FRB Strength Distribution Challenges the Cosmological Principle

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ABSTRACT
The distribution of FRB fluxes and fluences is characterized by a few very bright events and a deficiency of fainter events, compared to expectations for a homogeneous space-filling distribution. I define a metric to quantify this, and apply it to the 17 presently known Parkes FRB, products of a comparatively homogeneous search. With 98% confidence we reject the hypothesis of a homogeneous distribution in Euclidean space. Possible explanations include a reduction of fainter events by cosmological redshifts or evolution or a cosmologically local concentration of events. The former is opposed by the small value of the one known FRB redshift. The latter contradicts the Cosmological Principle, but may be explained if the brighter FRB originate in the Local Supercluster.

Key words: radio continuum: transients, large-scale structure of Universe

1 INTRODUCTION

From the discovery of the first Fast Radio Burst (Lorimer, et al. 2007), it has been noticed that there is a deficiency of weaker bursts compared to the number $N \propto S^{-3/2}$ expected in a Euclidean Universe. In order to have a reasonably homogeneous statistical sample we consider only the 17 bursts observed at Parkes out of the 23 bursts in the FRB Catalogue (Petroff, et al. 2016). The more recent discoveries of very bright bursts by UTMOST (Caleb, et al. 2017) and ASKAP (Bannister, et al. 2017) were made with instruments of lower sensitivity and cannot be commingled with Parkes observations in a homogeneous data set. We wish to test the hypothesis that FRB are homogeneously distributed in a Euclidean Universe. This cannot be exactly correct because we know that the Universe is not Euclidean and evolves. However, we are ignorant of the evolution of the FRB source population and of their spectra (required to calculate K-corrections), so the Euclidean model is as good as any we could choose, and has the advantage of specificity. The one known FRB redshift is small (0.193; Tendulkar, et al. 2017), suggesting that the Euclidean model is in fact a fair approximation. The statistics of the fainter Parkes FRB (Katz 2016a,b) are approximately consistent with the Euclidean model, but this has not been demonstrated quantitatively. This paper develops a quantitative metric and applies it to the Parkes dataset.

2 THE METRIC

The assumption of sources homogeneously distributed in Euclidean space makes definite predictions. We consider the $\alpha$-th moment of the received signal $S$, where $S$ may be any quantity that satisfies an inverse square law and has a definite detection threshold $S_0$. Examples include flux, fluence and flux times the square root of the pulse width, as for UTMOST (Caleb, et al. 2017). This last quantity is appropriate when the signal must be distinguished from detector thermal noise. The detection threshold may depend non-trivially on the pulse shape and other quantities that (if cosmological redshift is small) do not depend on distance.

The normalized $\alpha$-th moment of $S$ is defined

$$f(\alpha) \equiv \langle S^\alpha \rangle = \frac{\int S^\alpha dN}{\int dN}, \quad (1)$$

where $N$ is the number of sources in a catalogue. For a homogeneous and continuous distribution of sources of number density $n(\mathcal{L})$ per unit source strength $\mathcal{L}$, $dN = 4\pi n(\mathcal{L}) R^2 dR d\mathcal{L}$, where $R$ is their distance and $\mathcal{L}$ is luminosity for steady sources, energy for temporally unresolved bursts and something more complicated but still following the inverse square law (if cosmologically local, and ignoring any effect of intergalactic dispersion on detectability) for temporally resolved bursts. Then $S = S_0 R_0^2 / R^2$, where $R_0(\Omega, \mathcal{L})$ is the distance at which a source of strength $\mathcal{L}$ in the direction $\Omega$ is at the detection threshold. $R_0(\Omega, \mathcal{L})$ depends on the unknown distance of the source from the axis.

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of the telescope beam. Integrating,

\[
\frac{d\Omega}{d\Omega} \frac{dS^2}{dN} = \int d\Omega \int dL \frac{R_0(\Omega, L)^2}{R_0(\Omega, L)} \frac{S_0^2}{R_0^2(\Omega, L)} \frac{S_0^2}{R_0^2(\Omega, L)} \frac{d\Omega}{d\Omega} \frac{dS^2}{dN} \frac{dR}{dR} = \frac{3}{3 - 2\alpha} S_0^2.
\]

The angular dependence of telescope sensitivity cancels, provided \( S_0 \) is, at least statistically, independent of \( \Omega \). This is expected because the signals are processed and analyzed without knowledge of \( \Omega \). \( f(\alpha) \) is meaningful only for \( \alpha < 3/2 \) and useful only for \( \alpha > 0 \).

In general, \( S_0 \) is poorly known because of its complex dependence on pulse width and profile, so that it is not possible to use Eq. 2 directly. Instead, define a metric

\[
F(\alpha_1, \alpha_2) \equiv \frac{f(\alpha_1)}{f(\alpha_2)^{1/\alpha_2}} = 3^{1-\alpha_1/\alpha_2} (3 - 2\alpha_2)^{\alpha_1/\alpha_2} \frac{3 - 2\alpha_1}{3 - 2\alpha_2}
\]

This is dimensionless and independent of our knowledge or ignorance of \( S_0 \). A simple and intuitively appealing choice of parameters is \( \alpha_1 = 1 \) and \( \alpha_2 = 1/2 \), for which a homogeneous source distribution in Euclidean space yields

\[
F(1,1/2) = \frac{4}{3}.
\]

3 FINITE SAMPLE STATISTICS

The preceding results apply to continuously distributed sources. In practice, sources are discrete and catalogues are finite, so the predicted values of \( F \) and their distribution must be calculated by Monte Carlo methods. \( N \) sources are randomly but statistically uniformly distributed within a sphere whose outer radius is their detection limit. The mean value of \( F \) as a function of \( N \) is shown in Fig. 1 based on \( 10^6 \) realizations. It approaches \( 4/3 \) only slowly as \( N \to \infty \) because the inverse square law gives the poorly sampled small fraction of sources close to the observer a disproportionate influence.

The distribution of \( F(1,1/2) \) over \( 10^6 \) Monte Carlo realizations of 17 sources, corresponding to the FRB catalogue used in Sec. 4, is shown in Fig. 2. The distribution is narrowly peaked but very skew, with a maximum at 1.06, a mean of 1.21 and a standard deviation of 0.12, although it is far from Gaussian. As implied by Fig. 1, this distribution is only weakly dependent on \( N \).

For testing the significance of a value of \( F \) larger than the predicted mean the long tail of the cumulative distribution must be examined in more detail. This is shown in Fig. 3.

4 APPLICATION TO FRB

We consider three possible definitions of the Parkes FRB detection threshold: the flux, the fluence times the square root of the pulse width \( W \) (as expected when detection is only limited by thermal noise in the detector) and the fluence. The results are shown in Table 1.
Comparing to Fig. 3 and more detailed tabular data, the Euclidean hypothesis may be rejected at the 98% level for a fluence threshold and at even higher levels of confidence for other assumed threshold functions. However, if the two bright outliers are removed from the sample, the distribution of the remaining 15 FRB is consistent with the Euclidean hypothesis.

5 DISCUSSION

5.1 Uncertainties

This result is subject to the caveat that the uncertainties in the measured fluxes and widths have not been allowed for. This is difficult with the present Catalogue because uncertainties are missing for several of the bursts, and the meaning of those uncertainties that are in the Catalogue is unclear. For example, some uncertainty ranges are very asymmetric about the nominal values; the implied likelihood distributions must be far from Gaussian, but are unquantified. The fluences and their uncertainty ranges in the Catalogue are the products of the fluxes, widths and the limits of flux and width uncertainty ranges (with small deviations for the lower bound on fluence for a few bursts). This is questionable because the flux, width and fluence measurements are not independent; fluence is constrained independently of the flux and width, but its uncertainty is not given independently in the Catalogue. The maximum possible fluence is overestimated if the maximum flux is multiplied by the maximum width. Large uncertainty ranges for a few bursts (particularly FRB 130729, whose width is given as 15.61 $^{+9.98}_{-6.27}$ ms) may introduce spurious large uncertainties in average quantities. This problem might be addressed by removing bursts with large uncertainties from the database, but the decision of which to remove would necessarily be subjective.

The discrepancy with the Euclidean model is insensitive to uncertainties in the signal strengths of either the two very bright bursts or the remaining 15 because it is attributable to the absence or deficiency of bursts with signal strengths intermediate between these two widely separated groups. For FRB 150807 the quoted uncertainties are small, while for FRB 010724 the Catalogue contains only lower bounds that we used as actual values; if the true values were greater than these lower bounds, the discrepancy would be even larger.

5.2 Sensitivities

A separate caveat arises from the possibility that different values of $S_0$ were effectively used in the data analyses, which would invalidate the derivation in Eq. 2. Even though all Parkes bursts were observed with the same telescope, data reduction algorithms and acceptance criteria might have varied. Data with different $S_0$ cannot be combined because in a combined dataset the relation between $N$ and $R$ (equivalently, between $N$ and $S$) would then no longer be that implied by homogeneity and the inverse square law. Data from the less sensitive (larger $S_0$) UTMOST and ASKAP cannot be commingled with the Parkes data because that would introduce a spurious excess of strong bursts.

We can simulate the effect of variable sensitivity by artificially raising the threshold for acceptance to exclude the weaker bursts in the Catalogue, as if a stricter criterion (larger $S_0$) were applied to their detection. This could be done for any signal strength parameter that follows an inverse square law, but we choose $\text{Flux} \times W^{1/2}$ as most closely describing the detection threshold of observations limited by detector thermal noise. The 15 weaker Parkes bursts in the Catalogue have values of $\text{Flux} \times W^{1/2}$ ranging from 0.47 to 3.08 Jy-ms$^{1/2}$, with only one of the 15 above 2 Jy-ms$^{1/2}$.

We therefore repeat the analysis with thresholds rang-
As $S_0$ increases from 0 to 3.5 Jy-ms$^{1/2}$ (any threshold below 0.47 Jy-ms$^{1/2}$ admits the full dataset, while a threshold above 3.08 Jy-ms$^{1/2}$ reduces the dataset to the two very bright bursts). The results are shown in Fig. 4. The function $F(1,1/2)$ for each of the three measures of FRB signal strength remains above 2, corresponding to a 98% significant discrepancy with the homogeneous Euclidean model, for any threshold below 1.08 Jy-ms$^{1/2}$, at which eight of the 15 weaker bursts are excluded. The slowness of the decrease of $F(1,1/2)$ with increasing threshold implies that the result is insensitive to possible inconsistencies in the acceptance criteria for bursts ($S_0$). This result does not require knowledge of the instrumental sensitivity, unlike the argument of Lorimer, et al. (2007) that the deficiency of weak bursts was significant because the detection threshold was far below the signal strength of the (then) one observed burst.

We also simulate the effects of possible burst-to-burst variation in detection threshold $S_0$ by substituting for the 15 faint bursts a synthetic set consisting of 15 bursts randomly chosen from the 15 actual faint bursts. In the synthetic set one or more of the actual bursts may be omitted and others represented more than once. Unlike the thresholded samples of Fig. 4, all sets have a total of 17 bursts so there are no trends resulting from the reduction in number of weak bursts as the threshold is raised. Averaging over $10^5$ realizations, we find the results shown in Table 1 as 17 Randomized. The simulated uncertainties are not large, and do not affect our conclusions.

5.3 Bias

A final caveat arises from possible bias introduced by the fact that this study was performed not long after the discovery (Ravi, et al. 2016) of the bright burst FRB 150807. If that discovery motivated this study (a question unanswerable because it depends on human thought processes), the sample was biased to include a maximal fraction of very bright bursts. To estimate this bias the analysis was repeated excluding FRB 150807, with results shown in Table 1 for 16 bursts. The weakest constraint is now obtained from the flux data, and still indicates a 98% significant rejection of the homogeneous Euclidean hypothesis. As the number of observed bursts increases, any such bias will have less effect.

6 CONCLUSION

The distribution of FRB in space appears to violate the cosmological principle that, averaged over sufficiently large scales, the Universe is homogeneous. This conflict is resolved if “sufficiently large” means on scales greater than the unknown distances to the two very bright bursts.

If FRB are, roughly, standard candles, and if the repeating FRB 121102 at $z = 0.193$ is representative of FRB distances in general, then we may roughly estimate the redshifts of the bright FRB as $z \sim 0.03$. The assumption that FRB 121102 is representative is unproven; despite Ockham’s Razor to the contrary, it might be a different class of object, as SGR were distinguished from GRB only many years after their discovery in 1979. If FRB 121102 is representative, then the density of FRB sources at $z \lesssim 0.03$ is greater than their mean density at $z \sim 0.2$ by $\sim 50$, the ratio of the volumes (the $3/2$ power of the ratio of their signal strengths) times the ratio of the numbers ($2/15$) in the FRB Catalogue. These redshifts are small enough that cosmological evolution should have only a minimal effect on FRB density and the non-Euclidean geometry of space only a minimal effect on FRB statistics.

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