Dynamic Three-Way Concept Learning Model Based on PS-Rough Set

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Abstract. The PS-rough set adds the dynamic characteristics of the elements moving in and moving out of the set, and introduces it into the three-way concept learning, and establishes the dynamic three-way concept learning model based on the PS-rough set. Firstly, the related definitions of dynamic three-way concept learning are introduced, and the calculation method of conditional probability function is given. Secondly, the related dynamic characteristics of the dynamic three-way concept learning model are proposed and proved, and the dynamic three-way concept learning algorithm is given, and the validity of the algorithm are verified by an example. Finally, the dynamic three-way concept learning model solves the dynamic change problem of the sample subset, reduces the computation of the probability of the condition, and gives a simple three-way concept division method.

1. Introduction

Three-way decision [1-2] is a new decision-making mode in accordance with human cognition. When people can make an accurate judgment of the situation, give acceptance or rejection, but when the information is not sufficient to determine whether it has been accepted or rejected, the delay decision is considered a third decision behavior. Qi et al. proposed three-way concept analysis for the binary information table by introducing three-way decision theory [3]. Shivhare et al. extended formal concept analysis based on bidirectional associative memory to three-way formal concept analysis, adding an additional negative operator [4]. Compared to the concept, the three-way concepts contain more information, which can mean both "common have" and "common do not have ". Concept learning [5] is the process of learning unknown concepts based on specific methods from well-known information. On this basis, the various relationships between concepts are transformed into human cognitive processes, which are called concept cognitive learning. In the current data context, a variety of research results on concept cognitive learning are presented [6-9]. Li et al. improved under the condition of cognitive operator, put forward three-way concept learning [10], which characterizes the general three-way concepts by axiomatic method. Compared with the three-way concept analysis methods, it has more extensive application.

The data processed by the three-way concepts in previous studies are statically invariant and do not consider the move in and move out of elements in the set. However, in real life, the data of dynamic change is more common, for example, in the phase of examining partners, the choice of inter-firm partners will give up cooperation with some enterprises because of certain factors, and would also join new enterprises with the opportunity to cooperate, so that the data of current investigation have the existence of moving in and moving out. In the specific application, part of the subset in the...
information system needs to be learned, and the data information of the subset is dynamically changed, and the subset of the dynamic change may lead to the classification change of the subset, and the previous methods are not applicable for such cases.

Zhang et al considered the elements moving in and out of the set synthetically. And bidirectional probabilistic PS-rough set \([11–12]\) is proposed, which can deal with the subset of dynamic changes.

For this purpose, aiming at the learning problem of dynamic change subset, this paper proposes a dynamic three-way concept learning model. The expression of conditional probability function is given, and the method of calculating the change quantity of conditional probability produced by the subset of dynamic change is given. According to the relevant definition of probability function, the classification rules of dynamic change data are generated, and the three-way concepts of different moments are obtained, and the corresponding three-way granular concepts are formed under different attribute characteristics, so that the three-way concepts of dynamic change subset can be learned.

2. Three-Way Concept

Definition 1 Given two mappings: \(\sigma : 2^{\ell(A)} \to T(U), \tau : T(U) \to 2^{\ell(A)}\). The three-way decision based on PS-rough sets are recorded as \(\{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\}\). For \(\{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\} \in T(U)\) and \(\forall B_i, B_j \in 2^{\ell(A)}\), if satisfied:

\[
B_i \subseteq B_j \Rightarrow \sigma(B_i) \preceq \sigma(B_j), \sigma(B_i \cup B_j) \preceq \sigma(B_i) \cup \sigma(B_j)
\]

\[
\tau\{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\} = \left\{A \in Q(A) \mid \sigma\{\{A\}\} \preceq \{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\} \right\}
\]

Then \(\sigma\) and \(\tau\) are called three-way cognitive operators. \(T(U)\) is a set of three-way decision of \(U\), \(Q(A)\) is the quotient set of \(A\), \(2^{\ell(A)}\) is the power set of \(Q(A)\).

Definition 2 Three-way concept cognitive operators are \(\sigma\) and \(\tau\), For \(\{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\} \in T(U)\), \(\forall B \in 2^{\ell(A)}\), if satisfied: \(\sigma(B) = \{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\}\), \(\tau\{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\} = B\). The three-way concepts are defined as:

\[
\{\{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\}, B\}
\]

(1)

\(\{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\}\) and \(B\) are regarded as the extension and intension.

Definition 3 Three-way concept cognitive operators are \(\sigma\) and \(\tau\), and attribute set \(\{A\} \in 2^{\ell(A)}\). The concept of three-way granular based on different attributes is:

\[
\langle \sigma(A), \tau\sigma(A) \rangle
\]

(2)

Definition 4 Since the three-way granular concept is obtained under a single attribute and the three-way concept is obtained under multiple attributes. Therefore, the relationship between the two can be recorded as: \(\langle \{\text{POS}_{(a,\beta)}, \text{NEG}_{(a,\beta)}\}, B\rangle = \bigvee_{A \in \ell(A)} \langle \sigma(A), \tau\sigma(A) \rangle\). The set definition of the concept of three-way granular is as follows: \(G_{\sigma\tau} = \{\langle \sigma(A), \tau\sigma(A) \mid A \in Q(A) \rangle\}\)

3. Dynamic Three-way Learning Based on PS-rough Set

In information systems, a subset is sometimes learned, and the data in the subset is constantly updated and changed over time, and there will be new elements moving in and moving out of the subset, but the traditional three-way learning cannot deal with dynamic information. For this reason, this paper
proposes the corresponding expression of probability function for the subset of dynamic change at different times, that is, the time when elements move in or out, and defines the classification rule of data according to the probability function. The related properties of dynamic three-way concept learning are given, and a dynamic three-way concept learning algorithm is proposed. \( X \) is selected as a subset from the information system \( U \). \( X^{*} \) is a new subset of objects that move from \( U \) to \( X \) or \( X \) to \( U \).

3.1. Probability Function and Classification Rules

**Definition 5** Set \( p \) as a classification probability function of three-way learning, and the expression of conditional probability under each equivalence class is as follows:

\[
p = p(X^{*}|D_{i}) = \frac{|X^{*} \cap D_{i}|}{|D_{i}|}(3)
\]

Where \( X^{*} \) a new subset of objects that move from \( U \) to \( X \) or \( X \) to \( U \), and \( D_{i} \) is the \( i \)th equivalence class in information system.

**Definition 6** Let \( t_{0} \) exist \( X \subseteq U \), and the conditional probability is \( p \). If there is a subset of \( X_{a}(X_{m}) \) moving into (moving out) the \( X \), the two direction S-sets \( X \) of the \( t_{1} \) is \( X^{*}_{a}(X^{*}_{m}) \), and \( X \subseteq X^{*}_{a} \) \((X^{*}_{m} \subseteq X)\). Then the formula of conditional probability is:

\[
p^{*}_{a} = p + \Delta p \ (p^{*}_{m} = p - \Delta p)^{4}
\]

\[
\Delta p = p(X^{*}_{a}|D_{i}) = \frac{|X^{*}_{a} \cap D_{i}|}{|D_{i}|}, \quad \Delta p' = p(X^{*}_{m}|D_{i}) = \frac{|X^{*}_{m} \cap D_{i}|}{|D_{i}|}(5)
\]

According to the variation of subsets, the three regions of PS-rough set correspond to three parts of three-way learning. This defines three classification rules for three-way learning when objects move in and move out.

**Definition 7** Let the dynamically changing two direction S-sets be \( X^{*} \), and the three regions of the PS-rough set based on the parameter \((\alpha, \beta)\) are \( POS_{(\alpha, \beta)}(X^{*}) \), \( NEG_{(\alpha, \beta)}(X^{*}) \), \( BND_{(\alpha, \beta)}(X^{*}) \), respectively. The following classification rules are available: Classification rules P: \( p \geq \alpha, x \in X^{*}, [x]_{R} \cap X^{*} \subseteq POS_{(\alpha, \beta)}(X^{*}) \); Classification rules N: \( p \leq \beta, x \in X^{*}, [x]_{R} \cap X^{*} \subseteq NEG_{(\alpha, \beta)}(X^{*}) \); Classification rule B: \( \beta < p < \alpha, x \in X^{*}, [x]_{R} \cap X^{*} \subseteq BND_{(\alpha, \beta)} \)

3.2. Dynamic Properties Based on PS-rough Sets

**Inference 1** Let \( t_{i} \) exist \( X \subseteq U \), if there is a migration of elements, the two direction S-sets \( X \) of the \( t_{i} \) is \( X^{*}_{i} \), and \( X \subseteq X^{*}_{i} \((X \supseteq X^{*}_{i})\), the two direction S-sets \( X \) of the \( t_{i} \) is \( X^{*}_{i} \), and \( X \subseteq X^{*}_{i} \((X \supseteq X^{*}_{i})\), and so on, the two direction S-sets \( X \) of the \( t_{i} \) is \( X^{*}_{i} \), \( X^{*}_{i} \subseteq X^{*}_{i} \((X^{*}_{i} \supseteq X^{*}_{i})\), then

\[
Pr(X^{*}_{0}[x]_{R}) \leq \cdots \leq Pr(X^{*}_{i}[x]_{R}) \leq \cdots \leq \underbrace{Pr(X^{*}_{n}[x]_{R})}_{\geq \cdots \geq \cdots \geq Pr(X^{*}_{n}[x]_{R})}
\]

**Proof:** By \( X^{*}_{0} \subseteq X^{*}_{1} \subseteq \cdots \subseteq X^{*}_{i} \subseteq \cdots \subseteq X^{*}_{n} \subseteq \cdots \subseteq X^{*}_{n} \subseteq [x]_{R} \), then \( X^{*}_{i} \cap [x]_{R} \subseteq X^{*}_{i} \cap [x]_{R} \subseteq \cdots \subseteq X^{*}_{i} \cap [x]_{R} \), so \( X^{*}_{i} \), \( X^{*}_{i} \subseteq \cdots \subseteq X^{*}_{i} \subseteq \cdots \subseteq X^{*}_{i} \subseteq \cdots \subseteq X^{*}_{i} \subseteq \cdots \subseteq X^{*}_{i} \subseteq [x]_{R} \).

**Inference 2** Let \( t_{0} \) exist \( X \subseteq U \), \( Pr(X^{*}[x]_{R}) \geq \alpha \ (Pr(X^{*}[x]_{R}) \leq \beta) \), \( x \in POS_{(\alpha, \beta)}(X) \) \((x \in NEG_{(\alpha, \beta)}(X))\), if there is a migration of elements, the two direction S-sets \( X \) of the \( t_{i} \) is \( X^{*}_{i} \), and \( X \subseteq X^{*}_{i} \((X \supseteq X^{*}_{i})\), the two direction S-sets \( X \) of the \( t_{i} \) is \( X^{*}_{i} \), \( X^{*}_{i} \subseteq X^{*}_{i} \((X^{*}_{i} \supseteq X^{*}_{i})\), and so on, the
two direction S-sets $X$ of the $t_n$ is $X^*_n$, $X^*_{n-1} \subseteq X^*_n$ ($X^*_{n-1} \supseteq X^*_n$), it should choose $x \in \text{POS}_{(\alpha, \beta)} \left( X^*_n \right)$ ($x \in \text{NEG}_{(\alpha, \beta)} \left( X^*_n \right)$) at every $t_n$ in the future.

**Proof:** From inference 1, $\Pr \left( X^*_n \left| \mathcal{R}_n \right. \right) \geq \cdots \geq \Pr \left( X^*_{n-1} \left| \mathcal{R}_n \right. \right) \geq \Pr \left( X^*_{n-2} \left| \mathcal{R}_n \right. \right) \geq \alpha$

Therefore, it should choose $x \in \text{POS}_{(\alpha, \beta)} \left( X^*_n \right)$ at every $t_n$ in the future.

### 3.3. Dynamic Three-way Concept Learning Algorithm Description

**Input:** Data set $U$, Attribute set $Q(A)$, Subset $X$, Equivalence class $D_i$, Moving in subsets $n$, Moving out subsets $m$, threshold $(\alpha, \beta)$.

**Output:** Three-way concept and three-way granular concept at different times.

**Step 1:** Calculate of a subset of initial moments under the conditional probability $p_i$;

**Step 2:** One-direction moving in object set $X^*$, form a new two-direction S-set $X^*_{n-1}$, count $\Delta p_s = p_i + \Delta p$;

**Step 3:** One-direction moving out object set $X^*$, form a new two-direction S-set $X^*_{m-1}$, count $\Delta p_s = p_i - \Delta p$;

**Step 4:** Two-direction moving in object set $X^*$ and moving out object set $X^*$, form a new two-direction S-set $X^*_{s}$, count $\Delta p$ and $\Delta p'$, $p_s = p_i + \Delta p - \Delta p'$;

**Step 5:** Output

**Step 6:** End

### 4. Application Examples and Results Analysis

#### 4.1. Application Examples

In the information system given in table 1, $U = \{x_1, x_2, \cdots, x_{20}\}$ is the sample set, representing 20 selectable partners and $Q(A) = \{A_1, A_2, A_3\}$ is the attribute set, representing the influential factors that determine whether or not to cooperate with it. There are three-way decision classifications under each attribute $A_i (1 \leq i \leq 3)$, which are 1, 0, null values, respectively.

**Table 1.** Information system

|        | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $A_1$  | 1     | 1     | 0     | 1     | 1     | 1     | 0     | 0     | 0     | 0     |
| $A_2$  | 0     | 1     | 1     | 1     | 1     | 1     | 0     | 0     | 0     | 0     |
| $A_3$  | 1     | 1     | 1     | 1     | 0     | 1     | 1     | 0     | 0     | 0     |

|        | $x_{11}$ | $x_{12}$ | $x_{13}$ | $x_{14}$ | $x_{15}$ | $x_{16}$ | $x_{17}$ | $x_{18}$ | $x_{19}$ | $x_{20}$ |
|--------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| $A_4$  | 1          | 1          | 0          | 1          | 0          | 0          | 1          | 1          | 1          | 1          |
| $A_2$  | 0          | 1          | 1          | 1          | 0          | 0          | 1          | 1          | 1          | 1          |

According to the data in the information system, there are three equivalent classes under $A_1, A_2, A_3$, respectively:

- $D_{11} = \{x_1, x_2, x_4, x_5, x_7, x_{11}, x_{12}, x_{16}\}$, $D_{12} = \{x_3, x_4, x_9, x_{10}, x_{14}, x_{17}\}$, $D_{13} = \{x_6, x_7, x_{15}, x_{18}, x_{19}, x_{20}\}$
- $D_{21} = \{x_1, x_2, x_5, x_6, x_{13}, x_{15}, x_{19}, x_{20}\}$, $D_{22} = \{x_3, x_4, x_{10}, x_{12}, x_{17}, x_{18}\}$, $D_{23} = \{x_7, x_8, x_{11}, x_{14}, x_{16}\}$
- $D_{31} = \{x_1, x_2, x_4, x_5, x_7, x_{12}, x_{13}, x_{14}, x_{19}, x_{20}\}$, $D_{32} = \{x_3, x_{10}, x_{11}, x_{16}, x_{17}\}$, $D_{33} = \{x_6, x_8, x_{15}, x_{18}\}$
Set the initial moment \( t_0 \) selects a subset to learn as \( X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\} \). According to the probabilistic function expression (3), the conditional probability in different equivalence classes under each attribute is calculated as follows:

\[
A_1: p_{11} = P(X | A_1) = \frac{5}{8}, p_{12} = P(X | A_2) = \frac{2}{6}, p_{13} = P(X | A_3) = \frac{1}{6}
\]

\[
A_2: p_{21} = P(X | A_1) = \frac{4}{8}, p_{22} = P(X | A_2) = \frac{1}{6}, p_{23} = P(X | A_3) = \frac{3}{6}
\]

\[
A_3: p_{31} = P(X | A_1) = \frac{5}{10}, p_{32} = P(X | A_2) = \frac{1}{5}, p_{33} = P(X | A_3) = \frac{2}{5}
\]

When the probability threshold is \((0.5, 0.3)\), it is concluded that the three-way concepts at the initial time \( t_0 \) are:

\[
\begin{align*}
\{ &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \}, A_1 \\
\{ &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \}, A_2 \\
\{ &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \}, A_3
\end{align*}
\]

At the moment \( t_0 \) the three-way granular concept is:

\[
G_{str} = \{ \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}, A_1 \}, \{ \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}, A_2 \}, \{ \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}, A_3 \}
\]

At the initial moment \( t_0 \), the considered subset is:\n
\[
X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}
\]

The quantity of conditional probability change caused by moving in and moving out of the subset can be calculated according to formula (5):

\[
\Delta p_{11} = \frac{1}{8}, \Delta p_{12} = \frac{2}{6}, \Delta p_{13} = 0 ; \Delta p_{22} = 0, \Delta p_{23} = \frac{2}{6}, \Delta p_{31} = 0, \Delta p_{32} = \frac{2}{5}, \Delta p_{33} = \frac{1}{5}
\]

Then the conditional probability of \( t_1 \) time can be calculated according to formulas (4):

\[
p_{11} = p_{11} + \Delta p_{11} = \frac{4}{8}, p_{12} = \frac{3}{6}, p_{13} = \frac{1}{6}, p_{21} = \frac{3}{8}, p_{22} = \frac{2}{6}, p_{23} = \frac{3}{5}, p_{31} = \frac{3}{10}, p_{32} = \frac{3}{5}, p_{33} = \frac{2}{5}
\]

The re-acquisition of the three-way concepts of the \( t_1 \) moment is:

\[
\begin{align*}
\{ &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \}, A_1 \\
\{ &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \}, A_2 \\
\{ &x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \}, A_3
\end{align*}
\]

At the initial moment \( t_0 \), the considered subset is:

\[
X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}
\]

4.2. Interpretation of Result

At the initial moment \( t_0 \), the considered subset is:

\[
X = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}
\]
When the intension is $A_1$, the obtained concept of three-way granular is $\{\{x_1, x_2, x_4, x_5, x_7\}, 1\}$, indicating that it can be taken to accept cooperation with sample $x_1, x_2, x_4, x_5, x_7$, and refuse to cooperate with $x_6$. Take the delay to cooperate with the sample $x_1, x_6$ that belongs to the subset $X$. The same can be analyzed when the intension is $A_2, A_3$.

When considering two or three attributes at the same time, for example, when the intension is $\{A_1, A_2\}$, the obtained concept of three-way granular is $\{\{x_1, x_3, x_5, x_7\}, \emptyset, \{A_1, A_2\}\}$, indicating that it can be taken to accept cooperation with sample $x_1, x_3, x_5, x_7$, and delay cooperation for sample $x_2, x_3, x_6, x_8$ that belongs to the subset $X$. Compared with considering the attributes $A_i$ and $A_j$ separately, considering that the information contained in $\{A_1, A_2\}$ is more abundant and the three-way concepts obtained are more accurate. This is from the attribute multi-granularity angle, obtains the cooperation result under the different granularity according to the three-way granular concept.

Elemental migration occurs at the $t_i$ moment. At this point, when the intension is $A_1$, because of the effect of moving in and moving out of the subset, the three newly moved samples $x_9, x_{10}, x_{11},$ take the acceptance cooperation. The original sample $x_8$, which takes the delay cooperation, becomes the acceptance cooperation at this time, and the remaining samples remain unchanged. The change of whether the outgoing sample cooperation or not is not considered. When the intension is $A_2, A_3$, or consider two or three attributes at the same time, the analysis is the same and is not repeated.

The data processed by the previous three-way concept are static invariant, regardless of the move in and move out of elements in the set. However, the data with dynamic changes is more common. The experimental results show that the proposed algorithm can effectively deal with the data of dynamic change and reduce the calculation of conditional probability, which is a simple three-way concept division method.

5. Conclusions
The dynamic three-way concept learning proposed in this paper, according to the relevant definition of probability function, the classification rules of dynamic change data are generated, and the three-way concepts of different moments are obtained, and the corresponding three-way granular concepts are formed under different attribute characteristics, so that the three-way concepts of dynamic change subset can be learned. It provides a method to solve the data system of dynamic change in life. The next step will focus on the processing of uncertain objects in the learning of three-way concepts, the selection of probability thresholds and the specific application methods in different fields.

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