Grid-based Network Architecture for Distributed Computation in Wireless Sensor Networks

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Abstract—Wireless Sensor Networks (WSNs) are used to perform distributed sensing in various fields, such as health, military, home etc. In WSNs, sensor nodes should communicate among themselves and do distributed computation over the sensed values to identify the occurrence of an event. This paper assumes the no memory computation model for sensor nodes, i.e. the sensor nodes only have two registers. This paper presents an optimal architecture for the distributed computation in WSN and also claims that this architecture is the optimal for the described computation model.

Index Terms - Algorithms, Design and Performance.

I. INTRODUCTION

WSN consists of a set of sensor nodes that are deployed in a field and interconnected with a wireless communication network. Each of these scattered sensor node has the capabilities to collect data, fuse that data and route the data back to the sink/base station [1], [2]. To collect data, each of these sensor nodes makes decision based on its observation of a part of the environment and on partial a-priori information. As larger amount of sensors are deployed in harsher environment, it is important that the distributed computation should be robust and fault-tolerant. The identification of event in wireless sensor network should be done as fast as possible thus the computations are done in parallel.

Here we investigate the problem of design of optimal parallel distributed computation architecture. In distributed system components located on networked computers communicate and coordinate by passing messages to perform the specified task. Similarly distributed computation is done on distributed nodes connected over the network with defined computation model. A model of computation [3] is a formal description of a particular type of computational process. This paper assumes the no memory computation model for sensor nodes. No memory computation model means the sensor node just have registers to store two values, whenever sensor node receives any value from other sensor node it simply compute the function with its own measured value and the received value and pass the results to other sensor node(s).

The distributed architecture for WSN needs to be optimal from most of the following points [4]:

- Fault Tolerance
- Computation complexity
- Transmission delay required for the computations
- Deployment / Reconfiguration

II. DISTRIBUTED NETWORK ARCHITECTURE

A. Problem Statement

The problem is to define a globally optimal data structure for calculating the defined fusion function over the sensor field. The architecture should be as optimal as possible from the point of view of all the performance measures discussed in section I. The computation model considered is also important while defining the suitable distributed architecture. The paper assumes the no memory computation model as discussed in section I. Thus we are redefining the problem statement. To find the globally optimal architecture, we need to fix some of the performance measures and try to optimize the other measures. The modified problem statement is:

- Given the maximum allowed delay \( D_0 \), define the globally optimum data structure of wireless sensor network for the distributed computation of fusion functions of sensed values in no memory computation model.

B. Grid based Network Architecture

This sections discusses the optimal distributed architecture for homogeneous and heterogeneous wireless sensor networks. Homogeneous WSN consists of sensor nodes with same ability while heterogeneous WSN consists of sensor nodes with different ability such as different computing power.

1) Homogeneous WSN: The solution for the above defined problem is a grid like architecture as shown in figure I.

In this section, we first discuss the computation complexity of the minimum/maximum like fusion function where only one comparison is need at every node, later we show that the same comparisons can be done for other functions. Assume that the total number of processors \( P \) is equal to the number of sensor nodes \( N \) in the network. Calculations are as follows:

\[
D_0 = \sum_{n=1}^{N} (D_0 - 1)
\]

Computation complexity in each branch is \( D_0 \).
Here again the number of comparisons is equal to the minimum required comparisons. Also this architecture is very suitable for the sensor field from the point of view of deployment and coverage.

2) Heterogeneous WSN: Clustering of sensor networks has proved to be very effective in conserving energy in heterogeneous networks. For each cluster, one Cluster Head (CH) is selected using [5], [6]. CH is responsible for collecting, fusing and transmission of data for the cluster, and also only cluster heads participate in routing the other cluster data to the base station/sink. Thus in heterogeneous WSN, we consider three type of nodes:

1) Sensor Nodes
2) Cluster Heads
3) Base Station/Sink

The proposed architecture for this kind of network consists of hierarchy of grid architecture as shown in figure 2. In the hierarchical grid architecture, each node of the final grid architecture is actually a cluster head and this cluster head is connected to other sensor nodes of the cluster using other grid based architecture.

III. PRIMITIVE RECURSIVE FUNCTIONS

This section discusses the class of functions, i.e. primitive recursive functions, which can be computed optimally on grid like architecture. The basic primitive recursive functions are given by these axioms [7]:

- **Constant function:** The 0-ary constant function 0 is primitive recursive.
- **Successor function:** The 1-ary successor function \( S \), which returns the successor of its argument, is primitive recursive. That is, \( S(k) = k + 1 \).
- **Projection function:** For every \( n \geq 1 \) and each \( i \) with \( 1 \leq i \leq n \), the \( n \)-ary projection function \( (P_i)^n \), which returns its \( i^{th} \) argument, is primitive recursive.

Similarly for more complex networks, multiple hierarchy of clusters can be done and in such cases multi-hierarchical grid based architecture can be used.
More complex primitive recursive functions can be obtained from the initial functions by means of composition and primitive recursion.

- **Composition**: If \( g \) is a function of \( m \) arguments, and each of \( h_1, \ldots, h_m \) is a function of \( n \) arguments, then the function \( f(f(x_1, \ldots, x_n)) = g(h_1(x_1, \ldots, x_n), \ldots, h_m(x_1, \ldots, x_n)) \) is definable by composition from \( g \) and \( h_1, \ldots, h_m \). We write \( f = [g \circ h_1, \ldots, h_m] \), and in the simple case where \( m = 1 \) and \( h_1 \) is designated \( h \), we write \( f(x) = [g \circ h](x) \).

- **Primitive Recursion**: A function \( f \) is definable by primitive recursion from \( g \) and \( h \) if:
  
  \[
  f(x, 0) = g(x) \\
  f(x, s(y)) = h(x, y, f(x, y))
  \]

  We write \( f = PR[g, h] \) when \( f \) is definable by primitive recursion from \( g \) and \( h \). Here \( s \) is the successor function, which when given an argument \( n \) returns its immediate successor \( s(n) \).

The primitive recursive functions are the basic functions and those obtained from the basic functions by applying these operations a finite number of times. In WSN domain, simple aggregation techniques i.e., maximum, minimum, and average have been used to save energy while monitoring [8]. Incase of more complex fusion functions also, the fusion function can be represented using primitive recursive function class. Some of the examples of primitive recursive functions which can be used in fusion are:

- Addition
- Multiplication
- Exponentiation
- Factorial
- Proper subtraction defined as ‘If \( a \geq b \) then \( a - b \) else \( 0 \).’
- Minimum \( (a_1, \ldots, a_n) \)
- Maximum \( (a_1, \ldots, a_n) \)
- Absolute value
- Mean
- Weighted Mean
- Weighted Energy, i.e. \( w_1x_1^2 + w_2x_2^2 + \cdots + w_mx_m^2 \)

**IV. Fault Tolerance of Proposed Architecture**

Fault tolerance of a network is a measure of its performance to do the intended job if some node(s), link(s) or both fail. To increase the fault tolerance of the proposed architecture we included the row links as shown in figure 3. In this architecture if a node or link goes down, then other path is available to send the value computed so far which can be used while calculating the fusion function of the other branch.

Also we can do identification of error in this architecture. The steps for error detection are as follows:

1) Calculate row wise fusion function
   - Calculate the row wise fusion function for each row.

2) Similarly Calculate column wise fusion function
   - Calculate the column wise fusion function for each column.
   - Calculate the fusion function with each column’s calculated value.

3) If the result of step 1 and 2 are not same than there is some error in the computation.

A. Example of fault tolerance

This example consider the ‘maximum’ as the fusion function to be computed. Figure 4 shows the correct functionality without any link or node failure.

Figure 5 shows the node failure scenario of the example. Node failure is considered where the reading from a particular node is not available. The node failure doesn’t provide the reading for the respective node. If the failure node is the one giving maximum value as shown in the figure 5(a), than this affects the distributed computation and can’t be detected. But if some other node(s) (not giving the maximum value) fails than the computation is correct and hence the node failure can’t be detected. It is not such a restrictive idea to
assume that the maximum over the sensor field is sensed by a single node. But multiple nodes could also sense the maximum value. Here again if all the node giving maximum value fails than only the computation is incorrect otherwise the computation is correct.

![Fig. 5. Fault Tolerance Example: Node Failure Scenario](image)

Figure 5 shows the node failure scenario of the example. If the failure node is the one passing the maximum value as shown in the figure 5(a), then this can be detected. But if some other link(s) (not passing the maximum value) fails than the computation is correct and hence the link failure can’t be detected.

From this example it is clear that the one link failure (if the one is passing the maximum value) can be detected in the architecture.

**B. Probability error in maximum calculation**

This subsection calculate the probability of error in grid based architecture for maximum calculation.

Total number of sensor nodes = \( N \)

Let’s assume nodes fail independently with probability = \( P_f \)

![Fig. 6. Fault Tolerance Example: Link Failure Scenario](image)

And the number of nodes containing the maximum value = \( M \)

So the probability of error is:

\[
P_e = \frac{1}{\binom{N}{M}} P_f^M (1 - P_f)^{(N-M)}
\]

Here if only one node contain the maximum value, then the probability of error is \( \frac{1}{N} P_f (1 - P_f)^{(N-1)} \). In grid the probability of no error is \( (1 - P_e) \).

**V. Conclusion**

This paper discusses the architecture for distributed computation in wireless sensor network. The result for computation complexity for fixed maximum allowed delay is calculated in section [II-B] which shows that this architecture is the solution for given computation model. Section [IV] shows that the proposed architecture is very efficient with respect to faults in the network.
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