An Eigenvalues Method for Stability Analysis of Power Grid with Energy Storage Devices Based on Discrete Time Domain Model

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Abstract. The eigenvalue analysis based on state space model is an important method to study stability of power grid. With the large-scale access of energy storage devices and power electronics converters in power grid, the conventional state space modelling method suffers from complicated process of determining state variables and large calculation amount of eigenvalues, and nonlinear stability characteristics of power grid cannot be accurately reflected. To obtain the eigenvalues of power grid with energy storage devices more efficiently, this paper proposes a method for constructing state space model of power grid based on discrete time domain model (DTDM) and presents the corresponding stability criterion. Compared with the conventional state space modelling method, the nonlinear characteristics of components under proposed method can be linearized, and the constructed state space model can capture a wider range of system information including energy storage devices. Besides, the complete state space model of power grid can be obtained only according to the connection between component's DTDM and grid. Moreover, because eigenvalue solution under proposed method relies on the sparse grid node voltage matrix, the calculation of eigenvalues is simple and efficient, and it visually presents variation trend of eigenvalue and investigates the stability of system. Simulation results in Simulink/Matlab verify the correctness and effectiveness of proposed method.

1. Introduction
With the rapid development of distributed renewable power sources and power electronics technology, the stability analysis and control issue become more and more important for modern power grid with the large-scale access of renewable energy sources, energy storage devices and power electronics converters [1-2]. To study the stability of power grid, the eigenvalue analysis method based on state space model is a commonly used and effective method [3].

However, with the large-scale access of renewable energy sources, energy storage devices and power electronics converters in modern power grid, the conventional state space modelling and analysis method based on continuous time domain faces problems such as difficulty in selecting state variables, complicated state space optimization and inefficient eigenvalue solution. For the power grids with large number of switches, power electronics converters, energy storage devices, diverse loads and varied operating modes, the state space modeling based on continuous time domain needs to be linearization.
artificially, leading to the truncation errors in modeling process and reduced accuracy of analysis [4-5]. In addition, with the expanded scale of power grid, the dimensionality of state space method based on continuous time domain is increased, and the efficiency and effectiveness of eigenvalue analysis solution are greatly reduced, further reducing its applicability in stability analysis. Besides, although the electromagnetic transient simulation based on numerical integration simulates the operation of complicated power grid, electromagnetic transient simulation requires a large number of computing resources. In addition, electromagnetic transient simulation cannot provide parameter indicators and information to improve system performance, and it can only be verified after parameter changes by enumeration, seriously increasing the workload of engineers [6-7].

Although the electromagnetic simulation cannot directly solve stability analysis and improvement for power grid, it is still an important tool that provides a research idea for the state space modelling and eigenvalue analysis in power grid with energy storage devices. By using discrete time domain to solve nonlinear problem of power grid and construct state space of system in each unit time domain, its eigenvalues can judge the stability of system in each unit time domain. In this way, the trend of stability of power grid is obtained, and the overall stability of system can be studied [8-10].

Based on this framework, this paper proposed a method for constructing state space model of power grid with energy storage devices based on discrete time domain model (DTDM), and the corresponding eigenvalue solution method is also discussed. Based on trapezoidal numerical integration method, the relationship between discrete time domain and continuous time domain state space is analyzed, and the relationship between corresponding eigenvalues is derived. The discrete time domain models of resistance, inductance, capacitance, and energy storage devices are analyzed, and a normalized state space construction method based on DTDM is proposed. The simulation of a power grid with energy storage devices verifies the correctness and effectiveness of proposed method.

2. The Stability Criterion of State Space Equation in Discrete Time Domain

Assuming the state space equations of power grid in continuous time domain and discrete time domain are:

\[
\begin{aligned}
\dot{x}(t) &= [A]^c x(t) + [B]^c u(t) \\
y(t) &= [C]^c x(t) + [D]^c u(t)
\end{aligned}
\]

\[
\begin{aligned}
x(t + \Delta t) &= [A]^D x(t) + [B]^D u(t) \\
y(t + \Delta t) &= [C]^D x(t + \Delta t) + [D]^D u(t + \Delta t)
\end{aligned}
\]

where \([A]^c - [D]^c\) is the coefficient matrix in continuous time domain and \([A]^D - [D]^D\) is the coefficient matrix in discrete time domain.

From (1), the following equation can be obtained:

\[
\begin{aligned}
\dot{x}(t + \Delta t) &= [A]^c x(t + \Delta t) + [B]^c u(t + \Delta t) \\
\dot{x}(t) &= [A]^D x(t) + [B]^D u(t)
\end{aligned}
\]

(2)

Under trapezoidal numerical integration, the relationship between state variable value \(x(t+\Delta t)\) and \(x(t)\) in discrete time domain is:

\[
\begin{aligned}
x(t + \Delta t) &= x(t) + \frac{\Delta t}{2} [\dot{x}(t + \Delta t) + \dot{x}(t)]
\end{aligned}
\]

(3)

\[
\begin{aligned}
x(t + \Delta t) &= \frac{\Delta t}{2} \dot{x}(t + \Delta t) + \frac{\Delta t}{2} \dot{x}(t) + x(t)
\end{aligned}
\]

(4)

Substituting (2) into (4), the following equations can be obtained:
\[
\begin{align*}
\begin{cases}
 x(t + \Delta t) = \frac{KT + [A]^C}{KT - [A]^C} x(t) + \frac{[B]^C}{KT - [A]^C} [u(t + \Delta t) + u(t)] \\
 K = \frac{2}{\Delta t}
\end{cases}
\end{align*}
\]

where \( I \) is the identity matrix; \( K \) is the coefficient related to \( \Delta t \), when \( \Delta t \) is constant, \( K \) is a constant.

Assuming that the external input variable does not change, the (5) can be rewrite as:

\[
x(t + \Delta t) = \frac{KT + [A]^C}{KT - [A]^C} x(t) + \frac{2[B]^C}{KT - [A]^C} u(t)
\]

From the state space equation in discrete time domain in (1) and (6), the following equations can be obtained:

\[
\begin{align*}
[A]^D &= \frac{KT + [A]^C}{KT - [A]^C} \\
[B]^D &= \frac{2[B]^C}{KT - [A]^C}
\end{align*}
\]

Besides, since the output variable and the input variable are at the same time, the following equation can be obtained:

\[
\begin{align*}
[C]^D &= [D]^C \\
[D]^D &= [D]^C
\end{align*}
\]

From the state space coefficient matrix in discrete time domain and continuous time domain in (7) and (8), according to relationship between eigenvalues and coefficient matrix, the following equation can be obtained:

\[
\lambda_i^D = K \frac{1 + \lambda_i^C}{1 - \lambda_i^C}
\]

where \( \lambda_i^D \) is the eigenvalue in discrete time domain, and \( \lambda_i^C \) is the corresponding eigenvalue in continuous time domain. Therefore, the eigenvalues in discrete time domain and continuous time domain are corresponding.

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Fig. 1. The DTDM of inductor, capacitor, resistor, and energy storage devices.
According to (9) and the condition of Lyapunov stability in continuous time domain, the system stability criterion in discrete time domain is: in the coordinate reference system with the real part as abscissa and imaginary part as ordinate, draw the position of all the eigenvalues divided by $K$. When all points are within the range of unit circle, the system is asymptotically stable; when at least one point is outside the range of unit circle, the system is unstable; when at least one point is on the unit circle and the rest points are in the unit circle, the stability of system needs to be further analyzed. Besides, the closer the point representing the characteristic value is to the unit circle, the lower the stability margin of system obtains.

3. The Proposed Method of Normalized State Space in Discrete Time Domain

3.1. State space equations of inductance, capacitance, resistance, and energy storage in discrete time domain

Based on trapezoidal numerical integration, the discrete time domain models of inductance, capacitance, resistance, and energy storage device are shown in Fig. 1. In the figure, $\Delta t$ is the discrete time domain unit time period, $R_L(t)$, $R_C(t)$ and $R_R(t)$ are equivalent resistance of inductor, capacitor, and resistor in discrete time domain, respectively, and the energy storage device and its power converter can be equivalent to an ideal voltage source with a resistor in series:

$$
\begin{align*}
R_L(t) &= \frac{2L(t)}{\Delta t} \\
R_C(t) &= \frac{\Delta t}{2C} \\
R_R(t) &= R(t)
\end{align*}
$$

(10)

Besides, the $h_{is_L}(t)$, $h_{is_C}(t)$ and $h_{is_R}(t)$ are the equivalent current sources in discrete time domain at time $t$:

$$
\begin{align*}
\text{his}_L(t) &= -i_L(t-\Delta t) - \frac{\Delta t}{2L(t-\Delta t)}[v_1(t-\Delta t) - v_2(t-\Delta t)] \\
\text{his}_C(t) &= \frac{2C(t-\Delta t)}{\Delta t}[v_1(t-\Delta t) - v_2(t-\Delta t)] + i_C(t-\Delta t) \\
\text{his}_R(t) &= 0
\end{align*}
$$

(11)

From (11), the equivalent current sources of inductance, capacitance, and resistance at time $(t+\Delta t)$ are:

$$
\begin{align*}
\text{his}_L(t+\Delta t) &= -i_L(t) - \frac{\Delta t}{2L(t)}[v_1(t) - v_2(t)] \\
\text{his}_C(t+\Delta t) &= \frac{2C(t)}{\Delta t}[v_1(t) - v_2(t)] + i_C(t) \\
\text{his}_R(t+\Delta t) &= 0
\end{align*}
$$

(12)

From Fig. 1, the following equation can be obtained:

$$
\begin{align*}
i_L(t) &= \frac{\Delta t}{2L(t)}[v_1(t) - v_2(t)] - \text{his}_L(t) \\
i_C(t) &= \frac{2C(t)}{\Delta t}[v_1(t) - v_2(t)] - \text{his}_C(t) \\
i_R(t) &= \frac{1}{R(t)}[v_1(t) - v_2(t)] - \text{his}_R(t)
\end{align*}
$$

(13)

Substituting (12) into (11), the state spaces of inductance, capacitance, and resistance in discrete time domain are:
$$\begin{align*}
\text{his}_L(t + \Delta t) &= \text{his}_L(t) - 2G_L(t)[v_1(t) - v_2(t)] \\
\text{his}_C(t + \Delta t) &= -\text{his}_C(t) + 2G_C(t)[v_1(t) - v_2(t)] \\
\text{his}_R(t + \Delta t) &= 0 \times \text{his}_R(t) + 0 \times 2G_R(t)[v_1(t) - v_2(t)]
\end{align*}$$

(14)

where $G_L(t)$, $G_C(t)$ and $G_R(t)$ are the equivalent conductance values of inductance, capacitance, and resistance at time $t$ in discrete time domain, respectively:

$$\begin{align*}
G_L(t) &= \Delta t/2L(t) \\
G_C(t) &= 2C(t)/\Delta t \\
G_R(t) &= 1/R(t)
\end{align*}$$

(15)

Under the discrete time domain, when the switch device is in the on state, it is equivalent to a small value resistance (0.001 ohm or less); when the switch device is in off state, it is equivalent to a very large resistance (10^6 ohm or more). Under above mentioned equivalence, the state equation of switching device in time domain of same unit is the same as resistance. When the switching state changes, the equivalent resistance value changes in next unit time domain.

3.2. Construction method of normalized state space for power grid with energy storage in discrete time domain

For power grid, by replacing all devices with equivalent models, the DTDM and its state space equation of the key components can be used to derive the state space of the entire power grid. The process is as follows:

First of all, based on DTDM of inductance, capacitance, resistance and switches, the DTDM of power grid can be obtained. Based on network structure in discrete time domain model, the node voltage equation at $(t + \Delta t)$ is:

$$G_{t(\Delta t)} V_{t(\Delta t)} = I_{\text{his}(t+\Delta t)}$$

(16)

where $G_{t(\Delta t)}$ is the network DTDM model node admittance matrix at time $(t + \Delta t)$; $V_{t(\Delta t)}$ is the grid node voltage vector at time $(t + \Delta t)$; $I_{\text{his}(t+\Delta t)}$ is the grid node injection current vector at time $(t + \Delta t)$.

Besides, according to the calculation principle of electrical network, the relationship between the node injection current vector $I_{\text{his}(t+\Delta t)}$ and the current source of each branch is:

$$I_{\text{his}(t+\Delta t)} = L_{t(\Delta t)} I_{\text{his}(t+\Delta t)}$$

(17)

where $L_{t(\Delta t)}$ is the node-branch association matrix of the power grid DTDM model at $(t + \Delta t)$, and $I_{\text{his}(t+\Delta t)}$ is the current source vector composed of current sources in power grid DTDM model at $(t + \Delta t)$.

At the same time, the principle of obtaining elements in $L_{t(\Delta t)}$ is: if the node is directly connected to branch, and the current source is in the direction of node, the corresponding element is set to 1; if the node is directly connected to branch, and the current source is out of the node, the corresponding element is -1; if the node is not directly connected to branch, the corresponding element is 0.

From (16) and (17), the voltage of each node at $(t + \Delta t)$ in the DTDM of power grid can be expressed as:

$$V_{t(\Delta t)} = G^{-1}_{t(\Delta t)} L_{t(\Delta t)} I_{\text{his}(t+\Delta t)}$$

(18)

where $G^{-1}_{t(\Delta t)}$ is inverse matrix of node admittance matrix of power grid DTDM model at $(t + \Delta t)$.

Then, based on the equations of inductance, capacitance, resistance, and switch in (13), the current source of each branch at $(t + \Delta t)$ can be expressed as:

$$\text{his}(t+\Delta t) = k_{o(t)} \text{his}(t) + g_{o(t)} V_{\text{line}(t)}$$

(19)

where $k_{o(t)}$ is the branch modification matrix of power grid DTDM model at $t$, and it is dimensionless; $g_{o(t)}$ is also the branch modification matrix of power grid DTDM model at $t$, and the dimensions and admittance are the same; $V_{\text{line}(t)}$ is the branch voltage vector of power grid DTDM model at $t$. Besides, the principle of obtaining elements in $k(t)$ is: the dimension is the diagonal matrix of number of branches, and the element on diagonal can only be one of 0, -1 and 1; when the branch is inductive, the diagonal
element is 1; when the branch is capacitance, the diagonal element is -1; when the branch is resistive, 
the diagonal element is 0. The principle of obtaining elements in $g(t)$ is: the dimension is a diagonal 
matrix with number of branches, and the elements on the diagonal can only be 0, $-(\Delta t/L(t))$ and $(4C(t)/\Delta t)$; 
when the branch is inductive, the diagonal element is $-(\Delta t/L(t))$; when the branch is of capacitive, the 
diagonal element is $(4C(t)/\Delta t)$; when the branch is of resistive, the diagonal element is 0.

Based on the network relationship of power grid and (18), the branch voltage vector $V_{\text{line}}(t)$ at time $t$ 
has the following relationship with the current source $h(t)$ at time $t$:

$$V_{\text{line}}(t) = L^T(t) G^{-1}(t) L(t) h(t)$$  

where $L^T(t)$ is the transposed matrix of node-branch incidence matrix of power grid DTDM model at 
time $t$, that is, the branch-node incidence matrix.

Substituting (20) into (19):

$$V_{\text{line}}(t) = L^T(t) G^{-1}(t) L(t) h(t)$$

According to (21), if the current source is used as a state variable in DTDM model of power grid, a 
new state space architecture can be constructed, and the matrix $A(t)$ can be expressed as:

$$A(t) = [k(t) + g(t) L^T(t) G^{-1}(t) L(t)]$$

Each discrete-time unit can calculate the matrix $A(t)$ according to the corresponding parameters in 
power grids, thereby calculating the eigenvalues of each unit in time domain, and then analyze the 
stability of entire system.

The stability analysis method for power grid with energy storage devices based on the discrete time 
domain model is given in Fig. 2. According to the figure, the proposed stability analysis method based 
on the discrete time domain model mainly relies on the matrix calculation, and all matrices are real-
number matrices and some matrices are very sparse. Therefore, it has a better calculation speed and 
practicality. Besides, when the grid connection structure is determined and the properties and connection 
structure of each branch will not change, $k(t)$ and $L(t)$ and $L^T(t)$ in (22) will not change during the 
calculation process, it only needs to be calculated once, and $G(t)$ and $g(t)$ only need to modify the varied 
branch, further accelerating the matrix calculation speed. Moreover, under the proposed method, when 
the unstable eigenvalues appear in the system, it can quickly locate the moment when unstable 
eigenvalues appear and point out the factors affecting stability, which is convenient for engineers to find 
and solve unstable problem. and it has a good engineering application prospect.
4. Simulation Verification

To verify the theories analysis and proposed methods, the equivalent circuit of a power grid with energy storage devices is shown in Fig. 3, and the parameters are: $\nu = 8.5V$, $\Delta t = 50$ us; $R = 0.1$ ohm, $L = 5$ H, $C = 250$ uF; $R_1 = 10$ ohm, $L_1 = 4$ H, $C_1 = 20$ uF, $R_2 = 1$ ohm, $L_2 = 5$ H. The state of switch S1 changes from turn-off to turn-on in the 20th unit time, and the state of switch S2 changes from turn-off to turn-on in the 40th unit time domain.

Fig. 3. The study case of power grid with energy storage devices.
4.1. Verification of eigenvalue relationship between continuous time domain and discrete time domain

From the RLC series circuit in red dotted line in Fig. 3, the eigenvalue relationship in (9) is verified. The continuous time domain eigenvalues and the discrete time domain eigenvalues are calculated with parameters: \( R = 100 \text{ohms} \), \( L = 5 \text{H} \), and \( C = 250 \text{uF} \).

For the RLC series circuit in red dotted line, the inductor current and capacitor charge are selected as state variables under state space in continuous time domain, and the matrix \([A]_C\) can be obtained:

\[
[A]_C = \begin{bmatrix}
0 & 1 \\
-1 & -\frac{R}{L} \\
-\frac{1}{LC} & -\frac{1}{L}
\end{bmatrix} = \begin{bmatrix}
0 & 1 \\
-800 & -20
\end{bmatrix}
\]

(23)

Therefore, the eigenvalues in continuous time domain can be described as:
\[
\begin{bmatrix}
\lambda^c
\end{bmatrix} = \begin{bmatrix}
-10.0000 + j26.4575 \\
-10.0000 - j26.4575
\end{bmatrix}
\]

(24)

In discrete time domain, the coefficient matrices in state space are:
\[
\begin{align*}
k &= \begin{bmatrix}
-10.0000 + j26.4575 \\
-10.0000 - j26.4575
\end{bmatrix} \\
L &= \begin{bmatrix}
-10.0000 + j26.4575 \\
-10.0000 - j26.4575
\end{bmatrix} \\
g &= \begin{bmatrix}
-10.0000 + j26.4575 \\
-10.0000 - j26.4575
\end{bmatrix} \\
G &= \begin{bmatrix}
-10.0000 + j26.4575 \\
-10.0000 - j26.4575
\end{bmatrix}
\end{align*}
\]

(25)

Therefore, the matrix \([A]^D\) and eigenvalues in discrete time domain are:
\[
\begin{align*}
[A]^D &= \begin{bmatrix}
0 & 0 & 0 & 0 \\
0.0020 & 0.9980 & -0.0000 & 0 \\
0.0020 & 1.9980 & 1.0000 & 0 \\
0.9990 + j0.0026 & 0.9990 - j0.0026 & 0
\end{bmatrix} \\
\lambda^D &= \begin{bmatrix}
0.9990 + j0.0026 \\
0.9990 - j0.0026 \\
0
\end{bmatrix}
\end{align*}
\]

(26)

Comparing the eigenvalues in (24) with that in (26), it can be seen that the conversion relationship in (9) is correct, which also verifies that the stability criterion for power grid with energy storage devices based on discrete time domain eigenvalues is correct.

4.2. Validation of Power Grid Stability Analysis Method Based on Discrete Time Domain Model

According to the model of power grid with energy storage devices in Fig. 3 and the stability analysis flowchart in Fig. 2, the simulation results of stability analysis program are shown in Fig. 4.

Fig. 4 (a) shows the variation curve of system eigenvalues in discrete time domain. From the figure, the system has 5 eigenvalues at the time before S1 switch turn-on, after S1 switch turn-on and after S2 switch turn-on. However, due to the variation of circuit structure, these 5 eigenvalues do not completely overlap, as shown in Fig. 4 (b). Therefore, from Fig. 4 (a) and Fig. 4 (b), it can be seen that the proposed method can capture the variations of system eigenvalue and calculate the system eigenvalues, and that’s important for engineering applications. Besides, the simulation results also reflect that, under the proposed method, the variations of system eigenvalue can intuitively reflect by the variations in calculation results, thus the influence from circuit design and control on the system stability can be observed.

Moreover, the Fig. 4 (c) shows the relationship between all eigenvalues and the unit circle. It can be seen that the system is stable regardless of the operation of switch S1 or S2. The Fig. 4 (c) also shows that the stability of power grid with energy storage devices can be directly analyzed under the proposed method based on relationship between eigenvalues and the unit circle.

5. Conclusion

This paper proposes a eigenvalues analysis method and stability criterion for power grid with energy storage devices based on the discrete time domain model. In the proposed method, the components are considered to be linear in each calculation time domain, and the eigenvalues in calculation time domain is calculated. The stability of system in each time domain can be analyzed, and the overall stability of system is obtained. The proposed method can capture all stability information of system at one time, and is not limited by the model of devices. The construction and calculation of state space model are
simple and efficient. The proposed method has engineering application value for the power grid with renewable power sources and energy storage devices.

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