Large Field Inflation in Supergravity

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Abstract

We present a supergravity inflationary scenario in which the inflaton field takes values considerably larger than the Planck scale. It is based on a class of inflationary potentials which can be derived from “singular” Kähler potentials assuming simple superpotentials of the type $W \sim S^n$. To this class belong, among many others, all potentials which are even infinitesimally smaller than the one derived from the minimal Kähler potential. Our scenario allows for a detectable gravitational wave contribution to the microwave background anisotropy.
Most successful inflationary scenarios [1] invoke a very weakly coupled gauge singlet scalar field, the inflaton, in order to account for the tiny temperature fluctuations $\Delta T/T$ in the cosmic microwave background radiation. Nevertheless, the fine tuning that such a weak coupling entails can be avoided in an ingenious model constructed by Linde [2] and studied in detail soon afterwards [3]. Linde’s model is a hybrid of chaotic inflation [1] and the usual theory of spontaneous symmetry breaking involving a possibly gauge nonsinglet field. During inflation the non-inflaton field is trapped in a false vacuum state and the universe is dominated by the false vacuum energy density. Inflation ends with (or just before) a phase transition taking place when the non-inflaton field rolls very rapidly to its true vacuum state (“waterfall”). In the hybrid model the smallness of $\Delta T/T$ is not directly related to the smallness of the self-couplings of the inflaton but can be obtained by exploiting the smallness of the false vacuum energy density in Planck scale units together with an appropriate slope along the inflationary trajectory. Thus, one has the option of forbidding the inflaton self-couplings through appropriate symmetries. This is most naturally implemented in the context of global supersymmetry by imposing R-symmetries [4]. Of course, one is still left with the problem of generating the necessary slope along the inflationary trajectory. One possibility is that this slope, or at least a significant part of it, is generated when global supersymmetry is promoted to local [5], [6], [7].

To investigate the consequences that supergravity has on hybrid inflationary models we confine ourselves to the inflationary trajectory and use the simple superpotential

$$W = -\mu^2 S$$

(1)

involving just the gauge singlet superfield $S$. $W$ is the most general superpotential respecting the continuous R-symmetry $S \to e^{i\theta}S$, $W \to e^{i\theta}W$. In the context of global supersymmetry it gives rise to a slopeless potential $V_{gl} = \mu^4$ consisting entirely of the false vacuum energy density $\mu^4$ which plays the role of a “temporary cosmological constant”.

Let us now replace global supersymmetry by $N = 1$ supergravity with a choice of a minimal Kähler potential $K = |S|^2$ leading to canonical kinetic terms for the inflaton
σ. [Through R-symmetry transformations we bring the scalar component of the superfield $S$, for which the same symbol $S$ is employed, to the form $S \equiv \frac{1}{\sqrt{2}}\sigma$, where $\sigma$ is a real scalar field. Throughout our discussion we restrict ourselves to $\sigma \geq 0$ and we make use of units in which the reduced Planck scale $m_{pl} \equiv \frac{M_{pl}}{\sqrt{8\pi}} \simeq 2.4355 \times 10^{18}$ GeV is equal to 1 ($M_{pl} \simeq 1.221 \times 10^{19}$ GeV is the Planck mass).] Then, the “canonical” potential $V_{can}$ acquires a slope and becomes $\Box$, $\Box$, $\Box$

$$V_{can} = V_{gl}(1 - x + x^2)e^x = \mu^4(1 - x + x^2)e^x = \mu^4 \sum_{m=0}^{\infty} \frac{(m-1)^2}{m!}x^m,$$  \hspace{1cm} (2)

where $x \equiv |S|^2 = \frac{1}{2}\sigma^2$. $V_{can}$ of eq. (2) does not allow inflation unless $x \ll 1$. From the expansion of $V_{can}$ as a power series in $x$ we see that, due to an “accidental” cancellation, the linear term in $x$ is missing and therefore no mass-squared term is generated for $\sigma$.

Small deviations from the minimal form of the Kähler potential respecting the R-symmetry lead to a Kähler potential $\Box$

$$K = x - \frac{\beta}{4}x^2 + \cdots.$$ \hspace{1cm} (3)

The potential $V$ generated from such an almost-minimal Kähler potential has an expansion in powers of $x$ of the form

$$V = \mu^4(1 + \beta x + \cdots)$$ \hspace{1cm} (4)

in which a linear term proportional to the small parameter $\beta > 0$ is now generated. All higher powers of $x$ are still present in the series with coefficients which are only slightly different from the corresponding ones of eq. (2). In particular, there is a choice of the coefficients in eq. (3) for which the resulting potential corresponds to the one of eq. (2) with just the addition of the term $\beta x$ $\Box$. Again we naively expect inflation to be allowed only for $x \ll 1$.

Thus, one is tempted to conclude that the inflaton field variation in hybrid inflation with canonical $\Box$, $\Box$ or quasi-canonical $\Box$ supergravity is forced to be small in $m_{pl}$ units and that such a scenario necessarily allows only a limited number of e-foldings. Moreover, as a
consequence of the small inflaton field variation the gravitational wave contribution to the cosmic microwave background anisotropy is expected to be undetectable [8]. This conclusion is correct for canonical supergravity because the Kähler potential is known exactly. For the quasi-canonical case, however, one cannot safely decide by simply knowing a few terms in the expansion of eq. (3).

It would certainly be very interesting if we could arrange for a scenario in which the inflaton field takes values considerably larger than $m_{Pl}$ during the period of inflation relevant to the presently observable universe. Naively, this has a chance to be achieved if the Kähler potential differs substantially from the minimal one, such that the resulting potential is a much more slowly increasing function than the potential $V_{can}$ of eq. (2). It is then natural to expect the coefficients in the expansion of eq. (3) to be all large and in particular the coefficient $\beta$ of the first correction term to be of order unity thereby forbidding inflation at small $\sigma$ values. Surprisingly enough our naive expectations will prove wrong in the sense that, as we will shortly see, even infinitesimally small deviations from the canonical potential of eq. (2) could be sufficient to allow for an inflationary phase above the Planck scale.

Before going into a more technical discussion let us briefly explain the reason for this unexpected result. The important point is that when the potential differs from the canonical one and consequently the Kähler potential is non-minimal the kinetic terms of the field $\sigma$ acquire “corrections” which affect its equation of motion. Because of these complications the flatness of $V(\sigma)$ is not directly related to inflation. To be able to easily decide whether inflation is allowed we should find the canonically normalized field $\sigma_{infl}$ which obeys a “conventional” equation of motion and express the potential $V$ as a function of it. It is the flatness of $V(\sigma_{infl})$ that is directly related to inflation since the usual “slow-roll” parameters involve derivatives of $V$ with respect to the canonically normalized field $\sigma_{infl}$. It turns out that, even for small deviations from the canonical potential, $\sigma_{infl}$ differs significantly from $\sigma$ at large field values although it almost coincides with it at small field values. Actually in the cases considered below $\sigma_{infl}$ diverges as $\sigma$ tends to a finite value $\sigma_0$ with $V(\sigma_0)$ remaining finite. Then, a tiny variation of $\sigma$ in the vicinity of $\sigma_0$ results to an infinite variation of $\sigma_{infl}$. 
Consequently, $V(\sigma_{\text{infl}})$ at large $\sigma_{\text{infl}}$ values is much flatter than $V(\sigma)$ in the vicinity of $\sigma_0$. Thus, large field inflation in our scheme is due to the fact that even infinitesimally small deviations from the canonical potential, which are obviously unable to significantly alter its value, are able to dramatically change the behavior of $\sigma_{\text{infl}}(\sigma)$ leading to a function which blows up in the vicinity of a finite point $\sigma_0$.

In order to achieve our goal we shall adopt the following procedure. We start by choosing the potential $V(x) > 0$ (actually the supergravity corrections $V_{gl}$ to the potential $V_{gl}$ of the globally supersymmetric model) instead of the Kähler potential $K(x)$ and we attempt to subsequently determine $K$. Since the derivation of the potential $V$ from the Kähler potential $K$ involves partial differentiations of $K$ this inverse procedure is, in general, very difficult. In our case, however, the determination of the Kähler potential is greatly simplified by the fact that only the superfield $S$ is important during inflation and by the existence of the R-symmetry which forces the Kähler potential to be a function of only one variable $x \equiv |S|^2$ instead of being a function of the two variables $S$ and $S^*$. Then, assuming the linear superpotential $W = -\mu^2 S$ (and using the relations $S \frac{\partial K}{\partial S} = S^* \frac{\partial K}{\partial S^*} = x \frac{dK}{dx}$ and $\frac{\partial^2 K}{\partial S \partial S^*} = \frac{d}{dx} \left( x \frac{dK}{dx} \right)$), $V$ can be written in terms of $K$ in the form

$$V = V_{gl} \left[ \left( 1 + x \frac{dK}{dx} \right)^2 \left( \frac{d}{dx} \left( x \frac{dK}{dx} \right) \right)^{-1} - 3x \right] e^K$$ (5)

which does not involve partial derivatives. This relation can be regarded as an ordinary differential equation for $K(x)$

$$\frac{d}{dx} \left( x \frac{dK}{dx} \right) = \left( 1 + x \frac{dK}{dx} \right)^2 \left( 3x + \frac{V}{V_{gl}} e^{-K} \right)^{-1}$$ (6)

whose solution $K(x)$ satisfies the boundary conditions

$$K \simeq x, \quad \frac{dK}{dx} \simeq 1, \quad \text{for } x \ll 1.$$ (7)

The boundary conditions on $K$ do not really constrain $V$ since they simply require that $V$ tends to $V_{gl}$ as $x$ tends to 0. Thus, it seems that in our case choosing the potential is equivalent to choosing the Kähler potential.
However, one does not have the freedom to further assume that the field $\sigma$, as a function of which $V$ is chosen, has canonical kinetic terms and consequently obeys a “conventional” equation of motion because, as we mentioned earlier, the kinetic terms in supergravity obtain “corrections” involving derivatives of the Kähler potential. For this reason it is not easy to decide whether an input potential $V(x)$ allows inflation without determining the Kähler potential. An equivalent description is the one already adopted of determining the canonically normalized candidate inflaton field $\sigma_{infl}$ which satisfies the differential equation

$$
\frac{d}{d\sigma} \sigma_{infl} = \left[ \frac{d}{dx} \left( x \frac{dK}{dx} \right) \right]^{\frac{1}{2}} = \left( 1 + x \frac{dK}{dx} \right) \left( 3x + \frac{V}{V_{gl}} e^{-K} \right)^{-\frac{1}{2}}
$$

with the boundary condition

$$
\sigma_{infl} \simeq \sigma, \text{ for } x \ll 1.
$$

We see that determination of $\sigma_{infl}$ again necessitates determination of $K$. The advantage of this description, however, is that $\sigma_{infl}$ obeys a “conventional” equation of motion and therefore flatness of $V(\sigma_{infl})$ is sufficient for inflation to be allowed.

It should now be obvious that if the input function $V(x)$ is not singular anywhere but the Kähler potential $K(x)$ and the canonically normalized field $\sigma_{infl}(x)$ are singular at a finite point $x_0$, then $V(\sigma_{infl})$ becomes flat as $\sigma_{infl} \to \infty$ (i.e. as $x \to x_0$). Indeed, the variation of $\sigma_{infl}(x)$ in a small interval $(x_0 - \epsilon, x_0)$, with $0 < \epsilon \ll 1$, is infinite whereas, according to our assumption, the variation of $V(x)$ in the same interval remains finite. This is the essence of our inflationary scenario.

Such a scenario motivates us to assume the existence of potentials and Kähler potentials for which $\frac{V}{V_{gl}} e^{-K}$ sooner or later tends to zero. Then, once $\frac{V}{V_{gl}} e^{-K}$ becomes negligible, the above equations simplify considerably and the solutions follow the asymptotic forms

$$
K = -3 \ln |A - \ln x| - \ln x + B,
$$

$$
\sigma_{infl} = -\sqrt{\frac{3}{2}} \ln |A - \ln x| + C,
$$

6
which obviously depend on $V$ only through the integration constants $A$, $B$ and $C$. Eq. (6) with $V(x) > 0$ easily leads (for $x > 0$) to $\frac{dK}{d\ln x} = x \frac{dK}{dx} > 0$, from which it follows that $A \equiv \ln x + 3 \left( 1 + x \frac{dK}{dx} \right)^{-1} > \ln x$ when eqs. (10) and (11) start being applicable. Both $K$ and $\sigma_{\text{inf}}$ tend to infinity when $x$ tends to its largest allowed value $x_0 = e^A$ with $V$ tending to the finite, as we assume, value $V(x_0)$. Then $V$, viewed as a function of $\sigma_{\text{inf}}$, soon becomes very flat thereby allowing for a very long inflationary era. Indeed, for $x = x_0 - \delta x$ (with $0 < \delta x \ll 1$) $\frac{dV}{d\sigma_{\text{inf}}} = \frac{dV}{dx} \frac{dx}{d\sigma_{\text{inf}}} \simeq \sqrt{\frac{2}{3}} \frac{dV}{dx} \delta x$ which, for any finite value of $\frac{dV}{dx} \simeq \frac{dV}{dx}(x_0)$, is as small as one wants provided $x$ is sufficiently close to $x_0$ or, equivalently, $\sigma_{\text{inf}}$ sufficiently large ($\sigma_{\text{inf}} \simeq \sqrt{\frac{2}{3}} \ln \frac{1}{\delta x} + \sqrt{\frac{2}{3}} A + C$). We see that our scenario is actually consistent with eqs. (6) to (9) and consequently with supergravity. Whether a given $V$ belongs to the above class of potentials which, as we will shortly demonstrate, is non-empty can be easily tested numerically.

We first consider potentials $V$ which are only infinitesimally smaller than the canonical potential $V_{\text{can}}$

$$V = V_{\text{can}} + \delta V, \quad (12)$$

where $\delta V$ is negative and small. Equivalently, we consider potentials whose series expansion in powers of $x$ has coefficients equal or slightly smaller than the corresponding ones of eq. (2). Such potentials could arise perturbatively from the minimal Kähler potential as a result of small quantum corrections. It turns out that all potentials resulting from such perturbations do belong to the desired class just described.

As an example, we consider the perturbation to $V_{\text{can}}$ given by $\delta V/\mu^4 = -0.001 x^2$. In fig. 1 the numerical solution $K(\sigma)$ is plotted. We see that $K(\sigma)$ practically coincides with the canonical form $K(\sigma) = \frac{1}{2} \sigma^2$ for $\sigma \lesssim 6$, it remains close to it until $\sigma$ approaches the largest allowed value $\sigma_0 \simeq 7.913$ and suddenly tends to infinity following eq. (10) as $\sigma$ tends to $\sigma_0$. The canonically normalized inflaton $\sigma_{\text{inf}}(\sigma)$ is plotted as well. Again $\sigma_{\text{inf}}$ practically coincides with $\sigma$ for $\sigma \lesssim 6$, it remains close to it until $\sigma$ approaches $\sigma_0$ and quickly follows the asymptotic behavior of eq. (11) as $\sigma$ tends to $\sigma_0$. In fig. 2 the potential $V(\sigma_{\text{inf}})$ is
plotted. An enormous almost flat region for $\sigma_{infl} \gtrsim 12$ is apparent. Notice that this region corresponds to a variation of $\sigma$ in the tiny interval $(\sigma_0 - \varepsilon, \sigma_0)$ with $\varepsilon \approx 0.017$.

The following arguments could help us “understand” this unexpected result. Let us assume that at some point $x = x_i$ (with $x_i \ll 1$) $K(x_i) = x_i$ and $\frac{dK}{dx}(x_i) = 1$. Then, from $\frac{V_{gl}}{V_{can}} < \frac{V_{can}}{V_{gl}}$ and eq. (5) one can easily show that $\frac{d^2K}{dx^2}(x_i) > 0$. This means that there is an interval $(x_i, x_1)$ where $K(x) > x$. In this interval $\frac{V_{gl}}{V_{can}} e^{-K} < \frac{V_{can}}{V_{gl}} e^{-x} = (1 - x + x^2)$ and consequently

$$\left(1 + x \frac{dK}{dx}\right)^{-2} \frac{d}{dx} \left(x \frac{dK}{dx}\right) > (1 + x)^{-2}. \tag{13}$$

Integrating this relation from $x_i$ to $x$ (with $x_i < x < x_1$) we obtain $\frac{dK}{dx} > 1$ in the whole interval $(x_i, x_1)$ meaning that the interval where $K(x) > x$ can be extended beyond $x_1$. Repeating this procedure we may conclude that $K(x) > x$ and $\frac{dK}{dx} > 1$ hold for all meaningful values of $x > x_i$. Eq. (13), which now is assumed to hold for all $x > x_i$, can be easily rewritten as

$$\frac{d^2K}{dx^2} > x \left(\frac{dK}{dx} - 1\right) \left(\frac{dK}{dx} - \frac{1}{x^2}\right) (1 + x)^{-2}, \tag{14}$$

from which it follows that $\frac{dK}{dx} > 1$ implies $\frac{d^2K}{dx^2} > 0$ for $x \geq 1$. Thus, we expect that $\delta K \equiv K(x) - x$, which satisfies $\delta K > 0$, $\frac{d\delta K}{dx} > 0$ for $x > x_i$ and $\frac{d^2\delta K}{dx^2} > 0$ for $x \geq 1$, will eventually grow sufficiently fast forcing $\frac{V_{gl}}{V_{can}} e^{-K} < \frac{V_{can}}{V_{gl}} e^{-\delta K} = (1 - x + x^2) e^{-\delta K}$ to tend to zero. Then, $K(x)$ and $\sigma_{infl}(x)$ will be described by eqs. (10) and (11), respectively and $V(\sigma_{infl})$ will become asymptotically flat. Obviously, the asymptotic flatness of $V(\sigma_{infl})$ could not be due directly to the tiny perturbation $\delta V$ which is clearly unable to significantly alter the value of $V_{can}$. The important effect of the small perturbation is that it changes dramatically the behavior of $\sigma_{infl}(x)$ leading to a function which blows up as $x$ approaches a finite value $x_0$ (with $V(x)$ remaining finite as $x \to x_0$). Notice that the whole argument depends crucially on the boundary condition that the Kähler potential should tend to the minimal one at small field values.

It is worth emphasizing that the previous arguments do not involve the magnitude of the (negative) perturbation which only determines the rate of growth of $\delta K(x)$ with $x$ or,
equivalently, the point \( x_0 \) in the vicinity of which eqs. (10) and (11) start being applicable (the smaller \( |\delta V| \), the larger \( x_0 \)). Thus, we are led to the important conclusion that all potentials \( V \) with

\[
0 < \frac{V}{V_{gl}} < \frac{V_{can}}{V_{gl}}
\]

allow inflation above the Planck scale.

The class of potentials allowing inflation above the Planck scale is actually larger than the class of potentials resulting from the perturbations just described. Thus, one could consider perturbations \( \delta V \) which are not necessarily negative for all values of \( x \). For instance, one could add to \( V_{can} \) a small positive linear term in \( x \) provided he adds higher powers of \( x \) with negative coefficients of appropriate magnitude as well. Moreover, such perturbations do not have to be necessarily small. Thus, there are potentials which could be regarded as large deviations from \( V_{can} \) which still belong to the desired class.

As an illustration we consider potentials of the type

\[
V = \mu^4 \left[ 1 + (\beta - \gamma)x + \alpha x^2 \right] e^{\gamma x} = \mu^4 (1 + \beta x + \cdots)
\]

involving the three (non-negative) real parameters \( \alpha, \beta, \gamma \) in addition to the false vacuum energy density \( \mu^4 \). The choice \( \alpha = \gamma = 1, \beta = 0 \) corresponds to \( V_{can} \). For \( \beta > 0 \) all such potentials are larger than \( V_{can} \) for sufficiently small values of \( x \). Such potentials with \( \alpha = 1, \gamma \leq 1 - \beta \) or \( \alpha = \gamma \leq 1 - \frac{\beta}{2} \) do belong to the desired class not only for \( \beta \ll 1 \) but also for larger values of \( \beta \). Moreover, this class contains potentials with \( \alpha = \beta = 1, \gamma \lesssim 0.565 \) or \( \alpha = \beta = \gamma \lesssim 0.738 \). Even more surprisingly it contains potentials with \( \beta = \gamma \sim 1 \) provided \( \alpha \) is sufficiently small. More specifically, \( \beta = \gamma = 1 \) is allowed provided \( \alpha \lesssim 0.187 \) or \( \beta = \gamma \) could be as large as 1.253 if \( \alpha = 0 \). Clearly, the allowed values of \( \alpha \) and \( \gamma \) are not determined sharply by the value of \( \beta \) but instead they belong to quite large regions in the \( \alpha - \gamma \) plane. Table 1 gives the values of the integration constants \( A, B \) and \( C \) appearing in the asymptotic solutions for \( K \) and \( \sigma_{inf} \) for several values of the parameters \( \alpha, \beta \) and \( \gamma \) which give rise to potentials belonging to the class we are interested in. In fig. 3 the
numerical solutions for $K(\sigma)$ and $\sigma_{infl}(\sigma)$ are plotted for the choice of parameters $\alpha = 1$, $\beta = 1$, $\gamma = 0.5$. Deviations from the corresponding canonical forms are now certainly larger than the ones in fig. 1. Still, the potential $V(\sigma_{infl})$ is flat for $\sigma_{infl} \gtrsim 8$ as seen from fig. 4.

A potential of the type $V = \mu^4 e^{\frac{4}{3} \sigma^2} \ (\text{with } 0 < \frac{4}{3} \ll 1)$ along the inflationary trajectory in supergravity has been suggested earlier in connection with “tilted hybrid inflation” [9]. This is our potential of eq. (16) with $\alpha = 0$, $\beta = \gamma = \frac{4}{3} \ll 1$. However, such a potential has not been shown in [9] to be derivable from a Kähler potential and for this reason the fact that the canonically normalized inflaton really differs from $\sigma$ has been overlooked. Consequently, inflation seemed forbidden for large $\sigma$ values which had as a result a limited total number of e-foldings. Even if $\beta = \gamma = \frac{4}{3}$ is chosen sufficiently small and inflation is naively expected to take place at relatively small $\sigma$ values, the non-canonical kinetic terms could play an important role if strong radiative corrections [4] to the slope of the inflationary trajectory are present. In our opinion, as we already emphasized, small values simultaneously for $\alpha$, $\beta$ and $\gamma$ are rather unnatural because they suggest a small first correction to the canonical Kähler potential with all the higher ones in the expansion of eq. (3) being rather large. From our earlier discussion follows that $\delta$ could be quite large ($\frac{4}{3} \lesssim 1.253$). Then inflation, which is now forbidden at small $\sigma$, must necessarily take place at $\sigma$ values close to the largest allowed value $\sigma_0$.

Barring the unnatural situation of a very flat potential with $\beta \ll 1$, inflation for $x \gtrsim 1$ is not allowed unless $\frac{d}{dx} \sigma_{infl}$ becomes large i.e. unless $\sigma_{infl}$ is described by eq. (11) with $x$ close to $x_0 = e^A$. Then, the number of e-foldings $\Delta N(x_{in}, x_f)$ for the time period that $x$ varies between the values $x_{in}$ and $x_f \ (x_{in} \geq x_f)$ is given, in the slow roll approximation, by

$$\Delta N(x_{in}, x_f) = - \int_{x_{in}}^{x_f} V \left( \frac{dV}{dx} \right)^{-1} \left( \frac{d}{dx} \sigma_{infl} \right)^2 dx \simeq - \frac{3}{2} \left( \frac{d\ln V}{d\ln x} \right)^{-1}_{x_0} (A - \ln x)^{-1} _{x_f} \left( x_{in} \right). \quad (17)$$

With $x_H$ being the value of $x$ when the scale $\ell_H$, corresponding to the present horizon, crossed outside the inflationary horizon and $x_{end}$ its value at the end of inflation, $N_H \equiv \Delta N(x_H, x_{end})$ is estimated to be
\[ N_H \simeq \frac{3}{2} \left( \frac{d \ln V}{d \ln x} \right)^{-1} x_0 (A - \ln x_H)^{-1}. \] (18)

Using this relation we obtain estimates for the slow-roll parameters \( \frac{V'}{V} \) and \( \frac{V''}{V} \) (where the prime refers to differentiation with respect to \( \sigma_{infl} \)) at the scale \( \ell_H \)

\[
\left( \frac{V'}{V} \right)_{x_H} \simeq \sqrt{\frac{3}{2}} N_H^{-1}, \quad \left( \frac{V''}{V} \right)_{x_H} \simeq -N_H^{-1} \] (19)

and for the differential spectral index \( n_H \)

\[ n_H \simeq 1 + 2 \left( \frac{V''}{V} \right)_{x_H} - 3 \left( \frac{V'}{V} \right)_{x_H}^2 \simeq 1 - 2N_H^{-1}. \] (20)

For the quadrupole anisotropy \( \Delta T^2 \) we employ the standard formula [10]

\[
\left( \frac{\Delta T}{T} \right)^2 \simeq \frac{1}{720\pi^2} \left[ \frac{V^3}{V'^2} + 6.9V \right]_{x_H}, \] (21)

from which the parameter \( \mu \) is estimated

\[
\mu \simeq (1080\pi^2)^{\frac{1}{4}} \left( N_H^2 + 10.35 \right)^{-\frac{1}{2}} \left( \frac{V}{V'^2} \right)^{-\frac{1}{8}} \left( \frac{\Delta T}{T} \right)_{x_0}^{\frac{1}{2}}. \] (22)

The first term in eq. (21) is the scalar component \( (\Delta T^2)_{S} \) of \( (\Delta T^2) \) whereas the second is the tensor one \( (\Delta T^2)_{T} \) which represents the gravitational wave contribution. Their ratio \( r \) is

\[
r \equiv \left( \frac{\Delta T}{T} \right)^2_{T} / \left( \frac{\Delta T}{T} \right)^2_{S} \simeq 6.9 \left( \frac{V'}{V} \right)^2_{x_H} \simeq 10.35N_H^{-2}. \] (23)

Taking \( N_H \simeq 60 \) in the above formulas we obtain \( n_H \simeq 0.97 \) and \( r \simeq 3 \times 10^{-3} \). Therefore, the gravitational wave signal is undetectably small [11] in this scheme.

Our inflationary scenario, which is based on “singular” Kähler potentials, resembles the scenario considered in [12]. An important difference between the two is that in our case the superpotential during inflation is the typical one encountered in models of false vacuum inflation whereas in [12] conditions are imposed on the superpotential which are not satisfied in the simplest models.

The above formulas suggest that if the value \( N_H \simeq 12 \) is employed then \( r \simeq 7 \times 10^{-2} \) could be obtained leading possibly to a detectable gravitational wave signal [11]. Of course,
such a low value of $N_H$ is allowed only if the inflationary stage just described is followed by a second one at values of $x \ll 1$ producing the additional number of e-foldings necessary for the solution of the cosmological problems. This, in turn, becomes possible on the same inflationary trajectory provided $\beta \ll 1$ and the potential is not too steep for $0.1 \lesssim x \lesssim x_0$ such that a second stage of inflation complementary to the first one does take place. An example of such a potential is

$$V = V_{can} + \delta V$$

(24)

with

$$\frac{\delta V}{\mu^4} = f(x) = \beta x - 0.04x^2 - 0.2x^3 - 0.245x^4 - 0.13x^5 - 0.034x^6 - 0.007x^7 - 0.001x^8$$

(25)

and $0.03 \lesssim \beta \lesssim 0.035$. It is derived from a Kähler potential $K$ which for $x \ll 1$ admits an expansion in powers of $x$

$$K = \sum_{n=1}^{\infty} a_n x^n$$

(26)

with $|a_n| \lesssim 10^{-2}$ for $n \neq 1$. The values of the first few $a_n$’s are given in table 2. Fig. 5 gives the plots of $K(\sigma)$ and $\sigma_{infl}(\sigma)$ whereas fig. 6 gives the plot of $V(\sigma_{infl})$. Everywhere the choice $\beta = 0.03$ was made.

Our discussion so far assumes a linear superpotential $W = -\mu^2 S$ leading to a potential which tends to a constant $\mu^4$ at small field values. Consequently, during inflation the universe is trapped in a false vacuum state and the inflationary stage has to be followed by a phase transition in order for the false vacuum energy density $\mu^4$ to be cancelled. Our results can be easily extended to the case of inflation taking place with the universe being in its true vacuum state with a superpotential

$$W = \frac{\lambda}{n} S^n, \quad n > 1$$

(27)

respecting the continuous R-symmetry $S \to e^{i\theta} S$, $W \to e^{i\theta} W$. In the context of global supersymmetry the superpotential of eq. (27) gives rise to a potential
leading to the usual chaotic inflation \[ |\lambda|^2 |S|^{2n-2} = |\lambda|^2 x^{n-1} \] for \( x \gg 1 \). In N=1 supergravity the potential becomes

\[
V = V_{gl} \left[ \left( 1 + \frac{x}{n} \frac{dK}{dx} \right)^2 \left( \frac{d}{dx} \left( x \frac{dK}{dx} \right) \right)^{-1} - \frac{3}{n^2} x \right] e^K,
\]

which for the minimal Kähler potential \( K(x) = x \) gives

\[
V_{can} = V_{gl} (1 + \frac{2n-3}{n^2} x + \frac{1}{n^2} x^2) e^x = V_{gl} \sum_{m=0}^{\infty} (n + m)^2 - 4m \frac{1}{m!} x^m
\]

forbidding inflation for all values of \( x \). However, there are again non-singular potentials, including all potentials satisfying \( 0 < \frac{V}{V_{gl}} < \frac{V_{can}}{V_{gl}} \), for which \( \frac{V}{V_{gl}} e^{-K} \) tends to zero as \( x \) tends to a finite point \( x_0 \) leading to diverging \( K(x) \) and \( \sigma_{infl}(x) \) and consequently to a large field inflationary scenario. In particular eq. (11) remains unaltered as do eqs. (17) to (23) (with the exception of eq. (22) in which \( \mu \) should be replaced by \( \sqrt{|\lambda|} \)). Of course, with \( n > 1 \) inflation in two stages on the same trajectory is not possible since, as is well known, \( V_{gl} \) of eq. (28) does not allow inflation at small field values.

To summarize, we presented an inflationary scenario in supergravity taking place at inflaton field values considerably larger than \( m_{pl} \) and based on potentials derived from “singular” Kähler potentials. Of particular interest are cases in which the Kähler potential is very close to the minimal one for small inflaton field values. In some cases our scenario predicts a detectable gravitational wave signal in the cosmic microwave background anisotropy. Whether Kähler potentials similar to the ones required for the realization of our inflationary scenario are obtainable in the context of more fundamental constructions giving effectively rise to supergravity remains a challenging open issue.

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| $\alpha$ | $\beta$ | $\gamma$ | $A$     | $B$     | $C$    |
|-----|-----|-----|-------|-------|-------|
| 0   | 1   | 0   | 1.70765 | 2.60533 | 1.97641 |
| 1   | 1   | 0.1 | 1.95456 | 3.22173 | 2.08914 |
| 1   | 1   | 0.3 | 2.11480 | 3.78278 | 2.19302 |
| 1   | 1   | 0.5 | 2.52255 | 5.82376 | 2.55717 |
| 0.738 | 0.738 | 0.738 | 3.48979 | 20.80807 | 4.64869 |
| 0   | 0.5  | 0.5 | 1.61063 | 2.47728 | 1.95865 |
| 0   | 0.75 | 0.75 | 1.72307 | 2.72645 | 2.00278 |
| 0   | 1   | 1   | 1.90022 | 3.24982 | 2.09902 |
| 1   | 0.03 | 0.97 | 2.87459 | 13.93121 | 3.77610 |
| 1   | 0.05 | 0.95 | 2.74657 | 11.78929 | 3.48537 |
| 1   | 0.07 | 0.93 | 2.65533 | 10.40770 | 3.28897 |
| 0.985 | 0.03 | 0.985 | 3.04168 | 17.09767 | 4.18074 |
| 0.975 | 0.05 | 0.975 | 2.92584 | 14.82190 | 3.89277 |
| 0.965 | 0.07 | 0.965 | 2.84305 | 13.33466 | 3.69675 |

Table 1. The values of the integration constants $A$, $B$ and $C$ for various values of the parameters $\alpha$, $\beta$ and $\gamma$ in the potential of eq. (16).
Table 2. The values of the first few coefficients appearing in the expansion of the Kähler potential $K$ of eq. (26).

| $n$ | $a_n$       |
|-----|-------------|
| 1   | +1.         |
| 2   | -0.007500   |
| 3   | +0.010378   |
| 4   | +0.002090   |
| 5   | -0.001044   |
| 6   | +0.000705   |
| 7   | -0.000401   |
| 8   | +0.000278   |
| 9   | -0.000221   |
| 10  | +0.000188   |
Fig. 1. The Kähler potential $K$ and the canonically normalized inflaton $\sigma_{infl}$ are plotted as functions of $\sigma$ for the choice $\delta V/\mu^4 = -0.001x^2$ of the perturbation to $V_{can}$. Their “canonical” forms are plotted as well.

Fig. 2. The potential $V$ is plotted as a function of the canonically normalized inflaton $\sigma_{infl}$ for the choice $\delta V/\mu^4 = -0.001x^2$ of the perturbation to $V_{can}$.

Fig. 3. The Kähler potential $K$ and the canonically normalized inflaton $\sigma_{infl}$ are plotted as functions of $\sigma$ for the choice $\alpha = 1, \beta = 1$ and $\gamma = 0.5$ of the parameters in the potential of eq. (16). Their “canonical” forms are plotted as well.

Fig. 4. The potential $V$ is plotted for the choice $\alpha = 1, \beta = 1$ and $\gamma = 0.5$ of the parameters in the potential of eq. (16) as a function of the canonically normalized inflaton $\sigma_{infl}$.

Fig. 5. The Kähler potential $K$ and the canonically normalized inflaton $\sigma_{infl}$ are plotted as functions of $\sigma$ for the choice $\delta V/\mu^4 = f(x)$ of the perturbation to $V_{can}$. Their “canonical” forms are plotted as well.

Fig. 6. The potential $V$ is plotted as a function of the canonically normalized inflaton $\sigma_{infl}$ for the choice $\delta V/\mu^4 = f(x)$ of the perturbation to $V_{can}$. 
\[ \frac{\delta V}{\mu^4} = -0.001x^2 \]
$\frac{\delta V}{\mu^4} = -0.001 x^2$
\[ \alpha = 1, \beta = 1, \gamma = 0.5 \]

- \[ K(\sigma) \]
- \[ K(\sigma) = \sigma^2 / 2 \]
- \[ \sigma_{\text{infl}}(\sigma) \]
- \[ \sigma_{\text{infl}}(\sigma) = \sigma \]
\[ \frac{V}{\mu^4} \quad \sigma_{\text{infl}} \]

\[ \alpha = 1, \beta = 1, \gamma = 0.5 \]
$K(\sigma) = \sigma^2 / 2$

$\sigma_{inf}(\sigma) = \sigma$

$\delta V / \mu^4 = f(x)$
\( \frac{\delta V}{\mu^4} = f(x) \)