Research on the Stability of an Air-to-Air Missile Compound Control System with a Pulse Modulator

Wang JianQi¹, Wang DongMei¹*, Yan Liang²

¹Guilin University Of Aerospace Technology, Guilin, Guangxi, 541004, China
²China Airborne Missile Academy, Luoyang, Henan, 471000, China
*E-mail: wangjianqi@guat.edu.cn

Abstract. Based on pulse modulator, a method of lateral force and aerodynamic compound control system design in air-to-air missile and the stability is studied. First, the lateral force is assumed to be continuous and blended with aerodynamic fins according to a certain proportion, and the compound control is designed based on three-loop control method. Then the switch order of direct force device is obtained based on the pulse modulator. At last, the stability of control system is analysed based on describing function method, and then the parameter of pulse modulator is properly chosen. Simulation results show the validity of the proposed design.

1. Introduction
In the future air combat, the increasingly advanced fighter puts forward higher requirements for the agile turning ability and terminal precision strike ability of air-to-air missile. Single pneumatic rudder is difficult to meet these requirements, so it is necessary to add direct force device to provide control force, to make up for the lack of pneumatic rudder. Due to the switching characteristics of the direct force device, the sliding mode control method was adopted in the design of the direct/air compound control system in literature [1-3]. In Literature [4], the direct force is assumed as a continuous quantity for design, and then the impulse equivalent method is adopted for discretization design to obtain switching instructions. However, the stability of the system during discretization design is not considered in this paper. In Literature [5], for flexible aircraft, pulse width modulator is used to design the nozzle switch instruction, and nonlinear description function method is used to analyze the stability of the system. In Literature [6], the direct force/aerodynamic double feedback compound control structure is adopted to design the control law for the air-to-air missile, and the stability of the direct force component with dead zone characteristics is analyzed. In Literature [7], the hybrid control strategy of direct force and aerodynamic force is designed by adopting fuzzy control method in consideration of the economy of control dosage. In literature [8], the equivalence method of pulse modulation is proposed, and the structure and principle of four kinds of pulse modulators are given. Literature [9] designed a nonlinear link that could avoid limit cycles without loss of performance, and proposed a hybrid control logic of direct lateral force and aerodynamic rudder surface.

To the air-to-air missile, this paper will first use direct force as a continuous control, combined with pneumatic rudder, as a separate control input, using the classic three circuit design method to design the pitch \ yaw compound control, then assigned to the aerodynamic rudder and direct force device, direct force device by pulse modulator, switch instructions. The stability of the control system with pulse modulator is discussed in this paper because pulse modulator is a nonlinear link with dead zone, hysteresis and saturation characteristics. In this paper, the direct force is assumed as a continuous
quantity and the aerodynamic rudder is mixed, and the compound control system design is converted into a single input control system design, which has a strong engineering application. By analyzing the stability of the control system, this paper puts forward the reference basis for the parameter selection of pulse modulator, which provides theoretical support for the engineering application of pulse modulator.

2. Direct/air compound control system model

The direct force device in the form of 4-nozzle gas generator, as shown in Figure 1, is located at the front end of the missile, and the direct lateral force is perpendicular to the longitudinal axis of the missile body in a "ten" shape layout. In the figure, c.g represents the center of mass of the missile, and xyz represents the missile body coordinate system. The direct force device in the form of a gas generator can generate direct forces that can be turned on or off at any time, with no response delay, and the direct force size is 2000N.

\[\begin{align*}
\dot{\theta} &= a_4 \alpha + a_5 \delta + a_5' \delta_{rcs} \\
\dot{\omega}_x &= a_4 \omega_x + a_2 \alpha + a_3 \delta + a_3' \delta_{rcs} \\
\dot{\alpha} &= \omega_x - \dot{\theta}
\end{align*}\]

Among them,

\[\begin{align*}
a_5' &= \frac{F_{rcs}}{mV} \\
a_3' &= \frac{L_{rcs}}{J_z}
\end{align*}\]

\( \alpha \) is the Angle of attack, \( \theta \) is trajectory inclination angle, \( \delta \) is Pneumatic rudder deviation, \( m \) is the quality of the missile, \( V \) is missile velocity, \( F_{rcs} \) is the lateral thrust produced by a direct force device, \( \delta_{rcs} \) is switching instruction for direct force device, \( \delta_{rcs} \in [0,1] \), \( L_{rcs} \) is a moment arm of lateral thrust produced by a direct force device, \( \omega_x \) is the angular velocity of pitch, \( J_z \) is rotational inertia, \( a_1, a_2, a_3, a_4, a_5 \) is the commonly used pneumatic parameters.

Assuming that the direct force device generates a continuous control quantity, at a certain feature point, the direct force switch command is equivalent to the pneumatic rudder in a certain proportion, and let

\[\delta_{rcs} = K \cdot \delta\]

Equation (1) becomes:

\[\dot{\theta} = a_4 \alpha + (a_5 + Ka_5') \delta\]

Letting

\[a_{5blend} = a_5 + Ka_5'\]

then

\[\dot{\theta} = a_4 \alpha + a_{5blend} \delta\]

Similarly, if Equation (6) is substituted into Equation (2), then there is

\[\dot{\omega}_x = a_1 \omega_x + a_2 \alpha + (a_3 + Ka_3) \delta\]

Letting

\[a_{3blend} = a_3 + Ka_3'\]

then

\[\dot{\omega}_x = a_1 \omega_x + a_2 \alpha + a_{3blend} \delta\]

In this way, the relevant transfer function of the pitching channel can be obtained as follows:

\[\frac{\omega_x(s)}{\delta(s)} = \frac{R_{kr}(a_{1}\alpha + a_{2}\alpha + a_{3}\alpha + a_{4}\alpha + a_{5}\alpha)}{b_{12}\alpha + b_{11}\alpha + b_{10}\alpha + b_{0}}\]
Among them, \( K_{\omega z} = \frac{a_2 a_{5\text{blend}} - a_3 a_{4}}{a_2 + a_4} a_4 \), \( a_{3\text{blend}} = -\frac{a_3 a_4}{a_2 a_{5\text{blend}} - a_3 a_4} \), \( K_{\omega y} = K_{\omega z} V \), \( a_{12} = \frac{a_{\text{blend}}}{a_2 a_{5\text{blend}} - a_3 a_4} a_1 \), \( b_{12} = \frac{1}{a_2 + a_4} b_{11} = \frac{a_4}{a_2 + a_4} \).

The control gain is designed according to the three-loop design method [10]. The three-loop schematic diagram of the pitching channel is shown below.

The control gain is designed according to the three-loop design method [10]. The three-loop schematic diagram of the pitching channel is shown below.

![Figure 2. Schematic diagram of three-loop of pitch channel](image)

The fixed proportion in Equation (6) can be selected according to Equation (13)

\[ K = \frac{K_{\text{controller}}}{a_{3/4}} \times 57.3 \]  

3. Pulse modulator

The control quantity designed in the above section is a time domain continuous function, while the direct force device needs discrete impulse control quantity, which requires the continuous control signal to be transformed into discrete impulse control signal. The modulation from continuous quantity to pulse quantity can be realized by pulse modulator. Common pulse modulators are Schmidt flipflop, Pulse Width-modulated Frequency Modulation (PWPF) modulator, Pseudo-rate (PSR) modulator and integral pulse modulator. In this paper, Schmidt flip-flop and PWPF modulator are used to modulate the continuous quantity.

The Schmidt trigger structure is shown in Figure 3. Schmidt flip-flop is a relay with dead zone and hysteresis. This modulation method is very simple. Before Schmidt flip-flop is a continuous control input, and the control U obtained through modulation is a pulse train, which is used to control the switch of pulse engine.

![Figure 3. Schmidt trigger](image)

The PWPF modulator is shown in Figure 4. PWPF modulator is composed of first order inertial link and Schmidt trigger.

![Figure 4. PWPF modulator](image)

After replacing the pulse modulator link in Fig. 2 with the modulator in Fig. 3 and Fig. 4, the stability of the system is analyzed.
4. The descriptive function method is used to analyze the stability of the system
Because the pulse modulator is used to pulse modulate the continuous quantity, and the pulse modulator contains nonlinear link, the stability of the system with the pulse modulator should be considered. The stability of the system is analyzed by the descriptive function method of nonlinear system.

The basic idea of the descriptive function method is that when the system satisfies certain assumptions, the output of the nonlinear link in the system under the action of sinusoidal signal can be approximated by the first harmonic component, thus the approximate equivalent frequency characteristics of the nonlinear link can be derived, that is, the descriptive function. Thus, the frequency response method of linear system can be extended to nonlinear system.

It is assumed that the nonlinear system can be transformed into A structure as shown in Fig. 5, A unit negative feedback system consisting of A nonlinear link \(N(A)\) and A linear part \(G(s)\). Here, the nonlinear link may be the total nonlinear equivalent link of several physical components. The limit cycle phenomenon often appears because of the nonlinear link in the system. If there is a limit cycle in the system, the system of all the signals must be cycle, as a periodic signal, linear link in figure 7 input can show as the sum of a number of harmonic, and the linear link is generally low pass filtering characteristics, can filter out high frequency signal, and its output is mainly composed of lowest harmonics. Therefore, it is appropriate to assume that the signal in the entire system is of fundamental form. In this way, we can assume that there is some limit cycle of unknown amplitude and frequency in the system, and then confirm the existence of such solution in the system, and then determine the amplitude and frequency of the limit cycle.

In this paper, Schmidt flip-flop and PWPF modulator are taken as examples to analyze the stability of the closed-loop system by using the descriptive function method.

The description function of Schmidt trigger is:

\[
N(A) = \frac{2M}{\pi A} \left[ \sqrt{1 - \left( \frac{m e_0}{A} \right)^2} + \sqrt{1 - \left( \frac{e_0}{A} \right)^2} \right] - \frac{2Me_0}{\pi A^2} (m-1) \quad A \geq e_0
\]  

In figure 6 \(e_0\) represents \(U_{on}\) of figure 3, 4, 5, 6, \(me_0\) represents \(U_{off}\).

When the nonlinear link in Fig. 3 is used, the linear part \(G(s)\) is the open-loop transfer function of the pitching channel.
If the nonlinear link in Figure 4 is used, a transformation is required. By replacing the nonlinear link in Figure 5 with the PWPF modulator in Figure 4, the linear part is the open-loop transfer function equation (15) of the pitching channel, which can be obtained:

\[
HG = I_d \frac{a_3(s+a_4)}{s^2+B_0s+C_0} (1 + \frac{K_0}{s+A_4}) - I_d \frac{v(t)}{s(s^2+B_0s+C_0)}
\]  

(15)

The linear part right here is \((HG + 1)G_1\). For two different pulse modulators, whether the system produces limit cycles is analyzed according to Nyquist diagram, and if so, the stability of limit cycles is analyzed.

Figure 7. Schematic diagram of closed-loop system with PWPF modulator

Figure 7 shows that

\[
e = -c - y
\]

(16)

\[
G_1 \cdot N(A) \cdot e = y
\]

(17)

\[
HG \cdot y = c
\]

(18)

Among them, \(G_1 = \frac{K_m}{T_m+1}\). It can be obtained from Equations (15), (16) and (17)

\[
1 + (HG + 1)G_1 \cdot N(A) = 0
\]

(19)

The linear part right here is \((HG + 1)G_1\).

For two different pulse modulators, whether the system produces limit cycles is analyzed according to Nyquist diagram, and if so, the stability of limit cycles is analyzed. Fig. 8b is the local amplification of Fig. 8a at the point (-1, j0). The blue curve is the frequency response curve of the linear part, and the green part is the description function curve of Schmitt flip-flop. As can be seen from the figure, the frequency curve of the linear part does not contain the descriptive function curve of Schmitt flip-flop, and the nonlinear system does not generate limit cycles.

Figure 8. Description function of Schmitt trigger and Nyquist curve of linear link

Take different values of \(K_m, T_m\) in the PWPF modulator \(G_1 = \frac{K_m}{T_m+1}\) for simulation, the following results can be obtained. Fig. 9b is the local amplification of Fig. 9a at the point (-1, j0), the blue curve is the frequency response curve of the linear part, and the green part is the description function curve of the PWPF flip-flop. It can be seen from the figure that there are three points of intersection between the frequency curve of the linear part and the description function curve of the nonlinear part. Among them, unstable limit cycles are generated at point A, and stable limit cycles are generated at points B and C.
Figure 9. Description function of PWPF trigger and Nyquist curve of linear link, $K_m=1$, $T_m=0.1$

The blue curve in Fig. 10 is the frequency response curve of the linear part, and the green curve is the description function curve of the PWPF flip-flop. As can be seen from the figure, there is no intersection point between the frequency curve of the linear part and the description function curve of the nonlinear part, and the system is stable and will not generate limit cycles.

Figure 10. Description function of PWPF trigger and Nyquist curve of linear link, $K_m=1$, $T_m=0.01$

Fig. 11b is the local amplification of Fig. 11a at the point (-1, j0), the blue curve is the frequency response curve of the linear part, and the green part is the description function curve of the PWPF flip-flop. As can be seen from the figure, there are two points of intersection between the frequency curve of the linear part and the description function curve of the nonlinear part. At point A, an unstable limit cycle is generated, while at point B, a stable limit cycle is generated.

Figure 11. Description function of PWPF trigger and Nyquist curve of linear link, $K_m=10$, $T_m=0.1$
After analyzing the simulation results, the following conclusions can be drawn:

1. The Schmidt trigger is used for pulse modulation, and the system does not generate limit cycles.
2. When using the PWPF modulator for pulse modulation, the system will have different states if different values of $K_m, T_m$ in the continuous link of the modulator $G = \frac{K_m}{T_m+1}$, and there will be no limit cycle in the system through the selected values.

5. Simulation results and discussion

Choose $a_1 = 3.0s^{-1}, a_2 = 40.0s^{-2}, a_3 = 289.0s^{-2}, a_4 = 1.2s^{-1}, a_5 = 0.58s^{-1}$. Missile altitude $H = 2000m$, Missile velocity $V = 272m/s$, the lateral force generated by a direct force device $F_{rcs} = 2000N$, the moment arm of a direct force device $L_{rcs} = 1.2m$, weight of missile $m = 100Kg$, rotational inertia $J_z = 200Kg \cdot m^2$, $U_{on} = 0.45$, $U_{off} = 0.15$, two sets of values $K_m = 1$, $T_m = 0.01$ and $K_m = 1$, $T_m = 0.1$ were selected in the first order inertia link of PWPF trigger.

On the basis of limit cycle analysis, the acceleration command applied to the pitching channel is mainly considered on the projectile body. The results of the two pulse modulators are shown in the figure below. Figure 12-14 is the simulation results of the system using Schmidt trigger; Figure 15 and 16 are the simulation results of the PWPF modulator when $K_m = 1$ and $T_m = 0.1$; Figure 17-19 is the simulation results of the PWPF modulator when $K_m = 1$ and $T_m = 0.01$.

As can be seen from the results in Figure 12-14, the control system is stable and the direct force device acts in the rising segment of acceleration response, which improves the rapidity of acceleration response.

![Figure 12. Acceleration response curve using Schmidt trigger](image1)

![Figure 13. Pneumatic rudder deflection curve using Schmidt trigger](image2)

![Figure 14. Switching command of direct force device using Schmidt trigger](image3)

As can be seen from the results in Figure 15 and 16, the system generates stable limit cycles and the direct force device switches on and off frequently. This corresponds to the analysis of system stability in the previous section, that is, when $K_m = 1$ and $T_m = 0.1$, the system has a stable limit cycle.
As can be seen from Fig. 17-19, the control system is stable, and the direct force device acts in the rising and adjusting segments of the acceleration response, which improves the rapidity of the acceleration response.

From the above simulation results, it can be seen that the control system using Schmidt trigger does not generate limit cycles, and can improve the speed of acceleration response to a certain extent, and the consumption of direct force is less; In the control system with PWPF modulator, the limit cycle can be eliminated through the selection of modulator parameters, and the acceleration response has a shorter rise time than that with Schmidt trigger, but the consumption of direct force increases. Therefore, the appropriate modulator can be selected by considering the acceleration response and direct force consumption in practical application.
6. Conclusion
In this paper, a pulse modulator is proposed for the design of air-to-air missile direct/air compound control system. The direct force is assumed as a continuous quantity, and then the aerodynamic rudder is mixed in a fixed proportion. On this basis, the control gain is obtained according to the classical three-loop design method. Then, the direct force of continuous pulse modulator is used for modulation, the direct force device switch command, and make use of nonlinear describing function method for the stability of the control system was analyzed, and the analysis results show that the control system adopts the schmidt modulator won't generate limit cycle, and stability control system, control system using PWPF modulator can eliminate the limit cycle, through the modulator parameter selection and control system is stable. Finally, the rationality of the proposed method is proved by simulation.

Acknowledgments
This work is funded by the National Natural Science Foundation of China. The project number is 61966010. Thanks for the help and support of the National Natural Science Foundation of China.

Reference
[1] Ma KeMao, Zhao Hui, Zhang DeCheng. Design and Implementation of Direct Lateral Force and Aerodynamics Compound Control for Missile[J]. Journal of Aerospace. 2011, 32(2): 310-316.
[2] Yan Liang, Zhao YuJie, ZhaoYanhui. Design of Lateral Motion Controller for Air-to-Air Missile over – Shoulder Launch [J]. Navigation, Positioning and Timing, 2015,1:11-15.
[3] Yan Liang, Ma KeMao, Dong JiPeng. Over-shoulder Launch Control Design of Air-to-Air Missile with Direct Force [J]. Aviation Weapon, 2013,6:3-8.
[4] Zhang YouAn, Wu HuaLi, Liang Yong. Forward Interception Guidance Law Considering Dynamic Characteristics of Compound Control System [J]. Journal of Astronautics. 2015, 36(2):158-164.
[5] Geng YunHai, Cui HuTao, Yang Di. Pulse Modulation Control for Flexible Aircraft [J]. Journal of Astronautics. 1997,18(4):37-43
[6] Fan GuoLong, Yang Jun. Stability Analysis of Direct Lateral Force/Aerodynamic Compound Control System[J], Computer Simulation. 2011, 28(6):96-100.
[7] Zhao YanHui, Zhang GongPing, Yang YuRong. Design of Fuzzy Logic Autopilot for Direct Force/Aerodynamics Compound Control Missile [J]. Journal of Projectiles, Rockets, Missiles and Guidance. 2014,34(3):33-36
[8] Guo QinChen. Research on Constant Thrust Attitude Control Method [D]. Master Dissertation of Engineering, Harbin Institute of Technology, June 2006.
[9] Lu Yanhui, Zhang ShuGuang. Improvement of PWPF Modulation Mode and Design of Hybrid Control Logic for Discrete RCS [J]. Journal of Aerospace. 2012,33(9):1561-1570.
[10] Zarchan P. Tactical and strategic missile guidance [M]. AIAA,INC, 2007.