of dark matter (DM) and indirectly measuring DM particle properties (Goldman & Nussinov 1989). Multimessenger observations have been explored to constrain the structure of DM-admixed NSs (DANSs), which in turn puts limits on the DM parameter space. In this context, there are many studies of how DM may impact various observable properties of NSs, such as the mass–radius relation (Narain et al. 2006; Sandin & Ciarcualli 2009; Ciarcualli & Sandin 2011; Leung et al. 2011; Leung et al. 2012; Li et al. 2012, 2012; Goldman et al. 2013; Xiang et al. 2014; Kouvaris & Nielsen 2015; Li et al. 2015; Mukhopadhyay et al. 2017; Panotopoulos & Lopes 2017; Rezaei 2017; Motta et al. 2018; Wang et al. 2019), the surface temperature (Bertone & Fairbairn 2008; Kouvaris 2008; de Lavallaz & Fairbairn 2010; Gonzalez & Reisenegger 2010; Kouvaris & Tynakoy 2010; Fan et al. 2011), the kinematics and rotation properties (Pérez-García & Silk 2013), the tidal deformability (Ellis et al. 2018; Das et al. 2019; Nelson et al. 2019; Das et al. 2020; Fasano et al. 2020; Quddus et al. 2020; Das et al. 2021; Husain & Thomas 2021; Lee et al. 2021; Sagun et al. 2021; Sen & Guha 2021; Dengler et al. 2022; Di Giovanni et al. 2022; Rafiei Karkevandi et al. 2022), or the gravitational-wave waveform during the postmerger stage (Kopp et al. 2018; Bezares et al. 2019; Bauswein et al. 2020).

In general, DM can form either a dense core inside the NS or an extended halo surrounding the NS, which would result in different or even opposite effects on some properties of DANSs. Since the detection of the GW170817 binary NS merger (Abbott et al. 2017), the study of DANSs through tidal deformability has gained much attention. For example, it has been found that an extended dark halo could dramatically enhance the stellar tidal deformability $\Lambda$ (Nelson et al. 2019), while a dense dark core would affect it in the opposite way (Ellis et al. 2018; Sagun et al. 2021). The upper bound of tidal deformability for an NS with $1.4 M_\odot \Lambda_{1.4} \leq 800$ (Abbott et al. 2017) reported by the LIGO/Virgo collaboration has been used to provide new constraints on DM properties (Ellis et al. 2018; Quddus et al. 2020; Husain & Thomas 2021; Sagun et al. 2021). It has been suggested that the future detection of a large tidal deformability that exceeds the expected range of an NS could be interpreted as a dark halo (Nelson et al. 2019). The possibility of the $2.6 M_\odot$ compact object in the binary merger GW190814 (Abbott et al. 2020) being a DANS has also been investigated in the literature (Das et al. 2021; Lee et al. 2021; Wüsteb et al. 2021).

The formation of a dark halo or dark core should be determined by the nature of the DM and the amount of the DM accumulated in the DANS. As illustrated in, e.g., Ivanovtsyki et al. (2020) and Rafiei Karkevandi et al. (2022), light DM particles tend to form a dark halo, while heavy ones prefer to form a dark core. In the scenario of self-interacting DM, the strength of the DM self-interaction also plays an important role. Qualitatively, the larger the strength of the self-repulsive interaction, the easier it is for DM to occupy a larger volume; that is, the easier it is to form a dark halo. However, there is no general criterion in the current literature to determine whether it is a DM halo or a dark core when a specific DM model is assumed. This paper aims to perform an in-depth analysis of the formation criterion for dark cores/dark halos. As deduced in later sections, we point out that the central enthalpy is the key factor. For example, if the central enthalpy of the baryonic matter ($h_0^b$) is larger than that of the DM ($h_0^\chi$), then a dark core forms; otherwise, there should be a dark halo. For the critical case where the radii of the DM and the baryonic matter are the...
same, the amount of DM in the DANS, represented by the DM fraction of the DANS \( f_{\chi} \) at a fixed DANS total mass \( (M_p) \), should depend on a specific DM model. We further derive an analytical formula to describe the dependence of \( f_{\text{crit}} \) on the DM particle mass \( (m_{\chi}) \) and the strength of the DM self-interaction.

X-ray oscillations have been observed from a few millisecond pulsars. The shape of the pulsation in the X-ray flux (i.e., the pulse profile) encodes information about the surface properties of the NS, as well as its exterior spacetime. Therefore, when a dark halo is formed in a DANS, the pulses emitted from the surface of the baryonic matter should suffer an additional gravitational potential, and the modification of the pulse profile is expected. Di Giovanni et al. (2022) first mentioned such an effect in their discussion, but, so far, no detailed numerical calculations have been performed on how the dark halo would modify the pulse profile emitted from the surface of the baryon component. Such an analysis is carried out for the first time in the present work. As we will see later, this modification depends sensitively on the ratio of the halo mass to the radius of DM component, \( M_{\text{halo}}/R_{p} \). For a compact dark halo with \( M_{\text{halo}}/R_{p} \approx 0.01 \), the modification of the pulse profile can reach about 10%. Therefore, modified pulse profile modeling is necessary when one applies the Neutron Star Interior Composition Explorer (NICER) measurements to study DANSs. As an example, we show how we incorporate recent NICER data to study DANSs and constrain the dark DM particle properties.

This paper is organized as follows. In Section 2, we present the formalism of self-interacting fermionic DM. In Section 3, we study the structure of DANSs. In particular, we focus on deriving the criterion for the formation of cores/halos and estimating the critical amount of DM. In Section 4, we discuss the effects of dark halos on the DANS pulse profiles. Then, in Section 5, we perform Bayesian parameter estimation on the DM properties, by using the NICER results from X-ray pulse profile modeling. We summarize in Section 6.

## 2. DM Equation of State

In this work, we consider DM to be made of self-interacting fermions with masses from MeV to GeV. We assume that the DM particles are nonannihilating, as in asymmetric DM (Zurek 2014). Neglecting finite temperature effects, the energy density of DM with particle mass \( m_{\chi} \) is given by (Narain et al. 2006; Nelson et al. 2019):

\[
\epsilon_{\chi} = \epsilon_{\text{kin}} + m_{\chi} n_{\chi} + \frac{(\hbar c)^2 n_{\chi}^2}{m_f^2},
\]

where \( n_{\chi} \) is the DM number density and \( \epsilon_{\text{kin}} \) is the DM kinetic energy. \( m_f \) represents the energy scale of the self-interaction via a Yukawa type in the repulsive case (see the further discussions in Kouvaris & Nielsen 2015 and Gresham & Zurek 2019), providing stabilization with respect to the gravitational collapse (Nelson et al. 2019; Ivanovski et al. 2020). For weak interaction, \( m_f \sim 300 \text{ GeV} \), as mediated by W or Z bosons, while for strongly interacting DM particles, \( m_f \) is assumed to be \( \sim 100 \text{ MeV} \), according to chiral perturbation theory (Narain et al. 2006).

The kinetic energy can be calculated using the model of an ideal Fermi gas:

\[
\epsilon_{\text{kin}} = \frac{1}{\pi^2 (\hbar c)^3} \int_0^{k_F} dk k^2 (\sqrt{k^2 + m_{\chi}^2} - m_{\chi}),
\]

where \( k_F = \hbar c (3\pi^2 n_{\chi})^{1/3} \) is the Fermi momentum. By denoting \( x = k_F/m_{\chi} \) and \( y = m_{\chi}/m_f \), the energy density and pressure are determined as:

\[
\epsilon_{\chi} = \frac{m_{\chi}^4}{8\pi^2 (\hbar c)^3} \left[ (2x^3 + x) \sqrt{1 + x^2} - \sinh^{-1}(x) \right]
\]

\[
+ \frac{m_{\chi}^4 y^2 x^6}{(3\pi^2)^2 (\hbar c)^4},
\]

\[
P_{\chi} = \frac{m_{\chi}^4}{24\pi^2 (\hbar c)^3} \left[ (2x^3 - 3x) \sqrt{1 + x^2} + 3 \sinh^{-1}(x) \right]
\]

\[
+ \frac{m_{\chi}^4 y^2 x^6}{(3\pi^2)^2 (\hbar c)^4}.
\]

The DM equation of state (EOS), i.e., the pressure–density relation, is then obtained.

## 3. DANS Structure

The DANS stable configuration in hydrostatic equilibrium is obtained by solving the two-fluid Tolman–Oppenheimer–Volkoff (TOV) equations (Kodama & Yamada 1972; Sandin & Ciarculli 2009; Ciarculli & Sandin 2011) for pressure and enclosed mass:

\[
\frac{dP_B}{dr} = - \frac{GM(r) \epsilon_B(r)}{r^2} \times \left( 1 + \frac{4\pi r^3 P(r)}{M(r)} \right) \times \left( 1 + \frac{P_B(r)}{\epsilon_B(r)} \right) \left( 1 - \frac{2GM(r)}{r} \right)^{-1},
\]

\[
\frac{dP_D}{dr} = - \frac{GM(r) \epsilon_D(r)}{r^2} \times \left( 1 + \frac{4\pi r^3 P(r)}{M(r)} \right) \times \left( 1 + \frac{P_D(r)}{\epsilon_D(r)} \right) \left( 1 - \frac{2GM(r)}{r} \right)^{-1},
\]

\[
\frac{dM_B}{dr} = 4\pi r^2 \epsilon_B(r),
\]

\[
\frac{dM_D}{dr} = 4\pi r^2 \epsilon_D(r),
\]

where \( M(r) = M_B(r) + M_D(r) \) and \( P(r) = P_B(r) + P_D(r) \), with the subscript indexes “B” and “D” standing for the baryonic matter and DM components, respectively. Given the central pressures of the two components \( P_{B,c} \) and \( P_{D,c} \) as initial conditions, we solve Equations (5)–(8) outward by employing the Runge–Kutta fourth-order method. In each iteration step, we calculate the densities of the two components from their pressures by using the corresponding EOSs. We integrate Equations (5)–(8) outward, until the pressures of the baryonic matter and DM vanish. This gives the radii of the baryon component \( R_B \) and the DM component \( R_D \). The gravitational masses of the two components are then \( M_B(R_B) \) and \( M_D(R_D) \), respectively. Hereafter, we denote \( f_{\chi} = M_D(R_D)/M_F \) as the DM fraction, where \( M_F = M_B(R_B) + M_D(R_D) \) is the total mass of the DANS.
gravitational mass. For the description of the baryonic matter, we use the NL3\_zp EOS, developed in relativistic effective field theories to fit nuclear matter saturation and the properties of finite nuclei (Horowitz & Piekarewicz 2001). The DM EOS can be varied by changing the particle mass \( m_\chi \) and the interaction strength \( y \). Note that we have assumed that the baryonic matter and DM components couple essentially only through gravity, since the nongravitational interactions between them could be negligibly small (Marrodán Undagoitia & Rauch 2016; Rrapaj et al. 2017).

### 3.1. Mass–Radius Relation

In Figure 1, we show the mass–radius relation of DANSs with a DM fraction \( f_\chi = 0.05 \) (Ellis et al. 2018). The calculations are done for two cases of DM interaction strengths \((y = 0.1, 1000)\) and two cases of DM particle mass \((m_\chi = 0.1, 1 \text{ GeV})\). We see that in the case of the small interaction strength \((y = 0.1)\), the DM effects on the mass–radius relation are dominated by the DM particle mass, namely light DM particles \((m_\chi = 0.1 \text{ GeV})\) tend to increase the maximum mass. In contrast, heavy DM particles \((m_\chi = 1 \text{ GeV})\) would instead reduce the maximum mass of NSs. In the case of the large interaction strength \((y = 1000)\), regardless of the DM particle mass, one observes an increase in the total gravitational mass, but nearly unchanged radii for the baryon components.

The density profiles of \( M_T = 1.4 \, M_\odot \) DANSs are further reported in Figure 2, for both cases of DM interaction strengths and particle masses. The DM fraction is again fixed at \( f_\chi = 0.05 \). The influences of the DM properties on the mass–radius relations of DANSs observed above in Figure 1 can then be understood as follows.

1. In the case of the small interaction strength \((y = 0.1)\), light DM particles \((m_\chi = 0.1 \text{ GeV})\) would form a dark halo, i.e., \( R_D > R_b \). Therefore, most DM gathers into the halo, leaving only a small mass fraction inside the DANSs to impose a negligible influence on the stellar structure. As a result, one observes an increase in the total gravitational mass, which comes from the halo mass, but nearly unchanged radii for the baryon components. The situation changes for heavy DM particles with \( m_\chi = 1 \text{ GeV} \), as they form a dense core in the NS interior. Under this circumstance, the gravitational effect of the core cannot be neglected, and, consequently, the radius of the baryon component shrinks.

2. In the case of the large interaction strength \((y = 1000)\), the repulsion could force the DM to form a halo structure. In this case, we would again expect an increase in the total mass and an unchanged radius for the baryon component. As a result, the mass–radius relation could be indistinguishable for different DM particle masses. Nevertheless, it is possible to probe the \( m_\chi \) value from the radius of the DM component \( R_D \)—for example, through measurements of the tidal deformability \( \Lambda \) (Abbott et al. 2017)—as it approximately scales with the fifth power of the stellar radius, i.e., \( \Lambda \propto R_D^5 \).

### 3.2. Dark Cores and Dark Halos

As noted earlier, the distributions of DM in DANSs present two scenarios, namely dark halos \((R_D > R_b)\) and dark cores \((R_D < R_b)\). The formation of a dark core/halo depends not only on the DM properties, but also on the DM amount. The actual amount of DM accumulated in NSs should depend on the evolutionary history and the external environments of the NSs. See, e.g., Kain (2021) for detailed discussions of the mixing of DM with ordinary matter in NSs. However, a thorough exploration of the accumulation mechanisms is still lacking, and would be an interesting topic to explore in the future.

For our present purpose, we take two representative values of the DM fraction, \( f_\chi = 0.05 \) (Ellis et al. 2018) and \( f_\chi = 10^{-4} \) (Nelson et al. 2019).

In Figure 3, we show the \((m_\chi, y)\) parameter spaces for DM particles forming a dark core/halo. The calculations are done with a fixed total gravitational mass \( M_T = 1.4 \, M_\odot \). As expected, for both cases of DM fraction, light/heavy DM particles tend to form a dark halo/core, and large/small interaction strengths tend to create a dark halo/core. The solid black lines in Figure 3 mark the borders of the purple dark halo regions and the pink dark core regions. At small interaction strengths, the

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Figure 1. Total gravitational mass as a function of the radius of the baryon component. The solid black curve is for pure NSs from the NL3\_zp EOS (Horowitz & Piekarewicz 2001), without DM, whereas the other curves are for DANSs with a DM fraction \( f_\chi = 0.05 \) (Ellis et al. 2018). The NICER mass–radius measurements for PSR J0030+0451 (Miller et al. 2019) and PSR J0740+6620 (Miller et al. 2021; Riley et al. 2021) from X-ray pulse profile modeling are also shown in the shaded regions.

Figure 2. Energy density as a function of the stellar radius for \( M_T = 1.4 \, M_\odot \) DANSs. The DM fraction is fixed at \( f_\chi = 0.05 \) (Ellis et al. 2018). The solid curves show the density profiles for the DM components, while the dashed curves show those for the baryon components.
solid black lines are almost parallel to the y-axis, while at large interaction strengths, there is \( m_\chi \propto y^{1/2} \). When decreasing the DM fraction from \( f_\chi = 0.05 \) (in the upper panel) to \( 10^{-4} \) (in the lower panel), the dark halo (dark core) region shrinks (expands), with the black border lines shifting slightly to the left.

For given DM properties, the critical DM fraction, \( f_\chi^{\text{crit}} \), at which the radii of the baryonic matter and DM components coincide, can be calculated, i.e., \( R_B = R_D \). In Figure 4, we report the critical DM fractions as functions of the DM particle mass (left panel) and the interaction strength (right panel). In the left panel, we see that there exists an excellent correlation between \( m_\chi \) and \( f_\chi^{\text{crit}} \), as \( f_\chi^{\text{crit}} \propto m_\chi^{-4} \). The right panel shows that \( f_\chi^{\text{crit}} \approx \text{const} \) for \( y \ll 1 \), but \( f_\chi^{\text{crit}} \propto y^{-2} \) when \( y \gg 1 \). In the following, we further demonstrate and examine in detail the dependence of \( f_\chi^{\text{crit}} \) on the DM properties.

We first raise a new criterion for the formation of a dark core/halo. We begin with the chemical potential in the proper frame, \( \mu_c \), which should be a constant throughout the equilibrium configuration (Kodama & Yamada 1972), i.e.:

\[
\mu_c = \mu \sqrt{g(r)} = \text{const},
\]

where \( \mu = d\epsilon / dn \) is the chemical potential in the local Lorentz frame and \( g(r) \) is the \((t,t)\) component of the metric. By defining the relativistic specific enthalpy \( h \) as

\[
h = \int \frac{dP}{\epsilon + P} = \int \frac{d\mu}{\mu},
\]

we find that the derivative,

\[
\frac{dh}{dr} = -\frac{d \ln \sqrt{g(r)}}{dr},
\]

is only a function of \( r \). Therefore, the decreases of the enthalpies of the two components, \( h_B \) and \( h_D \), must be equal in the same radial interval \([r, r + dr]\). Combining this with the fact that the decrease of \( h_B / h_D \) will terminate once \( h_B / h_D \) vanishes at the surface of the baryonic matter/DM, we conclude that the formation of a dark core/halo is determined by the central enthalpies of the two components, i.e.:

\[
\begin{align*}
& h_B^c < h_B^c \Rightarrow \text{dark core}, \\
& h_B^c > h_B^c \Rightarrow \text{dark halo},
\end{align*}
\]

where \( h_B^c \) and \( h_D^c \) are the central enthalpies of the baryon and DM components, respectively.

With this criterion at hand, we proceed to derive the critical DM fraction of a DANS. In such critical cases, we denote the central enthalpy as \( h_c = h_B^c = h_D^c \) and the radius as \( R = R_B = R_D \). For the fermionic DM considered in the present paper, we have

\[
h_c = \ln \left( \frac{\sqrt{1 + x^2} + 2 y^2}{3 \pi^2 x^3} \right).
\]

The asymptotic solutions of \( x \) can then be found analytically for small and large interaction strengths, i.e.:

\[
x = \begin{cases} 
\sqrt[3]{x^2 - 1} + \frac{3\pi^2 x^2}{2 y^2}, & y \ll 1, \\
\frac{3\pi^2 x^2}{2 y^2}, & y \gg 1.
\end{cases}
\]

For current observed NSs with \( M \sim 1 \sim 2 M_\odot \), the central enthalpy is \( h_c \sim 0.1 \sim 0.3 \). Under such a range of \( h_c \), the DM energy density \( \epsilon_\chi \) is dominated by the rest-mass term (see Appendix A for detailed discussions), i.e., \( \epsilon_\chi \approx m_\chi n_\chi \approx m_\chi^4 x^2 / (3 \pi^2) \). The amount of DM could then be estimated as \( M_D \approx \epsilon_\chi R^3 = x^3 m_\chi^4 R^3 / (3 \pi^2) \), or, equivalently,

\[
f_\chi^{\text{crit}} \approx 5 \times 10^{-7} \left( \frac{x}{0.1} \right)^3 \left( \frac{m_\chi}{100 \text{ MeV}} \right)^4 \times \left( \frac{1.4 M_\odot}{M_T} \right) \left( \frac{R}{12 \text{ km}} \right)^3.
\]

We here mention that Equation (14) together with Equation (13) can be used to explain all the features previously observed for \( f_\chi^{\text{crit}} \) in Figures 3 and 4:

\[\text{footnote}^3\] Another conclusion from this derivative is that the ratio of the two chemical potentials, \( \mu_B / \mu_D \), is a constant throughout the star; see, e.g., Goldman et al. (2013) and Ivanytskyi et al. (2020).
1. From Equation (14), \( m_x \propto x^{-3/4} \) at fixed \( f_{\text{crit}} \); then from Equation (13), \( m_x \) does not rely on the value of \( y \) if \( y \ll 1 \), whereas \( m_x \propto y^{1/2} \) if \( y \gg 1 \), as shown in Figure 3.

2. Obviously, from Equation (14), \( f_x \propto m_x^4 \), as shown in Figure 4 (left panel); and

3. Substituting \( x \) in Equation (14) for Equation (13), one can easily find that \( f_x \) is independent of \( y \) for \( y \ll 1 \), while \( f_x \propto y^{-2} \) for \( y \gg 1 \), as shown in Figure 4 (right panel).

4. Dark Halo Effects on the Pulsar Pulse Profile

Based on the oblate Schwarzschild approximation for the NS spacetime (Morsink et al. 2007), the pulse profile modeling technique has been used by NICER (see Watts et al. 2016 and Watts 2019, and references therein) to deliver tight constraints on the mass and radius of NSs (Miller et al. 2019; Riley et al. 2019; Miller et al. 2021; Riley et al. 2021) and to pin down the unknown dense matter EOS (Miller et al. 2019; Raaijmakers et al. 2019; Raaijmakers et al. 2021). Since the pulse profile may be modified due to the extra gravitational potential of the dark halo (Di Giovanni et al. 2022), in this section, we numerically calculate the pulse profiles of DANSs and investigate the effects resulting from the existence of a dark halo.

In this first attempt at studying the dark halo effects on pulse profiles, several simplifications are in order. We assume that radiation emitted from two point-like spots sitting oppositely on the baryon surface (i.e., at \( r = R_B \)). And the specific intensity of the radiation is assumed to be isotropic (Miller & Lamb 1998). We do include the Doppler boost and relativistic aberration in our calculation, but neglect the frame dragging and the stellar deformation due to rotation. We also assume that the distance between the DANS and the observer is large enough to be treated mathematically as infinity. Below, we introduce the necessary formalism for the calculations.

The observed differential flux at distance \( D \) from a point-like spot is given by (see, e.g., Poutanen & Beloborodov 2006 for a detailed derivation):

\[
dF = \frac{\delta g(R_B)I'(\nu')d\nu' \cos \alpha}{d \cos \psi} \frac{d \cos \alpha}{d \cos \psi} \frac{dS'}{D^2},
\]

where \( \delta \) is the Doppler factor, \( I'(\nu') \) is the radiation intensity at the comoving frame, \( \alpha \) is the emission angle, and \( \psi \) is the angle between the local radial direction and the line of sight. \( dS' \) is the proper differential area where the photons are emitted (see Figure 1 in Poutanen & Beloborodov 2006 for an illustration).

For further analysis, we use the bolometric flux normalized by \( \int I'(\nu')d\nu'dS'/D^2 \):

\[
F = \int \frac{dF}{dS'} = \frac{\delta g(R_B)\cos \alpha}{d \cos \psi} = \frac{d \cos \alpha}{d \cos \psi}.
\]

Here, Equation (16) accounts for the special relativistic effects (the Doppler boost and relativistic aberration), the gravitational redshift, and light bending.

To obtain the normalized flux, one needs to determine the value of \( \psi \) and the relation between \( \psi \) and \( \alpha \). The latter can be derived from the photon geodesic equation as (see Appendix B for the derivation):

\[
\psi = \int_{R_0}^{\infty} \sqrt{\frac{b^2}{r^2} - \left(1 - \frac{8}{r^2}ight)^{-1/2}} dr,
\]

where \( b = R_B \sin \alpha \sqrt{g(R_B)} \) is the impact parameter and \( f(r) \) is the \( (r, r) \) component of the metric.

On the other hand, \( \psi \) can be obtained from the geometry relation

\[
\cos \psi = \cos i \cos \theta + \sin i \sin \theta \cos(2\pi \nu t),
\]

where \( i \) is the inclination angle of the spin axis to the line of sight, \( \theta \) is the spot colatitude, and \( \nu \) is the rotation frequency. Here, \( t = 0 \) is chosen when the spot is closest to the observer.

With \( \psi \) at hand, we invert Equation (17) to get \( \alpha \). We can then calculate the Doppler factor \( \delta \) and \( d(\cos \alpha)/d(\cos \psi) \) and obtain the normalized flux. Finally, the time-delay effects caused by the different travel paths of the emitted photons should be considered (see Appendix B for the derivation):

\[
\Delta t = \int_{R_0}^{\infty} \sqrt{\frac{f}{g} \left(1 - \frac{b^2g}{r^2}\right)^{-1}} dr.
\]

We correct the observer time as \( t_{\text{obs}} = t + \Delta t \).
For the following study, we define two new masses, namely: (i) $M_f(R_h) \equiv M_b(R_h) + M_d(R_h)$, which corresponds to the total gravitational mass inside the baryon radius $R_b$; and (ii) $M_{halo} \equiv M_f - M_T$, which corresponds to the mass of the dark halo. It is obvious that $M_f(R_b) = M_T$ in the absence of a dark halo; otherwise, $M_f(R_b) < M_T$. Notice that the original Schwarzschild spacetime outside a pure NS with no halo would be altered when adding a dark halo. The change can be characterized by the increase of $g(R_D)$ as

$$\Delta g(R_D) = \left( 1 - \frac{2M_T(R_b)}{R_D} \right) - \left( 1 - \frac{2M_f}{R_D} \right) = \frac{2M_{halo}}{R_D}. \quad (20)$$

Hereafter, we choose $M_{halo}/R_D$ as a characteristic parameter, and we will show that the modification of the pulse profile depends sensitively on $M_{halo}/R_D$.

In Table 1, we construct two DANSs by fixing $M_f(R_h) = 1.4 M_{\odot}$ and $M_{halo} = 0.15 M_{\odot}$, but varying the DM particle properties. Their corresponding pure NSs, which share the same $M_f(R_b)$ and $R_b$ as them, are also listed. It should be emphasized that the values of $M_{halo}/R_D$ are quite different for the two constructed DANSs. In Figure 5, we plot the metric as a function of the radius. We note that the metric functions of the DANSs deviate from their corresponding pure NS ones. Meanwhile, the deviations close to the star can be characterized by $M_{halo}/R_D$, since they are of the same order of magnitude.

In Figure 6, we show the pulse profiles for the DANSs and the pure NSs listed in Table 1. We take the spot colatitude $\theta = \pi/4$, the inclination angle $i = \pi/4$, and the rotating frequency $\nu = 200$ Hz in the calculations. It is noticeable that the pulse profiles of the DANSs deviate from the pure NS cases. The difference appears mainly in the amplitudes of the normalized flux, whereas that in the phase shift is almost negligible. The peak flux deviation is about 6% for the DANS II model ($M_{halo}/R_D = 7.7 \times 10^{-3}$), but only 0.05% for the DANS I model ($M_{halo}/R_D = 8.0 \times 10^{-3}$). The difference of nearly two orders of magnitude between the peak flux deviations is close to the difference between $M_{halo}/R_D$, implying that the peak flux deviation may strongly depend on $M_{halo}/R_D$.

To clarify this dependence, in Figure 7, we show in detail the peak flux deviation as a function of $M_{halo}/R_D$ for dozens of DANS models. These DANSs are generated with parameters chosen randomly throughout the parameter spaces of $\log(m_\gamma/\text{MeV}) \in [0, 6]$, $\log(y) \in [-2, 3]$, $M_f \in [1, 2.5] M_{\odot}$, $f_i \in [0, 0.5]$, $i \in [\pi/6, \pi/2]$, $\theta \in [\pi/6, \pi/2]$, and $\nu \in [100, 400]$ Hz. We observe an approximate linear correlation between the peak flux deviation and $M_{halo}/R_D$ in logarithmic space, i.e.,

$$\log \left( \frac{|\Delta F_{\text{peak}}|}{F_{\text{peak}}} \right) = 0.998 \log (\frac{M_{\text{halo}}}{R_D}) + 0.854, \quad (21)$$

with the corresponding coefficient of determination $R^2 = 0.995$. It is seen that for compact dark halos with $M_{halo}/R_D \simeq 0.01$, the deviations of peak flux can reach about 10%. Therefore, this may be relevant for the NICER analysis of the mass and radius of X-ray pulsars for the study of the dense matter EOS, and we also expect that such effects could be detected with future X-ray data. We mention here that although our study provides a connection between the observed flux and the microscopic physics of DM and baryonic matter, it is only a first, crude, approximation of the phenomena occurring in the observed flux of X-ray pulsars (see the discussions in, e.g., Lo et al. 2013, Psaltis et al. 2014, and Miller & Lamb 2015). A more detailed analysis incorporating, e.g., the number of photon counts and

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**Table 1**

Properties of the Two Constructed DANS Models, by Fixing $M_f(R_h) \equiv M_b(R_h) + M_d(R_h) = 1.4 M_{\odot}$ and $M_{halo} \equiv M_f - M_T = 0.15 M_{\odot}$, but Varying the DM Particle Mass $m_\gamma$ and the Interaction Strength $y$

| Model          | $m_\gamma/\text{GeV}$ | $\log(y)$ | $M_f(R_h)/M_{\odot}$ | $M_{halo}/M_{\odot}$ | $M_T/M_{\odot}$ | $f_i$ | $R_b/\text{km}$ | $R_D/\text{km}$ | $M_{halo}/R_D$ |
|----------------|------------------------|------------|------------------------|------------------------|----------------|------|----------------|----------------|----------------|
| DANS I         | 0.1                    | 1.5        | 1.4                    | 0.15                   | 1.55            | 0.096 | 13.75         | 2771.4         | $8.0 \times 10^{-5}$ |
| Pure NS I      | ...                    | ...        | 1.4                    | 0.15                   | 1.55            | 0.096 | 13.75         | 2771.4         | ...            |
| DANS II        | 1                      | 1.5        | 1.4                    | 0.15                   | 1.55            | 0.173 | 13.23         | 28.7           | $7.7 \times 10^{-3}$ |
| Pure NS II     | ...                    | ...        | 1.4                    | 0.15                   | 1.55            | 0.173 | 13.23         | 28.7           | ...            |

*Note.* The corresponding pure NS cases, which share the same $M_f(R_h)$ and $R_b$ as the two DANSs, are also listed.

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**Figure 5.** Metric components $g(r)$ and $f(r)$ as functions of the radius $r$ for the DANS models listed in Table 1. The Schwarzschild metrics outside a pure NS with $M_f = 1.4 M_{\odot}$, $r = 1-2M_f/r$, and $f = (1-2M_f/r)^{-1}$ are also shown with the solid black curves.
The empirical relation is shown with the solid black line. The blue and red dots are the results from the two DANSs listed in Table 1, with parameters chosen randomly throughout a wide parameter space. The corresponding pure NS cases are not the same, due to the small difference in the radii of the baryon component $R_B$ (see Table 1).

The obtained PDFs of the DM particle mass are quite different for the two cases. Assuming $M_{\text{halo}}/R_D \leq 10^{-3}$, the observational data prefer light DM particles, and we can place a 90% upper limit of $m_\chi \leq 1.5$ GeV. Heavier particles are mostly excluded, because they tend to reduce the maximum mass of the DANS below the masses of PSR J0030+0451 or PSR J0740+6620, whereas assuming $M_{\text{halo}}/R_D = 0$, there is a prominent peak at $m_\chi \sim 0.6$ GeV. The peak can be understood by considering the requirements of a dark core: light DM particles more easily form a dark halo, and thus they are more likely to be rejected in the nested sampling process. At the same time, to support the masses of the observed pulsars, $m_\chi$ favors those values that are not too large. Nevertheless, the interaction strength $y$ is poorly constrained by the observations of the pulsar pulse profile; due to that, within the current modeling of DM particles, the DM rest-mass term dominates the DM energy and pressure (see Appendix A). Nevertheless, as mentioned above, since the interaction strength could affect the tidal deformability, by affecting the radius of the DM component, future tidal deformability measurements from gravitational waves hold the promise of providing a complementary constraint on the interaction strength.

**5. DM Particle Properties**

In this section, we perform Bayesian parameter estimation on the unknown DM parameters, using the available NICER mass–radius measurements of two X-ray pulsars, i.e., PSR J0030+0451 (Miller et al. 2019; Riley et al. 2019) and PSR J0740+6620 (Miller et al. 2021; Riley et al. 2021). As illustrated above (see Section 3.2), $h_c$ plays an important role in determining the core/halo configuration. Therefore, it is convenient to combine the central enthalpy of baryonic matter $h_{Gc}$ and the ratio of the two enthalpies $q = h_{Gc}/h_{Gp}$, as well as the DM parameters $m_\chi$ and $y$, into a vector $\theta$. The total likelihood function in our analysis is expressed as:

$$p(\{M_i, R_i\}|\theta) = \prod_i p(\{M_{T,i}(R_B), R_{B,i}|\theta),$$

where we equate the individual likelihood $p_i$ of the observation $i$ to the joint posterior distributions of $M_i$ and $R_i$ reported by NICER (Riley et al. 2019, 2021), since the pulse profile modeling of NICER should be modified for DANSs and one cannot apply the NICER measurements to the study of DANSs directly. We simply treat $M_{T,i}(R_B)$ and $R_{B,i}$ as the measured mass and radius. Such treatments should be valid as long as $M_{\text{halo}}/R_D$ is sufficiently small, and we consider two cases consistent with it, i.e.: (i) $M_{\text{halo}}/R_D \leq 10^{-3}$, where the corresponding flux deviations are smaller than $\sim 1\%$; and (ii) $M_{\text{halo}}/R_D = 0$, which is equivalent to a dark core scenario.

We take logarithmic uniform distributions for the DM particle mass $(\log(m_\chi/\text{MeV}) \sim U[0, 6])$ and for the interaction strength $(\log(y) \sim U[-2, 3])$. The priors of $h_{Gc}$ are uniformly distributed in the ranges of $[0.1, 0.5]$ and $[0.2, 0.6]$ for PSR J0030+0451 and PSR J0740+6620, respectively. We also set uniform distributions for both cases of $M_{\text{halo}}/R_D \leq 10^{-3}$ $(q \sim U[0, 3])$ and for the case of $M_{\text{halo}}/R_D = 0$ $(q \sim U[0, 1])$. Note that $q \leq 1$ corresponds to a dark core.

With the priors and the likelihood function at hand, we sample from the posterior density using the nested sampling software PYMULTINEST (Buchner et al. 2014). The resulting probability density functions (PDFs) of the DM particle mass and interaction strength are shown in Figure 8.

The background emission will be performed in the future, for the observability related to DM halos.

**Figure 6.** Pulse profiles from two point-like spots for the DANSs listed in Table 1 (dashed curves). The spot colatitude at the northern hemisphere is $\theta = \pi/4$ and the inclination angle of the spin axis to the light of sight is $i = \pi/4$. The rotating frequency is taken to be $\nu = 200$ Hz. The corresponding pure NS cases, with same $M_B(R_B)$ and $R_B$, are also shown (solid curves). Note that the two pure NS cases are not the same, due to the small difference in the radii of the baryon component $R_B$ (see Table 1).

**Figure 7.** Peak flux deviation as a function of $M_{\text{halo}}/R_D$. The gray dots correspond to the results from dozens of DANS models, which are generated with parameters chosen randomly throughout a wide parameter space (see the text for details). The blue and red dots are the results from the two DANSs shown in Figure 6. The empirical relation is shown with the solid black line.
6. Summary and Conclusions

In this paper, we study how DM, admixed with ordinary baryonic matter in different distributions, changes the structure and observations of NSs, and confront the DANS properties with NS observational data. We consider DM as MeV–GeV self-interacting fermions, and explore in detail two possible scenarios: namely, the dark halo and the dark core.

Although the formation of a dark core/halo depends complicatedly on the DM particle mass ($m_\chi$) and self-interacting strength ($y$), as well as the fraction of DM accumulated ($f_\chi$), we here show that these dependencies can be understood in a unified manner by means of a newly proposed criterion for identifying the formation of a dark core or a dark halo. That is, the formation of a dark core/halo is essentially determined by the central enthalpies of the baryonic and dark components. We further provide a general formula for the critical amount of DM, $f_\chi^{\text{crit}}$, which is well consistent with the numerical results of solving the two-fluid TOV equations. In particular, $f_\chi^{\text{crit}}$ correlates linearly with the fourth power of the DM particle mass, i.e., $f_\chi^{\text{crit}} \propto m_\chi^4$; also, $f_\chi^{\text{crit}}$ is insensitive to $y$ in the weak interacting case ($y \ll 1$), but proportional to $y^{-2}$ in the strong interacting case ($y \gg 1$).

For the first time, we also estimate the dark halo effects on NS pulse profiles, which are observed in NICER-like X-ray missions. Our analysis reveals that the effects depend sensitively on the value of $M_{\text{halo}}/R_D$, and the modification in the peak flux deviation can reach up to ~10% if the dark halo is dense enough with $M_{\text{halo}}/R_D \approx 0.01$. Furthermore, we show that the existence of a dark halo could increase the maximum mass, while a dense dark core could dramatically decrease it, which is in agreement with previous works.

In addition, we apply the available NICER data to the study of DANSs, and present a Bayesian parameter estimation of the DM properties. In the dark core scenario ($M_{\text{halo}}/R_D = 0$), we find a peak PDF for the DM particle mass at $m_\chi \approx 0.6$ GeV, while in the dark halo scenario, there is a lower upper limit of $m_\chi \leq 1.5$ GeV at the 90% confidence level, when assuming $M_{\text{halo}}/R_D \leq 10^{-5}$. No new constraint is found for the interaction strength between DM particles by analyzing the present pulse observations of PSR J0030+0451 and PSR J0740+6620 from NICER.

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Software: PyMultiNest (Buchner et al. 2014; https://github.com/JohannesBuchner/PyMultiNest).

Appendix A

The Dimensionless Fermi Momentum and Energy Density of DM

In Figure 9, we report the exact solutions of the dimensionless Fermi momentum $x$ from Equation (12), together with the asymptotic solutions of Equation (13). It is shown that the asymptotic solutions are sufficiently accurate for $y \lesssim 0.1$ or $y \gtrsim 10$. For the considered range of $h_c \approx 0.1$–0.3 relevant to the present study, $x$ always lies in the range from $O(0.01)$ to $O(0.1)$.

Substituting $x$ into Equation (3), we can calculate the energy density of DM. In Figure 10, we plot the kinetic and interaction energy density as a function of the interaction strength. We find that in the considered range of $h_c$, the kinetic and interaction energy density would not exceed ~20% of the rest-mass density. This justifies the approximation $\epsilon_\chi \approx m_\chi n_\chi$ that we use in this context.

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Figure 8. Posterior distributions of the DM particle mass (upper panel) and interaction strength (lower panel). The corresponding priors are shown as the black dashed lines.

Figure 9. Dimensionless Fermi momentum $x$ as a function of the interaction strength $y$ for $h_c = 0.1$ and $h_c = 0.3$. Both the exact solutions of Equation (12) and the asymptotic solutions of Equation (13) are shown.

Figure 10. Dimensionless Fermi momentum $x$ as a function of the interaction strength $y$ for $h_c = 0.1$ and $h_c = 0.3$. Both the exact solutions of Equation (12) and the asymptotic solutions of Equation (13) are shown.
The spherical symmetry allows us to confine the photon geodesic to the equatorial plane (i.e., $\theta = \pi/2$). One can then choose an affine parameter $\lambda$, such that
\[
\frac{dt}{d\lambda} = \frac{1}{g}, \quad \frac{dr}{d\lambda} = f^{-1/2} \left(1 + \frac{b^2 g}{r^2}\right)^{1/2}, \quad \frac{d\theta}{d\lambda} = 0, \quad \frac{d\phi}{d\lambda} = \frac{b}{r^2},
\]
where $b$ is the impact parameter. The impact parameter is related to the emission angle at the baryon surface as $b = R_b \sin\alpha \sqrt{g(R_b)}$, which can be derived from the geometry $\tan\alpha = \left(U_b U_v \right)^{1/2} / \left(U_b U_w \right)^{1/2}$ (Beloborodov 2002).

Combining Equations (B3) and (B5), we find
\[
\frac{d\phi}{dr} = b \left(\frac{fg}{r^2 \left(1 - b^2 g/r^2\right)^{1/2}}\right).
\]
Finally, by integrating Equation (B6) from $R_b$ to infinity, we obtain the angle between the local radial direction and the line of sight as
\[
\psi = \int_{R_b}^{\infty} \frac{d\phi}{dr} dr = \int_{R_b}^{\infty} \frac{\sqrt{fg}}{r^2} \left(1 - \frac{b^2 g}{r^2}\right)^{-1/2} dr.
\]
We now turn to calculating the time delay, which is caused by the different travel paths of the emitted photons. The time delay can be calculated by combining Equations (B2) and (B3):
\[
t = \int_{R_b}^{\infty} \frac{dt}{dr} dr = \int_{R_b}^{\infty} \left(\frac{f}{g}\right)^{1/2} \left(1 - \frac{b^2 g}{r^2}\right)^{-1/2} dr.
\]
However, this integral diverges as $r \to \infty$. To avoid divergence, we define a relative time delay as (Pechenick et al. 1983; Poutanen & Beloborodov 2006):
\[
\Delta t = t(b) - t(0) = \int_{R_b}^{\infty} \left(\frac{f}{g}\right)^{1/2} \left(1 - \frac{b^2 g}{r^2}\right)^{-1/2} - 1 \right) dr,
\]
which is defined with respect to a photon emitted from the spot closest to the observer. As a result, the observer time should be corrected as $t_{\text{obs}} = t + \Delta t$.

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