Vehicle Driving State Estimation Using an Improved Adaptive Unscented Kalman Filter

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Vehicle driving state estimation using an improved adaptive unscented Kalman filter

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Abstract: This paper proposes an improved adaptive unscented Kalman filter (iAUKF)-based vehicle driving state estimation method. A three-degree-of-freedom vehicle dynamics model is first established, then the varying principles of estimation errors for vehicle driving states using constant process and measurement noises in the standard unscented Kalman filter (UKF) are compared and analyzed. Next, a new type of normalized innovation square-based adaptive noise covariance adjustment strategy is designed and incorporated into the UKF to derive our expected vehicle driving state estimation method. Finally, a comparative simulation investigation using CarSim and MATLAB/Simulink is conducted to validate the effectiveness of the proposed method, and the results show that our proposed iAUKF-based estimation method has higher accuracy and stronger robustness against the standard UKF algorithm.

Keywords: vehicle driving states; state estimation; adaptive noise covariance; unscented Kalman filter

1. Introduction

Currently, automotive active safety control systems (ASCSs) including rollover prevention system (RPS) \cite{1}, adaptive cruise control (ACC) \cite{2}, and lane-departure avoidance system (LDA) \cite{3}, have been extensively investigated and developed in response to the continuously growing demand on vehicle driving safety and dynamics performances \cite{4}. It is well known that the accurate vehicle states like sideslip angle, yaw rate, and longitudinal speed can significantly affect an ASCS’s performance. How to accurately obtain these vehicle states has become a core technical premise in the development of automobile ASCSs \cite{5}. However, considering the limits and costs of sensors, only a few of vehicle driving states can be directly measured. Consequently, it is necessary to propose an appropriate estimation method to achieve accurate and effective estimation of vehicle driving states using fewer on-board sensors \cite{6}.
At present, there exists two types of vehicle state estimation methods: kinematics and dynamics methods [7]. For the former one, the vehicle states are usually predicted by integrating the related vehicle states measured by sensors [8]. Note that the common kinematics methods include the closed-loop nonlinear observer [9] and neural network methods [10], etc. However, these methods have high dependency on the sensor accuracy, and the cumulative errors caused by continuous integration also limit their applications [11–12]. For the latter one, a large number of driving state estimation methods such as Kalman filter (KF) family algorithms, particle filters, and robust observers have been proposed by lots of researchers and scholars. In those methods, the KF family approaches are widely employed to estimate vehicle states due to its conveniences of providing optimal solutions and suppressing the effect of measurement and sensor noises. For instances, the extended Kalman filter (EKF) was usually used to estimate vehicle slip angle and lateral tire force, and a variable model covariance method was introduced to design a stable estimator in order to improve local observability of the observer [13]. In the work of [14], an effective state estimation was proposed for a four-wheel-drive vehicle based on the minimum model error criterion and the EKF algorithm. Besides, an augmented EKF algorithm was utilized to simultaneously estimate the common vehicle states and the lateral stiffness of tires, thus the impacts of the time-varying parameters in this vehicle system on the estimation accuracy can be eliminated [15].

Although the EKF algorithm maintains the elegant and efficient recursive update form by computing Jacobian matrices, while the local linearization may unavoidably lead to large cumulative estimation errors and divergence [16–18]. Fortunately, the unscented Kalman filter algorithm (UKF) can avoid this problem by approximating the nonlinear probability distribution using a sampling method instead of evaluating the Jacobian [19]. Recently, the UKF has been broadly used to estimate vehicle driving states. In [20], both EKF and UKF algorithms were applied to estimate the sideslip angle and the tire lateral force using real car test data. It was conformed that the performance of the UKF algorithm was far superior to that of the EKF algorithm. Moreover, a new type of online estimation method based on joint UKF was presented in [21] to estimate vehicle sideslip angle, body mass, along with moment of inertia, and the effectiveness of this estimation method were verified using the results from real car measurements.
Similarly, an adaptive state estimator was proposed for all-wheel-drive electric vehicles based on UKF algorithm [22], and this estimator could provide an accurate estimation of the longitudinal and lateral speed, tire slip angle, as well as tire friction coefficients. Different from the above literatures, a multi-sensor optimal data fusion method was proposed in [23] to estimate vehicle states, in which the integrated Inertial Navigation System (INS)/Global Navigation Satellite System (GNSS)/Celestial Navigation System (CNS) were integrated with the common UKF algorithm together. Afterwards, the global optimal state estimation of vehicle states was achieved according to the linear minimum variance principle.

Compared with the EKF, the UKF can greatly improve the estimation accuracy of the state variables for a nonlinear system. However, since the noise covariances of the EKF and UKF are often set as constant values that may be not accorded with the practical situation, the estimation accuracy of vehicle states cannot be guaranteed. Additionally, it is difficult to obtain accurate noise covariances under the changeable working conditions, which will reduce the robustness of common UKF algorithm. Therefore, it is an interesting and challenging issue to derive an improved UKF algorithm in conducting vehicle driving state estimation.

For example, in the work of [24], an adaptive EKF algorithm approach was proposed to achieve state-of-charge estimation for lithium-ion battery packs used in electric vehicles, in which the normalized innovation square (NIS) was used to validate the effectiveness of the designed estimation method. To compensate for the uncertainties caused by model errors, an adaptive sideslip angle observer was developed to fulfill vehicle body estimation [25] by combing the adaptive technology with the UKF along with a sensor-based integration approach. The simulation and real-car experimental results demonstrated that this method exhibited better estimation performances than the common UKF. Furthermore, in [26], a composite estimation procedure of the side slip angle was presented to reduce the interference of noise using an adaptive neuro-fuzzy inference system and the UKF. A novel adaptive UKF-based state estimation method was also presented in [27], wherein the measured signals were categorized into different road levels such that the noise covariance of different roads can be adaptively adjusted. Moreover, in the work of [28], an adaptive square-root UKF approach
was proposed for the state estimation/detection of nonlinear systems, in which process noise and measurement noises were unknown.

By summarizing the above related research work, this paper proposes an improved adaptive unscented Kalman filter (iAUKF)-based estimation method of vehicle driving states with adjustable noise covariance. The main contributions of this work are summarized as follows:

(1) By comparing and analyzing the influences of process and measurement noises on the estimation accuracy of vehicle states using the UKF algorithm, the varying principles of estimation errors for vehicle driving states are obtained.

(2) A NIS-based adaptive noise covariance adjustment strategy is designed and combined with the UKF algorithm to adaptively adjust process and measurement noise covariances, thus the proposed estimation method can improve the accuracy and adaptability of vehicle driving state estimation.

The rest of this paper is organized as follows. In Section 2, a three-degree-of-freedom (3-DOF) vehicle dynamics model is constructed, and the problem statement of vehicle driving states is described. In Section 3, an adaptive adjustment strategy of the noise covariance is designed, and based on this, a novel iAUKF-based vehicle driving state estimation method is proposed. In Section 4, simulation investigations based on CarSim and MATLAB/Simulink software are presented to illustrate the effectiveness of the proposed method under different working conditions. Finally, the concluding remarks are summarized in Section 5.

2. Vehicle dynamics modeling and problem formulation

2.1 Vehicle dynamics modeling

In this section, a 3-DOF “bicycle” or “single-track” model is used to describe the motion characteristics of vehicle in the yaw, lateral, and longitudinal directions, as shown in Fig. 1. This 3-DOF dynamics model can reflect the vehicles dynamics behaviors in real driving conditions, which has been extensively utilized in the previous literature [29–31]. The tire is here assumed to exhibit linear elastic behavior and complies to the small-angle approximation rule.

The dynamics equations of this vehicle are governed by
\[
\dot{\beta} = \frac{K_f + K_r}{mv_x} \beta + \frac{(aK_f - bK_r)}{mv_x^2} r - \frac{K_f}{mv_x} \delta_f, \\
\dot{r} = \frac{aK_f - bK_r}{I_z} \beta + \frac{a^2K_f + b^2K_r}{I_zv_x} r - \frac{aK_f}{I_z} \delta_f, \\
\dot{v}_x = a_x + v_x r = a_x + \beta v_x r, \\
a_y = \frac{K_f + K_r}{m} \beta + \frac{aK_f - bK_r}{mv_x} r - \frac{K_f}{m} \delta_f, \\
\]

where \(a\) and \(b\) are the distances from the vehicle’s center of gravity (CG) to the front axle and the rear axle, respectively; \(K_f\) and \(K_r\) are the equivalent cornering stiffness coefficients for the front axle and the rear axle, respectively; \(m\) is vehicle body mass; \(I_z\) is the moment of inertia for the yaw motion; \(v_x\) is the longitudinal velocity; \(r\) is the yaw rate; \(\beta\) is the sideslip angle of CG; \(\delta_f\) is the steering angle of the front wheel; and \(a_y\) is the lateral acceleration.

### 2.2 Problem formulation of vehicle state estimation

Generally, \(K_f\) and \(K_r\) are constant in the vehicle’s lateral dynamics modeling. However, in the practical operating conditions of vehicles, \(K_f\) and \(K_r\) are continuously varied with the change of its internal parameters. Therefore, \(K_f\) and \(K_r\) are herein treated as variable parameters and recursively adjusted by estimator.

Considering the relationship between \(\beta, r, v_x, a_y\), as well as \(K_f\) and \(K_r\), the nonlinear vehicle dynamics model can be constructed by the state transition equation and the measurement equation \([32]\), which are given as follows:

\[
\begin{align*}
\dot{x}(t) &= f(x(t), u(t)) + \omega(t) \\
\dot{z}(t) &= h(x(t), u(t)) + v(t)
\end{align*}
\]

where \(x(t)\) is the state vector; \(z(t)\) is the measurement vector; \(u(t)\) is the input vector; \(f(.)\) is the state evolution function; \(h(.)\) is the measurement function; \(\omega(t)\) and \(v(t)\) represent for the process and measurement noise, respectively, which satisfy Gaussian distributions with zero mean. The corresponding continuous time covariances of the noise functions are expressed by \(Q\) and \(R\), respectively.

Herein, define \(x\) as \(x = [\beta \ r_v \ \dot{v}_x \ K_f \ K_r \ r \ a_y]^T\), \(u\) as \(u = [\delta_f \ a_x]^T\), and \(z\) as \(z = [r_g \ a_y]^T\), wherein \(r_g\) is the measured yaw rate, and \(a_y\) is the measured lateral acceleration. Thus, Eq. (5) can be discretized as:

\[
\begin{align*}
x_{k+1} &= f(x_k, u_k) + \omega_k \\
z_{k+1} &= h(x_{k+1}, u_{k+1}) + v_{k+1}
\end{align*}
\]
In order to employ the first-order Euler approximation with sampling time $\Delta t$ to discretize Eq. (5) [15], both $f(.)$ and $h(.)$ can be represented as follows:

$$
\begin{align*}
  f_1 & : \dot{x}_{k+1} = x_k + \Delta t \cdot \dot{r}_k \\
  f_2 & : r_{k+1} = r_k + \Delta t \\
  f_3 & : v_{x,k+1} = v_{x,k} + \Delta t \cdot \dot{r}_k \\
  f_4 & : \dot{\beta}_{f,k} = \frac{K_{f,k} + K_{r,k}}{mv_{x,k}} \beta_k + \left( \frac{aK_{f,k} - bK_{r,k}}{mv_{x,k}^2} \right) r_k - \frac{K_{f,k}}{mv_{x,k}} \delta_{f,k} \\
  f_5 & : \dot{r}_k = \frac{aK_{f,k} - bK_{r,k}}{I_z} \beta_k + \frac{a^2K_{f,k} + b^2K_{r,k}}{I_z v_{x,k}} r_k - \frac{aK_{f,k}}{I_z} \delta_{f,k} \\
  f_6 & : \dot{v}_{x,k} = \beta_k r_k v_{x,k} + a_{x,k} \\
  f_7 & : K_{f,k+1} = \dot{K}_{f,k} \\
  f_8 & : K_{r,k+1} = \dot{K}_{r,k}
\end{align*}
$$

$$
\begin{align*}
  h_1 & : r_{g,k+1} = r_{k+1} \\
  h_2 & : a_{g,k+1} = \frac{K_{f,k+1} + K_{r,k+1}}{m} \beta_{k+1} + \frac{aK_{f,k+1} - bK_{r,k+1}}{mv_{x,k}} r_{k+1} - \frac{K_{f,k+1}}{m} \delta_{f,k+1}
\end{align*}
$$

The state-space form of Eq. (7) and Eq. (8) can be written as

$$
\begin{align*}
  \dot{x}_{k+1} &= A_x x_k + B_x u_k \\
  z_{k+1} &= C_x x_{k+1} + D_x u_{k+1},
\end{align*}
$$

where $A_x$, $B_x$, $C_x$, and $D_x$ are provided in the Appendix.

3. The iAUKF-based vehicle driving state estimation method

3.1 Standard unscented Kalman filter algorithm

The unscented transformation (UT) in the standard UKF is used to approximate the nonlinear probability distributions of the covariance and the mean values of vehicle states. For the 3-DOF vehicle model, the estimation procedure of vehicle driving states using the standard UKF is summarized as follows:

(1) Initialization

$$
\begin{align*}
  \hat{x}_0 &= E[x_0] \\
  P_0 &= E\left\{[x_0 - \hat{x}_0][x_0 - \hat{x}_0]^T\right\},
\end{align*}
$$

where $x_0$ is the initial values of $x$.

(2) Time updating
**Step-a**: Creating the Sigma points.

Based on the symmetric sampling strategy, the Sigma points \( \xi_k \) are created using the estimated \( \hat{x}_{k-1} \) and its corresponding covariance \( P_{x,k-1} \) at time \( k-1 \), then the weight coefficients of \( W^{(m)}_i \) and \( W^{(c)}_i \) for each Sigma point and their corresponding covariances are calculated, which are expressed as follows:

\[
\xi_k(i) = \left\{ \hat{x}_{k-1}, \hat{x}_{k-1} + (\sqrt{(n+\lambda)} P_{x,k-1})_j, \hat{x}_{k-1} - (\sqrt{(n+\lambda)} P_{x,k-1})_j \right\}.
\]

(11)

\[
W^{(m)}_i = \begin{cases} 
\lambda, & i = 0 \\
\frac{1}{2(n+\lambda)}, & i = 1, \ldots, 2n 
\end{cases}
\]

(12)

\[
W^{(c)}_i = \begin{cases} 
W^{(m)}_0 + (1-\alpha^2+\gamma) & i = 0 \\
\frac{1}{2(n+\lambda)} & i = 1, \ldots, 2n 
\end{cases}
\]

(13)

where \( n \) is the length of \( x \), and it is set as \( n = 8 \) in this work; \( \lambda = \alpha^2(n+k_p) - n \), and it is used to adjust the distance between the Sigma points and \( \hat{x}_{k-1} \); \( \alpha \) is a small positive constant determining the dispersion degree of the Sigma points; \( k_p \) is a scaling parameter, which is set as 0 or 3; \( n \) is the number of columns of matrix \( \sqrt{(n+\lambda)} P_{x,k-1} \), and \( \gamma \) describes the distribution information of \( \hat{x}_{k-1} \), and its optimal value is 2 under the Gaussian distribution.

**Step-b**: Computing the predicted values of \( \xi_i \).

The nonlinear transformation of \( \xi_i \) is performed by using

\[
\hat{x}_{k+1} = A_k x_k + B_k u_k
\]

in Eq. (9), and the predicted values of \( \xi_i \) are then obtained as

\[
\hat{\xi}_i(i) = A_k \xi_i(i) + B_k u_k, \quad i = 1, 2, \ldots, 2n+1.
\]

(14)

**Step-c**: Computing the priori estimation of \( \hat{x}_{k} \) and \( P_{x,k} \).

\[
\hat{x}_{k} = \sum_{i=0}^{2n} W^{(m)}_i \xi^-(i),
\]

(15)

\[
P_{x,k} = \sum_{i=0}^{2n} W^{(c)}_i [\xi^-(i) - \hat{x}^-] [\xi^-(i) - \hat{x}^-]^T + Q_k,
\]

(16)

where \( Q_k \) is the process noise covariance.

**Step-d**: Computing the priori estimation of \( z_k \).
Return to Step-a, and recreate new Sigma points, afterwards, substitute these points into $z_{k+1} = C_k x_{k+1} + D_k u_{k+1}$ in Eq. (9) to calculate the prior measurements $\hat{z}_k^-$. To facilitate the subsequent derivation process, the calculation formulas are summarized as follows:

$$
\chi_k(i) = \left\{ \hat{x}_k^-, \hat{x}_k^+ + \left( \sqrt{(n+\lambda)P_{x,k}} \right)_i, \hat{x}_k^- - \left( \sqrt{(n+\lambda)P_{x,k}} \right)_i \right\},
$$
(17)

$$
\chi^-_k(i) = C_k \chi_k(i) + D_k u_k,
$$
(18)

$$
\hat{z}_k^- = \sum_{i=0}^{2n} W_i^{(m)} \chi^-_k(i),
$$
(19)

$$
P_{z,k} = \sum_{i=0}^{2n} W_i^{(c)} [\chi_k(i) - \hat{z}_k^-] [\chi_k(i) - \hat{z}_k^-]^T + R_k,
$$
(20)

$$
P_{xz,k} = \sum_{i=0}^{2n} W_i^{(c)} [\chi_k(i) - \hat{x}_k^-] [\chi_k(i) - \hat{z}_k^-]^T,
$$
(21)

where $\chi_k(i)$ and $\chi^-_k(i)$ represent the recreated Sigma points and their corresponding prediction values, respectively; $P_{z,k}$ and $P_{xz,k}$ are the self-covariance and the cross-covariance of $\hat{z}_k^-$, respectively; and $R_k$ is the measurement noise covariance.

(3) Measurement updating

Using the actual measurements $z_k$ and Kalman gain $K_k$, the posteriori estimation of vehicle driving states and the related covariances at moment $k$ are updated by

$$
\begin{align*}
K_k &= P_{xz,k} P_{z,k}^{-1} \\
\hat{x}_k &= \hat{x}_k^- + K_k [z_k - \hat{z}_k^-] \\
\hat{P}_{x,k} &= P_{x,k}^- - K_k P_{z,k} K_k^T
\end{align*}
$$
(22)

3.2 Robustness analysis of standard UKF algorithm

In the standard UKF algorithm, $Q_k$ and $R_k$ are usually set as constant values to quantify the process and measurement noises. However, the actual process and measurement noises are often varied with the different driving conditions. Therefore, it is difficult to obtain the accurate $Q_k$ and $R_k$, moreover, using the inaccurate $Q_k$ and $R_k$ will result in greater estimation error and may reduce the robustness of standard UKF algorithm.

To assess the influence of process noise and measurement noise on the robustness of the UKF, the estimation errors of $\beta$, $r$, $v_\alpha$, and $a_y$ are taken as evaluation indices and the additional noise $Q_z$ and $R_z$ are used to simulate the
change of process noise and measurement noise. The related simulation experiments are as follows:

**Case 1:** The additional process noise $Q_z$ is set to $0$, $Q_0$, and $3Q_0$, and $Q_k = Q_k + Q_{z}$, where $Q_0 = 0.0045 \times I_{8 \times 8}$, and the initial values of $P$, $Q_k$, and $R_k$ are set as $P_0 = I_{8 \times 8}$, $Q_{k0} = 0.001 \times I_{8 \times 8}$, and $R_{k0} = \text{diag} ([0.01,0.5])$.

**Case 2:** The additional measurement noise $R_z$ is set to $0$, $R_0$, and $3R_0$, and $R_k = R_{k0} + R_{z}$, where $R_0 = \text{diag} ([0.02,0.2])$, and the initial values of $P$, $Q_k$, and $R_k$ are set as $P_0 = I_{8 \times 8}$, $Q_{k0} = 0.001 \times I_{8 \times 8}$, and $R_{k0} = \text{diag} ([0.01,0.5])$.

The estimation errors of $\beta$, $r$, $v_x$, and $a_y$ using standard UKF in Case 1 and Case 2 are shown in Fig. 2(a) and Fig. 2(b), respectively.

To further demonstrate the estimation errors of $\beta$, $r$, $v_x$, and $a_y$ using the standard UKF, the root mean squared error (RMSE) [33] is used to evaluate the related estimation errors, which is defined by

$$X_{\text{RMSE}} = \sqrt{\frac{1}{n} (X_i - \hat{X}_i)^2}, \quad (23)$$

where $n$ is the size of $X$; $X_i$ is the true value; $\hat{X}_i$ is the estimated value, where $i = 1, 2, \ldots, n$.

The RMSEs of $\beta$, $r$, $v_x$, and $a_y$ for the standard UKF under different process and measurement noises are shown in Table 1. It can be concluded from Fig. 2 and Table 1 that the absolute error and RMSE of $\beta$, $r$, $v_x$, and $a_y$ for the standard UKF increase gradually with the increase of $Q_z$ and $R_z$ when the driving conditions are fixed. That is to say, when $Q_k$ and $R_k$ are set as constants, the change of the actual process and measurement noises significantly affects the estimation accuracy of vehicle driving states, which implies that the robustness of the standard UKF becomes worse. Therefore, it is necessary to allow both $Q_k$ and $R_k$ to be tuned for the UKF algorithm if we expect that the UKF-based estimator can accurately estimate the vehicle states under different driving conditions.

### 3.3 The iAUKF-based vehicle driving state estimation

As described in Section 3.2, the variations of $Q_k$ and $R_k$ have a significant impact on the accuracy of the estimated vehicle states with the standard UKF. Therefore, by referring to a related study [24], an adaptive noise covariance adjustment strategy incorporating the UKF algorithm is herein proposed to conduct vehicle driving state estimation. This approach can adaptively adjust $Q_k$
and $R_k$ in terms of the errors between the prior and actual measurements, which can reduce the estimation error and the divergence possibility.

First, in order to approximate the process noise, the discrete $Q_k$ is set as $Q_k \approx \Delta t B \Sigma Q B^T \Sigma$ [34], rather than simply setting it as a diagonal matrix with $n$ dimension.

$$
B_o = \begin{bmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
K_{f,k} & 0 & 0 & 0 \\
\frac{mv}{m_v} & 0 & 0 & 0 \\
aK_{f,k} & 0 & 0 & 0 \\
\frac{I_z}{I_z} & 0 & 1 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
\end{bmatrix}.
$$

Moreover, $Q$ is the continuous process noise matrix. Since $Q$ is difficult to measure, it is set to be greater than the noise error of the common steering angle sensor and the accelerometer, as well as the standard deviation of the front-wheel and the rear-wheel lateral stiffness coefficients. Thus, we set $Q = \text{diag}(0.6,0.1,500,500)$ [15]. In addition, the initial value of $R_k$ is set as $R_{k,0} = \text{diag}(0.01,0.5)
$.

Second, the innovation $v_k$ is defined as the error between the prior measurements and actual measurements, i.e.,

$$
v_k = z_k - \hat{z}_k = z_k - \sum_{i=0}^{2n} W_i^{(n)} \chi^- (i).
$$

The theoretical innovation covariance is obtained by the UKF algorithm, which is equal to $P_{z,k}$. Therefore, the theoretical innovation covariance $C_k$ is denoted by

$$
C_k = P_{z,k} = \sum_{i=0}^{2n} W_i^{(c)} \{ \chi^- (i) - \hat{z}_k \} \{ \chi^- (i) - \hat{z}_k \}^T + R_k.
$$

Due to the influences of modeling and measurement errors, there usually exists a certain deviation between the actual and theoretical innovation covariance. The actual innovation covariance is obtained by the definition of the error covariance, which is defined as
\[ \hat{C}_k = \frac{1}{M} \sum_{i=1}^{M} [v_{k-i}v_{k-i}^T], \]  

where \( M \) is the length of the sliding window.

Considering that the vehicle driving states is often varied under different driving conditions, it is difficult to balance the steady-state error and dynamic response if using fixed \( M \) to calculate \( \hat{C}_k \). Therefore, \( M \) should be adaptively adjusted according to vehicle running conditions under the following constraints:

\[
\begin{cases}
M = M_{\min}, & d \geq \alpha_{\max} \\
M = M_{\max}, & d \leq \alpha_{\min} \\
M = \text{int}\left( \frac{(d-\alpha_{\min})(M_{\max}-M_{\min})}{\alpha_{\max}-\alpha_{\min}} \right) + M_{\min}, & \alpha_{\min} < d < \alpha_{\max}
\end{cases}
\]

where \( d \) is the adjustment factor of \( M \); \( \alpha_{\max} \) and \( \alpha_{\min} \) are the preset upper and lower thresholds, respectively; ‘int’ is an integral function; and \( M_{\min} \) and \( M_{\max} \) are the minimum and maximum values of \( M \), respectively. In order to avoid \( k-i < 0 \) when using Eq. (26) to calculate \( \hat{C}_k \), according to [35], \( M_{\min} \) and \( M_{\max} \) are chosen as:

\[
\begin{cases}
M_{\min} = 1, M_{\max} = k - 1, & k \leq 100 \\
M_{\min} = 20, M_{\max} = 100, & k \geq 100
\end{cases}
\]

In this adjustment strategy, when \( d > \alpha_{\max} \), the vehicle is considered to be running in a maneuver operation where vehicle states change rapidly, and \( M \) needs to be decreased to weaken the influence of state changes and then to reduce the steady-state error. When \( d < \alpha_{\min} \), it is considered that the vehicle is running in a steady working condition where vehicle states change slowly, and \( M \) should be increased such that the dynamic response performances of the adjustment strategy can be guaranteed. When \( \alpha_{\min} < d < \alpha_{\max} \), a simple linear extension method is adopted to replace the traditional exponential function to reduce the calculation burden and improve the real-time performance of the proposed adjustment strategy.

Essentially, \( d \) should reflect the changes of vehicle states correctly. When \( d \) is greater than a certain threshold, the vehicle states are considered to change rapidly. In contrast, the vehicle states change slowly, and then \( M \) can be adjusted.
adaptively according to the value of \(d\). Therefore, to evaluate the variation rates of vehicle states, the value of \(d\) is defined as the NIS that is commonly used in the Inertial Navigation System–Global Positioning System [24], which is expressed by

\[
d = v^T_k C_k^{-1} v_k,
\]

When vehicle running in a certain maneuver operation, the results estimated by the UKF algorithm will inevitably have large errors with the rapid change of vehicle driving states, which will eventually lead to the increase of the innovation and its NIS. Therefore, Eq. (29) is used as the judgment criterion of the UKF algorithm, and the adaptive noise covariance adjustment strategy is used to adjust \(Q_k\) and \(R_k\).

Here, the noise adjustment factor \(\alpha_k\) is introduced to adjust \(Q_k\) and \(R_k\) by comparing \(C_k\) and \(\hat{C}_k\). According to literature [36], \(\alpha_k\) is defined as

\[
\alpha_k = \max \left\{1, \frac{\text{trace}(\hat{C}_k)}{\text{trace}(C_k)}\right\},
\]

After correcting \(Q_k\) and \(R_k\) through \(\alpha_k\), both \(P_{z,k}\) and \(P_{x,k}\) can be expressed as

\[
P_{z,k} = \sum_{i=0}^{2n} W_{i}^{(c)} [\chi_{k}^{(i)}(i) - \hat{\chi}_{k}^{(i)}] [\chi_{k}^{(i)}(i) - \hat{\chi}_{k}^{(i)}]^T + \alpha_k R_k, \tag{31}
\]

\[
P_{x,k} = \sum_{i=0}^{2n} W_{i}^{(c)} [\chi_{k}^{(i)}(i) - \hat{\chi}_{k}^{(i)}] [\chi_{k}^{(i)}(i) - \hat{\chi}_{k}^{(i)}]^T + \kappa Q_k / \alpha_k, \tag{32}
\]

where \(\kappa\) is the multiplier coefficient of \(Q_k\).

In summary, the proposed iAUKF-based vehicle driving state estimation can be established if the above adaptive noise covariance adjustment strategy is combined with UKF algorithm, and the flowchart of the iAUKF-based vehicle driving state estimator is shown in Fig. 3.

Specifically, the inputs and measurements are obtained from the CarSim or the on-board sensors in terms of the steering wheel angle, and the prior estimation of \(x\) is then carried out, thus the time updating is completed. Subsequently, the adaptive noise covariance adjustment strategy is designed to realize the adaptive adjustment of \(Q_k\) and \(R_k\) in standard UKF, which will improve the accuracy and adaptability of vehicle driving state estimation. Finally, the posterior states and the corresponding covariance are updated.
4. Simulation investigation

In this section, simulation investigation of this 3-DOF vehicle dynamics model is conducted using the CarSim and MATLAB/Simulink software. The flowchart of the simulation based on CarSim and MATLAB/Simulink is shown in Fig. 4.

First, the 3-DOF vehicle dynamics model established in CarSim is employed to obtain the output responses of $\delta_f$, $\beta$, $r$, and $v_x$ along with $a_x$ and $a_y$ during a sinusoidal maneuver and two fishhook maneuvers. The obtained $\delta_f$ and $a_y$ are then taken as the input signals, and both $r$ and $a_y$ are determined as the pseudo measurements. The UKF, Sage-Husa-based AUKF (SHAUKF) and our proposed iAUKF algorithm are employed to estimate vehicle driving states in the MATLAB/Simulink software. Finally, the results of the three related UKF algorithms and the CarSim reference model are compared to evaluate the accuracy and the effectiveness of the proposed iAUKF algorithm in case of different working conditions. The developed simulator with the proposed iAUKF algorithm is shown in Fig. 5. Note that the initial value of matrix $P$ is set as $P_0 = I_{8\times8}$, and the related parameters in the simulation are listed in Table 2.

4.1 Sinusoidal maneuver test

In the first test, the proposed iAUKF is implemented in case of a sinusoidal maneuver. The estimated $\beta$, $r$, $v_x$, and $a_y$ obtained by the UKF, SHAUKF and our designed iAUKF algorithm are compared to the corresponding four vehicle state curves produced by CarSim vehicle model.

The mathematical expression of $\delta_s$ in CarSim is given by

$$\delta_s(t) = \begin{cases} 20\sin(0.5t - 10), & 8 < t < 32 \\ -40\sin(0.5t - 20), & 40 < t < 80 \\ 0, & \text{otherwise} \end{cases}$$

(33)

The curves of $\delta_f$ and $a_x$ simulated by CarSim in sinusoidal maneuver are displayed in Fig. 6. Besides, Fig. 7 shows the curves of $\beta$, $r$, $v_x$, and $a_y$ using the UKF, SHAUKF and our designed iAUKF algorithm in the same sinusoidal maneuver. These three UKF algorithms have certain local errors, yet the estimation effect of SHAUKF and the proposed iAUKF algorithm are better than those of the traditional UKF, and the estimation results of the iAUKF are closer to the reference values than that of SHAUKF. Although the error of $v_x$ generated by
the iAUKF is larger than that generated by UKF and SHAUKF in 10-50 s, the entire error of $v_x$ generated by the iAUKF exhibits a decreasing tendency, which makes its estimated curve fitting the simulation results of CarSim well.

Additionally, from the enlarged subplots A, B, and C in Fig. 7, it is clear that the corresponding vehicle states estimated by proposed iAUKF algorithm are closer to the reference values obtained by the CarSim, in compared to those obtained by the UKF and SHAUKF algorithm.

Fig. 8 displays the absolute errors of $\beta$, $r$, $v_x$, and $a_y$ curves with the UKF, SHAUKF and iAUKF against the CarSim curves in sinusoidal maneuver. The estimation errors of $\beta$, $r$, and $a_y$ obtained by the SHAUKF and iAUKF are always smaller than these of the UKF algorithm, and the estimation errors of $\beta$, $r$, and $a_y$ with the iAUKF algorithm are smaller than those of the SHAUKF. Additionally, the estimated error of $v_x$ is larger in the early stage, and it gradually decreases in the later stage. This is in good agreement with the conclusion above, and the entire error of $v_x$ exhibits a decreasing tendency.

Table 3 lists the RMSE values of $\beta$, $r$, $v_x$, and $a_y$ with the UKF, SHAUKF and iAUKF algorithms in sinusoidal maneuver. Compared with the related results with the UKF, the RMSE reduction rates of $\beta$, $r$, $v_x$, and $a_y$ using the SHAUKF are 12.8%, 8.4%, 37.1%, and 11.7%, respectively, while the RMSE reduction rates using the proposed iAUKF are 23.4%, 40.9%, 3.0% and 24.1%, respectively. Although the RMSE of $v_x$ using the iAUKF is larger than that of the SHAUKF, the RMSE reduction effects of proposed iAUKF are more obvious than those of the SHAUKF for other states.

4.2 Fishhook maneuver test

Two Fishhook maneuvers are considered, and they are separately discussed in this section.

4.2.1 Fishhook maneuver I

The mathematical expression of $\delta_s$ under fishhook maneuver I is expressed by
The simulated curves of $\delta_f$ and $a_x$ using CarSim are provided in Fig. 9.

Fig. 10 shows the curves of $\beta$, $r$, $v_x$, and $a_y$ estimated by the UKF, SHAUKF and iAUKF algorithms. Compared to the UKF and SHAUKF, it is obvious that the responses of $\beta$, $r$, $v_x$, and $a_y$ generated by the iAUKF are closer to the corresponding ones simulated by CarSim no matter from the subplot A, B, and C, which demonstrates that the proposed iAUKF maintains a relatively higher accuracy during fishhook maneuver I.

Additionally, Fig. 11 shows the absolute error comparisons of $\beta$, $r$, $v_x$, and $a_y$ in fishhook maneuver I. It is easily seen that the estimation errors of $\beta$, $r$, $v_x$, and $a_y$ using the iAUKF are flatter and smaller than using the other two UKF algorithms. Especially, the peak values of the estimation errors for these four vehicle states using our iAUKF algorithm are much lower than those using the UKF and SHAUKF algorithms.

Table 4 summarizes the RMSE values of $\beta$, $r$, $v_x$, and $a_y$ using the UKF, SHAUKF and iAUKF algorithms in fishhook maneuver I. Regarding to the UKF, the RMSE reduction rates of $\beta$, $r$, $v_x$, and $a_y$ using the iAUKF are 33.1%, 85.1%, 88.7%, and 33.4%, respectively, while the RMSE reduction rates using SHAUKF are only 18.4%, 50.7%, 56.0%, and 18.3%, respectively. It can be observed that the improvements of vehicle states estimation with our proposed iAUKF are entirely greater than those of vehicle states with the SHAUKF, which demonstrates that the proposed iAUKF holds superior estimation performances.

### 4.2.2 Fishhook maneuver II

The $\delta_s$ under fishhook maneuver II is expressed by

$$
\delta_s(t) = \begin{cases} 
0, & 0 < t < 1 \\
100(t-1), & 1 < t < 2 \\
100, & 2 < t < 4 \\
100-50(t-4), & 4 < t < 8 \\
-50, & 8 < t < 12 \\
\end{cases}
$$

The output curves of $\delta_f$ and $a_x$ from CarSim are provided in Fig. 12.
Fig. 13 and Fig. 14 show the estimated curves of $\beta$, $r$, $v_x$, and $a_y$ and the corresponding estimation errors, respectively, using the UKF, SHAUKF and iAUKF algorithms. As shown in Fig. 13, it is obvious that the proposed iAUKF could generate a more accurate estimation of vehicle driving states in comparison with the UKF and SHAUKF algorithm when facing a large change of $\delta_s$. Moreover, from the subplot A, B, and C, it is seen that the responses of $\beta$, $r$, $v_x$, and $a_y$ generated by the iAUKF are closer to the reference values of CarSim compared to the other two UKF algorithms.

Additionally, as shown in Fig. 14, the absolute errors of $\beta$, $r$, $v_x$, and $a_y$ generated by the iAUKF are far lower than those by the UKF and SHAUKF algorithm. Meanwhile, the maximum errors of $\beta$, $r$, $v_x$, and $a_y$ generated by the iAUKF are smaller than those of the other two UKF algorithms. Moreover, the estimation errors of the vehicle driving states by the iAUKF exhibit a gradually decreasing tendency, i.e., the error curves of the four vehicle states get flatter gradually, which also illustrates the superior robustness of the proposed iAUKF.

Table 5 lists the comparisons of the RMSE values of $\beta$, $r$, $v_x$, and $a_y$ by the UKF, SHAUKF and iAUKF algorithms in fishhook maneuver II. It is observed that the iAUKF could generate lower RMSE values compared to the UKF, and the reduction rates of $\beta$, $r$, $v_x$, and $a_y$ by the iAUKF are about 34.7%, 71.9%, 73.6%, and 34.8%, respectively, while the reduction rates of RMSE by the SHAUKF are only 24.6%, 42.0%, 49.5% and 24.7%, respectively. This demonstrates that the proposed iAUKF can achieve more accurate estimation than the UKF and SHAUKF, and it is more reliable for estimating the vehicle states when facing large changes of $\delta_s$.

### 4.3 Robustness analysis of the iAUKF algorithm

To further demonstrate the robustness of the designed iAUKF algorithm when encountering the variations of process and measurement noises, the same simulations are performed using the UKF and the proposed iAUKF for three different process and measurement noises. The absolute estimation errors of each driving state by the UKF and the iAUKF are provided in Fig. 15.

Under the three different process and measurement noises, the estimation errors of $\beta$, $r$, $v_x$, and $a_y$ by the iAUKF are much lower than those by the UKF algorithm. Moreover, with the gradual increase of $Q_z$ and $R_z$, the increase
magnitudes of the estimation errors of vehicle states by the proposed iAUKF are far less than those by the UKF algorithm. Besides, the estimated error of $v_x$ exhibits a downward trend with the increase of $Q_z$ and $R_z$, which further illustrates the higher robustness of the proposed iAUKF compared to the UKF.

To highlight the robustness of the proposed iAUKF algorithm more visually, the RMSEs of each vehicle driving state estimated by the UKF and iAUKF under different process and measurement noises are compared in Table 6, and its graphical presentation is provided in Fig. 16.

It is clear from Table 6 that the RMSEs of vehicle driving states obtained by the iAUKF are all less than those obtained by the UKF under different process and measurement noises, and the proposed iAUKF could retain a higher global accuracy, even when the process and measurement noises changed significantly.

Furthermore, as shown in Fig. 16, except for the RMSE of $r$ at $Q_z = 0$, the RMSEs of each driving state obtained by the iAUKF are all smaller than those by the UKF. In addition, based on the variation tendency of the RMSEs for the four vehicle states, as the process and measurement noises increased, the increase magnitudes of these four states estimated by the proposed iAUKF are very small, and even present a decreasing appearance in some cases, whereas the RMSEs for the UKF algorithm are all increasing. This further shows that the proposed iAUKF could lower the negative effects of the process and measurement noise changes on the estimation of vehicle driving state.

5. Conclusions and future work

In this paper, an improved adaptive unscented Kalman filter-based vehicle driving state estimation method is proposed. First, the impacts of the process and measurement noises on the standard UKF-based estimation of vehicle driving states are investigated and analyzed, and it is obtained that with the increase of process and measurement noises, the estimation errors of vehicle states become greater. Second, by incorporating the UKF algorithm, a NIS-based adaptive noise covariance adjustment strategy is developed to adaptively adjust process and measurement noise covariances, thus the accuracy and robustness of the proposed vehicle driving state estimation can be guaranteed. Finally, the simulation investigations based on CarSim and MATLAB/Simulink software are conducted, and the results show that compared to the UKF approach, our proposed iAUKF-
based estimation method has various degrees of improvements for the estimation accuracy of vehicle driving state under three different maneuvers.

In future work, this designed iAUKF approach will be employed to estimate the driving states of distributed-motor-driven electric vehicles, and those estimated states will be taken as the inputs of direct yaw-moment control. Meanwhile, the side slip angle is determined to be the control goal to fulfill the stability control of the electric vehicle under a turning maneuver.

**Appendix**

The state matrices of $A_k$, $B_k$, $C_k$, and $D_k$ in Eq. (9) are provided here:

\[
A_k = \begin{bmatrix}
1 & 0 & 0 & \Delta t & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & \Delta t & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & \Delta t & 0 & 0 \\
A_{k,1}^4 & A_{k,2}^4 & 0 & 0 & 0 & 0 & 0 & 0 \\
A_{k,1}^5 & A_{k,2}^5 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
B_k = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
B_{k,1}^4 & B_{k,1}^5 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
\]

where $A_{k,1}^4 = \frac{K_{f,k} + K_{r,k}}{m v_k^2}$; $A_{k,2} = aK_{f,k} - bK_{r,k} - 1$; $A_{k,1}^5 = aK_{f,k} - bK_{r,k} I_z$; $A_{k,2}^5 = \frac{a^2 K_{f,k} + b^2 K_{r,k}}{I_z v_k}$; $B_{k,1}^4 = -\frac{K_{f,k}}{m v_k}$; $B_{k,1}^5 = -\frac{aK_{f,k}}{I_z}$.

\[
C_k = \begin{bmatrix}
K_{f,k} + K_{r,k} & m \frac{aK_{f,k} - bK_{r,k}}{mv_{x,k}} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, D_k = \begin{bmatrix}
0 & 0 \\
-\frac{K_{f,k}}{m} & 0
\end{bmatrix}.
\]

CRediT authorship contribution statement

**Hui Pang**: Writing - review & editing, Supervision and Validation. **Peng Wang**: Conceptualization, Methodology, Software and Writing - original draft. **Zijun Xu**: Investigation and Data curation. **Gang Wang**: Software and review.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
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Data Availability Statements

The datasets generated and analysed during the current study are available from the corresponding author on reasonable request.

References

[1] Li, L., Lu, Y., Wang, R., Chen, J.: A Three-Dimensional Dynamics Control Framework of Vehicle Lateral Stability and Rollover Prevention via Active Braking With MPC. IEEE Trans. Ind. Electron. 64, 3389–3401 (2017). https://doi.org/10.1109/TIE.2016.2583400
[2] Li, S., Li, K., Rajamani, R., Wang, J.: Model predictive multi-objective vehicular adaptive cruise control. IEEE Trans. Control Syst. Technol. 19, 556–566 (2011). https://doi.org/10.1109/TCST.2010.2049203
[3] Minoiu Enache, N., Mammar, S., Netto, M., Lusetti, B.: Driver steering assistance for lane-departure avoidance based on hybrid automata and composite Lyapunov function. IEEE Trans. Intell. Transp. Syst. 11, 28–39 (2010). https://doi.org/10.1109/TITS.2009.2026451
[4] Xu, Y., Deng, B., Xu, G.: Estimation of vehicle states and road friction based on DEKF. 2015 6th Int. Conf. Power Electron. Syst. Appl. Electr. Transp. - Automotive, Vessel Aircraft, PESA 2015. (2016). https://doi.org/10.1109/PESA.2015.7398885
[5] Chen T., Cai Y., Chen L., Xu X., Jiang H., Sun X.: Design of vehicle running states-fused estimation strategy using Kalman filters and tire force compensation method, IEEE Access. 7 (2019) 87273–87287. https://doi.org/10.1109/ACCESS.2019.2925370.
[6] Zhang, B., Du, H., Lam, J., Zhang, N., Li, W.: A Novel Observer Design for Simultaneous Estimation of Vehicle Steering Angle and Sideslip Angle. IEEE Trans. Ind. Electron. 63, 4357–4366 (2016). https://doi.org/10.1109/TIE.2016.2544244.
[7] Selmanaj, D., Corno, M., Panzani, G., Savarese, S.M.: Vehicle sideslip estimation: A kinematic based approach. Control Eng. Pract. 67, 1–12 (2017). https://doi.org/10.1016/j.conengprac.2017.06.013
[8] Li, X., Chan, C.Y., Wang, Y.: A Reliable Fusion Methodology for Simultaneous Estimation of Vehicle Sideslip and Yaw Angles. IEEE Trans. Veh. Technol. 65, 4440–4458 (2016). https://doi.org/10.1109/TVT.2015.2496969
[9] Liu, W., Liu, W., Ding, H., Guo, K.: Side-slip angle estimation for vehicle electronic stability control based on sliding mode observer. Proc. 2012 Int. Conf. Meas. Inf. Control. MIC 2012. 2, 992–995 (2012). https://doi.org/10.1109/MIC.2012.6273468.
[10] Guo, H., Chen, H., Cao, D., Yang, S.: Vehicle Dynamic States Estimation: State of the Art and Perspectives. IEEE/CAA Journal of Automatica Sinica. 5, 418–431 (2014). https://doi.org/10.1109/JAS.2017.7510811.
[11] Piyabongkarn, D., Rajamani, R., Grogg, J.A., Lew, J.Y.: Development and experimental evaluation of a slip angle estimator for vehicle stability control. IEEE Trans. Control Syst. Technol. 17, 78–88 (2009). https://doi.org/10.1109/TCST.2008.922503.

[12] Kim, D., Min, K., Kim, H., Huh, K.: Vehicle sideslip angle estimation using deep ensemble-based adaptive Kalman filter. Mech. Syst. Signal Process. 144, 106862 (2020). https://doi.org/10.1016/j.ymssp.2020.106862.

[13] Naets, F., Van Aalst, S., Boulkroune, B., Ghouti, N. El, Desmet, W.: Design and experimental validation of a stable two-stage estimator for automotive sideslip angle and tire parameters. IEEE Trans. Veh. Technol. 66, 9727–9742 (2017). https://doi.org/10.1109/TVT.2017.2742665.

[14] Liu, W., He, H., Sun, F.: Vehicle state estimation based on Minimum Model Error criterion combining with Extended Kalman Filter. J. Franklin Inst. 353, 834–856 (2016). https://doi.org/10.1016/j.jfranklin.2016.01.005.

[15] Reina, G., Messina, A.: Vehicle dynamics estimation via augmented Extended Kalman Filtering. Meas. J. Int. Meas. Confed. 133, 383–395 (2019). https://doi.org/10.1016/j.measurement.2018.10.030.

[16] Zhao, Z., Chen, H., Yang, J., Wu, X., Yu, Z.: Estimation of the vehicle speed in the driving mode for a hybrid electric car based on an unscented Kalman filter. Proc. Inst. Mech. Eng. Part D J. Automob. Eng. 229, 437–456 (2015). https://doi.org/10.1177/0954407014546918.

[17] Strano, S., Terzo, M.: Constrained nonlinear filter for vehicle sideslip angle estimation with no a priori knowledge of tyre characteristics. Control Eng. Pract. 71, 10–17 (2018). https://doi.org/10.1016/j.conengprac.2017.10.004.

[18] Julier, S.J., Uhlmann, J.K.: Corrections to “Unscented filtering and nonlinear estimation.” Proc. IEEE. 92, 1958 (2004). https://doi.org/10.1109/JPROC.2004.837637.

[19] Julier S.J., Uhlmann J.K., Durrant-Whyte H.F.: A new method for the nonlinear transformation of means and covariances in filters and estimators. IEEE Trans. Automat. Contr. 45, 477–482 (2000). https://doi.org/10.1109/9.847726.

[20] Doumiati, M., Victorino, A.C., Charara, A., Lechner, D.: Onboard real-time estimation of vehicle lateral tire-road forces and sideslip angle. IEEE/ASME Trans. Mechatronics. 16, 601–614 (2011). https://doi.org/10.1109/TMECH.2010.2048118.

[21] Wielitzka, M., Dagen, M., Ortmaier, T.: Joint unscented Kalman filter for state and parameter estimation in vehicle dynamics. 2015 IEEE Conf. Control Appl. CCA 2015 - Proc. 1945–1950 (2015). https://doi.org/10.1109/CCA.2015.7320894.

[22] Heidfeld H., Schunemann M., Kasper R.: Experimental validation of a GPS-aided model-based UKF vehicle state estimator. 2019 IEEE Int. Conf. Mechatronics, ICM 2019. 537–543 (2019). https://doi.org/10.1109/ICMECH.2019.8722942.

[23] Gao B., Hu G., Gao S., Zhong Y., Gu C.: Multi-sensor optimal data fusion for INS/GNSS/CNS integration based on unscented Kalman filter. Int. J. Control Autom. Syst. 16, 129–140 (2018). https://doi.org/10.3390/s18020488.

[24] Zhang, Z., Jiang, L., Zhang, L., Huang, C.: State-of-charge estimation of lithium-ion battery pack by using an adaptive extended Kalman filter for electric vehicles. J. Energy Storage. 37, 102457 (2021). https://doi.org/10.1016/j.est.2021.102457.
[25] Chen, J., Song, J., Li, L., Jia, G., Ran, X., Yang, C.: UKF-based adaptive variable structure observer for vehicle sideslip with dynamic correction. IET Control Theory Appl. 10, 1641–1652 (2016). https://doi.org/10.1049/iet-cta.2015.1030.

[26] Boada, B.L., Boada, M.J.L., Diaz, V.: Vehicle sideslip angle measurement based on sensor data fusion using an integrated ANFIS and an Unscented Kalman Filter algorithm. Mech. Syst. Signal Process. 72–73, 832–845 (2016). https://doi.org/10.1016/j.ymssp.2015.11.003.

[27] Wang, Z., Qin, Y., Gu, L., Dong, M.: Vehicle System State Estimation Based on Adaptive Unscented Kalman Filtering Combing with Road Classification. IEEE Access. 5, 27786–27799 (2017). https://doi.org/10.1109/ACCESS.2017.2771204.

[28] Mohammadi Asl, R., Shabbouei Hagh, Y., Simani, S., Handroos, H.: Adaptive square-root unscented Kalman filter: An experimental study of hydraulic actuator state estimation. Mech. Syst. Signal Process. 132, 670–691 (2019). https://doi.org/10.1016/j.ymssp.2019.07.021.

[29] Tagne, G., Talj, R., Charara, A.: Design and Comparison of Robust Nonlinear Controllers for the Lateral Dynamics of Intelligent Vehicles. IEEE Trans. Intell. Transp. Syst. 17, 796–809 (2016). https://doi.org/10.1109/TITS.2015.2486815.

[30] Yu, Z., Huang, X., Wang, J.: A least-squares regression based method for vehicle yaw moment of inertia estimation. Proc. Am. Control Conf. 2015-July, 5432–5437 (2015). https://doi.org/10.1109/ACC.2015.7172189.

[31] Pauca, O., Caruntu, C.F., Lazar, C.: Predictive control for the lateral and longitudinal dynamics in automated vehicles. 2019 23rd Int. Conf. Syst. Theory, Control Comput. ICSTCC 2019 - Proc. 797–802 (2019). https://doi.org/10.1109/ICSTCC.2019.8885839.

[32] Chen, T., Xu, X., Chen, L., Jiang, H., Cai, Y., Li, Y.: Estimation of longitudinal force, lateral vehicle speed and yaw rate for four-wheel independent driven electric vehicles. Mech. Syst. Signal Process. 101, 377–388 (2018). https://doi.org/10.1016/j.ymssp.2017.08.041.

[33] Geng, G., Wei, B., Duan, C., Jiang, H., Hua, Y.: A strong robust observer of distributed drive electric vehicle states based on strong tracking-iterative central difference Kalman filter algorithm. Adv. Mech. Eng. 10, 1–12 (2018). https://doi.org/10.1177/1687814018779682.

[34] Franklin G., Powell D., Workman M.: Digital Control of Dynamic Systems, Addison-Wesley, New York (1997).

[35] Hao, Y.L., Guo, Z., Sun, F., Gao, W.: Adaptive extended Kalman filtering for SINS/GPS integrated navigation systems. Proc. 2009 Int. Jt. Conf. Comput. Sci. Optim. CSO 2009. 2, 192–194 (2009). https://doi.org/10.1109/CSO.2009.429

[36] Kim, K.H., Lee, J.G., Park, C.G.: Adaptive two-stage extended kalman filter for a fault-tolerant INS-GPS loosely coupled system. IEEE Trans. Aerosp. Electron. Syst. 45, 125–137 (2009). https://doi.org/10.1109/TAES.2009.4805268.
Figures

Fig. 1 A 3-DOF vehicle dynamics model

(a) Estimation errors of $\beta$, $r$, $v_x$, and $a_y$ using standard UKF in Case 1

(b) Estimation errors of $\beta$, $r$, $v_x$, and $a_y$ using UKF in Case 2

Fig. 2 Estimation errors of $\beta$, $r$, $v_x$, and $a_y$ using standard UKF under three different noises
The iAUKF-based Estimator

The iAUKF-based Estimator

Initialization
- Initial state value
- Initial covariance matrix

Calculating the prior state
- State value and covariance $x_{k}^{\text{m}}$, $P_{k}^{\text{m}}$

Calculating the prior measurement
- Sigma points and its predicted value $\bar{z}_{i}(i)$, $\tilde{z}_{i}(i)$

Prior state and covariance $x_{k}^{\text{m}}$, $P_{k}^{\text{m}}$

Adaptive noise covariance

Adaptive sliding window

Optimal state

Measurement updating
- Self-covariance and cross-covariance $P_{k+1}^{\text{m}}$, $P_{k+1}^{\text{m}}$

Kalman gain $K_{k}$

Posterior state and covariance $\hat{x}_{k}^{\text{m}}$, $P_{k+1}^{\text{m}}$

Fig. 3 Flowchart of the iAUKF-based vehicle driving state estimation

Fig. 4 Flowchart of the simulation based on CarSim and MATLAB/Simulink software

CarSim
- CarSim Input $[\delta_{s}, a_{s}]$
- CarSim Output $\hat{\delta}_{s}, \hat{a}_{s}$

MATLAB/Simulink
- iAUKF Algorithm $\hat{\beta}, \hat{r}, \hat{v}, \hat{a}$
- SHAUKF Algorithm $\hat{\beta}, \hat{r}, \hat{v}, \hat{a}$
- UKF Algorithm $\hat{\beta}, \hat{r}, \hat{v}, \hat{a}$

Vehicle States

Compare

CarSim Model
- CarSim Input $\delta_{s}, a_{s}$
- CarSim Output $\hat{\delta}_{s}, \hat{a}_{s}$

CarSim S-Function

The Proposed iAUKF Algorithm

CarSim Input
- Interpreted MATLAB Fcn Sine Manoeuvre
- Interpreted MATLAB Fcn Fishhook Case

CarSim Output
- Interpreted MATLAB Fcn Fishhook Case

MATLAB Function

Fig. 5 The developed simulator with the proposed iAUKF algorithm
Fig. 6 Simulated curves of δf and αs using CarSim in sinusoidal maneuver

Fig. 7 The comparison curves of β, r, vx, and ay in sinusoidal maneuver

Fig. 8 The absolute errors of β, r, vx, and ay in sinusoidal maneuver
Fig. 9 Simulated curves of $\delta_f$ and $a_x$ using CarSim in fishhook maneuver I

Fig. 10 The comparison curves of $\beta$, $r$, $v_x$, and $a_y$ in fishhook maneuver I

Fig. 11 The absolute errors of $\beta$, $r$, $v_x$, and $a_y$ in fishhook maneuver I
Fig. 12 Simulated curves of $\delta_f$ and $a_x$ using CarSim in fishhook maneuver II

Fig. 13 The comparison curves of $\beta$, $r$, $v_x$, and $a_y$ in fishhook maneuver II

Fig. 14 The absolute errors of $\beta$, $r$, $v_x$, and $a_y$ in fishhook maneuver II
(a) Estimation errors of $\beta$, $r$, $\nu_x$, and $\alpha_y$ using standard UKF and iAUKF in Case 1

(b) Estimation errors of $\beta$, $r$, $\nu_x$, and $\alpha_y$ using standard UKF and iAUKF in Case 2

Fig. 15 Estimation errors of $\beta$, $r$, $\nu_x$, and $\alpha_y$ using UKF and the proposed iAUKF under different noises
(a) RMSEs of $\beta$, $r$, $v_x$, and $a_y$ using standard UKF and iAUKF in Case 1

(b) RMSEs of $\beta$, $r$, $v_x$, and $a_y$ using standard UKF and iAUKF in Case 2

Fig. 16 RMSEs of $\beta$, $r$, $v_x$, and $a_y$ using UKF and the proposed iAUKF under different noises
## Tables

### Table 1. RMSEs of the standard UKF under different noises

| Parameter / Unit | Value | Parameter / Unit | Value |
|------------------|-------|------------------|-------|
| \( Q_z = 0 \)   | 0.0047| \( Q_z = Q_0 \)  | 0.0055(17.02%↑) |
| \( Q_z = 3Q_0 \) | 0.0058(18.97%↑) | \( R_z = 0 \)    | 0.0047 |
| \( R_z = Q_0 \)  | 0.0057(21.28%↑) | \( R_z = 3Q_0 \) | 0.0072(53.19%↑) |
| \( R_z = 3Q_0 \) | 0.0072| \( R_z = 3Q_0 \) | 0.0072 |

### Table 2. Related parameters in the simulation

| Parameter / Unit | Value | Parameter / Unit | Value |
|------------------|-------|------------------|-------|
| \( m \) /kg      | 1530  | \( L_z \) /kg m²  | 4605  |
| \( a \) /m       | 1.141 | \( b \) /m        | 1.639 |
| \( \alpha_{\text{min}} \) | 2    | \( \alpha_{\text{max}} \) | 10   |
| \( M_{\text{min}} \) | 20   | \( M_{\text{max}} \) | 100  |
| \( \Delta t \)   | 0.01  | \( \kappa \)      | 0.01  |

### Table 3. The RMSE values of \( \beta \), \( r \), \( v_x \), and \( a_y \) in the sinusoidal maneuver

| Parameter / Unit | Value | Parameter / Unit | Value |
|------------------|-------|------------------|-------|
| \( \beta \) (rad) | 0.0047| \( r \) (rad/s)  | 0.0083 |
| \( v_x \) (m/s)  | 0.5350| \( a_y \) (m/s²) | 0.6503 |

- **UKF**: 0.0047, 0.0083, 0.5350, 0.6503
- **SHAUKF**: 0.0041(12.8%↓), 0.0076(8.4%↓), **0.3363**(37.1%↓), 0.5745(11.7%↓)
- **iAUKF**: **0.0036**(23.4%↓), **0.0049**(40.9%↓), 0.5191(3.0%↓), **0.4937**(24.1%↓)
Table 4 The RMSE values of $\beta$, $r$, $v_x$, and $a_y$ in the Fishhook maneuver I

|        | RMSE                  |
|--------|-----------------------|
|        | $\beta$ (rad)        | $r$ (rad/s)   | $v_x$ (m/s) | $a_y$ (m/s²) |
| UKF    | 0.0163                | 0.0497        | 1.4796      | 2.2957       |
| SHAUKF | 0.0133 (18.4%↓)       | 0.0245 (50.7%↓) | 0.6506 (56.0%↓) | 1.8757 (18.3%↓) |
| iAUKF  | **0.0109** (33.1%↓)  | **0.0074** (85.1%↓) | **0.1676** (88.7%↓) | **1.5284** (33.4%↓) |

Table 5 The RMSE values of $\beta$, $r$, $v_x$, and $a_y$ in the fishhook maneuver II

|        | RMSE                  |
|--------|-----------------------|
|        | $\beta$ (rad)        | $r$ (rad/s)   | $v_x$ (m/s) | $a_y$ (m/s²) |
| UKF    | 0.0167                | 0.0264        | 0.6984      | 2.3463       |
| SHAUKF | 0.0126 (24.6%↓)       | 0.0153 (42.0%↓) | 0.3526 (49.5%↓) | 1.7670 (24.7%↓) |
| iAUKF  | **0.0109** (34.7%↓)  | **0.0074** (71.9%↓) | **0.1841** (73.6%↓) | **1.5292** (34.8%↓) |

Table 6. RMSEs of $\beta$, $r$, $v_x$, and $a_y$ using the UKF and the proposed iAUKF under different process and measurement noises.

|        | $\beta$ (rad)        | $r$ (rad/s)   | $v_x$ (m/s) | $a_y$ (m/s²) |
|--------|-----------------------|
| $Q_z = 0$ | UKF                  | 0.0047        | 0.0067      | 0.9052      | 0.6733     |
|        | iAUKF                | **0.0036**    | 0.0079      | **0.3744**  | **0.5000** |
| $Q_z = Q_0$ | UKF                  | 0.0055        | 0.0083      | 1.4989      | 0.7752     |
|        | iAUKF                | **0.0038**    | 0.0077      | **0.3008**  | **0.5349** |
| $Q_z = 3Q_0$ | UKF                  | 0.0058        | 0.0098      | 1.8547      | 0.8181     |
|        | iAUKF                | **0.0040**    | 0.0078      | **0.3367**  | **0.5718** |
| $R_z = 0$ | UKF                  | 0.0047        | 0.0067      | 0.9052      | 0.6733     |
|        | iAUKF                | **0.0017**    | 0.0079      | **0.3744**  | **0.2325** |
| $R_z = R_0$ | UKF                  | 0.0057        | 0.0073      | 0.9963      | 0.8161     |
|        | iAUKF                | **0.0015**    | 0.0079      | **0.3758**  | **0.2042** |
| $R_z = 3R_0$ | UKF                  | 0.0072        | 0.0083      | 1.1075      | 1.0224     |
|        | iAUKF                | **0.0014**    | 0.0076      | **0.3539**  | **0.1905** |