Robust optical-levitation-based metrology of nanoparticle’s position and mass

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(Dated: February 7, 2020)

Light has shown up an incredible capability in precision measurement based on opto-mechanics interaction in high vacuum by isolating environment noises. However, there are still obstructions, such as displacement and mass estimation error, highly hampering the improvement of absolute accuracy at the nanoscale. Here, we present a nonlinearity based metrology to precisely measure the position and mass of a nanoparticle with optical levitation under 10⁻⁵ mbar, 6-order of magnitude lower than the electrostatic-force and stochastic-force-based counterparts. By precisely controlling the amplitude of the levitated nanoparticle at the nonlinear regime, we realized a feasible sub-picometer-level position measurement with an uncertainty of 1.0% without the prior information of mass, which can be further applied to weigh the femtogram-level mass with an uncertainty of 2.2%.

It will also pave the way to construct a well-calibrated opto-mechanic platform in high vacuum for high sensitivity and accuracy measurement in force and acceleration at the nanoscale and the study in quantum superposition at the mesoscopic scale.

Light has been the most powerful tool for precision metrologies in time, frequency, and distance [1–3]. Based on opto-mechanical interaction in high vacuum, the gravitational wave has also been successfully detected [4]. Recently, the compact optical levitation in vacuum, which joins the fields of optomechanics [5] and optical trapping [6–8], is being put into the spotlight of high precision metrology for modern science, especially at the nanoscale. It extends the optical precision metrology in force [9–12], mass [13, 14], charge [15] and acceleration [16]. Such a system has also been considered as a promising platform for the investigation of quantum superposition at the mesoscopic scale [17–19], which may further enhance the performance of precision metrology due to the nature of quantum superposition and entanglement [20].

It had been regarded that the optical levitation has a high precision in position detection with state-of-the-art techniques, which required a priori knowledge of the particle mass [21] or the assistance of stochastic [21–23] or extra electrostatic [14, 21] forces at a moderate vacuum to calibrate the experimental output. However, in the experiment, the mass of the nanoparticle and stochastic and electrostatic forces can be hardly estimated or measured precisely at the nanoscale [24–26], which severely reduces the accuracy of the position and other metrologies with optical levitation system.

Here, we present an all-optical metrology for position and mass measurement with optical levitation in high vacuum. The geometry shape of the optical potential is regarded as a calibration gauge. The deviation from quadratic shape potential will induce a nonlinear natural frequency shift [27], which acts as the ruler mark to precisely read out the position and the motion of the nanoparticle without a priori knowledge of mass. Furthermore, we are able to weigh the mass and measure the density of the optically levitated nanoparticle. This high accuracy position and mass measurement can help to construct a well-calibrated opto-mechanic platform for high sensitivity and accuracy measurement in force and acceleration.

An optically levitated sensor realizes its functions by analyzing the motions of the trapped particle. The optical potential is harmonic and the oscillation has a fixed natural frequency \( \Omega_0 \) (shown as the purple dashed potential shape in Fig. 1) when the particle is trapped very near the equilibrium point. However, in an optical trap built by a laser beam in Gaussian mode [2, 27], the optical potential will become anharmonic when the oscillator has a large amplitude and can move far from the equilibrium point (shown as the red solid potential shape in Fig. 1). This anharmonic potential features a Duffing nonlinearity [27]. Ignoring the interaction between different spatial degrees of freedom, the motion of the trapped particle with mass \( m \) in this nonlinear regime can then be described as

\[
\ddot{q} + \Gamma_0 \dot{q} + \Omega_0^2 \left( q + \xi q^3 \right) = F_{\text{therm}}/m, \quad (1)
\]

in one degree of freedom, where \( q(t) \) is the position of the trapped particle. \( \Gamma_0 \) is the damping rate induced by air. \( \Omega_0 \) is the natural angular frequency when the oscillation is in the linear regime. \( F_{\text{therm}} \) is the Brownian stochastic force. \( \xi \) describes the nonlinear coefficient of the trap. For a Gaussian distribution optical potential, \( \xi = -2/w^2 \), where \( w \) is the 1/e² beam intensity radius. The position signal \( q(t) = c \cdot V(t) \) is expected to be experimentally

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measured by a photodetector output voltage $V(t)$ and a calibration constant $c$.

It is noticed that we can get the calibration constant $c$ with only the voltage signal $V(t)$. Due to the nonlinearity, the oscillator has got a natural frequency shift $\Delta \Omega / \Omega_0$, which can be obtained from the spectrum analysis of the voltage signal of detector $V(t)$, is related to the actual oscillation amplitude $q_A$ as $q_A = w \sqrt{-4 \Delta \Omega / (3 \Omega_0)}$. With the respective measured voltage signal amplitude $V_A$, the calibrated factor $c = q_A / V_A$ is got with only the information about $V(t)$. Then the position signal can be obtained as $q(t) = c \cdot V(t)$.

![FIG. 1. Scheme of position measurement without the information of particle mass. The measured nonlinearity induced relative natural frequency shift $\Delta \Omega / \Omega_0$, which can be obtained from the spectrum analysis of the voltage signal of detector $V(t)$, is related to the actual oscillation amplitude $q_A$ as $q_A = w \sqrt{-4 \Delta \Omega / (3 \Omega_0)}$. With the respective measured voltage signal amplitude $V_A$, the calibrated factor $c = q_A / V_A$ is got with only the information about $V(t)$. Then the position signal can be obtained as $q(t) = c \cdot V(t)$.

double-frequency parametric driving [27, 29, 30], and optical tweezer phonon laser [31], the imprecise amplitude control severely limits the performance and reliability of such methods for calibration.

Here, we introduce a parametric feedback amplitude locking modulation to precisely control the oscillation amplitude, which corresponds to the center of mass (c.m.) motion temperature. As shown in Fig. 2(a), when the amplitude is higher or lower than a target amplitude, a parametric feedback cooling [32, 33] or heating [1, 33] process is applied to drive the amplitude to the target. In detail, assuming the short term time trace of the oscillation’s voltage signal shown in Fig. 2(b) can be described as $u_{\sin}(t) = V_A \sin(\Omega_0 t + \varphi)$, where $\varphi$ is the phase of the oscillation. And a $\pi/2$-phase-shifted signal is $u_{\cos}(t) = V_A \cos(\Omega_0 t + \varphi)$. The applied trapping laser intensity for feedback amplitude locking modulation is

$$I(t) = I_0 \left[1 + A(t) \times 0.5 \eta_m \times \text{sign}(u_{\sin}(t) u_{\cos}(t))\right],$$

where $I_0$ is the laser intensity without modulation and $\eta_m$ is the modulation depth [1]. $A(t) = \text{sign}(V_A^2 - V_{\text{target}}^2)$ is the amplitude criterion. When $V_A$ is larger (smaller) than the target voltage amplitude $V_{\text{target}}$, $A$ is positive (negative) and Eq. (3) will be a feedback cooling (heating) modulation to decrease (increase) the amplitude of

![FIG. 2. (a) Schematics of the parametric feedback locking. When the measured amplitude ($V_A$) of the trapped particle is larger (smaller) than the target amplitude ($V_{\text{target}}$), modulations will be applied with feedback cooling (heating) to reduce (increase) the amplitude. The solid curves represent the nonlinear Gaussian potential which deviate from harmonic potential (dashed curves). (b) Part of the recorded signal time trace from an amplitude locked particle at a pressure of $10^{-5}$ mbar. (c) Simplified experimental setup. A 1064 nm laser was focused by an objective which was mounted in vacuum chamber to form the optical potential for particle trapping. The forward scattering light from optical trap was collected and sent to c.m. motion detection unit. The detector signal was processed to generate the feedback amplitude locking by modulating the trapping laser intensity with an acousto-optical modulator (AOM).]
The levitated oscillator [1].

In the experiment, as shown in Fig. 2(c), an objective (NA = 0.9) was mounted in the vacuum chamber to create an optical potential by focusing the Gaussian trapping laser. To avoid the interaction between motional degrees of freedom when more than two axes of oscillation reach the nonlinear regime [27, 29], only one axis motion will be amplitude-locked at a time, and the motions of the other two axes will be cooled to sub-Kelvin [1]. The modulation depth $\eta_m$ was set to be 0.5% during the experiment.

Figure 3(a) shows the measured signal voltage time traces of a trapped particle under different locked amplitudes and feedback cooling along the Y axis at a pressure of $10^{-5}$ mbar. The relative standard deviation of the voltage amplitude was lower than 0.5%, as shown in Fig. 3(b). And the power spectral densities (PSD) with different locked amplitudes confirmed that the natural frequency of the oscillator in an anharmonic optical potential decreased with increasing amplitude, as shown in Fig. 3(c).

Based on the feedback amplitude locking technique at the nonlinear regime, the oscillator frequency was measured while the amplitude locking target was swept from 0.4 V to 1.6 V. And the measurement results were averaged from tens of sweep cycles to eliminate the noise and low-frequency drift due to the system instability.

This measurement procedure was applied to both X and Y axes to verify the reliability of the calibration process. The dependence of relative frequency shift on voltage amplitude is shown in Fig. 3(d). By fitting it with Eq. (2), we can get that the nonlinear coefficients of X-axis and Y-axis are $\alpha_X = -0.002978 \text{ V}^{-2}$ and $\alpha_Y = -0.002669 \text{ V}^{-2}$, respectively.

To complete the high precision calibration, the intensity distribution in the $X-Y$ plane was simulated with Debye integral [2, 27], as shown in Figs. 3(e) and (f). By fitting the intensity along $X$ and $Y$ axes with Gaussian function, we get the $1/e^2$ intensity radius $w_X = 703 \pm 7 \text{ nm}$ and $w_Y = 551 \pm 22 \text{ nm}$, respectively. The errors come from the uncertainty of optical component parameters (see Supplementary Information for details [36]). With $\alpha_X$, $\alpha_Y$, $w_X$, and $w_Y$, we finally extracted the calibration constants of $c_X = 44.3 \pm 0.5 \text{ nm/V}$ along X axis and $c_Y = 32.9 \pm 1.3 \text{ nm/V}$ along Y axis. In Table 1, we compare the uncertainties of calibration constants with different calibration methods. It indicates that the uncertainty of position measurement was about 1.0% at the sensitivity level of 0.44 pm/√Hz (X-axis), which was much lower than that from other methods [14, 21]. Also, the operation pressure is 6 orders of magnitude lower, which can be applied to enhance the sensitivity due to high environment noises isolation.

Since we can get the calibrated trajectory of the
trapped particle with high accuracy, it is possible to extract more information about the levitated oscillator from its thermal motion such as the calibration constants of other axes, mass and density of the trapped nanoparticle. For a nano-mechanical resonator sensor, the force measurement uncertainty highly depends on the accuracy of the mass of the trapped nanoparticle. Based on the calibration constant, we can obtain the mass of the nanoparticle from the mean square displacement along one axis in a thermal equilibrium, which follows

\[ m = \frac{k_B T}{\langle q_X^2 \rangle / \Omega_{X0}}, \]

where \( k_B \) is the Boltzmann constant and \( T \) is the temperature of surrounding environment.

In the experiment, the thermal motion properties, such as mean square signal voltage \( \langle q_X^2 \rangle = \langle X^2 \rangle / c_X^2 \), natural angular frequency \( \Omega_{X0} \) and damping coefficient \( \Gamma_0 \) along X, Y, and Z axes, were measured at a pressure between 20 to 50 mbar by fitting the PSD. The pressure range we selected is to avoid the measurement errors due to the nonlinear distortion of PSD, effective bath temperature elevating under high vacuum and fitting error near overdamped regime. Before measurement, the vacuum chamber was exhausted below 0.1 mbar for a while and re-inflated to minimize the influence of particle properties varying at low gas pressure like the evaporation of adsorbed solvent. From the experiment measurement, we got the mass of the trapped nanoparticle was \( m = (3.63 \pm 0.08) \times 10^{-18} \) kg with Eq. (6) [36].

Moreover, the density of nanoparticle is an important parameter for its industrial and technological applications. Also the density of a nanoparticle is related to its refractive index [37] which is one of the key parameters for optical levitation properties. However, it is difficult to measure the density of an individual nanoparticle. And for the most popular Stöber silica nanospheres, its density can be smaller than a bulk glass due to its porosity [13, 38]. As for the density measurement, when the Knudsen number \( K \gg 1 \), where \( K = L / R \), \( L \) is the mean free path of air molecules, and \( R \) is the radius of the nanosphere, the air damping coefficient of a nanosphere follows [9]

\[ \Gamma_0 = \frac{8}{3m} \left( 1 + \frac{a \pi}{8} \right) \sqrt{\frac{2\pi m g}{k_B T}} p R^2, \]

where \( p \) is the air pressure, \( m_g \) is the mass of the air molecule, and \( a \) is the momentum accommodation coefficient. With the measured damping coefficients in Fig. 4 and Eq. (5), we can get that the radius of the trapped nanosphere was \( R = 75.4 \pm 1.5 \) nm and the density was \( \rho = 2.02 \pm 0.10 \) g/cm\(^3\) [36]. This result agreed with the value provided by the manufacturer (\( \rho_{\text{Bungs}} \approx 2.0 \) g/cm\(^3\)). As for a non-spherical nanoparticle, the dependence of the damping rate on the pressure can be obtained by the direct simulation with Monte Carlo method [40].

In conclusion, we have introduced a nonlinear fre-

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**TABLE I. Uncertainties of calibration constants with different calibration methods.**

| Calibration criterion                  | Primary uncertainty source | Operational pressure | Relative uncertainty |
|----------------------------------------|-----------------------------|----------------------|----------------------|
| Thermal stochastic force \[21\]       | Mass                        | 10 mbar to 1 atm     | 15%                  |
| Electrostatic force \[21\]            | Mass                        | < 10 mbar            | 30%                  |
| Stochastic and electrostatic force \[14\] | Electric-field strength     | ~ 50 mbar            | 1.2%<sup>a</sup>     |
| Potential nonlinearity [This work]    | Potential geometry shape    | 10<sup>−5</sup> mbar<sup>b</sup> | 1.0%                |

<sup>a</sup> The relative uncertainty is derived from its mass measurement uncertainty.

<sup>b</sup> The feedback amplitude locking is operational when \( \delta \Gamma \gg \Gamma_0 \), where \( \delta \Gamma \) is the feedback introduced damping following \( \delta \Gamma = \eta m \Omega_0 / (2\pi) \) [1] and \( \Gamma_0 \) is the air damping. It can be used in a pressure < 10<sup>−3</sup> mbar.

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**FIG. 4.** (a) Measured mass under different air pressures. The error bars came from the systematic errors. The dashed line is a linear fitting. The error bars came from the variance of \( \Gamma_0 \) between different axes.
frequency shift based measurement of the position, mass and density of a nanoparticle with optical levitation in high vacuum. We are able to control the amplitude of an optically levitated oscillator with tiny deviation and make it possible to deploy a precise nonlinear frequency shift measurement. Such a method does not require the mass of nanoparticle or an assistance from an external force. The absolute precision is then mainly limited by the error from the tight focusing light field estimation, which can be further measured with high precision in experiment [3–5]. Moreover, an amplitude locked nanoscillator can be regarded as a nearly ideal harmonic oscillator for classical and quantum investigation. It is possible to transform a position or velocity depended static interaction, such as Casimir force, electric field gradient, and Lorentz force, into a harmonic force, since an optically levitated nanoparticle sensor has an incredible sensitivity for resonant force measurement. Such an amplitude locked optically levitated nanoparticle can also be applied in the study of nonequilibrium physics and thermodynamics at the nanoscale [35, 44].

METHODS

Experimental setup. An objective (NA = 0.9) was mounted in the vacuum chamber to focus the 1064 nm Gaussian beam laser (~200 mW). Before the objective, the beam diameter was 4.5 mm, which was larger than the back pupil diameter (3.6 mm) of the objective to make full use of the numerical aperture. An acousto-optic modulator (AOM) was mounted to modulate the laser intensity based on the feedback control signal. The forward scattering light was collected by an aspheric lens (NA = 0.546) and sent to three sets of balanced photodetectors to measure the positions of the trapped nanoparticle along three motional degrees (set as X, Y, Z axis) of freedom. A silica nanosphere (nominal radius 82 ± 10 nm, Bangs labs Inc.) dispersed in ethanol was sent to the optical trap with a nebulizer. The position voltage signals were recorded by a digitizer on computer and simultaneously sent to a field programmable gate array (FPGA) board (Xilinx Spartan-6 XC6SLX16 Core Board) [1] to generate the feedback modulation signal that can control the oscillation amplitude along each axis.

ACKNOWLEDGMENT

This work is supported by the Science Challenge Project (No. TZ2018003), the National Natural Science Foundation of China (Nos. 91536219, 61522508, and 91850102), the Anhui Initiative in Quantum Information Technologies (No. AHY130000).

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Supplementary Information

I. EXPERIMENTAL SETUP

As shown in Fig. 5, a 1064 nm laser was used to trap the nanoparticle in vacuum chamber. Its intensity was modulated with an acousto-optical modulator (AOM). The laser beam was expanded to fulfill the back pupil of the objective (Nikon CFI LU Plan Fluor EPI 100X). The forward scattering light from the optical trap was collected by an aspheric lens (NA= 0.546) and sent to three sets of balanced photodetectors to measure the positions of the trapped nanoparticle along three motional degrees (set as X, Y, Z axis) of freedom.

The position voltage signals were sent to a FPGA board to generate the feedback control signal. The digitized signal first passed a Kalman filter to suppress the out-band noise. And then, to eliminate the influence of feedback-loop time delay, two suitable delays were added to the voltage signal to generate a phase-matched signal \( u_{\text{sin}} \) and a \( \pi/2 \) phase-shifted signal \( u_{\text{cos}} \). \( u_{\text{sin}} \) and \( u_{\text{cos}} \) were sent to generate the feedback cooling signal with \( S_{\text{cool}} = \text{sign}(u_{\text{sin}},u_{\text{cos}}) \) [1]. Simultaneously, the square of signal voltage amplitude \( V_{A}^{2} \) was calculated with \( V_{A}^{2} = u_{\text{sin}}^{2} + u_{\text{cos}}^{2} \). The amplitude was processed in square form because it is not convenient to do square root with FPGA. Then \( V_{A}^{2} \) was compared with the target amplitude \( V_{\text{target}}^{2} \). If \( V_{A}^{2} \geq V_{\text{target}}^{2} \), the cooling signal \( S_{\text{cool}} \) would not be changed. Else if \( V_{A}^{2} < V_{\text{target}}^{2} \), \( S_{\text{cool}} \) would be reversed and become a feedback heating signal. The feedback control signals along three axes were merged based on majority rule [1]. Finally, based on the modulation depth \( \eta_{m} \), the merged three-dimensional control signal was processed and converted to an analog voltage signal to control the laser intensity with AOM.

During the calibration, to obtain the nonlinear coefficient \( \alpha \), the square of voltage amplitude was swept between 0.16 \( V^{2} \) to 2.56 \( V^{2} \). This voltage sweeping was applied to X and Y axis motions as shown in Fig. 6. The frequency shift versus amplitude voltage was averaged from tens of sweeping cycles to reduce noises. Such calibration was not applied to Z-axis because the non-linear property along Z-axis was different from X and Y axes due to the nonlinearity of the scattering force and tremendously affected by changes in equilibrium position. The equilibrium position was related to the radius and refractive index of the silica nanoparticle which were unable to obtain accurately.

II. LIGHT FIELD ESTIMATION

The Debye integral [2] was utilized to make the simulation of the tightly focused light field around trapping position. The simulation condition we used was that the input laser beam has a Gaussian intensity distribution with a beam diameter of 4.5 mm which was measured with a CCD camera beam profiler (newport LBP2). The back pupil diameter of the objective was 3.6 mm. The laser wavelength was 1064 nm. And the laser was linearly polarized along X axis. the numerical aperture (NA) of the objective was 0.9. The equilibrium position along Z-axis was \( z_{\text{eq}} \approx 100 \) nm. We fitted the intensity distribution at \( z_{\text{eq}} = 100 \) nm with Gaussian function along X and Y axes and with \( I(z) = A/[1 + (z + z_{\text{0}})^{2}] \) along Z axis. The fitted 1/\( e^{2} \) intensity radius along X and Y axes were \( w_{X} = 696.6 \) nm and \( w_{Y} = 530.0 \) nm, and the Rayleigh range was \( z_{\text{0}} = 1080.0 \) nm.

According to the optical force with Rayleigh approximation and the light field estimation result, we were able to calculate the natural frequency ratio between each axis. Based on the above light field estimation results, the natural frequency ratio between X, Y, and Z should be 1.31 : 1 : 2.93, comparing with the experimental result 1.27 : 1 : 3.07.

There were two possible reasons for the differences between the estimation and experimental results. First, the objective we used was designed for visible light. It introduced a slight deterioration of the focal length when it was used for near-infrared light. Second, the actual intensity distribution of the input laser was deviated from a perfect Gaussian distribution and there might be a slight misalign between the input laser and backpupil of the objective. It would introduce an error of the effective input laser diameter. The above reasons introduced simulation condition errors about the NA and the fill factor (input laser diameter/back pupil diameter). By tuning the simulation conditions, when the NA=0.875 and the input laser diameter was 4.2 mm, the simulation result of the natural frequency ratio will be as same as the experimental result. Therefore, the simulated 1/\( e^{2} \) intensity radius along X and Y axes were estimated to \( w_{X} = 703 \pm 7 \) nm and \( w_{Y} = 551 \pm 22 \) nm. The errors demonstrated the difference between the experiment condition simulation and natural frequency ratio corrected simulation. Those errors can be further reduced in experiment [3-5].

III. ERROR ANALYSIS OF THE MASS, RADIUS, AND DENSITY MEASUREMENT

To measure the mass, radius, and density of the trapped nanosphere, a total of 12 data points at a pressure between 20 to 50 mbar are recorded. The recorded information of each data point was derived by fitting the power spectral density (PSD) of position signals, including the mean square signal voltage, natural frequency, and damping coefficient along X, Y, and Z axes.

Because the calibration error along X axis was less than Y axis, the mean square signal voltage \((V_{X}^{2})\) and natural angular frequency \((\Omega_{X0})\) along X axis was utilized to obtain the mass. Here,

\[
m = \frac{k_{B}T}{c_{X} \langle V_{X}^{2} \rangle \Omega_{X0}^{2}},
\]
FIG. 5. Experimental setup

FIG. 6. Part of the measured time trace of the swept voltage amplitude and natural frequency. (a) Sweep along X-axis. (b) Sweep along Y-axis.

where $k_B$ is the Boltzmann constant, $T$ is the environment temperature, and $c_X$ is the calibration factor along X. Based on the uncertainties shown in Table. II and the propagation of uncertainty, we can get that

$m = (3.63 \pm 0.08) \times 10^{-18} \text{ kg}$.

As for the radius, the most often used formula to estimate the damping coefficient in optically levitation is [6, 7]

$$\Gamma_0 = \frac{6\pi \eta R}{m} \frac{0.619}{0.619 + Kn} (1 + c_K),$$

where $R$ is the radius of the nanosphere, $c_K = 0.31Kn / (0.785 + 1.152Kn + Kn^2)$, $\eta$ is the viscosity coefficient of air, and $Kn = l/R$ is the Knudsen number. $l = k_BT/\sqrt{2\pi d^2p}$ is the mean free path of air molecules, where $p$ is the air pressure and $d$ is the collision diameter of air molecules. And the correctness of Eq. (7) is under the condition that the momentum accommodation coefficient $a$ of the target sphere is equal to 1. Some experiments have recommended that $a \sim 0.9$ [8].

So we turn to the Epstein’s work that the damping coefficient in the free-molecule regime ($Kn \gg 1$) is [9]

$$\Gamma_0 = \frac{8}{3m} \left(1 + \frac{\pi a}{8}\right) \sqrt{\frac{2\pi m_g}{k_BT} p R^2},$$

where $m_g$ is the mass of the air molecule. We can obtain
TABLE II. Uncertainties table.

| Quantity        | Value $z_i$   | Error $\delta z_i/z_i$ |
|-----------------|---------------|-------------------------|
| $c_X$           | 44.3 nmV⁻¹   | 0.010                   |
| $\Omega X₀/(2\pi)$ | 135450 Hz   | 0.001(stat)              |
| $\langle V_X^2 \rangle$ | 0.7980 V²    | 0.005(stat)              |
| $\Gamma_0$      | (0.942 ± 2.42) × 10⁵ s⁻¹ | 0.010⁵                  |
| $\langle P \rangle$ | (1.92 ± 4.99) × 10³ Pa | 0.002⁵                  |
| $a$             | 0.9           | 0.111                   |
| $T$             | 298 K         | 0.003(b)                |
| $m_p$           | 4.8089 × 10⁻²⁶ kg | c                      |
| $k_B$           | 1.3806 × 10⁻²³ JK⁻¹ | c                      |
| $m$             | 3.63 × 10⁻¹⁸ kg | 0.022(sys), 0.004(stat) |
| $R$             | 75.4 × 10⁻⁹ m  | 0.020(sys), 0.004(stat) |
| $\rho$          | 2.02 g/cm³    | 0.052(sys), 0.013(stat) |

*a* This error comes from the inconsistent damping coefficients between different axes due to the deviation from perfect sphere.

*b* From manufacturer datasheets.

*c* The error is less than 0.001.

the radius of the nanosphere with

$$ R = \left( \frac{\Gamma_0}{p} \right)^{\frac{1}{3}} \left( \frac{8 + \pi a}{3m} \right)^{-\frac{1}{2}} \left( \frac{2\pi m_p}{k_B T} \right)^{-\frac{1}{4}}. \tag{9} $$

Based on the uncertainties shown in Table. II and the propagation of uncertainty, the radius of the trapped nanosphere was estimated to be $R = 75.4 ± 1.5$ nm.

As for the density, with Eq. (9), we have

$$ \rho = \frac{3}{4\pi} m^{-\frac{1}{2}} \left( \frac{\Gamma_0}{p} \right)^{-\frac{1}{2}} \left( \frac{8 + \pi a}{3} \right)^{\frac{1}{2}} \left( \frac{2\pi m_p}{k_B T} \right)^{\frac{1}{4}}. \tag{10} $$

We can get the density of the trapped nanoparticle was $\rho = 2.02 ± 0.10$ g/cm³.

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