Fluorescence control through vacuum induced coherences

H B Crispin and R Arun

Department of Physics, School of Basic & Applied Sciences, Central University of Tamilnadu, Thiruvarur
610005, Tamilnadu, India

E-mail: benhurcris@yahoo.com and rarun@cutn.ac.in

Received 18 September 2018, revised 5 February 2019
Accepted for publication 20 February 2019
Published 19 March 2019

Abstract

The resonance fluorescence of a four-level atom in $J = 1/2$ to $J = 1/2$ transition driven by two coherent fields is studied. We find that the incoherent fluorescence spectrum shows a direct indication of vacuum-induced coherence in the atomic system. We show that such coherence manifests itself via an enhancement or suppression of the spectral peaks in the $\pi$-polarized fluorescence. The effect of the relative phase of the driving fields on the spectral features is also investigated. We show that phase-dependent enhancement or suppression of the fluorescence peaks appears in the incoherent spectrum emitted along the $\sigma$ transitions. It is found that this phase dependence occurs because of the polarization-detection scheme employed for the observation of the fluorescence light. We present an analytical explanation, based on dressed-states of the atom-field system, to interpret the numerical results.

Keywords: atomic coherence, spontaneous emission, resonance fluorescence, vacuum induced coherence

(Some figures may appear in colour only in the online journal)

1. Introduction

In the past few decades, considerable effort has been devoted to the study of coherence and interference effects arising from the spontaneous emission of atoms and the subject has been reviewed in detail by Ficek and Swain [1, 2]. It is well understood how the spontaneous decay of closely lying energy states coupled by common vacuum modes leads to a new type of coherence between the states [3–19]. Even in the absence of an external driving field, a coherence can be induced between the excited levels of an atom due to the vacuum field. This type of coherence is known as vacuum induced coherence (VIC) in the literature [2]. The effects of VIC depend on the level structure of the atom and the dipole moments of the atomic transitions involved in the dynamics. Early studies focussed on three-level atoms (V- or $\Lambda$-type configurations) and found modifications of the fluorescence, absorption, and dispersion properties of the atomic medium due to VIC, such as quenching of fluorescence [3], disappearance of coherent population trapping state [4], spectral line narrowing and dark lines in the spectrum [5], ultrasharp spectral lines [6], probe light amplification with and without population inversion [7], phase control of pulse propagation [8] and population dynamics [9], and enhancement of squeezing [10]. The role of VIC has been explored in four-level atomic systems as well [11–19]. Many interesting features such as spectral line elimination and spontaneous emission cancellation [11], phase control of spontaneous emission [12], fluorescence suppression and line narrowing [13], interference-assisted squeezing in resonance fluorescence [14], inhibition of two-photon transparency [15], interference in cascade spontaneous emission [16], enhancement of self-Kerr nonlinearity [17] and nonlinear dispersion [18], and superluminal light propagation [19] have been reported.

In all these publications, the VIC effects are studied based on the assumption that the dipole transition moments in atoms are non-orthogonal. This condition, being a stringent requirement for the existence of VIC in atoms, is difficult to achieve in practice. To overcome this difficulty, many alternative methods have been proposed to bypass the requirement of non-orthogonal dipole transitions [20]. In a remarkable paper, Kiffner et al proposed yet another scheme to realize
In the present work, we extend this analysis to include an atom. The value of the second order correlation function \( \sigma_2 \) is evaluated using the time-dependent density matrix formalism and the expressions for the incoherent fluorescence spectra of the \( \pi \) and \( \sigma \) transitions are then derived in section 4. In section 5, we present the numerical results of the fluorescence spectrum and analyze the new features using a dressed-state description. Finally, the main results are summarized in section 6.

2. Atomic system and density matrix equations

The atomic model under consideration has two degenerate excited and ground levels which can be realized with \( J = 1/2 \) to \( J = 1/2 \) transition (see figure 1(a)). Each of the excited atomic states \( (1\{1\} and 2) \) decays via spontaneous emission to both the ground states \( (3\} and 4) \). The transitions \( 1\{1\} \leftrightarrow 3\) \( and 2\) \( \leftrightarrow 4 \) are referred to as the \( \pi \) transitions, whereas the cross transitions \( 1\{1\} \leftrightarrow 4\) \( and 2\) \( \leftrightarrow 3 \) in the atom are designated as \( \sigma \) transitions. Direct transitions between the excited states \( (1\{1\} \leftrightarrow 2) \) as well as between the ground states \( (3\} \leftrightarrow 4) \) are assumed to be dipole forbidden. The \( \pi \) transitions have antiparallel dipole moments and can couple with light linearly polarized along the \( z \)-direction \( (e_z) \). The \( \sigma \) transitions \( 1\{1\} \leftrightarrow 4\) \( and 2\) \( \leftrightarrow 3 \) can couple to \( \sigma^- \) and \( \sigma^+ \) polarized light, respectively. The transition dipole moments can be calculated from the matrix elements of the electric-dipole moment operator \( \hat{d} \). Using the Wigner–Eckart theorem [28], they are obtained as:

\[
\begin{align*}
\hat{d}_1 &= \langle 1 | \hat{d} | 3 \rangle = -\frac{1}{\sqrt{3}} D e_z, \\
\hat{d}_2 &= \langle 2 | \hat{d} | 4 \rangle = -\hat{d}_1, \\
\hat{d}_3 &= \langle 2 | \hat{d} | 3 \rangle = \frac{2}{\sqrt{3}} D e^{(-)}, \\
\hat{d}_4 &= \langle 1 | \hat{d} | 4 \rangle = \hat{d}_3^*,
\end{align*}
\]

where \( e^{(-)} = (e_z - i e_x)/\sqrt{2} \) denotes the circular polarization vector and \( D \) is the reduced dipole matrix element.

We are interested in the situation in which two coherent fields of equal frequencies drive the atom. The coherent fields propagate in perpendicular directions and interact with the atomic system as shown in figure 1(b). The transitions \( 1\{1\} \leftrightarrow 3\) \( and 2\) \( \leftrightarrow 4 \) are coupled by a linearly polarized field (amplitude \( E_\alpha \), phase \( \phi_\alpha \), polarization \( e^{\alpha} \)) travelling in the \( x \)-direction. A circularly polarized field (amplitude \( E_\beta \), phase \( \phi_\beta \), polarization \( e^{(-)} \)) propagating along the \( z \)-direction is set to drive the transitions \( 1\{1\} \leftrightarrow 4\) \( in the atom. The Rabi frequency of the linearly (circularly) polarized field driving the atom is denoted as \( \Omega_\alpha (\Omega_\beta). \) The Hamiltonian for this atom-field system is given in the dipole and rotating-wave approximations to be

\[
H = \hbar \omega_\alpha (A_{11} + A_{22}) + h \{ \Omega_\alpha (A_{13} - A_{24}) e^{-i(\omega_\alpha t + \phi_\alpha)} - \Omega_\beta A_{14} e^{-i(\omega_\beta t + \phi_\beta)} + h.c.\},
\]

where \( \omega_\alpha = \omega_{13} = \omega_{24} \) is the atomic transition frequency, \( \omega_\beta \) is the frequency of both the applied fields, \( A_{mn} = |m\rangle \langle n| \) denotes the atomic transition operators for \( m \neq n \) and population operators for \( m = n \), and the Rabi frequencies are given by \( \Omega_\alpha = D E_\alpha / (\sqrt{3} \hbar) \) and \( \Omega_\beta = \sqrt{2} D E_\beta / (\sqrt{3} \hbar). \)
The time evolution of the system is studied using the density matrix formalism. The spontaneous emissions in the atom are included via master equation approach. The master equation for the reduced density operator $\rho$ of the atomic system in the Schrödinger picture is given by

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H, \rho] + \mathcal{L}\rho. \quad (3)$$

Here the Liouville operator $\mathcal{L}\rho$ describes the damping terms due to spontaneous decay processes. We choose the following unitary transformation to remove the fast-oscillating as well as phase-dependent exponential terms in the interaction

$$U = \exp\{i[\omega t + \phi_0](A_{11} + A_{22}) + i[\phi_0 - \phi_0](A_{22} + A_{44})\}.$$  

The transformed master equation for the density operator $\rho = U \rho U^\dagger$ in the interaction picture becomes

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H_I, \rho] + \mathcal{L}\rho. \quad (4)$$

In equation (4), the Hamiltonian of the atom-field system is given by

$$H_I = -\hbar \Delta(A_{11} + A_{22}) + \hbar (i\Omega_a(A_{13} - A_{24}) - \Omega_b A_{14} + h.c.), \quad (5)$$

where $\Delta = \omega_2 - \omega_0$ is the detuning of the applied fields from the atomic resonance frequency. The damping term $\mathcal{L}\rho$ is given by

$$\mathcal{L}\rho = -\frac{\gamma}{2}(\rho A_{11} + A_{11}\rho - 2A_{11}\rho A_{11}) \quad (6)$$

where $\gamma_1 = \gamma_2 = \gamma/3$ and $\gamma_\sigma = 2\gamma/3$ are the decay rates of the $\pi$ and $\sigma$ transitions, respectively (see figure 1(a)). Note that $\gamma = \gamma_1 + \gamma_\sigma = \gamma_2 + \gamma_\sigma$ gives the total decay rate of each of the excited atomic states. The cross-damping term $\gamma_{12}$ in equation (6) is responsible for VIC effects in the atom and arises because the spontaneous decays along the transitions $|1\rangle \rightarrow |3\rangle$ and $|2\rangle \rightarrow |4\rangle$ occur via common vacuum modes. It is given by

$$\gamma_{12} = (d_1 \cdot d_2^*/|d_1||d_2|) \sqrt{\frac{\gamma_1}{\gamma_2}} = -\sqrt{\frac{\gamma_1}{\gamma_2}},$$

where the minus sign comes from the anti-parallel dipole moments $d_1$ and $d_2$. If the $\gamma_{12}$-term is ignored ($\gamma_{12} = 0$), then there is no VIC effect in spontaneous emission.

To study the dynamical behavior of the driven atom, we use the master equation (4) in the interaction picture. The equations of motion of the density matrix elements in the atomic-state basis then take the form

$$\dot{\rho}_{11} = -\gamma_1 \rho_{11} + i\Omega_a(\rho_{13} - \rho_{31}) - i\Omega_b(\rho_{14} - \rho_{41}), \quad (7)$$

$$\dot{\rho}_{33} = \gamma_1 \rho_{11} + \gamma_\sigma \rho_{22} - i\Omega_a(\rho_{13} - \rho_{13}), \quad (8)$$

$$\dot{\rho}_{44} = \gamma_\sigma \rho_{11} + \gamma \rho_{22} + i\Omega_a(\rho_{24} - \rho_{42}) + i\Omega_b(\rho_{14} - \rho_{44}), \quad (9)$$

$$\dot{\rho}_{12} = \left(-\frac{\gamma_1 + \gamma_2}{2} - \gamma_\sigma\right)\rho_{12} - i\Omega_a(\rho_{32} + \rho_{42}) + i\Omega_b\rho_{12}, \quad (10)$$

$$\dot{\rho}_{13} = \left(-\frac{\gamma_1 + \gamma_2}{2}+ i\Delta\right)\rho_{13} + i\Omega_a(\rho_{11} - \rho_{33}) + i\Omega_b\rho_{13}, \quad (11)$$

$$\dot{\rho}_{14} = \left(-\frac{\gamma_1 + \gamma_2}{2}+ i\Delta\right)\rho_{14} - i\Omega_a(\rho_{12} + \rho_{34}) - i\Omega_b(\rho_{11} - \rho_{44}), \quad (12)$$

$$\dot{\rho}_{23} = \left(-\frac{\gamma_1 + \gamma_2}{2}+ i\Delta\right)\rho_{23} + i\Omega_a(\rho_{21} + \rho_{43}), \quad (13)$$

$$\dot{\rho}_{24} = \left(-\frac{\gamma_1 + \gamma_2}{2}+ i\Delta\right)\rho_{24} - i\Omega_a(\rho_{22} - \rho_{44}) - i\Omega_b(\rho_{21}), \quad (14)$$

$$\dot{\rho}_{34} = \gamma_1 \rho_{11} - i\Omega_a(\rho_{32} + \rho_{44}) - i\Omega_b\rho_{33}. \quad (15)$$

In writing equations (7)–(15), we have assumed that the constraint $\rho_{11} + \rho_{22} + \rho_{33} + \rho_{44} = 1$ is satisfied at all times. Note that the VIC term ($\gamma_{12}$) couples the ground-state and excited-state coherences as seen in equation (15). This term plays a crucially important role in modifying the fluorescence properties of the atom [21, 22].

### 3. Steady-state populations

We first study the population distribution in the atomic levels by solving the density matrix equations. For this purpose, we rewrite the equations (7)–(15) in a compact form as

$$\frac{d}{dt} \hat{\psi}(t) = \hat{M} \hat{\psi}(t) + \hat{\tilde{C}}, \quad (16)$$

where $\hat{\psi}$ is a column vector of density matrix elements

$$\hat{\psi} = (|A_{11}\rangle, |A_{33}\rangle, |A_{44}\rangle, |A_{12}\rangle, |A_{21}\rangle, |A_{13}\rangle, |A_{31}\rangle, |A_{23}\rangle, |A_{14}\rangle, |A_{41}\rangle, |A_{24}\rangle, |A_{42}\rangle, |A_{32}\rangle, |A_{43}\rangle)^T, \quad (17)$$

with $\hat{\tilde{C}}$ also a $15 \times 1$ column vector with non-zero elements $\tilde{C}_i = \gamma_1, \tilde{C}_i = \gamma_2, \tilde{C}_i = i\Omega_a, \tilde{C}_i = -i\Omega_b$. The inhomogeneous term $\hat{\tilde{C}}$ arises because of the elimination of the population $\rho_{22}$ in equations (7)–(15) using the trace condition $\text{Tr} \rho = 1$. In equation (16), $\hat{M}$ is a $15 \times 15$ matrix whose elements are independent of the density matrix elements and can be obtained explicitly by using $\hat{\psi}$ in equations (7)–(15).
The stationary solution of equation (16) is obtained by setting \(\dot{\psi}/dt = 0\) in the long-time limit. Solving the resulting equation \(\dot{\psi}(\infty) = -\hat{M}^{-1}\hat{C}\) gives the steady-state values of the density matrix elements as

\[
\rho_{11} = \rho_{22} = \frac{4\Omega_1^2}{2\Omega_0^2(\gamma^2 + 4\Delta^2 + 8\Omega_0^2)} + \frac{4\Omega_0^2}{\Omega_0^2 + (\Omega_0^2 + \Omega_0^2)(\gamma^2 + 4\Delta^2)},
\]
\[
\rho_{33} = \frac{2\Omega_0^2(\gamma^2 + 4\Delta^2 + 8\Omega_0^2) + \Omega_0^2(\gamma^2 + 4\Delta^2)}{2\Omega_0^2(\gamma^2 + 4\Delta^2 + 8\Omega_0^2) + \Omega_0^2(\gamma^2 + 4\Delta^2)},
\]
\[
\rho_{44} = \frac{4\Omega_2^2(\gamma^2 + 4\Delta^2 + 8\Omega_0^2) + \Omega_0^2(\gamma^2 + 4\Delta^2)}{4\Omega_2^2(\gamma^2 + 4\Delta^2 + 8\Omega_0^2) + \Omega_0^2(\gamma^2 + 4\Delta^2)},
\]
\[
\rho_{12} = \rho_{14} = 0.
\]

(18)

As seen in equations (18), the steady-state results for the populations and coherences are independent of the VEC parameter \((\gamma_1, 2)\) and the phases \((\phi_1, \phi_0)\) of the applied fields. For \(\Omega_0 = 0\), the results (18) become identical to those of Kiffner et al. [21, 22]. The coherences \(\rho_{23}\) and \(\rho_{34}\) are non-zero only when the additional \(\sigma\)-polarized field \((\Omega_0 \neq 0)\) drives the atom. Also, it is easy to see that \(\rho_{33} > \rho_{44}\) for \(\Omega_0 \neq 0\). An important point is that the two-photon coherence \(\rho_{34}\) is non-zero even though the one-photon coherence \(\rho_{14}\) is zero. This implies that the population in the state [4] can be pumped into the state [3] by two-photon transitions \([4] \rightarrow [1] \rightarrow [3]\) under the action of both the applied fields \((\Omega_0, \Omega_0 = 0)\). One thus expects the steady-state population in the ground state [3] to be greater than that of the state [4] unlike the results of Kiffner et al. [21, 22] as mentioned above. This feature is illustrated in figure 2, where we compare the population distribution in the atom for the cases with and without the additional field.

4. Resonance fluorescence spectrum

We proceed to analyze the resonance fluorescence from the driven atom and derive analytic expressions suitable for numerical computation of the fluorescence spectra. Since the atom is driven by two coherent fields, the fluorescence fields generated by the \(\pi\) and \(\sigma\)-transitions in the atom consist of coherent as well as incoherent components. We focus only on the incoherent parts of the fluorescence spectra. In the far-field zone, the electric field operator of the fluorescence field at an observation point \(\mathbf{r}\) can be written as \(\hat{E}(\mathbf{r}, t) = \hat{E}^{(+)}(\mathbf{r}, t) + \hat{E}^{(-)}(\mathbf{r}, t)\), where \(\hat{E}^{(+)}(\hat{E}^{(-)}) \equiv \hat{E}^{(+)}(\hat{E}^{(-)})\) represents the positive (negative) frequency part of the field. The positive frequency parts of the electric field operator for the fluorescence fields from the \(\pi\) and \(\sigma\)-transitions are found to be, respectively, [29]

\[
\hat{E}^{(+)}_\pi(\mathbf{r}, t) = -\frac{\omega_r^2}{c^2r}[\mathbf{\hat{r}} \times (\mathbf{\hat{r}} \times \mathbf{\hat{d}}_3)]A_{31}(t')
+ [\mathbf{\hat{r}} \times (\mathbf{\hat{r}} \times \mathbf{\hat{d}}_2)]A_{42}(t')e^{-i(\omega_{r} t' + \phi_b)},
\]
\[
\hat{E}^{(+)}_\sigma(\mathbf{r}, t) = -\frac{\omega_r^2}{c^2r}[\mathbf{\hat{r}} \times (\mathbf{\hat{r}} \times \mathbf{\hat{d}}_3)]A_{31}(t')
+ [\mathbf{\hat{r}} \times (\mathbf{\hat{r}} \times \mathbf{\hat{d}}_2)]A_{42}(t')e^{-i(\omega_{r} t' + \phi_b)}
\times A_{32}(t')e^{-2(i\omega_{r} t' + \phi_b)},
\]

(19)

where \(t' = t - r/c\) and \(\mathbf{\hat{r}} = \mathbf{\hat{r}}/r\) is the unit vector along the direction of observation. By choosing the direction (\(\mathbf{\hat{r}}\)) of detection of the fluorescence light to be along the \(\gamma\)-direction (see figure 1(b)), it is seen from equations (19) that the fluorescence field of the \(\pi\) transitions will be polarized along \(\mathbf{e}_r\), and the light emitted from the \(\sigma\)-transitions will be linearly polarized along \(\mathbf{e}_x\). With this choice of the detection scheme, the fluorescence light from the \(\pi\) and \(\sigma\)-transitions can be differentiated by means of a polarization filter.

The incoherent spectrum of resonance fluorescence is defined as

\[
S(\omega) = \frac{1}{\pi} \text{Re} \int_0^\infty \lim_{t \to -\infty} \langle \Delta\hat{E}^{(+)}(t + \tau)\Delta\hat{E}^{(+)}(t) \rangle e^{-i\omega \tau} d\tau,
\]

(20)

where \(\Delta\hat{E}^{(\pm)} = \hat{E}^{(\pm)} - \langle \hat{E}^{(\pm)} \rangle_{\text{stat}}\) are the deviations of the electric field operators \(\hat{E}^{(\pm)}\) from their steady-state average values \(\langle \hat{E}^{(\pm)} \rangle_{\text{stat}}\). Substituting for \(\hat{E}^{(\pm)}\) from equation (19) into

---

**Figure 2.** Steady-state results for the atomic-level populations as a function of the Rabi frequency \(\Omega_0\) for \(\Delta = 8\gamma\) and (a) \(\Omega_0 = 0\), (b) \(\Omega_0 = 12\gamma\). All the parameters are in units of \(\gamma\). In all figures in this paper, we assume that \(\gamma_2 = \sqrt{\gamma_2^2 - \gamma^2 / 3}\) unless specified otherwise.
equation (20), we get the following expressions for the incoherent spectra of the fluorescence light emitted on the τ and σ-transitions:

\[
S^\tau(\tilde{\omega}) = \frac{f_{\pi}}{\pi} \text{Re} \int_0^\infty \lim_{t \to -\infty} \{ \gamma_1 (\delta A_{13}(t + \tau) \delta A_{31}(t)) \\
+ \gamma_2 (\delta A_{23}(t + \tau) \delta A_{42}(t)) \\
+ \gamma_1 (\delta A_{33}(t + \tau) \delta A_{43}(t)) \\
+ \gamma_2 (\delta A_{34}(t + \tau) \delta A_{42}(t)) \} e^{-i\tilde{\omega} \tau} d\tau,
\]

(21)

\[
S^\sigma(\tilde{\omega}) = \frac{f_{\sigma}}{\pi} \text{Re} \int_0^\infty \lim_{t \to -\infty} \{ (\delta A_{14}(t + \tau) \delta A_{41}(t)) \\
+ (\delta A_{23}(t + \tau) \delta A_{32}(t)) \\
+ e^{2i\tilde{\omega}} (\delta A_{24}(t + \tau) \delta A_{42}(t)) \\
+ e^{2i\tilde{\omega}} (\delta A_{33}(t + \tau) \delta A_{41}(t)) \} e^{-i\tilde{\omega} \tau} d\tau.
\]

(22)

Here \( \phi = \phi_a - \phi_b \) represents the relative phase of the applied fields, \( \tilde{\omega} = \omega - \omega_i \) is the difference between the frequencies of the observed radiation and the applied lasers, \( \delta A_{ij}(t) = A_{ij}(t) - \langle A_{ij}(t) \rangle \) are the fluctuations of the atomic operators about their steady-state mean values, and \( f_{\pi} \) and \( f_{\sigma} \) are common prefactors. In what follows, we set the prefactors \( f_{\pi}, f_{\sigma} \) to unity for convenience.

To evaluate the correlation functions in equations (21) and (22), we define a column vector of two-time correlation functions given by

\[
\hat{U}^{mn}(t, \tau) = [\langle \delta A_{11}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{23}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{44}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{12}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{13}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{33}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{34}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{43}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{44}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{43}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{44}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{43}(t + \tau) \delta A_{mn}(t) \rangle, \langle \delta A_{44}(t + \tau) \delta A_{mn}(t) \rangle] \hat{I}.
\]

(23)

According to the quantum regression theorem [30], the column vector (23) satisfies the following equation

\[
\frac{d}{d\tau} \hat{U}^{mn}(t, \tau) = \hat{M} \hat{U}^{mn}(t, \tau),
\]

(24)

where the matrix \( \hat{M} \) is defined as in equation (16). Solving the above equation and substituting the results for the correlation functions in equations (21) and (22), the incoherent fluorescence spectra can be obtained as

\[
S^\tau(\tilde{\omega}) = \frac{\gamma}{3\pi} \text{Re} \left\{ \sum_{j=1}^{15} \lim_{t \to -\infty} [\hat{N}_{1j} \hat{U}_{j}^{31}(t, 0) \\
+ \hat{N}_{1j} \hat{U}_{j}^{42}(t, 0) \right\} - \hat{N}_{6j} \hat{U}_{j}^{32}(t, 0) \\
- \hat{N}_{1j} \hat{U}_{j}^{31}(t, 0)] \},
\]

(25)

\[
S^\sigma(\tilde{\omega}) = \frac{2\gamma}{3\pi} \text{Re} \left\{ \sum_{j=1}^{15} \lim_{t \to -\infty} [\hat{N}_{1j} \hat{U}_{j}^{41}(t, 0) \\
+ \hat{N}_{6j} \hat{U}_{j}^{32}(t, 0) \right\} - e^{2i\tilde{\omega}} \hat{N}_{6j} \hat{U}_{j}^{32}(t, 0) \\
+ e^{2i\tilde{\omega}} \hat{N}_{6j} \hat{U}_{j}^{41}(t, 0)] \},
\]

(26)

where \( \hat{N}_{ij} \) is the \((i, j)\) element of the matrix \( \hat{N} = (i\tilde{\omega} \hat{I} - \hat{M})^{-1} \) with \( \hat{I} \) being the \(15 \times 15\) identity matrix.

5. Numerical results and dressed-state analysis

In this section we present numerical results of the fluorescence spectra and then provide an understanding of the role of VIC using dressed-state descriptions. The detection scheme mentioned in the previous section enables us to study the fluorescence spectrum of π and σ-transitions separately. In the numerical calculations, the spectra \( S^\tau(\tilde{\omega}) \) and \( S^\sigma(\tilde{\omega}) \) are obtained using equations (25) and (26). All the parameters such as Rabi frequencies, detuning, and the decay rates are scaled by the total decay rate \( \gamma \).

5.1. Resonance fluorescence spectrum—π transitions

We first consider the spectrum of resonance fluorescence emitted on the π transitions. The numerical results for the
spectra are presented for both weak and strong-driving field limits in figure 3. For comparison, the results are also displayed without considering the additional $\sigma^+$-polarized field ($\Omega_b = 0$) driving the atom. In the absence of $\sigma^+$-polarized field, the spectra are the same as those of Kiffner et al [21, 22] as shown by the dashed curves in figure 3. It is seen that the spectrum changes significantly when the additional field ($\Omega_b = 0$) drives the atom. A sharp spectral peak with a width smaller than the decay rate $\gamma$ can be seen at the laser frequency in the presence of weak driving fields (solid curve in figure 3(a)). For stronger excitations ($\Omega_a, \Omega_b \gg \gamma$), the additional field may cause splitting of the sidebands in the spectrum (compare solid and dashed curves in figure 3(b)).

To study the role of VIC in the fluorescence of $\pi$ transitions, we compare the spectra with and without VIC terms in figure 4. The spectrum without considering VIC is obtained by setting $\gamma_{12} = 0$ in equations (16) and (21). Note that the last two terms with a minus sign in the spectrum (25) arise from $\gamma_{12}$-terms in equation (21) and hence these terms do not contribute when the VIC effect is not considered. The $\pi$-fluorescence spectrum in the absence of VIC is thus given by

$$S^\pi(\bar{\omega}) = \frac{\gamma}{3\pi} Re \left\{ \sum_{j=1}^{15} \lim_{t \to +\infty} [\hat{N}_a \hat{U}_j^{\dagger} (t, 0) + \hat{N}_2 \hat{U}_j^{\dagger} (t, 0)] \right\}.$$  \hspace{1cm} (27)

It is clear that the VIC modifies all the peaks of the fluorescence spectrum (see figure 4). When VIC is included ($\gamma_{12} = -\sqrt{1/2} \gamma$, the central peak is enhanced, whereas alternate sidebands are reduced or enhanced in comparison to the case without VIC. For a suitable set of parameters, a complete cancellation is also possible for those sidebands which got reduced due to VIC, as shown in figure 5 (compare solid and dashed curves in figure 5).

A physical understanding of these numerical results can be obtained if we employ the dressed-state description of atom-field interactions. The dressed atomic states are defined as eigenstates $|\Psi\rangle$ of the Hamiltonian in the interaction picture (5), i.e. $H_I |\Psi\rangle = \lambda_\Psi |\Psi\rangle$. For simplicity, we consider only the case in which the frequencies of the driving fields are tuned to the atomic transition frequency ($\Delta = 0$). Under this condition, the dressed states $|\Psi\rangle$ ($\Psi = \alpha, \beta, \kappa, \mu$) can be expanded in terms of the bare atomic states as

$$|\alpha\rangle = c_a |1\rangle + c_{a2} |2\rangle + c_{a3} |3\rangle + c_{a4} |4\rangle,$$

$$|\beta\rangle = c_b |1\rangle + c_{b2} |2\rangle + c_{b3} |3\rangle + c_{b4} |4\rangle,$$

$$|\kappa\rangle = c_a |1\rangle + c_{a2} |2\rangle + c_{a3} |3\rangle + c_{a4} |4\rangle,$$

$$|\mu\rangle = c_b |1\rangle + c_{b2} |2\rangle + c_{b3} |3\rangle + c_{b4} |4\rangle,$$

and the eigenvalues of these dressed states are, respectively,

$$\lambda_\alpha = \frac{\Omega_a}{2}, \quad \lambda_\beta = -\frac{\Omega_a}{2}, \quad \lambda_\kappa = \frac{\Omega_b}{2}, \quad \lambda_\mu = -\frac{\Omega_b}{2},$$  \hspace{1cm} (28)

where $\Omega_a = \sqrt{4\Omega_a^2 + \Omega_b^2 + \Omega_a} \Omega_b$ and $\Omega_b = \sqrt{4\Omega_a^2 + \Omega_b^2 - \Omega_b}$ are the effective Rabi frequencies of the driving fields. The expansion coefficients $c_{ij}$ in equation (28) are given in appendix A. The central peak and the different sidebands that appear in the fluorescence spectrum can be explained in terms of transitions between the dressed states $|i\rangle \rightarrow |j\rangle$ ($i, j = \alpha, \beta, \kappa, \mu$). The peaks in the spectrum are centered at the frequencies $\lambda_j = \lambda_{\Psi} - \lambda_j$ due to the dressed-state transitions $|i\rangle \rightarrow |j\rangle$. The central peak at $\bar{\omega} = 0$ is due to transitions that occur between adjacent manifolds of the same dressed-states. The peaks at $\pm \Omega_1$ and $\pm \Omega_2$ come from the transitions $|\mu\rangle \leftrightarrow |\alpha\rangle$ and $|\kappa\rangle \leftrightarrow |\beta\rangle$, respectively, whereas the transitions $|\mu\rangle \leftrightarrow |\beta\rangle$ and $|\kappa\rangle \leftrightarrow |\alpha\rangle$ are coupled to each other and contribute to the outer sidebands located at $\pm (\Omega_1 + \Omega_2)/2$. The innermost sidebands at $\pm (\Omega_1 - \Omega_2)/2$ originate from the coupled dressed-state transitions $|\mu\rangle \leftrightarrow |\kappa\rangle$ and $|\beta\rangle \leftrightarrow |\alpha\rangle$.

The dressed atomic states (28) can be used as basis states to solve for the atomic dynamics including the spontaneous decay processes. To this end, we first rewrite the density matrix equations using the states (28). When the driving fields are intense ($\Omega_a, \Omega_b \gg \gamma$), it is appropriate to invoke the secular approximation in which the couplings between populations and coherences can be ignored in the dressed-state basis. Following
closely the procedure as in the work of Narducci et al. [31], we recast the equations of motion of the density matrix elements (7)–(15) using the basis of dressed atomic states (28) as

\[
\begin{align*}
\dot{\rho}_{aa} &= -\Gamma_0 \rho_{aa} + \Gamma_3 \rho_{bb} + \Gamma_{kk} \rho_{kk} + \Gamma_{pp} \rho_{pp}, \\
\dot{\rho}_{bb} &= -\Gamma_0 \rho_{bb} + \Gamma_3 \rho_{aa} + \Gamma_{kk} \rho_{kk} + \Gamma_{pp} \rho_{pp}, \\
\dot{\rho}_{kk} &= -\Gamma_0 \rho_{kk} + \Gamma_3 \rho_{aa} + \Gamma_{bb} \rho_{bb}, \\
\dot{\rho}_{pp} &= -\Gamma_0 \rho_{pp} + \Gamma_3 \rho_{aa} + \Gamma_{bb} \rho_{bb}.
\end{align*}
\]

with

\[
\begin{align*}
\Gamma_0 &= \frac{\gamma(9\Omega^2 + 2\Omega_k^2) + 3\gamma_2 \Omega_k^2}{6(4\Omega^2 + \Omega_k^2)}, \\
\Gamma_3 &= \frac{\gamma(6\Omega^2 + \Omega_k^2) + 6\gamma_2 \Omega_k^2}{12(4\Omega^2 + \Omega_k^2)}, \\
\Gamma_1 &= \frac{\gamma(15\Omega^2 + 4\Omega_k^2) - 3\gamma_2 \Omega_k^2}{6(4\Omega^2 + \Omega_k^2)}, \\
\Gamma_2 &= \frac{\gamma(11\Omega^2 + 5\Omega_k^2) - 3\gamma_2 \Omega_k^2}{6(4\Omega^2 + \Omega_k^2)}, \\
\Gamma_4 &= \frac{2\gamma_2 \Omega^2 - 3\gamma_2 (2\Omega^2 + \Omega_k^2)}{12(4\Omega^2 + \Omega_k^2)}, \\
\Gamma_5 &= \frac{\gamma(13\Omega^2 + 3\Omega_k^2) + 3\gamma_2 \Omega_k^2}{6(4\Omega^2 + \Omega_k^2)}.
\end{align*}
\]

In steady state, all coherences between dressed states vanish and only the populations of the dressed states are non-zero. After solving equations (30) in the steady-state limit \( t \to \infty \), the populations of the dressed-states are found to be

\[
\rho_{aa} = \rho_{bb} = \rho_{kk} = \rho_{pp} = \frac{1}{4}.
\]

In order to understand why certain peaks are enhanced whereas others peak are diminished by VIC in the \( \pi \)-fluorescence, we calculate the spectrum (21) using the dressed states (28). Under the secular approximation, the incoherent fluorescence spectrum can be worked out in a compact form as

\[
S^\pi(\bar{\omega}) = \frac{A_{x1}}{\pi} \left[ \frac{\gamma^2}{\omega^2 + \gamma^2/4} + \frac{A_{x2}}{\pi} \frac{\Gamma_1}{(\bar{\omega} - \Omega_1)^2 + \Gamma_1^2} \right]
\]

\[
+ \frac{A_{x3}}{\pi} \frac{\Gamma_2}{((\bar{\omega} - \Omega_2)^2 + \Gamma_2^2)}
\]

\[
+ \frac{A_{x4}}{\pi} \left\{ \frac{\gamma_3 + \Gamma_1}{(\bar{\omega} + (\Omega_1 + \Omega_2)/2)^2 + (\Gamma_3 + \Gamma_4)^2} \right\}
\]

\[
+ \frac{\gamma_3 + \Gamma_2}{((\bar{\omega} + (\Omega_1 - \Omega_2)/2)^2 + (\Gamma_3 + \Gamma_4)^2)}
\]

\[
+ \frac{\gamma_3 + \Gamma_1}{((\bar{\omega} + (\Omega_1 + \Omega_2)/2)^2 + (\Gamma_3 + \Gamma_4)^2)}
\]

\[
+ \frac{\gamma_3 + \Gamma_2}{((\bar{\omega} + (\Omega_1 - \Omega_2)/2)^2 + (\Gamma_3 + \Gamma_4)^2)}
\]

\[
\frac{A_{x1}}{6(4\Omega^2 + \Omega_k^2)},
\]

\[A_{x2} = \frac{\gamma - 3\gamma_2 \Omega_k^2}{24(4\Omega^2 + \Omega_k^2)},\]

\[A_{x3} = \frac{(\gamma - 3\gamma_2 \Omega_k^2) + 6\gamma_2 \Omega_k^2}{24(4\Omega^2 + \Omega_k^2)},\]

\[A_{x4} = \frac{\gamma_2 + \Omega_2^2}{4(\gamma + 3\gamma_2 \Omega_k^2) + 2\gamma \Omega_k^2},\]

\[A_{x5} = \frac{(\gamma + 3\gamma_2)(4\Omega^2 + \Omega_k^2)}{4(\gamma + 3\gamma_2 \Omega_k^2) + 2\gamma \Omega_k^2}.
\]

The upper and lower signs in equation (33) give the positive \( (\bar{\omega} > 0) \) and negative \( (\bar{\omega} < 0) \) parts of the spectrum along the \( \bar{\omega} \)-axis, respectively. It is seen that the spectrum (33) is composed of nine spectral curves, consistent with the results in figure 4, with widths depending upon the decay rates (\( \Gamma \) terms) of the dressed-state coherences. In the absence of VIC \( (\gamma_2 = 0) \), the spectral components, located at \( \bar{\omega} = \pm (\Omega_1 + \Omega_2)/2 \) and \( \bar{\omega} = \pm \Omega_k \), consist of a sum of two Lorentzians with widths \( 2\Gamma_3 + \Gamma_4 \) and \( 2(\Gamma_3 + \Gamma_4) \), respectively (see the terms inside the curly brackets in equation (33)). The weights \( \mathcal{W}_1 \) and \( \mathcal{W}_2 \) of these two Lorentzians are normalized so that \( \mathcal{W}_1 + \mathcal{W}_2 = 1 \). When the effects of VIC are considered \( (\gamma_2 = -\gamma/3) \), then \( \mathcal{W}_1 = 1 \), \( \mathcal{W}_2 = 0 \), each spectral curve is represented by a single Lorentzian as seen from equations (33) and (34).

The weight of each spectral curve \( (A_{xk} \ k = 1, 2, 3, 4, 5) \) in the spectrum (33) can be calculated by knowing the transition rates between the dressed states. The rate for dressed-state transition \( \left| i \right> \to \left| j \right> \) \( (i, j = a, \beta, k, p) \) is given by

\[
\Gamma_{ij} = \left| \left< j \right| P_x^+ \left| i \right> \right|^2
\]

where

\[
P_x^+ = \sqrt{\gamma_1} A_{31} - \sqrt{\gamma_2} A_{42} \ e^{-i\omega t + \phi_k}.
\]
In general, the weight of the Lorentzian line originating from a single dressed-state transition $|i\rangle \rightarrow |j\rangle$ can be obtained in the dressed-state picture as \[ W_j^i = \Gamma_j^i \rho_{ji}, \quad (36) \]

where $\rho_{ji}$ denotes the steady-state population of the dressed state $|i\rangle$. The derivation of the weights of the spectral peaks \( (34) \) using equation \( (36) \) is outlined in appendix B. Since the transition rates satisfy $\Gamma_j^i = \Gamma_j^i$ and all the dressed-state populations are equal in steady state (see equation \( (32) \)), the spectrum $S^\sigma(\dot{\omega})$ in equation \( (33) \) is symmetric about $\dot{\omega} = 0$, confirming the numerical results in figures 4 and 5. It is clear that VIC modifies all the fluorescence peaks through $\gamma_{12}$ terms in equation \( (34) \). As seen from equations \( (33) \) and \( (34) \), the central peak and those of the sidebands at $\lambda_{a3}$, $\lambda_{b3}$ are enhanced due to VIC (note that $\gamma_{12}$ is negative), whereas the peaks located at $\lambda_{a3}$ ($= \lambda_{b3}$) and $\lambda_{b3}$ ($= \lambda_{a3}$) get diminished on comparing the case with $\gamma_{12} = 0$ (see figure 4).

5.2. Resonance fluorescence spectrum—$\sigma$ transitions

We now investigate the spectrum of the fluorescence light emitted on the $\sigma$ transitions. When there is no additional field ($\Omega_b = 0$), the cross-correlation terms $\langle \delta A_{14}(t + \tau) \delta A_{12}(t) \rangle$ and $\langle \delta A_{13}(t + \tau) \delta A_{14}(t) \rangle$ in equation \( (22) \) turn out to be zero as reported by Kiffner et al. [22]. However, in the presence of the additional field ($\Omega_b \neq 0$), these cross terms are non-zero and the spectrum \( (22) \) becomes dependent on the relative phase $\phi$ of the applied fields. We first show how the spectral profile can be controlled by changing this relative phase in figure 6. In the case of weak driving fields ($\Omega_a, \Omega_b < \gamma$), the results are shown in figure 6(a). The spectrum is a five-peaked structure when both fields are in phase ($\phi = 0$). The outer sidebands in the spectrum are located at $\dot{\omega} \approx \pm 4.5 \gamma$ and have a small intensity. As we change the relative phase to $\phi = \pi/4$, it is seen that the central peak is enhanced and the inner sidebands get reduced. However, the outer sidebands do not change much as the relative phase $\phi$ is changed (not shown in figure 6(a)). For relative phase $\phi = \pi/2$, a complete elimination of the inner sidebands occurs with a corresponding enhancement in the central peak (compare the graphs in figure 6(a)). In other words, the incoherent emissions at laser frequencies can be enhanced by changing the relative phase of weak driving fields. For strong-field excitation ($\Omega_a, \Omega_b \gg \gamma$), it can be shown that the central peak can be enhanced along with alternate sidebands in the spectrum by adjusting the relative phase. This feature is illustrated in figure 6(b) where we compare the spectra in the high-field limit for the relative phases $\phi = 0$ and $\phi = \pi/2$.

To explore the reasons for the phase control of spectral features in the $\sigma$-fluorescence, an analytical formula for the fluorescence spectrum \( (22) \) is obtained in the dressed-state formalism as

\[
S^\sigma(\dot{\omega}) = A_{s1} \frac{\gamma}{\pi} \frac{\gamma}{[\dot{\omega}^2 + \gamma^2/4]} + A_{s2} \frac{\Gamma_1}{\pi} \frac{\Gamma_1}{[\dot{\omega} + \Omega_b]^2 + \Gamma_1^2} + A_{s3} \frac{\Gamma_2}{\pi} \frac{\Gamma_2}{[\dot{\omega} + \Omega_a]^2 + \Gamma_2^2} + A_{s4} \frac{\Gamma_3 + \Gamma_4}{\pi} \frac{\Gamma_3 + \Gamma_4}{[\dot{\omega} + \Omega_a]^2 + (\Gamma_3 + \Gamma_4)^2} + A_{s5} \frac{\Gamma_3 + \Gamma_4}{\pi} \frac{\Gamma_3 + \Gamma_4}{[\dot{\omega} + \Omega_b]^2 + (\Gamma_3 + \Gamma_6)^2},
\]

with

\[
A_{s1} = \frac{\gamma_0}{4} \left( \frac{4\Omega_a^2 \sin^2 \phi + \Omega_b^2}{4\Omega_a^2 + \Omega_b^2} \right), \quad A_{s2} = A_{s3} = \frac{\gamma_0}{4} \left( \frac{4\Omega_a^2 \sin^2 \phi + \Omega_b^2}{4(4\Omega_a^2 + \Omega_b^2)} \right), \quad A_{s4} = A_{s5} = \frac{\gamma_0}{4} \left( \frac{2\Omega_a^2 \cos^2 \phi + \Omega_b^2}{4(4\Omega_a^2 + \Omega_b^2)} \right).
\]

The various terms in the spectrum given above have their meanings as explained following equation \( (33) \). From the analytic expressions \( (37) \) and \( (38) \), it is evident that the central
peak and the sidebands peaked at ±Ω₁ and ±Ω₂ in the spectrum get enhanced as the phase ϕ is changed from 0 to π/2 (see figure 6(b)). Note that the weights of the Lorentzians (38) depend only on the relative phase of the applied fields and are independent of the VIC parameter γ₁₂. This is in agreement with the numerical calculations of the spectrum in the strong-field limit. The derivation of the formulas (38) follows from calculating the weight of the spectral line \[ W_{ij} = \Gamma_{ij} \rho_{ii} \] (similar to equation (36)) and is given in appendix B. Here \( \Gamma_{ij} = |\langle j | P | i \rangle |^2 \) represents the transition rate between the dressed states with the atomic polarization operator defined for the σ-fluorescence as

\[
P^±_σ = [\sqrt{\alpha} A^1_{41} + \sqrt{\alpha} A^2_{32} e^{-2i\phi}] e^{-i(\omega t + \phi_σ)}. \tag{39}
\]

Finally, we consider the effect of VIC in the σ transitions. In figure 7, the spectra \( S'(\tilde{\omega}) \) with and without VIC are plotted for a fixed relative phase \( \phi = \pi/2 \). The numerical results show that the central peak in the spectrum is not affected by VIC terms in the density matrix equations (16) as can be verified with the analytical result (37). However, all the sideband peaks in the spectrum get reduced by VIC (compare solid and dashed curves in figure 7). The reduction is more prominent for certain sidebands, whereas the influence of VIC is comparatively lesser for the other sidebands. This can be understood from the widths (Γ) terms of the Lorentzians in the spectrum (37). For the parameters of figure 7, the widths (Γ₁, Γ₂) of the spectral curves peaked at ±Ω₁ and ±Ω₂ increase due to VIC (see equation (31)), thereby reducing the peak heights.

6. Conclusions

In conclusion, we have studied theoretically the effects of VIC in the resonance fluorescence spectrum of \( J = 1/2 \) to \( J = 1/2 \) system driven by two coherent fields. We have found that VIC affects all the peaks of the spectrum of π transitions, where certain peaks are enhanced whereas others are diminished. For a suitable choice of parameters, certain sidebands may be eliminated due to VIC. The enhancement and suppression of the spectral peaks can be explained using the weight of the line in which the VIC term changes the transition rates of the dressed states. The fluorescence spectrum emitted on the σ transitions is found to be dependent on the relative phase of the applied fields even though the steady-state atomic populations are phase-independent. We find that the phase-dependence of the spectral profiles occurs because the detection process involves only the e- polarized field in the fluorescence light from the σ transitions. The results of this paper can be verified experimentally using a single laser-cooled \(^{199}\text{Hg}^+\) ion in optical trap as in the experiment of Eichmann et al [24]. Further, it would be worthwhile to investigate the effects of VIC in photon correlations and squeezing spectra in the system studied here. Detailed descriptions of such studies will be published elsewhere.

Appendix A. Coefficients in the dressed states

The coefficients in equation (28) are

\[
c_{\sigma 1} = -\frac{1}{2} \frac{\Omega}{\Omega_1 - \Omega_0}, \quad c_{\sigma 2} = -\frac{\Omega}{\sqrt{\Omega_1 (\Omega_1 - \Omega_0)}},
\]

\[
c_{\sigma 4} = -\frac{1}{2} \frac{\Omega}{\Omega_1 - \Omega_0}, \quad c_{\sigma 5} = -\frac{\Omega}{\sqrt{\Omega_1 (\Omega_1 - \Omega_0)}},
\]

\[
c_{\sigma 4} = -c_{\sigma 1} = c_{\sigma 2} = c_{\sigma 3} = c_{\sigma 2} = c_{\sigma 3} = c_{\sigma 2} = c_{\sigma 3} = c_{\sigma 2}. \tag{A1}
\]

Appendix B. Calculation of the weights of the spectral lines

The purpose of this appendix is to outline how the weights of the Lorentzians in the spectra (33) and (37) can be derived. For that we use equation (36) to find the contribution of each dressed-state transition \( |i⟩ \rightarrow |j⟩ \) (i, j = α, β, κ, µ) to a spectral curve peaked at \( \tilde{\omega} = \lambda_1 - \lambda_2 \). The total weight of the spectral curve is obtained by adding the contributions from all dressed-state transitions that give rise to the peak. Thus, using equation (36), the weights are found to be

\[
A_{\alpha 1} = \Gamma_{\alpha \beta} P_{\alpha \alpha} + \Gamma_{\beta \beta} P_{\beta \beta} + \Gamma_{\alpha \kappa} P_{\alpha \kappa} + \Gamma_{\mu \mu} P_{\mu \mu},
\]

\[
A_{\alpha 2} = \Gamma_{\beta \alpha} P_{\beta \alpha} = \Gamma_{\alpha \beta} P_{\alpha \beta},
\]

\[
A_{\alpha 3} = \Gamma_{\beta \kappa} P_{\beta \kappa} = \Gamma_{\kappa \beta} P_{\kappa \beta},
\]

\[
A_{\alpha 4} = \Gamma_{\beta \mu} P_{\beta \mu} + \Gamma_{\kappa \mu} P_{\kappa \mu},
\]

\[
A_{\alpha 5} = \Gamma_{\mu \beta} P_{\mu \beta} + \Gamma_{\mu \kappa} P_{\mu \kappa} + \Gamma_{\alpha \beta} P_{\alpha \beta}, \tag{B1}
\]

where \( k = \pi (\sigma) \) refers to the spectrum of the π (σ) transitions.

On using the polarization operator (35) in the transition rates \( \Gamma_{ij} = |\langle j | P_{\pi} | i \rangle |^2 \) for the π-fluorescence, the weights are
obtained from equations (B1) to be

\[
A_{x1} = \rho_{\sigma\sigma} (c_{1a}^2 + c_{2a}^2) + 2\gamma_{12} c_{1a} c_{2a} c_{3a} c_{4a} \\
+ \rho_{\sigma\sigma} (c_{1b}^2 + c_{2b}^2) + 2\gamma_{12} c_{1b} c_{2b} c_{3b} c_{4b} \\
+ \rho_{\sigma\sigma} (c_{1c}^2 + c_{2c}^2) + 2\gamma_{12} c_{1c} c_{2c} c_{3c} c_{4c} \\
+ \rho_{\sigma\sigma} (c_{1d}^2 + c_{2d}^2) + 2\gamma_{12} c_{1d} c_{2d} c_{3d} c_{4d}.
\]

\[
A_{x2} = \rho_{\sigma\sigma} (c_{1a}^2 + c_{2a}^2) + 2\gamma_{12} c_{1a} c_{2a} c_{3a} c_{4a},
\]

\[
A_{x3} = \rho_{\sigma\sigma} (c_{1b}^2 + c_{2b}^2) + 2\gamma_{12} c_{1b} c_{2b} c_{3b} c_{4b},
\]

\[
A_{x4} = \rho_{\sigma\sigma} (c_{1c}^2 + c_{2c}^2) + 2\gamma_{12} c_{1c} c_{2c} c_{3c} c_{4c},
\]

\[
A_{x5} = \rho_{\sigma\sigma} (c_{1d}^2 + c_{2d}^2) + 2\gamma_{12} c_{1d} c_{2d} c_{3d} c_{4d}.
\]

(B2)

A similar calculation using the polarization operator (39) in the transition rates \(\Gamma_{\sigma\phi} = \frac{1}{2} |\langle j | P_{\sigma}^{\phi} | i \rangle|^2\) for the \(\sigma\)-fluorescence and the equations (B1) leads to

\[
A_{x1} = \rho_{\sigma\sigma} \gamma_{0a} (c_{1a}^2 + c_{2a}^2 + c_{4a}^2) + 2\gamma_{12} c_{1a} c_{2a} c_{3a} c_{4a} \cos 2\phi \\
+ \rho_{\sigma\sigma} \gamma_{0a} (c_{1b}^2 + c_{2b}^2 + c_{4b}^2) + 2\gamma_{12} c_{1b} c_{2b} c_{3b} c_{4b} \cos 2\phi \\
+ \rho_{\sigma\sigma} \gamma_{0a} (c_{1c}^2 + c_{2c}^2 + c_{4c}^2) + 2\gamma_{12} c_{1c} c_{2c} c_{3c} c_{4c} \cos 2\phi \\
+ \rho_{\sigma\sigma} \gamma_{0a} (c_{1d}^2 + c_{2d}^2 + c_{4d}^2) + 2\gamma_{12} c_{1d} c_{2d} c_{3d} c_{4d} \cos 2\phi,
\]

\[
A_{x2} = \rho_{\sigma\sigma} \gamma_{0a} (c_{1a}^2 + c_{2a}^2 + c_{4a}^2 + 2\gamma_{12} c_{1a} c_{2a} c_{3a} c_{4a} \cos 2\phi),
\]

\[
A_{x3} = \rho_{\sigma\sigma} \gamma_{0a} (c_{1b}^2 + c_{2b}^2 + c_{4b}^2 + 2\gamma_{12} c_{1b} c_{2b} c_{3b} c_{4b} \cos 2\phi),
\]

\[
A_{x4} = \rho_{\sigma\sigma} \gamma_{0a} (c_{1c}^2 + c_{2c}^2 + c_{4c}^2 + 2\gamma_{12} c_{1c} c_{2c} c_{3c} c_{4c} \cos 2\phi),
\]

\[
A_{x5} = \rho_{\sigma\sigma} \gamma_{0a} (c_{1d}^2 + c_{2d}^2 + c_{4d}^2 + 2\gamma_{12} c_{1d} c_{2d} c_{3d} c_{4d} \cos 2\phi).
\]

(B3)

Substituting the coefficients in the dressed states (A1) into the expressions (B2) and (B3) and simplifying, we get the weights (34) and (38) in the spectra.

ORCID iDs

R Arun ORCID: https://orcid.org/0000-0003-1740-0492

References

[1] Ficek Z and Swain S 2002 J. Mod. Opt. 49 3
[2] Ficek Z and Swain S 2005 Quantum Interference and Coherence (New York: Springer)
[3] Cardimona D A, Raymer M G and Stroud C R 1982 J. Phys. B 15 55
[4] Javanainen J 1992 Europhys. Lett. 17 407
[5] Zhu S Y, Chan R C F and Lee C P 1995 Phys. Rev. A 52 710
[6] Zhou P and Swain S 1996 Phys. Rev. Lett. 77 3995
[7] Evers J, Bullock D and Keitel C H 2002 Opt. Commun. 209 173
[8] Gong S Q, Paspalakis E and Knight P L 1998 J. Mod. Opt. 45 2433
[9] Paspalakis E, Gong S Q and Knight P L 1998 Opt. Commun. 152 293
[10] Dong P and Tang S H 2002 Phys. Rev. A 65 033816
[11] Xu W H, Wu J H and Gao J Y 2006 J. Phys. B 39 1461
[12] Bortman-Ashiv D, Wilson-Gordon A D and Friedmann H 2001 Phys. Rev. A 63 043818
[13] Macovei M, Evers J and Keitel C H 2003 Phys. Rev. Lett. 91 233601
[14] Gao S Y, Li F L and Zhu S Y 2002 Phys. Rev. A 66 043806
[15] Zhu S Y and Scully M O 1996 Phys. Rev. Lett. 76 388
[16] Paspalakis E and Knight P L 1998 Phys. Rev. Lett. 81 293
[17] Li F L and Zhu S Y 1999 Phys. Rev. A 59 2330
[18] Arun R 2013 Phys. Lett. A 377 200
[19] Li F L, Gao S Y and Zhu S Y 2003 Phys. Rev. A 67 063818
[20] Hou B P, Wang S J, Yu W L and Sun W L 2004 Phys. Rev. A 69 053805
[21] Arun R 2008 Phys. Rev. A 77 033820
[22] Yan X A, Wang L Q, Yin B Y and Song J P 2009 J. Opt. Soc. Am. B 26 1862
[23] Si L G, Liu X Y, Hao X and Li J H 2010 J. Phys. B 43 065403
[24] Arun R 2016 Phys. Rev. A 94 043843
[25] Li A J, Song X L, Wei X G, Wang L and Gao J Y 2008 Phys. Rev. A 77 053806
[26] Ficek Z and Swain S 2004 Phys. Rev. A 69 023401
[27] Zhou P and Swain S 2000 Opt. Commun. 179 267
[28] Paspalakis E, Keitel C H and Knight P L 1998 Phys. Rev. A 58 4868
[29] Kiffner M, Evers J and Keitel C H 2006 Phys. Rev. Lett. 96 100403
[30] Kiffner M, Evers J and Keitel C H 2006 Phys. Rev. A 73 063814
[31] Polder D and Schuurmans M F H 1976 Phys. Rev. A 14 1468
[32] Eichmann U, Bergquist J C, Bollinger J J, Gilligan J M, Itano W M, Wineland D J and Raizen M G 1993 Phys. Rev. Lett. 70 2359
[33] Das S and Agarwal G S 2008 Phys. Rev. A 77 033850
[34] Tan H T, Xia H X and Li G X 2009 J. Phys. B 42 125502
[35] Schmied S I and Evers J 2010 Phys. Rev. A 81 063805
[36] Sakurai J J 1994 Modern Quantum Mechanics (Reading, MA: Addison-Wesley)
[37] Scully M O and Zubairy M S 1997 Quantum Optics (Cambridge: Cambridge University Press)
[38] Lax M 1963 Phys. Rev. 129 2342
[39] Narducci L M, Scully M O, Oppo G L, Ru P and Tredicce J R 1990 Phys. Rev. A 42 1630
[40] Manka A S, Doss H M, Narducci L M, Ru P and Oppo G L 1991 Phys. Rev. A 43 3748
[41] Cohen-Tannoudji C, Dupont-Roc J and Grynberg G 1998 Atom-Photon Interactions (New York: Wiley)