Supporting Information

Rapidly and accurately shaping the intensity and phase of light for optical nanomanipulation

Xionggu Tang,* Fan Nan, and Zijie Yan*

Note 1. Relationship between uniformity and distortion
As we know, there exists Fourier transform relationship between optical fields at input and output planes, which can be expressed as follows,

\[ U(x_o, y_o) = \mathcal{F}[H(x_i, y_i)] \]  

(S1)

where \( \mathcal{F} \) denotes Fourier transform; \( U(x_o, y_o) \) is target optical fields at output planes, as given in Eq. (2); \( H(x_i, y_i) \) is optical fields at input planes, as shown in Eq. (3), if SLM can simultaneously modulate amplitude and phase of light. Actually, the commercial SLM just modulate the wavefront of optical beam, so Eq. (S1) is changed into,

\[ U'(x_o, y_o) = \mathcal{F}[H'(x_i, y_i)] \]  

(S2)

where \( H'(x_i, y_i) = \exp[j\phi'_h(x_i, y_i)] \) is optical field reflected by phase-only SLM at input planes; and \( U'(x_o, y_o) \) is real optical field at output plane. Then, by using linearity theorem, we can get,

\[ U(x_o, y_o) - U'(x_o, y_o) = \mathcal{F}[H(x_i, y_i) - H'(x_i, y_i)] \]  

(S3)

where \( H(x_i, y_i) - H'(x_i, y_i) \) stands for information loss of optical field at input plane, and \( U(x_o, y_o) - U'(x_o, y_o) \) represents the difference of optical fields at output plane. According to Parseval’s theorem, we can obtain,

\[ \iint_{-\infty}^{\infty} |U(x_o, y_o) - U'(x_o, y_o)|^2 \, dx_o \, dy_o = \iint_{-\infty}^{\infty} |H(x_i, y_i) - H'(x_i, y_i)|^2 \, dx_i \, dy_i \]  

(S4)

The Eq. (S4) can be rewritten using discrete expression,

\[ \sum_{k=-K/2 \atop s=-S/2}^{K/2 \atop S/2} |U(k,s) - U'(k,s)|^2 \Delta x_o \Delta y_o = \sum_{k=-K/2 \atop s=-S/2}^{K/2 \atop S/2} |H(k,s) - H'(k,s)|^2 \Delta x_i \Delta y_i \]  

(S5)

i.e.,

\[ \sum_{k=-K/2 \atop s=-S/2}^{K/2 \atop S/2} |U(k,s) - U'(k,s)|^2 = c_0 \sum_{k=-K/2 \atop s=-S/2}^{K/2 \atop S/2} |H(k,s) - H'(k,s)|^2 \]  

(S6)

where \( c_0 = \Delta x_j \Delta y_j / \Delta x_i \Delta y_i \). Finally, we can get,

\[ \sum_{k=-K/2 \atop s=-S/2}^{K/2 \atop S/2} |U(k,s) - U'(k,s)|^2 = c_0 \sum_{k=-K/2 \atop s=-S/2}^{K/2 \atop S/2} |a_h(k,s) - 1|^2 \]  

(S7)
where \( a_h(k,s) \) stands for \( a_h(k\Delta x_s,s\Delta y_s)/a_{mean} \), and \( a_{mean} \) is the mean amplitude. In Eq. (S7), the left value denotes the distortion degree between real optical field and target one. Now, we use standard deviation (SD) to describe the uniformity of amplitude \( a_h(k,s) \), as written below,

\[
SD = \left[ \frac{1}{(K+1)(S+1)} \sum_{k=-K/2}^{k=K/2} \sum_{s=-S/2}^{s=S/2} |a_h(k,s) - 1|^2 \right]^{1/2} \quad (S8)
\]

Then,

\[
\sum_{k=-K/2}^{k=K/2} \sum_{s=-S/2}^{s=S/2} [U(k,s) - U'(k,s)]^2 = c_9(K+1)(S+1)SD^2 \quad (S9)
\]

Obviously, the distortion degree between real optical field and target one is determined by the uniformity of amplitude \( a_h(k,s) \). If we can improve the uniformity of amplitude \( a_h(k,s) \), the quality of optical field at output plane can be promoted. In our method, the random phase is introduced in Eq. (5) to reduce interference among plane waves at input plane, which are helpful for promoting the uniformity of amplitude \( a_h(k,s) \).

Here, we present a simulation example of line optical pattern to demonstrate uniformity variation of amplitude \( a_h(k,s) \) when using random phase and not using random phase in Eq. (5), as shown in Fig. S1. The standard deviation in a(I) and b(I) are 1.3702 and 0.5547, respectively. It finds that the standard deviation greatly decreases when using random phase, and the quality of optical field at output plane are well improved, as given in (II-III) of Fig. S1.

![Simulation results of line optical pattern](image)

**Fig. S1.** The simulation results of ling optical pattern without (a) random phase and with (b) random phase. Note that panels I-III correspond to amplitude \( a_h(k,s) \) at input plane, reconstructed intensity and phase profile at output plane. Note that scale bar is 10 μm.

**Note 2.** The reconstructed optical intensity of other spot arrays
Furtherly, the other optical spot arrays are simulated by using our method. The reconstructed optical intensity distributions are shown in Fig. S2. Here, the non-uniformity is used to describe the quality of optical spot arrays. It is calculated by $\left\{\frac{1}{N} \sum_{n=1}^{N} [I_n - I_{mean}]^2\right\}^{1/2}$, where $N$ denotes number of spots, and $I_n$ is intensity of the spot $n$, and $I_{mean}$ is the mean intensity. The non-uniformity in (a-d) of Fig. S2 are 0.051, 0.0176, 0.0298 and 0.0434, respectively. It reveals that the high uniformity can be obtained when using our method to generate holograms for other optical spot arrays. Followingly, the trapping experiment are performed, in which optical power is 45 mW. The experimental results of a trapped metal nanoparticle in one of circle spot arrays are given in Fig. S3. Its standard deviations of fluctuation amplitudes are 40 nm in $x$ direction, and are 44 nm in $y$ direction, respectively. It demonstrates that the optical spot generated by our method has an excellent performance of trapping.

Fig. S2. The various reconstructed optical spot arrays. (a) $5 \times 5$ spot array. (b) Circle spot arrays. (c) Hybrid spot arrays. (d) Random spot arrays. Note that scale bar is 10 μm.
Note 3. Effects of phase noise on reconstructed intensity and phase profiles

In our method, the introduction of random phase factor is helpful for weakening crosstalk among plane waves in order to generate a desired optical field at the output plane. In this case, the phase factor will induce phase noise on the generated hologram. Here we discuss the effect of phase noise on the reconstructed intensity and phase profiles. We assume that the phase distribution of an ideal hologram is expressed as \( P_i(x_i, y_i) \), and the phase distribution of generated hologram by our method is written by \( P_o(x, y) \). The phase noise is regarded as \( P_3(x, y) \), which originates from the random phase factor employed in Eq. (5). As a result, the relationship among them is given below,

\[
P_2(x, y) = P_1(x, y) + P_3(x, y).
\]

(S10)

Similarly, the ideal amplitude and reconstructed amplitude at the output plane are written by \( A_i(x, y) \) and \( A_o(x, y) \), and the ideal phase and reconstructed phase at the output plane are expressed by \( P_i(x, y) \) and \( P_o(x, y) \). By using Fourier transform, we can obtain the following formulas

\[
A_i(x, y) \exp(jP_i(x, y)) = F \left[ \exp(jP_1(x, y)) \right],
\]

(S11)

\[
A_o(x, y) \exp(jP_o(x, y)) = F \left[ \exp(jP_2(x, y)) \right],
\]

(S12)

where \( j \) is equal to \( \sqrt{-1} \). From the Eq. (S10), (S11) and (S12), we can get

\[
A_o(x, y) \exp(jP_o(x, y)) = F \left[ \exp(jP_1(x, y)) + jP_3(x, y) \right],
\]

(S13)

i.e.,

\[
A_o(x, y) \exp(jP_o(x, y)) = F \left[ \exp(jP_1(x, y)) \right] * F \left[ \exp(jP_3(x, y)) \right],
\]

(S14)

where * stands for convolution. While modulation depth \( d_m \) is reasonably choosen, \( \exp(jP_3(x, y)) \) is a weak phase noise caused by the introduction of random phase factor. Its bandwidth is very narrow, which can be regarded to be approximately equal to the Dirac delta function \( \delta(x, y) \), as written below,

\[
F \left[ \exp(jP_3(x, y)) \right] \approx \delta(x, y)
\]

(S15)

Consequently, we have

\[
A_o(x, y) \exp(jP_o(x, y)) \approx F \left[ \exp(jP_1(x, y)) \right],
\]

(S16)

Comparing with Eq. S11, we get

\[
A_o(x, y) \approx A_i(x, y).
\]

(S17)
\[ \exp(jP_{22}(x_o, y_o)) \approx \exp(jP_{11}(x_o, y_o)). \]  

(S18)

It reveals that the phase noise induced by random phase factor nearly has no negative affect on the reconstructed intensity and phase.

We provide an example to further verify that our analysis above is reasonable. Here, the holograms, amplitudes and phases of ring traps with and without phase noise are shown in Fig. S4. These images show that the intensity and phase reconstructed from the hologram generated by our method with phase noise are nearly identical to those from the ideal hologram without phase noise. The Fourier transform of phase noise is almost equal to the Dirac delta function, which reveals that the phase noise has no substantial influence on the reconstructed intensity and phase. We thus conclude that accurate intensity and phase can be reconstructed from the hologram generated by our method.

Fig. S4. Ring traps with linear phase profiles. (a) Hologram for generating an ideal ring trap calculated using the method introduced by Roichman and Grier,¹ (b) intensity profile reconstructed from the ideal hologram, and (c) phase profile reconstructed from the ideal hologram. (d) Hologram with phase noise calculated by our method, (e) intensity profile reconstructed from our hologram, (f) phase profile reconstructed from our hologram, (g) the phase noise, and (h) Fourier transform of the phase noise. Note that scale bar is 10 μm.

**Note 4. Optimization of optical patterns**

If the reconstructed optical pattern is relatively poor due to a complex shape, we can pre-compensate its intensity distribution at the weak positions of the target pattern to achieve a more uniform pattern. Here, a rectangle pattern is shown as an example in Fig. S5. The intensity of target pattern is uniform, but its reconstructed intensity strongly fluctuates along its orbit as shown in Fig. S5(a). According to its profile feature, we simply modify \( U(k, s) \) by employing \( U(k, s)/(k^2 + s^2) \) in Eq. (5), i.e., pre-compensating its intensity profile. The improved intensity distribution is presented in Fig. S5(b). The results show that its intensity quality is obviously improved by the optimization process. Specially, its phase profile is not affected by our optimization, which is very helpful for
optical manipulation. In addition, the modulation depth, $d_m$, of the random phase factor should be reasonably chosen. An example is given in Fig. S6. It finds that its intensity and phase profiles is more acceptable when the modulation depth $d_m$ is $\pi$.

**Fig. S5.** Rectangle optical patterns (a) without optimization and (b) with optimization. The patterns in column I denote intensity profiles, and patterns in column II denote phase profiles. Note that scale bar is 5 $\mu$m.

**Fig. S6.** Line traps under different modulation depths of random phase. (a) 0, (b) 0.5 $\pi$, (c) $\pi$, (d) 1.9 $\pi$. The patterns in column I denote intensity profiles, and patterns in column II denote phase profiles. Note that scale bar is 10 $\mu$m.
**Note 5. Comparison of reconstructed optical patterns by different methods**

The reconstructed optical patterns generated by different methods are compared. Here, the rectangle-shaped optical patterns with linear phase gradient are generated by using modified GS algorithm, integral method and our method. The reconstructed intensity and phase profiles are given in Fig. S7. Obviously, it reveals that our method has an ability to achieve optical patterns with higher quality.

![Figure S7](image-url)

*Fig. S7.* The reconstructed intensity and phase profiles of rectangle optical patterns generated by (a) modified G-S algorithm, (b) Integral method and (c) our method. Note that panels I-II correspond to intensity and phase profiles, respectively. Note that scale bar is 5 μm.

**References**

1. Y. Roichman and D.G. Grier, "Three-dimensional holographic ring traps", Proc. SPIE 6483, 64830F (2007).
2. Zhanzhong Yuan and Shaohua Tao, “Generation of phase-gradient optical beams with an iterative algorithm”, J. Opt. 16, 105701 (2014).