The Jamming point street-lamp in the world of granular media

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The Jamming of soft spheres at zero temperature, the J-point, has been extensively studied both numerically and theoretically and can now be considered as a safe location in the space of models, where a street lamp has been lit up. However, a recent work by Ikeda et al.1 reveals that, in the Temperature/Packing fraction parameter space, experiments on colloids are actually rather far away from the scaling regime illuminated by this lamp. Is it that the J-point has little to say about real system? What about granular media? Such a-thermal, frictional, systems are a-priori even further away from the idealized case of thermal soft spheres.

In the past ten years, we have systematically investigated horizontally shaken grains in the vicinity of the Jamming transition. We discuss the above issue in the light of very recent experimental results. First, we demonstrate that the contact network exhibits a remarkable dynamics, with strong heterogeneities, which are maximum at a packing fraction \( \phi^* \), distinct and smaller than the packing fraction \( \phi_J \), where the average number of contact per particle starts to increase. The two crossovers converge at point-J in the zero mechanical excitation limit. Second, a careful analysis of the dynamics on time scales ranging from a minute fraction of the vibration cycle to several thousands of cycles allows us to map the behaviors of this shaken granular system onto those observed for thermal soft spheres and demonstrate that some light of the J-point street-lamp indeed reaches the granular universe.

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I. INTRODUCTION

In a loose sense, Jamming describes everyday situations where particles, objects, or people become dense, slow and rigid: one thinks of systems as different as sand piles, foams, or traffic jams as jammed systems.2 Significat progress was achieved in the field about ten years ago, when frictionless soft spheres at zero temperature were introduced as a minimal and seminal model for Jamming.3 This system has been extensively studied4–6 and now serves as a point of reference7 for which Jamming has a precise meaning. Specifically, for models for which forces are represented by particle overlaps, the Jamming transition occurs when the system can only be compressed further by allowing overlaps between particles. From that point of view, it is essentially a matter of satisfying geometric constraints, and indeed, a formal identification with an algorithmic description has been established3–8. For athermal systems, the Jamming transition is instrinsically out-of-equilibrium, and requires a precise characterization of the protocol used to prepare the system. However, many features of the transition appear to be protocol independent11, and for a given protocol on an infinite system, the Jamming transition is entirely controlled by the packing fraction. The transition occurs at the so-called "point \( J^* \)", and coincides with the onset of isostaticity11, i.e., the number of steric and mechanical constraints imposed at the contacts exactly matches the number of degrees of freedom available to the particles. A number of geometrical and mechanical quantities exhibit clear scaling laws with the distance to point-J7. One prominent signature of Jamming for systems of frictional particles is the singular behavior of the average number of contacts per particle \( z - z_J \sim (\phi - \phi_J)\alpha \), where \( z_J \) is equal to \( 2d \), where \( d \) is the space dimension, \( \phi_J \) is the packing fraction at point \( J \), and \( \alpha \approx 0.5 \). The distribution of the gaps between particles displays a delta function at zero and a square root decay for increasing gaps, which is at the root of the singular behavior of the average contact number6–12,14.

This framework has provided key physical insights into the nature of rigidity, and the structure/mechanics of disordered soft matter systems, such as emulsions15, foams16,17 and grains16,18. Of course, this idealized model misses some of the key features of real systems, such as friction for dry systems, interface effects for multiphase systems, or hydrodynamic interaction for suspensions. In particular, several works have shown that the Jamming scenario for static packings becomes more complex when friction comes into play19–21.

Furthermore, many systems of interest are not purely static: colloidal suspensions undergo thermal agitation; vibrated or flowing granular systems undergo mechanical agitation. Whether the Jamming framework is relevant in the presence of agitation remains an open, hotly debated issue2,6,22. On the one hand, one expects the singular nature of the Jamming point to be blurred. On the other hand, an anomalous dynamics is expected to occur, because particle motion, driven by agitation, may be influenced by the proximity of the singular point (see
In the zero mechanical excitation limit.

Frequency and observed that these two cross-overs merged to increase. Furthermore, by varying the vibration frequency, the average number of contact per particle started to decrease, and this feature blurs the picture. Contacts and overlaps are always present.

At zero temperature, below Jamming, there is always a way to pack the particles without overlaps and the energy of the system is strictly zero. Above Jamming, there is no packing without overlaps and the energy, purely potential, is greater than zero. At finite temperature, the kinetic energy is never zero, and this feature blurs the picture. Contacts and overlaps are always present.

A recent numerical study of harmonic spheres, in the presence of temperature, focuses on the dynamics in the region very close to the $T = 0$ Jamming point [1]. The authors demonstrate that there is no singularity at finite temperature and identify a critical region in the vicinity of the Jamming point, where vibrational dynamics is maximally heterogeneous. They also report crossover lines, in the temperature-packing-fraction parameter space, between harmonic and non-harmonic regimes, originating at point J. Finally, on the basis of the dynamical behavior reported in the literature, they place existing colloidal experiments in the temperature-packing-fraction parameter space. Their main conclusion is that these experiments actually sit rather far from the critical regime of point J.

In the past ten years, we have systematically investigated horizontally shaken grains in the vicinity of the Jamming transition [23–26]. Starting with rigid brass disks, we observed very large heterogeneities of the dynamics when focusing on minute displacements on the order of $5 \times 10^{-3}$ grain diameters [23–25]; it was conjectured that these heterogeneities were connected to the dynamics at the contact scale. This was later confirmed using soft photo-elastic disks [26]. In the latter case, the signature of the dynamical heterogeneities was not as sharp, but we clearly demonstrated that the contact network exhibited a remarkable dynamics, with strong heterogeneities, which are maximum at a packing fraction $\phi^*$, distinct and smaller than the packing fraction $\phi^\dagger$, where the average number of contact per particle started to increase. Furthermore, by varying the vibration frequency and observed that these two cross-overs merged in the zero mechanical excitation limit.

The strong similarities shared by the above experimental results with those reported in the numerical study of thermal soft spheres [1] call for further investigation. Indeed, one would like to know to the extent of overlap between models of thermal harmonic spheres and the dynamical criticality of the granular packings.

To address these questions, we present novel results spanning the short time (inner vibration cycle) dynamics of the photo-elastic soft disks using stroboscopic dynamics, and the longer times studied in previous studies [23–26]. In order to provide a background, we present a concise and reasonably complete picture of the dynamics, the forces and the contacts close to Jamming in the presence of mechanical agitation. Within this context, we are able to: (i) conciliate hard and soft grain experiments, (ii) locate the granular experiment into a “temperature”-packing fraction phase diagram and, by so doing, discuss the relevance of the Jamming framework for describing granular systems. We conclude that our granular experiments do probe the same critical regime as those described by [1]. This, in turn, validates the use of soft sphere model to describe such systems close to Jamming.

The paper is organized as follows. In section III, we describe the experimental set up in detail, emphasizing the two modes of data acquisition, a fast one and a slower stroboscopic one, which allow us to explore the dynamics over six orders of magnitude in the timescales. Section III demonstrates that the force network is essentially isotropic and Section IV focuses on the dynamics of the contact network. This section summarizes the results already reported in [26] and supplements these results with the dynamical properties of the contacts at short timescales. Section V is devoted to the study of the mean square particle displacements. This study explicitly details the data processing required to obtain a meaningful computation of these displacements. The quantitative results obtained in this section are the key elements of the discussion. Section VI analyzes the dynamical heterogeneities of the displacement field, relates them to those of the contact dynamics and show that they are embedded in the structural properties of the contact network. Finally, section VII synthesizes our observations, relates them to the previous study of brass disk experiments [23–25] performed in the same set-up, and discusses the issue raised in the introduction, regarding the correspondence between thermal soft-sphere models and experiments on vibrated grains, in terms of dynamical behavior in the vicinity of point J.

II. SETUP AND PROTOCOL

We first review the details of the experimental set-up, which was adapted from [23] in order to allow for the use of photo-elastic grains and the detection of contacts. We also review the different acquisition techniques, emphasizing in particular, the fast image acquisition which allows us to characterize the dynamics within one vibration cycle, as opposed to the previous studies, for which
FIG. 2: Sketch of experimental setup. (color online) (a): photo-elastic grains lighted by transmission by a polarized backlight. (b): confining cell. (c): wall piston. (d): force sensor. (e): micrometric stepper motor. (f): vibrating frame. (g): stepper motor ensuring vibration. (h): notched belt transmitting vibration. (i): shelf. (j): wall. (k): translation stages. (l): stainless steel bars. (m): optical table. (n): CCD camera. (o): analyzers located on a rotating wheel. (p): shelf isolated from vibrations.

one image per cycle was acquired in phase with the vibration.

A. Setup

The experimental setup is sketched in figure. A bidisperse mixture of \( \sim 8000 \) 4 mm and 5 mm photo-elastic disks (PSM-4) (a) lies on a glass sheet, and is confined in a cell (b), whose area can be tuned with a piston (c). The piston is attached to a force sensor (d) and a micrometric stepper motor (e). The packing fraction, \( \phi \), can be fine-tuned from 0.795 to 0.83, with a resolution of \( \delta \phi = 5 \times 10^{-6} \). Below the glass sheet, an LED backlight device, covered with a polarizing sheet, provides an intense, large, thin and uniform source of circularly polarized light. The glass sheet and the light are embedded in a frame (f), which vibrates horizontally with an amplitude \( a = 1 \) cm and frequencies \( f = 6.25, 7.5 \) and 10 Hz. The oscillation is driven by a stepper motor (g), a notched belt (h) and an eccentric revolving shaft, which are attached to a shelf (i), the stability of which is ensured by 300 kg of lead bricks ballast and a rigid bracket to the wall (j). The confining cell is mechanically decoupled from the vibration devices. It is embedded in a larger frame, which in turn is attached to four manual micrometric translation stages (k). This ensures a precise leveling of the confining cell with respect to the oscillating board. The translation stages are attached to stainless steel bars (l), which are screwed to an optical table (m). Also attached to the optical table is a trigger. The trigger is made of a reflection phototransistor/photo-diode device, together with a Schmitt trigger electronic circuit. The device is placed in front of the revolving shaft, where a piece of black tape has been taped; when the sensor is in front – respectively outside of – the tape, it delivers a 5 V, –respectively 0 V signal. The phase of the trigger fall is chosen to be when the velocity of the plate is minimum and the Mark-to-Space ratio is adjusted in such a way that the transients of the stepper and the exposure times occur separately.

B. Data acquisition

We want to investigate the dynamics, both at short times, namely within the vibration cycles, and at long times, that is over several thousands cycles. Altogether, the experiment covers seven decades of time steps and, apart from the force sensor (d), all our data comes from image acquisition. We thus need to conduct two separate series of experiments, one with a fast camera, running continuously, and one with a standard CCD camera, triggered by the motion of the oscillating plate. In both cases, we access both the position of the grains and the photo-elastic pattern inside the grains. This cannot be achieved simultaneously, and we need to adapt the acquisition schema in order to be as close as possible to this ideal situation. The camera is fixed on a shelf (p), lying on an optical table and isolated through a rubber gasket in order to reduce the transmission of vibrations and minimize blur on the pictures.

To record the displacements and the force network dynamics at short times, we use a fast camera (2000 frames per sec) with a resolution of \( 1024 \times 1024 \) pixels, which record 1361 frames during up to 6 cycles of vibration, for the largest vibration frequency of 10 Hz. We successively acquire two movies, with and without introducing an analyzer in the field of view of the camera. Only a few tens of vibration cycles separate the two acquisitions. Since the dynamics is completely frozen (see below), the packing barely moves, and synchronizing the two movies, we associate the photo-elastic pattern and the grains captured on the white-light (no crossed polarizers) images.

The long time dynamics is recorded with a high resolution (\( 2048 \times 2048 \)) CCD camera (n) triggered in such a way that the images are taken in phase with the motion of the oscillating board. Analyzers (o) located on a rotating wheel, with minimal inertia, are inserted in the field of view of the camera once every two cycles, using a triggered stepper motor, so that white-light (re-
respectively cross-polarized) pictures are taken every odd (respectively even) vibration cycle. It is then straightforward to match the photoelastic pattern to the white-light images of the grains. In order to minimize blur, the pictures are taken at the phase for which the board velocity is minimal, that is when it reverses direction. This is also when the acceleration is maximal. We shall see in the following that this has direct consequences on the contact number measurement. Also, the stepper motor that switches polarizers position is attached to the ceiling, to avoid the transmission of its vibrations to the camera. Finally, in order to prevent thermal expansion of the grains due to heating, the LED backlight is also triggered on the vibration and flashes only during 6 ms, which is also the time exposure of the camera.

From the white-light images, we extract grain positions, and diameters (black circles in fig.3(b)), on which we perform Delaunay triangulation (red lines in fig.3(b)) and Voronoi tessellation (blue lines in fig.3(b)). The grain positions are obtained with a resolution of 0.5% of $d$. Once the grains have been detected, an estimate of the pressure within each grain is obtained by integrating the square gradient of the cross-polarized light intensity over the disc area. We denote by $G^2_i$ this estimate of the pressure in grain $i$. The resolution in each grain is not good enough to carry out a force inverse algorithm for the photo-elastic problem [27], and compute the forces at contacts. However, we can estimate them as follow. For each inter-particle contact, we use the two particle positions and the positions of their two common Voronoi vertices to build a pattern of triangles, which we call $a$, $b$, $c$ and $d$ (see figure 3(b)). We then compute the spatial gradient of the associated cross-polarized image (see figure 3(a)), and we sum this signal within each of the triangles (see figure 3(c)). This defines $G^2_a$, $G^2_b$, $G^2_c$, and $G^2_d$, their associated photoelastic signal [28]. We then estimate the normal force of each link, $F_N$, by $F_N = (G^2_a + G^2_b + G^2_c + G^2_d)/2$. In the same vein, we estimate the tangential force of each link, $F_T$, by $F_T = (G^2_a - G^2_b + G^2_c - G^2_d)/2$.

C. Calibration and units

We compute $G^2$, the average of $G^2_i$ over space, and compare it with the force $F$, measured by the force sensor (d), normalized by $Mg$, the total weight of the grain assembly (figure 4(a)). One observes a linear relationship between the force measured at the piston and the sum of $G^2_i$ over the entire picture, $G^2$. In the following, we use this same linear relation to calibrate the local $G^2_i$. Below, all pressures and forces computed using the photo-elastic images, are expressed in units of $Mg$. Lengths are expressed in units of the small grain diameter and time is expressed in units of the microscopic time determined by the stiffness of two compressed discs: $t_0 = (k/m)^{-1/2}$, where $m$ is the mass of a grain ($\sim 3.75 \times 10^{-5}$ kg) and $k$ is the stiffness of two compressed discs ($\sim 1.5 \times 10^3$ N/m).

D. Protocol: Obtaining a granular glass

As already emphasized in the introduction, the Jamming transition is intrinsically a $T = 0$, out-of-
equilibrium transition, and therefore depends on the protocol followed to prepare the packing of interest. The situation need not be made simpler by the introduction of thermal or mechanical vibration. Indeed, for the packing fractions of interest, most systems become naturally dynamically arrested in non-equilibrium glassy states. The steep increase of the relaxation times associated with glassy behavior seriously hampers experimental work. samples brought to the high packing fractions of Jamming are deep into the glass phase and are difficult to manipulate on reasonable timescales. For athermal granular media, the situation is similar: they need some mechanical energy to be maintained in a non-equilibrium steady-state (NESS). As for thermal systems, this requires extremely slow compaction of the sample in order to avoid aging dynamics on the experimental timescales. For that reason, most granular experiments actually probe the glass transition and not the Jamming transition.

Here, we perform an annealed compaction (figure 4b), i.e., we increase packing fraction by constant amounts of \( \delta \phi = 3 \times 10^{-4} \), with exponentially increasing time steps. Then, the packing fraction is stepwise decreased, and measurements are performed between the decompression steps (figure 1a-b)). Lechenault et al. checked that the dynamics is reversible and stationary on experimental time scales during these decompression steps. As suggested by figure 5, the structure of the packing we obtain following the above protocol is frozen: the superimposition of two Voronoi tessellations, separated by a time lag of 5000 vibration cycles, display very few rearrangements, even for the lowest packing fraction. Such rearrangements are further quantified by \( Q^{nn}(\tau) \), the average fraction of neighbor relationships surviving in a time interval \( \tau \). Plotted with respect to the lag time, \( \tau \), \( Q^{nn} \) remains larger than 95\% even for the loosest packing fraction, and barely departs from 1 for the densest ones (figure 4b)). In the language of the glass community, “there is no \( \alpha \) relaxation”, meaning that the density profile survives on the experimental time-scale and the system can safely be considered as a glass, the structure of which is essentially frozen.

Finally, note that despite the fact that we perform the same protocol for each experiment, the initial conditions are still different for each run. Also, the system size is finite, and therefore, the Jamming transition of each packing will fluctuate from one realization to another. It is important to keep this in mind when comparing independent experimental runs.

III. PRESSURE AND CONTACT FORCES

For the ideal case of soft spheres at zero temperature, the pressure inside the packing exhibits the same basic features as the energy: below Jamming, it is strictly zero and above Jamming it grows with the packing fraction, according to the interaction force between particles. It is thus of interest, as a first sight at the transition in a system with dynamics, to look at the dependence of the pressure with the packing fraction.

Figure 6 displays the pressure measured at the wall as a function of the packing fraction. \( P_{TOT} \) (respectively \( P_{STAT} \)) is the pressure measured when the vibration is applied (vibration on) or not (vibration off). \( P_{STAT} \) corresponds to the static pressure sustained by the packing whereas \( P_{DYN} = P_{TOT} - P_{STAT} \) is the dynamic part of the pressure that comes from the vibration. One observes a smooth crossover from a constant, but nonzero pressure, to an pressure that increases with the packing fraction. On the large packing fraction side of the crossover, \( P_{TOT} \approx P_{STAT} \) and the pressure, which is

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**FIG. 5:** Obtaining a granular glass. (a): Superposition of the Voronoi cells computed at times \( t = 1 \) and \( t = 5000 \) for the loosest packing \( (\phi = 0.8031) \). (b): Average fraction of neighbors \( Q^{nn}(\tau) \) which have not changed between two images separated by a time interval \( \tau \), for different packing fractions. The vibration frequency is 10 Hz and the packing fraction takes 13 values in the range \([0.80 - 0.82]\). The color code spans from blue (low packing fractions) to red (high packing fractions).

**FIG. 6:** Wall pressure vs. packing fraction: (○): \( P_{TOT} \), (□): \( P_{STAT} \), (△): \( P_{DYN} \), as defined in the text for the present PSM-4 disks experiment. Note: the finite stiffness of the piston has been calibrated and removed from the data. The vibration frequency is \( f = 10 \) Hz.
mostly static, follows what is expected from the zero temperature prediction: it increases with packing fraction, according to the particle stiffness. On the low packing fraction side, there is an irreducible kinetic part of the pressure, induced by the vibration. To zeroth order, the crossover corresponding to Jamming can be identified with the packing fraction where the static pressure becomes larger than the kinetic one. Note that the static part of the pressure is not strictly zero below the crossover. We attribute this to the mobilization of the friction at the contacts, when the vibrating board is stopped. We return to the possible roles of friction in the discussion section.

One must realize that the kinetic part of the pressure, which is observed on the loose side of the Jamming crossover, does not strictly speaking come from collisions of the grains with the wall. Indeed, the instantaneous dynamics is very different from that of a thermal liquid, where the pressure has a collisional origin. Here, the forcing is periodic and a priori strongly anisotropic. The particles are accelerated along the vibration axis, then compressed along one wall, before being accelerated back in the reverse direction. A clearer idea of this process comes from the dynamics of the average inter-particles forces during a few vibration cycles, and by a decomposition of these forces into the vibration and transverse directions:

\[ F_X = \sqrt{\langle (\vec{f}_{ij} \cdot \vec{e}_X)^2 \rangle} \quad \text{and} \quad F_Y = \sqrt{\langle (\vec{f}_{ij} \cdot \vec{e}_Y)^2 \rangle}, \]

where \( \langle \cdot \rangle \) is the average over space, and where \( \vec{e}_X \) and \( \vec{e}_Y \) are unit vectors along the vibration and the transverse directions.

One observes in figure 7(a) that for the low packing fractions, there are strong oscillations at the vibration frequency. These oscillations correspond to the compression of the grains on the side walls. Interestingly, these oscillations are in phase, within the temporal resolution of the acquisition: the transfer of momentum, from the direction of vibration to the transverse direction, is instantaneous, as compared to the time scales considered here.

For the packing fraction above the kinetic to static crossover, the oscillations are much less pronounced, and the periodicity not so clearly defined: in that regime, the global motion of the grains with respect to the oscillating plate is reduced. Finally, averaging temporally those signals, and plotting the averages as functions of the packing fraction (see figure 7(b)), the same trends occur as those observed for the pressure measured at the wall, albeit with larger fluctuations, since the temporal sampling is much smaller. Note that the average force in the direction of vibration is only slightly larger than the average force in the transverse direction, indicating that the redistribution of momentum ensures the formation of a rather isotropic force network. We confirm this by a direct inspection of the pressure field inside each grain \( G_i \), interpolated on a cartesian grid, and the computation of the spatial auto-correlation function (figure 8). They both confirm a good level of isotropy of the pressure distribution in the packing.

We conclude this section by inspecting the probability distribution of the normal and tangential inter-particle forces \( F_N \) and \( F_T \) defined by \( \vec{f}_{ij} = F_N \vec{r}_{ij} + F_T \vec{t}_{ij} \), where \( \vec{r}_{ij} \) (respectively \( \vec{t}_{ij} \)) is the normal (respectively tangential) vector between the two grains \( i \) and \( j \). To be precise, we consider the distribution of \( \tilde{F}_N = [F_N(i,t)]/[G(i,t)] \langle G(i,t) \rangle_{i,t} \) and \( \tilde{F}_T = [F_T(i,t)]/[G(i,t)] \langle G(i,t) \rangle_{i,t} \), where \( G_{ij} \) is the instantaneous average over the particles and \( \langle \cdot \rangle_{i,t} \) is the average over time and space. Such a normalization has the advantage of capturing the shape and width of the distributions, without including the temporal variability of the packings \( \phi \), hence avoiding spuriously large tail distributions. Also, since the normalization is the same for \( F_N \) and \( F_T \), the ratio of \( \tilde{F}_T/\tilde{F}_N = F_T/F_N \), which ensures a correct interpretation in terms of friction coefficient. The distributions (figure 9(a) and (b)) have exponential tails at all packing fractions, and widen as the packing fraction is increased. This is consistent with existing works on granular packings [38, 39]. Note, however, that the existing consensus on the exponential tails of force distributions is not founded on any unambiguous

FIG. 7: Short time photo-elastic response. (a): Average interparticle force decomposed into the vibration \( F_X \) and the transverse \( F_Y \) directions vs. time \( t \) for packing fractions \( \phi = 0.8079 \) (blue), 0.8123 (green) and 0.8196 (red). (b): Time averaged quantities: \( \tilde{F}_X \) (blue), \( \tilde{F}_Y \) (red) and \( \tilde{F} = (\tilde{F}_X + \tilde{F}_Y)/\sqrt{2} \) (green) vs. packing fraction \( \phi \). The vibration frequency is \( f = 10 \) Hz.

FIG. 8: Isotropy of the force network. (a): Interpolated instantaneous \( G_i \)’s on a cartesian grid and (b) its associated 2d-autocorrelation. The packing fraction is \( \phi = 0.82 \). The vibration frequency is \( f = 10 \) Hz.
arguments, and that some studies report non-exponential tails [27,40,41]. Leaving aside this debate, we choose to focus on the joint distributions of $F_T$ and $F_N$ (figure 9(c) and (d)). The ratio $F_T/F_N$ is close to 0.2, on average, and always smaller than 0.7, which provides a good estimate for the static friction between the PSM-4 disks. One also notices an accumulation of contacts close to the threshold value $\mu_s$, especially at low forces, where a gap in the distribution clearly separates a majority of contacts with $F_T/F_N \approx 0.2$ from a secondary peak of contacts with $F_T/F_N \leq \mu_s = 0.7$. These so-called “critical contacts” are on the verge of slipping. Whether these slipping events are trivial fluctuations, or contain some interesting correlations in the vicinity of the Jamming crossover, was the central issue discussed recently by the present authors [26]. This issue will be recast in section IV.B.

IV. DYNAMICS OF THE CONTACT NETWORK

In order to measure $z_i(t)$, the number of contacts of particle $i$ at a given time $t$, one must identify the potential contacts of particle $i$ with its neighbors by thresholding the normal force $F_N$ and the inter-particle distance, $s$. We have shown [26] that the overall behavior, including the statics and the dynamics, remains unchanged when varying the thresholds within a reasonable range. Here, we shall keep the threshold fixed, and focus on the dynamics of the contact network both at short and long times.

A. Statics and short time dynamics

Figure 10(a) shows the average number of contacts $z$, computed over the images acquired in a stroboscopic way at a vibration frequency of 10 Hz, vs. the packing fraction. These data exhibit a clear cusp at any given packing fraction, $\phi^\dagger$. For packing fractions larger than $\phi^\dagger$, $z$ increases with the packing fraction, in a way that is similar to what is reported for zero-temperature soft spheres. By contrast, for packing fractions smaller than $\phi^\dagger$, $z$ is non-zero constant, and there is no discontinuity across $\phi^\dagger$, in contrast with the zero temperature behavior of soft spheres.

In terms of the pressure signal, a more precise pic-
tune of the mechanisms at play behind the shape of the $z(\phi)$ dependence can be obtained by examining the dynamics during a vibration cycle. Figure 10(b) displays the instantaneous contact number, $z(t) = \frac{1}{N} \sum_{i=1}^{N} z_i(t)$, acquired with the fast camera for three different packing fractions. Here, $N$ is the number of particles. For lower packing fractions, strong oscillations at the vibration frequency are clearly visible, while they are reduced and not so well defined at larger packing fractions. The similarity with the force signals reported in figure 11(a) is striking, and one easily understands that the number of contacts is temporarily larger when the grains are compressed against the wall. As a result, the average number of contacts computed from the stroboscopic data depends on the precise phase at which the acquisition is performed and this dependence is most significant when the packing fraction is low. This is illustrated in figure 11(c), where temporal averages of the contacts number, acquired at different phases, are plotted as a function of the packing fraction. In green, is the contacts number averaged over time frames which are in phase with the minimal acceleration: the grains are "away" from the walls, and the number of contact is minimal too. This situtation corresponds to the vertical green dotted lines in figure 10(b). There is no longer any evidence of a cusp, and the crossover is only indicated by an inflexion point that is barely discernable. In red, is the contact number averaged over time frames where the grains are compressed on one of the walls and the number of contacts is maximal; this situation corresponds to the vertical red dotted lines in figure 10(b). One recovers the cusp observed in figure 10(a), for which the stroboscopic acquisition was indeed performed in phase with the maximal acceleration and the minimal velocity of the plate. As explained in section 4 this choice of phase minimizes blur in the images.

Altogether, the presence of the cusp is related to the vibrational forcing and to the specific phase at which the stroboscopic acquisition is performed. There is no singularity in the dependence of the average contact number as a function of the packing fraction. As for the pressure, the signature of the Jamming transition is replaced by a crossover, the precise location of which depends on the details of the measure. Here, we were lucky enough to capture the images at the phase of the vibration, for which the crossover $\phi^+$ is most easily identified (red curve of figure 10(a)). One should however keep in mind that the green curve in the same figure is actually a more realistic dependence of the average number of contacts with the packing fraction $\phi^-$.

**B. Long times dynamics**

The results of this section, which concern the long time dynamics of the contact network and the nontrivial correlations that it contains, have been reported previously [26]. Here, we recast the important message they convey, together with new results.

To characterize the dynamics of the contact network, we introduce the contact overlap function, which evaluates how much the contacts have fluctuated between $t$ and $t + \tau$:

$$Q^z(t, \tau) = \frac{1}{N} \sum_{i=1}^{N} Q^z_i(t, \tau),$$

where

$$Q^z_i(t, \tau) = \left\{ \begin{array}{ll} 1 & \text{if } |z_i(t + \tau) - z_i(t)| \leq 1 \\ 0 & \text{if } |z_i(t + \tau) - z_i(t)| > 1 \end{array} \right.$$ (2)

Other choices of overlap functions are possible, and have been tried: the present results do not crucially depend on the particular choice. Figure 11(a) displays the temporal average of $Q^z(t, \tau)$ for a vibration frequency of 10 Hz and the same set of packing fraction $\phi \in [0.80-0.82]$ as in the above sections. At rather large packing fractions, $Q^z(\tau)$ is constant, with a plateau value, which depends weakly on the packing fraction. Hence, there is no relaxation on long time scales of the contact network. The relaxation, which occurs at short times, and is responsible for the plateau value, cannot be observed in the present stroboscopic data. However, it is apparently related to

![Figure 11: relaxation dynamics of the contact network.](color online)

(a): Temporal average of the overlap function $Q^z(\tau)$. Packing fractions as in figure 5. (b): Plateau value of $Q^z$, defined by $Q^z = Q^z(\tau = 4)$ (▲). (c): Relaxation time of the contact network, $\tau^+_z$ (see text for definition) (■, left axis), and average contact number, $z$ (x, right axis), vs. packing fraction $\phi$. The plain red line is a fit of the form $\tau^+_z \sim (\phi^+ - \phi)^{-2.0}$. The dashed line indicates $\phi^+ = 0.8151$. The vibration frequency $f = 10$ Hz.
the motion of the rattling particles, i.e., particles having less than 2 contacts, as suggested by the very strong correlation observed between the fraction of non-rattling particles and the value of the plateau at short-times (see figure 11b)). At lower packing packing fractions, a long time decorrelation sets in. We define the relaxation time of the contact network, $\tau^*_\alpha$, such that $Q^2(\tau^*_\alpha) = 0.9$. Note that this value of 0.9 is rather large as compared to the most commonly used value of 0.5. However, it is the smallest one which allows the measure of $\tau^*_\alpha$ for a broad range of packing fractions. We note that relaxation times measured in a standard way would be orders of magnitude larger. As shown in figure 11(c), left axis, $\tau^*_\alpha$ increases sharply with the packing fraction, and possibly diverges at the packing fraction $\phi^1 = 0.8151$, where the average number of contact starts to increase with the packing fraction.

Interestingly, the dynamics of the contact network below $\phi^1$ exhibits strong fluctuations and dynamical heterogeneities, albeit of a different kind from those reported in the literature, when studying the dynamics of supercooled liquids close to their glass transition (see 13). Here, the heterogeneities are relative to the degrees of freedom describing the contacts, not the position of the particles. To quantify such heterogeneities, one can compute the dynamical susceptibility which estimates the range of spatial correlations in the dynamics of the contact network:

$$\chi^*_\alpha(t, \tau) = N \frac{\text{Var}(Q^2(t, \tau))}{\langle \text{Var}(Q^2(t, \tau)) \rangle_i},$$

where $\text{Var}(.)$ denotes the variances sampled over time and $\langle . \rangle_i$ denotes the average over the grains. $\chi^*_\alpha(\tau)$ has a maximum for $\tau = \tau^*$ (not shown here, see 26), and we have studied how the maximum $\chi^*_\alpha$ of $\chi^*_\alpha(\tau)$ depends on both the packing fraction and the vibration frequency. To do so, it was necessary to prepare different packings, and run independent experiments at three different vibration frequencies: $f = 6.25, 7.5$ and 10 Hz. As emphasized in section 11 the precise value of the Jamming transition, and certainly those of the crossovers reported here, depend on the specific packing. Hence, following the methodology of section 11A for each frequency: we identified the structural crossover $\phi^1$, from which we define a reduced packing fraction $\epsilon = (\phi - \phi^1) / \phi^1$, in order to compare the different experimental runs. Note that for frequencies smaller than $f_0 = 4.17$ Hz, the grains do not slip on the driving plate, and the mechanical excitation is effectively null. Accordingly, we introduce $\gamma = (f - f_0) / f_0$, $\gamma = 0.5, 0.8$, and 1.4, to quantify the level of mechanical excitation.

The results are summarized in figure 12 and can be found in more detail in 20. $\chi_4^*$ is non-monotonic with respect to the reduced packing fraction, and has a maximum value at a negative reduced packing fraction $\epsilon^*$. This indicates the existence of a dynamical crossover corresponding to a maximally collective relaxation of the contact network at a packing fraction lower than the structural crossover. Also, when $\gamma$ is decreased one observes that (i) $\epsilon^*$ vanishes, i.e., the location of the dynamical crossover moves towards $\phi^1$, and (ii) the magnitude of the maximum $\chi_4^*$ significantly increases as $1 / \gamma$. Hence, we can safely conjecture that in the limit of no effective mechanical excitation the structural and dynamical crossovers merge, while the length scale associated with the dynamical crossover diverges. This strongly suggests, that we have probed the vicinity of a critical point, which in the present case ought to be the Jamming transition in the absence of dynamics. As a matter of fact, a similar phenomenology occurs for equilibrium systems close to a thermodynamic critical point: at the critical point, thermodynamic susceptibilities diverge and away from it, in the supercritical region, they exhibit finite maxima. These are the so-called Widom lines 11. Recent experiments 44 probing the dispersion of nano-metric acoustic waves report a crossover of the acoustic dispersion along one of such Widom lines and demonstrate the existence of a dynamical crossover involving subtle mechanisms at the particle scale in the supercritical region a thermodynamical critical point 46.

The above results clearly indicate that the mechanical agitation blurs the singular nature of the Jamming transition. This is a similar effect to one reported in the presence of thermal agitation for the Jamming transition of soft spheres 1. One of the remarkable results of that work is that the authors demonstrate in a convincing manner that all the physics of the soft sphere systems close to Jamming can be captured by a careful examination of the mean square displacement of the particles as a function of time. They can then use this measure as an effective thermometer to locate existing colloidal experiments in the Temperature-Packing fraction parameter space. Is it possible to extend the approach to the present case of vibrated granular media? If yes, where does the present experiment sits in an equivalent parameter space?
V. DISPLACEMENTS FIELDS

In order to answer the above questions, one needs to extract the mean square displacement of the particles on the largest possible range of timescales. While this is a straightforward but CPU costly task in numerical simulations, we shall see that it requires rather intricate data analysis in the present experiment. The reasons are twofold. First, the short time and long dynamics are acquired independently and in very different ways. While the long-time dynamics is acquired in phase with the vibration, the short time dynamics are acquired within a vibration cycle. The long-time acquisition naturally filters the “trivial” motion of the plate, but the short time does not, and we will have to filter it out. Second, we shall see that on long time scales, low-amplitude convection occurs. Although the resulting flow is never large—it mostly consists of a non-monotonic solid body rotation—we shall remove it before computing the mean square displacement.

A. Short time oscillations

The motion of the center of mass \((X_b(t), Y_b(t)) = ((X_i(t))_i, (Y_i(t))_i)\) provides a good indication of the way the energy is injected in the system at large scale. Figures 13(a) shows that the center of mass oscillates periodically, with a period equal to the forcing frequency. The amplitude of the motion is much larger in the direction of vibration, but part of the forcing is transferred to the transverse direction too. As shown in figure 13(b), the amplitudes of the oscillations, \(A_b^X\) and \(A_b^Y\), depend on the packing fraction: they are constant at low packing fractions, typically when \(\phi < \phi^*\), and they decrease for larger packing fractions, suggesting that energy injection is less efficient at large packing fractions.

In order to investigate the way the energy is transferred to smaller scales, we compute the averaged spectral density of the positions fluctuations. Specifically, we define \((\widehat{X}_i(t) = X_i(t) - X_b(t), \widehat{Y}_i(t) = Y_i(t) - Y_b(t))\), corresponding to the grain trajectories in the frame of reference of the oscillating center of mass. We next compute \(\widehat{X}_i^2(f) = ESD(\widehat{X}_i(t) - (\widehat{X}_i(t))_t)\), and similarly \(\widehat{Y}_i^2(f)\), where \(ESD\) denotes the Fourier energy spectral density (ESD). We then average over the grains to obtain the spectra \(\bar{X}_i^2 = \langle \widehat{X}_i^2(f) \rangle_i\) and \(\bar{Y}_i^2 = \langle \widehat{Y}_i^2(f) \rangle_i\), displayed in figure 14(a).

The energy cascades down to high frequencies, which unfortunately preserves the signature of the periodic forcing in the form of strong harmonics. This indicates that considering the motion of the grains in the frame of the center of mass is not sufficient to completely filter out the periodic motion induced by the moving plate. We thus further filter the grain trajectories by applying a band-cut Butterworth filter centered on each harmonic (up to the fifth) and a low-pass Butterworth filter with a cut-off frequency of 5 times the vibration frequency, on \(\bar{X}_i(t)\) and \(\bar{Y}_i(t)\). The spectra of the filtered trajectories, \(\Delta_i^2 = \bar{X}_i^2 + \bar{Y}_i^2\) (figure 14(b)) confirm that the harmonics have been successfully filtered out. The resulting motion \(\Delta_0^2\) at the lowest frequency \((f_0 = 0.3)\), corresponding to a timescale of a few vibration cycles, is a good indicator of the typical cage size in which the particle vibrates. It strongly depends on the packing fraction and sharply decreases as the Jamming crossovers are crossed. The absolute magnitude of \(\Delta_0^2\) (10^{-6} to 10^{-4}) corresponds to a typical cage size of \(\sim 10^{-3}\) to \(10^{-2}\) grain diameter.

In the remainder of the paper, we will apply the filtering procedure described here on each grain trajectory prior to computing any statistical property for the fast camera data.
where

\[ \text{One defines:} \]

This solid body rotation from the grain displacements of the cell (figure 15(b)). It is fairly easy to remove the distance \( \mathcal{D}_r \) (c): (d) (a): (b): Orthoradial displacements vs. distance to center for a lag time \( \tau = 6 \times 10^6 \) at \( \epsilon = -0.0948 \), and \( \gamma = 1.4 \).

(c): \( \Omega/\tau \) vs. time, \( t \), for different lag times \( \tau = 10^9 \) (green), \( \tau = 105 \) (red) at a reduced packing fraction, \( \epsilon = -0.0948 \).

(e): Rotational drift coefficient \( \Omega \) vs. reduced packing fraction, \( \epsilon \). The vibration frequency \( f = 10 \text{ Hz} \), i.e \( \gamma = 1.4 \).

B. Long time rotation

We now turn to the stroboscopic trajectories of the grains. Figure 15(a) displays the displacement of all grains in the region of interest (ROI), integrated over a lag time \( \tau = 6 \times 10^6 \). The inset provides a zoom on the trajectories of a few grains at the edge of the ROI. One observes a clear global rotation, which, curiously and fortunately, is essentially solid body motion, as demonstrated by the linear dependence of the orthoradial displacement \( R_i(t, t + \tau)\left(\theta_i(t + \tau) - \theta_i(t)\right) \) with the distance to the center of the cell (figure 15(b)). It is fairly easy to remove this solid body rotation from the grain displacements \( \Delta_r \tilde{r}_i(t) = \tilde{r}_i(t + \tau) - \tilde{r}_i(t) \).

One defines:

\[ \Delta_r \tilde{r}_i(t) = \Delta_r \tilde{r}_i(t) - \Delta_r \tilde{r}_i(t), \]

where

\[ \Delta_r \tilde{r}_i(t) = \begin{pmatrix} 0 & \Omega(t) \tau & 0 \end{pmatrix} \begin{pmatrix} \tilde{r}_i(t + \tau) - \tilde{r}_i(0, t) \end{pmatrix}, \]

is the solid rotation deformation field. The values of the angular velocity \( \Omega(t) \) and the center of rotation \( \tilde{r}_i^0(t) \) are explicitly computed from the displacements \( \Delta_r \tilde{r}_i(t) \), by minimizing \( \langle \|\tilde{r}_i(t + \tau) - \tilde{r}_i(t) - \Delta_r \tilde{r}_i(t)\|^2 \rangle_i \), with respect to \( \Omega(t) \) and \( \tilde{r}_i^0(t) \). One finds:

\[ \Omega(t) = -\frac{\sum_{i=1}^N \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \left( \tilde{r}_i(t + \tau) - \tilde{r}_i(t) \right) \cdot \tilde{r}_i(t + \tau) - \tilde{r}_i(t) \right)^2}{\tau \sum_{i=1}^N \left\| \tilde{r}_i(t + \tau) - \tilde{r}_i(t) \right\|^2} \]

and

\[ \tilde{r}_i^0(t) = \Omega(t) \tau \sum_{i=1}^N \left( \begin{array}{cc} 0 & 1 \\ -1 & 0 \end{array} \right) \left( \tilde{r}_i(t + \tau) - \tilde{r}_i(t) \right). \]

Figure 15(c) reveals that \( \Omega \times \tau(t) \), the angular rotation between times \( t \) and \( t + \tau \), fluctuates around zero, meaning that the solid body rotation has no preferred direction. As a result, there is no statistically systematic drift in any direction. However, for any finite time interval, \( [t, t + \tau] \), there is a finite angular displacement, the magnitude of which is controlled by \( \langle \Omega^2 \rangle \). As shown in Figure 15(d), it sharply decreases as the packing fraction increases across the Jamming crossovers.

C. Resulting vibrating dynamics

Now that both the short time “trivial” oscillating motion and the long time convection have been filtered out, we are in a position to characterize the vibrating dynamics of the grains in the frozen structure of the packing on time scales ranging from a hundredth of a cycle to several thousands cycles. We compute the following estimator of the mean square displacement:

\[ MSD = \frac{\pi}{2} \langle \langle \Delta_r \tilde{r}^{-1} \rangle \rangle \]

where \( \langle \ldots \rangle \) denotes the average over times and particles, and \( \Delta_r \tilde{r} \) is the particle displacement obtained from the filtering procedures described in the previous section. The choice of this estimator is motivated by the fact that it ensures a lower statistical weight to very large moves, such as those of the rattling particles. The factor \( \pi/2 \) ensures quantitative matching with the proper mean square displacement in the case of gaussian statistics. Alternatively, one could remove the rattling particles, but that strategy requires additional filtering and or thresholding. Figure 16 displays the mean square displacement over the full timescale interval probed in this experimental study. We again emphasize that the data at short times, shorter than \( 10^3 \), were obtained from the fast recording of the grain motion within the vibrating cycles, while those at long times were obtained performing stroboscopic acquisition in phase with the oscillating driving plate. Each type of acquisition were performed during independent experimental runs. The color codes the packing fraction.
The good overlap of the mean square displacement at intermediate time scales is not enforced and is remarkably good.

Altogether, one observes three regimes: a ballistic regime at short time $\tau < \tau_{En}$, a plateau at intermediate time scales, $\tau_{En} < \tau$, and for low enough packing fraction a crossover towards a diffusive regime at long time scales. The plateau regime characterizes the vibrational dynamics we are interested in. The height of the plateau, $\Delta^2$, measures the square of the average vibration amplitude of the grains within their cage. It decreases from $10^{-4}$ to $10^{-5}$ for increasing packing fractions (figure 16(b)), and it is consistent with the first estimate of the cage size, we had obtained in section IV A, from the low frequency limit of the Fourier spectral density of the position fluctuations, $\Delta^2 f$. The short time entrance to the plateau, estimated by $\tau_{En} = (\Delta^2/K)^{1/2}$, where $K \simeq 10^{-8}$ is obtained from the analysis of the ballistic regime, typically occurs at $\tau_{En} \sim 100$ and slightly decreases as the packing fraction is increased (figure 16(c)-left axis); the larger the packing fraction, the sooner the grains feel their neighbors and enter the vibrational regime.

The above vibrational dynamics is very similar to the one reported for thermal harmonic sphere systems close to Jamming [1, 47]. In this later study, a ballistic regime occurs at short time, followed by a plateau regime, the height of which decreases strongly with the packing fraction when crossing the Jamming point. A plateau exit is also reported in [47], where the authors show that the plateau exit time increases when the quench rate used to prepare the packing is decreased. This plateau exit is not reported in [1]. However, the maximum lag time there was $10^4$, and the systems were carefully equilibrated, so that the plateau exit, if it existed, was probably much larger than the simulated timescales. Before addressing the more quantitative comparison, which will allow us to discuss whether thermal soft spheres are a good model for mechanically excited grains, we will finish the description of the dynamics by characterizing its heterogeneities. Note that these heterogeneities, first reported in the brass grains experiment [23] and more recently in the harmonic spheres simulation [1] are distinct from those encountered in super-cooled liquids when approaching the glass transition [48]. Here, the structure is frozen, hence, the heterogeneities are not related to the relaxation of the structure. The next section will show how they are related to the heterogeneities of the contact dynamics described in section IV B.

VI. DYNAMICAL HETEROGENEITIES

In this section, we investigate the heterogeneities of the particle displacements. To do so, we focus on the long time stroboscopic data, once the convective motion has been subtracted. We will show that these heterogeneities take place at very small scales and are temporally correlated to the heterogeneities of the contact dynamics. Finally, a closer look at the organization of the contacts at short time will demonstrate that these heterogeneities take their root in the short time organization of the contact network, namely in the vibrational dynamics of the structure.

A. Heterogeneous non-affine dynamics

The characterization of dynamical heterogeneities has now become a standard tool in the study of the dynamical slowing down of super-cooled liquids and/or colloids approaching their glass transition [48]. It is much less frequently used when probing the Jamming transition, but relies on the same procedure [10]. In order to characterize the dynamics, and in particular to probe collective effects, one defines a dynamical structure factor for the
FIG. 17: **Dynamical heterogeneities.** (color online) (a): Dynamic susceptibility of the displacements \( \chi_4^\ast(a^\ast, \tau) \) vs. lag time \( \tau \). Same packing fractions as in figure 5 (b): Maximal dynamical susceptibility of the displacements \( \chi_4^\ast \) vs. reduced packing fraction \( \epsilon \). (c): \( \tau^\ast \) vs. reduced packing fraction \( \epsilon \). (d): \( a^\ast \) vs. reduced packing fraction \( \epsilon \). Dashed lines in frames (b,c,d) indicate \( \epsilon^\ast \). The vibration frequency \( f = 10 \) Hz, i.e, \( \gamma = 1.4 \).

Displacements, \( \Delta \vec{r}_i(t) \) (defined in eq. 4):

\[
Q^\ast_i(t, \tau, a) = \frac{1}{N} \sum_i Q_i^\ast(t, \tau, a),
\]

where

\[
Q_i^\ast(t, \tau, a) = \exp(-||\Delta \vec{r}_i(t)||^2/2a^2).
\]

This dynamical structure factor probes the dynamics at scale, \( a \), and time \( \tau \): \( Q_i^\ast(t, \tau, a) \approx 0(1) \), when the particle \( i \) has moved more (less) than \( a \), during \( \tau \). One then defines the dynamic susceptibility:

\[
\chi_4^\ast(a, \tau) = \frac{1}{\langle V ar(Q_i^\ast(t, \tau, a)) \rangle} \langle V ar(Q^\ast(t, \tau, a)) \rangle,
\]

where \( V ar \) denotes the temporal variance. It provides an estimate of the average number of particles which move up to the distance \( a \) during a time \( \tau \) in a correlated manner. In general \( \chi_4^\ast(a, \tau) \) has an absolute maximum \( \chi_4^\ast \) for \( a = a^\ast \) and \( \tau = \tau^\ast \) (see for instance 22).

Figure 17 illustrates the way the dynamical heterogeneities depend on the packing fraction. The most important effect is that \( \chi_4^\ast \) is nonmonotonous and exhibits a clear maximum at precisely the reduced packing fraction \( \epsilon^\ast \) (figure 17(b)). The magnitude of \( \chi_4^\ast \) close to \( \epsilon^\ast \) is close to 100, roughly a tenth of the total number of particles. (Even closer to \( \epsilon^\ast \), one may note the data point indicated by \( \square \), which is anomalously low compared to the trend given by the other data points. We believe that this is a signature of the lack of statistics necessary to resolve much larger heterogeneities.) The timescale, \( \tau^\ast \), where this maximum occurs, is not very sharply defined (note the logarithmic scale for \( \tau \), as can be seen from the dependance of \( \chi_4^\ast(a^\ast, \tau) \) on \( \tau \) (figure 17(a))). But it clearly increases significantly when the packing fraction increases and certainly is larger than the timescales for packing fractions larger than \( \phi^\ast \) (figure 17(c)). The length scale, \( a^\ast \), over which the particles move while building up the heterogeneities, decreases with the packing fraction and is of the order of \( 10^{-2}d \) (figure 17(d)). The same observation made in the case of the brass disks, 23, lead the authors to conclude that the dynamical heterogeneities observed close to Jamming have their origin in the dynamics of the contacts. We are now in position to confirm this intuition.

B. Relation to contact dynamics

The fact that the heterogeneities observed in the dynamics of the contact and in the displacement field are both maximal at the same value of the reduced packing fraction \( \epsilon^\ast \) is already a strong indication that they have a common origin. This is further confirmed by the quantitative comparisons of \( \chi_4^\ast \) to \( \chi_4^\ast \) and of \( \tau^\ast \) to \( \tau^\ast \) provided in figure 18(a) and (b). \( \chi_4^\ast \) and \( \tau^\ast \) are respectively proportional to \( \chi_4^\ast \) and \( \tau^\ast \) confirming a strong correlation between the two aspects of the dynamics. As in figure 17(b), the \( \square \) data point is way off the trend given by the other data points, because of a lack of statistics at the Jamming crossover (see previous section). Whereas the timescales are essentially identical, the dynamical susceptibility associated with the displacements is 20 times larger than that associated with the contacts. One must remain cautious in the interpretation of such a factor, since the dynamical susceptibilities are only an indicator of the number of elements correlated, even when they are properly normalized by the intrinsic fluctuations: the shape of the spatial correlator also enters into play. With that caveat, such a large factor suggests that the spatial organization of the dynamics is different in the two cases. This is indeed confirmed in figure 18(c) and (d), which shows the snapshots of respectively \( Q_i^\ast(t, \tau^\ast) \) and \( Q_i^\ast(t, \tau^\ast, a^\ast) \), taken at the same time. Whereas the dynamical heterogeneities of the displacements are organized in well identified large clusters, those of the contacts seem more scattered in smaller chain-like clusters. The dynamical correspondance is not simply that the particles moving more than \( a^\ast \) lose or gain contacts. On the contrary, it suggests that the loss of contact at some place induces motions on the scale of \( a^\ast \) further away, and in turn, the loss of other contacts.
frequency is reduced towards the zero mechanical excitation limit. One would expect the same to happen for the heterogeneities of the displacements. However, we have also seen that the system size limits the largest heterogeneities of the displacements. And indeed, when we reduce the vibration, there is saturation of the displacement heterogeneities, while those for the contacts increase fourfold (figure 18(c) and (d)).

FIG. 18: Dynamical heterogeneities. (a): Maximal dynamical susceptibility of the displacements $\chi_4^\ast$ vs. maximal dynamical susceptibility of the contacts $\chi_4^\ast$ and (b): $\tau^\ast$ vs. $\tau^\ast$ in parametric plots, where each point correspond to a different packing fraction (same color code as in figure 9). The vibration frequency $f = 10$ Hz, i.e $\gamma = 1.4$. (c-d): Maps of $Q^\ast_i(t, \tau)$ (c) and $Q^{\ast\ast}_i(t, \tau, a_{1/2})$ (d), for $\epsilon = -0.0013$. (d): Color code spans from yellow ($Q^\ast_i(t, \tau = 0)$ to red ($Q^\ast_i(t, \tau = 1$). (d): Color code spans from blue ($Q^{\ast\ast}_i(t, \tau, a_{1/2} = 0$) to red ($Q^{\ast\ast}_i(t, \tau, a_{1/2} = 1$). The vibration frequency $f = 10$ Hz, i.e $\gamma = 1.4$. (e): Same plot as in (a) but for different values of $\gamma = 0.5$ (blue), 0.8 (green), and 1.4 (red). (f): Peak of the dynamical susceptibility of the displacements max($\chi_4^\ast$) and peak of dynamical susceptibility of the contacts max($\chi_4^\ast$) vs. $\gamma$.

C. Short time origin of the heterogeneities

Figure 17(a) indicates that the nonmonotonic dependence of $\chi_4^\ast$ on the packing fraction applies not only at timescales of $\sim \tau^\ast$, but is also manifested at the shortest timescales of the data acquired stroboscopically, that is for $\sim$ one cycle or $10^4$ microscopic times. The same holds true for the contacts. Figure 19(a) and (b) respectively display $\chi_4^\ast(t)$ and $\chi_4^{\ast\ast}(\tau)$, together with $\chi_4^\ast = \chi_4^\ast(\tau^\ast)$ as functions of the packing fraction: both are nonmonotonic, suggesting that the dynamical heterogeneities of the contact dynamics have roots in the structure of the contact network. Still, $\chi_4^{\ast\ast}$ is smaller than $\chi_4^\ast$, indicating that the heterogeneities, present at short time, build up progressively via a process which remains to be explained.

The above results suggest that the contact network itself is heterogeneous. Whereas a number of papers discuss the heterogeneities of the force network in terms of the force intensities, we are not aware of a detailed examination of the spatial correlations in the contact network. A map of the instantaneous contact network for packing fractions lower than $\phi_0$ is provided in figure 20(a). One immediately notices a rather heterogeneous organization, with rather large holes where there are very few contacts. After interpolating the contact number on a grid, we compute the radial dependence of its spatial autocorrelation $G_2^\ast(r)$. This quantity decays exponentially towards zero (figure 20(b)), with a typical decay length $\xi_2^\ast$ defined as $G_2^\ast(\xi_2^\ast) = 0.2$. $\xi_2^\ast$ is non-monotonic with respect to packing fraction (figure 20(c)), and has a small maximum at $\epsilon^\ast$: the spatial correlations of the contacts are maximal at $\epsilon^\ast$. An alternative and stronger evidence is provided by the static susceptibility

$$\chi_2^\ast = \frac{N}{\frac{1}{N} \sum_{i=1}^{N} \var r_i \var z_i(t)} \var r_i \var z_i,$$

(12)
rather singular in the sense that they concern very small displacements, of the order of $10^{-2}$ grain diameters, and occur for very large packing fraction as compared to those observed in other granular systems. There exist no other experimental evidence of such dynamical heterogeneities, except perhaps in one colloidal experiment, and in other quasi-static experiments by the authors elsewhere, but it remains unclear whether these different experiments probe the same physics. Only recently, similar observations have been reported in numerical simulations of soft spheres, a system that is a priori rather different from vibrated granular media.

Our first set of experiments conducted with soft photo-elastic disks confirmed results for a system consisting of brass disks, and lead to the observation that the similarities with the simulations of thermal soft spheres were stronger than expected. Since the authors of that numerical study argued that existing colloidal experiments are rather far from the critical regime of Jamming, either because the packing fractions are too loose, or because the temperature is too high, we chose to decrease the vibration frequency in our system and to explore the vicinity of the zero excitation limit. Indeed, one would like to know to what extent thermal harmonic spheres have anything to say about the dynamical criticality of the granular packings, and conversely, whether granular experiments can provide physical insight into the ideal system of harmonic spheres.

The discussion section below is organized as followed. After a brief synthesis of the results, we first compare and reconcile the observations performed for the hard (brass) and the soft (photo-elastic disks), before discussing the analogy between the thermal soft spheres and our experimental systems.

### A. Synthesis

We have conducted systematic experiments of horizontally vibrated grains, decreasing the packing fraction over a very small range of high packing fractions where the dynamics of both the contacts and of the positions of the grains is frozen. Despite a strongly anisotropic mechanical forcing at large scales, the system at the scale of the grain is isotropic: nonlinear mechanisms, together with disorder, redistribute the energy at small scales, causing the system to progressively lose any memory of the forcing anisotropy. This is roughly analogous to the energy cascade in turbulence.

As previously noted, by using fast stroboscopic acquisition, we computed the average displacements over more than six temporal decades, once the short term oscillating dynamics and long term convection have been removed. We clearly identify a ballistic regime, followed by a long plateau, eventually followed by a crossover to a very long time diffusive regime for low enough packing fractions. These observations allowed us to measure the size of the cages, $\Delta$, as a function of the packing fraction in several

## VII. DISCUSSION

We recall here the motivations which lead us to conduct this comprehensive study of vibrated photo-elastic disks.

First we sought to confirm our first observations of dynamical heterogeneities in a very dense system of vibrated brass disks. These heterogeneities are...
Within the timescales where the grains are trapped in their cage, two distinct crossovers are observed. One is “structural” in the sense that it is revealed by the average number of contacts, which starts increasing sharply at the packing fraction $\phi^s$. The second is “dynamical” in the sense that it is indicated by a maximum of the dynamical heterogeneities of both the contacts and the displacements at a packing fraction $\phi^d < \phi^s$. We have demonstrated that the “dynamical” crossover is rooted first in the structure of the contact network, and second, that it is related to the spatial fluctuations of the contact number. By contrast, the “structural” crossover is given by its average value. Both signatures converge to a unique packing fraction when the excitation is reduced towards the zero excitation limit. We interpret this packing fraction as the Jamming transition for the present experimental system and compression protocol. The critical nature of the transition is suggested by the sharp increase of the dynamical susceptibilities when the vibration is reduced towards the zero excitation limit. We also note that the range of packing fractions over which the crossovers are observed are very different. The crossovers occur for lower packing fractions and on a broader range in the case of the soft disks than in the case of the hard particles. This is not so surprising, given

B. Soft vs. Hard

In an earlier experiment, within the same apparatus but with hard (brass) disks [23, 24], the authors reported the first experimental evidence of dynamical heterogeneities involving very small displacements of grains, within a structure almost completely frozen. These dynamical heterogeneities were rather different from those observed close to the glass transition, and the authors correctly attributed their observation to Jamming. However, they could not precisely identify the underlying mechanism responsible for these heterogeneities. The present study has clearly demonstrated that the heterogeneities have their origin in the dynamics of the contact network. Also, the existence of this maximum in dynamical heterogeneities suggests that the experiment probed both sides of the Jamming transition, a puzzling conclusion given the stiffness of the brass disks. The present study with soft disks solves this apparent contradiction in the following way. We have seen that there are several signatures of point J at finite mechanical excitation, $\gamma$, and that the one associated with the dynamical heterogeneities occurs at a lower packing fraction, $\phi^*(\gamma)$, than the one at which the average number of contact increases, $\phi^1(\gamma)$.

In the case of the brass disks, the authors reported (see figure 21a)) that the maximum of the dynamical heterogeneities occurs for the packing fraction where $P_{\text{DYN}}(\phi)$ and $P_{\text{STAT}}(\phi)$ intersect. This is also the case for the soft disks (see figure 21b)): the experiment with the brass disks actually probed the dynamical crossover $\phi^s$, both sides of which lie below the structural signature of the Jamming transition. In the case of brass disks, it is not possible to measure the average number of contacts. However, assuming Hertz’ law, the stiffness of two compressed 4 mm height cylinders made of brass (Young modulus, $E = 100$ GPa) is $k_{\text{brass}} \sim 3 \times 10^8$ N/m. By comparison, the stiffness of the force sensor and piston system is $k_{\text{piston}} \sim 6.10^5$ N/m and the brass grains can be considered as hard. In that case, Jamming is the point at which the pressure diverges [42], and the packing fraction at which the pressure sharply increases (see figure 21a)), provides a good estimate of the structural crossover $\phi^1$.

We also note that the range of packing fractions over which the crossovers are observed are very different. The crossovers occur for lower packing fractions and on a broader range in the case of the soft disks than in the case of the hard particles. This is not so surprising, given

![FIG. 21: Hard vs. Soft. (color online) Piston force (top) and Maximal dynamical susceptibility of the displacements (bottom) vs. reduced packing fraction, $\epsilon$, for (a): hard brass disks [23] and (b): soft photo-elastic disks. (\( \square \)). $P_{\text{TOT}}$. (\( \square \)): $P_{\text{DYN}}$. (\( \bigtriangleup \)): $P_{\text{STAT}}$. (\( \bigtriangleup \)): $P_{\text{DYN}}$ as in figure [\( \square \)]. The vibration frequency $f = 10$ Hz, i.e. $\gamma = 1.4$. Dashed lines indicate $\epsilon^*$ and $\epsilon = 0$.](image)

![FIG. 22: Towards zero vibration (color online) (a): Maximum of the dynamical susceptibility of the contact maximum($\chi^*_4$) for soft grains (\( \times \)) and hard grains (\( \square \)), estimated from $\text{max}(\chi^*_4)/20$, versus the split $|\epsilon^*|$ between static and dynamics signatures of Jamming. (b): MSD Plateau vs. density $\epsilon$, for $\gamma = 0.5$ (\( \square \)), $\gamma = 0.8$ (\( \times \)), $\gamma = 1.4$ (\( \bigtriangleup \)) and for hard brass disks at $\gamma = 1.4$ (\( \bigtriangledown \)).](image)
that the friction coefficient between the grains and between the grains and the glass plate are different. The soft disks have indeed a larger friction coefficient, so that their Jamming transition is expected for lower values of the packing fraction [53, 54]. They also have a larger friction coefficient with the glass plate shaking them so that the energy transfer and dissipation are different. It is remarkable that, despite these differences, when plotting the peak of the maximal dynamical susceptibility of the displacements as a function of the split separating the dynamical and the structural crossovers (see figure 22(a)), the experiment conducted with brass disks align with those conducted with the soft disks at different vibration frequencies. This suggests that the hard disks vibrated at a frequency \( f = 10 \text{ Hz} \), i.e. \( \gamma = 1.4 \) behave as soft disks with a much smaller effective value of \( \gamma \): the injection of energy is much less efficient in the case of the hard, less frictional, disks. It also indicates that friction plays a role in the absolute value of the packing fraction \( \phi_J \), as well as in the efficiency of the mechanical excitation, but not in the physics observed at finite vibration.

We can further confirm that the hard disks behave like the soft disks at a lower level of excitation, by comparing the mean square displacement of the grains \( \Delta^2 \) in the plateau regime, also called the Debye-Waller factor. For the brass disks, no fast camera acquisition were conducted, but one can take the displacements over one vibration cycle as an upper bound of the plateau value. Figure 22(b) displays this Debye-Waller factor for the brass disks for the three different vibration frequency, as reported in the present paper, and for the experiment with the hard brass disks. The value of \( \Delta^2 \sim 10^{-6} \) is significantly lower for the brass disks, confirming that they sit closer to the zero vibration limit.

C. A-thermal vs. Thermal

We have just seen that the physics of the Jamming transition of granular media in the presence of vibration is robust with respect to the specific properties of the grains. However, as stated in the introduction, the Jamming transition is precisely defined and well characterized for thermal soft spheres, not for frictional grains. To what extent does it describe our present observations? In other words, does the street lamp of thermal Jamming illuminate the granular world?

To answer this question we return to the recent numerical simulations [1] in which the authors studied the dynamical behavior of thermal soft spheres, close to Jamming, when approaching the limit of zero temperature. They report the existence of dynamical heterogeneities of the displacements when probed at very small scales. The peak of the maximal heterogeneities increases when the temperature is decreased to the \( T = 0 \) limit, and the packing fraction at which this peak occurs decreases in the same limit. All these observations are identical to those reported in the present work. Unfortunately, in the experiments, we don’t have access to a well defined and unique value of the Jamming packing fraction at strictly zero vibration: it varies from one realization to another. Also, comparing the scalings with the distance to point \( J \) would require a proper definition of an effective temperature, a notoriously difficult, if not impossible, task.

However, we can follow Ikeda et al. [1], who use the mean square displacement in the plateau regime as a sensitive thermometer close to Jamming, and when they compare their observations to experimental colloidal systems [55–57]. From \( \Delta^2(\phi) \) computed for several temperatures, and knowing the range of packing fraction explored by the colloidal experiments, it is straightforward, using the mean square displacements reported for such experiments, to locate them in the Temperature-packing fraction parameter space. The advantage of such a method is that is does not require any information about the details of the interaction potential, nor the knowledge of the kinetic energy. We follow exactly the same procedure. The authors report that the mean square displacement in the vicinity of Jamming decreases from \( 10^{-3} \) to \( 10^{-6} \) particle diameters when the temperature is decreased from \( T = 10^{-5} \) to \( T = 10^{-8} \). As observed in figure 22(b), for the soft photo-elastic disks experiments, \( \Delta^2 \in [10^{-4}10^{-5}] \), corresponding to temperatures of \( [10^{-6}10^{-7}] \) and, for the hard brass disks one, \( \Delta^2 \sim 10^{-6} \), corresponding to a temperature \( T \sim 10^{-8} \). Of course, these temperatures have no thermodynamic meaning, they are essentially a measure of the kinetic energy at short times. Figure 23 summarizes the above discussion: the granular experiment indeed probes the critical regime of Jamming at finite temperature.

One must realize the impact of such a conclusion. Shaken granular systems and thermal soft spheres are very different. In large part, due to the effect of dissipa-

![FIG. 23: Temperature-density phase space](image-url)
tional friction, shaken granular media are out of equilibrium systems for which detailed balance does not hold. Energy is injected at large scale and dissipated at small scales. In the present case, this ensures the isotropy of the displacements at short time, but it is also responsible for the large convection pattern that we have removed.

Such effects would never have existed in an equilibrium glass. Despite these significant differences, the dynamics seems to obey the same physics as soon as a small amount of vibrations or thermal agitation is present.

VIII. CONCLUSION

In this work, we have demonstrated that, in the presence of agitation, the Jamming transition's singular features are blurred into two crossovers, a structural one indicated by the increase of the contact number, which is directly inherited from the zero excitation limit case, and a dynamical one, specific to the presence of agitation. The contact network develops heterogeneous dynamics, which in turn induce heterogeneous displacements at very small scale. These heterogeneities take place within the vibrational regimes, while the structure of the glass remains essentially frozen, and they are related to structural heterogeneities in the contacts network itself.

These observations match the recent results reported in numerical simulations of harmonic soft spheres [1] very well, and we demonstrated that the critical regime of point J is indeed probed by our granular experiments. The street lamp of Jamming illuminates the granular world. This strongly suggests that similar experiments be conducted in other systems, such as foams or emulsions, provided one finds a way to "vibrate" them. Note that in all cases, it is key to reach a spatial resolution of the order of a thousandth of the size of the elementary component. At present this limitation has for instance prevented colloidal experiment from probing the critical regime, although in principal, colloidal suspension are the closest systems to the thermal soft spheres.

Finally, one may wonder whether similar conclusions apply in the presence of an external driving force. Conducting shear experiments in the limit of very weak vibrations, in the spirit of [58], while monitoring the contact dynamics is a promising perspective.

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