**Heterotic Matrix theory with Wilson lines on the lightlike circle**

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We develop a matrix model for the $SO(32)$ Heterotic string with certain Wilson lines on the lightlike circle. This is done by using appropriate T-dualities. The method works for an infinite number of Wilson lines, but not for all. The matrix model is the theory on the D-string of type I wrapped on a circle with a $SO(32)$ Wilson line on the circle. The radius of the circle depends on the Wilson line. This is a 1+1 dimensional $O(N)$ theory on a circle. $N$ depends not only on the momentum around the lightlike circle but also on the winding and $SO(32)$ quantum numbers of the state. Perspectives for obtaining a matrix model for all Wilson lines are discussed.

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1. Introduction

During the last year a matrix model for the $E_8 \times E_8$ Heterotic string theory has been developed $[1,2,3,4,5,6,7,8,9,10,11]$. The model can be derived by following Sen’s and Seiberg’s prescription $[12,13]$ for M-theory on $S^1/\mathbb{Z}_2$. The result is that Heterotic $E_8 \times E_8$ is described by the decoupled theory of D-strings in type II string on a circle with a Wilson line on the circle breaking $SO(32)$ to $SO(16) \times SO(16)$. It is important to note that this matrix theory is a description of the $E_8 \times E_8$ Heterotic string with a Wilson line on the lightlike circle, which breaks $E_8 \times E_8$ down to $SO(16) \times SO(16)$.

Recently a matrix model for the $SO(32)$ Heterotic string was derived $[14]$. Here the theory had no Wilson line on the lightlike circle. So the situation, by now, is that two 10 dimensional Heterotic matrix models are known: The $E_8 \times E_8$ theory with a Wilson line on the lightlike circle breaking $E_8 \times E_8$ to $SO(16) \times SO(16)$ and the $SO(32)$ theory with unbroken gauge group. Matrix models for Heterotic strings compactified on tori and K3 are also known $[8,10,15]$. In this paper we will only be interested in the 10 dimensional case.

The purpose of this paper is to find matrix models for other Wilson lines of the $SO(32)$ theory. The motivation to search for such theories come from recent developments in M theory on $T^2$ $[16,17,18,19,20,21,22]$, where a matrix model was derived in the case of a non-zero 3-form potential, $C_{-12} \neq 0$. The index $-$ denotes the lightlike direction and 1, 2 the $T^2$. The important feature is that background fields are turned on along the lightlike circle. The resulting matrix model is a SYM theory on a noncommutative torus. The advantage of noncommutative geometry is that it works for all values of $C_{-12}$ and that it is continuous in $C_{-12}$. The drawback of the noncommutative geometry approach is that the theories are nonlocal and it is not clear whether they make sense as quantum theories.

However for a dense subset of values of $C_{-12}$ the resulting matrix model can be obtained using standard T-dualities without use of noncommutative geometry. One might argue that this is good enough for physical applications. The problem, however, is that the involved T-dualities depend crucially on the value of $C_{-12}$ giving a description of the physics which has no manifest continuity in $C_{-12}$.

It is natural to believe that the Heterotic theories behave similarly. This is especially so for the $SO(32)$ theory, where the description without a Wilson line is known $[14]$. In this paper we will show that for a countable set of Wilson lines of the $SO(32)$ Heterotic string theory standard T-dualities will provide us with a matrix description of the theory.
Furthermore we will briefly speculate on perspectives for obtaining a noncommutative model for the Heterotic string.

The organization of the paper is as follows. In section 2 we briefly discuss the case of M theory on $T^2$ because of its similarity with the Heterotic case. In section 3 we discuss T-dualitites of the SO(32) Heterotic string and show how to obtain a matrix model for a countably infinite number of Wilson lines. In section 4 we discuss perspectives for obtaining a noncommutative model for all Wilson lines. We end with a conclusion in section 5.

2. M theory on $T^2$

Let us first recall how to obtain the matrix model for M theory on $T^2$ following [12], [13]. For the moment we set $C_{-12} = 0$. Let the lightlike circle have radius $R$, the Planck mass be $M$ and momentum $\frac{N}{R}$. The radii of the $T^2$ are called $R_1, R_2$. For simplicity we take the torus to be rectangular. By Seiberg’s boost and rescaling [13] we are led to consider a theory on a spatial circle of radius $\tilde{R}$, Planck mass $\tilde{M}$ and radii $\tilde{R}_1, \tilde{R}_2$ in the limit $\tilde{R} \to 0$

This turns into $N$ D0-branes in type IIA with string mass $m_s$ and coupling $\lambda$ on a $T^2$ with radii $\tilde{R}_i$ where

$$m_s^2 = \tilde{M}^3 \tilde{R}$$
$$\lambda = (\tilde{M} \tilde{R})^{3/2}$$

Now the transverse radii $\tilde{R}_i$ shrink so we perform a T-duality on both circles. This finally gives a theory with $N$ D2-branes on a $T^2$ with radii

$$R_i' = \frac{1}{m_s^2 \tilde{R}_i} = \frac{1}{M^3 R \tilde{R}_i}$$

which are finite. This is a $U(N)$ gauge theory. The crucial step was the T-duality on the two transverse circles. A T-duality turns a circle with radius going to zero into a circle with a radius going to infinity. This is so for fixed string mass. In our case the string mass goes to infinity so the final circle actually has finite size.

Let us now turn on a background field $C_{-12} \neq 0$. Following the same procedure as before we get to type IIA with a B field, $B_{12} \neq 0$. The radii of the torus are shrinking exactly as before. We would like to perform a T-duality which makes the radii large. The
standard T-duality, used above, does not do the job. When $B_{12} \neq 0$ it relates a shrinking torus to another shrinking torus as we explain now.

First we must recall how T-duality works for type IIA on a $T^2$. The compactification is, among others, specified by 4 real parameters. These are the complex structure, $\tau$, of the torus, the volume, $V$, and the integral of the B-field over the torus, $B$. $V$ and $B$ are conveniently collected into a complex number in the upper halfplane, $\rho = B + iV$. Here $V$ is measured in string units, so $\rho$ is dimensionless. The T-duality group \[^{[23]}\] is now $SL(2, \mathbb{Z}) \times SL(2, \mathbb{Z})$ where the first factor acts on $\tau$ and the second on $\rho$. There is also a T-duality which exchanges $\tau$ and $\rho$, but that will take us from type IIA to type IIB. We will only be concerned with $\rho$ and the associated $SL(2, \mathbb{Z})$. $V$ goes to zero in Seiberg’s limit, so we would like to make a T-duality that takes us to large $V$. In the following we will work at fixed string mass. When needed the string mass can easily be reinstated.

Let us perform the T-duality given by

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, \mathbb{Z}) \tag{2.4}$$

The transformed $\rho$ is

$$\rho' = \frac{a\rho + b}{c\rho + d} = \frac{a(B + iV) + b}{c(B + iV) + d} \tag{2.5}$$

giving

$$B' = \frac{(aB + b)(cB + d) + acV^2}{(cB + d)^2 + c^2V^2} \tag{2.6}$$

$$V' = \frac{V}{(cB + d)^2 + c^2V^2}$$

We see that $V' \rightarrow 0$ unless $cB + d = 0$ in which case $V' \rightarrow \infty$. We conclude that exactly for rational values of $B$ can we obtain a matrix model in this way. Let us look more closely at what model we obtain in the case $cB + d = 0$. Suppose we start in a sector with $N$ D0-branes and $N_2$ membranes wrapped around the $T^2$. After the T-duality we have

$$\begin{pmatrix} N' \\ -N_2' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} N \\ -N_2 \end{pmatrix} \tag{2.7}$$

Especially $N_2' = -cN + dN_2$ so we do not get a $U(N)$ theory but a $U(| - cN + dN_2|)$ theory. The volume of the torus is $V' = \frac{1}{c^2V}$ which is reduced by a factor of $\frac{1}{c^2}$ compared to the case $B = 0$. Furthermore there is a background field $B' = \frac{a}{c}$. We see that for various rational values of $B = -\frac{d}{c}$ we get gauge theories with different unitary groups on
tori of different size. The achievement of noncommutative geometry is that it can give a continuous description of the physics.

Before we go on to the main interest of this paper, namely the Heterotic string, let us understand in another way why any rational value of B can be coped with by T-dualitites. This will prove useful for understanding the Heterotic T-dualities in the next section. The group $SL(2, Z)$ is generated by the two elements S and T.

\[
S = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \quad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}
\]  

(2.8)

S is the usual volume inversion when $B = 0$ and T shifts the B field by one. We are considering $\rho = B + iV$ in the limit $V \to 0$. We now observe that setting $V = 0$ and then acting with S or T or acting with S or T and then setting $V=0$ amounts to the same thing with one exception. This exception is acting with S in the case $B = 0$. This observation follows easily from the explicit expressions

\[
T : \quad B + iV \to B + 1 + iV
\]

\[
S : \quad B + iV \to \frac{-B}{B^2 + V^2} + i \frac{V}{B^2 + V^2}
\]  

(2.9)

This means that we can use the following strategy given a B. We put $V = 0$ and act with a product of powers of S and T until $B=0$. Finally acting with S we get to infinite volume as we want. Let us illustrate this with an example. Suppose $B = \frac{2}{3}$. We then have the following group actions.

\[
B = \frac{2}{3} \xrightarrow{T^{-1}} B = -\frac{1}{3} \xrightarrow{S} B = 3 \xrightarrow{T^{-3}} B = 0
\]  

(2.10)

In other words the matrix

\[
ST^{-3}ST^{-1} = \begin{pmatrix} -1 & 1 \\ -3 & 2 \end{pmatrix}
\]  

(2.11)

does the job. We did not learn much new from this since we solved the problem above. However the equivalent method will prove useful in the Heterotic case, where the T-duality group is more complicated.

3. SO(32) Heterotic String theory

In this section we will first review various well known facts about T-duality in Heterotic theories [24,25]. We will then use this information to show that a matrix model for the SO(32) Heterotic string can be obtained for a countable number of Wilson lines on the lightlike circle. This extends the result in [14] where the matrix model was derived in the case of no Wilson line.
3.1. Heterotic T-dualities

The moduli space for Heterotic string theory compactified on a circle is

\[ O(17,1,Z)\backslash O(17,1,\mathbb{R})/O(17) \times O(1). \]  

There is no distinction between \( E_8 \times E_8 \) and \( SO(32) \) as soon as we compactify on a circle. We will think of this moduli space in \( SO(32) \) terms, since this is the case of relevance for us. Let us try to understand the moduli space more concretely. Consider the space \( \mathbb{R}^{17,1} \) with metric \( g_{\mu\nu} = \text{diag}(1, \ldots, 1, -1) \). In this space we place the even selfdual lattice \( \Gamma^{17,1} = \Gamma^{16} + \Gamma^{1,1} \). \( \Gamma^{16} \) is the weight lattice of \( \text{spin}(32)/\mathbb{Z}_2 \) which can be described as the set of points \((a_1, \ldots, a_{16})\) where either all \( a_i \in \mathbb{Z} \) or all \( a_i \in \mathbb{Z} + \frac{1}{2} \) and in both cases \( \sum_i a_i \) is even. \( \Gamma^{1,1} \) is the lattice consisting of points \( \frac{1}{\sqrt{2}}(m+n, m-n) \) for \( m, n \in \mathbb{Z} \). Consider the basis vectors \( e_i = (0, \ldots, 0, 1, 0, \ldots, 0) \) of \( \mathbb{R}^{17,1} \). The first 17 of these, \( e_i \quad i = 1, \ldots, 17 \) span a 17 dimensional subspace of positive signature and \( e_{18} \) span a 1 dimensional subspace of negative signature. Let \( g \in O(17,1,\mathbb{R}) \), then \( ge_i, i = 1, \ldots, 17 \) span a 17 dimensional subspace of positive signature and \( ge_{18} \) span an orthogonal 1 dimensional subspace of negative signature. Every such subspaces of \( \mathbb{R}^{17,1} \) can be obtained for appropriate \( g \). If \( h \in O(17) \times (1) \), then \( gh \) obviously gives the same subspaces as \( g \); the basis have just changed. This gives an identification of \( O(17,1,\mathbb{R})/O(17) \times O(1) \) with the space of 17 dimensional subspaces of positive signature. Note that the 1 dimensional subspace is determined as the orthogonal complement of the 17 dimensional space.

Now let \( P \in \Gamma^{17,1} \). \( P \) represents, together with other quantum numbers, a state in the theory. Let also a \( g \in O(17,1,\mathbb{R})/O(17) \times O(1) \) be given. \( g \) determines two subspaces as explained above. Decompose \( P \) as \( P_L + P_R \), where \( P_L \) is in the 17 dimensional subspace and \( P_R \) is in the 1 dimensional subspace. This is a unique decomposition. In terms of the basis \( e_i \),

\[ (P_L)_i = \langle ge_i, P \rangle \quad i = 1, \ldots, 17 \]

\[ (P_R) = - \langle ge_{18}, P \rangle \]  

where \( \langle, \rangle \) denotes the inner product in \( \mathbb{R}^{17,1} \).

The space \( O(17,1,\mathbb{R})/O(17) \times O(1) \) is 17 dimensional. It is not hard to see that the space can be parametrized by a vector \( A \in \mathbb{R}^{16} \) and a number \( R > 0 \). As explained in \[25\] \( R \) can be understood as the radius of the circle and \( A \) is the Wilson line. Given a lattice
Let us now consider the T-duality group $O(17, 1, Z)$. This is the group of $O(17, 1)$ matrices which preserve the lattice $\Gamma^{17,1}$. The $Z$ does not mean that the entries in the matrix are integers, but that the lattice $\Gamma^{17,1}$ is preserved. A T-duality transformation by an element $t \in O(17, 1, Z)$ means that we map the lattice into itself before we project into left- and right-movers. Since the theory is determined by the full set of states and not by how we label them, this is a symmetry of the theory. Let $P \in \Gamma^{17,1}$. The T-duality transformation maps it into $tP$. The decomposition into left- and right-movers is

\begin{equation}
(P_L)_i = \langle ge_i, tP \rangle = \langle t^{-1} ge_i, P \rangle \label{eqn:3.3}
\end{equation}

\begin{equation}
(P_R)_i = \langle ge_{18}, tP \rangle = \langle t^{-1} ge_{18}, P \rangle \label{eqn:3.4}
\end{equation}

where we used that the adjoint of $t$ is $t^{-1}$ for $t \in O(17, 1, Z)$. This shows that $g$ and $t^{-1}g$ define the same point in the moduli space.

Consider a point $P \in \Gamma^{17,1}$ and a $g \in O(17, 1)$. $P$ is specified by $(Q, m, n)$. $g$ corresponds to $(R, A)$ as above. Consider also the point $tP \in \Gamma^{17,1}$ specified by $(Q', m', n')$ and $tg \in O(17, 1)$ corresponding to $(R', A')$. Since $t$ is in $O(17, 1, Z)$ the decomposition of $P$ into $(P_L, P_R)$ with respect to $g$ is the same as the decomposition of $tP$ into $(P'_L, P'_R)$ with respect to $tg$. We might thus think that if we plug into eq.(3.3) with primed and unprimed variables we get the same result. This is not true since the set $(R, A)$ corresponds to an element in $O(17, 1)/O(17) \times O(1)$ and not an element in $O(17, 1)$. In comparing $(P_L, P_R)$ with $(P'_L, P'_R)$ obtained from eq.(3.3) we should thus allow for a $O(17) \times O(1)$ transformation. This might seem a bit complicated but we are saved by the fact that $O(1)$ is a very simple group. It consists of $\{\pm 1\}$. We thus conclude that for a T-duality given by $t \in O(17, 1, Z)$

\begin{equation}
\frac{m - \frac{1}{2}A^2 n - AQ}{R} - nR = \pm \left( \frac{m' - \frac{1}{2}A'^2 n' - A'Q'}{R'} - n'R' \right) \label{eqn:3.5}
\end{equation}

The sign is actually easily determined from $t$. It is equal to the sign of the matrix entry $t_{18,18}$. 

\section*{References}

[20]
Formula (3.5) will be important to us. We will use it to determine $A', R'$ as a function of $A, R$ for a given T-duality. For a given T-duality, $(m', n', Q')$ are known in terms of $(m, n, Q)$. The relation is a linear function. Collecting the coefficients of $(m, n, Q)$ in eq.(3.5) will give equations which determine $(A', R')$ in terms of $A, R$.

Let us now go on to consider the structure of $O(17, 1, Z)$. We will list 3 different types of elements which we will use to obtain matrix models.

**Case1.**

$$T_U = \begin{pmatrix} U_{16 \times 16} & 0 \\ 0 & 1_{2 \times 2} \end{pmatrix}$$

where $U \in O(16, Z)$. Here $Z$ means that $\Gamma^{16}$ is preserved. This T-duality transformation acts as follows

$$Q' = UQ$$

$$m' = m$$

$$n' = n$$

Solving eq.(3.5) we get

$$R' = R$$

$$A' = (U^t)^{-1}A$$

**Case2.**

$$T_P = \begin{pmatrix} 1_{16 \times 16} & \frac{P}{\sqrt{2}} & \frac{-P}{\sqrt{2}} \\ \frac{-P}{\sqrt{2}} & 1 - \frac{1}{4}P^2 & \frac{1}{4}P^2 \\ \frac{P}{\sqrt{2}} & -\frac{1}{4}P^2 & 1 + \frac{1}{4}P^2 \end{pmatrix}$$

with $P \in \Gamma_{16}$. This transformation acts as follows

$$Q' = Q + nP$$

$$m' = m - \frac{1}{2}nP^2 - QP$$

$$n' = n$$

Solving eq.(3.5) we get

$$R' = R$$

$$A' = A - P$$

**Case3.**

$$S = \begin{pmatrix} 1_{16 \times 16} & 0 \\ 0 & \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \end{pmatrix}$$
Here

\[ Q' = Q \]
\[ m' = -n \]
\[ n' = -m \]  \hspace{1cm} (3.13)

Solving eq. (3.5) we get

\[ R' = \frac{R}{R^2 + \frac{1}{2}A^2} \]
\[ A' = \frac{A}{R^2 + \frac{1}{2}A^2} \]  \hspace{1cm} (3.14)

### 3.2. Matrix models

Now we will consider the task of finding a matrix model for the SO(32) Heterotic string with a Wilson line on the lightlike circle. Let us first briefly review the method for obtaining this matrix model in the case of no Wilson line [14]. We start with the SO(32) Heterotic theory with string mass \( M \), coupling \( \lambda \) and radius of the lightlike circle \( R \). We consider the sector with momentum \( N_R \). First we follow Seiberg [13] and relate it to a spatial compactification on a shrinking circle. We now perform the T-duality called \( S \) above. In the case \( A = 0 \) it sends \( R \to \frac{1}{R} \) and exchanges momentum and winding. This takes us to a sector with winding \( N \) on a large circle. Finally we employ the Heterotic SO(32)-type I duality. This produces a theory on \( N \) D-strings of type I wrapped on a circle. This is an O(N) gauge theory.

If \( A \neq 0 \) this chain of dualities will not work, since the T-duality transformation, \( S \), only maps a small circle into a large circle when \( A = 0 \). However by analogy with M theory on \( T^2 \) we could try another T-duality transformation. If we can find a T-duality transformation that makes the circle large we will have obtained a matrix model, since the type I- SO(32) Heterotic duality works in any case. The type I coupling does go to zero and the theory on the D-strings decouples.

Let us now show that for a countable number of Wilson lines such a T-duality transformation actually exists. We will follow a procedure similar to the one described in the end of section 2 for M theory on \( T^2 \).

First we notice that the limit \( R \to 0 \) commutes with the T-duality elements \( T_U, T_P \) and \( S \) except for \( S \) when \( A = 0 \). Given a Wilson line \( A \) we should find a product of \( T_U, T_P \) and \( S \) which takes \( A \) to zero. All this should be done for \( R = 0 \). The resulting T-duality transformation succeeded by \( S \) will then do the job. We could also reverse the process and
start with any product of $T_U, T_P$ and $S$, not ending with $S$. The inverse of that will then map $A = 0$ into some $A$. This $A$ will then be an $A$ for which a matrix model exist. We see that this exactly produces a countable set of Wilson lines for which a matrix model can be obtained using this approach. They certainly all have rational entries. Unlike the case of M theory on $T^2$ it is not all Wilson lines with rational entries which can be solved in this way.

Let us take an explicit example. Start with Wilson line $A = (\frac{1}{2}, 0, \ldots, 0)$ and a shrinking spatial circle, $R_s \to 0$. This Wilson line breaks the gauge group down to $U(1) \times SO(30)$. Performing the duality $ST_PS$ with $P = (4, 0, \ldots, 0)$ we easily calculate that the resulting radius and Wilson line are $R'_s = \frac{1}{8 R_s}, A' = -A$. After dualizing to type I it becomes an $O(N')$ theory on a circle of radius $\frac{1}{8}$ of what was the case for $A = 0$ and with a Wilson line $A'$ around the circle. The number $N'$ in $O(N')$ is

$$N' = -(n + 8m - Q \cdot (4, 0, \ldots, 0)) \quad (3.15)$$

We see that it is not just equal to the momentum $m$.

Now we have seen, that lots of Wilson lines can be coped with in this way, it is natural to ask whether the $SO(16) \times SO(16)$ point is one of these. This would be strange since the resulting matrix model would be the theory on D-strings of type I wound on a circle with a Wilson line breaking $SO(32)$ to $SO(16) \times SO(16)$. This model is known to be the matrix model for $E_8 \times E_8$ Heterotic string theory on the $SO(16) \times SO(16)$ point. It would lead to a contradiction if this model was also a matrix model for the $SO(32)$ theory. We will now argue that, in fact, the $SO(16) \times SO(16)$ Wilson line cannot be coped with using the method described above. In this method a chain of T-dualities is employed which maps a set $(R_s, A)$ into a set $(R'_s, A')$ such that $A' \to 0$ in the limit $R_s \to 0$. This T-duality is then succeeded by $S$ which maps to a large circle. However T-duality also preserve the gauge group and there does not exist any Wilson lines close to zero which breaks $SO(32)$ to $SO(16) \times SO(16)$. The Wilson lines doing that form a discrete set. The same argument could be used to rule out many other unbroken gauge groups.

4. Perspectives for obtaining a matrix model for all Wilson lines

So far we have seen that it is possible to obtain a matrix model for a countable number of Wilson lines. What about the general case? Unfortunately we do not have such a model. It would presumably be similar in spirit to the models for M-theory on $T^2$
with a background three-form potential \[16\]. This model was obtained by generalising the procedure developed in \[27,28\].

What could be the equivalent for \(SO(32)\) Heterotic matrix theory. Without a Wilson line the model is the theory living on the D-string of type I wrapped on a circle. This model is equivalent to zero-branes in type IIA on a \(S^1\) modded out by a \(Z_2\) group \(\{1,\Omega R_9\}\), where \(\Omega\) is worldsheet orientation reversal and \(R_9\) is reflection in the 9'th direction. We can view the \(S^1\) as \(\mathbb{R}/\mathbb{Z}\) where \(\mathbb{Z}\) is a group of translations. This is the same as 0-branes in type IIA modded out by the group generated by the \(Z\) and the \(Z_2\) above. Doing it in the usual way would just give us the theory on the D-string of type I on a dual circle. However in the process one needs to introduce a “twisted” sector, namely the strings going from 0-branes to 8-branes. Finding the most general way of modding out with this group, allowing extra noncommutativity in some way, might give us a candidate for Heterotic matrix theory with a Wilson line on the lightlike circle. Methods for modding out 0-brane mechanics have been considered in \[18,19\]. What makes the situation harder in this case is that the noncommutativity is in the twisted sector, so one needs an improvement of the methods considered there.

Unfortunately there does not seem to be an equivalent of the methods employed in \[17,21,22\] for the Heterotic case.

5. Conclusions

Using T-dualities we have obtained a matrix model for the \(SO(32)\) Heterotic string theory with certain Wilson lines on the lightlike circle. We do not know a method to obtain a model which works for all Wilson lines.

What about the \(E_8 \times E_8\) theory? The matrix model in this theory is for the Wilson line breaking \(E_8 \times E_8\) down to \(SO(16) \times SO(16)\). It would be nice to have a larger unbroken gauge group, especially \(E_8 \times E_8\). The methods described here can not be used to obtain that. The reason is that the matrix model for the \(E_8 \times E_8\) theory is obtained by T-dualizing to an \(SO(32)\) theory on a large circle. Since the unbroken gauge group is the same and there are no enhanced gauge symmetries for very small or very large circles the unbroken gauge group has to be a subgroup of both \(E_8 \times E_8\) and \(SO(32)\) which means that \(SO(16) \times SO(16)\) is the best one can do using these methods.

It will be an interesting challenge for the future to find matrix models for all Wilson lines for both the \(SO(32)\) and \(E_8 \times E_8\) theories.
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