Rashba spin torque in an ultrathin ferromagnetic metal layer

Xuhui Wang and Aurelien Manchon

Physical Science & Engineering Division, KAUST, Thuwal 23955-6900, Kingdom of Saudi Arabia

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In a two-dimensional ferromagnetic metal layer lacking inversion symmetry, the itinerant electrons mediate the interaction between the Rashba spin-orbit interaction and the ferromagnetic order parameter, leading to a Rashba spin torque exerted on the magnetization. Using Keldysh technique, in the presence of both magnetism and a spin-orbit coupling, we derive a spin diffusion equation that provides a coherent description to the diffusive spin dynamics. The characteristics of the spin torque and its implication on magnetization dynamics are discussed in the limits of large and weak spin-orbit coupling.

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I. INTRODUCTION

By transferring angular momentum between the electronic spin and the orbital, spin-orbit coupling fills the need for electrical manipulation of spin degree of freedom. Outstanding examples are the electrically generated bulk spin polarization\textsuperscript{12} and the well-known spin Hall effect (SHE)\textsuperscript{2} in a two dimensional electron gas where the spin-orbit interaction, particularly of the Rashba-type,\textsuperscript{2} plays the leading role. Rashba spin-orbit interaction not only introduces an effective field perpendicular to the linear momentum but also provides the backbone to the spin-relaxation through the so-called D'yakonov-Perel mechanism,\textsuperscript{2} which is dominant in a two-dimensional system. Besides its prominent role in semiconductors, Rashba spin-orbit coupling is believed to exist at ferromagnetic/heavy metal as well as ferromagnetic/metal-oxide interfaces, in which the inversion symmetry breaking offers a potential gradient empowering the spin-orbit coupling.

Meanwhile, magnetism continuously stimulates the industrial and academic appetite. In the pursuit of fast magnetization switching, Slonczewski-Berger spin transfer torque\textsuperscript{6} employs a polarized spin current instead of a cumbersome magnetic field. This celebrated scheme demands non-collinear magnetic textures in forms of, for example, spin valves or domain wall structures.\textsuperscript{2}

In the presence of inversion symmetry breaking (such as asymmetric interfaces), a ferromagnetic metal layer assembles both magnetism and spin-orbit coupling, hence offering an alternative switching mechanism.\textsuperscript{10,11} Spin-orbit coupling transfers the orbital angular momentum carried by an electric current to the electronic spin, thus creating an effective magnetic field (Rashba field). As long as the effective field is mis-aligned with the magnetization direction, the so-called Rashba torque emerges, thus exciting the magnetization.

Current-driven magnetization dynamics by spin-orbit torque has been demonstrated by several experiments on metal-oxide based systems.\textsuperscript{12–14} In fact, the Rashba torque can be categorized into a broader family of spin-orbit interaction induced torque that has been observed in diluted magnetic semiconductors\textsuperscript{15–18}. Recently, Miron et al.\textsuperscript{15} has demonstrated the current-induced magnetization switching using a single ferromagnet in Pt/Co/AlO\textsubscript{x} trilayers, which further consolidates the feasibility of the Rashba torque. The same type of spin-orbit coupling induced torque is predicted to improve current-driven domain wall motion\textsuperscript{19,20}, which is supported by experimental observations.\textsuperscript{20} At this stage, we are aware of an alternative explanation, as pointed out by Liu et al.\textsuperscript{20} in terms of the spin Hall effect (SHE) occurring in the underlying heavy metal layer, such as Pt or Ta. The distinction between the spin Hall induced effect and the Rashba one is discussed in the last section of this article.

In searching for a general form of the Rashba torque in ferromagnetic metal layers,\textsuperscript{10} we found an expression that consists of two components.\textsuperscript{22} An in-plane torque ($\propto m \times (\hat{y} \times m)$) and an out-of-plane one ($\propto \hat{y} \times m$), given $\hat{y}$ is the in-plane direction transverse to the injected current and $m$ is the magnetization direction. Numerical solution on a two-dimensional nano-wire with one open transport direction has been carried out to appreciate the significance of diffusive motion on the spin torque. We found that the in-plane component of the torque increases when narrowing the magnetic wire.\textsuperscript{22}

In the present article, we give a full theoretical derivation of the coupled diffusive equation for spin dynamics in a ferromagnetic metal layer and describe the form of the Rashba torque in both weak and strong Rashba limits. In Sec. \textsuperscript{III} we combine the Keldysh formalism and the gradient expansion technique to derive a coupled diffusion equation for charge and non-equilibrium spin densities. To demonstrate that the diffusion equation provides a coherent framework to describe the spin dynamics, we dedicate Sec. \textsuperscript{IV} to the spin diffusion in a ferromagnetic metal, which shows an excellent agreement to early result on the same system. In Sec. \textsuperscript{V} we illustrate that the absence of magnetism (in our diffusion equation) describes the well-known phenomenon of electrically induced spin polarization. The cases of a weak and a strong spin-orbit coupling are discussed in Sec. \textsuperscript{VI} and Sec. \textsuperscript{VII}, respectively, where we provide an analytical form of the Rashba torque in an infinite medium. In Sec. \textsuperscript{VII} we discuss the implication of the Rashba torque on...
magnetization dynamics as well as its distinction from spin Hall effect induced torque.

II. DIFFUSION EQUATIONS

The system of interest is defined as a quasi-two-dimensional ferromagnetic metal layer rolled out in the xy-plane. Two asymmetric interfaces provide a confinement in z-direction, along which the potential gradient generates a Rashba spin-orbit coupling. Therefore a single-particle Hamiltonian for an electron of momentum $k$ is ($\hbar = 1$ is assumed throughout)

$$
\hat{H} = \frac{\hat{p}^2}{2m} + \alpha \mathbf{\sigma} \cdot (\mathbf{k} \times \hat{z}) + \frac{1}{2} \Delta_{xc} \mathbf{\sigma} \cdot \mathbf{m} + H_i
$$

(1)

where $\mathbf{\sigma}$ is the Pauli matrix, $m$ the effective mass, and $\mathbf{m}$ the magnetization direction. The ferromagnetic exchange splitting is given by $\Delta_{xc}$ and $\alpha$ represents the Rashba constant (parameter). Hamiltonian $\hat{H} = \sum_{j=1}^N V(r - r_j)$ sums the contribution of the non-magnetic impurity scattering potential $V(\mathbf{r})$ localized at $r_j$.

To derive a diffusion equation for the non-equilibrium charge and spin densities, we employ Keldysh formalism. Using Dyson equation, in a 2x2 spin space, we obtain a kinetic equation that assembles the retarded (advanced) Green’s function $\hat{G}^R (\hat{G}^A)$, the Keldysh component of the Green function $\hat{G}^K$, and the self-energy $\hat{\Sigma}^K$, i.e.,

$$
[\hat{G}^R]^{-1} \hat{G}^K - \hat{G}^K [\hat{G}^A]^{-1} = \hat{\Sigma}^K \hat{G}^A - \hat{G}^R \hat{\Sigma}^K,
$$

(2)

where all Green’s functions are full functions with interactions taken care of by the self-energies $\hat{\Sigma}^{A,K}$. The retarded (advanced) Green’s function in momentum and energy space is

$$
\hat{G}^{R(A)} (\mathbf{k}, \epsilon) = \frac{1}{\epsilon - \epsilon_k - \mathbf{\sigma} \cdot \mathbf{b}(\mathbf{k}) - \hat{\Sigma}^{R(A)} (\mathbf{k}, \epsilon)},
$$

(3)

where $\epsilon_k = k^2/(2m)$ is the single-particle energy. We have introduced a k-dependent effective field $\mathbf{b}(\mathbf{k}) = \Delta_{xc} m/2 + \alpha (\mathbf{k} \times \mathbf{z})$ of the magnitude $b_k = |\Delta_{xc} m/2 + \alpha (\mathbf{k} \times \mathbf{z})|$ and the direction $\hat{b} = \mathbf{b}(\mathbf{k})/b_k$.

Neglecting localization effect and electron-electron interactions, we assume a short-range $\delta$-function type impurity scattering potential. At a low concentration and a weak coupling to electrons, the second-order Born approximation is justified, i.e., the self-energy is

$$
\hat{\Sigma}^{R,A,K} (\mathbf{r}, \mathbf{r'}) = \frac{\delta(|\mathbf{r} - \mathbf{r'}|)}{m \tau} \hat{G}^{R,A,K} (\mathbf{r}, \mathbf{r})
$$

(4)

where the momentum relaxation time reads

$$
\frac{1}{\tau} = 2\pi \int \frac{d^2k'}{(2\pi)^2} |V(\mathbf{k} - \mathbf{k'})|^2 \delta(\epsilon_k - \epsilon_{k'}),
$$

(5)

where $V(\mathbf{k})$ is the Fourier transform of the scattering potential and the magnitude of momentum $\mathbf{k}$ and $\mathbf{k'}$ is evaluated at Fermi vector $k_F$.

The quasi-classical distribution function $\hat{g} \equiv \hat{g}_{k,v}(T, \mathbf{R})$, defined as the Wigner transform of the Keldysh function $\hat{G}^K (\mathbf{r}, t; \mathbf{r'}, t')$, is obtained by integrating out the relative spatial-temporal coordinates while retaining the center-of-mass ones $\mathbf{R} = (\mathbf{r} + \mathbf{r'})/2$ and $T = (t + t')/2$. As long as the spatial profile of the quasi-classical distribution function is smooth at the scale of Fermi wave length, we may apply the gradient expansion technique on Eq. (2) which gives us a transport equation associated with macroscopic quantities. The left-hand side of the kinetic equation in gradient expansion becomes

$$
[\hat{G}^R]^{-1} \hat{G}^K - \hat{G}^K [\hat{G}^A]^{-1}
$$

$$
\approx \{\hat{g}, \hat{\sigma} \cdot \mathbf{b}(\mathbf{k})\} + \frac{i}{\tau} \hat{g} + \frac{i}{\tau} \frac{\partial \hat{g}}{\partial T}
$$

$$
+ \frac{i}{2} \left\{ \frac{\mathbf{k}}{m} + \alpha (\hat{z} \times \hat{\sigma}), \nabla \mathbf{R} \hat{g} \right\},
$$

(6)

where $\{ \cdot, \cdot \}$ denotes the anti-commutator. The relaxation time approximation indolus the right-hand side of Eq. (2) as

$$
\hat{\Sigma}^K \hat{G}^A - \hat{G}^R \hat{\Sigma}^K
$$

$$
\approx \frac{1}{\mathcal{C}} \left[ \hat{\rho}(\epsilon, T, \mathbf{R}) \hat{G}^A(\mathbf{k}, \epsilon) - \hat{G}^R(\mathbf{k}, \epsilon) \hat{\rho}(\epsilon, T, \mathbf{R}) \right]
$$

(7)

where we have introduced the density matrix by integrating out the momentum $\mathbf{k}$ in $\hat{g}$, i.e.,

$$
\hat{\rho}(E, T, \mathbf{R}) = \frac{1}{2\pi N_0} \int \frac{d^2k'}{(2\pi)^2} \hat{g}_{k',v}(T, \mathbf{R}).
$$

(8)

For the convenience of discussion, time variable is changed from $T$ to $t$. At this stage, we have a kinetic equation depending on $\hat{\rho}$ as well as on $\hat{g}$

$$
\frac{i}{\tau} \{\hat{\sigma} \cdot \mathbf{b}(\mathbf{k}), \hat{g}\} + \frac{1}{\tau} \hat{g} + \frac{\partial \hat{g}}{\partial t} + \frac{i}{2} \left\{ \frac{\mathbf{k}}{m} + \alpha (\hat{z} \times \hat{\sigma}), \nabla \mathbf{R} \hat{g} \right\}
$$

$$
= \frac{i}{\tau} \left[ \hat{G}^R(\mathbf{k}, \epsilon) \hat{\rho}(\epsilon) - \hat{\rho}(\epsilon) \hat{G}^A(\mathbf{k}, \epsilon) \right].
$$

(9)

A Fourier transformation on temporal variable to the frequency domain $\omega$ leads to

$$
\Omega \hat{g} - b_k \hat{U}_k \hat{g} = \hat{i} \hat{K},
$$

(10)

where $\Omega = \omega + i/\tau$ and the operator $\hat{U}_k \equiv \hat{\sigma} \cdot \hat{b}$ satisfies $\hat{U}_k \hat{U}_k = 1$. The right hand side of Eq. (10) is partitioned according to

$$
\hat{K} = -\frac{1}{\tau} \left\{ \frac{\mathbf{k}}{m} + \alpha (\hat{z} \times \hat{\sigma}), \nabla \mathbf{R} \hat{g} \right\}
$$

$$
+ \frac{i}{\tau} \left[ \hat{G}^R(\mathbf{k}, \epsilon) \hat{\rho}(\epsilon) - \hat{\rho}(\epsilon) \hat{G}^A(\mathbf{k}, \epsilon) \right].
$$

(11)
The equilibrium part is denoted by \( \hat{K}^{(0)} \) while the gradient term \( \hat{K}^{(1)} \) is regarded as perturbation. Functions \( \hat{g} \) and \( \hat{\rho} \) are both in frequency domain. We solve Eq. (10) formally to find a solution to \( \hat{g} \)

\[
\hat{g} = \frac{(2b_k^2 - \Omega^2)\hat{K} + 2b_k^2\hat{U}_k\hat{K} - \Omega b_k[\hat{U}_k, \hat{K}]}{\Omega(4b_k^2 - \Omega^2)} = \mathcal{L}[\hat{K}],
\]

(12)

An iteration procedure to solve Eq. (12) has been outlined by Mishchenko et al., in Ref. [24]. We follow this procedure here: According to the partition scheme on \( \hat{K} \), we insert \( \hat{K}^{(0)} \) to obtain the zero-th order approximation as \( \hat{g}^{(0)} = \mathcal{L}[\hat{K}^{(0)}(\hat{\rho})] \), which replaces \( \hat{g} \) in \( \hat{K}^{(1)} \) to generate a correction due to the gradient term, i.e., \( \hat{K}^{(1)}(\hat{g}^{(0)}) \). We further insert \( \hat{K}^{(1)}(\hat{g}^{(0)}) \) back to Eq. (12) to obtain a correction given by \( \mathcal{L}[\hat{K}^{(1)}(\hat{g}^{(0)})] \), then we obtain the first order approximation to the quasi-distribution function,

\[
\hat{g}^{(1)} = \hat{g}^{(0)} + \mathcal{L}[\hat{K}^{(1)}(\hat{g}^{(0)})],
\]

(13)

The above procedure is repeated to any desired order, i.e.,

\[
\hat{g}^{(n)} = \hat{g}^{(n-1)} + \mathcal{L}[\hat{K}^{(1)}(\hat{g}^{(n-1)})],
\]

(14)

In this paper, the second order approximation is sufficient. The full expression of the second order approximation is tedious thus not included in the following. A diffusion equation is derived by an angle averaging in momentum space, which allows all terms that are odd order in \( k_i \) \((i = x, y)\) to vanish while the combinations such as \( k_ik_j \) contribute to the averaging by a factor \( k_F^2 \delta_{ij} \), given \( k_F \) the Fermi wave vector. Further more, a Fourier transform from frequency domain back to the real time brings a diffusion type equation for the density matrix,

\[
\frac{\partial}{\partial t} \hat{\rho}(t) = D\nabla^2 \hat{\rho} - \frac{1}{\tau_{xc}} \hat{\rho} + \frac{1}{2\tau_{xc}} (\hat{\rho}(\hat{\sigma} \cdot \hat{\sigma}) \cdot \hat{\rho}(\hat{\sigma} \times \hat{\sigma}) + iC [\hat{\rho}(\hat{\sigma} \times \hat{\sigma}), \nabla \hat{\rho}] - B \{ \hat{\rho}(\hat{\sigma} \times \hat{\sigma}, \nabla \hat{\rho} \}
\]

\[
+ \Gamma [(m \times \nabla)_z \hat{\rho} - \hat{\rho} \cdot m \nabla \hat{\rho}] + (\hat{\rho}(\hat{\sigma} \cdot m \delta \hat{\rho} - i\Delta_{xc}(\hat{\sigma} \cdot m, \hat{\rho}) - 2R \{ \hat{\rho} \cdot m, (m \times \nabla)_z \hat{\rho} \},
\]

(15)

where all quantities are evaluated at Fermi energy \( \epsilon_F \). In a two-dimensional system, the diffusion constant \( D = \tau v_F^2/2 \) is given in terms of Fermi velocity \( v_F \) and momentum relaxation time \( \tau \). The renormalized exchange splitting reads \( \Delta_{xc} = (\Delta_{xc}/2)/(4\xi^2 + 1) \) where \( \xi^2 = (\Delta_{xc}^2/4 + \alpha^2 k_F^2)^2 \). The other parameters are

\[
\begin{align*}
C &= \frac{\alpha
\end{align*}
\]

\[
= \frac{\alpha}{(4\xi^2 + 1)^2},
\]

\[
\Gamma = \frac{\alpha \Delta_{xc} v_F k_F^2 \tau_r}{2(4\xi^2 + 1)},
\]

\[
R = \frac{\alpha \Delta_{xc}^2 \tau_r}{2(4\xi^2 + 1)},
\]

\[
\frac{1}{\tau_{xc}} = \frac{2\alpha^2 k_F^2 \tau_r \tau}{4\xi^2 + 1},
\]

\[
B = \frac{\alpha^2 k_F^2 \tau_r \tau}{4\xi^2 + 1},
\]

\[
\Omega = \frac{\Delta_{xc} \tau_r}{4\xi^2 + 1},
\]

\[
\tau_{xc} \text{ is the relaxation time due to the so-called D'yakonov-Perel mechanism. Equation (15) is valid in the dirty limit } \xi \ll 1, \text{ which enables the approximation } 1 + 4\xi^2 = 1. \text{ Charge density } n \text{ and the non-equilibrium spin density } S \text{ are introduced by the vector decomposition on the density matrix } \hat{\rho} = n/2 + S \cdot \hat{\sigma}. \text{ In a real experimental setup, \text{ spin transport in ferromagnetic layers suffers from random magnetic fluctuations, for which we introduce an isotropic spin-flip relaxation } S/\tau_{sf} \text{ phenomenologically.}}
\]

Eventually, we obtain a set of diffusion equations for the charge and spin densities, i.e.,

\[
\frac{\partial n}{\partial t} = D\nabla^2 n + B\nabla_z \cdot S
\]

\[
+ \Gamma \nabla_z \cdot mn + R\nabla_z \cdot m(S \cdot m),
\]

(16)

and

\[
\frac{\partial S}{\partial t} = D\nabla^2 S - \frac{1}{\tau ||} S || - \frac{1}{\tau_\perp} S_\perp
\]

\[
- \Delta_{xc} S \times m - \frac{1}{T_{xc}} m \times (S \times m)
\]

\[
+ B\nabla_z n + C\nabla_z \times S + 2R(m \cdot \nabla_z m)
\]

\[
+ \Gamma \{ m \times (\nabla_z \times S) + \nabla_z \times (S \times S) \},
\]

(17)

where \( \nabla_z \equiv \hat{z} \times \nabla \). The spin density \( S_\parallel \equiv S_\parallel \hat{x} + S_\parallel \hat{y} \) is relaxed at a rate \( 1/\tau || \equiv 1/\tau_{xc} + 1/\tau_{sf} \) while \( S_\perp \equiv S_\perp \hat{z} \) has a rate \( 1/\tau_\perp \equiv 2/\tau_{xc} + 1/\tau_{sf} \).

For a broad range of the relative strength between spin-orbit coupling and the exchange splitting, i.e., \( \alpha k_F/\Delta_{xc} \), Eq. (16) and Eq. (17) describe the spin dynamics in a ferromagnetic layer. When the magnetism vanishes (\( \Delta_{xc} = 0 \)), the B-term provides a source that generates spin density electrically. On the other hand, when the spin-orbit coupling is absent (\( \alpha = 0 \)), the first two lines in Eq. (17) describe a diffusive motion of spin density in a ferromagnetic metal, which, to be shown in the next section, agree excellently with early results in the corresponding limit. C-term describes the coherent precession of the spin density around the effective Rashba field. The precession of the spin density (induced by the Rashba field) around the exchange field is described by the G-term, thus a higher order (compared to C) in the
dirty limit for $\Gamma = \Delta_{xc} \tau C/2$. The $R$-term contributes to the magnetization renormalization.

III. SPIN DIFFUSION IN A FERROMAGNET

Spin diffusion in a ferromagnet has been discussed actively in the field of spintronics.\textsuperscript{26-29} In this section we show explicitly that, by suppressing the spin-orbit coupling, Eq. (17) describes the spin diffusion equation in the corresponding limits.

In the present model, vanishing Rashba spin-orbit coupling means $\alpha = 0$, then Eq. (17) reduces to

$$\frac{\partial}{\partial t}S = D \nabla^2 S + \frac{1}{\tau_\Delta} \frac{m \times S}{\tau_s f} + \frac{m \times (S \times m)}{\tau_{sf}},$$

(18)

where $\tau_\Delta \equiv 1/\Delta_{xc}$ is the time scale of the coherent precession of the spin density around the magnetization. This equation differs from the result of Zhang et al.\textsuperscript{22} only by a dephasing of the transverse component of the spin density that is set by the time scale $T_{xc}$. In a ferromagnetic metal, we may divide the spin density into a longitudinal component that follows the magnetization direction adiabatically, and a deviation that is perpendicular to the magnetization, i.e., $S = s_0 m + \delta S$ where $s_0$ is the local equilibrium spin density. Such a partition, after restoring the electric field by $\nabla \to \nabla + eE \partial_z$, gives rise to

$$\frac{\partial}{\partial t} \delta S + \frac{\partial}{\partial t} s_0 m$$

$$= s_0 D \nabla^2 m + D \nabla^2 \delta S + D eP_F N_F E \cdot \nabla m$$

$$- \frac{\delta S}{\tau_{s f}} - \frac{s_0 m}{\tau_{s f}} - \frac{\delta S}{\tau_{xc}} m \times \delta S,$$

(19)

where the magnetic order parameter is allowed to be spatial dependent, i.e., $m = m(r,t)$. The energy derivative is treated as $\partial_t S \approx P_F N_F m$ given $P_F$ the spin polarization and $N_F$ the density of state, both at Fermi energy.

For a smooth magnetic texture in which the characteristic length scale of the magnetic profile is much larger than the length scale for electron transport, we discard the contribution $D \nabla^2 \delta S$.\textsuperscript{26} The diffusion of the equilibrium spin density follows $s_0 D \nabla^2 m \approx s_0 m/\tau_{sf}$. In this paper, we retain only terms that are first order in temporal derivative, which simplifies Eq. (19) to

$$- \frac{1}{\tau_\Delta} m \times \delta S + \left( \frac{1}{\tau_{sf}} + \frac{1}{T_{xc}} \right) \delta S =$$

$$- s_0 \frac{\partial}{\partial t} m + D eP_F N_F E \cdot \nabla m.$$

(20)

The last equation can be solved exactly

$$\delta S = \frac{\tau_\Delta}{1 + \zeta^2} \left[ \frac{P_F}{e} m \times (j_e \cdot \nabla) m + \zeta \frac{P_F}{e} (j_e \cdot \nabla) m \right.$$

$$- s_0 m \times \frac{\partial m}{\partial t} - \zeta s_0 \frac{\partial m}{\partial t} \right]$$

(21)

where $\zeta = \tau_\Delta (1/\tau_{sf} + 1/T_{xc})$ and the electric current $j_e = e^2 n \tau E / m$ is given in terms of electron density $n$. Apart from the inclusion of the dephasing of transverse component as implemented in parameter $\zeta$, the non-equilibrium spin density Eq. (21) agrees excellently with Eq. (8) in Ref.\textsuperscript{26}.

Given the knowledge of the spin density, the spin torque, defined as

$$T = - \frac{1}{\tau_\Delta} m \times \delta S + \frac{1}{T_{xc}} \delta S,$$

(22)

is given by

$$T = \frac{1}{1 + \zeta^2} \left[ - \eta s_0 \frac{\partial m}{\partial t} + \beta s_0 m \times \frac{\partial m}{\partial t}$$

$$+ \eta \frac{P_F}{e} (j_e \cdot \nabla) m - \beta \frac{P_F}{e} m \times (j_e \cdot \nabla) m \right]$$

(23)

where $\eta = 1 + \zeta \tau_\Delta / T_{xc}$ and $\beta = \tau_\Delta / \tau_{sf}$. Assuming a long dephasing time of the transverse component (i.e., $T_{xc} \to \infty$), then $\eta \approx 1$ and Eq. (23) reproduces the Eq.(9) in Ref.\textsuperscript{26}. On the other hand, a short dephasing time (of the transverse component) enhances parameter $\eta$ therefore increases the temporal spin torque (i.e., the first term in Eq. (23)).

IV. ELECTRICALLY GENERATED SPIN DENSITY

The effect of an electrically generated non-equilibrium spin density due to spin-orbit coupling\textsuperscript{2} can be extracted from Eq. (17) by setting exchange interaction zero (i.e., $\Delta_{xc} = 0$). Retaining D’yakonov-Perel as the only spin relaxation mechanism and letting $\tau_{sf} = \infty$, Eq. (17) ends up in

$$D \nabla^2 S + \frac{1}{\tau_{xc}} (S + S \hat{z})$$

$$+ 2 C(\hat{z} \times \nabla) \times S + B(\hat{z} \times \nabla) n = 0$$

(24)

which reduces to the results in the well-known spin Hall effect\textsuperscript{24,30,31} In the case of an infinite medium along transport direction, i.e., $\hat{x}$-direction, Eq. (24) gives rise to a solution to the spin density

$$S \approx \tau_{xc} B e E \frac{1}{e_F n} \hat{y} = \frac{eE \zeta}{\pi \nu_F} \hat{y},$$

where only the linear term in electric field has been retained. On the right hand side, we have used the charge
density in a 2D system $n = k_F^2/(2\pi)$ and introduced the parameter $\zeta = \alpha k_F \tau$ as used in Ref. [24].

In the following sections, we explore the spin torque in the presence of both exchange and Rashba field in an infinite medium. The primary focus is on two cases: Weak and a strong spin-orbit coupling, when comparing to the magnitude of exchange splitting. In general, Eq. (17) is applicable through a broad range of relative strength between spin-orbit coupling and exchange splitting. A full scale numerical simulation on the diffusion equation is beyond the scope of this paper, we refer the readers to Ref. [22] for further interests.

V. WEAK SPIN-ORBIT COUPLING

A weak Rashba spin-orbit coupling implies a small D’yakonov-Perel relaxation rate $1/\tau_{xc} \propto \alpha^2$, such that $\tau_{xc} \gg \tau_{sf}, \tau_\Delta$, which allows spin relaxation to be dominated by random magnetic impurities. In this regime, when comparing to the magnitudes of $C$ and $\Gamma$, the contribution from $B$ and $R$ are at a higher order in $\alpha$, thus to be disregarded. We consider a stationary state where $\partial S/\partial t = 0$. An electric field applied along $\hat{x}$-direction, i.e., $E = E\hat{x}$. In an infinite medium, all the spatial derivatives vanishes ($\nabla \to 0$) and the dynamic equation reads

$$-\frac{1}{\tau_\Delta} \textbf{m} \times \textbf{S} + \frac{1}{T_{xc}} \textbf{m} \times (\textbf{S} \times \textbf{m}) + \frac{1}{\tau_{sf}} \textbf{S} = 2eE \textbf{C} \hat{y} \times \partial_t \textbf{S} + eE(\hat{y} \times (\textbf{m} \times \partial_x \textbf{S}) + \textbf{m} \times (\hat{y} \times \partial_y \textbf{S})).$$

In addition to the spin density induced by exchange splitting, a weak spin-orbit interaction leads to a deviation in spin density that can be considered as a perturbation. Therefore, we may well apply the partition $\textbf{S} = \textbf{S}_\perp + \textbf{S}_\parallel \textbf{m}$ to separate the longitudinal and the transverse components. Eq. (25) is thus reduced to

$$\frac{1}{\tau_\Delta} \textbf{m} \times \textbf{S}_\perp - \frac{1}{T_{xc}} \textbf{S}_\perp - \frac{1}{\tau_{sf}} \textbf{S}_\parallel \textbf{m} = -2eE \textbf{C} \hat{y} \times \textbf{m} + eE(\hat{y} \times (\textbf{m} \times \partial_x \textbf{S}) + \textbf{m} \times (\hat{y} \times \partial_y \textbf{S})).$$

VI. STRONG SPIN-ORBIT COUPLING

The opposite limit to SecV is a strong spin-orbit coupling. In this case, we consider the scenario that $\alpha k_F \gg \Delta_{xc}$ and the D’yakonov-Perel relaxation mechanism is dominating, i.e., $1/\tau_{xc} \gg 1/\tau_{sf}$, due to the fact $1/\tau_{xc} \propto \alpha^2$. Therefore, it is not physical to simply assume that the direction of spin density is dominantly aligned along the magnetization direction, as what is treated in the case of a weak spin-orbit coupling. A self-consistent solution from Eq.(17) to the spin density is more justified.

Again, as in SecV, we consider an infinite system where an electric field $E$ is applied at $\hat{x}$-direction. The magnetization direction is left arbitrary. We approximate the energy derivative by $\partial_\parallel \approx 1/\epsilon_F$. The above
assumptions simplify Eq.\(\text{(17)}\) to
\[
\frac{1}{\tau_\Delta} \mathbf{S} \times \mathbf{m} + \frac{1}{\tau_{xc}} \mathbf{m} \times \left( \mathbf{S} \times \mathbf{m} \right) - \frac{2 e E C}{\epsilon_F} \hat{y} \times \mathbf{S} + \frac{1}{\tau_{xc}} (\mathbf{S} + S_z \hat{z}) = \frac{e E}{\epsilon_F} n B \hat{y}, \quad (32)
\]
where a strong spin-orbit coupling renders \(\Gamma\) and \(R\) terms negligible. By considering \(\tau_{xc} \gg \tau_\Delta, \tau_{xc}\), Eq. \(32\) reduces to
\[
\frac{1}{\tau_\Delta} \mathbf{S} \times \mathbf{m} + \frac{1}{\tau_{xc}} (\mathbf{S} + S_z \hat{z}) - \frac{2 e E C}{\epsilon_F} \hat{y} \times \mathbf{S} = \frac{e E}{\epsilon_F} n B \hat{y},\quad (33)
\]
which is a set of linear equations for the non-equilibrium spin density. We are interested in the linear response regime, which implies that at the distance as defined by the Fermi wave length \(1/k_F\), we have \(e E/k_F \ll \alpha k_F\). Therefore, up to the first order in exchange splitting, we extract the spin density from the above equation to be
\[
\mathbf{S} = \frac{e E}{\epsilon_F} n_{\tau_{xc}} B \left( \hat{y} - \chi \hat{y} \times \mathbf{m} - \frac{\chi}{2} m_z \hat{z} \right) \quad (34)
\]
where \(\chi \equiv \tau_{xc}/\tau_\Delta\) we have used the identity \(\hat{y} \times \mathbf{m} = m_z \hat{z} - m_x \hat{z}\). This yields a spin torque
\[
T = \frac{\alpha n_{\tau_{xc}}}{\epsilon_F} j_e \left( \hat{y} \times \mathbf{m} + \chi \mathbf{m} \times (\hat{y} \times \mathbf{m}) - \frac{\chi}{2} m_z \hat{z} \times \mathbf{m} \right). \quad (35)
\]
This torque is slightly different from the weak Rashba limit and has a strong implication in terms of magnetization dynamics. The torque is dominated by a field-like torque along \(\hat{y}\), similarly to the weak Rashba case. First, in contrast to the weak Rashba case [see Eq. \(30\)], the sign of the in-plane torque remains positive. Secondly, the anisotropic spin relaxation coming from D’yakonov-Perel mechanism yields an additional component of spin accumulation that is oriented along \(\hat{z}\). The implication of this torque on the current-driven magnetization dynamics is discussed in the next section.

VII. DISCUSSION

Current-induced magnetization dynamics in a single ferromagnetic layer has been observed in various structures that involve interfaces between transition metal ferromagnets, heavy metals and/or metal-oxide insulators. Existing experimental systems are Pt/Co/AIO\(_x\),\(^{12,13,15,20}\) Ta/CoFeB/MgO,\(^{14}\) Pt/CoFe and Pt/Co bilayers.\(^{21}\) Besides the structural complexity in such systems, an unclear form of spin-orbit coupling in the bulk and interfaces places a challenge to understand the nature of the torque.

A. Validity of Rashba model

The celebrated Rashba-type effective interfacial spin-orbit Hamiltonian was pioneered by E. I. Rashba to model the influence of asymmetric interfaces in semiconducting 2DEG\(^{2}\). A sharp potential drop, emerging at the interface (say, in the \(xy\)-plane) between two materials, gives rise to a potential gradient \(\nabla V\) that is normal to the interface, i.e., \(\nabla V \approx \zeta (r) \hat{z}\). In case a rotational symmetry exists in the interface plane, a spherical Fermi surface assumption allows the spin-orbit interaction Hamiltonian to have the form \(\hat{H}_R = \alpha \sigma \cdot (\mathbf{p} \times \hat{z})\), where \(\alpha \approx (\zeta)/4 m^2 c^2\). As a matter of fact, in semiconducting interfaces where the transport is described by a limited number of bands around a high symmetry point, the Rashba form can be recovered through \(k \cdot p\) theory.\(^{32}\)

As far as metallic interfaces are concerned, a spin-orbit splitting of the Rashba-type in the conduction band has been observed at Au surfaces,\(^{33}\) Gd/GdO interfaces,\(^{34}\) Bi surfaces and compounds\(^{35}\) and metallic quantum wells.\(^{36}\) The presence of a Rashba interaction in graphene\(^{37}\) and at oxide hetero-interfaces\(^{38}\) has also been reported recently. It is quite interesting to notice that the symmetry breaking-induced spin splitting of the conduction band seems rather general and might not be restricted to heavy metal interfaces.\(^{36}\)

In the case of transition metals, however, the free electron approximation fails to characterize the band structure accurately due to both a large number of band crossing at the Fermi energy and a strong hybridization among \(s, p\) and \(d\) orbitals. Density functional theory (DFT) is a successful tool to investigate the nature of spin-orbit interaction at metallic surfaces. For example, in Refs.\(^{39}\), the authors observe a band splitting that possesses similar properties as Rashba spin-orbit interaction and decays exponentially away from the surface.\(^{40}\) Alternatively, the spin-orbit interaction at metallic surfaces has been addressed using tight-binding models for the \(p\) orbitals.\(^{40,41}\) At such sharp interfaces, the magnitude of the orbital angular momentum (OAM) is considered to play a dominant role at the onset of a Rashba-type spin splitting.

This finding is consistent with the long standing work on interfacial magnetic anisotropy at a ferromagnet/heavy metal\(^{42}\) and more recently, ferromagnetic/metal-oxide interface.\(^{43}\) In such systems, a perpendicular magnetic anisotropy arises from the orbital overlap between the \(3d\) states of the ferromagnets and the spin-orbit coupled states of the normal metal. The observation of perpendicular magnetic anisotropy at Co/metal-oxide interfaces tends to support the major role of large interfacial OAM in the onset of interfacial spin-orbit effects.\(^{41,43}\) The presence of interfacial Rashba spin-orbit coupling has also been shown to produce interfacial perpendicular magnetic anisotropy.\(^{44}\)

All these previous theoretical and experimental studies strongly suggest that the interfacial spin splitting exists in the presence of a large OAM and potential gradient. However, a microscopic description of realistic interfaces
is still missing. Although the Rashba spin-orbit interaction is a convenient Hamiltonian to extract qualitative behaviors, its applicability to realistic metallic interfaces with complex band structures remains to be tested.

**B. Spin Hall effect versus Rashba torque**

Recently, Liu et al.\(^2\) proposed to manipulate the magnetization of a Pt/Co or Pt/NiFe bilayer using the spin current generated by spin Hall effect in the underlying Pt layer. When injecting a charge current \(j_c\) into a normal metal accommodating a strong spin-orbit coupling, the asymmetric spin scattering induces a transverse pure spin current that has the form \(J = (\alpha_H/e)\hat{x} \times \hat{\mu} \otimes \hat{m}\), where \(\alpha_H\) is the spin Hall angle and \(\hat{\mu}\) is the spin direction.\(^2\) When impinging on the ferromagnetic layer deposited on top of the Pt layer, the spin current transverse to the local magnetization is absorbed and generates a torque \(T_{SHE} = (b_H/e)(1 - \beta m) \times (\hat{\mu} \times m)\) (to be called SHE torque thereafter). Here, \(b_H = \alpha_H \mu_B/e\) is the spin torque amplitude where the regular spin polarization \(P\) is replaced by the spin Hall angle \(\alpha_H\). \(\beta\) is the nonadiabaticity parameter proposed by Zhang and Liu\(^2\) and it stems from the presence of spin-flip scattering in the system. In the configuration adopted by Liu et al., the charge current is injected along \(\hat{x}\) and the torque is given by

\[
T_{SHE} = \alpha_H \mu_B \frac{j_c}{e} (m \times (\hat{y} \times m) + \beta \hat{y} \times m). \tag{36}
\]

Note that a more realistic model should account for spin diffusion in Co and Pt, as discussed in Ref. \(^{48}\). An important conclusion is that, besides the correction in the case of a strong Rashba coupling, both Rashba and SHE produce the same type of torque, see Eq.\(^{23}\) and Eq.\(^{55}\) in this article.

Nevertheless, distinctions can be made. First, in the absence of the corrections due to spin-flip and spin precession, the Rashba torque reduces to the field-like term, \(\hat{y} \times m\), whereas the SHE torque reduces to the (anti-)damping term \(m \times (\hat{y} \times m)\). This assertion must be scrutinized carefully since the actual relative magnitude between the field-like and the damping torques depends on the width of the magnetic wire as well as on the detailed spin dynamics in presence of spin-flip and precession.\(^2\) Furthermore, for such an ultra-small system the spin-flip scattering giving rise to the non-adiabaticity parameter (\(\beta\)) might be significantly different from the one measured in a more conventional thin film.

A second important difference arises from the fact that the Rashba torque arises from spin-orbit fields generated by *interfacial* currents, whereas the SHE torque is due to the current flowing in the *bulk* of the Pt layer. Therefore, for a constant external electric field, varying the thickness of Pt layer shall enhance the SHE torque, while keeping the Rashba torque unchanged.

The torques as a function of the Co layer thickness is more difficult to foresee. Although one could claim that Rashba spin-orbit interaction is expected to be localized at the interface, where the potential gradient is large, numerical simulations show that the Rashba-type interaction survives a few monolayers\(^28\) (which is typically the thickness of the Co layer under consideration). In addition, the presence of quantum well states might also modify the nature of the spin-orbit interaction in the ultrathin magnetic layer in a system such as Pt/Co/AlO\(_x\)\(^{36}\).

The same is true for the SHE torque. The injection of spin current into a Co layer is accompanied by spin precession that takes place over a very short decoherence length. This decoherence length has been studied experimentally and theoretically in spin valves and found to be of the order of a few monolayers.\(^{21}\) In the typical case of 3 or 4 monolayer-thick ferromagnets, the SHE torque can not be considered as a purely interfacial phenomenon.

**C. Magnetization Dynamics**

In Pt/Co/AlO\(_x\) trilayers, Miron et al have observed a current-driven domain wall nucleation\(^1\)\(^2\) and an enhanced current-driven domain wall velocity\(^1\)\(^3\) and a current-driven magnetization switching.\(^1\)\(^4\) The symmetry of the spin torque required to explain the experimental findings agree well with Rashba torque proposed in Ref. \(^{11}\). On a similar structure, Pi et al.\(^{21}\) and Suzuki et al.\(^{14}\) also observed an effective field torque that could be interpreted in terms of the Rashba torque. Recently, Liu et al.\(^{21}\) interpreted their experiments on Pt/NiFe and Pt/Co bilayers using SHE in the underlying Pt layer.

1. **Magnetization switching**

According to our previous discussions, both Rashba torque and SHE torque have a general form \(T = T_{\parallel} \hat{y} \times m + T_{\perp} \hat{m} \times (\hat{y} \times m)\). The first term acts like a field oriented along the direction transverse current direction whereas the second term acts like an (anti-)damping term, mimicking a conventional spin transfer torque that would arise from a polarizer pointing to \(\hat{y}\).

As a consequence, both Rashba torque and SHE torque possess the appropriate symmetry to excite the magnetization of a single ferromagnet and induce switching, as observed by Miron et al.\(^15\) and Liu et al.\(^2\). In the case of a large Rashba spin-orbit coupling, the torque acquires an additional component that acts like an effective magnetic field along \(\hat{z}\), vanishing as the magnetization component \(m_z\) is zero (see Section \(^{17}\)), which provides an additional torque that helps destabilize the magnetization.
2. Current-driven domain wall motion

The influence of Rashba/SHE torque on a domain wall can be illustrated within the rigid Bloch wall approximation. The perpendicularly magnetized Bloch wall is parameterized by \( \mathbf{m} = (\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta) \) where \( \phi = \phi(t) \) and \( \theta(x, t) = 2 \tan^{-1}[e^{(x-x_c(0))/\Delta}] \), where \( x_c \) refers to the center of the domain wall and \( \Delta \) is defined as the domain wall width. To describe the dynamics of a Bloch wall, Landau-Lifshitz-Gilbert (LLG) equation

\[
\partial_t \mathbf{m} = -\gamma \mathbf{m} \times \mathbf{H}_{\text{eff}} + \alpha_G \partial_t \mathbf{m} \times \mathbf{m} + \mathbf{\tau} \tag{37}
\]

has to be augmented by the current induced torque \( \mathbf{\tau} \)

\[
\mathbf{\tau} = b_j \nabla \mathbf{m} - \beta b_j \mathbf{m} \times \nabla \mathbf{m} + b_j(\tau_\parallel \hat{\mathbf{y}} \times \mathbf{m} + \tau_\perp \mathbf{m} \times (\hat{\mathbf{y}} \times \mathbf{m}) + \tau_z m_x \hat{\mathbf{z}} \times \mathbf{m}) \tag{38}
\]

The torque \( \mathbf{\tau} \) is written in the most general form, where the first two terms are the regular adiabatic and the so-called non-adiabatic torques; the next two terms \( (\tau_\parallel \) and \( \tau_\perp \) \) emerge from the presence of Rashba and/or spin Hall effect and the last term \( \tau_z \) appears only in large Rashba limit (see Sec. VIII). The magnitude of the adiabatic torque is \( b_j = \mu_B P_J/e \). The effective field is given by

\[
\mathbf{H}_{\text{eff}} = \frac{2A}{M_s} \nabla^2 \mathbf{m} + H_K m_z \hat{\mathbf{x}} + H_\perp m_z \hat{\mathbf{z}}. \tag{39}
\]

Parameter \( \gamma \) in LLG is the gyromagnetic ratio, \( \alpha_G \) is the Gilbert damping, \( A \) is the exchange constant, \( M_s \) is the saturation magnetization, \( H_K \) is the in-plane magnetic anisotropy and \( H_\perp \) is the combination of an out-of-plane anisotropy and a demagnetizing field. The magnetization dynamics can be obtained readily from Eqs. (37)-\( \text{(39)} \) by integrating over the magnetic volume

\[
\partial_t \phi + \alpha_G \partial_t x_c \frac{\Delta}{\Delta} = \left[ \frac{\Delta \pi}{2} (\tau_\parallel - \tau_\perp) \cos \phi - \beta \right] \frac{b_j}{\Delta} \tag{40}
\]

\[
\alpha_G \partial_t \phi - \alpha_G \partial_t x_c \frac{\Delta}{\Delta} = -\gamma H_K \frac{2}{2} \sin 2\phi + \left( 1 + \frac{\Delta \pi}{2} \tau_\perp \cos \phi \right) \frac{b_j}{\Delta}. \tag{41}
\]

We observe that the in-plane torque \( \tau_\parallel \) distorts the domain wall texture, while the perpendicular torque \( \tau_\perp \) drives the domain wall motion. The additional torque \( \tau_z \), arising in the large Rashba limit, only contributes to the in-plane torque. Therefore, in the following, we will refer to the in-plane torque as \( \tau_\parallel = \tau_\| - \tau_z/2 \). Below the Walker breakdown \( (\partial_t \phi = 0) \), the velocity is given by

\[
\partial_t x_c = -\left( \beta - \frac{\Delta \pi}{2} \tau_\parallel \cos \phi \right) \frac{b_j}{\alpha_G} \tag{42}
\]

\[
\gamma \frac{H_K}{2} \sin 2\phi = \left[ \alpha_G - \beta + \frac{\Delta \pi}{2} (\alpha_G \tau_\perp + \tau_\parallel^*) \cos \phi \right] \frac{b_j}{\alpha_G \Delta}. \tag{43}
\]

where the tilting angle \( \phi \) is given by the competition between the magnetic anisotropy, the non-adiabatic torque, and the Rashba/SHE torque. In the case of weak Rashba \( (\tau_z = 0) \), assuming \( \tau_\parallel = \beta \tau_\perp \) and omitting the correction to the spin precession, we recover the results of Ref. [58]. When neglecting the in-plane torque and accounting for the perpendicular Rashba torque \( (\tau_\parallel = 0) \), the Rashba torque only acts like an effective transverse field and enhances the Walker breakdown limit [see Eq. (43)].

Accounting for the in-plane component \( \tau_\parallel \) arising either from corrections to Rashba torque or from the SHE, this torque appears to modify the domain wall velocity. Therefore, depending on the strength and the sign of Rashba/SHE torque as well as on the resulting tilting angle \( \phi \), it is possible to obtain a vanishing or even a reversed domain wall velocity, as has been shown numerically in Ref. [48] and illustrated in Eq. (22). A full scale numerical investigation is beyond the scope of this article, but it will help understand the profound effect of Rashba and SHE torque on the domain wall structures.

VIII. CONCLUSION

Using Keldysh technique, in the presence of both magnetism and a Rashba spin-orbit coupling, we derive a spin diffusion equation that provides a coherent description to the diffusive spin dynamics. In particular, we have derived a general expression for the Rashba torque in the bulk of a ferromagnetic metal layer, at both weak and strong Rashba limits. We find that the torque is in general composed of two components, a field-like torque and the other (anti-)damping one. Being aware of the recent alternative interpretation on the current-induced magnetization switching in a single ferromagnet, we have discussed the difference between the Rashba and the SHE torques. While exploring the common features, we found that the magnetization dynamics driven by the Rashba torque presents several interesting similarities to that induced by SHE torque. Nevertheless, further investigation involving structural modification of the system is expected to provide a deeper knowledge on the nature of the interfacial spin-orbit interaction as well as the current-induced magnetization switching in a single ferromagnet.

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