Surface Kinetics And Generation Of Different Terms In A Conservative Growth Equation

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A method based on the kinetics of adatoms on a growing surface under epitaxial growth at low temperature in (1+1) dimensions is proposed to obtain a closed form of local growth equation. It can be generalized to any growth problem as long as diffusion of adatoms govern the surface morphology. The method can be easily extended to higher dimensions. The kinetic processes contributing to various terms in the growth equation (GE) are identified from the analysis of in-plane and downward hops. In particular, processes corresponding to \( h \to -h \) symmetry breaking term and curvature dependent term are discussed. Consequence of these terms on the stable to unstable transition in (1+1) dimension is analyzed. In (2+1) dimensions it is shown that an additional asymmetric term is generated due to the in-plane curvature associated with mound-like structures. This term is independent of any diffusion barrier differences between in-plane and out-of-plane migration. It is argued that terms generated in the presence of downward hops are the relevant terms in a GE. Growth equation in the closed form is obtained for various growth models introduced to capture most of the processes in experimental Molecular Beam Epitaxial Growth. Effect of dissociation is also considered and is seen to have stabilizing effect on growth. It is shown that for uphill current the GE approach fails to describe the growth since a given GE is not valid over the entire substrate.

I. INTRODUCTION

Growth of solid phase from vapor is studied over many years due to its applications in various fields. In particular, epitaxial growth from vapor has given way to many technological advances through the development of solid state devices. Such a growth is known to be far from equilibrium [1,2]. It therefore offers to study the non-equilibrium (NE) phenomenon in a controlled fashion. Under these conditions, on the growing interface, processes such as island formation, dissociation, nucleation do not equilibrate, being limited by the insufficient mass transport. Resultant interface described by height function \( h(r,t) \), develops characteristic correlations on the surface. Study of space time evolution of these correlations constitute major aspect of understanding the NE behavior for such a growth. A continuum equation approach is used to understand this phenomenon [1,2]. Under most of the conditions used in growth from vapor, evaporation and the vacancy formation are negligible [3]. A conservative growth equation satisfying Langevin equation \( \partial_t h + \nabla \cdot J = F \) where \( J \) is current due to adatom relaxation on the growing surface and \( F \) is the average flux with white noise, should describe the time evolution of the height function. In order to understand the growth behavior via GE approach, it is necessary to establish the correspondence between surface kinetics and the terms appearing in the GE. This correspondence should corroborate with experimental observations. Experiments have produced results in a great variety [1,2]. However adjusting experimental parameters to desired accuracy seems a daunting task so that effect of initial surface roughness, impurities in the flux and similar phenomena [4] lead to results with less confidence to apply any particular GE, since there is a possibility of substantial contribution from these uncontrolled inputs. Under such conditions, computer simulations are expected to produce results that can be related to the physical processes used as inputs to the simulation giving more confidence in establishing the growth equation. In the present article, we have used computer simulations to verify the predictions of GE. The application of these results to real experiments is discussed in section (IV) under the presumption that the experimental conditions are same as assumed in the simulations.

Based on the experimental observations and computer simulations [5,6], when relaxation is mainly by surface diffusion of adatoms, minimum three terms are expected to contribute to the current: a slope dependent term, an asymmetric term and a curvature dependent term [6,7]. One of the early efforts [8] used the Master Equation approach for the model involving desorption and column diffusion as the relaxation mechanisms for adatoms. This approach yielded slope dependent term due to desorption, and also the asymmetric term along with fourth ordered Mullin [9] type term. The terms are obtained under small slope approximation so that the behavior at large slopes is not clear. Further, it is not possible to separate the kinetic processes, responsible for different terms. In the next section, we refer to some of the results from this work and suggest the contribution to GE with respect to in-plane and downward hops. Kinetic approach involving Arrhenius model or Burton, Cabrera and Frank (BCF) [10] approach including nucleation and step edge barriers, could not yield a closed form for the growth equation [2]. It could however relate the slope...
dependent term to diffusion of adatoms on the terrace of a step. To obtain asymmetric term additional assumption that density of adatoms should have quadratic slope dependence, was needed. Both the terms are obtained under small slope approximation and behavior at large slopes is not clear. The curvature dependent term is interpreted as due to the step detachment [5] or due to the slopes is not clear. The curvature dependent term is under small slope approximation and behavior at large dependence, was needed. Both the terms are obtained that density of adatoms should have quadratic slope of GE for DT model for the first time and also shows how the rules in the LD model produce a term in growth equation that has similar behaviour as the Lai- Das Sarma-Villain equation.

II. EQUATION FOR GROWTH WITH SURFACE DIFFUSION

Consider growth on a 1 dimensional flat substrate with lattice constant \( a \) and steps developed after some initial growth. We will consider the situation depicted in Fig. 1 for obtaining various contributions to the current where steps are such that positive slope is obtained. Here the underlying assumption is that the rough or unstable surface will essentially consist of stepped regions mainly. Adatoms are randomly deposited on the substrate. Let \( D_1 \) be the diffusion constant on the terrace, \( \ell_c \) be the average distance that an adatom travels on a terrace before encountering another adatom. Detachment from steps or nuclei on the terrace is assumed negligible. Contribution of a hop to the current is considered only when nearest neighbor (nn) configuration changes, during hop. The configuration is not restricted to number of neighbors, but as required by the relaxation rules, it can be heights of neighbors, or local discrete slope or any such derivative. In the present context, detachment is not allowed, hence the number of nn matters. In fact we treat every individual process, so that when dissociation is allowed, its contribution is considered separately to the current while contribution to current in the absence of dissociation is retained. We further differentiate between the current due to downward and in-plane hops. Since the \((1+1)\) dimensional surface essentially consists of such steps, kinetics of adatoms on such a surface will suffice to provide the essential description of growth in terms of a continuum GE for the given set of rules for relaxation of adatoms. Adatoms hopping down the descending steps contribute to the downward current \( j_d \) while those hopping on the terrace can get attached to an ascending step contribute to the in-plane current \( j_i \). Referring to Fig. 1, the adatoms reaching site \( A \) and hopping down the step to the left constitute \( j_d \) and those reaching site \( B \) and hopping to the right toward the ascending step constitute \( j_i \). The net current is \( j_d + j_i \). These are obtained in a mean field approach as follows,

\[
j_{(d)} = \left( \text{local density of site A(B)} \right) \cdot \text{flux of adatoms approaching A(B)} \cdot \left( \text{probability for hopping across A(B)} \right)
\]

\[ (1) \]

Since the relaxation of adatoms is through the diffusion on the terrace or across the step edges, the density of sites \( A \) and \( B \) is same as density of steps which is approximately given by \( \frac{\text{surface area}}{\text{step length max}} \), where \( m \) is the local slope. In the limit \( |m| \to 0 \), this density approaches zero. However, Elkinani and Villain have shown [16] that a 'plane'
substrate will actually consist of terraces of an average length say \( l_{av} \). This will introduce an additional small factor in the numerator of the expression for density. We will however consider the diffusion and deposition rates such that length \((D_s/F)^{1/4} < l_{av} \) [2] so that after growth of few MLs terraces are shorter than \( l_{av} \) and density of steps is \( \frac{|m|}{\sqrt{2l_{av}^2}} \).

In the absence of nucleation, lateral flux (LF) approaching site B or A is \( \frac{\hat{n}F}{2l_{av}^2} \), \( a|m|^{-1} \) being the average local terrace width and \( n \) is unit vector in the \( x \) direction. When terrace size is large enough, this flux is restricted due to the nucleation. The nucleation process will restrict the diffusion to an average length \( l_c \) on a large terrace. As a result, even for very large terraces, the approaching flux LF is almost constant. The effect of nucleation is incorporated by introducing \( l_c \) in the expression for LF as \( \frac{\hat{n}F}{2(l_c^{-1} + |m|a^{-1})} \), so that for small terraces the expression is reduced to a constant value. Let \( P_A \) and \( P_B \) represent the probabilities of hopping across sites A and B respectively. The Schwoebel length \([17,18] l_s \propto (P_B - P_A) \). In ref. [16] it is shown that, if there is large asymmetry between the sticking coefficients, distribution of diffusing adatoms on the terrace depends upon terrace width. This suggests that \( P_A \) and \( P_B \) may depend on \( m \) for larger asymmetry and shorter terraces. However when \( P_A \rightarrow 0 \) and \( P_B \rightarrow 1 \) i.e. the case of large SE barrier, the nucleation becomes significant being proportional to the density \([19]\). As a result, the asymmetry of the density on the terrace is reduced, rendering \( P_A \) and \( P_B \) almost independent of the terrace width. The flux will depend only on the terrace width, so that \( P_A \) and \( P_B \) will be independent of the width. Thus it suggests that the said dependence will be weak. We will however neglect this dependence since most of our discussion will be around \( P_A \approx P_B \). Further, such an asymmetry is significant when \( P_B \gg P_A \) since it will produce uphill current and large local slopes with shorter terraces. It will be shown that under these conditions, the continuum equation approach fails \([20]\), so that these probabilities could be considered independent of \( m \) where GE is valid. Hence from Eq.(1) the current is

\[
j_x = \frac{\hat{n}|m|F(P_B - P_A)}{2(1 + |m|)(l_c^{-1} + |m|a^{-1})}
\]

(2)

The LF approaching sites A and B is however modified due to the relative motion of the steps. Consider the situation in Fig.1 where \( v \) is the velocity of the step bearing the terrace while \( v' \) that of the higher one on the positive slope. For \( v' > v \) the terrace width is reduced, depleting the LF approaching sites A and B. The reduction in flux is \( \propto \delta v \) where \( \delta v = (v' - v) \). Adatoms hopping across upper step as well as those attaching in-plane, both contribute to the velocity of the step. Thus the velocity \( v \propto j_x \) except that the coefficient \((P_B - P_A)\) is replaced by \((P_A + P_B)\). Hence \( \delta v \propto \frac{(P_A + P_B)}{l_c^{-1} + |m|a^{-1}} \partial_x \frac{|m|}{2(1 + |m|)(l_c^{-1} + |m|a^{-1})} \).

Corresponding current will not depend on \((P_B - P_A)\) since this part of the flux is removed from the LF. The current is therefore obtained by multiplying the LF by density of steps. In the process described, there is relative advancement of higher step. With respect to the in-plane current, the step front advancement increases relative velocity. Thus effectively in-plane current will increase while downward current will decrease. It is this difference in currents represented by the current due to the lost flux. The relative movement of steps also causes increase in the local slope \([21]\). This shows that the direction of the current due to the lost flux is same as uphill current which also tends to increase the local slope. \( P_A \) and \( P_B \) are relative probabilities so that \( P_A + P_B = 1 \). For \( v' < v \) the terrace size is increased. This situation is encountered for positive curvature. This produces a term assisting the LF. Using the same arguments as above, corresponding current has the same analytical form as the one for the case of \( v' > v \) but with the coefficient \((P_B - P_A)\) and in the direction opposite to it. Accounting for this effect the expression for the current becomes,

\[
j(x) = \frac{\hat{n}|m|F(P_B - P_A)}{2(1 + |m|)(l_c^{-1} + |m|a^{-1})} + \frac{\hat{n}F(1 - (P_B - P_A))}{4} \partial_x \left( \frac{|m|}{(1 + |m|)(l_c^{-1} + |m|a^{-1})} \right)^2
\]

(3)

In the limit of small \( m \), the current reduces to,

\[
j(x) = \frac{\hat{n}F(P_B - P_A)|m|l_c^2/2}{2} \partial_x (m^2)
\]

(4)

The second term reduces for small slope conditions to \( \nabla(\nabla h)^2 \). This term was derived using the BCF theory and assuming that at small slopes the particle density on the terraces depends on the even powers of local gradient \([2,6]\). Further, in ref. [15] it was conjectured that such a term can arise due to the differences in the velocities of the steps near the top and the bottom of a profile. In the limit of large slope, \( |m|a^{-1} \gg l_c^{-1} \) the second term is proportional to \( \frac{|m|}{m} \partial_x m \), which was derived in the large slope limit in reference [6]. Thus the geometrical dependence of the symmetry breaking term in Eq.(3) exactly matches with previously derived two terms in the small and large slope limits. This shows that the asymmetric term in equation (3) appropriately interpolates through the limits of small and large slopes implying the correct analytical form of the term. Further, under the infinite SE barrier, i.e. \( P_B = 1 \) and \( P_A = 0 \), second term in above equation becomes zero. From computer simulations in \((1+1)-d\) and in \((2+1)-d\) it is seen that for the former case \([22,23]\) growth morphology is symmetric with respect to \( h \rightarrow -h \) transformation while for the later case it is asymmetric. It has been suggested [24] that asymmetric
term does not go to zero, it however decreases faster than slope dependent term which dominates to show symmetric profile in (1+1) dimensions under infinite SE barrier. Our analysis however shows that, in (1+1) dimensions, the asymmetric term is absent. In (2+1) dimensions, an additional effect of in-plane curvature gradient generates the asymmetric term. Under most of the circumstances, one obtains finite structures on the surface. In particular when mound like structures are formed, steps on the mounds have in-plane curvature. The in-plane curvature is given as \( \kappa(x, y) = \frac{h_x h_y^2 - 2 h_x h_y h_x h_y + h_y^2}{(h_x^2 + h_y^2)^{3/2}} \), where, \( h_x, h_y \) are derivatives of the height function \( h(x, y) \) w.r.t. \( x \) and \( y \) respectively. Consider a region where steps are forming concentric arcs. \( P_B \geq P_A \) and surface diffusion is isotropic. Under such conditions, the inward flux is proportional to \( R^{-1} \), where \( R^{-1} = |\kappa| \) is the radius of curvature at the point under consideration. In such region, the velocity along \( \hat{n} \) is along the radius of curvature. In the presence of \( R^{-1} \) dependence, velocity gradient is created due to the gradient in curvature. Thus, by applying previous considerations for obtaining the current, we find that current due to the difference in in-plane curvature across consecutive planes is

\[
\mathbf{j}_{\text{cur}} = \frac{\nabla h Fa(\nabla h \cdot \nabla)|\kappa(x, y)|}{2(1 + |\nabla h|)(\alpha + 1 - a^{-1}|\nabla h|)} \tag{5}
\]

This term will be present in addition to the one due to the curvature in the height profile. Further, since for mound like structures, in-plane curvature gradient is always positive implying that flux is removed. Hence, in the expression for \( \mathbf{j}_{\text{cur}} \), \( P_A \) or \( P_B \) do not appear explicitly. For small slope conditions, term in growth equation corresponding to the \( \mathbf{j}_{\text{cur}} \) is \( \nabla \cdot (\nabla h Fa(\nabla h \cdot \nabla))|\kappa| \). Under scaling transformations, this term gives \( z - 4 \), which is same as \( \nabla^2 h \) term, but differs in the \( h \rightarrow -h \) symmetry. This shows that for the growth near tilt independent current in (2+1) dimensions, mounds grow with \( t^{1/4} \) while \( h \rightarrow -h \) symmetry is broken. Thus, our analysis shows that, for an infinite barrier, in (1+1) dimensions, asymmetric term is zero while in (2+1) dimensions in-plane curvature gradient generates asymmetric term. In fact, in almost all (2+1) dimensional growths, asymmetry due to this term is unavoidable if mound formation occurs.

We further argue that a curvature dependent current must be present in any adatom relaxation process that involves downward hops across the descending step edges. This argument is based on the observation that, in a (1+1) dimensional simulation, if adatoms are restricted completely to the in-plane hops (infinite SE barrier) then correlations do not grow beyond the diffusion length. This results in to the ‘wedding cake’ type morphology with fixed size of the ‘cakes’ that do not grow in time [22,23] as discussed above. On the other hand, when such hops are allowed, correlation length for stable growth and mound size for unstable growth increases in time [12]. This observation allows one to conclude that height-height (h-h) correlations increase only in the presence of downward hops. The microscopic theory [8] predicts fourth ordered and asymmetric term to be present irrespective of in-plane or out-of-plane hops. Thus, accordingly downward hops produce these terms, however in plane hops also generate these terms. In order to differentiate between the role of in-plane and downward hops in generating these terms, consider a rough surface, obtained after deposition of several monolayers. We consider an in-plane hop and a downward hop. Let \( W_i \) and \( W_f \) be the initial and final widths with \( W = (1/N) \sum_i (h_j - \bar{h})^2 \). Also let \( G_2(i - k) \) and \( G_2([-l - k]) \) be initial and final height-height correlations. Consider a hop from site \( i \) to \( i+1 \). The in-plane hop gives \( W_i \rightarrow W_f \), while downward hop gives \( W_i - W_f = 4a^2 \). Thus, the width is reduced due to the downward hop. In order to see the effect on h-h correlations, we consider only contributions from participating sites. Thus, contribution to \( G_2(i - j) \) will be from sites at \( h_i, h_{i+1}, h_j, h_{j+1} \) and corresponding reflection sites in \( i \) and \( i+1 \). The difference \( G_2(i - j) - G_2([-l - j]) \) due to one set of sites is \( 2a^2 - 2a(h_j - h_{j+1}) \). The ensemble average for this process will yield \( 2a^2 \) as the difference. Same contribution will appear from reflection sites. Thus, \( G_2(i - j) \) is reduced by \( 4a^2 \) by a downward hop. On the other hand, in-plane hop does not change \( G_2([-j - j]) \), as can be verified by applying same procedure. This shows that, in order that correlations grow in time, downward hops are necessary. The in-plane hops reduce the deposition noise in a plane. Consider a large flat surface with very small coverage. Every event of an adatom getting captured by a nucleus, or another adatom causes decrease in \( G_2(1) \). For an island of \( m \) atoms on a 1- dimensional substrate, \( G_2(1), G_2(2), \ldots G_2(m) \) decrease from its value for \( m \) isolated atoms. This indicates reduction of noise in a plane. In fact, this process develop correlations over the diffusion length \( L_d \). In ref. [8], fourth ordered and asymmetric terms are generated whenever near neighbor configuration is changed in a hop. This is consistent with the reduction in \( G_2(m) \), \( m \leq L_d \) with hops leading to increase in near neighbors. Thus in-plane hops do generate fourth order and asymmetric terms, however, these exclusively operate within \( L_d \), reducing deposition noise in a plane. The processes, such as nucleation and step attachment or detachment [17,7] are suggested to generate fourth order term. Above discussion leads to the conclusion that such a term would operate only within the plane and not across different planes. This shows that in a growth equation, terms generated by the kinetic processes involving downward hops are the relevant terms. This also suggest that upward hops will decrease the correlations. We do not consider in the present work associated effects, as we restrict to analyzing low temperature growth. Note that adatoms crossing site \( A \) are hopping downward and lead
to the EW type term. Hence if the downward hops are al-
lowed but the current is tilt-independent, then it may be
expressed as a linear combination of $a_1 \nabla^3 h + a_2 \nabla^5 h + ...$ in-
cluding the nonlinear terms of the form $\nabla (\nabla^3 h)^2$. Here, $a_1, a_2, ...$ are such that the growth is stabilized. We will
retain only $\nabla^3 h$ in the current corresponding to our min-
imal GE. Thus the form of the current corresponding to
the minimal GE in $(1+1)$ dimensions is

$$j(x) = \frac{\hat{n}|m|F(P_B - P_A)}{2(1 + |m|)(l^{-1} + |m|a^{-1})} + \frac{\hat{n}|m|F(1 - P_B + P_A)}{4} + \partial_x \left( \frac{|m|}{(1 + |m|)(l^{-1} + |m|a^{-1})} \right)^2 + \nu \frac{\partial^3 h}{\partial x^3} \quad (6)$$

The first term has been studied widely as the stable
growth mode [25,26] and as the unstable growth mode
[27]. We aim to study tilt independent current models
here. This will allow exclusive effect of asymmetric term
to be studied. With this view we have performed simu-
lations of a $(1+1)$ dimensional solid-on-solid model with
no diffusion bias. This will produce growth with tilt in-
dependent (TI) current. A fourth ordered equation was
earlier proposed by Villain [17] for similar situation. Un-
der this condition $P_A = P_B$ and first term vanishes in
the Eq(3). The resultant GE in the moving frame with
growth front is of the form,

$$\partial_t h = -\nu \frac{\partial^4 h}{\partial x^4} + \nu_0 \partial_x^2 \left( \frac{m}{(1 + |m|)(l^{-1} + |m|a^{-1})} \right)^2 + \eta \quad (7)$$

where, $\nu_0$ is the appropriate constant for the asymmetry
term and $\eta$ is white noise associated with deposition flux
with $< \eta(x', t')\eta(x, t) > = D \delta(x' - x) \delta(t' - t)$. In
the limit of small slopes, renormalization group (RG) anal-
ysis shows that the roughness exponent $\alpha = 1$ and the
roughness evolves with the exponent $\beta = 1/3$ [15]. For
large slopes, when terrace width is very small (of the or-
der $a$), the multiplying length in the expression $\frac{m}{(l^{-1} + a^{-1}|m|)}$ is no more represented by $(l^{-1} + a^{-1}|m|)$ but is to be re-
placed by $a$. This leads to asymmetric term with $z = 4$
that describes DT model [13] in the absence of noise re-
duction. We discuss the GE for DT model in section III
A. Here we note that, asymptotically Eq. 7 will reduce to
a GE describing DT model, hence exponent $\beta$ should
cross over from a value of $1/3$ to $3/8$. In the next section
we describe a solid-on-solid growth model that mimics
the relaxation by surface diffusion. The relaxation rules
are consistent with the processes giving rise to different
terms in Eq. (6). These results will help establish the
relationship between process $\rightarrow$ term in GE. In section
V, we apply this method to predict GE for other models.

Extension of this equation to $(2+1)$ dimensions is pos-
sible by similar kinetic considerations. For isotropic dif-
fusion, same form as Eq.(6) is obtained with $\hat{n} = \nabla h/|\nabla h|$

and replacing length-derivative product in asymmetric
term by $\frac{\nabla h \nabla}{(|\nabla h|)}$ and adding the asymmetry term
due to the in-plane curvature gradient. For small slopes
one obtains $\nabla (\nabla h)^2$ term [15] along with $\nabla \cdot (\nabla h (\nabla h \cdot
\nabla))$, which seems to describe many experimentally
observed growth roughness measurements from vapor
[28]. In particular, large number of experiments show the
roughness exponent $\alpha$ between 0.65-1.0 predicted by
these two terms.

III. GROWTH MODEL

A. Growth with surface diffusion and dissociation

Corresponding model is as follows. Atoms are rained
on a 1-d substrate of length $L$ randomly with constant
flux. On deposition a given adatom is allowed to hop
n times, as in a random walk. The hops can be biased
through a parameter $p$. Thus, $p = 1.0$ is the growth with
infinite positive SE barrier, while $p = 0.0$ is with infinite
negative barrier. We have however kept $p = 1/2$, i.e.
no bias condition for most of the cases. If the hopping
adatom acquires two or more $nn$, before n hops are ex-
hausted, the adatom stays there permanently. If n hops
are exhausted without any encounter, it stays perma-
nently at the last position occupied after n hops. If a
single $nn$ is acquired before n hops are exhausted, an-
other parameter $q$ is called. $q$ decides fraction of such
events, where adatom will dissociate from its neighbor.
For $q = 0$, detachment is completely suppressed. Under
this condition detailed balance is not obeyed. As usual, $q$
is compared with the random number to decide whether
detachment can take place or not. We have extended the
same model in $(2+1)$ dimensions as well. Besides the
parameters $p$ and $q$ controlling the hops across the step
edge and away from the edge respectively, additionally
directional diffusion has been included [29]. It is considered
to be in plane process. We also employ the noise re-
duction method [30] where ever needed. In this method,
after deposition an adatom is allowed to make hops as per
the rules until it finds the location for the incorporation.
However, instead of actually incorporating the atom at
that position, a counter at that position is increased by
unity. A given position is filled only when the counter
exceeds certain pre decided number. The method has
been successful in bringing out the correct nature of the
growth at earlier times in simulation [31]. We find that
when number of allowed hops are large enough, noise re-
duction occurs in the diffusion process during initial
growth.

In $(2+1)$ dimensions, it may be noted that the slope
independent current is not obtained for $P_A = P_B$ due to
the different number of configurations for in-plane and
downward hops [31]. Thus for the same set of rules, re-
results in (1+1) dimensions will differ from those in (2+1) dimensions as noted in ref. [31].

The present model includes the physical processes dependent on the surface diffusion. It however differs from kinetic Monte Carlo (KMC) method, usually employed in such simulations. Firstly the detailed balance is accounted in KMC since dissociation from the steps or nuclei is allowed as per the activation barrier for that event. The dissociation from steps is opposite to attachment from site B in Figure 1. Hence, these events constitute additional downhill current. This current is however independent of terrace width and depends only on the density of edges (the details are discussed in sec. IV B). As a result, the slope dependent first term in equation decreases with slope while dissociation dependent term doesn’t. This leads to tilt independent current. We will illustrate this effect in the next section. KMC also allows accordingly, upward hops and edge diffusion. Thus based on the considerations of contributing currents, KMC method would tend to TI current growth rather than a true uphill current. The present model is however computationally convenient and allows variation of parameters in such a way that isolation of processes and their effect on GE can be studied. By adjusting parameters in our model, downhill, zero or uphill current can be maintained during growth simulation. For comparing the predictions of GE with simulations, we have measured width \( w_2 \), height-height correlations \( G(r, t) \) and skewness \( \sigma \) where, \( w_2 = \frac{1}{N} \sum_i (h_i - \bar{h}) \sim t^{2\beta} \) and \( G(r, t) = \frac{1}{N^2} \sum_{r'} (h(r + r', t) - h(r', t))^2 \). The skewness \( \sigma = w_3/w_2^{3/2} \), where \( w_3 = \frac{1}{N} \sum_i (h_i - \bar{h})^3 \) [32].

IV. RESULTS

A. TI Current Without Dissociation

From the derivation of the expression (6), it is clear that TI current is obtained when \( p = 0.5 \) and \( q = 0 \), allowing \( P_A = P_B \). Thus, mainly two terms are expected to contribute in GE, the \( \nabla^4 h \) and the asymmetric term. Presence of \( \nabla^4 h \) term is verified from the flatness of the saturated width for small \( L \). We have chosen, \( n=10 \) giving \( l_e \approx 3 \). The saturated width is flat almost up to \( 3l_e \), showing that \( \nabla^4 h \) dominates at small lengths [6]. Fig. 2(a) shows the morphology of the interface after 80000MLs are grown. As predicted by the Eq. (7), the asymmetry is evident in the figure with \( \sigma = -0.31 \pm 0.05 \). Fig. 2(b) shows plot of \( w_2 \) in time. We obtain initially \( \beta \) around 0.33 that attains a value of \( 0.35 \pm 0.015 \). Initial value of 0.33 is due to the small slope region of Eq. 7 that predicts a value of 1/3 (compare data in the region from 10 ML to 200 ML in the figure with the line having slope of 2/3). Correspondingly h-h correlations lead to the roughness exponent \( \alpha \) that increases from 0.5 to 0.75 \pm 0.01 over a growth of 10^3 to 4.10^6 MLs. Clearly, the \( \alpha \) tends to unity asymptotically on large substrates. The value of \( \alpha \) from saturated width and for small \( n \) is 1.35 \pm 0.1. These results indicate that most of the morphological features of the growth with diffusion without dissociation are captured by the current expression in Eq.(6). As is mentioned in section II, a slow cross over from \( \beta = 1/3 \) to \( \beta = 3/8 \) is observed. The exponent \( \alpha \) from \( W_{sat} \) is also close to the predicted value of 1.5 [13]. Thus the model in section III represents the GE given by Eq. 7 confirming the association of kinetic processes with the terms in GE. It also shows that diffusion of the adatoms roughens the growing surface. Diffusion bias causes additional effects in terms of stability or instability of growth. In particular, if the bias is varied from extreme -ve SE barrier to extreme +ve SE barrier, a stable→ unstable transition is observed. In this transition however \( h \rightarrow -h \) symmetry is broken asymptotically. Note that for -ve SE barrier \( \nu_2 \nabla^4 h \) term dominates with +ve value of \( \nu_2 \) [25], so that asymptotically, asymmetric term becomes irrelevant rendering \( \sigma = 0 \). At exactly zero SE barrier, finite value of \( \sigma \) is obtained. \( \sigma \) can be regarded as the asymmetry parameter, that changes abruptly at the transition point. Thus the growth transition is like 2nd order phase transition. This transition is however, a result of not complying with detailed balance.

As mentioned earlier, in (2+1) dimensions, TI current growth could not be obtained by putting \( P_A = P_B \). We could attain a situation close to TI current growth by setting the parameter \( p \) to a value 0.54, and without edge diffusion. Results in Fig3(a) show the morphology, where mound like structures are evident while Fig3(b) shows plot of position of first maximum in \( G(x, t) \) Vs. time. The \( \beta = 0.26 \pm 0.03 \) while slope of the curve representing exponent 1/\( z \), in Fig3(b) is 0.23 \pm 0.02 and \( \sigma = -1.12 \pm 0.1 \). Clearly, an indication of dominant fourth ordered term with asymmetry. As has been discussed previously, it is the term generated from the t
current \( j_{cur} \) in Eq. 5, governing the growth dynamics.

B. Effect of Dissociation

As we have mentioned earlier, dissociation from the steps introduces a term of the form \( -\beta \frac{F|m|}{1+|m|} \) in the current, as a downhill contribution. \( c' \) is fraction of the incident flux contributing to dissociation. This current does not decrease with \( m \). Hence, in the presence of SE barrier and/or edge diffusion, with increasing average local slopes, zero tilt current is attained and analysis of previous section applies. To illustrate this point, in our (1+1) dimensional model, we introduce, SE barrier by assigning \( p = 0.7 \). In order to simulate the dissociation effect, we take \( q = 0.01 \), i.e. one hundredth of the total encounters with adatoms having single in-plane
neighbors are allowed to hop in-plane or downward across the step edge. The results are displayed in Fig. 4a and b) showing morphology and roughness evolution respectively. For the sake of comparison, curve corresponding to \( q = 0.0 \) is also included. As seen from the figure 4b, the \( \beta \) value for \( q = 0.01 \) is same as the one for zero tilt current case within statistical error. The \( \beta \) for \( q = 0.0 \) increases to 1/2 showing the instability. The argument is true for higher dimension as well. However, present model is not designed to account the effect of detailed balance. Thus, TI current is obtained for a certain set of parameters only and does not evolve as in the case of KMC simulation accounting for all the processes relevant to the detailed balance. Hence arriving at TI current is possible in (1+1) dimensions with our model that includes these processes controlled through parameters \( p \) and \( q \). In (2+1) dimensions, the attachment-detachment processes are many due to the different possible configurations. Present model does not allow all such processes. Thus, tilt independence becomes difficult to attain through variation of model parameters. Hence in (2+1) dimensions we illustrate the dissociation effect mainly in the form of stable logarithmic growth. Fig. 5(a) and (b) shows morphology for the case with and without dissociation in (2+1) dimensions respectively. The effect of dissociation is seen as the logarithmic growth as against a mounding with \( \beta = 1/2 \) in the absence of it. The behavior of (2+1) dimensional model under TI current conditions can be predicted from the form of growth equation. If the steps are straight, then LDV type term will dominate giving, \( \alpha = 2/3 \), and \( \beta = 1/5 \) [15]. However, if mound like structures are formed, asymmetric term due to the in-plane curvature gradient (Eq. 5) will be operative leading to \( z = 4 \) and \( \beta = 1/4 \). Siegent and Plischke [33] have considered a symmetric term to explain the pyramid like structures giving \( z = 4 \) and \( \beta = 1/4 \).

V. GROWTH EQUATIONS FOR OTHER MODELS

A. DT Model

As mentioned earlier, the present method for obtaining current from kinetic considerations appropriately brings out the geometrical dependence in GE. We have applied this method to one of the stochastic growth models proposed to capture the essential features of low temperature molecular beam epitaxy (MBE) the DT model [13]. Based on noise reduction technique, the simulations of this model [31] confirm that, 1) exponent \( \beta = 3/8 \) with noise reduction factor unity, while \( \beta = 1/3 \) with reduction factor 10, 2) the morphology is asymmetric with \( \sigma \approx -0.5 \), 3) the current is tilt independent and 4) \( \alpha = 1.4 \) and 1.0 with noise reduction factor unity and 10 respectively [2,31]. The relaxation rules for adatom in this model allow it to hop only when it is deposited at site A or B (see Fig. 1). Also only downward hop is allowed, if deposited at A and toward the step, if deposited at B. If it has choice of sites A,A or B,B or A,B etc. on two neighboring sites, then it will hop randomly to the left or right. Applying the considerations for obtaining current for this situation shows that the LF approaching sites A or B in the present set of rules is \( \frac{m}{a+|m|} \). For this model, \( \lambda_c = a \) since only single hop in definite direction is permitted. Further, the LF is affected by relative motion of steps only when sites A and B differ by a lattice constant. In (Eq.3), the velocity gradient is considered over a length of \( (l_c^{-1} + a^{-1}|m|^{-1}) \) which is terrace width. For the DT model, this length is \( a \), the lattice constant, since the relative motion of steps for above set of rules can affect the LF only for this short terrace or lower. However lower terrace sizes are not possible, introducing effect of discretization as length cannot be reduced further. If \( n=1 \) in our model, described in section III, the situation is not discriminated by the present analysis. We expect that exactly same equation governing DT model should be applicable. In fact, when local slopes increase in time, the effect of discretization shows up and exponents pertinent to DT model characterize the model asymptotically as seen in section IV A. Accounting for this effect, the expression for the current is then,

\[ J_{DT}(x) = \frac{\nu'}{\nu} \frac{\delta^2 m}{\delta x^2} + \frac{nF a^2}{2} \frac{m}{1 + |m|} \frac{\partial_t}{\partial_x} \frac{m}{(1 + |m|)^2} \]  

(8)

From the rules it is clear that \( P_A = P_B \). The GE corresponding to this current in the moving frame will be,

\[ \partial_t h = -\nu' \frac{\partial^2 h}{\partial x^2} + \nu' \frac{m}{1 + |m|} \frac{\partial_x}{\partial t} \frac{m}{(1 + |m|)}} \] + \eta  

(9)

where, \( \nu' \) accounts for various constants in the corresponding expression for current. The power counting in this equation leads to, \( z = 4 \) from the first term, and \( z = 1 + 2\alpha \) for large slopes corresponding to the second term which is expected to be operative mainly over large local inclinations. The relation obtained from the second term is exactly same as the one obtainable from the noise term \( \eta \). For \( z = 4 \) all the terms are marginal. This implies that \( z = 4 \) and \( \beta = 3/8 \). The second term is symmetry breaking. Hence, above equation accounts for all the observed facts mentioned above in the simulation of DT model. Since the GE for DT model is obtained in the discretization limit of surface diffusion model, the asymptotic limit of surface diffusion model without dissociation is DT model. Fig. 6 shows plot of \( w_2 \) Vs time for our (1+1) dimensional model for different values of \( n \). As is expected, for \( n=1 \), \( \beta = 3/8 \) while as \( n \) increases,
it crosses over to this value at later time. Thus these results clearly demonstrate the effect of discretization in growth.

With large noise reduction factor however, the observed behaviour of this model corresponds to LDV type GE [31]. We have seen that the form of GE with large enough terraces is indeed LDV type, as in Eq.7. Effect of noise reduction technique is to reduce the nucleation noise. In the process, longer terraces are created and maintained during growth. Thus, discretization limit is never reached leading to LDV type behaviour. In both cases, the current is TI, so the universality of this model is ‘zero current universality’. It is a degenerate case since \( z = 3 \) and \( z = 4 \) are both possible for the same model.

In \((2+1)\) dimensions, with above rules for adatom relaxation, the local density of sites A and B need not be equal since, fluctuations in step edges render configurations that show bias for sites A or B. As a result, slope dependent current will dominate the growth changing the universality class with dimensions [31]. In this case, the noise reduction technique helps establish the sign of the current on tilted substrate. Without noise reduction, the nucleation noise obscures the real sign of the current and hence the universality of the model in \((2+1)\) dimensions. In particular, for DT model, it has been shown that [31] configurations favor downward hops. Thus in spite of intrinsic randomness in selecting the neighboring site for a hop, a down-hill current is produced on tilted substrate leading to EW type universality.

**B. WV Model**

This model was introduced by Wolf and Villain [14] to simulate low temperature MBE growth. In this model, relaxation rules require that an adatom will hop to a nearest site if number of \(nn\) increases. Thus as far as \((0)mn \rightarrow (1)nn\) hops are concerned, the model is same as DT model. However, it allows \((1)nn \rightarrow (2)nn\) hops, that cause an adatom to dissociate from a step and hop into the surface. Thus clearly, WV model will follow the DT model equation above, in addition due to dissociation, downhill current is produced as has been discussed in section IV B. The current on tilted substrate has been measured for this model and is confirmed to be downhill [34].

In \((2+1)\) dimensions, hops from lower \(nn\) to higher \(nn\) imply edge diffusion. This can compensate the dissociation induced down hill current. Das Sarma et.al. [31] have observed mound formation in \((2+1)\) dimensional WV model.

**C. LD Model**

This model was introduced by Lai-Das Sarma [15] in connection with the LDV equation. The rules were decided, based on the geometric interpretation of the term \(\nabla^2 (\nabla h)^2\). Accordingly, a zero neighbor adatom follows same rules as depicted for DT model. If the adatom is deposited at a kink site with a single lateral \(nn\), it is allowed to move to the nearest kink site with smaller step height. Thus, an upward or downward hop is permitted to satisfy the rule. The rule suggests that a hop from one kink site to the neighboring one is allowed from smaller to larger local slope. Thus flux in expression (1) is proportional to \(\frac{m}{mn} \frac{\partial h}{\partial x}\). The factor \(\frac{m}{mn}\) ensures proper direction. The probability for hopping, once the appropriate configuration is attained is unity by relaxation rules for the model. Thus the term in the GE due to the movement of adatom at kink is \(\partial_x (\frac{m}{mn} \frac{\partial h}{\partial x})\). The term is consistent with the requirement of invariance under \(x \rightarrow -x\). For small \(m\), it reduces to \(\frac{m}{mn} \partial_x (\frac{\partial h}{\partial x})^2\). However this term is expected to contribute mainly for larger slopes when the appropriate configurations (steps with terraces of unit length appearing consecutively) are large enough in number. Under these conditions, the term reduces to, \(\partial_x (\frac{m}{mn} \frac{\partial h}{\partial x})\). This term under scaling hypothesis, \(x \rightarrow bx\) and \(t \rightarrow bt^\beta\) gives exponent \(z = 3\). If this term is not renormalized, it leads to the same scaling exponents as given by \(\frac{\partial^2}{\partial x^2} (\frac{m}{mn})^2\) i.e. \(z = 3, \alpha = 1\) and \(\beta = 1/3\).

**VI. DISCUSSION**

Above results show that, a growth situation where kinetics of adatoms is well defined can be understood using proposed methodology for obtaining the GE. It is therefore perfectly suited for computer models with well defined relaxation rules. In real experiments as has been pointed out by Krug [2] effects of surface roughness, evaporation and impurities need to be addressed carefully since these can lead to different results for the same system under growth [35,36]. With this caveat in mind, implications of our results in real MBE like growth are discussed.

The results indicate that in real MBE growth, within a low temperature range where evaporation is still negligible, one can expect different behavior for different materials on a singular surface. The processes to monitor are in ascending order for activation barrier, SE barrier, edge diffusion and dissociation. SE barrier is encountered by a single adatom on the surface at the edge while edge diffusion is expected to be activated at relatively higher temperature due to higher co-ordination around the diffusing adatom. Similarly, dissociation is expected to be operative at relatively higher temperatures. Thus, for materials with very small or zero SE barrier, at low
temperatures TI current will dictate the morphology evolution. In many cases it will be with \( z = 4 \) and \( \beta = 1/4 \). At higher temperature edge diffusion is activated causing instability. This will lead to \( \beta = 1/2 \) asymptotically. At higher temperatures, dissociation will reduce the uphill current to TI current. However, this situation has to compete with the step flow, that leads to EW type growth \([17]\). This scenario is well fitted to growth of Cu \([37]\). If the SE barrier is high, at low temperature, unstable growth will appear with \( \beta = 1/2 \) \([36]\). At higher temperature, edge diffusion will not change the exponent. But once the dissociation is activated, TI current will reduce \( \beta \). In this study we have neglected any effects of upward hops. These can further add more scenarios \([12]\). It has been mentioned \([12]\) that with in-plane hops \( \beta \) cannot exceed the value 1/2. The transients in the growth are however known to produce higher apparent values of \( \beta \) \([2]\). Also, upward hops can give a value as high as 1 for \( \beta \) \([12]\).

So far, we have focused our discussion near the TI current. Above arguments suggest that the GE that we have obtained is specific to the stepped region on the surface. From the kinetics of adatoms on top or base terraces it is clear that the GE will change \([20]\). This will lead to the breakdown of spatial invariance. We argue that regions that offer restricted types of kinetics will support fewer terms in GE than the ones that offer wider possibilities. For the sake of argument we will restrict only to (1+1) dimensions, however the argument is easily extended in (2+1) dimensions. By inspection, a top terrace, defined between two down going step edges, a base region defined between two up going step edges, and stepped regions, are three distinct regions. Over time scales \( \ll \tau_{ML} \), a stepped region offers, downward hops, relative step motion and in-plane hops. Thus equation 6, including all the three terms is valid over this region. The top terrace in the absence of nucleation allows downward hops. By symmetry, the current must be TI, so that it will support only fourth ordered term. On the base region, only in-plane hops are possible. Again by symmetry the current must be TI. This region offers Poisson type growth with no apparent term to build the h-h correlations. In order that such a description is valid on reasonable time scales, it is necessary that these regions maintain their identity over appropriate time intervals. The corresponding time interval be at least \( \tau_{ML} \) which is the minimum time for a height fluctuation at a given site. Accordingly, if a base region is created locally, then its life time at the given place decides whether it will act as an independent region or not. In order to get a qualitative idea of such time stability of base regions, we have performed simulations of an isolated base region as depicted in Fig. 7(a) and (b).

We grow few layers allowing dynamics of adatoms as per the growth model described in section III, and compute the time correlations for height for different values of \( p \). The width of the base region is of the order of diffusion length. The number of hops \( n \) are chosen accordingly. We have also employed noise reduction technique to reduce the nucleation noise. Fig. 7(a) and (b) show typical development of base region for the parameter \( p = 0.1 \) and 0.9 respectively. Fig.7(c) shows the time correlations \( G_i (\tau) = \langle h(x,0) h(x, \tau) \rangle \), for various values of \( p \). The base region in real growth can occur in various surrounding configurations. Although stability times for each such configuration will differ, the trend depicted in Fig.7(c) is similar with respect to the parameter \( p \). The nature of these curve shows that a) the characteristic time \( \tau_0 \) for the decay depends on \( p \) such that, \( \tau_0 \) increases with \( p \), and b) for \( p = 0.9, \tau_0 \to \infty \). We find that as long as \( p > 0.5 \) there is a base region with finite depth such that \( G_i (\tau) \) does not decay. For \( p = 0.9 \) this happens for a single step depth. Such region offers no kinetics that can grow h-h correlations in the vicinity. Since no correlations can be built in this region, it reduces in size. It acts as a discontinuous region with respect to the adjoining stepped regions. Corresponding simulations under these conditions will always result in the formation of deep ridges. For such growth the current on tilted substrates is always observed to be positive. Above discussion suggests that, when ever, \( p \neq 0 \), initially, there would be regions on the substrate during growth separated by local base regions. However, as long as base region decays in time with finite \( \tau_0 \), the lateral growth of h-h correlations continues as per the GE on stepped region. In this sense, the GE is valid over the entire substrate. However, as growth proceeds, deeper base regions will be created by fluctuations. If these base regions do not decay in time, which is certain when current on tilted substrate is uphill, ridges are formed. Lateral growth of the regions separated by a ridge is then governed by the dynamics of adatoms across the ridge and not by the growth equation on the stepped region. Thus the GE approach fails to describe the growth in such cases. As a result, in (1+1) dimensions the mounds grow as \( \ln(t) \) asymptotically while in (2+1) dimensions even slower than \( \ln(t) \). Power law dependence in time is observed only for TI current and downhill current growth \([20]\).

In section II we have discussed growth under infinite SE barrier. The existence of three regions w.r.t. the GE on a growing interface is consistent with the observed growth for infinite SE barrier in (1+1) dimensions \([22]\). For the model growth in (1+1) dimensions with the rules in section III, unstable growth occurs for \( 1.0 > p > 0.5 \). Three regions are significant w.r.t. the growth under this condition. According to the analysis of these regions, the base region supports only Poisson type growth while top region can generate \( \frac{\partial^2 h}{\partial x^2} \) term. Therefore the top regions are flat while base regions have sharp ridges breaking \( h \to -h \) symmetry. However, for infinite SE barrier, i.e., for \( p = 1.0 \), top region cannot generate fourth ordered smoothing term due to the absence of downward hops.
VII. CONCLUSION

In conclusion, we have proposed a simple method for obtaining current in a solid-on-solid growth in (1+1) dimension. The resultant growth equation shows that presence of diffusion alone is responsible for roughening of a singular surface. It induces an asymmetric term in the continuum equation. The velocity gradient of steps on a growing surface is responsible for such a term. In (2+1) dimensions, in-plane curvature gradient also generates an asymmetric term. This term is independent of SE barrier and is responsible for asymmetry in the growth on a two dimensional substrate with infinite SE barrier. Role of in-plane hops is seen to smoothen out the deposition noise in a plane, within diffusion length. The corresponding terms generated are operative only within the plane. A curvature dependent term is seen to arise from downward hops. Study of zero bias model manifests effects of discretization and violation of detailed balance. A stable→ unstable transition with symmetry breaking results from such a violation. In this connection, present study brings out the effect of dissociation on the asymptotic behavior of growth. In the absence of upward hops, dissociation introduces a downhill current. The condition of detailed balance requires dissociation as a part of the process toward equilibration. Thus at high enough temperatures, a zero tilt current is expected to dictate the growth morphology. Considering the processes in a KMC simulation it is conjectured that these simulations are close to TI current even when SE barriers are included in simulations. The method is successfully applied to various models in the literature and provides an insight in the model growth. In DT model, in particular it supports the dimensional dependence of universality class for growth under DT rules. The GE approach is however seen to be restricted to zero or downhill current on tilted substrates. For uphill current, disjoint regions following different GEs are obtained.

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[1] See e.g. A.L. Barabasi and H.E. Stanley, Fractal Concepts in Surface Growth (Cambridge University Press, New York, 1995).
[2] J. Krug, Adv. Phys. 46, 141 (1997).
[3] R. Ghez and S.S.Iyer, IBM J. Res. Dev. 32, 804 (1988).
[4] P. Smilauer, M. Rost, and J. Krug, Phys. Rev. E 59, R6263 (1999); C. Castellano and J. Krug, Phys. Rev. B 62, 2879 (2000); J. Krug and M. Rost, Phys. Rev. B 60, R16334 (1999).
[5] J.A. Stroscio, D.T. Pierce, M.D. Stiles, A. Zangwill, and L.M. Sander, Phys. Rev. Lett. 75, 4246 (1995); similar references are obtainable from ref. 1 and 2 above.
[6] P. Politi and J. Villain, Phys. Rev B 54, 5114 (1996).
[7] P. Politi, G. Grnet, A. Marty, A. Ponchet, and J. Villain, Phys. Rep., 324, 271 (2000).
[8] D.D. Vvedensky, A. Zangwill, C.N. Luse and M.R. Wilby, Phys. Rev. E 48, 852 (1993).
[9] W.W. Mullin, J. Appl. Phys. 30, 77 (1959).
[10] W.K. Burton, N. Cabrera, and F.C. Frank, Phil Trans. R. Soc. (Lond.) A243, 299 (1951); A.K Myers-Beaghton and D.D. Vvedensky, Phys. Rev. A44, 2457 (1991).
[11] A. Madhukar and S.V. Ghaisas, CRC Crit. Rev. Solid State Mater. Sci., 14, 1 (1987).
[12] S.V. Ghaisas, Phys. Rev. E63, 062601 (2001); S.V. Ghaisas, unpublished.
[13] S. Das Sarma and P. Tamborenea, Phys. Rev. Lett. 66, 325 (1991).
[14] D.E. Wolf and J. Villain, Europhys. Lett. 13, 389 (1990).
[15] Z.W. Lai and S. Das Sarma, Phys. Rev. Lett. 66, 2348 (1991).
[16] I. Elkiniani and J. Villain, J. Phys. I (France), 4, 949 (1994).
[17] J. Villain, J. Phys. I 1, 19 (1991).
[18] G. Ehrlich and F. Hudda, J. Chem. Phys. 44, 1039 (1966); R.L. Schwoebel, J. Appl. Phys. 40, 614 (1969).
[19] A.K. Myers - Beaghton and D.D. Vvedensky, Phys. Rev. B 42, 5544 (1990); A.K. Myers - Beaghton and D.D. Vvedensky, Phys. Rev. A 44, 2457 (1991).
[20] S.V. Ghaisas, Phys. Rev. E, 67, R010601 (2003).
[21] * The current sign should be positive in reference 20 above, in e-prints cond-mat/0202210 and cond-mat/0207630.
[22] J. Krug, J. Stat. Phys., 87, 505 (1997).
[23] P.I. Cohen, G.S. Petrich, P.R. Pukite, G.J. Whaley, and A.S. Arrott, Surf. Sci., 216, 222 (1989); M. Bott, T. Michely, and G. Comsa, Surf. Sci. 272, 161 (1992); J. Vrijmoeth, H.A. Van Der Vegt, J.A. Meyer, E. Vlieg, and R.J. Behm, Phys. Rev. Lett. 72, 3843 (1994).
[24] P. Politi, J. de Physique I, 7, 797 (1997).
[25] S.F. Edwards and D.R. Wilkinson, Proc. R. Soc. London A381, 17 (1982).
[26] F. Family, J. Phys. A10, L441 (1986).
[27] M.D. Johnson, C. Orme, A.W. Hunt, D. Graff, J. Sudi-jono, L. M. Sander, and B.G. Orr, Phys. Rev. Lett. 72, 116 (1994).
[28] J. Krim and G. Palasantzas, Int. J. Mod. Phys. B 9, 599 (1995).
[29] O. Pierre-Louis, M.R. D’Orsogna and T.L. Einstein, Phys. Rev. Lett. 82, 3661 (1999); M.V. RamanMurty and B.H. Cooper, Phys. Rev. Lett. 83, 352 (1999).
[30] J. Kertez, D.E. Wolf, J. Phys. A 21, 747 (1988); D.E. Wolf and J. Kertez, Europhys. Lett. 4, 651 (1987).
[31] S. Das Sarma, P. Punyindu and Z. Toroczkai, e-print, cond-mat/0106495; P. Punyindu, Z. Toroczkai, and S. Das Sarma, Phys. Rev B 64, 205407 (2001); P. Punyindu and S. Das Sarma, Phys. Rev. E, 57, R4863 (1998).
FIG. 1. A typical step structure formed during growth along positive slope. $v$ and $v'$ are velocities of the steps.

FIG. 2. (a) Morphology of the surface after 80000 number of layers. (b) Plot of width as a function of time. Straight lines with slope $3/4$ and $2/3$ are drawn for reference. The substrate size is 10000 and SE barrier parameter $p$ is 0.5 (i.e. no SE barrier).
FIG. 3. The growth model with small +ve SE barrier but without edge diffusion. The growth is over 200X200 substrate size with SE barrier parameter \( p = 0.54 \) showing small +ve barrier to compensate for the larger number of configurations, available for downward hops. (a) Morphology of the surface after 2000 number of layers. (b) Plot of measure of mound size as a function of time. The slope obtained is 0.23\( \pm \)0.02.

FIG. 4. Comparison of growth model results, with dissociation and without, in (1+1) dimensions. Parameter \( q \) decides the fraction of adatoms with single neighbor dissociated if encountered during hopping. (a) Morphology of the surface after 50000 number of layers. Dotted curve represents morphology in the absence of dissociation while solid one is in the presence of it. (b) Plot of width as a function of time. The \( \beta \) value with dissociation is 0.377 \( \pm \) 0.007, while that without dissociation increases to 1/2.
FIG. 5. Comparison of growth model results, with dissociation and without in (2+1) dimensions. Parameter $q$ decides the fraction of adatoms with single neighbor dissociated if encountered during hopping. (a) Morphology of the surface after 2000 number of layers with dissociation ($q = 0.4$). Note the absence of mounds in this case. (b) Morphology of the surface after 2000 number of layers without dissociation ($q = 0.0$). Note the mounds in this case.

FIG. 6. Plot of width as a function of time for the (1+1) dimensional model described in section III for different number of hops $n$. Straight lines with slope $3/4$ and $2/3$ are drawn for reference as top curve and bottom curve respectively. In between, the curves from top correspond to $n = 1, 10$ and 25 number of maximum hops. The substrate size is 10000 and SE barrier parameter is 0.5 i.e. no SE barrier.
FIG. 7. (a) Time development of a base region of width 10 units, bounded by single steps of unit height. Figure shows the morphology for four layers grown with the parameter $p = 0.1$ and $n = 100$ for the model described in section III. (b) Time development of the base region for four MLs as in (a), but the model parameter $p = 0.9$ implying large SE barrier. The base region is seen to be stable in this case. (c) Plot of time correlation function $\langle h(0)h(t) \rangle$ for the growth over base region depicted in (a) and (b). The top most curve correspond to the model parameter value $p = 0.9$ while curves corresponding to $p = 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1$ appear below it in the descending order.