Bouncing universe from a modified dispersion relation

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Abstract

In this paper we argue that modified Friedmann equations with a bounce solution can be derived from a modified dispersion relation by employing a thermodynamical description of general relativity on the apparent horizon.
I. INTRODUCTION

One remarkable achievement of general relativity is linking the geometric structure of spacetime with the distribution of matter sources. Or in another word, the nature of matter sources determines which kind of spacetime geometry we may obtain through the Einstein equation. Once people know in first principle all the matter sources should be described by quantum theory, it is realized that the geometry of spacetime should be quantized as well. Before a complete and consistent quantum theory of gravity can be established, one widely accepted belief is that the combination of general relativity and quantum mechanics will provide a fundamental minimal length or maximal energy that can be detected or measured, which is always termed as Planck length or Planck energy. If this is true, then it is expected that the longstanding cosmological singularity problem may be solved in the quantum theory of gravity. Semi-classical theory of quantum gravity provides such an intuitive picture to replace the cosmological singularity in standard cosmology by a big bounce[1, 2], which is implemented by considering modifications due to the quantum corrections to the standard Friedmann equations at the semi-classical limit. Recently, the big bounce as a solution to the modified Friedmann equations has been extensively investigated in various approaches[3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14]. Moreover, the bounce solutions even exist in some special models in the Einstein gravity[15, 16, 17]. In this paper we intend to drive the modified Friedmann equations with a big bounce solution from a modified dispersion relation in a heuristic manner.

Our motivation directly comes from the fact that modified dispersion relations may provide a bound for both the energy and the momentum of particles, and such effects have greatly been studied on the fate of Lorentz symmetry at extremely high energy level, as well as quantum gravity phenomenology[18, 19, 20]. Based on the spirit of general relativity, if all the matter sources are described by modified dispersion relations which prevent the energy density from diverging at high energy level, then the curvature of the corresponding spacetime should also be bounded such that it is singularity free. Previously the quantum gravity effects of particles on spacetime have been investigated in gravity’s rainbow formalism[11, 21, 22, 23, 24, 25, 26, 27, 28, 29]. In this paper we intend to derive a modified Friedmann equation with a bounce solution from a modified dispersion relation by employing the thermodynamical description of the cosmological equations on the apparent horizon.
of the universe.

Originally the thermodynamical description of the Einstein equation is proposed by Jacobson\cite{30}. In this way the Einstein equation is an equation of state and can be derived from a fundamental thermodynamical relation, namely Clausius relation $\delta Q = T dS$, which connects heat, entropy and temperature for all the local Rindler causal horizons, where $\delta Q$ and $T$ are interpreted as the energy flux and the Unruh temperature detected by the accelerated observer inside the horizon, respectively. Recently, this proposal has been testified in various gravity theories and cosmological models\cite{31,32,33,34,35,36,37}. In this context, it turns out that the apparent horizon plays an appropriate role in deriving the Friedmann equations from Clausius relation. Subsequently, an analogy of the first thermodynamical law can also be established on the apparent horizon. In particular, a corrected entropy-area relation may give rise to a modified Friedmann equation has been studied in\cite{38,39}. However, along this direction deriving a modified equation with a bounce solution has not been succeeded in all previous papers. In this paper we argue that with the help of modified dispersion relations this can be realized by applying it to the energy flux passing through the apparent horizon.

Our paper is organized as follows. In next section we propose a specific form of modified dispersion relation in which both energy and momentum are bounded at the order of Planck scale. Then in section three we derive the modified Friedmann equations from the Clausius relation on the apparent horizon based on this modified dispersion relation. The corresponding entropy-area relation is also obtained with a logarithmic correction term. The conclusion and discussion are given in section four.

II. A MODIFIED DISPERSION RELATION

In this paper we propose a modified dispersion relation at quantum gravity phenomenological level, which has a form

$$\frac{1}{\eta l_p} \sin(\eta l_p E) = \sqrt{p^2 + m_0^2},$$

(1)

where $E$ and $p$ are the energy and momentum of a particle with mass $m_0$ respectively, and $\eta$ is a dimensionless parameter. The Planck length $l_p$ as well as the Planck mass $M_p$ is defined as $l_p \equiv \sqrt{8\pi G} \equiv 1/M_p$ with $\hbar = c = 1$. Obviously, to keep the magnitude of the momentum
positive definite, we require that the energy $E$ takes the value in the range $[0, \pi/\eta l_p]$. First of all, at low energy limit with $E \ll M_p$, the Taylor expansion of this relation leads to

$$
(1 - \eta^2 l_p^2 E^2/3)E^2 - p^2 = m_0^2,
$$

(2)

which is nothing but the standard dispersion relation when $l_p \to 0$. The impacts of the deformed dispersion relation with form Eq.(2) on black hole thermodynamics have been extensively studied in [23, 40, 41]. Now to obtain the modified Friedmann equations with bounce solutions, we insist to use the deformed dispersion relation Eq.(1) rather than Eq.(2), and the reasons to do so will be presented later.

One feature of the modified dispersion relation Eq.(1) is the peculiar relation between the energy and the momentum at Planck energy level. From (1) we easily see that in the energy range of $[0, \pi/2\eta l_p]$, as usual the energy $E$ is a monotonously increasing function of the momentum $p$. In particular as $E = \pi/2\eta l_p$, the momentum $p$ reaches its maximal value $1/\eta l_p$. However, as the energy climbs up further, the momentum $p$ decreases monotonously until it reaches zero at $E = \pi/\eta l_p$. This property can become more transparent when we consider the differential relation of the energy and momentum, which can be derived as

$$
\delta E = \pm \frac{1}{\sqrt{1 - \eta^2 l_p^2 p^2}} \delta p,
$$

(3)

where the positive sign “+” corresponds to the energy range $[0, \pi/2\eta l_p]$, while the negative sign “−” to $[\pi/2\eta l_p, \pi/\eta l_p]$. We point out that both cases are essential to derive the modified Friedmann equations with bounce solutions, especially to control the universe evolution around the bouncing point, and this is what we intend to do in next section.

III. MODIFIED FRIEDMANN EQUATION FROM THERMODYNAMICAL DESCRIPTION OF GR

Firstly we briefly review the thermodynamical description of the ordinary Friedmann equations in cosmology. Without loss of generality in this paper we only consider the spatially flat universe. Given the flat Friedmann-Robertson-Walker metric

$$
ds^2 = -dt^2 + a^2(t)(dr^2 + \bar{r}^2 d\Omega^2) = h_{ab} dx^a dx^b + \bar{r}^2 d\Omega^2,
$$

(4)

where $x^0 = t$, $x^1 = r$, $\bar{r} = a(t)r$, $h_{ab} = diag(-1, a^2)$ and $d\Omega^2$ is the metric of a 2-dimensional sphere with unit radius. Then it is straightforward to obtain the radius of the apparent
horizon which is defined as \( h^{ab} \partial_a \tilde{r} \partial_b \tilde{r} = 0 \). For the spatially flat universe, it turns out that

\[
\tilde{r}_A = \frac{1}{H},
\]

which coincides with the Hubble horizon. Recently a lot of work shows that an analogy of the first law of thermodynamics could be constructed on the apparent horizon if one assumes that the apparent horizon has an associated entropy \( S \) and temperature \( T \) which are respectively identified as

\[
S = \frac{A}{4G}, \quad T = \frac{1}{2\pi \tilde{r}_A},
\]

where \( A = 4\pi \tilde{r}_A^2 \) is the area of the apparent horizon. More explicitly, the Friedmann equation on the apparent horizon can be rewritten as the Clausius relation

\[
\delta Q = T dS.
\]

Conversely, starting from the Clausius relation on the apparent horizon, one can derive the standard Friedman equation

\[
H^2 = \frac{8\pi G}{3} \rho,
\]

through an integration

\[
\frac{8\pi G}{3} \rho = -\frac{\pi}{G} \int S'(A)(\frac{4G}{A})^2 dA,
\]

where \( S' = \frac{dS}{dA} \) defines the variation of the entropy with the area of the apparent horizon. In this case it is nothing but a constant which is \( \frac{1}{4G} \).

Next we consider how the modified dispersion relation will give rise to a modified Friedmann equation by employing the heuristic analysis which has been previously applied to Hawking radiation of black holes. A similar consideration for the \( FRW \) universe has also been presented in \[39\]. Consider a single radiation particle with quantum energy \( \epsilon \) passes through the apparent horizon, then from the Clausius relation we obtain the minimal change of the entropy corresponding to the minimal change of the horizon area as

\[
\delta S_{\text{min}} \sim dS = \frac{1}{T} \delta Q \simeq \frac{1}{T} \frac{1}{\delta x},
\]

where we have employed the standard dispersion relation and Heisenberg’s uncertainty relation, namely identifying the changes \( \delta Q \sim \epsilon \sim \delta E \sim \delta p \sim \frac{1}{\delta x} \). Furthermore, we identify the
uncertainty of particle position as its Compton wavelength which is inversely proportional to its Hawking temperature, namely $\delta x \sim \tilde{r}_A$ [42, 43, 44]. Given the assumption that the minimal change of the horizon area $\delta A$ of a quantum system is $l_p^2 = 8\pi G [45, 46, 47]$, we find that

$$S' = \frac{dS}{dA} \cong \frac{\delta S_{\text{min}}}{\delta A_{\text{min}}} = \frac{1}{4G},$$

leading to the standard Bekenstein-Hawking entropy. It is worthwhile to point out that the analysis based on the thermodynamical description here obtains the same result as the one based on the Bekenstein entropy assumption for black holes, as discussed in [40].

Now if we take the modified dispersion relation into account, using Eq.(3) we find the minimal change of the entropy is corrected as

$$\delta S_{\text{min}} \sim dS_M \simeq \pm \frac{1}{\sqrt{1 - \frac{4\pi\eta l_p^2}{A}}} \frac{1}{T} \frac{1}{\delta x} \equiv \pm 2\pi f_M(A),$$

where we have set $p \sim \delta p \sim \frac{1}{\delta x} \sim \frac{1}{\tilde{r}_A}$ since the maximal momentum uncertainty of a quantum particle $\delta p$ is of order of $p$, which will lead to the minimal change of the entropy [40, 41, 42]. Thus

$$S' = \frac{dS_M}{dA} \cong \frac{\delta S_{\text{min}}}{\delta A_{\text{min}}} = \pm \frac{1}{4G} f_M(A).$$

Since the positive sign “+” corresponds to the energy range $[0, \pi/2\eta l_p]$, which covers all the classical and semi-classical region below the order of Planck energy, we consider it firstly. Integrating Eq.(13) we obtain the entropy-area relation with corrections due to the modified dispersion relation as

$$S_M = \frac{A}{4G} \sqrt{1 - \frac{4\pi\eta^2 l_p^2}{A}} + \frac{\pi\eta^2 l_p^2}{2G} \ln[A + \sqrt{1 - \frac{4\pi\eta^2 l_p^2}{A}} - 2\pi\eta^2 l_p^2] + C,$$

where an integral constant $C$ has been set to $4\pi^2 \eta^2 (1 - \ln 2)$ simply requiring that $S_M$ has a classical limit $A/4G$ as $A \gg 4\pi\eta^2 l_p^2$. Notice that in this formalism the correction term has a logarithmic form but the factor is $4\pi^2 \eta^2$ which is always positive, different from the previous results for entropy correction of black holes [40, 48, 49, 50]. On the other hand, when the area $A$ is small enough to be comparable with the Planck scale, the famous factor $1/4$ in Bekenstein-Hawking entropy formula is changed. Moreover, we find that the area of the apparent horizon is bounded, namely $\tilde{r}_A \geq \eta l_p$, implying a finite temperature on the horizon even at the Planck scale.
Now given a corrected entropy-area relation, one can obtain the modified Friedmann equation by substituting the result of Eq. (13) into Eq. (9)

\[
\frac{8\pi G}{3} \rho = -\frac{2}{\eta^2 l_p^2} \sqrt{1 - \eta^2 l_p^2 H^2} + C_1,
\]

where \( C_1 \) is an integral constant we need to set. Obviously, we require that this equation should return to the standard Friedmann equation at low energy limit \( l_p \to 0 \). Thus we have

\[
C_1 = \frac{2}{\eta^2 l_p^2} - \frac{\Lambda}{3},
\]

where \( \Lambda/3 \) is a cosmological constant term independent of \( l_p \). Eq. (15) can be rewritten as

\[
H^2 = \frac{8\pi G}{3} \rho_t (1 - \frac{\rho_t}{\rho_c}),
\]

with \( \rho_c = 12/\eta^2 l_p^4 \) and the total energy density \( \rho_t = \rho + \frac{\Lambda}{8\pi G} \). Subsequently using the conservation equation \( \dot{\rho} + 3H(\rho + P) = 0 \), one has

\[
\dot{H} = -4\pi G(\rho + P)(1 - 2\frac{\rho_t}{\rho_c}).
\]

The above two equations can also be obtained in semiclassical loop quantum cosmology with holonomy corrections and some effective theories [3, 5, 6, 10], where the critical energy density \( \rho_c \) has the same order as Planck energy density. Indeed the big bounce solution to Eq. (17) exists whenever \( H = 0 \) and \( \dot{H} > 0 \), which implies \( \rho_t = \rho_c \). However, this solution is \textit{not} contained in Eq. (15). This can be easily seen if one sets \( H = 0 \) in Eq. (15), the unique solution to the energy density is \( \rho_t = 0 \). As a matter of fact, in this equation \( \rho_t \) only values in the range of \([0, 6/\eta^2 l_p^4]\). So, in order to obtain the bounce solutions, we need consider the differential relation of energy and momentum at higher energy level which is Eq. (3) with a “−” sign. Repeat the derivation above, we obtain the modified Friedman equation as

\[
\frac{8\pi G}{3} \rho = \frac{2}{\eta^2 l_p^2} \sqrt{1 - \eta^2 l_p^2 H^2} + C_2,
\]

where the constant \( C_2 \) can be determined by keeping \( H^2 \) as a single-valued and continuous function of the energy density \( \rho_t \). Namely, at \( H^2 = 1/\eta^2 l_p^2 \), we should obtain the same energy density through Eq. (15) and Eq. (19), which leads to

\[
C_2 = C_1 = \frac{2}{\eta^2 l_p^2} - \frac{\Lambda}{3}.
\]
In Eq. (19) \( \rho_t \) only values in the range of \( [6/\eta^2 l_p^4, 12/\eta^2 l_p^4] \). But combining Eq. (15) and Eq. (19) together, we find they cover all the evolution range controlled by Eq. (17). Therefore, we conclude that the modified Friedmann equations with bounce solutions can be obtained from the modified dispersion relation presented in Eq. (1) through a thermodynamical description of general relativity on the apparent horizon.

However, at the end of this section we need to point out that the entropy-area relation obtained in the energy density range of \( [6/\eta^2 l_p^4, 12/\eta^2 l_p^4] \) is problematic. As from Eq. (13) we find the entropy would be

\[
S_M = -\frac{A}{4G} \sqrt{1 - \frac{4\pi \eta^2 l_p^2}{A}} - \frac{\pi \eta^2 l_p^2}{2G} \ln\left[A + A \sqrt{1 - \frac{4\pi \eta^2 l_p^2}{A}} - 2\pi \eta^2 l_p^2\right] + C_3, \tag{21}
\]

where constant \( C_3 \) is set to have the same entropy at \( \rho_t = 6/\eta^2 l_p^4 \) through Eq. (14) and Eq. (22). It turns out that

\[
C_3 = 4\pi^2 \eta^2 [1 + 2 \ln(8\sqrt{2\pi^2 \eta^2})]. \tag{22}
\]

Obviously, the entropy would be negative as the scale factor approaches to the bounce point, where \( H \to 0 \) and \( A \to \infty \). This results from the peculiar differential relation between the energy and the momentum at Planck scale in the modified dispersion relation, implying that when evolving back to the big bounce, the entropy continuously decreases while the area of the apparent horizon will increase after reaching its minimum value \( \tilde{r}_A = \eta l_p \). This sounds very unreasonable, if one insists that entropy is defined as a measure of the number of microscopic quantum states. Previously negative entropy also appeared in the context of black holes and more recently phantom cosmology, and some relevant discussions can be found for instance in [51, 52, 53, 54, 55]. We think the appearance of negative entropy near the bounce point is analogous to the situation in phantom dominated universe, since the null energy condition is violated in the classical framework with a bounce universe. In our paper the presence of negative entropy strongly implies that one need find a new way to define the entropy of the universe near the bounce point. As a matter of fact, the entropy problem is a well-known open question in bouncing or cyclic universe scenarios if we respect the second law of the thermodynamics, stating that the total entropy of the universe should not decrease forever. Then the entropy should not have a proportional relation with the area of the apparent horizon for a contracting universe or any phase during which the area of the apparent horizon is decreasing, for instance after the bounce point of the
universe. Our picture implies that the Clausius relation maybe has not a thermodynamical interpretation when the system is far from the equilibrium state, or the entropy in such a thermodynamical relation has no longer a statistical interpretation. In statistics, the appearance of negative entropy usually implies a metastable state and some kind of phase transition could occur. We expect that near the big bounce point, the universe should be described by a genuine quantum theory of gravity, thus as the energy scale falling down, it might undergo a phase transition from a quantum phase to semi-classical phase through some decoherence mechanism.

IV. SUMMARY AND DISCUSSIONS

In this paper we have proposed a specific modified dispersion relation in which both energy and momentum of matter sources are bounded, and with the assumption that the Clausius relation holds on the apparent horizon, we have derived a modified Friedmann equation with a big bounce solution. Comparing with previous work on the modification of Friedmann equations from corrected entropy-area relation or generalized uncertainty principle, the novelty in our scenario can be summarized as the following two points. First of all, those modified Friedmann equations do not contain a big bounce solution. Mathematically we think the reason may be that the correction terms taken into account in literature are just the approximate expansion of all the corrections at some inappropriate limit. An obvious example can also be seen in loop quantum cosmology. If one takes the limit of the lattice parameter $\mu$ vanishing and consider some low orders of the expansion, as naively supposed in usual semi-classical approach, then it is easily found that the resulted Friedmann equation does not contain any bounce solution. The similar case happens in our paper. If we started from the modified dispersion relation Eq.(2), then the big bounce solution could not be obtained. Secondly, at Planck scale the scenario described by the modified dispersion relation Eq.(1) might be analogous to T-duality in string theory. In the context of T-duality, it is believed that there is a critical radius of the compactification, and physics on length scales below this radius can equally well be described by physics on length scales larger than that. So the temperature is also invariant under a T-duality transformation (see Brane Gas Cosmology for example). Similarly, like the critical radius of the compactification in string theory, here the critical size of the scale factor is reached at $\tilde{r}_A = \eta l_p$. Before that moment,
the temperature is increasing from 0 to a critical value $\frac{1}{2m_P^4}$. Inversely, in low energy region it will decrease monotonously to be vanished. Moreover, if we obtain the entropy of the universe after the bounce point, it is found that the entropy is monotonously increasing such that the second law of thermodynamics is respected, this can be seen from Eq(13). However the magnitude of the entropy is negative near the bounce point, the possible reason is that the semi-classical theory breaks down at so high energy scale, and the universe should be described by a genuine quantum gravity theory which is expected to be founded in future.

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